Heavy Baryon Mixing in Chiral Perturbation Theory

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Abstract

We discuss the SU(3) and heavy quark spin-symmetry breaking mixing between the $\Xi_c$ and $\Xi'_c$ charmed baryons. Chromomagnetic hyperfine interactions are the leading source of spin-symmetry breaking and together with the SU(3) breaking mass differences between the lightest pseudo-Goldstone bosons gives the leading contribution to the mixing. Such contributions are computed in chiral perturbation theory and compared to quark model expectations. We also compute the leading contribution to the semileptonic decay $\Xi_b \to \Xi'_c \ell \bar{\nu}_\ell$ at zero recoil, and find that it is an order of magnitude smaller than naive power counting would suggest. It appears that $\Xi_b \to \Xi'_c \ell \bar{\nu}_\ell$ is dominated by incalculable counterterms, and we discuss the implications for quark models based on the essential role of hyperfine interactions.

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The ongoing experimental efforts in charmed baryon physics have established the masses and dominant decay modes of most of the lowest lying charmed baryons. The $\Lambda^+_c$, $\Xi^+_c$, $\Sigma^+_c$, and $\Omega_c$ baryons with $J^G = \frac{1}{2}^+$ have been observed and studied \cite{[1],[2]}, while just recently the $J^G = \frac{3}{2}^+$ $\Sigma^*_c$ and $\Xi^*_c$ have been observed \cite{[3],[4]} (we note that a $J^G = \frac{1}{2}^+$ $\Xi'_c$ baryon candidate has also been identified \cite{[5]} but needs to be verified). The observed hyperfine mass splitting between the $\Sigma^*_c$ and the $\Sigma_c$ of $\delta_c \sim 67$ MeV is consistent with the naive estimate of $\Lambda^*_c/QCD/m_c$ from heavy quark considerations, but the $\Sigma^*_b - \Sigma_b$ mass splitting that has recently been hinted at \cite{[2]} seems to be larger than heavy quark symmetry predicts. To address this issue, Falk \cite{[6]} has proposed a reassignment of the observed resonances that would maintain the expected mass splittings. We will assume $\delta_b \sim \delta_c/3$, consistent with heavy quark symmetry predictions.

Despite the mass of the charm quark being only $m_c \sim 1.5$ GeV it appears appropriate to treat charmed baryons at leading order as being composed of an infinitely massive charm quark surrounded by light degrees of freedom with mass of order $\Lambda_{QCD}$. Corrections to the infinite mass limit can be computed or estimated in a systematic expansion in $1/M_c$. In the heavy quark limit the spin of the light degrees of freedom becomes a good quantum number because the spin dependent interactions between the charm quark and the light degrees of freedom are suppressed by inverse powers of $m_c$. Hence, the charmed baryons can be classified by the spin of the light degrees of freedom, $s_l$. The baryons can be further classified by their transformation properties under the light quark flavor symmetry SU(3). The lowest lying baryons have the quantum numbers of two light quarks with no orbital excitation and consequently form two irreducible representations of flavor SU(3), the $\mathbf{3}$ and the $\mathbf{6}$. The baryons in the $\mathbf{3}$ must have $s_l = 0$ while those in the $\mathbf{6}$ must have $s_l = 1$ by the antisymmetry of the state. Combining $s_l$ with the heavy quark spin produces baryons in the $\mathbf{6}$ with $J^G = \frac{3}{2}^+$, denoted $\mathbf{6}^*$, and $J^G = \frac{1}{2}^+$, as well as baryons in the $\mathbf{3}$ with $J^G = \frac{1}{2}^+$. In the infinite charm mass or light quark SU(3) limit of QCD the flavor eigenstates become the mass eigenstates. However, in the real world neither symmetry is exact and there is mixing between the two multiplets. Mixing between the $\Lambda^+_c$ member of the $\mathbf{3}$ and the $\Sigma^+_c$ member of the $\mathbf{6}$ is strongly suppressed by isospin symmetry and heavy quark symmetry and we will neglect it. Also, the $\Omega^0_c$ member of the $\mathbf{6}$ does not have a corresponding state in the $\mathbf{3}$ with which to mix. However, the $\Xi^*_{c3}$ of the $\mathbf{3}$ and the $\Xi^*_{c6}$ states of the $\mathbf{6}$ can mix via SU(3) breaking and heavy quark spin symmetry breaking interactions to form the mass eigenstates $\Xi_c$ and $\Xi'_c$. The origin of this mixing and its implications for $\Xi_b$ semileptonic decay is the subject of this paper.

