Effect of Fluid Saturation on Acoustic Wave Velocity for Sandstone Reservoirs

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Abstract. Compressional and shear velocities were measured at room temperature and ambient pressure on cylindrical samples. The p-wave and s-wave velocities have been measured at ultrasonic frequencies of 63 kHz and 33 kHz, respectively. Compressional and shear waves have different behaviour in rocks depending on difference in porosity, fluid saturation, fluid viscosity, rock density, lithologic laminations, fracturing, clay content, mineralogy, compaction and pore space framework. The velocity ratio of compressional and shear waves ($V_p/V_s$) varies in crystalline and metamorphic rocks within a very narrow range (from 1.7 to 1.9). In sedimentary rocks, it varies in a wider range from 1.5 to 14.0 due to the very low shear strength of highly porous rocks ($\phi > 25\%$). The technique used to measure acoustic wave velocity is the pulse first arrival technique, in which the travel times are determined for a pulse of compressional and/or shear waves to pass a known measured thickness of the rock (sample length). P-wave and s-wave velocities values (at different saturation) have been determined on a subset of 67 sandstone core samples. Only 26 sandstone samples are belonging to the upper Cretaceous in age (Bahariya Fm, Egypt) and the rest of samples belongs to the Miocene, Lower Pliocene and Upper Pliocene in age (Endrod gas field, Hungary). The p-wave and s-wave velocities were measured with the sample fully saturated with air (dry, $S_w = 0$) and partially saturated with brine water ($S_w = 25\%, 50\%$ and $75\%$) and fully saturated with brine water ($S_w = 100\%$). The effect of sandstone sample water saturation either partial or full water saturation on acoustic parameters were investigated. Some empirical approaches in obtained data analysis are developed based on both Wyllie and Raymer equations in order to predict seismic velocities ($V_p$ and $V_s$) and Poisson’s ratio for either dry or saturated sandstones.

1. Introduction

P-wave and s-wave velocities values (at different saturation) have been determined on a subset of 67 sandstone core samples. Only 26 sandstone samples are belonging to the upper Cretaceous in age (Bahariya Fm, Egypt) and the rest of samples belongs to the Miocene, Lower Pliocene and Upper Pliocene in age (Szolnok Fm, Hungary). The Bahariya Formation is a sedimentary sequence, which most likely deposited in a Tidal flat - lagoon environment at the beginning of the Upper Cretaceous (Cenomanian) transgression in the Western Desert of Egypt. Barrier bar, stream mouth bar, point bar, and distributary channel sand bodies were detected [1] in the Bahariya Formation encountered in the Salam and Khalda oil fields. [2] investigate poison’s ratio versus fluid saturation relations. Regarding to its hydrocarbons potentiality, the Bahariya Formation is counted among the most important hydrocarbon reservoirs in the Western Desert. Thus, a better understanding of the Petrological and petrophysical characteristics of the Bahariya reservoir sandstones is inquiry [3].

[1] [2] [3]
The Bahariya studied samples (see Figure 1) display fine grained sandstones with different numbers of black lamination or flaser bedding, parallel to the sedimentary bedding and partly imbedded particles. Subsequently, many authors [4, 5, 6&7] have studied the petrology and some petrophysical properties of the Bahariya core samples. The analyzed core samples of the Endrod gas field (Hungary) are belonging to the Miocene, Lower Pliocene and Upper Pliocene in age. The Endrod field is situated in the eastern part of the Pannonian Basin System. This field consists of several individual deep water fan-type rock bodies which belong to the Szolnok Formation. It contains sediments deposited in turbidity systems which fill in the deepest parts of the basin or pelitic sediments from the calm period. Its distal part mainly consists of siltstones and Clay stones, and its proximal part made of fine sands, that can be separated to channel and lobe facies [8]. Endrod gas field has been recently studied by [8] concluded that porosity and permeability of the analyzed sandstone reservoirs have bimodal character, while the reservoir lithology is intercalated with siltstone and claystone.

