Single production of the doubly charged scalar in the littlest Higgs model

Chong-Xing Yue, Shuang Zhao, Wei Ma

Department of Physics, Liaoning Normal University, Dalian 116029, China

(Dated: February 1, 2008)

Abstract

Single production of the doubly charged scalars $\Phi^{\pm\pm}$ via $e\gamma$, $ep$ and $pp$ collisions is studied in the context of the little Higgs (LH) model. Our numerical results show that the new particles $\Phi^{\pm\pm}$ can be abundantly produced and their possible signatures should be detected in future high energy linear $e^+e^-$ collider (ILC). The cross section for single production of $\Phi^{\pm\pm}$ at the LHC is much smaller than that at the ILC or the THERA.

PACS numbers: 11.30.Er, 12.60.Jv, 14.80.Cp

Keywords: doubly charged scale, little higgs, collider

*E-mail:cxyue@lnnu.edu.cn
I. Introduction

It is widely believed that the mechanism of electroweak symmetry breaking (EW SB) and the origin of the particle mass remain prominent mystery in current particle physics in spite of the success of the standard model (SM) tested by high energy experimental data. There has been no experimental evidence of the SM Higgs boson existing. Furthermore, the neutrino oscillation experiments have made one believe that neutrinos are massive, oscillate in flavor, which presently provides the only experimental hints of new physics (NP) [1]. Thus, the SM can only be an effective theory below some high energy scales. Other EW SB mechanisms and extended Higgs sectors have not been excluded in the theoretical point of view.

Doubly charged scalars arise in many new physics models with extended Higgs sectors, such as left-right symmetric models [2], Higgs triplet models [3], 3-3-1 models [4], and little Higgs models [5]. In these new physics models, doubly charged scalars appear typically as components of the SU(2) triplet representations, which do not couple to quarks and their couplings to leptons break the lepton number by two units. As a result, these new scalar particles have a distinct experimental signature, namely a same sign pair of leptons. Thus, discovery of a doubly charged scalar particle in future high energy colliders would be a definite signal of NP beyond the SM, which would help us to understand the Higgs sector and more importantly what lies beyond the SM.

Since charge conservation prevents doubly charged scalars from decaying to a pair of quarks and their Yukawa couplings violate lepton number conservation, this kind of new particles would have a distinct experimental signature, which has lower backgrounds. In addition, the presence of doubly charged scalars provides a simple explanation to the lightness of neutrinos via the see-saw mechanism [6], contents with recent data on neutrino oscillations [1]. This fact has lead to many studies involving production and decay of doubly charged scalars at present or future high energy collider experiments within specific popular models beyond the SM. For example, studies of production for doubly charged scalars have been given in the context of some specific popular models at hadron colliders [7,8] and different modes of operation of lepton colliders such as $e^+e^-$ mode [9], $e^-e^-$ mode [10], and $e\gamma$ or $\gamma\gamma$ mode [11].

Although there are lot of works on production and decays of doubly charged scalars in
the literature, it is need to be further studied in the context of the little Higgs models. There are several motivations to perform this study. First, little Higgs theory can be seen as one of the important candidates of the NP beyond the SM, most of the little Higgs models predict the existence of the doubly charged scalars. However, in previous works on studying the phenomenology of the little Higgs models, studies about doubly charged scalars are very little. Second, three types of colliders related to particle physics research seem to be promising in the next decade. Namely, they are the CERN Large Hadron Collider (LHC), the high energy linear $e^+e^-$ collider (ILC), and the linear-ring type $ep$ collider ($LC \otimes LHC$) called THERA. The phenomenology of the new physics models should be analyzed taking into account all three types of colliders. So far, a complete study on single production of the doubly charged scalars has not been presented in the context of the little Higgs models. Third, studying the possible signals of the doubly charged scalars in future high energy colliders can help the collider experiments to test little Higgs models, distinguish different NP models, and further to probe the production mechanism of the neutrino mass. Thus, in this paper, we will consider single production of the doubly charged scalars $\Phi^{\pm\pm}$ predicted by the littlest Higgs (LH) model [12] and see whether their signals can be detected in future three types of high energy colliders (ILC, THERA, and LHC).

There are several variations of the little Higgs models, which differ in the assumed higher symmetry and in the representations of the scalar multiples. In matter content, the LH model is the most economical little Higgs model discussed in the literature, which has almost all of the essential feature of the little Higgs models. The severe constraints on its free parameters coming from the electroweak precision data can be solved by introducing the T-parity [13]. Using of the fact that the LH model contains a complex triplet scalar $\Phi$, Refs.[14,15] have discussed the possibility to introduce lepton number violating interactions and generation of neutrino mass in the framework of the LH model. It has been shown [16] that the neutrino masses can be given by the term $\nu Y_{ij}$, in which $\nu$ is the vacuum expectation value (VEV) of the complex triplet scalar $\Phi$ and $Y_{ij}$ is its Yukawa coupling constant. As long as the VEV $\nu$ is restricted to be extremely small, the value of $Y_{ij}$ is of naturel order one. So, the doubly charged scalars $\Phi^{\pm\pm}$ predicted by the LH model might produce large contributions to some of lepton flavor violating (LFV) processes [17,18].

