Distributional Energy-Momentum Tensor of the Kerr-Newman Space-Time Family

Herbert BALASIN
Institut f"ur Theoretische Physik, Technische Universit"at Wien
Wiedner Hauptstra\ss e 8–10, A - 1040 Wien, AUSTRIA

and

Herbert NACHBAGAUER
Laboratoire de Physique Théorique ENSLAPP
Chemin de Bellevue, BP 110, F - 74941 Annecy-le-Vieux Cedex, France

Abstract

Using the Kerr-Schild decomposition of the metric tensor that employs the algebraically special nature of the Kerr-Newman space-time family, we calculate the energy-momentum tensor. The latter turns out to be a well-defined tensor-distribution with disk-like support.

PACS numbers: 9760L, 0250

ENS\textsc{L}APP-A-453/93
TUW 93 – 28
gr-qc/9312028

** e-mail: hbalasin @ email.tuwien.ac.at
*partially supported by the “Hochschuljubiläumsstiftung d. Stadt Wien“ project H-00013
††e-mail: herby @ lapphp1.in2p3.fr
URA 14-36 du CNRS, associée à l’E.N.S. de Lyon, et au L.A.P.P. (IN2P3-CNRS)
d’Annecy-le-Vieux
Introduction

Singular space-times present one of the major challenges in general relativity. Originally it was believed that their singular nature is due to the high degree of symmetry of the well-known examples ranging from the Schwarzschild geometry to the Friedmann-Robertson-Walker cosmological models. However, Penrose and Hawking [1] have shown in their celebrated singularity theorems that singularities are a phenomenon which is inherent to general relativity. Since the standard approach allows only for smooth space-time metrics, one has to exclude the so-called singular regions from the space-time manifold.

In a recent work [2] the authors advocated the use of such distributional techniques to calculate the energy-momentum tensor of the Schwarzschild geometry. It turns out that it is possible to include the singular region (i.e. the space-like line $r = 0$ with respect to Schwarzschild coordinates) in the space-time which now no longer is a vacuum geometry, and to identify it with the support of the energy-momentum tensor. The latter becomes a tensor-distribution [3, 4] with delta-like shape.

This reasoning puts the Schwarzschild geometry on the same footing with its ultrarelativistic limit the Aichelburg-Sexl shock-wave geometry [5], where the energy-momentum tensor has a delta-like support on a null line and is interpreted as being generated by a particle moving with velocity of light. Adopting this line of arguments one might consider the Schwarzschild geometry as being generated by a tachyon which hides behind the event horizon thus providing a new interpretation of cosmic censorship.

The aim of the present work is to extend our calculation to the general axisymmetric, stationary space-time family discovered by Kerr and Newman [6, 7]. This family also contains the Schwarzschild geometry and its charged extension the Reissner-Nordstrøm solution as special cases of spherical symmetry. We will show that the distributional techniques developed in [4] apply to this family too thereby allowing to calculate its energy-momentum tensor.
One of the main features of the Kerr-geometry (and its charged version) which eventually led to its discovery is the existence of a geodetic, null vector-field $k^a$ which gives rise to a corresponding congruence of geodesics. Taking advantage of this fact it is possible to decompose the metric into the so-called Kerr-Schild form \[ g_{ab} = \eta_{ab} + f k^a k_b, \] where $\eta_{ab}$ denotes a flat (background) metric and $f$ a scalar field. This geometrical decomposition greatly facilitates the calculation not only from a technical point of view but also from a conceptual one since it identifies $k^a$ as being an integral part of the geometry which will be kept fixed during an eventual regularisation of $g_{ab}$.

Our work is organised in the following way: The first chapter is devoted to the calculation of the Ricci-tensor and the curvature scalar of an arbitrary metric of Kerr-Schild form. In the second chapter we will rederive our previous results concerning the Schwarzschild geometry and extend them to the Reissner-Nordstrøm case. Finally, in the third section we calculate the distributional energy-momentum tensor of the Kerr- and Kerr-Newman geometries.

