Broken Flavor $2 \leftrightarrow 3$ Symmetry and phenomenological approach for universal quark and lepton mass matrices

Koichi MATSUDA
Graduate school of Science, Osaka University, Toyonaka, Osaka 560-0043, Japan

Hiroyuki NISHIURA
Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata, Osaka 573-0196, Japan

(Dated: December 26, 2005)

Abstract

A phenomenological approach for the universal mass matrix model with a broken flavor $2 \leftrightarrow 3$ symmetry is explored by introducing the $2 \leftrightarrow 3$ antisymmetric parts of mass matrices for quarks and charged leptons. We present explicit texture components of the mass matrices, which are consistent with all the neutrino oscillation experiments and quark mixing data. The mass matrices have a common structure for quarks and leptons, while the large lepton mixings and the small quark mixings are derived with no fine tuning due to the difference of the phase factors. The model predicts a value $2.4 \times 10^{-3}$ for the lepton mixing matrix element square $|U_{13}|^2$, and also $\langle m_\nu \rangle = (0.89 - 1.4) \times 10^{-4}$ eV for the averaged neutrino mass which appears in the neutrinoless double beta decay.

PACS numbers: 12.15.Ff, 14.60.Pq, 11.30.Hv
I. INTRODUCTION

It has been established through the discovery of neutrino oscillation [1] that neutrinos have finite masses and mix one another with near bimaximal lepton mixings \( \sin^2 2\theta_{12} \sim 1, \sin^2 2\theta_{23} \simeq 1 \) which are in contrast to small quark mixings. In order to explain the large lepton mixing and small quark mixing, mass matrix models with various structures have been investigated in the literature [2]–[12]. For example, it is argued that the large lepton mixing can be explained by mass matrices with a flavor 2 ↔ 3 symmetry [13]–[28]. We think that quarks and leptons should be unified. Therefore, it is interesting to investigate a possibility that all the mass matrices of the quarks and leptons have the same matrix form, which leads to the large lepton mixings and the small quark mixings. The mass matrix model with the universal form for quarks and leptons is also useful when it is embedded into a grand unified theory (GUT).

In this paper, we discuss a Hermit mass matrix model with a universal form given by

\[
M = \begin{pmatrix}
0 & a e^{-i\phi} & a e^{-i\phi''} \\
\bar{a} e^{i\phi} & b & \bar{c} e^{-i\phi'} \\
\bar{a} e^{i\phi''} & \bar{c} e^{i\phi'} & b
\end{pmatrix},
\]

(1.1)

where \( a, b, \) and \( c \) are real parameters and \( \phi, \phi', \) and \( \phi'' \) are phase parameters. It is important from a phenomenological point of view to parameterize the texture components of the mass matrix as the first step to make a GUT scenario. Assuming that neutrinos are the Majorana particles, we present the texture components of the universal mass matrices which will lead to the Cabibbo–Kobayashi–Maskawa (CKM) [29] quark mixing and the Maki–Nakagawa–Sakata (MNS) [30] lepton mixing which are consistent with the present experimental data. Here we explore a phenomenological mass matrix model base on the flavor 2 ↔ 3 symmetry. Our mass matrices has a broken flavor 2 ↔ 3 symmetry for quarks and charged leptons by introducing the 2 ↔ 3 antisymmetric parts of their mass matrices. We assume that this broken flavor 2 ↔ 3 symmetry is due to the 120 Higgs scalar in the SO(10) GUT model, while mass matrices contributed from 10 and 126 Higgs scalars are 2 ↔ 3 symmetric.

This article is organized as follows. In Sec. II, our mass matrix model is presented. In Sec. III, we discuss the diagonalization of mass matrix of our model. The analytical expressions of the quark and lepton mixings of the model are given in Sec. IV. Sec. V is devoted to a summary.
II. MASS MATRIX MODEL

In this paper, we propose the following mass matrices:

\[ M_u = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} e^{-i\phi_u} A_u & \frac{1}{\sqrt{2}} e^{-i\phi_u} A_u \\ \frac{1}{\sqrt{2}} e^{i\phi_u} A_u & B_{u+D_u} & B_{u-D_u} \\ \frac{1}{\sqrt{2}} e^{i\phi_u} A_u & B_{u-D_u} & B_{u+D_u} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & iC_u \\ 0 & iC_u & 0 \end{pmatrix}, \]

(2.1)

\[ M_d = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} e^{-i\phi_d} A_d & \frac{1}{\sqrt{2}} e^{-i\phi_d} A_d \\ \frac{1}{\sqrt{2}} e^{i\phi_d} A_d & B_{d+D_d} & B_{d-D_d} \\ \frac{1}{\sqrt{2}} e^{i\phi_d} A_d & B_{d-D_d} & B_{d+D_d} \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & iC_d \\ 0 & iC_d & 0 \end{pmatrix}, \]

(2.2)

\[ M_e = \begin{pmatrix} 0 & \frac{1}{2} A_e & \frac{1}{2} A_e \\ \frac{1}{2} A_e & B_{e+D_e} & -C_e \\ \frac{1}{2} A_e & -C_e & B_{e+D_e} \end{pmatrix} + \begin{pmatrix} 0 & -\frac{1}{2} A_e & i\frac{1}{2} A_e \\ i\frac{1}{2} A_e & 0 & \frac{i}{2} B_{e-D_e} \\ -i\frac{1}{2} A_e & -i\frac{i}{2} B_{e-D_e} & 0 \end{pmatrix}, \]

(2.3)

\[ M_\nu = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} A_\nu & \frac{1}{\sqrt{2}} A_\nu \\ \frac{1}{\sqrt{2}} A_\nu & B_{\nu+D_\nu} & B_{\nu-D_\nu} \\ \frac{1}{\sqrt{2}} A_\nu & B_{\nu-D_\nu} & B_{\nu+D_\nu} \end{pmatrix}, \]

(2.4)

\[ M_D = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} A_D & \frac{1}{\sqrt{2}} A_D \\ \frac{1}{\sqrt{2}} A_D & B_{D+D_D} & B_{D-D_D} \\ \frac{1}{\sqrt{2}} A_D & B_{D-D_D} & B_{D+D_D} \end{pmatrix}, \]

(2.5)

\[ M_R = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} A_R & \frac{1}{\sqrt{2}} A_R \\ \frac{1}{\sqrt{2}} A_R & B_{R+D_R} & B_{R-D_R} \\ \frac{1}{\sqrt{2}} A_R & B_{R-D_R} & B_{R+D_R} \end{pmatrix}, \]

(2.6)

where \( M_u, M_d, M_e, \) and \( M_\nu \) are mass matrices for up quarks \( (u, c, t) \), down quarks \( (d, s, b) \), charged leptons \( (e, \mu, \tau) \), and neutrinos \( (\nu_e, \nu_\mu, \nu_\tau) \), respectively. The mass matrices \( M_D \) and \( M_R \) are, respectively, the Dirac and the right-handed Majorana type neutrino mass matrices, from which with the seesaw mechanism \( [31] \) we derive \( M_\nu \). Here \( A_f, B_f, C_f, \) and \( D_f \) are real parameters and \( \phi_f \) and \( \phi'_f \) are phase parameters with \( f = u, d, e, \) and \( \nu \).

