Analysis of the Machining Stability in Milling Thin-walled Plate*

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Abstract—The thin-walled plate parts are widely used in the aeronautical industry. Research on the machining stability in end milling of thin-walled plate is of great significance to improve the materials remove rates and optimize milling parameters. Analyzing the influence of milling parameters on the milling stability is conducive to choosing appropriate cutter and milling machine in actual milling. In this study, statistical variances of the dynamic displacements are employed as a chatter detection criterion to acquire the stability lobe diagram. The milling experiment results show that the obtained stability lobe diagram can predict the stability domain well. Based on this stability theory, the influences of modal parameters and tooth number on the machining stability are analyzed systematically by simulation, which is significant for choosing an appropriate cutting condition.

Keywords: thin-walled plate; machining stability; stability lobe diagram; modal parameters

I. INTRODUCTION

Nowadays, more and more aircraft parts are composed of thin-walled plates. Due to the low rigidity of this type of structures, milling of thin-walled plates faces the risk of instability of the machining process at high removal rate conditions. The instability of the process is an undesirable vibration phenomenon known as chatter, which not only causes poor machining quality but also limits high productivity. Figure 1 shows the comparison of the finished surfaces between chatter-free and chatter operations.

![Comparison of the finished surfaces under different milling conditions](image)

This has demanded the development of new techniques and models for chatter model in milling thin-walled plates. Thus, analyzing the machining stability is critical for monitoring milling process and realizing chatter suppression. A stability lobe diagram (SLD) can be used for selecting milling parameters to avoid chatter and then be treated as a constraint in optimizing milling parameters.

Various methods, including analytical, numerical and experimental, have been put forward to predict the SLD which distinguishes chatter-free operations from the instability. Altintas and Budak [1] developed the stability method by adopting the zeroth order approximation and keeping only the average item. The critical axial depths of cut were acquired based on frequency sweeping around the milling system's natural frequencies. Insperger and Stepan [2,3] presented the semi-discretization method for stability analysis, where the delayed terms are discretized and the non-delayed terms are unchanged. Ding presented a full-discretization method based on the direct integration scheme to predict milling stability [4] and the numerical integration method where approximating the delayed differential equation by a series of algebraic equations [5]. These methods have high computational efficiency and don’t lose any numerical precision. Quo [6] updated the full-discretization method by a third-order Newton’s interpolation theory. Bayly et al. [7] investigated the milling stability by taking advantage of the temporal finite element analysis method. The Chebyshev polynomial based method [8] and the Chebyshev collocation method [9] are also effective numerical methods for analysis of milling stability. Sims et al. [10] researched the milling stability limits with the help of a fuzzy logic algorithm in order to accommodate uncertainty or variability in the model input parameters. Fansen and Junyi [11] adopted fuzzy methods to interpret the onset of stability in experimental machining tests. However, the present contribution is primarily concerned with the prediction of chatter stability, rather than experimental identification methods. Fuzzy stability lobes were mainly used for the case of single-degree-of-freedom structural dynamics, with a fuzzy-valued damping ratio. Yao et al. [12] proposed a method based on wavelets and support vector machines for chatter identification. Al-Regib and Ni [13] developed a normalized chatter detection index based on the Wigner time-frequency distribution. Li et al. [14] put forward a complete discretization scheme for milling stability prediction, where all time-dependent items of the DDE were discretized. Khasawneh and Mann [15] presented a spectral element approach for stability analysis of delay systems. Compean et al. [16] investigated the enhanced multistage homotopy perturbation method for milling...
stability analysis. Cao et al. [17] investigated an effective chatter identification method for the end milling process based on the study of two advanced signal processing techniques, i.e., wavelet package transform (WPT) and Hilbert-Huang transform (HHT). In this study, HHT with WPT as a preprocessor was introduced to detect the chatter in the milling process. The vibration signals were first decomposed by WPT, and then the wavelet packets with rich chatter information were selected for HHT. The time-frequency analysis (TFA) methods which map the one-dimensional signal to a two-dimensional time-frequency plane have a great potential to detect the chatter in the machining process. It can determine not only the time of the chatter occurring but also the frequency ranges of the chatter. Obviously, HHT does not involve the concept of frequency resolution or time resolution as in traditional TFA methods but represents the instantaneous frequency. Thus, the Hilbert-Huang spectrum has uniform resolution in the full frequency range. In addition, Quintana et al. [18] identified the stability lobe diagram using the experimental method.

