Testing the time dependence of the fundamental constants in the spectra of multicharged ions

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Abstract

A new method for measuring a possible time dependence of the fine-structure constant (α) is proposed. The method is based on the level-crossing in two-electron highly-charged ions facilitating resonance laser measurements of the distance between the levels at the point of crossing. This provides an enhancement factor of about 10³ in Helium-like Europium and thus reduces the requirements for the relative accuracy of resonance laser measurements at about 10⁻¹².

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Questions about the constancy of fundamental coupling and mass parameters in physics, first addressed by Dirac [1], now become of particular interest in connection with new developments towards unified field theories, such as string theories, D-brane models, etc. These theories, in principle, allow for a space and time dependence of fundamental constants (see Ref. [2] for a comprehensive review on the subject). Presently, the situation regarding this problem appears to be very stringent, since spectroscopical studies of the spectra of quasars indicated a deviation of the fine-structure constant from its standard value of about \( \Delta \alpha/\alpha = (-0.76 \pm 0.28) \times 10^{-5} \) for values of the redshift parameter \( z \) within the range \( z = 1.8 - 3.5 \) [3]. This deviation is related to the logarithmic time derivative \( \dot{\alpha}/\alpha = (-0.38 \pm 0.14) \times 10^{-14} \text{yr}^{-1} \) and suggests that \( \alpha \) may have been smaller in the past. From the other side, the results obtained from quasar data strongly contradict with those deduced from the natural fission reactor in Oklo, which yield \( \dot{\alpha}/\alpha = (-0.2 \pm 0.8) \times 10^{-17} \text{yr}^{-1} \) [4,2]. This contradiction may be also interpreted as indication for a spacial variation of \( \alpha \). Accordingly, laboratory atomic measurements are highly necessary for clarifying the situation.

The best empirical bounds are set by the comparison of the two-photon \( 1s - 2s \) resonance frequency in hydrogen to a Cs clock during 4 years which yield the result \( \dot{\alpha}/\alpha = (-0.9 \pm 2.9) \times 10^{-15} \text{yr}^{-1} \) [5], and by the comparison of an optical transition in Yb\(^+\) ion to Cs clock during 2.8 years \( \dot{\alpha}/\alpha = (-0.3 \pm 2.0) \times 10^{-15} \text{yr}^{-1} \) [6].

Within this paper we propose a new type of atomic experiments for the search for variations of the fine-structure constant employing the crossings of atomic levels as a function of the nuclear charge number \( Z \) in two-electron highly-charged ions (HCl). These energy levels including QED corrections were recently evaluated with high accuracy in [7]. The effect of the possible time-variation of \( \alpha \) becomes strongly enhanced when it is investigated close to such crossing points.

We consider the energy difference \( \Delta E \) between two energy levels in Helium-like HCl as a function of the two parameters \( \alpha \) and \( Z \), respectively. Accurate calculation of the energy levels within the framework of QED should account for a number of energy corrections.
corresponding to various effects. In terms of these corrections the function $\Delta E$ can be represented as the sum

$$\Delta E(\alpha, Z) = \Delta E^{NS}(Z) + \Delta E^{1\text{ph}}(\alpha, Z) + \Delta E^{\text{SE+VP}}(\alpha, Z) + \Delta E^{2\text{ph}}(\alpha, Z) + \Delta E^{\text{Scr(SE+VP)}}(\alpha, Z), \tag{1}$$

where the nuclear-size correction is of order $\alpha^0$. In the following, we shall focus on the two-electron levels $(1s, 2s)$ and $(1s, 2p)$, respectively. In case of a point nucleus the Dirac energies of $2s$- and $2p$-states are equal, correspondingly, finite-nuclear size induces a shift $\Delta E^{NS}$ which is supposed to be independent on $\alpha$. The contributions $\Delta E^{1\text{ph}}$ and $\Delta E^{2\text{ph}}$ refer to one- and two-photon exchange corrections, respectively. The term $\Delta E^{\text{SE+VP}}$ represents the sum of the first-order (one-electron) self-energy (SE) and vacuum-polarization (VP) corrections. Finally, the term $\Delta E^{\text{Scr(SE+VP)}}$ denotes the corresponding screening corrections. Each individual energy shift can be decomposed into a power series with respect to $\alpha$ and $Z$ with coefficients $C_{\text{correction powers of } \alpha, Z}$. Accordingly, the one-photon exchange correction is taken into account up to terms