In the context of the heavy quark effective theory, the masses of the flavor eigenstates can be expanded in powers of $1/M_c$ in terms of nonperturbative matrix elements $\lambda, \lambda_1, etc.$, Mixing between the $\Xi^*_{c3}$ and $\Xi^*_{c6}$ breaks the heavy quark spin symmetry and vanishes in the SU(3) symmetry limit. Since this mixing contributes to the masses of the $\Xi_c$ and $\Xi'_c$ at $O(1/M^2_c)$, it appears unlikely its contribution can be determined from a spectroscopic study of the charmed baryons (this disagrees with the conclusion of reference \cite{[7]}). In principle (though not in practice) the one $M^0_c$, two $1/M_c$ and six $1/M^2_c$ parameters in the $\Xi_c, \Xi'_c$ baryon mass expansions can be extracted from inclusive semileptonic decay spectra \cite{[8]} and substituted into the baryon mass expansions to yield the off-diagonal element in the baryon mass-matrix. Since some of these nonperturbative parameters are scheme-dependent quantities, we expect that naive definitions of a mixing angle based on this off-diagonal element
will be similarly scheme-dependent. For this reason it is important to consider physical observables, such as semileptonic decays, when discussing mixing in a field-theoretic context.

In this paper we compute the leading contributions to $\Xi_b \rightarrow \Xi'_c l \bar{\nu}_l$ and $\Xi_b \rightarrow \Xi^*_c l \bar{\nu}_l$ in heavy hadron chiral perturbation theory. We contrast this with the nonrelativistic quark model, where these decays arise entirely from mixing. It turns out that while the physical mechanisms underlying both the quark model and our calculation are the same, their implications for observables can be quite different.

In the non-relativistic quark model the mixing between the $\Xi^6_c$ and $\Xi^3_c$ arises from the chromomagnetic hyperfine interaction between the charmed quark and the light degrees of freedom of the form

$$H_{\text{int}} = A \sum_i \frac{s_c \cdot s_j}{M_C M_i},$$

(1)

where $M_c$ and $M_i$ are the constituent quark masses and $A$ is a model dependent constant. This gives rise to a mixing angle $\theta_c^{QM}$ that relates the mass and flavor eigenstates

$$|\Xi_c\rangle = \cos \theta^{QM}_c |\Xi_3\rangle + \sin \theta^{QM}_c |\Xi_6\rangle$$
$$|\Xi'_c\rangle = -\sin \theta^{QM}_c |\Xi_3\rangle + \cos \theta^{QM}_c |\Xi_6\rangle.$$  

(2)

This definition is universal in the sense that any SU(3) and heavy quark spin symmetry violating process involving these states resulting from a single insertion of a quark bilinear will depend only upon $\theta^{QM}_c$. For instance, radiative and semileptonic decay processes between these states that are forbidden in the heavy quark and flavor symmetry limit depend on $\theta^{QM}_c$. However, in field theory, the effects of SU(3) and heavy quark spin symmetry breaking interactions cannot be described by one such parameter as there are both wavefunction and vertex modifications that need not be the same for all processes.

We consider the semileptonic decay $\Xi_b \rightarrow \Xi'_c l \bar{\nu}_l$, which vanishes in the heavy quark ($m_{b,c} \rightarrow \infty$) and SU(3) limits of QCD. For our purposes it is sufficient to examine only the zero recoil point. This decay has the nice property that the matrix element of the vector current vanishes when $m_b = m_c$ due to the orthogonality of the states at this symmetry point (there is an exact global SU(2)$_H$ for the heavy quarks when $m_c = m_b$). In the naive quark model one finds that the relevant matrix element is

$$\langle \Xi'_c | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Xi_b \rangle = -\left( \sin \theta^{QM}_c - \sin \theta^{QM}_b \right) \bar{U}_{\Xi_c} \gamma_\mu U_{\Xi_b}$$
$$+ \left( \sin \theta^{QM}_c + \frac{1}{3} \sin \theta^{QM}_b \right) \bar{U}_{\Xi_c} \gamma_\mu \gamma_5 U_{\Xi_b},$$

(3)

where $\theta^{QM}_b \sim M_C/M_B$ $\theta^{QM}_c$ is the corresponding angle in the $b$-sector. With the observation of the $\Sigma^*_c$ and a measurement of the mass difference to the $\Sigma_c$ we have a measure of the size of spin-symmetry breaking interactions in the charmed baryon sector. Fitting to a naive quark model (such as used in [2]) gives a mixing angle of $\theta^{QM}_c \sim -2^\circ$. A quark model calculation utilizing SU(4) flavor symmetry [11] yields an angle of $\theta^{QM}_c \sim -4^\circ$.

