Shape Optimization and Analysis of a Low-resistance Mast

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Abstract. Aiming at the problem that the airfoil of WGN affects the motion resistance and noise, a set of general WEG airfoil design and optimization method is explored. First, the parameter design based on computational fluid dynamics computing method of WGN appearance is done. Then, the WGN appearance hydrodynamic characteristics was calculated with the airfoil design software XFOIL, and finally, the iterative optimization of the initial design of WGN appearance is done by the genetic algorithm, finally the fold down type SJZZWGN curve shape with low resistance and low noise is obtained to satisfy the mechanical properties. Finally, CFD software is used to analyse the optimization results and verify the rationality of the optimization results.

1. Introduction
The shape parameters of folded SJZZ WGN are the important factors that determine the resistance, flow around noise and structural mechanical vibration of WGN. How to construct WGN airfoil is the key problem in the design process of folded SJZZ. WGN airfoil is constructed based on CST method \cite{1-2}. The parameterized characterization method can not only ensure high contour curve fitting accuracy, but also facilitate the parameter optimization design of the contour. Because all the loads in the kinematic process of the inverted SJZZ mechanism act on WGN, the design of WGN should consider both the hydrodynamic characteristics and its own mechanical characteristics.

The design process of WGN airfoil is divided into three stages: optimization design, verification and analysis. In the optimization design stage, the computational fluid dynamics method and the airfoil design software XFOIL \cite{3} are used as the calculation tool for the hydrodynamic characteristics of WGN shape. Using genetic algorithm toolbox as optimization tool, the shape curve of SJZZWGN with low resistance and low noise was obtained and verified. In the verification phase, the CFD software FLUENT and the acoustic analysis software LMS Virtual Lab were used to compare and analyze the optimized results with the original design, which verified the rationality of the optimized results. In the analysis stage, the stiffness, strength, modal and other mechanical properties of WGN were analyzed by finite element method, and its vibration and noise characteristics were evaluated. The design optimization method and the design flow are universal and can be applied to other types of WGN design and optimization.
2. Shape Optimization of the Mast

2.1. Mathematical Model of Shape Optimization

The entire optimization process is realized on the MATLAB platform. By integrating the CST airfoil geometric design program written by MATLAB, XFOIL airfoil aerodynamic parameter calculation program (resistance coefficient), and optimization design toolbox, the automatic iterative optimization design of airfoil is realized. The optimization process is shown in Fig. 2-1. The main steps and specific parameters are set as follows.

The optimization model is as follows:

\[
\begin{align*}
\min & \quad dC_u(h_u) \\
\text{st} & \quad \zeta(\psi=0.5) > 425/2000 \\
& \quad \zeta(\psi=0.5) < 460/2000
\end{align*}
\]

Where, \( h_u \) is the design variable of the upper and lower surfaces of the airfoil, \( dC_u \) is resistance coefficient. \( \psi \) is the dimensionless X-axis coordinate of the airfoil, \( \zeta \) is the dimensionless Z-axis coordinate of the airfoil. The objective function is the minimum resistance coefficient and the constraint condition is the geometric maximum and minimum design space determined by the assembly relation of the mechanism.

2.2. Optimization Process

The main process of using CFD technology and multi-objective optimization method to optimize the design of folded SJZZWGN is as follows:

1. CATIA software was used to build the 3D model of WGN and extract the cross-sectional curve.
2. The extracted cross-section curve was imported into MATLAB in the form of data points, and NURBS curve fitting was carried out to obtain the control point coordinate parameters (70 control points) of the initial design contour curve, and realize the parameterization of each control point coordinate.
3. The NURBS parametric curve is taken as the contour curve, and the foil analysis software XFOIL is adopted to calculate the hydrodynamic parameters of WGN airfoil. In this stage, two important performance parameters, namely, fluid resistance and the position of the lowest pressure point are mainly concerned.
4. The multi-objective genetic algorithm tool in the MATLAB optimization toolbox was used to optimize and iterate the control point parameters of NURBS curves until the resistance change rate was less than 2%, and the optimization process was completed.
5. FLUENT flow-field model of original shape and optimized shape of WGN was established to verify the optimized design results. The resistance parameters, flow noise and other performance parameters are analyzed.

In summary, the main steps can be briefly expressed as shown in Fig. 1.
2.3. **Parameterization of WGN shape curve**

Generally, there are three kinds of parametric methods for airfoil design: polynomial fitting, spline curve fitting and type function fitting. [4-5] parametric airfoil (PARSEC) method is a commonly used polynomial fitting method. The fitting methods of spline curve include B-spline, non-uniform rational spline (NURBS) and Bezier curve. Model function fitting methods include e model function method and Wagner model function method. The CST method adopted in this paper is the one proposed by Kulfan et al. [6], which uses 1category function and one shape function to describe the airfoil shape. This method can accurately describe the complex aerodynamic shape with a set of fewer parameters, and has high fitting accuracy and simple and intuitive fitting. Due to the small number of parameters, it is suitable for the optimal design of airfoil.

