Spin-wave non-reciprocity in magnetization-graded ferromagnetic films

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Abstract

A theoretical approach has been developed to study the spin-wave dynamics of magnetization-graded ferromagnetic films, where the magnetic properties change along the film thickness. The theory is based on a multilayer approach, where the influence of both long-range dipolar interactions and interlayer exchange coupling between sublayers is included. This allows for instance to describe films with a continuous variation of the saturation magnetization along the thickness. A systematic study is carried out in order to analyze different profiles of the saturation magnetization, which is checked through a test of convergence. It is found that the spin-wave dispersion is significantly modified when the strength of the magnetization changes in the bulk film, where a notable frequency non-reciprocity of two counter propagating spin waves is predicted. This is associated with heterosymmetric mode profiles and a modification of the conventional quantization condition associated to perpendicular standing spin-wave modes. Micromagnetic simulations have been carried out to validate the model, where a perfect agreement is reached between both methods. These results show that magnetization-graded ferromagnetic films can be used to channel and control spin waves, thus promoting different kinds of functionalities for magnon-based devices.

1. Introduction

Spin-wave (SW) dynamics within nanoscale ferromagnetic (FM) films has been a topic of extensive research in the last decades [1–4]. One of the scientific interests is based on promising applications for signal processing devices, since the SW wavelength is shorter than those of electromagnetic waves in the GHz frequency range [2, 5], allowing for better prospects for the miniaturization of magnonic devices [6]. This, added to the fact that the magnon bandstructure can be enriched with the artificial modulation of the magnetic media [7–18], has stimulated experimental and theoretical advances in the emerging field of magnonics. Recently, it has been theoretically predicted and subsequently proven experimentally, that such waves in FM thin films can couple to a heavy-metal layer with strong spin–orbit coupling, as evidenced by a considerable frequency non-reciprocity due to the interfacial Dzyaloshinskii–Moriya interaction [19–26]. This behavior opens new functionalities for magnon-based devices, such as SW logic [27], SW caustic with highly non-reciprocal focusing patterns induced by a single point source [28], non-trivial localized magnetic excitations [29, 30], as well as the possible realization of isolators and circulators [31, 39]. Magnetic thin films with engineered lateral asymmetry, leading to an anisotropy gradient, has been recently proposed for skyrmion control [32], and for field-free magnetization reversal, owing to a new field-like spin–orbit torque [33]. The frequency non-reciprocity can be also induced by dipolar interaction, as has been predicted in arrays of coupled magnetic nanopillars [31], magnetic nanotubes [34], and FM films where the spatial symmetry is broken along the thickness [35–38]. The latter systems suggest
that the dipolar interaction, which induces non-reciprocity, becomes prominent in FM layers with some kind of gradation along the thickness, which is the focus of interest in this paper.

Ferromagnetic thin films with graded properties have been experimentally fabricated by alloying magnetic and non-magnetic materials via co-sputtering. The desired depth-dependent composition can be achieved by tuning the relative rate of deposition of each material during growth \([40-45]\). The above mentioned technique is nowadays a well-established strategy to obtain depth-dependent magnetic anisotropy within a film (exchange spring systems), motivated by the demand of the magnetic recording industry of bitcells with a combination of low coercivity, to facilitate data writing, while ensuring good thermal stability \([46-48]\). Besides anisotropy, the Curie temperature and saturation magnetization can be controlled as a function of depth introducing different concentrations of Cr or Ru dopants, allowing to manipulate the magnetization reversal process \([45]\). However, the influence of a continuous variation of the magnetic properties (along the film thickness) on the SW spectra and the discussion about the nature of the modes have not been addressed so far, and constitutes the main topic investigated here.

In this paper, the SW dynamics on magnetization-graded FM films is studied by means of a theoretical model and micromagnetic simulations. The SW spectrum is obtained by splitting the graded film into \(N\) sublayers, in such a way that after a given quantity of sublayers it is demonstrated that the model is able to describe the SW dynamics of a FM film with a continuous variation of the saturation magnetization along the thickness. Different profiles are analyzed, where the localization as well as non-reciprocal features are discussed. The manuscript is organized as follows. In section 2 together with the appendix, the details of the theory are presented, while the micromagnetic simulations are introduced in section 3. Section 4 is devoted to the discussion of the main results and concluding remarks are outlined in section 5.

