Matrix Quantum Mechanics for Supermembrane on $AdS_7 \times S^4$

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Abstract

We explore the light-cone gauge formulation of a closed supermembrane on $AdS_7 \times S^4$. We obtain the action of matrix quantum mechanics with large $N$ $U(N)$ gauge symmetry for the light-cone supermembrane. We show that this action reproduces leading order terms in $\alpha'$-expansion of the non-abelian Born-Infeld action of $N$ D0-branes propagating near the horizon of D4-branes. The matrix quantum mechanics obtained in this paper, therefore, has an interpretation as Matrix theory in the near-horizon of D4-branes.
1 Introduction

Supermembranes [1] play important roles in M-theory. For example, they lead to the equations of motion of the low-energy effective field theory of M-theory (i.e. eleven-dimensional supergravity) [1][2], provide string states in type IIA string theory [2] and in $E_8 \times E_8$ heterotic string theory [3], and become D2-branes in type IIA string theory [4][5]. Besides these facts, in Matrix theory [6], infinitely many D0-branes are conjectured to capture all physical degrees of freedom of M-theory in the infinite momentum frame and the resulting D0-brane action [7][8] coincides with that of a closed light-cone supermembrane in the flat space [9]. Supermembranes are therefore expected to play a pivotal role in pursuit of the microscopic description of M-theory.

Soon after Matrix theory was proposed, its curved-space extension was investigated from the viewpoint of D0-brane physics in e.g. refs.[10][11][12]. We may naturally expect that as well as these analyses, the investigation of light-cone supermembranes in curved spaces should provide another promising approach to this problem, taking into account the above-mentioned coincidence in the flat space between the action of D0-branes and that of a light-cone supermembrane. The analyses along this line were initiated by ref.[13].

In ref.[13], quite general results are obtained regarding the actions of bosonic light-cone membranes on curved backgrounds and those of the corresponding matrix theories, and then contributions of the fermionic coordinate $\Theta$ of supermembranes are included up to quadratic order. The actions of supermembranes in curved spaces are described in terms of the supervielbeins and the three-form superfields in the eleven-dimensional $\mathcal{N} = 1$ superspace [1]. Since the fermionic coordinate $\Theta$ in the superspace is an $SO(10,1)$ Majorana spinor, the superfields are, in general, 32nd order polynomials of $\Theta$. In generic situation, it becomes an intricate task to determine the superfields in all order of $\Theta$ [13]. This fact leads us to specific cases in which the backgrounds possess large isometry, because the restriction of the symmetry is expected to facilitate the task. In fact, for the maximally supersymmetric solutions of eleven-dimensional supergravity: Minkowski space, $AdS_4 \times S^7$, $AdS_7 \times S^4$ and the pp-wave solution, the full-order forms of the supervielbeins, super spin-connections and the three-form superfields are determined. In the $AdS_4 \times S^7$ and the $AdS_7 \times S^4$ cases, the superfields are obtained in refs.[14][15][16]. These superfields are reduced to those of the pp-wave background in the Penrose limit [17][18].

Related to the flat case, in refs.[19][20][21], a light-cone supermembrane is considered in the eleven-dimensional Minkowski space with non-trivial periodicity from which the ten-dimensional Kaluza-Klein Melvin background is derived. In refs.[22][23], the light-cone gauge formulation of a supermembrane on the pp-wave background is constructed and it is shown that the resulting system becomes the matrix model proposed by Berenstein, Maldacena and Nastase (BMN) [24]. Several properties of the BMN matrix model are studied also by refs.[25][26][27].

In this paper, we study the light-cone gauge formulation of a closed supermembrane on $AdS_7 \times S^4$. Similarly to the flat and the pp-wave cases, we obtain the matrix quantum mechanics for the supermembrane. We show that the matrix action of this system takes the same form as that of D0-branes propagating near the horizon of D4-branes. This result is
consistent with the fact that $AdS_7 \times S^4$ is obtained as near-horizon geometry \cite{28,29} of the M5-brane solution.

M-theory on $AdS_7 \times S^4$ is proposed to be dual to six-dimensional superconformal field theory on the M5-brane world-volume \cite{30,31,32,33}. This is an example of the $AdS/CFT$ correspondence conjectured by Maldacena \cite{29}. To obtain insights into this $AdS/CFT_6$ correspondence, (semi-)classical analyses of supermembranes on $AdS_7 \times S^4$ are carried out in refs. \cite{34,35,36,37}. Our result should give a new approach to the study of this correspondence.

We make a comment on light-cone supermembranes on $AdS_4 \times S^7$. In the $AdS_4 \times S^7$ solution, the three-form gauge potential $C_{\hat{m}\hat{n}\hat{p}}$ does not satisfy the relation $C_{+-}\hat{m} = 0$. Hence, the $X^-$-dependence is not straightforwardly eliminated from the light-cone hamiltonian of the supermembrane \cite{13}. It might deserve more study \cite{13}. In this paper, we will not pursue this case further.

It is worth mentioning that light-cone Green-Schwarz superstrings on $AdS_5 \times S^5$ are studied \cite{38,39,40}. Our investigation is therefore an extension of these string analyses into the supermembrane case.

This paper is organized as follows: In Section 2, we construct light-cone gauge formulation of a supermembrane on $AdS_7 \times S^4$. The resulting action may be regarded as that of quantum mechanics with area-preserving diffeomorphism gauge symmetry. We employ the matrix regularization \cite{11,9,12} for this action. The system consequently becomes matrix quantum mechanics with large $N$ $U(N)$ gauge symmetry. In Section 3, we show that the matrix action obtained in Section 2 coincides with that of large $N$ D0-branes propagating near the horizon of D4-branes. This implies that the light-cone supermembrane on $AdS_7 \times S^4$ has an interpretation in Matrix theory, similarly to the flat case. Section 4 is devoted to conclusions and discussions. In Appendix A, we provide conventions of the $SO(10,1)$ gamma matrices and spinors used in this paper. In Appendices B and C, rearrangements and detailed forms of several equations are presented.

## 2 Light-Cone Gauge Formulation

### 2.1 Supermembrane action on $AdS_7 \times S^4$

The action of a supermembrane on curved space-time is given in ref.\cite{11} as

$$S = \int d^3\xi \mathcal{L} = -T \int d^3\xi \left[ \sqrt{-\det h_{ij}} + \frac{\epsilon^{ijk}}{3!} \Pi_i A^j \Pi_k C_{ABC} \right], \quad (2.1)$$

where $T$ is the membrane tension, which is described in terms of the eleven-dimensional Planck length $l_p$ as

$$T = \frac{1}{(2\pi)^{2/3} l_p^3}, \quad (2.2)$$

$\xi^i = (\tau, \sigma^i) \ (i = 1, 2)$ are the world-volume coordinates on the membrane, and $\epsilon^{ijk}$ and $h_{ij}$ are respectively the anti-symmetric tensor density and the induced metric on the world-volume.
defined as
\[ \epsilon^{a_1 a_2} = -1 ; \quad h_{ij} = \Pi^i \Pi^j \eta_{a b} , \quad \Pi^A = \partial_i Z^M E^A_M \ (A = \dot{a}, \dot{a} ; \ M = \dot{m}, \dot{m}) . \] (2.3)

Here \( Z^M = (X^\dot{m}, \Theta^\dot{a}) \) are the coordinates on the eleven-dimensional curved superspace, \( E^A_M \) denotes the supervielbein and \( \eta_{a b} \) is the eleven-dimensional Minkowski metric in the local Lorentz frame. The fermionic coordinate \( \Theta^\dot{a} \) is a Majorana spinor of the \( SO(10,1) \) local Lorentz group. The conventions of \( SO(10,1) \) spinors and the gamma matrices used in this paper are presented in Appendix A.

The three-form
\[ C^{(3)} = \frac{1}{3!} \Pi^A \wedge \Pi^B \wedge \Pi^C C_{ABC} , \quad \Pi^A = dZ^M E^A_M , \] (2.4)
is the potential for the four-form field-strength
\[ F^{(4)} = dC^{(3)} = \frac{1}{4!} \Pi^A \wedge \Pi^B \wedge \Pi^C \wedge \Pi^D F_{ABCD} . \] (2.5)

In the superspace geometry of eleven-dimensional supergravity, there exists a closed four form
\[ F^{(4)} = \frac{1}{4!} \left( \Pi^\dot{a} \wedge \Pi^\dot{b} \wedge \Pi^\dot{c} \wedge \Pi^\dot{d} F_{\dot{a} \dot{b} \dot{c} \dot{d}} + 12 i \dot{\Pi} \Theta_{\dot{a} \dot{b} \dot{c} \dot{d}} \wedge \Pi^{\dot{a}} \wedge \Pi^{\dot{b}} \wedge \Pi^{\dot{c}} \right) . \] (2.6)

We consider the \( AdS_7 \times S^4 \) solution of eleven-dimensional supergravity. The non-vanishing components of the Riemannian tensor and the four-form field-strength of this solution are
\[ R_{abcd} = - \frac{1}{L^2} (\eta_{ac} \eta_{bd} - \eta_{ad} \eta_{bc}) , \]
\[ R_{\alpha' \beta' \gamma' \delta'} = \frac{4}{L^2} (\delta_{\alpha' \gamma'} \delta_{\beta' \delta'} - \delta_{\alpha' \delta'} \delta_{\beta' \gamma'}) , \]
\[ F_{\alpha' \beta' \gamma' \delta'} = - \eta \frac{6}{L} \varepsilon_{\alpha' \beta' \gamma' \delta'} , \quad \eta = \pm 1 , \] (2.7)
where \( a, b, c, d \) and \( \alpha', \beta', \gamma', \delta' \) are local Lorentz indices in the \( AdS_7 \) and the \( S^4 \) directions respectively, and \( L \) is the radius of \( AdS_7 \), which is related to the radius \( L_{S^4} \) of \( S^4 \) by \( L_{S^4} = \frac{1}{2} L \). Via a supercoset construction, the supervielbein one-form \( \Pi^A = dZ^M E^A_M \) and the superconnection one-form \( \Omega^{\dot{a} \dot{b}} = dZ^M \Omega^{\dot{a} \dot{b}}_M \) for this background are obtained \[14\] in the all order of the fermionic coordinate \( \Theta \) as\(^1\)
\[ \Pi^\dot{a} = \sum_{k=0}^{15} \frac{1}{(2k+1)!} (\mathcal{M}^{2k})^{\dot{a} \dot{b}} (\tilde{D} \Theta)^{\dot{b}} , \]
\[ \Pi^{\dot{a}} - e^{\dot{a}} - 2i \sum_{k=0}^{15} \frac{1}{(2k+2)!} \tilde{\Theta} \Gamma^{\dot{a} \dot{b}} \mathcal{M}^{2k} \tilde{D} \Theta , \]
\[ \Omega^{\dot{a} \dot{b}} = \omega^{\dot{a} \dot{b}} + \frac{i}{72} \sum_{k=0}^{15} \frac{1}{(2k+2)!} \tilde{\Theta} \left( \Gamma^{\dot{a} \dot{b} \dot{c} \dot{d} \dot{e}} \partial_{\dot{c} \dot{d} \dot{e}} F_{\dot{a} \dot{b} \dot{c} \dot{d} \dot{e}} + 24 \Gamma_{\dot{a} \dot{b} \dot{c} \dot{d} \dot{e}} \mathcal{M}^{2k} \tilde{D} \Theta \right) . \] (2.8)

\(^1\)The same result as eq. \[23\] is obtained in ref. \[16\] by using only eleven-dimensional supergravity torsion and curvature constraints.
Here \( e^\hat{a} = dX^{\hat{m}} e_{\hat{m}} \) and \( \omega^{\hat{a}\hat{b}} = dX^{\hat{m}} \omega_{\hat{m}}^{\hat{a}\hat{b}} \) are the vielbein and the spin-connection one-forms of \( AdS_7 \times S^4 \) respectively, and the one-form \( \tilde{D} \Theta \) and the matrix \( \mathcal{M}^2 \) are defined as

\[
\begin{align*}
\tilde{D} \Theta &= \left( d + \frac{1}{4} \omega^{\hat{a}\hat{b}} \Gamma_{\hat{a}\hat{b}} + e^\hat{a} T_{\hat{a} \hat{b} \hat{c} \hat{d}} F_{\hat{b} \hat{c} \hat{d}} \right) \Theta , \\
(\mathcal{M}^2)^{\hat{a}}_{\hat{b}} &= -2i \left( T_{\hat{a} \hat{b} \hat{c} \hat{d}} F_{\hat{c} \hat{d}} \right) \hat{\alpha} \left( \tilde{\Theta} \Gamma_{\hat{a}} \right)_{\hat{b}} + \frac{i}{288} (\Gamma_{\hat{a}} \Theta)_{\hat{b}} \left( \tilde{\Theta} \Gamma^{\hat{a}} \hat{b} \hat{c} \hat{d} \right)_{\hat{b}} = 1 \left( \Gamma_{\hat{a}} \Gamma^{\hat{b}} - \delta_{\hat{a}}^{\hat{b}} \Gamma_{\hat{c} \hat{d}} \right) , \end{align*}
\]

where \( T_{\hat{a} \hat{b} \hat{c} \hat{d}} \) denotes

\[
T_{\hat{a} \hat{b} \hat{c} \hat{d}} = \frac{1}{288} \left( \Gamma_{\hat{a} \hat{b} \hat{c} \hat{d}} \right) .
\]

For the \( AdS_7 \times S^4 \) solution, the closed four-form \( \Pi^4 \) is integrated and the three-form potential \( C^{(3)} \) is obtained explicitly \( \text{[1]} \) as follows:

\[
C^{(3)}(Z) = \frac{1}{3!} e^\hat{a} \wedge e^\hat{b} \wedge e^\hat{c} C_{\hat{a} \hat{b} \hat{c}}(X) + i \int_0^1 dt \tilde{\Theta} \Gamma_{\hat{a} \hat{b} \hat{c}} \Pi^4(X, t \Theta) \wedge \Pi^4(X, t \Theta) \wedge \Pi^4(X, t \Theta) .
\]

**2.2 Light-cone gauge supermembrane**

In this subsection, we impose the light-cone gauge conditions to fix the reparametrization invariance on the world-volume and the fermionic gauge symmetry (\( \kappa \)-symmetry) \( \Pi \) of the action \( (2.1) \).

