Timing Attack Resilient Decoding Algorithms for Physical Unclonable Functions

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Abstract—This paper deals with the application of list decoding of Reed–Solomon codes to a concatenated code for key reproduction using Physical Unclonable Functions. The resulting codes achieve a higher error-correction performance at the same code rate than known schemes in this scenario. We also show that their decoding algorithms can be protected from side-channel attacks on the runtime both by masking techniques and by directly modifying the algorithms to have constant runtime.

Index Terms—Physical Unclonable Functions, Reed–Solomon Codes, List Decoding, Side-Channel Attacks, Timing Attacks

I. INTRODUCTION

A Physical Unclonable Function (PUF) is a digital circuit that possesses an intrinsic randomness resulting from process variations. This randomness is exploited to generate random keys for cryptographic applications. An advantage of PUFs over other true random number generators is their ability to reproduce a key on demand. Thus, no embedded physically secure non-volatile memory is needed.

However, the regeneration of a key is not perfect due to environmental factors such as temperature variations and aging effects of the digital circuit. These variations can be seen as an erroneous channel and channel coding increases the reliability of key regenerations. Error-correction methods for this purpose were considered in [1] (repetition, Reed-Muller (RM), Golay, BCH and concatenated codes), [2], [3] (concatenation of a repetition and BCH code), [4]–[6] (generalized concatenated codes using RM and Reed–Solomon (RS) codes).

So far, most publications about error correction for PUFs have tried to find codes with low-complexity decoding methods (in time, area, etc.) and high decoding performance. However, as for most other hardware security devices, PUFs need to be resistant against side-channel attacks. Their purpose is to obtain information about the secret by measurements, such as timing, energy consumption or electromagnetic fields. Throughout this paper, we only deal with side-channel attacks on the runtime, often called timing attacks.

We consider RS codes in a concatenated coding scheme, where we use list decoding in order to increase the decoding radius beyond half the minimum distance. In this way, smaller block error probabilities¹ than the codes/decoders proposed in [1]–[6] can be achieved.

¹In PUF literature, block error probability is often called failure rate.

In addition, we protect the decoding algorithm from timing attacks. We prove that the masking technique introduced in [7] is information-theoretically secure and propose methods for preventing attacks on decoders with unmasked inputs.

Section II deals with preliminaries. We propose to use list decoding of RS codes in error correction for PUFs in Section III and analyze their performance. Sections IV, V, and VI present ideas of preventing timing-attacks on the list decoding algorithm and Section VII concludes the paper.

II. PRELIMINARIES

In this paper, \( C = \mathcal{C}(q; n, k, d) \) is a linear code over a finite field \( \mathbb{F}_q \) (q prime power) of length n, dimension k and minimum distance d. If the field is clear from the context, we write \( \mathcal{C}(n, k, d) \). We use the classical Shannon entropy

\[
H(X) = -\sum_x f_X(x) \log_2(f_X(x)),
\]

where the input X is considered to be a random variable. E.g., if \( e \) is a codeword that is drawn uniformly at random from a code \( \mathcal{C}(n, k, d) \), its entropy \( H(e) \) is \( k \).

A. Error-Correction in PUF-based Key Reproduction

We briefly describe key reproduction using PUFs with the code-offset method² [8], as illustrated in Figure 1. A comprehensive overview of PUFs and how to use error-correction for key reproduction can be found in [3], [9], [10].

![Figure 1. Key Generation and Reproduction based on PUFs [5].](image-url)

²We consider only linear codes in this paper while the code-offset method generally also works for non-linear codes.
An initial response \( r \in \mathbb{F}_q^2 \) with entropy \( H(r) \approx n \) is generated by the PUF (I) and a random codeword \( c \in \mathcal{C} \) of a binary linear code \( \mathcal{C}(n, k, d) \) is subtracted from \( r \) in the Helper Data Generation (II). The resulting helper data \( h \) is then stored in the Helper Data Storage (III) and can be made publicly available since knowing \( h \) leaves the attacker with an uncertainty of the choice of the codeword. More precisely, for a uniformly drawn codeword, we obtain

\[
H(r | h) = H(r, c) - H(h) = H(r) + H(c) - H(h) \\
\geq H(r) - (n - k) \approx k.
\]

In the reproduction phase, the PUF outputs a response \( r' \in \mathbb{F}_q^2 \), which differs from \( r \) by an error \( e \) whose physical causes are environmental conditions such as temperature and aging, and we can write \( r' = r + e \), where \( e \) is often modeled as a binary symmetric channel (BSC) with crossover probability \( p \) (e.g., \( p = 0.14 \) in [2]).

