Instability of de-Sitter black hole with scalar field coupled to Gauss-Bonnet invariant

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Abstract

The black hole scalarization in a special Einstein-scalar-Gauss-Bonnet (EsGB) gravity has been widely investigated in recent years. Especially, the spontaneous scalarization of scalar-free black hole in de-Sitter (dS) spacetime possesses interesting features due to the existence of cosmological horizon. In this work, we focus on the massive scalar field perturbation on Schwarzschild dS (SdS) black hole in a special EsGB theory. We study the (in)stability of SdS in frequency domain and verify the results in time domain. Then we figure out the unstable/stable regions in ($Λ, α$)-plane as well as in ($m, α$)-plane for various perturbation modes, where $Λ$, $α$ and $m$ denote the cosmological constant, the GB coupling strength and the mass of scalar field, respectively. Our study could be a good preparation for one to further understand the black hole scalarization in dS spacetime.

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I. INTRODUCTION

It is widely accepted that the effects of higher-order curvature terms are significant as we are exploring the strong gravity regime via detections of gravitational waves and black hole shadows. In theoretical framework, the inclusion of such terms usually involves the well-known ghost problem [1]. A counterexample which can be ghost-free is including the Gauss-Bonnet (GB) correction, however, it becomes a topological term in four-dimensional spacetime and has no dynamics when minimally coupled with Einstein-Hilbert action. One way to make this term contribute to the dynamic in four-dimensional spacetime is to introduce a coupling between the GB term and scalar field [2]. The theory which includes this kind of coupling is dubbed Einstein-scalar-Gauss-Bonnet (EsGB) gravity, which has attracted plenty of attention as it admits hairy black holes. Various black hole solutions and compact objects in four-dimensional EsGB theories were studied in the literatures [3–8] and therein. More recently, the spontaneous scalarization of scalar-free black hole with particular coupling functions in EsGB theory was proposed. It was addressed that below a certain mass the Schwarzschild black hole background may become unstable in regions of strong curvature, and then a scalarized hairy black hole emerges when the scalar field backreacts to the geometric. The precess evades the well-known no-hair theorems [9–11]. This proposal on spontaneous scalarization has inspired wide generalizations in the literatures [12–35].

For various reasons, physicists have great interest in de Sitter (dS) spacetime. On one hand, the theoretical model of our expansion universe consists on assuming positive cosmological constant, implying that the physical universe is asymptotically dS. Besides, dS spacetime plays an important role in primordial inflation theory, which is now part of the standard cosmological model [36, 37]. On the other hand, the holographic duality between quantum gravity in dS spacetime and a conformal field theory on it boundary sheds remarkable application on the asymptotically dS spacetime [38, 39].
Due to the above consideration, the spontaneous scalarization in EsGB theory has been soon extensively studied in dS spacetime \cite{40, 41}. It was found that the positive cosmological constant does not change the local conditions for a tachyonic instability of the background black hole, Schwarzschild dS (SdS) in this case, to emerge. In details, in \cite{40}, the authors claimed that a regular black hole horizon with a non-trivial hair may be always formed after an analysis in the near-horizon asymptotic regime. But the complete hairy solution was absent, the deep reason of which is not clear. Later in \cite{41}, it was addressed if the scalar field is confined between the black hole and cosmological horizons, then it is not likely to form scalarised black hole solution; while a new hairy black hole was numerically constructed if the scalar field is permitted to extend beyond the cosmological horizon. Thus, it is obvious that the existence of cosmological horizon introduces particular situation in the black hole scalarization in dS spacetime, which deserves further study.

It is noted that though the analysis of (in)stability on the background black hole under the massless scalar field perturbation was present in \cite{40, 41}, however, the quasi-normal mode (QNM) frequencies and the dynamical evolution of the scalar field in the SdS background is still missing. This paper intends to fill this gap. We shall consider a massive scalar field as a probe field on the SdS black hole and admit the scalar field can extend beyond the cosmological horizon. By computing its QNM frequencies and the linear time domain dynamical evolution, we fix the unstable/stable parameter regions in \((\Lambda, \alpha)\)-plane and \((m, \alpha)\)-plane, respectively for various angular momentum modes. Those unstable regions could constraint the possible parameters for which the hairy black hole form, which is helpful to further construct the scalaried hairy dS solutions in the special EsGB theory.

