Heavy Hybrid Mesons Masses

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Abstract
We estimate the ground state masses of the heavy hybrid mesons using a phenomenological QCD-type potential. $0^{-+}, 1^{-+}, 0^{++}, 1^{++}$ and $0^{+-} J^{PC}$ states are considered.

1 Introduction
Quantum Chromodynamics, acknowledged as the theory of strong interactions, allows that mesons containing gluons ( q̅qg hybrids ) may exist. The physical existence of these “exotic” particles (beyond the quark model) is one of the objectives of experimental projects [1]. These programs would contribute significantly to the future investigation of QCD exotics, and should improve our understanding of hybrid physics and on the role of the gluon in QCD. From experimental efforts at IHEP [2], KEK [3], CERN [4] and BNL [5], several hybrid candidates have been identified, essentially with exotic quantum numbers $J^{PC} = 1^{++}$ at 1400 and 1600 MeV. In the charm sector we can consider the recently observed $Y(4260)$[6] candidate.

We propose estimates for the masses of the hybrid mesons considering the c and b flavors, within the context of a quark-gluon constituent model using a phenomenological potential motivated by QCD and taking into account some relativistic effects.

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2 The model

We introduce a model in which the gluon is considered as massive constituent particle. The constituent gluon mass \( m_g \simeq 800 \text{ MeV} \) is chosen as an order of magnitude, taking in account the mass of the gluball candidate (1.6 GeV). Furthermore, the authors of [7] are generating constituent quarks and gluon masses in the context of Dynamical Quark Model employing BCS vacuum, and obtained a constituent gluon mass of 800 MeV. As proved in [9], the impact of the \( m_g \) in the results of hybrid masses is weak and the order of magnitude remains the same.

The Hamiltonian is constructed, containing a phenomenological potential which reproduce the QCD characteristics; its expression has the mathematical “Coulomb + Linear” form.

The basic hypothesis is to use a Schrödinger-type wave equation\[^8\]:

\[
\left\{ \sum_{i=1}^{N} \left( \frac{\vec{p}_i^2}{2M_i} + \frac{m_i^2}{2M_i} \right) + V_{eff} \right\} \Psi(\vec{r}_i) = E \Psi(\vec{r}_i) ;
\]

where \( M_i \) are some “dynamical masses” satisfying the conditions:

\[
\frac{\partial E}{\partial M_i} = 0 ;
\]

\( V_{eff} \) is the average over the color space of chromo-spatial potential\[^15\] :

\[
V_{eff} = \langle V \rangle_{\text{color}} = \left\langle - \sum_{i<j=1}^{N} \mathbf{F}_i \cdot \mathbf{F}_j \; v(r_{ij}) \right\rangle_{\text{color}}
\]

\[
= \sum_{i<j=1}^{N} \alpha_{ij} v(r_{ij}).
\]

The spatial term \( v(r_{ij}) \) is a potential motivated by QCD, which have the form:

\[
v(r_{ij}) = -\frac{\alpha_s}{r_{ij}} + \sigma \; r_{ij} + c;
\]

the \( \alpha_s, \sigma, \) and \( c \) may be fitted by experimental data.
3 The hybrid mesons

For the classification of hybrid mesons in a constituent model we will use the notations of [9]:

\( l_g \): is the relative orbital momentum of the gluon in the \( q \bar{q} \) center of mass;

\( l_{q \bar{q}} \): is the relative orbital momentum between \( q \) and \( \bar{q} \);

\( S_{q \bar{q}} \): is the total quark spin;

\( j_g \): is the total gluon angular momentum;

\( L \) : \( l_{q \bar{q}} + j_g \).

The parity and charge conjugation of the hybrid are given by:

\[ P = (-)^{l_{q \bar{q}} + l_g}, \]
\[ C = (-)^{l_{q \bar{q}} + S_{q \bar{q}} + 1}. \]

We have to solve the wave equation relative to the Hamiltonian:

\[ H = \sum_{i=q, \bar{q}, g} \left( \frac{\vec{p}_i^2}{2M_i} + \frac{M_i}{2} + \frac{m_i^2}{2M_i} \right) + V_{eff}; \quad (5) \]

with, for the hybrid meson:

\[ \alpha_{q \bar{q}} = -\frac{1}{6}, \]
\[ \alpha_{gq} = \alpha_{g \bar{q}} = \frac{3}{2}. \]

We define the Jacobi coordinates:

\[ \vec{\rho} = \vec{r}_{\bar{q}} - \vec{r}_q, \]
\[ \vec{\lambda} = \vec{r}_g - \frac{M_q \vec{r}_q + M_{\bar{q}} \vec{r}_{\bar{q}}}{M_q + M_{\bar{q}}}. \]

