One-loop renormalisation of the NMSSM in SloopS 2: the Higgs sector

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May 8, 2017

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Abstract

We present a full one-loop renormalisation of the Higgs sector of the Next-to-Minimal-Supersymmetric-Standard-Model (NMSSM) and its implementation within SloopS, a code for the automated computations of one-loop processes in theories beyond the Standard Model. The present work is the sequel to the study we performed on the renormalisation of the sectors of the NMSSM comprising neutralinos, charginos and sfermions thereby completing the full one-loop renormalisation of the NMSSM. We have investigated several renormalisation schemes based on alternative choices (on-shell or DR) of the physical parameters. Special attention is paid to the issue of the mixing between physical fields. To weigh the impact of the different renormalisation schemes, the partial widths for the decays of the Higgs bosons into supersymmetric particles are computed at one-loop. In many decays large differences between the schemes are found. We discuss the origin of these differences. In particular we study two contrasting scenarios. The first model is MSSM-like with a small value for the mixing between the doublet and singlet Higgs superfields while the second model has a moderate value for this mixing. We critically discuss the issue of the reconstruction of the underlying parameters and their counterterms in the case of a theory with a large number of parameters, such as the NMSSM, from a set of physical parameters. In the present study this set corresponds to the minimum set of masses for the implementation of the on-shell schemes.

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1 Introduction

The discovery of a Standard Model-like Higgs boson at the LHC [1, 2] has raised some concerns with one of the favourite extensions of the Standard Model, SM, the minimal supersymmetric standard model (MSSM). A Higgs mass of 125 GeV, near the maximum value achievable in this model, requires some fine-tuning [3]. In a minimal singlet extension of the model with a $Z_3$ symmetry, the Next-to-MSSM or NMSSM, additional Higgs quartic couplings allow to raise the tree-level mass of the SM-like Higgs [4, 5] hence reducing both the amount of radiative corrections required from the top/stop sector and the amount of fine-tuning [6, 7, 8]. In addition, this model provides a natural explanation for the scale of the higgsino parameter $\mu$, by relating it to the vacuum expectation value, or vev, of the scalar singlet, thus solving the little hierarchy problem of the MSSM.

The computation of higher-order corrections to Higgs production at the LHC as well as its decay rates has been a field of intense activity for the last two decades, in particular for the SM Higgs, for an update see [9, 10, 11, 12]. The bulk of the corrections having to do with QCD corrections. For the MSSM, the best example for the importance of the higher order corrections in the Higgs sector is the correction to the Higgs mass [13, 14]. Without these corrections driven by the top mass, the MSSM would not have survived for so long [15, 16, 17, 18]. Many of these computations have been extended and/or adapted to the case of the NMSSM, in particular for the Higgs masses [19, 20, 21, 22, 23, 24, 25] with improvements including several two-loop effects [26, 27, 28, 29, 30]. Adaptation of the computations of higher order corrections for Higgs production at the LHC from the SM and the MSSM to the NMSSM have been performed [31]. One-loop corrections to Higgs decays in the NMSSM have also been considered with varying degree of generalisation and approximation depending on the final state. Full QCD/SUSY-QCD corrections to decays to SM fermions [32] have been performed, as well as electroweak and QCD corrections to channels such as the decays of CP-odd Higgses into stops [33] and Higgs self-couplings [34], while many other channels like decays to neutralinos and charginos have been adapted from the MSSM [35]. Some electroweak corrections are still not fully systematically included for all decays. Many of these one-loop (or in the case of masses beyond one-loop corrections) have been incorporated in several public codes for the NMSSM: NMSSMTools [36, 37], SPheno [38, 39], NMSSMTools [40], SoftSUSY [41], NMSSMCALC [35] and FlexibleSUSY [42]. Most of these computations are based on a $\overline{\text{DR}}$ scheme or on a mixed $\overline{\text{DR}}$/On-Shell (OS) scheme as in NMSSMCALC [35]. In principle an automated implementation of two-body decays in $\overline{\text{DR}}$ of the NMSSM could be attempted for the NMSSM along the lines described in [43]. A full OS scheme at one-loop for the NMSSM has not been studied.

One of the aims of this paper is to precisely implement different renormalisation schemes including a few variants of a full OS scheme in order to perform complete one-loop corrections for any process in the NMSSM. We have shown in a previous paper [44] how such a programme is applied to the renormalisation of the neutralino/chargino and sfermion sectors of the NMSSM, we extend it here to cover the Higgs sector. Because of the rôle played by the effective $\mu$ parameter in the NMSSM, or in other words, the doublet-singlet $\lambda$ mixing, there is a strong interconnection between the chargino/neutralino sector and the Higgs sector which warrants a common and overall coherent approach to the com-
plete one-loop renormalisation of the NMSSM. This work is a natural extension of the work done for the MSSM in [45, 46] where, after performing the complete renormalisation of the MSSM, one-loop corrections to masses, two-body decays and production cross sections at colliders were computed together with one-loop corrections for various dark matter annihilation processes [47, 48, 49]. The fact that one is able to perform one-loop corrections to a host of processes is made possible by the implementation of the theoretical set-up for the one-loop renormalisation in SloopS, a code for the automated generation and evaluation of any cross section. The one-loop theoretical set-up will be detailed here. As a pre-requisite for SloopS, one first needs to read a model-file. The model file, the NMSSM in this case, is obtained automatically with an improved version of LanHEP [50, 51, 52] that allows for the generation of the counterterms and the corresponding Feynman rules. The code then relies on FeynArts [53], FormCalc [54] and LoopTools for the automatic computation of one-loop processes [55] including both electroweak and QCD corrections. Preliminary applications of SloopS to the NMSSM dealt with computing one-loop induced decays into photons: i) neutralino annihilations into photons, the gamma-ray lines for Dark Matter indirect detection [56, 57], ii) Higgs decays to photons at the LHC [58, 59]. These processes do not call for counterterms or renormalisation at one-loop yet an important part of the machinery of SloopS is called for.

The NMSSM is a typical beyond the SM theory with many parameters, fields and mixings where the different sectors are intertwined. The vast majority of its parameters, as they appear at the level of the Lagrangian, are not directly measurable in experiments in the sense that there is not a straightforward linear mapping between these parameters and an observable such as a mass. The reconstruction of these parameters is a real challenge even when attempted at tree-level. Renormalisation being tightly linked to the choice of input parameters to be extracted from experimental measurements, having currently no sign of supersymmetry leaves this choice with no clear guidance. However, the current extensive program of precision measurements of the Higgs couplings at the LHC requires nonetheless precise theoretical predictions making the renormalisation of the Higgs sector of the NMSSM highly desirable. As for the neutralino/chargino sector that we studied in [44], different schemes are possible. We use mostly on-shell schemes where input parameters are taken from the masses of the neutralino/chargino and from the Higgs sectors. In such schemes, based on two-points functions only, the task of renormalising the model boils down to choosing a minimal/sufficient set of physical masses as input parameters. In this work, we have adopted several sets of input parameters and discussed how efficiently each set can constrain the needed counterterms to keep the radiative corrections under control. Moreover, the renormalisation procedure induces additional mixing, not only among the Higgs physical states but also new gauge-Higgs and Goldstone-Higgs transitions in the pseudoscalar and charged sector appear. Such mixing must vanish for on-shell physical states by imposing appropriate conditions on the wave function renormalisation constants which have to satisfy Ward-Slavnov-Taylor (WST) identities. In doing so we have re-derived the WST identities governing the $A_1^0Z^0/H^±W^±$ mixing in the NMSSM.

The dependence on the renormalisation scheme is then illustrated in numerical computations of observables. Full one-loop electroweak corrections to decays of Higgs particles are computed. The scheme dependence for Higgs to Higgs decays and for decays involving charginos/neutralinos is carefully examined. Note that while the Higgs mass computation
is not as accurate as in other codes (only one-loop corrections are included), we nevertheless stress that our approach allows for a consistent treatment of on-shell renormalisation and one-loop corrections to masses, decays and scattering processes. This is of importance given the very precise experimental measurements achieved both for the Higgs and for Dark Matter observables.

The paper is organised as follows. Section 2 contains a description of the Higgs sector of the NMSSM and enumerates the number of fields and parameters which will need renormalisation. The needed counterterms to obtain ultraviolet finite results are introduced in Section 3 and in the following section the issue of mixing in the Higgs sector through the self-energies is discussed. Section 5 presents the different renormalisation schemes which enable a reconstruction of the counterterms of the underlying parameters the NMSSM with a special attention to those of the Higgs sector. Section 6 presents how the numerical results are checked and how the scheme dependence can be quantified in order to gain insights on the theoretical uncertainties. In section 7 we present two scenarios for which, in Section 8, we compute numerically several Higgs partial widths and discuss their scheme dependence. Finally our conclusions are made in Section 9.

2 The Higgs sector of the NMSSM

2.1 Fields and potential

The NMSSM contains three Higgs superfields: two $SU(2)_L$ doublets $\hat{H}_u$ and $\hat{H}_d$, as in the MSSM, and one additional gauge singlet $\hat{S}$

$$\hat{H}_u = \left( \begin{array}{c} \hat{H}_u^+ \\ \hat{H}_u^0 \end{array} \right), \quad \hat{H}_d = \left( \begin{array}{c} \hat{H}_d^0 \\ \hat{H}_d^- \end{array} \right), \quad \hat{S}. \quad (2.1)$$

In the $\mathbb{Z}_3$ implementation that we will assume, the Higgs superpotential is made up of two operators, associated with the dimensionless couplings $\lambda$ and $\kappa$,

$$W_{\text{Higgs}} = -\lambda \hat{S} \hat{H}_d \cdot \hat{H}_u + \frac{1}{3} \kappa \hat{S}^3, \quad (2.2)$$

where $\hat{H}_d \cdot \hat{H}_u = \epsilon_{ab} \hat{H}_d^a \hat{H}_u^b$ and $\epsilon_{ab}$ is the two dimensional Levi-Civita symbol with $\epsilon_{12} = 1$. The two parameters $\lambda$ and $\kappa$ of the superpotential will, by construction, affect the phenomenology of both the Higgs and chargino/neutralino sectors. The five neutralinos of the NMSSM will be an admixture of the $i) SU(2)$, $\tilde{w}$, and $U(1)$, $\tilde{b}$, neutral gauginos, $ii)$ the two neutral higgsinos, $\tilde{h}_u,d$, the fermionic components of two superfields $\hat{H}_{u,d}$ and $iii)$ the singlino, $\tilde{s}$, the fermionic component of $\hat{S}$. $\lambda$ in particular is crucial, not only it is necessary in order to induce the $\mu$ term but it also gives rise to mixing in the neutralino sector as well as in the Higgs sector between the Higgs doublets and the new singlet. In passing we recall that $\mu$ sets the mass scale for the higgsinos, see [44] for more detail. For the purpose of parameter counting and of the renormalisation of the Higgs sector at one-loop there is no need to go over the Yukawa superpotential which we have given in the previous paper [44]. However we do need to clearly specify again the soft SUSY breaking
Lagrangian, in particular the part relating to the Higgs sector.

\[-\mathcal{L}_{\text{soft, scalar}} = m_{H_u}^2 |H_u|^2 + m_H^2 |H_d|^2 + m_\xi^2 |S|^2 + (\lambda A H_u \cdot H_d S + \frac{1}{3} \kappa A \kappa S^3 + h.c)\]  \hspace{1cm} (2.3)

The first two terms in the first line represent the soft mass terms for the Higgs doublets and the third, not present in the MSSM, of the singlet. The second line, not present in the MSSM also, represents the NMSSM trilinear Higgs couplings $A$, $\lambda$. $A$ affects the mixing between the Higgs doublets and the singlet, beside the mixing introduced by $\lambda$. This parameter plays an important role in the phenomenology of the Higgs sector in the NMSSM, note that it gives rise to a Higgs tri-linear coupling $H_u H_d S$. No source of CP violation is assumed.

We are now in a position to write the Higgs potential whose parameters will need to be renormalised. With $g, g'$ being respectively the $SU(2)$ weak and $U(1)$ hypercharge gauge couplings and specifying the components of the doublets,

$$H_d = \begin{pmatrix} H_0^d \\ H_d^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_0^u \\ H_u^+ \end{pmatrix},$$  

the potential reads

$$V_{\text{Higgs}} = |\lambda(H_u^+ H_d^- - H_0^u H_d^-)|^2 + (m_{H_u}^2 + |\lambda S|^2)(|H_0^u|^2 + |H_u^+|^2)$$

$$+ (m_{H_d}^2 + |\lambda S|^2)(|H_0^d|^2 + |H_d^-|^2) + \frac{g^2 + g'^2}{8} (|H_u^0|^2 + |H_u^+|^2 - |H_0^u|^2 - |H_d^-|^2)^2$$

$$+ \frac{g^2}{2} |H_u^+ H_d^0 - H_u^0 H_d^-|^2 + m_\xi^2 |S|^2 + (\lambda A (H_u^+ H_d^- - H_0^u H_d^-) S + \frac{1}{3} \kappa A \kappa S^3 + h.c).$$  \hspace{1cm} (2.4)

Electroweak symmetry breaking occurs for appropriate values of the soft terms. The Higgs fields are expanded around their vacuum expectation values,

$$H_d = \begin{pmatrix} v_d + \frac{h_0^d + ia_0^d}{\sqrt{2}} \\ h_d^- \end{pmatrix},$$  \hspace{1cm} (2.5)

$$H_u = \begin{pmatrix} h_u^+ \\ v_u + \frac{h_0^u + ia_0^u}{\sqrt{2}} \end{pmatrix},$$  \hspace{1cm} (2.6)

$$S = s + \frac{h_0^s + ia_0^s}{\sqrt{2}}.$$  \hspace{1cm} (2.7)

The vacuum expectation values, $v_u, v_d, s$ are chosen to be real and positive. As in the MSSM we define

$$v^2 = v_u^2 + v_d^2, \quad \tan \beta \equiv t_\beta = v_u/v_d, \quad (v_{u,d} = v s_\beta, v c_\beta)$$  \hspace{1cm} (2.8)

such that the $W$ mass is

$$M_W^2 = g^2 v^2/2.$$  \hspace{1cm} (2.9)
The non vanishing value of the vev of $S$ also gives a solution to the so-called $\mu$-problem of the MSSM by generating this parameter dynamically:

$$\mu_{eff} = \lambda s.$$  \hfill (2.10)

We define $\mu_{eff} = \mu$ in the following and will take it as an independent parameter, comparison with the MSSM will then be easier. In addition to $\mu$, we take $\lambda$ and $\kappa$ as independent parameters while $s$ is kept as a shorthand notation for $\mu/\lambda$ in the same way as we use $c_W$ as a short-hand notation for $M_W/M_Z$. It is useful to introduce the combinations

$$\Lambda_v = \lambda v \quad \text{and} \quad m_\kappa = \kappa s = (\kappa/\lambda)\mu.$$  \hfill (2.11)

With these parameters, the MSSM limit is obtained by taking $\kappa, \lambda (\Lambda_v) \rightarrow 0$, while keeping $\mu$ fixed such that the mass of the higgsinos is $|\mu|$. The reason we take $m_\kappa$ is that the mass of the singlino-like neutralino is a substitute for $m_\kappa$. Indeed, in the MSSM limit, the singlino mass is, see [44]

$$m_\tilde{s} = 2m_\kappa.$$  \hfill (2.12)

At the minimum of the potential, the part of the potential linear in any of the CP-even Higgs field has to vanish, $\partial V_{Higgs}/\partial h_i^0 = 0$. It can be written in terms of the tree-level tadpoles,

$$\mathcal{F}_{h_d^0}^{h_u^0} = -\mu t_\beta (A_\lambda + m_\kappa) + \Lambda_v^2 s_\beta^2 + (m_{H_d}^2 + \mu^2) + \frac{M_Z^2}{2} c_{2\beta},$$

$$\mathcal{F}_{h_d^0}^{h_u^0} = -\mu t_\beta (A_\lambda + m_\kappa) + \Lambda_v^2 c_\beta^2 + (m_{H_u}^2 + \mu^2) - \frac{M_Z^2}{2} c_{2\beta},$$

$$\mathcal{F}_{h_s^{\pm}} = m_\kappa (A_\kappa + 2m_\kappa) + \Lambda_\kappa^2 (1 - s_\beta^2 \frac{A_\lambda + 2m_\kappa}{\mu}) + m_s^2.$$  \hfill (2.13)

The conditions on the vanishing of the three tadpoles allow us to express the soft mass terms $m_{H_d}^2, m_{H_u}^2, m_s^2$ in terms of the tadpoles.

The quadratic part of the Higgs potential, bilinear in the fields, gives rise to the mass terms for the Higgs sector:

$$V_{mass} = \frac{1}{2} (h^0)T \cdot M^2_S \cdot h^0 + \frac{1}{2} (a^0)T \cdot M^2_P \cdot a^0 + h^- \cdot M^2_{\pm} \cdot h^+,$$  \hfill (2.14)

with

$$\begin{align*}
(h^0)^T &= (h_d^0 \quad h_u^0 \quad h_s^0) \\
(a^0)^T &= (a_d^0 \quad a_u^0 \quad a_s^0) \\
(h^{\pm})^T &= (h_d^{\pm} \quad h_u^{\pm})
\end{align*}$$  \hfill (2.15-17)

and where $M^2_S$, $M^2_P$ and $M^2_{\pm}$ are respectively the mass matrices for the CP-even, the CP-odd, and the charged Higgs bosons.

