Generation of current vortex by spin current in Rashba systems

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Employing unbiased large-scale time-dependent density-matrix renormalization-group simulations, we demonstrate the generation of a charge-current vortex via spin injection in the Rashba system. The spin current is polarized perpendicular to the system plane and injected from an attached antiferromagnetic spin chain. We discuss the conversion between spin and orbital angular momentum in the current vortex that occurs because of the conservation of the total angular momentum and the spin-orbit interaction. This is in contrast to the spin Hall effect, in which the angular-momentum conservation is violated. Finally, we predict the electromagnetic field that accompanies the vortex with regard to possible future experiments.

The interconversion of charge and spin degrees of freedom is a key issue in spintronics. Noteworthy phenomena in this regard are the spin Hall effect, which describes the generation of a transverse spin current by a charge current, and its inverse. These effects are due to a spin-asymmetry of conduction electrons by the spin-orbit coupling. A typical model for studying the spin-charge interconversion is the two-dimensional electron gas with Rashba spin-orbit coupling. Various effects due to the Rashba spin-orbit coupling have been extensively investigated, including the spin-orbit torque and the Edelstein effect. While the spin Hall conductivity actually vanishes in the Rashba model with quadratic dispersion, spin Hall physics may still be observed in mesoscopic Rashba systems. It was shown, for example, that charge current in a nanowire can induce spin accumulation at the lateral edges.

In this work, we investigate a junction, in which a spin current is transmitted into a Rashba system from an antiferromagnetic spin-1/2 Heisenberg chain. The spin current in the spin chain is carried by elementary excitations called spinons. Our goal is to demonstrate the conversion of this spinon spin current into a conduction-electron spin current in the Rashba system, and in particular to investigate the charge-current signal caused by the interplay of the spin injection and spin-orbit coupling. Although the junction is an interacting quantum system, it can nevertheless be efficiently simulated by using matrix-product-state methods combined with a Lanczos transformation of the Rashba system, allowing us to obtain unbiased numerical results for the current dynamics. Most notably, we show that when a spin current is injected into the Rashba system, where it causes the formation of a charge-current vortex (red and blue arrows). The orange segments denote the coupling between the spin chain and the lead and Rashba systems.

![Sketch of the setup described by Eqs. (1)-(4). A spin current (purple arrow) is induced in the spin chain by switching on a spin voltage in the lead. This spin current is injected into the Rashba system, where it causes the formation of a charge-current vortex (red and blue arrows). The orange segments denote the coupling between the spin chain and the lead and Rashba systems.](image)

FIG. 1. Sketch of the setup described by Eqs. (1)-(4). A spin current (purple arrow) is induced in the spin chain by switching on a spin voltage in the lead. This spin current is injected into the Rashba system, where it causes the formation of a charge-current vortex (red and blue arrows). The orange segments denote the coupling between the spin chain and the lead and Rashba systems.

The direction of the current is not uniform and the system instead has a rotational symmetry around the injection point. The junction thus has a conserved total angular momentum, and it turns out that the injected spin angular momentum is mostly converted to orbital angular momentum of the current vortex. At the end, we will discuss the relevance of our results for possible experiments.

Let us first introduce the setup in more detail. We consider a Rashba model in the $xy$-plane on an infinite square lattice with sites $r \in \mathbb{Z}^2$:

$$\hat{H}_R = -\mu \sum_{r} \sum_{\sigma=\uparrow,\downarrow} \hat{c}_r^{\dagger} \sigma \hat{c}_r_{\sigma} - t \sum_{(rr')} \sum_{\sigma=\uparrow,\downarrow} \hat{c}_r^{\dagger} \sigma \hat{c}_{r'}_{\sigma}$$

$$- i\lambda \sum_{r} \left( \hat{c}_r^{\dagger} \sigma^y \hat{c}_{r+e_y} - \hat{c}_r^{\dagger} \sigma^x \hat{c}_{r+e_x} - H.c. \right), \quad (1)$$
where $\mu$ is the chemical potential, $t$ is the hopping, $\lambda$ is the spin-orbit coupling strength, $\sigma^x$ and $\sigma^y$ are Pauli matrices, and $\hat{c}_r = (\hat{c}^{\uparrow}_r, \hat{c}^{\downarrow}_r)^T$ are fermion annihilation operators. One site \( r_0 \) shall be coupled to another system that is used to inject a spin current polarized in the \( z \)-direction (see Fig. 1). Specifically, we employ an antiferromagnetic spin-1/2 Heisenberg chain of length \( N_S \):

