Astrophysics of Strange Matter

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The QCD phase transition has important consequences in the context of both the early universe as well as compact stars. Such transitions are being studied for high temperature and small chemical potential scenario in the laboratory. There are also plans to study systems with large chemical potential and small temperatures. Here we have reviewed the role of strange quark matter and the phase transition in all the above scenarios.

PACS numbers: 12.38.Aw, 12.38.Mh, 12.39.-x

I. INTRODUCTION

The study of strongly interacting system is an active area of research. The existence of quark-hadron phase transition, order of transition, signature and consequences of such transitions in different scenarios are some of the topics which are being studied both experimentally as well as theoretically. The fact that the QCD perturbative series shows poor convergence except for very small coupling at very high temperatures($\alpha_s < 0.5 \times 10^5 T_c$), makes the calculations more difficult. On the other hand, the fact that nature has provided us with two extreme scenario in the phase diagram, makes this area of research more intriguing as well as interesting.

One of the natural scenario is the large $T$ with vanishingly small chemical potential - the scenario which is supposed to have existed in the Early Universe and a similar situation is supposed to arise in the ultrarelativistic heavy ion collisions - the central collision in the central rapidity region, ideal hydrodynamics and longitudinal expansion mimicking the hubble expansion.

Off course there are differences, for example, Hubble expansion time scale is much larger than strong interaction scale whereas in ultrarelativistic collision expansion rate is comparable to the strong interaction.

The second natural scenario is the large chemical potential and small temperature limit which is expected to exist inside neutron stars. At present we do not know whether quark matter does exists inside the neutron stars. But with so many observational programmes going on it is really worth to study the possibilities of existence of such matter in the universe and especially inside the neutron star. This may lead us to that unique signature, the observational detection of which will resolve our doubt.

At present there exist large number of observational data on mass-radii of neutron stars. But, except very soft equation of states most of the other equation of states can explain these static properties within the error bars [1]. So it is necessary to study the different dynamical features of quark matter in its various forms.

II. COMPACT STARS

The theoretical study of role of strange matter in the context of compact stars has many facets. For example, it is important to have a good understanding of the all possible symmetry structures and the corresponding different forms of quark matter at very high densities. It is necessary to study the consequences of the existence of strange quark matter inside the neutron star and finally the consequences of hadron quark phase transition itself, such as the possibility of observation of drastic change, during phase transition, in some physical observables like breaking index. These consequences will also depend on how the conversion of nuclear matter to quark matter occurs i.e. the mechanism of phase transition itself.

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A. Symmetry and different quark matter phase

At very high densities and small temperatures, because of asymptotic freedom interaction is very weak. For higher momenta and near fermi surface quarks are almost free. In general at zero temperature, in absence of interaction, fermi energy is same as the chemical potential and adding or subtracting a particle would not cost anything. So if there is an attractive potential then one can add a pair of particles with quantum number of this attractive channel. The potential energy of this attraction will then lower the free energy and it will become more favourable. Many such pairs will then be created near the fermi surface. For QCD, the colour coulomb interaction is attractive between quarks having antisymmetric colour wave function. So the pairing in colour space is really a natural consequence of QCD theory itself. Since pair of quarks can not be colour singlet the cooper pair condensation in colour space will spontaneously break the color symmetry and gluons will acquire mass.

Again since, quarks also have flavour and spin along with colours, there will be many variety of patterns due to quark cooper pairing. The scenario is explained in figure 1. In general the strange quark mass is higher than u and d quark masses. So in matter fermi momenta of s quark is lower. From the charge neutrality condition number of d quarks and hence their fermi momenta is larger. So when both neutrality and flavour equilibrium is satisfied all the quarks tend to have different fermi momenta for the same baryon and electron chemical potential. For small densities fermi momenta of different flavours are too far apart to have any form of pairing and we get unpaired quarks. At higher densities, the difference between the fermi momenta decreases. As a result, we initially will have pairing of u and d which have the same small mass. At even larger densities, the fermi momenta of all the three flavours become very close and the matter ends up in colour-flavour locked (CFL) phase, in which, all the colour and flavour pair with each other. This is only possible if the energy cost to make the difference in fermi momenta zero can be compensated by the energy released due to the formation of the cooper pairs i.e. $\Delta_{CFL} > M s^2/\mu$. So only for very high chemical potential or density, such a scenario becomes possible.

