CP-violating inflation and its cosmological imprints

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Abstract

We study models with several SU(2) scalar doublets where the inert doublets have a non-minimal coupling to gravity and play the role of the inflaton. We allow for this coupling to be complex, thereby introducing CP-violation - a necessary source of the baryon asymmetry - in the Higgs–inflaton couplings. We investigate the inflationary dynamics of the model and discuss how the CP-violation of the model is imprinted on the particle asymmetries after inflation in the hot big bang universe.

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1 Introduction

The Standard Model (SM) of particle physics has been extensively tested and is in great agreement with experimental data, with its last missing particle – the Higgs boson – discovered by ATLAS and CMS experiments at the CERN Large Hadron Collider (LHC) \[1,2\]. Although the properties of the observed scalar are in agreement with those of the SM-Higgs boson, it may just be one member of an extended scalar sector. Even though so far no signs of new physics have been detected, it is well understood that the SM of particle physics is incomplete.

Cosmological and astrophysical observations imply a large dark matter (DM) component in the energy budget of the universe. Within the particle physics setting, this would be a particle which is stable on cosmological time scales, cold, non-baryonic, neutral and weakly interacting \[3\]. A particle with such characteristics does not exist in the SM. Another shortcoming of the SM is the lack of an explanation for the origin of the observed matter-antimatter asymmetry in the universe. One of the most promising baryogenesis scenarios is electroweak baryogenesis (EWBG) \[4\], which produces the baryon excess during the electroweak phase transition (EWPT). Although the SM in principle contains all required ingredients for EWBG, it is unable to explain the observed baryon excess due to its insufficient amount of CP-violation \[5–7\] and the lack of a first-order phase transition \[8\].

Furthermore, in its current form, the SM fails to incorporate cosmic inflation in a satisfactory manner. Inflation is a well-motivated theory predicting a period of exponential expansion in the early universe which explains the generation of primordial density fluctuations seeding structure formation, flatness, homogeneity and isotropy of the universe \[9–12\]. The simplest models of inflation in best agreement with observations are those driven by a scalar field, the inflaton, with a standard kinetic term, slowly rolling down its smooth potential. At the end of inflation, the inflaton which naturally is assumed to have couplings with the SM-Higgs, dumps its energy into the SM bath during the reheating process which populates the universe with SM particles.

Scalars with non-minimal couplings to gravity are well-motivated inflaton candidates since they acquire fluctuations proportional to the inflationary scale and can drive the inflation process in the early universe, as in the Higgs-inflation model \[13\] where the SM-Higgs plays the role of the inflaton, and s-inflation models \[14,15\] where the SM is extended by a singlet scalar. Extensive studies have been carried out in simple one singlet or one doublet scalar extensions of the SM (see e.g. \[16,19\] and references therein). These models, however, by construction can only partly provide a solution to the main drawbacks of the SM. For example, to incorporate both CP-violation and DM into the model one has to go beyond simple scalar extensions of the SM \[20\]; see also e.g. \[21–26\].

It is therefore theoretically appealing to have a more coherent setting where different motivations of beyond SM (BSM) frameworks could be simultaneously investigated. For example, in non-minimal Higgs frameworks with conserved discrete symmetries one can accommodate stabilised DM candidates. Moreover, the extended scalar potential could provide new sources of CP-violation and accommodate a strong first order phase transition \[27\]. Collider searches can constrain these model frameworks by excluding or discovering the existence of the spectrum of new states.

In this paper we introduce a model where a source of CP-violation originates from the
couplings of the inflation. Through the process of reheating this is transmitted to an asymmetry within the SM and can furthermore seed the generation of an excess of matter over antimatter during the evolution of the early universe. We describe these dynamics in the context of a $Z_2$ symmetric 3-Higgs Doublet Model (3HDM) with a CP-violating extended dark sector, which also provides a viable DM candidate, new sources of CP-violation and a strong first-order EWPT [20–25]. We study the inflationary dynamics of this set-up and outline its main consequences. In a future work we aim to continue to complement this study by more thorough analysis of EWBG and DM observables as well as a phenomenological analysis towards LHC searches for new physics.

The paper is organized as follows. In Section 2 we present the scalar potential and explore the inflationary dynamics. In Section 3, we discuss the inflationary imprints of our novel CP violating inflation phenomena. In Section 4, we discuss the inflaton decay into the SM particles and possible consequences. In Section 5 we draw our conclusions and discuss the outlook for further work.

