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Inverse Directional Simulation: an environmental contour method providing an exact return period

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Abstract. This study presents an improved approach for drawing environmental contours, derived from the inverse First Order Reliability Method (iFORM). It is shown that the approach used in the iFORM method is not accurate as it only takes into account a fraction of the probability space behind the contour line – which means that the iFORM method provides inherently non-conservative environmental contours. iDS, an alternative solution based on the chi-square distribution is suggested, and its adequacy is demonstrated with a Monte Carlo simulation. An expression is derived for computing sector-constrained environmental contours where the exceedance probability is contained within specific range of interest for one or more variables. The newly suggested environmental contour methods are applied on measured wind and turbulence time series and compared with the iFORM method.

1. Introduction

Environmental contours [1] are widely used in the wind, offshore and marine industries as means for characterizing the joint statistical distribution for extreme combinations of stochastic environmental conditions such as wind speed, turbulence, wave heights and periods, and gust magnitudes [2][3][4][5]. The approach typically consists of establishing the joint statistical distribution of the variables in physical space, after which a Rosenblatt transformation [6] is applied to map the physical variables into a set of i.i.d. (independent and identically distributed) variables. Due to the properties of the i.i.d. space, it is straightforward to draw a contour with constant exceedance probability, which is then converted to a contour in physical space by applying an inverse transformation. The iFORM (inverse First-Order Reliability Method)[7][2] is the first published method for generating environmental contours and is one of the most widely used methods nowadays. In iFORM, the i.i.d. space is based on the standard normal distribution, and the environmental contours are drawn as circles with radius computed through the use of multivariate normal distribution probability. iFORM is also the basis for the definition of the Extreme Turbulence Model (ETM) load case defined by the IEC61400-1, ed.3 design standard [8], meaning that it has an impact on wind turbine design.

Environmental contours, and specifically the joint distributions of variables in physical space, are normally derived from measured data spanning a significantly shorter period than the extreme event return periods that are required in structural design. As a result there is uncertainty associated with the contour predictions, and it is challenging to validate the models. There are multiple studies (see [3] for a detailed overview) which offer various modifications and improvements of the environmental contour method. Many of them focus on the data-related side of the problem and how to define accurate joint
distributions. A more overlooked problem is the computation of the contour radius in transformed (standard normal) space. Ref. [5] notes that the iFORM-based contours are non-conservative and suggest a modification to rectify this, while [4] suggests a method to avoid the transformation to i.i.d. space entirely. In the present study, the focus is on improving the accuracy and versatility of the procedure for computing the contour radius in i.i.d. space. It is shown that the approach used in the iFORM method is not accurate as it only takes into account a fraction of the probability space behind the contour line – which means that the iFORM method provides inherently non-conservative environmental contours. An alternative solution based on the chi-square distribution is suggested and its adequacy is demonstrated with a Monte Carlo simulation. An expression is derived for computing environmental contours where the exceedance probability is contained within a specific range of interest for one or more of the individual variables. Finally, the newly suggested environmental contour methods are applied on measured wind and turbulence time series and compared with the iFORM method.

2. The iFORM method
The iFORM (inverse First-Order Reliability Method)[7] is a useful statistical tool which in the last 30 years has been applied in wind energy and other industries for defining return period contours for extreme combinations of stochastic environmental conditions. The main idea behind the method is that a certain critical event which goes beyond a defined boundary (i.e., a return period contour), is considered analogous to a failure in a structural reliability problem. This means that the size of the return period contour can be considered analogous to the reliability index $\beta$, defined as the distance from the origin (the point where all variables have their mean values) to the limit state (failure) surface. The classical FORM (First Order Reliability Method) approach [9] employs a useful transformation where all variables are converted to standard normal space:

$$U_i = \Phi^{-1}(F(X_i)), \quad i = 1, \ldots, M,$$

where $\Phi()$ denotes the standard normal cumulative distribution function (cdf) and $\Phi^{-1}()$ the inverse cdf, $X_i, i = 1, \ldots, M$ is the $M$-dimensional vector of stochastic variables, and $F(X)$ denotes the cdf of the variables in physical space. The transition from the joint distribution in physical space to i.i.d., standard normal space is carried out with either the Rosenblatt transformation [6] or the Nataf transformation [10]. In the standard normal space, the reliability index $\beta$ is computed by finding the design point, i.e., the point on the limit state surface which has highest probability and – taking a linear approximation to the limit state surface in the vicinity of the design point. Under these conditions, the probability of failure is straightforward to compute as

