Type IIA String and Matrix String on PP-wave

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Abstract

We study type IIA string theories on the pp-waves with 24 supercharges. The type IIA pp-wave backgrounds are derived from the maximally supersymmetric pp-wave solution in eleven dimensions through the toroidal compactification on the spatial isometry directions. The associated actions of type IIA strings are obtained by using these metrics and other background fields of the type IIA supergravities on the one hand. On the other hand, we derive these theories from D=11 supermembrane on the pp-wave via double dimensional reduction for the spatial isometry directions. The resulting actions agree with those of type IIA strings obtained in the study of the supergravities. Also, the action of the matrix string is written down. Moreover, the quantization of closed and open strings is discussed. In particular, we study Dp-branes allowed in one of the type IIA theories.

Keywords: supermembrane, matrix theory, M-theory, pp-wave double dimensional reduction, matrix string
1 Introduction

The maximally supersymmetric pp-wave background in eleven dimensions is a classical solution (Kowalski-Glikman (KG) solution) [1] of the eleven-dimensional supergravity and is considered as one of the candidates for supersymmetric background of M-theory [2]. This pp-wave background is obtained from $AdS_7 \times S^4$ or $AdS_4 \times S^7$ via Penrose limit [3]. Recently, the maximally supersymmetric type IIB pp-wave [4] has been found and it has been shown that the type IIB string on this pp-wave is exactly solvable in the Green-Schwarz (GS) formulation [5–7] with a light-cone gauge. This pp-wave background [4] is also obtained from the $AdS_5 \times S^5$ via Penrose limit [3]. With this progress, the intensive studies of strings on the pp-waves are initiated. In particular, this type IIB string has been combined with the $AdS/CFT$ correspondence and the almost BPS sector of a large N gauge theory has been studied [8]. Moreover, the matrix model on the KG background has been proposed [8]. This model is often referred as the Berenstein-Maldacena-Nastase (BMN) matrix model. As the de Wit-Hoppe-Nicolai (dWHN) supermembrane [9–11] is closely related to the Banks-Fischler-Shenker-Susskind (BFSS) matrix.
model [12] in the flat space, the BMN matrix model is also intimately related to a supermembrane on the pp-wave [13–15]. In our previous works [14,15], we have shown that the algebra of supercharges in the supermembrane theory on the pp-wave agrees with that of the BMN matrix model in the same manner as the flat space [16]. We have also discussed BPS conditions in the supermembrane on the pp-wave. BPS multiplets in the BMN matrix model are also widely studied [13, 17]. Moreover, the classical solutions of the BMN matrix and the supermembrane are intensively researched [18,19]. In particular, we have lately investigated the quantum stability of giant gravitons [19], which are classical solutions of the BMN matrix model and exist due to the presence of the constant 4-form flux [20].

With recent progress, less supersymmetric type IIB and IIA pp-wave backgrounds, or strings on these pp-waves are greatly focused [21–28]. The maximal and less supersymmetric pp-wave backgrounds of the eleven-dimensional supergravity, type IIA and IIB theories are listed in Table 1 as far as we know. Motivated by these attempts, we consider the type IIA strings on the pp-waves from two viewpoints in this paper. On the one hand, we study the type IIA pp-waves and strings from the supergravity side through the toroidal compactification. On the other hand, we use the double dimensional reduction (DDR) [29] for the supermembrane action on the maximally supersymmetric pp-wave. Both results are equivalent as expected. We show that both compactifications are done for a spatial isometry direction, which can be found in the same way as in the type IIB case [21]. When we compactify this spatial direction, 8 supercharges are inevitably broken. Therefore, the resulting type IIA theory has 24 supercharges, not maximally supersymmetric. The type IIA string on this pp-wave is also

| SUGRA | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 |
|-------|----|----|----|----|----|----|----|----|----|
| 11 dim | o  | o  | o  | o  | o  | o  | ×  | ×  | unique |
| type IIA | o  | o  | o  | o  | o  | o* | ×  | ×  |        |
| type IIB | o  | o  | o  | o  | o  | o  | o  | unique |
exactly-solvable but it is different from the one obtained from a type IIB string theory via the T-duality [28]. This comes from the fact that the type IIA pp-wave with 24 supercharges is not unique and the type IIA pp-wave considered in this paper is different from the one in [28]. Moreover, the matrix string theory is considered. We also discuss the quantization of closed and open strings in our type IIA theory. There we study the allowed Dp-branes in the theory. The values $p = 2, 4, 6, 8$ are allowed but the directions of D-branes are restricted as in the case of type IIB string on the pp-wave [30–32].

This paper is organized as follows. In section 2 we consider the type IIA pp-wave backgrounds and actions of strings from two viewpoints. One is based on the analysis in the supergravity and the other is based on the double dimensional reduction. We will show both results are equivalent. In section 3 we consider the matrix string on the pp-wave and formally write down the action of the matrix string from the supermembrane action on the pp-wave in eleven dimensions. In section 4 we will discuss the mode-expansions and quantization of closed and open strings in the type IIA theory. We also discuss Dp-branes and investigate the allowed value $p$ and the direction of D-branes. Section 5 is devoted to conclusions and discussions. In Appendix we will briefly explain the compactification on an $SO(3)$-direction. The different points from the $SO(6)$-case considered in the text are summarized.

2 Type IIA Strings on PP-wave

2.1 Type IIA PP-wave Solution from KG Solution

We can consider the toroidal compactification of a spatial isometry direction in the eleven-dimensional maximally supersymmetric pp-wave (KG solution) given by

$$\begin{align*}
\frac{ds^2}{2} &= -2dX^+dX^- + G_{++}(X', X')(dX^+)^2 + \sum_{r=1}^9(dX^r)^2, \\
G_{++}(X', X') &\equiv -\left[ \left( \frac{\mu}{3} \right)^2 \sum_{i=1}^3(X^i)^2 + \left( \frac{\mu}{6} \right)^2 \sum_{i'=4}^9(X'^{i'})^2 \right],
\end{align*}$$

(2.1)

where the constant 4-form flux for $+, 1, 2, 3$ directions,

$$F_{+123} = \mu, \quad (\mu \neq 0)$$

(2.2)
is equipped. It is a unique pp-wave solution with 32 supercharges in eleven dimensions. The Killing vectors of the KG solution are constructed as follows [2]:

$$\xi_{e+} = -\partial_+ , \quad \xi_{e-} = \partial_-, $$

$$\xi_{e_I} = -\cos \left( \frac{\mu}{3} X^+ \right) \partial_I + \frac{\mu}{3} X^I \sin \left( \frac{\mu}{3} X^+ \right) \partial_-, \quad (I = 1, 2, 3), $$

$$\xi_{e_I'} = -\frac{\mu}{3} \sin \left( \frac{\mu}{3} X^+ \right) \partial_I - \left( \frac{\mu}{3} \right)^2 X^I \cos \left( \frac{\mu}{3} X^+ \right) \partial_-, $$

$$\xi_{e_{I'}} = -\cos \left( \frac{\mu}{6} X^+ \right) \partial_{I'} + \frac{\mu}{6} X^{I'} \sin \left( \frac{\mu}{6} X^+ \right) \partial_-, \quad (I' = 4, \ldots, 9), $$

$$\xi_{e_{I'}*'} = -\frac{\mu}{6} \sin \left( \frac{\mu}{6} X^+ \right) \partial_{I'} - \left( \frac{\mu}{6} \right)^2 X^{I'} \cos \left( \frac{\mu}{6} X^+ \right) \partial_-, $$

$$\xi_{M_{IJ}} = X^I \partial_J - X^J \partial_I , \quad (I, J = 1, 2, 3), $$

$$\xi_{M_{I'J'}} = X^{I'} \partial_{J'} - X^{J'} \partial_{I'} , \quad (I', J' = 4, \ldots, 9). $$

We utilize the procedure used in deriving the type IIA pp-wave from the maximally supersymmetric type IIB pp-wave through the T-duality [21]. The spatial isometries are given by

$$\xi_{e_j} \pm \frac{3}{\mu} \xi_{e_j'} \quad \text{and} \quad \xi_{e_{j'}} \pm \frac{6}{\mu} \xi_{e_{j'}}^*. $$

It is sufficient to consider only two cases; $\xi_{e_1} + (3/\mu)\xi_{e_2}$ ($SO(3)$-direction) and $\xi_{e_4} + (6/\mu)\xi_{e_5}$ ($SO(6)$-direction) due to the $SO(3) \times SO(6)$ symmetry of the KG background. The resulting type IIA pp-wave background has 24 supercharges since 8 supercharges are inevitably broken in the toroidal compactification on the spatial isometry direction in the same way as the construction of type IIA pp-wave from type IIB via T-duality. In below, we will discuss mainly the $SO(6)$-direction case. The case of $SO(3)$ is discussed in Appendix A, since the story is very similar to the $SO(6)$ case though there are a few differences, such as the field contents.