We choose the \( AdS_7 \times S^4 \) coordinates in terms of which the metric takes the form

\[
ds^2 = G_{\hat{m}\hat{n}} dX^{\hat{m}} dX^{\hat{n}} = \frac{2r}{L} \eta_{\hat{m}\hat{n}} dX^{\hat{m}} dX^{\hat{n}} + \left( \frac{L}{2r} \right)^2 d\hat{r}^2 + \frac{L^2}{4} d\Omega_4^2 ,
\]

where \( (X^{\hat{m}}, \hat{r}) \) \( \hat{r} = 0, \ldots , 4, 5 \) are the horospherical coordinates of \( AdS_7 \) and \( d\Omega_4^2 \) is the metric of a unit \( S^4 \). We denote the coordinates on \( S^4 \) by \( X^{m'} \) \( (m' = 6, 7, 8, 9) \). The metric \( (2.14) \) can directly be obtained as near-horizon geometry \( \text{[28]} \) \( \text{[29]} \) of the M5-brane solution \( \text{[27]} \) of eleven-dimensional supergravity,

\[
ds_{M5}^2 = \left( 1 + \frac{Q^{13} \pi}{r^3} \right)^{\frac{3}{2}} \eta_{\hat{m}\hat{n}} dX^{\hat{m}} dX^{\hat{n}} + \left( 1 + \frac{Q^{13} \pi}{r^3} \right)^{\frac{3}{2}} (d\hat{r}^2 + r^2 d\Omega_4^2) ,
\]

where \( Q \) denotes the number of the M5-branes, \( X^{\hat{m}} \) denote the directions parallel to the M5-branes and \( (r, X^{m'}) \) denote the polar coordinates parametrizing the directions perpendicular to the M5-branes. The near-horizon limit taken in ref. \( \text{[29]} \) is

\[
l_p \to 0 \quad \text{with} \quad \frac{r}{l_p^3} = \text{finite} .
\]

One can find that the radius of the \( AdS_7 \) is expressed \( \text{[29]} \) as

\[
L = 2l_p (\pi Q)^{\frac{1}{3}} .
\]
We introduce the light-cone coordinates \( X^\pm = (X^+, X^-, X^{m_\perp}) \) with
\[
X^\pm = \frac{1}{\sqrt{2}}(X^2 \pm X^0), \quad X^{m_\perp} = (X^\tilde{m}, r, X^{m_\perp}) \quad \tilde{m} = (1, \ldots, 4).
\] (2.17)

We denote the local Lorentz indices corresponding to these coordinates by \( \hat{a} = (\hat{+}, \hat{\bar{+}}, a_\perp) \)
with \( a_\perp = (\hat{a}, \tilde{a}, a') \) and \( \tilde{a} = (\bar{1}, \ldots, \bar{4}) \). In terms of the coordinates (2.17), the metric (2.8) is recast into
\[
ds^2 = \frac{2r}{L} (2dX^+dX^- + \delta_{\tilde{m}\tilde{n}}dX^{\tilde{m}}dX^{\tilde{n}}) + \left( \frac{L}{2r} \right)^2 dr^2 + \frac{L^2}{4} d\Omega_4^2.
\] (2.18)

We may choose the vielbein of \( AdS_7 \) in a diagonal form,
\[
e^i_+ = e^i_- = e^i_1 = e^i_2 = e^i_3 = e^i_4 = \sqrt{\frac{2r}{L}}, \quad e^r_+ = \frac{L}{2r}.
\] (2.19)

The non-vanishing elements of the spin-connection in the \( AdS_7 \) sector become
\[
\omega^+_{\hat{a}} = \omega^-_{\hat{a}} = \omega_1^\hat{a} = \omega_2^\hat{a} = \omega_3^\hat{a} = \omega_4^\hat{a} = \sqrt{\frac{2r}{L^3}}.
\] (2.20)

This yields \( \omega^+_{\hat{a}} = \frac{1}{L} e^i_+ \), \( \omega^-_{\hat{a}} = \frac{1}{L} e^i_- \) and \( \omega_{\tilde{m}}^\tilde{a} = \frac{1}{L} e^i_{\tilde{m}} \).

In order to fix gauge symmetries of the supermembrane action (2.1), we impose the conditions\(^2\)
\[
X^+ = \tau, \quad \Gamma^+\Theta = 0.
\] (2.21)

Introducing the projection operator \( P_{(\text{LC})}^{(\pm)} = \frac{1}{2}(1 \pm \Gamma^\pm) \), we separate the fermionic coordinate \( \Theta \) into two pieces:
\[
\Theta = \Theta_+ + \Theta_-, \quad \Theta_\pm = P_{(\text{LC})}^{(\pm)} \Theta.
\] (2.22)

The gauge condition (2.21) imposed on \( \Theta \) is equivalent to
\[
\Theta_+ = 0.
\] (2.23)

We note that the ‘chirality’ associated with the projection operator \( P_{(\text{LC})}^{(\pm)} \) flips under the Dirac conjugation:
\[
\bar{\Theta}_\pm = \Theta_\pm P_{(\text{LC})}^{(\pm)}.
\] (2.24)

Applying the condition (2.23) to eq. (2.10), we obtain
\[
\mathcal{M}_7^6[\Theta_-]^{\hat{a}}_{\hat{b}} = -2i \left( \Gamma^a_{\mp b\mp c\mp d\mp} F_{b' c' d' c'} \Theta_- \right)^\hat{a} \left( \Gamma^a_{\perp} \Theta_{\perp} \right)^\hat{b} + \frac{i}{144} \Theta^\hat{a} \left( \Theta_\perp \Gamma^a_{\mp b\perp} \Theta_{\perp} \right)^\hat{b} + \frac{i}{288} \left( \Gamma^a_{\perp} \Theta_{\perp} \right)^\hat{a} \left( \Theta_\perp \Gamma^a_{\mp b\perp} \Theta_{\perp} \right)^\hat{b} + \frac{i}{12} \left( \Gamma^a_{\mp b\perp} \Theta_{\perp} \right)^\hat{a} \left( \Theta_\perp \Gamma^a_{\mp b\perp} \Theta_{\perp} \right)^\hat{b}.
\] (2.25)

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\(^2\) In this paper, we use the convention in which the world-volume time coordinate \( \tau \) has dimensions of (length)\(^3\) in the space-time sense. This convention is possible because of the invariance of the action (2.1) under the transformation \( \xi^i \mapsto l^{(i)} \xi^i \) for arbitrary constants \( l^{(i)} \), which is a part of the reparametrization invariance of the membrane world-volume.
Here we have used the relations
\[ \Gamma^i \Theta_- = \bar{\Theta}_- \Gamma^i = 0 . \] (2.26)

In what follows, we will not use the explicit form of \( F_{a'b'c'd'} \) given in eq.(2.24) in order to trace the \( F_{a'b'c'd'} \) dependence. In Appendix C, we substitute the explicit form of \( F_{a'b'c'd'} \) into several quantities which will appear in the following. By using the relations
\[ P^{(LC)}_\pm \Gamma^i = \Gamma^i P^{(LC)}_\pm , \quad P^{(LC)}_\pm \Gamma^j = \Gamma^j P^{(LC)}_\pm , \quad P^{(LC)}_\pm \Gamma^m \Gamma^j = \Gamma^m P^{(LC)}_\pm , \] (2.27)
we find that
\[ \mathcal{M}^2[\Theta_-] = P^{(LC)}_- \mathcal{M}^2[\Theta_-] P^{(LC)}_+ , \] (2.28)
and thus
\[ \mathcal{M}^4[\Theta_-] = 0 , \quad \bar{\Theta}_- \Gamma^{a\perp} \mathcal{M}^2[\Theta_-] = 0 . \] (2.29)

It follows that in the gauge (2.21) the supervielbein and the superconnection become at most quartic order polynomials of the fermionic coordinate \( \Theta_- \). With the condition (2.23), the pull-back of the covariant derivative onto the membrane world-volume \( \tilde{D}_i \Theta \) becomes
\[
\tilde{D}_i \Theta_- = \partial_i \Theta_- + \partial_i X^{\bar{m}} \left( \frac{1}{4} \omega^{\bar{a} \bar{b}}_m \Gamma_{\bar{a} \bar{b}} + e_{\bar{a}} \Gamma^b \delta_{\bar{b} \bar{d}} \right) \Theta_-. \\
= \partial_i \Theta_- + \partial_i X^{\bar{m}} e_{\bar{a}} \Gamma^b \left( \frac{1}{2L} \Gamma_p + \frac{1}{288} \Gamma^{b'c'd'} F_{b'c'd'} \right) \Theta_-. \\
+ \partial_i X^{m_\perp} \left( \frac{1}{4} \omega^{m_\perp \perp} \Gamma_{a_\perp b_\perp} + e_{m_\perp} \Gamma_{a_\perp b_\perp} \right) \Theta_- .
\] (2.30)

Here we have used the relation \( \omega^{+ \pm \mp} = \frac{1}{2} e^{+ \pm} \).

If we plugged the explicit form (2.27) of \( F_{a'b'c'd'} \) into eq.(2.30), we would find that \( \tilde{D}_i \) contains the projection operator \( \mathcal{P}_{\pm}^{(M5)} \) \( \text{[14][15][16]} \) defined as
\[ \mathcal{P}_{\pm}^{(M5)} = \frac{1 \pm \eta \tilde{\gamma}'}{2} \tilde{\gamma}' = \frac{1}{4!} e^{a'b'c'd'} \Gamma_{a'b'c'd'} . \] (2.31)

We note that the projection operator \( \mathcal{P}^{(M5)}_\pm \) commutes with \( \mathcal{P}^{(LC)}_\pm \) and \( \mathcal{P}^{(LC)}_- \). We might further decompose the fermionic coordinate \( \Theta_- \) into two pieces as
\[ \Theta_- = \Theta_+^- + \Theta_-^-, \quad \Theta_-^{\pm} = \mathcal{P}_{\pm}^{(M5)} \Theta_- . \] (2.32)

In Appendix C, we describe several quantities in terms of \( \Theta_-^{\pm} \) after using the explicit form of \( F_{a'b'c'd'} \).

Taking into account the bosonic condition in eq.(2.21) as well as fermionic one, we obtain the following relations from eqs.(2.28) and (2.30),
\[
\mathcal{M}^2[\Theta_-] \tilde{D}_i \Theta_- = 0 , \quad \bar{\Theta}_- \Gamma^{a\perp} \tilde{D}_i = 0 , \\
\mathcal{M}^2[\Theta_-] \tilde{D}_i \Theta_- = e_{\perp}^i \mathcal{M}^2[\Theta_-] \Gamma^i \left( \frac{1}{2L} \Gamma_p + \frac{1}{288} \Gamma^{b'c'd'} F_{b'c'd'} \right) \Theta_- .
\] (2.33)
Gathering the relations obtained above, we find that in the gauge \([2.21]\) the pull-back of the supervielbein onto the membrane world-volume \(\Pi^a_i\) becomes as follows: the bosonic components \(\Pi^\pm_i\) take the forms

\[
\Pi^\pm_i = \partial_i X^m e^\pm_m - i\Theta^- \partial_i \Theta - \frac{i}{12} e^\pm_m \Theta^- \Gamma^- \left( \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} + \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} \right) \Theta^- ,
\]

where \(F\) denotes four-Fermi terms and the fermionic components \(\Pi^\parallel_i\) take the forms

\[
\Pi^\parallel_i = \partial_i X^m e^\parallel_m - i\Theta^- \partial_i \Theta - \frac{i}{12} e^\parallel_m \Theta^- \Gamma^- \left( \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} + \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} \right) \Theta^- ,
\]

\[
\Pi^\perp_i = \partial_i X^m e^\perp_m - i\Theta^- \partial_i \Theta - \frac{i}{12} e^\perp_m \Theta^- \Gamma^- \left( \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} + \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} \right) \Theta^- .
\]

We thereby obtain the induced metric on the membrane world-volume,

\[
h_{rr} = 2\partial_r X^- G_{++} + \partial_r X^m \partial_r X^n G_{m_{\perp} n_{\perp}} - 2ie^\pm \Theta^- \partial_r \partial_r \Theta - \frac{i}{12} e^\pm \Theta^- \Gamma^- \left( \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} + \frac{1}{2L} \Gamma^r + \frac{1}{288} \Gamma^a \Gamma^b \Gamma^c \Gamma^d F_{a'b'c'd'} \right) \Theta^- ,
\]

\[
\bar{h}_{ij} = \partial_i X^m \partial_j X^n G_{m_{\perp} n_{\perp}} ,
\]

where \(F\) denotes four-Fermi terms
Following the steps presented in Appendix B we may recast \( F \) into

\[
F = -(e_+^*)^2 \left[ \tilde{\Theta}_- \Gamma^a \left( \frac{1}{2L} \rho + \frac{1}{288} \Gamma^a \right) \right] \times \\
\left[ \tilde{\Theta}_- \Gamma^a \left( \frac{1}{2L} \rho + \frac{1}{288} \Gamma^a \right) \right] \times \\
\left[ \tilde{\Theta}_- \Gamma^a \left( \frac{1}{2L} \rho + \frac{1}{288} \Gamma^a \right) \right].
\]  

Substituting eqs. (2.34) and (2.35) into the lagrangian (2.31), we obtain

\[
\mathcal{L} = -T \sqrt{\Delta} h + T e_i^j \frac{\delta}{\delta r^i} X^m \frac{\delta}{\delta r^j} X^m_i \frac{\delta}{\delta r^j} X^m \left( \frac{1}{2} \omega_{m} \right) + \frac{1}{4L} e^h_{m} \Gamma^a \Gamma^b \Gamma^c \Gamma^d + \frac{1}{128} e^h_{m} \Gamma^a \Gamma^b \Gamma^c \Gamma^d \right) \Theta_+ \].
\]  

where

\[
\Delta = -h_{\tau} + u_i h^i u_j, \quad h_{ij} = \delta^i_j, \quad \tilde{h} = \det h_{ij}, \quad \epsilon^{ij} = -\epsilon^{ji}.
\]  

Here we have used the relation (3.39). From the lagrangian (2.39), the canonical momenta are determined by

\[
P_- = \frac{\partial \mathcal{L}}{\partial (\partial_r X^-)} , \quad P_{\pm} = \frac{\partial \mathcal{L}}{\partial (\partial_r X^\pm)} , \quad \tilde{P}_\Theta = \partial \mathcal{L} / \partial (\partial_r \Theta_+).
\]  

This yields

\[
P_+ = T \sqrt{\frac{h}{\Delta}} G_{+}, \quad P_m = T \sqrt{\frac{h}{\Delta}} G_{m \tilde{n}} \left( \partial_r X^\tilde{n} - u_i h^i \partial_r X^\tilde{n} \right),
\]

\[
P_r = T \sqrt{\frac{h}{\Delta}} G_{r \tilde{n}} \left( \partial_r X^\tilde{n} - u_i h^i \partial_r X^\tilde{n} \right),
\]

\[
P_m = T \sqrt{\frac{h}{\Delta}} G_{m n'} \left( \partial_r X^n' - u_i h^i \partial_r X^\tilde{n} \right) + T \frac{e_i^j}{2} \partial_r X^m \partial_r X^{m'} C_{m n' l} + T \frac{e_i^j}{2} \partial_r X^m \partial_r X^{m'} C_{m n' l}
\]

\[
- T \sqrt{\frac{h}{\Delta}} \epsilon^+ \tilde{\Theta}_- \Gamma - \left( \frac{1}{4} \omega_{m} \epsilon^{a'} \Gamma^a' \Gamma_{a'b'} - \frac{1}{2L} e^a_{m} \Gamma^a' \Gamma^a' \right) \Theta_+ ,
\]

\[
\tilde{P}_\Theta = -i T \sqrt{\frac{h}{\Delta}} e^+ \tilde{\Theta}_- \Gamma .
\]

