Observation of long-range quantum interferences of d-wave Andreev pairs in graphene

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Recent experiments have shown that superconducting correlations can be induced in graphene by proximity with high-temperature (cuprate) superconductors. Here we demonstrate that such correlations can propagate hundreds of nm into large-scale grown graphene, thereby allowing for the unusual observation of long-range interferences between d-wave Andreev pairs. This phenomenon is shown in planar YBa\textsubscript{2}Cu\textsubscript{3}O\textsubscript{7}/graphene devices that behave as Fabry-Perot interferometers, and it manifests in a series of pronounced conductance oscillations analogous to the De Gennes-Saint James resonances originally predicted for ultrathin metals backed by superconductors. The interpretation of the experimental results is supported by numerical simulations based on an extended Blonder-Tinkham-Klapwijk model that qualitatively and quantitatively reproduces the observed behavior.

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The superconducting proximity effect in graphene has remained a topic of much interest since the pioneering experiments of Ref. [1]. Much work has followed, motivated by the fact that the underlying mechanism –the Andreev reflection and propagation of electron-hole pairs [2]– is strongly affected by the graphene’s electronic structure. This yields a phenomenology significantly different from that observed in conventional metals. One distinctive feature is the strong dependence of the proximity behavior on the graphene’s doping level, which dramatically changes the Andreev reflection across the interface [3,4]. Other unique features include the transition from bulk to edge transport in mesoscopic samples driven by gate voltages [5] or magnetic fields [6,7]. In addition to their intrinsic interest, fundamental studies on graphene have paved the way for understanding the proximity effect in other Dirac materials, such as topological insulators [8,9].

Most of experimental research on the superconducting proximity effect in graphene has been carried out using low critical temperature ($T_C$) superconductors with conventional s-wave pairing. Although early theoretical studies showed that the d-wave character of high-$T_C$ superconductivity enriches the proximity behavior through the emergence of directional effects [10,11] or exotic pairing [12], evidence for high-$T_C$ superconducting correlations induced in graphene has been found only recently [13,14]. Namely, scanning electron tunneling microscopy (STM) of a single-layer graphene on Pr$_{2-x}$Ce$_x$CuO$_4$ (PCCO) revealed spectral features related to the opening of a superconducting gap [13], which led to the conclusion that a (p-wave) superconducting density of states was induced in graphene. However, those STM experiments did not determine the length scale over which the induced correlations decay. Some of the present paper’s authors reported recently experiments on solid-state devices based on YBa$_2$Cu$_3$O$_7$ (YBCO) and chemical-vapor-deposited (CVD) graphene, which showed transparent superconductor/graphene interfaces and clear evidence of Andreev reflection [14]. Interestingly, it was also found that the transmission of Andreev
electron/holes pairs can be modulated by tuning the Fermi wave-vector through the application of a back-gate voltage—an effect analogous to the Klein tunneling of non-superconducting electrons across graphene heterojunctions [15]. In these experiments the penetration length of superconducting correlations into graphene could not be probed either.

