A Proportional Derivative (PD) Controller for Suppression the Vibrations of a Contact-Mode AFM Model

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ABSTRACT The nonlinear dynamics control of a contact-mode atomic force microscopy (AFM) system with multi forces (harmonic and parametric excitation force) utilizing the time delay proportional derivative (PD) controller is investigated. The perturbation method is utilized to calculate the first-order approximate solutions for the AFM system. The stability of the AFM system is investigated at the worst resonance case by Lyapunov’s first method. We also show the bifurcation diagrams of response curves using frequency response equations before and after control are performed. Furthermore, we focus on the effect of the time-delayed control and returns signals gain on the vibration amplitude and the stability analysis of the controlled system in primary, sub-harmonic resonance for different parameter variations. In addition, the numerical results are procured using MATLAB program. Eventually, validation curves are presented to estimate the nearness degree between the analytical predictions also numerical simulation. The obtained results exhibited that, the time-delayed PD control efficiency to put down the nonlinear oscillations of the system.

INDEX TERMS A contact-mode AFM model, vibrations, proportional derivative control (PD).

NOMENCLATURE

\( u, \dot{u}, \ddot{u}, \mu_1, \beta, \)
\( \beta_1, \beta_2, \Omega_1, \omega_1, \Omega_2, f_1 \)
Dimensionless displacement, velocity, acceleration, damping coefficient, nonlinear parameters, natural and excitation frequencies and force amplitude of AFM model.

\( p, d \)
Dimensionless proportional and derivative control gains.

\( a \)
Dimensionless amplitude of AFM model.

\( \tau_1, \tau_2, t \)
Dimensionless signals of time-delayed controller and time.

\( \sigma_1, \sigma_2, \varepsilon \)
Detuning and perturbation parameters (0 < \varepsilon \ll 1).

I. INTRODUCTION

The Atomic force microscopy (AFM) is a powerful method that can picture almost any kind of surface, inclusive glass, polymers, ceramics, composites, and biological models. AFM is applied to measurement and locate many excitations, inclusive adhesion strength, magnetic forces, and mechanical characteristic. The AFM has three major abilities: topographic imaging, force measurement, and manipulation. Binning and Quate [1] concerned with the measure of ultrasmall excitations on particles as tiny as unattached atoms. They proposed to do this by monitoring the elastic distortion of different kinds of springs with the wiping tunneling microscope. Berg and Briggs [2] investigated intermittent contact system AFM from the viewpoint that the nonlinearity inserted by the drastic increasing the hardness on impact from that of the cantilever to that of the tip sample connect is important to its dynamics. Rützel et al. [3] investigated the near-resonant, nonlinear dynamic response of microcantilevers in AFM through numerical methods and simulations of discretized systems interactive with a sample through a Lennard-Jones potential. Moreover, the outcomes predict a broad range of nonlinear dynamic phenomena, many of that have been shown up on experimental AFM. A theoretical framework and an experimental procedure are presented by Abdel-Rahman and Nayfeh [4] to extend the
applicability of acoustic/ultrasonic AFM to public contact conditions. They used multiple perturbation method to get approximating expressions of the response of the probe to one-half subharmonic resonance. Arafat et al. [5] used AFM to estimate material and surface properties. The multiple scales technique is applied to obtain approximate solution to the probe response in the existence of 2:1 autoparametric resonance between the second and third modes. They found that the effectiveness of the interaction extends over a considerable domain of the tip-sample model hardness. Yamasue and Hikihara [6] proposed stabilization of the chaotic microcancillever vibrations applying the method of time-delayed feedback control, that has the ability to stabilize unstable periodic orbitals embedded in chaotic attractants. Besides, they discuss an improved transient response of vibration that allows us to accelerate the scanning average of the microscopes without decreasing their amplitude sensitivity. Active control of a tapping mode AFM system via delayed feedback technique is illustrated by Salarieh and Alasty [7]. They obtained the gain of feedback and adapted according to a minimize entropy algorithm. Moreover, results simulation explains the feasibility of the proposed technique in using the method of delayed feedback for chaos control of the AFM system. Yamasue et al. [8] illustrated the stabilization of first experimental of non-periodic and irregular vibration of cantilever in the modulation amplitude AFM applying the time-delayed feedback controller. Bahrami and Nayfeh [9] investigated the nonlinear dynamics of an AFM cantilever through different numerical simulations achieved for a large domain of the amplitude and excitation frequency.

