Wellbore instability management using geomechanical modeling and wellbore stability analysis for Zubair shale formation in Southern Iraq

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Received: 23 June 2021 / Accepted: 27 August 2021 / Published online: 7 September 2021
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Abstract
Wellbore instability problems cause nonproductive time, especially during drilling operations in the shale formations. These problems include stuck pipe, caving, lost circulation, and the tight hole, requiring more time to treat and therefore additional costs. The extensive hole collapse problem is considered one of the main challenges experienced when drilling in the Zubair shale formation. In turn, it is caused by nonproductive time and increasing well drilling expenditure. In this study, geomechanical modeling was used to determine a suitable mud weight window to overpass these problems and improve drilling performance for well development. Three failure criteria, including Mohr–Coulomb, modified Lade, and Mogi–Coulomb, were used to predict a safe mud weight window. The geomechanical model was constructed using offset well log data, including formation micro-imager (FMI) logs, acoustic compressional wave, shear wave, gamma ray, bulk density, sonic porosity, and drilling events. The model was calibrated using image data interpretation, modular formation dynamics tester (MDT), leak-off test (LOT), and formation integrity test (FIT). Furthermore, a comparison between the predicted wellbore instability and the actual wellbore failure was performed to examine the model’s accuracy. The results showed that the Mogi–Coulomb failure and modified Lade criterion were the most suitable for the Zubair formation. These criteria were given a good match with field observations. In contrast, the Mohr–Coulomb criterion was improper because it does not match shear failure from the caliper log. In addition, the obtained results showed that the inappropriate mud weight (10.6 ppg) was the main cause behind wellbore instability problems in this formation. The optimum mud weight window should apply in Zubair shale formation ranges from 11.5 to 14 ppg. Moreover, the inclination angle should be less than 25 degrees, and azimuth ranges from 115 to 120 degrees northwest-southeast (NE–SW) can be presented a less risk. The well azimuth of NE–SW direction, parallel to minimum horizontal stress (Shmin), will provide the best stability for drilling the Zubair shale formation. This study’s findings can help understand the root causes of wellbore instability in the Zubair shale formation. Thus, the results of this research can be applied as expenditure effectiveness tools when designing for future neighboring directional wells to get high drilling performance by reducing the nonproductive time and well expenses.

Keywords Wellbore instability · Geomechanical model · Horizontal stresses

List of symbols
BIT Bit size
MW Mud weight
MIN MW Minimum mud weight
MAX MW Maximum mud weight

Introduction
The study area is located in southern Iraq (in Thi-Qar Governorate) with 310 m² (31 km long and 10 km wide). The stratigraphy scheme used in the G field was based on the terminology presented in exploration and appraisal well reports (G-1, G-2 and G-3) and various literature on the geology of Iraq, such as (Jassim and Goff 2006; Nairn and Alsharhan 1997), and Abdelkarim et al. (2009). In this field, hydrocarbons were encountered in Mishrif, Zubair, Ratawi, and Yamama reservoirs. All formations were formed during the Cretaceous. Mishrif and Yamama are limestone formations, while Zubair and Ratawi are clastic formations. The Zubair...
formation mainly consists of sandstone and shale intercalation with small streaks of limestone and siltstone. These formations were subdivided into several reservoir units based on pressure and fluid information. Figure 1 shows the general G field stratigraphy based on G-1, G-2, G-3 combined with G-4.

The problems of wellbore instability are frequently reported in several fields of southern Iraq while drilling in the Zubair formation. Nonproductive time (NPT) in this field resulted from wellbore instability caused by hole collapse, hole enlargement, lost circulation, stuck pipe, and bad cement jobs, as presented in Fig. 2. Knowing geomechanical parameters, including the in situ stresses and the mechanical properties, can reduce the high cost resulting from NPT (Al-Wardy and Portillo 2010; Mohammed and Selman 2020). The operating mud weight is mainly selected to protect the well from the kick without considering geomechanical principles (Alsaahwai et al. 2017). Thus, shear failures will occur due to the imbalance between stress and rock strength (Mansourizadeh et al. 2016; Neamy and Selman 2020).

