Susceptibility Amplitude Ratio for Generic Competing Systems

C.F. Farias · Marcelo M. Leite

Received: 18 November 2011 / Accepted: 27 July 2012 / Published online: 12 August 2012
© Springer Science+Business Media, LLC 2012

Abstract We calculate the susceptibility amplitude ratio near a generic higher character Lifshitz point up to one-loop order. We employ a renormalization group treatment with $L$ independent scaling transformations associated to the various inequivalent subspaces in the anisotropic case in order to compute the ratio above and below the critical temperature and demonstrate its universality. Furthermore, the isotropic results with only one type of competition axes have also been shown to be universal. We describe how the simpler situations of $m$-axial Lifshitz points as well as ordinary (noncompeting) systems can be retrieved from the present framework.

Keywords Field theory · Renormalization group · Generic Lifshitz critical behaviors · Amplitude ratio

1 Introduction

The occurrence of $m$-axial Lifshitz phase transitions [1, 2] in various real physical systems (e.g., magnetic modulated materials [3–8], high-$T_c$ superconductors [9–11], liquid crystals [12–14], etc.) has increased the interest in the field-theoretic description of this subject in the last few years [15–20]. According to modern renormalization group arguments, critical phenomena of $m$-axial Lifshitz competing systems have their universality classes characterized by $(N, d, m)$, namely, the number of components of the (field) order parameter $N$ dwelling in $d$ space dimensions with $m$ ($= m_2$) space directions presenting...
alternate (repulsion-attraction) interactions among the fields [21–23]. In particular, a Lifshitz point arises at the confluence of the disordered, a uniformly ordered and a modulated phase. Other types of “competing axes” can be defined: if the alternate couplings are of the type attractive-repulsive-attractive and take place along $m_3$ spatial dimensions, one refers to $m_3$-axial third-character Lifshitz critical behavior (for the realization of the $m_3 = 1$ case, see [24]).

More generally, a $m_L$-fold $L$-th character Lifshitz behavior appears whenever short-ranged alternate interactions with $L$ couplings of the type repulsion-attraction-repulsion-attraction-… are allowed [25–27]. If all sorts of the aforementioned competing axes are present simultaneously in the critical system under consideration, its phase transitions are governed by the generic higher character Lifshitz critical behavior. The universality classes of these arbitrary competing systems are defined by the set $(N, d, m_2, \ldots, m_L)$ [28–30].

The simplest realization of the usual Lifshitz universality class $(N, d, m)$ can be encountered in uniaxial systems ($m = 1$ critical behavior). It can be understood in terms of a modified Ising model ($N = 1$) [31], namely the axial next-nearest-neighbor Ising (ANNNI) model [32, 33] with the usual ferromagnetic couplings (among all nearest-neighbors) with the inclusion of antiferromagnetic exchange interactions among second neighbor spins only along a single axis. Models with alternate couplings up to $L$ neighbor spins coupled through competing interactions (ferro-antiferro-ferro-antiferromagnetic-, etc.) along $m_L$ spatial dimensions have also been proposed [25–27]. They present $m_L$-axial Lifshitz critical behavior of $L$-th character either anisotropic when $m_L \neq d$ or isotropic if $m_L = d$. The anisotropic Lifshitz point of generic $L$-th character is described by the competing exchange coupling Ising (CECI) model [28], which explains arbitrary competing systems.

The theoretical mean-field prediction of $m$-fold isotropic behavior in copolymer-homopolymer ternary blends [34, 35] and its experimental identification in mixtures of block copolymer-homopolymer [36] caused a certain enthusiasm, but subsequent work on the subject showed a microemulsion phase incompatible with the existence of the Lifshitz point [37]: the fluctuation destroys its mean-field value, although the presence of the lamellar (modulated ordered) phase was a clear indication of Lifshitz behavior. The theoretical effect of fluctuations was incorporated immediately afterward using a self-consistent field theory (SCFT), and the modulated phase was understood utilizing a one-component order parameter ($N = 1$) [38]. This result confirmed the previous discussion from Ref. [37] and also located loci in the mean field phase diagram with third character isotropic Lifshitz point. Later, mean field studies using SCFT indicated the existence of up to 6th character isotropic critical behavior in blends of diblock copolymers [39].

