Assisted inflation in Bianchi VI\textsubscript{0} cosmologies

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Abstract

Exact models for Bianchi VI\textsubscript{0} spacetimes with multiple scalar fields with exponential potentials have been derived and analysed. It has been shown that these solutions, when they exist, attract neighbouring solutions in the two cases corresponding to interacting and non-interacting fields. Unlike the results obtained in a previous work dealing with the late-time inflationary behaviour of Bianchi VI\textsubscript{0} cosmologies, the knowledge of exact solutions has made possible to study in detail the occurrence of inflation before the asymptotic regime. As happened in preceding works, here as well inflation is more likely to happen with a higher number of non-interacting fields or a lower number of interacting scalar fields.

1 Introduction

The cosmological inflationary scenario, as pioneerely proposed by Guth [1], has been much favoured due to its troubleshooting capabilities [2]. Furthermore, it has recently gained the support of observational evidence [3, 4]. It is usually assumed that an epoch of accelerated expansion in the Universe was driven by one scalar field slowly rolling down its potential. Even though several scalar fields should enter the general picture, it is commonplace to make the additional assumption that only one of them played a significant dynamical role.

Recently, Liddle, Mazumdar and Schunck [5] put forward an alternative model representing a slight departure from those assumptions. By means of exact FRW examples, they showed that multiple scalar fields with exponential potentials can assist each other in the realisation of inflation, even if the individual fields are not flat enough to produce inflation on their own. This is the reason why they called this behaviour \textit{assisted inflation}. 
The work by Liddle and collaborators was generalised by Copeland, Mazumdar and Nunes [6] along two different lines. On the one hand, they introduced cross-coupling terms in the potential and showed that this seems to hinder inflation rather than assist it. On the other hand, they analysed cases with multiple exponential potentials and showed that, by choosing the slopes adequately, the expansion rate can be augmented. Motivation for the use of exponential potentials is can be found in dimensionally reduced supergravity theories (see for instance [6], [7], and the references therein).

Assisted inflation presents remarkable novel features from the dynamical point of view. One would normally expect that a single scalar field should eventually dominate the dynamics, but peculiarly inflationary solutions turn out to be late-time attractors. This fact has been reflected in several studies concerned with the dynamics of assisted inflation [8, 9, 10].

Kanti and Olive [11, 12] explored another interesting aspect of the problem using a realisation of the assisted inflation proposal based on the compactification of a fifth-dimensional Kaluza-Klein theory. In particular, they showed that assisted inflation can be a remedy to the initial conditions problem in the chaotic inflation scenario [13]. Along this line, Liddle and Kaloper [14] have found that in this kind of inflation the spacetime does retain some memory of the conditions that gave rise to it, even though a homogeneous and spatially flat universe is generated.

Although most of the references in the literature are concerned with FRW models, there is no reason to assume that at the onset of inflation the universe was as isotropic as it seems to be today. In fact, it is commonly believed that it was precisely an epoch of accelerated expansion that isotropised the universe. This is the reason why it is of much relevance to extend the studies on assisted inflation to Bianchi spacetimes.

Our starting point is a recent collaboration of one of us [15], which focused on the study of assisted inflation at late times in FRW and Bianchi I cosmologies by using exact solutions and in Bianchi VI\textsubscript{0} spacetimes by means of asymptotic approximate solutions. We construct here a family of exact Bianchi VI\textsubscript{0} spacetimes with exponential potentials which include the exact counterparts of the mentioned approximate solutions. By resorting to these exact, yet particular, solutions we have been able to perform a detailed analysis of the occurrence of inflation in this framework.

Let us recall that if the models behave as desirable they will undergo accelerated expansion in their early epochs, preferably for a limited period of time. Clearly, asymptotic studies, as those usually done up to the date, have the drawback of being unable to determine whether that occurs.