$$
\hat{H}_S = J \sum_{j \geq 1} \hat{S}_j \hat{S}_{j+1}, \quad J > 0.
$$

To generate a spin-current flow, the other end of the spin chain is connected to a one-dimensional semi-infinite tight-binding chain that serves as a spin reservoir:

$$
\hat{H}_L(\tau) = -t_L \sum_{j \geq 1} \sum_{\sigma} \left( \hat{c}^\dagger_{j,\sigma} \hat{c}^{\sigma\downarrow}_{j+1,\sigma} + \text{H.c.} \right)
- \Theta(\tau) \frac{V}{2} \sum_{j \geq 1} \left( \hat{c}^\dagger_{j,\uparrow} \hat{c}^{\uparrow\downarrow}_{j,\uparrow} - \hat{c}^\dagger_{j,\downarrow} \hat{c}^{\downarrow\uparrow}_{j,\downarrow} \right). \quad (3)
$$

The second term in Eq. (3) describes a spin voltage that is switched on at time \( \tau = 0 \). Finally, the coupling between the subsystems is given by

$$
\hat{H}_C = J' \sum_{\nu=x,y,z} \hat{S}^\nu_{N_S} \hat{c}^\dagger_{0,\nu} \hat{c}^{\nu\downarrow}_{0,\nu} + J'' \sum_{\nu=x,y,z} \hat{S}^\nu_1 \hat{c}^\dagger_{1,\nu} \hat{c}^{\nu\downarrow}_{1,\nu}
$$

with \( J', J'' > 0 \), i.e., an antiferromagnetic Heisenberg interaction. The complete Hamiltonian then becomes

$$
\hat{H}(\tau) = \hat{H}_R + \hat{H}_S + \hat{H}_L(\tau) + \hat{H}_C.
$$

It is assumed that the composite system is initially in the ground state of \( \hat{H}(\tau < 0) \) until the spin voltage is switched on. Throughout this paper, we set \( N_S = 12 \), \( J/t = t_L/t = 2 \), \( \mu/t = -3.5 \), and \( V/t = 0.5 \). Since \( \hat{H}(\tau) \) conserves the particle number in each tight-binding system, no charge current is injected in addition to the spin current.

We are interested in the charge-current that instead develops as a consequence of the injected spin current and the spin-orbit coupling. Here, the charge-current-density operators for neighboring sites \( r \) and \( r + e_{x,y} \) are defined by

$$
\hat{J}_{r,r+e_x}^c = \hat{c}^\dagger_{r,e_x} (-itI + \lambda \sigma^y) \hat{c}_{r+e_x} + \text{H.c.}
$$

and

$$
\hat{J}_{r,r+e_y}^c = \hat{c}^\dagger_{r,e_y} (-itI - \lambda \sigma^x) \hat{c}_{r+e_y} + \text{H.c.,}
$$

with \( J \) being the unit matrix in spin space, so that the total current at site \( r \) is

$$
\hat{J}_{r}^c = \frac{1}{2} \left[ \hat{J}_{r,r+e_x}^c + \hat{J}_{r-r+e_y}^c + (\hat{J}_{r-r+e_x}^c + \hat{J}_{r-r+e_y}^c) e_y \right]. \quad (5)
$$

In order to simulate the above model numerically, we use a Lanczos transformation that maps the two-dimensional Rashba system to a chain representation \[20,23\]. The Hamiltonian then becomes purely one-dimensional and matrix-product-state techniques can be used to calculate the ground state and simulate the time-evolution with high accuracy \[17,19\]. To be precise, we utilize a tensor-network representation in which each tight-binding chain is split into two branches corresponding to different spin indices (pseudospin indices for the Rashba system \[20,21,23\]. This significantly reduces the numerical effort compared with a regular matrix-product state. Figure 2 displays the tensor network in the usual graphical notation.

