Jet mixing optimization using machine learning control

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Abstract We experimentally optimize mixing of a turbulent round jet using machine learning control (MLC) following Li et al (2017). The jet is manipulated with one unsteady minijet blowing in wall-normal direction close to the nozzle exit. The flow is monitored with two hotwire sensors. The first sensor is positioned on the centerline 5 jet diameters downstream of the nozzle exit, i.e. the end of the potential core, while the second is located 3 jet diameters downstream and displaced towards the shear-layer. The mixing performance is monitored with mean velocity at the first sensor. A reduction of this velocity correlates with increased entrainment near the potential core. Machine Learning Control (MLC) is employed to optimize sensor feedback, a general open-loop broadband frequency actuation and combinations of both. MLC has identified the optimal periodic forcing with small duty cycle as the best control policy employing only 400 actuation measurements, each lasting for 5 seconds. This learning rate is comparable if not faster than typical optimization of periodic forcing with two free parameters (frequency and duty cycle). In addition, MLC results indicate that neither new frequencies nor sensor feedback improves mixing further—contrary to many of other turbulence control experiments. The optimality of pure periodic actuation may be attributed to the simple jet flapping mechanism in the minijet plane. The performance of sensor feedback is shown to face a challenge for small duty cycles. The jet mixing results demonstrate the untapped potential of MLC in quickly learning optimal general control policies, even deciding between open- and closed-loop control.

Keywords Jet mixing · machine learning control

1 Introduction

The enhancement of jet mixing is important to many industrial applications. One example is the production of polymers where additional substances are added in the main stream by jets and have to be rapidly mixed downstream. Similar mixing examples can be found in food industry. Another application is combustion in aeroengines: The first row of dilution jets enhances combustion by mixing while the second downstream row cools the fluid to prevent thermal damage to the turbine. Staying with aeroengines, the targeted homogeneous mixing of injected fuel in the airstream is affected by two-phase jet mixing. Carrier airplanes reply on enhanced jet mixing with flapping to prevent the burning of their tail after landing.

Consequently, jet mixing control has drawn significant attention in the past few decades. Jet mixing control can be classified into passive control and active control based on whether additional energy input is needed. For example, the use of non-circular nozzles (Gutmark and Grinstein 1999) or the deployment of tabs at the nozzle exit (Bradbury and Kadem 1975; Zaman et al. 1994) are passive control techniques. These devices can display impressive performance for the
design conditions. Yet, the techniques are typically permanent fixtures which are not readily modified or removed. In addition, the control performance may deteriorate departing from the design condition. In contrast, the active control may potentially achieve higher performance for a large range of operating conditions. Examples are acoustic excitation (Zaman and Hussain 1980), plasma actuators (Sammy et al 2007), synthetic jet (Ho and Gutmark 1987), oscillating boundaries based on piezo-electric actuators (Wiltse and Glezer 1993), steady and unsteady control jets (Davis 1982; Yang and Zhou 2016; Zhou et al 2012). From an industrial perspective, actuators and sensors become increasingly more reliable and cheaper, i.e. more attractive for applications. Thus, active flow control enjoys increasing progress from many fronts, like hardware development, control logic and modeling.

Active control techniques can be performed in an open-loop or closed-loop manner. By definition, closing the loop with sensors increases the opportunity space of actuation and—properly set up—should improve performance. The loop may be closed for in-time response to coherent structures or for adapting in response to changing flow conditions. Note that, ‘in-time’ means that the actuation responds on a time-scale much smaller than that of the physical process, while ‘adapting’ means that the change of the actuation parameter is slow as compared to the physical process time-scale (Brunton and Noack 2015). Many closed-loop control schemes have been proposed and investigated, as discussed in references Brunton and Noack (2015), Choi et al (2008), Collis et al (2004). Closed-loop control may be classified into model-based or model-free approaches, depending on whether the law is derived from a plant model or only based on the plant response. Many computational flow control studies are based on local linearization of a Navier-Stokes based model. Linear models may be also identified in black-box manner from input-output data sequences (Rapoport et al 2003) or via a reduced-order model of the fluid dynamics (Choi et al 2008). The challenge to this approach is the nonlinear dynamics of turbulence displaying a rich set of spatial and temporal scales. In the current work, we optimize jet mixing with a single minijet actuator and two downstream hotwire sensors. As cost function, the averaged centerline velocity after the potential core is taken. For the control logic, we employ MLC based on linear genetic programing incorporating sensor-based feedback and multi-frequency forcing as well as combinations thereof. We follow a methodologically similar MLC study for drag reduction of an Ahmed body (Li et al 2017). MLC significantly increases the range of possible control laws as compared to ESC. Moreover MLC does not rely on the qualitative knowledge of steady-state maps and could find the global optimal value even when multiple extrema exist.

The manuscript is structured as follows. Section 2 describes the experimental setup. The control logic and associated MLC algorithm is outline in Sect. 3. In Sect. 4 and Sect. 5 we detail the experimental control with sensor-based feedback and multi-frequency forcing, respectively. In Sect. 6 the mixing performance of different control schemes is physically explained. The conclusions Sect. 7 summarize the MLC study and preview future developments.

