Minimal SO(10) GUT in 4D and its extension to 5D

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Abstract. The problems of renormalizable minimal SUSY SO(10) GUT in 4D are discussed. Its highly predictivity has been charged with many observations, which urges further progresses. We show why and how broad data fittings and conceptual problems drive us to 5D and how it improves the model.

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MINIMAL SUPERSYMMETRIC SO(10) GUT

SUSY GUT is the most promising candidate beyond the Standard Model (SM). The SM is a very powerful theory but it has the application limit like the other great theories. Among many SUSY GUT models, a renormalizable minimal SUSY SO(10) GUTs (minimal SO(10) GUTs) have been considered to be very promising because of their high predictivity. Minimal implies that $10$ and $16$ Higgs are incorporated into Yukawa coupling. This model was first applied to neutrino oscillation data by [1]. Since that time, we have developed the following critical points among the other groups.

• The phase factors were proved to be indispensable for the neutrino oscillation data [2],
• RGE effect was incorporated, which enables us to match up with the low energy data from GUT relations [3].
• The complete symmetry breaking pattern from GUT to the SM was shown [4] etc.

Yukawa coupling is given by

$$W_Y = Y_1^{ij} 16_i H_{10} 16_j + Y_2^{ij} 16_i H_{126} 16_j,$$

where $16_i$ is the matter multiplet of the $i$-th generation, $H_{10}$ and $H_{126}$ are the Higgs multiplet of $10$ and $126$ representations under SO(10), respectively. Providing the Higgs VEVs, $H_u = v \sin \beta$ and $H_d = v \cos \beta$ with $v = 174$GeV, the quark and lepton mass matrices can be read off as

$$M_u = c_{10} M_{10} + c_{126} M_{126}, \quad M_d = M_{10} + M_{126},$$
$$M_D = c_{10} M_{10} - 3 c_{126} M_{126}, \quad M_e = M_{10} - 3 M_{126},$$
$$M_L = c_L M_{126}, \quad M_R = c_R M_{126},$$

where $M_u$, $M_d$, $M_D$, $M_e$, $M_L$, and $M_R$ denote the up-type quark, down-type quark, Dirac neutrino, charged-lepton, left-handed Majorana, and right-handed Majorana neutrino mass matrices, respectively.

In [2] and [3], we set $c_L = 0$ and $c_R$ is real (type I seesaw). We do not discuss Type II seesaw dominant model simply because of lack of space.

Together with real $c_R$ which is used to determine the overall neutrino mass scale, this system fixes all mass matrices, very strong predictability to the fermion mass matrices. The reasonable results we found are listed in Table 1. Thus we can fix neutrino mixing angles, absolute neutrino masses, four CP phases (one in the CKM and three in the MNS matrices). Moreover it fixes Dirac $M_D$ and $M_R$. The former (latter) is crucial for lepton flavour violation (leptogenesis mainly via $M_R$ decay). In the basis where both of the charged-lepton and right-handed Majorana neutrino mass matrices

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1 This is a talk at GUT2012 held on March 15-17 at Kyoto.
LFV effect most directly emerges in the left-handed slepton mass matrix through the RGEs such as [5]

We are now reconsidering data fitting with the update experimental data and new RGE results. It gives little bit different values from (2) but the LFV results are not essentially changed.

After Kamland, the fitting is not good for $\text{sol}^{23}$, and we can calculate LFV and related phenomena unambiguously [6].

It is important that this data fitting was essentially good before Kamland data appeared [8] except for fast proton decay [14]. After Kamland, the fitting is not good for $\theta_{13}$ and $\Delta m^2_{32}$. However, this data fitting was performed to show how minimal SO(10) GUT is predictive, and we have not exhausted parameter searching.

