FORMATION SCENARIO FOR WIDE AND CLOSE BINARY SYSTEMS

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ABSTRACT

Fragmentation and binary formation processes are studied using three-dimensional resistive MHD nested grid simulations. Starting with a Bonnor-Ebert isothermal cloud rotating in a uniform magnetic field, we calculate the cloud evolution from the molecular cloud core ($n = 10^4$ cm$^{-3}$) to the stellar core ($n \sim 10^{22}$ cm$^{-3}$), where $n$ denotes the central density. We calculated 147 models with different initial magnetic, rotational, and thermal energies and the amplitudes of the nonaxisymmetric perturbation. In a collapsing cloud, fragmentation is mainly controlled by the initial ratio of the rotational to the magnetic energy, regardless of the initial thermal energy and amplitude of the nonaxisymmetric perturbation. The cloud rotation promotes fragmentation, while the magnetic field delays or in some cases suppresses fragmentation through all phases of cloud evolution. The results are categorized into three types. When the clouds have larger rotational energies in relation to magnetic energies, fragmentation occurs in the low-density phase ($10^{12}$ cm$^{-3} \leq n \leq 10^{15}$ cm$^{-3}$) with separations of 3–300 AU. Fragments that appeared in this phase are expected to evolve into wide binary systems. On the other hand, when initial clouds have larger magnetic energies in relation to the rotational energies, fragmentation occurs only in the high-density phase ($n \geq 10^{17}$ cm$^{-3}$) after the clouds experience a significant reduction of the magnetic field owing to the ohmic dissipation. Fragments appearing in this phase have mutual separations of $\leq 0.3$ AU and are expected to evolve into close binary systems. No fragmentation occurs in the case of sufficiently strong magnetic field, in which single stars are expected to be born. Two types of fragmentation epoch reflect wide and close separations. We might be able to observe a bimodal distribution for the radial separation of the protostar in extremely young stellar groups.

Subject headings: binaries: general — ISM: clouds — ISM: magnetic fields — MHD — stars: formation — stars: rotation

1. INTRODUCTION

Stars are born as binary or multiple systems. Observations have shown that, although the multiplicity depends on the stellar mass and surrounding environment, about 60%–80% of field stars are members of binary or multiple systems (Heintz 1969; Ait & Levy 1976; Ait 1983; Duquennoy & Mayor 1991; Fischer & Marcy 1992). Thus, it has been established that a majority of main-sequence stars consist of binary or multiple systems. Between multiples, the fraction of binary ($b$), triple ($t$), and higher order multiple ($h$) systems is $b : t : h = 0.75 : 0.175 : 0.04$ (Duquennoy & Mayor 1991; Tokovinin & Smekhov 2002), which indicates that binary system account for a large percentage of these systems (Goodwin et al. 2007). Observations of star-forming regions (e.g., Mathieu 1994) have shown that the multiplicity of pre-main-sequence stars is larger than that of main-sequence stars. For example, in the Taurus star-forming region, it is expected that almost 100% of pre-main-sequence stars are members of binary or multiple systems (Leinert et al. 1993; Köhler & Leinert 1998). Recently, extremely young protostars (i.e., Class 0 protostars) have been observable with radio interferometer (Looney et al. 2000; Reipurth 2000) and wide-field near-infrared cameras (Haisch et al. 2002, 2004; Duchêne et al. 2004). These observations have shown that stars already have a high multiplicity at the moment of their birth. In contrast, Lada (2006) pointed out that the multiplicity of low-mass main-sequence stars ($\leq 0.5 M_\odot$) is less than $\sim 50\%$. The multiplicity of the field stars tends to decrease with decreasing stellar mass. Duchêne et al. (2007) denoted that the multiple system decays by dynamical disruptive interaction in a short timescale ($\leq 1$ Myr), and these systems evolve into an ejected single star or stable binary system. This mechanism is effective in multiple systems composed of lower mass stars because the binding energy of these systems is small. Thus, it is considered that stars are born as binary or multiple systems, and some systems disrupt into single stars as time goes on. For this reason, binary and multiple frequencies decrease with time, especially in low-mass systems.

Distribution of binary separation is known to be very wide ($0.01 \, \text{AU} \leq r \leq 10^4 \, \text{AU}$) and flat, usually modeled as a lognormal with a mean $\sim 30$ AU (Goodwin et al. 2007). For field main-sequence stars, roughly one-third of all companions have close separation ($r \leq 1$ AU). In addition, some binary systems have rotation periods of $P < 10$ days, which corresponds to $r \lesssim 0.1 \, \text{AU}$ of the separation when the total mass of $1 \, M_\odot$ is assumed (Mathieu 1994), and they are classified as close binary. These are the same for stars in the star-forming region. Mathieu (1994) showed that the binary separation extends from 0.01 up to 1000 AU also in pre-main-sequence stars. This indicates that both wide and close binary systems are formed in the star formation process. However, the mechanism that determines the binary frequency and separation is still unknown. The frequency and separation distributions of binaries are important clues to understanding the formation process of such stars. It is, however, difficult to observe

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4 In this paper, for convenience, we define the binary system with separation of $r < 0.1 \, \text{AU}$ as close binary and the binary system with separation of $r \geq 0.1 \, \text{AU}$ as wide binary.
the formation process of binary systems directly because these systems are embedded in dense gas clouds, and star (or binary) formation timescales are expected to be extremely short. Thus, numerical calculations are necessary to understand the binary formation process including binary frequency and separation distributions.

Under the spherical symmetry, Larson (1969), Tohline (1982), and Masunaga & Inutsuka (2000) numerically investigated the evolution of a cloud from the molecular cloud core stage to that of protostar formation including the detailed radiative transfer. They found that the gas cloud collapses self-similarly to reach stellar density. According to the thermal evolution derived in their studies, cloud evolution is divided into four phases: the isothermal \((n_c \lesssim 10^{11} \text{ cm}^{-3})\), adiabatic \((10^{11} \text{ cm}^{-3} \lesssim n_e \lesssim 10^{16} \text{ cm}^{-3})\), second collapse \((10^{16} \text{ cm}^{-3} \lesssim n_e \lesssim 10^{21} \text{ cm}^{-3})\), and protostellar phases \((n_c \gtrsim 10^{22} \text{ cm}^{-3})\), where \(n_e\) means the number density at the center of the cloud. The cloud collapses isothermally for \(n_e \lesssim 10^{11} \text{ cm}^{-3}\) (the isothermal phase or first collapse phase), then the gas around the center of the cloud becomes optically thick and collapses adiabatically after the central density exceeds \(n_e \gtrsim 10^{11} \text{ cm}^{-3}\) (the adiabatic phase). In the adiabatic phase, the adiabatic core (or the first core) surrounded by a shock is formed. When the central density reaches \(n_e \approx 10^{16} \text{ cm}^{-3}\), the molecular hydrogen begins to dissociate and the cloud collapses rapidly (the second collapse phase). After the molecular hydrogen is completely dissociated, the gas collapses adiabatically again and the protostar (or second core) is formed at \(n_e \approx 10^{21} \text{ cm}^{-3}\). In these spherically symmetric calculations, however, cloud fragmentation or binary formation cannot be investigated because these calculations only include the isotropic forces of the gravity and thermal pressure. Cloud fragmentation is controlled by the anisotropic forces caused by the cloud rotation, magnetic field, and turbulence. Thus, to investigate the fragmentation of the cloud, three-dimensional calculations are needed.

The evolution of rotating clouds in three dimensions has been investigated by many authors (e.g., Bodenheimer et al. 2000; Goodwin et al. 2007). Miyama et al. (1984) and Tsuribe & Inutsuka (1999) calculated the evolution of spherical clouds with initially uniform density and rigid-body rotation law in the isothermal regime. They have shown that such clouds fragment if \(\alpha_0 / \beta_0 < 0.12 - 0.15\), where \(\alpha_0 \equiv E_T/E_g\) is the initial ratio of thermal to gravitational energy and \(\beta_0 \equiv E_R/E_g\) is the initial ratio of rotational to gravitational energy. When fragmentation occurs in the isothermal phase, the separation of the fragments has a scale of \(100 - 10^4 \text{ AU}\), which corresponds to the Jeans length in this phase. However, observations have shown that molecular clouds are centrally condensed and often exhibit density profiles resembling those of the Bonnor-Ebert sphere (e.g., Alves et al. 2001; Kandori et al. 2005). Boss (1993) calculated the evolution of such clouds with exponential density profiles in the isothermal regime, and he found that fragmentation occurs only in highly unstable clouds of \(\alpha_0 \approx 0.3\). Kandori et al. (2005) found, from their near-infrared imaging observations, that many starless cores are near the stable state. Tachihara et al. (2002) observed 179 cores in the star-forming regions and showed that many of the cores are nearly in virial equilibrium. Namely, their observations indicate that \(\alpha_0\) is not so small but is close to \(\alpha_0 \approx 1\). For this reason, it is expected that fragmentation rarely occurs in the isothermal phase. The evolution of rotating clouds with centrally condensed density profiles from the isothermal to the adiabatic phase is investigated by Matsumoto & Hanawa (2003b) and Cha & Whitworth (2003). They found that cloud rotation promotes fragmentation in the adiabatic phase. Matsumoto & Hanawa (2003b) showed that fragmentation occurs in the adiabatic phase when the cloud core has a rotation energy of \(\beta_0 \gtrsim 3 \times 10^{-3}\), which is sufficiently smaller than the observations (Goodman et al. 1993; Caselli et al. 2002). Caselli et al. (2002) showed that the molecular clouds have \(10^{-4} \lesssim \beta_0 \lesssim 0.02\) with the mean value of \(\beta_0 \approx 0.02\). Thus, their results indicate that the major fraction of molecular clouds might fragment in the adiabatic phase.

Many observations have shown that cloud cores have a significant nonthermal motion, which can be attributed to turbulence. The turbulence is thought to dissipate as a cloud collapses (e.g., Larson 1981). In the very early phase of cloud collapse, however, the cloud acquires the angular momentum from turbulence, in addition to the large-scale ordered rotation motion of the host cloud. Thus, turbulence promotes fragmentation, as for the cloud rotation. Numerical studies have shown that more fragments appear in a cloud with larger turbulent energy in the initial state (Goodwin et al. 2007). The cloud evolution with larger turbulent energy, in which an initial cloud has turbulent energy comparable to the gravitational energy, is investigated by Bate et al. (2002, 2003), Bate & Bonnell (2005), and Delgado-Donate et al. (2004a, 2004b). In their calculations, fragmentation frequently occurs in an isolated cloud core, and many binary or multiple systems appear. Goodwin et al. (2004a, 2004b) calculated the evolution of clouds with smaller turbulent energy. They found that fragmentation occurs and a binary system is born when the initial cloud has \(~5\%\) turbulence energy exceeding gravitational energy. They have suggested that almost all stars are born as binary or multiple systems, even if a cloud core has a low level of turbulence. However, binary frequencies found in these turbulence calculations are higher than in observations. Since the magnetic fields were ignored in these calculations, these high binary frequencies may be caused by neglect of the magnetic effect.