As will be shown in the following sections, we have found the necessary and sufficient conditions to be fulfilled by the slopes of the individual potentials for the expansion to become accelerated. In the light of the results, we have been able to illustrate the cumulative effect of non-interacting fields in the occurrence of inflation to which assisted inflation owes its name. What is more, our results exemplify as well the hindering effect of direct interactions between the fields. For the sake of completeness we have also included an analysis showing the
asymptotic stability of the solutions as well as an study of the isotropization of our models. Summarising, our results confirms previous studies and give additional support to the assisted inflation proposal, providing particular exact solutions that attract neighbouring solutions and show inflation at the start of their life span.

2 Exact solutions to the multiple scalar field problem in a Bianchi VI₀ spacetime

In what follows we are going to consider the problem of n scalar fields minimally coupled to gravity\(^1\) and driven by exponential potentials in a Bianchi VI₀ spacetime. First, it will be assumed that the scalar fields interact through a product of exponential potentials; then, we will turn to consider the alternative case of uncoupled fields evolving via individual exponential potentials. Nevertheless, it should be pointed out that, although we will be loosely speaking of interacting and non interacting models, the geometry will be responsible for some interaction among the fields even in the examples devoid of direct couplings in the potential.

Before we enter the details of the two different cases to be studied, let us recall the form of the line element of a Bianchi VI₀ spacetime:

\[
ds^2 = -e^{f(t)}(dt^2 - dz^2) + G(t)(e^z dx^2 + e^{-z}dy^2).
\]

For convenience we define two vectors belonging to an n-dimensional Euclidean space as follows:

\[
\vec{\phi} = (\phi_1, \phi_2, \ldots, \phi_n),
\]

\[
\vec{k} = (k_1, k_2, \ldots, k_n).
\]

Moreover, we also demand \(\vec{k}\) be constant with respect to an orthonormal basis of the space.

2.1 Non-interacting fields

In these models we will take the potential responsible for the interaction among the fields to be of the form

\[
V(\vec{\phi}) = \sum_{i=1}^{n} V_i e^{-k_i \phi_i}.
\]

Motivated by the fact that all scalar fields in these kind of configurations seem to tend to a common limit\(^3\), we will make the simplifying assumption of having

\(^1\)Even though most of the papers on the subject deal with models in which the fields are minimally coupled to gravity, inflation can also be assisted in the case of non-minimally coupled multiple scalar fields with exponential potentials\(^4\).

\(^3\)
\( \phi_1 = \phi_2 = \ldots = \phi_n \equiv \phi, \ k_1 = k_2 = \ldots = k_n \equiv k \) and \( V_{o1} = V_{o2} = \ldots = V_{on} \equiv V_o \). Additional motivation for this assumption is that it simplifies considerably the task of finding exact solutions.

It will be possible to determine the model’s features upon the resolution of the system formed by the Einstein equations

\[
e^f = \frac{\ddot{G}}{2VG},
\]

\[
\ddot{G} - \frac{1}{2} \left( \frac{\dot{G}}{G} \right)^2 - \dot{\phi} \dot{f} + \frac{k}{2} = -n\phi^2
\]

and the Klein-Gordon equation

\[
\ddot{\phi} + \frac{\dot{G}}{G} \dot{\phi} - \frac{k}{n} e^f V = 0.
\]

A first integral to (3) is given by

\[
\dot{\phi} = \frac{k\dot{G}}{2nG} + \frac{m}{nG},
\]

where \( m \) is an integration constant. It is straightforward to realise that by means of this first integral and eqs. (3) and (4) the problem reduces to finding the solution to a single ODE, namely

\[
G\ddot{\phi}^2 - \ddot{G}\ddot{\phi} + \left( \frac{1}{2} - \frac{k^2}{4n} \right) \ddot{G}^2 + \frac{1}{2} \frac{m^2}{n} \ddot{G} = 0.
\]

