Slow oscillating dynamics of a two-level system subject to a fast telegraph noise: beyond the NIBA approximation

V. V. Mkhitaryan and M. E. Raikh
Department of Physics and Astronomy, University of Utah, Salt Lake City, UT 84112

We study the dynamics of a two-site model in which the tunneling amplitude between the sites is not constant but rather a high-frequency noise. Obviously, the population imbalance in this model decays exponentially with time. Remarkably, the decay is modified dramatically when the level asymmetry fluctuates in-phase with fluctuations of the tunneling amplitude. For particular type of these in-phase fluctuations, namely, the telegraph noise, we find the exact solution for the average population dynamics. It appears that the population imbalance between the sites starting from 1 at time \( t = 0 \) approaches a constant value in the limit \( t \to \infty \). At finite bias, the imbalance goes to zero at \( t \to \infty \), while the dynamics of the decay governed by noise acquires an oscillatory character.

I. INTRODUCTION

The central problem in the field of dissipative dynamics\(^{1–9}\) is formulated as follows. Consider a two-site system described by the Hamiltonian

\[ H = \frac{1}{2} \left( \Delta \sigma_x + \varepsilon \sigma_z \right) , \]

where \( \Delta \) denotes the tunneling amplitude between the sites, while \( \varepsilon \) is the detuning between the on-site levels, i.e. the bias. Without interaction with bath, the dynamics of the system contains a single frequency \( \omega = (\Delta^2 + \varepsilon^2)^{1/2} \). Dissipative dynamics studies how the interaction with the bath in the form of the random modulation of \( \varepsilon \) slows down the oscillations of the population, \( \sigma_z(t) \), of the left site occupied at \( t = 0 \).

Original results\(^{1–9}\) on dissipative dynamics were obtained within non-interacting blip approximation (NIBA). The accuracy of NIBA, see e.g. Refs. \(^{10–17}\), is evaluated by comparison of the numerical results for bath-averaged \( \langle \sigma_z(t) \rangle \) obtained within NIBA with numerical results obtained within different versions of the master equations.

Certainly there is always a question: in what domain of the bath parameters NIBA is applicable. A related question is: are there physical effects which are not captured by NIBA.

In order to simplify the analysis as much as possible the authors of Ref. \(^{18}\) considered the case of very high temperatures when the bath can be viewed as a fast classical noise, \( b_z(t) \). Then the equations of motion, \( \frac{dS}{dt} = B \times S \), where \( B = (\Delta, 0, b_z(t)) \), i.e. at zero bias, can be reduced to the following closed equation for \( \sigma_z(t) \):\(^{18}\)

\[ \frac{d\sigma_z}{dt} = -\Delta^2 \int_0^t dt_1 \cos \left[ \int_{t_1}^t dt_2 b_z(t_2) \right] \sigma_z(t_1) , \]

which applies for an arbitrary realization of the noise. We are interested in the noise-averaged \( \langle \sigma_z(t) \rangle \). NIBA approach corresponds to the decoupling of average of the product in the integrand of Eq. \(^{2}\) into the product of averages

\[ \langle \cos \left[ \int_{t_1}^t dt_2 b_z(t_2) \right] \sigma_z(t_1) \rangle = \langle \cos \left[ \int_{t_1}^t dt_2 b_z(t_2) \right] \rangle \langle \sigma_z(t_1) \rangle . \]

For the Gaussian white noise one has

\[ \langle \cos \left[ \int_{t_1}^t dt_2 b_z(t_2) \right] \rangle = \exp[-\Gamma(t-t_1)] . \]

The magnitude, \( b_z \), and the short correlation time, \( \tau_z \), are encoded into parameter \( \Gamma \sim b_z \tau_z \). The integral equation Eq. \(^{2}\) reduces to the second-order differential equation for the average \( \langle \sigma_z(t) \rangle \)

\[ \frac{d^2 \langle \sigma_z \rangle}{dt^2} + \Gamma \frac{d\langle \sigma_z \rangle}{dt} + \Delta^2 \langle \sigma_z \rangle = 0 . \]

It should be noted that the white-noise assumption, \( b_z \tau_z \ll 1 \), justifies NIBA. Indeed, the dynamics described by Eq. \(^{1}\) has two characteristic times, \( 1/\Gamma \) and \( \Gamma/\Delta^2 \gg 1/\Gamma \). For white noise, these times are much longer than \( \tau_z \).

