Externally excited planar dust acoustic shock waves in a strongly coupled dusty plasma under microgravity conditions

A Usachev$^1$, A Zobnin$^1$, O Petrov$^1$, V Fortov$^1$, M H Thoma$^2$, H Höfner$^3$, M Fink$^3$, A Ivlev$^3$ and G Morfill$^3$

$^1$Joint Institute for High Temperatures RAS, Moscow, Russian Federation
$^2$Institute of Experimental Physics I, Justus Liebig University, Heinrich-Buff-Ring 16, 35392 Giessen, Germany
$^3$Max-Planck Institute für Extraterrestrische Physik, Garching, Germany
E-mail: usachev@ihed.ras.ru

Received 31 December 2013, revised 24 March 2014
Accepted for publication 11 April 2014
Published 14 May 2014

New Journal of Physics 16 (2014) 053028
doi:10.1088/1367-2630/16/5/053028

Abstract

The formation and dissipation of an externally excited planar dust acoustic shock wave in a three-dimensional uniform dust cloud has been observed under microgravity conditions. The experiment has been performed in the dc gas discharge chamber ‘Plasma Kristall-4’ (Fortov et al 2005 Plasma Phys. Control. Fusion 47 B537) on board the A300 Zero-G airplane. The shock Mach number and compression factor reached 3.5 and 6, correspondingly, with a shock width of about the interparticle distance. Due to the utilization of the polarity-switching dc discharge mode and application of the Rankine–Hugoniot relations, the dust particle electrostatic pressure was determined and the Hugoniot percussive adiabat for the dust subsystem was derived. The obtained data were simulated using thermodynamic properties of highly nonideal Debye–Hückel (Yukawa) systems. Comparison of the experimental and simulated data has demonstrated that the screening length in a dense dusty plasma is not determined by the total ion number density, but rather by those ‘effective’ ions which are not bounded by negatively charged dust grains. Thus, this work presents a new experimental approach for the investigation of the dense dusty plasma clouds.
1. Introduction

Dusty plasmas attract our attention as a convenient example of a highly nonideal many-particle system [1]. They can be easily generated in laboratory setups and observed on the kinetic level. Dusty plasmas consist of electrons, ions, neutral atoms or molecules, and charged dust grains. Due to the high electrical charges of the dust grains, they intensively interact with each other and with the electrical fields and fluxes in the plasma, resulting in a high non-ideality and a rich variety of dusty plasma collective phenomena—waves and instabilities [2]. The presence of dust particles in a plasma modifies the already known plasma waves (for example, ion-acoustic waves, solitons, and shock waves) and creates completely new acoustic modes. The first theoretical prediction of a dust density acoustic wave (DAW) mode was discussed by P K Shukla [3] and then published by Rao, Shukla, and Yu in [4]. Due to the large grain masses, the characteristic frequencies of such waves are extremely low, i.e., from a few to about ten hertz. By laser light illumination, the DAWs can be visible even with the naked eye [5, 6]. Besides DAWs, a wide spectrum of other kinds of wave modes has been discovered in dusty plasmas: for example, dust acoustic solitary waves (solitons) [7], dissipative rarefaction (‘dark’) solitons [8], and dust acoustic shock waves (DA SWs) [9–11]. The characteristic details of these modes strongly depend on the particular dusty plasma parameters—grain masses, charges, and number densities; concentrations and temperatures of electrons, ions, and neutrals; distribution of space electric potential and plasma fluxes; gust grain charges; nonideality parameters, etc. Clearly the variety of such wave phenomena is very wide.

To be specific, in the present work we restrict our consideration to low-pressure gas discharge dusty plasmas with micrometer-sized dust particles. As a rule, such dusty plasmas are characterized by large values of the plasma nonideality parameter [1]

$$\Gamma_d \approx \frac{Q_d^2}{\lambda_{scr}^2} \exp \left( -\frac{\bar{r}}{\lambda_{scr}} \right) > 1,$$

(1)

where $Q_d$ and $T_d$ are the dust particle charge and kinetic dust particle temperature, respectively, $\varepsilon_0$ is the dielectric constant, $\bar{r}$ is the mean interparticle distance, $k_B$ is the Boltzmann constant, and $\lambda_{scr}$ is the screening length of the dust particles in the plasma. In this article we consider monodisperse spherical dust particles with a radius of $a$. If the radius of the dust particles $a$ is much smaller than the plasma Debye’s length, which is equal to

$$\lambda_D = \frac{\lambda_{Di} \lambda_{De}}{\sqrt{\lambda_{Di}^2 + \lambda_{De}^2}}$$

(2)

where $\lambda_{Di(e)} = \sqrt{\varepsilon_0 k_B T_i(e) / n_e(e) \varepsilon_0}$ is the electron (ion) Debye radius, $n_{e(0)} (n_{i(0)})$ and $T_e (T_i)$ are the electron (ion) number density and temperature, respectively, then $\lambda_{scr} \approx \lambda_D$ [12, 13]. In low-pressure gas discharge plasmas, usually $T_e \gg T_i$ holds, implying $\lambda_D \approx \lambda_{Di} \approx \lambda_{scr}$.

