A Fast Calculation Method for Bus Body Based on Equivalent Stiffness Element Method

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Abstract. In view of complex modeling and inefficient calculation at existing stage finite element analysis method in bus body skeleton analysis. This paper presents a method on simulating complex large section beams by equivalent stiffness element. Equivalent stiffness element model is analyzed theoretically. Experiments are carried out to verify that the equivalent stiffness element method can be used in rapid analysis and calculation for girder, body frame and body optimization. It not only reduces modeling complexity, but also improves calculation efficiency. To a certain extent, it can provide a better analysis method for large-scale vehicle modeling and simulation.

Keywords: Equivalent stiffness; Fast calculation; Aluminium alloy bus; Finite element analysis.

1. Introduction
The finite element method has a wide application in analysis of bus body skeleton [1-3]. For frame type bus body, the body skeleton modeling usually adopts shell model, beam-beam hybrid model and beam model [4-7]. The plate and shell models have good precision, but the modeling is complex and the calculation efficiency is low. It is suitable for verification of design scheme and difficult to be used for forward development of vehicle body [8-11]. In skeletal structure of all-aluminum bus body skeleton, there are complex large-section beams, which are overlapped with multi-layered beam [12]. If traditional beam element modeling method is used to model complex girders, only a beam axis exists, and it may lose intersection of such and other beams. Rigid elements were used to connect those beams which lost intersections, but it changed the bus body skeleton [13].

In order to make finite element beam frame model consistent with actual body frame and easy to modify and solve quickly, this paper proposes a method about simulating complex large section beam with equivalent stiffness. Based on it, rapid design calculation and optimization methods for body frame are established. To some extent, the finite element model for aluminum alloy beam element structure is further constructed to realize rapid, efficient and accurate modeling calculation and optimization for vehicle structure.

2. Material and Method
In researched and simulation, the material is 6061-T6 aluminum alloy. Its elastic modulus is 69GPa. Its shear modulus is 14.6GPa. Its yield strength is 205MPa. Its tensile strength is 309MPa.
The sub-section should satisfy the following conditions. Firstly, each sub-section is a closed section. Secondly, the sum of the shape and area for each sub-section is the same as that of the original complex large section. Thirdly, each sub-section beam has a separate beam axis, which ensures intersection with other beams.

It is assumed that the section and the joint division of combined structural unit conform to geometric section equivalent principle. For original beam element, the equivalent load is applied to joint structural unit.

The coordinate system for equivalent stiffness element model is established as shown in Figure 1. From Figure 1, the original girder unit $OO'$ is numbered 0. The length on girder unit is $L_i (i = 0 \sim 6)$. The local coordinate system for girder is $\vec{x}, \vec{y}$. The direction of girder length is $\vec{x}$. Under external force $F_{x_o}$, $F_{y_o}$, $F_{x_o'}$, $F_{y_o'}$, and bending moment $M_O$, $M_O'$, displacement $u_O$, $v_O$, $u_O'$, $v_O'$ and rotation $\theta_O$, $\theta_O'$ occur at the girder joints.

The original girder is split into three sub-section beams ①, ② and ③, which are connected by end beams ④, ⑤, ⑥ and ⑦, respectively. The numbering for each element and joint is shown in Figure 1(b). Among them, the length for original beam element $OO'$ is $L_o$, both the distances between joint 3 and the original beam axis end point $O$, and the distance between joint 4 and the original beam axis end point $O'$ are $L_e$.

![Figure 1. Deformation of original beam element and equivalent stiffness element under external force.](image)

According to plane assumption for beam bending and torsion deformation, joint 1, joint 3, joint 5 and joint 2, joint 4 and joint 6 are still on same plane after bending and torsion load. That is to say, in order to satisfy plane assumption of material mechanics, the beams for equivalent stiffness element end surface should be rigid beams.

The element number is $e_i (i = 0 \sim 3)$. The element number on the original cross-section beam is $e_0$. The joint number is $m, n (m, n = 1 \sim 6)$. When $i = 0 \sim 3$, the element stiffness matrix is formed because local coordinate system on original beam and each cross-section beam coincides with the global coordinate system.

$$K_i = \begin{bmatrix}
\frac{EA}{L_i} & 0 & 0 \\
0 & \frac{12EI}{L_i^3} & \frac{6EI}{L_i} \\
0 & \frac{6EI}{L_i} & \frac{4EI}{L_i} \\
0 & \frac{-12EI}{L_i^3} & \frac{-6EI}{L_i} \\
0 & \frac{-6EI}{L_i} & \frac{-2EI}{L_i} \\
 \end{bmatrix}$$

$$j = 0, 1, 2, 3$$

$$K_i' = \begin{bmatrix}
\frac{EA}{L_i} & 0 & 0 \\
0 & \frac{12EI}{L_i^3} & \frac{6EI}{L_i} \\
0 & \frac{6EI}{L_i} & \frac{4EI}{L_i} \\
0 & \frac{-12EI}{L_i^3} & \frac{-6EI}{L_i} \\
0 & \frac{-6EI}{L_i} & \frac{-2EI}{L_i} \\
 \end{bmatrix}$$

$$j = 0, 1, 2, 3$$

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Where $K^{ei}$ is element stiffness matrix in local coordinate system, $A_i$ is section area of beam, $L_i$ is length of beam element, $E$ is elastic material modulus, and $EI_i$ is flexural stiffness of cross-section beam. According to equivalence principle, the following conditions are satisfied between area of each sub-section and flexural rigidity.

$$
\begin{align*}
A_0 &= A_1 + A_2 + A_3 \\
EI_0 &= EI_1 + EI_2 + EI_3
\end{align*}
$$

