Name Independent Fault Tolerant Routing Scheme*

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Abstract

We consider the problem of routing in presence of faults in undirected weighted graphs. More specifically, we focus on the design of compact name-independent fault-tolerant routing schemes, where the designer of the scheme is not allowed to assign names to nodes, i.e., the name of a node is just its identifier. Given a set $F$ of faulty (or forbidden) edges, the goal is to route from a source node $s$ to a target $t$ avoiding the forbidden edges in $F$.

Given any name-dependent fault-tolerant routing scheme and any name-independent routing scheme, we show how to use them as a black box to construct a name-independent fault-tolerant routing scheme. In particular, we present a name-independent routing scheme able to handle any set $F$ of forbidden edges. This has stretch $O(k^2 |F|^{3/4} + \log^2 n \log D)$, where $D$ is the diameter of the graph. It uses tables of size $\tilde{O}(kn^{1/k} + k \deg(v))$ bits at every node $v$, where $\deg(v)$ is the degree of node $v$. In the context of networks that suffer only from occasional failures, we present a name-independent routing scheme that handles only 1 fault at a time, and another routing scheme that handles at most 2 faults at a time. The former uses $\tilde{O}(k^2 n^{1/k} + k \deg(v))$ bits of memory per node, with stretch $O(k^3 \log D)$. The latter consumes in average $\tilde{O}(k^2 n^{1/k} + \deg(v))$ bits of memory per node, with stretch $O(k^2 \log D)$.

1 Introduction

A routing scheme is a decentralized mechanism that allows to route messages between any pair of nodes in a network. Typically, messages are provided with headers that contain some information for routing. When an intermediate node receives a message, the latter is delivered to the appropriate neighbor by processing both the header and the routing table. The quality of a routing scheme is essentially measured by the stretch, and the size of the routing table stored at each node.

The stretch of a routing scheme is defined as the maximum, taken over all graphs and all pair of nodes $(s, t)$ in these graph, of the ratio between the length of the path from $s$ to $t$ designed by the scheme, and the length of a shortest path between $s$ and $t$. The size of a routing scheme is defined as the maximum, taken over all nodes of all graphs, of the memory-space required to store the routing table at those nodes.

The problem of designing compact routing schemes with low stretch has been addressed in many works $[2, 4, 6, 7, 9, 11, 12, 13, 15]$. Two main models were considered in the literature,

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namely, name-dependent and name-independent routing schemes, as first introduced in [4]. In the former, the designer is allowed to give specific names to nodes, while, in the latter, the name of a node is its identifier. Essentially, in a name-dependent routing scheme, a source node is given the name of the target node \( t \), \( \text{name}(t) \), that holds some information about the routing path. For example, the name of a node may contain the identifiers of few special nodes through which to pass for delivering the message. Instead, in a name-independent routing scheme, \( \text{name}(t) = t \), that is the source node knows just the identifier of the target node.

The design of compact low stretch fault-tolerant routing schemes recently attracted interest (e.g. [17, 18, 19]). In this context, the scheme must be able to handle edge failures, providing alternative paths to reach the destination node. Two models were considered in the literature, depending on whether or not the source node knows the names of the endpoint vertexes of the forbidden edges. Our focus is in models where the source node is not aware of the faulty edges. The stretch of a fault-tolerant routing scheme is defined in a slightly different manner. Given any undirected weighted graph \( G = (V, E, w) \) and a set \( F \subseteq E \) of forbidden edges, the stretch of a fault-tolerant routing scheme is given by the maximum, taken over all pair of nodes \((s, t)\), of the ratio between the length of the path from \( s \) to \( t \) designed by the scheme, and the length of a shortest path between \( s \) and \( t \) in \( G' = (V, E \setminus F, w) \). Hence, the stretch is calculated with respect to the length of shortest paths in \( G' \), that may be longer than the shortest paths in \( G \). In the context of name-dependent fault-tolerant routing schemes where the source node does not know the set of forbidden edges, different mechanisms were shown in the literature, depending on the number of failures tolerated. Specifically, [20] showed a name-dependent routing scheme that handles up to 2 faults at a time. They were able to bound the overall memory used by all nodes in the graph, but not the size of the memory per node. Later, [19] presented a name-dependent routing scheme tolerating an arbitrary number of failures, while bounding the size of the memory required at any vertex. Then, [21] improved the size of memory per node at the cost of tolerating just one fault at a time, providing a name-dependent scheme that is suitable for networks with occasional failures.