Let us mention a particular feature of these mass matrices with respect to the flavor \( 2 \leftrightarrow 3 \) symmetry. We assume that the neutrino mass matrix has only \( 2 \leftrightarrow 3 \) symmetric part. In the mass matrices for quarks and charged leptons, the \( 2 \leftrightarrow 3 \) anti-symmetric terms (the second terms) are added as broken \( 2 \leftrightarrow 3 \) symmetric parts, in addition to the \( 2 \leftrightarrow 3 \) anti-symmetric parts.
symmetric terms (the first terms). This structure is motivated by the SO(10) GUT model in which 10, 120, and 126 Higgs scalars contribute to the fermion mass matrices, together with the following assumptions: (i) The contribution from the 120 Higgs scalar is $2 \leftrightarrow 3$ anti-symmetric, while those from 10 and 126 Higgs scalars are $2 \leftrightarrow 3$ symmetric for quarks and charged leptons. (ii) There exists the contribution to the Dirac type neutrino mass matrix $M_D$ from only the 10 and 126 Higgs scalars. and (iii) The texture components of the broken $2 \leftrightarrow 3$ symmetric parts are assumed to have different form between quarks and charged leptons, which derives a difference between the small quark mixing and the large lepton mixing. Namely, we assume that the mass matrices $M_u$ and $M_d$ are superpositions of the common real symmetric matrices $S$ and $S'$ and pure imaginary anti-symmetric one $A$ and that $M_e$, $M_D$, and $M_R$ consist of the common real symmetric matrices $S''$ and $S'''$ and pure imaginary anti-symmetric one $A'$, as follows.

$$M_u = \alpha_u S + \beta_u S' + \gamma_u A,$$  \hspace{1cm} (2.7)

$$M_d = \alpha_d S + \beta_d S' + \gamma_d A,$$  \hspace{1cm} (2.8)

$$M_e = \alpha_e S'' + \beta_e S''' + \gamma_e A',$$  \hspace{1cm} (2.9)

$$M_D = \alpha_D S'' + \beta_D S''',$$  \hspace{1cm} (2.10)

$$M_R = \beta_R S''',$$  \hspace{1cm} (2.11)

$$M_\nu = -M_D^T M_R^{-1} M_D,$$  \hspace{1cm} (2.12)

where the matrices $S$, $S'$, $S''$ and $S'''$ are $2 \leftrightarrow 3$ symmetric too, and $A$ and $A'$ are $2 \leftrightarrow 3$ anti-symmetric too. Here $\alpha_i$, $\beta_i$, $\gamma_i$ ($i = u, d, e$), $\alpha_D$, $\beta_D$, and $\beta_R$ are real coefficient parameters. Note that the $2 \leftrightarrow 3$ symmetry of the model is broken through only $A$ in the quark sector and $A'$ in the lepton sector.

Some semi-empirical approaches for mass matrices with the similar structure to the above Eqs. (2.7) - (2.12) have been proposed in the literature. For example, Gronau, Johnson, and Schechter [4] have discussed a model which consists of combining the Fritzch [2] and Stech [3] ansatz for quarks. They use the combination of symmetric mass matrix with antisymmetric one, although they don’t use the $2 \leftrightarrow 3$ symmetry. An extension to leptons based on an SO(10) GUT model has been investigated with use of the type I and type II seesaw mechanism for neutrino masses [7, 8]. In the present paper, we use the $2 \leftrightarrow 3$ symmetry for a common origin of the small quark and the large lepton mixings. This is the large difference between our model and the other $2\leftrightarrow3$ symmetry models [13-21].
The mass matrix $M_f$ ($f = u, d, e,$ and $\nu$) given in Eqs. (2.1)–(2.4) has common structure when it is expressed with a unitary matrix $Q_f$ as follows:

$$M_f = Q_f \hat{M}_f Q_f^\dagger,$$

for $f = u, d,$ and $e$

$$M_f = Q_f \hat{M}_f Q_f^T,$$

for $f = \nu$  \hspace{1cm} (2.13)

where $\hat{M}_f$ ($f = u, d, e,$ and $\nu$) is one of the seesaw-invariant type of mass matrix defined by [32]

$$\hat{M}_f = \begin{pmatrix}
0 & A_f & 0 \\
A_f & B_f & C_f \\
0 & C_f & D_f
\end{pmatrix}.$$  \hspace{1cm} (2.14)

Here the unitary matrices $Q_f$ are given by

$$Q_u = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} e^{i\phi_u} & \frac{1}{\sqrt{2}} e^{i\phi_u} \\
0 & \frac{1}{\sqrt{2}} e^{i\phi_u} & -\frac{1}{\sqrt{2}} i e^{i\phi_u}
\end{pmatrix},$$  \hspace{1cm} (2.15)

$$Q_d = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} e^{i\phi_d} & \frac{1}{\sqrt{2}} i e^{i\phi_d} \\
0 & \frac{1}{\sqrt{2}} e^{i\phi_d} & -\frac{1}{\sqrt{2}} i e^{i\phi_d}
\end{pmatrix},$$  \hspace{1cm} (2.16)

$$Q_e = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} e^{i\pi} & \frac{1}{\sqrt{2}} i e^{i\pi} \\
0 & \frac{1}{\sqrt{2}} e^{-i\pi} & -\frac{1}{\sqrt{2}} i e^{-i\pi}
\end{pmatrix},$$  \hspace{1cm} (2.17)

$$Q_\nu = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} i \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} i
\end{pmatrix}.$$  \hspace{1cm} (2.18)

Note that the structure of $Q_f$ mentioned above is the same for all the quarks and leptons except for the phase factors in it. It should be also noted that the Eq. (2.13) implies that the mass matrix $M_f$ is transformed to $\hat{M}_f$ by using a rebasing of the quark and lepton fields respectively.
III. DIAGONALIZATION OF MASS MATRIX

