Dependencies of lepton angular distribution coefficients on the transverse momentum and rapidity of $Z$ bosons produced in $pp$ collisions at the LHC

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High precision data of lepton angular distributions for $\gamma^* / Z$ production in $pp$ collisions at the LHC, covering broad ranges of dilepton transverse momenta ($q_T$) and rapidity ($y$), were recently reported. Strong $q_T$ dependencies were observed for several angular distribution coefficients, $A_3$, including $A_0 - A_4$. Significant $y$ dependencies were also found for the coefficients $A_1$, $A_3$ and $A_4$, while $A_0$ and $A_2$ exhibit very weak rapidity dependence. Using an intuitive geometric picture, we show that the $q_T$ and $y$ dependencies of the angular distributions coefficients can be well described.

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I. INTRODUCTION

The angular distribution of leptons produced in the Drell-Yan process [1] remains a subject of considerable interest. The original Drell-Yan model offered a specific prediction of a transversely polarized virtual photon for collinear quark-antiquark annihilation, resulting in a $1 + \cos^2 \theta$ lepton angular distribution [1]. This prediction was in good agreement with the earliest data, which were dominantly from dileptons with low transverse momentum ($q_T$) [2, 3]. As the dilepton's transverse momentum becomes large, due to QCD effects involving emission of partons of large transverse momenta, the angular distribution would no longer be azimuthally symmetric. A general expression for the lepton angular distribution in the Drell-Yan process becomes [4]

$$\frac{d\sigma}{d\Omega} \propto 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \nu \frac{1}{2} \sin^2 \theta \cos 2\phi, \quad (1)$$

where $\theta$ and $\phi$ refer to the polar and azimuthal angles of $l^-$ ($e^-$ or $\mu^-$) in the rest frame of $\gamma^*$. The azimuthal dependencies of the lepton angular distributions are described by the parameters $\mu$ and $\nu$. While $\lambda = 1$, $\mu = 0$, and $\nu = 0$ in the original Drell-Yan model [1], the presence of the intrinsic transverse momentum and QCD effects would allow $\lambda \neq 1$ and $\mu, \nu \neq 0$. However, it was predicted [4] that the deviation of $\lambda$ from unity is precisely correlated with the coefficient of the $\cos 2\phi$ term, namely, $1 - \lambda = 2\nu$. This so-called Lam-Tung relation, expected to be insensitive to QCD corrections [5–8], was found to be significantly violated in pion-induced Drell-Yan experiments [9, 10]. The unexpectedly large violation of the Lam-Tung relation inspired many theoretical work [11–14], including the suggestion [14] that a nonperturbative effect originating from the novel transverse-momentum-dependent (TMD) Boer-Mulders function [15] can account for this violation. This suggestion was found to be consistent with the existing pion and proton induced Drell-Yan data [16]. It also led to first extractions of the Boer-Mulders functions from the $\cos 2\phi$ dependence of the unpolarized Drell-Yan data [17, 18].

The azimuthal angular distributions of leptons in unpolarized or polarized Drell-Yan process are now regarded as an important tool for accessing the novel TMDs [14–19, 21].

At collider energies, measurement of lepton angular distributions in $W$ and $Z$ boson productions has long been advocated as a sensitive tool for understanding the production mechanism of these gauge bosons [22, 23]. The first measurement of the lepton angular distribution in $\gamma^* / Z$ production was reported by the CDF Collaboration for $pp$ collision at $1.96 \text{ TeV}$ [24]. Very recently, the CMS [25] and ATLAS [26] Collaborations at the LHC reported high-statistics measurements of the lepton angular distribution of $\gamma^* / Z$ production in $pp$ collision at $\sqrt{s} = 8 \text{ TeV}$. Strong $q_T$ dependencies were observed for the $\lambda$, $\mu$, and $\nu$ parameters. Moreover, violation of the Lam-Tung relation was found for these data at large $q_T$. Since the effects of TMD are expected to be negligible at large $q_T$, the presence of the Boer-Mulders function cannot explain the striking violation of the Lam-Tung relation at LHC energies.

In a recent paper [27], we showed that the observed $q_T$ dependence of $\lambda$ and $\nu$, as well as the violation of the Lam-Tung relation, can be well described by a geometric picture. While it is important to compare perturbative QCD calculations with these data, it is also instructive to understand the essential features of these data in terms of an intuitive geometric picture. In this paper, we extend the previous work, which focuses on the $\lambda$ and $\nu$ parameters and the Lam-Tung relation, to other angular distribution parameters. We also compare the striking $q_T$ and rapidity ($y$) dependencies of the angular distribution coefficients measured at the LHC with our intuitive geometric picture. We find that many salient features of the data can be well understood within the framework of this simple and intuitive approach.

