Comments on CKM Elements

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Abstract

The sensitivity of determination of elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix to experimental inputs (particularly $|V_{cb}|$) is discussed and caution in assigning probable errors is urged.

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While experiments on magnitudes and phases of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements have become increasingly precise, significant uncertainties remain in relating hadron information to fundamental quark couplings. We stress the effect of some of these uncertainties, estimate the allowed parameter space, and conclude that it is consistent with recent measurements of the CP asymmetry in $B \to J/\psi K_S$ [1, 4].

We parametrize the CKM matrix [3] in terms of

$$
\lambda \equiv V_{us} \approx 0.22, \quad A \equiv |V_{cb}|/\lambda^2, \quad \rho \equiv \text{Re}(V_{ub})/(A\lambda^3), \quad \eta = -\text{Im}(V_{ub})/(A\lambda^3).
$$

The elements $V_{ud}, V_{us}, V_{cd},$ and $V_{cs}$ are well-determined [4, 5]. Our concern will lie primarily with $V_{cb}, V_{ub},$ and $V_{td}$. Unitarity predicts $V_{ts} \approx -V_{cb}$ and $V_{tb} \approx 1$.

Determination of $V_{cb}$ using inclusive semileptonic decays requires an understanding of kinematic effects associated with $m_b$ and $m_c$. The difference $m_b - m_c$ is constrained by spectroscopy to be close to 3.34 GeV [6], but a residual dependence on $m_b$ exists. A free-quark estimate yields $|V_{cb}| = 0.0384 - 0.0008[(m_b - 4.7 \text{ GeV})/(0.1 \text{ GeV})]$, for $\tau_b = 1.6 \text{ ps}$ and a branching ratio $B(B \to X_c \ell\nu) = 10.2\%$. Thus if $m_b$ is uncertain by 0.3 GeV (my guess), $|V_{cb}|$ is uncertain by $\pm 0.0024$.

Determination of $V_{cb}$ using exclusive semileptonic decays (see, e.g, [7]) introduces a strong correlation between its value and that of the form factor slope parameter $\rho^2$, so that values between 0.038 and 0.044 seem credible. This is the range we shall adopt. It is considerably broader than that which appears in several reviews and in two presentations [8] at this Conference.

The determination of $|V_{ub}|$ or $|V_{ub}/V_{cb}|$ relies upon semileptonic charmless $B$ decays either giving rise to leptons beyond the charm limit (and hence a tiny fraction of the spectrum), or a very precise modeling of the leptons due to semileptonic $b \to c\ell\nu$ decay. Values of $|V_{ub}|$ ranging from 0.003 to 0.0045 have appeared in recent analyses. In accord with [3] and [9], we therefore take $|V_{ub}/V_{cb}| = 0.09 \pm 0.025$ and hence $(\rho^2 + \eta^2)^{1/2} = 0.41 \pm 0.11$.

Loop diagrams describing CP-violating $K^0-\overline{K}^0$ mixing and the mixing of neutral nonstrange and strange $B$ mesons provide constraints on the elements $V_{td}$ and $V_{ts}$. We learn $\text{Im}(V_{td}^2), |V_{td}|^2$, and $|V_{ts}/V_{td}|$ from $\epsilon_K, \Delta m_d$, and $\Delta m_s/\Delta m_d$, respectively.

Using standard expressions for loop diagrams [10] and QCD corrections [11], the parameter $\epsilon_K$ leads to the constraint

$$
\eta \left[ 1 - \rho + B \left( \frac{m_c}{1.4 \text{ GeV}} \right)^2 \right] = \frac{0.8}{B_K}, \quad (1)
$$

1
where $B = (0.46, 0.39, 0.28)$ and $C = (0.51, 0.38, 0.28)$ for $|V_{cb}| = (0.038, 0.041, 0.044)$. A recent estimate of the vacuum-saturation factor $B_K$ [12] gives $0.87 \pm 0.13$. The value of $C$ changes by nearly a factor of 2 over the range we consider possible for $|V_{cb}|$. We plot the region of $(\rho, \eta)$ allowed by the simpler constraint $\eta(1 - \rho + 0.39) = 0.35 \pm 0.12$.

