An Integrated Numerical Hydrodynamic Shallow Flow-Solute Transport Model for Urban Area

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Abstract. The rapidly changing on land profiles in the some urban areas in Malaysia led to the increasing of flood risk. Extensive developments on densely populated area and urbanization worsen the flood scenario. An early warning system is really important and the popular method is by numerically simulating the river and flood flows. There are lots of two-dimensional (2D) flood model predicting the flood level but in some circumstances, still it is difficult to resolve the river reach in a 2D manner. A systematic early warning system requires a precisely prediction of flow depth. Hence a reliable one-dimensional (1D) model that provides accurate description of the flow is essential. Research also aims to resolve some of raised issues such as the fate of pollutant in river reach by developing the integrated hydrodynamic shallow flow-solute transport model. Presented in this paper are results on flow prediction for Sungai Penchala and the convection-diffusion of solute transports simulated by the developed model.

1. Introduction
Numerous studies in modelling flow in open channel have been done, however there are still deficiencies in the models. As it is difficult to resolve the problematic river reach in a 2D manner, a 1D component with an ability to deal with the complex flow hydrodynamics in complex topography is essential. Enhancement of computer technology also raised the 2D models to be more prevalent, yet it is still computationally difficult to signify the small streams in a larger scale. This brought to the necessity of having an intensive 1D flow model when the 2D also tends to be more data demanding [1]. The computational cost of 1D model is also lower than the 2D models. For a large scale river network, still it consents closely to real time analysis [2]. However, there is still a subject to debate on the 1D model. Most of the 1D models were developed based on the solution to the approximation of the 1D shallow water formula ([3],[4]) and popular for its straightforwardness. This ignored the spatially hydraulics variables and caused inability to represent the full dynamics of a flow. Thus, for broader applications it is desired to have a high ability of 1D model in solving the more complex hydrodynamics flow pertaining to flood in urban areas.

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2. Governing equations and the numerical scheme

It is common to assume that the 1D channel attached to a floodplain has a rectangular cross-section ([5],[6]). Dealing with complex geometry, the breadth variation is then included in order to accurately calculate the flow depth \( h \) and discharge \( q = uh \). The final equations can be written as:

\[
\frac{\partial h}{\partial t} + \frac{\partial (uh)}{\partial x} = - \frac{uh}{b} \frac{\partial b}{\partial x} \tag{1}
\]

\[
\frac{\partial (uh)}{\partial t} + \frac{\partial \left( u^2 h \right)}{\partial x} + \frac{g}{2} \frac{\partial}{\partial x} \left( h^2 \right) = - \frac{u^2 h}{b} \frac{\partial b}{\partial x} - C_f u^2 |u| - gh \frac{\partial Z_b}{\partial x} \tag{2}
\]

where \( u \) is velocity, \( Z_b \) is bed elevation, \( g \) is the gravity acceleration, \( b \) is channel width, \( C_f = g n^2 h^{1/3} \) is the roughness coefficient of the river bed and \( n \) is the Mannings' coefficient. Equation (1) allows varying channel width and bottom, while the source terms in equation (2) includes breadth variations, bottom friction and bed slope. Adding the equation (3) into the model, solute transport and shallow waters flow is simulated simultaneously.

\[
\frac{\partial (ch)}{\partial t} + \frac{\partial (uch)}{\partial x} = S_c \tag{3}
\]

Godunov-type finite volume schemes in conjunction with Harten-Lax-vanLeer (HLL) approximate Riemann solver is chosen to solve the fully SWE’s. Riemann solvers are of significant interest (e.g., [7],[8],[9],[10]) as widely implemented to modelling flows with abrupt and discontinuous gradients. The final expression for the scheme is:

\[
u^{i+1}_k = u^i_k - \Delta t \left( f_{i+1/2}^k - f_{i-1/2}^k \right) + \Delta t s_i \tag{4}
\]

