Resummation of scalar correlator in higher spin black hole background

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\textbf{ABSTRACT:} We consider the proposal that predicts holographic duality between certain 2D minimal models at large central charge and Vasiliev 3D higher spin gravity with a single complex field. We compute the scalar correlator in the background of a higher spin black hole at order $O(\alpha^5)$ in the chemical potential $\alpha$ associated with the spin-3 charge. The calculation is performed at generic values of the symmetry algebra $\mathfrak{hs}[\lambda]$ parameter $\lambda$ and for the scalar in three different representations. We then study the perturbative data in the large $\lambda$ limit and discover remarkable regularities. This leads to formulate a closed formula for the resummation of the leading and subleading terms that scale like $O(\alpha^n \lambda^{2n})$ and $O(\alpha^n \lambda^{2n-1})$ respectively.
1 Introduction

In the striking paper [1], Vasiliev higher spin theory on $AdS_3$ coupled to a complex scalar [2, 3] has been shown to be holographically dual to the conformal 2D coset conformal minimal models $W_N$ at large $N$. The precise limit that is considered has a large central charge and is parametrised by a single constant $\lambda$ whose gravity meaning is that of labelling a family of $AdS_3$ vacua. The symmetry content of the two sides of the correspondence matches. In particular, the asymptotic symmetry algebra of the higher spin gravity theory is $W_\infty[\lambda]$ [4–7] which is also the classical symmetry of the dual CFT [1, 4].

The bulk theory admits higher spin generalisations of the BTZ black hole [8, 9] as reviewed in [10]. The thermodynamical properties of these very interesting objects are
in agreement with the proposed duality \cite{11, 12}. However, these tests do not probe the dynamics of the higher spin gravity complex scalar that plays no role in matching the black-hole entropy. Indeed, black holes in $AdS_3$ are universal and explore the CFT thermodynamics at high temperature, which in two dimensions is only determined by the chiral algebra and is insensitive to the microscopic features of the conformal theory.

The quantum fluctuations of the complex scalar have been studied in \cite{13} by computing the bulk-to-boundary scalar correlator in the background of the 3D higher spin black hole analysed in \cite{11}, at least to first order in the spin-3 charge chemical potential $\alpha$ and for a special value of the $\lambda$ parameter. Later, the authors of \cite{14}, extended the calculation to $O(\alpha^2)$ and, remarkably, demonstrated explicitly the agreement with a CFT calculation.

In this paper, we considered again the correlator for the scalar field transforming in three different representations of the higher spin algebra. To be precise, this means that one assigns a higher representation $\Lambda_+$ of the symmetry algebra to the scalar dual CFT primary. In the bulk, the Vasiliev master field $C$ obeys the equation $dC + A C - C \bar{A} = 0$ where the higher spin gauge connections $A, \bar{A}$ are in the representation $\Lambda_+$. This implies that, in the bulk, the master field $C$ actually corresponds to an infinite tower of fields with different masses. The details of the spectrum can be worked out by decomposing $\Lambda_+$ into irreducible representations of $sl(2)$ as discussed in \cite{15}. In each case, we computed the propagator at order $O(\alpha^5)$, by applying the powerful methods introduced in \cite{14, 16}. The reason for such a brute-force calculation is the idea that some regularity could be observable once we have enough perturbative data to inspect. In particular, we looked for special features of the large $\lambda$ regime in the spirit of \cite{17}. The idea is that, beyond the large $N$ limit that defines the correspondence, there is another parameter that it is meaningful to take large. Indeed, for $\lambda = -N$, the higher spin calculation truncates to that of a $sl(N) \oplus sl(N)$ Chern-Simons 3D gravity theory. Taking $N$ to be large could lead to some simplifications in the structure of the perturbative corrections as it happens in $SU(N)$ gauge theories. This kind of reasoning proved to be successful in the study of the black hole partition function \cite{17}.

Although optimistic, this simple attitude turns out to be correct, even beyond the leading order. For the three considered representations of the scalar, we show that all corrections of the order $O(\alpha^n \lambda^{2n})$ and $O(\alpha^n \lambda^{2n-1})$ can be resummed in closed form. Also, the extension of the results to a generic $K$-antisymmetric representation $(\Box \otimes K)_A$ appears to be straightforward, once one recognises that the representation dependent feature of the resummation formula is encoded in the spin-3 zero mode eigenvalue of the highest weight of $(\Box \otimes K)_A$.

The plan of the paper is the following. In Sec. (2), we review the formulation of higher spin black holes in $hs[\lambda] \oplus hs[\lambda]$ Chern-Simons 3D gravity. In Sec. (3), we summarise the methods that allow to evaluate the scalar correlator. In Sec. (4), we present the general structure of the correlator in perturbation theory and in Sec. (5) we give some details of the actual $O(\alpha^5)$ calculation. The results are non trivially checked in Sec. (6). Finally, Sec. (7) is devoted to our proposed resummation formula.
2 Higher spin black hole in \( hs[\lambda] \oplus hs[\lambda] \) Chern-Simons gravity

The gauge fields of \( hs[\lambda] \oplus hs[\lambda] \) Chern-Simons gravity in \( AdS_3 \) are \( A \) and \( \overline{A} \), and obey the equation of motion

\[
dA + A \wedge \star A = 0, \quad (2.1)
\]
as well as its conjugate. The generators of \( hs[\lambda] \) are usually denoted \( V^m_n \) with \( |m| < s \).

Space-time is described by a radial coordinate \( \rho \) and Euclidean torus coordinates \( (z, \overline{z}) \) with periodic identification

\[
(z, \overline{z}) \sim (z + 2\pi, \overline{z} + 2\pi) \sim (z + 2\pi \tau, \overline{z} + 2\pi \overline{\tau}). \quad (2.2)
\]

As shown in [18], it is possible to choose a gauge where the black hole solution can be written in the form

\[
A(\rho, z, \overline{z}) = b^{-1} a b^{-1} + b^{-1} db, \quad \overline{A}(\rho, z, \overline{z}) = b \overline{a} b^{-1} + b \overline{db}^{-1}, \quad (2.3)
\]

where \( b = e^{\rho V^3_0} \) and \( a, \overline{a} \) are constant connections without radial components \( a_\rho = a_{\overline{\rho}} = 0 \).

The black hole solution with higher spin charges found in [11] has the following explicit expression of the holomorphic connection

\[
a_z = V^2_1 - \frac{2\pi}{k} \mathcal{L} - \frac{\pi}{2} \mathcal{W} V^2_2 + \sum_{n=4}^\infty \mathcal{J}_n V^n_{-n+1}, \quad (2.4)
\]

\[
a_\tau = -\frac{\alpha}{\pi} a_z \star a_z - \text{trace.} \quad (2.5)
\]

