MOKA: a new tool for strong lensing studies

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ABSTRACT

Strong gravitational lensing is a powerful tool that can be used to probe the matter distribution in the cores of massive dark matter haloes. Recent and ongoing analyses of galaxy cluster surveys – such as the Massive Cluster Survey (MACS), the Canada–France–Hawaii Telescope Legacy Survey (CFHTLS), the Sloan Digital Sky Survey (SDSS), the Sloan Giant Arcs Survey (SGAS), the Cluster Lensing and Supernova Survey with Hubble (CLASH) and the Local Cluster Substructure Survey (LoCuSS) – have addressed the question of the nature of the dark matter distribution in clusters. Using N-body simulations of cold dark matter haloes, it is consistently found that haloes should be characterized by a concentration–mass relation, which decreases monotonically with halo mass, and that they should be populated by a large amount of substructures, representing the cores of accreted progenitor halos. It is important for our understanding of dark matter that we test these predictions. We present MOKA, a new algorithm for simulating the gravitational lensing signal from cluster-sized haloes. It implements the most recent results from numerical simulations to create realistic cluster-scale lenses with properties independent of numerical resolution. We perform systematic studies of the strong lensing cross-section as a function of halo structures. We find that the strong lensing cross-sections depend most strongly on the concentration and on the inner slope of the density profile of a halo, followed in order of importance by halo triaxiality and the presence of a bright central galaxy.

Key words: gravitational lensing: strong – methods: analytical – galaxies: haloes – cosmology: theory – dark matter.

1 INTRODUCTION

Galaxy clusters can be used as an important probe for dark matter (DM) properties. According to the standard scenario of structure formation, galaxy clusters as a population are still in their formation process. Because gas cooling cannot substantially compress DM haloes, their density profiles are dominated by DM. Strong lensing by clusters is sensitive to their internal structure–mass distribution within the Einstein radius,1 which includes the presence of substructure, the asymmetry in the gravitational potential well, the ellipticity, the presence of a massive and bright central galaxy and the inner slope of the DM density profile. Apart from weak and strong lensing, the halo density profile can also be constrained from the velocity dispersion profile of the central galaxy (Sand et al. 2004; Newman et al. 2009) and X-ray emission from the hot intracluster gas. While gravitational lensing does not rely on any equilibrium or dynamical assumptions, the methods based on galaxy or gas dynamics do, potentially biasing the estimates of mass and concentration.

In this paper, we quantify the importance of the structural parameters of DM haloes for the strong lensing signal, using our new

1 For an axially symmetric lens, this is defined as the radius of a circle enclosing a mean convergence of 1.
and fast algorithm, matter density distribution code for gravitational lenses (MOKA), to create realistic maps of substructured triaxial DM haloes, which are in perfect agreement with the measurements performed on galaxy clusters extracted from numerical simulations.

In Section 2, we present the halo properties on which our algorithm relies. The halo lensing properties are presented in Section 2.2 and the dependence of the strong lensing signal on the halo modelling is discussed in Section 3. We discuss our conclusions in Section 4.

We adopt a ΛCDM model with Ω_m = 0.3, Ω_Λ = 0.7, h = 0.7 and σ s = 0.9 that is consistent with the MARENOSTRUM UNIVERSE with which we compare some of our results.

2 CONSTRUCTION OF REALISTIC LENSES

Strong gravitational lensing depends on the matter distribution near halo centres where limited numerical resolution can affect the particle distribution. Here, we present a new algorithm, MOKA, which analytically creates surface mass density distributions of triaxial lenses (Giocoli et al. 2010b). We use this density profile to model the DM distribution in the clusters produced by MOKA.

(ii) The halo concentration is a decreasing function of the host halo mass. This numerical result is explained in terms of hierarchical clustering in CDM and the different halo formation histories. Following the extended theory of Press & Schechter (1974), Bond et al. (1991) and Lacey & Cole (1993) have found that the collapse time of DM haloes depends on the halo mass and that their assembly history is hierarchical: small systems collapse at higher redshifts than larger systems (Sheth & Tormen 2004a; Giocoli et al. 2007). This trend is reflected in the mass–concentration relation (i.e. at a given redshift, smaller haloes are more concentrated than larger haloes). Different fitting functions for numerical mass–concentration relations have been given (Bullock et al. 2001; Neto et al. 2007; Duffy et al. 2008; Gao et al. 2008). In this paper, we use the relation proposed by Zhao et al. (2009), which links the concentration of a given halo with the time (t_{0.04}) at which its main progenitor assembles 4 per cent of its mass:

$$c_{\text{vir}}(M_{\text{vir}}, z_i) = 4 \left(1 + \frac{1}{3.75t_{0.04}^{0.4}}\right)^{1/8}.$$  (5)

The model of Zhao et al. (2009) fits the numerical simulations well, even with different cosmologies. It seems to be of reasonably valid.

Because of their different assembly histories, haloes with the same mass at the same redshift can have different concentrations (Navarro et al. 1996; Jing 2000; Wechsler et al. 2002; Zhao et al. 2003a,b). At fixed host halo mass, the distribution in concentration is well fitted by a lognormal distribution function with a variance σ_{ln} between 0.1 and 0.25 (Jing 2000; Dolag et al. 2004; Sheth & Tormen 2004b; Neto et al. 2007).

