Determination of the scale factor in the physical modeling of reinforced concrete structures exposed to emergency loads

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Abstract. The article proposes a solution to the problem of more accurately taking into account the influence of scale factors on the qualitative and quantitative indicators of the stress-strain state obtained by testing both individual structures and structural systems made of reinforced concrete. Methodical approaches to modeling tests for static and dynamic loads are considered, with the aim of taking into account the scale effects that occur during the manufacture of prototypes of models of individual structures and structural systems made of reinforced concrete. Comparative analysis estimated effect of different geometric proportions for free opera statically determinate beams and for a fragment of a monolithic reinforced concrete frame multiistory buildings, experimental studies which were perform by the authors earlier. Analytical patterns were established to assess the impact of large-scale factors on the particular static and dynamic deformation of structures and structural systems of reinforced concrete. The results can be taken into account when conducting experimental studies of model reinforced concrete structures for dynamic emergency loads

1. Introduction
Modeling is one of the main types of structural tests conducted for research purposes [1-5]. Despite the fact that the most complete information about the operation of structures can be obtained in the process of field tests, their implementation in many cases involves high material costs and insurmountable difficulties of a methodological nature. Such difficulties include the achievement in natural conditions of the required measurement accuracy or the exclusion of influencing factors, namely, the provision of a controlled experiment. While modeling, we can distinguish only the main factors, the study of which is the goal of this experiment, and while constructing a model, we can provide for the variation of these factors at given levels. In addition, in laboratory conditions it is much easier to provide the required measurement accuracy of all the studied parameters.

It should be noted that testing of structures of trusses with a span of 10 meters [6], for example, or beams with a span of 5 meters [7] in the laboratory, also refers to modeling in a 1:1 scale, since in this case the loading and bearing parameters of the structures are modeled. At the same time, model tests are largely complemented by field tests. A thorough study of these effects on the structural elements of
buildings or structures is successfully studied on models, for example, when blowing high-rise structures in a wind tunnel [8,9] or when simulating sea waves in trays [10].

The study of special emergency impacts on structural elements of buildings and structures is an urgent task aimed at improving the safety of designed and reconstructed objects in accordance with the current design standards [11-15].

2. Research methods

As an example, we will consider a pivotally supported reinforced concrete beam [16.17]. It is well known that the maximum value of the bending moment will be in the middle of the span (point B) (Figure 1a).

\[ M_b = \frac{ql^2}{8} \]  

Let us write the equation of equilibrium of internal forces for a reinforced concrete beam with a single reinforcement located in the stretched zone (Figure 1b):

\[ M_c \leq M_{ub} = N_b \cdot z = R_b \cdot b \cdot x(h_b - 0.5x) \]  

Here \( M_{ub} \) – the bearing section capacity, the height of the compressed zone \( x \) is determined from the condition of equality forces \( N_b \) and \( N_s \):

\[ x = \frac{R_s \cdot A_s}{R_b \cdot b} \]  

\( R_s \) - design tensile strength of reinforcement, \( R_b \) design resistance of concrete to compression, \( b \) - beam width.

\[ \frac{ql^2}{8} \leq R_b \cdot b \cdot x(h_b - 0.5x) \]  

Figure 1. To the calculation of a pivotally supported reinforced concrete beam: a - design scheme, b - internal forces scheme

To ensure accurate behaviour of the studied reinforced concrete structure in the stress strain state throughout all the stages of its operation, model material features shall be identical to the features of the full-scale specimen [19], i.e.

\[ \begin{cases} \sigma_c = \sigma_c^m = \sigma_c^m \cdot E_c^m = E_c^m = E_m \\ \sigma_m = \sigma_m^m \cdot E_m^m = E_m^m = E_m^m \end{cases} \]  

Here \( \sigma_c \) – concrete stress; \( \sigma_m \) – reinforcement strength; \( E_c \) – concrete modulus of elasticity; \( E_m \) – reinforcement modulus of elasticity.

In case of beam extension and deflection, the following conditions are to be ensured:
- with monoaxial extension, the following rigidity ratio is to be observed
\[
\frac{A_n E_n}{A_m E_m} = \text{const}; \tag{6}
\]

- in case of deflection, the following conditions are to be complied with

\[
\frac{E_n l_n}{E_m l_m} = \text{const.} \tag{7}
\]

Let us choose the linear scale factor for the model. It must be only one to provide for compliance with the geometric similarity in general. However, sometimes a number of model creation conditions in the strict geometric similarity cannot be observed. That is why, in particular cases, we can deviate from the purely geometric observation to evaluate the stress strain state of the structure.

We will use two linear scale factors:

- geometric scale factor of the model in the longitudinal direction of the beam (spans, spacings, floor height)

\[
m_{l1} = \frac{l_{n1}}{l_{m1}}; \tag{8}
\]

- for evaluation of the structural element cross sections (collar beams, columns)

\[
m_{l2} = \frac{l_{n2}}{l_{m2}}. \tag{9}
\]

For the considered example of experimental studies [16,17] \( m_{l1} = 4 \), \( m_{l2} = 2 \).

