Elimination of information redundancy of hyperspectral images using the “well-adapted” basis method

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Abstract. The paper considers some issues of eliminating information redundancy of hyperspectral images (HSI). The characteristic properties of the HSI are listed, a brief description of the existing HSI compression methods is given. The possibility of using local, homogeneous "well-adapted" basis functions (LHWABF) to eliminate information redundancy and adaptive compression of the HSI is considered. An algorithm for constructing a LHWABF system for the HSI based on the Chebyshev approximation is proposed. The results of computational experiments, including the use of a graphics processor, are presented. The effectiveness of the proposed method of adaptive compression HSI is shown.

1. Introduction
Currently, aircraft and satellite-based tools for Earth remote sensing (ERS) are quite effective in solving various problems of the Earth's surface monitoring [1]. Most modern systems are multichannel in a broad sense: the images of the same surface area are produced using several frequencies (wavelengths), polarizations, viewing angles, etc. One of the promising types of multichannel Earth remote sensing systems are hyperspectral imaging tools that cover the optical and near infrared ranges of electromagnetic waves with high spectral resolution (on the order of several nanometers) and good spatial resolution (from several meters to tens of meters) [1] and produce hundreds of virtually aligned images. Some examples of such systems and hyperspectral sensors include AVIRIS, HYDICE, Hyperion, CASI, CHRIS-PROBA, etc. The obtained hyperspectral images (HSI) are used to solve a wide range of various applied thematic problems [1, 2, 20, 21, 23, 25].

When transmitting hyperspectral data to the Earth and to users, when processing hyperspectral images (HSI) and extracting useful information from these documents [22], as well as when storing them, a number of problems arise [1-3, 24]. One of the key problems is related to the enormous amount of data to be transmitted via communication channels and to be processed [1, 3-7]. Therefore, the task of developing the existing methods and searching for new methods for HSI compression is highly relevant.

The monograph [8] gives a fairly complete overview of the existing methods of image compression in operational Earth remote sensing systems. It provides an overview of specific features of image processing in such systems and the requirements for onboard processing of the video data obtained. Among these requirements, one should note a high degree of data compression and low computational...
and structural complexity of the compression methods used. At the same time, the problem of HSI compression is also relevant when organizing HSI raster data storages and sending them over communication channels between different users.

When encoding large amounts of experimental data, methods based on dispersion and factor analysis are widely used. In this case, one finds basis functions that are in some sense adapted to the coding of the data in question. The basis functions obtained by the method of orthogonal components are optimal in the sense of the average statistical RMS error, and with preliminary normalization of the encoded data in terms of duration and energy they are optimal in the sense of the minimum of the decomposition coefficients [17].

However, from a practical point of view, these methods are computationally expensive and require certain memory resources, since it is necessary to calculate the eigenvectors of the covariance matrices obtained from the set of the HSI.

At the Nizhny Novgorod State University, within the framework of the scientific school of Prof. Yu.G. Vasin, Honoured Worker of Science of the Russian Federation (1940–2017), research work has been carried out for more than 40 years to develop software and hardware suites for processing large-format complex structured graphic information. Raster data of Earth remote sensing, in particular, HSI, belong to exactly this type of graphic information. Some recent publications of this school in the field of compression of multi- and hyperspectral images include the works [18, 24, 25].

2. Characteristic properties of ERS hyperspectral images

ERS hyperspectral images are characterized by a number of specific properties that distinguish them from ordinary black and white and color optical images:

1. HSI have a high degree of redundancy, since images in two adjacent spectral channels usually have a correlation coefficient close to unity [1, 4, 15, 24], while the correlation coefficients of the color components R, G and B of color images are usually close to 0.8, that is, they are significantly smaller [16]. This difference is due to the fact that for the neighboring HSI channels the differences between wavelengths are much smaller than between the wavelengths in the RGB representation of color optical images;

2. For RGB color images, the signal-to-noise ratio (SNR) in component images is usually assumed to be identical or close enough; [16]. For the HSI components, the SNR usually significantly differs. As follows from practice, for a relatively small number of spectral channels, the SNR is within several dB, but for most spectral channels, the SNR is of the order of 20 ... 30 dB. This is primarily due to the significant difference in the dynamic ranges of the data and the difference in the level of interference in different spectral channels [1, 13];

3. The dynamic range for each n-th spectral channel can be characterized by the difference of the maximum \( P_{\text{max}}(n) \) and the minimum \( P_{\text{min}}(n) \) values. The values \( P_{\text{max}}(n) \) can vary from several tens to tens of thousands, while the values \( P_{\text{min}}(n) \) are of the order of units and tens, and they can be both positive and negative. Therefore, for the representation of the HSI Earth remote sensing, usually 2 bytes are used per sample, where one of the bits is signed.