In this work, we report on the observation of d-wave Andreev-pair interferences enabled by the long-range propagation of non-conventional superconducting correlations into graphene. These interferences manifest through oscillations in the differential conductance of YBCO/graphene planar devices, and they are caused by the confinement of Andreev pairs within a graphene “cavity” whose length $L$ can be as long as hundreds of nm. Predicted in theoretical studies of proximitized graphene heterojunctions [10,11], their origin is analogous to that of the De Gennes-Saint James [16] and McMillan-Rowel resonances [17] found in the electronic density of states of ultrathin (~nm) normal-metal films backed by superconductors. A cartoon of the mechanism responsible for the oscillations is shown in Fig. 1 (a). Electrons injected in the cavity are Andreev-reflected as holes at the interface with the superconductor, retrace their path to the opposite end of the cavity where they are normal-reflected, and travel back towards the interface with the superconductor where they are once again Andreev-reflected, now as electrons. This process results in destructive/constructive interferences, as dictated by the energy-dependent phase accumulated along the described loop. As discussed below, this phenomenon manifests through oscillations of the conductance as a function of the voltage bias $V_{BIAS}$ across the YBCO/graphene device. In the case of a d-wave superconductor, Andreev pairs stemming from the process depicted in Fig. 1 (a) have d-wave symmetry, and therefore they decay away from the interfaces over the mean free path $l$ [18,19]. In other words, the quantum interference can only occur if the distance between the two interfaces is not larger than $l$. Thus, our measurements confirm ballistic transport over hundreds of nanometers. Consistently, the Andreev-pair interferences are accompanied by Fabry-Perot
electron resonances that result from normal-reflections at both interfaces of the cavity [sketch in Fig. 1(b)]. These resonances are due to commensurability between the cavity’s length $L$ and the electrons’ wavelength [20–24], and manifest as conductance oscillations as a function the top-gate voltage $V_G$ and bias voltage $V_{BIAS}$ because both of these two knobs tune the conducting electron’s wavelength. As discussed below, oscillations caused by normal electrons can be distinguished from those produced by Andreev pairs by their distinct periodicity and from the different voltage regime in which they dominate. The concurrent observation of both types of resonances is to the best of our knowledge a novelty, which is linked to the characteristics of superconductivity in YBCO. First, YBCO has a large superconducting gap (tens of meV) that can enclose various orders of Andreev-pair interferences. This makes their observation easier than with low-temperature (low-energy gap) superconductors. Second, YBCO has d-wave pairing which, as discussed above, leads to the necessary condition $L < l$ that is simultaneously satisfied for both types of oscillations.

The planar devices [Fig. 2 (a)] consist of four superconducting (SC) YBCO$_{50nm}$/Au$_{4nm}$ electrodes enclosed by an insulating (INS) YBCO matrix. The electrodes are separated by a gap (insulating YBCO), whose length $L$ ranges between 100 nm and 800 nm. The gap is bridged by single-layer CVD grown graphene (G) [25] that connects the four superconducting electrodes. Each of the four YBCO electrodes terminates in a large gold pad [labelled 1-4 in Fig. 2 (b)], which allows wire-bonding the devices for electrical measurements. The whole structure is covered by a 45 nm-thick ALD deposited AlO$_x$ layer [26,27], which is used as the top-gate dielectric, with a top-gate of Au defined over an area where the four YBCO electrodes meet [Fig. 2 (c)] and that covers the graphene entirely. Details on the sample fabrication steps can be found elsewhere [14].

In order to investigate the interference effects, we perform three-probe measurements in which the injected current $I$ flows from contact 1 to contact 4 and the voltage $V_{BIAS}$ is
measured between contacts 1 and 3. In this configuration, we probe the resistance of the YBCO/Au/graphene interface in series with the graphene cavity, as shown in the equivalent circuit sketched Fig. 2 (d). The differential conductance $G(V_{BIAS}) = dI/dV_{BIAS}$ is measured using a Keithley 6221 current source coupled to a Keithley 2182.

Representative examples of the lowest temperature (3.2 K) differential conductance measurements are shown in Fig. 3 (a), 3(d) and Fig. 4 (a) for different devices. In all cases, $G(V_{BIAS})$ shows a low-bias feature that stands out from the higher bias conductance level. They appear either as a conductance decrease (“dip”) [Fig. 3 (d)] or an enhancement (“peak”) [Fig. 4 (a)] within a typical bias range $|V_{BIAS}| < \delta_{exp} \sim 20-70$ mV as highlighted by the grey shade in the figures. These features are reminiscent of the conductance decrease (increase) observed for $e|V_{BIAS}| < \Delta$ in superconducting/normal-metal junctions with low (high) interface transparency [28]. For some devices the dip/peak feature extends over a bias range $\delta_{exp}$ that exceeds the maximum superconducting energy-gap expected for YBCO, $\Delta_{YBCO} \sim 30$ meV. As sketched in the bottom of Fig. 2(d). This is because $V_{BIAS}$ does not solely drop across the YBCO/Au/graphene interface, but also along the graphene in series with it.

In addition to the central feature, and regardless of whether it is a “peak” or a “dip”, an oscillation pattern appears superposed to the background conductance [see e.g. Fig. 3 (a) and 3 (d)]. The oscillations extend over a bias range well above $\delta_{exp}$. A close inspection of the curves allows distinguishing two type of oscillations: those having a longer period $\Delta V_{long}$ which is predominant for $|V_{BIAS}| > \delta_{exp}$ (highlighted by vertical black lines), and another those having shorter period $\Delta V_{short}$ which is more prominent at $|V_{BIAS}| < \delta_{exp}$ (vertical red lines).

