Bursts of Gravitational Waves due to Crustquake from Pulsars

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ABSTRACT

We explore here a possible consequence of crustquake, namely, the generation of bursts of gravitational waves (GWs) due to a sudden change in the quadrupole moment (QM) of a deformed pulsar as a result of crustquake. The occurrence of crustquake in a rotating neutron star can play many important roles in neutron star (NS) dynamics. Here we propose that if a pulsar undergoes crustquake, then the generation of bursts of GWs is an inevitable consequence of crustquake. We have estimated the strain amplitudes ($h_0$) for such bursts of GWs and compared with the strain amplitudes for GWs produced in various other scenarios for isolated pulsars as suggested earlier in the literature. The values we obtain are comparable to those suggestions. We also estimate the order of magnitude for characteristic strain ($h_c$) and signal to noise ratio (SNR) for such bursts. For exotic quarks stars, a multifold enhancement of strain amplitudes is expected, which makes quark stars a potential source of gravitational waves as a result of crustquake. The absence of such waves may put constraints on such hypothetical stars.

Keywords: Neutron Star, Pulsar, Crustquake, Gravitational wave.

1. INTRODUCTION

The remarkable first ever direct detection of GWs in 2015 by LIGO (Abbott & et. al (2016)) opened a new era in gravitational astronomy. Observations supplemented with numerical simulations identified a black hole merger as the source for ‘ripples’ in the space-time. Since then there have been quite a few significant detections of GWs. The peak strain amplitude ($h_0$) for all these detections have been in the range $10^{-21} \rightarrow 10^{-22}$. In view of these detections, we should be hopeful that the gravitational waves produced by compact isolated astrophysical objects like isolated pulsars can be measurable in near future by more sensitive upcoming ground-based advanced detectors, namely, aLIGO, VIRGO, the third generation Einstein Telescope (ET) etc. Prior to the GWs detections as mentioned above, there have been a few attempts for GWs searches from isolated pulsars as well. The attempt for GWs search associated with the timing glitch in the Vela pulsar in August 2006 (Abadie & et. al. (2011)) was one among those which is worth mentioning. The motivation of above searches were based on suggestions made by several authors (see the reference Abadie & et. al. (2011) and the list of references therein) through their works on this area. As per those suggestions, several sources namely the flaring activity, the formation of hypermassive NS following coalescence of binary neutron stars etc. are capable of exciting quasinormal modes of a pulsar and hence may emit GWs. The timing glitch can be one of these sources, which has the potential to excite quasinormal modes in the parent pulsar. Although the searches for GWs during August 2006 Vela pulsar (Abadie & et. al. (2011)) timing glitch produced no detectable GWs (Abadie & et. al. (2011)), with the improving sensitivity of advanced detectors, continuous attempts in this direction may produce more conclusive results in near future.

In the literature, there have been a few other theoretical works that advocate gravitational wave astronomy in the context of isolated neutron stars. There have been discussions on emission of GWs from deformed neutron stars (Zimmermann & Szedenits (1979)), crustal mountains in radio pulsars or in low mass X-ray binaries (Haskell et al. (2015)), continuous emission of GWs from a triaxially symmetric rotating neutron star due to permanent ellipticity
Further details on the above mentioned sources of GWs from isolated pulsars can be found in a detailed review by Lasky (2015). In the context of bursts of GWs from an isolated NS, there has also been an interesting suggestion by Bagchi et al. (2015), where authors have proposed a unified model for glitches, antiglitches and generation of GW from isolated pulsars. According to their suggestions, various phase transitions (quark-hadron phase transition, CFL phase transitions etc.) inside the core of a pulsar can produce density inhomogeneities, which may increase/decrease the moment of inertia (MI) of the star and can also produce quadrupole moment $Q$ in a very short time scale determined by phase transition time scale (Bagchi et al. (2015)). The authors claim that their model can explain glitches, anti-glitches and the density inhomogeneities arising in their model can also be a possible source of gravitational waves. Their estimate of strain amplitude of GW is encouraging from the context of detectability.

