Quantum induced $\omega = -1$ crossing of the quintessence and phantom models

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Abstract
Considering the single scalar field models of dark energy, i.e. the quintessence and phantom models, it is shown that the quantum effects can cause the system crosses the $\omega = -1$ line. This phenomenon does not occur in classical level. The quantum effects are described via the account of conformal anomaly.

1 Introduction
One of the most important aspects of the present universe is its accelerated expansion. Since two independent observations based on redshift-distance relation of type Ia supernovas in 1998 [1, 2], numerous observations [3] consistently indicate that our universe is dominated by a perfect fluid with negative pressure, dubbed dark energy, which constitutes two third of the present universe.

The first candidate which has been introduced for dark energy is a cosmological constant $\Lambda$ of order $(10^{-3}$ eV)$^4$, with equation of state parameter $\omega = p/\rho = -1$. This model suffers from fine tuning and coincidence problems [4]. As an alternative to cosmological constant, the dynamical models have been introduced. In these models, the equation of state parameters $\omega(z)$ is one of the main parameters which is usually used in studying the time variation of dark energy. In accelerating universe, $\omega$ satisfies $\omega < -1/3$.

In the quintessence model of dark energy, which consists of one normal scalar field $\phi$ [5], $\omega$ is always $\omega > -1$. In phantom model, which is a scalar field theory with a field $\sigma$ with negative kinetic energy, $\omega$ always satisfies $\omega < -1$ [6]. But some astrophysical data seem to slightly favor an evolving dark energy and show a recent $\omega = -1$, the so-called phantom-divide-line, crossing [7]. This phenomenon can not be explained by none of these two models, the quintessence or phantom models. A possible way to overcome this problem is to consider two scalar fields in the models known as hybrid models. One of these models is the model consists of one quintessence and one phantom field, the so-called quintom model [8]. Recently, it has been shown that the $\omega > -1$ to $\omega < -1$ transition always occurs in the quintom models with slowly-varying potentials [9]. Also

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if one considers one scalar field, but with suitable interaction with background dark matter, again this transition can be occurred [10].

In present paper, we study the contribution of quantum effects in $\omega = -1$ crossing of single scalar field models of dark energy, that is the quintessence and phantom models. Due to a No-Go theorem proposed in [11], a single scalar field which minimally couples to Einstein gravity can cross the phantom-divide-line only when the higher derivative terms of scalar field, like $\varphi \Box \varphi$ [12], exist in the Lagrangian. But, as we see, it is not the case at the quantum level. This transition can be induced quantum-mechanically, with no need to higher derivative terms. The quantum effects are described via the account of conformal anomaly, reminding about anomaly-driven inflation [13]. The contribution of this quantum effect in preserving the most of the energy conditions of phantom matter has been discussed in [14] and its influence in moderating the sudden future singularity (Big Rip) of phantom model has been studied in [15].

The scheme of the paper is as follows. In section 2 we briefly review the quintessence and phantom models and introduce the perturbative method of studying the phantom-divide-line crossing of these models. The energy density and pressure resulting from the conformal anomaly are also quoted. In section 3 we apply our method to quintessence and phantom models and show that the system, except for very special initial conditions, has a transition from $\omega < -1$ to $\omega > -1$, or vise versa, resulting from quantum effects. In special free pure phantom model, it is shown that this transition always occurs from $\omega < -1$ region to $\omega > -1$ region.

We use the units $\hbar = c = G = 1$ throughout the paper.