According to the stability theory developed by Schmitz [19], statistical variances are considered as chatter detection criteria. Based on the theory, statistical variances of the dynamic displacements of the workpiece are used for judging whether chatter occurs in milling thin-walled plates. The time-domain stability limits in milling thin-walled plates are obtained by applying the chatter detection criteria. However, the influence of milling parameters on the stability can not be expressed explicitly. So, the influence of modal parameters on the machining stability is analyzed by simulation, which can provide reference for choosing appropriate cutter and milling machine in actual milling.

II. STABILITY LOBES AND EXPERIMENT VERIFICATION

A. Chatter Model in Milling Thin-walled Plate

![Figure 2. Thin-walled plate in end milling](image)

The whole tool system is more rigid than the thin-walled plate (Figure 2), so the workpiece-holder system is considered flexible and can be reduced to a 2-degree-of-freedom as shown in Figure 3, where the rigidity in the Z direction is supposed to be high when compared to the other two directions. \( X \) and \( Y \) are the directions of feed and perpendicular to the machined surface, respectively.

Figure 3. Dynamic model of the system in milling thin-walled plate

\[
m_a, c_m, k_m (a = x, y) \text{ represent the modal mass, modal damping and modal stiffness of the workpiece-holder system in the two directions, respectively. The dynamics of the workpiece-holder system are described by following two second order ordinary differential equations for each degree-of-freedom:}
\[
\begin{align*}
mx''(t) + c_m x'(t) + k_m x(t) &= F_x(f, a_f, r, A, \omega, t) \\
m_y''(t) + c_m y'(t) + k_m y(t) &= F_y(f, a_f, r, A, \omega, t)
\end{align*}
\]

(1)

where \( f \) is the feed per tooth, \( a_f \) is axial depth of cut, \( r \) is the radius of the cutter, \( A \) is the phase angle between two adjacent teeth, \( \omega \) is the spindle angular velocity, \( t \) is the time; \( F_x \) and \( F_y \) are the cutting forces in the \( X \) and \( Y \) directions, respectively, which are the functions of the chip thickness referring to the document [20].

With the help of the software MATLAB, the fourth order of precision Runge-Kutta method is used to solve Eq. (1) in time domain solution by integrating the differential equations. Then, the dynamic displacements of the workpiece in the \( X \) and \( Y \) directions can be figured out. Statistical variances of the dynamic displacements are employed as a criterion to detect chatter. The expressions of statistical variances are:

\[
\sigma_x^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1} ; \sigma_y^2 = \frac{\sum_{i=1}^{n} (y_i - \bar{y})^2}{n-1}
\]

(2)

where \( \sigma_x^2 \) and \( \sigma_y^2 \) are the statistical variances in the \( X \) and \( Y \) directions. \( n \) is the number of time increments, \( x_i \) and \( y_i \) are the displacements of the workpiece in the \( X \) and \( Y \) directions calculated from Eq.(1). \( \bar{x} \) and \( \bar{y} \) are the average displacements in the two directions. They are depicted as:

\[
\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}; \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}
\]

(3)

It is considered that chatter occurs when the statistical variances are bigger than the value of 1 \( \mu m^2 \). In other words, if the statistical variance in the \( X \) or \( Y \) direction reaches 1 \( \mu m^2 \), the system is in the critical state and the stability domain is determined. Based on this stability...
theory, the stability lobe diagram in milling thin-walled plate is determined.

B. Identification of Modal Parameters and Experiment Verification

The part is machined on ME650 three-dimensional vertical machining center. The cutter used in experiments is a four-flute flat-end milling cutter (diameter 16 mm and helix angle 30 degrees), which is made of solid carbide and coated with TiSiN. The size of the thin-walled plate in this study is $160 \times 100 \times 6$ mm. The material of the workpiece selected for experiments is die steel NAK80, the hardness of which is HB 344-400. The physical properties are shown in Table 1.

| Tensile strength $\sigma_b$/MPa | Elongation $\delta$/% | Contraction $\psi$/% | Yield $\sigma_y$/MPa | Modulus of elasticity $E$/GPa |
|-------------------------------|----------------------|----------------------|----------------------|-----------------------------|
| 1319                          | 14.6                 | 51.2                 | 1186                 | 199                         |

Modal testing techniques can be used to measure the frequency response functions (FRF) of the mechanical structures, and then modal mass, modal damping, modal stiffness of the workpiece-holder system in the $X$ and $Y$ directions can be deduced by analyzing FRF. The dynamic signal analysis system CRAS is used to measure FRF in the impact test. The basic flow chart applied to identify modal parameters is presented in Figure 4.

