Neural-Based Command Filtered Backstepping Control for Trajectory Tracking of Underactuated Autonomous Surface Vehicles

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ABSTRACT
This paper is concerned with the problem of trajectory tracking control of underactuated autonomous surface vehicles subject to parameter uncertainties and nonlinear external disturbances. A robust control scheme is presented by employing backstepping method, neural network and sliding mode control. In addition, the overall signals are guaranteed the uniformly ultimate boundness by the Lyapunov stability theory. These advantages are highlighted as follows: (i) The derivations of virtual variables are obtained by a second-order filter. A compensation loop is proposed to reduce the filtered errors between the filtered variables and virtual variables. (ii) The neural network is combined with low-frequency learning techniques to estimate and approximate unknown functions of system. (iii) An anti-windup design is employed to restrict the amplitude of control inputs. Finally, simulation results show the strong robustness and tracking effectiveness of the designed control scheme under the nonlinear external disturbances.

INDEX TERMS Autonomous surface vehicle, trajectory tracking, neural network, low-frequency learning techniques, anti-windup design.

I. INTRODUCTION
In the past decades, the motion control of underactuated autonomous surface vehicles (ASVs) has aroused extensive attention owing to their wide applications [1]–[5]. In the practical engineering, the trajectory tracking of underactuated ASVs is applicable in enemy tracking, ocean salvage operation, marine supply, and so on. Actually, most of the ASVs are underactuated, which means that some degrees of freedom lack control inputs. The situation indicates that strong couplings are caused between the degrees of freedom. Moreover, the ASVs are susceptible under the strong nonlinear external disturbances which are induced by ocean currents, waves, and wind [6]–[10]. Therefore, the trajectory tracking control is a challenging problem to underactuated ASVs [11]–[13].

Many researchers have made great works to solve the trajectory tracking problem of the underactuated ASVs, included PID control, sliding mode control, cascaded control and so on [14]–[17]. In [18], a backstepping method was presented, and it has obvious advantages to solve the control problem of nonlinear system. In [19], the backstepping method was employed to realize the trajectory tracking of the underactuated ASVs. However, the explosion of complexity was caused in the traditional backstepping method. In [6], dynamic surface control was designed to solve the problem of computational complexities in the traditional backstepping method. The derivations of virtual control signal was obtained by a first-order filter such that avoid computational complexities of direct derivation. Unfortunately, the principle of dynamic surface control is a differential process, which means the measuring noise was amplified such that the performance of controller can be reduced. In [20], the second-order filter was presented to obtain the derivations of virtual control signals by the integral progress. In [21], The command filtered backstepping method was designed to solve the tracking problem of the underactuated ASVs.

The model parameters of underactuated ASVs are greatly difficult to obtain accurately due to harsh marine environment. In [22], a robust control scheme was proposed to track a desired trajectory for the underactuated ASVs based on all known parameters. In [23], an robust controller was designed to realize the trajectory tracking under the unknown ocean
current disturbances, and an adaptive law was employed to effectively compensate the unknown damping terms. However, the closed-loop control system only considered the unknown damping terms such that the controller was difficult to put into practice. The neural network has wide applications owing to its inherent ability of approximating unknown functions in the nonlinear system. In [24], a neural network adaptive control scheme was presented to solve the problem of trajectory tracking of the underactuated ASVs. A single-layer RBFNN was used to estimate and approximate the unknown functions of system. However, the high frequency oscillation and low-frequency learning techniques were effectively estimated and approximated based on neural filtered errors between virtual control signal and the filtered variables, and a compensation loop is presented to reduce the order filter is employed to obtain the derivations of virtual contributions of this paper are described as follows: uncertainties and nonlinear external disturbances. The main model.

In the practice, the integral saturation of control inputs was not solved. In [29], a low-pass command filter was designed to solve the problem of trajectory tracking. A three-layer neural network was used to estimate and approximate the unknown functions of system. However, the high frequency oscillation can be induced by nonlinear external disturbances. In [28], a neural network adaptive control scheme was designed to solve the trajectory tracking problem of underactuated ASVs based on backstepping method and dynamic surface control. However, the problem of high frequency oscillation was not solved. In [30], a low-pass command filter was combined with adaptive method to filter high frequency oscillation.

