A Criterion for Physically Acceptable Extra Dimensions with Boundaries

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Abstract

We present a criterion for deciding which compact extra dimensional spaces yield physically reliable Newton’s law corrections. We study compact manifolds with boundary and without boundary. The boundary conditions which we use on the boundaries are Dirichlet or Neumann. We find that compact connected Riemannian manifolds with Dirichlet boundaries are completely excluded as extra dimensional spaces.

Introduction

One of the main themes in the last century history of physics has been the unification of all forces [1–3]. This direction in research was motivated by the electromagnetism paradigm. Likewise, Kaluza and Klein, independently worked towards the unification idea, and tried to unify gravity and electromagnetism within general relativity by adding a spatial extra dimension to our four dimensional world. Nowadays, theories with extra dimensions have refined representatives, such as superstring theories [5–21] and several UV extensions (see for example, [22–30]). Yet, string theories give predictions that energetically are way above from the current experimental abilities that colliders provide. On that account, the next step was string inspired low energy effective quantum field theory which can be built on the basic assumption that our world has extra spatial dimensions. These extra dimensions should be compact and too small to be (until now, 2011) detected [31]. Actually, if the compactification scale (that is, the inverse of the radius of the compact dimension) is of the order TeV, then it is possible that these extra dimensions could be experimentally detected in the near future since their imprint may be present in many physical laws and effects. In colliders, if a particle gains energy of the order of the compactification scale,
then it could be possible to move along that extra dimensions, and this would imply lost energy in our world. Along with collider experiments we expect to see the imprint of extra dimensions indirectly and to other physical laws, such as in Newton’s law experiments. The first correct scientific and mathematical formulation of gravity is the Newton’s law. It is very well established and is, up to now, experimentally tested at distances of the order 0.05 mm \[32\]. There exist experiments that test Newtonian gravity such as the Eötvös-type and also Cavendish type experiments. If extra dimensions actually exist, it is possible that their imprint could be found in the Newton’s \(1/r\) law when tested at small distances. Many theoretical studies have been done towards this direction, see for example \[33\]–\[36\], \[40\]–\[43\] and references therein. The form of the modifications of the Newtonian potential, that the large compact extra dimensions models predict is:

\[
V(r) = -G_N \frac{M M'}{r} \left(1 + ae^{-r/\lambda}\right),
\]

(1)

which are Yukawa type. In the above, \(G_N\), \(a\) and \(\lambda\) are the gravitational constant, strength and the range of the gravitational force. The current experimental work constrains these two parameters, concerning Yukawa type corrections.

Depending on the geometrical and topological features of the extra dimensions, the strength and range will vary. The Yukawa type corrections are essential to most large compact extra-dimensional spaces (but this does not apply to all compact spaces, as we shall see).

In view of the Eötvös-type and also Cavendish type experiments, it is necessary to know the form of corrections that each compact space induces. There is a plethora of compact spaces that can be extra dimensional spaces, which can vary in their global topology, shape and their size.

However not all compact spaces give Yukawa type corrections, for example the Dirichlet disk yields completely unphysical Newton’s law corrections (see \[42\]). These spaces have been used in the literature to solve several phenomenological problems, however although interesting phenomenologically, these must be excluded as extra dimensions candidates.

It is necessary to have a criterion in order to distinguish among all compact spaces, these that give physical acceptable results. In this article we shall give such a criterion for compact spaces with or without boundary. The later is less involved in comparison to the former as we shall see, but we shall present it for completeness.

This paper is organized as follows: We review the technique for obtaining the Yukawa corrections to the Newton’s law in arbitrary dimensions. Next, we present the criterion that determines which spaces can give reliable corrections to the Newton’s law. The conclusions follow at the end of the article.

\[1\] Nonetheless, warped extra dimensional models of Randall-Sundrum type \[36\], give rise to modifications of the form,

\[
V(r) = -G_N \frac{M M'}{r} \left(1 + \frac{C}{(kr)^2}\right),
\]

(2)

with \(C\) a constant inherent to the model (of order one) and \(k\) the constant appearing in the warp factor of the metric (see also \[37\]–\[38\] and for an alternative supersymmetric version see \[39\]).
Connection of Yukawa-type corrections with the zero modes of the Laplace-Beltrami operator

We start with the very well known techniques for calculating gravitational corrections to Newton’s law (which is applicable to all compact manifolds) [33, 40–42]. Based on reference [33] we shall briefly present it.

