6-D model with scalar field condensation at brane and zero 4-D Cosmological Constant

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Abstract

In a previous publication we have shown that the gauge theory of relativistic 3-Branes can be formulated in a conformally invariant way if the embedding space is six-dimensional. The implementation of conformal invariance requires the use of a modified measure, independent of the metric in the action. We here generalize the theory to include conformal invariance breaking and a dynamical scalar field with a non-trivial potential. The non conformal invariance contribution can be interpreted as originating from a continuous "non ideal brane fluid" that exists between two singular branes. The scalar field potential also breaks the conformal invariance. At singular brane locations, conformal invariance is restored and the dynamics of the scalar field is frozen at a certain fixed value of the scalar field which depends on an arbitrary integration constant. Spontaneous Symmetry breaking can take place due to such boundary condition without the need of invoking tachyonic mass terms for the scalar field. In these Brane-world scenarios, zero 4-D cosmological constant is achieved without the need of invoking a fine tuned cosmological constant in 6D. Thus, no “old” cosmological constant problem appears. The use of a measure independent of the metric is crucial for obtaining all of the above results.
I Introduction

In recent years a great deal of work has been done on the notion that extended objects could play important roles in particle physics and cosmology. In the context of string theory for example, among the various kind of branes a unique role is played by $D$–branes [1] as they can trap the end-points of open strings. $D$–branes fit quite nicely with the idea, being studied since the 80’s, that our universe contains one or more branes embedded in some higher dimensional space. These “brane–universe” models, are currently under investigation as there is the hope that they will be of use in the solution of longstanding hierarchy problems in gauge theories.

The gauge theory formulation of $p$–branes, proposed some years ago as an alternative to the standard description of relativistic extended objects [2],[3], is well suited to describe this new type of cosmological scenario. Furthermore, the description of $p$–branes in terms of associated gauge potentials offers a vantage point to study some specific problem as the one concerning the fine tuning of the cosmological constant.

We have shown [4] that for 3-branes considered in an embedding 6D space the gauge theory formulation of 3-branes allows a conformally invariant realization. An essential element necessary to implement conformal invariance is the introduction of a measure of integration in the action which is independent of the metric [5],[6],[7],[8]. We use then such a formulation to construct a new type of brane world scenario.

Brane world scenarios in general are concerned with the possibility that our universe is built out of one or more 3-branes living in some higher dimensional space, plus some bulk component, [9],[10],[11],[12], [13]. In particular, the possibility of 3-branes embedded in 6D space has been studied in [14], [15],[16],[17],[18]. In this case the effect of the tension of the branes is to induce curvature only in the extra dimensions. In these models there is still a question of fine tuning that has to be addressed, since although the branes themselves do not curve the observed four dimensions, the bulk components of matter do, and they have to be fine tuned in order to get (almost)zero four dimensional vacuum energy. This very special feature of 3-branes is a 6D embedding spacetime is related to the fact that such matter content, even coupled to gravity, has a conformal invariance associated to it.

In our previous publication [4], in order to solve this problem, we incorporated the “brane-like features” that are quite good in what concerns the cosmological constant problem into the “bulk” part of the brane scenario as well. In this way both bulk and singular brane contributions shared the fundamental feature of curving only the extra dimensions.
Indeed, in Ref.[4], we saw that the GFF3B6D allows us to understand, extend and give a “pure brane interpretation” of the results of [19], where a “square root gauge theory”, coupled to 3-branes in 6D was considered. This model has conformal invariance and there is no need to introduce a 6D cosmological constant. The “fundamental physics” behind the model was not so clear however and its different matter elements: gauge fields, 3-branes appear rather disconnected from each other. Now with the bulk being an ‘ideal fluid of branes, the bulk and singular brane appear in a unified framework. We showed in [19] that GFF3B6D allowed us also to interpret a more general set of brane-world solutions, where the solutions presented in [19] appear as very particular cases.

The objective of this paper is to show how the above picture survives, even in the presence of conformal breaking terms. This is of course very important since our physical universe does not satisfy the requirements of conformal invariance, since the existence of massive particles is evidence against such a symmetry. However, even departing from the conformally invariant version of the GFF3B6D, we can maintain the basic feature that the four dimensional part of the manifold does not get curved, provided the use of the measure of integration independent of the metric is maintained.

