Skyrme Strings

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Abstract

We construct nontopological string solutions with $U(1)$ Noether charge in the Skyrme model with a pion mass term, and examine their stability by taking linear perturbations. The solution exhibits a critical angular velocity beyond which the configuration energetically prefers to decay by emitting pions. This critical point is observed as a cusp in the relation between energy and charge. We find that the maximum length for the string to be stable is comparable to the size of one skyrmion. Beyond the length, it is unstable to decay. This instability raises the possibility of dynamical realization of Skyrme strings from monopole strings inside a domain wall.

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§1. Introduction

The detection of the signal of chiral symmetry breaking in the early Universe or corresponding high-energy experiments is one of the most intriguing topics of research aimed at understanding quantum chromo dynamics (QCD). The spontaneous breaking of the chiral symmetry accompanies the production of massless Nambu-Goldstone bosons known as mesons. At low energies, QCD can be reduced to an effective theory described by only the meson degrees of freedom, called the sigma model.

The linear sigma model contains heavy sigma particles as well as mesons so as to respect $SU(2) \times SU(2) \sim O(4)$ symmetry. In the low-energy limit, one can integrate out the heavy sigma particles, and the nonlinear sigma model described by only pion fields is obtained. In this model, the sigma particles are dynamically generated as a bound state of two pions. Taking into account the terms up to the 4th-order derivative (Skyrme term) in the nonlinear sigma model, one can obtain the Skyrme model where topological soliton solutions, called skyrmions, are interpreted as baryons, and hadrons are described in a unified manner.

A generic phenomenon expected as a consequence of the chiral phase transition is the formation of topological or nontopological defects via the Kibble mechanism. In fact, it has been shown that the linear sigma model yields nontopological string solutions composed of neutral pions and sigma particles. They are not topologically stable and hence decay into pions and sigma particles which subsequently decay into pions. The detailed study of the decay process will give insight into the chiral phase transition observed in, for example, heavy-ion collision experiments.

The Skyrme model has been also known to possess nontopological string-like solutions. The solutions are topologically unstable to decay. They are formed by the separation of the baryon density and may be closely related to QCD strings of quark-antiquark pairs. The possible decay modes of the string are many, and produce different numbers of mesons and baryon-antibaryon pairs. If pion strings in the linear sigma model are to be produced during the chiral phase transition, it is natural to expect that the Skyrme strings would also be produced in the low-energy regime. Although the Skyrme model describes baryons only approximately, we consider it to be a very convenient framework to incorporate baryons in the study of the chiral phase transition, and the string solutions would be worth further investigation.

In this study, we extend the idea of the strings in the Skyrme model obtained by Jackson to those with steady $U(1)$ rotation in the internal space, obtain numerical Q-string solutions, and examine their stability. We find the critical length of the string is of the order of the effective length of one skyrmion. For the dynamical decay process of the
Skyrme string, our solutions may be more interesting because the Q-string would decay into rotating baryon-antibaryon pairs, which are more realistic states than the static one.\textsuperscript{13\textsuperscript{13}} \textsuperscript{13}

Let us mention that string solutions in the baby-Skyrme model were previously obtained\textsuperscript{14} and discussed\textsuperscript{15}. The former indicates that the string solution is stable against decay into single baryons as it contains the energy per unit length less than the energy of an isolated baryon. The latter shows the possibility of the reconnection of the strings.

\section{String solutions in Skyrme Model}

The Skyrme Lagrangian with the pion mass is defined by

$$\mathcal{L} = \frac{F_{\pi}^2}{4} \text{tr}(R_{\mu} R^\mu) + \frac{1}{32e^2} \text{tr}[R_{\mu}, R_{\nu}]^2 + \frac{1}{2} m_{\pi}^2 F_{\pi}^2 \text{tr}(U - 1), \quad (2.1)$$

where $R_{\mu} = U^\dagger \partial_\mu U$ and $U$ is an SU(2)-valued chiral field given by

$$U = \phi_0 + i\vec{\phi} \cdot \vec{\tau} \quad \text{with} \quad \phi_0^2 + \vec{\phi}^2 = 1 \quad (2.2)$$

and $F_{\pi} \sim 93\text{MeV}$ is the pion decay constant, $m_{\pi}$ is the pion mass and $e$ is a free parameter whose value is about 5.45 as given in Ref\textsuperscript{16}, for example.