This paper is organized as follows. In Sec. II we present our model and derive some expressions relevant for understanding the lepton angular distributions for $\gamma^* / Z$ production. We then compare calculations using this model with data on the $q_T$...
and rapidity dependencies in Secs. III and IV, respectively. We conclude in Sec. V.

II. LEPTON ANGULAR DISTRIBUTION COEFFICIENTS

The lepton angular distribution in the $\gamma^*/Z$ rest frame is expressed by both the CMS and ATLAS Collaborations as

$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) + A_0 \frac{A_0}{2} (1 - 3 \cos^2 \theta) + A_1 \sin 2\theta \cos \phi + A_2 \sin^2 \theta \cos 2\phi + A_3 \sin \theta \cos \phi + A_4 \cos \theta + A_5 \sin^2 \theta \sin 2\phi + A_6 \sin 2\theta \sin \phi + A_7 \sin \theta \sin \phi,$$  

(2)

where $\theta$ and $\phi$ are the polar and azimuthal angles of $l^-$ (e$^-$ or $\mu^-$) in the rest frame of $\gamma^*/Z$ like in Eq. (1). Compared to Eq. (1), Eq. (2) contains several additional terms ($A_0, A_1, A_2$), due to the presence of parity-violating coupling for the Z boson. It is clear that $\lambda, \mu, \nu$ in Eq. (1) are related to $A_0, A_1, A_2$ via

$$\lambda = \frac{2 - 3A_0}{2 + A_0}; \quad \mu = \frac{2A_1}{2 + A_0}; \quad \nu = \frac{2A_2}{2 + A_0}.$$  

(3)

Equation (3) shows that the Lam-Tung relation, $1 - \lambda = 2\nu$, becomes $A_0 = A_2$.

While Eq. (2) can be derived from the consideration of the general form of the lepton and hadron tensors involved in the $\gamma^*/Z$ production, we present a derivation based on an intuitive geometric picture. We first define three different planes, i.e., the hadron plane, the quark plane, and the lepton plane, shown in Fig. 1. For nonzero $q_T$, the beam and target hadron momenta, $\vec{P}_B$ and $\vec{P}_T$, are no longer collinear in the rest frame of $\gamma^*/Z$, and they form the "hadron plane" shown in Fig. 1. Various coordinate systems in the $\gamma^*/Z$ rest frame have been considered in the literature, and the Collins-Soper (C-S) frame was used by both the CMS and ATLAS Collaborations. For the Collins-Soper frame, the $x$ and $z$ axes both lie in the hadron plane, while the $z$ axis bisects $\vec{P}_B$ and $-\vec{P}_T$ with an angle $\beta$. It is straightforward to show that

$$\tan \beta = q_T/Q,$$  

(4)

where $Q$ is the mass of the dilepton. Figure 1 also shows the "lepton plane" formed by the momentum vector of $l^-$ and the $\hat{z}$ axis. The $l^-$ and $l^+$ are emitted back-to-back with equal momenta annihilate into a $\gamma^*/Z$.

In the $\gamma^*/Z$ rest frame, a pair of collinear $q$ and $\bar{q}$ with equal momenta annihilate into a $\gamma^*/Z$, as illustrated in Fig. 1. We define the momentum unit vector of $q$ as $\hat{z}'$, and the "quark plane" is formed by the $\hat{z}'$ and $z$ axes. The polar and azimuthal angles of the $\hat{z}'$ axis in the Collins-Soper frame are denoted as $\theta_1$ and $\phi_1$. The $q - \bar{q}$ axis, called the "natural" axis, has the important property that the $l^-$ angular distribution is azimuthally symmetric with respect to this axis, namely,

$$\frac{d\sigma}{d\Omega} \propto 1 + a \cos \theta_0 + \cos^2 \theta_0,$$  

(5)

where $\theta_0$ is the angle between the $l^-$ momentum vector and the $\hat{z}'$ axis (see Fig. 1), and $a$ is the forward-backward asymmetry originating from the parity-violating coupling to the Z boson. We recently showed that Eq. (2) can be derived from Eq. (5) by noting that

$$\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos (\phi - \phi_1).$$  

(6)

