Parity doubling in the high baryon spectrum: near-degenerate three-quark quartets

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Abstract

We report on the first calculation of excited baryons with a chirally symmetric Hamiltonian, modeled after Coulomb b-gauge QCD (or upgraded from the Cornwall meson potential model to a chiral theory in all of Fock-space) showing the insensitivity to chiral symmetry breaking. As has recently been understood, this leads to doubling between two hadrons of equal spin and opposite parity. As a novelty we show that three-quark states group into quartets with two states of each parity, all four states having equal angular momentum. Diagonalizing the chiral charge expressed in terms of quarks we show that the quartet is slightly split into two parity doublets by the tensor force, all splittings decreasing to zero high in the spectrum.

Our specific calculation is for the family of axial spin excitations of the Delta baryon. We provide a model estimate of the experimental accuracy needed to establish Chiral Symmetry Restoration in the high spectrum. We suggest that a measurement of mass of high partial wave resonances with an accuracy of 50 MeV should be sufficient to unambiguously establish the approximate degeneracy, and test the concept of running quark mass in the infrared.

The idea of chiral symmetry restoration has been around for a while, for example parity doubling was examined for the proton in the context of the linear sigma model in [1]. By current ideas we believe that this restoration should occur for higher excitations. Glazman and collaborators [2, 3, 4, 5, 6, 7, 8, 9] have theoretically examined (qg) mesons, and also shown preliminary evidence for chiral symmetry restoration in both meson and hadron spectra, that rekindles interest in intermediate energy resonances. Chiral symmetry restoration, or more precisely, Spontaneous Chiral Symmetry Breaking Insensitivity high in the spectrum, is established as a strong prediction of the symmetry breaking pattern of QCD, and such prediction in an energy region where little else can be stated, needs to be confirmed or refuted by experiment.

The baryon spectrum is a more difficult theoretical problem given the minimum three-body wave function (as opposed to only quark-antiquark for mesons) and in this paper we provide the necessary theoretical background to understand parity doubling, in agreement with a prior study by Nefediev, Ribeiro and Szczepaniak [10], and give the first model estimate of what the experimental target-precision should be. This should help quantify what "high enough" in the spectrum means, to assist experimental planning.

We customarily employ a truncation of Coulomb b-gauge QCD by ignoring the Faddeev-Popov operator

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with a strong kernel containing a linear potential \( V_L \), with string tension \( = 0.135 \text{ GeV}^2 \), coupled to the color charge density \( \rho(x) = \gamma(x)T^a(x) \). In our past work we have solved the BCS gap equation to spontaneously break chiral symmetry. This model has the same chiral structure of QCD, satisfying the Gel’fand-Yaglom-Renner relation, the low-energy theorem for pion scattering and allowing computations of static pion-nucleon observables. We have employed it in studies of gluodynamics shown at this workshop that agree with lattice gauge theory and are of qualitative phenomenological interest. In any case, these play a minor role in the topic of this article, as the decreasing of the splittings is dominated by chiral symmetry breaking alone. For a reduced baryon sector application we are going to perform two on simplifications. We employ only the \( V_L \) linear potential, and neglect all magnetic interactions. This makes the \( - \) nucleon mass splitting too small, but does not affect the spectrum much.

We truncate the Fock space variationally, as customary, to the \( \text{qqqq} \) minimum wavefunction. Since radial excitations of this system compete with multiquark excitations, we concentrate instead on maximum angular momentum excitations \( J = 3L + 1 \). Chiral forces are too weak to compensate large centrifugal forces and can hardly maintain \( l \approx 3 \) or \( l \approx 4 \); so one hopes to reduce the molecular component by studying the ground state in each \( J \)-channel, so that the \( \text{qqqq} \) correlation remains important high in the spectrum.

A rule of thumb is: one needs to keep in the Fock-space expansion \( \text{qqqq} \), \( \text{pppp} \), \( \text{ppqq} \), \( \text{pppp} \) as many states as will be competitive by phase space considerations, considering the quark and gluon dynamical mass gaps established by lattice and Dyson-Schwinger studies. When pentaquark correlations are more abundant than three-quark correlations (see figure 3), the typical quark \( m \) on entum will be lower than extrapolated from the ground-state baryons, so that chiral symmetry restoration will not be quite so fast.