2. Theoretical model

The main idea behind this work is sketched in figure 1. Here, an FM film with graded magnetization (b) is emulated by a multilayer system (a), where a variation of the saturation magnetization along the thickness is considered. By increasing the number of sublayers, a film with a continuous variation of the saturation magnetization is regained. The SW dynamics of a multilayer system has been studied theoretically by Henry et al. \([37]\), where different slabs along the thickness were connected through dipolar and exchange interactions. They discussed in particular Fe/Pt bilayers, where a non-reciprocal behavior of the SWs was obtained. In the present manuscript, the idea is to apply a similar procedure to describe the SW dynamics, but assuming general expressions for all effective fields in such a way that any profile of the graded magnetization can be addressed. To do this, a relation between the strength \(J\) of the interlayer exchange coupling, which is defined from the discrete definition of the exchange, is linked with the exchange constant \(A_{\text{ex}}\), defined for a continuous variation of the magnetic moments (see \([49]\) for details). This relation can be obtained by analyzing the discrete definition of the interlayer exchange interaction through the thickness, since if the number of sublayers increases the variation of the dynamic magnetization along the thickness behaves like continuous. Thus, it can be demonstrated that \(J = 2A_{\text{ex}}/d\) \([49]\), where \(d\) is the thickness of the sublayer, so that once the magnetic material is defined, both \(A_{\text{ex}}\) and \(J\) are given. Besides, since the goal is to describe the continuous variation, a convergence test is used to effectively describe a system with these particular continuous characteristics, which are supported by micromagnetic simulations.

The dynamics of the system is described in the framework of the Landau–Lifshitz Gilbert (LLG) equation \([2]\), which for the \(n\)th sublayer reads \(\dot{M}_n(t, t) = -\gamma\mu_0 M_n(t, t) \times \mathbf{H}_n(t, t) + \alpha/M_n(t, t) \times \dot{M}_n(t, t),\) where the dot represents the time derivative, \(M_n\) is the saturation magnetization of sublayer \(n\), \(\alpha\) is the dimensionless Gilbert damping parameter, \(\gamma\) is the magnitude of the gyromagnetic ratio and \(\mathbf{H}_n(t, t)\) is the effective field acting on sublayer \(n\), which takes into account all the relevant energetic contributions. Note the damping term \(\alpha\) does not significantly influences the bandstructure of the SWs, therefore it will be left aside in the rest of the
4. Results

MuMax3

To validate the theoretical model, micromagnetic simulations were performed using the GPU-accelerated code MuMax3 [50]. Here \( d_i \) is the thickness of the total film. Periodic exchange boundary conditions were applied along the \( x \)- and \( z \)-directions in order to mimic an extended film, while the dipole–dipole interaction was taken into account by using the macro geometry approach (see [50] and references therein). The cell size of the system was \( 4.88 \times 4.00 \times 2.00 \) nm\(^3\). A damping constant of \( \alpha = 0.01 \) has been used, while the material as well as the geometry have been chosen according to the parameters used in the theory. By applying the field \( h = \frac{\sin(k_0 z) \sin(2\pi \nu t)}{2k_0^2} \) SWs are excited with a sinc-pulse in space with the cut-off wavelength \( \lambda_0 = 2\pi/k_0 = 9.77 \) nm, and a cut-off frequency of \( f_0 = 50 \) GHz. Note that the \( y \)-direction is perpendicular to the equilibrium magnetization, which points along \( z \) (see figure 1). The SW dispersion relation was calculated by storing the magnetization configuration every 10 ps for 25 ns and afterwards transforming the data using a two-dimensional fast-Fourier transform.