On the other hand, it has been long expected to uncover the symmetry breaking pattern from GUT to the SM. The simplest Higgs superpotential at the renormalizable level is given by [9], [10], [11]

$$W = m_1\Phi^2 + m_2\overline{\Delta} \Delta + m_3 H^2 + \lambda_1 \Phi^3 + \lambda_2 \Phi \overline{\Delta} \Delta + \lambda_3 \Phi \Delta H + \lambda_4 \Phi \overline{\Delta} H,$$

where $\Phi = 210$, $\Delta = 126$, $\overline{\Delta} = 126$ and $H = 10$. The interactions of $210$, $126$, $126$ and $10$ lead to some complexities in decomposing the GUT representations to the MSSM and in getting the low energy mass spectra. Particularly, the CG coefficients corresponding to the decompositions of SO(10) $\rightarrow$ SU(3)$_C$ $\times$ SU(2)$_L$ $\times$ U(1)$_Y$ have to be found. This problem was first attacked by X. G. He and S. Meljanac [12] and further by J. Sato [13] and D. G. Lee [10]. But they did not present the explicit form of mass matrices for a variety of Higgs fields and also did not perform a formulation of the proton life time analysis. This is very labourious work and it is indispensable for the data fit of low energy physics. We completed that program in [4] (See also [15], [16], [17]). This construction is only possible for the minimal SO(10) GUT. So far many models have suggested the intermediate energy scales between GUT and the SM like seesaw scale and Peccei-Quinn symmetry breaking scale etc. The minimal SO(10) GUT explicitly gives these intermediate energy scales. However, these scales give rise to a trouble in the gauge coupling unification [18]. Thus we have mainly two problems; one is on the data fitting and another is on the gauge coupling unification.

\[ \tan \beta \mid m_s(M_Z) \mid \delta \mid \sigma \mid \sin^2 2\theta_{12} \mid \sin^2 2\theta_{23} \mid \sin^2 2\theta_{13} \mid \Delta m^2_{21}/\Delta m^2_{32} \]

| tan $\beta$ | $m_s(M_Z)$ | $\delta$ | $\sigma$ | $\sin^2 2\theta_{12}$ | $\sin^2 2\theta_{23}$ | $\sin^2 2\theta_{13}$ | $\Delta m^2_{21}/\Delta m^2_{32}$ |
|-----------|-----------|--------|-------|-----------------|-----------------|-----------------|-----------------|
| 40        | 0.0718    | 93.6°  | 3.190 | 0.738           | 0.900           | 0.163           | 0.205           |
| 45        | 0.0729    | 86.4°  | 3.198 | 0.723           | 0.895           | 0.164           | 0.188           |
| 50        | 0.0747    | 77.4°  | 3.200 | 0.683           | 0.901           | 0.164           | 0.200           |
| 55        | 0.0800    | 57.6°  | 3.201 | 0.688           | 0.878           | 0.152           | 0.198           |

are diagonal with real and positive eigenvalues, the neutrino Dirac Yukawa coupling matrix at the GUT scale is found to be \(^2\)

\[
Y_{\nu} = \begin{pmatrix}
-0.000135 - 0.00273i & 0.001113 + 0.0136i & 0.0339 + 0.0580i \\
0.00759 + 0.0119i & -0.0270 - 0.00419i & -0.272 - 0.175i \\
-0.0280 + 0.00397i & 0.0635 - 0.0119i & 0.491 - 0.526i
\end{pmatrix},
\]

\[ (\text{2}) \]

where the first term in the right hand side denotes the normal MSSM term with no LFV. We have found $Y_{\nu}$ explicitly and we can calculate LFV and related phenomena unambiguously [6].

It also gives proton decay ratio unambiguously [7].

\[ \mu \frac{d}{d\mu} (m_{\nu}^2)_{ij} = \mu \frac{d}{d\mu} (m_{\nu}^2)_{ij} \mid_{\text{MSSM}} + \frac{1}{16\pi^2} (m_{\nu}^2 Y_{\nu} Y_{\nu} + Y_{\nu} Y_{\nu} m_{\nu}^2 + 2Y_{\nu}^2 Y_{\nu} m_{\nu}^2 + 2m_{\nu}^2 Y_{\nu} Y_{\nu} + 2A_{\nu} Y_{\nu})_{ij}, \]

\[ (\text{3}) \]

\(^{2}\) We are now reconsidering data fitting with the update experimental data and new RGE results. It gives little bit different values from (2) but the LFV results are not essentially changed.
Here let me explain this: SUSY invariant action is assumed to be invariant under global supersymmetry as we consider in this review.