Unless explicitly specified, when we talk about routing schemes we refer to those that do not tolerate faults.

To the best of our knowledge, fault-tolerant routing schemes have been studied only in the context of the name-dependent model. Hence, a natural extension of the problem of routing in presence of faults is the design of name-independent fault-tolerant routing schemes. Name-independent mechanisms are of extremely interest, since they model the routing in the real world. In fact, everyday users of Internet route messages in the network knowing just the host-names (identifiers) and not the IP addresses (names) of websites.

### 1.1 Our contribution

Our main contribution is stated in the following theorem.

**Theorem 1** Let \( NI \) be a name-independent routing scheme, and let \( NDFT \) be a name-dependent routing scheme tolerating up to \( |F| \) faults, assigning names of size \( |\text{name}(NI)| \) to nodes. Then there exists a name-independent routing scheme \( NIFT \) tolerating up to \( |F| \) faults, with stretch

\[
\text{stretch}(NIFT) \leq \text{stretch}(NI) + 2\eta |F| \text{stretch}(NDFT)
\]

and space-complexity

\[
\text{size}(NIFT) \leq \text{size}(NI) + \text{size}(NDFT) + \Delta |\text{name}(NI)|
\]

in graphs with maximum degree \( \Delta \), where \( \eta \) is the maximum number of times a same edge can be traversed by a same message following \( NI \).
It follows from the theorem that, for example, we can achieve a name-independent routing scheme that can handle an arbitrary number of faults with stretch $\tilde{O}(\eta k |F|^4)$, using $\tilde{O}(kn^{1/k}(k + \deg(v)))$ bits of memory per node\(^1\). We also show that $\eta \leq 2k \log D$ for a name-independent routing scheme presented in [3], where $D$ is the weighted diameter of the graph, i.e., the maximum distance between any pair of nodes in the graph. This allows, for example, to construct a name-independent routing scheme, tolerating an arbitrary number of failures, where each node stores $\tilde{O}(kn^{1/k}(k + \deg(v)))$ bits, achieving a stretch of $\tilde{O}(k^2 |F|^{4})$. An overview of our results is presented in Table 1.

| stretch                  | size                                      | faults handled |
|--------------------------|-------------------------------------------|----------------|
| $O(k^3 \log D)$          | $\tilde{O}(k^2 n^{1/k} + k \deg(v))$ per node | 1              |
| $O(k^2 \log D)$          | $\tilde{O}(k^2 n^{1/k} + \deg(v))$ per node, in average | 2              |
| $O(k^2 |F|^3(|F| + \log^2 n) \log D)$ | $\tilde{O}(k^{1/k}(k + \deg(v)))$ per node | arbitrary      |

Table 1: A summary of fault-tolerant name-independent routing schemes presented in this paper.

Although our result is very general and it works given any name-dependent fault-tolerant routing scheme, we will mainly consider the ones where the source node does not know the set $F$ of failures. It is worth mentioning that, a positive aspect of our general result is that any improvement on name-independent routing schemes or any progress on fault-tolerant name-dependent routing mechanisms, directly leads in an improvement of our name-independent fault-tolerant routing scheme.

1.2 Related work

Routing in the distributed model of computation has been widely studied. In particular, Thorup and Zwick [14] showed a name-dependent routing scheme with stretch $4k - 5$ using $\tilde{O}(n^{1/k})$ bits per vertex, for any $k \geq 2$. Chechik improved later this result in [5], showing a name-dependent routing scheme that uses tables of size $\tilde{O}(n^{1/k})$ and stretch $ck$ for some absolute constant $c < 4$, obtaining improved results for every $k \geq 4$.

