Hydrodynamic and thermal mechanisms of filtration combustion inclinational instability based on non-uniform distribution of initial preheating temperature

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Abstract. Filtration combustion (FC) is one style of porous media combustion with inert matrix, in which the combustion wave front propagates, only downstream or reciprocally. In this paper, we investigate the FC flame front inclinational instability of lean methane/air mixtures flowing through a packed bed as a combustion wave front perturbation of the initial preheating temperature non-uniformity is assumed. The predicted results show that the growth rate of the flame front inclinational angle is proportional to the magnitude of the initial preheating temperature difference. Additionally, depending on gas inlet gas velocity and equivalence ratio, it is demonstrated that increase of gas inlet gas velocity accelerates the FC wave front deformation, and the inclinational instability evolves faster at lower equivalence ratio. The development of the flame front inclinational angle may be regarded as a two-staged evolution, which includes rapid increase, and approaching maximum value of inclinational angle due to the quasi-steady condition of the combustion system. The hydrodynamic and thermal mechanisms of the FC inclinational instability are analyzed. Consequently, the local propagation velocity of the FC wave front is non-uniform to result in the development of inclinational angle at the first stage of rapid increase.

1. Introduction

Filtration combustion (FC) is one style of porous media combustion with inert matrix[1], in which the combustion wave front propagates, only downstream or reciprocally. Compared with the conventional free flame combustion regimes, it may be realized superadiabatic combustion due to the heat recirculation of porous media[2, 3], theoretically[4], experimentally[5]. However, the unstable FC, especially the flame front inclinational instability within inert porous media, is still an obstacle to restrict widespread application of porous media combustion technology. A lot of investigations of the FC flame front inclinational instability were conducted, experimentally, numerically, and theoretically. Dobrego et al.[6] investigated the dynamical behavior of the FC inclinational instability experimentally, numerically. However, the original perturbation of non-uniform preheating temperature distribution wasn’t explicitly considered as an influencing factor of the inclinational instability in their studies. Theoretically, Kakutkina[7] studied the instabilities of both the hot spot and the combustion wave front inclination using a thermal model. Considering the effects of the curvature and local inclination of the wave front, a expression of the propagation velocity of a combustion wave front was presented. Yang et al.[8] studied experimentally the FC characteristics of lean methane/air mixtures within inert, high-porosity micro-fibrous media packed bed. Two peculiar instability phenomena, such as the flame anchoring effect, combustion split wave, were observed, but the
phenomena could not be interpreted using the model of the conventional volume-averaged treatment. Fursenko et al.[6] numerically investigated the dynamical behavior of the FC wave front propagation using two-dimensional model. A potential physical explanation of the dynamical behavior leading to the flame anchoring effect and split wave was provided on account of the effect of hydrodynamic instability. Besides, the FC characteristics of lean methane/air mixtures within inert, high-porosity micro-fibrous media packed bed were demonstrated in Fan et al. numerical work[9], and the calculated standing wave regime coincides well with the experimental results.

Zheng et al.[10] numerically studied the development of the flame front inclinational instability in an inert porous medium. An initial inclined band with a thickness of 34 mm, a temperature of 1400 K and an angle of 36.25° was utilized as an igniting perturbation condition. Their numerical results showed that the evolution of inclinational instability causes flow non-uniformity, and so, it can further result in a separation of the flame front to form the multiple flame zones. Shi et al.[11] studied experimentally and numerically the influences of non-uniform inlet gas velocity, and equivalence ratio on the flame front inclinational instability. Additionally, the effect of the hot-spot instability on the characteristics of the flame front inclinational deformation was experimentally investigated in our prior work[12].

In the present paper, the main objective is to reveal the hydrodynamic and thermal mechanisms of the FC flame front inclinational instability in packed bed as the initial preheating temperature is non-uniform. A simplified perturbation model of the initial preheating temperature non-uniformity is established reasonably. The effect of some parameters, such as gas inlet gas velocity, equivalence ratio, and initial preheating temperature non-uniformity, on the development of the inclinational instability is numerically investigated. The potential dynamical behavior of the flame front inclinational deformation is analyzed. The hydrodynamic and thermal mechanisms of the flame front inclinational instability are discussed.

