Four-Frequency Small-Signal Model for High-Bandwidth Voltage Regulator With Current-Mode Control

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ABSTRACT High-bandwidth buck voltage regulator with peak current-mode control is a commonly-used scheme, especially for CPU power applications. However, the conventional single-frequency and three-frequency models are inaccurate in modeling the loop gain stability when the control bandwidth is close or higher than one-third of the converter switching frequency. A more accurate four-frequency small-signal model for a high-bandwidth buck converter with peak current-mode control is proposed in the present paper. The describing function approach is used to model the duty-cycle modulator. The effects of the four-frequency components, including the modulating frequency, the two sideband harmonics, and the switching frequency, are considered. Simulations and experiments verified the results.

INDEX TERMS Buck converter, describing function (DF), high bandwidth, modeling, voltage regulators (VRs), current-mode control.

I. INTRODUCTION

In recent years, tremendous progress has been made about the voltage regulator (VR) for powering the latest computer central processor units (CPU), which is often called the “V-Core” regulator. The processing speed that the CPU operates with is becoming faster and faster. Therefore, the load requirement for V-Core would be more stringent. There are mainly two developing tendencies to satisfy stringent V-Core requirements and to deal with the heat dissipation problem of V-Core: multi-phase interleaving buck converter and adaptive voltage positioning (AVP) control strategies. Compared to traditional constant-frequency voltage mode (CFVM) control, current-mode control can be easily integrated with the above two crucial techniques required by V-Core application [1]–[3]. Therefore, a constant-frequency peak current mode (CFPCM) is widely used in the industry.

Besides the needs mentioned above, the selection of switching frequency is usually a trade-off between efficiency and physical size/transient response in V-Core design. Furthermore, the selection of control bandwidth is also important. Higher switching frequency design can lead to wider control bandwidth. Designs with wider control bandwidth improve transient responses or reduce the number of output capacitors required to save both space and cost of V-Core regulators [4]–[13]. For example, a PCM-controlled converter with over 40% bandwidth-to-switching frequency ratio is demonstrated in [14].

However, to achieve a high control bandwidth by designing feedback compensators, a highly accurate small-signal model is required to predict loop stability. Although some prior works such as discrete-time modeling methods [14]–[18] target stability issues, they generally deviate from the small-signal concept. Furthermore, some modeling methods [16]–[18] are stability-oriented and do not contain frequency-domain information for transient response behavior estimation. Therefore, those methods are not as widely adopted in the industry by engineers as frequency-domain small-signal models do. Previous single-frequency small-signal models of CFPCM control have been proposed for many years [19], [20], but it is only accurate for the case with a small control bandwidth/switching frequency ratio.
For applications with high control bandwidth/switching frequency ratio, a two-frequency model for CFVM control was proposed in [21]. However, a significant model error was observed in the two-frequency model for V-Core applications when the bandwidth/switching-frequency ratio is 40% [22]. It is due to the lack of some frequency components that may affect the loop behavior when the bandwidth/switching frequency ratio is high. To solve this issue, a four-frequency model has been proposed and verified to achieve small-signal model accuracy [22]. Nevertheless, the four-frequency model proposed in [22] is only valid for CFVM control, but not for CFPCM control. Similarly, the models proposed by some other works [14], [23]–[25] that apply a similar approach to CFVM control cannot be directly extended to CFPCM. The involved current loop makes the whole control loop model much more complex than CFVM control. One recent work proposed a three-frequency model for CFPCM control [25]. However, similar to the three-frequency CFVM control, it is not accurate under certain conditions. Besides, the three-frequency model may have the wrong prediction in stability, as will be shown in Fig. 10(d). Therefore, it is practically essential to develop a small-signal model with high accuracy for CFPCM control.

In this paper, previous single-frequency small-signal models for CFPCM control will be reviewed first, and the small-signal modeling concept will be mentioned. A novel multi-frequency small-signal model for CFPCM control will then be proposed. Describing function (DF) method [20], [22], [26], [27] will be used to derive transfer functions required for the proposed small-signal model. Finally, simulations and experimental results will be used to verify the accuracy of the proposed four-frequency model.

II. REVIEW OF PREVIOUS SMALL-SIGNAL MODELS FOR CFPCM CONTROL

A. CIRCUIT EXPLANATION OF CFPCM FEEDBACK BEHAVIOR FROM SPECTRUM PERSPECTIVE

Fig. 1 shows the circuit diagram of a CFPCM controlled buck converter where \( v_{\text{cs}} \) is the inductor-current ramp signal, \( v_c \) is the external ramp compensation signal added to the PWM comparator. CFPCM control uses the inductor current to modulate the PWM duty cycle. There are two loops for a CFPCM controlled buck converter, which are the outer voltage loop and inner current loop, respectively. To explain the circuit feedback behavior of CFPCM control, the frequency-domain scenario is as shown in Fig. 2.

As shown in Fig. 2, various frequency components affect the modulation of CFPCM. Therefore, CFPCM is a dynamic nonlinear system. When a perturbed signal with frequency \( f_m \) exists in control voltage \( v_c \), various frequency components would appear in the output of PWM comparator, including fundamental component \( f_m \), switching frequency component \( f_s \), harmonic components of switching frequency \( n \cdot f_s \), sideband components \( f_s \pm f_m \) and \( n \cdot f_s \pm f_m \). These high-frequency components would not be attenuated in the inner current loop as the inductor current ramp signal is directly feedbacked to the current-mode modulator. Therefore, all high-frequency components would be coupled and affect each other through the current-mode modulator. Single-frequency small-signal model for CFPCM control will be mentioned in the following section.