A lower bound on the size of the routing schemes in general graphs was proved by Peleg and Upfal in [12]. They showed that a routing scheme with stretch $k \geq 1$ requires $\Omega(n^{1+\frac{1}{n+1}})$ bits of overall memory. On the other hand, a conjecture by Erdős of 1963, shown to be true for $k = 1, 2, 3, 5$, says that any routing scheme with stretch less then $2k + 1$ needs $\Omega(n^{1+1/k})$ bits of memory.

In [16], Awerbuch and Peleg presented a name-independent routing scheme with stretch $O(k^2)$ using $\tilde{O}(n^{1/k})$ memory per vertex. Arias et al. [3] improved this result showing a name-independent routing scheme with stretch of $\min\{1+(2k-1)(2k-2), 64k^2+8k\}$ using $\tilde{O}(k^2 n^{1/k})$ bit memory per vertex. Later, [1] presented a name-independent routing scheme with linear stretch $O(k)$ using $\tilde{O}(k^2 n^{1/k})$ bit memory per node. For an overview on compact routing and compact network data structures, see [8, 10].

Fault-tolerant routing schemes have been studied in [18], where Courcelle and Twigg described a name-dependent mechanism that assumes that the source knows the set $F$ of forbidden

\(^1\)The notation $\tilde{O}$ ignores polylogarithmic factors.
edges. More specifically, they show a routing scheme that uses tables of size \(O(k^2 \log^2 n)\) bits per node for graphs with treewidth \(k\). They consider this problem also for bounded cliquewidth graphs, achieving the same results in terms of size and stretch of the routing scheme. In the context where the source node does not know the forbidden edges, [20] presented a \(O(k)\) name-dependent routing scheme for general graphs, tolerating of at most 2 faults (i.e., \(|F| \leq 2\)). Their scheme uses an overall memory of \(\tilde{O}(k n^{1+1/k} \log W)\) bits, where \(W\) is the maximum weight of the edges. Notice that [20] bounds the overall memory used by all the ensemble of nodes in the graph, and not the size of the routing tables per vertex. Later, Chechik presented a name-dependent routing scheme in [19] that tolerates an arbitrary-size set of forbidden edges \(F\). Its stretch is \(O(|F|^2 + \log^2 n)\) and the name given to the nodes are of size \(O(\log(n W))\) bits. The size of the scheme at each node \(v\) is of \(\tilde{O}(k \deg(v) + n^{1/k})\) bits of memory per vertex.

2 Model and Definitions

The distributed network is modeled by an undirected weighted connected graph \(G = (V, E, w)\), where \(V\) is the set of nodes, \(E\) is the set of communication links and \(w\) is a function that assigns weights to edges. Let \(\mathcal{G}\) be a family of such graphs.

2.1 Size

Given any distributed routing scheme, its size depends on the size of the memory that each node uses. In fact, each node \(v \in V\) stores a routing table, \(\text{table}(v)\) that it processes while delivering a message. The size of a routing scheme is given either by bounding the total amount of memory of all nodes in the graph, or by bounding the memory per node, taking the worst case, i.e., \(\max_{v \in G} |\text{table}(v)|\), for all \(G \in \mathcal{G}\).

2.2 Stretch

Consider a routing scheme \(R\) (name-dependent or name-independent) for a graph \(G = (V, E, w) \in \mathcal{G}\), and consider a source and a target \(s, t \in G\). We denote by

\[
d(s, t) = \text{the weighted length of a shortest path between } s \text{ and } t \text{ in } G.
\]

We also define as \(\text{length}(R, s, t)\) the weighted length of the path between \(s\) and \(t\) designed by the routing scheme \(R\). The stretch of \(R\) is defined as the worst case, taken over all graphs \(G \in \mathcal{G}\) and all pairs of nodes \(s, t \in G\), of the ratio between the length of the path of \(s\) and \(t\) designed by the routing scheme and the length of a shortest path of \(s\) and \(t\), that is

\[
\text{stretch}_R = \max_{s, t \in V} \frac{\text{length}(R, s, t)}{d(s, t)}.
\]

