TOWARD A DETERMINISTIC MODEL OF PLANETARY FORMATION. III. MASS DISTRIBUTION OF SHORT-PERIOD PLANETS AROUND STARS OF VARIOUS MASSES

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ABSTRACT

The origin of a recently discovered close-in Neptune-mass planet around GJ 436 poses a challenge to the current theories of planet formation. On the basis of the sequential accretion hypothesis and the standard theory of gap formation and orbital migration, we show that around M dwarf stars, close-in Neptune-mass giant planets may be relatively common, while close-in Jupiter-mass giant planets are relatively rare. The mass distribution of close-in planets generally has two peaks at about Neptune mass and Jupiter mass. The lower mass peak takes the maximum frequency for M dwarfs. Around more massive solar-type stars (G dwarfs), the higher mass peak is much more pronounced. Planets around G dwarfs undergo orbital migration after fully accreting gas, while those around M dwarfs tend to migrate before starting rapid gas accretion. Close-in Neptune-mass planets may also exist around G dwarfs, although they tend to be mostly composed of silicates and iron cores and their frequency is expected to be much smaller than that of Neptune-mass planets around M dwarfs and that of gas giants around G dwarfs. We also show that the conditions for planets' migration due to their tidal interaction with the disk and the stellar mass dependence in the disk mass distribution can be calibrated by the mass distribution of short-period planets around host stars with various masses.

Subject headings: planetary systems: formation — solar system: formation — stars: statistics

1. INTRODUCTION

In an attempt to place quantitative constraints on models of planet formation, we developed an algorithm to simulate the kinematic properties of gas giants formed in isolation (Ida & Lin 2004a, hereafter Paper I). This prescription is based on the sequential accretion model in which we assume that Jupiter-mass gas giant planets formed through (1) grain condensation, (2) runaway planetsesimal coagulation (Greenberg et al. 1978; Wetherill & Stewart 1989; Aarseth et al. 1993; Kokubo & Ida 1996), (3) oligarchic growth of protoplanetary embryos (Kokubo & Ida 1998, 2000), and (4) gas accretion onto solid cores (embryos; Mizuno 1980; Bodenheimer & Pollack 1986; Pollack et al. 1996; Ikoma et al. 2000). On the basis of a distribution of (1) dust disk masses ($M_d$) inferred from millimeter data (Beckwith & Sargent 1996), (2) a range (1–10 Myr) of disk depletion timescale ($\tau_{\text{dep}}$) inferred from the observed decline in the IR (Haisch et al. 2001) and millimeter data (Wyatt et al. 2003), and (3) three different growth termination criteria (Lin & Papaloizou 1993; Bryden et al. 1999), we simulated a distribution of protoplanetary masses. In addition, we considered the effect of postformation type II orbital migration due to planet-disk interaction (Lin & Papaloizou 1985), which is an important process in relocating protoplanets away from their birthplaces. The last process has been invoked (Lin et al. 1996) to account for the origin of a population of Jupiter-mass short-period planets such as 51 Peg b (Mayor & Queloz 1995).

We compared the results of our simulation with the available data of extrasolar planets. We suggested that around solar-type stars (1) there may be a deficit of intermediate mass ($\sim 20$–$100 M_\oplus$) and intermediate period ($\sim 0.1$–$1$ yr) in their mass-period distribution (Paper I), (2) the frequency of gas giant planets may be an increasing function of their host stars' metallicity [Fe/H], (Ida & Lin 2004b, hereafter Paper II), and (3) a large fraction of the planets that migrated to the proximity of their host stars may have perished (Paper I, Paper II). The first conclusion results directly from the expectation that (1) the growth of protoplanetary cores is limited by dynamical isolation in the inner regions of planetary systems and a slow coagulation rate in the outer regions, (2) even in the most favorable locations, 0.1–1 Myr is needed for the formation of protoplanetary embryos (cores), anywhere in the disk, with masses $M_c \gtrsim M_{\text{crit}}$, (3) cores with mass larger than $M_{\text{crit}}$, around several Earth masses can undergo runaway gas accretion, and (4) orbital migration occurs on a timescale ($\tau_{\text{mg}}$) similar to that of gas depletion in the disk ($\tau_{\text{dep}}$).

The model we have presented so far provides the first step in the construction of deterministic properties of planetary formation. Its simplistic assumptions must be reexamined in the wider context of the protoplanetary environment. For example, planets are assumed to form independently and in isolation in the model. Dynamical isolation in this context means that there are no other major planets around the same host stars. Most importantly, postformation dynamical interaction between the planets has not yet been considered. While these effects will be examined in a future investigation, here we consider the mass function of dynamically isolated planets and its dependence on the mass of the host star.

On the observational side, most of the known planets are found around solar-type stars (G dwarf stars) because the most successful radial velocity surveys have been conducted for these target stars, which have cool and well-defined atmospheric spectroscopic features. However, the search window is rapidly expanding to lower mass stars as detection techniques are being refined in both radial velocity and transit searches. Figure 1

1 The data are taken from The Extrasolar Planets Encyclopaedia, http://cfa-www.harvard.edu/planets.
shows the distributions of semimajor axis ($a$) and mass ($M_p \sin i$) of discovered extrasolar planets around M, K, G, and F stars. The planets around subgiants are excluded because the relation between stellar spectral type and its mass is different from that for main-sequence stars. The planets discovered by transit survey are also excluded because the transit survey has a different observational bias in planetary periods from that of the Doppler survey. Although many more planets have been discovered around G stars than around F and K stars, the detection probability (after correction for metallicity dependence) is similar among these stars (Fischer & Valenti 2005). However, the detection probability may be 1 order of magnitude lower around M stars. Since their signals are the most conspicuous, close-in planets are expected to be the first to be uncovered (Narayan et al. 2005). However, until recently, the only planets discovered around M stars have been two Jupiter-mass planets with moderate semimajor axis (0.13 and 0.21 AU) around Gl 876. No Jupiter-mass close-in planets have been discovered around M stars.

Recently, a short-period (2.6 days) Neptune-mass ($21 M_J$) planet was found to be orbiting around an M dwarf ($M_s = 0.4 M_\odot$), GJ 436 (Butler et al. 2004). With a mass between those of gas giants and the Earth, this finding signifies a transition in the quest to search for terrestrial planets. In the solar system, two ice giants, Uranus and Neptune, have masses in this range. These planets are primarily composed of icy cores with a modest gaseous envelope. In accordance with the core accretion scenario (e.g., Wuchterl et al. 2000), the cores in the outer solar system take a long time to emerge, and when they finally acquired $M_{\text{acc}}$, the solar nebula was already so severely depleted that they could accrete only a small amount of gaseous envelope (Hayashi et al. 1985). It is natural to extrapolate that GJ 436b may also have attained $M_{\text{acc}}$ but failed to accrete much gas. The main challenge to such a scenario is to account for the origin of both its low mass and short period.

The period of the first extrasolar gas giant planet (51 Peg b) discovered (Mayor & Queloz 1995) around a main-sequence star outside the solar system is comparable to that of GJ 436b. That planet and dozens of others like it are thought to have formed through sequential accretion beyond the ice boundary and migrated to their present locations (Lin et al. 1996) as a consequence of their tidal interaction with their nascent gaseous disks (Goldreich & Tremaine 1980; Lin & Papaloizou 1985). However, the much lower mass $M_p$ of GJ 436b implies that gas accretion onto it may have been greatly suppressed prior to, during, and after its migration.
Two other short-period Neptune-mass planets have been found around G dwarfs (a planet with a 2.8 day period and 15 $M_{\oplus}$ mass around 55 Cnc and one with a 9.6 day period and 14 $M_{\oplus}$ mass around HD 160691; McArthur et al. 2004; Santos et al. 2004). Since 55 Cnc and HD 160691 have three and two other giant planets (maybe gas giants), respectively, these Neptune-mass planets could form in situ by the accumulation of rocky materials caused by a sweeping mean motion resonance associated with the migration of a giant planet (e.g., Malhotra 1993; Ida et al. 2000) or sweeping secular resonance associated with disk gas depletion (e.g., Ward 1981; Nagasawa et al. 2000, 2005). Since we do not include interaction from other major planets, the formation of these Neptune-mass planets is beyond the scope of this paper. On the other hand, no gas giant planets have been found around GJ 436. GJ 436b is dynamically isolated, so its formation must be considered without the help of other giant planet(s) and our calculations in this paper can address its formation.

Although in this paper we propose a scenario that type II migration of a Neptune-mass planet occurs without significant gas accretion onto the planet around an M star, there are several other potential scenarios for the origins of dynamically isolated, close-in Neptune-mass planets. Boss et al. (2002) suggested that, along with Jupiter and Saturn, Uranus and Neptune were formed through gravitational instability, in a massive disk. Heavy elements settled to form the cores and the gas envelope (the clump) was greatly depleted by the photoevaporation due to the UV flux from nearby OB stars before it contracted to planetary size. Since gravitational instability is unlikely at the present location (0.028 AU) of GJ 436b, the planet must have migrated from the outer region by tidal interaction of the disk. However, the gravitational potential of disks around M stars is shallower than that around G stars, such that the evaporation of the envelope of collapsing clumps would also eliminate all the residual gas in the disk. It would be difficult for the collapsing fragments to lose a large fraction of their mass and migrate extensively to form a GJ 436b–like planet. On the other hand, under some extreme circumstances, the sporadic UV and X-ray irradiation from its host star could evaporate the envelope of a gas giant planet that has migrated from the outer region during an M star’s main-sequence lifetime. However, the overall impact of the photoevaporation process on a planet’s envelope and mass has not been determined.

Under the general concept of a sequential accretion scenario, dynamically isolated close-in planets may also form in situ (Bodenheimer et al. 2000). This scenario requires a concentration of planetesimals in proximity to the star. One possible mechanism that might lead to such a situation is through embryo-disk interaction, commonly known as type I migration (Ward 1986, 1997a). The accumulation of building blocks for close-in planets also requires the termination of their migration process and the interaction of the embryos with their host stars, as well as residual planetesimals.

