Study on Passenger Flow Assignment Method of Urban Rail Transit Based on Improved Logit Model

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Abstract. With the advantages of small pollution, punctuality, safety and large capacity, urban rail transportation provides good ideas for solving China’s urban transportation problems and gradually becomes an indispensable part of China’s urban public transportation system. According to the current development of urban rail transportation, the era of networked operation has arrived, and the connectivity between different stations in the metro network and the mobility of operation and management have greatly increased. And for passengers, considering the convenience of seamless transfers, the complexity of the line network structure, and the rapid feedback of real-time passenger flow information on the line, it will cause the passengers’ travel behavior to be highly random. Different passengers are choosing the route selection scheme to reach the target station will also make the spatial and temporal distribution within the rail transit network more diverse and complicated. This paper adopts the method of analyzing the behavior of passenger travel route selection under the established urban rail transit network to establish a generalized cost function and Logit model, and contributes to the planning, design and operation management of urban rail transit.

1. Introduction

In order to effectively alleviate the problem of urban road traffic congestion, many large and medium-sized cities have adopted scientific measures to vigorously build public transportation. Among them, rail transit has gradually become an important aspect of prioritizing the development of public transportation by virtue of its own high degree of comfort, less environmental pollution, safety performance, and large traffic volume, which is conducive to sharing the pressure of ground transportation. After the rail transit network is perfected, the original passenger flow distribution will acquire new characteristics according to the increase in the route. For example, for the two very important factors of transfer and waiting time, some passengers try to minimize transfers when considering routes. The number of times, and some passengers prefer to reduce travel time. Passengers’ different travel route choices will lead to changes in the distribution of passenger flow within the network. Transfer and waiting time are important factors in the distribution of public transportation. Some passengers may choose routes to minimize the number of transfers, while others may minimize travel time (or the time between the two).

Considering that the research on passenger flow distribution at home and abroad is a process of constant adjustment and change. At the beginning, scholars conducted research on the basis of urban road traffic network flow distribution theory and conventional bus passenger flow distribution theory. With further development, many researchers found that traditional road distribution models could not effectively solve the problems in the field of urban rail transit. For some problems, the basic theories
and distribution methods that are more suitable for urban rail transit have been constructed, and the traditional model has been optimized and improved. This paper analyses the factors influencing the travel behaviour of urban rail transit passengers, establishes the generalized cost function of rail transit travelers, and constructs a more reasonable path generalized cost function and Logit model. Taking the rail transit of a certain city as an example, the Logit model is combined with the OD data of a certain evening rush hour. While studying the factors affecting passenger flow distribution, the model passenger flow distribution calculation ratio is compared with the actual ratio.

2. Generalized cost function of rail transit
In order to better quantify the passenger's route choice behavior, it is necessary to define a generalized cost for each route to reflect the comprehensive cost of passengers' choice of route. [1] It can be seen from the above that, in addition to the passenger's normal rail transit travel expenses, the passenger's travel expenses also include congestion fees, transfer fees, walking time, and congestion caused by too many people on the train. Time cost. This paper constructs the path generalized cost function from two aspects: basic cost and penalty cost, and fully considers the cost of train transit time, train stop time, transfer walking time, transfer waiting time and so on.

2.1. Basic cost
Travel time is the basic part of the generalized cost function. It generally consists of the travel time of the train on the line and the stay time of each station in the train's online network. Therefore, the passenger travel time can be quantified by the total length of the OD route selected by the passengers and the train speed.

\[ T_{ij}^l = \begin{cases} t_{ij}^l & \text{Station} \, i \, \text{is the starting station}; \\ s_j^l + t_{ij}^l & \text{otherwise} \end{cases} \]

- \( T_{ij}^l \) — Passenger travel time on line \( l \) in section \( (i, j) \);
- \( t_{ij}^l \) — The travel time of the train on line \( l \) in the interval \( (i, j) \);
- \( s_j^l \) — Average stop time of trains on line \( l \) at station \( i \).

2.2. Penalty fee
Transfer time refers to the total time that passengers spend on cross-line trains without purchasing additional tickets without passing through the exit gates. Even if other factors are involved, such as transfer walking, waiting, and transfer penalties, transfer time can be used to successfully reflect the passenger transfer experience.