B. SINGLE-FREQUENCY [20] AND THREE-FREQUENCY [25] SMALL-SIGNAL MODELS FOR CFPCM CONTROL

A small-signal modeling technique for current-mode control was presented in [20]. It uses the DF method to establish a small-signal model for the non-linear current-mode modulator to acquire the control voltage-to-inductor current transfer function. Through the technique mentioned, the sideband effect of the inner current loop can be considered in the small-signal model. However, it is assumed in [20] that the compensator (which is a low-pass filter) can attenuate high-frequency components in the outer-voltage loop. Thus, it is enough to consider only the fundamental component \( f_m \) of the control voltage in the outer loop. Therefore, the small-signal model presented in [20] is also called the single-frequency small-signal model. Fig. 3(a) shows a control block diagram of single-frequency small-signal model for CFPCM controlled buck converter, where \( DF_{v_c}(f_m) \) is the control voltage-to-inductor current transfer function, \( DF_{vo}(f_m) \) is the output voltage-to-inductor current transfer function, \( Z_p(f_m) \) is inductor current-to-output voltage transfer function, \( H_i(f_m) \) is the transfer function of feedback compensator and \( v_p(f_m) \) is perturbed signal with frequency \( f_m \).
FIGURE 3. Control block diagram of (a) single-frequency model and (b) three-frequency model for CFPCM Controlled Buck Converter.

Besides, outer loop gain \( T(f_m) \) of the single-frequency small-signal model can be expressed as (1), as shown in Fig. 3(a).

\[
T(f_m) \equiv -\frac{v_o(f_m)}{v_p(f_m) + v_o(f_m)} = DF_{vc}(f_m) \cdot \frac{Z_p(f_m)}{1 - Z_p(f_m) \cdot DF_{vo}(f_m)} \cdot H_v(f_m) \tag{1}
\]

where (2)–(4), as shown at the bottom of the page.

Note here that \( R_o \rightarrow \infty \) when modelling the load as an ideal current source. Besides, \( S_e \) is the slope of external ramp compensation signal, \( S_n = R_l \cdot (V_{in} - V_o) / L \), \( S_n = R_l \cdot V_o / L \), \( L \) is the inductance of inductor, \( R_l \) is inductor current sensing gain.

While single-frequency modeling is simple and intuitive, it may inevitably lead to error since it only considers \( f_m \) in Fig. 2, especially with higher bandwidth where the sideband components cannot be neglected. Recently, a three-frequency modeling technique for CFPCM controlled buck converter was presented in [25]. It corrects the original single-frequency modeling by inserting a sideband component \( f_s - f_m \) into the loop model. Also, a switching frequency component \( f_s \) is considered such that it can be applied to a converter with a large ripple. Fig. 3(b) shows a control block diagram of the three-frequency small-signal model for the CFPCM controlled buck converter. Where the loop gain equation is shown as follows:

\[
T(f_m) = \frac{T_{LJ}(f_m)}{1 + T_{LJ}(f_m - f_s)}
\]

where

\[
T_{LJ}(f_m) = \frac{E_m(f) \cdot G_p(f) \cdot H_v(f)}{f_m \cdot f_s \cdot (S_n + S_e) + (S_f - S_e) \cdot e^{-j2\pi f_s T_s}}
\]

\[
G_p(f) = \frac{V_{in}}{2\pi f L \cdot Z_p(f)}
\]

\[
E_m(f) = K(f) \cdot \frac{f_s \cdot (1 - e^{-j2\pi f_s T_s})}{(S_n + S_e) + (S_f - S_e) \cdot e^{-j2\pi f_s T_s}}
\]

\[
K(f) = \frac{K_2}{(S_n + S_e - \gamma) + (S_f - S_e + \gamma) \cdot e^{-j2\pi f_s T_s}}
\]

\[
\gamma = 2\pi f_s V_u \cos(2\pi D - \theta_a)
\]

where \( V_u \) and \( \theta_a \) are phase and frequency for switching-frequency components.

For two CFPCM controlled converter designs with different control bandwidth, their small-signal models of loop gain \( T(f_m) \) are plotted against simulated results in Fig. 4. Table 1 lists the circuit parameters and operating conditions. Two cases of control bandwidths (45kHz and 145kHz) are achieved using similar compensator types but different compensating parameters. As can be seen from Fig. 4(a), the accurate result is obtained from both single-frequency and three-frequency small-signal models up to 100kHz when the control bandwidth/switching frequency ratio is 12\%. And from a practical point of view, those small-signal models are useful as long as it is accurate within the bandwidth. In other words, single-frequency and three-frequency small-signal models for CFPCM control are enough for this case, where low control bandwidth/switching frequency ratio is
selected. From Fig. 4(b), however, when the control bandwidth is increased to 145kHz, which indicates the control bandwidth-to-switching frequency ratio is 40%, the single-frequency small-signal model starts to deviate from the simulated result, both in frequency lower than 400 Hz and those higher than 500 kHz. From the above figure, system control bandwidth with an error of 41kHz appeared between the model and simulation, causing a 16.8° difference in phase margin. This control bandwidth/switching frequency ratio (i.e. 40%) is not an exaggerated number but is indeed used in practical V-Core applications. For the three-frequency small-signal model, a 2dB DC gain error has occurred, causing its gain to be inaccurate for frequencies lower than 500Hz. This also happens in the single-frequency model. Therefore, a 6.5° phase error can be found around 500Hz for both models due to the DC gain error in low frequency. Besides, the three-frequency model may have a wrong prediction for stability, as will be shown in Fig. 10(d). To achieve better accuracy for the small-signal model, sideband components \(f_m-f_s\) and \(f_s+f_m\) and switching frequency \(f_s\) must be considered in the model. This will be described in the following section.

