A Novel Method of Improved Three-dimensional Fixed Target Multi-station Passive Time Difference Positioning

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Abstract—In this paper, a three-dimensional fixed target multi-station passive time difference positioning method based on multiple observations is proposed. This method is aim to improve the four-station positioning ambiguity problem of the three-dimensional multi-station passive time difference positioning method. The improvement is that the main station does not move and the auxiliary stations move, and the observation values of multiple moments are used to solve the fixed target position in the three-dimensional space. Then the three-dimensional spatial positioning of the target to be positioned is realized. The advantage is that the complexity of the observatory equipment is greatly reduced. The simulation results suggest the effectiveness and feasibility of the algorithm.

1. INTRODUCTION (HEADING 1)
Multi-station passive time difference positioning technology is an important means of electronic reconnaissance in electronic support systems [1]. It has the advantages of high positioning accuracy and good concealment, which has been a research hotspot in the field of passive positioning technology in recent years. In the three-dimensional space, the time difference of arrival (TDOA) of the target signal of the two observing stations determines a pair of hyperboloids focusing on the two observing stations. The three-dimensional spatial positioning requires at least four stations to generate three pairs of hyperboloids [2]. The intersection of the hyperboloids is the location of the radiation source target to be located. However, in the process of positioning and parsing, it will produce multi-value phenomena, that is the positioning ambiguity.

There are roughly five methods for eliminating positioning ambiguity [3]. One is to add a secondary station to make the positioning solution have a unique solution. The second is to increase the azimuth information of the radiation source, add a direction measuring function to an observing station, detect the azimuth angle $A_i$ of the target to be located. Then, the azimuth angle $A_i$ is calculated from the solved ambiguous positioning point, and the position corresponding to $\min_{i=1,2} |d-A_i|$ can be considered as the unambiguous positioning point. The third is to increase the height information of the radiation source, and the target height $z_i$ of the radiation source to be located estimated by some prior information, compared with the obtained height value $z_i$ of the positioning ambiguity point, and the position corresponding to $\min_{i=1,2} |z_i|$ can be regarded as the positioning ambiguity point. The fourth is to utilize some a prior information of the radiation source target position to exclude the false positioning point of the target. For example, if the target cannot be below the surface, the point of the height value $z_i < 0 (i=1,2)$ in the solution result can be determined as a false positioning point. The
fifth is to associate the current positioning ambiguity point with the previous positioning point, and
determine the positioning point where the radiation source target does not have a sudden change of the
motion track, that is no ambiguity positioning. The above method for eliminating the location
ambiguity has problems such as increasing the complexity of the device, insufficient prior information,
and incomplete ambiguity point elimination.

In this paper, we propose a method which can improve the existing three-dimensional multi-station
passive time difference positioning analysis algorithm. The main station does not move and the
auxiliary station moves, and the observation values at multiple moments are used to solve the fixed
target position in the three-dimensional space. The algorithm eliminates the positioning ambiguity
without increasing the complexity of the device, and implements the multi-station passive time
difference positioning of the fixed target in three-dimensional space.

The rest of this paper is organized as follows. Section II provides three-dimensional space time
difference positioning principle and the positioning solution and positioning ambiguity problem. The
improved three-dimensional fixed target multi-station passive time difference positioning method has
been proposed in section III. Section IV briefly describe the simulation experiment. Section V is the
summary of this paper.

2. BRIEF DESCRIPTION OF TIME DIFFERENCE POSITIONING ALGORITHM

2.1 Three-dimensional space time difference positioning principle

As shown in Fig. 1, taking a three-dimensional passive time difference positioning system consisting of
one primary station and three auxiliary stations as an example, the position of the radiation source is
\((x, y, z)\), and the distance difference between the target to the primary station \(R_0\) and the three
auxiliary stations \(R_i, i=1,2,3\) is then the positioning equations can be expressed as

\[
\begin{align*}
\Delta R_i &= R_i - R_0, \\
R_i &= \sqrt{(x_i-x)^2 + (y_i-y)^2 + (z_i-z)^2}, \quad (i=1,2,3) \\
R_0 &= \sqrt{(x_0-x)^2 + (y_0-y)^2 + (z_0-z)^2}
\end{align*}
\]  

Fig.1 Three-dimensional spatial time difference positioning spatial geometric relationship

where \(\Delta R_i = \Delta T_i \cdot c (i=1,2,3)\), \(\Delta T_i\) is the arrival time difference of the target signal between primary station
0 and the arrival of the auxiliary station \(i\), and \(c\) is the propagation rate of the electromagnetic wave in
the air, \(R_0\) is the distance from the target to the primary station, and \(R_i\) is the distance from the target to the
\(i\)-th secondary station, respectively. Bringing the latter two formulas of equation (1) into the first
formula, and taking \(i\) values 1, 2, and 3 respectively, a ternary equation system can be obtained. The
solution of the nonlinear equation group represents the target position information in the geometric
space.
2.2 Time difference positioning solution and positioning ambiguity problem

The time difference positioning equations transform the passive localization problem into the solving problem of nonlinear equations. The scholars have proposed many algorithms for the time difference positioning solution [4-6]. The Chan algorithm is a non-recursive algorithm that uses analytical expressions to solve the time difference positioning equations [7]. The algorithm is simple, the calculation is fast, and it is widely used, but there is a problem of positioning ambiguity.

