Quantum computation using the Aharonov-Casher set up

Marie Ericsson and Erik Sjöqvist
Department of Quantum Chemistry, Uppsala University, Box 518, S-751 20 Sweden

It is argued that the Aharonov-Casher set up could be used as the basic building block for quantum computation. We demonstrate explicitly in this scenario one- and two-qubit phase shift gates that are fault tolerant to deformations of the path when encircling two sites of the computational system around each other.

PACS numbers: 03.65.Vf, 03.67.Lx

Efficient and reliable quantum computation of arbitrarily long duration is possible even with faulty components, if the errors can be corrected faster than they occur [1, 2]. Even more desirable would be to implement quantum computation that is intrinsically fault tolerant as this would prevent errors to occur. Topological ideas arise naturally in this context, as they are robust against deformations of the physical path, such as what happens when a small amount of noise is added.

Motivated by this intuition, fault tolerant quantum computation based upon topological phases in various anyonic systems has been suggested recently [3, 4, 5, 6, 7], [8, 9, 10, 11, 12, 13]. Related to these ideas is to achieve fault tolerance by using the geometric phase [14] to be particular cases of the notion of computation in noiseless quantum subsystems [15].

In this Letter, we propose the two dimensional Aharonov-Casher (AC) set up [16] as the basic building block for topological one- and two-qubit phase shift gates. This implementation would be fault tolerant as the AC effect is only dependent upon the winding number of the physical path. We suggest that combining the present phase shift gates with appropriate nontopological one-qubit gates would be a realisation of universal quantum computation [17, 18] based upon the AC set up.

Let us first briefly review the AC effect. In the two dimensional AC set up, a magnetic moment \( \mu \) and a point charge \( q \) are free to move in the \( x - y \) plane, say. This system is described by the Galilean and gauge invariant Lagrangian

\[
\mathcal{L} = \frac{1}{2} m \dot{v}^2 + \frac{1}{2} M \dot{v}^2 + q \mathbf{A} (\mathbf{r} - \mathbf{R}) \cdot [\mathbf{v} - \mathbf{V}],
\]

where \( m, r, v \) denote the mass, position, and velocity, respectively, of the charge, and \( M, R, V \) the corresponding quantities of the dipole. In this two dimensional set up, the curl \( \partial_x A_y - \partial_y A_x \) of the vector potential \( \mathbf{A} = (A_x, A_y) \) vanishes except at the origin, but when encircling the two particles \( n \) times around each other along the path \( C \) there is a phase of the form

\[
\gamma = \frac{q}{\hbar} \oint_C \mathbf{A} (\mathbf{l}) \cdot d\mathbf{l} \propto n \mu q, \quad (2)
\]

where \( \mathbf{l} = \mathbf{r} - \mathbf{R} \). This phase is topological in the sense that it only depends upon the winding number \( n \), which makes it insensitive to small deformations of the path \( C \).

This AC scenario may be used to implement fault tolerant one- and two-qubit phase shift gates as follows. Let us store the \( j \)th qubit on the two spatial sites \( a \) and \( b \) of a single AC set up. The computational basis consists of \( |0\rangle_j = |q(a)\mu(b)\rangle_j \) with the magnetic moment localised at site \( a \) and the charge localised at site \( b \), respectively, and the reverse for the orthogonal state, i.e. \( |1\rangle_j = |\mu(a)q(b)\rangle_j \), see Fig. 1. We assume that the states are sufficiently localised so that \( |0\rangle_j = 0 \) for each \( j \). The computational system is build up from such AC-qubits arranged along a line in the two dimensional plane and let us assume for simplicity that all the qubits contain the same magnetic moment \( \mu \) and the same charge \( q \).

A controlled phase shift gate \( B(\gamma) \) can be achieved by encircling along the path \( C \) the particle at site \( b \) around the particle at site \( a \) from two different AC-qubits \( j \) and \( j' \). This results in

\[
\begin{align*}
B(\gamma) : |00\rangle_{jj'} & \rightarrow e^{i\gamma}|00\rangle_{jj'} \\
B(\gamma) : |01\rangle_{jj'} & \rightarrow |01\rangle_{jj'} \\
B(\gamma) : |10\rangle_{jj'} & \rightarrow |10\rangle_{jj'} \\
B(\gamma) : |11\rangle_{jj'} & \rightarrow e^{i\gamma}|11\rangle_{jj'},
\end{align*}
\]

as illustrated in Fig. 2. This gate is topological as it is insensitive to any deformations of the path \( C \) under the...
assumption that charge-charge and dipole-dipole interactions can be neglected between all pairs of AC-qubits. Except for the trivial case where $\gamma$ is an integer multiple of $\pi$, $B(\gamma)$ may entangle the qubits it acts on.

Universal quantum computation can be achieved by combining $B(\gamma)$ with the one-qubit logic gates

\[
U(\gamma) = \exp \left[ -i \frac{\gamma}{2} \sigma_z \right], \quad \text{(phase shift gate),}
\]
\[
U_{SWAP}(\theta_j) = \exp \left[ -i \frac{\theta_j}{2} \sigma_y \right], \quad \text{(partial swap gate),}
\]

where $\sigma_z^i = |0\rangle_j \langle 0|_j - |1\rangle_j \langle 1|_j$ and $\sigma_y^j = -i|0\rangle_j \langle 1|_j + i|1\rangle_j \langle 0|_j$. That is, any $N$-qubit logic operation can be simulated to any precision with an appropriate set of $U(\gamma), U_{SWAP}(\theta_j)$, and $B(\gamma)$ gates. The one-qubit phase shift gate $U(\gamma)$ is achieved by encircling the two sites within a single AC-qubit around each other depending upon whether the state is $|0\rangle_j$ or $|1\rangle_j$. This could for example be achieved by addressing the site $a$, say, in such a way that the particle there is only taken around site $b$ if it is charged. This would result in $U(\gamma)$ up to an unimportant overall phase factor $e^{i\gamma/2}$. The $U_{SWAP}(\theta_j)$ gates could be realised in principle by exposing beam-splitters to each of the AC set ups. The parameter $\theta_j$ determines the transmission probability $T$ of such a beam-splitter according to $T = \cos(\theta_j/2)$.

