Stochastic Optimization Approaches for an Operating Room and Anesthesiologist Scheduling Problem

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We propose combined allocation, assignment, sequencing, and scheduling problems under uncertainty involving multiple operation rooms (ORs), anesthesiologists, and surgeries, as well as methodologies for solving such problems. Specifically, given sets of ORs, regular anesthesiologists, on-call anesthesiologists, and surgeries, our methodologies solve the following decision-making problems simultaneously: (1) an allocation problem that decides which ORs to open and which on-call anesthesiologists to call in, (2) an assignment problem that assigns an OR and an anesthesiologist to each surgery, and (3) a sequencing and scheduling problem that determines the order of surgeries and their scheduled start times in each OR. To address uncertainty of each surgery’s duration, we propose and analyze stochastic programming (SP) and distributionally robust optimization (DRO) models with both risk-neutral and risk-averse objectives. We obtain near-optimal solutions of our SP models using sample average approximation and propose a computationally efficient column-and-constraint generation method to solve our DRO models. In addition, we derive symmetry-breaking constraints that improve the models’ solvability. Using real-world, publicly available surgery data and a case study from a health system in New York, we conduct extensive computational experiments comparing the proposed methodologies empirically and theoretically, demonstrating where significant performance improvements can be gained. Additionally, we derive several managerial insights relevant to practice.

Key words: Operating rooms, surgery scheduling, mixed-integer programming, stochastic programming, distributionally robust optimization

1. Introduction

Operating room (OR) planning and scheduling has a significant impact on costs for hospital management and the quality of the health care that a hospital is able to provide. ORs typically generate 40–70% of hospital revenues and incur 20–40% of operating costs (Cardoen et al. 2010, Zhu et al. 2019). In addition, it is common for 60-70% of patients admitted to a hospital to require surgery (Guerriero and Guido 2011). As a result, OR planning and scheduling significantly influences overall patient flow, and whether or not they operate efficiently has a large influence on the quality of care that a hospital is able to provide.
On top of their critical nature, OR planning and scheduling problems are extremely complex since they require the coordination of multiple hospital resources, including ORs themselves, anesthesiologists, surgical equipment, and so on. Their complexity is compounded by the fact that, in addition to limited OR capacity and time, there is an overall shortage in terms of the physicians and anesthesiologists that are required to perform surgeries (De Simone et al. 2021, Shanafelt et al. 2016). Consequently, hospital managers could benefit greatly from advanced methodologies to improve OR utilization, surgical care, and quality, as well as to minimize OR operational costs.

Motivated by these important issues and our collaboration with a large health system in New York, we propose new optimization formulations of a scheduling problem in a surgical suite involving multiple parallel ORs, anesthesiologists, and elective surgeries. Specifically, given sets of ORs, regular anesthesiologists, on-call anesthesiologists, and elective surgeries (each of which requires an OR and an anesthesiologist to be performed), our formulations aim to solve the following decision-making problems simultaneously: (a) an allocation problem that determines which ORs to open and which on-call anesthesiologists to call in, (b) an assignment problem that assigns an OR and an anesthesiologist to each surgery, and (c) a sequencing and scheduling problem that determines surgery order and scheduled start time. We call this combination an operating room and anesthesiologist scheduling problem (ORASP). The objective is to minimize the sum of fixed costs for opening ORs and calling in on-call anesthesiologists along with a weighted average of costs associated with the idling and overtime of anesthesiologists and ORs, as well as the surgery waiting time.

The ORASP is a challenging problem in practice as it requires a significant amount of time for OR managers to make these decisions. Mathematical formulations of the problem are also challenging to solve for various reasons. First, it is a complex multi-resource scheduling problem with critical limits in terms of available ORs and anesthesiologists (Liu et al. 2018, Rath et al. 2017). Some types of surgeries require specialized anesthesiologists, whereas each anesthesiologist might have a different combination of specializations. This heterogeneity in the set of anesthesiologists increases the complexity of the assignment problem of anesthesiologists to surgeries. Second, different surgery types have different durations, and even surgery durations of the same type can vary significantly. To illustrate this variability, we provide Figure 1, which presents a box plot corresponding to a dataset of surgery durations (in minutes) categorized by surgical specialty. This data has been provided by our collaborating health system based on half-of-a-year’s worth of data. The figure illustrates clearly that there is significant variability in durations within and across surgery types. Ignoring such a variability in the ORASP may lead to substantial overtime, idling, and/or surgery delays, amongst other schedule deficiencies. Third, the ORASP is subject to a great deal of symmetry in the solution space, which can lead to computational inefficiencies (see Section 7).
By building high-quality schedules through solving the ORASP while accounting for the variability in surgery durations, there are substantial opportunities to improve resource utilization (equivalently, reduce overtime and idling time), improve patient and provider satisfaction, reduce delays and costs, and even achieve better surgical care. In this paper, we propose methodologies for accomplishing these goals by considering two methodologies for handling surgery duration uncertainty: stochastic programming (SP) and distributionally robust optimization (DRO).

SP has been a popular approach for optimization under uncertainty over the past decades (Zhu et al. 2019, Rahimian and Mehrotra 2022). In the SP approach, one essentially needs to assume that decision-makers know the distributions of the durations of each surgery type, or they possess a sufficient amount of high-quality data to estimate these distributions. Accordingly, one can formulate a two-stage SP model. The first-stage problem corresponds to determining the allocation, assignment, sequencing, and scheduling decisions while the second-stage problem corresponds to evaluating the performance metrics (i.e., overtime, idle time, and waiting time).

In practice, however, one might not have access to a sufficient amount of high-quality data to estimate surgery duration distributions accurately. This is especially true when data is limited during the planning stages when the OR schedule is constructed (Shehadeh 2022, Wang et al. 2019). As pointed out by Kuhn et al. (2019), even if one employs sophisticated statistical techniques to estimate the probability distribution of uncertain problem parameters using historical data, the estimated distribution may significantly differ from the true distribution. Moreover, future surgery durations do not necessarily follow the same distribution as in the past. Thus, optimal solutions to an SP model that is formulated using an estimated distribution may inherit bias. As such, implementing the (potentially biased) optimal decisions from the SP model may yield disappointing performance in practice, i.e., under unseen data from the true distribution (Smith
and Winkler 2006); in the context of the ORASP, this may correspond to significant overtime, delays, and under-utilization, amongst other negative consequences. While improving estimates of surgery durations may be possible, as pointed out by Kayis et al. (2012) and Shehadeh and Padman (2021), the inherent variability in such estimates remains high, necessitating caution in their use when optimizing OR schedules.

One approach to address the above challenges is DRO. In such an approach, one constructs ambiguity sets consisting of all distributions that possess certain partial information (e.g., first- and second-order moments) about the surgery durations. Using these ambiguity sets, one can formulate a DRO problem to minimize the worst-case expectation of the second-stage cost over all distributions residing within the ambiguity set, which effectively means that the probability distribution of the duration of each surgery type is a decision variable (Rahimian and Mehrotra 2022). DRO has received substantial attention recently in healthcare applications (Liu et al. 2019, Shehadeh et al. 2020, Wang et al. 2019) and other fields (Huang et al. 2020, Kang et al. 2019, Pflug and Pohl 2018, Shang and You 2018) due to its ability to hedge against unfavorable scenarios under incomplete knowledge of the underlying distributions.

1.1. Contributions

In this paper, we propose the first risk-neutral and risk-averse SP and DRO models for the ORASP, as well as methodologies for solving these models. We summarize our main contributions as follows.

1. Uncertainty modeling and optimization models.
   
   (a) We propose the first SP and DRO models for the ORASP. These models consider many of the costs relevant to our collaborating health system: fixed costs related to opening ORs and calling in on-call anesthesiologists, as well as the (random) operational costs associated with OR and anesthesiologist overtime, idle time, and surgery waiting time. Depending on the risk preference of a decision-maker, these models determine optimal ORASP decisions that minimize the fixed costs plus a risk measure, either expectation or conditional value-at-risk (CVaR), of the operational costs.

   (b) In the proposed SP model, we minimize the fixed costs plus a risk measure of the operational costs assuming known distributions of the surgery durations. This SP model generalizes recent SP models proposed for multiple OR scheduling problems by incorporating a larger set of important objectives, integrating allocation, assignment, sequencing, and scheduling problems, and modeling a decision maker’s risk preference. Moreover, it generalizes that of Rath et al. (2017) (a recent SP model for a closely related problem) by incorporating surgery waiting and OR and anesthesiologist idle time decision variables, constraints, and objective terms in the second stage, as well as by considering the decision-maker risk preferences.
We show that this generalization can offer more realistic schedules as compared with these models.

(c) The proposed DRO model provides an alternative formulation for cases when surgery duration distributions are ambiguous. The model seeks ORASP decisions that minimize the fixed costs plus a worst-case risk measure (either expectation or CVaR) of the operational costs over all surgery duration distributions defined by mean-support ambiguity sets. Note that mean and support are two intuitive statistics that capture distribution centrality and dispersion, respectively. Thus, practitioners could easily adjust the DRO input parameters based on their experience.

2. Solution Methodologies.

(a) We derive equivalent solvable reformulations of the proposed mini-max nonlinear expectation and CVaR DRO models, and propose a computationally efficient column-and-constraint generation (C&CG) method to solve the reformulations. We also derive valid and efficient lower bound inequalities that efficiently strengthen the master problem in C&CG, thus improving convergence.

(b) We obtain near-optimal solutions of our SP model using sample average approximation. We also derive valid lower bounding inequalities to improve the solvability of the SP model.

(c) We derive new symmetry-breaking constraints, which break symmetry in the solution space of the ORASP’s first-stage decisions and thus improve the solvability of the proposed SP and DRO models. These constraints are valid for any deterministic or stochastic formulation that employ the first-stage decisions and constraints of the ORASP.

3. Computational and Managerial Insights. Using real-world, publicly available surgery data and a case study at our collaborating health system, we conduct extensive computational experiments comparing the proposed methodologies empirically and theoretically. Our results show the significance of integrating the allocation, assignment, sequencing, and scheduling problems and the negative consequences associated with (i) adopting existing non-integrated approaches (see Section 8.6) and (ii) ignoring uncertainty and ambiguity of surgery duration (see Section 8.5). In addition, our results demonstrate the computational efficiency of the proposed methodologies (see Sections 8.2 and 8.3) and the potential for impact in practice.

1.2. Structure of the Paper

The remainder of the paper is organized as follows. In Section 2, we review relevant literature. Section 3 details our problem setting. In Sections 4 and 5, we present and analyze our proposed SP and DRO models for the ORASP, respectively. In Section 6, we present our solution strategies of our SP and DRO models, followed by a presentation of our symmetry-breaking constraints in Section 7. Finally, we present our numerical experiments and corresponding insights in Section 8.
2. Literature Review

For decades, much work has been done on formulating and solving OR and other healthcare planning and scheduling problems. For comprehensive surveys, we refer to Ahmadi-Javid et al. (2017), Cardoen et al. (2010), Gupta and Denton (2008), Guerriero and Guido (2011), Samudra et al. (2016), Zhu et al. (2019). In this section, we review recent literature most relevant to our work, namely, studies that propose and analyze stochastic optimization approaches to solve OR planning and scheduling problems.

SP is a useful tool to model uncertainty in surgery duration when distributions are known, and it has been widely applied in the OR planning and scheduling literature (Birge and Louveaux 2011, Zhu et al. 2019). Denton et al. (2007) proposed the first SP for a single-OR surgery sequencing and scheduling (SAS) problem and heuristic methods to solve it. Recently, Shehadeh et al. (2019) proposed a new SP model for the SAS problem that can be solved efficiently. Their results indicate that remarkable computational improvement can be achieved with their model when compared with those proposed by Mancilla and Storer (2012) and Berg et al. (2014). Khaniyev et al. (2020) discuss the challenges of obtaining exact solutions to the SAS problem under general duration distributions. They proposed an SP model that finds the optimal scheduled times for a given sequence of surgeries that minimize the weighted sum of expected patient waiting times and OR idle time and overtime. They derived an exact alternative reformulation of the objective function that can be evaluated numerically and proposed several scheduling heuristics.

Beyond the single OR setting, Denton et al. (2010) introduced an SP model that decides the OR-opening and surgery-to-OR assignments. An adapted L-shaped algorithm was proposed to solve the model. Wang et al. (2014) proposed an SP model that extends that of Denton et al. (2010) by considering emergency demand. They presented several column-generation-based heuristic methods and compared their computational performances. There have also been several studies on parallel and multi-resource OR scheduling problems (e.g., assigning multiple resources, such as surgical staff, to each surgery). Batun et al. (2011) proposed an SP model for an OR and surgeon assignment problem with random incision times. Their numerical results show that OR pooling is beneficial in reducing the operational costs. Assuming that surgery durations are normally distributed, Guo et al. (2014) proposed an SP model for a nurse assignment problem. Unlike in Batun et al. (2011), the surgery-to-OR assignment is assumed to be predetermined. Latorre-Núñez et al. (2016) generalized the work of Guo et al. (2014) by considering an OR scheduling problem with surgeon and other necessary resources (e.g., nurse and anesthesiologist). To overcome the computational difficulties, they developed a metaheuristic method based on a genetic algorithm. Vali-Siar et al. (2018) investigated an OR scheduling problem that considers the needs for nurses and anesthesiologists. They developed a genetic algorithm to solve their model.
While most existing studies on multiple-OR scheduling problems focus on OR-opening and surgery-to-OR assignment decisions, recent works also consider sequencing and scheduling decisions. In fact, various empirical and optimization studies have demonstrated the benefits of integrated approaches that incorporate sequencing decisions, including improving OR performance and reducing costs compared to fixed-sequence approaches (Cardoen et al. 2010, Cayirli et al. 2006, Denton et al. 2007). Freeman et al. (2016) proposed the first SP model incorporating these decisions. To deal with the computational challenges associated with solving their model, they proposed a two-step solution approach that reduces the set of surgeries and restricts the maximum number of surgeries in each OR by solving a knapsack problem. Tsai et al. (2021) proposed an SP model with chance constraints on overtime and waiting time and developed two approximation algorithms to solve their model.

SP provides an excellent basis for modeling and solving the ORASP if the distributions of surgery durations are known or one has a sufficient amount of high-quality data to estimate them. However, high-quality data is often unavailable in most real-world settings, such as for the ORASP. Accordingly, the distributions are often hard to characterize and subject to ambiguity. If one solves an SP model with a particular set of training data (i.e., the empirical distribution), the resulting schedule may have disappointing performance (e.g., excessive overtime and waiting time) in practice. Various studies have shown that decision-makers tend to be averse to ambiguity in distribution (Eliaz and Ortoleva 2016, Halevy 2007). In the context of the ORASP, some OR managers may err on the side of caution and prefer robust scheduling decisions that could safeguard the operational performance in adverse scenarios and mitigate the direct and indirect costs of operations (e.g., overtime, surgery delays, and quality of care).

Robust optimization (RO) is an alternative way to model uncertainty when the distributional information is limited. In this approach, one assumes that the random parameters lie in some uncertainty set consisting of possible scenarios and minimizes the worst-case costs over realizations in the uncertainty set. This could give a more robust solution, potentially reducing surgery waiting time and resource overtime. Examples of RO approaches for OR and surgery scheduling include Bansal et al. (2021b), Denton et al. (2010), Addis et al. (2014), Marques and Captivo (2017), Moosavi and Ebrahimnejad (2020) and references therein. Recently, Breuer et al. (2020) proposed an RO model for a combined OR planning and personnel scheduling problem that decides the number of elective surgeries and assigns staff (e.g., nurse, anesthetist, etc.) to surgeries. Unlike our ORASP, their model does not include decisions related to surgery sequences and start times.

Notably, Rath et al. (2017) proposed the first and, as far as we are aware, so far the only RO model for an integrated OR and anesthesiologist scheduling problem that is similar to our ORASP. They employed the uncertainty set of Bertsimas and Sim (2004) that characterizes surgery duration
lower and upper bounds with a tolerance on the maximum number of perturbations with respect to the nominal surgery duration. They solved their RO model using a decomposition algorithm and discussed the computational challenges of solving large instances. The model by Rath et al. (2017) only considers OR and anesthesiologists fixed and overtime costs in the objective, ignoring surgery waiting times and OR and anesthesiologists idle times. This is notable since, as we later show, by ignoring surgery waiting times, their model could lead to a schedule with multiple surgeries assigned to the same OR scheduled to start at the same time. Moreover, ignoring the idle times can lead to poor utilization of the ORs and anesthesiologists. In this paper, we incorporate both waiting times and idle times, which yield realistic schedules, reduce surgery delay, and improve utilization compared with Rath et al. (2017)’s schedules.

Focusing on hedging against worst-case scenarios, RO often yields overly conservative decisions (Roos and den Hertog 2020). One alternative is DRO, an approach that dates back to Scarf (1958) and has been of growing interest in recent years (Delage and Ye 2010, Rahimian and Mehrotra 2022). Specifically, in DRO, one assumes that the distribution of random parameters resides in some ambiguity set, i.e., a family of distributions (Delage and Ye 2010, Goh and Sim 2010, Rahimian and Mehrotra 2022). Accordingly, one minimizes the worst-case expected behavior over distributions in the ambiguity set. This reduces conservatism as compared to the RO approach while relaxing the stringent assumption in the SP approach that distributions are known with certainty. Despite these attractive features, the use of DRO models in the OR scheduling literature has been relatively sparse. Wang et al. (2019) extended the study by Denton et al. (2010) using the DRO approach, where the ambiguity set captures the support, mean, and mean absolute deviation of surgery durations. While the model can be reformulated as a mixed-integer linear program (MILP), to efficiently solve large-scale instances, they proposed a heuristic method to solve their model. Shehadeh (2022) studied an OR scheduling problem with emergency surgery capacity under a Wasserstein distance-based ambiguity set of surgery duration. An equivalent MILP reformulation was derived and the results indicate the benefits of the DRO approach. We emphasize that the models by Wang et al. (2019) and Shehadeh (2022) do not consider the need to assign both an OR and an anesthesiologist to each surgery, and do not consider the surgery sequencing and scheduling decisions that are part of our ORASP. Deng et al. (2019) proposed a DRO model with chance constraints that integrates surgery-to-OR assignment, sequencing, and scheduling decisions in multiple ORs. Dean et al. (2022) proposed a DRO model for the single-OR scheduling problem in Denton and Gupta (2003) that decides the surgery schedule times for a fixed surgery sequence. The ambiguity set captures quantiles of surgery durations predicted from quantile regression forests. For other recent DRO approaches in healthcare scheduling, see, e.g., Bansal et al. (2021a), Keyvanshokooh et al. (2022), and the references therein.
Finally, several studies have proposed approaches for physician and medical professional scheduling problems, but did not integrate OR and anesthesiologist scheduling decisions. We lastly discuss recent studies on anesthesiologist scheduling and refer to Abdalkareem et al. (2021) and Erhard et al. (2018) for recent surveys on the state of the art in general healthcare and physician scheduling problems. From an operational perspective, recent advances focus on developing implementable decision-support tools that automate the anesthesiologist scheduling process to replace traditional manual scheduling (Hoefnagel et al. 2020, Joseph et al. 2020). However, such tools and the underlying models do not consider the ORASP decisions. Other works include empirical studies investigating different anesthesiologist scheduling paradigms (e.g., Tsai et al. 2017, 2020). These studies also do not incorporate the ORASP decisions. Finally, on the optimization end, Rath and Rajaram (2022) proposed an anesthesiologist scheduling model that decides the number of anesthesiologists that are on regular duty and on call by minimizing the explicit costs (e.g., hiring cost) and implicit costs (e.g., idle cost). These studies do not consider optimizing the ORASP decisions.

3. Problem Setting

We start by introducing our ORASP setting. For a given day, we suppose that there is a set $I$ of elective surgeries to schedule, a set $R$ of available operating rooms (ORs), and a set $A$ of anesthesiologists. Each OR $r \in R$ is dedicated to only one type of surgical specialty (e.g., cardiothoracic, neurosurgery, etc.). There can be multiple ORs for the same specialty. Each OR has a pre-allocated length of time $T_{\text{end}}$ with service hours $[0, T_{\text{end}}]$. Many health systems implement a dedicated-OR policy, including our collaborating health system, to better manage elective surgeries. Hence, this policy has been widely adopted in the literature (see, e.g., Aringhieri et al. 2015, Bovim et al. 2020, Fügener et al. 2014, Makboul et al. 2022, Marques and Captivo 2015, Neyshabouri and Berg 2017, Shehadeh 2022), and is the one considered here. Our proposed models can be also used to solve ORASP instances where the OR can accommodate multiple surgery types.

In practice, there are two types of anesthesiologists: regular and on-call (Becker et al. 2019, Rath and Rajaram 2022, Rath et al. 2017). Each of the former type of anesthesiologist is scheduled to work on the given day, whereas each of the latter type is effectively on standby, ready to be called to work, if necessary. Assigning an on-call anesthesiologist to a surgery produces a high cost in some hospitals. We use the parameter setting $h_{\text{reg}}^a = 1$ to indicate that anesthesiologist $a \in A$ is on regular duty ($h_{\text{reg}}^a = 0$ otherwise) and the setting $h_{\text{call}}^a = 1$ to indicate that this anesthesiologist is on call ($h_{\text{call}}^a = 0$ otherwise). Each regular-duty anesthesiologist has a preassigned work shift $[t_{\text{start}}^a, t_{\text{end}}^a]$, where overtime occurs if/when they work beyond the scheduled end of their shift. In practice (e.g., at our collaborating hospital), some anesthesiologists are dedicated to cover a specific specialty, whereas some can cover a wide range of specialties. We refer to A.1 for an example.
Each surgery $i \in I$ has a type (e.g., cardiothoracic, breast, etc.), to which any OR dedicated to that surgery type can be assigned. Similarly, the assignment of an anesthesiologist to a surgery must respect the specialty required for the surgery. We assume that the surgery-surgeon combination is already known to mimic the current practice in many hospitals. (This is also a common assumption in the literature; see, e.g., Doulabi et al. 2014, Marques et al. 2014, Rath et al. 2017). Thus, one can think of each $i \in I$ as a surgery-surgeon unit. However, this assumption does not prevent surgeons from working in any of the ORs dedicated to their specialty. Surgery durations are random and depend on the surgery type. We use $d_i$ to denote the duration of surgery $i$ and let $d := [d_1, \ldots, d_I]^{\top}$ be the vector of all of the surgery durations. We assume that a lower bound $\underline{d}_i$ and upper bound $\overline{d}_i$ of surgery duration $d_i$ are known, which is a realistic assumption recommended by our collaborators and commonly used in healthcare scheduling (Denton et al. 2010, Shehadeh and Padman 2021, Wang et al. 2019). Mathematically, the random surgery duration $D$ is a measurable function $D : \Omega \to S$ with measurable space $(\Omega, \mathcal{F})$, where $S \subseteq \mathbb{R}^I$ is the bounded support defined as $S = \{d \in \mathbb{R}^I \mid \underline{d}_i \leq d_i \leq \overline{d}_i \text{ for all } i \in I\}$. We use $d$ to denote a realization of $D$.

Given $I$, $R$, and $A$ for each day, our ORASP models solve the following decision problems simultaneously: (a) an allocation problem in which we decide which OR to open, (b) an assignment problem assigning each surgery to an OR and anesthesiologist, and (c) a sequencing and scheduling problem that determines surgery order and scheduled start time. The objective is to minimize the sum of ORs and anesthesiologists fixed costs and a weighted average of the idling and overtime of anesthesiologists and ORs, and the surgery waiting time. For notational convenience, we define the following sets to be used in our formulations. The sets $\mathcal{F}^A$ and $\mathcal{F}^R$ consist of all feasible surgery-OR and surgery-anesthesiologist assignments. The sets $A_i$ and $R_i$ are, respectively, the sets of anesthesiologists and ORs to which a surgery can be assigned for $i \in I$. The sets $I_a$ and $I_r$ are surgeries that could be performed by anesthesiologist $a \in A$ and in OR $r \in R$, respectively. Mathematically, we let $\kappa^A_{i,a} = 1$ indicate that anesthesiologist $a$ can cover surgery $i$, and $\kappa^R_{i,r} = 1$ indicate that surgery $i$ can be scheduled in OR $r$. Then, we define $\mathcal{F}^A = \{(i,a) \in I \times A \mid \kappa^A_{i,a} = 1\}$, $\mathcal{F}^R = \{(i,r) \in I \times R \mid \kappa^R_{i,r} = 1\}$, $A_i = \{a \in A \mid (i,a) \in \mathcal{F}^A\}$, $R_i = \{r \in R \mid (i,r) \in \mathcal{F}^R\}$, $I_a = \{i \in I \mid (i,a) \in \mathcal{F}^A\}$, and $I_r = \{i \in I \mid (i,r) \in \mathcal{F}^R\}$. A complete list of our notation can be found in A.2.

\section{Stochastic Programming Models}

In this section, we present our proposed two-stage SP formulation of the ORASP, which assumes that the probability distributions of surgery durations are known. First, let us introduce the variables, parameters, and functions defining our first-stage SP model. For each $r \in R$, we define a binary decision variable $v_r$ that equals 1 if OR $r$ is opened, and is 0 otherwise. Similarly, for each $a \in A$, we define a binary variable $y_a$ that equals 1 if on-call anesthesiologist $a$ is called in, and is 0
We define binary decision variables $x_{i,a}$ and $z_{i,r}$ taking value 1 if surgery $i$ is assigned to anesthesiologist $a$ and OR $r$ respectively, and are 0 otherwise. To determine the surgery sequence, we proceed as in Rath et al. (2017) and define binary variables $u_{i,i'}$, $\alpha_{i,i',a}$, and $\beta_{i,i',r}$ to represent precedence relationships. Specifically, we define $u_{i,i'}$ that takes value 1 if surgery $i$ precedes surgery $i'$, and is 0 otherwise. Variables $\alpha_{i,i',a}$ and $\beta_{i,i',r}$ take value 1 if surgery $i$ precedes surgery $i'$ for anesthesiologist $a$ and in OR $r$ respectively, and are 0 otherwise. For each $i \in I$, we let nonnegative continuous variable $s_i$ represent the scheduled start time of surgery $i$.

