Finite-size critical fluctuations in microscopic models of mode-coupling theory

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Abstract. Facilitated spin models on random graphs provide an ideal microscopic realization of the mode-coupling theory of supercooled liquids: they undergo a purely dynamic glass transition with no thermodynamic singularity. In this paper we study the fluctuations of dynamical heterogeneity and their finite-size scaling properties in the $\beta$ relaxation regime of such microscopic spin models. We compare the critical fluctuation behavior for two distinct measures of correlations with the results of a recently proposed field theoretical description based on quasi-equilibrium ideas. We find that the theoretical predictions perfectly fit the numerical simulation data once the relevant order parameter is identified with the persistence function of the spins.

Keywords: finite-size scaling, dynamical heterogeneities (theory), structural glasses (theory), slow relaxation and glassy dynamics
Facilitated spin models (FSM) have been proposed as an elegant way to rationalize the idea that the glass transition is purely kinetic in nature [1]. This contrasts with the view that glassy phenomena in liquids can be understood in terms of a rugged free-energy landscape, as is the case for systems with complicated static interactions. Recent results have shown, however, that a common mechanism could be the basis of the slowing down of relaxation both in FSM and in landscape dominated systems. Important insight has come from the study of cooperative FSM on random graphs, as they provide a suitable mean-field limit in which the possibly sharp dynamic crossover observed in finite-size systems becomes an actual transition in the thermodynamic limit. In [2, 3] it has been shown that relaxational dynamics of FSM on random graphs verifies the universal relations predicted by mode-coupling theory (MCT) of supercooled liquids [4, 5]. In fact, the formal structure of MCT singularities is closely related to that appearing in the bootstrap percolation analysis of FSM [6] and $k$-core problems on random graphs. In some cases, it is known that rugged landscapes and kinetic constraints provide a dual description of slow relaxation [7, 8]. The recently studied case of the random XOR-SAT optimization problem of information theory [9] explicitly shows the emergence of a random first-order transition scenario and the highly nonlocal nature of the relation between such dual descriptions. Unfortunately, in the general case, a detailed analytic comprehension of the long-time dynamics is lacking. Although the bootstrap percolation analysis enables us to compute the phase diagram and the arrested part of the correlation, other important characteristics of relaxation, like the origin of the feedback mechanism leading to MCT behavior and the prediction of relaxation exponents, remain out of reach. On general grounds, MCT describes dynamical arrest as a critical phenomenon driven by long-lived dynamic heterogeneities with increasing correlations. Within the mean-field scenario this transition has universal characteristics that can be rationalized, postulating random critical temperature variations due to disorder [10]. In the so-called $\beta$ relaxation regime—i.e. the time window in which the correlation function approaches its plateau value corresponding to the glass transition and the system explores the configuration space near the ideal arrested state—it is possible to derive from first principles the postulated random critical point, and to describe overlap fluctuations in terms of a cubic field
theory in a random field [11]. This theory, initially formulated for systems with nontrivial Hamiltonian-like spin glasses and liquids, identifies the overlap with the initial condition as the order parameter in terms of which a field theoretical description of fluctuations is possible.

In this context it is natural to ask whether FSM conform to the expectations of this quasi-equilibrium field theory. The answer is not obvious as metastability and dynamic arrest in FSM are induced by purely kinetic constraints and the Hamiltonian is generally trivial (there is neither quenched disorder nor frustrated static interaction in the problem). As we shall see, the overlap fails to specify long-lived metastable states and one has therefore to turn to other possible correlation functions that can be used to characterize metastable states. The correspondence with the bootstrap percolation problem in fact suggests using the core and its time dependent analogue, the persistence function, as an appropriate order parameter for the glass transition. In this paper we study numerically the finite-size properties of both the fluctuations of the persistence function and of the spin overlap. We find that while the former follows scaling laws with the exponents suggested by the cubic random field theory, the scaling of the overlap fluctuations does not.

The scheme of the paper is the following. In section 2 we briefly review the theory of [11]. In section 3 we introduce the model and compare the numerical simulation results to the theory predictions for two different measures of correlations. In section 4 we discuss the main implications of our findings and conclude the paper.

