Curvature Injected Adaptive Momentum Optimizer for Convolutional Neural Networks

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Abstract—In this paper, we propose a new approach, hereafter referred to as AdaInject, for the gradient descent optimizers by injecting the curvature information with adaptive momentum. Specifically, the curvature information is used as a weight to inject the second order moment in the update rule. The curvature information is captured through the short-term parameter history. The AdaInject approach boosts the parameter update by exploiting the curvature information. The proposed approach is generic in nature and can be integrated with any existing adaptive momentum stochastic gradient descent optimizers. The effectiveness of the AdaInject optimizer is tested using a theoretical analysis as well as through toy examples. We also show the convergence property of the proposed injection based optimizer. Further, we depict the efficacy of the AdaInject approach through extensive experiments in conjunction with the state-of-the-art optimizers, i.e., AdamInject, diffGradInject, RadamInject, and AdaBeliefInject on four benchmark datasets. Different CNN models are used in the experiments. A highest improvement in the top-1 classification error rate of 16.54% is observed using diffGradInject optimizer with ResNeXt29 model over the CIFAR10 dataset. Overall, we observe very promising performance improvement of existing optimizers with the proposed AdaInject approach.

I. INTRODUCTION

Deep learning has shown a great impact over the performance of the neural networks for a wide range of problems [1]. In recent past, convolutional neural networks (CNNs) have shown very promising results for different computer vision applications, such as object recognition using AlexNet [2], VGG [3], ResNet [4], SENet [5], ResNeXt [6], DenseNet [7]; object detection using Fast RCNN [8], Faster RCNN [9], YOLO [10]; face recognition using FaceNet [11], DeepFace [12], and many more. CNNs have also been used as basic building blocks in other networks like Autoencoder [13], [14], Siamese Network [15], [16], Generative Adversarial Networks [17], [18], etc.

The training of different types of deep neural networks (DNNs) is mainly performed with the help of stochastic gradient descent (SGD) based optimization [19]. SGD optimizer updates the parameters of the network based on the gradient of objective function w.r.t. the corresponding parameters [20]. The vanilla SGD optimization suffers from three problems, including 1) zero gradient in local minima and saddle regions leading to no update in the parameters, 2) a jittering effect along steep dimensions due to the inconsistent changes in the loss caused by the different parameters, and 3) noisy updates due to the gradient computed from the batch of data. SGD with momentum (SGDM) [21] considers the first order moment (i.e., velocity) as an exponential moving average (EMA) of gradient for each parameter while training progresses [22]. The parameter is updated in SGDM based on the EMA of gradient which resolves the problem of zero gradient.

Several SGD based optimization techniques have been proposed in the recent past [23], [24], [25], [26], [27], [28], [29], and etc. AdaGrad [23] controls the learning rate by dividing it with the root of the sum of the squares of the past gradients. However, it makes the learning rate very small after certain iterations and kills the parameter update. AdaDelta [24] resolves the diminishing learning rate issue of AdaGrad by considering only a few immediate past gradients. However, it is not able to exploit the global information. In another attempt to resolve the problem of AdaGrad, RMSProp [25] divides the learning rate by the root of the exponentially decaying average of squared gradients. In 2015, Kingma and Ba [26] investigated the adaptive momentum based Adam optimizer. Adam combines the ideas of SGDM and RMSprop and uses first order and second order moments. Adam computes the first order moment as the EMA of gradients and uses it to update the parameter. Adam also computes the second order moment as the EMA of the square of gradients and uses it to control the learning rate. Adam performs well in practice to train the convolutional neural networks (CNNs) [26]. However, it suffers from overshooting and oscillations near minimum and varying gradient variance due to batch wise computation. diffGrad [27] resolved the issues as posed by Adam by introducing a friction term in parameter update using the rate of change in gradients. Radam [28] resolved the variance issue as posed by Adam by rectifying the variance of gradients during parameter update. AdaBelief [29] uses the belief in gradients to compute the second order moment. The belief in gradients is computed as the difference between the gradient and the average of the gradient of the corresponding batch. Other recently proposed and notable gradient descent optimizers are Proportional Integral Derivative (PID) [30], Nesterov’s Momentum Adam (NADAM) [31], Nostalgic Adam (NosAdam) [32], YOGI [33], Adaptive Bound (AdaBound) [34], Adaptive and Momental Bound (AdaMod) [35], etc.
Algorithm 1: Adam Optimizer

**Initialize:** $\theta_0, m_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$

**Hyperparameters:** $\alpha, \beta_1, \beta_2$

**While** $\theta$ not converged

$t \leftarrow t + 1$

$g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$

$m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$

$v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$

**Bias Correction**

$m_t \leftarrow m_t / (1 - \beta_1^t), \hat{v}_t \leftarrow v_t / (1 - \beta_2^t)$

**Update**

$\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \hat{m}_t / (\sqrt{\hat{v}_t} + \epsilon)$

Algorithm 2: AdamInject (i.e., Adam + AdaInject) Optimizer

**Initialize:** $\theta_0, s_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$

**Hyperparameters:** $\alpha, \beta_1, \beta_2, k$

**While** $\theta$ not converged

$t \leftarrow t + 1$

$g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$

If $t = 1$

$s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot g_t$

Else

$\Delta \theta \leftarrow \theta_{t-2} - \theta_{t-1}$

$s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot (g_t + \Delta \theta \cdot g_t^2) / k$

$v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$

**Bias Correction**

$s_t \leftarrow s_t / (1 - \beta_1^t), \hat{v}_t \leftarrow v_t / (1 - \beta_2^t)$

**Update**

$\theta_t \leftarrow \theta_{t-1} - \alpha \hat{s}_t / (\sqrt{\hat{v}_t} + \epsilon)$

Aggregated Momentum (AggMo) [36], Lamb [37], Adam Projection (AdamP) [38], Gradient Centralization (GC) [39], AdaHessian [40], and AngularGrad [41].

