Letter

Controlling the entropic uncertainty lower bound in two-qubit systems under decoherence

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Abstract
The uncertainty principle is an inherent characteristic of quantum mechanics. This principle can be formulated in various forms. Fundamentally, this principle can be expressed in terms of the standard deviation of the measured observables. In quantum information theory, the preferred mathematical quantity to express the entropic uncertainty relation is Shannon’s entropy. In this work, we consider the generalized entropic uncertainty relation in which there is an additional particle as a quantum memory. Alice measures on her particle \(A\) and Bob, with memory particle \(B\), predicts Alice’s measurement outcomes. It has a wide range of applications, for example, entanglement witnessing and quantum key distribution. We study the effects of the environment on the entropic uncertainty lower bound in the presence of weak measurement and measurement reversal. The dynamical model that is intended in this work is as follows: first the weak measurement is performed, second, the decoherence affects the system and finally the measurement reversal is performed on the quantum system. Here, we consider the generalized amplitude damping channel and depolarizing channel as environmental noises. We will show that in the presence of weak measurement and measurement reversal, despite the presence of environmental factors, the entropic uncertainty lower bound drops to an optimal minimum value. In fact, weak measurement and measurement reversal enhance the quantum correlation between the subsystems \(A\) and \(B\), and thus the uncertainty of Bob regarding Alice’s measurement outcomes reduces. Based on the entropic uncertainty relation, one can also show that the weak measurement and measurement reversal can preserve the quantum secret key rate lower bound from decoherence.

Keywords: entropic uncertainty relation, weak measurement, measurement reversal, decoherence

(Some figures may appear in colour only in the online journal)

1. Introduction
One of the inherent features of quantum mechanics is the uncertainty principle. This principle sets a limit on our ability to precisely predict the measurement outcomes of two incompatible observables simultaneously. According to the Heisenberg uncertainty principle, it is not possible to measure the position and momentum of a particle simultaneously with
high precision [1]. In [2], based on the Heisenberg uncertainty principle, Kennard formulated the first uncertainty relation of position $\hat{x}$ and momentum $\hat{p}$ as
\begin{equation}
\Delta \hat{p} \Delta \hat{x} \geq \frac{\hbar}{2}.
\end{equation}

