A Distributed-MPC Framework for Voltage Control Under Discrete Time-Wise Variable Generation/Load

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Abstract—This article presents a distributed predictive design for real-time voltage control under changing load and generation profiles at discrete intervals. The existing control designs include centralized approach, providing an optimal solution but less scalable and susceptible to single-point failures/attacks, as well as decentralized or localized approach, having increased scalability and attack resilience but lacking optimality. The proposed distributed solution offers the attractive features of both approaches, where the neighboring nodes share their local information to attain an optimal solution while retaining scalability and resilience to single-point failures/attacks. We first introduce the centralized version of the voltage control problem assuming grid observability and then transfer it to the distributed versions based on both bus-wise and area-wise decompositions of the network. The distributed version is solved via alternating direction method of multipliers (ADMM) that, for bus-wise decomposition, needs a full set of local measurements, whereas only a partial set of local measurements (that guarantee area-wise grid observability for each area) is needed for area-wise decomposition, along with neighbor-to-neighbor communications. Additionally, leveraging the availability of measurement data, the framework includes a distributed method to estimate the admittance matrix \( Y \) of the underlying network graph. The proposed framework is validated against IEEE-30, IEEE-57 bus transmission systems, and IEEE-123 bus distribution systems and can tolerate certain levels of generation/load prediction uncertainties, modeling errors, and communication failures; plus, its in-built redundancy supports attack detection.

Index Terms—Distributed control, data-driven estimation, local communication, model predictive control (MPC), voltage control.

NOMENCLATURE

A. Power Systems Quantities

- \( \gamma \): Network graph \( (\mathbb{N}, \mathbb{E}) \), where \( \mathbb{N} := \text{set of buses} \{0, \ldots, N\} \) \( \mathbb{E} := \text{set of lines} \).
- \( V_i, \theta_i \): Voltage magnitude and angle at bus \( i \).
- \( Y \): Bus admittance matrix: \( [Y_{ik}] \in \mathbb{C}^{(N_B) \times (N_B)} \).
- \( G_{ik}, B_{ik} \): Real and imaginary part of \( Y_{ik} \).
- \( P_i^G, Q_i^G \): Active and reactive power generations.
- \( P_i^D, Q_i^D \): Active and reactive power demands.
- \( \mathbb{N}_i \): Set of neighboring buses of bus \( i \).

- \( P_{\Delta i}, Q_{\Delta i} \): Predicted generations at bus \( i \).
- \( D_{\Delta i}, Q_{\Delta i} \): Predicted demands at bus \( i \).
- \( E_i \): Net injections at bus \( i \).
- \( \hat{D}_{\Delta i}, \hat{Q}_{\Delta i} \): Predicted net injections at bus \( i \).
- \( u_i \): Predicted changes of injections at bus \( i \).
- \( \Delta V_i, \Delta \theta_i \): Change in voltage magnitude and angle at bus \( i \).
- \( V_{\text{ref}}, u_{i, \text{min}}, u_{i, \text{max}} \): Voltage reference. Lower and upper limit of control inputs at bus \( i \).
- \( V_{i, \text{min}}, V_{i, \text{max}} \): Lower and upper limit of voltage magnitude at bus \( i \).
- \( \Delta \theta_{i, \text{min}}, \Delta \theta_{i, \text{max}} \): Lower and upper limit of change of angle at bus \( i \).

B. Distributed Optimization Quantities

- \( x_i \): Local-copy of public variables of bus \( i \): \( [x_{i,k}] \mid k \in \{i\} \cup \mathbb{N}_i \) \( \in \mathbb{R}^{2 \times ((N_B)+1)} \).
- \( z_i \): Bus \( i \)'s public variables (true-copy).
- \( J_i(\cdot) \): Objective function of bus \( i \).
- \( \lambda_i \): Lagrangian multiplier of bus \( i \).
- \( L_{E_i}(\cdot) \): Augmented Lagrangian for bus \( i \).
- \( E_i \): Consistency constraints for bus \( i \).
- \( N_{\gamma}, N_{\gamma}^B, N_{\gamma}^{NB} \): Set of buses, boundary buses, and non-boundary buses of area \( \gamma \), respectively.
- \( \tilde{\gamma} \): Set of neighboring areas of area \( \gamma \).
- \( X_{\gamma} \): Local-copy of public variables of area \( \gamma \).
- \( Z_{\gamma} \): Area \( \gamma \)'s public variables (true-copy).
- \( U_{\gamma} \): Area \( \gamma \)'s private variables.
- \( J_{\gamma}(\cdot) \): Objective function of area \( \gamma \).

I. INTRODUCTION

A. Motivation and Related Works

In order to achieve carbon-free-energy by 2050 [1], energy regulators all over the world are pushing independent system operators (ISOs) and power utilities to adopt energy generation from renewable sources such as solar and wind, making the energy sources volatile and also distributed throughout the grid. With this increased penetration of variable and intermittent electricity generation coupled with the load uncertainties (for example, owing to added loads such as plug-in electric vehicles), voltage regulation issues are a challenging problem in power system operation. The prevalent control designs include centralized designs [2], [3], as well as decentralized ones based...
on localized information [4]. While centralized processing can attain an optimal solution, those are (i) prone to single-point failures and cyber-attacks, (ii) lack scalability with an increase in the number of decision variables, and (iii) require robust communication infrastructure and computation resources capable of handling high volume of data. In contrast, decentralized control is scalable and not susceptible to single-point failures/attacks, but it provides non-optimal solutions. Also, at times the solution provided by the localized droop-control scheme is infeasible for the control problem at hand [5], [6], [7]. In contrast, the distributed solution involves local computations based on information shared among neighbors and offers scalability and resiliency like the decentralized case and optimality like the centralized case. This has resulted in recent studies [8], [9], [10], [11], showing that distributed controls are emerging as potential alternatives in various sectors of power system operation.

1) Existing Distributed Approaches and Their Limitations: The distributed algorithms for voltage control can be classified into two types [10], namely, (a) static (“offline”) vs. (b) dynamic (“online”). Static (or offline) approaches are generally open-loop (in feed-forward nature) and do not use real-time measurements. Among static approaches, [12] utilized a semi-definite programming (SDP)-relaxed OPF (optimum power flow) formulation in conjunction with the dual-ascent method for distributed voltage regulation. Another SDP-relaxed distributed formulation with ADMM-based decomposition can be found in [13] for optimizing active and reactive power set-points of PV inverters. In [14], distributed voltage control is achieved by combining ADMM with second-order cone programming (SOCP)-relaxed OPF. [15] presented an ADMM-based distributed reactive power compensation problem. This work utilized convex-relaxed OPF, where the nonlinear terms were held constant and updated periodically based on the desired operating point. A distributed voltage control problem minimizing total power losses with the Linearized DistFlow (LinDistFlow) model of radial-distribution network is presented in [16]. In [17], a distributed algorithm named proximal atomic coordination (PAC) is presented for OPF problems in a single-phase distribution network.

