Dynamical symmetry breaking of massless $\lambda\phi^4$ and renormalization scheme dependence

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Through two loop effective potential we demonstrate that the $\overline{MS}$ scheme of dimensional regularization and Jackiw’s prescription in cut-off regularization allow for the dynamical breaking solution in massless $\lambda\phi^4$ theory, while the Coleman-Weinberg prescription does not. The beta function of the broken phase is negative, like in the one loop effective potential, but the UV fixed point is not zero, i.e., not an asymptotic freedom solution, unlike the one loop case. Some related issues were briefly discussed.

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1 Introduction

In spite of the magnificent success of the standard model, the genuine symmetry breaking mechanism remains elusive as the Higgs sector is unsatisfactory due to hierarchy\(^1\), triviality\(^2\) and other problems. In the pursuit of more satisfactory models for symmetry breaking, supersymmetric field theories\(^3\) (including string theories and their descendants\(^4\)) and technicolor model and its variants\(^3\) have been intensively studied. Although these models already differ among themselves, they share a common feature: the elementary Higgs scalar fields are excluded. And it is widely believed that the solution to the hierarchy and triviality problem must be in non-perturbative regime\(^3\).

On the other hand there are also some efforts to revive the \(\lambda \phi^4\) interaction through the asymptotic free renormalization\(^6,7\) proposal, i.e., \(\beta(\lambda) < 0\) with \(\lambda = 0\) as the UV fixed point, in contrast to the perturbative renormalization where \(\beta(\lambda) > 0\) (leading to triviality), i.e., the renormalization prescription is physically relevant.

In this report we wish to investigate the existence of the dynamical symmetry breaking in the two loop effective potential. For simplicity, we will consider the simplest scalar model, the massless \(\lambda \phi^4\) model with \(Z_2\) symmetry, the first example with which the dynamical symmetry breaking was demonstrated\(^8\). There is also a technical concern in choosing massless scalar theory: there is no non-convexity associated with the tachyon mass term in massless models unlike the Higgs model\(^3\) and the configuration of the expectation value of scalar field automatically becomes homogeneous. In the meantime we also wish to examine the relevance of renormalization prescription.

The paper is organized as follows. The bare and renormalized two loop effective potentials will be given in several prescriptions in section two. Then in section three the symmetry breaking solution is shown to exist in several renormalization prescriptions but not in the Coleman-Weinberg\(^8\) prescription. Some discussions and the summary will comprise the last section.

2 The intermediate renormalization of the two loop effective potential of massless \(\lambda \phi^4\)

As is explained in the introduction, we consider the massless \(\lambda \phi^4\) model with \(Z_2\) symmetry: invariance under the transformation of \(\phi \rightarrow -\phi\). The algorithm for two loop effective potential is well known according to Jackiw\(^10\)

\[
L = \frac{1}{2} (\partial \phi)^2 - \lambda \phi^4, \quad (1)
\]

\[
V_{(2l)} \equiv \lambda \phi^4 + \frac{1}{2} I_0(\Omega) + 3 \lambda I_1^2(\Omega) - 48 \lambda^2 \phi^2 I_2(\Omega), \quad (2)
\]
\[ \Omega \equiv \sqrt{12\lambda \phi^2}; \quad (3) \]

\[ I_0(\Omega) = \int \frac{d^4k}{(2\pi)^4} \ln(1 + \frac{\Omega^2}{k^2}); \quad (4) \]

\[ I_1(\Omega) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + \Omega^2}; \quad (5) \]

\[ I_2(\Omega) = \int \frac{d^4kd^4l}{(2\pi)^8} \frac{1}{(k^2 + \Omega^2)(l^2 + \Omega^2)((k + l)^2 + \Omega^2)^2}; \quad (6) \]

Here we have Wick rotated all the loop integrals into Euclidean ones. As these integrals have already been calculated in literature both in dimensional regularization \[11\] and in cut-off scheme \[10\], we only list the results here. The bare forms in the two regularization schemes read:

\[ V^{(D)}_{(2l)}(\Omega) = \Omega^4 \{ \frac{1}{144\lambda} + \frac{-\frac{1}{\epsilon} + \mathcal{T} - \frac{3}{2}}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4} [ (-\frac{1}{\epsilon} + \mathcal{T} - 1)^2 + (\mathcal{T} - 1)^2 + 2(-\frac{1}{\epsilon} + \mathcal{T} - \frac{3}{2})^2 + 2(\mathcal{T} - \frac{3}{2})^2 + 7 + 6S - \frac{5}{3}\zeta(2)] \}; \quad (7) \]

