Effects of staggered fermions and mixed actions on the scalar correlator

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Abstract

We provide the analytic predictions for the flavor non-singlet scalar correlator, which will enable determination of the scalar meson mass from the lattice scalar correlator. We consider simulations with 2+1 staggered sea quarks and staggered or chiral valence quarks. At small \( u/d \) masses the correlator is dominated by the bubble contribution, which is the intermediate state with two pseudoscalar mesons. We determine the bubble contribution within Staggered and Mixed Chiral Perturbation Theory. Its effective mass is smaller than the mass of \( \pi\eta \), which is the lightest intermediate state in proper 2+1 QCD. The unphysical effective mass is a consequence of the taste breaking that makes possible the intermediate state with mass \( 2M_\pi \). We find that the scalar correlator can be negative in the simulations with mixed quark actions if the sea and valence quark masses are tuned by matching the pion masses \( M_{\text{val,val}} = M_\pi \).

1 Introduction

The Nature of the lightest observed scalar resonances is still not revealed yet. In this paper we consider the flavor non-singlet scalar meson and the problems related to its simulation on the lattice. Below 2 GeV there are two experimentally well established scalar resonances with isospin \( I = 1 \): \( a_0(980) \) and \( a_0(1450) \). It is still not clear which of the two corresponds to the lightest \( \bar{q}q \) scalar meson. Most of the models and lattice simulations have difficulty relating the mass and decay properties of \( a_0(980) \) with the \( \bar{q}q \) state. If \( a_0(980) \) belongs to the lightest \( \bar{q}q \) nonet, it does not have an obvious strange partner since the mass of \( K_0(1430) \) is relatively high, while \( \kappa(800) \) is experimentally very controversial and probably too light to be the partner. This suggests the possibility that \( a_0(1450) \) might be the lightest \( \bar{q}q \) state with \( I = 1 \), while \( a_0(980) \) could be exotic state such as tetraquark \( \bar{q}qqq \) or mesonic molecule.
The issue could be settled if the mass of the lightest \( \bar{q}q \) state with \( I = 1 \) would be reliably determined on the lattice. We refer to this resonance as \( a_0 \) and we consider \( \bar{d}u \) scalar meson for concreteness. In order to determine \( a_0 \) mass, lattice simulations evaluate the connected scalar correlator

\[
C(t) = \sum_{\vec{x}} \langle 0 | \bar{d}(\vec{x}, t) u(\vec{x}, t) \, \bar{u}(\vec{0}, 0) d(\vec{0}, 0) | 0 \rangle . \tag{1}
\]

If \( a_0 \) is the lightest state of the Hamiltonian with quantum numbers \( J^P = 0^+ \) and \( I = 1 \), then \( C(t) \propto e^{-m_{a0} t} \) at large \( t \) and extraction of \( m_{a0} \) is straightforward. Multi-hadron states with \( J^P = 0^+ \) and \( I = 1 \) also propagate between the source and the sink of the scalar correlator in Fig. 1 and they may shadow the interesting part \( e^{-m_{a0} t} \). The most important multi-hadron state is the intermediate state with two pseudoscalars \( P_1 P_2 \), we call it the bubble contribution \( B \) and display it in Fig. 2. The state \( P_1 P_2 \) has discrete energies \( E_n \) on the lattice with spatial extent \( L \)

\[
M_{P_1} + M_{P_2}, \quad \sqrt{M_{P_1}^2 + \left(\frac{2\pi}{L}\right)^2} + \sqrt{M_{P_2}^2 + \left(\frac{2\pi}{L}\right)^2}, \ldots, \sqrt{M_{P_1}^2 + \left(\frac{2\pi}{L}\right)^2} + \sqrt{M_{P_2}^2 + \left(\frac{2\pi}{L}\right)^2} \tag{2}
\]

and gives rise to a sum of decaying exponentials \( e^{-E_n t} \) in the correlation function. If the masses of \( P_1 \) and \( P_2 \) are small, the bubble \( B \) presents a sizable contribution to the scalar correlator (Fig. 1) and it has to be incorporated in the fit of the lattice correlator (1)

\[
C(t) = Ae^{-m_{a0} t} + B(t) + \cdots . \tag{3}
\]

The dots represent contributions of excited physical scalar meson and other multi-hadron intermediate states, which are both less important.

Figure 1: The scalar correlator with \( I = 1 \) receives the contribution from the propagation of the physical scalar meson \( a_0 \) as well the propagation of multi-hadron states. The most important multi-hadron state is the two-pseudoscalar intermediate state, which gives rise to the so-called bubble contribution.

Let us review the importance of two-pseudoscalar intermediate state in theories with various numbers of dynamical quarks:

- The interest of this paper is \( 2 + 1 \) flavor QCD \((m_u = m_d \neq m_s)\), where the intermediate states are \( \pi \eta, K \bar{K} \) and \( \pi \eta' \) (\( \pi \pi \) is not allowed by Bose symmetry and conservation of \( J^P \) and \( I^G \) in the strong decay). In Nature, all these intermediate states are lighter than \( a_0(1450) \), whereas \( \pi \eta \) is also lighter than \( a_0(980) \). Hence, two-pseudoscalar intermediate states dominate correlation function at large \( t \) if simulations are performed at small
$u/d$ masses. This makes it difficult to extract the mass of $a_0$ from \eqref{eq:mass}. For the same reason, the scalar correlator is very sensitive to the possible unphysical approximations undertaken in the simulations, like the application of the staggered fermions or mixed quark actions. In the present paper we calculate the bubble contribution for the case of simulations with staggered sea and staggered valence quarks, as well as for simulations with staggered sea and chiral valence quarks. The bubble contribution incorporates the physical intermediate states $\pi\eta$, $K\bar{K}$, $\pi\eta'$. It also gives the size of the unphysical effects of staggered fermions and mixed quark actions in the scalar correlator. We find that these unphysical effects are sizable. Our results are relevant for the existing simulations of scalar mesons with staggered sea and valence quarks by MILC \cite{MILC} and UKQCD \cite{UKQCD}. They are also relevant for the simulations with staggered sea quarks and chiral valence quarks, currently performed by LHPC, NPLQCD and UKQCD \cite{LHPC, NPLQCD, UKQCD}.

