Topical Review

Wavelength-versatile optical vortex lasers

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Abstract
The unique properties of optical vortex beams, in particular their spiral wavefront, have resulted in the emergence of a wide range of unique applications for this type of laser output. These applications include optical tweezing, free space optical communications, microfabrication, environmental optics, and astrophysics. However, much like the laser in its infancy, the adaptation of this type of laser output requires a diversity of wavelengths. We report on recent progress on development of optical vortex laser sources and in particular, focus on their wavelength extension, where nonlinear optical processes have been used to generate vortex laser beams with wavelengths which span the ultraviolet to infrared. We show that nonlinear optical conversion can be used to not only diversify the output wavelength of these sources, but can be used to uniquely engineer the wavefront and spatial properties of the laser output.

Keywords: optical vortex, nonlinear optics, singular optics

(Some figures may appear in colour only in the online journal)

1. Introduction
Optical vortices are helical modes of light which exhibit a spiral wavefront. These modes have a characteristic topological charge $\ell$, whereby there is a $2\pi\ell$ azimuthal phase change of the wavefront around the propagation axis (figure 1).

Optical vortices exhibit several unique features: an annular spatial form with a central null region (dark core) and an orbital angular momentum (OAM) of $\ell\hbar$ (where $\hbar$ is the reduced Planck constant) per photon [1–7]. These unique features enable optical vortices to be applied to a diverse range of fields, including optical manipulation [8–11], in which the OAM property of optical vortices are used to force micrometer-sized particles to revolve; super resolution fluorescence microscopy (STED) [12–15], in which optical vortices are used to quench fluorescence (by stimulated emission) signals around an outer ring defined by the annular spatial profile of the vortex beam; and space division multiplexing optical communications with ultra-high data density beyond 100 TB s$^{-1}$ [16, 17]. More recently, optical vortices have pioneered a new materials processing approach, in which these beams are used to twist materials to form monocrystalline needles and chiral structures on the micro- or nanoscale [18–25]. New applications for optical vortices include use in environmental optics and free space telecommunications, because they have the potential to propagate through turbulence with less degradation than conventional light beams [26]. Uniquely, rotational Doppler effects arising from interaction of optical vortex and rotating structured targets may pave the way towards new techniques in astrophysics, e.g. the remote sensing of rotating bodies in an astronomical setting [27, 28].

While the above-mentioned applications for vortex beams exploit their unique spatial and OAM properties, in many of these applications, it is still necessary to have versatility in the wavelength of the vortex beam for exactly the same reason that wavelength versatility is demanded for applications of more conventional laser beams.
For instance, optical vortex sources in the visible and ultraviolet (UV) regions, in which many materials exhibit strong absorption bands, will be potentially utilized in research as well as industrial applications, such as photo-chemistry [29], photolithography [30], and microfabrication [31]. Vortices in the UV might also open a new avenue towards high field physics, such as a structured filament with a phase dislocation in air [32].

Tunable optical vortex sources with moderate energy levels in the near-infrared and mid-infrared regions, in which the eigen frequencies of the vibrational modes of many molecules lie [33], are strongly desired for applications in molecular sciences, such as molecular spectroscopy, where high spatial resolution exceeding the diffraction limit may be achieved; and applications where organic materials processing can be accomplished without destruction of chemical structures.

A typical method which is used to generate optical vortices involves conversion of a standard Gaussian laser beam to a vortex beam through the use of phase elements, e.g. an azimuthally segmented spiral phase plate (SPP) for an $2\pi\ell$ azimuthal phase shift of the wavefront [34, 35], a $q$-plate [36], or a SLM programmed to produce a desired wavefront [37, 38]. Such phase elements are typically designed for a specific laser wavelength and this inherently constrains the wavelength versatility of the vortex laser sources generated using these techniques. Furthermore, conventional SPPs provide only an azimuthal $2\pi\ell$ phase shift to the incident wavefront; therefore, the resulting optical vortices include undesired higher-order radial Laguerre–Gaussian (LG) modes with a radial index $p$ and an azimuthal index (topological charge) $\ell$, which will also transform spatially during their propagation, owing to individual Gouy phase shifts, given by $\phi_{p,\ell}(z) = (2p + |\ell| + 1)\tan^{-1}\left(\frac{z}{z_R}\right)$, where $z_R$ is the Rayleigh length, and $z$ is the propagation distance [39]. Such characteristics are highly undesired for some applications.

In addition to the use of external elements, it is also possible to directly generate optical vortex modes from a laser cavity. This is a very effective way to produce high power, high beam quality optical vortices with high efficiency, and has been demonstrated by employing several methods, including using an annular pump beam for the laser medium [40], centrally damaged cavity mirrors [41], thermal lensing of the laser medium [42], and using an SLM as a back cavity reflector for computer-designing the laser wavefront [43, 44].

Wavelength conversion of vortex laser beams is achieved using the same methods used to convert Gaussian laser beams, and here both second order and third order nonlinear wavelength conversion has been demonstrated. Second order processes including sum-frequency generation and second harmonic generation [45], and optical parametric down-conversion of optical vortices [46] have been demonstrated, along with third order stimulated Raman scattering (SRS) [47]. What is interesting to note is that both conservation of energy and momentum must be observed in these nonlinear processes, and this results in quite interesting sharing of the topological charge (associated with OAM) of the vortex beams involved in the nonlinear process. As a result, the nonlinear conversion results in changes to not only the wavelength of the vortex fields but also the wavefront and spatial profiles of the beams. What we find is that in some instances, we have the capability to control how the topological charge is transferred during wavelength conversion.

