Elastic and quasi-elastic electron scattering off isotopic and isotonic chains

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Abstract. We present theoretical predictions for electron scattering on $Z = 20$ isotopic and $N = 20$ isotonic chain. The calculations are done within the framework of the distorted-wave Born approximation for elastic scattering and the plane-wave Born approximation for the QE one. The proton and neutron density distributions are evaluated adopting a relativistic Dirac-Hartree model. We present results for the elastic and quasi-elastic cross sections and for the parity-violating asymmetry and investigate the evolution of nuclear properties as a function of the neutron and proton number. We also present a comparison with the parity-violating asymmetry parameter obtained by the PREX Collaboration on $^{208}$Pb.

1. Introduction
The nuclear response to an external probe is a powerful tool to investigate the structure of hadron systems such as atomic nuclei and their constituents. In particular, electron-scattering reactions have provided the most complete and detailed information on nuclear and nucleon structure [1, 2, 3, 4, 5, 6]. A lot of experimental and theoretical work on elastic and inelastic electron scattering at different energies has provided detailed information on the charge density distribution of the nuclear ground state and on the energy, strength, and quantum numbers of the excited states produced by single particle (s.p.) or collective excitation mechanisms [2, 7, 8, 9].

The use of the electron probe can be extended to exotic nuclei. The detailed study of the properties of nuclei far from the stability line and the evolution of nuclear properties with respect to the asymmetry between the number of neutrons and protons is one of the major topics in modern nuclear physics. Of particular interest is the behavior of the s.p. properties, with a consequent modification of the shell model magic numbers.

The study of the evolution of nuclear properties along isotopic and isotonic chains requires a good knowledge of nuclear matter distributions for protons and neutrons separately. The ground state densities reflect the basic properties of effective nuclear forces and provide fundamental nuclear structure information. Elastic electron scattering allows to measure with excellent precision only charge densities and therefore proton distributions. It is much more difficult to measure neutron distributions. A model-independent probe of neutron densities is provided by
parity-violating elastic electron scattering, where direct information on the neutron density can be obtained from the measurement of the parity-violating asymmetry $A_{pv}$ parameter $[10, 11, 12]$, that is related to the radius of the neutron distribution $R_n$.

In addition, quasi-elastic (QE) electron scattering can give information on the momentum distribution of nuclei. Precise measurements have been carried out from light to heavy nuclei on a wide range of kinematics. A lot of theoretical efforts have been devoted to the analysis of the data $[5, 6]$.

In this contribution we present and discuss results of calculations of elastic and QE electron scattering and $A_{pv}$ performed for the calcium isotopic chain and the N=20 isotonic chain. In addition, we compare our theoretical predictions for $A_{pv}$ with the experimental results obtained by the PREX collaboration on $^{208}$Pb.

2. Theoretical framework

2.1. Elastic electron scattering

In the one-photon exchange approximation and neglecting the effect of the nuclear Coulomb field on incoming and outgoing electrons, i.e. in the plane-wave Born approximation (PWBA), the differential cross section for the elastic scattering of an electron with momentum transfer $q$ off a spherical spin-zero nucleus is given by

$$\left( \frac{d\sigma}{d\Omega'} \right)_{EL} = \sigma_M |F_p(q)|^2, \tag{1}$$

where $\Omega'$ is the scattered electron solid angle, $\sigma_M$ is the Mott cross section $[4, 5]$ and $F_p(q)$ is the charge form factor for a spherical nuclear charge (point proton) density $\rho_p(r)$.

The PWBA is, however, not adequate for medium and heavy nuclei where the distortion produced on the electron wave functions by the nuclear Coulomb potential $V(r)$ from $\rho_p(r)$ can have significant effects. The distorted-wave Born approximation (DWBA) cross sections are obtained from the numerical solutions of the partial wave Dirac equation.

Another interesting quantity that can be measured in elastic electron scattering is the parity-violating asymmetry, which is defined as the difference between the cross sections for the elastic scattering of electrons longitudinally polarized parallel and antiparallel to their momentum. This difference arises from the interference between photon and $Z^0$ exchange and represents an almost direct measurement of the Fourier transform of the neutron density $[10, 13]$. In fact, in Born approximation, neglecting strangeness contributions and the electric neutron form factor, the parity-violating asymmetry can be expressed as $[14]$

$$A_{pv} = \frac{G_F Q^2}{4\sqrt{2} \pi \alpha} \left[ 4 \sin^2 \Theta_W - 1 + \frac{F_n(q)}{F_p(q)} \right]. \tag{2}$$

Since $4 \sin^2 \Theta_W - 1$ is small and $F_p(q)$ is known, we see that $A_{pv}$ provides a practical method to measure the neutron form factor $F_n(q)$ and hence the neutron radius. For these reasons parity-violating electron scattering (PVES) has been suggested as a clean and powerful tool for measuring the spatial distribution of neutrons in nuclei.