- Two flavor QCD allows only $\pi\eta'$ intermediate state, where $\eta'$ is two-flavor singlet with mass $M_{\eta'}^2 = M_{\pi}^2 + \frac{2}{3}m_0^2$. The $\pi\eta'$ state is relatively heavy and does not shadow so significantly the interesting part of the correlation function given by $e^{-m_0 t}$. The mass of $a_0$ was extracted from the conventional exponential fit in the simulation with two flavors of dynamical Domain Wall Fermions \cite{DWF}. The resulting mass was unaffected if the $\pi\eta'$ intermediate state was also taken into account in the analysis of the correlation function \cite{DWF}. The SCALAR \cite{SCALAR} and UKQCD \cite{UKQCD} Collaborations also simulated $a_0$ in two-flavor QCD. Partially quenched scalar correlator with two dynamical Domain Wall Fermions was simulated in \cite{DWF}. The prediction of the bubble contribution within Partially Quenched ChPT describes well the striking effect of partial quenching, and the scalar meson mass was extracted by fitting the partially quenched correlators to \cite{DWF, SCALAR, UKQCD}.

- The scalar correlator in quenched QCD is dominated by $\pi\eta'$ intermediate state at light quark masses \cite{DWF}. This has large unphysical effect which can be attributed to the breakdown of unitarity in quenched QCD. The scalar correlator is negative and has the effective mass $2M_{\pi}$ at large $t$. The scalar meson mass $m_{a0}$ has been determined by fitting the quenched scalar correlator to the sum of $a_0$ exchange and $\pi\eta'$ exchange\footnote{The quenched result of \cite{DWF} agrees with two-flavor dynamical result \cite{DWF} if the effect of quenching is incorporated at the leading order in the chiral expansion. The quenched $m_{a0}$ from \cite{DWF} is somewhat lower since the quenching effect is incorporated at the next-to-leading order, given by the diagrams in Fig. 8 of \cite{DWF}.} in \cite{DWF, SCALAR, UKQCD}.

So the bubble contribution (Fig. 2) to the scalar correlator is sizable and has to be incorporated in the fit of the lattice correlator at light quark masses. Our analytic prediction for the bubble contribution $B(t)$ offers a way to determine the scalar meson mass $m_{a0}$ by fitting the lattice scalar correlator to Eq. \eqref{eq:mass}. The extraction of $m_{a0}$ in this way is possible for the range of quark masses and times, where $e^{-m_{a0} t}$ is not negligible with respect to the
bubble contribution\(^2\). We address simulations with 2+1 staggered sea quarks and staggered or chiral valence quarks:

- In the case of staggered valence quarks, we determine the bubble contribution using Staggered Chiral Perturbation Theory (SChPT) \(^\text{[14]}\). All the parameters entering our prediction, have been determined by MILC simulations. We find positive bubble contribution with effective mass \(2M_{\pi_5} < m_{\text{eff}} < M_{\pi_5} + M_{\eta_5}\) at large \(t\). This offers a natural explanation why the MILC \(^\text{[5]}\) and UKQCD \(^\text{[6]}\) Collaborations observed the effective mass well below \(M_{\pi_5} + M_{\eta_5}\), which would be the lightest mass in proper 2+1 QCD. We propose that this unphysical result is due to the taste breaking.

- In case of chiral (Ginsparg-Wilson) valence quarks we apply Mixed Chiral Perturbation Theory (MChPT) \(^\text{[16]}\). The result is relevant for simulations of LHPC, NPLQCD and UKQCD \(^\text{[7]}\) which employ staggered sea and chiral valence quarks. The bubble contribution depends on one unknown parameter \(a^2\Delta_{\text{Mix}}\), which gives magnitude of the taste breaking in the mass of the pion composed of one sea and one valence quark: \(M_{\text{val,sea}}^2 = B_0(m_{\text{val}} + m_{\text{sea}}) + a^2\Delta_{\text{Mix}}\). We find that the use of the mixed quark actions together with the taste breaking can have sizable unphysical effects on the scalar correlator and they can make the scalar correlator negative. Namely, the bubble contribution is found to be negative for \(a^4\Delta_{\text{Mix}} \gtrsim -0.01\) if sea and valence quark masses are tuned by imposing \(M_{\text{val,val}} = M_{\pi_5}\) (applied by LHPC Collaboration \(^\text{[7]}\)). The bubble contribution is positive for the reasonable values of \(a^4\Delta_{\text{Mix}}\) if \(M_{\text{val,val}} = M_{\pi_5}\) is imposed.

The bubble contribution to the scalar correlator is calculated in Section 2. It is evaluated using Staggered and Mixed ChPT in subsection 2.1 and 2.2, respectively. The implications of our analytical results for the lattice simulations are also discussed in this section. Conclusions are given in Section 3.

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\(^2\)The bubble contribution completely dominates the correlator for very small quark masses and large times.
2 Contribution of two-pseudoscalar intermediate state

In this section we calculate the contribution of two-pseudoscalar intermediate state to the scalar correlator with a point source and a point sink \([1]\). Our calculation is carried out at the lowest order of the effective field theory and the relevant bubble diagram is shown in Fig. 2. For this purpose we only need the coupling of the scalar current to a pair of pseudoscalar mesons \(P_1P_2\) and the propagators for pseudoscalars \(P_1\) and \(P_2\).

The point scalar current is expressed in terms of two pseudoscalar fields \(\Phi\) using the Chiral Perturbation Theory \([11]\) in the Appendix A:

\[
\bar{d}(x)u(x) \sim B_0[\Phi(x)^2]_{ud}
\]

So the coupling of the point scalar current to two pseudoscalars is equal to the slope parameter \(B_0 = M^2_\pi/(2m_q)\) which can be determined from the lattice data. The same coupling applies also in the Staggered as well as in the Mixed version of ChPT at the lowest order.