![Flow chart](image)

Table 1 Physical properties of NAK80

In order to ensure the accuracy and efficiency of the method to obtain the stability, axial depth of cut increment $\Delta a_p$ is 0.02 mm and spindle speed increment $\Delta n_0$ is 100 rev/min in this study. In addition, the feed per tooth $f$ is 0.15 mm. Based on the theory, the stability lobe of the spindle speed and axial depth of cut is presented in Figure 5 by using MATLAB software. The stability is derived in the spindle speed range from 2000 to 16000 rev/min.

Several groups of up milling experiments are performed from 2000 to 10000 rev/min spindle speed with increment of 1000 rev/min and constant feed per tooth of 0.15 mm. And the radial depth of cut $a_p$ is 1 mm. The cutting forces in the $X$ and $Y$ directions are measured by Kistler 9257B and analyzed by the fast Fourier transform (FFT) in the frequency domain. Subsequently, we can assess whether chatter occurs by analyzing the frequency spectra distributions of the cutting forces. Figure 5 presents the initial axial depths of cut when chatter happens determined experimentally at different spindle speeds when feed per tooth is 0.15 mm.

![Stability lobes](image)

The comparison between the proposed theoretical chatter model and the experimentally acquired stability limits is presented in Figure 5. It can be seen that the initial axial depths of cut determined experimentally when chatter happens correspond well with the predicted stability lobes.

III. ANALYSIS OF INFLUENCE FACTORS ON MILLING STABILITY

It can be seen from the process to obtain the stability that both the geometric parameters of the cutter and the modal parameters of the milling system affect the stability. And the influence cannot be expressed explicitly. Hence, we can analyze the influence of tooth number and modal parameters on the machining stability by simulation. It contributes to acquiring larger stability domain and improving milling efficiency. The following simulations are to study the
influences of single model parameters and tooth number on machining stability.

A. Influence of Natural Frequency on Milling Stability

Three simulations are conducted by changing the natural frequency $\omega_n$ in the $X$ direction and keeping the remaining modal parameters fixed. The results are shown in Figure 6. It can be seen from Figure 6 that the stability curves shift rightwards with the increase of natural frequency whereas the values of peaks and valleys of the stability curves remain unchanged. That is to say, natural frequency makes the stability lobes shift horizontally.

![Figure 6. Influence of natural frequency on the stability](image)

B. Influence of Damping Ratio on Milling Stability

Similarly, the influence of damping ratio on milling stability is investigated by merely changing the damping ratio $\zeta$ in the $Y$ direction. The simulation results in Figure 7 show that both the values of peaks and valleys of the stability curves are advanced with the increase of damping ratio, nevertheless, the peaks have less increase than the valleys. Moreover, the horizontal positions of the peaks and valleys still stay the same.

![Figure 7. Influence of damping ratio on the stability](image)

C. Influence of Stiffness on Milling Stability

Under the condition of changing stiffness $k$ in the $X$ direction and identical remaining modal parameters, several simulations results in Figure 8 are used to reveal the influence of stiffness on milling stability. As the stiffness increases, the stability curves shift upwards and the critical axial depths of cut increase whereas the horizontal positions of the stability lobes keep invariant. So it is deduce that stiffness makes the stability lobes shift vertically.

![Figure 8. Influence of stiffness on the stability](image)

D. Influence of Tooth Number on Milling Stability

It is assumed that the modal parameters and the milling condition remain unchanged, several simulations are performed by changing the tooth number of the cutter $N$ and the results are depicted in Figure 9. In Figure 9, as the tooth number $N$ decreases, the minimal value of critical axial depth of cut $a_{plim}$ increases and the acquired stability lobes move to the right. Besides, the stability domain becomes larger and the possibility of chatter falls. Namely, chatter are more prone to happen when the cutter has more teeth with other milling condition constant.

![Figure 9. Influence of tooth number on the stability](image)

IV. CONCLUSIONS

1. A method to predict the stability lobe diagram in the milling of thin-walled plate is presented. The statistical variances of the dynamic displacement of the workpiece are employed as chatter detection criteria to determine the stability limits.

2. The acquired stability lobe diagram is validated by
milling thin-walled plates. Results show acceptable agreement between the theoretically predicted and experimentally measured stability lobe diagrams.

