Finite Renormalization II*

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* Based on speculation concerning CODATA’s delayed 1999-2000? Report.

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Finite Renormalization II*

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Abstract. This paper is an updated version of the paper of similar title published in September 1998 modified to take into account recent experimental results and recommendations from CODATA and also to incorporate a correction. The original abstract follows and is still valid. A 1960’s suggestion by R. P. Feynman, concerning the possibility of carrying out a finite renormalization procedure in quantum electrodynamics, is here implemented using a newly discovered formula for $\alpha$, the fine structure constant.

1 Introduction

Quantum electrodynamics, QED is widely accepted as being the most successful theory of fundamental processes that has been assembled up to the present time. Assembled is perhaps a better term than discovered or created because it reflects it evolutionary development during much of this century. Its success lies in its power to predict measurable characteristics of a wide and very important range of physical systems. It is limited essentially to the interactions between electronic systems and the electromagnetic field. It is certainly difficult to overstate its significance in the general context of physical theory and in the numerical accuracy of its predictions it is preeminent, giving some results to twelve places of decimals. It has been the main source of ideas as to how the more general theories necessary in the high energy context might be constructed. However, there is a negative aspect of this success which stems from the very techniques that have been employed in the assembling of QED. It is no exaggeration to remark that the development of the QED structure has been plagued with infinities. To see how this has come about we shall now very briefly describe the form of calculational algorithm that QED consists of at its present stage of development. The essential basic computational machine is a series expansion, the S-matrix, in terms of a small numerical parameter called the fine structure constant, $\alpha \approx 1/137$. The size of this parameter is the essential ingredient that gives the series some sort of convergence. The coefficients of this series are complicated integrals over momentum space. These coefficients

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* See section 5 of this article
together with their appropriate power of $\alpha$ are the mathematical representatives of the various Feynman diagrams giving information about some physical process involving a specific power of $\alpha$. However, most of these coefficient are complex additions of a variety of integrals over momentum space and among the integrals are found quite a few that diverge to infinity in varying degrees. Much of the past work in QED research has been into finding techniques for evaluating the difficult integrals and into finding ways of making sense of the fact that they occur within the general QED structure. This has been accomplished by the introduction of methods, largely due to the work of F. J. Dyson, R.P. Feynman, and J. Schwinger for the evaluation of the rogue integrals while leaving undisturbed any real physical significance that they might have. This step is followed by renormalization, or the absorption of the offending parts into the fine structure constant power that goes along with that particular integral. The effect of the renormalization procedure is to produce a renormalized series in powers of the renormalized fine structure constant and with coefficients with no infinities and which represent only recognizable physical effects.

The preceding description of the state of affairs is exceeding over simplified but motivates a rather uncomplicated application of recent work by the present author to the evolution of QED. Greatly detailed discussion of the complications of renormalization can be found in references. Thus it is that the great success of QED which there is no wish to minimize is accompanied with the virulent question of the validity of the mathematics process that neatly throws away infinities and incorporates them into the fine structure constant which when used to generate the initial expansion was thought to be a definite numerical quantity approximately equal to $1/137$. This dilemma is almost certainly what Feynman had in mind when he made a suggestion in a book published in 1961. Essentially, he suggested that if a formula could be found for the fine structure constant in some future theory, not necessarily QED, it would be likely that a renormalization program could be carried through by finite steps. He was writing there specifically about charge renormalization though clearly if such a procedure can be accomplished with regard to charge renormalization its extension to the more general situation would likely follow without great difficulty. Thus in this short article we shall concentrate on the question of finding a finite renormalization scheme for charge. The result from QED that we require is the logarithmically divergent expression for the $Z_3$ renormalization factor which is the factor multiplying the theoretical charge $e$ in

$$e_R = \left(1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}\right)^{1/2} e \quad (1.1)$$

and is

$$Z_3 = \left(1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}\right)^{1/2} e. \quad (1.2)$$

