WalkingTime: Dynamic Graph Embedding Using Temporal-Topological Flows

David Bayani

Computer Science Department
Carnegie Mellon University, Pittsburgh, PA 15213, USA
dcbayani@alumni.cmu.edu

Abstract. Increased attention has been paid over the last four years to dynamic network embedding. Existing dynamic embedding methods, however, consider the problem as limited to the evolution of a topology over a sequence of global, discrete states. We propose a novel embedding algorithm, WalkingTime, based on a fundamentally different handling of time, allowing for the local consideration of continuously occurring phenomena; while others consider global time-steps to be first-order citizens of the dynamic environment, we hold flows comprised of temporally and topologically local interactions as our primitives, without any discretization or alignment of time-related attributes being necessary.

Keywords: dynamic networks · representation learning · dynamic graph embedding · time-respecting paths · temporal-topological flows · temporal random walks · temporal networks · real-attributed knowledge graphs · streaming graphs · online networks · asynchronous graphs · asynchronous networks · graph algorithms · deep learning · network analysis · datamining · network science

1 Introduction and Related Work

Graph embeddings are a collection of techniques for converting aspects of a network into a vector based (at least partially) on network topology, typically considered as mapping the high-dimensional structure of the network itself into a significantly lower dimensional space. This broad endeavor most commonly is concerned with representation of individual nodes [17, 6].

The embedding task is often performed by using matrix factorization methods and neural-network based approaches. In addition to diversity resulting from problem-specific modifications, embedding techniques also vary in regard to the balance they strike between reflecting the structures of community membership

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1 We note that the vast majority of the work and writing presented in this paper was done in September to November of 2018 (see, for instance, the hashes provided at [4]). For the most part, recent modifications to this writeup only streamlined portions (e.g., cut content) in order to enable more immediate presentation. Some visuals were mildly improved.
Neural network based approaches have seen an increase of popularity over the last several years, spurred in part by their demonstrations of superior performance in numerous areas. These approaches can generally be divided into random-walk-based and non-random-walk based methods. The former saw recent popularity ushered in by DeepWalk \[31\] and then node2vec \[16\]; both methods borrow advances in language modeling, generating an embedding for a given node by feeding fixed-length random walks starting from that node into the Skip-Gram word embedding model \[29\], using the paths as context windows surrounding the specified start node. Unlike DeepWalk, node2vec allows for biased random walks controlled by interpretable, user defined parameters, and the two methods differ in how they perform the non-convex optimization required by Skip-grams. Of particular note are these methods’ scalability (being trivially parallelizable and memory-efficient), which allows for considerations of higher-order proximity as desired, and — in the case of node2vec and its derivatives — allows for shifting emphasis between reflecting community membership and structural equivalence \[16\].

Among the notable extensions of DeepWalk and node2vec, HARP is a meta-scheme that augments sophisticated embedding techniques to intelligently avoid the many local-optima common in non-convex objectives, doing so by providing initial conditions that are more favorable for the base method. HARP constructs a multi-granular hierarchy of networks by carefully merging nodes going from one level to the next, and then produces embeddings for each level going from most-granular to least-granular (i.e., from the least detailed network to the most detailed), using the final embedding from the previous layer as initial conditions of the next layer \[8\]. HARP has been shown to consistently boost performance for a variety of representation-learning techniques \[17\].

Meta-schemes have also been utilized to incorporate attribute information into node embeddings, metapath2vec \[11\] being a chief example. Working over networks that may have node and edge attributes, metapath2vec builds off of node2vec by biasing the random walk based on user-provided templates which specify sequences of attribute-values which may occur during a random walk. For instance, if $A, B, C$ and $D$ are attribute values, a user may require that all random walks consist of paths who sub-sequences have nodes with attributes $< A, B, C >$, $< C, D, D >$, or $< D, A >$ ($< \cdots >$ being sequence constructor notation), in which case the random walk could only visit a node with attribute $D$ at most twice in a row.