4. Micromagnetic simulations

To validate the theoretical model, micromagnetic simulations were performed using the GPU-accelerated code MuMax3 [50]. For this, a magnetic stripe with 20 \( \mu m \) length, 64 nm width and \( d_i = 60 \) nm thickness was considered. Here \( d_i \) is the thickness of the total film. Periodic exchange boundary conditions were applied along the \( x \)- and \( z \)-directions in order to mimic an extended film, while the dipole–dipole interaction was taken into account by using the macro geometry approach (see [50] and references therein). The cell size of the system was \( 4.88 \times 4.00 \times 2.00 \) nm\(^3\). A damping constant of \( \alpha = 0.01 \) has been used, while the material as well as the geometry have been chosen according to the parameters used in the theory. By applying the field \( h = \frac{\sin(k_0 z) \sin(2\pi \nu t)}{2k_0^2} \) SWs are excited with a sinc-pulse in space with the cut-off wavelength \( \lambda_0 = 2\pi/k_0 = 9.77 \) nm, and a cut-off frequency of \( f_0 = 50 \) GHz. Note that the \( y \)-direction is perpendicular to the equilibrium magnetization, which points along \( z \) (see figure 1). The SW dispersion relation was calculated by storing the magnetization configuration every 10 ps for 25 ns and afterwards transforming the data using a two-dimensional fast-Fourier transform.

4. Results

To describe the SW spectra of graded-magnetization films, typical parameters for Permalloy were used; \( A_{ex} = 11 \) pJ m\(^{-1}\), and \( \gamma/2\pi = 28.02 \) GHz T\(^{-1}\). In the cases where the saturation magnetization is varied, it will change from \( M_s = 800 \) kA m\(^{-1}\) up to \( M_s = 1600 \) kA m\(^{-1}\). On the other side, for the case of constant \( M_s \) a saturation magnetization of 800 kA m\(^{-1}\) will be assumed. To analyze the influence of the thickness, an FM film with \( d_i = 60 \) nm is assumed. In order to corroborate the theoretical results, the case of uniform saturation magnetization \( (M_s = 800 \) kA m\(^{-1}\)) will be analyzed first. Thus, by means of a convergence method it will be demonstrated that the theory is able to describe the SW dynamics of a thick ferromagnetic film. The case of uniform saturation magnetization is depicted in figure 2(a), where the low-frequency mode is illustrated for an external field \( \mu_0 H_0 = 1.5 \) mT applied along the \( z \)-direction. This field value is chosen as the minimal stabilization field in the micromagnetic simulations. It is worth mentioning that if the external field increases in the range of hundreds of mT, the frequency shift is not notoriously modified. Nevertheless, if the external field is much higher (comparable with \( \mu_0 M_s \)) the Zeeman energy dominates the dynamic behavior, in such a way that the non-reciprocal effects are reduced. Here, the FM film has been modeled as a single layer with \( N = 1 \) (dotted line), two layered couples with \( N = 2 \) (dotted–dashed line), etc. The different cases quickly converge as \( N \) increases, as shown in the inset in figure 2(a), where the frequency reaches a constant value around \( N = 5 \). At \( N = 1 \), the dispersion assumes constant dynamic magnetization across the film. However, when the modulation along the thickness is allowed by increasing \( N \), the dispersion changes. The origin for this is that the out-of-phase first higher-order mode becomes less energetic than the coherent one, due to a smaller dipolar contribution. Thus, while at low wave vectors the uniform mode is the less energetic one, at higher wave vectors a kink...
develops (around $|k| > 3 \text{ rad} \mu \text{m}^{-1}$) above which the low-frequency mode is now the out-of-phase one, where the precession of the outer sublayers is out of phase. Note that the kink represents the point around which the uniform and out-of-phase mode have the same energy, and the critical point at which the kink occurs is strongly dependent of the film thickness, since at higher (lower) thicknesses this kink point is given by lower (higher) frequency.

