Beyond the best-fit parameter: new insight on galaxy structure decomposition from GALPHAT

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Abstract. We introduce a novel image decomposition package, GALPHAT, that provides robust estimates of galaxy surface brightness profiles using Bayesian Markov Chain Monte Carlo. The GALPHAT-determined posterior distribution of parameters enables us to assign rigorous statistical confidence intervals to maximum a posteriori estimates and to test complex galaxy formation and evolution hypotheses. We describe the GALPHAT algorithm, assess its performance using test image data, and demonstrate that it has sufficient speed for production analysis of a large galaxy sample. Finally, we briefly introduce our ongoing science program to study the distribution of galaxy structural properties in the local universe using GALPHAT.

1. What and Why is GALPHAT?

Large photometric and spectroscopic surveys of galaxies [e.g. SDSS\(^5\) and 2MASS\(^6\)] continue to provide vast ensembles of galaxy images, but rigorous conclusions about galaxy formation and evolution hypotheses based on the full volume of information present serious computational and algorithmic challenges. For example, parametric models are widely used to derive galaxy structural parameters: brightness, size, profile shape and ellipticity. A recent study\(^1\) presented a bulge-disc decomposition for \(10^4\) nearby galaxies. However, an accurate decomposition is stymied by degeneracies in the parameter estimation itself. A Sersic profile has 8 parameters, and the topology of the likelihood function in this high-dimensional parameter space is too complex to visualize and hard to characterize robustly. In most previous galaxy decomposition analyses, the uncertainties of derived model parameters have not been carefully characterized. The correlations of physical properties and structural parameters of galaxies are usually assessed through scatter plots of the best-fit parameters (e.g. maximum a posteriori estimates with formal inverse Hessian variance estimates). These correlations are subject to strong contamination by underlying systematic correlations of each model parameter.

A Bayesian approach to parameter inference and hypothesis testing naturally addresses these difficulties. The probability of model parameters \(P(M)\) for a given data set \(P(D|M)\), is related to the probability of the data given the model (the likelihood), \(P(D|M)\), and prior probability of the model, \(P(M)\), through Bayes theorem:

\[
P(M|D) = \frac{P(D|M)P(M)}{P(D)} = \frac{P(D|M)P(M)}{\int P(D|M)P(M)dM}
\]

In our galaxy decomposition problem, \(M\) is the vector of parameters describing the model and \(D\) is the image data. For a high-dimensional model space, direct integration is infeasible, and one resorts to Markov Chain Monte Carlo (MCMC) methods. Although costly, sampling the posterior using MCMC returns a robust estimate of the model parameters. From this distribution, one may also define a best-fit parameter value and confidence bounds, but the real power in this approach is the natural description by the posterior distribution of any intrinsic correlation between model parameters. This power comes at a cost: Bayesian MCMC requires much more computation time than \(\chi^2\) minimization. To make this feasible for present-day surveys with large numbers of galaxies, we present a novel image decomposition package GALPHAT (GALaxy
PHotometric ATtributes\(^1\) which uses a state-of-the-art Bayesian computation software package BIE (Bayesian Inference Engine\(^2\)) for sampling MCMC efficiently and incorporates an optimized image processing algorithm to reduce the likelihood evaluation time. We will introduce the algorithm, test its performance, and present science using GALPHAT in the following sections.

2. The GALPHAT Algorithm

Given a parameter vector \(M\), GALPHAT produces a likelihood function for an image (pixel data, mask, PSF, background) and the BIE samples the posterior for a prior distribution using an MCMC algorithm. The BIE provides a choice of MCMC algorithm depending on the complexity of the problem. To date, we have found that a multiple chain algorithm with tempering \(^5\) is a good compromise between speed and flexibility. For the tests described here, GALPHAT models the galaxy surface brightness using multiple 2D Sérsic profiles with arbitrary ellipticities and position angles. For computational efficiency, GALPHAT pre-generates two-dimensional cumulative distributions of Sérsic profile with many different \(n\) using a rigorous error tolerance. By assigning pixel values by table interpolation, GALPHAT generates a model image with arbitrary axis ratio \(b/a\). Image rotation is done in Fourier space using three shear operations; a rotation matrix is decomposed into three sequential shear matrices in the X,Y, and X directions and then each shear operation is carried out using the Fourier shift theorem \(^3\). Then, the model image is convolved with a given PSF and an adjustable flux pedestal with a spatial gradient is added to model the sky background. GALPHAT uses a Poisson likelihood function to describe the photon counting process. A more detail description of the algorithm will be published in upcoming method papers \(^6\) and \(^9\).

3. GALPHAT performance

For testing, we generate an ensemble of synthetic galaxy images over a wide range of signal-to-noise ratios \(S/N\) and sizes using the IDL program by Haußler\(^2\). GALPHAT successfully recovers the input parameters with robust statistical confidence intervals (CL).