$e_R$ is the renormalized charge which will be taken to be the actual measured physical value. This simple formula is obtained from QED by quite an elaborate series of steps and involves the addition of an infinite sum of Feynman diagrams as explained in references. The quantity $\Lambda$ is the cutoff value for mass in the
momentum space integrals and would go to infinity if the integral it is related to were fully evaluated according to the tenets of QED. However, it is kept at some unspecified but finite value greater than \( m \) on the understanding that it should go to infinity when its integral is suitably absorbed into the renormalized charge. Clearly it needs to be finite if sense is not to be violated while manipulations take place using equation (1.1). It should be emphasized that the result (1.1) arises from summing all relevant so called bubble graphs so that in a sense it is an exact result and not just an approximation associated with some power of \( \alpha \) or some term in the S-matrix series expansion. It is more convenient to use the fine structure constant version of equation (1.1) which is obtained by squaring both sides of (1.1) and then multiplying both sides by \( 1/(4\pi\epsilon_0\hbar c) \) by which means we obtain

\[
\alpha_R = \left(1 - \frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}\right) \alpha
\]

with \( \alpha_R \) being the renormalized fine structure constant and plain \( \alpha \) being the theoretical one. The value of the renormalized fine structure constant \( \alpha_R \) will also simultaneously coincide with the value from the physical definition,

\[
\alpha_R = e^2_R/(4\pi\epsilon_0\hbar c).
\]

This section will now be completed with my interpretation of a perspective on the renormalization factors such as \( Z_3 \) for which I am indebted to Professor C. W. Kilmister.

The introduction of the cutoff in the \( Z_3 \) multiplier is clever because it enables this potentially or actually divergent quantity to be manipulated. However, manipulated with what could be a false sense of security. This is because \( Z_3(\Lambda) \) is, as here indicated, defined as a function of \( \Lambda \) which when convenient is assumed to be finite but which has no defined definite finite values within the pattern of QED ideas. The definition of \( Z_3(\Lambda) \) as a function of \( \Lambda \) has the very restricted domain meaningful in QED of just \( \Lambda \) larger than any finite quantity. Further, there seems no prospect of assigning finite values to the parameter \( \Lambda \) for any QED originated reason. Thus some structure external to QED is needed in order to give some credence to an extension of the domain in which \( \Lambda \) can vary and have definite numerical values. Only then will it be possible to identify, at the very least, a conceptual range in which the manipulations of \( Z_3(\Lambda) \) are meaningful. A related reason why great care is needed in working with such potentially divergent quantities is that any finite quantities that might occur additively with them will have the ambiguous status of being present in some sense but numerically quenched and of uncertain relevance.

In the following sections, it will be shown that full account can be taken of this caveat. Equations (1.3) and (1.2) are all that is needed from QED to give firstly a very simple description of a finite renormalization scheme. Possible complications that might be contemplated will be discussed in section 4. With regard to the question of what Feynman exactly or precisely meant by his rather enigmatic remarks in the sixties suggesting the possibility of finite renormalization consequent on the finding of a formula for \( \alpha \) we cannot today be entirely certain. However, in section 3 of this paper a finite approach possibility for the electrical charge problem
is demonstrated in detail and seemingly conforms closely to what he apparently thought would follow the discovery of a formula for $\alpha$. That it has now been possible to construct a finite renormalization scheme seems to strongly confirm the correctness of Feynman’s conjecture and also the soundness of his intuition.

2 The formula for $\alpha$

The present author has recently found, from theory a QED independent formula for the fine structure constant which depends on two integer parameters which are denoted by $N$ and $N_b$. The formula is

$$\alpha(N, N_b) = N_b \cos(\pi/N) \tan(\pi/(NN_b))/\pi.$$  

This formula arises from the authors alternative theory for the quantum process and is entirely independent from the QED structure. This alternative theory will be referred to as GT in the following work. GT has a Schrödinger equation and special relativity basis. The integer 137 plays a special role in this theory so that for this and other reasons GT has some common conceptual ground with the work of Eddington and the more recent works of Bastin and Kilmister on the Combinatorial Hierarchy. The general validity of the formula (2.1) has now been strongly reinforced by showing that it has a fundamental and inevitable significance for the structure of the first Bohr orbits of the whole family of hydrogen like atoms.