In order to reproduce the original sequence \( r \), the Key Reproduction unit (IV) subtracts the helper data from \( r' \) and decodes the resulting word

\[
r' - h = c + h + e - h = c + e
\]

using a decoder of \( \mathcal{C} \) and obtains a codeword \( \hat{c} \). If the number of errors \( \Delta_H(e) \) is within the error-correction capability of the decoder, \( \hat{c} = c \) and we can compute the original sequence as \( r = \hat{c} + h \). The result is then usually hashed (V) in order to obtain keys of length \( m \leq k \), which ideally are uniformly distributed over \( \mathbb{F}_q^n \).

B. Reed–Solomon Codes and List Decoding

Reed–Solomon (RS) codes are algebraic codes with a variety of applications, the largest possible minimum distance, and efficient decoding algorithms, both for decoding up to and beyond half the minimum distance. Let \( q \) be a prime power and \( \mathbb{F}_q \) be the finite field of size \( q \) and let \( \mathbb{F}_q[x] \) denote the polynomial ring over \( \mathbb{F}_q \).

**Definition 1.** Let \( \alpha_1, \ldots, \alpha_n \in \mathbb{F}_q \) be distinct. An \((n, k)\) RS code of dimension \( k \leq n \) and length \( n \) is given by the set

\[
\mathcal{C}_{RS} = \{ (f(\alpha_1), \ldots, f(\alpha_n)) : f \in \mathbb{F}_q[x], \deg f < k \} \subseteq \mathbb{F}_q^n.
\]

It can be shown that the minimum distance of an \((n, k)\) RS code is \( d = n - k + 1 \). There are several algorithms for uniquely decoding up to \((d - 1)/2\) errors with RS codes, see e.g., [13]. List decoding generalizes the concept for \( \tau > [(d - 1)/2] \) errors. A list decoder guarantees to return a list of all codewords \( c \) that fulfill \( d_H(c, e) \leq \tau \) for a given decoding radius \( \tau \) and the received word \( r \). For RS codes, the Guruswami–Sudan decoding algorithm [14] accomplishes list decoding in polynomial time for any \( \tau < n - \sqrt{n(k - 1)} \). The algorithm is based on the following interpolation problem.

\[
\text{Problem 1. Given } r = (r_1, \ldots, r_n) \in \mathbb{F}_q^n, \text{ find a non-zero bivariate polynomial } Q(x, y) \in \mathbb{F}_q[x, y] \text{ of the form } Q(x, y) = \sum_{j=0}^{\ell} Q_j(x)y^j, \text{ such that for given integers } s, \tau \text{ and } \ell:\n
1) (\alpha_i, r_i) \text{ are zeros of } Q(x, y) \text{ of multiplicity } s, \\
   \forall i = 1, \ldots, n, \\
2) \deg Q_i(x) \leq s(n - \tau) - 1 - j(k - 1), \forall j = 0, \ldots, \ell.
\]

The multiplicity \( s \) can always be chosen large enough such that any \( \tau < n - \sqrt{n(k - 1)} \) can be achieved. The Guruswami–Sudan algorithm returns a list of all polynomials that satisfy \( (y - f(x))Q(x, y) \). It was proven in [14] that this list of polynomials includes all evaluation polynomials \( f(x) \), which generate codewords with \( d_H(c, r) \leq \tau \). The size of this list is bounded by a polynomial function in the code-length whenever \( \tau < n - \sqrt{n(k - 1)} \). The algorithm consists of two main steps: the interpolation step and the root-finding step. There are several efficient implementations, for both, the interpolation step [15, 16] and the root-finding step [17]. Also, efficient VLSI implementations exist, e.g. [18].

C. Reed–Muller Codes

A Reed–Muller (RM) code \( \mathcal{R}_M(r, m) \) of order \( r \leq m \) is a binary linear code with parameters \( n = 2^m, k = \sum_{i=0}^{r} \binom{m}{i} \) and \( d = 2^m - r \). It can be defined recursively using the Plotkin Construction [13]:

\[
\mathcal{R}_M(r, m) := \left\{ (a, a+b) : a \in \mathcal{R}_M(r, m-1), b \in \mathcal{R}_M(r-1, m-1) \right\}
\]

with \( \mathcal{R}_M(0, m) := \mathcal{C}(2^m, 1, 2^m) \) (Repetition code) and \( \mathcal{R}_M(m-1, m) := \mathcal{C}(2^m, 2^{m-1}, 2) \) (Parity Check code) for all \( m \). RM codes have been proposed for PUF key reproduction in [4, 5, 19, 20], and an efficient implementation of the decoding algorithm in FPGAs was presented in [6].

D. Concatenated Codes

Concatenation [21] of two linear codes is a technique for generating new codes from existing ones, while keeping encoding and decoding complexities small.