Also, it is important for our further consideration to distinguish between self-excited drift dust density waves and externally excited elastic dust density waves. In the first case, the drift waves are driven (energized) by the plasma fluxes and electrical forces. This is the general case for laboratory dusty plasmas—they are always restricted by the chamber walls, which create an
ambipolar electric field and corresponding plasma fluxes (for example, [6, 14]). Furthermore, a steady electric field is present in the dc discharge plasmas. As a rule, if the gas pressure is not too high (less than about 50 Pa), these fields stimulate dust acoustic instabilities in laboratory setups [5, 15]. In the second case, dust grains are kept motionless in an isotropic plasma. To create a dust wave perturbation, an experimentalist excites externally some local part of a dusty plasma cloud by magnetic [10], electrical [7–9], or laser [16] manipulators and investigates the propagation of the dissipative inertial elastic disturbance in the dust cloud. Such active experiments provide much more flexibility in the experiments and permit one to scan the necessary range of the wave amplitudes and frequencies.

Usually, the dust grains in the laboratory dc [10] and rf [7, 9, 16] plasmas are suspended by a permanent electric field. Hence, in such setups the elastic DA perturbations can exist only in the direction perpendicular to the direction of the suspending electric field. Such experiments were performed in a two-dimensional (2D) dust particle monolayer suspended in an rf discharge [7, 9, 16]. Therefore investigations of the elastic DAWs in the three-dimensional (3D) dusty plasmas have to be performed under microgravity conditions in the central part of a discharge chamber, far from the walls creating ambipolar plasma fluxes. Such an experiment was performed at the ‘Plasma Kristall-3’ setup on board the International Space Station in 2003 [17]. The dust density perturbation in a dust cloud of the rf discharge dusty plasma was excited by a gas impulse from the inlet gas port. The perturbation had a sharp front, propagated with a Mach number of $M = 1.2$, and was classified as a DA SW. A comparison of the wave live time $T_{SW} \sim 0.5$ sec and the dust drag time $\tau_{dn} \sim 3$ ms testifies that this wave was driven by some additional source of energy (most probably by the ambipolar electric field) and should be classified as a drift wave.

Up to now no active experiment on excitation and observation of pure elastic DA SWs in a 3D dusty plasma cloud has been performed. However, a simple analysis of easily measurable parameters of the elastic DA SW, such as the SW front, dust grain velocities, and the dust grain number densities, can give us important information on the dusty plasma compressibility and intergrain interaction potential.

In this work we report results of the experimental investigation of the inertial elastic DA SW in a strongly coupled system of charged dust particles in a low-pressure gas discharge plasma. In order to prevent the growth of different dusty plasma parametric instabilities, the experiment was performed in a symmetrically driven dc gas discharge plasma with alternating polarity under microgravity conditions. Such an experimental configuration provides a minimum of stored potential energy in the dust particle subsystem. The obtained data on dust cloud compressibility have been analyzed on the basis of the highly nonideal Debye–Hückel systems.

The paper is organized as follows. In section 2 we describe the experimental apparatus, experimental procedures, and experimental results. Section 3 introduces the method of determination of the electrostatic dust particle pressure behind the DA SW using the Rankine–Hugoniot relations. In section 4 we calculate the electrostatic dust particle pressure, assuming a Debye–Hückel interaction model between the charged dust particles, and compare it with the experimental findings. Three different physical approaches for the calculation of the screening length in the dense dust cloud are discussed in section 5 and the main physical findings are stated in section 6.
2. Setup and experiment

The present experiment on the excitation of elastic DA SWs in a low-pressure gas discharge dusty plasma was performed using the ‘Plasma Kristall-4’ (‘PK-4’) facility [18] during the 49th European Space Agency (ESA) parabolic flight campaign in November 2008 onboard the A300 Zero-G plane under microgravity conditions. The PK-4 chamber provides a particular advantage for investigation of wave phenomena in dusty plasma. A scheme of the setup and experiment is presented in figure 1. The experimental arrangement consists of the Π-shape glass discharge tube of 30 mm inner diameter with a total length of 85 cm filled by neon at a pressure of 15 Pa. The tube is equipped with two dc cylindrical stainless steel electrodes installed at the ends of the tube. The direct current (dc) gas discharge in the tube was operated at $I_{DC} = 1$ mA in two different discharge modes: usual ‘pure’ dc mode (dc-mode) and dc polarity switching mode (dc/ps-mode). The dc/ps-mode was operated at 1 kHz with the rectangular symmetrical waveform. Such a mode was necessary for the formation of a stable isotropic dust cloud without a dust-acoustic instability [13] at the low gas pressure and to eliminate the mean electrical force acting on the charged dust particles, both due to the axial electric field in the usual ‘pure’ dc-mode. A uniform positive column filled almost the entire chamber volume in both discharge modes. In addition to the dc electrodes, the ‘PK-4’ discharge chamber was equipped with an additional circular electrical manipulating electrode (em-electrode) for electrical manipulation of the dust particles inside the chamber. The em-electrode was made of perforated stainless steel tape of 7 mm width and installed into the discharge chamber as shown in figure 1. The em-electrode diameter was equal to the inner discharge tube diameter. So, under a floating plasma potential, the em-electrode does not disturb the dusty plasma in the discharge tube.