This paper uses the following model to represent the transfer time \( E_{ilm}^{im} \) for passengers to transfer from line \( l \) to line \( m \) at transfer station \( i \):

\[ E_{ilm}^{im} = \alpha \cdot \left( n_{ik}^{rs} \right) \beta (w_{i}^{lm} + 0.5 \cdot f_m) \quad \forall l, m, i \]

- \( n_{ik}^{rs} \) — The cumulative number of transfers occurring at transfer station \( i \) on the \( k \)-th path between OD and r-s;
- \( w_{i}^{lm} \) — Passenger's transfer walking time from line \( l \) to line \( m \) at transfer station \( i \);
- \( f_m \) — Train departure time interval of line \( m \);
- \( \alpha, \beta \) — Parameters to be calibrated.

2.3. Construction of generalized cost function
This paper adopts the method of using time value to measure the comprehensive travel impedance of passengers' choice of route, and helps to construct the generalized cost function of urban rail transit. Considering that there are many factors influencing the choice of passenger routes, including not only quantitative factors such as travel time, stay time, and transfer time, but also qualitative factors such as congestion and transfer convenience. Add these costs to get the total cost consumed by passengers when they choose a valid path \( k \), namely:
In the formula, $\delta_{ij,k}^rs$, $\emptyset_{lk}^rs$, $\eta_{lk}^rs$, $\eta_{m,k}^rs$, respectively represent the stations and lines in the subway line network. The correlation between the three and the interval. If the corresponding station, line, section meets the condition of the effective path $k$ between path $r$-$s$, then the value is 1, otherwise it is 0.

3. Path selection model based on improved logit

3.1. Analysis of traditional logit model

In the urban rail transit network, there are multiple lines between some OD pairs. In order to sort the tickets, the operating department needs to use some models to estimate the passenger's choice of the route, but to reduce the error between the estimated result and the actual selection result. Among them, the routes within the scope of passenger travel are called effective routes, and their generalized costs are usually accepted by passengers, so they can be expressed by the following formula:

$$C_{rs} \leq (1 + H)C_{rs}^{\text{min}} \quad k \in K_{rs}$$

$C_{rs}^{\text{min}}$—Minimum path cost between OD and $r$-$s$;

$H$—Path extension coefficient, generally a non-negative constant;

$K_{rs}$—Collection of effective paths between OD and $r$-$s$.

Generally speaking, when passengers choose a path between OD pairs, they will not consider all connected paths, but only a part of them is considered "reasonable." And these parts should also be within the limit of travel expenses, if the limit is exceeded, it will be disregarded. Obviously, "reasonable" or "effective" paths should not actually have cycles. In other words, no traveler will start at a node and then return to that node again during the trip.

The following formula can be used to express the random utility of passengers choosing the effective path $k \in K_{rs}$ between OD and $r$-$s$:

$$U_{rs} = V_{rs} + \epsilon_{rs} \quad k \in K_{rs}$$

$U_{rs}$—Random utility of effective path $k \in K_{rs}$;

$V_{rs}$—Utility value that travelers can determine;

$\epsilon_{rs}$—Random error term.

The determinable utility $V_{rs}$ can be expressed by the path cost:

$$V_{rs} = -\theta C_{rs}^k \quad k \in K_{rs}$$

Among them, $\theta$ is a constant, which plays a role in converting the cost into effect.

Then the random utility $U_{rs}$ can be expressed as the following form:

$$U_{rs} = -\theta C_{rs}^k + \epsilon_{rs} \quad k \in K_{rs}$$

Since the passenger’s path selection problem is essentially a probability problem, the process of their path selection between the same OD is the process of selecting the largest path generalized cost within the range of all alternative paths, so the selection probability of this path is as follows Show:

$$p_{rs}^k = Pr(U_{rs}^k > U_{rs}^n), n \neq k \quad k \in K_{rs}$$

$p_{rs}^k$—The probability that OD chooses the effective path $k \in K_{rs}$ for passengers between $r$-$s$.

