AdS and pp-wave D-particle superalgebras

Machiko Hatsuda

Theory Division, High Energy Accelerator Research Organization (KEK),
Tsukuba, Ibaraki, 305-0801, Japan

E-mails: mhatsuda@post.kek.jp

Abstract

We derive anticommutators of supercharges with a brane charge for a D-particle in $\text{AdS}_2 \times S^2$ and pp-wave backgrounds. A coset $GL(2|2)/(GL(1))^4$ and its Penrose limit are used with the supermatrix-valued coordinates for the AdS and the pp-wave spaces respectively. The brane charges have position dependence, and can be absorbed into bosonic generators by shift of momenta which results in closure of the superalgebras.

PACS: 11.30.Pb;11.17.+y;11.25.-w

Keywords: Superalgebra; Wess-Zumino term; AdS background; pp-wave background
1 Introduction

Recently brane dynamics in curved backgrounds are examined extensively motivated by the AdS/CFT correspondence [1] and its pp-wave limit [2] and [3]. The study of superstrings in AdS spaces has been started in earlier works [4, 5, 6], and further developed to pp-wave spaces [7] and to D-branes [8, 9]. Branes are characterized by brane charges in superalgebras which are topological [10, 11]. Superalgebras manifest non-perturbative dualities [12] and present solitonic solutions systematically [13]. In general a superalgebra is uniquely obtained coordinate independently, while Killing spinor equations are coordinate dependent where there are several different coordinate systems in curved spaces. So it will give better outlook to examine superalgebras with brane charges in curved spaces also. For a flat background p-brane charges are center [12], and classified by non-trivial elements of the Chevalley-Eilenberg (CE) cohomology [14]. However it is unclear whether these properties are common in curved spaces; any p-form charges (except scalars) cannot be centers in these backgrounds, and string charges in AdS space are given as trivial elements of CE cohomology [15, 16, 17, 18, 19].

There are several attempts to evaluate brane charges in superalgebras in the matrix theory approach for pp-wave backgrounds in earlier works [20] and in [21] and in the Green-Schwarz type action approach for the AdS space [19], where new contributions to brane charges are obtained in addition to the flat part. It is desirable to have background dependence of a brane charge, since a brane charge is proportional to a brane volume which depends on a background. However these brane charges are not allowed by the central extension of the superalgebra. Possible extension of the super-AdS algebra is also discussed [22]. In order to clarify background dependence of a brane charge and consistency of superalgebras, we analyze a simple case and show how these issues incorporate. We use the supermatrix-valued coordinates introduced in “supertwistor formulation” [16] and compute brane charges analogously to [19]. We take a D0-brane case in the AdS$_2 \times$S$^2$ [6] which is the near horizon geometry of the Reissner-Nordstr"om black hole in four dimensions. Its pp-wave limit and a flat limit are also examined.

The organization of this paper is as follows. In section 2.1, we explain the coset construction of the Wick rotated AdS$_2 \times$S$^2$ space which is expressed by a coset $GL(2|2)/(GL(1))^4$ [16]. Its Penrose limit [25] is taken for the bosonic part of AdS$_2 \times$S$^2$ in section 2.2 and for whole part in section 2.3. In section 3 an action for a D0-brane in the AdS$_2 \times$S$^2$ space is given. The pseudo-supersymmetric WZ term is confirmed to satisfy three criteria; 1. its exterior derivative to be correct two-form field strength, 2. kappa-invariance of the action, 3. correct flat limit. Its pp-wave limit and a flat limit are also given. In section 4 supercharges are obtained and brane charges are calculated by anticommutators of these supercharges. The BPS condition and the BPS mass are also discussed.


2 Super-\(\text{AdS}_2\times S^2\) and its Penrose limit

Branes in the super-\(\text{AdS}_2\times S^2\) background is expressed by the coset

\[
SU(1,1|2)/SO(1,1) \times SO(2) \rightarrow GL(2|2)/(GL(1))^4
\]

(2.1)

where Wick rotations are performed and scaling degrees of freedom are introduced. An element of the coset is denoted by \(Z_M^A\) with indices \(M\) for \(GL(2|2)\) and indices \(A\) for \((GL(1))^4\). Two \(GL(1)\)'s are Lorentz groups of two 2-dimensional pseudospheres and other two \(GL(1)\)'s are dilatations. At first we examine the bosonic part of the \(\text{AdS}_2\times S^2\) space, \([GL(2)/(GL(1))^2]^2\), in section 2.1 and take the Penrose limit to derive the pp-wave metric in section 2.2. The Penrose limit of the whole supergroup is presented in section 2.3.

2.1 \(\text{AdS}_2\times S^2\)

The bosonic part of the \(\text{AdS}_2\times S^2\) space is \([GL(2)/(GL(1))^2]^2\). Coset elements \(z_m^a\) and \(z_{\bar{m}}^{\bar{a}}\) correspond to a coset \(GL(2)/(GL(1))^2\) for \(\text{AdS}_2\) and another one for \(S^2\) respectively. The left-invariant (LI) one forms are given by

\[
j_{a}^{b} = z_{m}^{m} d z_{m}^{b}, \quad j_{\bar{a}}^{\bar{b}} = z_{\bar{m}}^{m} d z_{\bar{m}}^{\bar{b}}
\]

(2.2)

where \(z_{m}^{m}\) and \(z_{\bar{m}}^{\bar{a}}\) are inverse of \(z_{m}^{a}\) and \(z_{\bar{m}}^{\bar{a}}\). These indices are contracted with the Lorentz invariant metric \(\delta^{ab}\) and \(\delta^{\bar{a}}{}^{\bar{b}}\) and the Lorentz \(GL(1)\) is assigned to \(SO(2)\). A LI current \(j_{ab}\) is decomposed into three parts; a trace part \(\delta_{ab} j^{ab}\), a traceless symmetric part \(j_{\langle ab \rangle}\) and an antisymmetric part \(j_{[ab]}\);

\[
j_{ab} = \frac{1}{2} \delta_{ab} j + j_{\langle ab \rangle} + j_{[ab]}.
\]

(2.3)

If the following parameterization is chosen

\[
z_{m}^{a} = R \delta_{m}^{a} + \begin{pmatrix} x^0 \\ x^1 \\ -x^0 \\ x^1 \end{pmatrix}
\]