The susceptibility of a homopolymer-diblock copolymer blend (polybutadiene and polystyrene) has been investigated recently using small angle neutron scattering. Some amplitudes above the Lifshitz temperature were estimated for fixed values of the diblock copolymer composition [40]. The closest we can get to this system using field theory techniques is to look for universal quantities related to the susceptibility, i.e., the amplitude ratio above and below the Lifshitz critical temperature [41].

In this paper, the universal susceptibility amplitude ratio for generic competing systems is computed using field theory and renormalization group arguments up to one-loop order. By restricting the number of competition axes we can assess particular universality classes, e.g., the $m$-axial anisotropic Lifshitz criticalities. We show that the uniaxial result can be retrieved from the arbitrary anisotropic competing systems. The isotropic amplitude ratios for isotropic critical behaviors are calculated for the first time. We compare our field-theoretic $m = 3$ isotropic output with experimental results from homopolymer-diblock copolymer mixtures and show very good agreement among them.
The paper is organized as follows. In Sect. 2 we present the one-loop effective potential. We highlight a brief explanation of the several subspaces which occur in the problem along with the independent renormalization group transformations in the anisotropic cases. A simpler analogous discussion for calculating the isotropic amplitude ratio is explicited in Sect. 3. Section 4 presents the discussion of the results and conclusions.

2 Anisotropic Amplitude Ratio for Generic Competing Systems

We begin with the bare Lagrangian density associated with anisotropic generic competing systems described by the CECI model, which is given by

\[
L = \frac{1}{2} \left| \nabla_{(d-\sum_{n=2}^{L} m_n)} \phi_0 \right|^2 + \sum_{n=2}^{L} \frac{\sigma_n}{2} \left| \nabla_{m_n} \phi_0 \right|^2 + \sum_{n=2}^{L} \delta_{0n} \left( \left| \nabla_{m_n} \phi_0 \right|^2 + \sum_{n' = 2}^{L-1} \sum_{n'' = 2}^{n-1} \tau_{nn'} \left| \nabla_{m_n} \phi_0 \right|^2 \right) + \frac{1}{2} t_0 \phi_0^4 + \frac{1}{4!} \lambda_0 \phi_0^4.
\]  

(1)

The parameters which correspond to the physical situations are the coefficients of the derivatives of the bare field \(\phi_0\) (order parameter of the phase transition), the bare reduced temperature \(t_0\) (\(\propto T - T_L\), where \(T_L\) is the Lifshitz critical temperature) and the bare coupling constant \(\lambda_0\).

The Lifshitz critical region is defined for particular combinations of the exchange interactions among all the neighbors. In terms of the parameters appearing on the above mentioned Lagrangian density, this critical region is equivalent to set \(\delta_{0n} = \tau_{nn'} = 0\) in Eq. (1).

At the Lifshitz critical region, the temperature is close but not equal to \(T_L\). We can set \(\sigma_n = 1\) provided we perform a dimensional redefinition in the momentum characterizing the \(n\)-th \(m_n\)-dimensional competition subspace. If \(\Lambda\) is a momentum scale, we take the engineering dimension of the competing subspace as \([k(n)] = \Lambda^{\frac{1}{n}}\).

Consider any Feynman integral. For example, an explicit integral that shall be used is the one-loop contribution to the four-point function, namely

\[
I(P, K_{(2)}, \ldots, K_{(L)}) = \int \frac{d^{(d-\sum_{n=2}^{L} m_n)} q \prod_{n=2}^{L} d^{m_n} k_{(n)}}{\left[\sum_{n=2}^{L} ((k_{(n)} + K_{(n)})^2)^n + (q + P)^2 \left(\sum_{n=2}^{L} (k_{(n)}^2)^n + q^2 \right)\right]}.
\]  

(2)

Using normalization conditions in the computations of the vertex parts required in the present paper it suffices the exact calculation of this integral at the special symmetry point characterized by the following values of external momenta: \(P^2 = \kappa^2 = 1\), \(K_{(n)} = 0\), \(n = 2, \ldots, L\). For further details, the reader is advised to consult Ref. [29].