For the objective function, we define $f_r$ as the nonnegative fixed cost of opening OR $r$ and $f_a$ as the nonnegative fixed cost of calling in on-call anesthesiologist $a$. The remaining term in the objective function is a risk measure of the second-stage function (see more below), which, for a given realization $d$ of surgery durations represented by the random variable $D$, is a weighted average of idle time, overtime, and waiting time. Our first-stage SP model can now be stated as follows:

\[
\text{minimize} \quad \sum_{r \in R} f_r v_r + \sum_{a \in A} f_a y_a + \varrho_D(Q(x, y, z, v, u, s, D)) \quad (1a)
\]

\[
\text{subject to} \quad \sum_{a \in A_i} x_{i,a} = 1, \quad \sum_{r \in R_i} z_{i,r} = 1, \quad \forall i \in I, \quad (1b)
\]

\[
z_{i,r} \leq v_r, \quad \forall (i,r) \in F^R, \quad (1c)
\]

\[
x_{i,a} \leq h_{a}^{\text{reg}} + y_a, \quad \forall (i,a) \in F^A, \quad (1d)
\]

\[
y_a \leq h_{a}^{\text{call}}, \quad \forall a \in A, \quad (1e)
\]

\[
s_i \geq t_{a}^{\text{start}} - M(1 - x_{i,a}), \quad s_i \leq T_{\text{end}}, \quad \forall (i,a) \in F^A, \quad (1f)
\]

\[
\alpha_{i,i',a} \leq u_{i,i'}, \quad \forall \{ (i,a), (i',a) \} \subseteq F^A, \quad (1g)
\]

\[
\beta_{i,i',r} \leq u_{i,i'}, \quad \forall \{ (i,r), (i',r) \} \subseteq F^R, \quad (1h)
\]

\[
u_{i,i'} + u_{i',i} \leq 1, \quad \forall \{ i, i' \} \subseteq I, \quad (1i)
\]

\[
u_{i,i'} + u_{i',i''} \leq u_{i,i'} + u_{i',i''} - 1, \quad \forall \{ i, i', i'' \} \subseteq I, \quad (1j)
\]

\[
\alpha_{i,i',a} + \alpha_{i',i,a} \leq x_{i,a}, \quad \alpha_{i,i',a} + \alpha_{i',i,a} \leq x_{i',a}, \quad \forall \{ (i,a), (i',a) \} \subseteq F^A, \quad (1k)
\]

\[
\alpha_{i,i',a} + \alpha_{i',i,a} \geq x_{i,a} + x_{i',a} - 1, \quad \forall \{ (i,a), (i',a) \} \subseteq F^A, \quad (1l)
\]

\[
\beta_{i,i',r} + \beta_{i',i,r} \leq z_{i,r}, \quad \beta_{i,i',r} + \beta_{i',i,r} \leq z_{i',r}, \quad \forall \{ (i,r), (i',r) \} \subseteq F^R, \quad (1m)
\]

\[
\beta_{i,i',r} + \beta_{i',i,r} \geq z_{i,r} + z_{i',r} - 1, \quad \forall \{ (i,r), (i',r) \} \subseteq F^R, \quad (1n)
\]

\[
\alpha_{i,i',a} \geq x_{i,a} + x_{i',a} + \beta_{i,i',r} - 2, \quad \forall \{ (i,a), (i',a) \} \subseteq F^A, \quad (1o)
\]

\[
\beta_{i,i',r} \geq z_{i,r} + z_{i',r} + \alpha_{i,i',r} - 2, \quad \forall \{ (i,a), (i',a) \} \subseteq F^A, \quad (1p)
\]

\[
x_{i,a}, y_a, z_{i,r}, u_{i,i'}, v_r, \alpha_{i,i',a}, \beta_{i,i',r} \in \{ 0, 1 \}, \quad s_i \geq 0, \quad \forall i \in I, a \in A, r \in R. \quad (1q)
\]
The objective (1a) aims to find first-stage decisions \( (x, y, z, v, u, s, \alpha, \beta) \) that minimize the sum of the fixed cost of opening ORs (first-term), the fixed cost of employing on-call anesthesiologists (second term), and the risk measure \( \varrho_P \) of the random second stage function \( Q \) (third term). A risk-neutral decision-maker may opt to set \( \varrho_P(\cdot) = E_P(\cdot) \), i.e., the expected total operational costs, which is standard in the OR scheduling literature and intuitive for OR managers. In contrast, a risk-averse decision-maker might set \( \varrho_P(\cdot) = \text{CVaR}_\gamma(\cdot) \), i.e., the CVaR of the total operational costs. For simplicity, we let \( \text{SP-E} \) and \( \text{SP-CVaR} \) denote the risk-neutral and risk-averse SP models.

Constraints (1b) ensure that every surgery is assigned to exactly one anesthesiologist and one OR. Constraints (1c) ensure that surgeries are assigned to open ORs. Constraints (1d) indicate that an anesthesiologist can be assigned to surgeries if they are on regular duty (i.e., \( h^\text{reg}_a = 1 \)) or on call (i.e., \( y_a = 1 \)). Constraints (1e) ensure that \( y_a \) may equal 1 if anesthesiologist \( a \) is listed as an on-call anesthesiologist (i.e., \( h^\text{call}_a = 1 \)). Constraints (1f) enforce that the scheduled start time of the surgery assigned to anesthesiologist \( a \) is greater than or equal to his/her scheduled start time \( t^\text{start}_a \) and it is scheduled within the planned service hours \([0, T^\text{end}]\). Constraints (1g)–(1j) define precedence variables \( u_{i,i'} \). Constraints (1g)–(1h) ensure that if surgery \( i' \) follows surgery \( i \) in either an anesthesiologist or an OR schedule, then \( u_{i,i'} \) equals 1. Constraints (1i)–(1j) maintain the precedence and transitivity relationships that prevent non-implementable schedules (see an example in A.3.1). If surgeries \( i \) and \( i' \) are assigned to anesthesiologist \( a \) (i.e., \( x_{i,a} = x_{i',a} \)), then constraints (1k) ensure that either \( i' \) follows \( i \) (i.e., \( \alpha_{i,i',a} = 1 \)) or vice versa (i.e., \( \alpha_{i',i,a} = 1 \)), but not both. Constraints (1l) ensure that the sequencing constraints on \( \alpha \) only apply to surgeries assigned to the same anesthesiologist. Moreover, they enforce that either \( \alpha_{i,i',a} \) or \( \alpha_{i',i,a} \) takes value one if both surgeries \( i \) and \( i' \) are assigned to anesthesiologist \( a \). Constraints (1m)–(1n) enforce similar precedence and sequencing rules on surgeries assigned to the same OR. Constraints (1o) enforce \( \alpha_{i,i',a} = 1 \) if \( i \) and \( i' \) are performed by the same anesthesiologist while surgery \( i' \) follows surgery \( i \) in the same OR, and similarly for constraints (1p) with the role of anesthesiologist and OR swapped. Finally, constraints (1q) specify feasible ranges of the first-stage decision variables.

Remark 1. Our proposed model allows practitioners to accommodate special scheduling requests. For example, suppose that anesthesiologist \( a \) must perform a given set of surgeries \( I' \). In that case, one can set \( x_{i,a} = 1 \) for all \( i \in I' \). If surgery \( i \) must be performed in a particular OR, one can set \( z_{i,r} = 1 \). Similarly, one can set \( v_r = 1 \) if a specific OR \( r \in R \) must be open and \( y_a = 1 \) if anesthesiologist \( a \) must be called in. These are special cases and simplifications of our model.

Next, we introduce our second-stage (recourse) problem. For a given set of first-stage decisions \( (x, y, z, v, u, s) \) corresponding to a feasible solution of (1) and a realization of surgery durations \( d \), the following second-stage linear program (LP) computes costs related to anesthesiologist and OR
idle time and overtime (i.e., the first and second terms in (2a) respectively) and waiting time of surgeries (i.e., third term in (2a)).

In this problem, variable $q_i$ represents the actual start time of surgery $i$ and $w_i$ represents the waiting time of surgery $i$. We define the nonnegative continuous variables $o_r$ ($o_a$) and $g_r$ ($g_a$) respectively to represent the overtime and idle time of OR $r$ (anesthesiologist $a$). We define $c^a_o$ ($c^a_a$) as the per-unit overtime penalty for OR $r$ (anesthesiologist $a$), $c^a_g$ ($c^a_g$) as the per-unit idling penalty for OR $r$ (anesthesiologists $a$), and $c^w_i$ as the per-unit surgery waiting penalty. Finally, $M_{seq}$, $M_{anes}$ and $M_{room}$ are big-$M$ parameters (see A.3.2 for a discussion on these parameters). For a given realization of surgery duration $d$, our second-stage problem is as follows:

$$Q(x, y, z, v, u, s, d) := \min \sum_{a \in A} (c^a_g a + c^a_o a) + \sum_{r \in R} (c^a_g r + c^a_o r) + \sum_{i \in I} c^w_i w_i$$

subject to

$$q_i \geq q_i + d_i - M_{seq}(1 - u_{i,i'}) \forall \{i, i'\} \subseteq I, i \neq i'$$

$$q_i \geq s_i \forall i \in I$$

$$o_a \geq q_i + d_i - t^{\text{end}} - M_{anes}(1 - x_{i_a} + y_a) \forall (i, a) \in F^A$$

$$o_r \geq q_i + d_i - T^{\text{end}} - M_{room}(1 - z_{i,r}) \forall (i, r) \in F^R$$

$$w_i \geq q_i - s_i \forall i \in I$$

$$g_a \geq \left(t^{\text{end}}_a - t^{\text{start}}_a - \sum_{i \in I_a} d_i x_{i,a}\right) h^{\text{reg}} a + o_a \forall a \in A$$

$$g_r \geq T^{\text{end}} r - \sum_{i \in I_r} d_i z_{i,r} + o_r \forall r \in R$$

$$q_i, o_a, o_r, w_i, g_a, g_r \geq 0 \forall i \in I, a \in A, r \in R$$

Constraints (2b)–(2c) ensure that the actual start time of a surgery is not earlier than the scheduled start time and the completion time of the previous surgeries. Constraints (2d) and (2e) yield the overtimes of the anesthesiologists and ORs, respectively. Note that constraints (2d)–(2e) are relaxed if an on-call anesthesiologist is hired or an OR is not open (i.e., overtime is zero in these two cases). Constraints (2f) give the waiting time of each surgery $i \in I$ as the time from the scheduled start time of a surgery to its actual start time. Constraints (2g)–(2h) give the idle times of anesthesiologists and ORs, respectively. Note that the idling cost of on-call anesthesiologists and ORs that are not open are zero. It is easy to verify that formulation (2) is feasible for any feasible first-stage decisions. Thus, we have a relatively complete recourse.

Our model (1)–(2) generalizes recent SP models proposed for multiple-OR scheduling problems. For example, the models in Denton et al. (2010) and Wang et al. (2019) aim to decide the optimal number of ORs to open and surgery assignments to open ORs by minimizing the weighted sum of OR-opening and overtime-penalty costs. Our SP model generalizes these models by (a)
incorporating constraints, variables, and objectives related to regular and on-call anesthesiologist scheduling; (b) incorporating a larger set of important objectives; (c) integrating allocation, assignment, sequencing, and scheduling problems; and (d) modeling a decision-maker’s risk preference.

Furthermore, our SP model generalizes an existing model for a closely related OR-anesthesiologist scheduling problem presented by Rath et al. (2017). Key differences between our model (1)–(2) and Rath et al. (2017)’s model include the following. First, while we use similar sets of first-stage variables and constraints, we add constraints \((1f)\) to the first-stage model to restrict the scheduled surgery time to be within the planning horizon \([0, T^{\text{end}}]\), which is common in practice. Second, our second-stage model (2) generalizes that of Rath et al. (2017) by considering waiting and idling metrics and the related variables and constraints. In particular, incorporating surgery waiting time is essential to minimize delays and avoid scheduling many surgeries at the start of the day (i.e., time zero) or at the same time in the same OR. Rath et al. (2017)’s model can yield schedules with surgeries scheduled to start simultaneously in the same OR, which is not possible in practice (see Section 8.6). In addition, we incorporate OR and anesthesiologist idle time in the second-stage objective, which is essential to improve the utilization of these expensive resources (Cardoen et al. 2010). Moreover, different from Rath et al. (2017), we propose a CVaR model that allows for a decision-maker’s risk aversion. Finally, we derive a DRO counterpart of our SP model to address distributional ambiguity, which is the topic of the next section. In Section 8.6, we provide examples, results, and detailed discussions demonstrating the importance of incorporating these elements to produce realistic and implementable solutions with superior performance in practice.

5. Distributionally Robust Models

In this section, we present our proposed DRO formulation of the ORASP, which does not assume that the probability distributions of surgery durations are known. That said, we assume that the mean \(m := (m_1, \ldots, m_I)^T\) and support \(S = \{d \in \mathbb{R}^I \mid \underline{d}_i \leq d_i \leq \overline{d}_i, i \in I\}\) of surgery durations are known. These parameters can be estimated based on clinical expert knowledge. Moreover, when data on patient characteristics and medical history are available, one could build statistical and machine-learning models (e.g., regression models) to estimate the mean and support.

We first introduce additional sets and notation defining our ambiguity set. Let \(\mathcal{D} = \mathcal{D}(S)\) be the set of all probability measures on \((S, \mathcal{B})\) where \(\mathcal{B}\) is the Borel \(\sigma\)-field on \(S\). Elements in \(\mathcal{D}\) can be viewed as probability measures induced by the random vector \(D\). Using this notation, we construct the following mean-support ambiguity set:

\[
P(m, S) = \left\{ \mathbb{P} \in \mathcal{D}(S) \bigg| \mathbb{E}_\mathbb{P}(D) = m \right\}.
\]
Using the ambiguity set (3), we formulate our DRO model of the ORASP as

\[
\begin{align*}
\text{minimize} & \quad \sum_{r \in R} f_r v_r + \sum_{a \in A} f_a y_a + \left\{ \sup_{\varphi \in \mathcal{P}(m,S)} \varphi \left( Q(x,y,z,v,u,s,D) \right) \right\} \\
\text{subject to} & \quad (1b)-(1q). 
\end{align*}
\]

(4a)

(4b)

Formulation (4) finds first-stage decisions \((x,y,z,v,u,s)\) that minimize the first-stage cost and the worst-case of a risk measure of the second-stage cost over distributions residing in \(\mathcal{P}(m,S)\). In what follows, we refer to model (4) with \(\varphi(\cdot) = \mathbb{E}_{\varphi}(\cdot)\) as the DRO-E model, and to model (4) with \(\varphi(\cdot) = \mathbb{P}\text{-CVaR}_\gamma(\cdot)\) as the DRO-CVaR model. Note that formulation (4) is a mini-max problem, which is not straightforward to solve in its presented form. Therefore, our goal is to derive an equivalent solvable formulation of (4). For brevity, we relegate detailed proofs to A.4.

5.1. DRO-E Model Reformulation

In this section, we derive an equivalent reformulation of the DRO-E model (i.e., model (4) with \(\varphi(\cdot) = \mathbb{E}_{\varphi}(\cdot)\)). First, in Proposition 1, we present an equivalent reformulation of the inner maximization problem in (4).

**Proposition 1.** For \((x,y,z,v,u,s)\) satisfying (1b)-(1q), the inner problem in (4), namely, to solve

\[
\sup_{\varphi \in \mathcal{P}(m,S)} \mathbb{E}_{\varphi}[Q(x,y,z,v,u,s,D)],
\]

is equivalent to

\[
\min_{\rho \in \mathbb{R}} \left\{ \sum_{i \in I} \rho_i m_i + \sup_{d \in S} \left( Q(x,y,z,v,u,s,d) - \sum_{i \in I} \rho_i d_i \right) \right\}. 
\]

(5)

Again, the problem in (5) involves an inner max-min problem that is not straightforward to solve in its presented form. However, in Proposition 2, we present an equivalent MILP formulation of the inner problem in (5) that is solvable.

**Proposition 2.** Let \(\Delta d_i = \tilde{d}_i - \bar{d}_i\) for all \(i \in I\). Then, for \((x,y,z,v,u,s)\) satisfying (1b)-(1q), there exist \(\varrho_{i,a}\) for all \((i,a) \in \mathcal{F}^A\), \(\bar{\varrho}_{i,r}\) for all \((i,r) \in \mathcal{F}^R\), and \(\overline{\lambda}_{i,i'}\) for all \(\{i,i'\} \subseteq I\) such that solving the inner problem in (5), namely, solving

\[
\sup_{d \in S} \left( Q(x,y,z,v,u,s,d) - \sum_{i \in I} \rho_i d_i \right),
\]

is equivalent to evaluating the following function, which can be done by solving the presented MILP:

\[
H(x,y,z,v,u,s,\rho) = 
\]

\[
\begin{align*}
\text{maximize} & \quad \sum_{a \in A} c^a (\ell_{a}^{\text{end}} - \ell_{a}^{\text{start}}) h_a^{\text{reg}} + \sum_{r \in R} c_r^{\text{T}^{\text{end}}} v_r \\
& \quad + \sum_{i \in I} \left[ \sum_{i' : i' \neq i} (\lambda_{i,i'} - \lambda_{i',i}) + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r} \right] s_i \\
& \quad - M_{\text{seq}} \sum_{i \in I} \sum_{i' : i' \neq i} \lambda_{i,i'} (1 - u_{i,i'}) - \sum_{a \in A_i} \mu_{i,a} \left[ \ell_{a}^{\text{end}} + M_{\text{anes}} (1 - x_{i,a} + y_a) \right] 
\end{align*}
\]
For any tional efficiency if they are set too large. Therefore, in Proposition 3, we derive tight upper bounds following equivalent reformulation of the DRO-E model.

Replacing the inner maximization problem in (5) by its equivalent MILP reformulation in (6), and combining with the outer minimizing problem in (4), we derive the following equivalent reformulation of the DRO-E model.

**Proposition 3.** For any \((x, y, z, v, u, s)\) satisfying (1b)–(1q), the following bounds are valid.

\[
0 \leq \mu_{i,a} \leq c_a^w + c_a^s, \quad \forall (i, a) \in \mathcal{F}^A; \quad 0 \leq \theta_{i,r} \leq c_r^w + c_r^s, \quad \forall (i, r) \in \mathcal{F}^R;
\]

\[
0 \leq \lambda_{i,i'} \leq \sum_{i \in I} c_i^w + \sum_{a \in A} (c_a^w + c_a^s) + \sum_{r \in R} (c_r^w + c_r^s), \quad \forall \{i, i'\} \subseteq I.
\]

Replacing the inner maximization problem in (5) by its equivalent MILP reformulation \(H(x, y, z, v, u, s, \rho)\) in (6), and combining with the outer minimizing problem in (4), we derive the following equivalent reformulation of the DRO-E model.

\[
\begin{align*}
\text{minimize} & \quad \sum_{r \in R} f_r v_r + \sum_{a \in A} f_a y_a + \sum_{i \in I} \rho_i m_i + \delta \\
\text{subject to} & \quad (1b) - (1q), \quad \delta \geq H(x, y, z, v, u, s, \rho).
\end{align*}
\]

**5.2. DRO-CVaR Model Reformulation**

In this section, we derive an equivalent reformulation of the DRO-CVaR model. In Proposition 4, we present an equivalent reformulation of the inner problem in (4) with \(\rho(\cdot) = \mathbb{P}\cdot \text{CVaR}_\gamma(\cdot)\).
Proposition 4. For \((x, y, z, v, u, s)\) satisfying (1b)–(1q), the inner problem in (4), namely, to solve \(\sup_{P \in \mathcal{P}(m, S)} \mathbb{P} \cdot \text{CVaR}_\gamma (Q(x, y, z, v, u, s, D))\), is equivalent to

\[
\begin{align*}
\text{minimize} & \quad \rho_0 + \frac{1}{1 - \gamma} \sum_{i \in I} \rho_i m_i + \sup_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\} \\
\text{subject to} & \quad \rho_0 + \sum_{i \in I} \left( d_i \psi_i - d_i \psi_i \right) \geq 0, \\
& \quad \psi_i - \overline{\psi}_i = \rho_i, \quad \overline{\psi}_i \geq 0, \quad \psi_i \geq 0, \quad \forall i \in I.
\end{align*}
\]

Note that the inner maximization problem in (9) is the same as the inner problem in (5). Therefore, we apply the same techniques in the proof of Proposition 2 to derive the following equivalent reformulation of the DRO-CVaR model:

\[
\begin{align*}
\text{minimize} & \quad \sum_{r \in R} f_r v_r + \sum_{a \in A} f_a y_a + \frac{1}{1 - \gamma} \rho_0 + \frac{1}{1 - \gamma} \sum_{i \in I} \rho_i m_i + \delta \\
\text{subject to} & \quad (1b) - (1q), \quad (9b) - (9c), \quad \delta \geq H(x, y, z, v, u, s, \rho).
\end{align*}
\]

6. Solution Methods

In this section, we first propose valid inequalities to improve the solvability of the SP-E and SP-CVaR models (Section 6.1). Then, we propose a column-and-constraint generation (C&CG) method to solve our DRO-E and DRO-CVaR models (Section 6.2). Also, we propose several valid inequalities to improve the convergence of our C&CG method (Section 6.3). Finally, we discuss the separability of the models (Section 6.4).

6.1. Valid Inequalities for the SP-E and SP-CVaR Models

Note that the idle times of anesthesiologists and operating rooms are nonnegative. Thus, we add the following valid inequalities to the SP-E and SP-CVaR models, which, as we later show, tightens its linear relaxation.

\[
\begin{align*}
\left( t_{a, \text{end}} - t_{a, \text{start}} - \sum_{i \in I_a} d_i x_{i,a} \right) t_{a, \text{reg}} + o_a \geq 0, \quad T_{r, \text{end}} v_r - \sum_{i \in I_r} d_i z_{i,r} + o_r \geq 0, \quad \forall a \in A, \ r \in R.
\end{align*}
\]

6.2. The C&CG Method for the DRO-E and DRO-CVaR Models

Note that the DRO-E model in (8) and DRO-CVaR model in (10) involve inner maximization problems, specifically defining the function value \(H(x, y, z, v, u, s, \rho)\) in constraints (8b) and (10b), respectively. Thus, formulations (8) and (10) cannot be solved directly using standard techniques. In this section, we develop a C&CG method to solve our DRO-E model, and note that a similar C&CG method can be employed to solve our DRO-CVaR model. The motivation of this algorithm
is as follows. Consider the inner maximization problem in (5). The \( i \)th element of the optimal solution component \( d^* \in \mathcal{S} \) only takes value \( d_i \) or \( \bar{d}_i \). As a result, the inner maximization problem in (5) over \( d \in \mathcal{S} \) only needs to consider the \( 2^{|I|} \) combinations, i.e. \( \times_{i \in I} \{d_i, \bar{d}_i\} \), to determine \( d^* \). Instead of solving an MILP with exponentially many constraints, we use a C&CG method to identify scenarios in an iterative manner to obtain an optimal solution.

Algorithm 1 presents our C&CG method. In each iteration, we first solve the master problem (12), which only employs the scenario-based constraints (2b)–(2i) corresponding to a set of scenarios indexed by \( \mathcal{K} \), to obtain a solution (i.e., surgery assignment, sequence, and schedule). By considering only a subset of surgery durations, the master problem is a relaxation of the original problem. Thus, it provides a lower bound on the optimal value of the DRO-E model. Then, given the optimal solution from the master problem, we identify a duration vector \( d^* \) by solving subproblem (6). Given that solutions of the master problem are feasible to the original problem, we obtain an upper bound by solving the subproblem using these solutions. Next, we introduce second-stage variables and constraints associated with the identified duration scenario to the master problem. We then solve the master problem again with the new information (in the enlarged set \( \mathcal{K} \)) from the subproblems. This process continues until the gap between the lower and upper bound obtained in each iteration satisfies a predetermined termination tolerance \( \varepsilon \geq 0 \). Given the relatively complete recourse property of our second-stage problem, feasibility cuts are not needed. Moreover, since there are only a finite number of scenarios (i.e., \( 2^{|I|} \)) in total, the algorithm terminates in finite number of iterations (see Tsang et al. 2023, Zeng and Zhao 2013).

6.3. Valid Inequalities for the DRO-E and DRO-CVaR Models

Inequalities (11) are also valid in the master problem (12) for each scenario \( d^k \) with \( k \in \mathcal{K} \). Here, we present three more sets of valid inequalities for the master problem. First, observe that the Dirac measure on \( m \) lies in \( \mathcal{P}(m, \mathcal{S}) \). Therefore, we have

\[
\sup_{p \in \mathcal{P}(m, \mathcal{S})} \rho_p \left( Q(x, y, z, v, u, s, D) \right) \geq Q(x, y, z, v, u, s, m) =: L,
\]

where the lower bound is a deterministic problem with a single scenario \( d = m \). This means we can impose the constraint \( \sum_{i \in I} m_i \rho_i + \delta \geq L \) and \([ (1 - \gamma)^{-1} - 1 ] \rho_0 + (1 - \gamma)^{-1} \sum_{i \in I} m_i \rho_i + \delta \geq L \), which respectively serves as a global lower bound on the objective of the DRO-E and DRO-CVaR models. Second, from (13), for any given first-stage decision, the recourse value with \( d = m \) provides a lower bound. Therefore, in the initialization step of the C&CG method, we could include the scenario \( d = m \) (see A.5.1 for details). Third, the dual variable \( \rho \) is unrestricted in both the DRO-E and DRO-CVaR models; see (8) and (10). We derive valid inequalities that provide a lower and upper bound on \( \rho \) in Proposition 5 (see A.5.2 for a proof).
Algorithm 1: A column-and-constraint-generation method for the DRO-E model

Initialization: Set $LB = 0$, $UB = \infty$, $\varepsilon > 0$, $K = \emptyset$, $j = 1$.

1. Master problem. Solve the master problem

$$\begin{align*}
\text{minimize} & \quad \sum_{a \in A} c_a v_a + c_q \sum_{r \in R} y_a + \sum_{i \in I} \rho_i m_i + \delta \\
\text{subject to} & \quad (1b) - (1q), \quad \{ (2b) - (2i), k \in K \},
\end{align*}$$

(12a) (12b) (12c)

Record the optimal solution $(x^*, y^*, z^*, v^*, w^*, \alpha^*, \beta^*, \rho^*, \delta^*)$ and value $Z^*$. Set $LB = Z^*$.

2. Subproblem. Solve (6) with fixed $(x^*, y^*, z^*, v^*, w^*, \alpha^*, \beta^*, \rho^*, \delta^*)$.

2.1 Record the optimal solution $b^*$ and value $Y^*$. Set $UB = \min \{ UB, (Z^* - \delta^*) + Y^* \}$.

2.2 If $(UB - LB)/UB < \varepsilon$ or $\delta^* \geq Y^*$, then terminate; else, go to step 3.

3. Column-and-constraint generation routine.

3.1 Using $b^*$ from step 2, compute $d^i = d_i + b_i (\bar{d}_i - d_i)$ for all $i \in I$.

3.2 Add variables $(\alpha^*_r, \beta^*_r, w^*_r, g^*_r, k^*_r)$ and the following constraints to the master problem:

$$\delta \geq \sum_{a \in A} \left( c^*_a g^*_a + c^*_a \alpha^*_a \right) + \sum_{r \in R} \left( c^*_r g^*_r + c^*_r \alpha^*_r \right) + \sum_{i \in I} c^*_i w^*_i - \sum_{i \in I} \rho_i d^i, \quad \{ (2b) - (2i) \} \text{ with } d = d^i \}.$$ 

Update $j \leftarrow j + 1$ and $K \leftarrow K \cup \{ j \}$. Go back to step 1.

Proposition 5. The following bounds are valid lower and upper bounds on $\rho_i$ for all $i \in [I]$:

$$- \sum_{a \in A_i} c^a g^a x^a_{i,a} - \sum_{r \in R_i} c^r z^r_{i,r} \leq \rho_i \leq \min \left\{ \sum_{i' \in I, i' \neq i} u_{i,i'}, 2 \right\} \bar{\lambda} + \sum_{a \in A_i} c^a x^a_{i,a} + \sum_{r \in R_i} c^r z^r_{i,r}
\quad \text{(14)}$$

Although including (14) reduces the search space for $\rho_i$ in our preliminary experiments, we found that this might worsen the computational performance in some cases. This may follow from the increased model complexity by (14) due to the presence of first-stage variables. Therefore, in our experiments, we include (13) along with the following variable-free version of (14):

$$- \max_{a \in A} c^a_a - \max_{r \in R} c^r_r \leq \rho_i \leq 2 \bar{\lambda} + \max_{a \in A} c^a_a + \max_{r \in R} c^r_r.
\quad \text{(15)}$$

In A.8.2, we provide results comparing solution times for solving the DRO-E model using either the variable-free version (15) or the variable-dependent version (14). We observe that solution times under the variable-free version are generally similar to or shorter than those under the variable-dependent version. In particular, solution times under the variable-free version are significantly shorter for large instances of the ORASP.