(2.4)

whose inverse is

\[
z_{a}^{m} = \frac{1}{R^2 - (x^0)^2 - (x^1)^2} [R \delta_{a}^{m} - \begin{pmatrix} x^0 & x^1 \\ x^1 & -x^0 \end{pmatrix}],
\]

(2.5)

then the LI one forms are given by

\[
j_{\langle ab \rangle} = \frac{R}{R^2 - \sum_{\mu} (x^\mu)^2} \begin{pmatrix} dx^0 & dx^1 \\ dx^1 & -dx^0 \end{pmatrix}.
\]

(2.6)

The metric of this Wick rotated \(\text{AdS}_2\times S^2\) is given by

\[
ds_{AdS_2 \times S^2}^2 = -\frac{1}{2} (j_{\langle ab \rangle})^{\langle ab \rangle} - \frac{1}{2} (j_{\langle ab \rangle})^{\langle ab \rangle} - \frac{1}{2} (j_{\langle ab \rangle})^{\langle ab \rangle}.
\]

(2.7)

expressing two 2-dimensional pseudospheres with the same unit curvature, where the supersymmetry is imposed to restrict curvatures to be same.
2.2 Penrose limit of AdS$_2$×S$^2$

Under the global $GL(2)$ which is generated by $G_m^n$‘s

$$G_m^n = z_m^a \pi^a_n, \quad [z_m^a, \pi_b^n] = \delta_b^a \delta_m^n, \quad [G_m^n, G_l^k] = -\delta_l^m G_m^k + \delta_m^k G_l^n \quad (2.8)$$

with the conjugate momenta $\pi$’s, a contravariant vector

$$w_{(mn)} = z_m^a z_n^a \quad (2.9)$$

is transformed as

$$\delta_\lambda w_{mn} = \left[ w_{mn}, \lambda^l G^k_l \right] = \lambda_m^l w_{ln} + \lambda_n^l w_{lm} . \quad (2.10)$$

If we parameterize $z_m^a$ as (2.4), $w_{(mn)}$ becomes

$$w_{(mn)} = 2R \begin{pmatrix} x^0 & x^1 \\ x^1 & -x^0 \end{pmatrix} . \quad (2.11)$$

So the $GL(2)$ generators are parameterized as

$$G_{mn} = \begin{pmatrix} \bar{D} + \bar{P}_0 & \bar{P}_1 + \bar{M}_{01} \\ \bar{P}_1 - \bar{M}_{01} & \bar{D} - \bar{P}_0 \end{pmatrix} \quad , \quad (2.12)$$

where $D, P, M$ are dilatation, translations, Lorentz generators respectively. The same parameterization is chosen for another $GL(2)$ generators $\bar{G}_{mn}$ with replacing $P_0, P_1, M_{01}$ by $P_3, P_2, M_{32}$.

The Penrose limit is taken as follows: Introduce the lightcone variables and scale them with the parameter $\Omega$ then take a limit $\Omega \to 0$

$$P_{\pm} = \frac{1}{\sqrt{2}} (P_3 \pm P_0) \quad (2.13)$$

$$P_+ \to \Omega^{-2} P_+ , \quad (P_1, M_{01}, M_{32}) \to \Omega^{-1} (P_1, M_{01}, M_{32}) \quad (i=1, 2) , \quad (P_-, D) \to (P_-, D)$$

and therefore

$$x^\pm = \frac{1}{\sqrt{2}} (x^3 \pm x^0) , \quad x^+ \to \Omega^2 x^+ , \quad x^i \to \Omega x^i , \quad x^- \to x^- . \quad (2.14)$$

The metric (2.1), which is scaled as $ds^2 \to \Omega^2 ds'^{2}$ and omit prime from now on, becomes

$$ds_{pp}^2 = \Omega^{-2} \left(-\frac{1}{2} \right) \left( [j^{ab}]^2 - (j^{a\bar{b}})^2 \right) \bigg|_{\text{rescaling (2.14)} \Omega \to 0} \quad (2.15)$$

$$= \frac{R^2}{[R^2 - \frac{1}{2} x^2]^2} \left\{ 2dx^+ dx^- + \sum_{i=1, 2} (dx^i)^2 + \frac{2dx^+ dx^- + \sum_{i=1, 2} (x^i)^2}{R^2 - \frac{1}{2} (x^-)^2} (dx^-)^2 \right\}$$
with \( x^i = (ix^1, x^2) \). After field redefinition

\[
\begin{align*}
X^i &= \frac{R}{R^2 - \frac{1}{2} (x^-)^2} x^i \\
X^+ &= \frac{R}{R^2 - \frac{1}{2} (x^-)^2} \left( x^+ - \frac{1}{2} \frac{x^-}{R^2 - \frac{1}{2} (x^-)^2} (x^i)^2 \right) , \\
X^- &= \frac{1}{2} \ln \frac{\sqrt{2} R + x^-}{\sqrt{2} R - x^-}
\end{align*}
\]

(2.15) recast into the familiar form

\[
d s_{\text{pp}}^2 = 2dX^+X^- + \sum_{i=1,2} (dX^i)^2 + \frac{2}{R^2} \sum_{i=1,2} (X^i)^2 (dX^-)^2 .
\]

(2.16)

A flat limit is taken by rescaling \( x^\mu \to (1/R)x^\mu \) and \( ds^2 \to (1/R^2)ds^2 \) then \( R \to \infty \). The metric (2.7) has already dilatation scale \( R \) exists which is identified to the radius of the curvature,

\[
d s_{\text{flat}}^2 = R^2 \left( -\frac{1}{2} \right) \left[ j_{(ab)} j^{(ab)} - j_{(\bar{a}\bar{b})} j^{(\bar{a}\bar{b})} \right] |_{R \to \infty}
\]

\[
= \left\{ - \left[ 1 - \sum_{\mu} (x^\mu / R)^2 \right]^2 \sum_{\mu=0,1} (dx^\mu)^2 + \left[ 1 - \sum_{\mu} (x^\mu / R)^2 \right]^2 \sum_{\mu=2,3} (dx^\mu)^2 \right\} |_{R \to \infty}
\]

\[
= \sum_{\mu=0,1,2,3} (dx^\mu)^2 .
\]

