Interacting Induced Dark Energy Model

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Abstract

Following the idea of the induced matter theory, for a non–vacuum five–dimensional version of general relativity, we propose a model in which the induced terms emerging from the extra dimension in our four–dimensional space–time, supposed to be as dark energy. Then we investigate the FLRW type cosmological equations and illustrate that when the scale factor of the fifth dimension has no dynamics, in early time the universe expands with deceleration and then in late time, expands with acceleration. In this case, the state parameter of the effective dark energy has a range of $-1 < \bar{w}_X < 0$ and it has the value $-1/2$ for present time. The results for current acceleration impose that $\Omega_{X,0} > 2\Omega_{M,0}$ which is in agreement with the measurements. We show that the effective energy density of dark energy have been having the same order of magnitude of the effective energy density of matter from the early time in the decelerating epoch of the universe expansion until now. The model avoids the cosmological coincidence problem.

Keywords: Induced–Matter Theory; FLRW Cosmology; Dark Energy; Cosmological Coincidence Problem.

1 Introduction

Observations of the brightness of distant type Ia–supernovas indicate that the expansion of the universe is presently accelerating [1]–[5]. Hence, in a school of thought, the universe should mainly be filled with what usually has been called dark energy [6], [7]. Therefore, a considerable amount of researches has been performed in the literature to explain the accelerated expansion of the universe, such as the quintessence [8]–[11], the k-essence [12]–[14] and the chaplygin gas [15], [16] models. However, most of them presuppose minimally coupled scalar fields with different potentials or kinetic energies, which have been added in priori by hand, and therefore, their origins are not clearly known. Though, some efforts based on the Brans–Dicke theory in which the scalar field is non–minimally coupled to the curvature, has been performed to explain this acceleration [17]–[20]. Nonetheless, in the recent two decades, explaining the accelerated expansion of the universe through the fundamental theories has been a great challenge [21].

On the other hand, in search of an improved theory of gravitation, attempts for a geometrical unification of gravity with other interactions have begun by using higher dimensions beyond our conventional four–dimensional (4D) space–time. After Nordstrøm [22], who was the first established a unified theory based on extra dimensions, Kaluza [23] and Klein [24] built a five–dimensional (5D) version of general relativity (GR) in which electrodynamics rises from an extra fifth dimension. After that, an intensive amount of works have been focused on this regard either via different mechanism for compactification of extra dimension or generalizing it to non–compact scenarios [25] such as the space–time–matter (STM) or induced–matter (IM) theories [27], [28] and the Brane World scenarios [26]. The significant of the IM theories is that inducing 5D field equations without matter sources leads to

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the $4D$ field equations with matter sources. In another word, the matter sources of $4D$ space–times can be viewed as a manifestation of extra dimensions.

In this work, we follow the approach of the IM theory for a non–vacuum $5D$ version of GR and propose a model in which, neither the baryonic nor dark matter, but just only dark energy is considered as an effect of existence of the fifth dimension. We employ a generalized $5D$ Friedmann–Lemaître–Robertson–Walker (FLRW) type metric for the model and investigate its cosmological implications. In section 2, we give a brief review of the $5D$ version of GR and induce the non–vacuum $5D$ field equations in a $4D$ hypersurface and then collect the extra terms emerging from the fifth dimension as dark energy component of the induced energy–momentum tensor in addition to the common energy–momentum tensor of the matter (the baryonic and dark matter). In section 3, we derive the FLRW cosmological equations by considering a generalized FLRW metric in a $5D$ space–time. Then, we obtain the total energy conservation equation and separate it into two interacting energy conservation equations one for all kind of the matter and the other one for dark energy. In the follow, we investigate three simple and important cases. First, we assume the fifth–dimension energy conservation equations one for all kind of the matter and the other one for dark energy. In section 3, we also take the $5\alpha\beta$ energy–momentum tensor in the form of

$$\sigma = \sqrt{|g|} \left( \frac{1}{16\pi G} (5) R + L_m \right) d^5 x,$$

where $c = 1$, $(5) R$ is $5D$ Ricci scalar, $(5) g$ is the determinant of $5D$ metric $g_{\alpha\beta}$ and $L_m$ represents the matter Lagrangian. Hence, the Einstein field equations in five dimension can be written as

