A Survey of Label-noise Representation Learning: Past, Present and Future

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Abstract—Classical machine learning implicitly assumes that labels of the training data are sampled from a clean distribution, which can be too restrictive for real-world scenarios. However, statistical learning-based methods may not train deep learning models robustly with these noisy labels. Therefore, it is urgent to design Label-Noise Representation Learning (LNRL) methods for robustly training deep models with noisy labels. To fully understand LNRL, we conduct a survey study. We first clarify a formal definition for LNRL from the perspective of machine learning. Then, via the lens of learning theory and empirical study, we figure out why noisy labels affect deep models’ performance. Based on the theoretical guidance, we categorize different LNRL methods into three directions. Under this unified taxonomy, we provide a thorough discussion of the pros and cons of different categories. More importantly, we summarize the essential components of robust LNRL, which can spark new directions. Lastly, we propose possible research directions within LNRL, such as new datasets, instance-dependent LNRL, and adversarial LNRL. Finally, we envision potential directions beyond LNRL, such as learning with feature-noise, preference-noise, domain-noise, similarity-noise, graph-noise, and demonstration-noise.

Index Terms—Label-noise Learning, Representation Learning.

1 INTRODUCTION

“H ow can a learning algorithm cope with incorrect training examples?” This is the question raised in Dana Angluin’s paper entitled “Learning From Noisy Examples” in 1988 [1]. She made the statement that, “when the teacher may make independent random errors in classifying the example data, the strategy of selecting the most consistent rule for the sample is sufficient, and usually requires a feasibly small number of examples, provided noise affects less than half the examples on average”. In other words, she claimed that a learning algorithm can cope with incorrect training examples, once the noise rate is less than one half under the random noise model. Over the last 30 years, her seminal research opened a new door to machine learning, since standard machine learning assumes that the label information is fully clean and intact. More importantly, her research echoed the real-world environment indeed, as labels or annotations are often noisy and imperfect in real scenarios.

For example, the surge of deep learning comes from 2012, because Geoffrey Hinton’s team leveraged AlexNet (i.e., deep neural networks) [2] to win the ImageNet challenge [3] with obvious margin. However, due to the huge quantity of data, the ImageNet-scale dataset was necessarily annotated by crowdsourced workers in Amazon Mechanical Turk [1]. Due to the limited knowledge, crowdsourced workers cannot annotate specific tasks with 100% accuracy, which naturally brings noisy labels. Another vivid example locates in medical applications, where datasets are typically small. However, it requires domain expertise to label medical data, which often suffers from high inter- and intra-observer variability, leading to noisy labels. We should notice that, noisy labels will cause wrong model predictions, which might further influence decisions that impact human health negatively. Lastly, noisy labels are ubiquitous in speech domains, e.g., Voice-over-Internet-Protocol (VoIP) calls [4]. In particular, due to unstable network conditions, VoIP calls are easily prone to various speech impairments, which should involve the user feedback to identify the cause. Such user feedback can be viewed as the cause labels, which are highly noisy, since most of users lack the domain expertise to accurately articulate the impairment in the perceived speech.

All the above noisy cases stem from our daily life, which cannot be avoided. Therefore, it is urgent to build up a robust learning algorithm for handling noisy labels with theoretical guarantees. In this survey paper, we term such a robust learning paradigm label-noise learning, and the noisy training data \((x, \bar{y})\) is sampled from a corrupted distribution \(p(X, \bar{Y})\), where we assume that the features are intact but the labels are corrupted. Note that such assumption can limit the scope of the study. As far as we know, label-noise learning spans over two important ages in machine learning: statistical learning (i.e., shallow learning) and representation learning (i.e., deep learning). In the age of statistical learning, label-noise learning focused on designing noise-tolerant
losses or unbiased risk estimators [5]. However, in the age of representation learning, label-noise learning has more options to combat with noisy labels, such as designing biased risk estimators or leveraging memorization effects of deep networks [6], [7].

1.1 Motivation and Contribution

Label-noise representation learning has become very important for both academic and industry. There are two reasons behind. First, from the essence of the learning paradigm, deep supervised learning requires a lot of well-labeled data, which may require too much cost, especially for many start-ups. However, deep unsupervised learning (even self-supervised learning) is too immature to work very well in complex real-world scenarios. Therefore, as deep weakly-supervised learning, label-noise representation learning naturally has attracted much attention and has become a hot topic. Second, from the aspect of data, many real-world scenarios lack purely clean annotations, such as financial data, web data, and biomedical data. This has directly motivated researchers to explore label-noise representation learning.

As far as we know, there indeed exist three pioneer surveys related to label noise. Frenay and Verleysen [8] focused on discussing label-noise statistical learning, instead of label-noise representation learning. Although Algan et al. [9] and Karimi et al. [10], focused on deep learning with noisy labels, both of them only considered on image (or medical image) classification task. Moreover, their surveys were written from the applied perspective, instead of discussing methodology. To compensate for them and go beyond, we want to contribute to the label-noise representation learning area as follows:

- From the perspective of machine learning, we give the formal definition for label-noise representation learning (LNRL). The definition is not only general enough to include all existing LNRL, but also specific enough to clarify what the goal of LNRL is and how we can solve it.
- In contrast to [9], [10], via the lens of learning theory, we provide a deeper understanding why noisy labels affect the performance of deep models. Meanwhile, we report the generalization of deep models under noisy labels, which coincides with our theoretical findings.
- We perform extensive literature review from the age of representation learning, and categorize them in a unified taxonomy in terms of data, objective and optimization. The pros and cons of different categories are analyzed. We also present a summary of insights for each category.
- Based on the above observations, we summarize and discuss the essential components of robust label-noise representation learning. These can help to spark new directions in label-noise representation learning.
- Beyond label-noise representation learning, we propose several promising future directions, such as learning with noisy feature, preference, domain, similarity, graph, and demonstration. We hope they can provide some insights.

1.2 Position of the Survey

The position of this survey is explained as follows. Frenay and Verleysen [8] mainly summarized the methods of label-noise statistical learning (LNSL), which cannot be used for deep learning models directly. Note that although both LNSL and LNRL approaches address the same problem setting, they are fundamentally different. First, the underlying theories should be different due to different hypothesis space (e.g., Section 3.5.2); Second, the potential solution should be different due to different models, e.g., Section 6. Meanwhile, LNSL may fail to handle large-scale data with label noise, while LNRL is good at handling such data.

Although Algan et al. [9] and Karimi et al. [10] respectively summarized some methods of label-noise representation learning, both of them discussed from the perspective of applications, i.e., (medical) image analysis. Recently, Song et al. [11] summarized some methods of label-noise representation learning from the view of methodology. However, their categorization is totally different from us in philosophy. In our survey, we first introduce label-noise representation learning from three general views: input data, objective functions and optimization policies, with more theoretical understanding.

1.3 Organization of the Survey

The remainder of this survey is organized as follows. Section 2 provides the related literature of label-noise learning, and the full version can be found in Appendix. Section 3 provides an overview of the survey, including the formal definition of LNRL, core issues, and a taxonomy of existing works in terms of data, objectives and optimizations. Section 4 is for methods that leverage the noise transition matrix to solve LNRL. Section 5 is for methods that modify the objective function to make LNRL feasible. Section 6 is for methods that leverage the characteristics of deep networks to address LNRL. In Section 7, we propose future directions for LNRL itself. Meanwhile, beyond LNRL, the survey discloses several promising future directions with conclusions in Section 8.

2 RELATED LITERATURE

We divide the development of label-noise learning into three stages as follows.

2.1 Early Stage

Before delving into label-noise representation learning, we give a brief overview of some milestone works in label-noise statistical learning. In 1988, Angluin et al. [1] proved that a learning algorithm can handle incorrect training examples robustly, when the noise rate is less than one half under the random noise model. Lawrence and Schölkopf [12] constructed a kernel Fisher discriminant to formulate the label-noise problem as a probabilistic model. Bartlett et al. [13] justified that most loss functions are not completely robust to label noise. This means that classifiers based on label-noise learning algorithms are still affected by label noise.
During this period, a lot of works emerged and contributed to this area. For example, Crammer et al. [14] proposed the online Passive-Aggressive perceptron algorithm to cope with label noise. Nataraajan et al. [5] formally formulated an unbiased risk estimator for binary classification with noisy labels. This work was very important to the area, since it is the first work to provide guarantees for risk minimization under random label noise. Meanwhile, Scott et al. [15] studied the classification problem under the class-conditional noise model, and proposed a way to handle asymmetric label noise. In contrast, van Rooyen et al. [16] proposed the unihinge loss to tackle symmetric label noise. Liu and Tao [17] proposed a method of anchor points to estimate the noise rate, and further leveraged importance reweighting to design surrogate loss functions for class-conditional label noise.

In 2015, research of label-noise learning has been shifted from statistical learning to representation learning, since deep learning models have become mainstream due to its better empirical performance. Therefore, it is urgent to design label-noise representation learning methods for robustly training deep models with noisy labels.

### 2.2 Emerging Stage

There are three seminal works in label-noise representation learning with noisy labels from 2015. For example, Sukhbaatar et al. [18] introduced an extra but constrained linear “noise” layer on top of the softmax layer, which adapts the network outputs to model the noisy label distribution. Reed et al. [19] augmented the prediction objective with the notion of consistency via soft and hard bootstrapping. Intuitively, this bootstrapping procedure provides the learner to disagree with an inconsistent training label, and re-label the training data to improve its label quality. Azadi et al. [20] proposed an auxiliary image regularization technique, which exploits the mutual context information among training data, and encourages the model to select reliable labels.

Followed by seminal works, Goldberger et al. [21] introduced a nonlinear “noise” adaptation layer on top of the softmax layer. Patrini et al. [22] proposed the forward and backward loss correction approaches simultaneously. Both Wang et al. [23] and Ren et al. [24] leveraged the same philosophy, namely data reweighting, to learn with label noise. Jiang et al. [6] is the first to leverage small-loss tricks to handle label noise. However, they trained only a single network iteratively, which inherits the accumulated error. To alleviate this, Han et al. [7] trained two deep neural networks and backpropagated the data selected by its peer network and updates itself.

In the context of representation learning, classical methods, such as estimating the noise transition matrix, regularization and designing losses, are still prosperous for handling label noise. For instance, Hendrycks et al. [25] leveraged trusted examples to estimate the gold transition matrix, which approximates to the true transition matrix well. Han et al. [26] proposed a “human-in-the-loop” idea to easily estimate the transition matrix. Zhang et al. [27] introduced an implicit regularization called mixup, which constructs virtual training data by linear interpolations of features and labels in training data. Zhang et al. [28] generalized both the categorical cross entropy (CCE) loss and mean absolute error (MAE) loss by the negative Box-Cox transformation. Ma et al. [29] developed a dimensionality-driven learning strategy, which can learn robust low-dimensional subspaces capturing the true data distribution.

### 2.3 Mature Stage

Since 2019, label-noise representation learning has become mature in the top conference venues. Arazo et al. [30] formulated clean and noisy samples as a two-component (clean-noisy) beta mixture model on the loss values. Hendrycks et al. [31] empirically demonstrated that pre-training can improve model robustness against label corruption for large-scale noisy datasets. Under the criterion of balanced error rate (BER) minimization, Charoenphakdee et al. [32] proposed the Barrier Hinge Loss. In contrast to selected samples via small-loss tricks, Thulasidasan et al. [33] introduced the abstention-based training, which allows deep networks to abstain on confusing samples while learning on non-confusing samples. Following the re-weighting strategy, Shu et al. [34] parameterized the weighting function adaptively as a one-layer multilayer perceptron called Meta-Weight-Net.

Menon et al. [35] mitigated the effects of label noise from an optimization lens, which naturally introduces the partially Huberised loss. Nguyen et al. [36] proposed a self-ensemble label filtering method to progressively filter out the wrong labels during training. Li et al. [37] modeled the per-sample loss distribution with a mixture model to dynamically divide the training data into a labeled set with clean samples and an unlabeled set with noisy samples. Lyu et al. [38] proposed a provable curriculum loss, which can adaptively select samples for robust stagewise training. Han et al. [39] proposed a versatile approach called scaled stochastic integrated gradient underweighted ascent (SUGIA). SUGIA uses stochastic gradient descent on good data, while using scaled stochastic gradient ascent on bad data rather than dropping those data. 5 years after the birth of Clothing1M, Jiang et al. [40] proposed a new but realistic type of noisy dataset called “web-label noise” (or red noise).

### 3 Overview

In this section, we first provide the notation used throughout the paper in Section 3.1. A formal definition of LNRL problem is given in Section 3.2 with concrete examples. As LNRL problem relates to many machine learning problems, we discuss their relatedness and difference in Section 3.3. In Section 3.4, we reveal the core issues that make LNRL problem hard. Then, according to how existing works handle the core issues, we present a unified taxonomy in Section 3.5.

