A model of inflation independent of the initial conditions, with bounded number of e-folds and $n_s$ larger or smaller than one

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Abstract

We study a supergravity model of inflation essentially depending on one parameter which can be identified with the slope of the potential at the origin. In this type of models the inflaton rolls at high energy from negative values and a positive curvature potential. At some point defined by the equation $\eta = 1$ inflation starts. The potential curvature eventually changes to negative values and inflation ends when $\eta = -1$. No spontaneous symmetry breaking type mechanism for inflation of the new type to occur is here required. The model naturally gives a bounded total number of e-folds which is typically close to the required number for observable inflation and it is independent of the initial conditions for the inflaton. The energy scale introduced is fixed by the amplitude of the anisotropies and is of the order of the supersymmetry breaking scale. The model can also accommodate a spectral index bigger or smaller than one without extreme fine tuning. We show that it is possible to obtain reasonable numbers for cosmological parameters and, as an example, we reproduce values obtained recently by Tegmark et.al., from WMAP and SDSS data alone.

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1 Introduction

We are certainly living very promising days for cosmology. In a few years we have witnessed an explosion of cosmological data recollection coming from telescope, balloon and satellite experiments looking at the large scale structure distribution of galaxies as well as cosmic microwave background radiation anisotropies [1]. In particular projects like the Wilkinson Microwave Anisotropy Probe WMAP [2] and the Sloan Digital Sky Survey SDSS [3], in spite of having yet to complete their full schedule of observations, have already provided a significant amount of data which has been used, together with some reasonable assumptions, to determine cosmological parameters in a variety of ways. In particular this should eventually make possible to discriminate among several competing models of inflation.

Here we study a supergravity model of inflation essentially depending on one free parameter which can be identified with the slope of the potential at the origin and an energy scale fixed by the amplitude of the anisotropies, of the order of magnitude of the supersymmetry breaking scale $10^{11}\text{GeV}$. The model naturally gives a total number of e-folds which is typically close to the required number for observable inflation. The model can also accommodate a spectral index bigger or smaller than one without extreme fine tuning. We show that it is possible to obtain reasonable numbers for cosmological parameters and, as an example, we reproduce values obtained recently by Tegmark et al. [4], particularly the data provided in the sixth column of Table 4 of that paper, where cosmological parameters are determined from a six-parameter model using WMAP and SDSS data alone. We choose this sample set of parameters for simplicity.

The model described here actually defines a new type of models where the inflaton rolls at high energy from negative values and a positive curvature potential. There is no inflation yet, however at some point defined by the equation $\eta \equiv V''/V = 1$ inflation starts. The potential curvature eventually changes to negative values and inflation ends when $\eta = -1$, (see Fig.1). No spontaneous symmetry breaking type mechanism for inflation of the new type to occur is here required. As a consequence the total number of e-folds is bounded independently of the initial conditions for the inflaton and close to the required number for observable inflation, see Table 1. The spectral index can take values bigger than one at the expense of increasing the slope of the potential and thus decreasing the total number of e-folds.

In section 2 we give a complete description of the model and obtain some general expressions related to inflation in the slow-roll approximation. Section 3 presents the main results in the form of a table. We give a brief description of the ways the essentially unique free parameter of the model is fixed and calculate all other relevant quantities. Finally we conclude in section 4 briefly discussing the main results.