While cloud rotation and turbulence promote fragmentation, the magnetic field suppresses fragmentation. The evolution of magnetized clouds from the isothermal to the adiabatic phase is investigated by Hosking & Whitworth (2004), Machida et al. (2004, 2005a), Ziegler (2005), Fromang et al. (2006), Price & Bate (2007), Hennebelle & Fromang (2008), and Hennebelle & Teyssier (2008). In their studies, fragmentation rarely occurs in strongly magnetized clouds. This is because the angular momentum, which promotes fragmentation, is removed by the magnetic effect (e.g., magnetic braking, outflow, and jets). Machida et al. (2005a) showed that fragmentation depends only on the ratio of angular rotation speed to the magnetic field strength of the initial clouds, and hence it is necessary for strongly magnetized clouds to rotate rapidly for fragmentation.

Although fragmentation processes in the isothermal and adiabatic phases \((n \lesssim 10^{16} \text{ cm}^{-3})\) are investigated by many authors, there are few studies of fragmentation in the second collapse and protostellar phases \((n \gtrsim 10^{16} \text{ cm}^{-3})\). When fragmentation occurs in the isothermal or adiabatic phases \((n \lesssim 10^{16} \text{ cm}^{-3})\), fragments have separations of \(10 - 10^4 \text{ AU}\), which correspond to the Jeans length in these phases. Fragments formed in these phases are expected to evolve into wide binary systems, when the binary systems maintain separation at their formation epoch. However, about 20\%–30\% of observed binaries have separations of \(< 10 \text{ AU}\) (Mathieu 1994). In the collapsing cloud, the Jeans length becomes \(< 10 \text{ AU}\) after the central density exceeds \(n \gtrsim 10^{13} \text{ cm}^{-3}\). Thus, fragments with the separation of \(< 10 \text{ AU}\) and thus close binary systems are expected when fragmentation occurs for \(n > 10^{13} \text{ cm}^{-3}\). Bate (1998) and Whitehouse & Bate (2006) investigated the evolution of unmagnetized clouds from \(10^6\) to \(10^{21} \text{ cm}^{-3}\).
According to their calculations, fragmentation does not occur in the high-density region because the nonaxisymmetric structure that appears in the adiabatic stage effectively removes angular momentum from the center of the cloud. Thus, they concluded that fragmentation does not occur for $n \geq 10^{16}$ cm$^{-3}$. However, they investigated the cloud evolution in a few parameters. Goodwin et al. (2007) pointed out that cloud evolution in high-density gas is complicated, hence a large number of calculations and a statistical approach are needed to understand the fragmentation process.

Banerjee & Pudritz (2006) investigated the cloud evolution of magnetized clouds for $10^{6}$ cm$^{-3} \leq n \leq 10^{20}$ cm$^{-3}$ and found that fragmentation occurs in the high-density gas via a ring structure. They expected that this ring structure evolves into a close binary with the separation $3 \times 10^{12}$ cm. However, they adopted the ideal MHD approximation, which is not valid in high-density gas as $n \geq 10^{12}$ cm$^{-3}$. A significant magnetic flux is lost by ohmic dissipation in the density range of $10^{12}$ cm$^{-3} \leq n \leq 10^{15}$ cm$^{-3}$ (Nakano et al. 2002), and hence they overestimated the magnetic flux in a collapsing cloud, especially in the high-density gas regions. In addition, they investigated the cloud evolution with only one parameter.

In this study we investigate the evolutions of magnetized cloud cores ($n_c \approx 10^{4}$ cm$^{-3}$, $r \approx 4.6 \times 10^{4}$ AU) until a protostar is formed ($n_c \approx 10^{22}$ cm$^{-3}$, $r \approx 1 R_\odot$) using three-dimensional resistive MHD nested grid simulations. We calculated 147 models with different magnetic field strengths, rotation speeds, initial amplitudes of nonaxisymmetric perturbation, and thermal energies. We found that the formation conditions for close and wide binary systems are related to the ratio of the rotational to the magnetic energy of the cloud core.

The structure of the paper is as follows: The framework of our models is given in § 2, and in § 3 the numerical method of our computations is shown. The evolution of typical models is presented in § 4. We show the results of the parameter survey in § 5. The fragmentation conditions are discussed in § 6.

2. MODEL
2.1. Basic Equations

Our initial settings are almost the same as those of Machida et al. (2006a, 2006b, 2007a, 2007b). To study the cloud evolution, we use a three-dimensional resistive MHD nested grid code. We solve the resistive MHD equations including the self-gravity:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$

$$\rho \frac{\partial \mathbf{v}}{\partial t} + \rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla P - \frac{1}{4 \pi} \mathbf{B} \times (\nabla \times \mathbf{B}) - \rho \nabla \phi,$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},$$

$$\nabla^2 \phi = 4 \pi G \rho,$$

where $\rho$, $\mathbf{v}$, $P$, $\mathbf{B}$, $\eta$, and $\phi$ denote the density, velocity, pressure, magnetic flux density, resistivity, and gravitational potential, respectively. The resistivity $\eta$ in equation (3) is a function of the density and the temperature (Machida et al. 2007a). We quantitatively estimate the resistivity $\eta$ according to Nakano et al. (2002) and assume that $\eta$ is a function of density and temperature as

$$\eta = \frac{740}{X_e} \sqrt{\frac{T}{10^4 K}} \left[1 - \tanh \left( \frac{n}{10^{15} \text{ cm}^{-3}} \right) \right] \text{cm}^2 \text{s}^{-1},$$

where $T$ and $n$ are the gas temperature and number density and $X_e$ is the ionization degree of the gas, described as

$$X_e = 5.7 \times 10^{-4} \left( \frac{n}{\text{cm}^{-3}} \right)^{-1}.$$

We added the second term on the right-hand side of equation (5) to smoothly decline the diffusivity at $n \approx 10^{15}$ cm$^{-3}$ which means that ohmic dissipation becomes ineffective for $n > 10^{15}$ cm$^{-3}$ (for details see Machida et al. 2006b).

To mimic the temperature evolution calculated by Masunaga & Inutsuka (2000), we adopt a piecewise polytropic equation of state as

$$P = \begin{cases} c_s^2 \rho, & \rho < \rho_d, \\ c_s^2 \rho_d \left( \frac{\rho}{\rho_d} \right)^{7/5}, & \rho_d < \rho < \rho_e, \\ c_s^2 \rho_e \left( \frac{\rho}{\rho_e} \right)^{1.1}, & \rho_e < \rho < \rho_f, \\ c_s^2 \rho_f \left( \frac{\rho}{\rho_f} \right)^{5/3}, & \rho > \rho_f, \end{cases}$$

where $c_s = 190$ m s$^{-1}$, $\rho_d = 3.84 \times 10^{-13}$ g cm$^{-3}$ ($n_d = 10^{11}$ cm$^{-3}$), $\rho_e = 3.84 \times 10^{-8}$ g cm$^{-3}$ ($n_e = 10^{16}$ cm$^{-3}$), and $\rho_f = 3.84 \times 10^{-6}$ g cm$^{-3}$ ($n_f = 10^{21}$ cm$^{-3}$). For convenience, we define “the protostar formation epoch” as that at which the central density ($n_c$) reaches $n_c = 10^{21}$ cm$^{-3}$. We also call the period of $n_c < 10^{11}$ cm$^{-3}$ “the isothermal phase,” the period of $10^{11}$ cm$^{-3} < n_c < 10^{16}$ cm$^{-3}”$ “the adiabatic phase,” the period of $10^{16}$ cm$^{-3} < n_c < 10^{21}$ cm$^{-3}”$ “the second collapse phase,” and the period of $n_c > 10^{21}$ cm$^{-3}”$ “the protostellar phase.”

2.2. Initial Condition

As the initial condition of the cores, we use the density profile increased by a factor $f$ (density enhancement factor) from the critical Bonnor-Ebert sphere (Ebert 1955; Bonnor 1956). Note that the critical Bonnor-Ebert sphere has a density contrast of 14 between the center and the surface of the cloud. The density enhancement factors are fixed as $f = 1.08$ in all models except for five models shown in § 5.1. In § 5.1 we adopt the density enhancement factors of $f = 1.4, 1.8, 2.8, 8.41, \text{and 84.1}$. The density enhancement factor is related to the stability of the initial cloud. We discuss the relation of the density enhancement factor and the stability of the cloud in § 5.1. For the critical Bonnor-Ebert sphere, the central density is equal to $n_{c,0} = 10^{4}$ cm$^{-3}$ ($\rho_0 = 3.82 \times 10^{-20}$ g cm$^{-3}$), and the initial temperature is 10 K. The radius of the critical Bonnor-Ebert sphere $R_c = 6.45 c_s^2 f^{1/2} (\rho_0 m_p)^{-1/2}$ corresponds to $R_c = 4.58 \times 10^{4}$ AU. Outside this radius, we assume a uniform gas with a density of $n_{hi}(R_c) = 711$ cm$^{-3}$. The total mass contained in the critical Bonnor-Ebert sphere is $M_c = 4.5 M_\odot$, and our initial core is $f$ times as massive as this value.

Initially the cloud rotates rigidly $\Omega_0$ around the $z$-axis and has a uniform magnetic field $B_0$ parallel to the $z$-axis (rotation axis). The initial model is characterized by three nondimensional parameters: the magnetic-to-thermal pressure ratio,

$$b = \frac{B_{z,0}^2}{4 \pi \rho_0 c_s^2 f_{z,0}}.$$

the angular rotation velocity normalized by the free-fall timescale,

$$
\omega = \frac{\Omega_{c,0}}{\sqrt{4\pi G\rho_{c,0}}},
$$

and the initial amplitude of the nonaxisymmetric perturbation $A_{\phi,0}$. We add $m=2$ mode nonaxisymmetric density perturbation to the spherical initial cores. Then, the density profile of the core is denoted as

$$
\rho(r) = \begin{cases} 
\rho_{BE}(r)(1 + \delta_r) f_r & , r < R_c, \\
\rho_{BE}(R_c)(1 + \delta_r) f_r & , r \geq R_c,
\end{cases}
$$

where $\rho_{BE}(r)$ is the density distribution of the critical Bonnor-Ebert sphere and $\delta_r$ is the axisymmetric density perturbation. For the $m=2$ mode, we chose

$$
\delta_r = A_{\phi}(r/R_c)^2 \cos 2\phi,
$$

where $A_{\phi}$ represents the amplitude of the perturbation. The radial dependence is chosen so that the density perturbation remains regular at the origin ($r=0$) at one time step after the initial stage.