Borehole collapse (hole enlargement) occurs when hydrostatic pressure (mud weight) is lower than prospective. In other words, the hole enlargement happens when drilling fluid density is less than the strength of the rocks. Thus, shear failure happens at a low drilling fluid density while tangential stress raises high. Stress concentration causes an elliptic borehole shape because a rock piece breakdown the wellbore wall (Aadnoy and Kaarstad 2010). Shear failure causes poor cementing, insufficient hole cleaning, stuck pipe, and difficult logging. Sand production and an influx of the formation fluid can be caused by poor cementing. Moreover, when the borehole wall begins to collapse, parts rock fragment slips down into the borehole and then settling on the bottom hole assembly (BHA). This settling may stop the BHA from moving up and down and leading to a mechanical stuck pipe. As an outcome, the drilling operations will stop.

Likewise, enlargements in the borehole in a specific direction knows as borehole breakout. Therefore, the orientation of minimum horizontal stress can know from the breakout direction (in the same direction). Practically, the 4–6 arm caliper tool, resistive image log, optical imaging log, and acoustic image log are used to determine the breakout direction (Bell and Gough 1979; Jaeger et al. 2007; Zoback et al. 1985). The wellbore enlargement pattern via four arm caliper tools is shown in Fig. 3.

Figure 3a shows a standard inside the hole the reading of C1 and C2 equal to bit size, but Fig. 3c explains the wellbore with severe washout, and it can set for contrast in C1 and C2 reading.

The geomechanical study is rarely applied in the Iraqi oil fields except for some studies in the Zubair oil field. Therefore, this study is considered one of the best studies that can be relied upon in later studies. In addition, this research is the first in this G oilfield to identify the problems of instability and to develop appropriate solutions. Thus, the present work was successful in reducing unproductive time and cost.

This study evaluated the magnitude and direction principal horizontal stresses that it was governing element in geomechanical modeling. Wellbore stability was analyzed under stress alteration caused by drilling and predicts wellbore behavior.

**Geomechanical modeling**

The workflow is the steps for building and calibrating the modeling to estimate the nonproductive time causing a consequent cost of the wells. In addition, it will help understand the relationship between borehole failure, suitable mud weight, and the borehole inclination and orientation. Figure 4 represents a summary of all steps of the workflow.
The first step was collecting the appropriate data for the Zubair formation to build the modeling. The essential data required to build the modeling included well logs (density, compression wave velocities, porosity shear wave velocities, gamma ray, resistivity, formation microimager (FMI), caliper), and measurement data leak-off tests (LOT), and mini-frac tests. The second step is to check the quality of the collected data against the defined standards. After that, the model was constructed based on the data from the open hole wireline logs. The developed model was calibrated using drilling events, mini-frac tests, and modular formation dynamics tester (MDT). Finally, an appropriate criterion chose to use the model parameters and find proper drilling fluid density corresponding to the trajectory of wells in future drilling operations.
Time distribution

The most of wellbore instability problems are mentioned in Sect. 8.5 inches, as mentioned earlier in Fig. 2. This section initiates from the top of the Shuaiba formation to the top of the Sulaiy formation with an average gross vertical thickness of about 950 m (± 100 m) that constitutes about 30% of the total depth. Analyzing the operation time of drilling indicates that the total consumed days to drill all intervals for well G-4 is about 41.03, as shown in Fig. 5.