Robertson [3] and Schrodinger [4] have shown that for arbitrary pairs of incompatible observables $\hat{Q}$ and $\hat{R}$, the uncertainty relation is introduced in terms of the standard deviation as
\begin{equation}
\Delta \hat{Q} \Delta \hat{R} \geq \frac{1}{2} |\langle \psi \mid [\hat{Q}, \hat{R}] \mid \psi \rangle|,
\end{equation}
where $\Delta \hat{Q} = \sqrt{\langle \psi \mid \hat{Q}^2 \mid \psi \rangle - \langle \psi \mid \hat{Q} \mid \psi \rangle^2}$ and $\Delta \hat{R} = \sqrt{\langle \psi \mid \hat{R}^2 \mid \psi \rangle - \langle \psi \mid \hat{R} \mid \psi \rangle^2}$ are the standard deviations and $[\hat{Q}, \hat{R}] = \hat{Q} \hat{R} - \hat{R} \hat{Q}$. The lower bound of this uncertainty relation is dependent on the state of the system. It becomes trivial if the expectation value of the commutator $[\hat{Q}, \hat{R}]$ on state $|\psi\rangle$ is zero. The uncertainty relation is expressed in various forms. In quantum information theory the preferred mathematical quantity to express the uncertainty relation is Shannon’s entropy [5]. With regard to the concept of Shannon’s entropy, which indicates the amount of awareness about the measurement outcomes, it is quite logical to express the uncertainty relation in terms of Shannon’s entropy. In [6], one of the most famous uncertainty relations was presented by Kraus. It was proved by Maassen and Uffink [7]
\begin{equation}
H(\hat{Q}) + H(\hat{R}) \geq \log_2 \frac{1}{c},
\end{equation}
where $H(\hat{O}) = -\sum_i p_i \log_2 p_i$ is the Shannon entropy of the measured observable $\hat{O} \in \{\hat{Q}, \hat{R}\}$, $p_i$ represents the possibility that the result of the measurement of the observable $\hat{O}$ on the system $\rho$ is $\hat{O}$, and $c = \max_{(|j\rangle, |r\rangle)} |\langle j\mid r \rangle|^2$ quantifies the ‘complementarity’ between the observables, where $|j\rangle$ and $|r\rangle$ are the eigenstates of the Hermitian observables $\hat{Q}$ and $\hat{R}$, respectively. In [8], Berta et al examined the situation in which Bob has an extra particle as a quantum memory (particle $B$), which is entangled with the particle that is available for Alice (particle $A$). They showed that when Alice measures $\hat{Q}$ and $\hat{R}$, the uncertainty of Bob, that has access to the memory particle, regarding Alice’s measurement outcomes is bounded by
\begin{equation}
S(\hat{Q}|B) + S(\hat{R}|B) \geq \log_2 \frac{1}{c} + S(A|B),
\end{equation}
where $S(A|B) = S(AB) - S(B)$ is the conditional Von Neumann entropy, $S(\rho) = -\text{tr}(\rho \log_2 \rho)$ denotes the Von Neumann entropy and $S(\hat{O}|B) = S(\hat{O}^B) - S(\hat{O}^A)$, $O \in \{\hat{Q}, \hat{R}\}$ shows the conditional Von Neumann entropies of the post measurement states
\begin{equation}
\rho^B = \sum_i (|a_i\rangle\langle a_i| \otimes I) \rho^B (|a_i\rangle\langle a_i| \otimes I),
\end{equation}
where $\{|a_i\rangle\}$’s are the eigenstates of the observable $O$, and $I$ is the identity operator. Berta et al’s entropic uncertainty relation has vast applications in the field of quantum information, such as witnessing entanglement and cryptographic security [8–10]. Note that much effort has been made to improve Berta et al’s entropic uncertainty relation [11–20]. In the real world, quantum systems interact with their surroundings, thus investigation of open quantum systems from various perspectives has been the subject of intense research in recent years. However, in a realistic quantum world, entanglement is inevitably affected by the interaction between the system and its environment, which leads to degradation. Given the importance of the entanglement between particle $A$ and $B$ in Berta et al’s uncertainty relation to predict the measurement outcomes, it seems obvious to protect entanglement from environmental noise. Much effort has been made to achieve this purpose, such as dynamical decoupling [21–23], decoherence free subspaces [24–26], a quantum error correction code [27–29], an environment-assisted error correction scheme [30] and quantum Zeno dynamics [31, 32]. In the literature, it is mentioned that weak measurements and quantum measurement reversals can protect the single qubit system from decoherence [33–36]. This important issue has also been developed on two-qubit systems for protecting the entanglement from decoherence [37–39]. Moreover, the weak measurement and measurement reversal protocol can preserve the teleportation fidelity [40] and quantum secret key rate [41] in the presence of amplitude damping noise. In this work, Berta et al’s entropic uncertainty lower bound (EULB) in equation (4) is investigated in the presence of environmental noise, and we consider the generalized amplitude damping channel and depolarizing channel as environmental noises. As expected, due to the environmental effect, an increase of the EULB is inevitable. Here, we control the EULB by using weak measurements and measurement reversals. We will show that the EULB can be reduced to an optimal value by performing weak measurements, measurement reversal and regulating measurement parameters. This work is organized as follows: in section 2, we will review the concept of the generalized amplitude damping channel and depolarizing channel respectively. In section 3, we show how to use weak measurement to control the EULB in the presence of environmental noise. We will examine a few examples in section 4. The manuscript closes with a conclusion and outlook in section 5.

2. Environmental noises

2.1. Generalized amplitude damping channel

When a system interacts with an environment at zero temperature, its evolution can be described by an amplitude damping (AD) channel as follows
\begin{equation}
\rho(t) = \sum_{i=0}^{\infty} E_i \rho(0) E_i^\dagger,
\end{equation}
where
\begin{equation*}
E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1 - p} \end{pmatrix}, \quad E_1 = \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix},
\end{equation*}
are Kraus operators and $p$ represents the probability of transition from excited $|1\rangle$ to ground state $|0\rangle$. At zero temperature,
the only transition is the transition from a high energy level to a low energy level. It should be noted that having an environment at zero temperature is not possible in practice. When the temperature of the environment is non-zero, the conditions are completely different. In this situation, in addition to losing excitation, the system can obtain excitation, as the result of interaction with the environment. Such an interference with the environment can be described by the generalised amplitude damping (GAD) channel. By considering \( r \) as a probability of losing excitation and \( 1 - r \) as the probability of obtaining the excitation, the Kraus operators of the GAD channel for two-dimensional quantum systems are given by

\[
E_0 = \sqrt{r} \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1 - p} \end{pmatrix}, \\
E_1 = \sqrt{r} \begin{pmatrix} 0 & \sqrt{p} \\ 0 & 0 \end{pmatrix}, \\
E_2 = \sqrt{1 - r} \begin{pmatrix} \sqrt{1 - p} & 0 \\ 0 & 1 \end{pmatrix}, \\
E_3 = \sqrt{1 - r} \begin{pmatrix} 0 & 0 \\ \sqrt{p} & 0 \end{pmatrix}.
\]