Let \( M^4 \times M^n \) be the total spacetime, with \( M^n \) a n-dimensional compact manifold describing the extra space dimensions and \( M^4 \) the four dimensional Minkowski spacetime. There exists a complete set of orthogonal harmonic functions on \( M^n \), \( \Psi_m \), satisfying the orthogonality condition:

\[
\int_{M^n} \Psi_n(x)\Psi_m^*(x) = \delta_{n,m},
\]

and the completeness relation:

\[
\sum_m \Psi_m(x)\Psi_m^*(x') = \delta^{(n)}(x - x').
\]

In the above the variable \( x \) describes the compact space for now. The harmonic functions \( \Psi_m \) are eigenfunctions of the \( n \)-dimensional Laplace-Beltrami operator \( \Delta_n \) corresponding to the manifold \( M^n \), with eigenvalues \( \mu_m^2 \):

\[
-\Delta_n\Psi_m = \mu_m^2\Psi_m.
\]

Let \( V_{n+4} \), be the gravitational potential of the \( n+4 \) spacetime. It satisfies the Poisson equation in \( n+3 \) spatial dimensions, when the Newtonian limit is taken:

\[
\Delta_{n+3}V_{n+4} = (n + 1)\Omega_{n+2}G_{n+4}M\delta^{(n+3)}(x)
\]

with \( M \), the mass of the system, \( G_{n+4} \) the Newton’s gravitational constant in \( n+4 \) dimensions and,

\[
\Omega_{n+2} = \frac{2\pi^{n+3}}{\Gamma(n+3/2)}.
\]

The large compact radius solution (6) is equal to:

\[
V_{n+4} = -\frac{G_{n+4}M}{r_{n+1}^3}.
\]

with \( r_n \) the radius of the \( n+3 \)-dimensional space in spherical coordinates. Denoting by \( r \) the radius of the 3-dimensional space of our world, then \( r_n = \sqrt{r^2 + x_1^2 + x_2^2 + \ldots + x_n^2} \).

In our case the compact space’s radius is too small. Therefore we can write the harmonic expansion of \( V_{n+4} \) in terms of the eigenfunctions of the product space \( M^4 \times M^n \), as follows:

\[
V_{n+4} = \sum_m \Phi_m(r)\Psi_m(x),
\]
where \( r \) denotes the coordinates describing \( M^4 \) and \( x \) stands for all the coordinates parameterizing \( M^n \). Accordingly, the \( \Phi_m \) obey:

\[
\Delta_3 \Phi_m - \mu_m^2 \Phi_m = (n + 1) \Omega_{n+2} \Psi_m^*(0) G_{n+4} M^3(x). \tag{10}
\]

The solution of equation (10) is equal to:

\[
\Phi_m(r) = -\frac{\Omega_n G_{n+4} M \Psi_m^*(0) e^{-|\mu_m|r}}{2}. \tag{11}
\]

and lastly the gravitational potential reads:

\[
V_{n+4} = -\frac{\Omega_n G_{n+4} M}{2r} \sum_m \Psi_m^*(0) \Psi_m(x) e^{-|\mu_m|r}. \tag{12}
\]

The physical quantities of 4-dimensional point particles have no coordinate dependence on the internal compact space \( M^n \), thus we take \( x = 0 \) in (12). This way the four dimensional gravitational potential can be cast as:

\[
V_4 = -\frac{G_4 M}{r} \sum_m \Psi_m^*(0) \Psi_m(0) e^{-|\mu_m|r}, \tag{13}
\]

Not All Compact Spaces Yield Physically Acceptable Results

As it is obvious from equation (13), the Yukawa type corrections are obtained when the compact space has a zero mode. On that account, the most dominant contribution to the gravitational potential comes from the first Kaluza-Klein (KK) excitation of the extra compact space (or equivalently from the lowest non-zero eigenvalue of the Laplace-Beltrami operator). Most compact spaces do have a zero mode and therefore give the expected Yukawa type correction to the potential.

