Speed sensorless control employing adaptive sliding mode adjustable model MRAS for induction motors at low speed range

Jie Li¹, Dong Wang and Xiaoxiao Yang
Xi’an University of Technology, No. 5, Jinhua Southern Road, Xi’an, China

¹E-mail: lijie@xaut.edu.cn

Abstract. For the sensorless speed control of induction motors, at low speed range, there exist the common problems such as low speed estimation accuracy, weak loading capacity and poor dynamic performance. When the flux-based model reference adaptive system methods are adopted, the performance of the flux estimation will determine the accuracy of the speed estimation to a large extent. In this paper, an adaptive sliding mode observer was introduced to replace the adjustable model of the traditional model reference adaptive system. The adaptive gain is designed in order to estimate the flux more accurately, and a new speed adaptive law is proposed to improve the dynamic performance and the accuracy of the speed estimation at low speed range. Simulation results verify the correctness and effectiveness of the proposed method.

1. Introduction
The speed sensorless control of induction motors has been widely used in industrial applications due to the advantages of low maintenance cost, simple structure, and strong robustness. However, the high performance sensorless control at low speed range is still a challenge task due to the accurate speed estimation difficulty. There are mainly two kinds of low speed estimation algorithms: one is based on signal injection, the other is based on the mathematical model of induction motors. The former causes torque ripple, the precision and the robustness of the latter depend primarily on the accuracy of the model.

For the model-based method, the speed estimation is more sensitive to the internal and the external interference at low speed range because of the low signal-to-noise ratio, it may lead to the low accuracy. To overcome the problem, the excitation current error and the correction factor were proposed to strengthen the speed estimation in [1]. Benefit from the improved error feedback matrix and the improved speed adaptation law, the error of the flux estimation is reduced [1]. As the result, the control performance of the motor is enhanced at low speed. However, the performances of starting up and load variation are not evaluated in [1]. A novel model reference adaptive system (MRAS) was proposed in [2] to estimate the speed by the difference between the effective circuit impedance of the d-axis and the q-axis, instead of estimating the rotor flux but, thus achieved satisfactory dynamic performance and steady-state performance at very low speed without considering load. An improved voltage-model-based rotor flux observer was proposed in [3] to enhance the performances of the algorithm at standstill and low speed. The dynamic response is fast, however, it can only offer half load. A novel sliding mode observer with the optimized constant speed reaching law was proposed in [4] to improve the dynamic performance and suppresses chattering. Compared with the single constant rate reaching law, it achieved strong robustness at very low speed. For the speed sensorless control at the ultra-low-speed range, an
adaptive sliding mode observer was proposed in [5] to simultaneously estimate the stator current, the rotor flux and the rotor speed. The dynamic performance of the proposed method was satisfactory at zero and very low speed with full load.

For the MRAS based on the flux estimation, the flux estimation performance determines the performance of the speed estimation to a large extent. In this paper, the traditional model reference adaptive adjustable model is replaced by the adaptive sliding mode observer. The proposed method improves the low-speed dynamic performance of speed sensorless control of induction motor. A correction gain adaptive law is introduced in order to achieve more accurate flux estimation. In addition, the speed adaptive law is redesigned and the flux error term is used to improve the dynamic performance and robustness of the proposed method. Simulation results demonstrate the feasibility and effectiveness of the proposed speed estimation method.

2. Mathematical model of induction motor

The mathematical model of induction motors in the stationary reference frame can be described as follows,

\[
\begin{align*}
\frac{d i_{as}}{dt} &= b u_{as} + a_{11} \dot{i}_{as} + a_{12} \dot{\psi}_{ar} + a_{13} \omega \dot{\psi}_{at} + K_{1} l_a \\
\frac{d i_{bs}}{dt} &= b u_{bs} + a_{11} \dot{i}_{bs} + a_{12} \dot{\psi}_{br} - a_{13} \omega \dot{\psi}_{bt} + K_{1} l_b \\
\frac{d \psi_{ar}}{dt} &= a_{11} \dot{i}_{as} - \omega \dot{\psi}_{ar} - a_{13} \dot{\psi}_{at} + K_{2} \dot{l}_{a} \\
\frac{d \psi_{br}}{dt} &= a_{11} \dot{i}_{bs} + \omega \dot{\psi}_{ar} - a_{13} \dot{\psi}_{bt} - K_{2} \dot{l}_{b}
\end{align*}
\]

Where \( a_{11} = \left( \frac{R_{r}}{\sigma L_{t}} + \frac{1 - \sigma}{\sigma \tau_{s}} \right) \), \( b = \frac{1}{\sigma L_{t}} \), \( a_{12} = \frac{1}{\sigma \tau_{s}} \), \( c = \frac{\sigma L_{r} L_{t}}{L_{n}} \), \( a_{13} = \frac{1}{c} \), \( a_{41} = \frac{L_{n}}{\tau_{s}} \), \( a_{42} = \frac{1}{\tau_{s}} \), and \( \tau_{s} = \frac{L_{n} R_{r}}{a_{11}} \).

