Stability Analysis of Pitch-regulated, Variable Speed Wind Turbines in Closed Loop Operation Using a Linear Eigenvalue Approach

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Abstract. A multi-body aeroelastic design code based on the implementation of the combined aeroelastic beam element is extended to cover closed loop operation conditions of wind turbines. The equations of a controller for variable generator speed and pitch controlled operation in high wind speeds are combined with the aeroelastic equations of motion for the complete wind turbine, in order to provide a compound servo-aeroelastic system of equations. The control equations comprise linear differential equations for the pitch and generator torque actuators, the control feedback elements (PI control) and the various filters acting on the feedback signals. In its non-linear form the dynamic equations are integrated in time to provide the reference state, while upon linearization of the system and transformation in the non-rotating frame, the linear stability equations are derived. Stability results for a multi-MW wind turbine show that the coupling of the controller dynamics with the aeroelastic dynamics of the machine is important and must be taken into account in view of defining the controller parameters.

1. Introduction
The increasing size of the wind turbines and the requirement for maximizing energy capture and reducing loads, calls for integrated control strategies covering both objectives. Until very recently the design and optimization of controllers focused on the increase of power performance. During the last years a lot of research effort has been put in designing control strategies aiming at reducing fatigue loads of certain components of the wind turbine and enhancing stability by increasing the damping of the lowest damped wind turbine modes [1]. The adjustment of the generator torque for the alleviation of the drive-train and tower sideward bending loads [2], the collective variation of the blades pitch for enhancing the damping of the tower fore-aft motion [3],[4], the cyclic variation of the blades pitch for reducing 1P blade flapwise loads [4],[5] are some examples of the so-called “aeroelastic” control concepts that have been recently investigated. A thorough assessment of all the abovementioned control strategies has been also very recently concluded under the EU funded project STABCON [6],[7]. Some of these, to some extent, have been adopted by the wind industry and implemented at a commercial level.
Available design methods of control loops rely on simplified linear models of the wind turbine system [4], [8]. While these methods model the dynamics of the controls, they lack a detailed representation of the wind turbine aerodynamics and its structural dynamics. Usually the dynamic behaviour of the wind turbine is simulated by only a limited number of DOFs that include the basic dynamics of the drive train, the tower top motion and rarely the flap deflections of the blades. Rotor aerodynamics is accounted for through lookup tables that provide the mean performance of the rotor in steady-state conditions and so wake effects and unsteadiness of the local to the blade flow are completely suppressed. Consequently the interactions between the natural frequencies of the controller and the aeroelastic frequencies of the wind turbine cannot always be accurately predicted. Because such interactions can be responsible for unstable behaviour [9], the suitability of simplified models is questioned.

On the other hand, most of the existing aeroelastic design tools assume classic scalar control (or Proportional Integral Differential control; PID-design) resulting in only one scalar control design equation for each control objective. Such an approach is simple enough and solid, provided that the control loops do not interact. However, even mild interaction may significantly affect the simultaneous realisation of all the control objectives.

The above remarks indicate that in order to fully assess the efficiency and stability of a control concept, coupled servo-aeroelastic tools are needed, that account for the complete aerodynamics and structural dynamics of the wind turbine. The servo-aeroelastic couplings, which occur in closed loop operation, alter the frequencies and damping characteristics of the wind turbine structural modes. Since non-linear analysis is carried out in the time domain, identification of such coupling effects can only be performed on the time signals. This is not always obvious, mainly due to the large number of the system frequencies involved that are usually closely spaced. So, a more straightforward identification procedure has to be applied and this is offered by linear eigenvalue analysis. By performing consistent linearisation of the servo-aeroelastic equations, such an approach can provide a map of the stability characteristics of the complete system modes for different values of the control system parameters. Furthermore, through transfer function analyses on the linearised set of equations, valuable information on the frequency response of the full servo-aeroelastic system can be obtained. Therefore, linear eigenvalue analysis can be used as a supplementary tool in the design and customisation of different control algorithms.