1) Algebraically special geometries and Kerr-Schild structure

All geometries we are going to consider in this work are algebraically special geometries, which allow a Kerr-Schild decomposition of the metric \[ g_{ab} = \eta_{ab} + f k^a k_b, \] where $k^a = \eta^{ab} k_b$ denotes a null vector (field) with respect to the metric $\eta_{ab}$, which in turn implies its nullity with respect to $g_{ab}$. The above decomposition provides two metrical structures associated with a given manifold $\mathcal{M}$, $g_{ab}$ and $\eta_{ab}$ respectively. In what follows $\nabla_a$ denotes the derivative operator associated with $g_{ab}$ and $\partial_a$ its commuting counterpart with respect to $\eta_{ab}$. An important consequence of the nullity of $k^a$ and the decomposition (1) is

\[ k^a \nabla_a k^b = k^a \partial_a k^b, \]
which can be derived from the explicit form of the difference operation \( C^{a}^{bc} \) of the derivative operator

\[
\nabla_{a}v^{b} = \partial_{a}v^{b} + C^{b}^{ad}v^{d}, \quad v^{a} \in \Gamma(TM),
\]

\[
C^{a}^{bc} = \frac{1}{2}g^{ad}(\partial_{b}g_{dc} + \partial_{c}g_{db} - \partial_{d}g_{bc})
\]

\[
= \frac{1}{2}(\partial_{b}(fk^{a}k_{c}) + \partial_{c}(fk^{a}k_{b}) - \partial^{a}(fk_{b}k_{c}) + fk^{a}(k\partial)(fk_{b}k_{c})) ,
\]

where all index-raising and lowering is done with respect to \( \eta_{ab} \). Equation (2) tells us that if \( k^{a} \) is geodetic with respect to \( g_{ab} \) the same is true with respect to \( \eta_{ab} \) and vice-versa. Kerr and Schild \[8\] have shown that the vacuum Einstein-(Maxwell)-field equations require \( k^{a} \) to be geodetic, and we can assume \( k^{a} \) to be affinely parametrised, i.e. \( (k\partial)k^{b} = 0 \). This condition is an essential property of the geometry and thus we will strictly maintain it, even in the course of a regularisation procedure.

The calculation of the Ricci-tensor using the conventions \[3\]

\[
R^{cd} = \partial_{d}C^{c}{}_{ad} - \partial_{a}C^{c}{}_{bd} + C^{c}{}_{fa}C^{f}{}_{ad} - C^{c}{}_{fa}C^{f}{}_{bd},
\]

\[
R_{ab} = R_{acb}{}^{c} = \partial_{c}C^{c}{}_{ab} - \partial_{a}C^{c}{}_{cb} + C^{c}{}_{cf}C^{f}{}_{ab} - C^{c}{}_{af}C^{f}{}_{cb},
\]

together with the form (3) of \( C^{a}^{bc} \), which implies \( C^{a}{}_{ab} = 0 \), yields

\[
R^{a}{}_{b} = \frac{1}{2}\left(\partial^{a}\partial_{c}(fk^{c}k_{b}) + \partial_{b}\partial^{c}(fk_{c}k^{a}) - \partial^{2}(fk^{a}k_{b})\right),
\]

\[
R = \partial_{a}\partial_{b}(fk^{a}k^{b}).
\]

Let us again remind the reader that from now on all indices are raised and lowered with the help of \( \eta_{ab} \). (4) has the remarkable property of being a sum of second derivatives linear with respect to \( f \) which is neither the case for the upper nor the lower index parts. This property will allow a distributional evaluation of (4) whenever \( fk^{a}k^{b} \) is itself a well-defined distribution and therefore possesses a natural second derivation.
2) Schwarzschild geometry and Reissner Nordstrøm extension

