Analysis of shock calculation based on Newmark integral method

Benlan Chen 1*

1China Ship Development and Design Center National key Laboratory on Ship Vibration & Noise
*Corresponding author’s e-mail: noise701@126.com

Abstract. In this paper, the time domain method for the calculation of impact resistance is studied. Firstly, the central difference method and the Newmark integration method based on the analysis of the linear equations and the solution of the nonlinear equations are educed. Then, the advantages and disadvantages of these two methods are compared and analyzed, and the conclusion is that the Newmark integral method is suitable for the calculation of the non-dynamic equation.

1. Introduction
At present, there are two methods are used to study the impact resistance of vibration isolation device[1]. The one is the Dynamic Design Analysis Method (DDAM) which based on Shock spectrum, the other is shock response of time domain method. This paper will take the time domain method as the research object.

The analysis model of anti-shock device is very complex. So if we use the traditional analysis method, there may be a greater error. With the rapid development of computer technology, a variety of finite element method also have been improved. The research of this paper will be based on the finite element method.

2. The solution of linear equations
The anti-shock calculation is a kind of transient response analysis, its essence is to solve the dynamic equation. According to different models, there are linear and nonlinear analysis methods, the linear method is the basis of nonlinear method.

In the finite element method with the displacement as fundamental unknown quantity, according to the mechanical model, finite element disperse, unit analysis, system group and introducing the boundary conditions, then the following form of motion equation can be obtained[2,3,4,5].

\[ M \ddot{x} + C \dot{x} + K x = f \]  

(1)

M is the mass matrix, C is the damping matrix, K is the stiffness matrix, x is the nodal displacement vector, and F is the equivalent nodal force vector.

For the statics problem, the equation is simplified to

\[ Kx = f \]  

(2)

When K is a constant matrix and f is a vector, the equations are linear algebraic equations. The equations’ solution is the basis of solving nonlinear equations.
According to the equations (2), the linear equations solving method that used commonly can be divided into two kinds: direct solution and iterative method. The direct solution is based on the Gauss elimination has high efficiency, but in the process of calculation, with the increase of the order of equations, the rounding error will constantly accumulate. So this method is suitable for the equations whose order is not very high. Gauss elimination method is the basis of all direct methods. The most frequently used direct solutions include triangular decomposition, blocking solution and so on. When the order of equations is high, iterative method can be used, such as Gauss-seidel iterative method, successive over-relaxation method (SOR method) and so on.

3. The solution of nonlinear equations
In the equations (2), if the K or f is x or the function of time derivative about x, these equations are nonlinear equations. In general, nonlinear equations cannot be solved directly, we have to use the solutions of some linear algebraic equations to approximate. Different linear approximation methods are used for solving different nonlinear equations. At present, the methods used commonly for solving nonlinear equations contain direct iteration method, Newton-Raphson method, modified Newton method, quasi-Newton method, load increment method, arc-length method and so on[6,7].

The direct iteration method is the simplest and the most intuitive method. Because of the slow convergence, iterative process instability, depending heavily on the selection of the initial value, direct iteration method is seldom used in practical applications. The Newton-Raphson method used for solving nonlinear equations has fast convergence, the modified Newton method and the quasi-Newton method can improve the computational efficiency. The arc-length method can overcome the shortcoming of the Newton method which cannot cross the extreme points, it is more suitable for analyzing the problem of structural softening.

This paper will use the Newton-Raphson method. Rewrite the equations (2) as follow,

\[ \phi(x) = K(x)x - f = 0 \]  \hspace{1cm} (3)

By solving this equation, we can obtain \( x^{(n+1)} \) which is the new approximate value of x.

\[ x^{(n+1)} = x^{(n)} - (K^{(n)}_{T})^{-1}\phi(x^{(n)}) \]  \hspace{1cm} (4)

In this equation, \( K^{(n)}_{T} = \frac{\partial \phi}{\partial x} |_{x=x^{(n)}} \) represents the tangential stiffness matrix of the structure in the finite element analysis, it consists of the corresponding element tangent stiffness matrix. By comparing the relative error of the results obtained with the last iteration, we can control the convergence process.

