An Improved Ant Lion Optimization Algorithm and Its Application*

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Abstract—In order to solve the limitations of ant lion optimization algorithm such as local optimum slow convergence speed and so on this paper applies optimization strategy Logistic chaotic map and reverse learning to improve the selection of random ant lion update ant position and elite ant lion respectively. The improved ant lion optimization algorithm is compared and analyzed by selecting standard test functions, and the optimization accuracy and convergence speed of the algorithm are obviously improved. The improved ant lion optimization algorithm combined with a series of pipe routing engineering rules is applied to the pipe routing of aero-engine. The simulation of a pipe routing example is carried out by Siemens NX11.0 Grip and MATLAB, and the effectiveness of the proposed method is verified again.

Keywords—ant lion optimization algorithm, optimization strategy, Logistic chaotic map, reverse learning, pipe routing

I. INTRODUCTION

With the development of artificial intelligence, many complex optimization problems have appeared in the fields of communication, computer and automation. Swarm intelligence algorithm provides a new research method for solving complex optimization problems. In recent years, swarm intelligence optimization algorithm has been developed rapidly: particle swarm optimization algorithm (PSO) [2] is a random search algorithm based on swarm cooperation developed by Kennedy J et al. in 1995 to simulate the predation behavior of birds. As one of the classical algorithms, this algorithm has been studied by scholars all the time, and it also gives enlightenment to the development of various subsequent swarm intelligence algorithms; Ant lion optimization algorithm (ALO) [3] and whale optimization algorithm (WOA) [4] are new intelligent optimization algorithms proposed by Mirjalili S in 2015 and 2016 respectively, which are inspired by the daily predation behavior of ant lions and whales. Their advantages are simple and easy to implement, loose requirements on objective function conditions and less parameter control. Sparrow search algorithm (SSA) [5] is a new heuristic algorithm proposed by Jiankai Xue et al. according to the foraging behavior and anti-predator behavior of sparrows. The algorithm is novel and has strong optimization ability and convergence speed.

Swarm intelligence optimization algorithm has been applied to many fields to solve optimization problems, and ant lion optimization algorithm has been widely used. For example, ADPRP [6] studies the influence of distributed generation configuration on distribution system, and uses ant lion optimization algorithm to determine the optimal distributed generation, and obtains ideal results; Amritha K et al. [7] uses ant lion optimization algorithm to extract the best value of frequency and voltage PI gain, and simulates the control algorithm of wind power plant. The algorithm has the advantages of suppressing current harmonics and regulating voltage, and achieves the purpose of balancing load and compensating neutral current; Yao Y et al. [8] improved the ant lion algorithm by introducing the strategies of redistributing ants, dynamically reducing the number of ant lions, randomly walking boundary contraction factor strategy and limiting the range of ants, and applied the algorithm to wireless sensor networks, which significantly enhanced the coverage of wireless sensor networks. However, this kind of algorithm is random, so it is difficult to find the global optimal solution. In addition, swarm intelligence optimization algorithm also has certain application in pipe routing: Daniel Baeza et al. [9] compares ant colony algorithm with Dijkstra, and obtains a better pipe layout scheme; Yu Jiapeng et al. [10] introduced adaptive variable step size to longicorn beard algorithm for aero-engine pipe routing. In order to save routing space, different modeling methods were adopted for pipes and accessories, which effectively improved pipe routing efficiency.

In order to solve the limitations of local optimum and slow convergence speed in ant lion algorithm, this paper makes some improvements in many aspects, and verifies the effectiveness of the improved effect by using classical test functions. Then, the improved ant lion optimization algorithm combined with engineering rules and constraints is applied to aero-engine pipe routing, and the routing example is simulated by MATLAB and CAD software Siemens NX11.0.