The experiment was performed in the following manner. At the beginning of the microgravity period, with a duration of 22 sec on board the A-300 Zero-G plane, monodisperse plastic (melamine formaldehyde) microspheres (dust particles) with a radius of...
a = 1.7 μm and a mass of $m_d = 3.1 \cdot 10^{-14}$ kg were injected into the dc-mode discharge plasma in the vicinity of the cathode. Being injected, the charged dust particles drifted to the anode due to the dc axial electric field of the discharge of about 2 V cm$^{-1}$ (figure 1). The injected particles were illuminated by a laser sheet of 150 μm width and recorded by a CCD camera with an image resolution of 640 x 480 pixels and a frame rate of 60 frames per second (16.7 ms per frame). The camera field of view (FoV) was 45 x 35 mm$^2$ near the em-electrode. The injected particles formed an elongated drifting uniform dust cloud with a diameter of 5 mm and a particle density of $n_d \sim 40$ mm$^{-3}$, which was confined near the tube axis by the radial electric field in the cylindrical dc-positive column. As soon as the dust particles filled the FoV, an experimentalist manually changed the discharge mode from the dc to the dc/ps one, and the movement of the particles was stopped. After 2–3 sec for cloud relaxation, a negative potential $U_{EM}$ on the em-electrode was increased step by step. The value of this potential was chosen to obtain a proper value of the cloud disturbance. The corresponding em-current was found to be $I_{EM} = 0.3$ mA.

The formation of an elastic DA SW in this experiment is presented in figure 2. Under the action of negative potential loaded on the em-electrode the dust particles, which were close to the electrode (left side of the figure 2(b)), an initial speed of about 7–10 mm s$^{-1}$ along the tube axis was obtained. The dust particles at the central and the right parts of Pic. 2(b) remained undisturbed. This indicates the localized influence of the biased em-electrode on the dust cloud. During the next 100 ms from the beginning of the electric pulse (figures 2(c), (d)), the disturbance moved through the center of figure 2, forming a sharp front (figure 2(d)). After 200 ms the speed of disturbance decreased, the front dissipated, and the disturbance vanished. Using the video data in figure 2, we have determined the distribution of the dust particle number density along the tube axis within the white dashed rectangle in figure 2(a). The size of the strip (15 x 2 mm$^2$) and its position on the dust cloud were chosen to cover the area of interest. The position of the rectangle is slightly (on 1 mm) shifted upward with respect to the chamber axis following the cloud axis, which was shifted due to residual accelerations (~0.02 g–0.04 g) onboard the A300 Zero-G. The dotted rectangle in figure 2 has been divided into 65 vertical strips, and the brightness of all pixels within one strip was summarized. In this, all the pixels had an unsaturated brightness and the image backlighting was taken into account. Using the undisturbed part of the rectangle with distinguishable dust particles, and therefore with a defined dust number densities and known laser sheet width, we normalized the obtained relative profiles of the dust number densities $n_d(x)$ along the tube axis x. The obtained dust grain number density profiles for the five consequent moments are presented in figure 3. The dashed line in figure 3(a) indicates the profile of the unperturbed dust number density distribution $n_{d0}(x)$.

The dust particle number density profile $n_d(x)$ of the observed disturbance (figure 3) can be tentatively divided into three parts—the area of undisturbed concentration of dust particles (3), the area under influence of the em-electrode (1), and the transition region (2). As the number of dust particles in the elementary digitization segment was small, the results of the sampling distribution contain a dispersion of the experimental data (the points in figure 3), which has a statistical origin, but not a physical meaning. Therefore, the experimental data on the distribution $n_d(x)$ has been interpolated by a power polynomial in areas 1 and 3 (curves), and by a linear function in area 2 (lines). Using the interpolation curves $n_d(x, t)$, the following parameters of the wave were determined: the wave velocity $D$ as the velocity of the center of the transitional area 2; the dust particle number density just before ($n_{d1}$) and behind ($n_{d2}$)
transitional region 2 as the points of interception of the interpolative curves 2–3 and 1–2, respectively (figure 3(b)); and the width $w$ of the transitional area 2. In this, the widths $w$ measured from the profiles $n_d(x)$ (figure 3(c)) were corrected by subtraction of the wave shift $\delta w = D\tau_v$ during the video frame exposure $\tau_v = 15$ ms, i.e., $h = (w - \delta w)$. The results of the calculation of the dust density wave velocity $D$ and the actual front width $h$ during the wave propagation are presented in figure 4. The observed evolution of the dust density wave $n_d(x,t)$ can be divided into the three subsequent periods. During the first $\sim 70–80$ ms under the action of the em-pulse, the amplitude of the dust density disturbance $n_{d2}/n_{d1}$ (compression factor) grew to

Figure 2. Formation and dissipation of the shock wave in a dusty cloud in the central axial part of discharge tube; the em-electrode is at the left side of these pictures. (a) Dust cloud in the uniform positive dc/sp column just before the em-pulse. Two horizontal dashed lines indicate the area used for the determination of the dust number density distributions in the all pictures (a)–(e). (b) 50 ms after the starting of the em-pulse ignition: the left side of the cloud is accelerated due to the em-pulse, while the central and left parts of the cloud are still unperturbed; the arrow shows the direction of movement of the dust particles. (c), (d) During a period of 100–150 ms after the starting of the em-pulse ignition the front of the induced dust density wave becomes steeper. (e) 200 ms after the starting of the em-pulse ignition, the wave disappears due to dissipation processes. Areas shown are $23 \times 8 \text{ mm}^2$. 

Figure 2.
its maximum of about 6. Thereby the wave width $w$ quickly decreased to 0.2–0.3 mm, which is about the interparticle distance in the undisturbed dusty cloud. During the next 100 ms the amplitude of the dust density disturbance $n_{d2}/n_{d1}$ and the wave speed $D$ fell down, but the narrow width $w$ was kept at about 0.2–0.3 mm. During the final period the wave speed $D$ stabilized at about of 2.5 cm s$^{-1}$, but the wave width $w$ started to grow. After 200 ms from the em-pulse ignition the wave disturbance has disappeared due to dissipation processes of which the friction between the dust particles and the neutral neon atoms is the most important one.

