Quasi Non-linear Evolution of Stochastic Bias

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ABSTRACT

It is generally believed that the spatial distribution of galaxies does not trace that of the total mass. The understanding of the bias effect is therefore necessary to determine the cosmological parameters and the primordial density fluctuation spectrum from the galaxy survey. The deterministic description of bias may not be appropriate because of the various stochasticity of galaxy formation process. In nature, the biasing is epoch dependent and recent deep survey of the galaxy shows the large biasing at high redshift. Hence, we investigate quasi non-linear evolution of the stochastic bias by using the tree level perturbation method. Especially, the influence of the initial cross correlation on the evolution of the skewness and the bi-spectrum is examined in detail. We find that the non-linear bias can be generated dynamically. The small value of the initial cross correlation can bend the $\delta_g-\delta_m$ relation effectively and easily lead to the negative curvature ($b_2 < 0$). We also propose a method to predict the bias, cross correlation and skewness at high redshift. As an illustration, the possibility of the large biasing at high redshift is discussed. Provided the present bias parameter as $b = 1.5$ and $\Omega = 1.0$, we predict the large scale bias as $b = 4.63$ at $z = 3$ by fitting the bi-spectrum to the Lick catalog data. Our results will be important for the future deep sky survey.

Subject headings: cosmology:theory—large scale structure of universe

1. Introduction

The large scale structure in the universe has evolved from the primordial mass fluctuations according to the gravitational instability. In standard picture of structure formation, the density fluctuations are produced by the quantum fluctuation during the inflationary stage of the early universe. The CMB anisotropy observed by COBE reflects such fluctuations at the last scattering surface. Subsequently, the growth of the mass fluctuation leads to formation of galaxies and evolution of galaxy distribution which is observed by the galaxy survey such as APM, CfA survey, etc. The most important issue in modern cosmology is to construct a consistent history of our universe from inflation to galaxy clustering.
Statistical quantities of large scale structure provide us many important information to understand the evolution of our universe. From the power spectrum or the two-point correlation function of galaxies, we can evaluate the density parameter $\Omega_0$ by comparing the observations in the redshift space with those in the real space (Hamilton 1997). In general, the $n$-point galaxy correlation functions characterize the large scale structure of the universe. These quantities play a significant role to test the prediction of inflation theory. However, there exists an uncertainty in the relation between the distribution of galaxy and that of total mass in the universe. Galaxies have been formed at the high density regions of the mass fluctuations. Due to the lack of our knowledge about galaxy formation process, we do not have the definite answer how much fraction of the total mass is the luminous matter. This uncertainty is quantified by the bias parameter of the galaxy distribution, which significantly affects the determination of cosmological parameters and the tests of inflation theory.

To proceed further, it is necessary to model the galaxy distribution in the given mass distribution. The linear bias gives a simple relation between the galaxy and mass when both fluctuations are small enough. Denoting the fluctuation of total mass as $\delta_m$ and that of the galaxy mass as $\delta_g$, we have

$$\delta_g = b \, \delta_m,$$

where $b$ is the linear bias parameter. The assumption (1) yields a degeneracy of the parameters, $\beta(\Omega_0) = \Omega_0^{0.64}/b$, for determining $\Omega_0$ (Hamilton 1997). We need another observation in order to lift up this degeneracy. Moreover, we should keep in mind that the relation (1) cannot be static. Indeed, the bias parameter changes in time by gravitational force even if the galaxy formation turned off. The recent observation of the high redshift galaxy survey suggests that the galaxies at $z \simeq 3$ are largely biased, whose bias parameter is evaluated as $b \simeq 6$, even if we do not require the bias at present (Peacock 1998). It is necessary to consider the bias effect by taking into account the time evolution.

The linear bias is appropriate as long as the linear theory is a good approximation to the observation. A naive extension of biasing to the non-linear regime is the deterministic non-linear bias. Suppose that the galaxy distribution $\delta_g$ is a local function of $\delta_m$, we express $\delta_g$ in powers of $\delta_m$ (Fry & Gaztanaga 1993):

$$\delta_g = f(\delta_m) = \sum_n \frac{b_n}{n!} (\delta_m)^n.$$

The non-linear bias parameter $b_n$ appears when we compare the higher order statistics of the galaxy distribution with that of the mass fluctuation such as the skewness ($S_3$) and the kurtosis ($S_4$) defined by $S_3 \equiv \langle \delta^3 \rangle / \langle \delta^2 \rangle^2$ and $S_4 \equiv \langle \langle \delta^4 \rangle - 3 \langle \delta^2 \rangle^2 \rangle / \langle \delta^2 \rangle^2$, respectively. The relation becomes

$$S_{3,g} \equiv \frac{\langle \delta^3_g \rangle^2}{\langle \delta^2_g \rangle^2} = b^{-1} (S_{3,m} + 3 \frac{b_2}{b}) + \mathcal{O}(\langle \delta^2_m \rangle),$$

$$S_{4,g} \equiv \frac{\langle \delta^4_g \rangle - 3 \langle \delta^2_g \rangle^2}{\langle \delta^2_g \rangle^3} = b^{-2} (S_{4,m} + 12 \frac{b_2}{b} S_{3,m} + 12 \frac{b_3}{b^2} + 4 \frac{b_4}{b}) + \mathcal{O}(\langle \delta^2_m \rangle).$$
where we put $b_1 = b$. Subscript $g$ and $m$ denote the statistics of the galaxies and the mass fluctuation, respectively. Other than the above one-point functions, we can calculate the amplitude of the galaxy three point correlation function (bi-spectrum) $Q_g$ which also has the relation $Q_g = Q_m/b + b_2/b^2$ (Fry 1994). Since the statistical quantities for mass distribution can be obtained analytically under the assumption of gravitational instability, we can determine the bias parameters $b, b_2, b_3, \cdots$, by combining the observation of $S_{g,3}, S_{g,4}$ and with that of $Q_g$ (Frieman & Gaztañaga 1994, Gaztañaga & Frieman 1994). As for the time evolution of the bias parameter, Fry has discussed its influence on the skewness and the bi-spectrum (Fry 1996). He found that $S_{3,g}$ and $Q_g$ asymptotically approach the constant values of $S_{3,m}$ and $Q_m$, respectively.