Other aspects or ‘principles’ can be relaxed. In the first place we allow conformal invariance breaking which we introduce, to demonstrate the basic mechanisms that the theory provides (other generalizations are likely to produce similar results), in two different ways: First in the GFF3B6D, we allow for ‘non ideal’ or ‘interacting’ behavior in the fluid of 3 branes (between the singular branes) and second, we introduce a scalar field, with a non trivial potential which will break also the conformal invariance.

As we will see, singular branes, as opposed to a continuous distribution of 3-branes, is necessarily conformal invariant. Beyond the good behavior of the theory in relation to the cosmological constant problem, another, rather amazing effect appears here: it turns out that when considering a braneworld where say two singular branes are considered and the space in between is filled with ‘non ideal’ or interacting fluid of branes plus a scalar field, one finds then that at the brane itself, the scalar field is frozen at a particular fixed value, determined by an arbitrary integration constant. This brane is indeed a D brane, not just speaking from the possible underlying string theory, but also from the point of view of the scalar field expectation value, which gets fixed at a certain constant value in the singular branes.

This effect makes it possible for the scalar field to break spontaneously the symmetry, without having to rely on some tachyonic mass.

The paper is organized as follows. In Sect.II, we review the GFFB6D and and display its conformal invariant formulation when a modified measure is introduced, also the dual picture to this formulation is introduced; in Sect.III, the equations of motion in this dual picture are studied. In section IV we study the generalized GFFB6D, which includes conformal symmetry breaking introduced through a ‘non ideal’ brane fluid behavior in a space between singular branes and a scalar field with a non trivial potential and how such formulation introduces Dirichlet boundary conditions for the scalar field in the singular brane, while the four dimensional space still does not get curved. We end up with a brief discussion and conclusions.
II Conformally Invariant Realization in 6D

In a previous publication \[4\] we have shown that a fluid of 3-branes interacting with gravity can be formulated in a conformally invariant fashion provided the embedding space is 6-D. The relevant action is (for a full treatment see ref. \[4\])

\[
S = - \frac{1}{16 \pi G^{(5+1)}} \int d^{5+1} x \, \Phi \, g^{AB} \, R_{AB} \, (\Gamma) + e^2 \int d^{5+1} x \, \sqrt{-g} \int \frac{1}{2 \times 4!} g_{AE} \ldots g_{DH} \, W^{ABCD} \, W^{EFGH} \\
- \frac{1}{4!} \int d^{5+1} x \, \Phi \, \sqrt{-g^{(5+1)}} \, W^{EFGH} \, \partial_{[E} \, B_{FGH]} \\
\Phi \equiv \epsilon^{A_1 \ldots A_6} \epsilon_{a_1 \ldots a_6} \partial_{A_1} \phi^{a_1} \ldots \partial_{A_6} \phi^{a_6}
\]

(1)

where, \(\phi^{a_1}, \ldots, \phi^{a_6}\) are six scalar fields treated as independent degrees of freedom and we consider the gravitational action in the first order formulation, i.e. \(g^{AB}\) and \(\Gamma_{DE}^C\) are treated as independent variables. The connection \(\Gamma_{DE}^C\) is torsion-free, i.e. \(\Gamma_{DE}^C = 3D \Gamma_{ED}^C\). Thus, \(\partial_{[E} \, B_{FGH]} \equiv \nabla_{[E} \, B_{FGH]}\) where \(\nabla_M\) is the covariant derivative.

In Eq.1 \(R_{AB} \equiv R_{ABC}^A\) and \(R_{BCD}^A = \Gamma_{B,C,D}^A - \Gamma_{B,D,C}^A + \Gamma_{K,D}^A \Gamma_{B,C}^K - \Gamma_{K,C}^A \Gamma_{B,D}^K\).

\(\Phi d^{5+1} x\) is a scalar as well as \(\sqrt{-g^{(5+1)}} \, d^{5+1} x\) under x-coordinates transformation, while under scalar fields re-definitions:

\[
\phi^{a_j} \longrightarrow \phi'^{b_k} (\phi^{a_j}) \\
\Phi \longrightarrow \Phi' = J \Phi, J \equiv \det \left( \frac{\partial \phi'^{a_j}}{\partial \phi^{b_k}} \right)
\]

(3)

\(W^{ABCD} = 3\)-brane slope field, it assigns a tangent (hyper)plane to each spacetime point; the \(W\) field is totally anti-symmetric in the four indices. This field can describe a fluid of 3-branes \[2\],\[3\].