II. IMPROVED ANT LION OPTIMIZATION ALGORITHM

A. Principle of Ant lion Optimization Algorithm

The ant lion optimization algorithm is a hunting mechanism that simulates the natural ant lion preying on ants. The process is that the ant lion prepares traps in advance and waits for ants to fall in. After the ant lion catches ants, the ant lion will continue to dig traps and wait for the next ant to arrive, and cycle in turn. Because ants move randomly, random walk is chosen to simulate ant behavior. Random walk is defined as:

\[
x(t) = [0, cumsum(2r(t) - 1), cumsum(2r(t) - 1),\ldots, cumsum(2r(t) - 1)]
\]

Key Projects of Liaoning Natural Science Foundation Guidance Plan (20170540589)

2022 IEEE International Conference on Networking, Sensing and Control (ICNSC) | 978-1-6654-7243-2/22/$31.00 ©2022 IEEE | DOI: 10.1109/ICNSC55942.2022.10004110
cumsum represents the cumulative summation term, \( t \) represents the current number of iterations, \( n \) represents the maximum number of iterations, \( r(t) \) represents random function, the \( r(t) \) function expression is as follows:

\[
r(t) = \begin{cases} 
1 & \text{if } \text{rand} > 0.5 \\
0 & \text{if } \text{rand} \leq 0.5 
\end{cases}
\]  

(2)

\text{rand} \) is a random function of \([0,1]\).

Ants update their position by random walk in each step of random walk, and each search space has a boundary range. In order to make ants walk randomly within the boundary range, it is necessary to normalize the random function:

\[
x'_i = \frac{(x'_i - a_i) \times (d'_i - c'_i)}{(b'_i - a'_i)} + c'_i
\]  

Where \( a_i \) represents the minimum value of the \( i \) random walk, \( b_i \) represents the maximum value of the \( i \) random variable, \( d'_i \) represents the maximum value of the \( i \) random variable iteration, and \( c'_i \) represents the minimum value of the \( i \) random variable iteration. Ant random walk will be affected by ant lion trap. In order to achieve this goal, the following equation is established:

\[
c'_i = Antlion'_{j} + c'
\]  

(4)

\[
d'_i = Antlion'_{j} + d'
\]  

(5)

In the above formula, \( c' \) represents the minimum value of all variables in the \( t \) iteration, \( d' \) represents the maximum value of all variables in the \( t \) iteration, and \( Antlion'_{j} \) represents the position of the \( j \) ant in the \( t \) iteration.

According to the proposed mechanism, ant lions can build traps that are adaptive or proportional to them, and ants need to move randomly. But when an ant lion realizes that an ant has fallen into a trap, they will spray sand at the center, and the ant trying to escape will slide down from it. The mathematical model of this behavior is as follows:

\[
c' = \frac{c'}{T}
\]  

(6)

\[
d' = \frac{d'}{T}
\]  

(7)

Where \( T \) is the boundary contraction factor, which is defined as:

\[
\omega = \begin{cases} 
2 & t > 0.1T \\
3 & t > 0.5T \\
4 & t > 0.75T \\
5 & t > 0.9T \\
6 & t > 0.95T 
\end{cases}
\]  

(8)

Elite ant lion is an important feature of evolutionary algorithm. Elite ant lion is the most suitable ant lion, which should affect the movement of all ants in the iterative process. Suppose each ant randomly chooses an ant lion in roulette and chooses the elite at the same time.

\[ungt = \frac{R'_t + R'_e}{2}
\]  

(9)

\( R'_t \) represents the random walk of ant lion in roulette in iteration \( t \), \( R'_e \) represents the random walk around elite in iteration \( t \), and \( \text{Ant}'_t \) represents the position of \( i \) ant in iteration \( t \).

B. Improved Ant lion Optimization Algorithm

The basic ant lion optimization algorithm has achieved good results in solving general problems. Although the convergence speed is very fast in solving complex problems, it is easy to fall into local search and it is difficult to find the global optimal solution. In this section, the selection of ant lion, the update of ant position and the update of elite ant lion are improved by integrating optimization strategy, Logistic chaotic map and reverse learning respectively.

1) Selection of random ant lion by optimal selection strategy

Adaptive roulette has been used to select ant lions randomly in the ant lion optimization algorithm, but this method has a high probability that it will wander around the poor ant lions. Therefore, the roulette wheel in the initial algorithm can be used to sort and select ants and lions according to the adaptive size\([11]\).

\[
T < T \times p \times \text{rand}
\]  

(10)

\( T \) represents participating in roulette, otherwise not participating in roulette, \( p \) represents the average value of fitness, \( \text{rand} \) represents random number, and the value range is \((0,1)\). This can not only ensure that most ant lions are preserved, but also ensure that ants walk around better ant lions at random. Improving the algorithm can get a better solution and improve the running speed of the algorithm.