Given the $\Sigma^*_c - \Sigma_c$ mass difference, we can study the effects of the chromomagnetic hyperfine interaction in the model-independent framework of heavy hadron chiral perturbation theory. In particular, the one operator at leading order in the chiral expansion that gives the $\Sigma^*_c - \Sigma_c$ mass difference will also contribute to $\Xi_b \rightarrow \Xi'_c l \bar{\nu}_l$ decay through one loop graphs
involving a $K, \eta$ or $\pi$. Other heavy quark symmetry violating effects, such as current and coupling modifications, will also feed into $\Xi_b \to \Xi'_c e \nu$ decay through one loop graphs, but we will see that they are formally sub-leading. While numerically the sub-leading contributions may be comparable to the hyperfine contribution, one expects the contribution of the leading term to be indicative of the size of the actual decay rate. Quark model expectations that the hyperfine interaction dominates over all other effects reinforce this idea. We will see that neither expectation seems to be realized.

In the infinite mass limit the leading order interactions of the charmed baryons with the pseudo-Goldstone bosons associated with chiral symmetry breaking are given by

$$L^{(0)} = i T v \cdot D T - i S_{\alpha} v \cdot D S^\alpha - \Delta_0 S_\alpha S^\alpha +$${

\begin{align*}
+ i g_2 \xi_\mu \xi_\nu \sigma_\mu S^\nu + g_3 (T A_\mu S^\mu + \text{h.c.})
\end{align*}

$$,$$ (4)$$

where we are using heavy baryon fields with four-velocity $v_\mu$. The mass difference between the triplet $T$ and sextet $S$ baryons is $\Delta_0 \approx 210$MeV and the axial vector field of mesons, denoted by $A_\mu$, is

$$A_\mu = \frac{i}{2} \left( \xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi \right)$$

$$\xi = \exp \left( iM/f \right)$$,

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where $M$ is the usual octet of pseudo-Goldstone bosons, and $f \approx 132$MeV is the pion decay constant. The axial couplings $g_2$ and $g_3$ are not determined by the symmetries and must be fit to data. The recent measurements of the $\Sigma_c^*$ width ($\Gamma(\Sigma_c^{*+}) = 17.9^{+3.8}_{-3.2} \pm 4.0$ MeV, $\Gamma(\Sigma_c^{*0}) = 13.0^{+3.7}_{-3.0} \pm 4.0$ MeV) [4] determines that $|g_3| = 0.9 \pm 0.2$, approximately 30% smaller than one would have estimated from large $N_c$ arguments. The above Lagrange density embodies the spin symmetry of the heavy quark limit of QCD and therefore the baryons in the $6'$ and $6^{''}$ representations are degenerate at this order. The observed mass difference is described by the inclusion of the explicit spin-symmetry breaking term

$$L^{(1)} = \frac{i}{3} \delta_c \sigma_\mu S^\mu$$

for the charmed baryons and an analogous term for the $b$-baryon. This operator contains precisely the same physics as the quark model operator Eq. (1), namely the chromomagnetic hyperfine interaction, so by computing the contribution of $L^{(1)}$ to $\Xi_b \to \Xi'_c e \nu$ we capture the essential physics underlying the quark model in a field-theoretic context.

Matrix elements of flavor-changing currents have a chiral representation that in general involves unknown, nonperturbative Isgur-Wise form functions. At zero recoil heavy quark symmetry normalizes these form factors to unity allowing us to compute the contribution from $L^{(1)}$ in terms of $g_2, g_3$, and physical masses.

At leading order in chiral perturbation theory, the amplitude for $\Xi_b \to \Xi'_c e \nu$ receives contributions from the mixing graph of fig. 1 (wavefunction renormalization), as well as the vertex modification in fig. 2. The contribution from the wavefunction renormalization is process-independent, and can be identified with the contribution from the mixing angle in the quark model. However, the process-dependent vertex contribution, which is naively the same size as the contribution from wavefunction renormalization, has no analogue in
the quark model. Therefore we expect the prediction of chiral perturbation theory to differ substantially from that of quark models, even if the hyperfine interaction captures all the essential physics underlying the \( \Xi_b \rightarrow \Xi_c' e^- \nu_l \) decay (that is, even if the operators \( L^{(1)}_i, \mathcal{H}_{\text{int}} \) dominate the rate).