Let us discuss the Killing spinor in the above type IIA pp-wave. The Killing spinor in the KG solution is given by [2]

$$\epsilon(\psi_+, \psi_-) = \exp \left( -\frac{\mu}{4} X^+ I \right) \psi_- + \exp \left( -\frac{\mu}{12} X^+ I \right) \psi_+ $$

$$+ \frac{\mu}{6} \left[ \sum_{j=1}^{3} X^j \Gamma_j - \frac{1}{2} \sum_{j'=4}^{9} X^{j'} \Gamma_{j'} \right] I \exp \left( +\frac{\mu}{12} X^+ I \right) \Gamma_- \psi_+ , \quad (2.3)$$

where $\Gamma_j$’s are $32 \times 32$ gamma matrices and $I \equiv \Gamma_{123}$ obeys $I^2 = -1$. The spinors $\psi_+$ and $\psi_-$ with 32 components satisfy the conditions

$$\Gamma_+ \psi_+ = 0 , \quad \Gamma_- \psi_- = 0 , \quad (2.4)$$
hence they have 16 non-vanishing components. If \( X \) is a Killing vector, we can define an associated Lie derivative \( \mathcal{L}_X \) on any spinor \( \psi \) by

\[
\mathcal{L}_X \psi = X^M \nabla_M \psi + \frac{1}{4} \nabla_{[M} X_N] \Gamma^{MN} \psi,
\]

where \( \nabla \) is defined by

\[
\nabla_M \equiv \partial_M + \frac{1}{4} \omega^q_M \Gamma_{ab}.
\]

This has the following properties:

1. If \( X \) is a Killing vector field, \( f \) is any smooth function and \( \psi \) is any spinor, then

\[
\mathcal{L}_X (f \psi) = (Xf) \psi + f \mathcal{L}_X \psi.
\]

2. When the symbol “.” (dot) denotes the Clifford action of vector fields on spinors, then

\[
\mathcal{L}_X (Y \cdot \psi) = [X, Y] \cdot \psi + Y \cdot \mathcal{L}_X \psi.
\]

3. If \( X, Y \) are Killing vector fields and \( \psi \) is any spinor, then

\[
\mathcal{L}_X \mathcal{L}_Y \psi - \mathcal{L}_Y \mathcal{L}_X \psi = \mathcal{L}_{[X,Y]} \psi.
\]

The Lie derivatives for the Killing vector fields are given by [2]

\[
\begin{align*}
\mathcal{L}_{\xi_{-}} \epsilon(\psi_+, \psi_-) &= 0, & \mathcal{L}_{\xi_{+}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{\mu}{12} \Gamma_+ \psi_+, -\frac{\mu}{4} \Gamma_- \psi_- \right), \\
\mathcal{L}_{\xi_{I}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{\mu}{6} I \Gamma_+ \psi_+, 0 \right), & \mathcal{L}_{\xi_{J}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{\mu}{12} I \Gamma_- \psi_+, 0 \right), \\
\mathcal{L}_{\xi_{I}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{\mu^2}{18} \Gamma_+ \psi_+, 0 \right), & \mathcal{L}_{\xi_{J}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{\mu^2}{72} \Gamma_- \psi_+, 0 \right), \\
\mathcal{L}_{\xi_{M_1}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{1}{2} \Gamma_1 \psi_+, 0 \right), & \mathcal{L}_{\xi_{M_2}} \epsilon(\psi_+, \psi_-) &= \epsilon \left( -\frac{1}{2} \Gamma_2 \psi_+, 0 \right).
\end{align*}
\]

By the use of the above results, we can count the remaining unbroken supersymmetries. For an example, in the case of \( \xi_{I} + (3/\mu) \xi_{J} \), we obtain the following expression

\[
\mathcal{L}_{\xi_{I} + (3/\mu) \xi_{J}} = \epsilon \left( -\frac{\mu}{3} Q I \Gamma_+ \psi_+, 0 \right),
\]

\[
Q^{IJ} \equiv \frac{1}{2} (1 + \Gamma_1 \Gamma_2).
\]
Clearly, 16 spinors are annihilated by $\Gamma_-$. Furthermore, the constant matrix $Q$ plays the role of the projection operator and so annihilates additional 8 spinors in the same manner as the type IIB string [21]. For another example, in the case of $\xi_{e,j'} + (6/\mu)\xi_{\bar{e},j}$, the Lie derivative is given by

$$\mathcal{L}_{\xi_{e,j'}+(6/\mu)\xi_{\bar{e},j}} = \epsilon \left( -\frac{\mu}{6} Q \Gamma_{j'} \Gamma_- \psi_+, 0 \right),$$

$$Q^{j'j'} = \frac{1}{2} (1 + \Gamma_{j'} \Gamma_{j'}). \quad (2.7)$$

In the same way as the case of $\xi_{e,l} + (3/\mu)\xi_{\bar{e},l}$, 24 supersymmetries are preserved. In conclusion, the above two cases of the type IIA pp-wave backgrounds preserve 24 supersymmetries.

2.2 Type IIA String from PP-wave Solution of D=11 Supergravity

Here, we shall consider the toroidal compactification on an $SO(6)$ direction. Let us transform the variables $X^r$, $(r = +, -, 1, \ldots, 9)$ into $x$’s

$$X^+ = x^+, \quad X^- = x^- + \frac{\mu}{6} x^4 x^5,$$

$$X^I = x^I, \quad (I = 1, 2, 3) \quad X^a = x^a, \quad (a = 6, 7, 8, 9)$$

$$X^4 = x^4 \cos \left( \frac{\mu}{6} x^+ \right) - x^5 \sin \left( \frac{\mu}{6} x^+ \right), \quad X^5 = x^4 \sin \left( \frac{\mu}{6} x^+ \right) + x^5 \cos \left( \frac{\mu}{6} x^+ \right), \quad (2.8)$$

then the metric is rewritten as

$$ds^2 = -2dx^+dx^- + G'_{++}(x^I, x^a)(dx^+)^2 + \sum_{r=1}^{9} (dx^r)^2 - \frac{2}{3} \mu x^5 dx^+ dx^4,$$

$$G'_{++}(x^I, x^a) \equiv - \left[ \left( \frac{\mu}{3} \right)^2 \sum_{I=1}^{3} (x^I)^2 + \left( \frac{\mu}{6} \right)^2 \sum_{a=6}^{9} (x^a)^2 \right], \quad (2.9)$$

but the constant 4-form flux is still expressed in Eq.(2.2). We can easily read off from the above metric (2.9) that the $x^4$-direction is a manifest spatial isometry direction [21] and obtain the metric of the type IIA by the standard technique of the dimensional reduction from the eleven-dimensional supergravity to the type IIA supergravity in ten dimensions,

$$ds_{11}^2 = e^{-\frac{2}{3} \phi} g_{\mu\nu} dx^\mu dx^\nu + e^{\frac{4}{3} \phi} (dy + dx^\mu A_\mu)^2, \quad (2.10)$$
where $g_{\mu\nu}$ is a ten-dimensional metric, $A_\mu$ is a Kaluza-Klein gauge field (R-R 1-form) and $\phi$ is a dilaton. The ten-dimensional metric $g_{\mu\nu}$ is given by

$$g_{\mu\nu}dx^\mu dx^\nu = -2dx^+dx^- + g_{++}(x^a, x^b)(dx^+)^2 + \sum_{a=1}^{4}(dx^a)^2 + \sum_{b=5}^{8}(dx^b)^2, \quad (2.11)$$

$$g_{++}(x^a, x^b) \equiv -\left(\frac{\mu}{3}\right)^2\sum_{a=1}^{4}(x^a)^2 + \left(\frac{\mu}{6}\right)^2\sum_{b=5}^{8}(x^b)^2, \quad (2.12)$$

where we have made rearrangement of 8 coordinates $x^1, x^2, x^3, \ldots, x^8$ into $x^1, \ldots, x^8$. The Kaluza-Klein gauge field $A_\mu$ is expressed as

$$A_+ = -\frac{\mu}{3}x^4, \quad A_i = 0, \quad (i = 1, \ldots, 8), \quad (2.12)$$

and the R-R 3-form $C_{\mu\nu\rho}$ is given by

$$C_{+IJ} = -\frac{\mu}{3}\epsilon_{IJK}x^K, \quad (I, J, K = 1, 2, 3). \quad (2.13)$$

The dilaton $\phi$ and NS-NS 2-form $B_{\mu\nu}$ vanish.

Next, we discuss the action of the type IIA string theory on the above pp-wave. In below, we shall construct the action of bosonic and fermionic sectors, respectively.