\[ V^{(\Lambda)}_{(2l)}(\Omega) = \Omega^4 \{ \frac{1}{144\lambda} + \frac{L^\Lambda - \frac{1}{\mathcal{T}}}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4} [ (L^\Lambda)^2 - 2 + 2(L^\Lambda - 1)^2] + \Omega^2 \{ \frac{2\Lambda^2}{(8\pi)^2} + \frac{3\lambda \Lambda^2 L^\Lambda}{(4\pi)^4} - \frac{8\lambda \Lambda^2}{(4\pi)^4} \}; \quad (8) \]

where \[ S = \sum_{n=0}^{\infty} \frac{1}{(2+3n)^2}, \mathcal{T} = L + \gamma - \ln 4\pi, L = \ln \frac{\Omega^2}{\mu^2}, \] and \[ L^\Lambda = \ln \frac{\Omega^2}{\Lambda^2}. \] Here we have omitted all the field independent terms.

In literature the renormalization had been done in the \(\overline{MS}\) scheme for \(V^{(D)}_{(2l)}(\Omega)\) (Cf.\[11\]), while for \(V^{(\Lambda)}_{(2l)}(\Omega)\) the renormalization had been done in Jackiw’s prescription\[10\] and Coleman-Weinberg’s prescription\[8\]. The results read

\[ V^{(\overline{MS})}_{(2l)}(\Omega) = \Omega^4 \{ \frac{1}{144\lambda} + \frac{\mathcal{L} - \frac{3}{2}}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4} [ 3\mathcal{L}^2 - 10\mathcal{L} + 11 + 12S - \frac{8}{9}\pi^2] \}; \quad (9) \]

\[ V^{(Jackiw)}_{(2l)}(\Omega) = \Omega^4 \{ \frac{1}{144\lambda} + \frac{\hat{L}}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4} [ 3\hat{L}^2 - \hat{L}] \}; \quad (10) \]

\[ V^{(CW)}_{(2l)}(\Omega) = \Omega^4 \{ \frac{1}{144\lambda} + \frac{\tilde{L}}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4} [ 3\tilde{L}^2 - \tilde{L} + \frac{205}{12}] \}; \quad (11) \]

with \[ \hat{L} = \ln \frac{\Omega^2}{12\lambda \mu^2_{Jackiw}} \] and \[ \tilde{L} = \ln \frac{\Omega^2}{12\lambda \mu^2_{CW}} - \frac{25}{6}. \] In all the above formulæ the scheme (or prescription) dependence of field strength and coupling constant are understood. Note that the Jackiw and Coleman-Weinberg prescriptions were applied to the same bare effective potential, i.e., that calculated in the cut-off scheme.
Upon appropriate rescaling of the subtraction scales, all versions of the effective potential take the following form (we will drop all the dressing symbols)

\[ V_{(2)}(\Omega) = \Omega^4 \left\{ \frac{1}{144\lambda} + \frac{L - 1/2}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4}L^2 + 2(L - 1)^2 + \alpha \right\} \]  \hspace{1cm} (12)

with \( L \equiv \ln \frac{\Omega^2}{\mu^2} \). Now we can see the explicit scheme dependence represented by the constant \( \alpha \) that is summarized in the following tabular:

| Value of \( \alpha \) | Reg and/or Ren Scheme |
|-----------------------|-----------------------|
| \(-2.6878\)           | MS                    |
| \(-\frac{1}{2}\)      | Jackiw                |
| \(16\frac{1}{4}\)     | Coleman-Weinberg      |

Such scheme dependence of the effective potential differs from the case of truncating the perturbative scattering matrix\(^{[12]}\): The difference in \( \alpha \) could not be removed by redefinition of coupling constant (and perhaps of field strength) without changing the functional form of the effective potential.

### 3 Effective potential and symmetry breaking

Now let us look at the dynamical symmetry breaking. Working with the general parametrization form of Eq. (12), the task is to solve the following equation:

\[ \frac{dV_{(2)}(\sqrt{12\lambda\phi^2})}{d\phi} = 24\lambda\phi\Omega^2\left[ \frac{2V_{(2)}(\Omega^2)}{\Omega^4} + \frac{1}{(8\pi)^2} + \frac{3\lambda}{(4\pi)^4}(6L - 4) \right] = 0. \]  \hspace{1cm} (13)

An obvious solution is \( \phi = 0 \) which is the symmetric solution in perturbative (weak coupling) regime while the symmetry breaking solution is determined by the following algebraic equation

\[ 3L^2 + \left( \frac{4\pi^2}{3\lambda} - 1 \right)L + \alpha + \frac{16\pi^4}{27\lambda^2} = 0. \]  \hspace{1cm} (14)

Here it is obvious that existence of real number solution depends on both \( \alpha \) and \( \lambda \).

