Multiple tools for visualizing equipotential surfaces: Optimizing for instructional goals

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Curriculum developers are interested in how to leverage various instructional tools - like whiteboards, Mathematica notebooks, and tangible models - to maximize learning. Instructional tools mediate student learning and different tools support learning differently. We are interested in understanding how the features of instructional tools influence student engagement during classroom activities and how to design activities to match tools with instructional goals. In this paper, we explore these questions by examining an instructional activity designed to help advanced undergraduate physics students understand and visualize the electrostatic potential. During the activity, students use three different tools: a whiteboard, a pre-programmed Mathematica notebook, and a 3D surface model of the electric potential. We discuss how the tools may be used to address the instructional goals of the activity. We illustrate this discussion with examples from classroom video.

I. RESEARCH QUESTIONS & METHODS

Different features of instructional tools support learning in different ways and to various degrees [1]. For example, current online homework systems are able to provide immediate feedback to students on their answers and offer immediate, individualized tutoring resources. However, they do not provide feedback on the details of student solutions and have difficulty handling the many forms student input, such as diagrams and paragraphs. Online homework systems support instructional goals related to students being able to solve short calculations and answer conceptual questions, and are often used in large introductory physics courses. In contrast, with paper-based homework, students may provide detailed solutions that include algebra, diagrams, and text. Instructors can provide detailed feedback about the solution process, but this feedback is not immediate. Paper-based homework is used in more advanced physics courses where physics problems are longer and more complex and class sizes are smaller. In designing instruction, instructional tools should be selected carefully and used strategically to achieve the desired learning goals [2, 3].

In classroom activities, instructional tools might include whiteboards for students to draw on, laptop computers with relevant software, or other models/equipment. Information is represented with these tools to aid learning. For example, equations or graphs might be drawn on whiteboards. Equations might be typed into a Mathematica notebook and solved or plotted. Manipulatives might be observed from many angles and labeled. In order to understand how an instructional tool supports learning, it is important to consider: how information is encoded with/in the tool; how learners access, transform, and share information with the tool; and what cognitive processing is needed when using the tool [2].

Classroom activities may include multiple tools or representations. Being able to understand and use multiple representations is an important learning goal in STEM [4]. Additionally, multiple representations can support learning when representations complement each other, when they constrain a learner’s interpretation of other representations, or when learners integrate information from multiple different representations [2].

The discussion in this paper focuses on the ways in which tools used in classroom activities support student learning. Specifically, we discuss the tools used in an activity about superposition of electrostatic potential. We consider the form of the tools, what information is represented with the tools, what transformations/manipulations students can do with the tools, and how many students can participate in using each tool. This study is part of a larger project to design activities for advanced physics courses that leverage physical models and other representations effectively to support student learning and engagement.

Our analysis is informed by observations of students doing the activity in class. Author EG used the activity in an upper-division E&M course with 13 groups of 3 students. Video recordings were made of 4 of these groups. After class, she wrote a reflection of her classroom observations and shared it with the co-authors. She also had a half-hour long debriefing session with the two teaching assistants who assisted in class that day. Author RW also used the activity in an upper-division E&M course with 8 students. She also wrote a reflection that was shared with each of the co-authors, but no video recordings were made of her class.

Each author viewed the classroom videos at least once and made notes. One of the videos did not record audio and was excluded from the data set. Transcripts were made of the other videos and we identified episodes that illustrated how students thought about the tools and how they used the tools during the activity.

II. THE INSTRUCTIONAL ACTIVITY: DRAWING EQUIPOTENTIAL SURFACES

A. Instructional goals

The primary goals of the instructional activity are for students to be able to:

Goal 1 Superposition: find an electric potential field by adding the potential from each discrete charge.
Goal 2 **3D Function**: explain that electrostatic potential is a function of three spatial variables and equipotential surfaces are surfaces in 3D space.

Goal 3 **Contour Plot**: create a contour plot of potential due to multiple discrete sources; explain where contour lines should be more or less dense; explain shape of contours very close/far from point charges.

Goal 4 **Graphical Representations of Potential**: interpret and compare various representations of a 3D scalar field and 2D slices of the field.

Goal 5 **Inquiry**: investigate physical phenomena.