2. Numerical models
2.1 Physical model
The two-dimensional model of burner is showed in Figure 1. The burner consists of three sections included the inlet zone (50 mm), the preheating zone (50 mm), and the combustion wave propagating zone (400 mm). The initial thermal perturbation is assumed in the preheating zone. The preheating zone is divided into two equal-sized parts as shown in Figure 1. The solid phase temperature in zone 1 is $T_1$, while the temperature in zone 2 is $T_2$. As an initial preheating temperature difference is $\Delta T = T_1 - T_2$ between the two preheating zones, it may be considered as an original perturbation which causes the filtration combustion inclination instability. In this paper, the temperature in zone 1 is always set to $T_1 = 1700 \text{ K}$, and the temperature in zone 2 is less than the one in zone 1.

![Figure 1. Schematic diagram of the porous burner model.](image)

2.2 Governing equations
The gas mixtures are considered as the ideal incompressible fluid, and the gas radiation is neglected. The inert porous medium is non-catalytic, homogeneous, and optically thick. Meanwhile, the wall effect of
porosity variation is neglected. The conservation equations of continuity, momentum and gaseous species are expressed as follows.

1. **Continuity equation**

\[
\frac{\partial (\varepsilon \rho_g)}{\partial t} + \nabla \cdot (\varepsilon \rho_g \mathbf{u}) = 0
\]

where \(\varepsilon\), \(\rho_g\), \(\mathbf{u}\) are the packed bed porosity, the gas mixtures density, the velocity vector, respectively.

2. **Momentum equation**

\[
\frac{\partial (\varepsilon \rho_g \mathbf{u})}{\partial t} + \nabla \cdot (\varepsilon \rho_g \mathbf{u} \mathbf{u}) = -\frac{\partial p}{\partial x} + \nabla (\mu \nabla \mathbf{u}) - S_p
\]

where \(p\) is the pressure. \(S_p\) is the pressure loss source, and it can be calculated by Darcy’s law[13].

3. **Gas phase energy equation**

\[
\frac{\partial (c_g T)}{\partial t} + \varepsilon \nabla (c_g \rho_g \mathbf{u} T) = \varepsilon \nabla (\lambda_{\text{eff-g}} \nabla T) + h_v (T_s - T_g) - \varepsilon \sum W_i
\]

where \(c_g\) is the specific heat capacity of gas mixture, \(h_v\) is the volumetric convective heat transfer coefficient between gas and solid phases, and its expression is \(h_v = \frac{6 \varepsilon}{\alpha d} \nu \lambda\), where \(\nu\) is described as \(\nu = 2 + 1.1 \Pr^{1/3} \Re^{1/2}\).[14] \(\lambda_{\text{eff-g}}\) is the effective thermal conductivity of gas phase and its expression is \(\lambda_{\text{eff-g}} = \lambda_s + \rho_s C_p D'\), here, \(D'\) is the thermal dispersion[15], \(W_i\) is the molecular weight, \(W_i\) is the reaction production rate of the \(i^{th}\) species.

4. **Solid phase energy equation**

\[
(1 - \varepsilon) \frac{\partial (\rho_s T_s)}{\partial t} = \nabla (\lambda_{\text{rad}} \nabla T_s) + h_v (T_s - T_g)
\]

where \(\rho_s\), \(c_s\) are the solid density and specific heat capacity, respectively. \(\lambda_{\text{rad}}\) is the effective conductivity of porous matrix, and its expression is \(\lambda_{\text{rad}} = (1 - \varepsilon) \lambda_s + \lambda_{\text{eff}}\), where \(\lambda_{\text{eff}}\) is the equivalent radiative conductivity of alumina particles, and its expression is \(\lambda_{\text{eff}} = \frac{32 \cdot \varepsilon \cdot \sigma \cdot d \cdot T_s^4}{9 (1 - \varepsilon)^2}\), decided by the Rosseland approximation method[16], assuming an optically thick region around the high temperature zone in packed bed.