Using the current experimental upper limits on the branching ratios $Br(l_i \to l_j \gamma)$ and $Br(l_i \to l jl_k l_k)$, Ref.[18] obtains constraints on the relevant free parameters of the LH
model. Taking into account these constraints, we further consider the contributions of $\Phi^{\pm\pm}$ to the LFV processes $e^+e^+ \rightarrow l_i^+l_j^+$ and $e^+e^- \rightarrow l_i^+l_j^+$ ($l_i$ or $l_j \neq e$) in Ref.[18]. Based on Ref.[18], in this paper, we will discuss all possible processes for single production of the doubly charged scalars $\Phi^{\pm\pm}$ in the future high energy collider experiments. The rest of this paper is organized as follows. The relevant couplings of the doubly charged scalars $\Phi^{\pm\pm}$ are summarized in Section II. Sections III, IV, and V are devoted to the computation of the cross sections for single production of the doubly charged scalars $\Phi^{\pm\pm}$ at the ILC, THERA, and LHC, respectively. Some phenomenological analysis are also included in the three sections. Our conclusions and simple discussions are given in section VI.

II. The relevant couplings of the doubly charged scalars $\Phi^{\pm\pm}$

The LH model [3] consists of a non-linear $\sigma$ model with a global $SU(5)$ symmetry and a locally gauged symmetry $[SU(2) \times U(1)]^2$. The global $SU(5)$ symmetry is broken down to its subgroup $SO(5)$ at a scale $f \sim TeV$, which results in 14 Goldstone bosons (GB's). Four of these GB's are eaten by the new gauge bosons ($W_H^\pm$, $Z_H$, $B_H$), resulting from the breaking of $[SU(2) \times U(1)]^2$, giving them masses. The Higgs boson remains as a light pseudo-Goldstone boson, and other GB's give masses to the SM gauge bosons and form the complex triplet scalar $\Phi$. Thus, the remaining ten GB's can be parameterized as:

$$\Pi = \begin{pmatrix} 0 & \frac{H^+}{\sqrt{2}} & \Phi^+ \\ \frac{H^-}{\sqrt{2}} & 0 & \frac{H^0}{\sqrt{2}} \\ \Phi & \frac{H^0}{\sqrt{2}} & 0 \end{pmatrix}$$

with

$$\Phi = \begin{pmatrix} \Phi^{++} \\ \frac{\Phi^+}{\sqrt{2}} \\ \frac{\Phi^0}{\sqrt{2}} \end{pmatrix}.$$  

(1)

Where $H$ is the SM Higgs doublet, $\Phi^{++, \pm, 0}$, and $\Phi^p$ are the components of the complex triplet scalar $\Phi$, which are degenerate at lowest order with a common mass $M_\Phi$. The couplings of the doubly charged scalar $\Phi^{--}$ to other particles, which are related to our calculation, can be written as [15,19]:

$$\Phi^{--}W_\mu^+W_\nu^+ : 2i \frac{e^2}{\left(\frac{\sqrt{2}}{2}g_{\mu\nu}\right)} g' / g_{\mu\nu},$$  

(2)

$$\Phi^{--}W_\mu^+W_\nu^+ : 2i \frac{e^2}{\left(\frac{\sqrt{2}}{2}g_{\mu\nu}\right)} (c^4 + s^4) / g_{\mu\nu},$$  

(3)

$$\Phi^{--}W_\mu^+W_\nu^+ : 2i \frac{e^2}{\left(\frac{\sqrt{2}}{2}g_{\mu\nu}\right)} (c^2 - s^2) / g_{\mu\nu},$$  

(4)

$$\gamma\Phi^{++}\Phi^{--} : -2ie(p_1 - p_2)_\mu,$$  

(5)
\[ \Phi^{-}l_i^+l_j^+ : 2iY_{ij}^+ C P_R, \]  
\[ \Phi^{-}W^+_\mu \Phi^+ : -i \frac{e}{S_W} (p_1 - p_2)_\mu, \]  
\[ \Phi^{-}\Phi^+W^+_{R\mu} : i \frac{e}{S_W} \frac{(c^2 - s^2)}{2sc} (p_1 - p_2)_\mu. \]

Where \( S_W = \sin \theta_W \), \( \theta_W \) is the Weinberg angle. The free parameter \( c \) (\( s = \sqrt{1 - c^2} \)) is the mixing parameter between \( SU(2)_1 \) and \( SU(2)_2 \) gauge bosons. \( P_R = (1 + \gamma_5)/2 \) is the right-handed projection operator and \( C \) is the charge conjugation operator. The indices \( i \) and \( j \) represent three generation leptons \( e, \mu, \text{ or } \tau \). Eq.(6) gives the flavor-diagonal\( (FD) \) couplings and the flavor-mixing\( (FX) \) couplings of the doubly charged scalar \( \Phi^{-} \) to leptons for \( i = j \) and \( i \neq j \), respectively.

In the \( LH \) model, the neutrino mass matrix can be written as \( M_{ij} = Y_{ij}^\nu \nu' \) [15,16]. Considering the current bounds on the neutrino mass [19], we should have \( Y_{ij}^\nu \nu' \sim 10^{-10}\text{GeV} \). If we assume that the value of the triplet scalar \( VEV \nu' \) is smaller than \( 1 \times 10^{-5}\text{GeV} \), then there is the Yukawa coupling constant \( Y_{ij} > 1 \times 10^{-5} \), which does not conflict with the most severe constraints on the \( LH \) model from the \( LFV \) process \( \mu \rightarrow eee \) [18]. In this case, the decay channel \( \Phi^{-} \rightarrow W^-W^- \) can be neglected. As a result, the total decay width \( \Gamma_\Phi \) of the doubly charged scalar \( \Phi^{-} \) can be approximately written as [15,18]:

\[ \Gamma_\Phi \approx \frac{3M_\Phi Y_{ii}^2}{8\pi}. \]

In the following sections, we will assume \( Y_{ij} > 1 \times 10^{-5}\text{GeV} \), which might produce large contributions to some \( LFV \) processes, and take \( M_\Phi \) and \( Y_{ij} \) as free parameters to calculate the single production cross sections of the doubly charged scalar \( \Phi^{-} \) in future three types of high energy collider experiments.

