Variation Analysis of Vibration Phase Difference between the Horizontal and Vertical Directions during Run-up Process

Qixiang Wang, Lin Wang, Bin Sun, Chunlei Zhou and Chunyan Li
Jiangsu Frontier Electric Technology Co. Ltd., 58 Suyuan Road, Nanjing, 210010, China
Correspondence should be addressed to Lin Wang; 13913918185@163.com

Abstract. Unbalance excitation can cause rotor vibrations in both horizontal and vertical directions. The vibration phase difference between the two directions during the run-up process under unbalance excitation is analyzed. The change of vibration phase difference with rotational speed is studied. Results show that, the vibration phase difference changes in a large range during the run-up process, which is related to bearing anisotropy, system damping, natural frequency and rotational speed. When the system parameters such as damping and natural frequency are determined, the vibration phase difference is a function of rotational speed. It changes greatly when the working speed is close to the critical speed of each direction. Based on the vibration amplitudes and phase difference between the two directions, the damage of cross-section vibration to the equipment can be evaluated accurately.

1. Introduction
In order to ensure the safe operation of large rotating machinery such as turbo-generator unit, two vibration sensors are arranged at the upper left 45° (x-direction) and the upper right 45° (y-direction) of bearing pedestal, respectively. The vibrations in x- and y-direction are monitored, and their values are used as vibration protection signal sources [1-2]. From the perspective of unit operation reliability, single vibration signal protection mode has an advantage of timely response. However, it is easy to cause mis-operation (“over” protection). It is a common and popular method to make full use of adjacent bearing monitoring signals to design protection logics of TSI system [2]. The amplitudes of unbalance force acting on x- and y- direction are equal, with a difference of 90° in the phase. However, the unbalance force induced vibrations in the two directions are different due to different support characteristics, and the vibration phase difference is not 90°. Su et al. [3] studied the influence of support stiffness on vibration difference between the horizontal and vertical directions. Tian and Zhao [4] indicated that the rotor vibration phase difference between x- and y-direction is related to stiffness and damping coefficient of the system, and it is irrelevant to the magnitude and angle of the unbalance force. The rotating machinery fault diagnosis can be carried out based on the characteristics of vibration phase difference [5-7]. The journal center orbit is elliptical due to support anisotropy. Single vibration signal could not reflect the maximum cross-section vibration. Considering multi-sensor signal fusion, the square root of square sum of vibrations in the two directions is used for vibration protection. The maximum cross-section vibration could not be reflected accurately without considering the vibration phase difference.

In this study, the simplified rotor-bearing system model is established. The variation of vibration phase difference between the horizontal and vertical directions is analyzed. The maximum cross-section vibration obtained with considering vibration phase difference can reflect its damage to the...
equipment more accurately. Meanwhile, the vibration of a turbo-generator unit is measured and analyzed.

2. Vibration phase difference between the horizontal and vertical directions

Figure 1 depicts the vibration phase difference analysis model. The bearing is simplified as stiffness and damping in both horizontal and vertical directions. The support anisotropy is taken into consideration, and the coupling effect of vibrations between the two directions is ignored. The model can be used for qualitative analysis of vibration phase difference.

The rotor vibration equations in the horizontal and vertical directions can be expressed as:

\[
mx\ddot{x} + cx\dot{x} + kx x = F \cos(\omega t),
\]
\[
m\ddot{y} + cy \dot{y} + ky y = F \cos\left(\frac{\pi}{2} - \omega t\right),
\]

where \(m\) is rotor mass. \(k_x, k_y, c_x\) and \(c_y\) are stiffness and damping in the horizontal and vertical directions, respectively. \(F\) is unbalance force amplitude. \(\omega\) is rotation angular speed. \(x\) and \(y\) are displacements in the two directions, respectively, which can be described as:

\[
x = A_x \cos(\omega t - \phi_x),
\]
\[
y = A_y \cos(\omega t - \phi_y),
\]

where \(A_x, A_y, \phi_x\) and \(\phi_y\) are vibration amplitudes and phases in the horizontal and vertical directions, respectively. Then \(\phi_x\) and \(\phi_y\) can be given by derivation:

\[
\phi_x = \tan^{-1}\left(\frac{2\xi_x \omega_x \omega}{\omega_x^2 - \omega^2}\right),
\]
\[
\phi_y = -\frac{\pi}{2} + \tan^{-1}\left(\frac{2\xi_y \omega_y \omega}{\omega_y^2 - \omega^2}\right),
\]

where \(\omega_{nx}, \omega_{ny}, \xi_x\) and \(\xi_y\) are natural frequencies and damping coefficients in the horizontal and vertical directions, respectively.

\[
\omega_n = \left(\frac{k}{m}\right)^{1/2}, \quad \omega_n = \left(\frac{k_y}{m}\right)^{1/2},
\]
\[
\xi_x = \frac{c_x}{2m\omega_n}, \quad \xi_y = \frac{c_y}{2m\omega_n}.
\]
The vibration phase difference is

\[ \Delta \phi = \varphi_x - \varphi_y = \frac{\pi}{2} + (\tan^{-1} \frac{2\xi_x \omega_x \omega_y}{\omega_x^2 - \omega_y^2} - \tan^{-1} \frac{2\xi_y \omega_x \omega_y}{\omega_y^2 - \omega_x^2}). \]  

(4)

It can be observed from Equation (4) that:

1. When the bearing dynamic characteristics are approximately equal \((\xi_x \approx \xi_y, \omega_{nx} \approx \omega_{ny})\) in the two directions, or the system damping is extremely small \((\xi_x \approx 0, \xi_y \approx 0)\), the vibration phase difference \(\Delta \phi \approx 90^\circ\). The vibration phase difference is a complex function of damping, natural frequency and rotational speed under other circumstances. When the system parameters such as damping and natural frequency are determined, the vibration phase difference is a function of rotational speed.