We now discuss a diagonalization of the mass matrix $M_f$ given in Eq. (2.13). First let us discuss the diagonalization of the mass matrix $\hat{M}_f$ given in Eq. (2.14), which appears as a part of $M_f$. This $\hat{M}_f$ is diagonalized by an orthogonal matrix $O_f$ as discussed in Refs. [23] and [24]:

$$O_f^T \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix} O_f = \begin{pmatrix} -m_{1f} & \ & \ \\ & m_{2f} & \ \\ & & m_{3f} \end{pmatrix}.$$  \hspace{1cm} (3.1)

Here $m_{1f}, m_{2f},$ and $m_{3f}$ are eigenvalues of $M_f$. Explicit expressions of the orthogonal matrix $O_f$, and components $A_f, B_f, C_f,$ and $D_f$ in terms of $m_{1f}, m_{2f},$ and $m_{3f}$ are presented in Appendix A. Namely, the mass matrix $M_f$ is diagonalized as

$$U_{Lf}^\dagger M_f U_{Lf} = \begin{pmatrix} -m_{1f} & \ & \ \\ & m_{2f} & \ \\ & & m_{3f} \end{pmatrix}$$  \hspace{1cm} for $f = u, d,$ and $e$,  \hspace{1cm} (3.2)

$$U_{Lf}^\dagger M_f U_{Lf}^* = \begin{pmatrix} -m_{1f} & \ & \ \\ & m_{2f} & \ \\ & & m_{3f} \end{pmatrix}$$  \hspace{1cm} for $f = \nu$.  \hspace{1cm} (3.3)

where the unitary matrix $U_{Lf}$ is given by

$$U_{Lf} = Q_f O_f.$$  \hspace{1cm} (3.4)

Here we list the expressions for $O_f$ and $Q_f$ in order:

$$O_f \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_{1f}}{m_{2f}}} & \sqrt{\frac{m_{1f} m_{3f}}{m_{2f}^2}} \\ -\sqrt{\frac{m_{1f}}{m_{2f}}} & 1 & \sqrt{\frac{m_{1f} m_{3f}}{m_{2f}^2}} \\ -\sqrt{\frac{m_{2f}^2 m_{3f}}{m_{1f}}} & -\sqrt{\frac{m_{1f} m_{3f}}{m_{2f}^2}} & 1 \end{pmatrix}$$  \hspace{1cm} for $f = u, d,$ and $e$,  \hspace{1cm} (3.5)

$$O_\nu = \begin{pmatrix} \sqrt{\frac{m_2}{m_1+m_1}} & \sqrt{\frac{m_1}{m_1+m_1}} & 0 \\ -\sqrt{\frac{m_1}{m_1+m_1}} & \sqrt{\frac{m_2}{m_1+m_1}} & 0 \\ 0 & 0 & 1 \end{pmatrix},$$  \hspace{1cm} (3.6)
\[ Q_f = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} e^{i\phi_f} & \frac{1}{\sqrt{2}} i e^{i\phi_f} \\
0 & \frac{1}{\sqrt{2}} e^{i\phi_f} & -\frac{1}{\sqrt{2}} i e^{i\phi_f}
\end{pmatrix} \quad \text{for } f = u \text{ and } d , \]  
(3.7)

\[ Q_e = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} e^{i\pi/4} & \frac{1}{\sqrt{2}} i e^{i\pi/4} \\
0 & \frac{1}{\sqrt{2}} e^{-i\pi/4} & -\frac{1}{\sqrt{2}} i e^{-i\pi/4}
\end{pmatrix} , \]  
(3.8)

\[ Q_\nu = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} i \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} i
\end{pmatrix} . \]  
(3.9)

Here, \( m_{iu}, m_{id}, m_{ie}, \) and \( m_{i\nu} (i = 1, 2, 3) \) are, respectively, the masses of up quarks, down quarks, charged leptons, and neutrinos, which we shall denote as \( (m_u, m_c, m_t), (m_d, m_s, m_b), (m_e, m_\mu, m_\tau) \) and \( (m_1, m_2, m_3) \).

Furthermore, the neutrino mass matrix is diagonalized as
\[ U'_{L\nu} M_f U'^*_{L\nu} = \begin{pmatrix}
m_1 \\
m_2 \\
m_3
\end{pmatrix}, \]  
(3.10)

where the unitary matrix \( U'_{L\nu} \) is given by
\[ U'_{L\nu} = U_{L\nu} P_\nu = Q_f O_f P_\nu. \]  
(3.11)

Here, in order to make the neutrino masses to be real positive, we introduced a diagonal phase matrix \( P_\nu \) defined by
\[ P_\nu = \text{diag}(i, 1, 1). \]  
(3.12)

IV. CKM QUARK AND MNS LEPTON MIXING MATRICES

Next we discuss the CKM quark mixing matrix \( V \) and the MNS lepton mixing matrix \( U \) of the model, which are given by
\[ V = U_{Lu}^T U_{Ld} = O_u^T Q_u^T O_d, \]  
(4.1)

\[ U = U_{Le}^T U_{L\nu} = O_e^T Q_e^T O_\nu P_\nu. \]  
(4.2)
From Eqs. (3.7) – (3.9), we obtain

\[
Q_u^\dagger Q_d = \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i(\phi_d - \phi_u)} & 0 \\
0 & 0 & e^{i(\phi_d - \phi_u)}
\end{pmatrix}, \tag{4.3}
\]

\[
Q_e^\dagger Q_\nu = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}. \tag{4.4}
\]

It should be noted that \(Q_e^\dagger Q_\nu\) takes quite different structure from that of \(Q_u^\dagger Q_d\) in our model. Namely, \(Q_u^\dagger Q_d\) is a diagonal phase matrix, while \(Q_e^\dagger Q_\nu\) represents a mixing matrix with a maximal lepton mixing between the second and third generations. Therefore, the large lepton mixing is realized with no fine tuning in our model.

Let us discuss the quark and lepton mixing matrices in detail.