We shall here employ the one variable subfunction,

$$\alpha(N_b) = \alpha(137, N_b) = N_b \cos(\pi/137) \tan(\pi/(137N_b))/\pi,$$  

in which $N$ is kept fixed at the value 137 leaving only $N_b$ to range over positive integral values. $\alpha(N_b)$ or some multiple of it will, for the purposes of this article, be taken to be our theoretical fine structure constant. Other than the fact that it is now a variable quantity depending on $N_b$ it corresponds with the plain $\alpha$ used earlier and it is to be regarded as the initial small parameter expander or unrenormalized $\alpha$ that is used in the S-matrix series. Also in that context the parameter $N_b$, would be expected to have some definite integral value from its allowable range. The choice of $N_b$ used in the initial expansion can be regarded as representing a decision as to how well we know the physical coupling before the S-matrix complications are turned on or, in other words, a selection of a less dressed coupling constant.

The 1999-2000 CODATA recommended value and reliability range for $\alpha$ is substantially different from the 1986 value, the new range being outside the old. In table 1 below the values of $\alpha(N_b)$ are given that lie in the experimental and recommended CODATA 1986 range $\alpha_{\text{Nucmin}}$ to $\alpha_{\text{Nucmax}}$ with center at the best experimental value $\alpha_{\text{Nuc}}$. The nearest value from GT taken by the function $\alpha(N_b)$ to this center value is given by taking $N_b = 25$ and this value differs from the center value by approximately $2.3 \times 10^{-11}$. I have decided to leave this information intact but italicised in this version II* of this article for ease of reader reference and as an illustration of the nature of the predictive character of the formula and its dependence on having definite experimental information concerning the range of

*See section 5 of this article
the possible values. What was thought to be definite information concerning range is now likely to have changed as a result of the forthcoming CODATA* report and speculation based on *rumour* concerning the contents of this yet unpublished report follows after the paragraph involving the old $\alpha_b$, equation (2.3).
Table 1  

N\text{\textsubscript{b}} solutions with 1986 CODATA \(\alpha\) values and bounds

| \(N\text{\textsubscript{b}}\) | \(\alpha\)-values |
|---|---|
| \(N\text{\textsubscript{uc}}\text{max}\) | \(\alpha = 0.007297353410\) |
| 24 | \(\alpha(24) = 0.007297353232\) |
| \(N\text{\textsubscript{uc}}\text{min}\) | \(\alpha = 0.007297353080\) |
| 25 | \(\alpha(25) = 0.007297353057\) |
| 26 | \(\alpha(26) = 0.007297352903\) |
| 27 | \(\alpha(27) = 0.007297352766\) |

The next nearest value to the center value \(\alpha_{N\text{uc}}\) is given by taking \(N\text{\textsubscript{b}} = 24\) but this differs from the center value by approximately \(15.2 \times 10^{-11}\). Thus taking the simplistic but possible reasonable view that the center of the range is better than the more removed parts, \(N\text{\textsubscript{b}} = 25\) is better than \(N\text{\textsubscript{b}} = 24\) by a factor of about 6. There is another reason\textsuperscript{16} that 25 might be preferred to 24 and that is because 25 has the simple relation \(25 = k_2^2/4\) with the second combinatorial\textsuperscript{16} special number \(k_2 = 10\), the 4 relating to the value of the total solid angle \(4\pi\). However, this connection may be dismissed as pure numerology! Taking account of the latest measured and theoretically adjusted value\textsuperscript{9} ascribed to the fine structure constant, the best value from GT for the fully dressed coupling constant is obtained from taking \(N\text{\textsubscript{b}} = 25\) and is given by,

\[
\alpha_{\text{b}} = \alpha(25) = 0.007297353057
\]

(2.3)

to twelve places of decimals. It is emphasized that the value of \(\alpha(N\text{\textsubscript{b}})\) at \(N\text{\textsubscript{b}} = 25\) given in (2.3) is to be regarded as the value obtained by measurement and corresponds to the renormalized \(\alpha_R\) used earlier.

