Search for Violation of the Equivalence Principle in the Solar-Reactor Neutrino Sector as a Next to Leading Order Effect

G. A. Valdivieso† M. M. Guzzo‡ and P. C. Holanda§
Instituto de Física Gleb Wataghin - IFGW, University of Campinas - UNICAMP
(Dated: September 3, 2009)

A model for the violation of the equivalence principle (VEP) on solar and reactor neutrinos is investigated. New limits for the VEP are obtained considering the mass-flavor mixing hypothesis and the VEP model. The search for an upper bound for VEP effects led to a possible solution where usual parameters $\tan^2 \theta$ and $\Delta m^2$ are practically unchanged. The set of best fit point parameters is given by $|\phi \Delta \gamma| = 6.0_{-2.1}^{+3.1} \times 10^{-20}$, $\tan^2 \theta = 0.478_{-0.038}^{+0.040}$ and $\Delta m^2 = 7.20_{-0.27}^{+0.35} \times 10^{-5} eV^2$. This possible solution has increased the confidence level when compared with the MSW-only solution ($\tan^2 \theta = 0.462_{-0.036}^{+0.043}$ and $\Delta m^2 = 7.75_{-0.12}^{+0.16} \times 10^{-5} eV^2$). A superior limit for VEP parameters has also been obtained: $|\phi \Delta \gamma| \leq 3.9 \times 10^{-20}(3\sigma)$.

I. INTRODUCTION

Two decades ago, Gasperini introduced the idea of neutrinos mixing between eigen-states of flavor and gravitational interaction, leading to a Violation of the Equivalence Principle (VEP) [1]. The purpose of such model was to find a solution to the solar neutrino problem through an oscillation mechanism “à la” Pontecorvo supposing the neutrino field with a linearized gravitational Newtonian constant as its mass when coupling with gravity. Later, Halprin and Leung [2, 3] introduced independently a coupled scalar approximated neutrino field with a linearized space-time metric, such that $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ where $\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)$ and $h_{\mu\nu} = -2\phi(x)\delta_{\mu\nu}$.

Although these hypotheses have been formulated for massive neutrinos, no experimental data available at that time could distinguish between mass-flavor and gravity-flavor oscillations. It was much simpler therefore to consider only one of these two different effects. As a consequence, experimental confrontation made before the first KamLAND results [5] considered this simple case of massless neutrinos, mixed only via gravitational interaction (however see [6, 7, 8] for a treatment with mass and VEP effects). In fact, a “just so” vacuum solution could explain all solar data. On the other hand, the increasing evidence of neutrino disappearance at short distances ($\geq 180 km$) is not described by this kind of solution, which leads to neutrino oscillation lengths of the order of the Sun - Earth distance.

With the increasing statistics on solar and reactor neutrinos and the accumulated data from all other sources, one could ask what limits can be now imposed to neutrino’s VEP when we assume the mass-flavor mixing and MSW mechanism adding gravitational VEP interaction in the system. In other words, would the neutrinos be good probes for VEP-like effects? (Other effects could behave with similar phenomenology and thus be indistinguishable from VEP, as will be discussed later).

The VEP phenomenon manifests as a difference in the gravitational coupling for different states. In order to parameterize its effects we will adopt the Post-Newtonian Parameterization [4], where any difference from known gravitational Newtonian constant $G_N$ is measured with a $\gamma$ factor, so that $G_N = \gamma m G N$, where $\gamma m \equiv \gamma(m)$ depends on the mass of the system. As the $G_N$ constant is already been considered in the definition of the gravitational potential $\phi(r)$, one may also define the $\gamma$ factor as:

\[
\phi' = \gamma_m \phi ,
\]

where $\phi$ is defined to be positive. For macroscopic bodies $A$ and $B$, the difference between their $\gamma_A$ and $\gamma_B$ factors $\Delta \gamma = \gamma_A - \gamma_B$ has been measured with free fall experiments. Several gravitational sources are considered as the Sun, Earth, and the galactic center, obtaining $\Delta \gamma < 10^{-12}$ [10] for astrophysical sources and $\Delta \gamma < 10^{-9}$ [11] for terrestrial experiments. Interesting enough, some astrophysical events like pulsars with peculiar frequencies, could be explained if the neutrinos were experiencing VEP [12, 13]. By other hand, neutrinos cannot violate the equivalence principle by more than 1 part in 100 (90% C.L.) [14] since one would be observing more erratic pulsars.

II. VEP MODEL FOR MASSIVE NEUTRINOS

We start stating that the model will apply only to weak gravitational fields, so that no spin-gravity effects will be considered here. By doing so, one may use the Klein-Gordon equation to describe the neutrino field: $(g_{\mu\nu} \partial^\mu \partial^\nu + m^2) \Psi = 0$, where $g_{\mu\nu}$ is the metric tensor and $\Psi$ represents a scalar approximation to the neutrino field.

