Management of elastic displacements of a milling technological system based on the study of the main deformation axes

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Abstract. The article describes some issues of optimization of technological system parameters and processing parameters to ensure compliance of cutting forces and main deformation axes. As a result there appears a decrease in elastic displacements and an increase in dynamic stability.

1. Introduction
One of the most important factors that affects the efficiency of machine tools is rigidity. The rigidity affects both the machining accuracy [1] and performance, since machining performance is highly dependent on a vibration resistance of the machine and in turn, it is mainly determined by rigidity. In most studies concerning this topic, the focus is on issues of its own and contact rigidity of elements of the technological system. Meanwhile, the system arrangement also influences the rigidity of the entire system; it determines how elements characterized by their own and contact rigidity will perceive loads. This topic is not sufficiently developed in research, but at the same time relevant, because there is a significant reserve for ensuring the accuracy and dynamic stability of machines [2].

2. Optimization of the technological system arrangement by rigidity

2.1. Possibilities of problem solving on optimization of a technological system considering axes of rigidity
Rigidity axes or as they are called in the theory of elasticity axes of principal deformations can be taken as geometrical characteristics, which help to estimate the rigidity of a technological system [3]. The axis of maximum rigidity is a direction in the geometric space of the technological system, where the static elastic deformation is minimal. According to it, the axis of minimum rigidity is a direction in which static elastic deformations are maximal [4].

In terms of computer-integrated “digital” production, finite element modeling (FE modeling) of a technological system can be used to determine axes, which also includes the machine carrier system, cutting and auxiliary tools, the device and the workpiece. The results of finite element modeling are processed by analytical dependencies developed in the theory of elasticity. According to these dependencies with the help of FE modeling the strain tensor of an elementary elastic element (in this case conditional cutting rigidity) can calculate values of principal strains and direction cosines to determine principal axes of deformation [5].

You can solve the following problems, using the results of this calculation:
1. To build the optimal arrangement of the technological system, i.e. to select the relative position of the tool and the workpiece which can be achieved by using the design of the tool or turning axes in five coordinate machines to ensure the maximum rigidity axis approaches resultant cutting forces. This reduces static elastic deformations and increases a dynamic stability. Thus, here is solved the constructive task – the change in the position of axes of rigidity.

2. To change the position of resultant cutting forces. Knowing the arrangement of rigidity axes relative to reference axes of the machine, one can direct the resultant cutting force along the axis of maximum rigidity, then find projections of this vector on the machine coordinate axis. You can implement this method by changing tool parameters and cutting conditions. This method is easier to use in production conditions.

2.2. The solution of the technological problem

Consider the second task on the example of removing the allowance by end mill.

The components of cutting forces along the axes of coordinates depend on technological parameters; in particular, in our case the following analytical expressions are available for components \( F_x, F_y \) and \( F_z \) [6]:

\[
F_x = P_{\text{cutting}} \sin \omega; F_y = P_{\text{cutting}} \cos \omega; F_z = P_{\text{cutting}} \sin \omega_0
\] (1),

where \( P_{\text{cutting}} \) is the resultant of cutting forces, \( \omega_0 \) is the ascent angle of a helical line of the end mill,

\[
\omega = \frac{\pi}{4} + \rho' - \beta
\] (2),

where \( \rho' = \frac{\tan(\beta - \gamma)}{\tan(\beta - \gamma) + 2} \) (3),

\[
\beta = \frac{\pi}{2} - \arctg \frac{1.08(\eta E_1 a b)}{\cos \gamma \sin \gamma}
\] (4),

where \( \eta = \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2} \) (5),

\( E_1, E_2 \) are elastic moduli of the processed and tool materials; \( \mu_1, \mu_2 \) are Poisson’s ratios; \( \gamma \) is the average rake angle of the tool with the rounding of the edge; \( a, b \) are the depth and width of the cutting.

Each of projections can be expressed through parameters of a cutting condition or a tool [7, 8].

To solve this problem it is not necessary to determine the exact value of the cutting force. It is enough to take it equal to one. The main thing in this case is to determine relations between projections of the resultant cutting force on coordinate axes, i.e. to specify components of the vector \( \zeta \) along which the resultant should pass. In our case it turned out that the minimum deformation is \( \varepsilon_2 \), therefore, components of the vector \( \zeta \) take the form:

\[
\xi_1 = 0; \quad \xi_2 = 0; \quad \xi_3 = 0
\] (6).