Seismic properties of reservoir rocks in both dry and fluid saturated conditions are of great importance for hydrocarbon exploration. Seismic velocities of reservoir gases, water and hydrocarbons are discussed by many investigators [9]. Some of these studies concluded that fluid saturation and porosity influence Poisson's ratio. It is computed from p-wave velocity (Vp) and S-wave velocity (Vs) by the following relation [2] as:

$$\nu = \frac{[0.5 - (Vs/Vp)^2]}{[1 - (Vs/Vp)^2]}$$

By applying equation (1) in case of water, where Vs = 0.0 km/s, then Poisson's ratio should be equal 0.5. Poisson's ratio is stress dependent and various over a wide range 0.0 > ν < 0.5 for all types of dry and saturated rocks, including negative values [10]. In the present work, our target is to investigate the possible relations among measured and calculating seismic parameters (V_p, V_s &ν) and pore structures using Wyllie equation while, empirical modelling are suggested for calculating seismic velocities for some reservoir sandstone samples obtained from both Bahariya (Egypt) and Szolnok (Hungary) formations at different fluid saturations.

2. Methodology
All samples were analyzed at laboratories of Ain Shams University and the Egyptian Petroleum Research Institute, Egypt. Compressional wave velocity (P-wave velocity) and shear wave velocity (S-wave velocity) were measured at room temperature and ambient pressure on cylindrical samples using two channels Sonic Viewer (OYO - 170). The P-wave and S-wave velocities have been measured at ultrasonic frequencies of 63 kHz and 33 kHz, respectively [2].
3. Compressional wave velocity and water saturation based on Wyllie – equation

Under dry conditions with water saturation $S_W = 0$, we get

$$\frac{1}{V_{p(dry)}} = \frac{\Phi}{V_A} + \frac{(1-\Phi)}{V_{SP}}$$  \hspace{1cm} (1a)

With: $V_{PS} = V_A$, $V_A = $ velocity in air.

With: $V_w = $ velocity in water. \hspace{1cm} (1b)

Under fully saturated condition with $S_W = 1$, we get

$$\frac{1}{V_{p(sat.)}} = \frac{\Phi}{V_W} + \frac{(1-\Phi)}{V_{SP}}$$  \hspace{1cm} (2)

Considering partial saturation with $0 < S_W < 1$, the Wyllie – equation is extended as;

$$\frac{1}{V_p(S_W)} = \left[ \frac{\Phi S_W}{V_W} + \frac{\Phi (1-S_W)}{V_A} \right] + \frac{(1-\Phi)}{V_{SP}}$$  \hspace{1cm} (3)

or

$$\frac{1}{V_p(S_W)} = \frac{\Phi S_W}{V_W} - \frac{\Phi S_W}{V_A} + \left[ \frac{\Phi}{V_A} + \frac{(1-\Phi)}{V_{SP}} \right].$$  \hspace{1cm} (4)

Considering equation (1a), the relation between velocity and saturation can be expressed by;

$$V_p(S_W) = \frac{1}{\Phi S_W \left( \frac{1}{V_W} - \frac{1}{V_A} \right) + \frac{1}{V_{p(dry)}}}.$$  \hspace{1cm} (5)

For the numerical calculation, the following values are considered:

$V_{p(dry)} \equiv $ average velocity of dry samples,

$\Phi \equiv $ average porosity of samples,

$V_W = $ velocity in water (1500 m/s),

$V_A = $ velocity in air (330 m/s).

The resulting equation can be written in the form:

$$V_p(S_W) = \frac{1}{\frac{1}{V_{p(dry)}} - 0.0023636.\Phi S_W}$$  \hspace{1cm} (6)

With the limits for $S_W = 0$

Therefore: $V_p(S_W = 0) = V_{p(dry)}$ and for $S_W = 1$,

$$V_p(S_W = 1) = \frac{1}{\frac{1}{V_{p(dry)}} - 0.0023636.\Phi}$$  \hspace{1cm} (7)
Compressional wave velocity values are extrapolated from dry samples at different saturation levels according to equation (6). The obtained results show that the extrapolated compressional wave velocities calculated at different saturation levels exhibits higher values than the measured ones. Starting from fully saturated samples $V_p(S_w = 1)$ (see equation 6), the saturation dependent velocity can be derived using the following equation:

$$V_p(S_w) = \frac{1}{\frac{1}{V_p(sat.)} + \frac{\Phi(1-S_w)}{330}}.$$  \hspace{1cm} (8)

With the limits for $S_w = 1$: $V_p(S_w = 1) \neq V_p(sat.)$.

