Numerical modelling for the optimization of multi-element, capacitive, ultrasonic, air-coupled transducer

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Abstract. This paper presents an analytical method to predict the acoustic pressure field produced in the air by concentric, annular sources. The sources correspond to a multi-element, ultrasonic, air-coupled array, which should be used further for non-destructive testing (NDT) purposes. After validation of the model based on the comparison of the predicted acoustic pressure with experimental results, a numerical study of the consequence of the annular sources arrangement on the acoustic field is realized. This helps in optimizing the design of the array, so that the intended ultrasonic field is produced. For high spatial resolution to be obtained in further NDT process this field should be focused and, ideally, the frequency used for inspecting the tested piece should be tuned so that wavelengths in the inspected material take suitable values in regards of piece thickness and defect sizes. Numerical simulations show that the multi-element array allows dynamic focusing to be made by judiciously adjusting delay laws of the excitation signals applied to each individual element.

1. Introduction
The application of ultrasounds in both biomedical imaging and industrial non-destructive testing (NDT) has been greatly advanced recently. Air-coupled techniques for the generation-detection of ultrasonic waves in tested materials are very advantageous because they avoid the use of coupling liquids as gel (contact technique) or water (jets or immersion tanks) for instance, which render much more complicated online testing procedures. Today’s available air-coupled techniques for sending/receiving ultrasounds to/from tested samples are based on the use of either specific ultrasonic transducer [1][2], laser sources and/or probes [3] or electromagnetic acoustic transducers (EMATs) [4]. Laser-based techniques are quite expensive and rarely appropriate for on-line inspection because they require high level of accuracy in measurement set-up; EMAT-based techniques are suitable for conductive or magnetostrictive components only, thus limiting the field of applications. Consequently, as an alternative economical solution, air-coupled ultrasonic transducers could be used for the inspection of any metallic or polymer-based material pieces. However, one difficulty arising when using air-coupled ultrasonic transducers is the huge energy transmission losses at each solid-air interface, such as those between the transducers and the air, or those between tested materials and the
air, e.g. up to -54 dB loss at one air-aluminium interface [5]. Moreover, the attenuation of ultrasounds in air increases with the frequency according to the following law $1.64 f^2 \times 10^{-10} \text{dB/m}$ [6], where $f$ is the frequency in Hertz. For example, the amplitude loss for a 1 MHz frequency wave will be equal to about 85 dB after a distance equal to 50 cm. Consequently, for usual NDT applications, frequency must be kept down to reasonably low values, i.e. below 0.5 MHz, and transducers efficiency must be as large as possible. Furthermore, for NDT purposes, transducers should be of large frequency bandwidth to make frequency tuning possible, so that optimum frequency is set according to the thickness of the inspected piece and/or to the size of sought defects.

Capacitive ultrasonic transducers have been developed over the last two decades [7][8][9][10], and successfully used for the contact-less inspection of composite structures, for instance [11]. Their main advantages include a very large frequency bandwidth [12] and a sufficiently high sensitivity [13] for general non-contact NDT. Several manufacturing techniques have been reported, including series of coarse polishing [14] machining V-shaped grooves [15][16] or circular holes through laser ablation [8], photolithography [12], or sand blasting [9] so that obtained roughness or cavities at the surface of the metallic back-plate determine the frequency properties of the capacitive ultrasonic transducer [17]. Also, these elements require a few-micron thick membrane deposited on the back-plate, with metallised external surface. This must be stretched for the transducer to operate, thus producing locally plane wave front and introducing some difficulties in allowing the focusing of the ultrasonic beam, causing low spatial resolution, as a serious drawback for some NDT applications. In order to circumvent this problem, a method has been proposed using a micro-machined Fresnel zone-plate placed in the acoustic field [18]. A flexible back-plate as been developed as an alternative solution, allowing the realization of concave capacitive air-coupled transducers [19]. With micromachining fabrication technology on the silicon back-plate a micro-electromechanical system for cMUT (capacitive micromachining ultrasonic transducer) has been developed [20] recently. However, these solutions offer fixed focusing features and as many transducers as intended focusing conditions will be required for satisfying several NDT inspections.