For the numerical calculations, the tight-binding chain and the Lanczos representation of the Rashba system are each truncated to 500 sites. The time evolution is carried out using the time-evolving block decimation with a second-order Suzuki-Trotter decomposition and a time step of \( 0.1t^{-1} \). For all simulated times the truncation error is kept below \( 10^{-7} \). In the Supplemental Material \[25\], the Lanczos transformation is described in further detail and the accuracy of the numerical results is checked by confirming that they are in accordance with the spin continuity equations.

When the spin voltage is switched on in the first lead, a spin current starts to flow at the interface with the spin chain. The perturbation spreads through the chain, approximately with the spinon velocity \( J\pi/2 \), and finally reaches the Rashba system. At low temperatures, the efficiency of the spin injection into the Rashba system depends strongly on the coupling \( J'/J = 2.15 \) and \( J''/J = 1.70 \) in order to maximize the spin current in the steady state. For these parameters, the spin current into the Rashba system quickly saturates to a value slightly below \( V/(4\pi) \), which is the current corresponding to the expected linear spin conductance of the junction with ideal contacts. In the following, we analyze the charge current induced by this continuous spin-current injection. We assume that the spin current is polarized in the \( z \)-direction. Results for an \( x \)-polarized spin current are presented and briefly discussed in the Supplemental Material \[25\].

Figure 3 shows the numerically calculated charge-current profile for spin-orbit coupling parameters \( \lambda/t = 0.1 \) and 0.2, and different simulated times \( \tau \). Clearly, multiple rings with circular charge-current develop and then persist for long times. Neighboring rings have opposite orientation, i.e., the current alternates between clockwise...
and counterclockwise. This behavior can be understood qualitatively as follows: A spin current in the Rashba system generates a transverse charge current via the inverse spin Hall effect [29]. Here, the spin current points in the radial direction relative to the injection point, which leads to the observed circular charge current. Because of the Rashba spin precession, the spin current oscillates as a function of the distance \(r\) from the injection point, so that the charge current eventually changes direction as \(r\) is increased.

To make analytical predictions for the induced charge current that can be compared with the numerical results, it is more convenient to work with the continuous Rashba Hamiltonian

\[
\hat{H}_R = \hat{p}^2/(2m) + \alpha(\sigma^x \hat{p}_y - \sigma^y \hat{p}_x).
\]

By setting \(m = 1/(2t)\) and \(\alpha = -2\lambda\), \(\hat{H}_R\) can be used to analyze the lattice version Eq. (1) in the long-wavelength limit \(k \to 0\). The continuum results are therefore applicable if the spin-orbit-coupling strength \(\lambda/t\) is small and the Fermi energy \(\varepsilon_F\) is close to the bottom of the electron bands (working at zero temperature, \(\mu\) becomes the Fermi energy \(\varepsilon_F\)). In this regime, the wavenumber of the Rashba precession is \(k_R = 2\lambda/t\), which agrees with the widths of the observed current rings.

Figure 4 shows the radial dependence of the current for the largest simulated time \(\tau t = 45\) in more detail. Here, the charge current is separated into two parts, \(\hat{j}_r^c\) and \(\hat{j}_\phi^c\), which are the terms proportional to \(t\) and \(\lambda\), respectively. Namely, we define

\[
\hat{j}_{r,t}^c = \frac{it}{2} \left[ \hat{c}_r^\dagger (\hat{c}_{r-\varepsilon_x} - \hat{c}_{r+\varepsilon_x}) - \text{H.c.} \right] e_x \]
\[
+ \frac{it}{2} \left[ \hat{c}_r^\dagger (\hat{c}_{r-\varepsilon_y} - \hat{c}_{r+\varepsilon_y}) - \text{H.c.} \right] e_y,
\]
\[
\hat{j}_{\lambda,r}^c = \frac{\lambda}{2} \left[ \hat{c}_r^\dagger \sigma^y (\hat{c}_{r+\varepsilon_x} + \hat{c}_{r-\varepsilon_x}) + \text{H.c.} \right] e_x
\]
\[
- \frac{\lambda}{2} \left[ \hat{c}_r^\dagger \sigma^x (\hat{c}_{r+\varepsilon_y} + \hat{c}_{r-\varepsilon_y}) + \text{H.c.} \right] e_y.
\]

