Global Structure of a Multi-Fluid Cosmology

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Abstract

Fluid cosmologies are consistent with the generally accepted observational evidence during intermediate and late times, and they need not have singular behavior in primordial times. A general form for fluid cosmology consistent with Einstein’s equation is demonstrated, and a dynamic metric that incorporates fluid scale is developed. The large scale causal structure of a multi-fluid cosmology exemplary of standard cosmology is then examined. This is done through developing coupled rate equations for radiation, dust, and dark components. The beginning of the dissolution of the primordial fluid into the other components is singularity-free, since the fluid provides a non-vanishing scale for the cosmology. Penrose diagrams are developed for cosmologies both with and without a final state dark energy density.

1 Introduction

In the most generally accepted form of standard cosmology, the universe transitions from some earliest primordial state into a hot, radiation dominated epoch, then through a residual dust dominated epoch which eventually yields to domination by a remnant dark energy density. The equations that govern a spatially flat expansion satisfy spatial scale invariance, but not temporal scale invariance, due to the behavior of the intensive energy densities that drive the dynamics. However, the existence of any horizon due to dark energy introduces a persistent scale to the macroscopic cosmological dynamics.

Evidence for the existence of persistent dark energy comes from several independent observations. The luminosities of type Ia supernovae show that the rate of expansion of the universe was decelerating in the distant past, but has been accelerating for about 6 giga-years. This conclusion is independently supported by analysis of the Cosmic Microwave Background (CMB) radiation. Both the standard candle luminosity and independent CMB structure results are in quantitative agreement with a (positive) cosmological constant fit to the data.

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existence of a persistent dark energy density that might be described in terms of a cosmological constant defines a length scale to the large scale structure of the cosmology.

Whole sky, deep field observations yield additional interesting properties. The horizon problem examines the paradox of the observed large scale homogeneity and isotropy of the macro-physical properties of the universe beyond regions of causal influence. Uniformity across the whole sky of the temperature of and angular correlations of the fluctuations in the CMB have been accurately measured by several experiments[3]. These correlations provide evidence for space-like coherent phase associations amongst the cosmological fluctuations reflected in the CMB anisotropies.

Quantum fluids exhibit some behaviors similar to those previously mentioned. For example, liquid $^4$He undergoes a cessation of boiling as it is cooled through its “lambda” point and develops a non-vanishing superfluid component. This is due to complete temperature homogeneity maintained by the macroscopic quantum component[5]. Therefore, the dissolution of a primordial quantum fluid should exhibit paradoxical signs of its space-like coherent properties. Since a primordial fluid might have a finite length (density) scale, the cosmological scale need not singularly vanish at the beginning of dissolution.

It is expected that during the earliest of epochs, the quantum coherence of gravitating subsystems should qualitatively altered the dynamics of the cosmology. The entangled nature of co-gravitating quantum states with space-like separations should manifest as some form of spatial coherence in the large scale structure of the geometrodynamics. It is therefore of interest to examine geometric constraints on the causal relationships in a multi-component universe. This paper will explore the large-scale structure of cosmologies consisting of interrelated fluid components that evolve exemplary of what is expected from standard cosmology. A cosmological model with multiple dynamic scales, one of which replaces an apparent cosmological “constant”, is shown to reproduce standard cosmology during intermediate times, while making the exploration of the early and late time dynamics more accessible.

## 2 Fluid Cosmology

### 2.1 Dynamic space-time description

The inclusion of a cosmological constant into Einstein’s equation

$$ G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = - \left( \frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu} \right). $$

(2.1)
provides a convenient parameterization for the phenomena described using dark energy as a substantial constituent of the energy content of the universe. However, if the geometric and dynamic conservation principles are strictly valid, the constant \( \Lambda \) cannot have evolved from a primordial parameter, or be evolving towards a remnant value. For the present discussion, the cosmology will be assumed to evolve in the absence of any true cosmological constant \( \Lambda_{\text{true}} = 0 \). Big bang cosmology is consistent with that of a gravitating fluid that is homogeneous and isotropic on large scales. For an ideal fluid (one with negligible dissipation), the energy-momentum tensor generally takes the form

\[
T_{\mu\nu} = P \, g_{\mu\nu} + (\rho + P)u_\mu u_\nu,
\]

where the four velocity of the fluid satisfies the consistency condition

\[
u_\mu g^{\mu\nu} u_\nu = -1.\]  

