Modeling and analysis of the material removal rate for ultrasonic vibration–assisted polishing of optical glass BK7

Yingdong Liang¹ · Chao Zhang¹ · Xin Chen¹ · Tianqi Zhang¹ · Tianbiao Yu¹ · Ji Zhao¹

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Abstract
The emergence of ultrasonic vibration–assisted polishing technology has effectively improved the machining accuracy and efficiency of hard and brittle materials in the modern optical industry; however, the material removal mechanism of ultrasonic vibration–assisted polishing (UVAP) still needs to be further revealed. This paper focuses on the material removal mechanism of ultrasonic vibration–assisted polishing of optical glass (BK7), the application of ultrasonic to axial vibration, and the atomization of the polishing slurry; the material removal model was established. Based on the analysis of the relationship between the nominal distance d of the polishing pad and the actual contact area distribution, the prediction of the material removal profile is realized. In addition, the effects of different parameters on the material removal rate (MRR) were analyzed, including polishing force, spindle speed, abrasive particle size, ultrasonic amplitude, feed rate, and flow rate of polishing slurry. Based on the motion equation of abrasive particles, the trajectory of abrasive particles in the polishing slurry was simulated, and the simulation results show that the introduction of the ultrasonic vibration field changes the motion state and trajectory of embedded and free abrasive particles. The new model cannot only qualitatively analyze the influence of different process parameters on MRR, but also predict the material removal depth and MRR, providing a possibility for deterministic material removal and a theoretical basis for subsequent polishing of complex curved surfaces of optical glass.

Keywords Ultrasonic vibration–assisted polishing · Ultrasonic atomization · Material removal rate · Trajectory simulation · optical glass (BK7)

1 Introduction

Optical glass (BK7) is widely used in optoelectronics, diffractive optical elements, biomedicine, and other fields because of its excellent mechanical properties and chemical stability [1]. Therefore, the demand for this material is increasing, and the requirement for precision is higher and higher. However, due to its high hardness and brittleness, it is difficult to obtain high precision machined surface with conventional machining methods [2]. Scholars have studied processing methods including ultrasonic vibration–assisted polishing (UVAP), jet polishing [3, 4], laser polishing [5], and magnetorheological polishing [6]. Among these technologies, UVAP can greatly reduce the limitation of hard and brittle materials on machining, and accelerate the removal of subsurface damage, which has attracted more and more attention and research of scholars [7].

Polishing is an essential process for precision and ultra-precision machining, the quality of the workpiece is affected by the quality of polishing directly. Based on Preston equation [8], many experiments and simulations have been done on the mechanism of conventional polishing. Zhang et al. [8] modeled the mechanics of contact and friction mechanisms during polishing, then obtained the contact state, load distribution, and spatial polishing trajectory of the contact surface in the polishing process through the finite element simulation analysis of the small spherical optical lens. Lin et al. [9] obtained the pressure distribution in the polishing process by finite element, and proposed a model to describe the relationship between polishing parameters and MRR based on the simulation results. Zhang et al. [10, 11] studied the material removal of fixed abrasive polishing in detail. A material
removal model considering the wear law of polishing tools based on Hertz contact theory was proposed, which suggested that the material removal profile is related to the surface geometry and mechanical properties of the polishing tools and polishing conditions. Wang et al. [12] develop a prediction model of material removal depth of the workpiece surface, and present an approach to achieve the material removal profile, which considered the influence of abrasive size, established a linear removal intensity model according to material removal amount, and the material removal depth was calculated by integrating the contact path. Zhong et al. [13] conducted in-depth research on the chemical mechanical polishing (CMP) of optical silicon substrates, using two-body abrasion CMP to replace the traditional three-body abrasion. Comparative experiments show that the MRR is largely dependent on head load and table speed would give the same effect, which can greatly improve the MRR efficiency. Fan et al. [14] analyzed the influence of various process parameters (normal force, angular spindle velocity, angular feed rate, abrasive particle size, polishing slurry properties, polishing pad morphology parameters, and tool path curvature) on material removal, defined the removal depth per unit length of polishing path as the material removal index, and calculated the material removal profile model through the material removal index along the tool path.