#### 3.1 Determinants of symmetry breaking

Since we must start from a stable micro potential the coupling \( \lambda \) must be a positive real number. Now let us closely examine Eq. (14). For Eq. (14) to possess a finite real number solution, we must impose the following inequality in terms of \( \alpha \) and \( \lambda \)

\[ \Delta \equiv (\frac{4\pi^2}{3\lambda} - 1)^2 - 12(\alpha + \frac{16\pi^4}{27\lambda^2}) = \frac{1}{3}(4 - 36\alpha - (1 + \frac{4\pi^2}{3\lambda})^2) \geq 0. \]  \hspace{1cm} (15)

\(^1\)Rigorously speaking, this property has been firmly established only in massless gauge theories or in cases where threshold effects are negligible. But the relevance of renormalization prescriptions has been recently emphasized in theories with unstable elementary particles (like \( W^\pm, Z^0 \) bosons in electro-weak theory)\(^{[13]}\).
This inequality is only valid for certain ranges of $\alpha$ and $\lambda$,
\[
\alpha < \frac{1}{12}, \quad (16)
\]
\[
\lambda \geq \lambda_{cr} \equiv \frac{4\pi^2}{\sqrt{4 - 36\alpha - 1}}. \quad (17)
\]

Then the solutions to Eq. (14) can be found with the above two requirements,
\[
L_{\pm}(\lambda) = \frac{1}{6}[1 - \frac{4\pi^2}{3\lambda} \pm \sqrt{\Delta}], \quad (18)
\]
from which and the definitions $L \equiv \ln \frac{\Omega^2}{\mu^2}$, $\Omega \equiv \sqrt{12\lambda \phi^2}$ we can find the nonzero solutions of $\phi$ which read
\[
\phi^2_{\pm}(\lambda; [\mu, \alpha]) = \frac{\mu^2}{12\lambda} \exp\left\{\frac{1}{6}[1 - \frac{4\pi^2}{3\lambda} \pm \sqrt{\Delta}]\right\}. \quad (19)
\]

But the solutions corresponding to $L_{-}(\lambda)$ are local maxima (tachyonic), only the $L_{+}(\lambda)$ solutions are local minima, this can be seen from the second order derivative of the effective potential at $\Omega^2_{\pm}$ (which is exactly the effective mass),
\[
m_{eff;\pm}(\lambda) \equiv \frac{\partial^2 V(2\lambda)}{(\partial \phi)^2} |_{\phi^2 = \phi^2_{\pm}} = \pm \frac{18\lambda^2\Omega^2_{\pm}}{(2\pi)^4} \sqrt{\Delta}, \quad (20)
\]
as each factor is positive definite.

The inequality (16) shows the relevance of renormalization prescriptions at two loop level: the Coleman-Weinberg prescription could not lead to dynamical symmetry breaking at all as the critical inequality (16) is violated: $\alpha_{CW} = 16\frac{1}{3} = \frac{196}{12} \gg \frac{1}{12}$, similar to the one loop case[6,7].

### 3.2 Stability of symmetry breaking and criterion for coupling constant

Using Eq. (13) we have
\[
E_{\pm}(\lambda, \mu) \equiv V(2\lambda)(\sqrt{12\lambda \phi^2})|_{\phi^2 = \phi^2_{\pm}} = \frac{(12\lambda \phi^2_{\pm})^2}{2(8\pi)^2}[1 + \frac{3\lambda}{2\pi^2}(3L_{\pm} - 2)]. \quad (21)
\]

For the symmetry breaking states to be stable, the vacuum energy density must be less than zero. Then from Eq. (21) we find that,
\[
L_{\pm} \geq \frac{2}{3} - \frac{2\pi^2}{9\lambda}, \quad (22)
\]
which rejects the $L_{-}$ solution and leads to the following requirement in $L_{+}$,
\[
\lambda \geq \lambda_{cr} \equiv \frac{4\pi^2}{\sqrt{4 - 36\alpha - 27} - 1} (> \lambda_{cr} = \frac{4\pi^2}{\sqrt{4 - 36\alpha - 1}}). \quad (23)
\]
The values of the critical couplings are exhibited in the following tabular.

| Scheme | $\lambda_{cr}$ | $\hat{\lambda}_{cr}$ |
|--------|----------------|------------------|
| MS     | 4.368          | 5.2024           |
| Jackiw | 6.5797         | 10.698           |

We also note that the stable condition (22) amounts to the following more stringent requirement on renormalization prescriptions:

$$\alpha \leq -\frac{2}{3}. \quad (24)$$

### 3.3 RG invariance of vacuum energy and beta function

Since the vacuum energy is a physical entity, it must be renormalization group invariant, i.e., insensitive to the choice of subtraction point within a scheme[7]. Therefore we have

$$\mu \frac{dE_{+}(\lambda, \mu)}{d\mu} = 0. \quad (25)$$

We must stress that this important condition in fact defines a fundamental physical scale as input, corresponding to the necessary step after intermediate renormalization is done, i.e., to confront the renormalized amplitudes with experiments or to fix the amplitudes via physical conditions.