\(B_{FGH}\) 3-brane gauge potential; the \(B\) field is totally anti-symmetric in the three indices. In the last term the invariant integration measure is written in terms of \(g^{(5+1)}\), instead of \(\Phi\) to make the action invariant under (3). One must in this case assume the following Weyl rescalings also

\[
g_{A_1 A_2} \longrightarrow J \, g_{A_1 A_2} \\
g_{B_1 B_2} \longrightarrow J^{-1} \, g_{B_1 B_2} \\
g^{(5+1)} \longrightarrow J^0 \, g^{(5+1)} \\
W^{ABCD} \longrightarrow J^{-3} \, W^{ABCD} \\
B_{FGH} \longrightarrow B_{FGH}, \quad \Gamma_{BC} \longrightarrow \Gamma_{BC}^A
\]

(4)

(5)

(6)

(7)

(8)

(9)
Notice that this symmetry holds only in the case the embedding space in 6D. Let us remark that if we define $W$ as a “contravariant” object (upper indices) and $B$ as a covariant field (lower indices), then the last term in the action $S$ depends on the metric only through $\sqrt{-g^{(5+1)}}$.

We can define the Dual Representation of the theory by changing variables

$$W^{ABCD} = \frac{1}{2} \epsilon^{ABCDEF} \omega_{EF}$$  \hspace{1cm} (10)

$$S = -\frac{1}{16\pi G_{(5+1)}} \int d^{5+1}x \Phi R_{(5+1)} + e^2 \int d^{5+1}x \Phi \sqrt{\frac{1}{4} g^{AE} g^{DH} \omega_{AD} \omega_{EH}}$$

$$-\frac{1}{6!} \int d^{5+1}x \epsilon^{ABCD} \omega_{[AB} \partial_C B_{DEF]}$$ \hspace{1cm} (11)

III Field Equations

We will work out the equations of motion in the dual picture first and afterwards we will review the brane interpretation of these solutions.

To start let us notice the following facts concerning the action (11). First it can be written in the form

$$S = \int d^{5+1}x \Phi (L_G + L_m) - \frac{1}{6!} \int d^{5+1}x \epsilon^{ABCD} \omega_{[AB} \partial_C B_{DEF]}$$ \hspace{1cm} (12)

where

$$L_G = -\frac{1}{16\pi G_{(5+1)}} g^{AB} R_{AB} (\Gamma)$$ \hspace{1cm} (13)

$$L_m = e^2 \sqrt{\frac{1}{4} \omega_{AB} \omega_{CD} g^{AC} g^{BD}}$$ \hspace{1cm} (14)

are homogeneous of degree one in $g^{AC}$, that is
\[ g^{AB} \frac{\partial L_m}{\partial g^{AB}} = L_m, \quad g^{AB} \frac{\partial L_G}{\partial g^{AB}} = L_G \] (15)

this property is intimately related to the fact that the action (11) has the symmetry under \( g^{AB} \rightarrow J^{-1} g^{AB}, \Phi \rightarrow J \Phi \).

The equations of motion which result from the variation of the fields \( \phi^a \) are

\[ \mathbf{A}^M_a \partial_M \left( L_G + L_m \right) = 0 \] (16)

where

\[ \mathbf{A}^M_m \equiv \epsilon^{MBCDEF} \epsilon_{mbcdef} \partial_B \phi^b \partial_C \phi^c \partial_D \phi^d \partial_E \phi^e \partial_F \phi^f \] (17)

Since \( \det(\mathbf{A}^M_m) = 6^{-6} \Phi^6/6! \) Then we have that if \( \Phi \neq 0 \), this means that (16) implies

\[ L_G + L_m = M = \text{const.} \] (18)

The equation of motion obtained from the variation of \( g^{AB} \) is

\[ -\frac{1}{16\pi G_{(5+1)}} R_{AB} + \frac{\partial L_m}{\partial g^{AB}} = 0 \] (19)

by contracting (19) with respect to \( g^{AB} \) and using the homogeneity property of \( L_m \), we obtain that the constant of integration \( M \) equals zero. Evaluating (19) we find

\[ R_{AB} = 4\pi e^2 G_{(5+1)} \frac{\omega_{AC} \omega_B^C}{\sqrt{\frac{1}{4} \omega_{MN} \omega^{MN}}} \] (20)

Eq.(20) is also consistent with the Einstein form

\[ R_{AB} - \frac{1}{2} g_{AB} R = -8\pi G_{(5+1)} T_{AB} \] (21)

\[ T_{AB} = -2 \frac{\partial L_m}{\partial g^{AB}} + g_{AB} L_m \] (22)

which for \( L_m \) is given by

\[ T_{AB} = \frac{e^2}{2} \frac{\omega_{AC} \omega_B^C}{\sqrt{\frac{1}{4} \omega_{MN} \omega^{MN}}} - e^2 g_{AB} \sqrt{\frac{1}{4} \omega_{MN} \omega^{MN}} \] (23)
as one can easily check that solving from $R$ by contracting both sides of (21) with $T_{AB}$ given by (23) and then replacing $R$ into (21) gives (19).