2 The Model

We construct a model from the F-term part of the $N = 1$ supergravity potential for a single scalar field $z$. From now on we will take $M (\equiv M_P/\sqrt{8\pi} \approx 2.44 \times 10^{18}\text{GeV}) = 1$,
where $M$ is the normalized Planck mass. The supergravity potential is given by
\[ V = e^K [F_z(K_{zz^*})^{-1}F_{zz^*} - 3|W|^2] + D - \text{terms}, \] (1)
where
\[ F_z \equiv \frac{\partial W}{\partial z} + \left( \frac{\partial K}{\partial z} \right) W, \quad K_{zz^*} \equiv \frac{\partial^2 K}{\partial z \partial z^*}, \] (2)
and $K$ is the Kähler potential. To first approximation we take it to be of the canonical form
\[ K(z, z^*) = zz^* + \ldots, \] (3)
and the superpotential $W(z)$ is determined imposing some reasonable physical assumptions as follows [6]. The sectors of the theory in charge of supersymmetry breaking, inflation and the visible sector interact among themselves only gravitationally. In particular, this allow us to study the inflationary sector independently of the others. Requiring that supersymmetry remain unbroken in the global minimum and that the contribution to the cosmological constant vanishes we find that, at the minimum, the superpotential and its first derivative should vanish. Thus a first contribution to $W(z)$ in a Taylor expansion around the minimum at $z = z_0$ is
\[ W(z) = f(z_0)(z - z_0)^2, \] (4)
where $f(z_0)$ is a constant term with dimensions of mass. We take the simplest choice
\[ f(z_0) = \Lambda. \] (5)
We then write
\[ z = \frac{1}{\sqrt{2}}(\phi + i\chi), \] (6)
and set $\chi = 0$, being a stable direction of the full potential (11) which is then given by
\[ V = \Lambda^2 e^{\phi^2/2}(\phi - \phi_0)^2(2 + (\phi - \phi_0)(6\phi_0 + \phi(2 + \phi^2 - \phi\phi_0))/8). \] (7)
Finally we redefine the minimum at $\phi_0$ by
\[ \phi_0 = \sqrt{2} + c/8, \] (8)
where $c$ is a constant which will be determined by requiring agreement with cosmological parameters as given in [4]. The parameter $c$ is essentially the only free parameter of the model, it dictates the shape of the potential and thus the dynamics of the inflaton while $\Lambda$ is just an overall scale specifying the height of the potential during inflation $V \sim \Lambda^2 M^2$ which will be fixed by the amplitude of the anisotropies. The choice Eq.(5) makes easy to interpret $c$: it gives (to first order) the slope of the potential at the origin since $V'(0) = c(1 + 3c\sqrt{2}/32 + c^2/256)$. Thus $c = 0$ (the value used in all previous models [7], [8]) makes the potential flat at the origin. Here no extreme fine tuning in $c$ is required, being $O(10^{-4})$, with a modest tuning of the inflaton mass $m_\phi \approx \sqrt{-c} \approx O(10^{-2})$. For
Figure 1: The inflationary potential (in units of $\Lambda^2 M^2$) Eq.(7) for $c = -1.18 \times 10^{-4}$ is shown as a function of $\phi$, the real part of the scalar field $z$, Eq.(6). For this value of $c$ the slope of the potential around the origin is always negative, being equal to $c$, to first order. Initially the would be inflaton rolls at high energy from negative values without inflating. When the condition $\eta = 1$ is reached inflation starts. We can easily solve for the start of inflation at $\phi_s \approx -0.094$. At some point the potential curvature changes sign from positive to negative until inflation ends when $\eta = -1$ is satisfied. In this example the end of inflation occurs at $\phi_e \approx 0.169$ giving a total number of e-folds $N_T = 137$, all other relevant parameters are shown in Table 1. In this type of models we do not assume a previous epoch where the inflaton is taken to the origin and kept there by thermal effects waiting for spontaneous symmetry breaking to occur and inflation of the new type to start. Rather the inflaton rolls from high energy to its global minimum with an inflationary epoch in between. The modest tuning of $c$ is not a tuning on initial conditions (for large $|\phi|$ the potential is dominated by the exponential term alone and is insensitive to $c$.) but a requirement for an intermediate stage of observable inflation to occur.

negative $c$ the potential will always have negative slope (except at the global minimum where, of course, it vanishes becoming positive afterwards). This is shown in Fig.1. For small positive $c$ the potential develops a local minimum and then a maximum close to the origin. Because for negative $c$ the potential slope is always negative we can precisely determine not only the end but also the beginning of inflation, i.e., the total number of e-folds is bounded. Thus we can also take the model as a realization of a recent proposal by Banks and Fischler [8] where it is claimed that a universe which asymptotically becomes de Sitter is consistent with inflationary models with a bound on the total number of e-folds $N_T$ (see, however, [9]). Our model has a bounded number of e-folds independently of the initial conditions for the inflaton. The supergravity potential shows that for large $|\phi|$-values $V(\phi) \approx e^{\phi^2}$. Thus the would be inflaton might start rolling from anywhere at high energy without having any effect on inflation. In fact the beginning of inflation starts at $\eta = 1$ and expanding $\eta$ as a function of $\phi$ and
\[ c \text{ and keeping the lowest order one has } \eta = -3c/2\sqrt{2} - 6\sqrt{2}\phi + O(c^2, \phi^2) \simeq 1 \text{ giving a value } \phi_s = -1/6\sqrt{2} \simeq -0.1. \] 
This means that no fine tuning on the value \( \phi_i \) is needed and for any \( |\phi_i| \) larger than \( |\phi_s| \simeq 0.1 \) the number of e-folds of inflation will be the same (in general one expects at high energies an initial value \( \phi_i \) of the order of the Planck mass). The evolution of \( \phi \) is given by the usual equation of motion