The objective of this paper is to present a model and its use to optimize the design of ultrasonic air-coupled transducers. These should allow the generation of an ultrasonic beam with dynamic focusing properties, with a large frequency bandwidth and with high efficiency. The proposed solution is a multi-element, capacitive, array, so that the capacitive part may bring the large frequency range, and the multi-element feature will allow for dynamic focusing capabilities under appropriate phase delay control. This is made up of one central circular element and several concentric, annular elements. Two cases are studied, with different spatial configurations. In the first case, the difference between the outer and inner diameters is kept constant for every annular element. Thus, the active surface of the rings increases as rings get away from the axis of the assembly. In the second case, the elements have dimensions such that their respective active surface remains constant. In section 2, the analytical model, which is based on the Kirchhoff integral, is presented. It allows numerical predictions of ultrasonic fields radiated by either single or multi-element transducers to be made. Section 3 presents a laser-based, experimental set-up and its use for measuring the ultrasonic pressure field produced by a circular mono-element for validating the analytical model. Then the model is used for predicting the ultrasonic field produced by one single annular element, and experiments are preceded for validation purpose. Agreements and discrepancies are discussed. Finally, in section 4 the multi-element arrays are numerically simulated. The focusing properties of two configurations of element arrangement are compared, demonstrating that dynamic focusing is possible if adequate delay laws are used for exciting each element of the array.

2. Modelling the acoustic field pressure generated by air-coupled transducers
We consider the wave propagation of ultrasound in air radiated by an air-coupled transducer. The model lies on the assumption that the vibration of the transducer surface is spatially uniform. The expression uses Kirchhoff integral to calculate the pressure in the frequency domain \( p(r, z; \omega) \). In order to save computer resources and computational time, the model is built as a 2D axial-symmetric model, under polar coordinate system \((r, z)\) (Figure 1).

![Figure 1](image.png)

**Figure 1.** Definition of polar coordinate axes for the transducer acoustic field model.

Starting from a harmonic and uniform motion of the transducer surface, the normal velocity of the transducer surface expresses as \( V(t) = V_0 e^{j\omega t} \), where \( V_0 \) is the uniformed amplitude of the normal velocity and \( \omega \) is the angular frequency of the excitation signal applied to the transducer. The steady-state pressure at a fixed point in the medium \( p(r, z; \omega) \) could be derived using the following equation:

\[
p(r, z; \omega) = -j\omega \rho \hat{V} H(r, z; \omega)
\]

where \( \hat{V} \) is the Fourier transform of \( V(t) \), \( \rho \) is the density of air; \( H(r, z; \omega) \) is the Fourier transform of the impulse response \( h(r, z; t) \), which could be derived as [21],

\[
H(r, z; \omega) = \frac{a c_0}{j \omega \pi} \int_0^\pi \frac{r \cos \psi - a}{r^2 + a^2 - 2ar \cos \psi} \times (e^{-j\omega \sqrt{r^2 + a^2 - 2ar \cos \psi}} - e^{-ja \omega / c_0})
\]

in which \( a \) is the radius of the circular transducer, \( c_0 = \sqrt{\frac{K_f}{\rho_0}} \) is the sound speed in air, and where \( K_f \) is the incompressibility coefficient. In order to take into account the absorption of ultrasounds in air, it is possible to introduce an effective complex value of the incompressibility coefficient \( \tilde{K}_f = K_f(1 + j\delta) \), where \( \delta \) is a dimensionless parameter that represents the damping effect, regardless to the physical mechanism involved.

As the pressure calculus is based on the assumption that every single spot at the surface of the piston radiates a hemi-spherical wave into the medium (i.e. Huygens principle), the field radiated by an annular-shaped source can be deduced by subtracting the field radiated by a disc, whose diameter is equal to the inner diameter of the ring, to the field radiated by a second disc, whose diameter is equal to its outer diameter. Summing up all pressure fields thus calculated for each element of the array, including the central circular piston plus all annular concentric elements, would then provide the acoustic pressure generated by the whole multi-element array, in the frequency domain.
3. Measurement of ultrasonic pressure fields produced by capacitive transducers

In order to characterize the acoustic field generated by an air-coupled transmitter, the experimental technique presented in [9] is used. A 5 µm thick membrane is stretched and placed perpendicularly to the acoustic field at a given distance \( z \) away from the source. The incident acoustic wave induces a mechanical movement of the membrane, the normal velocity of which, \( V_n \), can be measured using a Laser Doppler velocimeter. This experimental set up provides the velocity of a point at the surface of the membrane. Then, assuming an incident plane wave and knowing the plane-wave transmission coefficient, \( T \), of the membrane immersed in air, the acoustic pressure \( P \) at the insonified surface of the membrane is deduced as follows:

\[
P_a = \frac{Z_a}{T} V_n
\]

where \( Z_a = 420 \) Rayl is the acoustic impedance in air. The transmission coefficient \( T \) expresses as:

\[
T = \frac{4Z_a Z}{(Z_a + Z)^2 \exp(-ikh) - (Z_a - Z)^2 \exp(ikh)}
\]

where \( Z \) is the acoustic impedance of the membrane, \( Z = 1.82 \) MRayl, \( h \) is the membrane thickness and \( k \) is the wave number of longitudinal wave in the membrane.