A. CKM quark mixing matrix

We obtain the CKM quark mixing matrix \(V\) as follows:

\[
V = O_u^T Q_u^\dagger Q_d O_d \\
= \begin{pmatrix}
1 & \sqrt{\frac{m_u}{m_c}} & \sqrt{\frac{m_u m_d^2}{m_t^2}} \\
-\sqrt{\frac{m_d}{m_s}} & 1 & \sqrt{\frac{m_d}{m_t}} \\
\sqrt{\frac{m_s^2}{m_c m_t}} - \sqrt{\frac{m_d}{m_t}} & \sqrt{\frac{m_d}{m_t}} & 1
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & e^{i(\phi_d - \phi_u)} & 0 \\
0 & 0 & e^{i(\phi_d - \phi_u)}
\end{pmatrix} \\
\times \begin{pmatrix}
1 & \sqrt{\frac{m_d}{m_s}} & \sqrt{\frac{m_d m_s^2}{m_b^2}} \\
-\sqrt{\frac{m_d}{m_s}} & 1 & \sqrt{\frac{m_d}{m_b}} \\
\sqrt{\frac{m_s^2}{m_c m_b}} - \sqrt{\frac{m_d}{m_b}} & \sqrt{\frac{m_d}{m_b}} & 1
\end{pmatrix}. \tag{4.5}
\]

The explicit magnitudes of \((i, j)\) elements of \(V\) are obtained as

\[
|V_{12}| \approx \left| \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{i(\phi_d - \phi_u)} \right| = |0.224 - 0.06 e^{i(\phi_d - \phi_u)}|, \tag{4.7}
\]

\[
|V_{23}| \approx \left| \sqrt{\frac{m_d}{m_b}} - \sqrt{\frac{m_u}{m_t}} \right| = 0.0336, \tag{4.8}
\]

\[
|V_{13}| \approx \left| \sqrt{\frac{m_d m_s^2}{m_b^2}} - \sqrt{\frac{m_u m_d}{m_c m_b}} e^{i(\phi_d - \phi_u)} \right| \\
= |0.00022 - 0.0021 e^{i(\phi_d - \phi_u)}|. \tag{4.9}
\]
Here we have used the following numerical values for the quark masses estimated at the unification scale $\mu = M_X$, which are presented in Appendix A.

$$
m_u(M_X) = 1.04^{+0.19}_{-0.20} \text{MeV}, \quad m_d(M_X) = 1.33^{+0.17}_{-0.19} \text{MeV},
$$

$$
m_c(M_X) = 302^{+25}_{-27} \text{MeV}, \quad m_s(M_X) = 26.5^{+3.3}_{-3.7} \text{MeV},
$$

$$
m_t(M_X) = 129^{+196}_{-40} \text{GeV}, \quad m_b(M_X) = 1.00 \pm 0.04 \text{GeV}.
$$

By using the rephasing of the up and down quarks, Eq. (4.6) is changed to the standard representation of the CKM quark mixing matrix,

$$
V_{\text{std}} = \text{diag}(e^{i\zeta_1}, e^{i\zeta_2}, e^{i\zeta_3}) \cdot \text{diag}(e^{i\zeta_u}, e^{i\zeta_d})
$$

$$
= \begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{i\delta} \\
-c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}.
$$

(4.11)

Here $\zeta_i$ comes from the rephasing in the quark fields to make the choice of phase convention. The $CP$ violating phase $\delta$ in Eq. (4.11) is predicted with the expression of $V$ in Eq. (4.6) as

$$
\delta = \arg \left( \frac{V_{us}V_{cs}^*}{V_{ub}V_{cb}^*} \right) + \frac{|V_{us}|^2}{1 - |V_{ub}|^2} \simeq \phi_u - \phi_d + \pi.
$$

(4.12)

The predicted values of $|V_{12}|$, $|V_{23}|$, $|V_{13}|$, and $\delta$ are functions of a free parameter $\phi_u - \phi_d$ as shown in Eqs. (4.7)–(4.9) and (4.12). They are roughly consistent with the following numerical values at $\mu = M_X$, which are estimated from the experimental data observed at electroweak scale $\mu = M_Z$ by using the renormalization group equation and presented in Appendix B:

$$
|V_{12}^0| = 0.2226 - 0.2259,
$$

$$
|V_{23}^0| = 0.0295 - 0.0387,
$$

$$
|V_{13}^0| = 0.0024 - 0.0038,
$$

$$
\delta^0 = 46^\circ - 74^\circ.
$$

(4.13)

(4.14)
B. MNS lepton mixing matrix

We obtain the MNS lepton mixing matrix $U$ as follows:

\[ U = O_e^T Q_e^T Q_\nu O_\nu P_\nu \]  
\[ = \begin{pmatrix}
1 & \sqrt{\frac{m_e}{m_\mu}} & \sqrt{\frac{m_e}{m_\tau}} \\
\sqrt{\frac{m_\mu}{m_\tau}} & 1 & \sqrt{\frac{m_\mu}{m_\tau}} \\
\sqrt{\frac{m_\tau}{m_e m_\tau}} & -\sqrt{\frac{m_\tau}{m_\tau}} & 1 \\
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
\end{pmatrix}
\begin{pmatrix}
\sqrt{\frac{m_2}{m_2 + m_1}} & \sqrt{\frac{m_3}{m_2 + m_1}} & 0 \\
-\frac{m_1}{m_2 + m_1} & \frac{m_2}{m_2 + m_1} & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\begin{pmatrix}
c_1 i & s_1 & -\frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}} \\
-\frac{1}{\sqrt{2}} s_1 i & \frac{1}{\sqrt{2}} c_1 & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} s_1 i & -\frac{1}{\sqrt{2}} c_1 & \frac{1}{\sqrt{2}} \\
\end{pmatrix}, \]  
\[ (4.15) \]

with

\[ s_1 \equiv \sqrt{\frac{m_1}{m_2 + m_1}}, \quad c_1 \equiv \sqrt{\frac{m_2}{m_2 + m_1}}. \]  
\[ (4.17) \]

The explicit magnitudes of $(i, j)$ elements of $U$ are

\[ |U| \approx \begin{pmatrix}
\sqrt{\frac{m_2}{m_2 + m_1}} & \sqrt{\frac{m_3}{m_2 + m_1}} & \frac{1}{\sqrt{2}} \sqrt{\frac{m_e}{m_\mu}} \\
\frac{1}{\sqrt{2}} \frac{m_1}{m_2 + m_1} & \frac{1}{\sqrt{2}} \frac{m_2}{m_2 + m_1} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} \frac{m_3}{m_2 + m_1} & \frac{1}{\sqrt{2}} \frac{m_2}{m_2 + m_1} & \frac{1}{\sqrt{2}} \\
\end{pmatrix}, \]  
\[ (4.18) \]