Through a case study, in this paper we consider the origin of dynamically isolated close-in planets of roughly Neptune mass around stars with various masses. With the model developed in Papers I and II, we show that Neptune-mass close-in planets may be abundant around M stars. In § 2, we briefly recapitulate the sequential accretion hypothesis. Using our prescription, we first compare the mass function of dynamically isolated close-in versus modest- to long-period planets around solar-type stars (G dwarfs) in § 3. This comparison is useful because we have the most observational data and constraints for planets around solar-type stars at the present moment. On the basis of the best available observed properties of pre–main-sequence evolutionary tracks of different-mass stars and accretion rates onto their host stars, we construct disk models around main-sequence stars with various masses. In § 4, we construct a conventional model of giant planet formation around the lowest mass stars and show that icy planets with 10–20 $M_{\oplus}$ accrete from planetesimals at ~1 AU without any significant gas accretion onto the planets. These Neptune-mass planets can also readily migrate to the proximity of the stellar surface, where gas accretion is quenched at Neptune mass. Through a series of simulations, we show, in § 5, that the disk mass dependence on the host star’s mass and the criteria of tidally induced migration in the disk may be quantitatively constrained by the mass distribution of short-period planets. We also place constraints, in § 6, on the dependence of disk mass on the stellar mass. Finally, in § 7, we summarize our results and discuss their implications.

2. CORE ACCRETION IN DISKS AROUND LOW-MASS STARS

The detailed description of the sequential accretion scenario and our prescription to simulate the formation of planets are given in Paper I. We briefly recapitulate the central features of our approach and define various quantities that are used in the discussions of our results.

2.1. Growth of Protoplanetary Embryos

In the sequential accretion scenario, planetesimals grow into protoplanetary embryos (cores), which affects the velocity dispersion $\sigma$ of nearby planetesimals and modifies their own growth (Ida & Makino 1993; Aarseth et al. 1993; Rafikov 2003). However, $\sigma$ is also affected by gas drag (Kokubo & Ida 2002). In a disk with a surface density of dust ($\Sigma_d$) and gas ($\Sigma_g$) around a host star with mass $M_*$, protoplanetary embryos’ mass at any location $a$ and time $t$ is

$$M_c(t) \approx \left( \frac{t}{0.48 \text{ Myr}} \right)^3 \left( \frac{\Sigma_d}{10^3 \text{ g cm}^{-2}} \right)^3 \left( \frac{\Sigma_g}{2.4 \times 10^3 \text{ g cm}^{-2}} \right)^{6/5} \times \left( \frac{m}{10^{20} \text{ g}} \right)^{-2/5} \left( \frac{a}{1 \text{ AU}} \right)^{-9/5} \left( \frac{M_*}{M_\odot} \right)^{1/2} M_\oplus, \tag{1}$$

where $m$ is the typical mass of the planetesimals accreted by the embryos. Since accretion time $t_{\text{grow}} = M_*/M_c$ increases with $M_c$, $M_c(t)$ does not depend on its initial value $M_{c,0}$ as long as $M_c(t) \gg M_{c,0}$.

In the limit of small $\sigma$, the full width of the embryos’ feeding zone ($\Delta a_\sigma$) is limited to $\approx 10\sigma_{11}$ (Lissauer 1987; Kokubo & Ida 1998), where $\sigma_{11}$ is the embryos’ Hill radius $[= (M_*/3M_\oplus)^{1/3}a]$. When all the residual planetesimals in an embryo’s feeding zone have coagulated with it, the embryo attains an isolation mass

$$M_{c, \text{iso}} \approx 0.16 \left( \frac{\Sigma_d}{10^3 \text{ g cm}^{-2}} \right)^{3/2} \left( \frac{a}{1 \text{ AU}} \right)^{3} \left( \frac{\Delta a_{\sigma}}{10\sigma_{11}} \right)^{3/2} \left( \frac{M_*}{M_\odot} \right)^{-1/2} M_\oplus. \tag{2}$$

2.2. Protostellar Disk Properties

Equations (1) and (2) indicate that both $M_c(t)$ and $M_{c, \text{iso}}$ are determined by the distribution of $\Sigma_d$ and $\Sigma_g$. In Papers I and II, we introduced a multiplicative factor ($f_d$ and $f_g$) to globally scale
disks with the minimum mass nebula model for the solar system (Hayashi 1981) such that

\[
\begin{align*}
\Sigma_d &= 10 \eta_{\text{ice}} f_d h_d \left( \frac{a}{1 \text{ AU}} \right)^{-3/2} \text{(g cm}^{-2}\text{)}, \\
\Sigma_g &= 2.4 \times 10^3 f_g h_g \left( \frac{a}{1 \text{ AU}} \right)^{-3/2} \text{(g cm}^{-2}\text{)},
\end{align*}
\]

where the step function \(\eta_{\text{ice}} = 1\) inside the ice boundary at \(a_{\text{ice}}\) and \(4.2\) for \(a \geq a_{\text{ice}}\). (Note that the latter can be slightly smaller \(\sim 3.0\); Pollack et al. 1994). The minimum mass model corresponds to \(f_d = f_g \approx 1\). Here we introduced a new scaling factor, \(h_d\) and \(h_g\), representing the dependence on stellar mass (see below).

If the disk is optically thin and heated by stellar irradiation only (Hayashi 1981),

\[
a_{\text{ice}} = 2.7 \left( \frac{L_\star}{L_\odot} \right)^{1/2} \text{AU}.
\]

The stars’ luminosity \(L_\star\) is generally a function of their mass \(M_\star\) and age \(t\). Additional heating due to viscous dissipation enlarges this boundary (Lin & Papaloizou 1980). This ad hoc phenomenological prescription provides a useful working hypothesis for comparative analysis between solar system architecture and extrasolar planetary systems. Since \(L_\star\) is generally an increasing function of \(M_\star\), \(a_{\text{ice}}\) is small in disks around low-mass stars.

Around solar-type T Tauri stars, the observationally inferred total mass of dust, \(M_d\), in the protostellar disks ranges from \(10^{-5}\) to \(10^{-3} M_\odot\) (Beckwith & Sargent 1996), which corresponds to a range of \(f_d \sim 0.1\)–10. The disk mass determinations are somewhat uncertain because of the poorly known radiative properties of the grains. Nevertheless, their divergent dust content provides reasonable evidence for a greater than 1 order of magnitude dispersion in \(M_d\). In addition, the radio image of the disks is not well resolved in many cases. A rough magnitude of \(f_d\) can be inferred from the total mass of the disk under the assumption that all disks have similar sizes (a few tens to 100 AU). In this paper we follow Paper II and generate a set of \(f_d\) with a unit variant of Gaussian logarithmic distributions and a range of 0.1–10 with cutoff of high \(f_d h_d\) tails at 30\((M_\star/M_\odot)\) (see Fig. 4). If \(f_d h_d \geq 30(M_\star/M_\odot)\), the disk is gravitationally unstable at a greater than around a few AU (see discussion in \(\S\) 7), and the total mass of heavy elements in the disk would be a significant fraction of that in the host star.

In equation (3), the dependence of \(\Sigma_d\) on \(M_\star\) is incorporated in a mass scaling function \(h_d\). In general, a relatively weak IR excess is associated with low-mass stars, which suggests that \(h_d\) may be an increasing function of \(M_\star\), at least in the inner regions of the disk. For host stars with \(M_\star > M_\odot\), \(h_d = 1\).

Equation (1) indicates that \(M_\star\) is also a function of \(\Sigma_y\) through damping of \(\sigma\) due to gas drag. The planetary migration rate that we mention later is also dependent on \(\Sigma_y\). Around other stars, very little information is available on \(\Sigma_y\). Since there is no indication of a divergent depletion pattern between molecular hydrogen and millimeter-size dust emission (Thi et al. 2001), we follow the same prescription for \(\Sigma_y\) with a disk mass scaling parameter \(f_y\) and a stellar mass dependence function \(h_y\) (eq. [3]).

Similar to Papers I and II, we adopt the conjecture that \(f_y\) does not change except in those regions where they have been totally accreted by the cores and

\[
f_y = f_{y,0} \exp \left( \frac{-t}{\tau_{\text{dep}}} \right),
\]

where \(\tau_{\text{dep}}\) is the disk depletion timescale discussed below. The assumption of the uniform exponential decay is for simplicity.

The effects of the detailed decay pattern as a result of the viscous evolution of disks will be discussed in a separate paper. For computational simplicity, we assume \([\text{Fe}/\text{H}] = 0\), i.e., a solar composition for all stars so that \(f_d = f_{y,0} h_d = h_y\). The dependence of planetary \(M_p-a\) distribution on \([\text{Fe}/\text{H}]\) for solar-type stars has already been discussed in Paper II.

We now consider a prescription for the stellar mass dependence. The parameters are \(h_d = (h_y)^2\) and \(\tau_{\text{dep}}\). In young clusters, the fraction of stars with detectable IR (Haisch et al. 2001) and millimeter continuum (Wyatt et al. 2003) from circumstellar disks around \(T\) Tauri stars declines on timescales of 1–10 Myr. Although this decline may be due to dust growth and planetesimal formation rather than the depletion of heavy elements (D’Alessio et al. 2001; Tanaka et al. 2005), the correlation between the intensity of millimeter dust continuum and the decline in gas emission (Thi et al. 2001) and the UV veiling for ongoing gas accretion suggest that gas is depleted as the dust signature fades. The dust signature also persists for up to 10 Myr for disks around brown dwarfs (Mohanty et al. 2003). Thus, in this paper, we assume \(\tau_{\text{dep}}\) in the range of 1–10 Myr for all stellar masses.