We describe code concatenation as in [22]. Let \( \mathcal{B}(q; n_b, k_b, d_b) \) (inner code) and \( \mathcal{A}(q^n; n_a, k_a, d_a) \) (outer code) be two linear codes for a suitable choice of \( q, n_b, k_b, d_b, n_a, k_a \) and \( d_a \). We use an encoding mapping for the code \( \mathcal{A} \), i.e., an \( \mathbb{F}_q \)-linear map \( \Theta : \mathbb{F}_q^{n_a} \rightarrow \mathcal{B}^{n_b} \). We can extend the mapping to matrices by applying it row-wise:

\[
\Theta : \mathbb{F}_q^{n_a} \rightarrow \mathcal{B}^{n_b} \quad \begin{bmatrix} a_1 \\
\vdots \\
\vdots \\
a_{n_a} \end{bmatrix} \mapsto \begin{bmatrix} \theta(a_1) \\
\vdots \\
\vdots \\
\theta(a_{n_a}) \end{bmatrix}.
\]

**Definition 2** (Concatenated Code). Let \( \mathcal{A}, \mathcal{B}, n_a \) and \( \Theta \) be as above. The corresponding concatenated code is given as

\[
\mathcal{C}_C = \Theta(\mathcal{A}) \subseteq \mathcal{B}^{n_b}
\]

We call the set of positions containing the \( i \)-th inner codeword \( \theta(a_i) \) the \( i \)-th inner block. Codewords are often

\[\text{In practice, each PUF bit exhibits a unique individual bit error rate due to the imperfect behavior of the digital circuit. Some papers therefore consider different channel models, cf. [11, 12].}\]
represented as matrices, where the \(i\)th row contains the \(i\)th block. Due to its construction, a concatenated code is \(\mathbb{F}_q\)-linear. The code has \((q^k)^{k_a} = q^{k_b \cdot k_a}\) codewords, each of it consisting of \(n_a\) many codewords from \(B\), resulting in a code-length of \(n_a \cdot n_b\) elements of \(\mathbb{F}_q\). Thus, the code has parameters
\[
C_C(q; n_C = n_a \cdot n_b, k_C = k_b \cdot k_a, d_C),
\]
where \(d_C \geq d_b \cdot d_a\) is the minimum distance. Although \(d_C\) might be small, often a lot more errors than half-the-minimum distance can be corrected. Concatenation of codes and standard decoders have been suggested for the PUF scenario in [1], [2].

The construction can be extended to generalized concatenated codes [23], see also [13]. Generalized concatenated codes were proposed for error correction in key reproduction using PUFs in [4], [5] and a low-complexity decoding design for FPGAs was presented in [6].

III. CODE CONSTRUCTIONS AND LIST DECODING IN THE PUF SCENARIO

Choosing codes and decoders for error correction in key reproduction using PUFs is subject to many constraints that arise from their physical properties. Typical design criteria [2], [5] are listed below.

- Choose a dimension that fulfills \(H(\text{key}) \leq H(\text{r}) - n + k\), where \(H(\text{key})\) is the desired entropy of the extracted key.
- Minimize the code-length \(n\).
- Obtain a block error probability \(P_{err}\) that is below a certain threshold (e.g., \(10^{-9}\)).
- Find efficient decoders (in time, area, memory, etc.).
- Provide resistance to side-channel attacks (with respect to time, energy, electro-magnetic radiation, etc.).

In the following, we recall one of the code constructions in [5] and show that by using list decoding we can improve the error-correction performance of this scheme.

A. Code Construction

As in [5], we choose the inner code to be a binary Reed–Muller code \(B(2; n_b, k_b, d_b) = \mathcal{R}_M(r, m)\) (cf. Section II-C) and a Reed–Solomon code \(A(2^k; n_a, k_a, n_a - k_a + 1) = \mathcal{C}_{RS}(n_a, k_a)\) (cf. Section II-B) as outer code.

B. Decoding

Decoding works in two steps. First, the inner blocks of the received word are decoded using the inner RM code \(B\). The respective decoding result either corresponds to an element in \(\mathbb{F}_q^{n_b}\) or to an erasure. Afterwards, the vector containing the decoding results of the inner blocks is decoded in the RS code.

If a list decoder (cf. Section II-B) is used in this step, more errors can be corrected than with power decoding, which was proposed in [5]. The following example compares our coding scheme with known ones for the scenario considered in [1]–[5] (BSC with \(p = 0.14\), \(H(\text{r}) \geq 128\), goal \(P_{err} < 10^{-3}\)).