2 The scalar potential

2.1 General definitions

A 3HDM scalar potential which is symmetric under a group $G$ of phase rotations, can be written as the sum of two parts: $V_0$ with terms symmetric under any phase rotation, and $V_G$ with terms symmetric under $G$ [28,29]. As a result, a $Z_2$-symmetric 3HDM can be written as

\[ V = V_0 + V_{Z_2}, \]

\[ V_0 = -\mu_1^2(\phi_1^\dagger \phi_1) - \mu_2^2(\phi_2^\dagger \phi_2) - \mu_3^2(\phi_3^\dagger \phi_3) + \lambda_{11}(\phi_1^\dagger \phi_1)^2 + \lambda_{22}(\phi_2^\dagger \phi_2)^2 + \lambda_{33}(\phi_3^\dagger \phi_3)^2 + \lambda_{12}(\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) + \lambda_{23}(\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) + \lambda_{31}(\phi_3^\dagger \phi_3)(\phi_1^\dagger \phi_1), \]

\[ V_{Z_2} = -\mu_{12}^2(\phi_1^\dagger \phi_2) + \lambda_1(\phi_1^\dagger \phi_2)^2 + \lambda_2(\phi_2^\dagger \phi_3)^2 + \lambda_3(\phi_3^\dagger \phi_1)^2 + h.c. \]

where the three Higgs doublets, $\phi_1, \phi_2, \phi_3$, transform under the $Z_2$ group, respectively, as

\[ g_{Z_2} = \text{diag}(-1, -1, +1). \]

The parameters of the $V_0$ part of the potential are real by construction. We allow for the parameters of $V_{Z_2}$ to be complex, using the following notation throughout the paper

\[ \lambda_j = |\lambda_j| e^{i\theta_j} \quad (j = 1, 2, 3), \quad \text{and} \quad \mu_{12}^2 = |\mu_{12}| e^{i\theta_{12}}. \]

The composition of the doublets is as follows:

\[ \phi_1 = \left( \begin{array}{c} H_1^+ \\ H_1^+ + iA_1 \\ \sqrt{2} \end{array} \right), \quad \phi_2 = \left( \begin{array}{c} H_2^+ \\ H_2^+ + iA_2 \\ \sqrt{2} \end{array} \right), \quad \phi_3 = \left( \begin{array}{c} G^+ \\ v + h + i\xi v \sqrt{2} \end{array} \right), \]

\[ ^1\text{We ignore additional } Z_2\text{-symmetric terms that can be added to the potential, e.g., } (\phi_3^\dagger \phi_1)(\phi_3^\dagger \phi_3), (\phi_1^\dagger \phi_3)(\phi_3^\dagger \phi_3), (\phi_1^\dagger \phi_2)(\phi_3^\dagger \phi_1) \text{ and } (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_2), \text{ as they do not change the phenomenology of the model [23].} \]
where $\phi_1$ and $\phi_2$ are the $Z_2$-odd inert doublets, $\langle \phi_1 \rangle = \langle \phi_2 \rangle = 0$, and $\phi_3$ is the one $Z_2$-even active doublet, which at low energy attains a vacuum expectation value (VEV) $\langle \phi_3 \rangle = v/\sqrt{2} \neq 0$. The doublet $\phi_3$ plays the role of the SM Higgs doublet, with $h$ being the SM Higgs boson and $G^\pm, G^0$ the would-be Goldstone bosons. Note that according to the $Z_2$ generator in Eq. (2) the symmetry of the potential is respected by the vacuum $(0, 0, v/\sqrt{2})$. In this paper we consider the scenario where the components of the inert doublets act as inflation candidates and reheat the universe at the end of inflation through their interactions with the SM-Higgs and gauge bosons. Note that at the scales relevant for inflation we can take the VEV of the active doublet to be zero, $\langle \phi_3 \rangle = 0$.

Furthermore, CP-violation is only introduced in the inert sector which is forbidden from mixing with the active sector by the conservation of the $Z_2$ symmetry. As a result, the amount of CP-violation is not limited by electric dipole moments [21]. The lightest particle amongst the CP-mixed neutral fields from the inert doublets is a viable DM candidate and stable due to the unbroken $Z_2$ symmetry. In this paper, we focus on the inflationary dynamics of the model and shall not discuss DM implications of the model any further.