Therefore, we obtain

\[
\tan^2 \theta_{\text{solar}} = \frac{|U_{12}|^2}{|U_{11}|^2} \approx \frac{m_1}{m_2}, \quad (4.19)
\]
\[
\sin^2 2\theta_{\text{atm}} = 4|U_{23}|^2 |U_{33}|^2 \approx 1, \quad (4.20)
\]
\[
|U_{13}|^2 \approx \frac{m_e}{2m_\mu}. \quad (4.21)
\]

In the following discussions we consider the normal mass hierarchy $m_1 < m_2 \ll m_3$ for the neutrino masses. Then the evolution effects which only give negligibly small correction effects can be ignored. Scenarios in which the neutrino masses have the quasi degenerate or the inverse hierarchy will be denied from Eqs. (4.19) and (4.24).
It can be seen from Eq. (4.16) that the large lepton mixing angle between the second and third generation is well realized with no fine tuning in the model. It should be noted that the present model leads to the same results for $\theta_{\text{solar}}$ and $\theta_{\text{atm}}$ as the model in Ref. [25], while a different feature for $|U_{13}|^2$ is derived.

On the other hand, we have [33] an experimental bound for $|U_{13}|^2_{\text{exp}}$ from the CHOOZ [34], solar [35], and atmospheric neutrino experiments [1]. From the global analysis of the SNO solar neutrino experiment [33, 35], we have $\Delta m^2_{12}$ and $\tan^2 \theta_{12}$ for the large mixing angle (LMA) Mikheev-Smirnov-Wolfenstein (MSW) solution. From the atmospheric neutrino experiment [1, 33], we also have $\Delta m^2_{23}$ and $\tan^2 \theta_{23}$. These experimental data with $3\sigma$ range are given by

$$|U_{13}|^2_{\text{exp}} < 0.054,$$  \hspace{1cm} (4.22)

$$\Delta m^2_{12} = m^2_2 - m^2_1 = \Delta m^2_{\text{solar}} = (5.2 - 9.8) \times 10^{-5} \text{ eV}^2,$$  \hspace{1cm} (4.23)

$$\tan^2 \theta_{12} = \tan^2 \theta_{\text{solar}} = 0.29 - 0.64 ,$$  \hspace{1cm} (4.24)

$$\Delta m^2_{23} = m^2_3 - m^2_2 \simeq \Delta m^2_{\text{atm}} = (1.4 - 3.4) \times 10^{-3} \text{ eV}^2,$$  \hspace{1cm} (4.25)

$$\tan^2 \theta_{23} \simeq \tan^2 \theta_{\text{atm}} = 0.49 - 2.2 .$$  \hspace{1cm} (4.26)

Hereafter, for simplicity, we take $\tan^2 \theta_{\text{atm}} \simeq 1$. Thus, by combining the present model with the mixing angle $\theta_{\text{solar}}$, we have

$$\frac{m_1}{m_2} \simeq \tan^2 \theta_{\text{solar}} = 0.29 - 0.64.$$  \hspace{1cm} (4.27)

Therefore we predict the neutrino masses as follows.

$$m^2_1 = (0.48 - 6.8) \times 10^{-5} \text{ eV}^2 ,$$

$$m^2_2 = (5.7 - 16.6) \times 10^{-5} \text{ eV}^2 ,$$  \hspace{1cm} (4.28)

$$m^2_3 = (1.4 - 3.4) \times 10^{-3} \text{ eV}^2 .$$

Let us mention other predictions in our model. Our model imposes a restriction on $|U_{13}|$ as

$$|U_{13}|^2 \simeq \frac{m_e}{2m_\mu} = 2.4 \times 10^{-3} .$$  \hspace{1cm} (4.29)

Here we have used the running charged lepton masses at the unification scale $\mu = \Lambda_X$ [36]: $m_e(\Lambda_X) = 0.325 \text{ MeV}$, $m_\mu(\Lambda_X) = 68.6 \text{ MeV}$, and $m_\tau(\Lambda_X) = 1171.4 \pm 0.2 \text{ MeV}$. The value in Eq. (4.29) is consistent with the present experimental constraints Eq. (4.22).
Next let us discuss the CP-violation phases in the lepton mixing matrix. The Majorana neutrino fields do not have the freedom of rephasing invariance, so that we can use only the rephasing freedom of $M_e$ to transform Eq. (4.16) to the standard form

$$U_{\text{std}} = \text{diag}(e^{i\alpha_1^e}, e^{i\alpha_2^e}, e^{i\alpha_3^e}) \ U$$

$$= \begin{pmatrix}
    c_{\nu 13} c_{\nu 12} & c_{\nu 13} s_{\nu 12} & s_{\nu 13} e^{i(\gamma - \delta)} \\
    (-c_{\nu 23} s_{\nu 12} - s_{\nu 23} c_{\nu 23} s_{\nu 13} e^{i\delta}) e^{-i\gamma} & c_{\nu 23} c_{\nu 12} - s_{\nu 23} s_{\nu 12} s_{\nu 13} e^{i\delta} & s_{\nu 23} s_{\nu 13} e^{i(\gamma - \delta)} \\
    (s_{\nu 23} s_{\nu 12} - c_{\nu 23} c_{\nu 12} s_{\nu 13} e^{i\delta}) e^{-i\gamma} & (-s_{\nu 23} c_{\nu 12} - c_{\nu 23} s_{\nu 12} s_{\nu 13} e^{i\delta}) e^{-i(\gamma - \beta)} & c_{\nu 23} c_{\nu 13}
\end{pmatrix}.$$  

(4.30)

Here, $\alpha_i^e$ comes from the rephasing in the charged lepton fields to make the choice of phase convention. The CP-violating phase $\delta$, the additional Majorana phase $\beta$ and $\gamma[37, 38]$ in the representation Eq. (4.30) are calculable and obtained as

$$\delta = \arg \left[ \frac{U_{12} U_{22}^*}{U_{13} U_{23}^*} + \frac{|U_{12}|^2}{1 - |U_{13}|^2} \right] \simeq \pi,$$

$$\beta = \arg \left( \frac{U_{12}}{U_{11}} \right) \simeq -\pi/2,$$

$$\gamma = \arg \left( \frac{U_{13} e^{i\delta}}{U_{11}} \right) \simeq \pi/2,$$

(4.31)

by using the relation $m_e \ll m_\mu \ll m_\tau$.