The remaining of this paper is organized as follows. In section II, we briefly present the EsGB model with a positive cosmological constant, and then show the covariant equation for a probed massive scalar field. Then in section III, we show the preliminary instruction on the study of QNM frequencies and linear time domain, respectively. We analyze the (in)stability of the SdS black hole according to the numerical results of QNM frequencies and the time domain in section IV. The last section contributes to our conclusion and discussion. We shall work with the units \(\hbar = G = c = 1\).

### II. MODEL AND SCALAR EQUATION OF MOTION

The action of EsGB gravity in dS spacetime with the scalar field coupled with the GB invariant is given by

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R - 2\Lambda - 2\nabla_\mu \phi \nabla^\mu \phi - m^2 \phi^2 - f(\phi) \mathcal{L}_{GB} \right],
\]

where \(R\) is the Ricci scalar, \(\Lambda\) is a positive cosmological constant, \(\phi\) is scalar field with mass \(m\), \(f(\phi)\) is the coupling function, and

\[
\mathcal{L}_{GB} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}.
\]

Noted that different forms of \(f(\phi)\) shall give different properties of the EsGB theory. As mentioned in \cite{11}, to admit Schwarzschild black hole as background solution, \(f(\phi)\) could satisfy the conditions \(\frac{df(\phi)}{d\phi} |_{\phi=0} = 0\) and \(\frac{d^2f(\phi)}{d\phi^2} |_{\phi=0} = b^2 > 0\), where \(b\) is a constant. Moreover,
one usually assumes that the scalar field vanishes at infinity and normalizes the constant $b$ to be unity. Thus, to fulfill the requirement, we shall follow [41] and choose the simplest form of the coupling function as

$$f(\phi) = a_0 - \alpha \phi^2,$$

where $a_0$ and $\alpha$ are the coupling parameters.

With $\phi = 0$, the above action admits the Schwarzschild de-Sitter (SdS) black hole solution

$$ds^2 = -g(r)d^2t + \frac{d^2r}{g(r)} + r^2\left(\sin^2 \theta d^2\theta + d^2\phi\right)$$

where

$$g(r) = 1 - \frac{2M}{r} - \frac{\Lambda r^2}{3}$$

with the constant $M$ is the black hole mass. In this background, the GB term is evaluated as

$$\mathcal{L}_{\text{GB}} = \frac{48M^2}{r^6} + \frac{8\Lambda^2}{3}.$$  \tag{6}

Depending on the model parameters, $g(r) = 0$ could have two positive real roots, $(r_e, r_c)$, and one negative real root, $r_o$. The positive roots, $r_e$ and $r_c$, represent the black hole event horizon and cosmological horizon, respectively, of the SdS black hole. Note that these two horizons only hold when $3M\sqrt{\Lambda} < 1$, known as the Nariai limit, within which their analytic formulas are

$$r_e = \frac{2}{\sqrt{\Lambda}} \cos\left(\frac{1}{3} \cos^{-1}\left(3M\sqrt{\Lambda}\right) + \frac{\pi}{3}\right) , \quad r_c = \frac{2}{\sqrt{\Lambda}} \cos\left(\frac{1}{3} \cos^{-1}\left(3M\sqrt{\Lambda}\right) - \frac{\pi}{3}\right).$$  \tag{7}

Approaching the Nariai limit, the event horizon and the cosmological horizon tend to merge into one horizon; and beyond the limit, no black hole horizon exists.

Considering $r_c > r_e > r_o$, the metric function $g(r)$ can be further expressed as,

$$g(r) = \frac{\Lambda}{3r} (r - r_e)(r_c - r)(r - r_o).$$  \tag{8}

Introducing the surface gravity $\kappa_i = \frac{1}{2} |g'(r)|_{r = r_i}$ associated with each root $r = r_i \ (i = e, c, o)$, one could calculate the tortoise coordinate $r_\ast = \int g^{-1}(r) dr$ in an analytic form

$$r_\ast(r) = \frac{1}{2\kappa_e} \ln \left(\frac{r}{r_e} - 1\right) - \frac{1}{2\kappa_c} \ln \left(1 - \frac{r}{r_c}\right) + \frac{1}{2\kappa_o} \ln \left(\frac{r}{r_o} - 1\right)$$

which will play an important role in our later numerical study.