#### 3.1 Notation

Let $x$ be features and $y$ be labels. Consider a supervised learning task $T$, LNRL deals with a data set $D = \{\mathcal{D}^t, \mathcal{D}^e\}$ consisting of training set $\mathcal{D}^t = \{(x^{(i)}, \bar{y}^{(i)})\}_{i=1}^N$ and test set $\mathcal{D}^e = \{x^{ke}\}$, where training set $\mathcal{D}^t = \{(x^{(i)}, \bar{y}^{(i)})\}_{i=1}^N$ is drawn from a corrupted distribution $p(\bar{Y}|X)$ ($\bar{Y}$ denotes label corruption). Note that $(X, \bar{Y})$ denotes the variable, while $(x, \bar{y})$ denotes its selected value. For the corrupted distribution $p(\bar{Y}|X)$, we assume that the features are intact but the labels are corrupted. Let $p(X, \bar{Y})$ be the ground-truth
As LNRL is naturally a sub-area in machine learning, before defining LNRL in Definition 2, we should recall how machine learning is defined literately. We borrow Tom Mitchell’s definition here, which is shown in Definition 1.

**Definition 1.** (Machine Learning [41], [42]). A computer program is said to learn from experience $E$ with respect to some classes of task $T$ and performance measure $P$ if its performance can improve with $E$ on $T$ measured by $P$.

The above definition is quite classical, which has been widely adopted in machine learning community. It means that a machine learning problem is defined by three key components: $E$, $T$, and $P$. For instance, consider speech recognition task ($T$, e.g., Apple Siri [4]), machine learning programs can improve its recognition accuracy ($P$) via training with large-scale speech data set ($E$) offline. Another example of $T$ is the hot topic in the security area, called empirical defense [43]. In a high level, machine learning algorithms can make deep neural networks defend against malicious cases. Specifically, a stop sign crafted by malicious people may cause an accident autonomous vehicles, which employ deep neural networks to recognize the sign. However, after adversarially training with adversarial examples ($E$), the robust generalization ($P$) of deep neural networks can improve a lot, which may avoid the above accident with large probability.

The above-mentioned classical applications of machine learning require a lot of “correctly” supervised information $\{(x^{(i)}, y^{(i)})\}_{i=1}^{N}$ for the given tasks. However, this may be difficult and even impossible. As far as we know, LNRL is a special case of machine learning, which belongs to weakly supervised learning [44]. Intuitively, LNRL exactly targets at acquiring good learning performance with “incorrectly” (a.k.a., noisy) supervised information provided by data set $D$, i.i.d. drawn from a corrupted distribution $p(\tilde{Y} | X)$. The noisy supervised information refers to training data set $D^p$, which consists of the intact input features $x^{(i)}$ but with corrupted labels $\tilde{y}^{(i)}$. More important, LNRL focuses on training deep neural networks robustly, which has many special characteristics, such as memorization effects. Formally, we define LNRL in Definition 2.

**Definition 2.** (Label-Noise Representation Learning (LNRL)). LNRL is a special but common case of machine learning problems (specified by $E$, $T$ and $P$), where $E$ contains noisy supervised information for the target $T$. Meanwhile, deep neural networks will be leveraged to model the target $T$ directly.

To understand this definition better, let us show three classical scenarios of LNRL (Table [1]):

- **Crowdsourcing**: Large-scale image data (e.g., ImageNet [45]) is the key factor to drive the second surge of deep learning from 2012. Note that it is impossible to annotate such scale data individually, which motivates us to leverage crowdsourcing technique (e.g., Amazon Mechanical Turk). However, the quality of crowdsourced data is normally low with a certain degree of label noise. Therefore, an important task ($T$) is to robustly training deep neural networks with crowdsourced data ($E$), and the trained deep models can be evaluated via test accuracy ($P$).
- **Healthcare**: Healthcare is highly related to each individual, whose data requires machine learning technique to analyze deeply and intelligently. However, intelligent healthcare ($T$) requires domain expertise to label medical data first, which often suffers from high inter- and intra-observer variability, leading to noisy medical data ($E$). We should notice that, noisy labels will cause the high error rate ($P$) of deep model predictions, which might further influence decisions that impact human health negatively.
- **Speech**: In speech task (e.g., Apple Siri), machine learning programs can improve its recognition accuracy via training with large-scale speech data set offline. However, noisy labels are ubiquitous in speech domains, e.g., the task of rating for service calls ($T$). Due to the difference of personal mood and understanding, service calls are easily prone to various rating ($E$) for the same service. Such rating can be viewed as labels, which are highly noisy since most of users lack the domain expertise to accurately rate the speech service. Therefore, it is critical to robustly train deep neural networks with the user rating ($E$), and evaluate the trained model via the quality rate of service calls ($P$).

As noisy supervised information related to $T$ is directly contained in $E$, it is quite natural that common deep supervised learning approaches will fail on LNRL problems. One of the recent findings (i.e., memorization effects [46]) in deep learning area may explain this: due to the high model capacity, deep neural networks will eventually fit and memorize label noise. Therefore, when facing the noisy data $E$, LNRL methods make the learning of the target $T$ feasible by leveraging the intrinsic characteristics of deep neural networks, e.g., memorization effects and non-linearity.

### 3.3 Relevant Learning Problems

In this section, we discuss the relevant learning problems of LNRL. The relatedness and difference with respect to LNRL are clearly clarified as follows.

- **Semi-supervised Learning** ([SSL] [47]) learns the hypothesis $f$ from experience $E$ consisting of both labeled and unlabeled data, where unlabeled data will be normally annotated by pseudo labels. Since the labeling process may be not fully correct and noisy, SSL has some relatedness with LNRL. However, SSL assumes that labeled data are fully clean,
which is different with LNRL, where labeled data are still noisy to some degree. To address SSL, there are several typical algorithms, such as [27], [48], [48], [49], [50].

- **Positive-unlabeled Learning (PUL)** [51] learns the hypothesis $f$ from experience $E$ consisting of only positive labeled and unlabeled data. Similar to SSL, unlabeled data will be normally annotated by pseudo labels. However, PUL assumes that labeled data are fully clean and only positive. To address PUL, there are several typical algorithms, such as [52], [53], [54].

- **Complementary Learning (CL)** [55] specifies a class that a pattern does NOT belong to. Namely, CL learns the hypothesis $f$ from experience $E$ consisting of only complementary data. Since the labeling process cannot fully exclude the uncertainty, namely belonging to which categories, CL has some relatedness with LNRL. However, CL requires that all diagonal entries of the transition matrix are zeros. Sometimes, the transition matrix may be not required to be invertible in empirical. To address CL, there are several typical algorithms, such as [55], [56], [57], [58].

- **Unlabeled-unlabeled Learning (UUL)** [59] is a recently proposed learning paradigm, which allows us to train a binary classifier only from two unlabeled datasets with different class priors. Different to SSL and PUL, there are two sets of unlabeled data in UUL instead of one set. To address UUL, there are two typical algorithms, including [59], [60].

### 3.4 Core Issue

When machine learning in an ideal environment, the data should be with clean supervision. Therefore, $\ell$-risk under the clean distribution should be as follows.

$$R_{\ell,D}(\theta) := \mathbb{E}_{(X,Y) \sim D}[\ell(f_{\theta}(X), Y)],$$  \hfill (1)

where $(X, Y)$ is the clean example i.i.d. drawn from clean distribution $D$, $f_{\theta}$ is a learning model (e.g., a deep neural network) parameterized by $\theta$ and $\ell$ is normally cross-entropy loss. In this survey, we consider the classification problem.

However, when machine learning is in the real-world environment, the data will be with noisy supervision. Namely, $\ell$-risk under the noisy distribution should be $\mathbb{E}_{(X,Y) \sim \tilde{D}}[\ell(f_{\theta}(X), Y)]$. Furthermore, under the limited data, empirical $\ell$-risk under the noisy distribution should be as follows.

$$\hat{R}_{\ell,D}(\theta) := \frac{1}{n} \sum_{i=1}^{n} \hat{\ell}(f_{\theta}(X_i), \tilde{Y}_i),$$  \hfill (2)

where $(X_i, \tilde{Y}_i)$ is the (observed) noisy example i.i.d. drawn from noisy distribution $\tilde{D}$ (with noise rate $\rho$). Note that $\hat{\ell}$ is a suitably modified loss, which is noise-tolerant. Here, we empirically demonstrate the generalization difference between $\ell$ and $\hat{\ell}$ under label noise (Figure 1).

Generally, the aim of LNRL is to "construct" such noise-tolerant $\ell$ that the learned $f_{\theta}$ in (2) approximates to the optimal $f_{\theta}$ in (1) well. Specifically, via a suitably constructed $\hat{\ell}$, we can learn a robust deep classifier $f_{\theta}$ from the noisy training examples that can assign clean labels for test instances. Before delving into constructing $\hat{\ell}$, we first take a theoretical look at label-noise learning, which will help us build $\ell$ more effectively.

### 3.5 Theoretical Understanding

In contrast to [9], [10], via the lens of learning theory, we provide a systematic way to understand LNRL. Our focus is to explore why noisy labels affect the performance of deep models. To figure it out, we should rethink the essence of learning with noisy labels. Normally, there are three key ingredients in label-noise learning problems, including input data, objective function and optimization policy.

In high level, there are three rule of thumbs, which explain how to handle noisy labels effectively via deep models.

- **For data**, the key is to discover the underlying noise transition pattern, which directly links the clean class posterior and the noisy class posterior. Based on this insight, it is critical to design unbiased estimator to estimate noise transition matrix $T$ accurately.

- **For objective function**, the key is to design noise-tolerant $\hat{\ell}$, which enjoys the statistical consistency guarantees. Based on this insight, it is critical to learn a robust classifier on noisy data, which can provably converge to the learned classifier on clean data.

- **For optimization policy**, the key is to explore the dynamic process of optimization policies, which relates to memorization. Based on this insight, it is critical to trade-off overfit/underfit in training deep networks, such as early stopping and small-loss tricks.
3.5.1 Perspective of Data

Early works on noisy labels studied random classification noise (RCN) for binary classification [11, 61]. In the RCN model, each instance has its label flipped with a fixed noise rate $\rho \in [0, 1]$. A natural extension of RCN is class-conditional noise (CCN) for multi-class classification. In the CCN model, each instance from class $i$ has a fixed probability $\rho_{i,j}$ of being assigned to class $j$. Thus, it is possible to encode some similarity information between classes. For example, we can expect that the image of a “dog” is more likely to be erroneously labelled as “cat” than “boat”. Until 2020, most of LNRL methods share an implicit assumption on noise model, namely class-conditional noise (CCN) model, which is formulated as $p(\bar{y}|y)$. Both RCN and CCN study instance-independent label noise.

Specifically, from the perspective of input data, the focus is to build up noise transition matrix, which models the process of label corruption. In general, there are two types of label noise: instance-dependent label noise (e.g., $p(\bar{y}|y, x)$) [62] and instance-independent label noise (e.g., $p(\bar{y}|y)$) [22]. For instance-dependent label noise, the noise transition matrix can be represented as $T(X)$, which depends on features. However, it can be ill-posed to learn the transition matrix $T(X)$ by only exploiting noisy data, i.e., the transition matrix is unidentifiable [62, 63]. Therefore, we emphasize instance-independent label noise here, and the noise transition matrix can be represented as $T$, which is independent of features.

In this case, the noise transition matrix $T$ approximately models the process of label corruption. Given an instance $x^i$ is an anchor point of the $i$-th clean class, if $p(y = e_i|x^i) = 1$, where $y = e_i$ means $y$ belongs to the $i$th class. The transition matrix can be obtained via:

$$
p(\bar{y} = e_j|x^i) = \sum_{k=1}^{C} p(\bar{y} = e_j|y = e_k, x^i)p(y = e_k|x^i),
= p(\bar{y} = e_j|y = e_i, x^i)p(y = e_i|x^i),
= p(\bar{y} = e_j|y = e_i, x^i) = T_{ij},
$$

Note that if anchor points are hard to identify, we can use $x^i = \arg \max_y p(\bar{y} = i|x)$ [17]. This transition matrix is very important, since it can bridge noisy class posterior and clean class posterior, i.e., $p(y|x) = T^{-1}p(\bar{y}|x)$. In practice, this transition matrix has been employed to build risk-consistent estimator via loss correction or classifier-consistent estimator via hypotheses correction. Besides, for inconsistent algorithms, the diagonal entries of this matrix are used to select reliable examples for further robust training.