The adaptive SGD optimization techniques have led to a promising performance on deep CNN models. The majority of the above mentioned adaptive gradient descent optimizers miss utilizing the curvature information to precisely control the parameter update. However, it is evident that the curvature in optimization plays an important role in gradient descent optimization [42]. We resolve the above mentioned issues by injecting the second order moment in first order for the parameter update. The injection is weighted by the curvature information in the form of the short-term parameter update history. The major contribution of this work is summarized as follows:

- We propose AdaInject for the adaptive optimizers by injecting the curvature weighted second order moment in EMA used for parameter update.
- We conduct theoretical analysis to depict the effectiveness of the proposed AdaInject in different curvature scenarios.
- We show the effect of the proposed approach using toy examples. The convergence analysis is also conducted using regret bound which shows the convergence property of the proposed approach.
- We validate the superiority of the proposed injection concept with the recent state-of-the-art optimizers, including Adam [26], diffGrad [27], Radam [28] and AdaBelief [29] using a wide range of CNN models for image classification over four benchmark datasets.
- The proposed concept is generic and can be easily integrated with any existing adaptive momentum based gradient descent optimizers.

The remaining paper is structured by presenting the proposed injection based optimizers in Section 2; theoretical and empirical analysis in Section 3; convergence analysis in Section 4; experimental analysis in Section 5; and concluding remarks in Section 6.

II. PROPOSED INJECTION BASED OPTIMIZERS

As per the conventions used in Adam [26], the aim of gradient descent optimization is to minimize the loss function $f(\theta) \in \mathbb{R}$ where $\theta \in \mathbb{R}^d$ is the parameter. The gradient ($g_t$) at $t^{th}$ step is computed as $g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$. Adam computes the first order moment ($m_t$) and second order moment ($v_t$) as the exponential moving average (EMA) of $g_t$ and $g_t^2$, respectively, which can be written as,

$$m_t = \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t \quad (1)$$

$$v_t = \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2 \quad (2)$$

where $\beta_1$ and $\beta_2$ are the smoothing hyperparameters, typically set as $\beta_1 = 0.9$ and $\beta_2 = 0.999$. The $g_t^2$ is computed as $g_t \cdot g_t$ in the Adam. A bias correction is performed as $\hat{m}_t = m_t / (1 - \beta_1^t), \hat{v}_t = v_t / (1 - \beta_2^t)$ to avoid very large step size in the initial iterations. The parameter update rule in Adam [26] is given as,

$$\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \hat{m}_t / (\sqrt{\hat{v}_t} + \epsilon) \quad (3)$$

where $\alpha$ is the learning rate and $\epsilon = 1e^{-8}$ is a small number for numerical stability to avoid division by zero. A detailed algorithm of Adam optimizer is summarized in Algorithm 1. The first order moment $m_t$ is used to update the parameters in Adam. Whereas, the second order moment $v_t$ is used to control the learning rate. It can be noticed that Adam majorly relies on the gradients.

In order to utilize the curvature information during optimization, we propose a novel concept of AdaInject. Basically, we inject the curvature information guided (weighted) second order moment into first order moment to compute the injected moment using the EMA of $(g_t + \Delta \theta \cdot g_t^2) / k$ as,

$$s_t = \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot (g_t + \Delta \theta \cdot g_t^2) / k \quad (4)$$

where $k$ is an injection controlling hyperparameter, typically set to 2 and $\Delta \theta = \theta_{t-2} - \theta_{t-1}$ is the short-term change in parameter $\theta$ to utilize the curvature information. The injection of curvature-guided second order moment helps the optimizers to perform the larger parameter update in the scenarios, where the optimization landscape exhibits the low gradient (i.e., saddle regions) and low curvature (i.e., saddle and monotonic increasing/decreasing regions). This phenomenon is depicted
in Fig. 1 with a detailed analysis in the next section. We perform the bias correction of injected moment and second order moment as \( \hat{s}_t \leftarrow s_t/(1-\beta_1^t) \) and \( \hat{v}_t \leftarrow v_t/(1-\beta_2^t) \), respectively.

The parameter (\( \theta \)) update of AdamInject optimizer is given as,

\[
\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \hat{s}_t / (\sqrt{\hat{v}_t} + \epsilon)
\]

(5)

where \( \alpha \) is the learning rate and \( \epsilon = 1e^{-8} \) is a small number for numerical stability to avoid the division by zero. We refer to Adam optimizer with the proposed second order moment injection as AdamInject optimizer. A detailed algorithm of AdamInject optimizer is presented in Algorithm 2 with highlighted changes in blue as compared to Adam optimizer which is shown in Algorithm 1.

Basically, we use the proposed AdaInject concept with four existing state-of-the-art optimizers, including Adam [26], diffGrad [27], Radam [28] and AdaBelief [29], and propose the corresponding AdamInject (i.e., Adam + Inject), diffGradInject (i.e., diffGrad + AdaInject), RadamInject (Radam and AdaInject) and AdaBeliefInject (i.e., AdaBelief + AdaInject) optimizers, respectively. The algorithms for different optimizers (i.e., without and with AdaInject), such as diffGrad, diffGradInject, Radam, RadamInject, AdaBelief, and AdaBeliefInject, are provided in Supplementary. Though we test the proposed injection concept with four optimizers, it can be extended to any EMA based gradient descent optimization technique. In the next section, we analyze the property of the proposed approach.

III. THEORETICAL AND EMPIRICAL ANALYSIS

In this section, we present the theoretical analysis using a one dimensional optimization curvature having three scenarios and empirical analysis using toy examples.

A. Theoretical Analysis

The existing gradient descent optimizers such as Adam, diffGrad, Radam, etc. only consider the EMA of gradient for parameter update. However, the consideration of curvature is important as the gradient behavior is different for different regions of loss optimization landscape [42]. We perform the theoretical analysis between the optimizers without and with the proposed injection under three different scenarios (i.e., S1, S2 and S3) as depicted in Fig. 1.