6.4. Separability of the Models

We note that the recourse function $Q$ can be decomposed into smaller problems based on the surgery type; i.e., each surgery type with specialized anesthesiologists and the class of surgery types

$\begin{align*}
\text{minimize} & \quad \sum_{a \in A} c_a v_a + \sum_{r \in R} y_a + \sum_{i \in I} \rho_i m_i + \delta \\
\text{subject to} & \quad (1b) - (1q), \quad \{ (2b) - (2i), k \in K \},
\end{align*}$

(12a) (12b) (12c)
with a common pool of anesthesiologists. Mathematically, let $\ell \in \{1, \ldots, L^{\text{spec}}\}$ be the surgery types with dedicated anesthesiologists. We can decompose the recourse function as

$$Q(x, y, z, v, u, s, d) = \sum_{\ell=1}^{L^{\text{spec}}} Q^\ell(x^\ell, y^\ell, z^\ell, u^\ell, s^\ell, d^\ell) + Q^g(x^g, y^g, z^g, v^g, u^g, s^g, d^g),$$

where the summation corresponds to each specialized surgery type $\ell \in \{1, \ldots, L^{\text{spec}}\}$ and the second term corresponds to the remaining surgery types. Thus, we can leverage this decomposable structure when solving the SP-E, DRO-E, and DRO-CVaR models; see A.6 for discussions on the separability of the DRO-E and DRO-CVaR models. In contrast, the SP-CVaR model does not admit such a decomposition due to the subadditivity of $\mathbb{P}$-CVaR$_\gamma(\cdot)$. However, our experimental results show that the difference in out-of-sample costs between the SP-CVaR model with and without decomposition is very small (less than 3% in most cases). This indicates that the SP-CVaR model with decomposition could produce near-optimal performance. Hence, we adopt the decomposition approach when solving large instances using the SP-CVaR model.

### 7. Symmetry-Breaking Constraints

Symmetry allows the existence of multiple equivalent solutions, which leads to a wasteful duplication of effort in the branch-and-bound tree. In practice, at our collaborating hospital, managers consider identical waiting costs for surgeries of the same type. Also, the idling and overtime costs of ORs and anesthesiologists of the same type are the same. Therefore, there is a great deal of symmetry in the ORASP. Although breaking symmetry is standard in scheduling problems to avoid exploring equivalent solutions, previous studies did not address the issue of symmetry in the ORASP. In this section, we present strategies to break the symmetry in the first-stage decisions.

To facilitate the discussion, let $I_\ell = \{i_{1,\ell}, \ldots, i_{|I_\ell|,\ell}\}$ and $R_\ell = \{r_{1,\ell}, \ldots, r_{|R_\ell|,\ell}\}$ be the sets of surgeries of type $\ell \in L$ and the sets of ORs that can perform surgeries of type $\ell \in L$, respectively. Here, we only present our new symmetry-breaking constraints specific to the ORASP, while in A.7.1 we also provide some existing symmetry-breaking constraints relevant to the ORASP.

#### 7.1. Surgery Assignment Order

Suppose we have three ORs and six surgeries of the same type. Since the surgery durations of the same type follow the same distribution, assignments 1 and 2 in Table 1 result in the same objective value. In other words, these solutions are equivalent as they both assign three surgeries to OR 1,
one surgery to OR 2, and two surgeries to OR 3. We can prevent such equivalent assignments by enforcing surgeries of smaller indices to be assigned to ORs with smaller indices using constraints

\[ z_{ij,\ell,r_{k,\ell}} \leq z_{ij-1,\ell,r_{k-1,\ell}} + z_{ij-1,\ell,r_{k,\ell}}, \quad \forall \ell \in L, \ j \in [2, |I_\ell|], \ k \in [|R_\ell|], \]  
\[ (16) \]

\[ z_{ij-1,\ell,r_{k,\ell}} \leq z_{ij,\ell,r_{k,\ell}} + z_{ij,\ell,r_{k+1,\ell}}, \quad \forall \ell \in L, \ j \in [2, |I_\ell|], \ k \in [|R_\ell|], \]  
\[ (17) \]

\[ u_{ij-1,ij} \geq z_{ij-1,1} + z_{ij,1} - 1, \quad \forall \ell \in L, \ j \in [2, |I_\ell|], \ k \in [|R_\ell|]. \]  
\[ (18) \]

Constraints (16) ensure that if surgery \( i_{j,\ell} \) is assigned to OR \( r_{k,\ell} \), then surgery \( i_{j-1,\ell} \) is assigned to OR \( r_{k,\ell} \) or \( r_{k-1,\ell} \), while constraints (17) ensure that surgery \( i_{j+1,\ell} \) is assigned to OR \( r_{k,\ell} \) or \( r_{k+1,\ell} \). Constraints (18) enforce that surgeries in the same OR are sequenced in ascending order of their indices. Although including either constraint (16) or (17) could break the symmetry, using both of them could tighten the LP relaxation (see A.7.2). In A.7.3, we propose an alternative set of symmetry-breaking constraints for the case when there are sub-types within each surgery type.

Note that constraints (16)–(18) are different from those proposed in Denton et al. (2010). The key differences are the following. First, we consider the sequencing decisions in our model as appears in constraints (18). In contrast, Denton et al. (2010) did not consider sequencing decisions. Second, in the ORASP, symmetry also exists in the surgery order. For example, consider that surgeries 1 to 5 are of the same type and let the surgery sequences in OR 1 and OR 2 be \( 1 \rightarrow 2 \rightarrow 4 \) and \( 3 \rightarrow 5 \), respectively. Then, the objective value remains unchanged if we swap surgeries 3 and 4. Our constraints prevent exploring such equivalent solutions. In addition, supposing that surgeries 1 to 5 are assigned to the first two ORs, constraints (16)–(17) ensure that surgery 6 will be assigned to OR 2 or OR 3, but not OR 1. In contrast, Denton et al. (2010)’s constraints allow assigning surgery 6 to OR 1.

7.2. Operating Room Indexing Order

Note that constraints (16)–(18) do not prevent symmetry in OR loads (i.e., the number of scheduled surgeries). To illustrate, consider assignments 2 and 3 in Table 1, where constraints (16)–(18) are satisfied. These solutions are equivalent since both have an OR with 3 surgeries, an OR with 2 surgeries, and an OR with 1 surgery. Therefore, to prevent exploring such equivalent solutions, we introduce constraints

\[ \sum_{j=1}^{|I_\ell|} z_{ij,\ell,r_{k,\ell}} \geq \sum_{j=1}^{|I_\ell|} z_{ij-1,\ell,r_{k,\ell}}, \quad \forall \ell \in L, \ k \in [2, |R_\ell|], \]  
\[ (19) \]

which enforces that an OR with a smaller index has a larger number of scheduled surgeries.
8. Numerical Experiments

In this section, we use sets of publicly available surgery data to construct various ORASP instances and perform a case study from our collaborating health system. We conduct extensive computational experiments comparing the proposed methodologies computationally and operationally, demonstrating where significant performance improvements can be obtained and deriving insights relevant to practice. In Section 8.1, we describe the set of ORASP instances constructed and discuss the experimental setup. In Section 8.2, we analyze solution times of the proposed models. We demonstrate the efficiency of the proposed valid inequalities and symmetry-breaking inequalities in Section 8.3. In Section 8.4, we compare the optimal solutions of the proposed models. Then, we compare their operational performance via out-of-sample simulation tests in Sections 8.5 and 8.6. Finally, in Section 8.7, we present a case study and derive managerial insights.

8.1. Test Instances and Experimental Setup

We develop diverse ORASP instances based on prior literature and a publicly available dataset from Mannino et al. (2010). This dataset consists of three years of actual surgery durations for six different surgical specialties. Table 2 summarizes the six ORASP instances we constructed based on this data. Each of these instances is characterized by the number of surgeries and their types, the number of ORs and their types, the number of anesthesiologists, and the master/block schedule. In A.8.2, we provide summary statistics of the datasets and details of the master schedule. Note that instances 1–2 are relatively small, instances 3–4 are medium-sized, and instances 5–6 are large. In A.8.3, we provide additional computational results for another set of six ORASP instances constructed based on another set of publicly available surgery data.

We obtain the parameters for each instance as follows. We estimate the mean $m_i$ and standard deviation $\sigma_i$ of the duration of each surgery type from Mannino et al. (2010). As in prior literature, we set the lower bound $d_i$ and upper bound $\bar{d}_i$ as the 20th and 80th percentiles of the data of that surgery type, respectively. For the SP-E and SP-CVaR models, we generate the in-sample scenarios using lognormal (logN) distributions with the estimated mean and variance clipped at $d_i$ and $\bar{d}_i$. The overtime costs per hour are set to $c_o^r = 450$ and $c_o^a = 150$ (Rath et al. 2017). We set the OR fixed cost as $f_r = 900$, which is equivalent to double the per-hour OR overtime cost (Denton et al. 2010). The on-call anesthesiologists fixed cost is set to $f_a = 1000$ (Rath et al. 2017). We set the waiting cost per hour as $c_w^r = 200$. We consider 3 different cost structures for the idling costs per
hour: cost 1 \((c_r^g = c_a^g = 0)\), cost 2 \((c_r^g = 300\) and \(c_a^g = 0)\), and cost 3 \((c_r^g = 300\) and \(c_a^g = 100)\). We maintain the ratio \(c^o/c^g\) as 1.5 as suggested in the literature (Shehadeh et al. 2019).

We solve the SP-E and SP-CVaR models via sample average approximation (SAA) with \(N\) scenarios, which replaces the true distribution by the empirical distribution from the data (see A.8.1 complete models). We pick \(\gamma = 0.95\) for the SP-CVaR model. To decide the number of scenarios \(N\), we employ the Monte Carlo optimization (MCO) procedure, which provides statistical lower and upper bounds on the optimal value of the ORASP based on the optimal solution to its SAA (Kleywegt et al. 2002, Lamiri et al. 2009). This in turn provides a statistical estimate of the approximated relative gap (see A.8.4 for a detailed description and corresponding results).

Applying the MCO procedure with \(N = 100\) in the SP-E model, the approximated relative gaps for the ORASP instances described in Table 2 range from 0.06\% to 1.05\%. Note that a larger \(N\) could result in longer solution times without significant improvements in the approximated relative gaps. Therefore, we select \(N = 100\) for our computational experiments.

We implemented our proposed models and algorithm in AMPL modeling language and use CPLEX (version 20.1.0.0) as the solver with default settings. We set the relative MIP tolerance to 2\%. We solve large DRO instances using an inexact version of our proposed C&CG method by imposing a time limit of 600s when solving the master problems (Tsang et al. 2023). Unless stated otherwise, we include the proposed symmetry-breaking constraints in all models and the proposed VIs to both SP and DRO models. We conducted all the experiments on a computer with an Intel Xeon Silver processor 2.10 GHz CPU and 128 Gb memory.

### 8.2. Computational time

In this section, we analyze the computational times for solving our proposed models. For each instance and cost structure, we solve the SP-E and SP-CVaR models with 20 generated SAA instances, while we solve the DRO-E and DRO-CVaR models using the lower and upper bounds of surgery durations with 10 different means generated from a uniform distribution on \([0.9m_i, 1.1m_i]\).

Table 3 presents the average solution times in seconds under cost 1. Throughout this section, for large instances marked with ‘†’ for the DRO-E model, we apply VIs (13) and (14) with initial scenario \(m\) that produces shorter computational times.

We first observe that solution time increases as the size of the ORASP instance increases. Second, we can solve all instances using the SP-E, DRO-E, and DRO-CVaR models within a reasonable time. In fact, we can solve medium-sized instances in less than three minutes while the solution times of large instances range from two minutes to an hour. Third, solution times of the DRO-E model are slightly longer than the SP-E model. This is reasonable as the master problem of large ORASP instances is a large-scale scenario-based MILP and its size increases with the number of
### Table 3

| Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|------------|------------|------------|------------|------------|------------|
| SP-E       | 1.53       | 9.41       | 3.90       | 21.01      | 109.25     | 1496.49    |
| SP-CVaR    | 12.54      | 491.83     | 18.14      | 971.94     | –          | –          |
| DRO-E      | 7.20       | 28.21      | 14.14      | 102.95     | 152.11     | 2505.44    |
| DRO-CVaR   | 1.64       | 2.31       | 2.53       | 5.61       | 16.67      | 99.59      |

C&CG iterations. Fourth, solution times of the DRO-CVaR model, ranging from 1 second to 100 seconds, are significantly shorter than other models. One possible explanation is that the worst-case scenarios that maximize the CVaR objective are always those with long surgery durations. Thus, our C&CG method quickly identifies these scenarios and terminates in a small number of iterations. On the other hand, solution times of the SP-CVaR model are the longest among all models, and it cannot solve larger instances (i.e., instances 5 and 6). This is expected since the SP-CVaR model is not separable (see Section 6.4). Moreover, solving SP problems with CVaR objectives is known to be challenging. Finally, we remark that solution times are generally longer under costs 2 and 3, where we also consider the idle time in the objective (see A.8.2 for detailed results). Nevertheless, the average solution times of the SP-E and DRO-E models is within 2 hours, and that of the DRO-CVaR model is within 3 hours. According to our clinical collaborators, these solution times are reasonable; i.e., our proposed models are tractable for practical purposes.

### 8.3. Efficiency of valid inequalities and symmetry-breaking constraints

In this section, we demonstrate the efficiency of the proposed valid inequalities (VIs) and symmetry-breaking constraints (SBCs). For brevity and illustrative purposes, we present results for the SP-E and DRO-E models only. First, we analyze the impact of including VIs (11) in the SP-E model. For each ORASP instance, we solve 20 generated SAA instances with (w/) and without (w/o) VIs (11). Table 4 presents the average solution time w/ and w/o these VIs, and the average ratio of the optimal objective values of LP relaxations of the SP-E model w/ VIs and w/o VIs. In general, solution times are longer on average w/o these VIs, and the differences in solution times are more significant for large instances. For example, the percentage increase in average solution time ranges from 45% to 75% for large ORASP instances. We attribute the difference in solution time to a weaker LP relaxation of the SP-E model w/o these VIs. It is clear from Table 4 the LP relaxations w/ these VIs are strictly tighter, and they can be up to 3 times the LP relaxations w/o these VIs. These results demonstrate the efficiency of VIs (11) for the SP-E model.

Next, we analyze the impact of including the proposed VIs in Section 6.3 in the master problem of the C&CG method for the DRO-E model. Table 5 presents the solution time and number of iterations of our C&CG method w/ and w/o these VIs. While solution times w/ and w/o VIs are
Table 4 Average solution time (in s) of the SP-E model with (w/) and without (w/o) VIs, and average ratio of optimal objective values of LP relaxations of the SP-E model w/ VI to that w/o VIs

| Instance   | Time (w/ VIs) | Time (w/o VIs) | LP Relaxation Ratio |
|------------|--------------|----------------|--------------------|
| 1          | 1.53         | 1.57           | 1.12               |
| 2          | 9.41         | 9.48           | 1.30               |
| 3          | 3.90         | 5.47           | 1.07               |
| 4          | 21.01        | 22.85          | 1.59               |
| 5          | 109.25       | 159.67         | 2.08               |
| 6          | 1496.49      | 2603.40        | 2.93               |

Table 5 Solution time (in s) of the DRO-E model with (w/) and without (w/o) VIs, and number of iterations w/ and w/o VIs (instance with †: apply (13) and (14) with initial scenario $m$)

| Instance   | Time (w/ VIs) | Time (w/o VIs) | No. Iter (w/ VIs) | No. Iter (w/o VIs) |
|------------|--------------|----------------|-------------------|--------------------|
| 1          | 8.92         | 7.94           | 22                | 23                 |
| 2          | 34.24        | 22.99          | 29                | 31                 |
| 3          | 17.48        | 21.25          | 34                | 46                 |
| 4          | 94.19        | 93.64          | 56                | 79                 |
| 5          | 115.28       | 179.84         | 56                | 93                 |
| 6          | 2501.97†     | 5300.02†       | 62†               | 150                |

Figure 2 Lower bound and relative gap over iteration with and without VIs in the DRO-E model (Instance 4)

similar for small to medium-sized instances, solution times w/ VIs are significantly shorter for large instances. This is because C&CG w/o these VIs takes a considerably larger number of iterations to converge w/o VIs. For example, the number of iterations w/o VIs is doubled for instance 6. In addition, we observe that the lower bound (LB) and optimality gap converges faster when we introduce our proposed VIs into the master problem. To illustrate this, we provide Figure 2, which presents the LB and optimality gap w/ and w/o these VIs in one subproblem of instance 4. We observe that due to the better bounding effect from these VIs, both LB and the optimality gap converge in a smaller number of iterations when we introduce the proposed VIs. We also note that without these VIs, some large instances cannot be solved. For example, instance 6 under cost 2 cannot be solved within 3 hours without these VIs.

Finally, we study the efficiency of the proposed SBCs. We only focus on the SP-E model for brevity as the results are similar for the DRO-E model. Given the challenges of solving ORASP instances w/o these SBCs, we use instance 1 in this experiment for illustrative purposes. We first generate 20 sets of scenarios for this instance with number of scenarios $N \in \{10, 20, 50, 100, 200\}$. Then, we separately solve the SP-E model w/ and w/o these SBCs. Figure 3 illustrates the solution
time for different $N$, where solid and dashed lines represent the average solution times w/ and w/o SBCs, respectively. The shaded regions are the corresponding 20th and 80th percentiles. We observe that solution times are significantly longer without SBCs. Specifically, using our SBCs, we can solve the generated instances 5 to 145 times faster. Indeed, without our SBCs, medium and large SP model instances with $N = 100$ cannot be solved within one hour. These results demonstrate the importance of breaking symmetry in the first-stage decisions and the effectiveness of our SBCs.

8.4. Analysis of optimal solutions

In this section, we compare the optimal solutions of the proposed SP-E, SP-CVaR, DRO-E, and DRO-CVaR models. Let us first analyze the number of ORs opened by these models presented in Figure 4. Under cost 1, the DRO-CVaR and SP-E models open the largest and smallest number of ORs, respectively. Similarly, we observe that the DRO-CVaR and SP-E models call in the largest and smallest number of on-call anesthesiologists, respectively. Under costs 2–3, which include the OR idling cost, all models open fewer ORs than under cost 1 to avoid excessive OR idle time. However, the DRO-E and DRO-CVaR models open more ORs than the SP-E and SP-CVaR models, leading to a smaller number of surgeries scheduled in each OR in general. As we show in the next section, by opening more ORs, scheduling fewer surgeries in each OR, and employing additional on-call anesthesiologists, the DRO-E and DRO-CVaR models intend to mitigate surgery delays that may accumulate due to a tighter schedule with fewer ORs and yield a shorter waiting time when compared with SP-E and SP-CVaR models.

Next, we analyze the structure of the optimal schedules obtained from each model. For brevity and illustrative purposes, we present results for OR 1 in instance 3 under cost 1, where all models schedule four surgeries. Figure 5 presents the time allocated to each surgery (i.e., the difference between the scheduled start time of a surgery and its subsequent surgery). The two dotted lines in this figure represent the minimum and maximum surgery durations.
Figure 4  Number of ORs opened for different instances under different costs

Figure 5  Illustration of the optimal schedules for OR 1 in instance 3. The two dotted lines indicate the minimum and maximum surgery durations.

We observe the following from Figure 5. First, the DRO-CVaR and DRO-E models intend to protect against the risk of surgery delays that may accumulate due to long surgery durations by allocating longer times to the first three surgeries than the other models (also reflected by shorter waiting times from the DRO-CVaR and DRO-E models reported in the next section). Specifically, the DRO-CVaR model allocates the maximum surgery duration to surgery 1–3, leaving a shorter time for the last surgery than the other models. The DRO-E model allocates the maximum surgery duration to surgery 1–2 and slightly less (more) time to surgery 3 (the last surgery) than the DRO-CVaR model. Second, the SP-CVaR model allocates a longer time to the first surgery than the SP-E model and a longer time to the last surgery than the other models, potentially leading to a smaller overtime. In contrast, time allowances in the SP-E schedule exhibit a zigzag pattern with less (more) time allocated to surgery 1 (the last surgery) than other surgeries.

8.5. Analysis of solutions quality

In this section, we analyze the operational performance of the optimal schedules via out-of-sample simulation under cost 1 (we provide similar results for other cost structures in A.8.2). Specifically,
we first generate four sets of $N' = 10,000$ out-of-sample scenarios under various distributions, summarized in Table 6. In setting I, we assume perfect distributional information. That is, we generate $N'$ samples from the same distribution (logN) that we use in the optimization. In settings II-IV, we vary the surgery duration distributions to study the impact of misspecified distributional information, i.e., when the in-sample scenarios do not accurately reflect the true distribution (see Shehadeh 2022, Wang et al. 2020). Specifically, in setting II, we use a truncated normal distribution with the estimated mean $m$ and variance $\sigma^2$ on $[(1 - \Delta)d_i, (1 + \Delta)d_i]$ with $\Delta \in \{0, 0.25, 0.5\}$. A larger value of $\Delta$ corresponds to a higher variation level and $\Delta = 0$ indicates that only the distribution is perturbed with the same support. In setting III, we use a uniform distribution $U[(1 - \Delta)d_i, (1 + \Delta)d_i]$ with $\Delta \in \{0, 0.25, 0.5\}$. We denote simulations under setting II (and similarly for setting III) with $\Delta = 0, \Delta = 0.25, \text{and } \Delta = 0.5$ as IIa, IIb, and IIc, respectively. Finally, in setting IV, we use a beta distribution with the same mean $\mu_i$ and variance $\sigma^2_i$ on $[0.5d_i, 1.5d_i]$. These settings are motivated by our clinical collaborators, who observe significant changes in distribution and range of actual surgery durations between different time frames (e.g., month, year). Our analysis of Mannino et al. (2010)’s data also suggest an annual change in the lower and upper bounds of actual surgery durations ranging from $-17\%$ to $31\%$. Second, we solve the second-stage problem with the generated scenarios to compute the out-of-sample performance metrics (i.e., overtime and waiting time). For the sake of brevity in our presentation, we discuss the results for instances 2 and 6; we have similar observations about the results for the other instances.

We first analyze the out-of-sample values of the operational metrics. Table 7 shows the average waiting time and OR overtime under different distributional settings (observations for anesthesiologist overtime are similar to those for OR overtime). It is clear that the DRO-E and DRO-CVaR schedules generally yield significantly shorter waiting times and slightly longer overtime than the SP-E and SP-CVaR models under all simulation settings. Furthermore, the SP-CVaR schedules lead to shorter waiting times than the SP-E model and the shortest overtime in most settings. Under misspecified distributional settings II–IV (i.e., when the true distribution is different from the in-sample distribution used to generate the data for optimization), the waiting time and overtime are generally longer than the perfect distributional setting I. Notably, the performance of the SP-E solutions significantly deteriorates with longer waiting times and OR overtime. Finally, when the true distribution significantly deviates from the in-sample distribution (e.g., setting IIIc),

### Table 6  Out-of-sample distributions

| Setting | Distribution of $d_i$ for $i \in I$ |
|---------|----------------------------------|
| I       | Same distribution as the in-sample scenarios, i.e., lognormal |
| II      | Truncated normal distribution with mean $m_i$, variance $\sigma^2_i$ on $[(1 - \Delta)d_i, (1 + \Delta)d_i]$, where $\Delta \in \{0, 0.25, 0.5\}$ |
| III     | Uniform distribution on $[(1 - \Delta)d_i, (1 + \Delta)d_i]$, where $\Delta \in \{0, 0.25, 0.5\}$ |
| IV      | Beta distribution with same mean $\mu_i$ and variance $\sigma^2_i$ on $[0.5d_i, 1.5d_i]$ |
Table 7: Average out-of-sample waiting time and OR overtime under settings I–IV (Instances 2 and 6)

| Waiting Time | Instance 2 | | | Instance 6 | | |
|--------------|-----------|-----------|-----------|-----------|-----------|
|              | SP-E | SP-CVaR | DRO-E | DRO-CVaR | SP-E | SP-CVaR | DRO-E | DRO-CVaR |
| Setting I    | 259 | 91 | 0 | 0 | 346 | 181 | 2 | 0 |
| Setting IIa  | 250 | 60 | 0 | 0 | 248 | 135 | 0 | 0 |
| Setting IIb  | 469 | 176 | 62 | 69 | 768 | 464 | 227 | 200 |
| Setting IIc  | 567 | 235 | 123 | 136 | 1175 | 760 | 491 | 439 |
| Setting IIId | 257 | 74 | 0 | 0 | 267 | 142 | 0 | 0 |
| Setting IIe  | 620 | 247 | 110 | 121 | 1093 | 669 | 381 | 336 |
| Setting IIc  | 567 | 235 | 123 | 136 | 1175 | 760 | 491 | 439 |
| Setting IIId | 257 | 74 | 0 | 0 | 267 | 142 | 0 | 0 |
| Setting IIe  | 620 | 247 | 110 | 121 | 1093 | 669 | 381 | 336 |
| Setting IV    | 1026 | 487 | 123 | 136 | 1175 | 760 | 491 | 439 |

| OR Overtime | Instance 2 | | | Instance 6 | | |
|-------------|-----------|-----------|-----------|-----------|-----------|
|              | SP-E | SP-CVaR | DRO-E | DRO-CVaR | SP-E | SP-CVaR | DRO-E | DRO-CVaR |
| Setting I    | 116 | 41 | 69 | 69 | 79 | 77 | 215 | 147 |
| Setting IIa  | 115 | 28 | 71 | 71 | 35 | 47 | 225 | 154 |
| Setting IIb  | 212 | 91 | 113 | 110 | 249 | 213 | 347 | 271 |
| Setting IIc  | 256 | 128 | 141 | 136 | 433 | 360 | 458 | 383 |
| Setting IIId | 118 | 29 | 70 | 70 | 41 | 51 | 221 | 151 |
| Setting IIe  | 280 | 134 | 140 | 136 | 433 | 360 | 458 | 383 |
| Setting IV    | 241 | 144 | 153 | 144 | 544 | 469 | 525 | 461 |

the DRO-E and DRO-CVaR models produce shorter or approximately the same overtime as the SP-CVaR model.