When $m_{\pi} = 0$, Lagrangian (2.1) is invariant under the chiral symmetry $SU(2)_L \times SU(2)_R$ defined by $U \to U^\prime = g_L U g_R^\dagger$ with $g_L \in SU(2)_L$ and $g_R \in SU(2)_R$. The pion mass term explicitly breaks $SU(2)_L \times SU(2)_R \to SU(2)_V$ with $g_L = g_R$. We consider the $U(1)$ subgroup of $SU(2)_V$ with the transformation

$$\begin{align*}
\phi_1 &\to \phi_1 \cos \alpha - \phi_2 \sin \alpha, \\
\phi_2 &\to \phi_1 \sin \alpha + \phi_2 \cos \alpha.
\end{align*}$$

The associated U(1) current is

$$J_\mu = \frac{\partial \delta \mathcal{L}}{\partial (\partial^\mu \alpha)} = \frac{F_{\pi}^2}{2} \text{tr}(R_{\mu} A) + \frac{1}{8e^2} \text{tr}([R_{\mu}, R_{\nu}][A, R_{\nu}]), \quad (2.3)$$

where

$$A = \begin{pmatrix}
\phi_1^2 + \phi_2^2 & (\phi_0 - i\phi_3)(\phi_1 - i\phi_2) \\
-(\phi_0 + i\phi_3)(\phi_1 + i\phi_2) & -i(\phi_1^2 + \phi_2^2)
\end{pmatrix}.$$ 

The conserved U(1) charge per unit length in the z-direction is given by the spatial integral of the zeroth component of the current,

$$Q = \int dx \, dy \, J^0. \quad (2.4)$$
To obtain string solutions with the $U(1)$ charge, let us consider the ansatz constructed by Jackson\cite{9} and induce steady rotation in the internal space by setting

$$\alpha = \alpha(t)$$  \hspace{1cm} (2.5)

in the $U(1)$ transformation of Eq. (2.3). Then we have

$$U = \begin{pmatrix} \cos f(r) & i \sin f(r) e^{-i(\theta + \alpha(t))} \\ i \sin f(r) e^{i(\theta + \alpha(t))} & \cos f(r) \end{pmatrix}$$ \hspace{1cm} (2.6)

in the cylindrical coordinate system with the metric

$$ds^2 = -dt^2 + dz^2 + dr^2 + r^2 d\theta^2,$$ \hspace{1cm} (2.7)

where $\hat{r}$ is a unit vector in the direction of $r$. This ansatz associates rotation in isospace with rotation in space.

Substituting ansatz (2.6) into Eq. (2.4), one obtains

$$Q = \frac{2\pi F}{e} \int d\rho \rho \sin^2 f (1 + f'^2) \dot{\alpha},$$ \hspace{1cm} (2.8)

where we have rescaled $\rho \equiv eF \pi r$ and $\tau \equiv eF \pi t$, and the prime and the dot denote differentiation with respect to $\rho$ and $\tau$, respectively.

To find the minimum of the string tension for fixed $Q$, we introduce a Lagrange multiplier $\omega$ and write the string tension in terms of $Q$ as\cite{17}

$$E_\omega = T + V + \hat{\omega} [\hat{Q} - 2\pi \int d\rho \rho \sin^2 f (1 + f'^2) \dot{\alpha}]$$

$$= V + \pi \int d\rho \rho \sin^2 f (1 + f'^2) (\dot{\alpha}^2 - 2\hat{\omega} \dot{\alpha}) + \hat{\omega} \hat{Q}$$

$$= V - \pi \dot{\omega}^2 \int d\rho \rho \sin^2 f (1 + f'^2) \pi \int d\rho \rho \sin^2 f (1 + f'^2) (\dot{\alpha} - \hat{\omega})^2$$

