Experimental and numerical analysis of damage and fracture mechanisms in metal sheets under non-proportional loading paths

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Abstract. New biaxial experiments and corresponding numerical simulations with specimens taken from metal sheets are discussed. Inelastic deformation behavior as well as damage and fracture mechanisms are examined in detail under different biaxial loading conditions with special focus on non-proportional loading paths. In this context, a continuum damage model is presented based on a yield condition and a damage criterion as well as evolution equations for plastic and damage strain rates. The damage criterion takes into account the effect of different processes on the micro-scale depending on the stress state. Experiments with biaxially loaded specimens with newly developed geometries have been performed using proportional and non-proportional loading histories. Strain fields are monitored by digital image correlation technique and fracture surfaces are analyzed with scanning electron microscopy. Numerical simulations of the experiments have been performed and numerical results are compared with available experimental data. In addition, based on the numerical calculations stress distributions in critical specimen’s areas are detected allowing prediction of damage and fracture modes. The results demonstrate the efficiency of the new geometries of the specimens covering a wide range of stress states as well as the effect of the loading paths on damage and fracture behavior in ductile metal sheets.

1. Introduction
Increasing demands of customers like requirements to enforce safety of structural elements or to design lightweight structures leading to cost efficiency or improved energy consumption have to be fulfilled by products of modern metal forming processes. Thus, these economic, environmental and material strength reasons require the increased use of high quality metals like high strength and advanced high strength steels or aluminum alloys. In addition, numerical analysis for optimization of metal forming processes receive remarkable attention. Therefore, practically applicable and highly predictive material models as well as corresponding accurate and efficient numerical theories have to be developed which are used to analyze microscopic damage and failure mechanisms causing macro-cracks and fracture in materials and structures. Many industrial processes like forging and rolling show non-proportional loading paths and, thus, damage and fracture mechanisms developing under non-proportional loading histories are here of special interest.

Damage and failure mechanisms on the micro-level depend on the current stress state acting in a material point. For example, hydrostatic stress states lead to formation and growth of
micro-voids whereas deviatoric stresses cause evolution of micro-shear-cracks. To investigate and to understand these complex stress-state-dependent damage and failure processes as well as their interactions detailed experimental and numerical studies on both the micro- and the macro-level have to be performed in order to develop accurate and realistic phenomenological continuum models. These constitutive theories are used to quantify damage at different stages of the deformation histories as well as to predict and to judge the quality of metal forming processes.

Different experiments with carefully designed specimens have been discussed in the literature to investigate the stress-state-dependent damage and fracture processes as well as their effect on the macroscopic deformation and failure behavior of ductile metals. In particular, uniaxial tension tests with differently notched and smooth unnotched specimens and corresponding numerical analyses have been presented to study the sheet metal behavior for positive stress triaxialities [1-5]. In addition, specimens with new geometries have been developed and tested under uniaxial loads to examine their deformation and failure behavior under nearly zero stress triaxialities causing micro-shear-crack dominated processes in their critical parts [1, 2, 4-6]. Furthermore, butterfly specimens with complex geometries have been proposed to analyze the ductile material behavior for various stress states. Using special experimental equipment they can be loaded in different directions causing combined tensile, shear and compression stress states [3, 6]. Alternatively, experiments with biaxially loaded cruciform specimens have been published [7]. Further two-dimensional tests with newly developed biaxial specimens have been developed [8-10] and corresponding numerical simulations have been performed to detect stress states in critical specimens areas to investigate stress-state-dependent damage and failure mechanisms in thin metal sheets. First results of biaxial experiments with non-proportional loading paths have been presented [11] and they show remarkable effect of the loading history on damage and fracture modes.

From theoretical point of view phenomenological anisotropic continuum damage approaches are most qualified to numerically simulate inelastic deformation behavior of ductile material elements affected by different stress-state-dependent damage mechanisms. On the other hand, their practical applicability is often limited by large number of material parameters and problems in their identification. Therefore, in the present paper an efficient anisotropic continuum damage model is briefly presented. Two-dimensional experiments with newly developed, biaxially loaded cruciform specimens undergoing non-proportional loading paths are presented. Evolution of strain fields in critical regions of the specimens are monitored by digital image correlation (DIC) technique while fracture surfaces are analyzed by scanning electron microscopy (SEM). Based on numerical simulations of the biaxial tests stress fields in critical regions of the specimens are predicted and are used to explain stress-state-dependent damage mechanisms.