The charged Higgs
The mass matrix for the charged Higgs reads

\[
M^2_{\pm} = \frac{1}{2} \begin{pmatrix}
\frac{\mathcal{F}_{A}}{v_d} & 0 \\
0 & \frac{\mathcal{F}_{V}}{v_u}
\end{pmatrix} + \left(\mu(A_\lambda + m_\kappa) + \frac{s_{2\beta}}{2}(M^2_W - \Lambda^2_v)\right) \begin{pmatrix} t_\beta & 1 \\ 1 & 1/t_\beta \end{pmatrix},
\]

(2.18)

With the tadpoles set to zero (using the tree-level condition in Eq. 2.13), we have \(\text{Det}M^2_{\pm} = 0\) which signals the presence of a massless charged Goldstone boson. The mass of the physical charged Higgs boson is given by the trace of \(M^2_{\pm}\),

\[
M^2_{H^\pm} = \frac{2\mu}{s_{2\beta}}(A_\lambda + m_\kappa) + (M^2_W - \Lambda^2_v).
\]

(2.19)

The MSSM limit is obtained by letting \(\Lambda_v\) to 0 in the above, while all other parameters are fixed. What is denoted \(M_A\) is the equivalent of the pseudoscalar mass in the MSSM limit. Note that if \(m_\kappa(\kappa), \Lambda_v(\lambda), \mu\) and \(t_\beta\) have been extracted from the chargino/neutralino sector, the measurement of the charged Higgs mass reconstructs \(A_\lambda\). As explained in [44], for this to work efficiently \(t_\beta\) should not be too large and in all cases should be well measured. The charged Higgs mass could also serve for the measurement of \(t_\beta\) if \(A_\lambda\) is determined from the other Higgs masses.

The diagonalising matrix, \(U(\beta)\), to obtain the Goldstone and physical charged Higgs is defined as

\[
U_\beta = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix},
\]

(2.21)

The Pseudoscalars

The pseudoscalar mass matrix decomposes into the following elements

\[
\begin{align*}
M^2_{P_{11}} &= \frac{\mathcal{F}_{A}}{2\mu s_{2\beta}} + \mu t_\beta(A_\lambda + m_\kappa), \\
M^2_{P_{22}} &= \frac{\mathcal{F}_{V}}{2\mu s_{2\beta}} + \mu(A_\lambda + m_\kappa), \\
M^2_{P_{33}} &= \frac{\mathcal{F}_{V}}{2\mu} + \Lambda_v^2 + \frac{A_\lambda + 4m_\kappa}{2\mu} s_{2\beta} - 3m_\kappa A_\kappa, \\
M^2_{P_{12}} &= M^2_{P_{21}} = \mu(A_\lambda + m_\kappa), \\
M^2_{P_{13}} &= M^2_{P_{31}} = \Lambda_v(A_\lambda - 2m_\kappa) s_\beta, \\
M^2_{P_{23}} &= M^2_{P_{32}} = \Lambda_v(A_\lambda - 2m_\kappa) c_\beta.
\end{align*}
\]

(2.22)

As expected, upon setting the tadpole to zero, \(\text{Det}M^2_P = 0\). This reveals the neutral Goldstone boson. With the \(2 \times 2\) submatrix, \(m^2_{12}\)

\[
m^2_{12} = s_{2\beta} c_\beta \frac{\mu^2_{A,MSSM}}{m^2_{12}} \begin{pmatrix} t_\beta & 1 \\ 1 & 1/t_\beta \end{pmatrix}, \quad \text{Tr}(m^2_{12}) = M^2_{A,MSSM},
\]

(2.23)

in the MSSM limit \((\Lambda_v \to 0)\) we have

\[
M^2_P \to \begin{pmatrix} m^2_{12} & 0 \\ 0 & -3m_\kappa A_\kappa \end{pmatrix}
\]

(2.24)
The Goldstone boson can be isolated through the $3 \times 3$ extension of the matrix $U_\beta$ encountered for the charged Higgs sector

$$U^{(3)}(\beta) = \begin{pmatrix} c_\beta & -s_\beta & 0 \\ s_\beta & c_\beta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (2.25)$$

In this new basis where the Goldstone boson is separated, the pseudoscalar mass matrix simplifies to

$$U^{(3)}(\beta)M^2_PU^{(3)}(\beta)\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \hat{M}_P^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.26)$$

where the $(2 \times 2)$ mixing matrix between the two pseudoscalar bosons is given by

$$\hat{M}_P^2 = \begin{pmatrix} M^2_A \\ \Lambda_v(A_\lambda - 2m_\kappa) \\ \Lambda_v(2A_\lambda + 4m_\kappa) \end{pmatrix} \begin{pmatrix} 1 \\ A_{\lambda} \end{pmatrix}. \quad (2.27)$$

The MSSM limit is clearly exhibited. Diagonalisation of this matrix is then performed through a $2 \times 2$ matrix $P_a$ which we can parameterise as

$$\hat{P}_a = \begin{pmatrix} c_p & -s_p \\ s_p & c_p \end{pmatrix} \quad (2.28)$$

Putting everything together the pseudoscalar mass matrix is diagonalised through the matrix $P_a$,

$$P_a = \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \hat{P}_a & 0 \\ 0 & 0 & \hat{P}_a^{(3)} \end{array} \right) U^{(3)}(\beta) = \begin{pmatrix} c_\beta & -s_\beta & 0 \\ c_p s_\beta & c_p c_\beta & -s_p \\ s_p s_\beta & s_p c_\beta & c_p \end{pmatrix}. \quad (2.29)$$

such that

$$\Phi^0 \equiv \begin{pmatrix} C^0_0 \\ A^0_1 \\ A^0_2 \end{pmatrix} = P_a a^0 = P_a \begin{pmatrix} a^0_d \\ a^0_u \\ a^0_s \end{pmatrix}, \quad (2.30)$$

It is important to remember that

$$(P_a)_{13} = (P_a^{-1})_{31} = 0 \quad \text{and that} \quad (P_a)_{ii} = (U^{(3)}(\beta))_{ii} \quad \text{for} \quad i = 1, 2. \quad (2.31)$$

We will also set $(\Phi^0)_1 \equiv A^0_0 \equiv G^0$ for the identification of the neutral Goldstone Boson.

The CP-even scalars
The elements of the scalar mass matrix read

\[
\begin{align*}
M_{S_{11}}^2 &= \frac{g_A^2}{2v^2} + M_Z^2 c_\beta^2 + M_A^2 s_\beta^2, \\
M_{S_{22}}^2 &= \frac{g_A^2}{2v^2} + M_Z^2 s_\beta^2 + M_A^2 c_\beta^2, \\
M_{S_{33}}^2 &= \frac{g_A^2}{2v^2} + \Lambda_v^2 A_\lambda^2 s_\beta^2 + m_\kappa (A_\lambda + 4m_\kappa), \\
M_{S_{12}}^2 &= M_{S_{21}}^2 = (2\Lambda^2 - M_Z^2 - M_A^2) s_\beta c_\beta, \\
M_{S_{13}}^2 &= M_{S_{31}}^2 = \Lambda_v (2\mu c_\beta - (A_\lambda + 2m_\kappa) s_\beta), \\
M_{S_{23}}^2 &= M_{S_{32}}^2 = \Lambda_v (2\mu s_\beta - (A_\lambda + 2m_\kappa) c_\beta),
\end{align*}
\]

To get the physical eigenstates, we introduce the orthogonal matrix \( S_h \), such that

\[
\begin{pmatrix}
h_1^0 \\
h_2^0 \\
h_3^0
\end{pmatrix} = S_h \begin{pmatrix}
h_1^0 \\
h_2^0 \\
h_3^0
\end{pmatrix}.
\]

We can make the MSSM limit more apparent by writing the diagonalising matrix of the CP-even Higgs mass matrix, \( S_h = S_h U^{(3)}(\beta) \). After rotation by \( U^{(3)}(\beta) \) an upper limit for the non-singlet and CP-even lightest neutral Higgs mass is contained in a element of the mass matrix. The upper bound on this mass is

\[
M_{h_1^0}^2 < M_Z^2 \left( c_{2\beta}^2 + \frac{\Lambda_v^2}{M_Z^2} s_{2\beta}^2 \right) \equiv M_Z^2 \left( 1 + \left( \frac{\Lambda_v^2}{M_Z^2} - 1 \right) s_{2\beta}^2 \right).
\]

To have a tree-level mass which is higher than \( M_Z \), \( \Lambda_v \) needs to be larger than \( M_Z \) \((\lambda > \sqrt{g^2 + g^2}/2)\). Moreover the largest tree level mass corresponds to moderate values of \( t_\beta \). We need to keep this in mind when we will discuss our benchmark points.

It is also useful to write

\[
M_{h_1^0}^2 = \sum_{j,k=1}^3 S_{h_{1j}} S_{h_{k1}} M_{S_{jk}}^2, \quad M_{h_1^0}^2 = \sum_{j,k=1}^3 P_{a_{ij+1}j} P_{a_{i+1}kj} M_{P_{jk}}^2.
\]

The properties of the physical states depend critically on the mixing matrixes \( S_h \) for the parity-even Higgses and on \( P_a \) for the parity odd Higgses. These mixing matrixes which stem from the non-diagonal nature of the mass matrices/bi-linear terms, introduce a highly non linear dependence of the couplings involving the Higgses on the underlying parameters of the theory whereas before mixing, in so to speak the current basis, the functional dependence of the Higgs couplings on the underlying parameters is quite simple, linear or quadratic. This can be seen from the Higgs potential in Eq. 2.7. For example, before these rotation matrices are introduced, the coupling between 3 different CP-even neutral Higgses \((h_1^0 h_2^0 h_3^0)\) is proportional to \( \lambda A_\lambda + 2 \kappa \mu = \Lambda_v (A_\lambda + 2m_\kappa)/v \) and hence directly proportional to \( \lambda \). After moving to the physical basis, the \( h_1^0 h_2^0 h_3^0 \) coupling is much more complicated since it involves the product of three \( S_h \). Therefore the dependence of this coupling on the underlying parameters is more difficult to track. Since the triple Higgs couplings will enter some of the decays we will study, we write them below for the CP-even Higgs sector,
\[
\sqrt{2}v h'^0 h'^0 = \frac{M^2}{2} \left( c_\beta (\Pi^S)_{i,j,k}^{1,1} + s_\beta (\Pi^S)_{i,j,k}^{2,2} \right) + \left( \Lambda^2 - \frac{M^2}{2} \right) (c_\beta (\Pi^S)_{i,j,k}^{1,2,2} + s_\beta (\Pi^S)_{i,j,k}^{2,1,1}) \\
+ \Lambda^2 \left( (c_\beta - \frac{m_\kappa}{\mu}) (\Pi^S)_{i,j,k}^{1,3,3} + (s_\beta - \frac{m_\kappa}{\mu}) (\Pi^S)_{i,j,k}^{2,3,3} \right) \\
+ \mu \left( (\Pi^S)_{i,j,k}^{3,1,1} + (\Pi^S)_{i,j,k}^{3,2,2} - (A_\lambda + 2m_\kappa) (\Pi^S)_{i,j,k}^{3,1,2} + \frac{m_\kappa A_\kappa}{\mu} + \frac{6m_\kappa}{3} (\Pi^S)_{i,j,k}^{3,3,3} \right)
\]

where the \( \Pi^S \) represent the product of three \( h \)

\[
(\Pi^S)_{i,j,k}^{a,b,c} = S_{h_a}(S_{h_b}S_{h_c} + S_{h_j}S_{h_k}) + S_{h_a}(S_{h_j}S_{h_b} + S_{h_c}S_{h_k}) + S_{h_c}(S_{h_j}S_{h_b} + S_{h_a}S_{h_b})
\]

In the case where mixing is neglected in \( S_h \) we have

\[
(\Pi^S)_{i,j,k}^{a,b,c} = \delta_{ia}(\delta_{bj}\delta_{kc} + \delta_{jc}\delta_{kb}) + \delta_{ib}(\delta_{ja}\delta_{kc} + \delta_{jc}\delta_{ka}) + \delta_{ic}(\delta_{ja}\delta_{kb} + \delta_{jb}\delta_{ka})
\]

\[
(\Pi^S)_{i,i,i}^{a,a,a} = 6\delta_{ia}\delta_{ib}\delta_{ic}
\]

\[
(\Pi^S)_{i,j,k}^{a,a,a} = 6\delta_{ia}\delta_{ja}\delta_{ka}
\]

\[
(\Pi^S)_{i,j,k}^{a,b,b} = 2\delta_{ia}\delta_{jb}\delta_{kb} + 2\delta_{ib}\delta_{ja}\delta_{kb} + \delta_{jb}\delta_{kb}
\]

It is important to realise that with our choice of the independent parameters all triple Higgs couplings involving the singlet are proportional to \( \lambda \) or \( \lambda^2 \). This should not be the case for the coupling between three singlets which gets contributions from \( S^3 \) and \( |S^2|^2 \) terms. In Eq. 2.35 this is proportional to \( \lambda m_\kappa/\mu = \kappa \). The fact that this coupling exhibits a \( \lambda \) dependence is due to our choice of inputs \( m_\kappa \) and \( \mu \) which are more directly related to the mass of the singlino and the higgsino, see Eqs. 2.11.

### 2.2 Counting parameters and fields

Let us take stock and summarise the situation as regards the number of (physical) parameters and fields in the Higgs sector of the NMSSM. The physical scalar fields consist of 3 neutral CP-even Higgs bosons, \( h^0_1, h^0_2, h^0_3 \), 2 CP-odd Higgs bosons, \( A^0_1, A^0_2 \) and a charged Higgs boson, \( H^\pm \). The NMSSM contains of course the SM gauge fields (and fermions). In particular the SM gauge parameters

\[
g, g' \quad \text{and} \quad v = v_u^2 + v_d^2
\]

are traded for the following physical input parameters

\[
e, M_W, M_Z
\]

For these parameters we will apply the usual On-Shell (OS) renormalisation scheme. In particular \( e \) will be defined in the Thomson limit. The Thomson limit, \( q^2 \to 0 \), may not be the most appropriate scale for the NMSSM processes whose loop corrections we will study, however one can easily quantify the effect of using a running \( a_{e.m.} \) at the scale of the process. Besides these standard model parameters, the NMSSM introduces an additional set of 9 parameters from the Higgs sector alone. From the Higgs potential, equation (2.4), it is clear that the Higgs sector of the NMSSM depends on the parameters:
where the first four parameters are also involved in the characterisation of the neutralino/chargino sector which we studied at length in the sister paper [44]. Alternatively the last three soft Higgs masses can be traded for the tadpoles of the neutral Higgs which need to be constrained to zero to impose that the potential is at its minimum. The latter are therefore considered as physical observables.

\[ t_{\beta}, \lambda, \kappa, \mu, A_{\lambda}, A_{\kappa}, m_{H_d}, m_{H_u}, m_S. \] (2.39)

The first six parameters above are not unambiguously defined in a simple mapping to an observable. We will discuss at length the choice and definitions of the input parameters that will construct the set of these six parameters. This issue is directly related to the renormalisation scheme.

The parameters and fields of the neutralino/chargino sector were described in [44]. The parameters of this sector are the \( U(1) \) and \( SU(2) \) gaugino soft masses, \( M_1, M_2 \) in addition to \( t_{\beta}, \lambda, \kappa, \mu \).

3 Renormalisation of the Higgs sector

3.1 A word about the gauge-fixing

In this work we have restricted ourselves to the simplest gauge-fixing, namely we take a linear gauge fixing constraint with a 'tHooft-Feynman parameter set to 1. Only the SM fields (including the necessary) Goldstone bosons appear, namely

\[ \mathcal{L}_{\text{GF}} = -|\partial^\mu W_\mu^+ + iM_W G^+|^2 - \frac{1}{2} |\partial^\mu Z_\mu^0 + iM_Z G^0|^2 - |\partial^\mu A_\mu|^2. \] (3.1)

It is important to stress that all fields and parameters in Eq. 3.1 are understood to be renormalised.

