Low metallicity natal environments and black hole masses in Ultraluminous X-ray Sources

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ABSTRACT

We review the available estimates of the masses of the compact object in Ultraluminous X-ray Sources (ULXs) and critically reconsider the stellar-mass versus intermediate-mass black hole interpretations. Black holes of several hundreds to thousands of $M_\odot$ are not required for the majority of ULXs, although they might be present in the handful of known hyper-luminous ($\sim 10^{41}$ erg s$^{-1}$) objects and/or some sources showing timing features in their power density spectra. At the same time, however, stellar mass BHs may be quite a reasonable explanation for ULXs below $\sim 10^{40}$ erg s$^{-1}$, but they need super-Eddington accretion and some suitable dependence of the beaming factor on the accretion rate in order to account for ULXs above this (isotropic) luminosity. We investigate in detail a 'third way' in which a proportion of ULXs contain $\approx 30 - 90 M_\odot$ black holes formed in a low metallicity environment and accreting in a slightly critical regime and find that it can consistently account for the properties of bright ULXs. Surveys of ULX locations looking for a statistically meaningful relationship between ULX position, average luminosity and local metallicity will provide a definitive test of our proposal.

Key words: accretion, accretion discs — black hole physics — X-rays: binaries

1 INTRODUCTION

When, at the beginning of the '80s, point-like, off-nuclear X-ray sources were first detected in the field of nearby galaxies (see, e.g., Fabbiano 1989, 2006), it was immediately recognised that the luminosity of a subset of these objects was unusually large. If physically associated with their host galaxies, these sources had an isotropic luminosity well in excess of the Eddington limit for spherical accretion onto a $10 M_\odot$ compact object. These apparently Super-Eddington sources, later called UltraLuminous X-ray sources (ULXs), were first noticed in Einstein data (Long & van Speybroeck 1983; Helfand 1984; Fabbiano 2006).

Nowadays well in excess of 150 candidate ULXs have been detected and catalogued by a variety of X-ray observatories (see e.g. Roberts & Warwick 2000; Colbert & Ptak 2002; Swartz et al. 2004; Liu & Bregman 2005). A small fraction of ULXs are now known to be X-ray luminous interacting supernovae such as those described by Immler (2007). A much larger fraction have subsequently been identified with background AGNs ($\sim 25\%$; Swartz et al. 2004; see also Foschini et al. 2002b; Masetti et al. 2003; Wong et al. 2008). This background contamination is stronger in ellipticals, where it accounts for $\sim 44\%$ of all the ULXs, than in spirals ($\sim 14\%$; Swartz et al. 2004). However, the majority of these puzzling Super-Eddington sources constitute a very interesting class, that remains yet to be fully understood.

The recent detection of a $\sim 62$ day modulation in the light curve of M 82 X-1 has been interpreted as the orbital period of the system (Kaaret et al. 2006a,b; Feng & Kaaret 2007). A periodic modulation (12.5 hrs) has also recently been detected in another ULX (NGC 3379; Fabbiano et al. 2006). These results support the notion that ULXs are X-ray binary systems, where mass transferred from a donor star falls onto a compact object via an X-ray luminous accretion disc. The high X-ray luminosities of ULXs suggest this compact object is most likely a black hole (BH).

The X-ray spectral properties of ULXs show similarities with those of X-ray binaries (XRBs) in our Galaxy (e.g. Foschini et al. 2002b). In many cases the spectrum can be well reproduced by a multicolour disc (MCD) blackbody, representing emission from an accretion disc, plus a power-law continuum (PL), with the latter nominally representing emission from a Compton-scattering corona. This is the same canonical model employed to describe the spectra of Galactic black hole X-ray binaries (cf. McClintock & Remillard 2006). Interestingly, the derived temperature of the MCD component in ULXs is often much lower than that observed in XRBs (e.g. Miller et al. 2003, 2004; Feng & Kaaret 2005). However, for some of the brightest ULXs,
a possible curvature above 2-3 keV has been reported and equally acceptable fits of their spectra may be obtained with (physically) different models, that suggest the presence of hitherto unusual features such as an optically thick corona, a fast ionised outflow or a slim disc (e.g. Stobbart et al. 2006; Gonçalves & Soria 2006; Mizuno et al. 2007).

Fast X-ray variability can also reveal much about accretion-powered sources. Although most ULXs show little variability on timescales of seconds to hours (Swartz et al. 2004; Roberts et al. 2004), the recent detections of quasi periodic oscillations in the power density spectra of M 82 X-1 and NGC 5408 X-1 has shed new light on the timescales in the inner accretion disc of these systems (Strohmayer & Mushotzky 2003; Fiorito & Titarchuk 2004; Mucciarelli et al. 2006; Strohmayer et al. 2007; Casella et al. 2008).

Stellar optical counterparts have been discovered to be associated with a number of ULXs (Roberts et al. 2001; Goad et al. 2002; Liu et al. 2002, 2004a; Kaaret et al. 2004a; Zampieri et al. 2004; Kaaret 2005; Mucciarelli et al. 2005; Soria et al. 2005; Pakull et al. 2006), although only some of them have been associated with stellar objects of known spectral type (e.g. Liu et al. 2002, 2004a; Kaaret et al. 2004a; Mucciarelli et al. 2005). In almost all cases, the counterparts appear to be hosted in young stellar environments (e.g. Ramsey et al. 2006; Pakull et al. 2006; Liu et al. 2007) and have properties consistent with those of young, massive stars. However, some ULXs appear to be associated to older stellar populations and at least one possible later-type stellar counterpart is now known (Feng & Kaaret 2008; Roberts et al. 2008). Some ULXs are also associated with very extended optical emission nebulae, that may provide important information on the energetics and lifetime of these systems (Pakull & Mirioni 2002; Roberts et al. 2003). These nebulae are also beginning to be detected as extended radio sources (Miller et al. 2005; Lang et al. 2007).

