Review of Multi-Agent Algorithms for Collective Behavior: a Structural Taxonomy

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Abstract: In this paper, we review multi-agent collective behavior algorithms in the literature and classify them according to their underlying mathematical structure. For each mathematical technique, we identify the multi-agent coordination tasks it can be applied to, and we analyze its scalability, bandwidth use, and demonstrated maturity. We highlight how versatile techniques such as artificial potential functions can be used for applications ranging from low-level position control to high-level coordination and task allocation, we discuss possible reasons for the slow adoption of complex distributed coordination algorithms in the field, and we highlight areas for further research and development.

Keywords: Autonomous mobile robots, Agents, Distributed Control, Decentralized Control

1. INTRODUCTION

Multi-agent robotic systems hold promise to enable new classes of missions in aerospace, terrestrial, and maritime applications, delivering higher resilience and adaptability at lower cost compared to existing monolithic systems. In particular, in the aerospace domain, multi-agent systems hold great promise for applications including multi-UAV patrolling, satellite formations for astronomy and Earth observation, and multi-robot planetary exploration. A number of algorithms have been proposed to control the collective behavior of such systems, ranging from low-level position control to high-level motion planning and task allocation algorithms.

Many excellent surveys of algorithms for collective behavior exist in the literature; however, such papers generally focus either on single applications (e.g., formation control (Oh et al., 2015) or coverage (Schwager et al., 2009)) or on specific control techniques (e.g., consensus (Garin and Schenato, 2010; Cao et al., 2013)). In contrast, in this paper, we survey the general family of collective behavior algorithms for multi-agent systems and classify them according to their underlying mathematical structure, without restricting our focus to specific tasks or individual classes of algorithms. In doing so, we aim to capture fundamental mathematical properties of algorithms (e.g. scalability with respect to the number of agents and bandwidth use) and to show how the same algorithm or family of algorithms can be applied to multiple tasks and missions.

In particular, the goal of this paper is threefold:

- to act as a guide to practitioners in the selection of control algorithms for a given task or application;
- to highlight how mathematically similar algorithms can be used for a variety of tasks, ranging from low-level control to high-level coordination;
- to explore the state-of-the-art in the field of control of multi-agent systems and identify areas for future research.

Tasks in multi-agent systems can be broadly categorized into the following classes (Brambilla et al., 2013):

1. Spatially-organizing behaviors, where agents coordinate to achieve a given spatial configuration and have negligible interactions with the environment. These tasks can be further classified into: (a) Aggregation: converging to one location. (b) Pattern Formation: achieving a desired formation. (c) Coverage: covering an area.

2. Collective explorations, where agents interact with the environment but have minimal interaction among themselves. These tasks can be classified into: (a) Area Exploration: exploring the environment for mapping or surveillance. (b) GoalSearching: searching for targets.

3. Cooperative decision making, where agents both coordinate among themselves and interact with the environment to accomplish complex tasks. These tasks can be further classified into: (a) Task Allocation: distributing tasks among agents. (b) Collective Transport: coordinating to transport large objects. (c) Motion Planning: finding paths in cluttered environments. (d) Distributed Estimation: estimating the state of one or multiple targets.

These simple tasks are the fundamental building blocks of many complex multi-agent applications.

Communication structure In centralized algorithms, all agents share their information with a central node, which computes and issues a joint set of control actions.
| Consensus | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Artificial Potential Functions (APF) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | F |
| Geometric Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Circumcenter Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Maze Searching Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Leader-Follower (LF) Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Velocity Obstacle (VO) based Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | F |
| State Machines and Behavior Composition | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Behavior Composition | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Petri Networks | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - | H |
| Game Theory based Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Resource Allocation Systems | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | - | S |
| Bio-Inspired Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Kilobot Self-Assembly Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Optimotaxis Source-Searching Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Beeclust Foraging Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Shepherding Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Termite-Inspired Collective Construction Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Fish-inspired Goal Searching Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Gillespie Self-Assembly Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Mergeable Modular Robots | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Density based Control | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Markov Chain-based Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Smoothed Particle Hydrodynamics (SPH) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Optimal Transport based Algorithm | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Distributed Optimization Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Distributed Linear Programming | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Distributed Convex Optimization | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Distributed Dynamic Programming | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Sequential Convex Programming | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Distributed Auction | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Local Optimization Algorithms for Global Behavior | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Decentralized Model Predictive Control (DMPC) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | H |
| Formal Methods | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Sampling-based Motion-Planning Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Centralized Optimization Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| MILPs and MINLPs | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Linear and Convex Optimization | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Markov Decision Processes (MDP) | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Multi-Agent Traveling Salesman Problems | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Multi-Armed Bandits | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Direct Methods for Optimal Control | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Multiagent Reinforcement Learning | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Frontier Techniques | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Network Flow Algorithms | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |
| Combinatorial Motion Planning | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | ✓ | S |