III. THE PROPOSED FOUR-FREQUENCY MODEL

While the single-frequency model shown in Fig. 3 assumes only fundamental component \(f_m\) exists in the loop, sideband components \(f_m-f_s\) and \(f_s+f_m\) and switching frequency \(f_s\) in the loop are neglected, as shown in Fig. 2. As control bandwidth pushes high, the sideband components that transfer over the converter’s cut-off frequency can no longer be suppressed sufficiently by the compensator, leading to the failure in the assumption of the single-frequency model. In this section, a four-frequency model that incorporates the factors mentioned above will be derived, rendering accurate predictions even for high-control bandwidth design.

In the proposed small-signal model for CFPCM control, sideband components \(f_m-f_s\) and \(f_s+f_m\) and fundamental frequency \(f_m\) of the outer voltage loop are included in the feedback block diagram. The beat-frequency component \(f_m-f_s\) took the place of \(f_s-f_m\) to help simplify the mathematical derivation, as shown in Fig. 5. This modeling concept of the outer voltage loop is an extension of the four-frequency small-signal model for CFVM control in [22]. In the control diagram, dotted lines inside the control voltage-to-inductor current \(DF_{vc}\) function block indicate the transfer functions between the control voltages \(v_c\) and inductor currents \(i_L\); whereas dotted lines inside the output voltage-to-inductor current \(DF_{vo}\) function block indicate the transfer functions between the output voltages \(v_o\) and inductor currents \(i_L\). Both the \(DF_{vc}\) and \(DF_{vo}\) can be expressed as gain matrices for the current-mode modulator. Notice that there are three different frequency components for each of the control voltages \(v_c\), output voltages \(v_o\), and inductor currents \(i_L\), which are modulation frequency \(f_m\) and two sideband frequencies \((f_s-f_m\) and \(f_s+f_m\)). Thus, there are nine \((3 \times 3)\) \(DF_{vc}\) transfer functions and nine \((3 \times 3)\) \(DF_{vo}\) transfer functions for such a system, representing eighteen different paths that these three frequency components travel through. The effect of the switching frequency \(f_s\) component, which is the fourth frequency component to the current-mode modulator, should be considered and incorporated into the small-signal model. Notice that \(v_c(f_s)\) operated onto any one of \(DF_{vc}\) function block or \(DF_{vo}\) function block is the products of three transfer functions \((a(f_s) \cdot G_{vc}(f_s) \cdot H_c(f_s)))\), where \(a(f_s)\) is the switching frequency component of duty cycle signal in its steady-state waveform. Because there are four frequency components to be considered in the proposed small-signal model for CFPCM.
control, this proposed model is called the four-frequency model for CFPCM control. The list of terms used during derivation process is given in Table 2.

A. DERIVATION OF DFVC TRANSFER FUNCTION

The DF method [20], [22], [26], [27] can be used to model the nonlinear current-mode modulator to obtain the relationship between frequency components of input and output signals. Therefore, the sideband effect of $DF_{VC}$ can be expressed as nine transfer functions. For example, a sinusoidal perturbation with a small magnitude at frequency $f_m$ is injected through $v_c$. Based on the perturbed inductor current waveforms, the $DF_{VC}$ transfer function can be found by mathematical derivation. Two assumptions are made for this derivation: (1) the magnitude of the perturbation signal is very small; (2) commensurable relation can be found between the perturbation frequency $f_m$ and the switching frequency $f_s$, that is, $T = M \cdot f_m = N \cdot f_s$, where $N$ and $M$ are positive integers.

In the DF method, the derivation consists of eight standard steps indicated below:

1) EXPRESSING THE SWITCHING-FREQUENCY COMPONENT IN OUTPUT VOLTAGE $V_{O4}(T)$ OF THAT IN THE CONTROL VOLTAGE $V_{C4}(T)$

The output voltage $v_o(t)$ consists of five terms, the dc component $v_o$, the switching-frequency component $v_o(f_s)$, the modulating-frequency component $v_o(f_m)$, and the two sideband-frequency components $v_o(f_m-f_s)$ and $v_o(f_m+f_s)$. For the control voltage $v_c(t)$, it is similar to $v_o(t)$, which is composed of the dc component $v_c$, the switching-frequency component $v_c(f_s)$, the modulating-frequency component $v_c(f_m)$, and the two sideband-frequency components $v_c(f_m-f_s)$ and $v_c(f_m+f_s)$.

For simplicity, $v_o(f_m)$ is designated as $v_{o1}(t)$, $v_o(f_m-f_s)$ as $v_{o2}(t)$, $v_o(f_m+f_s)$ as $v_{o3}(t)$, and $v_o(f_s)$ as $v_{o4}(t)$; also, $v_c(f_m)$ is designated as $v_{c1}(t)$, $v_c(f_m-f_s)$ as $v_{c2}(t)$, $v_c(f_m+f_s)$ as $v_{c3}(t)$, and $v_c(f_s)$ as $v_{c4}(t)$.

To find out $v_{o4}(t)$ and $v_{c4}(t)$ in steady-state, several steps are taken, which are described as follows. First, the steady-state duty cycle signal $D(t)$ can be expressed by:

$$D(t) |_{0 \leq t \leq T_s} = u(t) - u(t - T_{on})$$

where $u(t)$ is a unit step function.

