Mathematical Model of Cytomegalovirus (CMV) Disease

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Abstract. The article formed the mathematical model of cytomegalovirus (CMV) disease. Cytomegalovirus (CMV) is a type of herpes virus. This virus is actually not dangerous, but if the body’s immune weakens the virus can cause serious problems for health and even can cause death. This virus is also susceptible to infect pregnant women. In addition, the baby may also be infected through the placenta. If this is experienced early in pregnancy, it will increase the risk of miscarriage. If the baby is born, it can cause disability in the baby. The model is formed by determining its variables and parameters based on assumptions. The goal is to analyze the dynamics of cytomegalovirus (CMV) disease spread.

1. Introduction

Cytomegalovirus (CMV), a common virus, is one of the herpes viruses. The virus can infect people of all ages. Most people infected with CMV show no signs or symptoms. They are exposed to CMV at birth or as children. That’s because a healthy person’s immune system usually keeps the virus from causing illness. The virus stays in a person’s body for life and can reactive. A person can also be re-infected with a different strain of the virus.

CMV is spread by direct contact of body fluids, such as saliva, blood, urine, semen, vaginal fluids and breast milk from an infected person. CMV can also be transmitted through organs transplants, stem cell transplants, blood transfusion, breastfeeding, kissing, sexual contact, infection drug use (sharing needles). Not only that, CMV transmission can also occur during pregnancy through the placenta from the mother’s blood or from vaginal secretions at delivery to child (congenital CMV).

In some cases, infection in healthy people can cause mild illness that may include fever, sore throat, fatigue and swollen glands. Occasionally CMV can cause liver problem. However, CMV infection can cause serious health problems for people with weakened immune systems, as well as babies infected with the virus before they are born (congenital CMV). In people with weakened immune systems, CMV infection can attack different organs of the body and may cause blurred vision and blindness (CMV retinitis), lung infection (pneumonia), painful swallowing (esophagitis), diarrhea (colitis), inflammation of the liver (hepatitis), or inflammation of the brain (encephalitis), which may cause behavioral changes, seizures or coma. Babies born with CMV can loss hearing and vision, mental disability, seizures and growth problems.

Based on the problems then formed a mathematical model of the spread of CMV disease. The purpose is to look at the dynamics of disease spread and what to do in preventing or reducing the spread of the disease in the future.
2. Method and Design
The research is a basic (theoretical) research. The method used is the analysis of theories relevant to the issues discussed. The steps taken are to study the phenomenon of the problem, collect and link the theories related to the problem, determine the appropriate assumptions with the problems, define the variables and parameters used, to form and analyze the model, and interpret the results of model analysis obtained to answer the problem.

3. Findings and Discussion
3.1 Formation of Mathematical Model
The population consists of 3 groups. They are susceptible group (S), exposed group (E), and infected group (I). Assumption of the model: disease has latent period, every individual born of exposed and infected group is assumed to have CMV and move to exposed group, individual born in susceptible move to susceptible group, CMV transmission occur horizontally (direct contact between susceptible group and infected group (primary CMV)) and vertically (from mother to child during pregnancy (congenital CMV)), individual in exposed group moves to infected group due to the weakened immune system or reactivation of latent CMV infection, constant population and individual recovery moves to exposed group (doesn’t heal permanently). Parameter of the model: transmission rate (a), birth rate (b), death rate (d), recovery rate (f), and individual transfer rate from exposed group to infected group (c).

The general overview of model showed in flow diagram.

![Flow diagram of model](image)

Here, the mathematical model formula. That is system (1)
\[
\frac{ds}{dt} = (b - d)s - as \frac{I}{N}
\]
\[
\frac{dE}{dt} = as \frac{I}{N} + b(N - S) + fI - (c + d)E
\]
\[
\frac{dI}{dt} = cE - (f + d)I
\]

Where \(S + E + I = N\)
To simplify the system (1), let \(s = \frac{S}{N}, e = \frac{E}{N}, \) and \(i = \frac{I}{N}\) and obtained system (2)
\[
\frac{ds}{dt} = (b - d)s - asi
\]
\[
\frac{de}{dt} = asi + b(1 - s) + fi - (c + d)e
\]
\[
\frac{di}{dt} = ce - (f + d)i
\]

Where \( s + e + i = 1 \)