In the practice, the integral saturation of control signals was induced in the traditional second-order filter such that it may cause control performance degradation and instability of control system. Therefore, the integral saturation should be taken into the underactuated ASVs. In [30], the maximum and minimum values of the control inputs are set to limit amplitude of control inputs. However, the motion stability of control system can be guaranteed due to forced restriction in control input. In [31], a low-pass command filter was designed to overcome integral saturation problem. However, the proposed design procedures are only suitable for specific model.

In summary, an adaptive trajectory tracking control scheme is proposed for the underactuated ASVs subject to parameter uncertainties and nonlinear external disturbances. The main contributions of this paper are described as follows:

(i): To avoid the computational complexities, a second-order filter is employed to obtain the derivations of virtual variables, and a compensation loop is presented to reduce the filtered errors between virtual control signal and the filtered signal.

(ii): To easily apply into practice without any previous parameters, the uncertain dynamics of underactuated ASVs are effectively estimated and approximated based on neural network and low-frequency learning techniques.

(iii): To solve the problem of integral saturation of control input signals, An anti-windup design is employed to solve the problem of the integral saturation of control inputs for underactuated ASVs.

The other section of this study is arranged as follows. In section 2, the underactuated ASV model, errors equations and the control objective are listed. In section 3, a robust controller of underactuated ASV is presented by employing backstepping method, neural network, low-frequency learning techniques and sliding mode control. In section 4, the stability of closed-loop control system is proved by employing Lyapunov stability theory. In section 5, the simulation results and comparative analysis are shown to verify the trajectory tracking performance for proposed controller. In section 6, the conclusion of this paper is drawn.

II. PROBLEM FORMULATION

In this section, the underactuated ASV model, errors equations, control object, and the structure of second-order filter are listed.

A. UNDERACTUATED ASV MODEL

The kinematic and dynamic model are provided in this section. To simplify the formula derivation, the model is chosen that the mass and the damping matrices are diagonal, and the external disturbances are taken into consideration for underactuated ASV model.

The kinematic and dynamic model are as follows [20]:

\[
\begin{align*}
\dot{x} &= u \cos \psi - v \sin \psi \\
\dot{y} &= u \sin \psi + v \sin \psi \\
\dot{\psi} &= r
\end{align*}
\]

and

\[
\begin{align*}
\dot{m}_{11} &= m_{22}v_r - m_{11}f_v(u) + \tau_u + \tau_{eu} \\
m_{22} \dot{v} &= -m_{11}u_r - m_{22}f_v(v) + \tau_v \\
m_{33} \dot{r} &= (m_{11} - m_{22})u_r - m_{33}f_v(r) + \tau_e \psi < 0 \\
\end{align*}
\]

where \( x, y \) and \( \psi \) represent positions and orientations of underactuated ASVs in the geodetic fixed frame, respectively. \( u, v \) and \( r \) represent the surge velocity, sway velocity and yaw velocity in the body fixed frame, respectively. \( m_{ii}(i = 1, 2, 3) \) represent the combined inertia and added mass terms in the body fixed frame. \( f_v(u), f_v(v) \) and \( f_v(r) \) represent the unknown nonlinear dynamics of underactuated ASV including hydrodynamic damping and friction terms. \( \tau_u, \tau_v \) and \( \tau_e \) represent control inputs, including control force and control force moment. \( \tau_{eu}, \tau_{ev} \) and \( \tau_{er} \) represent the nonlinear external disturbances induced by ocean currents, waves and wind.

Based on the above kinematic and dynamic model, to clearly design controller, we make the most of the following assumptions:

**Assumption 1:** The sway velocity of underactuated ASV is time-varying bounded such that \( \sup_{t \geq 0} |v| \leq v_M \), where \( v_M \) is a positive unknown constant [27].

**Assumption 2:** The yaw angle of underactuated ASV is time-varying bounded such that \( 0 < |\psi| < \pi/2 \) [28].
**Assumption 3:** The nonlinear external disturbances of underactuated ASV are time-varying bounded such that satisfies $|\tau_{ek}| \leq \tau^*_e$, $k = u, v, r$, $\tau^*_e$ is a positive unknown constant [20].

**Remark 1:** There is not control input in sway velocity such that the range of sway velocity can be small. Therefore, we can make a assumption 1 in this paper. Based on assumption 2, the underactuated ASV can't turn at 90 degree, and it can prevents any possible singularity in the controller.

In order to design the controller effectively, we need to assume that the external disturbances are bounded.