The solving steps of Newton-Raphson method are shown below:

1. Set the initial value \( x(0) \), make \( n=0 \);
2. Calculating tangent stiffness matrix \( K^{(n)}_{T} = \frac{\partial \phi}{\partial x} |_{x=x^{(n)}} \)
3. Calculating unbalance quantity \( \phi^{(n)} = \phi(x^{(n)}) = K(x^{(n)})x^{(n)} - f \)
4. Solving equations \( K^{(n)}_{T}\Delta x^{(n)} = -\phi^{(n)} \) get
   \[ \Delta x^{(n)} = -(K^{(n)}_{T})^{-1}\phi^{(n)} \]
5. Calculating the approximate value after \( n+1 \) times iteration,
   \[ x^{(n+1)} = x^{(n)} + \Delta x^{(n)} \]
6. Judging whether it is convergent. If it is convergent, the iteration end, else make \( n=n+1 \), turn to step (2).
4. The solution of dynamic equations

The equation (1) is a group of second-order differential equations on time t. The solving methods of this kind of equations can be divided into two categories, mode superposition method and direct integration method. When the external load is impact load, the direct integration method is more suitable than the mode superposition method. But for the problems of nonlinear dynamic response, just the direct integration method can be used. In the direct integration method, the dynamic equations are discretized in time domain, are made approximate interpolation, are changed into difference scheme. According to the initial conditions, the response of the structure at each discrete time is obtained by using the discretized linear algebraic equations. Doing different interpolation processing in time domain, the different direct integral formulas can be obtained. At present, the following methods are commonly used in the direct integration method: central difference method, Wilson-θ method, Newmark method and so on. This paper will take the central difference method and the Newmark method as the research object.

4.1. The central difference method

If the balanced relationships of the equation (1) are considered as the differential equations with constant coefficients, the acceleration and velocity can be expressed by displacement approximation by using any finite difference method. Therefore, there are many different finite difference expressions can be used in theory. For solving some problems, the central difference method is a very effective method. This method assumes

\[
\ddot{x}_t = \frac{(x_{t+\Delta t} - 2x_t + x_{t-\Delta t})}{\Delta t^2}
\]

(5)

\[
\dot{x}_t = \frac{(x_{t+\Delta t} - x_{t-\Delta t})}{2\Delta t}
\]

(6)

Take equation (3) and equation (4) into equation(1) separately can get

\[
\left(\frac{M}{\Delta t^2} + \frac{C}{2\Delta t}\right)x_{t+\Delta t} = f_t \quad \left(\frac{M}{\Delta t^2} - \frac{C}{2\Delta t}\right)x_{t-\Delta t}
\]

(7)

The value of \(x_{t+\Delta t}\) can be get from equation (5). The integral method is explicit integral, Such an integral scheme does not need to decompose the effective stiffness in the step by step method. The solving steps of the central difference method are shown below:

a) Giving initial conditions \(x_0, \dot{x}_0, \ddot{x}_0\)

b) Giving integration time step \(\Delta t\)

c) Calculating integral constant \(C_0=1/\Delta t^2, C_1=1/2\Delta t, C_2=2C_0, C_3=1/C_2, \)

d) Calculating the displacement of the last time step \(x_{t-\Delta t}=x_0 + \Delta t\dot{x}_0 + T_0\)

e) Getting effective mass matrix \(M=0, M+C_1C\)

f) At any moment, calculating the effective load \(f\) of the current time

g) Calculating the displacement of the next time step \(x_{t+\Delta t}=\dot{x}-f\)

4.2. The Newmark method

In the direct integration method, there is an implicit integration method called Newmark method, its basic assumption is that:

\[
\ddot{x}_{t+\Delta t} = \dot{x}_t + [(1 - \delta)\ddot{x}_t + \delta\ddot{x}_{t+\Delta t}]\Delta t
\]

(8)

\[
x_{t+\Delta t} = x_t + \dot{x}_t\Delta t + [(1/2 - \alpha)\ddot{x}_t + \alpha\ddot{x}_{t+\Delta t}]\Delta t^2
\]

(9)

In these equations, \(\alpha\) and \(\delta\) are the parameters that can be adjusted according to the accuracy and stability of the integral.

When \(\delta = 1/2, \alpha = 1/4\), Newmark method is reduced to the average acceleration method, which is the most commonly used step-by-step integration method.
From the two equations above, we can obtain
\[
\ddot{x}_{t+\Delta t} = \frac{1}{a\Delta t^2} (x_{t+\Delta t} - x_t) - \frac{1}{a\Delta t} \dot{x}_t - \left(\frac{1}{2a} - 1\right)\ddot{x}_t \tag{10}
\]
\[
x_{t+\Delta t} = \frac{\delta}{a\Delta t} (x_{t+\Delta t} - x_t) + (1 - \frac{\delta}{a}) \dot{x}_t + (1 + \frac{\delta}{2a})\Delta t \ddot{x}_t \tag{11}
\]

Take the two equations above into equation (3), can get
\[
\bar{K}\ddot{x}^{k+1} = \ddot{f} \tag{12}
\]

