Nonequilibrium transport through parallel double quantum dots in the Kondo regime

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(Dated: December 29, 2017)

We extend a perturbative, nonequilibrium renormalization group approach to multi-orbital systems and apply it for studying transport through two parallel quantum dots coupled electrostatically. In general, the conductance shows pronounced Kondoesque peaks at three voltages. One of these peaks disappears if, as in some experiments, one of the dots is decoupled from one of the two leads. For an asymmetric coupling to the leads, also negative differential conductances are possible. This is a genuine nonequilibrium effect, accompanying the Kondoesque peaks. Moreover, a criterion to distinguish spin and orbital Kondo effect in such a system is discussed.

PACS numbers: 73.63.Kv,71.27.+a,72.10.Fk

I. INTRODUCTION

Kondo physics in quantum dots has been under intense investigation in recent years. As was predicted theoretically,\textsuperscript{12} the Kondo effect can lift the Coulomb blockade at low temperatures, leading to a conductance peak at zero bias. In such experiments, often more than one orbital is involved. This is unavoidable, if the temperature or Kondo temperature is larger or comparable to the mean orbital splitting. Some nanostructures are also intentionally constructed in such a way, for example, two parallel quantum dots coupled electrostatically.\textsuperscript{4,5,6} The latter allows for a controlled study of the interplay of spin and orbital (upper/lower dot) degrees of freedom.

On the theoretical side, the N-fold degenerate situation where all N (spin and orbital) levels of the Anderson model have the same one-particle energy ε, Coulomb interaction U, and hopping amplitude t to the leads is best understood. At strong coupling, the Schrieffer-Wolff transformation then leads to the Coqblin-Schrieffer model in terms of the exchange coupling J. This model has SU(N) symmetry for the rotation of orbitals and spins and, with increasing degeneracy N, leads to a strongly enhanced Kondo temperature $T_K \approx D e^{-1/(N \rho_0)}$ ($\rho_0$: density of states in the leads; D: bandwidth of the leads; we set $k_B = h = e = 1$ unless these constants appear explicitly in the equations).

Of course, we cannot expect a real quantum dot to have SU(N) symmetry, unless special arrangements are made.\textsuperscript{11} Hence, quite an effort was devoted in the literature\textsuperscript{12,13,14,15,16} to study the situation where either a magnetic field or a difference in orbital energy \(\delta\) splits \(N = 4\) levels into two doubly-degenerate subsets, hence breaking the SU(4) symmetry. If \(\delta \ll T_K^{SU(4)}\), the low-energy physics still resembles that of the SU(4)-symmetric model. In contrast for \(\delta \gg T_K^{SU(4)}\) one of the levels drops out of the scaling procedure; the low-energy physics is that of the usual SU(2)-symmetric Kondo model. In between, there is a crossover region.\textsuperscript{15}

It is also unavoidable that the coupling constants \(J\) are orbital-dependent in experiment. Numerical renormalization group (NRG) calculations\textsuperscript{14,16} suggest that for small enough \(\delta\) both orbitals are screened at the same energy, in agreement with scaling analyses showing a robust (marginal) SU(4) fix point.\textsuperscript{15,16,17}

Let us also distinguish here between orbital conserving and non-conserving dot-lead couplings. In our paper, we consider the former, experimentally realized by separate leads for each dot; see Figure 1. The non-conserving case is obtained if the quantum dots couple to a single lead like in Ref. \textsuperscript{18} Then the hopping processes do not conserve the orbital quantum number, which can give rise to SU(2) Kondo physics even if more than one degenerate level is involved, see Ref. \textsuperscript{18} and references therein.

So far, we only discussed equilibrium physics. A finite voltage \(V\) leads however to genuine nonequilibrium effects. For example, if \(V \approx \delta \gg T_K\) (the Kondo temperature) scattering between the two subsets of levels becomes relevant again. A Kondoesque resonance develops at \(V \approx \delta\), visible e.g. as a peak in the conductance. But, at the same time, a genuine effect of nonequilibrium is the presence of decoherence processes induced by the finite current. This decoherence cuts off the flow to the strong coupling fix point even for temperatures $T \ll T_K^{SU(4)}$.\textsuperscript{19,20}

Then, physical quantities such as the differential conductance depend strongly on the configuration of the system like the asymmetry of the hopping \(t\) between quantum dot and leads.

Nonequilibrium Kondo physics is not as thoroughly

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{Fig1.png}
\caption{Sketch of the parallel double quantum dot configuration. Each dot is coupled to its own leads; the coupling between the the dots is only via the Coulomb interaction (not indicated in the Figure).}
\end{figure}
investigated theoretically because standard approaches for the Anderson impurity model like NRG are not applicable. Rosch and coworkers recently developed a perturbative, nonequilibrium renormalization group (RG) method to address nonequilibrium Kondo physics. This approach is similar to Anderson's poor man's scaling approach but includes nonequilibrium effects caused by the finite current through the system. In nonequilibrium, the low energy physics cannot be absorbed properly by a pure renormalization of the bare couplings. It is necessary to include the frequency dependence of the vertex functions.

The outline of the paper is as follows: In Section II we extend the perturbative RG of Rosch et al. to many orbitals, supplemented by the calculation of the decoherence rates in the Appendix. In Section III we apply this method to the parallel double quantum dot system, with each dot being coupled to separate leads. Thereby, the first part, Section III.A, discusses briefly the simplification of the RG Eqs. for this special application. The following Sections present our results: Section III.B shows the renormalized vertex function, primarily of theoretical interest, Section III.C the increase of the decoherence rates with voltage, and Section III.D the change of the occupation in non-equilibrium. The reader who is mainly interested in the differential conductance may also directly turn to Sections III.E and III.F which present our results for this physical observable in the case of symmetric and asymmetric coupling to the two leads, respectively. In the case of symmetric couplings (Section III.E) we generally obtain two orbital Kondo peaks at $V \approx \pm \delta$ in the differential conductance of both dots and an additional spin Kondo peak at $V = 0$ for the dot which is lower in energy. Depending on the parameters, the peak at $V = 0$ can be strongly suppressed and hardly discernible. The main findings of Section III.E for asymmetric couplings are: (i) in very asymmetric situations the Kondo peaks at $V \approx -\delta$ can disappear and (ii) negative differential conductances are possible for $V \gtrsim \delta$. In addition to these non-equilibrium results, the linear (equilibrium) conductance is analyzed in Section III.G. A summary of the results can be found in Section IV.