In order to look at the SW precessional amplitudes at different sublayers of the film, the system is divided into $N = 30$ sublayers, in such a way that each sublayer is $d = 2$ nm thick. Thus, the sublayer-dependent precessional amplitudes can be clearly observed in figures 2(b), (c) for $k = \pm 20 \text{ rad} \mu \text{m}^{-1}$, a wave-vector magnitude that lies within the range of Brillouin light scattering measurements. These modes exhibit heterosymmetric profiles along the thickness [37, 51], together with a slightly different quantization condition of the standing waves in the perpendicular direction ($\gamma$-axis), while the SWs are propagating along the film’s plane. In other words, for the first standing mode, the single node does not lie exactly at the center of the film. This behavior agrees with recent results described in [51]. Besides, note that at $|k| = 20 \text{ rad} \mu \text{m}^{-1}$ the modes with $k < 0$ and $k > 0$ exhibit a different profile along the thickness, as they obtain a larger amplitude at the upper (lower) interface at negative (positive) wave vector. The standing character of the SWs can be explained by means of the dynamic dipolar interaction, since in essence this asymmetric localization appears due to the asymmetry of the dipolar energy within the film, which becomes prominent for thick films [36, 37, 52]. Thus, the dynamic magnetization amplitude has a tendency to compensate the dipole field asymmetry by increasing its amplitude on one side of the film [36]. Furthermore, since the dipolar interaction is $k$-dependent, then the node positions and amplitudes of the standing modes are also $k$-dependent, which indicates a connection between propagating (in-plane) waves and the perpendicular standing SWs. If the thickness is reduced (comparable with the exchange length), then the magnetization amplitudes on top and bottom are approximately equal, in such a way that the non-reciprocity is also reduced. This behavior suggests that any breaking of symmetry along the thickness should induce non-reciprocity in frequency for two opposing SWs [36], as long as the FM film is thick enough so that the SW amplitude is non-uniform along the thickness. To corroborate these results, micromagnetic simulations have been carried out as shown in figure 2(d), where the first three modes (labeled as Mode 1, Mode 2 and Mode 3) were calculated by means of both methods in a wider range of wave vectors and frequencies. An excellent agreement is obtained between simulations and the theoretical model, which validates the latter.

After having described the SW dynamics in case of an FM film, the one of a system with graded magnetization will now be addressed. Figure 3 shows the SW spectra for different systems with $N = 30$, where the breaking of symmetry along the thickness has been included through a variation on $M_\parallel$. Three cases are explored: Case 1: a smooth change of $M_\parallel$ whose profile is asymmetric with respect to the the film thickness. Case 2: a smooth change of $M_\perp$ around the center of the film thickness. Case 3: a linear variation of the saturation magnetization along the thickness. In all cases, the low frequency mode (Mode 1) at $k = \pm 20 \text{ rad} \mu \text{m}^{-1}$ corresponds to an out-of-phase one. Case 1 is illustrated in figure 3(a), where the smooth change in $M_\parallel$ clearly influences the SW dynamics, since the symmetry along the thickness has been broken. For waves propagating along $\hat{x}$, the frequency of Mode 1 is reduced in comparison to a counter-propagating wave, so that the localization of the SW is different as well, as shown in the left side of figure 3(a). Interestingly, the second mode...
also manifest non-reciprocity, which can be explained by analyzing the precessional amplitudes of Mode 2, as shown in the right side of figure 3(a). Here, it is easy to note that the second mode at $k = -20 \text{ rad } \mu \text{m}^{-1}$ manifests a coherent precession, with different magnetization amplitudes along the thickness, while for waves propagating at $k = 20 \text{ rad } \mu \text{m}^{-1}$, the mode is the one with two nodes. Thus, it can be inferred that the nature of the non-reciprocity of the second mode for Case 1 is mainly due to the fact that the nature of the SW localization has been radically modified by inverting $k$.

Case 2 is depicted in figure 3(b). For such saturation-magnetization profile, it is evident that the first mode manifests a slightly larger non-reciprocity as compared to Case 1. Nevertheless, Mode 2 is almost symmetric with respect to the wave-vector inversion, which can be understood by analyzing the SW amplitudes at $k = \pm 20 \text{ rad } \mu \text{m}^{-1}$, since for both counter-propagating waves the excitations are of the same nature with similar precession amplitudes. Case 3 is shown in figure 3(c), which is quite similar to Case 2. A possible explanation for this unexpected behavior may be the localization of the maxima of the precessional amplitude, since for both cases these maxima are given in the bottom and top sides of the film, where $M_s$ is the same for Case 2 and Case 3. Note that overall the frequency non-reciprocity is accomplished of a non-reciprocity of the magnetization precession amplitude as well (see top and bottom faces in figures 2(b) and (c)), so that the output signal amplitude (of a given measurement) will be asymmetric respect to the wave-vector inversion. This property is useful for SW logic and switch applications [27, 53] and SW isolators [55], where the SW amplitude is wave-vector dependent.