2 Perturbative method for studying the $w = -1$ crossing

Consider a spatially flat Friedman-Robertson-Walker space-time in co-moving coordinates $(t, x, y, z)$

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

where $a(t)$ is the scale factor. It is assumed that the universe is filled with (dark) matter and a single scalar field. The evolution equation of matter density $\rho_m$ is

$$\dot{\rho}_m + 3H\gamma_m \rho_m = 0,$$

in which $\gamma_m = 1 + \omega_m$. $\omega_m$, the equation of state parameter of matter field, is defined through $\omega_m = p_m/\rho_m$, in which $p_m$ is the pressure of matter field. $H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter and ”dot” denotes the time derivative. The dark energy consists of the quintessence field $\varphi$ (or phantom field $\sigma$), which in the case of homogenous field, its energy density $\rho_D$ and pressure $p_D$ (”D” denotes the dark energy) are [5,6]

$$\rho_{\text{quintessence}} = \frac{1}{2} \dot{\varphi}^2 + V(\varphi), \quad p_{\text{quintessence}} = \frac{1}{2} \dot{\varphi}^2 - V(\varphi),$$

or

$$\rho_{\text{phantom}} = -\frac{1}{2} \dot{\sigma}^2 + V(\sigma), \quad p_{\text{phantom}} = -\frac{1}{2} \dot{\sigma}^2 - V(\sigma).$$
The Friedman equations are
\[ H^2 = \frac{8\pi}{3} \rho_{\text{tot}}, \] (5)
and
\[ \dot{H} = -4\pi (\rho_{\text{tot}} + p_{\text{tot}}). \] (6)
The equation of state parameter \( \omega = p_{\text{tot}}/\rho_{\text{tot}} \) is found as
\[ \omega = -1 - \frac{2}{3} \frac{\dot{H}}{H^2}. \] (7)

For quintessence phase \( \omega > -1 \), we have \( \dot{H} < 0 \) and in phantom phase \( \omega < -1 \), \( \dot{H} \) obeys \( \dot{H} > 0 \). If \( \dot{H}(t_0) = 0 \) and \( H(t) \) has a relative extremum at \( t = t_0 \), the system crosses \( \omega = -1 \) line at time \( t = t_0 \).

If we restrict ourselves to \( t - t_0 \ll h_0^{-1} \), where \( h_0 = H(t_0) \) and \( h_0^{-1} \) is of order of the age of universe, \( H(t) \) can be expanded as
\[ H(t) = h_0 + h_1(t - t_0)^\alpha + h_2(t - t_0)^{\alpha+1} + O((t - t_0)^{\alpha+2}). \] (8)

\( \alpha \geq 2 \) is the order of first non-vanishing derivative of \( H(t) \) at \( t = t_0 \) and \( h_1 = \frac{1}{\alpha} H^{(\alpha)}(t_0) \). \( H^{(n)}(t_0) \) is the \( n \)-th derivative of \( H(t) \) at \( t = t_0 \). The transition from \( \omega > -1 \) region to \( \omega < -1 \) region occurs when \( \alpha \) is even positive integer and \( h_1 > 0 \). In reverse case, \( h_1 \) must be negative. In the case of quintom model, it has been shown that for slowly-varying potentials \( V(\varphi, \sigma) \), \( \alpha = 2 \) and \( h_1 > 0 \), and therefore \( \omega > -1 \) to \( \omega < -1 \) transition occurs [9].

To consider the quantum effects, one may use a standard method which leads to a closed form for quantum corrections. In this method, the interaction is considered between the quantum free matter field and classical gravitational field [13, 16]. It can be seen that the renormalization of effective action leads to some extra terms in the trace of energy-momentum tensor, which is known as trace/conformal anomaly. Note that in the classical level, the energy-momentum tensor is traceless. These extra terms are given by:
\[ T = b(F + \frac{2}{3} \Box R) + b'G + b'' \Box R, \] (9)

where \( F \) is the square of 4d Weyl tensor and \( G \) is Gauss-Bonnet invariant, given by:
\[ F = \frac{1}{3} R^2 - 2 R_{ij} R^{ij} + R_{ijkl} R^{ijkl}, \]
\[ G = R^2 - 4 R_{ij} R^{ij} + R_{ijkl} R^{ijkl}. \] (10)