Substituting Eq. (6) into Eq. (5), one obtains

$$\frac{d\sigma}{d\Omega} \propto (1 + \cos^2 \theta) + \sin^2 \theta_1 \cos \phi_1 (1 - 3 \cos^2 \theta) + \left(\frac{1}{2} \sin 2\theta_1 \cos \phi_1 \sin 2\theta \cos \phi\right) + \left(\frac{1}{2} \sin^2 \theta_1 \cos 2\phi_1 \sin^2 \theta \cos 2\phi\right) + \left(\sin \theta_1 \sin \phi_1 \sin \theta \cos \phi + (a \cos \theta_1) \cos \theta\right) + \left(\frac{1}{2} \sin^2 \theta_1 \sin 2\phi_1 \sin^2 \theta \sin 2\phi\right) + \left(\frac{1}{2} \sin 2\theta_1 \sin \phi_1 \sin 2\theta \sin \phi\right) + \left(\sin \theta_1 \sin \phi_2 \sin \theta \sin \phi\right).$$  

(7)

A comparison between Eq. (2) and Eq. (7) shows a one-to-one correspondence for all angular distribution terms. Moreover, the angular distribution coefficients $A_0 - A_7$ can now be ex-
pressed in terms of the quantities \(\theta_1, \phi_1\) and \(\alpha\) as follows:

\[
\begin{align*}
A_0 &= \langle \sin^2 \theta_1 \rangle \\
A_1 &= \frac{1}{2} \langle \sin 2\theta_1 \cos \phi_1 \rangle \\
A_2 &= \langle \sin^2 \theta_1 \cos 2\phi_1 \rangle \\
A_3 &= \langle \alpha \sin \theta_1 \cos \phi_1 \rangle \\
A_4 &= \langle \alpha \cos \theta_1 \rangle \\
A_5 &= \frac{1}{2} \langle \sin^2 \theta_1 \sin 2\phi_1 \rangle \\
A_6 &= \frac{1}{2} \langle \sin^2 \phi_1 \rangle \\
A_7 &= \langle \alpha \sin \theta_1 \sin \phi_1 \rangle.
\end{align*}
\]

The \(\langle \cdots \rangle\) in Eq. (8) is a reminder that the measured values of \(A_i\) at given values of \(q_T\) and \(y\) are averaged over events having different values of \(\theta_1, \phi_1\) and \(\alpha\), in general. Equation (8) is a generalization of an earlier work [20] which considered the special case of \(\phi_1 = 0\) and \(\alpha = 0\).

The values of \(A_0 - A_7\) are bounded by certain limits as a result of the properties of the trigonometric functions and \(|\alpha| < 1\). In particular, we obtain the following relations from Eq. (8):

\[
\begin{align*}
0 & \leq A_0 \leq 1 \\
-1/2 & \leq A_1 \leq 1/2 \\
-1 & \leq A_2 \leq 1 \\
-1 & \leq A_3 \leq 1 \\
-1 & \leq A_4 \leq 1 \\
-1/2 & \leq A_5 \leq 1/2 \\
-1 & \leq A_6 \leq 1/2 \\
-1 & \leq A_7 \leq 1.
\end{align*}
\]

The bounds on \(A_0, A_1, A_2\), together with Eq. (8), imply that

\[
-1/3 \leq \lambda \leq 1; \quad -1 \leq \mu \leq 1; \quad -1 \leq \nu \leq 1.
\]

Some inequality relations among the various coefficients \(A_i\) can also be obtained from Eq. (8). In particular, \(A_0\) and \(A_2\) satisfy the relation

\[
A_0 \geq A_2.
\]

Equation (8) shows that in the case of \(\phi_1 = 0\) or \(\pi\), i.e., the quark plane and hadron plane are coplanar, the Lam-Tung relation \(A_0 = A_2\) is obtained. When Lam-Tung relation is violated, \(A_0\) must be greater than \(A_2\) or, equivalently, \(1 - \lambda > 2\nu\).

While the values of \(\theta_1, \phi_1\), and \(A_i\) depend on the specific coordinate system chosen for the \(\gamma^*/Z\) rest frame, it is worth noting that the relations in Eqs. (8)-(11) are independent of this choice, as long as the \(\hat{x}\) and \(\hat{z}\) axes of the reference frame lie within the hadron plane. Examples of such reference frames include the Collins-Soper, Gottfried-Jackson, and the helicity frames. As a consequence, if the Lam-Tung relation is satisfied (or violated) in any of these frames, it will be satisfied (or violated) in all other frames.