The compressional wave velocity values extrapolated from the fully saturated state at different saturation levels from equation (8). The obtained results show that the extrapolated compressional wave velocity calculates at different saturation levels exhibit lower values than that measured ones.

Equations (6 and 8) with the starting point at the dry state with $V_p(S_w = 0) = V_p(dry)$ and the saturated state with $V_p(S_w = 1) \neq V_p(sat.)$ are generalized considering the aim that at the saturation level of 50% then the same velocity is obtained;

$$V_p(S_w) = \frac{1}{\frac{1}{V_p(dry)} - a\Phi S_w} \text{ Then, } V_p(S_w = 0.5) = \frac{1}{\frac{1}{V_p(dry)} - \frac{1}{2}a\Phi}.$$ \hspace{1cm} (9a)

$$V_p(S_w) = \frac{1}{\frac{1}{V_p(sat.)} + \Phi(1-S_w) \cdot b} \text{ Then, } V_p(S_w = 0.5) = \frac{1}{\frac{1}{V_p(sat.)} + \frac{1}{2}b\Phi}.$$ \hspace{1cm} (9b)

Equalizing equations 9a and 9b; $\frac{1}{V_p(dry)} - \frac{1}{2}a\Phi = \frac{1}{V_p(sat.)} + \frac{1}{2}b\Phi$ and $\frac{1}{V_p(dry)} - \frac{1}{V_p(sat.)} = \frac{1}{2}(b + a)\Phi$

Using the assumption that $(a = b)$, the free parameter $(a)$ can be determined by the following formula.

$$a = \frac{1}{\frac{1}{V_p(dry)} - \frac{1}{V_p(sat.)}}.$$

(10)

Using the parameter $(a)$, the saturation dependent velocity can be calculated by the following two equations:

$$V_p(S_w) = \frac{1}{\frac{1}{V_p(dry)} - a\Phi S_w} \text{ for } S_w = 0.0.....0.5,$$

\hspace{1cm} (11a)

$$V_p(S_w) = \frac{1}{\frac{1}{V_p(sat.)} + a\Phi(1-S_w)} \text{ for } S_w = 0.5.....1.0.$$ \hspace{1cm} (11b)

Therefore, the compressional wave velocity values interpolated from dry and fully saturated samples at different saturation levels can be estimated using equations 10a and 10b. Figure 2 shows
that the compressional wave velocities calculated at different saturation levels are close to the measured values.

![Figure 2](image-url)

**Figure 2.** Average of compressional wave velocity measured at $S_w = 0.0, 25, 50, 75 \& 100\%$ water saturation for Bahariya Fm. and $S_w = 0.0, 75\% \& 100\%$ water saturation for Szolnok Fm. Versus average compressional wave velocity calculated from equations 11a and 11b.

### 4. Shear wave velocity and water saturation

Considering isotropic conditions, the shear wave velocity can be determined from the shear modulus $\mu$ and the density $d$ of the rock according to the equation;

$$ V_S = \sqrt{\frac{\mu}{d}}. \tag{12} $$

Using this equation, the shear modulus $\mu$ can be determined from the known s-wave velocity and known density in dry state of the sample;

$$ \mu = V_{s(dry)}^2 \cdot d_{(dry)} $$

The saturation dependent bulk density can be determined using the mixing law;

$$ d(S_w) = d_{(grain)} (1 - \Phi) + d_a (1 - S_w) \cdot \Phi + d_{\mu} \Phi \cdot S_w $$

That can be simplified assuming a vanishing air density ($d_a = 0$):

$$ d(S_w) = d_{(grain)} (1 - \Phi) + d_{\mu} \Phi \cdot S_w $$

Under dry conditions with water saturation $S_w = 0$, we get:

$$ d(S_w = 0) = d_{(grain)} (1 - \Phi) = d_{(dry)} \tag{13} $$

Under fully saturated condition with $S_w = 1$, we get:

$$ d(S_w = 1) = d_{(grain)} (1 - \Phi) + d_{\mu} \Phi \cdot S_w. \tag{14} $$
Considering partial saturation with $0 < S_w < 1$, equation (12) is extended

$$ V_s(S_w) = \sqrt{\frac{\mu}{d_{(grain)}(1 - \Phi) + d_w \Phi S_w}}. \tag{15} $$

For the numerical calculation, the following values are considered:
- $d_{(grain)} \approx$ average grain density of samples ($\approx 2650$ kg/m$^3$),
- $\Phi \approx$ average porosity of samples,
- $d_w = \text{water density} (1000$ kg/m$^3$).