By taking the trace of the energy-momentum tensor, \( g^{\mu\nu} T_{\mu\nu} \equiv T^{\mu}_{\mu} = 3P - \rho \). one obtains forms of the pressure and density in terms of geometric quantities:

\[
P = T_\phi = T_\theta = -\frac{c^4}{8\pi G_N} G^\theta_\theta, \]
\[
\rho = 3P + \frac{c^4}{8\pi G_N} G_{\mu\mu}. \]

The components of the flow fields \( u^\beta \) can likewise be determined from the form of the Einstein tensor[6].

Dynamic geometries have been shown to manifest qualitatively different behaviors from their corresponding static analogies[7, 8]. Rotating systems (systems with angular dynamics) exhibit off-diagonal temporal-angular terms in the metric describing those systems. Likewise, radially dynamic black holes described using metrics with an off-diagonal temporal-radial term do not introduce physical singularities reflecting coordinate anomalies at the horizon[9]. Using the same reasoning, one is lead to consider the following hybrid metric, motivated by the so-called “river model” of various static space-time geometries[10, 11]:

\[
ds^2 = - \left( 1 - \frac{R^2(t)}{R^2(t_0)} \right) c^2 dt^2 - 2 \frac{R^2(t)}{R^2(t_0)} cdt \, dr \\
+ R^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \\
= -c^2 dt^2 + R^2(t) \left( dr - \frac{r}{R(t)} cdt \right)^2 \\
+ R^2(t) \left( r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \]

This metric was developed to incorporate scales that diagonalize towards a Robertson-Walker form[12] when maintaining the temporal coordinate, and towards a de Sitter form when maintaining the radial coordinate[13]. A de Sitter geometry[14], which manifests a horizon, is of interest for describing a cosmology with a positive cosmological constant. For the metric given in Eq. 2.5 the temporal coordinate
is the time of an observer located at the coordinate “center” \( r = 0 \). The cosmological principle asserts that this center and observer is not unique or special, and that spatial coordinates exist that can be freely translated and rotated. For this observer, the function \( R_{th}(ct) \) represents a scale for proper length measurements. The function \( R_c \) represents the scale in a de Sitter space-time in the static limit. For a de Sitter geometry \( (R_{th} = 1) \), the de Sitter scale is the radial coordinate of the horizon surrounding the observer.

The hydrodynamic parameters can be immediately calculated using Eq. 2.4:

\[
\rho = \frac{3c^4}{8\pi G_N} \left( \frac{1}{R_c} + \frac{R_{th}}{R_{th}} \right)^2,
\]

\[P = -\rho - \frac{c^4}{4\pi G_N} \frac{d}{dct} \left( \frac{1}{R_c} + \frac{R_{th}}{R_{th}} \right),
\]

where \( u_0 = -1 \), and the other components \( u_j \) vanish. The identifications in Eq. 2.6 then allows the dynamics to be expressed solely in terms of the energy content:

\[
\frac{d}{dct} \rho \equiv -\sqrt{\frac{24\pi G_N \rho c^4}{c^4}} (P + \rho).
\]

This relationship describes the expected dynamics of standard cosmology (as well as its various fractions\[15\]), once the appropriate equation of state relating the pressure to the density is incorporated.

### 2.2 Diagonal metric form

The metric form in Eq. 2.5 can be directly diagonalized using a coordinate transformation of the radial coordinate \( r \) while maintaining the temporal coordinate \( ct \). Examining Eq. 2.6 one can define the Robertson-Walker scale factor \( a(ct) \), and require angular isotropy of the metric expressed in either coordinate system:

\[
\frac{\dot{a}}{a} \equiv \frac{\dot{R}_{th}}{R_{th}} + \frac{1}{R_c} , \quad R_{th} r = a r_{RW} .
\]

A brief and straightforward calculation yields the form of the coordinate transformation,

\[
R_{th} dr = a \left[ \frac{r_{RW}}{R_c} dxt + dr_{RW} \right].
\]

Substitution of this form into Eq. 2.5 gives the expected metric

\[
ds^2 = -c^2 dt^2 + a^2(ct) \left[ dr_{RW}^2 + r_{RW}^2 d\theta^2 + r_{RW}^2 \sin^2 \theta d\phi^2 \right].
\]

The diagonal form obtained when the radial coordinate is maintained has been explored elsewhere\[13\]. The metric form in Eq. 2.10 explicitly demonstrates that the temporal parameter \( t \) being used to describe the dynamics in Eq. 2.5 is the same as that used in the Robertson-Walker metric, whose coordinates manifest spatial homogeneity and isotropy.