The hysteresis suppression and frequency response shift of contact mode AFM is illustrated by [10] applying parametric modification of the contact hardness. They used multiple perturbation technique and direct partition of motion to get the corresponding slow flow and the slow dynamic of the model, respectively. Kirrou and Belhaq [11], [12] studied analytically the effect of fast contact stiffness modulation on the frequency curve about 2:1 subharmonic resonance and primary resonance in contact-mode AFM. They give a comparison between the analytical and numerical solutions and provided example on the application to a real AFM. Bahrami and Nayfeh [13] investigated the nonlinear dynamics of an AFM microcantilever when it works in the tapping model and the AFM tip is at first existing in the bistable static zone. The differential quadrature techniques are applied to memorize the microcantilever equation and simulate the static, free vibration responses of the scanning probe. Kirrou and Belhaq [14] investigated the control of bistability in non-contact system AFM excited by a harmonic excitation and in the existence of time-delayed feedback. Vatankhah [15] illustrated the nonlinear oscillation of an AFM with an assembled cantilever probe. The results obtained shows the model becomes more nonlinear as increases the length of the vertical extension. Hsieh et al. [16] studied the control and nonlinear behavior of AFM system. The numerical results showed that the tools applied for the analysis give consistent results and that changes in frequency excitation have a major influence on system behavior. Mahmoudi et al. [17] used the harmonic balance technique to obtain the initial-boundary problem dominant the motion of microcantilever AFM system. Wagner [18] studied the effect of external excitations on the motion of the tip in dynamic AFM. Special emphasis is placed on discussing tip response in high damping environments, such as ambient or liquid environments. The authors [19]–[27] investigated the vibration control with stability analysis of many engineering systems, particularly from string-beam, rotor blade flapping, electromechanical oscillator, rotor seal, pitch-roll motion, articulated beam, buckled beam, a cantilever beam, mitigate lateral oscillations of a vertically Jeffcott rotor system. The authors [28]–[32] were analyzed vibration performance, stability and used the proportional derivative and positive position feedback controller to reduce the vibrations of the compressor blade, offshore wind turbine tower and vertical conveyor systems.

In this article, a PD controller is applied to eliminate the oscillations of a contact mode AFM system excited by multi excitation forces. A multiple scale perturbation technique is carried out to extract resonance cases and approximate nonlinear oscillations for the AFM model. Besides, the effects of proportional control and derivative control gains on the dynamical behaviors and stability of the AFM model are obtained. Verification curves show a good agreement between numerical simulation and analytical solution.

II. SYSTEM MODELLING

Figure 1, illustrates the lumped parameter single degree of freedom AFM model with modulation of fast contact hardness. The motion equation of the system is obtained from Ref. [11], [12] and described by the following equations:

\[
\ddot{u} + \varepsilon \mu_1 \dot{u} + \omega^2 u + \varepsilon \beta_1 u^2 + \varepsilon \beta_2 u^3 + \varepsilon \rho \left( \frac{3}{2} \beta - \beta u + \beta_1 u^2 + \beta_2 u^3 \right) \cos (\omega t) = \varepsilon f_1 \cos(\omega t) \tag{1}
\]
Applying the time delay with the proportional derivative controller (PD) to the motion equation (1), we get the modified normalized equation as follows:

\[
\ddot{u} + \varepsilon \mu_1 \dot{u} + \omega^2 u + \varepsilon \beta_1 \dot{u}^2 + \varepsilon \beta_2 u^3 \\
+ \varepsilon \tau \left( \frac{3}{2} \beta - \beta u + \mu_1 \dot{u} + \beta_2 u^2 \right) \cos(\Omega_2 t) \\
= \varepsilon f_1 \cos(\Omega_1 t) - \varepsilon pu (t - \tau_1) - \varepsilon \dot{d}u (t - \tau_2) \tag{2}
\]

III. MATHEMATICAL ANALYSIS

The multiple scale perturbation (MSP) [33], [34] is applied to find the approximate solutions in addition to frequent response equations respectively. To get the approximate solutions for equation (2) we applied this technique and the solution to be in the form:

\[
u(t; \varepsilon) = u_0(T_0, T_1) + u_1(T_0 - \tau_1, T_1 - \tau_1) + O(\varepsilon^2)
\]

\[
u(t - \tau; \varepsilon) = u_{0r}(T_0 - \tau, T_1 - \tau) + O(\varepsilon^2)
\]

\[
u(t - \tau; \varepsilon) = u_{0r}(T_0 - \tau, T_1 - \tau) + O(\varepsilon^2)
\]