To deeply understand the non-reciprocity induced by the graduation of $M_s$, the first mode of Case 1 will be analyzed from the point of view of the dipolar interaction. From the precessional amplitudes shown in figures 4(a) and (b), it can be inferred that the role of the bottom and top sublayers is preponderant for the occurrence of SW non-reciprocity. Therefore, the dipolar interaction between layer $n = 30$ and $n = 1$ will be analyzed. The components of the dipolar field acting in sublayer $n$ due to sublayer $p$ are given in equations (A.3), (A.4). Here, it is evident that the terms that induce non-reciprocity are the ones proportional to $ik$, in such a way that if the non-reciprocal parts of these fields are defined as $h_{n \rightarrow p} = -i \frac{k}{|k|} m_n^x(r) \langle \partial G_p / \partial y \rangle_0$ and $h_{p \rightarrow n} = i \frac{k}{|k|} m_n^y(r) \langle \partial G_p / \partial y \rangle_0$, then the relative orientation of

![Figure 3](image_url)

Figure 3. Figure (a) shows the spin-wave dispersion of a $d_t = 60$ nm film for Case 1, where a smooth change of $M_s$ is given in the top of the film. In (b) the Case 2 is depicted, where there is a smooth change of the saturation magnetization at the center of the film. Figure (c) illustrates the Case 3, where a linear variation of the saturation magnetization is given. The saturation-magnetization profiles are shown in the respective insets. The right and left figures show the precessional amplitudes for the respective cases at $k = \pm 20 \text{ rad } \mu \text{m}^{-1}$, where the time $t = 0$ has been highlighted (see filled black dots) to see the phase of the modes. The color code represents the numerical simulations, where the brighter color indicates a maximum of the response.
and the dynamic magnetization of sublayer \(30\) can be analyzed. Figures 4(c) and (d) show the field \( \mathbf{h}_{30}^{fr} \) (see text for details) for \( k > 0 \) and \( k < 0 \), respectively. The dipolar interaction between sublayer \( n = 1 \) and \( n = 30 \), as a function of the wave vector, is depicted in (c), where an evident reduction of the dipolar energy is observed for \( k > 0 \), in agreement with the relative orientations between the dynamic stray field and the dynamic magnetization shown in (c).

This field \( \mathbf{h}_{30}^{fr} \) and the dynamic magnetization of sublayer 30 can be analyzed. Figures 4(c) and (d) show the field \( \mathbf{h}_{30}^{fr} \) (non-reciprocal dipolar field acting on sublayer \( n = 30 \) due to sublayer \( p = 1 \)) and the dynamic magnetization of sublayer \( n = 30 \) at \( k = \pm 20 \text{ rad } \mu \text{m}^{-1} \). At \( k < 0 \), it is noted that the in-plane dynamic magnetization \( \left( m_{x}^{fr} \right) \) is opposite to the field \( \mathbf{h}_{30}^{fr} \) in such a way that the dipolar interaction due to in-plane components increases the energy of the system. Nevertheless, the out-of-plane components reduce the energy, since the dipolar interaction and the dynamic magnetization are parallel. On the other side, at \( k > 0 \) the opposite happens. However, by noting that the in-plane components of the dynamic magnetization and the dynamic stray field are the ones that dominate the interaction between sublayers 1 and 30 \((\chi_{1,30})\), then one can conclude that \( f(-k) < f(k) \). This is shown in figure 4(e), where an evident reduction of the dipolar interaction is given for \( k > 0 \).

Figure 5 shows the frequency shift \( \Delta f \) for Mode 1 as a function of the wave vector for Cases 1, 2 and 3. The insets illustrate the range of \( k \)-values probed by Brillouin light scattering measurements.

\[ \Delta f(k) = f_k - f_{-k} \]

Figure 5 shows the frequency shift \( \Delta f \) for Mode 1 as a function of the wave vector for Cases 1, 2 and 3. The insets illustrate the range of \( k \)-values probed by Brillouin light scattering measurements.