(iii) Halos are generally not spherical but they are triaxial because of their tidal interaction with the surrounding density field during their collapse (Sheth & Tormen 1999, 2002). The correlation of the halo shape with the surrounding environment is expressed as a function of the matter density parameter and of the typical collapse mass. Jing & Suto (2002) have performed a statistical study of halo shapes extracted from numerical simulations. They have derived that, if a, b and c represent the minor, median and major axes, respectively, then the empirical relations for a/c and a/b are given by the distribution

$$\rho(\lambda) \, d\lambda = \frac{1}{\sqrt{2\pi} \sigma_{\lambda}} \exp -\frac{\left(\lambda - 0.54\right)^2}{2\sigma_{\lambda}^2} \, d\lambda.$$  (6)

for \(\lambda = (a/c)(M_{\text{vir}}/M_{\text{vir}})^{0.072(t_{0.04})}\) with \(\sigma_{\lambda} = 0.113\) and the conditional probability for the axial ratios

$$p\left(\frac{a}{b} \mid \frac{a}{c}\right) = \begin{cases} \frac{1}{\lambda_{\text{min}}}^2 & \frac{1}{\lambda_{\text{min}}}^2 \leq \frac{a}{b} < \lambda_{\text{min}} \\ 0 & \frac{a}{b} \geq \lambda_{\text{min}} \end{cases}$$

where \(\lambda_{\text{min}} = 0.5\) if \(a/c < 0.5\) else \(\lambda_{\text{min}} = a/c\). As usual, \((M_{\text{z}}, z)\) is the non-linear mass at redshift \(z\). Simulations with gas cooling have produced haloes that tend to be more spherical than those in pure DM simulations (Kazantzidis et al. 2004). Then, the host
halo shape also depends on the morphology of the stellar mass component. In this paper, we use the prescriptions by Jing & Suto (2002), which are based on DM-only simulations. However, MOKA is a very flexible tool, and the input distributions of axial ratios can easily be modified.

(iv) Haloes are not smooth, but are characterized by a large number of substructures, which might or might not host satellite galaxies (Moore et al. 1999; Springel et al. 2000b; De Lucia et al. 2004). These substructures are cores of progenitor haloes accreted along the merger tree, which were not completely disrupted by tidal stripping (Springel et al. 2001b; De Lucia et al. 2004; Gao et al. 2004; van den Bosch, Tormen & Giocoli 2005; Shaw et al. 2007; Giocoli, Tormen & van den Bosch 2008). Because of the different assembly histories and time-scales for subhalo mass loss, more massive haloes retain more substructures than less massive haloes at a given redshift. Likewise, haloes with a lower concentration (and thus with a lower formation redshift) are, on average, more substructured than haloes with a larger concentration. Giocoli et al. (2010a) fitted the subhalo mass function by

\[
\frac{1}{M_{\text{vir}}} \frac{dN(M_{\text{vir}}, z)}{dm} = A(1 + z)^{1/2} \frac{\bar{c}}{c_{\text{vir}}} m^\alpha \exp\left(-\beta \left(\frac{m}{M_{\text{vir}}}\right)^3\right),
\]

(7)

where \(A = 9.33 \times 10^{-4}\), \(\beta = 12.2715\), \(\alpha = -0.9\) and \(\bar{c}\) is the mean concentration of a halo with mass \(M_{\text{vir}}\) at redshift \(z\). We adopt this model because it incorporates the dependences of the subhalo mass function on the mass \(M_{\text{vir}}\), the concentration \(c_{\text{vir}}\) and the redshift \(z\) of the host halo. To populate our haloes with substructures with mass \(m\), we randomly sample the distribution down to a minimal subhalo mass \(m_{\text{min}}\).

Subhalo density profiles are modified by tidal stripping because of close interactions with the main halo smooth component and because of close encounters with other clumps, gravitational heating and dynamical friction. Such events can cause the subhaloes to lose mass and can eventually result in their complete disruption (Hayashi et al. 2003; Choi, Weinberg & Katz 2007). The remaining self-bound subhaloes will have density profiles different from the NFW shape, in that they are truncated at the tidal radius (Bullock et al. 2001). In order to take the truncation into account, we model the DM density distribution in subhaloes using truncated singular isothermal spheres (SISs; Keeton 2003)

\[
\rho_{\text{sub}}(r) = \begin{cases} \frac{\sigma^2}{2\pi G M_{\text{vir}}} & r \leq R_{\text{sub}} \\ 0 & r > R_{\text{sub}} \end{cases},
\]

with velocity dispersion \(\sigma\), and with \(R_{\text{sub}}\) defined as

\[m_{\text{sub}} = \int_0^{R_{\text{sub}}} 4\pi r^2 \rho_{\text{sub}}(r) dr \Rightarrow R_{\text{sub}} = \frac{G m_{\text{sub}}}{2 \sigma^2}.
\]

(8)

The velocity dispersion \(\sigma\) is related to the subhalo temperature by

\[
\sigma = f(T)\sigma_v^{(K)} + [1 - f(T)]\sigma_v^{(\gamma)}(T),
\]

(9)

where

\[
f(T) = \left[1 + \left(\frac{kT}{\text{keV}}\right)^{2}\right]^{-1}.
\]

(10)

Based on the temperature definition in the spherical collapse model, we can write

\[kT = (kT_\gamma)^{1/3}(1 + z)\left[\frac{\Delta_{\text{vir}}}{187}\Omega_0(1 + z)\right]^{1/3},\]

(11)

where \(k\) is the Boltzmann constant and \(M_{\text{vir}}\) is the mass in units of \(10^{15}\ M_\odot\ h^{-1}\). For temperatures above 1 keV, equation (9) follows the relation observed by Wu, Fang & Xu (1998) between X-ray temperature and velocity dispersion:

\[
\sigma_v^{(T)} = 371.5 \left(\frac{kT}{\text{keV}}\right)^{0.56} \text{km s}^{-1}.
\]

(12)

For \(T \ll 1\ \text{keV}\), it reduces to the relation

\[
\sigma_v^{(K)} = \left(\frac{G m_{\text{sub}} H(z) \Delta_{\text{vir}}^{1/2}}{4}\right)^{1/3},
\]

(13)

which can be obtained by combining equations (8) and (1) that identify \(R_{\text{sub}} = R_{\text{vir}}\).

