Analysis of the resistance in p-SiGe over a wide temperature range

V. Senz\(^1\), T. Ihm\(^1\), T. Heinzel\(^1\), K. Ensslin\(^1\), G. Dehlinger\(^2\), D. Grützmacher\(^2\), U. Gennser\(^2\), E.H. Hwang\(^3\), and S. Das Sarma\(^3\)

\(^1\)Solid State Physics Laboratory, ETH Zürich, CH-8093 Zürich, Switzerland
\(^2\)Paul Scherrer Institute, CH-5234 Villigen PSI, Switzerland
\(^3\)Department of Physics, University of Maryland, Maryland 20742-4111, USA

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The temperature dependence of a system exhibiting a 'metal-insulator transition in two dimensions at zero magnetic field' (MIT) is still under discussion. We have recently analyzed the temperature dependence of the metallic phase in p-SiGe in terms of temperature dependent screening and quantum corrections to the conductivity \([5]\). At low temperatures the temperature dependence of the conductivity could be quantitatively described by three contributions, namely \(\sigma(T) = \sigma_D(T) + \delta\sigma_{WL}(T) + \delta\sigma_I(T)\), where \(\sigma_D(T)\) is the Drude conductivity, \(\delta\sigma_{WL}(T)\) and \(\delta\sigma_I(T)\) are the weak localisation (WL) and the interaction contributions, respectively. It was shown that the metallic behavior in the SiGe-systems stems from screening effects contributing to \(\sigma_D(T)\). In the present study we focus on the classical Drude part of the conductivity and extend the experimental range of temperatures up to 90 K. We aim at reproducing the non-monotonic temperature dependence of the experimental curves with a classical scattering description of the resistivity \([3]\). In this way we are able to make a quantitative comparison with the experimental data in a wide range of temperatures \((1.7K \leq T \leq 20 K)\) and densities \((1.5 \times 10^{11} \text{ cm}^{-2} \leq p \leq 5.1 \times 10^{11} \text{ cm}^{-2})\). Below 4K temperature dependent screening is the dominant contribution to the temperature dependence of the resistivity. At higher temperatures the smearing of the Fermi function and phonon scattering produce a non-monotonic temperature dependence. The good agreement of theory with experiment indicates that temperature dependent screening cannot only account for the metallic behavior close to the metal-insulator transition but is equally important in adjacent temperature and density regimes. The model cannot be applied in the insulating regime \((k_Fl_D < 1, \text{ where } k_F \text{ is Fermi wave vector and } l_D \text{ Drude mean free path})\).

In our MBE-grown samples the 2DHG resides in a 20nm Si\(_{0.85}\)Ge\(_{0.15}\) quantum well sandwiched between two undoped Si layers. Remote Boron doping was introduced at a distance of 15nm above the quantum well and a Ti/Al Schottky gate allowed tuning the hole density. For the transport measurements standard Hall-bars were used \((L=300\mu \text{m}, W=300\mu \text{m})\). The experiments were carried out in a \(^4\)He-system in a temperature range of 1.7K - 90K, using standard four terminal AC-technique. The hole density in the 2DHG was adjustable from complete depletion to a density of \(5.1 \times 10^{11} \text{ cm}^{-2}\), yielding a maximum mobility of \(7800 \text{ cm}^2/\text{Vs}\) at the highest density. The ratio of the Drude scattering time \(\tau_D\) and Shubnikov-de Haas relaxation time \(\tau_q\) is close to one, indicating that large angle scattering dominates the transport in such structures \([6,7]\). The insulating contributions of the weak localization effect and electron-electron interaction become significant at temperatures \(< 1K\). For a detailed study of these effects see Ref. \([3]\).