### III. Single production of \( \Phi^{-} \) at the \( ILC \)

It is widely believed that the hadron colliders, such as \( Tevatron \) and \( LHC \), can directly probe possible \( NP \) beyond the \( SM \) up to a few \( \text{TeV} \), while the \( \text{TeV} \) energy linear \( e^+e^- \) collider\( (ILC) \) is also need to complement the probe of the new particles with detailed measurement [21]. An unique feature of the \( ILC \) is that it can be transformed to \( \gamma\gamma \) or \( e\gamma \) collision with the photon beam generated by laser-scattering method. The effective luminosity and energy of the \( \gamma\gamma \) and \( e\gamma \) collisions are expected to be comparable to those of the \( ILC \).
In some scenarios, they are the best instrument for discovery of the NP signatures. The $e\gamma$ collision can produce particles, which are kinematically not accessible in the $e^+e^-$ collision at the same collider. For example, for the process $e\gamma \to AB$ with light particle $A$ and new particle $B$, the discovery limits can be much higher than in other reactions.

From discussions given in section II, we can see that the doubly charged scalar $\Phi^{--}$ can be produced via $e^-\gamma$ collision associated with a lepton. The relevant Feynman diagrams are shown in Fig.1, in which $l^+$ is the lepton $e^+$, $\mu^+$, or $\tau^+$.

For the process $e^-(P_1) + \gamma(k_1) \to l^+(P_2) + \Phi^{--}(k_2)$, the renormalization amplitude can be written as:

$$
M = -\frac{eY_{el}}{(P_1 + k_1)^2} v^T(P_2)C^{-1}P_R\gamma\gamma\mu u(P_1)\varepsilon_\mu(k_1) - \frac{2eY_{el}}{(k_1 - k_2)^2 - M_\Phi^2} v^T(P_2)C^{-1}P_Ru(P_1)(2k_2 - k_1)\varepsilon_\mu(k_1) - \frac{eY_{el}}{(P_1 - k_2)^2} v^T(P_2)\gamma\gamma\mu(P_1 - k_2)C^{-1}P_Ru(P_1)\varepsilon_\mu(k_1) \quad (10)
$$

After calculating the cross section $\hat{\sigma}(\hat{s})$ for the subprocess $e^-\gamma \to l^+\Phi^{--}$, the effective cross section $\sigma_1(s)$ at the ILC with the center-of-mass $\sqrt{s} = 2\text{TeV}$ can be obtained by folding the cross section $\hat{\sigma}_1(\hat{s})$ with the photon distribution function $f_{\gamma/e}[22]$:

$$
\sigma_1(s) = \int_{M_\Phi^2/s}^{0.83} dx \hat{\sigma}_1(\hat{s}) f_{\gamma/e}(x), \quad (11)
$$

where $x = \hat{s}/s$, in which $\sqrt{\hat{s}}$ is the center-of-mass energy of the subprocess $e^-\gamma \to l^+\Phi^{--}$.

The effective cross section $\sigma_1(s)$ for the process $e^-\gamma \to e^+\Phi^{--}$ is plotted in Fig.2 as a function of the FD coupling constant $Y = Y_{ee}$ for $\sqrt{\hat{s}} = 2\text{TeV}$ and three values of the $\Phi^{--}$ mass $M_\Phi$. From this figure, one can see that the doubly charged scalar $\Phi^{--}$ can...
be abundantly produced via $e\gamma$ collision at the ILC experiment with $\sqrt{s} = 2\text{TeV}$. For example, for $0.1 \leq Y \leq 0.9$ and $800\text{GeV} \leq M_\Phi \leq 1600\text{GeV}$, the value of $\sigma_1(s)$ is in the range of $42.8\text{fb} \sim 3474\text{fb}$. If we assume that the future ILC experiment with $\sqrt{s} = 2\text{TeV}$ has a yearly integrated luminosity of $L = 500\text{fb}^{-1}$, then there will be ten thousands up to several millions of the doubly charged scalar $\Phi^{--}$ associated with a positive electron $e^+$ to be generated per year.

In general, the doubly charged scalar $\Phi^{--}$ can decay to the modes $\Phi^-\Phi^-, W^-\Phi^-, W^-W^-$ and $l^-_i l^-_j$. However, degeneracy among the components of the triplet scalar $\Phi$ forbids appearance of the first two modes. Furthermore, the decay width $\Gamma(\Phi^{--} \rightarrow W^-W^-)$ is controlled by the triplet scalar VEV $\nu'$. It has been shown that, for $\nu' < 1 \times 10^{-5}\text{GeV}$ and $M_\Phi < 2000\text{GeV}$, $\Phi^{--}$ mainly decays to $l^-_i l^-_j$[10]. In this case, the signatures of $\Phi^{--}$ is the same-sign lepton pair, including lepton-number violating final states, which is the SM background free and has been investigated in Ref.[11]. Thus, as long as the doubly charged scalar $\Phi^{--}$ is not too heavy, its possible signals should be detected in future ILC experiments.

