Close Encounters of the Higher Kind
Emulating Constructor Classes in Standard ML

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We implement a library for encoding constructor classes in Standard ML, including elaboration from minimal definitions, and automatic instantiation of superclasses.

1. Introduction
In our recent work [4] on automating Isabelle proofs, we discovered that several proof search problems can be elegantly expressed as a monadic program. Unfortunately, Standard ML does not natively support the kinds of polymorphism required to easily express a Monad abstraction, nor similar abstractions such as Applicative and Functor [4]. In this paper, we present a technique for encoding constructor classes such as Monad, which relies only on the Standard ML module system.

Several others have attempted to enable constructor classes in Standard ML by changing the language. While it is tempting to customise the language by adding new features, new features tend to cause duplication [2] and inconsistency [3]. Furthermore, avoiding language extensions makes our approach transferable to all other ML dialects with a module system.

Our contributions are twofold: we develop a usable library for monads, monad transformers, applicatives, and more in Standard ML, and demonstrate an elegant technique using ML functors to elaborate minimal definitions of each abstraction to avoid code duplication. For example, given a minimal definition of the list monad, e.g. return and bind, our library derives other basic functions, such as map, join, and liftM automatically. Moreover, using the hierarchical relationship among constructor classes, our library automatically instantiates lists as a member of the parent classes, e.g. applicative and functor. Thus, for each monad, users can derive more than twenty functions from two manually written functions, i.e. return and bind.

2. Constructor Classes in Standard ML
Figure 1 shows the structure of the class hierarchy as it is implemented in our library. Each node represents a ML signature. Straight arrows stand for subtyping relations, whereas dashed arrows with labels stand for ML functors and their names. The ML functors expressed as vertical dashed arrows, e.g. mkMonad, produce full definitions of constructor classes from the corresponding minimal definitions; those expressed as horizontal dashed arrows, e.g. MonadToApp_MIN, generalise minimal definitions for a class to its superclass.

For example, the following code snippets show the specifications of MONAD_MIN and MONAD.

signature MONAD_MIN =

1 Not to be confused with an ML functor

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Figure 1. Automatic instantiation and function derivation.
functor Mona_Min_To_App_Min (Min:MONAD_MIN) =
  struct
  open Min;
  type 'a applicative = 'a monad;
  val pure = return;
  fun <*> (fs, xs) = bind fs (fn fs' =>
    bind xs (fn xs' => return (fs' xs')));
  end : APPLICATIVE_MIN

The ML functor Mona_Min_To_App_Min produces instances of APPLICATIVE_MIN in terms of MONAD_MIN functions. This is in contrast with the constructor classes in Haskell where return is defined as pure. It is this inversion that enables our library to derive superclass instances for a given type constructor.

The elaboration functor mk Мин is defined as follows.

functor mk Мин (Min : MONAD_MIN): MONAD =
  struct
  type 'a monad = 'a Min.monad;
  structure App_Min = Mona_Min_to_App_Min (Min);
  structure App = mk_Applicative (App_Min);
  open App_Min;
  fun liftM f m = bind m (fn m' => return (f m'));
  fun join n = ...; fun forever a = ...; fun ...
  end;

Apart from producing the various MONAD functions, mk Мин instantiates list as a member of APPLICATIVE by elaborating the result of the functor Mona_Min_To_App_Min with mk_Applicative, which in turn instantiates the FUNCTOR class similarly.

We formalise monad transformers as ML functors, too. For instance, the state monad transformer is a functor that takes two modules, the minimal definition of the base monad and a module containing just the type of the state, and produces a minimal definition of the transformed monad.

3. Corner Cases
Some functions in Haskell involve multiple classes, such as foldM:

foldM :: (Foldable t, Monad m)
  => (b -> a -> m b) -> b -> t a -> m b

We formalise these as ML functors that take multiple modules conforming to the appropriate signatures and return a module containing the function.

We can easily extend our approach to other constructor classes, even if they involve multiple inheritance. Figure 2 shows an example of such a case. Since our library is based on statically known mathematical properties, we avoid so-called diamond problems. For instance, given a type constructor of MONADOP in Figure 2 it does not matter semantically from which of ALTER and MONAD this type constructor inherits the methods of APPLICATIVE, as both of them have the same properties.

4. Comparison and Related Work
Our approach offers some benefits over traditional Haskell type classes. In particular, the ML module system allows more flexibility, as more than one instance can be provided for a given type. This flexibility is appreciated in constructor classes, too — for example, there are two perfectly valid Applicative instances for lists, one with a cartesian and one with a pairwise product operation. In Haskell, this necessitates the use of the newtype feature for one of the instances. In ML, both instances are equally natural.

Wehr et al. first introduced an approach to translate Haskell type classes in ML modules. They discussed that their scheme is not able to handle constructor classes, nor translate either recursive class constraints or default definitions into ML modules, while we addressed all of these. One example of a recursive class constraint would be:

instance (Monad f, Monad g) => Monad (f :*: g)

We express these using ML functors: in this case, we define a functor mk_ConsProd, which takes two modules of MONAD_MIN and returns a module of MONAD_MIN. Even though we can define mk_ConsProd parametrically, two concrete type constructors f and g must be supplied in order to instantiate MONAD_MIN for f :*: g.

Our approach is similar to the library code in Dreyer et al. however, we additionally support constructor classes, instance elaboration, and automatic instantiation of superclasses. We did not, however, extend the language as they did, as we did not wish to deviate from Standard ML, although we foresee no fundamental problems incorporating their implicit typing scheme into our library. Furthermore, we chose to express class hierarchies with flat module structures, while they did so hierarchically. Our choice allows users to avoid nested qualifiers, e.g. ListMonad.Applicative.Fmap.<$>, resulting in less verbose code in the absence of any implicit typing mechanism.

Scott seems to have employed a similar approach to ours, but in OCaml, suggesting that our technique is transferable to other ML dialects. There are also attempts to model type and constructor classes using features from the imperative object oriented programming paradigm. We purposefully avoided these deviations from Standard ML.

5. Current Status and Future Work
We previously developed a proof automation tool for Isabelle using this library, and our experience with it was positive. However, every library has room for improvement. We are working to include other constructor classes such as Arrow into this framework. In our approach, our MONAD module could also generate an instance of Arrow, once again eliminating the Haskell use of newtype for Kleisli arrows.

Furthermore, we plan to support multiple minimal definitions to instantiate some constructor classes. For example, we presented a minimal definition of MONAD with return and bind above, but we could provide a minimal definition of MONAD with return, fmap, and join instead. It is up to the user’s preference which minimal definition is easier to write. Since they are equivalent, we can write a functor that derives one from the other, providing multiple options to users.

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