Only a particular solution to the case \( m = 0 \) of eq. (3) has been known to date [17]. Remarkably, it also admits a rather simple family of solutions that has not been noticed before. In order to simplify the expression of these solutions we introduce a new constant defined as

\[
p = m \frac{\sqrt{k^2 - 2n}}{\sqrt{k^2 + 2n}}.
\]

Up to time-shifts and rescalings of the spatial coordinates \( x \) and \( y \), the mentioned solution to eq. (3) is given by

\[
G(t) = \left( \frac{1}{2} + p^2 \right) \sinh \frac{\sqrt{2n}(t + t_0)}{\sqrt{k^2 - 2n}} + \left( \frac{1}{2} - p^2 \right) \cosh \frac{\sqrt{2n}(t + t_0)}{\sqrt{k^2 - 2n}},
\]

and it exists provided that \( k^2 > 2n \). Note that the solutions with \( p \neq 0 \) have as their late time limit the \( p = 0 \) case, the already known solution.

At this point, the complete determination of the remaining metric function \( f \) and the scalar field \( \phi \) is a straightforward task. Nevertheless, we will omit the corresponding explicit expressions because they are rather lengthy and of little use for the forthcoming discussion.
The freedom to choose the value \( t_0 \) will allows us to avoid unwanted features in the solutions, in particular we will choose it carefully so that we prevent signature changes in the \( p = 0 \) cases. To this end we set:

\[
t_0 = \frac{\sqrt{k^2 - 2n}}{\sqrt{2n}} \arctanh \left( \frac{2p^2 - 1}{2p^2 + 1} \right),
\]

(12)

and thus guarantee \( G \geq 0 \) and \( \dot{G} \geq 0 \) in the whole range we are going to consider from now on: \( t \geq 0 \). Note that no signature changes arise in the \( p = 0 \) case, so there is in principle no restrictions upon the choice of time origin and we will set \( t_0 = 0 \) just out of utter convenience. A consequence of these choices is that unless \( p \neq 0 \) the curvature scalars will blow up at the beginning of times, i.e. there will be a spacelike singularity of big-bang type.

### 2.2 Interacting fields

In these other models the potential responsible for the interaction among the fields will be of the form

\[
V(\vec{\phi}) = \prod_{i=1}^{n} V_{oi} e^{-k_i \phi_i}.
\]

(13)

The system will then be completely determined upon the resolution of the set formed by the Einstein equations

\[
e^f = \frac{\ddot{G}}{2V G},
\]

(14)

\[
\frac{\ddot{G}}{G} - \frac{1}{2} \left( \frac{\dot{G}}{G} \right)^2 - \frac{\dot{G}^2}{G} + \frac{1}{2} = -\vec{\phi} \cdot \vec{\phi}
\]

(15)

and the Klein-Gordon equation

\[
\ddot{\phi} + \frac{\dot{G}}{G} \dot{\phi} - e^f V \vec{k} = 0.
\]

(16)

One can easily check that the vector

\[
\vec{\phi} = \frac{\dot{G}}{2G} \vec{k} + \vec{m}
\]

(17)

is a first integral to (16) and \( \vec{m} \) another constant vector belonging to the \( n \)-dimensional Euclidean space, that is, a set of integration constants. Again, by using this first integral and eqs. (14) and (15) the problem reduces to finding the solution to a single ODE, which does not differ much from the one for the previous case:

\[
GG\ddot{G}^2 - \dot{G} \dddot{G} G + \left( \frac{1}{2} - \frac{k^2}{4} \right) \dddot{GG}^2 + \frac{1}{2} \dddot{GG}^2 + m^2 \dddot{G} = 0.
\]

(18)
It follows that the solutions to eq. (18) can be obtained from those to eq. (9) by remembering that now $k^2 = |\vec{k}|^2$ and $m^2 = |\vec{m}|^2$ and performing the replacement $n \to 1$. Correspondingly, we introduce a new constant vector defined as

$$\vec{p} = \vec{m} \frac{\sqrt{k^2 - 2}}{\sqrt{2 + k^2}}.$$ (19)

We have then

$$G(t) = \left(\frac{1}{2} + p^2\right) \sinh \frac{\sqrt{2} (t + t_0)}{\sqrt{k^2 - 2}} + \left(\frac{1}{2} - p^2\right) \cosh \frac{\sqrt{2} (t + t_0)}{\sqrt{k^2 - 2}},$$ (20)

where we have used the shorthand $p^2 = |\vec{p}|^2$. Clearly, such solutions exist provided that $k^2 > 2$.