2. The field theoretical approach to fluctuations in the \( \beta \) regime: a mini-review

The theory proposed in [11] deals with equilibrium fluctuations of the local overlap, \( q_x(s, s_0) \), between an initial configuration, \( s_0 = s(0) \), and the one visited by the system at a certain time \( t \) during its evolution, \( s = s(t) \). The appropriate definition of overlap may be system dependent and some coarse graining is generally assumed. For example, in liquid systems one can divide the space into small cells centered around the position \( x \) and compare the local particle density of the current configuration with that of the initial configuration. In spin systems the most natural definition is the usual scalar product between the two spin configurations.

The theory suggests eliminating the temporal dependence of the relevant physical quantities from the description, starting from the observation that due to time scale separation, when the dynamical MCT transition is approached, the dynamics can be described as a random walk between metastable states; equilibration within a state establishes before a new state is found. In such conditions one can eliminate time in favor of the local overlap with the initial state and describe fluctuations in a restricted equilibrium ensemble with probability measure

\[
\mu(s|s_0) = \frac{1}{Z[s_0]} \exp \left( -\beta \mathcal{H}(s) + \int d x \nu(x) q_x(s, s_0) \right),
\]

where \( \mathcal{H}(s) \) is the system Hamiltonian and \( \beta = 1/k_B T \) is the inverse temperature. The Lagrange multiplier \( \nu(x) \) is fixed in such a way that the average values of the local overlap \( q_x(s, s_0) \) take some prescribed values close to the plateau value, \( q_{EA} \). The crucial assumption in the definition of the restricted Boltzmann–Gibbs measure, equation (1), is that the overlap provides a sufficient determination of the metastable states.
A deep analysis of such a theory close to the ideal MCT transition (when activated processes are neglected) shows the emergence of a universal description in terms of the local overlap fluctuation
\[ \phi(x) = q_x(s, s_0) - q_{EA}, \]
which is encoded in the effective action:
\[ S[\phi] = \int dx \left[ \frac{1}{2} (\nabla \phi)^2 + \epsilon \phi(x) + g \phi(x)^3 + h(x)\phi(x) \right], \]
where \( \epsilon \) is a parameter quantifying the distance from the glass state (e.g., \( \epsilon = T - T_d \)) and \( g \) is a coupling constant. The ‘external field’ \( h(x) \) accounts for the effects of heterogeneity in the initial condition: it is a random Gaussian variable with zero mean and variance
\[ E(h(x)h(y)) = \Delta \delta(x - y), \]
where \( \Delta \) is a system dependent parameter. The effective action can be used to study the finite-size corrections in mean-field glassy systems. In that case one only deals with global overlap fluctuations and so one just needs to replace the space integral with an overall volume factor \( N \). The field variance \( \Delta \) scales then as \( 1/N \) and the Gaussian theory suffices for understanding the finite-size scaling properties.

In order to characterize fluctuations, two kinds of four-point correlation functions can be introduced:
\[ \chi_{\text{th}}(t) = N \left[ \langle (\phi(t))^2 \rangle - \langle \phi(t) \rangle^2 \right], \]
\[ \chi_{\text{het}}(t) = N \left[ \langle (\phi(t))^2 \rangle - \langle \phi(t) \rangle^2 \right]. \]

We denote by the angular brackets, \( \langle \cdots \rangle \), the average over trajectories that start from the same initial condition. This was called the iso-configurational average in [13, 14]. In our stochastic dynamics the iso-configurational average represents the average over the thermal noise along the trajectories. The initial condition, denoted by \( s(0) = s_0 \), will always be chosen as an equilibrium configuration and the corresponding average will be denoted by the overbar \( \overline{\cdots} \). Thus, \( \chi_{\text{th}}(t) \) characterizes thermal fluctuations for fixed initial condition, while \( \chi_{\text{het}}(t) \) characterizes the fluctuations with respect to the initial condition. The sum of these two functions gives us back the total order parameter fluctuations
\[ \chi_A(t) = N \left[ \langle (\phi(t))^2 \rangle - \langle \phi(t) \rangle^2 \right]. \]

As is typical in random field systems, \( \chi_{\text{th}} \) and \( \chi_{\text{het}} \) have different scaling properties. If the quasi-equilibrium assumption of the theory is verified, one can then eliminate the explicit time dependence of the fluctuations in favor of \( \phi \), and it turns out that
\[ \chi_{\text{th}}(\phi, \epsilon, N) = N^{1/4} \mathcal{F}_{\text{th}}(\phi N^{1/4}, \epsilon N^{1/2}), \]
\[ \chi_{\text{het}}(\phi, \epsilon, N) = N^{1/2} \mathcal{F}_{\text{het}}(\phi N^{1/4}, \epsilon N^{1/2}). \]