Following Halprin’s approach, the metric tensor for a weak field can be written as $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x)$ where
\[
\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1) \quad \text{and} \quad h_{\mu\nu} = -2\gamma_m \phi(x)\delta_{\mu\nu} \]
where the redefinition (1) is being used from now on. So, the Klein-Gordon equation with weak gravitational field is
\[
[(\eta_{\mu\nu} - 2\gamma_m \phi(x)\delta_{\mu\nu})\partial^{\mu}\partial^{\nu} + m^2]\Psi = 0.
\]
Assuming a plane-wave solution of the form \(\Psi = \Psi_0 e^{i(\vec{p}\cdot\vec{x} - Et)}\), one arrives at the energy-momentum relation for this interacting system:
\[
E^2(1 - 2\gamma_m \phi) = p^2(1 + 2\gamma_m \phi) + m^2.
\]
Using the fact that for neutrinos \(m \ll p\) and ignoring terms with order higher than \(O(\phi^2)\), we finally have the energy-momentum relation for small masses and weak gravitational field:
\[
E \cong p(1 + 2\gamma_m \phi) + \frac{m^2}{2p}(1 + 4\gamma_m \phi). \quad (2)
\]
The above expression can be re-written as \(E = E_m + E_g\) so that \(E_m = p + \frac{m^2}{2p}\) is the free-particle energy-momentum relation (with \(m \ll p\)) and \(E_g = 2\gamma_m \phi \left(p + \frac{m^2}{2p}\right)\) is the gravitational contribution to the total energy.

In order to introduce the neutrino mixing, one has to define basis on which each phenomenon takes place. The most general scheme for this model would be a three basis system: a physical basis (states with definite mass), a weak base (states with definite flavor) and a gravitational base (states with definite gravitational couplings).

This would mean that the dynamical and gravitational contributions to the total energy, \(E_m\) and \(E_g\) could not be simply added anymore. Instead, the two physical quantities should be assigned to operators on different bases. Considering the further inclusion of weak interactions, and one third basis for it, the model will end with five free parameters (considering only two neutrino flavors). Although it is possible to carry on such analysis, it is interesting to test simpler models and, if any sign of VEP is found, a more complete analysis could be made in future works. To obtain a simpler model, we follow the hypothesis that the gravitational interaction takes place on the physical mass basis. This is exactly what has been done until now, when deriving the expression (2).

Considering only two neutrino flavors, each eigen-state of mass has total energies \(E_1\) and \(E_2\), given by expression (2), using \(m \rightarrow m_1\) and \(\gamma_m \rightarrow \gamma_1\) for \(E_1\), so that:
\[
E_1 = p(1 + 2\gamma_1 \phi) + \frac{m_1^2}{2p}(1 + 4\gamma_1 \phi), \quad (3)
\]
and \(m \rightarrow m_2\) and \(\gamma_m \rightarrow \gamma_2\) for \(E_2:\)
\[
E_2 = p(1 + 2\gamma_2 \phi) + \frac{m_2^2}{2p}(1 + 4\gamma_2 \phi). \quad (4)
\]
To describe a two level system, we introduce the Hamiltonian
\[
H^{(m)} = \frac{\Delta E}{2} \left( \begin{array}{cc} -1 & 0 \\ 0 & 1 \end{array} \right) \quad (5)
\]
where \(\Delta E = E_2 - E_1\) such that
\[
\Delta E = \frac{\Delta m^2}{2p} + 2p \phi \Delta \gamma \mathbf{\bar{\phi}} \mathbf{\bar{\phi}} \left(\bar{\gamma} \Delta m^2 + \bar{m}^2 \Delta \gamma\right), \quad (6)
\]
where \(\Delta m^2 = m_2^2 - m_1^2\), \(\Delta \gamma = \gamma_2 - \gamma_1\), \(\bar{\gamma} = (\gamma_2 + \gamma_1)/2\) and \(\bar{m}^2 = (m_2^2 + m_1^2)/2\).

Not all of these terms will contribute. Of the three terms with dependence on \(1/p\) in (6), the last two (between parentheses) are negligible, mainly because of the potential \(\phi\). Comparing all the sources of gravity that might have some effect here, as the Earth, the Sun, and larger scale structures such the Great Attractor [15, 16], the last contributes most, imposing a practically constant potential \(\phi \approx 3 \times 10^{-5}\) [17], that is at least one order of magnitude more intense than the other sources [3]. Using the definition of \(\gamma\) in (1) and other VEP tests already cited, then \(\bar{\gamma} \equiv 1\) with \(\Delta \gamma < 10^{-9}\). These statements assure that \((\bar{\gamma} \Delta m^2 + \bar{m}^2 \Delta \gamma) \ll \Delta m^2\), so that \(\Delta E\) may be considered only as
\[
\Delta E \cong \frac{\Delta m^2}{2p} + 2p \phi \Delta \gamma, \quad (7)
\]
\[
= \frac{\Delta G}{2E}, \quad (8)
\]
where the usual consideration for neutrinos \(p = E\) was used and \(\Delta G = \Delta m^2 + 4E^2 \phi \Delta \gamma\) is defined as an effective mass scale.