Second, the Fourier coefficient of the $f_s$ component, $\alpha(f_s)$ can be obtained by (6):

$$\alpha(f_s) = \frac{2}{T_s} \int_0^{T_s} d(t) \cdot e^{-j2\pi f_s t} dt$$

$$= \frac{2}{T_s} \int_0^{T_{on}} e^{-j2\pi f_s t} dt$$

$$= \frac{2}{j2\pi} \cdot (1 - e^{-j2\pi D})$$

Third, find out $v_o(f_s)$ and $v_c(f_s)$ in terms of $\alpha(f_s)$. Since the $v_o(f_s)$ and $v_c(f_s)$ signal propagates through $G_{\alpha}(f_s)$ and $H_{\alpha}(f_s)$, (7) and (8) can be obtained.

$$v_o(f_s) = \alpha(f_s) \times G_{\alpha}(f_s)$$

$$v_c(f_s) = \alpha(f_s) \times H_{\alpha}(f_s)$$

TABLE 1. Circuit parameters/conditions for simulated verification.

| Parameter          | Value         |
|--------------------|---------------|
| Input voltage $V_i$| 12 V          |
| Output voltage $V_o$| 0.9 V        |
| Load current $L$   | 10 A          |
| Switching frequency $f_s$ | 370 kHz    |
| Inductor $L$       | 360 mH        |
| DCR of inductor    | 0.968 mF      |
| Output capacitor $C$| 1.72 mF      |
| ESR of capacitor $R_C$ | 0.79 mΩ    |
| Current sensing ratio $R_i$ | 25 m      |
| External ramp slope $S_e$ | 185kV/s    |

TABLE 2. List of terms used in the equations.

| Term                | Equation |
|---------------------|----------|
| Crucial Terminologies                       |
| Fundamental frequency $f_m$                      |
| First-order sideband frequencies $f_s-f_m$ and $f_s+f_m$ |
| Switching frequency $f_s$                      |
| Control bandwidth $f_{BW}$                     |
| Control voltage $v_c$                          |
| Inductor current $i_l$                         |
| Output voltage $v_o$                           |
| Perturbed signal $v_p$                         |
| Duty cycle $d$                                 |
| Outer loop gain $T(f)$                         |
| Control voltage-to-inductor current transfer function $DF_{\alpha}(f)$ |
| Output voltage-to-inductor current transfer function $DF_{\delta}(f)$ |
| Inductor current-to-output voltage transfer function $Z_{\theta}(f)$ |
| Feedback compensator transfer function $H_{\alpha}(f)$ |
| Duty cycle-to-output voltage transfer function $G_{\delta}(f)$ |
| DC values                                         |
| Input voltage (V) $V_i$                        |
| Output voltage (V) $V_o$                       |
| Inductor (H) $L$                               |
| Output capacitor (F) $C$                       |
| Output load resistance $(\Omega) R_o$          |
| ESR of capacitor $(\Omega) R_C$                |
| Current sensing ratio $(V/A) R_i$              |
| External ramp slope $(V/V) S_e$                |
| Rising slope of the sensed inductor current $(V/V) S_r$ |
| Falling slope of the sensed inductor current $(V/V) S_f$ |
| On-time $(s) T_{on}$                           |
| Off-time $(s) T_{off}$                         |
| Switching period $(s) T_s$                     |
| Duty cycle $(s) D$                             |
\[ v_c(f_s) = \alpha(f_s) \times G_{vd}(f_s) \times [-H_v(f_s)] \]  \quad (8)

where

\[ G_{vd}(f_s) = \frac{V_{in} \cdot j2\pi \cdot f_s \cdot R_C \cdot C}{\Delta(f_s)} \]  \quad (9)

\[ \Delta(f_s) = 1 + j2\pi \cdot f_s \cdot \left( \frac{L}{R_o} + C \cdot R_C \right) + (j2\pi \cdot f_s)^2 \cdot L \cdot C \cdot \frac{R_C + R_o}{R_o} \]  \quad (10)

Finally, the frequency-domain equation of (7) and (8) can be rewritten as time-domain equations (11) and (12).

\[ v_{oa4}(t) = |v_o(f_s)^a| \cdot \sin(2\pi \cdot f_s \cdot t + \theta_5) \]  \quad (11)

\[ v_{c4}(t) = |v_c(f_s)^a| \cdot \sin(2\pi \cdot f_s \cdot t + \theta_6) \]  \quad (12)

where

\[ \theta_5 = -\tan^{-1}\left[ \frac{\text{Im}(v_o(f_s)^a)}{\text{Re}(v_o(f_s)^a)} \right] + \frac{\pi}{2} \]  \quad (13)

\[ \theta_6 = -\tan^{-1}\left[ \frac{\text{Im}(v_c(f_s)^a)}{\text{Re}(v_c(f_s)^a)} \right] + \frac{\pi}{2} \]  \quad (14)

Equations (11) and (12) would be used later in the derivation of \( DF_{vc} \) and \( DF_{vo} \) transfer functions. It should be noted that (11) and (12) play important roles in the outcome of the proposed small-signal model.

