A generation mechanism of chorus emissions using BWO theory

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Abstract: In this paper, discrete VLF chorus emissions recorded at low latitude ground station Jammu (geomag. Lat. = 22° 26' N, L = 1.17) are reported and their characteristics based on complete spectral analysis have been carried out. These discrete chorus emissions are generated during a strong geomagnetic storm period of 2-7 May, 1998. We have computed the sweep rate, repetition period, source region, and drift rate of the individual chorus elements. It is observed that the sweep rate increases with time. To explain the various temporal and spectral features of these emissions, a possible generation mechanism has been presented based on the backward wave oscillator (BWO) regime in the magnetospheric cyclotron maser. On the basis of this model, we have computed some discrete chorus emission parameters as well as magnetospheric parameters relevant to the generation process. A comparison of the computed and observed magnetospheric parameters has been presented. These results show a good agreement with the BWO model.

1. Introduction:
Chorus are characterized by a sequence of discrete elements depending upon their spectrum as intense tones, short duration, generally rising and often overlapping and occurring in association with disturbed magnetospheric conditions [1,2]. It is generally accepted that the generation mechanism of these emissions is connected with the cyclotron instability of whistler-mode waves and radiation belt electrons [3]. Detailed study of chorus emissions can be referred to Helliwell[4], Sazhin and Hayakawa[2]. The generation mechanism of these emissions are confirmed using active experimental and theoretical research [5-9]. Recently Santolik [10] has presented new results of investigations of whistler-mode chorus emissions.

Helliwell [3] was the first to suggest the idea of chorus emission and explained how the frequency spectrum of discrete elements is formed. He further developed a phenomenological model to study the generation of discrete chorus emissions. In this model, the idea of second – order cyclotron resonance of energetic electrons with whistler-mode waves first formulated and it explained about the numerous types of discrete emissions. Further analytical and computational calculations [11] confirmed the idea of the second order cyclotron resonance and permitted a connection to be made through non-linear currents, between the parameters of a triggered emission and quasi-monochromatic whistler wave.

Trakhtengerts [6] suggested a generation mechanism of discrete chorus emissions based on the backward wave oscillator (BWO) regime of magnetospheric cyclotron maser [12]. Trakhtengerts

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revealed that the step like deformation could be the cause of new generation regimes of the cyclotron instability, leading to a succession of discrete chorus signals. Recently Singh and Singh [8] have applied BWO theory successfully to explain the generation of discrete VLF emissions observed at low latitudes.

In the present paper, we present a detailed spectral analysis of the discrete VLF chorus emissions recorded at low latitude ground station Jammu (geomag. Lat = 22° 26' N; L = 1.17) during the routine recording of whistlers from January 1998 to December 1998. We also present the generation mechanism of these emissions based on the backward wave oscillator regime in the magnetospheric cyclotron maser [12, 6]. An attempt has been made to determine various discrete chorus emission parameters as well as magnetospheric parameters relevant to the generation process. Finally, derived and estimated parameters are compared and the results are discussed.

2. Experimental data and analysis:
At low latitude ground station Jammu (L = 1.17), the broad band discrete chorus emissions were received by a T-type antenna, pre and main amplifiers and tape recorder having band width of 50 Hz – 15 kHz. The VLF data were stored on the magnetic tapes, which were analyzed using a Raven Software. In this paper, the VLF discrete chorus emissions recorded in large numbers during the night hours of 4 – 5 May 1998 is analyzed. These emissions were recorded during strong geomagnetic period of 2 – 7 May 1998 (Dst Index -204 nT) as shown in figure 1. The emissions were recorded in

![Figure 1: Geomagnetic storm period of 2 – 7 May 1998. Occurrence of chorus emissions marked by arrow.](image-url)
recovery phase of geomagnetic storm period. The most intense discrete chorus emissions were started during the night of 4 – 5 May, when the magnetic activity was highest ($\Sigma K_p = 43$) during the recovery phase of sub-storm. Typical frequency-time spectrogram of discrete VLF chorus emissions recorded during nighttime at 2250 – 2330 hrs (IST), on 4 May 1998 at Jammu are shown in figure 2. Discrete VLF chorus emissions (fallers) in sufficient numbers in the frequency range 2.5 KHz to 5.5 KHz were observed for about 1 hour. For these chorus events the sweep rate $df/dt$, increases with increasing frequency. The repetition period of these events are not equal everywhere but increases with time. Figure 2 shows the chorus emissions observed in four different stage of time. Figure 2(a) shows the chorus emissions in the beginning of the observations at 22:50 IST. Figure 2(b) shows chorus emissions at 23:00 IST with the increased repetition period. Figure 2(c) shows the chorus emissions at 23:20 IST and figure 2(d) shows discrete chorus emissions at end of observations.