Obviously, the selection probability has the following properties:

$$0 \leq p_{rs}^k \leq 1; \sum_{k \in K_{rs}} p_{rs}^k = 1, \quad k \in K_{rs}$$

According to the above, the path selection probability $p_{rs}^k$ depends on the distribution of the random error term $\epsilon_{rs}$ and the path cost $C_{rs}^k$ can be determined. Assuming that $\epsilon_{rs}$ is independent of each other and obeys the Gumbel distribution, the probability that the $k$-th alternative path between OD and $r$-$s$ is selected can be expressed as the following Logit form:

$$p_{rs}^k = \frac{\text{exp}(-\theta C_{rs}^k)}{\sum_{k \in K_{rs}} \text{exp}(-\theta C_{rs}^k)} \quad k \in K_{rs}$$
The variance of $\theta$ and $\varepsilon^r_s$ is inversely proportional. Therefore, $\theta$ can be regarded as an index to measure the overall familiarity of travelers with the road network.

3.2. Limitations of traditional logit model and improvement measures

Because the traditional Logit model has the defect that "the absolute difference of utility determines the selection probability", that is, the size of the effective path passenger flow sharing rate is determined by the absolute difference of the path's comprehensive impedance, making the result of passenger flow distribution unreasonable. For example: there are two generalized travel cost paths between 105min, 110min and 5min and 10min between a certain OD pair, and assuming that they are path 1 and path 2, $\theta = 1$, then by the traditional Logit model, there are two paths The selection probability of the scheme is as follows:

i. When the generalized travel costs of Route 1 and Route 2 are 5min and 10min respectively:

$$
\begin{align*}
P_1 &= \frac{e^{-5}}{e^{-5} + e^{-10}} \approx 0.993 \\
P_2 &= 1 - P_1 \approx 0.007
\end{align*}
$$

ii. When the generalized travel costs of Route 1 and Route 2 are 105min and 110min respectively:

$$
\begin{align*}
P_1 &= \frac{e^{-105}}{e^{-105} + e^{-110}} \approx 0.993 \\
P_2 &= 1 - P_1 \approx 0.007
\end{align*}
$$

It can be seen from the two cases i. and ii that when the difference in the utility value determination item between the two paths is the same, the calculated path selection probability is also the same, but in this case, the Logit model calculates The result of is inconsistent with the actual situation: For the first case, when the generalized cost value of the route is doubled by the passenger, there is a high probability that the route with less cost will appear. Therefore, the probability of 99% and 1% respectively is It is in line with reality; and in scenario ii., when passengers face the two paths of 105min and 110min, since the difference between the cost of 5min and the actual cost base is large, the probability of choosing the two paths should not be too great. The degree of distinction between 99% and 1% is unreasonable. This fully reflects the defect that the traditional Logit model has "the absolute difference in utility determines the probability of selection".

In response to this defect, this paper uses the relative cost difference to calculate the effective path selection probability, and compares it with the shortest path cost. Assuming that the minimum generalized cost between OD pairs is $C_{min}^n$, [2] will improve the Logit model into the following form:

$$
p_k^{rs} = \frac{\exp\left(-\theta C_k^{rs}/C_{min}^r\right)}{\sum_{k \in K} \exp\left(-\theta C_k^{rs}/C_{min}^r\right)} \quad k \in K_{rs}
$$

Select the improved Logit model to re-analyze the above example:

i. When the generalized travel costs of Route 1 and Route 2 are 5min and 10min respectively:

$$
\begin{align*}
P_1 &= \frac{e^{-5}}{e^{-5} + e^{-10}} \approx 0.731 \\
P_2 &= 1 - P_1 \approx 0.269
\end{align*}
$$

ii. When the generalized travel costs of Route 1 and Route 2 are 105min and 110min respectively:

$$
\begin{align*}
P_1 &= \frac{e^{-105}}{e^{-105} + e^{-110}} \approx 0.512 \\
P_2 &= 1 - P_1 \approx 0.488
\end{align*}
$$

Comparing the calculation results of the above cases, improving the logit model to obtain different path allocation probability results can improve the problem that a certain path travel cost base continues to increase, and it is helpful to improve the defect caused by "the absolute cost difference determines the selection probability". Improve the accuracy of the Logit model to a certain extent.
3.3. Logit model maximum likelihood method parameter estimation

This paper chooses the maximum likelihood estimation method to calibrate the parameters of the passenger path selection model. By considering the possible configuration of n sampling points in the observation space, the problem of maximizing the log-likelihood function is studied. The possible configuration is essentially divided into complete separation, Quasi-complete separation and overlapping three mutually exclusive and exhaustive configurations. In each case, we focus on the finiteness of the maximum likelihood solution and the actual value of the maximum likelihood function. In this article, the absence of maximum likelihood estimation means the absence of a finite maximum. The basic steps are:

i. Calculate the likelihood function of the estimated parameter;
ii. Analyze the situation where the likelihood function reaches the maximum value, and obtain the estimated value of the parameter in combination with the actual statistical data.