2) DESCRIBING THE SINUSOIDAL PERTURBATION OF CONTROL SIGNAL \( v_c(t) \)

Fig. 5 shows that the control voltage is composed of five terms, as expressed in (15). The signals \( v_{c1}(t), v_{c2}(t), v_{c3}(t), \) and \( v_{c4}(t) \) are the four modulating signals considered in the proposed model.

\[ v_c(t) = r_0 + v_{c1}(t) + v_{c2}(t) + v_{c3}(t) + v_{c4}(t) \]  \quad (15)

where \( r_0 \) is the DC component, \( v_{c4}(t) \) has already been given in (12), and \( v_{c1}(t), v_{c2}(t), v_{c3}(t) \) can be expressed as (16) to (18), respectively.

\[ v_{c1}(t) = \hat{r}_1 \cdot \sin(2\pi \cdot f_m \cdot t + \theta_1) \]  \quad (16)

\[ v_{c2}(t) = \hat{r}_2 \cdot \sin(2\pi \cdot (f_m - f_s) \cdot t + \theta_2) \]  \quad (17)

\[ v_{c3}(t) = \hat{r}_3 \cdot \sin(2\pi \cdot (f_s + f_m) \cdot t + \theta_3) \]  \quad (18)

where \( \hat{r}_1, \hat{r}_2, \) and \( \hat{r}_3 \) are the amplitude in control voltage of the three corresponding frequency components.

3) CALCULATING THE ON-TIME PERTURBATION

Based on the modulation law for CFPCM control, the duty cycle signal and inductor current waveform can be described as Fig. 6, where the modulation law can be expressed as follows:

\[ v_c(t_i + T_{on(i-1)} - S_e \cdot T_{on(i-1)} - S_f \cdot (T_s - T_{on(i-1)}) = v_c(t_i + T_{on(i)}) - (S_n + S_e) \cdot T_{on(i)} \]  \quad (19)

where \( T_{on(i)} \) is the on-time of the \( i \)th cycle.

It is assumed \( T_{on(i)} = T_{on} + \Delta T_{on(i)} \), where \( T_{on} \) is the steady-state on-time and \( \Delta T_{on(i)} \) is on-time perturbation of the \( i \)th cycle. Also \( t_i \) can be expressed as: \( t_i = (i-1) \cdot T_s \), where \( T_s \) is the switching period in steady-state. From (19), the following equation can be obtained:

\[ (S_n + S_e) \cdot \Delta T_{on(i)} + (S_f - S_e) \cdot \Delta T_{on(i-1)} = v_c(t_i + T_{on(i)}) - v_c(t_{i-1} + T_{on(i-1)}) \]  \quad (20)

Finally, \( \Delta T_{on(i)} \) can be derived from (20) and thus can be obtained as (21):

\[ (S_n + S_e - S_{vcfs}) \cdot \Delta T_{on(i)} + (S_f - S_e + S_{vcfs}) \cdot \Delta T_{on(i-1)} = 2 \cdot \hat{r}_1 \cdot \cos \left\{ 2\pi \cdot f_m \cdot \left[ (i-1) \cdot T_s + \frac{T_{on} - T_{off}}{2} \right] + \theta_1 \right\} \cdot \sin(\pi \cdot f_m \cdot T_s) + 2 \cdot \hat{r}_2 \cdot \cos \left\{ 2\pi \cdot (f_m - f_s) \cdot \left[ (i-1) \cdot T_s + \frac{T_{on} - T_{off}}{2} \right] + \theta_2 \right\} \cdot \sin(\pi \cdot f_m - f_s) \cdot T_s + 2 \cdot \hat{r}_3 \cdot \cos \left\{ 2\pi \cdot (f_s + f_m) \cdot \left[ (i-1) \cdot T_s + \frac{T_{on} - T_{off}}{2} \right] + \theta_3 \right\} \cdot \sin(\pi \cdot f_s + f_m) \cdot T_s \]  \quad (21)

where,

\[ S_{vcfs} = 2 \pi \cdot f_s \cdot |v_c(f_s)^a| \cdot \cos(2\pi \cdot D + \theta_4) \]  \quad (22)

where \( D \) is the duty cycle in steady-state, \( v_c(f_s) \) is shown in (8) and \( \theta_4 \) is shown in (14).

4) FINDING THE FOURIER COEFFICIENT OF DUTY CYCLE SIGNAL \( D(F_M) \) AND \( I_L(F_M) \)

The duty cycle signal \( d(t) \) and inductor current \( i_L(t) \) can be expressed by (23) and (24), respectively,

\[ d(t) = \sum_{i=1}^{M} [u(t - t_i) - u(t - t_i - T_{on(i)})] \]  \quad (23)

\[ i_L(t) = \int_0^t \left( \frac{V_{in}}{L} \cdot d(t) - \frac{V_o}{L} \right) \ dt + i_{L0} \]  \quad (24)

where \( i_{L0} \) is the initial value of the inductor current.

Then, Fourier analysis can be performed on the inductor current signal, which results in:

\[ c_{m,il,vc} = \frac{j2}{T} \cdot \int_0^T i_L(t) \cdot e^{-j2\pi f_m t} \ dt \]

\[ = \frac{1}{j2\pi f_m} \cdot \frac{V_{in}}{L} \cdot \frac{j2}{T} \cdot e^{-j2\pi f_m T_{on}} \cdot \sum_{i=1}^{M} e^{-j2\pi f_m(i-1) T_s} \cdot \Delta T_{on(i)} \]  \quad (25)

where \( c_{m,il,vc} \) is its Fourier coefficient of perturbed frequency \( f_m \).
From the previous four steps, the Fourier coefficient \( c_{DF,v} \) of the perturbed signals can be found, where as shown by (26). By the same approach, Fourier coefficients can be applied to obtain the output voltage-to-inductor current relationship.