**Example 1.** We consider the construction in [5, Section IV-C], namely an inner RM code with parameters \(B(2; 32, 6, 16) = \mathcal{R}_M(1, 5)\) and an outer RS code \(A(2^6; 64, 22, 43)\). The resulting concatenated code has parameters \(C_C(2; 2048, 132, \geq 688)\). Using the algorithms proposed in [5], the resulting block error probability is \(P_{err} \approx 6.79 \cdot 10^{-37}\).

Maximum likelihood decoding of the inner RM code transforms the channel into a binary error and erasure channel with \(P(\text{error}) = 0.003170\) and \(P(\text{erasure}) = 0.017605\) [5]. Since the minimum distance of the RS codes is \(d = 43\), unique decoding is possible up to 21 errors and list decoding with the Guruswami–Sudan algorithm up to \([n - \sqrt{n(k-1)}] - 1 = 27\) errors. When erasures are present, the Guruswami–Sudan decoder simply considers only non-erased positions in the interpolation step. Let \(t\) and \(\varepsilon\) denote the number of errors and erasures, respectively. Then, the block error probability is
\[
P_{err} = \sum_{i=0}^{n} P(\varepsilon = i) P(t \geq n - i - \sqrt{(n-i)(k-1)})
\]
\[
\approx 3.5308 \cdot 10^{-46},
\]
which is significantly smaller than for unique decoding, cf. [5].

When replacing the outer code by the RS code \(\mathcal{C}_{RS}(2^6; 34, 22, 13)\), the concatenated code has parameters \(C_C(2; 1088, 132, \geq 208)\) and, using list decoding, the block error probability is
\[
P_{err} \approx 1.9981 \cdot 10^{-10} < 10^{-9},
\]
which is approximately the same as in [5] while reducing the length of the concatenated code from 1152 to 1088.

Using generalized concatenated codes in combination with list decoding, also the block error probability of the other code constructions in [5] can be decreased. Since the error correction schemes in [5] decreased the block error probabilities and code-lengths simultaneously compared to the constructions in [1]–[4], our results also improve upon them.

C. Optimal Rates in the PUFKY [2] scenario

In general, we would like to know how close to an optimal solution our error correction schemes are. When comparing it to the capacity of the binary symmetric channel,
\[
C = 1 - h(p) = 1 + p \log_2(p) + (1-p) \log_2(1-p),
\]
one will notice that the rates of most of the existing schemes are far away from this upper bound, which is expectable for finite block lengths. It was proven in [24, Theorem 52] that the maximal achievable rate \(R_n(p, P_{err})\) of a code of length \(n\) whose codewords are transmitted through a BSC with crossover probability \(0 < p < \frac{1}{2}\) with maximal block error probability \(P_{err}\) is
\[
R_n(p, P_{err}) = C - \sqrt{\frac{\log_2 n}{n} Q^{-1}(P_{err})} + \frac{\log_2 n}{2n} + O\left(\frac{1}{n}\right),
\]
where
\[
V = p(1-p) \log_2^2 \left(\frac{1-p}{p}\right), \quad Q(x) = \int_{x}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy.
\]

In [2], [4], [5], a BSC with crossover probability \(p = 0.14\) was considered and \(P_{err} < 10^{-9}\) was demanded. In this case,
the capacity of the BSC is $C \approx 0.5842$, but the actual maximal achievable rates $R^*$ are much smaller. Table I shows how close the rates of existing code constructions and of our new construction are to the optimal rates.

Table I

| A/B (ref.) | $P_{err}$ | $k$ | $n$ | $R$ | $R^*$ | $R/R^*$ |
|------------|-----------|----|----|-----|------|---------|
| BCH/RM [2] | 1.0 · 10$^{-5}$ | 174 | 2226 | 0.0782 | 0.3027 | 0.2582 |
| RS/RM$^a$ [5] | 1.2 · 10$^{-10}$ | 132 | 1152 | 0.1146 | 0.2506 | 0.4573 |
| RS/RM$^{A}$ | 2.0 · 10$^{-10}$ | 132 | 1088 | 0.1213 | 0.2481 | 0.4890 |

Legend: Outer code $A$, inner code $B$, block error probability $P_{err}$, rate $R = k/n$, maximal possible rate $R^*(n, 0.14, P_{err})$, ratio to optimality $R/R^*$. $^a$Decoder based on unique decoding of the RS code (cf. [5]). $^A$Decoder based on list decoding of the RS code (cf. Section II-B). $^*$This paper.

We conclude that using list decoding, the error-correction capabilities of (generalized) concatenated code constructions based on outer RS codes can be improved significantly. Also, the coding schemes achieve approximately half of the maximum possible rates in the scenario considered in [2], which is a large value for a practical coding scheme. However, this gain comes at the cost of increased time and space complexity and therefore a larger power and area consumption.