![Figure 2: Examples of VLF discrete chorus emissions recorded at Jammu on 4th May 1998 at 22:50 to 23:30 Hrs IST.](image)
Figure 2(a) shows about twenty five discrete VLF emissions, which are falling tones observed at 2250 hrs IST. The observed discrete emissions have the following parameters: \( f_{\text{min}} = 3.1 \text{ kHz} \), \( f_{\text{max}} = 4.4 \text{ kHz} \), average \( f_{\text{UB}} = 4.4 \text{ kHz} \), average frequency sweep rate \( df/dt = 9.5 \text{ kHz/sec} \) and average duration of each discrete emissions \( T = 0.15 \text{ sec} \). Figure 2(b) shows about fifteen discrete chorus emissions observed at 2300 Hrs IST in the frequency range of about 3.0 – 5.3 kHz. In this case the observed discrete VLF chorus emissions have the following parameters: \( f_{\text{min}} = 3.4 \text{ kHz} \), \( f_{\text{max}} = 4.9 \text{ kHz} \), average \( f_{\text{UB}} = 4.9 \text{ kHz} \), average frequency sweep rate \( df/dt = 7.5 \text{ kHz/sec} \) and average duration of each discrete emissions \( T = 0.23 \text{ sec} \). In third case which is shown in figure 2(c), we have shown frequency-time spectrogram of chorus emissions at 2320 Hrs IST in the frequency range of about 3.0 – 5.3 kHz. The observed discrete emissions have the following parameters: \( f_{\text{min}} = 3.4 \text{ kHz} \), \( f_{\text{max}} = 4.7 \text{ kHz} \), average \( f_{\text{UB}} = 4.7 \text{ kHz} \), average frequency sweep rate \( df/dt = 4.3 \text{ kHz/sec} \) and average duration of each discrete emissions \( T = 0.47 \text{ sec} \). In the end of the observation at 2327Hrs IST, figure 2(d) shows the chorus emissions which are in the frequency range of 3.5-6.2 kHz. The observed discrete emissions for the set have the following parameters: \( f_{\text{min}} = 4.8 \text{ kHz} \), \( f_{\text{max}} = 5.4 \text{ kHz} \), average \( f_{\text{UB}} = 5.4 \text{ kHz} \), average frequency sweep rate \( df/dt = 0.86 \text{ kHz/sec} \) and average duration of each discrete emissions \( T = 1.10 \text{ sec} \).

3. Generation Mechanism of Discrete Chorus Emissions:

Trakhtengerts [12, 6] suggested a new generation of MCM regime, the backward wave oscillator (BWO) regime, developed in electronic devices [13]. For a generator to operate in the BWO regime certain conditions have to be satisfied. The first condition requires that the interacting wave phase velocity component along the magnetic field should be opposite to the direction of the electron motion. The cyclotron resonance condition is written as

\[
\omega - \omega_b \gamma = kv_{||}
\]

where \( \omega_b \) is the electron gyro-frequency, \( \omega \) is the wave frequency, \( k \) is the wave vector, \( v_{||} \) is the field aligned component of the electron velocity and \( \gamma = (1 - v^2/c^2)^{-1/2} \) is the relativistic correction factor. The second condition is the existence of well-organized electron beam with small velocity dispersion in the region of discrete emission generation. The step-like deformation of energetic electrons distribution function ensures large growth rate (\( \gamma_{\text{HD}} \)) of whistler waves and transition to the BWO regime. For a dipolar field line, the interaction length \( l \) between the whistler waves and energetic electrons in limited conditions is given by [3, 6],

\[
l_{\text{min}} \leq l \leq l_{\text{max}} \tag{2}
\]

where,

\[
l_{\text{min}} = (R_e^{2/3} L^{2/3}/k^{1/3}) \tag{3}
\]

and

\[
l_{\text{max}} = \frac{2\sqrt{2}}{3} L R_e \left[ 1 + 4 \left( \frac{f_{\text{UC}} - f_{\text{LC}}}{f_b} \right) \right]^{1/2} - 1^{1/2} \tag{4}
\]

where \( L \) is the McLlwain parameter, \( R_e \) is the Earth radius, \( f_{\text{UC}} \) and \( f_{\text{LC}} \) is the upper and lower cut-off frequency of the discrete chorus emission, wave number \( k \) is given by the whistler dispersion relation as \( k = [(\omega \cdot \omega)^{1/2}/c] (\omega_b - \omega)^{1/2}) \), where \( c \) is the velocity of light.

The growth rate of BWO regime with periodic modulation, which is connected with the repetition period of discrete VLF chorus emissions, is estimated as [12],

\[
\gamma_{\text{BWO}} = \frac{2 p (p-1)}{T} \tag{5}
\]
here \[ T = 1.5\left( \frac{\gamma + \gamma_g}{\gamma^2} \right) \] (6)

and \[ p \approx 2 \]

The BWO generation regime starts when a quasi-monochromatic wavelets from the equatorial region and interacts with the energetic electron in the beam may trigger signal with a rising/falling frequency [6]. According to theory of Ommura et. al [14] and Trakhtengerts [6], maximum value of the non-linear growth rate can be written as

\[
\frac{df}{dt} = \frac{1}{2\pi} \frac{\omega}{\omega_H + \omega} \Omega_H^2 \left[ 2S + \frac{3v_m}{\Omega_H^2} \frac{d\omega_H}{dz} \right]
\] (7)

where the value of inhomogeneity factor \( S \) corresponding the maximum value of the non-linear growth rate lies between 0.2 \( \leq |S| \leq 0.8 \).

