Wrinkling Patterns and Stress Analysis of Tensile Membrane with Rigid Elements

Peng Sun, Jin Huang *, Jiaying Zhang and Fanbo Meng

Abstract: Heterogeneous membrane structures with rigid elements are often used in flexible electronic and aerospace structures. In heterogeneous membrane structures under tension, the disturbance stress caused by the rigid element changes the stress distribution of the membrane, and it is difficult to calculate the stress distribution of the heterogeneous membrane structure using the traditional stress functions method. In this article, we propose a method for calculating the non-uniform stress field based on the Eshelby elastic inclusion theory, which states that tension membrane structures contain square rigid elements. The wrinkle distribution of the rigid element at different positions is predicted by a stress analysis, and the influence of the position and size of the rigid element on the wrinkle distribution of the membrane is studied by a finite-element simulation. The research results show that the wrinkle pattern of the stretched membrane can be controlled by changing the position of the rigid element to meet some special needs.

Keywords: heterogeneous membrane structures; non-uniform stress field; elastic inclusion theory; wrinkle patterns

1. Introduction

Membrane structures are widely used in the field of aerospace engineering due to their lightweight characteristics [1–3]. However, the bending stiffness of the membrane is low, and wrinkles appear under compressive load, which has a significant negative impact on the mechanical properties and stability of the membrane structure [4,5]. Therefore, the wrinkling problem of the membrane structure has become a major research focus.

Previous studies mostly focused on uniform membrane structures [6–8], but current membrane structures are usually heterogeneous, such as membrane antenna structures with rigid components [9,10] or flexible electronic devices with rigid circuits in aerospace engineering [11,12]. Yan et al. studied the wrinkle characteristics of thin films with local microstructures, such as square rigid elements and circular holes [13,14], but the calculation method of the non-uniform stress field they proposed is based on infinite thin plates. Li et al. studied the stretch wrinkle characteristics of graphene membranes with frozen zones by the finite-element method [15], but they did not study the influence mechanism of the frozen zone on the membrane wrinkling. Fleurent-Wilson et al. studied the influence of the actuator on the membrane surfaces on wrinkling and found that the actuator changes the wrinkling morphology of the membrane; however, they did not study the relationship between the position of the actuator and the wrinkling [10].

In this article, we propose a method for calculating the non-uniform stress field based on the Eshelby elastic inclusion theory and analyze the stress distribution of a membrane with four square rigid elements. In this method, the perturbation stress of the square rigid element on the membrane is approximately calculated by the Eshelby cylindrical core, and the concentrated stress at the corner of the square element is fitted by the finite-element
method. The compressive-stress region where the second principal stress on the membrane is less than zero at different positions of the periodic rigid element is obtained.

2. Non-Uniform Stress Field

To explain the change in the wrinkling pattern of the tension membrane with the position of the square rigid element, in this paper, we propose a calculation method for the non-uniform stress field based on the Eshelby elastic inclusion theory, whose heterogeneous membrane structure is shown in Figure 1. When the rigid element is bonded with the membrane, since the thickness of the membrane is much smaller than the thickness of the rigid element, the strain distribution of the membrane structure can be expressed as:

\[ \sigma_{ij} = \sigma_{ij}^0 + \sigma_{ij}' \]  

(1)

where \( \sigma_{ij}' \) is the perturbation stress caused by rigid elements; \( \sigma_{ij}^0 \) is the stress distribution of the membrane without rigid elements. In order to obtain the stress distribution of a uniform membrane, its stress function can be expressed as:

\[ \Phi = A \rho \sin \theta \]  

(2)

Due to the stress concentration in the corners of the square rigid element, we introduce concentrated stress after finite-element analysis to correct the disturbance strain \( \varepsilon_{\psi\zeta} \) caused by cylindrical inclusions, and then obtain the stress distribution of the membrane as:

\[ \sigma_{ij} = \sigma_{ij}^0 + \sigma_{ij}' + \sigma_{ij}'' \]  

(29)

where \( \sigma_{ij}'' \) is the concentrated stress, can be expressed as:

\[ \sigma_{ij}'' = \frac{a_5}{\pi \eta} |x| - |y| \]  

(30)