In the present paper, such an eigenvalue approach is presented, with a view to identify the stability characteristics of a wind turbine in closed loop operation. The tool has been built on a non-linear modelling context in the context of the EU funded project STABCON [6]. Application of the method is performed to a pitch regulated variable speed multi-MW commercial wind turbine. Linear Proportional Integral (PI) controllers for generator-based speed adaptation in partial load and pitch-based regulation in full load are implemented in the aeroelastic model of the machine. Apart from the linear equation of the controllers themselves, their implementation in a servo-aeroelastic code involves the modelling of both the filters that act on the input signals, and the actuators that realise the control outputs. This approach adds value to the stability model since it refines the modelling of a wind turbine, while it is also of scientific importance since it can reveal physical interactions of the coupled servo-aeroelastic system through parametric studies.

Results in the paper include servo-aeroelastic damping and frequency distributions for two modes of the wind turbine controller operation: a) generator torque regulated constant speed operation at high wind speeds; b) pitch regulated constant speed operation in full load. Comparison between open and closed loop operation is presented and issues related to the implementation and the use of control algorithms are discussed. Also by varying the design parameters of the control equations, their effects on stability are investigated, indicating the design capabilities of the eigenvalue approach.

2. Coupled Servo-aeroelastic Stability Tool
A coupled servo-aeroelastic tool has been developed by extending the baseline aeroelastic stability tool presented in [10]. The extension concerns the implementation of the equations of the feedback
laws, as well as of the dynamic equations of the servo-actuators in closed loop operation. Stability computations are performed by solving the eigenvalue problem of the linearised system of the coupled structural dynamic and aerodynamic equations of motion for the complete wind turbine configuration.

In the baseline aeroelastic tool, structural dynamics is modelled using beam theory for all flexible components (i.e. the blades, the shaft and the tower) undergoing bending in two directions, torsion and tension. Approximation is based on the finite element method which results in beam elements of twelve degrees of freedom (DOFs). The dynamic and structural coupling of the different components is performed in the context of a multi-body analysis. Therefore in addition to their structural deformations, each deformable component is allowed to undergo rigid body motions under kinematic and load constraints specified by its connection to the remaining structure [11]. Rotor aerodynamics is modelled using based on the blade element momentum theory. Localised unsteady aerodynamics (including dynamic stall modelling) is accounted for by the Extended Onera Lift and Drag model [12].

For the combined treatment of the aerodynamics and the structural dynamics, additional, aerodynamic states, corresponding to the circulation DOFs of the ONERA model, are introduced. They are combined with the structural ones in the so-called ‘Aeroelastic Beam Element’ [13].

The baseline stability tool has been validated successfully against field measurements and other state-of-the-art stability tools in the framework of the EU co-funded STABCON project (contract NNK5-CT 2002-00627). Linear, eigenvalue results have been presented and discussed in [14], [15].

2.1. State-space Representation of the Coupled System

For more details concerning the approach followed in linearising the non-linear aeroelastic equations of motion the reader is referred to [10]. The linearised system of the coupled aeroelastic equations of motion is written in the following form:

\[
\begin{bmatrix}
    m_{uu} & m_{uq} \\
    m_{qu} & m_{qq}
\end{bmatrix}
\begin{bmatrix}
    \ddot{u} \\
    \ddot{q}
\end{bmatrix}
+
\begin{bmatrix}
    d_{uu} & d_{uq} \\
    d_{qu} & d_{qq}
\end{bmatrix}
\begin{bmatrix}
    \dot{u} \\
    \dot{q}
\end{bmatrix}
+
\begin{bmatrix}
    c_{uu} & c_{uq} \\
    c_{qu} & c_{qq}
\end{bmatrix}
\begin{bmatrix}
    u \\
    q
\end{bmatrix}
=
\begin{bmatrix}
    f_u \\
    f_q
\end{bmatrix}
\] (1)

In (1), \( u \) are the local structural and aerodynamic DOFs. In the case of a rotor blade \( u \) will include the local bending, tension and torsion DOFs plus the additional circulation states introduced by the Onera aerodynamic model [10]. The vector \( q \) contains all the rigid body translations and rotations that determine the origin and orientation of the local to the body co-ordinate system with respect to the inertial frame. In particular, vector \( q \) can both include large rotations and translations but also structural deflections (displacements and rotations) at the tower top and hub center attachment points. At tower top \( q \) will include the deformations of the tower as well as the yaw rotation of the nacelle while at the hub center \( q \) will additionally include the drive train deformations as well as the azimuth of the rotor and the pitch variation of each blade. In the context of a multi-body approach these are all treated uniformly as rigid body motions.