Generally for \( N \) scalars, \( N_{1/2} \) spinors, \( N_1 \) vector fields, \( N_2 (= 0 \text{ or } 1) \) gravitons and \( N_{HD} \) higher derivative conformal scalars (including phantom), \( b, b' \) and \( b'' \) are given by
\[ b = \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{HD}}{120 (4\pi)^2}, \]
\[ b' = - \frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{HD}}{360 (4\pi)^2}, \]
\[ b'' = 0. \] (11)
Using eq. (9), one can find the contributions due to conformal anomaly to $\rho$ and $p$ as follows \[17\]

$$
\rho_A = -\frac{1}{a^2} \left\{ b'(6a^4H^4 + 12a^2H^2) + \left( \frac{2}{3}b + b'' \right) \left[ a^4(-6H\ddot{H} - 18H^2\dot{H} + 3H^2) + 6a^2H^2 \right] - 2b + 6b' - 3b'' \right\},
$$

(12)

and

$$
p_A = b' \left[ 6H^4 + 8H^2\dot{H} + \frac{1}{a^2}(4H^2 + 8\dot{H}) \right] + \left( \frac{2}{3}b + b'' \right) \left[ -2\ddot{H} - 12H\dot{H} - 18H^2\dot{H} - 9\dot{H}^2 \right] + \frac{1}{a^2}(2H^2 + 4\dot{H}) \right] - \frac{-2b + 6b' - 3b''}{3a^4}.
$$

(13)

Now it looks reasonable to solve the Friedman equations with these quantum corrections and see if there exists any new result in the phantom-divide-line-crossing issue due to this correction.

## 3 The transition solutions

In this section we consider the expansion (8) for $H(t)$ and try to solve the equations (2), (5) and (6) with $\rho_{\text{tot.}} = \rho_m + \rho_D + \rho_A$ and $p_{\text{tot.}} = p_m + p_D + p_A$. We want to find any consistent solution of these equations with $\omega = -1$ crossing property.

### 3.1 The quintessence model

In the case of quintessence field, one has $N = 1$ and $N_{1/2} = N_1 = N_2 = N_{HD} = 0$. So

$$
b = -3b' = \frac{1}{120(4\pi)^2}.
$$

(14)

Eqs. (2) and (8) (with $\alpha \geq 2$ and $t_0 \equiv 0$) result in

$$
\rho_m(t) = \rho_m(0)[1 - 3h_0\gamma_m t + \frac{9}{2}\gamma_m h_0^2 t^2 + ...].
$$

(15)

By expanding both sides of eq.(5) near $t_0 = 0$, one finds

$$
h_0^2 + 2h_0h_1t^\alpha + ... = \frac{8\pi}{3} [\rho_{\text{tot.}}(0) + \dot{\rho}_{\text{tot.}}(0)t + \frac{1}{2}\ddot{\rho}_{\text{tot.}}t^2 + ...],
$$

(16)

in which

$$
\rho_{\text{tot.}}(t) = \rho_{\text{quintessence}} + \rho_m + \rho_A = \rho_{\text{cl.}} + \rho_A.
$$

(17)

In above equation, ”cl.” denotes ”classical”. Eq.(16) then results in the following two relations:

$$
h_0^2 = \frac{8\pi}{3} \rho_{\text{tot.}}(0) = \frac{8\pi}{3} \left[ \rho_{\text{cl.}}(0) + 2b \left( h_0^4 + 4h_0h_1\delta_{\alpha,2} + \frac{2}{a_0^2} \right) \right],
$$

(18)
\[ 0 = \dot{\rho}_{\text{tot.}}(0) = \dot{\rho}_{\text{cl.}}(0) + 2b \left( 12h_0 h_2 \delta_{\alpha,2} + 12h_0^2 h_1 \delta_{\alpha,2} + 12h_0 h_1 \delta_{\alpha,3} - \frac{8h_0}{a_0^4} \right), \]  
(19)
in which \( a_0 \) is the scale factor at transition time \( t = 0 \). The same expansion for the second Friedman equation \( (6) \) results in the following extra relation:

\[ 0 = \delta(0) = \rho_{\text{cl.}}(0) + p_{\text{cl.}}(0) - \frac{4b}{3} \left( 6h_0 h_1 \delta_{\alpha,2} + 6h_2 \delta_{\alpha,2} + 6h_1 \delta_{\alpha,3} - \frac{4}{a_0^4} \right), \]  
(20)
in which

\[ \delta(t) = \rho_{\text{tot.}} + p_{\text{tot.}} = \rho_{\text{cl.}} + p_{\text{cl.}} + \rho_A + p_A. \]  
(21)