We accept the dependence of the bearing capacity of the sections as their ratio and denote:

\[
\frac{M_{\text{ult},n}}{M_{\text{ult},m}} = \frac{m_{M,n}}{m_{M,m}} = m_{l2}^3 \tag{10}
\]

The external moment is proposed to be modeled on a longitudinal scale \( m_{l1} \). Then the scale of the external load will be:

\[
m_q = \frac{q_n}{q_m} = \frac{A_n l_n}{A_m l_m} = m_{l1}^2 \cdot \frac{1}{m_{l1}} = m_{l1} \tag{11}
\]

Let us consider the ratio of the moments of external forces for a pivotally supported beam, taking into account (2) and (11):

\[
m_q = \frac{q_n \cdot l_n^2}{8 \cdot l_m^2 \cdot q_m} = \frac{A_n l_n}{A_m l_m} = m_{l1}^2 \cdot m_{l1} \tag{12}
\]

external moment is modeled on the dependence:

\[
\frac{M_{\text{ult},n}}{M_{\text{ult},m}} = m_{l1}^3 \tag{13}
\]

then the ratio of external distributed loads will take the form:

\[
\frac{q_n}{q_m} \leq m_{l2} \tag{14}
\]

or
It may be seen from formula (15) that the ratio of length scales $m_{l1}$ significantly affects the value of the quantitative parameters obtained during model testing, as well as their interpretation on natural objects.

To obtain an accurate forecast of the ratio of the distributed external load with the ultimate moment that the cross section can take, it is necessary to recognize the equality of the geometric scale of the model in the longitudinal direction and the scale of the cross sections of the structural elements. Then:

$$m_{l1} = m_{l1} = m_l$$

(16)

Formula (15) will take the form:

$$q_u \leq q_m \cdot m_l$$

(17)

However, the conditions of the model experiment do not always allow a full geometric similarity. It is almost impossible to achieve the same scale of cross sections $m_{l2}$ and the scale of the geometric dimensions of structural elements $m_{l1}$, especially with regard to experimental studies of reinforced concrete structural systems under emergency impacts. In one case, when the scale $m_{l2}$ will be decisive, it is necessary to have sufficiently large volumes of test laboratories to accommodate a model of the structure or a fragment of the building’s load-bearing system, which is associated with difficulties in manufacturing, installation and increase of the load necessary to simulate the design load on the structural system elements [18]. On the other hand, if the scale $m_{l2}$ is taken as the determinant, the transverse dimensions of the test structure or fragment of the structural system will be so small that it will entail a distortion of the obtained qualitative and quantitative parameters due to the influence of the scale effect, and an unjustified increase in the complexity of manufacturing the structures under study or structural systems. In this case, a reliable result can be obtained if we fix the ratio of the scales of the geometric dimensions of the structural elements $m_{l1}$ and the scale of the cross sections $m_{l2}$ by introducing a dependence of their ratio:

$$\eta = \frac{m_{l1}}{m_{l2}}$$

(18)

i.e:

$$m_{l1} = \eta \cdot m_{l2}$$

(19)

In this case, the formula (15) will take the form:

$$q_u \leq q_m \cdot \frac{m_{l2}^3}{\eta^2 \cdot m_{l1}}$$

(20)

after conversion we will get:

$$q_u \leq q_m \cdot \frac{m_{l2}^2}{\eta^4}$$

(21)

Now let us consider a fragment of a multi-span continuous beam, which is a part of a monolithic reinforced concrete frame, the experimental studies of which were performed in [19,20] (Figure 2). For the considered example of experimental studies $m_{l1} = 4$, $m_{l2} = 1.5$. 
Taking the values of bending moments in the first approximation, taking into account their redistribution equal:

$$M_A = M_B = M_C = \frac{q \cdot l^2}{16}$$  \hspace{1cm} (22)

It is important to note that the bending moment on the support C, while simulating the switching off of the support, the moment diagram changes sign and will be equal to:

$$M_c^{m-1} = \frac{q \cdot l^2}{4} + R \cdot l$$  \hspace{1cm} (23)

Having done the transformations similar to (10) - (17) and taking into account that when identical materials are used in a model and a full-scale specimen, the material density is reduced when finding the scale factor, we will write down:

$$\frac{t_n^2}{t_m^2} = \frac{M_{n}^{m} t_m^5}{M_{m} t_n^5} = m_t^5 \cdot \frac{1}{m_n}$$  \hspace{1cm} (24)

We can simplify the equation (24)

$$m_t = \sqrt{m_n}$$  \hspace{1cm} (25)

Taking into account the obtained formulas and research [21], we analyze the influence of the scale factor on load parameter (Figure 3).

**Figure 2.** The design scheme of a fragment of a reinforced concrete beam: a- for the main combination of loads, b- for emergency loads

**Figure 3.** Dependence of the scale load of a linear scale
Let us analyze the influence of scale factors on the change of time scales, longitudinal forces and bending moments (Figure 4):

![Figure 4. Dependence of the scale time, scale of the longitudinal force and bending moment scale of a linear scale](image)

3. Conclusions
The wide use of the established criteria of mathematical and physical similarity, as well as the principles of modeling individual structures and structural systems made of reinforced concrete, make it possible to study the features of their static-dynamic deformation under special emergency influences comprehensively. This will reduce the amount of material and labor costs and improve the quality of the information obtained during experimental research.

Numerical studies of model tests have established the influence of the scale of cross sections \( m_{l2} \) and the scale of the geometric dimensions \( m_{l1} \) of structural elements on load parameters, the scale of taking into account the time of dynamic action \( m_t \), the scale of the longitudinal force \( m_N \) and bending moment \( m_M \). The obtained analytical laws should be taken into account when interpreting the results of model experimental studies of reinforced concrete structures under special emergency influences.

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