In view of these exciting theoretical proposals, we explore here another possible source of GWs from isolated pulsars, which may arise due to sudden change in elastic deformation ($\epsilon$) and hence, the sudden change in quadrupole moment ($Q$) of a pulsar as a result of crustquake. Nearby Crab pulsar exhibiting small size glitches of order $10^{-8}$ that can be explained successfully using the crustquake model, will be the most likely candidate to test our proposal. In this context, we should mention that Keer & Jones (2015) have earlier studied crustquake initiated excitation of various oscillation modes in a pulsar and hence the emission of possible GWs from such oscillations. They have made an order of magnitude estimate for the strain amplitude ($h_0$) of GWs arising due to the excitation of $l = 2, m = 0$ mode. Assuming the energy released in a glitch goes into oscillations, they have initially estimated the amplitude for such oscillations and hence estimated the order of magnitude for resulting GWs strain amplitude ($h_0 \sim 10^{-23}$ for a typical 2000 Hz frequency). The above estimated value of strain amplitude depends on various factors such as amplitude of oscillations, rotational frequency of the star etc. They have also performed a detailed numerical calculation by modeling the glitch resulting from crustquake and calculated the amplitudes of the oscillations associated with various modes. In such a numerical calculation, characteristic strain turned out be of order $10^{-24} Hz^{-1/2}$ for a normal NS. However as was mentioned by the authors, causality was not implemented in their calculation by assuming instantaneous energy loss in a glitch. On contrary, causality has been taken into account in this work.

In this work, we focus on spheroidal (oblate shape) pulsar, the shape/ellipticity ($\epsilon$) of which can be defined through, $I_{zz} \neq I_{xx} = I_{yy}$ and $\epsilon = \frac{I_{xx} - I_{yy}}{I_0}$. Here, $I_{zz} = \frac{2}{5}MC^2$, $I_{xx} = \frac{1}{5}M(a^2 + c^2)$ are the MI of the star about (symmetric) z-axis and x-axis, respectively ($c < a$). $I_0$ is the MI of the spherical star. Obviously, a spheroidal star does not radiate continuous gravitational waves due to its spherical symmetry. However, during the crustquake, there is a sudden change in the pulsar’s oblateness (from more oblate to less oblate) and hence it should emit short duration gravitational radiation (see also Ref. Bagchi et al. (2015) for a similar type of change of $Q$ as a result of phase transitions). Note that the strain amplitude $h_0$ depends on $(\dot{Q})^2$ and $Q$ changes in a very small time interval $\Delta t$ ($\Delta t$ characterizes the time during which $Q$ changes by an amount $\Delta Q$ as a result of crustquake). Hence, the generation of GWs with a significant strain amplitude ($h_0$) is expected in our scenario. Continuous emission of gravitational radiation due to the rotation of a deformed triaxially symmetric NS has already been discussed in the literature (Jones (2002)), where the change of $Q$ is determined by the rotational frequency ($\Omega$) of the NS and the NS emits continuous gravitational radiation. Coming back to our proposal, for spheroidal case, we use the terminology burst to signify that GWs caused by crustquake is of short duration in character similar to GWs burst due to supernova (SN) explosions (though a smaller magnitude of strain amplitude is expected in our case as compared to bursts from SN explosions). Here, we would also like to emphasize that the source of GWs in our case is the change in quadrupole moment ($Q$) in a finite time ($\Delta t$) as a result of crustquake contrary to the scenarios as suggested in Refs. Jones (2002) and Keer & Jones (2015).