4. A brief description of the existing HSI compression methods

Existing methods of image compression are generally divided into two groups: those that provide lossless and lossy compression.

In the case of lossless compression, the value of compression ratio (CR) of the HSI of about 3-4 [4] can be achieved, when taking into account the inter-channel correlation. When compressing each image separately, this value can be about 1.5-2. An obvious advantage of using this group of methods is that no distortions are introduced into HSI. However, the obtained CR values do not always satisfy the practical requirements. Therefore, it is important to develop and use compression methods for HSI with losses [5-7], which are capable of providing somewhat higher CR values compared with the methods of the first group. To be able to apply the methods of this group, the initial raster data of HSI will be represented in the form of vectors obtained as a result of sequential, line-by-line scanning of the initial HSI rasters.
These methods are based on the ideas of decomposition of the original signals in one or another system of basis functions (SBF) with a given accuracy of approximation $\varepsilon$. At the same time, the problem of optimal HSI coding is reduced to the search for such a SBF, which, for a given root-mean-square error $\delta$ provides the minimum (or close to minimum) number of such functions $\varphi^1(t), \varphi^2(t), \ldots, \varphi^m(t)$. Then the process $f(t)$ ($t_1 \leq t \leq t_2$) can be approximately represented as:

$$\tilde{f}(t) = \sum_{k=1}^{m} C_k \varphi^k(t)$$

of basis functions $\varphi^1(t), \varphi^2(t), \ldots, \varphi^m(t)$. The coefficients $C_1, C_2, \ldots, C_m$ in this case are considered as the code of the curve $f(t)$. The approximation error in this case is:

$$\varepsilon(t) = f(t) - \sum_{k=1}^{m} C_k \varphi^k(t).$$

Obviously, for various types of obtained HSI, various optimal SBFs will also be required.

When developing methods for compressing HSD with losses, a number of practical issues arise:

- The purpose of obtaining the HSD is to solve the final tasks of recognition and classification (classification of land cover types, detection and localization of contaminated sites, etc.). Quite often, there is no need to process data in all HSD channels, it may be sufficient to use a very limited set of the most informative channels for solving a particular thematic problem [2]. In each particular case, the set of such channels may be different, but it is clear that the user has to transmit instead of a full array of HSD only a part of it (usually no more than 8 ...12 channels), which is equivalent to compression.

- The use of lossy compression does not necessarily result in a lower efficiency when solving final tasks. Under certain conditions, the efficiency of solving final tasks with the use of HSD compressed with losses may remain almost at the same level compared to the case of using the original HSD (uncompressed or compressed without losses). [8, 9]

- Currently, there are no generally accepted quality metrics for HSI compression methods with losses [12-14].

Note that the main step in original signal approximation, the choice of informative SBF, is conducted semi-empirically using the researcher’s experience and intuition, or based on the external similarity of the original signal and SBF. This can lead to a situation when the optimal (or close to it) SBF cannot be found, especially in the case of a complex form of the original signal.

In this regard, of more interest are objective methods for selecting SBFs, where one proceeds only from the properties of the set of initial continuous data and does not rely on human knowledge. One of such optimal SBFs is the system of eigenvectors of the covariance matrix, calculated for a given set of input data, and the corresponding eigenvalues characterize the approximation accuracy [11]. However, this method is rather cumbersome in terms of computation. Besides, the SBF obtained in this way does not allow the encoding of every function of the original set of signals with a given accuracy, it only works on average over the set. In this case, the main part of the data will be encoded with a given approximation accuracy, however, the coding of the extremal data may occur with a significant error. Obviously, when encoding the HSI, this is an undesirable property of the resulting SBF.

In order to reduce the computational complexity of the algorithm for forming an optimal SBF, an algorithm was proposed for constructing a system of “well-adapted” basis functions (SWABF) [9].