S1: The saddle region with low curvature is very common in high dimensional optimization. An ideal optimizer is expected to perform large parameter updates in saddle regions. The gradient in saddle regions is small. Moreover, the change in gradient over iteration is also very small. Thus, the EMA of gradient \( (g_t) \) leads to the smaller values of the first order moment for parameter update. Whereas, the proposed injection based EMA of \( (g_t + \Delta \theta \cdot g_t^2)/k \) produces the higher values for parameter update, except the scenarios where \( \Delta \theta \) is zero. Thus, AdaInject leads to the larger parameter update in saddle regions.

S2: The monotonic increase and decrease is another scenario in optimization curvature. The gradient is higher in such regions, but a change in gradient over iterations is low. An ideal optimizer is expected to take the large parameter updates in such regions. The EMA of gradient \( (g_t) \) leads to the higher values of the first order moment for parameter update. However, the proposed injection based EMA of \( (g_t + \Delta \theta \cdot g_t^2)/k \) further boosts the parameter updates with even larger steps. The increase in the step size in terms of \( \Delta \theta \) for parameter update leads to the cascading effect for even higher parameter updates in the subsequent iterations.

S3: The third scenario is parameter updates near minimum. It is expected for an ideal optimizer to decrease the step size for parameter updates in this scenario to avoid the overshooting as well as to reduce the oscillation near minimum. The step size is reduced by the adaptive optimizers in such regions with the help of the EMA of the square of the gradient \( (g_t^2) \). Due to the high gradient in such regions, the EMA of gradient \( (g_t) \) leads to oscillations near minimum. However, as the EMA of squared gradient \( (g_t^2) \) reduces the step size for parameter update, i.e., \( \Delta \theta \) gets reduced, then it leads to the cascading effect to reduce the EMA of \( (g_t + \Delta \theta \cdot g_t^2)/k \). Ultimately, it reduces oscillations near minimum and avoids the overshooting. In order to show this effect, we conduct the empirical study with the help of synthetic, non-convex functions in the next subsection.

B. Empirical Analysis using Toy Examples

We perform the empirical analysis using three synthetic, one-dimensional, non-convex functions by following the protocol of diffGrad [27]. These functions are given as:

\[
F1(x) = \begin{cases} 
(x + 0.3)^2, & \text{for } x \leq 0 \\
(x - 0.2)^2 + 0.05, & \text{for } x > 0
\end{cases}
\]

(6)

\[
F2(x) = \begin{cases} 
-40x - 35.15, & \text{for } x \leq -0.9 \\
x^3 + x \sin(8x) + 0.85, & \text{for } x > -0.9
\end{cases}
\]

(7)
Fig. 2. The empirical results computed over three synthetic, non-convex functions as toy examples. Each row corresponds to a function. The 1st column shows the used functions. The 2nd and 3rd columns show the parameter value updates for 300 iterations using Adam and diffGrad optimizers, respectively, with and without the proposed second order injection. The regression loss based objective function is used to update the parameters. The initialization is done at $x = -1$.

$$F_3(x) = \begin{cases} 
  x^2, & \text{for } x \leq -0.5 \\
  0.75 + x, & \text{for } -0.5 < x \leq -0.4 \\
  -7x/8, & \text{for } -0.4 < x \leq 0 \\
  7x/8, & \text{for } 0 < x \leq 0.4 \\
  0.75 - x, & \text{for } 0.4 < x \leq 0.5 \\
  x^2, & \text{for } 0.5 < x 
\end{cases}$$

(8)

where $-\infty < x < +\infty$ is the input.

Functions $F_1$, $F_2$, and $F_3$ are illustrated in Fig. 2 in the 1st column and in the 2nd, 3rd, and 4th rows, respectively, for $-1 < x < +1$. The parameter $x$ is initialized at $-1$. The experiment is performed by computing the regression loss as the objective function. The 2nd column shows the parameter values at different iterations using Adam and AdamInject optimizers. Similarly, the 3rd column illustrates the parameter values at different iterations using diffGrad and diffGradInject optimizers. It can be noticed that Adam overshoots the minimum for both $F_1$ and $F_2$ functions, whereas AdamInject is able to avoid the overshooting due to the injection controlling hyperparameter $k$ and cascading effect in decreasing of $\Delta x$. In other cases, including Adam and AdamInject for $F_3$ function and diffGrad and diffGradInject for all three functions, the effect of the proposed second order injection can be easily observed in terms of the smooth parameter updates by accumulating the injected gradient in an accurate direction. It is noticed that AdaInject is more effective with Adam than diffGrad as diffGrad utilizes the short-term gradient change as friction coefficient. These results confirm that the proposed curvature-guided second moment injection leads to accurate and precise parameter updates.

IV. CONVERGENCE ANALYSIS

We use the online learning framework to show the convergence property of the proposed injection based AdamInject optimizer, similar to Adam [26]. Let’s represent the unknown sequence of convex cost functions as $f_1(\theta), f_2(\theta), ..., f_T(\theta)$. We want to estimate parameter $\theta_t$ at each iteration $t$ and assess over $f_t(\theta)$. The regret bound is commonly used in such scenarios to assess the algorithm where the information of the sequence is not known in advance. The sum of the difference between all the previous online guesses $f_t(\theta_t)$ and the best fixed point parameter $f_t(\theta^*)$ from a feasible set $\chi$ of all the previous iterations are used to compute the regret bound. The regret bound is given as,

$$R(T) = \sum_{t=1}^{T} [f_t(\theta_t) - f_t(\theta^*)]$$

(9)