As shown in Eq. (8), the \(q_T\) and \(y\) dependencies of the angular distribution coefficients, \(A_i\), are entirely governed by the \(q_T\) and \(y\) dependencies of \(\theta_1, \phi_1\) and \(\alpha\). We first consider the quantities \(\theta_1, \phi_1, \alpha\), ignoring the small intrinsic transverse momentum, \(k_T\), of the partons. At the leading-order in \(\alpha_s\) \((\alpha^o_s)\), the quark axis, \(\hat{z}'\), is collinear with the \(\hat{z}\) axis. Hence, the result \(\theta_1 = 0\) (or \(\theta_1 = \pi\)) is obtained, and Eq. (8) shows that all \(A_i\) except \(A_4\) vanish.

At the next-to-leading order (NLO), \(\alpha_s\), a hard gluon or a quark (antiquark) is emitted so that \(\gamma^*/Z\) acquires nonzero \(q_T\). Figure 2(a) shows a diagram for the \(q - \bar{q}\) annihilation process in which a gluon is emitted from the quark in the beam hadron. In this case, the momentum vector of the quark is modified such that it becomes opposite to the antiquark’s momentum vector in the rest frame of \(\gamma^*/Z\). Since the antiquark’s momentum direction is the same as the target hadron’s momentum direction, the \(\hat{z}'\) axis is along the direction of \(-\vec{p}_T\) (see Fig. 2(b)). From Fig. 1 it is evident that \(\theta_1 = \beta\) and \(\phi_1 = 0\) in this case. Similarly, for the case of Fig. 2(c), where a gluon is emitted from an antiquark in the target hadron, one obtains \(\theta_1 = \beta\) and \(\phi_1 = \pi\), as illustrated in Fig. 2(d). Analogous results with \(\theta_1 = \pi - \beta\) and \(\phi_1 = 0\) (or \(\phi = \pi\)) can be found when the roles of beam and target are interchanged, as illustrated in Fig. 3. Table I lists the values of \(\theta_1\) and \(\phi_1\) for the four cases considered above. Given \(\theta_1 = \beta\) (or \(\theta_1 = \pi - \beta\)) and tan \(\beta = q_T/Q\) in the Collins-Soper frame, we obtain the following results, relevant for the coefficients \(A_i\) in Eq. (8),

\[
\begin{array}{cccccc}
\text{Case} & \text{Gluon emitted from} & \theta_1 & \phi_1 & A_0 & A_1 & A_2 \\
1 & \text{Beam quark} & \beta & 0 & + & + & + \\
2 & \text{Target antiquark} & \beta & \pi & + & - & - \\
3 & \text{Beam antiquark} & \pi - \beta & 0 & + & + & + \\
4 & \text{Target quark} & \pi - \beta & \pi & + & + & -
\end{array}
\]
for the NLO $q - \bar{q}$ annihilation processes:

$$\sin \theta_1 = q_T/(Q^2 + q_T^2)^{1/2}$$
$$\cos \theta_1 = \pm Q/(Q^2 + q_T^2)^{1/2}$$
$$\sin^2 \theta_1 = q_T^2/(Q^2 + q_T^2)$$
$$\sin 2\theta_1 = \pm 2q_T Q/(Q^2 + q_T^2),$$

where the $+$ ($-$) sign corresponds to $\theta_1 = \beta$ ($\theta_1 = \pi - \beta$). Since $\phi_1 = 0$ or $\pi$, one can see from Table I and Eq. (8) that the Lam-Tung relation, $A_0 = A_2$, is satisfied. Moreover, $A_5 - A_7$ must vanish, since they are proportional to $\sin \phi_1$ or $\sin 2\phi_1$, which are identically zero.

We next consider the Compton process at NLO. Unlike the cases for the $q - \bar{q}$ initial state shown in Figs. 2 and 3 where a hard gluon is emitted, a hard quark or antiquark will now accompany the $\gamma^*/Z$ final state. Fig. 4(a) shows the diagram in which a gluon from the target hadron splits into a $q - \bar{q}$ pair and the quark from the beam hadron annihilates with the antiquark into a $\gamma^*/Z$. Since the momentum vector of the quark in the beam hadron is unchanged, $\theta_1 = \beta$ and $\phi_1 = \pi$, as shown in Fig. 4(b). This result is identical to that for the $q\bar{q}$ initial state shown in Fig. 2(d). Analogous results are obtained when gluon is emitted from the beam hadron, or when an antiquark replaces the quark in the initial state. However, a different situation is shown in Fig. 4(c), where the quark and gluon fuse into a quark, which then emits a $\gamma^*/Z$. As indicated in Fig. 4(d), $\theta_1$ must satisfy $\beta \leq \theta_1 \leq \pi - \beta$, since the momenta of the initial quark and gluon combine vectorially, resulting in

\[ A_f = \frac{2C_f^L}{(C_f^L)^2 + (C_A^L)^2}. \]