A simple illustration of the above concepts is provided by the Schwarzschild geometry and its Reissner-Nordstrøm extension. The Kerr-Schild form of the Schwarzschild geometry, whose line element reads in Schwarzschild coordinates

$$ds^2 = -dt^2 \left( 1 - \frac{2m}{r} \right) + dr^2 \left( 1 - \frac{2m}{r} \right)^{-1} + r^2 d\Omega^2,$$

is most easily displayed using the coordinate transformation

$$\bar{t} = t - 2m \log(2m - r), \quad r < 2m$$

which gives

$$ds^2 = -d\bar{t}^2 + dr^2 + r^2 d\Omega^2 + \frac{2m}{r} (d\bar{t} - dr)^2$$

and allows the immediate identification

$$f = \frac{2m}{r}, \quad k^a = (1, e_i^r), \quad i = 1, 2, 3$$

where $e_i^r$ is the unit vector with respect to $\eta_{ab}$ of the spherical-coordinate system. Due to the stationarity of the metric (3) the distributional evaluation of the Ricci-tensor and the curvature-scalar reduce to a 3-dimensional problem on the $\bar{t} = \text{const}$ surfaces. Let us evaluate the curvature scalar explicitly acting on an arbitrary test-function $\varphi \in C^\infty_0(\mathbb{R}^3)$,

$$\langle R, \varphi \rangle = \lim_{\epsilon \to 0} \int_{\mathbb{R}^3 - B_\epsilon} d^3x \ f k^i k^j \partial_i \partial_j \varphi(x) =$$

$$= \lim_{\epsilon \to 0} \left[ - \int_{\partial B_\epsilon} dA \ f k^i k^j N_i \partial_j \varphi(\epsilon e_i^r) + \int_{\partial B_\epsilon} dA \ \partial_i (f k^i k^j) N_j \varphi(\epsilon e_i^r) + \right.$$

$$\left. + \int_{\mathbb{R}^3 - B_\epsilon} d^3x \ \varphi(x) \partial_i \partial_j (f k^i k^j) \right]$$
where $B_\epsilon$ is the $\epsilon$-ball around the origin and $S_\epsilon$ its boundary. $N^i = r^2 \sin \theta e^i_r$ is the outward directed normal of $S_\epsilon$ and $dA$ denotes $d\theta d\phi$. The last term in (6) is just the curvature scalar which vanishes in the region $R^3 - B_\epsilon$, the first term is already of order $\epsilon$. Thus only the second term contributes and we get in the limit $\epsilon \to 0$

$$(R, \varphi) = \lim_{\epsilon \to 0} \int_{S_\epsilon} dA \frac{2m}{\epsilon^2} e^2 \sin \theta \varphi(\epsilon e^i_r) = 8\pi m \varphi(0)$$

which is precisely the result we obtained in [2] in Schwarzschild coordinates. With respect to Kerr-Schild coordinates the energy-momentum tensor becomes

$$8\pi T^a_b = R^a_b - \frac{1}{2} \delta^a_b R = -8\pi m \delta^{(3)}(x)(\partial_t)^a (df)_b.$$

The extension of this result to the Reissner-Nordström metric is easily achieved by observing that we only have to replace

$$f = \frac{2m}{r} \quad \to \quad f + \tilde{f} = \frac{2m}{r} - \frac{e^2}{r^2}.$$

Since expression (4) is linear in $f$ we necessarily obtain an additional contribution $\tilde{R}^a_b$ to the Ricci-tensor. Note that $\tilde{f}$ is still locally integrable, which is a necessary condition for the existence of the distribution $R$. Let us exemplify the calculation of $\tilde{R}^a_b$ by the evaluation of $\tilde{R}^0_0$.