IV. Preventing Timing Attacks

This section deals with securing the decoding algorithms of the code constructions considered in this paper against side-channel attacks on the runtime. A side-channel attacker tries to obtain information from the hardware implementation of the PUF, which includes runtime, power consumption, and electromagnetic radiation. For example, ring oscillator PUFs compare the frequencies of two ring oscillators and therefore inevitably induce an electro-magnetic emission depending on their frequencies that leads to side information [25]. The paper [25] deals with side-channel attacks on the helper data. In [7], it was proposed to add another random codeword (called codeword masking) on the helper data before the key reproduction. In [26], it was analyzed how much information is leaked from the power consumption when storing a codeword of a single-parity check code in a static memory.

However, to the best of our knowledge, there are no publications that focus on attacking the decoding process itself, e.g., the runtime and power consumption while executing the decoding algorithm. It is therefore important that a decoder has constant runtime and constant power consumption, independent of the received word. In the following, we design a list decoder with constant runtime.$^4$

We focus on side-channel attacks of the decoding algorithm. Therefore, assume that only parts of the key reconstruction functions are attackable, as illustrated in Figure 2.

Figure 2. Attacker Model.

The PUF (I) and the Helper Data Generation (II) are assumed to be secure here. As mentioned before, the Helper Data Storage (III) can be read by an attacker without obtaining more information about $r$ than contained in the random choice of $c$. Compared to the model in Figure 1, the Key Reproduction unit is subdivided into Preprocessing (IV), the Decoder (V), and the Post-Processing unit (VI). The latter also includes the hashing of the key here. Preprocessing (IV) is assumed to be not attackable. We distinguish two types of pre-processing:

1) Classical preprocessing: Compute $c + e = r' - h$ and hand it over to the Decoding (V) unit.

2) Masking: Choose random function $\varphi$ such that the decoder can map $\varphi(c + e)$ into $\varphi(c)$, but even if $\varphi(c + e)$ can be obtained by an attacker, the uncertainty about $r$ is not decreased.

The Decoder (V) can be attacked. In the Post-Processing unit (VI), we compute $\varphi^{-1}(\varphi(\hat{c})) + h = \hat{c} + h$. If decoding was successful, we obtain the original response $\hat{c} + h = c + h = r$. The key is then often computed as a hash of $r$ [2].

V. Attack Resistance by Masking

A. Codeword Masking

One method to hide the actual codeword $c$ from an attacker who can retrieve the processed data $\varphi(c + e)$ is the codeword masking technique proposed in [7], where a random codeword $c'$ is chosen and added to $c + e$, i.e.,

$$\varphi(c + e) = c' + c + e.$$ 

The technique is based on general masking schemes for preventing DPAs. In [7], no proof was given that the method actually masks well. The following theorem proves that even if an attacker is able to retrieve both the helper data $h$ and the masked word $c' + e + e$, the remaining uncertainty is still large enough.

Theorem 1. $H(r \mid (c' + c + e, h)) \geq H(r) - (n - k)$

Proof. We know that $r, c, c', e$ are pairwise independent. Also, $c$ and $c'$ are uniformly drawn from the code, so

$$H(c + e) = H(c + c' + e).$$  (1)
In general, it holds that
\[ H(c + c' + e, h) \leq H(c + c' + e) + H(h). \] (2)
Since we can compute \((r, c' + e + c, e)\) from \((r, c' + c + e, h)\) and vice versa, we have
\[ H(r, c' + c + e, h) = H(r, c' + e + c, e) \]
\[ = H(r | (c' + c + e)) + H(c' + e + c) \]
\[ = H(r) + H(c' + c + e | c) + H(e) \]
\[ = H(r) + H(c' + e) + H(e) \] (3)

Hence, we obtain
\[ H(r | (c' + c + e, h)) \]
\[ = H(r, c' + c + e, h) - H(c' + c + e, h) \]
\[ \geq H(r) + H(c' + e) + H(c) - H(c' + e) - H(h) \]
\[ = H(r) + H(c) - H(h) = H(r) + k - H(h) \]
\[ \geq H(r) - (n - k). \]

Note that if \(H(r) = n\), then \(H(r | (c' + c + e, h)) \geq k\).

### B. Alternative Masking Techniques

Other than adding a codeword to the processed word, the only masking operations that do not change the Hamming weight of the error (i.e., the hardness of the decoding problem) are the Hamming-metric isometries. Over \(\mathbb{F}_2^6\), those are exactly all permutations of positions since the other possibility, the Frobenius automorphism \(\alpha_2^2\), is the identity map in \(\mathbb{F}_2\).