**Bosonic Sector**

In general, the bosonic world-sheet action with non-zero NS-NS B-field is written as

$$S_B = -\frac{1}{4\pi\alpha'} \int d\tau \int_0^{2\pi L} d\sigma \left[ g_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu \eta^{ab} + B_{\mu\nu}\partial_\alpha x^\mu \partial_\beta x^\nu \epsilon^{ab} \right], \quad (2.14)$$

where the subscript $a$ denotes the coordinates of the string world-sheet $\sigma^a = (\tau, \sigma)$ and $\eta = \text{diag}(-1, 1)$ is the world-sheet metric. The $L$ is the arbitrary length parameter. The convention of the anti-symmetric tensor is taken as $\epsilon^{\tau\sigma} = 1$. The ten-dimensional metric obtained previously and the light-cone gauge condition $x^+ = \tau$ lead to the bosonic action of the type IIA string theory written as

$$S_B = \frac{1}{4\pi\alpha'} \int d\tau \int_0^{2\pi} d\sigma \left[ \sum_{i=1}^{8} \left[ (\partial_\tau x^i)^2 - (\partial_\sigma x^i)^2 \right] - \left(\frac{\mu}{3}\right)^2\sum_{a=1}^{4}(x^a)^2 - \left(\frac{\mu}{6}\right)^2\sum_{b=5}^{8}(x^b)^2 \right], \quad (2.15)$$
where we have rescaled the parameters $\tau$, $\sigma$ and $\mu$ as

$$\tau \rightarrow L\tau, \quad \sigma \rightarrow L\sigma, \quad \mu \rightarrow \frac{\mu}{L}.$$  

**Fermionic Sector**

In order to construct the fermionic action, we need explicit expressions of the covariant derivatives. In the celebrated work [5], the covariant derivatives are obtained by the coset construction in $AdS_5 \times S^5$. However, the resulting covariant derivatives become those appearing in the type IIB supergravity with non-trivial background fluxes when we take the light-cone gauge conditions. Hence, we could have a short-cut in our hand [6]. It would be sufficient to use the covariant derivatives in supergravity with background fluxes and the expressions of the covariant derivatives are already known. Following the work [28], we can derive the fermionic part of the action using the generalized covariant derivative $D$ defined by *

$$\left( D_a \right)_{pq} = \partial_a \delta_{pq} + \frac{1}{4} \partial_a x^\rho \left[ (\omega_{\mu \rho} \delta_{pq} - \frac{1}{2} H_{\mu \nu \rho} (\sigma_3)_{pq}) \gamma^{\mu \nu} \right. \\

\left. + \left( \frac{1}{2 \cdot 2!} F_{\mu \nu} \gamma^{\mu \nu} (i \sigma_2)_{pq} + \frac{1}{2 \cdot 4!} F_{\mu \nu \lambda \delta} \gamma^{\mu \nu \lambda \delta} (\sigma_1)_{pq} \right) \gamma_\rho \right], \quad (2.16)$$

where $\sigma_i$’s ($i = 1, 2, 3$) are Pauli matrices. The $H_{\mu \nu \rho}$ is the 3-form field strength of the NS-NS B-field. The $F_{\mu \nu}$ and $F_{\mu \nu \lambda \delta}$ are the 2 and 4-form field strengths of the R-R 1-form $A_\mu$ and 3-form $C_{\mu \nu \lambda}$, respectively. Here, we have ignored the contribution of higher Kaluza-Klein modes which has the spectrum tower with the energy difference $1/R$. Now, the energy difference is so large that we can ignore the $n \neq 0$ sectors. After this truncation, we are restricted to the $n = 0$ sector and the gauge coupling constant $n/R$ is effectively zero.

Using this covariant derivative, we can obtain the quadratic fermionic action of the type IIA described by

$$S_F = \frac{i}{2\pi} \int d\tau \int_0^{2\pi L} d\sigma \sum_{p,q,r=1}^2 (\eta^{ab} \delta_{pq} - \epsilon^{ab} (\sigma_3)_{pq}) \partial_a x^\mu \bar{\psi}_p \gamma_\mu (D_b)_{qr} \psi_r, \quad (2.17)$$

*It has been reported that the numerical coefficients in the covariant derivatives in the type IIA and the type IIB include some issues [24, 28]. But it should be remarked that these are based on the difference of the convention in Ref. [33], and not on the incorrectness. We thank C.N. Pope for the valuable comment on this point.
where $\theta^p$’s ($p = 1, 2$) are two 16-component spinors with different chiralities in ten dimensions. When we set the light-cone gauge conditions,

$$x^+ = \tau, \quad \gamma^+ \theta^p = 0,$$

then in the same way as the type IIB case [6] the above action can be rewritten as

$$S_F = - \frac{i}{2\pi} \int d\tau \int d\sigma \sum_{p,q,r=1}^2 \bar{\theta}^p \gamma_+ (\delta_{pq} (D_{\tau})_{qr} + (\sigma_3)_{pq} (D_{\sigma})_{qr}) \theta^r,$$  

(2.18)

where the length parameter should be fixed as $L = \alpha' |p^+|$ now. The covariant derivatives are also rewritten as

$$(D_{\tau})_{pq} = \partial_{\tau} \delta_{pq} + \frac{1}{4} \left[ (\omega_{\mu\nu} \delta_{pq} - \frac{1}{2} H_{\mu\nu} (\sigma_3)_{pq}) \gamma^{\mu\nu} \\
+ \left( \frac{1}{2} \cdot 2! F_{\mu\nu}^{\gamma} + \frac{1}{2} \cdot 4! F_{\mu\nu\lambda\delta}^{\gamma} (\sigma_1)_{pq} \right) \gamma_+ \right],$$

$$(D_{\sigma})_{pq} = \partial_{\sigma} \delta_{pq}.$$  

(2.19)

When we use the constant 2 and 4-form field strengths $F_{+4} = \frac{\mu}{3}$ and $F_{+123} = \mu$, the fermionic action can be rewritten as

$$S_F = \frac{i}{2\pi} \int d\tau \int d\sigma \left[ \psi^T \partial_{\tau} \psi + \psi^T \gamma_9 \partial_{\sigma} \psi + \frac{\mu}{4} \psi^T \left( \gamma_{123} + \frac{1}{3} \gamma_{49} \right) \psi \right],$$  

(2.20)

where we have introduced a 16 component spinor $\psi$ defined by

$$\psi = \begin{pmatrix} (\psi^1 + \psi^2)/\sqrt{2} \\ (\psi^1 - \psi^2)/\sqrt{2} \end{pmatrix} \equiv \begin{pmatrix} \Psi^1 \\ \Psi^2 \end{pmatrix}.$$  

The 8 component spinors $\psi^1$ and $\psi^2$ are given by

$$\theta^i \equiv \frac{1}{2^{1/4}} \begin{pmatrix} 0 \\ \psi^i \end{pmatrix}, \quad (i = 1, 2)$$

due to the light-cone conditions. Also, in the same way as in the bosonic sector, we have rescaled the parameters $\tau, \sigma, \mu$ and fermion $\psi$ as

$$\tau \to L \tau, \quad \sigma \to L \sigma, \quad \mu \to \frac{\mu}{L}, \quad \psi \to \frac{\psi}{L}.$$  

(2.21)
2.3 Type IIA String via Double Dimensional Reduction

By following the work [11] in the light-cone gauge in terms of an \( SO(9) \) spinor \( \psi \), we can write down the action of the supermembrane on the pp-wave [13, 14] as

\[
S = \frac{1}{\ell_M^3} \int d\tau \int_0^{2\pi L} d\sigma \int_0^{2\pi L} d\rho \mathcal{L},
\]

\[
w^{-1}\mathcal{L} = \frac{1}{2} \left[ D_\tau X^r D_\tau X^r - \frac{1}{2} (\{X^r, X^s\})^2 - \left( \frac{\mu}{3} \right)^2 \sum_{i=1}^3 X_i^2 - \left( \frac{\mu}{6} \right)^2 \sum_{i'=4}^9 X_i^{2'} \right. \\
- \frac{\mu}{3} \sum_{I,J,K=1}^3 \epsilon_{IJK} X^K \{X^I, X^J\} \left. \right] + i\psi^T \gamma^r \{X^r, \psi\} + i\psi^T D_\tau \psi + i\frac{\mu}{4} \psi^T \gamma_{123} \psi,
\]

where \((\sigma^0, \sigma^1, \sigma^2) = (\tau, \sigma, \rho)\) is the set of world-volume coordinates on the membrane and the \(\{,\}\) is a Lie bracket given by using an arbitrary function \(w(\sigma, \rho)\) of world-volume spatial coordinates \(\sigma^a\) \((a = 1, 2)\)

\[
\{A, B\} \equiv \frac{\epsilon^{ab}}{w} \partial_a A \partial_b B, \quad (a, b = 1, 2),
\]

with \(\partial_a = \partial/\partial \sigma^a\). This theory has the \(\tau\)-independent gauge symmetry called the area-preserving diffeomorphism (APD). It is a residual symmetry belonging to the reparametrization invariance of the membrane world-volume. When we use the gauge connection \(\omega\), the covariant derivative for this gauge symmetry is defined by

\[
D_\tau X^r \equiv \partial_\tau X^r - \{\omega, X^r\}, \quad (r = 1, 2, \cdots, 9).
\]