$$+ \hat{\omega} \hat{Q}$$ \hspace{1cm} (2.9)

where we have defined

$$T \equiv \pi \int d\rho \rho \sin^2 f (1 + f'^2) \dot{\alpha}^2,$$ \hspace{1cm} (2.10)

$$V \equiv \pi \int d\rho \rho \left[ \left( \frac{\sin^2 f}{\rho^2} \right) f'^2 + \frac{\sin^2 f}{\rho^2} + 2\tilde{m}_\pi^2 (1 - \cos f) \right]$$ \hspace{1cm} (2.11)

with $\tilde{m}_\pi \equiv m_\pi/eF_\pi$, $\hat{\omega} \equiv \omega/eF_\pi$, $\hat{Q} \equiv eQ/F_\pi$ and $E_\omega \equiv eE/F_\pi$. The third term in Eq. (2.9), which is the only time-dependent term, is positive definite and therefore should vanish at the minimum. We thus set $\alpha = \hat{\omega} \tau$ and the tension can be written as

$$E_\omega = V + \frac{\hat{Q}^2}{2T}$$ \hspace{1cm} (2.12)
with the moment of inertia for iso-rotation being
\[ I = 2\pi \int d\rho \rho \sin^2 f(1 + f'^2). \]

One can see that for fixed \( \hat{Q} \), the charge term \( \hat{Q}^2/2I \) plays the role of stabilizing string solutions as it is inversely proportional to the functional of the profile \( f \).

The field equation can be obtained by taking the variations of \( f \) in string tension (2.9),
\[
\left( 1 - \hat{\omega}^2 \sin^2 f + \frac{\sin^2 f}{\rho^2} \right) f'' + \left( 1 - \hat{\omega}^2 \sin^2 f - \frac{\sin^2 f}{\rho^2} \right) \frac{f'}{\rho} + \frac{\sin f \cos f}{\rho^2} (1 - \hat{\omega}^2 \rho^2)(f'^2 - 1) - \hat{m}_\pi^2 \sin f = 0.
\] (2.13)

Note that to find full solutions in our system, we must solve the equation of motion without imposing any ansatz. In this sense, the solution of Eq. (2.13) may not be the true minimum of the action. It, however, minimises the action within the cylindrical symmetric configuration.

The finiteness and regularity of string tension require the boundary conditions
\[
f(\infty) = 0, \quad f(0) = n\pi,
\] (2.14)
where \( n \) is any integer. In this paper, we only examine the case of \( n = 1 \). Other values of \( n \) will be reported elsewhere. Equation (2.13) is solved numerically subject to boundary conditions (2.14).

The asymptotic form of the profile \( f(\rho) \) as \( \rho \to \infty \) can be obtained by linearizing the field equation. Setting \( f = \delta f \), one can get
\[
\delta f'' + \frac{1}{\rho} \delta f' - \frac{1}{\rho^2} \delta f - (\hat{m}_\pi^2 - \hat{\omega}^2) \delta f = 0
\] (2.15)
with the solution of the Bessel function
\[
\delta f = CK_1(b\rho),
\]
where \( C \) is an arbitrary constant and
\[
b = \sqrt{\hat{m}_\pi^2 - \hat{\omega}^2}.
\] (2.16)
This restricts the value of \( \hat{\omega} \) as
\[
0 < \hat{\omega} < \hat{m}_\pi.
\] (2.17)
Thus, there exists a critical value \( \hat{\omega}_+ = \hat{m}_\pi \) beyond which soliton solutions cannot be found. This is because for \( \hat{\omega} = \hat{\omega}_+ \), \( f \sim 1/\rho \) and for \( \hat{\omega} > \hat{\omega}_+ \), \( f \sim CJ_1(b'\rho) \) with \( b' = \sqrt{\hat{\omega}^2 - \hat{\omega}_+^2} \)
as $\rho \to \infty$, resulting in the divergent string tension and inertia moment $I$. Physically, this oscillatory behaviour of $f$ signals the instability of the string against the emission of pions.