2 Experimental set-up

2.1 Jet facility and actuator system

The jet control platform, consisting of an axisymmetric main jet and a minijet assembly, is the same as that used in Fan et al (2017). Figure 1 shows the schematic diagram of the
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Fig. 1 Schematic of the experimental setup: (a) main-jet assembly; (b) minijet assembly.

The air for both main jet and minijet comes from the same compressed air supply with a 12 bar gauge pressure. The compressed air firstly passes through a mixing chamber in the case of particle image velocimetry (PIV) or flow visualization measurements. After that, it passes through a tube, a plenum chamber, a 300 mm–long diffuser of 15° in half angle, two fine screens (7 mesh/cm) and a cylindrical settling chamber of 400 mm in length and 114 mm in the inner diameter. The nozzle contraction contour follows the equation:

\[
R = 57 - 47\sin{1.5(90 - 9\frac{x}{8})} (\text{mm}),
\]

as used in Zhou et al (2012) and Wu et al (2016). The nozzle was extended with a 47 mm-long smooth tube of diameter \(D = 20 \text{ mm}\). The Reynolds number \(Re_D = U_j D/\nu\) of the main jet is fixed at 8000, where \(U_j\) is the jet centreline velocity measured at the exit of the nozzle extension and \(\nu\) is the kinematic viscosity of air. Figure 1b shows the schematic diagram of the pulsed minijet assembly. There are six orifices of 1 mm in diameter drilled radially for the minijets, 17 mm upstream of the jet exit. To minimize the resistance, the orifices are suddenly expanded to a diameter of 4 mm before being connected via a short plastic tube to electromagnetic-valves (Koganei K2-100SF-09-LL), which are used to produce a pulsed minijet (Fig. 1b). The electromagnetic-valves can generate the pulsed jet in ON/OFF mode within the frequency range \([0, 500] \text{ Hz}\). In the present study, we control only the ON/OFF of the electromagnetic-valves. The maximum frequency of the minijet is 500 Hz, exceeding \(3f_0\) at \(Re_D = 8000\), where \(f_0 = 135 \text{ Hz}\) is the dominant frequency of the uncontrolled jet. Only one single minijet injection was investigated presently. The mass flow rate of the minijet varied via a mass flow controller (FLOWMETHOD FL-802) with a range of 7 Standard Liter Per Minute (SLPM), whose experimental uncertainty is no more than 1%. The mass flow rate of the minijet is fixed at 1.3 l/min for all the current investigations, corresponding to \(C_m = 1.2\%\), where the \(C_m\) is the mass flow rate ratio of the main jet to the minijet.

2.2 Velocity measurement and flow visualization facilities

The origin of the coordinate system is defined at the centre of the jet exit, with the \(x\)- and \(z\)-axes along the streamwise and the radial minijet directions, respectively, and the \(y\) axis is normal to the \((x, z)\) plane, following the right-hand rule. The \((x, z)\) and \((x, y)\) planes are hereinafter referred to as the injection and non-injection planes, respectively. In this paper, an asterisk superscript denotes normalization by \(D\) or/and \(U_j\).

Two tungsten wires of 5 \(\mu m\) in diameter, operated on a constant temperature circuit (Dantec Streamline) at an overheat ratio of 1.8, is placed at \((x^*, y^*; z^*) = (3, 0, 25)\) and \((5, 0)\) to measure the streamwise velocities \(u_{3D}\) and \(u_{5D}\), respectively. Note that the slight deviation from centreline for hotwire 1, which is for measuring the \(u_{3D}\), is used to prevent its influence on hotwire 2. The output signal of the hotwire...
anemometer is offset, amplified, and filtered at a cut-off frequency of 500 Hz before being digitized and saved in a PC by a National Instrument multifunction I/O Device (PXIe-6356). The sampling frequency is 1 kHz for all experiments. The hotwires were calibrated at the jet exit using a pitot tube and a micromanometer (Furness Controls FCO510). The experimental uncertainty of the hotwire measurement is estimated to be less than 2%.

A planar high-speed PIV system, including a high-speed camera (LaVision ImagerproHS4M, 2016 × 2016 pixels resolution) and pulsed laser source (Litron LDY304-PIV, Nd:YLF, 120 mJ/pulse) is deployed for flow visualization in the (x, z) and (x, y) planes. An oil droplet generator (TSI MCM-30) is used to generate fog for seeding flow. The seeding particles are supplied into the mixing chamber (Fig. 1a) to mix with air. Flow illumination is provided by a laser sheet of 1 mm in thickness generated by a pulsed laser source of 120 mJ via a cylindrical lens. Particle images are captured at a sampling rate of 250 Hz.

2.3 Real-time system

The real-time control is realized by a National Instrument PXIe-6356 multifunction I/O Device running at a sampling rate of $f_{RT} = 1$ kHz, where a LabVIEW Real-Time module is used to process the program. Sensor data acquisition and control command generation for open- and closed-loop control are performed at the same sampling rate. For the effective working of the actuator, a verification is performed before sending the command to the actuators to ensure that the ON/OFF command lasts at least 1 ms.