where \( i \) denotes the cell index, \( \Delta x \) and \( \Delta t \) are the cell size and time step respectively, \( f_{i+1/2}^k \) and \( f_{i-1/2}^k \) are the interface fluxes vector. The HLL approximate Riemann solver [11] is adopted for flux calculation as it facilitate the wetting and drying cases. The Riemann states obtained from the face values that are reconstructed from the cell-centred flow variables. Herein the MUSCL (Monotone Upstream-centred Schemes for Conservation Laws) slope limited linear reconstruction (van Leer, 1979) is adopted to rebuild the face values in order to achieve 2nd-order accuracy in space. Combining the Godunov-type finite volume method with the second-order Runge-Kutta method gives:

\[
u^{i+1/2} = u^{i}_k + 0.5 \Delta t \left( K_i \left( u^{i}_k \right) + K_i \left( u^{i+1/2}_k \right) \right) \tag{5}
\]

The \( u^* \) is the intermediate flow variables while superscript \( k \) in equation (4) and (5) represent the present time level. \( K \) in equation (5) is the Runge-Kutta coefficient at predictor and corrector steps defined by:

\[
K_j = - \frac{\left( f_{i+1/2}^k - f_{i-1/2}^k \right)}{\Delta x} \tag{6}
\]

In reconstructing the Riemann states, the flow variables at the flux faces were first estimated from the central flow information using a slope limited linear interpolation to rebuild the face values to achieve second-order accuracy in space [12]. To preserves the accuracy and well-balance property, the source terms are split into two parts and discretized using central differences that are compatible to the flux calculation [13].

3. Model validation and implementation

The phenomenon of 1D surface flow is observed under various conditions experienced in open channel [13]. As the results are reliable, it is then implemented to the real case scenario. Sungai Penchala that is densely populated, surrounded by industrial and residential areas situated between Kuala Lumpur and Selangor is identified as the potential site studies. The river reach is divided into few sections and the readings of water level at predetermined locations were recorded. The time histories for flow depth along the Sungai Penchala were predicted numerically at 2000 m interval.
After validating its performance in open channels, the flow model was then integrated with the pollutant transport equation. Then, the model used to study the fate of pollutant.

4. Results and discussion
4.1. Flow hydrodynamic in Sungai Penchala
The proposed model has been validated against several benchmark tests and close agreement has been achieved. This confirms the effectiveness of the current 1D code [13]. The model used the channel geometry obtained from the field surveys done by Department of Irrigation and Drainage (DID). The bed profile is as plotted in figure 1.

![Figure 1. Bed profile of Sungai Penchala](image)

From the sketched, the Penchala catchment is a hilly terrain. Almost 80% of Sungai Penchala is a concrete channel with bed frictional coefficient equals to 0.015 s/m$^{1/3}$. The imposed outflow depth is 0.15 m as recorded during the survey. Results of simulation as in figure 2 (a-d) which describe the time histories of flow depth at five different locations, $x$. Simulation was run for 7200 s on a uniform grid.

![Figure 2 (a-d). Time histories of water depth in five points.](image)

The maximum flow depths at five locations numerically simulated and compared with the flow depth measured on site. Figure 3 illustrates the comparison between the maximum predicted and measured flow depth.
4.2. Fate of pollutant

The integrated model then used to study the fate of pollutant. The test involved a 1D convective–diffusion problem [14]. Figure 4 (a-b) exhibits the solute concentration at different simulation time.

![Figure 4](image)

(a) $t = 20s$  
(b) $t = 40s$

**Figure 4.** Comparison of concentration $C$ at different time histories.

The maximum concentration computed was 0.5982 mg/L and as the time of simulation increases, it reduced to 0.3537 mg/L. Results exhibit good accuracy in the convective-diffusion case with the global relative error, $\tilde{R} = 0.0949$ and $R = 0.10711$ for $t = 20$ s and 40 s respectively.

5. Conclusion and recommendations

The model has been validated by numbers of benchmark tests previously and good agreements were achieved. Model is then implemented on Sungai Penchala and gives sufficient results in flow prediction. This model is expected to be broadly tested on other rivers in urban area. Predictions on fate of pollution also exhibit good agreements. In future, it is suggested that the solver can include the advection-reaction-diffusion equation to model fate of pollutant as in reality the dynamics flow in urban areas are chemically reactive or biologically reproductive.

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