2. Continuum damage model

The continuum damage model published by Brünig [12] is briefly summarized which analyzes inelastic deformation behavior as well as the effect of damage and failure in ductile metals. In this phenomenological approach stress-state-dependent damage and failure processes on the micro-level are governed by the formation of macroscopic damage strains. In the thermodynamically consistent continuum framework different configurations are introduced: damaged and corresponding fictitious undamaged configurations. This is the basis for the kinematic model taking into account elastic, plastic and damage strain rate tensors. In the respective configurations free energy functions are introduced to model the elastic behavior of the undamaged matrix material and the damage-elastic behavior of the damaged material sample. Considering the undamaged configurations a yield criterion and a non-associated flow rule are formulated characterizing the plastic material behavior. In addition, in the damaged configurations stress-state-dependent damage behavior is modeled by a damage criterion and a
non-associated damage rule both depending on the stress triaxiality and the Lode parameter.

In particular, isotropic plastic behavior of the investigated aluminum alloy AlSiMgMn (EN AW 6082-T6) is characterized by the yield criterion

$$f_{pl} = a\bar{I}_1 + \sqrt{\bar{J}_2} - c = 0$$ (1)

with the first and second invariants, $\bar{I}_1$ and $\bar{J}_2$ of the effective Kirchhoff stress tensor [12]. The plastic hardening behavior is modeled by the power law

$$c = c_0 \left( \frac{H_0 \gamma}{n c_0} + 1 \right)^n$$ (2)

with the initial yield stress $c_0 = 162$ MPa, the hardening modulus $H_0 = 800$ MPa as well as the hardening exponent $n = 0.17$ and $\gamma$ denotes the equivalent plastic strain measure [12].

Furthermore, onset and continuation of damage is characterized by a damage surface formulated in stress space [12,13]. Thus, the damage criterion

$$f_{da} = \alpha I_1 + \beta \sqrt{J_2} - \sigma = 0$$ (3)

is written in terms of the first and second deviatoric stress invariants $I_1$ and $J_2$ of the Kirchhoff stress tensor. In addition, the damage threshold

$$\sigma = \sigma_0 - H_1 \mu^2$$ (4)

represents material toughness to micro-defect propagation with the initial equivalent stress $\sigma_0 = 250$ MPa and the modulus $H_1 = 400$ MPa where $\mu$ means the equivalent damage strain measure. In Eq. (3) the variables $\alpha$ and $\beta$ denote damage mode parameters corresponding to the different damage processes acting on the micro-level: void-growth-dominated modes for large positive stress triaxialities, shear modes for negative stress triaxialities and mixed modes (simultaneous growth of voids and formation of micro-shear-cracks) for moderate positive and nearly zero stress triaxialities. In the present model, the Lode parameter is also taken into account because it has been shown that its effect on the formation of the micro-structural effects can be remarkable especially in moderate positive and negative stress triaxiality regions [14]. Hence, the damage mode parameters $\alpha$ and $\beta$ in Eq. (3) depend on the stress intensity $\sigma_{eq} = \sqrt{3J_2}$ (von Mises equivalent stress), the stress triaxiality

$$\eta = \frac{\sigma_m}{\sigma_{eq}} = \frac{I_1}{3\sqrt{3J_2}}$$ (5)

defined as the ratio of the mean stress $\sigma_m = I_1/3$ and the von Mises equivalent stress $\sigma_{eq}$ as well as on the Lode parameter

$$\omega = \frac{2T_2 - T_1 - T_3}{T_1 - T_3} \quad \text{with} \quad T_1 \geq T_2 \geq T_3$$ (6)

written in terms of the principal stress components $T_1$, $T_2$ and $T_3$.