5. **Species transport equation**

\[
\varepsilon \frac{\partial (\rho_g Y_i)}{\partial t} + \varepsilon \nabla (\rho_g \mathbf{u} Y_i) = \nabla (\varepsilon \rho_g D_i Y_i) + \varepsilon W_i
\]

where \(Y_i\) is the mass fraction of the \(i^{th}\) species, \(D_i\) is the mass diffusivity of the \(i^{th}\) species.

6. **Ideal gas equation of state**

\[
p = \rho_g R T_s
\]

where \(R\) is the universal gas constant.

Considering that the flame speed predictions are much less sensitive to the choice of reaction mechanism[1], hence, in the present study, a single-step global chemistry mechanism of methane/air is used in the model.

2.3 **Boundary conditions**

- **Inlet condition:**

\[
T_s = 300K, u_g = u_{g,in}, v = 0, Y_{CH_4} = Y_{CH_4,in}, Y_{O_2} = Y_{O_2,in}
\]

\[
\lambda_s \frac{\partial T_s}{\partial x} = -\varepsilon_s \sigma (T_{in,in}^4 - T_s^4)
\]

where \(\varepsilon_s\) is the solid surface emissivity.

**Outlet condition:**
\[ \frac{\partial T_k}{\partial t} + \frac{\partial Y_{\text{ch}_4}}{\partial t} = \frac{\partial T}{\partial x} = \frac{\partial Y_{\text{O}_2}}{\partial x} = 0 \quad (9) \]

\[ \lambda_x \frac{\partial T_x}{\partial x} = -\varepsilon_x \sigma(T_{\text{out}}^4 - T_x^4) \quad (10) \]

Wall heat transfer with the ambient as follows:

\[ \lambda_{\text{st}} \frac{\partial T_{\text{st}}}{\partial y} = h_{\text{st}}(T_{\text{in}} - T_{\text{st}}) + \varepsilon_{\text{st}} \sigma(T_{\text{in}}^4 - T_{\text{st}}^4) \quad (11) \]

where \( \varepsilon_{\text{st}} = 0.92 \) is the emissivity of the quartz tube.

### 2.4 Numerical method

The governing equations are solved using the software package Fluent 6.3. The single-step reaction mechanism of methane/air is available. The solid radiation is treated using the Rosseland approximation. The user-defined function (UDF) and user-defined scalar (UDS) are used to establish the two-temperature model. SIMPLE algorithm is used to solve the coupling of the pressure and velocity.

### 3. Numerical results and discussions

#### 3.1 Development of FC inclinational instability

The development of the flame front inclinational instability is clearly shown in Figure 2 for an initial preheating perturbation of \( \Delta T = 300 \) K at \( u_{\text{g.in}} = 0.8 \) m/s, \( \phi = 0.4 \). The gas temperature variation is shown in Figure 2(a). The high temperature zone is small and centralized at the beginning of time of 300 s. The downstream packed bed is completely preheated while the temperature field already shows an original inclinational status. It implies that the initial preheating temperature difference could motivate the deformation of the combustion reaction front. As the combustion wave propagates downstream, the high temperature area gradually extends. At the time of 600 s, the maximum temperature zone mostly moves to the right side of burner. At the following combustion wave propagating downstream, the most high temperature area is mainly focused on the right side of burner. Figure 2(b) shows clearly the development of flame front shape as the inclinational instability. The reaction rate is used to show the flame front shape. At the time of 300 s, the flame front appears obviously concave on account of the heat recirculation of porous media. And then, at the time of 600 s, the flame front evolves into a primary S-shape. Soon after, the S-shaped flame front is greatly stretched. In Figure 2(b), it can be seen that the reaction rate is not uniform, which shows that the right reaction rate is faster than the left one. The local energy release of the flame front in the reaction is not uniform, which will cause the regional discrepancy of the combustion wave front propagation velocity[5]. Hence, the inclinational deformation of the flame front develops fast within the period of 300-1800 s. However, after the time of 2100 s, the inclinational flame front shape is not nearly changed any more. In essence, for the combustion system, when the gas flow, combustion reaction, heat and mass transfer are on the quasi-steady condition at the later stage of the combustion wave propagating downstream [10], the flame front inclinational angle approaches to a maximum value.