Table 1. Categorization of collective behavior algorithms according to their mathematical structure and applicability of each algorithm to common multi-agent tasks. The scalability, bandwidth use, and level of demonstrated maturity of each algorithm (formally defined in Section 1) are also reported.
In distributed algorithms, agents can only explicitly share information with their neighbors. Centralized algorithms can be implemented in a distributed fashion with a shared-world approach, discussed in Section 2.10.

Methodology We performed a thorough review of papers on multi-agent systems in major controls and robotics journals and conferences. It is not feasible to cite all existing works on control of multi-agent systems; accordingly, in this paper, we focus on identifying and classifying the key mathematical structures and techniques that drive coordination algorithms, as opposed to individual contributions.

We classify mathematical techniques according to their:

1. Scalability: Highly scalable algorithms have been demonstrated on systems with more than 50 agents (in simulations or hardware).
2. Bandwidth use: In low bandwidth algorithms, agents only communicate with their physical neighbors and do not exchange large messages.
3. Maturity: The three classes of algorithms are:
   - (i) only demonstrated in ‘simulation’ (S) (ii) demonstrated in ‘hardware’ (H) either in the lab or in technology demonstration missions (iii) demonstrated in ‘field’ (F) deployments (excluding technology demonstrator missions).

Organization Our key contribution is Table 1, which reports the proposed taxonomy of mathematical techniques for collective behavior, highlights the tasks that each mathematical technique can achieve, and lists relevant performance metrics. In Sections 2.1–2.10 we provide a synthetic description of the classification and relevant references. Finally, in Section 3 we draw conclusions and suggest directions for future research.

2. A STRUCTURAL TAXONOMY OF MULTI-AGENT COLLECTIVE BEHAVIOR ALGORITHMS

2.1 Consensus algorithms

Consensus is among the oldest and most widely used distributed algorithms. Each agent shares and averages its state with its neighbors (Tsitsiklis et al., 1986; Ren et al., 2007). Applications include synchronization (Li and Rus, 2006), flocking (Tanner et al., 2007; Olfati-Saber, 2006), formation flying (Chung et al., 2013), and distributed estimation (Rabbat and Nowak, 2004). In gossip algorithms (Boyd et al., 2006), each agent communicates with a single randomly-selected neighbor at each step. In cyclic pursuit algorithms (Marshall et al., 2004), the consensus algorithm is executed on a directed ring communication topology.

2.2 Artificial Potential Functions (APF)

APF algorithms synthesize agents’ control inputs using the gradient of a suitably-defined potential function (Khatib, 1986). These algorithms are very popular due to their simplicity, scalability, and ability to adapt to a number of tasks. Applications include pattern formation (Sepulchre et al., 2007), flocking (Zavlanos et al., 2007), path planning (Koditschek and Rimon, 1990), and task allocation (Weigel et al., 2002).

2.3 Distributed Feedback Control

Each agent is endowed with a feedback controller that uses the agent’s and its neighbors’ states as the input (Bamieh et al., 2002; Feddema et al., 2002). In particular, tools for synthesis of distributed LQG control are available that can adapt to noisy communication links (Sahai and Mitter, 2006), and packet losses (Liu and Goldsmith, 2004), with applications to formation flying (Ogren et al., 2002) and distributed estimation.

2.4 Geometric Algorithms

In geometric algorithms, agents leverage their neighbors’ location and speed information to perform spatially organizing tasks and path planning. Voronoi algorithms compute Voronoi partitions for coverage (Cortés et al., 2004), path planning (Zhou et al., 2017), and task allocation problems (Pavone et al., 2011). Other geometric algorithms include circumcenter algorithms for rendezvous (Cortés et al., 2006), bearing-only algorithms for formation control (Fredsund and Mataric, 2002) and rendezvous (Yu et al., 2008), maze searching algorithms for path planning (Lunelsky and Harinarayan, 1997), leader-follower algorithms for formation flying (Mesbahi and Hadaegh, 1999), and velocity obstacles for collision avoidance (van den Berg et al., 2008).

