Screening in (2+1)D pure gauge theory at high temperatures

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Abstract

We compute heavy quark potentials in pure gauge SU(3) at high temperatures in 2 + 1 dimensions and confront them with expectations emerging from perturbative calculations.

1 INTRODUCTION

The physical properties of the quark-gluon plasma still present a largely unsettled problem insofar as quantitative information about screening lengths and quasi-particle excitations is not really available yet. Lattice calculations provide interesting results, nevertheless, at very high temperatures one would like to make contact with perturbation theory. However, the domain of applicability of perturbative calculations is not known. The main difficulty originates from the infrared divergencies of the finite temperature theory. In the chromoelectric sector they are supposed to be regulated by the Debye mass, which appears in the resummation of the polarization diagrams. If these diagrams dominate, one would expect that the resummed perturbation theory gives a good description of the potential between heavy quarks.

In order to investigate these questions of principle, we turn to a simpler, but very similar theory, namely SU(3) gauge theory in 2+1 dimensions. This theory still has confinement at low temperatures, and a phase transition to a deconfined high temperature phase. In early investigations, d’Hoker both studied perturbation theory [1] and performed some Monte Carlo calculations on the analogous SU(2) model [2]. Other investigations of the SU(2) and SU(3) model in three dimensions have been devoted to the string tension at zero temperature [3] and the properties of the phase transition [4]. In the present contribution we describe the first results of our high statistics investigation of the SU(3) gauge theory in the high temperature phase. We test the relations expected from a naive resummation of perturbation theory. In contrast to the conclusions drawn by d’Hoker we will show below that the description based on naively resummed perturbation theory is not in agreement with the numerical data.

2 THE MODEL

The lattice version of the three dimensional pure SU(3) gauge theory, which we use, is given by the conventional Wilson action. The coupling constant \( g^2 \) in three dimensions has non-trivial dimension so that, in the scaling region, all physical quantities with dimension mass are expected to have a constant ratio to \( g^2 \).

To determine the screening mass as a function of temperature in the high temperature phase we study two quantities, namely the cyclic Wilson loop and the correlation between Polyakov loops. The cyclic Wilson loop \( W(R) \) is the colour-averaged trace of a closed rectangular loop of link matrices along the contour shown in Fig.1 where \( R = |\bar{x} - \bar{y}| \) ist the spatial distance between the two Polyakov-loop like fractions of it. From this quantity and the Polyakov loop \( L(R) \) we define the potentials

\[
V_{\text{cwl}}(R) = -\frac{< W(R) > - < L >^2}{< L >^2} T
\]  

and

\[
V_{\text{plc}}(R) = -\frac{< L(0)L^\dagger(R) > - < L >^2}{< L >^2} T
\] 


In perturbation theory, one can choose a gauge, where the spacelike links of the cyclic Wilson loops are set to one. Therefore, the lowest order contribution from perturbation theory comes from a single gluon exchange between the timelike sides of the rectangle. In contrast, for the correlation of the Polyakov loops, the lowest order is the exchange of two gluons, because the Polyakov loops are separately gauge singlets. Higher order polarization graphs are strongly infrared divergent, which can be taken into account by the selfconsistent introduction of a screening mass. In leading order this simple recipe leads to the following expressions for the potentials, in $SU(N)$,

$$V_{cwl}(r) = -\frac{N^2}{2N} - \frac{1}{2\pi} K_0(mr)$$ \hspace{1cm} (3)

and

$$V_{plc}(r) = -(V_{cwl}(r))^2/16$$ \hspace{1cm} (4)

where the screening mass $m$ is given by

$$m^2 = \frac{g^2 NT}{4\pi} \ln \frac{T}{g^2}$$ \hspace{1cm} (5)

and $K_0$ is the modified Bessel function. Note that we distinguish between lattice and continuum quantities, $R = r/a$ and $\mu = ma$.