3.5.2 Perspective of Objective Function

From the perspective of objective function, the focus is to derive the statistical consistency guarantees for robust $\ell_p$ [35]. Assume the Frobenius norm of the weight matrices $W_1, \ldots, W_d$ are at most $M_1, \ldots, M_d$. Let the activation functions be 1-Lipschitz, positive-homogeneous, and applied element-wise (such as the ReLU). Let $x$ is upper bounded by $B$, i.e., for any $||x|| \leq B$. With probability at least $1 - \delta$, if $\ell$ is classification-calibrated, there exists a non-decreasing function $\xi_\ell$ with $\xi_\ell(0) = 0$ such that:

$$
R_D(\hat{f}) - R^* \leq \xi_\ell(\min_{f \in \mathcal{H}} R_{\ell, D}(f) - \min_{f \in \mathcal{H}} R_{\ell, D}(f)) + 4L_p R(\mathcal{H}) + 2\sqrt{\log(1/\delta)/2n},
\leq \xi_\ell(\min_{f \in \mathcal{H}} R_{\ell, D}(f) - \min_{f \in \mathcal{H}} R_{\ell, D}(f)) + 4L_p B(2d \log^2(1/\delta)/2n),
$$

where $R_D(\hat{f}) = \mathbb{E}_{(x,y) \sim D}[1_{\{\text{sign}(\hat{f}(x)) \neq y\}}]$ denotes the risk of $\hat{f}$ w.r.t. the 0-1 loss; $R^* = R_D(f^*)$ denotes Bayes risk.
for Bayes optimal classifier \( f^* \) under the clean distribution \( D \). \( f = \arg \min_{f \in \mathcal{H}} \tilde{R}_{\ell,p}(f) \) and \( L_p \) is the Lipschitz constant of \( \ell \). When the function class \( \mathcal{H} \) is parameterized by deep neural networks, due to \([63]\), the Rademacher complexity \( R(\mathcal{H}) \) of the function class \( \mathcal{H} \) is upper bounded by \( B(\sqrt{2D \log 2 + 1}) \prod_{i=1}^d M_i / \sqrt{n} \).

Note that \( \min_{f \in \mathcal{H}} R_{\ell,D}(f) - \min R_{\ell,D}(f) \) denotes the approximation error for employing the hypothesis class \( \mathcal{H} \). According to the universal approximation theorem \([66]\), if a certain deep network model is employed, \( \mathcal{H} \) will be a universal hypothesis class and thus contains the Bayes classifier. Then, \( \min_{f \in \mathcal{H}} R_{\ell,D}(f) - \min R_{\ell,D}(f) = 0 \). This means that by employing a proper deep network, the upper bound will converge to zero by increasing the training sample size \( n \). Since \( R_D(f) \) is always bigger than or equal to \( R^* \), we have that \( R_D(f) \) will converge to \( R^* \). This further means that \( f \) learned from noisy data (i.i.d. drawn from \( D \)) will converge to Bayes optimal \( f^* \) defined by the clean data.

3.5.3 Perspective of Optimization Policy

From the perspective of optimization policy, the focus is to explore dynamic process of optimization policies. Take early stopping, which is a simple yet effective trick to avoid overfitting on noisy labels, as an illustrative example \([67]\).

Consider a dataset \( (x_i, y_i)^n_{i=1} \). Assume an initial weight matrix \( W^0 \sim \mathcal{N}(0,1) \) entries, and \( W^\tau \) update via stochastic gradient descent with step size \( \eta \), i.e., \( W_{\tau+1} = W_{\tau} - \eta \nabla L(W_{\tau}) \). If \( \varepsilon \leq \delta \lambda (C)/K^2 \) and \( \rho \leq \delta/8 \) with high probability, then after \( I \propto ||C||^2/\sqrt{s} \) steps, there are two conclusions \([67]\).

First, the model \( W^I \) predicts the true label function \( y(x) \) for all input \( x \) that lie within \( \varepsilon_0 \) neighborhood of a cluster center \( \{c_k\}^K_{k=1} \). Namely, \( y(x) = \arg \min_{y \in \varepsilon_0} |f(W^I,x) - \alpha_i| \), where \( \{\alpha_i\}^K_{i=1} \in [-1,1] \) denotes the labels associated with each class (cf. Definition 1.1 in \([67]\)). Second, for all training samples, the distance to initialization satisfies

\[
\|W^\tau - W^0\|_F \lesssim (\sqrt{K} + \tau^2 \kappa^2/||C||^2),
\]

where \( 0 \leq \tau \leq I \). The above conclusions demonstrate that gradient descent with early stopping (i.e., \( I \) steps) can be robust when training of deep neural networks. Moreover, the final network weights do not stray far from the initial weights for robustness, since the distance between the initial model and final model grows with the square root of the number of clusters \( \sqrt{K} \). Intuitively, due to memorization effects, deep neural networks will eventually overfit noisy training data. Thus, it is a good strategy to stop training early, when deep neural networks fit clean training data in first few epochs. This denotes the robust weights are not far away from the initial weights.

3.6 Taxonomy

Based on the above theoretical understanding, we categorize these works into three general perspectives:

1) **Data** \([68]\): From the perspective of data, we can construct \( \tilde{f} \) by leveraging the noise transition matrix, which explores the data relationship between clean and noisy labels. For example, we first model and estimate the noise transition matrix between latent \( Y \) and observed \( Y \). Then, via the estimated matrix, different techniques can generate \( \tilde{f} \) from the original \( f \). The key step here is to estimate the noise transition matrix. Mathematically, \( \tilde{R}_{\ell,p}(f_0) := \frac{1}{n} \sum_{i=1}^n \tilde{L}(f_0(X_i), \tilde{Y}_i) \), where \( \tilde{L} \) is a corrected loss transitioning from \( f \) via \( T \).

2) **Objective** From the perspective of objective, we can construct \( \tilde{f} \) by augmenting the objective function (i.e., the original \( f \)) via either explicit or implicit regularization. For instance, we may augment \( f \) by auxiliary example regularizer explicitly. Meanwhile, we may augment \( f \) by designing implicit regularization algorithms, such as soft-/hard-bootstrapping and virtual adversarial training (VAT). Meanwhile, we can construct \( \tilde{f} \) by reweighting the objective function \( f \). Lastly, motivated by some phenomenon or criteria, we can also construct and design \( \tilde{f} \) directly. Thus, \( \tilde{f} \) has three options:

- \( \tilde{f} = f + r \), where \( r \) denotes a regularization;
- \( \tilde{f} = \sum w_i \ell_i \), where \( \ell_i \) denotes ith sub-objective with the coefficient \( w_i \);
- \( \tilde{f} \) has a special format \( f' \) independent of \( f \).

3) **Optimization** \([46], [69]\): From the perspective of optimization, we can construct \( \tilde{f} \) by leveraging the memorization effects of deep models. For example, due to the memorization effects, deep models tend to fit the easy (clean) pattern first, and then over-fit the complex (noisy) pattern gradually. Based on this observation, we can back propagate the small-loss examples, which is equal to constructing the restricted \( \ell \) where \( \ell = sort(\ell,1,\tau) \), namely, sorting \( \ell \) from small, and fetching \( 1-\tau \) percentage of small loss (\( \tau \) is noise rate).

Accordingly, existing works can be categorized into a unified taxonomy as shown in Figure 2. We will detail each category in the sequel.

4 DATA

Methods in this section solve LNRL problem by estimating the noise transition matrix, which builds up the relationship between latent clean labels and observed noisy labels. The structure of this section is arranged as follows. First, we explain what is noise transition matrix and why this matrix is important. Then, we introduce three common ways to leverage noise transition matrix for combating label noise. The first common way is to leverage an adaptation layer in the end-to-end deep learning system to mimic the noise transition matrix, which bridges latent clean labels and observed noisy labels. The second common way is to estimate the noise transition matrix empirically, and further correct the cross-entropy loss by the estimated matrix. Lastly, the third common way is to leverage prior knowledge to ease the estimation burden.

4.1 Noise Transition Matrix

Before introducing three common ways, we first define what is the noise transition matrix, and explain why the noise transition matrix is important.

**Definition 3.** (Noise transition matrix \([68]\)) Suppose that the observed label \( y \) is noisy i.i.d. drawn from a corrupted distribution \( p(Y|X) \), where features are intact. Meanwhile, there exists a corruption process, transitioning from the latent clean label \( y \)
to the observed noisy label $\hat{y}$. Such corruption process can be approximately modeled via a label transition matrix $T$, where $T_{ij} = p(\hat{y} = e_j|y = e_i)$. We term this label transition matrix as noise transition matrix.

To further understand the transition matrix, we present two representative structures of $T$: (1) Sym-flipping [16]; (2) Pair-flipping [26]. The definition of the label transition matrix $T$ is as follows, where $\tau$ is the noise rate and $n$ is the number of the classes.

Sym-flipping: $T = \begin{bmatrix}
1 - \tau & \frac{\tau}{n-1} & \cdots & \frac{\tau}{n-1} \\
\frac{\tau}{n-1} & 1 - \tau & \cdots & \frac{\tau}{n-1} \\
\vdots & \ddots & \ddots & \vdots \\
\frac{\tau}{n-1} & \frac{\tau}{n-1} & \cdots & 1 - \tau
\end{bmatrix};$

Pair-flipping: $T = \begin{bmatrix}
1 - \tau & \tau & 0 & 0 \\
0 & 1 - \tau & \tau & 0 \\
0 & 0 & \ddots & \vdots \\
\tau & 0 & \cdots & 1 - \tau
\end{bmatrix}.$

Specifically, Sym-flipping structure models the most common classification task under label noise, where the class of clean label can uniformly flip into other classes. Meanwhile, Pair-flipping structure models the fine-grained classification task, where the class (e.g., Norwich terrier) of clean label can flip into its adjacent class (e.g., Norfolk terrier) instead of far-away class (e.g., Australian terrier). In the area of label-noise learning, we normally leverage the above two structures to generate simulated noise, and explore the root cause why the proposed algorithms can work on the simulated noise. Nonetheless, the real-world scenarios are very complex, where the noise transition matrix may not have structural rules (i.e., irregular). For example, ClothingIM is a taobao clothing dataset, where mislabeled clothing images often share similar visual patterns. The noise structure of ClothingIM is irregularly asymmetric, which is hardly estimated.

In mathematical modeling, the $ij$-th entry of the transition matrix, i.e., $[T(x)]_{ij} = p(\hat{y} = e_j|y = e_i, X = x)$, represents the probability that the instance $x$ with the clean label $Y = e_i$ will have a noisy label $\hat{y} = e_j$. The transition matrix has been widely studied to build statistically consistent classifiers, because the clean class posterior $p(Y|x) = [p(Y = e_1|X = x), \ldots, p(Y = e_n|X = x)]^T$ can be inferred by using the transition matrix and the noisy class posterior $p(\hat{Y}|x) = [p(\hat{y} = e_1|X = x), \ldots, p(\hat{y} = e_n|X = x)]^T$, i.e., we have the important equation $p(\hat{y}|x) = T(x)p(Y|x)$, where the noise transition matrix is a bridge between clean and noisy information. As the noisy class posterior can be estimated by exploiting the noisy training data, the key step remains how to effectively estimate the transition matrix and leverage the estimated matrix to combat with label noise. Based on this observation, we have three general ways as in Section 4.2.4.4

### 4.2 Adaptation Layer

Deep learning can be viewed as an end-to-end learning system. Therefore, the most intuitive way is to add an adaptation layer (Fig. 3) to estimate the transition matrix.

![Fig. 3. A general case of adaptation layer.](image-url)

#### 4.2.1 Linear Case

To realize this adaptation layer, Sukhbaatar et al. proposed a constrained linear layer inserted between base network and cross-entropy loss layer [18]. This linear adaptation layer is parameterized by $T$, which is equal to the function of noise transition matrix. Based on this idea, we can modify a classification model using a probability matrix $T$ that modifies its prediction to match the label distribution of the noisy data.

The training model consists of two independent parts: the base model parameterized by $\omega$ and the noise model parameterized by $T$. Since the noise matrix $T$ has been modeled as a constrained linear layer, the update of $T$ matrix can be easily finished by back propagating the cross-entropy loss. However, it is bare to achieve the optimal $T$ via minimizing cross-entropy loss, which is jointly parameterized by $\omega$ and $T$. To achieve the optimal $T$, Sukhbaatar et al. [18] leverage a regularizer on the $T$, e.g., a trace norm or a ridge regression, which forces it to diffuse. This work paves the way for deep learning with noisy labels, which directly motivates the following nonlinear case of adaptation layer.

#### 4.2.2 Nonlinear Case

Following the linear case, Goldberger et al. [21] proposed a non-linear layer inserted between base network and cross-entropy loss layer to realize this adaptation layer. Beyond the linear case, the training model consists of two independent parts: the base model parameterized by $\omega$ and the noise model/channel parameterized by $\theta$ (equal to the function of noise transition matrix). Since the outputs of the base model are hidden, they proposed to leverage EM algorithm to estimate the hidden outputs (E-step) and the current parameters (M-step). Different with linear case, nonlinear case is free of strong assumptions.