In contrast, dynamic (or online) approaches [18], [19], [20], [21], [22], [23], [24] are closed-loop (feedback based), and utilizing the local as well as neighbor-communicated measurements of current time instant decide the control actions at the next instant; where to decide on the control actions, the distributed controllers can go through multiple rounds of communications to reach their convergent decisions [24]. Within the class of dynamic approaches, [19] presented an ADMM-based distributed method for voltage control in a multi-phase distribution system with (i) the linearized sensitivity-based (linear q-v) model to avoid the computational challenges and (ii) a relaxation of hard voltage constraint by assigning a soft penalty. [20] presented a primal-dual-based distributed algorithm exploiting network sparsity with the Linearized DistFlow (LinDistFlow) model of the distribution system. A similar approach is taken in [21], which used asynchronous dual decomposition for faster convergence in the presence of asynchronous and delayed communication. Certain extensions of [20] and [21] can be found in [7] and [24], respectively, for multi-phase unbalanced distribution networks. A distributed micro-grid voltage control, with an assumption of a small phase angle difference among the neighboring buses and event-triggered communication, is presented in [22]. In another work [23], a dual-ascent-based distributed voltage control algorithm can be found with LinDistFlow model of the distribution network.

The following elaborate certain limitations in the existing state-of-the-art:

1) Solving optimal control problem with AC power flow constraints is known to be non-convex, and NP-hard [10], [25]. Static (or offline) approaches utilize SDP/SOCP-based relaxation to mitigate the associated computation issues. But these relaxations do not guarantee the feasibility of the obtained solution, requiring those to be verified separately [10], and also these methods are not computationally efficient to afford their real-time implementation [24].

2) Dynamic (or online) approaches, although amenable to real-time implementation, are mostly based on linearized power system models, e.g., Linearized DistFlow (LinDistFlow), or linear q-v model, that are applicable to only radial networks, and those do not consider cycle conditions (a set of nonlinear equations) required for general networks [8].

3) Although the existing dynamic (or online) methods use the current measurements, they do not incorporate the future estimates of generation and/or load profiles. This becomes limiting in the presence of variable renewable generations and load fluctuations.

4) Finally, the existing methods rely on the full knowledge of the system/network parameters. However, network updates/reconfiguration (e.g., in case of faults) alter the network admittances, and it may be possible to estimate some of those, exploiting system observability [26].

2) Predictive Frameworks: Driven by the availability of data logs, the importance of predictive control is growing and is supported by the recent advances in short-term forecasting [27], [28], [29], [30]. To this end, we leverage MPC, which computes the control variables iteratively at each control instant, optimizing the predicted future behavior of the underlying system by integrating the prediction of renewable generation/load profiles with the control computation (details can be found in Sections II and III). Among existing works, MPC-based emergency voltage control, secondary control are studied in centralized [31], [32] and distributed [33], [34], [35], [36] settings. A sensitivity-based distributed MPC with leader-follower consensus protocol is presented in [37], where the sensitivities are used to predict future trajectories utilizing linearized relationships between the control inputs and the controlled variables, are computed offline using a centralized protocol, as opposed to desired online protocol using local communication.

3) Data-Driven Approaches: Interests in data-driven estimation and control design are growing in power system applications [38], [39], [40], [41]. The authors in [38], [39] focused on measured-data-based sensitivity computation for centralized voltage control. [42] extended [38] to consider area-wise linear sensitivity models of a distribution network and further utilized those to determine power set-points of distributed energy.
resources (DERs), adopting ADMM-based distributed methods. In general, a sensitivity formulation ought to employ system-wide communication, but the proposed per-area sensitivity models in [42] only consider communications over adjacent areas. Compared to these approaches, our method (as discussed in Sections II to IV) can be implemented without estimating any sensitivity model, and we also perform distributed estimation of line admittances based on the availability of the measured data and communication among neighbors.

B. Our Approach and Contributions

We present here a novel distributed voltage control framework that integrates distributed optimization with predictive control. In contrast to existing approaches that use linearized version of the AC power flow model, which either assumes angle differences between connected buses to be small (hence, \( \sin(\theta_i - \theta_j) \approx \theta_i - \theta_j \) for adjacent buses \( i \) and \( j \)), or assumes relaxed DistFlow/LinDistFlow models without any constraint to ensure consistency among the voltage angles, our approach (i) takes into account the real-time measurement available at current time instant \( t \), (ii) linearizes the nonlinear AC power flow equations with respect to this measured operating point, (iii) incorporates the predicted values of renewable generations and loads based on short-term forecasting for time instant \( t + 1 \), (iv) formulates the voltage regulation problem for changes in the control variables (reactive power provided by DERs) considering hard constraints in voltage and reactive power, (v) decomposes and solves the optimization problem in a distributed manner, adopting ADMM and relying on communications among local neighbors. The distributed control framework supports both bus-wise as well as area-wise decomposition of the power network—the former fits with the futuristic smart-grid concept that will allow direct local measurements at each bus, whereas the latter is suitable for the current setting of sparsely placed measuring devices, so as to ensure observability of unmeasured nodes/buses within each area. The distributed framework also includes a method for data-driven estimation of the network parameters, namely, the admittance of the lines associated with the links used in communication, which for the bus-wise decomposition corresponds to all the lines, and for the area-wise decomposition amounts to all the tie-lines.

In summary, our main contributions are as follows:

1) We develop a dynamic (or online) distributed voltage control framework that is real-time and general: Unlike the existing works, it is not restricted to radial networks but rather can be applied to arbitrary transmission and distribution network topology.

2) The proposed framework is general enough to comply with bus-wise as well as area-wise decomposition, where the latter affords sparsely located measuring devices, as long as those ensure area-wise observability of the unmeasured buses.

3) The proposed distributed method is predictive and data-driven by integrating generation and load forecasting in control computation and can also perform data-informed estimation of network parameters (line impedances), exploiting the availability of the measurements.

4) Owing to operating pointwise linearization, the resulting optimization problem at each control instant is convex, possessing a unique optimum (not necessarily globally optimal for the nonlinear OPF problem). As a result, the ADMM-based distributed solution guarantees asymptotic convergence to the said optimum [43], [44], [45].

5) The proposed method is able to tolerate certain levels of prediction errors in generation and load forecasting, modeling errors associated with data-informed estimation, as well as communication failures and network attacks.

The distributed predictive control methodology is tested for IEEE-30 Bus and IEEE-57 Bus transmission networks and IEEE-123 Bus distribution network to achieve the desired voltage performance while adhering to the reactive power constraints and employing only the locally-exchanged information.

II. POWER SYSTEM MODEL AND CENTRALIZED CONTROL FORMULATION

A power system connectivity of buses via the lines can be represented by an undirected network graph \( G = (\mathbb{N}, \mathbb{E}) \), where \( \mathbb{N} = \{0, \ldots, N\} \) is the set of buses, and \( \mathbb{E} \) represents the set of lines connecting the buses, with line \( l_{ik} \in \mathbb{E} \) connecting buses \( i, k \in \mathbb{N} \). The magnitude and angle of voltage at bus \( i \) are denoted \( V_i \) and \( \theta_i \), respectively. We let bus 0 to be the slack/reference bus, and so by convention, \( V_0 = 1 \) p.u., \( \theta_0 = 0 \), and \( \mathbb{N} = \mathbb{N} - \{0\} \) denotes the set of buses except slack bus. The symmetric bus admittance matrix \( Y = [y_{ik}] \in \mathbb{C}^{N \times N} \) of the given network is defined as:

\[
y_{ik} = \begin{cases} y_i + \sum_{l=0, l\neq i}^{N} y_{il}, & \text{if } i = k \\ -y_{ik}, & \text{otherwise} \end{cases},
\]

where \( y_i \) denotes the (complex) admittance to the ground at bus \( i \), \( y_{ik} \) is the admittance of line \( l_{ik} \in \mathbb{E} \) (implying \( y_{ik} = y_{ki} \neq 0 \) if \( l_{ik} \in \mathbb{E} \) and otherwise \( y_{ik} = y_{ki} = 0 \)). We write the complex admittance as, \( Y_{ik} = G_{ik} + jB_{ik} \), with \( G_{ik} \) being the real part and \( B_{ik} \) the imaginary part.