There are however examples of extra dimensional spaces that yield completely unphysical results, as we already mentioned previously. For example, as is found in reference [42], when the extra space is a Dirichlet disk (that is, a disk with the extra dimensional field satisfying Dirichlet boundary conditions on the boundary), the correction to the potential is:

\[
V_4 \simeq -\frac{G_4 M}{r} \sum_n \frac{e^{-x_{0n}r}}{x_{0n} r}. \tag{14}
\]

with \( x_{0n} \) the \( n \)-th root of the \( J_0(x) \) Bessel function. By keeping the lowest eigenvalue in the above sum (which is \( x_{01} = 2.40483 \)) we have:

\[
V_4 \simeq -\frac{G_4 M}{r} e^{-2.40483 \frac{x_{01} r}{r}}. \tag{15}
\]

The above form is a Yukawa gravitational potential and not a Yukawa correction to the gravitational potential. As noted in [42] this form of potentials can also be found in the case the extra space is a hyperbolic manifold (a similarity that is probably accidental).
Although not frequently used in the literature, extra dimensions with Dirichlet or Neumann boundaries provide very interesting phenomenological features to field theory models with large extra dimensions. In particular the Dirichlet disk was used in reference [44] in a phenomenological six dimensional model, in which strong coupling is related with massless or light degrees of freedom in the spectrum. This is a frequent phenomenon in string theory and the authors of [44] achieved to construct a field theory model in which massless or very light degrees of freedom emerge upon turning on small couplings in the theory (the last initially had only heavy degrees of freedom in the spectrum). In the same way, the authors of [45] used the Neumann disk, in addition to the Dirichlet one. Obviously, the spaces with Dirichlet boundaries yield completely unphysical Newton’s law corrections. Therefore, even if the phenomenology these compactifications provide is quite interesting, these must be excluded from extra dimensions searches. Contrarily, if we use Neumann boundary conditions on the boundary circle, the result is different, since the corrections to the gravitational potential take the Yukawa type form [42]. Note that the absence of a zero mode in the Dirichlet case and the presence of a zero mode in the Neumann case, is responsible for these results. We shall see that this is by far not accidental. Specifically, the Dirichlet problem and the Neumann problem are actually interrelated. Therefore, we generalize the criterion for manifolds without boundary to the case the manifolds have Dirichlet or Neumann boundaries. We focus on the case that the boundary conditions are Neumann or Dirichlet. The compact space may have arbitrary topology but is assumed to be Riemannian.

**A Quick Mathematical Review**

In order to make the article self-contained, we briefly quote here some of the definitions of the well known mathematics we shall use. We start with the inclusion map, which is defined between a space $M$ and an open subset $A$ of it, as follows:

$$i : A \rightarrow M, \quad i(a) = a \text{ for } a \in A$$  \hspace{1cm} (16)

Let $M$ be a $n$-dimensional smooth Riemannian manifold. The exterior derivative $d$ maps a form $\omega$ that belongs to the space of smooth $p$-forms $\Omega(M)^p$ to the space of $p + 1$-forms, that is:

$$d : \Omega(M)^p \rightarrow \Omega(M)^{p+1}.$$  \hspace{1cm} (17)

The Hodge operation $\star$ is defined as the linear map,

$$\star : \Omega(M)^p \rightarrow \Omega(M)^{m-p}.$$  \hspace{1cm} (18)

The adjoint of $d$, denoted as $\delta$, maps $p$-forms to $p - 1$-forms, that is:

$$\delta : \Omega(M)^p \rightarrow \Omega(M)^{p-1}.$$  \hspace{1cm} (19)

We define the operator $\delta$:

$$\delta = (-1)^{m_0+m+1} \star d \star.$$  \hspace{1cm} (20)

Using the operators $d$ and $\delta$, we define the Laplacian $\Delta_p = d \cdot \delta + \delta \cdot d$, which is a map:

$$\Delta_p : \Omega(M)^p \rightarrow \Omega(M)^p.$$  \hspace{1cm} (21)
Zero Modes of the Laplace-Beltrami Operator, Betti Numbers and the Poincare-Lefschetz Duality

As we mentioned earlier, the zero modes of the Laplace-Beltrami operator corresponding to the extra dimensional space and the existence of Yukawa-type corrections due to compact space are strongly related. Indeed if no zero modes exist, then the compact space does not give the expected gravitational potential form.