From this equation we can determine the beta function of $\lambda$ as was did in ref.[7].

First let us rewrite the vacuum energy density as

$$E_{+} = -\frac{\mu^4}{2(8\pi)^2}\varepsilon_{+}(\lambda)e^{L_{+}(\lambda)}, \quad (26)$$

with

$$\varepsilon_{+}(\lambda) \equiv 1 + \frac{3\lambda}{2\pi^2}(3L_{+}(\lambda) - 2) = \frac{3\lambda}{4\pi^2}(\sqrt{\Delta} - 3). \quad (27)$$

Then we find from Eqs. (25) and (26) that,

$$\beta(\lambda) = -\frac{12\lambda\varepsilon_{+}(\lambda)}{\left\{\varepsilon_{+}(\lambda)(3 + \frac{4\pi^2}{3\lambda}) + 1 + 4\pi^2/\lambda\right\}} < 0. \quad (28)$$

This is true for all the two schemes allowing for symmetry breaking solution. When the coupling becomes infinitely strong, i.e., $\lambda \to \infty$, the beta function approaches to a straight line:

$$\beta(\lambda)|_{\lambda \to \infty} \sim -4\lambda, \quad (29)$$
while in the neighborhood of \( \hat{\lambda}_{cr} \),
\[
\beta(\lambda)|_{\lambda \to \hat{\lambda}_{cr}} \sim -4(\lambda - \hat{\lambda}_{cr}).
\] (30)

Our faith in the two loop effective potential is enhanced by the fact that all schemes (except the Coleman-Weinberg prescription) predict the same kind of running behavior of the coupling. As a rude approximation we use the following beta function,
\[
\beta_{\text{appr}}(\lambda) = -4(\lambda - \hat{\lambda}_{cr}), \forall \lambda : \lambda \geq \hat{\lambda}_{cr}
\] (31)
with the obvious solution
\[
\lambda - \hat{\lambda}_{cr} = \frac{\mu_0^4}{\mu^4}, \forall \mu : \mu \in (0, \infty)
\] (32)
which could also be obtained as a rude approximation of Eq. (21). In fact the RG invariant scale \( \mu_0^2 \) in the IR end differs from that in the UV end, but this fact does not affect the main properties described by the approximation (31) and (32).

Now it is clear that we obtained a nontrivial theory with a UV fixed point equal to the value of critical coupling \( \hat{\lambda}_{cr} \), a strong coupling as is clear from the tabular in last subsection. From Eq. (32) we can identify a pole in the IR region, which is quite different from the IR Landau pole in QCD. Since the coupling is strong in the entire broken phase, we found a solution entirely lives in strong coupling regime with the IR end being infinitely strongly coupled! Note that this property is true in several schemes (except the Coleman-Weinberg prescription) such as \( \overline{MS} \). In the following discussions, we refer to this solution as SCRDSB for Strong Coupling Regime Dynamical Symmetry Breaking.

The asymptotic behaviors of the effective mass defined in Eq. (21) can be obtained after some calculations (taking the vacuum energy density as fundamental physical quantity)
\[
m_{eff^2}(\lambda)|_{\lambda \to \infty} \sim \frac{\lambda^2}{\hat{\lambda}_{cr}^2}; \quad m_{eff^2}(\lambda)|_{\lambda \to \hat{\lambda}_{cr}^+} \sim (\lambda - \hat{\lambda}_{cr})^{-\frac{1}{2}}.
\] (33)
Or in terms of running scale
\[
m_{eff^2}(\mu^2)|_{\mu \to 0} \sim \frac{1}{\mu^6}; \quad m_{eff^2}(\mu^2)|_{\mu \to \infty} \sim \mu^2.
\] (34)