![FIG. 1. Temperature sweeps for different densities \(p = 5.1-2.0 \times 10^{11} \text{ cm}^{-2}\); the long range sweep (90K-1.7K, thin line) is done at B=0.5T, the short range sweep (10K-1.7K, thick line) at B=0T. The WL-contribution acts localizing and is suppressed at B=0.5T. \(\bullet\) represent the temperature \(T_c\) above which the system becomes non-degenerate, \(\blacksquare\) marks the Dingle temperature \(T_D\).](https://example.com/figure1.png)

Fig. 1 shows measurements of the longitudinal resistance \(\rho_{xx}\) as a function of temperature between 1.7K and 90K for different densities \(p\). A typical curve (e.g.
\( p = 2.3 \times 10^{11} \text{cm}^{-2} \), see Fig. 2) can be divided into four different temperature ranges:

i. \( T < 4 \text{K} \): the temperature dependence is metallic.

ii. \( 4 \text{K} < T < 14 \text{K} \): the temperature dependence is non-monotonic and a turnover to insulating behavior at higher temperatures occurs.

iii. \( 14 \text{K} < T < 60 \text{K} \): the dependence changes again to metallic behavior.

iv. \( 60 \text{K} < T < 90 \text{K} \): in this regime the 2DHG is not the only conducting channel in the sample (visible in the Hall density, not shown). This leads to a strong resistivity decrease as the temperature is raised from 60K to 90K.

\[
\frac{1}{\tau(E, T)} = \frac{m^*}{\pi \hbar^3} \int_0^\pi d\theta \frac{|V(q\theta)|^2}{\varepsilon^2(q\theta, T)} (1 - \cos \theta),
\]

where \( q = \sqrt{4m^*E(1 - \cos \theta)} / \hbar \) is the momentum transfer in a scattering event.

In the calculation we consider ionized impurity scattering \( \mathbb{I} \), interface roughness scattering \( \mathbb{I} \), alloy disorder scattering \( \mathbb{I} \), ionized background impurity scattering \( \mathbb{I} \) and acoustic phonon scattering \( \mathbb{I} \). They enter into \( < |V(q\theta)|^2 > \) which represents the sum of all individual mechanisms.

In this formulation the T-dependence of \( \rho_{xx} \) arises due to the energy averaging in Eq. 1 and due to the explicit T-dependence of Lindhard’s dielectric function

\[
\epsilon(q, T) = 1 + V(q) \Pi(q, T) F(q) [1 - G(q)],
\]

where \( V(q) = 2\pi e^2 / \varepsilon q \) is the interaction potential of the hole gas in two dimensions. The T-dependence in Eq. 3 again is due to energy averaging when calculating the polarizability function \( \Pi(q, T) \).

\[
\Pi(q, T) = \int_0^\infty dE \Pi(q, T = 0, E) \left( -\frac{df(E, \mu, T)}{dE} \right).
\]

In Eq. 3 \( F(q) \) is the form factor for the inversion layer \( \mathbb{I} \), and \( G(q) = q / g \sqrt{q^2 + k_F^2} \) with a degeneracy factor \( g \) is the local field correction in the Hubbard-approximation \( \mathbb{I} \).

Let us look in more detail into the Eqs. 1 and 2. We can assert two general effects for the temperature dependence: (1) the temperature dependence of the chemical potential \( \mu(T) = k_B T \ln(e^{E_F} / k_B T) - 1 \): this shifts the balance point around which the integral is performed to lower energies with increasing temperature (2) the integration itself performs an energy averaging around \( \mu(T) \) with width of about \( 4k_B T \) and depends strongly on the curvature of the integrand. In both cases the structure

FIG. 2. A closeup of the temperature dependence of the resistivity for the density \( p = 2.3 \times 10^{11} \text{cm}^{-2} \). Metallic and insulating behavior alternates depending on the temperature range.

In order to achieve a detailed understanding of these results we apply the standard semi-classical scattering model for the resistivity \( \mathbb{I} \), which is valid in the limit of \( k_F l_D > 1 \).

For finite temperatures and zero magnetic field the resistivity is expressed as

\[
\rho_{xx}(T) = \frac{m^*}{pe^2 \langle \tau(T) \rangle},
\]

with the effective mass of \( m^* = 0.25m_0 \) (obtained from the T-dependence of Shubnikov-de Haas oscillations) and the scattering time

\[
\langle \tau(T) \rangle = \frac{1}{E_F} \int_0^\infty dE E \tau(E, T) \left( \frac{df(E, \mu, T)}{dE} \right),
\]

where \( f(E, \mu, T) = (e^{E-\mu}/k_B T + 1)^{-1} \) is the Fermi distribution function (\( \mu \) temperature dependent chemical potential, \( k_B \) Boltzmann constant) and \( E_F \) is the Fermi energy.