In the formal chiral limit, where the chiral symmetry breaking scale \( \Lambda_\chi \sim \Delta_0 \gg M_K, M_\pi \), we expect the one-loop contribution to the \( \Xi_b \rightarrow \Xi_c' \) matrix element to be of order \( \sim g_2 g_3 \delta_c M_K^2 \log(M_K^2/\Delta_0^2) / (\Lambda_\chi^2 \Delta_0) \sim 0.02 \), yielding a mixing angle of a few degrees. In this limit the contributions of \( 1/M_c \) current and coupling modifications are easy to compute because they involve the same loop integral as the hyperfine interaction, except that the hyperfine mass splittings \( \delta_c, \delta_b \) may be set to zero. Explicit calculation shows that these integrals have no \( M_\pi^2 \log M_K^2 \) chiral logs, so they are sub-leading to the hyperfine contribution. Experimentally \( \Delta_0 \) is of the same order as \( M_K \), so we might instead consider the limit, \( \Delta_0 \sim M_K \sim m_{s/2} \) (In the large \( N_c \) limit of QCD, \( \Delta_0 \rightarrow 0 \), so we may imagine this as a combined large \( N_c \) and chiral limit). In this case, we expect to find a matrix element of order \( g_2 g_3 \delta_c M_K/\Lambda_\chi^2 \), yielding a similarly sized angle. Based on dimensional analysis, contributions from \( 1/M_c \) current or coupling modifications will be higher order, \( \sim (\Lambda_QCD/M_c)(M_K^2 \log M_K^2/\Lambda_\chi^2) \), so the hyperfine interaction is the leading contribution in this limit as well. Notice that in both cases these estimates are larger than what one would naively estimate from local counterterms, whose leading contribution is of order \( m_s/m_c \).

The graph in fig. 1 causes the \( \Xi_{c3} \) and \( \Xi_{c0} \) flavor eigenstates to mix in such a way that a non-unitary change of basis is required to diagonalize the Hamiltonian. Nevertheless, the off-diagonal element,

\[
A_{36} = -\frac{g_2 g_3 \delta_c}{2 \sqrt{6} 16 \pi^2 f^2 \Delta_0} \sum_i C_i \left[ K(m_i, 0, \delta_c) - K(m_i, \Delta_0, \Delta_0 + \delta_c) \right],
\]

is a scheme-independent, universal contribution to SU(3) and heavy quark spin symmetry violating processes involving these states, so it is tempting to identify it with the mixing angle of the quark model. The sum is over the mesons in the loop, with Clebsch-Gordan coefficients \( C_6 = 3, C_{K} = -2 \) and \( C_0 = -1 \). The loop integral function that arises, keeping only terms that survive the summation, is

\[
K(m, \delta_1, \delta_2) = \frac{2}{3} \left[ (\delta_1^2 + \delta_2^2 + \delta_1 \delta_2) \log \left( \frac{m^2}{\mu^2} \right) + \frac{(\delta_1^2 - m^2)^{3/2}}{\delta_1 - \delta_2} \log \left( \frac{\delta_1 - \sqrt{\delta_1^2 - m^2}}{\delta_1 + \sqrt{\delta_1^2 - m^2}} \right) \right] - \frac{(\delta_2 - m^2)^{3/2}}{\delta_2 - \delta_1} \log \left( \frac{\delta_2 - \sqrt{\delta_2^2 - m^2}}{\delta_2 + \sqrt{\delta_2^2 - m^2}} \right),
\]

where \( \mu \) is a subtraction scale that will cancel out after summing over SU(3) states. For physical mass values, comparing \( A_{36} \) with the naive quark model \( |\sin \theta_{cQM}^0| \) gives a mixing angle of \( 1.4 g_2 g_3 \) degrees, consistent with quark model expectations.

