Finite Element Modelling for Bending and Vibration Analysis of Composite and Sandwich Spherical Shells

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Abstract. Finite element modelling is used to illustrate the bending and vibration response of composite and sandwich spherical shells. ABAQUS is used to model the shell as a three-dimensional deformable solid part. Composites are modelled using mixed modelling technique. The quadrilateral continuum shell element, which has eight nodes, is used for finite element meshing. For a three-layered composite spherical shell with simply supported boundary conditions, the accuracy of the employed element is first evaluated. After that, three potential end conditions are evaluated for further analysis: cantilever (CFFF), two opposing sides clamped (CFCF), and all sides clamped (CCCC). Bending response is produced by applying two types of pressure loads to the exterior surfaces of composite and sandwich spherical shells: uniformly distributed load (UDL) and sinusoidal load (SINO). For various span to thickness ratios, free vibration response and forced vibration response (Time History) are provided under step pressure (UDL and SINO) loading. The element type being used is found to be quite precise and sturdy.

Keywords: FEM, ABAQUS, composite, sandwich, spherical shell, vibration, sinusoidal.

1. Introduction

Many mechanical and structural systems comprise of shell type geometrical features. There are different ways of manufacturing these shells. It depends upon their use and practical application. The Piezo laminated plates and shells are very common in smart structural applications. Composites and sandwich, as laminated structures, have received enormous research attention in present days. Such structures can be exposed to a variety of static and dynamics loads under different boundary and environmental conditions. A large variety of structural systems like aircraft wings, fuselage and empennage structures are generally modeled as shells, beams and panels. The composite and sandwich shell and beams also realize utilization as micro positioners, micro actuators and micro resonators that are used in, measuring apparatus, medical devices, robotics etc. These structures are generally made up of sandwich and composite laminates due to their light weight, high stiffness and good strength. They are often subjected to dynamic loading. Composite laminates are gradually emerging as a substitute to conventional materials and are nowadays frequently used in various applications subjected to dynamic loading such as heavy machines and vehicles, defense equipment, pressure vessels, satellites and aerospace structures. Vibration phenomenon may lead to resonance condition in mechanical structures. In addition to this it may also result in fatigue and fracture failures. Therefore, it is quite important to carry out vibration analysis of these structures in order to make them safe while in use. A number of FEM software packages are commercially available for modelling and analysis of composite and sandwich structures, such as...
ABAQUS, NASTRAN, ANSYS etc. ABAQUS is well recognized among academic, research and development organizations due to ease in use, more user-friendly menu driven interface, wide material modeling capability and variety of available modules. Kumar et al. [1] investigated the response of laminated composite shells, combined with piezoelectric devices and actuators, which are exposed to mechanical, electrical and thermal loads using FE formulation based on FSDT. They used spherical and cylindrical shell geometries and carried out both static and dynamic analysis in their study. Yasin and Kapuria [2] used a new efficient four node finite element based on ZIGT for shallow shells with laminated composite and piezoelectric layers. They take into account layer wise mechanics and electromechanical coupling in their formulation. At each node of the new element, there are seven degrees of freedom. Hossain et al [3] used an improved FEM using FSDT for the linear investigation of spherical and cylindrical shells/shell-panels and uses the approach of Mixed Interpolation of Tensorial Components (MITC). Vel et al. [4] found a precise solution for the three-dimensional deformation of thick plates subjected to surface mechanical and electrical loads. They considered laminated plates of rectangular planform that consists of embedded actuators. The plates are assumed in simply supported boundary condition at the edges. Yasin and Kapuria [5] extended, an earlier developed quadrilateral element for laminated composite plates, to laminated shells for the static and free vibration response. The results obtained using the new element for shallow spherical and cylindrical shells are well compared with the available elasticity solutions and finite element solutions. The element's performance is demonstrated to be straightforward, precise, and productive. Qatu et al. [6-7] reviewed the recent literature on static as well as dynamic behavior of composite shells and the theory applied for the analysis. Biswal, D. K., & Mohanty, S. C. [8] used first order shear deformation theory for vibration analysis of sandwich panels with spherical geometry. The configuration of sandwich panel has viscoelastic core and elastic face sheets. To arrive at the equation of motion, Hamilton's principle is used. Pang et al. [9] presented an analytical model based on multi-segment partitioning strategy and FSDT. Their results show that the methodology used yield a stable and rapid convergence. Tang et al [10] developed a new procedure for getting vibration response of stepped cylindrical shell with circular cross section under various boundary conditions. They utilized reverberation-ray matrix method in their formulation to determine frequencies of vibration and steady state responses. The method is used is proved to be precise and accurate for finding vibration response of stepped right circular cylindrical shells. Panda and Singh [11] presented a nonlinear finite element solution for vibration response of composite curved shells with large amplitudes of vibration. The higher order theory is used for laminate mechanics. To obtain the concerned governing equations, the variational principle is used. Shaukat Rafi S.M et al [12] performed FEM for vibration analysis of skew sandwich hybrid plates with piezoelectric top and bottom layers. Rafi et al [13] presented a detailed study on hybrid sandwich plate using commercial FE package ANSYS.

