A Note on the Implementation of Hierarchical Dirichlet Processes

Phil Blunsom∗
pblunsom@inf.ed.ac.uk
Sharon Goldwater∗
sgwater@inf.ed.ac.uk
Trevor Cohn∗
tcohn@inf.ed.ac.uk
Mark Johnson†
mark.johnson@brown.edu

∗Department of Informatics
University of Edinburgh
Edinburgh, EH8 9AB, UK
†Department of Cognitive and Linguistic Sciences
Brown University
Providence, RI, USA

Abstract
The implementation of collapsed Gibbs samplers for non-parametric Bayesian models is non-trivial, requiring considerable book-keeping. Goldwater et al. (2006a) presented an approximation which significantly reduces the storage and computation overhead, but we show here that their formulation was incorrect and, even after correction, is grossly inaccurate. We present an alternative formulation which is exact and can be computed easily. However this approach does not work for hierarchical models, for which case we present an efficient data structure which has a better space complexity than the naive approach.

1 Introduction
Unsupervised learning of natural language is one of the most challenging areas in NLP. Recently, methods from nonparametric Bayesian statistics have been gaining popularity as a way to approach unsupervised learning for a variety of tasks, including language modeling, word and morpheme segmentation, parsing, and machine translation (Teh et al., 2006; Goldwater et al., 2006a; Goldwater et al., 2006b; Liang et al., 2007; Finkel et al., 2007; DeNero et al., 2008). These models are often based on the Dirichlet process (DP) (Ferguson, 1973) or hierarchical Dirichlet process (HDP) (Teh et al., 2006), with Gibbs sampling as a method of inference. Exact implementation of such sampling methods requires considerable bookkeeping of various counts, which motivated Goldwater et al. (2006a) (henceforth, GGJ06) to develop an approximation using expected counts. However, we show here that their approximation is flawed in two respects: 1) It omits an important factor in the expectation, and 2) Even after correction, the approximation is poor for hierarchical models, which are commonly used for NLP applications. We derive an improved O(1) formula that gives exact values for the expected counts in non-hierarchical models. For hierarchical models, where our formula is not exact, we present an efficient method for sampling from the HDP (and related models, such as the hierarchical Pitman-Yor process) that considerably decreases the memory footprint of such models as compared to the naive implementation.

As we have noted, the issues described in this paper apply to models for various kinds of NLP tasks; for concreteness, we will focus on n-gram language modeling for the remainder of the paper, closely following the presentation in GGJ06.

2 The Chinese Restaurant Process
GGJ06 present two nonparametric Bayesian language models: a DP unigram model and an HDP bigram model. Under the DP model, words in a corpus \( w = w_1 \ldots w_n \) are generated as follows:

\[
G|\alpha_0, P_0 \sim \text{DP}(\alpha_0, P_0)
\]

\[
w_i|G \sim G
\]

where \( G \) is a distribution over an infinite set of possible words, \( P_0 \) (the base distribution of the DP) determines the probability that an item will be in the support of \( G \), and \( \alpha_0 \) (the concentration parameter) determines the variance of \( G \).

One way of understanding the predictions that the DP model makes is through the Chinese restaurant process (CRP) (Aldous, 1985). In the CRP, customers (word tokens \( w_i \)) enter a restaurant with an infinite number of tables and choose a seat. The table chosen by the \( i \)th customer, \( z_i \), follows the distribution:

\[
P(z_i = k|z_{-i}) = \left\{ \begin{array}{ll}
\frac{n_k}{i-1+\alpha_0}, & 0 \leq k < K(z_{-i}) \\
\frac{n_{K(z_{-i})}}{i-1+\alpha_0}, & k = K(z_{-i})
\end{array} \right.
\]

Proceedings of the ACL-IJCNLP 2009 Conference Short Papers, pages 337–340,
Suntec, Singapore, 4 August 2009. ©2009 ACL and AFNLP
where \( z_{i-1} = z_1 \ldots z_{i-1} \) are the table assignments of the previous customers, \( n_k^{z_{i-1}} \) is the number of customers at table \( k \) in \( z_{i-1} \), and \( K(z_{i-1}) \) is the total number of occupied tables. If we further assume that table \( k \) is labeled with a word type \( \ell_k \) drawn from \( P_0 \), then the assignment of tokens to tables defines a distribution over words, with \( w_i = \ell_{z_i} \). See Figure 1 for an example seating arrangement.