The stellar environment of ULXs can also provide interesting constraints on the properties of ULX binary systems (ULXBs). Several ULXs are located in groups or clusters of OB stars. Isochrone fitting of the cluster colour-magnitude diagram has been attempted and provides cluster ages of tens of millions of years, although there is some disagreement among different authors (Ramsey et al. 2006; Pakull et al. 2006; Liu et al. 2007; Grisé et al. 2008). These analyses translate into upper limits for the donor masses in ULXBs, assuming that they are coeval to their parent OB association. Typical values of these mass limits are in the range \( \sim 10 - 20 M_\odot \). Comparison of stellar evolutionary tracks of ULXs with the photometric properties of their optical counterparts on the colour-magnitude diagram may also be used to constrain the masses of their donor stars (e.g. Soria et al. 2005; Copperwheat et al. 2005, 2007). If accurate photometry is available, this approach may also provide interesting clues to the BH mass, once binary evolution and X-ray irradiation effects are taken into account (Patruno & Zampieri 2008).

ULXs play a fundamental role in the framework of X-ray source populations in nearby galaxies and can be detected and studied at larger distances than more 'normal' binary sources. An important tool to study the global properties of these populations is their X-ray Luminosity Function (XLF). Grimm et al. (2003) and Gilfanov et al. (2004) found that the XLF of high mass X-ray binaries in the Milky Way, Magellanic Clouds and nearby starburst galaxies has a smooth, single power-law behaviour in a broad luminosity range \( 10^{36} - 10^{39.5} \text{ erg s}^{-1} \); see also Kaaret & Alonso-Herrero 2008). This suggests that ULXs with luminosity up to a few \( 10^{39} \text{ erg s}^{-1} \) may simply represent the high luminosity tail of the high mass X-ray binary population.

These pieces of observational evidence, along with the long-term flux variability and the correlated luminosity/spectral variability (e.g. Kubota et al. 2001; La Parola et al. 2001; Zampieri et al. 2004), strongly suggest that a large fraction of ULXs are accreting BH X-ray binaries with massive donors. The very high luminosity demands that the accretion rate be very high, even in case of efficient, disc accretion. A massive donor is then needed to fuel persistent ULXs, irrespectively of the BH mass (e.g. Patruno et al. 2005; Rappaport et al. 2005; Patruno & Zampieri 2008), and the identification of blue, massive stars as the counterparts of some ULXs confirms this interpretation. However, transient ULXs associated with older stellar populations, may be fueled through the rapid accretion of material accumulated in the accretion disc over a relatively long period of time, and not necessarily be associated with massive companions, as is the case for, e.g. the Galactic BH candidate GRS 1915+105 (King 2002).

The critical issue is then understanding what is responsible for the exceptionally high (isotropic) luminosity of these sources. Two main scenarios have been proposed. Firstly ULXs could be relatively normal stellar-mass BHs \( (\lesssim 20 M_\odot) \) that are either anisotropically emitting X-ray binaries in a peculiar evolutionary stage (King et al. 2001; King & Pounds 2003), or are truly emitting above the Eddington limit via a massive, modified accretion disc structure (e.g. photon bubble dominated discs, Begelman 2002, two-phase super-Eddington, radiatively efficient discs, Socrates & Davis 2006) slim discs, Ebisawa et al. 2003, or perhaps via some combination of the two (Poutanen et al. 2007; King 2008). Secondly, the compact object could simply be bigger, and the accretion would be in the sub-Eddington regime. In this case the compact object would be an intermediate mass black hole (IMBH) with a mass in excess of \( 100 M_\odot \) (e.g. Colbert & Mushotzky 1999). Population synthesis calculations show that, in both scenarios, the mass transfer rates needed to supply the majority of the ULXs can be attained over a significant fraction of the life time of the systems and that the production efficiency of the two models are comparable (albeit with very large uncertainties on both), provided that stellar mass BHs can exceed the Eddington limit by a factor \( \gtrsim 10 \) (Podsiadlowski et al. 2003; Rappaport et al. 2005; Madhusudhan et al. 2006, 2008).

In this Paper we review the available estimates of the masses of the compact object in ULXs and present a critical re-evaluation of the current evidence regarding the stellar-mass versus intermediate-mass black hole interpretation. Here and in a companion investigation (Mapelli, Colpi & Zampieri 2009) we highlight an alternative formation scenario, already suggested before but never explored in depth, in which a proportion of ULXs contain \( \sim 30 - 90 M_\odot \) BHs formed in a low metallicity environment and accreting in a slightly critical regime. The plan of the paper reflects this approach. We start from a quite comprehensive review of the available estimates of the masses of the compact objects in ULXs (§2) and critically reconsider the 'traditional' in-
BH masses in ULXs

Table 1. Masses of the BH hosted in some ULXs estimated using X-ray spectroscopic methods.

| ULX          | $L_{\text{X, max}}$ | Edd. limit$^b$ | MCD fit$^c$ | Schwarzschild disc$^d$ | KERRBB fit$^e$ | Slim disc fit$^f$ |
|--------------|------------------|----------------|-------------|----------------------|----------------|------------------|
| M 81 X-1     | 0.66             | 130            |             |                      | 49$^{+25}_{-16}$ |                  |
| M 81 X-9$^*$ | 1.1              | 220            | 330 – 470   | 150 – 215            | 63$^{+127}_{-50}$ | 73$^{+57}_{-22}$ |
| M 101 X-2    | 0.41             | 82             |             |                      | 63$^{+113}_{-31}$ |                  |
| NGC 253 X-1  | 0.29             | 58             |             |                      |                 |                  |
| NGC 253 X-3  | 0.1              | 20             |             |                      |                 |                  |
| NGC 1313 X-1 | 2.5              | 500            | 400$^{+40}_{-40}$ | 200$^{+40}_{-40}$  | 16$^{+1}$ |                  |
| NGC 1313 X-2 | 1.5              | 300            | 200$^{+160}_{-60}$ | 100$^{+35}_{-80}$  | 74$^{+5}$ |                  |
| NGC 4559 X-7 | 2.1              | 420            | 2500$^{+1950}_{-1100}$ | 1200$^{+500}_{-500}$ | 31$^{+12}_{-9}$ |                  |
| NGC 4559 X-10| 1.2              | 240            |             |                      | 23$^{+3}$ |                  |
| NGC 5204 X-1 | 0.5              | 100            |             |                      |                 |                  |

$^a$ Also known as Ho IX X-1
$^b$ From eq. (1).
$^c$ From eq. (2).
$^d$ From eq. (2). The values of $D$ and $K_{BB}$ from Miller et al. (2004a); Cropper et al. (2004) and Lorenzin & Zampieri (2009) (cos $i = 1$).
$^e$ From eq. (2). The values of $D$ and $K_{BB}$ from Miller et al. (2004a); Cropper et al. (2004) and Lorenzin & Zampieri (2009) (cos $i = 1$).
$^f$ Computed from eq. (1).