As mentioned above, the stability of different forms of quark matter with different pairing depends on the gap energy $\Delta_{CFL}$. Depending on the value of $\Delta_{CFL}$, various form of pairing is expected to occur inside the compact star [2]. There are many consequences of these structures for the neutron star physics. For example the cooling rate is given by direct URCA and goes as $T^6$ for unpaired quark matter, nuclear matter at higher densities and some other forms of matter [3, 4]. On the other hand at very low densities, if proton fraction is lower than 0.1 it is modified URCA process which works here and cooling rate varies as $T^8$. In case of CFL matter, the emissivity is dominated by the Goldstone modes. But their emissivity is suppressed by the Boltzman factor. On the other hand, Neutrino emission from the process involving $\phi$, the massless Goldstone boson associated with breaking of $U(1)B$ symmetry goes as $T^{15}$ [2, 8]. The present observational scenario seems to indicate that some neutron stars cool much faster than others. So most probably it is the lighter neutron stars which cool following the modified URCA process whereas the heavier neutron stars contain some form matter which follows direct URCA process, e.g., the unpaired quark matter or high density nuclear matter with hyperons or one of the non-CFL superconducting phases [2].

B. Pulsating modes and Gravitation Wave

The other important aspect, for which observational efforts are being put, is the detection of gravitational waves from the neutron stars. Gravitational waves are ripples in the space-time curvature propagating through the space with velocity of light. Since neutron stars are very compact objects, i.e. they have high mass within a small radius, oscillating or more appropriately pulsating neutron stars can be very good source of gravitational wave that could be detected by the detectors.

In general neutron stars have a large number of families of pulsating modes with distinct characteristics. The one which is important in the present context corresponds to bulk flows in a rotating star and is known as Rossby mode or r-mode [2, 7]. The restoring force of these is the coriolis force and it transfers the star’s angular momentum into gravitational radiation. So for a rotating neutron star, there is a critical frequency above which the r-mode instability sets in, angular momentum gets transferred to gravitational wave and star spins down [5]. This r-mode instability is limited by the viscous damping. For a larger viscosity the critical spin at which r-mode becomes unstable is higher. Since the bulk viscosity of normal strange quark matter is larger than that of normal neutron star matter, an observation of newly born pulsar spinning near the Keplerian limit would provide the evidence for a strange quark star [5]. Similar situation is expected for 2SC quark star as well.

In contrast, CFL matter has been shown to have very small shear damping and bulk viscosity. So the heating effect due to the viscous dissipation is damped in CFL stars and r-mode damping becomes more important for their evolution. So except for first few hundred years, CFL stars will cool very slowly and can exist at higher temperatures for many years [10].

The unstable r-modes seems to affect the strange stars differently as compared to pure (no quark matter) neutron
FIG. 1: Pairing in colour space - extreme left shows the unpaired quarks at smaller densities or chemical potential $\mu$, with fermi momenta spacing $\delta k_f \sim \frac{M^2}{4\mu}$. For larger densities or $\mu$, 2-flavour pairing and then 3-flavour pairing or CFL phase becomes favourable.

stars [11]. Unlike neutron stars, the onset of r-mode instability, instead of leading to the thermo-gravitational runaway, results in the evolution of strange star to a quasiequilibrium state. Moreover, for strange stars, r-mode instability never grow to large amplitudes.

The gravitational wave bursts induced by r-mode spin down of hybrid stars has also been suggested. It has been proposed that continuous emission of gravitational waves due to r-mode instability from a star can induce a sudden variation in its structure and composition generating further bursts of gravitational waves. This scenario is more probable for hybrid stars due to the surface tension between the hadronic and quark matter [12].
The r-mode instability and the corresponding spin down will cause an increase in the central density of the star. This sudden increase may trigger a phase transition inside the core of neutron star. In strange quark matter, the strangeness fraction i.e. the ratio of strange quark and baryon number densities will be unity if one considers u, d and s masses to be same. Even for relativistic quark masses ($m_u > m_s \sim m_d$) the strangeness fraction at high density is not much smaller than unity. On the contrary, the strangeness fraction in hadronic matter is usually small. Even with hyperons the strangeness fraction is smaller compared to quark phase. Off course with kaon condensation, the situation may be different. But then with a kaon condensation inside neutron stars, the transition to quark matter is found to be pushed towards much higher densities [13]. So basically, the transition from hadronic to quark matter may be associated with large strangeness production. This can be explained in the following way. Initially hadronic matter, in terms of quark content consist predominantly of u & d and some s quark due to the hyperon population. The matter is certainly out of equilibrium. The weak interaction converts this chemically non-equilibrium matter to equilibrated matter with roughly equal number of u, d and s quarks. This conversion is associated with the release of large amount of energy in the form of neutrinos with average energy 100 Mev [14]. Simple energy consideration shows that amount of energy released in agreement with the observed energy release in gamma ray bursts.