![Figure 2](image-url)

(a) Contours of gas temperature  
(b) Contours of Arrhenius rate

Figure 2. A typical development of the flame front inclination instability at \( u_{\text{g.in}} = 0.8 \) m/s, \( \phi = 0.4 \),
3.2 Effect of preheating temperature difference, gas inlet gas velocity and equivalence ratio

Figure 3 shows the variation of the inclinational angle for various initial preheating temperature differences at $u_{g,in} = 0.8$ m/s, $\phi = 0.4$. The definition of the inclinational angle is same as the one in the literature[10]. It can be found that the growth variation of inclinational angle may be divided into two stages: rapid increase, approaching maximum value. This development of the flame front inclination instability is also visually shown in Figure 2. The developing trend of the inclinational angle is very similar to those in the literatures[10, 17]. The inclinational angle growth is an approximative power function of time. As shown in Figure 3, at the initial stage of 300 s, the inclinational angle is small for the initial preheating perturbation of $\Delta T = 100$ K, and the inclinational angle increases with the perturbation rise of the initial preheating temperature difference (200 K, 300 K). It manifests that the inclinational angle growth is proportional to the magnitude of the initial preheating temperature difference at the stage of the rapid development of the inclinational instability. However, at the later stage of maximum value of the inclinational angle, the effect of initial perturbation gradually disappears. The inclinational angle approaches to a maximum value of approximate 50°, while the combustion system is on the quasi-steady condition.

Figure 3. Variation of the inclinational angle $\theta$ with different preheating temperature differences at $u_{g,in} = 0.8$ m/s, $\phi = 0.4$.

The effects of both equivalence ratio and inlet gas velocity on the flame front inclination instability are demonstrated in Figure 4 for the perturbation of $\Delta T = 300$ K. The inclinational angle is monotonically increases approaching a final maximum value. The development of the flame front inclinational instability includes the rapid increase, and approaching maximum value as well, which is similar to Zheng et al. numerical results[10], whereas the initial thermal perturbation is the preheating temperature difference in our model. For equivalence ratio of $\phi = 0.4$, the growth rate of the inclinational angle increases with inlet gas velocity, and the final maximum angle is also proportional to the magnitude of inlet gas velocity. It indicates that the hydrodynamic instability intensifies the perturbation of the flame front inclinational instability as the combustion wave propagates downstream. Moreover, the hydrodynamic instability intensity is most weak for inlet gas velocity of $u_{g,in} = 0.4$ m/s among these cases, the development of the inclinational angle is slowest, and the combustion wave propagation velocity is also slowest (the wave propagation time is about 9000 s). Additionally, the growth rate of the inclinational angle is faster at lower equivalence ratio ($\phi = 0.3$). As a consequence, it is demonstrated that the initial preheating temperature difference as an original perturbation mainly influences on the developing behavior of the FC inclinational instability at the primary stage, while the final development of the flame front inclinational instability is determined by the hydrodynamic and thermal conditions.

Figure 4. Variation of inclinational angle for $\Delta T = 300$ K depending on inlet velocities and equivalence ratios.
3.3 Hydrodynamic mechanism of flame front inclinational instability

In Figure 5, the results of reaction rate, total pressure, velocities in y- and x-directions demonstrate the possible hydrodynamic mechanism of flame front inclinational instability at $u_{g,in} = 0.8$ m/s, $\varphi = 0.4$, $\Delta T = 300$ K at the first stage of 300 s. The reaction rate in Figure 5(a) is used to show the flame front shape, which obviously display the interface between the incoming gas mixtures and the hot reaction products. The dash lines in Figures 5(b-d) are plotted to denote the location of the reaction front. Figure 5(b) shows the pressure loss field near the combustion reaction zone. Obviously, the pressure gradient is non-uniform which shows that the right pressure gradient is lower than the left one. The right more kinetic energy forces the gas flow to the right combustion zone. Figures 5(c), (d) clearly reveal the dynamic behavior of the flow near the combustion reaction. In Figure 5(c), the maximum y-direction velocity is located on the right zone of burner behind the reaction front, while the minimum is located on the left zone before the reaction. Meanwhile, the x-direction velocity behind the reaction front is negative on the right zone of burner, while the x-velocity before the reaction front is positive as shown in Figure 5(d). As a consequence, the final generated flow performance is similar to the Zheng et al. numerical description [10]. Based on the hydrodynamic properties near the flame front, the right combustion wave velocity is faster than the left wave velocity. And thus, it indicates that the flame front will be stretched, accordingly, the inclinational angle will increase at the following combustion wave propagating downstream.