The arguments concerning possible signature changes given for the models in the previous section apply to these other ones too, so we set

$$t_0 = \frac{\sqrt{k^2 - 2}}{\sqrt{2}} \text{arctanh} \frac{2p^2 - 1}{2p^2 + 1}$$ (21)

if $\vec{p} \neq 0$, and $t_0 = 0$ otherwise.

Summarising, the two families of spacetimes we have constructed are quite analogous in their expressions. It remains to see, however, if these similarities can be extended to their behaviour.

### 3 Asymptotic behaviour and stability

The asymptotic behaviour of the kind of ODE leading to our two families of Bianchi VI$_0$ geometries is worth studying, as this analysis will enable us to discuss their stability. In terms of the variable $\Omega$ defined by

$$\Omega = \frac{\dot{h}}{h^2},$$ (22)

$$h = \frac{\dot{G}}{G},$$ (23)

eqs. (13) and (18) become

$$\dot{\Omega} + \left[\Omega + K - \frac{1}{2}h^2 - \frac{M^2}{G^2}\right](\Omega + 1)h = 0.$$ (24)

Here, we have introduced a couple of new parameters defined as

$$K = \frac{k^2}{4n} - \frac{1}{2}$$ (25)

and

$$M^2 = \frac{m^2}{n}$$ (26)
in the non-interacting case. The corresponding expressions for the interacting case are obtained by the replacement rules given above.

Equation (24) has the fixed point solution \( \Omega_1 = -1 \). In addition, if \( \dot{G} \to \infty \) asymptotically, then \( \Omega_2 = 0 \) is also a fixed point solution. They correspond to \( G \propto t \) and \( G \propto e^{t/\sqrt{2K}} \) respectively. Now, in order to determine whether these two solutions are stable on the \( G \to \infty \) regime we expand each of these solutions about fixed points by making either \( \Omega = \Omega_1 + \epsilon \) or \( \Omega = \Omega_2 + \epsilon \) with \( |\epsilon| \ll 1 \). For \( \Omega_1 \) we get

\[
\dot{\epsilon} = \frac{1}{2h} \epsilon, \quad (27)
\]

and it indicates that \( \Omega_1 \) is unstable. In fact, the corresponding solution \( G \propto t \) is spurious because it does not satisfy the Einstein equations (cf. eqs. (14) and (5)). The case \( \Omega_2 \) is more interesting because it corresponds precisely to the late-time approximation of the exact solutions given above, in this case we get

\[
\dot{\epsilon} = -\frac{1}{\sqrt{2K}} \epsilon, \quad (28)
\]

and this proves the asymptotic stability of our solutions for \( K > 0 \). Therefore, for a given \( m \) the solution \( \Omega_2 \) is the attractor for all the solutions that are close to it.