$$(5) G_{\alpha\beta} = 8\pi G (5) T_{\alpha\beta},$$

where the capital Latin indices run from zero to four, $(5) G_{\alpha\beta}$ is $5D$ Einstein tensor and $(5) T_{\alpha\beta}$ is $5D$ energy–momentum tensor. As a reasonable assumption we consider that $(5) T_{\alpha\beta}$ (the Greek indices go from zero to three) represents the same baryonic and dark matter sources of a $4D$ hypersurface, i.e. $T_{\alpha\beta}^{(M)}$. We also take the $5D$ energy–momentum tensor in the form of

$$(5) T_{\alpha\beta} = \begin{pmatrix} \rho_M & 0 & 0 & 0 & Q \\ 0 & p_M & 0 & 0 & 0 \\ 0 & 0 & p_M & 0 & 0 \\ 0 & 0 & 0 & p_M & 0 \\ Q & 0 & 0 & 0 & p_5 \end{pmatrix},$$

where $\rho_M$ and $p_M$ are energy density and pressure of the matter (baryonic and dark matter) and also $p_5$ and $Q$ respectively represent the pressure and momentum density in fifth dimension. For cosmological purposes we restricts our attention to the $5D$ warped metrics of the form

$$dS^2 = g_{\alpha\beta}(x^C) dx^A dx^B = g_{\mu\nu}(x^C) dx^\mu dx^\nu + g_{44}(x^C) dy^2 = (5) g_{\mu\nu}(x^C) dx^\mu dx^\nu + \epsilon b^2(x^C) dy^2,$$

in local coordinates $x^A = (x^\mu, y)$, where $y$ represents the fifth coordinate and $\epsilon^2 = 1$. By assuming the $5D$ space–time is foliated by a family of hypersurfaces, $\Sigma$, which are defined by fixed values of $y$, then, one can obtain the intrinsic metric of any typical hypersurface, e.g. $\Sigma_0(y = y_0)$, by restricting the line element (4) to displacements confined to it. Therefore, the induced metric on the hypersurface $\Sigma_0$ becomes

$$ds^2 = (5) g_{\mu\nu}(x^\alpha, y_0) dx^\mu dx^\nu \equiv g_{\mu\nu} dx^\mu dx^\nu,$$
thus the usual 4D space–time metric, $g_{\mu\nu}$, can be recovered. Therefore after some manipulations, equation (2) on the hypersurface $\Sigma_0$ can be written as

$$G_{\alpha\beta} = 8\pi G (T^{(M)}_{\alpha\beta} + T^{(X)}_{\alpha\beta}),$$

(6)

where we consider $T^{(X)}_{\alpha\beta}$ as dark energy component of the energy–momentum tensor that analogous to the IM theory [19] and [20], defined by

$$T^{(X)}_{\alpha\beta} \equiv T^{(IM)}_{\alpha\beta} \equiv \frac{1}{8\pi G} \left\{ \frac{b_{\alpha\beta}}{b} \partial \frac{b}{b^2} g_{\alpha\beta} - \frac{1}{2b^2} \left[ \frac{b'}{b} g'_{\alpha\beta} - g''_{\alpha\beta} + g''_{\mu\nu} g'_{\alpha\mu} g'_{\beta\nu} - \frac{1}{2} g''_{\mu\nu} g'_{\mu\nu} g_{\alpha\beta} \right] \right\},$$

(7)

in which the prime denotes derivative with respect to the fifth coordinate.

In the following section, we consider a generalized FLRW metric in a 5D universe and investigate its cosmological properties.