The vector $C_f^L$ and axial vector $C_A^L$ couplings for Z boson to fermion $f$ are, respectively, $P_W^f - 2Q \sin^2 \theta_W$ and $P_W^f$, where $P_W^f$ and $\theta_W$ denote the weak-isospin third component and the Weinberg angle. Using $\sin^2 \theta_W = 0.2315$, then Eq. (14) gives $a = 0.211$ for $u \bar{u} \rightarrow Z \rightarrow l^- l^+$, and $a = 0.299$ for $d \bar{d} \rightarrow Z \rightarrow l^- l^+$, where $l$ refers to $e$ or $\mu$. We note that $a$ has a positive value. Moreover, depending on the relative weight between the $u \bar{u}$ and the $d \bar{d}$ contributions, one expects the mean value of $a$ to vary between these two limits.
are observed. Figure 5 shows that the coefficients $A_i$ (the statistics compared to the CMS data in this work. The ATLAS data only consider the CMS data in this work. The ATLAS data cover a broad range in the dilepton’s $q_T$ versus $|y| < 1.0$.)

**III. TRANSVERSE MOMENTUM DEPENDENCIES OF ANGULAR DISTRIBUTION COEFFICIENTS**

We now compare the $\gamma^*/Z$ production data at the LHC with calculations based on the results obtained in Sec. II. The LHC data cover a broad range in the dilepton’s $q_T$ and rapidity $y$ ($0 < q_T < 600$ GeV and $0 < |y| < 3.5$). For simplicity, we only consider the CMS data in this work. The ATLAS data contain both the $\mu^-\mu^+$ and $e^+e^-$ dilepton events, doubling the statistics compared to the $\mu^-\mu^+$ data sample in CMS. However, the procedure of “regularization” adopted by the ATLAS Collaboration introduces model dependencies associated with the theoretical calculations used in the procedure. Although the tabulated uncertainties of the ATLAS data [26] are significantly smaller than that of the CMS data [25], it is difficult to assess the systematic uncertainties associated with the procedure of “regularization”. We therefore prefer to compare our calculations with the results of CMS, where a conventional analysis procedure without “regularization” is adopted.

Figure 5 shows the angular distribution coefficients $A_i$ at the mid-rapidity region $|y| < 1.0$ measured by the CMS Collaboration. Some salient features in the $q_T$ dependencies of $A_i$ are observed. Figure 5 shows that the coefficients $A_0 - A_3$ are consistent with zero at the smallest value of $q_T$. On the other hand, the coefficient $A_4$ is nonzero at $q_T \rightarrow 0$. The values of $A_5 - A_7$ are found by the CMS Collaboration to be consistent with zero [25]. In order to understand these general features of the angular distribution coefficients, Eq. (8) suggests that one could examine the properties of the quantities $\theta_1$ and $\phi_2$.

From Eqs. (8), (12), (13), noting that $\phi_1 = 0$ or $\pi$ and the $\gamma^*/Z$ cross sections are dominated by the NLO $qG$ processes depicted in Figs. 4 and 5, one can readily predict the following patterns for the $q_T$ dependencies of $A_0$ up to $A_4$:

1) As $q_T \rightarrow 0$, Eqs. (8), (12), (13) show that $A_0, A_1, A_2, A_3$ all approach zero, since $\theta_1 \rightarrow 0$. On the other hand, $A_4$ is at its maximal value, since it is proportional to $\cos \theta_1$. As $q_T \rightarrow \infty$, $\theta_1$ approaches the value of $\pi/2$, and $A_0, A_2, A_3$ reach their maximal values, while $A_1$ and $A_4$ approach zero. As shown in Fig. 5 the data are consistent with these expectations.

2) According to Eqs. (8), (12), (13) the values of $A_0$ would go from zero at $q_T = 0$ to unity as $q_T \rightarrow \infty$. At all values of $q_T$, one expects $A_2 \leq A_0$. In the case of $\cos 2\phi_1 = 1$, which occurs for the NLO processes as discussed above, the Lam-Tung relation, $A_0 = A_2$ is satisfied. When the Lam-Tung relation is violated, $A_0 \neq A_2$ (or $1 - \lambda \neq 2\nu$), it is expected that only $A_0 - A_2 > 0$ (or $1 - \lambda - 2\nu > 0$), not the alternative inequality $A_0 - A_2 < 0$, can occur. These expectations are
consistent with the data shown in Fig. 5.