The bubble contribution in Fig. 2 is equal to

\[
B = \langle 0|\bar{d}u\bar{d}u|0\rangle_{\text{bubble}} = B_0^2 \langle 0|\Phi^2|0\rangle_{ud}\langle \Phi^2|du|0\rangle.
\]

The Wick contractions lead to the products of two propagators for pseudoscalar fields. The propagators and the resulting bubble contribution will be given separately for Mixed and Staggered ChPT in two subsections that follow.

We note that the bubble contribution \([3]\) applies for the bare lattice point-point correlator \([1]\) via \([3]\), if the bare quark mass is inserted to \(B_0 = M^2_\pi/(2m_q)\). This is possible since the scalar current and \(B_0 \propto 1/m_q\) are multiplied by the same factor \(Z_S\) in the renormalization.

2.1 Staggered sea quarks and staggered valence quarks

First we consider the simulations with staggered valence quarks and three flavors of staggered sea quarks with \(m_u = m_d \neq m_s\). We study only the dynamical correlators, where the valence-quark mass is equal to the \(u/d\) sea-quark mass. Staggered quarks have four tastes and the scalar correlator is conventionally calculated using the taste-singlet source and sink. This correlator receives contribution from the physical scalar meson and the bubble contribution \([3]\). It also receives an oscillating contribution from a pseudoscalar meson\(^4\), but the oscillating contribution will not be considered here since it has been identified and removed in the analysis of the lattice data \([3, 6]\).

The bubble contribution \([3]\) in a theory with four tastes and three flavors of sea quarks \((4 + 4 + 4)\) is equal to

\[
B^{S\text{ChPT}}_{4+4+4} = \frac{1}{4} B_0^2 \sum_{t,t'} \langle 0|\Phi^2|ut\rangle dt \langle \Phi^2|dt'ut'|0\rangle,
\]

\(^3\)\(B_0\) was denoted by \(r_0\) in \([11]\) and by \(2\mu_0\) in \([8]\).

\(^4\)The operator with spin \(I\) and taste \(I'\) couples also to state with spin \(\gamma_5\gamma_4\) and taste \(\gamma_5\gamma_4\).
where 1/4 is appropriate normalisation. Here \( \Phi \) is 12 \times 12 pseudoscalar matrix of Staggered ChPT \[14\] and the subscripts \( u, d \) denote its flavor component, whereas \( t, t' = 1, \ldots, 4 \) denote its taste component. The field \( \Phi = \sum_{b=1}^{16} \frac{1}{2} T^b \phi^b \) is expressed in terms of mass eigenstates \( \phi^b \), where \( \phi^b \) is 3 \times 3 pseudoscalar matrix with flavor components \( \phi_{f}^{b} \), and \( T^b = \{ \xi_{5}, i \xi_{5} \xi_{\mu}, \xi_{\mu}, \xi_{\mu}, \xi_{b} \} \). The bubble contribution can be expressed in terms of the pseudoscalar propagators \( \langle \phi^b|\phi^b \rangle \) by applying the Wick contractions to \( \langle \rangle \).

\[
B_{4+4+4}^{\text{ChPT}} = \frac{1}{4} B_{0}^{2} \sum_{b=1}^{16} \left( \langle \phi^b_{us}|\phi^b_{su} \rangle \langle \phi^b_{sd}|\phi^b_{ds} \rangle + \langle \phi^b_{ud}|\phi^b_{du} \rangle \right)(\phi^b_{uu}|\phi^b_{uu} \rangle) + \langle \phi^b_{dd}|\phi^b_{dd} \rangle + 2\langle \phi^b_{uu}|\phi^b_{dd} \rangle) \right). \tag{7}
\]

![Figure 3: The quark level diagrams that correspond to the bubble contribution (Fig. 2) in simulations with dynamical \( u, d \), and \( s \) quarks. The diagram (c) represents also the diagrams with any number of quark loops inserted in the place of the dots.](image)

The propagators \( \langle \phi^b|\phi^b \rangle \) for the pseudoscalar fields of various tastes \( b \) are provided by SChPT \[14\] and are collected in the Appendix B. Propagators for all tastes \( I, V, A, T, P \) have connected part, while tastes \( I, V \) and \( A \) have also disconnected part. The bubble contribution \( \langle \rangle \) can be represented by the quark-level Feynman diagrams in Fig. 3 and it is calculated by inserting the mesonic propagators in Appendix B to expression \( \langle \rangle \). We get

\[
B_{4+4+4}^{\text{ChPT}}(t) = F.T.\left[ B_{4+4+4}^{\text{ChPT}}(p) \right]_{\vec{p}=\vec{0}} \quad \text{with} \quad (8)
\]

\[
B_{4+4+4}^{\text{ChPT}}(p) = B_{0}^{2} \sum_{k} \left\{ -4 \left[ \frac{1}{(k+p)^{2} + M_{U_{k}}^{2}} \right] \frac{1}{3} \left( \frac{k^{2} + M_{I}^{2}}{k^{2} + M_{U_{k}}^{2}}(k^{2} + \frac{1}{3} M_{U_{k}}^{2} + \frac{2}{3} M_{I}^{2}) \right) + \frac{1}{(k+p)^{2} + M_{U_{k}}^{2}} \right\}
\]

\[
+ C \sum_{b=1}^{16} \left[ \frac{1}{(k+p)^{2} + M_{b}^{2}} \right] \frac{1}{2} \left[ \frac{1}{k^{2} + M_{b}^{2}} + \frac{1}{(k+p)^{2} + M_{us_{b}}^{2}} \right] + \frac{1}{k^{2} + M_{us_{b}}^{2}} \right] \right\} \]

where

\[
C = 1, \quad M_{(i)_{V,A}} = m_{(i)_{V,A}}(a^{2}\delta_{V,A}^{(i)}) \quad \text{in} \; 2 \cdot 4 + 4 \; \text{theory} \tag{9}
\]

\[
C = \frac{1}{4}, \quad M_{(i)_{V,A}} = m_{(i)_{V,A}}(\frac{1}{4} a^{2}\delta_{V,A}^{(i)}) \quad \text{in} \; 2 + 1 \; \text{theory}
\]
and the functions \( m_q, m_{q'} \) are given by (23) in Appendix B. The four tastes per sea quark (2 · 4 + 4) are reduced to one taste per sea quark (2 + 1) by applying the \( \sqrt{\det} \) trick in staggered lattice simulations [5]. In SChPT [14] this is achieved by multiplying every sea quark loop by factor 1/4. The sea quark loops are present in diagrams (a,b) and in hairpin vertex of diagram (c) in Fig. 3. The corresponding factors of 1/4 are incorporated in (9). The taste breaking \( a^2 \Delta_b \) is flavor independent and vanishes in the continuum limit. The hairpin couplings of the taste-vector and taste-axial mesons were also determined by MILC for their coarse and fine lattices [15].