### 3 CRITICAL TEMPERATURE

The critical temperature was determined on lattices with time extent $N_\tau = 2, 4, 6$. A finite size scaling analysis from volumes $16^2, 32^2, 48^2, 64^2$ and $192^2$ gives, with $\beta = 6/\alpha g^2$,

$$\beta_c = 8.163(8) \quad N_\tau = 2$$ \hspace{1cm} (6)

$$\beta_c = 14.705(58) \quad \text{for} \quad N_\tau = 4$$ \hspace{1cm} (7)

$$\beta_c = 21.508(572) \quad N_\tau = 6$$ \hspace{1cm} (8)

Thus, we find that the critical temperature scales quite well, $T_c/g^2 = \beta_c/6N_\tau \simeq const$. By comparing with the string tension in the zero temperature three dimensional theory, we find

$$\sqrt{\sigma}/T_c = 1.070(2) \quad \text{for} \quad N_\tau = 4$$ 

This number is in astonishingly good agreement with the prediction from string theory, $\sqrt{\sigma}/T_c = \sqrt{\pi/3} = 1.023$.

### 4 SCREENING MASS

The location of the phase transition defines a temperature scale, which we use, under the assumption of scaling, to determine the potentials at various values of $T/T_c = 1.5, 2, 3, 6, 12, 24$. For this we use lattices with $N_\tau = 4$ and $N_\sigma = 32, 48$ and 64 where the larger spatial extent has been employed at the largest values of $T/T_c$, such that $N_\sigma \mu > 5$. As for the determination of the critical temperature, we performed typically six overrelaxation and one pseudoheatbath step per sweep. For each temperature we have accumulated between 50000 and 200000 sweeps. This data base resulted in small statistical errors of the correlation functions up
to distances of $rT \lesssim 4$. We first tried to fit the data with eqs. (3), (4), starting at $r_{\text{min}} = 1/T$, $\left( R_{\text{min}} = N_\tau \right)$. We found that these fits did not properly represent the shape of the correlation functions. The fit did not improve when we employed the finite lattice expression corresponding to the $K_0$-function. Therefore we applied a more general Ansatz

$$V_{\text{cwl}}(R) = \frac{\text{const}}{(\mu R)^\gamma} e^{-\mu R}$$  \hspace{1cm} (10)

Figure 2: The screening mass (in lattice units) from cyclic Wilson loops, $\mu_{\text{cwl}}$, as a function of temperature, $x = 1/\sqrt{T/T_c}$.

When we let both parameters $\gamma$ and $\mu$ vary freely, we obtain values of $\gamma$ very near to 0.25 while the asymptotic expansion of $K_0$ leads to $\gamma = 0.5$. Therefore we decided to employ eq. (10) keeping $\gamma = 0.25$ fixed to extract the values for $\mu$. In Fig. 2, we show $\mu$ plotted versus $x = 1/\sqrt{T/T_c}$. The straight line denotes the fit result

$$\mu = 0.0029(3) + 0.4395(6)x$$  \hspace{1cm} (11)

For $N_\tau$ fixed, this corresponds to a dependence of the physical mass $m$ proportional to $\sqrt{T}$, as given by eq. (3). We see, however, no sign of the logarithmic correction which is only expected to dominate for $\mu \ll 1$. The correlation of the Polyakov loops, $V_{\text{plc}}$, is again well described by a formula like eq. (10), with $\gamma = 0.5$ and $\mu$ replaced by $2\mu$ in the exponent. Also here eq. (10) is definitely less good.

Figure 3: The ratio between the screening masses $\mu_{\text{plc}}/\mu_{\text{cwl}}$ as a function of temperature. The three results at $T = 12T_c$ are obtained from three different lattices with $N_\tau = 32, 48$ and 64.

It is still very interesting to see whether the basic assumption regarding the exchange of one versus two single particles with fixed mass holds, even if the simple, resummed perturbative formulae, eqs. (3), (4), do not fit the data. Therefore, we check the screening masses from the cyclic Wilson loop and from the Polyakov loop correlation for equality. In Fig. 3, we plot the ratio between those two masses which should be equal to one if factorization holds. What we find, however, is a ratio of about 1/2 at $T$ values slightly bigger than $T_c$, and only a slow approach to one at very high temperatures, $\mu_{\text{plc}}/\mu_{\text{cwl}} \simeq 0.85$ at a temperature as high as $T = 24T_c$. 
Another test of the factorization hypothesis involves a direct comparison of the potentials, i.e. a check of eq. (4). In Fig. 4 we plot the ratio $16V_{plc}/V_{cwl}^2$ as a function of $r\sqrt{TT_c}$. At short distances and high temperatures this ratio goes to one, as given by eq. (4), but at larger distances this behaviour is no longer seen, although the various ratios seem to scale in the variable $r\sqrt{T}$. We therefore conclude that a more complicated mechanism, involving bound states between the gluons is at work. We are further investigating this possibility.

References

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