3) As \( A_1 \) is proportional to \( \sin 2\theta_1 \), it would first increase with \( q_T \), reaching a maximum, and then decrease. This is in contrast to \( A_0, A_2, \) and \( A_4 \), which are expected to increase with \( q_T \) monotonically. Similarly, \( A_4 \) would decrease monotonically with \( q_T \), as it is proportional to \( \cos \theta_1 \). The data are consistent with these expected trends.

4) The upper and lower bounds on \( A_i \), listed in Eq. (9), are well satisfied by the data.

We next compare the CMS data on the angular distribution coefficients \( A_0 \) to \( A_4 \) with calculations based on the intuitive geometric picture discussed above.

Figure 6(a) shows the values of \( A_0 \) versus \( q_T \) for \( |y| < 1.0 \). The dotted and dashed curves correspond to calculations using Eq. (8) and Eqs. (12), (13), respectively, for the \( q\bar{q} \) and \( qG \) processes, \( A_0 = \frac{q_T^2}{(Q^2 + q_T^2)} \) and \( A_0 = 5q_T^2/(Q^2 + 5q_T^2) \). Note that the \( q\bar{q} \) process alone underestimates \( A_0 \), while the \( qG \) process overestimates it. Since these two processes contribute incoherently to the \( \gamma^*/Z \) production due to their distinct initial and final states (see Figs. 2-4), the observed \( A_0 \) is the result of an incoherent sum of these two processes. A best fit to the data, shown as the solid curve in Fig. 6(a), is obtained with a mixture of 58.5 \( \pm \) 1.6% \( qG \) and 41.5 \( \pm \) 1.6% \( q\bar{q} \) processes. The excellent agreement between the data and the calculation lends support to the adequacy of this intuitive geometric picture. It also suggests that higher-order QCD processes do not affect the values of \( \theta_1 \) (and \( A_0 \)) significantly.

Figure 6(b) displays \( A_2 \) versus \( q_T \) for \( |y| < 1.0 \) data from CMS. Eq. (8) shows that the value of \( A_2 \) should be identical to that of \( A_0 \) if \( \phi_1 = 0 \) or \( \pi \). The dashed curve in Fig. 6(b) is identical to the solid curve in Fig. 6(a), obtained with a mixture of 58.5 \( \% \) \( qG \) and 41.5 \( \% \) \( q\bar{q} \). The deviation of the dashed curve from the data shows that the Lam-Tung relation, \( A_0 = A_2 \), is violated. From Eq. (8), it is evident that this violation is due to \( \phi_1 \neq 0 \) or \( \pi \), namely, the quark and hadron planes are not coplanar. This noncoplanarity is caused by higher-order processes, in which multiple partons accompany the \( \gamma^*/Z \) in the final state. The hadron plane then contains the vector sum of multiple partons, and is in general not coplanar with respect to the quark plane. The effect of the noncoplanarity is to reduce the value of \( A_2 \) with respect to that of \( A_0 \). The solid curve in Fig. 6(b), obtained with an overall reduction factor of 0.77, describes the CMS data well. This reduction factor, originating from the \( \cos 2\phi_1 \) factor, indicates that the effective value of the noncoplanarity angle, \( \phi_1 \), is around 20°. Figure 6(c) shows the \( q_T \) dependence of \( A_0 - A_2 \) for \( |y| < 1.0 \). The violation of the Lam-Tung relation, reflected by the nonzero values of \( A_0 - A_2 \), is well described by the solid curve taking into account the overall reduction factor of 0.77 for \( A_2 \).