5. Conclusions

In conclusion, the spin-wave dynamics of a ferromagnetic film with graded saturation magnetization has been studied by means of a theoretical model and micromagnetic simulations. It is demonstrated that the properties of the SWs in these films are notoriously modified in comparison to films with uniform magnetization. Interesting non-reciprocal and localization properties have been found, where a notable frequency shift is
predicted, which is attributed to the dipolar interaction that is enhanced when a breaking of spatial symmetry along the thickness is present. An arbitrary profile of the saturation magnetization or any other magnetic parameter (surface anisotropy, exchange constant, etc) can be described with the method developed here. Controlled by the graded profile, it is possible to obtain two counter-propagating waves where the phase of the mode can be radically different, depending on the SW propagation direction. Thus, this work proposes a way to tune the dynamic properties of SWs by introducing graduation of the magnetic properties, which is considered relevant for applications of magnon-based devices.

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Appendix. Effective fields and matrix elements

A.1. Dipolar field

The magnetic potential due to volumetric magnetic charges in the \( p \)th sublayer can be derived as follows. First, the volumetric magnetic charge density is given by \( \rho_p(r) = -\nabla \cdot \mathbf{M}_p(r) \), in such a way that the magnetic potential is

\[
\phi_p(r) = \frac{1}{4\pi} \int \frac{-\nabla \cdot \mathbf{M}_p(r')}{|r - r'|} d^3r' = -i k m_p(k) \frac{1}{4\pi} \int \frac{e^{ikr'}}{|r - r'|} d^3r',
\]

for SWs propagate along the \( x \)-axis, so that \( k = \hat{k} \mathbf{x} \), where \( k \) is the wave vector. Then, the integral in (A.1) is

\[
\int \frac{e^{ikr'}}{|r - r'|} d^3r' = 2\pi e^{ikr} \int_{\xi_p}^{\xi_p + d} e^{-|k||y - \xi|} \frac{|k|}{|k|} dy' = \frac{4\pi}{|k|^2} e^{ikr} G_p(y).
\]

Here, \( \xi_p = (p - 1)d \), with \( d \) being the thickness of the \( p \)th layer. Therefore, the magnetic potential can be written as

\[
\phi_p(r) = -i \frac{k}{|k|^2} m_p(k) e^{ikr} G_p(y), \quad (A.1)
\]

where \( G_p(y) = \sinh(|k|d/2)e^{-|k||y - (\xi_p + d/2)|} \) outside the \( p \)th layer at \( |y - (\xi_p + d/2)| > d/2 \); and

\[
G_p(y) = 1 - e^{-|k|d/2} \cosh(|k||y - (\xi_p + d/2)|) \) inside the \( p \)th layer at \( |y - (\xi_p + d/2)| < d/2 \). On the other side, the magnetic potential due to magnetic surface charges is expressed as

\[
\phi_p(r) = \frac{1}{4\pi} \int \frac{\mathbf{M}_p(r')}{|r - r'|} dS_p',
\]

where \( F_p(r) = \sinh(|k|d/2)e^{-|k||y - (\xi_p + d/2)|} \Theta(|y - (\xi_p + d/2)| - \Theta(|\xi_p + d/2| - |y|)) \) outside the \( p \)th layer at \( |y - (\xi_p + d/2)| > d/2 \); while \( F_p(r) = e^{-|k|d/2} \sinh(|k||y - (\xi_p + d/2)|) \) inside the \( p \)th layer at \( |y - (\xi_p + d/2)| < d/2 \). The element \( \Theta(\nu) \) is the Heaviside function, it is \( \Theta(\nu) = 0 \) for \( \nu < 0 \) and 1 for \( \nu > 0 \). Note that the magnetic potential \( \phi_p(r) \) outside the \( p \)th layer can be averaged into the \( n \)th order in order to take into account the dipolar interaction between the \( p \)th and \( n \)th layers. So that, if

\[
\langle f_p(r)n \rangle = \frac{1}{d} \int_{\xi_p}^{\xi_p + d} f_p(y) dy,
\]

the dynamic components of the dipolar field [derived from \( h_{dn}^{\text{dp}}(r) = -\partial \phi(r) \)] are

\[
h_{dn}^{\text{dp}}(r) = -i \frac{k}{|k|} m_p(k) \langle F_p(r)n \rangle - m_p(r) \langle G_p(r)n \rangle, \quad (A.3)
\]