The available periodic frequencies $f$ consistent with $f_{RT}$ can be derived from $f = f_{RT}/N_{sp}$, where $N_{sp}$ is the number of sampling points in one time period $1/f$. The working frequency range of actuators ($0, 500$ Hz) imposes a minimum value for $N_{sp}$, being $N_{sp} \geq 2$. For a given $f$, the possible duty cycle $DC$ can be deduced from $DC = m/N_{sp}$, $m = 1, \ldots, N_{sp} - 1$. The value of $m$ starts from 1 and ends at $N_{sp} - 1$ to ensure a response time of 1 ms for an effective working of the actuators. Thus, the number of possible duty cycles $N_{DC}$ for a given $f$ is $N_{DC} = N_{sp} - 1 = f_{RT}/f - 1$, which increases with $N_{sp}$ and decreases with $f$. This process is similar to that used in [1, 2017].

Figure 2 displays the permitted frequencies and duty cycles and shows the manually selected frequencies which allow for a locally maximum number of duty cycles. The number of possible duty cycles decreases as the frequency increases due to the limited sampling points in one period. Note that Fig. 2 includes all possible $DC$ and $f$ within the range [10, 500]. The red filled circles highlight the selected periodic forcing cases considered in the following, which contains the most interesting frequency range [20, 200] Hz.

3 Machine learning control (MLC)

The mixing enhancement of a turbulent jet involves a large range of temporal and spatial scales with complex nonlinear interactions. Model-based control as used for the stabilization of steady laminar flows faces fundamental challenges for such turbulent flow and has, to the best of the authors knowledge, not been presented for experimental mixing enhancement of turbulence. Instead, we follow Dracopoulos & Kent’s pioneering work [Dracopoulos and Kent 1997], in which control design is framed as regression problem and solved with one of the most powerful method of machine learning: genetic programming. This strategy, referred to as Machine Learning Control (MLC) in recent literature [Brunton and Noack 2015, Duriez and Brunton 2016] has been applied with large success for a range of turbulence control experiments [Noack 2017]. MLC has continually outperformed existing control strategies often exploiting surprising nonlinear frequency cross-talk mechanisms.

In this section, the employed MLC implementation is described. First (Sect. 3.1), the control problem is framed as optimization of a cost function. Then (Sect. 3.2), a general control ansatz is proposed. This ansatz comprises nonlinear sensor-feedback with signal history, multi-frequency forcing and combinations thereof. Now, the search for an optimal control law can be formulated as regression problem (Sect. 3.3). Finally (Sect. 3.4), the employed linear genetic programming is detailed as powerful regression solver, including the parameters of this evolutionary algorithm.
3.1 Cost function

Good jet mixing is associated with large entrainment of the ambient flow into the high-momentum jet fluid emanating from the orifice. This entrainment reduces the streamwise velocity on the centerline. Following earlier work (Wu et al. 2016), we take the streamwise velocity $u_{5D}$ five jet diameters downstream as mixing indicator. This location is approximately at the end of the potential core. The smaller this velocity, the better is the mixing between the high-momentum jet fluid and the surrounding ambient fluid. In the cost function, the centerline velocity is normalized with the maximum jet velocity at the orifice

$$J = \frac{u_{5D}}{U_j},$$

(1)

Here, the overbar denotes a time average. This non-dimensionalization allows to compare jet mixing for a range of operating conditions.

The minimization of the cost function corresponds to the maximization of the jet centerline decay rate,

$$\overline{K} = \frac{U_j - u_{5D}}{U_j} = 1 - J.$$  

(2)

The decay rate quantifies the streamwise velocity deficit on the centerline. $\overline{K}$ is correlated approximately with an equivalent jet half-width $R_{eq} = [R_H R_V]^{1/2}$, where $R_H$ and $R_V$ are the jet half-widths in two orthogonal planes (Zhou et al. 2012), that is, $\overline{K}$ is directly correlated to the entrainment rate.

3.2 Ansatz for control law

We search to optimize actuation in a very general ansatz for the control law comprising, for instance, multi-frequency forcing and sensor-based feedback with signal history. In the following the actuation command is denoted by $b$. It can take binary values 1 and 0 depending if minijet is on or off. The minijet velocity scales approximately with the inverse of the duty cycle.

3.2.1 Multi-frequency forcing

In similar experiments (Fan et al. 2017), a periodic forcing with a frequency $\omega^* = 2\pi f^*$, $f^* \approx 67$ Hz was found to be very effective. The corresponding open-loop control law reads

$$b(t) = H(\sin(\omega^* t) - k),$$

(3)

where $H$ represents the Heaviside function and $k$ controls the duty cycle. The larger $k \in (-1, 1)$, the smaller the duty cycle. Following Li et al. (2017), a much more general multi-frequency forcing is considered, generated here by 9 harmonic functions $h_i = \sin(2\pi f_i t)$, $i = 1, \ldots 9$. The frequencies are selected based on the data acquisition frequency of 1 kHz. With finite data acquisition frequency, only discrete frequencies with discrete duty cycles are possible as displayed in Fig. 2.