3 Multi-Fluid Cosmology

The generic form Eq. 2.8 will next be specialized in a manner that conveniently incorporates fluid scale into the cosmology. The radial rescale factor \( R_{\text{th}} \) will be taken to have the constant value of unity, providing a direct relationship between the radial coordinate \( r \) and that of the Robertson-Walker form, \( R_{\text{th}} = 1, r_{\text{RW}} = \frac{r}{a(ct)} \). The relation 2.6 then directly connects the dynamic scale \( R_c(ct) \) to the fluid density \( R_c \equiv R_\rho \) via

\[
\rho = \frac{3c^4}{8\pi G N R_p^2(ct)} \frac{1}{R_\rho} \quad , \quad \frac{1}{R_\rho} = \frac{\dot{a}}{a}.
\]

Thus, the fluid scale is directly represented in the metric form

\[
ds^2 = -\left(1 - \frac{r^2}{R_p^2(ct)}\right)c^2 dt^2 - 2\frac{r}{R_p(ct)} cdt \, dr + dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta \, d\phi^2.
\]

As previously stated, standard big bang cosmology models the evolution of a universe containing a remnant pressureless dust content (to which most familiar matter contributes, along with any dark matter), radiation (reflected in later stages via the thermal cosmic microwave background), and a now substantial component of dark energy:

\[
\rho = \rho_{DE} + \rho_{\text{rad}} + \rho_{\text{dust}},
\]

where for the present, primordial and remnant dark energies are included in the term \( \rho_{DE} = \rho_{\text{primordial}} + \rho_{\text{remnant}} \).

It is convenient to develop coupled rate equations to describe the evolution of constituent components that combine to give Eq. 2.7. A general form for a component rate equation is given by

\[
\frac{d\rho_{DE}}{dct} = -D_{DE\rightarrow\text{rad}}(ct) - \frac{3}{R_p} (P_{DE} + \rho_{DE}).
\]

The radiation will be assumed to “precipitate” from the primordial dark component in early times. The first term on the right of Eq. 3.4 represented by \( D_{DE\rightarrow\text{rad}} \) is a generic rate of the dissolution of the primordial dark energy into radiation and remnant dark energy, whose detailed form is determined by related microphysical processes. The second term generally incorporates the equation of state of the given component in a manner consistent with the composite rate equation.

Radiation has its energy component red-shifted during an expansion. Since the inverse fluid scale \( 1/R_\rho \) is the logarithmic derivative of the Robertson-Walker (RW) metric scale \( a \), and radiation density scales with the inverse 4\textsuperscript{th} power of the RW scale, the rate equation describing the radiation is expected to take the form

\[
\frac{d\rho_{\text{rad}}}{dct} = D_{DE\rightarrow\text{rad}}(ct) - \frac{4}{R_\rho} \rho_{\text{rad}} - \Theta(\rho_{\text{rad}} - \rho_{\text{threshold}}) \frac{\rho_{\text{rad}}}{c \tau_{\text{r}}}.
\]
The first term on the right of Eq. 3.5 incorporates the dissolution of primordial dark energy into radiation, the second incorporates the appropriate red shift (and equation of state) of radiation, and the third term assumes that above a threshold density, microscopic asymmetries or other processes generate remnant dust from the radiation. Since the nature of the transition of the primordial density into any dark matter component in the dust is not understood, for present purposes its dissolution is completely incorporated into the radiation equation. However, any partitioning of this dissolution into radiation and dark matter should not affect the global structure of the cosmology being explored.