There has been some confusions for the galactic bias effect since Kaiser provided a simple bias mechanism (Kaiser 1984). His original idea is to explain the enhancement of the two point correlation function of the rich clusters using the statistical feature of galaxy distribution. Assuming the rich clusters are formed where the fluctuation $\delta_g$ averaged over a larger scale exceeds a certain threshold, he derived a simple relation between the two-point correlation function of galaxies and that of rich clusters. It should be stressed that the bias parameter introduced by Kaiser is not defined by density fluctuations but correlation functions. In our case, the galaxy correlation function may relate with the mass correlation function, however, the deterministic relation between the variables $\delta_g$ and $\delta_m$ does not necessarily follow from the relation between correlation functions.

According to the inflationary scenario for the structure formation, the primordial density fluctuation is generated quantum mechanically and it is likely to have stochastic nature. (Linde 1990). Recently, the galaxy formation process is recognized as non-linear and stochastic one (Cen & Ostriker 1992). Therefore we should treat both $\delta_g$ and $\delta_m$ as independent stochastic variables. The stochastic treatment of the bias effect is referred to as the stochastic bias, which has been introduced by Dekel and Lahav (Dekel 1997, Lahav 1996). In stochastic bias, the relation between the galaxy and the total mass can be described by the correlation functions. When the fluctuations are small enough and the linear perturbation is valid, we need three parameters defined below:

$$
\sigma^2 = \langle \delta^2_m \rangle, \quad b^2 = \frac{\langle \delta_g^2 \rangle}{\langle \delta_m^2 \rangle}, \quad r = \frac{\langle \delta_m \delta_g \rangle}{\langle \delta_m^2 \rangle^{1/2}} \langle \delta_g^2 \rangle^{1/2}.
$$

(5)

The important ingredient of the stochastic bias is the cross correlation $r$ which is absent in the simple relation of linear bias (1). This is an extension of the Kaiser’s bias prescription. Matarrese et al. (Matarrese 1986) shows that the cross correlation between the regions with different thresholds of density peak naturally arises in general situation including both Gaussian and non-Gaussian spatial distribution. Mo & White discussed the spatial distribution of galactic haloes using the Press & Schechter formalism and evaluated the cross correlation by comparison with numerical simulation (Mo & White 1996). Recently, Catelan et al. extended this formalism (Catelan et al. 1998). We think that the cross correlation plays an important role to describe the galactic bias effect.
A naive introduction of cross correlation increases the unknown parameters and may complicate the determination of the cosmological parameters from the measurement of galaxy survey. Pen has discussed how to determine the parameters $b$, $r$, $\sigma$ and the other non-Gaussian variables. He concluded that the observation of redshift space distortion is useful to understand these parameters and the density parameter $\Omega_0$ (Pen 1997). However he did not consider the time evolution. On the other hand, Tegmark & Peebles studied the linear evolution of the stochastic bias after the galaxy formation epoch and on-going the galaxy formation (Tegmark & Peebles 1998). They found that the bias parameter $b$ and the cross correlation $r$ approaches unity even if galaxy formation never ends. However, their analysis is restricted to the linear evolution.

Purpose of the present paper is to understand the quasi non-linear time evolution of the stochastic bias by observing the three-point correlation functions (Fry 1994). The future development of galaxy survey such as Sloan Digital Sky Survey (SDSS) and two degree field survey (2dF) will provide us an enormous data of the large scale structure. In the future, the time evolution of galaxy distribution will be observed. Study of the bias evolution is of importance to separate the effect of the time evolution due to the galaxy formation process and the growth due to gravity. To investigate this, we must understand the qualitative and/or quantitative behavior of higher order statistics under the influence of gravity and recognize a role of the initial cross correlation which is important quantity in the stochastic bias.

In this paper, we shall pay an attention to the time evolution of skewness and bi-spectrum in the stochastic description of the bias effect. We discuss the basic formalism of stochastic bias and derive the evolution equations for galaxy and mass distribution in Sec.2. Based on this formulation, we first evaluate the bias parameter and the cross correlation at the tree level. In Sec.3, we develop the second order perturbation and study the time evolution of three-point function. The effect of the initial cross correlation is investigated by comparing with the various choices of the parameters. We find that the initial cross correlation significantly affects the non-linear relation between $\delta_m$ and $\delta_g$ which differs from the deterministic bias. Sec.4 is devoted to the discussion about the observation from the point of the stochastic biasing. Using the Lick catalog data and the present bias parameter inferred by the IRAS galaxy survey, we shall show that the high redshift galaxies are strongly biased, which will be observed by the future galaxy survey. We summarize the results briefly and present the future prospect in Sec.5.

2. Basic Formulation of Stochastic Bias

This section is devoted to a basic formulation of stochastic bias for further analysis of next section. We first consider the stochastic description of fluctuation for the galaxy distribution and the total mass distribution. In the case of weakly non-linear evolution, the generating functional for the stochastic variables of galaxy and total mass is constructed. Subsequently, we give the evolution equations and initial conditions. We will analyze these equations perturbatively and discuss the linear evolution of the stochastic bias.
2.1. Stochastic description

The basic quantities to treat the stochastic bias are the fluctuations of total mass and the galaxy distribution, $\delta_m$ and $\delta_g$. They are defined by

$$
\delta_m(x) \equiv \frac{\rho(x) - \bar{\rho}}{\bar{\rho}}, \quad \delta_g(x) \equiv \frac{n_g(x) - \bar{n}_g}{\bar{n}_g},
$$

where barred quantities means homogeneous averaged density. We shall investigate the skewness and the bi-spectrum of the galaxy distribution $\delta_g$. We evaluate them using the smoothed density field by the window function:

$$
\delta_{m,g}(R) \equiv \int d^3 x \cdot W_R(x) \delta_{m,g}(x).
$$

The window function $W_R(x)$ we adopt here is the spherical top-hat smoothing with the radius $R$ defined below:

$$
W_R(x) = \begin{cases} 1 & (|x| \leq R), \\ 0 & (|x| > R). \end{cases}
$$

As we are interested in the time evolution of bias effect under the influence of gravity, we ignore the galaxy formation process in consideration of the evolution of galaxy distribution. The stochastic bias is described by the stochastic variables $\delta_m$ and $\delta_g$. Their stochasticity is determined by the probability distribution functional $P[\delta_m(x), \delta_g(x)]$. Once $P[\delta_m(x), \delta_g(x)]$ is given, all the information of the distribution functional is encoded in the correlation functions of $\delta_m$ and $\delta_g$, and all of the correlation functions can be obtained from the following generating functional:

$$
Z[J_m, J_g] = \int \mathcal{D}\delta_m(x) \mathcal{D}\delta_g(x) \cdot P[\delta_m(x), \delta_g(x)] \\
\times \exp \left[ i \int d^3 x W_R(x) \{ J_m \delta_m(x) + J_g \delta_g(x) \} \right],
$$

where $J_m$, $J_g$ are external source terms. Using (9), the $n$-th moment of the filtered one-point function is written by

$$
\langle \{\delta_m(R)\}^j \cdot \{\delta_g(R)\}^{n-j} \rangle = i^{-n} \left. \frac{\delta^n Z}{\delta J_m^j \cdot \delta J_g^{n-j}} \right|_{J_m=J_g=0}.
$$