We comprise the harmonic functions into a vector-valued frequency generator

$$\mathbf{h}(t) = [h_1 h_2 \ldots h_9]^\dagger$$

(4)

where the $\dagger$ superscript denotes the transpose, $h_i = \cos(2\pi f_i t)$, and $f_i = 20, 50, 59, 67, 77, 91, 111, 143, 200$ Hz for $i = 1, \ldots 9$ respectively. We do not include sinusoidal functions at the same frequencies. In the open-loop actuation literature, phase differences are only found to be important for few frequency ratios, e.g. harmonic and subharmonic components of the mixing layer (Monkewitz 1988). Most Lissajous figures with $h_i, h_j$ densely fill out the square $[-1, 1] \times [-1, 1]$, indicating the phase difference cannot be expected to have an effect.

The open-loop multiple forcing actuation is performed with

$$b(t) = B(\mathbf{h}(t)).$$

(5)

If $B$ were linear function — putting the binary nature of actuation aside — the resulting actuation command can exhibit the input frequencies. If $B$ were a pure quadratic function, all different frequencies may appear. In case of a general non-linear function, a large range of frequencies can be generated. For instance, $h_i^{10} - 1/2$ generates a harmonic function with frequency $10 f_i$. Hence, the main frequency limitation of the general ansatz (5) is not caused by the ansatz but by the actuator performance.

3.2.2 Sensor-based feedback

The considered feedback is based on the hot-wire signal $u_{5D}$ used for the cost function (1) and on the slightly displaced hot-wire measurement three diameters downstream $u_{3D}$ monitoring the shear-layer vortices. The feedback signals are based on the Reynolds decomposition into a one-period average denoted by $\langle \cdot \rangle_T$ and a fluctuation, $u_{5D} = \langle u_{5D} \rangle_T + u_{5D}'$ and $u_{3D} = \langle u_{3D} \rangle_T + u_{3D}'$. The period $T$ is taken from the best periodic forcing frequency 67 Hz. In addition, the signals are normalized with the jet velocity at the nozzle exit to arrive at more robust control laws for a range of operating conditions, e.g. different jet velocities. The resulting feedback argument reads

$$\mathbf{s}(t) = \frac{1}{U_j} \begin{bmatrix} u_{3D}(t) \\
 u_{5D}(t) \\
 u_{3D}'(t) \\
 u_{5D}'(t) \end{bmatrix}.$$
Note that $J = \pi_0$.

The ansatz for the sensor-based feedback law has the form:

$$b(t) = B(s(t)).$$  \hspace{1cm} (7)

### 3.2.3 Generalized feedback control

A natural generalization of the previous approaches reads

$$b(t) = B(s(t), h(t)).$$  \hspace{1cm} (8)

The sensor-based feedback might, for instance, control the duty cycle of an actuation frequency. In Sect. 4 and 5 we will explore jet mixing performance for a variety of control laws.

### 3.3 Control design as model-free regression problem

The cost function $J$ evidently depends on the chosen control logic $B(s, h)$. We search for a control law which minimizes $J$:

$$B^* = \arg \min_B J[B(s, h)].$$  \hspace{1cm} (9)

It is important to realize that this optimization task is a regression problem of the second kind: the optimal actuation command $b$ for given sensor input $(s, h)$ is not known and hence can not be learned directly, as in a regression problem of the first kind. One can only judge the performance of a control law based on the cost function. This appears to be a very subtle distinction but practically excludes many known regression solvers of machine learning. For instance artificial neural networks and deep learning typically require ‘guidance’ for the optimal mapping output. Such actuation may be available from an optimal control using the full Navier-Stokes equations (Lee et al. 1997). For experiments, however, we rely on the control performance alone.

### 3.4 Linear genetic programming as regression solver

Following Li et al. (2017), the control optimization problem (9) is solved using linear genetic programming (LGP). LGP is a powerful regression technique of machine learning which can optimize general nonlinear mappings, like the control law. We refer to the exquisite textbook of Wahde (Wahde 2008) for a quick overview on a spectrum of evolutionary algorithms including LGP, to the detailed LGP textbook by Brameier & Banzhaf (Brameier and Banzhaf 2007), and to the first turbulence control applications (Li et al. 2017).

In this section, LGP is sketched and the key parameters are listed.

LGP control starts like a Monte-Carlo approach with $I$ random control laws, referred to as individuals:

$$b = B^{(1)}_i, \quad i = 1, \ldots, I.$$  \hspace{1cm} (10)

The superscript (1) represents the first generation. The jet mixing experiment grades the performance of each law $J^{(1)}_i$ in the plant. Figure 3 shows the plant, the fast inner control loop and the slow outer evaluation / learning loop. Then, the individuals are re-numbered and ranked in order of performance,

$$J^{(1)}_1 \leq J^{(1)}_2 \leq \ldots \leq J^{(1)}_I.$$  \hspace{1cm} (11)

The best $N_e$ individuals are adopted in the new generation,

$$B^{(2)}_i = B^{(1)}_i, \quad i = 1, \ldots, N_e.$$  \hspace{1cm} (12)

This operation is called elitism. The remaining $I - N_e$ individuals of the new generation are determined with a random
sequence of three genetic operations: crossover, mutation, and replication with probabilities $P_c$, $P_m$, and $P_r$, respectively. Crossover has two arguments (individuals) and breeds two new individuals exchanging equally sized part of ‘genes’ from both individuals. This operation tends to yield better individuals, e.g. exploit populated local minima. Mutation has a single argument and replaces part of the ‘genes’ rather randomly. This operation might explore new local minima. Replication has one argument which is copied unaltered into the new generation. Evidently, this operation has a memory effect. The argument of these operations is decided in a tournament. $N_i$ individuals of the graded generations are chosen with equal probability for the tournament. The genetic operation takes the best or the best two individuals, i.e. there is a bias towards processing better individuals but low performing individuals are not completely ignored.

