Theory of Piezoresistive Response of Unidirectional CFRP Under Spatial Stress

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Abstract. The piezoresistive mechanism of solid CFRP under uniaxial stress was revealed by decomposing the resistance response into the resistivity change and the size effect. Piezoresistive constitutive model was established for the conditions of spatial stress. The piezoresistive matrix is an asymmetric matrix which depends not only on the resistance properties of the composite, but also on its mechanical properties.

1. Introduction
Carbon fiber reinforced plastics (CFRP) are widely used as dominant advanced lightweight structural material due to its high strength, high modulus of elasticity and low density. In recent years, much focus has been concentrated on the piezoresistivity of such a composite, which refers to a phenomenon in which the electrical resistivity of a material changes with strain. Such a characteristic is valuable as it makes CFRPs possible to sense its own strain for the purpose of health monitoring. Since the piezoresistivity of CFRP discovered by Schulte in 1989[1], much work has been done on different kinds of CFRP to investigate the piezoresistive behavior and its mechanism[2-7], especially for the CFRPs with continuous carbon fibers.

However, most current studies on the piezoresistivity of CFRP are based on uniaxial stress state. The resistance change was usually only related with the strain in single direction. The resistance response of CFRP under multi-axial loading was preliminarily studied in Reference [8], but this studied is limited in plane stress state.

In this paper, based on Reference[8], the piezoresistive mechanism of solid CFRP under uniaxial stress was revealed by decomposing the resistance response into the resistivity change and the size effect, and the theory of piezoresistivity for unidirectional CFRP is expanded to spatial stress.

2. Theory of Piezoresistivity of solid CFRP under uniaxial stress
For solid CFRP, the change of resistance can be decomposed into the change of resistivity and the change of dimension.

\[
\frac{\Delta R}{R_0} = \frac{\Delta \rho}{\rho_0} + \frac{\Delta L}{L} - \frac{\Delta W}{W} - \frac{\Delta H}{H}
\]  

(1)

where \(\rho_0\) represents the initial resistivity, \(\Delta \rho\) represents the change of surface resistivity, \(L, W\) and \(H\) represent the length, width and thickness respectively.
Now based on the formula (1), the piezoresistivity under the nine kinds of unidirectional stress is analyzed herein:

When CFRP is loaded along the fiber direction, testing resistance changes along the fiber direction yields:

$$\frac{\Delta R_{11}}{R_{11}} = \frac{\Delta \rho_1}{\rho_1} + (1 + \nu_{12} + \nu_{13}) \varepsilon_i \quad (2)$$

In three-dimensional state, the constitutive relationship between the change of resistivity and the strain can be expressed as:

$$\begin{bmatrix}
\Delta \rho_1 \\
\rho_1 \\
\Delta \rho_2 \\
\rho_2 \\
\Delta \rho_3 \\
\rho_3
\end{bmatrix} =
\begin{bmatrix}
N_{11} & N_{12} & N_{13} \\
N_{21} & N_{22} & N_{23} \\
N_{31} & N_{32} & N_{33}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3
\end{bmatrix} \quad (3)$$

After substitution of formula (3) into formula (2), the piezoresistivity gage factor $q_{ij}$ is obtained as follows:

$$q_{11} = \frac{\Delta R_{11}}{R_{11} \varepsilon_i} = N_{11} - \nu_{12} N_{12} - \nu_{13} N_{13} + (1 + \nu_{12} + \nu_{13}) \quad (4)$$