$$p_f = \Phi(-\beta)$$

As the name suggests, the iFORM makes use of the above procedure, but in an inverse order. First, the probability of “failure” – i.e., probability of exceedance of some combination of environmental factors, is computed based on the required return period as $p_r = \frac{T_0}{T_R}$, where $T_0$ is the reference period (e.g. the time step of data or measurements) and $T_R$ is the return period converted to the same units as the reference period. Then, given the properties of the standard normal space, it follows that any return period contour can be represented as an ideal circle (Figure 1). Under the assumption that the linear approximation of the contour surface is sufficient (we will elaborate on this later), the radius of a return period contour circle in standard normal space can be defined as

$$\beta = \Phi^{-1}(1 - p_r)$$

Now, the contour can be drawn in standard normal ($U$-) space, by simply defining a variable $\theta \in [0,2\pi)$, and plotting $U_1 = \cos \theta$ against $U_2 = \sin \theta$. Finally, we can obtain the environmental contour in physical space by taking the inverse transformation to the one we used to get to $U$-space:

$$X_1 = F_{X_1}^{-1}(\Phi(U_1)), \quad X_2 = F_{X_2|X_1}^{-1}(\Phi(U_2))$$

In order to properly take the conditional dependence between the variables into account, we need to also use a Rosenblatt transformation, hence the distribution of $X_2$ is given as conditional on $X_1$. 


Figure 1. Illustration of the probability space outside an environmental contour, and the representation based on linear approximation using the First-Order Reliability Method.

Making use of the abovementioned linear approximation, both the FORM and iFORM have been successfully applied to numerous engineering problems [3]. In most reliability analysis problems, the limit state surface encloses a relatively small fraction of the probability space, and a linear approximation close to the design point is a fair representation. However, for a general case where we are interested in the contours over a wide range of conditions, the linear approximation of the limit state surface is not sufficient—which is clearly seen on Figure 1. In contrast to a “classical” limit state surface, the environmental contour is very nonlinear and encloses a large part of the space. Thus, when we are interested in the full environmental contour, applying the commonly used formula for the contour radius, \( \beta = \Phi^{-1} \left(1 - \frac{T_0}{T_{R}}\right) \), will not result in the desired return period. This is demonstrated on Figure 2, where the equivalent of 100 years of random pairs of wind speed and turbulence realizations are generated using a Monte Carlo simulation, assuming wind conditions corresponding to IEC Class 1A climate. The wind speed \( V \) is Weibull distributed with parameters \( A = 11.28 \) and \( k = 2 \), while the turbulence is lognormal and conditionally dependent on the wind speed, with mean value \( E(\sigma_V) = 0.16(0.75V + 3.8) \) and standard deviation \( \sigma_{\sigma_V} = 0.16 \times 1.4 \). For comparison, the 1-year return period contour as computed by the iFORM is drawn in red. Given the 100 years worth of samples, it is expected that approximately 100 realizations should fall outside the 1-year contour. However, in this case about 1100 samples are outside the contour, showing that the iFORM underestimates the exceedance probability by about an order of magnitude. This has implications for, among other, wind turbine design as the environmental contours are used for multiple purposes—one example is the extreme turbulence model (ETM) which is the basis of design load case DLC1.3 in the IEC61400-1 standard [8], and which represents the 50-year return period combination of wind and turbulence as computed by iFORM.