The probability functional $P[\delta_m(x), \delta_g(x)]$ can be determined in principle by the study of galaxy formation and evolution of fluctuations before galaxy formation. In our prescription of stochastic bias, we give $P[\delta_m(x), \delta_g(x)]$ as a parameterized function. A simple choice is Gaussian:

$$
P[\delta_m(x), \delta_g(x)] = \mathcal{N}^{-1} \exp \left[ - (\delta_m, \delta_g) \mathcal{G}^{-1} \left( \begin{array}{c} \delta_m \\ \delta_g \end{array} \right) \right],
$$

where $\mathcal{G}$ is the covariance matrix.
where $G$ is a $2 \times 2$ matrix and $N$ is a normalization constant. The non-vanishing off-diagonal component of $G$ represents the cross correlation between $\delta_m$ and $\delta_g$. We obtain

$$
\langle \delta_m(R)\delta_g(R) \rangle = \int d^3x d^3y \cdot W_R(x)W_R(y) \{G\}_{12}\delta_m(x)\delta_g(x).
$$

(12)

To take into account the non-linearity of gravitational evolution and the formation process of galaxies, the Gaussian statistics does not lead to a correct description. Pen has discussed the deviation from the Gaussian distribution by using Edgeworth expansion (Juszkiewicz et al. 1995), although his analysis is restricted to the static case (Pen 1997).

Since the density fluctuations are small on large scales, perturbative analysis is applicable to study the evolution of density fluctuation. In this case, the non-Gaussian stochastic distribution for $\delta_m$ and $\delta_g$ can be expressed in powers of the Gaussian variables $\Delta_m$ and $\Delta_g$ whose variances are small enough. We shall write the initial condition at the end of galaxy formation ($t = t_i$) as

$$
\delta_m = f(\Delta_m), \quad \delta_g = g(\Delta_g).
$$

(13)

If we know the function $f$ and $g$ and stochastic property for $\Delta_m$ and $\Delta_g$, the initial condition for $\delta_m$ and $\delta_g$ is determined. The time evolution may change the initial statistics. We can obtain the correlation functions after the galaxy formation epoch by solving the evolution equations for $\delta_m$ and $\delta_g$. Then the time-dependent generating functional becomes

$$
Z[J_m, J_g; t] = \int \int \mathcal{D}\Delta_m(x)\mathcal{D}\Delta_g(x) \cdot N^{-1} \exp \left[ -(\Delta_m, \Delta_g)\tilde{G}^{-1} \left( \begin{array}{c} \Delta_m \\ \Delta_g \end{array} \right) \right] \\
\times \exp \left[ i \int d^3x W_R(x) \left\{ J_m \delta_m(\Delta_m, \Delta_g, t) + J_g \delta_g(\Delta_m, \Delta_g, t) \right\} \right].
$$

(14)

The time dependence of $\delta_m$ and $\delta_g$ are obtained from the perturbative expansion:

$$
\delta_m(\Delta_m, \Delta_g, t) = \delta_m^{(1)} + \delta_m^{(2)} + \cdots, \quad \delta_g(\Delta_m, \Delta_g, t) = \delta_g^{(1)} + \delta_g^{(2)} + \cdots.
$$

(15)

Since the $n$-th order perturbed variables $\delta_m^{(n)}$ have the same order of magnitude as $(\Delta_m, \Delta_g)^n$, the initial condition given by (13) assigns the solutions (14) order by order. From (14) and (15), the lowest order non-vanishing $n$-th moment can be evaluated from the results up to the $(n-1)$-th order perturbation. It is composed of “tree” diagram with no loop, in graphical representation of the perturbation theory (Fry 1984). The tree-level third order moment for galaxy distribution becomes

$$
\langle [\delta_g(R)]^3 \rangle = i^{-3} \frac{\delta^3 Z}{\delta J_g^3 \mid J_g = J_m = 0} \simeq 3 \langle (\delta_g^{(1)}(R))^2 \delta_g^{(2)}(R) \rangle.
$$

(16)

From (14), in principle, we can obtain the joint distribution function of $\delta_m$ and $\delta_g$ by Laplace transformation.
2.2. Evolution equations and initial condition

The evolution of the galaxy and total mass distribution after galaxy formation is determined by gravity including the effect of cosmic expansion. The total mass density $\rho(x)$ is approximated by a non-relativistic pressureless fluid. The averaged homogeneous part $\bar{\rho}$ is obtained by solving the FRW equations:

\[
\left( \frac{\dot{a}}{a} \right)^2 \equiv H^2 = \frac{8\pi G}{3} \bar{\rho} + \frac{\Lambda}{a^2} ; \quad (K = 0, \pm 1), \tag{17}
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \bar{\rho} + \frac{\Lambda}{3}, \tag{18}
\]

where $a$ is the expansion factor, $K$ is the spatial curvature, and $\Lambda$ is a cosmological constant. Using these variables, the density parameter $\Omega$ is defined by

\[
\Omega \equiv \frac{8\pi G}{3} \frac{\bar{\rho}}{H^2}. \tag{19}
\]

Let us consider the fluctuating part of the mass distribution $\delta_m$. It obeys the equation of continuity and the peculiar velocity field $v$ is determined by the Euler equation in the presence of gravitational potential. On large scales, the assumption that the velocity field is irrotational would be valid. Then, the basic quantities for describing the dynamics are reduced to $\delta_m$ and $\theta \equiv \nabla \cdot v/(aH)$. The evolution equations become (Peebles 1980)

\[
\frac{\partial \delta_m}{\partial t} + H \theta + \frac{1}{a} \nabla \cdot (\delta_m v) = 0, \tag{20}
\]

\[
\frac{\partial \theta}{\partial t} + \left( 1 - \frac{\Omega}{2} + \frac{\Lambda}{3H^2} \right) H \theta + \frac{3}{2} H \Omega \delta_m + \frac{1}{a^2 H} \nabla \cdot (v \cdot \nabla) v = 0. \tag{21}
\]

As for the galaxy distribution, $\delta_g$ should satisfy equation of continuity as long as the galaxy formation is not efficient. After formation epoch, the galaxies move along the velocity field determined by the gravitational potential. Therefore we have (Fry 1996)

\[
\frac{\partial \delta_g}{\partial t} + H \theta + \frac{1}{a} \nabla \cdot (\delta_g v) = 0. \tag{22}
\]

Eqs. (20), (21), and (22) are our basic equations to determine the evolution of $\delta_m$ and $\delta_g$.

