Spin Accumulation in the Electron Transport with Rashba Interaction

Yi-Ying Chin,1 Jui-Yu Chiu,2 Ming-Che Chang,3 and Chung-Yu Mou1,4

1 Department of Physics, National Tsing Hua University, Hsinchu 30043, Taiwan
2 Dept. of Physics, University of California-San Diego, La Jolla, California 92093-0319, USA
3 Dept. of Physics, National Normal University, Taipei, Taiwan
4Physics Division, National Center for Theoretical Sciences, P.O.Box 2-131, Hsinchu, Taiwan

(Dated: June 26, 2018)

The non-equilibrium transportation of two-dimensional electrons through a narrow channel is investigated under the influence of the Rashba interaction. By introducing suitable lifetime in the Green’s function, the average spin values can be calculated from the ballistic regime to the diffusive regime. It is shown that the spin accumulation is a combined effect of the spin current and disorders. In the diffusive regime, disorders offer a mechanism to stop the spin current and generate the spin accumulation. In the ballistic regime, spins are more spread out and do not have definite signs. Further consideration indicates that the inclusion of ferromagnetic spin-spin interaction increases the spin accumulation near the edge.

PACS numbers: 72.25.-b, 72.25.Dc, 73.40.-c

Generation and transfer of spins, preferably without using external magnetic field or magnetic materials, is an important issue in spintronics[1]. In semiconductors, through the intrinsic spin-orbit interactions related to the Dresselhaus[2] or the Rashba[3] mechanism, it is possible to generate, or even manipulate spins using only an electric field. Recently, it is further proposed that the spin Hall current can be generated in bulk semiconductors[4] and semiconductor heterojunctions[5] because of the spin-orbit interactions. Unlike an earlier proposed spin Hall effect involving impurities[6], these two studies demonstrated that the possible spin Hall effect is possible in pure samples without any impurities. Experimentally, the spin Hall effect has been observed in a number of systems[7]. The key observation is the accumulation of opposite spin polarizations on two sides of the sample. While it is natural to associate the spin-accumulation with the spin Hall effect, for the intrinsic case, a satisfying theory of spin accumulation still does not exist.

In this paper, by introducing suitable lifetime in the Green’s function, the spin accumulation will be calculated from the ballistic regime to the diffusive regime. The origin of spin accumulation will be clarified.

We start by considering a two-dimensional electron system confined in a channel

\[ H = -\frac{\hbar^2}{2m} \nabla^2 + \frac{\alpha}{\hbar} \hat{\sigma} \cdot \hat{\mathbf{p}} + V_C + V_I. \]  

Here \( \alpha \) characterizes the Rashba interaction, \( \hat{\sigma} \) are Pauli matrices and \( V_C \) is the hard-wall potential that confines the electron to \(-W/2 < y < W/2\). \( V_I \) is the potential due to point disorders, \( V_I(r) = \sum_{i} n_i \varphi_i^0(r - R_i) \). We shall use \( \langle \rangle \) to denote averaging over disorders and \( n_i \) to denote the concentration of impurities. Since \( H \) possesses the time-reversal symmetry, the Kramer’s theorem implies that each eigenstate is doubly-degenerate. Physically, the degeneracy represents two opposite propagating directions along \( x \) axis. Therefore, we shall denote the Kramer degeneracy by \( \lambda = \pm \), representing propagating along \( \pm x \) respectively.

To find the spin-accumulation \( \langle S_z(r) \rangle \), we first note that because \( \sigma_y H(x, y) \sigma_y = H(x, -y) \), for each eigenstate \( \psi_{n\lambda}(x, y) \), one obtains \( \sigma_y \psi_{n\lambda}(x, y) = \psi_{n\lambda}(x, -y) \) and thus it follows that \( s_z(r) \) is antisymmetric in \( y \). This simple argument, however, cannot assign a definite sign for \( s_z(r) \). Numerics is then necessary to pin down the sign. It is important to note that for each energy, \( s_z(r) \) have opposite signs for opposite propagating directions. As a result, spin-accumulation can only happen when the system is not in equilibrium, i.e., there must be a biased potential between two electrodes. Therefore, one has \( s_z(r) = \sum_{\lambda} \langle \psi_{n\lambda}^\alpha(r) | S_z^\beta \psi_{n\lambda}^\beta(r) \rangle \), where \( \psi_{n\lambda}^\beta(r) \) denotes eigenstates with \( \lambda = \pm \) and the energy \( E_n \) is restricted to the regime \( \epsilon V_1 \leq E_n \leq \epsilon V_2 \) with \( V_1 \) and \( V_2 \) being the electric potentials of electrodes. By inserting \( \int_{\epsilon V_2} \int_{\epsilon V_1} d\epsilon dE \langle E - E_n \rangle \) and using the identity \( \text{Im}[1/(E - E_n + i\epsilon)] = -i\pi \delta(E - E_n) \), we find