The active and reactive power generations at any bus \( i \) are denoted by \( P_{G_i} \), \( Q_{G_i} \), respectively, while \( P_{D_i} \), \( Q_{D_i} \) represent the active and reactive power demands, respectively. Therefore the net injections of active and reactive powers at bus \( i \) are given by: \( P_{i}^\text{in} = P_{G_i} - P_{D_i} \), and \( Q_{i}^\text{in} = Q_{G_i} - Q_{D_i} \). Letting \( \mathbb{N}_i := \{k : l_{ik} \in \mathbb{E}\} \) denote the neighboring buses that are one hop away from bus \( i \), the power flow relations at bus \( i \in \mathbb{N} \) are given by:

\[
P_{i}^\text{in} = \sum_{k \in \{i\} \cup \mathbb{N}_i} V_i V_k \left[ G_{ik} \cos(\theta_i - \theta_k) + B_{ik} \sin(\theta_i - \theta_k) \right],
\]

\[
\equiv y_i^{G_i}(V_i, \theta_i, V_k, \theta_k \mid k \in \mathbb{N}_i),
\]

(2a)

\[
Q_{i}^\text{in} = \sum_{k \in \{i\} \cup \mathbb{N}_i} V_i V_k \left[ G_{ik} \sin(\theta_i - \theta_k) - B_{ik} \cos(\theta_i - \theta_k) \right]
\]

\[
\equiv y_i^{Q_i}(V_i, \theta_i, V_k, \theta_k \mid k \in \mathbb{N}_i).
\]

(2b)

With the proliferation of renewable energy generation and the variable nature of the loads, maintaining voltage trajectories close to the desired reference value of \( V_{\text{ref}} = 1.00 \) p.u. is challenging, requiring a well-defined control design. For this, we proposed to leverage the recent developments in data-driven
prediction techniques, through which an accurate prediction of up to an hour-ahead generation and load profiles has become possible for most utilities and system operators [27], [28], [29], [30]. The prediction error is typically within 3–5% for the data collected from California ISO [46]. Short-term forecasting (STF) of load and renewable generation has found successful application in day-to-day operations of power system, especially in unit commitment (UC), scheduling operations, system security and control of power systems [30]. As an example, [47] proposed a reactive power control strategy utilizing load and solar irradiance forecast, while [48] designed a real time automatic generation control (AGC) strategy based on short term load prediction. A forecast-based grid frequency control combining AGC and MPC can be found in [49].

In line with the above works, even in our work presented here, we employ the predicted generation and load profiles available at the utilities along with the latest measurements of the system variables: at any control time instant $t$, the bus voltages and angles $[V_{i,t}, \theta_{i,t}]$, as well as the active and reactive power injections $[P_{i,t}, Q_{i,t}]$ are either measured or inferred (exploiting grid observability —See Remark 1), so are known for all $i \in \mathbb{N}$. Additionally, the predicted generations $[\hat{P}_{i,t+1}, \hat{Q}_{i,t+1}]$ and predicted demand profiles $[\hat{P}^D_{i,t+1}, \hat{Q}^D_{i,t+1}]$, and thereby the predicted injections $[\hat{P}^m_{i,t+1}, \hat{Q}^m_{i,t+1}]$ for the next control time instant $t+1$ are also available from the forecast data.

Then the power flow (2) at bus $i$ for time $t+1$ under a reactive compensation of $u_{i,t+1}$ (to be decided through optimization) are:

$$
g^P_i(\hat{V}_{i,t+1}, \hat{\theta}_{i,t+1}, \hat{V}_{k,t+1}, \hat{\theta}_{k,t+1} | k \in \mathbb{N}_i) = \hat{P}^m_{i,t+1}, \tag{3a}$$

$$
g^Q_i(\hat{V}_{i,t+1}, \hat{\theta}_{i,t+1}, \hat{V}_{k,t+1}, \hat{\theta}_{k,t+1} | k \in \mathbb{N}_i) = \hat{Q}^m_{i,t+1} + u_{i,t+1}, \tag{3b}$$

where, $\hat{V}_{k,t+1} = V_{k,t} + \Delta V_{k,t+1}$ and $\hat{\theta}_{k,t+1} = \theta_{k,t} + \Delta \theta_{k,t+1}$ for $k \in \{i\} \cup \mathbb{N}_i$, are predicted values at $t+1$, and satisfy equation (3a)–(3b). Expressing the predicted changes in injected active and reactive powers as: $$\Delta P^m_{i,t+1} = \hat{P}^m_{i,t+1} - P_{i,t} , \quad \Delta Q^m_{i,t+1} = \hat{Q}^m_{i,t+1} - Q_{i,t} ,$$ and applying Taylor series approximation on (3), we obtain:

$$\left( \frac{\partial g^P_i}{\partial V_i} \right)_t \Delta V_{i,t+1} + \left( \frac{\partial g^P_i}{\partial \theta_i} \right)_t \Delta \theta_{i,t+1} + \sum_{k \in \mathbb{N}_i} \left[ \left( \frac{\partial g^P_i}{\partial V_k} \right)_t \right] = \Delta P^m_{i,t+1}, \tag{4a}$$

$$\left( \frac{\partial g^Q_i}{\partial V_i} \right)_t \Delta V_{i,t+1} + \left( \frac{\partial g^Q_i}{\partial \theta_i} \right)_t \Delta \theta_{i,t+1} + \sum_{k \in \mathbb{N}_i} \left[ \left( \frac{\partial g^Q_i}{\partial V_k} \right)_t \right] = \Delta Q^m_{i,t+1} + u_{i,t+1}. \tag{4b}$$

In the above, $u_{i,t+1}$ (and hence also $[\Delta V_{i,t+1}, \Delta \theta_{i,t+1}]$) is determined through the following proposed Centralized Predictive Voltage Control (C-PVC):

$$\begin{align*}
\min_{\Delta V_{i,t+1}, \Delta \theta_{i,t+1}, u_{i,t+1}} & \sum_{i=1}^{N} \left[ ||V_{i,t} + \Delta V_{i,t+1} - V_{ref}||_2 + ||u_i, u_{i,t+1}||_2 \right] , \\
\text{subject to:} & \forall i \in \mathbb{N}, \text{System Constraints: (4a)–(4b)}, \\
& u_{i,\min} \leq u_{i,t+1} \leq u_{i,\max}, \\
& V_{i,\min} \leq V_{i,t} + \Delta V_{i,t+1} \leq V_{i,\max}, \\
& \Delta \theta_{i,\min} \leq \Delta \theta_{i,t+1} \leq \Delta \theta_{i,\max}. 
\end{align*} \tag{5d}$$

In the C-PVC formulation of (5), the optimization variables correspond to: $\Delta V_{i,t+1} := \{\Delta V_{1,t+1}, \ldots, \Delta V_{N,t+1}\}^T, \Delta \theta_{i,t+1} := \{\Delta \theta_{1,t+1}, \ldots, \Delta \theta_{N,t+1}\}^T$, and $u_{i,t+1} := \{u_{1,t+1}, \ldots, u_{N,t+1}\}^T$, while $[u_{i,\min}, u_{i,\max}], [V_{i,\min}, V_{i,\max}], [\Delta \theta_{i,\min}, \Delta \theta_{i,\max}]$ represent the lower and upper bounds for control inputs (reactive compensation), the voltage values, and the changes in angles of bus $i$, respectively.

