Investigation of Long Baseline Neutrino Oscillations from Experiments at KAON

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Abstract

The proposed KAON factory at the Triumf Laboratory in Vancouver, Canada provides a unique opportunity for high statistics long baseline neutrino oscillation experiments. Several possibilities are under active consideration. In this paper we describe the theoretical expectations for a very long baseline experiment in which the neutrinos are directed towards, and detected at the Superkamiokande detector in Kamioka, Japan 7200 km away. We find that in the first year this experiment would probe oscillations down to about $\Delta m^2 = 9 \times 10^{-5} eV^2$ for maximal mixing, and if $\Delta m^2 \geq 5 \times 10^{-4} eV^2$ it would be sensitive to $\sin^2(2\theta_0) \geq 0.2$. These results are compared with a more modest proposal of a 100 km baseline with a 6300 tonne detector which would probe $\Delta m^2 \geq 1.3 \times 10^{-3} eV^2$ for maximal mixing, and if $\Delta m^2 \geq 0.02 eV^2$ it would be sensitive to $\sin^2(2\theta_0) \geq 0.01$. These experiments would either confirm or rule out the entire range of parameters allowed by Kamiokande and IMB to explain the deficit in the ratio of $\nu_\mu$ to $\nu_e$ in current atmospheric neutrino experiments. The Kamiokande experiment would also investigate matter enhanced neutrino oscillations (the MSW effect) down to about $\Delta m^2 = 3 \times 10^{-4} eV^2$. 
I. INTRODUCTION

In the past decade neutrino physics has become a very active field partly as a result of the Solar Neutrino problem. Several new detectors including KamioKande, SAGE, and Gallex have confirmed earlier results from the Homestake experiment that there is a deficit in the number of solar neutrinos arriving on the earth. More recently KamioKande and IMB have reported deficits in the expected ratio of muon to electron type neutrinos produced in the decays of pions and kaons in cosmic ray showers which occur in the upper atmosphere. This latter effect has been referred to as the Atmospheric Neutrino problem.

The deficit in the number of solar neutrinos may either require modification of the standard solar model or it may be due to new physics beyond the standard model of particle physics. It is clear [1] that minor changes to the standard solar model cannot account for the large deficits which are seen. Furthermore, as has been pointed out by Bachall [2,3], there is direct experimental evidence that the neutrino spectrum is affected by the neutrino’s journey from the core of the sun. KamioKande measures the high energy end of the 8B neutrino spectrum, and so assuming that the neutrino spectrum in 8B beta decay in the center of the sun is the same as that measured in the laboratory, one can determine the number of 8B neutrinos that the Homestake experiment should measure with its lower energy threshold. This calculation disagrees with the number measured at Homestake by at least 2σ. It thus follows that the spectrum of 8B neutrinos detected on the earth differs from that in the core of the sun.

The Atmospheric Neutrino problem stems from a deficit in the ratio of muon type to electron type neutrinos produced in the upper atmosphere [4]. When cosmic rays enter the upper atmosphere, they produce hadronic showers containing many pions and kaons. Nearly all of the positively charged pions decay via

\[ \pi^+ \rightarrow \mu^+ + \nu_\mu \]

\[ \leftrightarrow e^+ + \nu_e + \bar{\nu}_\mu \]  

and the negatively charged pions decay via the charge conjugate to this process. The majority of charged kaons either decay as the pions above do, or they decay to pions which follow the above decay.

Notice that in the decay above, there are two \( \nu_\mu \)’s produced and only one \( \nu_e \) (where \( \nu \) refers to both neutrinos and antineutrinos). Naively we would expect that the ratio \( R_{\text{calc}} = \frac{\#\nu_\mu}{\#\nu_e} \) in the decay of many positively and negatively charged pions and kaons would be equal to 2 [4]. In more detailed treatments employing Monte Carlo programs to simulate the hadronic showers in the upper atmosphere and the subsequent decay processes (taking the effect of muon polarization into account), this ratio is calculated to be about 1.8 [4]. Measurements by both IMB and KamioKande find that the ratio is much lower, and that the ratio of the measured to calculated ratios is \( R_{\text{meas}}/R_{\text{calc}} = 0.65 \) with deviations of about 0.1 depending on the range of energy used [4]. However, there is still some question as to whether or not there is such a deficit [4].