The value of $\Delta m_d$, whose present world average is $\Delta m_d = 0.487 \pm 0.014$ ps$^{-1}$ [13], is proportional to $f_B^2 B V_{td}|^2$, where $f_B$ is the $B$ meson decay constant and $B_B$ is the vacuum saturation factor. For $f_B \sqrt{B_B} = 230 \pm 40$ MeV [14] and $0.038 \leq |V_{cb}| \leq 0.044$, one then finds $0.66 \leq |1 - \rho - i\eta| \leq 1.08$.

The lower limit $\Delta m_s > 15$ ps$^{-1}$ may be interpreted in terms of a constraint on $|V_{ts}/V_{td}|$ if the effects of flavor-SU(3) breaking in matrix elements can be estimated. Lattice estimates of the parameter $\xi \equiv f_B \sqrt{B_B}/f_B \sqrt{B_B} = 1.1 \div 1.2$ are too restrictive in my opinion; a quark model obtains 1.25 for this ratio [3, 13], about 2$\sigma$ above the lattice range. With 1.25 as an upper limit one obtains the bound $|1 - \rho - i\eta| < 1.01$.

The constraints are plotted on the $(\rho, \eta)$ plane in Fig. 1. Also shown are the $\pm 1\sigma$ bounds on sin $2\beta \equiv \sin 2\phi_1$ from an average $0.49 \pm 0.23$ [3] of OPAL, ALEPH, CDF, BaBar, and BELLE values. There is no contradiction (yet)!

We comment further on decay constants. (1) The ratios $f_{B_s}/f_B$ and $f_{D_s}/f_D$ should be very similar [14, 16, 17]. The value of $f_{D_s}$ has been measured to be $251 \pm 30$ MeV [14]. The measurement of $f_D$ should be a first priority for a charm factory producing $\psi(3770) \to D^+ D^-$, in which one of the charged $D$'s is used for tagging and the other decays to $\mu\nu$. (2) The nonrelativistic quark model implies [18] $|f_M|^2 = 12|\Psi(0)|^2/M_M$ for the decay constant $f_M$ of a meson $M$ of mass $M_M$ composed of a quark-antiquark pair with relative wave function $\Psi(\vec{r})$. One estimates the ratios of $|\Psi(0)|^2$ in $D$ and $D_s$ systems from strong hyperfine splittings. Since $M(D^{*+}) - M(D^+) \approx M(D_s^{*+}) - M(D_s^+)$, one expects $|\Psi(0)|^2_{Q_d}/m_d \approx |\Psi(0)|^2_{Q_s}/m_s$ for mesons containing a heavy quark $Q$. In constituent-quark models [13] $m_d/m_s \approx 0.64$, so $f_{Q_d}/f_{Q_s} \approx \sqrt{0.64} = 0.8$. (3) The measurement of $F_B$ depends on determination of $B^+ \to \tau^+ \nu_\tau$ or $B^+ \to \mu^+ \nu_\mu$ to sufficient accuracy [20, 21]. One predicts

$$B \left\{ \begin{array}{l}
B(B^+ \to \mu^+ \nu_\mu) \\
B(B^+ \to \tau^+ \nu_\tau)
\end{array} \right\} = \left\{ \begin{array}{l}
2.5 \times 10^{-7} \\
5.7 \times 10^{-5}
\end{array} \right\} \left( \frac{f_B}{200 \text{ MeV}} \right)^2 \frac{V_{ub}}{0.003}^2.
\right.$$  \( (2) \)

The ratio $\Gamma(B^+ \to \ell^+ \nu_\ell)/\Delta m_d$ is proportional to $|V_{ub}/\sqrt{B_B} V_{td}|^2$; common
factors of $f_B$ cancel. Given $B_B$, this provides $r \equiv |V_{ub}/V_{td}|$. A 10% measurement of $r$ would require (e.g.) 25 $B^+ \to \mu^+\nu_\mu$ decays, or 100 million charged $B$’s (or $B\bar{B}$ pairs).

An estimate quoted in [20, 21] concludes that in the year 2003 the allowed region in $(\rho, \eta)$ space may involve errors of roughly $\pm 0.05$ on each parameter. In contrast to some of the more optimistic estimates [8], I do not think we are there yet.

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Figure 1: Region of \((\rho, \eta)\) specified by \(\pm 1 \sigma\) constraints on CKM parameters. Solid semicircles: \(|V_{ub}/V_{cb}|\). Dotted hyperbolae: \(\epsilon_K\). Dashed arcs: \(\Delta m_d\). Dash-dotted arc: 95\% c.l. lower limit on \(\Delta m_s\). Rays: \(\sin 2\beta \equiv \sin 2\phi_1\).