$$\Delta E^{1\text{ph}}(\alpha, Z) = C_{0,1}^{1\text{ph}} Z + C_{2,3}^{1\text{ph}} \alpha^2 Z^3 + C_{4,5}^{1\text{ph}} \alpha^4 Z^5. \tag{2}$$

The one-loop radiative effects are proportional to $Z(\alpha Z)^3$

$$\Delta E^{\text{SE+VP}}(\alpha, Z) = C_{3,4}^{\text{SE+VP}} \alpha^3 Z^4. \tag{3}$$

In comparison with $\Delta E^{1\text{ph}}$ the two-photon exchange correction $\Delta E^{2\text{ph}}$ is smaller by one factor of $Z$. Accordingly, it can be expanded up to terms $O(\alpha^5 Z^5)$

$$\Delta E^{2\text{ph}}(\alpha, Z) = C_{0,0}^{2\text{ph}} + C_{2,2}^{2\text{ph}} \alpha^2 Z^2 + C_{4,4}^{2\text{ph}} \alpha^4 Z^4. \tag{4}$$

The last term in Eq. (1) is a two-photon correction which accounts for the effect of screening of SE and VP

$$\Delta E^{\text{Scr(SE+VP)}}(\alpha, Z) = C_{3,3}^{\text{Scr(SE+VP)}} \alpha^3 Z^3. \tag{5}$$

While the terms proportional to $C_{0,1}^{1\text{ph}}$ and $C_{0,0}^{2\text{ph}}$ correspond to the pure Coulomb interaction between the two electrons, the terms with coefficients $C_{2,3}^{1\text{ph}}$ and $C_{2,2}^{2\text{ph}}$ represent
relativistic corrections to the interelectronic interaction, in particular, the Breit interaction. The terms with coefficients $C_{4,5}^{1\text{ph}}$ and $C_{4,4}^{2\text{ph}}$ account for higher-order relativistic corrections to the electron-electron interaction.

Since we are interested in the relative change of $\Delta E$ due to variations of the parameter $\alpha$, one may consider the quantity

$$\frac{\delta \Delta E(\alpha, Z)}{\Delta E(\alpha, Z)} = (\Delta E)^{-1} \frac{\partial \Delta E(\alpha, Z)}{\partial \alpha} \delta \alpha,$$

(6)

where $\delta \alpha$ is the change of the fine-structure constant during the time interval $\delta t$. Introducing now the relative variation $\delta \alpha/\alpha = (\dot{\alpha}/\alpha) \delta t$ yields

$$\frac{\delta \Delta E(\alpha, Z)}{\Delta E(\alpha, Z)} = \left[(\Delta E)^{-1} \alpha \frac{\partial \Delta E(\alpha, Z)}{\partial \alpha}\right] \frac{\dot{\alpha}}{\alpha} \delta t.$$

(7)

An estimate for the logarithmic derivative $\dot{\alpha}/\alpha$, measured in the units of yr$^{-1}$, follows from the measurement of the quantity on the left-hand side of Eq. (7). Recent atomic measurements can provide bounds at the level of accuracy $10^{-15}$ yr$^{-1}$, as stated above.