**B. ERROR DYNAMICS OF TRAJECTORY TRACKING**

In order to facilitate the problem formulation, the frame is designed to position tracking of underactuated ASV in Fig 1. $\{O_E, X_E, Y_E\}$ and $\{O_B, X_B, Y_B\}$ denote the geodetic fixed frame and the body fixed frame, respectively.

![FIGURE 1. The ASV frames in the trajectory tracking problem.](image)

where $\eta_e(t) = [x_c \ y_c \ \psi_c]^T$ represents the desired trajectory.

**Assumption 4:** The $\eta_e = [x_e \ y_e \ \psi_e]^T$ is a smooth desired trajectory and its derivatives is time-varying bounded [32].

According to assumption 4, it is obtained as

$$\begin{align*}
\left| \dot{x_e} \right| & \geq \zeta > 0 \\
\left| \dot{y_e} \right| & \geq \zeta > 0
\end{align*}$$

where $\zeta$ is a positive constant.

The control object is designing control force $\tau_u$ and control force moment $\tau_r$ to make trajectory tracking errors converge to a small neighbourhood of zero.

Firstly, the trajectory tracking errors are defined as

$$x_e = x - x_{cf} \quad y_e = y - y_{cf} \quad \psi_e = \psi - \psi_{cf}$$

The velocity tracking errors are defined as

$$u_e = u - u_{cf} \quad v_e = v - v_{cf}$$

Then, the derivation of the virtual control signal is obtained through a second-order filter

$$\ddot{v}_{cf} = -2\omega_n\dot{v}_{cf} - \omega_n^2(v_{cf} - v_c)$$

where $v_c$ denotes the virtual control signal. $v_{cf}$ denotes the filtered signal. $\omega$ denotes the damping ratio. $\omega_n$ denotes the frequency, and $\nu$ and $\nu_n$ are positive constants.

The structure of the second-order filter is given by Fig.2

**FIGURE 2. The structure of the second-order filter.**

The desired velocity and yaw angle are obtained by (9) yields

$$\begin{align*}
u_{xc} &= u_c \cos \psi - v_c \sin \psi \\
u_{yc} &= u_c \sin \psi + v_c \cos \psi
\end{align*}$$

According to (8), the desired positions can be defined as

$$\begin{align*}
u_{xc} &= u_c \cos \psi_c - v_c \sin \psi_c \\
u_{yc} &= u_c \sin \psi_c + v_c \cos \psi_c
\end{align*}$$

The desired velocity and yaw angle are obtained by (9) yields

$$\begin{align*}
u_{x}_{cf} &= u_{cf} \cos \psi_{cf} - v_{cf} \sin \psi_{cf} \\
u_{y}_{cf} &= u_{cf} \sin \psi_{cf} + v_{cf} \cos \psi_{cf}
\end{align*}$$

The errors variables are described as

$$\begin{align*}
u_{xe} &= u_x - u_{x_{cf}} \\
u_{ye} &= u_y - u_{y_{cf}}
\end{align*}$$