3.2 Parameters, fields and self-energies

What is also considered as renormalised are all the rotation matrices \( U(\beta), P_a, S_h \). This is an approach we have consistently applied in all our work on the renormalisation of supersymmetric models starting from the MSSM and imposed in all the sectors of the models where mixing between fields occurs, not only in the Higgs sector, but also in the sfermion, neutralino, chargino sectors [44, 45, 46]. While one-loop corrections do reintroduce mixing, the use of wave function renormalisation constants, with judicious choices of conditions imposed at the physical masses of the particles, will ensure that even at one-loop transitions between particles with the same quantum numbers will vanish when these particles are on their mass shell.
With the exception of $\mathcal{L}_{GF}, U(\beta), P_a$ and $S_h$ all fields and parameters encountered so far are bare quantities. All bare quantities ($X_0$) are then decomposed into renormalised ($X$) and counterterms ($\delta X$) quantities. First, the SM parameters are shifted such that

$$g \to g + \delta g, \quad g' \to g' + \delta g', \quad v \to v + \delta v,$$

which tantamount to

$$e \to e + \delta e, \quad M_W \to M_W + \delta M_W, \quad M_Z \to M_Z + \delta M_Z.$$  \hspace{1cm} (3.3)$$

The same procedure applies to the NMSSM parameters in Eq. 2.39 or equivalently Eq. 2.40 with

$$t_\beta, \lambda, m_\kappa, \mu, A_\lambda, A_\kappa \to t_\beta + \delta t_\beta, \lambda + \delta \lambda, m_\kappa + \delta m_\kappa, \mu + \delta \mu, A_\lambda + \delta A_\lambda, A_\kappa + \delta A_\kappa,$$

$$\mathcal{H}_d, \mathcal{H}_u, \mathcal{S} \to \mathcal{H}_d + \delta \mathcal{H}_d, \mathcal{H}_u + \delta \mathcal{H}_u, \mathcal{S} + \delta \mathcal{S}.$$ \hspace{1cm} (3.4)$$

For the fields all shifts are directly encoded in the wave function renormalisation constants, so that mass eigenstates are expressed in terms of the gauge eigenstates in the same manner at one-loop and at tree-level. For the gauge fields we perform $W_\mu \to Z_{W_\mu}^{1/2} W_\mu$ while the system ($A_\mu, Z_\mu^0$) involves a matrix with 4 entries $\delta Z_{\gamma \gamma}, \delta Z_{Z\gamma}, \delta Z_{ZZ}, \delta Z_{Z\gamma}$, see [60]. For the NMSSM Higgs sector, this entails that the three wave-function renormalisation matrices $Z_S, Z_P$ and $Z_C$, are introduced such that,

$$
\begin{pmatrix}
    h_0^0 \\
    h_2^0 \\
    h_3^0
\end{pmatrix} = Z_S^{1/2} \begin{pmatrix}
    h_0^0 \\
    h_2^0 \\
    h_3^0
\end{pmatrix}_0 \quad \text{and} \quad 
\begin{pmatrix}
    G_0^0 \\
    A_1^0 \\
    A_2^0
\end{pmatrix} = Z_P^{1/2} \begin{pmatrix}
    G_0^0 \\
    A_1^0 \\
    A_2^0
\end{pmatrix}_0 \quad \text{and} \quad 
\begin{pmatrix}
    G^\pm \\
    H^0 \\
    H^\pm
\end{pmatrix} = Z_C^{1/2} \begin{pmatrix}
    G^\pm \\
    H^0 \\
    H^\pm
\end{pmatrix}_0
$$

where the index 0 is attached to the bare fields while the renormalised fields do not have an index. The elements of the wave-function renormalisation matrices can be written as,

$$Z_C^{1/2} = \begin{pmatrix}
    1 + \frac{1}{2} \delta Z_{G^G} & \frac{1}{2} \delta Z_{G^H} & \frac{1}{2} \delta Z_{G^H}
\end{pmatrix} \quad \text{and} \quad 
Z_P^{1/2} = \begin{pmatrix}
    1 + \frac{1}{2} \delta Z_{A_1^G} & \frac{1}{2} \delta Z_{A_1^A} & \frac{1}{2} \delta Z_{A_1^A}
\end{pmatrix} \quad \text{and} \quad 
Z_S^{1/2} = \begin{pmatrix}
    1 + \frac{1}{2} \delta Z_{h_1^G} & \frac{1}{2} \delta Z_{h_1^h} & \frac{1}{2} \delta Z_{h_1^h}
\end{pmatrix} \quad \text{and} \quad (3.5)$$

$$Z_{A_1^G} \quad \text{and} \quad Z_{A_1^A} \quad \text{and} \quad Z_{h_1^G} \quad \text{and} \quad Z_{h_1^h}$$

$
3.3 \text{ One-point functions and tadpoles}$

Once shifts on all parameters of the models including the tadpole terms (Eq. 3.4) and wave function renormalisation of the fields have been performed, we concentrate on the terms which are linear in the scalar fields and combine them with the one-loop contribution to these tadpoles. The tree-level condition on the tadpole is now elevated to the one-loop level so that minimisation of the potential is realised. At one-loop, the linear part of the potential can be written:

$$V^{(1)}_{\text{lin}} = (-\mathcal{T}_0^{\text{loop}} + \delta \mathcal{T}_0^\prime) \frac{h_0^0}{\sqrt{2}} + (-\mathcal{T}_0^{\text{loop}} + \delta \mathcal{T}_0^\prime) \frac{h_0^0}{\sqrt{2}} + (-\mathcal{T}_0^{\text{loop}} + \delta \mathcal{T}_0^\prime) \frac{h_0^0}{\sqrt{2}},$$

$$\text{Eq. } (3.9)$$
where the first terms in the parenthesis are the pure loop contributions and the second ones the counterterms. We observe that because of the condition on the tree-level tadpoles, wave function renormalisation of the Higgses does not enter. Our first renormalisation condition is that these linear terms cancel. The loop contributions for the gauge eigenstates tadpoles are obtained from the mass eigenstates tadpoles with the use of the diagonalisation matrix $S_h$,

$$
\begin{pmatrix}
\mathcal{J}_{h_0}^{\text{loop}}
\mathcal{J}_{h_0}^{\text{loop}}
\mathcal{J}_{h_0}^{\text{loop}}
\end{pmatrix}
= S_h^{-1}
\begin{pmatrix}
\mathcal{J}_{h_0}^{\text{loop}}
\mathcal{J}_{h_0}^{\text{loop}}
\mathcal{J}_{h_0}^{\text{loop}}
\end{pmatrix}.
$$

(3.10)

The minimum condition then gives simply,

$$
\delta \mathcal{J}_{h_0}^i = \mathcal{J}_{h_0}^{\text{loop}}^i, \quad i = d, u, s.
$$

(3.11)

## 4 Bilinears and two-point functions self-energies

### 4.1 Mass counterterms for the Higgs sector

We now turn to the bilinear terms in the Higgs fields and perform shifts in the parameters according to Eq. 3.3 and Eq. 3.4. These shifts are performed on each of the underlying parameters (including the tadpoles) of the mass matrices $M_2^2, M_{D}^2, M_S^2$ in Eqs. 2.18, 2.22, 2.32. Since our approach is to use the same tree-level diagonalising mass matrices, namely $U_\beta, P_a, S_h$, to convert to the physical fields, after the shifts the "physical" fields will now mix. Therefore apart from the induced diagonal counterterms $\delta M_{H^\pm}, \delta M_{A_{1,2}}^2, \delta M_{h_{1,2,3}}^2$, spurious counterterms to the Goldstones $\delta M_{G^0,G^\pm}^2$ are generated as well as non-diagonal mass mixings such as $\delta M_{h_{1,2}}^2$ and transitions such as $\delta M_{G^0,H^\pm}^2$. Therefore, we need to enforce appropriate conditions to the one-loop wave function renormalisation matrices such that a correct on-shell definition and normalisation of the external particles is ensured. This is obtained by imposing that the residue of each (diagonal) propagator is equal to 1 (as is done in any theory without mixing). Strictly speaking, if we were only interested in having finite $S$-matrix elements and not finite Green's functions, wave function renormalisation would not be a must. Still, the residues of the propagators of the external particles must be set to 1 to correctly normalise the $S$-matrix, this can be achieved by introducing finite wave function correction normalisation factors to prevent non-vanishing transitions on the external legs.

Since the contributions of tadpoles is very important, let us summarise how the shifts in the underlying parameters affect the mass matrices. Generically the mass matrix in the three sectors ($M^2 = M_2^2, M_{D}^2, M_S^2$) can, in the bases $d, u, s$, be decomposed into a diagonal tadpole matrix $T_M$ and a non-tadpole matrix which we denote $M_M^2$, such that $M^2 = T_M + M_M^2$. At tree-level all $T_M$ are set to zero and the diagonalising matrix $U (U = U_\beta, P_a, S_h)$ is such that $M_D^2 = UM_M^2U^{-1}$ is diagonal with eigenvalues being the tree-level physical masses. In our notation, for the charged and pseudoscalar sectors the Goldstones are the $(11)$ entries, hence $(M_D)_{11} = 0$. The shifts entail the counterterm mass matrix

$$
\delta M^2 = \delta T_M + \delta M_M^2.
$$

(4.1)
In our approach, in all sectors of the NMSSM, to move to the (“physical”) basis \(A_i^0, G_i^0, \ldots\) we use the same matrix as the one at tree-level, namely (the sub-index \(Ph.\) below generically denotes the mass matrix in the “physical” basis)

\[
\delta M_{Ph.}^2 = U \delta T_M U^{-1} + U \delta M_{M}^2 U^{-1}. \tag{4.2}
\]

These are the mass counterterms which will be used to define the self-energies. In the code these counterterms are generated according to Eq. 4.2. However, for the sake of the discussion, it is more enlightening to express the above counterterms through the mass eigenvalues, \(M_D^2 = U M_{M}^2 U^{-1}\) (note that there is no tadpole here). It is important to realise that shifts in \(M_D\) now should be understood as shifts in all parameters that define \(M_D\) and in particular those entering \(U\). We can then write, using the unitarity of \(U\), \(\delta(U)U^{-1} + U \delta(U^{-1}) = 0\)

\[
\delta M_{Ph.}^2 = U \delta T_M U^{-1} + \delta M_D^2 + (U(\delta U^{-1})M_D^2 + M_D^2(\delta U)U^{-1})) \quad (\delta M_{Ph.})_{ij} = (U \delta T_M U^{-1})_{ij} + (\delta M_D)_{ij} + ((\delta U)U^{-1})_{ij} ((M_D)_{ij}^2 - (M_D)_{ij}^2) \tag{4.3}
\]

Eq. 4.3 shows that for the diagonal terms, the terms in \(\delta U\) vanish, in particular the counterterms to the Goldstones for both the charged and the pseudoscalar, \(\delta M_G^2\) (for \(i = j = 1\)), is a pure tadpole term as it should. From the properties of the \(P_i\) matrix of the pseudoscalars we even find that \(\delta M_G^2 = \delta M_G^2\). The mixing between a Goldstone and a non Goldstone field is proportional to the (tree-level) mass of the associated non Goldstone physical field. Because \(\delta M_{Ph.}^2\) is no longer diagonal, Goldstone-Higgs mixing mass counter-terms are generated,

\[
\delta M_{H^+G^0}^2 = \delta M_{G^+H^0}^2 = \frac{s_{2\beta}}{2} \left( \delta T + M_{H^0}^2 \frac{\delta t_3}{t_3} \right) \tag{4.4}
\]

\[
\delta M_{A^0G^0}^2 = \delta M_{G^0A^0}^2 = \frac{s_{2\beta}}{2} \left( \delta T + M_{A^0}^2 \frac{\delta t_3}{t_3} \right) \tag{4.5}
\]

where \(\delta T = \frac{1}{v s_{2\beta}} \sum_{i=1}^{3} (s_{\beta} S_{h,i1} - c_{\beta} S_{h,i2}) \delta \mathcal{F}_{h_i}\) and the need for the wave function renormalisation becomes evident in order to counterbalance the appearance of these transitions especially when the particles are on their mass shell.

### 4.2 Two-point functions from the Higgs potential

Implementing the wave function renormalisation directly in the physical basis, we can write the renormalised self-energies, with the non “hatted” expression as the result of the 1-loop unrenormalised self-energy, while the \(\delta Z\)’s are the result of the wave function renormalisation. The mass shifts correspond exactly to the elements of \(\delta M_{Ph.}^2\) (which include tadpoles). For the CP-even scalars we obtain \((i,j = 1, 2, 3)\),

\[
\hat{\Sigma}_{h_i^0h_j^0}(p^2) = \Sigma_{h_i^0h_j^0}(p^2) + \delta M_{h_i^0h_j^0}^2 - \frac{1}{2} (p^2 - M_{h_i^0}^2) \delta Z_{h_i^0h_j^0} - \frac{1}{2} (p^2 - M_{h_j^0}^2) \delta Z_{h_i^0h_j^0} \tag{4.7}
\]
while for the CP-odd scalars we get \((i, j = 1, 2)\),

\[
\begin{align*}
\hat{\Sigma}^{G\phi}(p^2) &= \Sigma^{G\phi}(p^2) + \delta M^2_{G\phi} - p^2 \delta Z^G_{\phi} \\
\hat{\Sigma}^{A_i\phi}(p^2) &= \Sigma^{A_i\phi}(p^2) + \delta M^2_{A_i\phi} - \frac{1}{2}p^2 \delta Z^G_{A_i\phi} - \frac{1}{2}(p^2 - M^2_{A_i}) \delta Z^{A_i\phi}_{\phi} \\
\hat{\Sigma}^{A_i\phi_j}(p^2) &= \Sigma^{A_i\phi_j}(p^2) + \delta M^2_{A_i\phi_j} - \frac{1}{2}(p^2 - M^2_{A_i}) \delta Z^{A_i\phi_j}_{\phi_j} - \frac{1}{2}(p^2 - M^2_{A_j}) \delta Z^{A_i\phi_j}_{A_j} 
\end{align*}
\]

(4.8)

and the charged scalars,

\[
\begin{align*}
\hat{\Sigma}^{G\pm G\pm}(p^2) &= \Sigma^{G\pm G\pm}(p^2) + \delta M^2_{G\pm} - p^2 \delta Z^{G\pm} \\
\hat{\Sigma}^{G\pm H\pm}(p^2) &= \Sigma^{G\pm H\pm}(p^2) + \delta M^2_{H\pm} - \frac{1}{2}p^2 \delta Z^{G\pm H\pm} - \frac{1}{2}(p^2 - M^2_{H\pm}) \delta Z^{H\pm G\pm} \\
\hat{\Sigma}^{H\pm H\pm}(p^2) &= \Sigma^{H\pm H\pm}(p^2) + \delta M^2_{H\pm} - (p^2 - M^2_{H\pm}) \delta Z^{H\pm}
\end{align*}
\]

(4.9)

The aim now is to determine all the counterterms entering these expressions. Note that the Goldstones will mix and we have singled out their appearance. Recall that the Goldstone bosons are not physical, they cannot appear in initial or final states of a physical process. Thus, we do not need to renormalise their wave function and we can set \(\delta Z^G = \delta Z^{G\pm} = 0\). Moreover, \(\delta Z^{A_i\phi_i}\) and \(\delta Z^{H\pm G\pm}\) can also be set to 0 since they also correspond to transitions where a Goldstone boson is on an external leg.

### 4.3 \(H^\pm W^\pm\) and \(A^0_{1,2}Z^0\) transitions

At tree-level the gauge fixing eliminates mixing between the gauge bosons and their corresponding Goldstone bosons therefore compensating for such a mixing that emerges from the gauge sector, \(\mathcal{L}^{GV}\) (the gauge covariant kinetic term of the Higgs fields). We follow an approach where the gauge-fixing is unrenormalised. As a result of shifting (both fields and parameters) in \(\mathcal{L}^{GV}\), the massive gauge bosons and the pseudoscalars (as well as the Goldstones) will mix. Gauge invariance relates the gauge-pseudoscalar transitions and the corresponding Goldstone-pseudoscalar transition which we studied previously. Therefore we need to consider these transitions.