Since \( (\phi_+ + \phi'_+ \equiv v_{120}) \) where parameters are increased and data fitting is improved and fast proton decay also remedied. This seems to be fine at least for data fitting of low energy.

\( \text{PROBLEMS OF MINIMAL SO(10) GUT} \)

model modifications in 4D

First we consider on the improvement of data fitting. More elaborate parameter searching including type II seesaw \((c_L \neq 0)\) was done by [19]. See also [20] incorporating the recent Daya-Bay result [21]. Another approach is to add 120 Higgs [22] where parameters are increased and data fitting is improved and fast proton decay also remedied.

Since 120 has two SM doublets \((1,2,2)\) and \((15,2,2)\), mass matrices become

\[
\begin{align*}
M_u &= c_{10}M_{10} + c_{120}^{(1)}M_{120} + c_{126}M_{126}, \quad M_d = M_{10} + M_{120} + M_{126} \\
M_D &= c_{10}M_{10} + c_{120}^{(2)}M_{120} - 3c_{126}M_{126}, \quad M_e = M_{10} + c_{120}^{(3)}M_{120} - 3M_{126} \\
M_L &= c_LM_{126}, \quad M_R = c_RM_{126}
\end{align*}
\]

Here

\[
\begin{align*}
c_{120}^{(1)} &= \frac{\langle \phi_+ \rangle + \langle \phi'_+ \rangle}{\langle \phi_+ \rangle + \langle \phi'_- \rangle}, \\
c_{120}^{(2)} &= \frac{\langle \phi_+ \rangle - 3\langle \phi'_+ \rangle}{\langle \phi_- \rangle + \langle \phi'_- \rangle}, \\
c_{120}^{(3)} &= \frac{\langle \phi_- \rangle - 3\langle \phi'_+ \rangle}{\langle \phi_- \rangle + \langle \phi'_- \rangle}
\end{align*}
\]

(5)

where \( \langle \phi_\pm \rangle \) are expectation values of \((1,2,2)\) of 120, and \( \langle \phi'_\pm \rangle \) are those of \((15,2,2)\) of 120.

This model has been extensively explored by [23]. In the original model, 126 takes part of Majorana neutrinos, as well as charged fermions (2). In other word, \( Y_{126} \) was of \( O(1) \) as \( Y_{10} \) to recover the wrong SUSY mass relation \( M_e = M_d \).

However, we have additionally many parameters and can use 126 for determining \( M_R \) and \( M_L \) independently on the determination of charged fermion mass matrices. That is \( Y_{126} \) is free from order one unlike the minimal case and vevs are free from having the intermediate energy scales and we may remedy the gauge coupling crisis mentioned later. This seems to be fine at least for data fittings of low energy.

The reason why the gauge coupling unification is broken is as follows. The renormalizable SUSY GUT with Higgs fields of high dimensional representation has many Standard Model vacua. However such intermediate energy scale is fixed by only single parameter as was shown in the Higgs superpotential (4) also

\[
\frac{c_{10}}{c_{126}} = \frac{3(\nu - 1)(\nu + 1)(2\nu - 1)(\nu^3 + 5\nu - 1)}{8\nu^6 - 27\nu^3 + 38\nu^4 - 70\nu^3 + 87\nu^2 - 31\nu + 3},
\]

(7)

\( \nu \equiv \frac{\phi_1}{\phi_{126}} \) with \( \phi_1 = (12 + 34)(56 + 78 + 90) \) and \( \mathcal{M}_i = 12 \left( \frac{\phi_i}{\phi_{126}} \right) \) (See [4] for notation).