String solutions are obtained by solving Eq. (2.13) numerically. Figure 1 shows the profile $f$ as a function of $\rho$ for several values of $\dot{\omega}$. As the value of $\dot{\omega}$ increases, the size of the soliton expands. This expansion is due to the centrifugal force effect. Figure 2 shows the string tension as a function of the charge. The tension increases as the charge and/or the pion mass increases. The approximate asymptotic formula of the tension can be deduced analytically as

$$\mathcal{E}_\omega \sim \text{const} + \frac{\hat{m}_\pi}{2} \dot{Q},$$

(2.18)

which holds in our numerical results within a few percent error. We found the cusp appears at the critical value of $\dot{\omega}$. In the context of Q-ball solutions, the second branch represents unstable solutions called Q-clouds. Consistently, our second branch represents string solutions which energetically favour decay by the emission of pions.

§3. Linear stability analysis

In this section, we shall examine the linear stability of our string solutions obtained in the previous section to see whether the $U(1)$ rotation changes their stability. To study the linear stability of the soliton solution, let us consider an infinitesimal fluctuation in the $x_3$-direction:

$$U = \begin{pmatrix} \cos f + i\delta_3 \\ i e^{i(\theta + \bar{\omega} \tau)} \sin f \\ i e^{-i(\theta + \bar{\omega} \tau)} \sin f \\ -i \delta_3 + \frac{1}{\rho} \right)$$

(3.1)

where $\delta_3$ is the fluctuation. The field $U$ in Eq. (3.1) is then unitary up to the first order in $\delta_3$. Note that one can show the fluctuations in other internal directions decouple from $\delta_3$ and contribute only to raising the total energy of the configuration.

The field equation for $U$ is derived as

$$\partial_\mu R^\mu_{\nu} + \partial_\mu [R^\mu_{\nu}(R^\nu_{\mu} - R^\nu_{\nu})] + \frac{1}{2} \hat{m}_\pi^2 \text{tr}(i \tau^4 U) = 0,$$

(3.2)

where we have defined $U^\dagger \partial_\mu U = i R^\nu_{\mu} \tau^4$. Inserting Eq. (3.1) into Eq. (3.2) and taking the first-order terms in $\delta_3$, one obtains the following equation for $\delta_3$:

$$\left(1 - \omega^2 \sin^2 f + \frac{\sin^2 f}{\rho^2} \right) \delta_3'' + \left[-2 \omega^2 \sin f \cos f + \delta_3 \left(2 f' \cos f - \frac{\sin f}{\rho} \right) \right] \delta_3' + \frac{1}{\rho^2} \left(1 + \omega^2 \sin^2 f \right) \delta_3 + \frac{1}{\rho} \left[\left(1 + \omega^2 \sin^2 f \right) \delta_3 + 2 \omega \sin^2 f \right] \delta_3' + \frac{1}{\rho^2} \left(1 + \omega^2 \sin^2 f \right) \delta_3' + \left[\left(1 + \omega^2 \sin^2 f \right) \delta_3' + \frac{1}{\rho^2} \left(2 \omega \sin^2 f \right) \delta_3 \right] \delta_3' + \left[\left(1 + \omega^2 \sin^2 f \right) \delta_3' + \frac{1}{\rho^2} \left(2 \omega \sin^2 f \right) \delta_3 \right] \delta_3'$$

(3.3)
Let us define the length of the string in the z direction as \( L \). Then the boundary conditions at \( z = \pm \frac{1}{2}L \) are given by
\[
\delta_3(\rho, \theta, z = \pm L/2) = 0.
\] (3.4)