\[ \langle S_z(r) \rangle = \int_{\epsilon V_1} dE \int_{\epsilon V_2} d\epsilon \left\{ -\frac{1}{\pi} \text{Im} \text{Tr}[S_z G^+(E, \mathbf{r}, r)] \right\}, \]  

where \( G^+(E, \mathbf{r}, r) \) is the retarded Green’s function in the \( \lambda = + \) channel and \( \text{Tr} \) is the trace over the spin space. In the Born approximation, \( G^+(E, \mathbf{r}, r) = \sum_\gamma \psi_\gamma^\alpha(r)^* \phi_\gamma^\beta(r) / (E - E_n + i\gamma N_\gamma(E, \mathbf{r}, r) + i\epsilon) \) with \( \gamma = n_i \varphi_i^0 \), \( \phi_{n\lambda}^\beta \) being the energy eigenstate and \( N_\gamma(E, \mathbf{r}, r) \) being the local density of states in the absence of disorders[8]. Because \( k_x \) is a good quantum number in the absence of disorders, \( \lambda = \pm \) corresponds to positive \( k_x \). For a given \( k_x \), \( \phi_{n\lambda}^\beta \) is a linear combination of four waves with \( k_y = \pm k_y^\pm \), where \( E \equiv \hbar^2 k_x^2/2m \pm \alpha k_y \) with \( k_x^\pm = k_x^2 + (k_y^\pm)^2 \). The hard-wall boundary condition
yields the following equation and further selects \( k_y^\pm \)
\[
1 + e^{2i(k_y^+ + k_y^-)W} - 4e^{i(k_y^+ + k_y^-)W} \frac{k_y^+ k_y^-}{k_y^+ k_y^- + k_y^2 + k_y^-} - (e^{ik_y^+ W} + e^{ik_y^- W}) k_y^+ k_y^- + k_y^2 - k_y^+ k_y^- = 0. \tag{3}
\]

Once \( k_y^\pm \) are found, \( \phi_{n+} \) and thus \( G_{+}^{\gamma}(E, r, r) \) are obtained. Clearly, \( k_y^\pm \) determine how \( s_z(r) \) oscillates for each state and in general, there is no obvious spin accumulation for each state. This is similar to the charge Hall effect in which there is no charge accumulation for single particle states and one needs Coulomb interaction to stop charges and generate charge accumulation. Fig. 1 shows the numerical results of \( s_z(r) \) for different \( \gamma \). By setting \( \gamma \to 0 \), one reaches the ballistic limit where spins are more spread out, and furthermore, as shown in the inset of Fig. 1, depending on parameters, \( s_z(r) \) may always switch the sign. Nonetheless, as one turns on disorders, \( s_z(r) \) begins to accumulate near the edge and always switches to be negative on the left hand side (LHS) and positive on the right hand side (RHS). Obviously, disorder is mechanism for generating spin accumulation. The reason can be found by exploring the continuity equation \( \partial s_z(r, t)/\partial t = -s_z(r, t)/\tau - \nabla \cdot J_z(r, t) \) where \( \tau \) is the effective diffusion time due to disorders and \( J_z \) is the spin current. In the steady state, one obtains \( s_z(r) = -\tau \nabla \cdot J_z(r, r) \), which implies that if \( J_z \) flows from right to left, \( s_z(r) \) is negative on LHS and positive on RHS, in consistent with numerical results. Thus the spin diffusion generates the spin accumulation that stops the spin current.

In addition to the spin diffusion, the spin-spin interaction, \( J \sum \sigma_i \sigma_j \), further enhances the spin accumulation. Fig. 2 shows our numerical results based on the self-consistent mean-field theory. Clearly, ferromagnetic coupling amplifies the oscillation of the spin density and increases the accumulation. For antiferromagnetic coupling, the oscillation has a shorter length scale because neighboring spins tend to be antiparallel.

In conclusion, we have investigated the spin accumulation from the ballistic limit to the diffusive regime. The origin of the spin accumulation is clarified. This research was supported by NSC of Taiwan.

[1] Igor Zutic, Jaroslav Fabian, S. Das Sarma, Rev. Mod. Phys. 76 (2004), p. 323.
[2] G. Dresselhaus, Phys. Rev. 100 (1955), p. 580.
[3] E. I. Rashba, Sov. Phys. Solid State 2 (1960), p. 1224.
[4] S. Murakami, N. Nagaosa, and S. C. Zhang, Science 01 (2003), p. 1348; Phys. Rev. B 69 (2004), 235206.
[5] Jairo Sinova, Dimitrie Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. 92 (2004), 126603.
[6] J.E. Hirsch, Phys. Rev. Lett. 83 (1999), p. 1834.
[7] Y.K. Kato, R.C. Myers, A.C. Gossard, and D.D. Awschalom, Science 306, 1910 (2004); J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. 94, 047204 (2005).
[8] G. D. Mahan, Many-particle Physics, p264, 2nd Edition, Plenum Press, New York (1990).