We assume \(m_2 > m_1\). Nevertheless, the same relation does not have to hold for \(\gamma_1\) and \(\gamma_2\). Previous models for VEP considered only gravitational states for massless neutrinos. Consequently \(\gamma_1\)’s could arbitrarily follow the hierarchy \(\gamma_2 > \gamma_1\) in the same way it was done for the masses. In a model with VEP and massive neutrinos, the \(\gamma\)’s are dependent on the masses and so it is clear from the definition of \(\Delta G\) that the hierarchy between \(\gamma_1\) and \(\gamma_2\) will have influence on the resulting phenomenology. So we must consider two possibilities: if \(\gamma_2 > \gamma_1\), following the same relationship defined for the masses, then \(\Delta \gamma > 0\); if \(\gamma_2 < \gamma_1\), then an inverted hierarchy on the VEP sector appears, and then \(\Delta \gamma < 0\). We consider
\[
\Delta G = \Delta m^2 + 4E^2 |\phi \Delta \gamma|, \quad (9)
\]
where \(|\phi \Delta \gamma|\) is one single parameter of the model and no further discussions about the single value of \(\phi\) are needed, as long as it is considered as a constant. The two possible relations between the \(\gamma\)’s will be referred simply as \(+\text{VEP}\) and \(-\text{VEP}\) for the plus and minus sign on (9), respectively.

When considering \(\Delta E\) as a function of \(E\), expression (7) may vanish in the \(-\text{VEP}\) case. If so, some region of the oscillated spectrum might behave as the mass states were degenerate, even the mass eigen-states being different. For the \(+\text{VEP}\) case, \(\Delta E\) can never be zero but it has a minimum value as its first derivative vanishes. Setting \(\frac{d}{dE}\Delta E = 0\) for \(+\text{VEP}\) and \(\Delta E = 0\) for \(-\text{VEP}\), one obtains
$$E_s = \frac{1}{2} \sqrt{\frac{\Delta m^2}{|\phi \Delta \gamma|}}.$$  

(10)

which is then a critical energy of the model, either for \( +VEP \) and \( -VEP \) cases.

As the energy \( E \) is a constant of motion, any previous solutions of the standard neutrino mixing model can be changed to accommodate the VEP hypothesis only by doing the substitution \( \Delta m^2 \rightarrow \Delta_G \). Furthermore, no mention about the weak basis mixing was needed until this point. In this simplified two-bases version of the VEP model, gravity has no influence over the vacuum mixing (which would not be the case if a three-bases model is considered \( \Delta \)).

The evolution of flavor states is given by the Schrödinger equation \( i \frac{\Delta}{\Delta t} \psi(f) = H(f) \psi(f) \), with

$$\psi(f) = U \psi(m) \quad \text{and} \quad H(f) = U^\dagger H(m) U$$  

(11)

(12)

where \( \psi(f) \) and \( H(f) \) represent the states and the Hamiltonian, written on the flavor base \( (f) \). In general, states and operators in both bases are related through expressions \( (11) \) and \( (12) \) respectively, where \( U \) is a \( SU(2) \) transformation. When dealing with only two bases, \( U \) has only one physical parameter \( \phi \) which can be expressed as

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}.$$  

(13)

In the flavor basis, one can introduce the effective weak potential \( \left( V_{ee} \right) \):

$$V_{ee}^{(f)} = \frac{\sqrt{2}}{2} G_F n_e \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$  

(14)

that describes the influence of a material medium on the neutrino conversion, known as the MSW effect \( \left( V_{ee} \right) \). In expression \( (14) \), \( G_F \) is the Fermi constant and \( n_e \equiv n_e(x) \) is the electron number density, used to describe globally neutral matter distributions. In our case, \( n_e \) describes the Sun’s and Earth’s density profiles. The complete Hamiltonian is then given by \( H(f) = H(f) + V_{ee}^{(f)} \), where the - sign denotes the presence of a material medium. Plane wave solutions of the Schrödinger equation for this system are of the form \( \psi(f)(t) = \exp(-i H(f) t) \psi_0 \). Considering the initial state \( \psi_0 \) to be constituted only by electron (anti-)neutrinos \( \psi_0 \), one arrives at the survival probability \( P_{ee}(t) \equiv |\psi_0^\dagger \psi(f)(t)|^2 \), which describes the probability that a neutrino is both produced and detected as an electron (anti-)neutrino. The simplest solution corresponds to the vacuum case \( (VAC) \), where \( n_e \equiv 0 \), and is given by:

$$P_{ee}(L, E) = 1 - \sin^2 2\theta \sin^2 \left( \frac{\Delta_G L}{4E} \right)$$  