In order to obtain the stress distribution of the membrane structures with rigid elements, we first deduced the stress function of the uniform membrane structure using the inverse method and solved the stress distribution \( \sigma_{ij}^0 \) of the uniform thin film by the stress function and the stress superposition principle. Next, the Eshelby tensor \( S_{ij\phi\kappa} \) of the membrane and the Eshelby tensor \( D_{ij\phi\kappa} \) of the rigid element were calculated according to the Eshelby elastic inclusion theory. The uniform membrane-stress distribution \( \sigma_{ij}^0 \) and the Eshelby tensor \( S_{ij\phi\kappa} \) of the thin film were substituted for the strain equilibrium equation to obtain the eigenstrain of the rigid element \( \varepsilon_{ij}^* \). According to the Eshelby tensor \( D_{ij\phi\kappa} \) and the eigenstrain \( \varepsilon_{ij}^* \) of the rigid element, the perturbation stress of the rigid element to the membrane structure was calculated. However, the elastic inclusion method of this paper is based on the cylindrical inclusion structure, and there was a large stress concentration at the apex of the square rigid element. Therefore, we calculated the stress distribution of the membrane structure containing the circular rigid element and the square rigid element, respectively, through finite-element simulation, and fitted the stress concentration factor at the apex of the rigid element by comparing the stress distribution of the two structures at the apex of the rigid element. Finally, the stress distribution of the membrane structure with square rigid elements was obtained by stress superposition.

Figure 1. Schematic diagram of heterogeneous film structure.

Through the compatibility equation \( \nabla^4 \Phi = 0 \), the stress component in the polar coordinate system can be obtained as:

\[
\begin{cases}
\sigma_\rho = \frac{2}{\rho} A \rho \sin \theta \\
\sigma_\theta = 0 \\
\tau_{\theta\rho} = 0
\end{cases}
\]  

(3)

Since there is no surface force on the membrane boundary, the stress boundary condition of the membrane can be expressed as:

\[ (\sigma_\theta)_{\theta=\pm \alpha, \rho\neq0} = 0, (\tau_\theta)_{\theta=\pm \alpha, \rho\neq0} = 0 \]  

(4)
From the stress boundary conditions, the equilibrium equation can be obtained as:

$$\int_{-\infty}^{\infty} \sigma_\rho d\theta \cos \theta = T$$

(5)

By substituting Equation (3) into Equation (5), and after integration, we can obtain:

$$A = \frac{2T \cos \theta}{\pi + 2}$$

(6)

Considering the thickness \(h_m\) of the membrane, the radial stress of the uniform membrane in the polar coordinate system can be obtained by substituting Equation (6) into Equation (3):

$$\sigma_\rho = \frac{4T \cos \theta}{\rho h_m(\pi + 2)}$$

(7)

According to the transformation formula from polar coordinates to rectangular coordinates, the radial stress given by Formula (6) is decomposed into the \(x\) and \(y\) directions:

$$\begin{align*}
\sigma_x &= \sigma_\rho \sin^2 \theta + \sigma_\rho \cos^2 \theta - 2\tau_{\theta \rho} \sin \theta \cos \theta \\
\sigma_y &= \sigma_\rho \cos^2 \theta + \sigma_\rho \sin^2 \theta + 2\tau_{\theta \rho} \sin \theta \cos \theta \\
\tau_{xy} &= (\sigma_\rho - \sigma_\theta) \sin \theta \cos \theta + \tau_{\theta \rho} (\sin^2 \theta - \cos^2 \theta)
\end{align*}$$

(8)

where \(\theta\) and \(\rho\) can be expressed in Cartesian coordinates as:

$$\cos \theta = \frac{R - y}{\rho}, \quad \sin \theta = \frac{x}{\rho}$$

(9)

Substituting Equations (7) and (9) into Equation (8), we can obtain:

$$\begin{align*}
\sigma_x &= \frac{4T x^2 (R - y)}{\rho^2 h_m (\pi + 2)} \\
\sigma_y &= \frac{4T (R - y)^3}{\rho^2 h_m (\pi + 2)} \\
\tau_{xy} &= \frac{4T x (R - y)^2}{\rho^2 h_m (\pi + 2)}
\end{align*}$$

(10)

The boundary stress at the top of the membrane due to the edge effect can be expressed as [16]:

$$T_c = -\frac{4T}{2\pi R}$$

(11)