Moving further to enrich the information carried in embeddings, dynamic graph embedding has received increase attention recently - for instance \[36, 12\].
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27, [15] 35, 33, 26, 28 all were published in major venues within the last two years, while [17] [7] 34, among others, noted the deficit of dynamic graph embedding methods just two years ago. In this embedding literature, the pervading models of dynamics are global, discrete, and strictly progress forward in time. That is, at a time \( T + 1 \), all areas of the network in question are updated to include changes since \( T \) - effectively every node receives the compiled changes that have occurred in the time window \((-\infty, T + 1]\). While these time-steps might have non-uniform window sizes or other sophisticated methods of selecting analysis times, the point remains that the network growth is cut into “snap-shots”, framing evolution as discrete and globally-effecting process; this is unnatural given that in many real systems, events are typically a function of local interactions, and many distinguishing behaviors may occur over time-scales much shorter than the collective behavior of the network as it evolves. Prior literature considered a number of time steps that was order of magnitudes smaller than either the node set or edge set - for example, in [36], only 13 time steps are considered in a graph with 200,000 nodes and close to two million edges. In addition to loss of fine-grain information, such an approach requires a global ordering of events relative to each other in order to construct the sequence of graphs considered. Further still, such schemes quite likely privilege relationships between events that happen to be co-located in the same discrete step over those that artificially land across a divide, despite the fact that events close to the separation point may be temporally closer to one another than to any other members of their respective partitions. In short, these models of dynamic graphs are, for many dynamic processes, unnatural and can fundamentally impact the view which down-stream embedding algorithms have of the network dynamics.

2 Proposed Method: WalkingTime

Our method, WalkingTime, extends node2Vec to the dynamic setting by adopting an event-time multi-graph representation and leveraging temporal information present on edges to inform the random walk. In contrast to prior work, WalkingTime has been built off of a dynamic network model that does not require global discretization, allowing time to remain a continuous value that can be considered locally between the nodes with which the interaction occurs. Further, our approach allows much of the temporal information to be used raw, removing several areas where performance-critical decisions previously had to be made up-front.

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4 The “two years” cited here are in respect to when this content was originally written; in respect to the time that this write-up has been released on ArXiv, it would be roughly five years ago.

5 See prior footnote in regard to the timeline.

6 Their is a question of scalability (i.e., time and memory limitations) and robustness (e.g., not washing-out desired signals) in increasing the number of time steps.
Fig. 1: Illustration comparing our approach to typical dynamic graph embedding approaches in an unattributed, unweighted 5-by-5 grid. In this scenario, we consider embedding the node highlighted in orange, node 3. In the figures, the edges that are traversable are shown in black, while edges that exist in the static graph but are not currently considered by the algorithms are dashed and highlighted in orange or yellow.

We consider as our input a multi-graph \( G^I_{\text{temporal}} = (V, E^I_{\text{intervals}} \cup E^I_{\text{timepoints}} \cup E^I_{\text{persist}}) \) where, letting \( \pi \) be the projection operator:

\[
\mathbb{R}^2_{\text{intervals}} = \{ w \in (\mathbb{R} \cup \{\infty, -\infty\}) \times (\mathbb{R} \cup \{\infty, -\infty\}) | \pi_1(w) < \pi_2(w) \}
\]

\( E^I_{\text{intervals}} \subset V \times V \times \mathbb{R}^2_{\text{intervals}}, \quad E^I_{\text{timepoints}} \subset V \times V \times \mathbb{R}, \quad E^I_{\text{persist}} \subset V \times V \)

Above, \( \mathbb{R}^2_{\text{intervals}} \) is the set of valid (including potentially indefinite) time intervals. For ease of future reference, we will define \( H = V \times V \times \mathbb{R}^2_{\text{intervals}} \). The semantics of these sets of edges are as follows:

- \( E^I_{\text{intervals}} \) represents interacts between nodes where beginning and end information are present in the data provided. For instance, a phone call may be recorded in such a fashion.

- \( E^I_{\text{timepoints}} \) represents interacts where a single time-point is presented in the data, owing to there being only one overt action that occurred, but that was presumably undertaken some period before and which will have impact for some unspecified period after. For example, an email may be recorded this way; while the sent-timestamp of an email is considered a single point in time, there is a latent interval of time stretching from when factors lend the author to write to when the recipient finishes reading.