Conventional polishing methods for hard and brittle materials with long processing cycles, low efficiency, and unsustainable surface quality have gradually failed to meet modern industrial requirements. In recent years, ultrasonic vibration has been introduced into polishing, which can improve the polishing quality and efficiency for hard and brittle materials. Ultrasonic machining was first proposed by Wood W.R et al. [15] in 1927, and was applied to actual machining in the 1920s. In recent decades, scholars have done a lot of research on ultrasonic vibration–assisted machining (UVAM). Agarwal S [16] analyzes the material removal mechanism and the material removal rate of UVAP of glass with a mild steel tool using boron carbide abrasive in water as the slurry. The propagation and intersection of radial and median cracks are induced due to repeated impacts of abrasive particles, which will lead to material removal. The models of shocking force and MRR are derived, which take into account the elastic properties and non-uniformity of abrasive particles, and the mechanical properties of workpiece and abrasive particle. Zhang et al. [17] considered the lateral extension of the contact area, the periodic changes of polishing force and contact radius, modeled the local surface profile and material removal distribution function of UVAP, and obtained that larger axial ultrasonic amplitude can improve the Preston coefficient through experiments. The model has good prediction results for local surface profile, maximum contact radius, material removal depth, and MRR. Han et al. [18] fixed the workpiece on a rectangle hexahedron ultrasonic sonotrode platform with optimized slots platform to realize the assisted ultrasonic vibration of workpiece, and carried out experimental research on austenitic stainless steel materials. It is concluded that the transverse ultrasonic vibration applied to the workpiece can reduce polishing force, improve machining accuracy, and enhance the proportion of plastic shear effect in material removal. When the vibration direction of the workpiece is consistent with the polishing path direction, the polishing force distribution is more uniform and the surface roughness is lower. When the included angle is 45°, the average polishing force can be reduced by 75.2%. Wang et al. [19] studied the mechanism of ultrasonic vibration–assisted grinding (UAG) of hard and brittle materials, established the mathematical model of UAG of brittle materials, conducted in-depth research on the influence of input variables on grinding force and prediction of surface roughness, and obtained the advantages of UAG through theoretical analysis and experimental verification.

In addition to the above studies of UVAP, ultrasonic has also been used for the ultrasonic atomization of polishing slurry. The introduction of ultrasonic atomization can make the abrasive particles homogeneous and decrease clustering phenomenon, effectively improve the trajectory of abrasive particles, which can improve machining efficiency. Ultrasonic atomization can produce droplets less than 100 nm, which have small temperature changes [20]. Zhang et al. [21] analyzed the effects of operating ultrasonic parameters such as input power and flow rate, and concluded that equipment parameters and liquid physicochemical properties have a great relationship with the mean particle size and particle size distribution of droplets. An effective ultrasonic atomization method was obtained through optimizing the parameters, and the ultrasonic atomization mechanism was established. Sekiguchi K [22] determined the relation between the ultrasonic frequency and the size distribution of the droplets, proposed the generation mechanism of ultrasonic atomization, and concluded that ultrasonic frequency has a negative correlation with atomized droplet diameter of atomized droplet, and is positively correlated with the number and concentration. It is considered that the atomized droplet diameter can be controlled by controlling ultrasonic frequency, power intensity, and density of ultrasonic energy transducer.