### B. Structure of the instructional activity

During the activity, groups of 2-3 students were given a large (2’x3’) whiteboard with 4 dots arranged in a square. The students were told these dots represented 4 positive charges and were asked to draw equipotential surfaces of the configuration. After they produced a drawing on the whiteboard, the instructor led a whole class discussion about strategies for drawing the surfaces.

Students were then provided with a Mathematica notebook pre-programmed with five different ways to visualize equipotential surfaces for five different distributions of point charges, including four positive charges arranged in a square. The ways of visualizing the potential (Fig. 1) in Mathematica include:

1. a 3D set of cross-sections parallel to the plane of charges, using color to represent the potential;
2. a 2D contour plot of one cross-section;
3. a 3D plot of the potential in the plane of the charges (or a parallel plane) using the 3rd axis to represent the value of the potential;
4. a movie showing the potential in planes parallel to the plane of the charges, using color to represent the value of the potential; and
5. a 3D contour plot of the potential function.

While demonstrating the Mathematica notebook for the students, the instructor led a discussion about the connection between the ways of visualizing potential in Mathematica.

The students were then asked to consider a quadrupole (two positive and two negative charges, with like charges on opposite corners of the square) and to draw equipotential surfaces on their whiteboards. During the discussion, students were given surface models representing the potential in the plane of the charges. The surfaces use height to represent the value of the potential, similar to the surface plot produced by Mathematica (Fig. 1, top right). The models have a base of 6.5”x6.5”, a height up to 5.5” and are transparent and dry-erasable (Fig. 2, left) [5]. The instructional activity ended with a whole class discussion, incorporating student ideas and leading to statements of the main instructional goals.

III. ANALYSIS

### A. Making decisions vs. explanations

Tools mediate learning by changing the nature of the instructional task. When using the whiteboard, all information that was added to the tool was added by the students (except the four dots that were drawn by the instructor), and we observed students making many decisions about how to draw the contours. Students generally started by considering the equation of the electric potential due to a point charge and limiting cases: what the equipotential surfaces look like very far from the charge distribution (a circle with V=0) and very close to the point charge (circles with a large positive or negative value of the potential). Then students typically used the equation to identify other locations where the potential might be zero and to estimated the potential at special locations like the midpoints between charges.

In contrast, the Mathematica notebook and the surface model does the superposition for the students. We observed that students used these tools in a looking-up-the-answer mode. When the Mathematica plots were different from the students’ drawings or if the students did not feel confident in
guessing a shape for the drawing, we observed that students tried to use physical reasoning to explain the correct Mathematica plot or contours determined with the surface model.

For example, one group became stuck while discussing how to determine the space of the equipotential curves inside the square. One of the students suggests using Mathematica to find out.

Anna: Yeah, so is this distance supposed to be bigger, or is this distance supposed to be bigger? That’s what I’m wondering. (pause) Can we try it over there? (points to the laptop)
Charlie: Yeah, why try to visualize when the computer can do the work for us?
[Some discussion about manipulating the code to make the charge distribution a quadrupole]
Anna: Does that? That looks right.
Charlie: Oh, hey. That’s exactly what it is.
Bailey: [chuckles] That’s funny. All right. So let’s just think of this picture then. [Anna evaluates a new Mathematica cell] What?
Charlie: Yeah, I was right! On the asymptotes it’s zero because along those lines, there’s equal push and pull.
Anna: Right. And then, yeah, so it is actually spaced farther out that way and closer this way [points to the computer screen]. So it’s the opposite of what you drew. [starts altering whiteboard drawing]
Bailey: So let’s think about why.

In this episode, we see that these students developed questions about the spacing of the contours, used the Mathematica notebook to find out what the contours are supposed to look like, and then began a process of trying to explain why the contours looked the way that they did. We also see Charlie was excited that the Mathematica visualization supported his idea that the symmetry lines (what he called asymptotes) have zero potential, and he described a physical reason for this result, albeit an incorrect one. We also find it interesting and encouraging that, in this case, the tools helped the students overcome their stuckness without an intervention from an instructor.

Therefore, we see evidence that the use of the whiteboard supported Goals 1, 3 & 5 (operationalizing superposition, understanding contour maps, and inquiry) by providing opportunities for decision-making that led to generating questions. The Mathematica visualizations and the surface model provided additional support when the students became stuck with their whiteboard drawings. The information provided by these tools acted as a foothold for further physical reasoning.