(2.41)
The Hamiltonian takes the form
\[
\mathcal{H}_0 \equiv P_- \partial_\tau X^- + P_m \partial_i X^{m_+} + \bar{P}_\Theta \partial_i \Theta_\tau - \mathcal{L}
\]
\[
= \frac{G^+}{2P_-} \left[ T^2 \mathcal{h} + P_m G^{n\bar{n}} P_{\bar{n}} + P_r G^{r\tau} P_\tau + Q_{m'} G^{m'n'} Q_{n'} \right]
- T \varepsilon^{ij} e^c_{m_+} \partial_i X^{m_+} i\tilde{\Theta}_- \Gamma^{-} \partial_\tau \left[ \partial_j \Theta_\tau + \partial_j X^{n_+} \left( \frac{1}{4} \omega^{c_\perp}_{n_+} \Gamma^{c_\perp}_{b_\perp L} \Gamma^{b_\perp c_\perp} \right) + \frac{1}{4L} \tilde{\Theta}_- \Gamma^{-} \partial_\tau \partial_j X^{n_+} \right]
+ \frac{1}{2P_-} \left[ \tilde{\Theta}_- \Gamma^{-} \partial_\tau \left( \frac{1}{2L} \Gamma_\tau + \frac{1}{144} \bar{\Gamma}_a \bar{\Gamma}_a \bar{F}_{a_1} \bar{F}_{a_2} \right) \right] \times
\left[ \tilde{\Theta}_- \Gamma^{-} \partial_\tau \left( \frac{1}{2L} \Gamma_\tau + \frac{1}{288} \bar{\Gamma}_a \bar{\Gamma}_a \bar{F}_{a_1} \bar{F}_{a_2} \right) \right] \times
- \frac{1}{96} \left[ \tilde{\Theta}_- \Gamma^{-} \partial_\tau \left( \frac{1}{144} \bar{\Gamma}_{a_1} \bar{\Gamma}_{a_2} \bar{F}_{a_1} \bar{F}_{a_2} \right) \right] \times
\left[ \tilde{\Theta}_- \Gamma^{-} \partial_\tau \left( \frac{1}{2L} \Gamma_\tau + \frac{1}{288} \bar{\Gamma}_{a_3} \bar{\Gamma}_{a_4} \bar{F}_{a_3} \bar{F}_{a_4} \right) \right] \Theta_-. \tag{2.42}
\]
where
\[
Q_{m'} \equiv \frac{P_{m'}}{2} \varepsilon^{ij} \partial_i X^{m_+} \partial_j X^{m_+} C_{m'm'_1'm'_2}
+ \frac{i}{e_-} \tilde{\Theta}_- \Gamma^{-} \left( \frac{1}{4} \omega^{a_1'} \bar{a}_1' \Gamma_a \Gamma_{a_1'} - \frac{1}{2L} \varepsilon^{a_1'} a_1' \Gamma_{a_1'} \Gamma_{a_1'} - \frac{1}{24} \varepsilon^{c_1'} c_1' \Gamma_{c_1'} \Gamma_{c_1'} \Gamma_{c_1'} \bar{F}_{a_1'} \bar{F}_{a_1'} \right) \Theta_-. \tag{2.43}
\]

Similarly to the flat case, the system has primary constraints,
\[
\Phi_i \equiv P_- \partial_i X^- + P_m \partial_i X^{m_+} + \bar{P}_\Theta \partial_i \Theta_\tau \approx 0, \tag{2.44}
\]
\[
\bar{\chi} \equiv \bar{P}_\Theta + \frac{i}{e_-} \bar{\Theta}_- \Gamma^{-} \approx 0. \tag{2.45}
\]

One can find that the constraint \( \Phi_i \) is first class and the constraint \( \bar{\chi} \) is second class. Following the prescription for the constrained Hamiltonian systems \cite{[48],[49]}, we introduce the total Hamiltonian
\[
\mathcal{H}_T = \mathcal{H}_0 + c^i \Phi_i, \tag{2.46}
\]
where \( c^i \) is a Lagrange multiplier. Since \( \bar{\chi} \) is a second class constraint, we solve this constraint and hence have not added this constraint in the total Hamiltonian \( \mathcal{H}_T \). We find that no further constraints emerge from the consistency conditions of the constraints \( \Phi_i, \bar{\chi} \), i.e., there are no secondary constraints in this system.

In order to fix the remaining gauge symmetry generated by the first class constraint \( \Phi_i \), we further impose conditions. Now we look for the gauge in which the dynamics of \( P_-(\tau, \sigma) \) becomes trivial, \( \partial_\tau P_- = 0 \). Since the Hamiltonian \( \mathcal{H}_T \) is independent of the coordinate \( X^- \) conjugate to \( P_- \) except for the constraint term\(^4 \) \( \Phi_i c^i \), we can achieve such a gauge, in the

\(^4\)Because of this fact, the field \( X^- \) can be eliminated from the resulting Hamiltonian \cite{[48],[49]]. This originates from the fact that all the components of the metric \( G_{\bar{m}\bar{n}} \) and the 3-form potential \( C_{\bar{m}\bar{n}\bar{p}} \) are independent of the coordinate \( X^- \) and the components \( C_{+\bar{m}} \) and \( C_{-\bar{m}} \) of the 3-form potential are vanishing in this background.
same way as the flat case \[41\], by imposing
\[ u_i \approx 0. \]  
(2.46)

We will henceforth refer to the gauge defined by eqs. (2.21) and (2.46) as the light-cone gauge. The condition (2.46) leads to \( c^i \approx 0 \) and thus the resulting hamiltonian becomes independent of \( X^- \). This leads to \( \partial_{\tau}P_- = 0 \) and we may set
\[ P_-(\sigma) = P_-^{(0)} \sqrt{w(\sigma)} , \]  
(2.47)

where \( \sqrt{w(\sigma)} \) is a scalar density in the spatial directions of the membrane world-volume, \( \Sigma_{(2)} \). Now that we are considering a closed supermembrane, \( \Sigma_{(2)} \) is a compact space. We normalize the area of \( \Sigma_{(2)} \) as
\[ \int d^2 \sigma \sqrt{w(\sigma)} = 1. \]  

\( P_-^{(0)} \) is therefore the zero mode of \( P_-(\sigma) \): \( P_-^{(0)} = \int d^2 \sigma P_- (\sigma) \). Since \( c^i \approx 0 \), the hamiltonian \( H_T \) introduced in eq. (2.45) turns out to take the same form as \( H_0 \) in eq. (2.42) with the identification (2.47).

It follows from the condition (2.46) that \( X^- \) is expressed in terms of other fields as
\[ \partial_{\tau}X^- = - \frac{1}{G_{++}} \partial_{\tau}X^{m=1} \partial_{i}X^{n=1} G_{m=1,n=1} + i \frac{1}{e_-} \Theta_1 \Gamma_- \partial_i \Theta_- 
+ i \frac{1}{e_-} \Theta_1 \Gamma_- \left( \frac{1}{4} \omega_{m'=a'b'} \Gamma_{a'b'} - \frac{1}{2L} e_{m'} \Gamma_{a'b} - \frac{1}{24} e_{m'} \Gamma_{c'd'} F_{a'b'c'd'} \right) \right) \Theta_1 \partial_i X^{m'} 
= - \frac{1}{P_-^{(0)} \sqrt{w}} P_{m=} \partial_{i}X^{m=} + i \frac{1}{e_-} \Theta_1 \Gamma_- \partial_i \Theta_- . \]  
(2.48)

The fields \( X^-(\tau, \sigma) \) and \( P_-(\tau, \sigma) \) are thus no longer independent physical degrees of freedom except for their zero modes \( q_-(\tau) \) and \( P_-^{(0)} \), where \( q_-(\tau) \) is defined as
\[ q_-(\tau) = \int d^2 \sigma \sqrt{w(\sigma)} X^- (\tau, \sigma) . \]  
(2.49)

From the original canonical commutation relation \( (X^-(\tau, \sigma), P_-(\tau, \sigma'))_P = \delta^{(2)}(\sigma, \sigma') \), where \( \delta^{(2)}(\sigma, \sigma') \) denotes the delta function on \( \Sigma_{(2)} \), one can find that \( q_-(\tau) \) and \( P_-^{(0)} \) obey the commutation relation
\[ \left( q_-(\tau), P_-^{(0)} \right)_P = 1. \]  
(2.50)

Now we make a brief comment on the relation between \( h_{\tau \tau} \) and \( \bar{h} \) in this gauge. Combined with the definition of \( P_- \) in eq. (2.41), the relation (2.41) yields
\[ h_{\tau \tau} = \left( \frac{T}{\sqrt{w(\sigma) P_-^{(0)}}} \right)^2 G_{++} \bar{h} . \]  
(2.51)

We note that the field dependent factor \( G_{++} = 2r/L \) appears in the proportional coefficient between \( h_{\tau \tau} \) and \( \bar{h} \). This implies that, unlike the flat case, we cannot accomplish the ‘conformally-flat-like’ world-volume metric in the \( AdS_7 \times S^4 \) case. The situation is quite
similar to the light-cone superstring on $AdS_5 \times S^5$ \cite{39}, in which conformally flat world-sheet metric is not allowed because it is incompatible with the equations of motion for the string coordinate $X^+$ in the light-cone gauge \cite{39}.

Using eq.(2.48), one can find that

$$\Phi_i = \bar{\chi} \partial_i \Theta_\cdot$$ (2.52)

This implies that the constraint $\Phi_i$ is reduced to the second class constraint $\bar{\chi}$ and need not be considered in the light-cone gauge. By using the second class constraint $\bar{\chi}$, we evaluate the Dirac brackets among the canonical variables,

$$(X^m_\perp (\tau, \sigma), P_{n_\perp} (\tau, \sigma'))_{DB} = \delta^m_{n_\perp} \delta^{(2)} (\sigma, \sigma') ,$$

$$\left( q^- (\tau), P_{-}^{(0)} \right)_{DB} = 1 ,$$

$$\left( \Theta^\alpha (\tau, \sigma), \bar{P}_{\partial_-} (\tau, \sigma') \right)_{DB} = \frac{1}{2} (\bar{P}_{-}^{(LC)} \hat{\alpha} \delta^{(2)} (\sigma, \sigma') ,$$

$$\left( q^- (\tau), \Theta^- (\tau, \sigma) \right)_{DB} = \frac{1}{2} \bar{P}_{-}^{(0)} \Theta^- (\tau, \sigma) ,$$

$$\left( P_{\tau} (\tau, \sigma), \Theta^\alpha (\tau, \sigma') \right)_{DB} = - \frac{1}{4r (\tau, \sigma)} \Theta^\alpha (\tau, \sigma) \delta^{(2)} (\sigma, \sigma') .$$ (2.53)

The third relation in the above is also expressed in the following way,

$$\left( \Theta^\alpha (\tau, \sigma), \Theta^- (\tau, \sigma') \right)_{DB} = - \frac{i}{4P_{-}^{(0)}} \frac{1}{\sqrt{w} (\sigma)} e^- \left( P_{-}^{(LC)} \Gamma^+ C \right) \delta^{(2)} (\sigma, \sigma') .$$ (2.54)

We note that the fermionic coordinate $\Theta^- (\tau, \sigma)$ does not commute with the bosonic canonical variables $q^- (\tau)$ and $P_{\tau} (\tau, \sigma)$.

In a similar way to the flat case \cite{41, 13, 9}, there still exists a constraint, i.e. the integrability condition for $\partial_{\tau} X^- \cdot$ given in eq.(2.48). The integrability condition is locally described by $P_{-}^{(0)} e^i \partial_i \partial_j X^- / \sqrt{w} = 0$, which reads

$$\varphi \equiv - P_{-}^{(0)} \left\{ \frac{1}{G_{+}} \partial_{\tau} X^m m G_{m_\perp n_\perp}, X^n_{\perp} \right\} + i P_{-}^{(0)} \left\{ \frac{1}{e^-} \Theta_-, \Gamma^\cdot \Theta_- \right\}$$

$$+ i P_{-}^{(0)} \left\{ \frac{1}{e^-} \bar{\Theta}_- \Gamma^- \left( \frac{1}{4} \omega^m_{a'b'} \Gamma_{a'b'} - \frac{1}{2L} e^a_{m'}, \Gamma_{a'} \Gamma_{r} - \frac{1}{24} e^a_{m'} \Gamma^{b'd'} F_{a'b'd'} \right) \Theta_-, X^m_\perp \right\}$$

$$= - \left\{ P_{m_\perp} / \sqrt{w}, X^m_\perp \right\} + i D_{-}^{(0)} \left\{ \frac{1}{e^-} \bar{\Theta}_-, \Gamma^\cdot \Theta_- \right\} \approx 0 ,$$ (2.55)

where the bracket \{*, *\} is defined as

$$\{ A, B \} = \frac{\epsilon_{ij}}{\sqrt{w}} \partial_i A \partial_j B$$ (2.56)
for arbitrary functions $A$ and $B$ on $\Sigma(2)$. By using the Dirac brackets (2.53) and (2.54), we can show that the constraint $\varphi$ generates area-preserving diffeomorphisms (APD),
\[
\int d^2\sigma' \sqrt{w(\sigma')} \{ f(\sigma'), X^{m_1}(\sigma), \varphi(\sigma') \} = \{ f(\sigma), X^{m_1}(\sigma) \},
\]
\[
\int d^2\sigma' \sqrt{w(\sigma')} \{ f(\sigma'), \Theta^a(\sigma), \varphi(\sigma') \} = \{ f(\sigma), \Theta^a(\sigma) \},
\]
where $f(\sigma)$ is an arbitrary function on $\Sigma(2)$. While we might impose a further condition to fix this residual APD gauge symmetry, we leave it unfixed, following the treatment of [9] to obtain quantum mechanics. When $\Sigma(2)$ is a Riemann surface of genus $g$, there exist $2g$ global integrability conditions (see e.g. [50]) in addition to the local one (2.55). These constraints generate APD transformations as well.