To get the remaining counterterms involving Goldstones, \(\delta Z^{G\phi, A_i}\) and \(\delta Z^{G\pm H\pm}\), one also has to deal with new transitions between gauge bosons and CP-odd or charged Higgses. The expansion of the covariant derivative in the kinetic part of the scalar Lagrangian gives the following interaction terms,

\[
\mathcal{L}^{GV} = M_Z \left( c_\beta \partial^\mu a^0_\mu - s_\beta \partial^\mu a^0_\mu \right) Z^0_{\mu} - M_W \left( i \left( c_\beta \partial^\mu h^-_d - s_\beta \partial^\mu h^-_u \right) W^+_{\mu} + h.c. \right) \tag{4.10}
\]

At tree-level the combination of the scalar fields makes up the Goldstone bosons, the first component of the corresponding Higgs fields, namely

\[
\begin{align*}
c_\beta a^0_d - s_\beta a^0_u &\equiv (P_a a^0)_1 = (\mathcal{P}^0)_1 = G^0 \\
c_\beta h^-_d - s_\beta h^-_u &\equiv (U(\beta) h^-)_1 = (\mathcal{H}^-)_1 = G^-
\end{align*}
\]

(4.11)

Take for example, the case of the pseudoscalar/neutral Goldstones. Before applying the wave function renormalisation, the shifts amount to

\[
(P_a a^0)_1 \rightarrow ((\delta P_a) a^0)_1 = ((\delta P_a) P_a^{-1} \mathcal{P}^0)_1 = ((\delta P_a) P_a^{-1})_1 (\mathcal{P}^0)_i \tag{4.12}
\]

14
This shift alone, prior to wave function renormalisation will introduce \( A^0Z^0 \) transitions, but not \( G^0Z^0 \). This is easy to see. With \( P_a = \tilde{P}^{(3)}_a U(\beta)^{(3)} \) and using the fact that \( \langle \tilde{P}^{(3)}_a \rangle_{1i} = \delta_{1i} \) allows to write

\[
\sum_{i=1}^{3} \left( \langle \delta P_a \rangle P_a^{-1} \right)_{1i} (\mathcal{O}^0)_{1i} = -2 \sum_{i=1}^{3} A_i^0 \frac{\sin^2 \beta}{2} \left( c_{\beta} P_{a,(i+1)2} + s_{\beta} P_{a,(i+1)1} \right) \delta t_{\beta} \frac{1}{t_{\beta}}
\]

\[
= -2 \sum_{i=1}^{2} A_i^0 \frac{\sin^2 \beta}{2} \tilde{P}_{a,ii} \delta t_{\beta} \frac{1}{t_{\beta}}
\]

(4.13)

Including the wave function renormalisation, we obtain

\[
\delta \mathcal{L}^{GV}_{\text{neutral}} = \frac{M_Z}{2} \left\{ \left( \delta Z_{ZZ} + \delta Z_{G^0} + \frac{\delta M_Z^2}{M_Z^2} \right) \partial^\mu G^0 Z^0_\mu + \delta Z_{Z^0} \partial^\mu G^0 A^0_\mu + \sum_{i=1}^{2} \left( \delta Z_{G^0 A^0_i} - s_{2\beta} \tilde{P}_{a,ii} \delta t_{\beta} \frac{1}{t_{\beta}} \right) \partial^\mu A^0_i Z^0_\mu \right\}
\]

(4.14)

The first line of the equation above is the usual SM term. However, there remains non-vanishing transitions between pseudoscalars and the \( Z^0 \) boson, leading to the following self-energies:

\[
\tilde{\Sigma}_{A^0 Z^0}(p^2) = \Sigma_{A^0 Z^0}(p^2) + \frac{M_Z}{2} \left( \delta Z_{G^0 A^0_i} - s_{2\beta} \tilde{P}_{a,ii} \delta t_{\beta} \frac{1}{t_{\beta}} \right).
\]

(4.15)

Following the same steps, the transition between the charged Higgs and the \( W^\pm \) boson is given by

\[
\tilde{\Sigma}_{H^\pm W^\pm}(p^2) = \Sigma_{H^\pm W^\pm}(p^2) + \frac{M_W}{2} \left( \delta Z_{G^0 H^\pm} - s_{2\beta} \tilde{P}_{a,ii} \delta t_{\beta} \frac{1}{t_{\beta}} \right).
\]

(4.16)

In the linear gauge we have used, there is a simple Ward identity which was derived in [45]. It can be readily extended to the NMSSM using the unitarity properties of the matrix \( \tilde{P}_a \). The starting point is the BRST variation on the correlator between the \( Z \)-boson ghost and the pseudoscalar \( \langle 0 | \bar{c}_Z(x) A_i^0(y) | 0 \rangle \). This induces an identity which sets the following strong constraints

\[
p^2 \tilde{\Sigma}_{H^\pm W^\pm}(p^2) + M_{W^\pm} \tilde{\Sigma}_{H^\pm G^\pm}(p^2) = -\frac{M_W}{2} \left( p^2 - M^2_{H^\pm} \right) \left( \delta Z_{H^\pm G^\pm} + s_{2\beta} \frac{\delta t_{\beta}}{t_{\beta}} \right)
\]

\[
p^2 \tilde{\Sigma}_{A^0 Z^0}(p^2) + M_{Z^0} \tilde{\Sigma}_{A^0 G^0}(p^2) = -\frac{M_{Z^0}}{2} \left( p^2 - M^2_{A^0} \right) \left( \delta Z_{A^0 G^0} + s_{2\beta} \frac{\delta t_{\beta}}{t_{\beta}} \right)
\]

where

\[
\mathcal{F}^\pm(p^2) = \frac{\alpha}{8\pi s_W^2} \sum_{i=1}^{3} \left( c_{2\beta} S_{h,i1} S_{h,i2} + \frac{s_{2\beta}}{2} (S_{h,i2}^2 - S_{h,i1}^2) \right) B_0(p^2, M_W^2, M^2_{h^0})
\]

(4.17)

\[
\mathcal{F}^0(p^2) = \frac{\alpha}{2\pi s_{2W}} \tilde{P}_{a,ii} \sum_{i=1}^{3} \left( c_{2\beta} S_{h,i1} S_{h,i2} + \frac{s_{2\beta}}{2} (S_{h,i2}^2 - S_{h,i1}^2) \right) B_0(p^2, M_Z^2, M^2_{h^0})
\]

(4.18)
with \( B_0(p^2, M^2_{V}, M^2_{A}) \) the scalar two point function \([61]\) and \( V = W, Z \).

The importance of these identities is that the \( H^\pm G^\pm \) and \( H^\pm W^\pm \) transitions (and their neutral counterparts) are not independent. In particular we will be interested in setting an on-shell renormalisation scheme whereby, on the mass-shell, a transition between the charged Higgs and the charged Goldstone, and any of the neutral pseudoscalars and the neutral Goldstone boson, does not occur at one-loop. In doing so, the identification that is made for these states at tree-level is maintained. The previous identities show that at \( p^2 = M^2_{H^\pm} \) and \( p^2 = M^2_{A_0} \), transitions between these physical scalars and the corresponding gauge bosons do not occur either. It means that one can simultaneously set

\[
\hat{\Sigma}^{H^\pm W^\pm}(M^2_{H^\pm}) = \hat{\Sigma}^{H^\pm G^\pm}(M^2_{H^\pm}) = 0, \\
\hat{\Sigma}^{A_0^i Z^0}(M^2_{A_0^i}) = \hat{\Sigma}^{A_0^i G^0}(M^2_{A_0^i}) = 0
\]

(4.19)

From Eqs (4.15, 4.16, 4.19) we derive

\[
\delta Z_{G^\pm H^\pm} = s_{23} \frac{\delta t_\beta}{t_\beta} - \frac{2}{M_{W^\pm}} \Sigma_{H^\pm W^\pm}(M^2_{H^\pm}) \\
\delta Z_{G^0 A_0^i} = s_{23} \tilde{P}_{a,i} \frac{\delta t_\beta}{t_\beta} - \frac{2}{M_{Z}} \Sigma_{A_0^i Z^0}(M^2_{A_0^i})
\]

(4.20)

The Higgs masses which appear as arguments of the two-point function in the equations above are taken as the tree-level masses in order to be consistent with a fully one-loop treatment. These equations allow to fix all wave function renormalisation constants pertaining to the Goldstone bosons, this will then leave us to deal with the system of the physical Higgses: the charged Higgs, the pair of pseudoscalars \( A_0^i \) and the three CP-even neutral Higgses \( h_0^i \). Likewise with the gauge bosons whose renormalisation goes now exactly the same way as the renormalisation of the gauge sector within the SM. For the latter we follow the on-shell scheme adopted in \([60]\). Note that although this procedure permits to decouple the Goldstones from the physical fields, to fully determine the values of \( \delta Z_{G^\pm H^\pm} \) and \( \delta Z_{G^0 A_0^i} \) we still need to define a renormalisation condition on \( \delta t_\beta \). This will need an input from the physical Higgses to which we now turn.

### 4.4 Renormalisation conditions from the Higgs self-energies

With the Goldstone bosons now set aside, the renormalised self energies of the Higgses in Eqs. (4.7, 4.8 and 4.9) take the same form, allowing for one-loop transitions between them. Again, we require that the mixing between any two particles of the same CP parity must vanish at the mass of any physical particle (on-shell condition). The mass is defined as the pole mass of the real part of the renormalised inversed propagator. In case of mixing this requires solving a \( 3 \times 3 \) (for the CP-even) and \( 2 \times 2 \) (for the CP-odd) Higgses system of an inverse propagator. At one-loop, these equations are linearised (see \([45]\)). In this case, starting with a tree-level Higgs mass, \( M_{i,\text{tree}} \) (\( i \) generically denotes \( h_0^{i,1,2,3}, A_0^{i,1,2}, H^{\pm} \)), the corrected one-loop mass is the solution of the equation

\[
p^2 - M_{i,\text{tree}}^2 - \text{Re} \tilde{\Sigma}_i(p^2) = 0 \quad p^2 = M_{i,\text{1loop}}^2,
\]

(4.21)
which, in the one-loop approximation, reads

\[ M_{i,1\text{loop}}^2 = M_{i,\text{tree}}^2 + \text{Re} \hat{\Sigma}_{ii}(M_{i,\text{tree}}^2) = M_{i,\text{tree}}^2 + \delta M_i^2 + \text{Re} \hat{\Sigma}_{ii}(M_{i,\text{tree}}^2) \] (4.22)

The above equation can be used in two ways. If all counterterms entering in \( \delta M_i^2 \) have been fixed (and hence are known), the above equation calculates the finite correction to the tree-level mass. The ultraviolet finiteness of the corrected mass is a very powerful check on the implementation of the one-loop set-up. We may also use one or some of the masses of the Higgses as input parameters in order to solve for one or some of the counterterms to the underlying parameters that enter in \( \delta M_i^2 \). In this case

\[ M_{i,1\text{loop}}^2 = M_{i,\text{tree}}^2 = M_{i,\text{input}}^2 \rightarrow \delta M_i^2 = -\text{Re} \hat{\Sigma}_{ii}(M_{i,\text{tree}}^2) \] (4.23)

For instance taking the charged Higgs mass \( M_{H^\pm} \) as an input parameter gives

\[ \delta M_{H^\pm}^2 = -\Sigma_{H^\pm H^\pm}(M_{H^\pm}^2) \] (4.24)

We also impose that the residue of the propagators for an on-shell physical field be equal to one such that

\[ \text{Re} \hat{\Sigma}_{ii}'(M_{i,\text{tree}}^2) = 1 \quad \text{with} \quad \frac{\partial \hat{\Sigma}_{ii}'(p^2)}{\partial p^2} = \hat{\Sigma}_{ii}'(p^2) \] (4.25)

which then fixes the diagonal entries of the wave function renormalisation constants such that

\[ \delta Z_i = \text{Re} \Sigma_{ii}^\prime(M_{i,\text{tree}}^2) \quad \text{(for example} \quad \delta Z_{h_0} = \text{Re} \Sigma_{h_0 h_0}^\prime(M_{h_0}^2) \text{)} \] (4.26)

We also impose that no mixing occurs between two same-parity field when any of them is on-shell. This condition translates into

\[ \text{Re} \hat{\Sigma}_{ij}'(M_{i,\text{tree}}^2) = \text{Re} \hat{\Sigma}_{ij}'(M_{j,\text{tree}}^2) = 0 \quad \text{for} \quad i \neq j \] (4.27)

which then fixes the non-diagonal elements of the wave-function renormalisation matrices such that

\[ \delta Z_{ij} = \frac{2 \text{Re} \Sigma_{ij}(M_{j,\text{tree}}^2) + \delta M_{ij}^2}{M_{j,\text{tree}}^2 - M_{i,\text{tree}}^2} \quad i \neq j. \] (4.28)

An example of the latter is

\[ \delta Z_{h_0 h_0} = \frac{2 \text{Re} \Sigma_{h_0 h_0}(M_{h_0}^2) + \delta M_{h_0}^2}{M_{h_0}^2 - M_{h_0}^2} \quad i \neq j \] (4.29)

### 5 Renormalisation schemes

The definition of the underlying counterterms involve solving a system of coupled equations which, moreover, depends crucially on the choice of the input parameters, for example which set of physical masses or other observables one chooses as input. In this respect, the wave-function renormalisation constants are somehow easy to evaluate. They
involve a one-to-one relation and are independent from each other. Their expression is independent of the scheme.

We have also already specified, see Section 2.2 how the SM parameters, \(g, g', v \leftrightarrow M_W, M_Z, e\) that enter also in the NMSSM, are renormalised.

The reconstruction of the counterterms of the 9 underlying parameters of the Higgs sector in Eq. 2.40 is more complicated. Indeed, most of these parameters contribute to more than one Higgs mass (and chargino/neutralino mass) or Higgs observable. Apart from the tadpoles, defined from Eq. 3.11, it is not obvious what the optimal set of the 9 input parameters should be in order to reconstruct these underlying parameters. Leaving the tadpole aside, there remains 6 parameters to determine in the Higgs sector. In principle, it is possible to use only masses as inputs since the Higgs sector does furnish 6 different Higgs masses, \(h^0_{1,2,3}, A^0_{1,2}, H^\pm\). Technically, this requires the computation of the self-energies. Note that four of these parameters, \(t_\beta, \lambda, \mu, \kappa\), could also just as well be determined from the neutralino/chargino sector. However, as we argued in detail in [44], one cannot solve for the latter 4 parameters alone since the chargino/neutralino system involves also the underlying parameters \(M_1, M_2\) (the \(U(1)\) and \(SU(2)\) gaugino soft masses).

The two parameters \(A_\lambda\) and \(A_\kappa\) can only be defined in the Higgs sector. The CP-even and CP-odd masses depend on both these parameters, while the charged Higgs mass depends on \(A_\lambda\) only. Because of the intrinsic interdependence of the NMSSM observables related to the Higgs sector (and the corresponding neutralino/chargino sector), in all generality we need to consider a system of counterterms for the 8 underlying parameters which in a vector notation reads

\[
p = \begin{pmatrix}
t_\beta, \lambda, \kappa(m_\kappa), \mu, A_\lambda, A_\kappa, M_1, M_2 \\
in \chi \text{ and Higgs sectors} & \text{Higgs} & \bar{\chi}
\end{pmatrix},
\]

(5.1)

for which we need to select 8 input parameters with at least two from the chargino/neutralino sector and at least two from the Higgs system. Let us recall the procedure and the most important points that we detailed in [44] for reconstructing all 8 counterterms. Injecting 8 input parameters we have to solve for

\[
\begin{pmatrix}
d_\text{input}_1 \\
\vdots \\
d_\text{input}_8
\end{pmatrix} = \mathcal{P}_{8,\text{param}} \cdot \delta p + \mathcal{R}_{8,\text{residual}},
\]

(5.2)

\(\mathcal{P}_{8,\text{param}}\) is a 8 \(\times\) 8 matrix, \(\mathcal{R}_{8,\text{residual}}\) contains other counterterms, such as gauge couplings, that are defined separately. Using the physical mass of one of the Higgs boson as an input, see Eq. 4.24, is a possible choice in an OS scheme. Not all inputs need to be OS. For an efficient resolution of Eq. 5.2, \(i.e.,\) determining the \(\delta p\) vector, one should break up the connecting matrix \(\mathcal{P}_{8,\text{param}}\) in as many, possibly smallest rank, matrices as possible

\[
\mathcal{P}_{8,\text{param}} = \mathcal{P}_{m,\text{param}} \oplus \mathcal{P}_{p,\text{param}} \oplus \cdots, \quad m + p + \cdots = 8
\]

(5.3)

The choice of the input parameters will determine how one can build the \(\mathcal{P}_{8,\text{param}}\) from smaller independent blocks, each choice will define a renormalisation scheme. It is important to seek a scheme where the determinant of each sub-matrix \(\mathcal{P}_{p,\text{param}}\) is not too small (\(\delta p \propto 1/\text{Det.P}\)) in order not to introduce large coefficient that would lead to large
radiative corrections solely from a bad choice of inputs. We will pursue the comparisons between different schemes in the applications to Higgs decays in Section 8.

5.1 Mixed OS-DR schemes

Realising that $t_\beta$ is ubiquitous, it even enters the determination of the wave-function renormalisation matrices of the Higgses, a practical possibility is to take a DR condition for $t_\beta$. In this case, the sectors for the Higgs, the neutralinos and the charginos can be solved independently. The counterterms for $\mu$ and the $SU(2)$ gaugino mass term $M_2$ can be extracted from both chargino masses. Then, the counterterms for the self-interacting singlet coupling $\kappa$ (through $m_\kappa$), the $U(1)$ gaugino mass term $M_1$ and the singlet-doublet coupling $\lambda$ can be obtained from the masses of the neutralinos that should be chosen to represent the mainly singlino, bino and higgsino neutralinos. We have commented at length about the issue of a knowledge of the nature (content) of the neutralinos in [44]. As concerns $A_\lambda, A_\kappa$, a possibility is for example to use both CP-odd Higgs bosons as inputs, $\delta M_{A_0}^2 = \text{Re} \Sigma_{A_0} A_0 A_0 (M_{A_0}^2), i = 1, 2$. This break-up corresponds to

$$\mathcal{P}_{8, \text{param}} = \mathcal{P}_{1, \text{param}} \oplus \mathcal{P}_{2, \text{param}} \oplus \mathcal{P}_{3, \text{param}} \oplus \mathcal{P}_{2, \text{param}}$$

The DR condition for $t_\beta$ in this scheme is an extension of the DCPR scheme[62, 63], used in the context of the MSSM, to the NMSSM[25],

$$\frac{\delta t_\beta}{t_\beta} = \frac{1}{2} (\delta Z_{H_u} - \delta Z_{H_d})|_\infty,$$

where $\delta Z_{H_u}$ and $\delta Z_{H_d}$ are the wave function renormalisation constants of the $H_u$ and $H_d$ doublets. The infinity symbol indicates that we only take the divergent part of the expression. $\delta Z_{H_u}$ and $\delta Z_{H_d}$ are related to the wave function renormalisation constants $\delta Z_{h_i}$ (Eqs. 3.8) through

$$\delta Z_{H_d} = \frac{1}{R} \sum_{i,j,k=1}^{3} \epsilon_{ijk} S_{h_{i,j,3}} S_{h_{i,k,2}} \delta Z_{h_{i,k}}, \quad \delta Z_{H_u} = \frac{1}{R} \sum_{i,j,k=1}^{3} \epsilon_{ijk} S_{h_{i,j,1}} S_{h_{i,k,3}} \delta Z_{h_{i,k}},$$

$$R = - \sum_{i,j,k=1}^{3} \epsilon_{ijk} S_{h_{i,1}}^2 S_{h_{j,2}}^2 S_{h_{k,3}}^2,$$

where $\epsilon_{ijk}$ is the fully antisymmetric rank 3 tensor with $\epsilon_{123} = 1$.