In conclusion, UVAP is proved to be an effective method for machining hard and brittle materials. However, the theory of UVAP of optical glass is still rarely studied, and the mechanism has not been fully explained so far. Therefore, this paper focuses on the mechanism of UVAP of BK7, a predicted model of the MRR based on ultrasonic atomization is proposed. The motion of abrasive particles of UVAP was discussed to understand the complex polishing process. The actual contact area between the polishing pad and the workpiece and its influence on the polishing accuracy are analyzed. At the same time, the influence of different process parameters on the MRR is analyzed, including not only the process
parameters, which refer to the polishing force, spindle speed and feed rate, but also the abrasive particle size, as well as ultrasonic amplitude and flow rate of the polishing slurry. This study can provide a theoretical basis for complex surfaces with UVAP.

2 Material removal rate model of the optical glass

2.1 The description of the ultrasonic vibration system

A schematic of the polishing principle is shown in Fig. 1. There are two ultrasonic vibration systems in this study: The first is an ultrasonic electric spindle system consisting of an ultrasonic generator I and an electric spindle, which can provide the ultrasonic amplitude of 0–10 μm and a maximum speed of 24,000 rpm, and the ultrasonic frequency of ultrasonic generator I is 25 kHz. The second is an ultrasonic atomization system consisting of ultrasonic generator II, ultrasonic nozzle and peristaltic pump, which can regulate the flow rate of polishing slurry (6–30 ml/min). Ultrasonic atomization can not only make the distribution of abrasive particles more homogeneous and reduce the clustering phenomenon, which can improve the surface quality, but also effectively avoid the oversupply of polishing slurry, reduce the amount of polishing slurry, and improve the economy.

2.2 Motion analysis of ultrasonic vibration

In this study, the cylindrical polishing tool is used to polishing the BK7 [23]. The polishing tool will leave a circular material removal area on the workpiece to be processed. In the polishing process, if the load is constant, the contact area is also constant [24]. Because the polishing head is directly driven by the ultrasonic electric spindle, the equation of motion of the polishing head is as follows:

\[ z = z_0 + A \sin(2\pi ft) \]
\[ \nu(t) = 2\pi fA\cos(2\pi ft) \]
\[ a(t) = -4\pi^2 f^2A \sin(2\pi ft) \]

where, \( z_0 \) is initial position of polishing head, \( f \) and \( A \) is ultrasonic frequency and ultrasonic amplitude, respectively.

2.3 Contact between the polishing pad and workpiece surface

It is known that the polishing pad surface is much rougher than the workpiece surface, so when the microscopic analysis is performed, the workpiece surface is often simplified to a flat surface and the polishing pad surface is considered as a rough surface, as shown in Fig 2. When the polishing pad touches the workpiece surface under a constant load, the distance between the reference surface and the workpiece surface is the nominal distance \( d \). The polishing pad microscopic surface is randomly distributed, so a Gaussian distribution is used to describe the polishing pad microscopic surface height distribution [25], as shown in Eq (2).

\[ \varphi(h) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left( -\frac{h^2}{2\sigma^2} \right) \]

where, \( h \) is the average distance between polishing pad and workpiece, \( \sigma \) is standard deviation of polishing pad surface height distribution, and \( \sigma = \sqrt{\frac{\varepsilon}{2}}R_a \).
Loose porous structure

Polishing tool

Fig. 2 Interactions between polishing tool, abrasive particle and workpiece

The actual contact area between the polishing pad and the workpiece is given by

\[
A = \int_{0}^{h} \eta A_{0} \pi R_{0} (h-d) \varphi (h) \, dh
\]  
where, \( \eta \) is the convexity density, \( A_{0} \) is the apparent area and \( R_{0} \) is the average convexity radius. The pressure between pad and workpiece is as follows:

\[
P = \int_{0}^{h} f \cdot N_{r} \varphi (h) \, dh = \frac{4}{3} N_{r} \sqrt{R_{0} E_{p}} \int_{d}^{h-d} \frac{3}{2} \varphi (h) \, dh
\]

where f is stress of single convex peak, \( N_{r} \) is the total number of abrasive particles involved in polishing. \( E_{p} \) is the contact modulus of the workpiece and the polishing pad given by 1/

\[
E_{p} = \left( 1-v_{p}^2 \right) / E_{p} + \left( 1-v_{p}^2 \right) / E_{w}.
\]

Thus, if the polishing pressure \( P \) is known, the nominal distance \( d \) can be calculated from Eq. (4).