Substituting (9), (11) and (12) into (7) yields, equation (7) is converted to

$$\begin{align*}
\dot{x} &= v_{xc} + u_{xe} + v_{x_{cf}} - v_{xc} \\
\dot{y} &= v_{yc} + u_{ye} + v_{y_{cf}} - v_{yc}
\end{align*}$$
The virtual control signals are defined as
\[
\begin{align*}
\eta_x &= -k_1 x_e + \dot{x}_e \\
\eta_y &= -k_2 y_e + \dot{y}_e
\end{align*}
\]  
(14)
where \(k_1\) and \(k_2\) are the positive constants.
Substituting (14) into (13) yields
\[
\begin{align*}
\dot{x}_e &= -k_1 x_e + \eta_x + \eta_{xf} - \eta_x \\
\dot{y}_e &= -k_2 y_e + \eta_y + \eta_{yf} - \eta_y
\end{align*}
\]  
(15)
According to [33], equation (15) can be converted to
\[
\begin{bmatrix}
\dot{x}_e \\
\dot{y}_e
\end{bmatrix} = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix} \begin{bmatrix} x_e \\
 y_e \end{bmatrix} + [R \, Qg(\psi_e)] \begin{bmatrix} \eta_x \\
 \eta_y \end{bmatrix} + \begin{bmatrix} \eta_{xf} - \eta_x \\
 \eta_{yf} - \eta_y \end{bmatrix}
\]  
(16)
where
\[
R = \begin{bmatrix} \cos \psi_{cf} & \sin \psi_{cf} \\
\sin \psi_{cf} & -\cos \psi_{cf} \end{bmatrix}, \quad Q = \begin{bmatrix} u_{cf} \sin \psi_{cf} - v \cos \psi_{cf} & u_{cf} \sin \psi_{cf} - v \cos \psi_{cf} \\
v \cos \psi_{cf} - u_{cf} \cos \psi_{cf} & u_{cf} \sin \psi_{cf} - v \cos \psi_{cf} \end{bmatrix}
\]
To reduce the approximation errors between the virtual variables and filtered variables, errors compensation loops are designed
\[
e_x = x_e - \sigma_x \quad e_y = y_e - \sigma_y
\]  
(17)
where \(\sigma_x\) and \(\sigma_y\) denote the compensation signals, and \(\sigma_x(0) = 0\) and \(\sigma_y(0) = 0\).
According to (16), the derivations of compensation signals are obtained as
\[
\begin{bmatrix}
\dot{\sigma}_x \\
\dot{\sigma}_y
\end{bmatrix} = \begin{bmatrix} -k_1 & 0 \\ 0 & -k_2 \end{bmatrix} \begin{bmatrix} \sigma_x \\
 \sigma_y \end{bmatrix} + [R \, Qg(\psi_e)] \begin{bmatrix} \sigma_x \\
 \sigma_y \end{bmatrix} + \begin{bmatrix} \eta_{xf} - \eta_x \\
 \eta_{yf} - \eta_y \end{bmatrix}
\]  
(18)
Combined (15), (17) and (18) yields, the derivations of position tracking errors are obtained as
\[
\begin{bmatrix}
\dot{e}_x \\
\dot{e}_y
\end{bmatrix} = \begin{bmatrix} -k_1 & 0 \\ 0 & -k_2 \end{bmatrix} \begin{bmatrix} e_x \\
 e_y \end{bmatrix} + [R \, Qg(\psi_e)] \begin{bmatrix} e_x \\
 e_y \end{bmatrix}
\]  
(19)
To stabilize the postion tracking errors, define a Lyapunov function as
\[
V_1 = \frac{1}{2}(e_x^2 + e_y^2)
\]  
(20)
Differentiating (20) along with (19) yields
\[
V_1 = \dot{e}_x e_x + \dot{e}_y e_y
= -k_1 e_x^2 - k_2 e_y^2 + e_x \dot{u}_e + e_y \dot{v}_e
\]
\[
= -k_1 e_x^2 - k_2 e_y^2 + [e_x, e_y] \begin{bmatrix} R \, Qg(\psi_e) \end{bmatrix} \begin{bmatrix} e_x \\
 e_y \end{bmatrix}
\]
\[
= -k_1 e_x^2 - k_2 e_y^2 + R^T \begin{bmatrix} e_x \\
 e_y \end{bmatrix} e_u + Qg(\psi_e) \begin{bmatrix} e_x \\
 e_y \end{bmatrix} e_\psi
\]  
(21)

\section*{B. YAW ANGLE CONTROL}
In this subsection, the yaw angle tracking errors are stabilized based on the Lyapunov stability theory and backstepping method.
Firstly, the desired signal is defined as
\[
r_e = -k_3 \psi_e + \dot{\psi}_e - \psi_{bs}
\]  
(22)
where \(k_3\) is a positive constant. \(\psi_{bs}\) is a robust term to be chosen later.
Combined (1) and (5) yields, it is obtained as
\[
\dot{\psi} = r_e + (r - r_{cf}) + (r_{cf} - r_e)
\]  
(23)
Substituting (23) into (4) yields
\[
\dot{\psi}_e = -k_3 \psi_e - \psi_{bs} + r_e + (r_{cf} - r_e)
\]  
(24)
The yaw angle filtered error compensation loop is designed as
\[
e_\psi = \psi_e - \sigma_\psi
\]  
(25)
Differentiating (25) along with (24) yields
\[
\dot{e}_\psi = \dot{\psi}_e - \dot{\sigma}_\psi
= -k_3 e_\psi - \psi_{bs} + e_r
\]  
(26)
where \(\sigma_\psi(0) = 0\)
To stabilize the yaw angle tracking error, define a new Lyapunov function as
\[
V_2 = V_1 + \frac{1}{2} e_\psi^2
\]  
(27)
Differentiating (27) along with (21) and (26) yields
\[
\dot{V}_2 = \dot{V}_1 + \dot{e}_\psi e_\psi
= -k_1 e_x^2 - k_2 e_y^2 + k_3 e_\psi^2 + e_r e_\psi - \psi_{bs} e_\psi
+ R^T \begin{bmatrix} e_x \\
 e_y \end{bmatrix} e_u + Qg(\psi_e) \begin{bmatrix} e_x \\
 e_y \end{bmatrix} e_\psi
\]  
(28)