The functional form of the two contributions can be explained using a semi-classical analysis in terms of wavepackets deflected by a spin-orbit force [30]. Let us consider the trajectory of an electron wavepacket at the Fermi energy \(\varepsilon_F\) that has average momentum \(\hat{p}\) and is initially centered at \(\mathbf{r} = 0\) with the spin pointing up. In addition to propagating in the direction of \(\mathbf{p}\), it experiences an effective transverse force proportional to the \(z\)-component of the spin and the magnitude \(p\) of the momentum. Since the spin oscillates with wavenumber \(k_R\) because of the spin-orbit coupling, so does the deflecting force. This transverse movement corresponds to the spin-orbit part \(\hat{j}_\phi^c\) of the charge current. Furthermore, it causes the momentum \(\hat{p}\) to no longer point in the radial direction \(e_r = (x,y)^T/r\), so that the regular part \(\hat{j}_r^c\) of the current obtains a finite component in the azimuthal direction \(e_\phi = (-y,x)^T/r\) as well. By assuming that the injected spin current is composed in equal parts of wavepackets for spin\(\uparrow\) electrons and spin\(\downarrow\) holes that are evenly distributed over all directions, one obtains the following prediction for the charge current for long times \(\tau\) and small \(\lambda/t\):

\[
\hat{j}_r^c(r) = j^c_r \frac{2A \sin^2(k_Rr/2)}{k_R} e_\phi,
\]
\[
\hat{j}_\phi^c(r) = -j^c_\phi A \frac{\sin(k_Rr)}{r} e_\phi,
\]

where \(A = 2\lambda/(v_F\pi)\) is a constant that depends on the Fermi velocity \(v_F = 2t\sqrt{4 + (\lambda/t)^2 + \varepsilon_F/t}\), and \(j^c_r\) is the
injected spin current. Inserting for \( j^z \) the time-averaged value from the numerical simulations, we obtain excellent agreement with the numerically calculated charge current for \( r \gtrsim 8 \) (see Fig. 4), without any adjustable parameters. Deviations for small \( r \) are likely due to the lattice discretization.

Since the continuous Rashba Hamiltonian \( \hat{H}_R \) is symmetric under a simultaneous rotation of space and spin, the \( z \)-component of the total angular momentum \( \hat{J}^z = \hat{M} + \hat{S}^z \), where \( \hat{M} = \hat{x}\hat{p}_y - \hat{y}\hat{p}_x \) is the orbital angular momentum, is conserved. While the lattice Hamiltonian \( \hat{H}_R \) does not have this symmetry, we may expect the conservation of the total angular momentum to hold approximately, when the Fermi energy is small and the lattice model behaves similar to the continuum model. To be concrete, we define the orbital angular momentum on the lattice as \( \hat{M} = \hat{x}\sin(\hat{p}_y) - \hat{y}\sin(\hat{p}_x) \). Using the first-quantized version of Eq. (1), \( \hat{H}_R = -2t[\cos(\hat{p}_y) + \cos(\hat{p}_x)]I - 2\lambda(\sigma^x\sin(\hat{p}_y) - \sigma^y\sin(\hat{p}_x)) \), one then obtains from the Heisenberg equation: \( d\hat{S}^z/d\tau = -2\lambda(\sin(\hat{p}_y)\sigma^y + \sin(\hat{p}_x)\sigma^x) \) and \( d\hat{M}/d\tau = 2\lambda[\cos(\hat{p}_x)\sin(\hat{p}_y)\sigma^y + \cos(\hat{p}_y)\sin(\hat{p}_x)\sigma^x] \). Obviously, \( \hat{S}^z + \hat{M} \) is approximately conserved if we confine our analysis to states with small momenta \( p \). To calculate \( \hat{M} \) in the interacting model numerically, we use the second-quantized expression