Remark 1: Note the optimization (5) depends on the voltage and angle values of all buses, but not all need to be measured in a centralized setting, since exploiting the notion of observability [50], the voltages and angles of unmeasured nodes/buses can be inferred from the measurements at the optimally placed minimal set of phasor measurement units (PMUs).

Remark 2: The C-PVC formulation (5) is an instance of convex optimization, with a convex objective and affine constraints; hence the problem can be solved for the unique global optimum using any efficient solver, e.g., CVX, or CPLEX.

III. DISTRIBUTED SOLUTION OF C-PVC

Our goal is to solve (5) distributively, employing distributed computation involving communication among the neighbors only. To achieve this, we proposed two possible network decomposition:

a) Bus-wise decomposition: This is the most granular decomposition framework, where each bus has its own control agent and solves the distributed control problem utilizing its bus-level local measurements and communications from its neighboring buses. This framework is aligned with the futuristic vision of the smart grid and distributed automation plan of energy policymakers, e.g., the United States Department of Energy (US DOE), that strives for the availability of PMU-based measurements at each bus.

b) Area-wise decomposition: This type of decomposition is more in line with the current practice of independently administered interconnected areas, each equipped with a minimal number of PMUs, whose placement ensures observability for the area [50], [51], [52]. Each area is associated with a single area-level control agent to decide the control inputs of all the controllers within its area, by way of distributed computation among other area control agents, in which the neighboring areas exchange information regarding their boundary nodes over their tie-lines and achieve consensus on their shared variables.
These two decompositions are formulated as follows.

A. Bus-Wise Decomposition

In the bus-wise decomposition, we cast the C-PVC problem (5) as an instance of N-agent distributed optimization, following the distributed estimation and consensus as in [53]. Here we consider the communication graph among the agents to be the same as the power system network graph \( G \). Subsequently, we present an ADMM-based algorithm to solve the distributed optimization problem.

We note that the constraints (4a)–(4b) of any bus \( i \) includes the changes of voltage and angle of bus \( i \) (\( \Delta V_{i,t+1}, \Delta \theta_{i,t+1} \)) as well as all of the neighboring buses \( k \in N_i (\Delta V_{k,t+1}, \Delta \theta_{k,t+1}) \), and that the \( i \)th control input \( u_{i,t+1} \) only appears in the model (4a)–(4b) of bus \( i \) (and not in the model equation of bus \( k \)). Keeping this in mind, we separate the optimization variables in (5) into two categories: public variables: \( \Delta V_{i,t+1}, \Delta \theta_{i,t+1} \), and private variables: \( u_{i,t+1} \).

To be able to reformulate (5) as a distributed optimization, we next introduce at each bus a local replica copy of associated public variables, namely, at each bus \( i \in \mathbb{N} \), we introduce a vector

\[
x_i := \left[ \Delta V_{i,t+1}, \Delta \theta_{i,t+1} \right] \quad | k \in \{i\} \cup N_i \in \mathbb{R}^{2(\mathbb{N}_i+1)}.
\]

In addition to \( x_i \), each bus \( i \) maintains a separate copy of its own local variables as the true values of the respective variables:

\[
z_i := [\Delta V_{i,t+1}, \Delta \theta_{i,t+1}] \in \mathbb{R}^2.
\]

The true variables in \( z := [z_i | i \in \mathbb{N}] \) and the local replica variables in \( x := [x_i | i \in \mathbb{N}] \) must together satisfy the self-consistency constraints expressed as follows:

\[
\forall i \in \mathbb{N} : x_i = E_i z_i, \quad \text{where}
\]

\[
\forall k \in \{i\} \cup N_i, l \in \mathbb{N} : E_i(k,l) := \begin{cases} I_{2 \times 2} & \text{if } x_{i(k)} = z_l \\ 0_{2 \times 2} & \text{otherwise} \end{cases}
\]

Note \( E_i \in \mathbb{R}^{2 \times (|\mathbb{N}_i|+1) \times 2 \times |\mathbb{N}|} \), and if we let \( E_i(k) \) denotes its \( k \)th column \( (k \in \mathbb{N}) \), (6a) can be rewritten as (10b). Besides, private variable for each bus \( i \) is the respective control input \( u_{i,t+1} \), and represented as \( u_i \).

1) Illustrative Example: The distributed computation framework involving the new set of variables defined above is illustrated using a 4-Bus system shown in Fig. 1. By the convention adopted above, bus 0 is the slack bus, so \( V_0 = 1 \) p.u. and \( \theta_0 = 0^\circ \), and will remain fixed, implying \( \Delta V_0 = 0 \), and \( \Delta \theta_0 = 0 \), hence are not considered in the variable description. Buses, neighbors

| Bus No | Neighbors \( N_i \) | Public Local Copy | Public True | Private |
|--------|------------------|-------------------|-------------|---------|
| Bus-1  | \( N_1 = \{2\} \) | \( x_1 = [x_{1(1)}, x_{1(2)}, x_{1(3)}] \top \) | \( z_1 \) | \( u_1 \) |
| Bus-2  | \( N_2 = \{1,3\} \) | \( x_2 = [x_{2(2)}, x_{2(3)}, x_{2(4)}] \top \) | \( z_2 \) | \( u_2 \) |
| Bus-3  | \( N_3 = \{2\} \) | \( x_3 = [x_{3(2)}, x_{3(3)}] \top \) | \( z_3 \) | \( u_3 \) |

(excluding slack bus only for variable declaration), private variables, and local and true copies of the public variables are given in Table I, with other details as described next.