C. Effect of Phase transition

Hadronic matter to quark matter phase transition can be studied for a rotating neutron stars. The production of equilibrated strange quark matter proceeds through non leptonic decay $u + d \leftrightarrow u + s$ and semi leptonic decays $d(s) \rightarrow u + e^- + \bar{\nu}_e$, $u \rightarrow d(s) + e^+ + \nu_e$ and reverse reactions. The transition from initial to final state will depend on the contribution of all possible rates.

$$\frac{dn_u(t)}{dt} = R_{d \rightarrow u}(e^-) + R_{s \rightarrow u}(e^-) - R_{u \rightarrow d}(e^-)R_{u \rightarrow s}(e^-) + R_{d \rightarrow u}(e^+) + R_{s \rightarrow u}(e^+) - R_{u \rightarrow d}(e^+) - R_{u \rightarrow s}(e^+)$$  \hspace{1cm} (1)

where $R_{d \rightarrow u}(e^-)$ is the reaction rate for the u quark production from d quark via electron process. The other rates in the above equation can be defined similarly.

The total number of neutrinos $N_\nu$ is obtained by the volume integration over the rotating star,

$$N_\nu = 2\pi \int r^2 d\mathbf{r} n_\nu n_B \frac{e^{2\alpha + \beta}}{\sqrt{1 - v^2}} \hspace{1cm} (2)$$

where $n_\nu$ is the number of neutrinos emitted per unit time per baryon, $n_B$ is the baryon number density, $\alpha$ and $\beta$ are the gravitational potential and $v$ corresponds to the rotational velocity. In figure 2 we have plotted the $N_\nu$ as a function of cosine of the polar angle $\mu(\cos\theta)$ for stars with two different central densities. For central density $6 \times 10^{14} gm/cm^3$ there is a sharp peak between $\mu = 0.1$ and $\mu = 0.24$ with a width of 12°. On the other hand for larger central density, $1 \times 10^{15} gm/cm^3$, matter concentration towards the polar region increases so that the beaming becomes less pronounced. This indicates that for stars with higher central density or larger quark matter region inside the compact star will show less beaming in the neutrino spectrum [10]. The neutrino beaming found here may be the missing link that causes the GRB. In general, cross section for the reaction $\nu + \bar{\nu} \rightarrow e^- + e^+$ is very small. It has been shown that the general relativistic effects may enhance this cross section substantially and more than 10% of the energy emitted in neutrinos may be deposited in $e^+e^-$. This enhancement is due to path bending of the neutrinos which in turn increases the probability of head on $\nu\bar{\nu}$ collision. So it is now necessary to calculate the energy deposition rate in the process $\nu + \bar{\nu} \rightarrow e^- + e^+$ for rotating compact stars. A detailed account of the rotation along with GTR effect is given in [16]. It has been shown that effect of rotation along with the general relativity enhances the energy deposition rate substantially. Hence it can provide a very efficient engine for the gamma ray bursts.

There are some other interesting approaches as well. For example, at a temperatures of 30-60 MeV, neutrino emission becomes collimated with a beaming angle $\theta$ around its magnetic axis, in the presence of surface magnetic field $\equiv 10^{14-17}G$. This happens in the early cooling evolution of strange star with colour superconducting quark matter core in the CFL phase [17].

D. Mechanism of transition

As mentioned earlier, we have really considered the phase transition to be a two step process [18]. In the first step, the nuclear matter, which is predominantly n, p, e$^-$ matter gets deconfined to u , d matter i.e. predominantly two flavour matter which then gets converted to three flavour equilibrated matter through weak interactions. For
neutron star environment, one should really study the conversion along with general relativistic effect. Our study shows that the GTR effect leads to qualitative changes in the result compared to special relativistic treatment for the conversion of hadronic matter to 2-flavour quark matter. In fact the GTR effect gives rise to different conversion fonts propagating with different velocities along different radial directions. A detailed study of these are given in [19]. The above result may have important physical consequences for the neutron star Physics.

III. QUARK MATTER IN EARLY UNIVERSE

Let us now move over to early universe scenario. According to WMAP data composition of the Universe is 5% baryonic matter 25% dark matter and 70% dark energy. This understanding is based on Λ CDM model [20]. The universe is expected to have undergone various changes at different epochs corresponding to different energy scales, starting from GUT scale of $E \approx 10^{19}$ GeV to recombination at around $E \equiv 10$ eV. In the present paper, we will concentrate on the QCD epoch only, at an energy scale of 100 MeV. It is believed that universe went through a phase transition from quark, gluon system to nucleonic system. This phase transition is important as it not only sets the
initial condition for nucleosynthesis, the relics generated at this transition might be observable today.

Most of the relics, proposed in the literature, forms during the QCD transition only if the transition is of first order. For example, strange quark matter as dark matter candidate, gravitational waves from colliding bubbles and generation of magnetic field.

Of course, a CDM candidate like QCD balls, as proposed by Zhitnitsky [21] does not require a first order phase transition. Such QCD balls are formed when under certain conditions axion domain walls trap a large number of quarks. This collapse is stopped by the fermi pressure of quarks which would settle in a colour superconducting phase.