and

\[
h_{dp}^{\text{dp}}(r) = -i \frac{k}{|k|} m_p(k) \langle \partial F_p/\partial y \rangle_n - m_p(r) \langle \partial G_p/\partial y \rangle_n, \quad (A.4)
\]
where
\[
\langle G_p \rangle_n = \begin{cases} 
\frac{2}{|k|d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k||n-p|d}, & n \neq p \\
1 - \frac{2}{|k|d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k||n|d}, & n = p
\end{cases}
\]
\[
\langle \partial F_p / \partial y \rangle_n = \begin{cases} 
\frac{2}{d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k||n-p|d}, & n \neq p \\
\frac{2}{d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k||n|d}, & n = p
\end{cases}
\]
\[
\langle \partial G_p / \partial y \rangle_n = \begin{cases} 
\frac{2}{d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k||n-p|d} \left[ \Theta(p-n) - \Theta(n-p) \right], & n \neq p \\
0, & n = p
\end{cases}
\]
\[
\langle F_p \rangle_n = \begin{cases} 
\frac{2}{|k|d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k||n-p|d} \left[ \Theta(n-p) - \Theta(p-n) \right], & n \neq p \\
0, & n = p
\end{cases}
\]

A.2. Interlayer exchange field
The nearest-neighbor interlayer exchange energy density can be written as
\[
e_{\text{inter}} = - \frac{J}{M_n M_p} M_n(r) \cdot M_p(r), \tag{A.5}\]
where \(J\) is the interlayer exchange coupling constant between two adjacent layers. Thus, the interlayer exchange field components can be readily derived from equation (A.5). These are
\[
h_{\text{inter}}^{xy} (r) = \frac{J}{M_n d} \sum_n m_n(r) \left( \delta_{n+1}^{p} + \delta_{n}^{p+1} \right), \tag{A.6}\]
\[
h_{\text{inter}}^{yz} (r) = \frac{J}{M_n d} \sum_n m_n(r) \left( \delta_{n}^{p+1} + \delta_{n+1}^{p} \right), \tag{A.7}\]
\[
H_{\text{inter}}^{x} = \frac{J}{M_n d} \sum_n \left( \delta_{n}^{p+1} + \delta_{n+1}^{p} \right). \tag{A.8}\]
Here, \(\delta_{n}^{p}\) is the Kronecker delta function (\(\delta_{n}^{p} = 0\) for \(p \neq n\) and \(\delta_{n}^{n} = 1\) for \(p = n\)).

A.3. Matrix elements
Now, in addition to dipolar and interlayer exchange fields, the Zeeman \((H^{\text{ext}} = H_z)\) and intralayer exchange \([h_{\text{ex}}^{\eta} (r) = -\epsilon^2 k^2 m_{\eta} (r)\), with \(\epsilon\) being the exchange length] fields are included into equations (1) and (2) of the manuscript. Thus, the matrix elements \(A_{\eta \eta'} (\eta = x, y)\) can be readily obtained. For \(n = p\), these matrix elements are:
\[
A_{x,x} = A_{y,y} = 0
\]
\[
A_{x,y} = -H \frac{J}{M_n d} \sum_{j} (\delta_{j}^{p-1} + \delta_{j+1}^{p}) - M_n \epsilon_n^2 k^2
\]
\[- \frac{2M_n}{|k|d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k|x|d/2} \]
\[
A_{y,x} = H \frac{J}{M_n d} \sum_{j} (\delta_{j}^{p-1} + \delta_{j+1}^{p}) + M_n \epsilon_n^2 k^2
\][ + M_n \frac{2M_n}{|k|d} \sinh \left( \frac{|k|d}{2} \right) e^{-|k|x|d/2},
\]
On the other side, for \( n = p \), the matrix elements are

\[
A_{x,y}^{p,p} = -A_{x,y}^{p,p} = -ik \frac{2M_s}{|d|} \sinh^2 \left( \frac{|k|d}{2} \right) e^{-|k||n-p|d} \sgn(n-p) \]

\[
A_{x,y}^{p,p} = \frac{2M_s}{|d|} \sinh^2 \left( \frac{|k|d}{2} \right) e^{-|k||n-p|d} \left( \frac{J_{n-1}^p + \delta_{n+1}^p}{M_n d} \right) \]

Here, \( \sgn(x) = x/|x|, d_i = d_i/N \) is the thickness of each sublayer and \( \epsilon' = \sqrt{2A_{ex}/\mu_0 M_n^2} \) is the exchange length, with \( A_{ex} \) being the exchange constant.

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