Field theories on spaces with linear fuzziness

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Abstract – A noncommutative space is considered, the position operators of which satisfy the commutativity relations of a Lie algebra. The basic tools for calculation on this space, including the product of the fields, inner product and the proper measure for integration are derived. Some general aspects of perturbative field theory calculations on this space are also discussed. One of the features of such models is that they are free from ultraviolet divergences (and hence free from UV/IR mixing as well), if the group is compact. The example of the group SO(3) or SU(2) is investigated in more detail.

Introduction. – During recent years much attention has been paid to the formulation and study of field theories on noncommutative spaces. The motivation is the natural appearance of noncommutative spaces in some areas of physics, for example recently in the string theory. In particular it has been understood that the longitudinal directions of D-branes in the presence of a constant B-field background appear to be noncommutative, as seen by the ends of open strings [1–4]. In this case the coordinates satisfy the canonical relation

\[ \hat{x}_a, \hat{x}_b = i \theta_{ab} 1, \]

in which \( \theta \) is an antisymmetric constant tensor and \( 1 \) represents the unit operator. The theoretical and phenomenological implications of possible noncommutative coordinates have been extensively studied; see [5].

In the present paper the case beyond the canonical one is investigated. In particular, a model is considered in which the (dimensionless) spatial positions operators satisfy the commutation relations of a Lie algebra [6]:

\[ [\hat{x}_a, \hat{x}_b] = f^c_{ab} \hat{x}_c, \]

where \( f^c_{ab} \)'s are structure constants of a Lie algebra.

An example of this kind is the algebra SO(3), or SU(2). A special case of this is the so-called fuzzy sphere [7], where an irreducible representation of the position operators is used, which makes the Casimir of the algebra, \((\hat{x}_1)^2 + (\hat{x}_2)^2 + (\hat{x}_3)^2\), a multiple of the identity operator (a constant, hence the name sphere). One can consider the square root of this Casimir as the radius of the fuzzy sphere. This is, however, a noncommutative version of a two-dimensional space (sphere).

In the present work a model is introduced in which the noncommutativity is again taken to be that of a group, but no specific irreducible representation is considered. In particular, we employ the regular representation of the group, which contains all representations. As a consequence and for the special case of the SU(2) group, in our model one deals with the whole 3-dimensional space, instead of a 2-dimensional subspace of it as in the fuzzy-sphere case. The space of the corresponding momenta is an ordinary (commutative) space, and is compact if the group is compact. In fact one can consider the momenta as the coordinates of the group. So a by-product of such a model would be the elimination of any ultraviolet divergence in any field theory constructed on such a space. One important implication of the elimination of the ultraviolet divergences, as we will see in more detail later, would be that there will not remain any place for the so-called UV/IR mixing effect, which is known as a common artifact one expects to face within the models with canonical noncommutativity, the algebra (1).

Here we consider the noncommutativity only among spatial coordinates. In [8–10] a situation is considered in which noncommutativity is introduced between spatial directions and time, that is

\[ [\hat{x}_a, \hat{t}] = \frac{i}{\kappa} \hat{x}_a, \]

\[ [\hat{x}_a, \hat{x}_b] = 0, \]

where \( \kappa \) is a constant.

The scheme of this paper is the following. In the next section, some basic aspects of the group algebra are

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reviewed, mainly to fix notations. In the section “The real scalar field” a model is investigated containing a real field with momenta in a compact group. In the section “An example: the group SU(2)” this case is specialized to the group SU(2) or SO(3). The last section is devoted to concluding remarks, and a discussion of the possible divergences of the theories is presented.

The group algebra. – For a compact group \( G \), there is a unique measure \( dU \) (up to a multiplicative constant) with the invariance properties

\[
\begin{align*}
\int d(VU) &= dU, \\
\int d(UV) &= dU, \\
\int d(U^{-1}) &= dU,
\end{align*}
\]

for any arbitrary element \( (V) \) of the group. These mean that this measure is invariant under the left translation, right translation, and inversion. This measure, the (left-right-invariant) Haar measure, is unique up to a normalization constant, which defines the volume of the group:

\[
\int_G dU = \text{vol}(G).
\]