\[
\hat{M} = -\frac{1}{2} \sum_\mathbf{r} \sum_\sigma \left[ i e^{-\frac{i}{\hbar} \mathbf{r} \cdot \mathbf{E}_\sigma} \frac{\partial}{\partial \mathbf{r}_\sigma} \hat{c}^\dagger_{\mathbf{r} \sigma} \hat{c}^{}_{\mathbf{r} + \mathbf{E}_\sigma \sigma} - i g \frac{\partial}{\partial \mathbf{r}_\sigma} \hat{c}^\dagger_{\mathbf{r} \sigma} \hat{c}^{}_{\mathbf{r} + \mathbf{E}_\sigma \sigma} + \text{H.c.} \right].
\]

(11)

Comparing with Eq. (7), one can see that \( \hat{M} \) is determined by the regular part \( \hat{J}^z_R \) of the charge-current-density operator \( \hat{J}^z \).

When the spin current is injected, it increases the total angular momentum \( \hat{J}^z_R = \hat{S}^z_R + \hat{M} \) in the Rashba system. One might then ask how \( \hat{J}^z_R \) is composed of the spin \( \hat{S}^z_R \) and the orbital contribution \( \hat{M} \). Figure 5 displays the numerical results for the time evolution of the angular-momentum expectation values. As noted above, the total angular momentum is not exactly conserved but the deviation is relatively small for \( \varepsilon_F/t = -3.5 \). Initially, \( \hat{M} = \hat{S}^z_R = 0 \) because the spin current has not entered the Rashba system yet. The delay before the angular momenta visibly change is in agreement with the expectation \( N_S/v_S \approx 3.8 \) based on the spinon velocity \( v_S = J\pi/2 \) in the infinite chain. For short times after the spin current has reached the Rashba system, \( \hat{S}^z_R \) makes up most of the angular momentum while \( \hat{M} \) remains approximately zero. On longer time-scales, however, \( \hat{S}^z_R \) can be seen to oscillate around zero, which means that eventually most of the injected spin angular momentum is converted to orbital angular momentum \( \hat{M} \). With the same assumptions used to derive Eqs. (9) and (10), one obtains that both the amplitude and the period of the oscillations are proportional to the wavenumber \( k_R \) of the Rashba precession. The numerical results roughly agree with these predictions, except that the oscillations in \( \hat{S}^z_R \) and \( \hat{M} \) also appear to decrease with time.

We estimate the magnetic field generated by the current vortex following the Biot-Savart law of electromagnetism. By assuming \( \lambda/t = 0.1 \), a lattice constant of 1 Å, a hopping parameter \( t = 1 \) eV and a linear dependence of the induced charge current on the spin voltage \( V \), we obtain a field strength \( B \approx V \cdot 10^{-5} \) T/eV at the center. Only a single spin chain was used in the model system, however, and we expect much larger fields if a bundle of chains is employed instead. If such a system is realized experimentally, the magnetic field may be observed by scanning tunneling microscopy methods. One could also consider injecting an AC spin current into the two-dimensional electron gas, in which case the current vortex would emit an electromagnetic field of similar strength.

In conclusion, a charge current vortex can be generated in a Rashba system by locally injecting a spin current. The formation of the current vortex is accompanied by
the conversion of the injected spin angular momentum to orbital angular momentum. We demonstrated these effects for a generic model in which the spin current is transferred from an antiferromagnetic Heisenberg spin chain to a square-lattice Rashba system. Accurate time-dependent density-matrix renormalization-group results for the charge current were found to agree well with predictions from semi-classical considerations. The charge-current vortex induces an electromagnetic field, which may be observed experimentally.