We next consider the coefficient \( A_1 \). From Eq. (8), the coefficient \( A_1 \) is related to the parameter \( \mu \) measured in fixed-target Drell-Yan experiments. In \( pp \) collision, \( A_1 \) is odd under \( y \leftrightarrow -y \) exchange. Figure 7(a) shows the \( q_T \) dependence of \( A_1 \) measured at CMS. The sign of \( A_1 \) measured at negative \( y \) is flipped before combining it with \( A_1 \) measured at positive \( y \). Equations (8) shows that \( A_1 \) is given as \( \frac{1}{2}(\sin 2\theta_1 \cos \phi_1) \). The values of \( \sin 2\theta_1 \) are given in Eqs. (12) and (13) for the \( q\bar{q} \) and \( qG \) processes, and \( \phi_1 = 0 \) (or \( \pi \)). For various cases as listed in Table I, one can calculate the values of \( A_1 \) for the four cases. Depending on the value of \( \phi_1 \), the sign of \( A_1 \) can be positive or negative, as shown in Table I. Hence, one expects a significant cancellation among contributions from processes with \( \phi_1 = 0 \) or \( \phi_1 = \pi \). The solid curve in Fig. 7(a) is obtained with the following expression

\[
A_1 = r_1\left[ f \frac{q_T Q}{Q^2 + q_T^2} + (1 - f) \frac{\sqrt{5}q_T Q}{Q^2 + 5q_T^2} \right],
\]

where \( f \) is the fraction of \( q\bar{q} \) process, \( f = 0.415 \), deduced from the \( A_0 \) data discussed earlier. The \( \sin 2\theta_1 \) values for the \( q\bar{q} \) and \( qG \) processes given in Eqs. (12) and (13) are weighted by \( f \) and \( 1 - f \), respectively. The reduction factor \( r_1 \) represents the combined effect of the partial cancellation discussed above and the deviation of \( \phi_1 \) from 0 or \( \pi \) due to higher-order QCD. The best-fit value of \( r_1 \) using Eq. (15) is \( r_1 = 0.0215 \). The small value of \( r_1 \) indicates the presence of a strong cancellation at small values of \( y \).

Similar considerations also apply to the coefficient \( A_4 \), which is also an odd function of \( y \) in \( pp \) collision. Both \( A_1 \) and \( A_3 \) are sensitive to \( \cos \phi_1 \). Table 1 shows the signs of \( A_3 \) for four different cases in \( q\bar{q} \) process. As a parity-violating ob-
servable, $A_3$ is also sensitive to the forward-backward asymmetry parameter $a$. The solid curve in Fig. 7(b) corresponds to the following expression

$$A_3 = r_3 \left[ f \frac{q_T}{(Q^2 + q_T^2)^{1/2}} + (1 - f) \frac{\sqrt{5q_T}}{(Q^2 + 5q_T^2)^{1/2}} \right].$$  \hspace{1cm} (16)

Equation (16) is analogous to Eq. (15), except that the reduction factor $r_3$ now includes an additional contribution from $a$. The best-fit value, $r_3 = 0.0163$, is obtained. As shown in Fig. 7(b), the agreement between the data and this simple calculation is reasonable.

Figure 7(c) shows $A_4$ versus $q_T$ for $|y| < 1.0$. Unlike all other coefficients, $A_4$ has a nonzero value as $q_T$ approaches zero. As discussed earlier, this is well explained by its dependence on $\cos \theta_1$, which has a maximal value at $q_T = 0$. The solid curve in Fig. 7(c) is obtained with the following expression

$$A_4 = r_4 \left[ f \frac{Q}{(Q^2 + q_T^2)^{1/2}} + (1 - f) \frac{Q}{(Q^2 + 5q_T^2)^{1/2}} \right].$$  \hspace{1cm} (17)

where the best-fit value for the reduction factor $r_4$ is 0.0183. Both $r_3$ and $r_4$ contain the parity violating parameter $a$. However, unlike $r_3$, $r_4$ does not contain the $\cos \phi_1$ term. This qualitatively explains the slightly larger value for $r_4$ than $r_3$. The calculation based on Eq. (17) is in very good agreement with the data shown in Fig. 7(c).

IV. RAPIDITY DEPENDENCIES OF ANGULAR DISTRIBUTION COEFFICIENTS

The CMS Collaboration has reported the rapidity dependencies of $A_i$ for two bins, $|y| < 1.0$ and $1.0 < |y| < 2.1$. In this Section, we compare the measured $y$ dependencies with expectations based on our intuitive geometric picture. Figure 8 shows that for $A_0$ and $A_2$, there are very weak, if any, rapidity dependencies. The solid curves in Fig. 8 are taken from the calculations shown in Fig. 6. It is evident that data at both rapidity bins are well described by a single curve. The weak rapidity dependence of $A_0$ reflects the fact that $A_0$ only depends on $\theta_1$, which, according to Eqs. (12) and (13), is independent of the rapidity $y$. However, higher-order QCD effects can introduce weak rapidity dependence for $A_0$. The weak rapidity dependence for $A_2$ shows that $\phi_1$ is weakly $y$ dependent. Indeed, at order $\alpha_s$, Table I shows that $\cos 2\phi_1$ is equal to unity for all four cases, independent of the value of $y$. Again, higher-order QCD will allow $\cos 2\phi_1$ to deviate from unity, but the deviation has a very weak $y$ dependence.