Under dry conditions with water saturation $S_w = 0$, $\mu(S_w = 0) = \mu_{(dry)}$, we get

$$ d(S_w = 0) = d_{(grain)}(1 - \Phi) = d_{(dry)} \cdot $$

$$ \mu_{(dry)} = V_{s(dry)}^2 d_{(grain)}(1 - \Phi) \cdot $$

The resulting equation can be written in the form:

$$ V_s(S_w) = \sqrt{\frac{V_{s(dry)}^2 d_{(grain)}(1 - \Phi)}{d_{(grain)}(1 - \Phi) + d_w \Phi S_w}}. \tag{16} $$

Shear wave velocity values extrapolated from dry samples at different saturation levels according to equation 17. It can be observed that the extrapolated shear wave velocities calculated at different saturation level show bad correlation. The calculated velocity values are higher than the measured velocity indicating that assumption of shear modulus independent with respect to fluid saturation is not valid.

Under fully saturated condition with $S_w = 1$, $\mu(S_w = 1) = \mu_{(sat.)}$, we get

$$ \mu_{(sat.)} = V_{s(sat.)}^2 \left(d_{(grain)}(1 - \Phi) + d_w \Phi S_w \right) \cdot $$

The resulting equation can be written in the form:

$$ V_s(S_w) = \sqrt{\frac{\mu(S_w)}{d_{(grain)}(1 - \Phi) + d_w \Phi S_w}} \cdot \tag{17a} $$

For linear equation: $\mu(S_w) = \mu_{(dry)} + (\mu_{(sat.)} - \mu_{(dry)}) (S_w)$

An interpolation of the shear modulus between the dry and fully saturated state is proposed in the form:

$$ \mu(S_w) = \mu_{(sat.)} + (\mu_{(dry)} - \mu_{(sat.)}) (1 - S_w)^n \cdot \tag{17b} $$

Shear wave velocity values at different saturation levels have been interpolated from dry and fully saturated state using equations 17a and 17b. The data is shown in figure 3.
Figure 3. Average of shear wave velocity measured at $S_w = 0.0, 25, 50, 75\&{100\%}$ water saturation for Bahariya Fm. and $S_w = 0.0, 75\&{100\%}$ water saturation for Szolnok Fm. Versus average shear wave velocity calculated from equations 17a and 17b.

5. Relation between measured and calculated Poisson’s ratio

Poisson’s ratio is computed from p-wave velocity $Vp$ and Vs-wave velocity (eq.1) and also by using the following relation:

$$\nu(S_w) = \frac{a(S_w) - 2}{2(a(S_w) - 1)}$$  \hspace{1cm} (18)

With

$$a(S_w) = \left(\frac{V_p(S_w)}{V_s(S_w)}\right)^2.$$  \hspace{1cm} (19)

The calculated Poisson’s ratio determined from compressional wave velocity data obtained from equations 11a and 11b and shear wave velocity data obtained from equations 17a and 17b is shown in ‘figure 4’. The calculated values give an intermediate correlation with that measured from compressional and shear wave velocity.
Figure 4. Average Poisson’s ratio measured at $S_w = 0.0$, 25, 50, 75 and 100% water saturation for Bahariya Fm. and $S_w = 0.0$, 75% and 100% water saturation for Szolnok Fm. Versus average Poisson’s ratio calculated from equations 11a, 11b for $V_p$ and 17a and 17b for $V_s$ calculation.

6. Conclusions

- The compressional wave velocity could be calculated for partially water saturated sandstone samples ($S_w$ ranges from 0.0 up to 50%) using equation (11a) while, it could be outlined as well if $S_w$ ranges from 50% up to 100%) using equation (11b).
- The shear wave velocity could be estimated for fully saturated ($S_w = 100\%$) sandstone samples by using equation (17a) while, for partially saturated sandstone samples ($S_w < 100\%$) the equation (17b) is more effective.
- The Poisson’s ratio is estimated by knowing both compressional and shear wave velocities, therefore the equations (11a, 11b, 17a and 17b) are used in order to outline it.

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