In a DR scheme only the divergent part of the counterterm is defined i.e, any finite term is set to 0. Nonetheless, the scheme and the one-loop result is still not fully defined unless one specifies the renormalisation scale $\bar{\mu}$. The latter is the remnant scale introduced by the regularization procedure, dimensional reduction. This class of renormalisation schemes will be denoted as $t_{ij,k,A_1,A_2}$. The letters $i, j, k$ refer to the neutralinos whose mass has been taken as input. $A_1 A_2$ is a reminder that the masses of the two physical pseudoscalars have been used as input also.

This disentangled scheme is rather simple to implement, but can lead to a poor extraction of $\delta \lambda$, since this parameter is present only in non diagonal entries of the neutralino mass matrix. A solution is to take another Higgs mass as input to get this counterterm, but in this case the neutralino and the Higgs sectors are no longer disassociated.
5.2 Fully on-shell schemes

For these schemes, the set of eight counterterms, including $t_\beta$, are obtained from OS conditions based on inputs taken from physical masses. As for the previous scheme, $\delta \mu$ and $\delta M_k$ will be mainly reconstructed from the two chargino masses but not fully since there is still some mixing introduced by $t_\beta$. In addition, it is natural to take the neutralino that is mainly bino to extract $\delta M_1$. The parameters $\delta \lambda$ and $\delta m_\kappa$ can be extracted either from the neutralino or the Higgs sector. As before, $\delta A_\lambda$ and $\delta A_\kappa$ have to be extracted from two masses from the Higgs sector including at least a mostly singlet Higgs. Finally, it is much better to obtain $\delta t_\beta$ from an additional Higgs input than from the neutralino masses, as was shown in [44]. We are then left with a system of eight equations, whose inversion will give all these counterterms.

Different classes of such (fully) on-shell renormalisation schemes are possible. For instance, one can take one example from the general class where the masses of the two charginos is exploited. This furnishes a system of two equations. On can then have variations on this scheme depending on which source provides the other parameters.

- One scheme could be take the masses of three neutralinos as well the masses of both pseudoscalars Higgses and the mass of the charged Higgs. This choice is referred to as $OS_{ijk\Lambda_1\Lambda_2 H^+}$ where $i, j, k = 1...5$ designates the three chosen neutralinos, usually a bino, a singlino and a higgsino.

- Another choice could be based on the masses of (only) two neutralinos, both pseudoscalars Higgses, the charged Higgs and an additional CP-even Higgs different from the SM-like Higgs. This subclass is labelled as $OS_{ij\hat{i}\Lambda_1\Lambda_2 H^+}$ where $i, j = 1...5$ and $\hat{i} = 1, 2, 3$.

- The third possibility is to use the mass of one neutralino, the bino, and the masses of 5 Higgses, in the obvious notation, $OS_{ih\hat{i}\hat{j}\Lambda_1\Lambda_2 H^+}$ where $i = 1...5$ and $\hat{i}, \hat{j} = 1, 2, 3$.

The numerical examples we will consider are based on the $OS_{ij\hat{i}\Lambda_1\Lambda_2 H^+}$ with an optimal choice for the neutralinos.

6 Checks on the results and tracking the scheme dependence

To check the validity of our numerical results, we perform several tests. The most powerful test consists in checking the absence of ultraviolet divergences on all the observables that we calculate. Ultraviolet divergences appear in many intermediate steps of the calculations and get cancelled out by the counterterms for the underlying parameters and/or among many diagrams. The ultraviolet divergences are encoded in the parameter $C_{UV}$ defined in dimensional reduction as $C_{UV} = 2/\epsilon - \gamma_E + \ln(4\pi)$ where $\epsilon = 4 - d$ ($d$ being the number of dimensions) and $\gamma_E$ is the Euler constant. We systematically check that the numerical results, for one-loop corrections to masses or to decay processes, are independent of $C_{UV}$ by varying this parameter from 0 to $10^7$. We require that the numerical results agree for at least seven digits ($SloopS$ uses double precision). Thus we ensure that physical processes are finite for all renormalisation schemes.
Finally, in schemes where at least one parameter is taken to be DR, a dependence on the renormalisation scale $\bar{\mu}$ remains, this can be used to quantify the scale dependence.

6.1 The $\beta$ functions and the scale dependence

In order to gain a qualitative understanding of the differences in the results for the one-loop corrections to the decay processes we have studied in different schemes, it is interesting to recall some simple arguments related to the counterterms. The infinite ($C_{\text{UV}}$) part of any counterterm is obviously the same regardless of the renormalisation scheme, this is one of the reasons our checks show finiteness for the calculated observable for all schemes. The finite part of the counterterm is however scheme dependent. The difference in this finite part for a particular parameter can be large between two schemes. This difference may get amplified in the computation of an observable that depends strongly on this particular parameter. This parametric dependence that can be derived from the study of the observable at tree-level is therefore an important ingredient also. Take a parameter $p_i$, its counterterm, within some scheme $Q_{p_i}$, reads

$$\delta_{p_i} = \beta_{p_i}(C_{\text{UV}} + \ln(\bar{\mu}^2/Q_{p_i}^2)) \quad (6.1)$$

$\bar{\mu}$ is the scale introduced by dimensional reduction. $\beta_{p_i}$ as defined here is the one-loop $\beta$ constant for the parameter $p_i$. It is scheme independent. $Q_{p_i}^2$ encodes the scheme dependence. In this notation, different schemes correspond to different values of $Q_{p_i}^2$. $Q_{p_i}^2$ is the square of some mass scale which represents both external momenta (corresponding to the choice of the subtraction points) and internal masses typical of two-point functions. All our renormalisation schemes are based on two-point functions. If $Q_{p_i}^2$ is dominated by a mass $m_P$ much larger than all other scales in the problem then $Q_{p_i}^2 \sim m_P$. Note however that these two-point functions can also involve a non-log constant terms, in our definition these non-log terms are lumped into $Q_{p_i}^2$. Within the same scheme, $Q_{p_i}^2$, the difference in the value of the counterterm due to a change in the regularisation scale $\bar{\mu}$ is a measure of $\beta_{p_i}$

$$\Delta_{\bar{\mu}_2-\bar{\mu}_1}\frac{\delta_{p_i}}{p_i} = \delta_{p_i}(\bar{\mu}_2)/p_i - \delta_{p_i}(\bar{\mu}_1)/p_i = \beta_{p_i} \ln(\bar{\mu}_2^2/\bar{\mu}_1^2) \quad (6.2)$$

In our code this is how we determine $\beta_{p_i}$ numerically. In our numerical analysis in Sec. 8 these $\beta$ constants have been checked against the $\beta$ functions given in [4]. Perfect agreement has been found when specialising to the one-loop result. This is a non-trivial check on our renormalisation procedure. For the so-called DR scheme, we set the corresponding counterterm to

$$\delta_{\text{DR}}_{p_i}/p_i = \beta_{p_i} C_{\text{UV}}, \quad (6.3)$$

which in effect corresponds to choosing a scale $Q_{p_i} = \bar{\mu}$.

The one-loop correction to an observable involves calculating all the virtual two-point, three-point, ...n-point functions which are specific for a given amplitude (regardless of the scheme) and then including the counterterms for all parameters on which the amplitude $\mathcal{O}$ depends on, to obtain a finite result. The dependence of the amplitude on a specific parameter $p_i$, the parametric dependence alluded to earlier, is obviously also very important. If at tree-level we slightly change the value of the parameter $p_i$ by an amount $\delta_{p_i}$,
we define the percentage change on the observable as

$$\frac{\delta \mathcal{O}}{\mathcal{O}} = \sum_i \kappa_{p_i} \frac{\delta p_i}{p_i}$$

(6.4)

The sum is over all the *independent* parameters of the model. If an observable is independent of a particular parameter $p_i$ then the corresponding $\kappa_{p_i}$ is $\kappa_{p_i} = 0^1$. In this definition of the parametric dependence we are assuming small, infinitesimal, $\delta p_i$, as is generally the case when $\delta p_i$ stand for one-loop counterterms or else that the dependence in the parameter $p_i$ is linear. With this proviso and to make the discussion simple, if all counterterms are defined on-shell, or at some subtraction point according to Eq. 6.1, then the virtual corrections for the amplitude can be written in a very compact way as

$$\frac{\delta \mathcal{O}}{\mathcal{O}} = \sum_i \beta_{p_i} \kappa_{p_i} \ln \left( \frac{Q^2}{Q_{p_i}} \right)$$

(6.5)

The correction can be large if some $\beta$ constants are large. The corrections could be also amplified if the parametric dependence on the parameter $\kappa_{p_i}$ is large and/or if there is a large difference between some subtraction scale in defining a particular parameter namely $Q_{p_i}^2$ and the scale that defines the observable $Q_\Delta$. $Q_\Delta$ can have contributions not only from two-point functions but also from n-point functions which we have lumped in its definition. In particular our parameterisation of $Q_\Delta$ can take into account non single logarithms (dilogarithms, \cdots). In particular some one loop dynamics may entail large genuine corrections which are then translated here as large values of $Q_\Delta$.

In a scheme where all counterterms are defined \`a la DR,

$$\frac{\delta \mathcal{O}}{\mathcal{O}} = \ln \left( \frac{Q^2}{\bar{\mu}^2} \right) \sum_i \beta_{p_i} \kappa_{p_i}.$$  

(6.6)

There is now a scale dependence which quantifies the uncertainty in the one-loop calculation. The correction is minimised for $\bar{\mu} \sim Q_\Delta$. Again if the virtual corrections are dominated by single logs with an argument corresponding to the largest scale/mass of the process, the corrections are minimised for $\bar{\mu}$ corresponding to this highest scale.

In a mixed-scheme like the one we have taken with $p_0 = t_\beta$ and with $Q_{p_0}$ the effective scale that defines the OS definition of $t_\beta$, the result for the correction to the same observable can be written as

$$\frac{\delta \mathcal{O}^{\text{mixed}}}{\mathcal{O}} = \delta \mathcal{O}^{\text{OS}} / \mathcal{O} + \beta_{p_0} \kappa_{p_0} \ln \left( \frac{Q_{p_0}^2}{\bar{\mu}^2} \right)$$

(6.7)

The $\beta$ functions can also be derived from an analysis of the renormalisation group. The system of coupled equations at 2-loop is given in [4]. Specialising to one-loop and

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1. Strictly speaking the sum applies to each Lorentz structure and/or helicity amplitude.
2. In a scheme where all parameters are defined on-shell according to Eq. 6.1, the numerical extraction of the $\beta$ constants through the $\mu$ variation is quite simple, it relies on the combination $\left(C_{\text{UV}} + \ln \bar{\mu}^2\right)$. In a mixed scheme like the one where $t_\beta$ is DR and the other underlying parameters are reconstructed from solving a coupled system based on on-shell quantities through masses as input, there may be a mismatch between the coefficient multiplying $C_{\text{UV}}$ and $\ln \bar{\mu}^2$. 


keeping only the dominant Yukawa coupling contributions, the system is rather simple and can help understand some features of the full one-loop calculation. With \( h_t \) the top-Yukawa coupling, the dominant contributions to the running of the underlying couplings of the NMSSM can be cast as

\[
\frac{1}{h_t^2} \frac{\mathrm{d} h_t^2}{\mathrm{d} \tau} = \frac{2}{\lambda^2} \frac{\mathrm{d} \lambda^2}{\mathrm{d} \tau} = \frac{2}{\mu^2} \frac{\mathrm{d} \mu^2}{\mathrm{d} \tau} = 6 h_t^2 \quad \text{and} \quad \frac{1}{A_\lambda} \frac{\mathrm{d} A_\lambda}{\mathrm{d} \tau} = 3 h_t^2 \frac{A_t}{A_\lambda}, \tag{6.8}
\]

with \( \tau = \ln \frac{\mu^2}{16 \pi^2} \). What this shows is that if \( A_t \gg A_\lambda \) then \( \beta_{A_\lambda} \) can be quite large.

Remember that a large \( A_t \) is needed for inducing a large one-loop correction to the MSSM-like CP-even Higgs (in the MSSM limit). Note that \( A_\kappa \) and \( s = \mu/\lambda \) do not have top-Yukawa enhanced running.

### 6.2 Infrared divergences

Many of the Higgs processes we will study at the one-loop level, will give rise to infrared divergences when electrically charged particles are involved in the external legs. The treatment of these divergences requires the computation of real photon emission as described in [44]. In a nutshell, for these \( 1 \to 2 \) decays it is sufficient to take an infinitesimally small photon mass as a regulator. For more details see [44].

### 7 The benchmark points

#### Table 1: Parameters for the benchmark point A (in GeV for all dimensionful parameters). \( Q_{\text{susy}} \) is calculated as \( Q_{\text{susy}} = \sqrt{M_{\tilde{t}_1} M_{\tilde{t}_2}} = 1117.25 \text{ GeV} \). The derived values for the tree-level masses of all Higgses, charginos and neutralinos are also given. For this benchmark, the top mass, crucial for the computation of the Higgs mass in particular, is taken as \( M_t = 175 \text{ GeV} \).

| Parameter | Value |
|-----------|-------|
| \( M_1 \) | 700 GeV |
| \( M_2 \) | 1000 GeV |
| \( M_3 \) | 100 GeV |
| \( t_\beta \) | 10 |
| \( \lambda \) | 0.1 |
| \( A_\kappa \) | 0 |
| \( m_{\tilde{Q}_3} \) | 1740 GeV |
| \( m_{\tilde{Q}_{1,2}} \) | 1000 GeV |
| \( m_{\tilde{U}_{3,1}} \) | 1000 GeV |
| \( m_{\tilde{L}_{3,1}} \) | 1000 GeV |
| \( m_{\tilde{D}_{3,1}} \) | 1000 GeV |
| \( m_{\tilde{L}_{1,2}} \) | 1000 GeV |

\((M_{h^+}, M_{A^0}; M_{\chi_1^0}, M_{\chi_2^0}, M_{\chi_3^0}, M_{\chi_4^0}, M_{\chi_5^0}) = (117.95, 1006.61; 112.77, 123.80, 241.57, 702.82, 1006.64) \)

\((M_{h^+}; M_{A^0}; M_{\chi_1^0}; M_{\chi_2^0}; M_{\chi_3^0}) = (577.33; 12.64, 572.06; 88.47, 240.07, 572.48) \)

The one-loop corrected SM-like Higgs mass is calculated to be 125.45 GeV in the \( \overline{\text{MS}} \) scheme and 126.47 GeV in the \( \overline{\text{DR}} \) scheme with a scale at \( Q_{\text{susy}} \).

We will concentrate on two quite distinct scenarios of the NMSSM. The first scenario, Point A, is chosen with a very small value of the mixing parameter \( \lambda \) in order to study the MSSM limit. Point B has a much larger value of \( \lambda \) exhibiting large mixing between the singlet and doublet components.

Point A is defined through practically the same parameters that we chose to set the benchmark Point 3 in our previous work [44] on the renormalisation of the chargino/neutralino.
sector. The defining parameters of Point A are listed in Table 1. An alert reader would have noticed that the difference between Point 3 in [44] and Point A is that the values of the stop masses were modified to ensure that the one-loop corrected mass for the SM-like Higgs is compatible with the value observed at the LHC. Because of the small value of $\lambda$, we are in the MSSM limit which requires to take a fairly large value of the trilinear parameter, $A_t$, in the stop sector. Observe that the value of $A_t$ is very large compared to $A_\lambda$ with $A_t/A_\lambda \sim 27$. This will have important side effects apart from giving large corrections to the lightest CP-even neutral Higgs. We have also listed the value of the one-loop corrected mass for the SM-like Higgs. We see that it is compatible with the mass of the Higgs discovered at the LHC. Table 1 gives the value of this mass in two schemes. The scheme difference is within 1 GeV. For more details on the correction to the Higgs masses and tuned comparisons with other calculations we refer to [64].

For the second benchmark, we borrowed parameters very similar to Point TP4 in [64]. Benchmark B is defined in Table 2.