According to the principle of stress superposition, the stress distribution \(\sigma^0_{ij}\) of the membrane can be obtained by superimposing the tensile force at the four vertices of the membrane and the boundary stress:

$$\begin{align*}
\sigma^0_{ij} &= -\frac{4T}{2\pi R} + \sum_{i=1}^{i=4} \frac{4T (x \cos \theta - y \sin \theta)^2 [R - (x \cos \theta - y \sin \theta)]}{\rho^2 h_m (\pi + 2)} \\
\sigma^0_{ij} &= -\frac{4T}{2\pi R} + \sum_{i=1}^{i=4} \frac{4T (R - (x \cos \theta - y \sin \theta))^3}{\rho^2 h_m (\pi + 2)} \\
\tau_{xy} &= -\frac{4T}{2\pi R} + \sum_{i=1}^{i=4} \frac{4T (x \cos \theta - y \sin \theta) (R - (x \cos \theta - y \sin \theta))^2}{\rho^2 h_m (\pi + 2)}
\end{align*}$$

(12)

where \(T\) is the tension, \(R\) is the radius of the circumscribed circle of the membrane, and \(h_m\) is the thickness of the film. According to the Eshelby elastic inclusion method, the stress field within rigid element can be expressed as:

$$C_{ijkl}^1 (\epsilon^0_{ij} + S_{ijkl} \epsilon^1_{ij}) = C_{ijkl}^1 (\epsilon^0_{ij} + S_{ijkl} \epsilon^1_{ij} - \epsilon^1_{ij})$$

(13)
where \( C_{ijkl} \) and \( C_{ijkl}^1 \) are the elastic stiffness matrices of the membrane and the rigid element, respectively. \( \varepsilon_0'_{ij} \) is the strain field of the membrane without rigid components, which can be obtained by Hooke's law from Equation (12). Here, \( \varepsilon'_ij \) is the disturbance strain caused by the rigid element. The disturbance strain of the rigid element is regarded as caused by the eigenstrain \( \varepsilon_{kl}^* \) generated in the inclusion, where \( \varepsilon'_ij \) can be expressed as:

\[
\varepsilon'_ij = \begin{pmatrix} S_{ijkl} \varepsilon_{kl}^* \\ D_{ijkl} \varepsilon_{kl}^* \end{pmatrix}
\]

(14)

\( S_{ijkl} \) is the Eshelby tensor of the rigid element, which can be obtained from the Somigliana equation. The Eshelby tensor in a cylindrical inclusion under plane stress can be calculated as [17]:

\[
S_{ijkl}(\eta) = \frac{1}{8\pi(1-\nu)} \left\{ \delta_{ij}\delta_{kl}[2\nu I_i(\eta) - I_i(\eta) + a^2 I_k(\eta)] + \left[ \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} \right][a^2 I_{ij}(\eta) - I_{ij}(\eta) + (1-\nu)(I_k(\eta) + I_m(\eta))] \right\}
\]

(15)

where \( \nu \) is the Poisson’s ratio of the membrane, \( \delta_{ij} \) is the unit second-order tensor, and \( I(\eta) \) is the ellipsoid integral, which can be expressed as:

\[
I_i(\eta) = \frac{2\pi a_1 a_2 a_3}{\eta} \int_\eta^\infty \frac{ds}{(a_i^2 + s)(a_j^2 + s)(a_k^2 + s)}
\]

(16)

\[
I_{ij}(\eta) = \frac{2\pi a_1 a_2 a_3}{\eta} \int_\eta^\infty \frac{ds}{(a_i^2 + s)(a_j^2 + s)(a_k^2 + s)(a_l^2 + s)}
\]

(17)

where \( \eta \) is the largest positive root of the following virtual ellipsoid equation:

\[
\frac{x^2}{a_1^2 + \eta} + \frac{y^2}{a_2^2 + \eta} + \frac{z^2}{a_3^2 + \eta} = 1
\]

(18)

Simplifying the elliptical inclusions into cylindrical inclusions, if we let \( a_1 = a_2 = a \) and \( a_3 \to \infty \), we can obtain:

\[
\eta = r^2 - a^2
\]

(19)

Integrating the ellipsoid integral Equations (16) and (17), and substituting them into Equation (15), the Eshelby tensor \( S_{ijkl} \) can be obtained:

\[
S_{1111}(r_n) = S_{2222}(r_n) = \frac{1}{8(1-\nu)} \left[ \frac{3a^4}{r_n^4} + (1-2\nu) \frac{2a^2}{r_n^2} \right]
\]

(20)

\[
S_{1122}(r_n) = S_{2211}(r_n) = \frac{1}{8(1-\nu)} \left[ \frac{a^4}{r_n^4} - (1-2\nu) \frac{2a^2}{r_n^2} \right]
\]