Setting
\[
\delta_3 = e^{i\Omega \tau} e^{im\theta} \cos(k_z z) R(\rho),
\] (3.5)
where \( m \) is an integer and
\[
k_z = \frac{(2n + 1)\pi}{L},
\] (3.6)
with \( n \) being an integer, and considering the most unstable mode \( m = n = 0 \), Eq. (3.3) is reduced to
\[
- \left( 1 - \hat{\omega}^2 \sin^2 f + \frac{\sin^2 f}{\rho^2} \right) R'' - \left[ -2\hat{\omega}^2 f' \sin f \cos f + \frac{1}{\rho} \left( 1 - \hat{\omega}^2 \sin^2 f + \frac{\sin f}{\rho} \left( 2f' \cos f - \frac{\sin f}{\rho} \right) \right) \right] R'
+ \left[ 1 + f'^2 - \hat{\omega}^2 \sin^2 f + \frac{\sin^2 f}{\rho^2} \right] \left( \frac{\pi}{L} \right)^2 - \left( 1 - 2\hat{\omega}^2 \sin^2 f + \frac{2\sin^2 f}{\rho^2} \right) f'^2 - \frac{\sin^2 f}{\rho^2} \left( 1 - \hat{\omega}^2 \rho^2 + \hat{m}_z^2 \cos f \right) \right] R
= \left( 1 + f'^2 + \frac{\sin^2 f}{\rho^2} \right) \Omega^2 R.
\] (3.7)

We introduce a new coordinate \( \tilde{\rho} \) such that
\[
\frac{d\tilde{\rho}}{d\rho} = \left[ \rho \left( 1 - \hat{\omega}^2 \sin^2 f + \frac{\sin^2 f}{\rho^2} \right) \right]^{-1}.
\] (3.8)

Then Eq. (3.7) takes the form of the Sturm-Liouville equation
\[
- \frac{d^2 R}{d\tilde{\rho}^2} + VR = \rho \left( 1 + f'^2 + \frac{\sin^2 f}{\rho^2} \right) \Omega^2 R,
\] (3.9)
where
\[
V = \rho \left[ \left( 1 + f'^2 - \hat{\omega}^2 \sin^2 f + \frac{\sin^2 f}{\rho^2} \right) \left( \frac{\pi}{L} \right)^2 + \left( 1 - 2\hat{\omega}^2 \sin^2 f + \frac{2\sin^2 f}{\rho^2} \right) f'^2 \right.
+ \frac{\sin^2 f}{\rho^2} \left( 1 - \hat{\omega}^2 \rho^2 - \hat{m}_z^2 \cos f \right).
\] (3.10)

The solution is linearly stable if there are no normalizable modes with negative energy (bound states), because such modes realise exponentially diverging \( \delta_3 \) owing to imaginary \( \Omega \).

It is straightforward to show that the normalizable wave function should take an asymptotic form of the Bessel function for large \( \rho \),
\[
R(\rho) \sim K_0(\kappa_1\rho) \quad \text{with} \quad \kappa_1 = \sqrt{\left( \frac{\pi}{L} \right)^2 + \hat{m}_z^2 - \Omega^2},
\] (3.11)
and for small \( \rho \),
\[
R(\rho) \sim J_0(\kappa_2\rho) \quad \text{with} \quad \kappa_2 = \sqrt{\frac{(1 + 2f_1^2)(\Omega^2 - \pi^2/L^2) + \hat{m}_z^2 + 2(1 + f_1^2)f_1^2}{1 + f_1^2}},
\] (3.12)
where we have defined $f_1 = f'(0)$.

In order for $\xi$ to be normalizable, $\kappa_1$ and $\kappa_2$ must be real, which gives an inequality for $\Omega^2$:

$$\left(\frac{\pi}{L}\right)^2 - \frac{1}{1 + 2f_1^2}[\hat{m}_\pi^2 + 2(1 + f_1^2)f_1^2] < \Omega^2 < \left(\frac{\pi}{L}\right)^2 + \hat{m}_\pi^2. \quad (3.13)$$

The condition that there exists no tachionic mode is $\Omega^2 > 0$, that is, the left-hand side in Eq. (3.13) is positive,

$$\left(\frac{\pi}{L}\right)^2 - \frac{1}{1 + 2f_1^2}[\hat{m}_\pi^2 + 2(1 + f_1^2)f_1^2] > 0. \quad (3.14)$$

This leads to the constraint for the length of the stable string:

$$L < \sqrt{\frac{(1 + 2f_1^2)\pi^2}{\hat{m}_\pi^2 + 2(1 + f_1^2)f_1^2}}. \quad (3.15)$$