The doubly charged scalar $\Phi^{--}$ can also be singly produced via the LFV processes $e^-\gamma \rightarrow l^+\Phi^{--}(l \neq e)$ induced by the lepton number violating interactions. However, the experimental upper limits on the LFV processes $\tau \rightarrow eee$ and $\mu \rightarrow eee$ can give severe constraints on the combination $|Y_{ii}Y_{ij}|^2/M_\Phi^4(i \neq j)$ [18], which make that the production
cross sections of the LFV processes $e^{-}\gamma \rightarrow l^{+}\Phi^{-}\Phi^{+} (l \neq e)$ are very small. For example, for $Y = 0.1$ and $M_{\Phi} = 1000\text{GeV}$, the effective cross section for the process $e^{-}\gamma \rightarrow \mu^{+}\Phi^{--}$ is smaller than $6.9 \times 10^{-4} \text{fb}$. Thus, it is very difficult to detect the signals of $\Phi^{--}$ via the LFV processes $e^{-}\gamma \rightarrow l^{+}\Phi^{-\Phi^{+}} (l \neq e)$ in future ILC experiments.

**IV. Single production of the doubly charged scalar $\Phi^{--}$ at the THERA**

Although the linear-ring-type $ ep $ collider ($ LC \otimes LHC $) with the center-of-mass energy $ \sqrt{s} = 3.7\text{TeV} $ and with the integral luminosity $ \mathcal{L} = 100\text{pb}^{-1} $ has a lower luminosity, it can provide better condition for studying a lot of phenomena, compared to the ILC due to the high center-of-mass energy and compared to the LHC due to clearer environment [23]. Thus, it can be used to detect the possible signals of some new particles. In this section, we consider single production of the doubly charged scalar $\Phi^{--}$ predicted by the LH model via $ ep $ collision at the THERA.

The doubly charged scalar $\Phi^{--}$ can be singly produced at the THERA via the processes:

\begin{equation}
    e^{-} p \rightarrow e^{+} X \Phi^{--}, \quad e^{-} p \rightarrow \mu^{+} X \Phi^{--}, \quad e^{-} p \rightarrow \tau^{+} X \Phi^{--},
\end{equation}

which were first studied by Ref.[24]. In terms of observability, the second and third processes are induced by the lepton number violating interactions. The corresponding subprocesses can be unitively written as $ e^{-}\gamma \rightarrow l^{+}\Phi^{--} (l = e, \mu \text{ or } \tau) $. The relevant Feynman diagrams are plotted in Fig.3. Since the production cross section for the process $ e^{-}\gamma \rightarrow \mu^{+}\Phi^{--} $ or $ e^{-}\gamma \rightarrow \tau^{+}\Phi^{--} $ is much smaller than that of the process $ e^{-}\gamma \rightarrow e^{+}\Phi^{--} $, we will use the equivalent photon approximation(\textit{EPA}) approach [25,26] to only consider the process $ e^{-} p \rightarrow e^{+} X \Phi^{--} $ in this section.

Using the \textit{EPA} method, the effective cross section $ \sigma_{2}(s) $ at the THERA with $ \sqrt{s} = 3.7\text{TeV} $ and $ \mathcal{L} = 100\text{pb}^{-1} $ can be folding the cross section $ \tilde{\sigma}_{2}(\tilde{s}) $ for the subprocess $ e^{-}\gamma \rightarrow e^{+}\Phi^{--} $ with the photon distribution function $ f_{\gamma/p} (x, \tilde{s}) $:

\begin{equation}
    \sigma_{2}(s) = \int_{M_{\Phi}^{2}/s}^{(1-m/\sqrt{s})^{2}} dx \tilde{\sigma}(xs) f_{\gamma/p} (x, xs),
\end{equation}

with $ x = \tilde{s}/s $, $ m $ is the proton mass and

\begin{equation}
    f_{\gamma/p} (x, xs) = f_{\gamma/p}^{el} (x) + f_{\gamma/p}^{inel} (x, xs).
\end{equation}
Where $f_{el}^{\gamma/p}(x)$ and $f_{in}^{\gamma/p}(x, xs)$ are the elastic and inelastic components of the equivalent photon distribution of the proton, which has been extensively studied in Refs.[25, 26, 27].

The effective cross section $\sigma_2(s)$ of the process $e^- p \to e^+ X \Phi^{--}$ is plotted in Fig.4 as a function of the FD Yukawa coupling constant $Y$ for $\sqrt{s} = 3.7$TeV and three values of the mass $M_\Phi$. One can see from Fig.4 that the cross section for single production of the doubly charged scalar $\Phi^{--}$ at the THERA is generally smaller than that at the ILC in most of the parameter space of the LH model. For $0.1 \leq Y \leq 0.9$ and $800GeV \leq M_\Phi \leq 1600GeV$, the value of the production cross section $\sigma_2(s)$ is in the range of $1.11 \times 10^2 fb \sim 5.9 \times 10^{-2} fb$. There will be several tens of the doubly charged scalar $\Phi^{--}$ to be generated per year at the THERA with $\sqrt{s} = 3.7$TeV and $\mathcal{L} = 100pb^{-1}$.

Similar to single production of $\Phi^{--}$ at the ILC, the process $e^- p \to e^+ X \Phi^{--}$ gives rise to
number of signal events with same-sign lepton pair and an isolated positive electron in the $l^-l^-+e^+$ signature, which is almost free of the SM backgrounds. Thus, possible signatures of the doubly charged scalar $\Phi^{--}$ might be detected in future THERA experiments.