Let us consider now the equation of motion for the connection coefficients $\Gamma^A_{BC}$. Defining

$$\bar{g}_{AB} = \left( \frac{\Phi}{\sqrt{-g(5+1)}} \right)^{1/2} g_{AB}$$

one can verify that

$$\Phi g^{AB} = \sqrt{-g(5+1)} \bar{g}^{AB}$$

Therefore, the equation of motion for $\Gamma^A_{BC}$ is obtained by the condition that the functional

$$I \equiv -\frac{1}{16\pi \pi G(5+1)} \int d^{5+1}x \sqrt{-\tilde{g}(5+1)} \bar{g}^{AB} R_{AB} (\Gamma)$$

is extremized under variation of $\Gamma^A_{BC}$. This is however the well known Palatini problem in General Relativity (but where the metric $\bar{g}_{AB}$ enters, not the original metric $g_{AB}$). Therefore $\Gamma^A_{BC}$ is the well known Christoffel symbol, but not of the metric $g_{AB}$ rather than the metric $\bar{g}_{AB}$:

$$\Gamma^A_{BC} = \left\{ b^{A} C \right\} | \bar{g}$$

(27)

Notice the interesting fact that $\bar{g}_{AB}$ is conformally invariant, i.e. invariant under the set of transformations (3), (4), (5).

Also, in the gauge $\Phi = \sqrt{-g(5+1)}$, the metric $g_{AB}$ equals the metric $\bar{g}_{AB}$ so one may call this the “Einstein gauge”, since here all non-Riemannian contributions to the connection disappear. Alternatively, without need of choosing a gauge one may choose to work with the conformally invariant metric $\bar{g}_{AB}$ in terms of which the connection equals the Christoffel symbol and all non-Riemannian structures disappear. Finally the equations of motion obtained from the variation of the gauge fields $\omega_{AB}$ and $B_{MNP}$ are

$$\Phi \left( \frac{\omega^{AB}}{\sqrt{\frac{1}{4} \omega_{MN} \omega^{MN}}} \right) = \frac{1}{6!} \epsilon^{ABCDEF} \bar{g}_{[C} B_{DEF]}$$

(28)

and

$$\epsilon^{ABCDEF} \partial_{[D} \omega_{EF]} = 0$$

(29)

taking the divergence of (28) we obtain

$$\partial_{A} \left( \Phi \left( \frac{\omega^{AB}}{\sqrt{\frac{1}{4} \omega_{MN} \omega^{MN}}} \right) \right) = 0$$

(30)
IV. Brane-world solutions in the Dual Picture

In this section we are going to consider the product spacetime

\[ ds^2 = g_{\mu\nu}(x_{||}) \, dx_{\mu} \, dx_{\nu} + \gamma_{ij}(\vec{x}_\perp) \, dx^i \, dx^j \]  

(31)

where \( \mu, \nu = 0, 1, 2, 3 \) and \( i, j = 4, 5 \). Furthermore, we consider a slope field \( W^{ABCD} \) with non-vanishing components only in the first four coordinates \( (0, 1, 2, 3) \), which means we are dealing with a set of parallel branes orthogonal to the extra-dimensions, if we use the brane interpretation requested of the field refs. [2],[3]. This means that the dual field \( \omega_{AB} \) has non-zero components in the \( 4, 5 \) directions only. In this case, we see from eq.(19) that the Ricci curvature induced in the four dimension \( 0, 1, 2, 3 \), is zero:

\[ R_{\mu\nu} = 0 \]  

(32)

Thus, the ordinary four dimensions (accessible to our experience) are not curved by this kind of matter. This is a very important remark, since there is no need to introduce a bare cosmological constant to cancel some contribution from the gauge field, no type of fine tuning, most usual in extra dimensional theories, is needed here. The simplest solution of (32) is flat, four dimensional spacetime

\[ g_{\mu\nu} = \eta_{\mu\nu} \]  

(33)