4 Inflation and other kinematic aspects

It is well known that those spacetimes for which the gradient of the scalar field is timelike can be reinterpreted as perfect fluid induced geometries. By taking advantage of this customary reinterpretation we will be able to study the kinematic features of our spacetimes. For our purposes we just need to know the expression of the four velocity of the fluid in terms of the scalar field’s gradient, namely

\[
u_\alpha = \frac{\Phi_\alpha}{\sqrt{-\Phi_\beta \Phi^\beta}} \quad \alpha, \beta = 0, \ldots, 3. \quad (29)\]

In order to find out whether the model inflates or not it is necessary to look at the sign of the deceleration parameter \( q = -\theta^{-2} \left(3 \theta_{,\alpha} u^\alpha + \theta^2\right) \) where the scalar \( \theta = u^\alpha \theta_\alpha \) is the expansion of the fluid (13). In the case of interacting fields, and after some algebra, we have arrived at the following expression, valid for any spacetime obtained from (20):

\[
q = 2 \frac{c_1 + c_2 \dot{G} + c_3 \dot{G}^2}{\left(c_4 + c_5 \dot{G}\right)^2}, \quad (30)
\]

where

\[
c_1 = 4p^2 \left[k^2 \left(k^2 + 2n\right) + 6n^2 \left(k^2 + 4n\right)\right], \quad (31)
\]
The equivalent expression for the interacting case could be obtained by making the replacements \( |p| \rightarrow p = |\vec{p}|, \ |k| \rightarrow k = |\vec{k}|, \ n \rightarrow 1 \) and \( \text{sign}(p k) \rightarrow \cos \lambda \), where

\[
\cos \lambda = \frac{\vec{k} \cdot \vec{n}}{k m} = \frac{\vec{k} \cdot \vec{p}}{k p},
\]  

and using

\[
c_1 = 4 p^2 \left[ k^2 (k^2 + 2) \cos^2 \lambda + 6 (k^2 + 4) \right]
\]  

instead of (31). Safely, the denominator of \( q \) never vanishes for \( t \geq 0 \), as can be easily tested.

Our first step in this analysis is to investigate the late time behaviour of \( q \). We get

\[
\lim_{t \rightarrow \infty} q = \frac{2 k^2 - 4 n}{k^2 + 4 n},
\]  

which is positive in the range of validity of our solutions, thus indicating that our models do not inflate at late times. This is a very useful result indeed and will be used later.

Let us try to find out now whether there is a period of time for which \( q \) becomes negative, recalling that this is the condition for the existence of a period of accelerated inflation. A quick look at the expression for \( q \) shows that \( \text{sign}(p k) \) and \( \cos \lambda < 0 \) are respectively the necessary conditions for inflation, since otherwise \( q \) would be definite positive. In general, the quantity \( q \) will either vanish at a couple of time instants which we will denote \( t_- \) and \( t_+ \), or will never become null at all. Moreover, from the fact that at late times \( q \) is necessarily positive it follows that \( q \) will be negative between those hypothetical \( t_- \) and \( t_+ \). The question to formulate is whether those instants exist and whether the sign change of the deceleration factor at \( t_+ \), which ends inflation, occurs during the model’s history \( t \geq 0 \).

For our purposes it suffices to give the implicit equations of \( t_- \) and \( t_+ \), in the interacting case these read

\[
\dot{G}(t_{\pm}) = \frac{2|p|}{(k^2 - 2n)^2} \left( \pm \frac{\sqrt{6} \sqrt{k^4 n - k^2 (k^2 - 2) n^2 - 2 k^2 n^3 + 8 n^4}}{\sqrt{k^2 + 4 n}} - |k| \text{sign}(p k) \sqrt{k^2 + 2 n} \right),
\]  

which can be solved provided that

\[
\frac{k^2}{n} + 1 \leq \sqrt{\frac{9 n - 1}{n - 1}}.
\]  

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Typically, in multifield models and for a given $k$, a minimum number of fields will be needed for inflation to occur in multifield models. Note that in the particular case $n = 1$ there will be inflation for any value of $k$. Let us assume now that we are dealing with a multifield case and that condition (40) is fulfilled.

Clearly, inflation will occur if $t_+ > 0$ and, under the assumptions made before, we can conclude that there will be an epoch of accelerated inflation if $\dot{G}(t_+) > \dot{G}(0)$. Remarkably, this condition follows from (10), so the latter is actually not only a necessary but also a sufficient condition for the existence of inflation. With these results at hand we conclude that our non-interacting models are examples of the assisted inflation phenomenon. As a matter of fact, in the cases in which the individual potentials are too steep inflation can still occur thanks to the cooperation of the fields if there are enough of them.