We also predict the averaged neutrino mass $\langle m_\nu \rangle$ which appears in the neutrinoless double beta decay $[38]$ as follows:

$$\langle m_\nu \rangle \equiv |m_1 U_{11}^2 + m_2 U_{12}^2 + m_3 U_{13}^2| = \frac{m_e m_3}{2 m_\mu}$$

$$= (0.89 - 1.4) \times 10^{-4} \text{ eV}.$$  

(4.32)

This value of $\langle m_\nu \rangle$ is too small to be observed in near future experiments $[39]$.

V. SUMMARY

We have investigated a Hermite mass matrix model given in Eqs. (2.1)-(2.6), in which the mass matrices for quarks and charged leptons are assumed to have a term in which the $2 \leftrightarrow 3$ symmetry is maximally broken. The mass matrices for up quarks, down quarks, charged leptons, and neutrinos have a common structure as shown by $\widehat{M}_f$ in Eq. (2.7) when it is expressed after rebasing of the quark and lepton fields. The large lepton mixing angle
between the second and third generation is realized with no fine tuning in our model. The model is almost consistent with the present data in the quark as well as lepton sectors. The model also predicts $|U_{13}|^2 \approx \frac{m_e}{2m_\mu} = 2.4 \times 10^{-3}$ for the lepton mixing matrix element $U_{13}$, and neutrino masses shown in Eq. (4.26) are obtained from the neutrino oscillation data for $\theta_{\text{sol}}$, $\Delta m^2_{23}$, and $\Delta m^2_{12}$. We also predict $\langle m_\nu \rangle = (0.89 - 1.4) \times 10^{-4}$ eV for the averaged neutrino mass which appears in the neutrinoless double beta decay.

Acknowledgments

This work of K.M. was supported by the JSPS, No. 15-3700.

APPENDIX A: DIAGONALIZATION OF MASS MATRIX $\hat{M}_f$

For the purpose of making this paper self-contained, here we summarize the diagonalization of mass matrix $\hat{M}_f$ (f=u,d,e and $\nu$) defined by

$$\hat{M}_f = \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix},$$

for up quarks, down quarks, charged leptons, and neutrinos.

1. mass matrix $\hat{M}_f$ for quarks and charged leptons

For quarks and charged leptons (f=u, d, and e), let us take a following choice for $\hat{M}_f$:

$$\hat{M}_f = \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix} = \begin{pmatrix} 0 & \sqrt{\frac{m_1^f m_2^f m_3^f}{m_3^f - m_1^f}} & 0 \\ \sqrt{\frac{m_1^f m_2^f m_3^f}{m_3^f - m_1^f}} & \sqrt{\frac{m_1^f m_3^f (m_3^f - m_2^f)}{m_3^f - m_1^f}} & m_2^f \\ 0 & \sqrt{\frac{m_1^f m_3^f (m_3^f - m_2^f)}{m_3^f - m_1^f}} & m_3^f \end{pmatrix}.$$ 

(A2)

This is diagonalized by an orthogonal matrix $O_f$ as (see Ref[23,24])

$$O_f^T \begin{pmatrix} 0 & A_f & 0 \\ A_f & B_f & C_f \\ 0 & C_f & D_f \end{pmatrix} O_f = \begin{pmatrix} -m_1^f & 0 & 0 \\ 0 & m_2^f & 0 \\ 0 & 0 & m_3^f \end{pmatrix}.$$ 

(A3)
Here $m_{i\nu}(i = 1, 2, 3)$ are eigenmasses and $O_f$ is given by

$$
O_f = \begin{pmatrix}
\frac{m_{2f}m_{2f}}{(m_{2f}+m_{1f})(m_{2f}^2-m_{1f}^2)} & \frac{m_{1f}m_{3f}(m_{3f}-m_{2f}-m_{1f})}{(m_{2f}+m_{1f})(m_{3f}^2-m_{1f}^2)} & \frac{m_{1f}m_{2f}}{(m_{3f}^2-m_{1f}^2)(m_{3f}^2-m_{2f}^2)} \\
\frac{m_{1f}m_{3f}}{(m_{2f}+m_{1f})(m_{3f}^2-m_{1f}^2)} & \frac{m_{1f}m_{2f}}{(m_{3f}^2-m_{1f}^2)(m_{3f}^2-m_{2f}^2)} & \frac{m_{1f}m_{3f}}{(m_{3f}^2-m_{1f}^2)(m_{3f}^2-m_{2f}^2)} \\
\frac{m_{1f}^2(m_{3f}-m_{2f}-m_{1f})}{(m_{2f}+m_{1f})(m_{3f}^2-m_{1f}^2)} & \frac{m_{1f}m_{2f}}{(m_{3f}^2-m_{1f}^2)(m_{3f}^2-m_{2f}^2)} & \frac{m_{1f}m_{3f}}{(m_{3f}^2-m_{1f}^2)(m_{3f}^2-m_{2f}^2)} \\
\end{pmatrix}
$$

Here $m_{iu}$, $m_{id}$, and $m_{ie}$ $(i = 1, 2, 3)$ are, respectively, masses of up quarks, down quarks, charged leptons, and neutrinos, which we shall denoted as $(m_u, m_c, m_t)$, $(m_d, m_s, m_b)$, and $(m_e, m_\mu, m_\tau)$.

2. mass matrix $\widehat{M}_\nu$ for neutrinos

For neutrinos ($f=\nu$) we choose :

$$
\widehat{M}_\nu = \begin{pmatrix}
0 & A_\nu & 0 \\
A_\nu & B_\nu & 0 \\
0 & 0 & D_\nu
\end{pmatrix} = \begin{pmatrix}
0 & \sqrt{m_1m_2} & 0 \\
\sqrt{m_1m_2} & m_2 - m_1 & 0 \\
0 & 0 & m_3
\end{pmatrix}.
$$

(A5)

Note we take $C_\nu = 0$. This $\widehat{M}_\nu$ is diagonalized as

$$
O_\nu^T \begin{pmatrix}
0 & A_\nu & 0 \\
A_\nu & B_\nu & 0 \\
0 & 0 & D_\nu
\end{pmatrix} \begin{pmatrix}
-m_1 \\
m_2 \\
m_3
\end{pmatrix} = \begin{pmatrix}
0 & \sqrt{m_2m_3} & 0 \\
-\sqrt{m_1m_2} & m_2 & 0 \\
0 & 0 & 1
\end{pmatrix}.
$$

(A6)

where $m_i(i = 1, 2, 3)$ are neutrino masses and the orthogonal matrix $O_\nu$ is given by

$$
O_\nu = \begin{pmatrix}
\sqrt{m_2m_3} & \sqrt{m_1m_2} & 0 \\
-\sqrt{m_1m_2} & m_2 & 0 \\
0 & 0 & 1
\end{pmatrix}.
$$