Let us define \(R_{t}\) as the radius at which the subhalo mean density is of the order of the mean density of the main halo within \(r\). Following Tormen, Diaferio & Syer (1998), we can write this as

\[
R_t = r \left\{\frac{15}{2\ln M_{\text{vir}}(r)/\ln r} M_{\text{vir}}(r)\right\}^{1/3},
\]

(14)

where \(m_{\text{sub}}\) and \(r\) represent the subhalo mass and its distance from the host halo centre, respectively, and \(M_{\text{vir}}(r)\) is the host halo mass profile. Truncating subhaloes at \(R_{\text{vir}}\) does not create discontinuities in the convergence map because \(R_t(r) \lesssim R_{\text{sub}}\). It preserves the total subhalo mass fraction in hosts, as found in numerical simulations (De Lucia et al. 2004; Gao et al. 2004; Giocoli et al. 2010a). If the host halo matter density distribution is described by an NFW profile, the subhalo tidal radius can be analytically estimated, as a function of the distance from the host halo centre. Fig. 1 shows the ratio between the subhalo tidal radius and the radius that encloses the subhalo mass as a function of the distance from the host halo centre. From the figure, we can see that for \(r < 0.75 R_{\text{vir}}\), \(R_t \ll R_{\text{sub}}\); however, when the subhalo is located near to the virial radius, the ratio tends to unity.

\[R_t = \frac{R_{\text{sub}}}{R_{\text{vir}}} = 1\]

for \(r \lesssim 0.75 R_{\text{vir}}\), and

\[R_t = \frac{R_{\text{sub}}}{R_{\text{vir}}} = 0.25\]

for \(r \gtrsim 0.75 R_{\text{vir}}\).

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for \(r \gtrsim 0.75 R_{\text{vir}}\).
The SIS profile represents the galaxy density profiles well on scales relevant for strong lensing. Previously, other authors have used this model to characterize the lensing signal by substructures after stripping (Metcalf & Madau 2001). None the less, additional subhalo density profiles are implemented in MOCA to allow the truncation of the subhalo profile to be more smooth.

(v) The spatial distribution of subhaloes tends to follow the smooth DM distribution of the host halo. However, clumps near the host halo centre are easily destroyed, because of their tidal interaction with the main halo. Thus, the spatial subhalo distribution is less concentrated than the NFW profile of the host halo. Gao et al. (2004) have studied this using a cosmological numerical simulation. They have found that the cumulative spatial density distribution of clumps in host haloes is well described by

\[
\frac{n(<x)}{N_{\text{tot}}} = \frac{(1 + c_{\text{vir}}^2)x^\beta}{(1 + c_{\text{vir}}^2)^{\alpha}}\, ,
\]

where \(x\) is the distance to the host halo centre in units of the virial radius, \(N_{\text{tot}}\) is the total number of subhaloes in the host, \(\alpha = 0.244\) and \(\beta = 2.75\). Fig. 2 shows equation (15) for different values of the host halo concentration \(c_{\text{vir}}\), as indicated in the key.

2.1.2 Dissipative baryonic component and its effects on dark matter

Strong lensing is sensitive to the matter distribution inside the centre of galaxy clusters (\(r \sim 100\) kpc). On such scales, the density of the baryonic component becomes comparable to that of DM. Meneghetti, Bartelmann & Moscardini (2003) have shown that the brightest central galaxy (BCG) on the strong lensing signal is generally moderately important for strong lensing, but it is potentially decisive for those haloes that are otherwise marginally supercritical strong lenses.

To populate haloes with a central galaxy of a certain stellar mass, we use the halo occupation distribution (HOD) technique. In the HOD, it is assumed that the stellar mass of a galaxy is tightly correlated with the depth of the potential well of the halo within which it formed. Thus

\[
M_{\text{star}} = \frac{2M_{\text{star,0}}}{(M_{\text{infall}}/M_\odot)^{-\alpha} + (M_{\text{infall}}/M_\odot)^{-\beta}}
\]

as estimated by Wang et al. (2006), who modelled this relation after the semi-analytical galaxy catalogue of the Millennium Simulation (Croton et al. 2006). In this relation, we include a Gaussian scatter in \(M_{\text{star}}\) at a given host halo mass with \(\sigma_{M_{\text{star}}} = 0.148\). For the central galaxy, we set \(\alpha'' = 0.39\), \(\beta'' = 1.96\) and \(\log (M_{\text{star,0}}) = 10.35\).

For the stellar component of the BCG residing in the halo centre, we adopt the Hernquist (1990) profile:

\[
\rho_{\text{star}}(r) = \frac{\rho_0}{(r/r_g)(1 + r/r_g)^{3}}.
\]

This has a scale radius \(r_g\) related to the half-mass (or effective) radius \(R_e\) by \(r_g = 0.551R_e\). As done by Keeton (2001), we define the effective radius to be \(R_e = 0.03R_{\text{vir}}\). The scale density \(\rho_0\) can be estimated by the definition of the total mass of a Hernquist model:

\[
\rho_0 = \frac{M_{\text{star}}}{2\pi \sigma^2_g}.
\]

The presence of a dissipative baryonic component influences the DM distribution near the host halo centre. Blumenthal et al. (1986) have described the adiabatic contraction analytically, finding good agreement with numerical simulations. The initial and final density profiles – characterized by an initial radius \(r_i\) and a final radius \(r_f\), when a central galaxy is present – are related by

\[
r \left[ M_{\text{star}}(r) + M_{\text{DM,i}} \right] = r_i M_{\text{DM,f}}(r_i).
\]

Here

\[
M_{\text{DM,f}} = M_{\text{DM,i}}(1 - f_{\text{cool}}),
\]

where \(f_{\text{cool}}\) is the baryon fraction in the halo that cools to form the central galaxy. To solve the adiabatic-contraction equation, we need to derive \(r\) from equation (19). With a Hernquist model,

\[
f_{\text{cool}} r^3 + (r + r_f)^2 [(1 - f_{\text{cool}})r - r_f] m_1(r_f) = 0.
\]

This equation has a single relevant real root. We note that \(m_1\) in equation (21) defines the initial mass profile normalized by the virial mass. Fig. 3 shows the density profile of a halo with virial mass \(M_{\text{vir}} = 10^{15} M_\odot h^{-1}\) and \(c_{\text{vir}} = 4\), populated by a galaxy with stellar mass \(M_{\text{star}} = 5 \times 10^{12} M_\odot h^{-1}\). The dotted and short-dashed curves refer to the initial DM and stellar density in the halo, respectively. The presence of a dissipative baryonic component contracts the DM distribution. Solving equation (21), the DM density profile changes to that shown by the long-dashed curve. The solid curve shows the total density distribution obtained by summing the short-dashed and long-dashed curves. We recall that, for DM-only realizations, \(M_{\text{star}}\) is given by the sum in smooth plus clumpy components, while \(M_{\text{vir}} = M_{\text{smooth}} + \sum_{i=1}^{N_{\text{tot}}} m_i + M_{\text{star}}\) if a BCG is present.