Let \(Z_{\phi^2}, Z_\phi\) be normalization functions and \(g^*\) be the renormalized coupling constant at the fixed point. The bare field and temperature are given by \(t_0 = Z_{\phi^2}^{-1} t, \phi_0 = Z_{\phi}^{-1} M\), where \(t, M\) are the renormalized reduced temperature and order parameter, respectively. The dimensionful renormalized coupling at the fixed point can be expressed in terms of the dimensionless entity \(u^*\) as \(g^* = u^* \kappa^* \epsilon_L\), where \(\epsilon_L = 4 + \sum_{n=2}^{L-1} \frac{(n-1)}{n} m_n - d\) is the perturbation parameter.
Using the symmetry point, the one-loop renormalized Helmholtz free energy density at the fixed point is given by

\[ F(t, M) = \frac{1}{2} t M^2 + \frac{1}{4!} g^* M^4 + \frac{1}{4} \left( t^2 + g^* t M^2 + \frac{1}{4} (g^* M)^2 \right) I_{SP} \]

\[ + \frac{1}{2} \int d^{(d-L)} m q \left[ \prod_{n=2}^L d m_n k(n) \right] \left[ \ln \left( 1 + \frac{t + \frac{1}{2} g^* M^2}{(\sum_{n=2}^L (k(n)^2)^n + q^2)} \right) \right] \]

\[ - \frac{g^* M^2}{2(\sum_{n=2}^L (k(n)^2)^n + q^2)} \],

where \( \tilde{q} \) is a \((d-m)\)-dimensional wave vector perpendicular to the competing axes, whereas \( \vec{k} \) is an \( m \)-dimensional wave vector whose components are parallel to the competition axes. The integral \( I_{SP} \) is defined by Eq. (2) at the convenient symmetry point above defined. Whenever a loop integral is performed, a typical geometric angular factor is produced, which can be factored out in a redefinition of the coupling constant in a standard way \[31\]. In our case this factor is given by the expression \( [S_{(d-\sum_{n=2}^L m_n)} \Gamma(2 - \sum_{n=2}^L \frac{m_n}{2})/(\prod_{n=2}^L \frac{\Gamma(\frac{m_n}{2})}{\Gamma(\frac{m_n+1}{2})})] \), such that it is going to be omitted whenever we report the result of any loop integral. The last integral at the chosen symmetry point was computed exactly in Ref. \[29\] (it involves no approximation whenever \( K(n) = 0 \) for \( n = 2, \ldots, L \)) and shown to be given by \( I_{SP} = \frac{1}{\epsilon_L}(1 + h_{mL} \epsilon_L) \),

where \( h_{mL} = 1 + \frac{\psi(1) - \psi(2 - \sum_{n=2}^L \frac{m_n}{2})}{2} \) and \( \psi(z) = \frac{\ln \Gamma(z)}{dz} \). It is worthy to stress that whenever \( m_3 = \cdots = m_L = 0 \) \((m_2 = m)\), \( h_{mL} = [i_2]_m = 1 + \frac{\psi(1) - \psi(2 - \frac{m}{2})}{2} \) and the usual anisotropic \( m \)-axial Lifshitz critical behavior is obtained from this more general competing situation in a rather simple manner (see also Ref. \[22\]).