There has been much activity in the past few years to explain both of these problems (i.e. the Solar Neutrino and the Atmospheric Neutrino problems) by the same minimal extension of the standard model of particle physics. If we assume that neutrinos are massive and that, like the quarks, their mass eigenstates differ from their weak eigenstates, then the
two sets of eigenstates are related by a unitary matrix similar to the CKM matrix in the quark sector. It is conventional to parameterize the oscillations between two neutrinos by two parameters $\Delta m^2_0$ and $\sin^2(2\theta_0)$ where $\Delta m^2_0 = m_{\nu_2}^2 - m_{\nu_1}^2$ is the difference in the squared masses of the neutrinos and $\theta_0$ is the mixing angle. Much work has been done in an attempt to account for the deficit of solar neutrinos by oscillations of the electron neutrino with either the muon or the tau neutrino as it travels through the sun. These oscillations can be enhanced by the presence of matter. The Atmospheric Neutrino problem can be explained by oscillations between the muon neutrino and another neutrino when it travels from the upper atmosphere to detectors several km below the surface of the earth. Experimental evidence puts quite strong constraints on what regions of the parameter space are able to explain these experimental measurements. However, regions of $\Delta m^2_0$ and $\sin^2(2\theta_0)$ which account for the Solar Neutrino problem do not overlap with those which account for the Atmospheric Neutrino problem. It is then proposed that matter enhanced $\nu_e - \nu_\mu$ oscillations account for the solar neutrino problem, while $\nu_\mu - \nu_\tau$ vacuum oscillations account for the deficit in the ratio $R_{\text{meas}}/R_{\text{calc}}$ in atmospheric neutrinos.

One of the main difficulties with extracting information about neutrino oscillations from the Solar and Atmospheric neutrinos is that the initial spectrum and composition of the neutrinos is not precisely known. A laboratory based long baseline experiment has the advantage of having a neutrino beam whose flux and spectrum can be measured at the source (by putting a small detector near the end of the muon decay tunnel). By comparing the spectrum and composition of the initial beam to that of the detected beam one can unambiguously determine whether the beam has changed while the neutrinos traveled from the source to the target. Also, in an accelerator based experiment we have some control over the spectrum and energy of the neutrino beam, and they can be tuned to investigate a particular energy of interest.

The purpose of this paper is to study the theoretical expectations for neutrino oscillations from a long baseline accelerator experiment at KAON, the proposed upgrade to TRIUMF. KAON will accelerate protons to about 30 GeV, and will have a flux on the order of 140 $\mu$A providing $8.74 \times 10^{14}$ protons on target per second. This high flux will allow very long baseline neutrino oscillation experiments to be done. Several such proposals are currently under active investigation by the Neutrino Group at Triumf. In this paper we begin by describing estimates for the neutrino fluxes and spectra which could be expected at KAON. We then compute numerically the rate at which neutrinos would be measured a large distance away for a variety of neutrino masses and mixing angles and including the effect of matter (the MSW effect). The long baseline experiments which we have studied would either confirm or rule out oscillations in the region $\Delta m^2_0 = 10^{-3} \text{ to } 10^{1} \text{eV}^2$ and $\sin^2(2\theta_0) > 0.4$. This is precisely the region which is studied by the Atmospheric Neutrino measurements. We also compute the minimum value of $\Delta m^2_0$ which can be explored with KAON for reasonable choice of the baseline and the size of the detector and we estimate the values of $\Delta m^2_0$ and $\sin^2(2\theta_0)$ which can be ruled out if no oscillations are found.

In section II, we briefly review the theoretical framework for the study of neutrino oscillations including both vacuum oscillations and oscillations in matter (the MSW effect). In Section III we describe the estimates of the neutrino fluxes expected for KAON and the methods used to obtain these estimates. We begin Section IV by describing an intermediate baseline experiment which has been proposed by the Neutrino Group at TRIUMF [8] using
KAON as a neutrino source and placing a detector 100 km away. This baseline, which is larger than the 20 km baseline proposed for the Brookhaven experiment is made possible by the very high neutrino fluxes at KAON. To allow comparison with the recent BNL-AGS proposal [10,11] we present results for expected event rates based on a 6300 tonne detector. We then proceed to our main result namely the calculation of the event rates for a very long baseline experiment in which the neutrino beam from KAON would be sent through the earth to the SuperKamiokande detector in Japan which is approximately 7200 km away. Matter effects are important in this region for some range of the parameters. Despite the low rates the extremely long baseline would allow much more stringent limits on $\Delta m^2$.