These iterations are performed until convergence of the best performing individuals or the end of the measurement time is reached (see Fig. 3b). Let $N_g$ be the number of the evaluated generations before termination. The best control law of the last generation $B_i^{(N_g)}$ is taken as the solution of the regression problem (9).

In LGP, each individual consists of a set of instructions using elementary operations on a register $r_j$, $j = 1, \ldots, N_i + N_b + N_c$, with the first $N_i$ values for the input signals, the next $N_b = 1$ values for the actuation commands, and $N_c$ constants. For sensor-based feedback (7) there are four input signals $N_i = 4$, for multi-frequency forcing (5) $N_i = 9$ and for the generalized feedback (8) $N_i = 4 + 9 = 13$. Before any operation, the input registers are initialized with the argument values and the actuation register is zeroed. The $k$th instruction is coded as integer matrix $B_{kl}, l = 1, \ldots, 4$. Here, $B_{kl}$ denotes the index of the operation, e.g. ‘1’ for ‘+’, $B_{k1}$, $B_{k2}$ represents the index of the input registers and $B_{k4}$ the index of the output registers (excluding the constants). Thus, $B_{k1} = 2$, $B_{k2} = 3$, $B_{k3} = 1$ and $B_{k4} = 5$ corresponds to $r_5 = r_1 + r_2$.

The employed parameters of LGP are listed in Table 1. The same or very similar parameters have been chosen in dozens of other turbulence control experiments, flow control simulations or dynamical systems control (Duriez and Brunton 2016). The performance of the resulting machine learning control was not critically dependent on any of these parameters.

### 4 Sensor-based feedback optimized with MLC

In this section, a sensor-based feedback control using MLC (8) is investigated. Experiments are performed at $Re_D = 8000$ and $C_m = 1.2\%$. The $C_m$ of 1.2% corresponds to the optimum jet mixing performance (Wu et al, 2016). Figure 4 shows the evolution of the cost $J_i$ as function of the individual index $i = 1, \ldots, 100$ for 6 generations $(n = 1, \ldots, 6)$. For visual clarity, only every 5th data is displayed, i.e. $i = 1, 6, 11, \ldots, 96$. The individuals of each generation are ordered by $J$ value following (11). Here, $i = 1$ corresponds to the smallest $J$ and the best control law. For the first generation $n = 1$, $J$ increases gradually with $i$ starting from the minimum of $J_1^{(n)} \approx 0.6$. With increasing number of generations, the curve converges to a plateau for the first half of the individuals. Genetic algorithm breeds more and more similar or even identical individuals with every generation. At some point,
the learning is converged while the last half of the individuals explore other new control laws but fail to find better minima.

This convergence is depicted in Figure 4b showing the performance $J(n)$ of the best individual for each generation $n = 1, \ldots, 6$. At $n \geq 4$ the $J$ value is converged to 0.588 corresponding to a decay rate of $K = 1 - J \approx 0.412$. At $n = 6$ the evolution is stopped and the best individual of the last generation $b_{\text{opt}}$ is taken as MLC law:

$$b_{\text{opt}} = H(-s_3)$$

(13)

The actuation fires when the streamwise velocity of the shear-layer sensor at $x/D = 3$ is below the average. Equation (13) is the simplified version of the equivalent LGP algorithm $b_{\text{opt}} = H(0.752s_3/(-0.576))$. The error bar in Fig. 4b displays the standard deviation of the repeated tests of the best control law in all the generations. The error bar significantly decreases with increasing time window. For MLC, only the approximate relative ordering needs to be preserved and we save significant measurement time by using a 5 second time window.

The MLC feedback forcing (13) leads to nearly periodic forcing as displayed in Fig. 5. The unforced $s_1$ signal from the shear-layer sensor (Fig. 5a) displays a dominant periodicity at $f_0 = 135$ Hz. This frequency corresponds to the coherent shear-layer structures. The forcing leads to a much lower frequency of 55 Hz corresponding to a flapping. This flapping frequency dominates the behavior of the shear-layer sensor at $x/D = 3$ and is also visible in the reading of the centerline sensor at $x/D = 5$. Note that $s_3$ follows $s_1$ modulo the short-time averaged mean value. Ditto for $s_4$ and $s_2$. The actuation fires for short periods of time when $s_3$ is negative following (13) (see Fig. 5b). In other words, the actuation fires whenever the control law (13) is larger than the threshold value $Th$ (see Fig. 5d and e).

The MLC discovery of $s_3$ as actuation trigger is visualized in Figure 6. This graph represents the percentage $P_{si}$ of having $s_j$ in the expression of all $I$ control laws for each generation $n$. With increasing number of generations $n$, the dominance of $s_3$ becomes more pronounced. In the last generation, $s_1$, $s_2$ and $s_4$ are neglected in most individuals as control law input. Note that multiple appearances of $s_3$ in one individual are counted only once. This behavior is easily explained by the high efficiency of periodic forcing for jet mixing via flapping and by the higher sensitivity of the shear-layer sensor to this frequency.