Similarly,

$$q_{12} = \frac{\Delta R_{12}}{R_{12} \varepsilon_i} = N_{11} - \frac{1}{\nu_{21}} N_{12} + \frac{\nu_{23}}{\nu_{21}} N_{13} - \frac{(\nu_{23} - \nu_{21})}{\nu_{21}} \quad (5)$$

$$q_{13} = \frac{\Delta R_{13}}{R_{13} \varepsilon_i} = N_{11} + \nu_{32} N_{12} - \frac{1}{\nu_{31}} N_{13} - \frac{(-\nu_{31} + \nu_{32})}{\nu_{31}} \quad (6)$$

$$q_{21} = \frac{\Delta R_{21}}{R_{21} \varepsilon_i} = -\frac{1}{\nu_{12}} N_{21} + N_{22} + \frac{\nu_{13}}{\nu_{12}} N_{23} - \frac{(\nu_{13} - \nu_{12})}{\nu_{12}} \quad (7)$$

$$q_{22} = \frac{\Delta R_{22}}{R_{22} \varepsilon_i} = -\nu_{21} N_{21} + N_{22} + \nu_{23} N_{23} + (1 + \nu_{21} + \nu_{23}) \quad (8)$$

$$q_{23} = \frac{\Delta R_{23}}{R_{23} \varepsilon_i} = \frac{\nu_{21}}{\nu_{32}} N_{21} + N_{22} - \frac{1}{\nu_{23}} N_{23} - \frac{(\nu_{23} - \nu_{21})}{\nu_{23}} \quad (9)$$

$$q_{31} = \frac{\Delta R_{31}}{R_{31} \varepsilon_i} = -\frac{1}{\nu_{31}} N_{31} + \frac{\nu_{12}}{\nu_{13}} N_{32} + N_{33} - \frac{(\nu_{12} - 1 - \nu_{13})}{\nu_{13}} \quad (10)$$

$$q_{32} = \frac{\Delta R_{32}}{R_{32} \varepsilon_i} = \frac{\nu_{21}}{\nu_{23}} N_{31} - \frac{1}{\nu_{23}} N_{32} + N_{33} - \frac{(\nu_{23} - 1 - \nu_{23})}{\nu_{23}} \quad (11)$$

$$q_{33} = \frac{\Delta R_{33}}{R_{33} \varepsilon_i} = -\nu_{31} N_{31} - \nu_{32} N_{32} + N_{33} + (1 + \nu_{31} + \nu_{32}) \quad (12)$$

Formula (4) $q_{11}$, (5) $q_{12}$ and (6) $q_{13}$ all reflect the sensitivity of the longitudinal resistance of CFRP $R_i$ to the longitudinal strain $\varepsilon_i$. The difference is: $\varepsilon_i$ in formula (4) is caused by longitudinal loading, $\varepsilon_i$ in formula (5) is caused by the Poisson effect in the transverse loading, and $\varepsilon_i$ in formula (6) is caused by the Poisson effect when CFRP is loaded in the thickness direction. In general, $q_{11}$, $q_{12}$ and $q_{13}$ are not equal to each other. This signifies that in the three-dimensional spatial stress state, analyzing the sensitivity of the longitudinal resistance to the longitudinal strain, $\Delta R_i$ and $\varepsilon_i$ can not be related directly, and the longitudinal strain $\varepsilon_i$ must be decomposed into three parts to consider: 1) the
longitudinal strain caused by loading in the longitudinal direction; 2) the longitudinal strain caused by loading in the transverse direction; 3) the longitudinal strain caused by loading in the thickness direction.

Similarly, \( q_{x1} \), \( q_{y2} \) and \( q_{z2} \) are not equal to each other, and \( q_{y3} \), \( q_{z2} \), \( q_{z3} \) are not equal to each other either. Therefore, analyzing the sensitivity of transverse resistance and the sensitivity of resistance in thickness direction, the strain must be decomposed into three parts: 1) the strain caused by the longitudinal loading; 2) the strain caused by the transverse loading; 3) the strain caused by the loading in the thickness direction.

3. Constitutive relation of piezoresistivity in spatial stress state
Under spatial stress state, the stress-strain relationship of unidirectional CFRP can be expressed as:

\[
\begin{bmatrix}
\sigma_L \\
\sigma_T \\
\sigma_Z \\
\sigma_{TZ} \\
\sigma_{ZL} \\
\sigma_{LT}
\end{bmatrix} =
\begin{bmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_L \\
\varepsilon_T \\
\varepsilon_Z \\
\gamma_{TZ} \\
\gamma_{ZL} \\
\gamma_{LT}
\end{bmatrix}
\]

(13)

Where \( L \) represents the direction of the fiber, \( T \) represents the transverse direction, and \( Z \) represents the thickness direction.