Although direct estimates of disk mass in both gas and dust are difficult to obtain for the inadequate sensitivity of existing observational instruments, the disk accretion rate \(\dot{M}\) can be inferred from the H\(\alpha\) line profiles (Muzerolle et al. 2003; Natta et al. 2004) such that \(M \propto M_\star^2\) with a large dispersion. If the angular momentum transfer and mass diffusion timescale is insensitive to \(M_\star\), we could infer \(h_d = (h_y)^2 \sim (M_\star/M_\odot)^2\). In view of the large uncertainty in the data, we consider three possible dependences on \(M_\star\),

\[
h_d = \left( \frac{M_\star}{M_\odot} \right)^{0.1-2},
\]

with \(h_d = (M_\star/M_\odot)^2\) as a standard case.

2.3. Core Growth, Isolation Mass, and Gas Giant Formation around Stars with Different Masses

With these prescriptions for disk parameters, we find that

\[
M_c(t) \simeq \left( \frac{t}{4.8 \times 10^5 \text{yr}} \right)^3 \eta_{\text{ice}} f_d h_d \left( \frac{a}{1 \text{ AU}} \right)^{-8/10} \left( \frac{M_\star}{M_\odot} \right)^{1/2 - 1/2} \left( \frac{M_\star}{M_\odot} \right)^{3/4} - 1/2 M_\odot,
\]

by assuming \(m = 10^{22} \text{ g}\). From equation (3), we estimate the core masses in protoplanetary systems with equation (2) such that

\[
M_{c,\text{iso}} \simeq 0.16 \eta_{\text{ice}} f_d h_d \left( \frac{a}{1 \text{ AU}} \right)^{3/4} \left( \frac{M_\star}{M_\odot} \right)^{-1/2} M_\odot.
\]

We have already pointed out in Papers I and II that the growth of planetesimals is limited by isolation at small \(a\) and the slow coagulation rate at large \(a\). Since for smaller \(M_\star\), \(r_{\text{iso}}\) is larger, \(M_{c,\text{iso}}\) is larger for the same mass disks. However, both \(h_d\) and \(h_y\) are increasing functions of \(M_\star\), so that the growth of \(M_c(t)\) and the isolation mass \(M_{c,\text{iso}}\) at any given \(a\) are actually smaller for lower mass stars. We do not explicitly include type I migration (Ward 1986, 1997a) of cores but take into account its effects in some runs (see Paper I and discussion in \(\S\) 5).
As the cores grow beyond a mass
\[ M_{\text{c, hydro}} \simeq 10 \left( \frac{\dot{M}_c}{10^{-6} \, M_\odot \, \text{yr}^{-1}} \right)^{0.25} M_\oplus, \quad (9) \]
their planetary atmosphere is no longer in hydrodynamic equilibrium and they begin to accrete gas (Stevenson 1982; Ikoma et al. 2000). In the above equation, we neglected the dependence on opacity (see Paper I). In regions where they have already attained isolation, the cores’ accretion \( \dot{M}_c \) is much diminished and \( M_{\text{c, hydro}} \) can be comparable to \( 1 \, M_\oplus \). However, the gas accretion rate is still regulated by the efficiency of radiative transfer such that
\[ \frac{dM_{p,g}}{dt} \simeq \frac{M_p}{\tau_{\text{KH}}}, \quad (10) \]
where \( M_p \) is the planet mass including gas envelope and the Kelvin-Helmholtz contraction timescale is (for details, see Paper II)
\[ \tau_{\text{KH}} \simeq 10^{10} \left( \frac{M_p}{M_\odot} \right)^{-3} \text{yr}. \quad (11) \]

Gas accretion onto the core is quenched when the disk is depleted either locally or globally. A protoplanet induces the opening of a gap when its rate of tidally induced angular momentum exchange with the disk exceeds that of the disk’s intrinsic viscous transport (Lin & Papaloizou 1985), that is, when the planet mass \( M_p \) exceeds
\[ M_{\text{g,vis}} \simeq \frac{40\nu}{a_0\Omega_k} M_\ast \simeq 40\alpha \left( \frac{h^2}{a} \right) M_\ast \]
\[ \simeq 3 \left( \frac{\alpha}{10^{-4}} \right) \left( \frac{a}{1 \, \text{AU}} \right)^{1/2} \left( \frac{L_\ast}{L_\odot} \right)^{1/4} M_\odot, \quad (12) \]

where we used an equilibrium temperature in optically thin disks (Hayashi 1981) and \( \alpha \)-prescription for the effective viscosity \( \nu \) (Shakura & Sunyaev 1973), in which \( T = 170 (a/d_{\text{iso}})^{1/2} \) K and \( \nu = \alpha h^2 \Omega_k \), where \( \alpha \) is a dimensionless parameter and \( h \) and \( \Omega_k \) are the disk scale height and Kepler frequency, respectively. Since \( h = c_s/\Omega_k \), where \( c_s \) is the sound velocity, \( h^2 \propto T/M_\ast \propto L_\ast^{1/4}/M_\ast \), so that \( M_{\text{g,vis}} \propto L_\ast^{1/4} \). Since \( L_\ast \) is an increasing function of \( M_\ast \), equation (12) indicates that \( M_{\text{g,vis}} \) is smaller around lower mass stars. The orbital evolution of planets is locked to the viscous evolution of the disk gas (type II migration) when their \( M_p \geq M_{p,\text{mig}} = A_p M_{\text{g,vis}} \), where \( A_p \) is the dimensionless factor \( \approx 3–10 \) for a laminar disk (Lin & Papaloizou 1985).

The type II migration rate \( \dot{a} \) is given by \( a/\tau_{\text{mig}} \) with
\[ \tau_{\text{mig}} = 10^6 f_{\text{am}} h_{\text{iso}}^{-1} \left( \frac{\alpha}{10^{-4}} \right) \left( \frac{M_p}{M_\odot} \right) \left( \frac{a}{1 \, \text{AU}} \right)^{1/2} \text{yr}, \quad (13) \]
where \( M_\odot \) is the Jupiter mass. We set a lower limit on \( \tau_{\text{mig}} \) at \( a^2/\nu \sim 4.3 \times 10^5 (\alpha/10^{-4}) (a/1 \, \text{AU}) \) yr (Paper I). We found that the formula for \( J_m \) (angular momentum flux at the radius of maximum viscous couple, \( r_m \)) given in Paper I (eq. [63] in that paper) must be multiplied by a factor of 2π. If we use \( J_m \) to evaluate the evolution of the planetary orbital radius \( a \), equation (13) is reduced by 2π as well (Paper I). However, since the planetary migration may be caused by a fraction of \( J_m \) and the fraction is uncertain, we use equation (13) in the present paper too. We adjust the migration timescale by the value of \( \alpha \), which is also uncertain, comparing with observational data. As shown in Paper II, in order to reproduce a period distribution of gas giant planets similar to that of observed extrasolar planets around solar-type stars, the disk viscous diffusion timescale \( \tau_\nu = r_m^2/\nu \sim 4 \times 10^4 (\alpha/10^{-4})(r_m/10 \, \text{AU}) \) yr must be comparable to the disk lifetime \( \tau_{\text{dep}} \). Hence, the \( \alpha \) viscosity in our model must be \( \sim 10^{-4} \). Although the regions at \( 1–10 \, \text{AU} \) could be a “dead zone” for MHD turbulence (Sano et al. 2000), resulting in a very small \( \alpha \) in these regions, the best-fit value \( \alpha \sim 10^{-4} \) may not necessarily reflect a realistic value because of rather simple assumptions for \( \Sigma_\ast \) distribution and its exponential decay in our model. We will carry out more detailed calculations coupled with disk viscous evolution in a separate paper.

The planets’ migration is terminated either when the disk is severely depleted \( (f_\nu \to 0) \) or when they reach \( a_{\text{stall}} \), which is set to be 0.04 AU in our calculations. There are potential mechanisms to stop migration at \( \leq 0.05 \, \text{AU} \) (Lin et al. 1996). However, Paper II suggests that only a small fraction \((\lesssim 10\%)\) of migrating planets can survive in the vicinity of their host stars. This fact should be kept in mind when the population of close-in planets is discussed with our model.

The gap becomes locally severely depleted when the planets’ Hill radius \( (r_H) \) exceeds the disk thickness \( h \) (Bryden et al. 1999), that is, when \( M_p \) exceeds
\[ M_{g,\text{th}} \simeq 120 \left( \frac{a}{1 \, \text{AU}} \right)^{3/4} \left( \frac{L_\ast}{L_\odot} \right)^{3/8} \left( \frac{M_\ast}{M_\odot} \right)^{-1/2} M_\odot. \quad (14) \]

Growth through gas accretion is quenched for planets with \( M_p \geq M_{p,\text{trunc}} = A_{\text{th}} M_{g,\text{th}} \). Numerical simulations show some uncertainties in the dimensionless parameter \( A_{\text{th}} \) (Bryden et al. 1999; Nelson et al. 2000). Planets with \( M_{p,\text{trunc}} > M_p > M_{p,\text{mig}} \) migrate with the disk while continuing to accrete gas, albeit at a reduced rate (e.g., Lubow et al. 1999). We use equation (10) without a reduction factor for simplicity because the reduction factor is quite uncertain and introduction of the factor does not affect the results significantly. Since gas accretion for \( M_p > M_{p,\text{mig}} \) is already very rapid (eq. [11]), the reduction does not change the total gas accretion timescale. Following Paper I, we adopt in this paper \( A_{\text{th}} = 1.5^3 \approx 3.4 \); that is, the truncation condition is \( r_H > 1.5 h \). Since \( L_\ast \) rapidly increases with \( M_\ast \), \( M_{p,\text{trunc}} \) is also smaller around lower mass stars.