We adopt $b$ as $b=0-4$ as summarized in Table 1, $\omega$ as $\omega = 0.003-0.2$ as summarized in Table 2, and $A_{\phi}$ as $A_{\phi} = 0.01, 0.2,$ and $0.4$. The ratios of rotational ($\gamma_0$) and magnetic ($\gamma_0$) energies to the gravitational energy are summarized in each table. In addition, we estimate the mass-to-flux ratio

$$
\frac{M}{\Phi} = \frac{M_{\text{crit}}}{\pi R_c^2 B_0},
$$

where $M$ is the mass contained within the critical radius $R_c$ and $\Phi$ is the magnetic flux threading the cloud. There exists a critical value of $M/\Phi$ below which a cloud is supported against the gravity by the magnetic field. For a cloud with uniform density, Mouschovias & Spitzer (1976) derived a critical mass-to-flux ratio

$$
\left(\frac{M}{\Phi}\right)_{\text{crit}} = \frac{5}{3\pi} \left(\frac{G}{\rho}\right)^{1/2},
$$

where the constant $\zeta = 0.53$ for uniform spheres ($\zeta = 0.48$ by recent careful calculation of Tomisaka et al. (1988a, 1988b). For convenience, we use the mass-to-flux ratio normalized by the critical value as

$$
\left(\frac{M}{\Phi}\right)_{\text{norm}} = \frac{(M/\Phi)}{(M/\Phi)_{\text{crit}}},
$$

The relation between parameter $b$ and the mass-to-flux ratio is also summarized in Table 1. We made 147 models by combining the above parameters.

### Table 1

| $b$ | $\gamma_0$ | $\beta_0$ | $(M/\Phi)_{\text{crit}}$ |
|-----|-----|-----|----------------|
| 0... | 0... | 0... | $\infty$ |
| 0.001... | 9.59 x 10^{-4} | 0.541 | 39.1 |
| 0.01... | 9.59 x 10^{-3} | 1.71 | 12.4 |
| 0.1... | 9.59 x 10^{-2} | 5.41 | 3.91 |
| 1... | 9.59 x 10^{-1} | 17.1 | 1.24 |
| 4... | 3.83 | 34.1 | 0.62 |

### Table 2

| $\omega$ | $\beta_0$ | $\Omega_0$ (10^{-14} s^{-1}) |
|-----|-----|----------------|
| 0.003... | 2.97 x 10^{-5} | 0.07 |
| 0.005... | 8.24 x 10^{-5} | 0.116 |
| 0.007... | 1.62 x 10^{-4} | 0.163 |
| 0.01... | 3.30 x 10^{-4} | 0.233 |
| 0.03... | 2.97 x 10^{-3} | 0.698 |
| 0.05... | 8.24 x 10^{-3} | 1.16 |
| 0.07... | 1.62 x 10^{-2} | 1.63 |
| 0.1... | 3.30 x 10^{-2} | 2.33 |
| 0.2... | 0.132 | 4.65 |

### 3. NUMERICAL METHOD

#### 3.1. Nested Grid and Boundary Condition

We adopt the nested grid method (for details see Machida et al. 2005b, 2006b) to obtain high spatial resolution near the center. Each level of a rectangular grid has the same number of cells ($=128 \times 128 \times 8$), although the cell width $h(l)$ depends on the grid level $l$. The cell width is reduced by a factor of $2^2$ as the grid level increases by 1 ($l \rightarrow l+1$).

The highest level of grids changes dynamically. A new finer grid is generated whenever the minimum local Jeans length $\lambda_J$ becomes smaller than $8h(l_{\text{max}})$, where $h$ is the cell width. The maximum level of grids is restricted to $l_{\text{max}} = 30$, in which the maximum density up to $n_c = 5 \times 10^{24} \text{ cm}^{-3}$ can be calculated safely. Note that since the central density stopped increasing at $n_c \leq 10^{22} \text{ cm}^{-3}$ in any model, the grids of $l = 29$ and 30 were not generated in our calculation. Since the density is highest in the finest grid, the generation of a new grid ensures the Jeans condition of Truelove et al. (1997) with a margin of a safety factor of 2.

We begin our calculations with four grid levels ($l = 1, 2, 3,$ and $4$). The box size of the initial finest grid $l = 4$ is chosen to be equal to $2 R_c$, where $R_c$ denotes the radius of the critical Bonnor-Ebert sphere. The coarsest grid ($l = 1$), therefore, has a box size equal to $2^4 R_c$. A boundary condition is imposed at $r = 2 R_c$, where the magnetic field and ambient gas rotate at an angular velocity of $\Omega_0$ (for details see Matsumoto & Tomisaka 2004). The large radius of this boundary condition eliminates the effect of artificial magnetic braking due to the boundary condition. We also assumed mirror symmetry with respect to $z = 0$ in order to reduce the computational cost.

#### 3.2. Numerical Schemes

The basic equations are solved by the finite volume approach on the nested grid described above. The scheme is composed of MHD and self-gravity schemes.

The hyperbolic terms of the MHD equations (1)–(3) are solved by the total variation diminishing (TVD) method. The numerical flux is obtained by the Roe scheme extended for solving the barotropic MHD (Fukuda & Hanawa 1999). A monotone upstream-centered scheme for conservation laws (MUSCL) approach and a predictor-corrector method are adopted here for integration over time in order to achieve second-order accuracy in space and time. The minmod limiter is employed in the MUSCL extrapolation, and the CFL number is set at 0.2. The adaptive time step is adopted in time integration, in which the time step of a coarse grid is larger than that of a fine grid. The hyperbolic cleaning of Dedner et al. (2002) is adopted in order to reduce $\nabla \cdot B$ error. The dissipation term $\eta \nabla^2 B$ in equation (3) is evaluated.
separately by an operator splitting approach; the magnetic field is dissipated in subcycles during a time step of the hyperbolic equations.

Self-gravity is updated in every time step across all grid levels by solving the Poisson equation (4) using the multigrid method for the nested grid developed by Matsumoto & Hanawa (2003a). In this method, a full multigrid scheme is employed to accelerate the convergence of the red-black Gauss-Seidel iteration. The method exhibits a spatial second-order accuracy in almost the entire computational domain.

4. EVOLUTIONS OF TYPICAL MODELS

In this section we show the cloud evolution until the protostar is formed. In our calculations, we observed cloud fragmentation only after the adiabatic phase \( (n > 10^{11} \text{ cm}^{-3}) \). For convenience, we call the models in which fragmentation occurs in the adiabatic phase \( (10^{14} \text{ cm}^{-3} < n_c < 10^{16} \text{ cm}^{-3}) \) “first fragmentation models,” the models in which fragmentation occurs after the second collapse \( (n_c > 10^{11} \text{ cm}^{-3}) \) “second fragmentation models,” and the models that never experience fragmentation “nonfragmentation models.” When fragmentation occurs in both the adiabatic \( (10^{11} \text{ cm}^{-3} < n_c < 10^{16} \text{ cm}^{-3}) \) and protostellar \( (n_c > 10^{16} \text{ cm}^{-3}) \) phases without merger, we classified this as “the first fragmentation model.”

Since we already reported the formation process for wide binary systems (i.e., first fragmentation models) in detail (Machida et al. 2004, 2005a), we focus on the formation process for close binary (i.e., second fragmentation models) and hierarchical stellar systems in this section. As typical models, we chose three models (models A, B, and C) that are summarized in Table 3.

### Table 3

| Model | \( b \) | \( \omega \) | \( A_c \) | \( \alpha_0 \) | \( \beta_0 \) | \( \gamma_0 \) | \( B \) (\( \mu \text{G} \)) | \( \Omega_0 \) (\( 10^{-14} \text{ s}^{-1} \)) |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|
| A     | 0.1    | 0.05   | 0.01   | 0.5    | 8.24 \( \times 10^{-3} \) | 9.59 \( \times 10^{-2} \) | 5.41   | 1.16   |
| B     | 0.01   | 0.05   | 0.2    | 0.5    | 8.24 \( \times 10^{-3} \) | 9.59 \( \times 10^{-3} \) | 0.541  | 1.16   |
| C     | 0.01   | 0.07   | 0.4    | 0.5    | 1.62 \( \times 10^{-2} \) | 9.59 \( \times 10^{-2} \) | 0.541  | 1.63   |

4.1. First Core Formation

Figure 1b shows the cloud structure when the central density reaches \( n_c = 7.3 \times 10^{14} \text{ cm}^{-3} \) (adiabatic stage). In this model, when the central density reaches \( n_c \approx 5.1 \times 10^{12} \text{ cm}^{-3} \), a shock appears \( (\rho \approx 2 \text{ AU}) \) and a first core is formed at the center of the cloud. The first core is represented by a thick red line in Figure 1b. The first core has an oblate shape as shown in the middle and bottom panels of Figure 1b. Since the cloud collapses along the \( z \)-axis, the central region becomes oblate before the first core formation. After first core formation, outflow appears near the first core. In the middle and bottom panels of Figure 1b, it is shown that the gas is outflowing from the central region inside the thick orange lines. After the gas is supported by the thermal pressure, the magnetic field lines are strongly twisted because the rotational timescale becomes shorter than the collapse timescale. The outflow is driven due to the twisted magnetic field lines and rotation of the first core. Outflows from the first cores are shown also in Tomisaka (1998, 2000, 2002), Banerjee & Pudritz (2006), Matsumoto & Tomisaka (2004), Fromang et al. (2006), Machida et al. (2004, 2005a, 2007a, 2007b), and Hennebelle & Fromang (2008).

Figure 1c shows the snapshot around the first core at the age of 263 yr after the first core formation. Although the first core has an almost axisymmetric structure at its formation epoch (Fig. 1b, top panel), the nonaxisymmetric perturbation grows and a spiral pattern appears inside the first core, as shown in the top panel of Figure 1c. The nonaxisymmetric perturbation can grow sufficiently in the quasi-static disk for the isothermal phase (Nakamura & Hanawa 1997; Matsumoto & Hanawa 1999; see also Machida et al. 2007c). Durisen et al. (1986) show that the bar structure develops with the bar mode instability, when the ratio of the rotational to gravitational energies \( (\beta) \) of the core exceeds \( \beta > 0.274 \). Saigo et al. (2007) found that the first core forms a bar or spiral pattern after exceeding \( \beta \geq 0.27 \). We found that the first core has \( \beta \approx 0.04 - 0.36 \). Thus, the bar mode instability can be induced in the first core by the dynamical instability (Durisen et al. 1986). As a result, the first core shown in the top panel of Figure 1c has a spiral structure, which is thought to be caused by nonmagnetic instability.

The top panel of Figure 1c shows two density peaks near the center of cloud in the 4 and 10 hr directions, which are caused by the fragmentation of the first core. In this model, fragmentation and subsequent merger are repeated twice in the range of \( 10^{15} \text{ cm}^{-3} \leq n_c \leq 10^{18} \text{ cm}^{-3} \). However, no apparent fragments

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**Note:**

6️⃣ Only a few models show fragmentation in both the adiabatic and second collapse or protostellar phases without merger because we stopped the calculation after the first fragmentation occurs for numerical limitation.