Mechanical properties

The determination of mechanical rock properties plays a significant role in any geomechanical analysis. The basic mechanical rock properties include elastic properties (Poisson's ratio ($\nu$), and Young's modulus (E)), and rock strength properties (internal friction angle (FANG), unconfined compressive strength (UCS), and cohesion). Continuous profiles of these properties can significantly indicate the natural variation of the strength and formation competence in all layers of the study area. In this study, the mechanical rock properties were estimated using derived equations suitable with five types of logs, including bulk density ($\rho_b$), compression wave velocity ($V_p$), shear wave velocity ($V_s$), gamma ray, and porosity (Fjar et al. 2008). The elastic parameters (Poisson's ratio and Young's modulus) estimated based on the shear acoustic wave velocity, compressional acoustic wave velocity, and bulk density logs (Abbas et al. 2018a, b), as follows:

$$G_{\text{dyn}} = 13474.45 \frac{\rho_b}{\Delta r^2_{\text{shear}}}$$  \hspace{1cm} (1)

$$K_{\text{dyn}} = 13474.45 \rho_b \left( \frac{1}{\Delta r^2_{\text{COMP}}} \right) - \frac{4}{3} G_{\text{dyn}}$$  \hspace{1cm} (2)
\[ v_{\text{Dyn}} = \left( \frac{V_p}{V_s} \right)^2 - 1 \]  
\[ E_{\text{Dyn}} = \frac{9G_{\text{dyn}} \times K_{\text{dyn}}}{G_{\text{dyn}} + 3K_{\text{dyn}}} \]  

where \( K_{\text{dyn}} \): dynamic bulk modulus (Mpsi), \( G_{\text{dyn}} \): dynamic shear modulus (Mpsi), \( v_{\text{Dyn}} \): dynamic Poisson’s ratio, and \( E_{\text{Dyn}} \): dynamic Young’s modulus (Mpsi).

The modified Morales’s equation was applied to determine the static Young’s modulus, as shown in Fig. 6 (at track number six under the name (YME STA 2)) (JJ Zhang and Bentley 2005). Furthermore, the static bulk and shear moduli are calculated based on the following expressions:

\[ G_{\text{sta}} = \frac{E_{\text{sta}}}{2(1 + v_{\text{sta}})} \]  
\[ K_{\text{sta}} = \frac{E_{\text{sta}}}{3(1 - 2v_{\text{sta}})} \]  

where \( G_{\text{sta}} \): static shear modulus (Mpsi), \( K_{\text{sta}} \): static bulk modulus (Mpsi), \( v_{\text{sta}} \): static Poisson’s ratio, and \( E_{\text{sta}} \): static Young’s modulus (Mpsi).

Also, several correlations were used to compute UCS for sandstone and shale rocks. Thus, these correlations were matched properties with lab tests to find the best fit model (Chang et al. 2006; Khakse et al. 2009; Nabaei et al. 2010). The McNally correlation was applied to calculate the UCS of the sandstones rock (McNally 1987), as shown in Fig. 6 (at track number seven under the name (UCS YME 2)) and Eq. 8.

\[ \text{UCS} = 1200\exp(-0.03\Delta t_c) \]  

where \( \Delta t_c \): compressional slowness of the bulk formation us/ft.

In addition, the ability of the rock to resist collapse is called the internal friction angle (FANG), and it is a significant parameter to predict the wellbore failure. Therefore, graphically, from the Mohr–Coulomb criterion plot, an inclination angle concerning the horizontal axis (normal stress) is friction angle. Most studies have recently found a relation between the rock’s hardness and the friction angle because of the high value of friction angle for some weak rock; if Young’s modulus for shale formation is high, the value of friction angle is high (Lama and Vutukuri 1978). Friction angle can be found from core lab analysis and special rock tables (M. Zoback et al. 2003). In addition, when unavailable core sample can use several empirical equations. It can estimate from correlation based on porosity (\( \Phi \)) and shale volume (\( V_{\text{shale}} \)). Dick Plumb proposed this correlation based on his data published in 1994, and it was modified in 2002. This new updated algorithm eliminates the non-physical increase of friction angle at low values of the volume of grain predicted by the former algorithm.

\[ \text{FANG} = 26.5 - 37.4\left(1 - \Phi - V_{\text{shale}}\right) + 62.1\left(1 - \Phi - V_{\text{shale}}\right)^2 \]  
\[ V_{\text{shale}} = \frac{GR - GR_{\text{min}}}{GR_{\text{max}} - GR_{\text{min}}} \]

where \( GR \): gamma ray log, \( GR_{\text{max}} \): maximum values of gamma ray log, and \( GR_{\text{min}} \): minimum values of gamma ray log. Figure 6 (at track number nine under the name (FANG PPC)) shows the internal friction angle (FANG).