Then we consider a small scalar field perturbation on the background of SdS black hole in the linear regime, which is governed by the covariant equation

$$\Box \phi = \frac{\partial f(\phi)}{\partial \phi} \mathcal{L}_{\text{GB}} + \frac{m^2}{2} \phi \equiv \mu_{\text{eff}}^2 \phi.$$  \tag{10}

Here the box denotes the d’Alembertian operator and the effective mass is

$$\mu_{\text{eff}}^2 = \frac{m^2}{2} - \alpha \left(\frac{8\Lambda^2}{3} + \frac{48M^2}{r^6}\right).$$  \tag{11}
The tachyonic instability may occur only when $\mu_{\text{eff}}^2 < 0$, which requires $\alpha > 0$. As addressed in [41], this instability may trigger the emergence of a hairy solution on the SdS background via the spontaneous scalarization process, though the authors only considered the massless $r$–dependent scalar field. The possible parameter regions which give unstable situation and how the scalar field grows up were not present. In our following study, we shall answer those questions by studying the dynamical perturbation of scalar field with various modes in both frequency domain and time domain.

III. PRELIMINARY PREPARATION

A. Frequency domain analysis

To analyze the (in)stability in frequency domain, one usually decompose the scalar field as

$$\phi(t, r, \theta, \psi) = \sum_{l, m} \frac{R_l m(r)}{r} Y_{l m}(\theta, \psi) e^{-i\omega t} \quad (12)$$

where $Y_{l m}(\theta, \psi)$ is the spherical harmonic function with angular momentum $l$ and azimuthal number $m$. Working under the tortoise coordinate $r_*$, we obtain that each wave function $R$ satisfies

$$\frac{\partial^2 R(r)}{\partial r_*^2} + (\omega^2 - V_{\text{eff}}(r)) R(r) = 0, \quad (13)$$

where the effective potential is

$$V_{\text{eff}}(r) = \frac{g(r)}{(l(l+1)} \frac{r^2}{r^2} + \frac{g'(r)}{r} + \mu_{\text{eff}}^2, \quad (14)$$

and it is not dependent of the azimuthal number $m$ because of the spherical symmetry. This potential could present a negative well between $r_e$ and $r_c$, which plays an important role in the instability of the SdS black hole under this perturbation as we will see soon. The asymptotic behavior of the perturbation near the horizons are

$$R(r \to r_e) \sim e^{-i\omega r_*} \quad \text{and} \quad R(r \to r_c) \sim e^{i\omega r_*} \quad (15)$$

which correspond to ingoing and outgoing boundary condition near the event horizon and cosmological horizon, respectively. It is known that only discrete eigenfrequencies $\omega = \omega_R + i\omega_I$ (QNM frequency), where $\omega_R$ and $\omega_I$ respectively denote the real part and imaginary part of the QNM frequency, satisfy the perturbation equation and the boundary conditions. Once $\omega_I > 0$, the amplitude of the perturbation will grow up, implying that the black hole is unstable under this perturbation. There are many methods developed to compute the QNM frequencies, for instance, WKB method, shooting method, Horowitz-Hubeny method, AIM method, spectral method, etc., and readers could refer to [42, 43] and therein for nice reviews. In this work, we will employ the spectral method, which has been well described in [44]. Moreover, we shall also testify our results with the time domain evolution.
B. Time domain analysis

To study the dynamical evolution of the perturbed scalar field, we decompose the scalar field as
\[
\phi(t, r, \theta, \psi) = \sum_{lm} \frac{R_{lm}(t, r)}{r} Y_{lm}(\theta, \psi). \tag{16}
\]
In the tortoise coordinate, the perturbation equation reduces to
\[
\left( -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial r^2} - V_{eff}(r) \right) R(t, r) = 0 \tag{17}
\]
where \( V_{eff}(r) \) has been defined in (14).

We have to solve the wave equation numerically since there is no analytic form of this time-dependent wave equation. We adopt the discretization method proposed in [45], and discretize the wave equation (17) by defining
\[
R(r^*, t) = R(j \Delta r^*, i \Delta t) = R_{j,i}, V(r) = V(j \Delta r^*) = V_j. \tag{18}
\]
Then it can be deformed as
\[
-\left( \frac{R_{j+1,i} - 2R_{j,i} + R_{j-1,i}}{\Delta t^2} \right) + \left( \frac{R_{j+1,i} - 2R_{j,i} + R_{j-1,i}}{\Delta r^2} \right) - V_j R_{j,i} + O(\Delta t^2) + O(\Delta r^2) = 0. \tag{19}
\]

With the initial Gaussian distribution \( R(r^*, t = 0) = \exp\left[-\frac{(r^*-a)^2}{2b^2}\right] \) and \( R(r^*, t < 0) = 0 \), we can derive the evolution of \( R \) by
\[
R_{j,i+1} = -R_{j,i-1} + \frac{\Delta t^2}{\Delta r^2} R_{j+1,i} + R_{j-1,i} + \left( 2 - 2 \frac{\Delta t^2}{\Delta r^2} - \Delta t^2 V_j \right) R_{j,i}. \tag{19}
\]

For the sake of the numerical precision, we shall fix \( \frac{\Delta t^2}{\Delta r^2} = 0.5 \) to fit the von Neumann stability conditions, and set \( a = 10, b = 3 \) in the profile of Gaussian wave.