\[ \ddot{\phi} + 3H\dot{\phi} + V' = 0 \quad (9) \]

and since \( V' \) is negative for \( \phi < \phi_0 \) we will have an increasing \( \dot{\phi} \) until it oscillates around \( \phi_0 \). The inflationary stage is controlled by the slope or shape parameter \( c \) which affects the potential only for small \( \phi \) (see Fig.1). Characteristics of inflation such as the number of e-folds, beginning and end of inflation, spectral index and so on are controlled by the value of \( c \) independently of the initial conditions for the inflaton [10]. We do not assume a previous epoch where the inflaton is taken to the origin and kept there by thermal effects waiting for spontaneous symmetry breaking to occur and inflation of the new type to start. Rather the inflaton rolls from high energy to its global minimum with a transient inflationary epoch in between (see Fig.1). The modest tuning of \( c \) is not a tuning on initial conditions but a requirement for an intermediate stage of observable inflation to occur. We calculate the scalar spectral index with the usual slow-roll expression [11]

\[ n_s = 1 + 2\eta - 6\epsilon \quad (10) \]

and its logarithmic derivative

\[ \frac{dn_s}{d\ln k} = 16\eta\epsilon - 24\epsilon^2 - 2\xi \quad (11) \]

where \( \eta, \epsilon \) and \( \xi \) are the slow-roll parameters

\[ \eta = \frac{V''}{V}, \quad (12) \]

\[ \epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2, \quad (13) \]

\[ \xi = \frac{V'V'''}{V^2}. \quad (14) \]

The amplitude of scalar fluctuations is given by [12]

\[ A_s = \frac{1}{2.95 \times 10^{-9}} \frac{25}{4} \delta_H^2, \quad (15) \]

where

\[ \delta_H^2 = \frac{1}{75\pi^2} \frac{V^3}{V'^2} = \left( 1.91 \times 10^{-5} \frac{e^{1.01(1-n_s)}}{\sqrt{1 + 0.75r}} \right)^2. \quad (16) \]
In the last equality of Eq. (16) we have used the fitting function of Bunn et al., [13]. The value of $\phi$ at horizon crossing $\phi_*$ when $k_* = aH$ during inflation is given by assuming a value for the number of e-folds $N$

$$N_* = - \int_{\phi_*}^{\phi_e} \frac{V(\phi)}{V'(\phi)} d\phi.$$  \hspace{1cm} (17)

or solving consistently using some other criteria. In Table 1 we use various criteria to determine $N_*$. In the expression above $\phi_e$ denotes the value of $\phi$ at the end of inflation, determined by the violation of the slow-roll $\eta = -1$. The scale $\Lambda$ obtained from Eq. (16) turns out to be $\mathcal{O}(10^{11} GeV)$ and could be related to the supersymmetry breaking scale while the inflationary scale $\Delta \equiv V^{1/4} \approx (\Lambda M)^{1/2}$ is of the order of $10^{14} - 10^{15} GeV$. The relative amplitude of the tensor to scalar modes $r$ is given by [12]

$$r = 16\epsilon.$$ \hspace{1cm} (18)

At the end of inflation the oscillations of the inflaton field would make it decay thus reheating the universe. The couplings of the inflaton to some other bosonic $\chi$ or fermionic $\psi$ MSSM fields occur due to terms $-\frac{1}{2}g^2\phi^2\chi^2$ or $-h\bar{\psi}\psi\phi$, respectively. These couplings induce decay rates of the form [14]

$$\Gamma(\phi \to \chi\chi) = \frac{g^4\phi_0^2}{8\pi m_\phi}, \hspace{0.5cm} \Gamma(\phi \to \bar{\psi}\psi) = \frac{h^2m_\phi}{8\pi}$$ \hspace{1cm} (19)

where $\phi_0$ is the value of $\phi$ at the minimum of the potential Eq. (8) and $m_\phi$ is the inflaton mass given by

$$m_\phi \approx \Lambda.$$ \hspace{1cm} (20)

