Ramp dynamics of phonons in an ion trap: entanglement generation and cooling

T. Dutta(1), M. Mukherjee(1,2), and K. Sengupta(3)

(1) Centre for Quantum Technologies, National University Singapore, Singapore 117543, Singapore.
(2) Physics Department, National University of Singapore, 2 Science Drive 3 Singapore 117551, Singapore.
(3) Theoretical Physics Department, Indian Association for the Cultivation of Science, Jadavpur, Kolkata-700032, India.

(Dated: May 27, 2013)

We show that the ramp dynamics of phonons in an one-dimensional ion trap can be used for both generating multi-particle entangled states and motional state cooling of a string of trapped ions. We study such ramp dynamics using an effective Bose-Hubbard model which describes these phonons at low energies and show that specific protocols, involving site-specific dynamical tuning of the on-site potential of the model, can be used to generate entangled states and to achieve motional state cooling without involving electronic states of the ions. We compare and contrast our schemes for these to the earlier suggested ones and discuss specific experiments to realize the suggested protocols.

PACS numbers: 03.75.Lm, 05.30.Jp, 05.30.Rt

Emulation of isolated strongly correlated quantum systems has been a subject of intense experimental and theoretical research in the recent past (1-3). The two most easily realizable models discussed in this context are the Ising and the Bose-Hubbard model (BHM). The experimental systems used to emulate these models falls into two distinct classes. The first involves ultracold neutral atoms in optical lattices and the second consists of trapped ions. While the former experimental systems allow easy realization of the BHM in higher dimensions, the latter allow better local control on the parameters of the model emulated (4). In such an ion trap based emulator for the BHM, phonons originating from the motional quanta of the ions play the role of bosonic degrees of freedom. It was pointed out in Ref. (5) that the low-energy behavior of these phonons can be described by an effective BHM. The ground state phase diagram of such a system displays a quantum phase transition between the Mott insulating and the superfluid phases (4,5). Moreover, such systems also allows us to study the non-equilibrium dynamics of the model emulated; such dynamics following a local quench of the on-site interaction can be observed by looking at specific experimentally relevant observables (4). The chief advantage of an ion trap emulator lies in the fact that it allows for site-specific tuning of the on-site interaction between the phonons; for example, this interaction can be selectively made negative at specific sites which may lead to several interesting phenomena (6,11). However, the consequence of turning on such a site-specific negative local interaction dynamically with a finite ramp rate has not been yet studied theoretically.

In this letter, we show that turning on a site specific attractive on-site interaction $-U_i$, where $i$ denotes the site index, with a finite ramp time $\tau$ can generate specific entangled pure states of the phonons in a single operation. We note that although in this letter, we shall limit the discussions to the generation of computationally important Bell-state between two sites, the method is not restricted to those states only. We also show that such dynamics involving local attractive potential provides a technique to cool a large string of ions to their transverse motional ground state. Since a real ion trap system always harbors a coupling between the axial and transverse modes, it is possible to cool the axial modes as well. We contend that such cooling is a viable alternate to resolved sideband and sympathetic cooling since it, unlike the latter techniques, does not require the presence of multiple species of ions and leads to a cooling time which is independent of the electronic structure of the ion species used. We compare and contrast our schemes of both dynamic generation of entangled states and the proposed cooling method with the earlier ones and also provide schematics of concrete experiments based on Barium ions in a linear Paul trap for realization of these schemes. To the best of our knowledge, our work constitutes the first concrete proposal of using non-equilibrium dynamics of the BHM for both generating computationally important pure entangled quantum many body states and achieving ground state cooling for ions in a trap; therefore it is expected to be of significant interest to both the experimentalists and theorists studying viable large scale quantum computation architecture as well as non-equilibrium dynamics of strongly correlated systems.

