New stellar constraints on dark photons

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Abstract

We consider the stellar production of vector states $V$ within the minimal model of "dark photons". We show that when the St"uckelberg mass of the dark vector becomes smaller than plasma frequency, the emission rate is dominated by the production of the longitudinal modes of $V$, and scales as $\kappa^2 m_V^2$, where $\kappa$ and $m_V$ are the mixing angle with the photon and the mass of the dark state. This is in contrast with widespread assertions in the literature that the emission rate decouples as the forth power of the mass. We derive ensuing constraints on the $(\kappa, m_V)$ parameter space by calculating the cooling rates for the Sun and horizontal branch stars. We find that stellar bounds for $m_V < 10$ eV are significantly strengthened, to the extent that all current "light-shining-through-wall" experiments find themselves within deeply excluded regions.
1 Introduction

The Standard Model of particles and fields (SM) can be naturally extended by relatively light neutral states. Almost all possible ways of connecting such states to the SM have been explored, and several of such ways stand out as the most economical/natural. One of the most attractive possibilities is the so-called "hypercharge portal", or "kinetic mixing" portal that at low energy connects the electromagnetic current with another massive photon-like state [1]. This model has been under intense scrutiny in the last few years, both experimentally and observationally. The interest to this model is fueled by attractive (yet speculative) possibilities: the dark vector can be a promising mediator of the dark matter-SM interaction [2], or form super-weakly interacting dark matter itself [3, 4]. Dark vectors were proposed as a possible solution to the muon $g - 2$ discrepancy [5], and have been searched for (so far with negative results), both at high energy and in medium energy high-intensity particle physics experiments.

The region of small vector masses, $m_V < eV$, can also be very interesting. On the theoretical side, there are speculations of dark photons contributing to dark matter (via an initial condensate-like state) [6] and dark radiation [7]. But perhaps more importantly, there are some hopes for the terrestrial detection of dark photons. So, far several avenues have been proposed: one can attempt observing a "visible-dark-visible" oscillation chain in "light-shining-through-wall" experiments (LSW) [8]. The quanta of dark photons emitted from the Sun can be searched for with "helioscopes" [9], neutrino- [10] and dark matter experiments [11, 12]. Some of these exciting possibilities have been summarized in the recent review [13]. We will refer to all proposals and experiments aimed at detection of dark vectors, produced astrophysically or in the laboratory, as direct searches.

At the same time, it is well-known that for many light ($m_V < keV$) and weakly-coupled exotic particles the astrophysical constraints are often far stronger than direct laboratory constraints [14]. The astrophysical constraints are very important for the dark vectors as well, as they determine a surviving fraction of the parameter space that can be explored in direct searches. The most important limits to reconcile with are the constraints on the emission of dark vectors from solar luminosity, from the horizontal branch stars, neutron star and supernovae cooling rates.

To date, the only in-depth analysis of astrophysical bounds on sub-keV dark vectors was performed by Redondo in [9], where the production of longitudinal modes of the dark photon is treated incorrectly. We trace the mistake traced to a wrong use of the in-medium polarization effects for longitudinal modes. In this paper we re-assess these bounds, provide correct calculations for the dark photon emission rates, and strengthen the astrophysical bounds in the LSW region by as much as ten orders of magnitude. Our findings significantly reduce the parameter space available for the direct searches and affect or completely change the conclusions of many papers written on this subject. In a separate forthcoming publication we will address new limits imposed by the most advanced WIMP detectors on the solar emission of dark vectors [15].

This paper is organized as follows. The next section introduces the minimal model of the dark vector, and explains the main scaling of its production rate with $m_V$. Section 3
contains technicalities of the in-medium production of the dark vector. Section 4 contains practical formulae for the stellar emission rates, in application to the Sun and horizontal branch stars, and sets the constraints on the mass-mixing parameter space. We reach our conclusions in section 5.

2 Dark photon production, in vacuum and in a medium

The minimal model of ”dark vectors” extends the SM gauge group SU(3)_c × SU(2)_L × U(1)_Y by an Abelian factor U(1)_V. Kinetic mixing of the hypercharge field strength $F^Y_{\mu\nu}$ with the field strength $V_{\mu\nu}$ of U(1)_V links the SM to the new physics sector, while SM fields are assumed to be neutral under U(1)_V. We are interested in processes far below the electroweak energy scale, for which the relevant low-energy Lagrangian takes the form

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} - \frac{1}{4}V^2_{\mu\nu} - \frac{\kappa}{2}F_{\mu\nu}V^{\mu\nu} + \frac{m_V^2}{2}V_{\mu\nu} + eJ^\mu em A_\mu.$$  

(1)

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the photon field strength and $V_\mu$ is the ”hidden photon” (also known as ”dark vector”, ”secluded vector”, ”dark photon” etc—an equivalent set of names). The coupling of $A_\mu$ and $V_\mu$ is regulated by the kinetic mixing parameter $\kappa$, redefined in an appropriate way to absorb the dependence on weak mixing angle. For all calculations in this paper we use $\kappa \ll 1$, and consider only leading order terms in the mixing angle. Finally $J^\mu em$ is the usual electromagnetic current with electric charge $e < 0$.