### 2.2 Potential for the inflaton

We start by rewriting the doublets in the unitary gauge and ignore the charged scalars (since they do not affect the inflationary dynamics).

\[
\phi_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\
 h_1 + i\eta_1 \end{pmatrix}, \quad \phi_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\
 h_2 + i\eta_2 \end{pmatrix}, \quad \phi_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\
 h_3 \end{pmatrix}.
\]

The action of the model in the Jordan frame is

\[
S_J = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} M_{pl}^2 R - D_\mu \phi_1^\dagger D^\mu \phi_1 - D_\mu \phi_2^\dagger D^\mu \phi_2 - D_\mu \phi_3^\dagger D^\mu \phi_3 
\right.
- V(\phi_1, \phi_2, \phi_3) - \left( \xi_1 |\phi_1|^2 + \xi_2 |\phi_2|^2 + \xi_3 |\phi_3|^2 + \xi_4 (\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1) \right) R ,
\]

where $R$ is the Ricci scalar, $M_{pl}$ is the reduced Planck mass and the parameters $\xi_i$ are dimensionless couplings of the scalar doublets to gravity. Note that, in principle, $\xi_4$ could be a complex parameter for which we use the notation

\[
\xi_4 = |\xi_4| e^{i\theta_4}.
\]

In Eq. (6) the covariant derivative, $D_\mu$, contains couplings of the scalars with the gauge bosons. However, for the dynamics during the inflation, the covariant derivative is reduced to the normal derivative $D_\mu \rightarrow \partial_\mu$. The minus sign in the kinetic terms follows the metric convention of $(-, +, +, +)$.

Since we identify the two inert doublets with inflaton, we assume that the energy density of $\phi_3$ is sub-dominant during inflation. Therefore, the part of the potential relevant for inflation is

\[
V = -\mu_1^2 (\phi_1^\dagger \phi_1) - \mu_2^2 (\phi_2^\dagger \phi_2) + \lambda_{11} (\phi_1^\dagger \phi_1)^2 + \lambda_{22} (\phi_2^\dagger \phi_2)^2 + \lambda_{12} (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) + \lambda_4 (\phi_1^\dagger \phi_1)^2 + h.c.
\]
Due to local SU(2) invariance, we can rotate away one of the CP-odd fields, say \( \eta_2 \). Such a transformation is equivalent to taking the \( \eta_2 \rightarrow 0 \) limit, and we assume this limit to be taken when writing the fields in terms of components in Eq. (5).

To facilitate the analysis, we apply a conformal transformation from the Jordan frame, which contains terms with scalar-gravity quadratic couplings, to the Einstein frame with no explicit couplings to gravity [30]. Physical observables are invariant under this frame transformation. The two frames are equivalent after the end of inflation when the transformation parameter equals unity.

The action in the Einstein frame can be written as

\[
S_E = \int d^4x \sqrt{-\tilde{g}} \left[ -\frac{1}{2} M_{pl}^2 \tilde{R} - \frac{1}{2} \tilde{g}^\mu\nu G_{ij} \partial_\mu \varphi_i \partial_\nu \varphi_j - \tilde{V} \right],
\]

where \( \tilde{V} = V/\Omega^4 \) is the potential in the Einstein frame following the conformal transformation

\[
\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad G_{ij} = \frac{1}{\Omega^2} \delta_{ij} + \frac{3 M_{pl}^2}{2 \Omega^4} \partial_{\varphi_i} \partial_{\varphi_j},
\]

where \( \varphi_k = h_1, h_2, \eta_1 \), and the transformation parameter

\[
\Omega^2 = 1 + \frac{\xi_1}{M_{pl}^2} (h_1^2 + \eta_1^2) + \frac{\xi_2}{M_{pl}^2} h_2^2 + \frac{2 |\xi_4|}{M_{pl}^2} \left( h_1 h_2 c_{\theta_4} + \eta_1 h_2 s_{\theta_4} \right)
\]

(11)

using the shorthand notation \( c_{\theta_k} = \cos \theta_k \) and \( s_{\theta_k} = \sin \theta_k \) throughout the paper.

The prefactor \( G_{ij} \) in Eq. (10) leads to mixed kinetic terms. We introduce the reparametrisation

\[
A = \sqrt{\frac{3}{2} M_{pl} \log(\Omega^2)} \quad \text{with} \quad \frac{\partial \Omega^2}{\partial \varphi_k} = \sqrt{\frac{2}{3}} \frac{\Omega^2}{M_{pl}} \frac{dA}{d\varphi_k}
\]

(12)

which reduces the kinetic terms to the diagonal form

\[
\tilde{g}_{\mu\nu} G_{ij} \partial_\mu \varphi_i \partial_\nu \varphi_j = \Omega^2 g_{\mu\nu} \left( \frac{\delta_{ij}}{\Omega^2} + \frac{\partial A}{\partial \varphi_i} \frac{\partial A}{\partial \varphi_j} \right) \partial_\mu \varphi_i \partial_\nu \varphi_j = \partial_\mu \varphi_i \partial_\mu \varphi_i + \Omega^2 \partial_\mu A \partial_\mu A
\]

(13)

To write the potential in the Einstein frame, we keep only terms in the potential in Eq. (8) which are quartic in \( h_{1,2} \) and \( \eta_1 \). This reduces the potential to

\[
\tilde{V} \approx \frac{1}{4 \Omega^4} \left[ \lambda_{11} (h_1^2 + \eta_1^2)^2 + \lambda_{12} h_2^4 + (\lambda_{12} + \lambda_{12}') (h_1^2 + \eta_1^2) h_2^2 \\
+ 2 |\lambda_1| \left( c_{\theta_1} (h_2^2 (h_1^2 + \eta_1^2)) + 2 s_{\theta_1} h_2 h_1 \eta_1 \right) \right]
\]

(14)

where \( \theta_1 \) is the CP-violating phase of the \( \lambda_1 \) parameter.

