Constraints on Coupling Constants through Charged $\Sigma$ Photoproduction

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Abstract

The few available data for the reactions $\gamma p \rightarrow K^0\Sigma^+$ and $\gamma n \rightarrow K^+\Sigma^-$ are compared to models developed for the processes $\gamma p \rightarrow K^+\Sigma^0$ and $\gamma p \rightarrow K^+\Lambda$. It is found that some of these phenomenological models overpredict the measurements by up to a factor of 100. Fitting the data for all of these reactions leads to drastically reduced Born coupling constants.
Most analyses of kaon electromagnetic production over the last several years have focused on the two processes $\gamma p \rightarrow K^+ \Lambda$ and $\gamma p \rightarrow K^+ \Sigma^0$. This is clearly due to the fact that almost all of the few available photokaon data have been taken for these two reactions, along with the related kaon radiative capture, $K^- p \rightarrow \Lambda \gamma$ and $K^- p \rightarrow \Sigma^0 \gamma$, and electroproduction processes. Despite the considerable effort spent in the last years, kaon photoproduction on the nucleon remains far from understood. Due to the limited set of data the proliferating number of models permit only some qualitative conclusions but do not yet allow the extraction of precise coupling constants and resonance parameters. Meanwhile, however, a basic understanding of these elementary reactions is required in order to predict cross sections for the photoproduction of hypernuclei.

Most models to date are based on diagrammatic techniques using hadronic degrees of freedom where a limited number of low-lying $s$-, $u$-, and $t$-channel resonances are employed in a fit to the data along with the standard set of Born terms. One general finding of all of these fits is that the leading hadronic coupling constant $g_{K\Lambda N}$ cannot be reconciled with the SU(3) value of $3.0 < |g_{K\Lambda N}/\sqrt{4\pi}| < 4.4$ that is consistent with other hadronic information such as $YN$ scattering. Instead, the value of $g_{K\Lambda N}$ extracted from photoproduction is too low unless a certain $t$-channel resonance is included or absorptive factors are applied to reduce the Born terms. The latter method also helps to eliminate the divergence of these models at higher energies. The SU(3) range for the leading coupling constant in $\Sigma$ electromagnetic production, $g_{K\Sigma N}$, is $0.9 < |g_{K\Sigma N}/\sqrt{4\pi}| < 1.3$ which is compatible with the range employed by the modern Nijmegen and Juelich $YN$-potentials. As discussed in Refs. the value of $g_{K\Sigma N}$ extracted from kaon photoproduction reactions varies widely and has remained very uncertain.

In this note we develop extensions of previous models in order to include the other four isospin channels listed in Table. For this purpose we employ the few available total cross section data for the charged $\Sigma$-photoproduction reactions, $\gamma p \rightarrow K^0\Sigma^\pm$ and $\gamma n \rightarrow K^+\Sigma^-$. The CEBAF Large Acceptance Spectrometer (CLAS) will detect neutral kaons and charged kaons with comparable efficiency and will measure kaon photoproduction on the neutron.
using deuterium. Clearly a theoretical study of the other isospin channels is called for.

In the electroproduction process for pseudoscalar mesons the transition matrix element can be written as:

\[ M_{f_i} = \bar{u}(p_Y) \sum_{j=1}^{6} A_j(k^2, s, t) M_j u(p_N) \]  

(1)

where \( s \) and \( t \) are the usual Mandelstam variables and \( k^2 \) denotes the virtual photon momentum squared. The gauge and Lorentz invariant matrices \( M_j \) are given in many references, while the amplitudes \( A_j \) can be obtained from Feynman diagrams. For the photoproduction reactions only the amplitudes \( A_1 - A_4 \) contribute. For the vertex factors and propagators, we follow Ref. \( \) with slight modifications in order to ensure gauge invariance in the electroproduction process. We use pseudoscalar (PS), rather than pseudovector (PV) since previous studies indicated the PS-coupling mode to be the preferred one. Chiral symmetry arguments that demand PV-coupling for pions most likely do not apply to kaons due to their larger mass.