- \( E^I_{\text{persist}} \) denotes those relations that have not observably changed over the period of study for which the data was collected. A family relationship in a social network is an example, which may predate and extend beyond the time span studied.

We transform this input graph into a uniform representation:

\[ G_{\text{temporal}} = (V, E_{\text{temporal}}) \quad \text{where} \quad E_{\text{temporal}} \subseteq H \]
Given a user-defined parameter, $\lambda \in \mathbb{R}^{\geq 0}$ — which we call the “symmetric window-extension” — the transformation is done as follows:

$$E_{\text{temporal}} = \left\{ e \in H \left| \exists e' \in E_{\text{timepoints}}, \pi_1(e) = \pi_1(e') \land \pi_2(e) = \pi_2(e') \land \pi_3(e) = \pi_3(e') \right. \right\}$$

where $H$ is the set of all reachable nodes.

Further, notice that our approach, compared to the approaches based on graph snapshots, alters both the set of nodes reachable during the random walk. This path construction resembles some of the numerous approaches falling under the heading of “time-respecting paths” [18].

As stated, our aim is to produce a random walk (and eventually a series of random walks) of form $< u_i \in V | i \in \mathbb{N} \cup \{0\}, i \leq l_{\text{walk}} >$, where, as similarly required by node2Vec, $l_{\text{walk}}$ is a user-defined non-negative integer specifying the number of steps in the walk. To produce our random walks using $G_{\text{temporal}}$, we introduce one additional structure, a set of active edges at each step in the walk.

In words, these are the multi-edges present in $E'$ which are incident on the nodes listed in the restriction. Note that $E' \mid (v, v)$ is only non-empty if node $v$ has explicit self-loops in $E'$.

We define the allowable random walks starting from a node $u_0$ recursively:

- $A_E(0) = E \mid u_0$.
- We require, for all $i \in \mathbb{N} \cup \{0\}$, that $A_E(i) \mid (u_i, u_{i+1}) \neq \emptyset$. This is a restriction on the possible choice of $u_{i+1}$ taken by the random walk.\(^7\)
- $\forall i \in \mathbb{N} \setminus \{0\}, A_E(i+1) = \{ e \in E_{\text{temporal}} \mid u_{i+1} \}
  \left| u_i, \exists e' \in A_E(i) \mid u_{i+1}, \pi_3(e') \cap \pi_3(e) \neq \emptyset \right.$

Here, $A_E(i)$ may be understood as the set of “active edges” on the $i^{th}$ step of the random walk. This path construction resembles some of the numerous approaches falling under the heading of ”time-respecting paths” [18]. Further, notice that our approach, compared to the approaches based on graph snapshots, alters both the set of nodes reachable during the random walk as well as the distance to the reachable nodes.

\(^7\) Note that this requirement in not equivalent to $A_E(i) \mid u_{i+1} \neq \emptyset$, which would allow revisiting $u_i$ even without $u_i$ having self-loops.
As done in node2Vec, we produce biased random walks that rely on parameters \( p, q \in \mathbb{R}_{>0} \) as follows ("A \leftarrow B" indicates assigning a value B to variable A):

- The set of nodes that may be visited at step \( i + 1 \) (i.e., the candidates for \( u_{i+1} \)) is \( U_{i+1} = \{ v \in V : \exists E(i) \mid (u_i, v_{i+1}) \neq \emptyset \} \).
- If \( u_{i-1} \in U_{i+1} \), then the probability of selecting \( u_{i+1} \leftarrow u_{i-1} \) is \( \alpha p^{-1} \).
- If \( w \in U_{i+1} \setminus \{ u_{i-1} \} \) is such that there exists \( e \in E_{temporal} \) where \( \{ \pi_1(e), \pi_2(e) \} = \{ w, u_{i-1} \} \) and there exists \( e' \in A_E(i) \mid (u_i, v_{i+1}) \) such that \( \pi_3(e') \cap \pi_3(e) \neq \emptyset \), then the probability of selecting \( u_{i+1} \leftarrow u_{i-1} \) is \( \alpha q^{-1} \).
- For all other members of \( U_{i+1} \), the probability of selection is 0.