(A7)
APPENDIX B: EVOLUTION EFFECT

We have estimated the evolution effects for the CKM matrix elements from the electroweak scale $\mu = m_Z$ to the unification scale $\mu = M_X$ by using the two-loop renormalization group equation (RGE) [minimal supersymmetric standard model with $\tan \beta = 10$ case] for the Yukawa coupling constants. In the numerical calculations, we have used the following running quark masses at $\mu = m_Z$ and at $\mu = M_X$ [36]:

\[
\begin{align*}
  m_u(m_Z) &= 2.33^{+0.42}_{-0.45}\text{MeV}, \quad m_d(m_Z) = 4.69^{+0.60}_{-0.66}\text{MeV}, \\
  m_c(m_Z) &= 677^{+56}_{-61}\text{MeV}, \quad m_s(m_Z) = 93.4^{+11.8}_{-13.0}\text{MeV}, \\
  m_t(m_Z) &= 181 \pm 13\text{GeV}, \quad m_b(m_Z) = 3.00 \pm 0.11\text{GeV}.
\end{align*}
\]

\[
\begin{align*}
  m_u(M_X) &= 1.04^{+0.19}_{-0.20}\text{MeV}, \quad m_d(M_X) = 1.33^{+0.17}_{-0.19}\text{MeV}, \\
  m_c(M_X) &= 302^{+25}_{-27}\text{MeV}, \quad m_s(M_X) = 26.5^{+3.3}_{-3.7}\text{MeV}, \\
  m_t(M_X) &= 129^{+196}_{-40}\text{GeV}, \quad m_b(M_X) = 1.00 \pm 0.04\text{GeV}.
\end{align*}
\]

We have calculated numerical values of the CKM mixing matrix elements at $\mu = M_X$ from their observed values at $\mu = m_Z$. Namely using as inputs the observed quark mixing angles and the $C\!P$ violating phase at $\mu = m_Z$ given by

\[
\begin{align*}
  \sin \theta_{12}(m_Z) &= 0.2243 \pm 0.0016, \quad \sin \theta_{23}(m_Z) = 0.0413 \pm 0.0015, \\
  \sin \theta_{13}(m_Z) &= 0.0037 \pm 0.0005, \quad \delta(m_Z) = 60^\circ \pm 14^\circ,
\end{align*}
\]

we obtain the following numerical values for the mixing angles and the magnitude of the mixing matrix elements at $\mu = M_X$ [28]:

\[
\begin{align*}
  \sin \theta_{12}^0 &= 0.2226 - 0.2259, \quad \sin \theta_{23}^0 = 0.0295 - 0.0383, \\
  \sin \theta_{13}^0 &= 0.0024 - 0.0038, \quad \delta^0 = 46^\circ - 74^\circ, \\
  |V^0| &= \begin{pmatrix}
    0.9741 - 0.9749 & 0.2226 - 0.2259 & 0.0024 - 0.0038 \\
    0.2225 - 0.2259 & 0.9734 - 0.9745 & 0.0295 - 0.0387 \\
    0.0048 - 0.0084 & 0.0289 - 0.0379 & 0.9993 - 0.9996
  \end{pmatrix}.
\end{align*}
\]

[1] Super-Kamiokande Collaboration, Y. Fukuda et al., Phys. Rev. Lett. \textbf{81}, 1562(1998); Phys. Rev. Lett. \textbf{86}, 5651(2001); SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. \textbf{87},
071301(2001); Phys. Rev. Lett. 89, 011301(2002); KamLAND Collaboration, K. Eguchi et al., Phys. Rev. Lett. 90, 021802(2003).

[2] H. Fritzsch, Phys. Lett. B73, 317 (1978); B85, 81 (1979); Nucl. Phys. B155, 189 (1979); L.F. Li, Phys. Lett. B84, 461 (1979); H. Georgi and C.V. Nanopoulos, Nucl. Phys. B155, 52 (1979); A.C. Rothman and K. Kang, Phys. Rev. Lett. 43, 1548 (1979); A. Davidson, V.P. Nair, and K.C. Wali, Phys. Rev. D29, 1513 (1984); M. Shin, Phys. Lett. B145, 285 (1984); H. Georgi, A. Nelson, and M. Shin, Phys. Lett. B150, 306 (1985); T.P. Cheng and L.F. Li, Phys. Rev. Lett. 55, 2249 (1985).

[3] B. Stech, Phys. Lett. B130, 189 (1983); G. Ecker, Z. Phys. C24, 353 (1984).

[4] M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. Lett. 54, 2176(1985); Phys. Rev. D33, 2641 (1986).

[5] L. Wolfenstein, Phys. Rev. Lett. 51, 1945(1983).

[6] K. Kang and M. Shin, Phys. Lett. B165, 383 (1985); B185, 163 (1987).

[7] A. Bottino, C.W. Kim, H. Nishiura, and W.K. Sze, Phys. Rev. D34, 862 (1986).

[8] R. Johnson, S. Ranfone, and J. Schechter, Phys. Lett. B179, 355 (1986); Phys. Rev. D35, 282 (1987).

[9] G.C. Branco, L. Lavoura, and F. Mota, Phys. Rev. D 39, 3443 (1989).

[10] P. Ramond, R.G. Roberts, and G.G. Ross, Nucl. Phys. B406, 19(1993).

[11] D. Du and Z.Z. Xing, Phys. Rev. D48, 2349 (1993); H. Fritzsch and Z.Z. Xing, Phys. Lett. B353, 114 (1995).