2.2 Halo lensing properties

In this section, we describe how we calculate lensing properties starting from the three-dimensional matter density of all components characterizing the haloes. For each component, we project the
density on a plane perpendicular to the line of sight,
\[ \Sigma(x, y) = \int_{-\infty}^{\infty} \rho(x, y, z) \, dz, \]
where \( r = \sqrt{x^2 + y^2 + z^2} \) with the coordinate origin put into the
host halo centre, and we define \( \xi = \sqrt{x^2 + y^2} \).

The projected mass density of the NFW, Hernquist and SIS pro-
files can be given analytically (Bartelmann 1996; Bartelmann &
Schneider 2001; Keeton 2001):
\[ \Sigma_{\text{NFW}}(x, y) = \frac{2 \rho_c r_s}{\xi^2 - 1} F(\xi); \]
\[ \Sigma_{\text{sub}}(x, y) = \frac{\sigma_x}{2G\xi}; \]
\[ \Sigma_{\text{DM}}(x, y) = \rho_c r_s (2 + \eta^2) G(\eta) - 3 \]
\[ \frac{1}{(\eta^2 - 1)^2} \]
\[ \Sigma_{\text{tot}}(x, y) = \frac{\Sigma_{\text{NFW}}(x, y) + \Sigma_{\text{sub}}(x, y) + \Sigma_{\text{DM}}(x, y)}{\Sigma_{\text{cr}}}, \]
\[ \kappa(x, y) = \frac{\Sigma(x, y)}{\Sigma_{\text{cr}}}, \]
\[ \kappa_{\text{DM}}(x, y) = \frac{c^2}{4\pi G D_l D_s} \frac{D_s}{D_l D_h}, \]
\[ \kappa_{\text{sub}}(x, y) = \sum_{i=1}^{N} \kappa_{\text{sub},i}(x - x_i, y - y_i). \]

We note that the subhalo density profile is truncated at the radius en-
closing its total (bound) mass. The convergence is the appropri-
ately scaled surface mass density \( \Sigma(x, y) \),
\[ \kappa(x, y) = \frac{\Sigma(x, y)}{\Sigma_{\text{cr}}}, \]
where
\[ \Sigma_{\text{tot}} = \frac{\Sigma_{\text{NFW}} + \Sigma_{\text{sub}} + \Sigma_{\text{DM}}}{\Sigma_{\text{cr}}}, \]
\[ \kappa_{\text{DM}} = \frac{c^2}{4\pi G D_l D_s} \frac{D_s}{D_l D_h}, \]
\[ \kappa_{\text{sub}} = \sum_{i=1}^{N} \kappa_{\text{sub},i}(x - x_i, y - y_i). \]

To introduce ellipticity into our model, we draw random numbers
from the Jing & Suto (2002) distributions for the axial ratios
\( a/\sqrt{bc} \) and \( bc/a \), requiring \( abc = 1 \). Once the axial ratios are
known, we deform the convergence map accordingly. To randomly
orient the halo, we choose a random point on a sphere identified by its
azimuthal and elevation angles, and we rotate the halo ellipsoid by
these angles. We assign the same projected ellipticity to the smooth
component, to the stellar density and to the subhalo spatial den-
sity distributions. For the DM density distribution in subhaloes,
we assume spherically symmetric models, because the subhalo typical
scale is much smaller than the virial radius of the host system in
which they are located. Having elliptical subhaloes will slightly af-
flect the total strong lensing cross-section. We have chosen to assign
the same ellipticity to the BCG and to the DM density distribution.
in the host, following the results of Fasano et al. (2010), who found
that the shape of the BCG tends to reflect that of the associated
DM halo. Fig. 5 shows the convergence maps of six galaxy clusters
generated with our algorithm. All haloes are located at redshift $z = 0.25$ and possess a virial mass equal to $10^{15} M_{\odot} h^{-1}$. The source
redshift is $z_s = 2$. The subhalo mass resolution is $10^{10} M_{\odot} h^{-1}$.
This ensures a substructure mass fraction compatible with Richard
et al. (2010), and consistent with the fact that systems with a lower
substructure fraction tend to form at higher redshift, and are thus
more concentrated (Smith & Taylor 2008).

From the convergence map, the effective potential and the scaled
deflection angle can be obtained using

$$\Phi(x, y) = \frac{1}{\pi} \int_{R^2} \kappa(\xi') \ln |\xi - \xi'| d^2\xi'$$ \hspace{1cm} (29)

and

$$\alpha(x, y) = \frac{1}{\pi} \int_{R^2} \kappa(\xi') \frac{\xi - \xi'}{|\xi - \xi'|} d^2\xi'.$$ \hspace{1cm} (30)

A source will be seen at the angular position $\theta$, related to its intrinsic
angular position $\beta$ by the lens equation

$$\beta = \theta - \alpha(\theta).$$ \hspace{1cm} (31)

