A simple analytical calculation of mean-field potential in Heavy Nuclei

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Abstract

In the many-body theory of particles with long range interaction potential, such as the potential of Newtonian gravity and its relativistic generalization, Newton-Birkhoff theorem plays a very important role. This theorem is violated in the case of short range interaction potential, such as the Yukawa potential in nuclear physics. We discuss the relevance of this in the many-body treatment of heavy nuclei. In particular, using techniques similar to those in Newtonian gravity, we calculate analytically the mean-field potential in heavy nuclei. The mean-field potential thus calculated depends on well known quantities such as inverse pion mass, pion-nucleon coupling constant, nuclear radius and density. The qualitative features of the mean-field potential thus obtained is similiar to the well known phenomenological Wood-Saxon potential.
In this paper, using simple analytical calculations, we intend to bring out some features of motion of nucleons in a heavy nucleus [1,2]. Our calculations are based on two phenomenologically well known facts: The density of nucleons in heavy nuclei is almost constant and the inter-particle interactions in nuclei is given by the Yukawa potential. These assumptions seem to be sufficient for the qualitative understanding of the motion of nucleons in heavy nuclei. The central ingredient in our calculations is that the Yukawa potential violates Newton-Birkhoff theorem. In order to explain the role played by the violation of this theorem by the Yukawa potential in nuclear physics, we first formulate and proof the theorem in Newtonian gravity. The proof reveals why such a theorem can not be valid for Yukawa potential. In the many-body treatment of heavy nuclei with inter-particle Yukawa interactions, we consider the nucleon density to be constant. In order to justify this assumption, in the following section, we argue why a heavy nucleus can be treated as system of constant density. We show that the constant density has more to do with the Pauli’s exclusion principle and short range repulsive core of the nucleons rather than any ergodicity or thermodynamic equilibrium [3–7]. In the next two sections, we derive the mean field potential and indicate how the surface tension of heavy nuclei emerges from the violation of Newton-Birkhoff theorem. The mean field potential thus obtained qualitatively agrees with the phenomenological Wood-Saxon potential [8]. At the end, we make some concluding remarks.

I. THE NEWTON-BIRKHOFF THEOREM

Newton-Birkhoff theorem in the theory of gravitation is a very important theorem. The violation of this theorem by Yukawa potential is the central ingredient in our calculation of the surface tension and the mean field of heavy nuclei. Therefore, in this paper we provide a discussion of this theorem in Newtonian gravity in detail. Let us consider a spherically symmetric ball of mass \( M \), mass density \( \rho \) and radius \( R \).

Theorem (Part-I): The potential due to the spherical ball at a distance \( r \), where \( r > R \) is the same as the potential due to point particle of mass \( M \) placed at the centre of spherical ball.

Let us consider a spherical shell of outer radius \( R \) and inner radius \( b \), having constant mass density \( \rho \).

Theorem (Part-II): The gravitational potential due the spherical shell at any point \( r \), where \( r < b \) is constant. Therefore, the force at the point \( r \) is zero.

This theorem also holds in general theory of relativity and is called the Newton-Birkhoff theorem. Below we provide proof of the theorem in the context of Newtonian gravity. The proof reveals why the theorem is so special to Newtonian gravitational potential and does not hold for other potentials.

We want to calculate the gravitational potential and potential energy of a spherically symmetric ball of radius \( R \) and constant density \( \rho \). The two-body Newtonian gravitational potential energy is given by,

\[
U_N = -\frac{Gm_1m_2}{|\vec{r} - \vec{r}'|}
\]
where \( \vec{r} \) and \( \vec{r}' \) are the locations of the point masses \( m_1 \) and \( m_2 \) respectively. The Newtonian gravitational potential at a point \( \vec{r} \) due to a point mass \( m \) at point \( \vec{r}' \) is given by

\[
\Phi_N(\vec{r}) = -\frac{Gm}{|\vec{r} - \vec{r}'|} \tag{2}
\]

For a mass distribution with density \( \rho(\vec{r}') \), the formulas above take the following forms,

\[
\Phi_N(\vec{r}) = -G \int \frac{\rho(\vec{r}')d^3\vec{r}'}{|\vec{r} - \vec{r}'|} \tag{3}
\]

\[
U_N = -G \int \int \frac{\rho(\vec{r})\rho(\vec{r}')d^3\vec{r}d^3\vec{r}'}{|\vec{r} - \vec{r}'|} = \int \Phi_N(\vec{r})\rho(\vec{r})d^3\vec{r} \tag{4}
\]