where $\theta^* = \text{arg min}_{\theta \in \chi} \sum_{t=1}^{T} f_t(\theta)$. The regret bound for the proposed injection based AdamInject is noticed as $O(\sqrt{T})$ which is the same as Adam and is comparable to general convex online learning approaches. We provide the proof in Supplementary. We consider $g_{t,i}$ as the gradient for the $i^{th}$ element in the $t^{th}$ iteration, $g_{t,i} = [g_{1,i}, g_{2,i}, ..., g_{T,i}] \in \mathbb{R}^t$ is
| CNN Models       | Classification error (%) using different optimizers without and with AdaInject |
|-----------------|--------------------------------------------------------------------------------|
|                 | Adam    | AdamInject | diffGrad | diffGradAdaInject | Radam   | RadamInject | AdaBelief   | AdaBeliefInject |
| VGG16           | 7.45    | 7.20 (↑ 3.36) | 7.24    | 7.04 (↑ 2.76)    | 7.06    | 6.88 (↑ 2.55) | 7.29        | 7.07 (↑ 3.02)   |
| ResNet18        | 6.46    | 6.20 (↑ 4.02) | 6.51    | 6.10 (↑ 3.60)    | 6.18    | 5.87 (↑ 5.02) | 6.37        | 6.30 (↑ 1.10)   |
| SENet18         | 6.61    | 6.29 (↑ 4.84) | 6.44    | 6.21 (↑ 3.37)    | 6.05    | 5.83 (↑ 3.64) | 6.59        | 6.23 (↑ 5.46)   |
| ResNet50        | 6.17    | 5.89 (↑ 4.54) | 6.19    | 5.73 (↑ 7.43)    | 5.86    | 5.29 (↑ 9.73) | 5.90        | 5.78 (↑ 2.03)   |
| ResNet101       | 6.90    | 6.01 (↑ 12.90) | 6.45    | 5.69 (↑ 11.78)   | 6.29    | 5.76 (↑ 8.43) | 6.37        | 6.03 (↑ 5.34)   |
| ResNeXt29       | 6.79    | 6.16 (↑ 10.28) | 6.83    | 5.70 (↑ 16.54)   | 6.00    | 5.67 (↑ 5.50) | 6.43        | 5.99 (↑ 6.84)   |
| DenseNet121     | 6.30    | 5.63 (↑ 10.63) | 5.90    | 5.43 (↑ 7.97)    | 5.25    | 5.10 (↑ 2.86) | 6.05        | 5.64 (↑ 6.78)   |

The experimental results of different CNNs in terms of top-1 classification error (%) over the CIFAR10 dataset using different optimizers, without and with the proposed AdaInject. The results with the proposed approach are highlighted in bold. The improvement in the error due to the proposed injection concept is also mentioned. The highest increase for an optimizer is also highlighted in bold. The symbols ↑ and ↓ represent the improvement and degradation in %, respectively, in the top-1 error. We follow the same convention in the results reported in Table II and III. These results are computed as the average over three independent trials.

For all $T \geq 1$, the proposed injection based AdamInject optimizer shows the following guarantee:

$$\frac{R(T)}{T} = O\left(\frac{1}{\sqrt{T}}\right).$$

Thus, $\lim_{T \to \infty} \frac{R(T)}{T} = 0$.

V. EXPERIMENTAL ANALYSIS

In this section, first we describe the experimental setup. Then, we present the detailed results using different optimizers. Finally, we analyze the effect of hyperparameter.

A. Experimental Setup

We use a wide range of CNN models (i.e., VGG16 [3], ResNet18, ResNet50, ResNet101 [4], SENet18 [5], ResNeXt29 [6] and DenseNet121 [7]) to test the suitability of the proposed AdaInject concept for optimizers. We follow the publicly available Pytorch implementation\(^1\) of these CNN models. For ResNeXt29 model, we set the cardinality as 4 and bottleneck width as 64. We train all the CNN models using all the optimizers under the same experimental setup. The training is performed for 100 epochs with a batch size of 64 for CIFAR10/100 and FashionMNIST and 256 for TinyImageNet dataset. The learning rate is set to 0.001 for the first 80 epochs and 0.0001 for the last 20 epochs. Different computers are used for the experiments, including Google colaboratory\(^2\). We perform a random crop and random horizontal flip over training data. The normalization is performed for both training and test data.

In order to demonstrate the efficacy of the proposed AdaInject based optimizers experimentally, we use four bench object recognition dataset, including CIFAR10 [43], CIFAR100 [43], FashionMNIST [44], TinyImageNet\(^3\). We use CIFAR and FashionMNIST datasets directly from the PyTorch library. CIFAR10 dataset consists of a total 60,000 images of dimension $32 \times 32 \times 3$ from 10 object classes with 6,000 images per class. The training set contains 50,000 images.

\(^1\)https://github.com/kuangliu/pytorch-cifar
\(^2\)https://colab.research.google.com/
\(^3\)http://cs231n.stanford.edu/tiny-imagenet-200.zip
| CNN Models | Classification error (%) using different optimizers without and with AdaInject |
|------------|--------------------------------------------------------------------------------|
| VGG16      | 32.71 | 31.81 († 2.75) | 31.81 | 30.80 († 3.18) | 29.31 | 30.07 († 2.59) | 31.08 | 30.04 († 3.35) |
| ResNet18   | 28.91 | 27.28 († 5.64) | 26.50 | 26.23 († 1.02) | 26.78 | 25.84 († 3.51) | 27.28 | 26.31 († 3.56) |
| SENet18    | 29.15 | 28.74 († 1.41) | 26.80 | 27.64 († 3.36) | 27.66 | 26.63 († 3.72) | 26.90 | 26.52 († 1.14) |
| ResNet50   | 28.12 | 25.44 († 9.53) | 24.94 | 24.18 († 3.05) | 25.05 | 24.13 († 3.67) | 24.47 | 24.25 († 0.90) |
| ResNet101  | 25.78 | 23.98 († 6.98) | 26.58 | 24.17 († 3.90) | 25.74 | 23.83 († 7.42) | 24.12 | 24.24 († 0.50) |
| ResNeXt29  | 28.78 | 24.96 († 13.27) | 25.47 | 24.53 († 3.69) | 24.66 | 22.74 († 7.79) | 24.61 | 23.63 († 3.98) |
| DenseNet121| 26.40 | 24.33 († 7.84) | 24.14 | 23.66 († 1.99) | 25.17 | 23.06 († 8.38) | 24.68 | 24.06 († 2.51) |