When the variable is a discrete random variable, the likelihood function can be obtained by multiplying the probability function according to the distribution law of the variable, and then the maximum likelihood estimation method is selected to estimate the variable. [3]

When the variable is a continuous random variable, the likelihood function needs to be calculated by the probability density function of each parameter independently observed:

\[ L(\theta) = L(\eta_1, \eta_2, \ldots, \eta_n; \xi) = f(\eta_1, \xi)f(\eta_2, \xi)\ldots f(\eta_n, \xi) \]

Take the logarithm of the likelihood function:

\[ \ln L(\xi) = \ln L(\eta_1, \eta_2, \ldots, \eta_n; \xi) = \sum_{i=1}^{n} \ln f(\eta_i, \xi) \]

Perform maximum likelihood estimation on the log-likelihood function, and take the parameter partial derivative equal to 0, when \( \xi = (\xi_1, \xi_2, \ldots, \xi_m) \), then

\[ \frac{\partial}{\partial \xi_k} \ln L(\eta_1, \eta_2, \ldots, \eta_n; \xi_1, \xi_2, \ldots, \xi_m) = \sum_{i=1}^{n} \frac{\partial}{\partial \xi_k} \ln L(\eta_i; \xi_1, \xi_2, \ldots, \xi_m) = 0 \]

The parameter value can be obtained by combining the sample to solve the function.

4. Case analysis

4.1. Overview of metro in a city

The operation structure diagram of rail transit in a city is shown in Figure 1 below. There are 3 operating lines and 31 stations. The stations with station numbers 1101, 1102, 1103 are transfer stations between the lines. In addition, the city’s entire network operating routes have adopted a single fare model, and the three routes can be seamlessly transferred.

![Figure 1. Operation structure of Metro Network.](image)

4.2. Passenger flow distribution instance verification

1) Basic data of subway network

   a) Basic line data
Table 1. Operation interval of each section of line 1 (unit: s)

| O/D | 101 | 102 | 103 | 1101 | 104 | 105 | 106 | 1102 | 107 | 108 |
|-----|-----|-----|-----|-------|-----|-----|-----|------|-----|-----|
| 101 | 0   | 120 | 240 | 420   | 600 | 720 | 840 | 960  | 1140| 1320|
| 102 | 120 | 0   | 120 | 300   | 480 | 600 | 720 | 840  | 1020| 1200|
| 103 | 240 | 120 | 0   | 180   | 360 | 480 | 600 | 720  | 900 | 1080|
| 1101| 420 | 300 | 180 | 0     | 180 | 300 | 420 | 540  | 720 | 900 |
| 104 | 600 | 480 | 360 | 180   | 0   | 120 | 240 | 360  | 540 | 720 |
| 105 | 720 | 600 | 480 | 300   | 120 | 0   | 120 | 240  | 420 | 600 |
| 106 | 840 | 720 | 600 | 420   | 240 | 120 | 0   | 120  | 300 | 480 |
| 1102| 960 | 840 | 720 | 540   | 360 | 240 | 120 | 0    | 180 | 360 |
| 107 | 1140| 1020| 900 | 720   | 540 | 420 | 300 | 180  | 0   | 180 |
| 108 | 1320| 1200| 1080| 900   | 720 | 600 | 480 | 360  | 180 | 0   |

