Experimental Study on a Standing Wave Thermoacoustic Prime Mover with Air Working Gas at Various Pressures

Ikhsan Setiawan¹, Wahyu N. Achmadin¹, Prastowo Murti², and Makoto Nohtomi³

¹Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Sekip Utara BLS 21, Yogyakarta 55281, Indonesia
²Department of Mechanical and Industrial Engineering, Faculty of Engineering, Universitas Gadjah Mada, Jl. Teknika Utara, Yogyakarta, Indonesia
³Graduate School of Environment and Energy Engineering, Waseda University, Nishi-tomita 1011, Honjo, Saitama, Japan

E-mail: ikhsan_s@ugm.ac.id

Abstract. Thermoacoustic prime mover is an energy conversion device which converts thermal energy into acoustic work (sound wave). The advantages of this machine are that it can work with air as the working gas and does not produce any exhaust gases, so that it is environmentally friendly. This paper describes an experimental study on a standing wave thermoacoustic prime mover with air as the working gas at various pressures from 0.05 MPa to 0.6 MPa. We found that 0.2 MPa is the optimum pressure which gives the lowest onset temperature difference of 355 °C. This pressure value would be more preferable in harnessing low grade heat sources to power the thermoacoustic prime mover. In addition, we find that the lowest onset temperature difference is obtained when \( r_h/\delta_k \) ratio is 2.85, where \( r_h \) is the hydraulic radius of the stack and \( \delta_k \) is the thermal penetration depth of the gas. Moreover, the pressure amplitude of the sound wave is significantly getting larger from 2.0 kPa to 9.0 kPa as the charged pressure increases from 0.05 MPa up to 0.6 MPa.

1. Introduction
Thermoacoustic prime mover is an alternative energy conversion device which converts thermal energy into acoustic work (sound wave) by applying thermoacoustic effect. The thermoacoustic effect is a mutual thermodynamic interaction between sound wave and solid surface that possess a temperature gradient. One of the advantages of thermoacoustic prime movers is that they can work with ordinary air as the working fluid, because it is free, safe, and we do not need to vacuum the device before we fill it with the working gas. Another benefit, the thermoacoustic prime movers do not produce any exhaust gases, so they would serve as environmentally friendly machines, even when there is a gas leak in the system, especially when air is used as the working gas. In addition, their structure are simple and they do not use any moving parts in generating the oscillating pressures of the acoustic wave. Hence, the combination of the simple structure and the use of air as the working gas makes the thermoacoustic prime mover can be built and maintained with a low cost.
We have constructed a standing wave thermoacoustic prime mover with air as the working gas. The schematic diagram of the prime mover is shown in Fig. 1. It consists of a straight closed resonator made of stainless-steel pipes, a stack, and a hot- and an ambient- heat exchangers which are installed at both ends of the stack to provide a large temperature gradient across the stack. When a temperature difference across the stack reaches an onset point, the stack starts to produce acoustic waves with a certain pressure amplitude. A low onset temperature difference is desired when low grade heat sources will be used as the thermal energy provider. Whereas high pressure amplitude is needed when we intend to apply it to drive a linear alternator to generate electricity, for instance.

In this paper, we describe our experimental study on the influence of various charged pressures of air inside the resonator on the onset temperature difference and pressure amplitude of the output sound wave. As will be shown later, the results recommend to apply 0.2 MPa pressure when we want the prime mover works with the lowest onset temperature difference, whereas when higher pressure amplitude is more desired, higher charged pressure should be used.

2. Air Properties at Various Pressures
Because we will use air with various pressures as the working gas, we need to know the sound speed and thermal diffusivity in air for different pressures. They are needed to calculate the Lautrec number ($N_L$), or $\omega \tau$ parameter ($= N_L^2$), which are non-dimensional quantities that describes the nature of heat exchange process between stack and working gas [1]. Here, $\omega$ is the angular frequency of the sound wave, $\tau$ is the thermal relaxation time in the cross section of the stack’s channel, and the Lautrec number is written as [2]

$$N_L = \frac{r_h}{\delta_k},$$

where $r_h$ is hydraulic radius of the stack and $\delta_k$ is thermal penetration depth of the working gas, as will be described later.