We can construct the lagrangian $L_{\text{APD}}$ which reproduces the light-cone gauge hamiltonian through the Legendre transformation, $L_{\text{APD}} = P_{m_\perp} \partial_\tau X^{m_\perp} + \bar{P}_\Theta \partial_\tau \Theta - H_T$. We find that $L_{\text{APD}}$ takes the form
\[
\frac{1}{\sqrt{w}} L_{\text{APD}} = \frac{P_{r_1}(0) G_{m_1 n_1}}{2 G_{++}} D_\tau X^{m_1} D_\tau X^{n_1} - \frac{G_{++}}{4 P_{r_1}(0)} T^2 \{ X^{m_1}, X^{n_1} \} \{ X^{p_\perp}, X^{q_\perp} \} G_{m_\perp p_\perp} G_{n_\perp q_\perp} + \frac{T}{2} C_{m'_1 n'_2} D_\tau X^{m'_1} \{ X^{m'_2}, X^{m'_3} \}
\]
\[
- \frac{P_{r_1}(0)}{e_-} i \bar{\theta} \gamma_5 D_\tau \theta + T e_i \bar{\theta} \gamma_5 \Gamma_a \epsilon_{a_{m_1}} {\{ X^{m_\perp}, \theta \}}
\]
\[
- \frac{P_{r_1}(0)}{e_-} D_\tau X^{m'_1} i \bar{\theta} \gamma_5 \Gamma_a \left( \frac{1}{4} \epsilon_{a'b'} \Gamma_{a'b'} - \frac{1}{2L} \epsilon_{a'} \Gamma_a \Gamma_f - \frac{1}{24} \epsilon_{a'} \Gamma_{a'c'd} F_{a'c'd} \right) \Theta_-
\]
\[
+ T e_i \epsilon_{m_{1}} {\{ X^{m_\perp}, X^{n_\perp} \}} \times
\]
\[
i \bar{\theta} \gamma_5 \Gamma_a \left( \frac{1}{4} \epsilon_{m_{1} n_{1}} b_{1} c_{1} \Gamma_{b_{1} c_{1}} + \frac{1}{4L} \epsilon_{n_{1}} b_{1} \Gamma_{b_{1} f} + \frac{1}{192} \epsilon_{n_{2}} \Gamma_{c_1...c_4} F_{b_{1}...b_{4}} \right) \Theta_-
\]
\[
- \frac{1}{2} P_{r_1}(0) \left[ \bar{\theta} \gamma_5 \Gamma_{a_{1}} \left( \frac{1}{2L} \Gamma_f + \frac{1}{144} \Gamma_{c_1...c_4} F_{b_{1}...b_{4}} \right) \right] \Theta_-
\]
\[
\times \left[ \bar{\theta} \gamma_5 \Gamma_{b_{1}} \left( \frac{1}{2L} \Gamma_f + \frac{1}{288} \Gamma_{c_1...c_4} F_{b_{1}...b_{4}} \right) \Theta_-
\right]
\]
\[
+ \frac{1}{96} P_{r_1}(0) \left( \bar{\theta} \gamma_5 \Gamma_{c_1...c_4} \Theta_+ \right) F_{b_1...b_4} \left[ \bar{\theta} \gamma_5 \Gamma_{c_1...c_4} \left( \frac{1}{2L} \Gamma_f + \frac{1}{288} \Gamma_{c_1...c_4} F_{b_1...b_4} \right) \Theta_+ \right].
\]

Here we have introduced the APD gauge field $v(\tau, \sigma)$ and replaced the $\tau$-derivative $\partial_\tau$ with the covariant derivative $D_\tau$ defined [9] as
\[
D_\tau X^{m_\perp} = \partial_\tau X^{m_\perp} - \{ v, X^{m_\perp} \}, \quad D_\tau \theta = \partial_\tau \theta - \{ v, \theta \}.
\]
The Hamiltonian $H_T$ can be interpreted as that in the ‘temporal gauge’, $v = 0$. The constraint $\varphi = 0$ may be obtained as the Gauss-law constraint in the temporal gauge:

$$0 = \frac{\delta S_{\text{APD}}}{\delta v} \bigg|_{v=0} = -\sqrt{w} \varphi ,$$

(2.60) where $S_{\text{APD}} = \int d^3\xi L_{\text{APD}}$.

The Lagrangian (2.58) does not explicitly depend on the geometry of $\Sigma_{(2)}$. This allows us to reinterpret the spatial directions of the membrane world-volume $\Sigma_{(2)}$ as an internal space on which the APD gauge transformations act and regard the Lagrangian

$$L_{\text{APD}} = \int d^2\sigma L_{\text{APD}}$$

(2.61) as that of quantum mechanics. We thus obtain the quantum mechanical system with APD gauge symmetry which describes the supermembrane on $AdS_7 \times S^4$.

2.3 Field redefinition and $SO(1,1) \times SO(9)$ decomposition in the fermionic sector

We remark on the field-dependent rescaling of the fermionic coordinate $\Theta_-$,

$$\Theta_- \mapsto \tilde{\Theta}_- = F \left[ X^{m_\perp}; P^{(0)}_\perp \right] \Theta_- ,$$

(2.62) where $F \left[ X^{m_\perp}; P^{(0)}_\perp \right]$ is an arbitrary scalar-valued functional of $X^{m_\perp}$ and $P^{(0)}_\perp$. By using eq. (A.5), one can readily show that this transformation does not generate in the Lagrangian (2.61) a new term which depends on the derivatives $\partial_i X^{m_\perp}$. It follows that, through the field redefinition (2.62), we may change the normalization of the fermionic coordinate $\Theta_-$ in the Lagrangian (2.61) even in a field-dependent way without altering the momenta $P_{m_\perp}$.

For later convenience, we decompose the $SO(10,1)$ gamma matrices $\Gamma^a$ into $SO(1,1) \times SO(9)$ gamma matrices following the manipulation given in Appendix A and describe several formulae obtained in the last subsection in terms of the $SO(9)$ spinors and the gamma matrices $\gamma_a$. Eq. (A.19) enables us to express the $SO(10,1)$ Majorana spinor $\Theta^a_-$ in terms of the $SO(9)$ Majorana spinor $\theta^\alpha$ as

$$\Theta_- = \frac{1}{2^{1/2} \sqrt{P^{(0)}_\perp}} \left( \frac{2r}{L} \right)^{1/4} \left( 0 \theta \right) .$$

(2.63)

Here we have made the field redefinition (2.62) with the rescaling factor $F = 2^{1/2} \sqrt{P^{(0)}_\perp} \left( \frac{L}{2r} \right)^{1/4}$ as well, in order that the new fermionic coordinate $\theta$ should commute with the bosonic canonical variables. In fact, the commutation relations (2.53) and (2.54) involving the fermionic coordinate are modified into

$$(\theta^\alpha(\tau, \sigma), \theta^\beta(\tau, \sigma'))_{\text{DB}} = -i \delta^{\alpha\beta} \frac{1}{\sqrt{w}} \delta^{(2)}(\sigma, \sigma') ,$$

and

$$(q^-(\tau), \theta^\alpha(\tau, \sigma))_{\text{DB}} = (P_r(\tau, \sigma), \theta^\alpha(\tau, \sigma'))_{\text{DB}} = 0 .$$

(2.64)
The APD constraint \( \varphi \) takes the form

\[
\varphi = -\frac{P(0)}{2} \left\{ \frac{L}{2r} \partial_{\tau} X_{m+} G_{m+ n-} X_{n-} \right\} - \frac{i}{2} \{ \theta, \theta \} - \frac{i}{2} \left\{ \theta \left( \frac{1}{4} \omega_m a' b' \gamma_{a'b'} - \frac{1}{2L} e_{m'}^{a'} \gamma_{a' \gamma_{\bar{r}}} + \frac{1}{24} e_{m'}^{a'} \gamma_{b' e'd'} F_{a'b'c'd'} \right) \theta , X^{m'} \right\} 
\]

(2.65)

The lagrangian \( L_{APD} \) is expressed as

\[
L_{APD} = \int d^2 \sigma \sqrt{w(\sigma)} \times \\
\times \left[ \frac{P(0)}{2} \frac{L}{2r} G_{m+ n-} D_{\tau} X_{m+} D_{\tau} X_{n-} - \frac{T^2}{4P(0)} \frac{2r}{L} \{ X_{m+} , X_{n-} \} \{ X_{p-} , X_{q-} \} G_{m+ p-} G_{n- q-} \\
+ \frac{T}{2} C_{m+ m' n} D_{\tau} X^{m'} \{ X_{m+} , X_{m'} \} \\
+ \frac{i}{2} \theta D_{\tau} \theta + \frac{T}{2P(0)} \frac{2r}{L} i \theta \gamma_{a+ m-} \{ X_{m-} , \theta \} \\
+ \frac{i}{2} D_{\tau} X^{m'} \theta \left( \frac{1}{4} \omega_m a' b' \gamma_{a'b'} - \frac{1}{2L} e_{m'}^{a'} \gamma_{a' \gamma_{\bar{r}}} + \frac{1}{24} e_{m'}^{a'} \gamma_{b' e'd'} F_{a'b'c'd'} \right) \theta \\
+ \frac{T}{2P(0)} \frac{2r}{L} e_{m+}^{a+} \{ X_{m+} , X_{n+} \} \times \\
\times i \theta \gamma_{a+} \left( \frac{1}{4} \omega_{a+ b' c' l} \gamma_{b' c' l} + \frac{1}{4L} e_{n+}^{b+} \gamma_{b+ \bar{r}} - \frac{1}{192} e_{n+}^{b+} \gamma_{c' l \cdots c' l} F_{c' l \cdots c' l} \right) \theta \\
- \frac{1}{8P(0)} \frac{2r}{L} \left[ \theta \gamma_{a+} \left( \frac{1}{2L} \gamma_{\bar{r}} - \frac{1}{144} \gamma_{b' \cdots b' l} F_{b' \cdots b' l} \right) \right] \theta \left[ \theta \gamma_{a+} \left( \frac{1}{2L} \gamma_{\bar{r}} - \frac{1}{288} \gamma_{c' l \cdots c' l} F_{c' l \cdots c' l} \right) \theta \right] \\
- \frac{1}{384P(0)} \frac{2r}{L} \left( \theta \gamma_{c' l \cdots c' l} \theta \right) F_{c' l \cdots c' l} \left[ \theta \gamma_{c' l \cdots c' l} \left( \frac{1}{2L} \gamma_{\bar{r}} - \frac{1}{288} \gamma_{a+ \cdots a+} F_{c' l \cdots c' l} \right) \theta \right] . \quad (2.67)
\]
2.4 Matrix regularization

The algebra generated by APD is approximated by the large $N$ $u(N)$ Lie algebra. This provides a prescription to regularize the two-dimensional continuum ‘internal space’ $\Sigma(2)$ by the $N^2$-dimensional vector space of the adjoint representation of the $U(N)$ group. Following the standard prescription [11] [9] [42], we replace the fields on the membrane-world volume with $\tau$-dependent $N \times N$ hermitian matrices, the APD gauge field $v$ with the $U(N)$ gauge field $A_\tau$, the integral over $\Sigma(2)$ with the matrix trace and the bracket $\{*, *\}$ with the matrix commutator $[*, *]$:

$$
X^{m_\perp} (\tau, \sigma) \xrightarrow{\ell \to \infty} X^{m_\perp} (\tau)^I_J \quad , \quad \theta^a (\tau, \sigma) \xrightarrow{\ell \to \infty} \theta^a (\tau)^I_J \quad ,
$$

$$
v (\tau, \sigma) \xrightarrow{\xi \to \infty} \frac{1}{2\pi N} \mathcal{A}_\tau (\tau)^I_J \quad , \quad P_{m_\perp} (\tau, \sigma) \xrightarrow{\xi \to \infty} N P_{m_\perp} (\tau)^I_J \quad ,
$$

$$
\{*, *\} \xrightarrow{\xi \to \infty} -i 2\pi N [*, *] \quad \int d^2 \sigma \sqrt{w(\sigma)} \xrightarrow{\xi \to \infty} \frac{1}{N} \text{Tr} \quad (2.68)
$$

where $I, J = 1, \ldots, N$ denote the matrix indices. Applying this prescription to the lagrangian (2.67), we obtain the action of the matrix model for the supermembrane on $AdS_7 \times S^4$,

$$
S_{\text{matrix}} = S_{\text{matrix}}^{(B)} + S_{\text{matrix}}^{(F2)} + S_{\text{matrix}}^{(F4)} \quad ,
$$

$$
S_{\text{matrix}}^{(B)} = \int d\tau \frac{P_+^{(0)}}{\sqrt{N}} \text{Tr} \left[ \frac{1}{2} \mathcal{L} \mathcal{D}_\tau X^{m_\perp} \mathcal{D}_\tau X^{n_\perp} G_{m_\perp n_\perp} \right. \quad
$$

$$
\left. + \frac{1}{4} \left( \frac{N}{P_+^{(0)}} \frac{1}{2\pi l_p^3} \right)^2 \frac{2r}{L} [X^{m_\perp}, X^{n_\perp}][X^{p_\perp}, X^{q_\perp}] G_{m_\perp p_\perp} G_{n_\perp q_\perp} \right. \quad
$$

$$
\left. - \frac{N}{P_+^{(0)}} \frac{1}{2\pi l_p^3} \frac{i}{2} C_{m_\perp m_\perp} D_\tau X^{m_\perp} [X^{m_\perp}, X^{m_\perp}] \right] \quad ,
$$

$$
S_{\text{matrix}}^{(F2)} = \int d\tau \frac{1}{\sqrt{N}} \left[ \frac{i}{2} \mathcal{D}_\tau \theta + \frac{1}{2} N \frac{P_+^{(0)}}{2\pi l_p^3} \frac{2r}{L} \theta \gamma_a \epsilon_{m_\perp} [X^{m_\perp}, \theta] \right. \quad
$$

$$
\left. + \frac{i}{2} \mathcal{D}_\tau X^{m_\perp} \theta \left( \frac{1}{4} \omega_m^{a' b' c'} \gamma_{a' b'} - \frac{1}{2L} \epsilon_m^{a' b' c'} \gamma_{a' b'} + \frac{1}{24} \epsilon_m^{a' b' c'} \gamma_{a' b'} F_{a' b' c'} \right) \theta \right. \quad
$$

$$
\left. + \frac{1}{2} N \frac{P_+^{(0)}}{2\pi l_p^3} \frac{2r}{L} \epsilon_{m_\perp} [X^{m_\perp}, X^{n_\perp}] \right. \quad
$$

$$
\left. \times \theta \gamma_a \left( \frac{1}{4} \omega_{n_\perp}^{b_\perp c_\perp} \gamma_{b_\perp c_\perp} + \frac{1}{4L} \epsilon_{n_\perp}^{b_\perp c_\perp} \gamma_{b_\perp c_\perp} - \frac{1}{192} \epsilon_{n_\perp}^{b_\perp c_\perp} \gamma_{c_\perp} F_{b_\perp c_\perp} \right) \theta \right] \quad ,
$$

$$
S_{\text{matrix}}^{(F4)} = - \int d\tau \frac{1}{\sqrt{N}} \left[ \frac{1}{8 P_+^{(0)}} \frac{2r}{L} \left[ \theta \gamma_a \left( \frac{1}{2L} \gamma_{b_\perp} - \frac{1}{144} \epsilon_{b_\perp}^{c_\perp} \gamma_{b_\perp} F_{b_\perp c_\perp} \right) \right. \quad
$$

$$
\left. \times \theta \gamma_a \left( \frac{1}{2L} \gamma_{b_\perp} - \frac{1}{288} \epsilon_{b_\perp}^{c_\perp} \gamma_{b_\perp} F_{b_\perp c_\perp} \right) \right] \quad ,
$$

15
\[ + \frac{1}{384 P_\perp^{(0)}} \frac{2r}{L} \left( \theta \gamma^e c^e_\perp \theta \right) F_{c^1 \cdots c_4} \left[ \frac{\theta \gamma^e c^e_4 \left( \frac{1}{2L} \gamma^a - \frac{1}{288} \gamma^{a_1 \cdots a_4} F_{a_1 \cdots a_4} \theta \right)}{2} \right] \]

where the covariant derivative \( D_r \) means

\[
(D_r X^m)_{I J}^I = \partial_r X^m_{I J} + i[A_r, X^m]_{I J}, \quad (D_r \theta^\alpha)_{I J}^I = \partial_r \theta^\alpha_{I J} + i[A_r, \theta^\alpha]_{I J}.
\]

Here we have expressed the membrane tension \( T \) in terms of the eleven-dimensional Planck length \( l_p \) by using eq. (2.2). The matrix regularization (2.68) entails converting the background fields which depend on the space-time coordinates into the functionals of the \( N \times N \) matrices \( X^m_{I J} \). This gives rise to ordering ambiguity. The problem to completely determine the ordering is beyond the scope of this paper. In this paper, we just adopt the ordering prescription in ref. [51], which is a combination of a symmetrized trace [8], a non-abelian Taylor expansion [52] and multipole moments of currents [12].