The motivation for theoretical studies on possible emission of GWs from isolated pulsars can be manifold. Firstly, the event rate of (number of events which can produce significant GWs in a galaxy per year) GWs emission from pulsars is expected to be quite high in comparison to the event rate from various other sources of gravitational radiation, namely mergers of neutron stars (NS) and/or black holes (BH). For example, as it has been mentioned in Ref. Riles (2013) that event rates lie in the range $2 \times 10^{-4}$ - 0.2 per year for initial LIGO detection of a NS-NS coalescence, and $7 \times 10^{-5}$ to 0.1 per year for a NS-BH coalescence etc. In contrast, there is a huge catalogue of pulsars and more than 450 glitch events (crustquake can account for small size $10^{-8} - 10^{-12}$ glitches) have been recorded $^1$ & analysed (Espinoza et al. (2011)) quite extensively. Thus crustquake can be quite frequent phenomenon when one considers a very large number of pulsars in our galaxy. Second motivation of the GWs study from pulsar is that it may help in

$^1$ http://www.jb.man.ac.uk/pulsar/glitches/gTable.html
understanding few features of the pulsar itself. For example, in this proposal the strain amplitude of GWs crucially depends on the change of quadrupole moment ($\Delta Q$) as a result of crustquake and the propagation time ($\Delta t$) of shear waves during which the elastic strain is released. Thus gravitational wave studies from pulsars perhaps can shed light on crustquake, possible mechanism of strain relaxation etc. So far to the best of our knowledge, no serious attempt has been made to determine strain relaxation time $\Delta t$ more precisely. This is because of the fact that in context of crustquake model for glitches, $\Delta t$ is not much relevant in determining glitch size and other features of glitches. However, as we show below, $\Delta t$ is very crucial to have significant value of $h_0$ in our model. Similarly, if a few small size glitches are caused by crustquake, then the glitches will be followed by little bursts of GWs can be a falsifiable prediction of a crustquake.

The paper is organized in the following manner. In section 2, we briefly review the basic features of crustquake. We shall also discuss here the possible values of ellipticity parameter ($\epsilon$) of a neutron star crust which can sustain the crustal stress. We’ll present our main proposal in section 3. Here, we shall provide the expression for strain amplitude ($h_0$) as a result of crustquake. For comparison, the strain amplitudes from triaxially symmetric pulsars will also be presented here. The order of magnitude estimate for strain amplitudes and the frequency content in the burst will be presented in section 4, followed by our concluding remarks in section 5.

2. CRUSTQUAKE : THE BASIC RELEVANT FEATURES

Crustquake was originally proposed by Ruderman (1968) as a possible explanation for the sudden spin-up (glitches) of otherwise highly periodic pulsar PSR 0833-45 (Vela). Currently, the superfluid vortex model (Anderson & Itoh (1975)) is the leading model to explain glitches. However, in the context of glitches or otherwise, crustquake model has been revisited repeatedly. There have been discussions in the literature suggesting the involvement of crustquake in NS physics, such as an explanation for the giant magnetic flare activities observed in magnetars (Thompson & Duncan (1995); Lander et al. (2015)), a trigger mechanism for angular momentum transfer by vortices (Eichler & Shaisultanov (2010); Melatos et al. (2008); Warszawski & Melatos (2008)) or to explain the change in spin down rate that continues to exist after a glitch event (Alpar et al. (1994)). In view of these discussions, crustquake seems to be an inevitable event for a large number of NS. This work will assume the occurrence of crustquake in a NS and study one of it’s many possible consequences, namely, the generation of short duration (bursts) GWs due to crustquake.

The crustquake in a neutron star is caused due to the existence of a solid elastic (Ruderman (1969); Smoluchowski & Welch (1970)) deformed crust of thickness about $10^5$ cm. The deformation parameter of the crust can be characterized by its ellipticity, $\epsilon = \frac{I_{zz} - I_{xx}}{I_{yy}}$ as defined earlier in Section I. Here, we briefly discuss the basic features of crustquake model (Baym & Pines (1971)). At an early stage of formation, the crust solidified with initial oblateness $\epsilon_0$ (unstrained value or reference value) at a much higher rotational frequency. As the star slows down, the ellipticity $\epsilon(t)$ decreases, leading to the development of strain in the crust due to the inherent crustal rigidity. Finally, once the breaking stress is reached, the crust cracks and releases its (partial) energy. The total energy of the pulsar can be written as (Baym & Pines (1971)),

$$E = E_0 + \frac{L^2}{2I} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2 \tag{1}$$