However, there are several potential drawbacks to the EM-based approach, such as local optima and scalability. To address these issues, Goldberger et al. proposed two noise modeling variants: c-model and s-model. Specifically, c-model predicts the noisy label based on both the latent true label and the input features; while s-model predicts the noisy label only based on the latent true label. Since the EM algorithm equals to the s-model, they regard both $\omega$ and $\theta$ as components of the same network and optimize them simultaneously. Moreover, s-model is similar to the linear case proposed by Sukhbaatar et al. [18], although they
proposed a different learning procedure. Note that the c-model depends on the input features, and they still leverage network training in the M-step to update $\theta$.

### 4.3 Loss Correction

Another important branch is to conduct loss correction via leveraging noise transition matrix, which can be also integrated into deep learning system. The aim of loss correction is that, training on noisy labels via the corrected loss should be approximately equal to training on clean labels via the original loss. Note that we also introduce a label smoothing method related to loss correction.

#### 4.3.1 Backward/Forward Correction

Patrini et al. [22] introduced two loss correction techniques, namely forward correction and backward correction. In high level, backward procedure corrects the cross-entropy loss by transition matrix $T^{-1}$; while forward procedure corrects the network predictions by transition matrix $T$. Both corrections share the formal theoretical guarantees w.r.t the clean data distribution.

**Theorem 1.** (Backward Correction, Theorem 1 in [22]) Suppose that the label transition matrix $T$ is non-singular, where $T_{ij} = p(y = e_j | y = e_i)$ given that corrupted label $y = e_j$ is flipped from clean label $y = e_i$. Given loss $\ell$ and network parameter $w_f$, Backward Correction is defined as

$$\ell^\rightarrow(\mathbf{x}, y; w_f) = T^{-1}\ell(\mathbf{x}, y; w_f).$$

Then, corrected loss $\ell^\rightarrow(\mathbf{x}, y; w_f)$ is unbiased, namely,

$$E_{y|x}\ell^\rightarrow(\mathbf{x}, y; w_f) = E_{y|x}\ell(\mathbf{x}, y; w_f), \forall x.$$

**Remark.** Backward Correction operates on the loss vector directly, where the loss is a vector. It is unbiased. LHS of (5) draws from corrupted labels, and RHS of (5) draws from clean labels. Note that the corrected loss is differentiable, but not always non-negative [68].

**Theorem 2.** (Forward Correction, Theorem 1 in [22]) Suppose that the label transition matrix $T$ is non-singular, where $T_{ij} = p(y = e_j | y = e_i)$ given that corrupted label $y = e_j$ is flipped from clean label $y = e_i$. Given loss $\ell$ and network parameter $w_f$, Forward Correction is defined as

$$\ell^\rightarrow(\mathbf{x}, y; w_f) = \ell(T^\top w_f(\mathbf{x}), y).$$

Then, the minimizer of the corrected loss under the noisy distribution is the same as the minimizer of the original loss under the clean distribution, namely,

$$\arg\min E_{\mathbf{x}, \bar{y}}\ell^\rightarrow(\mathbf{x}, y; w_f) = \arg\min E_{\mathbf{x}, y}\ell(\mathbf{x}, y; w_f).$$

**Remark.** Forward Correction also operates on the loss vector directly, where the loss is a vector. LHS of (7) draws from corrupted labels, and RHS of (7) draws from clean labels. Note that the property is weaker than unbiasedness of Theorem 1.

Normally, $T$ is unknown, which needs to be estimated (i.e., $\hat{T}$). Therefore, Patrini et al. proposed a robust two-stage training. First, they train the network with $\ell$ on noisy data, and obtain an estimation of $T$ via the output of the softmax. Then, they re-train the network with the corrected loss by $\hat{T}$. Note that the estimation quality of $T$ directly decides the learning performance via the loss correction.

#### 4.3.2 Gold Correction

Based on Forward Correction, Hendrycks et al. proposed Gold Correction to handle severe noise [25]. When severe noise, the transition matrix can not be estimated accurately by purely noisy data. The key motivation is to assume that a small subset of the training data is trusted. Normally, a large amount of crowdsourced workers may produce an untrusted set $\bar{D}$; while a small amount of experts can produce a trusted set $D$. In high level, Hendrycks et al. aim to leverage $D$ to estimate the noise transition matrix accurately, and employ Forward Correction based on the estimated matrix. Then, they train deep neural networks on $\bar{D}$ via the corrected loss, while training on $D$ via the original loss. This method is called as Gold Loss Correction (GLC).

Therefore, the key step in GLC is to estimate the noise transition matrix accurately. Mathematically, we can estimate the transition matrix $\hat{C}$ by $\hat{C}$ as follows.

$$\hat{C}_{ij} = \frac{1}{A_i} \sum_{x \in A_i} \hat{p}(\hat{Y} = e_j | Y = e_i, x),$$

where $\hat{p}(\hat{y}|x)$ can be modeled by deep neural networks trained on $D$. Empirically, the better estimation $\hat{C}$ will lead to the better GLC’s performance.

#### 4.3.3 Label Smoothing

The technique of label smoothing is to smooth labels by mixing in a uniform label vector [70], which is a means of regularization. Lukasik et al. relates label smoothing to a general family of loss-correction techniques [71], which demonstrates that label smoothing significantly improves performance under label noise. In general, both methods can be unified into a label smearing framework:

$$\ell_{SM}(f_0(X), Y) = M \cdot \ell(f_0(X), Y),$$

where $M$ is smearing matrix [72]. Such matrix is used for bridging original loss $\ell$ and smeared loss $\ell_{SM}$. Therefore, in this framework, there are three cases:

- **Standard training:** suppose $M = I$, where $I$ is identity matrix.
- **Label smoothing:** suppose $M = (1 - \alpha)I + \alpha J / L$, where $J$ is the all-ones matrix.
- **Backward correction:** suppose $M = 1/1-\alpha \cdot (I - \alpha J / L)$, where $M = T^{-1}$ in Theorem 1.

We can clearly see the closed connection between label smoothing and backward correction. Actually, label smoothing can have a similar effect to shrinkage regularization [73].

### 4.4 Prior Knowledge

As mentioned before, methods in this section solve LNRL problem by estimating the noise transition matrix. However, the accurate estimation can be hard in the real-world scenarios, which motivates researchers to incorporate prior knowledge for better estimation.

#### 4.4.1 Human-in-the-Loop Estimation

Han et al. proposed a human-assisted approach called “Masking” [26], which decouples the structure and value of transition matrix. Specifically, the structure can be viewed as a prior knowledge, coming from human cognition, since
human can mask invalid class transitions (i.e., cat ≈ car). Given the structure information, we can only focus on estimating the noise transition probability along the structure in an end-to-end system. Therefore, the estimation burden will be largely reduced. Actually, it makes sense that human cognition masks invalid class transitions and highlights valid class transitions, such as column-diagonal, tri-diagonal and block-diagonal. Therefore, the rest issue is how to incorporate such prior structure into an end-to-end learning system? The answer is generative model.

In particular, there are three steps in this generative model. First, the latent ground-truth label \( y \sim p(y|x) \), where \( p(y|x) \) is a Categorical distribution. Second, the noise transition matrix variable \( s \sim p(s) \) (in Bayes form) and its structure \( s_o \sim p(s_o) \), where \( p(s) \) is an implicit distribution modeled by neural networks without the exact form, \( p(s) = p(s) \frac{ds}{ds_o} \big|_{s_o = f(s)} \) and \( f(\cdot) \) is the mapping function from \( s \) to \( s_o \). Third, the noisy label \( \hat{y} \sim p(\hat{y}|y, s) \), where \( p(\hat{y}|y, s) \) models the transition from \( y \) to \( \hat{y} \) given \( s \). Based on the above generative process, we can deduce the evidence lower bound (ELBO) \([74]\) to approximate the log-likelihood of the noisy data.

### 4.4.2 Fine-tuning Revision

Xia et al. \([75]\) introduced a transition-revision method to effectively learn transition matrix, which is called Reweight T-Revision (Reweight-R). Specifically, they first initialize it by exploiting data points that are similar to anchor points, having high noisy class posterior probabilities. Thus, the initialized transition matrix can be viewed as a prior knowledge. Then, they fine-tune the initialized matrix by adding a slack variable, which can be learned and validated together with the classifier by using noisy data.

Specifically, given noisy training sample \( D^n \) and noisy validation set \( D^v \), there are two stages in Reweight-R. In the first stage, they minimize the unweighted loss to learn \( \hat{p}(\hat{Y}|X = x) \) without a noise adaption layer. Then, they initialize the noise transition matrix \( \hat{T} \), which can be viewed as a prior knowledge for further fine-tuning. Namely, they initialize \( T \) according to Eq. (1) in \([75]\) by using data with the highest \( \hat{p}(\hat{Y} = e_i|X = x) \) as anchor points for the \( i \)-th class. In the second stage, based on the prior \( \hat{T} \), they initialize the neural networks by minimizing the weighted loss with a noisy adaptation layer \( T^\top \). Furthermore, they minimize the weighted loss to learn classifier \( f \) and incremental \( \hat{T} \) with a noisy adaptation layer \( (\hat{T} + \Delta \hat{T})^\top \). Namely, the second stage modifies \( \hat{T} \) gradually by adding a slack variable \( \Delta T \), and learns the classifier and \( \Delta \hat{T} \) by minimizing the weighted loss. The two stages alternate until converges, namely minimum error on \( D^v \).

### 5 Objective

Methods in this section solve LNRL problem by modifying the objective function, and such modification can be realized in three different ways. The structure of this section is arranged as follows. First, we can directly augment the objective function by either explicit regularization or implicit regularization. Note that the implicit regularization tends to operate in algorithm level, which is equal to modifying the objective function. Second, we can assign dynamic weights to different objective sub-functions, and the more weights mean the more importance for the corresponding sub-functions. Lastly, motivated by some interesting observations and tricks, we can directly redesign new loss functions.

#### 5.1 Regularization

Regularization is the most direct way to modify the objective function. Mathematically, we need to add regularization term in original objective, i.e., \( \ell = \ell + r \). In label-noise learning, the aim of regularization is to achieve better generalization, which avoids or alleviates overfitting noisy labels. There are two intuitive ways as follows.

##### 5.1.1 Explicit Regularization

Azadi et al. propose a novel regularizer \( r = \Omega_{aux}(w) \) to exploit the data structure for combating label noise \([20]\), where \( \Omega_{aux}(w) = \|Fw\|_c \). Note that \( \|\cdot\|_c \) denotes the group norm, which induces a group sparsity that encourages the most coefficients to be zero. This operation will encourage a small number of clean data to contribute to learning of the model, while filtering mislabeled and non-relevant data. In other words, this regularizer enforces the features of the
good data to be used for modeling learning, while noisy additional activations will be disregarded.

It is worth noted that Berthelot et al. introduces MixMatch [50] for semi-supervised learning (SSL), and the empirical performance of MixMatch reaches the state-of-the-art. Most importantly, one of the key components in MixMatch is Minimum Entropy Regularization (MER), which belongs to explicit regularization in LNRL. Specifically, MER was proposed by Grandvalet & Bengio [85], and the key idea is to augment cross-entropy loss with an explicit term encouraging the classifier to make predictions with high confidence on the unlabeled examples, namely minimizing the entropy of $p_{\text{model}}(y|x; \theta)$ for unlabeled data $x$. Similar to MER, pseudo-label method conducts entropy minimization implicitly [86], which generates hard labels from high-confidence predictions on unlabeled data for further training. In particular, pseudo-label method (i.e., label guessing) first computes the average of the model’s predicted class distributions across all augmentations, and then applies a temperature sharpening function to reduce the entropy of label distribution.

### 5.1.2 Implicit Regularization

Recently, there are more and more implicit regularization, which takes the effects of regularization without the explicit form like Section 5.1.1. For example,

- **Bootstrapping** [19]: Reed et al. augment the prediction objective with a notion of consistency [19]. In high level, this provides the learner justification to “disagree” with a perceptually-inconsistent training label, and effectively relabel the data. Namely, the learner bootstraps itself in this way, which uses a convex combination of training labels and the current model’s predictions to generate the training targets. Intuitively, as the learner improves over time, its predictions can be trusted more. Such bootstrapping can avoid modeling the noise distribution.

Specifically, Reed et al. propose two ways to realize bootstrapping, such as soft and hard bootstrapping. Soft version uses predicted class probabilities $q$ directly to generate regression targets for each batch as follows:

$$\ell_{\text{soft}}(q, t) = \sum_{k=1}^{L} [\beta t_k + (1 - \beta)q_k] \log(q_k),$$

(10)

where $t$ denotes the training labels. The soft version equals to softmax regression with minimum entropy regularization, which encourages the model to have a high confidence in predicting labels.

Meanwhile, hard version modifies targets using MAP estimate of $q$ given $x$ as follows:

$$\ell_{\text{hard}}(q, t) = \sum_{k=1}^{L} [\beta t_k + (1 - \beta)z_k] \log(q_k),$$

(11)

where $z_k := 1[k = \arg \max_i q_i]$ and $i \in [1, \ldots, L]$. To solve the hard version via SGD, EM-like method will be employed. In the E-step, the approximate-truth confidence targets are estimated as a convex combination of training labels and model predictions. In the M-step, the model parameters are updated in order to predict those generated targets better.

