Frequency shift and dissipation of compound torsional oscillator at 10 mK

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Abstract. A compound torsion oscillator (TO), having two resonant mode frequencies (492 Hz and 1163 Hz) and high Q (0.5x10^6), has become an important tool in studying the supersolid properties of 4He loaded into the main TO bob. By simultaneously exciting both modes of this TO, observations could be made on how changes in the drive amplitude of one mode affect the sample response measured by the other mode. In order to separate out the effects arising from the sample, “background” characteristics due to the unloaded TO itself were studied. The resonant frequency and response amplitude of both modes were measured at 10 mK as the drive amplitude of one mode was varied while the other was held fixed at a low drive amplitude. Unexpectedly, the resonant frequency and the response amplitude of the fixed mode decreased as the drive amplitude of the varied mode was increased. The two modes interact and affect each other. The decrease in frequency could be due to a decrease in the stiffness of the torsion rods, while the decrease in amplitude could indicate an increase in internal friction.

1. Introduction
Torsional oscillator (TO) has been an important tool in low temperature physics experiments. Its utility came to fore by the discovery[1, 2] of an apparent superfluid behavior of solid 4He contained in TO. Conventional TO operates with a single high Q resonance mode determined by a torsion rod and an attached moment of inertia and cannot be used to study frequency dependent dynamical effects. Compound TO (CTO) can overcome this deficiency, if only incrementally, by endowing two resonant modes. CTO has become a powerful tool[3, 4, 5] in examining mechanical response of solid 4He at very low temperatures. The power of CTO arises from its ability to probe the sample properties at two distinct frequencies simultaneously.

Exciting two modes simultaneously opens up new methods of observing mechanical properties of solid 4He. For example, CTO makes it possible to drive the sample into a critical velocity state by driving one mode at high amplitude and to detect its response with the other mode driven at low amplitude. This is similar to measuring magnetic response to applying external magnetic field onto a superconducting sample. There are other intriguing questions regarding two modes being simultaneously excited. Before these questions can be addressed, however, it is important to understand the background behavior of simultaneously driven empty CTO without loading it with solid 4He sample. We carried out an extensive study of these background behaviors under various combinations of simultaneous drive amplitudes. Results of these background studies are briefly presented in this report.
2. Experiment and apparatus

Our CTO shown in Fig. 1(a) is made up of two (upper and lower) identically made torsion rods and two (upper and lower cell) blocks[5]. One end of the upper rod is fixed to a vibration isolation platform and the other end to the upper block (with moment of inertia $I_u$). The second rod connects the upper block to the lower (sample) block (with moment of inertia $I_l$). The system has two modes with resonant frequencies, $f_1$ and $f_2$, for the in-phase and out-phase motion of the two blocks, respectively. Angular displacements of the upper and lower blocks are detected by separately attached electrode “fins.” Let the angular displacement amplitudes of the upper and lower rod with respect to rest frame be, $\theta_i$ and $\gamma_i$, respectively, for the mode $i$ and define the relative displacement amplitude of the lower rod with respect to the upper rod as $\phi_i = \gamma_i - \theta_i$. Assuming the torsion constants of the rods are identical ($K$) and the motion occurs without dissipation, the resonant frequencies are:

$$f_i = \frac{\sqrt{K}}{2\pi} \sqrt{\frac{I_l + 2I_u \mp \sqrt{I_l^2 + 4I_u^2}}{2I_lI_u}}. \quad (f_1 < f_2).$$

The angular displacements are related to each other by:

$$\gamma_i = \left[ \frac{2K/I_u - (2\pi f_i)^2}{K/I_u} \right] \theta_i$$

Estimated TO parameters, $K = 1.08 \times 10^8$ dyncm, $I_u = 6.33$ gcm$^2$, and $I_l = 2.47$ gcm$^2$, give expected values: $f_1 = 534$ Hz, $f_2 = 1302$ Hz, $\phi_1/\theta_1 = 0.35$, and $\phi_2/\theta_2 = -2.9$. Measured values are $f_1 = 493.149$ Hz, $f_2 = 1165.007$ Hz, $\phi_1/\theta_1 = 0.23$, and $\phi_2/\theta_2 = -2.1$. Uncertainties in the estimated TO parameters are ±20 % and those in the measured angular displacements are ±30 %. The expected and measured values agree within the uncertainties. The frequencies and $Q$ of the resonant modes are tracked by an computer controlled data acquisition system.[5]