In the context of routing in presence of faults, the stretch is defined in a slightly different manner. Consider a fault-tolerant routing scheme \(\hat{R}\) (name-dependent or name-independent) for a graph \(G = (V, E, w) \in \mathcal{G}\), capable to handle a set of faults \(F \subseteq E\) of arbitrary size. Consider also the graph \(G'\) that has the same set of nodes \(V\) and the same weights as \(G\), but the set of edges includes only the ”good” ones, i.e., the set of edges is \(E \setminus F\). We define as

\[
\delta(s, t) = \text{a shortest path between } s \text{ and } t \text{ in } G' = (V, E \setminus F, w).
\]
In this context, differently from before, the stretch of \( \hat{R} \) does not depend on \( d(s, t) \), but is defined with respect to \( \delta(s, t) \). More precisely,

\[
\text{stretch}_{\hat{R}} = \max_{s, t \in V} \frac{\text{length}(\hat{R}, s, t)}{\delta(s, t)}.
\]

2.3 Message header

The goal of the routing mechanisms is to deliver messages among nodes in a network. Typically the message has two parts: the header and the body. When receiving a message, the node processes the information contained in the header along with the ones present in the routing table of that node, and chooses a neighbor, among all its neighbors, to which deliver the message. An intermediate node may also enrich the header by adding further information for the routing purpose. In general, the size of the header is reasonably small, typically \( O(\log n) \) for schemes that do not tolerate failures.

2.4 Name size

Another important parameter is the size of the names assigned by the name-dependent (fault-tolerant or not) routing schemes. Typically, this is \( O(\log n) \) in mechanisms that do not handle any faults. On the other hand, the size of the given names in name-dependent fault-tolerant schemes varies. For example, we have names of size \( O(\lceil \log(nW) \rceil \log n) \) bits for the name-dependent scheme in [19] that handles multiple faults, and names of \( O(\log n) \) bits for the scheme in [21] that handles 1 fault at a time.

2.5 Scheme aspects

At last, we define some aspects that characterize name-independent and fault-tolerant name-dependent routing schemes, that we will later use.

Given a name-independent routing scheme \( \mathbf{NI} \)

- \( \text{table}_{\mathbf{NI}}(v) \) is the routing table, stored at a node \( v \), while using \( \mathbf{NI} \);
- \( \eta \) denotes the maximum, taken over all edges of all graphs, of the number of times that \( \mathbf{NI} \) traverses the same edge.

Given a name-dependent fault-tolerant routing scheme \( \mathbf{NDFT} \)

- \( \text{table}_{\mathbf{NDFT}}(v) \) is the routing table, stored at a node \( v \), while using \( \mathbf{NDFT} \);
- \( F \subseteq E \) denotes the set of edge-failures handled by \( \mathbf{NDFT} \);
- \( \alpha \) represents the maximum, taken over all nodes \( v \), of \( |\text{name}(v)|\).
3 Name-dependent fault-tolerant routing scheme

Consider any name-independent routing scheme NI and any name-dependent fault-tolerant routing scheme NDFT that handles an arbitrary-size edge-failure set $F$. In this section we show how to use them as a black box to construct a name-independent fault-tolerant routing scheme NIFT, tolerating up to $|F|$ faults.

**Table Size.** In the name-independent fault-tolerant routing scheme, each node $v$ stores

1. the table $\text{table}_{NI}(v)$;
2. the table $\text{table}_{NDFT}(v)$;
3. $\text{name}(u)$, for each neighbor $u$ of $v$.