Then, the relative Hamiltonian is given by:

\[ H_R = \frac{\vec{p}_\rho^2}{2\mu_\rho} + \frac{\vec{p}_\lambda^2}{2\mu_\lambda} + V_{eff}(\vec{\rho}, \vec{\lambda}) + \frac{M_q}{2} + \frac{m_q^2}{2M_q} + \frac{M_{\bar{q}}}{2} + \frac{m_{\bar{q}}^2}{2M_{\bar{q}}} + \frac{M_g}{2} + \frac{m_g^2}{2M_g}; \quad (6) \]

with

\[ \mu_\rho = \left( \frac{1}{M_q} + \frac{1}{M_{\bar{q}}} \right)^{-1}, \]
\[ \mu_\lambda = \left( \frac{1}{M_g} + \frac{1}{M_q + M_{\bar{q}}} \right)^{-1}. \]
and

\[ V_{\text{eff}}(\bar{\rho}, \bar{\lambda}) = -\alpha_s \left( \frac{1}{6\rho} + \frac{3}{2} \frac{1}{|\bar{\lambda} + \frac{\bar{\rho}}{2}|} + \frac{3}{2} \frac{1}{|\bar{\lambda} - \frac{\bar{\rho}}{2}|} \right) \]

\[ + \sigma \left( \frac{1}{6} \rho + \frac{3}{2} \frac{1}{|\bar{\lambda} + \frac{\bar{\rho}}{2}|} + \frac{3}{2} \frac{1}{|\bar{\lambda} - \frac{\bar{\rho}}{2}|} \right) + \frac{17}{6} c + V_S. \]  (7)

The followed expansion representing the hybrid wave function in the cluster approximation:

\[ \Psi_{JPC}(\bar{\rho}, \bar{\lambda}) = \sum_{l_q l_g \mu \bar{\mu}} \rho_{l_q l_g} \lambda_{l_g} \chi_{S_{q\bar{q}}} \langle l_g m_g 1 \mu_g | J_g M'_g \rangle \]

\[ \times \langle l_q m_{q\bar{q}} J_g M'_g | L_m \rangle \langle L_m S_{q\bar{q}} | J 0 \rangle \]  (8)

where the summation runs over all possible values of \( l_q l_g \) \( L (M m \mu) \) for a given \( J^{PC} \) state, restricting ourselves to the first orbital excitations (\( l_q l_g \leq 2 \)) see Table 2,

and \( \varphi_{l_q l_g}(\bar{\rho}, \bar{\lambda}) \) are the Gaussian wavefunctions:

\[ \varphi_{l_q l_g}(\bar{\rho}, \bar{\lambda}) = \rho_{l-q} \lambda_{l-g} \exp \left( -\frac{1}{2} \beta (\rho^2 + \lambda^2) \right) \]

\[ Y_{l_q m_{q\bar{q}}}(\Omega_{\rho}) Y_{l_g m_{g\bar{g}}}(\Omega_{\lambda}); \]  (9)

The energy

\[ E_{JPC}(\beta, M_q, M_{\bar{q}}, M_g) = \frac{\langle J^{PC} | H_R | J^{PC} \rangle}{\langle J^{PC} | J^{PC} \rangle} \]  (10)

is minimized with respect to parameters \( \beta, M_q, M_{\bar{q}} \) and \( M_g \).

In Table 1 we give the parameters fitting to \( J^{PC} = 1^- - (c\bar{c}) \) and \( (b\bar{b}) \) spectrum.

| \( \alpha_s \) | \( \sigma \) (GeV\(^{-2}\)) | \( c \) (GeV) | \( m_c \) (GeV) | \( m_b \) (GeV) |
|---|---|---|---|---|
| 0.36 | 0.144 | -0.45 | 1.70 | 5.05 |

(Table 1: Heavy flavors potential parameters.)

We present in Table 2 our estimates of hybrid mesons masses for different \( J^{PC} \) states, we take 800 MeV for the mass of the gluon.
We compare our results with recent LQCD predictions\cite{10}(Table 3).

For the orders of magnitude of the masses, our results are in agreement with the recent LQCD predictions\cite{10}(Table 3), except the case of $0^- - $ state which is very more heavier than our results.

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| $J^PC$ | $S_{qar{q}}$ | $l_{qar{q}}$ | $l_g$ | $j_g$ | $L$ | $M_{car{c}}$ | $M_{bar{b}}$ |
|-------|--------------|--------------|------|------|----|--------------|--------------|
| $0^- - $ | 0             | 0.2          | 1    | 0.2  | 0  | 4.73         | 11.02        |
|        | 1             | 1            | 0.2  | 1.2  | 1  | 4.82         | 11.12        |
| $1^- - $ | 0             | 0.2          | 1    | 1.2  | 1  | 4.70         | 10.96        |
|        | 1             | 0.2          | 1    | 1.2  | 0  | 4.72         | 10.98        |
| $0^- +$ | 0             | 0.2          | 1    | 1.2  | 0  | 4.58         | 10.86        |
| $1^- +$ | 0             | 0.2          | 1    | 1.2  | 0  | 4.82         | 11.12        |
| $0^+ - $ | 0             | 2            | 2    | 0.1  | 0  | 5.88         | 11.02        |
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