The next CODATA* report\textsuperscript{19} is likely to gives a recommended value for \(\alpha\)

\[\alpha = 0.007297352534(13)\]

In contrast with the 1986 range, there is only one value given by the predictive formula (2.2) that lies in this new 1999-2000? CODATA* range and that is the value when \(N\text{\textsubscript{b}} = 29\) which is

\[\alpha_{\text{b}} = \alpha(29) = 0.007297352532\]

(2.3b)

This prediction differs from the speculated recommended value by approximately that 2 parts in \(10^{12}\) parts. This is certainly very impressive accuracy assuming the speculation is correct. Thus from now on in this version of the article we shall use the parameter value \(N\text{\textsubscript{b}} = 29\).

From (2.2) and (2.3b) it follows that

\[
\alpha_{\text{b}} = \frac{\text{29 tan}(\pi/(137 \text{29}))}{N\text{\textsubscript{b}} \text{tan}(\pi/(137N\text{b}))} \alpha(N\text{\textsubscript{b}}).
\]

(2.4)

Thus from the theoretically deduced \(\alpha(N\text{\textsubscript{b}})\) value we find theoretically deduced from GT a charge renormalization factor \(Z_3\) of form,

\[
Z_3 = \left(\frac{29 \text{tan}(\pi/(137 \text{29}))}{N\text{\textsubscript{b}} \text{tan}(\pi/(137N\text{b}))}\right)^{1/2}
\]

(2.5)

*See section 5 of this article
the square of which by multiplication converts the theoretical value of $\alpha$ to the measured or renormalized value. We note that the renormalization factor from QED given by (1.3) depends on the cutoff parameter mass $\Delta$ from the momentum space integrations. There is no compelling reason for this to be infinite other than for the reason that the QED formalism imposes no restriction on the integrations over momentum space. If there were such restriction in QED then the nature of the problem would change drastically. However, all the evidence up to date is that the possibly infinite integrals do not contribute to the physical information contained in that theory. This is the reason that the possible infinities can be dumped into the renormalized coupling constant and in effect disregarded. Of course, the conceptual difficulties are not removed by recognizing this aspect. Thus it would be quite acceptable and rather convenient if the finite cutoff $\Delta$ used in QED as an infinity remedial was in fact an actually finite limit for some hitherto unnoticed reason. As the possible infinite integrals are physically irrelevant anyway they could still be dumped into a renormalized coupling constant but conceptually rather more easily. This is the possibility that Feynman had in mind but then he had no theoretical formula for the fine structure constant. If we consider the theoretically deduced renormalization factor $Z_3$ from GT as given by (2.5), we see that it also contains a cutoff quantity, the value of $N_b$ at the specific value $N_b = 29$. The cutoff in GT arises from what seems to be an upper limit to the snap of bending of a wave in order for it to make a best fit to a curved contour along which it is moving. The formula (2.5) still makes sense for values above $N_b = 29$ but there seems to be this physical cutoff at the actual measured value of the fine structure constant. Thus we are motivated to investigate the possibility that the two cutoffs, the one from QED and the one from GT, a theory external to QED, are related functionally and in some physically determined sense.