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SUPPLEMENTAL MATERIAL

Block Lanczos transformation

We use a Lanczos transformation that maps the two-dimensional Rashba system to a chain representation [S1, S2], which can be more efficiently numerically simulated with tensor-network techniques. The method amounts to applying the usual block Lanczos recursion to the matrix $H_R$ of the single-particle Rashba Hamiltonian on the lattice, using as initial vectors the spin-$\uparrow$ and spin-$\downarrow$ states of the site $r_0$ coupled to the spin chain. In second quantization, the transformed Rashba Hamiltonian is [S2]

$$
\hat{H}_R = -\mu \sum_{j \geq 1} \sum_{\sigma = \uparrow, \downarrow} \hat{a}_{j,\sigma}^\dagger \hat{a}_{j,\sigma} + \sum_{j \geq 1} \sum_{\sigma = \uparrow, \downarrow} (\tilde{t}_j \hat{a}_{j,\sigma}^\dagger \hat{a}_{j+1,\sigma} + \text{H.c.}),
$$

(S1)

where $\tilde{t}_j$ are bond-dependent real hopping parameters, $\hat{a}_{j,\sigma}$ are fermion annihilation operators and $\hat{a}_{1,\sigma} = \hat{c}_{r_0,\sigma}$. Note that $\sigma$ is a pseudospin index that is equal to the physical spin only at the first site. The hopping parameters $\tilde{t}_j$ converge to $2t\sqrt{1 + \lambda^2/(2t^2)}$ as $j \to \infty$.

By applying the Lanczos transformation, the Hamiltonian becomes purely one-dimensional, consisting of a spin chain coupled to two semi-infinite tight-binding chains. For our numerical simulations, however, the system also needs to be truncated to a finite size. We do this by carrying out the Lanczos transformation for the first 150 sites and adding another 350 sites with the asymptotic hopping parameter as boundary conditions. Based on the calculated $\tilde{t}_j$, we estimate that the deviation from the exact hopping parameters is below 0.05%. The one-dimensional lead is truncated to 500 sites as well.

The correlation functions in the chain representation of the Rashba system are used to calculate the expectation values in the original two-dimensional system by applying the reverse Lanczos transformation. Details on this procedure are given in Ref. S2.

Spin continuity equation

In this section, we show the spin continuity equations for the Rashba model on a square lattice, and compare them against the numerical results presented in the main text. The Heisenberg equations of motion for the fermion operators $\hat{c}_r$ are

$$
\frac{d}{dt} \hat{c}_r = i\lambda \left( \hat{c}_{r+e_x} + \hat{c}_{r-e_x} + \hat{c}_{r+e_y} + \hat{c}_{r-e_y} \right) - \lambda \left[ \sigma^y (\hat{c}_{r+e_x} - \hat{c}_{r-e_x}) - \sigma^x (\hat{c}_{r+e_y} - \hat{c}_{r-e_y}) \right].
$$

(S2)

We ignored the chemical potential here, since it does not affect the time evolution of operators that conserve the total number of electrons. The time derivative of the total spin of a subset of sites $G$ is

$$
\frac{d}{dt} \hat{S}_G^\alpha = \sum_{r \in G} \frac{d}{dt} \hat{s}_G^\alpha = \sum_{r \in G} \frac{1}{2} \left[ \left( \frac{d}{dt} \hat{c}_r^\dagger \right) \sigma^\alpha \hat{c}_r + \hat{c}_r^\dagger \sigma^\alpha \left( \frac{d}{dt} \hat{c}_r \right) \right] = \hat{\Gamma}_{t,x}^\alpha + \hat{\Gamma}_{t,y}^\alpha + \hat{\Gamma}_{\text{soc},x}^\alpha + \hat{\Gamma}_{\text{soc},y}^\alpha,
$$

(S3)

where we have introduced

$$
\hat{\Gamma}_{t,x}^\alpha = -\frac{i\lambda}{2} \sum_{r \in G} \left[ (\hat{c}_{r+e_x}^\dagger + \hat{c}_{r-e_x}^\dagger) \sigma^\alpha \hat{c}_r - \hat{c}_r^\dagger \sigma^\alpha (\hat{c}_{r+e_x} + \hat{c}_{r-e_x}) \right],
$$

(S4)

$$
\hat{\Gamma}_{t,y}^\alpha = -\frac{i\lambda}{2} \sum_{r \in G} \left[ (\hat{c}_{r+e_y}^\dagger + \hat{c}_{r-e_y}^\dagger) \sigma^\alpha \hat{c}_r - \hat{c}_r^\dagger \sigma^\alpha (\hat{c}_{r+e_y} + \hat{c}_{r-e_y}) \right],
$$