Algorithm 1 summarizes how the pieces described above come together. Of the parameters listed, \( \lambda \) is the only additional one that \texttt{WalkingTime} uses that is not found in node2vec. The remaining parameters, found in the method we build off of, are as follows: \( d \) is the embedding dimension, \( n_{optIters} \) is the number of optimization iterations, and parameters \( p \) and \( q \) — introduced by node2vec when innovating over DeepWalk — control the emphasis between structural and proximal features.

2.1 Efficient Traversal

The original node2vec and its reference implementation called for biased random walks whose step-selection distribution required knowing, for each neighbor \( v \) of

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\[ A \leftarrow B \]

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As opposed to edges or paths on the random walk.
node \( u_t \), whether \( v \) was \( u_{t-1} \) or a neighbor of \( u_{t-1} \). Due to the potentially large neighborhoods and relatively small effective diameters of a many real graphs, this computation can be unnecessarily expensive when ultimately a single random selection will be made among the neighbors of \( u_t \). In order to improve the efficiency of the process, we conduct a form of rejection sampling, considering the neighbors of \( u_t \) in random order, and stopping when we select a node based, probabilistically, on its proximity to \( u_{t+1} \) when respecting time on the edges. Specifically, we reinterpret the parameters \( p \) and \( q \) from node2Vec — which were originally relative weight parameters, roughly speaking — and now take them to directly be probabilities of retention based on a nodes’ temporal-topological proximity to \( u_{t-1} \). In the case where all neighbors are sampled and none have been selected (as can happen in rejection-sampling), alias-sampling is used to ensure a neighbor is drawn with appropriate probability using the information which, at that point, would already be computed for each neighbor.

2.2 Illustration of Our Approach

An illustration comparing our approach (A) to snap-shot based approaches (B) can be seen in Fig. 1. We see that in (A), around node 8, all and only things in times \([1,3]\) may be active, while around node 12, anything from times \([-3, 7]\) may be active; note that, if a time window of \([-3,7]\) or larger was active around node 8, then more edges would be possibly activate than there are - this highlights the non-trivial local nature of our windowing, which other approaches fail to emulate. Importantly, notice that not only does (A) have a different reachable set than (B), but the distance and direction of travel is altered compared to the (simple) static graph: In order for one of WalkingTime’s random walks to reach node 6 after starting at node 3, it must visit — in order — nodes 8, 7, 12, and 11. Notice that the edge leading from node 6 to node 7 is active only after node 6 is visited from node 11; it is possible for a random walk starting at 3 to go from node 7 to node 6 after having visiting nodes 12 and 11 at least once.

In general, it is trivial to show that the length of allowable paths for WalkingTime are at least as long as the paths present in any time-step in a snapshot-based model, but not longer than the paths present in a static graph containing one edge to represent any that would eventually appear.

Examining the illustration for (B), we see that (1) the length and number of paths leading from node 3 are shorter and fewer, and (2) that there are seemingly many components to the graph. One can regard this second point as a loss of granular, local detail; seemingly, for instance, nodes 14, 15, 19, 20, 24, and 25 are temporally related, or at least more closely related to each other at this time than to node 3. Our method (A) suggests, however, that nodes 15 and 20 are transitively closer in relation to node 3 than the rest of the aforementioned nodes. Further, we see that in (A), node 15 and node 20 are considered immediately related, while nodes 14 and 15 are not - this is in contrast to the alternative, where node 14 and 15 are considered to be in closer proximity than node 20 is to node 15.
While aggregation across time might alter some of the trends shown here, the extensive segregation and lack of fine granularity shown in (B) would still play a role, and the nature of the relationships being compiled over time would still be fundamentally different than those we search for. For us in (A), flows are first-order citizens, while others consider static time-frames to be first-order citizens and flows as derived (second-order) objects.