The trivial case—Manifolds without boundary

Let $M^n$ be a compact $n$-dimensional space which is the extra dimensional space to $M^4$ (we follow the notation of the previous sections). We suppose that $M^n$ is compact, oriented and Riemannian. Then we can define the de-Rham complex and therefore cohomology classes on this manifold. Hence we expect all the known cohomology theorems to hold. Let $\Omega^p(M^n)$ be space of $p$-forms, $\omega$. The cohomology class $H^p(M^n)$ counts the forms that are closed closed but not exact, under the exterior derivative $d$ and it’s dual $\delta$. One can define the Laplacian $\Delta_p = d\delta + \delta d$ acting on $p$-forms (the Laplacian coincides with the Laplace-Beltrami operator when applied to 0-forms). It is very well known that on every compact Riemannian manifold without boundary, the following holds:

$$\dim\ker(\Delta_p) = \dim H^p(M^n).$$

(22)

Thereupon, the $p$-th Betti number, $b_p$, is equal to:

$$b_p = \dim H^p(M^n).$$

(23)

Thus, it is obvious that $b_p = \dim\ker\Delta_p$. Therefore, finding zero modes of the Laplace-Beltrami operators is equivalent to finding zero modes of 0-forms. Consequently we can trivially conclude that zero modes of the Laplace-Beltrami operator exist whenever the $b_0$ Betti number is non-zero. Particularly, the $b_0$ Betti number counts the number of different disconnected component of the manifold $M^n$. Hence if the manifold is connected (which is the usual case in studies of large extra dimensions), $b_0 = 1$. Indeed all the known compact Riemannian manifolds without boundary, have $b_0 = 1$, such as the $n$-torus, the sphere and so forth.

Manifolds with Boundary and Relative Cohomology

Consider now that the compact manifold $M^n$ is as before an $n$-dimensional oriented and Riemannian but with a smooth boundary. We want again to solve the problem of finding when there exist zero modes of the Laplace-Beltrami operator, with the 0-form $\omega$ obeying Dirichlet or Neumann boundary conditions on the boundary. Thus the problems at hand are:

$$\Delta\omega = 0$$

$$\omega|_{\partial M^n} = 0$$

(24)
for the Dirichlet case, and,
\[
\Delta \omega = 0 \quad (25)
\]
\[
\frac{\partial \omega}{\partial x} \mid_{\partial M^n} = 0
\]
for the Neumann case. Again, \(x\) denotes the coordinate describing the internal space \(M^n\).

Let us generalize to the case that \(\omega\) is a \(p\)-form. As before, the space of smooth \(p\)-forms is \(\Omega^p(M^n)\). The harmonic \(p\)-forms \(\omega\) are elements of \(\text{Harm}^p(M^n)\) and satisfy, \(d\omega = 0\) and \(\delta\omega = 0\) (or simply \(\Delta \omega = 0\)). Let \(i: \partial M^n \to M^n\) be the embedding (the inclusion map). Then the \(p\)-forms that are harmonic forms and satisfy Dirichlet boundary conditions, belong to the space \(\text{Harm}^p_D(M^n, \partial M^n)\) (the last is known as relative cohomology) which consists of forms that formally satisfy:
\[
\Delta \omega = 0 \quad (26)
\]
\[
i^* \omega = 0.
\]

The map \(i^*\) is the pullback of the embedding \(i\). It is not difficult to see that the problem \(26\) reduces to the one described by \(24\) when 0-forms are considered. In the Neumann case, the \(p\)-form Neumann problem reads,
\[
\Delta \omega = 0 \quad (27)
\]
\[
i^*(\star \omega) = 0.
\]