(2.17)

(2.18)

### 2.3 Penrose limit of super-AdS\(_2 \times S^2\)

Now we consider a coset \( GL(2|2)/(GL(1))^4 \) whose element is denoted by \( Z_M^A \), and the supergroup \( GL(2|2) \) is generated by

\[
G_{M}^N = Z_N^A \Pi_A^N , \quad [Z_M^A, \Pi_B^N] = (-1)^A \delta^A_B \delta_M^N \\
\left[ G_{M}^N, G_L^K \right] = (-1)^N (\delta^N_K G_M^K + \delta^K_M G_L^N)
\]

(2.19)

where conjugate momenta \( \Pi \)'s are introduced. Corresponding to that bosonic lightcone generators are defined as (2.13) for generators of \( GL(2) \) (2.12), fermionic lightcone generators are identified from the above algebra; for example, \( \{ G_{1}^1, G_{1}^1 \} \propto P_- \Rightarrow G_{1}^1, G_{1}^1 \in Q_- \), \( \{ G_{1}^2, G_{1}^1 \} \propto P_+ \Rightarrow G_{1}^2, G_{1}^2 \in Q_+ \) and so on. A center \( (D - \bar{D}) = \text{Str} G_{MN} \) can be neglected. Whole identification of lightcone generators with \( GL(2|2) \) is given as follows:

\[
G_{MN} = \begin{array}{cccc}
D + P_0 & P_1 + M_{01} & Q_- & Q_+ \\
P_1 - M_{01} & D - P_0 & Q_+ & Q_- \\
Q_- & Q_+ & D + P_3 & P_2 + M_{32} \\
Q_+ & Q_- & P_2 - M_{32} & D - P_3
\end{array}
\]
where \( P_0 = (P_+ - P_-)/\sqrt{2} \) and \( P_3 = (P_+ + P_-)/\sqrt{2} \). The Penrose limit is given by rescaling (2.13) and

\[
Q_+ \to \Omega^{-1} Q_+ , \quad Q_- \to Q_-, \quad \Omega \to 0.
\] (2.20)

or equivalently rescaling of \( GL(2|2) \) generators as follows

\[
G_{MN} \to \Omega^{-2} G_{MN} + \Omega^{-1} P_0, \quad P_+ \to P_+, \quad Q_+ \to Q_+, \quad P_3 \to P_3,
\] (2.21)

and taking the limit \( \Omega \to 0 \).

3 Super-D0-brane actions

The action of a super-D0-brane in \( AdS_2 \times S^2 \) is given in [6] and we reformulate this into the “supertwistor formulation” in this section. We must construct the WZ term with taking care of the boundary term to obtain brane charges. In section 3.1 we present a general argument of the pseudo-supersymmetric WZ term [19]. Then the super-D0-brane action is examined explicitly in section 3.2.

3.1 WZ term for a \( p \)-brane

In order to compute a brane charge by the anticommutator of supercharges we need to construct the pseudo-supersymmetric WZ term. For an group element, \( Z \), it can be parameterized generally as \( Z = 1 + X \) whose inverse is \( Z^{-1} = \sum_{n=0}^{\infty} (-X)^n \). A left-invariant (LI) one form current is given as

\[
J = Z^{-1} dZ = \sum_{n=0}^{\infty} (-X)^n dX,
\] (3.1)

satisfying the Maurer-Cartan equation, \( dJ = -JJ \). The right hand side of the integration of the Maurer-Cartan equation is calculated as

\[
- \int J J = \int d \left( \sum_{n=1}^{\infty} (-X)^n dX \right) = \sum_{n=1}^{\infty} (-X)^n dX + \text{boundary term}.
\] (3.2)
Its left-invariance requires “boundary term = dX”, so “− \int_{\text{L.I.}} JJ = J”. On the other hand the “pseudo”-left-invariant one form is
\[- \int_{\text{P.L.I.}} JJ = J - dX . \quad (3.3)\]

For super-p-branes the Wess-Zumino (WZ) term is given as the one-dimensional higher space integration of LI \((p + 2)\) form currents
\[\mathcal{L}_{WZ} = B_{[p+1]} = \int_{\text{P.L.I.}} H_{[p+2]} , \quad H_{[p+2]} = J^{p+2} , \quad (3.4)\]
which is pseudo-supersymmetry invariant producing the topological \(p\)-brane charge. This pseudo-invariance corresponds to the choice of the boundary term of the integration to be zero. If a LI \((p + 1)\) form current exists \(\tilde{B}_{[p+1]} = J^{p+1} \) with \(d\tilde{B}_{[p+1]} = H_{[p+2]}\), the pseudo-invariant \((p + 1)\)-form is given by \[19\]
\[\mathcal{L}_{WZ} = B_{[p+1]} = \tilde{B}_{[p+1]} - d(X(dX)^p) . \quad (3.5)\]

The super-AdS group and the nondegenerate supertranslation group are the cases where \(\tilde{B}_2 = (J)^2\) exists \[15, 16, 17, 18, 19\]. For the nondegenerate supertranslation group there exists the LI 2-form current
\[\tilde{B}_{[2]} = JQJQ , \quad d\tilde{B}_{[2]} = H_{[3]} = JQJPJQ \quad (3.6)\]
where \(\tilde{Q}\) is introduced to make a nondegenerate fermionic group metric \[21\] and satisfies \(dJ\tilde{Q} = JPJQ\). The WZ term for a superstring in a flat space is given by
\[\mathcal{L}_{WZ} = B_{[2]} = \tilde{B}_{[2]} - d(XQdXQ) . \quad (3.7)\]
The auxiliary variable \(X_Q\) is not contained in the kinetic term, so this can be gauged away in this case.