II. NONEQUILIBRIUM PERTURBATIVE RG

Starting point for modeling a quantum dot with $N$-levels should be the Anderson impurity model:

$$H_{\text{AIM}} = H_0 + \sum_{\lambda \mu \nu} (t_{\lambda \mu \nu} c_{\lambda \mu \nu}^+ d_{\lambda \mu \nu} + \text{H.c.}) + \sum_m \varepsilon_m d_m^+ d_m + \sum_{m \neq m'} U_{mm'} n_m n_{m'}.$$  \hspace{1cm} (1)

Here $d_m^+$ and $d_m (c_{m \lambda \mu}^+ \text{ and } c_{m \lambda \mu})$ are creation and annihilation operators for electrons in the dot (lead $\lambda$); $n_m = d_m^+ d_m$; $\varepsilon_m$ describes the one-particle energy of level $m$ with $m = \{ \sigma, i \}$ summing the spin and orbital index; $U_{mm'}$ is the Coulomb repulsion within the dot. The levels of the dot hybridize via $t_{\lambda m}$ with non-interacting leads $\lambda$ described by (later, we simplify $\epsilon_{\lambda m \nu} = \epsilon_k$)

$$H_0 = \sum_{\lambda m k} \epsilon_{\lambda m k} c_{\lambda m k}^+ c_{\lambda m k}.$$  \hspace{1cm} (2)

In this paper, we assume each quantum dot level $m$ to couple to its own lead channel which is for example the case if the different levels correspond to different quantum dots like in Ref. 4 (see also Figure II of the present paper as an illustration). But in other situations it is possible to move an electron phase coherently from one level $m$ via the leads to another level $m'$. This goes beyond Hamiltonian II.

While the Anderson impurity model allows us to make contact with experiment (estimating parameters), we address in the following the parameter regime $\epsilon_m \ll \mu_\lambda \ll U_{mm'} + \varepsilon_m$ for all $m,m',\lambda \langle \mu_\lambda \rangle$; chemical potential in lead $\lambda$ so that charge fluctuations are suppressed. Then, with $\sum_{m} n_m = 1$ electron within the dot, we can map Hamiltonian II onto the subspace with one electron in the dot by a Schrieffer-Wolff transformation, i.e., onto the general effective Kondo Hamiltonian (neglecting a potential scattering term)

\[
H = H_0 + \sum_m \epsilon_m X_{m m} + \sum_{\lambda m_1 k_1, \lambda m_2 k_2} (J_{m_1 m_2}^{\lambda \lambda_1 \lambda_2} X_{m_1 m_2} c_{\lambda m_1 k_1}^+ c_{\lambda m_2 k_2} + J_{m_1 m_2}^{\lambda \lambda_1 \lambda_2} X_{m_1 m_2} c_{\lambda m_1 k_1} c_{\lambda m_2 k_2}). \hspace{1cm} (3)
\]

Here, the Hubbard operators $X_{m_1 m_2}$ represent the scattering of the local state from level $m_2$ to level $m_1$. When the lead’s band edge $D > |\varepsilon_m|$, the local level $\varepsilon_m$ of the Anderson impurity model is renormalized to $\varepsilon_m$ in the Kondo model $J_{m_1 m_2}^{\lambda \lambda_1 \lambda_2} = t_{m_1 m_2} t_{m_2 m_1} (\frac{1}{\varepsilon_m - \varepsilon_d})$ and $J_{m_1 m_2}^{\lambda \lambda_1 \lambda_2} = (1 - 2\delta_{m_1 m_2}) t_{m_1 m_2}^* \frac{1}{\varepsilon_m - \varepsilon_d}$, where $\varepsilon_d$ is the average level energy, neglecting the level splitting which is much smaller than $U, \epsilon_d$. Note, that for $m_1 = m_2$ both $J$ and $\tilde{J}$ yield the same kind of contribution. The advantage of having this term twice (i.e., distributed to $j_{m_1 m_2}^{\lambda \lambda_1 \lambda_2}$ and $\tilde{j}_{m_1 m_2}^{\lambda \lambda_1 \lambda_2}$) is the simplification of the following equations. The same distribution of the $m_1 = m_2$ contribution is usually also employed for the COqblin-Schrieffer model which is the SU(N) symmetric version of Hamil-
The full line denotes the electron and the dashed the local-

The vertex depends only on one interval with the corresponding first (particle) diagram. For the vertex renormalization, the electron line is restricted to a small interval $|dD|$ at the band edges, as indicated by the dash.

For the nonequilibrium situation, the calculation of the diagrams of Figure 2 requires in principle the usage of Keldysh Green functions. Rosch et al., however that, to leading order in $1/\ln(V/T_K)$, it is sufficient to keep track of the real part of $g_{m_1m_2}^{\lambda_1\lambda_2}(\omega)$, as indicated by perturbation theory. This gives rise to the same kind of vertex renormalization per energy interval $\ln D$ as in the poor man’s scaling approach, i.e, at $T = 0$:

$$\frac{1}{2} \frac{\partial}{\partial \ln D} \int_{-D}^{D} \frac{\text{sign}(\epsilon)}{\epsilon - \Delta \omega} d\epsilon$$

Here, $\Delta \omega$ is given by the chemical potential and the change of energy of the localized state in Figure 2. A finite temperature, smears out the sharp step of the integrand in Eq. (3), cutting off the emerging logarithm.
in Section III with SU(2) spin symmetry, only two of the orbitals \( i \) \((m = \{\sigma, i\})\) can be different, and the vertices are symmetric w.r.t. the spin indices. We then actually use the intra-orbital rate \( \gamma = \max\{T, \gamma_i \equiv \gamma_1\} \) as a cut-off for diagrams where only one orbital is involved and the inter-orbital \( \gamma = \max\{T, \gamma_{12}\} \) whenever both orbitals are involved. The calculation of \( \gamma_{m_1m_2} \) via the susceptibilities is presented in the Appendix.

Having identified the cut-off \( \gamma_{m_1m_2} \), we can now write down the RG equation for the vertex functions \( \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega) \) and \( g_{m_1m_2}^{\lambda_1\lambda_2}(\omega) \), corresponding to Figure 2.

\[
\frac{\partial \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega)}{\partial \ln D} = \sum_{\lambda} g_{m_2m_1}^{\lambda_1\lambda}(\omega + \frac{\epsilon_{m_1} - \epsilon_{m_2}}{2}) \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega + \frac{\epsilon_{m_1} - \epsilon_{m_2}}{2}) \Theta^{\gamma_{m_2m_1}^{\lambda_1\lambda_2}} \nonumber
\]

\[
\frac{\partial g_{m_1m_2}^{\lambda_1\lambda_2}(\omega)}{\partial \ln D} = - \sum_{m_{\lambda}} g_{m_1m_2}^{\lambda_1\lambda}(\omega - \frac{\epsilon_{m_2} - \epsilon_{m_1}}{2}) \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega - \frac{\epsilon_{m_2} - \epsilon_{m_1}}{2}) \Theta^{\gamma_{m_2m_1}^{\lambda_1\lambda_2}} + \sum_{\lambda} \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega) \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega + \frac{\epsilon_{m_2} - \epsilon_{m_1}}{2}) \Theta^{\gamma_{m_2m_1}^{\lambda_1\lambda_2}} + \sum_{\lambda} \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega) \tilde{g}_{m_1m_2}^{\lambda_1\lambda_2}(\omega - \frac{\epsilon_{m_2} - \epsilon_{m_1}}{2}) \Theta^{\gamma_{m_2m_1}^{\lambda_1\lambda_2}}. \tag{6}
\]

