A Modified Method for Finding Initial Basic Feasible Solution for Fuzzy Transportation Problems Involving Generalized Trapezoidal Fuzzy Numbers

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Abstract. In this paper, we propose a modified method for finding an initial basic feasible solution(IBFS) for fuzzy transportation problem(FTP) involving generalized trapezoidal fuzzy numbers(GTFNS). Without disturbing the fuzzy nature, using the parametric representation of fuzzy numbers, the optimal solution of the FTP is obtained. A numerical illustration is given to demonstrate the suitability and efficiency of the proposed procedure and the solution is compared with the existing methods.

1. Introduction

Transportation problem is successfully applied in many areas such as investment, production, job scheduling, inventory management and so on. To solve such transportation problems, the transportation prices, sources and demands are need to be accurately defined. In certain real life scenarios, the decision conditions could not be understood for certain reasons. Therefore, in order to overcome real life issues, we need to deal with uncertainty. In such cases, the crisp coefficients are described in the problem may be considered as fuzzy numbers.

Many researchers have discussed transportation problems under fuzzy environment. In particular, Lai and Hwang [8] discussed the transportation problem under fuzzy environment. Chanas and Kuchta[2] proposed a modified method for the optimal solution of the transportation problem with fuzzy coefficients. Liu and Kao [9] have proposed a new procedure to extract the objective value of fuzzy transportation problem(FTP) with supplies are fuzzy numbers and obtained transportation cost for the given fuzzy transport problem by the concept of extension principle. A new method for finding the fuzzy transportation cost for the FTP was discussed by Ahlatioglu [1]. Gen [6] discussed genetic algorithm for finding the transportation cost of the solid FTP. Pandiyan and Natraajan [10] introduced the zero point method for finding an IBFS for the given FTP. A simplex method to find the transportation cost for the FTP was discussed by Edward [5]. Dinagar [3] used a new algorithm for solving FTP involving TFNS. Uma Maheswari and Ganesan [13] discussed the optimal solution of fuzzy transportation problem using pentagonal fuzzy number. A modified method to solve fuzzy transportation problem involving TFNS was discussed by Uma Mahewari [14]. Srinivasarao [12] proposed new method to find the fuzzy optimal solution for the generalized FTP. A new method for generalized FTP by harmonic mean method was discussed by Kumar[7]. In general, most of the...
researchers have obtained only a crisp solution of the given FTP. But the aim of this article is to find the fuzzy optimal solution of the given FTP without converting into its equivalent crisp transportation problem.

The rest of the paper is structured as follows. Section 2 recalls the generalized fuzzy numbers, and their arithmetic operations. Fuzzy transportation problem with GTFNS and its solution procedure is discussed in section 3. The numerical illustration to explain the suggested approach is provided in Section 4. Some comparative results are discussed in section 5. Finally, section 6 provides the conclusion.

2. Preliminaries

Some basic definitions and arithmetic operations of TPFNS are discussed for our further consideration.

**Definition 2.1:** A fuzzy set \( \tilde{G} \) defined on a set of real numbers \( R \) is said to be a generalized fuzzy number, if its membership function \( \mu_{\tilde{G}} : R \rightarrow [0, w] \) has following characteristics:

(i) \( \mu_{\tilde{G}}(x) : R \rightarrow [0, w] \) is continuous.

(ii) \( \mu_{\tilde{G}}(x) = 0 \), for all \( x \in (-\infty, g_1] \cup [g_4, \infty) \).

(iii) \( \mu_{\tilde{G}}(x) \) is strictly increasing on \([g_1, g_2]\) and strictly decreasing on \([g_3, g_4]\).

(iv) \( \mu_{\tilde{G}}(x) = w \), for all \( x \in [g_2, g_3] \), where \( 0 < w \leq 1 \).