We have also introduced a parameter \(\ell_M\), which is the M-theory scale related to the membrane tension \(T_M = 1/\ell_M^2\). It is associated to the string coupling \(g_s\) and the string scale \(\ell_s\) in ten-dimensional string theory (up to some numerical constant) with a relation \(\ell_M = g_s^{1/3} \ell_s\). We use a normalization

\[
0 \leq \sigma \leq 2\pi L, \quad 0 \leq \rho \leq 2\pi L, \quad \int d\sigma d\rho w(\sigma, \rho) = L^2,
\]

with \(L\) being an arbitrary length parameter. In our light-cone gauge, the time coordinate “\(\tau\)” is associated to the \(X^+\) as \(X^+ = (\ell_M^3/(2\pi L)^2)P_0^+ \tau\) and the longitudinal momentum \(P^+(\sigma, \rho)\) satisfies \(P^+(\sigma, \rho) = (P_0^+/L^2) w(\sigma, \rho)\). Hereafter we shall use a convention \(P_0^+ = 1\).
Here, we shall consider the double dimensional reduction (DDR) in the $SO(6)$-direction. It is considered that 11 dimensional supermembrane theory in the flat space should reduce to the type IIA string theory, at least classically. Based on this fact, we shall carry out the DDR of the supermembrane on the pp-wave. We will show that the type IIA string action on the pp-wave obtained in the previous subsection can be derived from the supermembrane action on the pp-wave (2.22) through the double dimensional reduction.

To begin, we rotate the variables $X^1, \cdots, X^9$ into $x$'s

$$X^I = x^I, \quad (I = 1, 2, 3), \quad X^a = x^a, \quad (a = 6, 7, 8, 9),$$

$$X^4 = x^4 \cos \left( \frac{\mu_6}{6} \tau \right) - x^5 \sin \left( \frac{\mu_6}{6} \tau \right), \quad X^5 = x^4 \sin \left( \frac{\mu_6}{6} \tau \right) + x^5 \cos \left( \frac{\mu_6}{6} \tau \right),$$

then an associated action is written down as follows:

$$S = \frac{1}{\ell_3^3} \int d\tau \int_0^{2\pi} d\sigma \int_0^{2\pi L_0} d\rho \mathcal{L},$$

$$w^{-1} \mathcal{L} = \frac{1}{2} \left[ (D_\tau x^r)^2 - \frac{1}{2} (\{x^r, x^s\})^2 - \left( \frac{\mu_6}{3} \right)^2 \sum_{i=1}^3 (x^i)^2 - \left( \frac{\mu_6}{6} \right)^2 \sum_{i'=6}^9 (x^{i'})^2 \right]$$

$$- \frac{\mu_3}{3} \sum_{i,j,k=1} \epsilon_{ijk} x^K \{x^r, x^s\} - \frac{2}{3} \mu_6 x^5 D_\tau x^4$$

$$+ i \psi^T \gamma^r \{x^r, \psi\} + i \psi^T d_\tau \psi + i \frac{\mu_4}{4} \psi^T \left( \gamma_{123} + \frac{1}{3} \gamma_{54} \right) \psi,$$

where it should be noted that the fermion mass term is modified compared with flat case. This contribution appears since we have moved to the rotated coordinate. In this time, the additional spin connection $\omega_{45}^{+} = \mu/6$ appears $\dagger$. Now let us consider the DDR in the $x^4$-direction. That is, we take $x^4 = \rho$. We choose the density function $w(\sigma, \rho)$ to be a constant so that $w = (2\pi)^{-2}$ and fix the parameter $L$ as $L = g_s \ell_s$. The resulting action is given by

$$S_{st} = \frac{1}{2\pi} \int d\tau \int_0^{2\pi} d\sigma \mathcal{L}_{st},$$

$$\mathcal{L}_{st} = \frac{1}{2\alpha'} \left[ \sum_{i=1}^8 \left[ (\partial_\tau x^i)^2 - (\partial_\sigma x^i)^2 \right] - \left( \frac{\mu_6}{3} \right)^2 \sum_{a=1}^4 (x^a)^2 - \left( \frac{\mu_6}{6} \right)^2 \sum_{b=5}^8 (x^b)^2 \right]$$

$$+ i \psi^T \left[ 1_{16} \cdot \partial_\tau - \gamma_9 \cdot \partial_\sigma + \frac{\mu_4}{4} \left( \gamma_{123} + \frac{1}{3} \gamma_{54} \right) \right] \psi,$$

$\dagger$We have modified the contribution of the spin connection in the revised version. This contribution is initially pointed out in Ref. [34] where the correct type IIA action is obtained.
where we have renamed the coordinate $x^4, x^5, \ldots, x^9$ as $x^9, x^4, x^5, \ldots, x^8$. It should be understood that the mass term of $x^4$ arises from the second term in Eq. (2.10). This term should describe the effect of the Kaluza-Klein 1-form. Also, the fermionic field and parameters has been appropriately rescaled as

$$
\sigma \to L\sigma, \quad \tau \to \frac{L}{(2\pi)^2} \tau, \quad \psi \to \frac{L^{3/2}}{L} \psi, \quad \mu \to \frac{(2\pi)^2}{L} \mu.
$$

The parameters of the resulting theory are related with those of M-theory

$$
\frac{1}{2\pi \alpha'} = \frac{(2\pi) L}{\ell_M^3}, \quad \ell_M = g_s^{1/3} \ell_s, \quad L = g_s \ell_s, \quad \ell_s = 2\pi \sqrt{\alpha'}.
$$

It should be noted that the above action is identical with the type IIA action derived in the previous subsection up to the sign of $\sigma$. Hereafter, we can use the following expression of $\gamma^\mu = (\gamma^i, \gamma^8, \gamma^9)$,

$$
\gamma^i = \tilde{\gamma}^i \otimes \sigma_2 = \begin{pmatrix} 0 & -i \tilde{\gamma}^i \\ i \tilde{\gamma}^i & 0 \end{pmatrix}, \quad (i = 1, \ldots, 7), \tag{2.26}
$$

$$
\gamma^8 = 1_8 \otimes \sigma_1 = \begin{pmatrix} 0 & 1_8 \\ 1_8 & 0 \end{pmatrix}, \quad \gamma^9 = 1_8 \otimes \sigma_3 = \begin{pmatrix} 1_8 & 0 \\ 0 & -1_8 \end{pmatrix}, \tag{2.27}
$$

where $\tilde{\gamma}^i$s ($i = 1, \ldots, 7$) are $SO(7)$ gamma matrices that obey commutation relations

$$
\tilde{\gamma}^i \tilde{\gamma}^j + \tilde{\gamma}^j \tilde{\gamma}^i = 2\delta^{ij}. \tag{2.28}
$$

The 16 component fermion $\psi$ is decomposed into two 8 component fermions $\Psi^1$ and $\Psi^2$ as

$$
\psi = \begin{pmatrix} \Psi^1 \\ \Psi^2 \end{pmatrix}.
$$

Moreover, we can decompose the 8 component fermion into two eigen-spinors of the matrix $R \equiv \tilde{\gamma}_{1234}$ as follows:

$$
\Psi^a = \left( \frac{1 + R}{2} \right) \Psi^a + \left( \frac{1 - R}{2} \right) \Psi^a \\
\equiv \Psi^{a+} + \Psi^{a-}, \quad (a = 1, 2). \tag{2.29}
$$

By definition, the spinor $\Psi^{a\pm}$ satisfies

$$
R \Psi^{a\pm} = \pm \Psi^{a\pm}. \tag{2.30}
$$
That is, $\Psi^{a\pm}$ is the eigen-spinor with eigen-value $\pm 1$. By the use of $\Psi^{a\pm}$, the fermionic Lagrangian can be rewritten as

$$L = i\Psi^1 \partial_{\tau} \Psi^1 + i\Psi^1 \tau \partial_{-} \Psi^1 - i\Psi^2 \tau \partial_{+} \Psi^2 + i\Psi^2 \partial_{\tau} \Psi^2 - \frac{i\mu}{3}\Psi^1 \Pi^r \Psi^2 - \frac{i\mu}{6}\Psi^1 \Pi^r \Pi^r \Psi^2 - \frac{i\mu}{6}\Psi^2 \Pi^r \Pi^r \Psi^1 + \frac{i\mu}{3}\Psi^2 \partial_{\tau} \Psi^1,$$

where $\Pi \equiv \tilde{\gamma}_{123}$, $\Pi^r \equiv \tilde{\gamma}_{321}$ and satisfies $\Pi \Pi^r = \Pi^r \Pi = 1$.