3.4 Thermal mechanism of flame front inclinational instability

In order to further understand the mechanism of flame front inclinational instability, the combustion wave velocities at various x-positions are analyzed to illuminate the development of flame front inclinational deformation. Figure 6 shows the variation of the combustion wave velocities at $u_{g,in} = 0.8$ m/s, $\varphi = 0.4$, $\Delta T = 300$ K. Here, we define the combustion wave velocities as the $u_{w-L}$ near the left burner wall, the $u_{w-R}$ near the right burner wall, the average combustion wave $u_{w-average}$ at the center of burner, and the wave velocity difference $\Delta u_w = u_{w-R} - u_{w-L}$. As shown in Figure 6, at the first stage of 1500 s, the combustion wave $\Delta u_w$ keeps an approximate constant, which implies that the flame front inclinational angle will increase continually. However, the local wave velocities on both the right and the left of burner present to decrease during this period. And then, the $\Delta u_w$ begins to decrease after the time 1500 s till it is almost zero at the time of 2700 s. The $u_{w,R}$ continues to decrease at the stage of 1500-2700 s, while the $u_{w,L}$ hardly changes. This means that the development of the flame front inclinational instability gradually slows down. At the later stage of the combustion wave propagating downstream, the $u_{w,R}$ is equal to the $u_{w,L}$, which indicates that the flame front is no longer stretched, and the inclinational angle approaches to a maximum value.
Furthermore, the thermal mechanism of the flame front inclination instability may be interpreted using the relationship of the combustion wave propagation velocity as follows[18],

$$u_w = u_t \cdot \left\{ 1 - \left[ \frac{\Delta T_a}{\Delta T_c} \right] - 4 \beta \lambda_{eff} \frac{|c(T,T)_{g eff}|}{\rho(T,T)_{g eff}} \right\}^{1/2} \tag{12}$$

where $u_t$ is the thermal wave velocity, $u_t = u_g \cdot (c(T,T)_{g eff}/\rho(T,T)_{g eff})$, $\Delta T_a$ is the adiabatic temperature rise of the gas mixture, $\Delta T_c$ is the combustion temperature rise in the filtration combustion wave, $\beta$ is the effective coefficient for heat exchange with the surroundings.

In the period of the inclinational angle increase, the flame front is significantly stretched. The enlargement of inclinational flame area causes the decrease of $\Delta T_c$[19], which results in the lower propagation velocity according to Eq. (14). As shown in Figure 6, the $u_{w-average}$ keeps decreasing. When the combustion system approaches to the quasi-steady condition at the later stage of combustion wave propagating downstream, the $u_{w-average}$ decreases to a constant, and the flame front inclinational deformation will no longer develops.

Figure 6. Variation of combustion wave velocities $u_w-L$, $u_w-R$, $\Delta u_w$, and $u_{w-average}$ at $u_{g,in} = 0.8$ m/s, $\varphi = 0.4$, $\Delta T = 300$ K.

4. Conclusion
In this paper, the FC flame front inclinational instability of lean methane/air mixtures in packed bed is investigated numerically. The effect of initial preheating perturbation, gas inlet gas velocity, equivalence ratio on the flame front inclinational instability is comparatively analyzed. The local hydrodynamic and thermal mechanisms of the flame front inclinational instability are discussed.