Further, we introduce another reparametrisation

\[
\eta_1 = \beta_1 h_1, \quad h_2 = \beta_2 h_1,
\]

(15)
with $\beta_1, \beta_2$ as field dependent values, to rewrite the potential as

$$\tilde{V} \approx \frac{h_1^4}{4\Omega^4} \left[ \lambda_{11}(1 + \beta_1^2)^2 + \lambda_{22} \beta_2^4 + \left( (\lambda_{12} + \lambda_{12}')(1 + \beta_1^2) + 2|\lambda_1| (c_{\theta_1}(1 - \beta_1^2) + 2 s_{\theta_1} \beta_1) \right) \beta_2^2 \right] \tag{16}$$

Using this reparametrisation, one can also simplify the $\Omega^2$ parameter in Eq. (11) as

$$\Omega^2 = 1 + \left( \frac{\xi_1}{M_{pl}^2}(1 + \beta_1^2) + \frac{\xi_2}{M_{pl}^2} \beta_2^2 + 2|\xi_4| \beta_2(c_{\theta_4} + \beta_1 s_{\theta_4}) \right) h_1^2 \equiv 1 + \frac{B}{M_{pl}^2} h_1^2. \tag{17}$$

From Eq. (12), recall that $\Omega_2 = \exp(\tilde{A})$ using the shorthand notation $\tilde{A} = \sqrt{\frac{2}{3} A_{M_{pl}}}$. One can then write the field $h_1$ in terms of the reparametrised field $\tilde{A}$

$$h_1^2 = \frac{M_{pl}^2}{B} \left( e^{\tilde{A}} - 1 \right). \tag{18}$$

Therefore, expressing $h_1^2$ and $\Omega^2$ in terms of $\tilde{A}$ allows us to write the potential in Eq. (16) in the form

$$\tilde{V} \sim (1 - e^{-\tilde{A}})^2 X(\beta_1, \beta_2). \tag{19}$$

We will be interested in the effect of the non-minimal coupling $\xi_4$ and the associated phase $\theta_4$. Therefore, we will set $\xi_1 = \xi_2 = 0$ and assume that the initial field values are such that $\Omega^2 > 0$ is guaranteed. Therefore, with these assumptions, the potential in Eq. (16) can be written as

$$\tilde{V} = \left( \frac{M_{pl}^2}{2|\xi_4|} \right)^2 \left( 1 - e^{-\tilde{A}} \right)^2 X(\beta_1, \beta_2) \tag{20}$$

where

$$X(\beta_1, \beta_2) = \frac{\lambda_{11}(1 + \beta_1^2)^2 + \lambda_{22} \beta_2^4 + ( (\lambda_{12} + \lambda_{12}')(1 + \beta_1^2) + 2|\lambda_1| (c_{\theta_1}(1 - \beta_1^2) + 2 s_{\theta_1} \beta_1) ) \beta_2^2}{4\beta_2^2 (c_{\theta_4} + \beta_1 s_{\theta_4})^2}. \tag{21}$$

Following the procedure in [16], to find the direction of inflation, we first minimise the $X(\beta_1, \beta_2)$ function with respect to $\beta_2$ which occurs at

$$\frac{\partial X(\beta_1, \beta_2)}{\partial \beta_2} = 0 \quad \Rightarrow \quad \beta_2^2 = \sqrt{\frac{\lambda_{11}}{\lambda_{22}}} (1 + \beta_1^2) \tag{22}$$

The second order derivative at this point is

$$\frac{\partial^2 X(\beta_1, \beta_2)}{\partial \beta_2^2} = \frac{2 \lambda_{22}}{(c_{\theta_4} + \beta_1 s_{\theta_4})^2} \tag{23}$$

which is always positive provided $\lambda_{22} > 0$, as shown in the left panel in Figure [1].

Using the $\beta_2$ value in Eq. (22), we can write the $X(\beta_1, \beta_2)$ function solely in terms of $\beta_1$,

$$X(\beta_1) = \frac{(1 + \beta_1^2) \Lambda + 2 \left( (1 - \beta_1^2)c_{\theta_1} + 2 \beta_1 s_{\theta_1} \right) |\lambda_1|}{4 (c_{\theta_1} + \beta_1 s_{\theta_1})^2}. \tag{24}$$
with $\Lambda = \lambda_{12} + \lambda'_{12} + 2\sqrt{\lambda_{11}\lambda_{22}}$.