In the equations
\[
\bar{K} = K + \frac{1}{a\Delta t^2} M + \frac{\delta}{a\Delta t} C
\]
\[
\ddot{f} = f_{t+\Delta t} + M\left[\frac{1}{a\Delta t}\dot{x}_t + \left(\frac{1}{2a} - 1\right)\ddot{x}_t\right] + C\left[\frac{\delta}{a\Delta t}\dot{x}_t + \left(\frac{\delta}{a} - 1\right)\ddot{x}_t + \left(\frac{\delta}{2a} - 1\right)\Delta t \ddot{x}_t\right]
\]

Solving the equations can obtain the displacement at \(t+\Delta t\) moment, take it into the two equations above, the velocity and acceleration at \(t+\Delta t\) moment can be obtained.

4.3. The advantages and disadvantages of the central difference method and the Newmark integration method

The amount of computation required by the central difference method and Newmark integration method is proportional to the number of steps required, so it is very important to select a suitable time step. On the one hand, in order to get the solution accuracy, the time step must be small enough. On the other hand, the time step cannot be too small, otherwise it will make the cost of solving much larger than the actual needs, and reduce the computational efficiency.

The advantages of the central difference method contain do not need to calculate the total stiffness matrix and the total mass matrix, the solving process is basically carried out at the unit level, High-speed storage required is less.

The disadvantages of the central difference method is \(\Delta t\) must be smaller than a critical value \(\Delta t_{cr}\). Its expression is as follow:
\[
\Delta t \leq \Delta t_{cr} = \frac{T_n}{\pi} \tag{13}
\]

\(T_n\) is the minimum period of finite element aggregate, \(n\) is the order of the unit system.

The central difference method requires \(\Delta t\) smaller than the critical time step, it is called conditional stability method. If using a time step larger than \(\Delta t_{cr}\), the integral will be unstable. That means the rounding error in the computer will increase. So for the accuracy of the calculation, the central difference method has no more choice.

Compared with the central difference method, in the Newmark integration method, \(\Delta t\) do not need to be smaller than a value, its time step can get a larger. It is only necessary to calculate accurately for the response which make important contributions to the whole structure, thus improve its efficiency. The other response components do not have to be calculated accurately, their error is not important.

From the above analysis, it can conclude that the Newmark integration method is more advantageous than the central difference method.

5. The solution of nonlinear dynamic equations

In the equations (1), if the \(K\) or \(f\) is \(x\) or the function of time derivative about \(x\), these equations are nonlinear equations. For the equations, Firstly, using the Newmark direct integration method or the central difference method to disperse the dynamic equations in time domain, it can get some equations that similar to the equation (2). The \(K\) or \(f\) in the equations is \(x\) or the function of time derivative about \(x\), so the equations (2) are nonlinear equations. According to these equations, the Newton-Raphson method is used to solve. Method modified or Newton method Newton-Raphson is used to analyze the solution in a time step. When a time step stiffness matrix is not changed, it is only necessary to modify the stiffness matrix at each time step.

By solving the above process, we can know that the difference between the method of solving the nonlinear dynamic equation is mainly that the method of the dynamic equation is different in the time domain, The difference is mainly due to the different result of the integration method. From the
aspects of stability, calculation accuracy and efficiency, the Newmark integration method is more suitable for solving nonlinear dynamic equations.

References
[1] Newmark N M. (1959) A method of computation for structural dynamics. J. Journal of Engineering Mechanics Division., 85(3):67-94.
[2] Sun HC. (1980) Solution of vibration equation of nonlinear structure-two step approximate acceleration integral method. J. Dalian Univ Technol.,19:1–18.
[3] R.Rajendren, K.Narasimhan. (2006) Deformation and Fracture Behaviour of Plate Specimens Subjected to Underwater Explosion—a Review. J. International Journal of Impact Engineering.,32:1945–1963.
[4] Suresh Menon, Mihir Lal. (1998) On the Dynamics and Instability of Bubbles Formed During Underwater Explosion. J.Experimental Thermal and Fluid Science.,16:305–321.
[5] R. Rajendren. (2001) Linear Elastic Shock Response of Plane Plates Subjected to Underwater Explosion. J.International Journal of Impact Engineering.,25:493–506.
[6] Cho-Chung L. (2006) Shock Responses of a Surface Ship Subjected to Noncontact Underwater Explosions. J.Ocean Engineering ., 33(7):48–77.
[7] J. Keith Clutter. (2004) Hydrocode Simulation of Air and Water Shock for Facility Vulnerability Assessments. J. Journal of Hazardous Materials.,106A:9–14.