and the space of \(\omega\)'s that satisfy the above, is denoted by \(\text{Harm}^p_N(M^n, \partial M^n)\). In the above \(\star\) is the well known Hodge operator. The two spaces \(\text{Harm}^p_N(M^n, \partial M^n)\) and \(\text{Harm}^p_D(M^n, \partial M^n)\) are interrelated by the Poincare-Lefschetz duality \([16]\) that states:
\[
\dim \text{Harm}^p_N(M^n, \partial M^n) = \dim \text{Harm}^{n-p}_D(M^n, \partial M^n) = b_p(M^n), \quad (28)
\]
with \(n\) the dimension of the space \(M^n\) and \(b_p\), the \(p\)-th Betti number. Let us see the impact of this duality on the zero modes of 0-forms that satisfy Dirichlet boundary conditions. We must find the quantity \(\dim \text{Harm}^0_N(M^n, \partial M^n)\), thus we set \(p = n\) in \(28\) and we have:
\[
\dim \text{Harm}^n_N(M^n, \partial M^n) = \dim \text{Harm}^0_D(M^n, \partial M^n) = b_n(M^n). \quad (29)
\]
In the case we are interested in Neumann 0-forms, we must set \(p = 0\) in \(28\) and thus,
\[
\dim \text{Harm}^0_N(M^n, \partial M^n) = \dim \text{Harm}^n_D(M^n, \partial M^n) = b_0(M^n). \quad (30)
\]

Relations \(29\) and \(30\) are really useful. Firstly we can see that the problem of finding the number of zero modes of the Laplace-Beltrami operator for compact spaces with boundaries simply reduces to a problem of algebraic topology, with the last being finding the Betti numbers of the manifold \(M^n\). Secondly the Dirichlet and Neumann problems are related. In particular, the number of zero modes that the Dirichlet Laplace-Beltrami operator has, is equal to the zero modes that the \(n\)-forms of the Neumann Laplace-Beltrami operator have and conversely.
Having the above results at hand we apply them in the case of the Neumann and Dirichlet disk, that we discussed in the previous section. For the disk manifold we have that $b_0(M^2) = 1$ and $b_2(M^2) = 0$, thus $\dim \text{Harm}^0_N(M^2, \partial M^2) = 1$ and $\dim \text{Harm}^0_D(M^2, \partial M^2) = 0$. It is obvious that no zero modes exist in the Dirichlet case while in the Neumann, exists one. In conclusion we sum up:

- Since any contractible space has trivial homology (which implies that $b_0 = 1$ and all the other Betti numbers are equal to zero), the homological trivial (with boundary) spaces are not likely to be extra dimensional spaces when Dirichlet boundary conditions are used on their boundary.

- Compact spaces for which the Laplacian of 0-forms on these has no zero modes, are not likely to be extra dimensional spaces.

Conclusions

Most known compact extra dimensional spaces yield Yukawa-type corrections to the $1/r$ four dimensional gravitational law. However not all compact spaces give reliable results, for example compact spaces with Dirichlet boundaries. In this article, we addressed the problem of finding a rule that tells us, which spaces can give reliable corrections to the 4-dimensional gravitational potential. We found that the problem is reduced to an algebraic topology one, since the number of the Laplace-Beltrami operator’s zero modes is related to the Betti numbers of the compact space. For manifolds without boundaries, all the connected compact spaces have a zero mode when the first Betti number is equal to, $b_0 = 1$. Recall that the first Betti number counts the disconnected components of the extra dimensional manifold. Still, extra dimensional spaces must be connected for consistency. So all compact connected boundary-less manifolds can yield Yukawa-type corrections. In the case that manifolds with boundaries are considered, things are somehow different. We found that connected compact manifolds with Dirichlet boundaries must be excluded from studies of extra dimensional physics. We related this result to the Betti number of the manifold. On the contrary, using similar arguments, we found that the spaces with Neumann boundaries always give the expected form of corrections to the gravitational potential, that is Yukawa-type form.

The exclusion of some spaces as extra dimensional spaces, can make the identification of extra dimensions somehow easier. However in theories with extra dimensions there is a problem that can make the identification of the exact topology and geometry of the extra dimensions very difficult. It is known as the shadowing effect (see for example [17, 18]). In some cases even the number of extra dimensions will be difficult to determine because results may coincide. In view of such difficulties in order to have a clear picture of the experimental results, we must combine the results coming from other physics areas, such as cosmic ray developments, [49–51], Casimir effect constraints [53–69] and alternative scenarios [63, 63, 70–80]. These scenarios, take into account compact extra dimensions and can pose restrictions on the number and even the size of the extra dimensions.
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