3 Experiments

We briefly demonstrate our approach on a discrete dynamical system that is well formalized, capable of sophisticated behavior, and which is well known to computer scientists: Conway’s Game of Life [13]. We consider a board with initial configuration shown in Fig. 2(a) where the relative coordinates are (1, 0), (2, 1), (0, 2), (1, 2), and (2, 2) for red, and (53, 5), (52, 6), (54, 7), (53, 7) and (52, 7) for blue. The colored positions indicated form two ”gliders”, structures which move themselves across the grid as time progresses. We run the simulation for 200 time-steps. In Fig. 2(b), we see the simulation after 113 steps: the darker cells are active at this step — note the two gliders still present — while the lighter blue and red indicate the path taken by the gliders up to this time (they are not part of the data for this time-step, but shown just to visualize the trajectories). The yellow is the section where the paths traced by the gliders share a location, although separated in times when they occupy it. Fig. 2(c) shows the states after 200 time-steps, using the same coloring convention as the previous time-step.

A temporal graph is produced from this system by assigning one node to each space in the grid that at some point was filled, and producing an edge labeled with time \( t \) between nodes \( v_{i,j} \) and \( u_{n,m} \) if and only if \((i, j)\) and \((n, m)\) are neighbors in the grid and are both filled at time \( t \). The graph produced in this fashion contains 404 nodes and 2200 temporal multi-edges.
We perform a node embedding into two dimensions and plot the results. Our baseline algorithms consist of two methods for static graphs - a scalable graph factorization based on SGD [1] (implementation courtesy of [14]) and standard node2vec - and two modern methods designed for the temporal setting - TNE [36] and DynamicTriad [35]. TNE performs an adjacency matrix factorization for each graph snap-shot, then produces an embedding using a loss that incorporates both reconstruction error and cross snap-shot consistency. DynamicTriad views network evolution through the eyes of triadic closure processes — taking a local understandings of structural evolution, then stitching together local views in order to share information globally; the latter feat of global integration is accomplished via joint regularization that considers social homophily and temporal smoothness. WalkingTime and node2vec’s parameters were $\lambda = 1$, $p = 10000$, $q = \frac{1}{p}$, and a walk-length of 480. All other parameters were the defaults as found on their authors’ websites.

Two dimensional embeddings resulting from this process are shown in Fig. 3. The red and blue represents nodes from the grid that only belonged to one of the trajectories shown in Fig. 3 while yellow was assigned to those that occurred in the cross-over section. As one can see, WalkingTime was the only method that properly sorted the colors.

Further information on experiments can be found in Appendix A and [4].
4 Discussion and Conclusion

We have introduced WalkingTime, a novel temporal graph embedding algorithm that views dynamic graphs through the lens of temporal-topological flows, as opposed to a series of discrete time-steps. Our demonstration shows that this avenue of investigation is worth further investigation.

Our approach trivially extends to directed graphs, and can be used in conjunction with metapath2vec ([11]) to handle heterogeneous networks. Further, weighted graphs can be treated by using the weight-distribution in a neighborhood to influence the node selection probability, similar to the methods mentioned in [16] for handling such an extension.

Parameter $\lambda$ may be chosen using a variety of means. A simple approach for informing the choice of lambda is to produce a histogram approximating the distribution of time-interval distances among neighboring edges; that is, one may pick a random node, $n$, and record the time-difference of two randomly select multi-edges incident on it (perhaps with bias to ensure the multi-edges are between distinct pairs of nodes). If $[a_1, b_1]$ and $[a_2, b_2]$ are the time-intervals on such a pair of multi-edges (where $a_i = b_i$ in the case of representing time-points), then the value $\frac{1}{2}(max(b_1, b_2) - min(a_1, a_2) - (b_1 - a_1) - (b_2 - a_2))$ may be used in a histogram if this value is zero or negative, the pair of edges already overlap in time, and if it is positive, the result provides the minimum $\lambda$ value that would cause the intervals to overlap in time. While this would provide information about the connectivity of immediate neighborhoods in the temporal graph, it does not necessarily indicate what impact $\lambda$ has on the broader graph structure. For richer analysis of $\lambda$’s impact on whole-graph statistics, a variety of sampling based approaches for approximating values of interest (e.g., effective radius, connectivity, etc.) may be used [19, 21, 23, 2, 20, 22], particularly random walks biased toward newly-discovered but not yet visited nodes [10]. Alternative to first producing a histogram to then inform a human decision for $\lambda$, for some measures of interest, it is possible for a user to determine a target value (and tolerance range) prior to sampling; random walks with mechanisms to vary or sample $\lambda$ may then more quickly determine a proper parameter choice that results in a statistic within tolerance of the target value.