The dependence of $\alpha$ and $\beta$ on stress state has been studied in detail for the investigated aluminum alloy. Numerical analyses on the micro-scale have been performed by Brünig et al. [14] considering the deformation and failure behavior of void-containing unit cells. Simplified functions have been proposed in [9] for practical applications still allowing accurate phenomenological modeling of the inelastic deformation as well as damage and failure behavior.
observed in experiments with biaxially loaded specimens. With these functions the parameters \( \alpha \) and \( \beta \) correspond to different damage and fracture mechanisms acting on the micro-level.

Based on these investigations [9, 14], the parameter \( \alpha \) is given by

\[
\alpha(\eta) = \begin{cases} 
-0.15 & \text{for } \eta \leq 0 \\
0.33 & \text{for } \eta > 0 
\end{cases}
\] (7)

whereas the parameter \( \beta \) is taken to be the non-negative function

\[
\beta(\eta, \omega) = -1.28\eta + 0.85 - 0.017\omega^3 - 0.065\omega^2 - 0.078\omega \geq 0 .
\] (8)

Furthermore, the damage strain rate tensor is given by the damage rule

\[
\dot{H}^{da} = \dot{\mu} \left( \tilde{\alpha} \frac{1}{\sqrt{3}} I + \tilde{\beta} N \right)
\] (9)

where \( \dot{\mu} \) represents a non-negative scalar-valued factor. In Eq. (9) the stress related deviatoric tensor \( N = \frac{1}{2\sqrt{2}} \text{dev} \mathbf{T} \) has been used and \( \dot{\mu} \) represents in the present continuum damage model the equivalent damage strain rate measure quantifying the amount of increase in irreversible damage. The parameters \( \tilde{\alpha} \) and \( \tilde{\beta} \) are kinematic variables denoting the portion of volumetric and isochoric damage-induced deformations. These parameters are also associated with various damage and fracture processes on the micro-level and, similar to the parameters in the damage criterion (3), they are based on numerical analyses with micro-defect-containing representative volume elements undergoing different three-dimensional loading scenarios [14] as well as on comparison of experimental data and results of numerical simulations of experiments with uniaxially and biaxially loaded specimens [9].

In particular, the stress-state-dependence of the parameter \( \tilde{\alpha} \) corresponding to the amount of volumetric damage strain rates caused by volume changes of micro-defects is expressed in the form

\[
\tilde{\alpha}(\eta) = \begin{cases} 
0 & \text{for } \eta \leq 0 \\
0.5714\eta & \text{for } 0 < \eta \leq 1.75 \\
1 & \text{for } \eta > 1.75 
\end{cases}
\] (10)

In addition, the stress-state-dependence of the parameter \( \tilde{\beta} \) representing the amount of anisotropic isochoric damage strain rates caused by formation of micro-shear-cracks is given by

\[
\tilde{\beta}(\eta, \omega) = \tilde{\beta}_0(\eta) + (0.0252 + 0.0378\eta) \tilde{\beta}_\omega(\omega)
\] (11)

with

\[
\tilde{\beta}_0(\eta) = \begin{cases} 
0.87 & \text{for } \eta \leq \frac{1}{3} \\
0.979 - 0.326\eta & \text{for } \frac{1}{3} < \eta \leq 3 \\
0 & \text{for } \eta > 3 
\end{cases}
\] (12)

and

\[
\tilde{\beta}_\omega(\omega) = \begin{cases} 
1 - \omega^2 & \text{for } \eta \leq \frac{2}{3} \\
0 & \text{for } \eta > \frac{2}{3} 
\end{cases}
\] (13)