Given the above definitions of \( u \) and \( q \), it follows that the off-diagonal elements in the mass, damping and stiffness matrices in (1) that are designated with the subscript ‘\( uq \)’, correspond to the coupling terms resulting from the relative motion of the local system with respect to the global inertial frame while those indicated by ‘\( qu \)’ correspond to the kinematic or structural dependency of the \( q \) DOFs on the local \( u \) DOFs.

In open loop operation, fixed value conditions are appointed to the \( q \) DOFs. They are associated with specific control output variables. For example, for a pitch regulated–variable speed wind turbine it is assumed that the generator speed and the pitch angle of the blades are time-invariant. The fixed values imposed to these DOFs are representative of the average operation conditions for a specific wind speed. Such an assumption will of course constrain the loads associated with the specific kinematic DOFs, i.e. the torque of the generator and the pitching torque at the root of the blades.

In closed loop operation, the generator and pitching torques can no longer remain constrained since they become input variables to the controlled system. So the system of equations shown in (1) must be
supplemented with additional states corresponding to these control input variables together with the
equations describing their dynamic response (servo-actuator dynamic equations). Furthermore, since
the system operates in closed loop, the equations of the feedback laws as well as the equations of the
various filters need to be defined (control equations). The additional equations are accommodated at
the end of (1) by introducing a new set of $q$ DOFs, denoted by $q_c$, that represent the system control
variables. The final step towards modifying (1) for active control operation is to replace the fixed
value conditions for the control output variables with dynamic equations providing their response
under the action of the control input variables $q_c$. The closed loop system then takes the form:

$$
\begin{bmatrix}
0 & m_{uu} & m_{uq} & m_{uc} \\
m_{uu} & 0 & m_{qu} & m_{qc} \\
m_{uq} & m_{qu} & 0 & m_{qc} \\
m_{uc} & m_{qc} & m_{qc} & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\dot{u} \\
q \\
\dot{q}
\end{bmatrix}
+
\begin{bmatrix}
d_{uu} & d_{uq} & d_{uc} \\
d_{uu} & 0 & d_{qc} \\
d_{uq} & d_{qc} & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\dot{u} \\
q \\
\dot{q}
\end{bmatrix}
+
\begin{bmatrix}
c_{uu} & c_{uq} & c_{uc} \\
c_{uu} & 0 & c_{qc} \\
c_{uq} & c_{qc} & 0
\end{bmatrix}
\begin{bmatrix}
u \\
\dot{u} \\
q \\
\dot{q}
\end{bmatrix}
= 
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
$$

(2)

2.2. Linear Eigenvalue Stability Analysis

To obtain the linearized system of equations (1) and (2) the non-linear servo-aeroelastic equations of
motion are linearised with reference to a periodic equilibrium (reference) state. For its definition, the
non-linear set of equations is integrated in time until a periodic response (with respect to the rotor
speed) is reached. In case of unstable conditions, where periodicity is never reached, the time domain
calculations provide a response that contains significant components in all basic system frequencies. In
such a case, by means of Fourier transformation, only the parts corresponding to the rotational
frequency (1 P) and its basic multiple $N$ P (where $N$ the number of blades) are retained. This provides
an approximation of the periodic conditions.

The linearised system (2) is reformulated into a first order system:

$$
\dot{x} = A(x_0, x_g) \cdot x + B
$$

(3)

Where, $x_0$ denotes the reference state and $x$ are perturbations of the state variables about this
reference state. The eigenvalues of matrix $A$ provide the natural frequencies and damping
characteristics of (3), on condition that $A$ is a constant coefficient matrix.