A maximum value for the decay is obtained when $m_{\chi,\psi} \approx m_\phi$. In this case we find

$$\Gamma \approx \frac{m_\phi^3}{8\pi\phi_0^2}.$$ \hspace{1cm} (21)

The reheat temperature at the beginning of the radiation-dominated era is thus [15]

$$T_{rh} \approx \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{\Gamma} \approx \frac{1}{\sqrt{16\pi}} \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \Lambda^{3/2},$$ \hspace{1cm} (22)

where $g_*$ is the number of relativistic degrees of freedom which for the MSSM equals 915/4.

3 Results

Our main results are presented in Table 1 where we show, in particular, that the scalar spectral index can have values bigger or smaller than one. We can understand this as
follows. For small $c$ the slow-roll parameter $\epsilon$ is much smaller than $\eta$ during inflation thus we have
\[ n_s \approx 1 + 2\eta. \] (23)
Thus we can have $n_s > 1$ or $n_s < 1$ depending on the curvature of the potential. It is easy to show that
\[ \eta_* \approx -\frac{3}{2\sqrt{2}}c - 6\sqrt{2}\phi_* + \frac{39}{2}\phi_*^2 + \ldots \] (24)
For negative $c$ the sign of $\eta_*$ depends on the relative size between the first and second terms of Eq.(24). If we increase the slope of the potential by increasing the (absolute) value of $c$ (but still $O(10^{-4})$) $\phi_*$ takes smaller and eventually negative values to satisfy the condition on the number of observable e-folds of inflation. At $\phi_* \approx -c/8$ curvature changes sign from negative to positive values and the spectral index can become bigger than one at $\phi_*$. This actually defines a new type of models where the inflaton rolls at high energy from negative values and a positive curvature potential without inflating due to the dominance of the $e^{\phi^2}$ term in the potential. For small $|\phi|$-values, at some point defined by the equation $\eta = 1$, inflation starts. The potential curvature eventually changes to negative values and inflation ends when $\eta = -1$. No spontaneous symmetry breaking type mechanism for inflation of the new type to occur is here required. We illustrate this behavior in Fig.1 and the rows of Table 1 where $c$ takes negative values. Examples of $n_s > 1$ are shown in the second and third rows where the spectral index takes the values 1.10 and 1.00 respectively. In both cases Eqs.(17) and (25) were solved consistently. We next show that it is possible to obtain reasonable numbers for cosmological parameters. As an example the first row shows cosmological parameters relevant to inflation. These were obtained from a 6-parameter model by Tegmark et al., (sixth column of Table 4 of [4]) using WMAP + SDSS data alone. We have taken that particular column for simplicity. In the fourth row ($c = -1.18 \times 10^{-4}$) we determine $c$ such that at $\phi_*$ we get the central value of the scalar spectral index such that the number of e-folds Eq.(17) be consistent with the recent upper estimate of Liddle and Leach [16] and Dodelson and Hui [17] for the maximum number of e-folds for the observable universe.
\[ N_* = 63.3 + \frac{1}{4} \ln \epsilon. \] (25)
Notice that this is not a bound on the total number of e-folds which, in general, could be much higher. In the fifth row we simply set $N_T = 50$ at $\phi_*$ and the sixth row requires a total number of e-folds to be equal to the upper estimate of Banks and Fischler [8], $N_T = 85$ giving $N_* = 39$ at $\phi_*$. Finally the last two rows show results for $c = 0$ (flat case [7]) and a positive $c$ where a local maximum develops close to the origin. In these two rows we do not impose that the spectral index be 0.977 at $\phi_*$ because that requirement violates the bound Eq.(25). Here we use Eq.(25) consistently with Eq.(17) to determine $\phi_*$ and then calculate all other quantities at $\phi_*$. 
Table 1: The first row shows cosmological parameters relevant to inflation obtained from a 6-parameter model by Tegmark et al., (sixth column of Table 4 of [4]) using WMAP + SDSS data alone. We have taken this set of values for simplicity. The next two rows show the scalar spectral index equal to 1.10 and 1.00 respectively. In both cases the number of e-folds was required to be consistent with the recent upper estimate [16], [17], given by Eq. (25) for the maximum number of e-folds for the observable universe. Notice that this is not a bound on the total number of e-folds which, in general, could be much higher, in both cases we found $\phi_\ast < 0$ and a positive curvature potential. In the next three rows we determine $c$ such that at $\phi_\ast$ we get the central value of the scalar spectral index 0.977 and such that (fourth row) the number of e-folds Eq. (17) be consistent with Eq. (25). In the fifth row we simply set $N_\ast = 50$ at $\phi_\ast$ and the sixth row requires a total number of e-folds to be equal to the upper estimate of Banks and Fischler [8], $N_T = 85$ giving $N_\ast = 39$ at $\phi_\ast$. Finally the last two rows show results for $c = 0$ (flat case [6], [7]) and a positive $c$ where a local maximum develops close to the origin. In these last two rows we do not impose that the spectral index be 0.977 at $\phi_\ast$ because that requirement violates the bound Eq. (25). We only use Eq. (25) consistently with Eq. (17) to determine $\phi_\ast$ and then calculate all other quantities at $\phi_\ast$. In all cases we see that the running of the spectral index is very small compared with the present sensitivity of observations. The scale of inflation $\Delta \equiv V(\phi_\ast)^{1/4} \approx (\Lambda M)^{1/2}$ and the reheat temperature $T_{rh}$ in columns sixth and seventh are given in GeV.