Using this model, the predictive probability of \( w_i \), conditioned on the previous words, can be found by summing over possible seating assignments for \( w_i \), and is given by

\[
P(w_i = w | w_{-i}) = \frac{n_{w-1}^{w_i} + \alpha_0 P_0}{i - 1 + \alpha_0}
\]

This prediction turns out to be exactly that of the DP model after integrating out the distribution \( G \). Note that as long as the base distribution \( P_0 \) is fixed, predictions do not depend on the seating arrangement \( z_{-i} \), only on the count of word \( w \) in the previously observed words (\( n_{w-1} \)). However, in many situations, we may wish to estimate the base distribution itself, creating a hierarchical model. Since the base distribution generates table labels, estimates of this distribution are based on the counts of those labels, i.e., the number of tables associated with each word type.

An example of such a hierarchical model is the HDP bigram model of GGJ06, in which each word type \( w \) is associated with its own restaurant, where customers in that restaurant correspond to words that follow \( w \) in the corpus. All the bigram restaurants share a common base distribution \( P_1 \) over unigrams, which must be inferred. Predictions in this model are as follows:

\[
P_2(w_i|h_{-i}) = \frac{n_{h_{-i}w_i}^{h_{-i}} + \alpha_1 P_1(w_i|h_{-i})}{n_{h_{-i}\star}^{h_{-i}} + \alpha_1}
\]

\[
P_1(w_i|h_{-i}) = \frac{t_{h_{-i}w_i} + \alpha_0 P_0(w_i)}{t_{\star h_{-i}} + \alpha_0}
\]

where \( h_{-i} = (w_{-i}, z_{-i}) \), \( n_{h_{-i}w_i}^{h_{-i}} \) is the number of tables labelled with \( w_i \), and \( t_{h_{-i}}^{h_{-i}} \) is the total number of occupied tables. Of particular note for our discussion is that in order to calculate these conditional distributions we must know the table assignments \( z_{-i} \) for each of the words in \( w_{-i} \). Moreover, in the Gibbs samplers often used for inference in these kinds of models, the counts are constantly changing over multiple samples, with tables going in and out of existence frequently. This can create significant bookkeeping issues in implementation, and motivated GGJ06 to present a method of computing approximate table counts based on word frequencies only.

### 3 Approximating Table Counts

Rather than explicitly tracking the number of tables \( t_{w} \) associated with each word \( w \) in their bigram model, GGJ06 approximate the table counts using the expectation \( E[t_{w}] \). Expected counts are used in place of \( t_{w}^{h_{-i}} \) and \( t_{w}^{h_{-i}} \) in (2). The exact expectation, due to Antoniak (1974), is

\[
E[t_w] = \alpha_1 P_1(w) \sum_{i=1}^{n_w} \frac{1}{\alpha_1 P_1(w) + i - 1}
\]
Antoniak also gives an approximation to this expectation:

\[ E[t_w] \approx \alpha P_1(w) \log n_w + \frac{\alpha P_1(w)}{\alpha P_1(w)} \]  

(4)

but provides no derivation. Due to a misinterpretation of Antoniak (1974), GGJ06 use an approximation that leaves out all the \( P_1(w) \) terms from (4).\(^1\) Figure 2 compares the approximation to the exact expectation when the base distribution is fixed. The approximation is fairly good when \( \alpha P_1(w) > 1 \) (the scenario assumed by Antoniak); however, in most NLP applications, \( \alpha P_1(w) < 1 \) in order to effect a sparse prior. (We return to the case of non-fixed based distributions in a moment.) As an extreme case of the paucity of the exact table assignments for customers are not required for prediction.

Teh et al. (2006) also note that the exact table assignments for customers are not required for prediction. Due to a misinterpretation of Antoniak (1974), GGJ06 use an approximation that leaves out all the \( P_1(w) \) terms from (4).\(^1\) Figure 2 compares the approximation to the exact expectation when the base distribution is fixed. The approximation is fairly good when \( \alpha P_1(w) > 1 \) (the scenario assumed by Antoniak); however, in most NLP applications, \( \alpha P_1(w) < 1 \) in order to effect a sparse prior. (We return to the case of non-fixed based distributions in a moment.) As an extreme case of the paucity of the exact table assignments for customers are not required for prediction.