To relate the coupling constants in the Born terms among the various isospin channels we first consider the hadronic vertices. Since the Lambda is an SU(3) isosinglet, one obtains

\[ g_\Lambda \equiv g_{K^+\Lambda p} = g_{K^0\Lambda n} \]  

(2)

Similarly for the \( N \rightarrow K^*\Lambda \) vertex

\[ g_{K^*+\Lambda p}^{V,T} = g_{K^*0\Lambda n}^{V,T} \]  

(3)

On the other hand the Sigma is an isovector, so the \( N \rightarrow K\Sigma \) couplings are related by the Clebsch-Gordan coefficients coupling isospin 1 plus isospin 1/2 to isospin 1/2.

\[ g_\Sigma \equiv g_{K^+\Sigma^0 p} = -g_{K^0\Sigma^0 n} = g_{K^0\Sigma^+ p}/\sqrt{2} = g_{K^+\Sigma^- n}/\sqrt{2} \]  

(4)

Note that there are different conventions used in expressing this relation. Hadronic reactions commonly employ the definition of, e.g., Refs. \( \), where the isospin state of
the $\Sigma^+$ is given as $\Sigma^+ = +|I = 1, I_3 = 1>$. In this paper, we define $\Sigma^+ = -|I = 1, I_3 = 1>$ which is consistent with $Y_{l,m}^* = (-1)^m Y_{l,-m}$ since $\Sigma^- = |I = 1, I_3 = -1>$. This convention is customarily used in all meson photoproduction reactions \cite{17}.

In $K^0$ photoproduction the vector meson exchanged in the t-channel is the $K^{*0}(896.1)$, hence the transition moment $g_{K^*K\gamma}$ in $K^+$ production case must be replaced by the neutral transition moment. The transition moment is related to the decay width by \cite{18}

$$\Gamma_{K^*\rightarrow K\gamma} = \frac{9.8\text{MeV}}{4\pi}|g_{K^*K\gamma}|^2$$

(5)

The measured decay widths are \cite{19}

$$\Gamma_{K^{*+}\rightarrow K^+\gamma} = 50 \pm 5\text{keV}$$

(6)

$$\Gamma_{K^{*0}\rightarrow K^0\gamma} = 117 \pm 10\text{keV}$$

(7)

Inserting these numbers in Eq. (5) we obtain

$$|g_{K^{*0}K^0\gamma}/g_{K^{*+}K^+\gamma}| = 1.53 \pm 0.20$$

(8)

However, the sign of this ratio is undetermined experimentally. For the phase of the neutral decay mode, we turn to a quark model prediction, in particular, the cloudy bag model computation by Singer and Miller \cite{20}, which accurately reproduces the experimental widths of Eqs. (6) and (7). The quark and pion cloud terms contribute in-phase to the $K^*$ photo decay, with the $K^{*0}$ amplitude of opposite sign as the $K^{*+}$ amplitude. Thus in our photoproduction amplitudes we use

$$g_{K^{*0}K^0\gamma} = -1.53 \ g_{K^{*+}K^+\gamma}$$

(9)

Each of the nucleon resonances will be excited by an anomalous magnetic moment $\mu^* = (e/2M_N)\kappa^*$, which can be written in terms of the helicity amplitude $A_{1/2}$. The $N^* \rightarrow N\gamma$ decay width can be used to relate the two quantities to each other,

$$\Gamma_{N^*\rightarrow N\gamma} = \frac{\alpha}{4\pi} (\kappa^* )^2 \frac{k_{c.m.}^3}{M^2} = \frac{k_{c.m.}^2}{\pi} \frac{M}{M^*} |A_{1/2}|^2$$

(10)
Thus

\[ \frac{\kappa_n^*}{\kappa_p^*} = \frac{A_{1/2}^n}{A_{1/2}^p} \]  

(11)

where we have used the quark model calculations by Koniuk and Isgur [21] to constrain
the magnitude of the neutron amplitudes (see Table I). Thus, the coupling constants
\( G_{K\Lambda N^*} = g_{K\Lambda N^*}g_{\gamma NN^*} \) for kaon production on the proton are adjusted to the data, while
for kaon production on the neutron the couplings are multiplied with the factor of Eq. (11)
and the appropriate isospin factors.