In eqs. \((18)-(20)\), \( \rho_m(t) \) is given by eq. \((15)\) and \( \rho_m + p_m = \gamma_m \rho_m \). \( \rho \text{quintessence} \) and \( p \text{quintessence} \) are those in eq. \((3)\), and \( \rho_A \) and \( p_A \) are given by eqs. \((12)\) and \((13)\) with \( H(t) = h_0 + h_1 t^\alpha + \ldots, b \) and \( b' \) from eq. \((14)\), and \( b'' = 0 \). Note that eqs. \((19)\) and \((20)\) indicate the energy conservation law \( \dot{\rho}_{\text{tot.}} + 3H(\rho_{\text{tot.}} + p_{\text{tot.}}) = 0 \) at \( t = 0 \). Since \( \delta(0) = \rho_{\text{tot.}}(0) + p_{\text{tot.}}(0) = 0 \), \( \dot{\rho}_{\text{tot.}} \) must satisfy \( \dot{\rho}_{\text{tot.}}(0) = 0 \).

Let us first consider the equation \((20)\). Since \( \rho_{\text{cl.}} + p_{\text{cl.}} = \dot{\varphi}^2 + \gamma_m \rho_m \), eq. \((20)\) for \( \alpha \geq 4 \) results in:

\[ \dot{\varphi}^2(0) + \gamma_m \rho_m(0) + \frac{16b}{3a_0^4} = 0, \quad (\text{for } \alpha \geq 4). \]  
(22)
The above equation has no solution except \( \dot{\varphi}(0) = 0, \rho_m(0) = 0, \) and \( a_0 \to \infty \), which is unphysical. So we have only two choices \( \alpha = 2 \) and \( \alpha = 3 \).

For \( \alpha = 2 \), three equations \((18)-(20)\) can be used to obtain the coefficients \( h_0, h_1 \) and \( h_2 \). \( h_0 \) is found as

\[ h_0 = -\frac{\ddot{\varphi}(0) + (dV/d\varphi)_0}{3\dot{\varphi}(0)}, \]  
(23)
which is nothing but the evolution equation

\[ \ddot{\varphi} + 3H \dot{\varphi} + \frac{dV(\varphi)}{d\varphi} = 0 \]  
(24)
at \( t = 0 \). \( h_1 \), in terms of \( h_0 \), is found from eq. \((18)\) as following

\[ h_1 = \frac{1}{8bh_0} \left\{ \frac{3}{8} h_0^2 \left( \rho_{\text{cl.}}(0) + 2bh_0 + \frac{4b}{a_0^4} \right) \right\}, \]  
(25)
in which \( \rho_{\text{cl.}}(0) = \frac{1}{2} \dot{\varphi}^2(0) + V(0) + \rho_m(0) \). Finally, \( h_2 \) can be expressed in terms of \( h_1 \) by using eqs. \((19)\) or \((20)\). The next-leading relations from Friedman equations, that is the coefficients of \( t^2 \) of eq. \((5)\) and \( t \) of eq. \((4)\), are:

\[ 2h_0 h_1 = \frac{1}{2} \rho_{\text{tot.}}(0). \]  
(26)
and

\[ 2h_1 = -4\pi \delta(0). \]  
(27)
Each of the above equations can be used to determine the parameter \( h_3 \) of expansion \((8)\), which are the same if one uses the field equation \((24)\).
It is interesting to note that the quantum correction terms (the third and fourth terms in the right-hand-side of eq.(25)) are much smaller than the classical terms:
\[ h_0^4 \sim \frac{1}{a_0^2} \ll h_0^2. \] (28)