Now let us go back to the curved background cases. The supercoset construction is obtained by replacing bosonic currents \[2, 2\] with supercurrents
\[J_A^B = Z_A^M dZ_M^B , \quad Z_M^A = \begin{pmatrix} 1 \\ X \end{pmatrix} + \begin{pmatrix} 0 \\ \Theta \end{pmatrix} \begin{pmatrix} \bar{X} \\ \Theta \end{pmatrix} . \quad (3.8)\]
All components are contained in \(X\) and \(X\) \[16, 18\], especially the antisymmetric part plays a role of auxiliary variables. The action for a \(Dp\)-brane is given by
\[I_{Dp} = T \int d^{p+1}\sigma (\mathcal{L}_{DBI} + \mathcal{L}_{WZ}) \quad (3.9)\]
where the Dirac-Born-Infeld term is given as
\[\mathcal{L}_{DBI} = -\sqrt{\det(g_{rs} + \mathcal{F}_{rs})} , \quad g_{rs} = (J^{(ab)}(a\bar{b})_{s} - J^{(a\bar{b})}_{(ab),s})/\text{tr} 1 \quad (3.10)\]
with \(J = d\sigma^r J_r , \quad r = 0, 1, \cdots, p + 1\), the tension \(T\) and the DBI field strength \(\mathcal{F}_{rs}\). The WZ term in super-AdS spaces has the form of \[19\] and it must satisfy following three criteria;
1. Its exterior derivative produces the correct field strength, \( d \mathcal{L}_{WZ,[p+1]} = H_{[p+2]} \) with \( dH_{[p+2]} = 0 \).

2. The action is kappa-symmetric.

3. Its flat limit reduces into the correct flat WZ term.

### 3.2 Super-D0-brane action

The action for a D0 brane is given as

\[
I_{D0} = \int d\tau (\mathcal{L}_{DBI,D0} + \mathcal{L}_{WZ,D0})
\]

\[
\mathcal{L}_{DBI,D0} = T \sqrt{g_{00}} , \quad g_{00} = (J_{0}^{(a\bar{b})} J_{(a\bar{b}),0} - J_{0}^{(a\bar{b})} J_{(\bar{a}\bar{b}),0})/2
\]

\[
\mathcal{L}_{WZ,D0} = \{ K\epsilon_{ab} (J_{0}^{ac} J_{c}^{b} + J_{a\bar{c}} J_{\bar{c}}^{\bar{b}}) + \bar{K}\epsilon_{\bar{a}\bar{b}} (J_{0}^{\bar{a}\bar{c}} J_{\bar{c}}^{\bar{b}} + J_{\bar{a}\bar{c}} J_{\bar{c}}^{\bar{b}}) \}
\]

where \( K \) and \( \bar{K} \) are determined by the kappa-symmetry as

\[
T^2 = K^2 + \bar{K}^2
\]

which agrees with Zhou’s result \(^6\).

This WZ term satisfies three criteria: The criterion 1. is confirmed easier. The two-form RR-field strength for a D0-brane is given by

\[
H_{[2]} = d\mathcal{L}_{WZ} = - \{ K\epsilon_{ab} (J_{0}^{ac} J_{c}^{b} + J_{a\bar{c}} J_{\bar{c}}^{\bar{b}}) + \bar{K}\epsilon_{\bar{a}\bar{b}} (J_{0}^{\bar{a}\bar{c}} J_{\bar{c}}^{\bar{b}} + J_{\bar{a}\bar{c}} J_{\bar{c}}^{\bar{b}}) \}
\]

where index 0 of \( J_{AB,0} \) has been omitted. The field strength \( H_{[2]} \) in a flat space contains bilinear of fermionic indices currents only. On the other hand the one in AdS\(^2\times S\(^2\) space contains bilinear of not only the fermionic indices currents but also the bosonic indices currents in order to make closed form \( dH_{[2]} = 0 \) \(^6\). For the expression of \( \mathcal{L}_{WZ} \) in \eqref{eq:3.11} this closure is trivial, i.e. \( dH_{[2]} = dd \mathcal{L}_{WZ} = 0 = ddJ \).

The criterion 2. is calculated as follows: If we introduce the following combination for an arbitrary variation \( \delta Z_M^A \)

\[
\Delta J^{AB} = Z^A M^B ,
\]

the kappa-variation is characterized by vanishing bosonic components

\[
\Delta_\kappa J^{ab} = 0 = \Delta_\kappa J^{a\bar{b}}
\]

as usual. In particular in this parameterization \( \Delta_\kappa J^{a\bar{b}} = \kappa^{a\bar{b}} \) and \( \Delta_\kappa J^{\bar{a}b} = \kappa^{\bar{a}b} \) \(^1\). Using\(^1\) in the canonical computation the fermionic constraints are given as \(^1\)

\[
\bar{D}_{ab} = D_{a\bar{b}} + \frac{\partial \mathcal{L}_{WZ}}{\partial \Theta^a_{m}} X_m^b = 0 \quad, \quad \bar{D}_{\bar{a}\bar{b}} = D_{ab} + \frac{\partial \mathcal{L}_{WZ}}{\partial \bar{\Theta}^a_{m}} \bar{X}_m^{\bar{b}} = 0
\]

with the local \( GL(2|2) \) generators, \( D_{AB} \)'s. The kappa-variation is generated by these fermionic constraints

\[
\delta_\kappa \mathcal{O} = \left[ \mathcal{O}, F_{a\bar{b}} \kappa^{a\bar{b}} + F_{a\bar{b}} \kappa^{\bar{a}b} \right] \quad, \quad \delta_\kappa Z_M^b = Z_M^a \kappa^{a\bar{b}} \quad, \quad \delta_\kappa Z_M^{\bar{b}} = Z_M^a \kappa^{\bar{a}b}
\]

with these parameters are projected into half.
the kappa-variation of currents,

\[ \delta_\kappa J^{ab} = J^{ac} \Delta_\kappa J^c_b - (\Delta_\kappa J^{ac}) J^c_b \]  

and the similar for the \( J^{\bar{a} \bar{b}} \), the kappa-variation of the D0-brane action becomes

\[ \delta_\kappa \mathcal{L}_{DBI} = \frac{T}{\sqrt{g_{00}}} \left\{ \Delta_\kappa J^{ab} \left( J^{ac} J^c_b - J^{\bar{a} \bar{b}} J^\bar{a}_{\bar{b}} \right) + \Delta_\kappa J^{\bar{a} \bar{b}} \left( J^{\bar{a} \bar{b}} J^\bar{c}_{\bar{c}} - J^{\bar{a} \bar{b}} J^\bar{b}_{\bar{b}} \right) \right\} \]  

\[ \delta_\kappa \mathcal{L}_{WZ} = -\Delta_\kappa J_{ab} \left( K \epsilon^{ab} J^b_\bar{b} + \bar{K} \epsilon^{\bar{a} \bar{b}} J^{\bar{a}}_a \right) - \Delta_\kappa J^{\bar{a} \bar{b}} \left( K \epsilon^{ab} J^a_\bar{a} + \bar{K} \epsilon^{\bar{a} \bar{b}} J^{\bar{b}}_b \right) \]  