(21)

\[
S_{1212}(r_n) = \frac{1}{2(1-\nu)} \left[ \frac{a^4}{r_n^2} + (1-2\nu) \frac{2a^2}{r_n^2} \right]
\]

(22)

When \( r = a \), by substituting into Equations (11)–(13), the eigenstrain \( \varepsilon_{ij}^* \) of the rigid element can be obtained.
where $I_{ij}(\eta)$, $I_{ij,k}(\eta)$, and $I_{ijkl}(\eta)$ can be obtained by taking the partial derivatives of Equations (6) and (7) with respect to $x_n$ and $y_n$ respectively, and, substituting them into Equation (23), the Eshelby tensor $D_{ijkl}$ can be obtained:

$$D_{ijkl}(r_n) = S_{ijkl}(r_n) + \frac{1}{2(1 - \nu)} \left\{(1 - 2\nu)x_n^2 + y_n^2 \right\} \frac{a^2}{r_n^3} - \left\{y_n^2 + \frac{1}{\alpha^2}x_n^2y_n^2 \right\} \frac{a^4}{r_n^6} + \frac{6a^4}{r_n^6} \delta_{ij} \delta_{kl}$$

(24)

$$D_{1111}(r_n) = S_{1111}(r_n) + \frac{1}{1 - \nu} \left\{(1 + \nu) \left[ \frac{4}{r_n^6} x_n^2 - \frac{3a^2}{r_n^6} x_n^2 y_n^2 - \frac{2a^2}{r_n^6} x_n^4 + \frac{3a^4}{r_n^8} \right] \right\}$$

(25)

$$D_{1122}(r_n) = S_{1122}(r_n) + \frac{1}{2(1 - \nu)} \left\{(1 - 2\nu)x_n^2 + y_n^2 \right\} \frac{a^2}{r_n^3} - \left\{y_n^2 + \frac{1}{\alpha^2}x_n^2y_n^2 \right\} \frac{a^4}{r_n^6} + \frac{6a^4}{r_n^6} \delta_{ij} \delta_{kl}$$

(26)

$$D_{2211}(r_n) = S_{2211}(r_n) + \frac{1}{2(1 - \nu)} \left\{(1 - 2\nu)y_n^2 + x_n^2 \right\} \frac{a^2}{r_n^3} - \left\{x_n^2 + \frac{1}{\alpha^2}x_n^2y_n^2 \right\} \frac{a^4}{r_n^6} + \frac{6a^4}{r_n^6} \delta_{ij} \delta_{kl}$$

(27)

$$D_{2222}(r_n) = S_{2222}(r_n) + \frac{1}{1 - \nu} \left\{(1 + \nu) \left[ \frac{4}{r_n^6} y_n^2 - \frac{3a^2}{r_n^6} y_n^2 x_n^2 - \frac{2a^2}{r_n^6} y_n^4 + \frac{3a^4}{r_n^8} \right] \right\}$$

(28)

When $r_n > a$, by substituting the eigenstrain $\varepsilon_{ij}^\ast$ and Equations (24)–(28) into the Equation (14), we can obtain the disturbance strain $\varepsilon_{ij}'$ caused by the cylindrical inclusion. Due to the stress concentration in the corners of the square rigid element, we introduce concentrated stress after finite-element analysis to correct the disturbance strain $\varepsilon_{ij}'$ caused by cylindrical inclusions, and then obtain the stress distribution of the membrane as:

$$\sigma_{ij} = \sigma_{ij}^0 + \sigma_{ij}' + \sigma_{ij}^\ast$$

(29)

where $\sigma_{ij}^\ast$ is the concentrated stress, can be expressed as:

$$\sigma_{ij}^\ast = \frac{\sigma^2}{\pi \eta} \frac{|x|}{r} \sigma_{ij}'$$

(30)