We begin by the description of a concrete experimental setup which may serve as test bed of our proposals. For definiteness, we propose a linear chain of barium ions trapped and Doppler cooled in a linear Paul trap as described in Ref. (4,12,13) as the quantum emulator for the BHM. The proposed trap is operated at 15 MHz radio-frequency and a trap stability parameter $q \sim 0.42$ is used for radial confinement. This generates a secular frequency of $\omega_z \approx 2.25$ MHz. The confinement in the axial direction is achieved by DC voltages applied to the endcap electrodes as shown in Fig. 1. This can be made shallow so that the axial frequency is $\sim 180$ kHz leading to an inter-ion mean distance $\sim 20 \mu$m (14).
which, in turn, leads to a tunneling strength for the transverse motional mode phonons of $J \approx 0.55$ kHz and $\beta_x = 2J/\omega_x \approx 5 \times 10^{-4}$. The inter-ionic distance in such a linear trap varies along the chain; however, such a variation can be neglected for $\beta_x \ll 1$. The reported heating rates in such a system is $\omega_{\text{heat}} \sim 3$ Hz [13]; thus both the ramp rate for phonon dynamics and the measurement cycle needs to be $\gg \omega_{\text{heat}}$ to avoid decoherence.

In the experimental setup, the barium ions are prepared by Doppler cooling using 493 nm and 650 nm (repumper) lasers. After being Doppler cooled, these ions have mean phonon number $\bar{N}_{\text{ph}} \leq 10$. The state of each ion in the string are defined by their internal (S and D states for our purpose) and external motional states (one axial and two radial states). Out of the two internal states, D denotes a meta-stable state with coherence time $\sim 80$ s. The external motional states are ideally decoupled from each other. In the parameter space of interest, the radial motional mode phonons at each site can be considered to be filling up levels of a harmonic oscillator at individual lattice sites (defined by the ion position). These phonons can be excited if the oscillator is anharmonic; such a local anharmonic potential is generated by a standing wave laser field of wavenumber $k$ interacting with the ions and leading to an on-site phonon interaction $U = F \cos^2(kx_i)$, where $x_i$ denote ion coordinates. Such an interaction term, along with the condition of phonon number conservation, as described in Ref. [9], leads to $U = 2(-1)^\delta F \Omega_i^2$, where $F$ and $\delta$ are the strength of the dipole and the phase of the standing wave formed by the laser at the ion’s position respectively. Here $F$ depends on the intensity of the laser; in a typical experimental setup, one has a 120mW argon ion laser focussed to a 5μm beam waist on individual ions which eventually allows individual addressing. In such a setup, it is easily possible to access the parameter range $2 \geq J/U \geq 0.1$. Also, most importantly, $U$ can be made repulsive or attractive by dynamically tuning the local laser phase at each individual site [14]. It is well-known [3, 10] that the effective Hamiltonian which determines the low-energy property of the phonons is given by the BHM

$$H = J \sum_{(ij)} (b_i^\dagger b_j + \text{h.c.}) + \sum_i U_i \hat{n}_i (\hat{n}_i - 1),$$

where $b_j$ denotes the annihilation operator of the bosons (phonons) at site $j$ and $\hat{n}_i = b_i^\dagger b_i$ is the local density operator. Note that the hopping term has a positive sign which is in contrast to the standard BHM realized with ultracold atoms in optical lattices.

To study the dynamics, we consider a linear time evolution of a system of $L$ Barium ions with $N$ phonons according to the protocol $U_i(t) = U_0 + (U_1 - U_0)t/\tau$. The time variation of $U_i$ starts at an initial time $t = 0$ with $U_i = U_0$ and continues till $t = \tau$ when $U_i = U_1$ and is characterized by the rate $\tau^{-1}$. The choice of sites $i$ at which the interaction parameter is dynamically changed depends on the protocol and shall be detailed later for specific cases.

At $t = 0$, we choose a fixed $J/U_0$ at each site and keep the total number of phonons(bosons) fixed to $N$. We use exact diagonalization method for the finite-size system keeping $n \leq N$ boson states per site to obtain the energy eigenstates $|\alpha\rangle$ and eigenvalues $E_\alpha$ for $H(t = \tau)$. In terms of these, one can express the initial ground state $|\psi_G\rangle$ as $|\psi_G\rangle = \sum_\alpha c_\alpha^0 |\alpha\rangle$, where the coefficients $c_\alpha^0$ denote the overlap of the initial ground state of the system (also obtained using exact diagonalization) with $|\alpha\rangle$. The time-dependent Schrödinger equation for the system wavefunction $|\psi(t)\rangle = \sum_\alpha c_\alpha(t)\alpha\rangle$ governing the dynamics of the system now reduces to equations for time evolution of $c_\alpha(t)$: $i\hbar \partial_t \sum_\alpha c_\alpha(t)|\alpha\rangle = H(t)\sum_\alpha c_\alpha(t)|\alpha\rangle$ with the boundary condition $c_\alpha(0) = c_\alpha^0$. To solve these equations, it is convenient to rewrite $H(t) = H(\tau) + \Delta H(t)$ where $\Delta H(t) = \sum_i [U_i(t) - U_1] \hat{n}_i (\hat{n}_i - 1)$. With this choice, one obtains

$$i\hbar \partial_t - E_\alpha c_\alpha(t) = \sum_\beta \Lambda_{\alpha\beta}(t)c_\beta(t)$$