In the final processing interval, the original discrete data $x^j = \{x_1^j, x_2^j, \ldots, x_N^j\}, j = 1, 2, \ldots, N$ can be represented as points in the $n$-dimensional space $\mathbf{R}_n$, with coordinates $\{x_1, x_2, \ldots, x_n\}$. Then a set of $X$ points $\mathbf{x}^j = \{x_1^j, x_2^j, \ldots, x_N^j\}, j = 1, 2, \ldots, N$ in the space $\mathbf{R}_n$ [9] will correspond to a certain set of experimental curves.

With such a geometric interpretation, an algorithm for constructing SWABF has been developed, based on the fact that the sets corresponding to the original descriptions of the experimental data have a relatively small dimension.

This fact was discovered experimentally, but is fairly general. If we proceed from the fact that the
set $X$ in the space $R^n$ has a small dimension, then we approximate the set $X$ with a certain linear manifold $L_m$ of dimension $m < n$ and choose an orthonormal basis of the linear manifold $L_m$ as a SWABF to encode the points of the set $X$. Then the problem of reducing redundancy in the original description of experimental data can be formulated as follows [10].

In the $n$-dimensional space $R^n$ with coordinates $\{x_1, x_2, ..., x_n\}$ a set of points $x^j \in X$, $j = 1, 2, ..., N$ is given. It is required to find an orthonormal basis $u_1, u_2, ..., u_m$ of some subspace $R_m$, in which, with a given accuracy $\varepsilon$, in the sense of some given metric $\rho$, all points $x^j \in X$, $j = 1, 2, ..., N$ are located. As soon as this basis is found, any point $x \in X$ can be represented with a given accuracy $\varepsilon$ using $m$ new features $C_k$, calculated according to the formula $C_k = \sum_{i=1}^{n} x_i u_i^k$, $k = 1, 2, ..., m$. The ordinates of the decoded point are: $\hat{x}_j = \sum_{i=1}^{n} C_k u_i^k$, $k = 1, 2, ..., m$.

Initially, SWABF coding based on Chebyshev approximations was proposed for processing medical curves (ECG, encephalograms, etc.) [9, 10].

Note also that when encoding, it is usually not so much important to achieve the possibly greater approximation accuracy $\varepsilon$ with a given number $m$ of basis vectors; it is more important to minimize the number of basis vectors $m$ for a given approximation accuracy $\varepsilon$. It has been proved that for a given accuracy $\varepsilon$, the proposed well-adapted basis does not include at most the last three unit vectors compared with the optimal coding on average [10].

The compression method considered here is based on Chebyshev approximations. It provides for a coding error that is not greater than the one specified for all points of the convex shell of the initial set of vectors, while the coding methods based on the principal component method provide for a sufficiently small error on the average over the entire original set.

The coding method under consideration is quasi-optimal, its advantage is that it allows a fairly simple practical implementation for large dimensions of the original data. This provided the starting point for the study of the possibility of HSI compression using the SWABF, based on Chebyshev approximations.

4. The algorithm for constructing SWABF based on Chebyshev approximations

Let $x^j \in X$, $j = 1, 2, ..., N$ be a set of initial points, $N$ is the size of the set $X$, $\delta$ is the required accuracy of the approximation (in %) and the maximum allowable number $L_{\max}$ of basis unit vectors of SWABF ($L_{\max} \leq P$), where $L$ is the number of SWABF unit vectors formed after the iteration of the algorithm, $N_0$ is the number of points of the set $X$, which by the given moment have already been decomposed with a given accuracy $\varepsilon$ using the already formed basis unit vectors $U^\ell$, $N$ is the number of unmarked points of the set participating in the formation of the next basis unit vector.

Step 1. Initialize the counters: $L=0$, $N_0=0$, $\overline{N}=N-N_0$.

Step 2. Linearly scan through $\overline{N}$ unmarked points $x^j \in X$ and find a point $x^jL \in X$ such that $\lambda_L = \max_j \|x^j\|$, $j = 1, 2, ..., \overline{N}$.

Step 3. If $L = 0$, we form the decomposition error $\varepsilon = \delta \ast \lambda_0$, otherwise Step 4.

Step 4. Increase the counter $L$ by one and find the next basis unit vector $U^\ell$: $U^L = \frac{x^jL}{\lambda_L}$.