The equations of motion of the rotating blades, being expressed in the rotating frame, involve
periodic coefficients. It is possible to eliminate these periodic coefficients and treat the full wind
turbine configuration in a linear eigenvalue context with reference to the non-rotating frame, by
introducing a multi-blade transformation of all the rotating DOFs. This co-ordinates transformation
capitalizes on the polar symmetry of rotors with identical blades and in the case of a three bladed rotor
is given by [16]:

$$
x_m = x_0 + x_c \cdot \cos \psi_m + x_s \cdot \sin \psi_m
$$

(4)

where $x_m$ is any rotating DOF of the $m$-th blade being at an azimuth position
$
\psi_m = \Omega \cdot t + (2\pi / N) \cdot (m-1), \ m = 1,2,3, \ N = 3 \ and \ x_g, \ x_c, \ x_s \ are \ the \ transformed \ co-ordinates \ (designated \ as \ collective, \ cyclic \ cosine \ and \ cyclic \ sine, \ respectively) \ expressed \ in \ the \ non-rotating \ frame. \ Of \ course \ besides \ transforming \ the \ rotating \ DOFs \ into \ the \ non-rotating \ frame, \ the \ same \ should \ be \ done \ to \ the \ equations \ of \ the \ blades \ which \ express \ the \ blade \ dynamics \ in \ the \ rotating \ frame. \ To \ this \ end \ the \ following \ operators \ are \ applied \ to \ the \ blade \ equations \ contained \ in \ (3):

$$
\begin{align*}
(non - rotating \ equation)_{1} &= \frac{1}{N} \sum_{m=1}^{N} (rotating \ equation)_{m} \\
(non - rotating \ equation)_{2} &= \frac{2}{N} \sum_{m=1}^{N} (rotating \ equation)_{m} \cdot \cos \psi_m \\
(non - rotating \ equation)_{3} &= \frac{2}{N} \sum_{m=1}^{N} (rotating \ equation)_{m} \cdot \sin \psi_m
\end{align*}
$$

(5)
The eigenvalue analysis on the transformed matrix $A$ will provide the natural frequencies of the rotor system with reference to the non-rotating frame. The ‘non-rotating’ frequencies are now associated with the corresponding non-rotating DOFs (i.e. collective and whirling). For an isolated three bladed rotor it can be shown [17] that the collective frequencies will be identical with the natural frequencies of the $m$-th blade while the two sets of cyclic frequencies, designated as backward whirling (or regressive) and forward whirling (or progressive), will differ by $-\Omega$ and $+\Omega$ respectively.

### 3. Application–Results

The coupled servo-aeroelastic stability tool is applied to a pitch regulated – variable speed 2.75 MW commercial wind turbine in closed loop operation. In the beginning of the section the different parts of the control system are first described. The resulting differential equations of the feedback loops constitute the set of the control equations in the coupled servo-aeroelastic system (2) presented in section 2.1. Predictions of the coupled system frequencies and damping characteristics, for the eight lowest wind turbine structural modes are provided. In order to highlight the effect on stability, these results are compared to those obtained in open loop (see Figure 1). Open loop operation results in positive damping to all structural modes. Of course high positive values are obtained for the modes that are excited by the rotor thrust (tower front-back bending and rotor flap modes) while the rotor lead-lag modes have substantially lower damping values. In addition to direct stability comparisons, parametric investigations of the feedback characteristics of selective control elements are performed aiming at identifying the parameters of the control system that optimise the stability of the coupled system.

#### 3.1. Description of the Wind Turbine Controller

The controller of this specific wind turbine consists of four distinct modes of operation. The first mode (Mode 1) is a constant speed mode at low wind speeds where the rotor speed is kept constant at its lowest bound ($\omega_{min}$) by adjusting the torque of the generator. For moderate wind speeds the controller enters the variable speed mode (Mode 2). In this mode the rotor speed is varied so that optimum performance (maximum $C_p$) is obtained. At high wind speeds (>10 m/s) the rotor speed is kept constant to its rated value ($\omega_R$) by varying the generator torque (Mode 3). In the first three modes of operation, the pitch angle is set equal to its minimum value and remains fixed. The fourth mode (Mode 4) is a constant speed variable pitch mode. Once the generator torque reaches the nominal value, the rotor speed is regulated by pitching the blades.