The sound speed ($a$) and thermal diffusivity ($\alpha$) in air at pressure $P$ can be estimated by assuming that the air is a gas mixture which is composed of 78% nitrogen (N$_2$), 21% oxygen (O$_2$), 0.9% argon (Ar), and 0.04% carbon dioxide CO$_2$, namely

$$a = a_{mix} = \sqrt{\frac{\gamma_{mix} P}{\rho_{mix}}}$$

Figure 1. The schematic diagram of the standing wave thermoacoustic prime mover. HHX and AHX are the hot- and ambient- heat exchangers, respectively. $T_H$, $T_A$ are temperature measurement locations of the hot-end and ambient-end of the stack, respectively. $P_1$ and $P_2$ is the mounting locations of pressure transducers.
Table 1. The calculated properties of air at different pressures.

| $P$ (MPa) | $\rho_{\text{mix}}$ (kg m$^{-3}$) | $c_{p,\text{mix}}$ (kJ kg$^{-1}$K$^{-1}$) | $\gamma_{\text{mix}}$ | $k_{\text{mix}}$ ($\times 10^{-3}$ W m$^{-1}$K$^{-1}$) | $a$ (m s$^{-1}$) ($\times 10^{-7}$ m$^2$s$^{-1}$) | $\alpha$ |
|-----------|---------------------------------|----------------------------------|----------------|---------------------------------|----------------|---------|
| 0.05      | 0.574                           | 1.060                           | 1.406          | 26.400                          | 349.20          | 456.94  |
| 0.10      | 1.149                           | 1.0068                          | 1.4015         | 26.424                          | 349.28          | 228.47  |
| 0.20      | 2.298                           | 1.0083                          | 1.4031         | 26.474                          | 349.44          | 114.25  |
| 0.30      | 3.448                           | 1.0099                          | 1.4048         | 26.523                          | 349.61          | 76.17   |
| 0.40      | 4.599                           | 1.0114                          | 1.4066         | 26.573                          | 349.78          | 57.13   |
| 0.50      | 5.750                           | 1.0130                          | 1.4083         | 26.622                          | 349.95          | 45.71   |
| 0.60      | 6.901                           | 1.0146                          | 1.4100         | 26.672                          | 350.12          | 38.09   |

and

$$\alpha = \alpha_{\text{mix}} = \frac{k_{\text{mix}}}{\rho_{\text{mix}}c_{p,\text{mix}}},$$

where $\gamma_{\text{mix}}$, $\rho_{\text{mix}}$, $k_{\text{mix}}$, and $c_{p,\text{mix}}$ are the ratio of specific-heats, density, thermal conductivity, and isobaric specific heat of the gas mixture at pressure $P$, respectively.

The density of the gas mixture can be expressed as

$$\rho_{\text{mix}} = \sum_{i=1}^{4} x_i \rho_i,$$

where $i$ represents the individual component of the gas mixture (i.e. N$_2$, O$_2$, Ar, and CO$_2$), and $x_i$ is the corresponding mole fraction. Similarly, the isobaric specific heat of the gas mixture is

$$c_{p,\text{mix}} = \sum_{i=1}^{4} x_i c_{p,i},$$

whereas the ratio of specific heats is calculated as

$$\gamma_{\text{mix}} = \frac{c_{p,\text{mix}}}{c_{v,\text{mix}}},$$

as can be found elsewhere [3]. Furthermore, the thermal conductivity of a gas mixture ($k_{\text{mix}}$) can be approximated by [4]

$$k_{\text{mix}} = \sum_{i=1}^{4} \frac{x_i k_i}{\sum_{j=1}^{4} x_j \phi_{k,ij}},$$

where

$$\phi_{k,ij} = \frac{1}{2\sqrt{2}} \left( 1 + \frac{M_i}{M_j} \right)^{-\frac{1}{2}} \left[ 1 + \left( \frac{k_i}{k_j} \right)^{\frac{1}{2}} \left( \frac{M_j}{M_i} \right)^{\frac{1}{2}} \right]^2$$

with $i$ and $j$ represent the individual component of the gas mixture, and $M_i$ is molar mass of the $i$-th component. The properties of the individual component of the gas mixture is obtained from NIST Chemistry Webbook [5]. The calculated values of the air properties within the pressure range of 0.05 MPa – 0.6 MPa are summarized in Table 1.
3. Experimental Condition and Method