Assume now that, instead of a constant \( \Delta \), we have some random \( b_z(t) \). This hypothetical situation implies that tunneling between the sites, constituting a two-state system, is exclusively due to noise. Then the NIBA ansatz prescribes two independent averagings in the integrand. We thus get

\[ \frac{d\langle \sigma_z \rangle}{dt} = -\int_0^t dt_1 \langle b_z(t)b_z(t_1) \rangle \cos \left[ \int_{t_1}^t dt_2 b_z(t_2) \right] \langle \sigma_z(t_1) \rangle , \]

where \( \langle b_z(t)b_z(t_1) \rangle = b_z^2 \exp[-(t-t_1)/\tau_z] \), we find that \( \langle \sigma_z(t) \rangle \) exhibits a simple exponential decay

\[ \langle \sigma_z(t) \rangle = \exp \left[ -b_z^2 \tau_z + b_z^2 \tau_z t \right] , \]

with a single characteristic time. Again, NIBA is justified when both \( b_z \tau_z \) and \( b_z \tau_z \) are small.

The central question addressed in the present paper is: what happens when the noise components, \( b_z(t) \) and \( b_z(t) \), being both fast, are strongly correlated? What makes this question non-trivial is the fact that the average of \( b_z(t)b_z(t_1) \) now contains, in addition to fast, a slow contribution. On the other hand, for
applicability of NIBA, this average should change faster than \(\langle \sigma_z(t) \rangle\). Thus we ask ourselves: what is the spin dynamics when the condition of applicability of NIBA is violated? Fortunately, the answer to this question can be obtained purely analytically. This is because, for the telegraph noise, the average spin dynamics can be found exactly. We obtain this result in Sect. II. Comparing the NIBA and exact results, we demonstrate that NIBA applies for \(b \ll b^*_z\). It appears that, unlike Eq. (6), with correlated \(b_x(t)\) and \(b_z(t)\), both NIBA and exact results saturate at long time. In Sect. III we establish that the saturation takes place only at zero bias, \(\varepsilon = 0\). At any finite bias, the average, \(\langle \sigma_z(t) \rangle\), decays with time.

II. ZERO BIAS

A. NIBA

As it was demonstrated in Ref. 20 (see also the Appendix), for \(b_x(t)\) and \(b_z(t)\) in the form of the telegraph noise, the kernel in the NIBA equation Eq. (5) has the following form

\[
K(T) = \left\langle b_z(t) b_z(t+T) \cos \left[ \int_t^{t+T} dt' b_z(t') \right] \right\rangle = \left( \frac{b_z^2}{\tau_s - \tau_f} \right) \left[ \tau_s \exp \left( -\frac{T}{\tau_f} \right) - \tau_f \exp \left( -\frac{T}{\tau_s} \right) \right],
\]

where \(\tau_f\) and \(\tau_s\) denote the fast and slow relaxation times defined as

\[
\tau_f = \frac{\tau}{1 + (1 - b^2 z^2)^{1/2}}, \quad \tau_s = \frac{\tau}{1 - (1 - b^2 z^2)^{1/2}}.
\]

The NIBA equation with the kernel Eq. (7) yields the solution

\[
\langle \sigma_z(t) \rangle = \exp \left\{ -\int_0^t dt_1 \int_0^{t_1} dt_2 K(t_1 - t_2) \right\} = \exp \left\{ -\frac{b_z^2 \tau_s \tau_f}{\tau_s - \tau_f} \left[ \tau_s \left( 1 - e^{-t/\tau_s} \right) - \tau_f \left( 1 - e^{-t/\tau_f} \right) \right] \right\}.
\]

At long times the solution Eq. (9) saturates at the value

\[
\langle \sigma_z(t) \rangle \bigg|_{t \to \infty} = \exp \left( -\frac{b_z^2 \tau_s \tau_f}{\tau_s - \tau_f} \right) = \exp \left( -\frac{b_z^2}{b^*_z} \right).
\]

B. Exact solution

The key observation, which allows to solve the problem exactly, is that, with telegraph noise, the eigenvectors of the time-dependent Hamiltonian

\[
H = \frac{1}{2} \left[ b_x(t) \sigma_x + b_z(t) \sigma_z \right]
\]

are time-independent. This is because the ratio

\[
\tan \alpha = \frac{b_x(t)}{b_z(t)}
\]

remains constant at all times. In terms of \(\alpha\), the eigenvectors are

\[
\psi_1 = \left( \frac{\cos(\alpha/2)}{\sin(\alpha/2)} \right), \quad \psi_2 = \left( -\frac{\sin(\alpha/2)}{\cos(\alpha/2)} \right).
\]