2.4 Total number of abrasive particles involved in polishing

In the UVAP, not all the abrasive particles distributed in the polishing slurry participate in the polishing process, and the abrasive particles participating in the polishing are called effective abrasive particles. In this paper, polishing slurry is sprayed by ultrasonic peristaltic pump [26], a calculation method of effective abrasive particles based on ultrasonic atomization is proposed. If \( \varepsilon \) is the utilization rate of effective abrasive particles and \( N \) is the total number of abrasive particles ejected by the slurry atomizing nozzle, the effective abrasive number \( N_{e} \) can be expressed as follows:

\[
N_{e} = \varepsilon N
\]

It is known that the mass of a single abrasive is

\[
m = \frac{3}{4} \rho \pi r^{3},
\]

\( Q \) is the flow rate per unit cycle, \( \rho \) is the density of polishing liquid, \( C_{m} \) is the percentage concentration of polishing liquid, \( M \) is the total mass of abrasive particles, and \( V_{m} \) is the volume of the polishing fluid that flows out of the nozzle within a cycle. In summary, the total mass of abrasive particles can be expressed as follows:

\[
M = \rho_{s} V_{m} C_{m} = \rho_{s} Q T C_{m}
\]

So, the total number of abrasive particles is as follows:

\[
N_{s} = \frac{M}{m} = \frac{3 \rho_{s} Q T C_{m}}{4 \rho_{m} \pi r^{3}}
\]

Assuming that the ratio of micro convex peaks on the surface of polishing head is \( \phi \), the effective number of abrasive particles \( N_{e} \) participating in the polishing process is as follows:

\[
N_{e} = \phi \cdot N_{s} = \frac{3 \rho_{s} Q T C_{m} \phi \varepsilon (t)}{4 \rho_{m} \pi r^{3}}
\]

2.5 Relative sliding speed of abrasive particles

As shown in Fig. 3, for any point \( M (x, y) \) on the polishing path, the speed \( V_{r} \) generated by the rotation of the polishing tool can be expressed as follows:

\[
V_{r} = \omega_{p} \sqrt{x^{2} + y^{2}}
\]

where \( \omega_{p} \) is spindle speed. When the feed speed of polishing tool is \( V_{f} \), the relative sliding speed of abrasive particles is:

\[
V_{s} = \sqrt{V_{r}^{2} + V_{j}^{2} - 2V_{r}V_{j} \cos (\pi - \theta)}
\]

and, \( \cos \theta = \frac{x}{\sqrt{x^{2} + y^{2}}} \)

2.6 The material removal of a single abrasive particle

The ultrasonic polishing process can be abstracted as a large number of abrasive particles high-frequency impact on the brittle material surface. As shown in Fig 4, based on the indentation fracture model [9], when the abrasive particles impact the workpiece, radial cracks appear on the surface of the workpiece, and transverse cracks appear when the abrasive particles are separated from the workpiece. Assuming that only the broken volume of the abrasive particles is considered in the ultrasonic polishing period, the material removal is determined by the length of the transverse cracks and the depth of the longitudinal cracks [27]. In the ultrasonic vibration period, the material removal amount is approximately equal to the elliptic cone volume shown in Fig. 4, which is the sum of the crushing volumes from the point when the abrasive particles directly contact the workpiece to the point when the abrasive particles just leave the workpiece.

Suppose that the length of transverse crack and the depth of longitudinal crack produced by abrasive
particles at the maximum pressure depth are respectively \( [19, 28] \):

\[
C_L = \xi_1 (P_{\text{max}}/Kd)^{3/4} \\
C_H = \xi_2 (P_{\text{max}}/H_v)^{1/2}
\]

where \( \xi_1 \) and \( \xi_2 \) are proportionality coefficient \( (\xi_1 = 1.2, \xi_2 = 1.5) \), \( P_{\text{max}} \) is maximum normal load applied.

