Abstract—The article presents the dynamic characteristics of a car with a two-stage spring suspension. The author investigates a system with asymmetric mass-inertial characteristics. Depending on the ratios of the mass-inertia parameters, possible solutions of differential equations are obtained. It follows from the calculations that a change in the system parameters significantly affects the linear and angular generalized coordinates.

Keywords—oscillations; mechanics; dynamic; two-stage spring suspension; displacement

I. INTRODUCTION

Several main directions can be traced in studies of the dynamics of rolling stock and its interaction with the track. These are analytical research methods, methods based on the numerical integration of differential equations, and a full-scale experiment. However, the requirements for increasing speeds and load-carrying capacity, which, in turn, pose new problems of wear [1] and stability, force us to consider these problems in a more systematic and fundamental way.

Studying vehicle dynamics is challenging. Even when moving along a direct route, when the movement is carried out at low speed, there are problems that arise due to hunting oscillations [2]. Dynamic forces that arise from the movement of the car in the train operation, as well as deviations from the equilibrium position, and inertial overloads, which affect passengers and cargo, are the result of oscillatory processes and other types of non-uniform movement of inertial masses that make up the mechanical system under consideration [3]. The magnitude and frequency of oscillations primarily determine the dynamic qualities of the car: overall safety, smoothness, stability in motion, as well as the magnitude of the forces that affect the strength of the elements of the car and railway track [4].

The purpose of studying the car's oscillations is to ascertain their physical nature and the reasons, to determine the permissible level of dynamic effects generated by these oscillations and to develop recommendations for choosing the design parameters of the car. These parameters which ensure its high dynamic qualities during operation, which can also be implemented using control [5–8].

Let us examine the fluctuations of railroad car in more detail on the model of crew with the two-stage spring suspension for purposes of the estimation of the smoothness of its motion and theoretical substantiation of the selection of mass-inertia characteristics, whose diagram is given in Figure 1 [9]. Sometimes good results can be obtained by using self-adjusting damping devices [9], but the system will be greatly complicated at that point.

II. RESULTS AND DISCUSSION

For the study of free oscillations of the sprung parts of the car without taking into account the friction forces, the
following symbols were agreed: \( m_k, m_{T1}, m_{T2} \) - the mass of the body and the bogies, respectively; \( I_K \) - moment of inertia of the body when pitching; \( c_{11}, c_{12} \) - vertical stiffness of the central suspension of the trolley; \( c_{21}, c_{22}, c_{31}, c_{32} \) - the vertical stiffness of the axle box suspension of the wheelset; \( z_K, z_{T1}, z_{T2} \) - the current vertical displacement of the center of gravity, respectively, of the body, the first and second bogies; \( \varphi_K, \varphi_{T1}, \varphi_{T2} \) - the current angular displacement of the body, the first and second bogies, respectively; \( L_1 + L_2 \) - body base; \( \beta_{21}, \beta_{22}, \beta_{31}, \beta_{32} \) - the resistance of the dampers of the axle box suspension sets of the wheelset.

Differential equations of oscillation of bouncing and pitching of the body, bouncing and pitching of the vehicle bogies without friction, are described by a system of six second-order differential equations (1).

\[
\begin{align*}
\[ m_k z_K + (c_{11} + c_{12}) z_K + (c_{11} L_1 - c_{12} L_2) \varphi_K - \\
-c_{11} z_{T1} + c_{11}(a_{21} - a_{22}) \varphi_{T1} - c_{12} z_{T2} - \\
-c_{12}(a_{21} - a_{22}) \varphi_{T2} = 0; \\
\end{align*}
\]

\[
\begin{align*}
\[ I_k \varphi_K + \left( c_{11} L_1^2 + c_{12} L_2^2 \right) \varphi_K + (c_{11} L_1 - c_{12} L_2) z_K - \\
-c_{11} L_1 z_{T1} + c_{11} L_1(a_{21} - a_{22}) \varphi_{T1} + \\
+c_{12} L_2 z_{T2} + c_{12} L_2(a_{21} - a_{22}) \varphi_{T2} = 0; \\
\end{align*}
\]