The energy components of the constituents of pressureless dust are not expected to red shift during the expansion. Therefore, the dust density scales with the inverse 3\textsuperscript{rd} power of \( a \), yielding a rate equation of the form

\[
\frac{d\rho_{\text{dust}}}{dt} = \Theta(\rho_{\text{rad}} - \rho_{\text{threshold}}) \frac{\rho_{\text{rad}}}{c_{\text{T,rad}}} - \frac{3}{R_{\rho}} \rho_{\text{dust}}.
\]  

(3.6)

The first term on the right of Eq. 3.6 generates the remnant dust in early times, while the second term insures proper scaling of this density during expansion.

The forms of Eqs. 3.4, 3.5, and 3.6 insure that the evolution of the total density given by Eq. 2.7 is consistent with the summed density components, as long as the pressure content is appropriate. The radiation component has been taken to satisfy \( P_{\text{rad}} = \frac{1}{3} \rho_{\text{rad}} \), and the dust has been assumed not to contribute to the pressure. The primodial form of the dark pressure will likely be that of a macroscopically coherent system undergoing the phase transition to a hot dense plasma. The equation of state for the dark energy gets consistently incorporated in the form of the rate equation for the dark energy component \( \dot{\rho}_{DE} \), as has been done for the radiation and dust components. For the popular model of an early state inflation with a form that would have generated a de Sitter space-time had it persisted, the primordial dark energy might be taken to satisfy the same equation of state that the remnant dark energy seems to satisfy, \( P_{DE} = -\rho_{DE} \). In that case, the form of the pressure can be taken as

\[
P = P_{\text{dust}} + P_{\text{rad}} + P_{DE} = \frac{1}{3} \rho_{\text{rad}} - \rho_{DE}.
\]  

(3.7)

However, quite generally, once the coupled rate equations 3.4, 3.5, and 3.6 have been developed, they can be solved to determine the fluid scale \( R_{\rho}(ct) \), from which the large scale structure of the cosmology can be explored.

### 4 Penrose Diagrams for Fluid Cosmologies

A Penrose diagram is a convenient tool for examining the large-scale causal structure of a given space-time[16]. Penrose diagrams are space-time diagrams with the following properties:
...the coordinates are conformal, which insure that outgoing light-like surfaces have a slope of +1, and ingoing light-like surfaces have a slope of -1,

...the domains of the coordinates are finite, which allows the whole geometry to be displayed in a finite image.

For isotropic systems, the angular coordinates \((\theta, \phi)\) are given arbitrary fixed values, from which one infers that any point on the remaining 2-dimensional diagram represents a spherical surface at a given time. This means that time-like relationships are vertical relative to the diagonal light-like curves, while space-like relationships are horizontal. Causal relationships always have relatively vertical or light-like orientations on the diagram. Likewise, regions that might contain systems with space-like coherence can be directly identified on such a diagram.

A set of conformal coordinates that might be used to construct the Penrose conformal coordinates can be directly obtained from the RW metric form Eq. 2.10

\[
ds^2 = a^2(ct) \left[ -\frac{c^2 dt^2}{a^2(ct)} + dr_{\text{RW}}^2 + r_{\text{RW}}^2 \sin^2 \theta d\phi^2 \right].
\]

The first term in the bracket of Eq. 4.1 is the differential form of the conformal time \(dct_s\). This then yields a metric whose null radial geodesics have slope \(\pm 1\) using differential coordinates \((dct_s, dr_s) = (dct/a(ct), dr_{\text{RW}})\). The Robertson-Walker scale factor \(a\) can be directly calculated once the fluid scale is known by using Eq. 3.1

\[
a(ct) = R_\rho(0) \exp \left[ \int_0^c \frac{dct'}{R_\rho(ct')} \right].
\]

This gives a generic form for conformal coordinates centered at fluid coordinate \((ct_o, r_o)\):

\[
ct_s = \int_{ct_o}^{ct} \frac{dct'}{a(ct')}, \quad r_s = \frac{r}{a(ct)} - \frac{r_o}{a(ct_o)}.
\]

The Penrose conformal coordinates will then be constructed using

\[
Y_+ = \left[ \tanh(\frac{ct + r}{\text{scale}}) - \tanh(\frac{ct - r}{\text{scale}}) \right]/2,
\]

\[
Y_\uparrow = \left[ \tanh(\frac{ct + r}{\text{scale}}) + \tanh(\frac{ct - r}{\text{scale}}) \right]/2.
\]