### 3 Matrix String Theory on PP-wave

We can also consider the matrix string theories [35] on the pp-wave \footnote{Matrix strings are also discussed in Refs. [37, 38] from different viewpoints from ours.} from the supermembrane by the use of the method in the work [36].

Let us start with the supermembrane action (2.22), and rotate the variables into $x$’s as given by (2.23). In this time, the gamma matrices are also transformed by this rotation. The resulting supermembrane action is given by

$$S = \frac{1}{\ell^3} \int d\tau \int_0^{2\pi} d\sigma \int_0^{2\pi L} d\rho \mathcal{L},$$

$$\mathcal{L} = \frac{1}{2} \left[(D_{\tau} x)^2 - \frac{1}{2} (\{x^r, x^s\})^2 - \left(\frac{\mu}{3}\right)^2 \sum_{i=1}^{3} (x^i)^2 - \left(\frac{\mu}{6}\right)^2 \sum_{i'=6}^{9} (x^{i'})^2 - \frac{\mu}{3} \sum_{i,j,k=1}^{3} \epsilon_{ijk} x^k \{x^i, x^j\} - \frac{\mu}{3} x^5 D_{\tau} x^4 \right]$$

$$+i\psi^T \gamma^{r} \{x^r, \psi\} + i\psi^T D_{\tau} \psi + i\frac{\mu}{4} \psi^T \left(\gamma_{123} + \frac{1}{3} \gamma_{54}\right) \psi,$$

where we have set $w = (2\pi)^{-2}$ and rescaled $\sigma$ as $\sigma \to (2\pi)^2 \sigma$. Now, the Lie bracket $\{,\}$ is simply defined by

$$\{A, B\} \equiv \partial_{\sigma} A \partial_{\rho} B - \partial_{\rho} A \partial_{\sigma} B.$$

Then, we rewrite $x^4$ as $x^4 \equiv Y$ and shift $Y$ as

$$Y \rightarrow \rho + Y.$$
The $Y$ is regarded as the compactified direction. As the result, the action is rewritten as

$$S = \frac{L}{\ell_M^3} \int d\tau \int_0^{2\pi} d\sigma \int_0^{2\pi} d\rho \mathcal{L},$$

(3.2)

$$\mathcal{L} = \frac{1}{2} \left[ F^2_{\sigma,\tau} + (D_\tau x^i)^2 - (D^\sigma x^i)^2 - \frac{1}{2L^2} \left( \{ x^i, x^j \} \right)^2 - \left( \frac{\mu}{3} \right)^2 \sum_{i=1}^{3} (x^i)^2 - \left( \frac{\mu}{6} \right)^2 \sum_{i'=5}^{8} (x^{i'}^2) \right]
$$

$$- \frac{\mu}{3L} \sum_{i,j,k=1}^{3} \epsilon_{ijk} x^K \{ x^i, x^j \} - \frac{2}{3} \mu x^4 F_{0,\sigma}
$$

$$+ i \frac{1}{L} \bar{\psi} \gamma_\tau \{ x^i, \psi \} + i \bar{\psi} \gamma_\sigma \gamma^9 D^\sigma \psi - i \bar{\psi} \gamma^7 D^\tau \psi + i \frac{\mu}{4} \bar{\psi} \left( \gamma_{123} + \frac{1}{3} \gamma_{49} \right) \psi,$$

where we have reassigned the variables $x^4, x^5, \ldots, x^9$ as $x^9, x^4, x^5, \ldots, x^8$ and rescaled $\rho \rightarrow L\rho$.

We have also introduced the following quantities,

$$F_{0,\sigma} \equiv \partial_\tau Y - \partial_\sigma \omega - \frac{1}{L} \{ \omega, Y \},$$

$$D_\tau x^i \equiv \partial_\tau x^i - \frac{1}{L} \{ \omega, x^i \},$$

$$D^\sigma x^i \equiv \partial_\sigma x^i - \frac{1}{L} \{ Y, x^i \},$$

where $A_0 \equiv \omega$ and $A_\sigma \equiv Y$. The inverse compactification radius $1/L$ plays a role of the gauge coupling constant. It seems that the action (3.2) is not explicitly invariant under the are-preserving diffeomorphism. But the action indeed has this symmetry under the transformation with an infinitesimal gauge parameter $\Lambda$

$$\delta \omega = L \partial_\tau \Lambda + \{ \Lambda, \omega \},$$

$$\delta Y = L \partial_\sigma \Lambda + \{ \Lambda, Y \},$$

$$\delta x^i = \{ \Lambda, x^i \}, \quad \delta \psi = \{ \Lambda, \psi \}.$$

The action (3.2) is very close to that of the matrix string theory. In fact, using the corresponding law of Ref. [36] in the large $N$ limit, it is straightforward to map the supermembrane action into matrix representations. Thus, we can formally obtain the matrix string action on
the pp-wave, up to $O(1/N^2)$ described by

$$ S = \frac{L}{\ell_3^3} \int d\tau \int_0^{2\pi} \frac{2\pi}{N} d\theta \mathcal{L}, $$

$$ \mathcal{L} = \frac{1}{2} \text{Tr} \left[ \left( F_{0,\theta}^2 + (D_\tau x^i)^2 - N^2 (D_\theta x^i)^2 + \frac{1}{2} \left( \frac{N}{2\pi L} \right)^2 ([x^i, x^j])^2 \right. \\
- \left. \left( \frac{\mu}{3} \right)^2 \frac{3}{2} \sum_{i=1}^3 (x^i)^2 - \left( \frac{\mu}{6} \right)^2 \frac{8}{5} \sum_{i'=5}^8 (x^{i'})^2 \right. \\
+ i \frac{\mu}{3} \left( \frac{N}{2\pi L} \right)^2 \sum_{i,i',i''=1}^3 \epsilon_{i,i',i''} x^{i''} [x^i, x^{i'}] - \frac{2}{3} \mu x^4 F_{0,\theta} \right] \\
+ \text{Tr} \left[ \frac{N}{2\pi L} \psi^T \gamma^i [x^i, \psi] + i \psi^T D_\tau \psi - N i \psi^T \gamma^9 D_\theta \psi + i \frac{\mu}{4} \psi^T \left( \gamma_{123} + \frac{1}{3} \gamma_{49} \right) \psi \right], $$

where the quantities in the action is replaced with

$$ F_{0,\theta} = \partial_\tau Y - N \partial_\theta \omega + i \frac{N}{2\pi L} \omega, Y, $$
$$ D_\tau x^i = \partial_\tau x^i + i \frac{N}{2\pi L} \omega, x^i, $$
$$ D_\theta x^i = \partial_\theta x^i + i \frac{1}{2\pi L} [Y, x^i]. $$

If we rescale some constants as

$$ \tau \rightarrow \frac{\tau}{N}, \quad \psi \rightarrow \sqrt{N} \psi, \quad L \rightarrow \frac{L}{2\pi}, \quad \mu \rightarrow N \mu, $$

then we can rewrite Eq.(3.3) as

$$ S = \frac{1}{\ell_3^3} \int d\tau \int_0^{2\pi} d\theta \mathcal{L}, $$

$$ \mathcal{L} = \frac{1}{2} \text{Tr} \left[ \left( F_{0,\theta}^2 - \left( \frac{\mu}{3} x^4 \right)^2 \right) + (D_\tau x^i)^2 - (D_\theta x^i)^2 + \frac{1}{2} ([x^i, x^j])^2 \right. \\
- \left. \left( \frac{\mu}{3} \right)^2 \frac{3}{2} \sum_{i=1}^3 (x^i)^2 - \left( \frac{\mu}{6} \right)^2 \frac{8}{5} \sum_{i'=5}^8 (x^{i'})^2 + i \frac{2}{3} \mu \sum_{i,i',i''=1}^3 \epsilon_{i,i',i''} x^{i''} [x^i, x^{i'}] \right. \\
+ \text{Tr} \left[ \psi^T \gamma^i [x^i, \psi] + i \psi^T D_\tau \psi - i \psi^T \gamma^9 D_\theta \psi + i \frac{\mu}{4} \psi^T \left( \gamma_{123} + \frac{1}{3} \gamma_{49} \right) \psi \right], $$

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where the field strength and covariant derivatives are given by

\[ F_{0,\theta} = \partial_\tau Y - \partial_\theta \omega + i[\omega, Y], \]
\[ D_\tau x^i = \partial_\tau x^i + i[\omega, x^i], \]
\[ D_\theta x^i = \partial_\theta x^i + i[Y, x^i]. \]

The action (3.4) includes the 3-point interaction and several mass terms and also the field strength of the gauge connection is shifted by \( x^4 \). Thus it seems that the action (3.4) is not invariant under the area-preserving diffeomorphism. However this action of the matrix string is actually invariant under the gauge transformation with an matrix parameter \( \Lambda \)

\[ \delta \omega = \partial_\tau \Lambda - i[\Lambda, \omega], \]
\[ \delta Y = \partial_\theta \Lambda - i[\Lambda, Y], \]
\[ \delta x^i = -i[\Lambda, x^i], \delta \psi = -i[\Lambda, \psi]. \]

The \( \tau \)-scaling leads to the \( N \) dependence of the physical light-cone time \( X^+ \) as

\[ X^+ = \frac{\ell_3^3 P_0^+ \tau}{N(2\pi L)^2}. \]

We also should rescale as \( P_0^+ \to N P_0^+ \) so that \( X^+ \) should be independent of \( N \). The diagonal elements of the matrix \( x^i \) describe a fundamental string bit in the large \( N \) limit and hence the total longitudinal momentum is proportional to the number \( N \) of string bits.