Civalek [14] performed static and dynamic analysis of shallow shells sitting on an elastic foundation. Donnell theory is used for formulation. Nonlinearity is considered while deriving the governing equations. Discrete convolution method and differential quadrature method are utilized to discretize the equations. Behera et al [15] examined the free vibration of different geometric aspects and cut-out or without central on spherical shell laminated composite panels using ABAQUS commercial finite element tool. Ma et al [16] presented two reinforcement methods to be applied at openings in spherical shells, to minimize effects of buckling and local weakening, when subjected to an external pressure. Their study includes both numerical analysis and experimentation. A good comparison of theoretical results with experimental data has been shown. Zhu et al [17] optimized the parameters of opening reinforcements used in spherical shells to have minimum instability in buckling. The optimization results were then used to fabricate experimental test models. Simulated buckling behaviour of shells was also obtained based on experimental data. Malavika et al [18] analyzed cracks in plates and evaluated the stress intensity factor using ABAQUS. They considered surface cracks as well as embedded cracks in their study. Under simply supported boundary conditions, Fantuzzi et al [19] investigated free vibration characteristics of spherical and cylindrical shells made-up of functionally graded material. Two types of FGM material configurations are considered for both shell geometries. A comparison of free vibration results is shown for exact and FE models.
Using finite element modelling, the bending and vibration response of composite and sandwich spherical shells is discussed in this paper. FE package ABAQUS is used to model the shell as a three-dimensional deformable solid part. A quadrilateral continuum shell element with 8 nodes is used for finite element meshing. First, the accuracy of the used element is evaluated for a three-layered composite spherical shell with clamped boundary conditions, and the findings are compared to those found in the literature. The results of free vibration, forced vibration and bending response are then presented for clamped-clamped (CFCF), cantilever (CFFF) and all side clamped (CCCC) boundary conditions for two different types of loads (UDL and sinusoidal load).

2. Finite Element Modeling using ABAQUS

2.1 Specifications of composite and sandwich spherical shells

Span to thickness ratio, \( S = a/b = (10, 20, 100) \).

Aspect ratio = \( a/b = 1, \frac{Rx}{a} = 5, \frac{Ry}{b} = 5 \),

where the dimensions in the \( x \)- and \( y \)-axes are ‘\( a \)’ and ‘\( b \)’, respectively.

\( h = (1) \) is the dimension in \( z \)-direction.

The radii of curvatures in the \( y \) and \( x \) directions, respectively, are \( R_y \) and \( R_x \).

Figure 1 shows configuration of shallow composite and sandwich spherical shells. A lay-up of \((0^\circ/90^\circ/0^\circ)\) is considered for the composite laminae. The layup order and thicknesses of the layers are mentioned in the figure. Table 1 shows the properties of the materials used in the configuration of shells. The finite element mesh is created in ABAQUS using a quad in-plane general-purpose continuum shell element (SC8R) with eight nodes. The geometric order is considered to be linear. The used elements have only displacement degrees of freedom and no rotational degrees of freedom. Upon modelling these elements look like 3-D continuum solids however their response, both kinematic and constitutive, is similar to conventional shell elements. The shallow composite spherical shell finite element model created in ABAQUS is shown in Figure 2.

![Composite lamina configuration for shallow composite (0/90/0) and sandwich spherical shells.](image)

**Table 1.** The properties of materials used [5]

| Properties of materials | \( Y_1 \) (GPa) | \( Y_2 \) (GPa) | \( Y_3 \) (GPa) | \( G_{12} \) (GPa) | \( G_{23} \) (GPa) | \( G_{31} \) (GPa) | \( \nu_{12} \) | \( \nu_{23} \) | \( \nu_{31} \) | \( \rho \) (kg/m\(^3\)) |
|-------------------------|----------------|----------------|----------------|------------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| Composite               | 172.5          | 6.9            | 6.9            | 3.45             | 1.38            | 3.45           | 0.25           | 0.25           | 0.25           | 1600           |
| Core                    | 0.276          | 0.276          | 3.45           | 0.1104           | 0.414           | 0.414          | 0.02           | 0.02           | 0.25           | 1600           |
Each elastic substrate mechanical node has three translational degrees of freedom: x, y, and z translations. The results are presented with a converged 12 x 12 mesh size. The boundary conditions used are cantilever at one edge (CFFF), clamped at opposite edges (CFCF) and clamped at all four edges (CCCC).

