Time evolution of the space-charge sheath in an rf hollow cathode

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Abstract. We have earlier shown that the inclusion of secondary and fast electrons, in addition to slow, thermalised electrons, give a wider space-charge sheath than with just slow electrons in an atmospheric pressure radio frequency hollow cathode. In this work we extend the previous work by including collisions between ions and neutrals by considering the motion of the ions to be mobility limited. To account for the slow and the fast electrons observed in hollow cathodes, we include two groups of Boltzmann distributed electrons with different temperatures. In addition, we include secondary electrons created at the electrode surface. We look at the time evolution of the potential in the sheath and we also compare the sheath structures for atmospheric and low gas pressures.

1. Introduction

Hollow cathodes are known as sources of high density plasmas. This characteristic is often ascribed to the so-called hollow cathode effect (HCE). A generally accepted principle of the HCE can be described as the oscillation of fast electrons between repelling potentials of space-charge sheaths at opposite walls inside the hollow cathode. This principle also explains the experimental fact that the HCE requires an optimal distance between the walls at different gas pressures, because the dimensions and properties of sheaths are pressure dependent [1]. The question is then what the dimension of the hollow cathode should be. Distances between the walls of 500 µm, and possibly even larger, have been reported in experiments [2, 3, 4].

In a previous work [5], we have shown that the space-charge sheath in a hollow cathode could have a thickness on the order of 100 µm. In this article, we develop our previous model further and include collisions, as well as extend it to two dimensions, to show that the sheath thickness still is on the order of 100 µm.

2. Sheath model

In the negative glow of the hollow cathode, one can identify two groups of electrons: slow and fast electrons [6]. The fast electrons are created at the cathode surface in secondary processes, and are subsequently accelerated in the sheath potential. They then enter the negative glow, make a few collisions, are reflected in the opposite sheath, and so they oscillate until they have lost most of their energies and join the group of slow electrons.

In our model, we also have positive argon ions and neutral argon atoms. The neutrals are assumed to be homogeneously distributed throughout the volume with a density \( N \). We take the computational domain to be in two dimensions and in the shape of a rectangle, with two sides (opposing each other) being the cathode and the other two sides being the anode.
Looking only at the sheath, we can discern four groups of particles: positive ions, and slow, fast, and secondary electrons [5]. The neutral gas is at atmospheric pressure, so the ions are modeled as highly collisional. Hence, the first and secondary moments of the Boltzmann equation for the ions, with density $n_i$, reduce to the drift-diffusion equations

$$\frac{\partial n_i}{\partial t} + \frac{\partial \Gamma_i}{\partial x} = S_i, \quad (1)$$

and

$$\Gamma_i = -D_i \frac{dn_i}{dx} - n_i \mu_i \frac{d\phi}{dx}, \quad (2)$$

where $D_i$ and $\mu_i$ are the ion diffusivity and mobility constants, respectively, and $S_i$ is the ion source function for ions. The source function is assumed to be homogeneous throughout the plasma volume and equal to the flux of ions out of the volume. A more accurate model should take into account the ionisation inhomogeneity in the plasma, depending mostly on the electron-neutral collisions. Instead of doing this by calculating the electron energy distribution from the Boltzmann equation, we assume homogeneous ionisation and that we have electrons with given energy distributions, and look at how the inclusion of fast electrons would change the sheath potential. The plasma is assumed to be quasi-neutral, and so the electron and ion densities are equal at the plasma/sheath boundary, $n_e = n_i = n_0$. The boundary condition for the ion flux at the surfaces is zero density gradient, $\frac{dn_i}{dx} = 0$, and so $\Gamma_i = -n_i \mu_i \frac{d\phi}{dx}$ there. If the ion velocity is away from the surface, $-n_i \mu_i \frac{d\phi}{dx} = 0$. All constants used in the equations are listed in table 1. Both the slow and the fast electrons are considered to be Maxwell distributed, following the Boltzmann relation in the sheath. For the slow electrons, we have

$$n_{\text{slow}} = n_{\text{slow},0} \exp \left( \frac{e\phi}{kT_{e,\text{slow}}} \right), \quad (3)$$

where $n_{\text{slow},0}$ is the slow electron density in the quasi-neutral plasma, $\phi$ is the potential, $k$ is the Boltzmann constant, and $T_{e,\text{slow}}$ is the slow electron temperature. The density profile of the slow electrons in the sheath showed no significant difference when the electrons were modeled with the drift-diffusion equations compared with when they were assumed to follow the Boltzmann relation.

The fast electrons are assumed to have a density $\beta$ times the total electron density, and a temperature $\Theta$ times the slow electron temperature, and so

$$n_{\text{fast}} = \beta n_0 \exp \left( \frac{e\phi}{kT_{e,\text{fast}}} \right) = \beta n_0 \exp \left( \frac{e\phi}{k\Theta T_{e,\text{slow}}} \right). \quad (4)$$

Adding the slow and the fast electron distributions together, we get a total distribution consisting of a group of slow electrons with a high energy tail (if $\Theta$ is sufficiently large).