The initial condition, \(\langle \sigma_z(0) \rangle = 1\), corresponds to the following form of the wavefunction at \(t = 0\)

\[
\psi(0) = \left( \begin{array}{c} 1 \\ 0 \end{array} \right) = \cos \left( \frac{\alpha}{2} \right) \psi_1 - \sin \left( \frac{\alpha}{2} \right) \psi_2.
\]

The time evolution of this wavefunction is given by

\[
\psi(t) = \cos \left( \frac{\alpha}{2} \right) \psi_1 \exp \left[ -i \frac{\phi(t)}{2} \right] - \sin \left( \frac{\alpha}{2} \right) \psi_2 \exp \left[ i \frac{\phi(t)}{2} \right],
\]

where the phase, \(\phi(t)\), is defined as

\[
\phi(t) = \frac{1}{\cos \alpha} \int_0^t dt' b_z(t').
\]

Using Eq. (15), we find the quantum-mechanical average of the operator \(\sigma_z\)

\[
\langle \psi(t) | \sigma_z | \psi(t) \rangle = \cos^2 \alpha + \sin^2 \alpha \cos \phi(t).
\]

The remaining task is to perform the averaging over the noise realizations. Note that only \(\cos \phi(t)\) has to be averaged. The details of this averaging are given in Appendix A. The final result reads

\[
\langle \sigma_z(t) \rangle = \frac{b_z^2}{b^2} + \frac{b_z^2}{b^2} e^{-t/\tau} \left[ \cosh \left( \frac{t}{\tau} \sqrt{1 - b^2 z^2} \right) \right] + \frac{1}{\sqrt{1 - b^2 z^2}} \sinh \left( \frac{t}{\tau} \sqrt{1 - b^2 z^2} \right),
\]

where \(b\) is the magnitude of the full field

\[
b = \left( b_x^2 + b_z^2 \right)^{1/2}.
\]

In the limit of a fast telegraph noise \(b \tau \ll 1\) the result Eq. (15) simplifies to

\[
\langle \sigma_z(t) \rangle = \frac{b_z^2}{b^2} + \frac{b_z^2}{b^2} \exp \left( - \frac{b_z^2 t}{2} \right).
\]

In the next subsection we compare this result with the NIBA prediction Eq. (9).

C. Comparison of exact and NIBA results

Expression Eq. (15) is our central result. It describes the evolution of \(\langle \sigma_z(t) \rangle\) for arbitrary relation between
FIG. 1: (Color online) Comparison of the exact and NIBA solutions for \( \langle \sigma_z(t) \rangle \). (a) \( \langle \sigma_z(t) \rangle \) is plotted versus the dimensionless time from Eq. (22) (solid lines) and Eq. (23) (dotted lines) for two values of \( \beta \) in the regime of fast noise \( b_z \tau \approx 0.1 \). (b) The same expressions are plotted for slow noise \( b_z \tau = 5 \).

The exact and the NIBA solutions coincide at small times. The difference at long times develops when \( \beta \) is large.

Then Eq. (18) assumes the form

\[
\tilde{t} = b_z^2 \tau t, \quad \beta = \frac{b_z^2}{b_z^2} = \tan^2 \alpha. 
\] (21)

Both expressions are plotted in Fig. 1 for different values of parameters \( \beta \) and different noise “strengths” \( b_z \tau \). We see that NIBA reproduces the exact result at small \( \beta \), i.e. at \( b_z \ll b_z \). This could be anticipated from the analytical expression Eq. (23) since it saturates at \( \exp(-\beta) \), while the exact saturation value is \( \langle \sigma_z(\infty) \rangle = \frac{1}{1 + \beta} \). At small \( \beta \) the net change of \( \langle \sigma_z \rangle \) from \( t = 0 \) and \( t = \infty \) is small. This justifies taking \( \langle \sigma_z(t) \rangle \) out of the integrand in the NIBA equation Eq. (19). It is worth noting that both solutions oscillate at large \( b_z \tau \). Emergence of these weakly decaying oscillations is not obvious a priori since they are noise-induced. The oscillations become possible because, for telegraph noise, the full field switches between two fixed values.