The important observation is that if we set \( b \to 0 \), then eq.(20) results in \( \dot{\varphi}(0) = 0 \) and \( \rho_m(0) = 0 \), from which eq.(27) reduces to \( h_1 = 0 \) (note that \( \delta(0) = \dot{\varphi}^2(0) + \gamma_m \rho_m(0) \) and \( \dot{\delta}(0) = 2\dot{\varphi}(0)\ddot{\varphi}(0) - 3h_0 \gamma_m^2 \rho_m(0) \)). So without quantum effects, there is no non-trivial solution for eqs.(5) and (6) with \( H(t) \) given by eq.(8), and therefore there is no phantom-divide-line crossing in quintessence model in classical level. But by considering the quantum effects, \( h_1 \) has non-trivial solution (25) which can be positive or negative, depending on the values \( h_0, a_0, \dot{\varphi}(0), V(0) \) and \( \rho_m(0) \). So quantum effects can induce the \( \omega = -1 \) crossing in quintessence models.

As pointed out earlier, it has been shown that the behavior of finite-time singularity in phantom models becomes rather milder if one considers the quantum corrections. This is because near the singularity, the curvatures and their time derivatives become larger, so the quantum corrections, which include the powers of these quantities, become large and important and can control the singularity. So one can expect that the conformal anomaly, which in our considered quintessence model is the only reason for transition from quintessence phase to phantom phase, can not itself cause any Big Rip.

Now consider the possibility of \( \alpha = 3 \). In this case, one again finds the field equation (23), which expresses the Hubble parameter \( H(t) \) at \( t = 0 \) in terms of \( \varphi(0), \dot{\varphi}(0) \) and \( \ddot{\varphi}(0) \). But now the first equation, eq.(18), also gives \( h_0 \) in terms of the field \( \varphi \) and its derivatives, the matter density \( \rho_m \), and the scale factor \( a(t) \) at \( t = 0 \):
\[
\begin{align*}
    h_0 &= \left\{ \frac{1}{2} \left[ \frac{3}{16\pi b} \pm \sqrt{\left( \frac{3}{16\pi b} \right)^2 - \frac{2}{b} \left( \rho_{cl.}(0) + \frac{4b}{a_0^2} \right)} \right] \right\}^{1/2} \\
    \text{(29)}
\end{align*}
\]
So \( \alpha = 3 \) solution exists only if the right-hand-sides of eqs.(23) and (29) are equal, which is a very special choice of initial values. Under these conditions, of course, there is no \( \omega = -1 \) transition. So except these fine-tuned initial values, the solution is \( \alpha = 2 \) and the quantum effects induce the \( \omega = -1 \) crossing in quintessence models.

It is worth noting that this quantum phenomenon may have a contribution equal to, or even more important than, the classical effects if one considers the early stages of the universe in which \( h_0 \) is large (see, for example, eq.(25)). This is, of course, consistent with our physical intuition about the role of quantum effects in gravitational phenomena.

3.2 The phantom model

To study the phantom model one must consider \( N = N_{1/2} = N_1 = N_2 = 0 \) and \( N_{HD} = 1 \) in eq.(11). So
\[
b' = -\frac{7}{6} b = \frac{7}{90(4\pi)^2}.
\] (30)
The procedure of previous subsection can be followed here. In this way we find equations similar to (18)-(20), but now \( \rho_D \) and \( p_D \) are those introduced in eq.(11) and \( \rho_A \) and \( p_A \) are given by eqs.(12) and (13) with \( b' = -(7/6)b \) and \( b'' = 0 \).