We proposed the derivatives in the format

\[
\frac{d}{dt} = D_0 + \varepsilon D_1 \\
\frac{d^2}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1
\]

where \(D_n = \frac{\partial}{\partial \tau_n} (n = 0, 1)\), where \(D_n\) are the derivatives and \(T_0 = \varepsilon^0 \tau\) are the time scales. Inserting equations (3) and (4) in equation (2), we equating the coefficients of powers of \(\varepsilon\) and we have:

\[
\text{order}(\varepsilon^0) \qquad \left(D_0^2 + \omega^2 \right) u_0 = 0
\]

\[
\text{order}(\varepsilon) \qquad \left(D_0^2 + \omega^2 \right) u_1 = -2D_0 D_1 u_0 - \mu_1 D_0 u_0 - \beta_1 u_0^2 - \beta_2 u_0^3 \\
+ \left( \beta_1 \omega^0 - \frac{3}{2} \beta - \beta_2 \right) u_1 \cos(\Omega_2 t) \\
+ f_1 \cos(\Omega_1 t) - pu_{0r} - d \dot{d}u_{0r} \tag{5}
\]

Can be formulating the general solution for equation (5) as:

\[
u_0 = A_0(T_1) e^{i\omega T_0} + \tilde{A}_0(T_1) e^{-i\omega T_0} \tag{7}
\]

According to equation (7), the time-delayed solutions can be written as:

\[
u_{0r} = A_0(T_1 - \varepsilon \tau_2) e^{i\omega(T_0 - \tau_1)} + \tilde{A}_0(T_1 - \varepsilon \tau_2) e^{-i\omega(T_0 - \tau_1)} \tag{8a}
\]

\[
u_{0r} = A_0(T_1 - \varepsilon \tau_2) e^{i\omega(T_0 - \tau_2)} + \tilde{A}_0(T_1 - \varepsilon \tau_2) e^{-i\omega(T_0 - \tau_2)} \tag{8b}
\]

where \(A_0(T_1)\) and \(\tilde{A}_0(T_1)\) are unknown functions in \(T_1\). Expanding \(A_{0r}\) and \(A_{0r}\) in Taylor-series as:

\[
A_{0r} = A_0(T_1) + \frac{\varepsilon \tau_1}{1!} D_1 A_0(T_1) + \ldots = A_0(T_1) \tag{9a}
\]

\[
A_{0r} = A_0(T_1) + \frac{\varepsilon \tau_2}{1!} D_1 A_0(T_1) + \ldots = A_0(T_1) \tag{9b}
\]

Taking equation (9) into account, substituting equations (7) and (8) into equation (6), we get:

\[
\begin{align*}
D_0^2 + \omega^2 u_1 &= \left( -2i \omega_1 D_1 A_0 - i \mu_1 \omega_1 A_0 - 3 \beta_2 A_0^2 \tilde{A}_0 \right) e^{i\omega T_0} \\
&+ 2i \omega_1 D_1 \tilde{A}_0 + i \mu_1 \omega_1 \tilde{A}_0 + \left( 3 \beta_2 A_0^2 \tilde{A}_0 \right) e^{-i\omega T_0} \\
&- p \left( A_0 e^{i\omega T_0} e^{-i\tau_1} + \tilde{A}_0 e^{-i\omega T_0} e^{i\tau_1} \right) \\
&- i \omega_1 \left( A_0 e^{i\omega T_0} e^{-i\tau_2} + \tilde{A}_0 e^{-i\omega T_0} e^{i\tau_2} \right) \\
&- \left( r \beta_1 A_0 \tilde{A}_0 + r \beta_2 \frac{3}{2} \beta \right) e^{i\omega T_0} + e^{-i\omega T_0} \\
&- \left( \beta_1 A_0^2 e^{i\omega T_0} + \beta_2 A_0^2 e^{-i\omega T_0} \right) \\
&- \left( \beta_2 A_0^2 e^{i\omega T_0} + \beta_2 A_0^2 e^{-i\omega T_0} \right) \\
&+ \frac{1}{2} \left( e^{i\omega T_0} + e^{-i\omega T_0} \right) - \frac{3}{2} r \beta_2 A_0 \tilde{A}_0 - \frac{r}{2} \beta_2 \tag{10}
\end{align*}
\]