The derivatives of the lensing potential are denoted by subscripts

$$\frac{\partial^2 \Phi(x, y)}{\partial \xi_i \partial \xi_j} = \Phi_{ij},$$ \hspace{1cm} (32)

where $\xi_i = x$ when $i = 1$ and $\xi_i = y$ when $i = 2$. We introduce the
pseudo-vector field of the shear $\gamma = (\gamma_1, \gamma_2)$ by its components

$$\gamma_1(x, y) = \frac{1}{2} (\Phi_{11} - \Phi_{22})$$ \hspace{1cm} (33)

and

$$\gamma_2(x, y) = \Phi_{12} - \Phi_{21}.$$ \hspace{1cm} (34)

Light bundles are deflected differentially and they are thus dis-
torted, as described by the Jacobian matrix

$$A = (\delta_{ij} - \Phi_{ij}),$$ \hspace{1cm} (35)

with the eigenvalues

$$\lambda_t = 1 - \kappa - \gamma$$ \hspace{1cm} (36)

and

$$\lambda_r = 1 - \kappa + \gamma.$$ \hspace{1cm} (37)

The conditions $\lambda_t = 0$ and $\lambda_r = 0$ define the location of the tangential
and radial critical curves in the lens plane, where the magnification
is formally infinite. If we map the critical curve back into the source
plane using the lens equation, we obtain the tangential and radial
causites.

Table 1 summarizes the list of parameters that control MOKA.

3 LENS STRUCTURAL PROPERTIES
AND STRONG LENSING SIGNALS

In this section, we discuss the strong lensing efficiency of the clus-
ters produced by MOKA. Most of the discussion is focused on the
lensing cross-section $\sigma$. This is defined as the region on the source
plane from where the sources are mapped into images with a certain
length-to-width ratio $l/w$.

For each cluster, we create a high-resolution deflection angle map
of $2048 \times 2048$ pixels centred on the cluster centre, with a side
length equal to the virial radius. From this, we numerically estimate
the lensing cross-section using ray-tracing methods (Meneghetti
et al. 2005a,b, 2011; Fedeli et al. 2006). A typical calculation takes
about 1 min on a 3.06 GHz single-core processor. Starting from
the observer, bundles of light rays are traced back to the source
plane. Using an adaptive grid, this is populated by elliptical sources
of a fixed equivalent radius of 0.5 arcsec. The number of highly
Table 1. List of parameters present in moka 1.0 (http://cgiocoli.wordpress.com/research-interests/moka).

| Parameter                      | Description                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|
| Halo concentration            | Drawn from a lognormal distribution around the mean value at fixed mass$^a$ |
| Halo virial radius            | Defined from the spherical collapse model                                   |
| Axial ratio                   | Drawn from the Jing & Suto (2002) model$^b$                                 |
| Halo orientation              | Random picking the point on a sphere                                        |
| Subhalo mass function         | Random sampling of the Giocoli et al. (2010a) model                         |
| Subhalo distribution          | Random sampling of the Gao et al. (2004) model (the NFW model has also been implemented) |
| Subhalo velocity dispersion   | Fixed by equation (9)                                                      |
| BCG stellar mass              | Sampling the Wang et al. (2006) model with a Gaussian scatter               |
| BCG effective radius          | Fixed to be 4 per cent of the host halo virial radius (Keeton 2001)         |

$^a$ moka is flexible, as it can work with different mass–concentration relation models.

$^b$ Ellipticity can be turned off.

Magnified images are increased by refining the spatial resolution on the source plane near caustics. By analysing the images individually and measuring their length-to-width ratio $l/w$, we can define the lensing cross-section for giant arcs $\sigma_{l/w}$ as the region on the source plane from where the sources are mapped into images with a certain $l/w$. Giant arcs are commonly defined as distorted images with a length-to-width ratio $l/w \geq 7.5$ or $l/w \geq 10$. Here, we discuss our results for the three different values $l/w = 5, 7.5$ and 10.

The integral of the strong lensing cross-section of a galaxy cluster for different $z_l$, weighted by a source redshift distribution, allows us to quantify the number of gravitational arcs that we can expect to see. Bartelmann et al. (1998) have carried out a strong lensing analysis of galaxy-cluster-size haloes using CDM numerical simulations, which has revealed that these haloes produce an order of magnitude fewer gravitational arcs than observed. By studying how strong lensing cross-sections change with structural halo properties, we might be able to understand the possible current limitations of numerical simulations, without needing to advocate cosmological models with dark energy (Bartelmann et al. 2003).

Recent observational studies of strong lensing clusters have revealed that observed clusters seem to be more concentrated than simulated clusters. This causes more prominent strong lensing features. In the following, we quantify how much the presence of a BCG and halo triaxiality influence the strong lensing cross-section for giant arcs. We have created a sample of 128 high-resolution convergence maps of haloes with mass $10^{15} \, M_{\odot} \, h^{-1}$ and sources located at a fixed redshift $z_s = 2$. This study is considered as a reference starting point to present our code MOKA. At present, we are performing a statistical analysis of haloes in a $\Lambda$CDM universe in order to give an estimate of the strong lensing cross-section as a function of redshift and of the halo abundance (Boldrin et al. in preparation).

3.1 BCG

Using a variety of gas-dynamical simulations, Puchwein et al. (2005) have studied the impact of the gas component on strong lensing signals. They have found that the formation of stars mainly in the central region of the cluster tends to increase the cross-section by about 20–30 per cent, depending on lens redshift. To consider an analogous case, we have generated a sample of haloes with the same structural properties ($M_{vir} = 10^{15} \, M_{\odot} \, h^{-1}$, $z_s = 0.25$ and $\alpha = 2$), with and without a central galaxy. We recall that for pure DM lenses the virial mass takes into account the sum of the smooth plus clump components, while in simulations with a BCG the total $M_{vir}$ also includes the presence of the stellar system. In both cases, the total masses are identical. For each halo, we have post-processed their deflection angle map and we have estimated the strong lensing cross-sections as a function of arc length-to-width ratio $l/w$. The left panel of Fig. 6 shows the median strong lensing cross-section as a function of $l/w$ for a sample of clusters with (solid) and without (dashed curve) a BCG. The shaded regions enclose 25–75 per cent of the data. The right panel of Fig. 6 shows for each cluster the ratio of the strong lensing cross-section as a function of the length-to-width ratio with and without a BCG, including the estimated median. This figure confirms that at redshift $z_l = 0.25$, the value of

![Figure 6](https://example.com/figure6.png)
σ for arcs with a length-to-width ratio \( l/w > 5 \) tends to be lower by 20 per cent when the BCG is not included.