**Definition 2.2:** A fuzzy number \( \tilde{G} \) on \( R \) is said to be generalized trapezoidal fuzzy number (TFN) its membership function \( \mu_{\tilde{G}} : R \rightarrow [0,1] \) has

\[
\mu_{\tilde{G}}(x) = \begin{cases} 
  w \frac{x-g_1}{g_2-g_1}, & g_1 \leq x \leq g_2 \\
  w, & g_2 \leq x \leq g_3 \\
  w \frac{g_1-x}{g_4-g_3}, & g_3 \leq x \leq g_4 \\
  0 & x > g_4.
\end{cases}
\]

**Definition 2.3:** A GTFN \( \tilde{G} = (g_1, g_2, g_3, g_4; w) \in F(R) \) can also be represented as a pair of \( \tilde{G} = (\underline{g}(\alpha), \overline{g}(\alpha)) \), is functions of \( \underline{g}(\alpha), \overline{g}(\alpha) \) for \( 0 \leq \alpha \leq w \) which satisfies the following assumptions:

1. \( \underline{g}(\alpha) \) is a bounded monotonic increasing left continuous function.

2. \( \overline{g}(\alpha) \) is a bounded monotonic decreasing left continuous function.

3. \( \underline{g}(\alpha) \leq \overline{g}(\alpha) \), \( 0 \leq \alpha \leq w \).
Definition 2.4: For an arbitrary trapezoidal fuzzy number, $\tilde{G} = \left( (g), (\bar{g}) \right)$, then
\[
g_0 = \left( \frac{g(w) + \bar{g}(w)}{2} \right)
\]
be the location index number of $\tilde{G}$. Then $g_* = \left( g_0 - g \right)$, $g^* = \left( \bar{g} - g_0 \right)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every trapezoidal fuzzy number $\tilde{G} = \left( g_1, g_2, g_3, g_4; w \right)$ can also be represented by $\tilde{G} = \left( g_0, g_*, g^*, w \right)$.

2.1. Ranking of Trapezoidal Fuzzy Numbers

For every $\tilde{G} = \left( g_1, g_2, g_3, g_4; w \right) \in F(R)$, the ranking function $\mathfrak{R}$ is defined from $\mathfrak{R} : F(R) \to R$ and it is defined by $\mathfrak{R} = \frac{w(g^* + 4g_0 - g_*)}{4}$, as for any two generalized trapezoidal fuzzy numbers $\tilde{G} = \left( g_1, g_2, g_3, g_4; w_1 \right) \in F(R)$ and $\tilde{H} = \left( h_1, h_2, h_3, h_4; w_2 \right) \in F(R)$, we have the following comparison:

(i). $\tilde{G} \succ \tilde{H}$ if and only if $\mathfrak{R}(\tilde{G}) \succ \mathfrak{R}(\tilde{H})$

(ii). $\tilde{G} \prec \tilde{H}$ if and only if $\mathfrak{R}(\tilde{G}) \prec \mathfrak{R}(\tilde{H})$

(iii). $\tilde{G} \cong \tilde{H}$ if and only if $\mathfrak{R}(\tilde{G}) = \mathfrak{R}(\tilde{H})$

2.2. Arithmetic Operations

For arbitrary fuzzy numbers $\tilde{G} = \left( g_1, g_2, g_3, g_4; w_1 \right)$ and $\tilde{H} = \left( h_1, h_2, h_3, h_4; w_2 \right)$, the arithmetic operations on the fuzzy numbers are defined by
\[
\tilde{G} * \tilde{H} = \left( g_0, g_*, g^*; w_1 \right) * \left( h_0, h_*, h^*; w_2 \right) = \left( g_0 * h_0, g_* \vee h_*, g^* \vee h^*, w_1 \wedge w_2 \right)
\]
\[
= \left( g_0 * h_0, \max\{g_*, h_*\}, \max\{g^*, h^*\}, \min\{w_1, w_2\} \right).
\]
In particular for any two fuzzy numbers $\tilde{G} = \left( g_0, g_*, g^*; w_1 \right)$ and $\tilde{H} = \left( h_0, h_*, h^*; w_2 \right)$

(i). Addition:
\[
\tilde{G} - \tilde{H} = \left( g_0, g_*, g^*; w_1 \right) - \left( h_0, h_*, h^*; w_2 \right) = \left( g_0 - h_0, g_* \vee h_*, g^* \vee h^*, w_1 \wedge w_2 \right)
\]
\[
= \left( g_0 - h_0, \max\{g_*, h_*\}, \max\{g^*, h^*\}, \min\{w_1, w_2\} \right).
\]