3 Finite Renormalization

It is possible to exploit the fact that the QED $Z_3(\Lambda)$ is a function of $\Lambda$ by equating the two expressions (1.2) and (2.5) for the $Z_3$ charge renormalization factors. Making this step we get the relation

$$
\left(1 - \frac{\alpha(N_b) \ln \Lambda^2(N_b)}{3\pi} m^2\right) = \left(\frac{29 \tan(\pi/(137.29))}{N_b \tan(\pi/(137N_b))}\right)
$$

where now the $\alpha$ in the QED factor (1.2) has been identified with the theoretical fine structure constant and the possible dependence of the QED cutoff, $\Lambda$, on $N_b$ has also been taken into account. We observe that when $N_b = 29$, $\Lambda = m$ as then both sides of the equation reduce to unity making the renormalized charge and theoretical charges equal in both QED and in GT. Solving equation (3.1) for $\Lambda(N_b)$, we obtain,

$$
\Lambda(N_b) = m \exp \left(\frac{3\pi}{2\alpha(N_b)} \left(1 - \frac{29 \tan(\pi/(137.29))}{N_b \tan(\pi/(137N_b))}\right)\right).
$$

The $Z_3(N_b)$ from GT is a definite and meaningful function of $N_b$. Thus identifying the two different $Z_3$ factors from QED and GT and the consequent generation of
the definite and finite relation (3.2) between \( N_b \) and \( \Lambda(N_b) \) adds to QED from the external system GT the possibility for rationally ascribing values to \( \Lambda(N_b) \) other than just infinity. It is then possible to make sensible a QED \( Z_3(\Lambda) \) functional dependence on an extended domain of \( \Lambda \) values. Thus effective account is taken of the Kilmister caveat. The relation between the two cutoff parameters given in equation (3.2) solves the problem of constructing a finite scheme for charge renormalization as originally conceived by R.P. Feynman. The prescription for its use is thus as follows:-

Expand the S-matrix as usual using a value for the fine structure constant in its theoretical form \( \alpha(N_b) \) at a specific value of \( N_b \). Cutoff the logarithmic divergent integrals arising in the coefficients at the corresponding \( N_b \) value as given by formula (3.2). Dump these unwanted finite integrals into the renormalized \( \alpha_R = \alpha_b \) as given by equation (1.3). Further, if the \( N_b \) value chosen at the outset is 29 then all the potentially logarithmically divergent integrals will evaluate to zero because \( \Lambda(N_b) = m \) in this case and so all such integrals can be ignored anyway. In all these situations the final S-matrix expansion will be in terms of the renormalized measured fine structure constant and will have only finite integral coefficients.

4 Complications

It has been shown how the charge renormalization process that plays the central role in QED of separating physically relevant information from possible divergent but otherwise redundant information can be carried through by finite steps according to a suggestion by R.P. Feynman. As we have noted, \( Z_3 \) is usually regarded as being in some sense a divergent quantity. Thus it might be argued that because its derivation in QED involves the loss of small quantities that would be important when in fact \( \Lambda \) is small as in the case in hand, the formula (1.1) for \( Z_3 \) may not be adequate in its simple form. This issue is another aspect of the Kilmister caveat. It will now be shown that a more general version of (1.1) covering all such possible small quantity omissions which has been obtained from QED can be used in place of (1.2). This more general \( Z_3 \) has the form

\[
Z_3 = \left( 1 - \frac{\alpha}{3\pi} (D(\frac{\Lambda}{m}) + G(\alpha)) \right)^{1/2}.
\] (4.1)

QED derived versions for the functions \( D(x) \) and \( G(x) \) are

\[
D(x) = \ln(x^2 + 1) + \frac{1}{x^2 + 1} - 1
\] (4.2)

and

\[
G(x) = -2/3.
\] (4.3)