2) Updating Ant Position by Logistic Chaotic Map

In ant lion optimization algorithm, the randomness of ant position update plays an important role, which affects the distribution position and diversity of population, resulting in weak search ability of the algorithm and easy to fall into local optimum. Therefore, Logistic chaotic mapping \([12-13]\) is carried out on ants to improve the overall optimization efficiency and local optimization ability. The Logistic chaotic mapping formula is shown in equation \((11)\):

\[
x_{n+1} = \alpha x_n (1 - x_n)
\]  

(11)
In the above formula, \( x_{n+1} \) represents the current position of the ant, \( x_i \) represents the position of the ant, and \( n \) represents the number of iterations, \( \zeta \in (0,4) \). The ant formula (9) is updated by using the above Logistic chaotic map formula as follows:

\[
Ant'_{i} = \left( \frac{R_{i}^{1} + R_{i}^{2}}{2} \right) x_{n+1}
\]

(12)

3) Reverse learning updates elite ant lions
Reverse learning [14-15], as its name implies, is to find the reverse solution of the problem, evaluate the current solution and the reverse solution, and choose the better solution as the next generation solution. Reverse learning is beneficial to increase the diversity of population, select the current optimal solution from the selected better solutions, and improve the efficiency of the algorithm. The reverse number is expressed as follows:

\[ x^{*} = a + b - x \]

(13)

Where \( x^{*} \) stands for the reverse learning number, \( x \in [a, b] \).

Let \( P = (x_1, x_2, x_3, \ldots, x_d) \) be any point in the \( d \)-dimensional space, where \( x_i \in [a_i, b_i] \), \( i = 1,2,\ldots,d \), then the reverse point is \( P^{*} = (x_1^{*}, x_2^{*}, x_3^{*}, \ldots, x_d^{*}) \), where:

\[ x_i^{*} = a_i + b_i - x_i \]

(14)

In this selection, the elite individual in the overall fitness between ants and ant lions is given to elite ant lions, and elite ant lions \( E_{p_{i}} = (E_{p_{1}}, E_{p_{2}}, \ldots, E_{p_{T}}) \) and \( E_{p_{i}} \in [u_{b}, l_{b}] \) are defined. According to the minimum and maximum fitness corresponding to elite inverse solution \( E_{p_{i}}^{*} = (E_{p_{1}}^{*}, E_{p_{2}}^{*}, \ldots, E_{p_{T}}^{*}) \) is as follows:

\[ E_{p_{i}}^{*} = u_{b} + l_{b} - E_{p_{i}} \]

(15)

The optimal fitness is compared with elite ant lion and elite inverse solution, and the optimal fitness is selected as the current optimal value, which can increase the population diversity and accelerate the convergence speed of the algorithm.

C. Implementation Steps of Improved Ant Lion Optimization Algorithm
Combined with the above improved method, the specific implementation steps of the improved ant lion algorithm are as follows:

Step1: Set the initial parameters of the algorithm, such as population size, iteration times, dimension of solution, upper and lower boundaries, etc.;

Step2: Initialize the position of ants and ant lions;

Step3: Calculate the fitness value by using the optimization strategy of formula (10), sort it, and select the first generation elite ant lion;

Step4: Update the maximum and minimum values of random walk of all ants in the current iteration times;

Step5: Updating the ant position according to the Logistic chaotic map of formula (12), and calculating the fitness function value of the ant;

Step6: Reverse learning the elite ant lion by using formula (15), selecting the optimal fitness value as the current optimal solution, and updating the position and fitness value of the elite ant lion;

Step7: Judge whether the maximum iteration number is reached. If the maximum iteration number is reached, end the loop and output elite individuals. If the maximum iteration number is not reached, return Step3.

III. SIMULATION EXAMPLE AND ANALYSIS
In this section, the effectiveness of the improved ant lion algorithm is verified by classical test functions and pipe routing simulation examples. The running environment of the computer is: the processor is Intel (R) Core (TM) i5-7200U CPU @ 2.50 GHz, and the memory is 2.70 G. The algorithm programming environment is MATLAB 2018 b, and the CAD software simulation environment is Siemens NX 11.0.