To further illustrate the impact of model misspecification, we compare out-of-sample costs under setting III with $\Delta \in \{0, 0.25, 0.5\}$ using instance 6. (Recall that a larger $\Delta$ corresponds to a larger extent of misspecification). First, Figure 6 shows the out-of-sample distributions of the operational (i.e., second-stage) costs for instance 6. When $\Delta = 0$, since the distributional change is mild, the operational costs of the SP-E and SP-CVaR schedules are generally lower than that of the DRO-E and DRO-CVaR schedules. When $\Delta \in \{0.25, 0.5\}$, i.e., deviations from the in-sample distribution are large, the SP-CVaR, DRO-E, and DRO-CVaR schedules yield significantly lower operational costs than the SP-E schedule on average and at all quantiles. Notably, the DRO-CVaR schedules lead to the lowest operational costs under large $\Delta$, and the DRO-E model leads to lower operational costs than the SP-CVaR model when $\Delta = 0.5$. These results demonstrate the robustness of the SP-CVaR, DRO-E, and DRO-CVaR solutions against distributional changes and the potential operational benefits of adopting the DRO-E and DRO-CVaR solutions when the true distribution significantly deviates from the in-sample distribution.

Second, Figure 7 shows the distributions of the total cost, as a sum of the fixed and operational costs, under setting III with $\Delta \in \{0, 0.25, 0.5\}$. When $\Delta = 0$, both SP-E and SP-CVaR schedules produce lower total costs than the DRO-E and DRO-CVaR schedules since the SP-E and SP-CVaR models open fewer ORs. However, when $\Delta \in \{0.25, 0.5\}$, i.e., deviations from the in-sample distribution are large, the DRO-E and DRO-CVaR models yield lower total costs at upper quantiles, especially when $\Delta = 0.5$. These results further illustrate that DRO-E and DRO-CVaR models can protect against distributional ambiguity and show the trade-off between fixed and operational costs. Specifically, by opening more ORs, the DRO-E and DRO-CVaR models incur higher fixed costs, but significantly smaller operational costs. We note that while the fixed cost is a one-time
cost (i.e., fixed once the ORs are open and on-call anesthesiologists are called in), the operational cost represents recurring long-run costs. With reference to our results, practitioners could decide which model to adopt based on their preferences and actual situations.

Finally, we investigate the value of distributional robustness from the perspective of out-of-sample disappointment, which measures the extent to which the out-of-sample cost exceeds the model’s optimal value (Van Parys et al. 2021). Let $V^{\text{opt}}$ and $V^{\text{out}}$ be the model’s optimal value and the out-of-sample objective value, respectively. That is, $V^{\text{opt}}$ and $V^{\text{out}}$ can be viewed as the estimated and actual costs of implementing the model’s optimal solutions, respectively. Then, we define the out-of-sample disappointment as $\max\{ (V^{\text{out}} - V^{\text{opt}})/V^{\text{opt}}, 0 \}$. A disappointment of zero implies that $V^{\text{out}} \leq V^{\text{opt}}$, which in turn indicates that the model is more conservative and avoids underestimating costs. Figure 8 presents the distributions of the out-of-sample disappointments for instance 6. Notably, the DRO-CVaR model yields significantly smaller out-of-sample disappointments at all quantiles. Moreover, the out-of-sample disappointment of the DRO-CVaR model is the most stable with the smallest standard deviation. On the other hand, disappointments of the SP-E model are
significantly larger than all models, especially at the upper quantiles (e.g., exceeding 100%). Both DRO-E and SP-CVaR models yield smaller disappointments than the SP-E model, with the DRO-E model having smaller disappointments than the SP-CVaR model under settings IIIc and IV. These results demonstrate that the SP-CVaR, DRO-E, and DRO-CVaR models provide a more robust estimate of the actual cost that the hospital will incur in practice. Thus, risk-averse OR managers who seek a robust financial plan and operational performance may prefer these solutions.

8.6. Comparison with Non-Integrated Approaches

In this section, we present results comparing the operational and computational performance of solutions to our proposed models with those of Rath et al. (2017)’s RO model and a sequential approach detailed later in this section. We focus on our SP-E and DRO-E models only for the sake of brevity in our presentation.

First, we compare the performance of our solutions with solutions to Rath et al. (2017)’s RO model. Recall that Rath et al. (2017)’s model does not include idle, waiting time components, and other components, and thus can schedule multiple surgeries to start simultaneously; see A.8.2 for details of this model. For a fair comparison, we include symmetry-breaking constraints for all models. To compare the operational performance of the optimal solutions, we solve instance 1 under cost 1 using the three models. (The observations are similar for other ORASP instances.) In particular, we solve Rath et al. (2017)’s RO model by their proposed decomposition method with different sizes $\tau \in \{0.1, 0.2, 0.4\}$ of the uncertainty set adopted in their paper. We provide a detailed comparison of the optimal solutions in A.8.2. We highlight that the RO model schedules all surgeries at time zero, while our models do not schedule surgeries performed by the same anesthesiologist or in the same OR to start at the same time. Moreover, as shown in A.8.2, the RO model assigns more surgeries to some anesthesiologists, which leads to the possibility of larger overtimes when compared with our proposed models. Figure 9 shows the histograms of the out-of-sample waiting
Table 8: Computational time (in s) of our proposed SP-E and DRO-E models along with Rath et al. (2017)’s RO model. Percentages in brackets show the relative gaps when terminating with maximum time limit (1 hour).

| Instance 1 Time | SP-E | DRO-E | RO | \(\tau = 0.1\) | \(\tau = 0.2\) | \(\tau = 0.4\) |
|-----------------|------|-------|----|----------------|----------------|----------------|
| 1               | 8    | 106   | 264| (11.6%)       | (87.2%)       | (88.6%)       |
| 2               | 10   | 1992  | 2431|(89.5%)       | (89.5%)       | (89.5%)       |

time associated with the optimal schedules under setting I. These plots indicate that solutions from Rath et al. (2017)’s RO model lead to significantly larger waiting times than our SP-E and DRO-E models. These results demonstrate the importance of our proposed generalization of the second-stage problem in the ORASP.

Next, to compare the computational performance of the three models, we solve instances 1, 3, and 5 (small, medium, and large-sized instances) using the three models under cost 1 with \(c^w_i = 0\). We also impose a one-hour time limit to solve each model. Table 8 shows the solution time of the three models. Using our proposed models, the instances can be solved to optimality within 2 minutes. In contrast, the RO model takes a longer time to solve instances 1 and 3, and it fails to solve instance 5. In particular, Rath et al. (2017)’s decomposition method terminates with a large relative MIP gap after one hour. These computational results indicate that even if we ignore the waiting time component of the ORASP, our proposed models are more computationally efficient to solve than Rath et al. (2017)’s RO model.

Second, we also compare our proposed SP-E model with a sequential approach that separates the OR assignment decisions from the remaining decisions (i.e., anesthesiologist assignment, sequencing, and scheduling decisions). Specifically, in the sequential approach, we first solve Denton et al. (2010)’s classical surgery-to-OR assignment model to obtain \((v^*, z^*)\); see A.8.2 for the formulation. Then, we solve our ORASP model by fixing \((v, z)\) to \((v^*, z^*)\). We follow the same experiment settings in Section 8.1 to solve instances 1–6 under cost 1 using the two approaches. Table A.22 in A.8.2 presents the number of ORs opened, and Table A.23 summarizes the associated average out-of-sample waiting time, OR overtime, and operational costs. We observe that the sequential approach opens the same or smaller number of ORs than our SP-E model. Moreover, the former results in a packed schedule, thus leading to longer waiting times, OR overtime, and, consequently, higher operational costs. In particular, the operational cost of the sequential approach is two times higher than that of our SP-E model for large instances (e.g., instances 5–6). These results suggest that our integrated approach could yield better operational performance than the sequential approach.
8.7. Case Study

In this section, we present a case study based on questions asked by our collaborators and information from their health systems. Specifically, we use the master schedule of a day in September 2021 to examine the sensitivity of the proposed models’ optimal solutions to the cost parameters in the objective (which they can easily adjust) and determine the number of surgeries to schedule. This master schedule consists of 29 ORs (all open) and 14 surgery types. There are 35 anesthesiologists, 8 of whom are on call. In what follows, we use the same experimental setup described in Section 8.1. We estimate $m$ and $\sigma$ of duration $d$ from the data provided by our collaborators.

First, we investigate the impact of the waiting cost parameter $c_{w_i}$ on the optimal schedule. Specifically, we keep other cost parameters as in cost 1 and solve the models with $c_{w_i} \in \{100, 200, 300, 400\}$. For brevity, in Figure 10, we illustrate the optimal time allowances for four colorectal surgeries assigned to one of the ORs. (We observe similar results for other surgery types and ORs). As $c_{w_i}$ increases, the SP-E and SP-CVaR models assign more time to each surgery except the last one. In contrast, the DRO-E and DRO-CVaR models appear to be less sensitive to $c_{w_i}$, and, in particular, time allowances are the same except for $c_{w_i} = 400$ in the DRO-E model. This is because the DRO-E and DRO-CVaR schedules already assign longer time to each surgery to hedge against waiting time. Given that the OR service hour is fixed, there is less or no room to assign more time to each surgery when $c_{w_i}$ increases in the DRO-E and DRO-CVaR schedules.

Second, we investigate the impact of $c_{o_i}$ on the optimal schedule. Again, we keep the other cost parameters as in cost 1 and consider $c_{o_i} \in \{50, 150, 250, 350, 450\}$. Figure 11 shows the time allocated to colorectal surgeries in the same OR we used in Figure 10. In general, when $c_{o_i}$ increases, the time allowances of the SP-E and SP-CVaR models decrease for surgeries in earlier positions (i.e., positions 1 and 2), and it increases in the last position. This is reasonable as the SP-E and SP-CVaR models tend to avoid overtime when $c_{o_i}$ increases. In contrast, the DRO-E and DRO-CVaR models are insensitive to the change of $c_{o_i}$. This makes sense because these models already schedule
Figure 10 Time allowance to each surgery in one of the ORs with different $c_w \in \{100, 200, 300, 400\}$.

Figure 11 Time allowance to each surgery in one of the ORs with different $c_o$ longer time for surgeries in earlier positions to avoid delays, which potentially mitigates overtime. Finally, we observe that when $c_o$ increases from 350 to 450 (resp. from 150 to 250), the SP-CVaR and DRO-E models (resp. the DRO-CVaR model) employ an extra on-call anesthesiologist for colorectal surgeries. This is to avoid excessive anesthesiologist overtime.

Finally, our collaborators are interested in determining the number of surgeries to schedule in each OR to maintain good operational performances that balance variable operational cost and revenue. Therefore, we use the same technique in Berg et al. (2014) to determine the number of surgeries to schedule that maximizes the difference between the profit from scheduling (i.e., performing) surgeries and the associated estimated cost (i.e., optimal value of our models). We use cost 1 and cardiac surgery as an example, for which there are six specialized anesthesiologists (one of whom is on call) and five ORs in the master schedule.

Figure 12 shows the revenue under different profits $\{2000, 2500, 3000\}$ per surgery. When the profit is relatively low (i.e., 2000 or 2500), all models schedule 5 surgeries (to maximize revenue), precisely one surgery per OR. This is because cardiac surgery has a long duration with a mean of 384 minutes, and each OR is available for 480 minutes. However, when the profit is larger (i.e., 3000), the SP-E model schedules 10 surgeries (i.e., two surgeries per OR) while the other models schedule 5 since these models tend to hedge against duration scenarios that may lead to high operational costs. We note that the resulting packed OR schedule of the SP-E model leads to poor
operational performance. For example, the waiting time and anesthesiologist overtime associated with the SP-E schedule are 511 and 1227 minutes, respectively, compared with zero waiting and overtime of 26 minutes in other models. Our collaborators indicate that they prefer a less packed schedule to ensure smooth operations with fewer delays and overtime.

9. Conclusion

In this paper, we study an operating room and anesthesiologist scheduling problem (ORASP) under uncertainty. We propose the first risk-neutral and risk-averse SP models to address uncertainty in surgery durations in the ORASP, which generalizes the state-of-the-art models by (a) incorporating a larger set of important objectives, (b) integrating allocation, assignment, sequencing, and scheduling problems, and (c) modeling the decision-maker’s risk preference. In addition, recognizing that high-quality data to estimate surgery duration distribution accurately is often not available, we propose DRO counterparts of our SP models based on a mean-support ambiguity set to account for distributional ambiguity. We derive equivalent solvable reformulations of these DRO models and propose a C&CG method that efficiently solves the reformulations.

Using publicly available surgery data and a case study from a large health system in New York, we construct various ORASP instances and conduct extensive computational experiments to compare the proposed methodologies, illustrating the potential benefits of our proposed integrated approach in practice. We observe significant differences in the optimal SP-E, SP-CVaR, DRO-E, and DRO-CVaR schedules and thus, substantial differences in these solutions’ impact on the operational performance and costs. Our results also show the significance of integrating the allocation, assignment, sequencing, and scheduling problems and the negative consequences of adopting existing non-integrated approaches and ignoring the uncertainty and ambiguity of surgery duration. We also conduct sensitivity analysis using the case study data from our collaborating health system to derive insights relevant to OR managers. Finally, our results demonstrate the computational efficiency of the proposed methodologies.
We suggest the following areas for future research. First, we would like to incorporate other sources of uncertainty, such as arrival of emergency surgeries. Second, our model can serve as a building block towards data-driven and robust OR planning. In particular, we aim to extend our models to more comprehensive OR and surgery planning models by considering all relevant organizational and technical constraints such as optimizing the block (master) schedule and incorporating recovery units. Finally, it would be interesting to explore advanced data-driven and machine learning methods that exploit, for example, patients’ characteristics to model surgery duration and other random factors and hence, to further optimize the OR planning process.

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Appendix

A.1. Anesthesiologists Specialty Example

Table A.1 provides an example of an anesthesiologist’s specialty and assignment at our collaborating health system. Some specialized anesthesiologists are dedicated to a specific specialty (e.g., cardiothoracic, obstetrics and pediatric). Some can perform a wide range of surgeries (i.e., cross-cover) such as general, orthopedic, neurosurgery and transplant surgeries.

| Specialty cross coverage | Primary        | General | ORTH | Pain Medicine | NSG | Transplant | CARD | OB | Pediatric |
|--------------------------|---------------|---------|------|---------------|-----|------------|------|----|-----------|
| General                  | General       | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| ORTH                     | ORTH          | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| Pain Medicine            | Pain Medicine | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| NSG                      | NSG           | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| Transplant               | Transplant    | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| CARD                     | CARD          | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| OB                       | OB            | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |
| Pediatric                | Pediatric     | NA      | NA   | NA            | NA  | NA         | NA   | NA | NA        |

A.2. Notation

Table A.2 summarizes the notation we use in the models, including parameters, sets, and decision variables.

A.3. Example and Discussions in Section 4

A.3.1. Example on Sequencing Variable Constraints

Example A.1. There are two possible types of non-implementable decisions without constraints \((1h)\) and \((1i)\). First, consider that surgeries 1 and 2 are assigned to the same anesthesiologist \(a\) and in the same OR \(r\), but we have that surgery 1 is performed followed by surgery 2 in the anesthesiologist schedule, but vice versa in the OR schedule, with both scheduled start time being 0. That is, the anesthesiologist perform surgery 1 first, but surgery 2 is scheduled first in the OR. Obviously, this schedule is not implementable and constraints \((1h)\) excludes such a possibility.

The second type of non-implementable decision corresponds to cycles. As an example, assume that we have four surgeries with the same scheduled start times, say 0, as follows.

- Anesthesiologist 1: \(1 \rightarrow 2\)  \(\text{OR } 1: 3 \rightarrow 1\)
- Anesthesiologist 2: \(4 \rightarrow 3\)  \(\text{OR } 2: 2 \rightarrow 4\)
Table A.2 Notation

| Indices | Description |
|---------|-------------|
| \(i\)   | index of surgery, \(i \in I\) |
| \(a\)   | index of anesthesiologist, \(a \in A\) |
| \(r\)   | index of operating room, \(r \in R\) |
| \(l\)   | index of surgery type, \(l \in L\) |

**Parameters and sets**

| \(I\) | set of surgeries |
| \(A\) | set of anesthesiologists |
| \(R\) | set of operating rooms |
| \(L\) | set of surgery types |

\[ F^A / F^R \] set of \((i, a) / (i, r)\) pairs such that surgery \(i\) can be performed by anesthesiologist \(a\) / in operating room \(r\)

\[ A_i / R_i \] set of anesthesiologists / operating rooms to which surgery \(i\) can be assigned

\[ I_a / I_r \] set of surgeries that could be performed by anesthesiologist \(a\) / in operating room \(r\)

\[ F^{start} / F^{end} \] start / end time of anesthesiologist \(a\)

\(T^{end}\) end time of the day

\(f_r\) fixed cost of using operating room \(r\)

\(f_a\) fixed cost of employing an on-call anesthesiologist

\(c^g_{a, r}\) idling cost of anesthesiologist \(a\) / operating room \(r\)

\(c^o_{a, r}\) overtime cost of anesthesiologist \(a\) / operating room \(r\)

\(c^w_i\) waiting cost of surgery \(i\)

\(\kappa^A_{i, a} / \kappa^R_{i, r}\) 1 if surgery \(i\) can be done by anesthesiologist \(a\) / in operating room \(r\), 0 otherwise

\(h_{reg, a}\) 1 if anesthesiologist \(a\) is on regular duty, 0 otherwise

\(h_{call, a}\) 1 if anesthesiologist \(a\) is on call, 0 otherwise

\(D_i\) random duration of surgery \(i\) \((d_i\) as a realization)\)

\(d_i / \bar{d}_i\) lower / upper bound of duration of surgery \(i\)

**First-stage decision variables**

| \(x_{i, a}\) | 1 if surgery \(i\) is performed by anesthesiologist \(a\), 0 otherwise |
| \(z_{i, r}\) | 1 if surgery \(i\) is performed in operating room \(r\), 0 otherwise |
| \(y_a\) | 1 if anesthesiologist \(a\) is assigned from on call, 0 otherwise |
| \(v_r\) | 1 if operating room \(r\) is used, 0 otherwise |
| \(s_i\) | scheduled start time of surgery \(i\) |
| \(u_{i, i'}, a\) | 1 if surgery \(i'\) follows surgery \(i\), 0 otherwise |
| \(a_{i, i', a}\) | 1 if surgery \(i'\) follows surgery \(i\) for anesthesiologist \(a\), 0 otherwise |
| \(b_{i, i', r}\) | 1 if surgery \(i'\) follows surgery \(i\) in operating room \(r\), 0 otherwise |

**Second-stage decision variables**

| \(q_i\) | actual start time of surgery \(i\) |
| \(o_{a, r}\) | overtime of anesthesiologist \(a\) / operating room \(r\) |
| \(w_i\) | waiting time of surgery \(i\) |
| \(g_{a, r}\) | idling time of anesthesiologist \(a\) / operating room \(r\) |

For anesthesiologist 1 to perform surgery 1, one needs to wait for the completion of surgery 3 in OR 1. However, to perform surgery 3 in OR 1, anesthesiologist 2 has to perform surgery 4 first, which follows surgery 2 in OR 2. Lastly, to perform surgery 2, anesthesiologist 1 needs to perform surgery 1. That is, a cycle occurs and no surgeries could be conducted. Note that with the transitivity constraints (1i), decisions with cycles can be excluded.

A.3.2. Choice of Big \(M\) Parameters

In the SP model (1)–(2), there are four big \(M\) parameters. We provide a suitable choice of these parameters for actual implementation. First, from constraints (1f), we have that \(M = \max_{a \in A} \{t^{start}_a\}\) is a suitable choice. To derive a suitable choice of the remaining three parameters which are related to the maximum of \(q_i + d_i\), we provide the following lemma. (Note that \(q_i\) is the second-stage variable).
Lemma A.1. Suppose that \( d_i \in [d_i, \bar{d}_i] \) for all \( i \in I \). Then, for any first-stage decision \((x, y, z, v, u, s)\) and realization \( d \), we have \( \max_{i \in I} \{ q_i + d_i \} \leq T^{\text{end}} + \sum_{i \in I} \bar{d}_i \).

Proof. The last possible actual surgery start time corresponds to the situation that the scheduled start time for all the surgeries are \( T^{\text{end}} \). Moreover, all surgeries are scheduled to one anesthesiologist only. This leads to the desired upper bound. \( \square \)

As a result of Lemma A.1, it suffices to set the big-\( M \) parameters as \( M_{\text{seq}} = T^{\text{end}} + \sum_{i \in I} \bar{d}_i \), \( M_{\text{anes}} = T^{\text{end}} + \sum_{i \in I} d_i - \min_{a \in A} \{ \mu^{\text{end}}_a \} \) and \( M_{\text{room}} = \sum_{i \in I} d_i \).

Remark A.1. The proof of A.1 is based on the hypothetical situation that an anesthesiologist conducts all the surgeries. In practical ORASP instances, we have anesthesiologists specialized for a single surgery type \( \ell \in \{1, \ldots, L^{\text{spec}}\} \) and a pool of anesthesiologists for general surgery types \( \{L^{\text{spec}} + 1, \ldots, L\} \). In this case, we could tighten the upper bound to

\[
\max_{i \in I} \{ q_i + d_i \} \leq T^{\text{end}} + \max \left\{ \max_{\ell=1,\ldots,L^{\text{spec}}} \{ \bar{d}_\ell | I_\ell | \}, \ \sum_{\ell=L^{\text{spec}}+1}^{L} \bar{d}_\ell | I_\ell | \right\},
\]

where \( \bar{d}_\ell \) and \( | I_\ell | \) are the maximum surgery duration and number of surgeries of type \( \ell \), respectively. Note that it is not optimal to schedule all surgeries at \( T^{\text{end}} \). Indeed, scheduling all surgeries at \( \max_{\ell \in A} t^{\text{start}}_\ell \) could produce a better solution by reducing both idle time and overtime. Therefore, it suffices to choose

\[
M_{\text{seq}} = \max_{a \in A} \{ t^{\text{start}}_a \} + \max \left\{ \max_{\ell=1,\ldots,L^{\text{spec}}} \{ \bar{d}_\ell | I_\ell | \}, \ \sum_{\ell=L^{\text{spec}}+1}^{L} \bar{d}_\ell | I_\ell | \right\}
\]
as in Rath et al. (2017). Note that we exploit the practical ORASP instance structures to result in tighter big-\( M \) parameters.

A.4. Proofs and Discussions Related to DRO-E and DRO-CVaR Models in Section 5

A.4.1. Proof of Proposition 1

Proof of Proposition 1 Note that \( \mathcal{S} \) is compact, \( \psi_0(d) := Q(x, y, z, v, u, s, d) \) and \( \psi_i(d) := d_i \) for \( i \in I \) are continuous functions in \( d \). From Proposition 6.68 of Shapiro et al. (2014), strong duality holds and the worst-case expectation equals

\[
\min_{\rho} \rho_0 + \sum_{i \in I} \rho_i m_i
\]
subject to \( Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \leq \rho_0, \ \forall d \in \mathcal{S}. \)

Since the constraint holds for all \( d \in \mathcal{S} \), we can rewrite it as

\[
\rho_0 \geq \sup_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\}.
\]

As we minimize \( \rho_0 \) in the objective, we have that the dual problem is equivalent to (5). \( \square \)
A.4.2. Proof of Proposition 2

Before proceeding to the proof of Proposition 2, we first derive the dual of the second-stage problem (2) in Lemma A.2.

**Lemma A.2.** The second-stage problem (2) is equivalent to

\[
\text{maximize } \lambda, \mu, \theta \quad \sum_{a \in A} c_a^e (t_a^e - t_a^{start}) + \sum_{r \in R} c_r^e t_r^e v_r
\]

\[
+ \sum_{i \in I} \left[ \sum_{i' \in I, i' \neq i} \lambda_{i,i'}(1 - u_{i,i'}) + \sum_{a \in A_i} \mu_i,a + \sum_{r \in R_i} \theta_{i,r} \right] s_i
\]

\[- M_{\text{seq}} \sum_{i \in I} \sum_{i' \in I, i' \neq i} \lambda_{i,i'}(1 - u_{i,i'}) - \sum_{i \in I} \sum_{a \in A_i} \mu_i,a \left[ t_a^{end} + M_{\text{anes}}(1 - x_{i,a} + y_a) \right]
\]

\[- \sum_{i \in I} \sum_{r \in R_i} \theta_{i,r} \left[ T_r^{end} + M_{\text{room}}(1 - z_{i,r}) \right]
\]

\[+ \sum_{i \in I} \left[ \sum_{i' \in I, i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c_a^h x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c_r^e z_{i,r}) \right] d_i \quad \text{(A.1a)}
\]

subject to

\[
\sum_{i \in I} \mu_{i,a} \leq c_a^e + c_a^o, \quad \forall a \in A, \quad \text{(A.1b)}
\]

\[
\sum_{i \in I} \theta_{i,r} \leq c_r^e + c_r^o, \quad \forall r \in R, \quad \text{(A.1c)}
\]

\[
\sum_{i' \in I, i' \neq i} (\lambda_{i,i'} - \lambda_{i',i}) + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r} + c_i^w \geq 0, \quad \forall i \in I, \quad \text{(A.1d)}
\]

\[
\lambda_{i,i'}, \mu_{i,a}, \theta_{i,r} \geq 0, \quad \forall i \in I, a \in A_i, r \in R_i, i' \in I \setminus \{i\}. \quad \text{(A.1e)}
\]

**Proof of Lemma A.2** First, from the second-stage problem (2), the optimal solution \( q_i, a_i, a_r \) and \( o_r \) give the actual surgery start time, the overtime of anesthesiologists and OR, respectively. From constraints (2c) and the minimization nature of the problem, we have \( w_i = q_i - s_i \). Moreover, constraints (2g)–(2h) achieve equality at optimality, which characterize the idle times of anesthesiologists and ORs, respectively. Therefore, the second-stage problem (2) is equivalent to

\[
\text{minimize } q', a, a', g \quad \sum_{a \in A} \left\{ c_a^e \left[ (t_a^{end} - t_a^{start}) - \sum_{i \in I} d_{i,a} x_{i,a} \right] h_a^{reg} + o_a \right\} + c^o a_o
\]

\[+ \sum_{r \in R} c_{r}^e \left[ T_r^{end} v_r - \sum_{i \in I} d_{i,r} z_{i,r} + o_r \right] + c^o r o_r + \sum_{i \in I} c_i^e (q_i - s_i) \quad \text{(A.2a)}
\]

subject to \( q_i \geq q_i + d_i - M_{\text{seq}}(1 - u_{i,i'}) \), \( \forall \{i, i'\} \subseteq I, i \neq i' \), \( q_i \geq s_i, \quad \forall i \in I, \quad \text{(A.2b)} \)

\( a_o \geq q_i + d_i - t_a^{end} - M_{\text{anes}}(1 - x_{i,a} + y_a), \quad \forall (i, a) \in \mathcal{F}^A, \quad \text{(A.2d)} \)

\( o_r \geq q_i + d_i - T_r^{end} - M_{\text{room}}(1 - z_{i,r}), \quad \forall (i, r) \in \mathcal{F}^R, \quad \text{(A.2e)} \)
Since we maximize $\lambda, \mu, \theta, \phi$, subject to (A.1b)–(A.1e)

$$\max_{\lambda, \mu, \theta, \phi} \{ \sum_{a \in A} c_a^g (t_a^\text{end} - t_a^\text{start}) h_a^\text{reg} + \sum_{r \in R} c_r^g T^\text{end} v_r + \sum_{i \in I} (\phi_i - c_i^o) s_i$$

$$- M_{\text{seq}} \sum_{i \in I} \sum_{i', i' \neq i} \lambda_{i,i'} (1 - u_{i,i'}) - \sum_{i \in I} \sum_{a \in A_i} \mu_{i,a} \left[ t_a^\text{end} + M_{\text{anes}} (1 - x_{i,a} + y_a) \right]$$

$$- \sum_{i \in I} \sum_{r \in R_i} \theta_{i,r} \left[ T^\text{end} + M_{\text{room}} (1 - z_{i,r}) \right]$$

$$+ \sum_{i \in I} \left[ \sum_{i', i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c_a^g h_a^\text{reg} x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c_r^g z_{i,r}) \right] d_i \} \quad (A.3a)$$

subject to

$$\sum_{i \in I} \mu_{i,a} \leq c_a^g + c_a^o, \quad \forall a \in A, \quad (A.3b)$$

$$\sum_{i \in I} \theta_{i,r} \leq c_r^g + c_r^o, \quad \forall r \in R, \quad (A.3c)$$

$$\sum_{i' \in I, i' \neq i} (\lambda_{i,i'} - \lambda_{i',i}) + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r} - \phi_i \geq -c_i^o, \quad \forall i \in I, \quad (A.3d)$$

$$\lambda_{i,i'}, \mu_{i,a}, \theta_{i,r}, \phi_i \geq 0, \quad \forall i \in I, \, a \in A_i, \, r \in R_i, \, i' \in I \setminus \{i\}. \quad (A.3e)$$

Note that from constraints (A.3d), we have

$$0 \leq \phi_i \leq \sum_{i' \in I, i' \neq i} (\lambda_{i,i'} - \lambda_{i',i}) + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r} + c_i^o, \quad \forall i \in I.$$ 

Since we maximize $\phi_i s_i$ with $s_i \geq 0$ in the objective, the optimal solution $\phi_i^*$ is the upper bound derived from constraints (A.3d). This shows the equivalence of (A.3) and (A.1).