B. 3D visualization of potential

Mathematica offered several ways of plotting functions that allowed students to visualize the 3D nature of the field, using either the ability to rotate 3D plots or using time to cycle through cross-sections (Fig. 1). One limitation of these representations was that, although they can be rotated on the screen, interpreting 3D representations on a 2D screen was sometimes difficult.

In contrast, the tangible surface model was easier for some students to interpret. However, the surface model only represents the potential in a single plane. Multiple surfaces, representing the potential in different planes, were made to be stacked on top of each other to capture the 3D nature of the field. Unfortunately, we find these surface stacks are also difficult to interpret, as height is used to represent which plane is being represented and height is used to represent the value of the potential.

The 2D nature of the whiteboard best supports 2D representations, like a contour plot of the potential in a plane. However, we saw that some students found surface plots more intuitive than contour plots. Before students were given the surface models, one student attempted a 3D perspective drawing of the surface. (Fig. 3).

Ethan: So, the only way I can actually, I can’t do this [the contour plot] visually. I go to this [surface perspective drawing].

FIG. 3. Ethan’s perspective drawing of the potential surface in the plane of the charges.

Therefore, we see that the Mathematica notebook best supported visualizing the potential as a 3D function (Goal 2).

C. Co-locating representations for making connections

The Mathematica representations included a digital version of the surface plot and the contour map (Fig 1, top right and center). However, the features of the surface model better supported students in understanding the connection between these two plots, particularly the spacing between contours. Equipotential contours could be drawn directly onto the surface model and the model placed over a contour map drawn on the whiteboard (Fig 2, right and center, and Fig. 4, bottom). The surface model is transparent and allowed students to perceive the connection between the spacing of the level
curves and the shape of the surface much more easily than viewing the surface plot and contour map separately. Therefore, the surface model provided more support than Mathematica in making a connection between the surface plot and the contour map (Goals 3 & 4).

D. Group manipulation of the tools

The different tools were differently accessible to the students within the groups. Each student had similar access to the whiteboard. The whiteboard was centrally located and each student had a pen and an eraser. Usually one student did most of the writing/drawing, but we observed occasions where multiple members of the group wrote simultaneously (Fig. 4).

Similarly, the surface model is large enough for all students to access and draw on it during discussions. It is lightweight and transparent and we observed students placing it centrally on top of the whiteboard.

Access to the Mathematica notebook was more limited. Typically, only one member of the group operated the laptop during the activity. Students tend to keep the laptop to one side of the whiteboard, not located centrally, and occasionally some students were not able to see the monitor easily (Fig. 4, top right). However, the students who could see the monitor sometimes made suggestions to the laptop operator and interpreted plots. Therefore, we find that the whiteboard and the surface supported interaction from all group members (Goal 5) more than the Mathematica notebook.

FIG. 4. Students using the instructional tools: three students writing on or pointing at whiteboard (top left), two students pointing to Mathematica notebook on a laptop screen (top right), and two student drawing on surface model while one student views contours from above (bottom).

IV. DISCUSSION

We have discussed how the different features of three instructional tools – a whiteboard, a Mathematica notebook, and a surface model – mediated student learning during an instructional activity. The whiteboard allowed for students to reason about a superposition of fields in order to make decisions about how to draw the contour lines. We discussed an example where this decision making process led to the students generating productive questions. In contrast, Mathematica and the surface perform the superposition for the student, and can provide support when students reason incorrectly or when they get stuck. Combinations of tools like this – one that leads students to ask questions and one that provides information that helps students address those questions – might positively impact a students’ confidence in doing inquiry and reduce the burden on the instructor to get around to every group, particularly in larger classes.

Tools that could be centrally located in the group better supported engagement for all students in the group. This is important for three reasons. First, when each student participates, more ideas may be brought to bear to the discussion and the discussion may be richer. Second, having equal access to tools might reduce the marginalization of students within groups. Third, if students are expected to learn certain skills or ways of reasoning during the activity, each student ought to have opportunity to practice those skills.

Mathematica has various options for visualizing the 3D nature of the field, but the surface model allows the surface plot and the contour maps to be co-located. This co-location highlights the relationship between these representations.

In this paper, we discuss the role of tools during one instructional activity in one classroom. Our analysis informs our understanding of this particular activity and how these tools might be used effectively in other instructional contexts. However, we are cautious in making generalized claims from these data alone. In future work, we would like to examine more closely how the features of the tools promote or suppress student inquiry and collaboration.

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