3 D0-branes Propagating Near the Horizon of D4-branes

In this section, we show that the action of matrix quantum mechanics obtained in the last section takes the same form as that of D0-branes propagating near the horizon of D4-branes.

Coincident Dp-branes on the curved backgrounds are described by supersymmetric non-abelian Born-Infeld actions coupled to background fields in curved space-times. Non-abelian extensions, supersymmetric extensions and/or curved-background extensions of the Born-Infeld actions have been carried out [8][53][54][55][56]. We use the result of ref. [51] for the bosonic non-abelian D0-brane action on curved backgrounds. A remarkable property of the Dp-brane actions proposed in ref. [51] is that Dp-branes can couple to the Ramond-Ramond (R-R) potentials with the ranks higher than \( p + 1 \), unlike the abelian case. For the fermionic sector of the D0-brane action, we use the result of ref. [53], where the explicit forms of the actions on bosonic curved backgrounds are presented up to quadratic order of the fermionic coordinate in the abelian case.

3.1 D4-brane solution

In supergravity theory, Dp-branes are described by black \( p \)-brane solutions [57][58]. The black 4-brane solution with \( Q \) unit R-R charge of type IIA supergravity takes the following form in the string-frame:

\[
\begin{align*}
\text{d} s^2 &= Z(r)^{-\frac{1}{2}} \, d\tilde{s}_{4+1}^2 + Z(r)^{\frac{1}{2}} \left( dr^2 + r^2 d\Omega_4^2 \right) , \\
e^{2\phi} &= Z(r)^{-\frac{1}{4}} , \\
H_{m_1 \cdots m_4} &= -\eta \, 3 \rho^3 \epsilon_{m_1 \cdots m_4}^{(4)} \sqrt{\det g(\Omega_4)} , \quad \eta = \pm 1 , \\
Z(r) &= 1 + \frac{\rho^3}{r^3} , \quad \rho = l_s(g_sQ\pi)^{\frac{1}{2}} ,
\end{align*}
\]

where \( d\tilde{s}_{4+1}^2 \) is the \( (4+1) \)-dimensional Minkowski metric for the directions \( (X^0, \ldots , X^4) \) along the D4-branes and \( d\Omega_4^2 \) is the metric on the unit \( S^4 \): \( d\Omega_4^2 = g_{m'n'}(\Omega_4) dX^{m'} dX^{n'} \).
(m′ = 6, 7, 8, 9). Here φ is the dilaton and \( H_{\mu_1...\mu_4} \) is the field-strength of the three-form R-R gauge field \( A^{(3)}_{\mu
u\rho} \). The parameters \( l_s \) and \( g_s \) denote the string length and the string coupling constant, respectively.

In order to see near-horizon geometry of the D4-brane solution (3.1), we take the limit

\[
l_s g_s^{-\frac{1}{2}} \rightarrow 0 , \quad \text{with} \quad \frac{r}{l_s g_s} = \text{finite} .
\]

This limit is essentially the same as that taken in eq.(2.15) to obtain the \( \text{AdS}_7 \times S^4 \) metric (2.13) from the M5-brane solution (2.14). This fact follows from the relation \[59\]

\[
l_p = l_s g_s^{\frac{1}{2}}, \quad R = l_s g_s ,
\]

where \( R \) is the radius of the direction compactified in obtaining ten-dimensional type IIA string theory from eleven-dimensional M-theory. In the limit (3.2), the D4-brane solution (3.1) becomes

\[
ds^2 \simeq g_{\mu\nu} dX^\mu dX^\nu = \left( \frac{r}{\rho} \right)^{\frac{3}{2}} d\tilde{s}^2 + \left( \frac{\rho}{r} \right)^{\frac{1}{2}} (dr^2 + r^2 d\Omega_4^2) ,
\]

\[
e^{2\phi} \simeq \left( \frac{r}{\rho} \right)^{\frac{3}{2}}, \quad H_{m'_1...m'_4} = -\eta \left( r^3 e_{m'_1...m'_4} \sqrt{\det g(\Omega_4)} \right) .
\]

We note that these three variables have the structure of \((l_s g_s)^2 \times \) (finite quantities).

### 3.2 Bosonic sector of the action for D0-branes near the horizon of D4-branes

We consider the action of \( N \) coincident D0-branes in the near-horizon geometry of D4-branes (3.4). In this subsection, we restrict our attention to the bosonic sector. We will discuss the fermionic sector in the next subsection.

We make the 1 + 9 split for the space-time coordinates: \( X^\mu = (X^0, X^{m_\perp}) \), with \( X^{m_\perp} = (X^1, \ldots, X^4, r, X^{m'}) \). The world-line theory of \( N \) coincident D0-branes is a non-abelian \( U(N) \) gauge theory \[59\]. We denote the gauge field by \( A_\tau \), where \( \tau \) denotes the world-line coordinate of the D0-branes. This gauge field is accompanied by 9 adjoint scalar fields \( \Phi^{m_\perp} \) with \( \mathcal{D}_\tau \Phi^{m_\perp} = \partial_\tau \Phi^{m_\perp} + i[A_\tau, \Phi^{m_\perp}] \). The fields \( \Phi^{m_\perp} \) have the dimensions of (length)^{-1} in the space-time sense and are related to the \((N \times N)\) matrix-valued collective coordinates of the D0-branes \( X^{m_\perp} \) by

\[
X^{m_\perp} = 2\pi l_s^2 \Phi^{m_\perp} .
\]

Substituting the background (3.4) into the non-abelian D-brane action given in ref.\[51\], we obtain the bosonic part of the action of the \( N \) D0-branes near the horizon of the D4-
branes\(^5\) in the static gauge \(X^0 = \tau\),

\[
S^{(B)}_{D0} = - T_{D0} \int d\tau \text{Tr} \left[ e^{-\phi} \sqrt{-\left( g_{00} + (2\pi l_s^2) D_0, \Phi^m \right) \Phi^m \left( \Phi^m \right)^\dagger} \right]
\]

\[
- \frac{I}{I_{D0}} \int d\tau \text{Tr} \left[ \frac{(2\pi l_s^2)}{2} A^{(3)}_{m'm'n'n'} \Phi^{m'm'} \left[ \Phi^{n'n'}, \Phi^{n'n'} \right] \right],
\]

(3.6)

where the matrix \(Q\) is defined as

\[
(Q)_{m'n'} = \delta_{m'n'} + i \frac{2\pi l_s^2}{2} (\Phi^{m'n'}, \Phi^{n'm'}) g_{p'n'},
\]

(3.7)

and \(T_{D0}\) and \(\mu_{D0}\) denote the D0-brane tension and charge respectively with

\[
T_{D0} = \mu_{D0} = \frac{1}{g_{s} l_s^2}.
\]

(3.8)

In the action (3.6), we choose the ordering prescription proposed in ref.\[51\]. Performing the \(\alpha'\)-expansion (with \(\alpha' = \frac{l_s^2}{g_{s} l_s^2}\)) in the action (3.6), we have

\[
S^{(B)}_{D0} = \frac{(2\pi l_s^2)^2}{g_{s}} \int d\tau \text{Tr} \left[ e^{-\phi} \sqrt{-g_{00}} \left( - \frac{1}{2g_{00}} D_\tau \Phi^m D_\tau \Phi^m g_{m'n'n'}
\right.ight.
\]

\[
+ \frac{1}{4} (\Phi^{m'n'}, \Phi^{n'm'}) (\Phi^{p'n'}, \Phi^{n'm'}) g_{p'n'n'},
\]

\[
- \frac{i}{2} A^{(3)}_{m'm'n'n'} D_\tau \Phi^{m'm'} \left[ \Phi^{n'n'}, \Phi^{n'n'} \right] + O(l_s^5).
\]

(3.9)

Here we have ignored the leading order contribution, because it becomes constant in the background (3.4).

Let us compare the action (3.9) with the bosonic part \(S^{(B)}_{\text{matrix}}\) of the action (2.69). For this purpose, we rewrite the variables of type IIA string theory in the action (3.9) into those of M-theory. Eqs.(2.16), (3.1) and (3.3) yield

\[
\rho = \frac{L}{2}.
\]

(3.10)

Hence, comparing eq.(2.18) with eq.(3.4), we find that (see e.g. \[2, 61\])

\[
g_{m'n'n'} = e^{\frac{\phi}{2}} C_{m'n'n'} = \left( \frac{2r}{L} \right)^\frac{1}{2} G_{m'n'n'},
\]

\[
H_{m'm'n'n'} = F_{m'm'n'n'} = C_{m'm'n'n'},
\]

(3.11)

\[5\] In ref.\[60\], D0-branes in the background (3.4) is considered. In that paper, Dielectric effects are studied and the electric four-form field strength associated with D2-brane charge is turned on. The Chern-Simons term in the resulting action is therefore different from ours (3.6), in which the magnetic four-form field strength is activated.
Using these relations and eq. (3.3), we may recast the action (3.9) into

\[ S_{D0}^{(B)} = \int d\tau \frac{1}{R} \text{Tr} \left[ \frac{1}{2} L \frac{1}{2} \mathcal{D}_\tau X^{m_\perp} \mathcal{D}_\tau X^{n_\perp} G_{m_\perp n_\perp} \right. \\
+ \frac{1}{4} \left( \frac{R}{2\pi l_p^2} \right)^2 \frac{2r}{L} [X^{m_\perp}, X^{n_\perp}] [X^{p_\perp}, X^{q_\perp}] G_{m_\perp p_\perp} G_{n_\perp q_\perp} \\
- \frac{R}{2\pi l_p^2} i \epsilon_{m'_1, m'_2, m'_3} \mathcal{D}_\tau X^{m'_1} [X^{m'_2}, X^{m'_3}] \right]. \quad (3.12) \]

This takes precisely the same form as the action \( S^{(B)}_{\text{matrix}} \) with the identification

\[ P^{(0)} = \frac{N}{R}. \quad (3.13) \]

Thus we have shown that in the bosonic sector the light-cone gauge supermembrane on \( AdS_7 \times S^4 \) provides the matrix theory action of infinitely many D0-branes propagating near the horizon of D4-branes.

### 3.3 Fermionic Sector

Now we turn to the fermionic sector. In the case of a single D0-brane, i.e. the abelian case, the explicit form of the fermionic sector of the Born-Infeld action on general bosonic backgrounds is given in ref. [63] up to the quadratic order of the fermionic coordinate. We will show that this action is indeed reproduced by the corresponding part of the action (2.69), i.e. the \( U(1) \) sector of \( S_{\text{matrix}}^{(F2)} \),

\[ S_{U(1)}^{(F2)} = \int d\tau \left[ \frac{i}{2} \theta \partial_\tau \theta + \frac{i}{2} \partial_\tau X^{m'} \theta \left( \frac{1}{4} \omega_{m'} \gamma^{m'} \gamma^{n'} - \frac{1}{2L} \epsilon_{m'} \gamma^{m'} \epsilon_{n'} + \frac{1}{24} \epsilon_{m'} \gamma^{n'} 
\right) \right]. \quad (3.14) \]

In order to write down the action given in ref. [63], we prepare notations. On the background (3.3), the supersymmetry transformation laws for the gravitino \( \psi_\mu \) and the dilatino \( \lambda \) of type IIA supergravity in the string-frame take the forms\(^6\)

\[ \delta \psi_\mu = \nabla_\mu \epsilon = \partial_\mu \epsilon + W_\mu \epsilon, \quad \delta \lambda = 2 \Delta \epsilon, \quad (3.15) \]

where

\[ W_\mu = \frac{1}{4} \omega_\mu^{ab} \Gamma_{ab} + \frac{1}{8 \cdot 4!} \epsilon^b \Gamma_{ab} \Gamma_{a'b'} \Gamma_{b'} H_{a'b'a'a'}, \]

\[ \Delta = \frac{1}{2} \Gamma^{ab} \partial_\mu \phi + \frac{1}{8 \cdot 4!} \epsilon^d \Gamma_{ab} \Gamma_{a'b'a'a'} H_{a'b'a'a'}, \quad (3.16) \]

with \( H_{a_1 a_2 a_3 a_4} = \epsilon_{a_1}^{m_1} \epsilon_{a_2}^{m_2} m_3^{m_4} m_4, H_{a^1 a^2 a^3 a^4} \). Here \( a = (0, a_\perp) \) with \( a_\perp = (\bar{a}, \tilde{r}, a') \), \( \bar{a} = (1, \ldots, 4) \) and \( a' = (6, \ldots, 9) \) denote the local Lorentz indices, and \( \omega^{ab} \) and \( \epsilon^a \) are the spin-connection and the vielbein for the metric (3.3). We introduce the pull-backs \( \nabla_\tau \) and \( \epsilon_\tau \)

\(^6\)We essentially follow the convention in refs. [62] [63].
of the covariant derivative $\hat{\nabla}_\tau$ and the gamma matrices $\Gamma_a$ onto the D0-brane world-line defined as

$$\hat{\nabla}_\tau = \hat{\nabla}_0 + 2\pi l_s^2 \partial_\tau \Phi^{m\perp} W_{m\perp},$$

$$\varrho_\tau = e^{\Phi}_{\theta_0} + 2\pi l_s^2 \partial_\tau \Phi^{m\perp} \epsilon_{m\perp} \Gamma_{a\perp}. \quad (3.17)$$

Here we have chosen the static gauge $X^0 = \tau$. We note that $\Phi^{m\perp} = \frac{1}{2\pi l_s^2} X^{m\perp}$ are functions commuting with each other and not matrices because we restrict ourselves to the $U(1)$ case. The matrix $\varrho_\tau$ satisfies

$$(\varrho_\tau)^2 = h_{\tau\tau}, \quad (3.18)$$

where $h_{\tau\tau}$ denote the induced metric on the D0-brane world-line defined as

$$h_{\tau\tau} = g_{00} + (2\pi l_s^2)^2 \partial_\tau \Phi^{m\perp} \partial_\tau \Phi^{m\perp} g_{m\perp n\perp}. \quad (3.19)$$

Here we have used the fact that $g_{0m\perp} = 0$ in the background (3.4). We denote the inverse of the metric $h_{\tau\tau}$ by $h^{\tau\tau}$: $h^{\tau\tau} = 1/h_{\tau\tau}$.

