Strength evaluation of the relocatable building frame made of the polymer composite material

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Abstract. The article reviews the application of composite materials in the construction of primary bearing rods of relocatable buildings. After loss of stability a rod made of composite materials continues to function with significant deflections, so it is possible to use such a property to increase the calculation value of buckling critical load. The obtained analytical expression allows us to calculate the collapsing force depending on the length of rods, form of section, stiffness characteristics of the material as exemplified by calculation of a multilayer rod made of composite materials.

1. Introduction
The issues of reducing the weight of load-bearing elements are particularly relevant to temporary buildings used in remote and seismic areas, permafrost regions, as well as developed areas, where the low mass of structures allows to ease transportation and erection. The base frame of relocatable buildings is prefabricated light-weight construction that allows its dismounting, relocation and reerection [1]. It also allows to fast-track building projects, simplifies and lowers labour input that goes into construction of foundations, transportation and erection of building and related structures making them more feasible. Strengths and rigidity of such constructions are often achieved by using framework made of wood and metal channels. Up until now there are no specific scientifically backed recommendations for the use of new materials in such structures.

In this regard, composite materials (CM), widely used in the aviation and space industry for primary bearing members, may find good prospects in construction [2, 3]. Currently CMs find wide application in civil construction for reinforcement of load-bearing elements, rebars, in bridging and erection of thin-walled enclosing structures. There are known works focused on production of beam elements and floors which are stable to corrosion [4, 5]. However, the use of composite materials in relocatable buildings is limited despite its potential. Some authors [6] indicated the prospects of using composite materials, such as bearing rib, for tent relocatable buildings, allowing lowering the mass of the overall construction without compromising rigidity. Following researches [7, 8] developed a project for tent relocatable buildings, where the framework was created using circular cross section pipe made of composite materials. However, CMs are rarely used as material for construction of load-bearing elements of relocatable buildings. This is hindered by the difficulties of calculation and design with account of CMs’ specifics, and high cost, which determines the rational use of the material.

The ability to function after the loss of stability and unwind after stress removal is one of the particular properties of thin-walled rods made of CM [9-11]. It allows increasing the survivability of relocatable buildings and other structures. Critical deformation was previously addressed in the
following works [12-14]. However there’s no simplified formula for determining the value of stability loss and its further collapsing force.

Figure 1. Relocatable buildings from complanar system of structural members.

2. Research methodology

As a calculation model we have considered the frame fragment – a rod made of layered CM, which is the main bearing element of a relocatable building (figure 2a). An axial compression load $P$ and a bond that prevents vertical movements of the supporting rod is applied at the point $B$ (figure 2a). Point $A$ is a pin support.

The elastic module of a multilayer package depends on the stiffness characteristics of the layers having a definite orientation and on the ratio of these layers thickness in the total thickness of the package.

The stiffness characteristics of $k$ (CM layer) are determined by the formulas [6, 9]:

$$E_{11}^k = E_1 c_1^4 + E_2 c_2^4 + G_{12} s_1^2 + \mu_{21} E_1 2s_1^2 c_1^2;$$

$$E_{22}^k = E_1 s_1^4 + E_2 c_2^4 + G_{12} s_1^2 + \mu_{21} E_1 2s_1^2 c_2^2;$$

$$E_{12}^k = (E_1 + E_2)s_1^2 c_1^2 - G_{12} s_1^2 + \mu_{21} E_1(c_1^2 + s_1^2);$$