We now provide a derivation for (4), which will allow us to obtain an \( O(1) \) formula for the expectation in (3). First, we rewrite the summation in (3) as a difference of fractional harmonic numbers:\(^2\)

\[ H(\alpha P_1(w) + n_w) - H(\alpha P_1(w) - 1) \]  

(5)

Using the recurrence for harmonic numbers:

\[ E[t_w] \approx \alpha P_1(w) \left[ H(\alpha P_1(w) + n_w) - \frac{1}{\alpha P_1(w)} \right] \]

\[ - H(\alpha P_1(w) + n_w) + \frac{1}{\alpha P_1(w)} \]  

(6)

We then use the asymptotic expansion, \( H_F \approx \log F + \gamma + \frac{1}{2F} \), omitting trailing terms which are \( O(F^{-2}) \) and smaller powers of \( F \):\(^3\)

\[ E[t_w] \approx \alpha P_1(w) \log n_w + \alpha P_1(w) + \frac{n_w}{2(\alpha P_1(w) + n_w)} \]

Omitting the trailing term leads to the approximation in Antoniak (1974). However, we can obtain an exact formula for the expectation by utilising the relationship between the Digamma function and the harmonic numbers: \( \psi(n) = H_{n-1} - \gamma \).\(^4\) Thus we can rewrite (5) as:\(^5\)

\[ E[t_w] = \alpha P_1(w) \cdot \left[ \psi(\alpha P_1(w) + n_w) - \psi(\alpha P_1(w)) \right] \]

(7)

\(^1\)The authors of GGJ06 realized this error, and current implementations of their models no longer use these approximations, instead tracking table counts explicitly.

\(^2\)Fractional harmonic numbers between 0 and 1 are given by \( H_F = \int_1^F \frac{1}{x} \, dx \). All harmonic numbers follow the recurrence \( H_F = H_{F-1} + \frac{1}{F} \).

\(^3\)Here, \( \gamma \) is the Euler-Mascheroni constant.

\(^4\)Accurate \( O(1) \) approximations of the Digamma function are readily available.

\(^5\)(7) can be derived from (3) using: \( \psi(x+1) - \psi(x) = \frac{1}{x} \).

### Efficient Implementation of HDPs

As we do not have an efficient expected table count approximation for hierarchical models we could fall back to explicitly tracking which table each customer that enters the restaurant sits at. However, here we describe a more compact representation for the state of the restaurant that doesn’t require explicit table tracking.\(^6\) Instead we maintain a histogram for each dish \( w_i \) of the frequency of a table having a particular number of customers. Figure 3 depicts the histogram and explicit representations for the CRP state in Figure 1.

Our alternative method of inference for hierarchical Bayesian models takes advantage of their

---

\(^6\)Teh et al. (2006) also note that the exact table assignments for customers are not required for prediction.
Algorithm 1 A new customer enters the restaurant
1: \( w \): word type
2: \( P_w \): Base probability for \( w \)
3: \( \text{HD}_m \): Seating Histogram for \( w \)
4: \textbf{procedure} INCREMENT\((w, P_w^m, \text{HD}_m)\)
5: \( p_{\text{share}} \leftarrow \frac{w^{-1} + \eta_0}{n_w^{-1} + \eta_0} \) \( \triangleright \) share an existing table
6: \( p_{\text{new}} \leftarrow \frac{w^{-1} + \eta_0}{n_w^{-1} + \eta_0} \) \( \triangleright \) open a new table
7: \( r \leftarrow \text{random}(0, p_{\text{share}} + p_{\text{new}}) \)
8: if \( r < p_{\text{share}} \) or \( n_w^{-1} = 0 \) then
9: \( \text{HD}_w[1] = \text{HD}_w[1] + 1 \)
10: \textbf{else}
11: Sample from the histogram of customers at tables
12: for \( c \in \text{HD}_m \) do \( \triangleright \) \( c \): customer count
13: \( r = r - (c \times \text{HD}_w[c]) \)
14: if \( r < 0 \) then
15: \( \text{HD}_w[c] = \text{HD}_w[c] - 1 \)
16: \textbf{if} \( c > 1 \) \textbf{then}
17: \( \text{HD}_w[c - 1] = \text{HD}_w[c - 1] + 1 \)
18: \textbf{break}
19: \( n_w = n_w^{-1} + 1 \) \( \triangleright \) Update token count