$$\langle \tilde{R}^0_0, \varphi \rangle = \lim_{\epsilon \to 0} \frac{1}{2} \int_{R^3 - B_\epsilon} d^3 x \tilde{f} \partial^2 \varphi =$$

$$\frac{1}{2} \lim_{\epsilon \to 0} \left[ - \int_{S_\epsilon} dA \tilde{f}(N \partial) \varphi + \int_{S_\epsilon} dA (N \partial) \tilde{f} \varphi + \int_{R^3 - B_\epsilon} d^3 x \partial^2 \tilde{f} \varphi \right] =$$

$$\frac{1}{2} \left[ \int_{S_\epsilon} dA \left( \frac{e^2}{\epsilon^2} \right) e^2 \sin \theta (e_r \partial) \varphi(0) + \int_{S_\epsilon} dA e^2 \sin \theta \left( \frac{2e^2}{\epsilon^2} \right) \varphi(0) + e(\partial_r) \varphi(0) \right]$$

$$- \int_{R^3 - B_\epsilon} d^3 x \left( \frac{2e^2}{r^4} \right) (\varphi(x) - \varphi(0)) - \varphi(0) \int_{R^3 - B_\epsilon} d^3 x \left( \frac{2e^2}{r^4} \right)$$

$$- e^2 \int_{R^3 - B_\epsilon} d^3 x \frac{1}{r^4} (\varphi(x) - \varphi(0)) = -\left( \frac{e^2}{r^4} \right),$$

5
where beginning with the third line of the calculation the limit \( \epsilon \to 0 \) is considered implicitly thereby dropping all terms of order \( \epsilon \). The calculation of the remaining components proceeds along the same lines so that we get in the end

\[
\tilde{R}^0_0 = -\left[ \frac{e^2}{r^4} \right], \quad \tilde{R}^0_i = 0, \quad \tilde{R}^i_j = \left[ \frac{e^2}{r^4}(\delta^{ij} - 2e^i_r e^j_r) \right], \quad \tilde{R} = 0.
\]

The electromagnetic part of the energy-momentum tensor is still traceless which tells us that the regularisation procedure did not destroy the conformal invariance of the theory.

With respect to Kerr-Schild coordinates the total energy-momentum tensor of the Reissner-Nordstrøm geometry is given by

\[
T^0_0 = -m\delta^{(3)}(x) - \frac{e^2}{8\pi} \left[ \frac{1}{r^4} \right], \quad T^0_i = 0, \quad T^i_j = \frac{e^2}{8\pi} \left[ \frac{1}{r^4}(\delta^{ij} - 2e^i_r e^j_r) \right].
\]

This result emphasizes the fact that the electromagnetic part of \( T^a_b \) is a well-defined tensor-distribution, which coincides with the classical result for \( r \neq 0 \), i.e.

\[
\left( \left[ \frac{e^2}{r^4} \right], \varphi \right) = \left( \frac{e^2}{r^4}, \varphi \right)
\]

for all \( \varphi \in C_0^\infty(\mathbb{R}^3) \) that vanish at \( r = 0 \).

3) Kerr and Kerr-Newman geometries

Passing from Schwarzschild to Kerr generalises the null vector-field \( k^a \) and the function \( f \) to

\[
k^a = (1, k^i) = (1, \frac{rx + ay}{r^2 + a^2}, \frac{ry - ax}{r^2 + a^2}, \frac{z}{r})
\]

\[
f = \frac{2mr}{\Sigma}, \quad \Sigma = \frac{r^4 + a^2z^2}{r^2},
\]

where \( r \) is implicitly given by

\[
r^4 - r^2(x^2 + y^2 + z^2 - a^2) - a^2z^2 = 0.
\]
The cartesian coordinate system \(\{x, y, z\}\) refers to the background metric \(\eta_{ab}\) of the Kerr-Schild decomposition. Taking again advantage of the stationarity of the metric reduces the distributional evaluation to a 3-dimensional problem. The constraint (7) may be solved by a change of coordinates from cartesian to spheroidal

\[
x = \sqrt{r^2 + a^2 \sin \theta \cos \phi},
\]
\[
y = \sqrt{r^2 + a^2 \sin \theta \sin \phi},
\]
\[
z = r \cos \theta.
\]

These coordinates represent a deformation of spherical coordinates expressed by the parameter \(a\). The \(r = \text{const}\) surfaces become confocal ellipsoids with focus on the ring \(\rho^2 = x^2 + y^2 = a^2, z = 0\), whereas the \(\theta = \text{const}\) surfaces are hyperboloids with an asymptotic cone of aperture \(\theta\). For \(r = 0\) the ellipsoid degenerates into a double cover of the disk \(\rho \leq a, z = 0\). In the latter region the coordinates essentially reduce to polar coordinates \((\rho, \phi)\) with radius \(\rho = a \sin \theta\).