The fact that \( \langle \sigma_z(t) \rangle \) can be calculated exactly for the telegraph-noise fluctuations of \( b_z(t) \), \( b_z(t) \) is long known in the literature on stochastic dynamics.\(^{23,24}\) In particular, in the chapter of the book Ref. 24 on stochastic differential equations, the following problem is considered. Suppose that the frequency of an oscillator takes only two values with equal probabilities. Then the noise-averaged dynamics of the oscillator, derived from the Fokker-Planck description, has the form closely resembling Eq. (18). In fact, our result Eq. (18) reduces to the result\(^{24}\) for two-frequency oscillator problem if we set \( b_z(t) = 0 \). This is the extreme “non-NIBA” limit. Pres-
III. FINITE BIAS

A. Spin dynamics within NIBA

In the presence of a finite bias, \( \varepsilon \), which adds to the random field \( b_z(t) \), the NIBA equation Eq. (5) assumes the form

\[
d\langle \sigma_z(t) \rangle = \frac{d\langle \sigma_z(t) \rangle}{dt} = F_0 - F_c(t) \cos \varepsilon t - F_s(t) \sin \varepsilon t
\]

(Eq. 24)

The solution of Eq. (24) is a straightforward generalization of Eq. (9)

\[
\langle \sigma_z(t) \rangle = \exp \left\{ - \int_0^t dt_1 \int_0^{t_1} dt_2 \cos [\varepsilon (t_1 - t_2)] K(t_1 - t_2) \right\}
\]

(Eq. 25)

With correlator Eq. (7), the integration can be performed analytically

\[
\langle \sigma_z(t) \rangle_{\text{NIBA}} = \exp \left\{ - \kappa t - F_0 - F_c(t) \cos \varepsilon t - F_s(t) \sin \varepsilon t \right\},
\]

(Eq. 26)

where

\[
\kappa = -\frac{b_z^2 \tau \varepsilon^2 (\tau_f^2 - \tau_s^2)}{2 \sqrt{1 - (b_z \tau)^2} (1 + \varepsilon^2 \tau_f^2) (1 + \varepsilon^2 \tau_s^2)}
\]

\[
F_0 = -\frac{b_z^2 \tau}{2 \sqrt{1 - (b_z \tau)^2}} \left[ \frac{\tau_f (1 - \varepsilon^2 \tau_f^2)}{(1 + \varepsilon^2 \tau_f^2)^2} \exp \left( -\frac{t}{\tau_f} \right) - \frac{\tau_s (1 - \varepsilon^2 \tau_s^2)}{(1 + \varepsilon^2 \tau_s^2)^2} \exp \left( -\frac{t}{\tau_s} \right) \right],
\]

\[
F_c = -\frac{b_z^2 \tau}{2 \sqrt{1 - (b_z \tau)^2}} \left[ \frac{\tau_f (1 - \varepsilon^2 \tau_f^2)}{(1 + \varepsilon^2 \tau_f^2)^2} \exp \left( -\frac{t}{\tau_f} \right) - \frac{\tau_s (1 - \varepsilon^2 \tau_s^2)}{(1 + \varepsilon^2 \tau_s^2)^2} \exp \left( -\frac{t}{\tau_s} \right) \right]
\]

(Eq. 27)

First we note that, at arbitrary nonzero bias, \( \langle \sigma_z(t) \rangle \to 0 \) instead of saturation at long times. The decay of \( \langle \sigma_z(t) \rangle \) is governed by the factor \( \exp(-\kappa t) \) in Eq. (26), in which \( \kappa \) is proportional to \( \varepsilon^2 \). Presence of the terms \( F_c \) and \( F_s \) in the exponent of Eq. (26) suggests oscillatory behavior of \( \langle \sigma_z(t) \rangle \) with the frequency \( \varepsilon \) at intermediate times. On the other hand, the prefactors \( F_c \) and \( F_s \) decay at long times. Thus it is not clear a priori whether these oscillations can be resolved. In Fig. 2 we present numerical curves plotted from Eq. (26) for different biases. We realize that oscillations are not developed in \( \langle \sigma_z(t) \rangle \) but are resolved in the derivatives, \( d\langle \sigma_z(t) \rangle/dt \). It is an interesting question whether these oscillations survive beyond NIBA. We address this question in the next section.

B. Beyond NIBA: small bias limit

Exact solution Eq. (11) applies not only to the telegraph noise but to arbitrary \( b_z(t) \) and \( b_x(t) \) as long as
the ratio $\frac{\theta}{x} = \tan \alpha$ remains constant. We will derive the evolution $\langle \sigma_x(t) \rangle$ assuming that the bias, $\varepsilon$, is much smaller than $b$. We start from the system of equations for “up” and “down” amplitudes of spin

$$i \frac{du_1}{dt} = \frac{1}{2} \left[ b_x(t) + \varepsilon \right] c_1 + \frac{1}{2} b_x(t) c_2,$$

$$i \frac{du_2}{dt} = -\frac{1}{2} \left[ b_x(t) + \varepsilon \right] c_2 + \frac{1}{2} b_x(t) c_1. \quad (28)$$