For classification of the observed dust density wave, the ratio between the observed wave velocity $D$ and the dust acoustic wave (DAW) velocity $C_{DAW}$ in the undisturbed dust cloud, i.e., the Mach number $M = D/C_{DAW}$, is very important. The value of $C_{DAW}$ in our case can be determined by using the formula [4] in the long wave limit.
where $\omega_{pd}$ is the plasma-dust frequency $\omega_{pd} = Q_d \sqrt{4\pi n_d/m_d}$, $Z_d$ is the dust particle charge number, $Z_d = Q_d/e$, and $e$ is the elementary electric charge. For this calculation we used the plasma parameters measured by Langmuir probe [18] and the ‘effective’ ion number density in the dense dust cloud $n_i^{eff}$ determined in section 5 of this article. The dust particle charge $Q_d$ was calculated for the case of slightly collisional plasma [13]. The results of the calculations of $Q_d$, $C_{DAW}$, and other plasma parameters are presented in table 1. The dust translational temperature $T_d$ was estimated by measuring isotropic random velocities of the dust particles.
An accuracy of these measurements was restricted by the CCD matrix spatiotemporal resolution and consisted of about $3 \text{ mm s}^{-1}$, which corresponds to $T_d \sim 0.8 \text{ eV}$. Dust particle random velocities with the stated instrumental accuracy were not detected. Therefore the dust translational temperature was taken as $T_d < 0.8 \text{ eV}$, which corresponds to the strongly coupled dusty plasma state with $\Gamma_d \gg 1$. As one can see in figure 4, just after the wave excitation ($t \approx 30–50 \text{ ms}$) the wave speed $D \approx 10 \text{ cm s}^{-1}$ corresponded to the Mach number $M \approx 4$. During the next period, $50 \text{ ms} < t < 120 \text{ ms}$, the wave speed $D$ reduced till the $D = 2.5 \text{ cm s}^{-1}$ ($M = 1$) and stabilized at this value during $120 \text{ ms} < t < 200 \text{ ms}$.

Thus, observing the supersonic dust disturbance propagation $D$ and the narrow wave width $h$, we can state that the observed disturbance is the inertial elastic DA SW transforming finally into the DAW.

3. Determination of dust pressure

The application of the polarity switching dc discharge mode (dc/ps-mode) in our experiment gives us the unique possibility to use the momentum conservation law to determine the electrostatic pressure of the dust component after the DA SW front. In the inertial coordinate system moving with the SW front, the mass conservation and the momentum conservation equations (Rankine–Hugoniot relations) can be written as

$$\rho_1 D = \rho_2 u,$$  

and

$$p_1 + \rho_1 D^2 = p_2 + \rho_2 u^2,$$

respectively, where $D$ and $u$ are the dust particle velocities with respect to the SW front just before and after the SW, respectively, and $\rho_i$ and $p_i$ are the specific mass density of the dust cloud and the pressure of the dust component before (after) the SW. Then the pressure jump on the SW front can be expressed as

$$\Delta p = p_2 - p_1 = D^2 \rho_1 \left(1 - \frac{\rho_1}{\rho_2}\right).$$

As soon as the dust particles passing the DA SW were carried along by the accompanying neon gas flow, the last one provided an additional dust pressure $\delta p_d$ after the SW. The neutral drag force $f_{dn}$ acting on a dust particle is

$$f_{dn} = m_d \nu_{dn} \nu_d,$$

where $\nu_{dn}$ is the characteristic frequency of the dust-neutral collisions and $\nu_d$ is the dust particle velocity with respect to the neutral gas. The friction time $\tau_{dn} = \nu_{dn}^{-1}$ for the used dust grains calculated by the Epstein formula was equal to 26 ms (under assumption of full accommodation). Then the neutral drag force $F_{dn}$ dragging the unit volume of the dust particles with the number density $n_d$ is given by

$$F_{dn} = m_d \nu_{dn} \nu_d n_d = \rho_d \nu_d \nu_{dn}.$$

Within the DA SW width $h$, the values $n_d$ and $\nu_d$ strongly depend on the axial coordinate $x$. Using the linear dependence $n_d(x)$ within the SW width and taking into account that the dust particle velocity with respect of the neutral gas linearly grows from 0 to $D(\rho_1/\rho_2)$, we can determine the pressure $\delta p_{dd}$ as
\[ \delta p_{dd} = \int_0^h \rho_d(x)u_d(x)dx = \frac{1}{2}D \left( \rho_{d1} - \rho_{d2} \right)u_{dd}h. \] (9)

Then, the full dust pressure jump \( \Delta p_d \) on the DA SW will be equal to

\[ \Delta p_d = \delta p_d + \delta p_{dd} = D^2 \rho_1 \left( 1 - \frac{\rho_{d1}}{\rho_{d2}} \right) + \frac{1}{2}D \left( \rho_{d1} - \rho_{d2} \right)u_{dd}h. \] (10)