The equation for \( \tilde{g} \) can be obtained from Figure 2. The first line of the equation for \( g \) corresponds to the right diagram of Figure 2; the second and third line stem from the particle and the hole diagram of Figure 2 with \( m = m_2 \) and \( m = m_1 \), respectively. Note that the different frequencies of Figure 2 are related by energy conservation and that \( \omega = (\omega_1 + \omega_2)/2 \). The \( k \)-integration (note that \( k \) is not conserved) over the band edge states results in a factor \( \rho_0 \) which is already included in \( g \). The electron states in the leads range to \( \pm D \) relative the the respective chemical potential \( \mu_\lambda \).

The renormalized vertex functions of Eq. 6 have to be calculated self-consistently together with \( \gamma_{m_1m_2} \). Eq. 6 of the Appendix determine the cut-offs \( \gamma_{m_1m_2m_3} \).

**Conductance**

To study the most important physical quantity, the conductance, we employ the scattering T-matrix \( T_{m_1m_2}^{\lambda_1\lambda_2}(k_1, k_2) \) for the process moving an electron from state \( \{k_1m_1\} \) in lead \( \lambda_1 \) to state \( \{k_2m_2\} \) in lead \( \lambda_2 \). This process necessarily also involves the local state which changes in the opposite direction. The change of the local state can be expressed via spin and orbital operators in the T-matrix (see e.g. Ref. 11). By \( \langle \hat{T}_{m_1m_2}^{\lambda_1\lambda_2}(k_1, k_2) \rangle^2 \), we then denote the thermal average of the T-matrix w.r.t. the local state configurations. The steady state condition requires in terms of this averaged T-matrix:

\[
\sum_{\lambda_1, \lambda_2; k_1, k_2} \langle \hat{T}_{m_1m_2}^{\lambda_1\lambda_2}(k_1, k_2) \rangle_f \lambda_f(\epsilon_k) \{1 - \lambda_f(\epsilon_k)\} = \sum_{\lambda_1, \lambda_2; k_1, k_2} \langle \hat{T}_{m_2m_1}^{\lambda_1\lambda_2}(k_2, k_1) \rangle_f \lambda_f(\epsilon_k) \{1 - \lambda_f(\epsilon_k)\}, \tag{7}
\]

where \( \lambda_f(\epsilon) = 1/(e^{\beta(\epsilon - \mu_\lambda)} + 1) \) is the Fermi function for lead \( \lambda \). A difference to Ref. 21 is that we do not take into account a \( \gamma \)-broadening of the spectral function. This is effectively described by a broadening of the Fermi functions in Ref. 21 resulting in corrections to subleading order in \( 1/\ln(V/T_K) \).

Since we consider the situation where either the voltage-induced decoherence rate or the temperature is larger than the Kondo temperature the renormalization process is cut off, instead of flowing to strong coupling. Therefore, we are at weak coupling and we can calculate the scattering matrix in lowest order: \( \hat{T}_{m_1m_2}^{\lambda_1\lambda_2}(k_1, k_2) = \langle \lambda_2m_2k_2 | H_{\text{int}} | \lambda_1m_1k_1 \rangle \), where \( H_{\text{int}} \) denotes the interaction between the lead electrons and the local states via \( J \) and \( \tilde{J} \), i.e., the last two terms of Eq. 6. Averaged over the local states, this yields:

\[
\langle \hat{T}_{m_1m_2}^{\lambda_1\lambda_2}(k_1, k_2) \rangle^2 = \left(1 - \delta_{m_1m_2}\right) \langle n_{m_2} \rangle \lambda_f(\epsilon_k) \lambda_f(\epsilon_k) \omega^2 / \rho_0^2 + \frac{\delta_{m_1m_2}}{\rho_0^2} \sum_{m \neq m_1} \langle n_{m} \rangle \lambda_f(\epsilon_k) \lambda_f(\epsilon_k) \omega^2 / \rho_0^2 + \frac{\delta_{m_1m_2}}{\rho_0^2} \langle n_{m_1} \rangle \lambda_f(\epsilon_k) \lambda_f(\epsilon_k) \omega^2 / \rho_0^2. \tag{8}
\]

Here, we have replaced the bare coupling by the renormalized one, i.e., \( J \rightarrow \bar{g}(\omega)/\rho_0 \) with \( \omega \equiv (\epsilon_k + \epsilon_{k'})/2 \); \( \langle n_{m} \rangle \) is the average occupation of local level \( m \).

In a first step \( g \) and \( \bar{g} \) are calculated by Eq. 6, which is solved self-consistently together with the cut-off \( \gamma_{m_1m_2} \) [Eq. A.4, 7]. With these \( g \)’s and \( \bar{g} \)’s, the occupations \( \langle n_{m} \rangle \) can be determined via Eqs. 7 and 8 and the constraint \( \sum_m \langle n_{m} \rangle = 1 \).

From the T-matrix [Eq. 8], we can then calculate the current from lead \( \lambda \) and orbital \( m \) as the difference
between scattering out of and into the lead:

\[ I_{\lambda m} = -2\pi \sum_{\lambda' m' \neq \lambda m} \delta(\epsilon_k - \epsilon_{k'} + \epsilon_{m'} - \epsilon_m) \]

\[
\left\{ f_x(\epsilon_k) [1 - f_x(\epsilon_{k'})] \langle \hat{T}_{m'm}^{\lambda\lambda'}(k, k') \rangle^2 \right. \\
- f_x(\epsilon_{k'}) [1 - f_x(\epsilon_k)] \langle \hat{T}_{m'm}^{\lambda\lambda'}(k', k) \rangle^2 \right\}. \tag{9}
\]

The differential conductance follows as

\[ G_{\lambda m} = \partial I_{\lambda m} / \partial V. \]

III. TWO PARALLEL QUANTUM DOTS

Let us now apply the perturbative RG equations to the situation of two parallel quantum dots each coupled to two leads, see Figure 1 of the introduction. For the Anderson impurity model, this dot-lead coupling is described by hopping processes \( v_{\lambda m} \): additionally there are intra-dot and inter-dot Coulomb repulsions \( U_{\alpha i} \). We study here the corresponding Kondo model which is restricted to the subspace with one electron in the dot. Then, \( g_{\lambda m}^{\lambda' \lambda} \) describes the combination of two hopping processes, leaving the number of electrons in the dot unchanged.