Equation 1 can be rewritten as:

\[(x_0 - x_i)x + (y_0 - y_i)y + (z_0 - z_i)z = k_i + R_0 \cdot \Delta R_i\]  

(2)

Where \(k_i = \frac{1}{2}\left[\Delta R_i + (x_i^2 + y_i^2 + z_i^2) - (x_i^2 + y_i^2 + z_i^2)\right].\)

When \(i\) takes different values, the above equation gives a system of nonlinear equations, considering \(R_0\) as a known quantity, and equation (2) can be rewritten as

\[AX = F\]  

Where \(A = \begin{bmatrix} x_0 - x_1 & y_0 - y_1 & z_0 - z_1 \\ x_0 - x_2 & y_0 - y_2 & z_0 - z_2 \\ x_0 - x_3 & y_0 - y_3 & z_0 - z_3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad F = \begin{bmatrix} k_i + R_0 \cdot \Delta R_i \\ k_i + R_0 \cdot \Delta R_i \\ k_i + R_0 \cdot \Delta R_i \end{bmatrix}, \) respectively.

The reason for the ambiguity is that in the positioning solution process, \(R_0\) is regarded as a known number, so that \(R_0\) is a nonlinear function of \(x, y,\) and \(z\), and then \(x, y,\) and \(z\) is substituted into the expression to find \(R_0\). According to the literature [8], the binary one-position equation obtained in the process of solving \(A\) can be expressed as

\[aR_0^2 + bR_0 + c = 0\]  

(4)

When \(a \neq 0\) and \(b^2 > 4ac\), equation (4) will have a double solution. As a result of \(B\) must be a positive number, when two roots are positive and negative, the negative root can be discarded, and the positive root is a non-ambiguity positioning solution. When the two are positive, the ambiguity solution appears.

Set the coordinates of the primary station, the secondary station 1, the secondary station 2, and the secondary station 3 to be (0, 0, 8) km, (0, 0, 6.01) km, (-4, 7, 6.01) km, (4, 7, 6.1) km, respectively. In the layout of the station, the schematic diagram of the location ambiguity area is shown in Fig. 2. In the figure, the red point is the no solution area, and the yellow point is the ambiguity area.
3 IMPROVED THREE-DIMENSIONAL FIXED TARGET MULTI-STATION PASSIVE TIME DIFFERENCE POSITIONING METHOD

In this section, we propose a method which can improve the existing multi-station passive time difference positioning algorithm in 3D space. The primary station does not move the auxiliary station motion, and uses multiple observations to solve the fixed target position in 3D space. This method eliminates positioning ambiguity without increasing device complexity.

Firstly, arrange the station mode of one main station and three auxiliary stations reasonably, and select the site to make rank(\(A\)) = 4. The primary station does not move, the auxiliary stations move at a constant speed in the three-dimensional space, and the signal arrival time difference and the position coordinates of the observation station are acquired at a plurality of time intervals after a certain time interval. Assume that the time difference between the target signal arriving at the primary station 0 and the auxiliary station \(i\) is \(\Delta T_i^\ast\) (\(i = 1, 2, 3\)) at time \(t_n\) (\(n = 1, 2, 3, \ldots\)). At this time, the primary station is marked as \((x_0, y_0, z_0)\), and the auxiliary stations are marked as \((x_i^*, y_i^*, z_i^*)\), \((x_i^*, y_i^*, z_i^*)\) and \((x_i^*, y_i^*, z_i^*)\), respectively.

The data acquired at multiple times is linked, and the time difference positioning equations can be expressed as

\[
\begin{align*}
(x_i^* - x_i) \cdot x + (y_i^* - y_i) \cdot y + (z_i^* - z_i) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast \\
(x_i^* - x_i) \cdot x + (y_i^* - y_i) \cdot y + (z_i^* - z_i) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast \\
(x_i - x_i^*) \cdot x + (y_i - y_i^*) \cdot y + (z_i - z_i^*) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast
\end{align*}
\]

The equations are shifted to square, and the simplification can be obtained by

\[
\begin{align*}
(x_i^* - x_i)^2 + (y_i^* - y_i)^2 + (z_i^* - z_i)^2 - R_i^2 &= \ell_i^2 \\
(x_i^* - x_i)^2 + (y_i^* - y_i)^2 + (z_i^* - z_i)^2 - R_i^2 &= \ell_i^2 \\
(x_i - x_i^*)^2 + (y_i - y_i^*)^2 + (z_i - z_i^*)^2 - R_i^2 &= \ell_i^2
\end{align*}
\]

Where \(\ell_i^2 = (1/2)[\Delta R_i^2 + (x_i^* + y_i^* + z_i^*)^2 - (x_i^* + y_i^* + z_i^*)^2]\) \(j = 1, 2, 3, n = 1, 2, 3, \ldots\), and \(\Delta R_i^2 = c \cdot \Delta T_i^\ast\) (\(n = 1, 2, 3, \ldots\)).