II. THEORETICAL BACKGROUND

If neutrinos have a nonzero mass their mass eigenstates will not, in general, coincide with the eigenstates that participate in weak interactions. These two sets of eigenstates will are then related by a unitary transformation. In the case of two neutrino flavors with mass eigenstates $|\nu_1\rangle$ and $|\nu_2\rangle$ and weak eigenstates $|\nu_\alpha\rangle$ and $|\nu_\beta\rangle$ the eigenstates are related by a single mixing angle:

$$
\begin{pmatrix}
\nu_\alpha \\
\nu_\beta
\end{pmatrix} = \begin{pmatrix}
cos(\theta_0) & sin(\theta_0) \\
-sin(\theta_0) & cos(\theta_0)
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
$$

(2)

The Schrödinger equation for propagation of a mass eigenstate of momentum $k$ is given by:

$$
\frac{id}{dt} \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} = \begin{pmatrix}
\sqrt{k^2 + m_1^2} & 0 \\
0 & \sqrt{k^2 + m_2^2}
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} \approx k \begin{pmatrix}
1 + \frac{m_1^2}{2k^2} & 0 \\
0 & 1 + \frac{m_2^2}{2k^2}
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
$$

(3)

where the last step is valid in the ultrarelativistic limit $k^2 \gg m_i^2$. It is customary to subtract $[k+(m_1^2+m_2^2)/4k]$ times the identity from the Hamiltonian (which has the effect of changing the overall phase of the wavefunction) to obtain

$$
\frac{id}{dt} \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix} = \frac{1}{4k} \begin{pmatrix}
-\Delta m_0^2 & 0 \\
0 & \Delta m_0^2
\end{pmatrix} \begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix}
$$

(4)

where $\Delta m_0^2 = m_2^2 - m_1^2$. Using the transformation (2) this can be expressed in terms of the weak eigenstates as follows:

$$
\frac{id}{dt} \begin{pmatrix}
\nu_\alpha \\
\nu_\beta
\end{pmatrix} = H \begin{pmatrix}
\nu_\alpha \\
\nu_\beta
\end{pmatrix} = \frac{\Delta m_0^2}{4k} \begin{pmatrix}
-cos(2\theta_0) & sin(2\theta_0) \\
-sin(2\theta_0) & cos(2\theta_0)
\end{pmatrix} \begin{pmatrix}
\nu_\alpha \\
\nu_\beta
\end{pmatrix}
$$

(5)

It was first noted by Wolfenstein [12] that when electron neutrinos pass through matter the $H_{ee}$ component of the Hamiltonian acquires an additional term. This term results from the fact that electron neutrinos can scatter off the electrons in matter via both the charged and neutral currents whereas muon and tau neutrinos have only neutral current interactions with those electrons. The term which is added to $H_{ee}$ is $+\sqrt{2}G_F N_e$, where $N_e$ is the electron density in the matter. In the case of electron and muon neutrinos, and after subtracting $\sqrt{2}G_F N_e/2$ times the identity matrix from $H$, equation (5) becomes:
\[
\frac{d}{dt}\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \frac{\Delta m^2_M}{4k} \begin{pmatrix} -\cos(2\theta_M) & \sin(2\theta_M) \\ \sin(2\theta_M) & \cos(2\theta_M) \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix}
\]

where the mass difference \(\Delta m^2_M\) and the mixing angle \(\theta_M\) in matter are given by:

\[
\Delta m^2_M = D_M \times \Delta m^2 
\]

\[
\sin(2\theta_M) = \frac{\sin(2\theta_0)}{D_M}
\]

and \(D_M\) is expressed in terms of the electron number density \(N_e\), the vacuum oscillation length \(L_0 = 4\pi E hc/\Delta m^2 c^4\) and neutrino interaction length \(L_e = \sqrt{2\pi hc/G_F N_e}\) as follows:

\[
D_M = \left[1 - 2 \left(\frac{L_0}{L_e}\right) \cos(2\theta_0) + \left(\frac{L_0}{L_e}\right)^2\right]^{1/2}
\]

The ratio \(L_0/L_e\) in units relevant for this problem is given by

\[
\frac{L_0}{L_e} = 1.52 \times 10^{-4} \left(\frac{E}{GeV}\right) \left(\frac{eV^2}{\Delta m^2}\right) \left(\frac{N_e}{2.1 mol/cm^3}\right)
\]

Thus the effective mass difference and mixing angle are affected by the presence of matter. This is the MSW effect. If a \(\nu_\mu\) is produced by some weak process, is allowed to propagate (as a linear combination of the mass eigenstates), and is then measured by some weak process a distance \(L\) away, then the probability that it will be measured as a \(\nu_\mu\) is

\[
P(\nu_\mu(0) \rightarrow \nu_\mu(L)) = 1 - \sin^2(2\theta_0) \sin^2\left(1.27 \frac{\Delta m^2 L}{E}\right)
\]

where \(\Delta m^2, L\) and \(E\) are measured in \(eV^2, km\) and \(GeV\) respectively.