Even for planets with \( M_p < M_{p,\text{trunc}} \), gas accretion may ultimately be limited by the diminishing amount of residual gas in the entire disk. For our disk models, the maximum available mass is
\[ M_{\ast,\text{noiso}} \sim \pi a^2 \Sigma_\ast \sim 290 f_\nu h_{\text{iso}} \left( \frac{a}{1 \, \text{AU}} \right) \exp \left( -\frac{t}{\tau_{\text{dep}}} \right) M_\odot. \quad (15) \]

When \( M_{\ast,\text{noiso}} \) becomes smaller than \( M_p \), gas accretion is terminated.

A similar global limit \( M_{c,\text{noiso}} = \pi a^2 \Sigma_\ast \) (see eq. [20]) is also imposed if equation (2) exceeds it. As the gas is severely depleted, the velocity dispersion \( \sigma \) of the embryos and residual planetesimals grows until they cross each other’s orbits (Iwasaki et al. 2002; Kominami & Ida 2002). Eventually, a few surviving embryos acquire most of the residual planetesimals and less massive cores during the late oligarchic growth stage. The
asymptotic embryos’ masses are given by equation (2) with \( \Delta a_c \approx V_{\text{surf}} / \Omega_K \),

\[
M_{c,\text{iso}} \approx 0.52 \eta_{\text{acc}} f_d^{3/2} h_d^{3/2} \left( \frac{a}{1 \text{ AU}} \right)^{3/2} \left( \frac{\rho_d}{1 \text{ g cm}^{-3}} \right)^{1/4} \left( \frac{M_c}{M_\oplus} \right)^{-3/4} M_\odot,
\]

where \( V_{\text{surf}} \) and \( \rho_d \) are the surface escape speed and internal density of the embryo. We use this enlarged asymptotic mass when \( f_d < 10^{-3} \).

In Papers I and II, we put all of these processes into a numerical scheme to simulate the formation and migration probabilities of planets around solar-type stars. Cores with \( M_p > M_{\text{acc}} \) emerge on timescales shorter than \( \tau_{\text{dep}} \) in disks with modest to large values of \( f_d \) around solar-type stars. In this limit, gas accretion and orbital migration lead to the formation of gas giants with kinematic properties similar to those observed.

3. EMERGENCE AND MIGRATION OF NEPTUNE-MASS PLANETS IN DISKS AROUND SOLAR-TYPE STARS

We now apply our numerical methods to studying the formation of planets around stars with various masses. Our objective is to simulate the mass function of close-in planets and assess the influence of formation and migration on it. We show here that this quantity can provide clues on the dominant processes that regulate planet formation, and it also can be used to distinguish between competing theories of planet formation. In all models, we choose \( \alpha = 10^{-4} \) on the basis of the assumption that the viscous evolution timescale for the disks is comparable to \( \tau_{\text{dep}} \) (see § 2.3). Using these models, we carry out Monte Carlo simulations. For simplicity, we generate a set of initial \( a \)-values of the protoplanets and \( \tau_{\text{dep}} \) with uniform distributions in log scale in the ranges of \( 0.1-100 \text{ AU} \) and \( 10^6-10^7 \text{ yr} \). The assumed \( a \)-distribution corresponds to orbital separations \( \Delta a \) that are proportional to \( a \), which may be the simplest choice. The distribution of \( f_d \) was discussed in § 2.2 (also see Fig. 4). In Papers I and II, we also assumed a distribution of \( M \) in a range of \( 0.7-1.4 M_\oplus \). In this paper, in order to make the \( M \), dependence clear, the value of \( M \) is fixed in each run.

In the following sections, we consider several sets of model parameters. We first discuss a standard model with \( A_p = 10 \) and \( h_d = (M_c/M_\oplus)^2 \) around a solar-type star with \( M_* = 1.0 M_\odot \) (model 1.0). The evolution of a total of 20,000 planets is calculated for each run.

3.1. Formation of Cores and Asymptotic Mass of Protoplanets

Model 1.0 is similar to the results that we have already presented in Paper I. In Figure 2a, we highlight the mass \( (M_{p,\text{fin}}) \) and semimajor axis \( (a_{\text{fin}}) \) distribution of planets at \( t = 10^9 \text{ yr} \) after they have attained their asymptotic mass and gone through the initial migration due to their tidal interaction with their natal disks. The main features to note in this panel are (1) a deficit of planets with intermediate masses \( (20-100 M_\oplus) \) at intermediate semimajor axis (0.1–1 AU) and (2) a large population of close-in \( (a_{\text{fin}} < 0.05 \text{ AU}) \) gas giants with \( 20 M_\oplus \leq M_p \leq 2 \times 10^3 M_\oplus \), although only a small fraction of them (\( \lesssim 10\% \)) may be able to survive (Paper II). We have already indicated in Paper I that these properties are due to (1) the runaway nature of dynamical gas accretion and (2) type II migration.

In order to distinguish between these two dominant effects, we trace back, in Figure 2b, the initial semimajor axis \( (a_{\text{fin}}) \) where the cores of both close-in (black circles) and intermediate- or long-period planets (gray dots) formed. These results clearly indicate a one-to-one mapping between the mass function of the close-in planets and the locations where they are formed. For illustrative purposes, we also mark the domain where some physical processes operate and dominate the evolution of protoplanets. For example, the thin solid lines indicate the upper limit of the isolation mass that cores can attain prior to gas depletion (it is obtained with \( f_d = 30 \) in \( M_{c,\text{iso}} \) and \( M_{c,\text{no iso}} \)). The transition at 2.7 AU in model 1.0 corresponds to the ice boundary. The thick solid lines indicate the critical mass for the onset of type II migration, \( M_{p,\text{crit}} = A_p M_{p,\text{vis}} \) with \( A_p = 10 \) according to equation (12). We also highlighted the asymptotic growth limit \( M_{p,\text{trunc}} = A_h M_{g,\text{th}} \) with \( A_h = 3.4 \) (eq. [14]; dashed lines).

At any given \( a_{\text{ini}} \), a fraction of terrestrial planets can form with \( M_p > M_{p,\text{vis}} \) through (1) gas accretion and (2) merger of residual planetesimals and other embryos after disk gas depletion. When the embryos reach their isolation mass, \( M_{\text{iso hyd}} \), their evolution of gas giants is so small that they induce clear gap formation when their mass \( M_p \approx 5-10 M_\oplus \) and they should not be referred to as hot Neptunes.

If their \( M_p > M_{p,\text{vis}} \) and \( M_{p,\text{vis}} > 10 M_\oplus \), the gas accretion is an increasing function of \( a \). For \( a \lesssim 0.7 \text{ AU} \), \( \tau_{\text{KH}} \) for embryos that can migrate is longer than their \( \tau_{\text{dep}} \). Although their \( M_{\text{iso hyd}} < M_{p,\text{vis}} \) and \( \tau_{\text{KH}} \) is set to be \( a_{\text{ini}} = 0.04 \text{ AU} \). Thus, the final mass of close-in planets is determined by the gas accretion to their asymptotic mass \( M_{p,\text{trunc}} \approx 30 M_\oplus \) at the stalling location, which is set to be \( a_{\text{stall}} = 30 M_\oplus \). Regardless of truncation \( M_{p,\text{trunc}} \) at the original locations \( a_{\text{ini}} \). These planets are formed interior to the ice boundary, and they are likely to be mostly composed of silicates and iron, in contrast to the ice giants in the solar system. Also, they cannot accrete large amount of gas because the aspect ratio of their nascent disk at 0.04 AU is so small that they induce clear gap formation when their mass reaches that of Neptune.

Plains formed at slightly larger \( a \) (\( \sim 1 \text{ AU} \)) must attain \( M_p \approx 20-30 M_\oplus \) before they acquire the mass to start migration, \( M_{p,\text{vis}} > 10 M_\oplus \). With this critical mass, embryos that can initiate migration there can also accrete gas efficiently with \( \tau_{\text{KH}} < \tau_{\text{dep}} \) and migrate to the disk. However, when they arrive close to their host stars, \( M_p \) may be \( > M_{p,\text{vis}} \) so they would not acquire any additional mass. Note that cores of these planets are also mostly made of silicates and iron rather than ice and they should not be referred to as hot Neptunes.

At even larger radius, \( M_{c,\text{iso}} > 5-10 M_\oplus \). Prior to reaching isolation, embryos’ gas accretion is suppressed by the bombardment of residual planetesimals, i.e., \( M_c > M_{\text{iso hyd}} \) for moderate \( M_* \). After isolation is reached, \( M_{\text{iso hyd}} \) becomes smaller than \( M_c \), their \( \tau_{\text{KH}} \) due to gas accretion is reduced below \( \tau_{\text{dep}} \), and they quickly evolve into gas giants. Thus, the final \( M_p \) in Figure 2b coincides with \( M_{p,\text{trunc}} \) at the initial locations. Beyond \( \sim 10 \text{ AU} \),
the timescale for the emergence of cores with \( M_p > M_{c,\text{acc}} \) or isolation is comparable to or longer than both \( \tau_{\text{mig}} \) and \( \tau_{\text{dep}} \). Although they may acquire \( M_p \) in the range of 10–100 \( M_e \) through mergers of residual embryos after the gas depletion, these planets generally do not migrate extensively.