The convergence of the learning process is displayed in Figure 4. Following Kaiser et al. (2017), we also want to learn the ‘control landscape’ of all tested individuals employing a proximity map. For that purpose, we rely on Multi Dimensional Scaling (MDS) (Mardia et al. 1979), a method classically used to visualize high-dimensional data in a low-dimensional feature space. Specifically, we employ classi-
cal multidimensional scaling (CMDS) which is originated from the works of Schoenberg (1935) and Young & Householder (1938) and optimally preserves the distances between the data in the projection from a high-dimensional space to a low-dimensional feature space. Our infinite-dimensional objects are $N = I \times 6 = 600$ control laws. The control laws are indexed in order of appearance, i.e. $i = 1, \ldots, 100$ belong to the first generation, $i = 101, \ldots, 200$ refer to the second generation, and so on. Next, we need to quantify the relative configuration of control laws with a distance matrix $D = (D_{lm})_{1 \leq l, m \leq N}$. Here, $D_{lm}$ denotes the difference between individuals $l$ and $m$. The square of the distance matrix $D^2$ is defined by

$$D^2_{lm} = \left< |h_l(x) - h_m(x)|^2 \right>_{1,m} + \alpha |J_l - J_m|$$  \hspace{1cm} (14)

The first term represents the difference between the $l$th and $m$th control laws averaged over the sensor readings of both actuated dynamics. Thus, the averaging takes into account the frequency and relevance of the sensor reading. The second term penalizes the difference of their achieved costs $J$ with coefficient $\alpha$. This penalization smooths the control landscape, i.e. the visualization of the cost $J$ in the feature space. The penalization parameter $\alpha$ is chosen so that the maximum variation of the first and second term of (14) are equal. Thus, the dissimilarities between control laws and between the cost functions have comparable weights in the distance matrix $D_{lm}$. For further details, please refer to Duriez and Brunton (2016). The aim of CMDS is to find a centred representation of points $\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_N]$ with $\gamma_1, \gamma_2, \ldots, \gamma_N \in \mathbb{R}^2$, such that the pairwise distances of the feature points are—in a well-defined sense—optimally close to the original distances, i.e. $\|\gamma_l - \gamma_m\|_2 \approx D_{lm}$.

Figure 7 shows the proximity map of all control laws ($N = 600$) in a two dimensional plane $[\gamma^1, \gamma^2] \in \mathbb{R}^2$ and the power spectral density of thee selected control laws $A$-$L$, respectively. The $\gamma^1$ and $\gamma^2$ represent the coordinates in the two dimensional plane. Each dot represents one control law and the corresponding $J$ value is color-coded. The optimal law $h_{opt}$ is indexed by $G$. All individuals are close to a V-shaped curve except continuous blowing with duty cycle $DC = 1$ as isolated point $A$. On the curve, the $DC$ increases constantly from 0 to 94.5% for control laws $B$ to $L$. The second feature coordinate $\gamma^2$ clearly correlates with the duty cycle (DC) of the control signal. The first feature coordinate $\gamma^1$ appears to correlate with spectral characteristics. The maximum frequency of the best performing law $G$ is on the rightmost side of the curve while the other individuals have lower dominant frequencies. The proximity map reveals that MLC explores multi-frequency actuation mechanisms and arrives at a dominant periodicity with low duty cycle of 26.3% and new flapping frequency of 55 Hz. This frequency of 55 Hz does not equal the optimal frequency (i.e. 67 Hz) achieved using periodic forcing method (Fan et al 2017). One possible reason is the influence of turbulence, which makes the the optimal frequency and duty cycle hardly be searched by sensor-based feedback control law (13). Detailed discussion will be given in Sect. 6.2.

5 Multi-frequency forcing optimized with MLC

The sensor-based feedback control results distill nearly periodic forcing as best actuation. However, the possibility of feedback control to give rise to strict periodic forcing is mitigated by the low-frequency drifts and high-frequency noise. Moreover, in some MLC studies, open-loop multi-frequency forcing has been shown to outperform both periodic forcing and sensor-based feedback (Li et al 2017). This motivates
the use of MLC to optimize open-loop multi-frequency actuation. Here, a range of harmonic functions are used as inputs of control laws. Table 2 provides the chosen harmonic functions $h_i(t)$.

Figure 8 illustrates the evolution of the cost $J$ as function of the individual index $i$ for 6 generations. As for the sensor-based feedback, we plot only every 5th individual for visual clarity. Interestingly, the increase of $J$ with $i$ is more steep than that for the sensor-based feedback, indicating that the minimum is less populated. One reason may be that the number of control law arguments has more than doubled and the search space is in some vague sense ‘larger’. Another reason is that the harmonic functions are by construction less correlated than the sensor signals. This reduced correlation has frequently been found to be associated with the increase of learning time. A particularly noteworthy characteristic of this MLC run is the sudden jump of the best cost value $J^{(n)}_1$ from 0.510 to the converged value of $J^{(4)}_1 = 0.458$ in the fourth generation (Fig. 8p), indicating that a new minimum may have been found from generation 3 to 4. This value is 24% better than 0.588 of the sensor-based feedback, and corresponds to a decay rate of $\gamma = 0.542$. The error bars of open- and closed-loop MLC are similar to each other.