Based on the study, the numerical results are concluded as follows.
1) The initial non-uniform preheating temperature distribution on the cross-section of burner is confirmed as an original perturbation to the primary combustion wave front inclinational deformation. The magnitude of the initial preheating non-uniformity is monotonically the growth rate of the flame front inclinational angle, but the development of the flame front inclinational instability becomes inconspicuous as the combustion system approaches to the quasi-steady condition at the later stage of the FC wave propagation.
2) The effect of gas inlet gas velocity, equivalence ratio indicates that the growth rate of the flame front inclinational angle becomes faster with increase of inlet gas velocity and decrease of equivalence ratio, which shows the same trend as the numerical results of Zheng et al.[10].
3) The dynamic behavior of the FC front inclinational instability is significantly influenced by the local hydrodynamic and thermal effects near the combustion reaction zone.
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References
[1] J.R. Howell, M.J. Hall, J.L. Ellzey, Combustion of hydrocarbon fuels within porous inert media, Prog. Energy Combust. Sci., 22(2) (1996) 121-145.
[2] A. Egerton, K. Gugan, F.J. Weinberg, The mechanism of smouldering in cigarettes, Combust. Flame, 7(1) (1963) 63-78.
[3] F.J. Weinberg, Combustion temperatures: The future, Nature, 233 (1971) 239-241.
[4] T. Takeno, K. Sato, K. Hase, A theoretical study on an excess enthalpy flame, Symposium (International) on Combustion, 18(1) (1981) 465-472.
[5] S. Zhdanok, L.A. Kennedy, G. Koester, Superadiabatic Combustion Of Methane Air Mixtures under Filtration In a Packed-Bed, Combust. Flame, 100(1-2) (1995) 221-231.
[6] (!!! INVALID CITATION !!!).
[7] N.A. Kakutkina, Some stability aspects of gas combustion in porous media, Combust. Explos. Shock Waves, 41(4) (2005) 395-404.
[8] H. Yang, S. Minaev, E. Geynce, H. Nakamura, K. Maruta, Filtration combustion of methane in high-porosity micro-fibrous media, Combust. Sci. Technol., 181(4) (2009) 654-669.
[9] Y. Liu, A. Fan, H. Yao, W. Liu, Numerical investigation of filtration gas combustion in a mesoscale combustor filled with inert fibrous porous medium, Int. J. Heat Mass Transfer, 91 (2015) 18-26.
[10] C.H. Zheng, L.M. Cheng, A. Saveliev, Z.Y. Luo, K.F. Cen, Numerical studies on flame inclination in porous media combustors, Int. J. Heat Mass Transfer, 54(15-16) (2011) 3642-3649.
[11] J.R. Shi, C.M. Yu, B.W. Li, Y.F. Xia, Z.J. Xue, Experimental and numerical studies on the flame instabilities in porous media, Fuel, 106 (2013) 674-681.
[12] Y.F. Xia, J.R. Shi, B.W. Li, C.M. Yu, Y.N. Xu, Z.J. Xue, Experimental investigation of filtration combustion instability with lean premixed hydrogen/air in a packed bed, Energy Fuels, 26(8) (2012) 4749-4755.
[13] I. Malico, X.Y. Zhou, J.C.F. Pereira, Two-dimensional Numerical Study of Combustion and Pollutants Formation in Porous Burners, Combust. Sci. Technol., 152(1) (2000) 57-79.
[14] N. Wakao, S. Kaguei, Heat and Mass Transfer in Packed Beds, Gordon and Breach Science Publications, New York, 1982.
[15] A.J. Barra, J.L. Ellzey, Heat recirculation and heat transfer in porous burners, Combust. Flame, 137(1-2) (2004) 230-241.
[16] V. Bubnovich, M. Toledo, Analytical modelling of filtration combustion in inert porous media, Appl. Therm. Eng., 27(7) (2007) 1144-1149.
[17] K.V. Dobrego, I.M. Kozlov, V.I. Bubnovich, C.E. Rosas, Dynamics of filtration combustion front perturbation in the tubular porous media burner, Int. J. Heat Mass Transfer, 46(17) (2003) 3279-3289.
[18] S.I. Foutko, S.I. Shabunya, S.A. Zhdanok, L.A. Kennedy, Superadiabatic combustion wave in a diluted methane-air mixture under filtration in a packed bed, Proc. Combust. Inst., 26(2) (1996) 3377-3382.
[19] S.S. Minaev, S.I. Potytnyakov, V.S. Babkin, Combustion wave instability in the filtration combustion of gases, Combust. Explos. Shock Waves, 30(3) (1994) 306-310.