\section*{C. VELOCITY AND ANGLE VELOCITY CONTROL}
In this subsection, the control inputs are designed based on neural network, adaptive techniques and sliding mode control, and the velocities tracking errors are stabilized based on the Lyapunov stability theory and backstepping method.
The velocities tracking errors are as follows
\[
m_{11} \dot{e}_u = -m_{22} v_r + m_{11} f_u(u) + m_{11} u_{cf} + \tau_u - \tau_{eu}
\]
\[
m_{33} \dot{e}_r = -(m_{11} - m_{22}) u_v + m_{33} f_r(r) + m_{33} e_{cf} + \tau_r - \tau_{er}
\]  
(29)
The unknown functions of underactuated ASVs are described as
\[
\chi_a = -m_{22} v_r + m_{11} f_u(u) + m_{11} u_{cf}
\]
\[
\chi_r = -(m_{11} - m_{22}) u_v + m_{33} f_r(r) + m_{33} e_{cf}
\]  
(30)
A radial basis function neural network (RBFNN) is used to estimate and approximate the unknown functions of underactuated ASVs owing to its linear parameterization and simplicity. A three-layer RBFNN is employed to solve the parameter uncertainties, whose structure is as follows

\[ f(x) = Wh(x) + e \]

\[ h(x) = \exp \left( \frac{|x - c_j|^2}{2b_j^2} \right) \]  
(31)

where \( f(x) = [\chi_x, \chi_r]^T \) denote unknown function of system. \( W \) denotes the weight adjustment. \( h(x) \) represents the Gaussian basis function. \( x = [u, v, r, \dot{u}_c, \dot{r}_c]^T \) denotes the input vector of the RBFNN. \( c_j \) represents an \( j \)-dimensional vector denoting the center of the \( j \)th basis function. \( b_j \) represents the standard deviation.

The ideal condition is described as

\[ f^*(x) = W^* h(x) + e \]  
(32)

\[ W^* = \arg \min \left\{ \sup \| f(x) - Wh(x) \| \right\} \]  
(33)

where \( W^* \) is the optimal weight adjustment.

The neural network output are described as

\[ \hat{f}(x) = \hat{W} h(x) + e \]  
(34)

where \( \hat{W} = \begin{bmatrix} \hat{W}_{u1} \hat{W}_{u2} \cdots \hat{W}_{uk} \\ \hat{W}_{r1} \hat{W}_{r2} \cdots \hat{W}_{rk} \end{bmatrix}^T \) and \( \hat{f}(x) \) denotes the NN output. \( \hat{W} \) denotes the estimation of the weight adjustment, and satisfies \( \hat{W} = W - W^* \). The ideal weight matrix \( W^* \) and the approximation error \( e \) are bounded so that there are \( W_M \) and \( \varepsilon_M \), \( \| W^* \| \leq W_M \) and \( \| e \| \leq \varepsilon_M \).

Assumption 5: There is a bounded function \( \alpha > 0 \), and satisfies \( |e| + |\tau_e| \leq \alpha \), that is to say, \( \alpha \) is the bounded function of nonlinear external disturbances and neural network approximation errors [28].

The control inputs are designed as

\[ \tau_u = -m_{11}(\lambda_ue_u + u_{bs}) + \hat{W}_u h(x_u) - \dot{\hat{u}}_u \tanh(e_u/\xi_u) \]

\[ \tau_r = -m_{33}(\lambda_re_r + r_{bs}) + \hat{W}_r h(x_r) - \dot{\hat{r}}_r \tanh(e_r/\xi_r) \]  
(35)

where \( \lambda_u, \lambda_r, \xi_u \) and \( \xi_r \) are positive constants. \( u_{bs} \) and \( r_{bs} \) are robust terms to be chosen later. \( x_u = [u, v, r, \dot{u}_c, \dot{r}_c]^T \), \( x_r = [u, v, r, \dot{r}_c]^T \).