(2)

where $\Lambda_{\alpha\beta}(t) = \langle \beta | \Delta H(t) | \alpha \rangle$. The set of coupled equations for $c_\alpha(t)$ are solved numerically leading to an exact numerical solution for the time-dependent boson wavefunctions.

We now discuss the specific dynamic protocols which leads to the entangled states. In order to generate a Bell-state of the transverse motional mode phonons involving any two sites (say $k$ and $l$; typically chosen to be the first and fourth sites of a linear trap of eight ions with site numbering starting from zero), we start from a fixed $J/U_0$ and ramp the interaction on sites $k$ and $l$ to

![Piezoelectric micromirror actuator](image_url)

FIG. 1: (Color online) A schematic of the experimental setup for the implementation of the ramp protocol. The blue standing wave laser phase is controlled by micro lenses placed on piezo stages.
$U_1 < 0$ (chosen to be $J/U_1 = -0.2$ for definiteness in all numerics) on these sites. The interaction on other sites are kept to $U_0$. Note that if this protocol is carried out adiabatically with total $N$ bosons, it would lead to the Bell state

$$|\psi_{\text{Bell}}\rangle = \frac{1}{\sqrt{2}} (|0N0000\rangle + |0000N0\rangle) ,$$

which is the ground state of the final Hamiltonian of the system. In what follows, we carry out the ramp with a definite rate $\tau^{-1}$ and monitor the time-dependent cross correlation $C_{kl}(t) = \langle \psi(t) | (\hat{b}_k^\dagger \hat{b}_l)^N | \psi(t) \rangle / N!$ between the sites $k = 1$ and $l = 4$. Since for the present case, the non-zero value of such a correlation is equivalent to presence of entanglement, a plot of $C_{14}$ as a function of time for several representative ramp rates provides us with a measure of entanglement generated at any time $t$ during or after the ramp. We find that as the ramp is switched on, $C_{14}$ initially grows and then saturates as shown in Fig. 2. For a fast ramp, $C_{14}$ increases fast but saturates to a value $C_{14}^{\text{Max}}$ which is much less than 0.5 signifying that the time needed for all the phonons to hop to the specified sites are longer than the ramp time. Such a non-adiabatic ramp therefore cannot produce a state which has significant overlap with $|\psi_{\text{Bell}}\rangle$. As our ultimate goal is to perform quantum gate operation using the protocol, we search for the shortest ramp time which maximizes such overlap leading to $C_{14}^{\text{Max}} \approx 0.5$ (the maximum for a Bell state). To this end, we consider a system with $L = 6$ ions and $N = 2$ total phonons, and vary initial value of $J/U_0$ at $t = 0$ to extract the dependence of $C_{14}$ on this parameter. The results are shown in Fig. 3 for three different representative ramp rates. For each ramp rate, we find that $C_{14}^{\text{Max}}$ attains a maximum value for an optimal $J/U_0$. Within the range of $\tau$ that we have studied, we find that $C_{14}^{\text{Max}} \geq 0.48$ is achieved for $J/U_0 \approx 0.18$ and ramp time $\tau \approx 500h/U_0$. We expect the presence of such an optimal $J/U_0$ to be qualitatively unaltered for larger $L$ and $N$ for the following reason. For $J/U_0 = 0$, $[H, \hat{a}_i] = 0$ and the system does not involve due to change of $U$; thus we expect the dynamics to be ineffective for small $J/U_0$. For $J/U_0 \geq 1$, the bosons would tend to delocalize before the system could attain the Bell state during the dynamics. Thus we expect the dynamics to yield optimal result for $0 < J/U_0 < 1$ for any $L$ and $N$. We note that a similar protocol may lead to the $W$ state where the desired state is $|\psi_W\rangle = \frac{1}{\sqrt{3}} (|00N00000\rangle + |0000N000\rangle + |00000N00\rangle)$. Here, the protocol would involve ramping the potential to $U_1$ with $J/U_1 < 0$ at three chosen sites (taken to be second, fourth and sixth sites of the chain for the state given above). A detailed analysis of the optimal ramp rates and cross correlation functions for such a state is left for future work.