3 Generalized FLRW Cosmology

For a 5D universe with an extra space–like dimension in addition to the three usual spatially homogenous and isotropic ones, metric (4), as a generalized FLRW solution, can be written as

$$dS^2 = -dt^2 + \tilde{a}^2(t, \tilde{y}) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right] + \tilde{b}^2(t, \tilde{y})d\tilde{y}^2.$$

(8)

Generally, the scale factors $\tilde{a}$ and $\tilde{b}$ should be functions of cosmic time and extra dimension coordinate. However, for physical plausibility and simplicity, we assume that they are separable functions of time and extra coordinate. Besides, the functionality of the scale factor $\tilde{b}$ on $\tilde{y}$ could be eliminated by transforming to a new extra coordinate $y$, hence metric (8) can be rewritten as

$$dS^2 = -dt^2 + a^2(t)l^2(y) \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta d\varphi^2) \right] + b^2(t)dy^2.$$

(9)

By considering metric (9) and assuming $H \equiv \dot{a}/a$, $B \equiv \dot{b}/b$ and $L \equiv l'/l$, the Einstein equations (2) reduce as follows. The time component, $A = 0 = B$, provides

$$H^2 = \frac{8\pi G}{3} \rho_M - HB + \frac{1}{b^2}(L' + 2L^2) - \frac{k}{a^2l^2} \equiv \frac{8\pi G}{3} \tilde{\rho} - \frac{k}{a^2l^2},$$

(10)

the spatial ones, $A = B = 1, 2$ or 3, give

$$\frac{\ddot{a}}{a} = -4\pi G p_M - \frac{1}{2} H^2 - HB - \frac{1}{2}(\dot{B} + B^2) - \frac{1}{2b^2}(2L' + 3L^2) - \frac{k}{2a^2l^2} \equiv -\frac{4\pi G}{3}(\dot{\tilde{\rho}} + 3\tilde{\rho}),$$

(11)

the $A = 4 = B$ component yields

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3} p_5 - H^2 + \frac{1}{b^2}L^2 - \frac{k}{a^2l^2}$$

(12)

and finally $A = 0$ and $B = 4$ (and also $A = 4$ and $B = 0$) component results

$$L(H - B) = \frac{8\pi G}{3} Q.$$

(13)
In equations (10) and (11) we have defined \( \tilde{\rho} \equiv \rho_M + \rho_X \) and \( \tilde{p} \equiv p_M + p_X \) in which the energy density and pressure of dark energy has specified by employing relation (6), as

\[
\rho_X \equiv T^{(X)}_{\nu\nu} = \frac{3}{8\pi G} \left[ \frac{1}{b^2} (L' + 2L^2) - HB \right]
\]

and

\[
p_X \equiv -T^{(X)}_{\mu
u} = \frac{1}{8\pi G} \left[ \dot{B} + B^2 + 2HB - \frac{1}{b^2} (2L' + 3L^2) \right].
\]

By substituting equation (10) in (12) and assuming \( p_M = 0 \) for current important components of the matter (baryonic and dark matter), one has

\[
b^2 \left[ \ddot{H} + H^2 - HB + \frac{8\pi G}{3} (\rho_M - p_M) \right] = -(L' + L^2) = -\lambda^2,
\]

where \( \lambda \) is a non-zero real constant and obtained from the fact that the left hand side of the equation is a function of cosmic time but the right hand side of the equation is a function of extra dimension coordinate, thus both sides of the equation must be equal to a constant. From the equation (16) one can get

\[
l(y) = l_\circ e^{ \pm \lambda (y - y_\circ) },
\]

with \( L = \lambda \) and \( L' = 0 \). Hence, the equations (10)–(15) on the hypersurface \( \Sigma_\circ(y = y_\circ) \) can be rewritten as

\[
H^2 = \frac{8\pi G}{3} \rho_M - HB + \frac{2\lambda^2}{b^2} - \frac{k}{a^2 l_\circ^2} \equiv \frac{8\pi G}{3} \tilde{\rho} - \frac{k}{a^2 l_\circ^2},
\]

\[
\frac{\ddot{a}}{a} = \frac{8\pi G}{3} p_5 - H^2 + \frac{\lambda^2}{b^2} - \frac{k}{a^2 l_\circ^2} \equiv -\frac{4\pi G}{3} (\tilde{\rho} + 3\tilde{p}),
\]

\[
H \equiv B = \frac{8\pi G}{3\lambda} Q,
\]

\[
\rho_X = \frac{3}{8\pi G} \left( \frac{2\lambda^2}{b^2} - HB \right)
\]

and

\[
p_X = \frac{1}{8\pi G} \left( \dot{B} + B^2 + 2HB - \frac{3\lambda^2}{b^2} \right),
\]

in which

\[
p_5 = \frac{1}{2} (\rho_M + \rho_X - 3p_X) - \frac{3\lambda^2}{8\pi G b^2}.
\]