\[
(a^4 \delta_A')_{\text{coarse}} = -0.04(1) , \quad (a^4 \delta_V')_{\text{coarse}} = -0.01(+3) , \quad (a^2 \delta_{A,V}')_{\text{fine}} \simeq 0.35 (a^2 \delta_{A,V}')_{\text{coarse}} .
\]  

These hairpin couplings mix the flavor neutral mesons of tastes \( V \) and \( A \) to the mass eigenstates \( \eta_{V,A} \) and \( \eta_{V,A} \) with masses given in (9,23).

a) Results in case of 4 tastes per quark

The theory with four tastes per sea quark (i.e. without \( \sqrt{\det} \) trick) is not relevant to QCD. In spite of that, it is illuminating to consider the properties of the bubble contribution in this case before proceeding to the single-taste theory.

The four-taste theory is a proper unitary quantum field theory even at finite lattice spacing and \( B_{2+4}^{\text{SChPT}}(t) \) is positive definite for any values of the input parameters. This is explicitly demonstrated for the simplified case of \( m_u = m_d = m_s \) and \( \Delta_b = 0 \) with general values of hairpin couplings \( a^2 \delta_{V,A} \) in Appendix C.

The \( B_{2+4}^{\text{SChPT}}(t) \propto e^{-2M_x t} \) at large time for zero and non-zero lattice spacing. This is in sharp contrast to three flavor QCD, where \( B(t) \propto e^{-M_{\eta}(t)} \) with \( M_{\eta}^2 = 1/3 M^2_{\pi} + 2/3 M^2_S \). It is easy to understand why an intermediate state with mass \( 2M_\pi \) is possible in \( B_{2+4}^{\text{SChPT}} \) at \( a = 0 \). The mass eigenstates that enter as intermediate states are \( \pi_0 \eta_b \) and \( \bar{K}_b K_b \). Only the mass of \( \eta_I \) is lifted by the anomaly to \( \sqrt{1/3 M^2_{\pi} + 2/3 M^2_S} \), while the etas of other tastes retain mass \( M_\pi \) at \( a = 0 \). So the lightest intermediate state with \( I = 1 \) has the mass \( 2M_\pi \).

b) Results in case of one taste per sea quark

Finally we study the properties of \( B_{2+1}^{\text{SChPT}} \), which is relevant to QCD since the four tastes of sea quarks have been reduced to one via (9).

In the continuum limit there is no taste breaking (\( a^2 \Delta_b \to 0 \), \( a^2 \delta_{V,A} \to 0 \)) and \( B_{2+1}^{\text{SChPT}} \) reduces to the contributions of \( \pi \eta \) and \( K \bar{K} \) from \( 2+1 \) ChPT

\[
B^{\text{ChPT}}(p) = B_0^3 \sum_k \left\{ \frac{2}{3(k+p)^2 + M^2_{\pi}} \frac{1}{k^2 + M^2_{\eta}} + \frac{1}{(k+p)^2 + M^2_K} \frac{1}{k^2 + M^2_K} \right\} \left( \frac{2}{3} \frac{e^{-(M_{\pi} + M_{\eta})t}}{M_{\pi} M_{\eta}} + \frac{e^{-2M_K t}}{M^2_K} \right) ,
\]

\[
\lim_{t \to \infty} B^{\text{ChPT}}(t) = \frac{B_0}{4L^3} \left( \frac{2}{3} \frac{e^{-(M_{\pi} + M_{\eta})t}}{M_{\pi} M_{\eta}} + \frac{e^{-2M_K t}}{M^2_K} \right) ,
\]  

(11)
where \( M_\pi^2 = \frac{1}{3} M_\pi^2 + \frac{2}{3} M_\eta^2 \) and \( M_K^2 = \frac{1}{2} M_\pi^2 + \frac{1}{2} M_\eta^2 \). The bubble contribution is positive and drops as \( e^{-(M_\pi + M_\eta)t} \) at large \( t \), as expected in QCD.

The fourth-root trick destroys the unitarity of the theory at finite lattice spacing and \( B^{SChPT}_{2+1}(t) \) can be negative for specific values of input parameters\(^5\). On top of that, the taste breaking at nonzero \( a \) allows for the intermediate state with \( 2M_\pi \), which is prohibited in proper QCD. This can be understood by considering the quark flow diagrams in Fig. 3. The diagram in Fig. 3a exhibits the expected time-dependence \( e^{-2M_Kt} \) in ChPT as well as in SChPT. The diagrams in Figs. 3b and 3c both contain terms with \( e^{-2M_\pi t} \), but these exactly cancel in ChPT with no taste breaking. This cancellation does not occur in the presence of taste breaking when pions of all tastes \( \pi_5,T,I,V,A \) flow in diagram 3b, but only the pions \( \pi_I,V,A \) flow in the diagram 3c. The taste breaking therefore significantly modifies the time-dependence

\[
\lim_{t \to \infty} B^{SChPT}_{2+1}(t) = \frac{B_0^2}{4L^3} \left( \frac{2}{16} \frac{e^{-2M_\pi t}}{M_{n_5}^2} + \text{terms with } e^{-2M_{n_5}t} + \pi\eta, \pi\eta', K\bar{K} \right)
\]

with respect to the expected behaviour in proper QCD (11). The coupling to the un-physical intermediate state with mass \( 2M_\pi \) vanishes only in the continuum limit.