This puts pentaquark correlations above 2 GeV, with the exception of possible meson-baryon resonances (as the Goldstone bosons avoid the mass gap). In any case it seems well established that three-quark correlations play an important role in baryon-phenomenology, so it is worth examining the effect of a chiral transformation on a three-quark variational form.

\[ H = \sum_{x} \frac{1}{2} (\partial \gamma)(x) \gamma(x) A(x) A(x) + \text{Tr} \sum_{x} (E E + B B) \]

\[ + \sum_{x} \chi_q(x)(i \gamma^3 m_q q(x) + \gamma^5 q(x)
\]

\[ + \frac{1}{2} \sum_{x \neq y} \rho(x) V_L(y) \gamma^a x \gamma^a y \]
wavefunction \[ \psi_i = F_{ijk} B_{ik}^i B_{kj}^j \phi_i. \]

We proceed variationally and employ several types of wavefunctions, rational and Gaussian, but the lowest energy (binding of the model’s J-ground state from above by the Rayleigh-Ritz principle) is obtained by employing the chiral limit pion-wavefunction rescaled with two variational parameters in terms of Jacobi coordinates, \( \sin (k = s) \sin (k = t) \psi_{ik}^s \psi_{ik}^t (\hat{k}) \).

We have found the angular excitation in \( F \) to be slightly higher in energy and neglect the correlation. Part of it though reenters the calculation upon (anti)symmetrizing the wavefunction, since quark exchange mixes the and variables. A typical variational search is represented in figure 1. Table 1 presents the interdoublet splittings. The interdoublet splittings, as well as improved precision on our three-body variational Monte Carlo method, will be given in an upcoming publication. As can be seen from the table, the model doublet splittings drop with the orbital angular momentum. This is easy to understand from the structure of the model Hamiltonian. The kernel for baryons is proportional to

\[
F_{s_i, s_2, s_3} (k_1; k_2) U_{s_i, s_1} U_{s_2, q_1} U_{s_3, q_2} U_{s_1, k_1} U_{s_2, k_2} U_{s_3, k_3}
\]

that, upon becoming insensitive to the gap angle, \( \sin (k \rightarrow 0) ! 0 \), turns into

\[
F_{s_i, s_2, s_3} s_i + (\hat{k}_1 \hat{k}_1 + q_1) s_i + (\hat{k}_2 \hat{k}_2 + q_2) s_i + F_{s_i, s_2, s_3}
\]

If instead of \( F_{s_i, s_2, s_3} \) one substitutes its chiral partner \( F_{s_i, s_2, s_3} \) (the same for the ket), the two states are seen to be degenerate. A less apparent in Eq. 3 is the role of the tensor force in enforcing chiral cancellations.

Finally, the first computation of the parity doubling for baryons is presented in figure 2.

Let us now show that there are indeed two closely separated baryon doublets, slightly split by tensor forces. We end convenient to employ the gap angle instead of the quark mass

\[
\sin (k) = \frac{M (k)}{M (k)^2 + k^2}
\]

Table 1: Experimental and computed doublet splittings. The entire quartet degenerates high in the spectrum, with the + parity doubling proceeding faster due to insensitivity to SB and the interdoublet splitting decreasing slower, as they are due to the tensor force and dynamical. We give a preliminary calculation of the interdoublet splitting (parity degeneracy). From the decreasing theory splittings we deduce that an experimental measurement of the parity splitting \( M_+ - M_- \) to an accuracy of 100, or better 50 MeV, should suffice to see the effect. Note that our excited splittings become compatible with zero within errors in the Monte Carlo 9-d integral.

| J       | Exp. | Theory |
|---------|------|--------|
| M_+     |      | 450(100) |
| M_-     |      | 400(100) |
| 3/2     | 470(40) | 450(100) |
| 5/2     | 70(90)  | 400(100) |
| 7/2     | 270(120)| 50(100)  |
| 9/2     | 50(250) | 200(100) |
| 11/2    | -     | 100(100) |
| 13/2    | -     | 100(100) |

and the Dirac spinors can be easily parametrized as

\[
\begin{align*}
U &= \frac{1}{P} \frac{1 + \sin \phi}{1 + \sin^2 \phi} \\uparrow \\downarrow \\
V &= \frac{1}{P} \frac{1 - \sin \phi}{1 + \sin \phi} \\uparrow \\downarrow \\
\end{align*}
\]