The kappa-invariance of the action leads to

\[ \delta_\kappa (\mathcal{L}_{DBI} + \mathcal{L}_{WZ}) = (\Delta_\kappa J^{ab} J_{\bar{c} \bar{d}} + \Delta_\kappa J^{\bar{c} \bar{d}} J_{ab}) \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} \]  

\[ \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} = \frac{T}{\sqrt{g_{00}}} \left( J^{ad} \delta^{bc} - \delta^{ad} J^{bc} \right) - (K \epsilon^{ad} \delta^{bc} + \bar{K} \delta^{ad} \epsilon^{bc}) \]  

\[ 0 = \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} \Delta_\kappa J_{ab} = \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} \Delta_\kappa J_{\bar{c} \bar{d}} \]  

Equivalently projected kappa-parameters \( \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} \kappa_{ab} = 0 = \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} \kappa_{\bar{c} \bar{d}} \) with the relation \( \mathcal{P}^{a \bar{b} \bar{c} \bar{d}} \).  

The criterion 3. is evaluated as follows: Under the flat limit which is obtained by rescaling, X, ński → X/R, X/R, Θ, ński → Θ/√R, Θ/√R and \( \mathcal{L}_{WZ} \) → \( \mathcal{L}_{WZ}/R \) and \( \mathcal{L}_{DBI,D0} \) → \( \mathcal{L}_{DBI,D0}/R \), and R → ∞, the WZ term \( \mathcal{L}_{DBI} \) reduces into:

\[ \mathcal{L}_{WZ,flat} = -K \left\{ \epsilon_{ab} \Theta^{ma} d\Theta_m^b + \epsilon_{ab} \Theta^{ma} d\Theta_m^{\bar{b}} \right\} \]  

where \( K = \bar{K} \) is imposed by requiring 4-dimensional covariance and this is consistent with \[26\] [27].

The WZ term for a D0-brane in the pp-wave background is obtained by taking the Penrose limit \[25\] given in \[22\]. The WZ term is scaled under the Penrose limit as \( \mathcal{L}_{WZ} \) → \( \Omega \mathcal{L}_{WZ} \) as well as the DBI term \( \mathcal{L}_{DBI} \) → \( \Omega \mathcal{L}_{DBI} \). Since the antisymmetric components of currents and parameters are scaled as \( (J_{[ab]}, J_{[\bar{a} \bar{b}]}) \) → \( \Omega (J_{[ab]}, J_{[\bar{a} \bar{b}]}) \), \([X, \bar{X}] \) → \( \Omega ([X, \bar{X}] \), all terms in \( \mathcal{L}_{WZ} \) are survived. The LI current after taking the Penrose limit, which is still complicated function of \( x^- = x_+ \) and \( \theta^- \), is denoted as \( J_{pp,AB} \). Then the WZ term is written as

\[ \mathcal{L}_{WZ,pp} = \left\{ K \epsilon_{ab} \left( J^{ab}_{pp} - d(\delta^{am} X^b_m) \right) + \bar{K} \epsilon_{\bar{a} \bar{b}} \left( J^{\bar{a} \bar{b}}_{pp} - d(\delta^{\bar{a}m} \bar{X}^\bar{b}_m) \right) \right\} \]  

4 D0-brane charges

The Noether charges of global GL(2|2) symmetry with parameters \( \Lambda^{MN} \) are modified under the existence of the WZ term \[11\]

\[ \Lambda^{NM} G_{MN} \rightarrow \Lambda^{NM} \tilde{G}_{MN} = \Lambda^{NM} G_{MN} - U_\Lambda \ , \ \delta_\Lambda \mathcal{L} = dU_\Lambda \]  

(4.1)
Because of \( \delta_L \mathcal{L}_{DBI} = 0 = \delta_T J \), only the total derivative terms in the WZ term in (3.1) contribute to \( U \) which is given as

\[
U_A = - \left( K \epsilon_{ab} \delta^{am} \delta_{L} X_m^b + \bar{K} \epsilon_{\bar{a}\bar{b}} \delta^{\bar{a}m} \delta_{L} \bar{X}_m^{\bar{b}} \right) ,
\]

(4.2)

\[
\delta_L Z^A_M = \Lambda_M^L Z^A_L .
\]

(4.3)

This modified \( GL(2|2) \) generators are then expressed as

\[
\tilde{G}_{MN} = G_{MN} + T_{MN}
\]

\[
\left\{ T_{mn} = - K (\delta^a_m + X^a_m) \epsilon_{ab} \delta^n_b , \quad T_{m\bar{n}} = - \bar{K} \Theta^a_m \epsilon_{\bar{a}\bar{b}} \delta^n_{\bar{b}} \right\}
\]

(4.4)

and also re-expressed as

\[
\tilde{G}_{MN} = Z^A_M \bar{\Pi}^A_{AN}
\]

\[
\left\{ \bar{\Pi}_m = \Pi_m - K \epsilon_{ab} \delta^n_b , \quad \bar{\Pi}_{m\bar{n}} = \Pi_{m\bar{n}} - \bar{K} \epsilon_{\bar{a}\bar{b}} \delta^n_{\bar{b}} \right\}
\]

satisfying the same algebra of \( GL(2|2) \). Under the existence of the WZ term Noether charges \( \tilde{G}_{MN} \) are conserved and generate the closed superalgebra. Fermionic parts are identified as supercharges as \( \tilde{G}_{mn} \equiv Q_\text{AdS,m} \), \( \tilde{G}_{m\bar{n}} \equiv Q_\text{AdS,m} \).