The amount of memory that node $v$ needs in order to store the routing tables of both schemes is $|\text{table}_{NI}(v)| + |\text{table}_{NDFT}(v)|$. Then, the names of each neighbor of node $v$ can be stored in $\deg(v)\alpha$ bits (recall that $\alpha$ is the size of the names assigned by the name-dependent fault-tolerant scheme). Hence, the total memory per node used by the name-independent fault-tolerant routing scheme NIFT is

$$\text{size}(\text{NIFT}) \leq |\text{table}_{NI}(v)| + |\text{table}_{NDFT}(v)| + \deg(v)\alpha.$$  

**Routing.** The idea is that we use the name-independent scheme to route the message until we find a broken link, and then use the fault-tolerant name-dependent scheme to avoid the faulty edge. More specifically, let $F$ be the set of forbidden edges and suppose we want to route a message from $s$ to $t$. The routing then proceeds as follows.

- If there are no faulty edges along the path between $s$ and $t$ designed by the routing scheme, then route using the name-independent mechanism NI by processing the information in $\text{table}_{NI}(v)$, already stored in $v$’s routing table.

- Otherwise, suppose that the message has arrived at a node $v$ and has to go through node $u$ that is a neighbor of $v$, but $e(v, u)$ is a broken link. In this case, use the name-dependent fault-tolerant scheme NDFT to route the message from $v$ to $u$. This can be done since $v$ has in its routing table the name of $u$, $\text{name}(u)$, and $\text{table}_{NDFT}(v)$.

**Stretch analysis.** Let $d(s, t)$ and $\delta(s, t)$ be respectively a shortest path between $s$ and $t$ in $G = (V, E, w)$ and in $G' = (V, E \setminus F, w)$. Notice that $d(s, t) \leq \delta(s, t)$. We first show the stretch analysis for $|F| = 1$ and then extend it for any arbitrary $|F|$.

The source $s$ starts to route to $t$ using the name-independent scheme. Suppose that the message arrived to an intermediate node $v$ and has to go to the neighbor $u$ of $v$, but $e(v, u) \in F$. Then, since $v$ has stored in its table $\text{name}(u)$, we use the name-dependent fault-tolerant routing scheme NDFT to deliver the message from $v$ to $u$ using $\text{name}(u)$. The routing from $u$ to $t$ is done by using again the name-independent scheme. In this case, the worst scenario is when we go back from $v$ to $s$, then from $s$ to $t$ and finally from $t$ to $u$. Hence, to route from $v$ to $u$ (see Figure 1) we pay at most

$$\left(\delta(v, s) + \delta(s, t) + \delta(t, u)\right) \text{stretch}(\text{NDFT}) \leq 2 \delta(s, t) \text{stretch}(\text{NDFT}).$$
Figure 1: Example of the name-independent fault-tolerant routing scheme for $|F| = 1$. The faulty edge is $e(v, u)$. The black path is the one walked by the name-independent scheme, while the green path represents the one walked using the name-dependent fault-tolerant routing scheme.

What remains, is to bound the length of the path used to route from $s$ to $v$ and from $u$ to $t$. Recall that for this purpose we use the name-independent scheme $NI$. Supposing that the path walked from $s$ to $t$ traverses $e(v, u)$ just once, we would pay at most

$$d(s, t) \text{stretch}(NI) + 2 \delta(s, t) \text{stretch}(NDFT).$$

Let $\eta$ be the maximal number of times a message traverses a given edge in the name-independent routing scheme. Then the routing from $s$ to $t$ has length at most

$$\ell \leq d(s, t) \text{stretch}(NI) + 2 \eta \delta(s, t) \text{stretch}(NDFT) \leq \delta(s, t) \text{stretch}(NI) + 2 \eta \delta(s, t) \text{stretch}(NDFT).$$

Then, the stretch of the name-independent routing scheme tolerating at most one fault, with respect to $\delta(s, t)$, is at most

$$\frac{\delta(s, t) \text{stretch}(NI) + 2 \eta \delta(s, t) \text{stretch}(NDFT)}{\delta(s, t)} = \text{stretch}(NI) + 2 \eta \text{stretch}(NDFT).$$