It is important to comment on the origin of $m_V$ in (1). The simplest possibility is that $m_V$ is a St"{u}ckelberg-type mass. Because of the conservation of the Abelian vector current, $m_V$ remains protected against sensitivity to UV scales, and such a model is technically natural even with very small $m_V$. An alternative generic possibility is a new scalar field(s) charged under U(1)_V that develops a vacuum expectation value that Higgses the hidden group. This introduces a new interaction term of the physical hidden Higgs with vectors, $g'm_Vh'V_{\mu}^2$, as well as $h'$ self-interaction (see e.g. [16]). It is well understood that in the limit of $m_V$ and $m_{h'}$ small compared to all energy scales in the problem, the production of dark sector states is dominated by the dark Higgsstrahlung [3, 16], or equivalently, by the pair-production of the U(1)_V-charged Higgs scalar fields. Importantly, this process is insensitive to the actual mass of $m_V$ in the small mass limit, and schematically

$$\text{Rate}_{SM \rightarrow V + h'} \propto \alpha' \kappa^2 (m_V)^0,$$  

(2)

where we show only the dependence on dark sector parameters, leaving the SM part of the $V + h'$ production process completely general; $\alpha' = (g')^2/(4\pi)$ is the square of the coupling of dark Higgs to $V_\mu$. For sub-keV dark vectors and Higgses, all previously derived constraints on ”millicharged particles” apply [17], and limit the $\kappa g'$ combination to be below $\sim 10^{-13}$. The technical reason for not having any small $m_V$ suppression of the rate (2) despite the interaction term $g'm_Vh'V_{\mu}^2$ being proportional to $m_V$ is of course tied to the production of the longitudinal modes of $V$ in $V + h'$ final state.

The models with the hard (i.e. St"{u}ckelberg) mass $m_V$ behave differently as the production rate of dark vectors has to decouple in the small $m_V$ limit. The easiest way to see this is to
restrict the interaction terms in (1) to on-shell $V_\mu$, using $\partial_\mu V^\mu = 0$ and to leading order in $\kappa$, $\partial_\mu V^{\mu\nu} = -m_V^2 V_\nu$, so that

$$L_{\text{int}} \rightarrow \left. \frac{\kappa}{2} F_{\mu\nu} V^{\mu\nu} + e J_{em}^\mu A_\mu \right|_{\text{on-shell}} \rightarrow -\kappa m_V^2 A_\mu V^\mu + e J_{em}^\mu A_\mu. \tag{3}$$

This expression is of course explicitly gauge invariant under $A_\mu \rightarrow A_\mu + \partial_\mu \chi$ due to the current conservation and on-shellness of $V_\mu$ conditions:

$$\partial_\mu J_{em}^\mu = 0; \quad \partial_\mu V^\mu = 0. \tag{4}$$

The appearance of $m_V^2$ in the coupling of $V_\mu$ and $A_\mu$ shows that two sectors are decoupled in $m_V = 0$ limit. The most important question in considering the production of $V_\mu$ states is the scaling of the production rate with $m_V$, in vacuum and inside a medium. The existing literature on the subject [9] and its subsequent follow-up papers claim that in-medium production decouples as $\text{Rate}_{SM\rightarrow V} \propto \kappa^2 m_V^4$ in the small $m_V$ limit. This inference is wrong.

To demonstrate our point we consider a generic production process $i \rightarrow f + V$ due to (3), where $i, f$ are any initial, final states of the SM particles. A schematic drawing of such a process is shown in Fig. 1. Without loss of generality we assume that $V$ is emitted in $z$-direction, so that its four-momentum $k_\mu$ is given by $(\omega, 0, 0, |\vec{k}|)$, with $\omega^2 - |\vec{k}|^2 = m_V^2$. Moreover, we assume that the energy of the emitted $V$ is much larger than its rest mass, $\omega \gg m_V$. Three polarization states can be emitted: two transverse states $V_T$ with polarization vectors $\epsilon_T^\mu = (0, 1, 0, 0, 0)$ and $(0, 0, 1, 0, 0)$, and one longitudinal mode $V_L$ with polarization vector $\epsilon_L^\mu = m_V^{-1}(|\vec{k}|, 0, 0, \omega)$. In all cases $\epsilon_\mu^2 = -1$ and $\epsilon_\mu k_\mu = 0$.