### 3.2 Triaxiality

Now, we consider spherical and triaxial haloes. Triaxiality affects strong lensing through the halo orientation. A cluster whose major axis is oriented along the line of sight will be a more efficient strong lens than if it is oriented otherwise. This orientation bias also influences mass and concentration estimates (Meneghetti et al. 2010b). To quantify how important triaxiality is for the appearance of giant arcs, we have created a sample of spherical haloes and we have compared their strong lensing cross-sections to those of our fiducial sample of triaxial haloes, whose mean ellipticity is \( \epsilon_{3D} = 0.15 \).

Fig. 7 shows the strong lensing cross-section as a function of \( l/w \) for the same sample of either triaxial or spherical galaxy-cluster haloes. Both samples contain BCGs. The right panel shows the median ratio of σ between the two samples: for \( l/w > 5 \), the strong-lensing signal tends to decrease by between 20 and 70 per cent in spherical haloes compared to triaxial haloes. This confirms the results obtained by Meneghetti et al. (2007) for a small sample of simulated haloes. Note also that arcs with small length-to-width ratios (between 2 and 5) are more abundant in spherical haloes because of their relatively lower shear.

### 3.3 Scatter

At a given host halo mass and lensing redshift, the strong lensing cross-sections scatter around its mean. This scatter is a result of halo triaxiality and the presence and distribution of substructures and concentrations. Fedeli et al. (2010) fit the scatter in \( y = \log (\sigma) \) for galaxy-cluster-sized haloes with a Gaussian distribution, or an Edgeworth expansion around it, including skewness. Fig. 8 shows the distribution of the strong lensing cross-sections for three length-to-width ratios \( l/w = 5, 7.5 \) and 10. To fit the distributions, we have considered both a Gaussian and an Edgeworth expansion, for \( y = \log (\sigma) \), which are superposed for corresponding values of \( \sigma_{l/w} \). The solid line shows a Gaussian with \( \sigma = 0.3 \).

\[
\sigma_{\log (\sigma)} = 0.56, 0.59 \text{ and } 0.61 \text{ for } l/w = 5, 7.5 \text{ and } 10, \text{ respectively.}
\]

These values are approximately twice the size of those found by Fedeli et al. (2010). We believe that the main reason for the larger standard deviation is the way in which the halo boundaries were defined in the simulations and with our algorithm: while we have included all matter of the main halo along the line of sight, Fedeli et al. (2010) have extracted boxes of 5-Mpc side length from their simulation. Another contribution to this difference is the fact that for each halo Fedeli et al. (2010) have estimated the strong lensing cross-section as a mean of the three projections: in the end, this procedure cuts the wings of their distribution.

### 3.4 Concentration

Haloes of the same mass have different concentrations, which reflect their different assembly histories. In Appendix A, we show how the rms scatter in the strong lensing cross-sections is related to the concentration scatter, and thus to the different host halo merger histories.

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\( \epsilon_{3D} \equiv (c - a)/[2(a + b + c)] \), where \( a, b \) and \( c \) define the minor, median and major axes, respectively.
Figure 9. Correlation between the strong lensing cross-section for arcs with length-to-width ratio $\geq 7.5$ and the host halo concentration. Filled triangles show the median of the correlation and the shaded region encloses 25–75 per cent of the data. The solid line is a least-squares fit to these data points, whose slope is $2.83 \pm 0.06$. The dashed curve shows the prediction for spherical and smooth NFW haloes (without a central galaxy). In this case, the sample of haloes is made up of 1024 systems with mass $M_{\text{vir}} = 10^{15} \, M_\odot \, h^{-1}$ at redshift $z_l = 0.25$.

Strong lensing depends mainly on the matter density in the cluster cores, where the DM density profile approaches a logarithmic slope near $-1$. The scale where the logarithmic density slope is $-2$ defines the scale radius $r_s$ and the host halo concentration by $c_{\text{vir}} = r_s/R_{\text{vir}}$. Higher concentrations cause larger strong lensing cross-sections. Fig. 9 shows this relation for our halo sample at $z_l = 0.25$. Filled triangles show the median of the relation, with error bars enclosing the quartile of the distribution in each $c_{\text{vir}}$ bin. The solid line shows a least-squares fit to the data whose slope is $2.83 \pm 0.06$. The dashed curve shows the predicted relation between the strong lensing cross-section and the host halo concentration for spherical smooth NFW haloes with mass $10^{15} \, M_\odot \, h^{-1}$. Note that our simulated lenses tend to have larger strong lensing cross-sections at low concentration compared to spherical NFW haloes. This is mainly because of the BCGs and triaxiality, which both tend to increase the projected central mass density.

Figure 10. Comparison between the median strong lensing cross-section $\sigma$ as a function of $l/w$ measured in analytically modelled haloes (solid line) and in haloes from the MARENOSTRUM UNIVERSE (filled circles) for a sample of cluster-sized haloes in the mass range $6-7 \times 10^{14} \, M_\odot \, h^{-1}$. The shaded regions enclose 25–90 and 90–100 per cent of the data for a sample of MOKA haloes in the same mass range, whose abundance has been weighted by the Sheth & Tormen (1999) mass function. The three solid curves show the same regions for the haloes in the MARENOSTRUM UNIVERSE. Our algorithm reproduces well the median of the strong lensing cross-sections measured in the numerical simulation. It does not reproduce the scatter of the lensing cross-sections, for the reasons discussed above.