The \( \Sigma \) photoproduction reactions allow \( \Delta \) resonance contributions whose various coupling
constants are related by

\[ G_\Delta \equiv G_{K^+\Sigma^0\Delta^+} = -\sqrt{2}G_{K^0\Sigma^0\Delta^0} = G_{K^0\Sigma^0\Delta^0} = \sqrt{2}G_{K^+\Sigma^-\Delta^0} \]  

(12)

Here we used the same isospin convention as the one in Eq. (4).

As a first step, we limit our analysis to the four \( \Sigma \) production channels. For the more
qualitative findings presented here this proves to be sufficient. A more complete quantitative
analysis that will include the \( \Lambda \) channels along with new upcoming data from Bonn [22] will
be presented in a future work. Here, we emphasize the fact that combining the \( K^+\Sigma^0 \)
photo- and electroproduction data [23] (86 and 96 points below 2.2 GeV, respectively) with
the few total cross section measurements of the \( K^+\Sigma^- \) and \( K^0\Sigma^+ \) channels in a common
fit leads to very strong constraints on the leading coupling constants. To our knowledge,
previous authors have not included the charged \( \Sigma \) channels in their analyses.

As in previous studies, our model includes the standard Born terms along with the
intermediate \( \Lambda \)- and \( K^* \)-exchange. Furthermore, we have incorporated the \( N^* \) resonances
\( S_{11}(1650) \) and \( P_{11}(1710) \) as well as the \( \Delta \) resonances \( S_{31}(1900) \) and \( P_{31}(1910) \) which can
only contribute to \( \Sigma \) photoproduction. Our choice of resonance was guided by our goal
to draw qualitative conclusions about the behaviour of coupling constants with a simple
model that contains as few parameters as needed to achieve a reasonable \( \chi^2 \). We found that
once a resonance with a particular spin-parity structure has been included in the fit, adding
additional states with the same quantum numbers does not significantly reduce the $\chi^2$ any more. The $S_{11}(1650)$ and $P_{11}(1710)$ states have considerable branching ratios into the $KY$ channels and, along with the $S_{31}(1900)$ and $P_{31}(1910)$ $\Delta$ resonances, yielded the smallest $\chi^2$ with a minimum number of parameters. We found that adding additional resonances like the $S_{31}(1620)$ or the hyperonic $\Lambda^*(1405)$ did not affect our conclusions. Furthermore, our fit does not contain the $K_1(1270)$ $t$-channel resonance. In contrast to $K^+\Lambda$ production where the inclusion of this state led to a $K\Lambda N$ coupling constant in agreement with SU(3), we found no sensitivity to this resonance in $K\Sigma$ production. In addition, due to the lack of information on the $K_1 \to K \gamma$ widths the $K_1$ contribution to the $K^0$ channel cannot easily be related to the $K^+$ channel. With the current data base there clearly remains an ambiguity as to which are the most important resonances contributing to the kaon photoproduction process. Future high precision data from CEBAF are expected to resolve this issue.

Fig. 1 compares the predictions of three different models for the total cross section of the four possible channels in $K\Sigma$ photoproduction. The simplest model shown is taken from Ref. [4], it contains only the Born terms plus one additional $\Delta$-resonance at 1700 MeV and was fitted solely to the $K^+\Sigma^0$ photoproduction data. Furthermore, two of our new fits are shown, one includes the $K^+\Sigma^-$ and $K^0\Sigma^+$ data, while the other one does not. The coupling constants of the three models are given in Table III.