| $c$       | $n_s$  | $dn_s$ | $A_s$     | $r$   | $\Delta$ | $T_{rh}$ | $N_\ast - N_T$ |
|-----------|--------|--------|-----------|-------|----------|----------|----------------|
| WMAP+SDSS | 0.977±0.025 | 0      | 0.81±0.05 | 0     | -        | -        | -              |
| -3.10×10^{-4} | 1.10   | -0.008 | 0.63       | 2×10^{-6} | 1×10^{15} | 9×10^{7} | 59-83          |
| -1.65×10^{-4} | 1.00   | -0.003 | 0.77       | 2×10^{-7} | 7×10^{14} | 2×10^{7} | 59-115         |
| -1.18×10^{-4} | 0.977  | -0.002 | 0.81       | 1×10^{-7} | 6×10^{14} | 2×10^{7} | 59-137         |
| -1.72×10^{-4} | 0.977  | -0.003 | 0.81       | 3×10^{-7} | 8×10^{14} | 3×10^{7} | 50-113         |
| -2.96×10^{-4} | 0.977  | -0.005 | 0.81       | 7×10^{-7} | 1×10^{15} | 6×10^{7} | 39-85          |
| 0         | 0.93   | -0.001 | 0.89       | 3×10^{-8} | 5×10^{14} | 7×10^{6} | 58-             |
| +1.18×10^{-4} | 0.90   | -0.0007 | 0.94       | 1×10^{-8} | 4×10^{14} | 3×10^{6} | 58-             |
4 Conclusions

We have studied a very simple model of inflation which essentially depends on one free parameter denoted by $c$ which controls the shape of the potential for small $\phi$ and thus the dynamics of inflation. The large-$|\phi|$ behavior, being dictated by the exponential term of the potential, is insensitive to $c$. In this sense characteristics of inflation are independent of the initial conditions for the inflaton. The parameter $c$ can be identified with the slope of the potential at the origin. For $c = 0$ we have the flat case [6, 7] where the slope and the curvature both vanish at $\phi = 0$. In this case the total number of e-folds grows without limit as we approach the origin and the spectral index can only take values lower than one. For negative $c$, however, the slope of the potential during inflation never vanishes and the amount of inflation is bounded, i.e., the total number of e-folds is bounded. Finite negative $c$ values actually define a new type of models where the would be inflaton rolls at high energy from negative values and a positive curvature potential without inflating. At some point defined by the equation $\eta = 1$ inflation starts. The potential curvature eventually changes to negative values and inflation ends when $\eta = -1$ (see Fig.1). Here the inflaton rolls all the way from high energy down to its global minimum. No pre-inflationary epoch where the inflaton is kept waiting at the origin for spontaneous symmetry breaking to occur is here required. The spectral index can take values even bigger than one at the expense of increasing the slope of the potential and thus decreasing the total number of e-folds. We illustrate this in the second and third rows of Table 1. We also show that it is possible to obtain reasonable numbers for cosmological parameters and, as an example, we reproduce values obtained recently by Tegmark et al. [4], where cosmological parameters are determined from a six-parameter model using WMAP and SDSS data alone. We see that the results are in excellent agreement. Also, the reheat temperature is sufficiently low to avoid overproduction of unwanted states such as gravitinos. The $\Lambda$ scale introduced in the superpotential can be related to supersymmetry breaking in the hidden sector being $O(10^{11}\text{GeV})$ and the resulting scale of inflation $\Delta \equiv V(\phi_*)^{1/4} \approx (\Lambda M)^{1/2}$ is close to the unification scale. In conclusion the good agreement with the data in such a simple model indicate that perhaps some of the underlying assumptions should be part of a more elaborated model coming from a more fundamental theory.