Here, \( \star \) is the lone star product [19], \( (\mathcal{L}, \mathcal{W}) \) are the stress tensor and spin-3 charges, while \( \mathcal{J}_n \) are higher spin charges. Finally, \( k \) is the Chern-Simons level. The explicit expansion of the charges in powers of the chemical potential \( \alpha \) associated with the spin-3 charge have been derived in [11, 17]. At order \( \mathcal{O}(\alpha^5) \), we shall need the following expressions

\[
\mathcal{L} = -\frac{k}{8(\pi \tau^2)} + \frac{a^2 k (\lambda^2 - 4)}{24 \pi \tau^6} - \frac{a^4 (k (\lambda^2 - 7) (\lambda^2 - 4))}{24 (\pi \tau^10)} + \mathcal{O}(\alpha^6), \quad (2.6)
\]

\[
\mathcal{W} = -\frac{ak}{3 (\pi \tau^5)} + \frac{10 a^3 k (\lambda^2 - 7)}{27 \pi \tau^9} - \frac{a^5 (k (5\lambda^4 - 85\lambda^2 + 377))}{9 (\pi \tau^{13})} + \mathcal{O}(\alpha^7), \quad (2.7)
\]

\[
\mathcal{J}_4 = \frac{7a^2}{36\tau^8} - \frac{7a^4 (2\lambda^2 - 21)}{36\tau^{12}} + \mathcal{O}(\alpha^6), \quad (2.8)
\]

\[
\mathcal{J}_5 = \frac{5a^3}{18\tau^{11}} - \frac{a^5 (44\lambda^2 - 635)}{54\tau^{15}} + \mathcal{O}(\alpha^7), \quad (2.9)
\]

\[
\mathcal{J}_6 = \frac{143a^4}{324\tau^{14}} + \mathcal{O}(\alpha^6), \quad (2.10)
\]

\[
\mathcal{J}_7 = \frac{182a^5}{243\tau^{17}} + \mathcal{O}(\alpha^7). \quad (2.11)
\]

From the explicit form of the charges, one can derive the thermal partition function and the black hole entropy [20, 21].
3 Scalar correlator in higher spin gravity

Let us briefly summarise how to compute the scalar bulk-boundary propagator in 3D higher spin gravity, see for instance [15, 16, 22–24]. Our presentation closely follows [14] whose notation we adopt. Assuming the gauge choice (2.3), we define
\[
\Lambda_0 = a \mu x^\mu, \quad \Lambda_\rho = b^{-1} \star \Lambda_0 \star b,
\]
\[
\bar{\Lambda}_0 = \bar{a} \mu x^\mu, \quad \bar{\Lambda}_\rho = b \star \bar{\Lambda}_0 \star b^{-1}.
\]
(3.1) (3.2)

Then, for a bulk scalar with mass \(m^2 = \Delta (\Delta - 2)\) transforming in a representation of \(hs[\lambda]\), the bulk-boundary propagator reads
\[
\Phi(z, \bar{z}, \rho; 0) = e^{\Delta \rho} \text{Tr} \left[ e^{-\Lambda_\rho} \star c \star e^{\bar{\Lambda}_\rho} \right],
\]
(3.3)

where \(c\) is a highest weight of \(hs[\lambda]\), i.e. an eigenstate of \(V_0^2\) under star product. The boundary two-point correlator between two dual fields at positive infinity can be extracted by taking the \(\rho \to +\infty\) limit giving from AdS/CFT duality
\[
\Phi(z, \bar{z}, \rho; 0) \sim e^{-\Delta \rho} \langle \varphi(z, \bar{z}) \varphi(0, 0) \rangle.
\]
(3.4)

In the following, we shall consider the defining representation of \(hs[\lambda]\) and its antisymmetric powers. Then, \(c\) is the projector onto the highest weight state \(c = |hw\rangle \langle hw|\) that can be built explicitly from the infinite-dimensional matrix realisation of \(hs[\lambda]\) (see Appendix A). Thus,
\[
\Phi(z, \bar{z}, \rho; 0) = e^{\Delta \rho} \langle hw|e^{\bar{\Lambda}_\rho} e^{-\Lambda_\rho}|hw\rangle,
\]
(3.5)

and in the large \(\rho\) limit we find
\[
\langle \varphi(z, \bar{z}) \varphi(0, 0) \rangle = \langle -hw|e^{-\Lambda_0}|hw\rangle \langle hw|e^{\bar{\Lambda}_0}| -hw\rangle.
\]
(3.6)

Due to the above factorisation, we shall consider the purely left-moving part without loosing any information. In particular, this means that \(a_z\) will play no role in the following.

4 Structure of the scalar correlator in the black hole background

For a scalar in a generic representation of \(hs[\lambda]\), we have [14, 16] \(^1\)
\[
\langle \varphi(z, \bar{z}) \varphi(0, 0) \rangle = \left( 4 \, \frac{\tau}{\pi} \sin \frac{z}{2\tau} \sin \frac{\bar{z}}{2\tau} \right)^{-\Delta} R(z, \bar{z}),
\]
(4.1)
\[
R(z, \bar{z}) = 1 + \sum_{n=1}^{\infty} \frac{\alpha^n}{\tau^{2n}} R^{(n)}(z, \bar{z}).
\]
(4.2)

The corrections \(R^{(1)}\) and \(R^{(2)}\) have been computed in [14] for the fundamental and \(\bar{B}\) representations. The calculation is done in the bulk and it is matched on the CFT side

\(^1\) The sum over images required to impose periodicity \((z, \bar{z}) \to (z + 2\pi, \bar{z} + 2\pi)\) is left implicit.
where the relevant dual quantity is the torus two-point function of a scalar primary in the presence of a deformation of the conformal theory by a holomorphic spin-3 operator.

Here, we perform an order $\mathcal{O}(\alpha^5)$ calculation including also the representation. In all cases, the general structure of $R^{(n)}$ turns out to be the following:

$$R^{(n)}(z, \overline{z}) = \frac{1}{\sin^{2n}(\frac{Z}{2})} \sum_{m=0}^{n} (Z - \overline{Z})^{m} \sum_{k=0}^{n} p^{(n)}_{m,k}(\lambda) \begin{cases} \sin(kZ), & n + m \text{ odd} \\ \cos(kZ), & n + m \text{ even} \end{cases}, \quad (4.3)$$

where $p^{(n)}_{m,k}(\lambda)$ are degree $2n$ polynomials and the variables $Z, \overline{Z}$ are

$$Z = \frac{z}{\tau}, \quad \overline{Z} = \frac{\overline{z}}{\overline{\tau}}. \quad (4.4)$$

5 Calculation of $\mathcal{O}(\alpha^5)$ correlators

The extension of the results obtained in [14] is fully straightforward. Here, we just point out a few details that can be useful in order to increase the efficiency of the calculation. First, one assumes that the functions $p^{(n)}_{m,k}(\lambda)$ are polynomials. Then, the corrections $R^{(n)}$ are evaluated at $\lambda = -N$ for $N = 3, 4, \ldots$ up to a point where $p^{(n)}_{m,k}(\lambda)$ can be consistently fixed. We always pushed the calculation some order further in order to confirm that $p^{(n)}_{m,k}(\lambda)$ are actually polynomials.

At $\lambda = -N$, we use the finite-dimensional matrix representation of the generators $V^s_m$. The matrix element $\langle -hw_k | e^{-\Lambda_0} | hw_K \rangle$ is evaluated for the scalar transforming in the $K$–antisymmetric representation $(\Box^\otimes K)_A$ using

$$|hw_K\rangle = \frac{1}{\sqrt{K!}} |1\rangle \otimes |2\rangle \otimes \cdots \otimes |K\rangle + \text{signed permutations}, \quad (5.1)$$

and computing

$$\langle -hw_k | e^{-\Lambda_0} | hw_K \rangle = \sum \langle N | e^{-\Lambda_0} | 1 \rangle \langle N - 1 | e^{-\Lambda_0} | 2 \rangle \cdots \langle N - k + 1 | e^{-\Lambda_0} | k \rangle, \quad (5.2)$$

where the sum is over the signed permutations of the labels $\{1, \ldots, k\}$ in the kets.