7️⃣ In this paper we use “first core formation” when the core surrounded by a clear shock boundary is formed in the adiabatic phase.
or protobinary stars) appear in this phase. The first fragmentation occurs at the density \(n_c \approx 2 \times 10^{15} \text{ cm}^{-3}\), and then the fragments are merged in a short timescale. The second fragmentation occurs at \(n_c \approx 8 \times 10^{16} \text{ cm}^{-3}\), and then the density exceeds the H\(_2\) dissociation density and the second collapse occurs in each fragment. The two fragments are merged to form a nearly spherical core at the density \(n_c \approx 2 \times 10^{18} \text{ cm}^{-3}\).

The two fragments are merged to form a nearly spherical core at the density \(n_c \approx 2 \times 10^{18} \text{ cm}^{-3}\). After merger, the central region changes its shape from spherical to barlike.

### 4.1.2. Protostar Formation

Figures 1d–1f show the cloud evolution after the protostar is formed. The nonaxisymmetric perturbation continues to increase in the protostellar phase, and an elongated bar is formed around the center of the cloud. When the number density reaches \(n_c \approx 1.8 \times 10^{17} \text{ cm}^{-3}\), the bar fragments into three pieces as shown in Figure 1d. When the fragmentation occurs, the separation between fragments is equal to \(R_{\text{sep}} \approx 0.031 \text{ AU}\). After fragmentation, fragments approach each other, and they are merged into a single core as shown in Figure 1e. After the merger, fragmentation occurs again through a ringlike structure as shown in Figure 1f. The final fragments survive separately, until we stop the calculation. At the end of the calculation, the separation between fragments is equal to \(R_{\text{sep}} \approx 0.03 \text{ AU}\), and each fragment has a mass of \(M = 3.1 \times 10^{-3} M_\odot\). As a result, a close binary system is expected to be formed in this model.

### 4.1.3. Magnetic Field

Figure 2 shows the evolution of the magnetic field strength normalized by the square root of the central density \(B_{\text{z}}/(4\pi c_s n_c)^{1/2}\). The normalized magnetic field strength. In the isothermal phase,
the normalized magnetic field strength is saturated at a constant level according to the magnetic flux--spin relation of Machida et al. (2005b, 2006b), and then the magnetic fields are dissipated by ohmic dissipation in the range of $10^{12}$ cm$^{-3} \leq n_c \leq 10^{15}$ cm$^{-3}$. The rapid decrease for $10^{12}$ cm$^{-3} \leq n_c \leq 10^{17}$ cm$^{-3}$ in Figure 2 corresponds to this ohmic dissipation. The first core, which is traced by two spiral arms occupying $r \leq 3$ AU in Figure 1c, has a large plasma beta $\beta \simeq 10^{-5}$. Thus, in the adiabatic phase, the magnetic field barely affects the cloud evolution inside the first core. In the second collapse phase, since the ohmic dissipation becomes ineffective for the dense (or high temperature) gas, the magnetic field begins to amplify again. However, since magnetic field is not too strong to activate the magnetic braking, fragmentation is possible in the second collapse and protostellar phases as shown in Figures 1d–1f.

4.2. Model B: Multiple Stellar System

Figure 3 shows the cloud structure for model B, which has parameters of $(b, \omega, A_{\varphi}) = (0.01, 0.05, 0.02)$. In this model, the initial cloud has a magnetic field strength of $B_0 = 0.541$ $\mu$G and an angular velocity of $\Omega_0 = 1.16 \times 10^{-14}$ s$^{-1}$, which correspond to the ratios of magnetic $\gamma_0 = 9.59 \times 10^{-3}$ and rotational energies $\beta_0 = 8.24 \times 10^{-3}$ to the gravitational energy. In model B, fragmentation occurs in both the second collapse ($10^{16}$ cm$^{-3} < n_c < 10^{21}$ cm$^{-3}$) and protostar formation ($n_c > 10^{21}$ cm$^{-3}$) phases, and four fragments with close separations appear.

In Figure 3, the structures around the center of the cloud with different scales are plotted for the final state of the calculation. The spatial scale of each successive panel differs by a factor of 4; thus, the scale between Figures 3a and 3f differs by a factor of 1024. In this model, fragmentation occurs in the second collapse phase, while fragmentation occurs in the adiabatic phase for the model with the same $b$ and $\omega$ but smaller $A_{\varphi} = 0.01$ (see Fig. 5 below). This is because the angular momentum in the model with $A_{\varphi} = 0.2$ is more effectively transferred from the center of the cloud than in the model with $A_{\varphi} = 0.01$. This stabilizes the first core for fragmentation.

The middle and bottom panels of Figures 3a and 3b show a thin disk, which is formed in the adiabatic phase and survives until the protostar stage. Red dotted lines in Figures 3a and 3b indicate a shocked region that corresponds to the first core. Since model B has a large amount of nonaxisymmetric perturbation $A_{\varphi} = 0.2$ in the initial state, the nonaxisymmetric pattern grows sufficiently and a spiral structure appears just after the first core formation. After that, the central compact core is separated from the ambient ringlike structure. The top panel of Figure 3b shows that the first core is composed of a central compact core and a surrounding distorted disk. There is a clear gap between the central core and ambient medium.

Figures 3c and 3d indicate that the central region has entered the second collapse phase because the density exceeds $n_c > 10^{16}$ cm$^{-3}$, while the surrounding ringlike structure stays in the adiabatic phase. Although we cannot see any internal structure in the compact core in Figures 3a and 3b, Figures 3c and 3d show that the central region forms a spiral structure. Similar structure is seen in Bate (1998), who calculated the evolution of an unmagnetized core having the parameter of $\omega = 0.07$ with an initial number density of $n_{c,0} = 3.6 \times 10^{2}$ cm$^{-3}$. He found that the cloud collapses to form a single star without fragmentation after the second collapse phase because the angular momentum is effectively removed from the center of the cloud by the spiral patterns. Although our model contains the magnetic field, the magnetic field becomes extremely weak in the second collapse phase due to the ohmic dissipation, as shown in § 4.1 and Machida et al. (2007a); thus, the magnetic field barely affects the cloud evolution in this phase. For this reason, both the unmagnetized and magnetized clouds are thought to follow a similar evolutional pattern in the second collapse phase.

In the top panels of Figures 3e and 3f we can see four fragments inside the spiral structure. In this model, the first fragmentation occurs when the central density reaches $n_c = 1.1 \times 10^{18}$ cm$^{-3}$, and two clumps appear. They are located at $(x, y) \simeq (\pm 0.01$ AU, $\pm 0.07$ AU) as shown in the top panel of Figure 3f. After that, fragmentation occurs again in each clump at $n_c \simeq 4 \times 10^{21}$ cm$^{-3}$, and as a result, four fragments are formed as shown in Figure 3f. Since the gas density of these four fragments exceeds $n_c \geq 10^{21}$ cm$^{-3}$, these fragments are protostars in our definition. Thus, we found a quadruple protostellar system. At the end of the calculation, the protostars have the furthermost separation of $R_{\text{sep}} \simeq 0.15$ AU.

At last, we comment on the protostellar outflow and jet. Orange lines in the middle and bottom panels of Figures 3a and 3b indicate a large-scale outflow with a radius $\approx 10$–20 AU, while those in the top and middle panels of Figures 3d and 3e indicate a small-scale jet with a radius $\approx 10$ R$_{\odot}$. The outflow appears just after the adiabatic core formation, while the jet appears after the protostar formation. In model B, two kinds of flows (outflow and jet) appear. Since we investigated in detail the evolution and mechanism of the outflow and jet in a separate paper (Machida et al. 2007b), we do not describe these further here.

4.3. Model C: Hierarchical Stellar System

Figure 4 shows the cloud structures for model C, which has parameters of $(b, \omega, A_{\varphi}) = (0.01, 0.07, 0.4)$. In this model, the initial cloud has a magnetic field strength of $B_0 = 0.541$ $\mu$G and an angular velocity of $\Omega_0 = 1.63 \times 10^{-14}$ s$^{-1}$, which correspond to respective ratios of the magnetic $\gamma_0 = 9.59 \times 10^{-3}$ and rotational energies $\beta_0 = 1.62 \times 10^{-2}$ to the gravitational energy. In model C, fragmentation occurs in both the adiabatic ($10^{11}$ cm$^{-3} < n_c < 10^{16}$ cm$^{-3}$) and second collapse ($10^{16}$ cm$^{-3} < n_c < 10^{21}$ cm$^{-3}$) phases, and the fragments appeared hierarchically.

In Figure 4 the structure around the center of the cloud is plotted with different scales at the end of the calculation. The spatial scale of each successive panel differs by a factor of 8, and thus the scale between Figures 4a and 4d differs by a factor of 512. In model C, the first core is formed at $n_c \simeq 3 \times 10^{12}$ cm$^{-3}$. The first core repeats fragmentation and merger three times in the adiabatic phase. As shown in Figure 4a, in a large scale, the high-density region is composed of a central compact core and a
surrounding ringlike structure. At the end of the calculation, the central compact core has a maximum density of \( n_c = 1.1 \times 10^{22} \text{ cm}^{-3} \) and a mass of \( M = 0.065 \, M_\odot \). The second collapse occurs in the central core at 459 yr after the first core formation epoch. Then, the central region fragments again at \( n_c = 8 \times 10^{21} \text{ cm}^{-3} \), and two protostars are formed, as shown in Figures 4c and 4d. Two fragments (or protostars) in Figure 4d have a mass of \( M = 4.4 \times 10^{-7} \, M_\odot \) and a separation of \( R_{\text{sep}} = 0.073 \, \text{AU} \) at the end of the calculation.

On the other hand, in the surrounding ringlike structure, there are two clumps at \((x, y) \approx (\pm 5 \, \text{AU}, \mp 25 \, \text{AU})\) as shown in Figure 4a. Each clump has mass of \( M \approx 0.036 \, M_\odot \). The clumps have a maximum density of \( n_c = 8 \times 10^{12} \text{ cm}^{-3} \), and they continue to collapse until the calculation ends. Thus, these clumps are expected to be more compact as time goes on, and then, a single or several protostars are formed in each clump.

As shown in Figure 4, four clumps are left in model C. Clumps formed in the adiabatic phase have a wide separation of \( R_{\text{sep}} \approx 26 \, \text{AU} \), and they seem to be a triple system in a large scale (Fig. 4a). The central core contains a close binary system; there are two fragments. This system is thought to evolve into a hierarchical stellar system, which is composed of two protostars with a large separation and two binary systems with a narrow separation, if the fragments do not merge with each other and separations remain almost unchanged.

5. PARAMETER SURVEY

To determine the fragmentation condition, we calculated the cloud evolution in a large parameter space. We used three parameters \( b, \omega, \) and \( A_\varphi \). In this section we compare the models with different \( b \) and \( \omega \) but a fixed \( A_\varphi \). In § 5.1 we show the final states obtained from the initially small nonaxisymmetric perturbation \( A_\varphi = 0.01 \). Then, the final states from the initially large nonaxisymmetric perturbations are shown in § 5.2 (\( A_\varphi = 0.2 \)) and § 5.3 (\( A_\varphi = 0.4 \)). In each model, to check fragmentation, we monitored the maximum and central densities at each time step.
Since we added $m = 2$ mode density perturbation to the sphere, the maximum density ($\rho_{\text{max}}$) exceeds the central density ($\rho_c$) when the cloud fragments. In other words, the density peak moves from the center. We defined “fragmentation epoch” at the time when two (or multiple) density peaks appear. We also confirmed fragmentation by eye.