The calculation proceeds in a fairly straightforward fashion, using the general formulas of chapter one. This time, however, we have to exclude a disk-shaped region \(r \leq \epsilon\) from the integrals and consider the limit \(\epsilon \to 0\) afterwards in order to do partial integrations. To illustrate the procedure explicitly let us calculate the curvature-scalar given by (6) where \(N_i\) denotes the surface normal of the \(r = \text{const}\) surface and \(dA\) the coordinate-surface area-element. Taking into account the identities

\[
N^i = r \sqrt{r^2 + a^2 \sin^2 \theta} e^i_\rho + (r^2 + a^2) \sin \theta \cos \phi e^i_z,
\]
\[
k^i = \frac{\sin \theta}{\sqrt{r^2 + a^2}} (r e^i_\rho - a e^i_\phi) + \cos \theta e^i_z, \quad (N k) = \Sigma \sin \theta,
\]
\[
(N \partial) k^i = \frac{a \sin^2 \theta}{\sqrt{r^2 + a^2}} (a e^i_\rho + r e^i_\phi),
\]
\[
(k \partial) f + f(\partial k) = \frac{2m}{\Sigma},
\]
where \(e^i_\rho, e^i_\phi, \) and \(e^i_z\) denote the unit vectors of the cylindrical coordinates of \(\mathbb{R}^3\), facilitates the evaluation of (6)

\[
(R, \varphi) = \int_{\mathcal{S}} \frac{2m}{\Sigma} \sum \sin \theta \varphi(\sin \theta \cos \phi, \sin \theta \sin \phi, 0) = \frac{4m}{a} \int_0^{2\pi} d\phi \int_0^a \frac{\rho d\rho}{\sqrt{a^2 - \rho^2}} \varphi(\rho \cos \phi, \rho \sin \phi, 0)
\]

which finally gives

\[
R(x) = \frac{4m}{a} \frac{\vartheta(a - \rho)}{\sqrt{a^2 - \rho^2}} \delta(z).
\]

The calculation of the components of the Ricci-tensor proceeds along similar lines and gives

\[
R^0_0 = 2m \left[ \frac{a \vartheta(a - \rho)}{\sqrt{a^2 - \rho^2}} \right] \delta(z) - \frac{2m}{a} \delta(\rho - a) \delta(z), \\
R^0_i = 2m \left[ \frac{\rho \vartheta(a - \rho)}{\sqrt{a^2 - \rho^2}} \right] \delta(z) e^i_\phi - \frac{m \pi}{a} \delta(\rho - a) \delta(z) e^i_\phi, \\
R^i_j = - \frac{2m}{a} \left[ \frac{\rho^2 \vartheta(a - \rho)}{\sqrt{a^2 - \rho^2}} \right] \delta(z) e^i_\phi e^j_\phi + \frac{2m}{a} \frac{\vartheta(a - \rho)}{\sqrt{a^2 - \rho^2}} \delta(z) e^i_z e^j_z + \frac{4m}{a} \delta(\rho - a) \delta(z) e^i_\phi e^j_\phi
\]

where

\[
\left( \frac{\vartheta(a - \rho)}{\sqrt{a^2 - \rho^2}} \right) \delta(z), \varphi := \int_0^{2\pi} d\phi \int_0^a \frac{\rho}{\sqrt{a^2 - \rho^2}} \left( \varphi(\rho \cos \phi, \rho \sin \phi, 0) - \varphi(a \cos \phi, a \sin \phi, 0) \right)
\]