The most time consuming part of the calculation is the construction of the exponential $e^{-\Lambda_0}$. We expand $-\Lambda_0 = \sum_{n=0}^{\infty} \alpha^n X_n$ and write the expansion of the exponential as

$$E(t) = e^{t \sum_{n=0}^{\infty} \alpha^n X_n} = \sum_{n=0}^{\infty} \alpha^n E_n(t). \quad (5.3)$$

From

$$E'(t) = -\Lambda_0 E(t), \quad (5.4)$$

we obtain

$$E_0(t) = e^{t X_0}, \quad (5.5)$$

$$E_n(t) = \int_0^t E_0(t - s) \sum_{m=1}^{n-1} X_m E_{n-m}(s) \, ds, \quad n \geq 1. \quad (5.6)$$
Finally, the desired exponential is recovered by setting \( t = 1 \). The building block \( E_0(t) \) is efficiently computed by noting that \( X_0 \) is diagonalised exploiting the relation

\[
e^{-\frac{i}{t}V_2}e^{2i \tau V_2} \left( V_1^2 + \frac{1}{4 \tau^2} V_{-1}^2 \right) e^{-2i \tau V_2} e^{\frac{i}{t}V_2} = \frac{i}{\tau} V_0^2, \tag{5.7}
\]

This change of basis can be done at the beginning and undone before taking matrix elements.

We have computed the polynomials \( p_{\mu,k}^{(n)}(\lambda) \) for \( n \leq 5 \) for the three representations \( \square, \square, \square \). The results for \( n \leq 4 \) are listed in Appendix B \(^3\).

6 Consistency checks

6.1 Representation constraints

Let us denote by \( \bar{R} \) the ratio \( R \) with the replacement \( \alpha \to -\alpha \). For simplicity, we can set \( Z = 0 \), since that variable is always paired with \( Z \) according to (4.3). We have

\[
\begin{align*}
\mathcal{N}' = -1 &: \quad R_{\square} = 1, \\
\mathcal{N}' = -2 &: \quad R_{\square} = R_{\square} = 1, \\
\mathcal{N}' = -3 &: \quad R_{\square} = 1, \quad R_{\square} = \bar{R}_{\square}, \\
\mathcal{N}' = -4 &: \quad R_{\square} = \bar{R}_{\square}, \\
\mathcal{N}' = -5 &: \quad R_{\square} = \bar{R}_{\square}.
\end{align*}
\]

These properties are simply understood in terms of equivalences of representations of \( sl(N) \) at the above special values of \( \mathcal{N}' \).

6.2 Zero temperature limit

An interesting regime is the zero-temperature limit where \( \mu = \kappa/\tau \) is fixed and \( \tau, \tau \to \infty \). This is the chiral deformation background discussed in [16, 24]. The constant flat connection reads in this limit

\[
a = V_1^2 \, dz - \mu V_2^3 \, d\zeta, \quad \bar{a} = V_{-1}^2 \, d\zeta. \tag{6.6}
\]

In [14], it has been proved that

\[
\lim_{\tau, \tau \to \infty \atop \text{fixed } \mu} R_{\square} = \sum_{n=0}^{\infty} \left( \frac{\mu \bar{\zeta}}{z^2} \right)^n \frac{\Gamma(2n + 1 + \lambda)}{n! \Gamma(1 + \lambda)}. \tag{6.7}
\]

\(^2\) Notice that (5.7) is consistent with the fact that the BTZ black hole holonomy is in the center of the group \( HS[\lambda] \) which is the formal exponential of \( hs[\lambda] \) [16]. Indeed, this condition requires \( \exp(2 \pi \tau \frac{i}{V_0^2}) = \exp(2 \pi i V_0^2) = 1 \) which is true since \( V_0^2 \) is diagonal with integer non zero elements.

\(^3\) The explicit polynomials for \( n = 5 \) are available upon request to the authors.
In Appendix C, we prove with a similar calculation that
\[
\lim_{\tau, \tau' \to \infty, \text{fixed } \mu} R_B = \sum_{n=0}^{\infty} \left( \frac{\mu \tau'}{2 \tau} \right)^n \sum_{m=0}^{n} \frac{(1 + \lambda)^m}{m!} \frac{(1 + \lambda)^{n-m}}{(n-m)!} \frac{\lambda + 1}{\lambda + n + 1 + 2(n-2m)^2},
\] (6.8)

where \((a)_n^+ = a(a+1) \cdots (a+n-1)\) is the ascending Pochhammer symbol. The explicit first terms are
\[
\lim_{\tau, \tau' \to \infty, \text{fixed } \mu} R_B = 1 + \frac{2(\lambda + 2)(\lambda + 4)\mu \tau}{\tau^2} + \frac{2(\lambda + 2)(\lambda + 3)(\lambda^2 + 9\lambda + 23)\mu^2 \tau^2}{\tau^4} + \frac{4(\lambda + 2)(\lambda + 3)(\lambda + 4)(\lambda + 6)(\lambda^2 + 9\lambda + 29)\mu^3 \tau^3}{3\tau^6} + \frac{2(\lambda + 2)(\lambda + 3)(\lambda + 4)(\lambda + 5)(\lambda^4 + 22\lambda^3 + 197\lambda^2 + 872\lambda + 1641)\mu^4 \tau^4}{3\tau^8} + \cdots
\] (6.9)

We have checked that our explicit expression for \(R_B\) obeys this limit at order \(O(\mu^5)\).

7 Leading and subleading order resummation at large \(\lambda\)

An interesting limit of the scalar correlator is the one where \(\lambda \to \infty\). This regime has been recently explored in [17] in the case of the higher spin black hole partition function. One finds that, with a suitable normalisation of \(hs[\lambda]\) generators, the large \(\lambda\) limit is smooth and non trivial. Indeed, all higher spin charges are turned on and contribute. Here, we shall also find that the correlators have a sensible large \(\lambda\) limit. The truncation at \(\lambda = -N\) suggest a possible treatment of this regime. The considered observables have a polynomial dependence on \(\lambda\), at least at each order of perturbation theory in \(a\). This means that the large \(\lambda\) limit in \(hs[\lambda]\) is the same as the large \(N\) limit in the \(sl(N) \oplus sl(N)\) Chern-Simons gravity. One expects that considering \(sl(\infty)\) could lead to some special properties as in the case of standard \(1/N\) expansion of \(SU(N)\) invariant gauge theories.

In the present case of the scalar correlator, we have investigated the large \(\lambda\) limit and found a remarkable regularities of the leading and subleading terms order by order in the expansion in powers of the spin-3 charge chemical potential. Indeed, from the explicit expressions of the ratio \(R_r\), we discover that for the three considered representations \(r = 0, B, \overline{B}\) it is possible to resum all the contributions at leading order \(O(a^n \lambda^{2n})\), as well as the next-to-leading terms \(O(a^n \lambda^{2n-1})\). To present the resummation, we introduce the following coefficient associated with the \(K\)-antisymmetric representation
\[
c_{r=(0\otimes K)\lambda} = \frac{K}{6}(\lambda + K)(\lambda + 2K),
\] (7.1)

and the two functions
\[
f_{LO}(\mathcal{Z}, \overline{\mathcal{Z}}) = \frac{3 \sin \mathcal{Z} - (2 + \cos \mathcal{Z})(\mathcal{Z} - \overline{\mathcal{Z}})}{\sin^2 \frac{\mathcal{Z}}{2}},
\] (7.2)
\[
f_{NLO}(\mathcal{Z}, \overline{\mathcal{Z}}) = 6 - 4 \cos \mathcal{Z} - 2 \cos(2 \mathcal{Z}) - (10 \sin \mathcal{Z} + \sin(2 \mathcal{Z}))(\mathcal{Z} - \overline{\mathcal{Z}})
\]
\[ +3 (1 + \cos Z) (Z - \overline{Z})^2. \] (7.3)