Next, we discuss a protocol for cooling. For this, we prepare a linear chain of $L = 8$ ions with $N$ transverse motional mode phonons in superfluid state with $J/U_0 \approx 0.5$ at all sites. The protocol here involves ramping $U$ to negative values at one of the sites (chosen to be the second site of the chain for clarity with $J/U_1 = -0.2$). This leads to migration of transverse motional mode phonons to that site and hence to their single site confinement leaving the rest of the chain in its motional ground state. This is demonstrated in Fig. 4 where $N_2/N$, where $N_2$ is the number of phonons on the second site, is plotted as function of time for $3 \leq N \leq 6$. At least for low total transverse phonon number, we find the the rate of cooling to be independent of the total number; thus we expect our result to hold for $N \geq 6$.
as well. We also find that it takes \( \sim 25(70)\text{ms} \) (with \( U_1 = -1.1\text{kHz} \)) for the system to have \( 90\%(97\%) \) overlap with the final ground state (for which \( N_2 = N \)). The cooling rate is a function of both \( J/U_0 \) and \( \tau^{-1} \); we thus optimize these parameters to obtain the best possible cooling which is shown in Fig. 4 for \( N = 3 \). We note that the ramp essentially leads to localization of excitation energies (phonons) to a single ion site, and hence to energy reduction of other sites without involving dissipative mechanism. This mechanism is therefore expected to be effective for a large chain of ions a part of which is used, for example, as a qubit since it may be used to remove transverse motional mode energy from the computationally important qubit states located at specific section of the chain.

The precise experimental steps for generation of the entangled Bell state and cooling are as follows. First, \( L = 8 \) ions of barium are loaded into a linear ion trap forming a chain along the axis of the linear trap. Second, these ions are then Doppler cooled to mean phonon numbers of about \( N = 6 \). Third, a standing wave laser at 476 nm is formed along the transverse direction of the trap with the ions at the anti-node. The laser power is adjusted such that the \( J/U_0 \sim 0.2 \) is obtained. Fourth, for the formation of one of the Bell-states as mentioned in Eq. [4] between sites \( k = 1 \) and \( l = 4 \), the 476 nm standing wave laser is phase shifted by \( \pi \) phase (node) by a piezo-mounted retro-reflecting mirror. Similar procedure is adapted for the ‘W’ state formation (cooling) with \( U_i \) changed for three (one) specific sites (site) as mentioned before. In all cases, the total time of the ramp, \( \tau^{-1} \), and \( J/U_0 \) shall determine the fidelity of the state obtained and the speed of the gate operation and/or cooling. The quantities can be varied, as shown in our numerical studies above, to obtain an optimal operating point for Bell state generation/cooling.

The main difference of our proposals for state preparation and cooling as compared to other proposals with trapped ions lies in its use of a dynamic ramp. Unlike the original Cirac and Zoller’s proposal [16], it is not necessary to apply sequence of laser pulses to generate a pure many body quantum state or to initialize the qubits. Though the Mølmer and Sørensen [17] type of quantum gate operation does not require initialization or ground state cooling, they require sequential pulses to be applied in order to create a many body pure quantum entangled state. In contrast, we do not need such elaborate sequence. Very recently, there has been a proposal to look for entanglement growth for 1D ultracold atom system in optical lattices after a quench [18]. However, such a proposal, in contrast to ours, do not provide deterministic entangled state formation. Regarding cooling, the most extensively used technique is the resolved side band cooling which requires addressing of all the ions [20]; in contrast, the ramp protocol described here addresses an individual site. Also, compared to cavity sideband cooling, it does not require complicated cavity setup. Moreover, in stark contrast to the available motional state cooling techniques, the ramp protocol is free from use of metastable states to resolve the motional sidebands. For such techniques, the cooling time strongly depends on metastable state lifetime and varies between \( 300 - 30\text{ms} \) (for Hg\(^+\) and Be\(^+\) ions) [20]; in contrast, our method leads to a cooling time which is independent of the ion’s electronic structure. Thus it constitutes an alternative ground state cooling method of a large ion string where all but one can be used as qubits.