2. Figure 2 depicts the vibration phase difference under different rotational speeds, which is calculated by Equation (4). In the calculation, the natural frequencies in \(x\)- and \(y\)-direction are 20 Hz and 22 Hz, respectively. The damping coefficients \(\xi_x\) and \(\xi_y\) are 0.06 and 0.12, respectively. Calculation results show that, when the working frequency is close to natural frequency of \(x\)- or \(y\)-direction, a slight speed change results in a large vibration phase change in the corresponding direction. It leads to vibration phase difference between the two directions a great change. This phenomenon also shows that the vibration phase changes obviously near the resonance points.

![Figure 2. Vibration phase difference under different rotational speeds.](image)

![Figure 3. Journal center orbit under different vibration phase difference.](image)

3. Method for calculating the maximum cross-section vibration

Figure 3 depicts the journal center orbit under different vibration phase difference \((A_x / A_y = 1\) in this case). When the vibration phase difference is large or small, the ellipticity of journal center orbit is large. When \(\Delta \phi = 0^\circ\) or \(180^\circ\), the ellipticity reaches the limit and transforms into a straight line; when \(\Delta \phi = 90^\circ\), the ellipticity is minimum.

The maximum cross-section vibration is denoted as \(z_2\), then

\[ z_2 = \max((x^2 + y^2)^{1/2}) = \left(\frac{1}{2}(A_x^2 + A_y^2) + \frac{1}{2}(A_x^4 + A_y^4 + 2A_x^2A_y^2\cos(2\Delta \phi))^{1/2}\right)^{1/2}. \]  

(5)

It can be observed from Equation (5) that:

\[ \begin{align*}
\Delta \phi &= 0 \text{ or } \pi, & z_2 &= \left(\frac{1}{2}(A_x^2 + A_y^2)\right)^{1/2}; \\
\Delta \phi &= \frac{3\pi}{4}, & z_2 &= \left(\frac{1}{2}(A_x^2 + A_y^2) + \frac{1}{2}(A_x^4 + A_y^4)\right)^{1/2}; \\
\Delta \phi &= \frac{\pi}{2}, & z_2 &= \max(A_x, A_y). 
\end{align*} \]  

(6)
The square root of square sum of vibrations in x- and y-direction is denoted as \( z_1 = (A_x^2 + A_y^2)^{1/2} \), then \( z_2 \in [\max(A_x, A_y), z_1] \) and \( z_2 < z_1 \). Without considering the vibration phase difference between the two directions, it could not reflect the maximum cross-section vibration accurately by using \( z_1 \) as vibration protection signal. It will cause “over” protection.

4. Analysis of vibration phase difference during run-up process

4.1. Calculation and analysis of vibration phase difference

Figure 4 depicts the rotor-bearing system of a 600 MW steam turbine. The rotor-bearing system is composed of a HP-IP rotor and two LP rotors, supported by 6 bearings, with a total length of 25.444 m and a total weight of 146.2 t.

In the calculation, a force is applied at the middle of LP-I rotor, and a couple is applied at both ends of the rotor impellers to simulate the first- and second-order unbalance, respectively. The third-order critical speed of LP-I rotor is much higher than its working speed. Therefore, only the influence of the first two order modes must be considered in the unbalance analysis. The unbalance response in the two directions during the run-up process is calculated by ARMD (a rotor dynamics analysis software).

Taking the bearing #3 as an example, Figure 5 depicts the vibration amplitudes and phase difference under different rotational speeds during the run-up process. It can be observed from Figure 5 that there are 2 peaks near 1380 rpm and 2820 rpm, which correspond to the first- and second-order critical speeds of LP-I rotor, respectively. During the run-up process, the vibration phase difference increases from -90° to 12°. When the working speed is close to its first-order critical speed, it drops to -114°. Then it gradually increases to -24°. When the working speed is close to its second-order critical speed, it drops to -114° again. During the run-up process, the vibration phase difference varies between -114° and 12°, with a large range of 126°. And it has a prominent change especially near the critical speeds.

![Figure 4. Rotor-bearing system of a 600 MW steam turbine.](image)

![Figure 5. Calculated vibration amplitudes and phase difference under different rotational speeds during the run-up process.](image)
4.2. Case study on vibration phase difference

The rotor vibrations of a turbo-generator unit during the run-up process were measured on site. Figure 6 depicts the rotor vibrations and phase difference at bearings #6 and #4 under different rotational speeds.

The vibration phase difference at bearing #6 decreased from 105° to 60° during the run-up process, as shown in Figure 6(a). The vibration phase difference changed obviously when passing through the critical speed (1300 rpm) of the generator, and dropped by about 30°. \( z_1 \) was always greater than \( z_2 \) during the run-up process. The maximum deviation was about 30% between the two methods.

During the run-up process, the vibration phase difference at bearing #4 increased from 200° to 345°, then dropped to 225°, as shown in Figure 6(b). The vibration phase difference dropped by about 100° near the critical speed (2000 rpm), which was a significant change.

Comparing the calculation results with the measured results, it can be found that the phenomena shown by calculation is basically consistent with that shown by measurement.

5. Conclusion

In this study, the vibration phase difference between the horizontal and vertical directions during the run-up process is analyzed. Two vibration sensors are arranged in the circumferential direction, which are deployed in an orthogonal way with an angle difference of 90°. However, the vibration phase difference between the output vibration signals of the two sensors is not a fixed value of 90° due to support anisotropy, which is related to rotational speed, natural frequency and damping. The vibration phase difference near the critical speed changes more greatly than in other speed regions. The maximum cross-section vibration obtained with considering vibration phase difference can reflect its damage to the equipment more accurately.

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