We next consider the initial condition given at the end of galaxy formation $t = t_i$. For the total mass fluctuation, we believe that $\delta_m$ is produced during the very early stage of the universe. In standard scenario of the inflationary universe, the density fluctuation is generated by the quantum fluctuation and may have the Gaussian statistics. We regard such fluctuations as $\Delta_m$. After the inflation, the gravitational instability incorporates the deviation from the Gaussian fluctuation. Since the galaxy formation does not affect the evolution of $\delta_m$ on large scales, the total mass fluctuation $\delta_m$ at the end of galaxy formation can be determined by the gravitational
instability. At that time, the growing mode is dominant. Therefore, we obtain the initial condition \( \delta_m = f(\Delta_m) \) from the evolution equations by dropping the decaying mode.

On the other hand, the fluctuation of galaxy number density is induced by the galaxy formation. To determine the function \( g(\Delta_g) \), we need to know halo formation processes. An analytic model for the spatial clustering of haloes is discussed by Mo & White and several authors (Mo & White 1996). Here, we rather treat \( g(\Delta_g) \) as a parameterized function, whose unknown parameters are determined by the observation of galaxy survey. Assuming \( g(\Delta_g) \) as a local function of \( \Delta_g \), we have

\[
g(\Delta_g) = \Delta_g + \frac{h}{6}(\Delta_g^2 - \langle \Delta_g^2 \rangle) + \cdots. \tag{23}
\]

The remaining task is to specify the stochastic property of \( \Delta_m \) and \( \Delta_g \). Since these are the Gaussian variables given by (14), their statistics can be characterized completely by the three parameters below:

\[
\sigma_0^2 = \langle \Delta_m^2 \rangle, \quad \tau_0^2 = \frac{\langle \Delta_g^2 \rangle}{\langle \Delta_m^2 \rangle}, \quad r_0 = \left( \frac{\langle \Delta_g \Delta_m \rangle}{\langle \Delta_m^2 \rangle \langle \Delta_g^2 \rangle} \right)^{1/2}, \tag{24}
\]

which is equivalent to determining the matrix \( \tilde{G} \). Notice that \(-1 \leq r_0 \leq 1\) can be deduced from the Schwarz inequality. We simply assume that the parameters \( b_0, r_0 \) are constant, which comes from the fact that there is no evidence of the scale-dependent bias on large scales (Mann et al. 1997). Therefore power spectra for the galaxy two-point correlation function and the galaxy-mass cross correlation are simply represented by the power spectrum \( P(k) \) of the total mass fluctuation given by

\[
\sigma_0^2(R) = \int \frac{d^3k}{(2\pi)^3} \tilde{W}_R(kR) P(k), \tag{25}
\]

where \( \tilde{W}_R(kR) \) is the Fourier transform of the top-hat window function,

\[
\tilde{W}_R(kR) = \frac{2}{(kR)^3} \left[ \sin (kR) - kR \cos (kR) \right]. \tag{26}
\]

### 2.3. Variance and covariance

We are in a position to study the evolution of stochastic bias. To begin with, it is necessary to study the variance and covariance. To calculate these quantity at the tree level, the linear perturbation is sufficient. In next section, we shall develop the second order perturbation and analyze the skewness and bi-spectrum.

Initial conditions and the evolution equations (20), (21), and (22) yield the linear order solutions:

\[
\delta_m^{(1)}(x,t) = \Delta_m(x) D_+(t), \tag{27}
\]

\[
\delta_g^{(1)}(x,t) = \Delta_m(x)(D_+(t) - 1) + \Delta_g(x), \tag{28}
\]
where the function $D_+(t)$ denotes the solution of growing mode by setting $D_+(t_i) = 1$, which satisfies

$$
\ddot{D}_+ + 2H\dot{D}_+ - \frac{3}{2}H^2\Omega D_+ = 0.
$$

(29)

In Einstein-de Sitter universe ($\Omega = 1$), we have $D_+(t) = a(t)/a(t_i)$.

Since the stochastic property is already given by (14) and (24), we can obtain the relation between $\delta_m$ and $\delta_g$ after galaxy formation. Similar to the expression (24), we define the time-dependent parameters as follows:

$$
\sigma^2(t) \equiv \langle \delta_m^2(t) \rangle, \quad b^2(t) \equiv \frac{\langle \delta_g^2(t) \rangle}{\langle \delta_m^2(t) \rangle}, \quad r(t) \equiv \frac{\langle \delta_g(t)\delta_m(t) \rangle}{\langle \delta_m^2(t) \rangle^{1/2}}
$$

(30)

Substituting (27) and (28) into (30), we get

$$
\sigma(t) = \sigma_0 D_+, \quad b(t) = \frac{\sqrt{(D_+ - 1)^2 + 2b_0r_0(D_+ - 1) + b_0^2}}{D_+}, \quad r(t) = b^{-1}(t) \left( \frac{D_+ - 1 + b_0r_0}{D_+} \right).
$$

(31)

These equations coincides with the result of Tegmark & Peebles in the absence of galaxy formation (Tegmark & Peebles 1998). The time dependent correlation functions (31) show that the bias parameter $b(t)$ and the cross correlation $r(t)$ approach unity asymptotically as the fluctuations grow. Note that the deterministic bias relation restricts on the parameter $r_0 = 1$, which leads to $r(t) = 1$. The value $r_0 = 0$ means that $\delta_m$ and $\delta_g$ have no correlation initially. Subsequent evolution of $\delta_g$ and $\delta_m$ develops the correlations and ultimately makes the complete correlation, i.e., $r = 1$ in the case of Einstein-de Sitter universe. This indicates that the cross correlation significantly affects the evolution of the higher order statistics. The functional form of the bias is non-linear, in general. This relation gives the linear bias relation (1) if we restrict us to the linear region, however, this is not sufficient obviously.