2.2. Inclusive quasi-elastic electron scattering

The inclusive $(e, e')$ scattering corresponds to an integral over all available nuclear states and consequently it is more directly related to the dynamics of the initial nuclear ground state.

The inclusive differential cross section for the quasi-elastic (QE) $(e, e')$ scattering on a nucleus is obtained from the contraction between the lepton and hadron tensors as $[5]$

$$\left( \frac{d\sigma}{d\varepsilon' d\Omega'} \right)_{QE} = \sigma_M [v_L R_L + v_T R_T], \tag{3}$$
where $\varepsilon'$ is the energy of the scattered electron and the coefficients $v$ come from the components of the lepton tensor that, under the assumption of the plane-wave approximation for the electron wave functions, depend only on the lepton kinematics [5]. All relevant nuclear structure information is contained in the longitudinal and transverse response functions $R_L$ and $R_T$. The response functions can be expressed in terms of suitable linear combinations of the components of the hadron tensor, which are given by products of the matrix elements of the nuclear current between initial and final nuclear states.

In the QE region the nuclear response is dominated by one-nucleon processes where the scattering occurs with only one nucleon, which is subsequently emitted from the nucleus by a direct knockout mechanism, and the remaining nucleons behave as spectators. Therefore, QE electron scattering can adequately be described in the relativistic impulse approximation (RIA) by the sum of incoherent processes involving only one nucleon scattering and the components of the hadron tensor are obtained from the sum, over all the s.p. shell-model states, of the squared absolute value of the transition matrix elements of the single-nucleon current [5].

A reliable description of final-state interactions (FSI) between the emitted nucleon and the residual nucleus is a crucial ingredient for the description of electron scattering data. The use of a complex optical potential (OP) in the distorted-wave impulse approximation (DWIA) [4, 5, 15, 16, 17, 18, 19, 20] provides a good description of exclusive $(e, e'p)$ data. In the case of exclusive scattering only the final lepton is detected, the final nuclear state is not determined and all elastic and inelastic channels contribute. This requires a different treatment of FSI where all final-state channels are retained and the flux, although redistributed among all possible channels, must be conserved. Different approaches have been considered to describe FSI in $(e, e')$ scattering. In the relativistic plane wave impulse approximation (RPWIA), FSI are simply neglected. In other approaches, FSI are incorporated using purely real OP.

In a different description of FSI, the relativistic Green’s function (RGF) model is adopted [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]: taking advantage of suitable approximation based on the IA, the explicit calculation of the single particle Green’s function is avoided and the components of the hadron tensor [22, 23] are expressed in terms of matrix elements of the same type as the RDWIA ones of the exclusive $(e, e'p)$ but which involve eigenfunctions of both the OP and its Hermitian conjugate, where the imaginary part has an opposite sign and gives in one case a loss and in the other case a gain of strength. The RGF formalism makes it possible to reconstruct the flux lost into nonelastic channels in the case of the inclusive response starting from the complex OP which describes elastic nucleon-nucleus scattering data and, moreover, it provides a consistent treatment of FSI in the exclusive and in the inclusive scattering.

2.3. Relativistic model for ground-state observables
In the standard representation of relativistic mean field approaches the nucleus is described as a system of Dirac nucleons coupled to the exchange mesons and the electromagnetic field through an effective Lagrangian. The isoscalar scalar-meson ($\sigma$), the isoscalar vector-meson ($\omega$), and the isovector vector-meson ($\rho$) build the minimal set of meson fields that, together with the electromagnetic one ($\gamma$), is necessary for a quantitative description of bulk and s.p. nuclear properties. The model is defined by the Lagrangian density

$$\mathcal{L} = \mathcal{L}_N + \mathcal{L}_m + \mathcal{L}_{\text{int}},$$

where $\mathcal{L}_N$ denotes the Lagrangian of the free nucleon, $\mathcal{L}_m$ is the Lagrangian of the free meson fields and the simplest set of interaction terms is contained in $\mathcal{L}_{\text{int}}$. Bare values are used for the masses of the $\omega$ and $\rho$ mesons: $m_\omega = 783$ MeV and $m_\rho = 763$ MeV. The DD-ME2 parametrization is determined by eight independent parameters, adjusted to the properties of symmetric and asymmetric nuclear matter, binding energies, charge radii, and neutron radii of spherical nuclei [32]. The interaction has been tested in the calculation of ground state
properties of a large set of spherical and deformed nuclei. When used in the relativistic random-phase approximation, DD-ME2 reproduces with high accuracy data on isoscalar and isovector collective excitations. For open-shell nuclei we employed a schematic ansatz: the constant gap approximation with empirical $\Delta$ given by the 5-point formula \[ \Delta^{(5)}(N_0) = \frac{1}{8} \left[ E(N_0 + 2) - 4E(N_0 + 1) + 6E(N_0) - 4E(N_0 - 1) + E(N_0 - 2) \right]. \] (5)