The schematic diagram of the standing wave thermoacoustic prime mover used in this study is depicted in Fig. 1. The length and inner diameter of the resonator is $L_{\text{res}} = 106$ cm and $D_{\text{res}} = 6.8$ cm, respectively. This resonator is filled with air with various pressures in the range of 0.05 MPa – 0.6 MPa. The fundamental resonance frequency of the resonator is estimated as

$$f_1 = \frac{a}{2L_{\text{res}}}. \quad (9)$$

By using the sound speed values in Table 1, the calculated resonance frequencies of the resonator within the pressure range mentioned above are almost the same, only range from 164.7 Hz to 165.2 Hz. On the other hand, the thermal penetration depth ($\delta_k$) of the air within the resonator is calculated as [6]

$$\delta_k = \sqrt{\frac{\alpha}{\pi f_1}}, \quad (10)$$

which is presented in Fig. 2 for various pressures from 0.05 MPa to 0.6 MPa. The figure shows that the thermal penetration depth is rapidly getting smaller when the pressure increases from 0.05 MPa to 0.3 MPa, and then decreasing more slowly as the pressure increases further.

The stack is made of a pile of stainless-steel wire-mesh screens with mesh number ($n$) of 16 mesh per inch and wire diameter ($D_{\text{wire}}$) of 0.35 mm. The stack has 5 cm length and is placed inside the resonator so that its center is located at a distance of 15 cm from the left end of the resonator in Fig. 1. The porosity ($\phi$) of the wire-mesh stack is estimated as $\phi = 1 - \pi n D_{\text{wire}} / 4$, whereas its hydraulic radius ($r_h$) is calculated as [6]

$$r_h = D_{\text{wire}} \frac{\phi}{4(1 - \phi)}, \quad (11)$$

and hence, our stack has $\phi = 0.83$ and $r_h = 0.42$ mm.

By knowing the $\delta_k$ and $r_h$, the Lautrec number $N_L$ is then calculated by using Eq. (1); the result is depicted in Fig. 3 which shows that the $r_h/\delta_k$ ratio is getting larger as the pressure increases. In addition, it is found that $N_L \gtrsim 1$ in our experimental condition, which is appropriate for the operation of standing wave thermoacoustic prime mover [1, 2].
The hot heat exchanger is comprised of a cable electric-heater which is wound around a cylindrical copper block which has many small holes parallel to the resonator axis. The heater has 400 W maximum electric power. The length of copper block is 4 cm. The ambient heat exchanger is composed of a water jacket outside the resonator pipe and cylindrical copper block which also has many small holes and is placed inside the resonator. Both copper blocks are installed tightly at both ends of the stack (see Fig. 1).

To measure the hot temperature $T_H$ and ambient temperature $T_A$ of the stack’s ends, a type-K thermocouple is mounted at each end of the stack. In addition, two pressure transducers (Kyowa PGM-10 KH) are mounted on the resonator wall at a distance of 12 cm and 35 cm from the right end of resonator and used to measure the dynamic pressures of the generated sound wave inside the resonator. The thermocouples and pressure transducers are connected to a data logger which is controlled by a computer.

The experiment is carried out by supplying heat through the 400 W electric heater and measuring the temperatures $T_H$ and $T_A$ as functions of time until a steady state is reached which is indicated by the constant temperature difference $\Delta T = T_H - T_A$. In the steady condition, the acoustic signals, namely the oscillating pressures, at $P_1$ and $P_2$ locations are recorded.