As in the case of zero bias, we introduce the linear combinations

$$c_1 = u_1 \cos \left( \frac{\alpha}{2} \right) - u_2 \sin \left( \frac{\alpha}{2} \right),$$

$$c_2 = u_1 \sin \left( \frac{\alpha}{2} \right) + u_2 \cos \left( \frac{\alpha}{2} \right). \quad (29)$$

The system of equations for the new variables $u_1$ and $u_2$ reads

$$i \frac{du_1}{dt} = \frac{1}{2} \left[ b_x(t) + \varepsilon \right] u_1 - \frac{1}{2} \varepsilon \sin \alpha \, u_2,$$

$$i \frac{du_2}{dt} = -\frac{1}{2} \left[ b_x(t) + \varepsilon \right] u_2 - \frac{1}{2} \varepsilon \sin \alpha \, u_1. \quad (30)$$

It is easy to see that at zero bias, $\varepsilon = 0$, the system Eq. (30) gets decoupled. Using the initial conditions

$$u_1(0) = \cos \left( \frac{\alpha}{2} \right), \quad u_2(0) = -\sin \left( \frac{\alpha}{2} \right), \quad (31)$$

the exact result Eq. (17) can be reproduced.

At finite $\varepsilon$, in Eq. (30) we make the substitution

$$u_1(t) = v_1(t) \exp \left[ -\frac{i}{2} \Phi(t) \right],$$

$$u_2(t) = v_2(t) \exp \left[ \frac{i}{2} \Phi(t) \right], \quad (32)$$

were we have introduced the short-hand notation

$$\Phi(t) = \frac{1}{\cos \alpha} \int_0^t dt' \left[ b_x(t') + \varepsilon \cos^2 \alpha \right]. \quad (33)$$

For $\varepsilon = 0$ we have $\Phi(t) = \phi(t)$, where $\phi$ is defined by Eq. (10). The substitution Eq. (32) yields the following system of coupled equations for the variables $v_1(t)$, $v_2(t)$

$$i \frac{dv_1}{dt} = -\frac{1}{2} \varepsilon v_2 \sin \alpha \exp \left[ i \Phi(t) \right],$$

$$i \frac{dv_2}{dt} = -\frac{1}{2} \varepsilon v_1 \sin \alpha \exp \left[ -i \Phi(t) \right]. \quad (34)$$

Substituting the second equation into the first, we arrive to the closed integral-differential equation for $v_1(t)$

$$i \frac{dv_1}{dt} = -\frac{i}{2} \varepsilon \sin \alpha \, v_2(0) \exp \left[ i \Phi(t) \right]$$

$$-\frac{\varepsilon^2}{4} \sin^2 \alpha \int_0^t dt' v_1(t') \exp \left[ i \left( \Phi(t) - \Phi(t') \right) \right]. \quad (36)$$

This equation applies for arbitrary bias. At this point we note that, since the right-hand side is proportional to $\varepsilon$, for small $\varepsilon$ the derivative $\frac{dv_1}{dt}$ is small. Thus, the function $v_1(t)$ changes slowly with time. This allows one to pull $v_1(t)$ out of the integrand. Then Eq. (36) turns into a first-order differential equation which can be readily solved yielding

$$v_1(t) = v_1(0) \exp \left[ -\frac{\varepsilon^2}{4} \sin^2 \alpha \int_0^t dt' G(t') \right] + i \frac{\varepsilon}{2} \sin \alpha \, v_2(0) \int_0^t dt' \exp \left[ i \Phi(t') - \frac{\varepsilon^2}{4} \sin^2 \alpha \left( G(t) - G(t') \right) \right]. \quad (37)$$

where the function $G(t)$ is defined as

$$G(t) = \int_0^t dt' \exp \left[ i \left( \Phi(t) - \Phi(t') \right) \right]. \quad (38)$$

Corresponding expression for $v_2(t)$ follows from Eq. (37) upon replacement $v_1(0) \rightarrow v_2(0)$, $v_2(0) \rightarrow v_1(0)$, and $\Phi(t) \rightarrow -\Phi(t)$.