A. Simplification of the RG Eq.

The two parallel quantum dot case corresponds to quantum numbers \( m = \{i, \sigma \} \) with \( \sigma = +, - \) denoting the spin and \( i = 1, 2 \) the two dots (orbitals) with level energy \( \epsilon_{i, \sigma} = -i/2 \) \((i = -i \) in the following).

Due to the SU(2) spin symmetry in each dot, we can reduce the number of vertices we have to deal with. Furthermore, some of the vertices are connected via scaling invariants. Altogether this allows for the following reduction of vertices:

\[ g_{\lambda i, \sigma, i' \sigma'}^{\lambda', \lambda} (\omega) \equiv g_{\lambda i, \sigma, i' \sigma'}^{\lambda', \lambda} (\omega) / 2, \quad g_{\lambda i, \sigma, i' \sigma'}^{\lambda', \lambda} (\omega) \equiv g_{\lambda i, \sigma, i' \sigma'}^{\lambda, \lambda} (\omega) / 2, \quad g_{\lambda i, \sigma, i' \sigma'}^{\lambda, \lambda} (\omega) \equiv -g_{\lambda i, \sigma, i' \sigma'}^{\lambda, \lambda} (\omega) / 4. \]

From the scaling invariants:

\[ g_{\lambda i, \sigma, i' \sigma'}^{\lambda', \lambda} (\omega) = (\tilde{g}_{\lambda i, \sigma, i' \sigma'}^{\lambda', \lambda} (\omega) - g_{\lambda i, \sigma, i' \sigma'}^{\lambda', \lambda} (\omega)) / 4. \]

The meaning of the redefined vertices becomes clear if we write the Hamiltonian in terms of the coupling constants \( J_{i(12)} \) and \( \tilde{J} \) corresponding to \( g_{i(12)}(\omega) \) and \( \tilde{g}(\omega) \):

\[ H = \sum_{\lambda \sigma \lambda' \sigma'} \left( \epsilon_k - \mu \right) c_{\lambda \sigma k}^{+} c_{\lambda' \sigma k} - \frac{\delta}{2} (n_1 - n_2) \tag{10} \]

\[ + \sum_{\lambda \lambda' \sigma, \bar{\sigma}, \sigma' \bar{\sigma}' k \bar{k'}} \left( \frac{J_{\lambda \lambda'}^{\lambda' \lambda}}{2} s_i \cdot c_{\lambda \sigma k}^{+} \tau_{\sigma \bar{\sigma}} c_{\lambda' \bar{\sigma} k'}^{+} \right) \]

\[ + \sum_{\lambda \lambda' \sigma, \bar{\sigma}, \sigma' \bar{\sigma}' k \bar{k'}} \left( \frac{\tilde{J}_{\lambda \lambda'}^{\lambda' \lambda}}{2} (X^{(1')}(2')) c_{\lambda \sigma k}^{+} c_{\lambda' \bar{\sigma} k'} + H.c. \right) \]

\[ + \frac{J_{\lambda \lambda'}^{\lambda' \lambda}}{2} \sum_{\lambda' \sigma, \bar{\sigma}, \bar{\sigma}' k \bar{k'}} \left( c_{\lambda \sigma k} c_{\lambda' \bar{\sigma} k'}^{+} c_{\lambda' \bar{\sigma} k'}^{+} + H.c. \right) \]

\[ + \frac{\tilde{J}_{\lambda \lambda'}^{\lambda' \lambda}}{4} \sum_{\lambda' \sigma, \bar{\sigma}, \bar{\sigma}' k \bar{k'}} \left( c_{\lambda \sigma k} c_{\lambda' \bar{\sigma} k'}^{+} c_{\lambda' \bar{\sigma} k'}^{+} + H.c. \right). \]

Here, \( S_i \) denotes the spin operators in orbital \( i \), \( \tau \) the vector of Pauli matrices, and \( n_i = \sum_\sigma n_{i, \sigma} \). Note that with only one electron in the dots, there is no need to consider Hund’s coupling.

Using the SU(2) symmetries in both orbitals and the above scaling invariants, the scaling Eq. (12) becomes particularly simple for a symmetric coupling to the leads (then, \( g_{i} \), \( \tilde{g}_{i} \), and \( g_{i+\lambda} \) do not depend on the lead indices; we can hence drop the \( \lambda, \lambda' \) indices):

\[ \frac{\partial g_{i}^{\lambda \lambda}}{\partial \ln D} = - \sum_{i, \lambda = \pm 1} \left[ g_{i}^{\lambda \lambda} (\omega) \Theta^{\lambda \lambda}_{\omega + i + \lambda \lambda} + \Theta^{\lambda \lambda}_{\omega + i + \lambda \lambda} \right], \]

\[ \frac{\partial g_{2}^{\lambda \lambda}}{\partial \ln D} = - \sum_{i, \lambda = \pm 1} \left[ g_{2}^{\lambda \lambda} (\omega) \Theta^{\lambda \lambda}_{\omega + i + \lambda \lambda} + \Theta^{\lambda \lambda}_{\omega + i + \lambda \lambda} \right], \]

\[ \frac{\partial g_{i}^{\lambda \lambda}}{\partial \ln D} = - \sum_{i, \lambda = \pm 1} \left[ g_{i}^{\lambda \lambda} (\omega) \Theta^{\lambda \lambda}_{\omega + i + \lambda \lambda} + \Theta^{\lambda \lambda}_{\omega + i + \lambda \lambda} \right]. \tag{11} \]

Eq. (11) has to be solved self-consistently together with the dephasing rates of the Appendix which simplify for the parallel double dot system to:

\[ \gamma_i = \pi \sum_{\lambda \lambda'} \int d\omega \left\{ g_{i}^{\lambda \lambda} (\omega) \left[ 1 - f_{\lambda\lambda}(\omega) \right] \right. \]

\[ \left. + g_{2}^{\lambda \lambda} (\omega) f_{\lambda\lambda}(\omega + i \frac{\delta}{2}) \left[ 1 - f_{\lambda\lambda}(\omega - i \frac{\delta}{2}) \right] \right\} \tag{12} \]

\[ \gamma_{12} = \frac{\pi}{2} \sum_{\lambda \lambda'} \int d\omega \left\{ g_{12}^{\lambda \lambda} (\omega) \left[ 1 - f_{\lambda\lambda}(\omega) \right] \right. \]

\[ \left. + g_{12}^{\lambda \lambda} (\omega + i \frac{\delta}{2}) f_{\lambda\lambda}(\omega - i \frac{\delta}{2}) \right\}. \tag{13} \]

These \( \gamma_i \), or \( T \) for \( T > \gamma_i \), cut off the \( \Theta \)-functions in Eq. (11).