In what follows in this section, these coordinates will be used to construct Penrose diagrams for cosmologies that evolve from a primordial “dark” fluid, through radiation and dust domination, and perhaps towards a remnant dark energy dominated geometry.
4.1 A cosmology with no remnant dark energy

At any given time, the particle horizon $r_{PH}(ct)$ for an observer located at $r = 0$ is the radial location on the space-like curve $t = 0$ whose ingoing light-like trajectory intersects the observer at that time. This particular surface on the past light cone represents the outermost region of the universe from which communications from the beginning can have been received by this observer. On a Penrose diagram, this location can be directly determined by extending an ingoing light-like trajectory (line of slope -1) from the time in question for the observer $(ct, r = 0)$ back to the time of dissolution $(0, r_{PH}(ct))$. There is also a surface corresponding to the set of observers for which the center $r = 0$ is contained on their particle horizons, which will be referred to as the particle out horizon. This outgoing light-like surface begins at the origin $(ct = 0, r = 0)$ and terminates on future infinity. Again, by the cosmological principle, none of the observers is spatially special or unique.

For the purposes of the constructions, the primordial dark fluid was assumed to “decohere” into radiation using a form $\rho_{DE}(ct) \sim \exp[-(t/T_{DE-rad})^{2}]$, and the initial Robertson-Walker scale has been chosen to be the same as the initial fluid scale $a(0) = R_\rho(0)$. In this subsection it will be assumed that there is no persistent dark energy, i.e., $\rho_{DE}(\infty) = 0$. Some of the features of such a fluid cosmology are displayed on the Penrose diagram in Figure 1. The origin of the conformal coordinates in Eq. 4.3 is chosen to coincide with that of the coordinates in the metric Eq. 3.2. Since there is a beginning to the dissolution of the primordial dark fluid, this time is represented as the non-singular space-like curve $t = 0$ bounding the lower portion of the diagram. The time-like trajectory of a stationary (inertial) observer at the center of this representation is labeled $r = 0$. The remaining boundary of this diagram is future infinity $skri^+$, which is an ingoing light-like surface.

The trajectories of several features are displayed on this diagram. The radial fluid scale $R_\rho(ct)$ from Eq. 3.1 is represented by the surface labeled (b). As has been stated, the Robertson-Walker scale $a(ct)$, labeled (a), is seen to take an initial value coincident with the fluid scale $R_\rho$, and to terminate at the upper corner of the diagram. The space-like volume labeled (c) is that of the cosmology as it transitions from primordial dark fluid domination to radiation domination. For the parameters chosen in this calculation, the space-like volume labeled (d) is that of the cosmology at the time of maximum dust density, and the space-like volume labeled (e) is that of the cosmology at the time of its transition from radiation to dust domination. The particle out horizon represents the surface of earliest causal influence of any constituent of the cosmology originally located at $r = 0$ upon other constituents in the cosmology.

Curves of constant temporal and radial components are superimposed on the previous Penrose diagram in Figure 2. The diagram on the left represents a coordinate grid $(ct, r)$ for the dynamic fluid metric form. From Eq. 3.2 one
Figure 1: Penrose diagram of the dynamic features of a multi-fluid cosmology with no final state dark energy. The vertical line bounding the diagram from the left is the time-like trajectory of the “center” $r = 0$. The horizontal line bounding the diagram from below represents the space-like beginning of dissolution $t = 0$. The particle out horizon is an outgoing light-like surface originating at the origin $(ct = 0, r = 0)$, terminating on future light-like infinity labeled $skri^+$. Trajectory (a) represents the Robertson-Walker scale $a(ct)$. Trajectory (b) represents the fluid density scale $R_\rho(ct)$. The time (c) is that of the transition from primordial dark energy to radiation domination, time (d) that of maximum dust density, and time (e) that of the transition from radiation domination to dust domination.
Figure 2: Radial coordinates for a cosmology with no remnant dark energy, superimposed on the features diagram Fig. Both Penrose diagrams show fixed temporal coordinate curves (green) \( ct = \text{constant} \) graded in tenths, then in the given units of the scale of the diagram, originating on the time-like curve \( r = 0 \), and terminating at the far right corner of the diagram. The diagram on the left shows fixed area radial coordinate curves (red) \( r = \text{constant} \), while that on the right shows fixed Robertson-Walker radial coordinate curves \( r_{RW} = \text{constant} \), each graded in tenths, then units of the given scale. The radial coordinate curves originate on the space-like curve \( ct = 0 \) and terminate at the uppermost corner of the diagram.
determines that any point on the diagram corresponds to the surface of a sphere of area $4\pi r^2$. Volumes of constant $ct$ are represented by the space-like (green) curves graded in tenths, then in units of the given scale. Each of these curves originate on the observational center $r = 0$ and terminate at the far right corner of the diagram. Surfaces of fixed area $4\pi r^2$ are represented by the (red) curves originating on the dissolution volume $t = 0$ and terminating at the top corner of the diagram, initially graded from $r = 0$ in tenths, then in units of the given scale. Future light-like infinity $skri^+$ is seen to correspond to the late-time / asymptotic-radial coordinate curve ($ct \to \infty, r \to \infty$). The particle out horizon is therefore seen to cross all temporal and radial coordinate curves at some point in the global cosmology.