In conclusion, we have shown that dynamic ramp of
phonons emulating the BHM in a linear chain of trapped ions is capable of producing computationally important maximally entangled state among large number of qubits. A very similar protocol can also perform ground state cooling of the transverse motional modes of a large chain of ions. We have provided details of the dynamic ramp protocol required for such operations and have also charted out the optimal parameter regime for implementing them. We have shown that both these processes can be implemented by relatively straightforward local protocols which are well within current experimental capability and provided a comparison of our proposal to the existing ones for both cooling and entangled state generation. We expect these protocols to be of use in future quantum computer architecture using these systems.

[1] M. Greiner, O. Mandel, T. Esslinger, T. W. Hensch and I. Bloch Nature 415, 39 (2002); C. Orzel, A. K. Tuchman, M. L. Fenselau, M. Yasuda and M. A. Kasevich, Science 291, 2386 (2001); T. Kinoshita, T. Wenger, and D. S. Weiss, Nature 440, 900 (2006); L. E. Sadler, J. M. Higbie, S. R. Leslie, M. Vengalattore and D. M. Stamper-Kurn, Nature 443, 312 (2006).
[2] D. Jaksch, C. Bruder, J. I. Cirac, C. W. Gardiner, and P. Zoller, Phys. Rev. Lett. 81, 3108 (1998).
[3] D. Porras and J. I. Cirac, Phys. Rev. Lett. 93, 263602 (2004); X.-L. Deng, D. Porras, and J. I. Cirac, Phys. Rev. A 77, 033403 (2008).
[4] E. E. Edwards et al., Phys. Rev. B 82, 060412(R) (2010); R. Islam et al., Nat. Comm. 2, 377 (2011).
[5] J. Simon et al., Nature 472, 307 (2011).
[6] S. Sachdev, K. Sengupta, and S. M. Girvin, Phys. Rev. B 66, 075128 (2002); K. Sengupta, S. Powell, and S. Sachdev Phys. Rev. A 69, 053616 (2004); M. Kobodurez, D. Pekker, B. K. Clark, and K. Sengupta, Phys. Rev. B 85, 100505 (2012).
[7] W. S. Bakr et al., Science 329, 547 (2010).
[8] S. Sachdev, Quantum Phase Transitions, Cambridge University Press, Cambridge, England, (1999).
[9] T. Dutta, M. Mukherjee and K. Sengupta, Phys. Rev. A 85, 063401 (2012).
[10] D. Porras and J. I. Cirac, Phys. Rev. Lett. 92, 207901 (2004); G.-D. Lin, C. Monroe, and L.-M. Duan, Phys. Rev. Lett. 106, 230402 (2011).
[11] S. Braun et al., Science 339, 52 (2013).
[12] A. V. Steele, L. R. Churchill, P. F. Griffin and M. S. Chapman, Phys. Rev. A 75, 053404 (2007).
[13] F. Dubin, D. Rotter, M. Mukherjee, C. Russo, J. Eschner, and R. Blatt, Phys. Rev. Lett. 98, 183003 (2007).
[14] D. F. A. James, Appl. Phys. B 66, 181190 (1998).
[15] G. Kirchmair, J. Benhelm, F. Zähringer, R. Gerritsma, C. F. Roos, and R. Blatt, New J. Phys. 11, 023002 (2009).
[16] J. I. Cirac and P. Zoller, Phys. Rev. Lett. 74, 4094-4097 (1995).
[17] K. Mølmer and A. Sørensen, Phys. Rev. Lett. 82, 1835 (1999).
[18] A. J. Daley, H. Pichler, J. Schachenmayer, and P. Zoller, Phys. Rev. Lett. 109, 020505 (2012).
[19] C. A. Regal, C. Ticknor, J. L. Bohn, and D. S. Jin, Nature 424, 47 (2003); M. Greiner, C. A. Regal, and D. S. Jin, Nature 426, 537 (2003).
[20] D. Leibfried, R. Blatt, C. Monroe, and D. Wineland, Rev. Mod. Phys. 75, 281 (2003).