The equations for the occupations \[ \{ Eq. (7), taking \( n_1 = n_2 \) in Eq. (3) \} and the current \[ \{ Eq. (9) \} \] reduce to:

\[ \langle n_i \rangle \sum_{\lambda \lambda'} \int d\omega f_{i/2}^{\lambda \lambda}(\omega + i \frac{\delta}{2}) \left[ 1 - f_{\lambda\lambda}(\omega + i \frac{\delta}{2}) \right] \tag{14} \]

and

\[ I_{\sigma} = \frac{\pi}{2} \int d\omega \left\{ 3 \langle n_i \rangle g_{i}^{\lambda \lambda}(\omega) f_{\lambda\lambda}(\omega) \left[ 1 - f_{\lambda\lambda}(\omega) \right] \right. \]

\[ \left. + 4 \langle n_i \rangle g_{12}^{\lambda \lambda}(\omega + i \frac{\delta}{2}) f_{\lambda\lambda}(\omega) \left[ 1 - f_{\lambda\lambda}(\omega - i \frac{\delta}{2}) \right] \right\} \tag{15} \]

respectively. In Eq. (15), the first line stems from the current from left channel \( i \) to right channel \( i \). The 2nd and 3rd (4th and 5th) line correspond to the current from right (left) channel \( i \). The latter are balanced by similar terms from the right lead so that the net current from one orbital to the other is zero.

B. Results: Renormalized vertex function

Let us now discuss the numerical solution for the renormalized vertex functions of Eq. (11) which is pre-
sent in Figure 5. This vertex function, a frequency-dependent renormalization of the interactions $J, J$ between lead electrons and local states in the quantum dot, is the fundamental theoretical quantity; it is not directly observable experimentally, but together with the occupations allows to calculate e.g. the current which will be discussed later. In the following, all energies are normalized to the (equilibrium) Kondo temperature for $\delta = 0$, i.e., to $T_K^0 = T_K(\delta = 0)$. Due to the finite voltage $V$ applied, the two leads are thermalized by two different Fermi functions with $\mu_L = \mu + V/2$ and $\mu_R = \mu - V/2$, respectively. The quantum dot is out of equilibrium.

Here and in the following figures, the initial coupling constants which are not explicitly mentioned follow from the ones of the Figure caption since they stem from the same $t$'s of the Anderson impurity. For example, $g_{LR}^{2(12)} = g_{12}^{L,R} \sqrt{g_1^{1R}/g_1^{1L}}$. We always set $\tilde{g}_i^{\lambda'\lambda} = 0$ initially.

To understand the four peak structure of Figure 5 it is instructive to integrate Eq. (11) analytically, dividing the Eqs. by the squared coupling constant from the left hand side (e.g. multiplying the first line by $1/g_i^2(\omega)$) and afterwards keeping the $g$'s on the right hand side constant (at the initial values). This is justified as long as the renormalization of the vertex functions stays small. But our mere aim here is to understand the positions and widths of the different peaks. From Eq. (11) we straightforwardly get in this way:

$$g_i(\omega) \sim \frac{1}{\sum_{\lambda} \ln \frac{|\omega + i(\delta + \lambda V/2)|_{12}}{T_{K,i}}},$$

$$g_{12}(\omega) \sim \frac{1}{\sum_{i,\lambda} \ln \frac{|\omega + i(\lambda V/2)|_{12}}{T_{K,i}}}$$

$$\tilde{g}_i(\omega) \sim \frac{1}{\sum_{\lambda} \ln \frac{|\omega + i(\lambda V/2)|_{12}}{T_{K,i}}}$$

where $|\omega|_i \equiv \sqrt{\omega^2 + \max^2}\{\gamma_i, T\}$, and factors of order one (like $g_{12}/g_i$) before the logarithmic terms are neglected.

By means of Eq. (18), we can now identify the origin of the four Kondoesque peaks in the $g_i$'s seen in Figure 5. When the frequency matches the difference to the left or right Fermi level ($\omega = \pm V/2$) a single dot (spin) Kondo effect develops; but the flow to strong coupling is cut off by the corresponding decoherence rate $\gamma_i$ (or $T$). The other two peaks in $g_i$ stem from the orbital Kondo effect. Here, the localized state changes to the other orbital and back to the original orbital. These processes are enhanced if the conduction electron scatters to the Fermi level of the left or right lead, requiring $\omega = -i \delta \pm V/2$. Hence the resonances [and those of Eq. (18)] are at $\omega = -i \delta \pm V/2$ and the cut-off is $\gamma_{12}$. Similarly, the peaks for $g_{12}(\omega)$ in Eq. (17) are at $\pm \frac{V}{\sqrt{2}}$ (with $\delta/2$ instead of $\delta$ since $\omega$ is the average of incoming and outgoing frequency which differ by $\delta$ as $g_{12}$ changes the orbital).

In part b) of Figure 5 the right lead of dot 2 was removed. As scattering to the removed lead is now prohibited, one out of four peaks disappears for $g_i$ and $g_{12}$. The missing peak is the one at the lowest value of $\omega$ in part a). We can hence identify the missing peak with spin ($g_2$) and orbital scattering processes ($g_i$ and $g_{12}$) via the right lead of dot 2. Part c) of Figure 5 is for the situation where also dot 1 is coupled asymmetrically. Some peaks are clearly suppressed.

C. Results: Decoherence rates

Figure 6 shows the three decoherence rates $\gamma_1$, $\gamma_2$, and $\gamma_{12}$ which need to be calculated self-consistently together with the RG Eqs. As in Eqs. (15) - (18) it is instructive to calculate $\gamma$ from the unrenormalized $g$'s (not self-consistently/in lowest order approximation). At large or-
bital splitting $|\delta| \gg \max\{T, |V|/4\}$, this yields for symmetric coupling to left/right lead (Figure 6) around zero:

$$\gamma_1 \approx 4\pi g_1^2 \max\{T, |V|/4\},$$
$$\gamma_2 \approx 4\pi g_{12}^2 |\delta|,$$
$$\gamma_{12} \approx 2\pi g_{12}^2 |\delta|.$$  \hfill (19)

The decoherence rate for dot 2, i.e., $\gamma_2$, and the intra-orbital rate $\gamma_{12}$ are both proportional to $|\delta|$ as there are many decoherence processes, reflecting the instability of the high energy level of dot 2. Decoherence processes for dot 1 on the other hand become only available at finite bias voltages. For very large $|\delta| \gg T_K^0$, the system shows the usual single-orbital (spin) Kondo effect. Note that, for small $V$, we have $\gamma_1 \prec T_K^0$. One might envisage that this signals a flow to strong coupling and, hence, a breakdown of the RG Eqs. However, for the large splitting $\delta$ the real Kondo temperature is very much reduced. It actually is only $\approx 1/100 T_K^0$ here, much smaller than the cut-off.