And

\[
C_{11} = \frac{1 - v_{TZ} v_{LT}}{E_T E_Z} ; \quad C_{12} = \frac{v_{LT} + v_{LT} v_{LT}}{E_T E_Z} ; \quad C_{13} = \frac{v_{LT} + v_{LT} v_{LT}}{E_T E_Z} ; \quad C_{22} = \frac{1 - v_{LT} v_{LT}}{E_T E_Z} ;
\]

\[
C_{23} = \frac{v_{LT} + v_{LT} v_{LT}}{E_T E_Z} ; \quad C_{33} = \frac{1 - v_{LT} v_{LT}}{E_T E_Z} ; \quad C_{44} = G_{TZ} ; \quad C_{55} = G_{ZL} ; \quad C_{66} = G_{LT} ;
\]

(14)

Where \( \Delta = \frac{1 - v_{LT} v_{LT} - v_{LT} v_{LT} - v_{LT} v_{LT} - 2 v_{LT} v_{LT} v_{LT}}{E_T E_Z} \).

\( \varepsilon_L, \varepsilon_T \) and \( \varepsilon_Z \) can be further decomposed into the strain caused by the stress of their own direction and the strain caused by the Poisson effect: (\( \varepsilon^L \) signifies the strain caused by loading in the \( L \) direction alone; \( \varepsilon^T \) signifies the strain caused by loading in the \( T \) direction alone; \( \varepsilon^Z \) signifies the strain caused by loading in the \( Z \) direction alone.)
Therefore,

\[
\begin{bmatrix}
\varepsilon_L \\
\varepsilon_T \\
\varepsilon_Z
\end{bmatrix}
= 
\begin{bmatrix}
\varepsilon_L^0 \\
\varepsilon_T^0 \\
\varepsilon_Z^0
\end{bmatrix}
+ 
\begin{bmatrix}
\varepsilon_L^t \\
\varepsilon_T^t \\
\varepsilon_Z^t
\end{bmatrix}
+ 
\begin{bmatrix}
S_{44}\tau_{TZ} \\
S_{55}\tau_{ZL} \\
S_{66}\tau_{LT}
\end{bmatrix}
\]

Then, the piezoresistivity of unidirectional CFRP in spatial stress state can be expressed as:

\[
\begin{bmatrix}
S_{11} \\
S_{22} \\
S_{33} \\
S_{44} \\
S_{55} \\
S_{66}
\end{bmatrix}
= 
\begin{bmatrix}
S_{11}\sigma_L \\
S_{22}\sigma_T \\
S_{33}\sigma_Z \\
S_{44}\sigma_{TZ} \\
S_{55}\sigma_{ZL} \\
S_{66}\sigma_{LT}
\end{bmatrix}
\]
The piezoresistive matrix of CFRP in spatial stress state can be simplified as:

\[
\begin{pmatrix}
K_{11} & K_{12} & K_{13} \\
K_{21} & K_{22} & K_{23} \\
K_{31} & K_{32} & K_{33}
\end{pmatrix} = \begin{pmatrix}
q_{11}S_{11} & q_{12}S_{12} & q_{13}S_{13} \\
q_{21}S_{21} & q_{22}S_{22} & q_{23}S_{23} \\
q_{31}S_{31} & q_{32}S_{32} & q_{33}S_{33}
\end{pmatrix}
\]

Formula (22) shows that the piezoresistive matrix is asymmetric. The value of each element in the matrix depends not only on the piezoresistive properties of the composite, but also on its mechanical properties.

### 4. Conclusion

1) The piezoresistive mechanism of CFRP in uniaxial stress state is analyzed.

2) Piezoresistive constitutive model was established for the conditions of spatial stress.

3) The piezoresistive matrix is an asymmetric matrix which depends not only on the resistance properties of the composite, but also on its mechanical properties.

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