For the self-consistency of the variables we need:

\[
\begin{align*}
x_{(1,1)} &= x_{(2,1)} = z_1, \\
x_{(2,2)} &= x_{(1,2)} = x_{(3,2)} = z_2, \\
x_{(3,3)} &= x_{(2,3)} = z_3.
\end{align*}
\]

Hence we can define \( E_i \)’s as follows:

\[
E_1 = \begin{bmatrix} E_{1(1)} & E_{1(2)} & E_{1(3)} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},
\]

\[
E_2 = \begin{bmatrix} E_{2(1)} & E_{2(2)} & E_{2(3)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},
\]

\[
E_3 = \begin{bmatrix} E_{3(1)} & E_{3(2)} & E_{3(3)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}.
\]

With the above introduction of variables, the (4a)–(4b) for bus \( i \) can be represented as follows:

\[
\begin{align}
\sum_{k \in \{i\} \cup N_i} \left[ \nabla x_{(i,k)} g_i^P \right] x_{(i,k)}^T &= \Delta P_{i,t+1}^\text{in}, \\
\sum_{k \in \{i\} \cup N_i} \left[ \nabla x_{(i,k)} g_i^Q \right] x_{(i,k)}^T &= \Delta Q_{i,t+1}^\text{in} + u_t,
\end{align}
\]

where for \( k \in \{i\} \cup N_i \):

\[
\begin{align}
\nabla x_{(i,k)} g_i^P &= \left( \begin{bmatrix} \partial g_i^P / \partial V_k \end{bmatrix}_t \right) \left( \begin{bmatrix} \partial g_i^P / \partial \theta_k \end{bmatrix}_t \right), \\
\nabla x_{(i,k)} g_i^Q &= \left( \begin{bmatrix} \partial g_i^Q / \partial V_k \end{bmatrix}_t \right) \left( \begin{bmatrix} \partial g_i^Q / \partial \theta_k \end{bmatrix}_t \right).
\end{align}
\]

Additionally, the constraints (5b)–(5d) can be transformed as follows:

\[
u_{i,\text{min}} \leq u_i \leq u_{i,\text{max}}, \quad x_{i,\text{min}} \leq x_i \leq x_{i,\text{max}}.
\]
Algorithm 1: Alternating Direction Method of Multipliers (ADMM).

1: Parallely ∀i ∈ N:
2: Initialize λᵢ = zᵢ = 0.
3: repeat
4: Update \( \{x_i^+, u_i^+\} = \arg \min_{(x_i,u_i) \in \mathcal{C}_i} L_\rho^i(x_i, u_i, z_i, \lambda_i) \)
5: Communicate \( x_i^+ \) to all neighbors \( k \in \mathcal{N}_i \)
6: Update \( z_i^+ = \frac{1}{|\mathcal{N}_i|} \sum_{k \in \mathcal{N}_i} E_k(i)^\top (x_k^+ + \frac{1}{\lambda} \lambda_k) \)
7: Communicate \( z_i^+ \) to all neighbors \( k \in \mathcal{N}_i \)
8: Update \( \lambda_i^+ = \lambda_i + \rho (x_i^+ - \sum_{k \in \mathcal{N}_i} E_k(k) z_k) \)
9: Communicate \( \lambda_i^+ \) to all neighbors \( k \in \mathcal{N}_i \)
10: until convergence

The affine equality (7) and the feasibility inequality (8) constraints together represent a convex set \( \mathcal{C}_i \) for each bus \( i \), and it must hold that: \((x_i, u_i) \in \mathcal{C}_i, \forall i \in \mathcal{N}\). Further the objective function (5a) can be referred as \( J(x, u) \), and is separable into those for individual buses \( i \in \mathcal{N} \), which can be written as:

\[ J_i(x_i, u_i) := ||V_{i,t} + \Delta V_{i,t+1} - V_{ref}||_2 + ||u_{i,t},u_{i,t+1}||_2 \]

so that:

\[ J(x, u) = \sum_{i=1}^{N} J_i(x_i, u_i). \]

Following [43], [53], the distributed version of optimization (5) can then be cast as a set of local optimizations, one at each bus \( i \in \mathcal{N} \):

\[ \min_{(x_i, u_i) \in \mathcal{C}_i} J_i(x_i, u_i) \quad \text{subject to,} \quad x_i = \sum_{k \in \{i\} \cup \mathcal{N}_i} E_i(k) z_k. \]

The augmented lagrangian for (10) can be written as in (11), where, \( \lambda_i \) is the lagrangian multiplier for bus \( i \), and \( \rho \) is a penalty constant:

\[ L_\rho^i(x_i, u_i, z_i, \lambda_i) = J_i(x_i, u_i) + \lambda_i^\top \left( x_i - \sum_{k \in \{i\} \cup \mathcal{N}_i} E_i(k) z_k \right) + \frac{\rho}{2} \left| \left| x_i - \sum_{k \in \{i\} \cup \mathcal{N}_i} E_i(k) z_k \right| \right|^2. \]

Then following [43], the dual problem of (5) or (10) collectively for all \( i \in \mathcal{N} \) can be solved by Algorithm 1, yielding the centralized optimum of (5) as explained below in Remark 3. Note the overall augmented Lagrangian for the centralized version can be written as:

\[ L_\rho(x, u, z, \lambda) = \sum_{i=1}^{N} L_\rho^i(x_i, u_i, z_i, \lambda_i). \]

Remark 3: Following [43], [44], whenever the local objective functions \( J_i(x_i, u_i), i \in \mathcal{N} \) are closed, proper, and convex and the overall unaugmented Lagrangian \( L_\rho(x, u, z, \lambda) \) for \( \rho = 0 \) has a saddle point, Algorithm 1 has the property that the residuals \( x_i - E_i(z) \) converge asymptotically to zero for all \( i \in \mathcal{N} \) and value of \( J(x, u) = \sum_{i=1}^{N} J_i(x_i, u_i) \) converges asymptotically to the primal optimum. In our case, (i) the functions \( J_i(x_i, u_i) \) defined in (9) are \( L_2 \)-norm of the optimization variables, hence they are closed, proper and convex, and (ii) by the convexity of the C-PVC problem (5), it can be shown that the unaugmented Lagrangian has a saddle point [53]. Therefore the distributed solution of Algorithm 1 converges asymptotically to the primal optimum.

B. Area-Wise Decomposition

For the area-wise decomposition, let the power network be comprised of \( \Gamma \) different interconnected areas, each independently administered, with the set of buses in area \( \gamma \in \Gamma \) denoted by \( \mathcal{N}_\gamma = \mathcal{N}^B_\gamma \cup \mathcal{N}^N_\gamma \), where \( \mathcal{N}^B_\gamma \) and \( \mathcal{N}^N_\gamma \) correspond to the set of boundary and non-boundary buses, respectively, of area \( \gamma \). The boundary buses are the points of connection with the neighboring areas, and we let the set neighboring areas of \( \gamma \) be denoted by \( \mathcal{N}_\gamma \). Similar to the bus-wise distributed formulation presented in Section III-A, each area \( \gamma \in \Gamma \) maintains \( X_\gamma \): local copies of public variables, \( Z_\gamma \): own public variables (as true values), and \( U_\gamma \): private variables, as follows:

\[ X_\gamma := [(\Delta V_{i,t+1}, \Delta \theta_{i,t+1}) | i \in \mathcal{N}^B_\gamma \cup \mathcal{N}^N_\gamma, \forall \beta \in \mathcal{N}_\gamma]^\top, \]

\[ Z_\gamma := [(\Delta V_{i,t+1}, \Delta \theta_{i,t+1}) | i \in \mathcal{N}^B_\gamma]^\top, \]

\[ U_\gamma := [(\Delta V_{i,t+1}, \Delta \theta_{i,t+1}) | i \in \mathcal{N}^B_\gamma \cup \{\cup_{\ell \in \mathcal{N}_\gamma} u_{\ell}\}]^\top. \]