Let us now discuss the large voltage range of Figure 6. For $|V| \gg |\delta|$, all curves show a similar asymptotic behavior:

$$\gamma_i \approx 4\pi \left[(g_i)^2 + (g_{12})^2\right] \max\{T, |V|/4\},$$
$$\gamma_{12} \approx 2\pi \sum_i \frac{3}{4}(g_i)^2 + (g_{12})^2 \max\{T, |V|/4\}.$$ \hfill (20)

While all $\gamma_i$’s are now proportional to $|V|$, we see in Figure 6 that nonetheless $\gamma_i \ll |V|$. This is due to the small prefactors $\sim g^2$ in Eq. (20). The current which has similar prefactors causes the decoherence processes. For sufficiently large $V$, all $\gamma_i$’s become larger than temperature; the current induced decoherence exceeds the temperature effect.

In between the two extremes $|V| \ll |\delta|$ and $|V| \gg |\delta|$, the transition from a strong orbital decoherence rate of dot 2 due to the high level energy to strong current induced decoherences in both dots is clearly visible at $V \approx \pm \delta$ in Figure 6.

Neglecting the finite orbital splitting, we can also get the higher order asymptotic behavior for the decoherence rates analogous to Ref. [12]:

$$\gamma_i \approx \frac{\pi}{4} \frac{V}{\ln T_K^0} \left[1 + \frac{2}{\ln T_K^0} + \ldots\right].$$ \hfill (21)

D. Results: Non-equilibrium occupation

The numerical results for the different electron occupation of the two orbitals is shown in Figure 7 for different orbital splittings $\delta$. To ensure that the RG approach is valid, i.e., that we stay in the weak coupling regime, we use here and in the following a rather high temperature $T/T_K^0 = 2$. Such temperatures are actually relevant in many experiments where often the lowest accessible temperatures are of the order of the Kondo temperature. At low bias voltage $V < \delta$, only the lower orbital is occupied for sufficient large $\delta$, as is to be expected for $\delta \gg T$. At high bias voltage $V > \delta$, the electrons tend to stay in both dots with the same probability; $\langle n_1 - n_2 \rangle$ decreases with increasing $V$.

E. Results: Conductance for symmetric coupling

Let us now turn to the physical observable of interest, the differential conductance through the two dots $G_i = \partial I_i(V)/\partial V$ ($i = 1, 2$), presented in Figure 8. The conductance consists of the intra-dot spin scattering and the inter-dot orbital scattering. For $\delta = 0$, a pronounced peak is found at $V = 0$, characteristic of the orbital enhanced Kondo effect. The flow to strong coupling is, however, cut off by temperature. If the orbital degeneracy is lifted, the peak at $V = 0$ is suppressed. For dot 2, it finally disappears at large $\delta$ when this dot is depopulated. For dot 1, we still have a spin Kondo effect at large $\delta$, but with a very much reduced Kondo temperature $T_K^{SU(2)} \ll T_K$. Because also $T_K^{SU(2)} \ll T$, the peak at $V = 0$ is very much smeared out. At finite $\delta$, two new peaks occur at $V = \pm \delta$, corresponding to orbital scattering. For $T/T_K^0 = 2$, we study an interplay between temperature-induced decoherence and current-induced decoherence. For small $\delta$ and $V$ the decoherence is solely due to temperature, whereas for larger the voltages it is mainly current noise. For the $\delta = 20$ and $\delta = 40$ curves in Figure 8 the $\gamma_1$ decoherence rate is at least at the smaller voltages temperature-induced. In contrast, the important inter-orbital decoherence rate $\gamma_{12}$ and also $\gamma_2$ are not even for smaller voltages.

If we decrease temperature, the main driving source for decoherence is the voltage-induced current noise. The
change with temperature is studied in the inset of Figure 8 showing the sharpening of the Kondo quantum peaks. At the lowest temperature $T/T_K = 0.1$, we reached the limit where the voltage-induced decoherence is dominating everywhere, except for the central peak in Figure 8.

In Figure 8 we show how the conductances change if the couplings of the two dots to the leads are different. In part c) and d), the stronger coupled dot (dot 1) is lower in energy. The emerging single-dot (spin) Kondo effect at $V \approx 0$ is somewhat suppressed, see Figure 9 c). In contrast in part a) and b), it is the weaker coupled dot which is lower in energy (dot 2). Then the spin Kondo peak for dot 2 is hardly discernible, see Figure 9 d).

In the experiments of Ref. 4, the inter-dot Coulomb interaction $U_{i2}$ is much smaller than the intra-dot Coulomb interaction $U_i$. Hence, the initial inter-dot coupling is stronger $g_{12} \approx \frac{1}{t_{12}} - \frac{1}{t_{12}}$ than $g_i \approx \frac{1}{U_i} - \frac{1}{U_i}$, as in Figures 8 and 9.

And, as in Figures 8 and 9 b), the single-dot (spin) Kondo peak at $V = 0$ is much weaker (if at all discernible) than the pronounced orbital Kondo quantum peaks at $V \approx \pm \delta$.

### F. Results: Conductance for asymmetric coupling

Let us now discuss the situation of strongly different (initial/unrenormalized) couplings $g_{m1,m2}^{\alpha \beta}$ to the right and the left lead: $g_{m1,m2}^{RR} \neq g_{m1,m2}^{LL}$. This is typically the case in experiments and can in extreme cases give rise to a different physical behavior compared to that of the symmetric coupling cases.

In Ref. 7, this asymmetry is even so strong that one of the dots is effectively decoupled from the right lead ($g_{m1,m2}^{RR} = g_{m1,m2}^{LL} = 0$), albeit still coupled to the left lead via $g_{m1,m2}^{LL}$. Such a situation is shown in Figure 10. Since dot 2 is decoupled from the left lead, the orbital Kondo effect is now only possible at $V = +\delta$, not any longer at $V = -\delta$. This can be understood by means of Figure 11. At $V = +\delta$, we can move an electron from the Fermi energy of the left lead into dot 2 and, conserving energy, the electron from dot 1 to the Fermi energy of the right lead, and vice versa. In the RG equations we hence get large couplings, resulting in an orbital Kondo quantum peak. If the coupling to the leads was symmetric we could have similar scattering processes moving the electron from dot 2 to the right lead’s Fermi energy and an electron from the left lead’s Fermi energy to dot 1 at $V = -\delta$. However, since $g_{m1,m2}^{RR} = 0$ this second scattering process is not possible, as was already argued in Ref. 4. Therefore, instead of two conductance peaks as in Figures 8 and 9, we see only one at $V = +\delta$ in Figure 10 which presents results for such strongly asymmetric couplings. The nearly indiscernible spin Kondo peak and the missing peak at $V \approx -\delta$ qualitatively agree with experiments. As for symmetric coupling, the height of the orbital Kondo peak at $V = \pm \delta$ decreases with increasing $\delta$.