So if we add another Higgs, if we retain renormalizability, 120, then by virtue of 120, \( c_{126} \) can be free [24].

However, it seems to be very difficult to recover gauge coupling renormalizability even in this case since there still remain four intermediate energy scales.

There are the other conceptual problems which become the obstacle towards the complete GUT in 4D.

The great advantage of minimal SO(10) model was its high predictivity, implying that all quark-leptons mass matrices including Dirac and Majorana neutrinos, are completely determined.

In order that such theory becomes the SM of next generation, we must also study Doublet-Triplet problem and SUSY breaking mechanism. We will see this point soon later.

One of the other approaches is to use Split Susy [25] with light gauginos and higgsinos in 100 TeV range and superheavy squarks and sleptons in energy scale close to GUT. However, it is essentially non SUSY and unnatural.

Of course, there is a choice of adopting nonsusy SO(10) GUT [26].

\( \text{NO-GO theorem in 4D} \)

However, there is arguments that it is impossible to construct a GUT in 4D with a finite number of multiplets that leads to the MSSM with a residual R symmetry [27], whose NO GO theorem is not applicable to extra dimensions. Let me explain this: SUSY invariant action is assumed to be invariant under global \( U(1)_R \) transformation (for \( N=1 \) supersymmetry as we consider in this review),

\[
\theta \rightarrow e^{i\alpha} \theta, \quad \theta^\dagger \rightarrow e^{-i\alpha} \theta^\dagger,
\]

(8)
impling that R-charge of $\theta$ and $\theta^\dagger$ are 1 and -1, respectively.

$$\Phi = \phi(y) + \sqrt{2} \theta \psi(y) + \theta \theta F(y)$$

(9)

with

$$y^\mu = x^\mu + i \theta^\dagger \sigma^\mu \theta.$$  (10)

Vector superfield is real and its R-charge = 0. Vector superfield in Wess-Zumino gauge is

$$V = \theta^\dagger \sigma^\mu \theta A_\mu + \theta^\dagger \theta^\dagger \lambda + \theta \theta^\dagger \lambda^\dagger + \frac{1}{2} \theta \theta \theta^\dagger \theta^\dagger D$$

(11)

and $A_\mu$, $\lambda$, $D$ have R-charge 0, 1, 0, respectively.

Nelson and Seiberg discussed the relation between R symmetry and SUSY breaking [28]. They showed under the condition

i) Superpotential is generic, and

ii) low energy theory can be described by a supersymmetric Wess-Zumino model

that

a) R symmetry is necessary for SUSY breaking, and

b) spontaneous R symmetry breaking is sufficient for SUSY breaking.

Thus if we have no U(1) symmetry we have appropriate SUSY vacuum, that is, U(1) symmetry is necessary for SUSY breaking (condition (a)).

If there is U(1) symmetry and it is spontaneously broken, SUSY is automatically broken (condition (b)).

So the problem is how to impose $U(1)_R$ symmetry in superpotential of GUT.

Reflecting these situations, Ratz et al. [27] concluded that no MSSM model with either a $Z_M^{R \geq 3}$ or $U(1)_R$ symmetry can be completed by a four dimensional GUT in the ultraviolet. The essential point is explained for SU(5) GUT as follows. SU(5) × $Z_M^{R}$ is broken to the SM × $Z_M^{R}$ by the vev of the SM singlet of 24. 24 has zero R-charge since $Z_M^{R}$ is unbroken, and

$$24 = (8, 1)_0 \oplus (1, 3)_0 \oplus (1, 1)_0 \oplus (3, 2)_{-5/6} \oplus (\bar{3}, 2)_{5/6}.  \quad \text{(12)}$$

Here $(1, 1)_0 \neq 0$, and $(3, 2)_{-5/6}$ and $(\bar{3}, 2)_{5/6}$ get absorbed to the longitudinal part of gauge bosons. The remaining $(8, 1)_0$ and $(1, 3)_0$ must be massive and therefore require mass term $m 24 \times 24$. However, it is prohibited because 24 has 0 R-charge but superpotential must be 2 R-charge. This is the case for more general multiplet and more general gauge group including SO(10). The detail should be referred with [27]. On the other hand in the case of Pati-Salam case, PS group to the SM need to reduce rank by one, which is done by (4,1,2) and break B-L quantum number and there give rise to no problem. Therefore, the minimum group subject to no-go theorem is SU(5).