Using this measure, one constructs a vector space as follows. Corresponding to each group element \( U \) an element \( \varepsilon(U) \) is introduced, and the elements of the vector space are linear combinations of these elements:

\[
f := \int dU f(U) \varepsilon(U).
\]

The group algebra is this vector space, equipped with the multiplication

\[
f g := \int dU dV f(U) g(V) \varepsilon(UV),
\]

where \( (UV) \) is the usual product of the group elements. \( f(U) \) and \( g(U) \) belong to a field (here the field of complex numbers). It can be shown that if one takes the central extension of the group \( U(1) \times \cdots \times U(1) \), the so-called Heisenberg group, with the algebra (1), the above definition results in the well-known star product of two functions, provided \( f \) and \( g \) are interpreted as the Fourier transforms of the functions.

So there is a correspondence between functionals defined on the group, and the group algebra. The definition (7) can be rewritten as

\[
(fg)(W) = \int dV f(WV^{-1}) g(V),
\]

\[
= \int dU f(U) g(U^{-1}W).
\]

Using Schur’s lemmas, one proves the so-called grand orthogonality relation which states that there is an orthogonality relation between the matrix functions of the group:

\[
\int dU U_{\lambda}^{a} b U_{\mu}^{-1} c = \frac{\text{vol}(G)}{\dim_{\lambda}} \delta_{\lambda\mu} \delta_{a}^{b} \delta_{c}^{d},
\]

where \( U_{\lambda} \) is the matrix of the element \( U \) of the group in the irreducible representation \( \lambda \), and \( \dim_{\lambda} \) is the dimension of the representation \( \lambda \). Exploiting the unitarity of these representations, one can write (9) in the more familiar form

\[
\int dU U_{\lambda}^{a} b U_{\mu}^{-1} c = \frac{\text{vol}(G)}{\dim_{\lambda}} \delta_{\lambda\mu} \delta_{a}^{b} \delta_{c}^{d}.
\]

Using this orthogonality relation, one can obtain an orthogonality relation between the characters of the group:

\[
\int dU \chi_{\lambda}(U) \chi_{\mu}(U^{-1}) = \frac{\text{vol}(G)}{\dim_{\lambda}} \delta_{\lambda\mu},
\]

or

\[
\int dU \chi_{\lambda}(U) \chi_{\mu}^{*}(U) = \frac{\text{vol}(G)}{\dim_{\lambda}} \delta_{\lambda\mu},
\]

where

\[
\chi_{\lambda}(U) := U_{\lambda}^{a} a.
\]

The delta distribution is defined through

\[
\int dU \delta(U) f(U) := f(1),
\]

where \( 1 \) is the identity element of the group. It is easy to see that this delta distribution is invariant under similarity transformations, as well as inversion of the argument:

\[
\delta(UV^{-1}) = \delta(U),
\]

\[
\delta(U^{-1}) = \delta(U).
\]

The regular representation of the group is defined through

\[
U_{\text{reg}} \varepsilon(V) := \varepsilon(UV),
\]

from which it is seen that the matrix element of this linear operator is

\[
U_{\text{reg}}(W,V) = \delta(W^{-1} UV).
\]

This shows that the trace of the regular representation is proportional to the delta distribution:

\[
\chi_{\text{reg}}(U) = \int dV U_{\text{reg}}(V,V),
\]

\[
= \text{vol}(G) \delta(U).
\]

So the delta distribution can be expanded in terms of the matrix functions (in fact in terms of the characters of irreducible representations). The result is

\[
\delta(U) = \sum_{\lambda} \frac{\dim_{\lambda}}{\text{vol}(G)} \chi_{\lambda}(U),
\]

or

\[
\delta(UV^{-1}) = \sum_{\lambda} \frac{\dim_{\lambda}}{\text{vol}(G)} U_{\lambda}^{a} b V_{\lambda}^{-1} a,
\]

\[
= \sum_{\lambda} \frac{\dim_{\lambda}}{\text{vol}(G)} U_{\lambda}^{a} b V_{\lambda}^{-1} a.
\]
This shows that other functions are also expandable in terms of the matrix functions:

\[ f(U) = \sum_{\lambda} \frac{\dim_\lambda}{\text{vol}(G)} U^*_\lambda a f_{\lambda a} b. \quad (21) \]

where

\[ f_{\lambda a} b : = \int dV V^{-1}_{\lambda a} f(V), \]

\[ = \int dV V^*_{\lambda a} b f(V). \quad (22) \]