In view of Eq. (7) and assuming that the value $\Delta E(\alpha, Z)$ almost tends to zero at some value of $Z$ (crossing-point), we find that the coefficient

$$\eta = (\Delta E)^{-1} \alpha \frac{\partial \Delta E(\alpha, Z)}{\partial \alpha},$$

(8)

plays the role of an enhancement factor when extracting the value for $\dot{\alpha}/\alpha$ provided the left-hand side of Eq. (7) is known. Indeed, differentiating $\Delta E(\alpha, Z)$ we find

$$\Delta E(\alpha, Z_0) = 0,$$

(9)

$$\alpha \frac{\partial \Delta E(\alpha, Z_0)}{\partial \alpha} \neq 0,$$

(10)

where $Z_0$ (in general, a real number) would correspond to the exact crossing point.

Note, that for ions the equality (9) holds only approximately, since the nuclear charge number $Z$ takes integer values, only. Nevertheless, the cancellation of different terms in the expression for $\Delta E(\alpha, Z_0)$ can reduce its value by several orders of magnitude, while such a cancellation does not occur in Eq. (10). For $Z = 66$ the energy difference $\Delta E_{21S_0-23P_0}(\alpha, Z)$
in two-electron ions reduces to 0.016eV, while the individual terms in Eq. (1) contributing to Eq. (10) are of the order $10^2eV$. Thus, we can expect a value for the enhancement factor $\eta$ of the order $10^3$ (see [7]).

It should be emphasized, that in order to make use of this almost level crossing, one has to be able to measure directly the energy difference $\Delta E(\alpha, Z)$, otherwise the gain of the enhancement factor $10^3$ will be lost by the requirement to measure the positions of each of the crossing levels with an accuracy better by 3 order of magnitude. Fortunately, such direct measurements are feasible when applying laser techniques for the spectroscopy of HCl. Energy differences of the order of $1eV - 0.1eV$ correspond to the optical or infrared frequency intervals. A number of experiments with lasers has already been performed for measurements of hyperfine-structure intervals in HCl, which lay in the optical region [8,9].

To become sensitive to temporal variations of $\dot{\alpha}/\alpha$ at the level of $10^{-15}yr^{-1}$ in laser experiments with HCl performed during a time interval $\delta t$ about 1yr, it will require a relative accuracy in the measurement of the energy difference of the order $10^{-12}$. Here we have taken into account the enhancement factor $10^3$. In principle, this accuracy could be achieved in laser resonant measurements. In particular, the two-photon transition for $1s-2s$ states in hydrogen has been measured with a relative accuracy of about $10^{-14}$ [10]. Such a level of precision has not yet been reached in experiments with HCl. Nevertheless, as a precondition of any experiments theory should provide most accurate values for $\Delta E(\alpha, Z)$ although the goal to achieve an experimental accuracy of $10^{-12}$ is still far beyond present abilities. In laser resonance measurements in neutral atoms the theoretical accuracy is much poorer than the experimental one. This is a usual situation, which should not be considered as a hindrance for the search of a temporal variation of $\alpha$ in HCl.

Let us consider in more detail the crossing of the levels $2^1S_0$ and $2^3P_0$ in two-electron ion with nuclear charge number $Z = 66$. Since the one-photon $0 - 0$ transition is forbidden, one should either investigate transitions between the hyperfine sublevels in the isotopes with the nonzero nuclear spin or look for the two photon $E1M1$ resonance. The $E1M1$ transitions $2^3P_0 - 1^3S_0$ in two-electron HCl were studied in Ref. [11–13]. We should stress that the
width of the mentioned resonance is quite large due to the $E1E1$ transition $2^1S_0 - 1^1S_0$: \[
\Gamma(2^1S_0 \rightarrow 2\gamma(E1) + 1^1S_0) \approx 1 \times 10^{12}\text{s}^{-1} = 4 \times 10^{-3}\text{eV} \] [14] which is only 4 times smaller than our value for the transition frequency $2^1S_0 - 2^3P_0$ for $Z = 66$.