Since we need the value of \( M \) in the coexistence curve above and below \( T_L \), let us compute the renormalized magnetic field, which is given by

\[ H_R = \frac{\partial F}{\partial M} = t M + \frac{1}{6} u^* M^3 + \frac{u^* M}{2} \left( t + \frac{u^* M^2}{2} \right) [I_{SP} - I], \]

where

\[ I = \int \frac{d^{(d-L)} m q \prod_{n=2}^L d m_n k(n)}{[\sum_{n=2}^L ((k(n)^2)^n + q^2)(\sum_{n=2}^L (k(n)^2)^n + q^2 + t + u^* M^2)/2}]. \]

Let us compute explicitly this integral. First we use a Feynman parameter. Second, we employ the identity

\[ \int d^d q \left[ \frac{1}{[q^2 + 2kq + m^2]^a} \right] = \frac{S_d \Gamma(\frac{d}{2}) \Gamma(\frac{d}{2} - \frac{d}{2})(m^2 - k^2)^{\frac{d}{2} - a}}{2 \Gamma(\alpha)}, \]

and get to

\[ I = \frac{1}{2} S_{(d-\sum_{n=2}^L m_n)} \Gamma \left( d - \sum_{n=2}^L m_n \right) \Gamma \left( 2 - \left( d - \sum_{n=2}^L m_n \right) \right) \int_0^1 dx \int \prod_{n=2}^L d m_n k(n) \]

\[ \times \left[ \frac{1}{[\sum_{n=2}^L ((k(n)^2)^n + x(t + u^* M^2))(2 - (d-\sum_{n=2}^L m_n))/2} \right]. \]

Now we have to perform the remaining integral with higher power of momentum. Indeed, in the integral

\[ i_n = \int \frac{d m_n k(n)}{[\sum_{n=2}^L ((k(n)^2)^n + m^2)]^n}, \]
perform the change of variables \( r_{(n)}^2 = k_{1(n)}^2 + \cdots + k_{m(n)}^2 \), call \( z = r_{(n)}^n \) and let \( z' = z^2 \). We obtain
\[
i_n = \frac{1}{2n \Gamma(n)} S_m \Gamma\left(\frac{m}{2n}\right) \Gamma\left(\gamma - \frac{m}{2n}\right) (m^2)^{-\gamma + \frac{m}{2n}}.
\]
Replacing this identity back into the expression for \( I \), using \( \epsilon_L = 4 + \sum_{n=2}^{L} (n-1)m_n - d \), developing the argument of the \( \Gamma \)-functions by using the identity \( \Gamma(a + bx) = \Gamma(a) \left(1 + bx \psi(a)\right) \) and recalling to absorb the angular factor already mentioned above, we can show that the integral \( I \) has the following singular structure
\[
I = \frac{1}{\epsilon_L} \left(1 + \epsilon_L \left[h_{m_L} - \frac{1}{2} \left(1 + \ln\left(t + u^* M^2\right)\right)\right]\right).
\]
We have to take another derivative of \( H_R \) with respect to \( M \), which will produce the inverse susceptibility
\[
\chi^{-1} = \frac{\partial H_R}{\partial M} = t + \frac{u^* M^2}{2} + \frac{u^*}{4} \left(t + \frac{3u^* M^2}{2}\right) \left[1 + \ln\left(t + \frac{u^* M^2}{2}\right)\right] + \frac{u^*^2 M^2}{4}.
\]
For \( T > T_L \) we substitute \( M = 0 \) and the coupling constant at the fixed point value \( u^* = \frac{2\epsilon_L}{\gamma} + O(\epsilon_L^2) \) into last equation, which produces the result
\[
\chi(T > T_L) = \left(1 - \epsilon_L\right) t^{-\left(1 + \frac{\epsilon_L}{6}\right)}.
\]
Note that \( \gamma_L = 1 + \frac{\epsilon_L}{6} \) and \( \chi(T > T_L) \) above is consistent with scaling in the neighborhood of the critical point.