2.5 State Machines and Behavior Composition

Automata-based algorithms leverage complex state machines and message-passing among agents to establish communication graphs and elect leaders for task allocation (Lynch, 1997; Rossi and Pavone, 2014). Behavior composition algorithms rely on composition of elementary behaviors for collective transport (Rus et al., 1995). Petri networks (King et al., 2003) and game theory (Arslan et al., 2007) algorithms are used for centralized task allocation. Resource allocation systems are used for multi-agent motion planning (Reveliotis and Roszkowska, 2011).

2.6 Bio-Inspired Algorithms

Bio-inspired algorithms mimic the behavior of swarms of animals such as insects and fish. We present a non-exhaustive list: the Kilobot algorithm achieves complex two-dimensional shapes and was demonstrated on a thousand-agent testbed (Rubenstein et al., 2014); the Optimotaxis source-searching algorithm is inspired by the run and tumble behaviors of bacteria (Mesquita et al., 2008); the Beeclust foraging algorithm is inspired by the behavior of honey bees (Hereford, 2011); Shepherding algorithms enable control of large numbers of uncontrolled agents with few controlled agents (Strömbom et al., 2014); a Termite-inspired algorithm generates low-level rules for construction of complex structures (Welf et al., 2014); a Fish-inspired goal-searching algorithm switches between individual and collective behavior based on confidence level (Wu and Zhang, 2012); the Gillespie self-assembly algorithm leverages chemical kinetics; Mergeable modular robots connect to form larger bodies or split into separate bodies, with self-healing properties (Mathews et al., 2017).

2.7 Density based Control

As opposed to the agent-based Lagrangian framework, density-based algorithms adopt an Eulerian framework by treating agents as a continuum and controlling their density. Markov chain based algorithms partition the workspace into disjoint cells and control the transition probabilities between cells for pattern formation and goal searching applications (Açıkmese and Bayard, 2012; Bandyopadhyay et al., 2017b). Smoothed particle hydrodynamics (SPH) (Zhao et al., 2011) and optimal transport (Bandyopadhyay et al., 2014) based algorithms are also used for swarm formation control.
Distributed optimization algorithms allow agents to jointly solve optimization problems through information exchange and local computations. Distributed linear programming (Bürger et al., 2012) is used for pattern formation and task allocation; distributed convex optimization can encode richer convex constraints (Boyd et al., 2011). Distributed dynamic programming (Bertsekas, 1982) is used for task allocation and motion planning. Sequential Convex Programming can solve non-convex motion planning problems through local convexification and iteration (Morgan et al., 2016). The above algorithms can also be used in a distributed model-predictive control framework (Scattolini, 2009). Market-based protocols like distributed auction (Gerkey and Mataric, 2002), mechanism design (Dias, 2004), and coalition formation (Shelory and Kraus, 1998) are widely used for task allocation.

### 2.8 Distributed Optimization Algorithms

Distributed optimization algorithms have been proposed for multi-agent systems: we refer the reader to (Sharon et al., 2015) for a thorough review. Centralized optimization algorithms can be implemented in a distributed fashion with a shared-world approach, where agents exchange their state and observations so that every robot has full knowledge of the entire system's state. However, shared-world algorithms have very onerous communication requirements (due to large messages and all-to-all communication) and high computation complexity, since each agent must solve the full centralized optimization problem.

### 3. CONCLUSION

The proposed taxonomy and the properties shown in Table 1 highlight some surprising characteristics of collective behavior algorithms. The majority of existing mathematical techniques is tailored to either low-level spatially organizing tasks (e.g., bio-inspired algorithms and density-based control) or high-level coordination applications (e.g., state machines and optimization-based algorithms). Only a small number of mathematical techniques (in particular, Artificial Potential Functions) can be adapted to a wide variety of tasks that include both low-level and high-level application. This prompts further research into non-APF algorithms for multi-agent systems that share APF’s key properties of simplicity, scalability, and high expressivity.
rying intentions and bids), and the communication topology induced by the algorithm (single-hop vs. multi-hop). Finally, we plan to further explore high-level multi-agent tasks, including adversarial “swarm vs. swarm” problems, and to assess the applicability and performance of collective behavior algorithms with respect to such tasks.

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