We include a boundary-free medium via some conducting plasma, characterized by the plasma frequency $\omega_p$. We consider two regimes, [almost] vacuum: $\omega_p \ll m_V \ll \omega$, and in-medium: $m_V \ll \omega_p \ll \omega$. The choice of $|\vec{k}|, \omega \gg \omega_p$ is not essential, and we consider all ranges of $\omega$ in the next section. The matrix element for the production process induced by (3) is given by

$$\mathcal{M}_{i \rightarrow f + V_{T(L)}(\omega)} = \kappa m_V^2 [e J_{em}]_{fi} \langle A^\mu, A^\nu \rangle \epsilon_{T(L)}^\nu,$$

where $\langle A^\mu, A^\nu \rangle$ stands for the photon propagator with input momentum $k_\mu$, and $[e J_{em}]_{fi}$ is the matrix element of the electromagnetic current. We disregard various $m_V$-independent phase factors and normalizations, as our goal in this section is to only consistently follow the powers of $m_V$. 

Figure 1: Illustration of the dark photon emission process by the electromagnetic current.
For convenience, we fix the photon gauge to be Coulomb, $\nabla_i A_i = 0$, but same results can be obtained in other gauge, of course. The photon propagator for the production of the transverse modes is given by (see, e.g. [18]):

$$\langle A_j, A_l \rangle = \frac{\delta_{jl}^\perp}{\omega^2 - |\vec{k}|^2 - \omega_p^2} = \frac{\delta_{jl}^\perp}{m_V^2 - \omega_p^2} \rightarrow \delta_{jl}^\perp \times \left\{ \begin{array}{ll} m_V^{-2} & \text{at } m_V \gg \omega_p, \\ -\omega_p^{-2} & \text{at } m_V \ll \omega_p, \end{array} \right.$$  \hspace{1cm} (6)

where $\delta_{jl}^\perp$ is the projector onto transverse modes. Because of the existence of two different regimes for the transverse modes of the photon, the amplitude of the $V^T$ production is $m_V$-independent in vacuum, and $m_V^2$-suppressed in medium.

The production of the longitudinal modes in Coulomb gauge is mediated by the $\langle A_0, A_0 \rangle$ part of the propagator. If $k \simeq \omega \gg \omega_p$, this propagator is unaffected by the medium [18],

$$\langle A_0, A_0 \rangle = \frac{1}{|\vec{k}|^2}. \hspace{1cm} (7)$$

The difference in the behavior of the two parts of the photon propagator can be readily understood. While the propagating transverse modes of the photon are close to mass shell, and plasma effects can affect them easily no matter how large $\omega$ is, the non-propagating piece $\langle A_0, A_0 \rangle$ cannot be changed in a causal manner if $|\vec{k}|$ is larger than e.g. the inverse distance between particles in the plasma. Plugging the expression (7) for the propagator together with the explicit form of $\epsilon_L^\mu$ into (5), we arrive at the conclusion that the amplitude for the production of the longitudinal mode is proportional to the first power of $m_V$.

Collecting all relevant factors in one expression, we get the all-important $m_V$ scalings for the production of both transverse and longitudinal modes:

$$\text{Rate}_{SM \rightarrow V_T} \propto \left\{ \begin{array}{ll} \kappa^2 & \text{in vacuum, } m_V \gg \omega_p, \\ \kappa^2 m_V^2 \omega_p^{-4} & \text{in medium, } m_V \ll \omega_p. \end{array} \right.$$ \hspace{1cm} (8)

Performing the same estimate for the production of the longitudinal mode, (and keeping $\omega \gg \omega_p$) we get

$$\text{Rate}_{SM \rightarrow V_L} \propto \kappa^2 m_V^2 \omega^{-2}, \text{ both in vacuum and in medium.} \hspace{1cm} (9)$$

Notice the quadratic, and not quartic dependence on the dark photon mass in (9), in contrast with conclusions of Ref. [9]. The $m_V$-scaling of the production rate for the transverse modes, Eq. (8), is of course a well-known result in the “dark photon” literature. It exhibits $m_V^4$ decoupling at small $m_V$, but as it turns out, this was incorrectly extended to the production of the longitudinal modes. (Thus we also learn that “nature does not like to skip an order,” and compared to the Higgsed case of an $O(m_V^0)$-rate [2] the emission of Stückelberg vectors occurs in lowest possible $O(m_V^2)$ order.) We also note in passing that for some processes, only the production of of the longitudinal modes of $V_\mu$ is actually possible. A well-studied process is the $K^+ \rightarrow \pi^+ V$ decay [5], which is forbidden for the transverse modes, but has the expected $m_V^2$ scaling at small $m_V$ for the longitudinal modes.