On the other hand, to obtain energy conservation equation, one can take the time derivative of the equation (10) and substitute the equation (11) into it and get

\[
\dot{\tilde{\rho}} + 3H (\tilde{\rho} + \tilde{p}) = 0.
\]

As the dark energy density is of the same order as the dark matter energy density in the present universe, one can imagine that there is some connection between dark energy and dark matter and
expect their conservation equations may not to be independent. Hence, we assume equation (24) can plausibly separate into two distinguished equations for $\rho_X$ and $\rho_M$ as

$$\dot{\rho}_X + 3H(\rho_X + p_X) = f(t)$$

(25)

and

$$\dot{\rho}_M + 3H(\rho_M + p_M) = -f(t),$$

(26)

where $f(t)$ is assumed to be the interacting term between dark energy and dark matter.

Now, substituting relations (21) and (22) into the equation (25), gets

$$b^2\left[ H + \frac{8\pi G f(t)}{3B} \right] = \frac{\lambda^2}{B}(3H - 4B),$$

(27)

which comparing it with the equation (16) results

$$f(t) = B(\rho_M - p_5) + \frac{3\lambda Q}{b^2}. $$

(28)

In the following, we investigate three cases. First, we assume the fifth–dimension momentum density be zero, i.e. $Q = 0$. Then we suppose the pressure in fifth dimension, $p_5$, equals to zero. Finally, we assume the scale factor of the fifth dimension to be a constant.

### 3.1 The Model with Zero Fifth–Dimension Momentum Density; $Q = 0$

By considering $Q = 0$, from the equation (20) one has $B = H$ with $b = (a_0/b_o)a$. Hence the equations (18) and (19) and the interacting term between matter and dark energy become

$$H^2 = \frac{4\pi G}{3} \rho_M + \frac{a_o^2 \lambda^2}{b_o^2 a^2} - \frac{k}{2l_5^2 a^2},$$

(29)

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} (\rho_M - 2p_5) - \frac{k}{2l_5^2 a^2}$$

(30)

and

$$f(t) = H(\rho_M - p_5).$$

(31)

In the case $f(t) = 0$, one has $p_5 = \rho_M$ and from the equations (26) gets

$$\rho_M = \rho_{M0} \left( \frac{a_0}{a} \right)^3$$

(32)

and hence equation (19) becomes

$$\frac{\ddot{a}}{a} = \frac{4\pi G \rho_{M0}}{3} \left( \frac{a_0}{a} \right)^3 - \frac{k}{2l_5^2 a^2}.$$ 

(33)

For spatially flat or open universe ($k = 0$ and $-1$), equation (33) illustrates eternal accelerated expansion, but for a closed one, it suggests that the expansion of the universe first accelerates and then decelerates. Therefore, the non–interacting case for all kind of the geometry has results contrary to the observations and hence it is not a suitable case.

When $f(t) \neq 0$, from the acceleration equation (30), it is clear that the first term containing energy density of matter, $\rho_M$, only contributes in deceleration of the universe expansion. However, the second and third terms are capable to contributes in the acceleration but, multiplying the equation (29) by $a^2$, shows that these terms also can not increase the velocity of the universe expansion, i.e. $\dot{a}$. Hence the model, when the fifth–dimension momentum density is zero, is not self consistent to having the accelerating solutions. Therefore, in the follow as the second case we assume the fifth–dimension pressure to be zero.
3.2 The Model with Zero Fifth–Dimension Pressure; \( p_5 = 0 \)

Taking \( p_5 = 0 \) with a mathematically simplification and physically plausible assumption as \( B = nH \) gives

\[
Q = \frac{3\lambda}{8\pi G} (1 - n)H 
\]

(34)

and

\[
f(t) = \left(n\rho_M + \frac{9\lambda^2}{8\pi G} \frac{1-n}{a^{2n}}\right)H, \tag{35}
\]

which substituting the second one in the equation (26) leads

\[
\rho_M = \rho_M \left(\frac{a_0}{a}\right)^{n+3} + \beta \frac{n-1}{3-n} a^{-2n}. \tag{36}
\]