Using this and (8), one arrives at

\[ (f g)_{\lambda a} b = f_{\lambda a} c g_{\lambda c} b. \quad (23) \]

Next, one can define an inner product on the group algebra. Defining

\[ \langle \epsilon(U), \epsilon(V) \rangle := \delta(U^{-1} V), \quad (24) \]

and demanding that the inner product be linear with respect to its second argument and antilinear with respect to its first argument, one arrives at

\[ (f, g) = \int dU f^*(U) g(U), \]

\[ = \sum_{\lambda} \frac{\dim_\lambda}{\text{vol}(G)} f^*_{\lambda a} b g_{\lambda a} b. \quad (25) \]

Finally, one defines a star operation through

\[ f^*(U) := f^*(U^{-1}). \quad (26) \]

This is in fact equivalent to the definition of the star operation in the group algebra as

\[ [\epsilon(U)]^* := \epsilon(U^{-1}). \quad (27) \]

It is then easy to see that

\[ (f g)^* = g^* f^*, \quad (28) \]

\[ (f, g) = (f^* g)(1). \quad (29) \]

Here a note is in order. While the results of this section were obtained for compact groups, in some cases the compactness is not necessary. It is easy to see that provided (4) holds, (6) to (8), (14) to (17), (24), the first equality in (25), and (26) to (29) are still true, even if the group is noncompact.

The real scalar field. – To give motivation for the particular form of the action which is going to be written for a real scalar field, let us first consider the real scalar field on an ordinary \( \mathbb{R}^D \) space.

\[ \text{The real scalar field: the Fourier transform picture.} \]

To be consistent with the notation used throughout this paper, the Fourier transform (only on space) of the field is denoted by \( \phi \), while the field itself is denoted by \( \dot{\phi} \). So,

\[ \tilde{\dot{\phi}}(r) = \int \frac{d^Dk}{(2\pi)^D} \phi(k) \exp(\text{i} r \cdot k). \quad (30) \]

An action for a scalar field is

\[ S = \int dt d^Dr \]

\[ \times \left\{ \frac{1}{2} \left[ \dot{\phi}(r) \dot{\phi}(r) + \tilde{\phi}(r) \tilde{\phi}(r) \right] - \sum_{j=3}^{n} \frac{g_j}{j!} [\tilde{\phi}(r)]^j \right\}, \quad (31) \]

where \( g_j \)'s are constants and \( \tilde{\phi}(r) \) is a differential operator. This action is translation invariant, that is invariant under transformations

\[ \tilde{\phi}(r) \rightarrow \tilde{\phi}'(r) := \tilde{\phi}'(r - a), \quad (32) \]

where \( a \) is constant.

One can write action (31) and transformation (32) in terms of the Fourier transforms:

\[ S = \int dt \frac{1}{2} \int d^Dk_1 d^Dk_2 \left| \dot{\phi}(k_1) \right| \dot{\phi}(k_2) \]

\[ + \phi(k_1) O(k_2) \phi(k_2) \right\} \]
where $g_j$'s are constants and $O$ is a linear operator from the group algebra to the group algebra. In a more explicit form,\[
S = \int dt \left\{ \frac{1}{2} \int dU_1 dU_2 \left[ \hat{\phi}(U_1) \hat{\phi}(U_2) + \hat{\phi}(U_1) O(U_2, U) \phi(U) \right] \delta(U_1 U_2) - \sum_{j=3}^n g_j \int \left[ \prod_{i=1}^j dU_i \phi(U_i) \right] \delta(U_1 \cdots U_j) \right\}. \tag{37}
\]
This action would have a symmetry under
\[
\phi(U) \to \det(U_{\lambda}) \phi(U), \tag{38}
\]
where $\lambda$ is a representation of the group, provided $O(U_2, U) = O(U) \delta(U_2 U^{-1})$. \tag{39}