5.1. Models with $A_\varphi = 0.01$

In this subsection we show the evolutions of clouds with a small nonaxisymmetric perturbation $A_\varphi = 0.01$ at the initial state. Figure 5 shows the cloud structures around the center of the cloud for each model at the end of the calculation. In this figure, all the models have the same $A_\varphi = 0.01$, but different $b$ and $\omega$. Each panel is placed according to the parameters $b$ (x-axis) and $\omega$ (y-axis). Models located in the upper right region have strong magnetic fields and rapid rotations, while models located in the lower left region have weak magnetic fields and slow rotations in the initial state. In the blue region, fragmentation occurs in the adiabatic (first core) phase ($10^{11}$ cm$^{-3}$ $\leq n_c \leq 10^{15}$ cm$^{-3}$; the first fragmentation models), while fragmentation occurs after the second collapse ($n_c > 10^{16}$ cm$^{-3}$; the second fragmentation models) in the red region. On the other hand, fragmentation is never seen in the gray region (nonfragmentation models).

In both the first and second fragmentation models, we stopped the calculations when the Jeans condition was violated outside the central deepest level grid. In these models, after the fragments escape from the finest grid, the Jeans condition is violated in the coarser grid. In nonfragmentation models, we stopped the calculations in the protostellar phase ($n_c > 10^{21}$ cm$^{-3}$; the second fragmentation models) in the red region. On the other hand, the cloud indicates oscillation around the initial state without any indication of collapse in model $(b, \omega) = (4, 0.2)$. Since this model has the strongest
magnetic field and most rapid rotation, the cloud does not collapse to form the first core.

In the first fragmentation models (blue region), the cloud fragments into several pieces in the adiabatic phase \(10^{11} \text{ cm}^{-3} \leq n_c \leq 10^{10} \text{ cm}^{-3}\). Typical separations between two fragments are \(R_{\text{sep}} \approx 20-200 \text{ AU}\) in these models. In general, these models have high rotational energies but low magnetic energies at the initial state. In the second fragmentation models, the cloud fragments after the second collapse phase \(n_c \approx 10^{10} \text{ cm}^{-3}\). The separations between the fragments in these models are \(R_{\text{sep}} \approx 0.01-0.5 \text{ AU}\). Compared with the first fragmentation models, the second fragmentation models have lower rotational energies and higher magnetic energies. In nonfragmentation models (gray region), protostars are formed without fragmentation. Compared with the fragmentation models, nonfragmentation models have lower rotational energies and higher magnetic energies in the initial state.

Figure 5 shows that fragmentation occurs in the lower density (or large scale; 20–200 AU) in the clouds having larger rotational energies and smaller magnetic energies, and fragmentation rarely occurs in clouds having smaller rotation energies or larger magnetic energies. In the models shown in Figure 5, the clouds evolve keeping an axisymmetry for a long time because the small nonaxisymmetric perturbations were added to the initial state. Thus, fragmentation is induced via ring structure in many models, and the formed protostars have outer axisymmetric disks.

5.2. Models with \(A_\varphi = 0.2\)

Figure 6 shows cloud structures around the center of the cloud at the end of calculations for models with \(A_\varphi = 0.2\). In the upper left region (weak magnetic field and fast rotation), fragmentation occurs in the adiabatic phase, and two fragments have wide separations \((20 \text{ AU} \leq R_{\text{sep}} \leq 100 \text{ AU})\). On the other hand,
fragmentation occurs in the second collapse or after the protostellar phase with narrow separations ($R_{\text{sep}} \lesssim 0.4$ AU) in the lower region (red region). Clouds having larger magnetic energies and smaller rotational energies at the initial state lead to formation of a protostar without fragmentation (gray region). Thus, similarly to models with small nonaxisymmetric perturbation $A_{\varphi} = 0.01$, the magnetic field suppresses fragmentation, but the rotation promotes it also in the models with large nonaxisymmetric perturbation $A_{\varphi} = 0.2$. However, three regions of first fragmentation (blue region), second fragmentation (red region), and nonfragmentation (gray region) are clearly divided in Figure 5, while the boundary between these three regions is ambiguous in Figure 6. Figures 5 and 6 show that, even though the initial clouds have the same magnetic field strength and rotation rate, fragmentation epoch depends on the initial amplitude of the nonaxisymmetric perturbation $A_{\varphi}$. For models of $(b, \omega) = (0, 0.07), (0, 0.05), (0.001, 0.05)$, and $(0.01, 0.05)$, fragmentation occurs in the adiabatic phase in the models with $A_{\varphi} = 0.01$ (Fig. 5), while fragmentation occurs in the second collapse or protostellar phases in the models with $A_{\varphi} = 0.2$ (Fig. 6). The epoch of fragmentation is delayed with increasing perturbation amplitude $A_{\varphi}$. In addition, some models in which fragmentation is found in Figure 5 (models with $A_{\varphi} = 0.01$) do not fragment when the initial cloud has a large nonaxisymmetric perturbation $A_{\varphi} = 0.2$. For models of $(b, \omega) = (1, 0.03), (1, 0.05), (1, 0.07)$, and $(1, 0.1)$, fragmentation occurs in the models with $A_{\varphi} = 0.01$ (Fig. 5), while fragmentation does not occur in the models with $A_{\varphi} = 0.2$ (Fig. 6).

As shown in Machida et al. (2005a, 2007b), there are two modes of fragmentation: ring and bar fragmentations. The fragmentation mode is related to the rotation rate and axis ratio of the central core. The axis ratio means the degree of the non-axisymmetry: the barlike structure is seen in the cloud with large

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**Fig. 6**—Same as Fig. 5, but for models with $A_{\varphi} = 0.2$. 

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8 The axis ratio is defined as $\epsilon = h_1/h_2 - 1$, where $h_1$ and $h_2$ are the major and minor axes, respectively, derived from the moment of inertia for the high-density gas of $\rho \geq 0.1 \rho_c$. 

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axis ratio. When the cloud has a small axis ratio and rapid rotation, fragmentation is induced via a ringlike structure. On the other hand, when the cloud has a large axis ratio ($\varepsilon_{\text{ar}} \gtrsim 10$), fragmentation occurs via a barlike structure, irrespective of the cloud rotation rate. Nonaxisymmetric structure has two distinct effects on fragmentation: suppression and promotion of fragmentations. When a core (the first core or second core) has a moderate axis ratio ($1 \lesssim \varepsilon_{\text{ar}} \lesssim 10$), the angular momentum is removed from the core by gravitational torque due to the nonaxisymmetric pattern and thus fragmentation is suppressed. In contrast, when the core has a large axis ratio ($\varepsilon_{\text{ar}} \gtrsim 10$), fragmentation occurs through the bar instability, irrespective of the cloud rotation rate. Thus, compared with models having small nonaxisymmetric perturbation $A_{\varphi} = 0.01$ (Fig. 5), in models with large nonaxisymmetric perturbation $A_{\varphi} = 0.2$ (Fig. 6), fragmentation is suppressed in some models [e.g., models ($b$, $\omega$) = (0.001, 0.05), and (0.01, 0.05)] by removal of angular momentum owing to the gravitational torque, although fragmentation is promoted in some models by the prominent bar mode ($b$, $\omega$) = (0.001, 0.03)]. However, comparison between Figures 5 and 6 indicates that only the models located near the boundaries between these three regions (first fragmentation, second fragmentation, and nonfragmentation) are influenced by the initial amplitude of the nonaxisymmetric perturbation. The trend that the magnetic field suppresses fragmentation while rotation promotes it is the same for both the models with $A_{\varphi} = 0.01$ and 0.2.

5.3. Models with $A_{\varphi} = 0.4$

Figure 7 shows the cloud structures around the center of the cloud at the end of the calculation for models with $A_{\varphi} = 0.4$. The fragmentation properties (fragmentation epoch, scale, number of fragments, and structure) shown in this figure are almost the same as those in Figures 5 and 6. In the following, we only describe the differences between the models $A_{\varphi} = 0.4$ (Fig. 7), $A_{\varphi} = 0.01$ (Fig. 5), and $A_{\varphi} = 0.2$ (Fig. 6).

First, we compare the models having the same parameters of ($b$, $\omega$) = (0.001, 0.07), which are located in the third row and
the second column in Figures 5, 6, and 7. Fragmentation occurs in the adiabatic phase with models having $A_{\varphi}$ = 0.01 and 0.2, while no fragmentation occurs through any of the phases of cloud evolution in the model with $A_{\varphi}$ = 0.4. In Figure 7, for this model, we can see that the central region around the protostar indicates a barlike structure in a large scale and a spiral structure in a small scale. Bate (1998) showed that the spiral structure effectively removes the angular momentum from the center of the cloud, and a single protostar is directly formed without fragmentation. Also in this model, since the angular momentum is effectively removed owing to the bar or spiral structure, the protostar is thought to form without fragmentation.

Next, we describe the models with $(b, \omega) = (0.001, 0.05)$ and $(0.01, 0.05)$, which are located at the fourth row and the second and third columns. Figures 5, 6, and 7 show that fragmentation occurs in the adiabatic phase with wide separations ($\sim$40 AU) for the models with $A_{\varphi}$ = 0.01 and 0.4, while fragmentation occurs with a narrow separation ($\sim$0.1 AU) in the second collapse or protostellar phase for the models with $A_{\varphi}$ = 0.2. According to the degree of nonaxisymmetric pattern (i.e., the axis ratio), these models experience three different types of evolutions: (1) if the core has a small axis ratio, fragmentation occurs via a ringlike structure in the adiabatic phase owing to the rapid rotation ($A_{\varphi}$ = 0.01); (2) if the core has a moderate axis ratio, fragmentation occurs in the second collapse phase because the angular momentum is effectively removed by the barlike structure in the adiabatic phase ($A_{\varphi}$ = 0.2); and (3) if the core has a large axis ratio, fragmentation occurs via a barlike structure in the adiabatic phase because a considerably elongated bar is formed in the adiabatic phase and such a structure is unstable ($A_{\varphi}$ = 0.4). Therefore, the separation between fragments does not represent the initial amplitude of the nonaxisymmetric perturbations.

Third, we compare the models having the same $(b, \omega) = (0.001, 0.03)$, which are located in the fifth row and the second column in Figures 5, 6, and 7. In these models, fragmentation occurs in the second collapse phase for the model with $A_{\varphi}$ = 0.01, while fragmentation occurs in the adiabatic phase for the models with $A_{\varphi}$ = 0.2 and 0.4. This difference is understood as follows: For the model with a small nonaxisymmetric perturbation ($A_{\varphi}$ = 0.01), since both the rotation rate and the nonaxisymmetric perturbation are small when the first core is formed, neither ring nor bar fragmentation occurs. On the other hand, fragmentation occurs in the model with larger nonaxisymmetric perturbations ($A_{\varphi}$ = 0.2 and 0.4) because a considerably elongated bar is formed at the first core formation epoch.