In this paper, we review methods of directly generating optical vortex laser beams using solid-state laser technologies, as well as the wavelength extension of vortex lasers, using both second and third harmonic nonlinear processes. Notably, second harmonic generation, optical parametric oscillation (OPC) and SRS are shown as techniques capable of producing vortex beams with wavelengths spanning the UV to infrared. It is clear that high-power vortex lasers in combination with nonlinear wavelength conversion will lead to new, novel applications exploiting the unique characteristics of vortex laser beams.

2. Optical vortex lasers

The direct generation of optical vortex modes from a laser cavity involves preferential oscillation of LG ($LG_{0,\ell}$) modes within the laser resonator, with the suppression of the lowest order Gaussian mode ($LG_{0,0}$) and the higher radial LG modes ($LG_{p,\ell}$), with high efficiency and power.

In this chapter, we review several techniques used to directly produce optical vortices from a solid-state laser cavity.

2.1. Annular beam pumping configuration

Annular beam pumping in an end-pumped solid state laser allows us to effectively achieve spatial intensity matching between the $LG_{0,\ell}$ modes and the gain volume in the laser resonator, resulting in generation of the $LG_{0,\ell}$ modes in various laser systems. In early days, Bisson et al demonstrated a miniaturized, hollow-shaped beam pumped 1.06 µm Nd:YAG laser, that provided $LG_{0,\ell}$ modes with a high value of the azimuthal index, $\ell$, of $\sim$200 [40].

Perhaps the most compelling demonstration of direct generation of a vortex laser beam via annular pumping is the work performed by Kim et al. They demonstrate the
robustness of this technique, to produce vortex outputs from Er:YAG [48] and Nd:YAG [49] by employing a capillary fiber to shape the pump beam into an annular beam. Importantly, they have been able to demonstrate both continuous-wave (cw) and Q-switched operation of their systems.

Figure 2(a) shows a schematic diagram of the 1.06 μm Q-switched Nd:YAG vortex laser. A diode pump beam was directed towards the annular waveguide of the silica capillary fiber with a hollow-core (air-hole). The shaped annular pump beam was imaged to an annular spot onto the Nd:YAG crystal, thereby generating the desired LG0,ℓ mode as this defined the spatial form of the gain volume. The laser resonator comprised a high-reflection plane input mirror (IC) and a plane output coupler (OC). A plano-convex lens was placed inside the cavity to ensure a stable resonator over the full range of the pump powers. An etalon and a Brewster plate were further placed inside the cavity so as to ensure selectively left- or right-handed vortex mode operation owing to azimuthal symmetry breaking.

By employing this pump geometry in the Nd:YAG laser, they demonstrated cw 1.06 μm laser output power of 1.5 W, for a pump power of 5.7 W, corresponding to a slope efficiency of 32%. The laser output exhibited an annular intensity profile (characteristic of an optical vortex) for all incident power levels, and the output beam propagation factor (M2) was measured to be ∼2.01 (this value is very close to the theoretical value of M2 = 2 for a LG0,1 mode), indicating that preferential oscillation of a high-quality LG0,1 mode was achieved in this laser system (figure 2(b)).

The laser system also acts at Q-switched operation by employing an acousto-optic Q-switch cell placed inside the cavity. At a pulse repetition frequency of 500 Hz, the laser system yielded Q-switched pulses with a pulse energy of ∼250 μJ and a pulse duration of ∼33 ns (the corresponding peak power was ∼7.6 kW).

Furthermore, it is interesting to note that the handedness of the vortex output (sign of the vortex beam topological charge) could be controlled merely by tilting the angle of the intracavity etalon. While this was demonstrated experimentally, the mechanism of the handedness control is not yet well understood. A novel handedness control technique, consisting of two nanoscale thickness aluminum stripes for azimuthal symmetry breaking, has been proposed for this laser system [50].

2.2. Central damaged cavity mirrors configuration

Another method of directly generating a vortex beam from a laser resonator is through the use of a cavity mirror with a region (of the appropriate size) of the reflective surface removed, herein referred to as a damage spot. This damage spot acts as a hard aperture, and when the laser mode oscillates on this damage spot, the lowest-order Gaussian mode can be suppressed, and the oscillation of LG modes can take place. Critical to the effectiveness of this technique is that the damage spot diameter is well matched to the resonator mode size. Generation of vortex laser output using a mirror with an appropriate damage spot has been demonstrated in a number of systems including He–Ne [41] and Nd:YAG lasers [51].

Ito et al demonstrated the generation of optical vortices and polarization vortices, i.e. radially (or azimuthally) polarized beams from a side-pumped Nd:YAG laser with a central damaged back reflector with a 0.25 mm diameter (this value was ∼0.2 that of the cavity LG0,1 mode) damage spot (figure 3(a)). Also, a Brewster plate was placed into a laser cavity, ensuring a linearly polarized output. This system produced preferentially LG0,ℓ modes with ℓ = 1–3 (figure 3(b)). The handedness of the vortex output could be controlled through slight changes to the cavity alignment. Further note that in the system, polarization vortex modes were encouraged to lase through the introduction of a mechanical aperture, to remove the higher modes, removing the Brewster plate and slightly tilting the back reflector or OC.