The value given here at (4.3) for the constant \( G(x) \) is the correction referred to in the abstract. The original value used was 5/6. This change in magnitude and sign makes no difference to the question of the validity of the idea of how to handle such constant terms that might have been missed in the QED renormalization argument. However, it does make some difference to numerical values of derived quantities such
as λ for example in equation (4.10). The wrong number was used earlier as a result of this author misinterpreting numerical results from published work on QED. If we use these two function in (4.1) we find that the quantity

$$C = -\frac{\alpha}{3\pi} (D(\frac{\Lambda}{m}) + G(\alpha))$$

(4.4)

does not have the value zero when $\frac{\Lambda}{m} = 1$ the value at which the potentially divergent integral actually evaluates to the value zero corresponding with the renormalized and theoretical value of the fine structure constant coinciding. This rather satisfactory behavior associated with the original $C' = -\frac{\alpha}{3\pi} \ln \frac{\Lambda^2}{m^2}$ in (1.2) can be restored for the renormalization factor (4.1) by making use of some freedom of choice that we have in selecting the theoretical fine structure constant which in this more general situation we shall denote by $\alpha_g(N_b)$. Clearly there is freedom in this respect because mathematically any small value could in principle be used to generate the original S-matrix expansion. In keeping with the usual thinking about the theoretical $\alpha_g$, although here it will be finite, it is still not a necessarily a physically measurable quantity. However, it is very important that the renormalized $\alpha_R$ which is physically measurable should have the most accurate numerical value attached that has been agreed from experimentation. Thus in the more general situation we define the theoretical fine structure constant $\alpha_g(N_b)$ by

$$\alpha_g(N_b) = \lambda \alpha(N_b),$$

(4.5)

where the multiplier $\lambda$ is chosen so that also in the more general situation

$$\alpha_g(N_b) = \lambda \alpha(N_b),$$

(4.6)

and $\alpha(N_b)$ is still given by the function defined in (2.2). Thus the new $Z_3$ factor from GT will be given by,

$$Z_3^2 = \frac{\alpha_R}{\alpha_g} = \frac{\alpha(29)}{\lambda \alpha(N_b)} = \left( \frac{29 \tan(\pi/(137 \times 29))}{\lambda N_b \tan(\pi/(137 N_b))} \right).$$

(4.7)

The relation between the two cutoff parameter $\Lambda$ and $N_b$ is now obtained by equating (4.1) squared with (4.7) and in this more general representation becomes

$$D(\frac{\Lambda(N_b)}{m}) = \frac{3\pi}{\lambda \alpha(N_b)} \left( 1 - \left( \frac{29 \tan(\pi/(137 \times 29))}{\lambda N_b \tan(\pi/(137 N_b))} \right) \right) - G(\alpha).$$

(4.8)

Applying the conditions (4.6) in equation (4.8) gives a quadratic equation for $\lambda$ with solutions

$$\lambda = \frac{2}{1 \pm \left(1 - \frac{4\alpha_g}{3\pi}(\ln(2) + \frac{\pi}{6})\right)^{1/2}}.$$  

(4.9)

The value of $\lambda$ given by the positive sign in (4.9) is near to unity. The value with the negative sign is $\lambda \approx -2728.52$. So that to keep to a value for the theoretical
fine structure constant, \( \alpha_g(N_b) = \lambda \alpha(N_b) \), near to \( 1/137 \) the positive sign solution is chosen. Thus giving the value of \( \lambda \) as

\[
\lambda \approx \frac{1}{1 - \frac{\alpha_g}{3\pi}(\ln(2) - \frac{7}{6})} \approx 0.999633635. \quad (4.10)
\]

With the value (4.10) for \( \lambda \) the cutoff quantity \( \Delta(20) \), evaluated at \( N_b = 20 \), assumes the value \( \Delta(20) = 1.000594024m \) just slightly greater than the electronic rest mass. The intrinsic formula (4.8) for \( \Lambda(N_b) \) in terms of \( N_b \) is not quite as simple as (3.2) but it will still give the cutoff in \( \Lambda(N_b) \) that arises from the values of \( N_b \) and apart from being a little more complicated the finite renormalization scheme is still effective.

5 Prediction of \( \alpha \)'s measured value

In this section, additional to the original version of this paper, it is explained why the change from \( N_b = 25 \) to \( N_b = 29 \) has become necessary in the light of long awaited next CODATA report on the measured values of the fundamental constants. This report is some years overdue and it is still uncertain whether it will appear this year 1999 or in the year 2000. In the light of this uncertainty, I have decided to anticipate its findings for the experimental numerical value of \( \alpha \) by speculation based on rumour.