[12] K. Kang and S.K. Kang, Phys. Rev. D56, 1511 (1997); H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D60, 013006 (1999); K. Matsuda, T. Fukuyama, and H. Nishiura, Phys. Rev. D61, 053001 (2000); K. Kang, S.K. Kang, C.S. Kim, and S.M. Kim, Mod. Phys. Lett. A16, 2169 (2001); H. Fritzsch and Z.Z. Xing, Phys. Rev. D61, 073016 (2000); C. Giunti and M. Tanimoto, Phys. Rev. D66, 113006 (2002); M. Frigerio and A.Yu. Smirnov, Nucl. Phys. B640, 233 (2002); Phys. Rev. D67, 013007 (2003); P.F. Harrison and W.G. Scott, Phys. Lett. B547, 219 (2002); Phys. Lett. B594, 324 (2004); E. Ma, Mod. Phys. Lett. A17, 2361 (2002); Phys. Rev. D66, 117301 (2002); K.S. Babu, E. Ma, and J.W.F. Valle, Phys. Lett. B552, 207 (2003); Z.Z. Xing, Int. J. Mod. Phys. A19, 1 (2004); M. Bando, S. Kaneko, M. Obara, and M. Tanimoto, Phys. Lett. B580, 229 (2004); O.L.G. Peres and A.Yu. Smirnov, Nucl. Phys. B680, 479 (2004); C.H. Albright, Phys. Lett. B599, 285 (2004); J. Ferrandis and
S. Pakvasa, Phys. Lett. B603, 184 (2004); S.T. Petcov and W. Rodejohann, Phys. Rev. D71, 073002 (2005); S.S. Masood, S. Nasri, and J. Schechter, Phys. Rev. D71, 093005 (2005); R. Dermišeka and S. Raby, Phys. Lett. B622, 327 (2005); F. Plentinger and W. Rodejohann, Phys. Lett. B625, 264 (2005).

[13] T. Fukuyama and H. Nishiura, [http://arxiv.org/abs/hep-ph/9702253](http://arxiv.org/abs/hep-ph/9702253) in Proceedings of the International Workshop on Masses and Mixings of Quarks and Leptons, Shizuoka, Japan, 1997, edited by Y. Koide (World Scientific, Singapore, 1998), p. 252.

[14] R.N. Mohapatra and S. Nussinov, Phys. Rev. D60, 013002 (1999).

[15] E. Ma and M. Raidal, Phys. Rev. Lett. 87, 011802 (2001).

[16] C.S. Lam, Phys. Lett. B507, 214 (2001); Phys. Rev. D71, 093001 (2005).

[17] K.R.S. Balaji, W. Grimus and T. Schwetz, Phys. Lett. B508, 301 (2001).

[18] W. Grimus and L. Lavoura, Acta Phys. Pol. B32, 3719 (2001); JHEP, 0107, 045 (2001); Euro. Phys. J. C28, 123 (2003); Phys. Lett. B572, 189 (2003); J. Phys. G30, 1073 (2004); Phys. Lett. B579, 113 (2004); JHEP, 0508, 013 (2005); W. Grimus A.S. Joshipura, S. Kaneko, L. Lavoura, and M. Tanimoto, JHEP, 0407, 078 (2004); W. Grimus A.S. Joshipura, S. Kaneko, L. Lavoura, H. Sawanaka, and M. Tanimoto, Nucl. Phys. B713, 151 (2005); W. Grimus, S. Kaneko, L. Lavoura, H. Sawanaka, M. Tanimoto, hep-ph/0510326.

[19] T. Kitabayashi and M. Yasue, Phys. Lett. B524, 308 (2002); Int. J. Mod. Phys. A17, 2519 (2002); Phys. Rev. D67, 015006 (2003); I. Aizawa, M. Ishiguro, T. Kitabayashi, and M. Yasue, Phys. Rev. D70, 015011 (2004); I. Aizawa, T. Kitabayashi, and M. Yasue, Phys. Rev. D71, 075011 (2005); Phys. Rev. D72, 055014 (2005); [http://arxiv.org/abs/hep-ph/0507332](http://arxiv.org/abs/hep-ph/0507332); T. Kitabayashi and M. Yasue, Phys. Lett. B607, 267 (2005); Phys. Lett. B621, 133 (2005); I. Aizawa and M. Yasue, [http://arxiv.org/abs/hep-ph/0510132](http://arxiv.org/abs/hep-ph/0510132).

[20] S. Kaneko, H. Sawanaka, M. Tanimoto, [http://arxiv.org/abs/hep-ph/0504074](http://arxiv.org/abs/hep-ph/0504074).

[21] R.N. Mohapatra, JHEP, 0410, 027 (2004); R.N. Mohapatra and S. Nasri, Phys. Rev. D71, 033001 (2005); R.N. Mohapatra, S. Nasri, and H. Yu, Phys. Lett. B617, 231 (2005); R.N. Mohapatra and W. Rodejohann, Phys. Rev. D72, 053001 (2005).

[22] H. Nishiura, K. Matsuda, T. Kikuchi, and T. Fukuyama, Phys. Rev. D65, 097301 (2002).

[23] Y. Koide, H. Nishiura, K. Matsuda, T. Kikuchi, and T. Fukuyama, Phys. Rev. D 66, 093006(2002).

[24] K. Matsuda and H. Nishiura, Phys. Rev. D 69, 053005 (2004).
[25] K. Matsuda and H. Nishiura, Phys. Rev. D 69, 117302 (2004).
[26] A. Ghosal, Mod. Phys. Lett. A19, 2579 (2004).
[27] Y. Koide, Phys. Rev. D 69, 093001 (2004).
[28] K. Matsuda and H. Nishiura, Phys. Rev. D 71, 073001 (2005); Phys. Rev. D 72, 033011 (2005).
[29] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).
[30] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962); B. Pontecorv, Zh. Éksp. Teor. Fiz. 33, 549 (1957)[Sov. Phys. JETP 26, 984 (1968)].
[31] P. Minkowski, Phys. Lett. B67, 421 (1977); T. Yanagida, in Proceedings of the Workshop on the unified theory and baryon number in the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979) p. 315; R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); D 25, 774 (1982).
[32] H. Nishiura, K. Matsuda, and T. Fukuyama, Phys. Rev. D 60, 013006 (1999).
[33] See, for a recent review, M. C. Gonzalez-Garcia, talk at NOON2004 [http://www-sk.icrr.u-tokyo.ac.jp/noon2004/].
[34] M. Apollonio et al., Phys. Lett. B466, 415 (1999).
[35] Q. R. Ahmad et al., Phys. Rev. Lett. 89, 011301 and 011302 (2002).
[36] H. Fusaoka and Y. Koide, Phys. Rev. D 57, 3986 (1998).
[37] S. M. Bilenky, J. Hosek, and S. T. Petcov, Phys. Lett. 94B, 495 (1980); J. Schechter and J. W. F. Valle, Phys. Rev. D 22, 2227 (1980); A. Barroso and J. Maalampi, Phys. Lett. 132B, 355 (1983).
[38] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett. 102B, 323 (1981); M. Doi, T. Kotani, H. Nishiura, and E. Takasugi, Prog. Theor. Phys. 69, 602 (1983).
[39] See, for a recent review, H. Ejiri, J. Phys. Soc. Jpn. 74, 2101 (2005).