Fig. 1 clearly demonstrates that different models which give an adequate description of the $\gamma p \to K^+\Sigma^0$ data can give drastically divergent predictions for the other isospin channels. The difference in the Born coupling constants listed in Table III helps to shed some light on these discrepancies. The model of Ref. [4] that overpredicts the charged $\Sigma$ cross sections by up to two orders of magnitude contains the largest Born coupling constant. The different predictions of our new model with set II and set III of the coupling constants illustrate the same point. Fitting all $K^+\Sigma^0$ photo- and electroproduction data (set II) leads very large discrepancies with the $K^+\Sigma^-$ and $K^0\Sigma^+$ total cross section data. Including those data into the fit yields a coupling strength $g_{K\Sigma N}$ that differs by almost a factor of 10 from the coupling constant in set II. Thus, fitting all data simultaneously reduces the Born couplings to very
small values, almost eliminating the Born terms. Clearly, the extracted couplings are significantly below their SU(3) range as well as the values obtained in hadronic reactions. This may be due to the neglect of hadronic form factors at the strong interaction vertices, thus affecting especially the non-resonant Born terms which are far off-shell even near threshold. The SU(3) predictions, on the other hand, relate on-shell couplings while determinations from low-energy hadronic scattering reactions generally include form factors explicitly. Future kaon photoproduction studies will have to address this question by including hadronic form factors in a gauge invariant fashion.

We have compared a wide variety of models that are available in the literature and always found the same pattern. For example, one of the more advanced models developed in Ref. [6] was fitted to photo- and electroproduction data of the $K^+\Lambda$, $K^+\Sigma^0$, and $K^+\Lambda^*(1405)$ final states, while neglecting the charged $\Sigma$ channels. Furthermore, they include crossing constraints to simultaneously reproduce the $K^-$ radiative capture branching ratios. Their fit [6] yielded significantly reduced couplings and its disagreement with the experimental $K^+\Sigma^-$ and $K^0\Sigma^+$ data is not as dramatic.

The underlying reason for the drastic differences in the various predictions is elucidated in Fig. 2. Analyzing the individual diagramatic contributions of the model of Ref. [4] in detail for the process $\gamma p \rightarrow K^+\Sigma^0$, reveals that the total cross section results from successive destructive interferences between the various diagrams. The basic Born terms consisting of the $K^+ t$-channel, the $\Sigma^0 u$-channel, and the $p s$-channel exchanges governed by $g_{K\Sigma N}$ diverge very quickly, adding the $\Lambda$ in the $u$-channel and the $K^*$ in the $t$-channel leads to cancellations that reduce the calculated cross section by up to an order of magnitude at higher energies. In contrast to the $K^+\Sigma^0$ channel, the other three processes do not exhibit successive destructive interferences, leading to large predictions for the total cross sections. This behavior can be traced to the relations between the coupling constants in Eqs.(4) and (9). The magnitude of the calculated charged $\Sigma$ cross sections is due in part to the fact that the intermediate $u$-channel $\Lambda$ diagram cannot contribute. Fitting all available data with one amplitude leads to the observed drastic reduction in the Born couplings, thus yielding
a resonance dominated process.

In conclusion, we have shown that existing models for $K^+\Lambda$ and $K^+\Sigma^0$ production can dramatically overpredict the few available total cross section data for $K^+\Sigma^-$ and $K^0\Sigma^+$ photoproduction. Including these data in the fit leads to drastically reduced Born coupling constants $g_{K\Sigma N}$ and $g_{K\Lambda N}$, yielding a description of the process that is resonance dominated. It is therefore imperative that future analyses include the complete data base and that ongoing and upcoming experimental efforts provide data for all isospin channels.