A. Example of Test Function
In order to verify the superiority of the improved ant lion algorithm, particle swarm optimization (PSO), ant lion optimization (ALO) and improved ant lion optimization (IALO) are selected. Using common standard test functions with different characteristics for testing. These test functions have a dimension of 30, a total size of \( N = 30 \), iterations of \( T = 100 \), and an optimal value of 0. Specific standard test function information is shown in Table 1, and representative iterative convergence curves selected from different functions through multiple tests are shown in Figure 1.

| Function | Scope       |
|----------|-------------|
| \( F_1(x) = \sum_{i=1}^{n} x_i^2 \) | \([-100,100] \) |
| \( F_2(x) = \sum_{i=1}^{n} |x_i| + \prod_{i=1}^{n} |x_i| \) | \([-10,10] \) |
| \( F_3(x) = \sum_{i=1}^{n} \left( \sum_{j=1}^{i} x_j \right)^2 \) | \([-100,100] \) |
| \( F_4(x) = \max \left[ |x_i| ; 1 \leq x \leq n \right] \) | \([-100,100] \) |
| \( F_5(x) = \sum_{i=1}^{n} \left[ 100(x_{i+1} - x_i^2)^2 + (x_i - 1)^2 \right] \) | \([-5.5] \) |
| \( F_6(x) = \sum_{i=1}^{n} \left[ (x_i + 0.5)^2 \right] \) | \([-100,100] \) |
| \( F_7(x) = \sum_{i=1}^{n} x_i^4 + \text{random}[0,1] \) | \([-1] \) |
| \( F_8(x) = \sum_{i=1}^{n} x_i^2 - 10\cos(2\pi x_i) + 10 \) | \([-5.5] \) |
| \( F_9(x) = \frac{1}{4000} \sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n} \cos \left( \frac{x_i}{\sqrt{i}} \right) + 1 \) | \([-0.5,0.5] \) |
In order to test the optimization performance of the improved ant-lion optimization algorithm, the above test functions $F_1(x) \sim F_9(x)$ are used for experiments. In order to ensure fairness, the three algorithms are run 10 times under the same condition with dimension 10, and the statistical function is run 10 times under the same condition. The specific information of the optimal solution, average value and standard deviation is shown in Table II.

| Function | Algorithm | Best        | Ave         | Std         |
|----------|-----------|-------------|-------------|-------------|
| $F_1(x)$ | PSO       | 6.47E+01    | 2.05E+02    | 1.10E+02    |
|          | ALO       | 1.32E+00    | 5.47E+00    | 8.65E+00    |
|          | IALO      | 0.00E+00    | 1.56E-32    | 4.84E-32    |
| $F_2(x)$ | PSO       | 5.93E+00    | 8.95E+00    | 5.45E+00    |
|          | ALO       | 3.10E+00    | 0.00E+00    | 0.00E+00    |
|          | IALO      | 0.00E+00    | 0.00E+00    | 0.00E+00    |
| $F_3(x)$ | PSO       | 3.12E+02    | 7.90E+00    | 4.67E+02    |
|          | ALO       | 2.46E+02    | 0.00E+00    | 4.10E+02    |
|          | IALO      | 0.00E+00    | 0.00E+00    | 0.00E+00    |
| $F_4(x)$ | PSO       | 3.12E+00    | 6.05E+00    | 3.55E+00    |
|          | ALO       | 7.66E+00    | 3.14E-10    | 6.70E-10    |
|          | IALO      | 9.10E-19    | 3.14E-10    | 6.70E-10    |
| $F_5(x)$ | PSO       | 5.46E+00    | 9.99E+00    | 3.21E+00    |
|          | ALO       | 2.46E+02    | 0.00E+00    | 4.10E+02    |
|          | IALO      | 0.00E+00    | 0.00E+00    | 0.00E+00    |
| $F_6(x)$ | PSO       | 9.20E+01    | 1.08E+03    | 1.55E+03    |
|          | ALO       | 1.37E-03    | 5.37E-01    | 1.18E+00    |
|          | IALO      | 2.87E+02    | 2.36E+02    | 1.26E+02    |
| $F_7(x)$ | PSO       | 4.80E+00    | 2.13E+01    | 5.82E+00    |
|          | ALO       | 4.80E+00    | 2.13E+01    | 5.82E+00    |
|          | IALO      | 5.17E-05    | 1.07E-03    | 8.50E-04    |
| $F_8(x)$ | PSO       | 9.20E-01    | 8.82E-02    | 4.63E-02    |
|          | ALO       | 6.97E-00    | 2.79E+01    | 1.65E+01    |
|          | IALO      | 0.00E+00    | 0.00E+00    | 0.00E+00    |
| $F_9(x)$ | PSO       | 4.21E+01    | 5.98E+01    | 1.08E+01    |
|          | ALO       | 1.61E+00    | 3.21E+00    | 1.40E+00    |
|          | IALO      | 8.40E-01    | 4.83E-01    | 3.94E-01    |