(S5)

and

$$
\hat{\Gamma}_{\text{soc},x}^\alpha = \frac{\lambda}{2} \sum_{r \in G} \left[ (\hat{c}_{r+e_x}^\dagger - \hat{c}_{r-e_x}^\dagger) \sigma^y \sigma^\alpha \hat{c}_r + \hat{c}_r^\dagger \sigma^\alpha \sigma^y (\hat{c}_{r+e_x} - \hat{c}_{r-e_x}) \right],
$$

(S6)

$$
\hat{\Gamma}_{\text{soc},y}^\alpha = \frac{\lambda}{2} \sum_{r \in G} \left[ (\hat{c}_{r+e_y}^\dagger - \hat{c}_{r-e_y}^\dagger) \sigma^x \sigma^\alpha \hat{c}_r + \hat{c}_r^\dagger \sigma^\alpha \sigma^x (\hat{c}_{r+e_y} - \hat{c}_{r-e_y}) \right].
$$

(S7)

In the continuum limit, $\hat{\Gamma}_{t,x}^\alpha + \hat{\Gamma}_{t,y}^\alpha$ corresponds to an integral of the divergence of the conventional spin current.
FIG. S1. Numerical test of the spin continuity equation for $\alpha = z$ and $\lambda/t = 0.1$. The black line indicates the time derivative of the $z$-component of the total spin $\hat{S}_z$, the green line shows the injected spin current from the spin chain to the Rashba system, and the blue line denotes the rest contributions of the right hand side of the spin continuity equation.

Equation (S3) can be used to check the accuracy of the numerical simulations. For this purpose, we evaluate the total spin $S_z$ (setting $G$ as the set of all sites) in the Rashba system at different times $\tau$, and then calculate the numerical derivative of $S_z(\tau)$. According to the continuity equation, we have $d\hat{S}_z/d\tau = \hat{j}_z + \hat{\Gamma}^{z}_{soc,x} + \hat{\Gamma}^{z}_{soc,y}$, where $\hat{j}_z$ is the spin current from the spin chain into the Rashba system (the terms $\hat{\Gamma}^{z}_{t,x}$ and $\hat{\Gamma}^{z}_{t,y}$ vanish for $\alpha = z$ when we sum over the whole system). As shown in Fig. S1, this is in good agreement with the numerical results.

Spin current polarized in-plane

In the main text, we considered an injected spin current polarized in the $z$-direction, which is perpendicular to the plane of the Rashba system. Here, we present results for a spin current polarized in the $xy$-plane. From the effective one-dimensional model obtained by the Lanczos transformation it is clear that the magnitude of the spin current injected into the system is independent of the polarization direction [S2]. The induced charge current, on the other hand, is different. Figure S2 displays the charge current for $x$-polarization, with other parameters the same as in Fig. 3 of the main text. Instead of a vortex, there is now a net current in the negative $y$-direction. Since there is no rotational symmetry, it is more difficult to make precise predictions for the charge-current profile, but one can understand the main features by noting that (i) the inverse Rashba-Edelstein effect causes a charge current in the $y$-direction due to a distortion of the Fermi surface [S3, S4], and (ii) the Rashba precession along the $x$-direction tilts the spin-current polarization out of the system plane, so that an inverse spin Hall current appears [S5]. The latter effect leads to oscillations of the charge current along the $x$-axis. An important difference to the $z$-polarized spin injection is that the charge current is symmetric instead of antisymmetric under inversion about the $y$-axis. The orbital angular momentum is therefore zero. We also note that there is no conserved total angular momentum unless the spin current is polarized in the $z$-direction.