Now we evaluate \( B^{SChPT}_{2+1}(t) \) for the case of MILC configurations in order to see if our observations agree with the results from the lattice simulations. Fig. 4 shows the magnitude of the bubble contribution evaluated for the \( 2 + 1 \) MILC coarse and fine lattices, using the pseudoscalar masses, the bare quark masses, the hairpins (10) and the lattice volumes from [5]. The Euclidean loop momentum \( k \) is summed over the discrete momenta in the finite box of the lattice\(^6\). The bubble contribution is larger for lighter \( u/d \) quark masses. It becomes the dominant contribution to the scalar correlator at large \( t \) if two-pseudoscalar state is lighter than \( a_0 \), which is realized\(^7\) for the \( u/d \) quark masses considered in Figs. 4 and 5.

The effective mass of \( B^{SChPT}_{2+1}(t) \) is shown by triangles in Fig. 5. It is compared with \( M_\pi + M_\eta, 2M_\pi \), and with the effective mass of the scalar correlator from MILC simulation [5]. The effective mass of the analytic prediction lies between \( 2M_{n_5} < m_{\text{eff}} < M_\pi + M_\eta \) at large \( t \). This agrees\(^8\) with the effective mass observed in lattice simulations by MILC [5] (shown in Fig. 5) and by UKQCD [6]. The exception are the highest quark masses, where the lattice simulations [5, 6] observe effective mass even below \( 2M_\pi \), which can not be understood within given theoretical framework. The unphysical effective mass \( 2M_\pi \) is lifted in the continuum limit to the physical value \( M_\pi + M_\eta \), which is the effective mass of \( B^{ChPT}_{2+1} \) and is represented by circles in Fig. 5.

\(^5\)For example, \( B^{SChPT}_{2+1}(t) \) is negative in the case of input parameters relevant for MILC simulations with sign of \( \alpha^2 \delta_{V,A} \) in (10) reversed.

\(^6\)The detailed expression for the finite sum in \( B(t) \) is given in Appendix B of [5] for analogous case of Partially Quenched theory.

\(^7\)This applies for observed \( m_{a_0} = 1450 \text{ MeV} \) as well as \( m_{a_0} \) obtained by MILC from heavier quark masses.

\(^8\)Fig. 5 presents the effective mass of \( B^{SChPT}_{2+1} \) at \( t = 20 \) for coarse and at \( t = 30 \) for fine lattice, while the MILC effective mass was extracted from a wider time-range \( t \sim 5 \sim 20 \) [5], so they are not expected to agree exactly.
2.2 Staggered sea quarks and chiral valence quarks

The simulations with chiral valence quarks on the available staggered MILC configurations present an appealing possibility. The LHPC and NPLQCD Collaborations use domain wall valence quarks, while UKQCD uses overlap valence quarks \cite{7}. The corresponding effective theory has been derived recently \cite{16} and we refer to it as Mixed ChPT (MChPT). It is a kind of Partially Quenched theory which takes into account also the taste breaking of the staggered sea quarks. The valence quarks $x = 1, 2$ and the ghost valence quarks $\bar{x} = \bar{1}, \bar{2}$ have degenerate mass $m_{val}$, while the staggered sea quarks $4u, 4d, 4s$ have masses $m_u = m_d$ and $m_s$. There is no unique recipe how to match the valence quark mass $m_{val}$ and the sea quark mass $m_u$, so the mixed theory always retains some features of partial quenching away from the continuum limit.

The bubble contribution is obtained by applying the Wick contractions to \cite{5}

$$B_{4+4+4}^{MChPT} = B_0^2 \left[ 2 \langle \Phi_{11} | \Phi_{22} \rangle \langle \Phi_{12} | \Phi_{21} \rangle + \sum_{x=1,2,\bar{1},\bar{2}} \langle \Phi_{1x} | \Phi_{x1} \rangle \langle \Phi_{x2} | \Phi_{2x} \rangle + \sum_{sea=4u,4d,4s} \langle \Phi_{sea} | \Phi_{sea} \rangle \langle \Phi_{sea} | \Phi_{sea} \rangle \right].$$

(13)

To reduce four tastes per sea quark to one taste, the last term will be multiplied by $1/4$ and the appropriate expressions for disconnected propagators from \cite{16} will be used.
Figure 5: The effective mass of the scalar correlator for MILC simulation with 2 + 1 staggered sea quarks and staggered valence quarks. The effective mass is multiplied by $r_1 \simeq 1.6 \text{ GeV}^{-1}$ as in [5] for easier comparison. The dependence on $am_{u/d}$ is shown, while $am_s$ is fixed to $am_s = 0.05$ and $am_s = 0.031$ on coarse and fine lattices, respectively. Triangles present effective mass of the bubble contribution in 2 + 1 Staggered ChPT [10] at $t = 20$ for coarse MILC lattice and at $t = 30$ for fine MILC lattice; the corresponding error-bar denotes the variation due to the uncertainty in the hairpin parameters [11]. In the continuum limit $B^{S\text{ChPT}}_{2+1}$ reduces to $B^{\text{ChPT}}_{11}$ and its effective mass is given by circles. Squares represent effective mass from MILC simulation [5]. The relevant masses $2M_{\pi_5}$ and $M_{\pi_5} + M_{\eta_5}$ from MILC simulations are also shown ($M^2_{\eta_5} \equiv \frac{1}{3}M^2_{U_5} + \frac{2}{3}M^2_{S_5}$).