defines the distribution denoted by the square brackets. Our result shows that the energy distribution is concentrated on a disk with radius \(a\) in the \(z = 0\) plane. For all test functions that vanish on the circle \(\rho = a\) our result coincides with that obtained in the classical work [10] on the source of the Kerr-geometry. Moreover,
it can be shown that the limit $a \to 0$ exists and coincides with the result obtained in the Schwarzschild case. Let us demonstrate this fact for the curvature scalar

$$\lim_{a \to 0} (R_K, \varphi) = \lim_{a \to 0} 4m \int_0^a 2\pi d\rho \int_0^{2\pi} d\phi \frac{\rho}{a \sqrt{a^2 - \rho^2}} \varphi(\rho e^i, 0) =$$

$$8\pi m \int_0^1 \frac{x dx}{\sqrt{1 - x^2}} \varphi(0) = 8\pi m \varphi(0) = (R_S, \varphi),$$

where $R_K$ and $R_S$ denote the curvature scalars of the Kerr and the Schwarzschild geometry respectively.

Passing from Kerr to its electromagnetic extension the Kerr-Newman geometry merely changes the function

$$f = \frac{2mr}{\Sigma} \to \bar{f} = \frac{2mr}{\Sigma} - \frac{e^2}{\Sigma}$$

in the Kerr-Schild decomposition (1), as it was the case with Schwarzschild and Reissner-Nordstrøm. The innocent looking additional contributions due to $\bar{f}$ turn out to require considerable computational effort, although all integrals that are involved can be done analytically. In order to spare the reader the unwieldy formulas we will just give some useful identities needed for the calculation

$$\partial_i k_j = \frac{r}{\Sigma} T_{ij} - \frac{a \cos \theta}{\Sigma} \epsilon_{ijk} k^k, \quad T_{ij} = \delta_{ij} - k_i k_j,$$

$$\left( k \partial \right) \bar{f} + \left( \partial k \right) \bar{f} = 0, \quad (N \partial) \bar{f} = \frac{2e^2 (r^2 + a^2) r \sin \theta}{\Sigma^2}$$

where $\epsilon_{ijk}$ the standard $\epsilon$-tensor of $\mathbb{R}^3$. Together with the identities (8), (10) leads to the following result for the $\bar{f}$-contribution $\bar{R}^a_b$ to the Ricci-tensor and to
the curvature scalar
\[ \tilde{R}^0_0 = -\frac{3\pi e^2}{4} \partial_i \left( \frac{\delta (\rho - a)}{a} \delta (z) e^i_{\rho} \right) - \left[ \frac{e^2 (r^2 + a^2 + a^2 \sin^2 \theta)}{\Sigma^3} \right], \] (11)

\[ \tilde{R}^0_i = -\frac{e^2}{a} \partial_k \left( \frac{\vartheta (a - \rho)}{\sqrt{a^2 - \rho^2}} \delta (z) (e^k_{\rho} e^i_{\rho} + e^k_{\phi} e^i_{\phi}) \right) - \frac{3\pi e^2}{4} \partial_k \left( \frac{\delta (\rho - a)}{a} \delta (z) e^k_{\rho} e^i_{\phi} \right) \]
\[ - \frac{\pi e^2}{2a} \left( \frac{\delta (\rho - a)}{a} \delta (z) (e^i_{\rho} - e^i_{\phi}) \right) + e^2 \left[ \frac{\rho \vartheta (a - \rho)}{a \sqrt{a^2 - \rho^2}} \delta (z) e^i_{\rho} \right] \]
\[ - 2e^2 \left[ \frac{a \sin \theta}{\Sigma^3} \sqrt{r^2 + a^2} e^i_{\phi} \right], \]