Fig. 4 illustrates the gating effects on the devices’ conductance. Fig. 4 (a) displays $G(V_{BIAS})$ with no applied gate voltage ($V_g=0$) for a device in which one can observe the same
general features as for those in Fig. 3. Fig. 4 (b) shows a color plot of a series of $G(V_{\text{BIAS}})$ measured at constant temperature and varying $V_G$. The central, vertical red region corresponds to the zero-bias conductance peak observed in Fig. 4 (a). One can see that the conductance is periodically modulated by $V_G$. In particular, a pattern of oblique lines (light-blue/green) is apparent, which indicates that $V_G$ gradually “shifts” the oscillations as a function of $V_{\text{BIAS}}$. Notice that the slope of the oblique lines gradually varies with $V_G$, and that the lines show a pronounced curvature over the periphery of the plot (low $V_G$ and/or high $|V_{\text{BIAS}}|$).

We discuss in what follows a theoretical model that accounts for the main experimental observations. Namely the model can explain i) the conductance increase/decrease around zero bias; ii) the origin and period of conductance oscillations; and iii) the connection between these features with the actual physical parameters (length $L$, energy-gap, interfaces transparency). The model is based on the Blonder-Tinkham-Klapwijk (BTK) formalism [28] extended to junctions between d-wave superconductors and normal metals [29] and to graphene homojunctions proximitized by d-wave superconductor [10,11]. A scheme of the model is shown in Fig. 2 (d). First, we consider the YBCO/Au/graphene interface. Because the Au thickness ($\sim 5$ nm) is well-below the mean free path $l_{\text{Au}}\sim 40$ nm [30] and the ballistic low-T coherence length $\xi_{\text{Au}} = \hbar v_F l/(6\pi K T) \sim 30$ nm [31], we characterize that interface (black region on Fig. 2(d)) via a single BTK barrier-strength parameter $Z$. $Z=0$ means a fully transparent interface with the conduction between YBCO and graphene being purely mediated by the Andreev reflection, which becomes less dominant as $Z$ increases. We then consider that the graphene “channel”, across which the Andreev pairs and normal electrons propagate, is divided in three regions A, B, C. Regions A and C correspond to graphene lying on superconducting YBCO/Au [yellow in Fig. 2 (d)]. These limit region B of length $L$ [dark in Fig. 2(d)] where graphene lies on insulating YBCO. The Fermi energy
$E_{F,B}$ of the B-region is expectedly different from $E_{F,A}, E_{F,C}$ as the graphene’s doping depends on the electronic properties of the substrate it lies on [32]. Thus, we assume a graphene homojunction defined by a gate-tunable Fermi energy step, as sketched in the upper part of Fig. 2 (d). As such, this setup behaves as a resonant cavity for electrons and Andreev pairs.

Within our model the devices’ conductance is calculated numerically by considering the in-series connection of the YBCO/Au/graphene interface [29,33] and the proximitized graphene homojunction [10,11]. The calculation details can be found in the Supplementary Material. The input parameters of our model are the barrier strength $Z$ at the YBCO/Au/graphene interface, the reduced Fermi energies in the different regions $\epsilon_{F,i} \equiv E_{F,i}/\Delta$ (with $i=$A, B, C and $\Delta$ the superconducting energy-gap induced in A by proximity effect), the reduced cavity (B-region) length $\Lambda \equiv L/\lambda_{F,A}$ (with the Fermi wavelength in A, $\lambda_{F,A}$, chosen as a reference following [10,11]), and the effective angle $\alpha$ between the nodes of the d-wave order parameter and the homojunction interfaces. From all of those, the key ones are the cavity length $\Lambda$ and Fermi energy $\epsilon_{F,A}$, because these determine the conductance oscillation periods $\Delta V_{\text{long}}$ and $\Delta V_{\text{short}}$. As shown below, $\alpha$ and (most importantly) $Z$ determine the shape of the conductance vs. bias background, and particularly whether there is a conductance enhancement or decrease around zero bias. Here, $\epsilon_{F,B}$ and $\epsilon_{F,C}$ play a secondary role as they only change the phase and amplitude of conductance oscillations, but have no effect on their period [14]. Thus, in order to reproduce the experimental curves via simulations, we first find $Z$ and $\alpha$ that allow mimicking the conductance background, and then we look for the values of $\Lambda$ and $\epsilon_{F,A}$ that yield the oscillations of the same reduced period $\Delta V/\delta$ as in the experiments. Finally, $\epsilon_{F,B}$ and $\epsilon_{F,C}$ are adjusted in order to modulate the oscillations’ amplitude. Examples of simulations are shown in Fig. 3 (b), (e) and Fig. 4 (c), which respectively match the experimental data in Fig. 3 (a), (d) and Fig. 4 (a) [further
examples and a table with the simulations’ parameters are included in the Supplemental Material]. The simulated curves closely reproduce the main experimental features.