The questions that we address in this paper are the following:

- Can dynamical fluctuations of FSM on random graphs be described in terms of the effective field theory (3)?
- What is the relevant measure of correlations that characterizes metastability in FSM on random graphs?
Some of the basic ingredients of the theory, namely time scale separation and quasiergodic exploration of metastable states, are manifestly true in FSM. However, the conventional spin overlap does not provide a sufficient determination of metastable states. The glass singularity of FSM is associated with a core or bootstrap percolation transition of frozen spins. The order parameter is therefore the fraction of frozen spins in the core. This suggests that the persistence function, \( p(t) \), i.e. the fraction of spins that have not flipped over the time interval \([0,t]\), is the natural correlation function that can be used to identify metastable states. Conversely, the conventional spin overlap, \( q(t) \), fails to identify metastable states: this is best understood in the frozen phase, where if one takes two equilibrium configurations with a value of the overlap corresponding to \( q_{EA} = \lim_{t \to \infty} q(t) \) and applies the bootstrap algorithm, one typically finds distinct core configurations.

Ideally, one could imagine that it is possible to derive a MCT equation for \( p(t) \), similarly to what has been done for the minimal size rearrangements in related models for the glass transition [12]. Even more optimistically, one could hope to write a field theory for persistence fluctuations like equation (3). Unfortunately, this is a nontrivial task because \( p(t) \) is a highly non-time-local quantity and its theoretical analysis is difficult. Rather than following this line of thought, in this paper we limit ourself to testing in numerical simulations the predictions, equations (8) and (9), obtained if one concentrates on the persistence function and the conventional overlap.

3. Numerical simulations of facilitated spin models on random graphs

We have studied numerically the dynamics of a facilitated system of \( N \) Ising spins, \( s_i = \pm 1 \) with \( i = 1, \ldots, N \), on a Bethe lattice with coordination number \( z \). The system has a trivial Hamiltonian,

\[
\mathcal{H} = -\frac{h}{2} \sum_{i=1}^{N} s_i,
\]

while spins evolve according to a Metropolis-like constrained dynamics: at each time step a randomly chosen spin, \( s_i \), is flipped with transition probability

\[
w(s_i \rightarrow -s_i) = \min \left\{ 1, e^{-h s_i / k_B T} \right\},
\]

if and only if at least \( f \) of its \( z \) neighboring spins are in the state \(-1\). Without loss of generality we set \( h = k_B \) hereafter. The Bethe lattice geometry is known to give a mean-field description of the system, with the advantage of preserving the notion of distance and that of finite connectivity. More precisely, Bethe lattices are intended here as random regular graphs, i.e. graphs chosen uniformly among the set of graphs with \( N \) vertices where all sites have exactly \( z \) neighbors. Since all sites have the same number of neighbors there is virtually no boundary in the system and the graphs converge locally to trees for large \( N \). It is this local tree-like structure, in fact, that allows us to derive exactly some properties of FSM.

We consider here the cooperative case \( f = 2 \) with \( z = 4 \), which is characterized by a discontinuous glass transition at a critical value \( T_d \approx 0.481 \) where the long-time limit of the persistence function, \( p(t) \), jumps from zero to a finite value \( p_{EA} \approx 0.673 \) [15]. In full analogy with the mixed discontinuous–continuous nature of the non-ergodicity parameter

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in MCT, the persistence has a square-root singularity coming from low temperatures and shares many other salient features of MCT [15, 2, 3, 6]. Microscopically, the discontinuity corresponds to the sudden emergence at $T_d$ of a giant cluster of permanently frozen spins (the core) with compact structure. From an algorithmic point of view this phenomenon is equivalent to the impossibility, below $T_d$, of generating an equilibrium configuration in which every spin is not permanently frozen. Metastable states can be identified with the set of spin configurations having the same core and their geometric organization is pretty much similar to that of TAP states in disordered $p$-spin systems [15].

Apart from their conceptual underpinning, such models offer the nontrivial advantage of requiring no thermal equilibration, therefore allowing the investigation of relatively large system sizes and samples with a moderate computational effort. Efficient simulations are performed by means of a faster-than-the-clock algorithm [16]. We have simulated spin systems of size ranging from $N = 128$ to 8192 for a number of samples of the order of about $10^3$–$10^4$, depending on the system size.