It can be seen from Figure 1 and Table II that comparing the improved ant lion algorithm with particle swarm optimization algorithm and basic ant lion optimization algorithm, the improved ant lion optimization algorithm obviously improves the convergence speed and effect of the algorithm, and can quickly jump out of the local optimal to
search for the global optimal solution, which further verifies the feasibility of the improved algorithm.

B. An Example of Pipe Routing Based on Improved Ant Lion Optimization Algorithm

1) Spatial model and rule description

The routing space of aero-engine is narrow and there are many pipes laid. There are many engineering rules for routing pipes. In addition to meeting the shortest pipe trajectory, the engineering constraints such as machinability, aesthetics and routing sequence should be met in the process of pipe routing. There are a large number of obstacles attached to the surface of aero-engine casing, such as high temperature area, equipment maintenance area and obstacles formed by arranged pipes to subsequent pipes, etc. For irregular obstacles, convex hull expansion treatment can be carried out. Siemens NX11.0 is used for CAD modeling of aero-engine, and the simplified CAD model of aero-engine is shown in Figure 2.

2) Routing simulation

In this section, the improved ant lion optimization algorithm is used to verify the effectiveness of the above-mentioned method by taking the conventional pipe laid at both ends as an example. The improved ant lion optimization algorithm sets population size \( N = 100 \), iteration times \( T = 50 \), for the objective function to make the path shortest, set point coordinates \((x_i, y_i)\) and \( i \) as pipe path nodes, and \( m \) as the number of path nodes. The expression of the objective function is shown in formula (16). When dealing with obstacle avoidance, the line segment intersection method is used to judge whether it intersects with obstacles. This method improves the running efficiency of the algorithm, and the average running time of the algorithm is 32s. Due to the limitation of the article, the pseudo-code of aero-engine pipe routing in this article is not shown here. At the same time, the ant lion optimization algorithm has randomness. After running for 15 times, the representative routing results are selected, as shown in Figure 3.

\[
L = \sum_{i=1}^{m-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}
\]

(16)

It can be seen from the routing result diagram that satisfactory results can be obtained by using this method to solve the conventional two-point pipe. Under the constraint conditions of engineering rules, the total length of pipe is minimized, which basically meets the requirements of aero-engine pipe routing. At the same time, the routing has achieved satisfactory results, which conforms to the engineering rules and verifies the feasibility of the proposed method.

IV. CONCLUSION

In this paper, we introduce the optimization strategy of randomly selecting ant lions, apply Logistic chaotic map to update the position of ant lions and apply reverse learning to update elite ant lions to improve the ant lion optimization algorithm. The improved ant lion optimization algorithm obviously improves the convergence speed and accuracy of the algorithm. The improved ant lion optimization algorithm is combined with a series of engineering rules, and is applied to the routing of conventional two-point pipe of aero-engine, and satisfactory results are obtained. In the next step, we will continue to improve the running speed of the ant lion optimization algorithm, and apply the improved ant lion
optimization algorithm to more fields and more complex NP-hard problems.

ACKNOWLEDGMENT

This study was supported in part by Major Basic Research Key Projects of Liaoning Provincial Natural Science Foundation (No. 20170540589).

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