In the case of RS codes, the decoder can handle a permutation \(\pi\) of positions since \(\pi(c)\) is also a codeword of an RS code with permuted code locators \(\alpha_i\). Thus, \(\pi(c + e) = \pi(c) + \pi(e)\) with \(\text{wt}_H(\pi(e)) = \text{wt}_H(e)\), we can obtain \(\pi(e)\) from \(\pi(c + e)\) using a decoder for Reed–Solomon codes whenever it is possible to correct \(e\) in \(c + e\).

Note that if \(\pi\) is not an element of the automorphism group of the code, the decoder must know the permutation \(\pi\). If it is in the automorphism group, then \(\pi(e) - c\) is a codeword and the method is equivalent to codeword masking.

### VI. ATTACK RESISTANCE BY CONSTANT RUNTIME (CLASSICAL PREPROCESSING)

#### A. Realizing Finite Field Operations

The codes used in Section III can be decoded using algebraic decoding algorithms that perform operations in finite fields. For error-correction in key regeneration using PUFs, we usually consider fields of characteristic 2, i.e., \(\mathbb{F}_{2^m}\) for some \(m \in \mathbb{N}\). Motivated by elliptic-curve cryptography, operations in these fields have recently been made resistant against timing-attacks while preserving sufficient speed in [27]. For small fields, lookup tables could be used. E.g., the field \(\mathbb{F}_{2^6}\) used in the construction in Section III would require tables of \(2^{6^2} = 4096\) entries.

Based on these considerations, we assume that field operations in \(\mathbb{F}_{2^m}\), also if a zero is involved, are constant in runtime.

#### B. Outer Code: List Decoding of RS Codes

1) **Interpolation step:** The interpolation step consists of finding a bivariate polynomial
\[ Q(x, y) = \sum_{\eta, \mu} Q_{\eta, \mu} x^{\eta} y^{\mu}, \]
where \(d_q = s(n - \tau) - 1 - \eta(k - 1)\), satisfying properties 1-2) of Problem 1. This corresponds to finding a non-zero solution \(Q_{\eta, \mu} \in \mathbb{F}_2\) for \(0 \leq \mu \leq d_q\) and \(0 \leq \eta \leq \ell\) of the system
\[ \sum_{\eta=0}^{\ell} \sum_{\mu=0}^{d_q} (\eta, \mu) = 0 \]
with \(i = 0, \ldots, n\) and \(h + j < s\).

There are many efficient algorithms for finding such a solution which are asymptotically faster than simply solving this system without considering its structure. However, these fast methods might reveal side-information about the processed data since their runtime depends on the received word \(r\).

Therefore, we propose to solve the system using “naive” Gaussian elimination where we always apply a row operation, even when an element is already zero (simply add a zero row to it). The resulting algorithm always performs the same number of field additions and multiplications and therefore its running time does not reveal any information about the processed data.

2) **Root-Finding Step:** Root-finding can be performed by a modification of the Roth–Ruckenstein algorithm [17]. The algorithm is outlined in Algorithm 1.

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**Algorithm 1**: RR \((M(x, y), g(x), i, \mathcal{L})\)

**Input:** \(M(x, y) = \sum_\ell M_\ell y^\ell, g(x), i, \text{global list } \mathcal{L}\)

1. if \(i = 0\) then return
2. \(M(x, y) \leftarrow Q(x, y)/x^r\) with \(r \in \mathbb{N}\) maximal
3. \(p(y) \leftarrow M(0, y)\)
4. Find roots of \(p(y)\)
5. Remove \(g(x)\) from the global list \(\mathcal{L}\)
6. for each root \(\gamma\) do
7. Add \(g(x) + \gamma x^i\) to the global list \(\mathcal{L}\)
8. \(\text{RR} \left(M(x, x(y - \gamma)), g(x) + \gamma x^i, i + 1, \mathcal{L}\)\)

---

We need to modify the algorithm slightly as follows:

- We compute the \(i\)-th recursion step of all recursive calls before starting to compute the \((i + 1)\)-th recursion depth.
- After finishing all recursion steps at depth \(i\), fill the list \(\mathcal{L}\) with random univariate polynomials of degree \(\leq i\) such that the list always contains \(\ell(k - 1)\) polynomials and mark them as \(\text{random}\) within the global list \(\mathcal{L}\).
- At depth \((i + 1)\), RR is called for all elements of \(\mathcal{L}\) with the corresponding bivariate polynomial \(M(x, x(y - \gamma))\). If the element is random, also call the algorithm with a random bivariate polynomial of \(y\)-degree \(\leq \ell\) but do not save the results in \(\mathcal{L}\).

The output \(\mathcal{L}\) of the modified algorithm without the \(\text{random}\) entries equals exactly the output of the Roth–Ruckenstein algorithm, so its correctness follows.