The propagators for valence-valence mesons with $x, x' = 1, 2, \tilde{1}, \tilde{2}$ and $m_0 \to \infty$ are [16]

$$\langle \Phi_{xx'} | \Phi_{x'x} \rangle = \frac{\epsilon_x}{k^2 + M^2_{\text{val,val}}} , \quad M^2_{\text{val,val}} = 2B_0 m_{\text{val}}, \quad \epsilon_{1,2} = 1 , \quad \epsilon_{\tilde{1},\tilde{2}} = -1$$

(14)

$$\langle \Phi_{xx'} | \Phi_{x'x} \rangle_{\text{disc}} = -\frac{1}{3} \left( \frac{k^2 + M^2_{\text{val,val}}}{k^2 + M^2_{\text{val,val}}} \right) \left( \frac{k^2 + M^2_{U_5}}{k^2 + \frac{1}{3}M^2_{U_5} + \frac{2}{3}M^2_{S_5}} \right) , \quad M^2_{U_5} = M^2_{U_5} + a^2 \Delta_I,
M^2_{S_5} = M^2_{S_5} + a^2 \Delta_I .$$

while the propagators for valence-sea mesons with $\text{sea} = u, d, s$ and $x = 1, 2$ are

$$\langle \Phi_{x \text{sea}} | \Phi_{\text{sea}x} \rangle = \frac{1}{k^2 + M^2_{\text{val,sea}}} , \quad M^2_{\text{val,sea}} = B_0 (m_{\text{val}} + m_{\text{sea}}) + a^2 \Delta_{\text{Mix}} .$$

(15)

The propagators (14,15) depend only on two taste breaking parameters $a^2 \Delta_I$ and $a^2 \Delta_{\text{Mix}}$. The taste-singlet breaking $a^2 \Delta_I$ of the sea-sea pion mass is taken from [5], while the taste breaking $a^2 \Delta_{\text{Mix}}$ of the valence-sea pion mass has not been determined yet and it is the only free parameter in the present paper.
The bubble contribution is obtained by inserting the propagators \( B^{MChPT}_{2+1} \) to \( B^{MChPT}_{2+1}(p) \) with

\[
B^{MChPT}_{2+1}(p) = B_0 \sum_k \left\{ -\frac{4}{3} \frac{1}{(k+p)^2 + M^2_{\text{val},\text{val}}} - \frac{1}{(k+p)^2 + M^2_{\text{val},\text{val}}} \frac{(k^2 + M^2_{U_2})(k^2 + M^2_{S_1})}{k^2 + \frac{1}{3} M^2_{U_2} + \frac{2}{3} M^2_{S_1}} \right. \\
\left. + 2 \frac{1}{(k+p)^2 + M^2_{\text{val},u}} + \frac{1}{(k+p)^2 + M^2_{\text{val},u}} \frac{1}{k^2 + M^2_{\text{val},s}} \frac{1}{k^2 + M^2_{\text{val},s}} \right\}.
\]

Our result agrees in appropriate limit\(^9\) with the scalar correlator from Ref. [17], which considers mixed quark actions but not staggered sea. In the continuum limit \((a^2 \Delta_t \rightarrow 0, a^2 \Delta_{Mix} \rightarrow 0)\) the expression \((16)\) reduces to \( B^{ChPT} \) once the valence and the sea quark masses are tuned as \( m_{\text{val}} = m_u \). Away from the continuum limit, the contributions \( e^{-2M_\pi t} \) from the diagrams in Figs. 3b and 3c do not cancel and the correlator has the effective mass \( 2M_\pi \) at large \( t \).

Fig. 4 represents the bubble contribution \((16)\) when valence and sea quark masses are tuned by matching \( M_{\text{val},\text{val}} = M_{U_2} \), which is used by LHPC Collaboration [7]. The expression \((16)\) is evaluated for MILC coarse lattices using \( M_{U_2} \) and \( V = 20^3 \times 32 \) from LHPC [7] and \( a^2 \Delta_t \) from [3]. The \( a^2 \Delta_{Mix} \) is the only unknown parameter and it is varied in the reasonable\(^10\) range \(-0.02 \leq a^4 \Delta_{Mix} \leq 0.08\). The bubble contribution is sizable for small \( m_{u/d} \) and we find it negative for \( a^4 \Delta_{Mix} \gtrsim -0.01 \). This negativity can be attributed to the fact that all the sea-sea pions (except \( \pi_5 \)) are heavier than the valence-valence pion when \( M_{\text{val},\text{val}} = M_{U_2} \) and the correlator goes negative for the same reason as in Partially Quenched QCD [8]. At small \( m_{u/d} \), the scalar correlator \((16)\) is dominated by the bubble contribution and is therefore negative for a large range of values \( a^2 \Delta_{Mix} \). At large \( m_{u/d} \), the scalar correlator is dominated by \( e^{-m_{\text{u/d}} t} \) for moderate \( t \), so it is positive irrespective of the sign of the bubble contribution. A lattice study of point-point scalar correlator at small quark masses offers a way to determine the unknown parameter \( a^2 \Delta_{Mix} \) via Eq. \((16)\).

If the valence and sea quark masses are tuned by matching the masses of the valence-valence pion and the heaviest sea-sea pion \( (M_{\text{val},\text{val}} = M_{U_1}) \), the bubble contribution is relatively small and positive for \( a^4 \Delta_{Mix} \lesssim 0.04 \) (see Fig. 7). Although the bubble contribution could be slightly negative for \( a^4 \Delta_{Mix} \gtrsim 0.04 \), we expect the lattice scalar correlator to be positive in this case, since the small negativity in the bubble contribution is most likely out-weighted by the positive contribution \( e^{-m_{\text{u/d}} t} \) in \((3)\).\(^11\)

Various choices of tuning the valence and sea quark masses have therefore large effects on the scalar correlator, although all the choices are equivalent in the continuum limit. UKQCD Collaboration in fact proposed to tune the valence and sea quark masses by considering the sign of the scalar correlator [7], but the proposal has not been used in practice yet.

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\(^9\)The results agree in the limit \( t \rightarrow \infty, a^2 \Delta_{Mix} \rightarrow 0 \) and \( m_u = m_d = m_s \).

\(^10\)The natural limit for the magnitude of taste breaking is \( |a^4 \Delta_{Mix}| < a^4 \Delta_t \approx 0.08 \). The additional restriction in case of negative \( a^4 \Delta_{Mix} \) is the positivity of the mass \( M_{\text{val},\text{sea}} \), which requires \( a^4 \Delta_{Mix} \gtrsim -0.03 \) for the simulation with \( a m_{u/d} / a m_s = 0.007/0.05 \).