In order to illustrate the separate contributions to the conductance we show in Figs. 3 (c) and (f) the conductance of a proximitized graphene homojunction [11] alone —that is, with no finite-Z junction in series. These simulations are made using the same values of $\varepsilon_{F,i}, \Lambda$ and $\alpha$ as for the full-model calculations. One can see, on the one hand, that in these new simulations the conductance shows a background dependence on $V_{BIAS}$ very different from the experiment. This evidences that the background conductance is strongly influenced by the YBCO/Au/graphene interface. In particular, finite values of $Z$ lead to the emergence of the “dip” around zero bias observed experimentally. On the other hand, Figs. 3 (c) and (f) do show the short and long period oscillations observed experimentally, which evidences that they originate within the graphene homojunction. Notice, however, that the ratio between the short and long oscillations period is different from that in the experiments (and full model simulations): in Fig. 3 (c) and (f), $\Delta V_{short}$ is clearly shorter (relative to $\Delta V_{long}$) than in Fig. 3 (a)-(b) and (d)-(e). This is because in the experiments (and full-model) the total voltage drop $V_{BIAS}$ is divided between the graphene homojunction and the YBCO/Au/graphene interface. Indeed, the strongly non-linear conductance leads to a voltage distribution that varies depending on the $V_{BIAS}$. In particular, for bias below the superconducting gap, $eV_{BIAS} < \Delta$, the conductance across the YBCO/Au/graphene interface decreases with respect to the background (this effect is more pronounced for higher $Z$ values). Contrarily, the homojunction conductance increases in that bias range [see Figs. 3 (c), (f)]. Consequently, $\Delta V_{short}$ (observed in the range $eV_{BIAS} < \Delta$) appears “stretched” relative to the $\Delta V_{long}$ (observed for $eV_{BIAS} > \Delta$).
The simulations are based on dimensionless parameters whose consistency can be quantitatively evaluated using fundamental relations and the actual device dimensions. In particular, from the definition of the parameters $\Lambda \equiv L/\lambda_{F,A}$ and $\epsilon_{F,i} \equiv E_{F,i}/\Delta$ used in the simulations, and considering the dispersion relationship in graphene $E_F = \hbar v_F k_F = \hbar v_F/\lambda_F$ (with $k_F$ and $v_F = 10^6$ m s$^{-1}$ the Fermi vector and velocity, and $\hbar$ the Planck constant), one can find the following expression for the superconducting gap $\Delta$:

$$\Delta = \left(\Lambda/\epsilon_{F,A}\right)\hbar v_F L^{-1}.$$  \hfill (Eq. 1)

Thus, Eq. 1 allows for estimates of the superconducting gap $\Delta$ based on the values of $\Lambda$ and $\epsilon_{F,i}$ used to reproduce the experimental curves and the nominal graphene cavity length $L_{device}$. The resulting values of $\Delta$ vary between 15 meV and 20 meV for most of the devices [see table in the Supplemental Material], well within the values expected for materials in proximity with YBCO [31].