By substituting (21) into (25), the Fourier coefficient can be expressed as follows:

\[
\begin{align*}
\mathcal{E}_{m,1L,vc} &= \frac{f_s \cdot (1 - e^{-2\pi f_m T_s})}{(S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi f_m T_s}} \\
&\quad \cdot \frac{1}{2\pi \cdot f_m} \cdot \frac{V_{in}}{L} \cdot \hat{v}_1 \cdot e^{j\theta_1} \\
&\quad + \frac{f_s \cdot (1 - e^{-2\pi (f_m - f_s) T_s}) \cdot e^{-2\pi D}}{(S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi (f_m - f_s) T_s}} \\
&\quad \cdot \frac{1}{2\pi \cdot f_m} \cdot \frac{V_{in}}{L} \cdot \hat{v}_2 \cdot e^{j\theta_2} \\
&\quad + \frac{f_s \cdot (1 - e^{-2\pi (f_m + f_s) T_s}) \cdot e^{2\pi D}}{(S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi (f_m + f_s) T_s}} \\
&\quad \cdot \frac{1}{2\pi \cdot f_m} \cdot \frac{V_{in}}{L} \cdot \hat{v}_3 \cdot e^{j\theta_3} \\
&= f_s \cdot (1 - e^{-2\pi f_m T_s}) \\
&\quad \cdot \frac{(S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi f_m T_s}}{2\pi \cdot f_m} \\
&\quad \cdot \frac{1}{L} \cdot \frac{V_{in}}{L} \cdot \hat{v}_1 \cdot e^{j\theta_1} \\
&\quad + (S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi f_m T_s} \\
&\quad \cdot \frac{1}{2\pi \cdot f_m} \cdot \frac{V_{in}}{L} \cdot \hat{v}_2 \cdot e^{j\theta_2} \\
&\quad + (S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi f_m T_s} \\
&\quad \cdot \frac{1}{2\pi \cdot f_m} \cdot \frac{V_{in}}{L} \cdot \hat{v}_3 \cdot e^{j\theta_3}
\end{align*}
\]

As a matter of convenience, a general expression of \( DF_{vc} \) transfer functions is defined as (28):

\[
DF_{vc}(f) = \frac{f_s \cdot (1 - e^{-2\pi f T_s})}{(S_n + S_e - S_{vcfs}) + (S_f - S_e + S_{vcfs}) \cdot e^{-2\pi f T_s}} \\
\cdot \frac{1}{2\pi \cdot f_m} \cdot \frac{V_{in}}{L}
\]

Finally, three transfer functions of describing functions \( DF_{vc,11}, DF_{vc,12}, \) and \( DF_{vc,13} \) can be obtained, as shown in (29) to (31).

\[
\begin{align*}
DF_{vc,11} &= \frac{i_{L}(f_m)}{v_c(f_m) \cdot v_c(f_m - f_s)} = DF_{vc}(f_m) = 0 \\
DF_{vc,12} &= \frac{i_{L}(f_m)}{v_c(f_m - f_s) \cdot v_c(f_m + f_m)} = DF_{vc}(f_m) \cdot e^{-2\pi D} \\
DF_{vc,13} &= \frac{i_{L}(f_m)}{v_c(f_m + f_m) \cdot v_c(f_m - f_s)} = DF_{vc}(f_m) \cdot e^{2\pi D}
\end{align*}
\]

6) FINDING OUT OTHER \( DF_{vc} \) TRANSFER FUNCTIONS

Besides \( DF_{vc,11}, DF_{vc,12}, \) and \( DF_{vc,13}, \) there are still six \( DF_{vc} \) transfer functions to be found. Following the same approach deriving those three transfer functions \( DF_{vc} \) above, other transfer functions can also be obtained. The whole multi-frequency control voltage-to-inductor current relationship can be formulated as matrix form:

\[
\mathbf{i}_L = DF_{vc} \times \mathbf{v}_c
\]

where,

\[
DF_{vc} = \begin{bmatrix}
DF_{vc,11} & DF_{vc,12} & DF_{vc,13} \\
DF_{vc,21} & DF_{vc,22} & DF_{vc,23} \\
DF_{vc,31} & DF_{vc,32} & DF_{vc,33}
\end{bmatrix}
\]

\[
\mathbf{i}_L = \begin{bmatrix}
i_{L}(f_m) \\
i_{L}(f_m - f_s) \\
i_{L}(f_m + f_m)
\end{bmatrix}, \quad \mathbf{v}_c = \begin{bmatrix}
v_c(f_m) \\
v_c(f_m - f_s) \\
v_c(f_m + f_m)
\end{bmatrix}
\]

where the complete \( DF_{vc} \) matrix can be referred to as (A.1) in Appendix.

7) FINDING OUT OTHER \( DF_{vo} \) TRANSFER FUNCTIONS

Besides the influence of control voltage on inductor current, the effect of output voltage on inductor current should be also incorporated into the small-signal model. Therefore, the same approach for deriving \( DF_{vc} \) transfer functions can also be applied to obtain the output voltage-to-inductor current transfer function \( (DF_{vo}) \). The whole multi-frequency output
where, \[ \mathbf{i}_L = [i_L(f_m), i_L(f_m - f_s), i_L(f_m + f_s)] , \quad \mathbf{v}_o = [v_o(f_m), v_o(f_m - f_s), v_o(f_m + f_s)] \]

where the complete \( \mathbf{DF}_{vo} \) matrix can be referred to as (A.3) in Appendix.