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References

[1] Some recent reviews: J. García-Bellido, hep-ph/0407111, D. Scott, F. Smoot, astro-ph/0406567, J.L. Feng, hep-ph/0405215, R.H. Sanders, astro-ph/0402065, M.Trodden and S. Carroll, astro-ph/0401547.
[2] C.L. Bennett et. al., ApJS, 148,1(2003); G. Hinshaw et. al., ApJS, 148,135(2003); A. Kogut et. al., ApJS, 148,161(2003); L. Page et. al., ApJS, 148,233(2003); H.V. Peiris et. al., ApJS, 148,213(2003); D.N. Spergel et. al., ApJS, 148,175(2003); L. Verde et. al., ApJS, 148,195(2003).

[3] D.G. York et. al., Astron.J., 120,1579(2000); C. Stoughton et. al., Astron.J., 123,485(2002); K. Abazajian et. al., Astron.J., 126,2081(2003).

[4] M. Tegmark, et. al., Phys. Rev. D69(2004)103501.

[5] E. Cremmer, S. Ferrara, L. Girardello, A. Van Proeyen, Nucl.Phys.B212:413 (1983)

[6] J. Ellis, D.V. Nanopoulos, K.A. Olive and K. Tamvakis, Phys. Lett. 120B(1983)331; D.V. Nanopoulos, K.A. Olive, M. Srednicki and K. Tamvakis, Phys. Lett. 123B(1982)41; 124B(1983)171; B.A. Ovrut and P.J. Steinhardt, Phys. Lett 133B(1983)161; R. Holman, P. Ramond and G.G. Ross, Phys. Lett. 137B(1984)343.

[7] G.G. Ross, S. Sarkar, Nucl. Phys. B461(1996)597.

[8] T. Banks, W. Fischler, ”An upper bound on the number of e-foldings”, astro-ph/0307459.

[9] D.A. Lowe and D. Marolf, hep-th/0402162. N. Kaloper, M. Kleban and L. Sorbo, astro-ph/0406099.

[10] For a discussion of a pre-inflationary phase followed by the onset of inflation see e.g., A.D. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic Press, 1990). T. Vachaspati and M. Trodden, Phys. Rev. D61(2000)023502, gr-qc/9811037.

[11] For a review and extensive references, see, D.H. Lyth and A. Riotto, Phys. Rep. 314 (1999) 1; A.R. Liddle, D. Lyth. Comological Inflation and Large Scale Structure, Cambridge U.P., 2000.

[12] L. Verde, Astrophys. J. Suppl. 148(2003)195. astro-ph/0302218. H.V. Peiris, Astrophys. J. Suppl. 148(2003)213, astro-ph/0302225.

[13] E.F. Bunn, A.R. Liddle, M. White, Phys. Rev. D54(1996)5917, astro-ph/9607038.

[14] A.D. Linde, Particle Physics and Inflationary Cosmology (Harwood Academic Press, 1990).

[15] P.J. Steinhardt, M.S. Turner, Phys. Rev. D29 (1984) 2162.

[16] A. Liddle and S.M. Leach, Phys. Rev. D68(2003)103503, astro-ph/0305263.

[17] S. Dodelson, L. Hui, Phys. Rev. Lett. 91(2003)131301, astro-ph/0305113.