### Pore pressure

The pore pressure is estimated using Eaton’s equation based on the wireline measurements (sonic log), as shown in Fig. 6 (at track number four under the name (PPRS EATON S)) (Eaton 1969; Zhang 2011). This equation is represented as:

\[ P_{pg} = OBG - \left( OBG - P_{pm} \right) \left( \frac{\Delta t_n}{\Delta t_o} \right)^x \]  

where \( OBG \): overburden gradient, \( P_{pm} \): gradient of normal hydrostatic pore pressure, \( \Delta t_n \): shale slowness at normal trend line, and \( \Delta t_o \): shale slowness derived from sonic log.

The exponent values are 1.5 when using resistivity log and 3 when using sonic log (or seismic data). The exponent values are derived from the Gulf of Mexico data, so globally, the exponent must be updated for the area of study (Azadpour and Shad Manaman 2015).

In addition, the estimated pore pressure is being calibrated using pressure points recorded from the modular formation dynamics tester (MDT), as shown in Fig. 6 (at track number four under the name (formation pressure psi)).
Overburden stress (vertical stress (Sv))

It is the stress at a certain point due to the weight of the upper layer’s formations. An increase in sediments in depth leads to an increase in the overburden pressure. The vertical stress for Zubair formation was estimated using rock densities, as shown in Fig. 6 (at track number four under the name (SVERTICAL.EXT*)). Bulk density was obtained from a density logging tool.
where \( Z \): depth, \( g \): acceleration due to gravity, and \( \rho \): bulk density.

**Horizontal stresses (minimum and maximum)**

The rock tends to move horizontally because of the overburden pressure impact on the rock vertically (Aadnoy and Looyeh 2019). The minimum and maximum stress tend to be the same in value when no tectonic activities, so only the vertical stress effect is present. Therefore, there are different values for the horizontal stresses and should be estimated when the faulting and tectonic activities have existed. Many geomechanical problems are solved by knowing the orientation and magnitude of the horizontal stresses.

The minimum horizontal stress can be estimated and matching with a leak-off and extended leak-off test (LOT and XLOT), mini-frac test (i.e., direct methods) (Yamamoto 2003; M. Zoback et al. 2003). Furthermore, the best method to determine the minimum horizontal stress is hydraulic fracturing because the mechanical properties of the rock do not need. There are many technical approaches developed to predict Maximum horizontal stress. The equation by Enever et al. (1996) derives using elasticity theory based on the slippage of the fault and Mohr–Coulomb criterion. Based on vertical stress, minimum horizontal stress and fault regime are derived as an equation by Peng and Zhang (2007).

The magnitudes of the minimum and maximum horizontal stresses are determined by using the poroelastic horizontal strain model (Thiercelin and Plumb 1994). Model equations are essentially based on Young’s modulus of the rock density, rock deformation, and regional pore pressure expressed in Eqs. 13 and 14.

\[
\sigma_h = \frac{v}{1-v} \sigma_v + \frac{1-2v}{1-v} \alpha P_r + \frac{E}{1-v^2} \epsilon_x + \frac{vE}{1-v^2} \epsilon_y \tag{13}
\]

\[
\sigma_H = \frac{v}{1-v} \sigma_v + \frac{1-2v}{1-v} \alpha P_r + \frac{E}{1-v^2} \epsilon_y + \frac{vE}{1-v^2} \epsilon_x \tag{14}
\]

\( \epsilon_x \) and \( \epsilon_y \): tectonic strains in maximum and minimum horizontal stress direction, respectively. The \( \epsilon_x \) and \( \epsilon_y \) can be an estimate based on overburden stress by using Eqs. 15 and 16.

\[
\epsilon_x = \frac{v \sigma_v}{E} \left( 1 - \frac{v^2}{1-v} \right) \tag{15}
\]

\[
\epsilon_y = \frac{v \sigma_v}{E} \left( \frac{1}{1-v} - 1 \right) \tag{16}
\]

where \( \sigma_H \): maximum principal stresses, \( \sigma_h \): minimum principal stresses, \( \alpha \): Biot’s coefficient (\( \alpha = 1 \), conventionally), \( v \): Poisson’s ratio, and \( E \): Young’s modulus.