It can be clearly seen that the macroscopic damage rule (9) contains a volumetric part (first term in Eq. (9)) associated with isotropic growth of voids on the micro-level as well as a deviatoric part (second term in Eq. (9)) corresponding to anisotropic development of micro-shear-cracks, respectively. Thus, the basic damage mechanisms discussed above (growth of isotropic voids and evolution of micro-shear-cracks) acting on the micro-level are incorporated in the macroscopic damage rule (9) of the phenomenological continuum model.
3. Experiments and numerical simulations
A new experimental program with biaxially loaded specimens has been proposed to investigate the effect of stress state on inelastic deformations as well as on damage and failure processes in ductile metal sheets [10]. Experiments with the X0-specimen for different load ratios undergoing proportional and non-proportional loading conditions have been presented in [11]. The central part of the geometry of the X0-specimen taken from sheets with 4 mm thickness is shown in Fig. 1(a). It contains four notched parts with minimum thickness of 2 mm where localization of inelastic deformations and damage will occur. In addition to the experiments [11], results of corresponding numerical simulations of the experiments are discussed in the present paper to elucidate the stress states in critical specimen’s regions leading to different damage mechanisms on the micro-level. As an example, the behavior of the X0-specimen is discussed for the load ratio \( F_1/F_2 = 1/−0.5 \) where numerical results are compared with available experimental data. Proportional (P) and non-proportional (NP) loading paths are considered (see Fig. 1(b)) and the effect of the load history on damage and failure mechanisms is discussed in detail. In the non-proportional case first loading is only by the force \( F_1 \) and at the stage \( F_1 = 4.8 \) kN additional loading in axis 2 by the force \( F_2 \) occurs. When this loading path reaches that one of the proportional case a final proportional part follows until final fracture of the specimen. In these experiments with the load ratio \( F_1/F_2 = 1/−0.5 \) the final loads at fracture for both loading scenarios are nearly identical.

Load-displacement-curves monitored during the experiments are compared with those predicted by the numerical simulations in Fig. 2 for loads and displacements in axis 1 (a1) and in axis 2 (a2). The displacements \( \Delta u_{ref} \) are the respective displacements in axis 1 and 2 between the red points shown in Fig. 1(a). Results for proportional (Fig. 2(a)) and non-proportional (Fig. 2(b)) loading histories are presented, respectively. In particular, in the proportional loading case (Fig. 2(a)) the load \( F_1 \) in axis 1 (a1) increases and at \( F_1 = 3.8 \) kN onset of plastic yielding can be observed. Further increase in load occurs until the specimen fails at \( \Delta u_{ref1} = 1.03 \) mm. Similar behavior can be seen in axis 2 (a2): Increase in the load level up to \( F_2 = −1.9 \) kN where inelastic behavior begins and the specimen fails at \( \Delta u_{ref2} = −0.92 \) mm. The experimental load-displacement-curves (EXP) are nicely predicted by the numerical simulation (SIM). On the other hand, in the non-proportional loading case (Fig. 2(b)) the load in axis 1 (a1) increases up to \( F_1 = 4.8 \) kN and no remarkable inelastic behavior is observed. Further displacements in axis 1 are caused by the increase in the compression load \( F_2 \) in axis 2 and remarkable further displacements in axis 1 occur during the final proportional loading step. The
specimen fails at \( \Delta u_{\text{ref}} = 1.00 \text{ mm} \) which is nearly identical with the final displacement after the proportional loading case. In addition, first loading by \( F_1 \) also leads to displacements in axis 2 \((a_2)\) up to \( \Delta u_{\text{ref}} = -0.04 \text{ mm} \). This displacement level further increases during additional loading by \( F_2 \) and at failure the displacement \( \Delta u_{\text{ref}} = -0.88 \text{ mm} \) is reached which is smaller compared to the proportional loading case. Again the experimental load-displacement-curves (EXP) are nicely predicted by the numerical simulation (SIM).

During the experiments strain fields are monitored by digital image correlation (DIC) technique. Fields of the first principal strain in one notched region of the specimen are shown in Fig. 3 based on the experiments (EXP) and the corresponding numerical simulations (SIM). In the experiments vertical bands of localized strains occur and the first principal strain reaches about 37\%. This final localized deformation behavior is nearly identical for the proportional and the non-proportional loading path. The experimentally obtained shape of this central part of the X0-specimen is accurately predicted by the numerical simulation for both the proportional and the non-proportional case. The localized principal strain distribution is also obtained whereas in the numerical simulation the maxima only reach about 34\% but no remarkable differences between the proportional and the non-proportional path can be seen.