Table 2. Operation interval of each section of line 2 (unit: s)

| O/D | 201 | 202 | 203 | 1101 | 204 | 205 | 206 | 1103 | 207 | 208 |
|-----|-----|-----|-----|------|-----|-----|-----|------|-----|-----|
| 201 | 0   | 180 | 300 | 480  | 600 | 720 | 840 | 960  | 1140| 1260|
| 202 | 180 | 0   | 120 | 300  | 420 | 540 | 660 | 780  | 960 | 1080|
| 203 | 300 | 120 | 0   | 180  | 300 | 420 | 540 | 660  | 840 | 960 |
| 1101| 480 | 300 | 180 | 0    | 120 | 240 | 360 | 480  | 660 | 780 |
| 204 | 600 | 420 | 300 | 120  | 0   | 120 | 240 | 360  | 540 | 660 |
| 205 | 720 | 540 | 420 | 240  | 120 | 0   | 120 | 240  | 420 | 540 |
| 206 | 840 | 660 | 540 | 360  | 240 | 120 | 0   | 120  | 300 | 420 |
| 1103| 960 | 780 | 660 | 480  | 360 | 240 | 120 | 0    | 180 | 300 |
| 207 | 1140| 960 | 840 | 660  | 540 | 420 | 300 | 180  | 0   | 120 |
| 208 | 1260| 1080| 960 | 780  | 660 | 540 | 420 | 300  | 120 | 0   |

Table 3. Operation interval of each section of line 3 (unit: s)

| O/D | 301 | 302 | 1103 | 303 | 304 | 305 | 306 | 1102 | 307 | 308 |
|-----|-----|-----|------|-----|-----|-----|-----|------|-----|-----|
| 301 | 0   | 180 | 300  | 420 | 540 | 660 | 900 | 1020 | 1200| 1380|
| 302 | 180 | 0   | 120  | 240 | 360 | 480 | 720 | 840  | 1020| 1200|
| 1103| 300 | 120 | 0    | 120 | 240 | 360 | 600 | 720  | 900 | 1080|
| 303 | 420 | 240 | 120  | 0   | 120 | 240 | 480 | 600  | 780 | 960 |
| 304 | 540 | 360 | 240  | 120 | 0   | 120 | 360 | 480  | 660 | 840 |
| 305 | 660 | 480 | 360  | 240 | 120 | 0   | 240 | 360  | 540 | 720 |
| 306 | 900 | 720 | 600  | 480 | 360 | 240 | 0   | 120  | 300 | 480 |
| 1102| 1020| 840 | 720  | 600 | 480 | 360 | 120 | 0    | 180 | 360 |
| 307 | 1200| 1020| 900  | 780 | 660 | 540 | 300 | 180  | 0   | 180 |
| 308 | 1380| 1200| 1080| 960 | 840 | 720 | 480 | 360  | 180 | 0   |

b) Basic train information

Table 4. Basic information of trains on each line

| line   | Number of seats | Maximum capacity |
|--------|-----------------|------------------|
| Line 1 | 240             | 312              |
| Line 2 | 240             | 312              |
| Line 3 | 240             | 312              |

c) Train stop time

For the convenience of description and calculation, this article assumes that the train stopping time at ordinary stations and transfer stations is 0.5min and 0.7min respectively.

d) Train departure time

Table 5. Train departure interval

| Departure interval | Line 1 | Line 2 | Line 3 |
|--------------------|--------|--------|--------|
| Low peak period    | 7'11"  | 6'30"  | 7'20"  |
| flat hump period   | 5'27"  | 5'6"   | 5'40"  |
| peak period        | 4'16"  | 3'9"   | 4'25"  |
e) Passenger transfer walking time

| Station number | Line i | Line j | Low peak (s) | Flat hump (s) | Peak (s) |
|----------------|--------|--------|--------------|--------------|---------|
| 1101           | 1      | 2      | 375          | 333          | 275     |
| 1101           | 2      | 1      | 391          | 344          | 308     |
| 1103           | 2      | 3      | 320          | 270          | 233     |
| 1103           | 3      | 2      | 375          | 333          | 275     |
| 1102           | 1      | 3      | 400          | 350          | 313     |
| 1102           | 3      | 1      | 311          | 264          | 228     |

f) Parameter value

The parameter value needs to be based on a large amount of data statistics and analysis. This example combines the findings of other researchers. The penalty coefficient for the number of transfers is usually 1 to 2. In general, $H$ is 0.15, and the constant $\theta$ is 3. The transfer time amplification factor generally fluctuates between 1.2 and 1.8.