3.4.2 Host halo concentration

Recent studies combining strong and weak lensing of galaxy clusters have disagreed with predictions of the number of arcs, and the observed clusters also seem to have Einstein radii larger than predicted in the ΛCDM cosmology (Zitrin et al. 2011a,b). This may emphasize that observed clusters are more concentrated than those found in numerical simulations (Oguri et al. 2005, 2009). However, overconcentrations in the observed haloes might also result from an orientation bias (Meneghetti et al. 2011).

We now quantify by how much the strong lensing signal changes if the normalization of the mass–concentration relation is increased. We perform different simulations of the same halo sample, assuming that the mass–concentration relation is given by equation (5), where we have included a normalization factor $c_0$. A reasonable choice for $c_0$ is a value between $-3.2$ and $6.4$, consistent with the simulation results for the mean concentration scatter at fixed halo mass.

In Fig. 11 we show the median strong lensing cross-section for three values of the normalization value $c_0$ of the concentration-mass relation: the two extreme cases $-3.2$ and $6.4$, and our reference model Zhao et al. (2009) where $c_0 = 0$. Fig. 12 shows the median ratio of strong lensing cross-sections with our fiducial sample as a function of $c_{\text{vir}} = c_{\text{vir}} + c_0$ for $l/w = 5, 7.5$ and 10. A higher normalization tends to increase strong lensing. Increasing the mean value of the concentration for cluster-size haloes from 4 to 8, the strong lensing cross-section increases by about a factor of 4.
Dependence of the median strong lensing cross-section on the normalization of the mass–concentration relation. The solid line shows the median cross-section for our fiducial cluster sample at $z_l = 0.25$. The dotted and dashed lines show the relation for higher and lower values, respectively, of the normalization in the mass–concentration relation: $c_{\text{vir}}(M_{\text{vir}}, z_l) + c_0$, $c_0 = 0$ for Zhao et al. (2009).

3.5 Subhalo abundance

Torri et al. (2004) have shown that merger events and substructures increase the strong lensing cross-sections, mostly if the latter are located near the cluster centre.

Our analysis has so far used cluster maps, whose subhalo mass function was obtained by sampling the equation (7) down to $10^{10} \, M_{\odot} \, h^{-1}$. To statistically analyse how the strong lensing signal depends on the minimum subhalo mass in our models, we have generated a sample of triaxial haloes with BCGs, sampling the subhalo mass function down to a variable minimum mass $m_{\text{min}}$. The total halo mass $M_{\text{vir}} = 10^{15} \, M_{\odot} \, h^{-1}$ is kept fixed.

Fig. 13 shows the strong lensing signal as a function of the minimum subhalo mass. The left panel shows the median ratio of runs with different resolutions compared to the fiducial case, whose subhalo mass is $\geq 10^{10} \, M_{\odot} \, h^{-1}$. The strong lensing signal increases when the minimum subhalo mass changes from $10^{10}$ to $10^{15} \, M_{\odot} \, h^{-1}$, because the smooth halo mass tends to increase for larger values of $m_{\text{min}}$, raising the projected mass density distribution near the host halo centre. The right panel shows the ratio $\langle \sigma_{\text{min}} / \sigma \rangle$ at $l/w = 7.5$ as a function of the minimum subhalo mass expressed in terms of the total mass. Again, the strong lensing signal increases because the smooth component contains more mass. This is also shown in the same figure, where the data points show the median of the smooth mass component in the host halo as a function of the minimum subhalo mass, rescaled with respect to the smooth mass component, with a minimum subhalo mass of $10^{10} \, M_{\odot} \, h^{-1}$.

Note that the strong lensing cross-section depends on the minimum subhalo mass. A decrease of the latter by a factor of 10 tends to increase $\sigma_{7.5}$ by 5 per cent, because the smooth mass increases. However, the impact of substructures can be different in different clusters, depending on the particular configuration of the lens.

Figure 11. Dependence of the median strong lensing cross-section on the normalization of the mass–concentration relation. The solid line shows the median cross-section for our fiducial cluster sample at $z_l = 0.25$. The dotted and dashed lines show the relation for higher and lower values, respectively, of the normalization in the mass–concentration relation: $c_{\text{vir}}(M_{\text{vir}}, z_l) + c_0$, $c_0 = 0$ for Zhao et al. (2009).

Figure 12. The median ratio of the strong lensing cross-section with respect to the fiducial sample as a function of the normalization $c_{\text{vir}} + c_0$ in the Zhao et al. (2009) mass–concentration relation for three different values of $l/w = 5$, 7.5 and 10 in the left, central and right panels, respectively. The shaded region is enclosed by the lower and upper quartiles.

Figure 13. The dependence of the strong lensing cross-section on the minimum subhalo mass. Left panel: median strong lensing cross-section scaled by the fiducial triaxial simulation with subhalo mass resolution $10^{10} \, M_{\odot} \, h^{-1}$. Right panel: median rescaled cross-section at $l/w = 7.5$ as a function of the minimum subhalo mass. The median rescaled strong lensing cross-sections for $l/w = 5$ and 10 as a function of $m_{\text{min}}/M_{\text{vir}}$ do not fall far away from the one estimated for $l/w = 7.5$. The data points connected with a dot-dashed line show the median of the host halo smooth mass component rescaled with respect to the smooth mass component when the minimum subhalo mass is $10^{10} \, M_{\odot} \, h^{-1}$, as a function of the minimum subhalo mass.
3.6 Generalized NFW density profile

One of the main potential problems of the CDM model regards the inner slope of the halo density profiles. Studying a sample of six strongly lensing clusters, Sand et al. (2004) concluded that, at 68 per cent confidence, the inner slope is consistent with $\beta \approx 0.52$, and that this is inconsistent with $\beta = 1$ at the 99 per cent level. Recent analyses by Newman et al. (2009, 2011) of Abell 611 and 383 have confirmed a flat central DM density profile. How much does the inner slope bias the strong lensing signal?