The measured dust particle mass densities \( \rho_{d1} \) and \( \rho_{d2} \) and the calculated dust pressure jump \( \Delta p_d \) for different time after the DA SW excitation are presented in figure 5. As soon as the relative compression \( \rho_{d2}/\rho_{d1} \) is high, the initial dust pressure \( p_{d1} \) is negligible with respect to the dust pressure \( p_{d2} \) after the DA SW and the jump of dust pressure \( \Delta p_d \approx p_{d2} \). A comparison of the values \( \delta p_d \) and \( \delta p_{dd} \) in figure 5 testifies to the minor role of the neutral drag forces on the formation of the DA SW pressure jump \( \Delta p_d \). Hence the DA SW can be considered as the inertial elastic wave. Note that the neutral gas pressure is orders of magnitude higher than the dust component pressure, so the dust component cannot involve neutral gas in any motion. Furthermore, the speed of sound in neon is about \( 10^4 \) faster than the DAW speed \( C_{DAW} \). Hence the dust density waves are almost independent of the gas acoustic waves. Finally, the experimentally determined dependence of the dust particle pressure \( p_{d2} \) after the DA SW front as a function of the specific mass density of the dust cloud \( \rho_{d2} \) is presented in figure 6. This is the Hugoniot percussive adiabat for the dust subsystem of the gas discharge dusty plasma.

4. Shock waves in a strongly coupled dissipative Yukawa system

The obtained experimental data on the dusty plasma shock compressibility, shown in figure 6, contain important information on the intergrain interaction potential \( U_{dd}(r) \). According to [19], the pressure of matter after the shock wave front \( p_z \) in the liquids or solids can be written as
\[
p_2 = p_2^c + \frac{G}{\nu_2} \left( \frac{\Delta p (\nu_1 - \nu_2)}{2} - \int_{\nu_2}^{\nu_1} p_2(v)dv \right), \tag{11}\]

where \( p_2^c \) is the so called ‘cool’ pressure after the SW front, corresponding to the matter’s elasticity at a low temperature, \( \Delta p = p_2 - p_1 \) is the pressure jump in the SW, \( \nu_1 \) and \( \nu_2 \) are the specific volumes of the matter before and behind the compression front, correspondingly, \( G \) is the Gruneisen coefficient, which characterizes the relation between thermal (‘hot’) pressure and thermal motion of atoms of condensed matter. The ‘cool’ pressure \( p_2^c \) is determined by the mutual repulsion of atoms (or molecules) in liquids or solids and can be calculated using the specific internal energy density \( u = u(V) \)

\[
p_2^c = -\left( \frac{\partial (uV)}{\partial V} \right)\]

where \( V \) is the considered volume of the media.

In this work we have adapted the general equation (11) for the system of the charged dust particles immersed into the low-pressure gas discharge plasma. In this case, taking into account the energy dissipation by the dust particles by gas friction during the passage of the wave front as \( \frac{1}{2} \alpha \rho d^2 (v_{d1} - v_{d2}) \), where \( \alpha = \delta p_d d \Delta p_d \) (section 3), we obtain

\[
p_{d2} = p_{dc2} + \frac{G_{d2}}{\nu_{d2}} \left( \frac{1}{2} \Delta p_{d_2} (v_{d1} - v_{d2}) - \int_{\nu_{d2}}^{\nu_{d1}} p_{dc_2}(v_d)dv_d - \frac{1}{2} \alpha \rho_{d2} (v_{d1} - v_{d2}) \right) \tag{11a}\]

where all the used parameters correspond to the dust subsystem. Neglecting the dust pressure \( p_{d1} \) before the DA SW in comparison with the pressure behind the wave front \( p_{d2} \), we obtain the
The following expression for the pressure of the dust component after the SW front

\[
p_{dc} = \frac{p_{dc} - G_{d} \frac{1}{v_{d2}} \int_{v_{d2}}^{v_{u}} p_{dc}(v_{d}) dv_{d}}{1 - \frac{1}{2} \frac{v_{d1}}{v_{d2}} (1 - \alpha)}.
\] (13)

The ‘cool’ dust pressure \(p_{dc}\) of the dust subsystem has been calculated using the inner energy density \(u_{d} = u_{d}(V)\) of the dust subsystem. According to the numerous theoretical [15, 16, 20] and experimental [21, 22] investigations the interaction potential of small charged dust grains \(U_{dd}(r)\) in a low-pressure gas discharge plasma up to the distances of a few Debye lengths is adequately approximated by the Debye–Hückel (or Yukawa) form

\[
U_{dd}(r) = \frac{Q_{d}^2}{4 \pi \varepsilon_{0} r} \exp \left( -\frac{r}{\lambda_{scr}} \right),
\] (14)

with two parameters—the dust particle charge \(Q_{d}\) and the screening length \(\lambda_{scr} \approx \lambda_{Dd}\). Strongly interacting particle systems with the interaction potential defined by equation (14) are often called the Yukawa systems.

Thermodynamic properties of Yukawa systems have been discussed in many papers [23–27]. The internal energy density of the Yukawa system is given by the expression

\[
u_{d} = 2 \pi n_{d} \int_{0}^{\infty} g(r) r^{2} U_{dd}(r) dr = \frac{Q_{d}^2 n_{d}^{2} \lambda_{scr}^2}{2 \varepsilon_{0}} + u_{corr},
\] (15)

where \(g(r)\) is the radial pair correlation function, the first term in the right side is the energy density in mean-field approximation [26], and the second term

\[
u_{corr} = 2 \pi n_{d} \int_{0}^{\infty} \left( g(r) - 1 \right) r^{2} U_{dd}(r) dr
\] (16)

is the correlation energy density [27]. For large coupling parameters \(\Gamma > 10\) the correlation energy \(u_{corr}\) can be approximated as [27]

\[
u_{corr} = -0.896 n_{d} \frac{Q_{d}^2}{4 \pi \varepsilon_{0} d} \exp \left( -0.604 \frac{d}{\lambda_{scr}} + 0.0005 \left( \frac{d}{\lambda_{scr}} \right)^{4} \right),
\] (17)

where \(d = \left( 4 \pi n_{d} / 3 \right)^{1/3} \) is the Wigner-Seitz radius.