Let us analyze now the additional field equations. It is convenient to choose gauge \( \Phi = \sqrt{-g_{(5+1)}} \), even if the conformally invariant metric \( \bar{g}_{AB} \) gives the same results. The two-dimensional metric \( \gamma_{ij} \) can always be put in a conformally flat form, i.e. one can always choose a coordinate system where

\[ \gamma_{ij} \, dx_i \, dx_j = \psi(x^4, x^5) \left[ (dx^4)^2 + (dx^5)^2 \right] \]  

(34)

As far as the dual slope field is concerned, its most general form along the extra dimension where it is non-zero, is dictated by its tensorial structure in two-dimensions, which is

\[ \omega^{ij} = \frac{\epsilon^{ij}}{\sqrt{\gamma}} \rho(x^4, x^5) , \quad \gamma \equiv \det(\gamma_{ij}) \]  

(35)

It turns out that the field equations do not determine the function \( \rho \) as

\[ \partial_i \left( \frac{\omega^{ij} \sqrt{\gamma}}{\sqrt{-\frac{1}{2} \omega^{kl} \omega_{kl}}} \right) = 0 \rightarrow \partial_i \epsilon^{ij} = 0 \]  

(36)
which is “trivially” satisfied \( \epsilon^{ij} \) being the totally anti-symmetric symbol in two-dimensions.

The function \( \rho (x^4, x^5) \) acts, however, as a source that determines the metric. The physical source of the arbitrariness in \( \rho \) can be understood by invoking the brane interpretation of the \( \omega \)-field. The function \( \rho \) is associated to the density of 3-branes being piled in the extra dimensions ref [4]. Since these branes do not exert any force one upon each other they can be accumulated with an arbitrary density at each extra-dimensional point. Recalling that the scalar curvature of (34) is \( R = -\psi^{-1}\nabla^2 \psi \), we have from \( R = 16\pi G_{(5+1)} L_m \):

\[
-\frac{1}{\psi} \nabla^2 \psi = 16\pi G_{(5+1)} \rho \tag{37}
\]

\( \rho \) is free to be taken any possible values, but once it is assigned \( \psi \) is determined by (37). The argument can be also reversed: for any \( \psi (37) \) gives the corresponding \( \rho \). An interesting case is obtained when rho consists of a constant part plus one or more delta function parts. Since \( R \) is a scalar a delta function part can appear only in combination \( \delta^{(2)}/\sqrt{\gamma} \). Let us define:

\[
\begin{align*}
 r &= \sqrt{(x^4)^2 + (x^5)^2} \tag{38} \\
 x^4 &= r \sin \phi \tag{39} \\
 x^5 &= r \cos \phi \tag{40}
\end{align*}
\]

which describe the metric close to \( r = 0 \), and take \( \psi = \psi(r) \), so

\[
\gamma_{ij} dx^i_\perp dx^j_\perp = \psi(r) \left( dr^2 + r^2 d\phi^2 \right) \tag{41}
\]

Then, using the representation of the delta-function (with integration measure \( r d\phi dr \))

\[
\delta^{(2)}(r) = \frac{1}{2\pi} \nabla^2 \ln r \tag{42}
\]

where \( \nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr} \). Then, for

\[
\rho = \sqrt{2} B_0 + T \frac{\delta^{(2)}(r)}{\psi} \tag{43}
\]
where \( B_0 \) and \( T \) are constants. By inserting (43) into (37) we obtain (similar equation was obtained in ref [20] in the context of 2 + 1 gravity)

\[
\psi = \frac{4\alpha^2 b^2}{r^2} \left[ \left( \frac{r}{r_0} \right)^\alpha + \left( \frac{r}{r_0} \right)^{-\alpha} \right]^{-2}
\]

(44)

where

\[
\alpha \equiv 1 - 4G_{(5+1)}T
\]

(45)

\[
b^2 \equiv \frac{\sqrt{2}}{16\pi G_{(5+1)}B_0}
\]

(46)

Such a metric can be transformed into the form

\[
\gamma_{ij} \, dx^i_\perp \, dx^j_\perp = b^2 \left( d\theta^2 + \alpha^2 \sin^2 \theta \, d\phi^2 \right)
\]

(47)

where \( \phi \) ranges from 0 to \( 2\pi \), or, equivalently,

\[
\gamma_{ij} \, dx^i_\perp \, dx^j_\perp = b^2 \left( d\theta^2 + \sin^2 \theta \, d\bar{\phi}^2 \right)
\]

(48)

where \( \bar{\phi} \) ranges from 0 to \( 2\alpha \pi < 2\pi \). A complete solution must contain two branes (in the coordinate system (38),(39),(40) we are able to display only one pole of the sphere, the other one is at the other pole of the sphere, where in \( (r,\phi) \) coordinates is at \( r \to \infty \)). Here the term “branes” means delta-functions contributions to \( \rho \).