Note that the conventional application of the MSW effect is to the Solar Neutrino problem in which electron neutrinos propagate through a medium of variable density possibly passing through a resonance at some point in their travels. The situation with long baseline terrestrial experiments is significantly different. In our case muon neutrinos travel through a region of nearly constant density. The MSW effect will be appreciable only for neutrinos whose energies lie close to the MSW resonant energy. This will occur only for a fraction of the neutrinos in the beam.

**III. NEUTRINO BEAM CALCULATIONS**

The neutrino group at TRIUMF \[9\] has developed Monte Carlo methods for estimating the neutrino fluxes at KAON using a Geant based program. Neutrino fluxes and energy spectra are generated by a Monte Carlo simulation of the magnetic horn focusing system used in E734 at Brookhaven \[13\]. The AGS beam at Brookhaven accelerates protons to 28.3 GeV, which is about the same energy as would be expected at KAON. In the simulation, the protons are incident upon a sapphire target rod where they produce secondary hadrons. The
positively charged secondaries, which consist mostly of pions and kaons, are focussed in the horns by fields of up to 5.3 T. They then enter a 200 m tunnel in which they decay to produce neutrinos. 3 km away the neutrinos pass through the front face of a hypothetical detector. Particle tracking, decay and energy loss are simulated by the Monte Carlo routine GEANT V3.15 [14], while hadronic production is handled by GHEISHA [15] which is interfaced with GEANT.

Once a neutrino is produced by the decay of a hadron or muon it is assumed that no interaction occurs before reaching the detector. Therefore it is only necessary to project the neutrino with its initial momentum to the plane containing the front face of the detector. If the projected neutrino coordinates place it inside an area of 8m by 8m at a distance of 3 km from the end of the last focusing horn, the neutrino is accepted. Momentum spectra for muon- and electron-neutrinos and their anti-particles were collected based on a sample of $1.2 \times 10^8$ protons incident upon the production target. The neutrino energy spectra were then fitted to the smooth curves shown in figure (1). In the calculations which follow, we use these fitted curves on the range from 1 to 6 GeV. At KAON, with a projected proton current on target of 140 $\mu$A, the muon-neutrino flux at 3 km would be $4.95 \times 10^5 \nu_\mu/\text{cm}^2/\text{s}$. Neutrino rates at larger distances are estimated by assuming a $1/L^2$ dependence of the neutrino flux. Monte Carlo flux calculations for detectors at 0.5, 1, 3, and 20 km indicate that the $1/L^2$ behavior is in fact closely followed for distances beyond 1 km.

IV. RESULTS

There have recently been several proposals for long baseline neutrino oscillation experiments using accelerator [10,11,16–19] and nuclear reactor [20] neutrinos. In a recent overview Parke [21] has considered several possible long baseline experiments. They fall into two main categories: experiments with neutrinos on the order of 10’s of GeV with baselines in the several 1000 km range, and experiments with neutrinos on the order of several GeV with baselines in the 10 to 100 km range. The higher energy neutrino beams allow longer baselines because the neutrino scattering cross section is proportional to its energy (which partly compensates for the $1/L^2$ reduction in neutrino flux). However, as can be seen from equation (11), an increase in the neutrino energy requires a proportionally longer baseline to probe the same $\Delta m^2$. Because of its high flux, KAON offers a unique opportunity to probe a very long (several thousand km) baseline with neutrino energies peaked at about 1.5 GeV. Table I compares the expected event rates (for ordinary neutrino scattering events) and the mass difference $\Delta m^2$ for which the baseline is exactly one half of an oscillation length at the peak energy of the accelerator’s neutrino beam. The table includes two possible configurations using the KAON factory. In one experiment, which has been proposed by the TRIUMF Neutrino Group, a neutrino beam from KAON is directed towards a large detector (we use 6300 tonne as a reference) which is 100 km from TRIUMF, and in the other the beam from KAON is directed towards SuperKamiokande which is about 7200 km away. A crude estimate of $\Delta m^2$ can be obtained by computing the distance to the first minimum in the oscillation in equation (11), so that

\[
(\Delta m^2)_{\lambda/2} = \frac{\pi}{2} \left( \frac{E_{\nu,\text{peak}}}{1.27L} \right)^2
\]
In practice, if the neutrino flux is large, it is possible to set more stringent limits on $\Delta m^2$ than those given above. Note that although KAON and the AGS at Brookhaven have similar proton energies, the peak energy of KAON is higher for a 200 meter decay tunnel resulting in more high energy neutrinos.