The physical meaning of the conductance oscillations emerges from the data representation in Fig. 5, which displays the short (red circles) and long (black squares) oscillation period as a function of the inverse of the nominal graphene cavity length $L_{device}^{-1}$ in the devices. Notice that we plot the period $\Delta\nu = \Delta\nu \cdot \Delta$ with $\Delta$ estimated via Eq. 1 and $\Delta\nu$ the period observed in simulations once the contribution of the YBCO/Au/graphene interface has been removed—as in Fig. 3 (c) and (e)—to avoid the $V_{BIAS}$ division artifacts discussed above. One can see that the long-period oscillations follow (black dashed line)

$$\Delta\nu_{th, long} = \hbar v_F/2L_{device}.$$  \hfill (Eq. 2)

This coincides with the period expected from the interference between electrons travelling back and forth from one cavity side to the other after being normal-reflected [Fig. 1 (b)]. As in a Fabry-Perot interferometer [15,17–20], Eq. 2 results from the condition for constructive
interference $2L_{\text{device}}k = 2n\pi$ (with $n$ an integer and $k$ the electron wavevector) and the graphene’s linear dispersion, which yields the ratio between the voltage period and corresponding electron wavevector increase $\Delta V = \hbar v_F \Delta k$ [34]. The observation of these interferences implies normal-electron coherence over the length $\sim L_{\text{device}}$.

On the other hand, the short period oscillations, clearly visible for $V < \Delta$, follow (red dashed line)

$$\Delta V_{\text{th,short}} = \hbar v_F / 4L_{\text{device}}$$

(Eq. 3)

This period is expected from the interference of electrons that are Andreev reflected as holes and travel back and forth across the graphene cavity [scheme in Fig. 1 (a)], as in the De Gennes-Saint James [16] and McMillan-Rowell [17] oscillations. In this case the resonance condition reads $2L_{\text{device}}|k_1 - k_2| = 2n\pi$, where $k_1$ and $k_2$ are the electron and reflected hole wavevectors, whose difference is established by the applied voltage $|k_1 - k_2| = 2V / \hbar v_F$ [17]. These resonances imply that the superconducting coherence is preserved over the length $\sim L_{\text{device}}$, which is up to hundreds on nm in the studied devices.

Our model also allows understanding the gating effects. Fig. 4 (d) displays a simulation of the conductance as a function of the bias voltage and the Fermi energy, which reproduces the main experimental features – particularly the pattern of oblique lines that characterize the conductance modulation by $V_G$ [Fig. 4 (b)]. The correspondence between the $y$-axis in simulations and experiments imply that, as in earlier experiments [14], the Fermi energy (and consequently the Fermi vector $k_F$) is varied $\sim$ proportionally to the applied voltage. The varying $k_F$ produces the periodic modulation of the conductance, via the same resonance condition $2L_{\text{device}}k_F = 2n\pi$ as for the bias oscillations, resulting in the oblique lines. We note that the model also reproduces the gradual change of slope of those lines, and their pronounced curvature over the periphery on the plot, which is obtained by including in
the model $\epsilon_{F,B}(V_G) \neq \epsilon_{F,A}(V_G) = \epsilon_{F,C}(V_G)$ to account for a partial pining of graphene Fermi energy on conducting YBCO.

In summary, we have realized planar devices in which graphene is proximitized by a d-wave cuprate superconductor. We find that the d-wave superconducting correlations propagate into graphene over distances of the order of hundreds of nm at temperatures $\sim 3$ K. This is proven in planar cuprate/graphene devices that allow the confinement of Andreev pairs and electrons in graphene homojunctions of submicrometric lateral dimensions, whose conductance show resonances as a function of the bias and gate voltages. These result from different quantum interferences. Those involving Andreev pairs are more prominent below the superconducting gap and thus analogous of the De Gennes – Saint James oscillations, predicted for normal metal backed by superconductors and difficult to observe in graphene/low-temperature (low energy-gap) superconductors devices. Here these are very clear due to the large energy-gap (tens of mV) in YBCO, which fits many orders of interference within. Furthermore, the unusual observation of the Andreev-pair interference effects is accompanied here by normal-electron resonance effects. The simultaneous observation of both effect is rare, and stems from the fact that, in the case d-wave superconducting correlations, the coherence length is limited by the electronic mean free path [19]. Thus, the necessary condition for both types of resonances is satisfied simultaneously, thanks to the large elastic mean free path and long coherence length found in the CVD graphene used in our experiments.