Then, equations (18) and (19) become

\[
H^2 = \frac{8\pi G \rho_M}{3(n+1)} \left(\frac{a_0}{a}\right)^{n+3} + \frac{n+3}{(n+1)(3-n)} \frac{\lambda^2}{a^{2n}} - \frac{k}{(n+1)\lambda^2 a^2} \tag{37}
\]

and

\[
\frac{\ddot{a}}{a} = -\frac{8\pi G \rho_M}{3(n+1)} \left(\frac{a_0}{a}\right)^{n+3} + \frac{n(1-n)}{(n+1)(3-n)} \frac{\lambda^2}{a^{2n}} - \frac{nk}{(n+1)\lambda^2 a^2}. \tag{38}
\]

The consistency condition between the equations (35) yields to \( n = 1 \), which gives the same unsuitable results of the previous case with \( Q = 0 \). Therefore, as the last assumption we consider that the scale factor of the fifth dimension does not have any dynamics.

3.3 The Model with Static Scale Factor of the Fifth–Dimension; \( b = cte \)

Assuming \( b = b_0 = cte \), gets \( B = 0 \) and from the equations (18)–(22) one has

\[
H^2 = \frac{8\pi G (\rho_M + \rho_X)}{3} - \frac{k}{\lambda^2 a^2}, \tag{39}
\]

\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\rho_M + \rho_X + 3p_X) \tag{40}
\]

and

\[
Q = \frac{3\lambda}{8\pi G} H, \tag{41}
\]

in which

\[
\rho_X = \frac{3\lambda^2}{4\pi G b_0^2} \tag{42}
\]

and

\[
p_X = -\frac{3\lambda^2}{8\pi G b_0^2}. \tag{43}
\]

Also, from the equations (28), (41) and (42) one gets

\[
f(t) = \frac{9\lambda^2}{4\pi G b_0^2} H = \frac{3}{2} \rho_X H, \tag{44}
\]

(44)
thereupon, substituting the relation (44) in the equation (26) gives

$$\rho_M = \rho_{M0} \left( \frac{a_0}{a} \right)^3 + \frac{3\lambda^2}{8\pi G b_{\lambda}^2} \left[ \left( \frac{a_0}{a} \right)^3 - 1 \right],$$

(45)

To have the common form of the evolution of energy density of matter, we define the effective energy densities of matter and dark energy as follows

$$\bar{\rho}_M \equiv \rho_{M0} \left( \frac{a_0}{a} \right)^3$$

(46)

and

$$\bar{\rho}_X \equiv \frac{3\lambda^2}{8\pi G b_{\lambda}^2} \left[ \left( \frac{a_0}{a} \right)^3 + 1 \right].$$

(47)

Hence, the equations (39) and (40) can be rewritten as

$$H^2 = \frac{8\pi G}{3} (\bar{\rho}_M + \bar{\rho}_X) - \frac{k}{l_5^2 a^2} \equiv \frac{8\pi G}{3} \rho_{M0} \left( \frac{a_0}{a} \right)^3 + \frac{\lambda^2}{b_{\lambda}^2} \left[ 1 + \left( \frac{a_0}{a} \right)^3 \right] - \frac{k}{l_5^2 a^2}$$

(48)

and

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} (\bar{\rho}_M + \bar{\rho}_X + 3\bar{p}_X) \equiv -\frac{4\pi G}{3} \rho_{M0} \left( \frac{a_0}{a} \right)^3 + \frac{\lambda^2}{b_{\lambda}^2} \left[ 1 - \frac{1}{2} \left( \frac{a_0}{a} \right)^3 \right]$$

(49)

in which

$$\bar{p}_X = p_X = \frac{3\lambda^2}{8\pi G b_{\lambda}^2}.$$  

(50)