A. A 100 km Baseline

It is clear from the results of this section that matter effects are negligible for the region of parameter space which can be studied by a 100km baseline experiment. If neutrino oscillations are present part of the produced $\nu_\mu$ beam will be converted either into $\nu_e$ or into $\nu_\tau$. In this paper we shall consider only oscillations between two neutrino species so that in our calculations we assume that the $\nu_\mu$ converts either into $\nu_e$ or into $\nu_\tau$. (In the case of $\nu_\mu \leftrightarrow \nu_\tau$ oscillations there is never a matter effect.)

We now take the $\nu_\mu$ flux from the previous section and use it to compute the rate of charged lepton production from charged current exchange for various values of $\Delta m^2_0$ and $\sin^2(2\theta_0)$. The total cross sections for neutrino-nucleon interactions summarized in Quigg [22] are

$$\frac{\sigma_T(\nu_lN \to l^-X)}{E} = 6.75 \times 10^{-38} \frac{cm^2}{GeV}$$

$$\frac{\sigma_T(\bar{\nu}_lN \to l^+X)}{E} = 3.25 \times 10^{-38} \frac{cm^2}{GeV}$$

These inclusive cross sections may be used if we are not interested in resolving the neutrino energy but only in identifying the interacting neutrino. (The usual method for determining the neutrino energy is to reconstruct the events in the exclusive interaction $\nu_l p \to l^- n$. The cross-section for this reaction flattens out at an energy of about 1 Gev.)

Figures (2)(a) and (b) show contours of constant probability that a neutrino or antineutrino produced at KAON would be measured as a neutrino or antineutrino of the same flavor in a 6300 tonne detector at 100 km. (We chose a 6.3 kiloton detector to allow a more direct comparison with the BNL proposal.) Since there are no matter effects at this distance the conversion rate for neutrinos is the same as the conversion rate for antineutrinos and the contours in figures (2)(a) and (b) are the same. In one year we expect about 490,000 events, so we may crudely estimate the standard deviation in the number of events per year to be $\sigma = 100/\sqrt{490,000} \approx 0.14\%$. Then in one year we exclude $\Delta m^2_0 \geq 1.3 \times 10^{-3}$ in the limit of large mixing and $\sin^2(2\theta_0) \geq 0.01$ for $\Delta m^2_0 \geq 0.02 eV^2$ at the 3$\sigma$ level. Figure (2)(c) shows the rate of $\mu^-$ production in the 6300 metric tonne detector per year assuming a 65% detector efficiency. Figure (2)(d) shows the rate of electron production assuming $\nu_\mu \to \nu_e$ oscillations. Such oscillations would result in electron appearance above the 3800 per year coming from $\nu_e$ contamination in the beam if no oscillations were present. If the $\nu_e$ beam could be well understood, then this appearance channel would provide very concrete evidence of neutrino oscillations. These results would improve on the present limits on $\Delta m^2_0$ by more than an order of magnitude, and cover the entire region allowed by the Kamiokande and IMB atmospheric neutrino results.
Another important feature of the 100 km baseline is the high event rate (as high as 1300 events per day using the total cross-section, of which about 730 are $\nu_\mu n \rightarrow \mu^- p$). The neutrino energy spectrum can be measured by reconstructing this latter exclusive cross section, and the neutrino oscillation length is dependent on the neutrino energy, as was previously noted by Barger et al [23]. The exclusive cross section for this process is approximately constant with $\sigma (\nu_\mu n \rightarrow \mu^- p) = 9.0 \times 10^{-39} \text{ cm}^2$ for energies on the range 1 to 6 GeV [11]. Thus if there are neutrino oscillations, the dependence of the neutrino deficit on energy may be used as a more sensitive measure of the values of $\Delta m^2_0$ and $\sin^2(2\theta_0)$.