For abounded solution, we eliminated the secular terms \(e^{\pm i\omega T_0}\) and the solutions of equation (10) obtained as:

\[
u_1 = A_1 e^{i\omega T_0} + \tilde{A}_1 e^{-i\omega T_0} - \frac{1}{(\omega_1^2 - \Omega_2^2)} \times \left( r \beta_1 A_0 \tilde{A}_0 + r \beta_2 \frac{3}{2} \beta \right) e^{i\omega T_0} \\
+ \frac{3}{2} \omega_1 \left( A_0^2 e^{2i\omega T_0} + \tilde{A}_0^2 e^{-2i\omega T_0} \right) + \frac{3}{8} \omega_1 \times \left( A_0^3 e^{3i\omega T_0} + \tilde{A}_0^3 e^{-3i\omega T_0} \right) \\
- \frac{r}{2} \left( \omega_1^2 - (\Omega_2^2 + \omega_1^2) \right)^2 \times \left( 3 \beta_2 A_0^2 \tilde{A}_0 - 3 \beta_0 A_0 \tilde{A}_0 \right) e^{i(\Omega_2 + \omega_1) T_0} \\
+ \left( 3 \beta_2 \tilde{A}_0^2 - 3 \beta_0 \tilde{A}_0 \right) e^{-i(\Omega_2 + \omega_1) T_0} \tag{10}
\]
Investigation, where the solution is only dependent on

\[ \frac{\dot{r}}{r} = \frac{2(\omega_1^2 - (\Omega_2 - \omega_1)^2)}{(3\beta_2A_0^2 - \beta A_0) e^{i(\Omega_2 - \omega_1)T_0}} + \frac{3\beta_2A_0^2 - \beta A_0 - \beta A_0 e^{-i(\Omega_2 - \omega_1)T_0}}{2(\omega_1^2 - (\Omega_2 + 2\omega_1)^2)} \]

Substituting equation (12) into equations (13), we obtained:

\[ r \beta_1 = \frac{2(\omega_1^2 - (\Omega_2 - \omega_1)^2)}{3\beta_2A_0^2 e^{i(\Omega_2 - \omega_1)T_0} + \beta A_0 e^{-i(\Omega_2 - \omega_1)T_0}} \]

where \( A_1 \) and \( \bar{A}_1 \) are complex functions in \( T_1 \). For stability investigation, where the solution is only dependent on \( T_0 \) and \( T_1 \) at primary, sub-harmonic cases \( \Omega_1 \equiv \omega_1, \Omega_2 \equiv 2\omega_1 \), we introduce detuning parameters \( \sigma_1 \) and \( \sigma_2 \) such that

\[ \Omega_1 = \omega_1 + \epsilon \sigma_1, \Omega_2 = 2\omega_1 + \epsilon \sigma_2 \]

From equations (10) the secular terms are removed by (equate the secular terms to zero), result in solvability conditions for the first order approach, like:

\[ (-2i\omega_1 D_1 A_0 - i\mu_1 \omega_1 A_0 - 3\beta_2 A_0^2 A_0) e^{i\omega_1 T_0} \]

Substituting equation (12) into equations (13), we obtained:

\[ (-2i\omega_1 D_1 A_0 - i\mu_1 \omega_1 A_0 - 3\beta_2 A_0^2 A_0) e^{i\omega_1 T_0} \]

Dividing equation (14) by \( e^{i\omega_1 T_0} \) we obtained:

\[ -2i\omega_1 D_1 A_0 - i\mu_1 \omega_1 A_0 - 3\beta_2 A_0^2 A_0 \]

Substituting the polar forms

\[ A_0 = \frac{1}{2} e^{i\theta_1(T_1)} \]

where a and \( \varphi \) are the amplitude and phase of the system. Inserting equation (16) in equation (15), we obtained the first order differential equations:

\[ \dot{\theta}_1 = \sigma_1 - \frac{3}{8\omega_1} \beta_2 \sigma_2 - \frac{1}{4\omega_1} r \beta_2 a^2 \cos 2\theta_1 + \frac{1}{4\omega_1} r \beta_3 \cos 2\theta_1 \]

where

\[ \theta_1 = \sigma_1 T_1 - \varphi, \theta_2 = \sigma_2 T_1 - 2\varphi, \dot{\varphi} = \sigma_1 - \dot{\theta}_1, \theta_2 = 2\dot{\theta}_1 \]