To answer this question, we have generated a sample of haloes with different $\beta$. We take into account the adiabatic contraction in the centre using equation (21), where $m_i(r_i)$ is estimated by integrating equation (38).

The generalized NFW density profile,

$$\rho_{\text{NFW}}(r) = \rho_0 \left(\frac{r}{r_i}\right)^{3(1+\beta)} (1 + \frac{r}{r_i})^{-\frac{3}{2}},$$

is taken to have an arbitrary inner slope $\beta$. We define $r_\text{c} \equiv (2 - \beta)r_i$, and the concentration $c_\text{c} = c_{\text{vir}}/(2 - \beta)$ at the radius where the density profile is isothermal.

Fig. 14 shows the median strong lensing cross-section for three samples of cluster-sized haloes. NFW denotes our fiducial sample with $\beta = 1$, while $\beta = 1.5$ and 0.5 illustrate a steeper profile and a shallower profile.

The three panels in Fig. 15 show the relation between the median ratio of the strong lensing cross-section at $lw = 5, 7.5$ and 10, compared to our fiducial sample, as a function of the inner slope $\beta$. In each panel, the shaded region encloses the lower and upper quartiles, while the dashed line shows the least-squares fit to the data:

$$\log \left(\frac{\langle \sigma(\beta)/\sigma_{\text{NFW}} \rangle}{\sigma_{\text{NFW}}} \right) = a\beta + b.$$

For this, we find

- $a = 0.572 \pm 0.005, \quad b = -0.560 \pm 0.005 \quad \sigma_5$,
- $a = 0.710 \pm 0.005, \quad b = -0.701 \pm 0.006 \quad \sigma_{7.5}$,
- $a = 0.768 \pm 0.009, \quad b = -0.750 \pm 0.009 \quad \sigma_{10}$.

A shallower (steeper) inner slope tends to decrease (increase) the strong lensing cross-section with $lw > 5$ by about 3 per cent. Considering triaxial haloes with $\beta = 1$ and $\beta = 1.5$, analogous values for the strong lensing cross-section for $lw = 7$ and $lw = 10$ and for sources with $z_s < 1.25$ have been found by Oguri, Lee & Suto (2003).

4 CONCLUSIONS

We have presented a new algorithm to study the lensing signal from cluster-sized haloes. The components that we use to build up our triaxial and substructured models take into account the most recent results from numerical simulations of structure formation. The possibility of turning these components on and off in our algorithm allows us to quantify the importance of all the halo properties for their strong lensing efficiency. Starting from the halo deflection angle maps, we estimate strong lensing cross-sections by ray-tracing.

We can summarize our main results as follows.

(i) Different structural halo properties affect strong lensing cross-sections in different ways.
(ii) Averaging over a sample of cluster-sized haloes, we find that a central galaxy and triaxiality tend to enhance the strong lensing signal by 20 per cent and 50–70 per cent, respectively.
(iii) The strong lensing cross-section monotonically increases with the host halo concentration; in a fixed mass bin, it is well characterized by a lognormal distribution.
(iv) An increase (decrease) of the normalization of the mass–concentration relation by a factor of $c_0 = 2$ increases (reduces) the strong lensing cross-section by a factor of 5 (0.03).
(v) Increasing the minimum subhalo mass by a factor of 10 slightly increases $\sigma_{7.5}$ by about 3 per cent.
(vi) The change in the inner slope of the density profile of the host halo is linearly related to $y = \log (\sigma_i/c_i)$; profiles shallower than the NFW profile are weaker strong lenses.

These results might help us to understand future observational data and the predictions for upcoming wide-field surveys (Refregier et al. 2010).
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APPENDIX A: SCATTER IN THE STRONG LENSING CROSS-SECTION

Haloes form hierarchically with the gravitational instability of DM density fluctuations. Small systems form first, and merge to form larger objects. The assembly history depends on the environment where a halo grows. Virialized structures with the same final mass might have experienced different growth histories (Gao, Springel & White 2005). Haloes with the same mass can thus have different
concentrations and subhalo mass functions. Numerical simulations have shown that the concentration distribution is well fitted by a lognormal distribution:

\[ p(c|M_{\text{vir}}) = \frac{1}{\sqrt{2\pi}\sigma_{\ln c}} \exp \left[ -\frac{(\ln c - \ln c_{\text{vir}})^2}{2\sigma_{\ln c}^2} \right] \]. \hspace{1cm} (A1)

We assume \( \sigma_{\ln c} = 0.25 \) in our algorithm. In order to relate the concentration scatter with the scatter of strong lensing cross-sections at fixed host halo mass, we have generated different halo samples with different \( \sigma_{\ln c} \), from which we randomly draw the host halo concentration. Fig. A1 shows the median ratio of the strong lensing cross-sections compared to our fiducial sample for different standard deviations \( \sigma_{\ln c} \). We find that smaller standard deviations reduce the median strong lensing signal.

For each halo sample, we have estimated the standard deviation \( \sigma_{\log \sigma} \) for length-to-width ratios \( l/w = 5, 7.5 \) and 10. Fig. A2 shows the relation between the standard deviation in the strong lensing signal for haloes with \( M_{\text{vir}} = 10^{15} \, M_{\odot} \, h^{-1} \), \( z = 0.25 \) and \( z = 2 \), and the scatter in concentration, rescaled to the scatter for the sample with \( \sigma_{\ln c} = 0.25 \). For the chosen \( l/w \) ratios, the correlation is well fitted by a straight line. Higher standard deviations in the concentration scatter cause larger strong lensing signals. The dashed lines in all panels show the least-squares fits to the data points:

\[ \sigma_{\log \sigma} = a \times \sigma_{\ln c} + b. \] \hspace{1cm} (A2)

Here, we obtain the slopes

\[ a = 0.714 \pm 0.049, \quad \text{for } \sigma_5, \]
\[ a = 0.640 \pm 0.091, \quad \text{for } \sigma_{7.5}, \]
\[ a = 0.506 \pm 0.053, \quad \text{for } \sigma_{10}. \]

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