Energy harvester for use on nearby current-carrying conductors

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Abstract. We report a concept of a new electret-based energy harvester that can extract energy from a nearby conductor carrying a time-varying current. In the harvester, a permanent magnet located near an electret is made move relative to the electret by the magnetic interaction of the circumferential time-varying magnetic field produced by the current flow and the static magnetic field of the permanent magnet. The advantage of this harvester over harvesters that employ piezoelectric elements is that piezoelectric materials are known to fatigue over time, whereas electrets having extremely long lifetimes are possible.

1. Introduction
Energy harvesters that couple to magnetic fields surrounding wires carrying AC current have been presented in previous publications [1, 2] using piezoelectric transduction. However, piezoelectric material has a problem of the material degradation that results to change the resonance of energy harvesters [3]. Recent development of long lifetime electrets [4-6] is attractive to explore electret-based energy harvesters for use nearby the AC-current carrying conductors.

![Figure 1](image)

Figure 1: Drawing concept of electret-based energy harvester (EH) near the AC current-carrying conductors. (a) A permanent magnet is mounted on a movable electrode, suspended by springs. The harvester is placed in an orientation of $45^\circ$. (b) A permanent magnet is rested on a rigid surface, inside a cavity, without spring supporting. The harvester is placed an orientation of $0^\circ$.

A proposed electret-based energy harvester (EH) for use nearby the AC-current carrying conductor is depicted in figure 1(a). The structure of the harvester is similar to an out-of-plane energy harvester from ambient motion [7], except for adding a permanent magnet on the movable electrode. The magnet is oscillated mechanically by the circumferential magnetic field around the conductor and...
actuates the EH. The device is placed in an orientation of 45° from the conductor center to obtain the strongest force in the vertical direction on the magnet [1, 2].

The presence of suspension such as cantilever beams or folder-springs could limit the harvester lifetime due to their mechanical fatigue in long term of oscillations. Moreover, more space is required to design the suspended systems. The elimination of suspension seems to be more interested in design of vibration EHs. A propose of EH without suspension is shown in figure 1(b). The magnet is rested to design the suspended systems. The elimination of suspension seems to be more interested in design lifetime due to their mechanical fatigue in long term of oscillations. Moreover, more space is required along the direction perpendicular to the line current and to the position vector r.

In this paper, we will report a concept of an electrostatic EH without suspension as illustrated in figure 1(b). The equations of motion and experiment to verify the device concept will be presented.

2. Device modelling

2.1. Forces and torques on the magnet

![Figure 2: Geometry of element magnets nearby an AC-current carrying conductor $i_c$, $\vec{H}$ is the circumferential magnetic field from the conductor. $\vec{B}$ is the flux density of the magnet.](image)

![Figure 3: Forces on the magnet. The magnet will rotate anti-clockwise around A when the current $i_c$ is in $-z$ direction, and clockwise around B when $i_c$ flips.](image)

We consider two elements of the magnet at the coordinate of $(x, y)$ and $(-x, y)$ (figure 2). The cable along the z-axis carries a current $i_C = I_C \sin (2\pi ft)$ in the $-z$ direction. We assume that the flux density $\vec{B}$ of the permanent magnet is uniform and aligns in the positive y-direction ($\vec{B} = iB_y$). A magnetic field at a distance $r$ from the origin, created by the conductor, has the magnitude of $H = i_C/(2\pi r)$ and the direction perpendicular to the line current and to the position vector $r$ as shown in figure 2.

The magnetic forces on the magnet elements are given by

$$dF_y = \frac{B_y i_C}{2\pi} \frac{2xy}{(x^2+y^2)^2}, \quad dF_x = -\frac{B_y i_C}{2\pi} \frac{y^2-x^2}{(x^2+y^2)^2},$$

(1)

The direction of the forces on the elements is shown in figure 3. The forces in $y$-direction will create a torque and rotate the magnet in the anti-clockwise (and in clockwise when $i_c$ changes to z direction) when the current is larger than the current threshold $I_C$. Assuming the magnet thickness $t$ is very small compared to its length $L$, the total magnetic torque due to the magnetic force is approximated by

$$\tau_{magnet} = 2 \iiint_{V/2} x \, dF_y \, dV \approx \frac{B_y i_C}{\pi} \frac{y}{r_0},$$

(2)

in which $V$ is the magnet volume and $r_0$ is the distance from the lower edge of the magnet to the cable center. At rest, the torque due to the gravitational force $mg\hat{y}$ and the electrostatic force $F_e$ will be
balanced by the torque of the reaction force from the rigid surface. The magnet starts rotating when the magnetic torque is larger than the torque of the gravitational and electrostatic forces. In the order word, to actuate the magnet, the current in the cable must be larger than the current threshold, given by

$$I_{CT} = \frac{2\pi r_0}{\eta_w} (mg + F_e)$$

(3)

where \(w\) is the magnet width.