Furthermore, based on the numerical simulations of the experiments the respective stress fields can be predicted. The stress triaxiality \( \eta \) (Eq. (2)) in the notched region of the X0-specimen is shown in Fig. 4 for the proportional case (P) and for the non-proportional loading path at the end of the load step with \( F_1 \) only (NP(1)) and at the end of the test (NP(12)).
In particular, in the proportional case (P) at the end of the experiment the stress triaxiality in the center is about $\eta = 0.15$ indicating shear behavior with slight superimposed tension. Smaller stress triaxialities can be seen in the boundaries of the central area corresponding to shear modes whereas high stress triaxialities up to about $\eta = 0.5$ are numerically predicted for the lower boundary. In the non-proportional case after first loading with $F_1$ only (NP(1)) the distribution of the stress triaxiality is nearly homogeneous in this area with values of about $\eta = 0.3$ indicating tension-shear behavior. After additional loading with $F_2$ (NP(12)) the stress triaxialities change in the area and the final distribution is very similar to that one after the proportional loading path (P).

Moreover, fracture surfaces are visualized by scanning electron microscopy (SEM) and the results after the proportional and non-proportional loading histories are shown in Fig. 5. After the proportional loading path (P) remarkable micro-shear-behavior can be seen with few very small voids caused by the slight superimposed tension behavior. These damage and failure modes excellently correspond to the numerically predicted stress triaxialities discussed above. After the non-proportional loading history (NP) micro-voids in combination with shear effects are visualized by scanning electron microscopy. Formation and growth of micro-voids occur during the first loading step with tension-dominated stress triaxiality distribution. With additional loading in axis 2 shearing of these voids with additional formation of micro-shear-cracks happens. Thus, the non-proportional path leads to more and larger voids compared to the proportional one.
4. Conclusions
A phenomenological damage model using stress-state-dependent damage criterion and damage rules has been briefly presented. The continuum approach has been validated by experiments with biaxially loaded specimens allowing analysis of different stress states as well as of proportional and non-proportional loading paths. Digital image correlation technique has been used to monitor current strain fields during the experiments in critical parts of the specimen and after the tests fracture surfaces have been visualized by scanning electron microscopy. In addition, numerical simulations of the experiments have been performed. Load-displacements curves as well as strain fields have been predicted and compared with experimental data. Analysis of stress fields allow interpretation of damage and failure mechanisms on the micro-level which can be justified by scanning electron microscopy of the fracture surfaces. Hence, the presented continuum damage model can be used to investigate and to optimize sheet metal forming processes. In addition, biaxial experiments with the X0-specimen are recommended to examine stress-state-dependent damage and fracture mechanisms in thin sheets.

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References
[1] Bai Y and Wierzbicki T 2004 Int. J. Mech. Sci. 46 81
[2] Gao X, Zhang G and Roe C 2010 Int. J. Damage Mech. 19 75
[3] Dunand M and Mohr D 2011 J. Mech. Phys. Sol. 59 1374
[4] Brüning M, Chyra O, Albrecht D, Driemeier L and Alves M 2008 Int. J. Plast. 24 1731
[5] Driemeier L, Brüning M, Micheli G and Alves M 2010 Mech. Mat. 42 207
[6] Bai Y and Wierzbicki T 2008 Int. J. Plast. 24 1071
[7] Kuwabara T 2007 Int. J. Plast. 23 385
[8] Brüning M, Brenner D and Gerke S 2015 Eng. Fract. Mech. 141 152
[9] Brüning M, Gerke S and Schmidt M 2016 Int. J. Fract. 200 63
[10] Gerke S, Adulyasak P and Brüning M 2017 Int. J. Solids Struct. 110 209
[11] Gerke S, Zistl M, Bhardwaj A and Brüning M 2019 Int. J. Solids Struct.
[12] Brüning M 2003 Int. J. Plast. 19 1679
[13] Chow C and Wang J 1987 Eng. Fract. Mech. 27 547
[14] Brüning M, Gerke S and Hagenbrock V 2013 Int. J. Plast. 50 49