IV. STABILITY INVESTIGATION

We put \( \dot{a} = \dot{\theta}_1 = 0 \), into equations (17) and (18), to get the steady state as:

\[ \frac{1}{2} \mu_1 a = -\frac{1}{8\omega_1} r \beta_2 a^3 \sin \theta_2 + \frac{1}{4\omega_1} r \beta a \sin \theta_2 \]

\[ + \frac{f_1}{2\omega_1} \sin \theta_1 + \frac{1}{2\omega_1} \mu_1 a \sin (\omega_1 T_1) - \frac{1}{2} d \cos (\omega_1 T_2) \]

\[ \dot{\theta}_1 = \sigma_1 - \frac{3}{8\omega_1} \beta_2 \sigma_2 - \frac{1}{4\omega_1} r \beta_2 a^2 \cos 2\theta_1 + \frac{1}{4\omega_1} r \beta_3 \cos 2\theta_1 \]

\[ + \frac{f_1}{2\omega_1} \cos \theta_1 - \frac{1}{2\omega_1} \mu_1 a \cos (\omega_1 T_2) - \frac{1}{2} d \sin (\omega_1 T_2) \]
To check the stability of the nonlinear solution based on Lyapunov’s direct method [34], we suppose that

\[ a = a_{10} + a_{11}, \quad \text{and} \quad \theta_1 = \theta_{10} + \theta_{11} \quad (21) \]

where \( a_{10} \) and \( \theta_{10} \) are the solutions of equations (17) and (18). Inserting equation (21) into equations (17),(18), and using only linear terms in \( a_{11} \) and \( \theta_{11} \), we obtain the system of differential equations

\[
\dot{a}_{11} = \left( -\frac{\mu_1}{2} - \frac{3r\beta_2}{8\omega_1}a_{10}^2\sin(2\theta_{10}) - \frac{r\beta}{4\omega_1}\sin(2\theta_{10}) + \frac{1}{2}\omega_1^2 \frac{1}{2} \sin(\omega_1\tau_1) - \frac{1}{2}d \sin(\omega_1\tau_2) \right) a_{11}
\]
the necessary and sufficient conditions that the system is stable, if the roots of equation (31) have negative real parts, and the following equation is verified:

\[ r_1 > 0, \quad r_1r_2 > 0 \]  

(27)

V. DISCUSSIONS AND NUMERICAL RESULTS

This section investigated the time history, bifurcation diagram and validation the numerical results with the analytical ones for the contact-mode AFM model before and after adding controller at the dimensionless system parameters of equations (2) as: \( \mu_1 = 0.05, \beta = 0.01, \beta_1 = 0.005, \beta_2 = 0.0004, \rho = 0.2, \omega_1 = 1, \Omega_1 = 1, \Omega_2 = 2, f_1 = 0.05, p = 2.0, d = 0.05. \)

A. SYSTEM WITH CONTROLLER

Figure 3 represents the controlled AFM time history at \( \Omega_1 \cong \omega_1, \Omega_2 \cong 2\omega_1. \) With this figure, the system amplitude has been suppressed from about 1 at steady state as in Figure 2 to about 0.02 as in Figure 3 at \( p = 2, d = 0.05 \) then, the controller efficiency \( E_a \) (\( E_a = \text{amplitude of uncontrolled system / amplitude of controlled system} \)) is about 50. Then the AFM vibration reduced by about 98%.

From Figure 4, we show that the amplitude has been suppressed to about 0.1 at \( \Omega_1 \cong \omega_1, \Omega_2 \cong 2\omega_1, p = 0.5, d = 0.05 \) then, the controller efficiency \( E_a \) is about 10 and the AFM amplitude is reduced by about 90%.

Figures 6 and 7 illustrate the controlled AFM response according to points A, B, C and D as in Figure 5. We observed
that the stable solutions located inside the stable region at the point A and Hopf curve occur at the boundary between the stable and unstable regions at B. In addition, the unstable solutions located inside the unstable region at the points C and D. Figure 6(a) shows stable periodic motion, quasi-periodic motion in Figure 6(b), and unstable motion in Figures 7(a, b). By comparing the data in Figure 5 with the results of Figures 6 and 7, we conclude that there is best validations between the numerical methods applied and the analytical ones.