Then, the material removal volume of a single abrasive particle in a single vibration period \( T \) is \( V_{\text{sg}} \):

\[
V_{\text{sg}} = \frac{1}{3} \pi C_L C_H L = \frac{1}{3} \pi C_L C_H v_T T
\]

Taking into account the tiny effects of other factors, set a correction factor \( K \) \( (K=1.2) \), the material removal volume of single abrasive particle on the micro convex peak and in the pore is respectively,

\[
V_{\text{mik}} = \frac{\pi}{3} K \xi_1 \xi_2 \left( \frac{4 \pi \rho^2 f^2 k_0 \sin(2\pi f) + F_0}{3 K d} \right)^\frac{3}{2} \left( \frac{4 \pi \rho^2 f^2 k_0 \sin(2\pi f) + F_0}{H_v} \right)^\frac{1}{2}
\]

\[
V_{\text{mik}} = \frac{\pi}{3} K \xi_1 \xi_2 \left( \frac{P_{\text{max}}}{3 K d} \right)^\frac{3}{2} \left( \frac{P_{\text{max}}}{H_v} \right)^\frac{1}{2}
\]
2.7 The material removal rate of numerous abrasive particles

The total material removal volume in the UVAP is equal to the sum of the material removal volume of the abrasive particles in the pores and in the micro convex peak of the polishing tool. Among them, the removal volume of abrasive particles in pores and micro convex peaks in a single cycle are expressed as $V_p$ and $V_v$ respectively.

$$V_p = \int_0^T N_{st} \left(\frac{V_{mlk}}{2}\right) \frac{T}{C12/C12/C12/C12} \, dt$$ (15)

$$V_v = \int_0^T \left((N_{st} - N_{st}) \cdot \frac{V_{mlk}}{2}\right) \frac{T}{C12/C12/C12/C12} \, dt$$ (16)

Total material removal volume in a single vibration period $V$ is as follows:

$$V = V_v + V_p$$ (17)

So, the per unit time MRR in the polishing process can be expressed as follows:

$$MRR = \frac{V}{T}$$ (18)

The material removal profile prediction is shown in Fig. 5.

2.8 Experimental method to verify the prediction model

To verify the novel prediction model, a series of UVAP experiments were conducted. The experimental process and test results are shown in Fig. 6. The detailed experiments parameters are listed in Table 3. The experimental results are shown in Table 1. From Table 1, it can be clearly seen that the predictions of the novel model are in good agreement with the measured data overall.

3 Analysis and discussions

Theories analyses are carried out in this section for the MRR prediction model, the abrasive motion characteristics in the process of UVAP is analyzed, and the relationship between different polishing process parameters (spindle speed, feed rate, abrasive particle diameter, ultrasonic amplitude, and the flow rate of polishing slurry) and MRR are analyzed in detail respectively to provide a theoretical basis for subsequent research.

3.1 The simulation parameters conditions

The detailed parameter is explained in this section. The associated material properties are listed in Table 2 and Table 3. The detailed simulation parameters are listed in Table 4.

3.2 Motion analyses of the abrasive particles

From the above analysis, it can be seen the part of the abrasive particles in the polishing slurry embedded the polishing pad micro-raised part with the polishing pad movement. Another part of the abrasive particles exists in the polishing pad pores, forming the free abrasive particles. The two kinds of abrasive particles movement form are slightly different.