The result is:

\[
h^2_0 = \frac{8\pi}{3} \left[ \rho_{\text{cl}}(0) - \frac{6b'}{7} \left( 7h^4 + \frac{10h^2_0}{a^2_0} + 8h_0h_1\delta_{\alpha,2} + \frac{9}{a^2_0} \right) \right], \tag{31}
\]

\[
0 = \rho_{\text{cl}}(0) - \frac{6b'}{7} \left( 24h_0h_2\delta_{\alpha,2} + 24h^2_0h_1\delta_{\alpha,2} + 24h_0h_1\delta_{\alpha,3} - \frac{20h^3_0}{a^5} - \frac{36h_0}{a^3} \right), \tag{32}
\]

and

\[
0 = \rho_{\text{cl}}(0) + p_{\text{cl}}(0) + \frac{8b'}{7} \left( 6h_0h_1\delta_{\alpha,2} + 6h_2\delta_{\alpha,2} + 6h_1\delta_{\alpha,3} - \frac{5h^2_0}{a^5} - \frac{9}{a^3} \right), \tag{33}
\]

in which

\[
\rho_{\text{cl}} = -\frac{1}{2} \dot{\alpha}^2 + V(\sigma) + \rho_m, \quad \rho_{\text{cl}} + p_{\text{cl}} = -\ddot{\alpha} + \gamma_m\rho_m. \tag{34}
\]

For \( \alpha \geq 3 \), eq.(31) does not depend on \( h_1 \), so \( h_0 \) is determined from two independent equations. The first one is the equation of motion of phantom field

\[
h_0 = -\frac{\ddot{\alpha}(0) - (dV/d\sigma)_0}{3\dot{\alpha}(0)}, \tag{35}
\]

and second one is eq.(31). These two expressions are equal only if a very specific initial conditions has been chosen. So the only typical solution, which always exists, is \( \alpha = 2 \).

In the case \( \alpha = 2 \), the equations (31), (33) can be used to calculate \( h_0, h_1 \) and \( h_2 \). \( h_0 \) is specified by eq.(35), and \( h_1 \) and \( h_2 \) can be expressed in terms of \( h_0 \). Using (31), \( h_1 \) is found as:

\[
h_1 = \frac{7}{48b'h_0} \left\{ \rho_{\text{cl}}(0) - \left[ \frac{3}{8\pi} h^2_0 + \frac{60b'h^2_0}{7a^2_0} + 6b'h^3_0 + \frac{54b'}{7a^2_0} \right] \right\}. \tag{36}
\]

Like the quintessence case, it can be easily shown that the resulting equations have no non-trivial solution in \( b' \to 0 \) limit, so there is no \( \omega = -1 \) crossing in phantom model in classical level. In \( b' \neq 0 \), \( h_1 \) becomes non-trivial and is determined by eq.(36).

In special case \( \rho_m(0) = 0 \) and \( V = 0 \), one has \( \rho_{\text{cl}} = -(1/2)\dot{\alpha}^2(0) \) and therefore eq.(36) clearly results in:

\[
h_1 < 0, \quad \text{for free pure phantom model} \tag{37}
\]

which proves the transition from \( \omega < -1 \) region to \( \omega > -1 \) region. In this case, the value of \( h_2 \) is determined by relation

\[
h_2 = \frac{1}{10368\pi b'\dot{\alpha}^4(0)a^4_0} \left\{ 2268\pi \dot{\alpha}^6(0)a^4_0 + 63\dot{\alpha}^2(0)(2268\pi \dot{\alpha}^6(0)a^4_0 + 112\pi b'\dot{\alpha}^4(0)a^4_0 + 2400\pi b'\dot{\alpha}^2(0)(2400\pi b'\dot{\alpha}^2(0)a^2_0 + 27216\pi b'\dot{\alpha}^4(0)) \right\} \tag{38}
\]

which is a positive quantity.
Therefore for arbitrary matter density and phantom potential, the sign of $h_1$ and $h_2$ can be positive or negative, depending on values $h_0$, $a_0$, $\dot{\sigma}(0)$, $V(0)$, and $\rho_m(0)$. But in free ($V = 0$) and pure ($\rho_m = 0$) phantom model, the sign of $h_1$ and $h_2$ are uniquely determined by Friedman equations. $h_1$ is negative and $h_2$ is positive.

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