Glassy features in FSM are intimately accompanied by a substantial growth of dynamical heterogeneity as the transition point is approached [15]. Previous studies [17] have stressed the importance of the emerging time-reparameterization invariance of soft mode fluctuations during the ageing dynamics [18]–[20]. Other manifestations of dynamic heterogeneity that have been studied include the Kovacs memory effect [21] and the four-point dynamical susceptibility [22]. We shall focus here on the system size dependence of correlation fluctuations at criticality. To measure $\chi_{\text{het}}$ and $\chi_{\text{th}}$ we generate, for each sample $\ell = 1, \ldots, M$, two independent dynamical trajectories $k = 1, 2$ starting from the same equilibrium configuration and evolving with different thermal noise. Then for each sample and each trajectory we measure the persistence function $\phi_{\ell,k}$, where the initial state is equal for the two dynamical trajectories. The two components of persistence fluctuations are thus estimated as

$$\chi_{\text{th}} = N \left[ \frac{1}{2M} \sum_{\ell,k} \phi_{\ell,k}^2 - \frac{1}{M} \sum_{\ell} \phi_{\ell,1} \phi_{\ell,2} \right],$$

$$\chi_{\text{het}} = N \left[ \frac{1}{M} \sum_{\ell} \phi_{\ell,1} \phi_{\ell,2} - \left( \frac{1}{2M} \sum_{\ell,k} \phi_{\ell,k} \right)^2 \right].$$

Notice that since quenched disorder is absent in such FSM there is no additional source of fluctuations apart from the one due to the topology fluctuations in the random generation of graphs. We have checked that such an effect is substantially negligible as expected from the self-averaging property of random graphs.

In figure 1 we illustrate what the two components of persistence fluctuations at temperature slightly above $T_d$ look like in the quasi-equilibrium description when they are expressed in terms of the persistence ‘clock’. We observe that heterogeneity fluctuations are almost one order of magnitude larger than thermal ones, though they present qualitatively the same behavior. The results are similar to those obtained in the $p$-spin spherical model. To better mirror realistic situations in which the system size is finite but very large we have disallowed, in the simulations of figure 1, initial configurations having a fraction of permanently frozen spins. This is reasonable as long as the dynamics is time reversible and occurs above $T_d$. This corresponds to the situation in which the amorphous
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Figure 1. Parametric plot of the two components of the persistence fluctuations $\chi_{\text{het}}(t)$ and $\chi_{\text{th}}(t)$ versus the average persistence, $p(t)$, at temperature $T = 0.5$ close to the glass transition ($T_d \simeq 0.481$) and for various values of the system size $N$.

solid has been formed by a slow enough cooling procedure such that irreversibility effects are negligible. In fact, permanently frozen spins are not the results of the microscopic dynamics which obeys the detailed balance condition. Rather, they can only be due to ‘external’ influences which are ultimately determined by the experimental conditions. For finite systems at temperature sufficiently close to criticality, or even below $T_d$, the absence of a core in a sample is an assumption that cannot be obviously guaranteed. For this reason we have considered two distinct situations. In the first one, the sampling of the initial condition is restricted to the subset of those equilibrium configurations which do not have permanently frozen spins. In the second situation, the initial equilibrium configurations are sampled without such a restriction. In this case, regions of permanently frozen spin may exist in a sample. Even though they do not directly contribute to fluctuations, such frozen spins may have an indirect influence on those spins located near the core boundary. A priori, such an influence might propagate to the entire system if the core has a special structure and is large enough. So it is important to assess the core contribution to the fluctuation scaling near criticality.

We have therefore studied the finite-size scaling properties of $\chi_{\text{th}}$ and $\chi_{\text{het}}$ at the glass transition temperature for these two distinct situations. At $T = T_d$ ($\epsilon = 0$) equations (8) and (9) read

$$\chi_{\text{th}}(\phi, 0, N) = N^{1/4} F_{\text{th}}(\phi N^{1/4}, 0),$$

$$\chi_{\text{het}}(\phi, 0, N) = N^{1/2} F_{\text{het}}(\phi N^{1/4}, 0),$$

where $\phi = p - p_{\text{EA}}$. We first consider the case in which the are no permanently frozen spins in the initial condition. We have checked that the curves of $\chi_{\text{th}} N^{-1/4}$ and $\chi_{\text{het}} N^{-1/2}$ as a function of $p$ obtained for various values of $N$ cross precisely at the exactly known value of $p_{\text{EA}}$. In figure 2 we see that, in agreement with the analytical predictions, the different curves of rescaled fluctuations plotted against the scaling variable $N^{1/4}\phi$ produce a pretty nice data collapse in a relatively wide region around the origin. We then consider the situation in which the initial condition is sampled from the equilibrium measure with no restriction. Even though the fluctuation curves are slightly different than those for the
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Figure 2. Scaling of persistence fluctuations in the $\beta$ relaxation regime at the critical glass temperature $T_d \simeq 0.481$ in the parametric plot representation. The initial condition of the dynamics is sampled from the equilibrium measure and configurations with permanently frozen spins are disallowed.