When \( T < T_L \), we have to use the value of \( M \) at the coexistence curve which is defined by the condition \( H_R = 0 \), namely
\[
M^2 = \frac{-6t}{u^*} + 3t \left[1 + \ln(-2t)\right].
\]
Replace this value at Eq. (11) and neglect \( O(u^*^2 \sim \epsilon_L^2) \). Next, expand the “logarithm of the logarithm” in the expression for \( \chi^{-1} \) using the expansion \( \ln(1 + x) = x + O(x^2) \). Employing the fixed point \( u^* = \frac{2\epsilon_L}{\gamma} + O(\epsilon_L^2) \) and neglecting \( O(\epsilon_L^2) \), we obtain
\[
\chi = (-t)^{-\left(1 + \frac{\epsilon_L}{6}\right)} \frac{1}{2} \left[1 - \frac{\epsilon_L}{6} (4 + \ln 2)\right].
\]
Consequently, the susceptibility amplitude ratio is given by
\[
\frac{C_+}{C_-} = 2 \left[1 + \frac{\epsilon_L}{6} (3 + \ln 2)\right] = 2^{\gamma_L - 1} \frac{\gamma_L}{\beta_L},
\]
where \( \beta_L = \frac{1}{2} - \frac{\epsilon_L}{6} \).

This expression is exact at one-loop level, its functional form in \( \epsilon_L \) for several universal classes is the same, but the latter encodes distinct universalities since \( \epsilon_L = \epsilon_L(d, m_2, \ldots, m_L) \).

### 3 Amplitude Ratio for Generic Isotropic Competing Systems

There are some minor modifications in the isotropic behaviors, but the trend to obtain the amplitude ratio follows the same script as in the anisotropic case. As there is only one
subspace, say along $d = m_n$ space directions coupling $n$ neighbors via alternate competing interactions, the bare density Lagrangian is given by

$$L = \delta_{0n} \frac{1}{2} |\nabla_{mn} \phi_0|^2 + \sum_{n'=2}^{n-1} \frac{1}{2} \tau_{nn'} |\nabla_{mn} \phi_0|^2 + \frac{\sigma_n}{2} |\nabla_{mn} \phi_0|^2$$

$$+ \frac{1}{2} t_0 \phi_0^2 + \frac{1}{4} \lambda_0 \phi_0^4.$$  \(16\)

As before, the isotropic critical region is defined by $\delta_{0n} = \tau_{nn'} = 0$ with $T \neq T_L$. There is only one renormalization group direction characterized by the $\xi_n$ correlation length. We perform a dimensional redefinition in the momentum just as done in the discussion of the anisotropic behavior. The expansion parameter is now $\epsilon_n = 4n - m_n$. The renormalized free energy at one loop can be written in the form

$$F(t, M) = \frac{1}{2} t M^2 + \frac{1}{4!} g^* M^4 + \frac{1}{4} \left( t^2 + g^* t M^2 + \frac{1}{4} \left( g^* M^2 \right)^2 \right) I_{SP}$$

$$+ \frac{1}{2} \int d^{m_n} k \left[ \ln \left( 1 + \frac{t + \frac{1}{2} g^* M^2}{k^{2n}} \right) - \frac{g^* M^2}{2 k^{2n}} \right],$$  \(17\)

and the nomenclature is almost the same as in the anisotropic case, except that now the integral $I_{SP}$ given by

$$I_{SP} = \left[ \int \frac{d^{m_n} k}{(k + K)^{2n} k^{2n}} \right].$$  \(18\)

is computed at the symmetry point $K^{2n} = k^{2n} = 1$. Performing a derivative with respect to $M$, we obtain

$$H_R = \frac{\partial F}{\partial M} = t M + \frac{1}{6} u^* M^3 + \frac{u^* M^2}{2} \left( t + \frac{u^* M^2}{2} \right) [I_{SP} - I],$$  \(19\)

with

$$I = \int \frac{d^{m_n} k}{k^{2n}(k^{2n} + t + \frac{u^* M^2}{2})}.$$  \(20\)

Let us compute this last integral by employing a Feynman parameter. We then discover that the resulting integral has the same pattern as Eq. (8) and can be solved along the same changes of variables in a identical manipulation which led to Eq. (9). The geometric angular factor which appears here is just the area of the $m_n$-dimensional unity sphere $S_{m_n}$ and shall be absorbed in a redefinition of the coupling constant as before. Carrying out this procedure, we get to

$$I = \frac{1}{\epsilon_n} \left[ 1 - \frac{\epsilon_n}{2n} \ln \left( t + \frac{u^* M^2}{2} \right) \right].$$  \(21\)