In spherical co-ordinate system the formula for \( \Phi_N(r) \) gives,

\[
\Phi_N(r) = -G\rho \int \int \frac{r'^2dr'd\theta'd\phi}{(r^2 + r'^2 - 2rr'\cos\theta)^{1/2}} \tag{5}
\]

In the expression above, the integral over \( \phi \) ranges from 0 to \( 2\pi \) and for \( \theta \), it ranges from 0 to \( \pi \). The integral over \( \phi \) gives a factor of \( 2\pi \). The integral over \( \theta \) can be carried out by making the following change of variable

\[
\xi^2 = r'^2 + r^2 - 2rr'\cos\theta; \ \xi d\xi = rr'\sin\theta d\theta
\]

The integration over \( \xi \) ranges from \( \xi = (|r - r'|) \) to \( \xi = (r + r') \). After carrying out integrations over \( \phi \) and \( \theta \), we obtain

\[
\Phi_N(r) = -2\pi G\rho \int [(r + r') - |r - r'|] \frac{r'}{r} dr' \tag{6}
\]

For \( r > a \) (> \( r' \)), and \( 0 \leq r' \leq a \)

\[
\Phi_N(r) = -2\pi G\rho \int_0^a [(r + r') - (r - r')] \frac{r'}{r} dr' = -4\pi \frac{G\rho}{r} \int_0^a r'^2 dr' = -\frac{4\pi a^3 \rho G}{3r} = -\frac{GM}{r} \tag{7}
\]

For \( r < b \) (< \( r' \)), and \( b \leq r' \leq R \)

\[
\Phi_N(r) = -2\pi G\rho \int_0^a [(r + r') - (r' - r')] \frac{r'}{r} dr' = -4\pi G\rho \int_b^R r' dr' = -2\pi G\rho (R^2 - b^2) \tag{8}
\]

which is constant independent of \( r \). Note that, in the equation above, the cancellation of \( r \) from the expression is due to the special form of the Newtonian potential. Since force is given by the gradient of the potential with respect to \( \vec{r} \), \( \vec{F} = -\vec{\nabla}_r \Phi_N(r) = 0 \), therefore there is no gravitational force within the spherical shell. This demonstrates that in a classical theory this constant potential is dynamically irrelevant. However, it should be noted that in a quantum theory this "constant potential well" is relevant and its’ physical effects can be observed.
II. CONSTANT DENSITY OF NUCLEONS IN HEAVY NUCLEI

In this section we discuss why the density of nucleons in heavy nuclei can be taken to be constant. We do not provide here a formal proof of our claim. We rather argue that it follows very naturally from the short range character of the attractive potential, Pauli’s exclusion principle and the repulsive hard core potential of nucleons. Our claim is essentially based on self consistency arguments. However, it is important to note that we do not require any ergodicity or thermodynamic equilibrium for the constancy of the density of nucleons in the nuclei.

In order to maintain simplicity in our exposition and to avoid complications, we consider only one type of nucleon and assume that it does not carry any electric charge. The attractive interaction potential between a pair of nucleons is given by the short range Yukawa Potential,

\[ V = -g \frac{e^{-\mu r}}{r} \]  \hspace{1cm} (9)

where \( g \) is the nuclear charge (\( \frac{e^2}{\hbar} \) is the dimensionless nuclear fine structure constant) and \( \mu \) characterizes the range of the attractive potential. We assume that \( d \) is the radius of the core of the hard core potential. We assume that, \( R \), is the radius of the nucleus and \( N \) is the number nucleons in it. At first, we consider the nuclear matter limit of the nucleus. This means that both \( R \) and \( N \) are large. Yukawa potential is a short range potential, and therefore, each nucleon interacts with nucleons in a small neighbourhood around itself. The number of nucleons in its neighbourhood is proportional to the density of nucleons \( n_p \). Let the constant of proportionality be \( f \). Note that \( f \) is the volume characterising the range of the potential and can be taken to be \( f \approx \frac{1}{\mu} \).