V. Single production of the doubly charged scalar $\Phi^{--}$ at the LHC

In this year, the LHC with $\sqrt{s} = 14$ TeV and $\mathcal{L} = 100$ fb$^{-1}$ will begin operation, which has an increase of a factor of seven in energy and a factor of 100 in luminosity over the Fermilab Tevatron. The LHC is expected to directly probe possible NP beyond the SM up to a few TeV, which might provide some striking evidence of NP.

From Eqs. (2) — (8) given in section II, we can see that the doubly charged scalar $\Phi^{--}$ can be singly produced at the LHC via the partonic processes:

\begin{align*}
\bar{q}q' &\rightarrow W^-\bar{s}, \ W^-\bar{s} \rightarrow W^+\Phi^{--}; \quad (15) \\
nq &\rightarrow W^-\bar{s}W^-\bar{s}q'q' \rightarrow \Phi^{--}q'q'; \quad (16) \\
\bar{q}q' &\rightarrow W_H^-\bar{s}, \ W^-\bar{s} \rightarrow W_H^+\Phi^{--}; \quad (17) \\
\bar{q}q' &\rightarrow W^-\bar{s}, \ W_H^-\bar{s} \rightarrow \Phi^+\Phi^{--} \ (q, q' = u, d, c, or s). \quad (18)
\end{align*}

However, the production cross sections of the partonic processes (15), (16), and (17) are strongly suppressed by the factor $\nu'^2 (\nu' \leq 1 \times 10^{-5}$ GeV). All of their values are smaller than $1 \times 10^{-8} fb$. So, in this section, we only consider the partonic processes $\bar{q}q' \rightarrow W^-\bar{s}, W_H^-\bar{s} \rightarrow \Phi^+\Phi^{--}$, as shown in Fig. 5.

Using Eq.(7), Eq.(8), and other relevant Feynman rules, we can write the scattering
amplitude for the partonic process $\bar{q}(P_1) + q'(P_2) \rightarrow \Phi^+(P_3) + \Phi^{--}(P_4)$:

$$M = -\frac{e^2}{\sqrt{2} s_{\nu W}} |V_{qq'}^{SM}| |\bar{q}(P_1) P_L \gamma^\mu u(P_2) g_{\mu\nu} (P_1 + P_2)^2 - M_W^2 | (P_3 - P_4)\nu$$

$$- \frac{ie^2 (c^2 - s^2)}{2\sqrt{2} s_{\nu W} s^2} |V_{qq'}^{SM}| |\bar{q}(P_1) P_L \gamma^\mu u(P_2) g_{\mu\nu} (P_1 + P_2)^2 - M_W^2 | (P_3 - P_4)\nu. \quad (19)$$

The cross section $\sigma_3(s)$ for single production of the doubly charged scalar $\Phi^{--}$ associated with a singly charged scalar $\Phi^+$ at the LHC with $\sqrt{s} = 14\text{TeV}$ can be obtained by convoluting the production cross section $\hat{\sigma}_3(\hat{s})$ of the partonic process $\bar{q}q' \rightarrow \Phi^+\Phi^{--}$ with the quark distribution functions (PDF's):

$$\sigma_3(s) = \sum_{ij} \int_r^1 dx_1 \int_{r/x_1}^1 dx_2 f_i(x_1, \mu) f_j(x_2, \mu) \hat{\sigma}(ij \rightarrow \Phi^+\Phi^{--}). \quad (20)$$

Where $i$ and $j$ stand for the partons (light quarks u, d, c, or s), $\tau = 4M_\Phi^2/s$ and $\hat{s} = x_1 x_2 s$ (We have assumed $M_\Phi = M_{\Phi^{--}} = M_{\Phi^+}$). In our numerical calculation, we will use CTEQ6L PDF's [28] for the quark distribution functions and assume that the factorization scale is of order $M_\Phi/2$.

From Eqs.(19) and (20), we can see that the production cross section $\sigma_3(s)$ is dependent on the free parameters $M_\Phi$, $M_{W_H}$, and $c$. It is well known that global fits to the electroweak precision data impose rather severe constraints on the free parameters of the LH model. However, if the SM fermions are charged under $U(1)_1 \times U(1)_2$, the constraints can become relaxed [29]. As numerical estimation, we will assume that the free parameters $M_\Phi$, $M_{W_H}$, and $c$ are in the ranges of $800\text{GeV} \sim 1600\text{GeV}$, $1\text{TeV} \sim 2\text{TeV}$, and $0.1 \sim 0.5$, respectively.

Our numerical results are summarized in Fig.6, in which we plot the production cross section $\sigma_3(s)$ as a function of the mass $M_\Phi$ for different values of the free parameter $M_{W_H}$. Since the contributions of the LH model to the process $pp \rightarrow \Phi^+\Phi^{--}X$ mainly come from $W$ exchange, the production cross section $\sigma_3(s)$ is insensitive to the free parameter $c$. So, we have taken $c = 0.3$ in Fig.6. One can see from Fig.6 that the cross section for single production of the doubly charged scalar $\Phi^{--}$ at the LHC is much smaller than that at the ILC or the THERA. This is because, at tree level, the doubly charged scalar $\Phi^{--}$ is singly produced at the LHC via the s-channel processes with highly virtual gauge boson propagators. For $800\text{GeV} \leq M_\Phi \leq 1600\text{GeV}$, $1\text{TeV} \leq M_{W_H} \leq 2\text{TeV}$, and $c = 0.3$, the value of $\sigma_3(s)$ is in the range of $7.4 \times 10^{-1} fb \sim 1.3 \times 10^{-4} fb$. Then, there will be several tens $\Phi^+\Phi^{--}$ events to be generated one year at the LHC with $\sqrt{s} = 14\text{TeV}$ and $L = 100 fb^{-1}$. \[11\]
FIG. 6: The cross section $\sigma_3(s)$ as a function of the scalar $\Phi$ mass $M_\Phi$ for three values of the mass $M_{W_H}$.

