Floquet-state Maser under Real-time Quantum Feedback Control

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We experimentally observe a multi-mode Floquet-state maser in a periodically driven two-level system of pumped 129Xe atoms with real-time feedback control. The demonstrated maser is based on the Floquet synthetic states and shows, in principle, unlimited observation time to overcome the fundamental decoherence effect. Further taking advantage of this kind of maser, we demonstrate the 129Xe Floquet spectroscopy with sub-millihertz resolution. The reported Floquet-state maser offers new capabilities to detect ultralow-frequency magnetic fields and search for ultralight axions and axion-like particles with rest energies below 10^-16 eV (Compton frequencies below 20 mHz).

Introduction.—Periodic driving, as implemented for example by laser and radio-frequency fields, is a powerful tool for coherent control of quantum systems [1–3]. For two-level quantum systems, a weak periodic driving field on resonance can induce the well-known Rabi oscillation [4–6]. Periodic driving provides a new “knob” to prepare exotic phases of systems, such as Floquet time crystals [7–9]. Quantum system immersed in a periodic driving field can reveal coherent effects beyond thermal equilibrium and linear response [10, 11]. Recently, ultracold quantum gases in optical lattices subjected to periodic driving have attracted much attention in the coherent manipulation of many-body systems [2]. However, quantum systems are fragile and inevitably suffer from decoherence due to the coupling to environment, which is a major challenge for practical realization of quantum experiments. A key question is: how to suppress or eliminate decoherence for periodically driven quantum systems, and what new phenomena can be observed if decoherence is eliminated?

Non-periodic driving of quantum systems has been employed to overcome decoherence via dynamical decoupling [12], decoherence-free subspaces [13], and real-time feedback control [14–18]. Real-time feedback control strategy, which is motivated by the classical phase-locked loop used for stabilizing an oscillator, uses the outcome of a continuous measurement to stabilize the system about a desired state. This strategy has been successfully demonstrated in masers and lasers [14–17] based on, for example, 29Al solid [19], liquid-state proton spins [20, 21], and 3He or 129Xe gas [22–25]. This strategy has been also applied to sensitive self-oscillating magnetometers [26, 27]. Real-time feedback control has also been used to stabilize Rabi oscillations in a superconducting qubit [18] and quantum states of a nanomechanical oscillator [28].

In this Letter, we report a novel Floquet-state maser, which is based on the synthetic dimensions supported by Floquet states of a periodically driven system. We present an experimental demonstration of spin-exchange pumped 129Xe Floquet-state maser with real-time feedback control, and illustrate ultrahigh-resolution Floquet spectroscopy beyond decoherence-limited resolution. As the spectral resolution is greatly increased, a different regime emerges where high-order Floquet sidebands become significant and complex Floquet spectra are expected. The reported maser paves the way for new quantum technology for measuring ultralow-frequency magnetic fields. The maser can also be used to search for ultralight axions and axion-like particles with rest energies below 10^-16 eV (Compton frequencies below 20 mHz) [29–32].

Floquet-state maser.—We consider a two-level spin system interacting with a static magnetic field and an oscillating field. Here, an oscillating field $B_{ac}\cos(2\pi\nu_{ac}t)$ is applied parallel to the static magnetic field $B_0\hat{z}$. The Hamiltonian for the two-level system can be represented as

$$H/2\pi = \frac{\gamma B_0}{2}\sigma_z + \frac{\gamma B_{ac}}{2}\cos(2\pi\nu_{ac}t)\sigma_z,$$  \hspace{1cm} (1)

