The Generally Covariant Locality Principle – A New Paradigm for Local Quantum Field Theory

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Abstract: A new approach to the model-independent description of quantum field theories will be introduced in the present work. The main feature of this new approach is to incorporate in a local sense the principle of general covariance of general relativity, thus giving rise to the concept of a \textit{locally covariant quantum field theory}. Such locally covariant quantum field theories will be described mathematically in terms of covariant functors between the categories, on one side, of globally hyperbolic spacetimes with isometric embeddings as morphisms and, on the other side, of \(\ast\)-algebras with unital injective \(\ast\)-monomorphisms as morphisms. Moreover, locally covariant quantum fields can be described in this framework as natural transformations between certain functors. The usual Haag-Kastler framework of nets of operator-algebras over a fixed spacetime background-manifold, together with covariant automorphic actions of the isometry-group of the background spacetime, can be re-gained from this new approach as a special case. Examples of this new approach are also outlined. In case that a locally covariant quantum field theory obeys the time-slice axiom, one can naturally associate to it certain automorphic actions, called “relative Cauchy-evolutions”, which describe the dynamical reaction of the quantum field theory to a local change of spacetime background metrics. The functional derivative of a relative Cauchy-evolution with respect to the spacetime metric is found to be a divergence-free quantity which has, as will be demonstrated in an example, the significance of an energy-momentum tensor (up to addition of scalar functions) for the locally covariant quantum field theory. Furthermore, we discuss the functorial properties of state spaces of locally covariant quantum field theories that entail the validity of the principle of local definiteness.

1. Introduction

Quantum field theory incorporates two main principles into quantum physics, locality and covariance. Locality expresses the idea that quantum processes can be localized in
space and time [and, at the level of observable quantities, that causally separated processes are exempt from any uncertainty relations restricting their commensurability]. The principle of Poincaré-covariance within special relativity states that there are no preferred Lorentzian coordinates for the description of physical processes, and thereby the concept of an absolute space as an arena for physical phenomena is abandoned. Yet it is still meaningful to speak of events in terms of spacetime points as entities of a given, fixed spacetime background in the setting of special relativistic physics.

In general relativity, however, spacetime points lose this a priori meaning (cf. the discussion of the “hole argument” in general relativity in [34]). The principle of general covariance forces one to regard spacetime points simultaneously as members of several, locally diffeomorphic spacetimes. It is rather the relations between distinguished events that have a physical interpretation.

This principle should also be observed when quantum field theory in the presence of gravitational fields is discussed. A first approximation to such situations is to consider quantum fields on a given, curved Lorentzian background spacetime where the sources of the gravitational curvature are described classically and independently of the dynamics of the quantum fields in that background. Due to the weakness of gravitational interactions compared to elementary particle interactions, this is expected to be a reasonable approximation which nevertheless has a range of applicability where nontrivial phenomena occur, like particle creation in strong, or rapidly varying, gravitational fields. The most prominent effects of that sort are the Hawking effect [24] and the Fulling-Unruh effect [19, 48].

For quantum field theory on Minkowski spacetime, one demands that quantum fields behave covariantly under Poincaré-transformations, and there are distinct states, like the vacuum state, or (multi-) particle states tied to the Wigner-type particle concept. Such states are natural reference states which allow to fix physical quantities in comparison with experiments. In contradistinction to this familiar case, a generic spacetime manifold need not possess any (non-trivial) spacetime symmetries (isometries), and thus there is in general no restrictive concept of covariance for quantum fields propagating on an arbitrary, but fixed curved background spacetime. (A similar problem arises already for quantum fields in flat spacetime coupled to outer classical fields, and most of what follows applies, mutatis mutandis, also to this case.)

This lack of covariance is a source of serious ambiguities in quantum field theory on curved spacetime, such as the lack of a natural candidate of a vacuum state or a Wigner-type particle concept. In turn, this leads to ambiguities in the concrete determination of physical quantities. This problem was observed some time ago by Wald [52] in his discussion of a renormalization prescription for defining the energy-momentum tensor of a quantized field on a curved spacetime $M$ with metric tensor $g = g_{\mu \nu}$.

One can define a renormalization procedure for the energy-momentum tensor of a free quantum field on a curved spacetime by picking a quasifree Hadamard state $\omega$ as “reference state” and normal ordering of creation and annihilation operators in the GNS-representation of $\omega$. In this way, one arrives at an expression for the quantized energy-momentum tensor as an operator valued distribution, but the problem is the dependence on the reference state $\omega$: On a generic spacetime without symmetries, there is in general no preferred quasifree Hadamard state, like the vacuum on Minkowski spacetime which is selected by invariance with respect to spacetime symmetries. In order to restrict this ambiguity, Wald imposed as a further requirement a principle of locality and covariance that states that the energy-momentum tensor should only locally depend on the spacetime metric; we will outline this condition further below.