while bosonic parts are interpreted as sum of the bosonic charges \( G_{MN} \) and topological charges \( T_{mn} \), \( T_{m\bar{n}} \) conventionally. Therefore commutators of supercharges for a D0-brane in the \( \text{AdS}_2 \times S^2 \) background are given by

\[
\{ Q_\text{AdS} , Q_\text{AdS} \} = 0 = \{ \bar{Q}_\text{AdS} , \bar{Q}_\text{AdS} \}
\]

\[
\{ Q_\text{AdS,m} , Q_\text{AdS,l} \} = \{ Q_\text{AdS,l} , Q_\text{AdS,m} \}
\]

(4.5)

\[
= - \delta^i_l (P^k_m + M^k_m + T^k_m) + \delta^k_m (P^i_n + M^i_n + T^i_n) + \delta^n_l \delta^k_m (D - D)
\]

where a center \( D - D = \text{Str} G_{MN} \) can be ignored. The D0-brane charges are \( T_{mn} \) and \( T_{m\bar{n}} \). They have position dependence so they are not center. The generators of this algebra are rather \( Q, \bar{Q}, \tilde{G}_{mn}, \tilde{G}_{m\bar{n}} \) than \( Q, \bar{Q}, P, M, T \).

Under the flat limit contributions from the WZ term are given as

\[
\left\{ T_{\text{flat},mn} = - K \delta^a_m \epsilon_{ab} \delta^n_b , \quad T_{\text{flat},m\bar{n}} = - \bar{K} \Theta^a_m \epsilon_{\bar{a}\bar{b}} \delta^n_{\bar{b}} \right\}
\]

(4.6)

and imposing four-dimensional symmetry leads to \( K = \bar{K} \). The commutators of supercharges reduce into as

\[
\{ Q_{\text{flat}}, Q_{\text{flat}} \} = 0 = \{ \bar{Q}_{\text{flat}}, \bar{Q}_{\text{flat}} \}
\]

\[
\{ Q_{\text{flat},m} , Q_{\text{flat},l} \} = \{ Q_{\text{flat},l} , Q_{\text{flat},m} \}
\]

\[
= - \delta^i_l (P^k_m + T_{\text{flat},m}^k) + \delta^k_m (P^i_n + T_{\text{flat},n}^i) .
\]

(4.7)

\[\text{We thank Kiyoshi Kamimura for pointing out that a total derivative term does not change the global symmetry algebra.}\]
This is the supertranslation algebra with a center for the D0-brane.

Under the Penrose limit contributions from the WZ term are given as
\[
\begin{align*}
\mathcal{T}_{pp, mn} &= -K(1 + X |_{x^+=0})_m^a \epsilon_{ab} \delta^n_b, \\
\mathcal{T}_{pp, m\bar{n}} &= -\tilde{K}(1 + \tilde{X} |_{x^+=0})_{\bar{m}}^{\bar{a}} \epsilon_{\bar{a}\bar{b}} \delta^\bar{n}_{\bar{b}}.
\end{align*}
\] (4.8)

The commutators of supercharges become
\[
\{Q_{pp}, \bar{Q}_{pp}\} = 0 = \{\bar{Q}_{pp}, Q_{pp}\}
\]
\[
\{ar{Q}_{pp, m}, Q_{pp, \bar{n}}\} = -\delta^n_l (G_{pp, m}^\bar{k} + \mathcal{T}_{pp, \bar{m}}^\bar{k}) + \delta^\bar{n}_m (G_{pp, \bar{l}}^m + \mathcal{T}_{pp, l}^m),
\] (4.9)

and the concrete expression is given by
\[
\begin{align*}
\{\bar{Q}_{pp, +, m}^n, Q_{pp, +, \bar{k}}^\bar{l}\} &= -\delta^n_l \left( \sqrt{2} P_+ - \sqrt{2} P_+ \right)_{m}^\bar{k} \\
\{\bar{Q}_{pp, -, m}^n, Q_{pp, -, \bar{k}}^\bar{l}\} &= -\delta^n_l \left\{ \left( \sqrt{2} P_- - \sqrt{2} P_- \right) \\
&\quad - \left( K x^1 - \bar{K} x^2 \right) \left( \begin{array}{cc} K x^1 - \bar{K} x^2 & 1 \\
-1 & 1 \end{array} \right) \right\}_{m}^\bar{k} + \delta^\bar{n}_m \delta^\bar{k}_m (D - \bar{D}) \\
\{\bar{Q}_{pp, +, m}^n, Q_{pp, -, \bar{k}}^\bar{l}\} &= -\delta^n_l \left\{ \left( P_2 - M_{32} \right) P_2 + M_{32} \right\} - \tilde{K} \left( \begin{array}{cc} 1 + \frac{x^-}{\sqrt{2}} & 1 - \frac{x^-}{\sqrt{2}} \\
-1 & 1 \end{array} \right) \right\}_{m}^\bar{k} \\
&\quad + \delta^\bar{n}_m \left\{ \left( P_1 - M_{01} \right) P_1 + M_{01} \right\} - K \left( \begin{array}{cc} 1 - \frac{x^-}{\sqrt{2}} & 1 + \frac{x^-}{\sqrt{2}} \\
-1 & 1 \end{array} \right) \right\}_{l}^n.
\end{align*}
\] (4.10)

The anticommutator \(\{Q_{pp, -, m}^n, Q_{pp, +, l}^\bar{k}\}\) satisfies the same relation with the last equation of (4.10) with opposite lightcone projection. This is the anticommutators of supercharges for a D0-brane in the 2+2 dimensional pp-wave background, where the super-pp-algebra part is consistent with [2, 23] by setting a center \(D - \bar{D} = \text{Str}G_{MN}\) to be zero. The D0-brane charge in the pp-wave background is independent on \(x^+\) but depend on \(x^i\)'s (\(i = 1, 2\)) and \(x^- (x^- = x_+ = p_+ \tau\) in the lightcone gauge). In the lightcone gauge all supersymmetries are broken except \(x^i = 0\), since \(Q_+\) are broken because of \(P_+ \neq 0\) and \(Q_-\) are also broken for \(x^i \neq 0\).