**Proof of Proposition 2**  Substituting (A.1) into (5), we have

$$\max_{\lambda, \mu, \theta, \phi, d_i} \sum_{a \in A} c_a^g (t_{a}^\text{end} - t_{a}^\text{start}) h_{a}^\text{reg} + \sum_{r \in R} c_r^g T^\text{end} v_r$$

$$+ \sum_{i \in I} \sum_{i', i' \neq i} \lambda_{i,i'} (1 - u_{i,i'}) + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r} \sum_{i \in I, r \in R_i} s_i$$

$$- M_{\text{seq}} \sum_{i \in I} \sum_{i', i' \neq i} \lambda_{i,i'} (1 - u_{i,i'}) - \sum_{i \in I} \sum_{a \in A_i} \mu_{i,a} \left[ t_a^\text{end} + M_{\text{anes}} (1 - x_{i,a} + y_a) \right]$$

$$- \sum_{i \in I} \sum_{r \in R_i} \theta_{i,r} \left[ T^\text{end} + M_{\text{room}} (1 - z_{i,r}) \right]$$

$$+ \sum_{i \in I} \left[ \sum_{i', i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c_a^g h_a^\text{reg} x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c_r^g z_{i,r}) - \rho_i \right] d_i \} \quad (A.4a)$$

subject to

$$\text{(A.1b)} - \text{(A.1e)}, \quad (A.4b)$$

$$d_i \leq d_i \leq \bar{d}_i, \quad \forall i \in I. \quad (A.4c)$$
Problem (A.4) is not linear due to the quadratic terms between the dual variables and \(d_i\) in the objective function. Note that we can first perform maximization over \(d\), i.e.,

\[
\begin{align*}
\text{maximize}_{d} & \quad \sum_{i \in I} \left[ \sum_{i' \in I, i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c^p_a h^\text{reg}_{a} x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c^p_r z_{i,r}) - \rho_i \right] d_i \\
\text{subject to} & \quad d_i \leq d_i \leq \bar{d}_i, \quad \forall i \in I.
\end{align*}
\] (A.5a)

Problem (A.5) is separable in \(i\) and the objective function is linear in \(d_i\). Therefore, either \(d_i\) or \(\bar{d}_i\) is an optimal solution. Hence, problem (A.5) is equivalent to the binary program

\[
\begin{align*}
\text{maximize}_{b} & \quad \sum_{i \in I} \left[ \sum_{i' \in I, i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c^p_a h^\text{reg}_{a} x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c^p_r z_{i,r}) - \rho_i \right] (d_i + b_i \Delta d_i) \\
\text{subject to} & \quad b_i \in \{0,1\}, \quad \forall i \in I,
\end{align*}
\] (A.6a)

where \(\Delta d_i = \bar{d}_i - d_i\). With the reformulation (A.6), problem (A.4) is equivalent to

\[
\begin{align*}
\text{maximize}_{\lambda, \mu, \theta, b} & \quad \sum_{a \in A} c^p_a (t^\text{end}_a - t^\text{start}_a) h^\text{reg}_a + \sum_{r \in R} c^p_r T^\text{end}_r v_r \\
& \quad + \sum_{i \in I} \left[ \sum_{i' \in I, i' \neq i} (\lambda_{i,i'} - \lambda_{i',i}) + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r} \right] s_i \\
& \quad - M_{\text{end}} \sum_{i \in I} \sum_{i' \in I, i' \neq i} \lambda_{i,i'} (1 - \mu_{i',i}) - \sum_{i \in I} \sum_{a \in A_i} \mu_{i,a} \left[ t^\text{end}_a + M_{\text{max}} (1 - x_{i,a} + y_a) \right] \\
& \quad - \sum_{i \in I} \sum_{r \in R_i} \theta_{i,r} \left[ T^\text{end} + M_{\text{room}} (1 - z_{i,r}) \right] \\
& \quad + \sum_{i \in I} \sum_{i' \in I, i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c^p_a h^\text{reg}_{a} x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c^p_r z_{i,r}) - \rho_i \right] (d_i + b_i \Delta d_i) \\
\text{subject to} & \quad (A.1b) - (A.1e), \\
& \quad b_i \in \{0,1\}, \quad \forall i \in I.
\end{align*}
\] (A.7a)

Finally, to reformulate it into an MILP, we introduce auxiliary variables \(\zeta^L_{i,i'} = \lambda_{i,i'} b_i, \zeta^M_{i,a} = \mu_{i,a} b_i\) and \(\zeta^T_{i,r} = \theta_{i,r} b_i\) with McCormick inequalities given by (6e)–(6g). The existence of the upper bounds of dual variables for McCormick inequalities will be shown in Proposition 3. □

A.4.3. Proof of Proposition 3

To obtain an upper bound for the dual variables, we first derive the complementary properties of the optimal dual solutions in Lemma A.3.

**Lemma A.3.** For a given first-stage decision \((x, y, z, v, u, s)\) and realization \(d\), let \((\lambda^*, \mu^*, \theta^*)\) be an optimal solution to (A.1). Then, we have the following equations:

\[
\mu_{i,a}^* (1 - x_{i,a} + y_a) = 0, \quad \forall (i,a) \in \mathcal{F}^A,
\] (A.8a)
\[
\theta^*_i (1 - z_{i,r}) = 0, \quad \forall (i, r) \in F^R, \quad (A.8b)
\]
\[
\lambda^*_i (1 - u_{i,i'}) = 0, \quad \forall i \in I, i' \in I \setminus \{i\}. \quad (A.8c)
\]

**Proof of Lemma A.3** For brevity, we only argue that \(\lambda^*_i (1 - u_{i,i'}) = 0\) and the remaining two equations can be derived in the same manner. If \(u_{i,i'} = 1\), then the equation holds immediately. Consider the case that \(u_{i,i'} \neq 0\). Then, by a sufficiently large choice of the big \(M\) parameters (see A.3.2), the slack variable associated to constraint (2b) is strictly positive. By the complementary slackness condition, the dual variable \(\lambda^*_i\) is zero. This completes the proof. \(\square\)

**Proof of Proposition 3** The bounds for \(\mu_{i,a}\) follow immediately from the its non-negativity constraint (A.3e) and the constraint (A.3b). Similarly, the bounds for \(\theta_{i,r}\) follow from constraints (A.3d) and (A.3e). For \(\lambda_{i,i'}\), the lower bound follows from (A.3e).

Next, we derive the upper bound for \(\lambda_{i,i'}\). Consider a given realization \(d\) and denote the actual surgery start time as \(q_i\) for \(i \in I\). Note that this is known when given a feasible first-stage decision. Without loss of generality, let \(q_1 \geq q_2 \geq \cdots \geq q_{|I|}\). Define \(C_i = c_i^w + \sum_{a \in A_i} \mu_{i,a} + \sum_{r \in R_i} \theta_{i,r}\) for \(i \in I\). From Lemma A.3, at optimality, we immediately have \(\lambda_{i,i'} = 0\) for any \(i' \geq i\) (since surgery \(i\) cannot precede surgery \(i'\)). Using this observation, the constraints (A.3d) read as follows:

\[
\sum_{i' \geq 2} \lambda_{i',1} \leq c_1^w + \sum_{a \in A_1} \mu_{1,a} + \sum_{r \in R_1} \theta_{1,r},
\]
\[
\sum_{i' \geq 3} \lambda_{i',2} \leq \lambda_{2,1} + c_2^w + \sum_{a \in A_2} \mu_{2,a} + \sum_{r \in R_2} \theta_{2,r},
\]
\[
\sum_{i' \geq 4} \lambda_{i',3} \leq \lambda_{3,1} + \lambda_{3,2} + c_3^w + \sum_{a \in A_3} \mu_{3,a} + \sum_{r \in R_3} \theta_{3,r},
\]
\[
\vdots
\]
\[
\sum_{i' \geq |I| - 1} \lambda_{i',|I| - 1} \leq \lambda_{|I| - 1,1} + \cdots + \lambda_{|I| - 1,|I| - 2} + c_{|I| - 1}^w + \sum_{a \in A_{|I| - 1}} \mu_{2,|I| - 1} + \sum_{r \in R_{|I| - 1}} \theta_{2,|I| - 1}.
\]

Note that the constraint for \(|I|\) (i.e., the last inequality) holds automatically since the summation on the left is zero. If we sum from the first to the \(j\)th inequalities, we obtain

\[
\sum_{i=1}^{j} \sum_{i' = j + 1}^{\min(\lambda_{i',i})} \lambda_{i',i} \leq \sum_{i=1}^{j} C_i, \quad \forall j \in \{1, \ldots, |I| - 1\}.
\]

Note that all \(\lambda_{i,i'}\) with \(i' < i\) appear in at least one of the above \(|I| - 1\) inequalities. Since \(\lambda_{i,i'} \geq 0\), this concludes that an upper bound for \(\lambda_{i,i}\) is given by

\[
\sum_{i=1}^{\min(\lambda_{i,i'})} C_i = \sum_{i \in I} c_i^w + \sum_{i \in I} \sum_{a \in A_i} \mu_{i,a} + \sum_{i \in I} \sum_{r \in R_i} \theta_{i,r} \leq \sum_{i \in I} c_i^w + \sum_{a \in A} (c_a^w + c_a^s) + \sum_{r \in R} (c_r^w + c_r^s),
\]

where the last inequality follows from constraints (A.3b) and (A.3c). \(\square\)
Remark A.2. From the proof of Proposition 3, we could obtain a tighter upper bound for \( \lambda \) by summing over only the largest \(|I| - 1\) constants \( C \). That is,

\[
\lambda_{i,i'} \leq \left( \sum_{i \in I} c_i^w - \min_{i \in I} c_i^w \right) + \sum_{a \in A} (c_a^g + c_a^o) + \sum_{r \in R} (c_r^g + c_r^o).
\]

In particular, if \( c_i^w = c^w \) for all \( i \in I \), \( c_a^g = c_A^g \), \( c_a^o = c_A^o \) for all \( a \in A \) and \( c_r^g = c_R^g \), \( c_r^o = c_R^o \) for all \( r \in R \), the upper bound is given by

\[
\lambda_{i,i'} \leq \left( |I| - 1 \right) c^w + |A|(c_A^g + c_A^o) + |R|(c_R^g + c_R^o).
\]

In Example A.2, we show that this bound is the tightest possible constant upper bound of \( \lambda_{i,i'} \).

Example A.2. Suppose we have one anesthesiologist, two ORs and five surgeries. Their schedules (i.e., first-stage decisions) and the surgery durations are given as follows.

Anesthesiologist 1: 5 → 4 → 3 → 2 → 1
Operating room 1: 5 → 4 → 3
Operating room 2: 2 → 1

\[ s = (400, 300, 200, 100, 0) \quad d = (150, 160, 170, 180, 190) \]

Assume the cost structure \( c^w = 100 \), \( c_A^g = 30 \), \( c_A^o = 150 \), \( c_R^g = 20 \) and \( c_R^o = 450 \). We can solve the second-stage model to obtain the optimal dual solutions, which are given by

\[
\mu = \begin{pmatrix} 180 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \quad \theta = \begin{pmatrix} 0 & 470 \\ 0 & 0 \\ 470 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad \lambda = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 750 & 0 & 0 & 0 & 0 \\ 0 & 850 & 0 & 0 & 0 \\ 0 & 0 & 1420 & 0 & 0 \\ 0 & 0 & 0 & 1520 & 0 \end{pmatrix}.
\]

Note that the upper bound on \( \lambda \) we derived in Proposition 3 is

\[
\left( |I| - 1 \right) c^w + |A|(c_A^g + c_A^o) + |R|(c_R^g + c_R^o) = 4(100) + (30 + 150) + 2(20 + 450) = 1520.
\]

This shows that the upper bound on \( \lambda \) we obtain in the proof of Proposition 3 is tight. It is straightforward to see that the upper bounds on \( \mu \) and \( \theta \) are tight as well.

A.4.4. Proof of Proposition 4

Proof of Proposition 4. For notational simplicity, we suppress the dependence of \( P \) in \( E_P(\cdot) \) and \( P\text{-CVaR}_\gamma(\cdot) \) in the proof. First, recall the definition of CVaR (Rockafellar et al. 2000):

\[
\text{CVaR}_\gamma(Z) = \min_{\tau \in \mathbb{R}} \left\{ \tau + \frac{1}{1 - \gamma} \mathbb{E} \left[ \max\{Z - \tau, 0\} \right] \right\}. \tag{A.9}
\]
With this definition, we have

\[
\sup_{P \in \mathcal{P}(m, \mathcal{S})} \text{CVaR}_\gamma (Q(x, y, z, v, u, s, d)) = \sup_{P \in \mathcal{P}(m, \mathcal{S})} \min_{\tau \in \mathbb{R}} \left\{ \tau + \frac{1}{1-\gamma} \mathbb{E}\left[ \max\{Q(x, y, z, v, u, s, d) - \tau, 0\} \right] \right\}.
\]  
(A.10)

The objective function is convex in \(\tau\) and concave in \(P\). Moreover, the set \(\mathcal{P}(m, \mathcal{S})\) is weakly compact (under the topology of weak convergence of probability measures) (see, e.g., Sun and Xu 2016). Then, by Sion’s minimax theorem (Sion 1958),

\[
\sup_{P \in \mathcal{P}(m, \mathcal{S})} \text{CVaR}_\gamma (Q(x, y, z, v, u, s, d)) = \min_{\tau \in \mathbb{R}} \left\{ \tau + \frac{1}{1-\gamma} \sup_{P \in \mathcal{P}(m, \mathcal{S})} \mathbb{E}\left[ \max\{Q(x, y, z, v, u, s, d) - \tau, 0\} \right] \right\}.
\]  
(A.11)

Applying the same argument for strong duality of the worst-case expectation problem in Proposition 1, we have that \(\sup_{P \in \mathcal{P}(m, \mathcal{S})} \mathbb{E}\left[ \max\{Q(x, y, z, v, u, s, d) - \tau, 0\} \right]\) is equivalent to

\[
\minimize_{\rho_0 \in \mathbb{R}, \rho \in \mathbb{R}^I} \rho_0 + \sum_{i \in I} \rho_i m_i \tag{A.12a}
\]

subject to \(\rho_0 + \sum_{i \in I} \rho_i d_i \geq Q(x, y, z, v, u, s, d) - \tau, \forall d \in \mathcal{S}, \tag{A.12b}\)

\(\rho_0 + \sum_{i \in I} \rho_i d_i \geq 0, \forall d \in \mathcal{S}, \tag{A.12c}\)

where (A.12b) and (A.12c) follows from the definition of \(\max\{\cdot, 0\}\). Note that these constraints hold for all \(d \in \mathcal{S}\). Therefore, we can reformulate (A.12) as

\[
\minimize_{\rho_0 \in \mathbb{R}, \rho \in \mathbb{R}^I} \rho_0 + \sum_{i \in I} \rho_i m_i \tag{A.13a}
\]

subject to \(\tau \geq \max_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\} - \rho_0, \tag{A.13b}\)

\(\rho_0 + \min_{d \in \mathcal{S}} \sum_{i \in I} \rho_i d_i \geq 0. \tag{A.13c}\)

Combining (A.13) with the outer minimization problem in \(\tau\) in (A.11), we derive the following equivalent reformulation of \(\sup_{P \in \mathcal{P}(m, \mathcal{S})} \text{CVaR}_\gamma (Q(x, y, z, v, u, s, d)):\)

\[
\minimize_{\tau \in \mathbb{R}, \rho_0 \in \mathbb{R}, \rho \in \mathbb{R}^I} \frac{1}{1-\gamma} \left( \rho_0 + \sum_{i \in I} \rho_i m_i \right) \tag{A.14a}
\]

subject to \(\tau \geq \max_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\} - \rho_0, \tag{A.14b}\)

\(\rho_0 + \min_{d \in \mathcal{S}} \sum_{i \in I} \rho_i d_i \geq 0. \tag{A.14c}\)
Remark A.3. From constraint (A.15), we note that the following reformulation of sup \( P(\rho) \) that

\[
\min_{\rho_0 \in \mathbb{R}, \rho \in \mathbb{R}^I} \left( \frac{1}{1-\gamma} - 1 \right) \rho_0 + \frac{1}{1-\gamma} \sum_{i \in I} \rho_i m_i + \max_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\}
\]

(A.15a)

subject to \( \rho_0 + \min_{d \in \mathcal{S}} \sum_{i \in I} \rho_i d_i \geq 0. \)

(A.15b)

Finally, by LP duality, we have

\[
\min_{d \in \mathcal{S}} \sum_{i \in I} \rho_i d_i = \max_{\psi \in \Psi(\rho)} \sum_{i \in I} (d_i \psi_i - d_i \overline{\psi}_i),
\]

where \( \Psi(\rho) = \{(\psi_i, \overline{\psi}_i) \in \mathbb{R}_+^I \times \mathbb{R}_+^I \mid \psi_i - \overline{\psi}_i = \rho_i, \forall i \in I\} \). Therefore, if there exists \( \psi \in \Psi(\rho) \) such that \( \rho_0 + \sum_{i \in I} (d_i \psi_i - d_i \overline{\psi}_i) \geq 0 \), constraint (A.15b) holds, and vice versa. Hence, we can derive the following reformulation of sup \( \tau_{\rho} \in \mathcal{P}(\mathcal{M}, \mathcal{S}) \) CVaR \( (Q(x, y, z, v, u, s, d)) \):

\[
\min_{\rho_0 \in \mathbb{R}, \rho \in \mathbb{R}^I, (\psi_i, \overline{\psi}_i) \in \mathbb{R}_+^I \times \mathbb{R}_+^I} \left( \frac{1}{1-\gamma} - 1 \right) \rho_0 + \frac{1}{1-\gamma} \sum_{i \in I} \rho_i m_i + \max_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\}
\]

(A.16a)

subject to

\[
\rho_0 + \sum_{i \in I} (d_i \psi_i - d_i \overline{\psi}_i) \geq 0, \quad (A.16b)
\]

\[
\psi_i - \overline{\psi}_i = \rho_i, \quad \forall i \in I, \quad (A.16c)
\]

\[
\psi_i \geq 0, \overline{\psi}_i \geq 0, \quad \forall i \in I. \quad (A.16d)
\]

This completes the proof. \( \square \)

**Remark A.3.** From constraint (A.15), we note that \( \rho_0^* = -\min_{d \in \mathcal{S}} \sum_{i \in I} \rho_i d_i \) is an optimal solution since the coefficient associated to \( \rho_0 \) in the objective is \( 1/(1-\gamma) - 1 \geq 0 \) for \( \gamma \in (0, 1) \). Therefore, an equivalent reformulation of sup \( \tau_{\rho} \in \mathcal{P}(\mathcal{M}, \mathcal{S}) \) CVaR \( (Q(x, y, z, v, u, s, d)) \) without going through LP duality is

\[
\min_{\rho \in \mathbb{R}^I} \left\{ \frac{1}{1-\gamma} \sum_{i \in I} \rho_i m_i - \left( \frac{1}{1-\gamma} - 1 \right) \min_{d \in \mathcal{S}} \sum_{i \in I} \rho_i d_i + \max_{d \in \mathcal{S}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\} \right\}. \quad (A.17)
\]

However, this does not facilitate the use of the C&CG algorithm (see Section 6.2) since we have two optimization problems inside the objective function. Nevertheless, we will use (A.17) to verify the separability of the DRO-CVaR model.

### A.5. Proofs and Discussions on Valid Inequalities for the DRO-E and DRO-CVaR Models

**A.5.1. Valid Inequalities for the Recourse Function**

**Lemma A.4.** The following lower-bounding inequalities are valid for the DRO-E model.

\[
\sum_{i \in I} m_i \rho_i + \delta \geq \sum_{a \in A} (c_a^o o_a^m + c_a^g g_a^m) + \sum_{r \in R} (c_r^o o_r^m + c_r^g g_r^m) + \sum_{i \in I} c_i^w w_i^m, \quad (A.18a)
\]
\[
q_i^m \geq q_i^m + m_i - M_{seq}(1 - u_{i,i'}) \quad \forall \{i, i'\} \subseteq I, i' \neq i, \tag{A.18b}
\]
\[
q_i^m \geq s_i, \quad \forall i \in I, \tag{A.18c}
\]
\[
o_a^m \geq q_i^m + m_i - l_{a}^{end} - M_{anes}(1 - x_{i,a} + y_a), \quad \forall (i, a) \in \mathcal{F}^A, \tag{A.18d}
\]
\[
o_r^m \geq q_i^m + m_i - T_{end} - M_{room}(1 - z_{i,r}), \quad \forall (i, r) \in \mathcal{F}^R, \tag{A.18e}
\]
\[
w_i^m \geq q_i^m - s_i, \quad \forall i \in I, \tag{A.18f}
\]
\[
g_a^m \geq \left[(l_{a}^{end} - l_{a}^{start}) - \sum_{i \in I_a} m_i x_{i,a} + o_a^m\right] g_a, \quad \forall (i, a) \in \mathcal{F}^A, \tag{A.18g}
\]
\[
g_r^m \geq T_{end} v_r - \sum_{i \in I_r} m_i z_{i,r} + o_r^m, \quad \forall (i, r) \in \mathcal{F}^R, \tag{A.18h}
\]
\[
q_i^m, o_a^m, o_r^m, w_i^m, g_a^m, g_r^m \geq 0, \quad \forall i \in I, \ a \in A, \ r \in R. \tag{A.18i}
\]

**Proof.** Note that we minimize \( \sum_{i \in I} m_i \rho_i + \delta \) in the master problem. This is equivalent to minimizing the right hand side expression of (A.18a), which is the second-stage cost. (A.18b)–(A.18h) are the second-stage constraints with the scenario \( d = m \) (i.e., the mean of the surgery duration). Therefore, in view of the inequality

\[
\sup_{\bar{p} \in \mathcal{P}(m, S)} q_{\bar{p}}(Q(x, y, z, v, u, s, d)) \geq Q(x, y, z, v, u, s, m),
\]

the inequalities are valid. \( \square \)

**Lemma A.5.** The following lower-bounding inequalities are valid for the DRO-CVaR model.

\[
\left(\frac{1}{1 - \gamma} - 1\right) \rho_0 + \frac{1}{1 - \gamma} \sum_{i \in I} \rho_i m_i + \delta \geq \sum_{a \in A} (c_a^o o_a^m + c_a^o g_a^m) + \sum_{r \in R} (c_r^o o_r^m + c_r^o g_r^m) + \sum_{i \in I} c_i^w w_i^m, \tag{A.19a}
\]

(A.18b) – (A.18i).

**Proof.** The validity of inequalities (A.19a)–(A.19b) can be easily verified using the same argument and techniques in the proof of Lemma A.4. \( \square \)

**A.5.2. Proof of Proposition 5**

**Proof of Proposition 5** We focus on the DRO-E model since the same argument applies to the DRO-CVaR model. For simplicity, we suppress the dependence of \( x, z \) and \( u \) in \( \rho \) and \( \bar{p} \). First, we consider the lower bound. For a fixed first-stage decision, suppose that there exists \( i \in I \) such that \( \rho_i^* < \rho \), where \( \rho^* \) is an arbitrary solution. Let \((\lambda^*, \mu^*, \theta^*, d^*)\) be an optimal solution to (A.4), i.e.,

\[
\max_{d \in \mathcal{D}} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i d_i \right\}.
\]

Since \( \rho_i^* < \rho \) in the objective function, the coefficient associated to \( d_i \) is positive. Indeed,

\[
\sum_{i' \in I, i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a}^* - c_a^{reg} h_a x_{i,a}) + \sum_{r \in R_i} (\theta_{i,r}^* - c_r^z z_{i,r}) - \rho_i^* \]
\[
\begin{align*}
\geq & - \sum_{a \in A_i} c_a^g h^\text{reg}_{i,a} x_{i,a} - \sum_{r \in R_i} c_r^z z_{i,r} - \rho_i^* \\
\geq & - \sum_{a \in A_i} c_a^g h^\text{reg}_{i,a} x_{i,a} - \sum_{r \in R_i} c_r^z z_{i,r} - \rho \\
= & 0.
\end{align*}
\]

Therefore, we must have \( d_i^* = \overline{d}_i \). Next, for any \( \epsilon \in (0, \overline{\rho} - \rho_i^* \), define another solution \( \overline{\rho} \) by

\[
\overline{\rho}_j = \begin{cases} 
\rho_j^* + \epsilon, & \text{if } j = i, \\
\rho_j^*, & \text{otherwise.}
\end{cases}
\]

Note that changing \( \rho_i^* \) to \( \overline{\rho}_i \) only affects the coefficient associated to \( d_i \) in the objective (A.4a). By construction of \( \epsilon \), the coefficient associated to \( d_i \) is still positive. Therefore, \( d_i^* = \overline{d}_i \) is still optimal.

After maximizing \( d_i \), the remaining program of (A.4) is the same. That is, we have

\[
\sup_{d \in S} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \overline{\rho}_i d_i \right\} = \sup_{d \in S} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i^* d_i \right\} - \epsilon \overline{d}_i. \tag{A.20}
\]

Hence, we have

\[
\sum_{i \in I} \overline{\rho}_i m_i + \sup_{d \in S} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \overline{\rho}_i d_i \right\} = \left( \sum_{i \in I} \rho_i^* m_i + \epsilon m_i \right) + \sup_{d \in S} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i^* d_i \right\} - \epsilon \overline{d}_i \]

\[
= \left( \sum_{i \in I} \rho_i^* m_i + \sup_{d \in S} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i^* d_i \right\} \right) + \epsilon (m_i - \overline{d}_i) \]

\[
\leq \sum_{i \in I} \rho_i^* m_i + \sup_{d \in S} \left\{ Q(x, y, z, v, u, s, d) - \sum_{i \in I} \rho_i^* d_i \right\}.
\]

The first equality follows from (A.20) while the second equality is obtained by re-arraigning the terms. The inequality follows from \( m_i \leq \overline{d}_i \) and \( \epsilon > 0 \). Therefore, without loss of optimality, we can assume that \( \rho_i \geq \rho_i^* \).