Step 5. Linearly scan through all unmarked points $x^j \in X$ and perform their coding using the newly constructed unit vector $U^\ell$, that is, for each such point, we obtain the value of the coefficient
\[ C^j = \sum_{i=1}^{n} (x_i^j, u_i^L), j = 1, 2, ..., \quad \overline{N} \] and restore the coordinates of the points \( x^j \) from the constructed unit vector \( U^j: \quad \tilde{x}_i^j = C^j u_i^L, i = 1, 2, ..., n, j = 1, 2, ..., \overline{N}. \)

**Step 6.** Adjust the metric of the points: \( x_i^j = x_i^j - \tilde{x}_i^j, i = 1, 2, ..., n, j = 1, 2, ..., \overline{N}. \)

**Step 7.** Linearly scan through all unmarked points \( x^j \in X \) and mark those ones for which the condition \( \|x^j\| < \varepsilon, j = 1, 2, ..., \overline{N} \) is satisfied, adjust the value of the counter \( N_L \) increasing it by the number of points \( K \), marked at this step (decomposed using the current unit vector \( U^j \) with the required accuracy \( \varepsilon \)), \( N_L = N_L + K_\varepsilon \), and of the counter \( \overline{N} = \overline{N} - N_L \).

**Step 8.** If \( \overline{N} = 0 \), there are no unmarked points \( x^j \in X, j = 1, 2, ..., \overline{N} \) left in the set, or the number of constructed unit vectors \( L > L^{max} \), then **THE END OF THE ALGORITHM**, otherwise go to **Step 2**.

By eliminating at each subsequent step of the algorithm of those points \( x^j \), which by the given moment have been already represented with a given accuracy \( \varepsilon \), it was possible to significantly reduce the computational complexity of the algorithm, which is especially important in the last iterations when a lot of calculations are made.

**5. Results of practical experiments**

Practical experiments on the coding of remote sensing HSI were carried out on a sample of 360 half-tone raster images (spectral channels data) with geometrical dimensions of each raster ~ \( 10^3 \times 10^3 \) and a color depth of 16 bits/pix. A raster image of each channel was converted by progressive scanning into a one-dimensional signal, with the dimension of ~ \( 10^6 \) counts. Thus, at the input of the SWABF construction algorithm, the dimension of the original space \( R_n \sim 10^6 \), and the size of the set of source vectors is \( V: H(V) = 360 \).

For each \( n \)-th spectral channel, its dynamic range was determined experimentally:

\[ D(n) = 10 \log_{10} \frac{P_{max}(n)+1}{P_{min}(n)+1} (dB), \] where \( P_{max}(n) \) and \( P_{min}(n) \) are the maximum and minimum brightness values of pixels in the channel \( n, n = 1, 2, ..., H(V) \). The minimum and maximum brightness values for all channels are \( D_{min} = 4dB, D_{max} = 24dB \). The resulting interval \( I = D_{max} - D_{min} = 20dB \) was evenly divided into 10 groups. The distribution of channels by groups is given in Table 1 and Figure 1.

**Table 1.** The distribution of channels in groups depending on the dynamic range.

| Group | Dynamic range, dB | Number of channels | Channel numbers |
|-------|-------------------|--------------------|----------------|
| 1     | 4–6               | 2                  | 4, 5           |
| 2     | 6–8               | 0                  | –             |
| 3     | 8–10              | 42                 | 3, 6, 16-27, 38-65 |
| 4     | 10–12             | 5                  | 2, 34-37      |
| 5     | 12–14             | 11                 | 1, 7, 12-15, 28-30, 66, 67 |
| 6     | 14–16             | 4                  | 31-33, 68     |
| 7     | 16–18             | 4                  | 8 – 11        |
| 8     | 18–20             | 40                 | 103, 199 – 203, 256 – 282, 329, 331 – 335, 338 |
| 9     | 20–22             | 126                | 69-71, 185-198, 204-255, 283-328, 330, 336, 337, 339-346 |
| 10    | 22–24             | 126                | 72-102, 104-184, 347-360 |
Figure 1. The distribution of the number of spectral channels depending on the group number.

Dividing into groups seems necessary for the following reasons: in the case when a SWABF is built based on source data with fundamentally different dynamic ranges of the channels $D(n)$, the decomposition error $\varepsilon$ cannot adequately describe the distortions introduced during compression with image losses in the $n$th spectral channel.

The same value of $\varepsilon$ in the channel with a small $D(n)$ will lead to an obvious degradation of image quality, while in the channel with a large dynamic range, the introduced distortions will not be noticeable. Dividing into groups eliminates the difference between channels in terms of dynamic range.