Of course there are a loophole of this no-go theorem. For instance it is for meta-stable supersymmetry breaking vacuum, where $U(1)_R$ is broken explicitly [29]. That is, let us consider

$$W = -k \Phi_1 + m \Phi_2 \Phi_3 + \frac{y}{2} \Phi_1 \Phi_2^2$$

(13)

which is $U(1)_R$ symmetric with R-charge, $R_{\Phi_1} = R_{\Phi_2} = 2, R_{\Phi_3} = 0$.

$$\Delta W = \frac{1}{2} \varepsilon m \Phi_2^2,$$

(14)

where $\varepsilon$ is a small dimensionless parameter. Thus we must explain this time why $\varepsilon$ is so small to satisfy longevity of metastable state $\Phi_1 = \Phi_2 = \Phi_3 = 0$ and we do not adopt this scenario.

On the otherhand, no-go theorem can not be applied in an extra dimensions, where new ways of GUT symmetry breaking mechanisms appear [30] [31] [32]. This is one of very strong motivations to proceed to extra dimension.

We may consider (8) from string theory. In string theory [33], it has originally global space-time SO(10) symmetry and is broken to SO(4) × SO(6) in 4D. This SO(6) is isomorphic to SU(4). The spinor in ten space-time dimensions has $16_L + 16_R$ components. (Do not confuse with flavour group so far discussed.) In the splitting from 10 to (4+6) dimensions, this spinor is divided into four 4-component spinor, $\theta_0^{(a)}, \theta_1^{(a)} \ (a = 1, 2), \ i = 1, 2, 3, 4$ So there is SU(4)$_R$ transformation

$$\theta^{(i)} \equiv U^i_j \theta^{(j)}.$$  (15)
SO(10) GUT IN 5D

From this chapter we will realize the new model compatible with No-Go theorem discussed in the last part of previous chapter.

Model Setup

The model is described in 5D and the fifth dimension is compactified on the orbifold $S^1/Z_2 \times Z_2$. A circle $S^1$ with radius $R$ is divided by a $Z_2$ orbifold transformation $y \to -y$ ($y$ is the fifth dimensional coordinate $0 \leq y < 2\pi R$) and this segment is further divided by a $Z_2'$ transformation $y' \to -y'$ with $y' = y + \pi R/2$. There are two inequivalent orbifold fixed points at $y = 0$ and $y = \pi R/2$. Under this orbifold compactification, a general bulk wave function is classified with respect to its parities, $P = \pm$ and $P' = \pm$, under $Z_2$ and $Z_2'$, respectively.

Assigning the parity $(P, P')$ the bulk SO(10) gauge multiplet suitably, only the PS gauge multiplet has zero-mode and the bulk 5D N=1 SUSY SO(10) gauge symmetry is broken to 4D N=1 SUSY PS gauge symmetry. Since all vector multiplets have wave functions on the brane at $y = 0$, SO(10) gauge symmetry is respected there, while only the PS symmetry is on the brane at $y = \pi R/2$ (PS brane).