$$E_{33}^k = (E_1 + E_2)s_2^2 c_2^2 + G_{12} s_2^2 + \mu_{21} E_1 2s_2^2 c_2^2;$$
\[ E^{k}_{13} = \tilde{E}_{1} s c^{3} \tilde{E}_{2} s c^{3} - G_{12} s c_{2} + \mu_{21} \tilde{E}_{1} (s c^{3} - sc^{3}); \]
\[ E^{k}_{23} = \tilde{E}_{1} s c^{3} - \tilde{E}_{2} s c^{3} - G_{12} s c_{2} + \mu_{21} \tilde{E}_{1} (s c^{3} - sc^{3}), \]
\[ c = \cos \phi; \quad c_{2} = \cos 2 \phi; \quad E_{1} = E_{1} (1 - \mu_{12} \mu_{21}); \quad E_{1} \mu_{21} = E_{2} \mu_{12}; \]
\[ s = \sin \phi; \quad s_{2} = \sin 2 \phi; \quad E_{2} = E_{2} (1 - \mu_{12} \mu_{21}); \]

where \( E_{1}, E_{2}, G_{12}, \mu_{12} \) – the stiffness characteristics of a monolayer (figure 2c).

With sufficient accuracy for engineering calculations the stiffness characteristics of the multilayer CM are found by the formulas [13]:

\[ E_{x} = A_{11} - \frac{A_{12}}{A_{22}}; \quad E_{y} = A_{22} - \frac{A_{12}}{A_{11}}; \quad G_{xy} = A_{33}; \quad \mu_{xy} = \frac{A_{11}}{A_{22}}; \quad \mu_{x} = \frac{E_{x}}{E_{x}}; \quad \mu_{i} = \frac{A_{12}}{A_{11}}, \quad (1) \]

where \( A_{ij} = \frac{1}{h} \sum_{k=1}^{n} h_{k} E_{ik}; i,j = 1,2,3. \) For example, for a rod made of CM with layers [0,1,90,45] \( \text{°} \), a thickness of a monolayer 0.5 mm, and stiffness characteristics \( E_{1} = 11600 \) MPa, \( E_{2} = 10000 \) MPa, \( G_{12} = 3800 \) MPa, \( \mu_{12} = 0.28 \) MPa we have obtained the stiffness values of the entire package by formulas (1) equal to \( \mu_{xy} = 0.245, \mu_{x} = 0.189, \mu_{y} = 0.245, G_{xy} = 1.818 \times 10^{4} \) MPa.

Using the energy method [15], we have obtained the critical load value, and studied the nonlinear behavior of the bearing rod made of layered CMs, for which the solution of the linear problem is known. For small but finite lateral deflections of the rod we can approximate the angle of rotation \( \theta_{1}(x) = \theta(x) \) by its own function \( \theta(x) = \cos \left( \frac{x}{l} \right) \). Then the solution of the nonlinear problem can be represented as: \( \theta(x) = \alpha \theta_{1}(x) \), where \( \alpha \) is a load-dependent coefficient [16].

The change in the total potential energy of the rod is determined by the equation:

\[ \Delta \mathcal{E} = \frac{\alpha}{2} \int_{0}^{l} E_{x} I \left[ \frac{d^{2} \theta(x)}{dx^{2}} \right]^{2} \left[ \frac{1}{2} \int_{0}^{l} \frac{\theta(x) dx \times \frac{a^{4}}{4!} \int_{0}^{l} \theta(x) dx + \frac{a^{6}}{6!} \int_{0}^{l} \theta(x) dx + \ldots} {min} \right] \]

\[ I_{min} \text{ - the axial moment of inertia; } E_{x} \text{ - the elastic modulus of the package across the fibers; } E_{y} \text{ - the elastic modulus of the package along the fibers; } E_{y} < < E_{x}. \text{ From the stationary condition } \frac{\partial \Delta \mathcal{E}}{\partial \alpha} \text{ we obtain} \]

\[ \alpha[P_{kp} - P(1 - \alpha^{2} B_{2} + \alpha^{4} B_{4} - \ldots)] = 0, \quad \text{where } B_{2} = \frac{1}{E_{0}} \int_{0}^{l} \theta(x) dx; \quad B_{4} = \frac{1}{120} \int_{0}^{l} \theta(x) dx; \quad \text{then} \]
\[ P_{kp} = \frac{E_{0} \alpha}{B_{2}} \frac{L_{min}}{r_{min}} \quad \text{or} \]

\[ P_{kp} = \frac{E_{0} \alpha}{B_{2}} \frac{L_{min}}{r_{min}} \quad \text{when } P < P_{kp} \text{ only one rectilinear stability mode of equilibrium is possible, which correspond to } \alpha = 0. \text{ And when } P > P_{kp}, \text{ a curvilinear mode of equilibrium becomes possible: } P = P_{kp} + \Delta P. \]