Fig. 5  Predict polished 3D-surface-topography
3.2.1 Motion analyses of the embedded abrasive particles

According to Fig. 3, the displacement equation of the contact embedded abrasive particle \( M(x, y) \) could be expressed as follows:

\[
X(t) = \left( \begin{array}{c}
(l \cos(\omega_q t) + v_j t) \\
l \sin(\omega_q t) \\
\text{Asin}(2\pi ft)
\end{array} \right)
\]

(19)

where, \( l \) is the distance from point \( M \) to point \( O \), \( \omega_q \) is spindle speed, \( A \) and \( f \) are ultrasonic amplitude and frequency respectively. The trajectory of abrasive particles embedded the polishing pad can be obtained from the above formula, as shown in the Fig. 7. As shown in Fig. 8, the instantaneous velocity of point \( M(x, y) \) is composed of the sub motions in \( X-, Y-, \) and \( Z- \)directions:

\[
V(t) = \left( \begin{array}{c}
-l \sin(\omega_q t) + v_j \\
(l \omega_q \cos(\omega_q t)) \\
2\pi f A \cos(2\pi ft)
\end{array} \right)
\]

(20)

From the above formula, the instantaneous velocity \( V \) at point \( M \) is as follows:

\[
V = \sqrt{v(t)^2 + V_s^2}
= \sqrt{(2\pi f A \cos(2\pi ft))^2 + (V_r^2 + V_j^2 - 2V_r V_j \cos(\pi - \theta))^2}
\]

(21)

As shown in Fig. 8, in the XOY plane, the magnitude and direction of \( V_r \) change with time due to the rotation of the spindle, and the axial movement changes periodically with the ultrasonic vibration, so the instantaneous speed and position also change periodically with time. When the angle between \( V_r \) and \( V_j \) is 0°, the speed \( V_r \) is horizontal and the value is largest, and when the angle is 180°, the sum speed is the smallest. During the polishing process, the feed speed \( V_j \) remains constant, \( V_r \) value and direction changes all the time, and \( V_r = \omega_q \times r \), the speed of the points at different distances from the \( O \) point also changes, as shown in Fig. 9, the M1, M2, M3 three points of the same plane and speed \( V_r \) is drawn, it can be seen that the same angle under the different positions of the speed curve trend is the same, the further away from the \( O \) point the greater the speed.

It can be seen from Eq. 20 that the factor that has the greatest influence on the law of abrasive particles motion is the spindle speed. The trajectories of abrasive particles at different spindle speeds are shown in Fig. 3. As can be seen from Fig. 8, the distribution of abrasive particles motion trajectory under different rotational speeds is very different. When the spindle speed is higher, the horizontal direction relative velocity is higher, the instantaneous velocity is higher, the frequency change of abrasive trajectory is faster, and the number of

### Table 1
Comparison of predicted and experimental on material remove depth

| No | Material remove depth (μm) | Relative error (%) |
|----|---------------------------|--------------------|
|    | Calculated | Measured |       |
| 1   | 3.2628      | 3.673   | 12.57 |
| 2   | 3.430       | 3.673   | 5.12  |
| 3   | 3.781       | 3.673   | 15.88 |
| 4   | 3.268       | 3.673   | 0.16  |
| 5   | 3.889       | 3.673   | 19.19 |
| Mean| 3.2628      | 3.6082  | 10.59 |

### Table 2
Material properties of BK7, diamond

| Material | Density (g/cm³) | Young modulus (GPa) | Poisson ratio | Vickers hardness (GPa) |
|----------|----------------|---------------------|--------------|-----------------------|
| BK7      | 2.51           | 82                  | 0.21         | 7.70                  |
| Diamond  | 3.52           | 1000                | 0.07         | 50                    |
scratches on the workpiece per unit time is increased, which helps to improve the polishing accuracy and efficiency.

3.2.2 Motion analyses of the free abrasive particles

The free abrasive particles are only affected by the axial ultrasonic vibration, so they do periodic simple harmonic vibration in the axial direction. The free abrasive particles motion trajectories under different amplitudes as shown in Fig. 10 and the free abrasive particle velocity curves at different frequencies are shown in Fig 11. From Fig. 10 and Fig. 11 that the greater the ultrasonic amplitude is, the greater the motion distance of abrasive particles is; the higher ultrasonic frequency is, the more times of scratching with the workpiece in unit time are, and the higher the MRR is.