3. Inverse Directional Simulation (iDS)

Despite the importance of this inaccuracy in the application of the iFORM method, the problem has not been widely studied in scientific literature. A study by Chai & Leira [5] points the issue out and suggests that a second-order surface (a hypersphere) approximation and a contour radius calculated using a chi-square \( (\chi^2) \) distribution can represent a conservative estimate of the environmental contours. In the present study, it is suggested that the linear limit state approximation is replaced by a hypersphere in standard normal space, and that this is in fact an exact solution for the return period. This approach is inspired by the Directional Simulation method used in structural reliability problems [11],[12], and therefore will be referred to as inverse Directional Simulation (iDS). Given that in standard normal space the variables \( U_i \) are independent and Normally distributed, the sum of their squares will by definition follow a chi-square \( (\chi^2) \) distribution. Then, for any “direction” – i.e., a unit
vector $\bar{\alpha} = \mathbf{u} / \| \mathbf{u} \|$ defined by the relative magnitudes of the components of the $M$-dimensional input variable vector $\mathbf{u} = (u_1, u_2, ..., u_M)$, the cdf of the chi-square distribution will correspond to the probability that the vector magnitude in this direction is smaller or equal than a given value. The exceedance probability (one minus the cdf) will correspond to the probability that the vector magnitude is larger or equal than the given value [11]. Since all variable distributions in the $U$-space are identical, for any direction $\bar{\alpha}$, a given vector magnitude $\beta = \| \mathbf{u} \|$ will always correspond to the same exceedance probability along that direction. An $M$-dimensional contour with exceedance probability $p_r$ will be a hypersphere with radius $\beta = \sqrt{\chi^\text{inv}_M(1 - p_r)}$ where $\chi^\text{inv}_M$ is the inverse chi-square distribution with $M$ degrees of freedom. Having computed the radius of the contour, the remaining calculations follow the procedure as in the iFORM method. The 1-year return period contour calculated with the above method is shown with blue on Figure 2. For this particular realization of the MC simulation there are 103 points outside this contour, which agrees with the target exceedance probability.

Figure 2. Comparison of a full Monte Carlo simulation with return period contours computed with the iFORM and iDS (chi-squared) approaches.

4. Sector-constrained environmental contours using iDS
In most design problems, only a part of the range of an environmental variable will be critical, while other values may have no relevance for the design. For example, rare combinations of low wind speeds and very low turbulence levels may be outside of an environmental contour. However, due to the very low loadings associated with such conditions, they have no significance for the structural design. It is therefore relevant to consider environmental contours where the target probability of exceedance is contained within a specific range of interest for the individual variables. Such a consideration is possible with the iDS method, by defining limits for a particular variable in terms of direction bounds.

Let us consider a variable space $\mathbf{x} = (x_1, x_2, ..., x_M)$, mapped to a standard normal space $\mathbf{u}$. We are interested in drawing an environmental contour for variable $x_1$ where only the range $(x_{1,lb}, x_{1,ub})$ is relevant. We denote the bounds mapped to standard normal space as $(u_{1,lb}, u_{1,ub})$. For a given exceedance probability associated with contour radius $\beta$, the directions corresponding to the bounds are $u_{1,lb}/\beta$ and $u_{1,ub}/\beta$. The directions can also be defined as trigonometric angles by:
\[
\theta_{1,lb} = \cos\left(\frac{u_{1,lb}}{\beta}\right), \quad \theta_{1,ub} = \cos\left(\frac{u_{1,ub}}{\beta}\right)
\]

In 2-dimensional standard normal space, the area between the upper and lower bound will have the shape of a sector from a circle (Figure 3, left), and the probability associated with it will be equal to \((\theta_{1,ub} - \theta_{1,lb})/2\pi\). For a contour radius \(\beta\) which corresponds to exceedance probability \(p_r\) contained within the sector \((\theta_{1,lb}, \theta_{1,ub})\), the following equation needs to be satisfied:

\[
\beta = \sqrt{x^2_{\chi^2}}\left(1 - \frac{p_r}{2\pi} (\theta_{1,ub}(\beta) - \theta_{1,lb}(\beta))\right)
\]

In the above, the contour radius \(\beta\) is the only unknown but appears on both sides of the equation, therefore a solution is obtained iteratively through a root-finding algorithm such as the Newton method. This sector-constrained approach is applied to the same MC simulation discussed in Sections 2 and 3, and the 1-year return period contour is obtained for combinations of wind speed and turbulence where the wind speed is bounded between 4m/s and 25m/s. The results are shown on Figure 3, in standard normal space (left), and in physical space (right). It is seen that the sector-constrained iDS contour is smaller than the total iDS contour, and the number of points outside its boundary agrees with the target exceedance probability. In this particular case, the sector-constrained contour comes very close to the original iFORM contour, however this is incidental since if other ranges of wind speed were chosen the sector-constrained contour would be of different size.