However, the hyperfine interaction induces not only mixing, but also the vertex modification of fig. 2. Combining the two and evaluating at zero-recoil \( (v \cdot v' = 0) \), we find a \( \Xi_b \rightarrow \Xi_c' e^- \nu_l \) amplitude of

\[
\langle \Xi_c' | \bar{e}_\mu (1 - \gamma_5) b | \Xi_b \rangle = \frac{g_2 g_3}{2 \sqrt{6} 16 \pi^2 f^2} \left[ F_V \bar{U}_{\Xi_c'} \gamma_\mu U_{\Xi_b} + F_A \bar{U}_{\Xi_c'} \gamma_\mu \gamma_5 U_{\Xi_b} \right]
\]
\[ F_V = \sum_i C_i \left[ K(m_i, \delta_c, \Delta_0 + \delta_b) - K(m_i, 0, \Delta_0) + \frac{\delta_c}{2\Delta_0} (K(m_i, 0, \delta_c) - K(m_i, \Delta_0, \Delta_0 + \delta_c)) \right. \\
\left. + \frac{\delta_b}{2\Delta_0} (K(m_i, 0, \delta_b) - K(m_i, \Delta_0, \Delta_0 + \delta_b)) \right] \]

\[ F_A = \frac{1}{2} \sum_i C_i \left[ \frac{2}{9} K(m_i, 0, \Delta_0) + \frac{16}{9} K(m_i, 0, \Delta_0 + \delta_b) - \frac{10}{9} K(m_i, \delta_c, \Delta_0 + \delta_b) - \frac{8}{9} K(m_i, \delta_c, \Delta_0) \right. \\
\left. - \frac{\delta_c}{3\Delta_0} (K(m_i, 0, \delta_c) - K(m_i, \Delta_0, \Delta_0 + \delta_c)) + \frac{\delta_b}{3\Delta_0} (K(m_i, 0, \delta_b) - K(m_i, \Delta_0, \Delta_0 + \delta_b)) \right] , \quad (9) \]

For physical values, the mixing and vertex contributions nearly cancel, resulting in an amplitude of

\[ \langle \Xi_c | \overline{c} \gamma_\mu (1 - \gamma_5) b | \Xi_b \rangle \approx 10^{-3} g_{2g3} \left[ 0.9 \overline{U}_{c \gamma_\mu U_{\Xi_0}} + 3.0 \overline{U}_{c \gamma_\mu \gamma_5 U_{\Xi_0}} \right] . \quad (10) \]

Notice that the amplitude given in Eq. (10) is not in the form expected by Eq. (3). The relative contributions of the four terms in the naive quark model result, Eq. (3), are a consequence of including one insertion of the hyperfine interaction, while Eq. (9) includes the effects of the hyperfine interaction to all orders. Treating the hyperfine splittings \( \delta_{c,b} \) as small compared to the other mass scales in the problem and expanding to linear order we can write the zero-recoil matrix element in the form of Eq. (3) with an effective (but non-universal) mixing angle of

\[ \sin(\theta_{c}^{\text{eff}}) = -\frac{g_{2g3}}{4\sqrt{6} \ 16\pi^2 f^2} \Delta_0 \delta_c \sum_i C_i \ G(m_i, \Delta_0) , \]

where

\[ \int \frac{d^m q}{(2\pi)^m} \frac{q^a q^b}{[q^2 - m^2][v \cdot q]^2[v \cdot q - \Delta_0]^2} = -i \ \frac{16\pi^2}{16\pi^2} \left[ G(m, \Delta_0) g^{ab} + V(m_i, \Delta_0) v^a v^b \right], \]

\[ G(m, \Delta_0) = \frac{2}{3\Delta_0} \left[ \sqrt{\Delta_0^2 - m^2}(2m^2 + \Delta_0^2) \ln \frac{\Delta_0 + \sqrt{\Delta_0^2 - m^2}}{\Delta_0 - \sqrt{\Delta_0^2 - m^2}} + \Delta_0^3 \ln \frac{m^2}{\mu^2} + 2m^3 \pi - 4\Delta_0 m^2 \right] . \quad (12) \]

Numerically, this gives \( \theta_{c}^{\text{eff}} = -0.06 g_{2g3} \) degrees.