Where $A$ and $B$ are two coefficients, the values of which are available (Baym & Pines (1971)). The equilibrium value of $\epsilon$ can be obtained by minimizing $E$ while keeping angular momentum ($L = I\Omega$) fixed. The value thus obtained is given by,

$$\epsilon = \frac{\Omega^2}{4(A+B)} \frac{\delta I}{\delta \epsilon} + \frac{B}{A+B} \epsilon_0 \simeq \frac{B}{A+B} \epsilon_0. \tag{2}$$

The crustquake causes the shift in the reference value of $\epsilon_0$ by $\Delta \epsilon_0$, which in turns changes the equilibrium value $\epsilon$ by $\Delta \epsilon$, where

$$\Delta \epsilon = \frac{B}{A+B} \Delta \epsilon_0. \tag{3}$$

For normal neutron star of 10 km radius & 1 km crust thickness, we have $A \simeq 6 \times 10^{32}$ erg and $B \simeq 6 \times 10^{47}$ erg (Baym & Pines (1971)) which in turn provides the value of $\epsilon$ as,

$$\epsilon = 10^{-5} \epsilon_0. \tag{4}$$
The upper limit of ellipticity can be constrained by noting that the crust has a critical strain, say, $\Delta_{cr}$ such that,

$$|\epsilon - \epsilon_0| = A \frac{B}{I} \epsilon < \Delta_{cr}. \quad (5)$$

The theoretical value of $\Delta_{cr}$ ($10^{-2} - 10^{-4}$) had been estimated earlier by Jones (2002). However in a recent work (Horowitz & Kadau (2009)) on crustal breaking strain of NS, the authors have done detailed molecular dynamics simulations by modeling the crust by single pure crystal and obtained the value, $\Delta_{cr} = 0.1$. As per their work, the value of $\Delta_{cr}$ remains around 0.1 even in the presence of impurities, defects etc. Note, this value is at least an order of magnitude higher than the maximum value quoted in Ref. Jones (2002). Here for the estimate of maximum GWs strain amplitude, we take $\Delta_{cr} = 0.1$ and we obtain the upper limit of $\epsilon$ as,

$$\epsilon < B \frac{A}{I} \Delta_{cr} = 10^{-6}. \quad (6)$$

We will take the change in ellipticity, $\Delta \epsilon = \eta \epsilon$ (where, $0 < \eta < 1$ is a numerical factor characterizing the fraction of strain released due to crustquake) to estimate the strain amplitude of gravitational waves. Note, the above estimate has been provided for normal NS. It has been proposed that more exotic quark star (see the reference Keir & Jones (2015) and the references therein) may exist having a very large solid core which can sustain a large ellipticity. For such a star, $A \simeq 8 \times 10^{52}$ erg & $B \simeq 8 \times 10^{50}$ erg and hence, the upper limit of $\epsilon$ can be as large as $10^{-3}$ (for same value of $\Delta_{cr} = 0.1$ as above) as obtained from Eq. 6. Thus, the quark star can also be a potential source of gravitational wave burst as a result of crustquake.

3. GRAVITATIONAL WAVES FROM CRUSTQUAKE

The quadrupole moment $Q_{xx} = \frac{2}{3}(c^2 - a^2) \equiv Q$ of oblate shaped rotating star is related to the ellipticity through, $Q = -2 I_0 \epsilon$. The negative value of $Q$ arises solely due to oblate shape of the star. From now onwards, we shall only assume the absolute values of $Q$, $\Delta Q$ and $\Delta \epsilon$. The change $\Delta \epsilon$ is equivalent to the change in quadrupole moment, $\Delta Q = 2I_0 \Delta \epsilon$. Here we propose that if this change $\Delta Q$ happens in a very short time scale ($\Delta t$) then the emission of burst of gravitational waves are expected as a consequence of crustquake. The strain amplitude ($h_0$) for such GWs due to crustquake from a pulsar a distance $d$ away from earth can be written as (Riles (2013)),

$$h_0 = \frac{2G}{c^4 d} \tilde{Q} \simeq 10^{-26} \left( \frac{1\text{ kpc}}{d} \right) \left( \frac{\Delta \epsilon}{10^{-8}} \right) \left( \frac{10^{-3} \text{s}}{\Delta t} \right)^2. \quad (7)$$