It is easily observed that the above action (3.4) becomes the usual matrix string action in the flat limit \( \mu \to 0 \). Moreover, let us consider the IR region. At the time, the matrix variables are restricted to the Cartan subalgebra. That is, the matrix becomes diagonal and so the term including commutator should vanish. Finally, integrating out the field strength \( F_{0,\theta} \) as the auxially field, one can find that the above action should reduce to the free type IIA string theory obtained in the previous section, as is expected.

Also, we should remark that the above action of the matrix string is included in the family of the work [37] where the action of the matrix string and supersymmetry have been more generally investigated from the viewpoint of the mass deformation of the Yang-Mills theory.

Finally, we comment on the classical solution. As in the BMN matrix model [8], for an
example, this matrix string theory has the static fuzzy sphere solution described by

\[ x^I = \frac{\mu}{3} J^I, \quad (I = 1, 2, 3), \]
\[ x^4 = \cdots = x^8 = Y = \omega = 0, \quad (3.5) \]

where the \( J^I \)'s are generators of an \( SU(2) \) algebra. The existence of the fuzzy sphere solution might be physically expected from the presence of the constant flux of R-R 3-form [20]. It would be possible to consider other classical solutions.

4 Quantization of Type IIA String on PP-wave

In this section we will consider the mode-expansions and quantization of closed and open strings in the type IIA on the pp-wave. In particular, we investigate D-branes living in the theory.

4.1 Closed Strings in Type IIA on PP-wave

In this subsection we will discuss the mode-expansions of closed bosonic and fermionic degrees of freedom and consider the quantization of the type IIA string.

First the variation of the action previously obtained leads to equations of motion given by

\[ \partial_+ \partial_- x^a + \frac{\mu^2}{9} x^a = 0, \quad (a = 1, 2, 3, 4), \quad (4.1) \]
\[ \partial_+ \partial_- x^b + \frac{\mu^2}{36} x^b = 0, \quad (b = 5, 6, 7, 8), \quad (4.2) \]
\[ \partial_+ \Psi^{2+} + \frac{\mu}{3} \tilde{\Pi} \Psi^{1-} = 0, \quad (4.3) \]
\[ \partial_- \Psi^{1-} - \frac{\mu}{3} \tilde{\Pi} \Psi^{2+} = 0, \quad (4.4) \]
\[ \partial_+ \Psi^{2-} + \frac{\mu}{6} \tilde{\Pi} \Psi^{1+} = 0, \quad (4.5) \]
\[ \partial_- \Psi^{1+} - \frac{\mu}{6} \tilde{\Pi} \Psi^{2-} = 0. \quad (4.6) \]
The mode-expansions of bosonic variables are described by

\[ x^a(\tau, \sigma) = x_0^a \cos \left( \frac{\mu}{3} \tau \right) + \left( \frac{3}{\mu} \right) \alpha^0_p \sin \left( \frac{\mu}{3} \tau \right) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{\omega_n} \left[ \alpha_n^a \hat{\phi}_n + \bar{\alpha}_n^a \tilde{\phi}_n \right], \quad (a = 1, 2, 3), \]

\[ x^b(\tau, \sigma) = x_0^b \cos \left( \frac{\mu}{6} \tau \right) + \left( \frac{6}{\mu} \right) \alpha^0_p \sin \left( \frac{\mu}{6} \tau \right) + i \sqrt{\frac{\alpha'}{2}} \sum_{n \neq 0} \frac{1}{\omega_n} \left[ \alpha_n^b \hat{\phi}_n + \bar{\alpha}_n^b \tilde{\phi}_n \right], \quad (b = 5, 6, 7, 8), \]

and those of fermionic variables are

\[ \Psi^1^-(\tau, \sigma) = \Psi_0 \cos \left( \frac{\mu}{3} \tau \right) + \bar{\Psi}_0 \sin \left( \frac{\mu}{3} \tau \right) + \sum_{n \neq 0} c_n \left[ \Psi_n \phi_n^p - \frac{3}{\mu} i(\omega_n^p - n) \bar{\Pi}^T \Psi_0 \hat{\phi}_n^p + \bar{\Psi}_0 \tilde{\phi}_n^p \right], \quad (4.7) \]

\[ \Psi^2^+(\tau, \sigma) = -\bar{\Pi} \Psi_0 \sin \left( \frac{\mu}{3} \tau \right) + \bar{\Pi} \bar{\Psi}_0 \cos \left( \frac{\mu}{3} \tau \right) + \sum_{n \neq 0} c_n \left[ \Psi_n \phi_n^p - \frac{3}{\mu} i(\omega_n^p - n) \bar{\Pi} \bar{\Psi}_0 \tilde{\phi}_n^p \right], \quad (4.8) \]

\[ \Psi^1^+(\tau, \sigma) = \Psi'_0 \cos \left( \frac{\mu}{6} \tau \right) + \bar{\Psi}'_0 \sin \left( \frac{\mu}{6} \tau \right) + \sum_{n \neq 0} c'_n \left[ \Psi'_n \phi'_n^p - \frac{6}{\mu} i(\omega_n^p - n) \bar{\Pi} \bar{\Psi}'_0 \tilde{\phi}'_n^p \right], \quad (4.9) \]

\[ \Psi^2^-\tau, \sigma) = -\bar{\Pi} \Psi'_0 \sin \left( \frac{\mu}{6} \tau \right) + \bar{\Pi} \bar{\Psi}'_0 \cos \left( \frac{\mu}{6} \tau \right) + \sum_{n \neq 0} c'_n \left[ \Psi'_n \phi'_n^p - \frac{6}{\mu} i(\omega_n^p - n) \bar{\Pi} \bar{\Psi}'_0 \tilde{\phi}'_n^p \right], \quad (4.10) \]
where we introduced several notations

\[ \omega_n^\mathrm{B} = \text{sgn}(n) \sqrt{n^2 + \left(\frac{\mu}{3}\right)^2}, \quad \omega_n^\prime = \text{sgn}(n) \sqrt{n^2 + \left(\frac{\mu}{6}\right)^2}, \quad (4.11) \]

\[ \phi_n^\mathrm{B} = \exp \left(-i (\omega_n^\mathrm{B} \tau - n \sigma)\right), \quad \tilde{\phi}_n^\mathrm{B} = \exp \left(-i (\omega_n^\prime \tau + n \sigma)\right), \]
\[ \phi_n^\prime = \exp \left(-i (\omega_n^\prime \tau - n \sigma)\right), \quad \tilde{\phi}_n^\prime = \exp \left(-i (\omega_n^\prime \tau + n \sigma)\right), \]
\[ \omega_n^\mathrm{F} = \text{sgn}(n) \sqrt{n^2 + \left(\frac{\mu}{3}\right)^2}, \quad \omega_n^\prime = \text{sgn}(n) \sqrt{n^2 + \left(\frac{\mu}{6}\right)^2}, \]
\[ \phi_n^\mathrm{F} = \exp \left(-i (\omega_n^\mathrm{F} \tau - n \sigma)\right), \quad \tilde{\phi}_n^\mathrm{F} = \exp \left(-i (\omega_n^\prime \tau + n \sigma)\right), \]
\[ \phi_n^\prime = \exp \left(-i (\omega_n^\prime \tau - n \sigma)\right), \quad \tilde{\phi}_n^\prime = \exp \left(-i (\omega_n^\prime \tau + n \sigma)\right), \]
\[ c_n = \left(1 + \left(\frac{3}{\mu}\right)^2 (\omega_n^\mathrm{F} - n)^2\right)^{-1/2}, \quad c_n' = \left(1 + \left(\frac{6}{\mu}\right)^2 (\omega_n^\prime - n)^2\right)^{-1/2}. \]