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Figure 1: Scheme of interferences in a cavity in proximity with a superconductor (a) The incoming electron is Andreev-reflected at the SC/cavity interface as a hole, then this hole propagates to the other end of the cavity where it is normal-reflected towards the SC/cavity interface. Here it is again Andreev-reflected as an electron on the, this last electron can interfere with the incoming electron. The path tracked by the particle is thus 4L with L the length of the cavity. (b) The incident electron can also be normally reflected at the SC/cavity. The resulting electron can then interfere after the incoming one after being reflected at the other end of the cavity. The distance traced by the particle is thus 2L. The weight between processes (a) and (b) depends on the ratio between Andreev reflection and normal reflection on the superconducting interface.
Figure 2 (a): Microscopic picture on a graphene/YBCO superconducting the device. The scale bar is 10 µm. (b) Zoom on the junction: YBCO superconducting electrodes (SC) are connected by nanometric graphene bridges (G) on top of insulating YBCO (INS). The scale bar is 100 µm. (c) Microscopic picture of a device with the top gate, the scale bar is 100 µm. (d) Scheme of the graphene homojunction used to model the nanodevices system. The graphene doping depend on whether it lies on insulating or superconducting YBCO, which gives rise to a three graphene regions A B, C with different Fermi energies as indicated. This creates the cavity in which the interferences iscussed in the main text occur. The lower scheme shows the measuring layout. The YBCO/Au/graphene interface, which is characterized by a barrier strength Z, is measured in series with the cavity.
Figure 3: (a,b) Experimental conductance versus applied $V_{\text{bias}}$ for two different devices, B4U and B3U measured at 4K. The cavity length $L$ is respectively 300 nm. The vertical lines point out the series of oscillations, and the horizontal double-headed arrows indicate the periods $\Delta\nu_{\text{long}}$ (black) and $\Delta\nu_{\text{short}}$ (red). The central shaded area indicates the width $2\delta$ of the superconducting-gap related feature, as discussed in the main text. (b) and (e) show the corresponding full-model calculations of the conductance, as discussed in the text and with the parameters indicated Z indicated in the legend (the other parameters are shown in the Supplemental Information). (e) and (f) shows simulations for a proximitized graphene cavity alone, that is, with no finite-Z junction in series.
Figure 4: (a) Experimental conductance versus applied $V_{bias}$ for device A5U, with cavity length $L = 150$ nm. Measured at $T=4$ K. (b) Experimental conductance (color scale) of the YBCO/graphene same device in (a) as function of bias voltage (horizontal axis) and gate voltage (vertical axis). (c) Corresponding full-model calculation of the conductance, as discussed in the text and with the parameter indicated $Z$ indicated in the legend (the other parameters are shown in the Supplemental Information). (d) Corresponding full-model calculation of the conductance as a function of bias and Fermi energy variation in the graphene cavity.
Figure 5: Conductance oscillations’ period as a function of the nominal length of the cavity $L_{device}$, which is the distance between YBCO electrodes in the measured devices. Black squares are for long-period oscillations and red circles are for short-period ones. The oscillations’ period are obtained by multiplying the superconducting gap estimated through Eq. 1 by the periods $\Delta \nu$ observed in the cavity simulations that fit the experimental results, as shown in Fig. 2 (c) and (f). The black and red dashed lines respectively correspond to the period theoretically expected for Fabry-Perot electron resonances (Eq. 2) and for Andreev-pair (De Gennes-Saint James) interferences (Eq. 3).
1. Theoretical model

We reproduce the experimental device conductance via numerical simulations based on combination of the models of Refs. [1] and [2].

The model of Ref. [1] applies to a graphene homojunction proximitized by a d-wave superconductor. The junction is divided into three different sections A, B, C connected in series. Each of them has a different Fermi energy and together form an energy quantum well which is tuneable upon application of a gate voltage (in the case studied here, we assume that the different gate capacitance on insulating and metallic YBCO lead to different gating effects in A, C and B). In order to allow for Andreev reflection at the A/B interface, the model [1] considers the electronic density of states in region A presents a superconducting energy-gap. The homostructure conductance is obtained by matching the electronic wave functions in the A/B and C/B interfaces. Given the transmission $T = (1 - r^2)$ and reflection $R = r_A^2$ coefficient, the full expressions for $r$ and $r_A$ are cumbersome and can be found in the appendix of Ref. [3]. The conductance across the A/B/C structure is given by:

$$G_1(V_1) = \int_0^\pi \frac{d\theta}{2} (1 - r^2) \cos(\theta) + r_A^2 \cos(\theta_A).$$