The correct scaling (9) will bring about momentous change in all estimates of stellar cooling rates. For the favorite LSW region with $m_V \sim 10^{-3}$ eV, one should expect that the
previous literature underestimates the solar cooling rates by as much as $m_V^2/\omega_p^2 \sim 10^{-10}$, and correspondingly, one expects the tightening of the constraints by the same large factor, once the production of the longitudinal mode of the dark vector is treated correctly. In the two subsequent sections we perform this analysis in some detail.

In closing this section, it is important to realize that the $m_V^2$-scaling of the $SM \rightarrow V_L$ rate derived in Eq. (9) is not going to change if one abandons the $\omega \gg \omega_p$ approximation. Indeed, in the Coulomb gauge the only ”sources” of $m_V$ (at small $m_V$) in the entire problem are the coupling in the Lagrangian and the expression for $\epsilon^L$. Therefore, the counting of powers of $m_V$ is the same, and the $m_V^2$-scaling is preserved even when the medium effects significantly modify the propagator (7) at $\omega \sim \omega_p$. At the same time, these effects will affect the normalization of $m_V^2$ in (9), and in order to derive an accurate expression for the rate, we have to go through a more detailed account of in-medium effects in the next section.

3 Plasma production rate of dark photons

Inside a medium, the propagation of the electromagnetic field is determined by the electromagnetic polarization tensor $\Pi^{\mu\nu} = e^2 \langle J_{\text{em}}^\mu, J_{\text{em}}^\nu \rangle$. Due to $k_{\mu} J_{\text{em}}^\mu = 0$ where $k = (\omega, \vec{k})$ is the four-momentum flow inside the polarization tensor, $\Pi^{\mu\nu}$ can be parameterized as

$$\Pi^{\mu\nu} = \Pi_T \sum_{i=1,2} \epsilon_T^i \epsilon_T^i + \Pi_L \epsilon_L^i \epsilon_L^i,$$

(10)

where $\epsilon_T^i, \epsilon_L$ are the transverse and longitudinal polarization vectors of a vector boson with momentum $k$. In particular,

$$\epsilon_L = \frac{1}{\sqrt{\omega^2 - |\vec{k}|^2}} (|\vec{k}|, \omega, \vec{k}) .$$

(11)

In the Coulomb gauge, the propagator of the electromagnetic field can be written as

$$\langle A^i, A^j \rangle = \frac{1}{\omega^2 - |\vec{k}|^2 - \Pi_T} \left( \delta^{ij} - \frac{k^i k^j}{k^2} \right),$$

$$\langle A^0, A^0 \rangle = \frac{1}{|\vec{k}|^2 - \frac{|\vec{k}|^2}{\omega^2 - |\vec{k}|^2} \Pi_L} ,$$

(12)

where $\epsilon^0_0 = |\vec{k}|/\sqrt{k_{\mu} k^{\mu}}$ has been used. Notice that while the definition of $\Pi_T$ is usually uniform across the literature, the definition of $\Pi_L$ varies, and e.g. in Ref. [18] it is defined differently, $\Pi_L^{\text{Ref. [18]}} = |\vec{k}|^2/\omega^2 - |\vec{k}|^2 \Pi_L^{\text{this work}}$. With the explicit expression of the photon propagator, the matrix element for the dark photon emission in Eq. (5) can further be written as

$$\mathcal{M}_{i \rightarrow f +V_T} = -\frac{k m_V^2}{m_V^2 - \Pi_T} [e J_{\text{em}}^\mu f_I \epsilon^T_\mu],$$

$$\mathcal{M}_{i \rightarrow f +V_L} = \frac{k m_V^2}{m_V^2 - \Pi_L |\vec{k}|^2} [e J_{\text{em}}^0 f_I \epsilon^L_\mu].$$

(13)
Using the condition \( k_{\mu} J_{\text{em}}^{\mu} = 0 \) and Eq. (11), it is easy to show that
\[
J_{\text{em}}^0 \epsilon_0 = -\frac{\hat{k}^2}{m_V^2} J_{\text{em}}^{\mu} \epsilon_{\mu}.
\] (14)

Therefore, Eq. (13) can be further simplified to
\[
\mathcal{M}_{i \rightarrow f + V_{T,L}} = -\frac{\kappa m_V^2}{m_V^2 - \Re \Pi_{T,L}} [e J_{\text{em}}^\mu]_f \epsilon^T_{\mu}.
\] (15)

The general expression for \( \Re \Pi_T \) and \( \Re \Pi_L \) can be found in Ref. [18]. Thus we find that the emission of dark vectors is given by the vacuum matrix element for the emission of massive photons, \([e J_{\text{em}}^\mu]_f \epsilon^T_{\mu}\), with fiducial photon mass \( m_{\gamma} = m_V \) and multiplied by the effective mixing angles, defined according to,
\[
\kappa_{T,L}^2 = \frac{\kappa^2 m_V^4}{(m_V^2 - \Re \Pi_{T,L})^2 + (\Im \Pi_{T,L})^2}.
\] (16)