\[ \tilde{R}^i_j = \frac{2e^2}{a} \partial_k \left( \frac{\rho \vartheta (a - \rho)}{a \sqrt{a^2 - \rho^2}} \delta (z) (e^k_{\rho} e^j_{\rho} + e^k_{\phi} e^j_{\phi}) \right) + 3\pi e^2 \partial_k \left( \frac{\delta (\rho - a)}{a} \delta (z) e^k_{\rho} e^j_{\phi} e^i_{\rho} e^i_{\phi} \right) \]
\[ - 2e^2 \left[ \frac{\rho^2 \vartheta (a - \rho)}{a \sqrt{a^2 - \rho^2}} \delta (z) e^i_{\rho} e^j_{\phi} \right] + 4e^2 \left[ \frac{\delta (\rho - a)}{a} \delta (z) e^i_{\rho} e^j_{\phi} \right] + \left[ \frac{e^2}{\Sigma^3} \left( r^2 + a^2 \right) \cos^2 \theta - r^2 \sin^2 \theta \right] \left( e^i_{\rho} e^j_{\rho} - e^i_{\phi} e^j_{\phi} \right) \]
\[ + \left[ \frac{e^2}{\Sigma^3} (r^2 + a^2 + a^2 \sin^2 \theta) e^i_{\phi} e^j_{\phi} \right] - \frac{4e^2}{\Sigma^3} r \sqrt{r^2 + a^2} \sin \theta \cos \theta e^i_{\rho} e^j_{\phi} e^j_{z}, \]

\[ \tilde{R} = \frac{2e^2}{a} \partial_k \left( \frac{\rho \vartheta (a - \rho)}{a \sqrt{a^2 - \rho^2}} \delta (z) e^k_{\phi} \right), \]

where the parenthesis around the indices of the base vectors denotes symmetrisation with unit weight. The non-vanishing trace of the Ricci-tensor is due to the length-scale \( a \) of the disk. However, conformal invariance is restored in the limit \( a \to 0 \), thus reproducing the result of the spherical symmetric case.

Let us finally comment on the issue of regularisation. Our calculation started from the Kerr-Schild decomposition of the metric. In the derivation of the Ricci-tensor and the curvature-scalar we implicitly assumed the validity of classical differential calculus and the existence of a smooth function \( f \). This might be
interpreted as regularising the intermediate quantities like the difference tensor of the covariant derivatives. Since the final result, the Ricci-tensor, turned out to be the second derivation of a distribution it was possible to directly evaluate it without referring to any regularisation procedure.

Conclusion

In the present work we have explicitly shown how to calculate the distributional energy-momentum tensor of the Kerr-Newman space-time family, thereby following closely the approach proposed in [2] which includes the singular regions of the geometry in the manifold. The Kerr-Schild-structure related to the algebraically special nature of this space-time family turned out to play a prominent role of the interpretation. Moreover, our results furnish a well-defined basis for the investigation of the so-called ultrarelativistic limit geometries, [3, 11] by boosting the energy-momentum tensor. Work in this direction is currently under progress.

Acknowledgement: The authors are greatly indebted to Prof. P. C. Aichelburg for many useful discussions.
References

[1] Hawking S W and Ellis G F R, *The large scale structure of space-time* Cambridge University Press, 1973.

[2] Balasin H and Nachbagauer H, *Class. Quantum Grav.* **10**, 2271 (1993).

[3] Lichnerowicz A, *Propagateurs, Commutateurs et Anticommutateurs en Relativité Générale*, IHES No. 10 (1961).

[4] Parker P E, Distributional Geometry, *J. Math. Phys.* **20** (1979) 1423.

[5] Aichelburg P and Sexl R, *J. Gen. Rel. Grav.* **2** (1971) 303.

[6] Kerr R P, *Phys. Rev. Lett.* **11** (1963) 237; Newman E T and Janis A I, *J. Math. Phys.* **6** (1965) 915.

[7] Boyer R H and Lindquist R W, *J. Math. Phys.* **8** (1967) 265.

[8] Debney G C, Kerr R P and Schild A, *J. Math. Phys.* **10** (1969) 183.

[9] Wald R, *General Relativity*, University of Chicago Press, 1984.

[10] Israel W, *Phys. Rev. D* **2** (1970) 641; Hamity V H, *Phys. Lett.* **A56** (1976) 77.

[11] Loustó C O and Sánchez N, *Nucl. Phys.* **B383** (1992) 377.