For each of the corrections Eqs. (2-5) the corresponding values for the coefficients $C^{\text{correction}}_{\text{powers of } \alpha, Z}$ in the vicinity of $Z = 66$, can be deduced via interpolation. Continuous curves $\Delta E^{\text{correction}}(Z)$ are generated employing the results of the calculations presented in [7]. The adjustment for the value $Z = 66$ yields the following results (in units of eV)

$$\Delta E^{\text{NS}} = 1.869,$$

$$C^{1\text{ph}}_{0,1} = 0.162, \quad C^{1\text{ph}}_{2,3} = -1.259, \quad C^{1\text{ph}}_{4,5} = -1.691,$$

$$C^{2\text{ph}}_{0,0} = -3.341, \quad C^{2\text{ph}}_{2,2} = 19.489, \quad C^{2\text{ph}}_{4,4} = -31.669,$$

$$C^{\text{SE+VP}}_{3,4} = 1.803, \quad C^{\text{Scr(SE+VP)}}_{3,3} = -0.761.$$

The relatively large values for the coefficients $C^{2\text{ph}}$ indicate the poor convergence of the series expansion (4). This does not mean, however, the inaccuracy in the result. One has to remember that the coefficients in (11) are obtained from the numerical all-order in $\alpha Z$ calculation and the poor convergence of $\alpha Z$ expansion for any correction in the left-hand side of Eq. (11) does not influence our conclusions. The fit of the exact numerical function by the few $\alpha Z$ expansion terms for the fixed value of $Z$ (in our case $Z = 66$) can be done as accurate as necessary and only the large values of the coefficients for $C^{2\text{ph}}$ correction compared to expansion coefficients to other corrections are puzzling. The minimum value for the total energy shift at $Z = 66$ turns out to be $\Delta E(\alpha, Z) = -0.016\text{eV}$. Calculation of the derivative according to Eqs. (1-5) leads to

$$\frac{\partial \Delta E(\alpha, Z)}{\partial \alpha} = 2C^{1\text{ph}}_{2,2} \alpha^2 Z^3 + 4C^{1\text{ph}}_{4,5} \alpha^4 Z^5 + 2C^{2\text{ph}}_{2,2} \alpha^2 Z^2 + 4C^{2\text{ph}}_{4,4} \alpha^4 Z^4 + 3C^{\text{SE+VP}}_{3,4} \alpha^3 Z^4 + 3C^{\text{Scr(SE+VP)}}_{3,3} \alpha^3 Z^3,$$

respectively after insertion of the coefficients (11) $\alpha(\partial \Delta E(\alpha, Z)/\partial \alpha) = -2.1 \times 10^1\text{eV}$. Hence the enhancement factor $\eta$ at $Z = 66$ is obtained as $\eta = 1.3 \times 10^3$. It was found that the inaccuracy of the employed scheme of interpolation is less than 6%.  


The feasibility of the proposed experiment depends crucially on whether tunable lasers within the frequency range of about 0.01 – 0.02eV, respectively, with wavelengths between 60 – 120µm are available. In particular, the Free Electron Lasers (FELs) satisfy these conditions, though they do not provide the required relative accuracy $10^{-12}$. In particular, the Forschungzentrum Rossendorf (Dresden) provides ELBE FELs [15] with wavelengths $\lambda = 3 - 25\mu m$ and it is supposed to reach the region $\lambda = 20 - 150\mu m$. Alternative methods may exploit relativistic Doppler tuning, multiphoton resonances and optical mixing techniques. E.g. CO$_2$ lasers have typical wavelengths in the region of about 10.6µm. Consider an ion with relativistic factor $\gamma = 1/\sqrt{1-\beta^2}$ colliding head-on with a photon of frequency $\omega_{\text{lab}}$ in the laboratory frame. In the ions rest frame the frequency $\omega$ is given by $\omega = \gamma (1 + \beta) \omega_{\text{lab}}$. The recent GSI ESR facility provides $\gamma = 1.048$ and up to $\gamma = 23$ at the new projected GSI storage ring [16]. Manipulating between these two values one can cover the range of wavelengths $10.6\mu m \leq \lambda \leq 488\mu m$. The same concerns tunable semiconductor-diode laser PbSnTe with wavelength $\lambda_{\text{lab}}$ within the range 6 – 30µm. Furthermore, one could think of exciting the levels under consideration via absorption of $n$ photons of frequency $\omega$ such that $\Delta E = n\omega$ (multiphoton resonance) [17,18]. Finally, we mention that the optical mixing techniques [19] can also help to achieve radiation sources with wavelengths within the range 1 – 100µm.