3.2. Mass Distribution of Short-Period Planets

Interior to the ice boundary, nearly all the planets with sufficient mass to initiate efficient gas accretion have migrated to the vicinity of the host star (see Fig. 2b). However, a majority of the planets that migrated to the vicinity of solar-type stars were formed beyond the ice boundary, as gas giants, prior to their migration. There is a narrow window in the range of \( a \) where the seeds of intermediate-mass planets may form and migrate to the proximity of their host stars. In Figure 3, theoretically predicted mass distributions are plotted. Since it is expected that most close-in planets may fall onto their host stars, we plot the distributions of close-in planets, reducing the amplitude \( N \) by a

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**Fig. 2.—Distributions of semimajor axis \((a)\) and mass \((M_p)\) of planets predicted by the Monte Carlo simulations. \((a)\) Final semimajor axes \((a_{\text{fin}})\) and masses \((M_{p,\text{fin}})\) at \( t = 10^9 \text{ yr} \); and \((b)\) initial semimajor axes \((a_{\text{ini}})\) and \( M_{p,\text{ini}} \). The values of \( x \) (0.2, 0.4, 0.6, 1.0, and 1.5) represent the host star mass scaled by the solar mass, \( M/M_\odot \). In \((b)\), close-in planets with \( a_{\text{ini}} < 0.05 \text{ AU} \) are marked by black crosses, while the other planets are marked by gray dots. The thin solid black lines indicate the isolation mass \( M_{p,\text{iso}} \) with \( f_d = 30 \). The thick solid black lines express the critical mass for radial migration, \( M_{p,\text{mig}} = A_{\text{r}} M_{p,\text{ini}} \) with \( A_{\text{r}} = 10 \). The dashed lines show the truncation mass for gas accretion, \( M_{p,\text{trunc}} = A_{\text{th}} M_{p,\text{ini}} \) with \( A_{\text{th}} = 3.4 \).
factor of 10. For comparison of the amplitude between close-in and distant planets, uncertainty in this calibration is noted. We also plotted observed distributions. Since the number of runs in each model does not reflect the number of targets for the current Doppler survey, we cannot compare the amplitude $N$ between the observed and the theoretically predicted distributions. Only the shape of the distributions should be compared. Also note that observed distributions do not exactly correspond to host stars’ mass of each model and numbers of observed planets are not large enough for statistical arguments for stars other than G stars (model 1.0). We show that around stars with $M_\ast \simeq 1 \, M_\odot$, the mass distribution for the close-in (with $a_{\text{fin}} < 0.05$ AU) planets is skewed toward $\sim 10^3 \, M_\oplus$ (model 1.0 in Fig. 3a). This distribution is more enhanced near $\sim 10^3 \, M_\oplus$ than that observed. The effect of postformation star-planet tidal interaction, which has not been taken into account in our model, may have caused the demise of a majority of the close-in planets, in particular massive planets (Gu et al. 2003; Paper II).

Fig. 3.—Calculated distribution of final mass of planets for (a) close-in planets at $a_{\text{fin}} < 0.05$ AU and (b) planets at $0.1 \, \text{AU} < a_{\text{fin}} < 1 \, \text{AU}$ (filled circles). The values of $x$ are the same as in Fig. 2. Since it is expected that most close-in planets might fall onto their host stars, the calculated amplitude $N$ in (a) is reduced by a factor of 10. Observed distributions are also plotted with open triangles. The $\sin i$ factor is neglected for simplicity (it enhances the observed values only by $4\pi$ on average). The number of runs in each model does not reflect the number of targets for the current Doppler survey. Also note that observed distributions do not exactly correspond to the host stars’ mass in each model.
In disks with modest masses, planets form in the advanced stages of evolution when gas depletion is well underway. Some of these planets may migrate interior to the ice boundary and become stalled, while others remain close to their place of birth. A majority of gas giant planets with 0.1 AU \( \lesssim a_{\text{iso}} \lesssim 1 \) AU have migrated but not extensively. In contrast to the close-in planets, model 1.0 in Figure 3b clearly shows a paucity of longer period (0.1 AU \( \lesssim a_{\text{iso}} \lesssim 1 \) AU) planets at \( M_p \sim 20-100 \, M_\oplus \). This distribution reflects the stringent prerequisite that gas accretion into gas giants must be preceded by the rapid formation of sufficient mass cores, whereas the buildup of terrestrial planets can continue well after the severe depletion of the disk.

4. DEPENDENCE ON THE STELLAR MASS

In a generalization of the solar nebula model, Nakano (1988) showed that the temperature distribution throughout the disk increases with \( M_\star \). However, he did not consider the dependence of \( \Sigma_d \) and \( \Sigma_g \) on \( M_\star \). In a recent paper, Laughlin et al. (2004a) considered a model in which the disk mass increases with the stellar mass. Their objective is to demonstrate the difficulties of forming gas giants around M dwarf stars. However, the effects of planetary migration, truncation of gas accretion due to gap opening, and the gradual depletion of the gas disk are neglected.

In this section, we consider the variation of three model parameters: (1) the stellar mass \( M_\star \), (2) the dependence of disk mass on the stellar masses, \( h_d \), and (3) the condition for the onset of type II migration. A standard series are model \( x \), where \( x = 0.2, 0.4, 0.6, 1.0, \) and 1.5 represents models with \( M_\star = x \, M_\odot \). In the standard series, we set \( A_e = 10 \) and \( h_d = \left( M_\star / M_\odot \right)^{2} \). Models \( xB \) (series B) and \( xC \) (series C) correspond to \( A_e = 1 \) and 100 with \( h_d = \left( M_\star / M_\odot \right)^{2} \); \( x \) in \( xB \) and \( xC \) expresses \( M_\star / M_\odot \) as well. Models \( xD \) (series D) and \( xE \) (series E) correspond to \( h_d = M_\star / M_\odot \) and 1 with \( A_e = 10 \).

In Figure 4, the distributions of \( f_d \) and \( h_d \) that we used for models \( x \) and \( xB \) are shown. They represent the relative mass distribution of disks around stars with various masses. The mean value of \( \Sigma_d \) is an increasing function of \( M_\star \) (see eq. [3]). In order to limit additional model parameters, we assume a zero-age main-sequence mass-luminosity relationship, \( L_\star / L_\odot = (M_\star / M_\odot)^{4} \). Since the timescale of the pre–main-sequence stage of lower mass stars is long, planet formation around these stars may proceed during their pre–main-sequence stage in which \( L_\star \) is rather large. In that case, the dependence of \( L_\star \) on \( M_\star \) is weaker, but it may still have a positive power-law dependence, so the trend of the \( M_\star \) dependence of planetary systems shown below does not change.

In the calculations in this section,

\[
a_{\text{ice}} = 2.7 \left( \frac{M_\star}{M_\odot} \right)^{2} \, \text{AU},
\]

\[
M_p(t) = \left( \frac{t}{4.8 \times 10^{5} \, \text{yr}} \right)^{3/2} a_{\text{ice}}^{3/2} f_{d}^{1/5} h_{d}^{6/5} \times \left( \frac{a}{1 \, \text{AU}} \right)^{-81/10} \left( \frac{M_\star}{M_\odot} \right)^{89/10} M_\odot,
\]

\[
M_{c, \text{iso}} = 0.16 a_{\text{ice}} f_{d}^{3/2} \left( \frac{a}{1 \, \text{AU}} \right)^{3/4} \left( \frac{M_\star}{M_\odot} \right)^{3/2} M_\odot,
\]

\[
M_{c, \text{iso}} \approx 1.2 a_{\text{ice}} f_{d} \left( \frac{a}{1 \, \text{AU}} \right)^{1/2} \left( \frac{M_\star}{M_\odot} \right)^{2} M_\odot.
\]

![Fig. 4.—Distribution of \( f_d h_d \) that we used for the standard model, which is a Gaussian distribution in terms of \( \log f_d \) with a center at \( \log f_d = 0 \) and dispersion of 1. In the standard model, we assume \( h_d = (M_\star / M_\odot)^{2} \). We omit the high \( h_d \) tail at \( >30(M_\star / M_\odot) \), since such heavy disks are self-gravitationally unstable. Filled circles, open circles, filled squares, open squares, and filled triangles represent the cases of \( M_\star = 0.2, 0.4, 0.6, 1.0, \) and 1.5 \( M_\odot \), respectively.](image-url)

\[
M_{c, \text{iso}} = 0.52 a_{\text{ice}} f_{d}^{3/2} \left( \frac{a}{1 \, \text{AU}} \right)^{3/2} \left( \frac{\rho_d}{1 \, \text{g cm}^{-3}} \right)^{1/4} \left( \frac{M_\star}{M_\odot} \right)^{9/4} M_\odot,
\]

\[
M_{p, \text{mig}} = A_v M_\star \theta_{\text{vis}} = 30 \left( \frac{A_v}{10} \right) \left( \frac{\alpha}{10^{-4}} \right) \left( \frac{a}{1 \, \text{AU}} \right)^{1/2} \left( \frac{M_\star}{M_\odot} \right) M_\odot,
\]

\[
M_{p, \text{trunc}} = A_{\text{th}} M_{\star, \text{th}} = 400 \left( \frac{A_{\text{th}}}{1.5^7} \right) \left( \frac{a}{1 \, \text{AU}} \right)^{3/4} \left( \frac{M_\star}{M_\odot} \right) M_\odot.
\]

4.1. Planets around M Dwarf Stars

In Figures 2 and 3, we also included the results of the simulations for models 0.2–0.6 and 1.5. We first present a low-mass model 0.2, since it is in strong contrast to model 1.0. Stars with \( M_\star \lesssim 0.2 \, M_\odot \) correspond to relatively light M stars. These stars are not only the most numerous, but they also contribute most to the initial stellar mass function. According to our prescription and model parameters, \( \Sigma_d \) and \( \Sigma_g \) around the host star in model 0.2 are 25 times smaller than those in model 1.0.

In the result of model 0.2 on the top panels of Figures 2 and 2b, we find that Jupiter-mass planets rarely formed around low-mass stars. This paucity is due to the slow growth rate of embryos and their low isolation mass such that little gas can be accreted by them prior to its depletion. This result confirms the conclusion reached previously by Laughlin et al. (2004a).