3.3 The relationship between nominal distance and material removal rate

It can be seen from the above analysis that the actual contact area between the polishing pad and the workpiece is directly related to the nominal diameter. Only the micro convex peak of the polishing pad higher than the nominal diameter can contact the workpiece, and this parameter directly affects the polishing accuracy and efficiency.

3.3.1 The relationship between nominal distance and actual contact area

Under the same conditions, the larger the nominal distance is, the smaller the actual contact area is, and the values of nominal distance and actual contact area can be fitted into a curve by equation, as shown in Fig. 12. It can be seen from Fig. 12 that there is a negative correlation between the nominal distance and the actual contact area. The curve shows that the attenuation rate of the actual contact area and the nominal distance is getting lower and lower, and when the nominal distance is $6.25 \times 10^{-5}$ m, the polishing pad is disconnected from the workpiece.

3.3.2 The relationship between nominal distance and material removal rate

It can be seen from the pressure formula $P = F/S$, when $F$ is constant, the smaller the actual contact area, the greater the pressure between the contact surfaces, and as shown in Fig. 13, the greater the nominal distance $d$, the smaller the actual contact area, the greater the pressure, and the greater the MRR. Figure 13 plots the relationship between the nominal distance $d$ and the MRR, and it can be seen from the figure that the value of $d$ is negatively correlated with the MRR, and when $d$ is greater than $22 \mu$m, the MRR decreases rapidly and approaches zero.

3.4 The relationship between different process parameters and MRR

3.4.1 The relationship between spindle speed and material removal rate

The MRR at different spindle speeds are shown in Fig. 14. In the simulation analysis, the spindle speed varies from 2000 to 8000 rpm, while the other process parameters are fixed as $d=0.5 \mu$m, $F=3$ N, $V_j=400$ mm/min, $f=25$ kHz, $A=5$ $\mu$m, $Q=12$ ml/min$^{-1}$. It can be seen the faster spindle speed the greater the MRR, and the two are proportional to each other.
It can be seen from Eqs 9, 10, and 12 that the spindle speed directly affects the relative speed $V_s$. The faster the spindle speed is, the faster the relative speed is, the more times the workpiece surface is scratched in unit time, the larger the removal volume of single abrasive particle is, and the greater the MRR is. However, in the actual polishing, the spindle speed is limited by the stiffness of the machine tool, and cannot be increased indefinitely. Only when the processing conditions are met, the spindle speed can be increased as much as possible under the processing conditions to improve the MRR.

3.4.2 The relationship between feed speed and material removal rate

MRR at different feed speeds is shown in Fig. 15, settings include $d=0.5\mu m$, $F=3N$, $\omega_p=8000$rpm, $f=25kHz$, $A=5\mu m$, $Q=12ml\cdot min^{-1}$. It can be seen that the faster the feed speed the greater the MRR. It can be seen from Eqs. 10 and 12 that the speed of the spindle directly affects the relative speed $V_j$. The higher the feed speed, the higher the relative speed, but the difference between the feed speed and the spindle speed is very large, and the feed speed has limited influence on the
relative speed. In the actual polishing, the feed rate should be selected appropriately to prevent too high feed rate leading to low polishing accuracy or too low feed rate leading to lower machining efficiency.

3.4.3 The relationship between abrasive particle size and material removal rate

The relationship between different abrasive particle diameters and MRR is shown in Fig. 16, which shows that the larger the abrasive particle diameter, the lower the MRR. When the abrasive particle diameter increases from 0.5 to 1 μm, the MRR decreases rapidly. The simulation settings are $F=3N$, $\omega_p=8000rpm$, $V_j=400mm/min$, $f=25kHz$, $A=5\mu m$, $Q=12ml\cdot min^{-1}$. When the abrasive particle size increases, the removal volume of a single abrasive particle increases, but the linear density of the abrasive decreases (when the mass concentration or volume concentration of the polishing slurry is a fixed value), and the number of abrasive particles embedded in the polishing tool per unit area decreases. That is to say, the number of effective abrasive particles decreases greatly, resulting in the decrease of MRR.