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### TABLE I. The six reactions of photokaon production with their threshold energies.

| Type                     | $E_{\gamma,lab}^{\text{thresh}}$ (MeV) | $E_{\text{tot.,c.m.}}^{\text{thresh}}$ (MeV) |
|--------------------------|----------------------------------------|-----------------------------------------------|
| $\gamma p \rightarrow K^+ \Lambda$ | 911.                                   | 1609                                          |
| $\gamma n \rightarrow K^0 \Lambda$  | 915.                                   | 1613                                          |
| $\gamma p \rightarrow K^+ \Sigma^0$  | 1046                                   | 1686                                          |
| $\gamma p \rightarrow K^0 \Sigma^+$   | 1048                                   | 1687                                          |
| $\gamma n \rightarrow K^+ \Sigma^-$   | 1052                                   | 1691                                          |
| $\gamma n \rightarrow K^0 \Sigma^0$  | 1051                                   | 1690                                          |

### TABLE II. $N + \gamma \rightarrow N^*(\frac{1}{2}^\pm)$ Amplitudes

| Resonance | $J^\pi$ | $A_{1/2}^p$ (GeV$^{-1/2}$) | $A_{1/2}^n$ (GeV$^{-1/2}$) | $\kappa_n/\kappa_p$ |
|-----------|---------|---------------------------|---------------------------|---------------------|
| $N(1650)$ | $\frac{1}{2}^-$ | +88.                      | -35.                      | -0.40               |
| $N(1710)$ | $\frac{1}{2}^+$ | -47.                      | -21.                      | +0.45               |
TABLE III. Coupling constants (CC) set I comes from Ref. [4], set II is generated by fitting to all but the charged Σ data, and set III comes from fitting all available data in $K\Sigma$ production.

| CC                                | I     | II    | III   |
|-----------------------------------|-------|-------|-------|
| $g_{K\Sigma N}/\sqrt{4\pi}$      | 2.72  | 1.30  | 0.130 |
| $g_{K\Lambda N}/\sqrt{4\pi}$     | -1.84 | -0.842| 0.510 |
| $G_{V}(K^*)/4\pi$                 | 0.104 | 0.053 | 0.052 |
| $G_{T}(K^*)/4\pi$                 | 0.005 | 0.019 | 0.053 |
| $G_{N1}(1650)/\sqrt{4\pi}$       | -     | -0.136| 0.111 |
| $G_{N2}(1710)/\sqrt{4\pi}$       | -     | -0.739| 0.807 |
| $G_{\Delta(1/2)}(1900)/\sqrt{4\pi}$| -    | 0.125 | 0.109 |
| $G_{\Delta(1/2)}(1910)/\sqrt{4\pi}$| -    | 0.746 | 0.457 |
| $G_{\Delta(3/2)}(1700)/4\pi$     | -0.069| -     | -     |
| $G_{\Delta(3/2)}(1700)/4\pi$     | 0.314 | -     | -     |
| $\chi^2/N$                        | 3.15  | 2.67  | 5.30  |
FIGURES

FIG. 1. Total cross section for the four isospin channels in $\Sigma$ photoproduction. The dash-dotted curve represents the model with coupling constants of set I (Ref. [4]) in Table III. The dashed curve represents set II, while the solid curve shows the result from set III. For the $n(\gamma, K^+)\Sigma^-$ and $p(\gamma, K^0)\Sigma^+$ graphs, the dash-dotted curve has been renormalized by a factor of 0.1 in order to fit on the scale. The experimental data are from [23].

FIG. 2. Contributions from the individual Born diagrams of the model from Ref. [4] in the total $K\Sigma$ cross section. The dotted curve shows the basic $N$ s-channel, $\Sigma$ u-channel and $K$ t-channel diagrams only. The dashed curve includes the intermediate $\Lambda$ u-channel (only for $\Sigma^0$ production), the dash-dotted curve includes the $K^*$, while the solid curve shows the full model. For the $n(\gamma, K^+)\Sigma^-$ and $p(\gamma, K^0)\Sigma^+$ graphs, all of the curves have been renormalized by a factor of 0.1 in order to fit on the scale.
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