IV. THE QNM FREQUENCIES AND INSTABILITY

We shall study how the coupling parameter and the mass of scalar field affect the (in)stability of SdS black hole. We will also consider different cosmological constants and angular momentums of the perturbation modes. To this end, we fix \( M = 1 \) without loss of generality.

A. \( \ell \)-dependence

In this subsection, we compute the fundamental QNM frequency of different \( \ell \)-modes perturbation around the SdS black hole. We focus on the effect of \( \ell \) on the GB coupling dependent QNM frequencies by fixing \( m = 0 \) and \( \Lambda = 0.1 \).

The results are shown in figure 1. For the \( \ell = 0 \) mode, both \( \omega_R \) and \( \omega_I \) are zero in minimal coupling case implying that the mode is prone to instability. As the coupling parameter, \( \alpha \), increases, the black lines shows that \( \omega_I \) becomes positive while \( \omega_R \) is still zero. This phenomena means that the \( \ell = 0 \) mode is always unstable. For the \( \ell \neq 0 \) modes, the left
plot shows that as \( \alpha \) increases, \( \omega_I \) first decreases and then increases. There exists a critical value, \( \alpha_c \), at which \( \omega_I = 0 \) for each mode. When \( \alpha > \alpha_c \), \( \omega_I \) becomes positive implying that the corresponding perturbation could grow up to destabilize the background SdS black hole. \( \alpha_c \) is larger for modes with larger \( \ell \) and the samples are listed in table I, which implies that under larger \( \ell \) mode perturbation, stronger GB coupling is called for to trigger the instability.

Moreover, comparing the two plots in figure 1, we see that the cases with decreasing \( \omega_I \) have non-vanishing \( \omega_R \), while the cases with increasing \( \omega_I \) possess purely imaginal QNM frequency. As addressed in [46], the former cases are dubbed photon sphere modes while the latter are dS modes. In our study, since the critical \( \alpha \) from photon sphere mode to dS mode is always smaller than \( \alpha_c \), so the unstable perturbation modes are all dS modes, which is an interesting phenomena deserving further study.

![FIG. 1: The QNM frequency as a function of the GB coupling for different \( \ell \) modes.](image)

| \( \ell \) | 0  | 1  | 2  | 3  | 4  | 5  | 6  |
|---------|----|----|----|----|----|----|----|
| \( \alpha_c \) | 4.63 | 12.75 | 23.78 | 37.84 | 54.93 | 75.09 |

**TABLE I:** The critical coupling for different momentum angular.

We also directly calculate the time evolution of the perturbation field and further reveal the instability of SdS in EsGB gravity. The pedagogy on evolutionary analysis has been shown in subsection III B. The evolutions of the perturbation with different \( \ell \) in log plot are shown in figure 2. For \( \ell = 0 \) mode, non-vanishing GB coupling makes the perturbation grow as time, meaning that the system is unstable. For \( \ell \neq 0 \), when \( \alpha < \alpha_c \), the perturbation will decay as the time evolves, but it grows up when \( \alpha > \alpha_c \). The phenomenon indicates that the system will become unstable under the perturbed modes once the GB coupling is larger than the corresponding \( \alpha_c \). These findings in time domain are consistent with those in frequency domain analysis. The effect of the GB coupling on the dynamical evolution is different from that of the non-minimally coupled to curvature studied in [47] which always makes the evolution decay in SdS black hole.
FIG. 2: The time evolution of the perturbation modes with $\ell = 0$ (upper-left), $\ell = 1$ (upper-right), $\ell = 2$ (lower-left) and $\ell = 3$ (lower-right).