From now on, it is assumed that this is the case. So
\[
S = \int dt \left\{ \frac{1}{2} \int dU_1 dU_2 \left[ \hat{\phi}(U_1) \hat{\phi}(U_2) + \phi(U_1) O(U_2) \phi(U_2) \right] \delta(U_1 U_2) - \sum_{j=3}^n g_j \int \left[ \prod_{i=1}^j dU_i \phi(U_i) \right] \delta(U_1 \cdots U_j) \right\}. \tag{40}
\]
A simple choice for $O$ is
\[
O(U) = c \chi_{\lambda}(U + U^{-1} - 2 \mathbf{1}) - m^2, \tag{41}
\]
where $\lambda$ is a representation of the group, and $c$ and $m$ are constants. An argument for the plausibility of this choice is the following. Consider a Lie group and a group element near its identity, so that
\[
U_{\lambda} = \exp(\kappa T_{a\lambda}), \approx 1 + \kappa T_{a\lambda} + \frac{1}{2} (\kappa T_{a\lambda})^2, \tag{42}
\]
where $T_{a\lambda}$'s are the generators of the group. One has
\[
O(U) \approx c \chi_{\lambda} (T_a \kappa) \kappa^a \kappa - m^2, \tag{43}
\]
which is a constant plus a bilinear form in $\kappa$, just as was expected for an ordinary scalar field. In fact, if one introduces a small constant $\ell$ so that $\kappa$ is proportional to $\ell$, and $c$ is proportional to $\ell^{-2}$, then in the limit $\ell \to 0$ expression (43) is exactly equal to a constant plus a bilinear form.

An action of the form (40) with the choice (41), has also a symmetry under
\[
\phi(U) \to \phi(V U V^{-1}), \tag{44}
\]
where $V$ is an arbitrary member of the group.

One can write action (40) in terms of the Fourier transform of the field in time:
\[
\phi(t, U) := \int \frac{d\omega}{2\pi} \exp(-i \omega t) \tilde{\phi}(\omega, U), \tag{45}
\]
to arrive at
\[
S = \frac{1}{2} \int d\omega_1 dU_1 d\omega_2 dU_2 \left[ \right. \left. -\omega_1 \omega_2 \phi(U_1) \phi(U_2) + \phi(U_1) O(U_2) \phi(U_2) \right] \times \left[ 2 \pi \delta(\omega_1 + \omega_2) \delta(U_1 U_2) \right] - \sum_{j=3}^n g_j \int \left[ \prod_{l=1}^j d\omega_l \phi(U_l) \right] \times \left[ 2 \pi \delta(\omega_1 + \cdots + \omega_j) \delta(U_1 \cdots U_j) \right]. \tag{46}
\]
The first two terms represent a free action, with the propagator
\[
A(\omega, U) := \frac{i h}{\omega^2 + O(U)}, \tag{47}
\]
while the third term contains interactions. Any Feynman graph would consist of propagators, and $j$-line vertices to which one assigns
\[
V_j := \frac{g_j}{i h^j} \sum_{\pi} \{ 2 \pi \delta(\omega_1 + \cdots + \omega_j) \delta(U_{\Pi(1)} \cdots U_{\Pi(j)}) \}, \tag{48}
\]
where the summation runs over all $j$-permutations. Also, for any internal line there is an integration over $U$ and $\omega$, with the measure $d\omega dU/(2\pi)$. As the group is assumed to be compact, the integration over the group is an integration over a compact volume. Hence there would be no ultraviolet divergences.

One can compare this model to a field theory on a group manifold. In the latter model, the integration in (37) or (40) would be on the position not on the momenta, and the operator $O$ would be the differentiation with respect to the coordinates. In a model on a group manifold, the position coordinates are still commuting but the momenta are not. Here the situation is reversed, and it is not only a matter of convenience. The operator $O$ determines which model is being investigated: it is an algebraic model in terms of the momenta and a differentiation one in terms of the position. For models on group manifolds with compact groups, there would be no infrared divergences while here there is no ultraviolet divergence. The fact that for noncommutative geometry based on Lie groups the momenta are still commuting, is the reason why here the momentum picture has been preferred to the position picture.