Appropriate selection of the station mode, taking at least four equations in the above equations, can solve the target three-dimensional space coordinates, the equation has a unique solution, which can eliminate the positioning ambiguity.

Assuming that the five sets of data at time \(t_1\), \(t_2\) and \(t_3\) are taken, the equation set can be expressed as

\[
\begin{align*}
(x_i^* - x_i) \cdot x + (y_i^* - y_i) \cdot y + (z_i^* - z_i) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast \\
(x_i^* - x_i) \cdot x + (y_i^* - y_i) \cdot y + (z_i^* - z_i) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast \\
(x_i - x_i^*) \cdot x + (y_i - y_i^*) \cdot y + (z_i - z_i^*) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast \\
(x_i - x_i^*) \cdot x + (y_i - y_i^*) \cdot y + (z_i - z_i^*) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast \\
(x_i - x_i^*) \cdot x + (y_i - y_i^*) \cdot y + (z_i - z_i^*) \cdot z - R_i \cdot \Delta R_i^\ast &= \ell_i^\ast
\end{align*}
\]

Equation set can be represented by a matrix as

\[
AX = B
\]

\[
A = \begin{bmatrix}
x_i - x_i^* & y_i - y_i^* & z_i - z_i^* & -\Delta R_i^\ast \\
x_i^* - x_i & y_i^* - y_i & z_i^* - z_i & -\Delta R_i^\ast \\
x_i - x_i^* & y_i - y_i^* & z_i - z_i^* & -\Delta R_i^\ast \\
x_i^* - x_i & y_i^* - y_i & z_i^* - z_i & -\Delta R_i^\ast \\
x_i - x_i^* & y_i - y_i^* & z_i - z_i^* & -\Delta R_i^\ast
\end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix}
\ell_i^\ast \\
\ell_i^\ast \\
\ell_i^\ast \\
\ell_i^\ast \\
\ell_i^\ast
\end{bmatrix}, \quad \text{and} \quad X = \begin{bmatrix}
x \\
y \\
z
\end{bmatrix}, \quad \text{respectively.}
\]
When the station site is properly selected to make \( \text{rank}(A) = 4 \), the equation has a unique solution, and the least squares solution of \( X \) is \( \hat{X} = \left(A' A\right)^{-1} A' B \).

It can be seen that the proposed method increases observation information and eliminates the ambiguity problem of four-station time difference positioning in three-dimensional space without increasing the complexity of the equipment.

4 SIMULATION

The simulation parameters are set as follows: the three-dimensional coordinate system is established with a certain point on the horizontal ground as the origin, and the position coordinates of the observation station in the three-dimensional space are obtained at time \( t_1 \). The coordinates of the primary station are \((0, 0, 8) \) km, and the coordinates of the three auxiliary stations are \((0, 0, 6.01) \) km, \((-4, 7, 6.01) \) km and \((4, 7, 6.1) \) km, respectively. The auxiliary stations perform uniform linear motion, and the speeds in the three directions of \( x \), \( y \) and \( z \) are \( v_x = 0 \), \( v_y = -500 \text{ m/s} \), \( v_z = 0 \), respectively. The time difference measurement accuracy of the observation station is 15 ns, and the station site error is 3 m. The primary station does not move, the auxiliary station moves at a constant speed, and it is observed once every 1 s. The positioning accuracy is measured by the Position Dilution of Precision \( GDOP = \sqrt{(x-\tilde{x})^2 + (y-\tilde{y})^2 + (z-\tilde{z})^2} \). The smaller the GDOP value, the higher the positioning accuracy.

Where \([\tilde{x}, \tilde{y}, \tilde{z}]\) is the estimate of the position of the target radiation source and \([x, y, z]\) is the true value of the position of the target radiation source. The unit of the GDOP contour is km.

Due to the existence of random error time difference measurement error and site error, 100 Monte Carlo simulation experiments were carried out to verify the effectiveness of the proposed method. The experimental results are shown in Fig.3.
Assuming that the data of all stations at time 1 are used to solve the positioning equation, Fig.3(a) shows the positioning result of the auxiliary station 1 data at time 2 and time 3, and Fig.3(b) shows the positioning result of the auxiliary station 2 data at time 2 and time 3, and Fig.3(c) shows the positioning result of the auxiliary station 3 data at time 2 and time 3. It can be seen that the data of different auxiliary stations leads to different spatial positioning accuracy, and the data can be reasonably selected according to actual conditions in the actual application process.

5. CONCLUSION
The method proposed in this paper had good application value in practical engineering. It acquired additional target space position information through observation station motion, and solved the ambiguity problem of four-station time difference positioning in three-dimensional space. From the simulation results, we can see the effectiveness of the proposed method. Whereas, due to the limitations of real conditions and own strength, there are still many problems need to be solved. In the future, we will further study how to quickly and effectively locate the moving target in three-dimensional space.

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