The diagram on the right represents a coordinate grid $(ct, r_{RW})$ for the RW metric form. The volumes of constant $ct$ are again represented by the same space-like (green) curves as in the diagram on the left. From Eq. 2.10 surfaces of area $4\pi[a(ct)]^2 r_{RW}^2$ are represented by the (red) curves originating on the dissolution volume $t = 0$ and terminating at the top corner of the diagram, initially graded from $r_{RW} = 0$ in tenths, then in units of the given scale. The curve $r_{RW} = 1$ is of course seen to correspond with the trajectory of the Robertson-Walker scale. Although in principle it is possible to maintain an accelerating trajectory that remains external to the region of causal influence of a constituent originally located at $(ct = 0, r = 0 = r_{RW})$, there are no inertial (time-like) observers that can remain exterior to the particle out horizon for this cosmology. Of course, all outgoing light-like communications originating on $(ct = 0, r > 0)$ reach future light-like infinity without ever crossing $r = 0$.

### 4.2 A cosmology with remnant dark energy

A cosmology with a non-vanishing final state dark energy manifests a horizon beyond which there cannot be any incoming communication to an inertial observer located at $r = 0$. This horizon will be an ingoing light-like surface separating the two regions of causal influence. For this reason, unlike the cosmology discussed in subsection 4.1, the intersection of the particle out horizon with the horizon $(ct_X, r_X)$ provides a unique scale for the Penrose diagram representing this cosmology. This scale will serve as the center of the conformal coordinates in Eq. 4.3 used to construct the diagram. The primordial dark energy density will be assumed to take a form similar to that discussed in the previous subsection, only relaxing towards a final non-vanishing value for the remnant dark energy density $\rho_{DE}(ct \to \infty) \Rightarrow \rho_{\Lambda}$. All other parameters were taken to be the same as for the previous cosmology. Some features of this multi-fluid cosmology are displayed on the Penrose diagram in Figure 3. The beginning of the dissolution of the primordial dark fluid is again represented by the non-singular space-like curve $t = 0$ bounding the diagram from below, originating at the origin.
Figure 3: Penrose diagram of the dynamic features of a multi-fluid cosmology with non-vanishing final state dark energy. The scale determined by the intersection of the outgoing light-like surface defining the particle out horizon with the ingoing light-like surface defining the horizon is set at the center of the conformal coordinates. The particle out horizon again begins at \((ct = 0, r = 0)\) labeled O, and terminates on the space-like curve \(r_\infty\) bounding the diagram from above. The horizon originates at a finite radial coordinate on the curve \(ct = 0\) and terminates at the future infinity termination point for fixed area radial curves \(t_\infty\). The time-like trajectory between the initiation of the particle out horizon at O and future infinity termination \(t_\infty\) bounding the diagram from the left is the “center” \(r = 0\). The space-like trajectory between the initiation of the particle out horizon at O and the far right corner \((1, 0)\) bounding the diagram from below is the beginning of dissolution \(t = 0\). Again, the curve (a) represents the Robertson-Walker scale \(a(ct)\), while curve (b) represents the fluid density scale \(R_p\). The times of transition from primordial dark energy to radiation domination and maximum dust density are barely distinguishable near the \(t = 0\) curve, while again the volume at the time of transition from radiation domination to dust domination is labeled (e). There is an additional time of transition from dust domination to remnant dark energy domination represented by the mid-time dashed space-like curve labeled (f).
(ct = 0, r = 0) (the source point O of the particle out horizon), and terminating
at the far right corner of the diagram, which has the extremal value for the Pen-
rose conformal coordinates (1, 0). There are no other coordinates on the Pen-
rose diagram associated with the limiting values (−1 ← Y → +1) of the conformal
coordinates (Y_, Y↑), or light-like surfaces between these extreme values. Because
of the exponential temporal behavior of the scale a(ct) in Eq. 4.3, propor-
tionate late-time/asymptotic-radial coordinates (ct → ∞, r → ∞) correspond to the
single conformal coordinate (ct∗(∞), −rX/a(ctX)), labeled t∗ on the diagram. The
time-like trajectory of a stationary observer at the center of this representation,
originating at the origin O and terminating at its future infinity t∞, is labeled
r = 0. The point t∞ is unique with regards to coordinates (ct, r). The diagram is
bounded from above by the space-like curve r∞ representing infinite static areas
with radial coordinate r → ∞. This future boundary is space-like despite being
represented by the asymptotic behavior of the radial coordinate.