Suppose now that multiple faults occur, i.e., $|F| \geq 2$. In this case, the routing would be

$$s \leadsto v_1 \leadsto u_1 \leadsto v_2 \leadsto u_2 \leadsto \cdots \leadsto v_{|F|} \leadsto u_{|F|} \leadsto t,$$

where $F = \{e(v_i, u_i) \mid 1 \leq i \leq |F|\}$. To route from a fixed $v_i$ to a fixed $u_i$, where $1 \leq i \leq |F|$, as before, we use the name-dependent fault-tolerant scheme $NDFT$, and pay at most

$$(\delta(v_i, s) + \delta(s, t) + \delta(t, u_i)) \text{stretch}(NDFT) \leq 2 \delta(s, t) \text{stretch}(NDFT).$$

Since we have to deal with $|F|$ faults and we traverse a given edge at most $\eta$ times, the routing from $v_i$ to $u_i$, for all $1 \leq i \leq |F|$, costs at most

$$2 \eta |F| \delta(s, t) \text{stretch}(NDFT).$$

What remains is to bound the distance that we pay when routing a message from $s$ to $t$ using the name-independent scheme. For this path, similarly as before, we pay at most $d(s, t) \text{stretch}(NI)$. Hence, the worst case length of a routing path using the name-independent fault-tolerant routing scheme is
\[ \ell \leq d(s, t) \text{stretch(NI)} + 2\eta |F| \delta(s, t) \text{stretch(NDFT)} \]
\[ \leq \delta(s, t) \text{stretch(NI)} + 2\eta |F| \delta(s, t) \text{stretch(NDFT)}. \]

At last, the stretch with respect to \( \delta(s, t) \) is
\[ \frac{\delta(s, t) \text{stretch(NI)} + 2\eta |F| \delta(s, t) \text{stretch(NDFT)}}{\delta(s, t)}, \]
which gives the stretch of the name-independent fault-tolerant routing scheme \( \text{NIFT} \), that is
\[ \text{stretch(NIFT)} \leq \text{stretch(NI)} + 2\eta |F| \text{stretch(NDFT)}. \]

Notice that the header size of the name-independent fault-tolerant routing scheme is equal to the sum of the header sizes of the name-independent and the name-dependent fault-tolerant schemes that it uses.

### 3.1 Examples of name-independent fault-tolerant routing schemes

In Table 2 and 3 is shown a summary of recent results on the design of name-independent routing schemes and fault-tolerant name-dependent routing schemes. The weights on the edges are assumed to be of polynomial size. We can combine any of these two mechanisms to construct our name-independent fault-tolerant routing scheme. For example, combining the results of [1] and [21], we can achieve a name-independent routing scheme tolerating an arbitrary number of faults, that consumes \( \tilde{O}(k n^{1/k} (k + \deg(v))) \) bits of memory per node, with stretch \( \tilde{O}(\eta k |F|^4) \). On the other hand, combining the results of [1] and [21], we can achieve a name-independent routing scheme that handles one fault at a time, with stretch \( O(\eta k^2) \) and tables of size \( \tilde{O}(k \deg(v) + k^2 n^{1/k}) \) bits per node.

| NI scheme | stretch | size |
|-----------|---------|------|
| [16]      | \( O(k^2) \) | \( \tilde{O}(n^{1/k}) \) per node |
| [3]       | \( \min\{1 + (2k - 1)(2^k - 2), 64k^2 + 8k\} \) | \( \tilde{O}(k^2 n^{1/k}) \) per node |
| [1]       | \( O(k) \) | \( \tilde{O}(k^2 n^{1/k}) \) per node |

Table 2: Summary of recent results present in the literature on name-independent routing schemes.