One can also write action (40) in terms of the matrix elements of the field defined in (22). One arrives at
\[
S = \int dt \sum_{\lambda} \frac{\text{dim}_{\chi_{\lambda}}}{\text{vol}(G)} \text{tr} \left[ \frac{1}{2} \left( \hat{\phi}_{\lambda}^2 + \phi_{\lambda} \hat{\phi}_{\lambda} \right) - \sum_{j=3}^n g_j \phi_{\lambda}^j \right], \tag{49}
\]
where $\lambda$ is a representation of the group.
where $\phi_\lambda$ is defined in (22), the summation goes over irreducible representations of the group, and one has
\[
\tilde{\phi}_\lambda a^b := \frac{\dim_\rho}{\text{vol}(G)} \sum_{\sigma \rho} \frac{\dim_\sigma}{\text{vol}(G)} C_{\lambda, \sigma \rho} a^b c d \sigma f \phi_\rho c f,
\]
where $C$ is the kernel appearing in the decomposition of the product of the two representations $\sigma$ and $\rho$:
\[
U a^c e U a^d f = \sum_{\lambda} C_{\lambda, \sigma \rho} a^b c d \sigma f U a^b\sigma.
\]
Perhaps the form (49) shows more clearly the role of all representations of the group in the model, compared to models based on a single representation.

**An example: the group SU(2).** For the group $SU(2)$, one has
\[
f^a_{bc} = \epsilon^a_{bc}.
\]
A group element $U$ can be characterized by the coordinates $(k^1, k^2, k^3)$ such that
\[
U = \exp(\ell k^a T_a),
\]
where $\ell$ is a constant. The invariant measure is
\[
dU = \frac{\sin^2(\ell k/2)}{(\ell k/2)^2} \frac{dk^i}{(2\pi)^3},
\]
where
\[
k := (\delta_{ab} a^b b^a)^{1/2}.
\]
The reason for this particular choice of normalization is that for small values of $k$, (54) reduces to the integration measure corresponding to the ordinary space. The integration region for the coordinates is
\[
k \lesssim \frac{2\pi}{\ell}.
\]
Of course this does not mean that we are dealing with functions on a three-dimensional ball of radius $(2\pi/\ell)$. The functions are defined on a three-sphere, $S^3$. The situation is very much like the case of functions defined on a circle. One can say that the argument of such a function is between 0 and $(2\pi)$, while it is understood that the values of the function for 0 and $(2\pi)$ are the same.

In the small-$k$ limit, one also has
\[
\delta(U_1 \cdots U_i) \approx (2\pi)^3 \delta(k_1 + \cdots + k_i),
\]
which ensures an approximate momentum conservation. The exact conservation law, however, is that at each vertex the product of incoming group elements should be unity. For the case of a three-leg vertex, one can write this condition as
\[
\exp(\ell k_1^a T_a) \exp(\ell k_2^a T_a) \exp(\ell k_3^a T_a) = 1,
\]
or a similar condition in which $k_1$ is replaced by $k_2$ and vice versa. One has
\[
\exp(\ell k_1^a T_a) \exp(\ell k_2^a T_a) := \exp[\ell \gamma^a(k_1, k_2) T_a],
\]
where the function $\gamma$ enjoys the properties
\[
\gamma(k_1, \gamma(k_2, k_3)) = \gamma[\gamma(k_1, k_2), k_3],
\]
\[
\gamma(-k_1, -k_2) = -\gamma(k_2, k_1),
\]
\[
\gamma(k, -k) = 0.
\]
So that (58) becomes one of the three equivalent forms
\[
k_3 = -\gamma(k_1, k_2),
\]
\[
k_2 = -\gamma(k_3, k_1),
\]
\[
k_1 = -\gamma(k_2, k_3).
\]
The explicit form of $\gamma$ is obtained from
\[
\cos \frac{\ell \gamma}{2} = \cos \frac{\ell k_1}{2} \cos \frac{\ell k_2}{2} - \frac{k_1 k_2}{k_1 k_2} \sin \frac{\ell k_1}{2} \sin \frac{\ell k_2}{2},
\]
\[
\sin \frac{\ell \gamma}{2} = \epsilon^a_{bc} \frac{k_1^b k_2^c}{k_1 k_2} \sin \frac{\ell k_1}{2} \sin \frac{\ell k_2}{2} + \frac{k_2^b}{k_1} \sin \frac{\ell k_1}{2} \cos \frac{\ell k_2}{2} + \frac{k_1^b}{k_2} \sin \frac{\ell k_2}{2} \cos \frac{\ell k_1}{2}.
\]
It is easy to see that in the limit $\ell \to 0$, $\gamma$ tends to $k_1 + k_2$, as expected.