Its Yukawa coupling is given by

$$W_Y = Y^{ij}_{1} F_{L_i} F_{R_j} H_1 + \frac{Y^{ij}_{15}}{M_5} F_{L_i} F_{R_j} (H'_{15} H_{15})$$

$$+ Y^{ij}_{2} F_{R_i} F_{R_j} (\phi \phi) + \frac{Y^{ij}_{2}}{M_5} F_{L_i} F_{L_j} (H_L H_L),$$

(16)

Here the notations are as follows: $M_5$ is the 5D Planck scale, $F_{L_i}$ and $F_{R_i}$ are matter multiplets of i-th generation in $(4, 2, 1)$ and $(\bar{4}, 1, 2)$ representations, respectively. $H_1 = (1, 2, 2)$, $H'_1 = (1, 2, 2)'$, $H_{15} = (15, 1, 1)_H$. $H_6 = (6, 1, 1)_H$. $\phi = (4, 1, 2)$, $\bar{\phi} = (\bar{4}, 1, 2)$, $H_L = (4, 2, 1)_H$, $H'_L = (\bar{4}, 2, 1)_H$ are Higgs multiplets.

The product $H'_1 H_{15}$, effectively works as $(15, 2, 2)_H$, while $\phi \phi$ and $H_L H_L$ effectively work as $(10, 1, 3)$ and $(\bar{10}, 3, 1)$, respectively, and are responsible for the left- and the right-handed Majorana neutrino masses. Providing VEVs for appropriate Higgs multiplets, fermion mass matrices are obtained.

$$M_u = c_{10} M_{1, 2, 2} + c_{15} M_{15, 2, 2}, \quad M_d = M_{1, 2, 2} + M_{15, 2, 2},$$

$$M_D = c_{10} M_{1, 2, 2} - 3c_{15} M_{15, 2, 2}, \quad M_e = M_{1, 2, 2} - 3M_{15, 2, 2},$$

$$M_L = c_L M_{10, 3, 1}, \quad M_R = c_R M_{10, 1, 3}.$$  

(17)

Two important remarks are in order.

1. $M_{15, 2, 2}$ is, in general, not symmetric unlike $M_{126}$. However, we imposed the L-R symmetry $4, 1, 2 \leftrightarrow \bar{4}, 2, 1$, which implies that both $M_{1, 2, 2}$ and $M_{15, 2, 2}$ matrices are symmetric and mass structure of charged Fermions and Dirac neutrino is same as that in SO(10).

2. $M_L$ and $M_R$ are independent on those of the charged Fermions and the Dirac neutrino unlike the SO(10) case (See Eq.(2)). So the precise data fitting becomes possible without changing $Y_V$. This is very important especially for LFV and leptogenesis.

$H_6$ is necessary to make the color triplet heavy. However, there arises no Doublet-Triplet problem since they are not involved in the same multiplet. There are sufficient numbers of free parameters to fit all the observed fermion masses and mixing angles.

SUSY breaking and Dark Matter

In the orbifold GUT model, we assume that the GUT model takes place at some high energy beyond the compactification scale. For the theoretical consistency of the model, the gauge coupling unification should be realized at some scale after taking into account the contributions of Kaluza-Klein modes to the gauge coupling running.
In our setup, the evolution of gauge coupling has three stages, $G_{321}$ (SM+MSSM), $G_{422}$ (whose energy scale is $v_{PS}$) and $M_c = 1/R$. From the model setting we adopted gaugino mediation mechanism as SYSY breaking scenario. First we simply assumed $v_{PS} = M_c$ [34]. In this case, stau becomes the lightest SUSY particle (LSP).

In order to remedy this trouble we next considered $M_c > v_{PS}$ and showed that neutralino becomes the LSP at [35]

$$M_c = 2.47 \times v_{PS} = 2.95 \times 10^{16} \text{GeV}.\quad (18)$$

We gives the gauge coupling running in both cases.

![FIGURE 1. Left pannel: Gauge coupling unification in the left-right symmetric case, taken from [34]. Each line from top to bottom corresponds to $g_3$, $g_2$ and $g_1$ for $\mu < M_c = v_{PS}$, while $g_3 = g_4$ and $g_2 = g_{2R}$ for $\mu > M_c = v_{PS}$. Right pannel:Gauge coupling unification for $M_c > v_{PS}$ from [35]. Each line from top to bottom corresponds to $g_3$, $g_2$ and $g_1$ for $\mu < v_{PS}$, while $g_3 = g_4$ and $g_2 = g_{2R}$ for $\mu > v_{PS}$. Here, we have taken $M_c = 2.47 \times v_{PS}$.](image)