In order to examine the BPS condition let us compare with the case of a D0-brane in a flat space. The supercharge \(Q_\alpha\) and the fermionic constraint \(\bar{D}_\alpha\) satisfy the following
\{Q_\alpha, Q_\beta\} = -2C(P_\mu \gamma^\mu - T \gamma^{11}) = -2M_{\text{BPS}} \mathcal{P}_{+,\alpha\beta}
\Rightarrow \begin{cases} Q_+ = Q \mathcal{P}_+ & \text{broken} \Rightarrow \theta^+ \text{ Nambu-Goldstone mode} \\ Q_- = Q \mathcal{P}_- & \text{unbroken} \Rightarrow \theta^- \text{ independent observables} \end{cases} (4.11)
\{\tilde{D}_\alpha, \tilde{D}_\beta\} = 2C(P_\mu \gamma^\mu - T \gamma^{11}) = 2M_{\text{BPS}} \mathcal{P}_{+,\alpha\beta}
\Rightarrow \begin{cases} \tilde{D}_+ = \tilde{D} \mathcal{P}_+ & \text{second class} \Rightarrow 1/2 \text{ (in phase space) } \theta^+ \text{ physical} \\ \tilde{D}_- = \tilde{D} \mathcal{P}_- & \text{first class, } \kappa \text{ symmetry} \Rightarrow \theta^- \text{ gauged away} \end{cases}

with \{Q_\alpha, \tilde{D}_\beta\} = 0. The right hand side of \{Q, Q\} is the BPS projection operator while the one of \{\tilde{D}, \tilde{D}\} is the kappa-symmetry projection operator. For a particle case they coincide, but in general they are similar but not equal \cite{27}. The BPS projection operator is \mathcal{P}_{+,\alpha\beta} and the BPS mass is \mathcal{M}_{\text{BPS}} \geq T.

In flat space (mass)^2 classifies BPS states. In curved space a quadratic Casimir operator \(c_{[2]}\), which is the background covariant Hamiltonian, corresponds to (mass)^2. The quadratic Casimir operator in the super-AdS and the super-pp-wave spaces are given as \(c_{[2],sAdS} = (P_\mu)^2 + (M_\mu)^2\) and \(c_{[2],spp} = (P_\mu)^2 + (P^*_i)^2\) where \(P, M, P^*_i\) are translation, Lorentz rotation, boost parts of the Lorentz rotation generators respectively. In cases of AdS_2 x S^2 and four-dimensional pp-wave space, \(c_{[2],sAdS} = c_{[2],spp}\). Analogously to (4.11) the BPS projection operator \(\mathcal{P}_+\) is obtained from the right hand side of the anticommutator of supercharges (4.5) in the Majorana supercharge basis \cite{6}
\{Q_\alpha, Q_\beta\} \equiv -2\sqrt{c_{[2]} P_{+,\alpha\beta}}, (4.12)

and the quadratic Casimir is bounded below
\(c_{[2]} \geq K^2 + \bar{K}^2 = T^2\). (4.13)

If a D0-brane in a ground state has only non-zero value in \(P_0\) component and is located at the origin, the BPS projection operator is given as
\[\mathcal{P}_{+,\alpha\beta} = 1 - \epsilon_{IJ} \left( \frac{K}{\sqrt{K^2 + \bar{K}^2}} \gamma_0 \otimes 1 + \frac{\bar{K}}{\sqrt{K^2 + \bar{K}^2}} \gamma_0 \gamma \otimes \bar{\gamma} \right)\] (4.14)

where \(I = 1, 2\) for \(N = 2\) \(\gamma_\mu\) and \(\bar{\gamma}_\mu\) are gamma-matrices for AdS_2 and S^2 spaces and \(\gamma = \gamma_0 \gamma_1\) and \(\bar{\gamma} = \gamma_2 \gamma_3\). It can be seen that a D0-brane is a 1/2 supersymmetry preserving BPS state. For the four dimensional pp-wave background limit this form stays the same for a D0-brane. For the flat limit two kappa-coefficients become equal \(K = \bar{K}\), and reduce into the similar form with (4.11).

5 Conclusions

We have shown that in curved backgrounds Noether charges are also modified by the WZ term, and anticommutators of fermionic Noether charges produce a brane charge. The
The commutators of supercharges for a D0-brane in the AdS$_2$×S$^2$ and in the pp-wave limit are given in (4.5) and (4.10), and brane charges are given in (4.4) and (4.8). These contributions from the WZ term are absorbed in conjugate momenta, and the resultant algebras are the same algebra with the original super-AdS$_2$×S$^2$/super-pp-wave algebras. The brane charges have position dependence; $x^\mu$ ($\mu = 0, 1, 2, 3$) dependence in the AdS, $x^-, x^i$ ($i = 1, 2$) dependence in the pp-wave limit and no position dependence in a flat limit. In the lightcone gauge, where $P_+ \neq 0$ then $Q_+$’s are broken, the dynamical supercharges $Q_-$ are preserved only for $x^i = 0$ as discussed in [8]. In the static gauge half supersymmetry condition is given by the BPS projection $P_+$ where the square of the quadratic Casimir is bounded below with the value $T = \sqrt{K^2 + \bar{K}^2}$. It is natural to expect the above properties for general $p$-branes.

It is also important to examine general brane configurations. This background covariant approach presents more general description including a D0-brane, which is excluded in the lightcone gauge approach since the lightcone gauge restricts $x^\pm$ to be Neumann boundary condition. Toward better understanding of D-branes, dualities, gauge theory correspondence and quantum theories further studies are required.

Acknowledgments

We thank Kiyoshi Kamimura, Makoto Sakaguchi and Warren Siegel for stimulating discussions and help of making colour tables.