3. Adaptive sliding mode observer

The adaptive sliding mode observer in Reference [5] is applied. When the stator currents as the system outputs, the SMO (sliding mode observer) in the stationary reference frame is given as follows,

\[
\begin{align*}
\frac{d \hat{i}_{as}}{dt} &= b u_{as} + a_{11} \dot{\hat{i}}_{as} + a_{12} \dot{\hat{\psi}}_{ar} + a_{13} \omega \dot{\hat{\psi}}_{at} + K_{1} \hat{l}_{a} \\
\frac{d \hat{i}_{bs}}{dt} &= b u_{bs} + a_{11} \dot{\hat{i}}_{bs} + a_{12} \dot{\hat{\psi}}_{br} - a_{13} \omega \dot{\hat{\psi}}_{bt} + K_{1} \hat{l}_{b} \\
\frac{d \hat{\psi}_{ar}}{dt} &= a_{11} \dot{\hat{i}}_{as} - \dot{\hat{\psi}}_{ar} - a_{13} \omega \hat{\psi}_{at} + K_{2} \dot{\hat{l}}_{a} \\
\frac{d \hat{\psi}_{br}}{dt} &= a_{11} \dot{\hat{i}}_{bs} + \dot{\hat{\psi}}_{ar} - a_{13} \omega \hat{\psi}_{bt} - K_{2} \dot{\hat{l}}_{b}
\end{align*}
\]

where \( \hat{l}_{a} = \text{sgn}(e_{a}), \hat{l}_{b} = \text{sgn}(e_{b}), e_{a} = \hat{i}_{as} - i_{as}, e_{b} = \hat{i}_{bs} - i_{bs} \). Defining a Lyapunov function candidate as:

\[
V = \frac{1}{2} e_{a}^{2} + \frac{1}{2} e_{b}^{2}, \quad V = \frac{1}{2} e_{\psi_{ar}}^{2} + \frac{1}{2} e_{\psi_{br}}^{2}, \quad \text{the values of adaptive are given as: } K_{1} = k_{1} - \left[ a_{12} e_{\psi_{ar}} + a_{13} \dot{\hat{\psi}}_{at} \right], \quad K_{2} = k_{2} - \left[ a_{12} e_{\psi_{br}} - a_{13} \dot{\hat{\psi}}_{bt} \right], \quad K_{3} = k_{3} + \left[ \frac{K_{2}}{a_{11}} \right], \quad K_{4} = k_{4} + \left[ \frac{K_{2}}{a_{11}} \right]. \]

Where \( k_{1}, k_{2}, k_{3}, k_{4} \) are the initial values of \( K_{1}, K_{2}, K_{3}, K_{4} \).
4. Proposed adaptive sliding mode MRAS

The block diagram of the induction motor sensorless control system based on the proposed adaptive sliding mode MRAS is shown in Figure 1.

![Block diagram of proposed scheme](image)

Equation (3) is the voltage model without speed information, Equation (5) is the improved adjustable model.

\[
\begin{align*}
\frac{d\psi_{ar}}{dt} &= \frac{L_n}{L_m} \left[ u_{ar} - \left( R_s + \sigma L_s p \right) i_{ar} \right] \\
\frac{d\psi_{fr}}{dt} &= \frac{L_n}{L_m} \left[ u_{fr} - \left( R_s + \sigma L_s p \right) i_{fr} \right]
\end{align*}
\]