There are several other surfaces of interest on this diagram. The fluid scale
Rρ(ct) from Eq. 3.1 is again represented by the surface labeled (b), which origi-
nates at a finite radial scale on the space-like surface t = 0, and terminates defining
the horizon at the future infinity point for r = const surfaces labeled t∞. The
ingoing light-like surface that originates on the dissolution volume and terminates
at this point globally constructs the horizon of this geometry. The Robertson-
Walker scale a(ct) labeled (a) on the diagram is a time-like surface that origi-
nates on the dissolution volume t = 0 at the fluid scale Rρ(0) and terminates on the
static infinite area surface r∞. The space-like volumes barely distinguishable in
the diagram from that at the time of dissolution t = 0 represent the time of trans-
ition from primordial dark energy domination to radiation domination and the
time of maximum dust density using the same parameters as used for constructing
the diagrams with no remnant dark energy density. The time of transition from
radiation domination to dust domination is labeled (e), and the time of transition
from dust domination to remnant dark energy domination is labeled (f). Again,
the particle out horizon is an outgoing light-like surface that represents the sur-
face of earliest causal influence of any constituent originally located at r = 0 upon
other constituents in the cosmology. The point of observational origin for the
representation, O, is the point of initiation for the particle out horizon, which
terminates on the static infinite area surface r∞. As previously mentioned, the
scale has been chosen so that the particle out horizon crosses the horizon at the
center of the conformal coordinates of the diagram (Y→X = 0, Y↑X = 0).

For this cosmology with a remnant dark energy, curves of constant temporal
and radial components are superimposed on the previous Penrose diagram in
Figure 4. The diagram on the left represents a coordinate grid of volumes of fixed
times ct and surfaces of fixed areas 4πr2 labeled (ct, r) using coordinates from the
dynamic fluid metric form Eq. 3.2. Volumes of constant ct are represented by
the space-like (green) curves graded in tenths, units, then decades of the given
Figure 4: Radial coordinates for a cosmology with a remnant dark energy density, superimposed on the features diagram Fig. Both Penrose diagrams show fixed temporal coordinate curves (green) $ct = constant$ graded in tenths, units, then decades of the given scale of the diagram, originating on the time-like curve $r = 0$, and terminating at the far right corner of the diagram. The diagram on the left shows static area coordinate curves (red) $r = constant$, graded in tenths, units, then decades of the given scale. The diagram on the right shows fixed Robertson-Walker radial coordinate curves $r_{RW} = constant$, graded in units, then decades of the given scale. The radial coordinate curves originate on the space-like curve $ct = 0$ and terminate on the future infinity of the coordinate.
scale. Each of these curves originates on the observational center $r = 0$ and terminates at the far right corner of the diagram, which has the extremal value for the Penrose conformal coordinates $(1, 0)$. Surfaces of fixed area parameterized by the radial coordinate $r$ are represented by the (red) curves originating on the dissolution volume $t = 0$ and terminating at the point $t_{\infty}$ of the termination of the radial fluid scale $R_\rho(\infty)$, initially graded from $r = 0$ in tenths, then in units and decades of the given scale. As can be extrapolated from the diagram, the space-like curve labeled $r_{\infty}$ bounding future trajectories is seen to correspond to the late-time/asymptotic-radial coordinate curve $(ct \to \infty, r \to \infty)$. The trajectory of a radial coordinate in the metric Eq. 3.2 can be space-like due to the non-orthogonal nature of the coordinates in the metric. Light-like radial trajectories follow null geodesics of the metric, satisfying