Then the cool pressure \(p_{dc}\) can be obtained by

\[
p_{dc} = -\frac{\partial (u_{d} V_{d})}{\partial V} = n_{d} \frac{\partial (u_{d} / n_{d})}{\partial n_{d}}
\]

\[
= \frac{Q_{d}^2}{4 \pi \varepsilon_{0} d} n_{d} \left[ 2 \pi n_{d} \lambda_{scr}^2 d - \frac{0.896}{3} \exp \left( -0.604 \frac{d}{\lambda_{scr}} + 0.0005 \left( \frac{d}{\lambda_{scr}} \right)^{4} \right) \right]
\times \left[ 1 + 0.604 \frac{d}{\lambda_{scr}} - 0.003 \left( \frac{d}{\lambda_{scr}} \right)^{4} \right].
\] (18)
The Gruneisen parameter $G_d$ for the dust component was calculated by the equation [19]:

$$G_d = -\frac{\partial \ln (\nu)}{\partial \ln (V)} \approx \frac{\partial \ln (\omega_{pd})}{\partial \ln (n_d)},$$

(19)

where $\nu$ is the characteristic frequency of the media atoms in their atomic cells replaced by the dusty plasma frequency $\omega_{pd}$ for the dusty plasma. Then we have

$$G_d \approx \frac{1}{2} + \frac{n_e}{Q_d} \frac{\partial Q_d}{\partial n_d}.$$  

(20)

Because the dust particle charge $Q_d$ decreases with dusty number density, the Gruneisen parameter for a dusty plasma is small and lies between 0.2 and 0.3 in our case.

5. Three approaches for calculation particle charges and screening length inside a dense dust cloud

To calculate the dust pressure $p_d$ behind the DA SW front we need to know the dust particle charge $Q_d$ in the dense dust cloud. The dust grain charge $Q_d$ was calculated by a solution of the balance equation for the electron $I_{de}$ and ion $I_{di}$ fluxes on a dust particle. The electron current $I_{de}$ was calculated for the Maxwell electron energy distribution function

$$I_{de} = \pi n_e a^2 v_e \exp \left(\frac{e\varphi}{T_e}\right)$$

(21)

where $n_e$ is the electron number density, $a$—is the gain radius, $e$ is the elementary charge, $\varphi$ is the surface potential of the grain, $T_e$ is the electron temperature, and $v_e = \sqrt{8k_B T_e/(\pi m_e)}$ is the mean electron velocity ($m_e$—electron mass). The electron number density $n_{e0} = 1.2 \cdot 10^8$ cm$^{-3}$ and temperature $T_e = 7$ eV in the uniform positive column of the dc discharge without the dusty cloud were measured in the laboratory by a Langmuir probe [18] for the used discharge regime. Due to the relatively high electron temperature, the electron number density inside the dust cloud $n_{ed}$ is close to the unperturbed electron number density in the positive column $n_{e0}$. During the experiment, as soon as the manipulating electrode was loaded by a negative potential and provided additional dc discharge current $I_{EM} = -0.3$ mA in addition to the basic discharge current $I_{DC} = 1$ mA, due to $I_{EM}$ the electron number density $n_{e0}$ was enlarged by about 30%. To cover a range of all possible electron concentrations $n_{e0}$ and to test the sensitivity of the calculated $p_d$ from the used values of $n_{e0}$, we have used two values of $n_{e0} = 1 \cdot 10^8$ cm$^{-3}$ and $4 \cdot 10^8$ cm$^{-3}$. The ion current on the grain $I_{di}$ was calculated by the approximation [13] for conditions of the weakly collisional plasma

$$I_{di} = \pi n_{i0} a^2 v_i \left(1 - \frac{e\varphi_i}{T_i}\right) \left(1 + d(l_r, a, \varphi_i)\right)$$

(22)

where $n_{id}$ is ion number density inside the dust cloud, $d(l_r, a, \varphi_i)$ is a function of the mean free path length of ions $l_r$, $a$ is the grain radius, and $\varphi_i$ is the grain surface potential.

Three approaches for the evaluation of the ion number densities $n_{id}$ inside the dense dust cloud were used with the aim of their subsequent comparative analysis. In the first, simplest approach (model I), we used a fixed (frozen) ion number density $n_{id(I)} = n_{i0} = n_{e0}$. In the second approach (model II) we used a spatially averaged ion number density resulting from the quasi-neutrality condition $n_{id(II)} = n_{e0} + |Q_d| n_{ed}/e$ with a fixed electron density and the particle charge.
calculated self-consistently for the particular dust particle number density \( n_d \). In our opinion, the most adequate approach for the real situation is the next, third one (model III). The ions in dusty clouds are distributed strongly non-uniformly. It is unlikely that (bounded) ions in the vicinity of one dust grain can essentially affect the ion flux and shielding of the other grains. In this case the effective ion concentration \( n_{id(III)} \) in the dust cloud has to be equal to those concentrations that would be present at the position of the dust particles after its removal, keeping the positions of all other dust particles the same, as shown in figure 7. According to the Debye–Hückel model the ion concentrations are determined by the electric potential \( \varphi \) as

\[
\begin{align*}
  n_{id(III)} &= n_{e0} \left( 1 - \frac{z_i \varphi}{k_B T_d} \right).
\end{align*}
\] (23)