(ii). Subtraction:
\[
\tilde{G} - \tilde{H} = \left( g_0, g_*, g^*; w_1 \right) - \left( h_0, h_*, h^*; w_2 \right) = \left( g_0 - h_0, g_* \vee h_*, g^* \vee h^*, w_1 \wedge w_2 \right)
\]
\[
= \left( g_0 - h_0, \max\{g_*, h_*\}, \max\{g^*, h^*\}, \min\{w_1, w_2\} \right).
(iii). Multiplication:
\[ \tilde{G} \times \tilde{H} = \left( g_0, g_\ast, g_\ast, w_1 \right) \times \left( h_0, h_\ast, h_\ast, w_2 \right) = \left( g_0 \times h_0, g_\ast \vee h_\ast, g_\ast \vee h_\ast, w_1 \wedge w_2 \right) \]
\[ = \left( g_0 \times h_0, \max \{ g_\ast, h_\ast \}, \max \{ g_\ast, h_\ast \}, \min \{ w_1, w_2 \} \right). \]

(iv). Division:
\[ \tilde{G} \div \tilde{H} = \left( g_0, g_\ast, g_\ast, w_1 \right) \div \left( h_0, h_\ast, h_\ast, w_2 \right) = \left( g_0 \div h_0, g_\ast \vee h_\ast, g_\ast \vee h_\ast, w_1 \wedge w_2 \right) \]
\[ = \left( g_0 \div h_0, \max \{ g_\ast, h_\ast \}, \max \{ g_\ast, h_\ast \}, \min \{ w_1, w_2 \} \right), \text{ provided } \Re(\tilde{H}) \neq 0. \]

3. Mathematical Formulation of Fuzzy Transportation Problem

Fuzzy transportation problem (FTP) deals with \( m \) sources and \( n \) destinations where \( S_1, S_2, S_3, \ldots, S_m \) the sources with supply \( \bar{g}_i \) (\( i = 1, 2, \ldots, m \)) and \( D_1, D_2, D_3, \ldots, D_n \) are destination with demand \( \bar{h}_j \) (\( j = 1, 2, 3, \ldots, n \)). Let \( \tilde{c}_{ij} \) represent the unit fuzzy transportation cost from source \( i \) to destination \( j \) and \( \tilde{y}_{ij} \) be the number of fuzzy units to be transported from source \( i \) to destination \( j \), then the mathematical formulation of the problem is stated as:

\[ \min \tilde{F} \approx \sum_{i=1}^{m} \sum_{j=1}^{n} \tilde{c}_{ij} \tilde{y}_{ij} \]

subject to \( \sum_{j=1}^{n} \tilde{y}_{ij} \approx \bar{g}_i, \quad i = 1, 2, \ldots, m \)

\[ \sum_{i=1}^{m} \tilde{y}_{ij} \approx \bar{h}_j, \quad j = 1, 2, \ldots, n \]

and \( \tilde{y}_{ij} \geq 0 \) for all \( i, j \).

The following table represent the general form of transportation problem:

|       | \( D_1 \) | \( D_2 \) | \ldots | \( D_n \) | Supply |
|-------|----------|----------|--------|----------|--------|
| \( S_1 \) | \( \tilde{c}_{11} \) | \( \tilde{c}_{12} \) | \ldots | \( \tilde{c}_{1n} \) | \( \bar{g}_1 \) |
| \( S_2 \) | \( \tilde{c}_{21} \) | \( \tilde{c}_{22} \) | \ldots | \( \tilde{c}_{2n} \) | \( \bar{g}_2 \) |
| \ldots | \( \ldots \) | \( \ldots \) | \ldots | \( \ldots \) | \( \ldots \) |
| \( S_m \) | \( \tilde{c}_{ml} \) | \( \tilde{c}_{m2} \) | \ldots | \( \tilde{c}_{mn} \) | \( \bar{g}_m \) |

| Demand | \( \bar{h}_1 \) | \( \bar{h}_2 \) | \ldots | \( \bar{h}_n \) |
3.1. Algorithm for finding an initial feasible solution:

*Step 1:* Convert all the fuzzy numbers into their location index and fuzziness index forms.

*Step 2:* Check whether the given FTP is balanced, go to step 4 if else go to step 3.