An interesting phenomena in the case of asymmetric couplings occurs if $g_{m2}^{RR} = 0$ as before but $g_{m1}^{RR} \gg g_{m2}^{LL}$. As shown in Figure 12 we find a negative differential conductance which is always present after the orbital Kondo peak, i.e., for $V > \delta$. Generally such negative conductances occur when the hopping parameters of the Anderson model fulfill $|t_{11}| > |t_{12}|$, $|t_{12}| \ll |t_{12}|$. How can we understand this unexpected negative conductance?
The basic picture is that—although higher in energy—dot 2 is increasingly occupied for \( V > \delta \). Counterintuitively, it becomes even more occupied than dot 1, see the lower part of Figure 11. The reason for this inversion of the occupation (in comparison to the equilibrium occupation) can be understood from the sketch in Figure 11. The scattering process shown in the Figure is energetically possible: Because \( V - \delta > 0 \), we can transfer an electron from the Fermi level of the left lead to the empty states above the Fermi level of the right lead and still pay the energy \( \delta \) to change the local state from dot 1 to dot 2. But the reverse process, starting with an electron from the right lead's Fermi level, is not possible since \( \delta - V < 0 \). All the other processes for changing the local state back from dot 2 to dot 1 are much smaller because of \( |t_{L1}|/|t_{R1}| \ll 1 \) and \( |t_{R2}|/|t_{L2}| \ll 1 \). Take for example the process involving the transfer of the dot 2 electron to the right lead and simultaneously that of an electron from the left lead to dot 1. The amplitude for this process is by a factor of \( |g_{21}^{LR}/g_{21}^{RR}| \left( \approx |t_{L1}/t_{R1}| |t_{R2}/t_{L2}| \ll 1 \right) \) smaller than that of Figure 10. Hence, the localized state is nearly trapped in dot 2, \( \langle n_1 - n_2 \rangle < 0 \), for large bias voltage.

Altogether the behavior of the conductance in Figure 11 can be explained as following: At \( V \approx \delta \) a new, inter-orbital channel opens for conductance to the left lead of dot 1, the one displayed in Figure 11. This explains the strong enhancement of \( G_1 \) for \( V \approx \delta \); the orbital Kondo peak in Figure 12. With a further increase of \( V \), however, there is a dramatic decrease of \( \langle n_1 - n_2 \rangle \) and hence of the number of electrons in dot 1 \( \langle n_1 \rangle \). Because of this, the contribution to the current involving solely dot 1, i.e.,
the first line in Eq. (9), rapidly decreases. This decrease dominates for $V > \delta$, resulting in a negative differential conductance.

Negative conductances for quantum dots with many levels have been observed experimentally and described theoretically, albeit in the sequential tunneling regime. Our results show that similar effects are also possible in the Kondo regime.

G. Results: Linear conductance

Substantial simplifications arise in the linear response regime for small voltages since there are no nonequilibrium effects any more like the current induced decoherence rate $\gamma_i$ and the frequency dependence of the vertex. This linear conductance can be used as an indicator whether we have a spin or an orbital Kondo effect. The reason for this dramatic decrease of the conductance is simply that the RG flow of the inter-orbital coupling (and that for the high energy dot) is cut-off at an energy scale $\sim \delta$, preventing a strong renormalization and, hence, suppressing the conductance at larger values of $\delta$.

IV. CONCLUSION

In conclusion, we reported on the extension of the nonequilibrium perturbative RG scheme to multi-orbital applications. The method keeps the simplicity of the poor man’s scaling approach, but can be used for nonequilibrium transport by including the frequency dependence of the vertex. However, it is restricted to the weak coupling regime. Either current or temperature has to cut off the flow to the strong coupling fix point.

Specifically, we calculated conductances for the case of two quantum dots coupled only electrostatically. For SU(2) spin symmetry one spin (at $V \approx 0$) and two orbital Kondoesque peaks (at $V \approx \pm \delta$) are found, in agreement with the expectations. For the typical experimental situation that the intra-dot Coulomb interaction is much stronger than the inter-dot Coulomb interaction (translating to initial couplings $g_{12} \gg g_1, g_2$) however, the spin...
FIG. 13: Part (a): Linear conductance $G_1(\delta)$ as a function of orbital splitting $\delta$ for symmetric coupling (see legend; $T/T_K = 2$, note that $T_K^0$ depends on $g$). Part (b) shows $\partial G_1(\delta)/\partial \delta$ vs. $\delta$ which is symmetric/antisymmetric for a predominantly spin/orbital Kondo effect.

Kondo peak is much less pronounced, if at all discernible.

For strongly asymmetric couplings with the extreme situation that one of the dots is decoupled from one of the two leads, the conductance peak at $V \approx -\delta$ disappears, as observed experimentally. An unexpected result was the negative differential conductance immediately after the remaining Kondoesspeak at $V \approx \delta$, occurring for initial couplings $g_1^{RR} \gg g_1^{LL}$ and $g_2^{RR} \ll g_2^{LL}$. This is a genuine nonequilibrium phenomenon.

Already in the the linear conductance regime, we can distinguish between spin and orbital Kondo effect via the derivative $\partial G_1(\delta)/\partial \delta$ which is symmetric/antisymmetric w.r.t. $\delta \rightarrow -\delta$, respectively.

Acknowledgments

We thank A. Hübels, A. A. Katanin, A. Rosch, D. Quirion, and J. Weis for discussions. This work was supported in part by the Deutsche Forschungsgemeinschaft through the Emmy Noether program.

APPENDIX: DECOHERENCE RATES

In this Appendix, we calculate the decoherence rates via the susceptibility or the corresponding correlation function:

$$\chi_{m_1m_2}(\omega) = \langle \langle X_{m_1m_2}^* X_{m_2m_1} \rangle \rangle_{\omega},$$

where $\langle \langle A; B \rangle \rangle = -i\Theta(t) \langle \langle A(t), B \rangle \rangle$ and $\langle \langle \ldots \rangle \rangle_{\omega}$ denotes the Fourier transformation to frequency space.

Near resonance $\omega_0$, $\chi_{m_1m_2}(\omega)$ behaves like:

$$\chi_{m_1m_2}(\omega) \sim \frac{1}{(\omega - \omega_0) + i\gamma_{m_1m_2}}$$

where $\gamma_{m_1m_2}$ is just the decoherence rate wanted.

For the exact diagrammatic evaluation at finite bias voltage, one should consider the whole Keldysh contour like in Ref. 20. However, for the lowest order contribution, it is sufficient to use only the equilibrium Green functions as in Ref. 21. Moreover, the proper decoherence rates to the lowest order in $J$ can be obtained from the equation of motion method like in Ref. 32 with a truncation of the Eqs. in second order in $J$.