The sought quantity, $\sigma_z(t)$, is expressed via the functions $v_1(t)$ and $v_2(t)$ as follows

$$\sigma_z(t) = \left( |v_1|^2 - |v_2|^2 \right) \cos \alpha$$

$$-\sin \alpha \left( v_1 v_2^* \exp \left[ -i \Phi(t) \right] + v_1^* v_2 \exp \left[ i \Phi(t) \right] \right). \quad (39)$$

A crucial step in performing averaging in Eq. (39) is that the second term in the expression for $v_1(t)$ proportional to $\varepsilon$ is small compared to the first term. The result of averaging over realizations of the telegraph noise reads

$$\langle \sigma_z(t) \rangle = \frac{b_x^2}{b^2} + \frac{b_x^2}{b^2} \exp \left( \frac{-b_x^2 \tau}{2} \right) \cos \left( \frac{\varepsilon b_x \tau}{b^2} \right) \exp \left( \frac{-2 \varepsilon^2 b_x^2}{b^4 \tau} \right). \quad (40)$$

The origin of the oscillating factor $\cos \left( \frac{\varepsilon b_x \tau}{b^2} \right)$ is the term $\sim \varepsilon$ in $\Phi(t)$ defined by Eq. (35). The common exponential factor originates from averaging of

$$\exp \left[ -\frac{\varepsilon^2}{4} \sin^2 \alpha \int_0^t dt' G(t') \right].$$

We can now compare the result Eq. (40) with the NIBA result Eq. (25). The decrement of $\langle \sigma_z(t) \rangle_{\text{NIBA}}$ in the limit $b_x \tau \ll 1$ can be cast in the form

$$\kappa = \frac{b_x^2 \tau \varepsilon^2 \tau_s^2}{2 \left( 1 + \varepsilon^2 \tau_s^2 \right)} \frac{\varepsilon^2 \tau_s b_x^2}{\left( 1 + \varepsilon^2 \tau_s^2 \right) b_x^2}. \quad (41)$$

where $\tau_s \approx \frac{2}{b_x^2}$ is defined by Eq. (35). In the limit of small bias considered above we have $\varepsilon \tau_s \ll 1$. Then the
decrement Eq. (11) reproduces the common exponential factor in Eq. (10) under the condition \( b_x \ll b_z \). This is the same condition under which NIBA applies at zero bias. Equally, expanding the exponent in the NIBA result with respect to \( F_z(t) \cos \epsilon t \) and assuming \( b_x \ll b_z \), we reproduce the oscillating part of Eq. (10).

Overall, the small-bias regime is quantified by the condition \( \varepsilon \tau_s \ll 1 \). This justifies the reduction of Eq. (30) to the first-order differential equation. Also the second term in Eq. (37) is of the order of \( \varepsilon \tau_s \).

IV. DISCUSSION

1. Physical situation considered in the present paper corresponds to the noise created by a fluctuator, see e.g. Ref. 25, rather than the noise created by the continuum of harmonic oscillators commonly considered in the literature on quantum dissipation.

2. Our central conclusion is that, when the components \( b_x(t) \) and \( b_z(t) \) are fully correlated, the average \( \langle \sigma_z(t) \rangle \) is the same condition under which NIBA applies at zero bias. This is the same condition under which NIBA applies. In the opposite limit, \( b_x \gg b_z \), fluctuations of \( \sigma_z(t) \) from realization to realization are strong.

Note that anomalously strong sensitivity of the spin dynamics to the finite bias is long known in the field of \( \varepsilon \rightarrow 0 \).

3. NIBA result Eq. (26) contains two times, \( \tau_s \) and \( \tau_f \). Exact result also contains two characteristic times but in stead of the combination \( \sqrt{1 - b_z^2 \tau_f^2} \) these times contain \( \sqrt{1 - (b_x^2 + b_z^2) \tau_f^2} \). NIBA applies in the limit \( b_x \ll b_z \) when these times are close to each other.

4. Unlike Ref. 18 \( \langle \sigma_z(t) \rangle \) is the average over the realizations of the telegraph noise. In fact, the derivation of the result Eq. (17) can be modified to find the variance

\[
\text{Var}(\sigma_z) = \sin^4 \alpha \left[ \langle \cos^2 \phi(t) \rangle - \langle \cos \phi(t) \rangle^2 \right].
\]

Analytical form of \( \langle \cos \phi(t) \rangle \) was found above. To find \( \langle \cos^2 \phi(t) \rangle \) one can use the same expression with \( b \) replaced by \( 2b \). This follows from the relation

\[
\cos \left[ 2\phi(t) \right] = \cos \left[ \frac{2}{\cos \alpha} \int_0^t dt' b_z(t') \right].
\]

In the long-time limit the variance saturates together with average

\[
\text{Var}[\sigma_z(t)] \bigg|_{t \to \infty} = \frac{1}{2} \sin^4 \alpha = \frac{b_z^4}{2b^4},
\]

Note that the fluctuations of \( \sigma_z(\infty) \) are much smaller than the average when \( b_x \ll b_z \), i.e. in the same domain where NIBA applies.