Finally, the strong magnetized models $b \geq 1$ are described. Figures 5, 6, and 7 clearly show that fragmentation becomes harder with increasing $A_{\varphi}$ especially in the strong magnetized models. For models with $A_{\varphi}$ = 0.01 (Fig. 5), fragmentation occurs in the second collapse phase for models having $\omega$ = 0.03, 0.05, 0.07, and 0.1 for $b$ = 1, and $\omega$ = 0.07 and 0.1 for $b$ = 4. In contrast, for models with $A_{\varphi}$ = 0.4 (Fig. 7), no fragmentation occurs in models having the same $b$ and $\omega$. The increase of the angular velocity is suppressed by the magnetic effect (e.g., magnetic braking and outflow; Machida et al. 2007a) in strongly magnetized models, and then these models manage to fragment in the second collapse or protostellar phase when the initial cloud has a small amount of the nonaxisymmetric perturbation of $A_{\varphi}$ = 0.01. In contrast, for models with large $A_{\varphi}$, since the angular momentum is removed not only by the magnetic effect but also by the gravitational torque as shown in Figures 3c–3f, fragmentation is more severely suppressed.

Figures 5, 6, and 7 show that the magnetic field strength, angular velocity, and nonaxisymmetric perturbation amplitude play an important role in fragmentation, and they are closely related. Thus, it is rather hard to derive the fragmentation condition from small numbers of calculations.

6. DISCUSSION

6.1. Initial Cloud Stability and Its Evolution

In the gravitationally contracting core, the thermal pressure, rotation, and magnetic field work against the self-gravity. Generally, in the initial clouds, the former three forces are characterized by the ratios to the gravitational energy, $\alpha_0$ (thermal energy), $\beta_0$ (rotational energy), and $\gamma_0$ (magnetic energy), and the clouds are specified by the combination of these three parameters. In the previous section we investigated the evolutions of clouds with the same $\alpha_0$ ($\alpha_0 = 0.5$) but different $\beta_0$ and $\gamma_0$. The magnetic field and rotation are important for cloud fragmentation because these forces cause the anisotropic patterns that promote fragmentation. However, the initial ratio of thermal to gravitational energy, which is related to the stability of the initial cloud, may affect cloud fragmentation. In this section we compare the evolutions of clouds with the same $\beta_0$ and $\gamma_0$ but different $\alpha_0$.

We investigate the evolutions of clouds with five different $\alpha_0$ ($\alpha_0 = 0.01, 0.1, 0.3, 0.5$, and 0.7). The cloud with $\alpha_0 = 0.01$, which has 1% of the thermal energy to the gravitational energy at the initial state, is highly gravitationally unstable, while that with $\alpha_0 = 0.7$ is initially nearly in equilibrium. We fixed the three other parameters as $(b, \omega, A_{\varphi}) = (0.01, 0.05, 0.01)$. Figure 8 shows the evolution of the oblateness $\varepsilon_0 b^3$ (Fig. 8a), axis ratio $\varepsilon_\varphi$ (Fig. 8b), normalized magnetic field strength $B_{zc}/(4\pi \rho c^2 \rho_0)^{1/2}$ (Fig. 8c), and normalized angular velocity $\Omega_{zc}/(4\pi G \rho_0)^{1/2}$ (Fig. 8d).

First, we show the evolution of clouds with large $\alpha_0$ ($\alpha_0 = 0.3, 0.5$ and 0.7). These models are initially more stable. In Figure 8, these models show similar evolutions for oblateness, normalized magnetic field strength, and normalized angular velocity. After the central densities exceed $n_c \approx 10^{12}$ cm$^{-3}$, the central regions have almost the same degrees of oblateness, magnetic fields, and angular velocities as shown in Figures 8a, 8c, and 8d. Shock fronts appear and the first cores surrounded by the shock waves are formed at $n_c \approx 10^{13}$ cm$^{-3}$ in these models. The first cores have almost the same properties except for the axis ratios. Comparing all five models, the formed first cores are shown to have larger axis ratios in the models with smaller $\alpha_0$. Hanawa &

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9 The oblateness is defined as $\varepsilon_0 b^3 = (h_0 h_3)^2 h_3$, where $h_0$, $h_3$, and $h_4$ are the major axis, minor axis, and z-axis, respectively, derived from the moment of inertia for the high-density gas of $\rho \geq 0.1 \rho_c$. 
Matsumoto (2000) and Lai (2000) showed that the growth rate of the nonaxisymmetric perturbation is larger in a more unstable cloud core from their linear analysis (see also Fig. 14 of Omukai et al. 2005). Since the cloud evolution converges to a self-similar solution as the cloud collapses even when the cloud is rotating and magnetized (Machida et al. 2005b, 2006b), the growth rate of the nonaxisymmetric perturbation is also converged to $C_{26}^{1/6}$ in the collapsing cloud as shown in the linear analysis (Hanawa & Matsumoto 2000; Lai 2000). Thus, the difference in the axis ratio that reflects the amplitudes of the nonaxisymmetric perturbation comes from the growth in the early phase of the contraction. For this reason, the initially more unstable cloud (i.e., the cloud with small $C_{11}$) has a large amplitude of nonaxisymmetric perturbation.

Figure 9 shows the cloud structures near the center of the cloud just after fragmentation. Fragmentation occurs via a ringlike structure in the models $C_{11} = 0.5$ and 0.7, while fragmentation occurs without the ring formation in the model with $C_{11} = 0.3$. This difference in the fragmentation pattern comes from the difference in the axis ratio of each model: the model with $C_{11} = 0.3$ has larger amplitude of nonaxisymmetric perturbation than those of models with $C_{11} = 0.5$ and 0.7 at the first core formation epoch. However, the scale and fragmentation epoch are almost the same for all models with $C_{11} \geq 0.3$, as shown in Figures 9b–9d. Thus, even though the fragmentation patterns are different, the fragmentation scale and its epoch are hardly changed for models $C_{11} \geq 0.3$.

Next is shown the evolution of the highly unstable cores with $C_{11} = 0.1$ and 0.01. Figure 8 indicates that the evolutions of oblateness, angular velocity, and magnetic field of the models with $C_{11} = 0.1$ and 0.01 differ appreciably from those of models with $C_{11} \geq 0.3$. In addition, the first core in the model of $C_{11} = 0.1$ is considerably smaller than those in models with larger $C_{11}$ (Fig. 9). In the model with $C_{11} = 0.01$, an extremely thin disk with $\frac{\alpha}{\text{ob}} \approx 50$ is formed in the isothermal phase (Fig. 8a). Tsuribe & Inutsuka (1999) expected these thin disks to fragment in the isothermal phase when the cloud collapses isothermally for a long time. In our calculation, however, fragmentation does not occur in the isothermal phase because the central region begins to behave adiabatically just after the oblateness reaches its peak. Then, an elongated bar is formed in the adiabatic phase, and the bar fragments at the central density $n_c \approx 10^{15}$ cm$^{-3}$. The fragmentation scale of the model of $C_{11} = 0.01$ is $\sim 2$ AU, which is considerably smaller than in models with $C_{11} > 0.3$. Thus, fragmentation size becomes smaller with decreasing $C_{11}$ for $C_{11} < 0.3$. 

![Image of cloud structures](image_url)
In summary, for models with $\alpha_0 > 0.3$, the clouds show similar evolutions, and the first cores have almost the same properties. On the other hand, clouds with $\alpha_0 < 0.3$ evolve in a considerably different way, and the first cores have different properties. However, observations have shown that molecular cloud cores are observed in nearly thermal equilibrium against gravity (e.g., Tachihara et al. 2002), which indicates $\alpha_0 \approx 1$. As a result, the evolution of molecular cloud cores is specified by only $\beta_0$ and $\gamma_0$ since clouds with $\alpha_0 > 0.3$ have the same evolutional properties when the clouds have the same $\beta_0$ and $\gamma_0$.

6.2. Scales and Epochs of Fragmentation

We calculated 147 models with different sets of initial cloud rotation, magnetic field, thermal energy, and amplitude of the nonaxisymmetric perturbation. In 102 out of 147 models, we observed fragmentation. To investigate the scales and epochs of fragmentation statistically, for all models that show fragmentation in Figures 5, 6, and 7, the number densities at the fragmentation epoch and the farthest separations between fragments at the end of the calculation are plotted in the main panel of Figure 10. Right and bottom panels in Figure 10 show the histogram of fragmentation scales and epochs measured in the central density, respectively. Although we calculated the cloud evolution from $n_c = 10^4 \text{ cm}^{-3}$, we observe no fragmentation in the isothermal phase ($n_c < 10^{11} \text{ cm}^{-3}$). Thus, only the range of $n_c < 10^9 \text{ cm}^{-3}$ is plotted. For convenience, we divide fragmentation models into three groups (A, B, and C) by the evolution stage when fragmentation occurs. Models in groups A, B, and C, respectively, show fragmentation in the adiabatic ($10^{11} \text{ cm}^{-3} < n_c < 10^{16} \text{ cm}^{-3}$), second collapse ($10^{16} \text{ cm}^{-3} < n_c < 10^{21} \text{ cm}^{-3}$), and protostellar phases ($n_c > 10^{21} \text{ cm}^{-3}$). The dotted line in the main panel shows the Jeans length, which is derived from the relation of the temperature and density assumed in our calculation. Figure 10 shows that points are distributed near the dotted line, which indicates that fragments have a separation nearly equal to the Jeans scale when they are born. Note that models are distributed slightly above the Jeans length in Figure 10 because a larger critical wavelength is expected for rotating magnetized gas, while the Jeans length plotted here is expected to be a spherical symmetry.

Figure 10 shows that there are two distinct fragmentation epochs, namely, $n_c \approx 10^{12}$ and $10^{21} \text{ cm}^{-3}$. The bottom panel shows models belonging to groups B and C to be smoothly distributed, but there is a clear gap between groups A and B for $10^{15} \text{ cm}^{-3} < n_c < 10^{16} \text{ cm}^{-3}$. Models in group A are distributed only in the range of $10^{12} \text{ cm}^{-3} < n_c < 10^{15} \text{ cm}^{-3}$, although the adiabatic phase lasts for $10^{11} \text{ cm}^{-3} < n_c < 10^{16} \text{ cm}^{-3}$. In group A, the fragmentation epochs are bunched into the first core formation epoch. The central density at the first core formation epoch is distributed for $3 \times 10^{15} \text{ cm}^{-3} < n_c < 10^{15} \text{ cm}^{-3}$. The first core is formed at lower density when the host cloud has a strong magnetic field or rapid rotation. In group A, models that fragment at relatively low density have strong magnetic fields or rapid rotations, while models that fragment at relatively high density have weak magnetic fields and slow rotations. In addition, no fragmentation occurs for $10^{15} \text{ cm}^{-3} < n_c < 10^{16} \text{ cm}^{-3}$. This means that the core fragments just after the first core formation, and fragmentation never occurs if the core fails at this epoch.