The $I_{SP}$ integral was already computed in Ref. [29] at the symmetry point and was shown to be given by the expression $I_{SP} = \frac{1}{\epsilon_n} \left[ 1 + D(n) \epsilon_n \right]$, where $D(n) = \frac{1}{2} [\psi(2n) + \psi(1)] - \psi(n)$. First, using Eq. (21) we find

$$I_{SP} - I = D(n) + \frac{1}{2n} \ln \left( t + \frac{u^* M^2}{2} \right),$$  \(22\)

which turns out to result in the following magnetic field

$$H_R = \frac{\partial F}{\partial M} = t M + \frac{1}{6} u^* M^3 + \frac{u^* M^2}{4n} \left( t + \frac{u^* M^2}{2} \right) \left[ 2n D(n) + \ln \left( t + \frac{u^* M^2}{2} \right) \right].$$  \(23\)
It is easy to show that the inverse susceptibility which follows can be written as
\[ \chi^{-1} = t + \frac{u^* M^2}{2} + \frac{u^*}{4n} \left( t + \frac{3u^* M^2}{2} \right) \left[ 2n D(n) + \ln \left( t + \frac{u^* M^2}{2} \right) \right] + \frac{u^{*2} M^2}{4n}. \] (24)

Hereafter we are going to use the coupling constant at the fixed point, i.e., \( u^* = \frac{2\epsilon_n}{3} \). Set \( M = 0 \) for \( T > T_L \) in the above equation in order to find the susceptibility in the following form
\[ \chi(T > T_L) = t^{-\gamma_n} \left( 1 - \frac{D(n) \epsilon_n}{3} \right). \] (25)

For \( T < T_L \) the value of \( M \) in the coexistence curve is given by
\[ M^2 = \frac{(-6t)}{u^*} + \frac{3t}{n} \left[ 2n D(n) + \ln(-2t) \right]. \] (26)

Substitution of this value into the expression for \( \chi^{-1} \) results in the following value for the susceptibility below \( T_L \)
\[ \chi(T > T_L) = (-2t)^{-\gamma_n} \left[ 1 - \frac{\epsilon_n}{2n} \left( 1 + \frac{2n D(n)}{3} \right) \right]. \] (27)

Using Eqs. (25) and (27), we finally obtain
\[ \frac{C_+}{C_-} = 2 \left[ 1 + \frac{\epsilon_n}{2n} + \frac{\epsilon_n}{6n} \ln 2 \right] = 2^{\gamma_n - 1} \frac{\gamma_n}{\beta_n}. \] (28)

4 Conclusion

Our results for the susceptibility amplitude ratios involve no approximation either in the anisotropic or isotropic behaviors. In the former, if we turn off all the competing interactions, i.e., by setting \( m_3 = \cdots = m_L = 0 \), keeping just alternate couplings among second neighbors and identify \( m_2 = m \), we obtain the result for the anisotropic \( m \)-axial universality class. Plus, it reproduces the uniaxial case \( m = 1 \) studied earlier [41]. For instance, three-dimensional systems have perturbative parameters \( \epsilon_1 = 1 + \frac{m}{2} \) which change at distinct values of \( m \). Besides, if go on and switch off the competing interactions among second neighbors (\( m = 0 \)) we obtain the result of the Ising-like universality class.

The isotropic amplitude ratios, on the other hand, possess their own version of universality class reduction. Different values of the number of neighbors coupled via alternate couplings (\( n \)) are responsible by the variation of the susceptibility ratio. The case \( n = 2 \) corresponds to the isotropic \( m \)-axial (\( d = m \) close to 8) universality class. Although the perturbative parameter is rather large (\( \epsilon_2 = 5 \)), we can extract meaningful results from our output in connection with three-dimensional systems such as copolymer mixtures [34, 35, 38, 40, 42].