2 MASS ESTIMATES IN ULXS: METHODS AND RESULTS

Thanks to the identification of the optical counterparts of a handful of ULXs, the measurement of the mass function of ULXs is now well on the way to becoming possible, and will provide direct constraints on the mass of individual sources. Unfortunately, given the observational difficulties associated with such measurements - most notably their faintness, with typical magnitudes in the range $m_v \sim 22–26$ (Roberts et al. 2008), and the contamination of the counterpart spectrum by nearby stars - only one very recent measurement of optical periodicity has been made (Liu, Bregman & McClintock 2009), and a mass function is yet to be constrained. However, this is where the observational effort is focussed at present and where the definitive answer to the question of whether intermediate or stellar mass BHs power ULXs will come from. Claims of a radial velocity shift of $\sim 300$ km s$^{-1}$ in the He II $\lambda 4686$ line have been reported for the optical counterpart of NGC 1313 X-2 by Pakull et al. (2006), but this measurement may be uncertain (see e.g. Mucciarelli et al. 2005).

Until these measurements are performed, we have to rely on indirect methods to estimate the BH mass. Assuming that the emission of ULXs originates from accretion and that it is stationary and isotropic, a lower limit for the BH mass $M_{BH}$ is obtained for Eddington-limited accretion ($L \approx L_{\text{Edd}}$):

$$M_{\text{BH}} \geq \frac{80}{M_{\odot}} \left( \frac{L}{10^{40} \text{ erg s}^{-1}} \right) \sim 200 \left( \frac{L_{\text{X}}}{10^{40} \text{ erg s}^{-1}} \right),$$

(1)

where $L_{\text{X}}$ typically refers to the [0.2-10] or [0.3-10] keV band and we consider a 'bolometric correction' of $\sim 2$ to account for the flux emitted outside this band. In fact, for the typical spectral parameters of a bright ULX (column density $\sim 3 \times 10^{21}$ cm$^{-2}$ and power-law photon index $\sim 1.7$) the bolometric flux is $\sim 2$ times larger than the flux emitted in the 0.2-10 keV band (e.g. Patruno & Zampieri 2008). This equation is also sensitive to the material accreted; the Eddington limit will rise by a factor $\sim 3$ for the accretion of helium (e.g. Grimm et al. 2003). Masses of some ULXs calculated from equation (1) are reported in Table 1. Estimates based on this argument may become more reliable if there is some other evidence that the emission is (almost) isotropic, as for example when ULXs appear to be responsible for the photo-ionization of their surrounding optical nebulae (e.g. Kaaret et al. 2004a; Abolmasov et al. 2007).

Spectroscopic estimates of the BH masses have also been attempted assuming that the soft component observed in ULX X-ray spectra can be ascribed to emission from an accretion disc (e.g. Miller et al. 2003, 2004a). Assuming that this spectral component can be modelled with the so-called multicolour disc blackbody model (MCD; Mitsuda et al. 1984), it is possible to express the BH mass $M_{BH}$ as (e.g. Lorenzin & Zampieri 2009):

$$M_{\text{BH}} \geq \frac{M_{\odot}}{D^2 67.5 \left( \frac{D}{1 \text{ Mpc}} \right) \left( \frac{K_{BB}}{\cos i} \right)^{1/2}},$$

(2)

where $D$ is the distance of the source, $f$ is a colour correction.
The interpretation of the soft component in terms of emission from a standard accretion disc suffers from a high degree of equivocality. Spectral fits with a disc component and a comptonizing corona to the best available X-ray spectral data indicate that ULXs display distinct spectral curvature above 2 keV (Stobbart et al. 2006; Gladstone et al. 2009). This is because the corona is optically thick and cool, and hence hides the inner part of the accretion disc, in what is perhaps an extreme form of the so-called very high state of Galactic BH candidates (Done & Kubota 2006; Stobbart et al. 2006). In this case, the soft component is produced by the visible outer regions of the accretion disc. Only upper bounds to the inner disc radius can be obtained and, similarly, the spectroscopic estimates for $M_{\text{BH}}$ reported in Table 1 should be considered as upper limits. However, as the thick corona is probably only launched at extreme accretion rates, this implies that this “ultraluminous” state is in the Super-Eddington regime, and hence the BH masses are relatively small ($< 100 M_\odot$; Roberts 2007). Further to this, Gladstone et al. (2009) consider the energy required to launch the thick corona, and from that calculate the intrinsic (corona-less) disc temperatures, mainly recovering temperatures in the correct regime for the discs around stellar-mass BHs (0.7 - 1 keV). A subset of ULXs retain apparently cool discs even after this correction; however, Gladstone et al. (2009) argue this is because these are the highest accretion rate stellar-mass BHs, in which a strong wind is launched from the central regions, creating a cool photosphere. In a similar vein, for other models characterising super-Eddington accretion, such as the slim disc (e.g. Watarai et al. 2000; Ebisawa et al. 2003) or photon bubble models (Finke & Böttcher 2007), the entire 0.2–10 keV spectrum is produced in the accretion disc, although its physical state is completely different with respect to that of a standard disc. However, indirect estimates of $M_{\text{BH}}$ can again be obtained from X-ray spectral fits and give values typically in the range $\approx 15 - 75 M_\odot$ (Vierdayanti et al. 2008; see again Table 1).

The mass estimates inferred from X-ray spectral fits of ULXs depend critically on the interpretation of their spectra which, as mentioned above, is not unique. The situation will improve in the future as our understanding of the spectral evolution of ULXs will increase and it will be possible to select spectral models on the basis of their consistency with the observed correlation patterns (see e.g. Feng & Kaaret 2009; Kajava & Poutanen 2009 for preliminary work on ULX spectral variability based on XMM-Newton data). Nonetheless, the available data points towards BH masses definitely smaller than those estimated from the early MCD spectral fits.