In group A, fragments have mutual separations of $3–300 \text{ AU}$. Since we stopped calculation in group A after fragmentation occurs, we do not follow the cloud evolution until the protostar is formed ($n_c \approx 10^{21} \text{ cm}^{-3}$). Thus, fragmentation may occur again for these models in the second collapse or protostellar phase. In that case, it is expected that hierarchical stellar systems are formed.
Some models that do not show fragmentation in the adiabatic phase show fragmentation in the second collapse phase (group B). The bottom panel of Figure 10 shows that the number of fragmentation models smoothly increases with the cloud density for \( n_c \gtrsim 10^{16} \text{ cm}^{-3} \). As shown in Figures 2 and 8, the central region has a considerably weak magnetic field in the second collapse phase owing to the ohmic dissipation (for details see Machida et al. 2007a). Thus, the magnetic braking is not so efficient that the central region can spin up as the cloud collapses. Due to this accelerated rotation, fragmentation is induced in the second collapse phase. Fragments in group B have a typical separation of 0.01–0.3 AU.

When the central density exceeds \( n_c \gtrsim 10^{21} \text{ cm}^{-3} \), the second core (or protostar) is formed. Even after the second core formation, fragmentation frequently occurs. In group C, fragmentation occurs just after the second core formation, similar to the fragmentation in the adiabatic phase. Fragments in group C have a typical separation of 0.005–0.07 AU.

The fragmentation process in both the adiabatic and protostellar phases is thought to differ from that in the second collapse phase. In both the adiabatic and protostellar phases, fragmentation occurs after the respective cores are formed. Fragmentation easily occurs in a quasi-static core, and the core has enough time to amplify the perturbation that induces fragmentation. On the other hand, fragmentation occurs owing to cloud rotation in the second collapse phase. Fragmentation does not occur in the isothermal phase when the clouds collapses in a self-similar fashion because the self-similar solution with \( \gamma = 1 \) exists even in a rotating collapsing cloud (Matsumoto et al. 1997; Matsumoto & Hanawa 1999). On the other hand, the clouds with \( \gamma > 1 \) form rotating disks and stop the contraction because no self-similar solution exists (Saigo et al. 2000). These clouds can fragment as shown in Saigo et al. (2004). In addition, since the cloud collapses with \( \gamma = 1.1 \), the nonaxisymmetric perturbation manages to evolve in this phase as shown in Hanawa & Matsumoto (2000) and Lai (2000). Thus, the fragmentation process in groups A and C is different from group B, which causes a gap between group A and group B. However, we cannot distinguish group B from group C because the number of fragmentation models smoothly increases after \( n_c \gtrsim 10^{16} \text{ cm}^{-3} \). As a result, the densities when fragments appear are divided into two groups: \( 10^{12} \text{ cm}^{-3} \leq n_c \leq 10^{15} \text{ cm}^{-3} \) and \( n_c \gtrsim 10^{16} \text{ cm}^{-3} \). The histogram of separation shows that the separations between fragments are clearly divided into two groups (Fig. 10, right panel): models anticipating wide (3–300 AU) separations and those anticipating narrow (0.005–0.3 AU) separations. This feature of two distinct groups of separation (i.e., bimodal distribution) may be smoothed out as long as the protostar evolves. However, we may observe a bimodal distribution for the radial separation of the protostar in young clusters composed of extremely young stars.

**6.3. Does Magnetic Field Suppress or Promote Fragmentation?**

In Figure 11, the fragmentation epochs are plotted against the ratio of the initial magnetic energy to the rotational energy \( E_{\text{mag}}/E_{\text{rot}} \). The right axis in this figure indicates the Jeans length that is calculated from the number density (right axis), in which the temperature is related to the number density by the spherical symmetric calculation (Masunaga & Inutsuka 2000). Only fragmentation models in the magnetized clouds are plotted in this figure. The figure shows that the fragmentation epochs shift to high density as \( E_{\text{mag}}/E_{\text{rot}} \) increases. This indicates that cloud rotation promotes fragmentation, while the magnetic field delays or in some cases suppresses fragmentation. This is valid through all the phases of cloud evolution from the isothermal to the protostellar phases. When the magnetic energy is lower than the rotational energy \( E_{\text{mag}} < E_{\text{rot}} \) in the initial cloud, many models result in fragmentation in the adiabatic phase. On the other hand, when the magnetic energy is predominant over the rotational energy \( E_{\text{mag}} > E_{\text{rot}} \), fragmentation occurs mainly in the second collapse and protostellar phases, and some models do not show fragmentation. In this figure, there are some exceptions. Two models show fragmentation in the second collapse phase even if \( E_{\text{mag}}/E_{\text{rot}} < 0.1 \). In these models, fragmentation does not occur in the adiabatic phase because the nonaxisymmetric perturbation is moderately grown and an induced spiral pattern effectively removes the angular momentum from the center of the cloud. Thus, fragmentation is postponed.

**6.4. Fragmentation Condition and Wide/Close Binary Formation**

We investigated the cloud evolutions controlled by four parameters. Each parameter corresponds to the magnetic energy \( b \), rotational energy \( \omega \), thermal energy \( y_{\text{th}} \), and the amplitude of the nonaxisymmetric perturbation \( \beta_{\text{NAP}} \). The initial cloud. As shown in the previous sections, fragmentation significantly depends on the magnetic field strength and rotation rate of the initial cloud, while it slightly depends on the thermal energy and the amplitude of the nonaxisymmetric perturbation. Thus, in Figure 12 we have plotted the final state of each cloud against the magnetic field strength and rotation rate of the initial cloud. In this figure, \( x \) and \( y \)-axes are parameters \( b \) and \( \omega \), respectively. Top and right axes indicate, respectively, the ratio of the magnetic \( (\gamma_0) \) and rotational \( (\beta_0) \) energies to the gravitational energy of the initial cloud. The bottom axis means the mass-to-magnetic flux ratio \( (M/\Phi_{\text{norm}}) \) normalized by the critical value (see eqs. [12]–[14]). In this figure, some models are located in the region of \( \gamma_0 > 1 \) or \( (M/\Phi_{\text{norm}}) < 1 \), which indicates that the cloud is magnetically supercritical. In these models, the initial cloud is magnetically subcritical as a whole, while the central region is magnetically supercritical since we adopted the Bonnor-Ebert density profile (for details see Machida et al. 2007a). Thus, any cloud can collapse in a sufficiently shorter timescale than the ambipolar diffusion timescale.

In Figure 12, three types of symbols (circles, diamonds, and plus signs) are plotted against the initial magnetic fields and angular velocities. Each symbol includes three models with different \( A_\varphi \) (\( A_\varphi = 0.01, 0.2, \) and 0.4). Circles indicate that at least one model out of three shows fragmentation in the adiabatic phase.
(10^{11} \text{ cm}^{-3} \lesssim n_c \lesssim 10^{16} \text{ cm}^{-3})$; diamonds indicate that in models that do not show fragmentation in the adiabatic phase ($n_c \lesssim 10^{16} \text{ cm}^{-3}$), at least one model out of three shows fragmentation in the second collapse or protostellar phase ($n_c \gtrsim 10^{16} \text{ cm}^{-3}$); and plus signs indicate that models show no fragmentation through any phases of cloud evolution ($10^4 \text{ cm}^{-3} \lesssim n_c \lesssim 10^{22} \text{ cm}^{-3}$). As shown in Figure 12, we can divide the parameter space into three regions, wide fragmentation, close fragmentation, and nonfragmentation, which correspond to the formation of wide binary, close binary, and single star. The separations between fragments are clearly divided into two classes (wide and narrow separation) according to the fragmentation epoch, as shown in Figure 10.

Figure 12 indicates that models with weak magnetic fields and rapid rotations are distributed in the wide binary region. Models in the wide binary region can fragment in the adiabatic phase, and fragments have separations of 3–300 AU. However, not all models in this region fragment in the adiabatic phase. Even among models in this region, some models show fragmentation only in the second collapse or protostellar phase, and some models show no fragmentation through any phases of cloud evolution. For example, models $(b, \omega) = (0.01, 0.05)$, which are located in the wide binary region in Figure 12, show fragmentation in the adiabatic phase for $A_\varphi = 0.01$ (Fig. 5) and 0.4 (Fig. 7), while fragmentation occurs only in the second collapse phase for the model with $A_\varphi = 0.2$ (Fig. 6). In models $(b, \omega) = (0.001, 0.07)$, fragmentation occurs in the adiabatic phase for the models with $A_\varphi = 0.01$ (Fig. 5) and 0.2 (Fig. 6), while the protostar is directly formed without fragmentation for the model with $A_\varphi = 0.4$ (Fig. 7). Thus, a small difference in the amplitude of the non-axisymmetric perturbation at the initial state has a possibility to...
induce a large difference in the fragmentation epoch. In some models located in the wide binary region, we stopped calculation before the protostar was formed when the fragmentation occurred in the adiabatic phase because the fragments are likely to escape from the finest grid. Thus, fragmentation can occur again in each fragment in the second collapse or protostellar phase, which seems to lead to a hierarchical stellar system.

Models distributed in the close binary region have a weaker magnetic field and slower rotation than models in the wide binary region. However, the tendency of rotation to promote fragmentation and the magnetic field to suppress fragmentation is the same. Models in the close binary region can fragment in the second collapse or protostellar phase, and fragments have separations of 0.005–0.3 AU as shown in Figure 10. However, similarly to wide binary models, even in models in the close binary region, some models do not fragment in either the second collapse or protostellar phases. For example, in models \((b, \omega) = (1, 0.05)\), fragmentation occurs in the protostellar phase for the model with \(A_\varphi = 0.01\) (Fig. 5), while protostars are directly formed without fragmentation for the models with \(A_\varphi = 0.1\) (Fig. 6) and 0.4 (Fig. 7).

Models in the single-star region have stronger magnetic fields and slower rotations than those in the wide and close binary regions. In all models in this region, the protostar is directly formed without fragmentation. As a result, a single compact sphere is formed at the center of the cloud, as shown in models \((b, \omega) = (4, 0.007)\) located in the bottom right corner of Figures 5, 6, and 7. These compact spheres are expected to evolve into single stars.

Models located in the wide/close binary regions have the possibility of forming the wide/close binary. As for fragmentation, it is difficult to forecast the cloud evolution from the initial conditions, and a small difference in the initial state sometimes results in different final outcomes. For example, even models located in the wide binary region have three possibilities: formation of a wide binary, a close binary, or a single star. That is, the parameters in wide/close binary regions are the necessary but not sufficient conditions for wide/close binary formation.

6.5. Comparison with Observation

As shown in Figures 11 and 12, our results indicate that cloud rotation promotes fragmentation, but the magnetic field suppresses fragmentation. In this section, to quantify the magnetic field and cloud rotation in the molecular cloud core, we compare our results with observations.