**Figure 3.** Application of the sector-constrained iDS contour method on a Monte Carlo simulation of the joint distribution of mean wind speed and turbulence. Left: standard normal space; Right: physical space

5. **Example application – Høvsøre wind and turbulence data**

To demonstrate the effect of using different environmental contour methods on measured data, ten years of wind speed and turbulence time series measured by a sonic anemometer at the Høvsøre site in northern Denmark [13] are analyzed. It has been shown earlier that applying iFORM to data from this site results in non-conservative environmental contours [14]. For the purpose of the present study, ten-minute statistics of mean wind speed, wind speed standard deviation, and wind direction data from a 3D sonic anemometer measured at 100m height over a time span of 10 years are considered. The data are filtered to remove invalid entries, and only measurements from a disturbance-free, westerly sector (230-300°) are considered. The total amount of data samples selected after filtering is 128571, or about 2.5 years. The joint distribution of wind speed \(V\) and turbulence \(\sigma_V\) is defined by considering the ten-minute mean wind speed as independent and Weibull distributed, and the turbulence as log-normally distributed. The
turbulence distribution parameters $\mu_\sigma$ and $\sigma_\sigma$ are both defined as second-order functions of the mean wind speed. The Weibull distribution parameters for the wind speed are obtained by the maximum-likelihood method. All data are binned according to wind speed in 1m/s bins, whereas the mean and standard deviation of the turbulence corresponding to each mean wind speed bin are obtained. Then, quadratic relations are fit for the mean and the standard deviation of turbulence as function of mean wind speed. Using the joint distribution of $V$ and $\sigma_V$ obtained in this way, the environmental contours corresponding to 1 year return period are drawn, using the iFORM and iDS methods, and with the sector-constrained iDS method for a wind speed range of 4m/s to 25m/s. The resulting contours are shown on Figure 4. All of them seem to be non-conservative, and some of the error can be attributed to the quality of the joint distribution fit, with a cluster of points with low turbulence at medium wind speed being outside the contours. However there is also a clear difference between the various contour methods, with the iFORM being significantly more non-conservative than the iDS.

6. Discussion
The environmental contour examples in the present paper focus on the joint distribution of wind speed and turbulence, as this is a key element of wind turbine design and there are also suitable datasets to support the discussion. It was shown that an environmental contour based on iFORM yields non-conservative results while the suggested iDS approach results in the correct exceedance probability. Specifically regarding the ETM turbulence model for IEC class 1A conditions, the iFORM contour with radius in normal space $\beta_{iFORM} = 4.945$ corresponds to a return period of 3.88 years, i.e., exceedances are an order of magnitude more frequent than the target period of 50 years. However, given that situations with low turbulence have no special significance for wind turbine extreme loads, it can be argued that only a portion of the environmental contour is relevant for design. Above, it was shown how contours can be drawn for partial probability spaces by the sector-constrained iDS method. If the analysis is limited to a typical operating range of wind speeds from 4 to 25m/s, the sector-constrained iDS method computes a 50-year return period contour size of $\beta_{sector-iDS} = 5.03$. Applying the same computations for the iFORM contour radius, it turns out that $\beta_{iFORM} = 4.945$ corresponds to a return period of 32.2 years for exceedance events where the wind speed is between 4 and 25m/s. Hence, in this case the full iFORM contour is quite similar to the sector-constrained iDS contour. However, we should not consider the iFORM as a reliable approximation, because if other range of wind speeds was chosen the results would differ.

The sector-constrained iDS approach was defined and demonstrated for a 2-dimensional case. The approach can be extended to cover additional dimensions if a multi-dimensional convex-shaped sector of interest can be defined in a way allowing that the probability mass within the sector can be computed.
As discussed earlier, obtaining environmental contours has two key elements – determining the joint statistical distribution of the variables, and determining the correct contour size using a transformation to standard normal space. The present study only deals with the latter, while deriving the joint distribution, and the uncertainty associated with it, is outside the scope of the paper. Nevertheless, it should be noted that the improved approaches for computation of contour sizes described here will only be adequate if the joint distribution of environmental variables is accurately defined.

7. Conclusions
The present study discussed the computation of environmental contours using the inverse First Order Reliability Method (iFORM), and potential improvements were formulated and demonstrated. The newly suggested environmental contour methods were applied on measured wind and turbulence time series and compared with the iFORM method. It was shown that

- The iFORM method, which has been widely used in the offshore wind and marine industries for more than 30 years, yields inherently non-conservative results. This is due to the approach used for computing the contour radius in standard normal space, where only a fraction of the probability behind the contour line is taken into account;
- As a consequence of the above, the Extreme Turbulence Model (ETM) contour defined in the IEC61400-1 standard actually corresponds to a return period of about 3.88 years rather than 50;
- iDS, an alternative solution based on the chi-square distribution was derived and its adequacy was demonstrated with a Monte Carlo simulation;
- An expression was derived for computing sector-constrained environmental contours where the exceedance probability is contained within specific range of interest for one or more variables.

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