Additional material regarding this work can be found at [3] and [4]. We hope to eventually have the code on GitHub at https://github.com/DBay-ani.

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A Appendix

—Below is a portion of the writing that was under construction for the description of the more comprehensive experiments. We leave a subset of a content outline and a few notes on what existed, but do not give the full details for now. We may return at a later time to provide this information.—

A.1 Baseline Algorithms

Static Embeddings

SDNE ([32]):
Laplacian Eigenmaps (LAP) ([5]):
Graph Factorization ([1]):
HOPE ([30]):
node2vec ([16]):

The implementations of our static baselines, with the exception of node2vec, come courtesy of [14].
Dynamic Embeddings
TNE ([36]):
Dynamic Triad ([35]):

A.2 Evaluation Tasks

**Node Classification:** A standard task for node embeddings, we feed the embeddings produced by our method and the baselines into classifiers as part of a node-classification task. We choose relatively simple classifiers to ensure that the performance reflects qualities of the embeddings and their ability to make desired structure apparent, as opposed to agility of a sophisticated classifier to pick up on latent signals that might be in otherwise confused/noisy data. In order to perform due diligence in ensuring the performance results are not simply artifacts from the specifics of one classifier, we use a variety of standard classifiers: KNN, Gaussian Naive Bayes, Logistic Regression, and an SVM.

**Two-Dimensional Node Embedding Plots:**

**Latent Graph Reconstruction:** Given a graph $G_{\text{observed}} = (V, E_{\text{observed}})$ to produce embeddings on, we evaluate how the local rankings of distances between node embeddings resemble relations in a latent graph, $G_{\text{latent}} = (V, E_{\text{latent}})$, which influences the dynamics. Specifically, for a node $v$ in $V$ and $k$-dimensional embedding mapping $\phi$, we rank the pairwise distances between $\phi(v)$ and members of $\{y \in \mathbb{R}^k | \exists u \in V \setminus \{v\}. \phi(u) = y\}$, and produce ROCs / AUCs to determine how well this ranking reflects the neighbor relationship in $E_{\text{latent}}$. Note that this reconstruction scheme is locally linear, not globally - this more appropriately handles potentially nonlinear scaling between areas of the embedding space than considering a single global distance cutoff below which edges are considered to exist. To avoid computation costs quadratic in the size of the graph’s nodes, we randomly sample 1000 nodes from $V$ to do full pair-wise comparisons against the rest of the set’s members.

**Link Forecasting:** Given a splitting time, $t_{\text{split}}$, we generate embeddings with all information available up to and including $t_{\text{split}}$, and train classifiers to determine if nodes $v_1$ and $v_2$ will have links present at some time after $t_{\text{split}}$. Under the hypothesis that identifying persistent edges are easier, we make sure in evaluation to highlight both total prediction performance and performance restricted to edges not present at time $t_{\text{split}}$. While this task is similar to traditional link prediction tasks, we emphasize that the desired edges are not the result of missing or partial topological information in the training set, but from lack of knowledge of how the topology evolves; that is, at any time for which it is available, the topology is not considered to be broken or downsampled for the sake of evaluation, in contrast to

A.3 Datasets

**Simulated Data:** In order to produce synthetic data for initial testing, development, and reasonable demonstration, we selected a discrete dynamical
system that is simple to implement while being capable of sophisticated behavior and which is well known to computer scientists: Conway’s Game of Life [13].

HiggsTwitter: This data was collected from Twitter following the announcement of the Higgs boson’s discovery on July 4th, 2012. In total, the data spans seven days: three days prior to the day of the announcement (the 1st of July) and three days after (the 7th of July). We consider two graphs where nodes are accounts: One static graph consisting of friendship / social relations between accounts, and one dynamic multi-graph where an edge between two nodes represents a retweet, reply, or mention, with a label indicating the time of such activity. We will refer to the former network as the social graph and the later as the activity graph.

DBLP:

CSTraces Weather: http://skuld.cs.umass.edu/traces/sensors/weather.tar.gz

Digg:

blogCatalog:

https://snap.stanford.edu/data/higgs-twitter.html