**Confrontation with Cosmology–Smooth hybrid inflation**

Original single-field inflaton theory suffered from fine tuning problem though observational check is due to its prediction on non-Gaussianity $f_{NL} \approx 0.02$ [36]. In this subsection we discuss the smooth hybrid inflation [37] in the context of a simple supersymmetric SO(10) GUT in 5D orbifold [38]. (For another hybrid model to solve monopole non-zero VEVs for $\langle \phi \rangle$ and $\langle \phi \rangle$, we have taken $M_c = 2.47 \times v_{PS}$)

We evaluated the spectral index, the tensor-to-scalar ratio and the running of the spectral index:

$$0.963 \leq n_s \leq 0.968, \quad 4.0 \times 10^{-7} \geq r \geq 3.1 \times 10^{-7}, \quad -8.4 \times 10^{-4} \leq \alpha_s \leq -6.1 \times 10^{-4} \quad (21)$$

for $1 \text{MeV} \leq T_{\text{th}} \leq 10^7 \text{GeV}$. The tensor-to-scalar ratio and the running of the spectral index are negligibly small. These results are consistent with the WMAP 5-year data [40]: $n_s = 0.960^{+0.014}_{-0.013}$, $r < 0.2$ (95% CL) and $\alpha_s = -0.032^{+0.021}_{-0.020}$ (68% CL) (consistent with zero in 95% CL). We also discussed on the non-thermal leptogenesis [41]. As the mass relation between charged fermions are same as minimal SO(10) and we can use $Y_f$ of Eq.(2), whereas we can not reproduce MNS uniquely grom model and assumed tri-bimaximal model [42]. The resultant baryon asymmetry is obtained as a function of the lightest mass eigenvalue of the light neutrinos, and we find that a suitable amount of baryon asymmetry of the universe can be produced in the normal hierarchical case, while in the inverted hierarchical case the baryon asymmetry is too small to be consistent with the observation.

Thus the advantageous points of minimal SO(10) are succeeded to the SO(10) model in 5D, which goes over the mismatches with observations as well as the conceptual trouble indicated by several no-go theorems [43].
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REFERENCES

1. K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. 70, 2845 (1993).
2. K. Matsuda, Y. Koide and T. Fukuyama, Phys. Rev. D 64, 053015 (2001); K. Matsuda, Y. Koide, T. Fukuyama and H. Nishiura, Phys. Rev. D 65, 033008 (2002) [Erratum-ibid. D 65, 079904 (2002)].
3. T. Fukuyama and N. Okada, JHEP 0211, 011 (2002).
4. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Eur. Phys. J. C 42, 191 (2005) [arXiv:hep-ph/0401213]
5. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, JHEP 0409, 052 (2004).
6. K. Eguchi et al. [KamLAND Collaboration], Phys. Rev. Lett. 90, 021802 (2003).
7. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, JHEP 0211, 011 (2002).
8. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, Eur. Phys. J. C 42, 191 (2005) [arXiv:hep-ph/0401213].
9. J.Hisano, T.Moroi, K.Tobe and M.Yamaguchi, Phys.Rev. D53, 2442 (1996).
10. T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, JHEP 0409, 052 (2004).
11. B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys.Rev. D 70, 035007 (2004). [arXiv:hep-ph/0402122].
12. C. S. Aulakh and A. Girdhar, Nucl. Phys. B 711, 275 (2005) [arXiv:hep-ph/0405074].
13. Y. Mimura, Private communication.
14. B.Bajc, I.Dorsner, and M.Nemevsek, JHEP 11 (2008) 007.
15. E.Komatsu, talk in this conference.
16. T. Fukuyama and N.Okada, Phys.Rev. D75, 051701 (2005).
17. P.F.Harrison, D.H.Perkins, W.G.Scott, Phys.Lett. B530, 167 (2002).
18. T.Fukuyama, Opening Address in this conference.