3. The influences of modal parameters and tooth number on the milling stability are analyzed. These parameters have effects on the milling stability, but there are differences among their effects. The natural frequency makes the stability lobes shift horizontally while the damping ratio and the stiffness affect the stability lobes vertically. Tooth number contributes to influencing the stability lobes in both the two directions. These findings may be utilized as a guide for design machining plan and choosing appropriate milling conditions.

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REFERENCES

[1] Y. Altintas, E. Budak, Analytical prediction of stability lobes in milling, CIRP Annals-Manufacturing Technology. vol. 44, pp. 357-362, 1995.
[2] T. Insperger, G. Stepan, Semi-Discretization Method for Delayed Systems, Int J Numer Methods Eng. vol. 55, pp. 503-518, 2002.
[3] Insperger, G. Stepan, J. Turi, On the Higher-Order Semi-Discretizations for Periodic Delayed Systems, J. Sound Vib, vol. 313, pp. 334-341, 2002.
[4] Y. Ding, L.M.Zhu, X.J. Zhang, H. Ding, A full-discretization method for prediction of milling stability, Int J Mach Tools Manuf, vol. 50 pp. 502-509, 2010.
[5] Y. Ding, L.M.Zhu, X.J. Zhang, H. Ding, Numerical integration method for prediction of milling stability, Journal of Manufacturing Science and Engineering, vol. 133, no. 031005, 2011.
[6] Q. Quo, Y. Sun, Y. Jiang, On the Accurate Calculation of Milling Stability Limits Using Third-Order Full-Discretization Method, Int. J. Mach. Tools Manuf., vol. 62, pp. 61-66, 2012.
[7] P.V. Bayly, J.E. Halley, B.P. Mann, M.A. Davies, Stability of Interrupted cutting by temporal finite element analysis, Journal of Manufacturing Science and Engineering, vol. 125, no. 220, 2003.
[8] E.A. Butcher, H. Ma, E. Bueler, V. Averina, Z. Szabo, Stability of Linear Time-Periodic Delay-Differential Equations Via Chebyshhev Polynomials, Int. J. Numer. Methods Eng., vol. 59, pp. 895-922, 2004.
[9] E.A. Butcher, O.A. Bobrenkov, E. Bueler, P. Nindjua, Analysis of Milling Stability by the Chebyshov Collocation Method: Algorithm and Optimal Stable Immersion Levels, ASME J. Comput. Nonlinear Dyn., vol. 4, no. 031003, 2009.
[10] N.D. Sims, G. Manson, B. Mann, Fuzzy stability analysis of regenerative chatter in milling, Journal of Sound and Vibration, vol. 329, pp. 1025 – 1041, 2010.
[11] F. Kong, J. Yu, Study of fuzzy stochastic limited cutting width on chatter, The International Journal of Advanced Manufacturing Technology, vol.33 (7), pp.677-683, 2007.
[12] Z.H. Yao, D.Q. Mei, Z.C. Chen, On-line chatter detection and identification based on wavelet and support vector machine, Journal of Materials Processing Technology, vol.210 (5), pp.713-719, 2010.
[13] E. Al-Regib, J. Ni, Chatter detection in machining using nonlinear energy operator, Journal of Dynamic Systems Measurement and Control-Transactions of the ASME. vol.132 (3), 2010.
[14] M. Li, G. Zhang, Y. Huang, Complete Discretization Scheme for Milling Stability Prediction, Nonlinear Dyn., vol.71(1), 2013.
[15] F.A. Butcher, O.A. Bobrenkov, E. Bueler, V. Averina, Z. Szabo, Stability of Linear Periodic Delay Differential Equations via Chebyshov Polynomials, Int. J. Numer. Methods Eng., vol. 59, pp. 895-922, 2004.
[16] E. Butcher, H. Ma, E. Bueler, V. Averina, Z. Szabo, Stability of Linear Time-Periodic Delay-Differential Equations Via Chebyshnev Polynomials, Int. J. Numer. Methods Eng., vol. 59, pp. 895-922, 2004.
[17] E. Butcher, O.A. Bobrenkov, E. Bueler, P. Nindjua, Analysis of Milling Stability by the Chebyshov Collocation Method: Algorithm and Optimal Stable Immersion Levels, ASME J. Comput. Nonlinear Dyn., vol. 4, no. 031003, 2009.
[18] N.D. Sims, G. Manson, B. Mann, Fuzzy stability analysis of regenerative chatter in milling, Journal of Sound and Vibration, vol. 329, pp. 1025 – 1041, 2010.