The resulting MLC law reads

$$b_{\text{opt}} = H \left( \log \left( \frac{1}{2} \left( \frac{-0.646 - \log(|h_4|^2)}{} \right) \right) \right)$$

and corresponds to periodic forcing with a frequency of $f_e = 67$ Hz and a duty cycle of 7%. Figure 9 illustrates the corresponding actuation command and the resulting sensor fluctuations in the shear layer ($s_3$) and on the centerline ($s_4$) in a few period interval. The actuation is strongly correlated to low $s_3$ values, that is, the mechanisms of MLC for open- and closed loops are similar. The $s_3$ signal tends to follow the actuation with a minor delay. The sensor $s_4$ associated with the cost function shows little response to actuation.

Figure 10 shows typical photographs in the injection (x,z) and non-injection (x,y) planes. The unforced jet (Fig. 10a) is, by symmetry, similar in both planes. The smoke clearly distills high-frequency shear-layer vortices which are consistent with the observed 135 Hz shear-layer signal of Fig. 5. Further visualizations in the (y,z) plane (not shown here) are consistent with axisymmetric ring vortices. Figures 10b and c show the flow visualization data under the optimal periodic forcing. The photograph in the injection plane displays a strong flapping motion which leads to large dispersion of smoke in the transverse direction. That in the non-injection plane does not show any significant dispersion.
Fig. 10 Photographs from flow visualization of the unforced benchmark and the optimal actuation. Flow is from the left to right. Comparison in the typical flow structure between the uncontrolled jet (a) and the controlled jet ($C_m = 1.2\%, f_e = 67$Hz, $DC = 7\%$) in the non-injection (b) and injection planes (c).

plane indicates no increase in the transversal mixing. Yet, the photographs in both planes indicate that the length of the potential core is about 2D. This value is to be compared with the continuous smoke on the centerline until at least 4 diameters for the unforced jet. These observations are consistent with the observed decrease of the chosen cost function. This flapping, characterized by greatly enhanced entrainment in the non-injection plane and very rapid spread in the injection plane, has also been observed in Yang et al (2016). In their study, the jet manipulation was performed with two asymmetrically arranged unsteady minijets. All results convincingly demonstrate that the flapping motion is responsible for the rapid decay of the centreline mean velocity (Fig. 10).

Similarly to Sect. 4, the control landscape is given in Fig. 11. Figure 11b shows the power spectral density functions for selected control laws A–I. The coordinate $\gamma^1$ is strongly correlated to $DC$. From A to I, the $DC$ varies from 97% (left) to 7% (right), while the cost $J$ decreases from 1 to 0.458. The optimal law $b_{opt}$ is indexed by I. These results indicate again that $DC$ plays an important role in control performance.

6 Discussion of open versus closed-loop control

Sections 4 and 5 reveal arguably surprising features of machine learning control and of the optimal actuation mecha-
nism. In the following, we present three aspects: the learning rate for periodic forcing (Sect. 6.1), the poor performance of feedback control (Sect. 6.2) and the convergence against pure periodic forcing without other frequency components (Sect. 6.3).

6.1 Learning rate of MLC for optimal periodic forcing

In Sect. 5, MLC performs a global search over many frequencies and many duty cycles and arrives at pure periodic forcing after testing only 400 individuals. From this and the previous study (Wu et al. 2016), the characterizing frequency and duty cycle of this periodic forcing could not be improved further by a systematic search.

Moreover, the decay rate $K$ is 8% larger as compared to the previously achieved mixing, where $DC$ was fixed at 15% and $C_m$ and $f_e$ were optimized using a dual-input/single-output extremum-seeking controller (Wu et al. 2018), yielding $C_m = 1.2\%$ and $f_e = 67$ Hz. This performance can be considered quite impressive for an evolutionary learning algorithm. These algorithms are powerful for exploration but perform less well for exploitation, the realm of gradient-based parameter optimization. Moreover, 400 individuals to convergence may be compared with a systematic parameter variation over two parameters. Let us assume 400 cost functions are evaluated with a systematic equidistant sampling using 20 frequencies in the range from 0 to 200 Hz and 20 duty cycles from zero to unity. The closest result $f_e = 60$ Hz and $DC = 10\%$ would be far away from the optimal parameters and yield a significantly worse performance.

MLC may also be compared with local gradient search—at the expense of potentially arriving in a suboptimal minimum. A two-parameter extremum seeking and a two-parameter simplex search can be expected to require $O(100)$ test runs worth of measurement time. Summarizing, MLC is a competitive two-parameter optimizer, a task which is not a typical application of an evolutionary learning algorithm. In addition, MLC has operated in much larger search space of multi-frequency laws. A side benefit of this search is that the performance advantages of sensor-based feedback and of non-periodic actuation have been assessed.