The upper limit of the \( M_{p, \text{fin}} \) distribution is determined by the \( M_{c, \text{iso}} \). This correlation arises because \( L_\star \) is a rapidly rising function of \( M_\star \). In our prescription, the ice boundary is located at \( a_{\text{ice}} \approx 0.11 \) AU (eq. [17]), which is 25 times closer to a host star with \( M_\star = 0.2 \, M_\odot \) than in model 1.0 with solar-type stars. Nearly all the cores formed around these low-mass stars are mostly composed of ice. Planets can acquire masses \( M_p > M_{c, \text{iso}} \).
masses increase with $M_p$. Peaks at $M_p$ necessary condition for the onset of type II migration is satisfied for relatively low mass planets (eq. [22]). When their $M_p > M_{p, mig}$, these cores undergo orbital decay. Similar to model 1.0, there is a population of close-in planets with $2 M_\odot \lesssim M_p \lesssim 10 M_\odot$. The results on the top panel of Figure 2b show that they indeed originated from a region between $a_{\text{acc}}$ and $\sim 3$ AU. They also indicate that planets with mass down to $2 M_\odot$ may migrate to the proximity of a 0.2 $M_\odot$ star. At the arbitrary $a_{\text{stat}} = 0.04$ AU around such a host star, the equilibrium temperature of these close-in Neptune-mass planets is similar to that of the Earth. The composition and structure of such planets have already been discussed by Léger et al. (2004).

Although a few planets formed with $M_p > 10 M_\oplus$, they are the exceptional cases resulting from the tails of the $f_d$ distribution. Consequently, the mass function of planets with modest or large $a_{\text{fin}} (>0.1$ AU) shows a sharp decline at $M_p \sim 10 M_\odot$ (Fig. 3b). It also extends well into the low-mass range. In contrast, the mass function for the close-in planets (with $a_{\text{fin}} < 0.05$ AU) shows a peak near $M_p \sim 10 M_\odot$ (Fig. 3a).

In model 0.4, we consider a host star with $M_* = 0.4 M_\odot$, which corresponds to a relatively massive M dwarf star. It is an analog of GJ 436, around which a single close-in Neptune-mass planet has been discovered (Butler et al. 2004). Figures 3a and 3b indicate that the frequency of planets observable with the current Doppler survey is significantly smaller for M stars than for K, G, and F stars (also see Fig. 8). This is consistent with the observation and the finding of Laughlin et al. (2004a).

Figure 5 indicates the mean mass and characteristic mass associated with the highest peak of the mass distribution given in Fig. 3, as a function of host stars’ mass $M_*$. (a) close-in planets with $a_{\text{fin}} < 0.05$ AU; (b) planets at $0.1$ AU $< a_{\text{fin}} < 1$ AU. The mean mass and the peak mass are plotted with filled circles and open squares, respectively. For (b), only planets with $M_p$ over a deficit ($M_p > 50 M_\oplus$) are considered (see Figs. 2a and 3b).

The planet’s growth is quenched at $\sim 14 M_\oplus$ as a clean gap is formed because $M_{p, \text{trunc}} \simeq 14 M_\oplus$ at 0.04 AU. Figures 2 and 3 show that many planets evolve in a similar way to acquire $M_{p, \text{fin}} \simeq 14 M_\oplus$. The composition of these planets is similar to that of Uranus and Neptune, and their day-side surface temperature may be $\sim 500$ K.

In the lowest mass model 0.2, the upper limit of $M_{c, \text{iso}}$ does not exceed $M_{p, \text{mig}}$ even outside the ice boundary. The migration of Neptune-mass planets requires a delicate balance between $M_{c, \text{iso}}$ and $M_{p, \text{mig}}$. Thus, the frequency of close-in Neptune-mass planets in model 0.4 is larger than that in model 0.2.

This frequency in model 0.4 is also larger than that in model 1.0. Although the upper limit of $M_{c, \text{iso}}$ is larger than $M_{p, \text{mig}}$ outside the ice boundary around solar-type stars, $M_p \gtrsim 30-40 M_\oplus$ at the onset of migration, so gas accretion is much more efficient onto such large cores (eq. [11]). Thus, most isolated planets tend to arrive at the proximity of a solar-type star as gas giants, and only a small fraction arrive as silicate and iron cores with limited gaseous envelopes.

Recently, Neptune-mass planets have been detected around 55 Cnc and HD 160691 (McArthur et al. 2004; Santos et al. 2004). Since these stars are G-type stars, if the result of Figure 3 is applied, the probability of formation of such planets is very
Fig. 6.—Example of the formation of a close-in Neptune-mass planet around an M star. A planet is initially at 0.74 AU in a disk with $f_{dep} = 7.2$ and $\tau_{dep} = 9.2$ Myr around a star with $M_* = 0.4 M_\odot$. The upper and middle panels show time evolution of the mass and semimajor axis of the planet. In the upper panel, the isolation mass of a core ($M_{c,iso}$), the core’s critical mass for migration ($M_{c,mig}$), and the truncation mass of gas accretion ($M_{g,trunc}$) are plotted for comparison. In the lower panel, the core accretion timescale ($\tau_{grow} = M_c/M_\dot{c}$), gas accretion timescale ($\tau_{KH}$), and migration timescale ($\tau_{mig}$) are plotted.

low, although it is not zero. Unlike GJ 436, there are three additional Jupiter-mass planets around 55 Cnc and two additional ones around HD 160691. In the system of 55 Cnc, sweeping mean motion resonance associated with the migration of the giant planet (Malhotra 1993; Ida et al. 2000) presently at 0.1 AU could bring rocky embryos/planetesimals to the vicinity of the host star so that a Neptune-mass rocky planet could accrete in situ. On the other hand, in the system HD 160691, sweeping secular resonance associated with disk gas depletion (Ward 1981; Nagasawa et al. 2000) could bring rocky embryos/planetesimals to inner regions (Nagasawa et al. 2005). In this paper, we do not include such interactions. We will present details elsewhere. In the system of GJ 436, however, no additional Jupiter-mass planet has been found. Our model accounts for the formation of the isolated Neptune-mass planet around an M star.

4.2. Planets around K Dwarfs

Figure 3b shows that in model 0.6 where $M_* = 0.6 M_\odot$, the intermediate-mass (20–100 $M_\oplus$) and intermediate-$a$ (0.1–1 AU) planets are more abundant than in either model 1.0 or model 0.4. The mass function of close-in planets also appears to be smoother with some $M_p$ in the range of $\sim 10–100 M_\oplus$ (Fig. 3a). These intermediate-mass planets formed just outside the ice boundary in disks with modest $f_{dep}$ where the upper limit of $M_{c,iso} \sim 50–100 M_\oplus$, while that for typical disks (with $f_{dep} \sim 1$) is $\sim M_\oplus$. The critical mass for starting planetary migration at the ice boundary is 20 $M_\oplus$ in model 0.6. A small fraction of emerged cores may accrete a modest amount of gas as they start to migrate.

The formation of the intermediate-mass and intermediate-$a$ planets, which tends to smooth the mass distribution, may be one of the characteristics of planets around K stars that distinguishes them from those around G and F stars; although it is much less pronounced than in the characteristics of planets around M stars. For illustration, we present, in Figure 7, the formation of a typical planet that formed with an intermediate mass and attained an intermediate semi-major axis during its migration. In this case, a seed embryo is formed at $a_{ini} = 1.8$ AU in a slightly massive $h_{df} = 2$ disk with $\tau_{dep} = 2.3$ Myr. Through planetesimal coagulation, this embryo attains a mass $M_p \sim 8 M_\oplus$ and becomes isolated in 1.5 Myr. The cessation of the planetesimal bombardment enables the embryo to grow through gas accretion. When its mass reaches $M_p = 30 M_\oplus$ at $\sim 8$ Myr, gas accretion is quenched by the severe depletion of gas near its orbit. The newly formed planet undergoes migration while gas is globally depleted. The orbital migration is eventually halted at an intermediate location, 0.4 AU.

The condition for growth to be quenched between 10 and 100 $M_\oplus$ is $a \sim 1$ AU. The above example shows that to halt migration at an intermediate location, both timescales of migration and growth due to gas accretion are required to be comparable to the gas depletion timescale. In general, such special circumstances have a small probability of being satisfied. However, they are more likely around K dwarfs than other types of stars because cores with $M_c \gtrsim 10 M_\oplus$ are more abundant at $a \sim 1$ AU around K dwarfs as a result of the $M_*$ dependence of $a_{ice}$ and
haboring giant planets with periods smaller than several years that are currently detectable by Doppler survey is rather smaller than that of G stars (Fig. 8), although the difference is within a factor of 1.5. The similar fraction within a factor of 2 among K, G, and F stars shown in Figure 8 is consistent with the observation (Fischer & Valenti 2005). The predicted mass distributions for M, K, G, and F stars are not inconsistent with the observed distributions. Since the numbers of observed planets are not enough for statistical discussion, in particular for stars other than G stars (Fig. 3), we cannot discuss the agreement between the predicted and observed distributions in more detail.

Strong winds and jets from further higher mass stars (A, B stars) may decrease \( v_{\text{dep}} \), which reduces the formation rate of gas giants. It is not clear how much the fraction of massive stars with gas giant planets is reduced. We will address planetary formation around massive stars elsewhere.

5. Dependence on the Migration Condition

The results of the standard models clearly indicate that the mass function of planets depends on the delicate balance between growth and migration timescales. There are some uncertainties concerning the migration process. In the standard series, we set \( A_0 = 10 \) (eq. [22]) in accordance with the results of previous numerical simulations (Lin & Papaloizou 1985). However, the additional contribution from a torque imbalance between the Lindblad resonances of low-mass embedded cores may also lead to type I migration while their masses are relatively small (Goldreich & Tremaine 1980; Ward 1986, 1997a), although turbulence in the disk (Nelson & Papaloizou 2004; Laughlin et al. 2004b) and self-induced secondary instability (Balmforth & Korycansky 2001; Koller et al. 2003) can also retard the rate of migration. In our simulations, we can partly take into account the effect of type I migration by lowering the value of \( A_0 \) (to unity). We also carried out simulations with \( A_0 = 100 \) such that the onset of migration is delayed until the cores have attained relatively large masses.