where we set $\hbar = 1$, $\sigma_{x,y,z}$ are Pauli operators for the spin-$\frac{1}{2}$ system, and $\gamma$ denotes the gyromagnetic ratio of the spin. The first term in Eq. (1) is the Zeeman interaction of the spin with the static field, and the Larmor precession frequency $\nu_0 = \gamma B_0$. The second term is the periodic driving term. We discuss the spectrum of a periodically driven two-level system within the framework of the Floquet theory. The Floquet synthetic dimensions of the periodically driven two-level system described by Eq. (1) consist of Floquet levels denoted as $|\pm, n\rangle$ [1, 33]. Here, $+(-)$ denotes the spin-up (down) state of the spin. In Fig. 1, the energies of the upper and lower Floquet levels are $E_{\pm, n}/2\pi = \ldots$
FIG. 1: (color online). Floquet levels and Floquet states of a periodically driven two-level system. The energy gap between the upper and lower Floquet states \(|+\rangle_n\) and \(|-\rangle_m\) is \(E_{n,m}/2\pi = (n - m)\nu_{ac} + \nu_0\). The relevant transition energy is \(E_{n,m}/2\pi = (n - m)\nu_{ac} + \nu_0\). In experiments, the measurement of the magnetization of the periodically driven system yields a Floquet spectrum, which exhibits a central peak at the Larmor frequency \(\nu_0\) and sidebands located at \(\nu_{\pm k} = \nu_0 \pm k\nu_{ac}\), where \(k = 1, 2, 3, \ldots\). The \(k\)th sideband’s amplitude is proportional to \(J_k(\gamma B_{ac}/\nu_{ac})\).

A Floquet-state maser can be built up by creating a non-Boltzmann population among the Floquet states \(|+\rangle_n\) and \(|-\rangle_m\) of the periodically driven system and a feedback control field \(B_\parallel\) proportional to that of the transverse polarization of the driven two-level system. The feedback field produces a torque (described by a damping rate, \(\gamma_d\)) on the spins that change the magnetization \([22–25, 34]\). This self-coupling can lead to stimulated Rabi oscillation of the Floquet states \(|+\rangle_n\) and \(|-\rangle_m\), and to steady-state maser oscillations \([23]\). Unlike conventional masers, the Floquet-state maser is based on a periodically driven system and creates the population inversion of its Floquet states albeit the system itself is a two-level quantum system. As shown in Fig. 1, each transition, which is relevant to the coherence between the Floquet states of the periodically driven system, corresponds to a specific Floquet-state maser mode.

Experiments. Floquet-state maser is performed using the spin-exchange pumped \(^{129}\)Xe system depicted in Fig. 2(a). We use a 0.5 cm\(^3\) cubic vapor cell made from pyrex glass, with 5 torr \(^{129}\)Xe, 250 torr N\(_2\), and a droplet of enriched rubidium-87 (\(^{87}\)Rb). The vapor cell is placed inside a five-layer cylindrical \(\mu\)-metal shield, and is resistively heated to 140 °C. \(^{129}\)Xe is polarized by spin-exchange collisions with \(^{87}\)Rb atoms \([35, 36]\), which are pumped with a circularly polarized laser beam at the D1 transition. Due to the Fermi contact interaction between \(^{129}\)Xe and \(^{87}\)Rb atoms \([37–39]\), the \(^{87}\)Rb atoms can also serve as a sensitive magnetometer (sensitivity \(\approx 1.1\) pT/\(\sqrt{\text{Hz}}\) in our experiment), which is primarily sensitive to the magnetic field along \(\hat{y}\), to detect the \(^{129}\)Xe \(y\)-magnetization. The effective magnetic field of \(^{129}\)Xe experienced by the \(^{87}\)Rb atoms is given by \(B_{\text{eff}} = \frac{2\gamma}{\gamma} M_y\), where \(M_y\) is the \(y\)-magnetization of \(^{129}\)Xe and enhancement factor \(\kappa_0\) is about 500 for \(^{129}\)Xe \([40]\). The effective field \(B_{\text{eff}}\) is measured via optical rotation of linearly polarized probe laser light at the D2 transition. The inset of Fig. 2(a) shows the magnitude of \(^{129}\)Xe free-decay signal, which is induced by applying a \(\pi/2\) pulse along \(\hat{x}\), as a function of applied bias magnetic field \(B_0\) along \(\hat{z}\). The signal-to-noise ratio (SNR) of a \(^{129}\)Xe measurement is about 5400 when the bias magnetic field is \(B_0 = 15\) \(\mu\)T.