The implicit equations of $t_-$ and $t_+$ in the interacting case are slightly different:

$$\dot{G}(t_\pm) = \frac{2p}{(k^2 - 2)^2} \left( \pm \sqrt{6}\sqrt{8 - k^2 (k^2 + 2) \sin^2 \lambda} \frac{\sqrt{k^2 + 4}}{\sqrt{k^2 + 4}} - k\sqrt{k^2 + 2 \cos \lambda} \right)$$

and they can be solved provided that

$$k^2 + 1 \leq \sqrt{\frac{9 - \cos^2 \lambda}{1 - \cos^2 \lambda}}. \tag{42}$$

By recalling that in this case $k^2 = k_1^2 + \ldots + k_n^2$ we can see that the more fields there are, the more difficult it becomes for (42) to be satisfied and the less likely inflation is. Note that in the particular case $\cos \lambda = -1$ there will be inflation regardless of the value of $k$. In general lines, the role played by the number of fields in also agreement with existing results. The standard interpreted of this behaviour is a direct consequence of the way the friction depends on the field population. Moreover, if (12) holds, then $G(t_+) > \dot{G}(0)$ is fulfilled too, and we draw the conclusion that the fulfillment of (12) is in fact a necessary and sufficient condition for inflation.

Finally, it is interesting to look at the late time degree of isotropy of the spacetimes under discussion. According to an intuitive criterion for a perfect fluid model to become isotropic at late times [19] the quotient between the shear and the expansion of its fluid must tend to zero on that very limit. We use here the standard definition for the shear scalar of the fluid [18], namely

$$\sigma^2 = \frac{1}{2} \sigma_{\alpha\beta} \sigma^{\alpha\beta} \tag{43}$$

where $\sigma_{\alpha\beta}$ is the shear tensor. In general, in the non-interacting case one has for our geometries

$$\sigma = \frac{\dot{G} - \dot{G}}{\sqrt{3}(\dot{G} + 2\dot{G})} \tag{44}$$

and at late times it becomes

$$\lim_{t \to \infty} \sigma = \frac{k^2 - 2n}{\sqrt{3}(k^2 + 4n)}. \tag{45}$$
The latter limit is a monotonically increasing function of \( k^2/n \). So the larger the number of non-interacting fields, the more isotropy at late times. Conversely, using \( n = 1 \) and \( k^2 = k_1^2 + \cdots + k_n^2 \) we see that a larger number of interacting fields will result asymptotically in a more anisotropic spacetime.

5 Conclusions

Families of exact solutions to the set of Einstein-Klein-Gordon equations for Bianchi VI\(_0\) geometries with multiple scalar fields with exponential potentials have been derived. The models obtained include cases with both interaction and non-interaction among the fields, and we have shown the asymptotic stability of them all. In particular this means that those spacetimes characterised by parameters compatible with assisted inflation act as attractors of other possible solutions with the same values of those parameters.

Performing a kinematic analysis based on exact, yet particular, examples like the ones used here has a clear advantage in comparison with studies just concerned with asymptotic features. The severe limitation of not having any information about early stages in the evolution of the model is overcome here, and therefore we can formulate and answer the more natural question of whether inflation exists soon after the beginning of times.

A thorough analysis of the conditions under which inflation occurs has yielded the result that in the interacting cases profusion in the number of fields redounds to the improbability of inflation; whereas in the cases without interaction all the contrary happens, this is, inflation is assisted. A discussion on the late time isotropisation of these spacetimes has been included as well.

In our opinion, it would be of interest to extend this work by performing an analogous in-depth analysis about the conditions for inflation in the setup of other isotropic and anisotropic cosmologies.

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