- **Mixup** [27]: Motivated by Vicinal Risk Minimization (VRM), Zhang et al. introduces a data-agnostic regularization method called Mixup [27], which constructs virtual training examples $(\tilde{x}, \tilde{y})$ as follows.

$$\tilde{x} = \lambda x_i + (1 - \lambda)x_j \quad \text{and} \quad \tilde{y} = \lambda y_i + (1 - \lambda)y_j,$$

(12)

where $(x_i, y_i)$ and $(x_j, y_j)$ are two examples randomly drawn from the training data. Intuitively, Mixup conducts virtual data augmentation, which dilutes the noise effects and smooths the data manifold. This simple but effective idea can be used in not only noisy labels but also adversarial examples, since the smoothness happen in both features and labels according to (12).

- **VAT** [49]: Miyato et al. also explore the smoothness for combating label noise [49], and they propose virtual adversarial loss, where a new measure of local smoothness of the conditional label distribution given input. Specifically, their method trains the output distribution to be isotropically smooth around each input data via selectively smoothing the model in its most anisotropic direction. To realize such smoothness, they first design virtual adversarial direction, which can greatly deviate the current inferred output distribution from the status quo without the label information. Based on such direction, they define local distributional smoothness, and then propose a training method called virtual adversarial training (VAT).

In mathematically, they first define local distributional smoothness (LDS):

$$\text{LDS}(x^*, \theta) := D[p(y|x^*, \hat{\theta}), p(y|x^* + r_{\text{adv}}, \theta)],$$

(13)

where $r_{\text{adv}} := \arg \max_{\|r\|_2 \leq r} D[p(y|x^*, \hat{\theta}), p(y|x^* + r)]$, and $x^*$ represent either labeled or unlabeled features. We use $\hat{\theta}$ to denote the vector of model parameters at a specific iteration step of the training process. Then, we have a regularization term:

$$R_{\text{adv}}(D_l, D_{ul}, \theta) := \frac{1}{N_l + N_{ul}} \sum_{x^* \in D_l, D_{ul}} \text{LDS}(x^*, \theta).$$

Therefore, the full objective function of virtual adversarial training (VAT) is given by

$$\ell(D_l, \theta) + \alpha R_{\text{adv}}(D_l, D_{ul}, \theta).$$

- **SIGUA** [39]: It is noted that, given data with noisy labels, over-parameterized deep networks can gradually memorize the data, and fit everything in the end [46, 69, 57]. Although equipped with corrections for noisy labels, many learning methods in this area still suffer overfitting due to undesired memorization. To relieve this issue, Han et al. propose stochastic integrated gradient underweighted ascent (SIGUA) [39]: in a mini-batch, we adopt gradient descent on good data as usual, and learning-rate-reduced gradient ascent on bad data; the proposal is a versatile approach where data goodness or badness is w.r.t. desired or undesired memorization given a base learning method. Technically, SIGUA is a specially designed regularization by pulling optimization back for generalization when their goals conflict with each other. A key difference between SIGUA and parameter shrinkage like weight decay is that SIGUA pulls optimization back on some data but parameter shrinkage does the same on all
data; philosophically, SIGUA shows forgetting undesired memorization can reinforce desired one, which provides a novel viewpoint on the inductive bias of neural networks.

5.2 Reweighting

Instead of augmenting the objective via explicit/implicit regularization, there is another typical solution to modify the objective function termed Reweighting. In high level, Reweighting is a way to assign different weights to different sub-objective functions. The weights should be learned, and the larger weights will bias to sub-objectives that can better overcome label noise. The procedure can be formulated as \( \ell = \sum_i w_i \ell_i \). Here, we introduce several Reweighting approaches as follows.

5.2.1 Importance Reweighting

Liu and Tao [7] introduce importance reweighting [8] from domain adaptation to label-noise learning by treating the noisy training data as the source domain and the clean test data as the target domain. The idea is to rewrite the risk w.r.t. the clean data by exploiting the noisy data. Specifically,

\[
R(f) = E_{(X, Y) \sim D}[\ell(f(X), Y)] \\
= \int \sum_i p_D(X = x, Y = i) \ell(f(x), i) dx \\
= \int \sum_i \frac{p_D(X = x, Y = i)}{p_D(X = x, Y = \tilde{Y})} \frac{p_D(Y = \tilde{X} = x)}{p_D(Y = \tilde{Y} = i|X = x)} \ell(f(x), i) dx \\
= \int \sum_i \frac{p_D(X = x, \tilde{Y} = i)}{p_D(Y = \tilde{Y} = i|X = x)} \ell(f(x), i) dx \\
= E_{(X, \tilde{Y}) \sim D}[\beta(X, \tilde{Y}) \ell(f(X), \tilde{Y})],
\]

where the second last equation holds because label noise is assumed to be independent of instances and \( \beta(X, \tilde{Y}) = p_D(\tilde{Y} = i|X = x)/p_D(\tilde{Y} = i|X = x) \) denotes the weights which plays a core part in importance reweighting and can be learned by either exploiting the transition matrix or a small set of clean data.

5.2.2 Bayesian Method

Wang et al. [23] propose reweighted probabilistic models (RPM) to combat label noise. The idea is simple and intuitive: down-weighting on corrupted labels but up-weighting on clean labels, which brings us Bayesian data reweighting. The mathematical formulations include three steps:

1) Define a probabilistic model \( p_\beta(\beta) \prod_{n=1}^N \ell(y_n | \beta) \).
2) Assign a positive latent weight \( w_n \) to each likelihood, and choose a prior on the weights \( p_w(w) \), where \( w = (w_1, \ldots, w_N) \). Thus, the RPM can be represented by

\[
p(y, \beta, w) = \frac{1}{Z} p_\beta(\beta) p_w(w) \prod_{n=1}^N \ell(y_n | \beta) w_n,
\]

where \( Z \) is the normalizing factor.
3) Infer the posterior of both the latent variables \( \beta \) and the weight \( w \) and \( p(\beta, w | y) \). The prior knowledge on the weights \( p_w(w) \) trades off extremely low likelihood terms, while the options of \( p_w(w) \) are bank of Beta priors, scaled Dirichlet prior and bank of Gamma priors. Note that, RPMs treat weights \( w \) as latent variables, which are automatically inferred.

5.2.3 Mixture Model

Arazo et al. introduce a two-component (clean-noisy) beta mixture model (BMM) for a mixture of clean and noisy data [30], which brings us bootstrapping loss. Specifically, the posterior probabilities under BMM are leveraged to implement a dynamically-weighted bootstrapping loss, robustly dealing with noisy samples without discarding them. Mathematically, the probability density function of a mixture model of \( K \) components on the loss \( \ell \) is defined as:

\[
p(\ell) = \sum_{k=1}^K \lambda_k p(\ell | k),
\]

where \( \lambda_k \) are dynamic weights, and \( p(\ell | k) \) can be modeled by beta distribution:

\[
p(\ell | \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} \ell^{\alpha-1}(1-\ell)^{\beta-1},
\]

The above BMM can be solved by Expectation Maximization (EM) procedure. Specifically, they introduce latent variables \( \gamma_k(\ell) = p(k|\ell) \). In the E-step, they fix \( \lambda_k, \alpha_k, \beta_k \) and update \( \gamma_k(\ell) \) via Bayes rule. In the M-step, given fixed \( \gamma_k(\ell) \), they estimate \( \alpha_k \) and \( \beta_k \) using the weighted moments. Meanwhile, the dynamic weights are updated in an easy way:

\[
\lambda_k = \frac{1}{N} \sum_{i=1}^N \gamma_k(\ell_i).
\]

Based on this BMM model, they further propose dynamic hard/soft bootstrapping losses, where the weight \( w_i \) of each sample is dynamically set to \( p(k = 1 | \ell_i) \).

5.2.4 Neural Networks

Shu et al. introduce Meta-Weight-Net (MW-Net) [34], which can adaptively learn an explicit weighting function from data. In high level, the weighting function is an MLP with one hidden layer, mapping from loss to weights. Note that MW-Net is theoretically a universal approximator for any continuous function. Mathematically, the optimal parameter \( w \) can be calculated by minimizing the weighted loss,

\[
w^*(\theta) = \arg \min_w \ell^\theta(w; \theta) = \frac{1}{N} \sum_{i=1}^N \mathcal{V}(\ell^\theta_i(w); \theta) \ell^\theta_i(w),
\]

where \( \mathcal{V}(\ell^\theta_i(w); \theta) \) denotes MW-Net, namely a MLP network with one hidden layer. Here, the parameters in MW-Net can be optimized by the meta learning idea. Given a small amount of meta-data set \( \{x_i^{\text{meta}}, y_i\} \), the optimal parameter \( \theta \) can be obtained by minimizing the meta-loss:

\[
\theta^* = \arg \min_{\theta} \ell^\text{meta}(w^*(\theta)) = \frac{1}{M} \sum_{i=1}^M \ell^\text{meta}_i(w^*(\theta)).
\]

They employ SGD to update between \( w \) (parameters of classifier network) and \( \theta \) (parameters of MW-Net) iteratively.

5.3 Redesigning

Besides augmenting and reweighting the objective function, there is another common solution called redesigning. In high level, redesigning, which generally replaces \( \ell \) with a special format \( \ell' \) independent of \( \ell \), is motivated by different observations and principles. Thus, these method diverse much for different scenarios. Here, we introduce several redesigning approaches as follows.
5.3.1 Loss Redesign

In recent years, there are a lot of new losses for combating label noise. Their designs are based on different principles, such as gradient clipping [35] and curriculum learning [89]. Here, we choose several representative losses to explain.

- Zhang et al. propose a generalized cross entropy loss called $\ell_q$ [29], which encompasses both mean absolute error (MAE) and categorical cross entropy (CCE) loss. $\ell_q$ loss can be well justified by theoretical analysis. More importantly, it empirically works well for both closed-set and open-set noise. Mathematically, they use the negative Box-Cox transformation as a $\ell_q$ loss function:

$$\ell_q(f(x), e_j) = (1 - f_j(x)^q)/q,$$

where $q \in (0, 1]$. The proposed loss equals to CCE when $\lim_{q \to 0} \ell_q(f(x), e_j)$, and becomes MAE when $q = 1$. Since a tighter bound can bring stronger noise tolerance, they propose the truncated $\ell_q$ loss:

$$\ell_{\text{trunc}}(f(x), e_j) = \begin{cases} \ell_q(k) & \text{if } f_j(x) \leq k \\ \ell_q(f(x), e_j) & \text{otherwise} \end{cases}, \quad (16)$$

where $0 < k < 1$, and $\ell_q(k) = 1 - k^q/q$. When $k \to 0$, the truncated $\ell_q$ loss equals to the original $\ell_q$ loss.

- Charoenphakdee et al. [32] theoretically justify the advantages of symmetric losses via the lens of theoretical tools, including classification-calibration condition, excess risk bound, conditional risk minimizer and AUC-consistency. The key idea is to design a loss that does not have to satisfy the symmetric condition everywhere, i.e., $\ell(z) + \ell(-z)$ is a constant for every $z \in \mathbb{R}$. Motivated by this phenomenon, they introduce a barrier hinge loss, which satisfies a symmetric condition not everywhere but gives a large penalty once $z$ is outside of the interval. Mathematically, a barrier hinge loss is defined as follows.

$$\ell(z) = \max(-b(r + z) + r, \max(b(z - r), r - z)), \quad (17)$$

where $b > 1$ and $r > 0$.

- Thulasidasan et al. propose to abstain some confusing examples during training deep networks [33]. Philosophically speaking, abstention has some relation with small-loss tricks, which implicitly abstain big-loss examples during training. In practice, abstention has some relation with loss redesign. Based on abstention, they introduce a deep abstaining classifier (DAC). DAC has an additional output $p_{k+1}$, which indicates the probability of abstention. The loss of DAC is as follows.

$$\ell(x_j) = -\hat{p}_{k+1} \sum_{i=1}^{k} t_i \log(p_i/\hat{p}_{k+1}) - \alpha \log \hat{p}_{k+1}, \quad (18)$$

where $\hat{p}_{k+1} = 1 - p_{k+1}$. If $\alpha$ is large, the penalty drives $p_{k+1}$ to zero, which denotes the model not abstaining. If $\alpha$ is small, the classifier may abstain on everything. They further propose an auto-tuning algorithm to tune the optimal $\alpha$. Note that DAC can be used for both structured and unstructured label noise, where DAC becomes a data cleaner.