The present analysis considers only the two last modes of operation (Mode 3 and Mode 4) corresponding to operation of the wind turbine beyond 10 m/s wind speed and at nominal rotational speed. Also the non-linear transition procedure that ensures smooth switching from one mode of operation to the other is not considered in the modelling. So, it is assumed that when the wind speed lies in the range between 10–14 m/s the wind turbine operates in Mode 3. For wind speeds higher than 14 m/s it is assumed that the wind turbine instantly enters Mode 4 and therefore pitch regulation of the rotor speed is on.

In Mode 3 of operation, the rotor speed is regulated through the torque of the generator. To this end, a feedback loop is introduced for determining the required generator torque $T_{gen}$ that takes as input the difference of the measured generator speed from the reference speed–speed set point $\omega_g - \omega_{ref}^g$ (see Figure 2). This feedback loop is realised through a proportional–integral (PI) element given by the following 1st order differential equations:

$$\dot{T}_{gen} = K_p^g \cdot \dot{\omega}_g + K_i^g \cdot (\omega_g - \omega_{ref}^g) \quad (6)$$

where $K_p^g, K_i^g$ are the proportional and integral gains of the PI element respectively.
In Mode 4 of operation, the torque of the generator reaches its rated value and remains constant. Therefore, the rotor speed in this mode is regulated through the pitch angle of the blades. In this case, a feedback loop is introduced for determining the collective pitch demand angle \( \theta_d \) (see Figure 3), based again on the deviation of the measured generator speed from the speed set point:

\[
\dot{\theta}_d = K_G \cdot \left( K_p^r \cdot \dot{\omega}_g + K_I^r \cdot (\omega_g - \omega_g^{ref}) \right)
\]

where \( K_p^r, K_I^r \) are the proportional and integral gains of the controller while \( K_G \) is a non-linear gain schedule parameter shown in Figure 4. This function accounts for the variation of the gradient of the rotor torque with respect to the pitch angle across the full load region. The decaying form of the gain schedule implies that the torque gradient increases with increasing wind speed and therefore lower gain values are required at higher wind speeds.

The collective pitch \( \theta_d \) and pitch speed \( \dot{\theta}_d \) demands are input to the pitch servo actuators of the three blades. The actuation torque of each blade is determined through a feedback loop of the pitch position plus an inner loop of the pitch speed \( \dot{\theta}_d \), towards the desired pitch and pitch speed values [18], as shown is Figure 5. The feedback laws in both the outer position loop and the inner speed loop are proportional-integral actions. Both feedback laws are extended with a first order low pass filter with a time constant that is ten times smaller than the time constants that apply in the respective actions. The reason for introducing these first order filters is to accomplish the smoothing property of the PI feedback laws. The actuating torque \( T_{pi} \) of the i-th blade is then provided by the following differential equations:

\[
\begin{align*}
\ddot{T}_{pi} + \tau_r^r \cdot \dot{T}_{pi} + \tau_r^r \cdot T_{pi} &= \tau_r^r \cdot K_p^r \cdot (\dot{\theta}_{qi,d} - \theta_d) + K_I^r \cdot (p_{qi,d} - p_i) \\
\ddot{p}_{qi,d} + \tau_s^s \cdot \dot{p}_{qi,d} &= \tau_s^s \cdot K_p^s \cdot (\dot{\theta}_{qi,d} - \theta_d) + K_I^s \cdot (p_{qi,d} - p_i)
\end{align*}
\]

where, \( K_r^r, K_r^s \) are the proportional gains and \( \tau_r, \tau_s \) are the time constants of the two PI elements, \( \tau_r^r, \tau_s^r, \tau_r^s, \tau_s^s \) are the time constants of the two first order filters with \( \gamma^r = \gamma^s = 10 \) and \( p_i \) is the actual pitch of the i-th blade.
In Mode 4 of the controller operation, the torque of the generator being constant will result in very low damping of the drive train system. In order to avoid undamped vibrations in the frequency of the drive-train mode, a drive train damper (DTD) is introduced. The drive train damper is a band-pass filter acting on the measured generator speed with a centre frequency around the free-free frequency of the drive-train mode (see Figure 2). The differential equation that provides the additional torque, $T_{DTD}$, introduced by the drive train damper is given by:

$$\ddot{T}_{DTD} + 2 \cdot d_{DTD} \cdot \dot{\omega}_{DTD} \cdot T_{DTD} + \omega_{DTD}^2 \cdot T_{DTD} = -2 \cdot K_{DTD} \cdot d_{DTD} \cdot \omega_{DTD} \cdot \dot{\omega}_g$$  \hspace{1cm} (10)$$

where $\omega_{DTD}$, $d_{DTD}$ are the centre frequency and the damping, while $K_{DTD}$ is the gain of the filter. It is noted that although the drive train damper is only necessary in Mode 4 where zero damping from the generator is anticipated, it also acts in Mode 3.

Furthermore in order to avoid unnecessary pitch action at the same frequency but also to avoid excitation of the blade natural frequencies (rotor second asymmetric flapwise natural frequencies lie close to the drive train free-free frequency), a band-stop (notch) filter is also introduced prior to the PI controller that provides the collective pitch demand. In this way the input signal to the pitch PI will be free of the response of the generator speed to the drive-train mode frequency (see Figure 3). The equation that provides the filtered input $e$ to the pitch PI is given by:

$$\ddot{e} + 2 \cdot d_{e2} \cdot \dot{e} + \omega_{e2}^2 \cdot e = \ddot{\omega}_g + 2 \cdot d_{e1} \cdot \dot{\omega}_g + \omega_{e1}^2 \cdot (\omega_g - \omega_g^{ref})$$  \hspace{1cm} (11)$$
where, as in the case of the band-pass filter, $\omega_a, d_a$ denote the frequency and damping characteristics of the filter.

The above set of equations corresponds to the control equations in the system of equations (2). Note that equations (6), (7), (10) and (11) are expressed in the non-rotating frame while (8) and (9) are expressed in the rotating frame.

3.2. Stability Results

The linear servo-aeroelastic tool presented so far is used to predict the damping characteristics of a wind turbine in closed loop operation. In the analysis presented in this section, the different elements of the control system are gradually added in an attempt to distinguish their effect on the stability of the full servo-aeroelastic system. First the basic dynamics of the generator are included in the full operation range considered. Then, pitch regulation is superimposed for the full load region (Mode 4). Finally the two filters (DTD and notch filter on the speed error) are added. The gains of the pitch PI controller and the centre frequencies of the filters are tuned through parametric analyses and the stability of the wind turbine equipped with the final integrated design is compared to the stability of the wind turbine in open loop operation.

The first comparison deals with the basic generator dynamics. This implies that in Mode 3, the rotor speed is regulated by the torque of the generator through the PI control on the speed error, while in Mode 4 the torque of generator remains constant. Despite the fact that above the rated speed, operation is regulated by pitch actions, in this first comparison constant pitch of the blades is assumed (pitch is fixed to the average value for given wind speed conditions). In this case the main difference of the closed loop simulations as compared to the open loop simulations consists of imposing a boundary condition for the electric torque instead of fixing the speed at the generator side.

The resulting servo-aeroelastic frequency and log decrement damping of the coupled system modes are presented in Figure 6 and Figure 7 respectively. The presence of the controller impacts on the frequency distribution of the first tower lateral bending mode which slightly increases, Figure 6. For this low-damped mode, the presence of the controller has a beneficiary effect (increase of the damping) in the wind speed range below the rated wind speed, Figure 7. This damping improvement is obtained as a result of the effect of the generator torque (generator torque PI) which directly acts in the sideways tower direction. A small increase in the (highly positive) damping of the front-back tower mode is also observed over the same wind speed range.