Here, $\Delta t$ characterizes the time during which $Q$ changes by $\Delta Q$ as a result of crustquake. The factor $\frac{1}{(\Delta t)^2}$ takes care of this change. If starquake is responsible for glitches, then $\Delta t$ is expected to be the spin-up time for glitches. As we will discuss in the next section, the frequency $f \sim \frac{\Delta \epsilon}{\Delta t}$ of GWs in the burst is also expected to be characterized by the time interval $\Delta t$. The Eq. (7) assumes slow motion approximation where the wavelength $\lambda$ of gravitational radiation must be much larger than the size of the star. Thus for a frequency range of Hz - kHz, $\lambda > 10^2$ km, which is much larger than the size of the source ($\sim 10$ km). We will now produce the expression for $h_0$ from a triaxially symmetric rotating pulsars (Jones (2002)) for comparison. For triaxial stars (for which $I_{xx} \neq I_{yy} \neq I_{zz}$ and the ellipticity, $\epsilon_{tr} = \frac{I_{xx} - I_{yy}}{I_{xx}}$), the time scale relevant for the change in $Q_{tr}$ is determined by the time period of rotation ($T$) of the star and the star is expected to emit continuous GW. The strain amplitude ($h_0)_{tr}$ from such a star can be estimated from (Jones (2002); Riles (2013)),

$$(h_0)_{tr} = \frac{4G}{c^4 d} I_0 T^2 \epsilon_{tr} \simeq 10^{-25} \left( \frac{1\text{ kpc}}{d} \right) \left( \frac{\epsilon_{tr}}{10^{-7}} \right) \left( \frac{10^{-3} \text{s}}{T} \right)^2. \quad (8)$$

The strain amplitude in this case depends on the rotational period, $T$ and the ellipticity, $\epsilon_{tr}$ of the star. In contrast, it depends on $\Delta \epsilon$ (caused by crustquake) and $\Delta t$ while being independent of the rotational frequency of the star for our case.

4. RESULTS

The strain amplitude in Eq. (7) of gravitational waves from the star in our scenario crucially depends on duration, $\Delta t$ of strain relaxation and to what extent strain is released, i.e., $\Delta \epsilon$. Following the arguments provided in Sec. 2, for a normal NS the maximum possible value of $\epsilon$ is in the order of $10^{-6}$ and consequently, the value of $\Delta \epsilon$ will be
determined by the prefactor $\eta$ ($0 < \eta < 1$). Note, the value of $\eta = 0.01$ (i.e., 1% release of strain) corresponds to glitch size $10^{-8}$ as per crustquake model for glitches, which is also a typical glitch size observed for Crab pulsar (Espinoza et al. 2011). Here, for consistency we should mention that for a Crab pulsar, the above value of $\Delta \epsilon$ corresponds to only few years waiting time for the next glitch to occur, which is much less than the spin-down time scale ($\approx 10^3$ years) of Crab pulsar and this fact is fairly consistent with the observations. The change in quadrupole moment of the star will be, $\Delta Q = 2 I_0 \Delta \epsilon = 2 \times 10^{37} \text{gm} - \text{cm}^2$, where $M$ of the star $I_0$ is taken as $10^{45} \text{gm} - \text{cm}^2$.