In the last few years, X-ray timing has provided a new opportunity to estimate BH masses in ULXs thanks in particular to the detection of broad band noise and quasi periodic oscillations (QPOs) in the power density spectrum (PDS) of some ULXs, as M 82 X-1 ($\nu_{\text{QPO}} = 54 - 166 \text{ mHz}$; Strohmayer & Mushotzky 2003; Mucciarelli et al. 2006) and NGC 5408 X-1 ($\nu_{\text{QPO}} \approx 20 \text{ mHz}$; Strohmayer et al. 2007). Extrapolating timing and spectral-timing correlations known to exist for similar timing features in BH binaries and assuming that the frequency of the QPO scales inversely to $M_{\text{BH}}$, various estimates have been obtained, which are consistent with a rather broad interval of values: $10 - 1000 M_\odot$ for M 82 X-1 (Fiorito & Titarchuk 2004; Mucciarelli et al. 2006; Feng & Kaaret 2007), 600 – 5000 for NGC 5408 X-1 (Strohmayer et al. 2007). Recently, a new timing approach has been presented to assess BH masses in ULXs, which is based on the so-called “variability plane”, populated by both Galactic black-hole candidates and active galactic nuclei. Assuming that the accretion flow in ULXs behaves in a similar way (which remains an open question) and taking into account the uncertainty on the efficiency of the accretion disc, Casella et al. (2008) find that $M_{\text{BH}}$ is in the interval $\sim 95 - 1300 M_\odot$ for M 82 X-1 and $\sim 115 - 1300 M_\odot$ for

| ULX     | $L_{\text{Edd}}^{\text{a}}$ ($10^{46}$ erg s$^{-1}$) | Edd. limit$^{b}$ ($M_\odot$) | QPO$^{c}$ ($M_\odot$) | Break/Lack of variability$^{d}$ |
|---------|-----------------------------------------------|-------------------------------|-----------------------|---------------------------------|
| Ho II X-1 | 1.7 | 340 | $\lesssim 100$ |
| M 82 X-1 | 17 | 3400 | 95–1300 |
| NGC 5408 X-1 | 0.85 | 170 | 115–1300 | $\sim 100$ |
| NGC 4559 X-7 | 2.1 | 420 | 38-1300 |

\textsuperscript{a} From Cropper et al. (2004); Kaaret et al. (2004a); Mucciarelli et al. (2006); Strohmayer et al. (2007).
\textsuperscript{b} $M_{\text{BH}}$ computed from eq. (1).
\textsuperscript{c} Casella et al. (2008).
\textsuperscript{d} Cropper et al. (2004); Dewangan, Titarchuk & Griffiths (2006); Goad et al. (2006); Soria et al. (2003).
NGC 5408 X-1 (see Table 2). In combination with QPOs, the scaling of the break frequencies of the broad band noise, by comparison with the corresponding timing features of Galactic BH candidates, has also been proposed for estimating \( M_{\text{BH}} \). This method has been applied to NGC 5408 X-1 and gives a similar range of masses to QPOs (see Table 2). A tentative identification of a break at \( \sim 28 \text{ mHz} \) in the PDS of NGC 4559 X-7 has also been reported (Cropper et al. 2004), although Barnard et al. (2007) called it into question. The inferred BH mass is reported in Table 2.

The problem with using timing properties of ULXs to directly infer masses is that it remains to date unclear how exactly these quantities are related. All the estimates are based upon tentative identifications of the timing features and the use of scaling laws that are known to hold only for a limited number of objects, and are therefore highly uncertain.

A further possibility is to use the non-detection of variability power in the PDS to limit the size of the BH, assuming that all power is at higher frequencies, as is seen in various XRB states. Adopting this approach Goad et al. (2006) found \( M_{\text{BH}} \lesssim 100M_\odot \) for Holmberg II X-1 (see again Table 2). In fact, Heil et al. (2009) have demonstrated that suppressed variability (compared to the PDS of classic XRBs and AGNs) appears a common feature of ULXs. There are several possible explanations of this - the variability is limited to higher frequencies, the data in the XMM-Newton band pass are disc-dominated (and so any variability is heavily diluted) or the ULXs are in a new, super-Eddington accretion state in which the X-ray emission is very stable (note this is predicted in the hydrodynamic simulations of highly super-Eddington accretion by Ohsuga 2007). Again, the common thread running through all these models is that the BH is relatively small.

### 3 Intermediate or Stellar-Mass BHs?

We have reviewed the mass estimates drawn from observations of individual ULXs; we now ask how these results fit into our more general understanding of the possible nature of their underlying engines. ULX models differ mainly in the assumptions on the physical state of the disc and its mode of accretion. If the accretion disc is in a standard regime, then emission is isotropic and the most straightforward interpretation of the exceptionally high luminosity of ULXs is that they contain BHs with large masses. Both the very high luminosity and low characteristic temperature (and high normalization, see previous Section) of the soft spectral component have been taken as evidence for this interpretation.