Since analysis is carried out in the non-rotating frame, rotor results refer to the distributions of the collective as well as the backward and forward whirling modes (denoted as CO, BW and FW, respectively hereafter). The presence on the controller only affects the collective modes, whereas the whirling flap and lead-lag modes are hardly changed. For the CO flap mode, a rather negligible increase in damping is observed mainly attributed to the different boundary conditions at the generator side. The differences observed in the CO lead-lag mode (both in terms of frequency and damping) reflect the fact that this mode turns from a fixed-free one in open loop to a free-free one in closed loop operation. It is noted that the CO lead-lag mode is strongly coupled to the dynamics of the drive train (shaft flexibility, generator dynamics). Therefore, the damping distribution of this mode features high
positive values in the region below the rated speed, where the generator torque controls the speed while takes near zero values beyond the rated speed where the torque of the generator remains constant. Furthermore in comparison to the open loop operation the CO lead-lag mode looses its damping dependence to the pitch angle (higher damping values for higher pitch angles).

Moving to the above rated wind speed region, the pitch PI controller and the pitch servo actuators of the three blades are added. While in the previous case all eight lowest modes of the wind turbine configuration are presented, to serve as reference, hereafter only those affected by the modifications performed will be shown. The goal is to identify the value of the proportional gain for optimised operation. In that respect a parametric study is conducted for 6 values of the proportional gain from 0 to 0.05 s using a 0.01 s step). In Figure 8 the frequency distributions of the tower front back mode and the rotor collective flap mode are shown. Also in Figure 9 the damping characteristics of the servo-aeroelastic system for the first tower lateral and front-back bending modes, as well as the collective flap and lead-lag modes are presented. The other structural modes are not affected by the inclusion of the pitch controller. In all cases, the value of the integral gain was equal to the one quarter of the value of the proportional.

While by means of increasing the value of the proportional gain, the values of the damping of the tower lateral bending and the collective lead-lag modes increase, this is not the case with the front-back tower bending mode and the collective flap mode. These two are the most affected modes by the
rotor thrust variations resulting from the variable pitch (collective in this case) operation. As depicted by Figure 9, starting from the zero-gain case (that corresponds to constant pitch for each specific wind speed), the damping characteristics of both modes increase only for very low values of the proportional gain (0.01 s). Then they gradually start to decrease as the gain increases. For the highest value of the proportional gain examined (0.05 s) the decrease in the damping is of the order of 40 and 80% for the tower front-back and the collective flap modes, respectively. Also as seen in Figure 8 the frequencies of these two modes considerably decrease by increasing the gain of the controller. This indicates that closed loop variation of the pitch artificially alters the flexibility of the modes that are closely linked to variations of the thrust.

A good compromise for acceptable damping levels of these modes and fast response of the controller is to set 0.02s for the proportional gain of the pitch PI controller. For this gain value, the increase in the damping of the tower lateral mode is still very high (an average value of 5.3% is obtained compared to the initial 3.3%), whilst the improvement in stability of the collective flap modes is restricted to wind speeds below 17 m/s. The frequency of the PI controller for this gain value is found to be around 0.3P.

The next step is to introduce a drive train damper in order to enhance the damping of the low damped collective lead-lag mode (see Figure 6). Maintaining a reasonable damping ratio value

Figure 7 (clockwise from upper left): Servo-aerelastic damping of the first tower (lateral and front-back bending), flap (collective and whirling), whirling lead-lag modes and collective lead-lag.
Figure 8: Servo-aeroelastic frequencies of the first tower front-back (left) and CO flap modes (right).

Figure 9 (clockwise from upper left): Servo-aeroelastic damping of the first tower lateral bending, tower front-back bending, CO lead-lag and CO flap modes.

$\Delta_{\text{DD}} = 0.1$, the parametric study focused on optimizing the centre drive train damper (DTD) frequency. In this connection, results are presented for two wind speeds: 16 and 18 m/s, Figure 10.
The initial values for the collective lead-lag mode are denoted as a solid line at the level of the value obtained so far. Since the DTD is modelled by a linear differential equation, the solution of the eigenvalue problem provides an additional mode, which is related to the operation of the drive train damper. So in the figures, two distributions appear; the one corresponding to the collective lead-lag mode and the other corresponding to the DTD. Results are shown over a specific range of centre DTD frequencies. At low values of the DTD centre frequency (<<2) the CO lead-lag mode curve converges to the baseline frequency (straight line), while the DTD mode curve converges to the DTD centre frequency (imagine a straight line with slope equal to 1). At high values of the DTD centre frequency (>>2.25) the opposite trend is observed. This indicates that at some point the two modes interchange frequencies. The same conclusion is also drawn from the damping plots where at a certain point the two damping curves cross and thereafter interchange damping characteristics. The intersection point of the damping curves determines the optimum value of the DTD centre frequency since at that point the maximum interaction between the two modes will occur. Optimum values for the centre frequency of the DTD, in terms of damping of the two coupled modes, are found in the range of [2.20, 2.25] Hz. In this range of the DTD centre frequency, the frequencies of the CO lead-lag mode and the DTD mode are about 2.30 Hz and 2.00 Hz respectively.