**Theorem 2.** Consider Algorithm 1 with above modifications. \(\text{RR} \left(Q(x, y), 0, 0, \{0\}\right)\) calls \(\text{RR}(\cdot)\) exactly \(\ell(k - 1)\) times.

**Proof.** The original Roth–Ruckenstein algorithm calls itself \(\leq \ell(k - 1)\) times [17], so the number of \(\text{non-random}\) entries of \(\mathcal{L}\)
Theorem 3. The number of multiplications and additions needed by Algorithm 1 for fixed parameters is independent of \(Q(x, y)\).

Proof. Due to lack of space, we only give the idea: We know that \(\deg p(y) \leq \ell\), so evaluation corresponds to \(\ell + 1\) multiplications and \(\ell\) additions of field elements. Root finding in \(p(y)\) can be done by evaluating it at all elements of \(\mathbb{F}_q\). In recursion depth \(i\), \(\deg M_i(x) \leq \deg Q + \ell_i\), so computing \(M_i(x, y-\gamma)\) can be done in constant time since we can treat \(M_i(x)\) as a polynomial of degree exactly \(\max\{\deg Q, \ell_i\}\). Finding \(r\) is a matter of data structures. Obtaining \(Q(x, y)/x^r\) and \(M(0, y)\) requires no computation.

Thus, the modified Roth–Ruckenstein algorithm always performs the same number of field operations and can be considered to be resilient against timing attacks, cf. Section VI-A.

C. Inner Codes: Reed–Muller Codes

For codes of small cardinality \(k_0\), as often used as inner codes, maximum likelihood decoding can be used, e.g., by finding the minimum of the Hamming distances \(h_i = d_H(e + c_i, e)\) of the received word \(e + c\), with \(e \in B\) and error \(e\), to all codewords \(c_i\) for \(i = 1, \ldots, 2^{k_0}\). In order to not reveal information about \(e\), the \(h_i\) must be carefully computed.

Let \(\pi\) be a random permutation of the indices \(\{1, \ldots, 2^{k_0}\}\) and \(h_{\pi(1)}, \ldots, h_{\pi(2^{k_0})}\) be the ordered list of Hamming distances of the received word to the permuted list of codewords. We can prove the following theorem that states that even if the ordered list of Hamming distances can be extracted by an attacker, the uncertainty of the codeword does not decrease.

Theorem 4. \(H(c) | h_{\pi(1)}, \ldots, h_{\pi(2^{k_0})} = H(c)\).

Proof. Since \(h_{\pi(i)} = d_H(e + c, c_{\pi(i)}) = d_H(e + c + e, c_{\pi(i)})\) for any codeword \(c' \in B\) and we can define another permutation \(\pi'\) such that \(c_{\pi'(i)} = c' + c_{\pi(i)}\) (adding a codeword is a bijection on the code), \(h_{\pi(i)} = d_H(e + c + e, c_{\pi'(i)})\), so the uncertainty of choosing a codeword \(c'\) remains.

VII. CONCLUSION

In this paper, we have presented decoding algorithms for key reproduction using PUFs that both achieve larger decoding performance than existing ones and are resistant against side-channel attacks on the runtime. Both, list recovery [28] and the Kötter–Vardy algorithm [29], a soft-decision variant of the Guruswami–Sudan algorithm, promise a further large gain in decoding performance. Investigating the capability to use them for PUFs is work in progress. Moreover, it is necessary to prevent differential power analysis (DPA) attacks on the decoding step, e.g., by combining our methods with DPA-resistant logic styles, see [30] and references therein.