3. Skewness and Bi-spectrum

Following the formulation of the stochastic bias in previous section, we investigate the bias effect on the skewness and the bi-spectrum. First of all, the solutions of second order perturbation are given. Using this result and the analysis in Sec.2.3, we evaluate the time evolution of the skewness and bi-spectrum of galaxies and analyze the influence of cross correlation $r$ on them in Sec.3.1 and Sec.3.2. To evaluate the skewness with the top-hat smoothing, the integration including the window function $W_R$ is tedious and complicated. However, the calculation is tractable in the Fourier space because the useful formulae have already been found (Bernardeau 1994a, Bernardeau...
Here, utilizing these formulae, we develop the second order perturbation in the Fourier space. The variables $\delta_m, \delta_g$ and $\theta$ are expanded as follows:

$$\delta_{m,g}(x,t) = \int \frac{d^3k}{(2\pi)^3} \hat{\delta}_{m,g}(k,t)e^{-ikx}, \quad \theta(x,t) = \int \frac{d^3k}{(2\pi)^3} \hat{\theta}(k,t)e^{-ikx}. \quad (32)$$

Hereafter, we denote the Fourier coefficient by attaching the hat.

The calculation in the Fourier space is thoroughly investigated by Bernardeau, although he did not consider the evolution of $\delta_g$ (Bernardeau 1994b). We display the results only. The second order solutions satisfying the initial conditions are

$$\hat{\delta}^{(2)}_m(k,t) = \int \frac{d^3k'}{(2\pi)^3} \left[ D_+^2(t) \left( \frac{6}{7} P(k', k-k') + \frac{1}{7} P(k-k', k') - \frac{3}{2} L(k', k-k') \right) + \frac{3}{4} E_+(t) L(k-k', k-k') \right] \hat{\Delta}_m(k-k') \hat{\Delta}_m(k'), \quad (33)$$

$$\hat{\delta}^{(2)}_g(k,t) = \hat{\delta}^{(2)}_m(k,t) - \hat{\delta}^{(2)}_m(k,t_i) + (D_+(t) - 1) \int \frac{d^3k'}{(2\pi)^3} P(k', k-k') (\hat{\Delta}_m(k') \hat{\Delta}_g(k-k') - \hat{\Delta}_m(k') \hat{\Delta}_m(k-k')) + \frac{h}{6} \int \frac{d^3k'}{(2\pi)^3} \left( \hat{\Delta}_g(k') \hat{\Delta}_g(k-k') - \langle \hat{\Delta}^2_g \rangle \delta_D(k') \right), \quad (34)$$

where

$$P(k_1, k_2) = 1 + \frac{(k_1 \cdot k_2)}{|k_1|^2}, \quad L(k_1, k_2) = 1 - \frac{(k_1 \cdot k_2)^2}{|k_1|^2 |k_2|^2}. \quad (35)$$

The solutions (33) and (34) contain the function $E_+(t)$ which satisfies $E_+(t_i) = 1$. This is the inhomogeneous solution of the following equation:

$$\ddot{E}_+ + 2H\dot{E}_+ - \frac{3}{2} H^2 \Omega E_+ = 3H^2 \Omega D_+^2 + \frac{8}{3} \dot{D}_+^2. \quad (36)$$

In Einstein-de Sitter universe, we have

$$E_+(t) = \frac{34}{21} D_+^2(t). \quad (37)$$

It is known that the $\Omega$ and $\Lambda$ dependences of $E_+/D_+^2$ is extremely weak (Bernardeau 1994b). Therefore, we proceed to analyze the skewness and bi-spectrum by replacing $E_+(t)$ with $(34/21)D_+^2(t)$.

### 3.1. Skewness

The second order solutions (33) and (34) together with the linear order solutions give the tree-level third order moment by substituting them into (16). From (14), the result of the
yields $\gamma$

Hereafter we assume that the power spectrum simply obeys the single power-law $P(k) \propto k^n$, which yields $\gamma = n + 3$ from (39). In Fig.1, we have plotted the evolution of $S_{3,g}$ as a function of the expansion factor $a$. For a specific choice of the parameters, we take $\Omega_0 = 1$, $n = -3$, $h = 3.0$, for convience. We have verified that the other choice of the parameters leads to the similar behavior of the time evolution of the skewness. The initial conditions at $a = 1$ for each lines are $b_0 = 2.0, r_0 = 1.0 (\text{solid line})$, $b_0 = 2.0, r_0 = 0.5 (\text{long-dashed line})$, $b_0 = 1.0, r_0 = 0.0 (\text{short-dashed line})$, and $b_0 = 0.5, r_0 = 0.0 (\text{dotted line})$. The figure manifests the deviation from the deterministic bias $r = 1$ (solid and long-dashed lines). The skewness with the smaller $r_0$ tends to become the larger value. We see that the gravitational instability changes the cross correlation largely and this enhances the non-linear growth of skewness $S_{3,g}$. The smaller bias parameter $b_0$ also leads to the larger skewness (short-dashed and dotted lines). Especially, the skewness in anti-biasing case ($b_0 < 1$) has a characteristic initial behavior, since the rare distribution of galaxies rapidly concentrates on that of total mass due to the attractive force of gravity.

For more various initial data, the resulting parameters which evolve from the high redshift are exhibited in Table.1. Provided that the initial parameters $b_0$, $r_0$ and $h(= S_{3,g})$ at $z = 3$, we obtain

The skewness of the galaxy distribution is obtained as follows:

$$S_{3,g} = \frac{\langle \delta^3(R) \rangle}{\sigma_0^6}; \quad \frac{\langle \delta^2(R) \rangle}{\sigma_0^2} = (D_+ - 1)^2 + 2b_0r_0(D_+ - 1) + b_0^2,$$  \hspace{1cm} (41)
the bias parameter \( b \) and the skewness \( S_{\delta_g} \) at present, which qualitatively have the same behavior as depicted in Fig.1. Although we obtain the skewness \( S_{\delta_g} \) in the treatment of the stochastic bias, the non-linear bias parameter \( b_2 \) which gives the deterministic \( \delta_g - \delta_m \) relation

\[
\delta_g = f(\delta_m) = b\delta_m + \frac{b_2}{2}(\delta_m^2 - \langle \delta_m^2 \rangle) + \cdots
\]