Firstly, we consider my theoretical formula for \( \alpha \) and the nature and significance of the prediction involved in its discovery. The formula (2.1) in question is

\[
\alpha(N, N_b) = N_b \cos(\pi/N) \tan(\pi/(N_b \times N))/\pi. \quad 5.1
\]

The two dimensional domain of this function is taken to be all positive integer pairs \((N, N_b)\). The value ascribed to the integer \( N \) is dominant in determining the function value and is involved as the angle \( \chi = \pi/N \) which is the size of the angular sector that a trapped electron wave would occupy in a circular Bohr orbit if it were moving with the quantized velocity \( N \alpha c \) with \( N = 137 \). The second integer \( N_b \) has a more subtle significance and its value controls very small variations from the value determined by \( N \). It measures the number of linear quantized subdivisions in the trapped wave that occur so that the wave can aligns itself as near as possible to a circular section of its orbit while its mean velocity coincides with the quantized velocity associated with the orbit. In a sense this is the requirement that the relativistic contraction factor for the whole wave body should have the usual relativistic dependence on the quantized velocity in orbit. However, this conformity imposes no restriction on the actual integral value of \( N_b \). The requirement of this conformity is simply a result of the quantization of the wave length into a finite number of linear segments. Thus the formula is predictive in that it says the value of the fine structure constant is determined by two integers but the two integers are not given by the theory and do have to be determined by detailed experimental knowledge of the range of values that the fine structure constant is considered to lie within. The integer value \( N = 137 \) is however inevitable as any change in this would give values for \( \alpha \) greatly outside the range of values generally accepted as inferred from experimental measurements. The choice of the integer pair has
to be made from the experimental information concerning the measured range of values that is assessed as contained the numerical value of $\alpha$ as near to certainty as possible. Thus if this range is large there are many possibilities and if the range is sufficiently small the possibilities could be reduced to a single unique value. The 1986 CODATA\textsuperscript{9} range within which the numerical value of $\alpha$ was claimed to reside was small but the predictive function (5.1) had four possibilities $N_b = 24, 25, 26, 27$ with 25 the value nearest to the centre as shown in table 1. Thus it seemed that the value $N_b = 25$ would give the best prediction and this was reinforced by the fact that the physical definition for $\alpha$ equation (1.4) gave a value only differing from this value by approximately 2 part in $10^{11}$. Thus $N_b = 25$ seemed an excellent option and this option was used by the author in \textsuperscript{21} on the fine structure constant. However, CODATA is expected to publish its next report in 1999 – 2000 and there are what can only be described as rumours that substantial changes are expected with possibly the new range being outside the old range.