A driving oscillating field is applied to the \(^{129}\)Xe vapor cell along \(\hat{z}\). A feedback control field \(B_\parallel\) proportional to that of the \(^{129}\)Xe transverse polarization is passively generated by using a set of \(\hat{y}\) coils around the vapor cell. In the absence of the feedback control, a \(\pi/2\) pulse along \(\hat{x}\) is used to rotate the \(^{129}\)Xe polarization away from the \(\hat{z}\) to \(\hat{y}\) axis. The recorded signals in Fig. 2(b) are well described by an exponentially decaying oscillation, which gives the coherence time of \(^{129}\)Xe \(T_2 \approx 20\) s. In contrast, when the feedback control loop turns on, the \(^{129}\)Xe transverse polarization shows characteristic initial transients, which subsequently level into a stationary oscillation shown in Fig. 2(c) (without periodic driving field) and Fig. 2(d) (with 0.900-Hz driving field, \(B_{ac} = 56.15\) nT). The transients can be well simulated by the modified Bloch equations \([23–25]\). A small transverse polarization component caused by misalignment or quantum fluctuation is sufficient for activating the driven system to establish the stationary oscillating signal \([34]\).

The observed phenomenon of continuous oscillation is similar to the self-organized masers such as \(^{3}\)He or \(^{129}\)Xe maser \([22–25]\) and room-temperature diamond maser \([41, 42]\), which are based on creating population inversion between Floquet states \(|+\rangle_n\) and \(|-\rangle_m\) (see Fig. 1). To verify this point, we suddenly invert the direction of the bias magnetic field corresponding to removing the population inversion, the established continuous oscillation then decays to zero amplitude \([34]\). To create the maser of periodically driven two-level systems, the damping rate should also satisfy the threshold condition: \(\gamma_d > 1/T_2\), which is same with that of conventional masers \([22–25]\). In our experiment, the damping rate satisfies \(\gamma_d \approx 0.16\) s\(^{-1}\) > \(1/T_2 \approx 0.05\) s\(^{-1}\). The measurement of the damping rate is presented in the Supplemental Material \([34]\).

We perform ultrahigh-resolution Floquet spectroscopy
using the Floquet-state $^{129}$Xe maser. We use the $^{87}$Rb magnetometer to acquire the $^{129}$Xe magnetization signal for 70 s, and then apply Fourier transform to the acquired data. Under a bias magnetic field and a periodic driving field along $\hat{z}$, the $^{129}$Xe atoms are fed back by its real-time signal along $\hat{y}$. PEM, photoelastic modulator. $B_t(t)$, feedback control field. The inset indicates that the amplitudes of $^{129}$Xe free-decay signal under different applied bias magnetic fields without feedback control. (b) Free-decay signal of $^{129}$Xe without feedback control. Signal of $^{129}$Xe using real-time feedback control with periodically driving (c) or with periodically driving (d). The signals with feedback control persist for much longer times than that without feedback control. The insets in (c) and (d) are zoom-in plots for the signal within a time window of 20 s. Plots (e), (f), (g) indicate the corresponding amplitude spectra of the signals or signals after eliminating the transient.