\(^11\)The rough size of the \( a_0 \)-exchange in the point-point scalar correlator can be estimated from \( C_{ab}(t) = (16B_{\pi}^2 f_{a0}^2 / m_{ab}) e^{-m_{\text{u/d}} t} \) where \( f_{a0} \approx 0.08(3) \) GeV and \( m_{a0} = 1.58(34) \) GeV.
Figure 6: The bubble contribution $B_{2+1}^{\text{MChPT}}(t)$ (16) for the simulation with chiral fermions on $2+1$ MILC staggered configurations with volume $20^3 \times 32$ and coarse $a$. The valence and sea quark masses are tuned by matching $M_{\text{val},\text{val}} = M_{U_5}$. The unknown parameter $a^4 \Delta_{\text{Mix}}$ is varied in the reasonable range $-0.02 \leq a^4 \Delta_{\text{Mix}} \leq 0.08$.

3 Conclusions

The Nature of the observed scalar resonance $a_0(980)$ with $I = 1$ is not revealed yet. The determination of $a_0$ mass on the lattice could indicate whether $a_0(980)$ is a conventional $\bar{q}q$ state or perhaps something more exotic.

The $a_0$ mass is conventionally determined from the flavor non-singlet scalar correlator on the lattice. However, the interesting contribution $e^{-m_{\text{val}} t}$ to the scalar correlator is accompanied by the contribution from multi-hadron intermediate states. The most important
multi-hadron intermediate state is the bubble contribution $B$, which is the intermediate state with two pseudoscalars. In this paper we determined the size of the bubble contribution for simulations with three sea quarks of mass $m_u = m_d \neq m_s$. This offers a way to determine $m_{a0}$ by fitting the lattice scalar correlator to $C(t) = Ae^{-m_{a0}t} + B(t)$.

We determined the bubble contribution for the case of simulations with staggered sea and staggered valence quarks, as well as for simulations with staggered sea and chiral valence quarks. We found that the bubble contribution dominates the scalar correlator for small masses of $u/d$ quarks. In proper QCD, the lightest two-pseudoscalar intermediate state is $\pi\eta$ and the bubble contribution drops as $e^{-(M_\pi+M_\eta)t}$ at large $t$. The effects of taste breaking or mixed quark actions make possible also the intermediate state with mass $2M_\pi$, so the correlator drops as $e^{-2M_\pi t}$ at large $t$. This unphysical contribution to the scalar correlator vanishes only in the continuum limit.

- We used Staggered ChPT for predicting the bubble contribution in case of staggered sea and staggered valence quarks. Its sign depends on the values of the input parameters, indicating the broken unitarity of the theory due to the $\sqrt{\text{Det}}$ trick. All the input parameters have already been determined for the case of MILC configurations and they render positive bubble contribution.

- The case of staggered sea and chiral valence quarks was explored using Mixed ChPT. The size and the sign of the bubble contribution depends on one unknown parameter $a^2\Delta_{Mix}$. The bubble contribution and the scalar correlator are negative (positive) for most of the values $-0.02 \leq a^2\Delta_{Mix} \leq 0.08$ if the valence and the sea quark masses
are tuned by matching the masses of valence-valence pion and the lightest (heaviest) sea-sea pion. The comparison of the point-point scalar correlator from lattice to our prediction offers a way to determine the unknown parameter $a^2 \Delta_{Mix}$.

Our analytic prediction for the bubble contribution $B(t)$ will be helpful for the determination of $m_{a0}$ by fitting the lattice scalar correlator to $C(t) = Ae^{-m_{a0}t} + B(t)$. Such extraction of scalar meson mass is possible for moderate quark masses and times where $e^{-m_{a0}t}$ is not negligible with respect to the bubble contribution.

**Note added:**

Some of the results given in the present paper were already presented in the conference proceedings [18] by the same author.

**Acknowledgments**

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Appendix A: Coupling of a scalar current to two pseudoscalars

Here we derive the coupling of a point scalar current $\bar{d}(x)u(x)$ to a pair of pseudoscalar fields at the lowest order of ChPT. The effective scalar current can be determined from the dependence of the QCD Lagrangian and the Chiral Lagrangian on the spurion field $\mathcal{M}_{du}$, where $\mathcal{M}$ is quark mass matrix. The quark current $\bar{d}(x)u(x)$ is given by $-\partial\mathcal{L}_{QCD}/\partial\mathcal{M}_{du}$, so the effective current is represented by $-\partial\mathcal{L}_{ChPT}/\partial\mathcal{M}_{du}$. The Chiral Lagrangian is

$$\mathcal{L}_{ChPT} = \frac{1}{2}f^2\mathrm{Tr}[\partial^\mu U \partial_\mu U^\dagger] + \frac{1}{2}B_0f^2\mathrm{Tr}[\mathcal{M}^2U + U^\dagger\mathcal{M}] + \ldots,$$

where $U = \exp[\sqrt{2}f\Phi/f]$ incorporates the $SU(3)_L \times SU(3)_R$ pseudoscalar field matrix $\Phi$, $f \sim 95$ MeV, while the dots indicate terms of higher order or terms which are independent of $\mathcal{M}_{du}$. The effective current is therefore equal to

$$\bar{d}(x)u(x) \sim -\frac{1}{2}B_0f^2[U(x) + U^\dagger(x)]_{ud} = B_0[\Phi(x)^2]_{ud} + \ldots,$$

so the coupling of the current $\bar{d}(x)u(x)$ to a pair of pseudoscalar fields $\Phi$ is equal to the slope parameter $B_0 = M^2_\pi/(2m_q)$. The result is derived within conventional ChPT here, but it applies also to various extensions of CHPT (Staggered, Mixed, Quenched and Partially Quenched) at the lowest order. In the extended case $\Phi$ in Eq. (18) represents the field matrix of the extended effective field theory.