*Step 3:* If the given FTP is unbalanced, then convert the given problem into balance by adding dummy rows or columns with zero fuzzy cost.

*Step 4:* Identify the smallest fuzzy cost in each row of the given cost table and subtract that from each fuzzy cost of the corresponding row, and in the reduced cost table identify the smallest fuzzy cost in each column and subtract that from each fuzzy cost of the corresponding column.

*Step 5:* Each column and row having at least one zero value. Identify the row/column with maximum number of zeros.

*Step 6:* In the identified row with supply (or column with demand); identify a cell with zero cost corresponding to the minimum demand (minimum supply).

*Step 7:* Allocate as much possible to the identified cell and adjust the supply and demand. Cross out the identified row or column.

*Step 8:* For the resulting table, repeat the process until all the source and demand are exhausted.

3.2. Fuzzy Version of MODI Method:

To check the optimality of the transportation problem, follow the steps

*Step 1:* Find the IBFS of the given problem by using the proposed method. Check the number of basic cells. If the number of occupied cell is less than $m+n-1$, then the problem possess the degeneracy and introduce the smallest cost $\varepsilon (\cong 0)$ at an independent position so that the number of basic cells is exactly equal to $m+n-1$.

*Step 2:* Assume either $\tilde{u}_i = \tilde{0}$ or $\tilde{v}_j = \tilde{0}$ corresponding to the row or column having maximum number of basic cells. Solve the equation $\tilde{u}_i + \tilde{v}_j = \tilde{c}_{ij}$ for each basic cell in the present table and find the values of $\tilde{u}_i$ and $\tilde{v}_j$.

*Step 3:* Find the value of $\tilde{d}_{ij} = \tilde{u}_i + \tilde{v}_j - \tilde{c}_{ij}$ for all non-basic cells and enter the values in the right corner of the respective non-basic cells.

*Step 4:* Check $\tilde{d}_{ij}$ for all non-basic cells.

(i). If $\tilde{d}_{ij} > \tilde{0}$ then the current IBFS is the optimal solution and the given transportation problem possess unique optimal solution and stop the process.
(ii). Suppose all \( \hat{d}_{ij} > \bar{0} \) and any one \( \hat{d}_{ij} \approx \bar{0} \) then the given transportation problem possess an alternative optimal solution. If anyone \( \hat{d}_{ij} < \bar{0} \) then the IBFS is not optimal then go to the next step.

**Step 5:** From the new table select the most negative (\( \hat{d}_{ij} < \bar{0} \)) value appeared in the non-basic cells. From that cell draw the closed loop by the vertical and horizontal lines begin and end with the same selected cell and all other corners of the loop ends at the occupied cells. Along this closed loop indicate \( +\theta \) and \( -\theta \) alternatively at all the corners. Identify the smallest entry in the cells with \( -\theta \). Add this minimum allocation to the cells with \( +\theta \) and subtract this minimum allocation to the cells with \( -\theta \).

**Step 6:** Repeat the step 2 to step 5 until optimality is reached.

4. **Numerical Example**

Consider a FTP was discussed by Srinivasarao [12].

| Source | \( S_1 \) | \( S_2 \) | \( S_3 \) |
|--------|------------|------------|------------|
| \( D_1 \) | (11, 13, 14, 18; 0.5) | (6, 7, 8, 11; 0.2) | (14, 15, 17, 18; 0.4) |
| \( D_2 \) | (20, 21, 24, 27; 0.7) | (9, 11, 12, 13; 0.2) | (15, 16, 18, 19; 0.5) |
| \( D_3 \) | (14, 15, 16, 17; 0.4) | (20, 21, 24, 27; 0.7) | (10, 11, 12, 13; 0.6) |
| Supply | 13 | 20 | 5 |