(15)

where \( L \) is the distance between the source and the detector. The resulting periodic pattern has an oscillation length \( \lambda = 4\pi E/|\Delta_G| \), where the absolute value of \( \Delta_G \) is used since it can become negative (in the \( -VEP \) case). In practice, \( L \) is fixed (a characteristic of the experiment) and we observe \( P_{ee} \) as a function of \( E \). If VEP is not present, \( \lambda \) depends linearly on \( E \) and the oscillation stops when \( E \gg L\Delta m^2/4\pi \). Otherwise, the dependence of \( \Delta_G \) on \( E \) prevents the oscillation from stopping as now \( \lambda \) has a maximum where \( |\Delta_G|/E \) has a minimum, at \( E = E_s \). In a particular case, \( \lambda \rightarrow \infty \) when \( E \rightarrow E_s \), for the \( -VEP \) scenario.

FIG. 1: Survival probability \( P_{ee}(E) \) for VAC\( \pm VEP \). Figure \( (a) \) corresponds to VAC\( +VEP \) and \( (b) \) to VAC\( -VEP \). The parameters corresponding to the usual neutrino mixing are taken to be \( \sin^2 2\theta = 0.86 \), \( \Delta m^2 = 8.0 \times 10^{-5}eV^2 \) and \( L = 180km \) (this last one corresponds to the average distances considered for KamLAND). Each line represents a specific value of VEP, \( |\phi \Delta \gamma| = 0 \) (no VEP), \( 10^{-21} \) (\( E_s = 140MeV \)) and \( 10^{-20} \) (\( E_s = 45MeV \)).
are those found in the literature \cite{23} for the standard mixing, and \( L \) is constant and refers to the KamLAND \cite{22} experiment average source-detector distance. On both figures, one can observe the new effect where the oscillations are restored for energies above \( E_s \). The presented values of \( |\phi \Delta \gamma| \) are chosen so that any predicted new effect will not be visible within the reactors’ spectra energy range (approximately \( E \leq 9 \text{MeV} \)). This gives us a visual “first limit” for VEP as \( |\phi \Delta \gamma| \leq 10^{-20} \), if the present data reject the hypothesis.

Solutions that describe solar neutrinos must consider the Sun’s matter profile, given by the Solar Standard Model (SSM) \cite{24,39}. The MSW effect predicts not only conversion between flavor states but also, under certain conditions, conversion of mass states referred to as non-adiabatic effects \cite{13,19}. To better understand these effects, one has to transform \( \tilde{H}(x) \) back to the mass basis, where it should be diagonal. The introduction of the weak potential \( V_W \) assures that this transformation is different from \( \tilde{U} \). It’s then necessary to define the effective mass basis so that \( \tilde{\Psi}^{(m)} = \tilde{U} \tilde{\psi}^{(x)} \) and \( \tilde{H}^{(m)} = \tilde{U} \tilde{H}(x) \tilde{U}^\dagger \) where \( \tilde{H}^{(m)} \) has a diagonal form. The transformation \( \tilde{U} \) is defined as \( {\psi}_\theta(x) \rightarrow \theta \). Requiring \( \tilde{H}^{(m)} \) to be diagonal, one arrives at the effective mixing in matter, given by

\[
\cos 2\tilde{\theta}(x) = \frac{\Delta_G \cos 2\theta - A(x)}{\sqrt{\Delta_G \cos 2\theta - A(x)}^2 + \Delta_G \sin^2 \theta}, \tag{16}
\]

where \( A(x) \equiv \sqrt{2} G_F E_{n,x}(x) \). The Schrödinger equation will not retain the same form under such transformation, since \( \tilde{U} \) has a dependence on the position \( x \). Transforming states and the Hamiltonian from the flavor to the effective mass basis results in 

\[
i \tilde{U}^\dagger \frac{d}{dx} \tilde{U} \tilde{\Psi}^{(m)} = \tilde{H}^{(m)} \tilde{\Psi}^{(m)},
\]

The resulting evolution operator has additional off-diagonal terms that come from the derivative \( \tilde{U}^\dagger \frac{d}{dx} \tilde{U} \), which together with the diagonal Hamiltonian \( \tilde{H}^{(m)} \) give us

\[
\frac{id}{dx} \tilde{\Psi}^{(m)} = \begin{pmatrix}
\tilde{E}_1 & \frac{d\tilde{\theta}}{dx} \\
\frac{d\tilde{\theta}}{dx} & \tilde{E}_2
\end{pmatrix} \tilde{\Psi}^{(m)}, \tag{17}
\]

where \( \tilde{E}_1 \) and \( \tilde{E}_2 \) are the eigen-values of \( \tilde{H}^{(m)} \) and \( \tilde{\theta} \) is implicitly given by \( \text{(16)} \). The off-diagonal terms in \( \text{(17)} \) result in a non-zero probability of conversion between effective mass states. The intensity of these non-adiabatic effects can be measured by the relation between the diagonal and the off-diagonal terms of \( \text{(17)} \), such that when