But how big is the BH mass? Early estimates based on equations (1) and (2) gave masses largely in excess of \( 100M_\odot \) (up to several thousands; see Table 1). The obvious question is then how big this massive may form. It has been proposed that remnants from the collapse of Population III stars formed in cosmological epochs (Madau & Rees 2001) may trigger ULX activity, if they can capture a donor star to accrete from. A second formation route for IMBHs may be in globular clusters, through repeated mergers of stellar mass BHs (Miller & Hamilton 2002), or in young, dense stellar super clusters, from the dynamical collapse of supermassive stars in their centres (e.g. Portegies Zwart et al. 2004). ULX activity would be sustained by binary companions captured in the cluster (Blecha et al. 2006). The latter may be a possible explanation for some ULXs (e.g. M82 X-1) but, in galaxies with starburst activity, the majority do not appear inside such supermassive clusters (e.g. Zezas et al. 2002; Kaaret et al. 2004b). Another difficulty with the IMBH interpretation is the apparent break in the power-law slope of the XLF of the high mass X-ray binary population in external galaxies at a luminosity \( \sim 2 \times 10^{36} \text{ erg s}^{-1} \). If there is a population of large IMBHs contributing to the XLF, the break suggests that they turn off at this luminosity; yet this is at a rather low fraction of the Eddington limit for putative large IMBHs (\( \sim 10\% \) for a \( 1000M_\odot \) BH). This would be rather unusual behaviour, as no other accreting class switches off at only a fraction of their Eddington limit (Roberts 2007). Finally, the co-location of ULXs with regions of star formation, such as those in the Antennae and Cartwheel galaxies, implies that they must be (relatively) short-lived, which requires successive generations of ULXs to be formed over the duration of the star formation event. Thus, if all ULXs in these regions were IMBHs, an unfeasibly large fraction of star forming mass would end up in IMBHs (King 2004; Mapelli et al. 2005). In principle, these arguments rule out all but a small minority of ULXs from being IMBHs bigger than \( 100M_\odot \).

So can we explain ULXs as stellar-mass BHs? If the accretion flow in ULXs is in a different regime, the situation may be different and either the isotropy and/or the Eddington limit may be circumvented. This occurs if the accretion rate is at or above Eddington, so that radial advection of thermal and radiative energy (slim disc; Abramowicz et al. 1988; Ebisawa et al. 2003) and/or radiation-driven instabilities (photon bubble model; Begelman 2002; 2006) set in. A slim disc can sustain larger accretion rates than a standard disc and modest super-Eddington luminosities (\( \lesssim 10L_{\text{Edd}} \)). As the emission is isotropic, luminosity scaling arguments similar to those discussed above give \( M_{\text{BH}} \gtrsim 20(L_X/10^{36} \text{ erg s}^{-1})M_\odot \). Therefore, the presence of slim discs would imply that only the brightest known ULXs (with \( L_X \gtrsim 3 \times 10^{36} \text{ erg s}^{-1} \)) need BHs significantly more massive than the stellar-mass BHs in our Galaxy (see also Table 1). Accretion discs dominated by photon bubble transport may also reach super-Eddington luminosities, while remaining geometrically thin. For a \( 20M_\odot \) BH, the maximum luminosity is \( \sim 30L_{\text{Edd}} \) (Begelman 2006). This may in principle account for the emission of all but the very brightest (\( \gtrsim 5 \times 10^{36} \text{ erg s}^{-1} \)) ULXs, but photon bubble-dominated discs are subject to the same thermal and viscous instabilities that characterize the inner region of radiation pressure dominated discs and hence may be significantly unsteady (with more than a factor of 10 variation in the emitted flux) on short time scales (e.g. Zampieri et al. 2001). However, bright (\( \gtrsim 10^{39} \text{ erg s}^{-1} \)) ULXs typically show more limited fluctuations in the observed X-ray luminosity. An alternative scenario for generating steady, super-Eddington luminosities from stellar-mass BHs, is the radiatively efficient, two-phase super-Eddington accretion disc model by Socrates & Davis (2006). In this model the gravitational potential energy is not trapped in the disc, but effectively removed from it through magnetic buoyancy and dissipated in a corona. Fields anchored in the disc and/or Compton drag in the
low density corona prevent the launching of a wind, keeping the radiative efficiency high and assuring super-Eddington luminosities. However, we note that there are many assumptions and theoretical uncertainties that need to be clarified in this model, and a quantitative estimate of the maximum attainable luminosity is not yet available.

If accretion becomes largely super-Eddington, other processes may be present that complicate the picture. A thick disc may form and the emission becomes beamed \( (\text{King et al. 2001; King 2002}) \). At the same time, outflows and powerful ejection of matter along the axis perpendicular to the accretion disc may be produced (as in SS433; e.g. \text{Poutanen et al. 2007}). In these assumptions, emission is no longer isotropic. If \( L_{\text{iso}} \) is the apparent isotropic luminosity, the BH mass inferred from equation (1) must then be corrected for the so called beaming factor \( b_f \), that represents the fractional opening angle of the beam: \( M_{\text{BH}} \simeq 20(b_f/0.1)(L_{2-10,\text{iso}}/10^{40}\text{erg s}^{-1})M_{\odot} \). Hence, simple beaming can not account for bright \( (\gtrsim 2 \times 10^{40} \text{ erg s}^{-1}) \) ULXs, unless one is willing to consider a beaming factor \( < 0.05 \) and therefore a very narrow opening angle \( (< 1^\circ) \), that appears to be more consistent with a jet rather than a geometrical funnel in a thick disc. However, in addition to having beamed emission, thick discs may also radiate at super-Eddington luminosities, reaching at most \( (1 + \ln(M/M_{\text{Edd}}))/L_{\text{Edd}} \) (e.g. \text{Poutanen et al. 2007; King 2008}). A hypothetical ULX in this state would bear some similarity to a Galactic BH candidate in the very high state, albeit with the ULX being in a much more extreme version of this state, with powerful winds carrying away the excess matter and energy, potentially thickening the corona and even producing a cool photosphere (consistent with the X-ray spectral modelling of \text{Gladstone et al. 2005}). A combination of a beaming factor \( b_f \simeq 0.3  \sim 0.5 \) and super-Eddington emission \( (\sim 10 L_{\text{Edd}}) \) may in principle explain ULXs with luminosities up to \( \sim 10^{40} \text{ erg s}^{-1} \), assuming accretion onto a stellar-mass BH. Furthermore, if a dependence of the beaming factor on the accretion rate is assumed \( (b_f \propto (M/M_{\text{Edd}})^{-2/3}; \text{King 2009}) \), it might be possible to account also for the high luminosity tail of the ULX population.