For a quark star, the values of $\Delta \epsilon$ can be of order $10^{-5}$ and hence $\Delta Q$ will be increased by about $10^3$ times from that of a normal neutron star. The value of $\Delta t$ depends on detailed mechanism through which the star re-adjusts towards a new unstrained value of oblateness. According to the standard scenario, this is determined by the speed of shear wave, $v = \sqrt{\frac{E}{\rho}} = 3 \times 10^8 \text{cm} \text{s}^{-1}$. Where $\mu = 10^{36} \text{dy}nec - \text{cm}^2$ is the shear modulus of the crust and $\rho \approx 10^{13} \text{gm} - \text{cm}^{-3}$ is the typical value of crust density. Thus, time for shear wave to propagate the stellar radius ($R = 10 \text{km}$) can be estimated as, $\Delta t = R/v \approx 10^{-3} \text{sec}$. This time scale is of the order of time period of Crab pulsar (i.e., Crab pulsar will complete one full rotation between the crust crack and the spin-up event). We shall use $\Delta t = 10^{-3} \text{sec}$ to estimate strain amplitudes for Crab like normal stars. However, for exotic quark stars, speed of shear wave is about one order of magnitude larger (shear modulus (Keer & Jones 2015)), $\mu \approx 4 \times 10^{32} \text{dy}nec - \text{cm}^2$ and $\rho \approx 10^{14} \text{gm} - \text{cm}^{-3}$.

Hence, $\Delta t$ for quark star can be taken as $10^{-4} \text{sec}$.

We estimated $h_0$ as a result of crustquake for Crab using Eq. (7). The results are provided in Table 1. It is obvious that shorter the time scale, $\Delta t$ for spin up events, larger the strain amplitude one expects. It also depends to what extent the strain is released ($\Delta \epsilon$) as a result of starquake. Both these factors are reflected in producing higher value of strain amplitude for a quark star. We have assumed $\Delta \epsilon = 10^{-8}$ (i.e., 1% release of strain) to be consistent with theoretical estimate on maximum strain a normal NS can resist and also this is the typical glitch size for Crab pulsar in crustquake model. Thus for Crab pulsar, a typical value of strain amplitude is expected to be of the order $10^{-26}$.

For comparison, we will now take the cases of GWs from triaxially symmetric rotating pulsars (Jones 2002), GWs due to starquake initiated oscillations (Keer & Jones 2015) etc. Although the triaxiality is not a criterion in our scenario, however even a triaxial star may undergo crustquake. In this case, the continuous emission of GWs will be followed by sudden emission of GW burst. Now before comparison, it’s important to discuss the magnitude of frequency content in the burst and the potential of detectability. The GW burst in our case lasts only about a millisecond and hence the bursts should come off with a frequency bandwidth in contrast to well defined frequency as expected in a continuous emission. For example, the frequency of GWs emitted from a triaxial NS or a star with crustal mountain is related to the spin frequency ($\Omega$) of the star. Similarly, in the case of starquake initiated oscillations (Keer & Jones 2015), the frequency of GWs is related to the oscillation frequency. Note, the authors in Ref. (Keer & Jones 2015) have assumed a frequency of 2 kHz to estimate the strain amplitude. For burst in our case, we expect the frequency of GWs to lie above $f \sim \frac{1}{2 \pi \Delta t} = \frac{1 \text{kHz}}{2 \pi} \sim 160 \text{Hz}$. As a guide, we should mention here that a frequency of GWs from a self gravitating body in general is related to its natural frequency, $f_0 = \sqrt{4 \pi G \rho_0}$ (within Newtonian approximation) (Sathyaprakash & Schutz 2009), where $\rho_0$ is the mean density of the object. For a typical neutron star of radius 10 km and mass 1.4 $M_{\odot}$, $f_0 \approx 2 \text{kHz}$. These are all suggestive to expect that the frequency of bursts in our case is more likely to lie in the range of a few hundred Hz to a few KHz.