Let us represent by \(-\sigma\) the potential energy of attractive interaction between two nucleons. Therefore, the potential energy of attractive interaction of a single nucleon with its neighbours is \(-fn_p\sigma\), and of \( N \) nucleons with their neighbours is \(-fNn_p\sigma\). In the next section, we will show that \( f\sigma = 4\pi g^2/\mu^2 \) (see the end of section-III).

We assume that the kinetic energy of the nucleons in the nucleus is solely due to the zero point energy. Therefore, using Pauli’s exclusion principle we obtain, 

\[ K.E. \approx \frac{\hbar^2}{2m} \times \frac{N^{5/3}}{R^2} \]

Taking \( n_p \approx \frac{N}{4\pi/3 \cdot R^3} \), we write the total energy \( E(R) \) as function of \( R \) as,

\[ E(R) \approx \frac{\hbar^2}{2m} \times \frac{N^{5/3}}{R^2} - fNn_p\sigma = \frac{\hbar^2}{2m} \times \frac{N^{5/3}}{R^2} - \frac{3N^2f\sigma}{4\pi R^3} \]  \hspace{1cm} (10)

We look for the variational (QM) ground state of the system. This can be done by finding the minimum of the function \( E(R) \). By imposing the condition, \( \frac{dE}{dR} = 0 \), we obtain all the extrema. It turns that the function has a minimum at \( R = 0 \) (\( E(R = 0) = -\infty \)) and a maximum when the interparticle distance (\( \approx \frac{R}{N^{1/3}} \)) is a few fermi. The function goes to zero at infinity. Therefore, there is no minima for finite value of \( R \). Such a system would either fly apart or collapse to a point. The ground state of the system is unstable. It is here that the hard core potential comes to play an important role. Because of the hard core potential the collapse would stop when \( \frac{R}{N^{1/3}} = d \). Therefore,

\[ E(R) = \frac{\hbar^2}{2m} \times \frac{N^{5/3}}{R^2} - \frac{3N^2f\sigma}{4\pi R^3} = \frac{\hbar^2}{2m} \times \frac{N}{d^2} - \frac{3Nf\sigma}{4\pi d^3} \]  \hspace{1cm} (11)
and the per particle binding energy

\[ E(R)/N = \frac{\hbar^2}{2m} \times \frac{1}{d^2} - \frac{3f\sigma}{4\pi d^3} \]  

(12)

This leads to the emergence of constant density. This picture resembles very much like those in packed granular material. However, our arguments are exact only in the limit of nuclear matter. For finite nuclei, however heavy it may be, volume occupied by the nucleons will be a bit larger than the hard core radii. Pauli’s exclusion principle adding a soft core in addition to the already existing hard core. This soft core is not a scattering potential. Because of Pauli’s exclusion principle, the nucleons rather avoid each other. It is a granular material in which the particle fail to touch other by the virtue of Pauli’s exclusion principle. But the gap is limited and this does not allow the particles to pile up at any one single place thereby mainintaining a constant density profile.

III. DERIVATION OF THE MEAN FIELD POTENTIAL IN HEAVY NUCLEI

We consider heavy spherical nucleus consisting of \( N \) nucleons and of radius \( R \). The density of nucleons in the nucleus, \( n_\rho \), is considered to be constant. Since the system is spherically symmetric, the potential at any point on a layer at \( r \) will be the same. In general the potential on any layer at \( r \) will consist of two parts- the potential due to the spherical ball of radius \( r - \epsilon \) and the potential due to the spherical shell of inner radius \( r + \epsilon \) and outer radius \( R \). We will calculate these two contributions separately and then add them up. This amounts to calculating the potential, \( \Phi_1 \), out side a spherical ball and the potential , \( \Phi_2 \), inside a spherical shell exactly in the way we carried out the calculations in Newtonian gravity. Let us write the general expression for the potential, \( \Phi \),

\[ \Phi(\vec{r}) = -g \int \frac{n_\rho e^{-\mu|\vec{r}-\vec{r}'|} d^3r'}{|\vec{r}-\vec{r}'|} = -gn_\rho \int \frac{e^{-\mu|\vec{r}-\vec{r}'|} r'^2 \sin\theta d\theta d\phi}{|\vec{r}-\vec{r}'|} \]  

(13)