\[
\left| \frac{d\tilde{\theta}}{dx} \right| \ll \left| \tilde{E}_2 - \tilde{E}_1 \right|,\tag{17'}
\]

is approximately diagonal. This condition can be summarized in the form of a Adiabaticity Coefficient \( \Gamma(x, E) \) defined as

\[
\Gamma(x, E) \equiv \left| \frac{d\tilde{\theta}}{dx} \right| \ll 1 , \text{ with } \Delta\tilde{E} = \tilde{E}_2 - \tilde{E}_1 , \tag{18}
\]

where \( \Delta\tilde{E} = \frac{1}{2\pi} \sqrt{\left[ \Delta_G \cos 2\theta - A(x) \right]^2 + \Delta_G^2 \sin^2 2\theta} \). To better appreciate non-adiabatic effects on the neutrino spectrum, it is useful to define \( \Gamma \) as a function of the energy only, eliminating the \( x \) dependence by taking the maximum value of \( \Gamma(x, E) \) for any \( E \), i.e.

\[
\Gamma(E) \equiv \max_{x} \left( \Gamma(x, E) \right).\tag{18'}
\]

FIG. 2: Adiabaticity Coefficient \( \Gamma(E) \) for MSW+VEP. Figure (2a) corresponds to MSW+VEP and (2b) to MSW-VEP. The parameters corresponding to the usual neutrino mixing are taken to be \( \sin^2 2\theta = 0.86, \Delta m^2 = 8.0 \times 10^{-5} \text{eV}^2 \) and the Sun’s matter profile is the one from BS05(OP) \cite{23}. Each line represents a specific value of VEP: \( |\phi \Delta \gamma| = 0 \) (no VEP), \( 10^{-20} \), \( 10^{-19} \), \( 10^{-18} \) and \( 10^{-17} \). For the +VEP case, the system is adiabatic. For -VEP, non-adiabatic effects occur when \( E \rightarrow E_s \) for any value of \( |\phi \Delta \gamma| \).
and $E_2$ to vanish. As a consequence, the off-diagonal terms of (17) become infinitely larger than the diagonal ones (as these go to zero), even when $\frac{d\theta}{dx}$ is naturally small, as they are expected to be in the Sun (as can be seen from Fig. [6] and [7]).

For those cases where condition (18) holds, equation (17) may be solved in the adiabatic approximation that leads to the following survival probability (18):

$$P_{ee}^{ad}(x) = \frac{1}{2} \left[ 1 + \cos 2\tilde{\theta}_o \cos 2\theta(x) \right. \right.$$  

$$+ \left. \sin 2\tilde{\theta}_o \sin 2\theta(x) \cos \alpha(x) \right] \quad (19)$$

where $\cos 2\tilde{\theta}_o$ is also given by (16) where $\cos 2\tilde{\theta}_o \equiv \cos 2\theta(x_0)$ being $x_0$ the neutrino production point. The factor $\cos \alpha(x)$ corresponds to the oscillating term with $\alpha(x) = \int_0^x \Delta E(x') \, dx'$. When the matter contribution vanishes, expression (19) corresponds to vacuum solution (18). Moreover, if $\Delta \gamma \gg 10^{-10} \, eV^2$, $\cos \alpha(x)$ rapidly oscillates (when compared to the Sun’s dimensions [12, 23]) and is ruled out by the average over the production point $x_0$. Without VEP, this condition is satisfied as $\Delta m^2 = 8.0 \times 10^{-5} \, eV^2$ [23], what leads to the useful simplified survival probability for solar neutrinos,

$$P_{ee}^{ad}(x) = \frac{1}{2} \left[ 1 + \cos 2\tilde{\theta}_o \cos 2\theta(x) \right] \quad (20)$$

From the studies of the adiabaticity coefficient, the above expression is expected to hold for the $MSW + VEP$ case and almost everywhere for $MSW - VEP$, except in the neighborhood of $E_\ast$. Fig. [3a] shows a comparison between expression (20) with no VEP and with $|\phi\Delta| = 5 \times 10^{-20}$, for the $MSW + VEP$ case. This value of VEP was chosen so that $E_\ast$ is just above the solar neutrino spectrum ($E_\ast = 20 \, MeV$). The survival probability with VEP is always greater than the one for MSW only, but this difference only becomes appreciable for energies above $E_\ast$. Fig. [3b] shows the same comparison for $MSW - VEP$. As expected, non-adiabatic effects occur near $E_\ast$. Fig. [3b] also shows a numerical solution of the equation (17), in which the non-adiabatic behavior can be seen in details. These effects are confined inside a narrow region around $E_\ast$ and they are not observable with the present data statistics. The adiabatic approximation (20) describes well the survival probability by any practical means, in both $\pm VEP$ cases.