Substituting these spinors, and in terms of Bogoliubov-rotated quark and antiquark normal modes \( B, D \), the chiral charge takes the form

\[
Q_a^5 = \frac{Z}{(2)^{a}} \frac{q^3 k}{2^{a} f^a} \frac{X}{Z} \frac{1}{2} \frac{\partial f^a}{\partial q^a} \frac{\partial f^a}{\partial X} \frac{\partial f^a}{\partial X}
\]

\[
(\hat{k}) = B_{k f c} B_{k f c} + D_{k f c} D_{k f c} \sin (k)
\]

\[
(\hat{l}) = B_{k f c} D_{k f c} + B_{k f c} D_{k f c} \sin (k)
\]

In the presence of Spontaneous Chiral Symmetry Breaking, \( \sin (k) \neq 0 \), and the two terms in the
second line are responsible for the non-linear realization of chiral symmetry in the spectrum. One can see this by applying the chiral charge on a hadron state to collect the same hadron state plus a pion. As in Ja e-Pirjol and Scardicchio [11],

\[ [Q_0^2; N_i] = v_0(2)_{abc} c_i \delta j N_j : \]  
\[ [Q_0^2; N_i] = c_i \delta j N_j : \]  

(Here, \( i \) and \( j \) are the chiral multiplet indices).

Eq. (4) is easy to derive because the \( i,j \) matrix couples the quark-antiquark pair to pseudoscalar quantum numbers, so the terms in the second line of Eq. (4) provide an interpolating field for the pion. In fact, if the vacuum is variationally chosen as the BCS ground state \( j_i = 0, D_j = 0 \), then (1) provides precisely the RPA pion wavefunction in the chiral limit, and the terms with \( \sin (k) \) become the RPA pion-creation operator.

If instead chiral symmetry was not spontaneously broken in QCD, the \( X \) would not change the particle content since the second line of Eq. (4) would vanish, and the first line is made of quark and antiquark number operators. Then chiral symmetry would be linearly realized in Wigner-Weyl mode where hadrons come in degenerate opposite-parity pairs

\[ \{Q_0^2; N_i\} = a_i \delta j N_j : \]  
\[ \{Q_0^2; N_i\} = a_i \delta j N_j : \]  

The parity change follows from the \( \hat{K} \) p-wave present in the first line of Eq. (4).

In fact, the contemporary realization is that both phenomena are simultaneously realized in QCD. The vacuum is not annihilated by the chiral charge, forcing spontaneous symmetry breaking, but the mass gap angle has a constant support and if, in a hadron, the typical quark momentum is high, as illustrated in figure 4, its wavefunction is insensitive to ChiralSymmetry breaking. Therefore, one asymptotically recovers degenerate G parities as parity doublets. We will in the following drop the isospin index.

If a given resonance is high enough in the spectrum so the quarks have a momentum distribution peaked higher than the support of the gap angle, as in figure 4, only the first line of Eq. (4) is active. \( Q_0 N_i \) contains also three quarks, but one of them is spin-rotated from \( B_k \) to \( \hat{K} B_k \). Successive application of the chiral charge spin-rotates further quarks, changing each time the parity of the total wavefunction. However, the sequence of states ends since \( \hat{K} \hat{K} = I \). In fact, starting with an arbitrary such wavefunction, one generates a quartet

\[ n_0^{+} = a_i \hat{K} B_k Y B_j \hat{K} Y \]  
\[ n_1^{+} = a_i \hat{K} B_k Y B_j \hat{K} Y \]  

that is the natural basis to discuss chiral symmetry restoration in baryons, through wavefunctions that are linear combinations \( N_i = c_i N_i \).

Because the Hamiltonian and the chiral charge commute, they can be diagonalized simultaneously.