The possible decay modes of the singly charged scalar $\Phi^+$ are $l^+\bar{\nu}_l$, $t\bar{b}$, $T\bar{T}$, $W^+Z$, and $W^+H$[15]. The decay widths $\Gamma(\Phi^+ \to W^+Z)$ and $\Gamma(\Phi^+ \to W^+H)$ are proportional to the factor $v^2$, which can be neglected. If we assume that the scalar $\Phi^+$ mass $M_\Phi$ is smaller than the vector-like top quark mass $M_T (M_\Phi \leq M_T)$, then the decay channel $\Phi^+ \to T\bar{T}$ is kinematically forbidden. In this case, the singly charged scalar $\Phi^+$ mainly decays into $l^+\bar{\nu}_l$ and $t\bar{b}$. If the singly charged scalar $\Phi^+$ decays into $l^+\bar{\nu}_l$, then the production of the doubly charged scalar $\Phi^{--}$ associated with a singly charged scalar $\Phi^+$ at the LHC gives rise to a number of signal events with like-sign di-leptons, with one jet and large missing energy, $l^-l^-+E+jet$, which has a relatively high detection efficiency and enjoys essentially negligible background from SM processes. The production rate of the signal event $l^-l^-+E+jet$ is shown in Fig.7 as a function of the $FD$ Yukawa coupling constant $Y$ for $f = 1$TeV and $M_\Phi = 1$TeV. One can see from Fig.7 that, for $0.1 \leq Y \leq 0.9$ and $1$TeV $\leq M_{W_H} \leq 2$TeV, the production rate of the signal event is in the range of $6.7 \times 10^{-2} fb \sim 1.4 \times 10^{-1} fb$. With the high luminosity option of the LHC, around $300 fb^{-1}$, there will be several tens signal events to be generated one year.

If we assume that the singly charged scalar $\Phi^+$ decays into $t\bar{b}$, then the partonic process $qq' \to \Phi^+\Phi^{--}$ will generate the $l^-l^-+E+jets$ signature, which is similar to that coming from production of the first or second generation T-odd quark pair in the context of the LH model with T-parity[30]. However, the production rate of this kind of signal events is too
FIG. 7: Production rate of the signal event $l^-l^- + E + jet$ as a function of the $FD$ coupling constant $Y$ for three values of the $W_H$ mass $M_{W_H}$.

small to be detected in future $LHC$ experiments.

VI. Conclusions and discussions

Doubly charged scalars appear in some popular $NP$ models beyond the $SM$, motivated by their usefulness in generating neutrino masses. This kind of new particles have distinct experimental signals through their decay to same-sign lepton pairs. Their observation in future high energy collider experiments would be a clear evidence of $NP$ beyond the $SM$. Thus, searching for doubly charged scalars is one of the main goals of future high energy collider experiments.

Little Higgs models have generated much interest as possible alternative to the $EWSB$ mechanism, which can be regarded as one of the important candidates of $NP$ beyond the $SM$. As the most economical little Higgs model, the $LH$ model can explain the observed neutrino mass by introducing the lepton number violating interaction of the triplet scalar to leptons. The neutrino mass is proportional to the triplet scalar $VEV\,\nu'$ multiplied by the Yukawa coupling constant $Y_{ij}$ without invoking a right-handed neutrino. This scenario predicts the existence of the doubly charged scalars $\Phi^{\pm\pm}$. In this paper, we consider single production of this kind of new particles in future three types of high energy collider experiments($ILC$, $THERA$, and $LHC$).
In future $ILC$ experiments, the doubly charged scalar $\Phi^{--}$ can be singly produced via $e^-\gamma$ collision. Our numerical results show that it can be abundantly produced in future $ILC$ experiments. In most of the parameter space of the $LH$ model, there will be a large number of the signal events with same-sign lepton pair and an isolated positive electron to be generated in the $ILC$ experiment with $\sqrt{s} = 2\text{TeV}$ and $\mathcal{L} = 100\text{fb}^{-1}$. Thus, the possible signals of the doubly charged scalar $\Phi^{--}$ should be detected in future $ILC$ experiments.

Using the $EPA$ method, we calculate the cross section for single production of the doubly charged scalar $\Phi^{--}$ at the $\text{THERA}$ with $\sqrt{s} = 3.7\text{TeV}$ and $\mathcal{L} = 100\text{pb}^{-1}$ via the subprocess $e^-\gamma \rightarrow e^+\Phi^{--}$. In our numerical estimation, we have included both contributions of the elastic photon and of the inelastic photon components from proton. We find that, for $0.1 \leq Y \leq 0.9$ and $800\text{GeV} \leq M_{\Phi} \leq 1000\text{GeV}$, the value of the production cross section $\sigma_2$ is in the range of $1.11 \times 10^2\text{fb} \sim 5.9 \times 10^{-2}\text{fb}$. As long as the doubly charged scalar $\Phi^{--}$ is not too heavy, its possible signatures might be detected via the process $e^-p \rightarrow e^+\Phi^{--}X$ in future $\text{THERA}$ experiments.