![Figure 1](Color online) (a)Schematic of compound torsional oscillator: (1) torsion rods, (2) upper block, (3) electrode fin, (4) sample chamber, (5) lower block, (6) fill line. (b) Changes in the mode frequencies at 10 mK as the ac drive amplitude of $f_1$ mode is varied while the drive amplitude of $f_2$ mode is fixed at low level: $\Delta f_1$ as the drive is increased (red up triangle) and decreased (red down triangles), and $\Delta f_2$ as the drive is increased (blue squares) and decreased (blue lozenges).
3. Results

Naively it would appear that the two high Q resonant modes of our compound oscillator behave independently of each other when both are driven simultaneously. Our measurements show that this is not the case and the motion of one mode is affected by that of the other in an intricate manner. If one mode is driven at sufficiently large amplitude, the frequency and dissipation of the other mode decreases and increases, respectively. These effects at 10 mK are illustrated by displaying changes in mode frequencies $\Delta f_i$ in Fig. 1(b) (Fig. 2), where the ac drive voltage amplitude of $f_1$ ($f_2$) mode is varied while the ac drive voltage amplitude of the $f_2$ ($f_1$) mode is held constant and low such that corresponding sample cell rim velocity amplitude is 15 (13) $\mu$m/s. The drive voltage amplitude may be converted to sample cell rim velocity amplitude by $v_i = A_i V_{ac}$, where $A_1 = 0.8$ $\mu$m/(s mV) and $A_2 = 1.8$ $\mu$m/(s mV). Results are qualitatively similar at higher temperatures up to 60 mK. Observed direction-dependent changes seen in Fig. 1(b) are likely results of errors in our electronic tracking system. The sudden change in slope of $\Delta f_2$ at 800 mV in Fig. 2 is reproducible but its origin is not known.

An important observation is that the frequencies of both the varied and fixed modes decrease as the drive amplitude of the varied mode is increased. The observed changes in frequencies are comparable to those associated with supersolid effects. Furthermore, critical velocity effects observed in supersolid phenomenon are in the same decreasing direction. It is clear that the changes in the resonant frequencies of the background empty cell are extremely important in interpreting observed frequency shifts when the cell is loaded with solid $^4$He samples.

The net dissipation of each mode is likely determined by the motion of both rods. Bantel and Newman[6] have shown in the studies of their BeCu fiber TO that dissipation was described by the angular strain in the torsion fiber. Extending their analysis, dissipation ($\equiv Q_i$) of each mode is plotted in Fig. 3 as functions of the total sum of angular displacements of the two rods, $\theta + \phi$, in the same measurements as those shown in Fig. 1 and 2. Here, $\theta \equiv |\phi_1 | + | \phi_2 |$. The results at higher temperatures up to 60 mK are similar.

It is clear from Fig. 3 that dissipation in our CTO increases as the oscillation amplitude is increased. Observed dissipation is dependent on oscillation amplitude at the lowest drive levels applied. Observed dependence of dissipation on $\theta + \phi$ are coincident for both modes within the uncertainties of measurement whether the CTO motion is produced by variation in the drive level of the first or the second mode. This coincidence does not occur if the dissipation are plotted as functions of the sum of angular velocities, $2\pi [f_1(\theta_1 + \phi_1) + f_2(\theta_2 + \phi_2)]$. This observation suggests that the dissipation are determined by the sum of angular displacements. There is little hysteresis in dissipation as the drive is increased up and then decreased back to the original low drive level. There is a sudden increase in $Q_1^{-1}$ at $\theta + \phi = 22 \times 10^{-6}$ rad. Since $\phi$ is dominant here, the increase implies a sudden onset of additional dissipation in the first mode at the particular

![Figure 2](image-url)
value of angular displacement amplitude in the lower rod. Accompanying frequency shift does not show this sudden break as \( \theta \) changes (see Fig. 1).

4. Conclusions
A compound torsional oscillator having two resonant modes with high quality factors was fabricated in order to study the dynamic response of solid helium at low temperatures. Background mechanical properties of the oscillator were studied when the two modes were excited simultaneously but without loading a solid helium sample. As the drive amplitude of one mode was increased, both the frequency and the quality factor of the other mode (whose drive amplitude was kept constant) decreased. This unexpected “interaction” between the two modes was found to depend on the sum of angular displacements of the two modes and likely originated in the mechanical properties of the torsion rods made of BeCu in the oscillator. It is clear that careful attention must be paid to the background effects of empty torsional oscillators in interpreting their response attributed to loaded solid \(^4\)He samples. A complete description of these studies will be reported elsewhere.

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6. References
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