It can be seen from Equation (1) that the differential equation of the rotor flux can also be expressed by speed as given by

\[
\begin{align*}
\frac{d\psi_{ar}^{ref}}{dt} &= a_{31} i_{as} - \omega \psi_{fr}^{ref} - a_{32} \psi_{ar}^{ref} \\
\frac{d\psi_{fr}^{ref}}{dt} &= a_{31} i_{fs} + \omega \psi_{ar}^{ref} - a_{32} \psi_{fr}^{ref} \\
\frac{d\dot{\psi}_{ar}}{dt} &= a_{31} \dot{i}_{as} - \dot{\omega} \dot{\psi}_{fr} - a_{32} \dot{\psi}_{ar} - K_{d} \dot{\omega}
\frac{d\dot{\psi}_{fr}}{dt} &= a_{31} \dot{i}_{fs} + \dot{\omega} \dot{\psi}_{ar} - a_{32} \dot{\psi}_{fr} - K_{d} \dot{\omega}
\end{align*}
\]

Subtract Equation (5) from Equation (4), and define the rotor flux error and speed error as

\[ e = \dot{\psi}_{r} - \dot{\psi}_{r}^{ref}, \Delta \omega = \dot{\omega}_r - \omega_r \], we have
Define a Lyapunov function:

\[ V = \frac{1}{2} \left( e_{\varphi_r}^2 + e_{\varphi_\beta}^2 + \Delta \omega_i^2 \right) \]  

Substituting Equation (6) into Equation (7), we get

\[ \dot{V} = -\Delta \omega \dot{\psi}_m e_{\varphi_{sr}} - a_{32} e_{\varphi_{sr}}^2 + K_i \dot{e}_{\varphi_m} e_{\varphi_{sr}} + \Delta \omega \dot{\psi}_m e_{\varphi_{\beta r}} - a_{32} e_{\varphi_{\beta r}}^2 + K_i \dot{e}_{\varphi_\beta} e_{\varphi_{\beta r}} + \Delta \omega_i \dot{\omega}_i \]  

If \( \dot{V} = V_1 + V_2 \), then

\[ \dot{V}_1 = -\frac{1}{\tau_r} e_{\varphi_{sr}}^2 + K_i e_{\varphi_m} e_{\varphi_{sr}} - \frac{1}{\tau_r} e_{\varphi_m} e_{\varphi_{\beta r}} + K_i e_{\varphi_\beta} e_{\varphi_{\beta r}} \]  

\[ \dot{V}_2 = -\Delta \omega \dot{\psi}_m e_{\varphi_{sr}} + \Delta \omega \dot{\psi}_m e_{\varphi_{sr}} + \Delta \omega_i \dot{\omega}_i \]  

If \( K_3, K_4 < 0 \); then, \( V_1 < 0 \). To this end, \( \dot{V}_2 = 0 \) is a necessary condition for system stability. 

Set \( \dot{V}_2 = 0 \), from Equation (11), we have

\[ \epsilon = \dot{\psi}_m e_{\varphi_{sr}} - \dot{\psi}_m e_{\varphi_{\beta r}} \]  

The adaptive law of ASM-MRAS is

\[ \dot{\omega}_i = k_p \epsilon + k_i \int \epsilon dt \]  

Where \( \dot{\omega}_i \) is the estimated speed, \( k_p \) is proportional gain, \( k_i \) is the integral gain.

5. Simulation results

The simulation model is built in the MATLAB/Simulink to verify the proposed scheme using the parameters given in Table 1. The parameters of the proposed speed estimator are listed in Table 2.

| Parameter                  | Value          |
|----------------------------|----------------|
| Rated power                | 3kW            |
| Rated voltage              | 380V           |
| Rated frequency            | 50Hz-70Hz      |
| Pole pairs                 | 2              |
| Rated torque               | 19Nm           |
| Stator self-inductance     | 0.1899H        |
| Rotor self-inductance      | 0.1899H        |
| Mutual inductance          | 0.1823H        |
| Stator resistance          | 2.055Ω         |
| Rotor resistance           | 2.02Ω          |

![Figure 2](image1.png)  
Figure 2. Speed estimation results of MRAS at 15rpm while load torque steps.

![Figure 3](image2.png)  
Figure 3. Speed estimation error of MRAS at 15rpm while load torque steps.
Figure 4. Speed estimation results of SM-MRAS at 15rpm while load torque steps.

Figure 5. Speed estimation error of SM-MRAS at 15rpm while load torque steps.

Figure 6. Speed estimation results of ASM-MRAS at 15rpm while load torque steps.

Figure 7. Speed estimation error of ASM-MRAS at 15rpm while load torque steps.