Then, we have the rather simple formula
\[
R_{\lambda \to \infty} = \exp \left( c_r f_{\text{LO}} \frac{\lambda}{2} \alpha^2 \right) \left( 1 + c_r f_{\text{NLO}} \frac{\lambda}{8} \alpha^4 \right) + \mathcal{O}(\alpha^n \lambda^{2n-2}). \] (7.4)

Resummation works as follows. The argument of the exponential captures all terms \( \sim \alpha^n \lambda^{2n} \) and introduces also some contributions of order \( \alpha^n \lambda^{2n-1} \). The correction in parenthesis completes the resummation of all contributions of order \( \alpha^n \lambda^{2n-1} \). From the CFT calculation of the \( \mathcal{O}(\alpha) \) contribution to \( R \), we identify the meaning of \( c_r \). It is the eigenvalue of the zero mode of the \( \mathcal{W}_\infty \) spin-3 generator on the dual primary. In terms of \( \text{hs}[\lambda] \), this is the eigenvalue of \( V_0^3 \) on the highest weight. A straightforward calculation gives indeed
\[
V_0^3 |\text{hw}_K\rangle = \frac{K}{6} (\lambda + K)(\lambda + 2K) |\text{hw}_K\rangle. \] (7.5)

The resummation formula (7.4) is our conjecture and we have tested it up to the order \( \mathcal{O}(\alpha^5) \) included.

8 Conclusions

In this paper, we have considered the duality between Vasiliev 3D higher spin gravity and \( \mathcal{W}_N \) minimal models. We have computed the correlator of the bulk complex scalar in the background of a higher spin black hole and we have determined the exact dependence on the symmetry parameter \( \lambda \) at order \( \mathcal{O}(\alpha^5) \) where \( \alpha \) is the chemical potential associated with the spin-3 charge of the black hole. We have analysed the perturbative data in the large \( \lambda \) limit where we had some expectations that some simplification could occur. We found special regularities and proposed a resummation of all the leading and subleading contributions that take the form \( \mathcal{O}(\alpha^n \cdot \{\lambda^{2n}, \lambda^{2n-1}\}) \).

We believe that such non-trivial property of the perturbative expansion could be explained, and possibly extended, in terms of a systematic \( 1/\mathcal{N} \) expansion of the \( \mathfrak{s}(\mathcal{N}) \oplus \mathfrak{s}(\mathcal{N}) \) Chern-Simons gravity theory. Alternatively, a suitable contraction of the symmetry algebra \( \text{hs}[\lambda] \) could be devised in order to capture the corrections considered in this paper.

A The infinite dimensional algebra \( \text{hs}[\lambda] \)

The \( \text{hs}[\lambda] \) algebra is generated by \( V^s_m \) with \( s \geq 2 \) and \( |m| < s \). The lone-star operation \([19]\) is the associative product defined by
\[
V^s_m \star V^t_n = \frac{1}{2} \sum_{u=1}^{s+t-1} \delta^{st}_u (m, n, \lambda) V^{s+t-u}_{m+n}, \] (A.1)

with
\[
g^{st}_u (m, n, \lambda) = \frac{(1/4)^{u-2}}{2(u-1)!} {}_3F_3 \left( \begin{array}{c} \frac{1}{2} + \lambda, \frac{1}{2} - \lambda, \frac{2-u}{2} \\ \frac{3}{2} - s, \frac{3}{2} - t, \frac{1}{2} + s + t - u \end{array} \right) \times \] (A.2)
\[
\sum_{k=0}^{u-1} (-1)^k \binom{u-1}{k} (s-1+m)^u \binom{u-1-k}{k} (t-1+n)^u \binom{t-1-k}{k} (t-1-n)^u -
\]

where \((a)_n = a(a-1) \cdots (a-n+1)\) is the descending Pochammer symbol. An infinite-dimensional matrix representation of \(hs[\lambda]\) is obtained by starting with the following infinite-dimensional matrix representation of \(sl(2)\) (we list the non-zero elements)

\[
(V_0^2)_{n,n} = -\frac{\lambda + 1}{2} - n,
\]

\[
(V_1^2)_{n+1,n} = -\sqrt{-\lambda - n} n,
\]

\[
(V_{-1}^2)_{n,n+1} = \sqrt{-\lambda - n} n,
\]

and building the other generators according to

\[
V_m^s = (-1)^{s-1-m} \frac{(s+m-1)!}{(2s-2)!} \text{Ad}_{V_1^2}^{s-m-1} (V_1^2)^{s-1}.
\]

A very important property of this representation is that setting \(\lambda = -N\), a negative integer, and restricting \(V_m^s\) to the set with \(s \leq N\), we obtain a \(N \times N\) dimensional representation of \(sl(N)\) by restricting the infinite dimensional matrices to their first \(N\) rows and columns. Also, the lone-star product becomes simply matrix multiplication.

**B Non-zero polynomials \(p_{m,k}^{(n)}(\lambda)\) for \(n \leq 4\)**

In this section we report the explicit expressions for the polynomials appearing in (4.3).

**B.1 Fundamental representation**

Let us define

\[
f = (\lambda + 1)(\lambda + 2).
\]

We have:

\[
p_{0,0}^{(1)}(\lambda) = \frac{f}{4}
\]

\[
p_{0,1}^{(1)}(\lambda) = -\frac{f}{6}
\]

\[
p_{1,1}^{(1)}(\lambda) = -\frac{f}{12}
\]

\[
p_{0,0}^{(2)}(\lambda) = \frac{f}{64} (\lambda + 4)(\lambda + 7)
\]

\[
p_{0,1}^{(2)}(\lambda) = -\frac{f}{12} (\lambda + 4)
\]

\[
p_{0,2}^{(2)}(\lambda) = -\frac{f}{192} (\lambda + 4)(3\lambda + 5)
\]

\[
p_{1,1}^{(2)}(\lambda) = -\frac{f}{24} (\lambda + 4)^2
\]

\[
p_{1,2}^{(2)}(\lambda) = -\frac{f}{96} (\lambda + 1)(\lambda + 4)
\]
\[
\begin{align*}
p^{(2)}_{2,0}(\lambda) &= \frac{f}{64} \left( \lambda^2 + 7\lambda + 14 \right) \\
p^{(2)}_{2,1}(\lambda) &= \frac{f}{144} \left( \lambda + 2 \right) \left( 2\lambda + 11 \right) \\
p^{(2)}_{2,2}(\lambda) &= \frac{f}{576} \left( \lambda + 1 \right) \left( \lambda + 2 \right) \\
\end{align*}
\]  
(B.3)