3. Results

In this section we present and discuss numerical predictions for elastic and QE electron scattering which can hopefully be useful for future measurements in experimental radioactive ion beam (RIB) facilities. We study the evolution of some electron scattering observables in the isotopic and isotonic chains of medium systems, which are exemplified by the cases of calcium isotopes and $N = 20$ isotons. Many of these nuclei lie in the region of the nuclear chart that is likely to be explored in future electron-scattering experiments. For each nucleus in an isotopic and isotonic chain, we compute and compare the associated elastic and QE cross sections and parity-violating asymmetry in order to obtain information on the effects of isospin asymmetry on nuclear structure.

3.1. Elastic electron scattering

The cross sections for elastic electron scattering have been calculated in the DWBA, where the distortion produced by the nuclear Coulomb field on the electron wave functions is explicitly included. We have checked that our calculations are in fair agreement with experimental data of elastic electron scattering cross sections of medium and heavy nuclei [29, 34].

Figure 1. (Color online) Panel (a): Differential cross section for elastic electron scattering on $^{36-56}\text{Ca}$ at $\varepsilon = 496.8$ MeV as a function of the scattering angle $\theta$. Panel (b): Differential cross section for elastic electron scattering on the $N = 20$ isotonic chain at $\varepsilon = 850.0$ MeV as a function of the scattering angle $\theta$. 

The differential cross sections for elastic electron scattering on various calcium isotopes ($^{36-56}\text{Ca}$) at an electron energy $\varepsilon = 496.8.5$ MeV are shown in panel (a) of figure 1. With increasing neutron number the positions of the diffraction minima shift towards smaller scattering angles, i.e., towards smaller values of the momentum transfer. The shift of the minima towards smaller $q$ is in general accompanied by a simultaneous increase in the height of
the maxima. In panel (b) of figure 1 we present the same differential cross sections at $\varepsilon = 850$ MeV along $N = 20$ isotones. With increasing proton number, the positions of the diffraction minima shift towards smaller scattering angles and, correspondingly, there is an enhancement in the height of the maxima of the cross sections. Some shell effects can be seen owing to the filling of the single-particle states.

3.2. Quasi-elastic electron scattering

In the calculations of the matrix elements for QE electron scattering, the s.p. bound nucleon states are obtained from the relativistic mean-field model with density-dependent meson-nucleon vertices and the DD-ME2 parametrization as described in section 2.3.

The RPWIA cross sections for the inclusive QE $(e, e')$ reaction on $^{36-56}$Ca isotopes at $\varepsilon = 560$ MeV and $\theta = 60^\circ$ are shown in panel (a) of figure 2. The results for $N = 20$ isotones at $\varepsilon = 1080$ MeV and $\theta = 32^\circ$ are shown in panel (b) of figure 2. Owing to the fact that FSI are neglected, the differences between the results for the various isotopes and isotones are entirely due to the differences in the s.p. bound state wave functions of each nucleus. While only the charge proton density distribution contributes to the cross section of elastic electron scattering, the cross section of QE electron scattering is obtained from the sum of all the integrated exclusive one-nucleon knockout processes, due to the interaction of the probe with all the individual nucleons, protons and neutrons, of the nucleus and contains information on the dynamics of the initial nuclear ground state.

The main role is played by protons, which give most of the contribution. Increasing the neutron number along the isotopic chain there is a proportional increase of the neutron contribution, but no significant increase of the proton one. Thus the enhancement of the cross sections in panel (a) of figure 2 can be mainly ascribed to the neutrons. In panel (b) of figure 2 we show the QE $(e, e')$ cross sections calculated in the RPWIA for isotones of the $N = 20$ chain at $\varepsilon = 1080$ MeV and $\theta = 32^\circ$. Increasing the proton number along the $N = 20$ chain, in panel (b) of figure 2, there is a proportional increase of the cross sections, owing to the enhancement of the proton contribution.