- Aditya et al. [35] leverage gradient clipping to design a new loss. Intuitively, clipping the gradient prevents over-confident descent steps in the scenario of label noise. Motivated by gradient clipping, they propose partially Huberized loss

$$\tilde{\ell}_q(x, y) = \begin{cases} -\tau p_0(x, y) + \log \tau + 1 & \text{if } p_0(x, y) \leq \frac{1}{\tau} \\ -\log p_0(x, y) & \text{otherwise} \end{cases}. \quad (19)$$

- Lyu et al. [38] propose curriculum loss (CL), which is a tighter upper bound of the 0-1 loss. Moreover, CL can adaptively select samples for stagewise training. In particular, giving any base loss function $\ell(u) \geq 1(u < 0)$, $u \in \mathbb{R}$, CL can be defined as follows.

$$Q(u) = \max \left( \min_{v \in \{0, 1\}^n} f_1(v), \min_{v \in \{0, 1\}^n} f_2(v) \right),$$

where

$$f_1(v) = \sum_{i=1}^{n} v_i \ell(u_i) \text{ and } f_2(v) = n - \sum_{i=1}^{n} v_i + \sum_{i=1}^{n} 1(u_i < 0).$$

To adapt CL for deep learning models, they further introduce the noise pruned CL.

5.3.2 Label Ensemble

- Laine & Aila [90] introduce self-ensembling, including $\pi$-model and temporal ensembling. Specifically, $\pi$-model encourages consistent network output between two realizations of the same input, under two different dropout conditions. Beyond $\pi$-model, temporal ensembling consider the network predictions over multiple previous training epochs. The loss function of $\pi$-model is:

$$\ell = \frac{1}{B} \sum_i \log z_i[y_i] + \frac{w(t)}{C} \sum_i \|z_i - \hat{z}_i\|^2, \quad (20)$$

where the first term handles labeled data, and the second term handles unlabeled data. Both $z_i$ and $\hat{z}_i$ are transformed from the same input $x_i$. The second term is also scaled by time-dependent weighting function $w(t)$.

Temporal ensembling goes beyond $\pi$-model by aggregating the predictions of multiple previous network evaluations into an ensemble prediction. Namely, the main difference to $\pi$-model is that the network and augmentations are evaluated only once per input per epoch, and the target $\hat{z}$ is based on prior network evaluations. After every training epoch, the network outputs $z$ are accumulated into ensemble outputs $Z$ by updating $Z \leftarrow \alpha Z + (1 - \alpha)z$ and $\hat{z} \leftarrow Z/(1 - \alpha)$.  

- Nguyen et al. [36] propose a self-ensemble label filtering (SELF) method to progressively filter out the wrong labels during training. In high level, they leverage the knowledge provided in the network’s output over different training iterations to form a consensus of predictions, which progressively identify and filter out the noisy labels from the labeled data. In filtering strategy, the model can determine the set of potentially correct labels $L_i$ based on agreement between the label $y$ and its maximal likelihood prediction $\hat{y}|x$ with $L_i = \{(y, x)|\hat{y}_x = y; \forall(y, x) \in L_0\}$, where $L_0$ is the label set in the beginning. In self-ensemble strategy, they maintain two-level ensemble. First, they leverage model ensemble with Mean Teacher, namely an exponential running average of model snapshots. Second, they employ prediction ensemble by collecting the sample predictions over multiple training epochs:
where \( \bar{z}_j = \alpha \bar{z}_{j-1} + (1 - \alpha) \tilde{z}_j \), where \( \bar{z}_j \) depicts the moving-average prediction of sample \( k \) at epoch \( j \) and \( \tilde{z}_j \) is the model prediction for sample \( k \) in epoch \( j \).

- Ma et al. [29] investigate the dimensionality of the deep representation subspace of training samples. Then, they develop a dimensionality-driven learning strategy, which monitors the dimensionality of subspaces during training and adapts the loss function accordingly.

In high level, they leverage local intrinsic dimensionality (LID) to discriminate clean labels and noisy labels during training deep networks. Then, they find two-stage of learning in this scenario: an early stage of dimensionality compression and dimensionality expansion, which corresponds to memorization effects implicitly. Based on such two stages, they propose Dimensionality-Driven Learning (D2L), which avoids the dimensionality expansion stage of learning by adapting the loss function. Mathematically, the estimation of LID can be defined:

\[
\text{LID} = - \left( \frac{1}{k} \sum_{i=1}^{k} \log \frac{r_i(x)}{r_{\text{max}}(x)} \right)^{-1},
\]

where \( r_i(x) \) denotes the distance between \( x \) and its \( i \)-th nearest neighbor, and \( r_{\text{max}}(x) \) denotes the maximum of the neighbor distance. They can update LID score via batch sampling for computation efficiency. Based on LID score, they can oversee the dynamics of deep networks.

Specifically, when learning with clean labels, LID score is consistently decreasing and the test accuracy is increasing with the increase of training epochs. However, when learning with noisy labels, LID score first decreases and then increases after few epochs. In contrast, the test accuracy is totally opposite, which first increases and then decreases. This corresponds to memorization effects of deep networks (Section 5.1). Based on observation in LID, they propose dimensionality-driven learning strategy. Intuitively, the larger LID denotes dimensionality expansion, vice versa. To avoid dimensionality expansion, they propose the adaptive LID-corrected labels:

\[
y^* = \alpha_i y + (1 - \alpha_i) \hat{y},
\]

where \( \alpha_i = \exp(-\lambda \text{LID}_{i}/\min_{j=1}^{k} \text{LID}_j) \) is a LID-based factor. Based on \( y^* \), they can robustly train deep networks.

### 5.4 Discussion

Based on the above observations, we can know that modifying the objective function is another mainstream method to solve LNRL problem. First, we can augment the objective via either explicit regularizer, e.g., Minimum Entropy Regularization [35], or implicit regularizer, e.g., Virtual Adversarial Training [49]. Second, instead of treating all sub-objective functions equally, we can leverage reweighting strategy to assign different weights to sub-objective functions. The more weights we assign, the more important these sub-objective functions are. We can realize reweighting strategy via different ways, e.g., importance reweighting, Bayesian method, mixture model and neural networks. Lastly, we can modify the objective function via redesigning the loss function, e.g., \( L_q \), barrier hinge loss, partial Huberized loss and curriculum loss. Moreover, we can conduct label ensemble, e.g., temporal ensembling and self-ensemble filtering.

Note that there are other related works from the objective perspective. For instance, online crowdsourcing greatly reduces the amount of redundant annotations, when crowdsourcing annotations such as bounding boxes, parts, and class labels [91]; undirected graphical model represents the relationship between noisy and clean labels, where the inference over latent clean labels is tractable and regularized using auxiliary information [92]; active-bias method trains more robust deep networks by emphasizing high variance samples [93]; model bootstrapped EM jointly models labels and worker quality from noisy crowdsourced data [94]; joint optimization framework corrects labels during training by alternating update of network parameters and labels [95]; iterative learning framework trains deep networks with open-set noisy labels [96]; deep bilevel learning is based on the principles of cross-validation, where a validation set is used to limit the model over-fitting [97]; symmetric cross entropy (CE) boosts CE symmetrically with a noise robust counterpart Reverse Cross Entropy (RCE) [98]; ubiquitous reweighting network learns a robust model from large-scale noisy web data, by considering five key challenges in image classification [99]; information-theoretic loss is a generalized version of mutual information, which is provably robust to instance-independent label noise [100]; peer loss enables learning from noisy labels without requiring a priori specification of the noise rates [101]; and normalized loss theoretically demonstrates that a simple normalization can make any loss function robust to noisy labels [102].

### 6 Optimization Policy

Methods in this section solve LNRL problem by changing optimization policy, such as early stopping. The effectiveness of early stopping is due to memorization effects of deep neural networks, which avoid overfitting noisy labels to some degree. To combat with noisy labels using memorization effects, there exist the other better way, namely small-loss tricks. The structure of this section is arranged as follows. First, we explain what are memorization effects and why this phenomenon is important. Then, we introduce several common ways to leverage such memorization effects for combating label noise. The first common way is to self-train single network robustly via small-loss tricks, which brings us MentorNet and Learning to Reweight. Furthermore, the second common way is to co-train double networks robustly via small-loss tricks, which brings us Co-teaching and Co-teaching+. Lastly, there are several ways to further improve the performance of Co-teaching, such as cross-validation, automated learning and Gaussian mixture model.

#### 6.1 Memorization Effects

Arpit et al. introduce a very critical work called “A closer look at memorization in deep networks” [45], which shapes a new direction towards solving LNRL. In general, memorization effects can be defined as the behavior exhibited by Deep Networks trained on noise data. Specifically, deep networks tend to memorize and fit easy (clean) patterns, and gradually over-fit hard (noisy) patterns. Here, we empirically reproduce a simulated experiment to justify this hypothesis.

From Fig. 4, we leverage MNIST dataset, and add random noise on its labels. The noise rate comes from the
range between 0% and 80%. Empirically, we train our deep networks on corrupted training data. Then, we test the trained networks on both training data and validation data, respectively. We can clearly see two phenomenon:

- The training curve will finally reach or approximate 100% accuracy eventually. All curves will converge the same.
- The validation curve will first reach near 100% accuracy in first few epochs, but drop gradually until convergence (after 40 epochs).

Under the data corruption, since deep networks tend to memorize and fit easy (clean) patterns in corrupted data, the validation curve will first reach a peak. However, such overparameterized model will gradually over-fit hard (noisy) patterns. The validation curve will drop gradually, since the validation data is clean. This simple experiment not only justifies the hypothesis of memorization effects, but also open a new door to LNRL problem, namely small-loss tricks.

Specifically, small-loss tricks mean deep networks regard small-loss examples as “clean” examples, and only back propagate such examples to update the model parameters. Mathematically, small-loss tricks is equivalent to constructing the restricted $\ell$, where $\ell = \text{sort}(\ell, 1 - \tau)$. Namely, $\ell$ can be constructed by sorting $\ell$ from small to large, and fetching $1 - \tau$ percentage of small loss ($\tau$ is noise rate).

6.2 Self-training

Based on small-loss tricks, the seminal works leverage self-training to improve the model robustness (left panel of Fig. 5). There are two works as follows.

6.2.1 MentorNet

Jiang et al. introduce MentorNet [6], which supervises the base deep networks termed StudentNet. The key focus of MentorNet is to provides a curriculum for StudentNet. Instead of pre-defining, MentorNet learns a data-driven curriculum dynamically.

Mathematically, MentorNet $g_m$ can approximate a predefined curriculum via minimizing the equation as follows.

$$\arg\min_{\theta} \sum_{(x_i, y_i) \in D} g_m(x_i; \theta)\ell_i + G(g_m(x_i; \theta); \lambda_1, \lambda_2).$$

To address the above objective, we can get the closed-form solution as follows.

$$g_m(z; \theta) = \begin{cases} 1(\ell_i \leq \lambda_1) & \text{if } \lambda_2 = 0, \\ \min(\max(0, 1 - \ell_i, -\lambda_2), 1) & \text{if } \lambda_2 \neq 0. \end{cases}$$

Intuitively, when $\lambda_2 = 0$, MentorNet only provides small-loss samples, where $\ell_i < \lambda_1$. When $\lambda_2 \neq 0$, MentorNet will not provide big-loss samples, namely, samples of loss larger than $\lambda_1 + \lambda_2$ will not be selected during training. Meanwhile, MentorNet can also discover new curriculums from data directly, which is unrelated to small-loss tricks.

6.2.2 Learning to Reweight

Ren et al. [24] employ meta-learning framework to assign different weights to training examples based on their gradient directions. In general, the small-loss examples are assigned to more weights, since small-loss examples are more likely to be clean. In general, Ren et al. believe that the best example weighting should minimize the loss of a set of unbiased clean validation examples. Namely, they perform validation at every training iteration to dynamically determine the example weights of the current batch. Mathematically, they hope to learn a reweighting of the inputs via minimizing a weighted loss:

$$\theta^*(w) = \arg\min_{\theta} \sum_{i=1}^N w_i f_i(\theta),$$

where $w$ is based on its validation performance:

$$w^* = \arg\min_{w} \frac{1}{M} \sum_{i=1}^M f_i^*(\theta^*(w)).$$

To realize “Learning to Reweight”, there are three technique steps. First, they forward and backward noisy training examples via training loss, which updates the model parameter $\theta$ and calculates $\nabla \theta$. Second, the incremental $\nabla \theta$ affects the validation networks, where they forward and backward clean validation examples via validation loss. Lastly, training networks leverage meta-learning to update example weights $w$ via backward on backward. Note that the same strategy can be also used for class imbalance problems, where big-loss tricks are preferred, since they are more likely to be the minority class.

6.3 Co-training

Although self-training works well, in the long term, the error will be definitely accumulated, which motivates us to explore Co-training based methods (right panel of Fig. 5).
6.3.1 Co-teaching/Co-teaching+  

Han et al. propose a new deep learning paradigm called “Co-teaching” [7]. In general, instead of training single deep network, they train two deep networks simultaneously, and let them teach each other given every mini-batch. Specifically, each network feeds forward all data and selects some data of possibly clean labels; then, two networks communicate with each other what data in this mini-batch should be used for training; lastly, each network back propagates the data selected by its peer network and updates itself. The selection criteria is still small-loss trick.