Of course, this solution is one out of a continuum of solutions, but is interesting because it allows us to connect to other works on the subject ref. [17] where similar effects are discussed.

Nevertheless, we stress the fact that the function \( \rho \) is totally free in this conformally invariant model. The situation can change once conformal breaking contributions are allowed. This will be the subject of the next section.
V. The Introduction of Conformal Symmetry Breaking

We consider now a generalization of (1),(2),

\[ S = \int d^{5+1}x \Phi \left( -\frac{1}{16\pi G_{(5+1)}} g^{AB} R_{AB} (\Gamma) + \frac{1}{2} g^{AB} \partial_A \alpha \partial_B \alpha - V(\alpha) \right) \]

\[ + \int d^{5+1}x \Phi F \left( \sqrt{-\frac{1}{2 \times 4!} g_{AE} \ldots g_{DH} W^{ABCD} W^{EFGH}} \right) \]

\[ - \frac{1}{4!} \int d^{5+1}x \sqrt{-g(5+1)} W^{EFGH} \partial_{[E} B_{FGH]} \]

\[ \Phi \equiv \epsilon^{A_1 \ldots A_6} \epsilon_{a_1 \ldots a_6} \partial_{A_1} \phi^{a_1} \ldots \partial_{A_6} \phi^{a_6} \]

We have now introduced a new (in principle) degree of freedom, the scalar field \( \alpha \). In the above expression, conformal symmetry is broken in two different ways: i) by the potential \( V \) of the scalar field \( \omega \) and ii) by the introduction of a function \( F \), which gives rise to conformal symmetry breaking in the case this function is a non linear function of its argument.

Once again, going to the dual picture

\[ W^{ABCD} = \frac{1}{2} \frac{\epsilon^{ABCDEF}}{\sqrt{-g(5+1)}} \omega_{EF} \]

We obtain now,

\[ S = \int d^{5+1}x \Phi \left( -\frac{1}{16\pi G_{(5+1)}} R_{(5+1)} + \frac{1}{2} g^{AB} \partial_A \alpha \partial_B \alpha - V(\alpha) \right) \]

\[ + \int d^{5+1}x \Phi F \left( \sqrt{-g} \frac{1}{4} g^{AE} g^{DH} \omega_{AD}, \omega_{EH} \right) - \frac{1}{6!} \int d^{5+1}x \epsilon^{ABCDEF} \omega_{[A} \partial_{C} B_{DEF]} \]

VI. Curvature and a Constraint Equation in the case of Conformal Symmetry Breaking

The equations of motion which result from the variation of the fields \( \phi^a \) are

\[ A^M_a \partial_M (L_G + L_m) = 0 \]

where

\[ A^M_m \equiv \epsilon^{MBCDEF} \epsilon_{mbcdef} \partial_B \phi^h \partial_C \phi^e \partial_D \phi^d \partial_E \phi^c \partial_F \phi^f \]

(54)
Since \( \det \left( A_m^M \right) = 6^{-6} \Phi^6 / 6! \). Then we have that if \( \Phi \neq 0 \), this means that (53) implies

\[
L_G + L_m = M = \text{const.} \quad (55)
\]

As opposed to the conformally invariant case, the constant \( M \) will not be determined to be zero, but can in principle remain undetermined. In fact, it could play a very important role in the spontaneous symmetry breaking of internal symmetries.

The equation of motion obtained from the variation of \( g^{AB} \) gives us the curvature equation:

\[
- \frac{1}{16\pi G_{(5+1)}} R_{AB} + \frac{\partial L_m}{\partial g^{AB}} = 0 \quad (56)
\]

by contracting (56) with respect to \( g^{AB} \), using also eq. (55), we get that

\[
g^{AB} \frac{\partial L_m}{\partial g^{AB}} = L_m - M \quad (57)
\]