In Figure (3) we compare the expected rate of neutrino events as a function of the neutrino energy with and without oscillations. In (a), (c), and (e) we plot the probability from equation (11) as a function of energy with $\sin^2(2\theta_0) = 0.25$ and $\Delta m^2_0 = 0.004, 0.02,$ and $0.1 \text{ eV}^2$ respectively. Notice that there are minima in the event rates whenever $E_n = 1.27 \Delta m^2 L (2n+1) \pi$ with $n = 0, 1, 2, 3, \ldots$. These minima correspond to “coherent” oscillations. They occur at energies $E_0, \frac{2}{3} E_0, \frac{3}{5} E_0$ and so on, corresponding to $1/2, 3/2, and 5/2$ oscillation lengths. Note that only the first few minima in the spectrum will be seen because as $n$ increases the peaks at $E_n$ get closer together and the detector will not be able to resolve them. In the low energy limit the rate will be $1 - \frac{1}{2} \sin^2(2\theta_0)$.

In figures (3)(b), (d), and (f) we fold in the expected KAON energy spectrum and then compare the neutrino spectra which would be observed with and without oscillations for the above values of $\Delta m^2_0$ and $\sin^2(2\theta_0)$. The rates shown are number of events per year assuming 65% detector efficiency and 1/2 GeV energy bins.

In summary, as has been previously noted [4], a 100 km baseline experiment at KAON can improve the present limit on $\Delta m^2_0$ by more than an order of magnitude. This is sufficient to study all of the regime of interest to the Atmospheric Neutrino experiments, and $\Delta m^2_0$ well below the BNL-AGS proposal, but it is insufficient to say anything about the Solar Neutrino experiments or matter effects. The high event rates may allow a comparison of the energy spectrum of the oscillating neutrinos.

**B. A 7200 km Baseline: KAON to SuperKamiokande**

A very exciting possibility would be to direct the KAON beam towards the SuperKamiokande (SK) detector in Japan. The decay tunnel would have to be aimed 35 degrees below the horizontal and the total length of the baseline would be approximately 7200 km. KAON’s high flux and relatively low energy combined with SuperKamiokande’s large size would allow a probe of much lower values of $\Delta m^2_0$ than other accelerator-detector configurations. With no oscillations the rate for the KAON-SK experiment would be about 2 events per day assuming high efficiency in the 32,000 tonne fiducial mass of SuperKamiokande (50,000 tonne total mass).

As in the case of the 100 km baseline experiment the idea is to do a $\nu_\mu$ disappearance experiment. The number of $\nu_\mu$ events at Kamiokande must be compared to the (presumably measured) initial neutrino flux. The absence of a reduction in the number of $\nu_\mu$ would set limits on $\Delta m^2_0$ and $\sin^2(2\theta_0)$. If the oscillations are between $\nu_\mu$ and $\nu_\tau$ then matter effects (the MSW effect) are not present. Figures 4(a) and (b) show contours of constant probability for neutrinos and antineutrinos not oscillating (ie. being measured with the same flavor with which they were produced) assuming vacuum oscillations. Figures 4(c) and (d)
show the number of muons produced per year as a function of $\Delta m^2_0$ and $\sin^2(2\theta_0)$. With approximately 700 events per year, an experiment which did not see a disappearance of 10% of the muon neutrinos would rule out $\Delta m^2_0 \geq 9 \times 10^{-5} \text{eV}^2$ in the limit of large mixing, and $\sin^2(2\theta_0) \geq 0.2$ for any $\Delta m^2_0 \geq 4 \times 10^{-4} \text{eV}^2$.

If the dominant oscillation is between $\nu_\mu$ and $\nu_e$ then the calculation of the expected $\nu_\mu$ deficit must be modified due to the matter effects. With this long baseline, the neutrinos pass through a slowly changing density which varies with distance from the center of the earth. In figure (5) we show the electron and muon neutrino components of the KAON neutrino beam at SuperKamiokande calculated assuming a constant density of 2.1 $\text{mol/cm}^3$. These results were found to be almost identical those calculated by numerically integrating equation (6) with $\Delta m^2_M$ and $\sin^2(2\theta_M)$ varying according to the earth’s density profile. Figures 5(a) and (b) show the probability that a neutrino and antineutrino arrive at SuperKamiokande with the same flavor as they had when they were produced. Matter effects enhance the mixing of neutrinos while they suppress the mixing of antineutrinos.