We have also demonstrated the direct generation of vortex laser output from a cw, diode-end-pumped 1.06 μm Nd:GdVO4 laser system [52]. The laser system consisted of a Nd:GdVO4 crystal with the input surface coated high reflectivity (HR) for 1064 nm, and a concave OC with a laser-micro-machined 340 μm circular damage spot. Careful alignment of the resonator mode on the damage spot resulted in the laser operating with a vortex mode (figure 4).

The optical vortex output had an annular spatial form (with a central dark core), i.e. LG0,1 mode, which lased at a pump power above 0.1 W; and its maximum output power at
1064 nm reached 0.4 W. When the wavefront of the output was characterized using a Mach–Zehnder interferometer, the wavefront of the vortex mode exhibited single-armed spiral fringes indicative of a first order phase-singularity. Even with increasing pump power, the order and direction of the topological charge remained unchanged. It was however possible to control the sign of the topological charge by altering the resonator alignment through slight adjustment of the tilt angle of the OC.

2.3. Thermally induced mode aperture in a side pumped vortex laser

Direct generation of a vortex laser beam from a laser resonator can be induced without the need for any special shaping or blocking elements and can be achieved within the laser by exploiting an often undesired characteristic, thermal lensing of the gain medium. This approach has been demonstrated in side pumped bounce lasers. In these systems, the cavity mode is reflected within the gain medium (which is transversely pumped by a diode array) through total internal reflection (bounced). These systems exhibit rather strong thermal lensing effects which can induce non-parabolic, higher-order thermal aberrations. Such thermal lensing effects force the Gaussian mode to be unstable, but still support the \( \text{LG}_{0,1} \) vortex mode due to mode overlap considerations [42].

This approach to direct generation of vortex laser emission has been demonstrated in a diode array side-pumped 1.06 \( \mu \text{m} \) bounce laser. As shown in figure 5(a), this system comprises an Nd:GdVO\(_4\) slab, and two flat mirrors with high
reflection and partial reflection for 1.06 \mu m. To maintain a good spatial overlap between the cavity mode and the pumped region, two cylindrical lenses were also placed inside the cavity. At a high pump level beyond the threshold, the system significantly suppresses the Gaussian mode lasing, resulting in the preferential vortex mode, i.e. the LG_{0,1} mode, operation (figure 5(b)). As shown in figure 5(c), the maximum vortex output power was measured to be 17.8 W at the pump power of 55 W. The handedness of the vortex output was also controlled by slightly tilting a focusing lens of the pump diode [53]. This work demonstrates the direct generation of ultra-high-power vortex output from a laser system without needing any additional beam shaping elements, for instance, damaged mirror or annular pump beam, and it can be extended to generate a high power 1.3 \mu m vortex output by employing a Nd:YVO_{4} slab crystal [54].

The Q-switch operation of this system is also achieved by employing the AOM as a Q-switch cell. Chard et al have successfully produced a Q-switch output with an average power of >16 W and a pulse duration of 45 ns within a pulse repetition frequency region of 150–400 kHz by employing a Nd:YVO_{4} bounce laser with a stigmatic cavity configuration without intracavity cylindrical lenses [55].

Alexandrite (Cr-doped chrysoberyl), a vibronic laser crystal [56], is attractive due to its broad tenability (700–850 nm) and excellent thermo-mechanical properties. The direct generation (output power \sim 2 W) of an optical vortex output from the red diode pumped Alexandrite slab lasers with a bounce geometry, in which the handedness of the vortex output is controlled merely by cavity adjustments, was also demonstrated [57]. What should be noted in these demonstrations of side pumped vortex lasers is that the characteristics of the beam output evolves with pump power as a sufficient thermal lens must develop in order to suppress the Gaussian resonator mode. The systems are, however, conducive to generating very high output power vortex beams.

2.4. Computer-controlled vortex lasers

It is possible to generate vortex laser beams with almost arbitrary wavefront properties through the application of spatial light modulators (SLMs) within the laser cavity. Here, a SLM can act as a resonator mirror, and through appropriate programming of the SLM it is possible to tailor the wavefront and spatial intensity profile of the laser mode to generate/ resonate LG modes. This negates the requirement of custom designed and fabricated optics and has been demonstrated in [43, 44]. In this work, a laser resonator was formed using Nd:YAG with a cavity configuration comprising a plane mirror and a back reflector, SLM, as shown in figures 6(a) and (b).

Ngcobø et al have successfully demonstrated the generation of on-demand laser modes, including Hermite–Gaussian (HG), radial and azimuthal higher-order LG_{p,q} and Airy modes (figure 6(c)) [58], i.e. non-diffraction beam with features of self-healing and self-acceleration. While such a computer-based laser system which offers generation of such arbitrary beams will open a new avenue of laser development, its power scaling capacity is still limited owing to optical damage of the SLM.

2.5. Other approaches including other transverse mode generations

A group led by Forbes proposed a laser system with an intracavity non-homogeneous polarization optical element (so-called q-plate), so as to couple the spin angular momentum to the OAM inside the laser cavity; it has successfully produced all states on the higher-order Poincaré sphere [59].