The effect of the GB coupling on the (in)stability of the scalar perturbation could be explained by analyzing the effective potential (14). It is straightforward to obtain that the effective potential with a fixed radius near the event horizon could be suppressed by larger $\alpha$ but enhanced by larger $\ell$. Then in figure 3, we explicitly show the profile of the potential between the event horizon and cosmological horizon. It is obvious that in each case, when $\alpha$ is smaller than a certain value, the effective potential is always positive. As $\alpha$ increases, a negative potential well would form, and becomes more deeper as $\alpha$ is further enlarged. It is noted that comparing $\alpha_c$ in the frequency as well as time domain analysis, the critical value of $\alpha$ for the emergence of negative potential well in figure 3 is smaller in all cases. This is because the negative potential well is not a sufficient condition for the instability. Only deep enough potential well could help the scalar to collect near the event horizon and finally trigger the instability of system.

FIG. 3: The profile of the effective potential between the event horizon and cosmological horizon for $\ell = 0, 1, 2$ from left to right.
B. $\Lambda$–dependence

We then study the effect of the cosmological constant $\Lambda$ on the fundamental QNM frequency. The results are shown in figure 4, which shows that the effect of $\Lambda$ on various $\ell$-modes is similar. Specifically, for small $\alpha$, $\omega_I$ is negative and larger for larger $\Lambda$. It means that the perturbation in the spacetime with larger $\Lambda$ can live longer and then decays, since the lifetime is connected with the QNM frequency via $\tau \sim 1/|\omega_I|$. As $\alpha$ increases, $\omega_I$ increases and there exists an intersection for different $\Lambda$. As $\Lambda$ increases, the critical value $\alpha_c$ at which $\omega_I$ transits from negative to positive increases. Then for large enough $\alpha$, $\omega_I$ becomes positive and is smaller for larger $\Lambda$. This indicates that in SdS black hole with larger $\Lambda$, the perturbation has longer relax time and then grows up. The above picture can also explicitly reflected in the time domain analysis, see figure 5 for the $\ell = 2$ mode with $\alpha = 10$ as an example.

FIG. 4: The QNM frequency as a function of GB coupling for different cosmological constant. The modes from left to right are $\ell = 1$, 2 and 3, respectively.

FIG. 5: The time evolution of perturbation mode with $\ell = 2$. Here the GB coupling parameters is fixed as $\alpha = 10$.

It is worthwhile to point out that for $\alpha = 0$, i.e., in Einstein gravity, our QNM frequencies with different $\ell$ and $\Lambda$ calculated via spectral method match well with the results computed by WKB method and Prony method in [48, 49]. Then in EsGB theory with $\alpha \neq 0$, by scanning the cosmological constant inside the Nariai limit, we extract $\alpha_c$ and figure out the unstable/stable region in $\Lambda - \alpha$ plane under massless scalar perturbation. The results for samples of $\ell$ are shown in figure 6. Under the perturbation mode with each $\ell$, the instability can be triggered in the region above each line (the shading region), below which the system
is stable. It is obvious that for the modes with larger $\ell$, the system could be stable in a wider parameters region in $\Lambda - \alpha$ plane.

![Graph showing unstable/stable region of SdS black hole under massless scalar perturbation with $\ell = 1, 2$ and 3.]

**FIG. 6:** The unstable/stable region of SdS black hole under massless scalar perturbation with $\ell = 1, 2$ and 3.

### C. $m$-dependence

In this subsection, with fixed $\Lambda = 0.1$, we shall turn on the mass of the scalar field and study its effect. The fundamental QNM frequencies for $\ell = 1$ mode with different $m$ is shown in figure 7. It shows that the rule is similar to that for massless case, namely, as the GB coupling increases, $\omega_I$ first decreases and then increases. And for larger $m$, the turning value of $\alpha$ from decreasing to increasing is larger, so is $\alpha_c$ at which $\omega_I$ crosses the horizontal axis. This indicates that for the scalar field with larger mass, stronger GB coupling is required to destabilize the SdS black hole.

![Graph showing fundamental QNM frequency for $\ell = 1$ mode as a function of GB coupling for different mass of scalar perturbation.]

**FIG. 7:** The fundamental QNM frequency for $\ell = 1$ mode as a function of GB coupling for different mass of scalar perturbation.

This conclusion is also testified by the dynamical evolution of the massive scalar field in figure 8. Each plot shows that when $\alpha$ is smaller than a certain $\alpha_c$, the scalar field would decay as the time evolves; while the scalar field will grow up when $\alpha$ is larger than $\alpha_c$. The growth of scalar field could finally destabilize the SdS black hole and trigger spontaneous scalarization. Comparing the value of $\alpha_c$ in each plot, it is obvious larger $m$ corresponds to
larger $\alpha_c$ which matches the findings in figure 7. More explicit effect of $m$ on the dynamical evolution with fixed $\alpha$ is shown in figure 9. This mass dependence is reasonable because even in Einstein gravity, increasing $m$ would enhance $\omega_I$ which is always negative and decrease the scalar field damping rate, such that the corresponding modes live longer [50].