In striking contrast to $A_0$ and $A_2$, the coefficients $A_1$, $A_3$...
and $A_4$ exhibit pronounced rapidity dependencies, as shown in Fig. 9. A common feature for $A_1$, $A_3$, and $A_4$ is that they all rise significantly as $y$ increases. An intuitive explanation for this strong $y$ dependence is as follows. Table I shows that the various contributions to $A_1$, $A_3$, and $A_4$ can be positive or negative, and each contribution is weighted by the corresponding density distributions for the interacting partons. At small values of $y$, the momentum fraction carried by the beam parton, ($x_B$), is comparable to that of the target parton, ($x_T$). Hence the weighting factors for various cases are of similar magnitude and the net contribution is small due to partial cancellations among them. On the other hand, as $y$ becomes large, $x_B$ becomes significantly larger than $x_T$. Hence, the weighting factors are now dominated by fewer terms, resulting in less cancellation and a larger net result. The various curves shown in Fig. 9 correspond to calculations using Eqs. (15), (16), (17), respectively, for $A_1$, $A_3$ and $A_4$. The CMS data are quite well described by the best-fit values of $r_1$, $r_3$, and $r_4$ listed in Table II.

### V. SUMMARY AND CONCLUSIONS

We have presented an intuitive interpretation for the lepton angular distribution coefficients for $\gamma^* / Z$ production measured at the LHC. We first derive the general expression [Eq. (7)] for the lepton polar and azimuthal angular distributions in the dilepton rest frame, starting from the azimuthally symmetric lepton angular distribution [Eq. (5)] with respect to the quark-antiquark axis. We show that the various angular distribution coefficients are governed by three quantities, $\theta_1$, $\phi_1$ and $a$ (Eq. 8). The upper and lower bounds [Eq. (9)] for the angular distribution coefficients are obtained as a result of the expressions in Eq. (8). Similarly, the inequality relation between $A_0$ and $A_2$, relevant for the violation of the Lam-Tung relation, is obtained [Eq. (11)].

We then consider the characteristics of the quantities $\theta_1$, $\phi_1$ and $a$. The expressions for $\theta_1$ and $\phi_1$ are obtained for both the $q\bar{q}$ and $qG$ processes at order $\alpha_s$. The $q_T$ dependence of $A_0$ is found to be very well described using the results for $\theta_1$. It also allows a determination of the relative fractions of these two processes. This result is noteworthy, since it shows that a measurement of the angular distribution coefficient $A_0$ alone could lead to important information on the dynamics of the production mechanism, namely, the relative contribution of the $q\bar{q}$ annihilation and the $qG$ Compton processes.

The CMS data clearly show that the Lam-Tung relation, $A_0 = A_2$, is violated. The origin of this violation is attributed in our approach to the deviation of $\cos 2\phi_1$ from unity, indicating the noncoplanarity between the hadron and quark planes. This noncoplanarity is caused by higher-order QCD processes. We show that the amount of noncoplanarity can be deduced from the $A_0 - A_2$ data directly. We have also compared our approach with the CMS data for other angular distribution coefficients, $A_1$, $A_3$, $A_4$, and found that their $q_T$ dependencies, governed by the $q_T$ dependence of $\theta_1$, can be well described.

We also show that the rapidity dependencies of the $A_i$ can be well understood in this intuitive approach. In particular, the weak rapidity dependencies of the $A_0$ and $A_2$, and the pronounced rapidity dependencies for $A_1$, $A_3$ and $A_4$ can be explained by the absence or presence of cancellation effects, which depend strongly on the rapidity.

We note that the intuitive approach presented in this paper is by no means a substitute for the perturbative QCD calculations. The goal of this work is to provide some intuitive explanation of some salient features present in the lepton angular distribution data. This could offer some useful insights on the origins of many interesting characteristics of the lepton angular distributions which are being measured at the LHC with high precision.

The present approach could also be extended to fixed-target Drell-Yan experiments. Some recent work [32] shows the importance of the perturbative QCD effects even at fixed-target energies. A comparison between this intuitive approach and the perturbative QCD calculations is also of interest. It is also promising to extend this intuitive approach to some other processes with hadron or lepton beams.

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