Further, we investigate the performance of Floquet-state maser under an ultralow-frequency magnetic field driving. For achieving distinct sideband peaks in $T_2$-limited condition, $\nu_{ac}$ should be larger than the spectral linewidth $1/(\pi T_2)$. Alternatively, Floquet-state maser provides the capability of achieving the spectral linewidth beyond $T_2$ limit, and thus provides the possibility of detecting sidebands in the regime of $\nu_{ac} \lesssim 1/(\pi T_2)$, in which high-order sidebands are significant. As an experimental benchmark, a 0.050 Hz field with magnitude of 56.15 nT drives the $^{129}$Xe, $\gamma B_{ac}/\nu_{ac} \approx 13.0$. In Fig. 3(a), the Floquet spectrum based on the free-decay signal (blue line) shows indistinct lines, and contrarily the spectrum based on Floquet-state maser (red line) exhibits at least 25 evident symmetric lines centered at the Larmor-frequency line. All lines are at regular intervals equalling to the periodic driving frequency $\nu_{ac}$. Similarly, there are at least 135 sidebands, obtained from 4000-s continuous oscillation, with the FWHM of 0.3 MHz in Fig. 3(b). The linewidth is partially due to the static magnetic field fluctuation coming from the current insta-
Floquet-state maser is promising for operating as an ultralow-frequency magnetic field sensor. As discussed before, the Floquet spectrum of periodically driven $^{129}$Xe shows that obvious sideband peaks locate at $\pm k\nu_{ac}$ about the central frequency. This indicates that the setup can be used to sense an external a.c. magnetic field, which is applied on the $^{129}$Xe atoms. We consider the measurement of a weak a.c. magnetic field, which corresponds to the regime of modulation index $\gamma B_{ac}/\nu_{ac} \ll 1$ in the intensity plots of Fig. 4. The $k$th sideband's amplitude is therefore proportional to $\gamma B_{ac}/\nu_{ac} \approx \gamma B_{ac}/(2\nu_{ac})^{k}$. We focus on the next most prominent first-order sidebands occurring at $\pm \nu_{ac}$ about the central frequency. As shown in Fig. 5(a), a $2.25$-nT magnetic field is applied along $\hat{z}$, and its frequency is swept from 1.000 Hz to 22.000 Hz. We Fourier-transform the individual measurement traces (with 60-s acquisition time) and record the corresponding sideband peak amplitudes. The experimental amplitudes of first-order sidebands depending on frequency are fitted to an inverse function $1/\nu_{ac}$, which is in good agreement with theoretical model. Similarly, we test the linear response of the a.c. magnetic field amplitude by applying 1.000 Hz magnetic fields with different amplitudes, as shown in Fig. 5(b).

Importantly, the Floquet-state maser exhibits unique advantage of measuring ultralow-frequency magnetic fields due to the $1/\nu_{ac}$ response, which is generally challenging in current magnetometers including atomic mag-
netometers [43, 44] and superconducting quantum interference devices [45]. We show that the reported maser can be used to search for axions and axion-like particles, which can be viewed as a background classical quasi-magnetic field oscillating at the Compton frequency corresponding to the particle’s mass [29–32]. Probing ultralight axions and axion-like particles with rest energies below $10^{-16}$ eV, which corresponds to Compton frequencies below 20 mHz, are typically difficult due to the diminishing sensitivity of magnetometers in this frequency regime. In contrast, our approach, i.e., a direct Floquet-state $^{129}$Xe-maser sideband search with an in-situ $^{87}$Rb magnetometer, provides an opportunity in the search for ultralight axions and axion-like particles below $10^{-16}$ eV level.

It is worth noting that the demonstrated Floquet-state maser technique can be applied to other existing masers. With a periodic driving, the conventional masers could emit multi-sideband radiation at regular controllable frequency intervals, which may be used to design radio- or microwave-frequency comb for frequency-comb excitation [46, 47]. Moreover, the further extension research on Floquet-state maser, which overcomes the decoherence effect, may provide an excellent platform for studying the physics of periodically driven systems, such as two-band Wannier-Stark ladder model [48], photon-assisted tunnelling [49] and Floquet Raman transition [50].

Conclusion.—We have experimentally demonstrated a novel Floquet-state maser based on the synthetic dimensions supported by Floquet states of the periodically driven $^{129}$Xe system. We show that it can overcome the fundamental decoherence effect with real-time feedback control, and further demonstrate the Floquet spectra of $^{129}$Xe with sub-millihertz resolution beyond the decoherence limit. The demonstrated Floquet-state $^{129}$Xe maser offers a new capability for operating as an ultralow-frequency magnetic field sensor, and opens a possibility for searching for ultralight axions and axion-like particles with rest energies below $10^{-16}$ eV.

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