The transport scattering rate is calculated according to

\[
\frac{1}{\tau(E, T)} = \frac{m^*}{\pi \hbar^3} \int_0^\pi d\theta \frac{|V(q\theta)|^2}{\varepsilon^2(q\theta, T)} (1 - \cos \theta),
\]
is the same: the integrand is a product of the symmetric (around $E = \mu$) function $df/dE$ with an asymmetric function, i.e. $E\tau$ and $\Pi$, which in total yields asymmetric functions but with opposite sign in the curvature. The curvature of $\Pi(q, T)$ is negative (see Fig. 3), meaning that $d\Pi/dT < 0 \Rightarrow dE/dT < 0 \Rightarrow d\tau/dT < 0$ due to energy averaging. The temperature dependence of the chemical potential leads to a slight damping of the energy averaging effect but can not change its overall behavior. In our case it is negligible: the effect becomes only important for temperatures higher than $T_c$ at which the chemical potential $\mu(T)$ deviates about 10% from its zero temperature value $\mu(0) = E_F$ (indicated by filled circles in Fig. 3).

Due to reduced screening $\tau(E, T)$ in Eq. (1) decreases with increasing temperature, but energy averaging counteracts this tendency since the curvature of $E\tau(E, T)$ is positive and therefore $d\langle\tau\rangle/dT > 0$. We illustrate this behavior of $E\tau(E, T)$ in Fig. 4. At the lowest temperatures the temperature dependence of the polarization function leads to a metallic behavior in the resistivity. In the special case of large angle scattering, i.e. $q = 2k_F$ which is dominant in SiGe samples [3], the temperature dependence of $\Pi(q, T)$ is strongest. Only at higher temperatures the insulating contribution of the energy averaging becomes significant and finally overcomes the metallic screening effect.

We now discuss the interplay of these effects in the temperature ranges (i)-(iii) mentioned in the beginning:

i. 2DHG is degenerate: the screening function dominates the temperature behavior. For large angle scattering (i.e. scattering for $q \approx 2k_F$) it has been shown that $\sigma_D(T) = \sigma_D(0) \left[ 1 - O(p) \frac{T}{T_F} \right] + O \left( \frac{T}{T_F} \right)^3$.

ii. the effect of energy averaging comes into play and counteracts temperature dependent screening. At higher temperatures energy averaging dominates, i.e. the temperature dependence is insulating.

iii. the system becomes non-degenerate: phonon scattering is the strongest scattering mechanism; the observed temperature dependence is due to phonon freeze out. This behavior is in agreement with the generally accepted linear temperature dependence of the acoustic deformation potential scattering for which a larger temperature coefficient of the resistivity is expected for lower carrier densities [8].

Deviations from this scenario occur at the two extreme densities. At high density the maximum in the resistivity disappears gradually and a purely metallic temperature dependence is observed over the entire T-range. At the lowest densities one enters the strongly localized regime where the presented model can no longer be applied.

We are now able to compare our calculations with the experiment. This is illustrated in Fig. 5. In order to achieve quantitative agreement between experiment and calculations we additionally included level-broadening effects on the polarization function characterized by the parameter $\Gamma = \hbar/\tau_B = k_B T_D$ ($T_D$ Dingle temperature, marked with filled squares in Fig. 1) [3]. This acts similar to thermal effects in rounding off the sharp corner of the polarization function visible in Fig. 3. The influ-

![Graph](image-url)
ence on the resistivity can be seen in the inset in Fig. 5: it reduces the effect of temperature dependent screening and the overall resistance rises.

The agreement of our simulations with the experiment is, even quantitatively, quite good. Only at the lowest densities the model fails in describing the experimental data, due to the reasons mentioned above.

In summary we have shown that the non-monotonicity in the temperature dependence of the resistivity of p-type SiGe samples showing a MIT can be described by the temperature dependence of the scattering time $\tau_D$. The metallic part stems from the energy averaging of the polarizability function $\Pi(q, T)$, the insulating part from the energy averaging when performing the integral for $\tau(E, T)$. This interplay is not significantly altered by the T-dependence of the chemical potential $\mu(T)$. Finally we compare our calculations with experimental curves and find quantitatively good agreement.

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