We can also compute the contribution of the hyperfine interaction to \( \Xi_b \to \Xi_c b \nu_l \),

\[ \langle \Xi_c | \overline{c} \gamma_\mu (1 - \gamma_5) b | \Xi_b \rangle = \frac{2}{9} \frac{g_{2g3}}{16\pi^2 f^2} \sum_i \left[ K(m_i, \Delta_0 + \delta_b, \delta_c) - K(m_i, \Delta_0, \delta_c) \\
+ 5K(m_i, \Delta_0 + \delta_b, 0) - 5K(m_i, \Delta_0, 0) + \frac{3\delta_b}{\Delta_0} [K(m_i, \delta_b, 0) - K(m_i, \Delta_0, \Delta_0 + \delta_b)] \right] T^\mu_{\Xi_c} U_{\Xi_b} , \quad (13) \]

where \( T^\mu_{\Xi_c} \) is the Sarita-Schwinger spinor for the \( \Xi_c^* \). Notice that this amplitude vanishes when \( \delta_b = 0 \), as required by angular momentum conservation. For physical values, the mixing and vertex contributions again nearly cancel, giving an amplitude of

\[ \langle \Xi_c^* | \overline{c} \gamma_\mu (1 - \gamma_5) b | \Xi_b \rangle = 5 \cdot 10^{-4} \ g_{2g3} \ T^\mu_{\Xi_c} U_{\Xi_b} , \quad (14) \]
again much smaller than naive quark model estimates.

The amplitudes arising from this calculation are roughly a factor of thirty smaller than naive expectations. In the limit $\Delta_0 \gg m_i, \delta_c$, the decay amplitude vanishes like $\delta_c/\Delta_0$, since the intermediate loop baryon becomes very massive, but we know of no a priori reason why the amplitude would vanish for $\Delta_0 \sim m_i$. Algebraically, the strong cancelations between mixing and vertex contributions arise because $K(m, \delta_1, \delta_2)$ in Eq. (8) is a slowly varying function. Away from zero recoil, the vertex and mixing contributions get multiplied by different nonperturbative form factors, so far from this kinematic point, the contribution from the hyperfine interaction could be an order of magnitude larger.

What can we conclude from the exceptionally small amplitudes at zero recoil? The formally leading contributions we have computed are clearly not the numerically dominant terms: Other operators, responsible for current and coupling corrections, determine the rate. The “estimate” provided by computing the hyperfine contribution cannot be trusted, even to an order of magnitude. Nevertheless, we can learn something interesting from the computation.

Firstly, naive power counting in the heavy baryon sector can fail rather spectacularly. This can be traced in part to the presence of the additional mass scale $\Delta_0$, which leads to powers of $\frac{m_\pi}{\Delta_0}$ and $\frac{\Delta_0}{m_K}$ that are important numerically. In addition, it is important to keep powers of $\frac{\delta_c}{m_K}$ (as we did in this calculation), because this ratio is neither parametrically small nor large in the most general combined heavy quark, chiral limits. This feature has been noted previously in the context of heavy mesons [12]. Care should be used when using chiral perturbation theory to compute properties of charmed and bottom baryons ( e.g. [13]).

Secondly, this computation reveals potential drawbacks to the naive quark model estimate. The quark model asserts that the chromomagnetic hyperfine interaction is responsible for $\bar{3} - 6$ mixing, and that this mixing is solely responsible for $\Xi_b \rightarrow \Xi'_c l \nu_l$. We have computed the contribution of the hyperfine interaction to this decay. We find a process and scheme-independent contribution from wavefunction mixing that can be identified with the quark model mixing angle, and is numerically comparable to quark model expectations. However, the hyperfine interaction also induces a compensating vertex modification to $\Xi_b \rightarrow \Xi'_c l \nu_l$ that is absent in the quark model, and nearly cancels the contribution from mixing. This suggests the hyperfine interaction plays an inessential role for this decay, at least near zero recoil. The decay may simply be dominated by operators that are not addressable within the context of a nonrelativistic quark model.

These cautionary notes for both quark model and chiral perturbation theory calculations highlight the difficulty of reliably estimating the $\Xi_b \rightarrow \Xi'_c l \nu_l$ decay rate. By illustrating the limitations inherent in both methods, we may aid their successful application elsewhere. It will be interesting to see if the remarkable cancelations plaguing the contributions of the leading operator to $\Xi_b \rightarrow \Xi'_c l \nu_l$ and $\Xi_b \rightarrow \Xi^*_c l \nu_l$ decay amplitudes are mirrored in other physical observables in the charmed baryon system.

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FIG. 1. Wavefunction mixing between the charmed baryon or b-baryon states. The dashed line denotes a pseudo-Goldstone boson. The spin-symmetry violating mass difference between the intermediate states of the $6^{(*)}$ representations prevents these graphs from vanishing.

FIG. 2. Vertex contribution to the decay $\Xi_b \rightarrow \Xi'_c e^{-\nu_e}$. The dashed line denotes a pseudo-Goldstone boson.