Derivation of Eqs. (9) and (10)

The eigenfunctions and corresponding eigenvalues of the continuous Rashba Hamiltonian are

$$\psi_{\pm,p}(r) = \frac{C}{\sqrt{2}} \left( \frac{1}{\pm ie^{i\varphi_p}} \right) e^{i p r}, \quad \epsilon_{\pm,p} = p^2/2m \pm \alpha p,$$

(S8)
FIG. S2. Charge current induced by an injected spin current polarized in the $x$-direction. For easier comparison, the other parameters are the same as in Fig. 3 of the main text, i.e., $N_S = 12$, $J/t = t_L/t = 2$, $\mu/t = -3.5$ and $V/t = 0.5$.

where $p = |\mathbf{p}|$, $\tan(\phi_p) = p_y/p_x$ and $C$ is a normalization constant. At fixed energy, the two bands are separated by a momentum difference $k_R = 2m\alpha$.

To describe the injection of a pure spin current, we consider pairs of spin-$\uparrow$ electron and spin-$\downarrow$ hole wavepackets that are initially localized at $r = 0$. An electron wavepacket at the Fermi energy that is polarized in the $z$-direction and has average momentum $\mathbf{p}$ may be constructed as a superposition of states near $\psi_+ \mathbf{p}(1 - m\alpha/p)$ and $\psi_- \mathbf{p}(1 + m\alpha/p)$, which have opposite spin orientation. As the wavepacket propagates, the momentum difference between the two contributions causes the spin to precess around an effective magnetic field proportional to $\mathbf{p} \times \mathbf{e}_z$. While the wavepacket of course spreads over time, we assume here that it can be treated as a classical particle with sharply defined momentum $\mathbf{p}$ and position $r$. Furthermore, we henceforth consider specifically a spin-$\uparrow$ electron wavepacket moving in the $x$-direction.

Owing to the spin precession, the expectation value of the the $z$-component of the spin is $S_z(x) = \cos(k_R x)/2$. The spin-orbit interaction makes the electron experience an effective transverse force $-4\alpha^2 m \mathbf{p} S_z \mathbf{e}_y$ [S6], which deflects it in the $y$ direction but does not alter the momentum. Using $x = v_F \tau$, where $v_F = p/m$ is the Fermi velocity, one obtains $y(x) = -\frac{1}{2} \frac{p}{m} [1 - \cos(k_R x)]$. This could also be derived by noting that the total angular momentum should be conserved, so that a change in $S_z$ has to be compensated by an opposite orbital angular momentum, i.e., $-yp = 1/2 - S^z$. The azimuthal and radial components of the electron’s velocity are

\[
\frac{d\mathbf{r}}{dt} = \frac{1}{r} \left( \frac{1}{2m} [1 - \cos(k_R x)] - \frac{k_R}{2m} x \sin(k_R x) \right)
\approx \frac{1}{mr} \sin^2(k_R r/2) - \frac{k_R}{2m} \sin(k_R r)
\]
and
\[
    e_r \cdot \frac{dr}{d\tau} = \frac{1}{r} \left( xv_F + \frac{k_R}{4pm} [1 - \cos(k_Rx)] \sin(k_Rx) \right)
\]
\[
    \approx v_F,
\]
(S10)
respectively. In the second lines of Eqs. (S9) and (S10), we assumed that \( y/x \ll 1 \), which is valid for a small spin-orbit force, or long-enough distances \( r \). For a spin-↓ hole, the radial component is the same while the azimuthal one is reversed. As described in the following, the above equations can be used to estimate the charge current induced by the spin injection.

In our approximation, the spin current \( j^z \) into the Rashba system is equal to the rate with which the pairs of spin-↑ electron and spin-↓ hole wavepackets are injected. Since the wavepackets are evenly distributed over all directions and move radially approximately with \( v_F \), their density is \( j^z/(2\pi rv_F) \). The charge-current density, taking both spin-↑ electrons and spin-↓ holes into account, is then
\[
    j^c(r) = \frac{j^z}{\pi v_F} \left[ \frac{1}{mr} \sin^2(k_Rr/2) - \frac{k_R}{2m} \sin(k_Rr) \right] e^\phi.
\]
(S11)
There is no radial component, since the electron and hole contributions cancel each other. The first term in Eq. (S11) can be identified as the regular part of the charge current, and the second term as the additional spin-orbit part. By setting \( m = 1/(2\hbar^2) \) and \( \alpha = -2\lambda \), we obtain Eqs. (9) and (10) of the main text.

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