At finite temperature \( T \neq 0 \), the imaginary parts of \( \Pi_{T,L} \) are related to the rates at which the respective distribution functions approach equilibrium [19]. Detailed balance equation allows this to be expressed exclusively in terms of the absorption rate \( \Gamma_{T(L)}^{\text{abs}} \) of in-medium transverse (longitudinal) massive photons,
\[
\Im \Pi_{T,L}(\omega, \hat{k}) = -\omega \left( 1 - e^{-\omega/T} \right) \Gamma_{T,L}^{\text{abs}}(\omega, \hat{k}).
\] (17)

One should note that the on-shell condition of the outgoing dark photon imposes the relation \( \omega^2 = \hat{k}^2 + m_V^2 \). Therefore, in this sense, \( \Gamma_{T,L}^{\text{abs}} \) can be understood as the absorption rate of the transverse and longitudinal mode of dark photon as if it couples to the electromagnetic current with the same coupling constant as the photon field.

It will be convenient to express \( \Gamma_{T(L)}^{\text{abs}} \) in terms of a differential production rate \( d\Gamma_{T,L}^{\text{prod}}/(d\omega dV) \) per frequency interval and volume. To this end we write a Boltzmann-type equation for the photon distribution function \( dn_{T,L}/d\omega \),
\[
\frac{dn_{T,L}}{d\omega dt} = \frac{d\Gamma_{T,L}^{\text{prod}}}{d\omega dV} g_{T,L} \frac{g_{T,L}}{1 - e^{-\omega/T}} - \frac{dn_{T,L}}{d\omega} \Gamma_{T,L}^{\text{abs}}.
\] (18)

Using detailed balancing and noting that the equilibrium dark photon distribution is given by \( dn_{T,L}/d\omega = g_{T,L} \omega |\hat{k}|/(2\pi^2) / (e^{\omega/T} - 1) \), where \( g_T = 2 \) and \( g_L = 1 \), are the degeneracies for the transverse and longitudinal modes respectively, this yields,
\[
\Gamma_{T,L}^{\text{abs}} = \frac{2\pi^2}{\omega |\hat{k}|} \frac{d\Gamma_{T,L}^{\text{prod}}}{d\omega dV} e^{\omega/T}.
\] (19)

Therefore, according to Eqs. (15), (16), we conclude that the differential production rate of transverse and longitudinal modes of dark vectors, \( d\Gamma_{T,L}^{\text{prod},V}/(d\omega dV) \), can be written as
\[
\frac{d\Gamma_{T,L}^{\text{prod},V}}{d\omega dV} = \kappa_{T,L}^2 \frac{d\Gamma_{T,L}^{\text{prod}}}{d\omega dV}.
\] (20)
Current conservation demands that both \( \text{Re} \Pi_L \) and \( \text{Im} \Pi_L \) are proportional to \( m_V^2 \). Therefore, \( \kappa_L^2 \) will remain \( m_V \)-independent in the small \( m_V \) limit as can be seen from [16] when expressed as

\[
\kappa_L^2 = \frac{\kappa^2}{(1 - \frac{\text{Re} \Pi_L}{m_V^2})^2 + (\frac{\text{Im} \Pi_L}{m_V^2})^2},
\]

which is independent of the details of the production processes.

From Eqs. [16] and [21] it is obvious that in the region \( m_V^2 \ll \text{Re} \Pi_T \), \( \kappa_T^2 \) scales as \( m_V^4 \), whereas \( \kappa_L^2 \) scales as \( m_V^2 \). However, \( \Gamma_L^\text{abs} \), \( \Gamma_L^\text{prod} \) scale as \( m_V^2 \) due to the current conservation. Therefore, the production rate of the transverse modes is suppressed by \( m_V^4 \) in the small \( m_V \) limit whereas the production rate of the longitudinal mode is only suppressed by \( m_V^2 \), in agreement with our qualitative discussion in the previous section. To avoid confusion we would like to stress again that \( \Gamma_L^\text{abs} \) describes the damping of the longitudinal photon modes, and not what one would call plasmons. The damping of plasmons will be \( m_V^2 \)-independent, and can be obtained from \( \Gamma_L^\text{abs} \) upon the appropriate rescaling. From Eq. [16] one can see that \( \Gamma_{T,L}^\text{prod,V} \) reaches resonance at \( m_V^2 = \text{Re} \Pi_{T,L} \), and can be written as

\[
\frac{d\Gamma_{T,L}^\text{prod,V}}{d\omega dV} = \frac{\kappa^2 m_V^4 \sqrt{\omega^2 - m_V^2}}{2\pi(\omega/T - 1)} \delta(m_V^2 - \text{Re} \Pi_{T,L}),
\]

which is independent of the details of the production processes.