The meanings and values of the relevant parameters are shown in Table 7:

| Parameter | Meaning                          | Value |
|-----------|----------------------------------|-------|
| $\alpha$  | Transfer time amplification factor | 1.3   |
| $\beta$   | Penalty parameter for transfer times | 1.8   |
| $\theta$  | Constant                         | 3     |
| $H$       | Stretch coefficient of path       | 0.15  |

2) Passenger flow distribution method of subway network

This paper is based on the OD data of urban rail transit evening peak passenger flow in a certain city on a certain day, and the steps of the passenger flow distribution algorithm based on the Logit model are as follows:

Step 1: Calculate the generalized travel cost $C_{k}^{rs}$ of each path between r-s and the minimum travel cost function $C_{\min}^{rs}$ through Dijkstra's algorithm and Dial algorithm to determine the effective path range between r-s;

Step 2: Combine the Logit model to obtain the selection probability $p_{k}^{rs}$ of different effective paths between r and s;

Step 3: Import the evening peak OD passenger flow matrix of the entire rail transit network, and define the passenger flow between OD and r-s as $q_{rs}$;

Step 4: Calculate the passenger flow $f_{k}^{rs}$; on each path through the formula $f_{k}^{rs} = q_{rs} * p_{k}^{rs}$;

Step 5: Compare the result with the actual result data.

3) Passenger flow distribution method of subway network

Effective path 1:

Travel time: $T_{1}^{ij} = 1020 + 360 = 1380$ (s)

Transfer time: $E_{1}^{in} = 1.3 * (228 + 0.5 * 132) = 362(s)$

Total cost: $C_{k}^{rs} = 1380 + 362 = 1742(s) \approx 29$(min)

Effective path 2:

Travel time: $T_{2}^{ij} = 300 + 480 + 900 = 1680(s)$

Transfer time: $E_{2}^{in} = 1.3 * 2^{1.8} * (275 + 308 + 0.5 * 189 + 0.5 * 1256) = 847(s)$

Total cost: $C_{k}^{rs} = 1680 + 847 = 2527(s) \approx 42.12$(min)

After calculation, the distribution ratio of the routes with station numbers 301 and 108 is shown in Table 4.8 below. The table contains the description of passenger travel route, transfer situation, travel time, total time cost, and selection probability.
Table 8. Passenger flow distribution results in peak hours between OD301-108

| Path | Path description | Number of transfers | Travel time (min) | Total cost (min) | Probability of choice |
|------|------------------|---------------------|-------------------|-----------------|----------------------|
| Effective path 1 | 301(Departure station)-1102 (Transfer station) -108 | 1 | 23 | 29 | 0.6994 |
| Effective path 2 | 301(Departure station)-1103 (Transfer station) -1101 (Transfer station) -108 | 2 | 28 | 42.12 | 0.3006 |

Table 9. Proportion of passenger flow distribution before and after using the model in peak hours between OD301-108

| Path | Path description | Model passenger flow distribution probability | Actual path selection probability |
|------|------------------|---------------------------------------------|----------------------------------|
| Path 1 | 301(Departure station)-1102 (Transfer station) -108 | 0.6994 | 0.7692 |
| | 301(Departure station)-1103 (Transfer station) -1101 (Transfer station) -108 | 0.3006 | 0.1923 |
| Path 2 | 301(Departure station)-1103 (Transfer station) -108 -103-108 | 0.0000 | 0.0384 |

5. Conclusion

Due to the increasing complexity of the operation of the developing urban rail transit network and the increase in the different nature of the routes that passengers can choose, it is difficult to establish a scientific and accurate passenger travel behavior probabilistic selection model and solve it through algorithms. Together with other constantly changing factors, it is necessary to further improve the urban rail system. The level of transportation management.

Passenger flow is an important foundation of rail transit operation organization. Its scale and distribution characteristics are closely related to the operation organization plan of the entire system. Under the condition of the one-ticket system of the subway, passengers have more and more choices of travel routes in the subway system. Therefore, this article starts from the many factors influencing the choice of passenger travel routes, and combines basic costs and penalty costs to construct a generalized path cost function, and It also analyzed the traditional Logit model, and established an improved Logit model by using the path search algorithm to solve the defect that "the absolute difference in utility determines the probability of selection". Finally, based on the analysis of actual calculation examples, this paper realizes the passenger flow distribution of the subway network based on the Logit model by importing the OD data of the urban rail transit evening rush hour in a certain city.

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