\[
\begin{align*}
\[ m_{T1} \varphi_{T1} + (c_{11} + c_{12} + c_{22}) z_{T1} - \\
\left[ c_{11}(a_{21} - a_{22}) - c_{22} a_{21} + c_{22} a_{22} \right] \varphi_{T1} - c_{11} z_{T1} - \\
-c_{11} L_1 \varphi_K = 0; \\
\end{align*}
\]

\[
\begin{align*}
\[ I_{T1} \varphi_{T1} + \left[ c_{11}(a_{21} - a_{22})^2 + c_{22} a_{21}^2 + c_{22} a_{22}^2 \right] \varphi_{T1} - \\
-\left[ c_{11}(a_{21} - a_{22}) - c_{22} a_{21} + c_{22} a_{22} \right] z_{T1} + \\
+c_{11}(a_{21} - a_{22}) z_K + c_{11} L_1(a_{21} - a_{22}) \varphi_{T1} = 0; \\
\end{align*}
\]

\[
\begin{align*}
\[ m_{T2} \varphi_{T2} + (c_{12} + c_{11} + c_{22}) z_{T2} + \\
\left[ c_{12}(a_{22} - a_{21}) + c_{22} a_{21}^2 - c_{22} a_{22}^2 \right] \varphi_{T2} - c_{12} z_{T2} + \\
+c_{12} L_2 \varphi_K = 0; \\
\end{align*}
\]

\[
\begin{align*}
\[ I_{T2} \varphi_{T2} + \left[ c_{12}(a_{22} - a_{21})^2 + c_{22} a_{21}^3 + c_{22} a_{22}^3 \right] \varphi_{T2} + \\
\left[ c_{12}(a_{22} - a_{21}) + c_{22} a_{21}^2 - c_{22} a_{22}^2 \right] z_{T2} - \\
-c_{12}(a_{22} - a_{21}) z_K + c_{12} L_2(a_{22} - a_{21}) \varphi_{T2} = 0. \\
\end{align*}
\]

(1)

It was previously shown that the motion of a system can be represented by differential equations independent of each other. In practice, the determination of the principal coordinates of system (1) is a task of the same order of difficulty as a complete study of the free oscillations of this system in generalized coordinates.

To obtain a analytical solution of equations (1) is quite difficult. Therefore, we use the mathematical software package MathCAD to study them.

As an example, let us take the following parameters of the system:

\[
\begin{align*}
\[ m_k = 3.6300 \text{ ts}^2/\text{m}, \quad m_{T1} = 0.3991 \text{ ts}^2/\text{m}, \quad m_{T2} = 0.3801 \text{ ts}^2/\text{m}, \quad I_K = 103.2820 \text{ tm/s}^2, \quad c_{11} = 118.6575 \text{ ts}/\text{m}, \quad c_{12} = 124.7425 \text{ ts}/\text{m}, \quad c_{21} = 237.0500 \text{ ts}/\text{m}, \quad c_{22} = 249.7500 \text{ ts}/\text{m}, \quad c_{31} = 237.7500 \text{ ts}/\text{m}, \quad c_{32} = 249.0500 \text{ ts}/\text{m}, \quad L_1 = 6.4580 \text{ m}; \quad L_2 = 6.1420 \text{ m}; \quad a_{21} = 0.5130 \text{ m}, \quad a_{22} = 0.4870 \text{ m}, \quad a_{31} = 0.5100 \text{ m}; \quad a_{32} = 0.4900 \text{ m}. \\
\end{align*}
\]

Calculation of the dependence of the lateral displacement of body \( z_K \), of the first \( z_{T1} \) and second \( z_{T2} \) bogies and angle of rotation of body \( \varphi_K \) and bogies \( \varphi_{T1}, \varphi_{T2} \) on the time indicate that the natural oscillations of the masses of body and the bogies, which are the sum of two harmonic oscillations, are non-damped, since while solving the problem we disregarded the action of the inelastic resisting forces of the dampers, which suppress natural oscillations.