6.2 No performance benefits of sensor-based feedback

The performance of sensor-based feedback is surprisingly low. In the following, we explore if sensor-based feedback $b = B(s_1, s_2, s_3, s_4)$ could have mimicked the best open-loop control. The MLC feedback law $b = B_{opt}(s_3)$ employs only the excitation frequency is demonstrated in Fig. 12b. Figure 13 evidences that, with a fixed $C_m$, a lower $DC$ yields larger changes of the main jet, especially for the standard deviation $u^* = u_{r.m.s.}$ at $x^* = 0.05$ of the unforced jet and manipulated jet at $DC = 7\%$ and $DC = 15\%$ in $x-z$ and $x-y$ plane, $C_m = 1.2\%$ and $f_e = 67$ Hz.
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6.3 No performance benefits from multi-frequency forcing

Literature contains many experimental flow control studies in which multi-frequency forcing has outperformed periodic forcing at the same or similar amplitude. In Li et al. (2017), MLC has identified a two-frequency forcing which has improved the drag reduction of a car model as compared to optimized periodic forcing. In Chovet et al. (2017), a similar observation was made for the mixing enhancement behind a backward-facing step. Numerous reports show how sub- and subharmonic forcing components increase the mixing layer width by triggering more early vortex pairing (Coats 1997, Monkewitz 1988). Hence, the performance of pure periodic forcing for optimal jet mixing is initially surprising.

However, forcing augments jet mixing by inducing a strong asymmetric flapping of the jet in the plane with the minijet actuator. This is a comparably simple mechanism, like the excitation of pendulum motion with a kick near the lower equilibrium point. It is difficult to perceive how scheduling the ‘firing’ non-periodically should improve such an inherently periodic phenomena. The mixing layer, for instance, is much more complex by incorporating multiple vortex merging and three-dimensional structures, making multiple frequencies more advantageous.

7 Conclusions

In this experimental study, we maximize jet mixing using one minijet actuator and two hotwire sensors—advancing past closed-loop control studies by the group (Wu et al. 2016). The control law ansatz comprises a multi-frequency forcing, sensor-based feedback and combinations thereof following Li et al. (2017). From this large search space, machine learning control (MLC) has identified periodic forcing with short duty cycle as optimal. The mixing is quantified by the averaged streamwise velocity decay rate at five diameters downstream on the symmetry axis. The achieved mixing is better than in a previous study by Wu et al. (2016) with ex-
tremum seeking control since a better (smaller) duty cycle was found. MLC performed optimization in only 4 generations with 100 control laws in each, i.e. 400 runs with 5 seconds evaluation for each run. The frequency and duty cycle identified by MLC could not be improved further with parametric studies. Summarizing, the learning time is comparable to alternative optimization of periodic forcing, e.g. testing 50 different frequencies for 20 different duty cycles.

In addition to identifying the optimal periodic forcing, MLC indicates that neither additional forcing frequencies nor employing sensor-based feedback improves mixing further. Both implications may initially be surprising but can—in hindsight—easily be explained. Numerous turbulence control experiments show how multi-frequency forcing outperforms periodic forcing. In [Li et al. (2017)], MLC identified that multi-frequency forcing is more effective for drag reduction of a car model than the optimized periodic forcing. MLC also found multi-frequency forcing to outperform optimized periodic actuation in the reduction of a recirculation zone behind a backward facing step (Chovet et al. 2017). The list of similar observations can easily be extended (Coats 1997). Multi-frequency forcing is a very large superset of periodic forcing. In case of jet mixing, the underlying mechanism is a flapping in the plane containing the minijet. It seems that this simple mechanism cannot be improved by other frequencies. Moreover, a single actuator may not trigger other mechanisms which may be based on other different frequencies.

The poor performance of sensor-based feedback for an oscillatory mechanism is also initially surprising in light of a common experience of turbulence control experiments: If periodic forcing improves a performance, feedback can generally improve it further. This feedback may adjust in-time the phase of actuation to flow events (Pastoor et al. 2008) or may perform a slow adaption of a forcing parameter. However, in case of the jet mixing neither phasor control nor parameter adaption can be expected to work. The mixing is critically depending on a short ‘firing’ time in a narrow time interval. Any feedback ‘firing’ policy will be mitigated by the low-frequency drifts and by high-frequency noise. In addition, the boolean on-off nature of control excludes the possibility of an amplitude adaptation. In hindsight, the poor performance of sensor-based feedback in comparison with the optimal periodic forcing can be expected if very short duty cycles are necessary for good actuation performance. We did improve sensor-based feedback by several measures, e.g. optimization of the sensor position or other sensor filters. However, in none of these experiments the performance of periodic forcing has been reached.

The current jet mixing study reveals that a simple periodic actuation appears to be best. MLC simultaneously optimizes the forcing parameters at a highly competitive learning rate and seems to exclude performance increases by feedback and new frequencies. Subsequent studies of the authors concern jet mixing enhancement in the same facility, but employing all six minijets. In this case, MLC is found to yield a dramatic performance increase with a non-harmonic and non-symmetric forcing not reported in literature so far. MLC has a large untapped potential in turbulence control applications. We actively pursue a MLC generalization which learns not only the optimal control law but also the corresponding control-oriented model within few hundred or few thousand short test runs.

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