In the expression above, the integral over \( \phi \) ranges from 0 to 2\( \pi \) and for \( \theta \), it ranges from 0 to \( \pi \). The integral over \( \phi \) gives a factor of 2\( \pi \). The integral over \( \theta \) can be carried out by making the following change of variable

\[ \xi^2 = r^2 + r'^2 - 2rr' \cos\theta; \ \xi d\xi = rr' \sin\theta d\theta \ ; \]

The integration over \( \xi \) ranges from \( \xi = (|r - r'|) \) to \( \xi = (r + r') \). After carrying out integrations over \( \phi \) and \( \theta \), we obtain

\[ \Phi(r) = -\frac{2\pi gn_\rho}{\mu r} \int dr' \ r'[e^{-\mu|r-r'|} - e^{-\mu(r+r')} ] \]  

(14)

For \( r > a \ (> r') \), and \( 0 \leq r' \leq a \)

\[ \Phi_1(r, a; \ r > a) = -\frac{2\pi gn_\rho}{\mu r} \int_0^a dr' \ r'[e^{-\mu(r-r')} - e^{-\mu(r+r')} ] \]  

(15)

After carrying out the integration, we obtain
\[ \Phi_1(r, a; r > a) = -\frac{4\pi gn_\rho}{\mu^2} \frac{e^{-\mu r}}{r} [a \cosh(\mu a) - \frac{1}{\mu} \sinh(\mu a)] \]  

(16)

This is the potential outside the spherical ball of radius \( a \) at a distance \( r \) from its' centre. Let

\[ A(a) = -\frac{4\pi gn_\rho}{\mu^2} [a \cosh(\mu a) - \frac{1}{\mu} \sinh(\mu a)] \]

This allows us to write the potential outside the spherical ball as

\[ \Phi_1(r, a; r > a) = -A(a) \frac{e^{-\mu r}}{r} \]

which has the same form as the Yukawa potential except for the fact that, now, the nuclear charge is \( A(a) \) and not \( g \). To obtain the same potential as that of the spherical ball at distance \( r, \ r > a \) we need to place at centre of the ball a point nucleus of nuclear charge equal to \( A(a) \). We can estimate the value of \( A(a) \) by writing the series expansion of hyperbolic cosine and sine functions in the expression of \( A(a) \). This gives,

\[ A(a) = Ng \left[ 1 + \frac{1}{10} (\mu a)^2 + \frac{1}{280} (\mu a)^4 + \sum_{n=4}^{\infty} \frac{3 \times 2n}{(2n + 1)!} (\alpha \mu)^{2n-2} \right] \]

(17)

where \( N = \frac{4\pi a^3 n_\rho}{3} \) is the number of nucleons inside the spherical ball of radius \( a \). When radius \( a \) is large, the quantity \( A \) is much much larger than the total nucleon charge \( Ng \) inside the spherical ball. Therefore, to obtain the same potential as that of the spherical ball containing the amount of nuclear charge \( Ng \) (at distance \( r, \ r > a \)), we need to place at centre of the ball a point nucleus of nuclear charge equal to \( A \) which is larger than \( Ng \). This clearly demonstrates that the part-I of the Newton-Birkhoff theorem is violated by the Yukawa potential. However, in the limit of \( \mu \to 0 \), we recover result similar to the case of gravity.

For \( r < b \) (\( < r' \)), and \( b \leq r' \leq R \)

\[ \Phi_2 (r, b ; r < b) = -\frac{2\pi gn_\rho}{\mu r} \int_b^R drr' [e^{-\mu(r'-r)} - e^{-\mu(r+r')}] \]

(18)

After carrying out the integration, we obtain

\[ \Phi_2 (r, b ; r < b) = -\frac{4\pi gn_\rho}{\mu^2} \frac{\sinh(\mu r)}{r} [-e^{-\mu R}(R + \frac{1}{\mu}) + e^{-\mu b}(b + \frac{1}{\mu})] \]

(19)

\( \Phi_2 (r, b ; r < b) \) is the potential inside a hollow spherical shell, \( \vec{F} = -\vec{\nabla} \Phi_2 (r, b ; r < b) \neq 0 \), and therefore, there is force inside a hollow spherical shell. This violates part-II of the Newton-Birkhoff theorem. However, in the limit of \( \mu \to 0 \), we recover result similar to the case of gravity.