It should be noted that the variables \( X_\gamma, Z_\gamma, U_\gamma \) are analogous to the variables \( x_i, z_i, u_i \) of Section III-A, and hence similar to (6), one can form the consistency matrix \( E_\gamma \) between variables \( X_\gamma \) and \( Z_\gamma \). Further, similar to (7a) and (7b), any bus \( \gamma \in \mathcal{N}_\gamma \) has two equality constraints \( \phi^2_\gamma (X_\gamma, U_\gamma) = 0 \) and \( \phi^2_\gamma (X_\gamma, U_\gamma) = 0 \), yielding \( 2 \times |\mathcal{N}_\gamma| \) linear equality constraints for each area \( \gamma \in \Gamma \). Also, \( \forall \gamma \in \Gamma : U_{\gamma,\min} \leq U_{\gamma} \leq U_{\gamma,\max} \), \( X_{\gamma,\min} \leq X_{\gamma} \leq X_{\gamma,\max} \). Accordingly, \( \forall \gamma \in \Gamma : (X_\gamma, U_\gamma) \in \mathcal{C}_\gamma \), where \( \mathcal{C}_\gamma \) is a convex set. The objective function within each area \( \gamma \in \Gamma \) can be written as:

\[ J_\gamma(X_\gamma, U_\gamma) := \sum_{i \in \mathcal{N}_\gamma} \left( ||V_{i,t} + \Delta V_{i,t+1} - V_{ref}||_2 + ||u_{i,t},u_{i,t+1}||_2 \right). \]

Finally, the area-wise distributed optimization takes the form similar to that of (10), and following Algorithm 1, control agents of various areas compute the control inputs for all their area controllers, communicating only their boundary variables to the their neighboring area control agents.

IV. DATA-DRIVEN ESTIMATION OF ADMITTANCE MATRIX

As an effort towards wide area monitoring, protection, and control (WAMPAC) [54], PMUs in the network are increasing in addition to the conventional SCADA/RTU devices, and further, the ongoing inclusion of field-RTUs and μ-PMUs in distribution network [55] are further increasing their visibility, providing a pathway to the data-driven estimation of the full admittance matrix \( Y \). In case every node is not equipped with a measurement device, then instead of a bus-wise distributed decomposition, an area-wise distributed decomposition is more meaningful, as described above, where each area is independently administered and has PMUs installed at its minimal set of buses to ensure the observability of the area. In the area-wise decomposition, then, the admittance of certain tie-lines can be estimated from...
the measurement data. This section details the data-driven distributed estimation of the line admittances corresponding to the underlying network links of the distributed computation and is an additional feature of our distributed framework.

In the distributed optimization framework above, the computation of the partial derivatives in (7) (that appear in (4a)–(4b)) are of the functions \( g^P_i(\cdot) \) and \( g^Q_i(\cdot) \) introduced in (2), and their values depend on the entries of the admittance matrix \( Y \), for which from (1), \( Y_{ii} = y_i + \sum_{k=0, k \neq i}^{N} \frac{1}{G_{ik}} \), and \( B_{ii} = b_i + \sum_{k=0, k \neq i}^{N} b_{ik} = b_i - \sum_{k=0, k \neq i}^{N} B_{ik} \). Note in general, \( g_i = 0 \), and \( b_i \) corresponds to all shunt connected elements (including line charging capacitance) in a particular node/bus. Hence \( b_i \) (if known) can be absorbed in net reactive injection \( Q^a \) with an appropriate equivalent value, so that we can also treat \( b_i = 0 \). With this convention, we have: \( G_{ii} = -\sum_{k=0}^{N} G_{ik} \) and \( B_{ii} = -\sum_{k=0}^{N} B_{ik} \). Consequently, (2) can be written as:

\[
g^P_i(V_i, \theta_i, V_k, \theta_k) = \sum_{k=0}^{N} \left[ \{ V_k V_k \cos(\theta_i - \theta_k) - V_i^2 \} G_{ik} + \right. \\
\left. + V_k V_k \sin(\theta_i - \theta_k) B_{ik} \right], \\
\tag{13a}
\]

\[
g^Q_i(V_i, \theta_i, V_k, \theta_k) = \sum_{k=0}^{N} \left[ \{ V_k V_k \sin(\theta_i - \theta_k) G_{ik} + \right. \\
\left. + \{ V_i^2 - V_k V_k \cos(\theta_i - \theta_k) \} B_{ik} \right]. \\
\tag{13b}
\]

We can now compute the required partial derivative terms of (4), and assemble them in the matrix form (14). Clearly the values of the partial derivatives for bus \( i \in N \) require, (i) the measurements of bus voltages and angles of buses \( \{ i \} \cup N \), and (ii) the knowledge of \( G_{ik} \) and \( B_{ik} \), \( k \in N \), which as shown next is estimated from the same measurement data.

\[
\begin{bmatrix}
\frac{\partial g^P_i}{\partial V_i} \\
\frac{\partial g^P_i}{\partial \theta_i} \\
\frac{\partial g^Q_i}{\partial V_i} \\
\frac{\partial g^Q_i}{\partial \theta_i}
\end{bmatrix} =
\begin{bmatrix}
V_k \cos(\theta_i - \theta_k) - 2V_i & V_k \sin(\theta_i - \theta_k) \\
-V_k V_k \sin(\theta_i - \theta_k) & V_k V_k \cos(\theta_i - \theta_k) \\
V_k \sin(\theta_i - \theta_k) & 2V_i - V_k \cos(\theta_i - \theta_k) \\
V_k \cos(\theta_i - \theta_k) & V_k V_k \sin(\theta_i - \theta_k)
\end{bmatrix}
\begin{bmatrix}
G_{ik} \\
B_{ik}
\end{bmatrix} \tag{14a}
\]

\[
\begin{bmatrix}
\frac{\partial g^P_i}{\partial V_k} \\
\frac{\partial g^P_i}{\partial \theta_k} \\
\frac{\partial g^Q_i}{\partial V_k} \\
\frac{\partial g^Q_i}{\partial \theta_k}
\end{bmatrix} =
\begin{bmatrix}
V_i \cos(\theta_i - \theta_k) & V_i \sin(\theta_i - \theta_k) \\
V_k V_k \cos(\theta_i - \theta_k) & -V_i V_k \sin(\theta_i - \theta_k) \\
V_i V_k \sin(\theta_i - \theta_k) & -V_i V_k \cos(\theta_i - \theta_k) \\
-V_i V_k \cos(\theta_i - \theta_k) & V_i V_k \sin(\theta_i - \theta_k)
\end{bmatrix}
\begin{bmatrix}
G_{ik} \\
B_{ik}
\end{bmatrix} \tag{14b}
\]