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REFERENCES

[1] C. Bösch, J. Guajardo, A.-R. Sadeghi, J. Shokrollahi, and P. Tuyls, “Efficient Helper Data Key Extractor on FPGAs,” in CHES, 2008.
[2] R. Maes, A. Herrewege, and I. Verbauwhede, “PUF KY: A Fully Functional PUF-Based Cryptographic Key Generator,” in CHES, 2012.
[3] R. Maes, Physically Unclonable Functions. Springer, 2013.
[4] S. Müelich, S. Puchinger, M. Bossert, M. Hiller, and G. Sigl, “Error Correction for Physical Unclonable Functions Using Generalized Concatenated Codes,” in ACCT, 2014, arXiv preprint arXiv:1407.8034.
[5] S. Puchinger, S. Müelich, M. Bossert, M. Hiller, and G. Sigl, “On Error Correction for Physical Unclonable Functions,” in IFIP SCC, 2015.
[6] M. Hiller, L. Kurzinger, G. Sigl, S. Müelich, S. Puchinger, and M. Bossert, “Low-Area Reed Decoding in a Generalized Concatenated Code Construction for PUFs,” in IEEE ISVLISI, 2015.
[7] D. Merli, F. Stumpf, and G. Sigl, “Protecting PUF Error Correction by Codeword Masking,” IACR Cryptology ePrint Archive, p. 334, 2013.
[8] Y. Dodis, R. Ostrovsky, L. Reyzin, and A. Smith, “Fuzzy Extractors: How to Generate Strong Keys from Biometrics and Other Noisy Data,” SIAM Journal on Computing, vol. 38, no. 1, pp. 97–139, Mar. 2008.
[9] C. Boehm and M. Hofer, Physical Unclonable Functions in Theory and Practice. Springer, 2013.
[10] C. Wachsmann and A. Sadeghi, Physically Unclonable Functions (PUFs): Applications, Models, and Future Directions. M&C. 2015.
[11] R. Maes, “An Accurate Probabilistic Reliability Model for Silicon PUFs,” in CHES. Springer, 2013, pp. 73–89.
[12] J. Delvaux, D. Gu, D. Schellekens, and I. Verbauwhede, “Helper Data Algorithms for PUF-based Key Generation: Overview and Analysis,” IEEE TCAD, vol. 34, no. 6, pp. 889–902, 2015.
[13] M. Bossert, Channel Coding for Telecommunications. Wiley, 1999.
[14] V. Guruswami and M. Sudan, “Improved Decoding of Reed-Solomon and Algebraic-Geometry Codes,” IEEE Trans. Inf. Theory, vol. 45, no. 6, pp. 1757–1767, Sep. 1999.
[15] M. Alekhnovich, “Linear diophantine equations over polynomials and soft decoding of Reed-Solomon codes,” IEEE Inf. Theory, vol. 51, no. 7, pp. 2257–2265, Jul. 2005.
[16] A. Zeh, C. Gittner, and D. Augot, “An Interpolation Procedure for List Decoding Reed-Solomon Codes Based on Generalized Key Equations,” IEEE Inf. Theory, vol. 57, no. 7, pp. 5946–5959, 2011.
[17] R. Roth and G. Ruckenstein, “Efficient Decoding of Reed-Solomon Codes Beyond Half the Minimum Distance,” IEEE Trans. Inf. Theory, vol. 46, no. 1, pp. 246–257, 2000.
[18] W. J. Gross, F. R. Kschischang, R. Koetter, and R. Gulak, “A VLSI Architecture for Interpolation in Soft-Decision List Decoding of Reed-Solomon Codes,” in IEEE SIPS, 2002.
[19] R. Maes, P. Tuyls, and I. Verbauwhede, “Low-Overhead Implementation of a Soft Decision Helper Data Algorithm for SRAM PUFs,” in CHES, 2009.
[20] M. Hiller, D. Merli, F. Stumpf, and G. Sigl, “Complementary IBS: Application Specific Error Correction for PUFs,” in IEEE HOST, 2012.
[21] G. D. Forney, Concatenated Codes. Citeseer, 1996, vol. 11.
[22] B. Sendrier, “On the Concatenated Structure of a Linear Code,” Applicable Algebra in Engineering, Communication and Computing, Springer, vol. 9, no. 3, pp. 221–242, 1998.
[23] E. Blokh and V. Zyablov, “Coding of Generalized Cascaded Codes,” Problemy Peredači Informatsii, vol. 1, no. 3, 1974.
[24] Y. Vatsyayan, H. V. Poor, and S. Verdú, “Channel Coding Rate in the Finite Blocklength Regime,” IEEE Trans. Inf. Theory, vol. 56, no. 5, pp. 2307–2359, 2010.
[25] D. Merli, D. Schuster, F. Stumpf, and G. Sigl, “Side-Channel Analysis of PUFs and Fuzzy Extractors,” in TRUST. Springer, 2011.
[26] J. Dai and L. Wang, “A Study of Side-Channel Effects in Reliability-Enhancing Techniques,” in IEEE DFT, 2009.
[27] D. Pamula and A. Tisserand, “Fast and Secure Finite Field Multipliers,” in IEEE DSD, 2012.
[28] V. Guruswami and A. Radu, “Limits to List Decoding Reed-Solomon Codes,” IEEE Trans. Inf. Theory, vol. 52, no. 8, pp. 3642–3649, Aug. 2006.
[29] R. Koetter and A. Vardy, “Algebraic Soft-Decision Decoding of Reed-Solomon Codes,” IEEE Trans. Inf. Theory, vol. 49, no. 11, pp. 2809–2825, 2003.
[30] A. Wild, A. Moradi, and T. Güneysu, “GliFred: Glitch-Free Duplication-Towards Power-Equalized Circuits on FPGAs.” IACR Cryptology ePrint Archive, vol. 2015, p. 124, 2015.