In addition to LG modes, it is possible to generate a range of interesting mode shapes from a laser cavity. For example, the longitudinal-transverse coupling in a high-Q laser resonator has been experimentally demonstrated [60]. The high-Q laser was formed of a thin Nd:YVO_{4} crystal (thickness: 1 mm), and two concave cavity mirrors. A fiber-coupled laser diode was used for a pump source, and its output was off-axially focused onto Nd:YVO_{4} crystal to be 25 \mu m spot (this was sufficiently smaller than a diameter of transverse cavity mode). The laser output exhibited not conventional HG modes but Lissajous shaped beam, so-called 3D coherent waves with phase singularities, owing to the longitudinal-transverse mode coupling, which transformed through the beam propagation as shown in figure 7.

The generation of Ince–Gaussian modes has also been demonstrated from an off-axial pumped solid-state laser with a high-Q cavity configuration, whereby there is a continuous transition between HG and LG modes, i.e. eigen modes in an elliptic coordinate system. Otsuka et al demonstrated the

![Figure 4. Experimental setup for Nd:GdVO_{4} vortex laser with a centrally damaged output coupler. Reproduced from [52]. CC BY 3.0.](Image 110x648 to 487x773)
generation of a single-longitudinal Ince–Gaussian mode from an end-pumped Nd:YAG ceramic laser by carefully adjusting the pump position and the angle between the laser and the pump axes [61, 62].

3. Nonlinear wavelength conversion of optical vortex sources

Nonlinear wavelength conversion is a powerful method for extending the wavelength of optical vortices; in particular, when implemented in an intracavity configuration, it enables the development of wavelength-variant vortex lasers with high pump to output conversion efficiency. As will be shown in the following section, it is not only a powerful way to manipulate the wavelength of generated beams, but it can also be used to manipulate the wavefront and spatial profile of the generated beams. Nonlinear conversion of vortex beams is an attractive approach to generating vortex beams in the UV and infrared wavelength range, particularly as there are very few external modulation devices such as SPPs and SLMs which operate in these ranges.

In this chapter, we cover nonlinear wavelength conversion techniques which have been applied to vortex laser systems. These processes include second order processes such as extra-cavity second and third harmonic generation, intracavity second harmonic generation and OPO, to produce wavelengths in the UV and infrared with high efficiency and wide tunability. Also, we detail the use of intracavity SRS, a third-order nonlinear process, to produce multi-color vortex laser output, which fills the wavelength space.

3.1. UV vortex generation

Early work concerning second harmonic generation of LG modes showed that while the frequency of the fundamental vortex beam was doubled, the azimuthal index of the mode, i.e. its topological charge, is also doubled, owing to conservation of momentum. In this investigation, it was also assumed that there is no walk-off of the fundamental beam and second harmonics arising from the birefringence of the nonlinear crystal, and that the depletion of the fundamental beam is also negligible [45].

However, noticeable walk-off effects of the nonlinear crystal will easily break the cylindrical interaction symmetry, resulting in changes to the vortex, such as the collapse of a higher order vortex, i.e. second harmonic vortex output transforms into singly charged vortices and laterally spatial separation of vortices perpendicular to the walk off direction [63].

LiB₃O₅ (LBO) [64] and periodically poled LiNbO₃ (PPLN) [65] crystals for second harmonic generation in the visible and near-infrared regions allow us to achieve noncritical phase matching (NCPM) between the fundamental vortex wave and the second harmonic vortex output. Thus, efficient second harmonic generation of the optical vortex beams in the visible and near-infrared regions has been demonstrated [66].

However, a conventional nonlinear crystal β-BaB₂O₄ (BBO) [67] in the UV region shows strong birefringence, and this results in the aforementioned spatial separation of single-charged vortices due to walk-off effects, and this impacts the production of UV vortices with high beam quality and conversion efficiency. A device formed using a series of BBO crystals with alternating orientations bonded together [68] has been proposed to compensate for walk-off effects and to improve the efficiency of UV vortex generation (figure 8). In fact, a high-quality UV (266 nm) vortex output with a moderate energy and without any spatial separation of the single-charged vortices has been successfully demonstrated with a
conversion efficiency of 13.7%, by employing a 2 mm length device consisting of four BBO crystals with lengths of 0.5 mm in combination with a Q-switched frequency-doubled Nd:YAG laser (wavelength, 532 nm; a maximum pulse energy, 6.7 mJ; a pulse duration, 25 ns) [69].

It is difficult to efficiently produce optical vortices in the deep-UV (DUV) region with wavelengths <250 nm, owing to a lack of efficient nonlinear crystals which operate in this range. DUV vortex generation has been successfully demonstrated via third harmonic generation by utilizing an intense femtosecond pulse-induced filament in air [70]. This setup involved the use of a femtosecond Ti:sapphire oscillator, a grating-based pulse-stretcher, two Ti:sapphire amplifiers, a SLM with a 4-f grating system, and a grating-based pulse-compressor, provided the LG_{0,1} mode with a pulse energy of 1.5 mJ, a pulse duration of 60 fs, and a central wavelength of 720 nm.

The femtosecond vortex pulses were focused in air by using a lens, so as to establish a filament as a third-order nonlinear material. The topological charge of the third harmonic vortex output (240 nm) was then tripled, as evidenced by a ring-spatial form and two dark fringes observed by employing an astigmatic focusing technique with a tilted focusing lens (figure 9). These indicate that such higher harmonic generation of an optical vortex still follows the OAM conservation law. The maximum conversion efficiency of $8 \times 10^{-6}$ was obtained at a fundamental vortex energy of 1 mJ.