In evaluating $f_1$ numerically, we have $f_1 \sim O(1)$. Roughly speaking, $\hat{m}_\pi = m_\pi/eF_\pi = 137/(5.45 \times 93) = 0.27 \sim O(10^{-1})$ and hence, $L \sim O(1)$. As the mass of one skyrmion is $M_s/eF_\pi = 1000/(5.45 \times 93) = 1.97 \sim O(1)$, the maximum length for the string to be stable is of the order of the length of one skyrmion. When the string becomes longer than that, it will decay by emitting pions. We have also checked this conclusion by solving Eq. (3.7) numerically for both vanishing and nonvanishing values of $\dot{\omega}$.

§4. Conclusions

In this paper we constructed string solutions with the $U(1)$ Noether charge in the Skyrme model and examined whether the $U(1)$ charge could stabilize the string solution.

The string solution is exponentially localized in the radial direction if the angular frequency is less than the pion mass. Otherwise it is oscillatory along the radial direction. We found that there exists a critical value of the angular velocity beyond which the solution energetically favours decay by the emission of pions. This unstable branch of the string is observed as a cusp in the relation between the energy and the charge, as shown in Figure 3.

The stability was examined by taking linear perturbations. We found that the maximum length for the string to be stable is comparable to the size of a skyrmion. Beyond that length, they are unstable to decay.

A similar instability is observed in the case of skyrmions when one performs the projection after $SU(2)$-isorotation (projection after rotation). Our string solutions have also been obtained by the projection after $U(1)$-isorotation. In this method, the configuration breaks down completely in the chiral limit ($m_\pi = 0$) owing to the radiation of pion waves.

When the dynamical decay process of the Skyrme string is considered, our solutions may be more interesting because these would decay into rotating baryon-antibaryon pairs, which are more realistic states than the static one. In particular, if the Skyrme strings are assumed to be produced during the chiral phase transition, as are pion strings, the study of the decay mode will provide important information on high-energy experiments.
Our solutions are considered to be the embedding of the (2+1)-dimensional spinning skyrmion in 3+1 dimensions and therefore, they are qualitatively similar to the solutions reported in. When topological or nontopological solitons are embedded in higher dimensional spacetime, the instability in the direction of worldvolume is not alleviated by the existence of a Noether conserved charge. Thus, an alternative mechanism of stabilizing the Skyrme strings should be considered, such as cosmological expansion, which is a technique of stabilizing $\sigma$-model lump strings. The Skyrme string might, however, be physically advantageous over the lump string in that the former is radially stable up to the effective length of one skyrmion while the latter is unstable to collapse. Thus it would be interesting to study them in the context of cosmic strings or to discuss the Skyrme strings in the context of the QCD strings since we are not yet sure how they are actually relevant to QCD strings.

Approximate skyrmion solutions are known to be obtained from a holonomy of the Yang-Mills instanton particles in five space-time dimensions. It has recently been shown that this situation can be realized by placing instanton particles inside a domain wall whose low-energy dynamics is described by the Skyrme model. In the same way, unstable Skyrme strings discussed in this paper may be approximately constructed from a holonomy of monopole strings inside the domain wall, because it has been shown by Eto et al. that a monopole and a domain wall cannot coexist as a Bogomol'nyi-Prasad-Sommerfield state and the configuration is unstable.

Finally, the possible realization of our solution in string theory, such as AdS/QCD where skyrmions are constructed will be very interesting to explore.
Fig. 1. Profile function $f$ as a function of $\rho$ with $\hat{m}_\pi^2 = 0.1$ for $\hat{\omega} = 0.00, 0.05, 0.10$.

Fig. 2. Tension $E_\omega$ of string as a function of charge $\hat{Q}$ for $\hat{m}_\pi^2 = 0.05, 0.10, 0.15$.

Fig. 3. Tension $E_\omega$ of string as a function of charge $\hat{Q}$ for $\hat{m}_\pi^2 = 0.10$. The cusp is observed at the critical value of $\hat{\omega}$ which is $\hat{\omega}_+ = \hat{m}_\pi$. 


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