### B. Experimental Results

We compare the performance using four recent state-of-the-art adaptive gradient descent optimizers (i.e., Adam [26], diffGrad [27], Radam [28] and AdaBelief [29]), without and with the proposed injection approach. We consider VGG16 [3], ResNet18, ResNet50, ResNet101 [4], SENet18 [5], ResNeXt29 [6] and DenseNet121 [7] CNN models. The experimental results over the CIFAR10 dataset are depicted in Table I in terms of the error rate. It is observed that the performance of all CNN models is improved with AdaInject based optimizers as compared to its performance with corresponding vanilla optimizers. The RadamInject optimizer leads to best performance using the DenseNet121 model with a 5.10% error rate in classification. The highest improvement is reported by the ResNeXt29 model using diffGradInject. Moreover, the performance of the ResNeXt29 model is also significantly improved using AdaBeliefInject. In general, we observe better performance gain by heavy CNN models.

The results over the CIFAR100 dataset are illustrated in Table II. The best performance of 77.26% accuracy is achieved by the RadamInject optimizer using the ResNeXt29 model. The performance of ResNeXt29 is improved significantly using the proposed injection for optimizers with highest improvement by AdaInject. The results due to the proposed injection based optimizers are improved using all the CNN models except RadamInject using VGG16 and AdaBeliefInject using ResNet101. Note that Radam does not use second order moment if rectification criteria is not met and AdaBelief reduces the second order moment. These could be the possible reasons that the performance of RadamInject and AdaBeliefInject is marginally down in some cases. A very similar trend is also noticed over FashionMNIST (FMNIST) dataset in Table III, where the performance using the proposed approach is improved in all the cases. The best accuracy of 95.44% is observed for the AdaBeliefInject optimizer using the DenseNet121 model. An outstanding improvement in top-1 error is perceived for the ResNeXt29 model over the FashionMNIST dataset using the optimizers with the proposed AdaInject concept. The performance of other models is also significantly improved due to the proposed injection approach.

We also perform the experiment over the TinyImageNet dataset using VGG16, ResNet18 and SENet18 models and show the results in terms of the classification accuracy in % in Table IV for different optimizers with and without the proposed injection concept. It is observed from this experiment that the proposed approach is able to improve the performance of the existing optimizers over large scale dataset as well. These results confirm that the proposed injection updates...
TABLE IV

| CNN Models | Accuracy (%) using different optimizers without and with AdaInject |
|------------|------------------------------------------------------------------|
|            | Adam | AdamInject | diffGrad | diffGradInject | Radam | RadamInject | AdaBelief | AdaBeliefInject |
| VGG16      | 44.05 | **44.58** (↑ 1.20) | 46.00 | **47.18** (↑ 2.57) | 45.92 | **46.38** (↑ 1.00) | 47.88 | **48.25** (↑ 0.77) |
| ResNet18   | 50.58 | **51.90** (↑ 2.61) | 52.04 | **52.37** (↑ 0.63) | 52.12 | **52.50** (↑ 0.73) | 52.05 | **52.74** (↑ 1.33) |
| SENet18    | 48.04 | **49.52** (↑ 3.08) | 49.51 | **50.28** (↑ 1.56) | 50.73 | **51.01** (↑ 0.55) | 51.76 | **51.94** (↑ 0.35) |

TABLE V

| Model      | Dataset | k = 1 | k = 2 | k = 3 |
|------------|---------|-------|-------|-------|
| AdaInject  | CIFAR10 | 93.71 | 92.80 | 93.88 |
|            | CIFAR100| 71.97 | 72.72 | 73.30 |
|            | FMNIST  | 95.15 | 95.26 | 95.19 |

TABLE VI

| Model      | Dataset | k = 1 | k = 2 | k = 3 |
|------------|---------|-------|-------|-------|
| AdaInject  | CIFAR10 | 93.89 | 94.11 | 94.41 |
|            | CIFAR100| 72.99 | 74.56 | 75.99 |
|            | FMNIST  | 95.14 | 95.24 | 95.22 |

C. Effect of Injection Hyperparameter (k)

In the previous results, we use the value of injection hyperparameter (k) as 2. We show a performance comparison by considering the value of k as 1, 2, and 3 in Table V using the AdamInject optimizer for ResNet18 and ResNet50 models. The results over the CIFAR10, CIFAR100, and FMNIST datasets are presented. The higher value of k is suitable for AdamInject optimizer with k = 3 on CIFAR10 and CIFAR100 datasets and k = 2 on FMNIST dataset. It is noticed that though we use k = 2 in earlier experiments, a careful selection of k for different optimizers for different models over different datasets can further improve the performance of the proposed AdamInject optimizer. The performance comparison by considering the value of hyperparameter k as 1, 2, and 3 is given in Table VI using the RadamInject optimizer for ResNet18 and ResNet101 models. The selection of the value of k as 2 is also justified from the results of RadamInject optimizer.

VI. Conclusion

In this paper, we present a novel and generic injection based EMA of gradients for parameter update by utilizing the curvature information along with the second order moment. Thus, it leads to the accurate update by performing higher updates in the saddle and monotonic regions and cascaded precise updates in the local minimum region. The effect of the proposed injection based optimizers is observed using toy examples. The convergence property of the proposed optimizer is analyzed. The object recognition results for different CNN models over benchmark datasets using four optimizers show the superiority of the proposed injection concept. It is observed that a careful selection of injection hyperparameter can further improve the performance. The theoretical, empirical, convergence, and experimental analyses are evident that the proposed injection based optimizers lead to better optimization of CNNs.