Let us first understand the difference between the laboratory and early universe scenario. A phase transition occurs in local thermodynamic equilibrium if the rate of equilibration which will be similar to collision rate is larger than the cooling rate which again is similar to expansion rate as cooling occurs through expansion. In the early universe at the time of phase transition expansion time scale is of the order of inverse of hubble expansion which is much larger than the strong interaction time scale 10^{-23} seconds. This implies that the transition, in the early universe is most likely to occur in thermodynamic equilibrium. On the other hand, in ultra-relativistic heavy ion collisions, expansion rate is comparable to the strong interaction scale and hence the nonequilibrium effects would be important for the transition.

The recent consensus seems to be that for the physical masses of quarks (two light u and d and one heavier s quark) there is a sharp crossover between the high temperature gas of quarks and gluon quasiparticles and a low temperature hadronic phase without any thermodynamic discontinuities [22, 23].

Let us now deliberate on the issue of the order of phase transition. In general a phase transition is said to be of order n, if the nth order derivative of the free energy with respect to the external field is discontinuous or divergent. More specifically, for a second order transition, second derivative of free energy with respect to temperature is discontinuous, first order derivative being continuous. Such a transition give rise to scale independent long wavelength fluctuations so that large regions behave coherently. On the other hand, for a first order transition, 1st order derivative of free energy with respect to external variables is discontinuous. The value of the order parameter and the position of the minimum of the free energy jump discontinuously at the critical temperature. The difference in the free energy between the local and global minimum yields a latent heat which is released upon the decay of the metastable state. The speed of sound \( \frac{d P}{d T} = c_s T^2 \) jumps from zero at \( T = T_c \) to a finite value at \( T > T_c \). Here the fluctuation are scale dependent and gives rise to the formation of domains.

Recent lattice calculations of QCD transition [24] have shown a sharp drop in the speed of sound as temperature decreases towards the critical temperature. This implies that the system going through a cross over transition may behave somewhat similar to first order transition. In figure 3 we have plotted the speed of sound along with the conformal measure [25] calculated form PNJL model [26]. The \( v_s^2 \) is close to its ideal gas value at the temperature of

![FIG. 3: Squared velocity of sound \( v_s^2 \) and conformal measure \( C = \Delta/\epsilon \) as function of \( T/T_c \). The arrow on the right shows the ideal gas value for \( v_s^2 \). For comparison with \( v_s^2 \) we also plot the ratio \( P/\epsilon \).](image-url)
about 2.5$T_c$. This is close to the results for pure glue theory on the Lattice as reported in Ref.\[27\] and also that with 2 flavour Wilson Fermions in Ref.\[28\], with 2+1 flavours of staggered quarks reported in Ref.\[29\], and with improved 2 flavour staggered fermions ($P/\epsilon$ was measured in this case). However, near $T_c$, the $v^2_s$ in Ref.\[27\] goes to a minimum value above 0.15, whereas we find the minima going close to 0.08, consistent with simulations with dynamical quark in Refs.\[28, 29\], and remarkably close to the softest point $P/\epsilon = 0.075$ in Ref.\[30\].

In case of a 1st order transition, one would expect the fomation of trapped false vacuum domains (TFVD) during the transition\[31\] some of which may become stable depending on their baryon number content\[32, 33\]. These quark nuggets may further evolve giving rise to massive compact objects\[34\]. The above process gives rise to an interesting phenomena when one considers the role of colour charges explicitly\[36\].

Let us assume the colour wave function of the universe to be colour singlet prior to the QCD phase transition, or in other words, wave function of all the coloured objects are completely entangled\[35\]. In such a situation the universe is characterized by the vacuum energy of perturbative QCD. During the transition, colour neutral configurations (Hadrons) arise which results in the gradual decoherence of the entangled colour wave function of the entire universe. This amounts to aproporationate reduction in the perturbative vacuum energy density which goes into providing the latent heat of the transition. The TFVDS formed during the transition would try to release the residual colours to become colour neutral. So at the end of the transition one would be left with coloured charges separated by space like distances. From the quantum mechanical point of view, the colour wave function of the largely separated quarks still remains entangled and the corresponding amount of perturbative energy would persist\[36\], which would then play the role of cosmological constant. Now, for a cross over transition, TFVDS may or may not evolve to stable quark nuggets. But the existence of scale dependent fluctuations is enough for the the formation of TFVDSs and the quarks separated by space like distance. In other words, even for a cross over transition, the QCD transition may provide a solution to the dark energy problem.

**IV. SUMMARY**

QCD transition is one of the mose interesting areas of research. We have discussed the impact of the QCD transition in natural scenario of neutron stars and early universe. Though, an observational signature of phase transition and strange quark matter in nature is still elusive, the study of these phenomena is extremely important for a better understanding of the physics of strongly interacting matter. Moreover, there are many questions remain to be answered before we can make any conclusions.
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