The expression of CST parameterization to describe airfoil is:

$$\zeta(\psi) = C(\psi)S(\psi) + \psi \xi$$

(2)

Where: $\psi = x / c$ is the dimensionless X-axis coordinate of the airfoil; $\xi = z / c$ is the dimensionless z-axis coordinate of the airfoil; $c$ is the chord length of the airfoil; $\zeta_T$ is the coordinate of the trailing edge relative to the z-axis; $C(\psi)$ is a shape function, denoted by

$$C(\psi) = (\psi)^{\nu_1}(1-\psi)^{\nu_2}$$

(3)

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**Fig. 1** Optimization design flow chart of WGN shape curve
when \( N_1, N_2 \) is taken for different values, different geometry categories can be defined. This article take \( N_1 = 0.5, N_2 = 1 \), The airfoil shapes of the leading edge of the circle and the trailing edge of the sharp edge are defined. \( S(\psi) \) is the shape function, denoted by

\[
S(\psi) = \frac{\xi(\psi) - \psi\zeta}{\sqrt{\psi(1-\psi)}} = \sum_{i=0}^{n} (b_i \psi_i)
\]  

(4)

\( S(\psi) \) can be expressed by different method, such as Bernstein Polynomial and B spline basis functions. The weight of Bernstein polynomials is used as \( S(\psi) \):

\[
\begin{cases}
S(\psi) = \sum_{i=0}^{n} b_i \left[ K_i \left( (1-\psi)^{n-i} \right) \right] \\
K_i = \frac{n!}{i!(n-i)!}
\end{cases}
\]

(5)

where: \( K_i \) is the combinatorial number of components of a shape function; \( n \) is the order of Bernstein polynomials; Shape function coefficient \( b_i \) can be used as a design variable, The initial value can be obtained by fitting the original airfoil with the least square method.

In a certain range, the use of higher-order Bernstein polynomials to define shape functions can effectively improve the accuracy of the CST parameterization method in the representation of geometric shapes, but too high order (more than order 10) polynomial order will make the parameterization process ill-conditioned. [7]

The 4th-order Bernstein polynomials are adopted, and the 4shape function coefficients of the upper and lower airfoil surfaces are \( b_{u_i} \) and \( b_{l_i} \) \((i = 1 \sim 4)\) respectively. The upper and lower surfaces of the airfoil are symmetrical, so the total design variable is set as 4. As shown in Fig. 2, the leading edge of the envelope curve of the initial design of WGN was oval, and the trailing edge was triangle. The upper and lower surfaces of the curve were fitted by the CST method with 71 interpolation points. The initial value of the shape function coefficient is:

\[
\begin{align*}
&b_{u_1} = 0.502230738580721; \ b_{u_2} = 0.647127407404529; \\
&b_{u_3} = 0.955300686359384; \ b_{u_4} = 0.222999333399693
\end{align*}
\]

Fig. 2 Initial shape of WGN and its CST fitting curve
2.4. Optimize solutions and results

GA method in MATLAB optimization toolbox was adopted to obtain the optimal shape curve of WGN as shown in Fig. 3 after 671 iterations. The CST parameters of the optimized airfoil were as follows:

\[ b_{u1} = 0.449114003398956; \quad b_{u2} = 0.5129473980787304; \]
\[ b_{u3} = 0.900212576325960; \quad b_{u4} = 0.201616460293874 \]

As can be seen from the figure, the leading-edge radius of the shape curve of WGN was slightly reduced after optimization, and the upper airfoil presented a smooth transition. As the chord length increased, the trailing edge showed a contraction of first fast then slow. Fig. 4 shows the change curve of resistance coefficient with the number of optimization iterations. It can be seen from the figure that the damping coefficient of WGN shape is improved greatly after optimization.

3. Analysis and verification of optimization results

In order to further verify the correctness of shape optimization design result of WGN quantitatively. The CFD "virtual experiment" technology was used to simulate the 2D steady turbulent flow outside WGN, so as to reveal its internal flow rules and predict the fluid resistance characteristics of WGN. The grid model of mast outflow field after optimization is shown in Fig. 5 and Fig. 6.
Inlet boundary condition: use velocity inlet. Assume the distribution of pressure on the inlet section is uniform, the axial velocity at the inlet is determined according to the law of mass conservation and the no rotation assumption.