The complex power flowing from bus \( i \) to \( k \) is \( S_{ik} = P_{ik} + jQ_{ik} = \bar{V}_i \bar{I}_k \) can be measured by measuring \( \{ \bar{V}_i, \bar{I}_k \} \) at each bus \( i \in N \) from which \( P_{ik} \) and \( Q_{ik} \) become known. For branch flow equations [56], we have, \( P_{ik} = \{ V_k V_k \cos(\theta_i - \theta_k) - V_i^2 \} G_{ik} + V_k V_k \sin(\theta_i - \theta_k) B_{ik} \) and \( Q_{ik} = V_k V_k \sin(\theta_i - \theta_k) G_{ik} + \{ V_i^2 - V_k V_k \cos(\theta_i - \theta_k) \} B_{ik} \). Therefore the least square estimate (LSE) [57] of \( G_{ik} \) and \( B_{ik} \) is given by the standard formula:

\[
\begin{bmatrix}
\hat{G}_{ik} \\
\hat{B}_{ik}
\end{bmatrix} = \hat{\Theta} = (A^T A)^{-1} A^T b. \tag{15}
\]

The above data-driven estimation of line impedances can be done distributively using neighbor-to-neighbor communication. For the case of bus-wise decomposition, the measurements are supposed to be available at each node, so the admittance of all lines can be estimated. In contrast, for area-wise decomposition, the line admittance of certain tie-lines can be estimated from the measured data. The complete steps of the proposed method considering a generic bus-wise decomposition are presented in Algorithm 2. Fig. 2 depicts the associated information flow for the illustrative example system of Fig. 1.

V. TEST RESULTS

We validate our proposed distributed control framework for (i) Transmission systems: IEEE-30 bus and IEEE-57 bus, and (ii) Distribution system: IEEE-123 bus system, by way of tracking a specified reference voltage \( V_{ref} \) under uncertain time-varying loads and renewable generations. For our validation purposes, we obtained real-world utility-scale data from CAISO [46] and created representative profiles for the discrete time-wise varying renewable generation and load at 15 min intervals over 24 hrs. period, as shown in Fig. 3. Since the predictions of load and renewable generation profiles are available at 15 min intervals [46], we considered control adjustments at 15 min intervals for 96 such intervals spanning 24 hrs. period. It is possible to decrease the control interval if needed to equal or exceed the control computation time, which for our examples, is less than 100 sec. (see Section V-C), and also increase the number of intervals, but those will not alter the steps performed. PyPower (or MatPower [58]) is used to solve the nonlinear AC power flow of the chosen power systems.
Algorithm 2: Distributed Predictive Voltage Control with Data-driven Estimation.

1: Parallely \( \forall i \in N \):
2: \textbf{for} each control instant \( t \in [0, T] \) \textbf{do}
3: \hspace{1em} Obtain latest measurement data \( \{V_i, I_{ik} \mid k \in N_i\} \), and compute \( P_{ik}, Q_{ik} \) (recall, \( V_i I_{ik} = P_{ik} + jQ_{ik} \)).
4: \hspace{1em} Communicate \( V_i, \theta_i, P_{ik}, Q_{ik} \) with neighboring buses/agents \( k \in N_i \), and receive the data \( V_k, \theta_k, P_{ki}, Q_{ki} \).
5: \hspace{1em} Using the collected data, estimate \( \hat{G}_{ik} \) and \( \hat{B}_{ik} \) for each \( k \in N_i \) employing (15).
6: \hspace{1em} Compute the required partial derivatives (14) using the estimated values \( \hat{G}_{ik} \) and \( \hat{B}_{ik} \), and the latest measurements of \( \{V_i, \theta_i, V_k, \theta_k \mid k \in N_i\} \).
7: \hspace{1em} Collect the predicted generation \( [\hat{P}_{G_i}^{t+1}, \hat{Q}_{G_i}^{t+1}] \) and demand profiles \( [\hat{P}_{D_i}^{t+1}, \hat{Q}_{D_i}^{t+1}] \).
8: \hspace{1em} Solve the MPC problem (10) by Algorithm 1.
9: \hspace{1em} From the output of the optimization, obtain the reactive compensation \( u_{d,i,t+1} \) and implement it for the next time step \( t + 1 \).
10: \textbf{end for}

A. Test Case Description

1) IEEE-30 Bus System: The standard test system with six generators located at buses 1, 2, 13, 22, 23, and 27 is utilized. We include renewable generations (solar and wind) in these generator buses, allowing a mix of conventional and renewable generations, with 50% coming from renewable sources during their peak generations. Under the representative profiles of Fig. 3, if no control compensation is exercised, the voltage values of multiple buses drop below the recommended \( V_{\text{min}} = 0.95 \) p.u. as can be seen in Fig. 4(a) (No Control case is in red), that has a minimum voltage of around 0.91 p.u.

2) IEEE-57 Bus System: We modified the standard IEEE-57 bus network, which has generators only at buses 3, 8, and 12, by adding generators at buses from 13 to 57. Here again, the generations are taken to be a mix of conventional and renewable ones, with renewable generations serving approximately 50% of the total load at its peak availability. The predictions of renewable generation and load profiles are same as shown in Fig. 3. Similar to the IEEE-30 bus network, the voltages violate the desired margin in the absence of any control as depicted in Fig. 4(b) (No Control case is in red), thereby necessitating some reactive control compensation for mitigating the voltage violations.

3) IEEE-123 Bus System: IEEE-123 is a large distribution system including overhead and underground line segments, transformers, breakers, capacitors banks, and voltage regulators. In this article, we utilized a balanced version of this test system from [59]: modified and added renewable generation from buses 27 to 56 and buses 84 to 114. The utilization of a balanced distribution system for proof-of-concept is also exemplified in earlier works [12], [18], [21], [23]. The predicted load profiles are the same as in Fig. 3; the predicted renewable generation profile is modified to reduce the wind part. Even then, it is observed that voltage values drop below the acceptable range under no control input (see Fig. 4(c) (No Control case is in red)).

B. Control Design

We implemented the proposed framework in Algorithm 2 to mitigate the voltage drops observed in all 3 test systems: IEEE-30, IEEE-57, and IEEE-123 bus systems. Reactive control is computed to (i) to maintain voltage values close to the given reference \( V_{\text{ref}} = 1 \) p.u., (ii) ensure that the voltage trajectories always remain within the [0.95, 1.05] p.u., while the reactive power compensation per controller at each control time step remains limited to \([-0.05, 0.05]\) p.u.

1) Control Under Uncertain Prediction and Modeling Errors: As mentioned earlier, the generation and load profiles shown in Fig. 3 are subject to prediction error of 3–5% in practice (as seen in the data collected from [46]), and therefore we evaluate the performance of the proposed method by introducing a prediction error of \( \pm 5\% \). Moreover, instead of relying on the
knowledge of the admittance matrix, we estimated it online, which brings in added estimation errors. The estimation errors in $G_{ik}$ and $B_{ik}$ are depicted in Fig. 5 for the IEEE-30 bus system. The voltage profiles with proposed distributed control in Algorithm 2 is overlayed in blue in Fig. 4(a) (IEEE-30 bus), Fig. 4(b) (IEEE-57 bus), and Fig. 4(c) (IEEE-123 bus). (It should be noted that the variation of voltages over all buses in represented by red shade in case of ‘No Control’ and by blue shade in case of proposed distributed control.)