The choice (41) for $O$ turns to be
\[
O = 2c \left\{ \frac{\sin \left[ \left( s + \frac{1}{2} \right) \frac{\ell k}{2} \right]}{2} - (2s + 1) \right\} - m^2,
\]
where $s$ is the spin of the representation. For small values of $k$, this is turned to
\[
O \approx -c s \left[ s + \frac{1}{2} \right] (2s + 1) \left( \frac{\ell k}{2} \right)^2 - m^2, \quad (\ell k) \ll 1.
\]
One chooses $c$ so that in the small-$k$ limit $O$ takes the ordinary form of the propagator inverse:
\[
O \approx -k^2 - m^2, \quad (\ell k) \ll 1.
\]
Choosing
\[
c = \frac{3}{s(s + 1)(2s + 1) \ell^2},
\]
the propagator becomes
\[\text{see eq. (69) on the next page}\]
It is easy to see that in the limit $\ell \to 0$, the usual commutative propagator is recovered.

Similar things holds for the group $SO(3)$. One only has to replace the integration region by
\[
k \lesssim \frac{\pi}{\ell}.
\]
Concluding remarks. – A real-scalar-field theory was investigated constructed on a noncommutative space, the commutation relations of which are those of a compact Lie group. To avoid explicit calculus on such a noncommutative space, everything was defined on the momentum space. This space is commutative and one can attribute well-defined (local) coordinates to it, so that ordinary differential and integral calculus (on manifolds) can be performed on it. As far as observables of field theories are concerned, this momentum representation is sufficient. The Feynman rules for perturbative field theory were performed on it. As far as observables of field theories are concerned, this momentum representation is sufficient. The Feynman rules for perturbative field theory were obtained for the noncommutative model, and it was seen that for small momenta these are the same as the corresponding rules for ordinary field theories, as expected. Another way to state this is that there is a length parameter in the noncommutative theory so that if this length tends to zero, one recovers the results of ordinary field theories. Some comments are in order. As the commutation relations for the space coordinates are the commutation relations of the generators of a compact Lie group, say $SU(2)$ or $SO(3)$, the eigenvalues of the space coordinates are discrete. Roughly speaking, such theories resemble theories defined on lattices rather than on continua. But generally in lattice theories the rotational symmetry is broken, while in a noncommutative theory based on the group $SO(3)$ this is not the case. This similarity between the noncommutative theories discussed here and lattice theories is directly related to the fact that in these noncommutative theories (which are based on compact groups) there are no ultraviolet (UV) divergences. This is simply a result of the fact that the integration region for loop integrations is not $\mathbb{R}^4$, but $\mathbb{R}$ times a compact manifold. (This compact manifold is the group manifold.) This UV-finiteness of the model is reminiscent of the old expectation that in noncommutative spaces the theory might be free from the divergences caused by the short-distance behavior of physical quantities. In this sense noncommutative theories based on compact groups resemble ordinary (commutative) theories with a momentum cutoff.

It would be interesting to mention the fate of the UV/IR mixing phenomena [11]. As a generic property of models defined on canonical noncommutative spaces (1), certain combinations of external momenta and noncommutativity parameter $\theta$ may appear as a dynamical cutoff in momentum space. For example, in two external-leg diagrams of the $\phi^4$ theory, the combination $(p \circ p)^{-1/2}$ with $p \circ p := (p^\mu \theta_\mu^2 p')$ acts as a cutoff, causing that the contribution of the so-called nonplanar diagram be UV-finite [11]. In the extreme IR limit of external momenta ($p \to 0$), this cutoff tends to infinity and the result diverges. In such a case, in the IR limit of the theory the UV divergences of the commutative (ordinary) theory are restored. This is the so-called UV/IR mixing. If the noncommutative theory had been based on a commutative theory with a momentum cutoff, there would be no UV divergence and no UV/IR mixing.

Theories discussed here are free from UV divergences, as the momentum space is compact. In this sense, they are based on commutative theories with a momentum cutoff. Hence there is no UV divergence in the original theory to be restored in some IR limit, and there is no place for UV/IR mixing.

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