APPENDIX

A. Convergence Proof

**Theorem 2.** Assume that the gradients for function \( f_t \) (i.e., \( \|g_t, \theta\|_2 \leq G \) and \( \|g_t, \theta\|_\infty \leq G_\infty \)) are bounded for all \( \theta \in \mathbb{R}^d \). Let also consider that the bounded distance is generated by the proposed optimizer between any \( \theta_i \) (i.e., \( \|\theta_n - \theta_m\|_2 \leq D \) and \( \|\theta_n - \theta_m\|_\infty \leq D_\infty \) for any \( m, n \in \{1, ..., T\} \)). Let \( \gamma \leq \frac{\sqrt{2}}{2\sqrt{2}} \), \( \beta_1, \beta_2 \in [0, 1] \) satisfy \( \frac{\beta_2}{\sqrt{2}} < 1 \), \( \alpha_t = \frac{\sqrt{t}}{\sqrt{t} + \epsilon} \), and \( \beta_1 = \beta_2 \lambda^{t-1}, \lambda \in (0, 1) \) with \( \lambda \) is around 1, e.g. \( 10^{-8} \). For all \( T \geq 1 \), the proposed injection based AdamInject optimizer shows the following guarantee:

\[
R(T) \leq \frac{D^2}{\alpha(1 - \beta_1)} \sum_{i=1}^d \sqrt{T}v^T_i + \frac{2\alpha(1 + \beta_1)G_\infty}{(1 - \beta_1)\sqrt{1 - \beta_2^2}(1 - \gamma)^2} \sum_{i=1}^d \|g_{T,i}\|_2 \]

\[
+ \sum_{i=1}^d D_\infty^2 G_\infty \sqrt{1 - \beta_2^2} + 4D_\infty G_\infty^2 \sum_{i=1}^d \|g_{T,i}\|_2^2
\]

(12)

**Proof.** Following can be written from Lemma 10.2 of Adam [26],

\[
f_t(\theta_t) - f_t(\theta^*) \leq g_{T}^T (\theta_t - \theta^*) = \sum_{i=1}^d g_{t,i}(\theta_{t,i} - \theta_{t,i}^*)
\]

Following can be also written by utilizing the update formula of the proposed injection with Adam (i.e., AdamInject) with \( k = 2 \) and after discarding \( \epsilon \),

\[
\theta_{t+1} = \theta_t - \frac{\alpha_{t+1}}{\sqrt{v_{t+1}^T}}
\]

\[
= \theta_t - \frac{\alpha_t}{(1 - \beta_1)} \left( \frac{\beta_1}{\sqrt{v_{t+1}^T}} s_{t+1} - \frac{(1 - \beta_1)\Delta g^2}{\sqrt{v_{t+1}^T}} \right)
\]

(13)
where $\beta_{t,i}$ is the 1st order moment coefficient at $t$th iteration.
We can write the following w.r.t. the $i$th dimension of parameter vector $\theta_t \in \mathbb{R}^d$,

\[
(\theta_{t+1,i} - \theta^*_i)^2 = (\theta_{t,i} - \theta^*_i)^2 - \frac{2\alpha_t}{1 - \beta_{1,t}^2} \left( \frac{\beta_{1,t}}{\sqrt{\hat{v}_{t,i}}} s_{t-1,i} \right)^2 + \frac{(1 - \beta_{1,t}) \left( g_{t,i} + \Delta \theta g_{t,i}^2 \right)}{2} (\theta_{t,i} - \theta^*_i) + \alpha_t^2 \left( \hat{s}_{t,i} \right)^2
\]

\[\text{(14)}\]

We can reorganize the above equation as,

\[
g_{t,i}(\theta_{t,i} - \theta^*_i) = \frac{(1 - \beta_{1,t}^2)}{\alpha_t (1 - \beta_{1,t})} \left( (\theta_{t,i} - \theta^*_i)^2 - (\theta_{t+1,i} - \theta^*_i)^2 \right) - \frac{2\beta_{1,t}}{1 - \beta_{1,t}^2} (\sigma_{t,i}^2 s_{t-1,i})\]

\[\text{+}\ \frac{\alpha_t (1 - \beta_{1,t}^2)}{(1 - \beta_{1,t})} \left( \hat{s}_{t,i} \right)^2 - \Delta \theta g_{t,i}^2 (\theta_{t,i} - \theta^*_i).\]

\[\text{(15)}\]

\[\text{We can rewrite it as follows:}\]

\[
g_{t,i}(\theta_{t,i} - \theta^*_i) \leq \frac{1}{\alpha_t (1 - \beta_{1,t})} \left( (\theta_{t,i} - \theta^*_i)^2 - \left( (\theta_{t+1,i} - \theta^*_i)^2 \right) \right) + \frac{\beta_{1,t}}{\alpha_t (1 - \beta_{1,t})} \left( \sigma_{t,i}^2 s_{t-1,i} \right) + \frac{\alpha_t (1 - \beta_{1,t}^2)}{(1 - \beta_{1,t})} \left( \hat{s}_{t,i} \right)^2 - \Delta \theta g_{t,i}^2 (\theta_{t,i} - \theta^*_i)\]

\[\text{(16)}\]

\[\text{Next, we use the Young’s inequality,} \ ab \leq a^2/2 + b^2/2 \text{ as well as the information that} \ \beta_{1,t} \leq \beta_1. \text{We also replace} \ \Delta \theta \text{ with} \ \theta_{t-1} - \theta_t. \text{Thus, we can write the above equation as,}\]

\[
g_{t,i}(\theta_{t,i} - \theta^*_i) \leq \frac{1}{\alpha_t (1 - \beta_{1,t})} \left( (\theta_{t,i} - \theta^*_i)^2 - \left( \theta_{t+1,i} - \theta^*_i \right)^2 \right) \sqrt{\hat{v}_{t,i}} + \frac{\beta_{1,t}}{\alpha_t (1 - \beta_{1,t})} \left( \sigma_{t,i}^2 s_{t-1,i} \right) + \frac{\alpha_t (1 - \beta_{1,t}^2)}{(1 - \beta_{1,t})} \left( \hat{s}_{t,i} \right)^2 - \Delta \theta g_{t,i}^2 (\theta_{t,i} - \theta^*_i)\]