In order to assess the viability of the different scenarios for the origin of ULXs, it is necessary to understand the possible evolutionary history of the various types of candidate binary systems and compare them with the available observations. Calculations of the evolutionary tracks of ULX binaries and model population studies of systems containing stellar-mass BHs and IMBHs have been carried out by several authors (e.g. \text{Podsiadlowski et al. 2003; Patruno et al. 2005; Rappaport et al. 2005; Madhusudhan et al. 2006}). Although calculations depend sensitively on uncertain parameters of the common envelope phase, it turns out that stellar-mass BHs accreting at super-Eddington rates may be able to account for most of the observed ULXs (except for the brightest), if they violate the Eddington limit by a factor \( \sim 10 - 30 \) (\text{Podsiadlowski et al. 2003; Rappaport et al. 2005}). Similarly, IMBH systems might produce bright \( (L_X \gtrsim 10^{40} \text{ erg s}^{-1}) \), persistent ULXs and have an acceptable production efficiency if the donor star is \( \gtrsim 10 M_{\odot} \) and the initial orbital separation is small \( (\lesssim 6 - 40 \text{ times the initial main sequence radius of the donor} \) \text{Patruno et al. 2005; Madhusudhan et al. 2006}).

Along the same lines, theoretical calculations of the color-magnitude (CM) diagrams for systems containing stellar-mass BHs and IMBHs are used to constrain ULX models and their evolutionary history by comparison with observations of their optical counterparts \text{Madhusudhan et al. 2008; Patruno & Zampieri 2008}. As already mentioned, in order to supply the mass transfer rates needed to fuel ULXs, rather massive donor stars are required. The evolutionary tracks of such systems are strongly affected by the binary interaction and the emission from the accretion disk, including X-ray irradiation. Numerical computations show that the regions of the CM diagram with the highest probability of finding ULX optical counterparts have \( B - V \) between \( \sim -0.1 \) and \( -0.3 \) and correspond to the early phases of the evolution of massive donors, while they are on the main sequence or the subgiant branch \text{Madhusudhan et al. 2008}. This result is consistent with the properties of the observed counterparts. Similarly, the most favourable orbital periods are between 1 and 10 days, corresponding again to the main sequence or subgiant phases. This is true for both stellar-mass BH and IMBH systems and depends on the fact that the donors spend most of their life time in these phases. In these conditions normal nuclear-driven mass transfer is effective and provides sufficiently high transfer rates to sustain the ULX emission, although in rare circumstances the evolution may be driven by the thermal timescale mass transfer during the giant phase.

4 A DIFFERENT INTERPRETATION

A critical revaluation of the available observational evidence presented in Sections 2 and 3 indicates that BHs of several hundreds to thousands \( M_{\odot} \) are not required for the majority of ULXs. However, it is not possible to rule out that they are present in the handful of known hyper-luminous \( (\sim 10^{41} \text{ erg s}^{-1}) \) objects and/or in the sources showing large amplitude broad band noise or QPOs in their PDS (such as M 82 X-1 and NGC 5408 X-1; see Table 2). At the same time, models with stellar mass BHs may work for a large fraction of the ULX population, if the accretion flow has some degree of beaming and is super-Eddington. Bright \( (\gtrsim 10^{40} \text{ erg s}^{-1}) \) ULXs may be accounted for if some form of modified beaming that accounts for a suitable dependence of \( b_f \) on the parameters of the accretion flow is allowed \text{King 2009}. Although none of these scenarios can be ruled out, the fact that the observational limits discussed in Section 2 are converging towards masses \( \lesssim 100M_{\odot} \), but bigger than stellar mass BHs, led us to consider an alternative interpretation. In our scenario bright ULXs may contain BHs with masses above 30-40 \( M_{\odot} \) and up to \( \sim 80 - 90M_{\odot} \), formed from ordinary stellar evolution of massive \( (30 - 120M_{\odot}) \) stars in a low metallicity natal environment. While this idea has already been suggested before (e.g. \text{Pakull & Mirioni 2002; Cropper et al. 2004; Zampieri et al. 2004}), it has not yet been explored quantitatively in detail.

Stars with mass \( \gtrsim 8 M_{\odot} \) produce compact remnants from the gravitational collapse of the iron core. For stars up to \( \sim 25 - 30M_{\odot} \), the collapse is halted when the core reaches nuclear densities: the star explodes and a neutron star forms. For larger main sequence masses, the early accretion of the inner mantle onto the core before shock passage and
the fallback of material afterwards cause the newly formed proto-neutron star to collapse to a BH after the supernova explosion (Zampieri 2002 and references therein). At solar metallicity, these fallback BHs reach at most $\sim 10 \, M_\odot$ as, for very massive stars with mass $\gtrsim 40 \, M_\odot$, the stellar envelope is in large part effectively removed through line-driven winds, while the remaining part is expelled by the supernova explosion.

For sub-solar metallicities, however, this mechanism becomes progressively less efficient and stars with masses above $\sim 30 - 40 \, M_\odot$ may retain rather massive envelopes at the time of explosion. The supernova shock wave then loses more and more energy in trying to unbind the envelope until it stalls and most of the star collapses to form a BH with a mass comparable to that of the pre-supernova star (Fryer 1999). These may be the BHs hosted in some ULXs. Their mass would not be significantly larger than $\sim 80 - 90 \, M_\odot$ as above $\sim 100 - 120 \, M_\odot$ a star undergoes pulsational pair-instability in its core and most of the envelope mass is expelled. We note that the possibility of forming BHs in this mass range through a different channel (binary mergers of massive components) was also proposed few years ago (e.g. Belczynski, Sadowski & Rasio 2004).