Figure \(^1\) compares the GALPHAT posterior median and 99.7\% CL with the true magnitude, effective radius \(r_e\), Sérsic index \(n\) and axis ratio \(b/a\) as a function of \(S/N\). We define \(S/N\) as the ratio of the average flux per pixel from the source within \(r_e\) to the average flux from noise [following Haußler et al.\(^2\)]. We use 10,000 converged MCMC samples for characterizing the posterior. As \(S/N\) decreases, the 99.7\% CL range increases. However GALPHAT is robust and encloses true parameter within the 99.7\% CL even in the extreme case of \(S/N\sim 1\).

To assess the performance for bulge-disk decomposition, we use simulated galaxies with a Sérsic bulge \((n = 4)\) and an exponential disk \((n = 1)\). Figure \(^2\) compares the GALPHAT parameter inferences to the exact values with varying bulge-to-total flux ratio \(B/T\). As the bulge fraction increases, the inference of bulge parameters becomes more reliable and the inference of disk parameters becomes less reliable, as expected. The marginalized posterior distribution for pairs of parameters show that correlations between parameters abound; the magnitude of the correlations depend on \(B/T\). Clearly, a characterization of this parameter correlation is necessary for interpreting the distribution of these parameters for the galaxy population overall and for testing hypotheses of galaxy formation and evolution. GALPHAT provides the full posterior probability distribution of model parameter and enables us to quote reliable uncertainties for each

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\(^1\)GALPHAT web page: [http://sites.google.com/site/galphat/galphat](http://sites.google.com/site/galphat/galphat)

\(^2\)For more information on BIE, see the BIE web page \(^8\) and \(^9\)
Figure 1. The differences between GALPHAT posterior medians and the exact values as a function of $S/N$ for Sérsic $n = 4$ profiles with $b/a = 1$. Error bars are 99.7\% CL. The GALPHAT inference is unbiased until $S/N \approx 3$. The systematically lower estimate of $b/a$ owes to the uniform prior with a range of $[0,1]$.

Figure 2. The differences between GALPHAT posterior medians and the exact values for bulge-disk galaxies as a function of $f_B = B/T$. Error bars are 99.7\% CL. From the top left to the bottom right panel, corresponding parameters are bulge magnitude, bulge $r_e$, bulge $n$, bulge $b/a$, disk magnitude, disk $r_e$, disk $b/a$, and sky background.
model parameter by including all the systematic correlations. Furthermore, the posterior distribution enables the computation of higher-order statistics and more general model comparisons.

Convergence may be slow in high-dimensional parameter spaces owing to the complexity of the likelihood function. We have found it productive to iteratively add image information using a hierarchy of successively aggregated images. Beginning with the most aggregated image (Level 0) one computes the posterior, \( P(\theta|D_0) \). The posterior for the next level (Level 1) is \( P(\theta|D_1) \propto P(\theta|D_0)[P(D_1|\theta)/P(D_0|\theta)] \) and so on. Using this hierarchical data structure, GALPHAT reduces the run time by factors of two, depending on the level of aggregation, by accelerating convergence. Also, the BIE checkpoints the full state of the simulation, efficiently enabling the posterior distribution to be augmented at a later date.

GALPHAT can be run on either a single CPU or in a cluster environment. Table 1 provides GALPHAT runtimes for a simple one-component decomposition. Of course, in addition to computing hardware, the time for your parameter estimate will depend on image size, characteristics of the model, the MCMC algorithm, the convergence diagnostic method, and the required number of posterior samples. More detailed results will be shown in the methods paper.

Table 1. Wall clock time for GALPHAT

| Image samples | CPU | Processors | Level | runtime |
|---------------|-----|------------|-------|---------|
| 200 by 200    | AMD Athlon(tm) MP 1800+ 1533 MHz | 8 | 1 | 2 hr |
| single Sérsic | 20,000 | 8 | 2 | 40 min |

4. Ongoing science program using GALPHAT

We are currently performing a bulge+disk model decomposition of 2MASS \( K_s \) band selected galaxies with \( 7 < K_s < 11 \) to derive luminosity functions for each component in the present day Universe. We will investigate the effect of environment on these distributions. From these, we can compare to mass assembly histories predicted from hierarchical galaxy formation theories.

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References

1. P. Allen et al., 2006, MNRAS, 371, 2
2. B. Häußler et al., 2007, ApJS, 172, 615
3. K. G. Larkin, M. A. Oldfield, H. Klemm, 1997, Optics Communications
4. M. Skrutskie et al., 2006, AJ, 131, 1163
5. C. J. Geyer, E. A. Thompson, 2005, JASA, 90, 909
6. I. Yoon, M. D. Weinberg, N. S. Katz, 2009, in preparation
7. D. York et al., 2000, AJ, 120, 1579
8. [http://www.astro.umass.edu/BIE](http://www.astro.umass.edu/BIE)
9. M. D. Weinberg et al., 2009, in preparation