In MentorNet [6], the error from one network will be directly transferred back to itself in the second mini-batch of data, and the error should be increasingly accumulated. However, in Co-teaching, since two networks have different learning abilities, they can filter different types of error introduced by noisy labels. Namely, in this exchange procedure, the error flows can be reduced by peer networks mutually. However, with the increase of training epochs, two networks will converge to a consensus and Co-teaching will reduce to the self-training MentorNet in function. Note that the principle of ensemble learning is to keep different classifiers diverged.

Yu et al. [103] introduce “update by disagreement” [104] strategy to keep Co-teaching diverged and named their method Co-teaching+. In general, Co-teaching+ consists of disagreement-update step and cross-update step. In disagreement-update step, two networks feed forward and predict all data first, and only keep prediction disagreement data. This step indeed keeps two networks diverged. The cross-update step has been explored in Co-teaching. Note that both Co-teaching and Co-teaching+ share the same dropping rate for big-loss examples, which is hand designed. Via both methods, we summarize three key factors in this line research: (1) using the small-loss trick; (2) cross-updating parameters of two networks; and (3) keeping two networks diverged.

6.3.2 Beyond Co-teaching  

After 2018, there are several important works to further improve Co-teaching. Here, we specify two representative works based on Co-teaching and go beyond.

- Chen et al. use cross-validation to randomly split noisy datasets, which identifies most samples that have correct labels [105]. In general, Chen et al. design Iterative Noisy Cross-Validation (INCV) method to select a subset of samples, which has much smaller noise ratio than the original dataset. Then, they leverage Co-teaching for further training over a selected subset. Apart from selecting clean samples, the INCV removes samples that have large loss at each iteration.

- Yao et al. [87] use automated machine learning (AutoML) [106], [107] to explore the memorization effect thus improve Co-teaching. It is noted that both Co-teaching and Co-teaching+ share the same dropping rate for big-loss examples, which is hand designed. However, such rate is critical in training deep networks. Specifically, Yao et al. [87] design a domain-specific search space based on such rate and propose a novel Newton algorithm to solve the bi-level optimization problem efficiently. To explore the optimal rate $R(\cdot)$, they formulate the problem as

$$R^* = \arg \min_{R(\cdot) \in \mathcal{F}} L_{\text{val}}(f(w^*; R), D_{\text{val}}),$$

s.t. $w^* = \arg \min_w L_{\text{tr}}(f(w^*; R), D_{\text{tr}})$,

where $\mathcal{F}$ is the search space of $R(\cdot)$ exploring the general pattern of memorization effect. Such a prior on $\mathcal{F}$ not only allows efficient bi-level optimization but also boost the final learning performance.

- Motivated by MixMatch [50], Li et al. [37] promote a novel framework termed DivideMix by leveraging semi-supervised learning techniques. In high level, DivideMix models the per-sample loss distribution with a mixture model, which dynamically divide the training data into two parts. The first part includes labeled data with clean labels; while the second part includes unlabeled data with noisy labels. During the semi-supervised learning phase, they leverage variants of co-training, such as co-refinement on labeled data and co-guessing on unlabeled data.

Specifically, Li et al. [37] use Gaussian Mixture Model (GMM) to better distinguish clean and noisy samples, due to its flexibility in the sharpness of distribution. They fit a two-component GMM to $\ell$ using the EM algorithm. For each sample, its clean probability $w_i$ is the posterior probability $p(g|\ell_i)$. Based on GMM, they propose co-divide, where the GMM for one network is used to divide training data for its peer network. The dividing criteria is to set a threshold $\tau$ on $w_i$. Namely, samples larger than $\tau$ will be regarded as clean samples. Once the data is divided into labeled and unlabeled data, they conduct co-refinement for labeled data, which linearly combines the ground-truth label with the network’s prediction and sharpens on the refined label. Then they conduct co-guessing for unlabeled data, which averages the predictions from both networks. After co-refinement and co-guessing, they follow the routine MixMatch to mix the data, and update the model parameters.

6.4 Beyond Memorization  

This section focuses on solving LNRL by leveraging overparameterized deep models, especially from their memorization effects. However, besides memorization, there are two new branches based on deep models as follows.

6.4.1 Pre-training  

In many CV and NLP applications, pre-training paradigm has become commonplace, especially when data is scarce in target domain. Hendrycks et al. [31] demonstrate that pre-training can improve model robustness and uncertainty, including adversarial robustness and label corruption.

Normally, pre-training occurs on a bigger dataset first, and fine-tuning the pre-trained model on a smaller target dataset. For example, if we design a LNRL method for image classification with label noise, we can pre-train a model on ImageNet via LNRL method first. Then, we can fine-tune a pre-trained model on target dataset via LNRL method. Note that pre-training approach has been demonstrated in many robustness and uncertainty tasks, including label noise, adversarial examples, class imbalance, out-of-distribution detection and calibration.
6.4.2 Deep k-NN
Bahri et al. propose Deep k-NN method [108], which executes on an intermediate layer of a preliminary deep model to filter mislabeled training data. In high level, deep k-NN filtering consists of two steps. In the first step, they train a model \( M \) with architecture \( A \) (e.g., 2-layer DNN with 20 hidden nodes) to filter noisy data \( D_{\text{noisy}} \) via k-NN algorithm, which identifies and removes examples whose labels disagree with their neighbors. After filtering \( D_{\text{noisy}} \), in the second step, they re-train a final model with architecture \( A \) on \( D_{\text{filtered}} \) \( \cup D_{\text{clean}} \).

6.5 Discussion
Based on the above observations, we can know that leveraging memorization effects is an emerging mainstream method to solve LNRL problem. First, we can combine self-training with memorization effects, which brings us self-paced MentorNet and learning to reweight. Second, we can combine co-training with memorization effects, which introduces Co-teacher, Co-teacher+, INCV Co-teacher and S2E. Lastly, we can combine co-training with the SOTA semi-supervised learning MixMatch, which provides DivideMix. Meanwhile, besides memorization, pre-training and deep k-NN are new branches using overparameterized models.

Note that there are other related works from the optimization policy perspective, namely changing training dynamic. For example, multi-task network jointly learns to clean noisy annotations and accurately classify images [109]; the unified framework of random grouping and attention effectively reduces the negative impact of noisy web image annotations [110]; decoupling trains two deep networks simultaneously, and only updates parameters on examples, where there is a disagreement between the two classifiers [104]; CleanNet is designed to make label noise detection and learning from noisy data with human supervision scalable through transfer learning [111]; CurriculumNet designs a training curriculum by measuring and ranking the complexity of data using its distribution density in a feature space [112]; co-mining combines co-teaching with Arcface loss [113] for face recognition task [114]; O2U-Net only requires adjusting the hyper-parameters of deep networks to make its status transfer from overfitting to underfitting (O2U) cyclically [115]; deep self-learning proposes an iterative learning framework to relabel noisy samples and train deep networks on the real noisy dataset, without using extra clean supervision [116]; label-noise information strategy proposes training methods that control memorization by regularizing label noise information in weights [117]; different from Co-teacher+, Co-regularization aims to reduce the diversity of two networks during training [118]; and data coefficient method wisely takes advantage of a small trusted dataset to optimize exemplar weights and labels of mislabeled data, which distills effective supervision for robust training [119].

7 Future Works
Starting from 1988, there are three decades in label-noise learning, evolving from statistical learning to representation learning. Especially from 2012, representation learning becomes increasingly important, which directly gives birth to above LNRL methods. Similar to other areas in machine learning, we hope to propose not only new methods, but also new research directions, which can broaden and boost the LNRL research in both academia and industry.

7.1 Build up New Datasets
In LNRL, the first thing we should do is to construct new datasets with real noise, which is critical to the rapid development of LNRL. To our best knowledge, most of researchers test their LNRL methods on simulated datasets, such as MNIST and CIFAR-10. To make further breakthrough, we should build up new datasets with real noise, such as Clothing1M [120].

Note that, similar to ImageNet, many researchers train deep networks to overfit Clothing1M via different tricks, which may not touch the core issue of LNRL. This motivates us to rethink real datasets in LNRL. After Clothing1M born in 5 years, Jiang et al. [40] propose a new but realistic type of noisy dataset called “web-label noise” (or real noise), which enables us to conduct controlled experiments systematically in realistic scenario. Another interesting point is that benchmark datasets with real noise mainly focus on image classification, instead of natural language and speech processing. Obviously, these directions also involve label noise, which need to be addressed further.

To sum up, we should build up new benchmark datasets with real noise, not only in image but also in language and speech. Normally, the better datasets can boost the rapid development of LNRL.

7.2 Instance-dependent LNRL
Previously in Section 3.5.1, we have seen that CCN is a popular assumption in LNRL. However, CCN model is only an approximation to the real-world noise, which may not work well in practice. To directly model the real-world noise, we should consider the features in the label corruption process. This motivates us to explore instance-dependent noise (IDN) model, which is formulated as \( p(\hat{Y} | X, Y) \).

Specifically, the IDN model considers a more general noise, where the probability that an instance is mislabeled depends on both its class and features. Intuitively, this noise is quite realistic, as poor-quality or ambiguous instances are more likely to be mislabeled in real-world datasets. However, it is much more complex to formulate the IDN model, since the probability of a mislabeled instance is a function of not only the label space but also the input space that can be very high dimensional. Moreover, without some extra assumption/information, IDN is unidentifiable.

Towards IDN model, there are three seminal explorations. For instance, Menon et al. propose the boundary-consistent noise model [62], which considers stronger noise for samples closer to the decision boundary of the Bayesian optimal classifier. However, such a model is restricted to binary and cannot estimate noise functions. Cheng et al. recently studied a particular case of the IDN model [63], where noise functions are upper-bounded. Nonetheless, their method is limited to binary classification and has only been tested on small datasets. Berthon et al. propose to tackle the IDN model from the source, by considering confidence scores to be available for the label of each instance. They term this new setting confidence-scored IDN (CSIDN) [121]. Based on CSIDN, they derive an instance-level forward correction algorithm.
We have envisioned three promising directions above, which are adversarially perturbed while the features and labels are intact (i.e., clean). Is it the optimal way to acquire adversarial robustness? In other words, shall we consider the scenario, where the features are adversarially perturbed while the labels are noisy? We term this adversarial LNRL.

Towards adversarial LNRL, there are two seminal works. For example, Wang et al. propose a new defense algorithm called misclassification aware adversarial training (MART) [122], which explicitly differentiates the misclassified examples (i.e., label noise) and correctly classified examples during the training. To address this issue, MART introduces misclassification aware regularization, namely $1/n \sum_{i=1}^{n} 1(h_\theta(x_i) \neq h_\theta(\hat{x}_i)) \cdot 1(h_\theta(x_i) \neq y_i)$. Intuitively, $1(h_\theta(x_i) \neq y_i)$ denotes the misclassified examples, which can be closely connected with label noise. Meanwhile, Zhang et al. consider the same issue, namely misclassified examples in adversarial training. Specifically, they propose friendly adversarial training (FAT) [123], which trains deep networks using the wrongly-predicted adversarial data minimizing the loss and the correctly-predicted adversarial data maximizing the loss. To realize FAT, they introduce early-stopped PGD. Namely, once adversarial data is misclassified by the current model, they just stop the PGD iterations early. In high level, the objective of FAT is min-min instead of min-max in standard adversarial training.

### 7.4 Beyond Labels: Noisy Data

We have envisioned three promising directions above, which belong to the vertical domain in LNRL. However, we hope to explore the horizontal domain more, namely noisy data instead of only noisy labels. Here, we summarize different formats of noisy data, and show some preliminary works.

- **Feature**: Naturally, label noise can arouse us to consider feature noise, where adversarial example is one of special cases in feature noise. The problem of feature noise is formulated as $\mathcal{p}(\hat{X}|Y)$, where features are corrupted but labels are intact. Therefore, adversarial training can be the main tool to defend adversarial examples. Note that, there exists another feature noise called random perturbation. To address this issue, Zhang et al. propose a robust ResNet [124], which is motivated by dynamic systems. Specifically, they characterize ResNet based on an explicit Euler method. This allows us to exploit the step factor $h$ in the Euler method to control the robustness of ResNet. They have proved that a small step factor $h$ can benefit its training and generalization robustness during back and forward propagation. Namely, controlling $h$ robustifies deep networks, which can alleviate feature noise.

- **Preference**: Han et al. [125] and Pan et al. [126] try to address preference noise in ranking problem, which plays an important role in our daily life, such as ordinal peer grading, online image-rating and online product recommendation. Specifically, Han et al. propose ROPAL model, which integrates the Plackett-Luce model with a denoising vector. Based on the Kendall-tau distance, this vector corrects $k$-ary noisy preferences with a certain probability. However, ROPAL cannot handle dynamic length of $k$-ary noisy preferences, which motivates Pan et al. to propose COUPLE, which leverages stagewise learning to break the limit of fixed length. To update the parameters of both models, they use online Bayesian inference.