Following the usual canonical quantization procedure, we can quantize the theory and obtain the commutation relations. The canonical momenta are given by

\[ p^a = \frac{1}{2\pi \alpha'} \partial_\tau x^a, \quad (a = 1, 2, 3, 4), \]
\[ p^b = \frac{1}{2\pi \alpha'} \partial_\tau x^b, \quad (b = 5, 6, 7, 8), \]
\[ S_\alpha = i \psi^\tau_\alpha, \]

and the canonical (anti-)commutation relations are represented as

\[ [x^i(\tau, \sigma), p^j(\tau, \sigma')] = i \delta^{ij} \delta(\sigma - \sigma'), \]
\[ \{\psi_\alpha(\tau, \sigma), S_\beta(\tau, \sigma')\} = \frac{i}{2} \delta_{\alpha\beta} \delta(\sigma - \sigma'), \]
\[ \left\{\psi_\alpha(\tau, \sigma), \psi^*_\beta(\tau, \sigma')\right\} = \frac{1}{2} \delta_{\alpha\beta} \delta(\sigma - \sigma'), \]

where the delta function is defined by

\[ \delta(\sigma - \sigma') = \frac{1}{2\pi} \sum_n e^{in(\sigma - \sigma')}. \]

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From the above results, we can obtain the commutation relations of bosonic modes as

\[
\begin{align*}
[\alpha^i_m, \alpha^j_n] &= [\alpha^i_m, \bar{\alpha}^j_n] = 0, \quad (i, j = 1, \ldots, 8), \\
[\alpha^a_m, \alpha^{a'}_n] &= \omega^a_m \delta_{m+n,0} \delta^{a a'}, \quad (a, a' = 1, 2, 3, 4), \\
[\bar{\alpha}^a_m, \bar{\alpha}^{a'}_n] &= \omega^a_m \delta_{m+n,0} \delta^{a a'}, \\
[\alpha^b_m, \bar{\alpha}^{b'}_n] &= \omega^b_m \delta_{m+n,0} \delta^{b b'}, \\
[x^i_0, p^j_0] &= i \delta^{ij}, \quad \text{otherwise is zero.}
\end{align*}
\]

and those of fermionic ones as

\[
\begin{align*}
\{(\Psi_m)_\alpha, (\bar{\Psi}_n)_\beta^T\} &= \{(\bar{\Psi}_m)_\alpha, (\Psi_n)_\beta^T\} = 0, \quad (4.13) \\
\{(\Psi_m)_\alpha, (\Psi_n)_\beta^T\} &= \{(\bar{\Psi}_m)_\alpha, (\bar{\Psi}_n)_\beta\} = \frac{1}{2} \delta_{m+n,0} \delta_{\alpha \beta}, \\
\{(\Psi'_m)_\alpha, (\bar{\Psi}'_n)_\beta^T\} &= \{(\bar{\Psi}'_m)_\alpha, (\Psi'_n)_\beta^T\} = 0, \\
\{(\Psi'_m)_\alpha, (\Psi'_n)_\beta^T\} &= \{(\bar{\Psi}'_m)_\alpha, (\bar{\Psi}'_n)_\beta\} = \frac{1}{2} \delta_{m+n,0} \delta_{\alpha \beta}.
\end{align*}
\]

Though further considerations will not be done here, we can obtain the quantum Hamiltonian or spectrum exactly with the standard procedure.

4.2 Open Strings and Dp-branes in Type IIA on PP-wave

In this subsection we shall discuss the mode-expansions of open strings in the type IIA string by imposing boundary conditions. In particular, we would like to consider D-branes, following Ref. [30]. (For more detailed studies, see Refs. [31, 32].) It has been shown in Ref. [30] that Dp-brane is not allowed for \( p = 1, 9 \) and there are some restrictions on directions of allowed D-branes. First we consider the open string action described by

\[
S_{st} = \frac{1}{2\pi} \int d\tau \int_0^\pi d\sigma L_{st},
\]

\[
L_{st} = \frac{1}{2\alpha'} \sum_{i=1}^8 \partial_+ x^i \partial_- x^i - \left( \frac{\mu}{3} \right)^2 \sum_{a=1}^4 (x^a)^2 - \left( \frac{\mu}{6} \right)^2 \sum_{b=5}^8 (x^b)^2 \\
+ i \sum_{i=1}^8 \partial_+ x^i \partial_- \Psi^1 + i \sum_{i=1}^8 \partial_+ \Psi^1 \partial_- x^i + i \sum_{i=1}^8 \partial_+ \Psi^2 + i \sum_{i=1}^8 \partial_+ \Psi^3 + i \sum_{i=1}^8 \partial_+ \Psi^4
\]

\[
- \frac{\mu}{3} \sum_{i=1}^8 \partial_+ \bar{\Psi}^1 i \partial_- \bar{\Psi}^1 + i \sum_{i=1}^8 \partial_+ \bar{\Psi}^2 + i \sum_{i=1}^8 \partial_+ \bar{\Psi}^3 + i \sum_{i=1}^8 \partial_+ \bar{\Psi}^4.
\]
Similarly, we obtain equations of motion (4.1)-(4.6) from the above action (4.14). In order to solve the above equations of motion we have to impose the following boundary conditions on bosonic coordinates \( x^i \)'s (\( i = 1, 2, \cdots, 8 \)),

\[
\text{Neumann} : \quad \partial_\sigma x^m = 0, \quad (m = +, -, \text{and some } p - 1 \text{ coordinates}),
\]

\[
\text{Dirichlet} : \quad \partial_\tau x^m = 0, \quad (m = \text{other } 9 - p \text{ coordinates}),
\]

where 8 transverse indices \( i = 1, \ldots, 8 \) are decomposed into \( a = 1, 2, 3, 4 \) (flux directions) and \( b = 5, \ldots, 8 \).

For fermionic coordinates, boundary conditions are imposed as

\[
\Psi^1 - |_{\sigma = 0, \pi} = \tilde{\Omega}^r \Psi^2^+ |_{\sigma = 0, \pi},
\]

\[
\Psi^2^+ |_{\sigma = 0, \pi} = \tilde{\Omega}^r \Psi^{-1} |_{\sigma = 0, \pi},
\]

\[
\Psi^{-1} |_{\sigma = 0, \pi} = \tilde{\Omega}^r \Psi^2 |_{\sigma = 0, \pi},
\]

\[
\Psi^2 |_{\sigma = 0, \pi} = \tilde{\Omega}^r \Psi^1 |_{\sigma = 0, \pi},
\]

where \( \tilde{\Omega} \) is defined by

\[
\tilde{\Omega} = \tilde{\gamma}_m \tilde{\gamma}_m \cdots \tilde{\gamma}_m \cdots
\]

The \( \tilde{\Omega} \) includes odd number of gamma matrices since the \( SO(8) \) chiralities of \( \Psi^1 \) and \( \Psi^2 \) must be opposite in the type IIA theory and hence \( p \) is restricted to even.

Under these boundary conditions we can obtain classical solutions for equations of motion, and mode-expansions of bosonic variables are given by

\[
x^a(\tau, \sigma) = x_0^a \cos \left( \frac{\mu}{3} \tau \right) + \frac{3}{\mu} 2\alpha' p_0^a \sin \left( \frac{\mu}{3} \tau \right) - i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{\omega_n^a} \alpha_n^a e^{-i\omega_n^a \tau} \cos(n\sigma), \quad (a = 1, 2, 3, 4)
\]

\[
x^a(\tau, \sigma) = \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{\omega_n^a} \alpha_n^a e^{-i\omega_n^a \tau} \sin(n\sigma), \quad (a = 5, 6, 7, 8)
\]

\[
x^b(\tau, \sigma) = x_0^b \cos \left( \frac{\mu}{6} \tau \right) + \frac{6}{\mu} 2\alpha' p_0^b \sin \left( \frac{\mu}{6} \tau \right) - i\sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{\omega_n^b} \alpha_n^b e^{-i\omega_n^b \tau} \cos(n\sigma), \quad (b = 5, 6, 7, 8)
\]

\[
x^b(\tau, \sigma) = \sqrt{2\alpha'} \sum_{n \neq 0} \frac{1}{\omega_n^b} \alpha_n^b e^{-i\omega_n^b \tau} \sin(n\sigma), \quad (b = 5, 6, 7, 8)
\]
where $\omega_{n}^{B,B'}$ has been defined by (4.11). The mode-expansions of fermionic variables are the same as in the closed string case. The quantization can be done in the same way as closed strings. The commutation relations of bosonic and fermionic modes are the same as (4.12) and (4.13). The quantum Hamiltonian and spectrum can be also studied with the standard procedure but we will not investigate them furthermore here.