Then, for each group of channels, a SWABF was constructed for different values of the required accuracy of the approximation $\delta$, and the compression ratio $K_{CR}(\delta)$ was determined for each value of $\delta$ using the formula $K_{CR}(\delta) = \left(1 - \frac{N_{ort}(\delta)}{P}\right) \times 100\%$, where $N_{ort}(\delta)$ is the number of SWABF unit vectors formed in all groups for a fixed value of $\delta$. The dependence of the overall compression ratio on the accuracy of the approximation $\delta$ is shown in Figure 2. The actual values are given in Table 2.

Table 2. The dependence of the number of formed unit vectors of the SWABF on the group number and the accuracy of approximation.

| Group | Accuracy of approximation $\delta$ (%) |
|-------|--------------------------------------|
|       | 1         | 1.5        | 2         | 3         | 5         | 7         | 15        |
|       | The number of formed unit vectors of the SWABF |
| 1     | 2         | 2          | 2         | 2         | 2         | 2         | 2         |
| 2     | 4         | 4          | 4         | 4         | 4         | 4         | 1         |
| 3     | 42        | 42         | 42        | 31        | 8         | 3         | 2         |
| 4     | 5         | 5          | 5         | 4         | 3         | 2         | 1         |
| 5     | 11        | 11         | 11        | 10        | 6         | 5         | 1         |
| 6     | 4         | 4          | 4         | 4         | 3         | 2         | 1         |
| 7     | 4         | 4          | 4         | 4         | 4         | 4         | 1         |
| 8     | 37        | 31         | 25        | 10        | 4         | 3         | 3         |
| 9     | 126       | 99         | 69        | 16        | 6         | 5         | 2         |
| 10    | 121       | 80         | 43        | 13        | 7         | 4         | 3         |
| $\Sigma$ | 352     | 278        | 205       | 94        | 43        | 30        | 16        |
| CR (\delta) (%) | 2         | 23         | 43        | 74        | 88        | 92        | 96        |

Figure 3 contains fragments of images of the 107th frequency channel and normalized histograms [19] of the distribution brightness of pixels in it. Figure 3a is the original image, Figure 3b is the restored image with $\delta = 3\%$ using 94 SWABF ors, Figure 3c is the inverse image of the recovery error with $\delta=3\%$ using 94 SWABF ors, figure 3d is the restored image with $\delta= 7\%$ using 30 SWABF ors, Figure 3e - recovery error with $\delta= 7\%$ using 30 SWABF ors.
Figure 2. The dependence of compression ratio on the accuracy of the approximation $\delta$ (%).

Figure 3. a) a fragment of the original image of channel 107 (with its normalized histogram); b) the recovered image of channel 107 for $\delta = 3\%$ using 94 SWABF orts; c) the inverse image of the recovery error; d) the recovered image of channel 107 for $\delta = 7\%$ using 30 SWABF orts; e) the inverse image of the recovery error.

As can be seen in Figure 3c, the channel image recovered using 94 SWABF orts does not contain any distortions compared to the original one. Figure 3e shows the result of decoding the original image using 30 SWABF orts. The analysis of the recovered signal histogram shows the presence of distortion in the recovered signal, which, however, is not visible. For the value $\delta = 15\%$, the compression ratio of 96% was obtained on this set of images, since using 16 SWABF orts it was possible to encode 360 raster channels images. The histogram of the brightness distribution of the recovery error is shown in Figure 3e.

6. Conclusions

In this paper, we have considered the problem of reducing information redundancy of raster remote sensing HSI data. It was noted that the use of traditional SBFs for these purposes (trigonometric, exponential systems, Lager functions, etc.) is not always justified, since in this case the choice of an informative SBF is most often based on the external similarity of the original signal and SBF, which can result in a situation when the optimal (or close to it) SBF may not be found, especially in the case of a complex form of the original signal. We have developed objective methods for selecting SBF, based only on the properties of the source data set. These methods include the encoding of the original information using SWABF. The paper describes the SWABF formation algorithm adapted for HSI processing and shows its rather high efficiency (up to 90–96%) in eliminating information redundancy on raster HSI with a low level of distortion introduced into the signal at the decoding stage.

The authors plan to continue exploring the possibility of further reducing the amount of computation when encoding HSI using SWABF, applying SWABF for noise suppression in HSI, and also exploring the possibility of constructing adaptive HSI compression algorithms based on SWABF.
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