Next the notch filter is introduced prior to the pitch PI, in the signal of the measured generator speed error $\omega_g - \omega_g^{cf}$. The purpose of the notch filter is two fold: first to limit the unnecessary pitch

Figure 10: Servo-aeroelastic frequencies (left) and damping (right) of the collective lead-lag and the drive train damper modes for wind speed 16 m/s (upper) and 18 m/s (lower).
actions at the frequency of the drive-train modes and second to prevent excitation of the rotor 2nd flapwise asymmetric modes which lie closely to the 2.30 Hz frequency of the CO lead-lag mode. The centre frequency of the DTD is then further tuned; now in the presence of the notch filter. A parametric analysis has been carried out, in terms of both the DTD and the notch filter centre frequencies, for the wind speed of 16 m/s. Keeping in mind that the centre frequency of the notch filter should be placed in between the two poles of the drive train, the range within which the notch centre frequency varies, is [1.8, 2.2] Hz. Again a certain damping ratio value is assumed ($d_{\alpha_2} = 0.4$).

**Figure 11:** Servo-aerodynamic frequencies (upper) and damping (lower) of the collective lead-lag and the drive train damper modes.
The results are shown in Figure 11. The notch filter results in a considerable reduction in both the DTD mode and the CO lead-lag mode frequencies, over the range of DTD centre frequencies maximum interaction between the two modes occurs. The frequency of the mode corresponding to the notch filter is almost constant with respect to the DTD centre frequency, but slightly higher compared to the imposed value due to the servo-aeroelastic couplings. As regards the damping of the two drive train modes also plotted in the same figure, the range of the DTD centre frequency over which maximum damping exchange is obtained, is also shifted to lower frequencies \([2.05, \, 2.10] \text{ Hz}\). The values: \(2.05\text{Hz}\) for the centre frequencies of the DTD and \(1.95\text{Hz}\) for the notch filter have been finally selected.

For the final configuration of the integrated controller, the damping distributions, over wind speed, for the first tower lateral and front-back bending, CO lead-lag, whirling lead-lag and flap modes, are presented in Figure 12. Again they are compared to the predictions obtained for open loop operation. A significant increase in damping is obtained for the two tower bending modes over the complete range of wind speeds. The whirling flap and whirling lead-lag modes are only negligibly affected. Taking advantage from the pitch regulation, the damping distribution of the CO flap mode highly increases above the rated speed, whilst the CO lead-lag mode takes high positive values, especially below the rated speed.

**Figure 12** (clockwise from upper left): Servo-aeroelastic damping of the first tower lateral and front back bending, CO lead-lag, whirling lead-lag and flap modes.

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4. Conclusions
A complete servo-aeroelastic tool for wind turbines in closed loop operation has been presented which upon linearization defines the appropriate framework for linear stability analysis based on the solution of the corresponding eigenvalue problem. Stability predictions for a 2.75MW commercial pitch regulated – variable speed wind turbine indicate that the natural frequencies and the damping characteristics of certain structural modes can be considerably altered as a result of the servo-aeroelastic couplings occurring in closed loop operation. This type of couplings can either increase or decrease the damping characteristics depending on the design parameters of the control algorithm. Therefore, it has been clearly identified that upgrading aeroelastic stability tools in view of incorporating the dynamics of the controllers and the servo-actuators is an essential step towards improving their prediction capabilities. This is especially true when new “aeroelastic” control concepts are tested and assessed. In this connection, linearized servo-aeroelastic tools of the complete wind turbine system form a rigid basis for the evaluation and optimisation of specific control strategies for a specific wind turbine.

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