Although the value of the amplitude above the critical temperature is not universal, let us compare it with our exact results for the isotropic case \( n = 2 \) using Table III from Ref. [40]. For diblock polymer composition \( \Phi_{DB} = 0.073 \) measured at temperature 69.5 ± 0.2 °C the amplitude value is \( C_+ = 0.94 \pm 0.07 \), which is consistent with the exact amplitude from Eq. (28) (using \( \epsilon_2 = 5 \) and \( D(2) = -\frac{1}{11} \)), namely \( C_+ = 1.13 \). Even though those authors confirmed the absence of the isotropic Lifshitz point, they considered the Lifshitz critical region with the inclusion of the fluctuations using SCFT (Ref. [38]) in their experimental fits of the susceptibility curves. Thus, this is the first solid indication that field-theoretic results
including the contribution of fluctuations are consistent with experiments in those sort of polymers.

Another aspect is the theoretical possibility of occurrence of up to 6th character Lifshitz points in $AB/BC$ mixtures of diblock copolymers [39]. If this system can be fabricated in the laboratory, our work represents a prevision of results for the susceptibility with increasing values of the perturbation parameter for three-dimensional systems.

The most interesting extension of the method proposed here is the study of the specific heat amplitude ratio for generic competing systems, generalizing the discussion carried out for the anisotropic $m$-axial critical behavior [43]. It would be nice to tackle the computation of other universal amplitude ratios either at one-loop level or to extend the method to compute amplitude ratios at two-loop order [44] for generic competing systems. The orthogonal approximation would be required in anisotropic criticalities, however, since it is the only analytical method available to evaluating higher-order Feynman diagrams in the literature.

Experimental techniques in order to determine the susceptibility amplitude ratio in magnetic systems such as MnP, Mn$_{0.9}$Co$_{0.1}$P [45–48], etc., would be most welcome in identifying (or ruling out) analogues of the critical systems described in the present work.

Acknowledgements CFF would like to thank CAPES for financial support.