Appendix B: Pseudoscalar propagators in Staggered ChPT

Here we list the propagators of pseudoscalar mesons in SChPT for three sea-quark flavors (with one or four tastes) in case of $m_u = m_d \neq m_s$. All the propagators in Euclidean space-time have the connected part

$$\langle \phi^b_{ff'}|\phi^b_{ff'} \rangle_{con} = \frac{1}{k^2 + M^2_{ff'}} ,$$

where $M_{ff'}$ is the mass of the un-mixed meson with flavor $f'f$ and taste $b = 1, \ldots, 16$

$$M_{ff'}^2 = B_0(m_f + m_{f'}) + a^2\Delta_b , \quad M_{U_b} = M_{S_b} \equiv M_{uu_b} , \quad M_{S_b} \equiv M_{ss_b} , \quad \Delta_5 = 0 .$$

The taste-vector, taste-axial and taste-singlet mesons have also the disconnected part. The disconnected parts of $\langle \phi^b_{uu}|\phi^b_{uu} \rangle$, $\langle \phi^b_{dd}|\phi^b_{dd} \rangle$ and $\langle \phi^b_{uu}|\phi^b_{dd} \rangle$ are all equal to

$$\langle \phi^b_{uu}|\phi^b_{uu} \rangle_{disc} = -\frac{4}{3}(k^2 + M^2_{S_f})(k^2 + \frac{4}{3}M^2_{U_f} + \frac{2}{3}M^2_{S_f}) \quad \text{for } m_0 \rightarrow \infty$$

$$\langle \phi^V_{uu}|\phi^V_{uu} \rangle_{disc} = -a^2\delta^V \frac{k^2 + M^2_{S_f}}{(k^2 + M^2_{U_f})(k^2 + M^2_{U_f})}$$

$$\langle \phi^A_{uu}|\phi^A_{uu} \rangle_{disc} = -a^2\delta^A \frac{k^2 + M^2_{S_f}}{(k^2 + M^2_{U_f})(k^2 + M^2_{U_f})} .$$

(21)
These hairpin couplings mix the flavor neutral mesons with \( I, V, A \) tastes and the resulting mass eigenstates are \[ (14) \]

\[
M_{\pi I} = M_{U I} \quad , \quad M_{\eta_I}^2 = \frac{1}{3} M_{U I}^2 + \frac{2}{3} M_{S I}^2 \quad , \quad M_{\eta'_I} = \infty \quad \text{for} \quad m_0 \to \infty
\]

\[
M_{\pi V,A} = M_{U V,A} \quad , \quad M_{\eta V,A} = m_\eta (C a^2 \delta'_{V,A}) \quad , \quad M_{\eta'_V,A} = m_{\eta'} (C a^2 \delta'_{V,A})
\] (22)
with \( C = 1 \) in four-taste theory \( 2 \cdot 4 + 4 \) and \( C = 1/4 \) in a single-taste theory \( 2 + 1 \). The functions \( m_\eta \) and \( m_{\eta'} \) are defined as

\[
m_\eta (\delta) = \sqrt{\frac{1}{2} [M_{U V}^2 + M_{S V}^2 + 3 \delta - Z(\delta)]}
\]

\[
m_{\eta'} (\delta) = \sqrt{\frac{1}{2} [M_{U V}^2 + M_{S V}^2 + 3 \delta + Z(\delta)]}
\]

with

\[
Z(\delta) = \sqrt{(M_{S V}^2 - M_{U V}^2)^2 - 2 \delta (M_{S V}^2 - M_{U V}^2) + 9 \delta^2}
\]
for the vector taste and analogously for the axial taste.

**Appendix C: The sign of bubble contribution in SChPT**

Here we explore the sign of the bubble contribution \( B_{\text{SChPT}} (t) \) given in (8). This is relevant since the positive definite bubble contribution to scalar correlator is expected in a proper unitary quantum field theory. We consider the unitary four-taste theory \( 2 \cdot 4 + 4 \) and a single-taste theory \( 2 + 1 \), where the later loses unitarity due to the fourth-root trick.

Since the sign depends mostly on the values of the hairpin couplings \( a^2 \delta'_{V} \) and \( a^2 \delta'_{A} \), we concentrate on a special case with \( m_u = m_d = m_s \), \( \Delta_b = 0 \) and \( a^2 \delta'_{V} = a^2 \delta'_{A} \equiv a^2 \delta' \) for simplicity. In this case

\[
\lim_{t \to \infty} B_{\text{SChPT}}^{2+1} (t) = \frac{B_0^2}{2 \pi L^3} \left[ (4C - \frac{4}{9}) \frac{3 \pi}{2 M_\pi^2} e^{-2 M_\pi t} - 8 a^2 \delta' \ f_{\text{dis}} (C a^2 \delta') \right]
\]

\[
f_{\text{dis}} (\delta) = \frac{\pi}{6 \delta M_\pi^2 M_{\eta'}} \left[ M_{\eta'} e^{-2 M_\pi t} - M_\pi e^{-(M_\pi + M_{\eta'}) t} \right] , \quad M_{\eta'} = \sqrt{M_\pi^2 + 3 \delta}
\]
with \( C = 1 \) in four-taste theory and \( C = 1/4 \) in a single-taste theory. It is easy to find a positive value of \( a^2 \delta' \), which gives \( B_{\text{SChPT}}^{2+1} (t) < 0 \) indicating the breakdown of unitarity due to the fourth root trick. On the other hand, we find that \( B_{\text{SChPT}}^{2+1} (t) \geq 0 \) for any values of \( a^2 \delta' \), \( M_\pi \) and \( t \). The positive definiteness of \( B_{\text{SChPT}}^{2+1} (t) \) can be demonstrated by noting that \( B_{\text{SChPT}}^{2+1} (0) > 0 \), \( B_{\text{SChPT}}^{2+1} (\infty) = 0 \) and \( dB_{\text{SChPT}}^{2+1} (t) / dt < 0 \) for all \( t \), which prohibits any extremum with a negative value. The positive definiteness of the scalar correlator in the four-taste theory is a consequence of the unitarity.

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