In terms of the quantities introduced above, the quadratic action in the fermionic sector of the D0-brane is described by

$$S_{D0}^{(F2)U(1)} = \frac{i}{2} T_{D0} \int d\tau e^{-\phi} \sqrt{-h_{\tau\tau}} \Sigma(1 - \tilde{\Gamma}_{D0}) h^{\tau\tau} \varrho_\tau \left( \hat{\nabla}_\tau - \varrho_\tau \Delta \right) \Psi, \quad (3.20)$$

where $\Psi$ is a $SO(9,1)$ Majorana spinor and $\tilde{\Gamma}_{D0}$ is defined as

$$\tilde{\Gamma}_{D0} = -\frac{1}{\sqrt{-h_{\tau\tau}}} \varrho_\tau \Gamma_z. \quad (3.21)$$

Carrying out the $\alpha'$-expansion in this action, we have

$$S_{D0}^{(F2)U(1)} = \frac{1}{g_{s} l_s} \int d\tau e^{-\phi} \sqrt{-g_{00}} \left[ \frac{i}{g_{00}} \Psi P_+^{(D0)} \epsilon_{0}^{0} \Gamma_0 \left( \partial_\tau + W_0 - \epsilon_{0}^{0} \Gamma_0 \Delta \right) \Psi ight. \\
+ \frac{2\pi l_s^2}{g_{00}} \partial_\tau \Phi^{m\perp} \epsilon_{0}^{0} \Gamma_0 \left( W_{m\perp} - \epsilon_{m\perp}^{a\perp} \Gamma_{a\perp} \Delta \right) \Psi \\
+ \frac{2\pi l_s^2}{g_{00}} \partial_\tau \Phi^{m\perp} \left( \partial_\tau + W_0 - \epsilon_{0}^{0} \Gamma_0 \Delta \right) \Psi \left. \right] + O(l_s^3). \quad (3.22)$$

where $P_+^{(D0)}$ is the projection operator defined as $P_+^{(D0)} = \frac{1}{2} (1 \pm \Gamma_0 \Gamma_z)$.

Let us compare the action (3.22) of the D0-brane with the action (3.14) of the supermembrane. For this purpose, we express the variables of type II A string theory in the action (3.22) by those of M-theory, as carried out for the bosonic sector in the last subsection. Eq. (3.11) enables us to choose the vielbein $e^\alpha_\mu$ such that

$$e^0_0 = e^{\frac{1}{2} \phi} e^0_0 = \left( \frac{2r}{L} \right)^{\frac{1}{4}} \epsilon^0_0, \quad e^{a\perp}_{m\perp} = e^{\frac{1}{2} \phi} e^{a\perp}_{m\perp} = \left( \frac{2r}{L} \right)^{\frac{1}{2}} \epsilon^{a\perp}_{m\perp}, \quad (3.23)$$

$$e^{a\perp}_{m\perp} = e^{\frac{1}{2} \phi} e^{a\perp}_{m\perp} = \left( \frac{2r}{L} \right)^{\frac{1}{2}} \epsilon^{a\perp}_{m\perp}, \quad (3.23)$$

20
where \( e^a_m \) denotes the vielbein of the \( AdS_7 \times S^4 \) metric (2.13), the \( AdS_7 \) components of which are given in eq. (2.19). This yields

\[
\frac{1}{4} \bar{\omega}^{0 aberrant} \Gamma_{ab} = \frac{1}{2} \bar{\omega}^{a \beta}_0 \Gamma^{\beta}_{0} = \frac{1}{2} \bar{\omega}^{0 aberrant}_0 \Gamma^0 \Gamma^r + \frac{1}{4} L \sqrt{2r} \Gamma_0 \Gamma^r ,
\]

\[
\frac{1}{4} \bar{\omega}^{a \beta}_m b^b \Gamma_{a b} \mid a b \rangle = \frac{1}{4} \bar{\omega}^{m aberrant}_m b^b \Gamma_{a b} \mid a b \rangle + \frac{1}{4} L e^a_m \Gamma_{a b} ,
\]

\[
H_{a' a' \perp a_4} = e^{-\frac{1}{4} L} F_{a' a' \perp a_4} ,
\]

(3.24)

where \( \omega^a \mid a \rangle \) denotes the spin-connection of \( AdS_7 \times S^4 \), the \( AdS_7 \) components of which are given in eq. (2.20). By using these relations, we find that

\[
W_0 - e^0 \Gamma_0 \Delta = W_m - e^a_m \Gamma_a \Delta = 0 \quad (m = 1, \ldots, 4) ,
\]

\[
W_r - e^r \Gamma_r \Delta = - e^r \frac{3}{4} L = - \frac{3}{8r} ,
\]

\[
W_m' - e^m' \Gamma_m \Gamma_a \Delta = \frac{1}{4} \omega^{m'} a b \Gamma_a b - \frac{1}{2} L e^a_m \Gamma_a \Gamma_r - \frac{1}{24} e^m_0 \Gamma_{a b} a_4 F_{a' a' \perp a_4} .
\]

(3.25)

By using eq. (A.18), we have

\[
\mathcal{P}_{(D0)}^{(1)} = \mathcal{P}_{(LC)}^{(1)} .
\]

(3.26)

Plugging eqs. (3.25) and (3.26) into the action (3.22), we obtain

\[
S_{D0}'^{(F2)U(1)} = - \int d \tau \frac{1}{L} \left( \frac{L}{2r} \right)^{\frac{3}{2}} i \left[ \bar{\Psi} \mathcal{P}_{(LC)}^{(1)} \Gamma_0 \partial \Psi
\right.
\]

\[
+ \partial_r X^{m'} \bar{\Psi} \mathcal{P}_{(LC)}^{(1)} \Gamma_0 \left( \frac{1}{4} \omega^{m'} a b \Gamma_a b - \frac{1}{2} L e^a_m \Gamma_a \Gamma_r - \frac{1}{24} e^m_0 \Gamma_{a b} a_4 F_{a' a' \perp a_4} \right) \Psi
\]

\[
+ \frac{1}{2} L \bar{\Psi} \mathcal{P}_{(LC)}^{(1)} \Gamma_0 \left( \partial \right) \partial_r X^{m'} \bar{\Psi} e_{a b} \Gamma_a \partial_r \Psi \right] .
\]

(3.27)

We note that the contribution of \( W_r - e^r \Gamma_r \Delta \) to this action becomes vanishing because of eq. (A.3). By using the relations in Appendix [A], we may describe the action (3.27) in terms of the \( SO(9) \) spinors \( \psi^{(t)}, \psi^{(d)} \) and the gamma matrices \( \gamma_{a \perp} \) as follows:

\[
S_{D0}'^{(F2)U(1)} = \int d \tau \frac{1}{L} \left( \frac{L}{2r} \right)^{\frac{3}{2}} i \left[ \psi^{(t)} \partial \psi^{(d)}
\right.
\]

\[
+ \partial_r X^{m'} \psi^{(d)} \left( \frac{1}{4} \omega^{m'} a b \psi^{(d)} a b - \frac{1}{2} L e^a_m \psi^{(d)} a \Gamma_r + \frac{1}{24} e^m_0 \Gamma_{a b} a_4 F_{a' a' \perp a_4} \right) \psi^{(d)}
\]

\[
- \frac{1}{2} L \bar{\psi} \partial_r X^{m'} e_{a b} \left( \psi^{(d)} \gamma_{a \perp} \partial_r \psi^{(d)} + \psi^{(d)} \gamma_{a \perp} \partial_r \psi^{(d)} \right) .
\]

(3.28)

Here we have decomposed the Majorana spinor \( \Psi \) as

\[
\Psi = \frac{1}{\sqrt{2}} \left( \begin{array}{c}
\psi^{(t)} \\
\psi^{(d)}
\end{array} \right) .
\]

(3.29)
The supersymmetric Dp-brane action possesses the fermionic gauge symmetry ($\kappa$-symmetry). In the present case, under the $\kappa$-transformation, the fermionic coordinate $\Psi$ transforms as
\[
\delta_\kappa \Psi = (1 - \tilde{\Gamma}_D) \kappa + \mathcal{O}(\kappa^3) = (1 + \Gamma_0 \Gamma_\tilde{2}) \kappa + \mathcal{O}(l_s^2, \kappa^3),
\]
where the transformation parameter $\kappa$ is a $\tau$-dependent $SO(9,1)$ Majorana spinor. We may therefore impose the following condition\(^7\) on the fermionic coordinate $\Psi$ to fix this gauge symmetry,
\[
P^+_{D0}\Psi = 0 \iff \psi^{(\uparrow)} = 0.
\]
In this gauge, the last term in the action (3.27) becomes vanishing. We thus find that the action (3.27) takes the same form as the action (3.14) with the identification
\[
\psi^{(\downarrow)} = \sqrt{\frac{R}{N}} \left( \frac{2r}{L} \right) ^{\frac{3}{8}} \theta,
\]
with $N = 1$. As addressed in Section 2.3, the normalization factor of the fermionic coordinate $\theta$ may be absorbed into conventions, even though it depends on the bosonic fields. In this way, we have verified that, as well as the bosonic sector, the fermionic sector of the matrix action of the light-cone supermembrane on $AdS_7 \times S^4$ has the interpretation as the Matrix theory action of D0-branes.

4 Conclusions and Discussions

In this paper, we obtained matrix quantum mechanics for a supermembrane on $AdS_7 \times S^4$ from the light-cone supermembrane on this background. We constructed light-cone gauge formulation for the supermembrane on $AdS_7 \times S^4$ in a similar way to the flat case [9]. Taking account of the fact that $AdS_7 \times S^4$ is obtained as near-horizon geometry of the M5-brane solution [28][29], we expect that the resulting matrix quantum mechanics (2.69) should govern the system of D0-branes propagating near the horizon of the D4-brane solution of type IIA supergravity. We verified that the action (2.69) has such a Matrix theory interpretation. We showed that the bosonic part $S_{\text{matrix}}^{(B)}$ of the matrix action (2.69) takes the same form as the leading order terms in the $\alpha'$-expansion of the non-abelian Born-Infeld action (2.69) of D0-branes. As for the fermionic sector, we compared the action (2.69) with the results in ref. [63], where the explicit form of the action is presented up to quadratic order in the fermionic coordinate $\theta$ for a single D$p$-brane on a general bosonic curved background. We showed that the corresponding part of our action (3.14) reproduces their results.

The fermionic sector of the action (2.69) also contains non-abelian parts and quartic order terms of $\theta$. From the fact that the action (2.69) coincides with the part of the D0-brane action stated in the above, we may expect that our result should give a hint as to the non-abelian extension of the fermionic sectors of the Born-Infeld actions for D0-branes on

\(^7\)From eq. (3.20), we find that the gauge condition (3.31) formally takes the same form as the light-cone gauge condition (2.23) imposed on the fermionic coordinate $\Theta$ of the supermembrane.
curved backgrounds. We may rewrite the action $S^{(F2)}_{\text{matrix}}$ in eq. (2.69) in terms of variables of ten-dimensional type IIA string theory into the following form in the gauge (3.31):

$$
S^{(F2)}_{\text{matrix}} = \frac{1}{g_s l_s} \int d\tau \text{Tr} \left[ e^{-\phi} \sqrt{-g_{00}} \times \right.
\times \left. i \left( \frac{1}{g_{00}} \Psi \mathcal{P}_+^{(D0)} e^0_0 r_0 (D_\tau + W_0 - \tilde{e}^a_0 \Gamma_0 \Delta) \Psi - \Psi \mathcal{P}_+^{(D0)} \Gamma_2 \Gamma_3 a_\perp \tilde{e}^a_\perp \Gamma_{0} \Phi_{a \perp}, \Psi \right) + 2\pi l_s^2 \mathcal{D}_\tau \Phi^{m_\perp} \Psi \mathcal{P}_+^{(D0)} e^0_0 \Gamma_0 \left( W_{m_\perp} - \tilde{e}^a_{m_\perp} \Gamma_{a_\perp} \Delta \right) \Psi
+ 2\pi l_s^2 \mathcal{D}_\tau \Phi^{m_\perp} \frac{1}{2} \tilde{e}^a_{m_\perp} \Gamma_{a_\perp} \left( D_\tau + W_0 - \tilde{e}^0_0 \Gamma_0 \Delta \right) \Psi
- 2\pi l_s^2 \tilde{e}^a_{m_\perp} i [\Phi_{m_\perp}, \Phi_{n_\perp}] \Psi \mathcal{P}_+^{(D0)} \Gamma_2 \Gamma_{a_\perp} W_{n_\perp} \Psi \right),
$$

(4.1)

with the identifications (3.13) and (3.32). We note that there are ambiguities in covariantizing the $SO(9)$ spinor $\theta$ and gamma matrices $\gamma_{a_\perp}$ into the $SO(9,1)$ spinor $\Psi$ and gamma matrices $\Gamma_0$ in eq. (4.1). The first, the third and the fourth terms on the r.h.s. in eq. (4.1) are directly obtained from the $U(1)$ part of the D0-brane action (3.22) by replacing $\partial_\tau \Phi^{m_\perp}$ with $\mathcal{D}_\tau \Phi^{m_\perp}$. This replacement is a part of steps in the non-abelian extension of the Born-Infeld actions [51]. We may naturally guess that the last term on the r.h.s. in eq. (4.1) should be derived from terms proportional to the field strength $F_{\mu\nu}$ of the gauge field $A_{\mu}$ in the $D_p$-brane action via T-duality. We make a comment on the four-Fermi terms $S^{(F4)}_{\text{matrix}}$ of the action (2.69). In Section 2.2 we used the Fierz relations (3.11) and (3.12) in the four-Fermi terms, before applying the matrix regularization to the membrane action. Whether we are allowed to use such Fierz relations also after the matrix regularization depends on the ordering prescription in the fermionic sector of the non-abelian Born-Infeld action. This point needs further investigations.