3.2. Multi-color vortex radiation in the visible generated by intra-cavity second harmonic generation

It is also possible to make use of intracavity sum-frequency mixing to generate a host of wavelengths from a laser system which resonates on multiple wavelengths. This has been demonstrated to great effect in a self-Raman laser (details which will be covered in latter parts of this chapter) to generate a range of wavelengths in the visible range.
A Nd:GdVO₄ laser system with a high Q cavity configuration was formed of a damaged concave OC and HR back reflector, and it acted as a self-Raman laser, which generated both a fundamental vortex (1063 nm) and its Stokes vortex (1173 nm) outputs, both with a topological charge of 1. The fundamental and Stokes outputs in this system were then affected by the same damaged OC; therefore, the Stokes output should retain the same topological charge as that of the fundamental one. The Stokes output exhibited a relatively high quality $M^2$ of 2.2–2.5 at any pump levels, indicating that this laser system produces the high-quality LG₀,₁ mode with less undesired Gaussian and higher-order modes owing to the beam cleanup effects via the stimulated Raman conversion process [71].

Here, a NCPM LBO crystal was placed inside the cavity of the Nd:GdVO₄ vortex laser. As this laser generated high intensity intracavity fields at 1063 and 1173 nm, it was possible to temperature-tune the LBO crystal to perform both second harmonic of the 1173 nm line (producing 586.5 nm) and sum-frequency mixing of the 1063 and 1173 nm lines to produce 559 nm emission. This system produced a yellow output (586.5 nm) output with a maximum power of 727 mW at an optical conversion efficiency from diode to yellow output of ~4%. The yellow output exhibited an annular near-field as the vortex mode with a doubled topological charge of 2, as its wavefront exhibited two-armed fork fringes. However, it transformed into a spot with a central bright core in the far-field. The spatial profiles of the yellow output in the near- and far-field are shown in figures 10(a) and (b).

Such beam transformation under propagation can be explained by decomposing the oscillating LG₀,₁ mode into its constituent HGₘₙ,n modes, and considering an additional small phase term that manifests within the cavity [72].

The intra-cavity Stokes output $LG_{cavity}$ is expressed as,

$$LG_{cavity} = HG_{1,0} + iHG_{0,1} e^{i\Delta} = x \cdot e^{-(x^2+y^2)} + iy \cdot e^{-(x^2+y^2)} \cdot e^{i\Delta},$$

where $\Delta$ is the small dephasing term.
Thus, the yellow output, $E_{\text{SHG}}$, i.e. the second harmonics of the intra-cavity Stokes output, is given as

$$E_{\text{SHG}} \propto L_{G_{\text{cavity}}} \cdot L_{G_{\text{cavity}}} = (x^2 - y^2)e^{2i\Delta} \cdot e^{-2(x^2+y^2)}$$

$$+ 2ixy \cdot e^{i\Delta} \cdot e^{-2(x^2+y^2)}$$

$$= HG_{2,0} - HG_{0,2} \cdot e^{2i\Delta} + \frac{1}{2}(1 - e^{2i\Delta})HG_{0,0}$$

$$+ 2ie^{i\Delta} \cdot HG_{1,1}. \quad (2)$$

The above equation includes the constituent Gaussian mode, and thereby provides a central bright core to its spatial profile in the far-field. In fact, the Gouy phase difference between $HG_{2,0}$ (or $HG_{0,2}$ or $HG_{1,1}$) and $HG_{0,0}$ is

$$\Delta \phi(z) = 2\tan^{-1}\left(\frac{z}{2R}\right) \sim \pi(z \gg z_R). \quad (3)$$

Thus, the constituent Gaussian mode interferes constructively (or destructively) with the vortex mode in the near- (or far-) field, resulting in a spatial transformation of the yellow output through beam propagation in figures 10(c) and (d). The lime vortex output (559 nm) exhibits a similar spatial transform to that of the yellow output, and its beam propagation can also be explained in the same way [73].

### 3.3. Tunable vortex OPO

#### 3.3.1. Pioneering works

OPOs, in which an initial photon, designated the ‘pump’, is split into two lower energy photons (with a longer wavelength), termed ‘signal’ (high-frequency) and ‘idler’ (low-frequency) photons [74, 75], are attractive for their ability to significantly increase the diversity of lasing wavelengths. They have been used to great effect to generate tunable laser outputs with tunable bands ranging across the visible to infrared.

One of the pioneering works in vortex OPOs is that of Smith et al where they proposed the use of a singly resonant OPO with a nonplanar ring cavity configuration formed using four plane mirrors, to produce optical vortex output owing to a 90° image rotation on each cavity pass (figure 11(a)) [76]. The nonlinear OPO crystal used was a 15 mm long KTiOPO4 (KTP) crystal, which allows phase-matching among the extra-ordinary 800 nm signal (e-wave), the ordinary 1.588 μm idler (o-wave), and the ordinary 352 nm pump beams. Cavity mirrors, $M_s$, and $M_b$, were HR for 800 nm. The plane containing $L_4$ and $L_1$ was perpendicular to that containing $L_1$ and $L_2$, ensuring that an e-wave was an s-polarized wave for mirror $M_1$ and a p-polarized wave for mirror $M_2$. An s-polarized wave for mirror $M_3$ was also a p-polarized wave for mirror $M_4$. With this system (where $L_1 = L_3, L_2 = L_4$, and $L_1/L_2 = \sqrt{2}$), a circulating cavity beam was rotated by 90° on each cavity pass. The off-axis CW seeding (seed power ~1 mW) in this cavity produced an optical vortex output with a central dark core, as shown in figure 11(b). Radially or azimuthally polarized beams, which may be expressed as a coherent superposition of circularly polarized optical vortex modes, will be potentially produced by using this cavity configuration.