### 2.2. Electret-energy harvesting modelling

Assuming the magnet rotates a small angle of \(\theta\) as shown in figure 4. The capacitance for a small tilt angle of \(\theta\) is approximated by

$$C = C_0 [1 + \frac{\varepsilon_0 L}{\varepsilon_c} \theta]$$

(4)

where \(C_0 = \varepsilon_0 \varepsilon_\varepsilon (\omega L/\varepsilon_c)\) is the initial capacitance (when \(\theta = 0\)), \(t_i\) is the dielectric thickness, \(\varepsilon_0\) and \(\varepsilon_c\) are the absolute permittivity and relative permittivity of the electret, respectively. The equations of motion for the energy harvester can be written as

$$\begin{align*}
I \ddot{\theta} + b \dot{\theta} + m g \frac{L}{2} \left(1 - \frac{\theta^2}{2}\right) + \tau_{elec} + \tau_{stopper} &= \frac{B_v V}{4\pi r_0} f(\theta) i_c \\
-q R &= \frac{\Gamma}{C_0} \theta + \frac{q}{C_0}
\end{align*}$$

(5)

(6)

Here, \(I\) is the moment of inertia of the magnet; \(b\) is the damping constant; \(\tau_{elec}\) is the torque of electrostatic force, and is a function of the charge on the capacitor \(q\) and the tilt angle \(\theta\); \(f(\theta)\) is a function of \(\theta\), expressing for the dependence of magnet torque on the tilt angle of \(\theta\); \(R\) is the load resistor; \(\Gamma = V_e C_0 (\varepsilon_0 L/\varepsilon_c)\) is the transduction factor; \(\tau_{stopper}\) is the stopper torque, using to express the situation of the magnet impacting the rigid surface. In the equations (5) and (6), \(\theta\) must be positive and \(i_c \geq I_{CT}\).

Deriving the closed-forms of \(\tau_{elec}\) and \(f(\theta)\) as well as solving the equations will be addressed in the other work. In this paper, experiments to verify the energy harvester concept will be focused and presented in the next section.

### 3. Experiment

Figure 5 shows the experimental set up to test the energy harvester. The output of the harvester is connected to a load resistor of 44 M\(\Omega\). In this experiment, a DC voltage, stimulating of an electret, is used to bias the harvester. The current in the cable is the 60-Hz power line. The output voltage \(V_0\) in the time and frequency domains is shown in the figure 6. The magnet plate rocks at the frequency of 60 Hz. Since lifting of the magnet on the left and the right have the same effect on the capacitance, \(V_0\) has a large component at the double frequency, 120 Hz.
Figure 6: Measurement of the output voltage in the time domain (left) and frequency domain (right), $R = 44 \, \text{M}\Omega$, $V_e = 67\, \text{V}$. The amplitude of the current in the cable is 30A.

Figure 7 shows the RMS output voltage when increasing the amplitude of the current in the conductor cable. The output voltage slowly increases when the current amplitude is larger than 20 A and sharply increases when reaching the threshold current (Eq. (3)) of 36 A. Beyond 40 A, the RMS output voltage does not increase more. The RMS output voltage when varying the bias voltage is shown in figure 8. In the bias voltage range of experiments, the RMS output voltage increases when the bias voltage increases.

Figure 7: RMS output voltage vs. amplitude current in the conductor cable, $R = 44 \, \text{M}\Omega$, $V_e = 9\, \text{V}$.

Figure 8: RMS output voltage vs. bias voltage, $R = 44 \, \text{M}\Omega$, the amplitude of the current is 30A.

4. Conclusion
A new concept of an electrostatic energy harvester without suspension for use on nearby an electrical conductor carrying an AC current has been presented. The experimental results of an energy harvester, using external bias voltage, have verified the working principle of the harvester. The RMS output voltage sharply increases when the current amplitude gets 36-40 A, and continuously increases when the bias voltage varies from 5V-67V.

References
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