**B. DERIVATION OF OVERALL LOOP GAIN TRANSFER FUNCTION**

The overall control block diagram can be obtained by substituting transfer functions in (A.1) and (A.3) into Fig. 5, as shown in Fig. 7. The overall control block diagram shown in Fig. 7 can be rearranged as a simple control block diagram, as shown in Fig. 8, to help derive the overall loop gain transfer function \( T \), where \( GM_{iL,11}, GM_{iL,12}, GM_{iL,13}, GM_{iL,21}, GM_{iL,22}, GM_{iL,23}, GM_{iL,31}, GM_{iL,32}, GM_{iL,33} \) are coefficients for matrix \( GM_{iL} \), which can be referred to as (A.6) in Appendix.

The definition of overall loop gain transfer function \( T \) is shown in (34):

\[
T(f_m) \equiv - \frac{v_o(f_m)}{v_p(f_m) + v_o(f_m)}
\]

Therefore, the overall loop gain transfer function can be expressed as (35):

\[
T(f_m) = - \frac{v_o(f_m)}{v_p(f_m) + v_o(f_m)} \frac{T_m(f_m)}{1 + T_m(f_m - f_s) + T_m(f_m - f_s + f_m)}
\]

where,

\[
T_m(f) = DF_{vc}(f) \cdot \frac{\Delta T_{FB,CL}}{1 + \frac{Z_o(\omega_f)}{Z_p(f)} \cdot Z_o(f)}
\]

where \( \Delta T_{FB,CL} \) can be referred to as (A.9) in Appendix and \( Z_p(f) \) can be referred to as (4).

**C. DISCUSSION ON SELECTION OF FREQUENCY COMPONENTS**

In this subsection, the reason for selecting four frequency components is briefly discussed. The reason of selecting four frequency components is proved in [22], that sideband components \( f_s - f_m, f_s + f_m \) and \( f_s \) should be presented simultaneously; otherwise an inevitable error would occur. For example, if \( f_s + f_m \) is neglected, significant error in both gain and phase can be found. On the other hand, if all other three frequency components are incorporated except for \( f_s \), a DC gain error would occur. A follow-up work [29] generalizes the four-frequency model by deriving equations for any \( n \)th ordered sideband components \( f_m - n \cdot f_s, n \cdot f_s + f_m \) and \( n \cdot f_s \) in CFVM controlled buck. And it further shows that \( f_m - n \cdot f_s, n \cdot f_s + f_m, \) and \( n \cdot f_s \) should be incorporated simultaneously to prevent the error mentioned before. Therefore, for a designer that would like to use high enough \( n \) to achieve a nearly error-free model, \( 1 + 3n \) components should be considered, where a four-frequency model has been enough for nowadays V-Core application.
TABLE 3. Circuit parameters/conditions for experimental verification.

| Parameter          | Value   |
|--------------------|---------|
| Input voltage $V_{in}$ | 19 V    |
| Output voltage $V_o$  | 3.1 V   |
| Load current $I_L$   | 3 A     |
| Switching frequency $f_s$ | 241.7 kHz |
| Inductor $L$         | 15 μH   |
| Output capacitor $C$ | 44.1 μF |
| Current sensing ratio $R_i$ | 111 m |
| External ramp slope $S_e$ | 0 V/s   |

FIGURE 9. Comparison of single-frequency model and the proposed four-frequency model when $f_{BW}$ of overall loop gain is selected: (a) $f_{BW} = 45$kHz, (b) $f_{BW} = 145$kHz.

IV. VALIDATION OF THE PROPOSED MODEL

In this section, both simulated results and experimental results will be presented to check the proposed theoretical model’s validity. The theoretical loop gain functions of the single-frequency, three-frequency and the proposed four-frequency models are plotted against the simulated or experimental results.

A. SIMULATED RESULTS

The circuit in Fig. 1 is simulated using the circuit simulation tool SIMPLIS for measuring overall loop gain $T$ [28]. The power-stage circuit parameters and working conditions listed in Table 1 are used. Designs of different control bandwidths are achieved by adjusting the circuit parameters of the compensator.

Fig. 9(a) shows the design results when narrower control bandwidth of 45kHz is selected, whose control bandwidth is about 12% of the switching frequency. All small-signal models agree well with the simulation result below control bandwidth. However, the requirement for the V-Core regulator is fairly stringent. Thus $f_{BW} / f_s$ ratio is usually higher than 12%.

Fig. 9(b) shows the design results when narrower control bandwidth of 145kHz is selected, whose control bandwidth is about 40% of the switching frequency. As can be seen, the proposed model agrees well with the simulation unto the switching frequency, while the errors of the single-frequency and three-frequency models are shown. There is about 41kHz error in control bandwidth, 16.8° of error in phase margin in single-frequency model, and a 2dB DC gain error in both single- and three-frequency models.

B. EXPERIMENTAL RESULTS

A commercially available evaluation board EV1586EN-00A, from Monolithic Power System, Inc., is selected to validate the accuracy of the proposed model, as shown in Fig. 10(a). Table 3 shows the circuit parameters and converter operating conditions. Gm-type compensation with a pole and a zero was used, resulting in loop gains with a minus-one slope crossing over at $f_{BW} = 23$kHz. As shown in Fig. 10(b), the proposed model fit well with the experimental results.
To further show the benefit of the proposed four-frequency model over the single-frequency and three-frequency models, two simulation cases will be used to prove that the proposed model can accurately predict the stability of a CFPCM controlled buck converter even when the single-frequency or three-frequency model fails. These verifications will only be shown by simulations. Assuming a designer would like to adjust the switching frequency $f_s$ to 70kHz. By using the single-frequency model, the designer suggests that he will get a stable loop gain with $f_{BW} = 25.5kHz$, which is around 36.5% of switching frequency, with a 26.6° phase margin. However, simulation in both time-domain and frequency-domain shows that the circuit is unstable, which are shown in Fig. 10(c) and (d), respectively. As shown by Fig. 10(d), the proposed four-frequency model predicts $f_{BW} = 42.7kHz$, which is about 61% of the switching frequency, and the stability phase margin that matches the result of simulation, showing that the loop is unstable.