2) Control Under Failure of Communication Link: Here the proposed distributed control algorithm is tested for random failures of communication links (recall the graph of the communication network is taken to be that of underlying power system). Under dynamic generation, load variation, prediction uncertainty and modeling error we simulated random failures of communication links $l_{ik}$ between any two neighboring buses $i$ and $k$. In case of such a communication failure, the local controllers at buses $i$ and $k$ eliminate the respective buses from their list of neighboring buses and then proceed with the local control computation. Fig. 6(a) shows the satisfactory voltage performance for the IEEE-30 bus test system (in blue with control vs. in red without control), validating our method’s robustness to random failure of communication links.

Remark 4: The proposed framework is based on steady-state power flow-based solution under discrete time-wise varying load and generation profiles, with control changes from $t$ to $t+1$. The control adjustments are only done upon the completion of the distributed control computation based on the measurements at $t$ and predicted estimates at $t+1$. Hence the convergence dynamics of ADMM computation has no impact on the actual system dynamics. The transients caused by the switching of reactive compensations for small-scale variations from $t$ to $t+1$ do not perturb the stability of the system in transitioning from its steady state at $t$ to that at $t+1$. Therefore, the short-term voltage transients (relative to the 15 min control interval) do not play a role.

C. Sensitivity to Optimization Parameter; Computation Time

We examine the effect of optimization parameter $\rho$ on the convergence speed of Algorithm 2. To balance the speed vs. accuracy, we use the stopping criteria of $||x - Ez||_\infty = \max_{i \in N} ||x_i - E_i z||_\infty \leq 3.5 \times 10^{-5}$ for IEEE-30 bus test case. The convergence speed under 4 different values of $\rho = 20, 40, 100, 1000$ is presented in Fig. 7(a) (for a typical scenario). It is found that $\rho = 100$ gives the fastest convergence: 380 iterations (approx.). The stopping criteria for IEEE-57 bus test case is set as, $||x - Ez||_\infty \leq 10^{-4}$. In this case, we did not observe much variations in convergence speed for $\rho = 100$ vs. $\rho = 1000$, both of which took approximately 2300 iterations. In the IEEE-123 bus test case, we selected $||x - Ez||_\infty \leq 5 \times 10^{-5}$ as the stopping criteria and studied the convergence...
speed for $\rho = 100, 500, 1000, 2000$ (as plotted in Fig. 7(b)). For the IEEE-123 bus test case, we found that $\rho = 2000$ solves the distributed optimization in the least number of iterations.

For our implementation, we run the program in a single machine (intel(R) Core(TM) i7-4790 CPU @ 3.60 GHz processor with 16 GB RAM). But, in a real-world implementation, the entire framework can be run in parallel with higher computational resources; therefore, respective computation time can be divided by the number of buses. Given that we observed the average maximum computation time to be 124 sec, 1427 sec, and 231 sec, respectively, for IEEE-30, IEEE-57, and IEEE-123 bus test cases, these can reduce to the range of 4-25 sec using parallelization under no communication latency, and 16-100 sec considering communication latency.

D. Comparison With Existing Local Volt-Var Control Scheme

For the sake of completeness, the efficacy of the proposed distributed control strategy is compared with the conventional local droop-based volt-var control (VVC) scheme [61]. The VVC design is done similar to [7], where there is no reactive power injection when bus voltage is in the dead-band range $[0.97, 1.03]$ p.u. On the other extreme, if the voltage goes below 0.94 p.u. (or beyond 1.06 p.u.), the reactive injection is $+Q_{\text{max}} = 0.05$ (or $-Q_{\text{max}} = 0.05$) p.u. In between, the reactive injection varies linearly from 0 to $+Q_{\text{max}}$ (or 0 to $-Q_{\text{max}}$) in the voltage range $[0.94, 0.97]$ (or $[1.03, 1.06]$) p.u. The voltage comparison for our proposed distributed control and VVC schemes are presented in Fig. 6(c) for IEEE-30 bus system, which clearly indicates the superiority of proposed method over VVC (blue vs. red plots).

E. Attack Resiliency of the Framework

One of the features of a distributed implementation is that it is attack resilient by being not susceptible to single point failures. To demonstrate this, we conducted a simple study and found that the proposed framework is promising in rapid anomaly detection using the approach in [62]. In IEEE-30 bus example, bus-2 and bus-6 are neighboring buses, so according to proposed protocol, bus-2 has the variable $x_{(2,2)} = \Delta V_{2,t+1}$, while bus-6 has $x_{(6,2)} = \Delta V_{2,t+1}$, meaning the two variables are replicas of the same true variable. Under normal condition, the difference $\kappa = (x_{(2,2)} - x_{(6,2)})$ gradually decreases to 0 as expected, and this difference $\kappa$ follows a distribution as shown in Fig. 7(c) (blue curve). But in case of a random bias injection attack in the communication channel between bus-2 and bus-6, the distribution drastically changes (red curve), which can then be used to alert the operator about possible attacks or unusual behavior. In summary, the proposed method has in-built redundancy through the incorporation of replica variables that can be leveraged to detect cyberattacks on communication channels corrupting the data.

F. Sparsely Placed PMUs: Grid-Observability; Area-Wise Decomposition

In Section III-B, we presented area-wise distributed formulation. For its proof-of-concept, we implemented an area-wise distributed control for IEEE-30 bus test case [63], by dividing it into three areas: Area-1 contains the buses $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$, Area-2 is formed with the buses $\{12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22\}$, while Area-3
includes remaining buses \{23, 24, 25, 26, 27, 28, 29, 30\}. The boundary buses are as follows: Area-1: \{4, 6, 8, 10\}, Area-2: \{12, 15, 17, 20, 21, 22\}, and Area-3: \{23, 24, 28\}. Each area-centric controller computes the compensations for the buses in the area equipped with reactive controllers, and in this process, it communicates with its neighboring area-centric controllers to exchange information about the boundary buses for consensus on the variables of the boundary buses. For the IEEE-30 bus, with the knowledge of area-centric admittance, we determined the minimal set of PMU locations besides the boundary locations using the integer programming method of [50]: For the self-observability of each area, we need PMUs at the area-wise boundary buses, and at the buses \{1, 5, 11\} for Area-1, the buses \{26, 27\} for Area-3, and no additional buses for Area-2. The voltage performance is shown in Fig. 6(b), where it is observed that the voltage profiles under bus-wise vs. area-wise distributed computation (blue vs. green) are similar, although the area-wise distributed optimization uses only 18 number of PMUs, as opposed to 30 PMUs for the bus-wise decomposition.

VI. CONCLUSION

The proposed distributed control utilizes local measurements and neighbor-to-neighbor communication, in determining the reactive controls in the presence of prediction and modeling errors, link failures, and also supports anomaly detection in the case of corrupted data through its in-built redundancy in the form of various replicated variables. The proposed methodology is applicable for both transmission and distribution systems and to general network topology. We also provided a method of distributed estimating of line admittances corresponding to the communication links of the distributed computation network. The test results applied to IEEE-30 bus, IEEE-57 bus transmission systems, and IEEE-123 bus distribution system validated the performance and robustness of the proposed framework against model/generation/demand/network uncertainties/failures/attacks. We also demonstrated that the proposed method offers better optimality than standard VVC-based approaches. Future research can explore the extension to unbalanced distribution networks in line with the extensions [21], [24], and also the integration of various robust learning-based methods within the proposed framework.

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