Martinelli et al investigated OAM conservation in a nondegenerate, triply resonant CW KTP-OPO. In OPOs, energy and momentum conservation between the pump, signal and idler photons must be established, i.e. $\omega_s + \omega_i = \omega_p$ and $k_s + k_i = k_p$, where $\omega_s$, $\omega_i$, and $\omega_p$ are the lasing frequencies of the signal, idler and pump photons; $k_s$, $k_i$, and $k_p$ are the wavenumbers of the signal, the idler and the pump photons, respectively. Optical vortex pumped OPOs also require OAM conservation, i.e. topological charge...
3.3.2. OAM selection rule in singly resonant OPO. The OAM selection rule from the pump to the signal or idler is well established in a singly resonant OPO cavity. The stability of a higher-order mode, such as a vortex mode, in the singly resonant OPO cavity for the signal can be characterized by the Fresnel number $F = (a^2/\lambda L)$ [39], where $a$ is the aperture size of the cavity, $\lambda$ is the wavelength of the signal beam, and $L$ is the effective cavity length. $F$ is inversely proportional to the effective cavity length. Therefore, the signal output can be made to lase in the vortex mode by employing a compact cavity with a larger $F$, leading to selective transfer of the topological charge of the pump beam to the signal beam. The resulting idler output has a Gaussian spatial form without any phase singularities.

In contrast, an extended cavity with a low $F$ provides a large diffraction loss for the higher-order mode, thereby forcing the signal beam to lase in the Gaussian mode. The resulting idler beam exhibits an annular spatial profile, indicating the topological charge transfer from the pump beam to the idler beam. Thus, the topological charges of the signal and idler outputs can be selectively switched simply by shortening or extending the linear cavity.

In fact, the parametric gain in the singly resonant OPO for the signal is governed by the spatial amplitude overlap efficiency, $\eta_{\ell_p \ell_s}$, of the pump and signal in nonlinear crystals, given by [75]

$$\eta_{\ell_p \ell_s} = \int_0^\infty \frac{E_p E_s}{2\pi} r \, dr \propto \int_0^\infty \left( r^{\ell_p + \ell_s} \exp \left( -\frac{r^2}{\omega_p^2} \right) \right) \cdot \left( r^{\ell_p} \exp \left( -\frac{r^2}{\omega_s^2} \right) \right) \cdot 2\pi r \, dr,$$

where $E_p$ and $E_s$ are the normalized amplitudes of pump and signal electric fields, $\ell_p$ and $\ell_s$ are the topological charges, and $\omega_p$ and $\omega_s$ are the electric mode field sizes of the pump and signal, respectively. We then assume that the topological charge conservation, $\ell_p + \ell_s = \ell_i$ is established.

A compact OPO cavity, in which the normalized gain factor for vortex mode $g$ defined as $\eta_{1,1}/\eta_{0,0}$, i.e. the parametric gain for the vortex mode versus the parametric gain for the Gaussian mode, sufficiently exceeds unity (i.e. 1), allows preferential vortex mode lasing (not Gaussian mode lasing) of the signal, thereby producing the idler output with a Gaussian mode. In contrast, an extended OPO cavity, in which the cavity mode fields expand, prevents the signal from lasing in a vortex mode due to high diffraction loss for the vortex mode, and instead produces a Gaussian signal and a vortex idler.

3.3.3. Tunable near-infrared vortex OPO. A wavelength-tunable optical vortex laser source operating in the near infra-red has been demonstrated, making use of a 532 nm nanosecond optical vortex pumped OPO system, as illustrated in figure 12 [77, 78]. The output from a conventional frequency-doubled Q-switched Nd:YAG laser with a wavelength of 532 nm and a pulse duration of 25 ns was shaped to be a first-order optical vortex with a topological charge of $\ell_p = 1$. This vortex beam was collimated and injected into an OPO formed using two cascaded NCPM LBO crystals mounted in a heater block, to produce high parametric gain and narrowband parametric emission. The wavelengths of the signal and idler outputs were tuned by adjusting the temperature of the cascaded LBO crystals and hence the phase matching conditions. Note that a singly resonant cavity was used for the signal output, consisting of a flat input mirror coated HR for 800 nm, and a partially reflective (80%) concave OC for 800 nm. This cavity configuration enables the selective transfer of OAM of the pump vortex beam to the signal output.

A stable compact cavity (effective cavity length ~215 mm) enabled the signal output to lase in a first-order vortex mode ($\ell_s = 1$) with an annular intensity profile due to a phase
singularity. The idler beam then exhibited a Gaussian spatial profile without any phase singularities. The wavelengths of the signal and idler outputs were then measured to be 900 nm and 1.3 μm, respectively.

When the cavity was extended, the signal and idler outputs lased in a mixed mode with a shallow central dip, indicating an incoherent spatial coupling between the Gaussian and first-order optical vortex modes. The signal output was converted into a Gaussian mode without any phase singularities by further extension of the cavity (effective cavity length ~435 mm). In this case, the idler beam transformed into an optical vortex mode with a first-order phase singularity. Figure 13 summarizes the spatial forms and wavefronts of the signal and idler outputs for various cavity lengths.