Figure 6 (at track number four under the name (SHMIN PHS) and (SHMAX PHS)) illustrates minimum horizontal stresses and maximum horizontal stresses magnitudes. In addition, the maximum and minimum horizontal stress was calibrated using closure and breakdown pressure measurement points, as shown in Fig. 6 (at track number four under the name (closure pressure) and (breakdown pressure)). The results showed that the Zubair formation is a normal faulting regime (i.e., \( \sigma_v > \sigma_H > \sigma_h \)).

**Orientation of horizontal stresses**

The direction of the principal stresses is a critical part of wellbore failure analysis (Barton et al. 1997). The well trajectory design improves by identifying stress direction and magnitudes to minimize wellbore instability (Alsahlawi et al. 2017).

Generally, the breakout happens around the wellbore at the same orientation as the minimum principal stress, with a high-stress concentration.

Image log observations in the well G-4 show several breakouts (Figs. 7 and 8), which give an average breakout orientation (minimum horizontal stress orientation) of 115 deg. Therefore, the orientation of maximum horizontal stress is 25 deg. The image log shows that the stress

![Fig. 7 Stereonet showing the orientation of all breakouts observed with a consistent NW–SE trend in G-4. Average breakout orientation is 115 deg](image-url)
orientation observed from the offset wells in all Well pads is approximately the same. Moreover, the seismic cross sections indicate a tectonically relaxed area, so no significant changes in stress orientation are expected. Therefore, the local minimum horizontal stress direction is about NW–SE, and the local maximum horizontal stress is orthogonal to this direction.

Wellbore instability

After completing the 1D-MEM, it was validated before its application. After the developed model is completed, the failure criterion can estimate a failure around the borehole by matching the predicted wellbore instability with shear failure from the image and caliper logs and the drilling events. Thus, there was a very agreement between the predicted shear failure and measured shear failure using caliper log. The tensile and collapse failures predicted along the wellbore trajectory based on the defined mud weight (10.6 ppg) of offset wells. Then, predicted wellbore failure matched the data from wellbore instability related to drilling events or log data.

This study was applied various failure criteria (modified Lade, Mohr–Coulomb, and Mogi–Coulomb) to predict the borehole instability. The outcomes showed that the Mogi–Coulomb and modified Lade criteria expected breakout regions correspond with the detected breakouts in the caliper log, as represented in Figs. 9, 10, 11, and 12. In addition, Figs. 12 and 13 (in "Appendix") show the result modified Lade and Mohr–Coulomb failure criteria. Therefore, Mogi–Coulomb and modified Lade can be considered the most suitable failure criterion for the Zubair shale formation. Often, these criterion results are good because they do not neglect the effect of the intermediate principal stress component in the failure analysis. (Al-Ajmi and Zimmerman 2006; Gholami et al. 2014). The Mohr failure criterion does not give a match and which considered unsuitable for this formation. The main reason is that the Mohr failure criterion neglects the effect of the intermediate principal stress component.

Evaluation of a safe mud weight window

The mud weight window was determined by relying on the earlier geomechanical modeling and drilling data examination. Stereographic plots were applied to determine the impact of azimuth and deviation on the breakdown and breakout, as shown in Fig. 10. The shale and weak sandstone points were conducted to find the minimum mud weight relative to orientation and inclination.