### 3.2 Equilibrium Point of Model

To find the solution of system (2), we can change system (2) become to be system (3)

\[
\frac{ds}{dt} = (b - d)s - asi
\]

\[
\frac{di}{dt} = c(1 - (s + i)) - (f + d)i
\]

(3)

According Perko [1996], point \( \hat{x} \in \mathbb{R} \) is called equilibrium point of system (3) if \( f(\hat{x}) = 0 \). Furthermore, we get

\[
(b - d)s - asi = 0
\]

\[
c(1 - (s + i)) - (f + d)i = 0
\]

(4)

(5)

Based on equations (4) and (5), we get 2 kinds equilibrium point: free and endemic equilibrium points. Free equilibrium point is \( E_0 = (\hat{s}, \hat{i}) = (1, 0) \) and endemic equilibrium point is \( E_1 = (\hat{s}, \hat{i}) = (1 - \left( \frac{c+f+d}{c} \right) \left( \frac{b-d}{a} \right), \frac{b-d}{a}) \)

### 3.3 Stability Equilibrium Point of Model

The stability of equilibrium point is investigated to study system dynamics. To facilitate an analytical calculation, then System (3) is linearized using the Jacobian matrix and obtained:

\[
J(\hat{s}, \hat{i}) = \begin{bmatrix}
    b - d - ai & -as \\
    -c & -(f + c + d)
\end{bmatrix}
\]

The stability of equilibrium point of the model is as follows:

#### 3.3.1. Stability of free equilibrium point \( E_0 = (\hat{s}, \hat{i}) = (1, 0) \)

The stability of this point is seen from the Eigen value of Jacobian matrix around the \( E_0 \) point, as follows:

\[
|\lambda I - J(f(E_0))| = 0
\]

\[
\begin{vmatrix}
    \lambda - (b - d) & a \\
    c & \lambda + (f + c + d)
\end{vmatrix} = 0
\]

\[
\lambda^2 + a_1 \lambda + a_2 = 0
\]

where, \( a_1 = (f + c + d) - (b - d) \) and \( a_2 = -(ac + (b - d)(f + c + d)) \)

Based on the Routh-Hurwitz criteria, \( E_0 \) point is stable when \( a_1 > 0 \) and \( a_2 > 0 \). This happens if and only if \( R_0 < 1 \) and \( R_0 < -\frac{ac}{(f+c+d)^2} \)

#### 3.3.2. Stability of endemic equilibrium point \( E_1 = (\hat{s}, \hat{i}) = (1 - \left( \frac{c+f+d}{c} \right) \left( \frac{b-d}{a} \right), \frac{b-d}{a}) \)

The stability of this point is seen from the Eigen value of Jacobian matrix around the \( E_1 \) point, as follows:

\[
|\lambda I - J(f(E_1))| = 0
\]
\[
\begin{vmatrix}
\lambda & a - \frac{(f + c + d)(b - d)}{c} \\
\frac{c}{\lambda} & \lambda + (f + c + d)
\end{vmatrix} = 0
\]

\[
\begin{vmatrix}
\lambda & a - \frac{(f + c + d)^2}{c} \left( \frac{b - d}{f + c + d} \right) \\
\frac{c}{\lambda} & \lambda + (f + c + d)
\end{vmatrix} = 0
\]

\[
\begin{vmatrix}
\lambda & a - \frac{(f + c + d)^2 R_0}{c} \\
\frac{c}{\lambda} & \lambda + (f + c + d)
\end{vmatrix} = 0
\]

\[
\lambda^2 + (f + c + d)\lambda + ((f + c + d)^2 R_0 - ac) = 0
\]

Based on the Routh-Hurwitz criteria, \( E_1 \) point is stable when \((f + c + d) > 0 \) and \((f + c + d)^2 R_0 - ac \). This happens if and only if \( R_0 > \frac{ac}{f + c + d} \).

4. Conclusion
The research found 2 kinds equilibrium points of model. It is free equilibrium point \((E_0 = (1,0))\) and endemic equilibrium point \((E_1 = (1 - \left( \frac{c + f + d}{c} \right) \left( \frac{b - d}{a} \right), \frac{b - d}{ac}) \). \( E_0 \) point stable if eligible \( R_0 < 1 \) and \( R_0 < -\frac{ac}{(f + c + d)^2} \). Furthermore, \( E_1 \) point stable if \( R_0 > \frac{b - d}{ac} \), where \( R_0 = \frac{b - d}{f + c + d} \).

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4