At the $LHC$, the doubly charged scalar $\Phi^{--}$ can be singly produced via several patronic processes. However, considering that the triplet scalar $VEV$ $\nu'$ is very small ($\nu' \leq 1 \times 10^{-5}\text{GeV}$), the production channels involving the gauge bosons $W$ or $W_H$ in the final states can be neglected. Thus, in this paper, we only consider the production of the doubly charged scalar $\Phi^{--}$ associated with a single charged scalar $\Phi^+$ via the patronic processes $\overline{q}q' \rightarrow W^{-*}, W_H^{-*} \rightarrow \Phi^+\Phi^{--}$ at the $LHC$. Since the partonic process $\overline{q}q' \rightarrow \Phi^+\Phi^{--}$ proceeds by the s-channel $W$ exchange and $W_H$ exchange, which have highly virtual gauge boson propagators, its production cross section is much smaller than that at the $ILC$ or the $\text{THERA}$. However, the partonic process $\overline{q}q' \rightarrow \Phi^+\Phi^{--}$ with $\Phi^+ \rightarrow l^+\nu_l$ and $\Phi^{--} \rightarrow l^-l^-$ can produce distinct experimental signature, which is almost free of the $SM$ background. If the mass of $\Phi^{--}$ is smaller than 1TeV, its possible signals might be detected at the high luminosity option of the $LHC$ experiments.

At the $ILC$, the doubly charged scalar $\Phi^{--}$ can also be produced associated with a photon via $e^-e^-$ collision, i.e. the subprocess $e^-e^- \rightarrow \Phi^{--}\gamma$. The cleanliness of the final state photon detection in future $e^-e^-$ collider can be very helpful in identifying the doubly charged scalar. However, this production process suffers from a large $SM$ background process $e^- + e^- \rightarrow \gamma + e^- + e^-$, which has to be carefully disposed by suitable technique [10].
Certainly, the doubly charged scalar $\Phi^{++}$ can also be singly produced at the *ILC*, the *THERA*, and the *LHC* via the charge-conjugation processes of the corresponding processes for the doubly charged scalar $\Phi^{--}$. Similar to above calculation, we can give the values of the production cross sections for these processes. Thus, the conclusions for the doubly charged scalar $\Phi^{--}$ also apply to the doubly charged scalar $\Phi^{++}$.

The *LH* model with T-parity also predicts the existence of the doubly charged scalars $\Phi^{\pm\pm}$. While they have T-odd parity and their $VEV$ is equal to zero, which can not generate the neutrino masses via introducing the lepton number violating interaction of the T-odd triplet scalar $\Phi$ to leptons. However, they can also be produced associated a T-odd new particle in future high energy collider experiments, which might be need to be further studied.

**Acknowledgments**

This work was supported in part by Program for New Century Excellent Talents in University(NCET-04-0290), the National Natural Science Foundation of China under the Grants No.10475037 and 10675057.

---

[1] M. C. Gonzalez-Garcia and Y. Nir, *Rev. Mod. Phys.* **75**, 345(2003); V. Barger, D. Marfatia, and K. Whisnant, *Int. J. Mod. Phys* E**12**, 569(2003); A. Y. Smimov, *hep-ph*/0402264.

[2] J. C. Pati and A. Salam, *Phys. Rev. D* **10**, 275(1975); R. N. Mohapatra and J. C. Pati, *Phys. Rev. D* **11**, 566(1975); G. Senjanovic and R. N. Mohapatra, *Phys. Rev. D* **12**, 1502(1975); T. G. Rizzo, *Phys. Rev. D* **25**, 1355(1982).

[3] G. B. Gelmini, M. Roncadelli, *Phys. Lett. B* **99**, 411(1981); H. Georgi and M. Machacek, *Nucl. Phys. B* **262**, 463(1985); J. F. Gunion, R. Vega, and J. Wudka, *Phys. Rev. D* **42**, 1673(1990).

[4] F. Pisano and V. Pleitez, *Phys. Rev. D* **46**, 410(1992); P. H. Frampton, *Phys. Rev. Lett.* **69**, 2889(1992); R. Foot, O. F. Hernandez, F. Pisano and V. Pleitez, *Phys. Rev. D* **47**, 4158(1993).

[5] For recent review see: M. Schmaltz and D. Tucker-Smith, *Ann. Rev. Nucl. Parti. Sci.* **55**, 229(2005); T. Han, H. E. Logan, and L. T. Wang, *JHEP* **0601**, 099(2006).

[6] H. Fritzsch, M. Gell-Mann, and P. Minkowski, *Phys. Lett. B* **59**, 256(1975); T. P. Cheng, *Phys. Rev. D* **14**, 1367(1976); M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity,*
Proceedings of the Workshop, Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; T. Yanagida, in Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan, edited by O. Sawada and A. sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; S. Weinberg, Phys. Rev. Lett. 43, 1566(1979); R. N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44, 912(1980).

[7] K. Huitu, J. Maalampi, A. Pietila, M. Raidal, Nucl. Phys. B487, 27(1997); A. Datta, A. Raychaudhuri, Phys. Rev. D62, 055002(2000); J. Maalampi, N. Romanenko, Phys. Lett. B 532, 202(2002); M. Muhlleitner, M. Spira, Phys. Rev. D68, 117701(2003).