The optical vortex output wavelength could also be readily tuned within a wavelength region of 735–990 nm for the signal and 1.13–1.903 μm for the idler merely by controlling the temperature of the LBO crystals. The resultant vortex pulse energy was then in the range of 0.24–2.36 mJ at a pump energy of 9 mJ, corresponding to an optical–optical efficiency of 0.3%–26%. It is worth noting that vortex output was possible within the wavelength region of 900 nm–1.13 μm (wavelength gap). This is because the signal and idler outputs lased in a mixed-mode, owing to a double resonance. Very recently, Araki et al successfully achieved the vortex output generation with a wavelength tunability of over 2 octaves (from 670 nm to 2.57 μm) by optimizing the cavity configuration and the pump source [79].

Recently, Aadhi et al reported that three different OAM states of the signal and idler outputs \( \ell_s = 2, \ell_i = 0; \ell_s = 1, \ell_i = 1; \ell_s = 0, \ell_i = 2 \) can be selectively produced in a cw doubly-resonant MgO-doped stoichiometric LiTaO\(_3\) (PPSLT) OPO pumped by a 2nd-order green optical vortex by modulating any cavity loss (for example by replacing an OC and tilting an intracavity wavelength separator for the signal and idler) (figure 14) instead of the cavity length, e.g. Fresnel number [80, 81].

### 3.3.4. Tunable 2 μm vortex OPO

Figure 15(a) shows a schematic diagram of a tunable 2 μm optical vortex source based on a 1 μm optical vortex pumped KTiOPO\(_4\) (KTP)-OPO [82, 83]. A first-order 1.06 μm nanosecond optical vortex with a topological charge of \( \ell = 1 \) was loosely focused into cascaded KTP crystals mounted on computer controlled Galvano stages, enabling type II (ordinary wave (o-wave) → ordinary wave (o-wave) + extraordinary wave (e-wave)) phase matching. A cascaded KTP crystal configuration in this setup can compensate for a lateral displacement of the idler output (extraordinary wave) arising from walk-off effects, thus allowing wide wavelength tunability of the signal and idler outputs [84]. The cavity, composed of a HR coated input mirror and a 50% reflective concave OC (a stable cavity configuration), allows a single resonance for the signal output (ordinary wave), resulting in topological charge of the pump beam to be selectively transferred to the signal output. The wavelength of the signal

![Figure 12](image-url) Experimental setup of the 532 nm first-order optical vortex pumped NCPM-LBO OPO with a linear cavity configuration. The cavity can be extended by a translator. Reproduced from [77]. CC BY 3.0.

![Figure 13](image-url) (a), (c) Transverse beam profile and (b), (d) self-referenced fringes of the signal (900 nm) and idler (1300 nm) beams, respectively, for a compact cavity configuration (~215 mm). (e), (g) Transverse beam profile and (f), (h) wavefronts of the signal and idler beams, respectively, for a cavity length of 315 mm. (i), (k) Transverse beam profile and (j), (l) self-referenced fringes of the signal and idler beams, respectively, for an extended cavity configuration (~435 mm). Reproduced from [77]. CC BY 3.0.
or idler output can also be tuned by controlling the angles of the KTP crystals.

The signal output for this system lased with a first-order optical vortex mode as shown in figure 15(b). The idler output then had a Gaussian profile without any phase singularities (figure 15(c)). The maximum energies of the signal (1.949 μm) and idler (2.339 μm) outputs were measured to be 3.5 mJ and 2.5 mJ at a pump energy of 31 mJ, respectively, corresponding to slope efficiencies of the signal and idler outputs of 14% and 8%.

The lasing wavelength of the signal and idler outputs were tunable across wavelength regions of 1.820–1.954 μm and 2.561–2.335 μm, respectively. When the cascaded crystal geometry was not used, the wavelength tunability of the signal and idler outputs was limited to 1.923–1.955 μm and 2.376–2.335 μm due to lower parametric gain and walk-off effects. In addition, the experimental energies of the signal (1.949 μm) and idler (2.339 μm) outputs were limited to 1.2 and 1.0 mJ at a maximum pump level.

The operation of an oscillator with a plane-parallel cavity configuration for degenerate operation (doubly resonant condition, in contrast to the stable resonator design discussed above), in which the signal and idler outputs have the same wavelength of 2.128 μm has been investigated. In this case,
the OAM of the pump beam was evenly shared between the signal and idler outputs [85]. As a consequence of this sharing of the topological charge, the signal output lased with as a fractional optical vortex with a non-integer topological charge [86, 87], as evidenced by a half-crescent shaped spatial profile as shown in figure 16. This is a highly significant finding as it demonstrates that splitting of a topological charge is possible.

3.4. Mid-infrared vortex source

The generation of optical vortices with wavelengths in the mid-infrared has been demonstrated by making use of combined processes of OPC and difference frequency generation (DFG) [83, 88]. The OAM conservation law for DFG dictates that \( \ell_d = \ell_s - \ell_i \), where \( \ell_s \), \( \ell_i \), and \( \ell_d \) are the topological charges of the signal, idler, and difference frequency photons. As described in the previous section, an optical vortex pumped OPO with a stable cavity configuration allows the topological charge \( \ell_p \) of the pump beam to transfer to the singly resonant signal output and the resulting relationship, \( \ell_d = \ell_p \), can be established.