\[\text{(17)}\]

\[\text{In order to compute the regret bound, we aggregate it as per the Lemma 10.4 of Aad} \text{m [26] across the dimensions for} \ i \in \{1, \ldots, d\} \text{ and the convex function sequence for} \ t \in \{1, \ldots, T\} \text{ in the upper bound of} \ f_t(\theta_t) - f_t(\theta^*) \text{ as,}\]

\[
R(T) \leq \sum_{i=1}^{d} \frac{1}{\alpha_1 (1 - \beta_1)} (\theta_{1,i} - \theta^*_i)^2 \sqrt{\hat{v}_{i}} + \sum_{i=1}^{d} \sum_{t=2}^{T} \frac{1}{\alpha_t (1 - \beta_1)} (\theta_{t,i} - \theta^*_i)^2 \sqrt{\hat{v}_{t,i}} - \frac{\sqrt{\hat{v}_{t-1,i}}}{\alpha_{t-1}} \]

\[\text{+} \ \sum_{i=1}^{d} \sum_{t=1}^{T} \frac{\beta_{1,t}}{\alpha_t (1 - \beta_{1,t})} (\theta_{t,i} - \theta^*_i)^2 \sqrt{\hat{v}_{t,i}}\]

\[\text{+} \ \sum_{i=1}^{d} \sum_{t=1}^{T} \frac{2\beta_{1,t} G_{\infty}}{(1 - \beta_1) \sqrt{1 - \beta_2 (1 - \gamma)^2}} \sum_{i=1}^{d} \|g_{1:T,i}\|_2\]

\[\text{+} \ \sum_{i=1}^{d} \sum_{t=1}^{T} \frac{2\alpha G_{\infty}}{(1 - \beta_1) \sqrt{1 - \beta_2 (1 - \gamma)^2}} \sum_{i=1}^{d} \|g_{1:T,i}\|_2\]

\[\text{+} \ \sum_{i=1}^{d} \sum_{t=1}^{T} (\theta_{t,i} - \theta^*_i) (\theta_{t,i} - \theta^*_i) g_{t,i}^2\]

\[\text{(18)}\]

\[\text{It can be further refined with the assumptions that} \ \alpha = \alpha_t \sqrt{T}, \ |\theta_t - \theta^*|_2 \leq D, \ |\theta_m - \theta_n|_\infty \leq D_{\infty} \text{ and } \Delta \theta (\theta_t - \theta_{t-1}) \text{ is very small. Moreover, } \Delta \theta \approx 0 \text{ when } t \text{ is large. Then, we can approximate } \Delta \theta \text{ with an upper bound of } 1/t^2. \text{ Thus, the above equation can be written as,}\]

\[
R(T) \leq \frac{D^2}{\alpha (1 - \beta_1)} \sum_{i=1}^{d} \sqrt{T} \hat{v}_{i} + \frac{2\alpha (1 + \beta_1) G_{\infty}}{(1 - \beta_1) \sqrt{1 - \beta_2 (1 - \gamma)^2}} \sum_{i=1}^{d} \|g_{1:T,i}\|_2\]

\[\text{+} \ \frac{D^2 \alpha}{\beta_1} \sum_{i=1}^{d} \sum_{t=1}^{T} \frac{1}{(1 - \beta_{1,t})} \sqrt{\hat{v}_{t,i}}\]

\[\text{+} \ \frac{D^2 G_{\infty} \sqrt{1 - \beta_2}}{\alpha} \sum_{i=1}^{d} \sum_{t=1}^{T} \frac{\beta_{1,t}}{(1 - \beta_{1,t})} \sqrt{t} \]

\[\text{+} \ \frac{D^2 G_{\infty}}{\alpha} \sum_{i=1}^{d} \sum_{t=1}^{T} \frac{\beta_{1,t} \sqrt{t}}{t}\]

\[\text{(19)}\]
As per the finding of Adam [26], i.e., $\sum_{i=1}^{T} \frac{\beta_i t}{1-\beta_i t} \sqrt{T} \leq \frac{1}{(1-\beta_t)(1-\gamma_t)}$, the regret bound can be further rewritten as,

$$R(T) \leq \frac{D^2}{\alpha(1-\beta_t)} \sum_{i=1}^{d} \sqrt{T\hat{v}_{T,i}}^2 \left[ + \frac{2\alpha(1+\beta_t)G_{\infty}}{(1-\beta_t)^2(1-\gamma_t)^2} \sum_{i=1}^{d} \|g_{1:T,i}\|_2 \right] \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. 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A. Algorithms

This section provides the Algorithms for different algorithms, including diffGrad (Algorithm 3), diffGradInject (Algorithm 4), Radam (Algorithm 5), RadamInject (Algorithm 6), AdaBelief (Algorithm 7) and AdaBeliefInject (Algorithm 8).

Algorithm 3: diffGrad Optimizer

Initialize: $\theta_0, m_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$

Hyperparameters: $\alpha, \beta_1, \beta_2$

While $\theta_t$ not converged

\[ t \leftarrow t + 1 \]
\[ g_t \leftarrow \nabla_0 f_t(\theta_{t-1}) \]
\[ \xi_t \leftarrow 1/(1 + e^{-|g_t - g_{t-1}|}) \]
\[ m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t \]
\[ v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2 \]

Bias Correction

\[ \tilde{m}_t \leftarrow m_t/(1 - \beta_1^t) \]
\[ \tilde{v}_t \leftarrow v_t/(1 - \beta_2^t) \]

Update

\[ \theta_{t+1} \leftarrow \theta_t - \alpha \xi_t \tilde{m}_t/\sqrt{\tilde{v}_t + \epsilon} \]