(43)
can be evaluated by using the result of the deterministic bias (3). The \( \delta_g - \delta_m \) relation has to do with the peak of the joint probability distribution for \( \delta_m \) and \( \delta_g \). The non-vanishing value \( b_2 \) represents the deviation from a simple linear relation \( \delta_g \propto \delta_m \). Table.1 says that the smaller cross correlation and the smaller bias parameter bend the \( \delta_g - \delta_m \) relation efficiently. The remarkable property of the stochastic bias is that \( b_2 \) could become negative for the small initial cross correlation, although it approaches zero asymptotically. This tendency can be interpreted in the following way. Assume the existence of the primary linear bias \( b \), it will approach unity asymptotically. However, this relaxation may not proceed monotonously. The larger fluctuation accelerates the relaxation due to the non-linearity of the gravitational force. Moreover, the small initial cross correlation enhances this acceleration. Hence, \( \delta_g - \delta_m \) relation with the negative \( b_2 \) can be obtained. Fry and Tegmark & Peebles found that both \( b \) and \( r \) approach to unity asymptotically (Fry 1996, Tegmark & Peebles 1998). Here, we found the general tendency of the non-linear bias which is determined by the gravitational dynamics. The tilted relation with a negative \( b_2 \) is consistent with the recent numerical simulation (Dekel and Lahav 1998).

Table.1 also gives the cases for the different power spectrum from the scale invariant one \((n = -3)\) and the different density parameter from \( \Omega_0 = 1 \). From the evolution of skewness with the spectral index \( n = -1 \), we observe that the initial non-linear bias parameter \( b_2 \) rapidly decreases to zero compared with the case with index \( n = -3 \). Since the spectrum with \( n = -1 \) has larger power than \( n = -3 \) on large scales, the result shows that the tilted spectrum could enhance the non-linear relation of \( \delta_g - \delta_m \). As for the low density universe\((\Omega_0 = 0.2)\), the growth of the density fluctuations becomes slow and the variations of \( b \) and \( b_2 \) are small. Accordingly, the deviations from \( b = 1 \) and \( b_2 = 0 \) in low density universe are larger than that in Einstein-de Sitter universe.

### 3.2. Bi-spectrum

In addition to the skewness, the bi-spectrum provides us the other information of three-point correlation function. In Fourier space, it is defined by

\[
\langle \delta_g(k_1)\delta_g(k_2)\delta_g(k_3) \rangle = \delta_D(k_1 + k_2 + k_3)B_g(k_1, k_2, k_3).
\]

(44)
The amplitude \( B_g \) relates with the third moments (38) as follows:

\[
\langle \delta_g^3(R_0) \rangle = \int \frac{d^3k_1}{(2\pi)^3} \int \frac{d^3k_2}{(2\pi)^3} \tilde{W}(|k_1 + k_2|R_0)\tilde{W}(k_1R_0)\tilde{W}(k_2R_0)B_g(k_1, k_2, k_3).
\]

(45)
Recall that $k_1 + k_2 + k_3 = 0$, we obtain

$$B_g(k_1, k_2, k_3) = \left\{ \begin{array}{l}
\frac{4}{7} \mathcal{L}(k_1, k_2) (D_+^2 - 1) (D_+ - 1 + b_0 r_0)^2 \\
+ \{ \mathcal{P}(k_1, k_2) + \mathcal{P}(k_2, k_1) \} (D_+ - 1 + b_0 r_0) \{ (b_0 r_0 - 1) D_+ + b_0^2 + 1 - 2 b_0 r_0 \} \\
+ \frac{h}{3} (b_0 r_0 (D_+ - 1) + b_0^2)^2 \mathcal{P}(k_1) \mathcal{P}(k_2) \\
+ \text{permutations.} \end{array} \right. (46)$$

$P(k)$ is the power spectrum of total mass fluctuations given by (25). A more convenient observable is the reduced bi-spectrum amplitude $Q_g$ which is briefly discussed in Sec. 1 (Fry 1994, Fry 1996):

$$Q_g \equiv \frac{B_g(k_1, k_2, k_3)}{P_g(k_1) P_g(k_2) + P_g(k_2) P_g(k_3) + P_g(k_3) P_g(k_1)}. \quad (47)$$

The power spectrum of the galaxies $P_g(k)$ is written by

$$P_g(k) = \left\{ (D_+ - 1)^2 + 2 b_0 r_0 (D_+ - 1) + b_0^2 \right\}^2 P(k). \quad (48)$$

The reduced bi-spectrum amplitude $Q_m$ for the mass fluctuation has a remarkable property that it is scale independent and constant in time. In our case, $Q_g$ is also scale independent due to the assumption (24), although it depends on time and approaches to $Q_m$. To evaluate the configuration shape of the bi-spectrum, we plot $Q_g$ as a function of the angle $\theta = \cos^{-1}(\hat{k}_1 \cdot \hat{k}_2)$ by setting $k_1/k_2 = 2$. Fig.2 shows the various cases of the present amplitude $Q_g$ ($z = 0$) whose initial conditions are set at $z = 3$ in the case of Einstein-de Sitter universe ($\Omega_0 = 1$). We also take the initial skewness parameter $h = 3.0$ and the spectral index $n$ is chosen as $n = -3$. In Fig.2a, we plot the bi-spectrum for the various initial values of the cross correlation when the initial bias parameter is fixed ($b_0 = 2.0$). Each solid line shows the result of the initial conditions $r_0 = 0.0, 0.25, 0.5, 0.75, \text{and 1.0(from up to bottom)}$, respectively. We see that the cross correlation affects the shape of the bi-spectrum. For the smaller $r_0$, the shape of $Q_g$ becomes steeper and it approaches $Q_m$ faster. This is the same behavior as the non-linear growth of $S_{3,g}$.

In cases the initial cross correlation is fixed, the shape of the bi-spectrum changes significantly depending on the bias parameters (Fig.2b). In the next section, we will show that the small shape dependence of the cross correlation plays an important role for explaining both the present observation of galaxies and the galaxies at high redshift.

4. Discussion

In previous section, we studied the influence of the cross correlation on the time evolution of skewness and bi-spectrum. In this section, using these results, we describe how the stochastic treatment of the bias affects the analysis of observational data for the galaxy distribution. We will
present a method for predicting the bias, cross correlation and skewness at high redshift from a present observational data.