3.4.4 The relationship between ultrasonic amplitude and material removal rate

The relationship between different ultrasonic amplitude and MRR is shown in Fig. 17, which shows that the ultrasonic amplitude and MRR are positively correlated, and the MRR increases with the increase of ultrasonic amplitude. And polishing settings are $d=0.5\mu m$, $F=3N$, $\omega_p=8000rpm$, $V_j=400mm/min$, $f=25kHz$, $Q=12ml\cdot min^{-1}$. The ultrasonic amplitude directly affects the motion characteristics of the abrasive particles during the polishing. The larger the amplitude, the larger the motion range, and the MRR increases in the same time.
3.4.5 The relationship between flow rate of polishing slurry and material removal rate

The relationship between different polishing slurry flow rates and MRR is shown in Fig. 18, simulation settings include \( d = 0.5 \mu m, F = 3N, \omega_p = 8000 \text{rpm}, V_j = 400 \text{mm/min}, f = 25 \text{kHz}, A = 5 \mu m \). It can be seen that the flow rate of polishing slurry and MRR are approximately proportional, the greater the polishing slurry flow rate, the greater the MRR. The effective abrasive particles increase with the increase of the slurry flow rate, which can improve the MRR. In the actual polishing, too little polishing slurry will result in insufficient abrasive particles, affecting polishing accuracy. However, excessive use of polishing slurry wastes resources, polishing fluid affects the environment, and does not conform to the green processing trend.

4 Conclusions

UVAP can effectively improve polishing efficiency. In this paper, the process parameters are qualitatively analyzed, and the MRR model is established. In order to better apply the UVAP technology to actual production, this paper draws the following conclusions:

1) The MRR model of UVAP of optical glass (BK7) based on ultrasonic atomization was established by analyzing the abrasive particle motion during the polishing, which can be used to predict the material removal profile and analyze the effect of different process parameters on the MRR.

2) In this paper, the influence of technological parameters (spindle speed, feed speed, abrasive diameter, ultrasonic
amplitude, polishing slurry flow rate) on MRR in the polishing process was analyzed qualitatively. Through a detailed analysis of a single variable, it is concluded that the MRR is approximately proportional to the spindle speed and the polishing slurry flow rate. The feed rate and ultrasonic amplitude are positively related to the MRR while negatively related to the diameter of the abrasive particles. Among them, the spindle speed has the greatest influence on MRR, followed by the abrasive diameter and polishing slurry flow rate, while the ultrasonic amplitude and feed speed have less influence on MRR.

3) The characteristics of abrasive particle motion are changed by the introduction of ultrasonic vibration. The trajectory of the embedded abrasive particles is a space sine curve, and the free abrasive particles do a simple harmonic motion in the vertical direction. Each abrasive particle makes intermittent contact with the BK7 specimen only the first half vibration period, which can reduce micro-polishing force and improve machining accuracy. Ultrasonic vibration can greatly improve the impact between the abrasive particles and the workpiece, which can improve the polishing efficiency. Ultrasonic vibration and polishing efficiency are directly affected by ultrasonic frequency and amplitude.

Fig. 18 Effect of flow rate of polishing slurry on material removal rate

Authors’ contributions Yingdong Liang: conceptualization, methodology, investigation, data curation, visualization, writing—original draft; Chao Zhang: investigation, writing—review and editing; Xin Chen: investigation, writing—review and editing; Tianqi Zhang: investigation, writing—review and editing; Tianbiao Yu: resources, supervision, project administration; Ji Zhao: resources, supervision, project administration.

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Declarations

Competing interests The authors declare no competing interests.

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