- **Domain**: Domain adaptation (DA) is one of the fundamental problems in machine learning, when the data volume in target domain is scarce. Previous DA methods assume that labeled data in source domain is purely clean. However, in practice, labeled data in source domain come from amateur annotators or the Internet due to its large volume. This issue brings us a new setting, where labels in source domain are noisy. We call this setting as *wildly domain adaptation* (WDA). There are two seminal works. Specifically, to handle WDA, Liu et al. [127] propose a Butterfly framework, which maintains four deep networks simultaneously. Butterfly can obtain high-quality domain-invariant representations (DIR) and target-specific representations (TSR) in an iterative manner. Meanwhile, Yu et al. [128] propose a novel Denoising Conditional Invariant Component (DCIC) framework, which provably ensures extracting invariant representations and estimating the label distribution in target domain with no bias.

- **Similarity**: Similarity-based learning is one of emerging problems. Compared to class labels, similarity labels are easier to obtain, especially for some sensitive issues, e.g., religion and politics. For example, for sensitive matters, people often hesitate to directly answer “What is your opinion on issue A?”; while they are more likely to answer “With whom do you share the same opinion on issue A?”. Therefore, similarity labels are easier to obtain. However, for some cases, people may not be willing to provide their real thoughts even facing easy questions. Therefore, noisy-similarity-labeled data are very challenging. Wu et al. employ a noise transition matrix to model similarity noise [129], which has been integrated into a deep learning system.

- **Graph**: Graph neural networks (GNNs) are very hot in machine learning community [130]. However, are GNNs robust to noise? For example, once the node or edge is corrupted, the performance of GNNs will deteriorate definitely. Since GNNs is highly related to discrete and combinatorial optimization, LNRL methods can not be directly deployed. Therefore, it is very meaningful to robustify GNNs under the node or edge noise, where noise can occur in label and features. Recently, Wang et al. propose a robust and unsupervised embedding framework called Cross-Graph [131], which can handle structural corruption in attributed graphs.

- **Demonstration**: The goal of imitation learning (IL) is to learn a good policy from high-quality demonstrations. However, the quality of demonstrations in reality should be diverse, since it is easier and cheaper to collect demonstrations from a mix of experts and amateurs. This brings us a new setting in IL called diverse-quality demonstrations, where low-quality demonstrations are highly noisy. To handle diverse-quality demonstrations, when experts provide additional information about the quality, learning becomes relatively easy, since the quality can be estimated by their confidence scores [132], ranking scores [133] and a small number of high-quality demonstrations [134]. However, without the...
availability of experts, these methods may not work well. Recently, Voot et al. push forward this line, and propose to model the quality with a probabilistic graphic model termed VILD [135]. Specifically, they estimate the quality along with a reward function that represents an intention of experts’ decision making. Moreover, they use a variational approach to handle large state-action spaces, and employ importance sampling to improve the data efficiency.

7.5 Summary
In future, we should have four research directions. Among them, the first three directions mainly focus on LNRL itself. Building up new datasets will boost the rapid development of LNRL; while instance-dependent LNRL and adversarial LNRL will push the knowledge boundary of INRL deeply. Lastly, beyond noisy labels, there are noisy data, including feature, preference, domain, similarity, graph and demonstration. Our research scope extends from a point to a surface.

8 Conclusions
In this survey, we thoroughly review the history of label-noise representation learning (LNRL), and formally define what is LNRL from the view of machine learning. Via the lens of representation learning theory and empirical experiments, we try to understand the mechanism of deep networks under label noise. Based on the above analysis, we categorize different LNRL methods into three perspectives, namely data, objective and model. Specifically, under this unified taxonomy, we provide thorough discussion of pros and cons across different categories. Moreover, we summarize the essential components of robust LNRL, which can enlighten new directions in LNRL. Lastly, we propose four possible research directions. The first three directions mainly focus on pushing the knowledge boundary of LNRL, including building up new datasets, instance-dependent LNRL and adversarial LNRL. The last direction is beyond LNRL, which learns from noisy data, such as preference-noise, domain-noise, similarity-noise, graph-noise and demonstration-noise. Ultimately, we hope to uncover the secret of data-noise representation learning, and formulate a general framework in the near future.

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Appendix

Early Stage. Before delving into label-noise representation learning, we should briefly overview some of milestone works in label-noise statistical learning. Starting from 1988, Angluin et al. prove that a learning algorithm can handle incorrect training examples robustly, when the noise rate is less than half under the random noise model [1]. Bylander further demonstrate that linear threshold functions are polynomially learnable in the presence of classification noise [136]. Lawrence and Schölkopf construct a kernel Fisher discriminant to formulate label-noise problem as a probabilistic model [12], which is solved by Expectation Maximization algorithm. Although the above works explore to tackle noisy labels theoretically and empirically, Bartlett et al. justify that most loss functions are not completely robust to label noise [13]. It means that classifiers based on label-noise robust algorithms are still affected by label noise.

During this period, a lot of works emerged and contributed to this area. For example, Crammer et al. propose an online Passive-Aggressive perceptron algorithm to cope with label noise [14]. Dredze et al. propose confidence weighted learning to weigh trusted labels more [137]. Freund propose a boosting algorithm to combat against random label noise [138]. To handle label noise theoretically, Cesa-Bianchi et al. propose an online learning algorithm, leveraging unbiased estimates of the gradient of the loss [139]. Until 2013, Natarajan et al. formally formulate an unbiased risk estimator for binary classification with noisy labels [5]. This work is very important to the area, since it is the first work to provide guarantees for risk minimization under random label noise. Moreover, this work provides an easy way to suitably modify any given surrogate loss function for handling label noise.

Meanwhile, Scott et al. study the classification problem under class-conditional noise model, and propose the way to handle asymmetric label noise [15]. In contrast, van Rooyen et al. propose the unhinge loss to tackle symmetric label noise [16]. Liu and Tao propose the method of anchor points to estimate the noise rate, and further leverage importance reweighting to design surrogate loss functions for class-conditional label noise [17]. Instead of designing ad-hoc losses, Patrini et al. introduce linear-odd losses, which can be factorized into an even and an odd loss function [64]. More importantly, they estimate the mean operator from noisy data, and plug this operator in linear-odd losses for empirical risk minimization, which is resistant to asymmetric label noise.

It is noted that, we move from label-noise statistical learning to label-noise representation learning after 2015. There are two reasons behind this phenomenon. First, label-noise statistical learning mainly focus on designing theoretically-robust methods for small-scale noisy data. However, such methods cannot empirically work well on large-scale noisy data in our daily life, such as Clothing1M [120] emerging from 2015. Second, label-noise statistical learning mainly applies to shallow and convex models, such as support vector machines. However, deep and non-convex models, such as convolutional and recurrent neural networks, have become trendy and mainstream due to the better empirical performance, not only in vision, but also in language, speech and video tasks. Therefore, it is urgent to design label-noise representation learning methods for robustly training of deep models with noisy labels.

Emerging Stage. There are three seminal works in label-noise representation learning with noisy labels. For example, Sukhbaatar et al. introduce an extra but constrained linear “noise” layer on top of the softmax layer, which adapts the network outputs to model the noisy label distribution [18]. Reed et al. augment the prediction objective with a notion of consistency via a soft and hard bootstrapping [19], where the soft version is equivalent to softmax regression with minimum entropy regularization and the hard version modifies regression targets using the MAP estimation. Intuitively, this bootstrapping procedure provides the learner to disagree with an inconsistent training label, and re-label the training data to improve its label quality. Azadi et al. propose an auxiliary image regularization technique [20]. The key idea is to exploit the mutual context information among training data, and encourage the model to select reliable labels.

Followed by seminal works, Goldberger et al. introduce a nonlinear “noise” adaptation layer on top of the softmax layer [21], which adapts to model the noisy label distribution. Patrini et al. propose forward and backward loss correction approaches simultaneously [22]. Based on the corrected loss, they explore a robust two-stage training algorithm. A very interesting point is, both Wang et al. and Ren et al. leverage the same philosophy, namely data reweighting, to learn with label noise. However, they tackle from different perspectives. Specifically, Wang et al. come from a view of Bayesian and propose robust probabilistic modeling [23], where the posterior of reweighted model will identify uncorrupted data but ignore corrupted data. Ren et al. come from a view of meta-learning [24], which assign weights to training samples based on their gradient directions. Namely, their method performs a meta gradient descent step on the current mini-batch example weights (initialized from zero) to minimize the loss on a clean unbiased validation set.

Besides the above works, there are many important works born in 2018, ranging in diverse directions. In high level, there are several major directions, such as estimating transition matrix, regularization, designing losses and small-loss tricks. Among them, small-loss tricks are inspired by memorization effects of deep neural networks, where deep models will fit easy (clean) patterns first but over-fit hard (noisy) patterns eventually. Namely, small-loss tricks regard small-loss samples as relatively “clean” samples, and back-propagate such samples to update the model parameters. For example, Jiang et al. is the first to leverage small-loss tricks to handle label noise [4]. However, they train only a single network iteratively, which is similar to the self-training approach. Such approach inherits the same inferiority of accumulated error caused by the sample-selection bias. To address this issue, Han et al. train two deep neural networks simultaneously, and back propagates the data selected by its peer network and updates itself [7].

In the context of representation learning, estimating transition matrix, regularization and designing losses are still prosperous for handling label noise. For instance, given that a small set of trusted examples are available, Hendrycks et al. propose gold loss correction. Namely, they leverage trusted examples to estimate the (gold) transition matrix...
Therefore, on noisy examples, they will train deep models via forward correction (by gold matrix); on trusted examples, they will train deep models normally. Han et al. propose “human-in-the-loop” idea to easily estimate the transition matrix [26]. Specifically, they propose a human-assisted approach called “Masking” that conveys human cognition of invalid class transitions and naturally speculates the structure of the noise transition matrix. Then, they regard the matrix structure as prior knowledge, which is further incorporated into deep probabilistic modeling.

Moreover, Zhang et al. introduce an implicit regularization called mixup [27], which constructs virtual training data by linear interpolations of features and labels in training data. Mixup encourages the model to behave linearly in-between training examples, which reduces the amount of undesirable oscillations when predicting outside the training examples. Zhang et al. generalize both categorical cross entropy (CCE) loss and mean absolute error (MAE) loss by the negative Box-Cox transformation [28]. Their proposed $\mathcal{L}_q$ loss not only has theoretical justification, but also work for both closed-set and open-set noisy labels. Motivated by a dimensionality perspective, Ma [2018] develop a dimensionality-driven learning strategy, which can effectively learn robust low-dimensional subspaces that capture the true data distribution.

**Mature Stage.** Starting from 2019, label-noise representation learning has become mature in the top conference venues. Arazo et al. formulate clean and noisy samples as a two-component (clean-noisy) beta mixture model on the loss values [30], where the posterior probabilities are then used to implement a dynamically weighted bootstrapping loss. To boost the performance of Co-teaching, Chen et al. introduce the Iterative Noisy Cross-Validation (INCV) method to select a subset of most confident samples (with correct labels) [105], while Yu et al. employ the “Update by Disagreement” strategy to keep two networks diverged [103]. Hendrycks et al. empirically demonstrate that pre-training (i.e., “pre-train then tune” paradigm) can improve model robustness against label corruption [31], which is for large-scale noisy datasets.

Under the criteria of balanced error rate (BER) minimization and area under curve (AUC) maximization, Charoenphakdee et al. find that symmetric losses have many merits in combating with noisy labels, even without knowing the noise information. Based on such observation, they propose Barrier Hinge Loss [32]. In contrast to selected samples via small-loss tricks, Thulasidasan et al. introduce the abstention-based training, which allows deep abstaining networks to abstain on confusing samples while learning on non-confusing samples [33]. Following the re-weighting strategy, Shu et al. parameterize the weighting function adaptively as one-layer multilayer perceptron called Meta-Weight-Net [34], which is free of manually pre-specifying the weighting function.

Entering 2020, Menon et al. mitigate the effects of label noise from an optimization lens, namely using composite loss-based gradient clipping, which naturally introduces the partially Huberised loss for training deep models [35]. Nguyen et al. propose a self-ensemble label filtering method to progressively filter out the wrong labels during training [36]. Li et al. model the per-sample loss distribution with a mixture model to dynamically divide the training data into a labeled set with clean samples and an unlabeled set with noisy samples [37]. Lyu et al. propose a provable curriculum loss, which can adaptively select samples for robust stage-wise training [38]. Han et al. propose a versatile approach called scaled stochastic integrated gradient underweighted ascent (SIGUA) [39]. SIGUA uses gradient decent on good data, while using scaled stochastic gradient ascent on bad data rather than dropping those data. After Clothing1M born in 5 years, Jiang et al. propose a new but realistic type of noisy dataset called “web-label noise” (or red noise) [40], which enables us to conduct controlled experiments systematically in more realistic scenario.