Substituting (35) into (29) yields

\[ m_{11}\dot{u}_u = -m_{11}(\lambda_ue_u + u_{bs}) + \kappa_u - \chi_u - \tau_{eu} \]

\[ m_{33}\dot{r}_r = -m_{33}(\lambda_re_r + r_{bs}) + \kappa_r - \chi_r - \tau_{er} \]  
(36)

According to (35) and (36), the equations are given as

\[ \kappa_u = \hat{W}_u h(x_u) - \dot{\hat{u}}_u \tanh(e_u/\xi_u) \]

\[ \kappa_r = \hat{W}_r h(x_r) - \dot{\hat{r}}_r \tanh(e_r/\xi_r) \]  
(37)

The desired velocities are obtained as

\[ \dot{u}_c = -\lambda_u u_e + \dot{u}_c - u_{bs} \]

\[ \dot{r}_c = -\lambda_r r_e + \dot{r}_c - r_{bs} \]  
(38)

The velocities tracking errors are described as

\[ u_e = u - u_c = -\lambda_u u_e - u_{bs} \]

\[ r_e = r - r_c = -\lambda_r r_e - r_{bs} \]  
(39)

Considering the signals \( \sigma_u \) and \( \sigma_r \) are 0, and the relations are obtained as

\[ e_u = u_e \]

\[ e_r = r_e \]  
(40)

The adaptive laws are designed as

\[ \dot{\hat{W}}_u = \delta_u(e_u h(x_u) - \eta_u (\hat{W}_u - \hat{W}_u)) \]

\[ \dot{\hat{W}}_r = \delta_r(e_r h(x_r) - \eta_r (\hat{W}_r - \hat{W}_r)) \]  
(41)

where \( \delta_u, \eta_u, \delta_r, \) and \( \eta_r \) are positive constants.

The high-frequency oscillations can be produced owing to the parameter uncertainties and external disturbances. To solve the problem of the high-frequency oscillations, the low frequency learning techniques [29] are introduced to the designed controller. The adaptive laws are improved as

\[ \dot{\hat{W}}_u = \delta_u(e_u h(x_u) - \eta_u (\hat{W}_u - \hat{W})) \]

\[ \dot{\hat{W}}_r = \delta_r(e_r h(x_r) - \eta_r (\hat{W}_r - \hat{W})) \]  
(42)

where \( \rho_u \) and \( \rho_r \) are filter gain parameters. \( \hat{W}_uf \) and \( \hat{W}_rf \) are the low-pass filter weight estimations of \( \hat{W}_u \) and \( \hat{W}_r \), respectively.

The \( \hat{W}_uf \) and \( \hat{W}_rf \) can be obtained by first-order filters, and the relations are given by

\[ \hat{W}_uf = \hat{W}_uf/ (\beta_u s_u + 1) \]

\[ \hat{W}_rf = \hat{W}_rf/ (\beta_r s_r + 1) \]  
(43)

where \( \beta_u, s_u, \beta_r \) and \( s_r \) are positive constants.

The sliding mode control is combined with adaptive techniques [34] to estimate the bounded vector consisting of external disturbances and approximation errors of neural network. The adaptive laws are designed as

\[ \dot{\hat{u}}_u = \gamma_u \left[ \tanh(e_u/\xi_u) - \theta_u (\hat{u}_u - \alpha_u) \right] \]

\[ \dot{\hat{r}}_r = \gamma_r \left[ \tanh(e_r/\xi_r) - \theta_r (\hat{r}_r - \alpha_r) \right] \]  
(44)

where \( \gamma_u, \gamma_r, \theta_u \) and \( \theta_r \) are positive constants. \( \alpha_u \) and \( \alpha_r \) are the prior estimations \( \alpha_u \) of and \( \alpha_r \), respectively.

To stabilize velocities tracking errors, define a new Lyapunov function as

\[ V_3 = V_2 + \frac{1}{2} (e_u^2 + e_r^2) \]  
(45)

Differentiating (45) along with (28) yields

\[ \dot{V}_3 = \dot{V}_2 + \dot{e}_u u_e + \dot{e}_r r_e \]

\[ = -k_1 e_u^2 - k_2 e_r^2 - k_3 \psi_{\dot{e}_e} - \lambda_u e_u^2 - \lambda_r e_r^2 \]

\[ + e_r e_r - \psi_{s_e} e_r - u_{bs} e_u - r_{bs} e_r \]
The expression is given as the integral saturation of control inputs for underactuated ASVs.