Figures 5(c) and (d) show the number of muons and electrons which would be observed per year as a function of $\Delta m^2_0$ and $\sin^2(2\theta_0)$. If no muon neutrino disappearance was observed at a level of 10%, this experiment could place limits of $\Delta m^2_0 \geq 3 \times 10^{-4} \text{eV}^2$ for large mixing and $\sin^2(2\theta_0) \geq 0.03$ if $\Delta m^2_0 \approx 10^{-3} \text{eV}^2$. This latter mass difference has a resonance at the KAON peak energy of 1.5 GeV when the neutrinos are passing through matter with electron density equal to that of the earth. In addition to the muon disappearance, one could look for electron appearance above the expected rate of about 5 per year from $\nu_e$ contamination in the KAON beam. Although this experiment is unable to probe the range of parameters which are of interest in the Solar Neutrino problem, it would still be very interesting because it probes a much lower neutrino mass difference than any other experiment to date.

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REFERENCES

[1] J.N. Bahcall and R.K. Ulrich, Rev. Mod. Phys. 60, 297 (1988); S. Turck-Chieze, S. Cahen, M. Cass, and C. Doom, Astrophysical Journal 335, 415 (1988); R. Sienkiewicz, J.N. Bahcall, and B. Paczynski, Astrophysical Journal 349, 641 (1990); J.N. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 64, 885 (1992); The apparent discrepancy between the Turck-Chieze result and the others is discussed in Bahcall and Pinsonneault.

[2] J.N. Bahcall, in Frontiers of Neutrino Astrophysics, Proceedings of the International Symposium on Neutrino Astrophysics, Takayama/Kamioka, 19-22 October, 1992.

[3] J.N. Bahcall and H.A. Bethe, Phys. Rev. D 47, 1298 (1992).

[4] K.S. Hirata et al., Phys. Lett. B 205, 416 (1988); R. Becker-Szendy et al., Phys. Rev. D 46, 3720 (1992).

[5] T.K. Gaisser, Todor Stanev, and Giles Barr, Phys. Rev. D 38, 85 (1988).

[6] Giles Barr, T.K. Gaisser, and Todor Stanev, Phys. Rev. D 39, R3532 (1989); Haeshim Lee and Yoon S. Koh, Nuov. Cim. B 105, 883 (1990); M. Kawasaki and S. Mizuta, Phys. Rev. D 43, 2900 (1991); M. Honda et al., Phys. Lett. B 248, 193 (1990).

[7] K.S. Hirata et al., Phys. Lett. B 280, 146 (1992). E.W. Beier et al., Phys. Lett. B 283, 446 (1992);

[8] Ch. Berger et al., Phys. Lett. B 245, 305 (1990); R. Becker-Szendy et al., Phys. Rev. Lett. 69, 1010 (1992);

[9] J. Beveridge, P. Gumplinger, R. Helmer, J. Henderson, D. Frekers, P. Fuchs, R. Meyer-Drees, J. Pouissou, D. Wright, to be published.

[10] M. Albert, A.K. Mann, and M.J. Murtagh, “Proposal for a Narrowly Aimed Long Baseline Neutrino Oscillation Experiment at the AGS” BNL Preprint June, 1992.

[11] I. Chiang et al., “Proposal for a Long Baseline Neutrino Oscillation Experiment at the AGS”, BNL Preprint August 30, 1993.

[12] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978); L. Wolfenstein, Phys. Rev. D 20, 2634 (1979).

[13] L.A. Ahrens et al., Phys. Rev. D34, 75 (1986).

[14] R. Brun, F. Bruyant, M. Maire, A.C. McPherson and P. Zanarini, GEANT 3.15, CERN (1992).

[15] F. Carminati and H. Fesefeldt, GHEISHA Hadronic Interaction Package, CERN (1987).

[16] James Pantaleone, Phys. Lett. B 246, 245 (1990).

[17] Robert H. Bernstein and Stephen J. Parke, Phys. Rev. D 44, 2069 (1991).

[18] G. Fiorentini and B. Ricci, INFN Preprint FE-05-93.

[19] K. Nishikawa, INS-Rep.-924, April, 1992.

[20] R.I. Steinberg, Bulletin Board Preprint hep-ph 9306282.

[21] Stephen Parke, Bulletin Board Preprint hep-ph 9304271.

[22] Chris Quigg, in Gauge Theories of the Strong, Weak, and Electromagnetic Interactions, (The Benjamin/Cummings Publishing Company, Inc., 1983).

[23] V. Barger, K. Whisnant, and R.J.N. Phillips, Phys. Rev. D 22, 1636 (1980).

[24] John N. Bahcall, Neutrino Astrophysics, (Cambridge Univerity Press, 1989).