3. Results and discussion

3.1 Validation

For a three-layer composite spherical shell of square planform with layup (0°/90°/0°), the correctness of the employed element for static and dynamic response is first tested. [5]. Linear perturbation frequency procedure is used for obtaining dynamic free vibration response. Table 2 shows the comparison of non-dimensional natural frequency associated with bi-symmetric modes of composite spherical panel with all sides clamped (CCCC) and Table 3 shows the comparison of non-dimensional central deflection for simply supported (SSSS) boundary condition under uniformly distributed load. The non-dimensionalization is done using Eqs (1) and (2) respectively.

\[
\tilde{\omega}_n = \omega_n * a * S * \sqrt{E_2 / \rho_2} \quad (1)
\]
\[
\tilde{w} = 1000\omega Y_0 / h S^4 P_0 \quad (2)
\]

where, \( S = a/h, P_0 = 1 \) for UDL and \( Y_2 \) is equal to \( E_2 \) (MPa).
Both the results are found in accordance with the published reference which shows the correctness of the used modelling technique and robustness of the used element.

| Table 2. Non-dimensional natural frequencies for all sides clamped boundary Condition. |
|----------------------------------|----------------|----------------|
| Natural frequency (\( \tilde{\omega}_n \)) [\( S = 10, a/b = 1, R/a = 5 \)] | Three-layer composite spherical shell (0°/90°/0°) | |
| \( S \) | Mode | Ref [5] | 3D FE (Abaqus) |
|---|---|---|---|
| 10 | 1 | 17.382 | 18.232 |
| 2 | - | - | 25.274 |
| 3 | - | - | 28.537 |
| 4 | - | - | 33.361 |

| Table 3. Non-dimensional central deflection for simply supported boundary Condition. |
|----------------------------------|----------------|----------------|
| Non dimensional Central deflection (\( \tilde{w} \)) | Three-layer composite spherical shell (0°/90°/0°) | |
| \( S = 10, a/b = 1, R/a = 5 \) | Ref [5] | 3D FE (Abaqus) |
|---|---|---|
| 10 | 11.145 | 11.157 |
3.2 Free vibration response

3D FE model for dynamic response is developed using ABAQUS for composite spherical shell. The natural frequencies have been obtained for span to thickness ratio, $S = 10$, 20 and 100 for cantilever and clamped boundary conditions. These results have been tabulated in Table 4 for first five modes. As observed from the table, the non-dimensional natural frequencies for all sides clamped (CCCC) boundary condition are higher than corresponding frequencies for cantilever (CFFF and CFCF) boundary condition. Further the Non-dimensional natural frequencies increase as the shell becomes thinner. The frequencies of vibration of thick composite spherical shells are smaller in comparison to their corresponding values in thick sandwich spherical shells for all the considered boundary conditions. This is due to higher stiffness to mass ratio in sandwich structures. However reverse is true in thin shells where the effect of span to thickness ratio becomes more dominant.

| $S$ | Mode | Composite | Sandwich | Composite | Sandwich | Composite | Sandwich |
|-----|------|-----------|----------|-----------|----------|-----------|----------|
| 10  | 1    | 4.1472    | 13.2227  | 15.0845   | 41.6402  | 18.2392   | 46.4546  |
| 2   | 4.6105| 14.5456   | 15.1601  | 42.0954   | 25.2748  | 63.4697   |
| 3   | 10.2822| 24.8051  | 18.0076  | 48.0286   | 28.5378  | 75.3438   |
| 4   | 11.2318| 27.7264   | 21.7825  | 51.9055   | 33.3618  | 86.9291   |
| 5   | 15.7915| 45.2056   | 26.7275  | 68.3596   | 37.4141  | 91.4909   |
| 20  | 1    | 4.7787    | 8.1756   | 25.6014   | 37.6061  | 29.5139   | 39.4342  |
| 2   | 5.3546| 9.0783    | 25.6213  | 37.6439   | 36.4591  | 47.8699   |
| 3   | 12.6256| 16.2424  | 27.4926  | 39.3102   | 48.8149  | 66.9353   |
| 4   | 20.2699| 24.6766  | 36.5619  | 47.7807   | 53.2585  | 67.0411   |
| 5   | 23.4403| 33.1214  | 43.5814  | 51.9388   | 54.6447  | 72.4134   |
| 100 | 1    | 5.8909    | 2.0147   | 70.7124   | 21.1278  | 77.4289   | 21.9927  |
| 2   | 6.2695| 2.0679    | 70.7245  | 21.1429   | 84.6020  | 22.2724   |
| 3   | 17.5155| 4.4112   | 73.3191  | 21.6827   | 94.6783  | 24.6101   |
| 4   | 30.6673| 8.4067   | 74.5832  | 21.6843   | 98.6973  | 29.7223   |
| 5   | 35.5603| 10.9812  | 85.3429  | 23.3263   | 98.7669  | 30.0247   |