It is now easy to calculate the potential at any \( r \) within the nucleus. Assuming,

\[ B(x) = \frac{4\pi gn_\rho}{\mu^2} (x + \frac{1}{\mu}) e^{-\mu x} \]

(20)
We can write the total potential at \( r, a < r < b \) as

\[
\Phi_{in}(r) = -A(a) \frac{e^{-\mu r}}{r} - B(b) \frac{\sinh(\mu r)}{r} + B(R) \frac{\sinh(\mu r)}{r} \tag{21}
\]

When the radius of the inner sphere, \( a \), approaches \( r \) from below and the inner radius of the shell, \( b \), approaches \( r \) from above, the first two terms of the equation above add up to a constant independent of \( r \), the third term remains unchanged and we obtain

\[
\Phi_{in}(r) = -\frac{4\pi g n}{\mu^2} \left[ 1 - \left( R + \frac{1}{\mu} \right) e^{-\mu R} \frac{\sinh(\mu r)}{r} \right]
\]

This is the mean field potential of the nucleons at any point \( r \) inside the nucleus. The mean potential outside the nucleus is given by

\[
\Phi_{out}(r) = -\frac{4\pi g n}{\mu^2} \frac{e^{-\mu r}}{r} \left[ R \cosh(\mu R) - \frac{1}{\mu} \sinh(\mu R) \right] \tag{22}
\]

The potential energy of a single nucleon in the mean mean field inside and outside the nucleus is given by the following equations,

\[
U(r < R) = -\frac{4\pi g^2 n}{\mu^2} \left[ 1 - \left( R + \frac{1}{\mu} \right) e^{-\mu R} \frac{\sinh(\mu r)}{r} \right] \tag{23}
\]

\[
U(r > R) = -\frac{4\pi g^2 n}{\mu^2} \frac{e^{-\mu r}}{r} \left[ R \cosh(\mu R) - \frac{1}{\mu} \sinh(\mu R) \right] \tag{24}
\]

In the formula above the values of the the physical constants are the following: pion-nucleon coupling constant, \( g^2/c\bar{h} = 0.081 \), conversion constant, \( c\bar{h} = 197.328 \), range of two-body Yukawa potential, \( 1/\mu = 1.4094 \) fm, radius of nucleus of atomic number \( A \), \( R = 1.07 \ A^{1/3} \) fm, density of nucleons in nucleus, \( n_\rho = 0.0165/fm^3 \). We provide plots of the potential for three different nuclei in Fig.(1). The qualitative feature of this potential is similar to the phenomenological Wood-Saxon potential [8]. The qualitative features is similar even near the boundary of the nucleus. This is a bit of a surprising result because the nucleon density near the boundary of the nucleus decreases drastically where as in our derivation we have assumed constant density through out the nucleus. This suggests that the mean field potential is not very sensitive to the distribution of nucleons near the boundary.

From eq.(23), it is now easy to infer that in the nuclear matter limit, i.e., when \( R \) is very large, \( U(r) = -\frac{4\pi g^2 n}{\mu^2} \). Therefore, the quantity, \( f\sigma \) introduced in section-II is equal to \( 4\pi g^2/\mu^2 \).

IV. THE ORIGIN OF SURFACE TENSION IN HEAVY NUCLEI

In this section we sketch some arguments to show that the surface tension in heavy nuclei has its’ origin in the violation of Newton-Birkhoff theorem. Actual calculation of the surface...
tension will require the true density profile of the nucleons in the nucleus. However, in the framework of the present paper the density is assumed to be constant, and therefore, we will not attempt to calculate the surface tension. We rather point out that, even under the assumption of constant density, one can observe the origin of surface tension. The true density profile influences only the numerical value of the surface tension.