4. Results and Discussion
Figure 4 shows the measurement results of temperatures $T_H$ and $T_A$, and the temperature difference between the stack ends $\Delta T$, which are obtained with air at 0.4 MPa pressure as the working gas. In the figure, the onset condition is indicated by the sudden change in temperatures. Before onset, the temperature of the stack’s hot-end $T_H$ is increasing as the heat is supplied by the electric heater, while the temperature of the ambient end $T_A$ is still remain at the room temperature because the heat from the hot end has not yet arrived there, and therefore the temperature difference $\Delta T$ is also increasing with almost the same rate as the increasing $T_H$. When the onset condition is reached, the stack starts to produce the acoustic wave. This makes the heat spreads out from the hot end quickly, and hence the temperature $T_H$ decreases, whereas $T_A$ increases. Consequently, the temperature different $\Delta T$ is drop soon after onset. However, because the heat is continuously given by the electric heater, the hot temperature $T_H$ is increasing again but more slowly than that before onset. At the same time, the ambient temperature $T_A$ is also increasing due to the heat transfer from the stack hot end by conduction through the stack.
Figure 5. The onset temperature difference ($\Delta T_{onset}$) for different pressures of air inside the resonator.

Figure 6. The onset temperature difference ($\Delta T_{onset}$) for various $r_h/\delta_k$ ratios in the standing wave prime mover.

material and by convection via the oscillating gas. Because the increasing rate of the $T_H$ and $T_A$ are almost the same, then the $\Delta T$ tends to be constant. After a long time, all temperatures and temperature difference are constant due to the thermal equilibrium among the parts of the thermoacoustic prime mover and the environment is established.

The influence of the charged pressure in the range of 0.05 MPa – 0.6 MPa on the onset temperature difference ($\Delta T_{onset}$) is shown in Fig. 5. It can be seen that the ($\Delta T_{onset}$) rapidly decreases from 460 °C to 355 °C as the air pressure increases from 0.05 MPa to 0.2 MPa, and then linearly rises to 428 °C when the pressure further increases until 0.6 MPa. In the other words, the charge pressure of 0.2 MPa is the optimum pressure which gives the minimum ($\Delta T_{onset}$) of 355 °C. At pressures lower than 0.2 MPa, the lower density of the working gas makes the ($\Delta T_{onset}$) higher, because the average distance among the gas molecules is too large, and hence it is more difficult to start the gas oscillations. Extremely, when the resonator is vacuum, the sound wave will not be generated even by a very large ($\Delta T_{onset}$). On the other hand, at pressures higher than 0.2 MPa, the gas density is higher so that the gas stiffness is larger and therefore the gas needs stronger force to oscillates. That is why the ($\Delta T_{onset}$) is larger when the charged pressure is higher.

We only focus within 0.05 MPa – 0.6 MPa pressure range because of the fact that the optimum pressure is found at 0.2 MPa giving the lowest ($\Delta T_{onset}$), and that the ($\Delta T_{onset}$) tends to rise linearly as the pressure increases, for pressures higher than its optimum value. If the pressure increases further, the ($\Delta T_{onset}$) might be constant, as obtained by Hao et al [7] and predicted by Ikhsan et al [3] via the critical temperature difference for gases other than air.

Because the increase of pressure also means the increase of $r_h/\delta_k$ ratio, as has been depicted in Fig. 3, then the results presented Fig. 5 can be viewed in another way, as shown in Fig. 6 which describes the influence of $r_h/\delta_k$ ratio on the onset temperature difference ($\Delta T_{onset}$). We can see that there is an "optimum" ratio of $r_h/\delta_k$ in the sense of giving the lowest onset temperature difference. In this case, the optimum $r_h/\delta_k$ ratio is around 2.85. It means that the thermoacoustic effect most effectively occurs in the stack to start the acoustic oscillations is when the hydraulic radius $r_h$ of the stack’s channels is 2.85 times the thermal penetration depth $\delta_k$ of the gas inside the stack’s channels.