(7)

It is worth noting that the amount of the \( r = 1 \) GAD channel is the same as the AD channel. We consider a quantum bipartite system \( AB \), such that each separate part interacts with the environment at non-zero temperatures independently. The evolution of this quantum system in the Kraus representation is given by

\[
\rho_{AB}(t) = \sum_{i,j=0}^3 (F_i \otimes F_j)\rho_{AB}(0)(F_i \otimes F_j)^\dagger,
\]

(8)

where \( \rho_{AB}(0) \) is the initial state of the two-qubit system.

### 2.2. Depolarizing channel

The depolarizing channel is a channel that depolarizes the state with probability \( r \) and leaves the state of the system unchanged with probability \( 1 - r \). The depolarizing channel converts the state of the single-qubit systems to a completely mixed state. The Kraus operators of the depolarizing channel for two-dimensional quantum systems are given by

\[
F_0 = \sqrt{1 - r} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \\
F_1 = \frac{\sqrt{r}}{\sqrt{3}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \\
F_2 = \frac{r}{\sqrt{3}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \\
F_3 = \frac{r}{\sqrt{3}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.
\]

(9)

such that \( r \) is the parameter of the depolarizing channel. Let us consider a quantum bipartite system \( AB \), such that each separate part interacts with the depolarizing channel independently. The evolution of this quantum system in the Kraus representation is given by

\[
\rho_{AB}(t) = \sum_{i,j=0}^3 (F_i \otimes F_j)\rho_{AB}(0)(F_i \otimes F_j)^\dagger,
\]

(10)

where \( \rho_{AB}(0) \) is the initial state of the two-qubit system.

### 3. Weak measurement and measurement reversal

Weak measurements are obtained by generalizing Von Neumann measurements. They are related to the positive operator valued measure [46]. In general, a weak measurement operator for a single-qubit system is given by

\[
M = \begin{pmatrix} 1 & 0 \\ 0 & m \end{pmatrix},
\]

(11)

where \( m \in [0, \infty) \). When \( m = 0 \), the weak measurement is the same as the projective measurement. For \( 0 < m < 1 \), the weak measurement is a measurement which partially projects the state on the ground state and for the case of \( 1 < M < \infty \) the weak measurement represents a measurement which partially projects the state on the excited state. The measurement reversal operator for single-qubit systems is given by

\[
N = \begin{pmatrix} n & 0 \\ 0 & 1 \end{pmatrix},
\]

(12)

where \( n \in [0, \infty) \).

### 4. Model

The fundamental method to control the EULB in the presence of environmental noise is illustrated in figure 1. In figure 1, \( M_A^w(M_B^w) \) represents weak measurement on subsystem \( A(B) \) and \( N_A^R(N_B^R) \) represents measurement reversal on subsystem \( A(B) \). Bob prepares a correlated bipartite quantum state \( \rho_{AB} \) and sends part \( A \) to Alice and holds the second part \( B \) as a particle memory. Then Alice and Bob will reach an agreement for measuring the two observable \( Q \) and \( R \) by Alice on her particle. Before the effect of the decoherence, the weak measurement \( M_{AB} = M_A^w \otimes M_B^w \) is performed on the quantum system

\[
M_{AB} = \begin{pmatrix} 1 & 0 \\ 0 & m_1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & m_2 \end{pmatrix}.
\]

(13)

The post weak measurement state of the system is given by

\[
\rho'_{AB} = \frac{M_{AB}\rho_{AB}M_{AB}^\dagger}{tr(\rho_{AB}M_{AB}^\dagger M_{AB})}.
\]

(14)

In the second step, quantum system \( AB \) is affected by the environment. The dynamics of such a system can be described by
\[ \rho''_{AB} = \sum_{i,j=1}^{n} (K_i \otimes K_j)\rho'_{AB}(K_i \otimes K_j)^\dagger, \]  

where \(K_{ij}\)'s are Kraus operators. Next, in the third step, the reversal measurement \(N_{AB} = N_{k}^B \otimes N_{k}^B\) is performed on the quantum system

\[ N_{AB} = \begin{pmatrix} n_1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \begin{pmatrix} n_2 & 0 \\ 0 & 1 \end{pmatrix}, \]

then the state of the system becomes

\[ \rho''_{AB} = \frac{N_{AB}\rho''_{AB} N_{AB}^\dagger}{\text{tr}(\rho_{AB} N_{AB}^\dagger N_{AB})}. \]