The signal and idler outputs, generated collinearly from the 2 \( \mu \)m OPO mentioned in section 3.3.4, were collimated so as to avoid walk-off effects in a ZnGeP\(_2\) (ZGP) crystal [89], and they were directed into the ZGP crystal for DFG (figure 17(a)). The ZGP crystal with a high transmission in the mid-infrared region enabled us to achieve collinear type-I phase matching (o–e → e) in the range 6.3–12 \( \mu \)m.

As shown in figure 17(b), the lasing wavelength of the difference frequency output was tuned within a region of 6.3–12 \( \mu \)m without any degradation of the beam quality. This tuning range was limited by the wavelength tunability of the 2 \( \mu \)m vortex pumped OPO. The maximum achieved pulse energy of the 6.5 \( \mu \)m vortex output was 390 \( \mu \)J at a pump energy (total energy of signal and idler output) of \(~4.6\) mJ (figure 17(c)). The corresponding conversion efficiency from the 2 \( \mu \)m output to the 6.5 \( \mu \)m output was 3.5\% [83].

Many applications of optical vortices often require not only wavelength versatility, but also handedness control. In the context of the DFG system, there are two methods to change the handedness of the difference frequency output. One way is to invert the handedness of the optical vortex pump beam for a 2 \( \mu \)m OPO by reversing the 1 \( \mu \)m SPP. However, this requires realignment of all the optical elements for the 2 \( \mu \)m OPO. An alternative is to swap the wavelengths of the signal and idler fields, in which the handedness of the difference frequency output can be selectively controlled by adjusting the ZGP angle [88]. In fact, it was demonstrated that both right- and left-handed vortex outputs can be obtained within a wavelength region of 6.0–12.5 \( \mu \)m by swapping the signal and idler wavelengths (figures 17(d) and (e)).

The resulting general relationship for the topological charge of the difference frequency output is given as follows:

\[
\ell_d = \ell_p \left( \frac{\omega_s - \omega_i}{\omega_s - \omega_i} \right).
\]

3.5. Multi-color vortex generation via stimulated Raman conversion

In the stimulated Raman conversion process, a fundamental photon excites a phonon associated with the vibration modes of specific materials, leading to energy coupling among the incident photon, the scattered photon (Stokes photon) and the phonon. Conservation of OAM under the process of stimulated Raman conversion should also be considered; however, it has not yet been fully established experimentally and theoretically.

As mentioned in section 2.2, cw self-Raman laser based on an end-pumped Nd:GdVO\(_4\) laser with a central damaged OC has been developed. The fundamental and Stokes outputs in this system were then affected by the same damaged OC; therefore, the Stokes output should retain the same topological charge as that of the fundamental one. In fact, it was mentioned that the topological charge of the photon in a stimulated scattering event remains unchanged. As detailed in section 3.2, though implementation of intracavity sum frequency mixing, it is possible to produce a range of cw visible vortex wavelengths from this laser configuration.

More recently, within the context of pulsed vortex laser outputs, Zhi et al investigated the interaction of femtosecond optical vortex pulses in a Raman active PbWO\(_4\) crystal [47, 90], in which the pump and first-Stokes pulses with specified topological charges of \( \ell_p \) and \( \ell_e \), respectively, interacted to generate cascaded higher-order Stokes (S) and anti-Stokes (AS) pulses. In this case, the topological charges, \( \ell_p^{\text{AS}} \), and \( \ell_e^{\text{S}} \), for the nth-order AS and S pulses should be predicted theoretically as follows:

\[
\ell_p^{\text{AS}} = \ell_p + n(\ell_p - \ell_e),
\]

and

\[
\ell_e^{\text{S}} = \ell_p - n(\ell_p - \ell_e).
\]

For instance, when \( \ell_p = 1 \) and \( \ell_e = 0 \), \( \ell_p^{\text{AS}} \) and \( \ell_e^{\text{S}} \) are given by \( n \pm 1 \).

A pump pulse with a pulse duration of 40 fs, a center wavelength of 806 nm and a pulse repetition frequency of 1 kHz and a Stokes pulse with a wavelength of 870 nm,
generated from a femtosecond laser system formed of a Ti:sapphire laser, regenerate amplifier, an optical parametric amplifier, and a frequency doubler. Both the pump and the Stokes pulses were injected into a thin Raman crystal PbWO4. The pump beam was shaped to be an optical vortex with \( \ell_S = \pm 1 \) by a SPP. Up to 8th AS (507 nm) sideband pulses were observed, and the topological charges of 1–5th AS pulses (751–589 nm) were assigned to be 2–6 by a tilted lens and interferometric techniques. There is good agreement between the experiments and theoretical predictions (figure 18).
4. Conclusion

We have reviewed recent progress concerning the direct generation of optical vortex beams from solid-state lasers, and wavelength extension of vortex lasers through processes of nonlinear frequency conversion. Wavelength conversion through second order processes such as second (or third) harmonic generation, OPOs and DFG, and third order processes such as SRS, and combinations of these, have proven effective at extending the wavelength span of vortex laser sources. Interestingly, in addition to wavelength conversion, these nonlinear processes offer unique methods of manipulating the OAM state of the vortex beams. It is apparent that the combination of direct generation of vortex beams and nonlinear conversion has significant potential to produce wavelength- versatile optical vortex laser sources with substantial energy and power levels in the UV, visible, infrared, and potentially the terahertz wavelength regions. It is anticipated that this will lead to opportunities for new research fields [91, 92], which exceed the capabilities of conventional optical technologies such as optical manipulation and optical telecommunication.

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