Taking the electric potential \( \varphi \) as the space averaged one

\[
\bar{\varphi} = n_d 4\pi \int_0^\infty \varphi (r) r^2 dr = \frac{Q_d n_d}{\varepsilon_0 \lambda_D^2}
\] (24)

and taking into account the equation (2), we obtain a quadratic equation for \( n_{id(III)} \) with the solution

\[
\begin{align*}
  n_{id(III)} &= n_{e0} \left( 0.5 + \sqrt{0.25 + \frac{Q_d n_d}{z_i n_{e0}}} \right).
\end{align*}
\] (25)

The results of the calculations of the dust particle charges \( Q_d \) for different dust particle concentrations \( n_d \) for the model II and model III are presented in figure 8.

6. Results and discussions

The cool pressures \( P_{dc2} \) and the Hugoniot pressures \( p_{d2} \) behind the DA SW front with a correction due to neutral gas friction calculated for two electron number densities (\( n_{ed} = 1 \cdot 10^8 \text{cm}^{-3} \) and \( 4 \cdot 10^8 \text{cm}^{-3} \)) and with the three approaches (models I, II, and III) for calculation of the ion number densities \( n_{id} \) inside the dust cloud are presented in figure 6. Two main conclusions can be stated here. First, one can see that the calculated data strongly depend...

Figure 7. Sketch of the ion number density distribution \( n_i(x) \) within the dust cloud and definition of the effective ion number density \( n_{i-eff} = n_{id(III)} \).
on the model chosen for calculation of the ion number densities $n_{id}$ inside the dust cloud. A comparison of the experimental data with the calculated data explicitly verifies the model III for calculation of the effective ion number density, the dust particle charge, and the actual screening length inside the dense dust cloud. At the same time, the calculated data do not critically depend on the plasma concentration for $n_{e0} = n_{i0}$. The best agreement between experimental and simulated data was found for $n_{e0} = 2 \cdot 10^8 \text{cm}^{-3}$, which is in excellent agreement with the probe measurements. Hence, the proposed method can be successfully used for diagnostics of the inter-grain interaction potentials in dusty plasmas under the conditions of a low-pressure neutral gas. Second, the calculated ‘cool’ pressures $p_{dc2}$ (equation (18)) and ‘full’ pressures $p_{d2}$ (equation (13)) are very close to each other for all considered conditions. The percussive adiabats $p_{d2}$ lie sometimes below the corresponding cool isotherm $p_{dc2}$, which can be explained by the inaccuracy of the determination of the parameter $\alpha$. The small value of the Gruneisen parameter $G \approx 0.2–0.3$ and neutral gas friction make the compression nearly isothermal. It is interesting to estimate the electric potential jump on the shock wave front. Assuming that the gain of dusty particle energy is provided by the electric field, we can estimate $\Delta \phi_{sw} = p_{d2} (v_1 - v_2) n_d/2Q_d \approx 2 \ldots 20 \text{mV}$. Such a potential jump cannot essentially change the electron density, so the latter can be taken as constant.

Finally, we justify here our approach for the repulsion potential $U_{dd}(r)$ depending on the interparticle distance $r$. The cool pressure of dust particles $p_{cd}$ for any repulsion potential $\Phi(r)$ can be expressed via the integral

$$p_{dc} = \frac{2\pi}{3} n_0^2 \int_0^\infty g(r) r^3 F_{dd}(r) dr,$$

(26)
where $F_{dd}(r)$ is the pair interaction force between charged dust particles. Since the potential (14) is the product of the electrical Debye–Hückel potential $U_d(r)$ created by one charged particle on the electrical charge $Q_p$ of the second charged particle, $U_{dd}(r) = U_d(r) \cdot Q_d$, the corresponding repulsion force $F_{dd}(r)$ will be determined by the derivative

$$F_{dd} = -\left( \frac{\partial U_{dd}}{\partial r} \right)_{Q_d=\text{const}, \lambda_{scr}=\text{const}},$$

assuming a constant dust particle charge $Q_d$ and screening length $\lambda_{scr}$. After the substitution of equation (27) into equation (26) and calculation of the work for a small isotropic expansion, one can obtain

$$p_{dc} = n_d^2 \frac{\partial (u/n_d)}{\partial n_d} \bigg|_{Q_d=\text{const}, \lambda_{scr}=\text{const}}$$

with the derivative taken at constant dust particle charge $Q_p$ and screening length $\lambda_{scr}$. The internal energy density $u$ is still defined by (15) and (14), but it is no longer the internal energy density of the system. In other words, the pressure $p_{dc}$, which is determined by the interaction (repulsion) between the particles in a given state of the system, is the same for the system with the interaction potential $U_{dd}(r)$ independent of the dust particle number density $n_d$ and for the system with the interaction potential $U_{dd}(r)$ parametrically dependent on the dust particle number density $n_d$—if the positions of the particles and the forces between them are the same for these two cases. Therefore, for the particle systems with a variable particle charge and screening length, one can use the results of the theory for Yukawa systems with constant potential parameters, if the differentiation in equation (18) is performed with constant interaction parameters $Q_d$ and $\lambda_{scr}$.