The equation of motion of the correlation functions for Hamiltonian (49) is written as:

$$\omega X_{m_1m_2}(\omega) = \langle \langle X_{m_1m_2}^* X_{m_2m_1} \rangle \rangle_{\omega} = \langle \langle X_{m_1m_2}^* X_{m_2m_1} \rangle \rangle_{\omega} + \sum_{\lambda k} J_{m_2m_1}^{\lambda}(\langle \langle X_{m_1m_2}^* e^{i\chi_{m_1m_2}^{k}c_{\lambda m_1}^{k}} c_{\lambda m_2}^{k} X_{m_2m_1} \rangle \rangle_{\omega}) + \sum_{\lambda k:\lambda' k'} (J_{m_2m_1}^{\lambda\lambda'} - J_{m_1m_2}^{\lambda\lambda'})(\langle \langle X_{m_1m_2}^* e^{i\chi_{m_1m_2}^{k}c_{\lambda m_1}^{k}} c_{\lambda m_2}^{k} X_{m_2m_1} \rangle \rangle_{\omega}).$$

Here, we used the fact that $X_{m_1m_2}^* X_{m_2m_1} = \delta_{m_2m_1} X_{m_1m_1} = \epsilon_{m_1m_1}$ in the single electron subspace. The terms on the right hand side can be expressed in a similar way by another equation of motion. We then decouple this equation of motion by using the approximation

$$\langle \langle X_{m_1m_2}^* c_1^+ c_2^+ c_3^+ c_4^+ X_{m_2m_1} \rangle \rangle_{\omega} \approx \langle \langle \epsilon_{1}^+ c_1^+ c_4^+ \rangle \rangle_{\omega} \langle \langle X_{m_1m_2}^* X_{m_2m_1} \rangle \rangle_{\omega} \approx 0.$$

This way, we neglect terms to order $J^3$, i.e., our decoherence rate is only valid to order $J^2$. The $\langle \langle \epsilon_{1}^+ c_1^+ c_4^+ \rangle \rangle_{\omega}$ term can be absorbed in a renormalization of the energies

$$\epsilon_m \rightarrow \epsilon_m \approx \epsilon_m + \sum_{\lambda k} |J_{m_2m_1}^{\lambda\lambda'}| f_\lambda(\epsilon_k) \sum_{m'} |J_{m_2m_1}^{\lambda\lambda'} f_{\lambda'}(\epsilon_{k}),$$

We can effectively include this renormalization by denoting with $\epsilon_m$ not the original level energy of the Hamiltonian but the renormalized $\epsilon_m$ which is also measured in the experiment. The RG Eqs. would include the same kind of change if higher order terms were included.

$$\omega \epsilon_m \epsilon_m = \langle \langle X_{m_1m_2}^* X_{m_2m_1} \rangle \rangle_{\omega} \approx \langle \langle X_{m_1m_2}^* X_{m_2m_1} \rangle \rangle_{\omega} - \sum_{m} C_{m_2m_1}(\omega) \langle \langle X_{m_2m_1} \rangle \rangle_{\omega} + i\pi \delta_{m_2m_1} C_{m_2m_1}(\omega) \langle \langle X_{m_2m_1} \rangle \rangle_{\omega} \langle \langle X_{m_2m_1} \rangle \rangle_{\omega}$$

plus an additional real part which typically results in a modification of $\epsilon$ but is of no interest for the decoherence rates since these correspond to the imaginary part given by
In our perturbative RG calculation, we replace the bare couplings \(g_{\text{bare}}\) with renormalized couplings \(g_{\text{ren}}\), i.e.
\[
\gamma_{m_1 m_2} = \pi B_{m_1 m_2}(\omega_0).
\]
This is the rate entering the RG Eq. (6) as a cut-off. In our perturbative RG calculation, we replace the bare couplings \(\rho_0 J\) by the renormalized \(g(\omega)\)'s in Eq. (A.5). Then, scaling corrections are included in a similar way as in the RG Eqs. themselves.

For \(m_1 \neq m_2\), we directly obtain the decoherence rate from Eq. (A.5), taking \(\omega \rightarrow \omega_0 \approx \tilde{\epsilon}_{m_2} - \tilde{\epsilon}_{m_1}\):
\[
\gamma_{m_1 m_2} = \pi B_{m_1 m_2}(\omega_0).
\]

As an example, the relaxation rate of the SU(2) model is
\[
\gamma = 2\pi N \rho_0^2 J^2 T,
\]
which can be obtained by simply substituting \(J_{m_1 m_2} = J\) and \(\tilde{J}_{m_1 m_2} = J/N\).

An additional check is the spin-\(\frac{1}{2}\) Kondo model. In a magnetic field and at a finite bias, we obtain from Eq. (A.5) and an analogous equation of motion for \(\langle X_{m_1 m_2}; X_{m_1 m_2}\rangle\) the transversal and longitudinal spin relaxation rates:
\[
\gamma_{\perp} = \frac{\pi}{2} \rho_0^2 J^2 \sum_{\lambda\nu} \int d\epsilon \left[ f_{\lambda}(\epsilon - \frac{s\delta}{2})(1 - f_{\lambda}(\epsilon + \frac{s\delta}{2})) + f_{\lambda}(\epsilon - f_{\lambda}(\epsilon)) \right],
\]
\[
\gamma_{\parallel} = \frac{\pi}{2} \rho_0^2 J^2 \sum_{\lambda\nu} \int d\epsilon \left[ f_{\lambda}(\epsilon - \frac{s\delta}{2})(1 - f_{\lambda}(\epsilon + \frac{s\delta}{2})) \right].
\]

This is just the lowest order results one expects from nonequilibrium perturbation theory, i.e.:
A. C. Hewson, *The Kondo Problem to Heavy Fermions*, Cambridge University Press (Cambridge, 1993).

Proposals to artificially arrive at an SU(4) symmetric con-

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A difference to Ref. 21 is the way how we deal with the frequency conservation at the two vertices of Figure 2. In Ref. 21, the conduction electron frequency of the intermediate state is set to the band edge \( \omega' = \pm D \) so that the pseudo Fermion line (dashed line in Figure 2) is off resonance, i.e., its frequency \( \neq \epsilon_m \) where \( m \) is the intermediate pseudo Fermion (localized) state. In contrast in Eq. (6), the localized state is on resonance at \( \epsilon_m \) so that the conduction electron frequency \( \omega' \neq D \). This is similar to the traditional poor man’s scaling approach where \( \omega' \neq D \) and the conduction electron momentum is such that \( \epsilon_k = \pm D \), see, e.g. Ref. [11] for details.

This is the Kondo temperature which the two dot system had if the orbital splitting was zero but the \( g \)'s were the same. The reason for taking \( T_K(\delta = 0) \) as the energy unit is to study changes w.r.t. \( \delta \), keeping the other parameters constant.

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