**Table 2**: Parameters for the benchmark Point B (in GeV for all dimensionful parameters). $Q_{\text{susy}}$ is calculated as $Q_{\text{susy}} = \sqrt{M_{t_1}M_{t_2}} = 753.55$ GeV. The derived values for the tree-level masses of all Higgses, charginos and neutralinos are also given.

| Parameter | Value |
|-----------|-------|
| $M_1$     | 120   |
| $\lambda$ | 0.67  |
| $A_\kappa$| 0     |
| $m_{Q_3}$ | 750   |
| $m_{D_3}$ | 750   |
| $m_{\tilde{\nu}_{1,2}}$ | 1500 |
| $M_2$     | 300   |
| $m_{Q_3}$ | 59.7  |
| $A_\kappa$| 0     |
| $m_{D_3}$ | 1000  |
| $m_{\tilde{\nu}_{1,2}}$ | 1500 |
| $M_3$     | 1500  |
| $a$       | 200   |
| $A_\tilde{\lambda}$ | 405  |
| $t_\beta$ | 1.92  |

($M_{\chi_1^+, \chi_2^+}, M_{\tilde{\chi}_1^0, \tilde{\chi}_2^0}, M_{\tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^0}, M_{\tilde{\chi}_6^0, \tilde{\chi}_7^0, \tilde{\chi}_8^0}, M_{\tilde{\chi}_9^0, \tilde{\chi}_10^0}) = (159.63, 342.70; 89.99, 143.90, 196.18, 235.64, 344.98)
($M_{H^\pm}, M_{\tilde{A}_1^0, \tilde{A}_2^0}, M_{\tilde{H}_1^0, \tilde{H}_2^0, \tilde{H}_3^0, \tilde{H}_4^0}) = (469.09, 111.59, 481.56; 102.92, 142.84, 479.00)

One loop-corrected SM-like Higgs mass is calculated to be 124.44 GeV in the OS$_{12}$4$A_1,A_2,H^+$ and 121.62 GeV in the $\overline{\text{DR}}$ scheme with a scale at $Q_{\text{susy}}$. To calculate the Higgs mass we have taken a running top mass at the scale $Q_{\text{susy}} = \sqrt{M_{t_1}M_{t_2}}$ with $M_t = 146.94$ GeV. One notable difference between the two benchmark points is the value of $\lambda$ (6 times larger for Point B), such that for Point B, $A_\kappa > M_Z$. As a consequence, for Point B the tree-level value for the mass of the SM-like Higgs is larger than $M_Z$ which is not the case for Point A. This is the reason why for Point B the value of $t_\beta$ is $\sim 2$ and more importantly $A_t/A_\lambda \sim 2.5$ only. Still, one needs radiative corrections to lift the mass of the SM-like Higgs from 103 GeV at tree-level to about 125 GeV.

Because we will study Higgs decays either to other Higgses or to neutralinos and charginos, the field content (in terms of the current, unmixed, fields) is very important. The field content or the purity of the physical fields, at tree-level, is given in Table 3. If we arrange the physical fields in terms of their dominant component, then for

Point A

$$ (\tilde{\chi}_1^0, \tilde{\chi}_2^0, \tilde{\chi}_3^0, \tilde{\chi}_4^0, \tilde{\chi}_5^+, \tilde{\chi}_6^+) \sim (\tilde{H}^0, \tilde{H}^0, \tilde{S}^0, \tilde{B}^0, \tilde{W}^0_3, \tilde{H}^+, \tilde{W}^+) $$

$$ (h_1^0, h_2^0, h_3^0, h_4^0, h_5^0, A_1^0, A_2^0) \sim (h_u^0, h_d^0, a_u^0, a_d^0). $$

(7.1)
For Point A the states have a very high degree of purity. Given the fact that $\tilde{\chi}_1^0$ is mostly higgsino, $\tilde{\chi}_3^0$ is mostly singlino and $\tilde{\chi}_4^0$ mostly bino, this justifies to use the $t_{134A_1A_2}$ mixed OS-DR renormalisation scheme, following the notation of Sec. 5.1 and the OS$_{34h_2A_1A_2H+}$ renormalisation scheme, following the notation of Sec. 5.2, to compute the one-loop corrections. Indeed, as discussed in [44], in choosing the input masses, on should preferably include the bino and singlino from the neutralino sector, and the higgsino when a third neutralino is to be used.

For point B, there is strong mixing and only the lightest physical pseudoscalar field can be described as pure, nonetheless we can write the dominant components

\begin{align*}
\text{Point B} \\
(h_1, h_2, h_3; A_1, A_2) &\sim (h_u, h_d, a_s, a_d). 
\end{align*}

We will therefore use the $t_{123A_1A_2}$ and OS$_{12h_2A_1A_2H+}$ renormalisation schemes to compute the radiative corrections.

### Table 3: Components the mass eigenstates for benchmark points A and B. The dominant component is highlighted.

|       | Point A   | Point B   |
|-------|-----------|-----------|
| $h_1^0$ | 1.1%      | 22.5%     |
| $h_2^0$ | 98.6%     | 67.4%     |
| $h_3^0$ | 0.3%      | 10.1%     |
| $h_4^0$ | 0.1%      | 0.0%      |
| $h_5^0$ | 99.6%     | 87.5%     |
| $h_6^0$ | 98.8%     | 77.5%     |
| $h_7^0$ | 1.1%      | 19.7%     |
| $h_8^0$ | 0.1%      | 2.8%      |
| $A_1^0$ | 0%        | 1.8%      |
| $a_1^0$ | 0%        | 0.5%      |
| $a_2^0$ | 100%      | 97.7%     |

|       | Point A   | Point B   |
|-------|-----------|-----------|
| $h_1^0$ | 56.6%     | -         |
| $h_2^0$ | 32.3%     | -         |
| $h_3^0$ | 98.4%     | 10.3%     |
| $h_4^0$ | 0.77%     | 0.8%      |
| $h_5^0$ | -         | 4.0%      |
| $h_6^0$ | -         | 2.6%      |
| $h_7^0$ | 99.5%     | 19.3%     |
| $h_8^0$ | -         | 74.0%     |
| $A_1^0$ | -         | 10.1%     |
| $a_1^0$ | -         | -         |
| $a_2^0$ | 0.9%      | 78.9%     |
| $a_3^0$ | 99.1%     | 11.0%     |
| $A_2^0$ | 99.6%     | 18.1%     |
| $a_2^0$ | -         | 12.3%     |
| $a_3^0$ | 0.69%     | 55.8%     |
| $a_4^0$ | -         | 13.7%     |
| $A_3^0$ | 99.3%     | 11.2%     |
| $a_4^0$ | 0.0%      | 35.7%     |
| $a_5^0$ | -         | 0.4%      |
8 Higgs decays

As an application to our set-up for the renormalisation of the Higgs sector and its implementation in SloopS we consider Higgs decays. This covers decays of Higgses into neutralinos and charginos, final states with a single gauge boson as well as decays into lighter Higgses. These channels also serve to test the most critical aspects of the renormalisation of the Higgs sector in the NMSSM. We have not computed decays involving sfermions since the sfermion sector does not introduce much novelty compared to the MSSM, nor did we consider here decays into SM fermions and pairs of gauge bosons. Note that we have computed decays of the neutral Higgs scalars to $\gamma\gamma$ and $Z\gamma$ in an earlier publication[59], however these loop induced decays do not require renormalisation.

The importance of the radiative corrections and the choice of the renormalisation scheme underline the importance of studying the parametric dependence. To gain an understanding, at least qualitatively, of the results of some of the radiative corrections for the most prominent decays of the Higgses, we will first show, for both Points A and B, how the value of the corresponding tree-level partial width changes when one of the underlying parameters is modified around each one of the reference points that define the model. As discussed in section 6.1 this will give us an insight on the parametric dependence and an approximate extraction of the coefficients $\kappa_{p_i}$ (see Eq. 6.4) when specialising to small variations. We will in fact only show the variations of the square of the coupling involved in the decay. This quantity represents the square of the amplitude for the partial width, leaving the phase space factor out. The rationale for doing this is that a variation of the underlying parameters changes also the values of the masses which in turn change the phase space and hence introduce another source of change in the partial width. In the renormalisation process some of the masses of the particles taking part in the process are taken as input parameters with a value fixed at all orders.

Before giving the results for the full one-loop corrections to the decays, we will first extract the universal $\beta_{p_i}$ for each parameter $p_i$ as explained in section 6.1. For each scheme we will also give the value of the finite term (see Eq. 6.1) of the counterterm to the parameter $p_i$

$$\left. \frac{\delta p_i}{p_i} \right|_{\text{finite}} = \beta_{p_i} \ln(\mu^2/Q^2_{p_i})$$

(8.1)

evaluated at a value of $\mu$ which we will specify. For later reference, observe that a large value of $\beta_{p_i}$ will most certainly entail a large value for the finite part of the corresponding counterterm, unless $\mu^2 \sim Q^2_{p_i}$. Remembering our discussion in section 6.1 (see also Eqs. 6.5 and 6.6), the $\beta_{p_i}$ and the finite part of the counterterms in a given scheme, together with what we will have learnt about the parametric dependence, will help gain some understanding of the results of the full one-loop corrections, the scheme dependence. One could also learn whether there may be large genuine corrections that stem from the two and three-point functions or even from the real corrections (brechmstrahlung).

8.1 Point A

For this point we will compute the full electroweak corrections to the partial decay widths of CP-even, CP-odd and charged Higgs into other Higgses and supersymmetric particles.
For the latter, only neutralinos and charginos are kinematically accessible. We will only include the channels for which the branching ratio is above 1% as they are the only potentially relevant ones. Note that the lightest CP-even and CP-odd Higgs decay only into SM particles and that the components of the heavy doublet Higgs ($h_0^3, A_0^2$) also decay mainly into SM particles, in particular $b\bar{b}$. Only the singlet $h_0^2$ decays dominantly in the pair of singlets $A_0^1 A_0^1$. The partial widths of all the channels considered are of the same order at tree-level, about $10^{-2}$ GeV.

### 8.1.1 Tree-level. Parameter dependence on some couplings in Higgs decays

**Figure 1:** Parameter dependence of (the square) of the couplings that enter some important decays which we will study at one-loop. We look at the variation in the parameters $t_\beta, \lambda, m_\kappa, A_\kappa, A_\lambda$ and $\mu$. Plotted is the percentage variation measured from the reference point, defined in Table 1. We allow variations of $\pm 20\%$ for these parameters apart from $A_\kappa$ whose reference value, $A_\kappa = 0$, is varied smoothly up to 40 GeV. The solid (blue) lines represents the $h_0^3 h_0^1$ coupling, the dash-dotted-dotted-dotted (purple) the $A_0^2 Z h_0^1$ coupling, the dotted (green) lines the $h_0^3 \tilde{\chi}_0^1 \tilde{\chi}_0^1$ coupling, the dashed (red) lines the $h_0^2 A_0^1 A_0^1$ coupling, the dash-dotted (turquoise) lines the $A_0^1 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^1$ and the long-dashed (gray) lines the $A_0^2 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^1$ coupling.

As promised, and before going to the loop results, we first look at the parametric dependence of some of the decays we will study. The analysis of the parametric dependence relies on the tree-level behaviour of the observables as the underlying parameters are varied. Results of these variations are shown in Fig. 1. The first general observation is the smooth and almost linear dependence on all the (independent) parameters, for all the coupling across the whole range of the variations, $\pm 20\%$. This can be easily understood if
we recall that this point is characterised by very small mixing \( \lambda \) where the physical states have a high degree of purity, whereby the Higgs states \( h_0, A_0^0 \) and the neutralino \( \chi^0_3 \) are essentially singlet states. With this small \( \lambda \) scenario, it is instructive to subdivide these decays into three classes of decays

\[ i) \] All particles involved in the decay are predominantly singlets, \( h_2^0 \rightarrow A_1^0 A_1^0 \).

\[ ii) \] None of the particle taking part in the decay is singlet-like with characteristics close to the MSSM and with very little dependence on \( \lambda \). Two examples are shown in Fig. 1, \( A_1^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \) and \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \). In the one-loop calculation we will also consider \( h_3^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \).

\[ iii) \] Decays involving both singlets and MSSM like particles. \( h_2^0 \rightarrow h_1^0 h_1^0, A_2^0 \rightarrow Z^0 h_2^0 \)

(and its \( SU(2) \) equivalent \( H^+ \rightarrow W^+ h_2^0 \)) and \( h_3^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_3^0 \). These decays are therefore sensitive to the mixing parameters due to the addition of the singlet in the NMSSM. For most decays this mixing parameter is essentially \( \lambda \) while for \( h_3^0 h_2^0 h_1^0 \) and \( A_2^0 \rightarrow Z^0 h_2^0 A_\lambda \) is also crucial.

We first look at the decays which involve only singlets and then none of them.

- \( h_2^0 \rightarrow A_1^0 A_1^0 \)

Similar to the self-coupling of the three CP-even neutral Higgs singlets in Eq. 2.35, with \( \kappa = 0 \) (at tree-level), the \( h_2^0 A_1^0 A_1^0 \) interaction stems from the term \( \kappa^2 S^4 \) of the Higgs potential, Eq. 2.4. This trilinear coupling is controlled by \( \kappa^2 s \propto m_\kappa^2 / s \propto \lambda / \mu \ m_\kappa^2 \). The relative variation of the square of the coupling

\[
\frac{\Delta G^2_{h_2^0 A_1^0 A_1^0}}{G^2_{h_2^0 A_1^0 A_1^0}} \sim 2 \left( \frac{\Delta \lambda}{\lambda} + 2 \frac{\Delta m_\kappa}{m_\kappa} - \frac{\Delta \mu}{\mu} \right)
\] (8.2)

is well rendered by this simple observation and corresponds very well to the variations shown in Fig. 1. The small \( \Delta A_\kappa \) variation in Fig. 1 can be explained similarly from the Higgs potential. Considering that this coupling is solely within the singlet sector, it is important to stress that the \( \lambda \) dependence here is due to our choice of \( m_\kappa \) (rather than \( \kappa \), \( \lambda \), \( \mu \)) as independent parameters.

- \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) and \( A_2^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \)

\( A_2^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \) is a MSSM-like decay with a very small dependence on the mixing \( \lambda \) scenario and practically independent of all other parameters. \( h_3^0 \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \) shares these same features. \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) is quite similar with however some small dependence that creeps in from the mixing between the neutralinos. Still, as seen from Fig. 1 the \( \lambda \) dependence is four times smaller, \( \Delta \lambda / 2 \lambda \), as compared to the situation when a singlet state is involved in the decay. One can also note a small dependence on \( \mu \) (recall that the lightest neutralino is higgsino-like) as well as some \( t_\beta \) dependence. Overall the parametric dependence for the decay \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) can be approximated as

\[
\frac{\Delta G^2_{A_2^0 A_1^0 A_1^0}}{G^2_{A_2^0 A_1^0 A_1^0}} \sim \frac{1}{2} \left( \frac{\Delta \lambda}{\lambda} + \frac{\Delta \mu}{\mu} - \frac{\Delta m_\kappa}{m_\kappa} - \frac{1}{2} \frac{\Delta t_\beta}{t_\beta} \right)
\] (8.3)
We now turn to the decays which involve a mixture of singlets/doublets states.

- \( h_3^0 \rightarrow h_5^0 h_2^0 \).
  This decay is triggered from the coupling \( h_5^0 h_2^0 h_3^0 \) whose strength is controlled by \( \lambda (A_\lambda + 2m_\kappa) \), see the term (3, 1, 2) in Eq. 2.35. With the values of \( m_\kappa \) and \( A_\lambda \), this dependence gives a relative variation
  \[
  \frac{\Delta G_{h_5^0 h_2^0 h_3^0}^2}{G_{h_5^0 h_2^0 h_3^0}^2} \approx 2 \left( \frac{\Delta \lambda}{\lambda} + \frac{A_\lambda}{A_\lambda + 2m_\kappa} \frac{\Delta A_\lambda}{A_\lambda} + \frac{2m_\kappa}{A_\lambda + 2m_\kappa} \frac{\Delta m_\kappa}{m_\kappa} \right)
  \approx 2 \left( \frac{\Delta \lambda}{\lambda} + 0.38 \frac{\Delta A_\lambda}{A_\lambda} + 0.6 \frac{\Delta m_\kappa}{m_\kappa} \right)
  \] (8.4)
  which is extremely well exhibited in Fig. 1.

- \( h_3^0 \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_3 \).
  This coupling is practically independent of the dimensionful parameters, it is not generated in the Higgs potential. The coupling is essentially dependent only on the mixing \( \lambda \), with a very small \( t_\beta \) dependence, as confirmed by Fig. 1. The variation, for the square of the coupling, can be parameterised as
  \[
  \frac{\Delta G_{h_3^0 \tilde{\chi}^0_1 \tilde{\chi}^0_3}^2}{G_{h_3^0 \tilde{\chi}^0_1 \tilde{\chi}^0_3}^2} \approx 2 \frac{\Delta \lambda}{\lambda}
  \] (8.5)

- \( A_2^0 \rightarrow h_2^0 Z^0 \).
  The coupling responsible for this decay derives in part from \( h_2^0 a_2^0 Z^0 \), therefore the singlet-doublet mixing is an essential ingredient. It is not a surprise that the \( \lambda \) dependence is the same as with all other decays in this class. Like with the Higgs self-couplings the \( A_\lambda \) mixing is not negligible as well as the \( t_\beta \) and \( \mu \) dependence which enter through the mixing in the CP-odd and CP-even Higgs sectors. Apart from the \( \lambda \) dependence, deriving an analytical formula for the relative variation is difficult. Approximately we get,
  \[
  \frac{\Delta G_{A_2^0 h_2^0 Z^0}^2}{G_{A_2^0 h_2^0 Z^0}^2} \approx 2 \left( \frac{\Delta \lambda}{\lambda} - 0.3 \frac{\Delta A_\lambda}{A_\lambda} - \frac{\Delta t_\beta}{t_\beta} + 0.4 \frac{\Delta m_\kappa}{m_\kappa} - 1.2 \frac{\Delta \mu}{\mu} \right)
  \] (8.6)
  This is in good agreement with the behaviour seen in Fig. 1. For later reference, observe also that it is this coupling and the \( h_1^0 h_2^0 h_3^0 \) that are most sensitive to a variation in \( A_\lambda \), albeit with opposite trends.