| Demand | 12 | 15 | 11 |

| Source | \( S_1 \) | \( S_2 \) | \( S_3 \) |
|--------|------------|------------|------------|
| \( D_1 \) | (13.5, 2.5 – 4\( \alpha \), 4.5–8\( \alpha \); 0.5) | (7.5, 1.5 – 5\( \alpha \), 3.5–15\( \alpha \); 0.2) | (16, 2–2.5\( \alpha \), 2–2.5\( \alpha \); 0.4) |
| \( D_2 \) | (22.5, 2.5–1.4286\( \alpha \), 4.5–4.2857\( \alpha \); 0.7) | (11.5, 2.5–10\( \alpha \), 1.5–5\( \alpha \); 0.2) | (17, 2–2\( \alpha \), 2–2\( \alpha \); 0.5) |
| \( D_3 \) | (15.5, 1.5–2.5\( \alpha \), 1.5–2.5\( \alpha \); 0.4) | (22.5, 2.5–1.4286\( \alpha \), 4.5–4.2857\( \alpha \); 0.7) | (11.5, 1.5–1.6667\( \alpha \), 1.5–1.6667\( \alpha \); 0.6) |
| Supply | 13 | 20 | 5 |

| Demand | 12 | 15 | 11 |
Table 4: IBFS by proposed method

| Source | \( D_1 \)                  | \( D_2 \)                  | \( D_3 \)                  |
|--------|-----------------------------|-----------------------------|-----------------------------|
|        | \((13.5, 2.5 – 4\alpha, 4.5 – 8\alpha; 0.5)\) | \((22.5, 2.5 – 1.4286\alpha, 4.5 – 4.2857\alpha; 0.7)\) | \((15.5, 1.5 – 2.5\alpha, 1.5 – 2.5\alpha; 0.4)\) |
|        | 7                           | 6                           | 6                           |
|        | \((7.5, 1.5 – 5\alpha, 3.5 – 15\alpha; 0.2)\) | \((11.5, 2.5 – 10\alpha, 1.5 – 5\alpha; 0.2)\) | \((22.5, 2.5 – 1.4286\alpha, 4.5 – 4.2857\alpha; 0.7)\) |
|        | 5                           | 15                          |                               |
|        | \((16, 2 – 2.5\alpha, 2 – 2.5\alpha; 0.4)\) | \((17, 2 – 2\alpha, 2 – 2\alpha; 0.5)\) | \((11.5, 1.5 – 1.6667\alpha, 1.5 – 1.6667\alpha; 0.6)\) |
|        | 5                           | 5                           | 5                           |

Initial Cost is = \((13.5, 2.5 – 4\alpha, 4.5 – 8\alpha; 0.5)\)\(\times 7\) + 
\((15.5, 1.5 – 2.5\alpha, 1.5 – 2.5\alpha; 0.4)\)\(\times 6\) + 
\((7.5, 1.5 – 5\alpha, 3.5 – 15\alpha; 0.2)\)\(\times 5\) + \((11.5, 2.5 – 10\alpha, 1.5 – 5\alpha; 0.2)\)\(\times 5\) 
\((11.5, 1.5 – 1.6667\alpha, 1.5 – 1.6667\alpha; 0.6)\)\(\times 5\) 
\(= (455, 1.5 – 1.6667\alpha, 1.5 – 1.6667\alpha; 0.2).\)
When using fuzzy version of MODI method to solve fuzzy transportation problem we obtained an optimal solution:

Table 5: Optimal solution for Modi method

| Source | D₁       | D₂       | D₃       |
|--------|----------|----------|----------|
| S₁     | (13.5, 2.5–4α, 4.5–8α; 0.5) | (22.5, 2.5–1.4286α, 4.5–4.2857α; 0.7) | (15.5, 1.5–2.5α, 1.5–2.5α; 0.4) |
|        |          |          |          |
| S₂     | (7.5, 1.5–5α, 3.5–15α; 0.2) | (11.5, 2.5–10α, 1.5–5α; 0.2) | (22.5, 2.5–1.4286α, 4.5–4.2857α; 0.7) |
|        | 7        | 5        | 6        |
| S₃     | (16, 2–2.5α, 2–2.5α; 0.4) | (17, 2–2α, 2–2α; 0.5) | (11.5, 1.5–1.6667α, 1.5–1.6667α; 0.6) |
|        |          |          |          |
|        | 5        |          |          |

The Optimal cost is = \((13.5, 2.5–4α, 4.5–8α; 0.5)\times 7 + (15.5, 1.5–2.5α, 1.5–2.5α; 0.4)\times 6 + (7.5, 1.5–5α, 3.5–15α; 0.2)\times 5 + (11.5, 2.5–10α, 1.5–5α; 0.2)\times 15 + (11.5, 1.5–1.6667α, 1.5–1.6667α; 0.6)\times 5 = (455, 1.5–1.6667α, 1.5–1.6667α; 0.2).