The resonant production can be understood as a thermal bath of photons which slowly transits into dark photons. The transition amplitude for this can written as

\[
\mathcal{M}_{T,L} = -\kappa m_V^2 \left(1 - \frac{\partial \text{Re} \Pi_{T,L}}{\partial \omega^2}\right)^{-1/2},
\]

where the factor \( (1 - \frac{\partial \text{Re} \Pi_{T,L}}{\partial \omega^2})^{-1/2} \) comes from the wave function renormalization of the photon field due to the thermal correction. Therefore, the transition rate for a single transverse photon or longitudinal plasmon with four-momentum \( k = (\omega, |\vec{k}|) \) can be written as

\[
\Gamma_{T,L}^\text{trans} = \frac{1}{2\omega} \int \kappa^2 m_V^4 \left(1 - \frac{\partial \text{Re} \Pi_{T,L}}{\partial \omega^2}\right)^{-1} d\Phi_1,
\]

where \( d\Phi_1 = (2\pi)^4 \delta^4(k - p) \frac{dp}{(2\pi)^3} \) is the one particle phase space, and \( p \) is the four-momentum of the outgoing dark photon satisfying the on shell condition \( p^2 = m_V^2 \). One finds,

\[
\Gamma_{T,L}^\text{trans} = \frac{\pi \kappa^2 m_V^4}{\omega|1 - \partial \text{Re} \Pi_{T,L}/\partial \omega^2|} \delta(\omega^2 - |\vec{k}|^2 - m_V^2).
\]

Therefore, the production rate of dark photon can be written as

\[
\frac{d\Gamma_{T,L}^\text{prod,V}}{dV} = \int d^3k \frac{1}{(2\pi)^3 e^{\omega/T} - 1} \Gamma_{T,L}^\text{trans}
= \int \frac{|\vec{k}| d|\vec{k}|^2}{4\pi \omega} \frac{\kappa^2 m_V^4 \delta(\omega^2 - |\vec{k}|^2 - m_V^2)}{e^{\omega/T} - 1} \left|1 - \frac{\partial \text{Re} \Pi_{T,L}}{\partial \omega^2}\right|
= \int d\omega d^3k \frac{|\vec{k}| d|\vec{k}|^2}{4\pi \omega} \frac{\kappa^2 m_V^4 \delta(\omega^2 - |\vec{k}|^2 - m_V^2)}{(e^{\omega/T} - 1)},
\]

where
Figure 2: Upper limits on the kinetic mixing parameter $\kappa$ vs. the mass of the dark photon $m_V$. The solid blue and red curves are the total constraints on the dark photon parameter space from the sun (blue) and horizontal branch stars (red). The dashed curves show constraints from retaining only the longitudinal resonance contributions. For comparison, the dot-dashed curve shows the upper limit by considering only the contribution from transverse modes and the dotted curve shows the constraint from the CAST experiment by only considering the contribution from the transverse mode $[9]$. The current bounds from the latest result of the LSW experiment by the ALPS collaboration $[25]$ is shown in gray, accompanied by the potential reach (yellow region) of the next generation of LSW experiments $[26]$.

where the on shell conditions for the transverse photon and longitudinal plasmon are used in the last step. Upon integration over $|\vec{k}|^2$ we reproduce Eq. (22).

Finally, for a finite size thermal system, the radiation power into the dark photons can be written as

$$ P = \int dV \int d\omega \left( \kappa_T^2 \frac{\omega d\Gamma_T^{\text{prod}}}{d\omega dV} + \kappa_L^2 \frac{\omega d\Gamma_L^{\text{prod}}}{d\omega dV} \right), $$

(27)

In the next section, we use (27) to calculate the $V$-emission rate as a function of $m_V$. 

Solar luminosity and horizontal branch stars constraints on dark vectors

Inside the sun, the electrons are non-relativistic and non-degenerate. In this limit, the leading order terms in a $T/m_e$ expansion of $\text{Re} \, \Pi_{T,L}$ can be written as

$$\text{Re} \, \Pi_T = \omega_p^2, \quad \text{Re} \, \Pi_L = \omega_p^2 \left( 1 - \frac{|\vec{k}|^2}{\omega^2} \right),$$

and the plasma frequency is given by

$$\omega_p^2 = \frac{e^2 n_e}{m_e} = \frac{4\pi \alpha n_e}{m_e},$$

where $n_e$ is the number density of electrons.