As the last step, according to the preliminary agreement between Alice and Bob, Alice performs a measurement on her particle. The post measurement state is given by

\[ \rho''_B = \sum_i \langle \phi_i | (\rho_{AB}^\dagger N_{AB} N_{AB}^\dagger \rho_{AB}^\dagger N_{AB} N_{AB}^\dagger)^{-1} | \phi_i \rangle \otimes I, \]

where \(|\phi_i\rangle\)'s are the eigenstates of the observable \(\hat{O} \in \{\hat{Q}, \hat{R}\}\), and \(I\) is the identity operator. Thus the evaluated EULB \(U_{LB}\) is given by

\[ U_{LB} = \log_2 \frac{1}{c} + S(\rho''_{AB}). \]

In the following, we will show that the weak measurement and measurement reversal is an appropriate tool for protecting EULB \(U_{LB}\) from enhancing under the decoherence. But before that, we consider one of the most important applications of uncertainty to illustrate the importance of using the weak measurement and measurement reversal technique in the dynamics of the entropic uncertainty relation in equation (4). The entropic uncertainty relation can be used to confirm the security of quantum key distribution protocols [42, 43]. In [44], it is shown that the amount of the key that can be extracted per state by Alice and Bob \(K\) is lower bounded by \(S(\hat{R}|E) - S(\hat{R}|B)\), where the eavesdropper (Eve) prepared a quantum state, \(\rho_{AB}\), and distributes the \(A\) and \(B\) parts to Alice and Bob respectively. Coles \textit{et al} reformulated their result, in equation (4), as \(S(\hat{R}|E) + S(\hat{R}|B) \geq \log_2 \frac{1}{c} [45]\). Taking these together, they obtain a new lower bound on the quantum secret key rate as

\[ K \geq \log_2 \frac{1}{c} - S(\hat{R}|B) - S(\hat{Q}|B). \]

In general, environmental noise can decrease the lower bound of the quantum secret key rate. In [41], the authors have used the weak measurement and measurement reversal technique to preserve the lower bound of the quantum secret key rate in the presence of decoherence. Thus, using the weak measurement and measurement reversal technique, one can also preserve the lower bound in equation (20) from decoherence. The lower bound of the quantum secret key rate under decoherence with weak measurement and measurement reversal \(K_{LB}\) has the following form

\[ K_{LB} = \log_2 \frac{1}{c} - S(\rho''_{BR}|B) - S(\rho''_{QR}|Q). \]

By performing weak measurements and measurement reversal, the quantum correlation between Alice and Bob can be protected from environmental noise. By preserving the quantum correlation, the uncertainty of Bob regarding Alice’s measurement, i.e. the second and third terms in equation (21), does not increase. Thus the lower bound of the quantum secret key rate is preserved in the presence of decoherence.

5. Examples

5.1. Bell diagonal state

As an initial example, let us consider the two-qubit Bell diagonal state with the maximally mixed marginal states as an initial state which is shared between Alice and Bob. This state can be written as

\[ \rho_{AB} = \frac{1}{4}(I \otimes I + \sum_{i=1}^{3} w_i \sigma_i \otimes \sigma_j), \]

where \(\sigma_i (i = 1, 2, 3)\) are Pauli matrices. By utilizing the singular value decomposition theorem, the matrix \(W = \{w_i\}\) can be diagonalized by a local unitary transformation, thus the Bell-diagonal states can be written as

\[ \rho_{AB} = \frac{1}{4}(I \otimes I + \sum_{i=1}^{3} c_i \sigma_i \otimes \sigma_i), \]

where \(\sigma_i (i = 1, 2, 3)\) are Pauli matrices. This density matrix is positive if \(\tilde{c} = (c_1, c_2, c_3)\) belongs to a tetrahedron defined by the set of vertices \((-1, -1, -1), (-1, 1, 1), (1, -1, -1)\) and \((1, 1, 1)\). Here we consider the case where \(c_1 = 1 - 2p\) and \(c_2 = c_3 = -p\), with \(0 \geq p \geq 1\). Thus the state in equation (23) can be written as

\[ \rho_{AB} = |\Psi^\prime\rangle\langle \Psi^\prime| + \frac{1 - p}{2}(|\Psi^\prime\rangle\langle \Psi^\prime| + |\Phi^\prime\rangle\langle \Phi^\prime|). \]

By following the process outlined in section 4, we can find \(\rho''_{AB}\). Then, we check how the EULB \(U_{LB}\) behaves under the decoherence, weak measurement and measurement reversal. We obtained numerical results for the EULB \(U_{LB}\) with and without utilizing the weak measurement and measurement reversal protocol. We select the weak measurement parameters in such a way that the weak measurement on the second part \(B\) does not exist \(m_2 = 1\) and we let \(m_1 = m\). One can get the minimum value of the EULB \(U_{LB}\) for an optimal value of \(m\). In this work, we use the genetic algorithm to obtain the optimal value of EULB \(U_{LB}\).