### 8.1.2 Point A: Finite part of the counterterms and their \( \beta \) constant

The \( \beta_{p_i} \) constants that we extract numerically (see section 6.1) are given in units of \( 10^{-3} \),

\[
\beta_\mu = 5.70 \quad \text{(8.7)}
\]

\[
\beta_{t_\beta} = 8.44 \quad \text{(8.8)}
\]

\[
\beta_\lambda = 5.83 \quad \text{(8.9)}
\]

\[
\beta_{m_\kappa} = 0.510 \quad \text{(8.10)}
\]

\[
\beta_{A_\lambda} = 548.74 \quad \text{(8.11)}
\]
Since $A_κ = 0$, $β_{A_κ}$ is not amenable to a numerical extraction. However we have checked that $δA_κ$ never plays a significant role, we will omit it from our discussion. The most striking observation is that $β_{A_λ}$ is very large, it is practically two orders of magnitude larger than the $β$ of all the other parameters. In sharp contrast, note the tiny $β_{m_κ}$. When we recall that for this point the ratio $A_t/A_λ$ is very large, $A_t/A_λ \sim 27$, these findings are not surprising, see Eqs. 6.8. We therefore expect a large scale dependence in the $\overline{\text{DR}}$ scheme and probably a large correction for those branching ratios that are most sensitive to $A_λ$. A glance at Fig. 1 indicates that $h_0^0 \to h_1^0 h_0^0$ and $A_0^0 \to Zh_0^0$ (and its SU(2) equivalent $H^+ \to W^+ h_0^0$) are two such observables. We have also derived the finite parts of the corresponding counterterms. For this benchmark we take $\bar{μ} = Q_{\text{susy}} = 1117.25$ GeV, see Eq. 8.1. As mentioned in Section 7, to ensure an a priori good extraction of the finite parts we computed these finite parts in the schemes $t_{134A_1A_2}$ and $OS_{34h2A_1A_2H^+}$. The results are given in Table 4.

| Scheme            | $δμ/μ$ | $δt_β/t_β$ | $δλ/λ$ | $δm_κ/m_κ$ | $δA_λ/A_λ$ |
|-------------------|--------|------------|--------|------------|-------------|
| $t_{134A_1A_2}$   | -2.42% | 0          | 62.26% | -0.67%     | -5.49%      |
| $OS_{34h2A_1A_2H^+}$ | -1.57% | -80.69%   | -7.88% | 0.3%       | 134%        |

We note that at the level of the counterterms, the scheme dependence in $μ$ and $m_κ$ is extremely small (less than one per-cent) and that in both schemes the values of these two counterterms are quite small. This is no surprise since $μ$ can be extracted almost directly from one of the chargino masses while $m_κ$ was chosen as an independent parameter precisely because it is an almost direct measure of the singlino mass in this small $λ$ limit. As for the counterterms for $A_λ$, $λ$ and $t_β$ we have large corrections. In the OS scheme $δA_λ/A_λ$ is more than 100% and $δt_β/t_β$ is also large. One would think that the use of $M_{H^0}$ as an input in the OS scheme would have constrained $δA_λ/A_λ$ far better. In fact, as can be derived from the expression of the charged Higgs mass, Eq. 2.19, in the limit of small $λ$ and rather large $t_β$, as is the case here, $δM_{H^0}^2/M_{H^0}^2 \sim (δA_λ/A_λ + 1.8δt_β/t_β)/2$. Therefore it is only the combination $(δA_λ/A_λ + 1.8δt_β/t_β)$ of these two counterterms which is well constrained. This is corroborated by the values of these two counterterms in Table 4. This issue with $t_β$ is similar to the one encountered in the MSSM where the Higgs masses alone are not efficient to reconstruct $t_β$ [45]. The good extraction of $δA_λ/A_λ$ in the $t_{134A_1A_2}$ scheme is therefore a result of the $\overline{\text{DR}}$ condition $δt_β/t_β = 0$. This said, the $t_{134A_1A_2}$ scheme gives a bad reconstruction of $δλ/λ$. As argued in [44], in this small $λ$ limit, the chargino/neutralino masses are not sensitive to $λ$. This leads to a large uncertainty on $δλ/λ$. The Higgs system with the inclusion of the singlet dominated $h_2^0$ in the OS scheme fares better as demonstrated in Table 4.

To summarise, we foresee i) in the $\overline{\text{DR}}$ scheme, a large scale variation for decays and couplings that feature a non negligible dependence on $A_λ$ ($h_3^0 \to h_1^0 h_0^0$, $A_0^0 \to Zh_0^0$ and $H^+ \to W^+ h_0^0$). The correction should minimise for $μ = Q_{\text{susy}}$, ii) in the $t_{134A_1A_2}$ scheme
the corrections should be mainly driven by $\delta \lambda$. Since $\beta_\lambda$ is small, the scale dependence in this mixed scheme is negligible. All decays are affected expect those not involving any singlet state, $\llbracket iii \rrbracket$ in the OS scheme one should pay a special attention to those observables where the $A_\lambda$ dependence is important and to a lesser extent the $t_\beta$ dependence.

8.1.3 Full one-loop results

| tree-level | $t_{134A_1A_2}$ | OS$_{3\text{susy}}$ | DR $Q_M$ | DR $Q_{\text{susy}}$ |
|------------|-----------------|------------------|----------|------------------|
| $h_0^0 \to A_0^0 A_1^0$ | 47.9 | 128% | -12% | 0.4% | -0.4% |
| $h_0^0 \to h_1^0 h_2^0$ | 22.1 | 116% | 79% | 52% | -1.7% |
| $h_0^0 \to \chi_1^0 \chi_3^0$ | 35.2 | 122% | -3% | 2% | 0.3% |
| $h_0^0 \to \tilde{\chi}_1^0 \tilde{\chi}_3^0$ | 33.8 | 126% | -35% | 3% | 1.1% |
| $h_0^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$ | 45.5 | 1% | -11% | -9% | -7.4% |
| $A_0^0 \to Z^0 h_2^0$ | 18.6 | 120% | 80% | -56% | -14.5% |
| $A_2^0 \to \tilde{\chi}_1^0 \chi_1^0$ | 33.0 | 28% | 13% | 0.3% | -1.6% |
| $A_2^0 \to \tilde{\chi}_1^0 \chi_3^0$ | 24.4 | 130% | -31% | 8% | 6.2% |
| $A_2^0 \to \tilde{\chi}_2^0 \chi_3^0$ | 30.2 | 122% | -5% | -0.4% | -1.9% |
| $A_2^0 \to \tilde{\chi}_3^0 \chi_1^0$ | 55.1 | -10% | -1.5% | -6% | -8% |
| $H^+ \to W^+ h_0^0$ | 20.1 | 119% | 79% | -56% | -16% |
| $H^+ \to \tilde{\chi}_1^+ \tilde{\chi}_3^0$ | 64.0 | 125% | -18% | 3% | 1.1% |

Full one-loop results for a variety of Higgs decays are shown in Table 5. For each decay we give the result for the mixed $t_{134A_1A_2}$ scheme, the on-shell scheme OS$_{3\text{susy}}$ and the full DR scheme. For the $t_{134A_1A_2}$ scheme we set the scale at $Q_{\text{susy}} = 1117.25$GeV. For the DR scheme we consider both an implementation with a scale at $Q_{\text{susy}}$ as well with a scale $Q_M$ that corresponds to the mass of the decaying Higgs. For the important decays of $h_0^0, A_0^0, H^+$, $Q_M \sim Q_{\text{susy}}/2$. A quick glance at the table reveals that the corrections in the DR scheme at $Q_{\text{susy}}$ are quite small in practically all channels. The same results at the scale $Q_M$ are within 2% except for the notable decays $h_0^0 \to h_1^0 h_2^0$, $A_0^0 \to Z h_2^0$ and $H^+ \to W^+ h_0^0$ as a consequence of the very large $\beta_{A_\lambda}$. The results in the mixed scheme show a very large and almost common correction of order 120%! (due to the large $\delta \lambda/\lambda \sim 62\%$ in this scheme) except for $h_0^0 \to \chi_1^+ \chi_1^-$, $A_0^0 \to \chi_1^+ \chi_1^-$ and $A_0^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$. The corrections in the OS scheme are small to moderate except for the same notable decays where the scale dependence in the DR scheme is large, $h_0^0 \to h_1^0 h_2^0$, $A_0^0 \to Z h_2^0$ and $H^+ \to W^+ h_0^0$. Following our discussion on the parametric dependence and the values of the counterterms, as well as the $\beta$ functions, these results are easily understood. In fact this is in perfect agreement with the arguments we summarised at the end of the preceding subsection.
The smallness of the radiative corrections in the DR scheme seems to indicate that $\bar{\mu} \sim Q_{\text{susy}}$ is the effective scale for all the decays in this model. This is also indicative that genuine corrections beyond the running of the parameters are quite small. We can be more quantitative about the differences between the schemes and the scale dependencies by combining the parametric dependencies derived from tree-level considerations in Eqs. 8.2-8.6 with the values of the finite parts of the counterterms in Table. 4, that is $\Delta p_i \to \delta p_i$. Let us go through the results of some of the decays.

- $h_2^0 \to A_1^0 A_1^0$
  The scheme dependence of the relative correction to the decay is contained in $2(\delta \lambda / \lambda + 2 \delta m_{\kappa} / m_{\kappa} - \delta \mu / \mu) \sim 2 \delta \lambda / \lambda$. This contribution from the finite part of the counterterms gives practically the full one-loop correction in all the schemes, for instance in the OS scheme this contribution returns a $-11.4\%$ correction by using the values given in Table 4. This is another manifestation that genuine corrections from three-point functions contributions are negligible. The scale dependence for the DR scheme is tiny, indeed with the negligible $\beta_{m_{\kappa}}$ and with $\beta_{\lambda} \sim \beta_{\mu}$ the difference in the correction between the scale $Q_{\text{susy}}$ and $m_{h_2^0}$ (using the general formula of Eq. 6.6 for the amplitude) is $2(\beta_{\lambda} + 2 \beta_{m_{\kappa}} - \beta_{\mu}) \ln(m_{h_2^0}^2 / Q_{\text{susy}}^2) \sim -0.7\%$. This difference is an excellent approximation to the full one-loop result.

- $A_2^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$, $A_2^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$ and $h_3^0 \to \tilde{\chi}_1^+ \tilde{\chi}_1^-$
  We classified these decays in the second category where all particles involved these decays are MSSM-like though with a very small singlet component for the neutralino case. We note that for these decays the scheme dependence is much smaller as compared to the other categories. In particular, these are the only decays where the relative one-loop corrections are under control in the $t_{134A_1A_2}$ scheme. The largest correction in this class shows up for $A_2^0 \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$ in the $t_{134A_1A_2}$ scheme driven essentially by the large value of $\delta \lambda$ despite the small parametric dependence of this parameter. Indeed, Eq. 8.3 when interpreted in terms of counterterms is an excellent explanation of the results we find for the full corrections and the scheme dependence.

- $h_3^0 \to h_2^0 h_0^0$
  Apart from the DR scheme with a scale at $Q_{\text{susy}}$ where the correction is modest, all other schemes lead to large corrections. We verify, based on the parametric dependence in Eq. 8.4 and on the fact that $\beta_{A_{\lambda}} \gg \beta_{\lambda} \gg \beta_{m_{\kappa}}$, that the difference between the scales in DR is indeed given by $2 \times 0.38 \beta_{A_{\lambda}} \ln(m_{h_3^0}^2 / Q_{\text{susy}}^2) \sim -54\%$. The values for the corrections in the OS scheme and the mixed scheme are also very well approximated by the parametric dependence upon replacing the variations by the corresponding counterterms found in Table 4. In the OS scheme the correction is driven essentially by the poor extraction of $A_{\lambda}$ while in the mixed scheme it is again essentially the imprecise input $\delta \lambda / \lambda$ which is behind the large correction.

- $A_2^0 \to Z^0 h_2^0$ and the equivalent $H^+ \to W^- h_2^0$
  Because of the large running $A_{\lambda}$, it is sufficient to consider $A_{\lambda}$ when comparing the results in the DR scheme at the two scales. The parametric dependence, Eq. 8.6, explains very well the $\sim 40\%$ difference between the two scales in the DR scheme. The difference between the OS scheme and the DR at $Q_{\text{susy}}$ is driven essentially
by the determination of $A_\lambda$ and $t_\beta$ both of which are badly derived in the OS scheme, whereas the discrepancy in the mixed scheme comes once again from a bad reconstruction of $\lambda$.

- All the other remaining decays of Table 4, are those where the Higgses (CP-even, CP-odd or charged) are decaying into neutralinos/charginos involving the mostly singlet $\tilde{\chi}_0^3$. These decays would vanish in the $\lambda \to 0$ limit. We studied the parametric dependence of a representative of these decays earlier, $h^0_3 \to \tilde{\chi}^0_1\tilde{\chi}^0_3$. We verified the strong $\lambda$ dependence, and noted a small $t_\beta$ dependence (which takes place in the neutralino/chargino sector). Translated in terms of counterterms, the parametric dependence is $2\delta\lambda/\lambda$ which again explains extremely well the almost uniform large correction, $\sim 120\%$, in the $t_{134A_1A_2}$ scheme. In the OS scheme, the corrections are much smaller but the residual $t_\beta$ dependence in some of these decays is not totally negligible.

Point A is somehow pathological in the sense that it is MSSM-like and the amount of mixing is small. This makes it difficult to reconstruct all the parameters rather precisely. The OS scheme would perform well if it were not penalised by a very imprecise reconstruction of $t_\beta$ which impacts badly on the reconstruction of $A_\lambda$. The mass of the charged Higgs as an input only constrains a very specific combination of these two parameters. The MSSM is fraught with the same problem of a reconstruction of $t_\beta$ from the Higgs masses alone. We have shown that in the MSSM a very good scheme for extracting $t_\beta$ relied on the decay $A^0 \to \tau^+\tau^-$ [45]. In the NMSSM, this issue needs to be investigated in depth and is left for a future work.

### 8.2 Point B

The most crucial features to keep in mind when reviewing the results for Point B, especially after what we have seen for Point A, is the fact that $\lambda$ is large (and $t_\beta$ small) and $A_\lambda$ is smaller than $A_1$ by only a factor of 2. The last observation should mean that $\beta_{A_\lambda}$ should not be excessively large. With such value of $\lambda$ (and $A_\lambda$) this point constitute a genuine example of the NMSSM, the branching ratios for the decays we consider here are at least an order of magnitude larger than in Point A. The difficulty now is that the notion of an almost singlet (and MSSM-like) state will be lost, couplings between the physical states will depend strongly on the pattern of the mixing matrices. The study of the parametric dependence of the couplings, and hence the decays, on the underlying parameters is here even more important. This is what we look at, at tree-level, for a few decays for which we have calculated the full one-loop corrections.

#### 8.2.1 Tree-level. Parameter dependence on some couplings

Fig. 2 shows the variations of some couplings when some of the underlying parameters are perturbed within $\pm 20\%$. For Point B we have also examined variations in $M_1$ and $M_2$ which we do not show in Fig. 2 to avoid clutter. The most striking difference with Point A is the fact that for the tri-linear Higgs couplings, $h^0_3h^0_1h^0_2$, $h^0_3h^0_2h^0_2$ and $h^0_1A^0_1A^0_2$, the dependence on almost all parameters is large and highly non linear even for parametric variations of about 10%. For example a 10% change in $A_\lambda$ around its reference value gives a variation of almost 100% on the tri-linear Higgs couplings. Recall here that $t_\beta$ is small,
Figure 2: Parameter dependence on (the square) of the couplings that enter some important decays which we will study at one-loop. We look at the variation in the parameters $t_\beta, \lambda, m_\kappa, A_\kappa, A_\lambda$ and $\mu$. Plotted is the percentage variation measured from the reference point. We allow variations of $\pm 20\%$ for these parameters apart from $A_\kappa$ whose reference value, $A_\kappa = 0$, is varied smoothly up to 40 GeV. The solid (blue) lines represents the $h_0^0 h_0^0 h_1^0$ coupling, the long dash-dotted (purple) the $H^+ W^- A_1^0$ coupling, the dotted (green) line the $A_2^0 A_1^0 h_1^0$ coupling, the dashed (red) line the $h_0^0 h_0^0 h_2^0$ coupling and the dash-dotted (turquoise) line the $A_2^0 \tilde{\chi}_1^+ \tilde{\chi}_1^-$. 

but we can see that small changes in this parameter give rise to dramatic changes in the tri-linear coupling. Decays involving two Higgses and a gauge boson, one such coupling is $A_1^0 H^+ W^-$, show also large deviations though not as dramatic as for couplings involving three Higgses. Decays of the Higgs into neutralinos/charginos such as $A_2^0 \to \chi_1^+ \chi_1^-$ show much more moderate variations with practically an almost linear dependence. Although not shown in these figures, for decays into charginos and neutralinos the effect of a change in the gaugino masses $M_1, M_2$ is not totally negligible. For most decays the parameters that lead to the largest changes are $A_\lambda, \lambda, \mu$ and $t_\beta$. Considering the highly non-linear behaviour of these variations, a linear parametrisation is justified only for small variations. We will therefore first check whether the finite parts of the counterterms in this model are small enough. If so we will give the parameterisation as befits a one-loop correction where the counterterms enter only at first order.

In view of these preliminary observations, the 2-body decays studied here can be classified into 3 categories depending on how many Higgses are taking part in the decay:

- a single Higgs for decays into neutralinos and charginos, the amplitude will then involve the matrix $S_h$. 

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• two Higgses and a gauge boson where in this case the Higgses are of opposite parity hence involving the product of the diagonalising matrices $S_h \times P_a$.