Computation of the evolution of massive stars up to advanced evolutionary stages for different metallicities and/or including mass loss have been performed by several authors (e.g. Hellings & Vanbeveren 1981; Maeder 1990, 1992; Portinari et al. 1998; Heger et al. 2003; Chieffi and Limongi 2004; Eldridge and Tout 2004; Hirschi 2007). These works adopted known empirical parameterizations of the mass loss rate for stars over the whole Hertzsprung-Russell diagram (e.g. de Jager et al. 1988; Nieuwenhuijzen & de Jager 1990). A certain degree of uncertainty is introduced if the star enters some peculiar evolutionary stages, such as the Wolf-Rayet (WR) stage (e.g. Langer 1989; Wellstein & Langer 1999). In fact, mass loss rates in WR and O stars may be significantly reduced if the wind is clumpy as a consequence of, e.g., supersonic turbulence (Moist & Carmelle 1994; Fuller et al. 2006). In particular, during the WR phase a decrease by a factor of $\gtrsim 3$ with respect to homogeneous wind models is attainable. This turns out to be compatible with some observational estimates and would clearly lead to more massive pre-supernova stars and hence more massive BHs. Additional uncertainty is caused by the dependence of mass loss on metallicity. A scaling law $\propto Z^{0.5}$ is often adopted for hot stars (see e.g. Kudritzki et al. 1989; Nugis & Lamers 2000). For example, at the end of main sequence, the mass of a star with an initial mass of $100 \, M_\odot$ is $\sim 25 \, M_\odot$ at solar metallicity and $\sim 75 \, M_\odot$ for $Z \approx 0.1 \, Z_\odot$. However, the mass lost during the He burning phase may be much more significant than that occurring during main sequence. According to the adopted mass loss history, the final mass of a star (after all nuclear burning stages are exhausted) may differ up to a factor of $\sim 2$ (or even more for clumpy winds) for a given metallicity. Considering again a star with an initial mass of $100 \, M_\odot$, Figure 1 shows that its final mass may be in the interval $\sim 3 - 6 \, M_\odot$ for $Z \approx Z_\odot$ and $\sim 30 - 70 \, M_\odot$ for $Z \approx 0.1 \, Z_\odot$.

As already mentioned above, the final evolutionary stages of a star and, in particular, the outcome of the final collapse depend critically on how massive is the envelope that it retains at the time of explosion. Therefore, owing to their larger final masses, the fate of stars with sub-solar metallicity is likely to be quite different from that of...
higher metallicity stars. Although different authors obtain different results for the mass of the compact remnant, it is not unreasonable to think that, if an envelope more massive than \( \sim 30 - 40 M_\odot \) is retained at the time of explosion, a low metallicity \((Z \approx 0.1 Z_\odot)\) star may collapse directly to form a BH of comparable mass. Looking again at Figure 1 it is possible to see that, already at metallicities \( \lesssim 0.22 Z_\odot \), stars with initial mass \( \gtrsim 60 M_\odot \) appear to possess final envelope masses that overcome this threshold for direct BH formation. Significant stellar rotation (hundreds of \( \text{km s}^{-1} \)) may change this picture somewhat, as it enhances the mixing of heavy elements throughout the star increasing the metal content of the envelope and consequently mass loss \(\text{e.g.} \) Maeder & Meynet 2001, Meynet & Maeder 2005. Also, ejection of part of the envelope during the final collapse can not be ruled out. However, if the core is not rapidly rotating, there is no good reason why most of the star should not collapse into a BH. Therefore, the formation of a \( \gtrsim 30 - 40 M_\odot \) BH throughout the evolution of a low metallicity, slowly rotating star of \( 40 - 120 M_\odot \) appears a viable possibility. The parameter space in the metallicity-main sequence plane where this formation channel may actually work corresponds to the black-shaded area between \( \sim 40 \) and \( \sim 100 M_\odot \) in Figure 1 of Heger et al. 2003 \(\text{(see also Figure 5 in Eldridge and Tout 2004)}\). At variance with intermediate mass BHs, the formation of these very massive stellar remnant BHs does not require an exotic, new mechanism but is referable to ordinary stellar evolution. Also, the unbroken power-law slope of the X-ray continuum distribution of BH masses from the stellar-mass BHs of \( 10 - 20 M_\odot \) up to the suggested BHs of \( \approx 40 - 90 M_\odot \) is consistent with the fact that variations in metallicity may produce a continuum distribution of BH masses from the stellar-mass BHs of \( 10 - 20 M_\odot \) up to the suggested BHs of \( \approx 40 - 90 M_\odot \). Given the size of these BHs, no difficulty with the fraction of star-forming mass in large starbursts ending up in BHs would arise, and so the objection of King 2001 for ULXs as \( \sim 1000 M_\odot \) IMBHs is circumvented. At the same time, only modest beaming (\( b_f \sim 0.5 \)) or slight violations of the Eddington limit (a factor of a few) would be needed to account for the luminosity of bright \( \gtrsim 10^{40} \text{erg s}^{-1} \) ULXs, at variance with the extreme accretion scenarios required by stellar mass BH models. Also the essentially isotropic irradiation of X-ray photoionised nebulae would find an explanation.

5 OBSERVATIONAL TESTS

Model population studies for our scenario are needed to determine the production efficiency of binary systems containing very massive BHs and to understand if they are in agreement, in a statistical sense, with the available X-ray and optical data of ULXs. However, there may be already some indications strengthening our present suggestion. In a parallel, preliminary investigation, we show that massive BHs formed in low metallicity environments might well explain most of the ULXs observed in the Cartwheel galaxy (Mapelli, Colpi & Zampieri 2009). Also, the optical luminosities of massive BH systems would be, on average, larger than that of stellar-mass BHs, as the former allow for more massive donors \( \gtrsim 25 M_\odot \) and have more extended accretion discs that dominate the optical emission. This would make them more consistent than stellar-mass BHs with the observed distribution of the luminosity of the ULX optical counterparts \(\text{e.g.} \) Madhusudhan et al. 2008, Patruno & Zampieri 2008.