We repeat the same treatment and minimise the $X(\beta_1)$ function with respect to $\beta_1$.

$$\frac{\partial X(\beta_1)}{\partial \beta_1} = 0 \Rightarrow \beta_1 = \frac{(\Lambda + 2|\lambda_1|c_{\theta_1})s_{\theta_4} - 2|\lambda_1|c_{\theta_4}s_{\theta_1}}{(\Lambda - 2|\lambda_1|c_{\theta_1})c_{\theta_4} - 2|\lambda_1|s_{\theta_4}s_{\theta_1}} \tag{25}$$

We check the positivity of the second order derivative at the minimum point which is satisfied for all $\theta_1, \theta_4$ values as shown in the right panel of Figure 1.

Replacing the $\beta_1$ value which minimises the $X(\beta_1)$ function back into the $X(\beta_1)$ function itself, yields the form of $X$ independent of $\beta_1$ and $\beta_2$ with only $\theta_1$ and $\theta_4$ as variables:

$$X(\theta_1, \theta_4) = \frac{\frac{1}{4}\Lambda^2 - \lambda_1^2}{\Lambda - 2\lambda_1 \cos(\theta_1 - 2\theta_4)} \tag{26}$$

The left panel in Figure 2 shows the $X(\theta_1, \theta_4)$ function for allowed values of $\theta_1$ and $\theta_4$. At each point in the plots, one can derive the values of $\beta_1$ and consequently $\beta_2$ using Eq. (22) for given values of $\theta_1$ and $\theta_4$. The right panel in Figure 2 shows the values of $\beta_1$ for varying values of $\theta_1$ and $\theta_4$. 

Figure 1: The second order derivative of the function $X(\beta_1, \beta_2)$ with respect to $\beta_2$ at the minimum ($\partial X/\partial \beta_2 = 0$) on the left and the second order derivative of the function $X(\beta_1)$ with respect to $\beta_1$ at the minimum ($\partial X/\partial \beta_1 = 0$) on the right (all $\lambda_i \sim 0.001$). The white area on the left panel corresponds to where the denominator in Eq. (23) becomes zero.
Figure 2: The $X(\theta_1, \theta_4)$ function on the left and the values of $\beta_1$ on the right for varying values of $\theta_1$ and $\theta_4$ (all $\lambda_i \sim 0.001$). The white region in the right panel shows a discontinuity where $\beta_1$ values tend to plus infinity approaching from the bottom and to minus infinity approaching from the top of the plot.

Figure 3: The inflationary potential for different values of $\theta_1$ and $\theta_4$ (all $\lambda_i \sim 0.001$).

3 Inflationary dynamics

With the procedure used in the previous section, the dynamics is essentially that of a single field inflation. The full inflationary potential in Eq. (20) can be written as

$$\tilde{V} = \left( \frac{M_{pl}^2}{2|\xi_4|} \right)^2 \left( 1 - e^{-\tilde{A}} \right)^2 X(\theta_1, \theta_4)$$

Figure 3 shows the inflationary potential for different values of $\theta_1$ and $\theta_4$. Note that the potential is almost flat at high field values which ensures a slow roll inflation.

For the usual slow roll parameters in this case the function $X$ is irrelevant, since it cancels
in the expressions for $\epsilon$ and $\eta$, which are

$$\epsilon = \frac{1}{2} M_{\text{pl}}^2 \left( \frac{1}{V} \frac{dV}{dA} \right)^2 = \frac{4}{3 (1 - e^{\tilde{A}})^2}, \quad (28)$$

$$\eta = M_{\text{pl}}^2 \frac{1}{V} \frac{d^2 V}{dA^2} = \frac{4(2 - e^{\tilde{A}})}{3 (1 - e^{\tilde{A}})^2}. \quad (29)$$