When arriving at night time, solar neutrinos cross several Earth layers. Again, the presence of matter alters the survival probability in a way that night time neutrinos have more chance to survive than those arriving at day time. This effect is called regeneration and is not observed on the solar neutrino data [26]. On the same way that new non-adiabatic effects were predicted for day neutrinos in the $MSW - VEP$ case, new regeneration signal may also be expected. To account for these possibilities a numerical solution of (17), for the Earth’s matter profile, is used. Fig. [4] is the equivalent to [31] for night neutrinos. In the neighborhood of $E_\ast$, the solar non-adiabatic effects are intensified by Earth’s matter. Fig. [6] shows a measure of the asymmetry between night and day for the $−VEP$ case. As it can be seen, an excess is expected in a region wider than the one where the solar non-adiabatic effect takes place. The absence of regeneration signs on the solar neutrinos data imposes a stronger limit on $E_\ast$ for the $−VEP$ case than for $+VEP$. This has direct consequences on the limits for $|\phi\Delta|$.

III. DATA ANALYSIS

For solar neutrinos, we consider data from Homestake [27], SAGE [28], Gallex/GNO [29], SuperKamiokande(SK) [30] and SNO (I [31] e II [32])
rates respectively and \( \chi \) analysis is done, where we define

\[
\chi^2 = \chi^2_{\text{KL}} + \chi^2_{\text{sun}}.
\]  

(21)

The solar neutrinos contributions \( \chi^2_{\text{sun}} \) is given by \(20\):

\[
\chi^2_{\text{sun}} = \sum_{i,j=1}^{119} \left[ R^{\text{the}}_{i,j} - R^{\text{exp}}_{i,j} \right] \left[ S^2 \right]^{-1}_{ij} \left[ R^{\text{the}}_{i,j} - R^{\text{exp}}_{i,j} \right],
\]  

(22)

where \( R^{\text{the}}_{i,j} \) and \( R^{\text{exp}}_{i,j} \) are the theoretical and experimental rates respectively and \( S^2 \) takes in account all the correlation between uncertainties \(32, 33\). Reactor neutrinos contribute through a Poisson statistics \(30\):

\[
\chi^2 = \sum_{i=1}^{24} \frac{N^{\text{th}}_i - N^{\text{exp}}_i}{N^{\text{exp}}_i} \ln \left( \frac{N^{\text{exp}}_i}{N^{\text{th}}_i} \right)
\]  

(23)

where \( N^{\text{th}}_i \) and \( N^{\text{exp}}_i \) are the theoretical and experimental countings.

The \( \chi^2 \) distribution is taken to be a function of the three parameters: \( \tan^2 \theta, \Delta m^2 \) and \( |\phi_\Delta\gamma| \). In order to identify local minima, we minimize \( \chi^2 \) with respect to two parameters, varying the third freely. This procedure is repeated for each parameter, for both +VEP and −VEP cases. Figure 6 shows \( \Delta \chi^2 \) as a function of \( |\phi_\Delta\gamma| \), where

\[
\Delta \chi^2 = \chi^2_{\min(\tan^2 \theta, \Delta m^2)} - \chi^2_{\min(\tan^2 \theta, \Delta m^2, |\phi_\Delta\gamma|)}.
\]  

(24)

When \( |\phi_\Delta\gamma| \leq 10^{-22} \), the analysis shows that the standard global solution for solar and reactor neutrinos is recovered: \( \tan^2 \theta = 0.462^{+0.043}_{-0.036} \) and \( \Delta m^2 = 7.75^{+0.16}_{-0.12} \times 10^{-5} eV^2 \). For the +VEP case, figure 6a shows a global minimum with better fit than the no-VEP case. Both solar and reactor neutrino alone shows a minimum, although in the solar case this is less than one sigma. So it is better suited to say that the solar data excludes +VEP with \( |\phi_\Delta\gamma| > 3.4 \times 10^{-19} (3\sigma) \). On the other hand, the fit with reactor neutrino data from KamLAND is enhanced by almost 3\( \sigma \) what is, to say the least, surprising. The VAC + VEP solution seems to almost exclude VAC alone by a 3\( \sigma \) factor (\( \Delta \chi^2 \approx 8 \)). This difference is smoothed when

FIG. 4: Survival probability \( P_{\text{ex}}(E) \) for neutrinos arriving at night, with MSW–VEP. A comparison between day and night probabilities shows an excess for the night time. The parameters used in this plot are the same as in fig. 3b.

FIG. 5: Day-Night asymmetry for MSW–VEP. The −VEP hypothesis predicts new regeneration effects due to Earth’s matter generating an excess of solar neutrinos arriving at night time, for energies close to \( E_i \).

