Analysis of electron beam dynamics in a rectangular dielectric waveguide

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Abstract. Wakefield acceleration of electron bunches is effective method developed for linear accelerator techniques. The main problem of this method is transverse beam dynamics which limits the propagation length of bunch. This work is dedicated to beam dynamics in dielectric waveguide with rectangular geometry. This type of waveguide has complex mode composition including LSM and LSE modes. The main purpose is a study of modes influence in rectangular dielectric waveguide on beam dynamics with different types of dismission of driver from waveguide’s axis. It is shown that the LSM asymmetric modes make a major contribution to wakefield and transverse dynamics.

1. Introduction

The wakefield scheme is based on the acceleration of charged particles in the field of a traveling wave (Vavilov-Cherenkov radiation) created by a bunch (driver) with large charge and low energy. Dielectric waveguides with cylindrical geometry are widely used as an accelerator structure [1]. Rectangular dielectric waveguides have become widespread [1,2] due to the simplicity of manufacturing technology. The transverse and longitudinal sections of a rectangular waveguide are shown in figure 1. The driver 4 moves in the vacuum cavity 1 of the waveguide along the z axis parallel to the dielectric plates 2. A dielectric waveguide covered with a metal shell 3 has vertical walls along the x axis without dielectric layers. The Vavilov-Cherenkov radiation generated by the driver has strong component $E_z$ using as accelerating field and transverse components of electromagnetic field which define deflecting force. In addition to the above, there is a possibility of changing the distance between dielectric plates in dechirpers [3] for tuning of wakefield spectrum. Also, rectangular waveguide is adapted to pass wide bunches with high charge through vacuum gap 1 [1].

The beam dynamics of bunches is calculated for determination of propagation distance of the driver before it touches the dielectric wall 2. This value allows to calculate the amount of energy that an accelerated bunch (witness) can receive. Numerical simulation of the beam dynamics is one of the problems for rectangular waveguide due to complex mode composition in compare with cylindrical one: symmetrical and asymmetrical LSM and LSE modes. In this article, we consider the contribution

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of all above modes to the dynamics of the bunch and clarify cases when only part of the modes can be considered.

![Figure 1](image.png)

**Figure 1.** Longitudinal (left) and transverse (right) views of a rectangular waveguide.

2. Methods
The calculations presented in this paper are based on the previously obtained analytical expression of the Green function for a rectangular waveguide [2]. The wakefield is calculated using an integral convolution of charge distribution function with a Green function which includes LSM (Longitudinal Section Magnetic, \(H_z=0\)) and LSE (Longitudinal Section Electric, \(E_y=0\)) modes. The contribution of these modes depends on the transverse coordinate of the bunch. Each of the mentioned types of modes is also classified into symmetric and asymmetrical with respect to the plane relative to the coordinate \(y = 0\).

The dynamics of a bunch of charged particles is calculated by the method of macroparticles [4]. This method is based on the representation of a bunch with a charge \(Q\) in the form of \(N\) particulates with a charge \(Q/N\). Such a partition allows us to carry out numerical simulation of the dynamics of a real bunch with savings in computational resources. The number of macroparticles is defined by geometrical dimensions of the bunch, its charge and transverse coordinates. It can take values from 100 to 5000. For optimization of the calculating time the dynamics, the number of LSE and LSM modes is selected based on the longitudinal length of the bunch.

The longitudinal wake field \(E_z\) has a braking effect on the central part of the bunch, while the deflecting forces \(F_x\) and \(F_y\) act on the tail part of the bunch. The sign of the deflecting force depends on the direction and magnitude of the displacement of the center of the bunch relative to the axis of the waveguide.

3. Results and Discussion
Table 1 presents the parameters of the waveguide and the leading electron bunch corresponding to the accelerator facility of the Argonne National Laboratory [5].

**Table 1.** Parameters of the waveguide (see figure 1) and an electron bunch for numerical simulation.

| Parameters of the waveguide | Parameter value | Parameters of the bunch | Parameter value |
|-----------------------------|-----------------|-------------------------|-----------------|
| a                           | 0.5 cm          | Charge                  | 100 nC          |
| b                           | 0.7 cm          | Longitudinal length     | 0.1 cm          |
| w                           | 1 cm            | Y length                | 0.035 cm        |
| Permittivity of plates      | 3.75            | X length                | 0.035 cm        |
| LSM\(_{11}\) asymmetric mode frequency | 6.93 GHz       | Energy                  | 50 MeV          |
The original Dielectric Waveguide program [6] was used to analyze the effect of individual types of modes on the transverse beam dynamics. This code allows to calculate Green function and beam dynamics. Table 2 present different initial transverse positions of bunch which can be dismissed in X and Y directions. Transverse force $F_r$ is acting on bunch in case of transverse dismission (offset). So, the position 4 coincides with waveguide center and allows bunch to pass for long distances. Figures 2 and 3 present the results of modeling the dynamics of bunch for position 1 (offset only in X direction). Because of symmetry of bunch relative to Y coordinate only two types of modes can be excited: LSM asymmetrical modes and LSE symmetrical modes. Both modes act on bunch as focusing force namely to center of waveguide. From figures 2 and 3 it can be seen that influence LSM asymmetrical modes is higher that LSE ones.

### Table 2. Variants of transverse positions of bunch in waveguide (figure 1) for beam dynamics simulation

| Positions  | X       | Y       |
|-----------|---------|---------|
| Position 1| 0.25 cm | 0 cm    |
| Position 2| 0.25 cm | 0.25 cm |
| Position 3| 0.5 cm  | 0.25 cm |
| Position 4| 0.5 cm  | 0 cm    |

**Figure 2.** The cross section of the bunch after beam dynamics simulation for position 1. Only LSM asymmetrical modes are considering. Solid circle shows initial position of bunch, blue point is center of waveguide (position 4).

**Figure 3.** The cross section of the bunch after beam dynamics simulation for position 1. Only LSE symmetrical modes are considering. Solid circle shows initial position of bunch, blue point is center of waveguide (position 4).
Figure 4. The longitudinal view (end of simulation) of the bunch passing from left to right for initial transverse position 3. Only LSE symmetrical modes are considering. Solid line shows initial position of bunch.

Figure 5. The longitudinal view (end of simulation) of the bunch passing from left to right for initial transverse position 3. Only LSM symmetrical modes are considering. Solid line shows initial position of bunch.

Positions 2 and 3 mean offsets in Y direction. Therefore, it is necessary to consider all types of LSE and LSM modes. The calculations show that LSM modes act on bunch stronger than LSE ones. In case of LSM mode macroparticles touch dielectric plate after 11 cm (figure 5) of passing in compare with 14 cm of LSE modes (figure 4). Also need to add that difference between position 2 and 3 is increase of contribution LSE modes in case of offset growth in coordinate X. In addition to the above that beam dynamics calculation with contribution of all modes shows that the tail of bunch touch dielectric on after passing of 6.3 cm.

4. Conclusion
In conclusion, it should be noted that the advantage of the rectangular wake structure consists not only in the simplicity of its manufacturing technology, but also in the ability to pass bunches of large charges by increasing their length along the X axis. The disadvantages of a rectangular structure compared to a cylindrical structure include the absence of symmetry about the central axis. This circumstance complicates the analytical expression and increases the time for calculating the dynamics due to the large number of modes that must be considered.

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