\[
\begin{align*}
p^{(3)}_{0,1}(\lambda) &= \frac{f}{1536} \left( \lambda + 3 \right) \left( \lambda + 4 \right) \left( \lambda + 6 \right) \left( 3\lambda + 35 \right) \\
p^{(3)}_{0,2}(\lambda) &= -\frac{f}{96} \left( \lambda + 3 \right) \left( \lambda + 4 \right) \left( \lambda + 6 \right) \\
p^{(3)}_{0,3}(\lambda) &= -\frac{f}{1536} \left( \lambda + 1 \right) \left( \lambda + 3 \right) \left( \lambda + 4 \right) \left( \lambda + 6 \right) \\
p^{(3)}_{1,0}(\lambda) &= -\frac{f}{1152} \left( 3\lambda^4 + 68\lambda^3 + 539\lambda^2 + 1942\lambda + 2488 \right) \\
p^{(3)}_{1,1}(\lambda) &= -\frac{f}{1536} \left( \lambda^4 + 2\lambda^3 - 89\lambda^2 - 842\lambda - 1592 \right) \\
p^{(3)}_{1,2}(\lambda) &= \frac{f}{384} \left( \lambda^4 + 20\lambda^3 + 145\lambda^2 + 418\lambda + 424 \right) \\
p^{(3)}_{1,3}(\lambda) &= \frac{f}{4608} \left( 3\lambda^4 + 36\lambda^3 + 149\lambda^2 + 226\lambda + 88 \right) \\
p^{(3)}_{2,1}(\lambda) &= \frac{f}{4608} \left( \lambda + 6 \right) \left( 17\lambda^3 + 236\lambda^2 + 1093\lambda + 1594 \right) \\
p^{(3)}_{2,2}(\lambda) &= \frac{f}{1152} \left( \lambda + 2 \right) \left( \lambda + 6 \right) \left( \lambda + 7 \right) \left( 2\lambda + 5 \right) \\
p^{(3)}_{2,3}(\lambda) &= \frac{f}{4608} \left( \lambda + 2 \right) \left( \lambda + 6 \right) \\
p^{(3)}_{3,0}(\lambda) &= -\frac{f}{10368} \left( 11\lambda^4 + 201\lambda^3 + 1412\lambda^2 + 4344\lambda + 4904 \right) \\
p^{(3)}_{3,1}(\lambda) &= -\frac{f}{13824} \left( 17\lambda^4 + 318\lambda^3 + 2093\lambda^2 + 5892\lambda + 5972 \right) \\
p^{(3)}_{3,2}(\lambda) &= -\frac{f}{3456} \left( \lambda + 2 \right) \left( \lambda + 4 \right) \left( \lambda + 7 \right) \\
p^{(3)}_{3,3}(\lambda) &= -\frac{f^3}{41472} \\
\end{align*}
\]  
(B.4)

\[
\begin{align*}
p^{(4)}_{0,0}(\lambda) &= \frac{f}{147456} \left( \lambda + 8 \right) \left( 9\lambda^5 + 345\lambda^4 + 4925\lambda^3 + 32135\lambda^2 + 100546\lambda + 117400 \right) \\
p^{(4)}_{0,1}(\lambda) &= -\frac{f}{4608} \left( \lambda + 8 \right) \left( 3\lambda^4 + 82\lambda^3 + 721\lambda^2 + 2762\lambda + 3656 \right) \\
p^{(4)}_{0,2}(\lambda) &= -\frac{f}{36864} \left( \lambda + 8 \right) \left( 3\lambda^5 + 99\lambda^4 + 1055\lambda^3 + 4925\lambda^2 + 9526\lambda + 5896 \right) \\
p^{(4)}_{0,3}(\lambda) &= \frac{f}{4608} \left( \lambda + 8 \right) \left( 3\lambda^4 + 50\lambda^3 + 305\lambda^2 + 778\lambda + 712 \right) \\
p^{(4)}_{0,4}(\lambda) &= \frac{f}{147456} \left( \lambda + 8 \right) \left( 3\lambda^5 + 51\lambda^4 + 319\lambda^3 + 877\lambda^2 + 1046\lambda + 392 \right) \\
p^{(4)}_{1,0}(\lambda) &= -\frac{f}{9216} \left( 3\lambda^6 + 124\lambda^5 + 2013\lambda^4 + 16904\lambda^3 + 76708\lambda^2 + 177768\lambda + 163952 \right) \\
p^{(4)}_{1,1}(\lambda) &= -\frac{f}{18432} \left( \lambda^6 + 7\lambda^5 - 287\lambda^4 - 5551\lambda^3 - 37282\lambda^2 - 109368\lambda - 117664 \right) \\
p^{(4)}_{1,2}(\lambda) &= \frac{f}{9216} \left( \lambda + 2 \right) \left( 3\lambda^5 + 34\lambda^4 + 443\lambda^3 + 2642\lambda^2 + 7368\lambda + 7656 \right) \\
p^{(4)}_{1,3}(\lambda) &= \frac{f}{36864} \left( \lambda + 1 \right) \left( \lambda^5 + 22\lambda^4 + 171\lambda^3 + 598\lambda^2 + 872\lambda + 352 \right) \\
p^{(4)}_{2,0}(\lambda) &= \frac{f}{221184} \left( 51\lambda^6 + 1955\lambda^5 + 30137\lambda^4 + 245921\lambda^3 + 1101812\lambda^2 + 2564444\lambda + 2406080 \right) \\
\end{align*}
\]
\[ p_{2,1}^{(4)}(\lambda) = \frac{f}{55296} \left( 6\lambda^6 + 169\lambda^5 + 1870\lambda^4 + 7807\lambda^3 + 1672\lambda^2 - 73748\lambda - 137360 \right) \]
\[ p_{2,2}^{(4)}(\lambda) = -\frac{f}{27648} \left( 6\lambda^6 + 229\lambda^5 + 3541\lambda^4 + 27937\lambda^3 + 118309\lambda^2 + 253066\lambda + 213808 \right) \]
\[ p_{3,1}^{(4)}(\lambda) = -\frac{f}{58296} (\lambda + 2) \left( 6\lambda^5 + 173\lambda^4 + 1876\lambda^3 + 935\lambda^2 + 21338\lambda + 18232 \right) \]
\[ p_{2,3}^{(4)}(\lambda) = -\frac{f^2}{221184} \left( 3\lambda^4 + 50\lambda^3 + 245\lambda^2 + 406\lambda + 160 \right) \]
\[ p_{3,2}^{(4)}(\lambda) = -\frac{f}{82944} (\lambda + 8) \left( 19\lambda^5 + 529\lambda^4 + 5869\lambda^3 + 31759\lambda^2 + 82816\lambda + 82876 \right) \]
\[ p_{3,3}^{(4)}(\lambda) = -\frac{f}{165888} (\lambda + 8) \left( 25\lambda^5 + 643\lambda^4 + 6199\lambda^3 + 28093\lambda^2 + 60556\lambda + 50140 \right) \]
\[ p_{3,4}^{(4)}(\lambda) = -\frac{f^3}{317776} (\lambda + 1)(\lambda + 8) \]
\[ p_{4,0}^{(4)}(\lambda) = \frac{f}{3981312} \left( 227\lambda^6 + 7659\lambda^5 + 107715\lambda^4 + 795357\lambda^3 + 3235542\lambda^2 + 6848460\lambda + 5873560 \right) \]
\[ p_{4,1}^{(4)}(\lambda) = \frac{f}{497664} \left( 38\lambda^6 + 1287\lambda^5 + 17562\lambda^4 + 124461\lambda^3 + 482406\lambda^2 + 968940\lambda + 787624 \right) \]
\[ p_{4,2}^{(4)}(\lambda) = \frac{f}{995328} \left( 25\lambda^6 + 765\lambda^5 + 9249\lambda^4 + 56655\lambda^3 + 187086\lambda^2 + 317556\lambda + 217496 \right) \]
\[ p_{4,3}^{(4)}(\lambda) = \frac{f}{497664} (\lambda + 2)^3 \left( 2\lambda^3 + 33\lambda^2 + 168\lambda + 299 \right) \]
\[ p_{4,4}^{(4)}(\lambda) = \frac{f^4}{3981312} \]

**B.2 2-Antisymmetric representation**

Let us define

\[ f = (\lambda + 2)(\lambda + 4). \]  