Both magnetic field and rotation rate are observed in molecular cloud TMC-1C. TMC-1C has \(\beta = 1.2 \times 10^{-3}\) (Goodman et al. 1993) and \(\gamma < 0.08\) (Crutcher 1999). For magnetic field strength, only the maximum value was estimated for observational limitation. When a cloud has \(\beta = 1.2 \times 10^{-3}\), Figure 12 indicates that (1) fragmentation occurs in the adiabatic phase and the wide binary can form when \(\gamma < 10^{-3}\); (2) fragmentation occurs in either the second collapse or protostellar phase, and the close binary can form when \(10^{-3} < \gamma < 0.45\); and (3) only a single star forms without fragmentation when \(\gamma > 0.45\). Since observation showed \(\gamma < 0.08\), it is possible to form a binary system in TMC-1C.

In the observations of the molecular cloud core, Crutcher (1999) showed that the magnetic energy is comparable to gravitational energy \((\gamma \sim 0.5)\), while Goodman et al. (1993) and Caselli et al. (2002) showed that rotational is much smaller than gravitational energy \((\beta \sim 0.02)\). Thus, these observations indicate that it is difficult to form a (wide) binary system. Except for TMC-1C, however, there are few clouds in which both magnetic field and rotation rate are observed. In addition, the observed magnetic and rotation energies might not reflect the distribution of these energies in the majority of cloud cores because we can only observe a limited range of the magnetic field strength and rotation speed: we cannot observe weak magnetic field strengths and slow rotation speeds. Furthermore, we have to measure the magnetic field strength and rotation rate at the same radius or the same number density in the molecular cloud core. To determine magnetic field strength and rotation rate precisely, we need a high-resolution observational facility such as the future ALMA.

As denoted above, observations have shown that molecular cloud cores have high magnetic energies but low rotational energies (Crutcher 1999; Caselli et al. 2002), which indicates \(E_{\text{mag}}/E_{\text{rot}} > 1\). Figure 11 indicates that these clouds barely fragment. This conflicts with observations that the majority of stars are born as binary (e.g., Duquennoy & Mayor 1991). In this study, we calculated the evolution of a magnetized cloud in which the initial magnetic field lines are aligned to the rotation axis. When magnetic field lines are not aligned to the rotation axis, cloud evolution may be changed. However, Machida et al. (2006b) showed that when the magnetic field lines are not aligned to the rotation axis at the initial state, the magnetic braking is more effective, and the angular momentum is effectively transferred outwardly. This implies that fragmentation is more suppressed. On the other hand, Price & Bate (2007) showed, in their ideal MHD calculations, that when the initial cloud has a considerably distorted structure, fragmentation can occur even in a strongly magnetized cloud.

To understand observational binary frequency, we may need to calculate the cloud evolution in a nonideal regime, parameterizing the cloud shape and the angle between the rotation axis and the magnetic field lines, as well as the nonaxisymmetric perturbation and the magnetic, rotational, and thermal energies.

6.6. Comparison with Previous Works

Fragmentation conditions in the adiabatic phase (i.e., the wide binary region) were investigated by Machida et al. (2005a), Price & Bate (2007), and Hennebelle & Teyssier (2008) using the ideal MHD approximation, which is not valid in the high-density gas region with \(10^{12} \text{ cm}^{-3} \leq n_c \leq 10^{15} \text{ cm}^{-3}\). In this paper we studied cloud evolution in the nonideal MHD regime. However, the wide binary region in Figure 12 corresponds well to results of Machida et al. (2005a) and Hennebelle & Teyssier (2008). As shown in Figure 10, many models located in the wide binary region fragment in the range of \(10^{12} \text{ cm}^{-3} \leq n_c \leq 10^{15} \text{ cm}^{-3}\). In addition, since the density outside a small central part of the cloud is \(n_c \lesssim 10^{12} \text{ cm}^{-3}\), the ohmic dissipation is not so effective at the fragmentation epoch. From this, there must be only a little difference between the ideal MHD and nonideal MHD calculations for fragmentation appearing in the adiabatic phase.

Price & Bate (2007) investigated the evolution of magnetized clouds and fragmentation conditions with their MHD-SPH code. Their results are qualitatively the same as ours: the magnetic field suppresses fragmentation. However, there is a small difference. They adopted a rotation energy of \(\beta_0 = 0.005\) for the initial clouds. Fragmentation occurs when \(M/\Phi_{\text{norm}} > 5\) in Price & Bate (2007), while fragmentation occurs when \(M/\Phi_{\text{norm}} \gtrsim 10\) in our calculations for models with \(\beta_0 = 0.005\), as shown in Figure 12. Thus, in Price & Bate (2007) fragmentation occurs in stronger magnetic fields than ours.

Fragmentation is promoted by rotation, but it is suppressed by the magnetic field. In strongly magnetized clouds, since the cloud rotation is removed from the central region by the magnetic braking and outflow, fragmentation rarely occurs. Outflow is closely related to the magnetic braking: both outflow and magnetic braking...
are caused by the torsional Alfvén wave or the magnetic tension force generated by rotation of the central core. Thus, the outflow is one of the proofs that the Alfvén wave is properly resolved. When the Alfvén waves are resolved, the outflow always appears after the adiabatic core (or first core) is formed. In the collapsing clouds, no outflow appears in Price & Bate (2007), while the outflow appears in the grid-based MHD simulations, such as Tomisaka (1998, 2002), Matsumoto & Tomisaka (2004), Machida et al. (2004, 2005a), Fromang et al. (2006), Banerjee & Pudritz (2006), and Hennebelle & Fromang (2008). Note here that an accurate description of (torsional) Alfvén waves is crucial in the study of the driving mechanism of outflow and magnetic braking. Thus, we suspect that the angular momentum transfer caused by the torsional Alfvén wave (or the magnetic tension force) may not be correctly resolved in Price & Bate (2007) and thus excess angular momentum around the center of the cloud is left. Therefore, fragmentation occurs more frequently in their calculation than in ours.

In addition, Price & Bate (2007) and Hennebelle & Teyssier (2008) have argued that fragmentation is possible even in a strongly magnetized cloud when the initial cloud is considerably distorted. However, in our calculation, even when the large non-axisymmetric perturbation is added \( A_p = 0.4 \), strongly magnetized clouds did not show fragmentation. There are two differences in the initial condition between ours and theirs: initial shapes of clouds (or the density distribution) and density perturbation. We adopted a Bonnor-Ebert density profile as the initial state, while Price & Bate (2007) and Hennebelle & Teyssier (2008) adopted

| Model | Density Profile | \( m = 2 \) Mode | \( A_p \) | \( b \) | \( \omega \) | \( \alpha \) |
|-------|----------------|-----------------|--------|-------|--------|--------|
| 1..... | Uniform sphere | Eq. (11)        | 0.1    | 0.001 | 0.1    | 0.5    |
| 2..... | Uniform sphere | Eq. (15)        | 0.1    | 0.001 | 0.1    | 0.5    |
| 3..... | Bonnor-Ebert sphere | Eq. (11) | 0.1    | 0.001 | 0.1    | 0.5    |
| 4..... | Bonnor-Ebert sphere | Eq. (15) | 0.1    | 0.001 | 0.1    | 0.5    |

![Model 1: Uniform Sphere with eq (11)](image1)

![Model 2: Uniform Sphere with eq (15)](image2)

![Model 3: B.E. Sphere with eq (11)](image3)

![Model 4: B.E. Sphere with eq (15)](image4)

**Fig. 13.** Cloud structures around the center of the cloud at the adiabatic core formation epoch for models 1–4. As the initial state, a uniform density distribution is adopted in models 1 and 2, while a Bonnor-Ebert density distribution is adopted in models 3 and 4. The initial perturbation of eq. (11) is added in models 1 and 3, while eq. (15) is added in models 2 and 4. The elapsed time \( t \), maximum number density \( n_{\text{max}} \), and arrow scale are listed in each panel. The level of the subgrid is shown in the upper left corner of each panel.
a uniform sphere. For the initial density perturbation, Price & Bate (2007) and Hennebelle & Teyssier (2008) adopted it as

$$\delta_0 = A_2 \cos 2\phi.$$  \hfill (15)

Compared with our initial condition, $\delta_0 = A_2 (r/R_c)^2 \cos 2\phi$ (eq. (11)), theirs contains a larger density perturbation than ours especially near the origin even for the same $A_2$.

To compare the evolutions with different initial configurations, we calculated four additional models (models 1–4) shown in Table 4. The initial clouds of models 1 and 2 have a uniform density profile, while those of models 3 and 4 have a Bonnor-Ebert density profile. We added the density perturbation of equation (11) to the initial state for models 1 and 3 and that of equation (15) for models 2 and 4. These four models have the same parameters of $(b, \omega, A_2) = (0.001, 0.1, 0.2)$. In addition, they have the same ratio of the thermal to gravitational energies $\alpha_0 = 0.5$ (see Table 4).

The density distributions for each model at the adiabatic core formation epoch ($\alpha_c \approx 10^{12} - 10^{13}$ cm$^{-3}$) are plotted in Figure 13. This shows that the initial density profile hardly affects the subsequent cloud evolution. That is, models 1 and 3 form a ring, while models 2 and 4 form a bar. Furthermore, the time evolutions of oblateness and axis ratio in model 1 (model 2) coincide with those in model 3 (model 4). Thus, the clouds experience almost the same evolution regardless of the initial density distribution, when the clouds have the same nonaxisymmetric perturbation in the initial state. This is natural because the Bonnor-Ebert sphere has a uniform density distribution near the center, and a small part of the central region collapses first and forms the adiabatic first core. Thus, the difference between ours and theirs (Price & Bate 2007; Hennebelle & Teyssier 2008) seems to come from the shape of the initial density perturbation assumed.

We consider that our results do not contradict theirs, if we take into account the fact that they imposed a larger perturbation than ours and that fragmentation occurs promptly due to their large perturbation. By virtue of the large density perturbation near the origin, the cloud fragments even in a strongly magnetized environment in their simulations. Since it is still hard to observe the degree of nonaxisymmetric pattern near the center of the molecular cloud core, we cannot precisely specify the initial amplitude of density perturbation. To construct more realistic initial settings, we need high-resolution observations with a new-generation radio telescope such as ALMA.

7. SUMMARY

We investigate the cloud evolutions for $10^4$ cm$^{-3} < n_c < 10^{22}$ cm$^{-3}$ in a large parameter space. In this study we systematically calculated 147 models with different magnetic field, rotation, thermal energies, and the amplitude of the nonaxisymmetric perturbation of the initial cloud. These calculations indicate the following:

1. Fragmentation significantly depends on the magnetic field and rotation but slightly depends on the thermal energy and the amplitude of the nonaxisymmetric perturbation of the initial cloud.
2. The magnetic field delays or in some cases suppresses fragmentation, and rotation promotes fragmentation through all phases of cloud evolution.
3. The distributions of the separations between fragments are clearly divided into two classes: fragments formed in the adiabatic phase have wide separations (3–300 AU), while fragments formed in the second collapse and protostellar phases have narrow separations (0.007–0.3 AU), which indicates a bimodal distribution for the residual separation of the protostar.

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