**Table 1.** Constants used in the model.

| Parameter | Value |
|-----------|-------|
| Ion diffusivity [7] | $ND_i = 0.8 \times 10^{18} \text{ (cm s)}^{-1}$ |
| Ion mobility [7] | $N\mu_i = 3.6 \times 10^{19} \text{ (V cm s)}^{-1}$ |
| Slow electron temperature [8] | $T_{e,\text{slow}} = 3 \text{ eV}$ |
| Secondary electron emission coeff. [9] | $\gamma = 0.2$ |
| Secondary electron emission temperature [10] | $E_w = 3 \text{ eV}$ |
| rf voltage | $V_{rf} = 500 \text{ V (1 kV peak-to-peak)}$ |
| rf frequency | $\omega_{rf}/2\pi = 13.56 \text{ MHz}$ |
We model the secondary electrons as a monoenergetic beam emanating from the cathode. The continuity equation for them, \( \frac{d}{dx}(n_{\text{sec}} v_{\text{sec}}) = 0 \), yields
\[
n_{\text{sec}} = \frac{j_{\text{sec}}}{e v_{\text{sec}}},
\]
where \( j_{\text{sec}} \) is the current density of secondary electrons at any point in the sheath and \( v_{\text{sec}} \) is the secondary electron velocity. The secondary electron current is related to gamma processes at the surfaces, e.g. to the impinging rate of ions, electrons, neutrals, metastables, photons, etc. However, to distinguish the contribution of each gamma process would be rather difficult. To determine the secondary electron current, we only take into account the effect of impinging ions, and take the secondary electron emission coefficient to be \( \gamma = 0.2 \).

Together with the energy conservation equation \( (m_e v_{\text{sec}})(d v_{\text{sec}}/dx) = e(d\phi/dx) \), equation (5) gives
\[
n_{\text{sec}} = \frac{j_{\text{sec}}}{e v_{\text{sec}}} \left(2e/m_e (\phi_w - \phi) + E_w/m_e\right)^{-1/2},
\]
where \( \phi_w \) is the potential at the surface, \( m_e \) is the electron mass, and \( E_w = m_e v_w^2/2 \) is the energy with which the secondary electrons are born. At the surfaces, we have that \( j_{\text{sec}} = \gamma n_i e v_w \). The secondary electron density in the quasi-neutral plasma is \( n_{\text{sec}0} = (j_{\text{sec}}/e)(2e/m_e \phi_w + E_w/m_e)^{-1/2} \). We can now write the slow electron density in the quasi-neutral plasma as \( n_{\text{slow0}} = n_0 (1 - \beta - n_{\text{sec0}}/n_0) \).

To close the equations (1) through (4) and (6), Poisson’s equation
\[
\frac{d^2\phi}{dx^2} = \frac{e}{\epsilon_0} [n_i - (n_{\text{slow}} + n_{\text{fast}} + n_{\text{sec}})]
\]
is employed with boundary conditions \( \phi_w = V_{\text{rf}} \sin(\omega_{\text{rf}} t) + \phi_{\text{bias}} \) at the cathode surfaces, and \( \phi_w = 0 \) at the anodes. \( V_{\text{rf}} \) is the rf voltage amplitude and \( \phi_{\text{bias}} \) is the self-bias, which ensures that the cathode is always negative with respect to the plasma in our model, and \( L \) is the hollow cathode dimension.

The equations were solved with the commercial software COMSOL Multiphysics.

3. Results
Figure 1 shows the potential at atmospheric pressure (760 Torr) for one rf period without any secondary and fast electrons. The plotted values are from a cross-section in the middle of the computational two-dimensional domain. The sheath thickness is, as seen, about 200 \( \mu \)m. A comparison between the potential distributions at maximum cathode potential for the cases without and with secondary and fast electrons is shown in figure 2. At lower pressures, the increase of the width of the sheath potential at the inclusion of secondary and fast electrons is more apparent, see figure 3 and 4.

4. Conclusions
In a previous model [5], we showed that the sheath thickness in an atmospheric pressure rf hollow cathode could be of the order of 100 \( \mu \)m. In this work, we have refined the model by making the ions mobility-limited, as they should be at high pressures, and we have extended the model to two dimensions in order to solve self-consistently for the hollow cathode geometry. The model presented here has shown that the sheath thickness is about 200 \( \mu \)m. Adding secondary and fast electrons to the population of ions and slow electrons increases the sheath thickness. This effect has been shown to increase with decreasing pressure.

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Figure 1. The potential distribution at the mid-point cross-section of the computational domain at 760 Torr, for a whole rf period, without secondary and fast electrons. \( n_0 = 10^{13} \text{ cm}^{-3} \).

Figure 2. The potential distribution at maximum cathode potential for the cases without (-----) and with secondary and fast electrons (- - - -) at atmospheric pressure (\( \beta = 0.08 \), \( \Theta = 130 \)), \( n_0 = 10^{13} \text{ cm}^{-3} \).

Figure 3. The potential distribution at maximum cathode potential for the cases without (-----) and with secondary and fast electrons (- - - -) at 100 Torr (\( \beta = 0.08 \), \( \Theta = 130 \), \( n_0 = 10^{12} \text{ cm}^{-3} \)). Note the change of scale.

Figure 4. The potential distribution at maximum cathode potential for the cases without (-----) and with secondary and fast electrons (- - - -) at 1 Torr (\( \beta = 0.08 \), \( \Theta = 130 \)), \( n_0 = 10^{11} \text{ cm}^{-3} \). Note the change of scale.

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