It is very important to notice that since \( L_m \) is not an homogeneous function of degree one of \( g^{AB} \), then, as anticipated, \( M \) will not necessarily vanish. As it is apparent, all non homogeneous of degree one pieces of the Lagrangian will enter into the above equation, these are exactly the conformal breaking terms. When inserting our specific lagrangian density, we obtain a very interesting constraint equation:

\[
u \frac{dF(u)}{du} - F(u) + V + M = 0 \quad (58)
\]

where

\[
u = \sqrt{\frac{1}{4} g^{AE} g^{DH} \omega_{AD} \omega_{EH}} \quad (59)
\]

**VII Zero 4-D Cosmological Constant and The fixed or ”D-Brane” boundary conditions for the scalar field on the singular branes**

After the study of the curvature equation and the constraint equation in the previous section, we are now in conditions to discuss the basic effects associated with the model with breaking of conformal invariance. In order to do so, it is useful to understand the meaning of the function \( F \). In the situation of conformal invariance, it is fundamental that the function \( F \) be a linear function. Even in the situation of scale invariance breaking, if we still wish to have as our solution, the singular brane case, then we must have that as \( u \) becomes very large, then \( F(u) \to C u \), where \( C \) is a constant. The linearity of
$F$ in $u$, or what is the same, the square root choice (in eq. 1 or in eqs. 12-14) is a requisite for the existence of singular brane (see also refs [2] and [3]). Then, the construction of the singular branes studied in section IV of the paper becomes possible. For example we get in this limit eq. (36) that a singularity in $\omega$ is cancelled. In this limit we clearly see that $u \frac{dF(u)}{du} - F(u) = 0$, which by equation (58) means that at the location of the singular brane, the scalar field is "frozen" at the values determined by

$$V + M = 0$$

Therefore the brane acts like a "D-Brane", giving the scalar field a boundary condition and in general an expectation value, or a non trivial average value in the bulk as well. This even without introducing tachyonic mass terms in general.

Now, concerning the question of the vanishing of the 4-D cosmological constant: If we just make the assumption that the compactified solutions are such that the field strength $\omega_{AB}$ gets an expectation value for the values of $A$ and $B$ in the extra dimensions only, and that the scalar field $\alpha$ has non trivial gradient only in the extra dimensions, then as a consequence, the matter lagrangian does not depend on the metric components $g_{\mu\nu}$ and eq. (56) tells us then immediately that

$$R_{\mu\nu} = 0, (\mu, \nu = 0, 1, 2, 3)$$

That is the four dimensional part of the manifold once again does not get curved.

X The complete system of equations

We have already discussed some equations of motion, in fact the equations of motion which are responsible for the most important effects, that is that only the extra dimensions aquire curvature and that the scalar field $\alpha$ is fixed at some value determined by an arbitrary constant of integration $M$. Those conclusions do not depend on the details of the solutions, but on very general features. A complete discussion requires however the consideration of all the equations of motion.

The equation for the connections gives the solution that the connections are the Christoffel symbols of a conformally transformed metric, exactly as it was in the conformally invariant case, i.e. the connections are the Christoffel symbols of the metric

$$\bar{g}_{AB} = \left( \frac{\Phi}{\sqrt{-g(5+1)}} \right)^{1/2} g_{AB}$$

Now, as opposed to the conformally invariant case, one does not have the freedom to choose that factor to be one. Since the Riemannian 4D curvature of the barred metric is zero, we have the right to consider solutions where
\[ \tilde{g}_{\mu\nu} = \eta_{\mu\nu} . \]  \hspace{1cm} (63)

while the extra dimensional part of the metric can (and in general must) be curved. Let us define the following notation,

\[ \tilde{g}_{ij} = \tilde{\gamma}_{ij} ; \, g_{ij} = \gamma_{ij} \]  \hspace{1cm} (64)

where of course the relation of the extra dimensional bar and unbarred metrics is

\[ \tilde{\gamma}_{ij} = \left( \frac{\Phi}{\sqrt{-g(5+1)}} \right)^{1/2} \gamma_{ij} \]  \hspace{1cm} (65)

here \( i, j = 4, 5 \). The gauge field equation (expressed in terms of the original metric, not the barred one) is

\[ \partial_A \left( \frac{dF}{du} \Phi \frac{\omega^{AB}}{\sqrt{\omega_{CD}\omega^{CD}}} \right) = 0 \]  \hspace{1cm} (66)