The safe mud weight window against a deviation begins to narrow at an inclination of 25° degrees and becomes very narrow in the horizontal wells. In contrast, the wellbore azimuth does not affect the breakout mud weight because of the low-stress contrast, as shown in Fig. 10c and d. Moreover, the highest mud weights of breakdown are expected with
Fig. 9  Breakout predict using Mogi–Coulomb criterion
inclined less than 50° degrees and at the same orientation of the minimum horizontal stress. On the contrary, when drilling a wellbore in the direction of maximum horizontal stress, as presented in Fig. 10a and b. Therefore, the results can be summarized by avoiding highly inclined wells or using sufficient high mud weight to avert breakout failure and incur limited mud loss.

**Wellbore stability expectations using new proposed mud weight**

Mogi–Coulomb failure criterion was applied to forecast safe mud weight window and to predict preferred well trajectory. The main goal planned to drill in shaly sand formation because the drilling in this formation is challenging with an increased risk of wellbore collapse. Therefore, the potential shear failure evaluation was for weak formations (sandstone and shale sections) along the proposed trajectory using the Mogi–Coulomb criterion.

Figure 11 shows the planned well analysis and the recommended drilling mud weight program determined. The old mud weight is 10.6 ppg, and the new mud weight should be in the range of 11.5–14 ppg to avoid drilling problems such as breakout or breakdown. Furthermore, good drilling practices like proper hole cleaning, tripping speed (pulling out and running in), and controlled penetration rate can help improve wellbore stability. In addition, monitoring the equivalent circulation density (ECD) is essential to avoid loss circulation of drilling mud when crossing the upper threshold of the mud weight (Allawi and Almahdawi 2019). Also, a high surge while tripping in may cause a breakdown because it increases instantaneous downhole pressure. Thus, it is crucial to monitor the tripping speed.

**Summary and conclusions**

In this study, a robust and accurate geomechanical model was based on in situ reservoir measurements, consistent with observations of wellbore failures. The results showed many essential points that should be considered in the upcoming development wells in the future:

- The Mogi–Coulomb failure and modified Lade criterion were the most suitable for the Zubair shale formation. These criteria were given a good match with field observations.
- The main reason is that the Mogi–Coulomb and modified Lade criterion does not neglect the intermediate principal stress effect on the predicted failure.
- Mogi–Coulomb failure and modified Lade criterion can be adopted as the basis for wellbore stability analysis of several wells drilled through this formation.
- Mohr–Coulomb criterion was improper because it does not match shear failure from the caliper log.
- The results showed most of the wellbore instability problems due to low mud weight (10.6 ppg).
- It is recommended that the mud weight (11.5–14 ppg) increase as required and according to the planned well path.
- As well as drilling wells with an angle of inclination less than 25°, they are safe and stable.
- The drilling orientation should be parallel to the minimum horizontal stress to reduce the risk of wellbore instability.
- There will be a significant challenge in tripping and hole cleaning in drilling inclined wells with an angle of more than 25° degrees.
- Monitoring the equivalent circulation density (ECD) is important to avoid loss circulation of drilling mud when crossing the upper threshold of the mud weight
- The swab and surge should be monitored with upper and lower limits of mud weight to avoid wellbore instability problems.
- Ultimately, controlled penetration rate, proper hole cleaning, and adequate mud conditioning can mitigate wellbore instability related to issues during drilling.
- The limitations of this study were that some wells do not have shear wave velocity, so elastic properties cannot be estimated, which consider essential in the geomechanical model.

**Appendix**

**Mohr–Coulomb criterion**

The friction and contact forces of the shearing resistance are related to physical bonds of the rock grains, which are presented by Mohr–Coulomb failure criteria as shown in Eq. 17

$$\tau = c + \sigma \tan \phi$$  \hspace{1cm} (17)

The coefficient equation of internal friction angle is:

$$\mu = \tan \phi$$  \hspace{1cm} (18)

