Magnetic Field Limits on SGRs

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Abstract.
We measure the period and spin-down rate for SGR 1900+14 during the quiescent
period two years before the recent interval of renewed burst activity. We find that
the spin-down rate doubled during the burst activity which is inconsistent with both
magnetic dipole driven spin down and a magnetic field energy source for the bursts. We
also show that SGRs 1900+14 and 1806-20 have braking indices of ∼1 which indicate
that the spin-down is due to wind torques and not magnetic dipole radiation. We
further show that a combination of dipole radiation, and wind luminosity, coupled
with estimated ages and present spin parameters, imply that the magnetic fields of
SGRs 1900+14 and 1806-20 are less than the critical field of 4×10^13 G and that the
efficiency for conversion of wind luminosity to x-ray luminosity is <2%.

I SPIN-DOWN HISTORY OF SGR 1900+14

The spin-down of SGR 1900+14 from 1966 September to 1999 April (Figure 1) is
characterized by three intervals of time for which the spin-down rate was essentially
constant within the interval. This characterization of the SGR 1900+14 spin-down
is based upon both direct measurements of ˙P as part of the period determination,
and upon differences in measured spin periods between two different observations.
The first interval begins with the RXTE observation in September of 1996 and ends
with the ASCA observation at the beginning of May, 1998. The mean spin-down
˙P ∼ 6×10^{-11} s/s [1–3]. The second interval begins with the onset of bursting on
May 26, 1998 and continues until mid-September 1998. The mean spin-down during
this time ˙P ∼13×10^{-11} s/s [4,1,5]. The third interval begins in mid-September 1998
and continues at least until March 30, 1999. The mean spin-down rate at that time
is again ∼ 6×10^{-11} s/s [8].

Woods et al. [8] (1999b) have suggested that the data may be consistent with a
discontinuous spin-down event during the second interval as a result of the giant
burst of August 27, 1998, as opposed to a doubling of ˙P during the entire second
interval [1]. This suggestion, however, appears to be at odds with the measurement
of ˙P=(11.0±1.7)×10^{-11} s/s in early June, 1998 [4], approximately 3 months before
FIGURE 1. The pulse period history for SGR 1900+14 versus time. All of the published values of the pulse period are given along with the three measurements of $\dot{P}$ made as part of the period determination analysis. The mean spin-down in the three time intervals are also given.

the Superburst and with the RXTE/ASCA determination of $\dot{P} \sim 10 \times 10^{-11} \text{ s/s}$ just after the event [5]. A $\dot{P}$ variation of more than a factor of 3 was also indicated in the timing measurements of SGR 1806-20 [9].

Very similar variations in the spin down rate $\dot{P}$ have been observed in AXPs 1E1048.1-5937 and 1E2259+58 (see review by Stella, Israel, and Mereghetti [6]). Such behavior argues strongly against SGR spin-down by magnetic dipole radiation, since that would require large increases (e.g. $\sim 100\%$) in the total magnetic field energy of the star. The SGR 1900+14 observations also strongly argue against magnetic fields as the source of energy for SGR bursts, since that should lead to a decrease in the field energy, and hence a decrease in the spin-down rate during bursting periods. The observations show the spin-down rate increases.

II SGR 1900+14 & 1806-20 SPIN-DOWN DUE TO RELATIVISTIC WINDS

The association of SGRs 1900+14 and 1806-20 with radio supernova remnants provides additional evidence as to the origin of their spin-down torques. Assuming that the spin-down torque is given by $\dot{\Omega} \propto \Omega^n$, the age of a pulsar with period $P$ and spin-down torque $\dot{P}$ is given by

$$t_{age} = P/[(n - 1)\dot{P}]$$
where the spin-down braking index, $n = 3$ for pure magnetic dipole radiation and $n \sim 1$ for wind torques. Taking a nominal age of 10 kyr for a detectable [7] supernova remnant, inverting the age equation yields

$$n = 1 + \left(\frac{P}{\dot{P}}\right)(t_{\text{age}})^{-1}.$$  

Using $P = 5.16$ s and the long-term $\dot{P} = 6\times10^{-11}$ s/s measured for SGR 1900+14, we find that

$$n = 1 + 0.27/(t_{\text{age}}/10^4 \text{yr}),$$

and $P = 7.47$ s and the long-term $\dot{P} = 8.3\times10^{-11}$ s/s measured [9] for SGR 1806-20

$$n = 1 + 0.29/(t_{\text{age}}/10^4 \text{yr}).$$

This indicates that the braking index for SGRs 1900+14 and 1806-20 must be $\sim 1$, and that the spin-down of SGR 1900+14 is dominated by torques due to the relativistic wind and not magnetic dipole radiation.