In order to obtain the stress distribution of the membrane structures with rigid elements, we first deduced the stress function of the uniform membrane structure using the inverse method and solved the stress distribution $\sigma_{ij}^0$ of the uniform thin film by the stress function and the stress superposition principle. Next, the Eshelby tensor $S_{ijkl}$ of the membrane and the Eshelby tensor $D_{ijkl}$ of the rigid element were calculated according to the Eshelby elastic inclusion theory. The uniform membrane-stress distribution $\sigma_{ij}^0$ and the Eshelby tensor $S_{ijkl}$ of the thin film were substituted for the strain equilibrium equation to obtain the eigenstrain of the rigid element $\varepsilon_{ij}^\ast$. According to the Eshelby tensor $D_{ijkl}$ and the eigenstrain $\varepsilon_{ij}^\ast$ of the rigid element, the perturbation stress of the rigid element to the membrane structure was calculated. However, the elastic inclusion method of this paper is based on the cylindrical inclusion structure, and there was a large stress concentration at the apex of the square rigid element. Therefore, we calculated the stress distribution of the membrane structure containing the circular rigid element and the square rigid element, respectively, through finite-element simulation, and fitted the stress concentration factor at the apex of the rigid element by comparing the stress distribution of the two structures.
at the apex of the rigid element. Finally, the stress distribution of the membrane structure with square rigid elements was obtained by stress superposition.

The above theory was realized by MATLAB programming, and the calculation results were compared with the calculation results of finite-element analysis. The static simulation of the non-uniform membrane structure shown in Figure 1 was carried out by using ABAQUS finite-element analysis software. The thin-shell element S4R with four nodes and six degrees of freedom was used for the membrane structure, and the hexahedral element C3D8R with eight nodes and three degrees of freedom was used for the rigid element. The rigid element and the membrane were connected by binding constraints. The material parameters of the membrane and rigid elements are shown in Table 1. $R_c$ is the distance between the boundary of the membrane and the boundary of the rigid element. When $R_c = 40$ mm, the analytical solution and finite-element results of the membrane structure with four rigid elements are shown in Figure 2. The analytical solution to the first principal stress of the membrane is in good agreement with the finite-element results. The membrane structure used in this paper is a polyimide membrane with a side length of 300 mm, and the rigid element is a square aluminum sheet with a side length of 10 mm. The material parameters of the membrane and the rigid element are shown in Table 1.

Table 1. Material parameters of membrane and rigid elements.

| Parameter          | Membrane | Rigid Element |
|--------------------|----------|---------------|
| Thickness (mm)     | 0.025    | 1             |
| Poisson’s ratio    | 0.34     | 0.34          |
| Young’s modulus (Mpa) | 2500     | 71,000        |

Figure 2. The first principal stress of a film with four rigid elements: (a) analytical solution; (b) finite-element solution.

When the rigid elements are in different positions, the superposition of the tensile stress of the membrane and the disturbance stress of the rigid elements changes the compressive-stress distribution area of the membrane. The compressive-stress distribution area of the membrane is the area of the first principal stress $\sigma_1 > 0$ and the second principal stress $\sigma_2 \leq 0$ of the membrane. Using the non-uniform stress-field calculation method mentioned above, the stress distribution of the membrane structures containing four rigid elements when the rigid element position $R_c$ is 30 mm, 40 mm, 50 mm, and 60 mm were studied. The results are shown in Figure 3. It can be seen from the figure that the compressive-stress distribution area of the membrane changes with the change in the position of the rigid element.
3. Experimental Observation

According to the theory of tension field, we can predict that when the position $R_c$ of the rigid element is 30 mm and 40 mm, the membrane will have two different wrinkle patterns of the upper side of the rigid element and on the left and right sides. When the position $R_c$ of the rigid element is 50 mm and 60 mm, we can predict that the membrane will only have a wrinkle pattern located on the upper side of the rigid element.

To verify the results of our prediction of the wrinkle mode of the rigid elements in different positions, the wrinkle pattern of the polyimide membrane structure of four rigid elements was studied through experiments. When the rigid element position $R_c$ was 30 mm, 40 mm, 50 mm, and 60 mm, we applied 15 N of tension in the four corners of the membrane, and captured this with the three-dimensional scanner OKIO-B-1000. The results are shown in Figure 4. It can be seen from Figure 4 that when the position $R_c$ of the rigid element is 30 mm and 40 mm, the wrinkling patterns of the upper and lower sides of the rigid element and the left and right sides with the rigid element appear on the membrane at the same time. When the position $R_c$ of the rigid element is 50 mm and 60 mm, the wrinkle pattern of the upper side of the rigid element appears on the membrane, but the wrinkles on the left and right sides are no longer visible. The experimental results shown in Figure 4 are in good agreement with the wrinkle pattern we predicted, but when the position $R_c$ of the rigid element is 30 mm and 40 mm, wrinkles also appear on the underside of the rigid element. This is because the rigid element is bonded with the membrane through the adhesive. When the adhesive is cured, a certain bonding stress is generated in the rigid element, resulting in a wrinkle pattern of the underside of the rigid element.
The eigenvalue buckling analysis was performed on the membrane to obtain the critical positions. In mode 2, the wrinkles in mode 2 are distributed over the upper side of the rigid element, which is similar to the wrinkle distribution of the uniform membrane. Mode 3 is the wrinkle distribution of the uniform membrane structure.