In this paper, we did not discuss the (super-)isometry of $AdS_7 \times S^4$. It is because the isometries of the target spaces of light-cone supermembranes become subtle after the matrix regularization. In fact, even in the flat case, the eleven-dimensional Lorentz symmetry is obscure in the matrix action [30] [61] [62]. Such obscurity originates in the fact that it is not known how to employ the matrix regularization for the coordinate $X^-$ and consequently for the Lorentz generators $M^{-a_\perp}$, since $X^-$ explicitly depends on the geometry of $\Sigma_{(2)}$ in the light-cone gauge.

As mentioned in Introduction, supermembrane theory on $AdS_7 \times S^4$ is supposed to be dual to six-dimensional superconformal field theory [30] [31] [32] [33]. We hope, therefore, that our matrix action should provide a useful new approach to the study of the $AdS_7/CFT_6$ correspondence beyond the supergravity level and shed light on this problem.

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**A  \( SO(10,1) \) and \( SO(9) \) gamma matrices**

The \( SO(10,1) \) gamma matrices \( \Gamma_{\hat{a}} \) (\( \hat{a} = \vec{0}, \ldots, \vec{9}, \vec{\tau} \)) satisfy the \( SO(10,1) \) Clifford algebra

\[
\Gamma_{\hat{a}} \Gamma_{\hat{b}} + \Gamma_{\hat{b}} \Gamma_{\hat{a}} = 2\eta_{\hat{a}\hat{b}} , \quad \eta_{\hat{a}\hat{b}} = \text{diag}(-1,1,\ldots,1) .
\]

(A.1)

The hermitian conjugation of \( \Gamma_{\hat{a}} \) is given by \( \Gamma_{\hat{a}}^\dagger = \Gamma_{\bar{\hat{a}}} \). The charge conjugation of the \( SO(10,1) \) spinor \( \Psi \) is defined as \( \Psi^c = \mathcal{C}\bar{\Psi}^T \), where \( \bar{\Psi} = \Psi^\dagger \Gamma_{\vec{0}} \) is the Dirac conjugate of \( \Psi \) and \( \mathcal{C} \) is the charge conjugation matrix defined by

\[
\Gamma_{\hat{a}}^T = -\mathcal{C}^{-1} \Gamma_{\hat{a}} \mathcal{C} , \quad \mathcal{C}^T = -\mathcal{C} .
\]

(A.2)

Eq.\( \text{(A.2)} \) yields

\[
(C^{-1} \Gamma_{\hat{a}_1 \cdots \hat{a}_n})^T = (-1)^{\frac{n(n+2)}{2}} C^{-1} \Gamma_{\hat{a}_1 \cdots \hat{a}_n} .
\]

(A.4)

This implies that the matrix \( C^{-1} \Gamma_{\hat{a}_1 \cdots \hat{a}_n} \) is symmetric for \( n \equiv 1, 2 \) (mod 4) and anti-symmetric for \( n \equiv 0, 3 \) (mod 4). Thereby we obtain

\[
\bar{\Psi}_M \Gamma_{\hat{a}_1 \cdots \hat{a}_n} \Psi_M = 0 \quad \text{for} \quad n \equiv 1, 2 \pmod{4} ,
\]

(A.5)

where \( \Psi_M \) is an arbitrary Majorana spinor.

A set of the matrices

\[
(C^{-1})_{\hat{a}\hat{\beta}} , \quad (C^{-1} \Gamma_{\hat{a}})_{\hat{a}\hat{\beta}} , \quad (C^{-1} \Gamma_{\hat{a}_1 \hat{a}_2})_{\hat{a}\hat{\beta}} , \quad \cdots , \quad (C^{-1} \Gamma_{\hat{a}_1 \cdots \hat{a}_5})_{\hat{a}\hat{\beta}}
\]

(A.6)

composes a complete basis of the \( 32 \times 32 \) matrices. The completeness relation reads

\[
\delta_{\hat{a}\hat{\alpha}} \delta_{\hat{\beta}\hat{\beta}} = \frac{1}{32} \sum_{p=0}^{5} \frac{1}{p!} (\Gamma_{\hat{a}_p \cdots \hat{a}_1})^{\hat{\delta} \hat{\epsilon}} (C^{-1} \Gamma_{\hat{a}_1 \cdots \hat{a}_p})_{\hat{a}\hat{\beta}} .
\]

(A.7)

Using this relation, we can prove the identities

\[
(C^{-1} \Gamma_{\hat{a}\hat{b}})_{\hat{a}\hat{\beta}} (C^{-1} \Gamma_{\hat{b}\hat{c}})_{\hat{\delta}\hat{\epsilon}} = 0 ,
\]

(A.8)

\[
(C^{-1} \Gamma_{\hat{a}\hat{b}})_{\hat{a}[\hat{\beta}} (C^{-1} \Gamma_{\hat{b}\hat{c}})_{\hat{\delta]\hat{\epsilon}]} - 6(C^{-1} \Gamma_{\hat{c}})_{\hat{a}[\hat{\beta}} (C^{-1}\Gamma_{\hat{c}})_{\hat{\delta]\hat{\epsilon}]} = 0 .
\]

(A.9)

In this paper, we choose the Majorana representation for the \( SO(10,1) \) spinors in which the Majorana spinors become real spinors. One can find that the following relations hold in this representation:

\[
\mathcal{C} = \Gamma_{\vec{0}} , \quad \Gamma_{\hat{a}}^* = \Gamma_{\hat{a}} ,
\]

(A.10)

where \( \Gamma_{\hat{a}}^* \) denotes the complex conjugate of \( \Gamma_{\hat{a}} \).

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In accordance with the light-cone coordinates (2.17), we introduce \((\Gamma^+, \Gamma^-)\) defined as

\[
\Gamma^+ = \Gamma_\perp = \frac{1}{\sqrt{2}} \left( \Gamma^5 + \Gamma^0 \right), \quad \Gamma^- = \Gamma_\perp = \frac{1}{\sqrt{2}} \left( \Gamma^5 - \Gamma^0 \right).
\] (A.11)

These matrices obey the relations

\[
\Gamma^+ \Gamma^- + \Gamma^- \Gamma^+ = 2\eta^{\perp \perp} = 2, \quad (\Gamma^+)^2 = \eta^{\perp \perp} = 0, \quad (\Gamma^-)^2 = \eta^{\perp \perp} = 0.
\] (A.12)

We decompose the gamma matrices \(\Gamma_\perp\) into \(SO(1,1) \times SO(9)\) gamma matrices as follows:

\[
\Gamma_0 = -i\sigma_2 \otimes \textbf{1}_{16} = \left( \begin{array}{cc} 0 & -\textbf{1}_{16} \\ \textbf{1}_{16} & 0 \end{array} \right), \quad \Gamma_5 = -\sigma_1 \otimes \textbf{1}_{16} = \left( \begin{array}{cc} 0 & -\textbf{1}_{16} \\ -\textbf{1}_{16} & 0 \end{array} \right),
\]

\[
\Gamma_a = \sigma_3 \otimes \gamma_a = \left( \begin{array}{cc} \gamma_a & 0 \\ 0 & -\gamma_a \end{array} \right) \quad a = (\bar{1}, \ldots, \bar{4}, \bar{r}, \bar{6}, \ldots, \bar{9}),
\] (A.13)

where \((\sigma_1, \sigma_2, \sigma_3)\) denote the standard Pauli matrices and \(\gamma_a\) denote \(SO(9)\) gamma matrices, which satisfy the \(SO(9)\) Clifford algebra,

\[
\gamma_a \gamma_b + \gamma_b \gamma_a = 2\delta_{ab}.
\] (A.14)

Using eqs. (A.12) and (A.10), we find that the \(SO(9)\) gamma matrices \(\gamma_a\) in eq. (A.13) are real and symmetric:

\[
\gamma_a^T = \gamma_a, \quad \gamma_a^* = \gamma_a.
\] (A.15)

The \(SO(9)\) charge conjugation matrix \(C_9\), defined by \(\gamma_9^T = C_9^{-1} \gamma_a C_9\) and \(C_9^T = C_9\), may hence be chosen to be unity, \(C_9 = 1\), in this representation.

In accordance with the decomposition (A.13) of the gamma matrices, an arbitrary \(SO(9,1)\) spinor \(\Psi\) is decomposed into two \(SO(9)\) spinors as

\[
\Psi_\hat{\alpha} = \left( \begin{array}{c} \psi_{\hat{\alpha}}^{(\uparrow)} \\ \psi_{\hat{\alpha}}^{(\downarrow)} \end{array} \right) \quad (\alpha = 1, \ldots, 16).
\] (A.16)

Eq. (A.13) yields

\[
\Gamma^+ = \left( \begin{array}{cc} 0 & 0 \\ -\sqrt{2} & 0 \end{array} \right), \quad \Gamma^- = \left( \begin{array}{cc} 0 & -\sqrt{2} \\ 0 & 0 \end{array} \right),
\]

\[
\mathcal{P}^{(LC)}_\perp = \frac{1 - \Gamma^-}{2} = \left( \begin{array}{cc} 0 & 0 \\ 0 & \textbf{1}_{16} \end{array} \right), \quad \mathcal{P}^{(LC)}_+ = \frac{1 + \Gamma^-}{2} = \left( \begin{array}{cc} \textbf{1}_{16} & 0 \\ 0 & 0 \end{array} \right),
\] (A.17)

where

\[
\Gamma^\perp = \frac{1}{2} (\Gamma^- \Gamma^+ + \Gamma^+ \Gamma^-) = \Gamma_0 \Gamma_5.
\] (A.18)

This leads to

\[
\Psi_- \equiv \mathcal{P}^{(LC)}_- \Psi = \left( \begin{array}{c} 0 \\ \psi^{(\downarrow)} \end{array} \right), \quad \Psi_+ \equiv \mathcal{P}^{(\uparrow)}_+ \Psi = \left( \begin{array}{c} \psi^{(\uparrow)} \\ 0 \end{array} \right).
\] (A.19)
B Derivation of eq. (2.38)

In this appendix, we derive eq. (2.38). In the following, we will frequently use the relations (2.20, 2.27) and (A.3), without mentioning it.

We begin by providing relations for later use. Multiplying eq. (A.8) by \( \Theta_\beta^\dagger \Theta_\delta^\dagger \Theta_\epsilon^\dagger \) and setting \( \hat{c} = - \), we have

\[
\left( \bar{\Theta}_- \Gamma^a b^1 \Theta_\perp \right) \bar{\Theta}_- \Gamma_{a \perp b \perp} = 0 . \tag{B.1}
\]

Multiplying eq. (A.8) by \( (\Gamma^\dagger \Theta_\perp)^\hat{a} \Theta_\delta^\dagger (\Gamma_{c_1 c_2 c_3 c_4} \Theta_\perp)^\delta F^{c_1' c_2' c_3' c_4'} \) and setting \( \hat{a} = c_1' \), we may show that

\[
\left( \bar{\Theta}_- \Gamma^{c_1' a \perp b \perp \Theta_\perp} \right) \bar{\Theta}_- \Gamma_{c_1' c_2' c_3' c_4'} \Gamma^{b_1 b_2} F^{c_1' \cdots c_4'} = -3 \left( \bar{\Theta}_- \Gamma^{c_1' c_2' c_3' c_4'} \Theta_\perp \right) \bar{\Theta}_- \Gamma_{c_1' c_2} F^{c_1' \cdots c_4'} . \tag{B.2}
\]

Combining the identities

\[
\Gamma^{a \perp b_1} \Gamma_{c_1' c_2' c_3' c_4'} = \Gamma^{a \perp b_1} c_1' \cdots c_2' - 8 \delta_{c_1'}^{[a} \Gamma_{c_2' c_3' c_4']} - 12 \delta_{c_2'}^{b_1} \delta_{c_1'}^{b_1} \Gamma_{c_3' c_4'} ,
\]

\[
\Gamma_{c_1' c_2' c_3'} \Gamma^{b_1} = -\Gamma^{b_1} c_1' c_2' c_3' + 3 \delta^{b_1}_{c_2} \delta_{c_1'}^{b_1} \Gamma_{c_3' c_4'} , \tag{B.3}
\]

we obtain

\[
\Gamma^{a \perp b_1} c_1' c_2' c_3' c_4' = \Gamma^{a \perp b_1} c_1' c_2' c_3' c_4' - 8 \delta_{c_1'}^{[a} \Gamma_{c_2' c_3' c_4']} \Gamma^{b_1]} + 36 \delta_{c_1'}^{b_1} \delta_{c_2'}^{b_1} \Gamma_{c_3' c_4'} . \tag{B.4}
\]

Eq. (2.23) yields

\[
\bar{\Theta}_- \Gamma^{i} M^2[\Theta_\perp] = \frac{i}{18} \left( \bar{\Theta}_- \Gamma^{i} b^i_4 b^i_4 F^{b_1' \cdots b_4'} \Theta_\perp \right) \bar{\Theta}_- \Gamma_{b^i_4} + \frac{i}{288} \left( \bar{\Theta}_- \Gamma^{i} a \perp b_1 \Theta_\perp \right) \bar{\Theta}_- \Gamma_{a \perp b_1} \Gamma_{c_1' c_2' c_3' c_4'} F^{c_1' \cdots c_4'} + \frac{i}{12} \left( \bar{\Theta}_- \Gamma^{i} c_1' c_2' \Theta_\perp \right) \bar{\Theta}_- \Gamma_{c_1' c_2} F^{c_1' \cdots c_4'} . \tag{B.5}
\]

Here we have used the following relation, which follows from eq. (A.5):

\[
\bar{\Theta}_- \Gamma^{i} a \perp b_1 \cdots b_4' \Theta_\perp = 0 . \tag{B.6}
\]

Substituting eq. (B.4) into the second term on the r.h.s. in eq. (B.5), we have

\[
\bar{\Theta}_- \Gamma^{i} M^2[\Theta_\perp] = \frac{i}{18} \left( \bar{\Theta}_- \Gamma^{i} b^i_4 b^i_4 F^{b_1' \cdots b_4'} \Theta_\perp \right) \bar{\Theta}_- \Gamma_{b^i_4} + \frac{i}{288} \left( \bar{\Theta}_- \Gamma^{i} a \perp b_1 \Theta_\perp \right) \bar{\Theta}_- \Gamma_{a \perp b_1} \Gamma_{c_1' c_2' c_3' c_4'} F^{c_1' \cdots c_4'}
\]

\[
- \frac{i}{36} \left( \bar{\Theta}_- \Gamma^{i} c_1' \perp b_1 \Theta_\perp \right) \bar{\Theta}_- \Gamma_{c_1' c_2' c_3'} \Gamma^{b_1} F^{c_1' \cdots c_4'} + \frac{i}{24} \left( \bar{\Theta}_- \Gamma^{i} c_1' c_2' \Theta_\perp \right) \bar{\Theta}_- \Gamma_{c_1' c_2} F^{c_1' \cdots c_4'} . \tag{B.7}
\]
By using eq. (B.6), we may recast this equation into

\[ \Theta_\gamma \Gamma^\gamma \mathcal{M}^2[\Theta_-] = \frac{i}{12} \left( \Theta_\gamma \Gamma^\gamma \mathcal{M}^2[\Theta_-] \right) \Theta_\gamma \Gamma_i \theta_i + \frac{i}{8} \left( \Theta_\gamma \Gamma^\gamma \mathcal{M}^2[\Theta_-] \right) \Theta_\gamma \Gamma_i \theta_i F^{\epsilon_i \cdots \epsilon_4} . \]  

(B.8)

By using eq. (B.6), we may recast this equation into

\[ \Theta_\gamma \Gamma^\gamma \mathcal{M}^2[\Theta_-] = \frac{i}{48} \left( \Theta_\gamma \Gamma^\gamma \mathcal{M}^2[\Theta_-] \right) \Theta_\gamma \Gamma_i \theta_i + \frac{i}{8} \left( \Theta_\gamma \Gamma^\gamma \mathcal{M}^2[\Theta_-] \right) \Theta_\gamma \Gamma_i \theta_i F^{\epsilon_i \cdots \epsilon_4} . \]  

(B.9)

Substituting this equation into the definition \( \mathcal{F} \) of \( \mathcal{F} \), we obtain eq. (2.38). Thus we have derived eq. (2.38).