• three Higgses where in the case of three neutral CP-even Higgs the elements of $(S_h)^3$ enter and in the case of a CP-even Higgs decaying into 2 pseudoscalars the amplitudes call for $S_h \times P_a^2$.

The dependence of these mixing matrices on the underlying parameters, in particular $\lambda, A_\lambda$, is highly non-linear when $\lambda$ is not small. In the latter case one expect that when more and more Higgses are involved, like for instance in the case of the Higgs self-couplings, the mixing matrices introduce highly non-trivial dependencies.

8.2.2 Point B: Finite part of the counterterms and their $\beta$ constant

We first list the $\beta$ functions, in units of $10^{-3}$ they are

\[
\begin{align*}
\beta_{\mu} &= 7.13 \\
\beta_{t} &= -8.81 \\
\beta_{\lambda} &= 10.22 \\
\beta_{m_\kappa} &= 6.19 \\
\beta_{A_{\lambda}} &= 61.36
\end{align*}
\]

As expected, since the ratio $A_t/A_\lambda$ has decreased by about a factor 10 compared to model A, the value of $\beta_{A_{\lambda}}$ has decreased by almost an order of magnitude, see Eq. 6.8. Yet, this is still the largest $\beta$ constant (6 % rather than 1% for the others).

| Scheme | $\delta \mu/\mu$ | $\delta t_\beta/t_\beta$ | $\delta \lambda/\lambda$ | $\delta m_\kappa/m_\kappa$ | $\delta A_{\lambda}/A_{\lambda}$ |
|--------|------------------|---------------------------|--------------------------|-----------------------------|-----------------------------|
| $t_{123A_1A_2}$ | -1.04% | 0 | 3.71% | -1.5% | 6.85% |
| $OS_{12h_2A_1A_2H^+}$ | -1.63% | 6.50% | 5.94 | -1.52% | 3.40% |

The finite part of the counterterms (evaluated at $Q_{\text{susy}} \sim 754$ GeV) that we extract are given in Table 6. All counterterms are of the same order, none exceeding 7% and in any case they are much smaller than some of the large values we found for Point A. This rather precise extraction has to do with the fact that, because of the not so small mixing, a large number of observables, in particular masses, are quite sensitive to the underlying parameters. The fact that these values are not very large justifies parameterising the variation at first order in the counterterm. The first derivative of the variation, at the origin, gives the infinitesimal parameterisation.

8.2.3 One loop results and analysis of scheme dependence.

A quick inspection of the results in Table 7 reveals that, if we leave out the decays of the category involving the tri-linear Higgs couplings, in particular the CP-even $h_3^0 \rightarrow$
DR schemes at two scales. The scheme dependence is quite small for all corrections for the problematic decays \( h^0 \rightarrow h^0 h^0, h^0 h^0 \), the radiative corrections are moderate, especially compared to Point A. Overall, the OS scheme performs quite well. In particular, the OS scheme returns the smallest observed at tree-level on the Higgs trilinear couplings. The differences between schemes are within about 10%, the smallest corrections are usually obtained in the mixed scheme where only \( t_\beta \) is taken \( \overline{\text{DR}} \) \( t_{123A_1 A_2} \) at the scale \( Q_{\text{susy}} \), the full OS scheme, and the full \( \text{DR} \) schemes at two scales. \( Q_M \) is taken to be the mass of the decaying particle.

| Tree-level \((\text{GeV})\) | \( t_{123A_1 A_2} \) | OS \( t_{12h_2 A_1 A_2 H^+} \) | \( \text{DR} \) \( Q_M \) | \( \overline{\text{DR}} Q_{\text{susy}} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( h_3^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0 \) | 0.726 | 13.3% | 14% | 5% | 3% |
| \( h_3^0 \rightarrow A_1^0 Z_1^0 \) | 0.613 | 13% | 3% | -3% | 8% |
| \( h_3^0 \rightarrow h_2^0 h_1^0 \) | 0.341 | -142% | -25% | -106% | -50% |
| \( h_3^0 \rightarrow h_2^0 h_1^0 \) | 0.514 | 51% | 6% | 13% | -28% |
| \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) | 1.523 | 9% | 7% | 2% | 1% |
| \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) | 0.723 | 19% | 32% | 2% | 2% |
| \( A_2^0 \rightarrow Z_1^0 h_2^0 \) | 0.638 | -10% | 12% | -16% | -9% |
| \( A_2^0 \rightarrow A_1^0 h_2^0 \) | 0.415 | -43% | -0.3% | -32% | -17% |
| \( H^+ \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) | 1.056 | 10% | 6% | 10% | 8% |
| \( H^+ \rightarrow W^+ h_2^0 \) | 0.609 | -11% | 11% | -18% | -10% |
| \( H^+ \rightarrow W^+ A_1^0 \) | 0.603 | 12% | 2% | -3% | 9% |
| \( H^+ \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) | 0.561 | 14% | 21% | 9% | 9% |

Decays into charginos and neutralinos \( (A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0, A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0) \)

First observe that the scale dependence in these decays is never larger than 2%. This is due in a large part to the fact that the couplings involved in these decays are insensitive to \( A_\lambda \), a parameter which comes with the largest \( \beta \). The \( \beta \) constants for the other parameters are all smaller than 1%. Besides, as Fig. 2 confirms for \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \), the parametric dependence is quite small for these decays compared to those where more than one Higgs is involved. We can write

\[
\frac{\delta \Gamma_{A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0}}{\Gamma_{A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0}} \sim 0.67 \frac{\delta \lambda}{\lambda} + 0.55 \frac{\delta \mu}{\mu} - 0.3 \frac{\delta t_\beta}{t_\beta} \tag{8.17}
\]

The differences between schemes are within about 10%, the smallest corrections are usually obtained in the \( \overline{\text{DR}} \) scheme. The scheme dependence is quite small for all the decays in this category apart from the special case of \( A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \). \( A_2^0 \) here is bino-dominated but with a large wino and higgsino component. In this case we have worked out the parametric dependence including the \( M_1 \) and \( M_2 \) counterterms

\[
\frac{\delta \Gamma_{A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0}}{\Gamma_{A_2^0 \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0}} \sim 1.1 \frac{\delta \lambda}{\lambda} - 2.4 \frac{\delta \mu}{\mu} + 0.9 \frac{\delta t_\beta}{t_\beta} + 2.6 \frac{\delta M_1}{M_1} - 0.9 \frac{\delta M_2}{M_2} - 0.9 \frac{\delta m_\kappa}{m_\kappa} \tag{8.18}
\]
The \( \sim 10\% \) difference between the \( t_{12A_1,A_2} \) and the OS scheme is essentially due to the \( t_{12} \) definition in the two schemes. The relatively large correction of 32% in the OS scheme is in fact due to the addition of many smaller contributions including \( M_1, M_2 \) (and \( \mu \)) which all affect the neutralino sector.

- Higgs decays into a vector boson and another Higgs (\( A_2 \to Z^0 h_2^0, H^+ \to W^+ h_2^0, H^+ \to W^+ A_1^0, h_3^0 \to A_1^0 Z^0 \))

The pattern of the corrections for these decays is quite similar. In all the schemes, the one-loop corrections are moderate. They are, in absolute terms, within 20% with a scheme difference that can attain 30%. In \( \overline{\text{DR}} \) the scale dependence is about 6% when we compare the values obtained at \( Q_{\text{susy}} \sim 754 \text{ GeV} \) and at a scale about the mass of the decaying Higgs \( \sim 480 \text{ GeV} \). For \( H^+ \to W^+ A_1^0 \), this is accounted for by the \( A_\lambda \) running \( \kappa_{A_\lambda}^{HW A} A_{A_\lambda} \ln(Q_{\text{susy}}^2/M_{H^\pm}^2) \) with \( \kappa_{A_\lambda}^{HW A} \sim 1 \) as can be inferred from Fig. 2.

- \( h_3^0 \to h_3^0 h_1^0 \)

With three Higgses involved in the process, the parametric dependence becomes very important. Because of the large value of \( \beta_{A_\lambda} \) compared to all other \( \beta \)'s, one still expects the running to be dominated by this parameter. Indeed, the corrections in the \( \overline{\text{DR}} \) schemes are not only quite large, \(-50% \) at \( Q_{\text{susy}} \sim 754\text{ GeV} \), but they are also very sensitive to the choice of scale since at the scale \( M_{H^+} \) which is not even half \( Q_{\text{susy}} \) the corrections more than doubles to \(-106\% \). This is driven by the large variation due to \( A_\lambda \) as observed in Fig. 2. One can approximate the variation as \( 10\beta_{A_\lambda} \ln(Q_{\text{susy}}^2/M_{h_3^0}^2) \sim 60\% \). In fact the dependence in the counterterms can be worked out more precisely

\[
\frac{\delta \Gamma_{h_3^0 h_2^0 h_1^0}}{\Gamma_{h_3^0 h_2^0 h_1^0}} \sim 1.3 \frac{\delta \lambda}{\lambda} + 16 \frac{\delta \mu}{\mu} + 11 \left(1.15 \frac{\delta t_\beta}{t_\beta} - \frac{\delta A_\lambda}{A_\lambda} \right) + 0.2 \frac{\delta m_\kappa}{m_\kappa} \quad (8.19)
\]

With the extracted values of the counterterms in the OS and mixed schemes (written for \( \mu = Q_{\text{susy}} \)) we recover the differences shown in Table 7 between the OS and \( \overline{\text{DR}} \) and \( t \) schemes. Note also the “compensation” between the variation in \( t_\beta \) and \( A_\lambda \) in the parametric dependence which is effective in keeping the correction in the OS scheme manageable.

Although the scheme dependence is well understood, the fact that the corrections in the \( \overline{\text{DR}} \) scheme at the scale \( Q_{\text{susy}} \) are large indicates that this scale is not the most appropriate effective scale which minimises the correction. In this respect the OS scheme gives a more “perturbative” prediction. Nonetheless a scale of \( Q_{\text{eff}} \sim 1.5Q_{\text{susy}} \) gives a correction of about only 1%. An effective scale \( \sim 1.2Q_{\text{susy}} \) reproduces the result of the OS scheme. This shows that small changes in the scale (around \( Q_{\text{susy}} \)) reduce the results quite significantly. This is driven mostly by the large sensitivity in some of the parameters, essentially \( A_\lambda \) whose \( \beta \) constant is the largest.

- \( h_3^0 \to h_3^0 h_1^0 \)

The parametric dependence that tracks the dominant variations are also very large here as we remarked earlier. It can be approximated as
\[ \frac{\delta \Gamma_{h^0_1 h^0_1 h^0_2}}{\Gamma_{h^0_1 h^0_1 h^0_2}} \sim 4 \frac{\delta \lambda}{\lambda} - 8 \left( \frac{\delta \mu}{\mu} + \frac{1}{2} \frac{\delta t_\beta}{t_\beta} - \frac{\delta A_\lambda}{A_\lambda} \right) + 0.3 \frac{\delta m_\kappa}{m_\kappa} \] 

(8.20)

We note that although the \( A_\lambda \) dependence is large it is not as large as the one found for the \( h^0_1 h^0_2 h^0_3 \) coupling. Indeed, before performing the diagonalisation to the physical basis, this coupling would stem from the \( h^0_3 h^0_2 h^0_2 \) part of the potential whose strength is \( \lambda^2 v_d \propto \lambda^2 c_\beta \). This also explains the quartic dependence on \( \lambda \) of the coupling. Even though the parametric dependence on \( A_\lambda \) is more moderate than for \( h^0_1 h^0_2 h^0_1 \), it remains a strong parametric dependence in this decay also. Add to this the fact that \( \beta_{A_\lambda} \) is the largest of all \( \beta \), the large scale dependence is driven essentially by \( \beta_{A_\lambda} \). For the coupling responsible for this decay, an effective scale about twice lower than what was found in \( h^0_1 h^0_1 h^0_1 \) is required to bring the corrections to a negligible level. Varying again by 30\% \( Q_{\text{susy}} \) brings the DR corrections in par with the correction in the OS scheme. Eq. 8.20 when combined with the values of the finite part of the counterterms in the OS scheme given in Table 6 reproduce very well the difference between the OS scheme and the DR scheme.

- \( A^0_2 \to A^0_1 h^0_1 \)

For this decay the parametric dependence can be approximated by

\[ \frac{\delta \Gamma_{A^0_2 \to A^0_1 h^0_1}}{\Gamma_{A^0_2 \to A^0_1 h^0_1}} \sim 0.9 \frac{\delta \lambda}{\lambda} + 8.4 \left( \frac{\delta \mu}{\mu} + \frac{9}{16} \frac{\delta t_\beta}{t_\beta} - \frac{7}{16} \frac{\delta A_\lambda}{A_\lambda} \right) - 1.1 \frac{\delta m_\kappa}{m_\kappa}. \] 

(8.21)

Once again, the correction for this tri-linear Higgs coupling is smallest in the OS scheme. The scale dependence that can be seen from the two values in the DR scheme is not small. As has been a pattern for other tri-linear Higgs couplings, the rather large scale dependence is also a sign of a large correction in the mixed \( t_{123 A_1 A_2} \) scheme. The scale dependence is driven essentially by \( \beta_{A_\lambda} \). \( Q_{\text{eff}} \sim 1.5 Q_{\text{susy}} \) is the effective scale where the corrections vanishes in the DR scheme. Note that this is the same effective scale we found for \( h^0_3 \to h^0_1 h^0_1 \). The small correction in the OS scheme is a result of a cancellation between the contribution of the \( A_\lambda \) and \( t_\beta \) counterterms. Because in the \( t \)-scheme, \( t_\beta \) is defined in DR, this cancellation is not operative and again the correction is dominated by \( A_\lambda \).

To summarise the results for model B, it is worth stressing that in decays of Higgs into Higgses the OS scheme performs quite well in the sense that it gives very small corrections. There is a large scale dependence for these decays, but the effective scale where the correction vanishes in DR is, after all, not that much different from \( Q_{\text{susy}} \). Although \( \beta_{A_\lambda} \) is about 6\%, a value 10 times smaller than in Model A, the parametric dependence in Model B is strong thus enhancing the loop correction. Decays into charginos and neutralinos being much less sensitive to \( A_\lambda \) do not show much scheme and scale dependence.

9 Conclusions

With the renormalisation of the Higgs sector, it is now possible to compute full one-loop electroweak and QCD corrections to masses, decays and scattering processes in the
NMSSM with $\texttt{SloopS}$. This is particularly relevant as experiments are improving the precision in the measurements of Higgs and Dark Matter observables. Our computation of partial decay widths illustrates the importance of pure electroweak corrections for Higgs decays into supersymmetric particles and highlights the choice of the renormalisation scheme. Our setup allows to choose between the $\text{DR}$ scheme, different on-shell schemes and “mixed” schemes whereby some conditions are imposed on on-shell quantities and others taken as $\text{DR}$. This variety of schemes has been implemented within $\texttt{SloopS}$. Comparing different renormalisation schemes is crucial to weigh the theoretical uncertainties and the possible necessity of higher order corrections. For this purpose, we discussed at length how the choice of the minimal set of physical masses to reconstruct the underlying parameters can affect the numerical results and their reliability. We have found large radiative corrections for some observables. Some of these large corrections appear only in certain renormalisation schemes. In this case, when the scheme dependence is large this can also be accompanied by a large scale dependence in the $\text{DR}$ scheme. These large scheme dependencies and large scale variations are due to a large value of some $\beta$ constants for some specific underlying parameters and/or are associated with a large parametric dependence of the observable upon this specific parameter. The latter situation occurs in a NMSSM with a moderate $\lambda$. In the small $\lambda$ scenarios, this parametric dependence is not as large, however many counterterms are poorly reconstructed precisely because they are extracted from a set of input masses with little sensitivity on some of the underlying parameters. It has to be stressed that, although easier to implement, taking only masses as inputs may not be the optimal choice to renormalise the model. When new particles are discovered, not only their masses will be measured but so will the strength of their production modes and their decays. These observables will thus offer new possibilities for reconstructing the fundamental parameters of the model that will not require the knowledge of the complete particle spectrum. It remains to be seen whether a more cleverly chosen renormalisation scheme, for example one that uses the partial width of a heavy Higgs decay as a renormalisation condition, would lead to better controlled corrections. In the MSSM we have shown[45] that the decay of the pseudoscalar Higgs to a pair of $\tau$’s is an excellent definition of $t_\beta$. Some of the large corrections we found are also pathological in the sense that they are due to a rather large value of $A_t$ compared to the NMSSM parameter $A_\lambda$. Such a discrepancy between $A_t$ and $A_\lambda$ is responsible for a large $\beta_{A_t}$ which will then propagate into the corrections of many Higgs observables, in particular those for the Higgs self-couplings. A natural NMSSM should not require very large values of $A_t$ as what is required for MSSM-like models (with $\lambda \ll 1$), in this case the one-loop corrections are contained and an on-shell scheme is a quite judicious choice.

Acknowledgements

This research was supported in part by the Research Executive Agency (REA) of the European Union under the Grant Agreement PITN-GA2012-316704 (“HiggsTools”), by “Investissements d’avenir, Labex ENIGMASS” and by the French ANR, Project DMAstroLHC, ANR-12-BS05-006. G.C is supported in part by the ERC advanced grant Higgs@LHC and the THEORY-LHC France initiative of CNRS/IN2P3. The authors would like to thank A. Djouadi for useful discussions regarding the project.
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