In order to check the validity of the model, the first experiment is carried out with a single, circular (radius \( a = 24 \) mm), capacitive transducer. The input electric signal is a Gaussian-windowed, 10-cycle, 100 kHz centre frequency, sine toneburst. The central frequency is chosen in order to create a Fresnel distance value equal to \( D_f = 168 \) mm that allows the observation of both near field and far field areas within the measurement range. Besides, the toneburst duration is chosen long enough in order to lead to a quasi-monochromatic regime, but short enough to avoid the temporal superposition of the incident wave and waves reflected between the membrane and the transducer surface.

Figure 2 compares the measured, on-axis, pressure distribution together with the corresponding distribution predicted with and without considering damping in air. The experimental Fresnel distance matches the theoretical one. By considering the experimental amplitude decrease in the far field area, one can observe that for such a frequency the damping effects are not negligible and should be considered in the model for proper prediction. Thus, in the following analysis, these effects are systematically incorporated in the model. The analytical far-field amplitude distribution matches well with the experimental results. Conversely, in the near-field area, discrepancies remain. The reason may lie on the non-uniform movement of the transducer membrane in our experiments. In order to investigate this assumption, the velocity of the surface of the capacitive transducer, i.e. of its membrane, has been scanned using the Laser Doppler velocimeter over a 30 mm long side square area. The measured velocity profile is presented in Figure 3. The capacitive transducer surface does not vibrate uniformly because: (1) the backing plate has been sand-blasted so that small cavities of various sizes resonate at its surface with different frequencies and amplitudes, and (2) the membrane does not move as a piston because of its thinness and flexibility and because it is clamped at its edge. Therefore, the velocity, which is measured by the laser probe, is not necessarily the normal-to-the-membrane component, but the projection along the direction of the laser beam of that normal component. This makes a difference with the assumption of piston-like motion of the source, which is introduced in the analytical model, so justifying the disagreement obtained between both measured and predicted pressure amplitude distributions, in the near field area. Moreover, for technical reasons, no measurement was possible between \( z = 0 \) mm and \( z = 30 \) mm, because placing the membrane too close to the transmitter was causing reflections between that membrane and the edges of that transmitter, which are protruding with respect to its active surface. Consequently, it was not possible to measure the very near field pressure produced by the transmitter, without strong perturbations.
caused by overlapping echoes. However, as the far field distribution is based on the total and global contribution of the vibration of the transducer, the experimental measurement is in good agreement with the prediction in far field, which in our case validates the model and provides confidence for its use for further developing NDT applications.

![Near and far field comparison](image1.png)

**Figure 2.** On-axis pressure amplitude radiated by a mono-element, circular transducer of r = 24 mm. Experimental (diamonds) and analytical results with (solid line) and without (dashed line) damping effects.

![2D scan of velocity profile](image2.png)

**Figure 3.** 2D scans of the velocity profile (measured using Laser Doppler velocimeter) of the membrane surface of the mono-element, circular, capacitive transducer ($x = r \cos \psi$ and $y = r \sin \psi$).

The case of a wave radiated by one annular element is also considered. This annular element, with inner and outer radius equal to 7 mm and 9 mm respectively, is excited by a Gaussian-windowed, 10-cycle, sine toneburst, while the centre frequency is now equal to 250 kHz. Analytically, the resulting pressure field produced by the annular element is obtained by subtracting the inner field from the outer...
one. The resulting on-axis pressure amplitude versus the propagation distance $z$ is compared to the experimentally measured one (Figure 4).

**Figure 4.** On-axis normalized-amplitude distribution of the pressure field radiated by a single annular element. Experimental (diamonds) and analytical (solid line) results.

**Figure 5.** Transverse distribution of pressure amplitude at a distance $z=150$ mm, radiated by the single annular element of inner and outer radius equal to 7 mm and 9 mm, respectively. Experimental (diamonds) and analytical (solid line) results.

**Figure 5** compares together both measured and calculated transverse distributions of the pressure amplitude at a distance equal to 150 mm, so in the far field area. The agreement is very good thus validating the modelling approach, which consists of subtracting both acoustic fields predicted for two circular piston sources. This will then be used for modelling the multi-element transducer made up of several annular, concentric elements.