With $\theta_A = \arcsin \left( \frac{(eV_1+E_F)}{(eV_1-E_F)} \sin(\theta) \right)$ and $\theta$ the electron angle of incidence with respect to the interfaces. Both $r$ and $r_A$ and thus the conductance $G_1(V_1)$ depend on the following parameters: the reduced channel length $\Lambda \equiv L/\lambda_{F,A}$ (with $\lambda_{F,A}$ the Fermi wavelength in A), the reduced graphene Fermi energy in the different regions ($\epsilon_{F,i} \equiv E_{F,i}/\Delta$, with $i=A$, B, C) the amplitude of the superconducting energy-gap, and the angle between the d-wave nodes of the superconducting order parameter and the interface $\alpha$. As discussed in the manuscript, this first block of the model explains the conductance oscillations as a function of gate and bias voltage. It is important to notice that the above result applies provided the electrons propagate ballistically within the B-region.

The second block of the model takes into account the finite transmission of the YBCO/Au/graphene interface that is model as a d-wave/metal interface as studied in Refs. [2] and [4]. Specifically, the conductance is given by:

$$G_2(V_2) = \int_0^\pi \frac{d\theta}{2} (16(1 + |\Gamma_+|^2) \cos(\theta)^4 + 4Z^2(1 - |\Gamma_+\Gamma_-|^2) \cos(\theta)^2)/(|4 \cos(\theta)^2 + Z^2(1 - \Gamma_+\Gamma_-)|^2),$$

where $\Gamma_{+/−} = (eV_2/|\Delta(\theta_{+/−})|) - \sqrt{(eV_2/|\Delta(\theta_{+/−})|) - 1}$ and $\theta_+ = \theta, \theta_- = \pi - \theta$. While $\Delta(\theta) = \Delta \cos[2(\theta - \alpha)]$ and $\alpha$ is the angle between the superconducting order parameter and the interface. The conductance $G_2(V_2)$ thus depends on the parameters $\alpha$ and on the BTK barrier strength $Z$. As discussed in the manuscript this block of the model leads to a proper description of the background conductance vs. the voltage bias.

Using $I = \int G(V) dV$ we obtain $G_1(I)$ and $G_2(I)$ and from these, we calculate the conductance when the two building blocks are connected in-series:
\( G(I) = 1/(1/G_1(I) + 1/G_2(I)) \)

Finally, using \( V = \int \frac{1}{G(I)} dI \), we obtain \( G(V) \).

In order to reproduce the experimental curves via simulations, we first find \( Z \) and \( \alpha \) that allow mimicking the conductance background, and then look for \( \Lambda \) and \( \epsilon_{F,A} \) that yield oscillations of the same reduced period \( \Delta V/\delta \) as in the experiments. Notice that \( \epsilon_{F,A} \) was chosen to be close to the values extracted in a previous study [5], around 400 meV. Finally, \( \epsilon_{F,B} \) and \( \epsilon_{F,C} \) are adjusted in order to modulate the oscillations’ amplitude. Examples of simulations are shown in Fig. 2 (b), (e) and Fig. 3 (c) as well as below in the S3. Simulation vs. experiment section.

2. Simulation parameters

| Device | \( L_{\text{device}} \) | Input parameters | \( \Delta_S \) from Eq. 1 |
|--------|-----------------|-----------------|------------------|
|        | mm              | \( Z \) \( \alpha \) \( \Lambda \) \( \epsilon_{F,A} \) \( \epsilon_{F,B} \) \( \epsilon_{F,C} \) | meV |
| A3D    | 100             | 2 0 10 20 400 17,5 20,7 |
| A3U    | 100             | 2 0 38 40 42,5 15,5 39,3 |
| A5U    | 150             | 0,25 0,39 10 14 150 9,5 19,7 |
| B3U    | 300             | 1 0,008 25 20 150 11,5 17,2 |
| B4U    | 300             | 2 0 21 20 400 15 14,4 |
| B4D    | 300             | 2,5 0,39 13 20 400 10 8,9 |
| E3D    | 800             | 2,5 0,15 80 25 80 20 16,5 |

3. Simulation vs. experiment

We present in the following the comparison between the experimental curves (in blue) and the numerical simulation based on the model detailed above.
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