In the absence of damping in the system, its natural oscillations are not damped, and the period, amplitude and frequency of oscillations of the body bouncing will be equal to \( T = 0.70 \text{ (sec)} \), \( \lambda_1 = 0.228 \text{ (m)} \) and \( n_1 = 1.350 \text{ (Hz)} \), respectively.

The characteristics of natural oscillations of the body pitching on the basis of the solution obtained are the following: \( T_2 = 0.66 \text{ (sec)} \), \( \lambda_2 = 0.009 \text{ (m)} \), \( n_2 = 1.505 \text{ (Hz)} \).

Since the periods of oscillation of bouncing and pitching of the body differ little \( T_1 \approx 0.7 \text{ (sec)} \), \( T_2 \approx 0.66 \text{ (sec)} \), the oscillations of the bogies \( z_{T1} \) and \( z_{T2} \) have the form of beatings with the period \( T_3 = T_4 \approx 0.60 \text{ (sec)} \) and amplitude \( \lambda_3 = \lambda_4 \approx 0.0429 \text{ (m)} \).
With symmetric mass-inertia, harmonic oscillations were obtained for all six degrees of freedom. The angular oscillations of the bogie one and the bogie two are of beating nature. The maximum amplitudes of oscillations were:

a) for the body: linear vibration is 0.028 m (Figure 2), angular is 0.69° (Figure 3);

b) the first and second bogies: the linear oscillation has the nature of beatings with a maximum amplitude of 0.021 m, the minimum is 0.014 (Figure 4-5), the angular one is 0.05° (Figure 6-7).

In case the symmetry is violated: the center of masses of the body has shifted 0.316 m forward. The moment of inertia about the axis $OY$ increased and became equal to $I_y = 103.282 \text{ tm}^2$. The nature of the vibrations of the body has changed. The movement has become of a beating nature.
Fig. 7. The angular displacement of the second bogie

From the frequency equation we determine the frequencies of free oscillations $k_1 = 57.3755$, $k_2 = 73.7959$, $k_3 = 121.6480$, $k_4 = 121.7362$, $k_5 = 1534.0870$, $k_6 = 1626.5658$. We find the shapes of the principal oscillations.

$$A_1(k) = (-1)^{2k}$$

$$A_2(k) = (-1)^{2k}$$

$$A_3(k) = (-1)^{2k}$$

$$A_4(k) = (-1)^{2k}$$

$$A_5(k) = (-1)^{2k}$$

$$A_6(k) = (-1)^{2k}$$

where $b_{21} = c_1L_1 - c_2L_2$, $b_{11} = -c_1$, $b_{11} = c_1(a_{21} - a_{22})$, $b_{21} = -c_1$, $b_{61} = -c_1(a_{31} - a_{32})$, $b_{22} = (c_1L_1^2 + c_2L_2^2)$. $b_{31} = c_1$, $b_{31} = c_1L_1(a_{32} - a_{31})$, $b_{25} = -c_1$, $b_{25} = c_1L_1$, $b_{33} = c_1 + c_2 + c_3$, $b_{31} = c_1(a_{31} - a_{21}) - c_2a_{21} + c_2a_{22}$, $b_{34} = c_1(a_{31} - a_{21})^2 + c_2a_{21}^2 + c_2a_{22}^2$, $b_{36} = c_1(a_{31} - a_{31}) + c_1a_{31} - c_2a_{31}a_{32}$, $b_{36} = c_1(a_{31} - a_{31})^2 + c_1a_{31}^2 + c_2a_{31}^2$.

Substituting successively all the roots into (2), we find the distribution coefficients.

III. CONCLUSION

According to the calculated distribution coefficients, it is possible to represent the forms of the principal oscillations in graphical form.

The ordinates of the points of these graphs usually depict the amplitudes of oscillations on a representative scale and mark the nodes, i.e. that kind of points in the system that remain motionless all the time in a given oscillation.

According to the results of the study, it was found that for an asymmetric system, in contrast to a symmetric one, oscillations in all coordinates are of the nature of a beating.

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