To derive an upper bound for \( \rho_i \), it suffices to determine a upper bound for the coefficient associated to \( d_i \), i.e.,

\[
\sum_{i' \in I, i' \neq i} \lambda_{i,i'} + \sum_{a \in A_i} (\mu_{i,a} - c_a^g h^\text{reg}_{i,a}) + \sum_{r \in R_i} (\theta_{i,r} - c_r^z z_{i,r}).
\]

We can then derive the upper bound by applying a similar argument when deriving the lower bound. Note, from the complementary slackness condition of the second-stage problem (2), \( \lambda_{i,i'} \neq 0 \) if the constraint (2b) achieves equality. Since we only have two resources (anesthesiologist and OR), at most two surgeries will immediately follow surgery \( i \). Hence, using the upper bound on \( \lambda \) provided in Proposition 3, we have \( \sum_{i' \in I, i' \neq i} \lambda_{i,i'} \leq \min \{ \sum_{i' \in I, i' \neq i} u_{i,i'}, 2 \} \lambda \).
Next, we consider the second term. For regular anesthesiologists (i.e., \( h_a^{\text{reg}} = 1 \)), by the complementary slackness condition in Lemma A.3, we have \( \mu_{i,a} = 0 \) if \( x_{i,a} = 0 \). Since \( \mu_{i,a}^l \leq c_a^e + c_a^o \) from Proposition 3, we have \( \mu_{i,a} - c_a^e h_a^{\text{reg}} x_{i,a} \leq c_a^e x_{i,a} \). For on-call anesthesiologists (i.e., \( h_a^{\text{reg}} = 0 \)), we have \( x_{i,a} = 0 \) if \( y_a = 0 \) and this follows that \( \mu_{i,a} = 0 \). On the other hand, if \( y_a = 1 \), then \( \mu_{i,a} = 0 \) from Lemma A.3. Hence, for on-call anesthesiologists, we have \( \mu_{i,a} - c_a^e h_a^{\text{reg}} x_{i,a} = 0 \leq c_a^o x_{i,a} \). Therefore, this results in the desired upper bound \( c_a^e x_{i,a} \). Finally, a similar argument using complementarity condition on the last term yields the desired upper bound for the last term. □

A.6. Separability of the Models

A.6.1. Discussion on Separability of the DRO-E Model

Note that for some instances of the ORSAP, the DRO-E model can be decomposed into a sum of sub-problems. In this appendix, we discuss how the DRO-E model could be decomposed into a sum of sub-problems. Let \( \ell \in \{1, \ldots, L^{\text{spec}}\} \) be the surgery types with dedicated anesthesiologists. First, we rewrite the recourse function as

\[
Q(x, y, z, v, u, s, d) = \sum_{\ell=1}^{L^{\text{spec}}} Q^l(x^l, y^l, z^l, u^l, s^l, d^l) + Q^S(x^S, y^S, z^S, u^S, s^S, d^S),
\]

where the first part of the summation corresponds to each specialized surgery type \( \ell \in \{1, \ldots, L^{\text{spec}}\} \) and the second part corresponds to the remaining surgery types. Here, \( d^l \) is the vector of surgery durations of type \( \ell \) surgeries for \( \ell \in \{1, \ldots, L^{\text{spec}}\} \) and \( d^S \) is the vector of surgery durations of the remaining surgeries. Recall the support of \( d \) is \( S = S^1 \times S^2 \times \cdots \times S^{L^l} \), where \( S^l \) is the support for the surgery of type \( \ell \in \{1, \ldots, L\} \). For notation simplicity, we write \( \chi = (x, y, z, v, u, s) \). Hence,

\[
\begin{align*}
\sup_{\rho \in F(S, m)} & \mathbb{E}_\rho \left[ Q(\chi, d) \right] \\
= & \min_{\rho} \left\{ \sum_{i \in I} \rho_i m_i + \sup_{d \in S} \left\{ Q(\chi, d) - \sum_{i \in I} \rho_i d_i \right\} \right\} \\
= & \min_{\rho} \left\{ \sum_{\ell=1}^{L^{\text{spec}}} (\rho^l)^\top m^l + (\rho^S)^\top m^S + \sup_{d \in S} \left\{ \sum_{\ell=1}^{L^{\text{spec}}} Q^l(\chi^l, d^l) + Q^S(\chi^S, d^S) - \sum_{\ell=1}^{L^{\text{spec}}} (\rho^l)^\top d^l - (\rho^S)^\top d^S \right\} \right\} \\
= & \min_{\rho} \left\{ \sum_{\ell=1}^{L^{\text{spec}}} (\rho^l)^\top m^l + (\rho^S)^\top m^S + \sup_{d \in S^l} \left\{ Q^l(\chi^l, d^l) - (\rho^l)^\top d^l \right\} + \sup_{d \in S^S} \left\{ Q^S(\chi^S, d^S) - (\rho^S)^\top d^S \right\} \right\} \\
= & \sum_{\ell=1}^{L^{\text{spec}}} \min_{\rho^l} \left\{ (\rho^l)^\top m^l + \sup_{d \in S^l} \left\{ Q^l(\chi^l, d^l) - (\rho^l)^\top d^l \right\} \right\} + \min_{\rho^S} \left\{ (\rho^S)^\top m^S + \sup_{d \in S^S} \left\{ Q^S(\chi^S, d^S) - (\rho^S)^\top d^S \right\} \right\} \\
= & \sum_{\ell=1}^{L^{\text{spec}}} \mathbb{E}_{\rho^l} \left[ Q(\chi^l, d^l) \right] + \mathbb{E}_{\rho^S} \left[ Q(\chi^S, d^S) \right].
\end{align*}
\]

Equation (A.21) follows from Proposition 1 while equation (A.22) follows from the recourse function decomposition. In particular, vectors \( \rho^l \) and \( m^l \) correspond to entries of surgery type \( \ell \) while vectors...
$\rho^g$ and $m^g$ correspond to entries of the remaining surgeries. Equation (A.23) makes use of the separability of $S = S^1 \times \cdots \times S^{L^{\text{spec}}} \times S^g$, where $S^g$ is the support of surgery durations for types $L^{\text{spec}}+1$ to $L$. Equation (A.24) follows from the separability of the objective function in $\rho$. Equation (A.25) is a direct consequence of Proposition 1.

The derivation also shows that worst-case distribution of the DRO-E model takes the form $\mathbb{P}_1 \times \cdots \times \mathbb{P}_L^{\text{spec}} \times \mathbb{P}_g^+$ for some $\mathbb{P}_\ell \in \mathcal{F}(S^\ell, m^\ell)$ and $\mathbb{P}_g^+ \in \mathcal{F}(S^g, m^g)$. That is, surgery durations of type $\ell$ and $\ell'$ are independent for any $\{\ell, \ell'\} \subseteq \{1, \ldots, L^{\text{spec}}\}$ while the surgery durations of types $\ell \in \{L^{\text{spec}}+1, \ldots, L\}$ are not necessarily independent (i.e., they may be correlated). This is because these surgeries (i.e., of type $L^{\text{spec}}+1$ to $L$) share a common pool of anesthesiologists. Hence, the anesthesiologist may perform surgeries of multiple types, and this leads to the possibility that the surgery durations of these types are not independent in the worst case.

### A.6.2. Separability of the DRO-CVaR Model

Next, we show that the DRO-CVaR model can be decomposed into a sum of sub-problems as in the DRO-E model. Note that $\mathbb{P}^-\text{CVaR}_\gamma(\cdot)$ is subadditive, i.e., $\mathbb{P}^-\text{CVaR}_\gamma(Z_1+Z_2) \leq \mathbb{P}^-\text{CVaR}_\gamma(Z_1) + \mathbb{P}^-\text{CVaR}_\gamma(Z_2)$ for any integrable random variables $Z_1$ and $Z_2$. Therefore, if we use $\mathbb{P}^-\text{CVaR}_\gamma(\cdot)$ directly as our objective, we may not be able to obtain such a decomposition.

To study the separability of the DRO-CVaR model, using the notation in A.6.1, we can rewrite the recourse function as

$$Q(x, y, z, v, u, s, d) = \sum_{\ell=1}^{L^{\text{spec}}} Q^\ell(x^\ell, y^\ell, z^\ell, u^\ell, s^\ell, d^\ell) + Q^g(x^g, y^g, z^g, v^g, u^g, s^g, d^g).$$

Recall the support of $d$ is $S = S^1 \times S^2 \times \cdots \times S^L$, where $S^\ell$ is the support for the surgery of type $\ell \in \{1, \ldots, L\}$. Again, for notation simplicity, we write $\chi = (x, y, z, v, u, s)$. Hence, we have

$$\sup_{\mathbb{P} \in \mathcal{F}(S, \mathbb{P}^-\text{CVaR}_\gamma(\cdot))} \mathbb{P}^-\text{CVaR}_\gamma(Q(\chi, d))$$

$$= \min_{\rho} \left\{ \frac{1}{1-\gamma} \sum_{i \in I} \rho_i m_i - \left( \frac{1}{1-\gamma} - 1 \right) \min_{d \in S} \sum_{i \in I} \rho_i d_i + \sup_{d \in S} \left\{ Q(\chi, d) - \sum_{i \in I} \rho_i d_i \right\} \right\} \quad (A.26)$$

$$= \min_{\rho} \left\{ \frac{1}{1-\gamma} \sum_{\ell=1}^{L^{\text{spec}}} (\rho^\ell)^\top m^\ell + \frac{1}{1-\gamma} (\rho^g)^\top m^g - \left( \frac{1}{1-\gamma} - 1 \right) \min_{d \in S} \left\{ \sum_{\ell=1}^{L^{\text{spec}}} (\rho^\ell)^\top d^\ell + (\rho^g)^\top d^g \right\} \right\}$$

$$\sup_{d \in S} \left\{ \sum_{\ell=1}^{L^{\text{spec}}} Q^\ell(\chi^\ell, d^\ell) + Q^g(\chi^g, d^g) - \sum_{\ell=1}^{L^{\text{spec}}} (\rho^\ell)^\top d^\ell - (\rho^g)^\top d^g \right\} \quad (A.27)$$

$$= \min_{\rho} \left\{ \frac{1}{1-\gamma} \sum_{\ell=1}^{L^{\text{spec}}} (\rho^\ell)^\top m^\ell + \frac{1}{1-\gamma} (\rho^g)^\top m^g - \left( \frac{1}{1-\gamma} - 1 \right) \left[ \sum_{d \in S^{\ell}} \min_{\ell=1}^{L^{\text{spec}}} (\rho^\ell)^\top d^\ell + \min_{d \in S^g} (\rho^g)^\top d^g \right] \right\}$$

$$\sum_{\ell=1}^{L^{\text{spec}}} \sup_{d \in S^{\ell}} \left\{ Q^\ell(\chi^\ell, d^\ell) - (\rho^\ell)^\top d^\ell \right\} + \sup_{d \in S^g} \left\{ Q^g(\chi^g, d^g) - (\rho^g)^\top d^g \right\} \quad (A.28)$$
Table A.3 Two different assignments of surgeries to anesthesiologists

| Assignment 1 | Anes 1 | Anes 2 | Anes 3 |
|--------------|--------|--------|--------|
| Assignment 2 | 1, 5, 6| 2, 3   | 4      |

\[
S = \rho \quad \text{decomposition. In particular, vectors } \rho \quad \text{Equation (A.26) follows from (A.17) while equation (A.27) follows from the recourse function A.7.1.}
\]

**Additional Symmetry-Breaking Constraints**

**A.7. Discussion and Examples on Symmetry-Breaking Constraints**

**A.7.1. Additional Symmetry-Breaking Constraints**

**Operating room opening order.** Suppose that there are three ORs of type \( \ell \). The solutions \( v^1 := (1, 0, 1)^T \), \( v^2 := (1, 1, 0)^T \) and \( v^3 := (0, 1, 1)^T \) are equivalent in the sense that they all open two out of the three ORs available for type \( \ell \). To avoid exploring such equivalent solutions, we assume that ORs of the same type are numbered sequentially, i.e., \( R_\ell := \{r_{1,\ell}, \ldots, r_{|R_\ell|,\ell}\} \), and add the following inequalities (see Denton et al. 2010) to avoid arbitrary opening of ORs:

\[
v_{r_{k-1,\ell}} \geq v_{r_{k,\ell}}, \quad \forall \ell \in L, k \in [2, |R_\ell|]. \tag{A.31}
\]

**Variable fixing on OR assignment.** Note that we always assign surgery \( i_{j, \ell} \) to an OR with index \( r_{k, \ell} \) where \( k \leq j \) (see, for example, Denton et al. 2010 and Vo-Thanh et al. 2018). Therefore, we impose the following constraints:

\[
z_{i_{j, \ell}, r_{k, \ell}} = 0, \quad \forall \ell \in L, j \in \min\{|I_\ell|, |R_\ell|\}, k \in \{j + 1, \ldots, |R_\ell|\}. \tag{A.32}
\]

**Anesthesiologist assignment order.** Symmetry also exists in anesthesiologists assignment. Recall that anesthesiologists in set \( A_{\ell}^{\text{spec}} = \{a_{1, \ell}, \ldots, a_{|A_{\ell}^{\text{spec}}|, \ell}\} \) are dedicated to a specific type of surgery \( \ell \in \{1, \ldots, L^{\text{spec}}\} \), while those in set \( A_{\ell}^{\text{gen}} = \{a_{1, \ell}, \ldots, a_{|A_{\ell}^{\text{gen}}|, \ell}\} \) can perform multiple surgery types. Suppose we have 3 homogeneous anesthesiologists (i.e., same type with a single shift) and
6 surgeries. Then, the two assignments in Table A.3 result in the same objective value. To partly address this kind of symmetry, we introduce constraints

\[ x_{i_1, l, a_1, l} = 1, \quad \forall l \in [L^{\text{spec}}] , \quad (A.33) \]

\[ x_{i_1, g, a_1, g} = 1. \quad (A.34) \]

Constraints (A.33) and (A.34) enforce that the first surgery is assigned to the anesthesiologist with the smallest index. Note that one can also fully break the anesthesiologists order symmetry by introducing lexicographical ordering. However, as pointed out in Vo-Thanh et al. (2018) for another context, this may lead to poor computational results, which we observe in our numerical experiments. For completeness, we also provide these ordering inequalities in A.7.4.

**On-call anesthesiologist order.** Suppose that there are three on-call anesthesiologists of the same type. Then, \( y^1 = (1, 0, 0)^\top \), \( y^2 = (0, 1, 0)^\top \) and \( y^3 = (0, 0, 1)^\top \) are equivalent since only one of the three anesthesiologists is called. To prevent such equivalent solutions, we add the following equivalent solutions, we add the following inequalities to the set of on-call anesthesiologists of the same type \( A^{\text{call}} := \{a_1, \ldots, a_{|A^{\text{call}}|}\} \):

\[ y_{a-1} \leq y_a, \quad \forall a \in [2, |A^{\text{call}}|]. \quad (A.35) \]

### A.7.2. Symmetry-Breaking Constraints on Surgery Assignment Order

We note that either the set of constraints (16) or constraints (17) would suffice to enforce the surgery index ordering. However, these two sets of constraints are not equivalent in the sense that there exist feasible solutions to the former one but not the latter one and vice versa (see Example A.3) in the LP relaxation. Therefore, using both of them could tighten the LP relaxation.

**Example A.3.** Suppose we have surgeries 1 to 5 and ORs 1 to 3 of same type. We write \( Z = (z_{i,r}) \) as a \( 5 \times 3 \) matrix to denote the set of \( z \) variables (i.e., surgery-to-OR assignments). Consider the matrices:

\[
Z^1 = \begin{pmatrix}
1 & 0 & 0 \\
0.6 & 0.4 & 0 \\
0.4 & 0.2 & 0.4 \\
0.2 & 0.4 & 0.4 \\
0.25 & 0.25 & 0.5
\end{pmatrix}, \quad Z^2 = \begin{pmatrix}
1 & 0 & 0 \\
0.6 & 0.4 & 0 \\
0.4 & 0.2 & 0.4 \\
0.25 & 0.25 & 0.5 \\
0.2 & 0.4 & 0.4
\end{pmatrix}.
\]

Note that \( Z^1 \) is feasible to constraints (17) but not (16) since we have \( z_{5,1} \not\leq z_{4,1} \). On the other hand, \( Z^2 \) is feasible to constraints (16) but not (17) since \( z_{4,3} \not\leq z_{5,3} \). This shows that using both sets of constraints could tighten the linear relaxation.
Table A.4  Two different assignments of surgeries to anesthesiologists

| Anes. | Shift 1 | Shift 2 |
|-------|---------|---------|
| 1     | 6 → 7 → 8 / |         |
| 2     | 1 → 2 → 3 / |         |
| 3     | 12 → 13 /  |         |
| 4     | / 14 → 15  |         |
| 5     | / 4 → 5    |         |
| 6     | / 9 → 10 → 11 |     |

(b) Assignment 2

| Anes. | Shift 1 | Shift 2 |
|-------|---------|---------|
| 2     | 6 → 7 → 8 / |         |
| 1     | 1 → 2 → 3 / |         |
| 3     | 12 → 13 /  |         |
| 6     | / 14 → 15  |         |
| 4     | / 4 → 5    |         |
| 5     | / 9 → 10 → 11 |     |

A.7.3. Alternative Symmetry-Breaking Constraints on Surgery Assignment Order

We can employ constraints (16)–(18) if durations of surgeries of the same type have the same distribution. In the following, we propose an alternative set of symmetry-breaking constraints when there are sub-types within a surgery type, assuming durations of surgeries belonging to the same sub-type have the same distribution. Mathematically, let $L_{\ell}$ be the set of sub-types for a given type $\ell \in L$. Then, we could impose the following constraints:

$$z_{i,j,\ell,t,r_{k,\ell}} \leq \sum_{k' = 1}^{k} z_{i_{j-1},\ell,t,r_{k',\ell}} \quad \forall \ell \in L, \ell' \in L_{\ell}, j \in [2, |I_{\ell}|], k \in [|R_{\ell}|],$$  \hspace{1cm} (A.36)

$$z_{i_{j-1},\ell,t,r_{k,\ell}} \leq \sum_{k' = k}^{R_{\ell}} z_{i,j,\ell,t,r_{k',\ell}} \quad \forall \ell \in L, \ell' \in L_{\ell}, j \in [2, |I_{\ell}|], k \in [|R_{\ell}|],$$  \hspace{1cm} (A.37)

$$u_{i_{j-1},\ell,t,i_{j},\ell,t} \geq z_{i_{j-1},\ell,t,r_{k,\ell}} + z_{i,j,\ell,t,r_{k,\ell}} - 1, \quad \forall \ell \in L, \ell' \in L_{\ell}, j \in [2, |I_{\ell}|], k \in [|R_{\ell}|].$$  \hspace{1cm} (A.38)

For a given type $\ell \in L$ and its sub-type $\ell' \in L_{\ell}$, constraints (A.36) enforce that if surgery $i_{j,\ell}$ is assigned to OR $r_{k,\ell}$, then surgery $i_{j-1,\ell}$ is assigned to any OR with index at most $k$. One the other hand, constraints (A.37) enforce that if surgery $i_{j-1,\ell}$ is assigned to OR $r_{k,\ell}$, then surgery $i_{j,\ell}$ is assigned to any OR with index at least $k$. Finally, constraints (A.38) ensure that surgeries of the same sub-type in the same OR are sequenced in ascending order of their indices.

A.7.4. Symmetry-Breaking Constraints on Anesthesiologist Assignment

We derive symmetry breaking constraints that could address the symmetry in anesthesiologist assignments in full. We first study the set of anesthesiologists $A_{\text{spec}}^{\ell}$ that are dedicated to only one surgery type $\ell \in \{1, \ldots, L_{\text{spec}}\}$. To illustrate the symmetry, suppose we have 6 anesthesiologists of two shifts and 11 surgeries to be assigned. Table A.4 shows two equivalent assignments that yield the same objective value. Symmetry exists since perturbing anesthesiologists from the same shift would not change the objective value.

Hence, we could impose an ordering on the anesthesiologist assignments. In particular, we make use of the descending lexicographical ordering on the assignment vector. For each set of surgery,
we associate the assignment decision with a vector of zeros except that at position $i$ where surgery $i$ is assigned, this entry takes value one. For example, for the set of surgeries $6 \to 7 \to 8$, the assignment vector is a vector of zeros with entries being ones at the 6th to 8th position. Write $A_{t}^{\text{spec}} = \cup_{t \in \mathcal{T}} A_{t}^{\text{spec},t}$ as the disjoint union of the sets of anesthesiologists belonging to shift $t \in \mathcal{T}$, where $\mathcal{T}$ is the set of possible shifts. Then, the constraints

$$
\sum_{j=1}^{[I_{t}]} 2^{[I_{t}]-j} x_{i_{j},t,a_{k_{j},t}} \geq \sum_{j=1}^{[I_{t}]} 2^{[I_{t}]-j} x_{i_{j},t,a_{k_{j},t}}^{\ell}, \quad \forall \ell \in [L^{\text{spec}}], \ t \in \mathcal{T}, \ k \in [2, [A_{t}^{\text{spec},t}]], \quad (A.39)
$$

where $A_{t}^{\text{spec},t} := \{a_{1,t}^{t}, \ldots, a_{[I_{t}]}^{t}\}$, enforces a descending order in the assignment vectors. Similarly, for set of anesthesiologists $A_{\text{gen}}$ that could perform multiple types, let $I_{\text{gen}} := \{i_{1,\text{g}}, \ldots, i_{[I_{\text{gen}}]}^{\text{g}}\}$ be the set of surgeries of general type. Write $A_{\text{gen},t} := \{a_{1,\text{g},t}^{t}, \ldots, a_{[I_{\text{gen},t}]}^{t}\}$. Then, we can impose the following set of constraints:

$$
\sum_{j=1}^{[I_{\text{gen}}]} 2^{[I_{\text{gen}}]-j} x_{i_{j,\text{g}},a_{k_{j},\text{g}}} \geq \sum_{j=1}^{[I_{\text{gen}}]} 2^{[I_{\text{gen}}]-j} x_{i_{j,\text{g}},a_{k_{j},\text{g}}}^{\ell}, \quad \forall t \in \mathcal{T}, \ k \in [2, [A_{\text{gen},t}]], \quad (A.40)
$$

### A.8. Additional details and results on numerical experiments

#### A.8.1. Sample average approximation of the SP-E and SP-CVaR models

We provide the sample average approximation (SAA) approach for the SP-E and SP-CVaR models. Given a set of finite scenarios $\{d^{n}\}_{n=1}^{N}$, the SAA approach is to solve the SP-E model with the empirical distribution based on $\{d^{n}\}_{n=1}^{N}$. That is,

$$
\begin{align*}
\text{minimize} & \quad \sum_{r \in R} c_{r} v_{r} + c_{y} \sum_{a \in A} y_{a} + \frac{1}{N} \sum_{n=1}^{N} \left[ c_{a}^{g} g_{a}^{n} + c_{a}^{q} q_{a}^{n} + \sum_{i \in I} c_{i}^{w} w_{i}^{n} \right] \\
\text{subject to} & \quad (1b) - (1q), \\
& \quad q_{i}^{r} \geq q_{i}^{a} + d_{i}^{a} - M_{\text{pen}} (1 - u_{i,r}), \quad \forall \{i, i'\} \subseteq I, \ i \neq i', \ n \in [N], \\
& \quad q_{i}^{a} \geq s_{i}, \quad \forall i \in I, \ n \in [N], \\
& \quad a_{i}^{n} \geq q_{i}^{n} + d_{i}^{n} - t_{\text{end}}^{a} - M_{\text{ana}} (1 - x_{i,a}^{n} + y_{a}), \quad \forall (i, a) \in F^{A}, \ n \in [N], \\
& \quad a_{i}^{n} \geq q_{i}^{n} + d_{i}^{n} - T_{\text{end}}^{a} - M_{\text{room}} (1 - z_{i,r}), \quad \forall (i, r) \in F^{R}, \ n \in [N], \\
& \quad w_{i}^{n} \geq q_{i}^{n} - s_{i}, \quad \forall i \in I, \ n \in [N], \\
& \quad g_{a}^{n} \geq \left( t_{\text{end}}^{a} - t_{\text{start}}^{a} - \sum_{i \in I_{a}} d_{i}^{n} x_{i,a} \right) h_{a}^{\text{reg}} + o_{a}^{n}, \quad \forall a \in A, \ n \in [N], \\
& \quad g_{r}^{n} \geq T_{\text{end}}^{r} v_{r} - \sum_{i \in I_{r}} d_{i}^{n} z_{i,r} + o_{r}^{n}, \quad \forall r \in R, \ n \in [N], \\
& \quad q_{i}^{n}, a_{i}^{n}, o_{i}^{n}, w_{i}^{n}, g_{a}^{n}, g_{r}^{n} \geq 0, \quad \forall i \in I, \ a \in A, \ r \in R, \ n \in [N].
\end{align*}
$$

(A.41a - A.41j)
Next, for the SP-CVaR model, using the definition of CVaR in (A.9), we can reformulate the SP-CVaR model as follows:

$$\begin{align*}
\text{minimize} & \quad \sum_{r \in R} f_r v_r + \sum_{a \in A} f_a y_a + \left\{ \tau + \frac{1}{1 - \gamma} \mathbb{E}_p \left[ \max \left\{ Q(x, y, z, v, u, s, d) - \tau, 0 \right\} \right] \right\} \\
\text{subject to} & \quad (1b) - (1q) \text{.} 
\end{align*} \quad (A.42a)$$

Given a set of finite scenarios \( \{d^n\}_{n=1}^N \), we can introduce auxiliary variable \( \eta_n \) to linearize the \( \max \{\cdot, 0\} \) operator in the objective. Therefore, we have the following reformulation of the SP-CVaR model.

$$\begin{align*}
\text{minimize} & \quad \sum_{r \in R} f_r v_r + \sum_{a \in A} f_a y_a + \left\{ \tau + \frac{1}{N(1 - \gamma)} \sum_{n=1}^N \eta_n \right\} \\
\text{subject to} & \quad (1b) - (1q) \text{,} \\
\eta_n & \geq 0, \quad \eta_n \geq Q(x, y, z, v, u, s, d^n) - \tau, \ \forall n \in [N] \text{.} \quad (A.43c)
\end{align*} \quad (A.43b)$$

Substituting \( Q(x, y, z, v, u, s, d^n) \) by the second-stage problem (2), we obtain the final formulation of the SP-CVaR model.

$$\begin{align*}
\text{minimize} & \quad \sum_{r \in R} c_r v_r + c_q \sum_{a \in A} y_a + \left\{ \tau + \frac{1}{N(1 - \gamma)} \sum_{n=1}^N \eta_n \right\} \\
\text{subject to} & \quad (1b) - (1q) \text{,} \\
\eta_n & \geq \sum_{a \in A} (c^g_a g^n_a + c^o_a o^n_a) + \sum_{r \in R} (c^g_r g^n_r + c^o_r o^n_r) + \sum_{i \in I} c_i w^n_i - \tau, \ \forall n \in [N] \text{,} \quad (A.44c) \\
\eta_n & \geq 0, \quad (2b) - (2i), \ \forall n \in [N] \text{.} \quad (A.44d)
\end{align*} \quad (A.44b)$$

### A.8.2. Details and Additional Results for Section 8

#### A.8.2.1. Instance Details

We first provide the summary statistics of the surgery duration for different specialties. In Table A.5, we present the mean, standard deviation, minimum, and maximum of the random surgery duration of each type for instances 1 to 6. The statistics are computed based on the data set from Mannino et al. (2010). The minimum and maximum surgery duration are chosen as the 20% and 80% percentiles of the surgery data respectively. Next, Table A.6 presents the details for instances 1 to 6 with six surgery types (CARD: Cardiology, ORTH: Orthopedics, GYN: Gynecology, MED: Medicine, GASTRO: Gastroenterology, URO: Urology).