Figure 8 shows the influnces of proportional control gain \( p \) on the stable solutions region in the \( \tau_1 - \tau_2 \) plane. From this figure, it is clear that for decreasing the proportional control gain \( p \), the stability region is increased. In addition, the region of stability in \( \tau_1 - \tau_2 \) plane is increasing, for increasing derivative control gain \( d \), as illustrated in Figure 9.
Figure 10 shows the detuning parameter $\sigma_1$ effects on frequency response curve for the system without and with control. We note that at the system without controller the maximum amplitude occurs when $\sigma_1 = 0$ ($\Omega_1 \approx \omega_1$, $\Omega_2 \approx 2\omega_1$) and the amplitude is about 1, while for system with control the amplitude at $\sigma_1 = 0$ is about 0.02. Then, the controller efficiency $E_a$ is about 50 and the AFM amplitude is reduced by about 98%. Figure 11 shows that for growing control gain $p$ the curve is moved to the right. We have inverse proportional in steady state amplitude with feedback gain $d$ and directed proportional to the excitation force $f_1$ respectively as shown in Figures 12, 13.

Figures 14, 15 shows that amplitude is inversely proportional to the natural frequencies $\omega_1$, and damping coefficient $\mu_1$. As will, to increasing natural frequency $\omega_1$ the curve is shifted to the right side. From growing values of nonlinear parameter $\beta_1$, the amplitude is increasing as seen in Figure 16. From Figure 17, we record that the maximum...
amplitude of is increasing with the increase at time delays \( \tau_1 \) values.

**B. VERIFICATION CURVES OF ANALYTICAL SOLUTIONS OF THE AFM SYSTEM**

From time response, Figures 18 and 19 shown the comparison among the numerical solution for the system equation (2) applying Rung-Kutta method and perturbation modulated amplitude of equations (17)-(18) using the multiple perturbation method with different values of natural and excitation frequencies at \( \Omega_1 \approx \omega_1, \Omega_2 \approx 2\omega_1 \). The stiff blue line agrees with the numerical integral, while the stiff red line show to perturbation solution. With these figures, we see that the analytical solutions are fully compatible with numerical solutions. Figure 20 explain a comparison among the frequency response curves utilizing MSP method in the uncontrolled system versus \( \sigma_1 \) also the numerical solution applying RKM in equation (2) at values of the parameters in Figure 2.

Figures 21 and 22 shows that the validated curves of close-ness between analytical and numerical solutions at different values of proportional and derivative control gains \( p, d \).

**VI. CONCLUSION**

In this work, the nonlinear vibrations of the AFM model in contact mode are reduced utilizing a time-delay PD controller. The effect of different parameters before and after applying the control on frequency response curves was investigated. The control and feedback signals gain effects respecting the vibration amplitude also stability behavior of the controlled system were studied with the case \( \Omega_1 \equiv \omega_1, \Omega_2 \equiv 2\omega_1 \). From this study, we remark the following:

- The efficiency \( E_a \) was about 50, and controller reduced the vibration by about 98%.
- The amplitude \( a \) of the system without control reached maximum value at \( \sigma_1 = 0 \) (\( \Omega_1 \equiv \omega_1, \Omega_2 \equiv 2\omega_1 \)) while after control the amplitude \( a \) of the system reached minimum value at \( \sigma_1 = 0 \), the controller is able to eliminate the vibrations.
- For increasing derivative control gain \( d \), the stability region in \( \tau_1 - \tau_2 \) plane is increasing.
- For increasing proportional control gain \( p \), the stability region in \( \tau_1 - \tau_2 \) plane is decreasing.
- The steady state amplitude grows with growing the values of excitation force \( f_1 \).
- The amplitude \( a \) has inverse proportional with the damping coefficient \( \mu_1 \) and the natural frequency \( \omega_1 \).
- For increasing the time delay \( \tau_1 \), the maximum amplitudes were increasing.
- Figures 18 to 22 show that a good agreement of close-ness between analytical and numerical solutions.

**CONFLICTS OF INTEREST**

The authors declare that there are no conflicts of interest associated with this publication.

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