We have:

\[ p_{0,1}^{(1)}(\lambda) = \frac{f}{2} \]
\[ p_{1,0}^{(1)}(\lambda) = -\frac{f}{3} \]
\[ p_{1,1}^{(1)}(\lambda) = -\frac{f}{6} \]  

\[ p_{0,0}^{(2)}(\lambda) = \frac{1}{16} (\lambda + 2)^2 \left( \lambda^3 + 14\lambda^2 + 74\lambda + 121 \right) \]
\[ p_{0,1}^{(2)}(\lambda) = -\frac{1}{6} (\lambda + 2)^2 \left( 2\lambda^2 + 12\lambda + 26 \right) \]
\[ p_{0,2}^{(2)}(\lambda) = -\frac{1}{48} (\lambda + 2)(\lambda + 5) \left( 3\lambda^2 + 19\lambda + 31 \right) \]
\[ p_{1,1}^{(2)}(\lambda) = -\frac{1}{12} (\lambda + 2)^2 \left( 2\lambda^2 + 25\lambda^2 + 112\lambda + 164 \right) \]
\[ p_{2,1}^{(2)}(\lambda) = -\frac{1}{24} (\lambda + 2)^2 \left( \lambda^3 + 11\lambda^2 + 38\lambda + 43 \right) \]
\[ p_{2,2}^{(2)}(\lambda) = \frac{1}{16} (\lambda + 2)^2 \left( \lambda^3 + 12\lambda^2 + 52\lambda + 74 \right) \]
\begin{align*}
  p_{21}^{(2)}(\lambda) &= \frac{f}{72} (\lambda + 4)(4\lambda + 17) \\
  p_{22}^{(2)}(\lambda) &= \frac{\rho^2}{144} \\
  p_{00}^{(4)}(\lambda) &= \frac{f}{192} (\lambda + 3)(\lambda + 6)\left(3\lambda^2 + 37\lambda + 167\right) \\
  p_{01}^{(4)}(\lambda) &= -\frac{f}{24} (\lambda + 3)(\lambda + 6)(\lambda + 8) \\
  p_{02}^{(4)}(\lambda) &= -\frac{f}{192} (\lambda + 3)(\lambda + 5)\left(\lambda^2 + 7\lambda + 13\right) \\
  p_{10}^{(4)}(\lambda) &= -\frac{f}{144} (3\lambda^4 + 61\lambda^3 + 503\lambda^2 + 1979\lambda + 2794) \\
  p_{11}^{(4)}(\lambda) &= -\frac{f}{192} \left(\lambda^4 + 10\lambda^3 - 8\lambda^2 - 457\lambda - 1070\right) \\
  p_{12}^{(4)}(\lambda) &= \frac{f}{48} \left(\lambda^4 + 19\lambda^3 + 145\lambda^2 + 497\lambda + 622\right) \\
  p_{13}^{(4)}(\lambda) &= \frac{f}{576} \left(3\lambda^4 + 46\lambda^3 + 248\lambda^2 + 581\lambda + 502\right) \\
  p_{20}^{(4)}(\lambda) &= \frac{f}{576} \left(17\lambda^4 + 322\lambda^3 + 2488\lambda^2 + 9026\lambda + 11988\right) \\
  p_{21}^{(4)}(\lambda) &= \frac{f}{288} (\lambda + 5) \left(4\lambda^3 + 51\lambda^2 + 224\lambda + 324\right) \\
  p_{22}^{(4)}(\lambda) &= \frac{f}{576} (\lambda + 2) \left(\lambda^3 + 12\lambda^2 + 44\lambda + 54\right) \\
  p_{30}^{(4)}(\lambda) &= -\frac{f}{2592} \frac{(22\lambda^4 + 399\lambda^3 + 2953\lambda^2 + 10212\lambda + 13072)}{2952} \\
  p_{31}^{(4)}(\lambda) &= -\frac{f}{1728} \frac{(17\lambda^4 + 312\lambda^3 + 2216\lambda^2 + 7284\lambda + 8936)}{1728} \\
  p_{32}^{(4)}(\lambda) &= -\frac{1}{864} f (\lambda + 4)^2 (\lambda + 5)(2\lambda + 7) \\
  p_{33}^{(4)}(\lambda) &= -\frac{f^3}{5184} \\
  p_{0,0}^{(4)}(\lambda) &= \frac{(\lambda + 2) (9\lambda^7 + 366\lambda^6 + 6632\lambda^5 + 68870\lambda^4 + 439976\lambda^3 + 1700624\lambda^2 + 3588743\lambda + 3127340)}{9216} \\
  p_{0,1}^{(4)}(\lambda) &= -\frac{1}{576} \frac{(\lambda + 2) (3\lambda^6 + 110\lambda^5 + 1712\lambda^4 + 14507\lambda^3 + 68774\lambda^2 + 167078\lambda + 159968)}{2304} \\
  p_{0,2}^{(4)}(\lambda) &= -\frac{(\lambda + 2) (3\lambda^7 + 114\lambda^6 + 1880\lambda^5 + 17234\lambda^4 + 93488\lambda^3 + 296192\lambda^2 + 505949\lambda + 360356)}{2304} \\
  p_{0,3}^{(4)}(\lambda) &= -\frac{1}{576} \frac{(\lambda + 2) (3\lambda^6 + 94\lambda^5 + 1216\lambda^4 + 8299\lambda^3 + 31366\lambda^2 + 62182\lambda + 50464)}{1152} \\
  p_{0,4}^{(4)}(\lambda) &= \frac{(\lambda + 2) (3\lambda^7 + 90\lambda^6 + 1144\lambda^5 + 8002\lambda^4 + 33304\lambda^3 + 82672\lambda^2 + 113389\lambda + 66148)}{9216} \\
  p_{1,1}^{(4)}(\lambda) &= -\frac{(\lambda + 2)(\lambda + 8) (6\lambda^6 + 181\lambda^5 + 2414\lambda^4 + 18004\lambda^3 + 77105\lambda^2 + 174640\lambda + 160172)}{1152} \\
  p_{1,2}^{(4)}(\lambda) &= -\frac{(\lambda + 2) (\lambda^7 + 21\lambda^6 + 78\lambda^5 - 1894\lambda^4 - 26049\lambda^3 - 139840\lambda^2 - 349987\lambda - 335548)}{1152} \\
  p_{1,3}^{(4)}(\lambda) &= -\frac{(\lambda + 2) (2\lambda^7 + 71\lambda^6 + 1106\lambda^5 + 9644\lambda^4 + 50411\lambda^3 + 156968\lambda^2 + 268548\lambda + 194208)}{1152} \\
  p_{1,4}^{(4)}(\lambda) &= \frac{(\lambda + 2) (\lambda^7 + 29\lambda^6 + 350\lambda^5 + 2298\lambda^4 + 8903\lambda^3 + 20448\lambda^2 + 25837\lambda + 13828)}{2304} \\
  p_{2,0}^{(4)}(\lambda) &= \frac{(\lambda + 2) (51\lambda^7 + 1870\lambda^6 + 30420\lambda^5 + 285351\lambda^4 + 1653324\lambda^3 + 5831100\lambda^2 + 11379570\lambda + 9326444)}{13824}
\end{align*}
Let us define