Another example shown in Fig. 10(e) shows the case where the three-frequency model fails to correctly predict the stability of the converter. The converter in Table 3 is redesigned by tuning the compensator parameters such that a stable loop with $f_{BW} = 74.8kHz$, which is 31% of switching frequency, with around 50° phase margin is achieved. However, the three-frequency model predicts an unstable loop gain with $f_{BW} = 139.4kHz$. The proposed four-frequency model matches the simulation, showing that the loop is stable.

By these two examples, it can be concluded that the proposed four-frequency model surpasses both single-frequency and three-frequency models, where it can predict the loop stability more accurately while the single-frequency or three-frequency models fail.

V. CONCLUSION

A four-frequency small-signal model for CFPCM control is developed and experimentally verified. The non-linear effects of the current-mode modulator are modeled by considering four frequency components of the outer loop, including the modulating frequency component ($f_m$), the two sideband frequency components ($f_m - f_s$ and $f_s + f_m$), and the switching-frequency component ($f_s$). In the past, the switching-frequency ripple effect is usually neglected in modeling the current-mode modulator, but it is indeed crucial for the accuracy of the multi-frequency small-signal model. Therefore, it is taken into consideration in the proposed four-frequency small-signal model.

Compared to single-frequency and three-frequency small-signal models, the proposed four-frequency small-signal model is more accurate and can predict stability correctly, especially when the control bandwidth/switching frequency ratio is high. Despite the slightly high model complexity for the proposed model, it can be incorporated into a simulation program to render accurate predictions within seconds. In summary, this model provides a useful feedback circuit design tool for system designers for the application of novel V-Core regulators in future generations.

APPENDIX

\[
DF_{vc} = \text{diag}_{DF_{vc}} \times \begin{bmatrix}
1 & e^{-j2\pi D} & e^{2\pi D} \\
e^{-j2\pi D} & 1 & e^{4\pi D} \\
e^{-j4\pi D} & e^{2\pi D} & 1
\end{bmatrix} \tag{A.1}
\]

where, (A.2) and (A.3), as shown at the bottom of the page, where,

\[
\beta(f) \equiv \frac{R_i \cdot V_{in}}{f \cdot f \cdot L} \cdot \frac{f_s \cdot (1 - e^{-j2\pi f \cdot T_i})}{(S_n + S_e - S_{vofs}) + (S_f - S_e + S_{vofs}) \cdot e^{-j2\pi f \cdot T_i}}
\]

\[
S_{vofs} = S_{vofs} + \frac{R_i}{L} \cdot |v_o(f_s)| \cdot \sin(2\pi \cdot D + \theta_5)
\]

where $v_o(f_s)$ is shown in (7) and $\theta_5$ is shown in (13).

\[
\text{diag}_{GM_{IL}} = \text{diag}_{GM_{IL}} \times DF_{vc} \tag{A.6}
\]

where,

\[
\text{diag}_{DF_{vc}} = \begin{bmatrix}
DF_{vc}(f_m) & 0 & 0 \\
0 & DF_{vc}(f_m - f_s) & 0 \\
0 & 0 & DF_{vc}(f_s + f_m)
\end{bmatrix} \tag{A.2}
\]

\[
DF_{vo} = \frac{\beta(f_m) - 1}{j2\pi \cdot f_m \cdot L} \cdot \frac{\beta(f_m) \cdot e^{-j2\pi D}}{\beta(f_m - f_s) \cdot e^{2\pi D}} \\
\frac{\beta(f_m - f_s) \cdot e^{-j2\pi D}}{\beta(f_m - f_s) \cdot e^{2\pi D}} \cdot \frac{\beta(f_m) - 1}{j2\pi \cdot f_m \cdot L} \\
\frac{\beta(f_m) - 1}{j2\pi \cdot f_s \cdot L} \cdot \frac{\beta(f_m) \cdot e^{-j2\pi D}}{\beta(f_m - f_s) \cdot e^{2\pi D}} \\
\frac{\beta(f_m - f_s) \cdot e^{-j2\pi D}}{\beta(f_m - f_s) \cdot e^{2\pi D}} \cdot \frac{\beta(f_m) - 1}{j2\pi \cdot f_s \cdot L}
\]

\[
\text{diag}_{GM_{IL}} = \begin{bmatrix}
\Delta T_{FB,CL} & 0 & 0 \\
0 & \Delta T_{FB,CL} & 0 \\
0 & 0 & \Delta T_{FB,CL}
\end{bmatrix} \tag{A.7}
\]
\[ Z_o(f) = \frac{Z_p(f)}{\sqrt{2\pi} \cdot f \cdot L} \quad \text{(A.8)} \]

\[ \Delta T_{FB,CL} = \frac{1}{1 - T_{FB,CL}(f_m) - T_{FB,CL}(f_m - f_s) - T_{FB,CL}(f_s + f_m)} \quad \text{(A.9)} \]

\[ T_{FB,CL}(f) = \beta(f) \cdot \frac{Z_o(f)}{1 + Z_o(f)} \quad \text{(A.10)} \]

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