References

[1] L. Maldacena, Adv. Theor. Math. Phys. 2 (1998) 231, hep-th/9711200
S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B248 (1998) 105, hep-th/9802109
E. Witten, Adv. Theor. Math. Phys. 2 (1998) 253, hep-th/9802150

[2] J. Kowalski-Glikman, Phys. Lett. B134 (1984) 194;
C.M. Hull, Phys. Lett. B139 (1984) 39;
P.T. Chrusciel and J. Kowalski-Glikman, Phys. Lett. B149 (1984) 107;
J. Figueroa-O’farrill and G. Papadopoulos, J. High Energy Phys. 0108 (2001) 036, hep-th/0105308
M. Blau, J. Figueroa-O’farrill, C. Hull and G. Papadopoulos, J. High Energy Phys. 0201 (2002) 047, hep-th/0110242
[3] D. Berenstein, J. Maldacena and H. Nastase, *J. High Energy Phys.* 4 (2002) 013, hep-th/0202021
[4] R.R. Metsaev and A.A. Tseytlin, *Nucl. Phys.* B533 (1998) 109, hep-th/9805028.
[5] For example,
  R. Kallosh and J. Rahmfeld, *Phys. Lett.* B443 (1998) 143, hep-th/9808038.
  J. Rahmfeld and A. Rajaraman, *Phys. Rev.* D60 (1999) 64014, hep-th/9809164.
  I. Pesando, *J. High Energy Phys.* 11 (1998) 002, hep-th/9808020; *J. High Energy Phys.* 2 (1999) 007, hep-th/9809145.
  J. Park and S-J. Rey, *J. High Energy Phys.* 1 (1999) 001, hep-th/9812062.
[6] J-G. Zhou, *Nucl. Phys.* B559 (1999) 92, hep-th/9906013.
[7] R.R. Metsaev, *Phys. Rev.* D65 (2002) 126004, hep-th/0202109.
[8] M. Billó and I. Pesando, *Nucl. Phys.* B536 (2002) 121, hep-th/0203028.
  A. Dabholkar and S. Parvizi, *Nucl. Phys.* B641 (2002) 223, hep-th/0203231.
  D. Bak, hep-th/0204033.
  K. Skenderis and M. Taylor, *J. High Energy Phys.* 6 (2002) 025, hep-th/0204054.
  P. Lee, J. Park, *Phys. Rev.* D67 (2003) 0206002, hep-th/0203257.
  P. Bain, P. Messen and M. Zamarkar, “Supergravity solutions for D-branes in Hpp wave backgrounds”, hep-th/0205106.
  O. Bergman, M.R. Gaberdiel, M.B. Green, D-brane interactions in type IIB plane-wave background, hep-th/0205183.
  Y. Michishita, *J. High Energy Phys.* 10 (2002) 048, hep-th/0206131.
[9] For example;
  M. Cvetic, H. Lu and C.N. Pope, hep-th/0203082, *Nucl. Phys.* B644 (2002) 65, hep-th/0203229.
  A. Kumar, R.R. Nayak and Sanjay, *Phys. Lett.* B541 (2002) 183, hep-th/0204025.
  H. Singh, *M5-branes with 3/8 supersymmetry in pp-wave background* hep-th/0205020.
  M. Alishahiha and A. Kumar, *Nucl. Phys.* B542 (2002) 130, hep-th/0205134.
  Y. Hikida and Y. Sugawara, *J. High Energy Phys.* 10 (2002) 67, hep-th/0205200.
[10] D. Olive and E. Witten, *Nucl. Phys.* B78 (1978) 97;
    E. Witten. *Nucl. Phys.* B443 (1995) 85, hep-th/9503124
[11] J.A. de Azcarraga and P.K. Townsend, Phys. Rev. Lett. 62 (1989) 2579.
[12] P.K. Townsend, “*M theory from its superalgebra*”, in “Cargese 1997, Strings, branes and dualities” 141-177, hep-th/9507048.
[13] P.K. Townsend, “*P-brane democracy*” in PASCOS/Hopkins 1995:0271-286 (QCD161:J55:1995), hep-th/9507048.
[14] J.A. de Azcarraga, J.M. Izquierdo and P.K. Townsend, *Phys. Lett.* B267 (1991) 366.
[15] N. Berkovits, M. Bershadsky, T. Hauer, S. Zhukov and B. Zwiebach, *Nucl. Phys.* B567 (2000) 61, hep-th/9907200.
[16] R. Roiban and W. Siegel, *J. High Energy Phys.* 11 (2000) 024, hep-th/0010104.
[17] M. Hatsuda, K. Kamimura and M. Sakaguchi, *Phys. Rev.* **D62** (2000) 105024, hep-th/0007009
[18] M. Hatsuda and K. Kamimura, *Nucl. Phys.* **B611** (2001) 77, hep-th/0106202
[19] M. Hatsuda and M. Sakaguchi, *Phys. Rev.* **D 66** (2002) 045020, hep-th/0205092
[20] K. Sugiyama and K. Yoshida, *Nucl. Phys.* **B644** (2002) 113, hep-th/0206070; *Phys. Lett.* **B546** (2002) 143, hep-th/0206132
S. Hyun, H. Shin, *Nucl. Phys.* **B543** (2002) 115, hep-th/0206090
[21] J-H. Park, *J. High Energy Phys.* **10** (2002) 32, hep-th/0208161
K.M. Lee, *Phys. Lett.* **B549** (2002) 213, hep-th/0209009
N. Nakayama, K. Sugiyama and K. Yoshida, “Ground state of supermembrane on pp wave”, hep-th/0209081
S. Hyun and J-H. Park, *J. High Energy Phys.* **10** (2002) 70, hep-th/0209219
[22] K. Kamimura and M. Sakaguchi, “osp(1|32) and extensions of super-AdS$_5 \times$S$^5$ algebra”, hep-th/0301083
[23] M. Hatsuda, K. Kamimura and M. Sakaguchi, *Nucl. Phys.* **B632** (2002) 114, hep-th/0202190; *Nucl. Phys.* **B637** (2002) 168, hep-th/0204002
[24] M.B. Green, *Phys. Lett.* **B223** (1989) 157;
W. Siegel, *Phys. Rev.* **D50** (1994) 2799, hep-th/9403144
[25] M. Hatsuda, K. Kamimura and M. Sakaguchi, *Nucl. Phys.* **B644** (2002) 40, hep-th/0207157
[26] M. Cederwall, A. von Gussich, B.E.W. Nilsson and A. Westerberg, *Nucl. Phys.* **B496** (1997) 163, hep-th/9610148
M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *Nucl. Phys.* **B490** (1997) 179, hep-th/9611159
M. Aganagic, C. Popescu and J.H. Schwarz, *Phys. Lett.* **B393** (1997) 311, hep-th/9610249; *Nucl. Phys.* **B495** (1997) 99, hep-th/9612080
E. Bergshoeff and P.K. Townsend, *Nucl. Phys.* **B490** (1997) 145, hep-th/9611173
[27] M. Hatsuda, K. Kamimura, *Nucl. Phys.* **B520** (1998) 493, hep-th/9708001