For field values $A \gg M_{\text{pl}}$ (or equivalently $\tilde{A} \gg 1$), both parameters $\epsilon, \eta \ll 1$ which satisfies the slow roll condition. Inflation ends when $\epsilon \simeq 1$. To calculate the values of $A$ at the beginning and end of inflation, $A_i$ and $A_f$ respectively, one needs to calculate the number of e-folds $N_e$, i.e. the number of times the universe expanded by $e$ times its own size. $N_e$ is calculated to be

$$N_e = \frac{1}{M_{\text{pl}}^2} \int_{A_i}^{A_f} \frac{\dot{V}}{V} dA = \frac{3}{4} \left[ \tilde{A}_f - \tilde{A}_i - e^{\tilde{A}_f} + e^{\tilde{A}_i} \right], \quad (30)$$

where $\dot{V} = \frac{dV}{dA}$ and $A_i$ ($\tilde{A}_i$) is the value of $A$ ($\tilde{A}$) at the beginning of inflation and $A_f$ ($\tilde{A}_f$) is the value of $A$ ($\tilde{A}$) at the end of the inflation. Since inflation ends when $\epsilon \simeq 1$, one can calculate $A_f$, which yields:

$$e^{\tilde{A}_f} = \exp \left( \sqrt{\frac{2}{3 M_{\text{pl}}^2}} \right) \simeq 2.1547 \quad \Rightarrow \quad \tilde{A}_f = \sqrt{\frac{2}{3 M_{\text{pl}}^2}} \approx 0.7676. \quad (31)$$

To calculate $A_i$, one could plug in the $A_f$ value into Eq. (30) assuming $N_e = 60$, which results in

$$\frac{3}{4} \left[ -\tilde{A}_i + e^{\tilde{A}_i} \right] - 1.0403 = 60, \quad \Rightarrow \quad \tilde{A}_i = \sqrt{\frac{2}{3 M_{\text{pl}}^2}} \approx 4.4524 \quad (32)$$

At this point we can also check the field values in terms of the original field $h_1$ using Eq. (18). This gives

$$h_{1f} = \frac{1.85 \times 10^{18}}{\sqrt{\xi_4 | \beta_2 (c_\theta + \beta_1 s_\theta_4) |}}, \quad h_{1i} = \frac{1.59 \times 10^{19}}{\sqrt{\xi_4 | \beta_2 (c_\theta + \beta_1 s_\theta_4) |}}. \quad (33)$$

In the case of Higgs-inflation where the non-minimal coupling to gravity, $\xi$, is forced to be of the order $\sim 10^4$ GeV, the $h$ field values during inflation are as large as $10^{16}$ GeV or so. In our case the situation is similar.

Having fixed $N_e$ to 60, and calculated the $A$ field value at the start of inflation, we can derive the scalar power spectrum, $P_s$, the tensor to scalar ratio $r$ and the spectral index $n_s$ as follows:

$$P_s = \frac{1}{12 \pi^2 M_{\text{pl}}^6} \left( \frac{\dot{V}}{V} \right)^3 = \frac{(1 - e^{\tilde{A}})^4}{128 \pi^2 e^{2\tilde{A}}} \frac{X(\theta_1, \theta_4)}{|\xi_4|^2} = 5.565 \times \frac{X(\theta_1, \theta_4)}{|\xi_4|^2}, \quad (34)$$

$$r = 16 \epsilon = 0.00296, \quad (35)$$

$$n_s = 1 - 6 \epsilon + 2 \eta = 0.9678, \quad (36)$$

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where $\tilde{V}'$ is the derivative of $\tilde{V}$ with respect to $\tilde{A}$ and both $\tilde{V}$ and $\tilde{V}'$ are calculated at the $A_i$. Figure 4 shows the slow roll parameters $N_e$, $n_s$ and $r$ with respect to $\tilde{A}$ with the grid-lines highlighting the $55 < N_e < 65$ values. We show the inflationary parameters over a range of $N_e$, since there is no reason for $N_e$ to be precisely 60. The values of $r$ and $n_s$ are well within the Plank bounds of $n_s = 0.9677 \pm 0.0060$ at 1σ level and $r < 0.11$ at 95% confidence level \cite{Planck1}. Note that the spectral index and the tensor to scalar ratio are in agreement with the Planck bounds over the full range of $N_e$. Figure 5 shows the 1σ and 2σ regions allowed by Planck observations in the $r$-$n_s$ plane and the theoretical predictions of our framework for $N_e$ values of 55 and 65.

Observations from WMAP7 \cite{WMAP7} constrain the scalar power spectrum which put a bound on the $\xi_4$ coupling and angles $\theta_1, \theta_4$,

$$P_s = (2.430 \pm 0.091) \times 10^{-9} = 5.565 \times \frac{X(\theta_1, \theta_4)}{|\xi_4|^2}.$$  \hspace{1cm} (37)
In the left panel of Figure 6, we show $P_s$ values for the fixed $\theta_1 = \pi/3$ angle and varying values of $\xi_4$ and $\theta_4$ up to $3\sigma$ standard deviation from the observed central value in Eq. (37). In the right panel, we fix $P_s$ to the WMAP central value for fixed values of $\lambda_i \sim 0.001$ to get

$$|\xi_4| = 4.785 \times 10^4 \sqrt{X(\theta_1, \theta_4)}$$

and show contours of $\xi_4$ for varying values of $\theta_1$ and $\theta_4$. Note that every point in the plot yields the exact $P_s$ central value.