Note that the $\delta_k$ in Fig. 6 are calculated by using the measured frequencies, they are slightly different from those in Fig.3 where the estimated frequencies (see Eq. 9) are used to calculated
them. The difference between the estimated and measured frequencies can be seen in Fig. 7 which shows that the measured frequencies are higher than the estimated ones approximately by 3.6% for each charged pressure. This frequency shift is mainly caused by the existence of the stack and heat exchangers inside the resonator. However, this frequency shift raised the $r_h/\delta_k$ ratio only by about 2%.

Figure 8 shows the influence of charged pressure on the pressure amplitude of the sound wave at P$_1$ and P$_2$ locations. It can be seen clearly that the pressure amplitudes increase along with the charged pressure increases. It implies that if we want to obtained the output with high pressure amplitude we should apply high mean pressure of the working gas inside the resonator. Furthermore, the pressure amplitude at P$_1$ is larger than that at P$_2$ because P$_1$ is located closer to the pressure antinode at the closed end of the resonator than the P$_2$ location. In addition, the difference in locations of P$_1$ and P$_2$ causes the pressure amplitude at P$_1$ is increasing faster than the increase of pressure amplitude at P$_2$ as the charged pressure increases, recalling that the distribution of the pressure amplitude along the resonator approximately follows a sinusoidal pattern.

From the results above, it can be said that air with 0.2 MPa pressure as the working gas is preferable in utilizing low-grade heat sources to power the thermoacoustic prime mover, by which the onset temperature difference is 355 °C. However, if we can provide more heat, which is capable to create temperature difference of 430 °C for example, it is recommended to use higher pressures up to 0.6 MPa due to a significant enhancement in pressure amplitude of the output acoustic work that would be obtained.

5. Conclusions
A standing wave thermoacoustic prime mover with air as the working gas has been experimentally studied. The onset temperature difference, pressure amplitude of the sound wave, and the thermal penetration depth of air working gas in a standing wave thermoacoustic prime mover are significantly influenced by the charged pressure of the working gas. The lowest onset temperature difference of 355 °C is obtained by using 0.2 MPa charged pressure. At this pressure, the ratio of the stack’s hydraulic radius to the thermal penetration depth of air is 2.85, which is the optimum ratio to have the lowest onset temperature difference. The onset...
temperature difference rises linearly as the pressure increases further. In addition, the pressure amplitude of the sound wave significantly gets larger from 2.0 kPa up to 9.0 kPa when the charged pressure increases from 0.05 MPa to 0.6 MPa.

Acknowledgments
The research is supported by Laboratory of Atomic and Nuclear Physics, Department of Physics, Faculty of Mathematics and Natural Sciences, Universitas Gadjah Mada, Indonesia, and Graduate School of Environment and Energy Engineering, Waseda University, Japan. The authors wish to acknowledge the assistance provide by technical staffs of the Laboratory of Atomic and Nuclear Physics: Mr. Rizki, Mr. Jamhari, Mr. Cipto, and Mr. Farid.

References
[1] Yazaki T et al 1998 Traveling wave thermoacoustic engine in a looped tube, Phys. Rev. Lett. 81 31283131.
[2] Garrett S L 2004 Resource letter: TA-1: Thermoacoustic engines and refrigerators, Am. J. Phys. 72 11.
[3] Ikhsan Setiawan et al 2015 Critical temperature difference of a standing wave thermoacoustic prime mover with various helium-based binary mixture working gases, J. Phys.: Conf. Ser. 622 012010.
[4] Mason E A and Saxena S C 1958 Approximate formula for the thermal conductivity of gas mixtures, Phys. Fluids 1 361.
[5] Lemmon E W et al 2015 “Thermophysical Properties of Fluid Systems” in NIST Chemistry WebBook, NIST Standard Reference Database Number 69, Eds. P.J. Linstrom and W.G. Mallard, National Institute of Standards and Technology, Gaithersburg MD, 20899, http://webbook.nist.gov, (retrieved December 17, 2015).
[6] Swift G W 2002 Thermoacoustics: A unifying perspective for some engines and refrigerators, Acoust. Soc. Am., New York.
[7] Hao X H et al 2011 Influence of working fluid on the performance of a standing-wave thermoacoustic prime mover, Cryogenics 51 559.