**FIGURE 4.** The structure of anti-windup control part.

The controller of anti-windup part is described as

\[
\begin{align*}
W_u & = W_u - W_u^* \\
W_r & = W_r - W_r^*
\end{align*}
\]

**D. ANTI-WINDUP DESIGN**

The anti-windup design [31] is presented to solve problem of integral saturation of control inputs under underactuated ASVs. The expression is given as

\[
U = U_a - K_a \int (U - U_a) dt
\]

where \( K_a \) is the positive gain constants.

The structure of anti-windup design is given by Fig. 4.

Figure 5 indicates a block diagram of designed trajectory tracking control system. In the next section, the stability of the closed-loop system is proved by Lyapunov stability theory.

**IV. STABILITY ANALYSIS OF CLOSED-LOOP SYSTEM**

**Theorem:** Consider the closed-loop system consisting of underactuated ASV dynamics (1) and (2), the control laws (34), the adaptive update laws (42), the second-order filter (6), the compensation loop (52), together with low-frequency learning techniques (43). The exist suitable control parameters \( k_1, k_2, k_3, \lambda_u, \lambda_r, \gamma_u, \gamma_r, \theta_u, \theta_r, \rho_u, \rho_r \) such that all the signals of control system are time-varying bounded and all error signals uniformly ultimately converge to a small neighborhood of zero.

**Proof:** Firstly, consider the following Lyapunov function for closed-loop system.

\[
V_4 = V_3 + \frac{1}{2\delta_u} \tilde{W}_u^T \tilde{W}_u + \frac{1}{2\delta_r} \tilde{W}_r^T \tilde{W}_r + \frac{\eta_u}{\rho_u} \tilde{W}_u^T \tilde{W}_u + \frac{\eta_r}{\rho_r} \tilde{W}_r^T \tilde{W}_r - \frac{\eta_r}{\rho_r} \tilde{W}_r^T \tilde{W}_r
\]

Differentiating (49) along with (46) yields

\[
\dot{V}_4 = -\frac{1}{\gamma_u} \tilde{u}_a \tilde{u}_a - \frac{1}{\gamma_r} \tilde{r}_a \tilde{r}_a
\]

\[
= -k_1 \dot{e}_u^2 - k_2 \dot{e}_r^2 - k_3 \dot{e}_r^2 - \lambda_u e_u^2 - \lambda_r e_r^2 - e_\psi e_r - b_\psi e_r - b_\psi e_u + \psi_e e_r + \psi_e e_u - b_\psi e_u + b_\psi e_u - b_\psi e_r
\]

The equation (50) can be converted to as

\[
\dot{V}_4 = -k_1 \dot{e}_u^2 - k_2 \dot{e}_r^2 - k_3 \dot{e}_r^2 - \lambda_u e_u^2 - \lambda_r e_r^2 - e_\psi e_r - b_\psi e_r - b_\psi e_u + \psi_e e_r + \psi_e e_u - b_\psi e_u + b_\psi e_u - b_\psi e_r
\]

To improve the performance of proposed controller, choosing the system robust terms as follows

\[
\psi_{bs} = g(\psi_e) \mathcal{Q} \left[ \begin{array}{c} e_x \\ e_y \end{array} \right] \quad \psi_{bs} = R^T \left[ \begin{array}{c} e_x \\ e_y \end{array} \right] \quad \psi_{bs} = e_\psi
\]

Substituting (52) into (51) yields

\[
\dot{V}_4 = -k_1 \dot{e}_u^2 - k_2 \dot{e}_r^2 - k_3 \dot{e}_r^2 - \lambda_u e_u^2 - \lambda_r e_r^2 - e_\psi e_r - b_\psi e_r - b_\psi e_u + \psi_e e_r + \psi_e e_u - b_\psi e_u + b_\psi e_u - b_\psi e_r
\]

\[
+ e_r (e_r + \tilde{\alpha}_r \tan(e_r / \xi_r) - \tau_{er})
\]

\[
+ e_u (e_u + \tilde{\alpha}_u \tan(e_u / \xi_u) - \tau_{eu})
\]

\[
+ e_r (e_r + \tilde{\alpha}_r \tan(e_r / \xi_r) - \tau_{er})
\]

**FIGURE 3.** Structure of the three-layer RBF neural network.
Finally, equation (58) can be converted to
\[ V \leq \frac{\Gamma}{2\sigma^2} + \left[ V(0) - \frac{\Gamma}{2\sigma^2} \right] e^{-2\sigma t} \tag{59} \]

The result proves overall error signals of controller are time-varying boundedness. The trajectory tracking errors of underactuated ASV converge to a small neighborhood of zero.