### III SPIN-DOWN TORQUES OF SGRS

Addressing these problems, Thompson et al [10] considered a magnetar driven Alven wind. The torque provided by the emission of such a relativistic wind is

$$I_s \dot{\Omega}_w = -\Lambda L_w/c^2 R_A^2 \Omega$$

where $I_s$ is the neutron star moment of inertia, $L_w$ is the mechanical luminosity of the wind, $\Omega \equiv 2\pi/P$ is the spin frequency, $\dot{\Omega}_w$ is the spin-down rate due to the wind, and $R_A$ is the Alven radius. $\Lambda$ is a constant equal to 2/3 for a magnetic dipole field aligned with the rotation axis. The Alven radius is given by:

$$\frac{L_w}{4\pi R_A^2 c} = \frac{B^2(R_A)}{8\pi}$$

where $B_0$ is the magnetic field of the neutron star. When the Alven radius is inside the light cylinder radius ($R_A < R_{lc}$, where $R_{lc} = c/\Omega$),

$$I_s \dot{\Omega}_w = -\Lambda B_0 R_A^3 \left(\frac{L_w}{2\pi c}\right)^{1/2} \Omega$$

where $R_0$ is the radius of the neutron star and dipole geometry is assumed. When the Alven is outside the light cylinder radius, the torque is limited to

$$I_s \dot{\Omega}_w = -\Lambda L_w \Omega^{-1}$$

The transition frequency between these two wind spin-down regimes is

$$\Omega_{tr} = 8.572 \left(\frac{L_w}{10^{36} \text{ergs/s}}\right)^{1/4} \left(\frac{B_0}{10^{14} \text{G}}\right)^{-1/2} \text{ radians/s}.$$  

The torque due to a rotating magnetic dipole is:

$$I_s \dot{\Omega}_{mdr} = -k \frac{B^2 R^6}{6c^3} \Omega^3$$

where $k = 1$ [11].

Once the total spin-down torque is specified as a function of $\Omega$, the age of the SGR can found by the integral of $d\Omega$ over the total torque divided by $I_s$, where the integration is performed from an initial frequency to the present-day angular frequency.
FIGURE 2. Age contours for SGRs 1900+14 and 1806-20 for constant magnetic fields and Alfvén wind luminosities. The cross-hatched areas denote the allowed regions of parameter space given the constraints provided by the age of the associated supernova remnant (solid lines) and long term present-day spin-down rate (dotted lines). The vertical dashed lines denote the 10\% and 1\% efficiencies of the wind in producing the observed x-ray flux of $\sim10^{35}$ ergs/s for SGR 1900+14 and $\sim2\times10^{35}$ ergs/s for SGR 1806-20.

IV MAGNETIC FIELD AND WIND LUMINOSITY LIMITS

Using the above model we explore a wide range of magnetic fields $B_*$ and wind luminosity $L_w$, shown in Fig. 2. We see in the upper panel that the presently observed period of $P = 5.157$ s, the spindown rate of $\dot{P} = 6 \pm 1 \times 10^{-11}$ s/s (dotted lines) of SGR 1900+14, and the 10 to 20 Kyr range of ages (solid lines) of its associated supernova remnant G42.8+0.6 [12], tightly constrain the allowable magnetic field to $B_* < 6 \times 10^{13}$ G and wind luminosities $L_w > 5 \times 10^{36}$ erg/s.
Compared to the quiescent 2-10 keV x-ray luminosity of $\sim 10^{35} \text{ erg/s}$ [5], this wind luminosity implies a < 2 % conversion efficiency of wind energy to x-rays in that band which is quite consistent with theoretical calculations [11,13,14]. We also see in the lower panel of Fig. 2, that a similar set of constraints can be obtained for SGR 1806-20 and its supernova remnant G10.0-0.3, using the present spin period [9] $P=7.47 \text{s}$ and $\dot{P} = 8.3 \times 10^{-11} \text{s/s}$. Again, a wind luminosity with <2% conversion efficiency to x-rays yields a sub-critical ($4 \times 10^{13} \text{ G}$) magnetic field. The limit for SGR 1806-20 is very similar that found from a comparable analyses by Harding et al. [11]. Thus, we see that with such winds the magnetic field limits are quite consistent with the limiting values inferred for normal radio pulsars, but not with those expected for magnetars.

V SUMMARY

We show that the large variations in the spin down rate $\dot{P}$ measured in SGRs and AXPs argue strongly against spin-down by magnetic dipole radiation, since that would require large increases (e.g. $\sim 100\%$) in the total magnetic field energy of the star. The SGR 1900+14 observations also strongly argue against magnetic fields as the source of energy for SGR bursts, since that should lead to a decrease in the field energy, and hence a decrease in the spin-down rate during bursting periods rather than the increase observed. We further show from the braking indices that the spin-down torque of the two SGRs is in fact due to winds, not magnetic dipole radiation, and that with such the magnetic fields are $< 6 \times 10^{13} \text{ G}$, which is quite consistent with normal pulsars, but not with magnetars.

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