FIG. 6: \( \Delta \chi^2 \) distribution as a function of \( |\phi_\Delta\gamma| \). The dashed curve represents \( \Delta \chi^2_{\text{KL}} \), while the dotted one refers to \( \Delta \chi^2_{\text{sun}} \). The continuous curve stands for the global fit. Figures 6a and 6b shows the +VEP and −VEP cases respectively.
of the data. If one discards for a moment the best fit point as a superior limit, strongly imposed by KamLAND, the KamLAND data, which clearly excludes the same, but the mass scales are $\Delta \gamma$ (within the 1σ range). Two minima are visible although not $\chi^2$ minimum in the global $\Delta \gamma$ of global solution.

Comparison with the just found $V_{AC} + V_{EP}$ for the b.f.p similar to figure 1a, but now shows the $V_{AC}$ solution in this region. This can also be seen in figure 8, which is larger "fall" in the left side than indicated by data, while the VEP effects makes an additional improvement of the data, one can see from this figure that the introduction of $V_{EP}$ alone generates a good fit of these experimental data, one can see from this figure that the introduction of VEP effects makes an additional improvement of the quality of the fit, especially in the higher energy region of the reactor spectrum. The VAC-alone model predicts a larger "fall" in the left side than indicated by data, while the VAC+VEP solution seems to be in better agreement in this region. This can also be seen in figure 8 which is similar to figure 1a, but now shows the VAC solution in comparison with the just found VAC+VEP for the b.f.p of global solution.

As for the $-V_{EP}$ case, figure 6b shows no local mininum in the global $\Delta \chi^2$. When only the solar data is taken in account, two minima are visible although not so evident. They correspond to $|\phi \Delta \gamma| = 9.0 \times 10^{-20}$ and $|\phi \Delta \gamma| = 2.0 \times 10^{-19}$. The mixing angle is roughly the same but the mass scales are $\Delta m^2 = 6.2 \times 10^{-5} eV^2$ and $\Delta m^2 = 1.6 \times 10^{-4} eV^2$ respectively. These values correspond to the same critical energy: $E_c = 14 MeV$ (within the 1σ range). But this is not consistent with the KamLAND data, which clearly excludes $-V_{EP}$ with $|\phi \Delta \gamma| > 3.9 \times 10^{-20}$ (3σ). So this can be regarded as a superior limit, strongly imposed by KamLAND data. If one discards for a moment the best fit point of the $+V_{EP}$ case, a superior limit is also obtained: $|\phi \Delta \gamma| \leq 1.5 \times 10^{-19}$ (3σ). The superior limit for presence of VEP represented by the $MSW \pm V_{EP}$ model (regardless of the sign case) is the less restrictive of these values, so that:

$$|\phi \Delta \gamma| < 1.5 \times 10^{-19} (3\sigma),$$

(25)

since the + and $-V_{EP}$ cases are obviously mutually excluding.

In order to compare with the macroscopic experiments on VEP, one has to consider an estimative of the gravitational potential $\phi$. It seems that, among several possible sources, the Great Attractor offers the largest contribution, with its best estimative given by $\phi = 3 \times 10^{-5}$. So the upper bound of $|\phi \Delta \gamma|$ given in (25) corresponds to a maximum value of the order $|\Delta \gamma| < 10^{-14}$, which is in agreement with the macroscopic experiments presented in the introduction ($|\Delta \gamma| < 10^{-12}$). The possible solution found with non-zero VEP value corresponds to $|\Delta \gamma| \approx 10^{-15}$.

IV. CONCLUSION

The results of our analysis suggest that VEP-like effects might be taking place in the neutrino sector. The model offers two theoretical possibilities: one in which greater mass represents greater gravitational coupling (here called $VAC/MSW + V_{EP}$) and an inverse situation, where greater mass implicates a smaller cou-
pling with the gravitational field \((VAC/MSW - VEP)\). With the latest statistics presented by the KamLAND collaboration and all known solar neutrino data, except Borexino and SNO-III, such evidence manifests itself as a surprising enhancement of the global fit for the \(VAC/MSW + VEP\) case. The analysis showed that the \(VAC\) model alone is almost excluded with \(3\sigma\). In the case of reactor neutrinos, the \(VAC/MSW + VEP\) model predicts a higher survival probability for high energy reactor neutrinos, which is shown to be in better agreement with the experimental data. The \(VAC/MSW - VEP\) option is excluded by KamLAND data within the present statistics. All results are summarized on table 1.

These evidences are enough to encourage further studying. Hopefully future results from the KamLAND collaboration, with increased statistics, would allow not only a better analysis of this version of the model but of more complex versions as well. In particular, this analysis shows that models with phenomenology similar to this one might enrich the neutrino sector of the Standard Model. The VEP hypothesis presented here is just one. Any model that presents a mixing scenario, with a Hamiltonian like (5) and with \(\Delta E\) given by an expression with the same momentum dependency as the one seen on (7), would lead to a better fit. As an example, the combination of Violation of Lorentz Invariance (VLI) models with mass-flavor mixing presents the same phenomenological behavior (37) as shown on (5) and (7), needing only a parameter reinterpretation: \(|\alpha| = 2|\phi|\Delta\gamma|\), where \(|\alpha| \neq 0\) implies VLI between neutrino flavors (as defined on section 2 of (37)). In general, the results of table 1 also apply to a possible Lorentz violation or a combination of both phenomena.