Let us determine values \( F_x, F_y \) and \( F_z \):

\[
F_x = \xi_1 \cdot \cos(l_1) + \xi_2 \cdot \cos(l_2) + \xi_3 \cdot \cos(l_3)
\]
\[
F_y = \xi_1 \cdot \cos(m_1) + \xi_2 \cdot \cos(m_2) + \xi_3 \cdot \cos(m_3)
\]
\[
F_z = \xi_1 \cdot \cos(n_1) + \xi_1 \cdot \cos(n_2) + \xi_1 \cdot \cos(n_3)
\] (7).
To solve the second problem, it is most convenient to take as an adjustable value the angle of flute helix of the milling cutter $\omega_0$, the depth $a$ and the width $b$ of the cutting.

To determine $\omega_0$ and $\beta$, we use expressions for $F_y$ and $F_z$ from equation (1) and found values of these components from equations (7):

$$
\cos\left(\frac{\pi}{4} + \frac{tg(\beta - \gamma)}{tg(\beta - \gamma) + 2 - \beta}\right) = F_x / \sin\omega_0 = F_x
$$

Solving this equation by numerical methods, for example, using the Levenberg-Marquardt algorithm, we obtain values $\omega_0$ and $\beta$. The latter is used to find the ratio between the quantities $a$ and $b$ from equation (4):

$$
\frac{\pi}{2} - \arctg\left[1.084\left(\frac{\eta E b}{a}\right)^{0.5} \right] = \beta
$$

This equation is solved by the same method.

### 3. Conclusions

Thus, we can propose the following method of controlling elastic displacements while cutting through the appointment of technological processing parameters:

1. Finite element modeling of the technological system is the elastic system of the machine, fixture, tool, and parts with the calculation of deformations.
2. Determination of the direction of axes rigidity [5].
3. Setting the direction of the unit vector of the resultant cutting force along the axis of maximum rigidity and the definition of projections of this vector on the axis of the machine (7).
4. Formation of analytical dependencies of cutting force projections through parameters of the cutting condition or the tool (1) – (5).
5. By solving these equations, we obtain necessary technological parameters (in this example, $\omega_0$, $a$ and $b$), which provide the necessary direction of cutting force (8), (9).

In this case a maximum approximation of the resultant cutting force to the axis of maximum rigidity is ensured, which causes a decrease in elastic displacements and an increase in the dynamic stability of the technological system.

### References

[1] Balabanov I P, Simonova L A and Balabanova O N 2015 Systematization of Accuracy Indices Variance when Modelling the Forming External Cylindrical Turning Process IOP Conference Series: Materials Science and Engineering (Bristol: IOP Publishing Ltd) vol. 86 p 012010

[2] Kasjanov S V, Kondrashov AG and Safarov DT 2017 Regulation of Geometrical Parameters Deviations of Automotive Components Parts through Diagnostic Measurements Organization Procedia Engineering (Elsevier) vol 206 pp 1508-14

[3] Kolesnikov K S, Aleksandrov D A, Astashev V K, etc.. 1994 Dynamics and Strength of Machines. Theory of Mechanisms and Machines. (Moscow: Mechanical Engineering) 534 p

[4] Timoshenko S P, Goodier J N. 1975 Theory of Elasticity. (Moscow: Science) 576 p

[5] Sabirov A R, Khusainov R M 2017 Calculation of the Direction of Deformation Axes in a Vertical Milling Machine Zone. Materials of the International Scientific and Technical Conference “Innovative Engineering Technology Equipment and Tools – 2017” (ITM-2017) 2 (Kazan : Foliant) p 228-32
[6] Grechishnikov V A, Petukhov Yu E, Pivkin P M, Romanov V B, Ryabov E A, Yurasov S Yu and Yurasova O I 2017 Trochoidal Slot Milling Russian Engineering Research. (New York: Allerton Press) Vol 37 No 9 pp 821-3

[7] Gruby V S, Zaitsev A M 2013 The Study of End Mills while Milling Aluminum-Alloy Body Parts. Science and Education No 12 P 31-54

[8] Golovko A N, Kondrashov A G and Yurasov S Yu 2017 Improved Design of a Worm Type Instrument for Final Machining of Evolvent Gear Teeth Procedia Engineering (Elsevier) Vol 206 pp 1333-6