\[ p_{2,1}^{(4)}(\lambda) = \frac{(\lambda + 2) (12\lambda^7 + 379\lambda^6 + 5118\lambda^5 + 36678\lambda^4 + 142599\lambda^3 + 260448\lambda^2 + 86208\lambda - 213256)}{6912} \]
\[ p_{2,2}^{(4)}(\lambda) = -\frac{f(38\lambda^6 + 1194\lambda^5 + 16485\lambda^4 + 127347\lambda^3 + 574173\lambda^2 + 1399308\lambda + 1395856)}{10368} \]
\[ p_{2,3}^{(4)}(\lambda) = -\frac{f^2(3\lambda + 4\lambda^2 + 26\lambda + 70)}{3456} \]
\[ p_{2,4}^{(4)}(\lambda) = \frac{(\lambda + 2)^2 (3\lambda^3 + 4\lambda^2 + 12\lambda + 169)}{6912} \]
\[ p_{3,1}^{(4)}(\lambda) = \frac{f(25\lambda^6 + 759\lambda^5 + 9822\lambda^4 + 282600\lambda^2 + 618708\lambda + 560402)}{10368} \]
\[ p_{3,2}^{(4)}(\lambda) = \frac{f(\lambda + 4) (2\lambda^5 + 46\lambda^4 + 419\lambda^3 + 1897\lambda^2 + 4295\lambda + 3880)}{3456} \]
\[ p_{3,3}^{(4)}(\lambda) = \frac{f^2(\lambda + 2) (\lambda^3 + 13\lambda^2 + 50\lambda + 65)}{20736} \]
\[ p_{3,4}^{(4)}(\lambda) = \frac{f(\lambda + 2) (227\lambda^7 + 7802\lambda^6 + 119208\lambda^5 + 1045128\lambda^4 + 5645904\lambda^3 + 18594528\lambda^2 + 34117144\lambda + 26560720)}{248832} \]
\[ p_{4,0}^{(4)}(\lambda) = \frac{(\lambda + 2) (152\lambda^7 + 5234\lambda^6 + 78852\lambda^5 + 676626\lambda^4 + 35588120\lambda^3 + 11403312\lambda^2 + 20392768\lambda + 15518560)}{124416} \]
\[ p_{4,1}^{(4)}(\lambda) = \frac{(\lambda + 2) (25\lambda^7 + 820\lambda^6 + 11607\lambda^5 + 92220\lambda^4 + 444648\lambda^3 + 1298208\lambda^2 + 2115848\lambda + 1476692)}{62208} \]
\[ p_{4,2}^{(4)}(\lambda) = \frac{f(\lambda + 4)^3 (8\lambda^3 + 102\lambda^2 + 420\lambda + 577)}{124416} \]
\[ p_{4,3}^{(4)}(\lambda) = \frac{f^4}{248832} \]
\[ (B.10) \]

### B.3 3-Antisymmetric representation

Let us define

\[ f = (\lambda + 2)(\lambda + 4). \]

We have:

\[ p_{0,0}^{(1)}(\lambda) = \frac{3f}{4} \]
\[ p_{0,1}^{(1)}(\lambda) = -\frac{f}{2} \]
\[ p_{1,0}^{(1)}(\lambda) = -\frac{f}{4} \]
\[ (B.12) \]

\[ p_{0,0}^{(2)}(\lambda) = \frac{3}{64} (\lambda + 3)(\lambda + 4) (3\lambda^2 + 41\lambda + 178) \]
\[ p_{0,1}^{(2)}(\lambda) = -\frac{1}{4} (\lambda + 3)(\lambda + 4)(\lambda + 14) \]
\[ p_{0,2}^{(2)}(\lambda) = -\frac{1}{64} (\lambda + 3)(\lambda + 4)(\lambda + 5)(9\lambda + 62) \]
\[ p_{1,0}^{(2)}(\lambda) = -\frac{1}{8} (\lambda + 3)(\lambda + 4) (3\lambda^2 + 38\lambda + 136) \]
\[ p_{1,1}^{(2)}(\lambda) = -\frac{1}{32} (\lambda + 3)(\lambda + 4) (3\lambda^2 + 35\lambda + 94) \]

\[ (B.11) \]
\[ p_{2,0}^{(2)}(\lambda) = \frac{3}{64}(\lambda + 3) \left(3\lambda^3 + 49\lambda^2 + 276\lambda + 508\right) \]
\[ p_{2,1}^{(2)}(\lambda) = \frac{f}{16}(\lambda + 6)(2\lambda + 9) \]
\[ p_{2,2}^{(2)}(\lambda) = \frac{f^2}{64} \]
\[ (B.13) \]
\[ p_{0,1}^{(3)}(\lambda) = \frac{3f}{512}(\lambda + 4) \left(9\lambda^3 + 182\lambda^2 + 1419\lambda + 3822\right) \]
\[ p_{0,2}^{(3)}(\lambda) = -\frac{3f}{32}(\lambda + 4) \left(\lambda^2 + 19\lambda + 74\right) \]
\[ p_{0,3}^{(3)}(\lambda) = -\frac{f}{512}(\lambda + 4) \left(9\lambda^3 + 150\lambda^2 + 811\lambda + 1454\right) \]
\[ p_{1,0}^{(3)}(\lambda) = -\frac{f}{128} \left(9\lambda^4 + 212\lambda^3 + 2009\lambda^2 + 8758\lambda + 14072\right) \]
\[ p_{1,1}^{(3)}(\lambda) = -\frac{f}{128} \left(9\lambda^4 + 150\lambda^3 + 583\lambda^2 - 1662\lambda - 8200\right) \]
\[ p_{1,2}^{(3)}(\lambda) = \frac{f}{128} \left(9\lambda^4 + 204\lambda^3 + 1825\lambda^2 + 7302\lambda + 10808\right) \]
\[ p_{1,3}^{(3)}(\lambda) = \frac{f}{512} \left(9\lambda^4 + 182\lambda^3 + 1319\lambda^2 + 4162\lambda + 4856\right) \]
\[ p_{2,1}^{(3)}(\lambda) = \frac{f}{512} \left(51\lambda^4 + 1154\lambda^3 + 10399\lambda^2 + 42712\lambda + 65204\right) \]
\[ p_{2,2}^{(3)}(\lambda) = \frac{f}{128} \left(\lambda + 5\right) \left(6\lambda^3 + 101\lambda^2 + 576\lambda + 1084\right) \]
\[ p_{2,3}^{(3)}(\lambda) = \frac{f}{512} \left(\lambda + 3\right) \left(3\lambda^3 + 49\lambda^2 + 252\lambda + 428\right) \]
\[ p_{3,0}^{(3)}(\lambda) = -\frac{f}{384} \left(11\lambda^4 + 243\lambda^3 + 2124\lambda^2 + 8424\lambda + 12456\right) \]
\[ p_{3,1}^{(3)}(\lambda) = -\frac{f}{512} \left(17\lambda^4 + 378\lambda^3 + 3221\lambda^2 + 12396\lambda + 17876\right) \]
\[ p_{3,2}^{(3)}(\lambda) = -\frac{f}{128} \left(\lambda + 4\right) \left(\lambda + 5\right) \left(\lambda + 6\right) \]
\[ p_{3,3}^{(3)}(\lambda) = -\frac{f^3}{1536} \]
\[ (B.14) \]
\( p^{(4)}_{2,0}(\lambda) = \frac{(\lambda + 3) (459\lambda^7 + 19635\lambda^6 + 371775\lambda^5 + 4021241\lambda^4 + 26675766\lambda^3 + 107760380\lambda^2 + 243267960\lambda + 234536384)}{24576} \)
\( p^{(4)}_{2,1}(\lambda) = \frac{(\lambda + 3) (18\lambda^7 + 709\lambda^6 + 11952\lambda^5 + 110323\lambda^4 + 592866\lambda^3 + 1814348\lambda^2 + 2841112\lambda + 1661984)}{2048} \)
\( p^{(4)}_{2,2}(\lambda) = -\frac{(\lambda + 3) (18\lambda^7 + 769\lambda^6 + 14541\lambda^5 + 156459\lambda^4 + 1026717\lambda^3 + 4079068\lambda^2 + 9022540\lambda + 8515840)}{1024} \)
\( p^{(4)}_{2,3}(\lambda) = -\frac{(\lambda + 3) (54\lambda^7 + 2175\lambda^6 + 37776\lambda^5 + 365993\lambda^4 + 2131686\lambda^3 + 7453988\lambda^2 + 1447456\lambda + 12027488)}{6144} \)
\( p^{(4)}_{2,4}(\lambda) = \frac{(\lambda + 3)^2 (9\lambda^6 + 302\lambda^5 + 4131\lambda^4 + 29650\lambda^3 + 118452\lambda^2 + 259684\lambda + 221120)}{8192} \)
\( p^{(4)}_{3,1}(\lambda) = \frac{(\lambda + 3) (19\lambda^7 + 797\lambda^6 + 14783\lambda^5 + 156299\lambda^4 + 1011478\lambda^3 + 3981140\lambda^2 + 8760984\lambda + 825288)}{1024} \)
\( p^{(4)}_{3,2}(\lambda) = \frac{(\lambda + 3) (25\lambda^7 + 1031\lambda^6 + 18493\lambda^5 + 186753\lambda^4 + 1144046\lambda^3 + 4240252\lambda^2 + 8776344\lambda + 7795072)}{2048} \)
\( p^{(4)}_{3,3}(\lambda) = \frac{-f(\lambda + 6) (3\lambda^5 + 81\lambda^4 + 871\lambda^3 + 4667\lambda^2 + 12470\lambda + 13280)}{1024} \)
\( p^{(4)}_{3,4}(\lambda) = \frac{-f^2(\lambda + 3) (\lambda^3 + 17\lambda^2 + 90\lambda + 160)}{4096} \)
\( p^{(4)}_{4,0}(\lambda) = \frac{(\lambda + 3) (227\lambda^7 + 9363\lambda^6 + 169953\lambda^5 + 1752177\lambda^4 + 11034384\lambda^3 + 42243432\lambda^2 + 90519984\lambda + 83227920)}{49152} \)
\( p^{(4)}_{4,1}(\lambda) = \frac{(\lambda + 3) (384\lambda^7 + 1569\lambda^6 + 282483\lambda^5 + 2872832\lambda^4 + 1778880\lambda^3 + 6687912\lambda^2 + 14078592\lambda + 12733680)}{6144} \)
\( p^{(4)}_{4,2}(\lambda) = \frac{(\lambda + 3) (25\lambda^7 + 1005\lambda^6 + 17427\lambda^5 + 169035\lambda^4 + 990348\lambda^3 + 3502032\lambda^2 + 6910848\lambda + 5858448)}{12288} \)
\( p^{(4)}_{4,3}(\lambda) = \frac{f(\lambda + 6)^5 (2\lambda^3 + 27\lambda^2 + 120\lambda + 177)}{6144} \)
\( p^{(4)}_{4,4}(\lambda) = \frac{f^4}{49152} \) \hspace{1cm} (B.15)