Considering only VEP effects, this analysis has improved the previous limits for \(\Delta\gamma\) in two orders of magnitude, lowering it from \(|\Delta\gamma| < 10^{-12}\) down to \(|\Delta\gamma| < 10^{-14}\), when it is assumed that neutrinos are mainly affected by the gravity of the Great Attractor.

We would like to thank FAPESP, CNPq and CAPES for several financial supports.

[1] M. Gasperini, Phys. Rev. D 38, 2635 (1988).
[2] A. Halprin and C. N. Leung, Phys. Rev. Lett. 67, 1833 (1991).
[3] A. Halprin, C. N. Leung, and J. Pantaleone, Phys. Rev. D 53, 5365 (1996).
[4] C. M. Will, Theory and Experiment in Gravitational Physics (Cambridge University Press, 1993), revised ed.
[5] T. K. Collaboration, Phys. Rev. Lett. 90, 021802, hep-ex/0212021 (2003).
[6] K. Iida, H. Minakata, and O. Yasuda, Mod. Phys. Lett. A8, 1037 (1993), hep-ph/9211328.
[7] H. Minakata and H. Nunokawa, Phys. Rev. D51, 6625 (1995), hep-ph/9405239.
[8] H. Minakata and A. Y. Smirnov, Phys. Rev. D54, 3698 (1996), hep-ph/9601311.
[9] C. W. Misner, K. S. Thorne, and J. A. Wheeler, Gravitation (Freeman, San Francisco, 1973).
[10] Y. Su, B. R. Heckel, E. G. Adelberger, J. H. Gundlach, M. Harris, G. L. Smith, and H. E. Swanson, Phys. Rev. D 50, 3614 (1994).
[11] J. H. Gundlach, G. L. Smith, E. G. Adelberger, B. R. Heckel, and H. E. Swanson, Phys. Rev. Lett. 78, 2523 (1997).
[12] R. Horvat, hep-ph/9806380v2 (1998).
[13] M. Barkovich, H. Casini, J. C. D’Olivo, and R. Montemayor, Phys. Lett. B 506 (2001).
[14] T. Damour and G. Schäfer, Phys. Rev. Lett. 66, 2549 (1991).
[15] D. Burstein, Rep. Prog. Phys. 53, 421 (1990).
[16] R. C. Kraan-Korteweg, astro-ph/0006199v1 (2000).
[17] I. R. Kenyon, Phys. Lett. B 237, 274 (1990).
[18] A. Y. Smirnov, hep-ph/0305106 (2003).
[19] B. P. Palash, International Journal of Modern Physics A 7, 5387 (1992).
[20] S. P. Mikheyev and A. Y. Smirnov, Nuovo Cimento, C9 17 (1986).
[21] I. Wolfenstein, Phys. Rev. D 17, 2369 (1978).
[22] S. Abe, hep-ex/0801-4589 (2008).
[23] P. D. Group, Journal of Physics G 30 (2006).
[24] J. N. Bahcall, A. M. Serenelli, and S. Basu, The Astrophysical Journal 621, L85 (2005).
[25] J. N. Bahcall, Neutrino Astrophysics (Cambridge Univ. Press, 1989).
[26] P. C. d. Holanda and A. Y. Smirnov, Astropart.Phys. 21, 287 (2004).
[27] B. T. Cleveland, T. Daly, and R. Davis, The Astrophysical Journal 496, 505 (1998).
[28] J. N. Abdurashitov, Journal of Experimental and Theoretical Physics 95, 181 (2002).
[29] M. A. Altmann, Phys. Lett. B 616, 174 (2005).
[30] J. Hosaka, Phys. Rev. D 73, 112001 (2006).
[31] A. W. P. Poon, hep-ex/0211013 (2002).
[32] B. Aharrnin, nucl-ex/0502021 (2005).
[33] B. Aharrnin, Phys. Rev. Lett. 101, 111301 (2008).
[34] C. Arpesella, Phys. Rev. Lett. 101, 091302 (2008).
[35] G. L. Fogli and E. Lisi, Astroparticle Physics 3, 185 (1995).
[36] P. C. d. Holanda and A. Y. Smirnov, JCAP 0302, 001 (2003).
[37] P. Arias, J. Gamboa, F. Mendez, A. Das, and J. Lopez-Sarrion, Phys. Lett. B 650, 401 (2007).
[38] This condition is equivalent to \(\Psi_0 = \begin{pmatrix} i \\ 0 \end{pmatrix}\).
[39] From some variations of the model, BS05OP is the one considered here.