4. Discussion

In this section, we use the finite-element method to analyze the eigenvalue buckling of the tension membrane with four square rigid elements, study the influence of the position of the rigid element on the critical buckling tension in the tension membrane, and analyze the influence mechanism of the position of the rigid element on the wrinkle pattern of the tension membrane. We selected a rigid element with a side length of 10 mm and a thickness of 1 mm and changed the position of the rigid element from 10 mm to 60 mm. The eigenvalue buckling analysis was performed on the membrane to obtain the critical buckling tension and buckling mode of the membrane. The results are shown in Figure 5.

As can be seen in Figure 5, the tensile membrane containing rigid elements has two distinct buckling modes. In mode 1, there are both wrinkling that passes through the rigid element and wrinkles that are distributed over both sides of the rigid element. The wrinkles in mode 2 are distributed over the upper side of the rigid element, which is similar to the wrinkle distribution of the uniform membrane. Mode 3 is the wrinkle distribution of the uniform membrane structure. As can be seen from Figure 5, the critical buckling tension in mode 1 is smaller than that of mode 3 when the rigid element is in the region of $R_c \leq 45$ mm, which indicates that the rigid element reduces the stability of the membrane.
in the region close to the membrane boundary. The critical buckling tension in mode 1 increases from the increase in position \( R_c \) of the rigid element, which means that under the same tension, the wrinkle amplitude decreases with the increase in position \( R_c \) of the rigid element. The buckling mode of the membrane changes from mode 1 to mode 2 when the position of the rigid element \( R_c \geq 50 \text{ mm} \), the wrinkles on both sides with the rigid element disappear, and the wrinkle distribution of the membrane is similar to that of mode 3. Therefore, the rigid element has no effect on the wrinkle distribution of the membrane in this area. However, when the position of the rigid element \( R_c \geq 50 \), the critical buckling tension in mode 2 is greater than that of mode 3, and the introduction of the rigid element increases the stability of the membrane.

It can be seen from the above analysis that with the change in the position of the rigid element, the superposition of the rigid element and the tensile stress places the membrane in different stress states, which leads to the transformation of the instability mechanism of the membrane, thus showing different types of wrinkle pattern. Therefore, the size of the rigid element also affects the stress distribution and buckling mode of the membrane. We can change the wrinkle mode and buckling mode of the membrane by designing the position and size of the rigid element. According to the eigenvalue buckling analysis, the relationship between the critical buckling mode of the membrane and the size and position of the rigid element is shown in Figure 6. When the rigid element is in the blue area, there are both wrinkles passing through the rigid element and wrinkles distributed over both sides of the rigid element. When the rigid element is in the black line and the red area, the membrane has a wrinkle distribution similar to that of the uniform membrane, and the stability of the heterogeneous membrane in this region is greater than that of the uniform membrane.

![Figure 6. Relationship between critical buckling mode of membrane and size and position of rigid element.](image)

This paper presents the wrinkle patterns of a tension membrane with rigid elements and describes how to control the wrinkle pattern by designing the size and position of the rigid elements, an effect that can be used for membrane and sheet structures with complex and ordered sensors and optical elements. At the same time, when using the membrane structure, a smooth and flat surface is required. We can change the size and position of the rigid element according to the stress analysis method proposed in this paper, so as to reduce and eliminate the wrinkles.
5. Conclusions

We proposed a method for calculating the stress field of heterogeneous membranes based on the Eshelby elastic inclusion theory, and the method was used to predict the wrinkle distribution of rigid elements at different positions. The influence of the position and size of the rigid element on the membrane wrinkle distribution was investigated by a finite-element simulation. When the rigid element is close to the center of the membrane, there are both wrinkles on the rigid element and wrinkle modes distributed over both sides of the rigid element. When the rigid element is close to the center of the membrane, there are both wrinkles passing through the rigid element and wrinkles distributed over both sides of the rigid element. The results show that we can bond rigid elements on the surfaces of membranes or customize the patterns of membranes through three-dimensional printing technology, combined with Eshelby elastic inclusion theory, to change and reduce the compressive-stress region of the membrane, to change the wrinkling mode of the membrane, and to suppress the wrinkling of the membrane.

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