Now whether such bursts of GWs have the potential to be detectable against signal noises depends on various factors such as sensitivity of interferometer, proper ‘template’ to analyse burst, etc. There are several ways to characterize the detectability of signal strength from a source against the background noise. One such commonly used method (Sathyaprakash & Schutz 2009; Moore et al. 2014)) is the comparison of square root of power spectral density (PSD) for the source, $\sqrt{S_h(f)} = \frac{h_c(f)}{\sqrt{f}}$, and for the noise, $\sqrt{S_n(f)} = \frac{h_n(f)}{\sqrt{f}}$. Here, $h_c(f)$ is the characteristic signal amplitude in the frequency domain and $h_n(f)$ is the effective noise that characterizes the sensitivity of a detector. Note, $S_h(f)$ & $S_n(f)$ both have a dimension of $Hz^{-1/2}$ (see Refs. Sathyaprakash & Schutz 2009; Moore et al. 2014) for details), whereas $h_c(f)$ & $h_n(f)$ are dimensionless. The characteristic signal amplitude $h_c(f)$ is defined as, $h_c(f) = f |\tilde{h}_c(f)|$. Where, $\tilde{h}_c(f)$ is the Fourier transform of the signal amplitude $h(t)$. In our case, $\tilde{h}_c(f)$ can be estimated (approximately) as (Sathyaprakash & Schutz 2009),

$$\tilde{h}_c(f) = \int_{-\infty}^{\infty} dt \ h(t)e^{i2\pi ft} = \frac{h_0}{\pi f} \sin(\pi f \Delta t)e^{i\pi f \Delta t}. \quad (9)$$
In the above equation, we have used the fact that the burst lasts for a duration $\Delta t$ and $h(t) = h_0$ is assumed to be constant during this interval. The characteristic strain then turns out to be, $h_0(f) = f |\dot{h}_c(f)| = \frac{h_0}{n} \sin(\pi f \Delta t)$. We thus get the maximum value of $h_0(f)$ as, $(h_c)_m = \frac{4h_0}{\pi}$. For $h_0(f)$, we utilize the data available in the literature (Sathyaprakash & Schutz (2009); Hild & Abernathy (2011)) for the upcoming Einstein Telescope 2. As per the data provided, the values of $h_0(f)$ for a frequency range $100 \text{ Hz} - 1 \text{ KHz}$ lies between $10^{-23} - 10^{-22}$. We can thus roughly estimate the signal to noise ratio, $SNR \sim \frac{h_c(f)}{h_n(f)}$ to check the potential of detectability in our case. Using $h_c(f) \simeq (h_c)_m = \frac{4h_0}{\pi}$ and the above data for $h_n(f)$, the values of SNR in our case lie in the range $(3 \times 10^{-4} - 3 \times 10^{-5})$ for the Crab pulsar. For quark stars, the values of SNR ($\sim 10\to 1$) is enhanced quite significantly. Though the numbers are not encouraging at this moment for Crab pulsar, we should mention that there are a few limitations in our calculation of $h_0$. For example, in our future work, we would like to model the time evolution of quadrupole moment $Q(t)$ during the time interval $\Delta t$ following crustquake. Such time evolution and the magnitude of $\Delta t$ should depend on the detailed mechanism through which the rearrangement of the shape of the star occurs. This type of modeling eventually helps to determine the time evolution of the strain amplitude $h(t)$ and the magnitude more precisely. Frequency distribution of GWs is also expected to be realized through this model, more reliably.

Within this uncertainty, we now compare the strain amplitude from a triaxial Crab pulsar using Eq. 8 and which is about of order $(h_0)_tr \sim 10^{-26} - 10^{-27}$. Here, we assume the value of $\epsilon_{tr} = 10^{-6} - 10^{-7}$. The lower value of $\epsilon_{tr} = 10^{-7}$ ensures that triaxiality sustains the crustquake events. The time period of the pulsar is taken as, $T = 33$ ms. Here we should mention that, $(h_0)_tr$ depends on time period of rotation and the magnitude can be even smaller for slowly rotating NS. Our estimate of strain amplitude for the burst of GW as a result of crustquake (refer table 1), turns out to be of similar order of magnitude when compared to the corresponding value for triaxial stars. Also, we should point out that GW bursts in our case is expected even from a spheroidal star for which continuous emission will be completely absent. We have also mentioned earlier in section 1 that Keer & Jones (2015) have also provided the value of strain amplitude resulting from neutron star oscillations initiated due to starquake. Their suggested values for a 2 KHz frequency are larger by a few order of magnitude, however this value can be modified by relaxing the assumption that all energy released in a crustquake goes into oscillations and transfer of energy is instantaneous.