Several Mohr’s circles are plotted to make the failure envelope. These circles represent a triaxial test, where a sample is compressed to side confining pressure ($\sigma_2 = \sigma_3$)
and axial stress ($\sigma_1$) at the beginning of failure. The plot of Mohr circles gave an envelope which is the basis of this failure criterion, and failure point ($\sigma$, $\tau$) can express as:

$$\tau = \frac{1}{2} (\sigma_1 - \sigma_3) \cos \phi$$  \hspace{1cm} (19)
Fig. 12 Prediction of breakout regions using the modified Lade failure criterion
Fig. 13 Prediction of breakout regions using the Mohr–Coulomb failure criterion
\[ \sigma = \frac{1}{2} (\sigma_1 + \sigma_3) - \frac{1}{2} (\sigma_1 - \sigma_3) \sin \phi \]  

The maximum and minimum principle can be expressed by Mohr failure criteria, as follows:

\[ \sigma_1 = 2c \frac{\cos \phi}{1 - \sin \phi} + \frac{1 + \sin \phi}{1 - \sin \phi} \sigma_3 \]  

\[ \text{UCS} = 2c \frac{\cos \phi}{1 - \sin \phi} \]  

where \( \sigma_1 \): maximum principal stresses, \( \sigma_2 \): minimum principal stresses, \( \phi \): angle of internal friction, \( \tau \): shear stress, and \( C \): cohesion (inherent shear strength).

**Mogi–Coulomb criterion**

Mogi–Coulomb criterion is based on the triaxial compression measurement on carbonate and silicate rocks introduced by Mogi (1971). He pointed that not only the least compressive (\( \sigma_3 \)) and maximum compressive stresses (\( \sigma_1 \)) had significantly affected the fracture and flow properties of rocks, but also by intermediate compression (\( \sigma_2 \)). The fracture and yielding are produced by the stress, which the relation can determine:

\[ \tau_{oct} = f_1(\sigma_1 + \sigma_3); \text{ for fracture} \]  

\[ \tau_{oct} = f_2(\sigma_1 + \sigma_2 + \sigma_3); \text{ for yielding} \]  

\[ \tau_{oct} = \left[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right]^{1/2} \]  

where \( f_1 \) and \( f_2 \) are monotonically increasing functions, and \( \tau_{oct} \) is the octahedral shear stress.

The criteria showed that increases monotonically with the effective mean pressure: \( (\sigma_1 + \sigma_2)/2 \) for fracture and \( (\sigma_1 + \sigma_2 + \sigma_3)/3 \) for yielding could be caused a fracture of yielding, especially when the distorted strain energy reaches a critical value. It is concluded that the impact of \( \sigma_2 \) and \( \sigma_3 \) is opposite on elasticity, so the permanent strain is just before fracture. The ductility relates with \( \sigma_3 \), so it increases with increasing \( \sigma_3 \), but it decreases with increasing \( \sigma_2 \) and can govern by the general relation:

\[ \varepsilon_n = f_3(\sigma_3 - a\sigma_2) \]  

where \( f_3 \), monotonically increasing function and \( \varepsilon_n \): strain.

**Modified Lade failure criteria**

Lade (1984) suggested a 3-D failure criterion for frictional materials that considered curved failure envelopes without effective cohesion. Therefore, Ewy (1998) was modified Lade failure criteria to include the effect of cohesion. This failure criterion defines as:

\[ \frac{(f'_1)^3}{f_1^3} = 27 + \eta \]  

\[ I'_1 = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \]  

\[ I'_3 = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S) \]  

\[ S = \frac{S_0}{\tan \phi} \]  

\[ \eta = \frac{4(\tan \phi)^2(9 - 7\sin \phi)}{(1 - \sin \phi)} \]  

where \( S \) is related to cohesion, \( \eta \) is related internal friction, and \( S \) and \( \eta \) are depended to Coulomb’s cohesion parameter.

**Acknowledgements** The authors would like to gratefully acknowledge Thi-Qar Oil Company (T.Q.C) in Iraq for providing technical data and permission to publish the results.

**Funding** No funding was received.

**Declarations**

**Conflict of interest** On behalf of all the co-authors, the corresponding author states that there is no conflict of interest.

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