Then, the state parameter of effective dark energy becomes

$$\bar{w}_X = \frac{\bar{p}_X}{\bar{\rho}_X} = -\frac{a^3}{a^3 + a_0^3},$$

(51)

which has a range of $-1 < \bar{w}_X < 0$. Fig. 1 shows the changes of state parameter of effective dark energy.
Since the measurements of anisotropies in the cosmic microwave background indicates that the universe must be very close to spatially flat one [29]–[31], then we consider \( k = 0 \). Equation (49) illustrates that the universe expansion, in the early epoch, decelerates and then in the late epoch, accelerates. Present acceleration imposes \( \frac{\lambda^2}{b^2} > \frac{8\pi G}{3} \rho_{M0} \), this yields to \( \Omega_{X0} > 2\Omega_{M0} \), or, \( \Omega_{X0} > 2/3 \) and \( \Omega_{M0} < 1/3 \), which is in agreement with the measurements, i.e. \( \Omega_{X0} \approx 0.7 \) and \( \Omega_{M0} \approx 0.3 \).

Also, one gets

\[
\frac{\bar{p}_X}{\bar{\rho}_M} = \frac{3\lambda^2}{8\pi G b^2 \rho_{M0}} \frac{1}{1 + \bar{w}_X} > \frac{1}{1 + \bar{w}_X} = \begin{cases} 
1 & \bar{w}_X \to 0 \quad (a \to 0) \\
2 & \bar{w}_X = -1/2 \quad (a = a_c) \\
\infty & \bar{w}_X \to -1 \quad (a \to \infty),
\end{cases}
\]

which shows that the effective energy density of dark energy dominates all the time of the universe expansion with the same order of magnitude of energy density of matter in the past and present time. Hence, the cosmological coincidence problem can be solved with this model. Note, this domination of dark energy does not mean eternally accelerating expansion in this model, the term, \( -\frac{\lambda^2}{b^2} \left( \frac{\dot{a}}{a} \right)^3 \), actually contributes in deceleration, so at early time beside the energy density of matter, the effective energy density of dark energy also contributes in deceleration.

4 Conclusions

The observations illustrate that the universe is presently in an accelerated expanding phase. Hence, the main content of the universe should be consisted of what commonly called dark energy. However, an enormous amount of work has been performed to explain this acceleration, but the origin and the nature of dark energy is unknown yet.

In this work, following the approach of the induced matter theory, we have investigated the cosmological implications of a non–vacuum five–dimensional version of general relativity in order to explain both deceleration and acceleration eras of the universe expansion and solve the cosmological coincidence problem. In this respect, in general we have considered a five dimensional energy–momentum tensor contains the elements, pressure and momentum density in fifth dimension, i.e., \( p_5 \) and \( Q \), in addition to the energy density and the pressure of the matter (baryonic and dark matter) ones. Then, on a \( 4D \) hypersurface, we have classified the energy–momentum tensor into two parts. One part represents the baryonic and dark matter together and the other one which contains every extra terms emerging from the fifth dimension, have been considered as the energy–momentum tensor of dark energy. Then, by considering a generalized FLRW metric in a \( 5D \) space–time, we have derived the FLRW cosmological equations on the \( 4D \) hypersurface and separated the total energy conservation equation into the two equation, one for the matter and the other one for dark energy with interacting term between them. Afterwards, we have investigated three cases.

In the first one, we assumed the fifth–dimension momentum density be zero, \( Q = 0 \). In this case, for non–interacting situation for all kind of the geometry the results are contrary to the observations and also for interacting one the equations are not self consistent. For the second case, we supposed the pressure in the fifth dimension equals to zero, i.e. \( p_5 = 0 \). The consistency condition between the equations yields to the same unsuitable results of the first case with \( Q = 0 \).

Finally, in the last case, we considered a constant scale factor of the fifth dimension, i.e. \( b = cte \). In this situation by defining the effective energy densities of matter and dark energy we have illustrated that in early time, the universe expands with deceleration and then in the late time, expands with acceleration. The present acceleration, for this model, yields to \( \Omega_{X0} > 2\Omega_{M0} \) which is in agreement with the measurements. The state parameter of the effective dark energy has the range \(-1 < \bar{w}_X < 0 \) and it is quale to \(-1/2 \) at the present time. We have also shown that the effective energy density of dark energy have has the same order of magnitude of the effective energy density of matter since the early time until now. Therefore, the model avoids the cosmological coincidence problem.
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