References

1. Hornreich, R.M., Luban, M., Shtrikman, S.: Phys. Rev. Lett. 35, 1678 (1975)
2. Hornreich, R.M.: J. Magn. Magn. Mater. 15–18, 387 (1980)
3. Becerra, C.C., Shapira, Y., Oliveira, N.F. Jr., Chang, T.S.: Phys. Rev. Lett. 44, 1692 (1980)
4. Yokoi, C.S.O., Coutinho-Filho, M.D., Salinas, S.R.: Phys. Rev. B 24, 5430 (1981)
5. Yokoi, C.S.O., Coutinho-Filho, M.D., Salinas, S.R.: Phys. Rev. B 29, 6341 (1984)
6. Becerra, C.C., Bindilatti, V., Oliveira, N.F. Jr.: Phys. Rev. B 62, 8965 (2000)
7. Hanawa, T., Shinkawa, K., Ishikawa, M., Miyatani, K., Saito, K., Kohn, K.: J. Phys. Soc. Jpn. 63, 2706 (1994)
8. Weitzel, H., Ehrenberg, H., Heid, C., Fuss, H., Burlet, P.: Phys. Rev. B 62, 12146 (2000)
9. Hayden, S.M., Aeppli, G., Mook, H., Ryz, D., Hundley, M.F., Fisk, Z.: Phys. Rev. Lett. 66, 821 (1991)
10. Keimer, B., Birgeneau, R.J., Cassanho, A., Endoh, Y., Erwin, R.W., Kastner, M.A., Shirane, G.: Phys. Rev. Lett. 67, 1930 (1991)
11. Sachdev, S., Ye, J.: Phys. Rev. Lett. 69, 2411 (1992)
12. Ramanavare, S.B., Pisipati, V.G.K.M., Wong, E.W.: Phys. Rev. Lett. 72, 3558 (1994)
13. Zalar, B., Gregorovič, A., Simšič, M., Zidanšec, A., Binc, R., Keast, S., Neubert, M.: Phys. Rev. Lett. 80, 4458 (1998)
14. Škarabot, M., Binc, R., Muševič, I., Rastegar, A., Rasing, Th.: Phys. Rev. E 61, 3961 (2000)
15. Mergulhão, C. Jr., Carneiro, C.E.I.: Phys. Rev. B 58, 6047 (1998)
16. Mergulhão, C. Jr., Carneiro, C.E.I.: Phys. Rev. B 59, 13954 (1999)
17. Albuquerque, L.C., Leite, M.M.: unpublished. cond-mat/0006462
18. Albuquerque, L.C., Leite, M.M.: J. Phys. A, Math. Gen. 34, L327 (2001)
19. Diehl, H.W., Shpot, M.: Phys. Rev. B 62, 12338 (2000)
20. Shpot, M., Diehl, H.W.: Nucl. Phys. B 612(3), 340 (2001)
21. Leite, M.M.: unpublished. hep-th/0109037
22. Leite, M.M.: Phys. Rev. B 67, 104415 (2003)
23. Carvalho, P.R.S., Leite, M.M.: Ann. Phys. 324, 178 (2009)
24. Selke, W.: Z. Phys. B 27, 81 (1977)
25. Selke, W.: Phys. Lett. A 61, 443 (1977)
26. Nicoll, J.F., Tuthill, G.F., Chang, T.S., Stanley, H.E.: Phys. Lett. A 58, 1 (1976)
27. Nicoll, J.F., Tuthill, G.F., Chang, T.S., Stanley, H.E.: Physica B 86-88, 618 (1976)
28. Leite, M.M.: Phys. Lett. A 326, 281 (2004)
29. Leite, M.M.: Phys. Rev. B 72, 224432 (2005)
30. Carvalho, P.R.S., Leite, M.M.: Ann. Phys. 325, 151 (2010)
31. For similar field-theoretic conventions in noncompeting systems, see Amit, D.J., Martin-Mayor, V.: Field Theory, the Renormalization Group and Critical Phenomena, 3rd edn. World Scientific, Singapore (2005)
32. Selke, W.: Phys. Rep. 170, 213 (1988)
33. Selke, W.: In: Domb, C., Lebowitz, J. (eds.) Phase Transitions and Critical Phenomena, vol. 15, p. 1. Academic Press, London (1992)
34. Broseta, D., Fredrickson, G.H.: J. Chem. Phys. 93, 2927 (1990)
35. Holyst, R., Schick, M.: J. Chem. Phys. 96, 7728 (1992)
36. Bates, F.S., Maurer, W., Lodge, T.P., Schulz, M.F., Matsen, M.W., Almdal, K., Mortensen, K.: Phys. Rev. Lett. 75, 4429 (1995)
37. Bates, F.S., Maurer, W.W., Lipic, P.M., Hillmyer, M.A., Almdal, K., Mortensen, K., Fredrickson, G.H., Lodge, T.P.: Phys. Rev. Lett. 79, 849 (1997)
38. Kielhorn, L., Muthukumar, M.: J. Chem. Phys. 107, 5588 (1997)
39. Olmstead, P.D., Hamley, I.W.: Europhys. Lett. 45, 83 (1999)
40. Pipich, V., Schwahn, D., Willner, L.: J. Chem. Phys. 123, 124904 (2005)
41. The case $m = 1$ was computed in Leite, M.M.: Phys. Rev. B 61, 14691 (2000).
42. Denesyuk, N.A., Gompper, G.: Macromolecules 39, 5497 (2006)
43. Leite, M.M.: Phys. Rev. B 68, 052408 (2003)
44. Bervillier, C.: Phys. Rev. B 14, 4964 (1976)
45. Becerra, C.C., Zieba, A., Oliveira, N.F. Jr., Jellvag, H.F.: J. Appl. Phys. 67, 5442 (1990)
46. Plackowski, T., Matusiak, M., Sznajd, J.: Phys. Status Solidi (b) (2011), doi:10.1002/pssb.201147287. arXiv:1105.2701 [cond-mat]
47. Becerra, C.C.: J. Phys. Condens. Matter 12, 5889 (2000)
48. Yamazaki, T., Tabata, Y., Waki, T., Nakamura, H., Matsuura, M., Aso, N.: J. Phys. Conf. Ser. 200, 32079 (2010)