Similarly, there is a proposal by Bagchi et al. (2015) that several phase transitions inside the core of a pulsar can generate quadrupole moment which can result in emission of GW bursts. Our estimated maximum strain amplitude turns out to be about two order of magnitude smaller as compared to the values obtained in Ref. Bagchi et al. (2015). This is mainly due to smaller value of $\Delta t$ (about one microsecond, typical value for a QCD phase transition time) used in their work. We have also presented our estimate of $h_0$ for a typical quark star (assumed to be at a same distance as Crab pulsar). The strain amplitude for such exotic star (about $10^{-21}$) is increased by about $10^5$ order of magnitude compared to a normal star. This happens due to high ellipticity (and hence larger $\Delta \epsilon$ released) and smaller $\Delta t$ (due to larger shear modulus $\mu$).

5. CONCLUSION

To conclude, we propose that crustquake of a pulsar should produce a burst of gravitational waves due to almost sudden change of QM. We have estimated the maximum possible value of strain amplitude for possible values of $\Delta \epsilon$ and $\Delta t$. We have compared our results with the results already suggested in the literatures in context of isolated pulsars and which to some extent resemble our suggestion. The order of magnitude estimate of $h_0$ in this work turns out to be reasonably comparable to those suggestions. Here we should mention that there are few limitations in our calculation of $h_0$ or estimating the frequency of GWs. We would like to address those issues in our future work.

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2 refer http://www.et-gw.eu/index.php/etsensitivities#references
However, the existing works we have mentioned also suffered from several limitations. For example, the reason behind triaxility (or equivalently the crustal mountain) which may be responsible for continuous emission of GWs still not well understood. In our case triaxiality is not a requirement. Similarly, in the work of Ref. (Keer & Jones (2015)), the authors have assumed the starquake initiated oscillations modes. However, one should note that there can be several other mechanisms (still not known with certainty) which can excite such modes and produce gravitational waves. So, at present it is quite impossible to isolate exact source of oscillations which might cause the emission of GWs.

So far detectability is concerned, it’s of course more challenging to detect short duration bursts than the continuous GWs. However, we hope such challenge will be overcome through proper analysis suitable for burst study and eventually by detecting with more sensitive third generation detectors. We also noticed that there is a multifold increase in the values of $h_0$, $h_c$ and SNR for exotic quark stars. Hence, non-detectability of such waves can put various constraints on such hypothetical stars. For example, the strain amplitude being dependent on distance of the source from earth (Eq. 7), non-detectability of such GW bursts from more advanced detectors can put constraint on possible distance of such stars. We would also like to emphasize that the time-scale $\Delta t$ during which quadrupole moment changes is very crucial in this work. To the best of our knowledge, no serious attempt has been made to determine the precise value of $\Delta t$. In the context of crustquake model for glitches, this is perhaps not necessary as the size of glitches and other relevant features of glitches are not sensitive to the value of $\Delta t$. If crustquake causes glitches, then uncertainty of such time scale may be resolved through pulsar timing by narrowing down to the characteristic time-scale for spin up events/glitches. So far, the best resolved time observed for spin-up of pulsar has been reported (McCulloch et al. (1990)) to be $\approx 2$ min, which is too far away from our requirement. However, we are hopeful that with upcoming next generation telescopes such as MeerKAT radio telescope, Giant Magellan telescope, Square Kilometre Array (SKA) etc. more details would be added to the glitch observations. The increased sensitivity and time resolution of the next generation telescopes may enhance the chances of direct glitch (as it happens) detections and could further constrain the characteristic spin-up time for pulsar glitches.

In case, glitches are not associated with crustquakes, it will be then worth studying in the direction to put constraint on such time scale from some other observational phenomena which might be sensitive to $\Delta t$. Finally we conclude, if crustquake occurs then bursts of gravitational waves as its consequence is a falsifiable prediction of our proposal.

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