### 4. Numerical study of multi-element capacitive annular arrays

The case of multi-element arrays is considered analytically, in order to test their focusing properties. Two types of array are studied. Both consist in an arrangement of 8 elements, but with different spatial characteristics. The first one (named “Type I” - Figure 6.a) has the following geometrical parameters:
a central circular element with diameter equal to 2 mm is surrounded by 7 annular elements: the spacing between two neighbouring elements is constant and equal to 2 mm, and the width of each element is equal to 2 mm. This leads to a total diameter of the array equal to 58 mm. The second array (named “Type II” -Figure 6.b) has the same total outer diameter as the first array, but its elements have all the same area. The central element is circular with a diameter equal to 7.87 mm, and the 7 surrounding annular elements have different widths each other, so that their area keeps the same. For a fixed wavelength, the Fresnel distance depends only on the emitting surface $S$ of the source. Thus, if a continuous wave is applied to each element of this same-area-element array, without any phase delay, the total pressure field will present a natural focal zone.

![Figure 6.](image6.png)

**Figure 6.** Schematic profile of annular array design a) Type I, where all the elements (in grey) have the same width, and b) Type II, where all the elements share the same area.

![Figure 7.](image7.png)

**Figure 7.** Predicted 2D distribution of acoustic pressure amplitude (in dB scale) produced in air by an 8-element annular array of type I (same width but different areas), focusing at distances: a) $z=100\text{mm}$, b) $z=150\text{mm}$, c) $z=200\text{mm}$. 

The pressure fields radiated by these two types of array are calculated. The exciting signal frequency is equal to 250 kHz. The fields radiated by these source (either array of type I or II) are deduced from the fields radiated by their respective central circular element and surrounding annular elements, and phase delays are applied before summing up these various fields together in order to investigate the dynamic focusing capabilities. Predicted distributions, in the \((r, z)\) plane, of the acoustic pressure amplitude radiated by the first array (Type I) are shown in Figure 7. Three focal distances are considered: \(z=100\) mm (Figure 7.a), \(z=150\) mm (Figure 7.b), and \(z=200\) mm (Figure 7.c). The pressure amplitude distributions calculated in the same conditions for the case of a Type II array are plotted in Figure 8. As expected, for both types of multi element arrays, a focal spot appears at the desired location in the air (Figure 7 and Figure 8). The two configurations of annular array design provide similar pressure amplitude at each focal distance, even if the array of type II provides slightly higher amplitude than that produced by the array of type I, at the focusing points. As the field is focused at a large distance, the pressure amplitude decreases and the lateral dimension of the focal spot increases, deteriorating the transverse spatial resolution for further use of the transducer as a materials scanner. Acoustic pressure amplitude distributions in transverse direction are plotted in Figure 9, for the three previous focusing distances: \(z=100\) mm, \(z=150\) mm and \(z=200\) mm. For the nearest focal distance (\(z=100\) mm), grating lobes are visible for the type I array, at \(r = \pm 35\) mm (Figure 9.a). The width of these grating lobes is about 10 mm. Although their amplitude is about 22 dB less than that of the focal spot amplitude, such grating lobes may induce artefacts if involved in NDT measurements. Conversely, for the array of type II, these grating lobes have significantly reduced amplitudes (Figure 9.b), thus making this type of array more suitable for NDT applications.

**Figure 8.** Predicted 2D distribution of acoustic pressure amplitude (in dB scale) produced in air by an 8-element annular array of type II (the same area but different widths) focusing at distances: a) \(z=100\) mm, b) \(z=150\) mm, c) \(z=200\) mm.
5. Summary

This paper introduced a multi-element capacitive annular array designed for non-destructive testing (NDT) purposes. An analytical model was built to simulate the acoustic field generated in air by circular or annular, individual elements or by an array made of one central circular element and several surrounding annular elements. The model has been validated through the comparison of several predicted distributions of pressure with measurements carried out using a Laser Doppler velocimeter. Then, using the model, two types of 8-element annular arrays have been numerically investigated and their performances compared together. The first one, was made of one central circular element plus seven annular elements all having the same width (and so different areas); the second one was also made up of one central and seven annular elements but these were designed so that they all have the same area. Dynamic focusing has been numerically shown possible for both arrays, but the array made of elements having all the same area was shown to have no (or very little) grating lobes. This geometrical configuration is therefore more suitable for further NDT applications. Future work will focus on the fabrication of the transducer arrays, and the optimized choice of the filling material between the elements, in order to minimize cross-talking effects.

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