FIG. 8: The dynamical evolution of massive scalar field in log plot. The upper-left, upper-right, lower-left and lower-right plots correspond to $m = 0.2, 0.3, 0.4$ and 0.5 respectively.

FIG. 9: The time evolution of the $\ell = 1$ mode with fixed $\alpha = 5.49$.

For the modes with $\ell > 1$, similar phenomena could be seen as for $\ell = 1$ mode. So instead of repeating the analysis, we collect $\alpha_c$ with different $m$ for $\ell = 1, 2$ and 3 modes, and then draw the unstable/stable region in $m - \alpha$ plane in figure 10. For each mode, the SdS black hole could be unstable when the parameters are in the regime above each line.
FIG. 10: The unstable/stable region of SdS black hole under massive scalar perturbation with \( \ell = 1, 2 \) and 3.

V. CONCLUSION AND DISCUSSION

In this paper, we studied the dynamics of the massive scalar field perturbation on Schwarzschild de-Sitter black hole in a special EsGB theory. In both frequency and time domains, we analyzed the (in)stability of scalar-free dS black hole. To make sure the precision, we first repeat the results in Einstein theory without the GB coupling. In frequency domain, for various \( \ell \) perturbation modes, there exists a critical GB coupling \( \alpha_c \) at which the imaginary part of QNM frequency, \( \omega_I \), is zero. When \( \alpha < \alpha_c \), \( \omega_I \) is negative indicating that the system is stable; while when \( \alpha > \alpha_c \), \( \omega_I \) turns to be positive implying that the system would undergo a tachyonic instability. Our calculation showed that larger angular momentum, cosmological constant and the mass of scalar field correspond to larger \( \alpha_c \), which means that in those cases, the SdS black hole is more difficult to be destabilized. The physical reason is that larger \( \alpha \) always give more deeper negative potential well which triggers the tachyonic instability and finally destroy the SdS black hole, while larger \( \ell \), \( \Lambda \) and \( m \) provide positive values into the effective potential and enhance the negative potential well. In time domain, for the case with \( \alpha < \alpha_c \), the scalar field perturbation would finally decay as time evolves while it would grow up for the case with \( \alpha > \alpha_c \). The growth of the perturbation could trigger the scalar-free black hole unstable and a scalarized hairy solution may emerge.

Then, by scanning the model parameters, we figured out the unstable/stable region in the \((\Lambda, \alpha)\)-plane and also in the \((m, \alpha)\)-plane for various perturbation modes. It is noted that though the scalarization of SdS black hole with backreaction has been investigated in [40, 41], there are still many open issues as their authors addressed. Here we analyzed the (in)stability in probe limit, but our findings could be helpful to further understand the black hole scalarization in dS spacetime. For example, it is only possible to construct the scalarized hairy solution for the parameters in the shading region in figure 6 and figure 10.

As a simple way to constraint the model parameters of possible spontaneous scalarization, it is interesting to extended our study into many cases. A straightforward case is the Schwarzschild or AdS black holes in EsGB theory as well as other modified gravities mentioned in the introduction. The second case is the dynamic of massive scalar filed perturbed on Kerr black hole in EsGB. Though this direction has been investigated in [51], but the authors only considered the \( \ell = 1 \) mode of massless scalar field. Since besides the tachyonic instability, the superradiant instability may occur in rotating black hole, so considering
the mass of the scalar field and different modes will introduce more physics on the fate of scalar-free background. The next but not the last case is to consider the dynamic of charged black hole before it was scalarized. The scalarization of charged black hole has been firstly proposed in [52] and soon been generalized in [53–59], and a complete dynamical analysis on the scalar-free charged black hole still deserves to be present.

Acknowledgments

This work is partly supported by Natural Science Foundation of China under Grant No. 11775036, Fok Ying Tung Education Foundation under Grant No. 171006 and Natural Science Foundation of Jiangsu Province under Grant No.BK20211601. Guoyang Fu is supported by the Postgraduate Research & Practice Innovation Program of Jiangsu Province (KYCX20 2973). Jian-Pin Wu is supported by Top Talent Support Program from Yangzhou University.

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