In order to illustrate the effects of the weak measurement and measurement reversal process, in figures 2–5 we plot EULB \(U_{LB}\) in terms of weak measurement parameter \(m\) for various initial states under generalized amplitude damping and the depolarizing channel. In figures 2 and 3, we plot EULB \(U_{LB}\) for the Bell diagonal initial state with two various state parameters \(p\) under the generalized amplitude damping channel characterized by decoherence parameters \((p_1, r_1)\) and \((p_2, r_2)\) for the first and second part of the quantum state respectively. In figure 2, we use the decoherence...
In this section, we consider the special class of two-qubit states that are called X-states as an initial state
\[ \rho_{AB} = p |\psi^\perp\rangle \langle \psi^\perp | + (1 - p) |11\rangle \langle 11|, \]
where \( |\psi^\perp\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle) \) is a maximally entangled state and \( 0 \leq p \leq 1 \). In figure 6, the initial state parameter is \( p = 0.5 \) and we use the GAD channel parameters (\( p_1 = 0.1, r_1 = 0.1 \)), (\( p_2 = 0.4, r_2 = 0.4 \)). As can be seen from figure 6, \( \text{EULB}_{ULB} \) reaches its minimum value 0.08 for (\( m = 0.63, n_1 = 9619.9, n_2 = 15365.6 \)). In this situation, without any existing weak measurements and measurement reversal, the \( \text{EULB}_{ULB} \) is equal to 1.18. Figure 7 shows the \( \text{EULB}_{ULB} \) for the initial state parameter \( p = 0.2 \) and the GAD channel parameters (\( p_1 = 0.1, r_1 = 0.1 \)), (\( p_2 = 0.2, r_2 = 0.2 \)). As can be seen from figure 7, \( \text{EULB}_{ULB} \) reaches its minimum value 0.008 for (\( m = 0.33, n_1 = 658.7, n_2 = 2006.9 \)). In this situation, without any existing weak measurements and measurement reversal, the \( \text{EULB}_{ULB} \) is equal to 1.25.

We now focus on the evolution of the \( \text{EULB}_{ULB} \) in terms of the weak measurement parameter \( m \) for two various initial X-states under the depolarizing channel.
In figure 8, we consider the dynamics of the EULB in terms of the weak measurement parameter \(m\) for the initial two qubit X-state parameters \(p = 0.5\) under the depolarizing channel with parameters \((r_1 = 0.4, r_2 = 0.1)\). As can be seen from figure 8, \(\text{EULB}\) reaches its minimum value 1 for \((m = 1.5, n_1 = 5.1 \times 10^{-9}, n_2 = 0.54)\). In this situation, without any existing weak measurements and measurement reversal, \(\text{EULB}\) is equal to 1.88. In figure 9, we use the depolarizing channel parameters \((r_1 = 0.1, r_2 = 0.1)\) and initial two qubit X-state parameter \(p = 0.2\) (blue solid line), \(\text{EULB}\) without weak measurement and measurement reversal (red dashed line).

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we find that the weak measurement and measurement reversal can decrease the EULB $U_{LB}$ under the decoherence.

6. Conclusion and outlook

In this work, we studied the effects of the environment on the entropic uncertainty lower bound in the presence of weak measurement and measurement reversal. First, the weak measurement is performed on a bipartite quantum system $AB$, then the decoherence affects each part of the system independently, and at last the measurement reversal is performed on the decohered system. Here, we considered the generalized amplitude damping channel and depolarizing channel as the environmental noises. We consider two various initial states, the two damping channel and depolarizing channel as the environment. In this paper, we observed that by regulating the weak measurement and measurement reversal under decoherence, the entropic uncertainty lower bound decreases to an optimal minimum value.

Based on the results shown in the literature, weak measurement and measurement reversal enhance the quantum correlation in bipartite open quantum systems [34, 46–49]. Thus, the result we have taken here is quite reasonable, because by enhancing the correlation between the memory particle (which is in Bob’s possession) with the measured particle (which is also in Bob’s possession) the entropic uncertainty lower bound decreases to an optimal value.

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