Next we will study D-branes. Though the mode-expansions of fermionic variables are the same as in the closed string case, in the open string case fermionic boundary conditions lead to further constraints

$$
\Psi_{0} = \tilde{\Omega}\tilde{\Pi}\Psi_{0}, \quad \tilde{\Omega}'\tilde{\Psi}_{0} = -\tilde{\Pi}\Psi_{0},
$$

$$
\tilde{\Psi}_{n} = \tilde{\Omega}\Psi_{n}, \quad (n \neq 0),
$$

$$
\Psi'_{0} = \tilde{\Omega}\tilde{\Pi}\tilde{\Psi}'_{0}, \quad \tilde{\Omega}'\tilde{\Psi}'_{0} = -\tilde{\Pi}\tilde{\Psi}'_{0},
$$

$$
\tilde{\Psi}'_{n} = \tilde{\Omega}\Psi'_{n}, \quad (n \neq 0).
$$

The self-consistency of these conditions gives us a condition

$$
\tilde{\Omega}\tilde{\Pi}\tilde{\Omega}\tilde{\Pi} = -1. \quad (4.20)
$$

This condition is peculiar to the pp-wave, and gives an additional constraint for the D$p$-branes in the theory. In fact, in the massive type IIB theory, D1- and D9-branes are forbidden and D3-, D5- and D7-branes can exist but those directions are limited. In the massive type IIA theory similar restrictions are imposed.

We shall list the possible D$p$-branes below:

- $p = 8$ : one of $I = 1, 2, 3$ is Dirichlet type,

$$
\tilde{\Omega} = \tilde{\gamma}',
$$

- $p = 6$ : 1) two of $I = 1, 2, 3$ and one of $I' = 4, \ldots, 8$ are Dirichlet types,

$$
\tilde{\Omega} = \tilde{\gamma}'\tilde{\gamma}'\tilde{\gamma}'',
$$

2) three of $I' = 4, \ldots, 8$ are Dirichlet types,

$$
\tilde{\Omega} = \tilde{\gamma}'\tilde{\gamma}'\tilde{\gamma}'',
$$
• $p = 4$:
  1) all of $I = 1, 2, 3$ and two of $I' = 4, \ldots, 8$ are Dirichlet types,

  $$\Omega = \tilde{\gamma}^I \tilde{\gamma}^J \tilde{\gamma}^K \tilde{\gamma}^{I'} \tilde{\gamma}^{J'},$$

  2) one of $I = 1, 2, 3$ and four of $I' = 4, \ldots, 8$ are Dirichlet types,

  $$\Omega = \tilde{\gamma}^I \tilde{\gamma}^{I'} \tilde{\gamma}^J \tilde{\gamma}^{J'} \tilde{\gamma}^{K'} \tilde{\gamma}^{L'},$$

• $p = 2$:
  two of $I = 1, 2, 3$ and all of $I' = 4, \ldots, 8$ are Dirichlet types,

  $$\Omega = \tilde{\gamma}^I \tilde{\gamma}^J \tilde{\gamma}^I' \tilde{\gamma}^J' \tilde{\gamma}^{K'} \tilde{\gamma}^{L'} \tilde{\gamma}^{M'}.$$

In conclusion, all $D_p$-branes can exist for $p = even$, but those directions are constrained as the case of the type IIB [30]. We note that zero-point energy varies for each direction of $D$-branes.

5 Conclusions and Discussions

We have considered the type IIA string theory on the pp-wave background from the eleven-dimensional viewpoint. To begin, we have discussed the type IIA pp-wave solution through the toroidal compactification of the maximally supersymmetric pp-wave solution in eleven dimensions on a spatial isometry direction. Next, we have derived the action of the type IIA string theory from the type IIA pp-wave solution of the supergravity. Moreover, we have derived the type IIA string action from the eleven-dimensional supermembrane theory on the maximally supersymmetric pp-wave background by applying the double dimensional reduction for a spatial isometry direction. The resulting action agrees with the one obtained from the supergravity side. In particular, the Kaluza-Klein gauge field induces a mass term of a bosonic coordinate in the type IIA theory. Furthermore, we have written down the action of the matrix string on the pp-wave. This action contains the 3-point interaction and mass terms. Also, the field strength of the gauge connection is shifted. However, this action is still gauge invariant, though this theory is not maximally supersymmetric. In particular, this theory is reduced to the matrix string theory in the flat space by taking the limit $\mu \to 0$. We also discussed the quantization of closed and open strings in the type IIA string. In particular, the allowed $D_p$-branes in this
theory has been investigated. The value $p = 2, 4, 6$ and $8$ are allowed but the directions of D-branes are constrained.

We can also consider compactifications along other isometry directions. In such cases the number of the remaining supercharges is less than $24$. It is nice to study the Type IIA pp-wave background preserving $26$ supercharges [27] or type IIA string theory from the eleven-dimensional supermembrane. It is an interesting work to discuss less supersymmetric type IIA string theories from the supermembrane. Moreover, the supersymmetric D-branes in such type IIA string theories are very interesting subject to study.

It is nice to study the matrix string theory written down here from several aspects. In particular, it would be interesting to study the relation between the matrix string theory on the pp-wave and “string bit” [39].
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Appendix

A Compactification on an $SO(3)$-direction

In the text we have considered the compactification on an $SO(6)$-direction. We can also compactify one of the directions $x^I$’s, $(I = 1, 2, 3)$. In this case, there are some different points from the compactification on an $SO(6)$-direction and we shall summarize these differences.

In this compactification on an $SO(3)$-direction the change of variables is given by

\[
X^+ = x^+, \quad X^- = x^- + \frac{\mu}{3} x^1 x^2, \quad X^a = x^a, \quad (a = 3, \ldots, 9)
\]

\[
X^1 = x^1 \cos \left( \frac{\mu}{3} x^3 \right) - x^2 \sin \left( \frac{\mu}{3} x^3 \right), \quad X^2 = x^1 \sin \left( \frac{\mu}{3} x^3 \right) + x^2 \cos \left( \frac{\mu}{3} x^3 \right),
\]

(A.1)

then the metric is rewritten as

\[
ds^2 = -2 dx^+ dx^- + G'_{++}(x^3, x^I')(dx^+)^2 + \sum_{r=1}^9 (dx^r)^2 - \frac{4}{3} \mu x^2 dx^+ dx^1,
\]

(A.2)

\[
G'_{++}(x^3, x^I') \equiv - \left[ \left( \frac{\mu}{3} \right)^2 (x^3)^2 + \left( \frac{\mu}{6} \right)^2 \sum_{r=4}^9 (x^r)^2 \right],
\]

and the constant 4-form flux is still written in Eq.(2.2). In this case the $x^1$-direction is a manifest spatial isometry direction. In the same way, we can obtain the type IIA solution from the above expression. The ten-dimensional metric $g_{\mu\nu}$ is given by

\[
g_{\mu\nu} dx^\mu dx^\nu = -2 dx^+ dx^- + g_{++}(x^i) (dx^+)^2 + \sum_{i=1}^8 (dx^i)^2,
\]

(A.3)

\[
g_{++}(x^i) \equiv - \left[ \left( \frac{2}{3} \mu \right)^2 (x^1)^2 + \left( \frac{\mu}{3} \right)^2 (x^2)^2 + \left( \frac{\mu}{6} \right)^2 \sum_{a=3}^8 (x^a)^2 \right],
\]

where we have relabelled the indices $x^2, \ldots, x^9$ as $x^1, \ldots, x^8$. The Kaluza-Klein gauge field $A_\mu$ is represented as

\[
A_+ = -\frac{2}{3} \mu x^1, \quad A_i = 0, \quad (i = 1, \ldots, 8),
\]

(A.4)
and non-zero NS-NS 2-form is given by

$$B_{+1} = \frac{\mu}{3} x^2, \quad B_{+2} = -\frac{\mu}{3} x^1.$$  (A.5)

In this case, the R-R 3-form $C_{\mu\nu\rho}$ and dilaton $\phi$ vanish.

To begin, let us discuss the bosonic sector. We can easily obtain the bosonic action from Eq. (2.14) using the type IIA metric. The resulting action is expressed as

$$S_B = \frac{1}{4\pi \alpha'} \int d\tau \int_0^{2\pi} d\sigma \left[ \sum_{i=1}^{8} \left[ (\partial \tau x^i)^2 - (\partial \sigma x^i)^2 \right] + \frac{2}{3} \mu x^2 \partial_\sigma x^1 - \frac{2}{3} \mu x^1 \partial_\sigma x^2 
- \left( \frac{2}{3} \mu \right)^2 (x^1)^2 
- \left( \frac{\mu}{3} \right)^2 (x^2)^2 
- \left( \frac{\mu}{6} \right)^2 \sum_{a=3}^{8} (x^a)^2 \right],$$  (A.6)

where the mass term for $x^1$ is induced from the Kaluza-Klein gauge field $A_\mu$ as the case of the compactification on an $SO(6)$-direction. It is also an easy exercise to derive the above action (A.6) by using the double dimensional reduction.

Next, we shall consider the fermionic sector. Now, in the study of the supergravity, the field strength of R-R 3-form is zero, but NS-NS 2-form is non-zero and it has the constant field strength proportional to $\mu$. Thus, this contribution induces the fermion mass term. However, there might be possibly an issue for the numerical constant and the fermion mass term obtained in the supergravity analysis is not identical with the one derived via double dimensional reduction if we naively use the expression of the covariant derivative in the text.
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