The sensitive dependence of the mass function of close-in planets on the migration condition makes it an ideal observable feature that can be used to calibrate the criteria and efficiency of migration. In a variation of the standard models, we consider two new series: (1) series B (model xB) with \( A_0 = 1 \) and (2) series C (model xC) with \( A_0 = 100 \), which have \( M_{\star} \) identical to that of the standard series with \( A_0 = 10 \). In Figures 9a and 9b, we show the final mass and semimajor axis distribution of planets in models xB and xC.

For models xB, cores undergo migration before they attain sufficient mass to engage in efficient gas accretion. Although the cores’ migration may terminate close to their host stars, the relatively small aspect ratio of the disk for small \( a \) implies low \( M_{\text{p, trunc}} \). At small \( a \), gap formation prevents the cores from accreting gas. The accumulation of cores and planetesimals in the proximity of their host star may promote their coagulation (Ward 1997b). Although we cannot rule out the possibility of a highly efficient migration on the basis of the mass function of the close-in planets, it does pose difficulties in accounting for the modest frequency of gas giants with periods longer than a few weeks. Even around solar-type stars, cores rapidly migrate to the stellar proximity before they have acquired sufficient mass to efficiently accrete gas, so the probability of gas giant formation is strongly suppressed.

For the low-stellar-mass models 0.2B and 0.4B, the isolation mass is only a few times larger than that of the Earth. Nevertheless, it is larger than \( M_{\text{p, mig}} \) for this low-\( A_0 \) case. Most cores migrate toward their host stars with \( M_{\star} \) less than a few \( M_{\odot} \). Although these masses are \(< M_{\text{p, trunc}} \), gas accretion is too slow.

\( M_{\text{c, iso}} \). In model 0.6, gas accretion may be quenched when the cores attain a mass \( 10 M_{\oplus} \leq M_{\text{c, iso}} \leq 100 M_{\oplus} \), while they migrate to \( 0.04 \text{ AU} \leq a \leq 1 \text{ AU} \). In comparison with model 1.0, the isolation mass \( M_{\text{c, iso}} \) at the ice boundary increases with \( M_{\star} \). Around a solar-type star, the isolation mass near the ice boundary is sufficiently large for efficient gas accretion to be initiated. Eventually runaway gas accretion leads to the emergence of the intermediate-mass deficit in the mass distribution of planets around relatively high mass stars. Figure 1 shows that some fraction of planets discovered around \( K \) stars may have the intermediate mass. However, the number of detected planets may be insufficient for statistical discussion. This stellar mass dependence in the extrapolated planetary characteristics can be tested with future observation.

4.3. Planets around Higher Mass Stars

In model 1.5 where \( M_{\star} = 1.5 M_{\odot} \), the range of \( a \) where gas giants are formed is more extended (Fig. 2, bottom). This arises primarily because the disks around more massive stars have relatively large \( \Sigma_a \). In this case, \( M_{\text{c, acc}} \) (the core mass required for rapid gas accretion) can be attained before gas depletion even at large \( a \). However, larger \( a_{\text{acc}} \) in this case leads to slow core growth beyond the ice boundary. Thus, most gas giants have cores composed of silicates and iron but not icy cores, in contrast with gas giants around lower mass stars. The very large \( a_{\text{acc}} \) also leads to less efficiency of formation of gas giants in the range of a few AU to 10 AU than that around \( G \) stars, although gas giants form in a broader range of \( a \). As a result, the fraction of \( F \) stars
(eq. [11]) for them to acquire any significant amount of mass prior to gas depletion. The detection of close-in Neptune-mass planets around M dwarf stars and the modest detection frequency of gas giants around solar-type stars are inconsistent with the results of series B in Figure 9a, and therefore, we suggest that $A_\nu$ is substantially larger than unity.

An upper limit on the magnitude of $A_\nu$ may be inferred from models xC. With $A_\nu = 100$, the condition for the onset of migration becomes much more stringent and most cores do not have sufficient mass to undergo migration. For solar-type stars, a few relatively massive cores can form rapidly in disks with very large $f_d$. These systems can migrate to form close-in Jupiter-mass gas giants. The mass distribution of close-in planets around solar-type stars (model 1.0C) is skewed to $10^3 M_\oplus$ with a lower cutoff below $\sim 0.5 M_\oplus$, which is inconsistent with the observed mass-period distribution of extrasolar planets (Fig. 1). For low-mass host stars (models 0.2C and 0.4C), many intermediate-mass planets can form during and after gas depletion. But they retain their initial semimajor axis. The mass distribution of the close-in planets peaks near the mass of Saturn and hardly any planets have masses comparable to that of Neptune. These simulation results are again inconsistent with the observed mass-period distribution of extrasolar planets. Furthermore, in series C, the deficit of planets with intermediate masses and periods is too
pronounced to be consistent with observations. Therefore, we infer $A_v \sim 10$.

6. DEPENDENCE ON THE DISK MASS

The expressions in equations (1) and (2) indicate that the growth rate and asymptotic mass of cores are an increasing function of $\Sigma_d$. In the standard series of models, we set $h_d = (M_* / M_\odot)^2$. With this prescription, $\Sigma_d$ of disks around low-mass stars is relatively small. Consequently, the emergence of gas giants occurs preferentially around massive stars.

The dependence of the disks’ $\Sigma_d$ on the $M_*$ of their host stars is poorly known. On the theoretical side, gravitational instability may limit the amount of mass that can be retained by the disks, especially those around low-mass stars. However, smaller $L_*$ values and lower intensity of ionizing photons may also reduce the influence of the magnetorotational instability (Gammie 1996) and the angular momentum transfer efficiency so that more mass may be stored in disks around low-mass stars. The best available observational data suggest $M \propto M_*^2$ (see § 2.2), but the dependence of $\Sigma_d$ on $M_*$ is poorly known. In view of these uncertainties, we introduce another two series of models.

The parameters of models xD and xE are identical to those of models x. The only difference is that we set $h_d = M_* / M_\odot$ in series D and it is set to be unity for all $M_*$ in series E. In
comparison with the standard models, disks around low-mass stars are less deficient in these new models while those around the solar type stars remain the same.

The predicted distributions for series D and E are shown in Figure 10. Since $M_{p \text{ mig}}$ and $M_{p \text{ trunc}}$ do not depend on $\Sigma_d$ or $\Sigma_g$, planets’ migration and gas accretion undergo the same paths as in standard models for planets of the same mass. As a result, the mass distribution of close-in planets is self-similar among models x, xD, and xE. Since massive disks exist around lower mass stars more frequently in series D (and even more frequently in series E), the amplitudes of mass distributions around lower mass stars are enhanced in these models. In series D and E, the inferred frequency of Jupiter-mass planets around M stars is comparable to those around G stars (Fig. 10). The sparse detection of close-in Jupiter-mass and Neptune-mass planets around M stars suggests that the disk masses are a rapidly increasing function of $M_*$ as we have assumed in the standard models with $h_d = (M_*/M_\odot)^2$.

7. SUMMARY AND DISCUSSION

The observational discovery of extrasolar planets is advancing rapidly. We now have sufficient data to carry out the statistical characterization of planetary properties and to place constraints not only on the dominant mode of planet formation but also the range of physical quantities that determine their growth and migration rates. In this paper, we focused our discussion on the mass function of close-in planets around stars with various masses, in particular masses lower than the solar mass, because its origin is determined by the delicate balance of various processes and they are the most conspicuous companions of nearby stars. Although many ($\approx 90\%$) planets that once migrated to the proximity of their host stars can be eliminated (Paper II), we can compare the mass distribution of close-in planets around stars with various masses. If the elimination factor is taken into account, rough comparison is also possible between close-in and more distant planets.

Our results are summarized as follows.

1. Dynamically isolated, close-in, Neptune-mass planets with silicate and iron cores can form in relatively massive disks around solar-type stars. But their frequency is expected to be an order of magnitude smaller than that of close-in Jupiter-mass planets.

2. Dynamically isolated, close-in, Neptune-mass ice giants can form in lower mass stars. Their frequency peaks around the M dwarfs. Since the luminosity of M dwarfs is weak, the ice boundary is located well inside 1 AU. These planets are formed at around 1 AU but outside the ice boundary, and their cores are primarily composed of volatile ices. Planets that migrated to the stellar proximity with masses in the range of 5–15 $M_\oplus$ may acquire, in situ, a limited amount of additional gas, but gas accretion is immediately quenched by gap formation because of the small aspect ratio of the disk in the proximity of the host star. Because these planets are composed mostly of icy material and M stars’ luminosity is relatively weak, they may have a water vapor atmosphere and water ocean (Léger et al. 2004).

3. Embryos with mass lower than 10 $M_\oplus$ cannot migrate to the proximity of their F, G, and K dwarf host stars through type II migration. The detection of dynamically isolated Earth-mass close-in planets may be attributed to type I migration of low-mass embryos (Ward 1997a), sweeping secular resonances (Nagasawa et al. 2005), or sweeping mean motion resonances. Around late M dwarfs, however, dynamically isolated planets of a few Earth masses can form with temperatures comparable to that of the Earth.

4. Around M dwarfs, the formation probability of gas giants is much reduced. The relatively low $\Sigma_d$ prevents the emergence of sufficiently massive cores prior to the severe depletion of gas in their nascent disks, which is consistent with the results by Laughlin et al. (2004a).

5. The mass function of close-in planets generally has two peaks at about Neptune mass and at Jupiter mass. The lower mass peak takes the maximum frequency for M stars, while the higher mass peak is much more pronounced around higher mass stars (F, G, K dwarfs). Planets tend to undergo type II migration after fully accreting gas in the systems around the higher mass, while planets around M dwarfs tend to migrate before starting rapid gas accretion. Unless the termination location of planetary migration is a decreasing function of $M_*$, close-in Neptune-mass planets around M dwarfs are easier to detect than those around G dwarfs.