To make the role played by the violation Newton-Birkhoff theorem more transparent, we briefly recall some of the derivations. The total potential at any point, \( r \), in the region between a sphere of radius \( a \) and a spherical shell of inner radius \( b \) and outer radius \( R \) ( \( a < r < b \) ) is given by

\[
\Phi(r) = -A(a)\frac{e^{-\mu r}}{r} - B(b)\frac{\sinh(\mu r)}{r} + B(R)\frac{\sinh(\mu r)}{r}
\]

When the radius of the inner sphere, \( a \), approaches \( r \) from below and the inner radius of the shell, \( b \), approaches \( r \) from above, the first two terms of the equation above add up to a constant independent of \( r \), the third term remains unchanged and we obtain

\[
\Phi_{\text{in}}(r) = -\frac{4\pi g n_\rho}{\mu^2} \left[ 1 - \left( R + \frac{1}{\mu} \right) e^{-\mu R} \frac{\sinh(\mu r)}{r} \right]
\]

Force \( \vec{F} \), on a single nucleon at any point \( r \), \( r < R \) is given by,

\[
\vec{F} = -g \vec{\nabla} \Phi_{\text{in}}(r) = -\frac{4\pi g^2 n_\rho}{\mu^2} \left( R + \frac{1}{\mu} \right) e^{-\mu R} \frac{\mu r \cosh(\mu r) - \sinh(\mu r)}{r^2} \vec{n}_r
\]

and for \( r \), \( r > R \)

\[
\vec{F} = -g \vec{\nabla} \Phi_{\text{out}}(r) = -\frac{4\pi g^2 n_\rho}{\mu^2} \frac{(1 + \mu r) e^{-\mu r}}{r^2} \left[ R \cosh(\mu R) - \frac{1}{\mu} \sinh(\mu R) \right] \vec{n}_r
\]

where \( \vec{n}_r \) is a unit vector directed along the radius towards the centre of the nucleus.

The depth of the mean field potential varies very slowly in the central region (\( r \approx 0 \)) of the nucleus (see Fig.(1)). However, near the outer surface, there is a sharp decline in the depth of the total potential. Therefore, the force will have very small in magnitude (practically zero) in the central region but a broad peak near the boundary of the nucleus. This is what we observe in Fig.(2), where we have plotted the total force, \( \vec{F} \) as a function of \( r \). The direction of this force at any point \( r \) near the boundary is towards the centre along the radius connecting the point. Every nucleon in the outer shell is, therefore, radially pulled towards the centre of the nucleus. Since the force is practically zero in the central part of the of the nucleus, there is a net pressure difference between the outer shell and the inner core. This is the origin of surface tension in nuclei. We believe that the surface tension in most normal liquids have similar origin. Our believe is based on the fact that potentials of the type \(-1/r^n\) which describe the properties liquid state very well, can be obtained by the exchange of continuum of Yukawa modes [9,10] with some spectral weight \( \rho(\mu) \). In fact, the most commonly used Leonard-Jones potential in liquids can be obtained with spectral weight \( \rho(\mu) = \mu^4 \).

\[
-\frac{1}{r^6} = -\frac{1}{4!} \int_0^\infty d\mu \mu^4 e^{-\mu r} \quad (27)
\]
V. CONCLUDING REMARKS

In this paper, we have tried to emphasise the importance of the violation of Newton-Birkhoff theorem by Yukawa interaction for the emergence of mean field shell model potential as well as the surface tension in heavy nuclei. The only input from the nuclear physics that we have used is the constant density profile of nucleons in heavy nuclei. Some very basic and general assumptions, such as Pauli’s exclusion principle, short range character of the attractive Yukawa potential and the existence of a repulsive hard core in nucleons, ensures the emergence of constant density profile in heavy nuclei. We have derived the mean field shell model potential analytically. This potential qualitatively agrees with the phenomenological Wood-Saxon potential. Our formula explicitly shows how the mean field potential depends on pion-nucleon coupling constant, range of the two-body Yukawa potential, nuclear size and the density. Solution of the Schroedinger equation with the mean field potential so derived leads to the emergence of shell model structure. We have also pointed out how the violation of the Newton-Birkhoff theorem leads to the emergence of surface tension in heavy nuclei. Our derivations, although semi-quantitative in nature, strongly suggest that the qualitative features which are attributed either to shell model or liquid drop model can be derived from the Yukawa interaction of nucleons having repulsive hard core, and some basic principles of quantum mechanics.

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Figure 1: Plot of potential energy of a single nucleon in the mean-field, inside and outside the nucleus, as a function of radial distance from the center of the nucleus.
Figure 2: Plot of mean-field force $F$ on a single nucleon inside and outside the nucleus as a function of radial distance $r$ from the centre of the nucleus. The minima in the plot occur at the radius $R=1.07A^{1/3}$ for the respective nuclei.