This is a very important feature of our framework. To satisfy the bounds on the scalar power spectrum, the function $X(\theta_1, \theta_4)$ allows for a wide range of $\xi_4$ values as shown in Figure 6. This is in contrast to the Higgs-inflation models where $P_s \propto \lambda/\xi^2$ with $\lambda$ the Higgs self-coupling which is fixed to be $\sim 0.12$ at the electroweak scale. Thus, for $P_s$ to agree with observations at the inflationary scale, $\xi$ will have to be very large $\mathcal{O}(10^4)$. In our set-up, a combination of parameters $\lambda_1, \lambda_{11}, \lambda_{22}, \lambda_{12}, \lambda'_{12}$ appears in the $X(\theta_1, \theta_4)$ function. The only constraint limiting these parameters is the stability of the potential requiring

$$\lambda_{ii} > 0, \quad \lambda_{ij} + \lambda'_{ij} > -2\sqrt{\lambda_{ii}\lambda_{jj}}, \quad |\lambda_i| \leq |\lambda_{ii}|, |\lambda_{ij}|, |\lambda'_{ij}|, \quad i \neq j = 1, 2, 3,$$

which allows for very small values of $\lambda_i \sim 0.001$ which, in turn, allows for much smaller values of $\xi_4$, at least one order of magnitude than the $\xi$ value in Higgs-inflation models.

4  Reheating and scalar asymmetries

At the end of inflation, the energy stored in the inflaton disperses as the inflaton decays/annihilates into the SM particles through processes mediated by the SM-Higgs and gauge bosons in our
case, during the so-called reheating phase $[33]$. There are numerous details on how the inflaton decays and creates the initial condition for the conventional hot early universe. Here our main interest is to discuss how the CP asymmetry originating from the non-minimal coupling, is transferred to the SM degrees of freedom.

For the discussion of the scalar asymmetries, let’s focus on the neutral components of the $\phi_1$ doublets acquiring an initial non-vanishing expectation value at the exit from inflaton. We write the field fluctuations around the initial conditions as

$$
\begin{aligned}
\phi_1 &\rightarrow \phi_1 - a_1 e^{i\alpha}, & \phi_1^\dagger &\rightarrow \phi_1^\dagger - a_1 e^{-i\alpha} \\
\phi_2 &\rightarrow \phi_2 - a_2, & \phi_2^\dagger &\rightarrow \phi_2^\dagger - a_2 \\
\phi_3 &\rightarrow \phi_3 - a_3. & \phi_3^\dagger &\rightarrow \phi_3^\dagger - a_3
\end{aligned}
$$

Eq. (40)

The phase $\alpha$ here is related to the CP-violating phases of inflation. Note that at the end of inflation the $h_1$ field has taken the value $h_{1f}$ according to Eq. (33) which is dependant on the inflationary dynamics, namely $\theta_1$, $\beta_1$ and $\beta_2$ which are dependant on $\theta_1$. Since $h_1$ is the real part of the complex field $\phi_1$, its value is what feeds the $a_1 \cos \alpha$ component of fluctuations in Eq. (40). The imaginary part of $\phi_1$, represented by $\eta_1$, takes a value proportional to $\eta_{1f} = \beta_1 h_{1f}$ as shown in Eq. (15), and feeds the $a_1 \sin \alpha$ component of the field fluctuations. Recall that one can obtain the values of $\beta_1$ and $\beta_2$ for any given value of $\theta_1$ and $\theta_4$ from Eq. (22) and Eq. (25).

Explicitly, one can write

$$
\tan \alpha = \frac{a_1 \sin \alpha}{a_1 \cos \alpha} = \frac{\eta_{1f}}{h_{1f}} = \beta_1 = \frac{(\Lambda + 2|\lambda_1|c_{\theta_1})s_{\theta_4} - 2|\lambda_1|c_{\theta_1}s_{\theta_4}}{(\Lambda - 2|\lambda_1|s_{\theta_1})c_{\theta_4} - 2|\lambda_1|s_{\theta_1}s_{\theta_4}},
$$

with $\Lambda = \lambda_{12} + \lambda_{13} + 2\sqrt{\lambda_{11}\lambda_{22}}$ as mentioned before. However, to keep the present discussion more transparent, we retain a generic phase $\alpha$ here.

To discuss the consequences of this complex phase, we now assume instant reheating. Since the field $\phi_3$ is light with respect to the inflaton degrees of freedom, we expect the latter to quickly decay to $\phi_3$. The asymmetry arising from the values of the fields in Eq. (40) will manifest in creation of unequal number of $\phi_3$ and $\phi_3^*$ quanta as follows.