C \hspace{1cm} \textbf{Zero temperature limit for the 2-antisymmetric representation}

In the 2-antisymmetric representation, the zero temperature limit of the scalar propagator is given by:
\[
\Phi = \lim_{\text{fixed } \mu} \lim_{\tau, r \to \infty} e^{\Delta \mu} \left( \frac{1}{2} 1_{\langle O | 1 \rangle (2 | O | 2) - \frac{1}{2} 1_{\langle O | 2 \rangle (2 | O | 1)} \right),
\]
where \( \Delta \) is the conformal dimension of the dual scalar operator, related to the bulk scalar mass by \( m^2 = \Delta (\Delta - 2) \), and \( O \) is the operator
\[
O = e^{\eta \tau V_2} e^{-\eta \tau V_1} e^{\eta \tau V_2} e^{\eta \tau V_1}.
\]

Using the fact that \( V_2^3 = V_1^2 \ast V_1^2 \), we need the following matrix elements:
\[
V_{1,1}(p, q) = \langle 1 | (V_1^2)^p (V_2^2)^q | 1 \rangle = \delta_{pq} \delta \Gamma(q + \lambda + 1) \Gamma(\lambda + 1),
\]
\[
V_{2,2}(p, q) = \langle 2 | (V_1^2)^p (V_2^2)^q | 2 \rangle = \delta_{pq} (q + 1)! \Gamma(q + \lambda + 2) \Gamma(\lambda + 2),
\]

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in the zero temperature (and large $\tau$) limit to:

$$\lim_{\tau, \mu \to \infty} R_0 = \frac{1}{2} \left[ \sum_{n=0}^\infty \left( \frac{\mu^2 \rho^2}{\sqrt{\bar{\rho}^2 + \mu^2}} \right)^n \frac{\Gamma(2n + 1 + \lambda)}{n! \Gamma(1 + \lambda)} \right] \left[ \sum_{n=0}^\infty \left( \frac{\mu^2 \rho^2}{\sqrt{\bar{\rho}^2 + \mu^2}} \right)^n \frac{(3 + 4n + \lambda) \Gamma(2n + 2 + \lambda)}{n! \Gamma(2 + \lambda)} \right] - \frac{1}{2} \left[ \sum_{n=0}^\infty \left( \frac{\mu^2 \rho^2}{\sqrt{\bar{\rho}^2 + \mu^2}} \right)^n \frac{\Gamma(2n + 2 + \lambda)}{n! \Gamma(1 + \lambda)} \right] \left[ \sum_{n=0}^\infty \left( \frac{\mu^2 \rho^2}{\sqrt{\bar{\rho}^2 + \mu^2}} \right)^n \frac{(1 + 4n + \lambda) \Gamma(2n + 1 + \lambda)}{n! \Gamma(2 + \lambda)} \right].$$

(C.5)

After some simple manipulation, we recover the result (6.8).

References

[1] M. R. Gaberdiel and R. Gopakumar, An AdS$_3$ Dual for Minimal Model CFTs, Phys.Rev. D83 (2011) 066007, [arXiv:1011.2986].

[2] M. A. Vasiliev, Higher spin gauge theories in four-dimensions, three-dimensions, and two-dimensions, Int.J.Mod.Phys. D5 (1996) 763–797, [hep-th/9611024].

[3] M. A. Vasiliev, Higher spin matter interactions in (2+1)-dimensions, hep-th/9607135.

[4] M. R. Gaberdiel and R. Gopakumar, Triality in Minimal Model Holography, JHEP 1207 (2012) 127, [arXiv:1205.2472].

[5] M. Henneaux and S.-J. Rey, Nonlinear $W_{\infty}$ as Asymptotic Symmetry of Three-Dimensional Higher Spin Anti-de Sitter Gravity, JHEP 1012 (2010) 007, [arXiv:1008.4579].

[6] A. Campoleoni, S. Fredenhagen, S. Pfenninger, and S. Theisen, Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields, JHEP 1011 (2010) 007, [arXiv:1008.4744].

[7] M. R. Gaberdiel and T. Hartman, Symmetries of Holographic Minimal Models, JHEP 1105 (2011) 031, [arXiv:1101.2910].

[8] M. Banados, C. Teitelboim, and J. Zanelli, The Black hole in three-dimensional space-time, Phys.Rev.Lett. 69 (1992) 1849–1851, [hep-th/9204099].

[9] M. Banados, M. Henneaux, C. Teitelboim, and J. Zanelli, Geometry of the (2+1) black hole, Phys.Rev. D48 (1993) 1506–1525, [gr-qc/9302012].

[10] M. Ammon, M. Gutperle, P. Kraus, and E. Perlmutter, Black holes in three dimensional higher spin gravity: A review, J.Phys. A46 (2013) 214001, [arXiv:1208.5182].