Figure 3. The first five non dimensional frequencies for cantilever composite and sandwich spherical ($S = 10$, $a/b = 1$, $a/h = 10$) shells.
3.3 Bending analysis

The bending study considered composite and sandwich spherical shells with cantilever (CFFF) and clamped boundary conditions (CFCF and CCCC) exposed to two load variations (UDL and SINO). Shells are modelled with span to thickness ratio, \( S = 10, 20 \) and 100. The loads are applied on the outer surfaces of the shells. Central deflection is recorded for the clamped shells and the tip deflection is recorded for the cantilever shells under both type of loads. The non-dimensional deflections have been tabulated in Table 5. A general decreasing trend of deflection is observed as the span to thickness ratio increases and the deflection due to UDL is found to be greater than deflection due to sinusoidal load.

### Table 5. Non-dimensional deflection for different boundary Conditions.

| LOAD | S   | composite | sandwich | Composite | Sandwich | Composite | Sandwich |
|------|-----|-----------|----------|-----------|----------|-----------|----------|
| UDL  | 10  | 55.6347   | 11.9646  | 1.21371   | 0.31271  | 0.32340   | 0.07030  |
|      | 20  | 20.8251   | 5.3734   | 0.29472   | 0.11204  | 0.29334   | 0.01173  |
|      | 100 | 0.8294    | 0.3453   | 0.00451   | 0.00526  | 0.00151   | 0.00027  |
| SINO | 10  | 20.9001   | 4.4878   | 0.69897   | 0.17079  | 0.32340   | 0.05112  |
|      | 20  | 7.9307    | 2.0338   | 0.17263   | 0.06039  | 0.06447   | 0.00876  |
|      | 100 | 0.3252    | 0.1326   | 0.00324   | 0.00262  | 0.00147   | 0.00023  |

3.4 Forced vibration Response

Forced vibration analysis is performed on composite spherical shells of span to thickness ratio, \( S = 10 \). The following load cases are considered.

1. Uniform Pressure \( P_z = -P_0 F(t) \) on outer surface.
2. Sinusoidal pressure \( P_z = -P_0 F(t) \) on outer surface.

For unit step load \( F(t) \) is taken as 1. The deflection results for the load cases are non dimensionalized as:

\[
\hat{w} = 100wY_0 / hS^4P_0
\]

where, \( S = a/h \) and \( Y_0 \) is taken as the value of \( Y_2 \).

![Figure 4](image-url)  
**Figure 4.** Time history of central deflection of all sides clamped (CCCC) composite spherical shell for (a) UDL and (b) SINO loads.
Forced vibration response is captured for 0.12 seconds for clamped (CCCC and CFCF) spherical shells under both uniform and sinusoidal loads (Figures 4, 5, 7 and 8) whereas for cantilever (CFFF) shells it is captured for 0.4 sec (figure 6 and figure 9). These figures show undamped time history response of the tip deflection, $\hat{w}(a/2, b, h/2, t)$, for cantilever and central deflection, $\hat{w}(a/2, b/2, h/2, t)$, for clamped boundary conditions. Almost 7 cycles are completed in 0.12 seconds under CCCC boundary conditions and 4 cycles are completed under CFCF boundary conditions for composite spherical shells. On the other hand in sandwich spherical shells, the number of cycles completed in the said time duration is almost 3.5 and 1.5 respectively. The time period for CFFF boundary condition is maximum for both composite as well as sandwich spherical shells. The amplitude of vibration is larger in case of UDL for all the boundary conditions.
Figure 7. Time history of the central deflection of all sides clamped (CCCC) sandwich spherical shell for (a) UDL and (b) SINO loads.

Figure 8. Time history of the central deflection of opposite two sides clamped (CFCF) sandwich spherical shell for (a) UDL and (b) SINO loads.

Figure 9. Time history of the tip deflection of only side is clamped (CFFF) Sandwich spherical shell for UDL and SINO loads from left to right.
Figure 10. Fifth mode of vibration for cantilever (CFFF) (a) composite and (b) sandwich spherical shells.

Figure 10 shows the fifth mode shape for cantilever composite and sandwich spherical shells.

4. Conclusion
Using finite modelling in the commercial FE tool ABAQUS, the static and dynamic behaviour of composite and sandwich spherical shells is examined. The developed 3D FE model yield fairly accurate results for bending, free and forced vibration response. The used element is found to be quite accurate and robust. The natural frequencies under various boundary conditions increase as the shell becomes thinner. A general decreasing trend of deflection is observed as the span to thickness ratio increase and the deflection due to UDL is found to be greater than deflection due to sinusoidal load. Different layup configuration provide different stiffness and resistance to deflection under a given load type and boundary condition. Thus a suitable configuration can be optimized depending upon the application of shell.

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