Therefore, the optimal cost for the given transportation problems is (453.5, 453.833, 456.1667, 456.5; 0.2). For the same problem Srinivasarao obtained the optimal cost (376, 436, 474, 543; 0.2).

5. Result and Discussion

The initial solution obtained by the proposed method is optimal while checking with fuzzy version of MODI method. That is the initial solution obtained by the proposed method will be optimal or very much close to the optimal. Comparing with the solution obtained by Srinivasarao method, our method gives the vagueness reduced minimum transportation cost and also our proposed method gives initial cost is same as the optimal cost comparing all other methods. The following table shows the comparison results between our proposed method to some other existing methods.
5.1. **Comparison Results:**

| METHOD USED                  | IBFS                              | OPTIMAL SOLUTION                   |
|------------------------------|-----------------------------------|-------------------------------------|
| North West Corner Rule       | $\{536.5, 536.833, 539.166, 539.5; 0.2\}$ | $\{453.5, 453.833, 456.166, 456.5; 0.2\}$ | $=107.6$ |
| Vogel’s method               | $\{481, 481.333, 483.666, 484; 0.2\}$ | $\{453.5, 453.833, 456.166, 456.5; 0.2\}$ | $=96.5$ |
| Least Cost method            | $\{488.5, 488.833, 491.166, 491.5; 0.2\}$ | $\{453.5, 453.833, 456.166, 456.5; 0.2\}$ | $=98$ |
| Srinivasarao method (ZAM)    | $\{376, 436, 474, 543; 0.2\}$ | $\{453.5, 453.833, 456.166, 456.5; 0.2\}$ | $=91.45$ |
| Our proposed method          | $\{453.5, 453.833, 456.166, 456.5; 0.2\}$ | $\{453.5, 453.833, 456.166, 456.5; 0.2\}$ | $=91$ |

6. **Conclusion**

In this paper, we have proposed a new method to find an IBFS of a FTP with GTFNS. Using proposed method, an IBFS is obtained without converting to an equivalent crisp form. It’s to be noted that the IBFS obtained by our method is better than the solutions obtained with existing methods and is nearer to the optimal solution. An example is solved by using the proposed method without converting to the given problem into its crisp equivalent problem.

**References**

[1] Ahlatcioglu M Sivri M and Güzel N 2002 Journal of Marmara for Pure and Applied Sciences Istanbul Turkey **18** 141-157
[2] Chanas S and Kuchta D 1996 Fuzzy Sets and Systems **82** 299-305
[3] Dinagar D S and Palanivel K 2009 Computing and Mathematics **2** 65-71
[4] Dinesh C S Bisht and Pankaj Kumar Srivastava 2019 International Journal of Mathematical, Engineering and Management Sciences 4(5) 1251–1263
[5] Edward Samuel A and Raja P 2017 International Journal of Pure and Applied Mathematics (IPAM) **5** 553-561
[6] Gen M Ida K and Li Y 1994 IEEE International Conference2 1200-1207
[7] Kumar SS Raja P Shanmugasundram P and Thota S 2020 Int J Chem Math Phys **4** 51-60
[8] Lai YJ and Hwang CL 1992 Springer-Verlag Berlin Germany ISBN-13
[9] Liu ST and Kao C 2004 Eur. J. Oper. Res. **153** 661-674
[10] Pandiyan Natrajan 2016 International Journal of Applied Engineering Research **11**(1) 498-502
[11] Samuel AE Raja P and Thota S 2020 Appl Math Inf Sci **14** 459-65
[12] Srinivasarao Thota and Raja P 2020 Asian Journal of Mathematical Sciences **4**(2) 19-24
[13] Uma Maheswari P and Ganesan K 2018 Journal of physics: Conf.Series **1000**
[14] Uma Maheswari P and Ganesan K 2020 AIP Conference Proceedings **2277**
[15] Zadeh L A 1965 Fuzzy sets Information and Control **8** 338-353