The element stiffness matrix is written into block form $K^{ei} = \begin{bmatrix} K^{e_{11}} & K^{e_{12}} \\
K^{e_{21}} & K^{e_{22}}\end{bmatrix}$. For $m$-end of beam element $e_i$, block $K^{e_{mm}}$ is near-end stiffness of beam. $K^{e_{mn}}$ is far-end stiffness. For $n$-end of beam element, $K^{e_{mn}}$ is near-end stiffness and $K^{e_{nn}}$ is far-end stiffness. In order to assemble global stiffness, the local numbers $m$ and $n$ of element joints are replaced by corresponding global numbers. The subscripts of sub-matrices in element stiffness matrix are also replaced by total numbers. From this, the stiffness matrices for each element are converted into total numbers.

$$
K^{ei} = \begin{bmatrix} K^{e_{11}} & K^{e_{12}} \\
K^{e_{21}} & K^{e_{22}}\end{bmatrix}, \quad K^{ei} = \begin{bmatrix} K^{e_{33}} & K^{e_{34}} \\
K^{e_{43}} & K^{e_{44}}\end{bmatrix}, \quad K^{ei} = \begin{bmatrix} K^{e_{55}} & K^{e_{56}} \\
K^{e_{65}} & K^{e_{66}}\end{bmatrix}
$$

According to plane assumption on beam bending and torsion deformation, the joint 1, joint 3, joint 5 and joint 2, joint 4, joint 6 are still in the same plane after bending and torsion loading, which means the beams on the end face of equivalent stiffness element are rigid beams. From each beam element stiffness matrix, according to assemble principle on whole stiffness matrix and considering the influence of rigid body, the overall stiffness matrix on equivalent stiffness composite structural element is obtained as follows.

$$
K = \sum K^{ei} = \begin{bmatrix} K^{e_{11}} & K^{e_{12}} & 0 & K^{e_{13}} & 0 & K^{e_{14}} \\
K^{e_{21}} & K^{e_{22}} & 0 & K^{e_{23}} & 0 & K^{e_{24}} \\
K^{e_{31}} & 0 & K^{e_{33}} & K^{e_{34}} & K^{e_{35}} & 0 \\
0 & K^{e_{42}} & K^{e_{43}} & K^{e_{44}} & 0 & K^{e_{46}} \\
K^{e_{51}} & 0 & K^{e_{53}} & 0 & K^{e_{55}} & K^{e_{56}} \\
0 & K^{e_{62}} & 0 & K^{e_{64}} & K^{e_{65}} & K^{e_{66}}\end{bmatrix}, i = 0, 1, 2, 3
$$

Since the deformation for each joint is $\delta_i (i = 1 \sim 6)$, there are three components of displacement $u_i$, $v_i$, on $x$ and $y$ axes, and angle $\theta_i$ around $z$ axis, and there are six joints in the frame.

3. Results and Discussion

3.1. Bending Conditions

The maximum displacement in $z$ direction appears in the middle body under bending condition. The maximum sink is 4.25mm, which is shown in Figure 2(a). As shown in Figure 2(b), the maximum equivalent stress is 41.20 MPa.

![Figure 2. Analysis on bending condition.](image)
3.2. Modality
Figure 3 shows the results of the first five orders of modal analysis and mode shape. The first 5 natural frequencies are 7.57 Hz, 13.92 Hz, 16.61 Hz, 21.22 Hz and 25.27 Hz, respectively. The first mode shows largest displacement occurs in the middle of the roof. The second mode shows largest displacement occurs in the tail of the roof. The third mode shows largest displacement occurs in the middle of the roof. The fourth mode shows largest displacement occurs in the middle of the roof. The fifth mode shows largest displacement occurs in the middle of the roof.

![Figure 3](image)

Figure 3. Analysis results on the first five modes.

3.3. Body Skeleton Optimization Design
The analysis results for body skeleton optimization are shown in Table 1. It can be seen that after equivalent beam replacement, the maximum equivalent stress of body frame under each working condition is less than experimental material yield stress of 205 MPa, the skeleton is safe. After the equivalent beam replacement, the weight loss is very significant, which reaches 10.19%. It can be seen that after applying equivalent stiffness calculation model of complex section beam, the beam element can be applied to frame finite element modeling, especially for frame with complex section beam.

|                | Before optimization | Optimized | Rate of change |
|----------------|---------------------|-----------|----------------|
| Bending condition | Maximum displacement in the middle/mm 4.25 | 5.08 | 19.53% |
|                 | Maximum equivalent stress/MPa 41.20 | 46.17 | 12.06% |
| Left torsion condition | Front maximum displacement/mm 20.29 | 21.63 | 6.60% |
|                 | Maximum equivalent stress/MPa 85.99 | 91.23 | 6.09% |
| Right torsion condition | Front maximum displacement/mm 18.15 | 19.29 | 6.28% |
|                 | Maximum equivalent stress/MPa 76.44 | 81.16 | 6.17% |
| Body skeleton quality/kg | 682 | 612.51 | -10.19% |

4. Conclusion
In this paper, an equivalent stiffness element method is proposed to simulate complex large cross-section beam elements in skeleton structure. The equivalent stiffness for composite structure to original complex beam element is realized. The method is applied to rapid analysis and calculation for girder, the whole vehicle body frame and body optimization. Then, the fast simulation results are validated by experiments.
Finally, an example of body optimization is analyzed. By simply replacing beam section, the weight of body frame is reduced by 10.19%. The method on equivalent stiffness element simulating complex large section in skeleton structure has obvious effect in bus body optimization. In our future work, we will focus on actual testing for bus body.

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