Table 2. Parameters of proposed ASM-MRAS.

| k_p | 400  |
|-----|------|
| k_i | 1500 |
| k_j | 0.4  |
| k_d | 0.005|

From Figure 2 to Figure 7 are the simulation results of the estimated speed and estimation error waveform for the traditional method (in Figure 2 and Figure 3, respectively), SM-MRAS (in Figure 4 and Figure 5, respectively) and the proposed ASM-MRAS (in Figure 6 and Figure 7, respectively). It can be seen that, compared with the traditional scheme and SM-MRAS, the proposed ASM-MRAS with the adaptive correction factor has stronger robust to load variation and less transient time.

Figure 8. Speed estimation results of MRAS at low speed while forward and reverse.

Figure 9. Speed estimation error of MRAS at low speed while forward and reverse.
To evaluate the performance of estimation under dynamic conditions, the root-mean-square errors of the three algorithms under different operating conditions are calculated, and their dynamic performance are compared as shown in Table 3. It can be seen from Table 3 that the dynamic performance of SM-MRAS is improved compared with the traditional MRAS. After the adaptive gain is introduced in the sliding mode observer, it can be seen that compared with SM-MRAS, the dynamic performance of proposed ASM-MRAS is significantly improved.

From Figure 8 to Figure 13 are the estimated speed and error waveforms of the proposed algorithm, the traditional algorithm and the SM-MRAS at very low speed, forward and reverse rotation. It can be seen that the sliding mode observer together with the adjustable model solves the problem of the poor low-speed dynamic performance of the traditional algorithm. Compared with the SM-MRAS, due to the introduction of an adaptive factor, the speed estimation performance of the proposed ASM-MRAS is better than that of SM-MRAS. It can be seen that the proposed ASM-MRAS has better low-speed dynamic performance and higher speed estimation accuracy.
Figure 14 and Figure 15 show the robustness to the parameter. It can be seen that the traditional MRAS fails to estimate the speed in the case of rotor resistance mismatch. However, the proposed method in this paper can still estimate the speed accurately, its robustness is stronger.

![Figure 14](image1.png)  
*Figure 14. Speed estimation results of ASM-MRAS at low speed when $R_r$ becomes $1.2R_r$.*  

![Figure 15](image2.png)  
*Figure 15. Speed estimation results of MRAS at low speed when $R_r$ becomes $1.2R_r$.*

6. Conclusions

The unsatisfactory dynamic performance of the flux estimation of the traditional model reference adaptive adjustable model causes the poor speed estimation at very low speed range. To improve the dynamic performance of the model reference adaptive speed sensorless control system of induction motors at low speed range, this paper replaces the traditional adjustable model to a sliding mode observer with adaptive gain in order to achieve more accurate flux estimation. The Lyapunov stability criteria is applied to redesign the speed adaptive law, the flux error term is introduced to provide higher precision of flux estimation. The dynamic performance of the proposed ASM-MRAS has significant improvement as compared to the traditional MRAS solution. Simulation comparison results demonstrate the feasibility and superiority of the proposed method.

Acknowledgment

The work is sponsored in part by Shaanxi Province R & D Key Project-General Project-Industrial Field (2018GY-165).

References

[1] Lv Y, Liu Z, Suo T, Huang X and Suo X 2019 Research of sensorless vector control performance for induction motor at very low-speed and zero-speed *Proceedings of the CSEE* vol. 39 no. 20 pp. 6095-6103+6190 (in Chinese)

[2] S Das, R Kumar and A Pal 2019 MRAS-based speed estimation of induction motor drive utilizing machines’ d-and q-circuit impedances *IEEE Transactions on Industrial Electronics* vol. 66 no. 6 pp. 4286-4295

[3] A Pal, S Das and A K Chattopadhyay 2018 An improved rotor flux space vector based MRAS for field-oriented control of induction motor drives *IEEE Transactions on Power Electronics* vol. 33 no. 6 pp. 5131-5141

[4] Zhang Y Q, Yin Z G, Zhang Y P, Liu J and Tong X Q 2020 A novel sliding mode observer with optimized constant rate reaching law for sensorless control of induction motor *IEEE Transactions on Industrial Electronics* vol. 67 no. 7 pp. 5867-5878

[5] M S Zaky, M K Metwaly, H Z Azazi and S A Deraz 2018 A new adaptive SMO for speed estimation of sensorless induction motor drives at zero and very low frequencies *IEEE Transactions on Industrial Electronics* vol. 65 no. 9 pp. 6901-6911