Assuming the \( \omega_{AB} \) to have expectation value only for \( A, B = i, j = 4, 5 \), we obtain that \( \omega_{ij} \) has only one independent component, because of the antisymmetry of such tensor, then in equation (66) the determinant of the internal metric appears. Such equation can be integrated to give

\[ \frac{dF}{du} \frac{\Phi}{\sqrt{\gamma}} = C \]  \hspace{1cm} (67)

where \( C \) is some constant. The above equation allows us to determine the measure \( \Phi \) in terms of \( \sqrt{\gamma} \) and \( u \), while \( u \) itself is determined in terms of the scalar field through eq. (58), except at the singular branes, when \( u \) is absent from such equation. This equation becomes instead an equation which fixes the value of the scalar field \( \alpha \) at such boundaries, that is, it gives a Dirichlet boundary condition for the scalar field. The \( i, j \) components of the gravitational equations are

\[ \frac{1}{16\pi G_{(5+1)}} R_{ij} = \frac{1}{2} u \frac{dF}{du} \gamma_{ij} + \frac{1}{2} \partial_i \alpha \partial_j \alpha \]  \hspace{1cm} (68)

All the system of equations appears then well defined. The two basic features that we have focused on, that is the fact that the ordinary dimensions do not get curved and the fact that the scalar field gets fixed at the bounday, do not depend on the details. Only on the fact that the scalar field has gradients only in the extra dimensions, which is certainly consistent since the boundary conditions are fixed by the branes, which have positions in the extra dimensions but which are totally homogeneous with respect to the 4 dimensions. The other assumption is of course that the gauge field \( \omega_{AB} \) gets vacuum expectation value in the extra dimensions only.
All details of the specifics of the solution are not going to change these facts, although they may be very important for the phenomenology of the theory. For example one may study how the scalar field $\alpha$ interpolates between the value determined by the boundary condition at the brane, i.e $V + M = 0$ and a value close to the minimum of the potential $V$ may be close to the middle region between the two branes.

The introduction of conformal breaking terms can be discussed in the context of “degenerate perturbation theory”: When conformal symmetry is present the four dimensional space is flat, while the extra dimensional part is largely arbitrary. This is because in the conformal symmetric case in the continuous distribution of branes, the branes do not interact with each other. Any particular brane does not suffer any force from the others and therefore those branes can be pilled with an arbitrary density in the extra dimension. This large degeneracy is broken once conformal symmetry is broken, one specific profile or density of branes appears singled out as the solution.

X Discussion and conclusions

In this paper we have discussed how the gauge formulation of branes can be used in the framework of “brane world” scenarios.

The formulation of 3-branes in a six-dimensional target spacetime can be made in a conformally invariant way. This is possible for extended objects in case the target spacetime has two more dimensions than the extended object itself.

This conformal invariance is intimately related to fact that the branes (or equivalently the associated gauge fields) only curve the manifold orthogonal to the brane, the extra-dimensions. No fine tuning of a $6D$ cosmological constant is needed in this case. Therefore, no “old cosmological constant problem” , as Weinberg has defined it [21], appears.

An interesting phenomenon is that the parallel 3-branes can be found with an arbitrary density for any value of $\vec{x}_\perp = (x^4, x^5)$. The density $\rho(\vec{x}_\perp)$ cannot be determined. This represents a large degeneracy and, therefore, a freedom in the possible ways the branes can be accounted in the extra dimensions.

The basic feature, that the matter curves only the extra dimensions is related to the fact that one is able to formulate the theory in terms of the measure $\Phi$, since then Eq.(19) follows automatically. Provided we adopt such formulation Eq.(19) tell us that if $L_m$ depends only from $\gamma_{ij}$, then only extra dimensions are curved. Conformal invariance holds if the embedding space is $6D$.

We then generalize to include conformal breaking terms. The terms which we include and which break the conformal invariance are of two types: one, which introduces a ”non ideal” fluid behavior for the branes, but leaves singular branes solutions unchanged and the other conformal breaking term is a scalar field potential. At the singular brane, the scalar field gets frozen at an expectation value determined by the eq. $V + M = 0$. This can be a mechanism that could introduce spontaneous breaking of internal symmetries, due to boundary conditions. Breaking of internal symmetries
by boundary conditions has been studied recently by several authors [22], although these authors use this effects to advocate the possibility of spontaneous symmetry breaking without a Higgs field. In our case, the existence of a scalar (that is a Higgs) still appears necessary, but the spontaneous symmetry breaking can be achieved by means of the boundary conditions at the wall $V + M = 0$, which will require a condensation of the scalar field at the branes, irrespective of the existence of a tachyonic mass term in $V$. Under very general conditions, the solutions do not curve the four dimensional space, but only the extra dimensional space.

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