V. SIMULATION AND COMPARATIVE ANALYSIS

In the section, the performance of designed controller is shown by comparing with the traditional backstepping method in the MATLAB software environment. In order to verify the tracking efficiency of the proposed controller, the simulations are applied into an underactuated ASV, which is designed by the university of western Australia. The model parameters are given by [36].

Firstly, the tracking trajectory of the underactuated ASV is assumed as
\[ x_c = 0.2m/s \]
\[ y_c = 6\cos(0.01t)m/s \tag{60} \]

The initial conditions of underactuated ASV are chosen as
\[ x(0) = -1m \quad y(0) = 1m \quad \psi(0) = 0rad \]
\[ u(0) = 0m/s \quad v(0) = 0m/s \quad r(0) = 0rad/s \tag{61} \]

To verify performance and effectiveness of the proposed controller, we increase the external disturbances of Liu’ study [37], and the nonlinear external disturbances are chosen as
\[ \tau_{eu} = 0.8\sin(0.5t)\sin(0.4t) + 0.4\sin(0.3t)\cos(0.4t) \]
\[ \tau_{ev} = 0.4\sin(0.1t) \]
\[ \tau_{er} = 0.1\sin(0.9t)\cos(0.4t) \tag{62} \]
The control parameters are given by $k_1 = 0.3, k_2 = 0.2, k_3 = 0.3, \lambda_u = 0.1, \lambda_r = 0.4, \gamma_u = 10, \gamma_r = 10, \theta_u = 0.2, \rho_u = 0.1, \rho_r = 0.1$.

Then, the parameters of RBF neural network with five hidden nodes designed in this paper are given by $\delta_u = 10, \delta_r = 10, \eta_u = 0.2, \eta_r = 0.2$. The Gaussian function width $b_j$ is an important factor affecting the mapping range of the network. The wider the Gaussian function, the greater the network’s ability to map inputs. For inputs, the closer the $c_j$, the more sensitive the Gaussian function. Based on above principles, according to [4], we first select the appropriate value, and through multiple attempts. The centre vector and the standard deviation are given as $c_j = [-1.0 -0.5 0 0.5 1.0]$ and $b_j = 5.0$, respectively.

Finally, to clearly present performance of designed controller, the simulation time is chosen as 1600s. The simulation results of dynamic surface control (DSC) [28], the traditional backstepping method (Backstepping), neural based command filtered backstepping (NBCFB) and comparative analysis are shown in Fig 6-10.

Fig 6 and Fig 7 indicate that the proposed method has better trajectory tracking performance and effectiveness than
The trajectory tracking control for underactuated ASV in presence of the parameters uncertainty and the strong nonlinear external disturbances are taken into consideration in this paper. A robust controller is proposed to solve the problem of trajectory tracking based on backstepping method, neuro-adaptive technology and sliding mode control. The second-order filter is employed to acquire the derivation of virtual variables such that effectively reduces computational complexities of the traditional backstepping method. The filtered errors between virtual variables and filtered variables are reduced by designing a compensation loop. The closed-loop stability of control system is proved based on Lyapunov stability theorem. The simulation results are combined with comparative analysis to verify that the designed control system has a good robustness and effectiveness.

VI. CONCLUSION

The trajectory tracking control for underactuated ASV in presence of parameter uncertainties and nonlinear external disturbances for underactuated ASV. We can clearly see that the trajectory tracking errors of proposed method are smaller than other two methods. Compared to other two methods, the actual velocities of proposed method can track the desired velocities more effectively in Fig 8.

The all unknown functions of underactuated ASV can be estimated and approximated accurately by employing the RBF neural network and low frequency learning techniques in Fig 9. Finally, the control force and control force moment are given in Fig 10.

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