Algorithm 4: diffGradInject (diffGrad + AdaInject) Optimizer

Initialize: $\theta_0, s_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$

Hyperparameters: $\alpha, \beta_1, \beta_2, k$

While $\theta_t$ not converged

\[ t \leftarrow t + 1 \]
\[ g_t \leftarrow \nabla_0 f_t(\theta_{t-1}) \]
\[ \xi_t \leftarrow 1/(1 + e^{-1|g_t - g_{t-1}|}) \]

If $t = 1$

\[ s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot g_t \]

Else

\[ \Delta \theta \leftarrow \theta_{t-1} - \theta_{t-1} \]
\[ s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot (g_t + \Delta \theta \cdot g_t^2)/k \]
\[ v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2 \]

Bias Correction

\[ \tilde{s}_t \leftarrow s_t/(1 - \beta_1^t) \]
\[ \tilde{v}_t \leftarrow v_t/(1 - \beta_2^t) \]

Update

\[ \theta_{t+1} \leftarrow \theta_{t-1} - \alpha \xi_t \tilde{s}_t/\sqrt{\tilde{v}_t + \epsilon} \]

Algorithm 5: Radam Optimizer

Initialize: $\theta_0, m_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$

Hyperparameters: $\alpha, \beta_1, \beta_2$

While $\theta_t$ not converged

\[ t \leftarrow t + 1 \]
\[ \rho_{\infty} \leftarrow 2/(1 - \beta_2) - 1 \]
\[ g_t \leftarrow \nabla_0 f_t(\theta_{t-1}) \]
\[ m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t \]
\[ v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2 \]
\[ \rho_t = \rho_{\infty} - 2t^2/(1 - \beta_2^t) \]

If $\rho_t \geq 5$

\[ \rho_u = (\rho_t - 4) \times (\rho_t - 2) \times \rho_{\infty} \]
\[ \rho_d = (\rho_{\infty} - 4) \times (\rho_{\infty} - 2) \times \rho_u \]
\[ \rho = \sqrt{(1 - \beta_2) \times \rho_u / \rho_d} \]
\[ \alpha_1 = \rho \times \alpha / (1 - \beta_2^t) \]

Update

\[ \theta_{t+1} \leftarrow \theta_{t-1} - \alpha_1 \times m_t/\sqrt{v_t + \epsilon} \]

Else

\[ \alpha_2 = \alpha / (1 - \beta_1^t) \]

Update

\[ \theta_{t+1} \leftarrow \theta_{t-1} - \alpha_2 \times m_t \]
Algorithm 6: RadamInject (i.e., Radam + Inject) Optimizer

Initialize: $\theta_0, s_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$
Hyperparameters: $\alpha, \beta_1, \beta_2, k$
While $\theta_t$ not converged
  $t \leftarrow t + 1$
  $\rho_{\infty} \leftarrow 2/(1 - \beta_2) - 1$
  $g_t \leftarrow \nabla_{\theta} f_t(\theta_{t-1})$
  If $t = 1$
    $s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot g_t$
  Else
    $\Delta\theta \leftarrow \theta_{t-1} - \theta_{t-2}$
    $s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot (g_t - \Delta\theta \cdot g_t^2) / k$
    $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$
    $\rho_t = \rho_{\infty} - 2t\beta_1^2/(1 - \beta_1^2)$
  If $\rho_t \geq 5$
    $\rho_u = (\rho_t - 4) \times (\rho_t - 2) \times \rho_{\infty}$
    $\rho_d = (\rho_{\infty} - 4) \times (\rho_{\infty} - 2) \times \rho_t$
    $\rho = \sqrt{(1 - \beta_2) \times \rho_u / \rho_d}$
    $\alpha_1 = \rho \times \alpha / (1 - \beta_1^2)$
    Update
    $\theta_t \leftarrow \theta_{t-1} - \alpha_1 \times s_t / (\sqrt{v_t} + \epsilon)$
  Else
    $\alpha_2 = \alpha / (1 - \beta_1^2)$
    Update
    $\theta_t \leftarrow \theta_{t-1} - \alpha_2 \times s_t$

Algorithm 7: AdaBelief Optimizer

Initialize: $\theta_0, m_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$
Hyperparameters: $\alpha, \beta_1, \beta_2$
While $\theta_t$ not converged
  $t \leftarrow t + 1$
  $g_t \leftarrow \nabla_{\theta} f_t(\theta_{t-1})$
  $m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$
  $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot (g_t - m_t)^2$
Bias Correction
  $\hat{m}_t \leftarrow m_t / (1 - \beta_1^2)$
  $\hat{v}_t \leftarrow v_t / (1 - \beta_2^2)$
Update
  $\theta_t \leftarrow \theta_{t-1} - \alpha\hat{m}_t / (\sqrt{\hat{v}_t} + \epsilon)$

Algorithm 8: AdaBeliefInject (AdaBelief + AdaInject) Optimizer

Initialize: $\theta_0, s_0 \leftarrow 0, v_0 \leftarrow 0, t \leftarrow 0$
Hyperparameters: $\alpha, \beta_1, \beta_2, k$
While $\theta_t$ not converged
  $t \leftarrow t + 1$
  $g_t \leftarrow \nabla_{\theta} f_t(\theta_{t-1})$
  If $t = 1$
    $s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot g_t$
  Else
    $\Delta\theta \leftarrow \theta_{t-2} - \theta_{t-1}$
    $s_t \leftarrow \beta_1 \cdot s_{t-1} + (1 - \beta_1) \cdot (g_t - \Delta\theta \cdot g_t^2) / k$
    $v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot (g_t - s_t)^2$
Bias Correction
  $\tilde{s}_t \leftarrow s_t / (1 - \beta_1^2)$
  $\tilde{v}_t \leftarrow v_t / (1 - \beta_2^2)$
Update
  $\theta_t \leftarrow \theta_{t-1} - \alpha\tilde{s}_t / (\sqrt{\tilde{v}_t} + \epsilon)$