The quartet then separates into two doublets connected by the chiral charge

\[ Q_5(N_0; N_2) = N_1 N_3 \]  
\[ Q_5(N_1; N_3) = N_0 N_2 \]  
\[ Q_5(N_0 + 3N_2) = 3(3N_1 + N_3) \]  
\[ Q_5(3N_1 + N_3) = 3(N_0 + 3N_2) \]

Since the quartet can be divided into two two-dimensional irreducible representations of the chiral group, (with different eigenvalues of \( Q_5 \), 1 and 9 respectively), the masses of the two doublets may also be different, and the interdoublet splitting becomes a dynamical question. However, the splitting within the doublet must vanish asymptotically. This is a prediction following from first principles understanding of QCD alone. Should it not be borne experimentally, it would falsify the theory.
...spectrum, the vector and the Thomas precessions \[20\]. However, higher in the spectrum, the vector potential comes forward, and it is known to present larger spin-orbit splittings than found to date. Therefore not all splittings in a given baryon shell will disappear alike; while parity splittings must decrease fast by chiral symmetry, other spin-orbit splittings will stay constant or even grow. This is demanded by a necessary cancellation between L S, centrifugal forces l(l+1) and tensor forces. This has been explicitly shown for mesons in \[21\].

It has also been pointed out \[6,22,13\] that the pion decouples from the very excited resonances due to the falling overlap between the wavefunctions and sinh(k) (the pion wavefunction in the chiral limit). This might already be observable in the known widths for pion decays, that decrease even with larger phase space see table 2. There are lattice calculations addressing low-excited baryons \[24\], but it is still a long way to go until highly excited states can be examined.

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\[\begin{array}{|c|c|c|}
\hline
J^P & n & \chi \\
\hline
1^- & 118(2) & 118(2) & 118(2) \\
0^+ & 300(100) & 50(30) & 190(90) \\
1^- & 330(60) & 42(18) & <80(20) \\
1^+ & 350(150) & 40(30) & - \\
1^- & 285(50) & 115(35) & 170(30) \\
1^+ & 400(150) & 30(20) & - \\
1^- & 400(150) & 30(20) & - \\
1^+ & 400(180) & 35(25) & - \\
1^- & 450(150) & 50(40) & - \\
1^+ & - & - & - \\
1^- & - & - & - \\
1^+ & 400(200) & 20(12) & - \\
1^- & 550(300) & 30(25) & - \\
\hline
\end{array}\]

\[\text{Table 2: Total width, exclusive pion-nucleon width and seminclusive pion width (decay to one pion plus any other particles excluding pions) for the ground state J resonances. All units MeV. Data adapted from PDG \[23\].}\]
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Figure 2: Parity doubling in the spin-excited spectrum. Top: with infrared quark mass as calculated in the model (probably too low). Bottom: quark mass rescaled to Landau-gauge lattice data. The model clearly displays parity doubling. The experimental situation is still unclear, the degeneracy can be claimed for the $9=2$ states alone, and the chiral partners higher in the spectrum are not experimentally known.

Figure 3: The typical momentum of a quark in a three-quark state is (by kinetic energy considerations alone, with a running mass-gap) $\frac{3}{M}$. Plotted is the typical momentum in a three quarks and five quark wavefunction. At the jump the phase space for ve-quark states is larger, so it is more likely that a baryon of that mass is in a ve-quark configuration, and the typical momentum is therefore smaller. Hence chiral symmetry restoration has to be somewhat slower than three-quark models would indicate.
The sine of the gap angle $M(k) = \sqrt{M(k)^2 + k^2}$ has limited support if the chiral-symmetry breaking quark mass remains of order $\mathcal{O}(QCD)$ or less. Top: we show the running mass from a model computation for a linear potential with string tension $\sigma = 0.135$ GeV$^2$, and its rescaling to match Landau-gauge data [12, 13] (no Coulomb-gauge lattice data for the quark mass is known to us). Bottom: Quark-momentum distributions for $J=3/2$ and $J=9/2$ with simple variational wave functions. The quark-momentum distribution for higher hadron resonances has a smaller overlap with this gap angle, and therefore the quarks in those hadrons behave effectively as if they were massless. Hence they become insensitive to the gap angle, and chiral symmetry is restored in Wigner-Weyl mode with degenerate multiplets.