As mentioned in Sec.1, the recent observation indicates that the high redshift galaxies at $z \simeq 3$ is largely biased, $b \simeq 6.2$ (Peacock 1998, see also Peacock et al. 1998). On the other hand, the correlation functions of the present optical galaxies are one half time larger than that of the IRAS galaxies, which means that the bias parameter for the optical galaxies can be evaluated as $b(z = 0) = 1.5$ if we require no bias to the IRAS galaxies (Hamilton 1997, Strauss et al. 1992). This suggests that the time evolution plays an important role for explaining the difference of the bias parameter. The present bias parameter is also evaluated by the observation of the higher order statistics for various galaxy survey, the Lick catalog, for example. Using the prediction from the perturbation theory, Fry examined the deterministic non-linear bias and obtained $b = 3.5$ and $b_2 = 14.7$ to explain the flat shape of $Q_g$ for the Lick catalog (Fry 1994). This contradicts with the result inferred from IRAS survey.

In the stochastic description of the bias effect, taking into account the cross correlation between $\delta_m$ and $\delta_g$, we can fit the bi-spectrum to the Lick catalog so as to be consistent with the bias parameter inferred from IRAS galaxy survey. Using the best fit parameters of the stochastic bias, we will see the evidence of large biasing for high redshift galaxies and evaluate the initial skewness at $z = 3$. Let us denote $b_0$ and $r_0$ as the initial bias parameter and the initial cross correlation set at $z = 3$, respectively. Providing the present bias parameter as $b = 1.5$, the initial bias parameter $b_0$ is written in terms of the cross correlation $r_0$:

$$b_0 = 3 \left( \sqrt{r_0^2 + 3} - r_0 \right). \quad (49)$$

Substituting the above equation into (31), $b(t)$ becomes a function of $r_0$. Thus we can know the initial bias parameter from the final condition $b(z = 0) = 1.5$ by varying the $r_0$. Fig.3 represents the time reversal evolution of $b$ for the initial cross correlation $0 \leq r_0 \leq 1$. In this inverse problem, we have the large bias parameter $b_0 = 5.20$ when $r_0 = 0.0$.

The unknown parameter $r_0$ can be determined from the present bi-spectrum of the Lick catalog. The initial amplitude and the configuration shape of the bi-spectrum at $z = 3$ change due to the evolution. Using the fact that the spectral index of the Lick catalog becomes $n = -1.41$ on large scales (Fry 1994), the present bi-spectrum $Q_g(z = 0)$ can be given as a function of $r_0$ and $h$. In Sec.3.2, we observed that the bias parameter $b_0$ affects the configuration shape of $Q_g$ (for the larger value $b_0$, the shape becomes flatter). In the present case, $b_0$ is associated with $r_0$ by (49). On the other hand, the initial skewness parameter $h$ can change the amplitude of the bi-spectrum. Therefore, adjusting $h$ and $r_0$, we can obtain the bi-spectrum $Q_g$ consistent with the Lick catalog.

In Fig.4, assuming the Einstein-de Sitter universe, the bi-spectrum $Q_g$ at $z = 0$ is depicted by varying the initial cross correlation $0 \leq r_0 \leq 1$ (solid lines). We use the spectral index $n = -1.41$ same as the Lick catalog. The dashed line corresponds to the bi-spectrum of the Lick catalog with the best fitting parameters $b = 3.5$, $b_2 = 14.7$ obtained by Fry (Fry 1994). The figure shows that
the configuration shape of the Lick catalog coincides with the bi-spectrum given by the initial cross correlation \( r_0 = 0.2 \). Therefore, the bias parameter at high redshift \( z = 3 \) is evaluated \( b_0 = 4.63 \) from \( [13] \). In this figure, the initial skewness is chosen as \( h = 6.96 \). This result indicates that the large-scale galaxy distribution significantly deviates from Gaussian distribution and has a large bias parameter. This is consistent with the recent observation of Lyman-limit galaxies, which has \( b \approx 6.2 \) on small scales \( \sim 7.5 \text{Mpc} \) (Peacock 1998). Note that the present skewness evaluated by the best fit parameters is \( S_{3,g} = 4.55 \) from \( [38] \) and \( [41] \), which also agrees with the observation of the Lick catalog (Gaztañaga 1992).

Another explanation of the flat shape of the bi-spectrum in the Lick catalog is that the non-linear evolution eliminates the shape dependence (Scoccimarro et al. 1998). If this is true, our tree-level analysis breaks and the next-to-leading order perturbations (loop correction) are needed (Scoccimarro 1997). However, the observation on large scales guarantees the quasi non-linear evolution. Thus the most appropriate answer would be the bias effect. We should remark that the analysis discussed here can also apply to an arbitrary high redshift galaxies as well as the galaxies at \( z = 3 \). We can predict the bias parameter, cross correlation and skewness at high redshift if we obtain the bias parameter, the cross correlation and the skewness at present consistent with the observation of the bi-spectrum. Although this prediction does not take into account the formation process of galaxies or morphology of the galaxy type, one can say the presence of cross correlation is necessary to lead to the consistent result with the observational data. Therefore, the stochastic bias is essential to describe the galaxy distribution.

5. Conclusion

In this paper, we have developed the formalism of stochastic bias and investigated the quasi non-linear evolution of the galaxy biasing under the influence of gravity. After deriving the evolution equations, we analyzed the influence of the cross correlation between \( \delta_m \) and \( \delta_g \) on the evolution of skewness and bi-spectrum of galaxy distribution. We observed that the stochastic description differs from the deterministic biasing. The small value of the cross correlation gives rise to the rapid growth of skewness and steep shape of the bi-spectrum. Our main results can be summarized as follows:

- We found the dynamical feature of the stochastic bias from the weakly non-linear analysis. The gravity enforces the \( \delta_g-\delta_m \) relation to become unity and the larger fluctuation accelerates this behavior. The small initial cross correlation enhances the acceleration and leads to the non-linear \( \delta_g-\delta_m \) relation with a negative curvature \( b_2 < 0 \). The result implies that the gravity significantly affects the stochastic property of the galaxy and the mass. Hence, the dynamical evolution of the stochastic bias should be taken into account when we interpret the observation of the large scale structure.

- A method for predicting the bias, cross correlation and skewness at high redshift was
presented. Using the present observation of the bi-spectrum and the bias parameter inferred from IRAS galaxy survey, the stochastic property of $\delta_m$ and $\delta_g$ at present can be determined. The parameters $b$, $r$ and $S_{3,g}$ at high redshift are obtainable by the time reversal evolution. We solved this inverse problem and predicted the large scale bias $b = 4.63$ at $z = 3$ by fitting the bi-spectrum to the Lick catalog. The method is easily applicable to an arbitrary high redshift survey and will provide a probe for investigating the clustering property of high redshift galaxies.

These results show that the cross correlation plays an important role and the consideration of the dynamical evolution of the stochastic bias is required to explain the observational data. Thus, it turns out that the stochastic bias and its dynamics are essential to explore the evolution of large scale structure.