### 4 An upper bound for \( \eta \)

The stretch of our name-independent fault-tolerant routing scheme depends on this value \( \eta \) that we defined before, that is a parameter of the name-independent routing scheme. An interesting question would be to figure out the order of magnitude of this variable \( \eta \). Of course, this depends on the details of the specific name-independent routing mechanism. For this purpose, we consider a name-independent routing scheme with stretch \( 64k^2 + 8k \), that uses \( \tilde{O}(k^2 n^{1/k}) \) bits of memory per node, shown by Arias et al. [3]. In this section, at first we briefly describe the routing mechanism introduced in [3], and then we bound the value of \( \eta \) for this scheme.
### Table 3: Summary of recent results present in the literature on name-dependent fault-tolerant routing schemes; \( \text{deg}(v) \), \( W \) and \( F \), denote respectively the degree of a node \( v \), the weight of the heaviest edge and the set of forbidden edges.

| NDFT scheme | stretch | size | faults handled | name size |
|-------------|---------|------|----------------|----------|
| [21]        | \( O(k^2) \) | \( \tilde{O}(k \text{deg}(v) + n^{1/k}) \) per vertex | 1 | \( O(\log n) \) |
| [20]        | \( O(k) \) | \( \tilde{O}(kn^{1+1/k}) \) overall | 2 | \( (1 + o(1)) \log n \) |
| [19]        | \( O(k|E|^2(|F| + \log^2 n)) \) | \( \tilde{O}(kn^{1/k}\text{deg}(v)) \) per vertex | arbitrary | \( O([\log(nW)] \log n) \) |

The name-independent scheme in [3] borrows some ideas and techniques from [16, 6, 14]. In fact, like in [16], they use an underlying topology dependent routing scheme, and build on top of that some techniques that we will describe later. They also use the results from [6, 14], which states that there exists a shortest path name-dependent routing scheme for any tree \( T \), using names and tables both of \( O(\log^2(n)/\log \log(n)) \) bits, in the fixed-port model, i.e., where the port numbers are arbitrarily assigned by the network.

**Cover and clusters.** Consider an undirected weighted graph \( G = (V, E, w) \in G \) where weights are of polynomial size. A *cover* of \( G \) is a collection of clusters, such that, the union of these clusters covers every node \( v \in V \). Let \( \hat{N}^r(v) \) be the set of nodes that are within distance \( r \) from \( v \) in \( G \). Also, denote with \( \text{rad}(v, G) \) the maximum distance between \( v \) and any node in \( G \). The radius of the graph \( G, \text{rad}(G) \), is the minimum, over all \( v \in V \), of \( \text{rad}(v, G) \). Given a cluster, \( C \), the vertex \( v \) such that \( \text{rad}(v, C) = \text{rad}(C) \) is said the *center* of that cluster. For each cluster \( C \), consider a shortest path tree \( T \), rooted at the center of \( C \), that spans all the nodes of that cluster. Then, for every \( i = 1, \ldots [\log D] \), as in [15], [3] construct a cover such that,

1. there exists a cluster in the cover that includes \( \hat{N}^{2^i}(v) \), for every \( v \in V \);
2. every node \( v \in V \) is contained in at most \( kn^{1/k} \) clusters;
3. the diameter of a shortest path tree \( T_i \), rooted at the center of a cluster is at most \( (4k+1)2^i \).

At every level \( i \), let \( C_i \) be the cluster that contains \( \hat{N}^{2^i}(v) \). Then \( C_i \) is said the *home cluster* of node \( v \) for level \( i \).

**Routing tables and prefix matching.** Suppose there are \( n^{1/k} \) different colors and each node \( v \) is assigned \( k - 1 \) colors, \( \text{color}_1(v), \ldots, \text{color}_{k-1}(v) \). These colors are assigned randomly, using some hash functions that map identifiers on colors. Notice that, with high probability, there are \( \tilde{O}(n^{1/k}) \) nodes that have the same \( k - 1 \) colors. Let \( \text{prefix}_i(v) \) be the first \( i \) colors of node \( v \). Each node \( v \) in the graph stores the routing table of the scheme in [6, 14], plus some additional information that we will describe in the following. Each vertex \( v \) stores the names of all nodes \( u \) such that \( \text{prefix}_{k-1}(v) = \text{prefix}_{k-1}(u) \), and, for each color \( c_j \), \( 1 \leq j \leq n^{1/k} \) and for each \( i \) such that \( 0 \leq i \leq k - 2 \), the name of the nearest node \( w \) such that
prefix_{i+1}(w) = prefix_i(v)c_j.