The effective coupling, defined as \( \lambda_{eff}(\lambda) \equiv \frac{\partial^4 V(\phi)}{\partial \phi^4}|_{\phi^2 = \phi^2_+} \) possesses the following asymptotic behaviors
\[
\lambda_{eff}(\lambda)|_{\lambda \to \infty} \sim 10^1 \lambda^3; \quad \lambda_{eff}(\lambda)|_{\lambda \to \hat{\lambda}_{cr}} \sim 10^2 \hat{\lambda}_{cr},
\] (35)
or equivalently
\[
\lambda_{eff}(\mu)|_{\mu \to 0} \sim \frac{1}{\mu^{12}}; \quad \lambda_{eff}(\mu)|_{\mu \to \infty} \sim 10^3.
\] (36)
Note that the effective coupling becomes more singular than the effective mass does in the IR limit.
4 Discussion and summary

Thus far we just made use of the well known two loop calculations to search for the symmetry breaking solutions. Our results here are new in two aspects: (1) First, the scheme dependence of the non-perturbative framework differs from that of the standard perturbative framework[12]; (2) Second, the broken phase is a new kind of nontrivial dynamics, i.e., a totally (non-perturbative) strong coupling dynamics regime with negative beta function (SCRDSB).

For the first aspect, we note that the standard perturbative scheme dependence relies on the following well known fact: in the complete sum of the one particle irreducible (1PI) loop diagrams at a given order, the real divergence after all subdivergences were removed is only a single log divergence, i.e., $\sim \ln \Lambda^2$ (or single power divergence, $\sim \frac{1}{\varepsilon}$) in gauge theories like QCD and QED. This key point then implies that for an experimentally interested quantity (like a reaction ratio $R_{\cdot \cdot}(Q)$), all the energy scale dependence (via $\ln^n \frac{Q^2}{\mu^2}, n \geq 1$) in the coefficients $(r_i(Q), i \geq 1)$ of the perturbative expansion of the quantity in terms of the coupling constant $(R_{\cdot \cdot}(Q) = R_0(1 + \sum_{i=1}^{N} r_i \alpha^i(\mu)))$ can be absorbed into the energy scale dependence of the coupling constant through RG improvement[12]. This property is ensured by the gauge invariance of these models.

But the situation is changed in massive $\lambda \phi^4$ due to the appearance of the double log term in the sum of two loop 1PI diagrams. In the previously mentioned example[14], the relevance of renormalization prescription arises because the method in use only assumes a subset of diagrams and therefore fails the redefinition of running parameters (this is also true of the one loop effective potential case[6,7]), here we found that, the problem is technically due to the interaction structures. Hence, this problem would persist in higher orders massive diagrams even in perturbative approaches. However, the massive loop diagrams considered here are non-perturbative by construction[10], but we must note that the sum of the 1PI diagrams in the potential is complete at given order and more importantly the method of effective potential is systematic. Of course in pure perturbative framework of massless $\lambda \phi^4$ (without dynamical masses), there is no double log terms in the sums of the 1PI diagrams and the perturbative scheme dependence is as usual[13].

Now we turn to the possible physical implications of SCRDSB. The one loop effective potential can be renormalized in non-perturbative way so that asymptotic freedom ensued consequently[6,7], while here we found a nontrivial solution entirely living in the strong coupling regime after symmetry breaking. Although this phenomenon is a two loop level result, at least there is one thing that is conspicuous: the two loop solution supports the existence of nontrivial dynamical symmetry breaking solution that is first discovered one loop case. We think nontrivial solution might persist after including still higher order contributions, with the running behaviors might be more complicated, perhaps with more stringent constraints on the scheme.
choices.

Since the entire broken phase is in the strong coupling regime, it is very difficult to find the elementary scalar fields in asymptotic final states, as far as the new nontrivial solution is concerned. This might somehow hint us an alternative possibility for the Higgs particles' hunting: it might be impossible to detect the Higgs bosons in the final states in high energy experiments. Maybe some sort of bound states of the scalar particles could be found, but such bound states might in turn add difficulties for us to discern their true origin: from fermion fields condensate (like in the technicolor model) or from some mysterious scalar fields, etc. Since the SCRDSB is entirely non-perturbative, it is more important to see the influence of still higher order contributions to dynamical symmetry breaking. At this moment it is not clear what our results here imply to the hierarchy or unnaturalness problem, but the predictions of the upper and/or lower bounds on Higgs masses based on perturbative triviality should be reexamined with the non-triviality solution in mind. It is also interesting to see how the non-perturbative result here could be applied to SUSY like contexts. We will make no comment about it before further study is done.

In summary, we performed a new investigation on the two loop effective potential of massless $\lambda\phi^4$ and found that the dynamical symmetry breaking solution entirely lives in the strong coupling regime. As in the one loop case, there is still nontrivial relevance of renormalization schemes or prescriptions at the two loop level. Some related issues were briefly touched.

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