Future sky surveys will provide us a lot of information of galaxy distribution. The time evolution of the statistical quantities of the large scale distribution will be observed. Since the stochasticity of galaxy and mass distribution cannot be removed, to use stochastic bias is inevitable for investigating the evolution of large scale structure. Therefore it is important to understand the stochastic property of the galaxy and mass distribution by taking into account the time evolution. The analytic study of the distribution function may give us the important information for the measurement of cosmological parameter and test of inflation theory. As for this direction, we have obtained the non-linear $\delta_m$-$\delta_g$ relation in $(\delta_m, \delta_g)$-plane although the evaluated relation is a kind of expectation value. The most relevant object for the observation is the peaks of the joint probability functional which gives the most probable galaxy-mass relation in the measurement of galaxy distribution. The primary goal is to clarify the dynamical evolution of the joint probability distribution function $P[\delta_m, \delta_g]$. Weakly non-linear analysis using the Edgeworth expansion would be useful to study the dynamics of the stochastic bias on large scales. Secondary the large scale motion of the galaxies should be explored. Since the velocity field of the galaxy distribution itself has stochasticity, it is important to examine the effect of stochastic bias on the POTENT analysis. Although we studied the relation between $\delta_m$ and $\delta_g$ in this paper, our formalism should be extended to include the velocity field. We will focus on these issues in the future work (Taruya, Koyama & Soda 1998).

The authors thank Masaaki Sakagami for helping us to do the numerical calculations and useful discussions. They are also grateful to Ofer Lahav and Paolo Catelan for useful comments. This work is partially supported by Monbusho Grant-in-Aid for Scientific Research 10740118.
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Table 1. The present parameters $b$, $b_2$ and $S_{3,g}$ for the various initial conditions given at $z = 3$.

| $\Omega_0$ | $n$ | $b_0$ | $r_0$ | $S_{3,g}$ | $(z = 3)$ | $b$    | $b_2$  | $S_{3,g}$ | $(z = 0)$ |
|------------|-----|-------|-------|-----------|-----------|--------|--------|-----------|-----------|
| 1.0        | -3  | 0.5   | 0.0   | 3.0       | 1.0       | 0.760  | -0.088 | 5.93      |           |
| 1.0        | -3  | 1.0   | 0.0   | 3.0       | 1.0       | 0.790  | -0.133 | 5.51      |           |
| 1.0        | -3  | 2.0   | 0.0   | 3.0       | 1.0       | 0.901  | -0.245 | 4.48      |           |
| 1.0        | -3  | 2.0   | 0.5   | 3.0       | 1.0       | 1.090  | -0.088 | 4.23      |           |
| 1.0        | -3  | 2.0   | 1.0   | 3.0       | 1.0       | 1.250  | 0.119  | 4.11      |           |
| 1.0        | -1  | 2.0   | 0.5   | 3.0       | 1.0       | 1.090  | 0.033  | 2.71      |           |
| 1.0        | -1  | 2.0   | 0.5   | 0.0       | 1.0       | 1.090  | -0.128 | 2.30      |           |
| 0.2        | -3  | 2.0   | 0.5   | 3.0       | 1.0       | 1.30   | -0.132 | 4.49      |           |
Figure Caption

Fig.1 The time evolutions of the skewness $S_{3,g}$ with the specific parameters $\Omega_0 = 1, n = -3, h = 3.0$ are plotted as a function of expansion factor $a$. The initial conditions for each lines are given at $a = 1$ as follows: $b_0 = 2.0, r_0 = 1.0$(solid line), $b_0 = 2.0, r_0 = 0.5$(long-dashed line), $b_0 = 1.0, r_0 = 0.0$(short-dashed line), $b_0 = 0.5, r_0 = 0.0$(dotted line). These lines asymptotically approach 34/7, which corresponds to the skewness of the total mass distribution.

Fig.2 The evolved results of the bi-spectrum amplitude at $z = 0$ with the spectral index $n = -3$. We plotted $Q_g$ as a function of $\theta = \cos^{-1}(\hat{k}_1 \cdot \hat{k}_2)$ setting $k_1/k_2 = 2$. Each line has the same initial skewness $h = 3.0$ which are given at $z = 3$ and the universe is assumed to be a Einstein-de Sitter universe($\Omega_0 = 1$): (a). Variation of $Q_g(\theta)$ when the initial bias parameter is fixed ($b_0 = 2.0$). The initial values of the cross correlation for each solid lines are $r_0 = 0.0, 0.25, 0.5, 0.75$ and 1.0(from top to bottom). We see that the smaller cross correlation makes the configuration shape of $Q_g$ steeper. (b). Shape and amplitude dependences of $Q_g(\theta)$ when the initial cross correlation is fixed ($r_0 = 0.5$). The results of the bi-spectrum have the initial bias parameters given by $b_0 = 0.0, 0.5, 1.0, 2.0$ and 3.0(from top to bottom). Compared with both figures, influence of the cross correlation is weaker than that of the bias parameter.

Fig.3 The time reversal evolution of the bias parameters $b$ when the present ($z = 0$) bias parameter is given by $b = 1.5$. The horizontal axis denotes the redshift parameter $z$. The results are shown for the cross correlation $r_0 = 0.0, 0.2, 0.4, 0.6, 0.8$ and 1.0(from top to bottom), which are the values set at $z = 3$. The figure shows that the bias parameter takes the values from 3 to 5.2 at $z = 3$.

Fig.4 The bi-spectrum $Q_g$ by setting the initial conditions at $z = 3$, all of which have the same bias parameter $b = 1.5$ at present (solid lines). We use the spectral index $n = -1.41$ same as the Lick catalog. Each lines denote the present value of bi-spectrum with the initial cross correlation $r_0 = 0.0, 0.2, 0.4, 0.6, 0.8$ and 1.0(from top to bottom), respectively. These lines have the same initial skewness $h = 6.96$. The dashed line corresponds to the bi-spectrum of the Lick catalog constructed by the time independent non-linear bias (Fry 1994). We can fit the bias parameter to the Lick catalog if the initial cross correlation is chosen as $r_0 = 0.2$, which leads to the initial bias parameter $b_0 = 4.63$. This indicates that the large-scale distribution of high redshift galaxies is largely biased and has large skewness.
Fig. 1
Fig. 2a

\[ Q_g \]

\[ n = -3 \]
Fig. 2b
Fig. 3
Fig. 4

\[ n = -1.41 \]