Figure 3. Scaling of the heterogeneity and thermal fluctuations in the situation in which the initial condition is sampled from the equilibrium measure with no restriction.

previous case, we see in figure 3 that the predicted finite-size properties are met, implying that the core has a negligible influence on the fluctuation scaling near criticality. We can thus summarize our first test by stating that the theory predictions are excellently verified when the correlations are measured in terms of the persistence function, no matter what choice of initial condition is made.

Next, we study equilibrium fluctuations of the conventional overlap, $q(t)$, that for our system should be more properly defined as

$$q(t) = \frac{1}{1 - m^2(0)} \left[ \frac{1}{N} \sum_{i=1}^{N} s_i(0)s_i(t) - m(0)m(t) \right], \quad (16)$$

where $m$ is the magnetization density. Notice that $q(t)$ exhibits features similar to the persistence function, i.e., for large system sizes its long-time limit jumps at $T_d$ from zero to $q_{EA} \simeq 0.4$ and has a square-root singularity below $T_d$. It can, consequently, play a legitimate role as an order parameter. However, as we have seen, this does not guarantee

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that metastable states are well characterized. Indeed, we find that two components of the spin overlap fluctuations do not obey the predicted scaling, equations (14) and (15). On one hand, we observe that the size dependence of the heterogeneity fluctuations, $\chi_{\text{het}}$, is substantially weaker than the expected $N^{1/2}$ one: a scaling form is recovered only if $\chi_{\text{het}}$ is rescaled by a factor $N^{-0.35}$, rather than $N^{-1/2}$; see figure 4 (left). On the other hand, we find that thermal fluctuations, $\chi_{\text{th}}$, are of order 1 over the whole range $q > q_{\text{EA}}$ and do not depend on the system size, as is observed in figure 4 (right). These results confirm that although the overlap can be used as an order parameter to describe the glass transition, it yields only a partial characterization of the metastability in FSM [9]. Therefore, when the nature of metastable states in the system under consideration is not exactly known, it is a rather delicate task to give a universal description of finite-size critical fluctuations.

Finally, we note that finite-size analyses of core fluctuations near the bootstrap percolation transition have been performed in [23, 24]. These works are concerned with an edge-deletion process (or leaf removal algorithm) that corresponds, unlike our case, to an irreversible dynamics. This stochastic process is endowed with an absorbing state and is described by a one-dimensional Langevin equation in a cubic potential and Gaussian noise. The absorbing state corresponds to the presence of a core and the dynamics occurs, in our context, below the critical temperature $T_d$, i.e. in the frozen phase. Our results for fluctuation exponents and scaling variables are in agreement with those found in [23, 24] when the time dependence of the various quantities is parametrically expressed in terms of the persistence function. This happens in spite of the deep difference between two dynamical processes: irreversible for the leaf removal algorithm and reversible for the facilitated dynamics. This suggests that once the persistence function is fixed, both dynamical processes sample the configuration space with the same law.

4. Discussion and conclusion

We have investigated the finite-size scaling properties of critical fluctuations in the $\beta$ relaxation regime of a cooperative facilitated spin model on regular random graphs for two different correlation measures. We tested our numerical simulation results against the predictions of a recent field theoretical approach that describes fluctuations in the $\beta$ regime.
in terms of a cubic field theory in a random field. We find it remarkable that although FSM have been purposely devised to lack any thermodynamical content, the predictions of the quasi-equilibrium approach are exactly matched when correlations are measured in terms of the persistence function. In contrast, when correlations are measured through the conventional overlap function we find that theory predictions are not verified. Such a discrepancy can be traced back to the fact that the conventional overlap function does not provide a sufficient characterization of metastable states in FSM. Thus, our findings confirm the universal character of fluctuations close to ideal MCT transitions, provided that the order parameter is correctly identified. In the presence of an incomplete knowledge about the very nature of the metastability, some caution is required when simulations or experiments are compared with theory.