#### A.8.2.2. Additional Results for Computational Time

In this section, we provide additional results for computational time of instances 1–6 under costs 2 and 3 (see Section 8.1 for detailed experiment settings). Table A.7 presents the average solution times in seconds under costs 2 and 3. For the SP-E, DRO-E, and DRO-CVaR models, we can
Table A.5  Summary statistics of surgery duration for instances 1 to 6 (Std. Dev.: standard deviation)

| Type            | CARD | ORTH | GYN | MED  | GASTRO | URO |
|-----------------|------|------|-----|------|--------|-----|
| Mean            | 99   | 142  | 78  | 75   | 132    | 72  |
| Std. Dev.       | 53   | 58   | 52  | 42   | 76     | 38  |
| Minimum         | 54   | 87   | 31  | 37   | 66     | 44  |
| Maximum         | 143  | 188  | 121 | 111  | 194    | 94  |

Table A.6  Instance details based on data from Mannino et al. (2010). Note: MED, GASTRO, and URO surgeries can be covered by the same pool of anesthesiologists. For example, in instance 1, there are two regular and one on-call anesthesiologists that can perform MED and GASTRO surgeries.

Instance 1
Surgery type
Number of surgeries
Number of ORs
Number of anesthesiologists (regular)
Number of anesthesiologists (on call)

Instance 2
Surgery type
Number of surgeries
Number of ORs
Number of anesthesiologists (regular)
Number of anesthesiologists (on call)

Instance 3
Surgery type
Number of surgeries
Number of ORs
Number of anesthesiologists (regular)
Number of anesthesiologists (on call)

Instance 4
Surgery type
Number of surgeries
Number of ORs
Number of anesthesiologists (regular)
Number of anesthesiologists (on call)

Instance 5
Surgery type
Number of surgeries
Number of ORs
Number of anesthesiologists (regular)
Number of anesthesiologists (on call)

Instance 6
Surgery type
Number of surgeries
Number of ORs
Number of anesthesiologists (regular)
Number of anesthesiologists (on call)

obtain near optimal solutions within 3 minutes for small to medium-sized instances and within 3 hours for most large instances. Also, we can solve the SP-CVaR model within 1 hour for small to medium-sized instances. While the observations are similar to those under cost 1, we note that the computational times under costs 2 and 3 are longer. We attribute this observation to the inclusion of the OR idle time and anesthesiologist idle time (for cost 3) in the objective function. That is, the objective is a weighted sum of multiple conflicting performance metrics: waiting time, overtime, and idle time.
Table A.7  Computational time (in s) with costs 2 and 3

| Cost 2   | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|----------|------------|------------|------------|------------|------------|------------|
| SP       | 1.17       | 2.72       | 3.81       | 16.67      | 153.63     | 2076.62    |
| CVaR     | 4.30       | 43.00      | 30.32      | 1744.67    | NA         | NA         |
| DRO      | 8.58       | 13.97      | 16.14      | 88.15      | 706.52     | 5573.29    |
| DRO-CVaR | 5.08       | 19.64      | 9.55       | 72.02      | 889.70     | 8194.48    |

| Cost 3   | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|----------|------------|------------|------------|------------|------------|------------|
| SP       | 1.23       | 2.94       | 3.62       | 14.46      | 211.80     | 2327.63    |
| CVaR     | 4.31       | 46.27      | 30.35      | 2655.75    | NA         | NA         |
| DRO      | 8.50       | 13.78      | 16.33      | 147.38     | 714.91     | 6239.39    |
| DRO-CVaR | 4.25       | 45.02      | 9.43       | 95.29      | 638.71     | 9275.96    |

Table A.8  DRO-E computational time (in s) using variable-free VIs (15) or variable-dependent VIs (14)

| Cost 1   | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|----------|------------|------------|------------|------------|------------|------------|
| Variable-Free | 7.20       | 28.21      | 14.14      | 102.95     | 152.11     | 2845.10    |
| Variable-Dependent | 6.53       | 60.66      | 9.23       | 45.62      | 186.91     | 4455.80    |

| Cost 2   | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|----------|------------|------------|------------|------------|------------|------------|
| Variable-Free | 8.58       | 13.97      | 16.14      | 88.15      | 706.52     | 5573.29    |
| Variable-Dependent | 6.56       | 11.33      | 9.61       | 68.32      | 646.44     | 9325.73    |

| Cost 3   | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|----------|------------|------------|------------|------------|------------|------------|
| Variable-Free | 8.50       | 13.78      | 16.33      | 147.38     | 714.91     | 6239.39    |
| Variable-Dependent | 6.95       | 13.54      | 10.55      | 191.87     | 681.04     | 9804.87    |

A.8.2.3. Comparisons of Valid Inequalities for the DRO-E Model

In this section, investigate the computational performance of the DRO-E model when solving it with the variable-dependent valid inequalities (VIs) (14) and the variable-free VIs (15). Specifically, we follow the same experiment settings detailed in Section 8.2 to solve the DRO-E model with either VIs (14) or the VIs (15). We keep VIs (13) in both cases. Table A.8 summarizes the average solution times under costs 1–3. We observe that solution times under the variable-free version are generally similar to or shorter than solution times under the variable-dependent version. In particular, solution times under the variable-free version are significantly shorter for the largest instance, instance 6. This may be explained by the increased model complexity when including VIs (14), which involve the first-stage variables.

A.8.2.4. Additional Results for Solution Quality

In this section, we present additional out-of-sample simulation results for instances 1–6 under costs 1–3. First, Tables A.9–A.11 present respectively the average waiting time, OR overtime, and anesthesiologists overtime under cost 1. (In the tables, we abbreviate DRO-CVaR as D-CVaR.) As discussed in Section 8.5, the DRO-E and DRO-CVaR models yield significantly shorter waiting times but slightly longer OR and anesthesiologist overtime compared with the SP-E and SP-CVaR models.

Next, Tables A.12–A.15 present respectively the average waiting time, OR overtime, anesthesiologists overtime, and OR idle time under cost 2. (Recall that in this cost structure, we additionally...
include the cost of the OR idle time in the objective). Similar to our observations under cost 1, the DRO-E and DRO-CVaR models yield significantly shorter waiting times than the other models. Moreover, since the DRO-E and DRO-CVaR models open more ORs and allocate more time to each surgery (to protect against potential long surgery durations and excessive delays), both the DRO-E and DRO-CVaR models yield slightly longer OR idle time compared with the SP-E and SP-CVaR models.

Finally, Tables A.16–A.20 present respectively the average waiting time, OR overtime, anesthesiologists overtime, OR idle time, and anesthesiologist idle time under cost 3. (Recall that in this cost structure, we additionally include the anesthesiologist’s idle time in the objective). Again, we have observations similar to those made for cost 1 regarding waiting time, overtime, and OR idle time. Also, we observe that DRO-E and DRO-CVaR models yield slightly longer anesthesiologist idle time when compared with the SP-E and SP-CVaR models, mainly because the DRO-E and DRO-CVaR models allocate more time to each surgery.

A.8.2.5. Additional Results for the RO Model in Rath et al. (2017)

We first present the details of the RO model proposed by Rath et al. (2017). The RO model minimizes the fixed costs and overtime costs under the worst-case scenario residing in the uncertainty set given by

$$D(\tau) = \left\{ d \in \mathbb{R}^{|I|} \mid d_i = m_i + f_i \hat{d}_i, i \in I, f \in F(\tau') \right\},$$

where $F(\tau') = \left\{ f \in \mathbb{R}^{|I|} \mid \sum_{i \in I} |f_i| \leq \tau', -1 \leq f_i \leq 1 \right\}$. That is, the surgery duration is a sum of $m_i$ and $\hat{d}_i$, while $f_i$ controls the deviation from the mean $m_i$. The parameter $\tau'$ controls the allowable amount of deviation. For the parameters, we choose $\tau' = \lceil \tau |I| \rceil$ as suggested in Rath et al. (2017) and $\hat{d}_i = \min\{\hat{d}_i - m_i, m_i - \hat{d}_i\}$ to mimic the lower and upper bounds in our DRO-E model.

Next, we provide further discussions and comparisons between our SP-E and DRO-E models, and the RO model proposed in Rath et al. (2017) based on the experiment setting in Section 8.6. We start with comparing the optimal assignments as shown in Table A.21. In the RO model, the worst-case surgery duration can either be the mean $m_i$ or the upper bound $m_i + \hat{d}_i$ (i.e., when $f_i = 1$), and $\tau' = \lceil \tau |I| \rceil$ controls the number of surgeries that can deviate from mean. Surgeries highlighted in blue in Table A.21 are those surgeries taking the maximum surgery duration $m_i + \hat{d}_i$.

First, we observe that the OR assignments are the same across different models. Moreover, the anesthesiologist assignments for SP-E and DRO-E models are the same. However, the optimal anesthesiologist assignments in the RO model are different from both SP-E and DRO-E models. In particular, when $\tau = 0.2$, the RO model assigns six surgeries to anesthesiologist 3, which may lead to huge overtime as we will show next.
### Table A.9  Average waiting time under cost 1

| Waiting Time | Instance 1 | Instance 2 | Instance 3 |
|--------------|------------|------------|------------|
|              | SP-E SP-CVaR DRO-E D-CVaR | SP-E SP-CVaR DRO-E D-CVaR | SP-E SP-CVaR DRO-E D-CVaR |
| Setting I    | 140 61 99 34 | 259 91 0 0 | 161 87 100 35 |
| Setting IIa  | 120 40 100 36 | 250 69 0 0 | 142 59 101 36 |
| Setting IIb  | 235 125 164 83 | 469 176 62 69 | 284 195 197 124 |
| Setting IIc  | 291 178 211 125 | 567 235 123 136 | 372 296 275 223 |
| Setting IIIa | 123 43 99 35 | 257 74 0 0 | 143 62 98 35 |
| Setting IIIb | 307 184 206 118 | 620 247 110 121 | 366 281 253 183 |
| Setting IIIc | 535 389 406 293 | 1026 487 350 376 | 707 648 559 538 |
| Setting IV   | 308 206 241 156 | 546 258 167 186 | 425 367 338 308 |

### Table A.10  Average OR overtime under cost 1

| OR Overtime | Instance 1 | Instance 2 | Instance 3 |
|-------------|------------|------------|------------|
|              | SP-E SP-CVaR DRO-E D-CVaR | SP-E SP-CVaR DRO-E D-CVaR | SP-E SP-CVaR DRO-E D-CVaR |
| Setting I    | 95 162 192 241 | 116 41 69 69 | 95 155 199 249 |
| Setting IIa  | 91 154 205 252 | 115 28 71 71 | 90 147 214 264 |
| Setting IIb  | 150 207 258 298 | 212 91 113 110 | 163 216 275 315 |
| Setting IIc  | 179 236 283 322 | 256 128 141 136 | 207 256 311 354 |
| Setting IIIa | 90 155 203 251 | 118 29 70 70 | 89 149 212 262 |
| Setting IIIb | 189 243 291 328 | 280 134 140 136 | 210 260 315 350 |
| Setting IIIc | 327 373 422 450 | 480 304 290 271 | 403 443 491 530 |
| Setting IV   | 186 245 279 320 | 241 144 153 144 | 221 270 316 363 |

### Table A.11  Average anesthesiologist overtime under cost 1

| Anes Overtime | Instance 1 | Instance 2 | Instance 3 |
|---------------|------------|------------|------------|
|               | SP-E SP-CVaR DRO-E D-CVaR | SP-E SP-CVaR DRO-E D-CVaR | SP-E SP-CVaR DRO-E D-CVaR |
| Setting I     | 95 162 192 241 | 116 41 69 69 | 95 155 199 249 |
| Setting IIa   | 91 154 205 252 | 115 28 71 71 | 90 147 214 264 |
| Setting IIb   | 150 207 258 298 | 212 91 113 110 | 163 216 275 315 |
| Setting IIc   | 179 236 283 322 | 256 128 141 136 | 207 256 311 354 |
| Setting IIIa  | 90 155 203 251 | 118 29 70 70 | 89 149 212 262 |
| Setting IIIb  | 189 243 291 328 | 280 134 140 136 | 210 260 315 350 |
| Setting IIIc  | 327 373 422 450 | 480 304 290 271 | 403 443 491 530 |
| Setting IV    | 186 245 279 320 | 241 144 153 144 | 221 270 316 363 |
### Table A.12  Average waiting time under cost 2

| Waiting Time | Instance 1 | Instance 2 | Instance 3 |
|--------------|------------|------------|------------|
| Setting I    | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
|              | 176        | 102        | 99         | 96         | 309        | 275        | 274        | 175        | 205        | 133        | 100        | 136        |
| Setting IIa  | 162        | 77         | 100        | 82         | 314        | 271        | 304        | 141        | 181        | 101        | 101        | 121        |
| Setting IIb  | 284        | 189        | 164        | 168        | 543        | 487        | 421        | 281        | 367        | 253        | 191        | 237        |
| Setting IIc  | 340        | 252        | 211        | 220        | 640        | 595        | 505        | 369        | 467        | 355        | 275        | 313        |
| Setting IIIa | 165        | 81         | 99         | 85         | 320        | 277        | 302        | 146        | 182        | 104        | 98         | 123        |
| Setting IIIb | 359        | 263        | 206        | 228        | 700        | 633        | 514        | 382        | 476        | 341        | 251        | 309        |
| Setting IIIc | 596        | 510        | 406        | 434        | 1113       | 1055       | 801        | 732        | 917        | 709        | 559        | 601        |
| Setting IV   | 352        | 281        | 241        | 242        | 605        | 592        | 510        | 406        | 552        | 422        | 341        | 351        |

### Table A.13  Average OR overtime under cost 2

| OR Overtime | Instance 1 | Instance 2 | Instance 3 |
|-------------|------------|------------|------------|
| Setting I   | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
|             | 85         | 127        | 192        | 202        | 99         | 135        | 298        | 277        | 89         | 125        | 199        | 170        |
| Setting IIa | 84         | 119        | 205        | 200        | 101        | 134        | 327        | 274        | 85         | 117        | 214        | 161        |
| Setting IIb | 140        | 175        | 258        | 265        | 198        | 235        | 412        | 351        | 163        | 191        | 271        | 227        |
| Setting IIc | 169        | 206        | 283        | 299        | 242        | 283        | 455        | 389        | 214        | 236        | 303        | 267        |
| Setting IIIa| 83         | 120        | 203        | 202        | 103        | 137        | 320        | 276        | 88         | 117        | 213        | 162        |
| Setting IIIb| 179        | 213        | 291        | 310        | 267        | 304        | 473        | 406        | 215        | 239        | 307        | 270        |
| Setting IIIc| 317        | 354        | 422        | 460        | 468        | 513        | 680        | 587        | 443        | 431        | 470        | 442        |
| Setting IV  | 175        | 214        | 279        | 302        | 226        | 273        | 427        | 382        | 235        | 252        | 306        | 285        |

### Table A.14  Average anesthesiologist overtime under cost 2

| Anes Overtime | Instance 1 | Instance 2 | Instance 3 |
|---------------|------------|------------|------------|
| Setting I     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
|              | 85         | 127        | 192        | 202        | 99         | 148        | 298        | 277        | 89         | 125        | 199        | 170        |
| Setting IIa   | 84         | 119        | 205        | 200        | 101        | 145        | 327        | 274        | 85         | 117        | 214        | 161        |
| Setting IIb   | 140        | 175        | 258        | 265        | 198        | 143        | 320        | 276        | 88         | 117        | 213        | 162        |
| Setting IIc   | 169        | 206        | 283        | 299        | 242        | 283        | 455        | 389        | 214        | 236        | 303        | 267        |
| Setting IIIa  | 83         | 120        | 203        | 202        | 103        | 137        | 320        | 276        | 88         | 117        | 213        | 162        |
| Setting IIIb  | 179        | 213        | 291        | 310        | 267        | 304        | 473        | 406        | 215        | 239        | 307        | 270        |
| Setting IIIc  | 317        | 354        | 422        | 460        | 468        | 513        | 680        | 587        | 443        | 431        | 470        | 442        |
| Setting IV    | 175        | 214        | 279        | 302        | 226        | 273        | 427        | 382        | 235        | 252        | 306        | 285        |
### Table A.15: Average OR idle time under cost 2

| OR Idle Time | Instance 1 | Instance 2 | Instance 3 |
|--------------|------------|------------|------------|
| SP-E         | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I    | 468        | 510        | 575        |
| Setting IIa  | 393        | 428        | 514        |
| Setting IIb  | 344        | 379        | 462        |
| Setting IIc  | 351        | 388        | 465        |
| Setting IIIa | 397        | 434        | 517        |
| Setting IIIb | 316        | 350        | 428        |
| Setting IIIc | 272        | 310        | 378        |
| Setting IV   | 450        | 498        | 563        |

### Table A.16: Average waiting time under cost 3

| Waiting Time | Instance 1 | Instance 2 | Instance 3 |
|--------------|------------|------------|------------|
| SP-CVaR      | DRO-E      | D-CVaR     |
| Setting I    | 179        | 108        | 99         |
| Setting IIa  | 165        | 86         | 100        |
| Setting IIb  | 288        | 195        | 164        |
| Setting IIc  | 344        | 255        | 211        |
| Setting IIIa | 168        | 90         | 90         |
| Setting IIIb | 362        | 266        | 206        |
| Setting IIIc | 599        | 501        | 406        |
| Setting IV   | 355        | 281        | 241        |

### Table A.17: Average OR overtime under cost 3

| OR Overtime | Instance 1 | Instance 2 | Instance 3 |
|-------------|------------|------------|------------|
| SP-E        | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I   | 84.3443    | 115        | 192        |
| Setting IIa | 83         | 107        | 205        |
| Setting IIb | 140        | 165        | 258        |
| Setting IIc | 169        | 194        | 229        |
| Setting IIIa| 82         | 108        | 203        |
| Setting IIIb| 178        | 203        | 291        |
| Setting IIIc| 316        | 340        | 422        |
| Setting IV  | 174        | 201        | 279        |

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### Table A.18 Average anesthesiologist overtime under cost 3

| Anes Overtime | Instance 1 | Instance 2 | Instance 3 |
|---------------|------------|------------|------------|
|               | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I     | 84         | 115        | 192        | 160        |
| Setting Ia    | 83         | 107        | 205        | 153        |
| Setting Ib    | 140        | 165        | 258        | 201        |
| Setting Ic    | 160        | 194        | 283        | 229        |
| Setting Ila   | 82         | 108        | 203        | 154        |
| Setting Iib   | 178        | 203        | 291        | 235        |
| Setting Iic   | 316        | 340        | 422        | 364        |
| Setting IV    | 174        | 201        | 279        | 236        |

### Table A.19 Average OR idle time under cost 3

| OR Idle Time | Instance 1 | Instance 2 | Instance 3 |
|--------------|------------|------------|------------|
|              | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I    | 467        | 498        | 575        | 543        |
| Setting Ia   | 392        | 416        | 514        | 462        |
| Setting Ib   | 344        | 369        | 462        | 405        |
| Setting Ic   | 351        | 376        | 465        | 411        |
| Setting Ila  | 396        | 421        | 517        | 468        |
| Setting Iib  | 315        | 340        | 428        | 372        |
| Setting Iic  | 271        | 296        | 378        | 319        |
| Setting IV   | 458        | 485        | 563        | 520        |

### Table A.20 Average anesthesiologist idle time under cost 3

| Anes Idle Time | Instance 1 | Instance 2 | Instance 3 |
|----------------|------------|------------|------------|
|                | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I      | 467        | 498        | 575        | 543        |
| Setting Ia     | 392        | 416        | 514        | 462        |
| Setting Ib     | 344        | 369        | 462        | 405        |
| Setting Ic     | 351        | 376        | 465        | 411        |
| Setting Ila    | 396        | 421        | 517        | 468        |
| Setting Iib    | 315        | 340        | 428        | 372        |
| Setting Iic    | 271        | 296        | 378        | 319        |
| Setting IV     | 458        | 485        | 563        | 520        |
### Table A.21  
Optimal assignments from SP-E, DRO-E, and RO models (surgeries highlighted in blue in the RO model are those attaining maximum surgery duration in optimization, i.e., \( f_i = 1 \))

| Anesthesiologists | SP-E, DRO-E | RO (\( \tau = 0.2 \)) | RO (\( \tau = 0.4 \)) |
|-------------------|-------------|-------------------------|-------------------------|
| Anes 1            | 1 \( \rightarrow \) 2 \( \rightarrow \) 3 | 1 \( \rightarrow \) 2 \( \rightarrow \) 3 | 1 \( \rightarrow \) 2 \( \rightarrow \) 3 |
| Anes 2            | 4 \( \rightarrow \) 5 \( \rightarrow \) 6 \( \rightarrow \) 7 | 4 \( \rightarrow \) 5 \( \rightarrow \) 6 \( \rightarrow \) 7 | 4 \( \rightarrow \) 5 \( \rightarrow \) 6 \( \rightarrow \) 7 |
| Anes 3            | 8 \( \rightarrow \) 9 \( \rightarrow \) 10 \( \rightarrow \) 11 \( \rightarrow \) 12 | 8 \( \rightarrow \) 13 \( \rightarrow \) 14 \( \rightarrow \) 15 \( \rightarrow \) 11 \( \rightarrow \) 12 | 8 \( \rightarrow \) 13 \( \rightarrow \) 14 \( \rightarrow \) 11 \( \rightarrow \) 12 |
| Anes 4            | 13 \( \rightarrow \) 14 \( \rightarrow \) 15 | 9 \( \rightarrow \) 10 | 9 \( \rightarrow \) 10 \( \rightarrow \) 15 \( \rightarrow \) 12 |
| Anes 5            | – | – | – |

| Operating Rooms   | SP-E, DRO-E | RO (\( \tau = 0.2 \)) | RO (\( \tau = 0.4 \)) |
|-------------------|-------------|-------------------------|-------------------------|
| OR 1              | 1 \( \rightarrow \) 2 \( \rightarrow \) 3 | 1 \( \rightarrow \) 2 \( \rightarrow \) 3 | 1 \( \rightarrow \) 2 \( \rightarrow \) 3 |
| OR 2              | 4 \( \rightarrow \) 5 \( \rightarrow \) 6 \( \rightarrow \) 7 | 4 \( \rightarrow \) 5 \( \rightarrow \) 6 \( \rightarrow \) 7 | 4 \( \rightarrow \) 5 \( \rightarrow \) 6 \( \rightarrow \) 7 |
| OR 3              | – | – | – |
| OR 4              | 8 \( \rightarrow \) 9 \( \rightarrow \) 10 \( \rightarrow \) 11 \( \rightarrow \) 12 | 8 \( \rightarrow \) 9 \( \rightarrow \) 10 \( \rightarrow \) 11 \( \rightarrow \) 12 | 8 \( \rightarrow \) 9 \( \rightarrow \) 10 \( \rightarrow \) 11 \( \rightarrow \) 12 |
| OR 5              | – | – | – |
| OR 6              | 13 \( \rightarrow \) 14 \( \rightarrow \) 15 | 13 \( \rightarrow \) 14 \( \rightarrow \) 15 | 13 \( \rightarrow \) 14 \( \rightarrow \) 15 |
| OR 7              | – | – | – |

**Figure A.1**  
Anesthesiologist overtime from SP-E, DRO-E, and RO models in instance 1 (\( \tau \) controls the size of RO uncertainty set)

Figures A.1 and A.2 show the out-of-sample anesthesiologist and OR overtime from the three models, respectively. We observe that our SP-E model generally produces a smaller anesthesiologist and OR overtime than the DRO-E and RO models. As discussed in Section 8.5, the DRO-E model produces a slightly larger overtime than the SP-E model, as well as the RO model. However, we observe that the RO model may yield an anesthesiologist or OR overtime significantly larger than both the SP-E and DRO-E models. This could be explained by the construction of the RO uncertainty set \( D(\tau) \), which consists of scenarios with exactly \( \tau' \) surgeries deviating from the mean only. That is, the uncertainty set may not be able to capture the variability of all surgery durations (since only part of the surgery durations are altered) as opposed to our SP-E and DRO-E models. In contrast, the out-of-sample overtime performances from the SP-E and DRO-E models are more stable (with a smaller standard deviation).

### A.8.2.6. Comparison with the Sequential Approach
In the section, we compare our integrated approach with a sequential approach that separates the OR assignment decisions from the remaining decisions (i.e., anesthesiologist assignment, sequencing, and scheduling decisions). Specifically, in the sequential approach, we first solve the following classical OR assignment model (Denton et al. 2010):

\[
\begin{align*}
\text{minimize} & \quad \sum_{r \in R} f_r v_r + \mathbb{E}_\psi \left[ \sum_{r \in R} c_r^a \left( \sum_{i \in I} d_i z_{i,r} - T_{\text{end}} \right) \right] \\
\text{subject to} & \quad \sum_{r \in R_i} z_{i,r} = 1, \quad \forall i \in I, \\
& \quad z_{i,r} \leq v_r, \quad \forall (i, r) \in \mathcal{F}^R, \\
& \quad v_r \in \{0, 1\}, \quad z_{i,r} \in \{0, 1\}, \quad \forall (i, r) \in \mathcal{F}^R.
\end{align*}
\]

Model (A.45) decides which OR to open and assigns surgeries to open ORs. It assumes that a scheduled surgery can start immediately after the preceding surgery (in the same OR) is completed. The objective is to minimize the sum of the fixed cost of opening ORs and the expected overtime cost. After solving model (A.45) to obtain the optimal decision \((v^*, z^*)\), we then solve our proposed model (1)–(2) by fixing \((v, z)\) to \((v^*, z^*)\).

We conduct the following experiment to compare the performance of our optimal solution obtained using our integrated approach and the sequential approach. Following the same experiment settings in Section 8, we solve instances 1–6 under cost 1 using the two approaches, employing the same set of symmetry-breaking constraints for a fair comparison. Table A.22 presents the number of ORs opened. Table A.23 summarizes the associated average out-of-sample waiting time, OR overtime, and operational cost under setting I (the perfect distributional setting) and setting IIIc (a mis-specified distributional setting); see Section 8.5 for descriptions of the settings.