The most severe problem which arises when performing the proposed experiment is how to achieve the required accuracy $10^{-12}$ in case of the broad resonances. As mentioned above the width of the $2^1S_0 - 2^3P_0$ resonance in a He-like ion with $Z = 66$ is about $10^{12}s^{-1}$. Accordingly, the problem consists in the observation of time-dependence derivation of natural line profile. The latter is characterized by the maximum frequency $\omega_{\text{max}}$ and the width $\Gamma$. Although the value $\omega_{\text{max}}$ may not coincide with $\Delta E$ [20] it exhibits the same enhancement behaviour due temporal variation of $\alpha$. Thus, one has to concentrate on the measurement of $\omega_{\text{max}}$, which, in principle, appears as a question of statistics. Suppose that the measurement occurs in the vicinity of $\omega_{\text{max}}$ within the laser bandwidth that is assumed to be much smaller than $\Gamma$. Then each photon absorption can be considered as statistical “event”, or elementary
act of measurement. An accuracy $r = 10^{-12}$ implies to register $r^{-2} = 10^{24}$ “events”.

The required intensity of the laser beam may be estimated according to the formula $r^{-2} = TN_{\text{ion}} \sigma I_{\text{ph}}$. Here $T$ denotes the time duration of the laser experiment (e.g. even $T = 3 \times 10^7$ s may be still realistic), $I_{\text{ph}}$ is the photon flux (in units photons/s) and $N_{\text{ion}}$ is the number of ions, meeting the laser beam per second. The latter value may be estimated according to $N_{\text{ion}} = N_{\text{tot}} N_{\text{rev}}$, where $N_{\text{tot}} = 10^8/0.01 \text{cm}^2$ is the total number of the ions in the storage ring (assuming a beam diameter of about 0.1 cm) and $N_{\text{rev}}$ is the number of revolutions per second. A rough estimate for the photon-absorption cross-section $\sigma$ is provided by $\sigma \simeq (a_0)^2/Z^2$, where $a_0$ is the Bohr radius. For $Z = 66$ this yields about $\sigma \simeq 10^{-20} \text{cm}^2$. Taking all together one obtains the necessary laser beam intensity of the order $I_{\text{ph}} \simeq 3 \times 10^{20} \text{photon/s}$. Although this is a very huge number one should keep in mind first, that the future GSI facilities will provide an ion beam being more intensive by several orders of magnitude and second, that the progress in laser techniques has been very fast during the last years.

Very stringent, in principle, is also the problem with Doppler broadening. The latter is defined by the expression $\Gamma_D = \gamma \beta (\Delta v/v) \omega_{\text{lab}}$. While the natural broadening is time-translational invariant the Doppler broadening depends on the beam temperature, which can also vary during the time period $T$. Fortunately, the Doppler broadening appears to be smaller than the natural one in case of $2^1S_0 - 2^3P_0$ transition. According to [21] the relative velocity spread in the beam is given by $\Delta v/v \simeq 5 \times 10^{-4}$, which yields $\Gamma_D \simeq 2 \times 10^{-4} \omega_{\text{lab}}$, i.e. much smaller than $\Gamma$. Once the problem with the natural broadening will be solved, the Doppler broadening problem will be solved as well.

It should be mentioned, that similar atomic experiments utilizing levels crossing with both HCI and neutral atoms in the presence of an external magnetic or electric field can also be proposed. However, the reliability of such experiments will strongly depend on the constancy of the strength of the external fields during the time interval $\delta t \approx 1\text{yr}$. In our case the “constancy” of the $Z$ value is assured by the charge conservation law.

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before completion of this investigation.

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