In general, interesting and peculiar effects are obtained in the evolution of QE inclusive cross section, but it is not easy to relate them to changes in the matter distribution, which can be
3.3. Parity-violating asymmetry

The calculation starts with the self-consistent relativistic ground state proton and neutron densities. The charge and weak densities are calculated by folding the point proton and neutron densities. The resulting Coulomb potential $V(r)$ and weak potential $A(r)$ are used to construct $U(r)$ potential as it follows: $U(r) = V(r) ± A(r)$. The cross sections for elastic electron scattering are obtained from the numerical solution of the Dirac equation for electron scattering in the $U(r)$ potential and includes Coulomb distortion effects [12, 35, 36, 37]. The cross sections for positive and negative helicity electron states are calculated and the resulting asymmetry parameter $A_{pv}$ is plotted as a function of the scattering angle.

In panel (a) of figure 3 we plot the parity-violating asymmetry parameters $A_{pv}$ for $^{36-56}$Ca nuclei for elastic electron scattering at $\varepsilon = 850$ MeV. At $\varepsilon = 850$ MeV the values of $A_{pv}$ are of the order of $10^{-5}$, with lower values for smaller angles and larger values for larger angles.

As suggested in Ref. [10] the asymmetry parameter $A_{pv}$ provides a direct measurement of the Fourier transform of the neutron density. This relation has been tested and confirmed in Ref. [13] comparing asymmetries and the squares of the Fourier transforms of the neutron densities.

In addition to predictions about calcium isotopic chains we also provide calculations for recent measurements. In panels (b) and (c) of figure 3 we show our theoretical predictions for the empirical values extracted from the first run of the PREX experiment on $^{208}$Pb at $\varepsilon = 1.06$ GeV. In Ref. [38] the weak charge density $(-\rho_W)$ has been deduced from the weak charge form factor. The error band (shaded area) represents the incoherent sum of experimental and model

significant, particularly in the center of the nucleus.

Significant distortion effects are produced by FSI in the RGF model, but the general trend and magnitude of the cross sections, and their evolution with respect to the neutron number in the considered isotopic chain or the proton number in the considered isotonic chain are generally similar in RPWIA and RGF [29, 34].
errors. Our prediction, plotted by the red line in the left panel, is in rather good agreement with empirical data. In fact, if we evaluate the corresponding asymmetry parameter $A_{pv}$ averaged over the acceptance function $\epsilon(\theta)$ [40]

$$
\langle A_{pv} \rangle = \frac{\int d\theta \sin \theta A_{pv}(\theta) \frac{d\sigma}{d\Omega} \epsilon(\theta)}{\int d\theta \sin \theta \frac{d\sigma}{d\Omega} \epsilon(\theta)}
$$

we find 0.712 ppm, in very good agreement with the empirical estimate $0.656 \pm 0.060(\text{stat}) \pm 0.014(\text{syst})$ ppm.

4. Conclusions

We have presented and discussed numerical predictions for the cross section and the parity-violating asymmetry in elastic and quasi-elastic electron scattering along $Z = 20$ isotopic and $N = 20$ isotonic chains with the aim to investigate their evolution with increasing neutron and proton number.

The understanding of the properties of exotic nuclei is one of the major topics of interest in modern nuclear physics. Large efforts in this direction have been done over last years and are planned for the future. Electron probes are powerful tools to achieve this goal because their interaction is well known and relatively weak with respect to the hadron force and can therefore more adequately explore the details of inner nuclear structures.

Our theoretical predictions will be useful for clarifying the different aspects of the electron scattering experiments off exotic nuclei that have been proposed in the ELISe experiment at FAIR and in the SCRIT project at RIKEN.

In this work, both elastic and inclusive quasi-elastic electron scattering have been considered. The elastic scattering can give information on the global properties of nuclei and, in particular, on the different behavior of proton and neutron density distributions. The inclusive QE scattering is affected by the dynamical properties, being the integral of the spectral density function over all the available final states, and, due to the reaction mechanism, preferably exploits the single particle aspects of the nucleus. In addition, when combined with the exclusive $(e, e'p)$ scattering, it is able to explore the evolution of the single particle model with increasing asymmetry between the number of neutrons and protons.

Our results show an evolution of the calculated cross sections without discontinuities as the asymmetry between protons and neutrons changes. The increase of the neutron number along $Z = 20$ chain enhances the nuclear and proton densities and flattens the charge densities. The increase of the proton number along each chain essentially produces an enhancement and an extension of the proton densities. The densities of the proton-rich isotones are significantly extended toward larger $r$ with respect to those of the proton-deficient ones. An interesting way to relate the differential cross section and parity-violating asymmetry parameter for elastic scattering to the matter distribution is to consider the evolution of their first minima positions along each isotopic or isotonic chain.

The parity-violating asymmetry parameter has been also calculated to investigate the neutron skin. We have compared our calculations with the results of the first measurement of the asymmetry parameter achieved by the PREX Collaboration on $^{208}$Pb and have obtained a good agreement with the empirical value.

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