$$\frac{dr_\gamma}{dct} = \frac{r_\gamma}{R_\rho(ct)} \pm 1,$$

(4.5)

for outgoing/ingoing trajectories $r_\gamma(ct)$. Therefore, ingoing light-like trajectories are momentarily stationary in the radial coordinate as they traverse the fluid scale $R_\rho(ct)$. This means that each of the static area, fixed $r$ curves have slope -1 as the fluid scale crosses that coordinate. Fixed radial coordinate curves are seen to be time-like surfaces left of the fluid scale, and space-like surfaces to the right of that scale. As was the case with no remnant dark energy, the particle out horizon is seen in Fig. 4 to cross all temporal and radial coordinate curves at some point in the global cosmology. However, for this cosmology there are clearly inertial trajectories (right of the horizon) external to having causal influence upon an inertial constituent originally located at $(ct = 0, r = 0)$.

The diagram on the right represents a coordinate grid $(ct, R_W)$ for the Robertson-Walker metric form. The volumes of constant $ct$ are again represented by the same space-like (green) curves as in the diagram on the left. Surfaces of fixed radial scale $R_W$ (with time dependent areas $4\pi [a(ct) R_W]^2$) are represented by the (red) curves originating on the dissolution volume $t = 0$ and terminating on the space-like curve labeled $r_{\infty}$, graded from $R_W = 0$ in units, then decades of the given scale. The curve $R_W = 1$ is again seen to correspond with the trajectory of the Robertson-Walker scale. Unlike fixed area radial coordinates $r$, the RW fixed radial coordinate curves $R_W$ terminate on differing points on the future bounding curve $r_{\infty}$.

Curves with fixed RW coordinate $R_W$ can represent the trajectories of co-moving (intertial) observers. There are clearly inertial observers that can originate on the volume $t = 0$ and terminate external to any causal influence upon/from an observer at $r = 0$ (e.g. all fixed radial RW coordinates originating and terminating to the right of the horizons).
5 Conclusions

The global causal structure of a spatially coherent dynamic fluid cosmology has been examined in this paper. A geometry with early spatial coherence can directly address the horizon problem as usually put forth in standard cosmology. The multi-fluid cosmologies examined evolve from a singularity-free initial dark fluid that goes through a dissolution into radiation and dust, with perhaps a non-vanishing final dark energy content, while satisfying all expected dynamic and geometric conservation principles.

In both cases examined, the Penrose diagrams were found to be bounded by three surfaces. For a fluid cosmology with no remnant dark energy, the bounding surfaces consist of a time-like “center”, a space-like beginning, and a light-like future infinity, as was expected. However, the Penrose diagram for a fluid cosmology with remnant dark energy had some unexpected characteristics. The bounding surfaces consist of one time-like and two space-like curves. There is only one point on the diagram where a Penrose conformal coordinate reaches an extremal value, which is the terminating (distant) infinity of the singularity-free space-like beginning of dissolution. The future infinity of time-like observers is a space-like static infinite area surface. The time-like “center” trajectory is an ordinary inertial world line that traverses between points in the beginning and future bounding volumes. In this cosmology, the existence of a horizon provides a natural scale for the diagram.

Within this discussion, no consideration has been given for quantum measurability constraints\cite{17, 18, 19}, thermodynamic or holographic considerations that might arise from the finite horizon scale (such as possible Poincare recurrences\cite{20}), or micro-physical specifics. Work on quantum behaviors in fluid cosmologies is presently underway. Further questions concerning self-consistent relationships between co-gravitating quantum components analogous to those developed for dynamic black holes\cite{9}, are being examined. Those relationships should give insight into constraints upon the temporal behavior of the primordial fluid during its dissolution\cite{18}. Results of these explorations will be presented in a subsequent paper.

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