[8] A. G. Akeroyd and M. Aoki, Phys. Rev. D72, 035011(2005); G. Azuelos, K. Benslama and J. Ferland, J. Phys. G32, 73(2006); J. E. Cieza Montalvo, N. V. Cortez Jr, J. Sá Borges and M. D. Tonasse, Nucl. Phys. B756, 1(2006); A. Hektor, M. Kadastik, M. Muntel, M. Raidal, L. Rebane, [arXiv:0705.1495[hep-ph]].

[9] S. Godfrey, P. Kalyniak, N. Romanenko, Phys. Lett. B545, 361(2002); E. J. Chun, K. Y. Lee, S. C. Park, Phys. Lett. B 566, 142(2003).

[10] F. Cuypers, M. Raidal, Nucl. Phys. B501, 3(1997); G. Barenboim, K. Huitu, J. Maalampi, M. Raidal, Phys. Lett. B394, 132(1997); M. Raidal, Phys. Rev. D57, 2013(1998); J. Maalampi, N. Romanenko, Phys. Lett. B474, 347(2000); B. Mukhopadhyaya, S. K. Rai, Phys. Lett. B633, 519(2006); O. Cakir, New J. Phys. 8, 145(2006)

[11] S. Chakrabarti, D. Choudhury, R. M. Godbole, B. Mukhopadhyaya, Phys. Lett. B434, 347(1998); E. M. Gregores, A. Gusso, S. F. Novaes, Phys. Rev. D64, 015004(2001); S. Godfrey, P. Kalyniak, N. Romanenko, Phys. Rev. D65, 033009(2002).

[12] N. Arkani-Hamed, A. G. Cohen, E. Katz, A. E. Nelson, JHEP 0207, 034(2002).

[13] H. C. Cheng and I. Low, JHEP 0309, 051(2003); JHEP 0408, 061(2004); I. Low, JHEP 0410, 067(2004)

[14] W. Kilian and J. Reuter, Phys. Rev. D70, 015004(2004); J. Y. Lee, JHEP 0506, 060(2005).

[15] T. Han, H. E. Logan, B. Mukhopadhyaya and R. Srikant, Phys. Rev. D72, 053007(2005);

[16] J. Schechter, J. W. F. Valle, Phys. Rev. D22, 2227(1980); G. B. Gelmini, M. Roncadelli, Phys. Lett. B99, 411(1981).

[17] S. R. Choudhury, N. Gaur, and A. Goyal, Phys. Rev. D72, 097702(2005); J. Goyal, Mod. Phys. Lett. A21, 1931(2006).
[18] Chong-Xing Yue and Shuang Zhao, Eur. Phys. J. C50, 897(2007).
[19] T. Han, H. E. Logan, B. M. Elrath, and L. -T. Wang, Phys. Rev. D 67, 095004(2003).
[20] G. L. Fogli et al, Phys. Rev. D70, 113003(2004); M. Tegmark et al. [SDSS Collaboration], Phys. Rev. D 69, 103501(2004); O. Elgaroy and O. Lahav, New J. Phys. 7, 61(2005).
[21] T. Abe et al. [American Linear Collider Group], hep-ex/0106057; J. A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group], hep-ph/0106315; K. Abe et al. [ACFA Linear Collider Working Group], hep-ph/0109166; G. Laow et al., ILC Technical Review Committee, second report, 2003, SLAC-R-606.
[22] I. F. Ginzbury et al., Nucl. Instrum. Meth. Phys. Res. Sec A 21, 5(1984); V. I. Telnov, Nucl. Instrum. Meth. Phys. Res. Sec A 294, 72(1990).
[23] S. Sultansoy, Eur. Phys. J. C33, s1064(2004), and reference therein.
[24] E. Accomando and S. Petrarca, Phys. Lett. B323, 212(1994).
[25] C. F. Von Weizsäcker, Z. Phys. 88, 612(1934); E. J. Williams, Phys. Rev. 45, 729(L)(1934).
[26] M. Glück, C. Pisano and E. Reya, Phys. Lett. B540, 75(2002); C. Pisano, Eur. Phys. J. C38, 79(2004), and reference therein.
[27] G. Altarelli, G. Martinelli, B. Mele and R. Rückl, Nucl. Phys. B262, 204(1985); B. A. Kniehl, Phys. Lett. B254, 267(1991); A. Mukherjee and C. Pisano, Eur. Phys. J. C30, 477(2003).
[28] J. Pumplin et al., JHEP 0207, 012(2002); D. Stump et al., JHEP 0310, 046(2003).
[29] J. L. Hewett, F. J. Petriello and T. G. Rizzo, JHEP 0310 062(2003); C. Csaki, J. Hubisz, G. D. Kribs, P. Meade and J. Terning, Phys. Rev. D67, 115002(2003); C. Csaki et al., Phys. Rev. D68, 035009(2003); R. Casalbuoni, A. Deandrea and M. Oertel, JHEP 0402, 032(2004); M. C. Chen and S. Dawson, Phys. Rev. D70, 015003(2004); Chong-Xing Yue and Wei Wang, Nucl. Phys. B683, 48(2004); W. Kilian and J. Reuter, Phys. Rev. D70, 015004(2004); T. Gregoire, D. R. Smith and J. G. Wacker, Phys. Rev. D69, 115(2004).
[30] A. Belyaev, C. -R. Chen, K. Tobe, C. -P. Yuan, Phys. Rev. D74, 11502(2006).