V. ACKNOWLEDGEMENTS

This work was supported by the Department of Energy, Office of Basic Energy Sciences, Grant No. DE-FG02-06ER46313.

Appendix A

In this Appendix we evaluate the average, \( \langle \cos \phi(t) \rangle \), where the phase \( \phi(t) \) is defined by Eq. (16). In the course of the noise, the magnetic field switches from \( -b \) to \( b \) at random time moments. The durations of intervals between the successive switchings are Poisson-distributed as \( \frac{1}{2} \exp(-\frac{t}{\tau}) \). Thus, averaging over the noise realizations reduces to averaging over these intervals

\[
\langle \cos \phi(t) \rangle = \text{Re} \sum_{n=0}^{\infty} \int_0^\infty \frac{dt_1}{\tau} e^{-\frac{t_1}{\tau}} \cdots \int_0^\infty \frac{dt_{n+1}}{\tau} e^{-\frac{t_{n+1}}{\tau}} \times e^{ib(t_1-t_2+\cdots+(1)^n t_{n+1})} \left[ \theta(t-\sum_{k=1}^{n+1} t_k) - \theta(-\sum_{k=1}^{n+1} t_k) \right].
\]

Here \( n \) is the number of field flips during the time \( t \). This is ensured by the difference of the \( \theta \)-functions which imposes the condition \( \sum_{k=1}^{n+1} t_k < t < \sum_{k=1}^{n} t_k \).

Taking the integral over \( t_{n+1} \) by parts yields

\[
\langle \cos \phi(t) \rangle = \text{Re} \sum_{n=0}^{\infty} \int_0^\infty \frac{dt_1}{\tau} \cdots \int_0^\infty \frac{dt_n}{\tau} \int_0^\infty \frac{dt_{n+1}}{\tau} e^{-\frac{t_{n+1}}{\tau}} \times e^{ib(t_1-t_2+\cdots+(1)^n t_{n+1})} \delta(t-\sum_{k=1}^{n+1} t_k). 
\]

The individual integrals over \( t_i \) can be taken upon using the integral representation, \( \delta(z) = \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{isz} \). This leads to

\[
\langle \cos \phi(t) \rangle = \text{Re} \sum_{k=1}^{\infty} \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{ist} \frac{2\tau + i(s+b)\tau^2}{(1 + is\tau)^2 + b^2\tau^2}.
\]

We next take the sum in Eq. (A3) and symmetrize the result with respect to the sign of \( b \) at \( t = 0 \). This yields

\[
\langle \cos \phi(t) \rangle = \text{Re} \int_{-\infty}^{\infty} \frac{ds}{2\pi} e^{ist} \frac{2\tau + is\tau^2}{(1 + is\tau)^2 + b^2\tau^2 - 1}.
\]

The latter integral is calculated by adding up the contributions of two poles, located at \( s = \frac{i}{\tau} \pm (b^2 - \frac{1}{\tau})^{1/2} \).
For $b\tau < 1$, we find
\[
\langle \cos \phi(t) \rangle = \exp \left( -\frac{t}{\tau} \right) \tag{A5}
\]
\[
\times \left\{ \cosh \left[ \frac{t}{\tau} \left( 1 - b^2\tau^2 \right)^{1/2} \right] + \frac{\sinh \left[ \frac{t}{\tau} \left( 1 - b^2\tau^2 \right)^{1/2} \right]}{(1 - b^2\tau^2)^{1/2}} \right\}.
\]
Substituting Eq. (A5) into Eq. (17) leads to our main result Eq. (18).
To find the average defining the kernel $K(T)$ through Eq. (7) we note the relation,
\[
K(t_2 - t_1) = \left( \frac{b^2}{b^2} \right) \frac{\partial^2}{\partial t_1 \partial t_2} \langle \cos \phi(t_2 - t_1) \rangle. \tag{A6}
\]
This relation yields
\[
K(T) = b_T^2 \exp \left( -\frac{T}{\tau} \right) \tag{A7}
\]
\[
\times \left\{ \cosh \left[ \frac{T}{\tau} \left( 1 - b^2\tau^2 \right)^{1/2} \right] - \frac{\sinh \left[ \frac{T}{\tau} \left( 1 - b^2\tau^2 \right)^{1/2} \right]}{(1 - b^2\tau^2)^{1/2}} \right\}.
\]
Note that two terms in $\{,\}$ in Eq. (A5) add up, while in Eq. (A7) they subtract.