Let us study the decay process $\phi_1 \rightarrow \phi_3^* \phi_3^*$ in detail. From the potential in Eq. (1), the amplitude of the tree-level process is proportional to

$$
|\mathcal{M}_{(\phi_1 \rightarrow \phi_3^* \phi_3^*)}| \propto -2a_1 \lambda_3 e^{i(\alpha + \theta_3)} \quad \text{and} \quad |\mathcal{M}_{(\phi_1 \rightarrow \phi_3 \phi_3)}| \propto -2a_1 \lambda_3 e^{-i(\alpha + \theta_3)}.
$$

The generation of the asymmetry is sensitive to the interference between the tree and loop diagrams $[34,35]$. Hence, we need to sketch what happens at loop level. At one loop level, there are many diagrams that contribute to this decay process. For the purpose of demonstration, we consider the bubble diagrams which convert $\phi_1$ to $\phi_3$ with only $\phi_1$ and $\phi_1^*$ in the loop, as shown in Figure 7. Clearly one needs to take into account all diagrams contributing to this decay process, specially since there may be interferences cancelling the CP asymmetry. However, since all triple scalar couplings in the potential can be different, one can ensure that such cancellation does not occur. More careful analysis of these effects is deferred to a future work.
Figure 7: The tree level decay process $\phi_1 \rightarrow \phi_3^* \phi_3^*$ and the one-loop bubble diagram with $\phi_1$ and $\phi_1^*$ in the loop.

The amplitude of the loop process with $\phi_1$ and $\phi_1^*$ running in the loop is proportional to

$$M(\phi_1 \rightarrow \phi_3^* \phi_3^*) \propto -4a_1 a_2^2 \lambda_{11} \lambda_{33} (\lambda_{31} + \lambda'_{31}) e^{-i\alpha},$$

(43)

$$M(\phi_1^* \rightarrow \phi_3^* \phi_3^*) \propto -4a_1 a_2^2 \lambda_{11} \lambda_{33} (\lambda_{31} + \lambda'_{31}) e^{i\alpha}.$$  

(44)

Due to the interference of the tree and loop diagrams, the decay processes are CP-violating and result in unequal number of $\phi_3$ and $\phi_3^*$ states. Consequently, we define the asymmetry $A_{\text{CP}}^{\phi_1}$ as the difference between the $\phi_1$ decay rate and its conjugate, and we find

$$A_{\text{CP}}^{\phi_1} = \Gamma_{\text{tree+loop}}^{\phi_1 \rightarrow \phi_3^* \phi_3^*} - \Gamma_{\text{tree+loop}}^{\phi_3^* \rightarrow \phi_3^* \phi_3^*} = -\frac{1}{16\sqrt{3}\pi^2} a_1^2 a_2^2 \lambda_{11} \lambda_{33} (\lambda_{31} + \lambda'_{31}) \sin(2\alpha + \theta_3).$$

(45)

This asymmetry in the scalar sector is then transferred to the fermion sector through the couplings of the Higgs field (the $\phi_3$ doublet) with the fermions. For example, assuming the existence of right-handed neutrinos, the Yukawa interactions between neutrinos and $\phi_3$ will generate an asymmetry between $\nu_L$ and $\bar{\nu}_R$, which would be further translated into baryon asymmetry by the electroweak sphalerons.

5 Conclusion and outlook

Scalar fields which have non-minimal couplings to gravity are well-motivated inflaton candidates. Paradigmatic examples are the Higgs-inflation \cite{13} and s-inflation models \cite{15}. In this paper we have considered a scenario where several non-minimally coupled scalars contribute to the inflationary dynamics. In particular we investigated a model where these scalars are electroweak doublets and therefore generalize the Higgs inflation. We focused on a setting where the dominant non-minimal coupling is allowed to be complex and investigated the effect that this would have on CP-violation in our universe. We determined the inflationary dynamics in the regime where the model essentially conforms to the predictions of single field inflation. The essential difference is that the inflaton obtains a non-zero phase representing possible source of CP-violation for subsequent post-inflationary evolution. At the end of inflation, the inflaton particle which is naturally assumed to have couplings with the SM Higgs, dumps its energy into the SM particle bath through the process of reheating, which populates the universe with the SM particles. We sketched how the complex value of the inflaton field leads to an asymmetry in the scalar sector decays, and how this asymmetry will further be transmitted to the fermion sector through the Yukawa couplings.
sector. There are numerous details in our scenario which can be investigated in more detail. These include the multi-field dynamics during the inflation as well as the details of reheating and subsequent particle decays. Also the detailed analysis of the effects on the generation of baryon asymmetry need to be addressed in more detail. We will consider these in future work on the model introduced in this paper.

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