Routing. To send a message from a source \( s \) to a target \( t \), for increasing values of \( i = 1, \ldots, \lfloor \log D \rfloor \), node \( s \) tries to route on the tree \( T_i \) rooted at the center of \( C_i \), where \( C_i \) is the home cluster of \( s \) at level \( i \). To route a message in \( T_i \in C_i \), the scheme in [3] uses a combination of the name-dependent routing in trees of [6, 14] with the prefix matching technique, that consist of the following. At first, \( s \) calculates \( \text{color}_1(t), \ldots, \text{color}_{k-1}(t) \) by applying the hash functions on the identifier of \( t \). Then, the routing in \( T_i \) proceeds as follows. From any intermediate node \( v \) (including \( s \)), the message is routed to a node in \( T_i \) that matches the largest prefix of \( \text{color}_1(t), \ldots, \text{color}_{k-1}(t) \), using the name-dependent routing scheme on trees ([6, 14]). If the destination node \( t \) does not exist in \( T_i \), then the message is returned to \( s \), that will try to route on \( T_{i+1} \in C_{i+1} \). Otherwise, the message reaches the target node in at most \( k \) such steps. Notice that the success is guaranteed since the tree rooted at the last level, spans all the nodes in the graph.

We show an upper bound of \( \eta \) for the name-independent routing scheme of [3]. Our result is stated in the following lemma.

**Lemma 2** Given any graph \( G = (V, E, w) \in G \) and any edge \( e \in E \), the name-independent routing scheme of [3] traverses \( e \) at most \( \eta \leq 2k \log D \) times, where \( D \) is the weighted diameter of \( G \).

**Proof.** In order to prove the lemma, we first have to bound \( \eta \) for the name-dependent routing scheme on trees [6, 14], since it is used in the name-independent routing scheme of [3]. Notice that, since the mechanisms in [6, 14] provide a shortest path routing on trees, then given any edge \( e \) of the tree, it holds that \( \eta \leq 1 \).

In each of the \( \log D \) phases of the name-independent scheme of [3], we walk at most \( k \) different shortest paths to try to reach the target, and in case of error we walk at most \( k \) different shortest paths to return to the origin. Since in the worst case we have to search for the target node in \( C_{\log D} \), then \( \eta \leq 2k \log D \). \( \square \)

Now we have all the ingredients to state the following theorems.

**Theorem 3** There is a name-independent routing scheme able to deal with an arbitrary-size set \( F \) of edge-failures, with stretch \( O(k^2 |F|^3(|F| + \log^2 n) \log D) \) and size \( \tilde{O}(kn^{1/k}(k + \deg(v))) \) bits per node.

**Theorem 4** There is a name-independent routing scheme handling at most \( 2 \) faults, with stretch \( O(k^2 \log D) \) and average memory per node of \( \tilde{O}(k^2 n^{1/k} + \deg(v)) \) bits.

**Theorem 5** There is a name-independent routing scheme that tolerates 1 fault at a time, that has size of \( \tilde{O}(k^2 n^{1/k} + k \deg(v)) \) bits per node and stretch \( O(k^3 \log D) \).

5 Conclusions and further work

We have shown a general name-independent fault-tolerant routing scheme, the design of which uses as a black box the combination of any name-independent routing scheme that does not handle any fault, with any name-dependent fault-tolerant routing mechanism. The stretch and size of our scheme depend on the ones of the name-independent scheme and fault-tolerant name-dependent routing scheme. An interesting open question would be whether we can design ”by
hand” a name-independent fault-tolerant routing scheme, achieving better results in terms of stretch and size than the ones presented in this paper. Also, our scheme depends on a parameter, \( \eta \), that is the maximum number of times that the name-independent scheme traverses the same edge. We showed that \( \eta \leq 2k \log D \) for the name-independent routing scheme of [3]. It would be interesting to analyze \( \eta \) also for other name-independent schemes, like, for example, for the ones shown in [1] and [16].

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