[25] K. Nakamura, in Proceedings of the 15th International Conference on Neutrino Physics and Astrophysics, Granada, Spain, 7-12 June, 1992, edited by A. Morales.

[26] P. Anselmann et al., Phys. Lett. B 285, 276 (1992)
[27] S. Turck-Chieze, in *Proceedings of the 15th International Conference on Neutrino Physics and Astrophysics*, Granada, Spain, 7-12 June, 1992, edited by A. Morales.

[28] The numbers given are based on the upgrade to KEK discussed in [24].

[29] The numbers given assume a 5,000 tonne fiducial mass for Soudan, as discussed in [21].
TABLE I. Parameters for various proposed long baseline neutrino oscillation experiments. The Charged Current (CC) events per day are given in the absence of neutrino oscillations. The parameter $(\Delta m^2)_{\lambda/2}$ shown is the value of $\Delta m^2$ for which a neutrino with energy equal to the peak energy of the accelerator would just reach its first minimum at the detector. Note that this $\Delta m^2$ is not necessarily the limit which the experiment can probe. Other factors such as the event rate will play an important role.

| Baseline       | $E_{\nu,\text{peak}}$ (GeV) | Baseline | Protons on Target per second | CC Events per day | $(\Delta m^2)_{\lambda/2}$ (eV$^2$) |
|----------------|-----------------------------|----------|-----------------------------|-------------------|-----------------------------------|
| BNL-AGS        | 1                           | 20       | $3.5 \times 10^{13}$        | 320               | $6.2 \times 10^{-2}$              |
| KEK-SuperK [28]| 1                           | 250      | $7.5 \times 10^{12}$        | 2.6               | $4.9 \times 10^{-3}$              |
| KAON 100 km    | 1.5                         | 100      | $8.74 \times 10^{14}$       | 1340              | $1.8 \times 10^{-2}$              |
| FNAL-Soudan [29]| 10                         | 710      | $2 \times 10^{13}$          | 9.5               | $1.0 \times 10^{-2}$              |
| CERN SPS-ICARUS| 6                           | 730      | $8.3 \times 10^{12}$        | 29                | $2.6 \times 10^{-4}$              |
| KAON-SuperK    | 1.5                         | 7200     | $8.74 \times 10^{14}$       | 2                 |                                   |
FIGURES

FIG. 1. Monte Carlo calculation of neutrino and antineutrino spectra at KAON using a 200 m decay tunnel. The spectra used in later calculations are the fitted curves. Note that the raw fluxes generated by the Monte Carlo were multiplied by 56.9 to convert them to $GeV^{-1}cm^{-2}s^{-1}$. Low statistics then account for the noisy data in the electron antineutrino spectrum.

FIG. 2. Contours of constant probability of nonconversion of neutrino and antineutrino flavors, and number of muons and electrons measured per year by a 6300 tonne detector 100 km away assuming 65% detector efficiency as a function of neutrino oscillation parameters. If no oscillations were present, then $N(\mu^-)$ would be $4.9 \times 10^5$ and $N(e^-)$ would be 1340.

FIG. 3. $P(\nu_\mu(0) \rightarrow \nu_\mu(L))$ and muon neutrino spectra at 100 km assuming $sin^2(2\theta_0) = 0.25$ and $\Delta m^2 = 0.004, 0.02, and 0.1eV^2$. If no oscillations were present the probabilities in (a), (c), and (e) would be equal to unity. The solid curves in equations (b), (d), and (f) are the spectra which would be measured if no oscillations were present while the dashed curves are the spectra if oscillations were present, and where the bin width is 1/2 GeV.

FIG. 4. Contours of constant probability of nonconversion of neutrino and antineutrino flavors and number of muons measured per year at SuperKamiokande as a function of neutrino oscillation parameters assuming vacuum oscillations. The rates assume high efficiency in the 32,000 tonne fiducial mass. If no oscillations were present, then $N(\mu^-)$ and $N(\mu^+)$ would be 700 and 11 respectively.

FIG. 5. Contours of constant probability of nonconversion of neutrino and antineutrino flavors and number of muons and electrons measured per year at SuperKamiokande as a function of neutrino oscillation parameters assuming matter enhanced $\nu_e - \nu_\mu$ oscillations. The rates assume high efficiency in the 32,000 tonne fiducial mass. If no oscillations were present, then $N(\mu^-)$ and $N(e^-)$ would be 700 and 5 respectively.