First, we observe that the sequential approach opens the same or smaller number of ORs than our SP-E model. This is because in the sequential approach, model (A.45) assumes that every
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Table A.22  Number of ORs open from our SP-E model and the sequential approach

|                  | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|------------------|------------|------------|------------|------------|------------|------------|
| SP-E             | 4          | 5          | 7          | 10         | 16         | 22         |
| Sequential       | 4          | 5          | 7          | 9          | 12         | 19         |

Table A.23  Average out-of-sample waiting time, OR overtime, and operational cost from our SP-E model and the sequential approach under Settings I and IIIc

| Setting I | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|-----------|------------|------------|------------|------------|------------|------------|
| Waiting   | 155        | 280        | 166        | 423        | 319        | 367        |
| Time      | 155        | 279        | 168        | 582        | 645        | 911        |
| OR        | 91         | 108        | 95         | 192        | 75         | 80         |
| Overtime  | Sequential | 91         | 108        | 96         | 309        | 301        | 317        |
| Cost      | 1429       | 2018       | 1506       | 3330       | 1815       | 2028       |
| Operational | 1427     | 2010       | 1514       | 5042       | 5172       | 6216       |

| Setting IIIc | Instance 1 | Instance 2 | Instance 3 | Instance 4 | Instance 5 | Instance 6 |
|--------------|------------|------------|------------|------------|------------|------------|
| Waiting   | 573        | 1082       | 749        | 1713       | 1700       | 2296       |
| Time      | 562        | 1080       | 754        | 2195       | 2613       | 3849       |
| OR        | 324        | 490        | 396        | 843        | 794        | 1104       |
| Overtime  | Sequential | 321        | 483        | 409        | 1077       | 1252       | 1737       |
| Cost      | 5146       | 8554       | 6454       | 14184      | 13684      | 18781      |
| Operational | 5089     | 8442       | 6605       | 18237      | 21477      | 30703      |

surgery can start immediately when the preceding surgery is completed and ignores the need for anesthesiologists to perform each surgery. This results in a packed schedule leading to longer waiting times, OR overtime, and consequently higher operational costs. In particular, the operational cost of the sequential approach could be two times higher than that of our SP-E model for large instances (e.g., instances 5–6). Finally, we note that the differences in the value of operational metrics and associated costs are more significant (in magnitude) under the mis-specified distributional setting.

A.8.3. Additional Computational Results

A.8.3.1. Instance Details

We first provide the summary statistics of the surgery duration for different specialties of this data set. In Table A.24, we present the summary statistics of the random surgery duration for instances 7 to 12. The mean and standard deviation are directly obtained from Min and Yih (2010) while the minimum and maximum surgery duration are computed as the (exact) 20% and 80% percentiles of lognormal distribution with given mean and standard deviation respectively (since we do not have the surgery data). Next, Table A.25 presents the details for instances 7 to 12 with various surgery types (ENT: Ear, nose and throat, CARD: Cardiology, VAS: Vascular, ORTHO: Orthopedics, NSG: Neurosurgery, GEN: General, OPHTH: Ophthalmology, URO: Urology).
### Table A.24
Summary statistics of surgery duration for instances 7 to 12 (Std. Dev.: standard deviation)

| Type       | ENT | OBGYN | ORTHO | Nuro | General | OPHTH | Vascular | Cardiac | Urology |
|------------|-----|-------|-------|------|---------|-------|----------|---------|---------|
| Mean       | 74  | 86    | 107   | 93   | 38      | 120   | 240      | 240     | 64      |
| Std. Dev.  | 37  | 40    | 44    | 49   | 19      | 61    | 103      | 103     | 52      |
| Minimum    | 44  | 54    | 71    | 98   | 54      | 23    | 71       | 156     | 27      |
| Maximum    | 99  | 113   | 138   | 212  | 125     | 51    | 160      | 312     | 90      |

### Table A.25
Instance details based on data from Min and Yih (2010)

Note: GEN, OPHTH, and URO surgeries can be covered by the same pool of anesthesiologists. For example, in instance 7, there are two regular and one on-call anesthesiologists that can perform GEN and OPHTH surgeries.

#### Instance 7
- Surgery type: ENT, ORTHO, GEN, OPHTH
- Number of surgeries: 4, 5, 4, 2
- Number of ORs: 1, 2, 2, 1
- Number of anesthesiologists (regular): 1, 2, 2, 1
- Number of anesthesiologists (on call): 0

#### Instance 8
- Surgery type: ENT, NSG, ORTHO, GEN, OPHTH
- Number of surgeries: 5, 2, 6, 8, 2
- Number of ORs: 1, 1, 2, 3, 1
- Number of anesthesiologists (regular): 1, 1, 1, 3
- Number of anesthesiologists (on call): 0

#### Instance 9
- Surgery type: ENT, CARD, VAS, GEN, OPHTH, URO
- Number of surgeries: 8, 3, 4, 9, 3, 3
- Number of ORs: 2, 1, 2, 2, 1
- Number of anesthesiologists (regular): 2, 1, 2, 4
- Number of anesthesiologists (on call): 0

#### Instance 10
- Surgery type: ENT, CARD, VAS, GEN, OPHTH, URO
- Number of surgeries: 10, 6, 6, 8, 4, 6
- Number of ORs: 2, 4, 2, 2, 1
- Number of anesthesiologists (regular): 2, 3, 2, 6
- Number of anesthesiologists (on call): 0

#### Instance 11
- Surgery type: ENT, CARD, VAS, ORTHO, GEN, OPHTH, URO
- Number of surgeries: 4, 9, 11, 13, 13, 2, 8
- Number of ORs: 1, 5, 3, 4, 3, 1, 3
- Number of anesthesiologists (regular): 1, 4, 2, 3, 6
- Number of anesthesiologists (on call): 0

#### Instance 12
- Surgery type: ENT, CARD, VAS, ORTHO, NSG, GEN, OPHTH, URO
- Number of surgeries: 7, 10, 9, 16, 10, 15, 3, 10
- Number of ORs: 2, 5, 3, 6, 3, 4, 2, 5
- Number of anesthesiologists (regular): 1, 4, 2, 6, 3, 12
- Number of anesthesiologists (on call): 1

### A.8.3.2. Computational Time

In this section, we provide the computational times for solving our proposed models under the three cost structures (see Section 8.2 for the experiment settings). Table A.26 presents the average solution times in seconds for instances 7–12 under costs 1–3. We note that the observations are similar to those for instances 1–6.
Table A.26  
Computational time (in s) for additional instances (instance with “†”: apply (13) and (14) with initial scenario \( m \); instance with “–”: cannot be solved within 10 hours)

| Cost 1 | Instance 7 | Instance 8 | Instance 9 | Instance 10 | Instance 11 | Instance 12 |
|--------|------------|------------|------------|-------------|-------------|-------------|
| SP-E   | 0.82       | 3.23       | 8.60       | 13.70       | 410.82      | 352.00      |
| SP-CVaR| 2.43       | 47.40      | 204.18     | 722.54      | --          | --          |
| DRO-E  | 5.22       | 10.73      | 20.74      | 54.51       | 1065.69     | 777.36      |
| DRO-CVaR| 1.55      | 2.63       | 3.45       | 5.13        | 20.02       | 32.23       |

| Cost 2 | Instance 7 | Instance 8 | Instance 9 | Instance 10 | Instance 11 | Instance 12 |
|--------|------------|------------|------------|-------------|-------------|-------------|
| SP-E   | 0.81       | 3.41       | 9.03       | 16.18       | 229.06      | 491.22      |
| SP-CVaR| 2.22       | 35.70      | 99.68      | 4907.91     | --          | --          |
| DRO-E  | 5.88       | 12.85      | 23.98      | 57.02       | 553.40      | 1005.93     |
| DRO-CVaR| 3.27      | 5.52       | 16.50      | 18.52       | 390.80      | 2688.92     |

| Cost 3 | Instance 7 | Instance 8 | Instance 9 | Instance 10 | Instance 11 | Instance 12 |
|--------|------------|------------|------------|-------------|-------------|-------------|
| SP-E   | 0.81       | 3.31       | 7.60       | 16.00       | 265.99      | 645.85      |
| SP-CVaR| 2.82       | 41.35      | 132.68     | 6087.30     | --          | --          |
| DRO-E  | 5.82       | 12.43      | 28.13      | 78.18       | 630.64      | 961.50      |
| DRO-CVaR| 3.25      | 7.20       | 15.16      | 19.77       | 547.40      | 2344.09     |

Figure A.3  
Number of ORs opened for different instances under different costs

A.8.3.3. Analysis of Optimal Solutions

In this section, we compare the number of ORs opened and the number of on-call anesthesiologists called in from our proposed models. Figure A.3 shows the optimal number of ORs opened by each model. The observations are similar to those in Section 8.4. In addition, we also observe that the DRO-CVaR and SP-E models call in the largest and smallest number of on-call anesthesiologists, respectively.

A.8.3.4. Analysis of Solution Quality

In this section, we present the average out-of-sample performance metrics for instances 7–12 under costs 1–3. Tables A.27–A.29 present respectively the average waiting time, OR overtime, and anesthesiologists overtime under cost 1; Tables A.30–A.33 present respectively the average waiting time, OR overtime, anesthesiologists overtime, and OR idle time under cost 2; Tables A.34–A.38 present respectively the average waiting time, OR overtime, anesthesiologists overtime, OR idle time, and anesthesiologist idle time under cost 3. We note that the observations are similar to those for instances 1–6.
### Table A.27 Average waiting time under cost 1

| Waiting Time | Instance 7 | Instance 8 | Instance 9 |
|--------------|------------|------------|------------|
|              | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I    | 5          | 4          | 0          | 0          | 15         | 14         | 0          | 0          | 168        | 72         | 33         | 9          |
| Setting IIa  | 2          | 1          | 0          | 0          | 4          | 3          | 0          | 0          | 140        | 60         | 36         | 5          |
| Setting Iib  | 29         | 38         | 36         | 31         | 75         | 64         | 69         | 79         | 342        | 185        | 150        | 91         |
| Setting Iic  | 78         | 74         | 75         | 64         | 142        | 124        | 145        | 176        | 491        | 304        | 290        | 194        |
| Setting IIIa | 2          | 1          | 0          | 0          | 4          | 3          | 0          | 0          | 140        | 59         | 36         | 5          |
| Setting IIIb | 59         | 57         | 55         | 48         | 110        | 94         | 107        | 120        | 424        | 238        | 201        | 133        |
| Setting IIIc | 176        | 168        | 170        | 150        | 297        | 274        | 337        | 389        | 839        | 550        | 555        | 402        |
| Setting IV   | 113        | 105        | 107        | 92         | 195        | 172        | 200        | 256        | 560        | 408        | 432        | 301        |

### Table A.28 Average OR overtime under cost 1

| OR Overtime | Instance 7 | Instance 8 | Instance 9 |
|-------------|------------|------------|------------|
|             | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I   | 0          | 1          | 5          | 5          | 1          | 1          | 14         | 14         | 14         | 67         | 67         | 133        | 163        |
| Setting IIa | 0          | 0          | 3          | 3          | 0          | 0          | 7          | 7          | 7          | 51         | 57         | 142        | 165        |
| Setting Iib | 9          | 9          | 11         | 11         | 26         | 25         | 31         | 45         | 159        | 133        | 210        | 224        |
| Setting Iic | 19         | 20         | 20         | 20         | 58         | 55         | 57         | 89         | 238        | 197        | 271        | 268        |
| Setting IIIa| 0          | 0          | 3          | 3          | 0          | 0          | 8          | 8          | 8          | 51         | 56         | 140        | 163        |
| Setting IIIb| 13         | 14         | 15         | 15         | 42         | 38         | 42         | 66         | 211        | 169        | 243        | 251        |
| Setting IIIc| 51         | 54         | 44         | 43         | 151        | 137        | 135        | 204        | 445        | 351        | 433        | 388        |
| Setting IV  | 27         | 28         | 26         | 26         | 83         | 79         | 78         | 119        | 266        | 241        | 302        | 283        |

### Table A.29 Average anesthesiologist overtime under cost 1

| Anes Overtime | Instance 7 | Instance 8 | Instance 9 |
|---------------|------------|------------|------------|
|               | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     | SP-E       | SP-CVaR    | DRO-E      | D-CVaR     |
| Setting I     | 0          | 1          | 5          | 5          | 1          | 1          | 14         | 14         | 14         | 67         | 68         | 70         | 100        |
| Setting IIa   | 0          | 0          | 3          | 3          | 0          | 0          | 7          | 7          | 7          | 51         | 57         | 75         | 98         |
| Setting Iib   | 9          | 9          | 11         | 11         | 21         | 22         | 31         | 41         | 155        | 136        | 115        | 130        |
| Setting Iic   | 19         | 20         | 20         | 20         | 48         | 40         | 57         | 81         | 235        | 205        | 160        | 159        |
| Setting IIIa  | 0          | 0          | 3          | 3          | 0          | 0          | 8          | 8          | 8          | 51         | 56         | 74         | 97         |
| Setting IIIb  | 13         | 14         | 15         | 15         | 33         | 33         | 42         | 58         | 206        | 174        | 134        | 143        |
| Setting IIIc  | 51         | 54         | 44         | 43         | 125        | 121        | 126         | 182        | 450        | 374        | 269        | 271        |
| Setting IV    | 27         | 28         | 26         | 26         | 70         | 71         | 76         | 109        | 265        | 252        | 189        | 176        |
### Table A.30 Average waiting time under cost 2

| Waiting Time | Instance 7 | Instance 8 | Instance 9 |
|--------------|------------|------------|------------|
| Setting I    | SP-E 116   | SP-E 144   | SP-E 353   |
| Setting Ia   | 111        | 144        | 353        |
| Setting Ib   | 203        | 264        | 673        |
| Setting Ic   | 252        | 475        | 1100       |
| Setting IIa  | 108        | 125        | 347        |
| Setting IIb  | 247        | 121        | 223        |
| Setting IIc  | 425        | 811        | 909        |
| Setting IV   | 272        | 411        | 650        |

### Table A.31 Average OR overtime under cost 2

| OR Overtime | Instance 7 | Instance 8 | Instance 9 |
|-------------|------------|------------|------------|
| Setting I    | SP-E 53    | SP-E 189   | SP-E 224   |
| Setting Ia   | 54         | 188        | 223        |
| Setting Ib   | 92         | 237        | 269        |
| Setting Ic   | 110        | 269        | 293        |
| Setting IIa  | 53         | 188        | 223        |
| Setting IIb  | 110        | 262        | 295        |
| Setting IIc  | 185        | 390        | 428        |
| Setting IV   | 113        | 266        | 321        |

### Table A.32 Average anesthesiologist overtime under cost 2

| Anes Overtime | Instance 7 | Instance 8 | Instance 9 |
|---------------|------------|------------|------------|
| Setting I     | SP-E 53    | SP-E 151   | SP-E 224   |
| Setting Ia    | 54         | 188        | 223        |
| Setting Ib    | 92         | 237        | 269        |
| Setting Ic    | 110        | 269        | 293        |
| Setting IIa   | 53         | 188        | 223        |
| Setting IIb   | 110        | 262        | 295        |
| Setting IIc   | 185        | 390        | 428        |
| Setting IV    | 113        | 266        | 321        |
### Table A.33 Average OR idle time under cost 2

| OR Idle Time | Instance 7 | Instance 8 | Instance 9 |
|--------------|------------|------------|------------|
| Setting I    | SP-E 768   | SP-CVaR 816 | SP-E 1010  |
| Setting IIa  | 727        | 766        | 941        |
| Setting IIb  | 691        | 722        | 736        |
| Setting IIc  | 600        | 720        | 732        |
| Setting IIla | 733        | 773        | 784        |
| Setting IIib | 678        | 707        | 720        |
| Setting IIic | 640        | 660        | 673        |
| Setting IV   | 753        | 785        | 792        |

### Table A.34 Average waiting time under cost 3

| Waiting Time | Instance 7 | Instance 8 | Instance 9 |
|--------------|------------|------------|------------|
| Setting I    | 145        | 79         | 100        |
| Setting IIa  | 136        | 54         | 103        |
| Setting IIb  | 250        | 161        | 164        |
| Setting IIc  | 309        | 223        | 206        |
| Setting IIla | 133        | 54         | 102        |
| Setting IIlb | 302        | 210        | 194        |
| Setting IIlc | 509        | 409        | 349        |
| Setting IV   | 329        | 254        | 232        |

### Table A.35 Average OR overtime under cost 3

| OR Overtime | Instance 7 | Instance 8 | Instance 9 |
|-------------|------------|------------|------------|
| Setting I   | 54         | 89         | 101        |
| Setting IIa | 53         | 88         | 105        |
| Setting IIb | 95         | 128        | 137        |
| Setting IIc | 115        | 155        | 152        |
| Setting IIla| 52         | 85         | 103        |
| Setting IIlb| 115        | 149        | 152        |
| Setting IIic| 194        | 251        | 218        |
| Setting IV  | 115        | 165        | 152        |
### Table A.36 Average anesthesiologist idle time under cost 3

| Anes Overtime | Instance 7 | Instance 8 | Instance 9 |
|---------------|------------|------------|------------|
|               | SP-E | SP-CVaR | DRO-E | D-CVaR | SP-E | SP-CVaR | DRO-E | D-CVaR | SP-E | SP-CVaR | DRO-E | D-CVaR |
| Setting I      | 54   | 89     | 101   | 118   | 149  | 195    | 182   | 224   | 222  | 348   | 168   | 346   | 145  |
| Setting IIa    | 53   | 86     | 105   | 114   | 155  | 193    | 190   | 222   | 159  | 359   | 160   | 375   | 136  |
| Setting IIb    | 95   | 126    | 137   | 148   | 212  | 268    | 248   | 266   | 266  | 507   | 289   | 466   | 233  |
| Setting IIc    | 115  | 148    | 152   | 169   | 246  | 312    | 285   | 294   | 292  | 592   | 369   | 518   | 302  |
| Setting IIIa   | 52   | 85     | 103   | 113   | 154  | 193    | 188   | 222   | 158  | 368   | 158   | 368   | 135  |
| Setting IIIb   | 115  | 145    | 152   | 165   | 241  | 303    | 278   | 289   | 287  | 571   | 345   | 507   | 277  |
| Setting IIIc   | 194  | 228    | 222   | 241   | 380  | 468    | 428   | 405   | 405  | 585   | 585   | 692   | 491  |
| Setting IV     | 115  | 153    | 153   | 175   | 262  | 323    | 290   | 308   | 308  | 581   | 381   | 503   | 321  |

### Table A.37 Average OR idle time under cost 3

| OR Idle Time | Instance 7 | Instance 8 | Instance 9 |
|--------------|------------|------------|------------|
|              | SP-E | SP-CVaR | DRO-E | D-CVaR | SP-E | SP-CVaR | DRO-E | D-CVaR | SP-E | SP-CVaR | DRO-E | D-CVaR |
| Setting I     | 768  | 804    | 816   | 833   | 1008 | 1054    | 1040  | 1083  | 974  | 1021 | 1451  | 1730  |
| Setting IIa   | 726  | 758    | 778   | 786   | 940  | 978     | 975   | 1006  | 873  | 910  | 1369  | 1610  |
| Setting IIb   | 694  | 727    | 736   | 747   | 867  | 921     | 904   | 921   | 831  | 863  | 1272  | 1517  |
| Setting IIc   | 695  | 735    | 732   | 749   | 869  | 926     | 909   | 918   | 846  | 876  | 1258  | 1522  |
| Setting IIIa  | 732  | 765    | 784   | 793   | 948  | 993     | 983   | 1017  | 887  | 925  | 1325  | 1629  |
| Setting IIIb  | 683  | 718    | 720   | 733   | 844  | 901     | 881   | 892   | 829  | 857  | 1248  | 1498  |
| Setting IIIc  | 649  | 706    | 673   | 696   | 795  | 855     | 843   | 820   | 818  | 835  | 1163  | 1441  |
| Setting IV    | 756  | 805    | 792   | 816   | 988  | 1034    | 1025  | 1008  | 1050 | 1418 | 1715  |

### Table A.38 Average anesthesiologist idle time under cost 3

| Anes Idle Time | Instance 7 | Instance 8 | Instance 9 |
|----------------|------------|------------|------------|
|                | SP-E | SP-CVaR | DRO-E | D-CVaR | SP-E | SP-CVaR | DRO-E | D-CVaR | SP-E | SP-CVaR | DRO-E | D-CVaR |
| Setting I      | 1468 | 1473   | 1611  | 1512  | 1578 | 1953    | 2033  | 2167  | 1360 | 1562 | 2694  | 2183  |
| Setting IIa    | 1294 | 1300   | 1468  | 1333  | 1323 | 1669    | 1861  | 1892  | 1013 | 1177 | 2432  | 1813  |
| Setting IIb    | 1167 | 1167   | 1343  | 1219  | 1250 | 1572    | 1744  | 1771  | 993  | 1158 | 2260  | 1736  |
| Setting IIc    | 1183 | 1187   | 1378  | 1251  | 1340 | 1645    | 1814  | 1826  | 1123 | 1296 | 2459  | 1844  |
| Setting IIIa   | 1321 | 1326   | 1480  | 1360  | 1350 | 1710    | 1887  | 1932  | 1305 | 1296 | 2470  | 1861  |
| Setting IIIb   | 1139 | 1139   | 1318  | 1198  | 1229 | 1543    | 1713  | 1736  | 988  | 1162 | 2226  | 1724  |
| Setting IIIc   | 1061 | 1084   | 1300  | 1161  | 1257 | 1518    | 1728  | 1679  | 1056 | 1250 | 2249  | 1740  |
| Setting IV     | 1469 | 1477   | 1661  | 1548  | 1727 | 2048    | 2171  | 2214  | 1590 | 1801 | 2858  | 2366  |
A.8.4. Monte Carlo Optimization

In this section, we describe the Monte Carlo optimization procedure to obtain near optimal solution for the SP-E model with possibly a small number of scenarios and provide the corresponding results. The procedure is as follows (see Jebali and Diabat 2015, Kleywegt et al. 2002, Lamiri et al. 2009, Shehadeh et al. 2021 for more detail explanations).

Step 1a. Generate scenarios \( d_{i}^{n,k} \) for \( i \in \mathcal{I}, m \in \{1, \ldots, K\} \) and \( n \in \{1, \ldots, N\} \).

Step 1b. Solve the SP-E model using SAA with scenarios \( \{d_{i}^{n,k}\}_{n=1}^{N} \) and obtain the optimal first-stage solution \( \chi^{k} := (x^{k}, y^{k}, z^{k}, v^{k}, u^{k}, s^{k}) \) with optimal value \( v^{k} \) for \( k \in \{1, \ldots, K\} \).

Step 1c. Generate new scenarios \( \tilde{d}_{i}^{n,k} \) for \( i \in \mathcal{I}, k \in \{1, \ldots, K\} \) and \( n \in \{1, \ldots, N'\} \).

Step 1d. Obtain the estimate of the true function value \( \hat{v}^{n,k} \) evaluated at \( \chi^{k} \) using samples \( \{\tilde{d}_{i}^{n,k}\}_{n=1}^{N'} \) by solving the second-stage problem (2) for \( m \in \{1, \ldots, K\} \) and \( n \in \{1, \ldots, N'\} \).

Step 2. Compute the estimates \( \hat{\mu} = K^{-1} \sum_{k=1}^{K} v^{k} \), \( \hat{\mu}^{k} = (N')^{-1} \sum_{n=1}^{N'} \hat{v}^{n,k} \),

\[
\hat{\sigma}^{2} = \frac{1}{K(K-1)} \sum_{k=1}^{K} \left( v^{k} - \hat{\mu} \right)^{2} \quad \text{and} \quad (\hat{\sigma}^{2})^{k} = \frac{1}{N'(N'-1)} \sum_{n=1}^{N'} \left( \hat{v}^{n,k} - \hat{\mu}^{k} \right)^{2}.
\]

Step 3. Obtain the estimated optimality gap \( \hat{\mu}^{k} - \hat{\mu} \) and its variance \( \hat{\sigma}^{2} + (\hat{\sigma}^{2})^{k} \) for \( k \in \{1, \ldots, K\} \).

The quantities \( \hat{\mu} \) and \( \hat{\mu}^{k} \) provide a statistical lower and upper bound on the optimal objective value of the SP-E model respectively (Kleywegt et al. 2002, Mak et al. 1999). Therefore, given a sample size \( N \), if the estimated optimality gap \( \hat{\mu}^{k} - \hat{\mu} \) and its variance are small, then the sample size \( N \) is sufficiently large for producing near optimal solutions. Otherwise, we could increase the sample size \( N \) and estimate the new optimality gap.

We apply the Monte Carlo optimization method to the ORASP with \( K = 20, N = 100 \) and \( N' = 10000 \). In Table A.39, we report the absolute value of the mean (over \( K = 20 \) replications) of the estimated gap and the standard deviation estimate. Moreover, we report the normalize gap, i.e., approximated optimality index (AOI) defined as \( \left| \sum_{k=1}^{K} (\hat{\mu}^{k} - \hat{\mu}) / \sum_{k=1}^{K} \hat{\mu}^{k} \right| \) (Shehadeh et al. 2021). It is clear from Table A.39 that almost all the AOIs are less than 1%. That is, the relative optimality gap is small. Also, the standard deviations of the estimated optimality gaps are small. Therefore, it is reasonable to solve the SP-E model using SAA with \( N = 100 \) scenarios to obtain near-optimal solutions.
Table A.39 Statistics from Monte Carlo optimization method with $K = 20$, $N = 100$ and $N' = 10000$

| Instance | Cost 1 | | Cost 2 | | Cost 3 |
|----------|--------|--------|--------|--------|--------|
|          | Gap    | Std.   | AOI    | Gap    | Std.   | AOI    | Gap    | Std.   | AOI    |
| 1        | 10.87  | 22.07  | 0.22%  | 34.35  | 22.82  | 0.47%  | 42.15  | 25.14  | 0.52%  |
| 2        | 15.34  | 20.36  | 0.24%  | 23.85  | 22.08  | 0.27%  | 29.14  | 25.92  | 0.28%  |
| 3        | 43.04  | 21.81  | 0.55%  | 68.01  | 26.75  | 0.50%  | 69.95  | 32.00  | 0.45%  |
| 4        | 58.89  | 31.64  | 0.48%  | 64.46  | 48.67  | 0.37%  | 65.95  | 51.62  | 0.31%  |
| 5        | 55.83  | 20.39  | 0.35%  | 96.96  | 32.03  | 0.42%  | 108.44 | 40.64  | 0.32%  |
| 6        | 102.50 | 19.82  | 0.47%  | 184.59 | 44.29  | 0.58%  | 209.68 | 42.90  | 0.40%  |
| 7        | 47.48  | 10.45  | 1.05%  | 54.53  | 17.64  | 0.65%  | 60.71  | 22.16  | 0.58%  |
| 8        | 5.95   | 1.71   | 0.08%  | 30.68  | 27.86  | 0.24%  | 26.20  | 32.54  | 0.18%  |
| 9        | 6.29   | 17.22  | 0.06%  | 34.81  | 34.02  | 0.21%  | 43.24  | 36.94  | 0.22%  |
| 10       | 20.23  | 24.85  | 0.18%  | 30.71  | 27.20  | 0.17%  | 27.92  | 30.06  | 0.12%  |
| 11       | 42.40  | 41.07  | 0.19%  | 110.49 | 60.77  | 0.36%  | 120.40 | 55.62  | 0.36%  |
| 12       | 134.56 | 37.73  | 0.47%  | 112.24 | 46.57  | 0.30%  | 132.14 | 54.92  | 0.28%  |

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