1. A. O. Caldeira and A. J. Leggett, “Influence of Dissipation on Quantum Tunneling in Macroscopic Systems,” Phys. Rev. Lett. 46, 211 (1981).
2. A. J. Bray and M. A. Moore, “Influence of Dissipation on Quantum Coherence,” Phys. Rev. Lett. 49, 1545 (1982).
3. H. Grabert and U. Weiss, “Quantum Tunneling Rates for Asymmetric Double-Well Systems with Ohmic Dissipation,” Phys. Rev. Lett. 54, 1605 (1985).
4. A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, “Dynamics of the dissipative two-state system,” Rev. Mod. Phys. 59, 1 (1987).
5. M. Grifoni and P. Hänggi, “Driven quantum tunneling,” Phys. Rep. 304, 229 (1998).
6. K. Le Hur, “Quantum Phase Transitions in Spin-Boson Systems: Dissipation and Light Phenomena,” in Understanding Quantum Phase Transitions, edited by Lincoln D. Carr (Taylor and Francis, Boca Raton, 2010).
7. U. Weiss, Quantum Dissipative Systems, 4th ed. (World Scientific, Singapore, 2012).
8. I. de Vega and D. Alonso, “Dynamics of non-Markovian open quantum systems,” Rev. Mod. Phys. 89, 015001 (2017).
9. L. M. Cangemi, V. Cataudella, M. Sassetti, G. De Filippis, “Dissipative dynamics of a driven qubit: interplay between non-adiabatic dynamics and noise effects from weak to strong coupling regime,” Phys. Rev. B 100, 014301 (2019).
10. L. M. Cangemi, G. Passarelli, V. Cataudella, P. Lucignano, and G. De Filippis, “Beyond the Born-Markov approximation: Dissipative dynamics of a single qubit,” Phys. Rev. B 98, 184306 (2018).
11. M. Wiedmann, J. T. Stockburger, and J. Ankerhold, “Time-correlated blip dynamics of open quantum systems,” Phys. Rev. A 94, 052137 (2016).
12. S. Bera, H. U. Baranger, and S. Florens, “Dynamics of a qubit in a high-impedance transmission line from a bath perspective,” Phys. Rev. A 93, 033847 (2016).
13. H. Shapourian, “Dynamical renormalization-group approach to the spin-boson model,” Phys. Rev. A 93, 032119 (2016).
14. P. P. Orth, A. Imambekov, and K. Le Hur, “Nonperturbative stochastic method for driven spin-boson model,” Phys. Rev. B 87, 014305 (2013).
15. F. Nesi, E. Paladino, M. Thorwart, and M. Grifoni, “Spin-boson dynamics beyond conventional perturbation theories,” Phys. Rev. B 76, 155323 (2007).
16. M. Thoss, H. Wang, and W. H. Miller, “Self-consistent hybrid approach for complex systems: Application to the spin-boson model with Debye spectral density,” J. Chem. Phys. 115, 2991 (2001).
17. C. H. Mak and R. Egger, “Quantum Monte Carlo study of tunneling diffusion in a dissipative multistate system,” Phys. Rev. E 49, 1997 (1994).
18. G. B. Lesovik, A. V. Lebedev, and A. O. Imambekov, “Dynamics of Two-Level System Interacting with Random Classical Field,” JETP Lett. 75, 474 (2002).
19. This reformulation of the problem was first proposed by H. Dekker, “Noninteracting-blip approximation for a two-level system coupled to a heat bath,” Phys. Rev. A 35, 1436 (1987).
20. V. V. Mkhitaryan, C. Boehme, J. M. Lupton, and M. E. Raikh, “Two-photon absorption in a two-level system enabled by noise,” Phys. Rev B 100, 214205 (2019).
21. P. W. Anderson and P. R. Weiss, “Exchange Narrowing in Paramagnetic Resonance,” Rev. Mod. Phys. 25, 269 (1953).
22. R. Kubo, “Note on the Stochastic Theory of Resonance Absorption,” J. Phys. Soc. Jpn. 9, 935 (1954).
23. J. R. Klauer and P. W. Anderson, “Spectral Diffusion Decay in Spin Resonance Experiments,” Phys. Rev. 125, 162 (1962).
24. N. van Kampen, Stochastic Processes in Physics and Chemistry (North-Holland, 1981).
25. E. Paladino, Y. M. Galperin, G. Falci, and B. L. Altshuler, “1/f noise: Implications for solid-state quantum information,” Rev. Mod. Phys. 86, 361 (2014).