C \ Explicit Forms of Several Quantities

In this appendix, we substitute the explicit form of the field \( F_{a'b'c'd'} \) in the \( AdS_7 \times S^4 \) solution (2.7) into several quantities in the text. As mentioned in Section 2.2, it is convenient to use the fermionic coordinates \( (\Theta_+^-, \Theta_-^+) \) introduced in eq. (2.32). We note that the ‘chirality’ associated with the projection operator \( P^{(M5)}_\pm \) flips under the Dirac conjugation:

\[ \Theta_\gamma^\pm = (\Theta_\gamma^\pm) P^{(M5)}_\mp . \]  

(C.1)

Plugging eq. (2.31) into eq. (2.25), we find that \( \mathcal{M}^2[\Theta_-] \) takes the form

\[ \left( \mathcal{M}^2[\Theta_-] \right)^\hat{\eta}_\beta = \eta \left[ \Gamma_{\hat{\gamma}} \Theta_\eta^\gamma \left( \Theta_\gamma \Gamma_{\gamma} \right) \right] \beta \alpha + (\Gamma_r \gamma \Theta_-) \hat{\alpha} \left( \Theta_\gamma \Gamma_{\gamma} \right) \beta - 2 (\Gamma_r \gamma \Theta_-) \hat{\alpha} \left( \Theta_\gamma \Gamma_{\gamma} \right) \beta \]

\[ - \Theta^\alpha \left( \Theta_\gamma^\eta \right) \beta - (\Gamma_r \gamma \Theta_-) \hat{\alpha} \left( \Theta_\gamma \Gamma_{\gamma} \right) \beta \]

\[ - \frac{1}{2} (\Gamma_r \gamma \Theta_-) \hat{\alpha} \left( \Theta_\gamma \Gamma_{\gamma} \right) \beta + (\Gamma_r \gamma \Theta_-) \hat{\alpha} \left( \Theta_\gamma \Gamma_{\gamma} \right) \beta , \]  

(C.2)

where \( \gamma^\prime \) is the matrix defined in eq. (2.31). From eq. (2.30), we obtain

\[ \tilde{D}_{\eta} \Theta_{-} = \partial_{\eta} \left( \Theta_{-} \right) + \partial_{\eta} \left( \Theta_{-} \right) \]

\[ + \partial_{\eta} X^k \tilde{e}_{\eta} \frac{1}{L} \Gamma_{\kappa} \Theta_{-} + \partial_{\eta} X^{\ell} \tilde{e}_{\eta} \frac{1}{L} \Gamma_{\ell} \Theta_{-} + \partial_{\eta} \frac{1}{4 r} \left( \Theta_{-} - \Theta_{+} \right) \]

\[ + \partial_{\eta} X^{\ell} \tilde{e}_{\eta} \left( \frac{1}{4} \omega_{\ell \omega} a' b' \Gamma_{a' b'} + \frac{\eta}{L} a' b' \Gamma_{a' b'} \right) \left( \Theta_{+} + \Theta_{-} \right) . \]

(C.3)

Using these relations, we obtain a detailed form of the pull-back \( \Pi_\gamma \) of the supervielbein. The component \( \Pi_\gamma \) is expressed as

\[ \Pi_\gamma = \partial_\gamma X^k \tilde{e}_\gamma - i \Theta_\gamma \Gamma^- \partial_\gamma \Theta_+ - i \Theta_\gamma \Gamma^- \partial_\gamma \Theta_- \]
\[ -i \partial_t X^m e_m^a \frac{1}{L} \Theta^{(+)} \Gamma^a \Gamma^\dagger \Theta^{(-)} \]
\[ -i \partial_t X^{m'} \left[ \left( \frac{1}{4} \omega_{m'm''}^{a'b'} \Gamma_{a'b'} + \frac{\eta}{L} e'_{m'} \Gamma^\dagger \right) \Theta^{(+)} \right. \]
\[ \left. + \Theta^{(-)} \left( \frac{1}{4} \omega_{m'm''}^{a'b'} \Gamma_{a'b'} + \frac{\eta}{L} e'_{m'} \Gamma^\dagger \right) \right] \]
\[ = \partial_t X^+ \imath \frac{1}{8L^2} \left[ 2 \left( \Theta^{(-)} \Gamma^a \Gamma^\dagger \Theta^{(-)} - \Theta^{(+)} \Gamma^a \Gamma^\dagger \Theta^{(+)} \right) \Theta^{(-)} \right. \]
\[ \left. + \Theta^{(-)} \left( \Theta^{(-)} \Gamma^a \Gamma^\dagger \Theta^{(-)} + \Theta^{(+)} \Gamma^a \Gamma^\dagger \Theta^{(+)} \right) \Theta^{(-)} \Gamma^a \Gamma^\dagger \Theta^{(-)} \right] . \] (C.4)

Here we have used eq. [B.3]. The components \( \Pi^\dagger \) and \( \Pi^{a'} \) take the forms

\[ \Pi^\dagger = \partial_t X^m e_m^a - i e_m^a \frac{1}{2L} \Theta^{(+)} \Gamma^a \Gamma^\dagger \Theta^{(-)} , \]
\[ \Pi^{a'} = \partial_t X^{m'} e_{m'}^{a'} - i e_{m'}^{a'} \frac{1}{2L} \Theta^{(-)} \Gamma^{a'} \Gamma^\dagger \Theta^{(-)} . \] (C.5)

The fermionic components \( \Pi^\dagger \) and \( \Pi^{a'} \) become

\[ \Pi^\dagger = \partial_t \left( \Theta^{(+)} + \Theta^{(-)} \right) \]
\[ + \frac{1}{2L \Gamma^a \Gamma^\dagger \Gamma^{a'} \Theta^{(-)}} \left( \Theta^{(+)} \Gamma^a \Gamma^\dagger \Theta^{(-)} \right. \]
\[ \left. - 2 \Gamma^a \Gamma^\dagger \left( \Theta^{(-)} - \Theta^{(+)} \right) \left( \Theta^{(-)} \Gamma^a \Gamma^\dagger \Theta^{(-)} \right) \right] , \]
\[ \Pi^{a'} = \partial_t \left( \Theta^{(+)} + \Theta^{(-)} \right) \]
\[ + \frac{1}{2L \Gamma^a \Gamma^\dagger \Gamma^{a'} \Theta^{(-)}} \left( \Theta^{(+)} \Gamma^a \Gamma^\dagger \Theta^{(-)} \right. \]
\[ \left. - 2 \Gamma^a \Gamma^\dagger \left( \Theta^{(-)} - \Theta^{(+)} \right) \left( \Theta^{(-)} \Gamma^a \Gamma^\dagger \Theta^{(-)} \right) \right] , \] (C.6)

The elements \( h_{\tau \tau} \) and \( u_i \equiv h_{\tau i} \) of the induced metric \( h_{ij} \) turn out to be

\[ h_{\tau \tau} = 2 \partial_\tau X^a \partial_\tau X^{a'} \partial_{\tau X}^n \partial_{\tau X}^{n'} \Gamma_{a} \Gamma^\dagger \Theta^{(-)}, \]
\[ - i 2 \left( \Theta^{(+)} \Gamma^a \partial_\tau \Theta^{(-)} + \Theta^{(-)} \Gamma^a \partial_\tau \Theta^{(-)} \right) . \] (28)
\[ u_t = \partial_t X^\alpha G_\alpha + \partial_t X^{m\pm} \partial_t X^{n\pm} G_{m\pm n\pm}, \]
\[ -i e^\tau_{\tau} \left[ \Theta_\tau^{(\pm)} \Gamma_{\tau} \left( \frac{1}{4} \omega_{m' a'} \Gamma_{a' b'} + \frac{\eta}{L} e_{m'} a' \Gamma_{a' \gamma'} \right) \Theta_\tau^{(\pm)} \right. \]
\[ + \left. \Theta_\tau^{(-)} \Gamma_{\tau} \left( \frac{1}{4} \omega_{m' a'} \Gamma_{a' b'} + 2 \frac{\eta}{L} e_{m'} a' \Gamma_{a' \gamma'} \right) \Theta_\tau^{(-)} \right] + F, \]

where \( F \) is the four-Fermi term introduced in eq. (2.37). From eq. (2.38), we find that this is described by

\[ F = -\left( \frac{e_{\tau}}{L} \right)^2 \left[ \left( \Theta^{(\pm)} \Gamma_{\tau} \Theta^{(-)} \right) \left( \Theta^{(\pm)} \Gamma_{\tau} \Theta^{(-)} \right) \right. \]
\[ + \frac{1}{2} \left( \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \left( \Theta^{(\pm)} \Gamma_{\tau} \Theta^{(-)} \right) \left( \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \]
\[ + \frac{1}{4} \left( \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \left( \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \left( \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \] .

Let us denote the \( \Theta \)-dependent part of the Wess-Zumino term by \( L^{(F)}_{WZ} \),

\[ L^{(F)}_{WZ} = -T_i \int_0^1 dt e_{\tau}^{ijk} \Theta_\tau \Gamma_{\tau} \Pi_i(X, t \Theta_\tau) \Pi_j(X, t \Theta_\tau) \Pi_k(X, t \Theta_\tau) \]
\[ = T_i e_{\tau}^{ijk} e_{m \pm}^{a \pm} \partial_t X^{m \pm} \Theta_\tau \Gamma_{\tau} \left[ \partial_j \Theta_\tau + \partial_j X^{n \pm} \left( \frac{1}{4} \omega_{n \pm b} b_{\pm c \pm} \Gamma_{b \pm c \pm} \right. \right. \]
\[ + \left. \left. \frac{1}{4L} e_{n \pm}^{b \pm} \Gamma_{b \pm c \pm} + \frac{1}{192} e_{n \pm}^{b \pm} \Gamma_{c \pm d \pm} \Gamma_{b \pm e \pm} F_{c \pm e \pm} \right) \Theta_\tau \right] . \]

This is expressed as

\[ L^{(F)}_{WZ} = T_i e_{\tau}^{ijk} \left[ \partial_t X^{n \pm} e_{m \pm}^{a \pm} \left( \Theta^{(\pm)} \Gamma_{\tau} \partial_j \Theta^{(-)} \right. \right. \]
\[ + \partial_t \partial_j e_{m \pm}^{a \pm} \left( \Theta^{(\pm)} \Gamma_{\tau} \partial_j \Theta^{(-)} \right) \left. \right. \]
\[ + \partial_t X^{m \pm} e_{m \pm}^{a \pm} \left( \Theta^{(\pm)} \Gamma_{\tau} \partial_j \Theta^{(-)} \right) \left. \right. \]
\[ + \partial_t X^{m \pm} \partial_j X^{n \pm} \left( \frac{3}{2L} e_{m \pm}^{a \pm} e_{n \pm}^{b \pm} \Theta^{(-)} \Gamma_{\tau} \partial_j \Theta^{(-)} \right) \]
\[ + \partial_t X^{m \pm} \partial_j X^{n \pm} \left( \frac{3}{L} e_{m \pm}^{a \pm} e_{n \pm}^{b \pm} \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \]
\[ + \partial_t X^{m \pm} \partial_j X^{n \pm} \left( \frac{3}{L} e_{m \pm}^{a \pm} e_{n \pm}^{b \pm} \Theta^{(-)} \Gamma_{\tau} \Theta^{(-)} \right) \]
Here we have used the relation $\omega_m \tilde{a}^\tilde{m} = \frac{1}{L} e^\tilde{m}$. Using the notation $\mathcal{L}^{(F)}_{WZ}$, we may write the hamiltonian (2.42) as

$$
\mathcal{H}_0 = \frac{G_{++}}{2P_-} \left[ T^2 \bar{h} + P_m \bar{G}^{\tilde{m}\tilde{n}} P_{\tilde{n}} + P_r G^{rr} P_r + Q_{m'} G^{m'n'} Q_{n'} \right] - \mathcal{L}^{(F)}_{WZ} - \frac{P_-}{2G_{++}} \mathcal{F}, 
$$

(C.11)

where $Q_{m'}$ is given in eq. (2.43), which is described as

$$
Q_{m'} = P_{m'} - \frac{T}{2} \epsilon^{ij} \partial_i X^{m'_1} \partial_j X^{m'_2} C_{m'_1 m'_2} + \frac{P_-}{e^-} \left[ \Theta^{(+)} \Gamma^{-} \left( \frac{1}{4} \omega_{m'} a^\dagger b^\prime \Gamma_{a'b'} + \frac{\eta}{L} e_{m'} \Gamma_{a'b'} \right) \Theta^{(+)} + \Theta^{(-)} \Gamma^{-} \left( \frac{1}{4} \omega_{m'} a^\dagger b^\prime \Gamma_{a'b'} + \frac{\eta}{L} e_{m'} \Gamma_{a'b'} \right) \Theta^{(-)} \right]. 
$$

(C.12)
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