Algorithm 2 A customer leaves the restaurant
1: \( w \): word type
2: \( \text{HD}_m \): Seating histogram for \( w \)
3: \textbf{procedure} DECREMENT\((w, P_w^m, \text{HD}_m)\)
4: \( r \leftarrow \text{random}(0, n_w^{-1}) \)
5: for \( c \in \text{HD}_w \) do \( \triangleright \) \( c \): customer count
6: \( r = r - (c \times \text{HD}_w[c]) \)
7: if \( r < 0 \) then
8: \( \text{HD}_w[c] = \text{HD}_w[c] + 1 \)
9: \textbf{if} \( c > 1 \) \textbf{then}
10: \( \text{HD}_w[c - 1] = \text{HD}_w[c - 1] - 1 \)
11: \textbf{break}
12: \( n_w = n_w^{-1} - 1 \) \( \triangleright \) Update token count

exchangeability, which makes it unnecessary to know exactly which table each customer is seated at. The only important information is how many tables exist with different numbers of customers, and what their labels are. We simply maintain a histogram for each word type \( w \), which stores, for each number of customers \( m \), the number of tables labeled with \( w \) that have \( m \) customers. Figure 3 depicts the explicit representation and histogram for the CRP state in Figure 1.

Algorithms 1 and 2 describe the two operations required to maintain the state of a CRP.\(^7\) When a customer enters the restaurant (Algorithm 1), we sample whether or not to open a new table. If not, we sample an old table proportional to the counts of how many customers are seated there and update the histogram. When a customer leaves the restaurant (Algorithm 2), we decrement one of the tables at random according to the number of customers seated there. By exchangeability, it doesn’t actually matter which table the customer was “really” sitting at.

\(^7\)A C++ template class that implements the algorithm presented is made available at: http://homepages.inf.ed.ac.uk/tcohn/

5 Conclusion

We’ve shown that the HDP approximation presented in GGJ06 contained errors and inappropriate assumptions such that it significantly diverges from the true expectations for the most common scenarios encountered in NLP. As such we emphasise that that formulation should not be used. Although (7) allows \( E[t_w] \) to be calculated exactly for constant base distributions, for hierarchical models this is not valid and no accurate calculation of the expectations has been proposed. As a remedy we’ve presented an algorithm that efficiently implements the true HDP without the need for explicitly tracking customer to table assignments, while remaining simple to implement.

Acknowledgements

The authors would like to thank Tom Griffiths for providing the code used to produce Figure 2 and acknowledge the support of the EPSRC (Blunsom, grant EP/D074959/1; Cohn, grant GR/T04557/01).

References

D. Aldous. 1985. Exchangeability and related topics. In École d’Été de Probabilités de Saint-Florent XIII 1983, 1–198. Springer.

C. E. Antoniak. 1974. Mixtures of dirichlet processes with applications to bayesian nonparametric problems. The Annals of Statistics, 2(6):1152–1174.

J. DeNero, A. Bouchard-Côté, D. Klein. 2008. Sampling alignment structure under a Bayesian translation model. In Proceedings of the 2008 Conference on Empirical Methods in Natural Language Processing, 314–323, Honolulu, Hawaii. Association for Computational Linguistics.

S. Ferguson. 1973. A Bayesian analysis of some nonparametric problems. Annals of Statistics, 1:209–230.

J. R. Finkel, T. Grenager, C. D. Manning. 2007. The infinite tree. In Proc. of the 45th Annual Meeting of the ACL (ACL-2007), Prague, Czech Republic.

S. Goldwater, T. Griffiths, M. Johnson. 2006a. Contextual dependencies in unsupervised word segmentation. In Proc. of the 44h Annual Meeting of the ACL and 21st International Conference on Computational Linguistics (COLING/ACL-2006), Sydney.

S. Goldwater, T. Griffiths, M. Johnson. 2006b. Interpolating between types and tokens by estimating power-law generators. In Y. Weiss, B. Schölkopf, J. Platt, eds., Advances in Neural Information Processing Systems 18, 459–466. MIT Press, Cambridge, MA.

P. Liang, S. Petrov, M. Jordan, D. Klein. 2007. The infinite PCFG using hierarchical Dirichlet processes. In Proc. of the 2007 Conference on Empirical Methods in Natural Language Processing (EMNLP-2007), 688–697, Prague, Czech Republic.

Y. W. Teh, M. I. Jordan, M. J. Beal, D. M. Blei. 2006. Hierarchical Dirichlet processes. Journal of the American Statistical Association, 101(476):1566–1581.

340