Outlet boundary conditions: pressure outlet. For the static pressure at the designated outlet, when there is reflux, the convergence result is better by using the pressure outlet boundary condition instead of the free outflow boundary condition.

Wall conditions: For flow near the wall, the $R_e$ value is low, the turbulence development is not sufficient, and the pulsation effect of turbulence is not as big as that of molecular viscosity. Therefore, the $k - \varepsilon$ model cannot be used in this area, the wall function method [8] can be used instead.

There is no slip condition on the solid wall, that is, the relative velocity $w = 0$, The pressure is taken as the second boundary condition, i.e. $\partial p / \partial n = 0$. The wall function boundary condition is adopted for the turbulent wall condition. In the region close to the solid wall, the wall forces the flow to produce a large velocity gradient, which is suitable to adopt the full development of turbulence $k - \varepsilon$ model modified in this area. Suppose the distance between the near wall $P$ to the wall is $y^+$, the values of the velocity $u_p$ of point $P$ $u_p$ and the dissipation of turbulent kinetic energy $\varepsilon_p$ are determined by the following wall function:

$$\frac{u_p}{u_*} = \frac{1}{\kappa} \ln \left( E y_p^+ \right)$$
\[ k_p = \frac{u^2}{\sqrt{C_p}} \]

\[ \varepsilon_p = \frac{u^3}{\kappa y_p} \]

\[ y_p^+ = \frac{\mu u_p y_p}{\mu} = \frac{\rho C^\mu_p k_p^{1/4} y_p}{\mu} \]

Wall friction factor \( u_* = \sqrt{\tau_w/\rho} \), \( \tau_w \) is the wall shear stress;

Where, the constant values of \( E \) and \( k \) are 9.011 and 0.419 separately.

Fig. 7 is the pressure distribution diagram and flow diagram of optimized WGN when the speed is 6Kn. It can be seen from the figure that:

1. The minimum pressure point of the optimized flow field is now in the back of the airfoil, so that more wall boundary layer to maintain laminar flow characteristics, can not only reduce the resistance, but also greatly reduce the laminar flow state fluctuations of flow field pressure, is conducive to reducing the flow noise.

2. The separation of WGN external flow field boundary layer is late after optimization, the pressure in the tail is large, the resistance is small, and there is no large eddy current, which is conducive to noise reduction.

Through FLUENT fluid dynamics simulation analysis, the optimized resistance coefficient is 0.2775, which is the same as the resistance coefficient obtained by XFOIL calculation, proving that the calculation results of both are highly reliable.

Fig. 7 The pressure distribution and flow diagram of optimized WGN at a speed of 6Kn (Cd = 0.2775)

4. Conclusion
This digital simulation method to optimize the shape of WGN is adopted, and the vibration characteristics of WGN structure is analyzed. The conclusions are as follows:
(1) The optimized WGN shell profile is a laminar flow airfoil, which can reduce both the drag and flow noise.
(2) The WEN shape parameter design and optimization method and process explored are universal and applicable to the design and optimization of another WGN.

References
[1] Gao Yong, Zhu Feixiang, Li Bing, et al. Application of improved CST method in airfoil optimization design [J]. Journal of Naval Aeronautical Engineering college, 2017(05):10-14.
[2] Bu Yuepeng, Song Wenping, Han Zhonghua, et al. Aerodynamic optimization design of airfoil based on CST parameterization [J]. Journal of Northwest Polytechnical University, 2013, 31(5):829-836.
[3] Lafountain C, Cohen K, Abdallah S. Use of XFOIL in Design of Camber-Controlled Morphing UAVs[J]. Computer Applications in Engineering Education, 2012, 20(4):673-680.
[4] Sun Qin, Zhang Junyan, Zou Ruhong. Three kinds of airfoil parameterization methods [J]. Journal of Natural Science of Xiangtan University, 2013, 35(3):34-39.
[5] Liao Yanping, Liu Li, Long Teng. Research on several airfoil parameterization methods [J]. Journal of Projectile and Guidance, 2011, 31(3):160-164.
[6] Kulfan, B. M. Universal Parametric Geometry Representation Method[J]. Journal of Aircraft, 2008, 45(1):142-158.
[7] Yu Fangyuan, Gao Yong, Wang Yunliang, et al. Optimal design of small unmanned airfoil [J]. Journal of Naval Aeronautical Engineering college, 2013, 28(3):307-310.
[8] Fang Pingzhi, Gu Ming, Tan Jianguo, et al. Research on methods to solve the problem of wall function in the numerical simulation of atmospheric boundary layer [J]. Vibration and Impact, 2015, 34(2):85-90.