Firstly let us dwell on the way the CODATA published information and recommendations for fundamental constant values is constructed out of the enormous amount and very diverse world wide experimental information they gather on the measured values of the various physical constants. A greatly simplified description of this very complex operation is as follows. As far as the fine structure constant was concerned, in the 1996 report there were eight main largely independent different type of experimental information sources each source having its own values and reliability range. The central value given by the various sources were very disparate with values for the inverse fine structure constant ranging from 137.0359 to 137.036 roughly. This is a large variation when we consider that we are attempting to get the error for $\alpha$ itself down to less than something of the order of 2 parts in $10^{12}$ parts. The experimental information that has been gathered for the next report is likely to be at least as widely varied as the earlier 1986 version. Thus the daunting problem for CODATA is firstly deciding on the comparative reliability and significance of the various sources. Then a very elaborate statistical analysis of the information to hand will hopefully generate a best value preferably the actual physical value for the fine structure constant together with a single range about this best value within which they can claim the true physical value is certain to lie. Whereas the final single range deduced by this analysis may be very reliable or even certain it seems unlikely that their central value will be the true physical value unless they are very lucky. The new CODATA report will have successfully had to perform this task and have had to come up with values that any predictions can be checked against. So that the coming report holds an exciting prospect for checking the accuracy of my predictive formula. As mentioned earlier, there are only rumours available at this moment of time. The hottest rumour at the moment of the date on this article is that the new range is much smaller than the old range and that the new recommended value for $\alpha$ is much nearer to $1/137.035999$ than was the old value. If this turns out to be the case, the value $N_b = 29$ could be very near the mark and hopefully it might be the only value predicted within the new range. This would strongly confirm the validity of formula (5.1). Such changes that might occur make no difference to questions of the validity or make changes of principle necessary in the theoretical structure that the formula was generated
from. However, numerical values that have been obtained for other physical quantities in reference\textsuperscript{21} will be slightly changed as these depend on the value of $N_b$ that is supposed to give the actual physical, \textit{renormalized} value. This aspect will effect the author’s application of the formula (5.1) to the problem of constructing a scheme of \textit{finite} renormalization\textsuperscript{2}. Very small numerical changes in renormalized quantities will occur if it is found necessary to replace $N_b = 25$ with $N_b = 29$. In particular, the physical or renormalized value of the fine structure constant will be given by $\alpha(137, 29)$ rather than by $\alpha(137, 25)$. These changes have now been made in this version of the original paper.

6 Conclusions

The conclusion regarding the \textit{definitely} finite terms that might be regarded as missing from the simple $Z_3$ formula (1.1) is that they can easily be absorbed into the \textit{bare} or theoretical fine structure constant. That is into initial $S$-matrix expander for which there is \textit{input} freedom of choice except that it should have a value near to $1/137$. Thus because of this freedom in the selection of the \textit{finite} theoretical fine structure constant our initial simple derivation of the \textit{finite} renormalization scheme in section 3 is not significantly unsound. Feynman’s remarks in reference\textsuperscript{4} were only about charge renormalization and here also only charge renormalization has been considered so far. In the more general divergency context, mass renormalization which as is well known can be carried through to all orders in the coupling constant, should be shown to fit into a finite renormalization scheme. In view of the ease with which \textit{finite} charge renormalization has been accomplished by using the new formula (2.1) for the fine structure constant there seems no obvious reason why a \textit{finite} mass renormalization scheme should not also be constructed. It is immediately obvious that if the cutoff in Feynman’s mass counter term $\delta m = m_R - m$ is taken to be the same quantity as the cutoff parameter $\Lambda$ used earlier and the formula for the theoretical fine structure constant (2.2) is used in its representation, we have

$$\frac{\delta m(N_b)}{m} = \frac{\alpha(N_b)}{2\pi} \left( \frac{3}{2} \ln\left( \frac{\Lambda^2(N_b)}{m^2} \right) + \frac{3}{4} \right). \quad (6.1)$$

This mass counter term is finite for positive integral values of $N_b$ and in particular for the physical value $N_b = 29$. Thus it can be manipulated without ambiguity in the renormalization operations. We note the physical and finite value

$$\frac{\delta m(29)}{m} = \frac{3\alpha(29)}{8\pi} \approx 0.00089106. \quad (6.2)$$

if (3.2) is used. As with the $Z_3$ factor, Feynman’s simple version of $\frac{\delta m(N_b)}{m}$ may not be considered adequate when $\Lambda$ is finite. However, this is also easily generalized. The main conclusion can be expressed as follows. An additional rule, equation (3.2) or more generally (4.8), from GT can be added to the QED formalism restricting the integration range of the divergent integrals so that the renormalization process can be made finite and therefore conceptually rational. The renormalization process can then be seen as a \textit{very} complex \textit{one} step iteration operation with only finite quantities. A numerically exact (unrenormalized) fine structure constant is the
initial input expansion parameter with some reasonable finite starting value. The whole QED formalism then, through the S-matrix expansion, generates a finite physically exact fine structure constant to replace the initial parameter value by a single iteration step under the control of an externally imposed constraint.

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