While the $U(\gamma)$ gate is topological and thereby fault tolerant to path deformations, the $U_{SWAP}(\theta_j)$ gate is essentially dynamical and nontopological as it relies upon the detailed interaction between the AC-qubit and the beam-splitter. This situation is expected since the present treatment of the AC effect is basically Abelian and there are therefore non-Abelian operations necessary to achieve universality that could not be obtained by the AC effect alone (see Ref. 3 for a similar case).

Quantum computation based upon the AC set up can be realised as follows. First, translate the quantum algorithm into a set of elementary one- and two-qubit gates. Prepare an initial state by spreading out a set of AC composites along a line in the plane of motion. Perform appropriate phase shifts and partial swaps on each AC set up and perform appropriate two-qubit controlled phase shifts by braiding sites from pairs of AC-qubits. The final answer of the computation is obtained by measuring the spatial location of the particles in the output.

Universal quantum computation using the AC set up only involves electromagnetic interactions between elementary systems and works even for distinguishable qubits. This should be compared with the suggestions for topological quantum computation in Refs. 3 4 5 6 7, that all involves collective effects such as anyons or spin systems with long-range correlations.

In principle, quantum computation based upon the AC set up could be realisable in combined atom-ion systems confined to a plane. A similar implementation could be achieved in three dimensions by replacing the point charge with a line of charge. However, to put this charged line in a coherent superposition would be difficult in practice and it is therefore unclear whether AC based quantum computation could have any relevance in the three-dimensional context. Moreover, it is important to keep in mind that the AC shift is essentially a relativistic effect and thus usually quite small also for such systems (see, e.g., 19 20), which means that the gates have to be repeated many times to achieve phase shifts of useful size. This may spoil the fault tolerance of the phase shift gates as the error probability is expected to increase with the winding number. Another challenge, associated with the implementation of the controlled phase shift gate, is the control of the nontopological charge-charge and dipole-dipole interactions that act between the various AC-qubits. These interactions could be made small under certain circumstances, but may add up when repeating the gate.

![FIG. 2: Controlled phase shift gate based upon the Aharonov-Casher set up.](image-url)
In conclusion, we have proposed to use the two dimensional Aharonov-Casher (AC) set up as the basic building block for quantum computation. We have argued that the AC set up could be useful in the implementation of one- and two-qubit phase shift gates that are fault tolerant to path deformations when two sites are encircled around each other. Universality is achieved by adding nontopological one-qubit partial swap gates. Although it seems hard to implement quantum computation based upon the AC set up with present day technology, we believe it has a conceptual value as it demonstrates topological quantum computation using electromagnetic interactions between elementary systems.

The work by E.S. was financed by the Swedish Research Council.

[1] P. Shor, Phys. Rev. A 52 (1995) R2493.
[2] A. Steane, Phys. Rev. Lett. 77 (1995) 793.
[3] A. Kitaev, quant-ph/9707021.
[4] R.W. Ogburn and J. Preskill, Lect. Notes Comput. Sc. 1509 (1999) 341.
[5] S. Lloyd, quant-ph/0004010.
[6] M.H. Freedman, A. Kitaev, M.J. Larsen, and Z. Wang, quant-ph/0101025.
[7] C. Mochon, quant-ph/0206128.
[8] P. Zanardi and M. Rasetti, Phys. Lett. A 264 (2000) 94.
[9] J. Pachos, P. Zanardi, and M. Rasetti, Phys. Rev. A 61 (2000) R010305.
[10] L.-M. Duan, J.I. Cirac, and P. Zoller, Science 292 (2001) 1695.
[11] J.A. Jones, V. Vedral, A. Ekert, and G. Castagnoli, Nature 403 (2000) 869.
[12] A. Ekert, M. Ericsson, P. Hayden, H. Inamori, J.A. Jones, D.K.L. Oi, and V. Verdal, J. Mod. Opt. 47 (2000) 2501.
[13] W. Xiang-Bin and M. Keiji, Phys. Rev. Lett. 87 (2001) 097901.
[14] P. Zanardi and S. Lloyd, quant-ph/0208132.
[15] E. Knill, R. Laflamme, and L. Viola, Phys. Rev. Lett. 84 (2000) 2525.
[16] Y. Aharonov and A. Casher, Phys. Rev. Lett. 53 (1984) 319.
[17] S. Lloyd, Phys. Rev. Lett. 75 (1995) 346.
[18] D. Deutsch, A. Barenco, and A. Ekert, Proc. Roy. Soc. London, Ser. A 449 (1995) 669.
[19] K. Sangster, E.A. Hinds, S.M. Barnett, E. Riis, and A.G. Sinclair, Phys. Rev. Lett. 71 (1993) 3641.
[20] A. Görlitz, B. Shuh, and A. Weis, Phys. Rev. A 51 (1995) R4305.