Confined by the fourth degrees of \( \theta \), we obtain the correlation between \( \alpha \) and the load \( P \) in the deformation zone beyond the range of stability. We find the dependence “a load – a convergence of the ends of the rod \( \lambda^{*} \), which is equal to the sum \( \lambda = \lambda_{0} + \lambda_{1} \), where \( \lambda_{0} \) is the shortening of the rod due to compression, \( \lambda_{1} \) is the shortening of the rod due to bending, \( F \) is the area of cross section.

\[ \lambda_{0} = \frac{P_{kp}}{E_{0} F} \lambda = \frac{P_{k}}{E_{0} F} \]
\[ \lambda_{1} = \alpha^{2} \int_{0}^{l} \left[ \frac{1}{2} \theta(x) \frac{a^{2}}{4!} \theta(x) + \ldots \right] dx \approx \alpha^{2} \int_{0}^{l} \theta(x) dx \text{ or } \lambda_{1} = \frac{a^{2}}{4} \]
The condition for convergence of the ends of the rod has the following form:

$$u^2 = \frac{4}{E_f} (P-P_{kr})$$

To assess the possible fracture propagation of the rod we have used the Griffith’s criterion $G=\Gamma$, where $G=\frac{\partial}{\partial l} (A-U)$ is the energy release rate, $\Gamma=4l\gamma$ — the resisting force, or the cracking resistance, and $\gamma$ — the fracture surface tension of the material [17]. The work of external forces is $A=\frac{P^2}{2E_f F^2}$.

The potential energy of the rod deformation, which consists of the potential energy of compression and bending, can be represented as

$$U=U_c+V,$$

where $U_c = \frac{P_{kr}^2}{2E_f F}$ is the potential energy of compression, and

$$V = \frac{1}{2} \int_0^l E_s I_{mn} (\theta')^2 \, dx = -\frac{\pi^2 E_s I_{mn}}{4l} \alpha^2$$

is the potential energy of bending. In view of the dependence $\alpha^2$ the potential energy of the rod deformation is

$$U = \frac{1}{2E_f F} (3P_{kr}^2 - 2PP_{kr})$$

Then the analytical expression for the generalized force of the rod fracture propagation has the form:

$$G = \frac{\partial (A-U)}{\partial l} = \frac{1}{2E_f F} (P^2 + 2PP_{kr} - 3P_{kr}^2)$$

From the equilibrium condition of the crack system, according to Griffith, we can obtain the dependence in the following form:

$$\frac{1}{2E_f F} (P_{raz}^2 + 2P_{raz} P_{kr} - 3P_{kr}^2) - \Gamma = 0$$

where $P_{raz}$ is the value of the collapsing force (N).

3. Conclusion
The calculated values of the critical load for rods and the collapsing force are presented in Figure 3. According to the presented diagram (figure 3), the rod under the load $P < P_{kr}$ is in a rectilinear stable position, when $P > P_{kr}$ a nonlinear form of stability is possible. The rod does not collapse (yellow part) until the load reaches the fracture curve $P_{raz}$, and after overloading the destruction occurs. The task of increasing the survivability of relocatable buildings of collapsible type can be solved by using building construction elements made of CM, since such bearing rod elements continue to function even with large non-linear deformations $P > P_{kr}$.
Figure 3. Diagram of the Euler’s critical load (N) (2) and the collapsing force (N) (3).

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