Outside the mean-field scenario, when one turns to finite-dimensional systems, the MCT transition is generally expected to become a crossover in which critical effects get mixed up with thermally activated hopping processes. In this case the notion of metastability is not sharply defined and does require the introduction of a suitable time scale, possibly dependent on the observation time. Evidently, in such a situation the appropriate choice of relevant correlations becomes even more crucial. For these reasons, it would certainly be interesting to extend the present investigation to finite-dimensional FSM. This would provide the possibility to assess, on one hand, the role of finite-size corrections to scaling, which in bootstrap percolation related problems are known to be anomalously large and, on the other hand, the relevance of the above scenario in systems in which the glass transition is avoided in the infinite-size limit. Also, for continuous glass transitions, such as those appearing in the simplest higher-order singularity scenario of MCT [4, 3], we expect the above field theoretical description be still valid provided that the action $S$ includes a quartic term, and the scaling behavior of fluctuations is modified accordingly [25, 26]. Finally, from a more speculative theoretical perspective, we observe that since the ageing dynamics of glassy systems can be interpreted in terms of quasi-equilibrium concepts, one would be tempted to generalize the field theoretical scenario that we tested above in a suitably defined out of equilibrium regime of length and time scales.

References

[1] Fredrickson G H and Andersen H C, 1984 Phys. Rev. Lett. 53 1244
[2] Sellitto M, De Martino D, Caccioli F and Arenzon J J, 2010 Phys. Rev. Lett. 105 265704
[3] Arenzon J J and Sellitto M, 2012 J. Chem. Phys. 137 084501
[4] Götze W, 2009 Complex Dynamics of Glass-Forming Liquids (Oxford: Oxford University Press)
[5] Reichman D R and Charbonneau P, 2005 J. Stat. Mech. P05013
[6] Sellitto M, 2012 Phys. Rev. E 86 030502(R)
[7] Newman M E J and Moore C, 1999 Phys. Rev. E 60 5068
[8] Garrahan J P and Newman M E J, 2000 Phys. Rev. E 62 7670
[9] Foini L, Krzakala F and Zamponi F, 2012 J. Stat. Mech. P06013
[10] Sarlat T, Billoire A, Birolı G and Bouchaud J-P, 2009 J. Stat. Mech. P08014
[11] Franz S, Ricci-Tersenghi F, Rizzo T and Parisi G, 2011 Eur. Phys. J. E 34 102
Franz S, Ricci-Tersenghi F, Rizzo T and Parisi G, Replica field theory of the dynamical transition in glassy systems, 2011 arXiv:1105.5230
[12] Montanari A and Semerjian G, 2006 J. Stat. Phys. 124 103
[13] Widmer-Cooper A, Harrowell P and Fynewever H, 2004 Phys. Rev. Lett. 93 135701
[14] Widmer-Cooper A, Perry H, Harrowell P and Reichman D R, 2008 Nature Phys. 4 711
[15] Sellitto M, Birolı G and Toninelli C, 2005 Europhys. Lett. 69 496

doi:10.1088/1742-5468/2013/02/P02025
Finite-size critical fluctuations in microscopic models of mode-coupling theory

[16] Krauth W, 2006 Statistical Mechanics: Algorithms and Computations (Oxford: Oxford University Press)
[17] Chamon C, Charbonneau P, Cugliandolo L F, Reichman D and Sellitto M, 2004 J. Chem. Phys. 121 10120
[18] Chamon C and Cugliandolo L F, 2007 J. Stat. Mech. P07022
[19] Castillo H E and Parsaeian A, 2007 Nature Phys. 3 26
[20] Chamon C, Corberi F and Cugliandolo L F, 2011 J. Stat. Mech. P08015
[21] Arenzon J J and Sellitto M, 2004 Eur. Phys. J. B 42 543
[22] Franz S, Mulet R and Parisi G, 2002 Phys. Rev. E 65 021506
[23] Dembo A and Montanari A, 2008 Annu. Appl. Probab. 18 1993
[24] Iwata M and Sasa S, 2009 J. Phys. A: Math. Theor. 42 075005
[25] Franz S, Parisi G and Ricci-Tersenghi F, 2013 J. Stat. Mech. L02001 (arXiv:1203.4849)
[26] Arenzon J J, Franz S and Sellitto M, in preparation

doi:10.1088/1742-5468/2013/02/P02025