The production rate of the transverse modes is thoroughly calculated in Ref. [9], which we have checked and agree with. As already mentioned multiple times, the production of longitudinal modes is treated incorrectly in [9]. We believe that one can trace the error to $\Pi_{L}^{\text{Ref}}$ defined as $\omega_p^2 - |\vec{k}|^2$ in Eq. (8) of Ref. [9]. This is unphysical, as $\Pi_L$ generated by the plasma must vanish in the $n_e \to 0$ limit. We can only speculate that this error crept in from an inaccurate translation of the results from Ref. [18] obtained in the Coulomb gauge to a generic covariant gauge.

The production rate of the longitudinal modes has contributions both from bremsstrahlung and from Compton scattering. Inside the Sun and a horizontal branch stars the bremsstrahlung processes are dominant. In the non-relativistic and non-degenerate limit the cross section for the bremsstrahlung process to produce the longitudinal massive photons at $k^2 = m_V^2$ can be written as

$$\frac{d\sigma}{d\omega} \bigg|_{\text{brem}} = \frac{\chi^2 Z_i^2 e^6 m_V^3}{3(2\pi)^3 m_e^2 \omega^4 v^2} \log \frac{v + \sqrt{v^2 - 2\omega/m_e}}{v - \sqrt{v^2 - 2\omega/m_e}},$$

where $v$ is the velocity of the incoming electron. Then, appropriately averaging over the electron energy distribution, the production rate (i.e. the production rate of longitudinal dark vectors, when multiplied by the $\kappa_L^2$ factor) can be written as

$$\frac{d\Gamma_{\text{prod}}}{dV d\omega} \bigg|_{\text{brem}} = \sum_i \frac{8 Z_i^2 \alpha^3 n_e n_{Z_i} m_V^2}{3 m_e^2 \omega^4} \sqrt{\omega^2 - m_V^2} \sqrt{8 m_e \pi T} f \left( \sqrt{\frac{\omega}{T}} \right),$$

where $n_{Z_i}$ is the number density of ions of charge $-Z_i e$, and

$$f(a) = \int_a^\infty dx \, xe^{-x^2} \log \left| \frac{x + \sqrt{x^2 - a^2}}{x - \sqrt{x^2 - a^2}} \right|.$$

In principle, inside the plasma, Eq. (30) should be modified to take into account the Debye screening effect characterized by the Debye screening length $\lambda_D^2 \equiv T/(e^2 n_e)$. However,
inside the sun, a numerical study shows that the square of the typical momentum transfer of electrons $|q|^2 \sim m_e T$ is much larger than $1/\lambda_D^2$, and therefore this effect is of little importance for our level of rigour.

Substituting the explicit form of $\text{Re} \Pi_{T,L}$ into the resonant condition $m_V^2 = \text{Re} \Pi_{T,L}$ discussed in Sec.3 we observe that the production rate of the transverse mode and longitudinal modes of dark vectors reach a resonance at $m_V = \omega_p$ and $\omega = \omega_p$, respectively. Therefore, for $m_V \ll \omega_p$ the longitudinal mode of dark photon can be produced on resonance at any temperature. The resonant contribution to the radiation power per unit volume and per unit frequency can be written as

$$\frac{dP_L}{dV d\omega} \approx \frac{1}{4\pi} \frac{\kappa^2 m_V^2 \omega_p^3}{e^{\omega_p/T} - 1} \times \delta(\omega - \omega_p),$$

which is independent of the details of the production processes. Therefore, the resonant contribution to power emitted into longitudinal dark vectors can be written as

$$P_\odot|_{\text{res}} \approx \kappa^2 m_V^2 \int_0^{R_\odot} r^2 dr \frac{\omega_p^3(r)}{e^{\omega_p(r)/T} - 1}.$$

where $R_\odot$ is the radius of the sun. The corresponding results in Ref. [9] contain an additional factor of $2/\pi \times m_V^2/\omega_p^2$ under the integral, which we argue is wrong.

We use the standard solar model BP05(OP) [20] to calculate the total power radiated into dark photons. Consistency of this model with observations, requires this additional "dark radiating power" be smaller than the actual measured solar luminosity [21, 22], which is $L_\odot = 3.83 \times 10^{26}$ Watt. This sets the limit on the parameter space of the model, that we plot in Fig. 2. The solid blue curve shows the resulting constraint, and everything above this curve is excluded. For comparison, we also show the breakdown of the constraint by different contributions. The dashed blue curve shows the constraint resulting from retaining only the resonant contribution in the longitudinal modes. We can see that in the small $m_V$ region ($m_V < 0.1$ eV), the dark radiation is indeed dominated by the longitudinal resonance contribution where the constraint can be simplified as

$$\kappa \times \frac{m_V}{\text{eV}} < 1.4 \times 10^{-11}.$$  

The dot-dashed brown curve shows the constraint by considering only the contribution from the transverse mode, which coincides with the solid blue curve in the large $m_V$ region, $m_V > 10$ eV, where we are also in agreement with [9]. However, in the small $m_V$ region, the emission of $T$-modes gives only a subdominant contribution, and indeed the solid blue curve positions itself much below the brown dot-dashed curve. Therefore, our paper greatly improves the bounds on dark photons below the mass range of a few eV.