6. The mass function of dynamically isolated close-in planets around stars with various masses can also be used to calibrate the sufficient condition for the onset of planetary migration and for the termination of gas accretion due to planet-disk tidal interaction.

The metallicity dependence on the frequency of extrasolar gas giant planets may not be easily accounted for by the gravitational instability scenario (e.g., Boss 2001), while it is naturally accounted for by the sequential core accretion scenario that our work is based on (Paper II). The condition of the gravitational instability is $1 > Q = \frac{c_s \Omega_k}{\pi G \Sigma_g} \propto M_\star^{1/2} M_{d}^{1/2} / \Sigma_g = M_\star^{1-\beta}$, and its radial wavelength $\lambda = 2 \pi \sqrt{G \Sigma_g / \Omega_k^2} \propto M_\star^{1-\beta}$ (Toomre 1964), where $\Sigma_g \propto M_\star^{1/2}$. Hence, the instability may be more limited and result in smaller clumps around lower mass stars, which could account for the above features 4 and 5 if $\beta = 2$ is assumed as in the standard series in this paper. However, if $\beta = 1$, the gravitational instability scenario cannot account for features 4 and 5, while the sequential core accretion scenario still shows the tendency for the features. More detailed study of the dependence of disk mass on stellar mass $M_\star$ is needed.

Our work is primarily motivated by the discovery of GJ 436b (Butler et al. 2004). Our theoretical extrapolations can be tested with the following statistical properties of close-in planets to be discovered by various techniques.

1. As mentioned above, the mass function of dynamically isolated close-in planets as a function of the spectral classes of their host stars is particularly useful in the determination of the growth, migration, and disk depletion timescales.

2. A comparison between the frequencies of gas giants with close-in orbits and those with extended orbits can provide constraints on the migration condition, survival criteria, and disk mass as functions of the host stars’ mass.

3. In systems with multiple giant planets, the dynamical architecture may provide clues on whether the migration of the close-in planets is driven by planet-disk tidal interaction or sweeping secular resonance.

4. A comparison of atmospheric properties of close-in Neptune-mass planets around G dwarfs to those around M dwarfs can verify the conjecture that the former have silicate and iron cores whereas the latter have ice cores.

The following observations of protostellar disks may provide useful input to the model:

1. A spatially resolved image of disks can directly provide information about $\Sigma_d$ and the temperature distribution around any given host star.

2. The dependence of $\Sigma_d$ on the mass of the host stars determines the functional form of $h_d$. 
3. A relation between the disk mass and accretion rate onto the host stars places a constraint on the rate of type II migration.
4. A direct measurement of the gas distribution is particularly important in determining $q_{dep}$ and $h_{g}$.

On the modeling side, we need to consider
1. The possibility of radiative feedback on the termination of gas accretion.
2. The stoppage of type II migration and the survival of short-period planets.
3. The rate (and direction) of type I migration of cores.
4. The enhanced probability of multiple planet formation.
5. The effect of dynamical interaction between multiple planets during and after gas depletion.

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REFERENCES

Aarseth, S. J., Lin, D. N. C., & Palmer, P. L. 1993, ApJ, 403, 351
Balbus, S. O., & Hawley, J. F. 1998, ApJ, 499, 389
Beckwith, S. V. W., & Sargent, A. I. 1996, Nature, 383, 139
Bodenheimer, P., Hubickyj, O., & Lissauer, J. J. 2000, Icarus, 143, 2
Bodenheimer, P., & Pollack, J. B. 1986, Icarus, 67, 391
Boss, A. P. 2001, ApJ, 551, L167
Boss, A. P., Wetherill, G. W., & Haghighipour, N. 2002, Icarus, 156, 291
Bryden, G., Xingming, C., Lin, D. N. C., Nelson, R., & Papaloizou, J. C. B. 1999, ApJ, 514, 344
Butler, P., Vogt, S., Marcy, G., Fischer, D., Wright, J., Henry, G., Laughlin, G., & Lissauer, J. 2004, ApJ, 617, 580
D’Alessio, P., Calvet, N., & Hartmann, L. 2001, ApJ, 553, 321
Fischer, D., & Valenti, J. 2005, ApJ, 622, 1102
Gammie, C. F. 1996, ApJ, 457, 355
Goldreich, P., & Tremaine, S. 1980, ApJ, 241, 425
Greenberg, R., Hartmann, W. K., Champman, C. R., & Wacker, J. F. 1978, Icarus, 35, 1
Gu, P., Lin, D. N. C., & Bodenheimer, P. H. 2003, ApJ, 588, 509
Haisch, K. E., Lada, E. A., & Lada, C. J. 2001, ApJ, 553, L153
Hayashi, C. 1981, Prog. Theor. Phys. Suppl., 70, 35
Hayashi, C., Nakazawa, K., & Nakagawa, Y. 1985, in Protostars and Planets II, ed. D. C. Black & M. S. Mathew (Tucson: Univ. Arizona Press), 1100
Ida, S., Bryden, G., Lin, D. N. C., & Tanaka, H. 2000, ApJ, 534, 428
Ida, S., & Lin, D. N. C. 2004a, ApJ, 604, 388 (Paper I)
Ida, S., & Makino, J. 1993, Icarus, 106, 210
Ikoma, M., Nakazawa, K., & Emori, E. 2000, ApJ, 537, 1013
Ivanov, P. B., Papaloizou, J. C. B., & Polnarev, A. G. 1999, MNRAS, 307, 79
Iwasaki, K., Emori, H., Nakazawa, K., & Tanaka, H. 2002, PASJ, 54, 471
Kokubo, E., & Ida, S. 1996, Icarus, 123, 180
———. 1998, Icarus, 131, 171
———. 2000, Icarus, 143, 15
———. 2002, ApJ, 581, 666
Koller, J., Li, H., & Lin, D. N. C. 2003, ApJ, 596, L91
Kominami, J., & Ida, S. 2002, Icarus, 157, 43
Laughlin, G., Bodenheimer, P., & Adams, F. C. 2004a, ApJ, 612, L73
Laughlin, G., Steinacker, A., & Adams, F. C. 2004b, ApJ, 608, 489
Léger, A., et al. 2004, Icarus, 169, 499
Lin, D. N. C., Bodenheimer, P., & Richardson, D. 1996, Nature, 380, 606
Lin, D. N. C., & Papaloizou, J. C. B. 1980, MNRAS, 191, 37
———. 1985, in Protostars and Planets II, ed. D. C. Black & M. S. Mathew (Tucson: Univ. Arizona Press), 981
Lin, D. N. C., & Papaloizou, J. C. B. 1993, in Protostars and Planets III, ed. E. H. Levy & J. I. Lunine (Tucson: Univ. Arizona Press), 749
Lissauer, J. J. 1987, Icarus, 69, 249
Lubow, S. H., Seibert, M., & Artymowicz, P. 1999, ApJ, 526, 1001
Malhotra, R. 1993, Nature, 365, 819
Mayor, M., & Queloz, D. 1995, Nature, 378, 355
McArthur, B., et al. 2004, ApJ, 614, L81
Mizuno, H. 1980, Prog. Theor. Phys. Suppl., 64, 544
Mohanty, S., Jayawardhana, R., & Barrado y Navascués, D. 2003, ApJ, 593, L109
Muzerolle, J., Hillenbrand, L., Calvet, N., Briceno, C., & Hartmann, L. 2003, ApJ, 592, 266
Nagasawa, M., Lin, D. N. C., & Thommes, E. W. 2005, ApJ, submitted
Nakano, T. 1988, MNRAS, 230, 551
Narayan, R., Cumming, A., & Lin, D. N. C. 2005, ApJ, 620, 1002
Natta, A., Testi, L., Muzerolle, J., Randich, S., Comeron, F., & Persi, P. 2004, A&A, 424, 603
Nelson, R. P., & Papaloizou, J. C. B. 2004, MNRAS, 350, 849
Nelson, R. P., Papaloizou, J. C. B., Masset, F., & Kley, W. 2000, MNRAS, 318, 18
Pollack, J. B., Holtenbach, D., Beckwith, S., Simonelli, D. P., Roush, T., & Fong, W. 1994, ApJ, 421, 615
Pollack, J. B., Hubickyj, O., Bodenheimer, P., Lissauer, J. J., Podolak, M., & Greenzweig, Y. 1996, Icarus, 124, 62
Rafikov, R. R. 2003, AJ, 125, 922
Sano, T., Miyama, S. M., Umebayashi, T., & Nakano, T. 2000, ApJ, 543, 486
Santos, N. C., et al. 2004, A&A, 426, L19
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 373
Stevenson, D. J. 1982, Planet. Space Sci., 30, 755
Tanaka, H., Himeno, Y., & Ida, S. 2005, ApJ, in press
Tanaka, H., & Ida, S. 1997, Icarus, 125, 302
Thi, W. F., et al. 2001, Nature, 409, 60
Toomre, A. 1964, ApJ, 139, 1217
Ward, W. 1981, Icarus, 47, 234
———. 1986, Icarus, 67, 164
———. 1997a, Icarus, 126, 261
———. 1997b, ApJ, 482, L211
Wetherill, G. W., & Stewart, G. R. 1989, Icarus, 77, 330
Wuchterl, G., Guillot, T., & Lissauer, J. J. 2000, in Protostars and Planets II, ed. V. Mannings, A. P. Boss, & S. S. Russell (Tucson: Univ. Arizona Press), 1081
Wyatt, M. C., Dent, W. R. F., & Greaves, J. S. 2003, MNRAS, 342, 876

6. The radial distributions of $\Sigma_d$ and $\Sigma_g$; the effects of a power-law index of $a$-dependence different from the $-1.5$ that we used; a more realistic time evolution of $\Sigma_g$.
7. The influence of a stellar companion on the emergence and survival of planets.

Some of these issues will be addressed in future discussions.