A crucial aspect of the interpretation of ULXs in terms of BHs from the direct collapse of low-\( Z \), massive stars is the metallicity of the environment in which ULXs form. The available estimates appear to favour a low metallicity scenario, although there are some discrepancies. Optical observations appear to provide evidence of sub-solar metallicity in the environment of some ULXs. The emission nebula surrounding Ho II X-1 has a spectrum resembling that of a high-excitation H II region typical for low metallicity \( (Z \sim 0.1 Z_\odot) \) star-forming regions \(\text{e.g.} \) Mirioni 2002, Pakull & Mirioni 2002. The stellar environment of NGC 4559 X-7 shows a blue-to-red supergiant ratio \((3 \pm 1)\) and colours of the red supergiant population consistent with a low metal abundance environment with \( Z = 0.1 - 0.4 Z_\odot \), similar to that in the SMC and other nearby dwarf galaxies \(\text{e.g.} \) Soria et al. 2005. Also the stellar field around NGC 1313 X-2 is characterised by a low metallicity \((10/\mathcal{H} = -1.9 \pm 0.3)\), as inferred from the intrinsic colour of the red giant branch \(\text{e.g.} \) Grisé et al. 2008; studies of the metal abundance of H II regions in NGC 1313 also give low values of \( Z \sim 0.008; \) Walsh & Roy 1997, Hadfield & Crowther 2007. Winter et al. 2007 \(\text{analysed high signal-to-noise XMM}\) spectra of a sample of 14 ULXs, trying to determine the Oxygen abundance from the detection of K-shell photoionization edges. They apparently find values that match the solar abundance. However, the comparison of the X-ray estimates with a compilation of \([O/\mathcal{H}]\) ratios determined through spectrophotometric studies of H II regions \(\text{Pilyugin} 2004\) and with the luminosity-metallicity relation derived from the Sloan Digital Sky Survey \(\text{Tremonti} 2004\) shows significant systematic differences, especially at low galaxy luminosities and sub-solar metallicities. We note that Ho II X-1 provides a clear example of this dichotomy for a ULX; its XMM-Newton RGS spectrum suggests a significantly higher metallicity \((\sim 0.6 \text{times solar;} \text{Goad} et al. 2006)\) than the optical data. At the same time, X-ray spectral fits in at least the case of NGC 4559 X-7 seem to provide evidence for a more subsolar metallicity \((Z \sim 0.3 Z_\odot; \text{Cropper} et al. 2004)\).

It will be possible also to test our proposal against the stellar-mass BH interpretation as it will lead to a different spatial distribution of bright ULXs. In fact, ULXs from stellar-mass BHs should essentially appear anywhere in regions of star formation or in young stellar environments, regardless of metallicity. Indeed there may even be a bias towards these objects appearing in low metallicity regions, as effects such as the low mass loss rate in stellar winds will tend to keep binaries close, allowing more high mass transfer systems. However, if we make the reasonable assumption that the modes of accretion and Eddington rates will be similar in both standard \( 10 - 20 M_\odot \) black holes, and the \( \sim 30 - 90 M_\odot \) black holes examined in this paper, then the latter black holes may simply be on average brighter. Hence, in our proposed scenario ULXs should show some evidence of correlation \(\text{in terms of position and average luminosity) with low metallicity environments. So, one of the definitive
tests of our proposal would be to survey ULX locations, and determine whether a relationship between ULX luminosity and local metallicity was evident in a large enough sample to provide statistically meaningful results.

We note that evidence supporting our argument is already available. Firstly, Swartz et al. (2008) have recently surveyed galaxies within the Local Volume and determined that the specific ULX frequency decreases with host galaxy mass above $\sim 10^{10.5} M_\odot$. This means that smaller, lower metallicity systems have more ULXs per unit mass than larger galaxies, consistent with the idea that BHs can at least form and/or feed more efficiently in low metallicity environments. Secondly, we note that there is interesting evidence from our own relative backyard that the brighter ULXs are more abundant in the late-type spiral galaxies we might expect to be low-metallicity systems. The ROSAT High Resolution Imager ULX survey of Liu & Bregman (2005) lists 15 ULXs within 5 Mpc, four of which have observed X-ray luminosities in excess of $5 \times 10^{39}$ erg s$^{-1}$. All four of the very luminous ULXs reside in galaxies of Hubble type Sd or later. This compares to only 3 of the 11 lower-luminosity ULXs being hosted by similarly late systems - the remainder are in galaxies of type between Sab - Scd. Although the sample of ULXs is small, this clearly supports the case that brighter ULXs preferentially occur in the smaller, lower-metallicity systems where we might expect to find the very massive stellar remnant BHs.

It is worth noting that, recently Prestwich et al. (2007) and Silverman & Filippenko (2008) succeeded in performing dynamical mass measurements using Gemini and Keck spectra of the Wolf-Rayet optical counterpart of IC 10 X-1, a variable X-ray source in the the Local Group metal poor starburst galaxy IC 10. They find a BH mass in the range $23 \sim 33 M_\odot$, which represents the most massive BH known to exist in a binary system and definitely corroborates our interpretation.

6 CONCLUSIONS

In the last few years, X-ray and optical observations have significantly boosted our understanding of ULXs. We are now confident that the majority of these sources are X-ray binaries in external galaxies and we suspect that many may have massive binary companions. Yet, the most fundamental questions on ULXs still remain to be definitively answered: do they contain stellar or intermediate mass BHs? How do they form?

A critical revaluation of the available evidence presented indicates that BHs of several hundreds to thousands $M_\odot$ are not required for the majority of ULXs, although they might be present in the handful of known hyper-luminous ($\sim 10^{41}$ erg s$^{-1}$) objects and/or some sources showing timing features in their power density spectra. At the same time, however, stellar mass BHs may be quite a reasonable explanation for ULXs below $\sim 10^{40}$ erg s$^{-1}$, but they need super-Eddington accretion and some suitable dependence of the beaming factor on the accretion rate in order to account for ULXs above this (isotropic) luminosity.

We investigated in detail an alternative scenario in which bright ULXs contain BHs with masses above $\sim 30 \sim 40 M_\odot$ and up to $\sim 80 \sim 90 M_\odot$, produced by stars with initial, main sequence mass above $\sim 40 \sim 50 M_\odot$. At sub-solar metallicity, the explosion energy of these stars is not sufficient to unbind the envelope and most of the star collapses to form a BH with a mass comparable to that of the pre-supernova star. These may be the BHs hosted in bright ULXs. Above $\sim 100 \sim 120 M_\odot$ pulsational instability becomes effective and most of the envelope is expelled from the star.

The formation of these very massive stellar remnant BHs does not require an exotic, new mechanism but is referable to ordinary stellar evolution. For luminosities $\sim 10^{40}$ erg s$^{-1}$, this would imply only modest violations of the Eddington limit, attainable through very modest beaming ($b_f \sim 0.5$) and/or slightly super-critical accretion.

Measurements of the metallicity of the environment of some ULXs appear to favour a low metallicity scenario, although there are some discrepancies. Surveys of ULX locations looking for a statistically meaningful relationship between ULX number, position, average luminosity and local metallicity will provide a definitive test of our proposal.

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