For the horizontal branch stars, we set the constraint by requiring that energy loss into dark photons should not exceed the nuclear energy generation rate. Thus, the radiation power into dark photons in the nuclear reaction region of the horizontal branch stars should not exceed $10^{-5}$ Watt gram$^{-1}$ [21, 23]. The average temperature and density of the nuclear reaction region of horizontal branch stars can be estimated as

$$T_{\text{HB}} = 10^8 K, \quad \rho_{\text{HB}} = 10^4 \text{gram/cm}^3.$$  

(36)
In this region, the electron gas can still be viewed as non-relativistic and non-degenerate\cite{24}. The solid red curve in Fig. 2 shows the upper limit on $\kappa$ by requiring the dark radiation power is smaller than the nuclear reaction power. The dashed red curve shows the upper limit by considering only the resonant longitudinal contribution. One can see that in the region $m_V < 100$ eV, the radiation is dominated by the longitudinal mode. The spike at $m_V \approx 2$ keV is caused by the resonant production of the transverse modes which appears at $m_V^2 = \omega_p^2$. Overall, we find that the solar luminosity provides a somewhat stronger constraint for all masses below several 100 eV.

5 Summary and Discussions

We have shown that the production rates of massive dark vectors $V$ with the St"uckelberg mass $m_V$, coupled to the SM via the kinetic mixing portal, scales as $m_V^2$ at small $m_V$ due to the emission of the longitudinal modes of $V$. This drastically change the strength of the stellar constraints in the small $m_V$ region. Thus, for the first time, and despite the large abundance of literature on dark photons, our paper sets correct stellar constraints on the dark photon parameter space in the whole mass range below a few eV. This turns out to be a region of special interest for LSW experiments. In Fig. 2, we show that even the most advanced ones (ALPS)\cite{25} find themselves inside a deeply excluded region. Recalling that the signal of dark photons in LSW experiments scales as $\kappa^4$, a two order of magnitude gap in $\kappa$ at $m_V \sim 0.01$ eV between stellar constraints and LSW region translates into a required eight orders of magnitude improvement in sensitivity before LSW experiments become competitive with stellar bounds. Indeed, a large part of the sensitivity region for the next generation of LSW experiments, deemed reachable in Ref. \cite{26}, is also excluded.

We conclude with several additional remarks pertaining to light dark photons:

- The correct scaling with $m_V$, the $m_V^2$-behaviour, is important not only for the production, but also for the detection of dark photons in the laboratory environment. Taking an example of dark matter detectors, made of some material with refractive index $n$, one can expect that in the region $m_V^2 \ll \omega^2 |1 - n|$, the absorption of the transverse mode scales as $m_V^4$, whereas the absorption of the longitudinal mode scales as $m_V^2$. Therefore, in this region one expect that the longitudinal mode will dominate the observed signal. The details of setting constraints on the solar dark photons with the use of the most sensitive low-energy threshold dark matter detectors will be addressed in our next paper\cite{15}.

- It has to be emphasized that in this paper we assume that, once produced, the dark photon freely escapes the stellar interior. It is possible that for large values of mixing angles, the energy loss process is quenched because of absorption. Determining whether such ”islands” actually exist deep inside the excluded region, and if so whether they survive other constraints goes outside the scope of the present paper.

- Stellar constraints derived in this paper have implications for the “dark CMB”\cite{7}. We do not expect that the emission of the longitudinal modes in processes like $\gamma_T + e \rightarrow$
$V_L + e$ in the primordial plasma will drastically change the estimates for the total energy density locked in $V$ modes, because of the very small value for $n_e/n_\gamma \simeq 6 \times 10^{-10}$. The combination of our new stellar constraints derived in this paper, and the constraints from spectral distortions of the normal CMB in the very small $m_V$ region prevents generating large deviations of the effective number of relativistic degrees of freedom $N_{eff}$ from its standard model value.

- Finally, we can go back to the picture of the “Higgsed” dark photon production, Eq. [2], and ask the question of whether one can avoid strong constraints on $\kappa$ by choosing very small $\alpha'$, and by having the solar interior restore the symmetry in U(1)$_V$ sector, so that $m_V = 0$ inside the sun. The symmetry restoration may occur for $\kappa \omega_p > m_V$, which can have an overlap with the LSW regions of interest. Such an unusual “chameleonic” scenario deserves a special analysis.

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