### 7. Conclusion

For the first time an externally excited elastic inertial DA SW propagating in an isotropic three-dimensional dusty plasma has been experimentally investigated in this study under microgravity conditions. The extended dust cloud was formed in the uniform cylindrical positive column of a dc discharge operated in the cylindrical discharge glass tube of the PK-4 setup. The axial isotropy of the dusty plasma was achieved by applying a dc discharge with the switchable polarity mode at a frequency of 1 kHz, which is much higher than the dusty plasma frequency $\omega_{pd}$. The dusty plasma was characterized as a strongly coupled system in the liquid phase.

The inertial elastic DA SW has been excited electrostatically with the help of an additional manipulating electrode installed inside the discharge chamber. The following parameters of the DA SW have been derived by analyzing images of the video camera—speed, thickness, compression factor, and dust particle number densities. Three main ingredients classify the observed disturbance as the shock wave—the high compression factor, the supersonic velocity, and the wave front steepening.

The shock-jump Hugoniot relations were used to determine the electrostatic pressure of the dust component after the DA SW front. Consistently applying these relations for subsequent time steps we have experimentally determined the percussive adiabat of the 3D dusty plasma cloud. The obtained data were compared with those calculated in the approximation of the
strongly coupled Debye–Hückel system with two variable parameters—the dust particle charge and the screening length. The particle charges and the screening lengths were calculated according to three physical approaches for ion number densities inside the dust cloud. The results of the comparison clearly indicate that the dust particle charge and the screening length inside a dense dust cloud in a plasma are not determined by the total ion number density \( n_{id} = n_e + |Q_d| n_d / e \), but rather by those ‘effective’ ions which are not bound to the negatively charged dust grains. In other words, it is necessary to take into account the strong correlation of ion positions with respect to the dust particle positions when one calculates the ion fluxes on the dust particles and the screening length. Thus, this work presents a new experimental approach for the investigation of dense dusty plasma clouds.

Acknowledgments

This work was supported by the German Aerospace Center (DLR) under grants 50 WM 0804 and 50 WM 1150, by the ESA at the 49th parabolic flight campaign, by the Russian Ministry of Education and Science, by the Program of Fundamental Researches of the Presidium of the Russian Academy of Sciences, and by the Russian Foundation for Basic Research Grant No. 13-02-01393 a. The authors highly appreciate the excellent engineering support throughout this work by Karl Tarantik, Sebastian Albrecht, Christian Deysenroth, and Christian Rau from the Max-Planck-Institut für extraterrestrische Physik.

References

[1] Fortov V E and Gregor E M (ed) 2010 Complex and Dusty Plasmas: From Laboratory to Space (Boca Raton, FL: CRC Press)
[2] Shukla P K and Mamun A 2002 Introduction to Dusty Plasma Physics (Bristol: Institute of Physics Publishing)
[3] Shukla P K 1989 Proc. 1st Capri Workshop on Dusty Plasmas (Capri, May–June 1989) pp 38–9
[4] Rao N N, Shukla P K and Yu M Y 1990 Planet. Space Sci. 38 543
[5] Barkan A, Merlino R L and D’Angelo N 1995 Phys. Plasmas 2 3563
[6] Fortov V E et al 2003 Phys. Plasmas 10 1199
[7] Samsonov D et al 2002 Phys. Rev. Lett. 88 095004
[8] Zhdanov S et al 2010 EPL 89 25001
[9] Samsonov D et al 2004 Phys. Rev. Lett. 92 255004
[10] Fortov V E et al 2005 Phys. Rev. E 71 036413
[11] Heinrich J, Kim S-H and Merlino R L 2009 Phys. Rev. Lett. 103 115002
[12] Kilgore M D et al 1993 J. Appl. Phys. 73 7175
[13] Zobnin A V et al 2008 Phys. Plasmas 15 043705
[14] Piel A et al 2006 Phys. Rev. Lett. 97 205009
[15] Ratynskaia S et al 2004 Phys. Rev. Lett. 93 085001
[16] Sheridan T E, Nosenko V and Goree J 2008 Phys. Plasmas 15 073703
[17] Samsonov D et al 2003 Phys. Rev. E 67 036404
[18] Fortov V E et al 2005 Plasma Phys. Control. Fusion 47 B537
[19] Zel’dovich Ya B and Raiser Yu P 1967 Physics of Shock Waves and High Temperature Hydrodynamics Phenomena vol 2 (New York: Academic Press)
[20] Khrapak S A, Klumov B A and Morfill G E 2008 Phys. Rev. Lett. 100 225003
[21] Konopka U, Morfill G E and Ratke L 2000 Phys. Rev. Lett. 84 891
[22] Fortov V E, Petrov O F, Usachev A D and Zobnin A V 2004 Phys. Rev. E 70 046415
[23] Rosenfeld Y 1993 Phys. Rev. E 47 2676
[24] Farouki R T and Hamaguchi S 1994 J. Chem. Phys. 101 9885
[25] Hamaguchi S, Farouki R T and Dubin D H E 1997 Phys. Rev. E 56 4671
[26] Henning C, Ludwig P, Filinov A, Piel A and Bonitz M 2007 Phys. Rev. E 76 036404
[27] Totsuji H 2006 J. Phys. A: Math. Gen. 39 4565