We can determine the value of wave magnetic field amplitude \( B \) as

\[
B_\perp = \left( \frac{32}{3\pi} \right)^2 \frac{\gamma^2_{BWO} B_L}{k\omega_b}
\] (8)

where \( B_L \) is the geomagnetic field.

In our case the generated signal propagates along the geomagnetic field line from the equator towards the increasing magnetic field, so \( S < 0 \) [14]. Putting \( S = -0.5 \) and substituting in equation (8) the value of \( \Omega_H \), we can write the frequency sweep rate as [6],

\[
\frac{df}{dt} = 1.5 \frac{\omega^2_{BWO}}{2\omega + \omega_H} \left( 1 + S_0 \right)
\] (9)

where

\[
f = \omega/2\pi, S_0 = (\nu/3\gamma^2_{BWO})(\partial\omega/\partial z)
\]

Using the above equations (1) to (9) we have computed various discrete chorus VLF emissions parameters and compared with our observations.

4. Results and Discussions:
The dynamic spectrum of discrete chorus VLF emissions observed at low latitude ground station Jammu shows that the occurrence rate of these emissions is low and sporadic. It is also seen that the number of discrete chorus emissions observed during strong magnetic storm period are large. The most frequent observations of chorus events by satellites near the geomagnetic equator [15, 16] support the idea that the source of chorus emissions are mostly localized near equatorial region. The generation of chorus emissions is derived by the injection of substorm electrons [15,17]that interact with whistler mode waves through the cyclotron resonance [18]. By analyzing POLAR satellite data Lauben et al. [19] studied various source characteristics of chorus emissions and indicated the source region lies near the magnetic equator.

The maximum of chorus intensity is found to move towards lower latitudes with the increase in the geomagnetic activity, which may be due to the motion of the propagating channels and source region towards the Earth [20, 21]. We have followed the upper boundary frequency (UBF) method...
developed by Smirnova [21], to find out the location of source for the recorded discrete VLF chorus emissions. The upper boundary frequency of the ground based observation of discrete chorus events is determined on the assumption of dipolar geomagnetic field configuration, by the half equatorial electron gyro-frequency in the generation region, irrespective of the latitude of the observation station. According to Smirnova [21], the L-value of the observed discrete VLF chorus emissions source is written as

\[ L = \left( \frac{440}{f_{UC}} \right)^{1/3} \]  

(10)

Where \( f_{UC} \) is the upper cut-off boundary frequency of the observed discrete chorus events in kHz. Using equation (10), the L-value of the source region for the reported emissions is found to be \( L_{\text{source}} = 4.62 \). The higher L-value of the source region compared to the observation station Jammu (\( L = 1.17 \)) shows that the wave may have propagated towards significantly lower latitudes [20].

To examine the generation mechanism of discrete VLF chorus emissions, we have computed various parameters of these emissions using the above equations (1) to (10). The discrete chorus emissions observed at Jammu, which is shown in figure 2 (a,b,c,) and the equatorial electron gyro-frequency is \( f_b = 8.92 \) kHz, 9.66 kHz and 9.28 kHz for set (a,b,c) respectively. According to Carpenter and Anderson [22], empirical equatorial electron density profile model the electron density is taken as \( \sim 16 \) electrons cm\(^{-3}\) and the corresponding plasma frequency is \( f_p \sim 36 \) kHz. Considering a frequency \( f = f_b/3 \) we find from the dispersion relation that \( k \sim 0.53 \) km\(^{-1}\). From equation (3) – (5) the interaction length \( l \) should lies between \( l_{\text{min}} = 1220 \) km, 1098 km and 1220 km and \( l_{\text{max}} = 15860 \) km, 14274 km and 13786 km for figure 2 (a,b,c) respectively. Singh and Patel [8] have reported maximum value of interaction length 1000 km and 14000 km for the Gulmarg and Maitri station respectively. From equation (1), using the resonance condition we obtained the group velocity \( v_g \sim 2\omega^2/(\omega k) \sim 2.33 \times 10^7 \) m/s, 2.54 \( \times \) 10\(^7\) m/s and 2.44 \( \times \) 10\(^7\) m/s, which is lower than the resonant electrons velocity \( v \sim 6.99 \times 10^7 \) m/s, 7.63 \( \times \) 10\(^7\) m/s and 7.32 \( \times \) 10\(^7\) m/s for set (a,b,c) respectively.

From figure 2 (a,b,c) the modulation period which determines the repetition period of discrete chorus elements is found to lie between \( T = 0.10 \) and 1.3 s for first set, 0.09 and 1.17 s for second set and for the last set, 0.1 and 1.13 s respectively. Our experimental value of \( T = 0.2 \) s, 0.31 s and 0.43 s is in this interval and suggest that the real interaction length is around 2500 km, 4000 km and 5000 km respectively. Using the value of \( T = 0.2 \), 0.31, 0.43 s and \( p = 2 \) for every case in equation (5), we find the growth rate \( \gamma_{\text{BWO}} = 20, 12.9 \) and 9.3 s\(^{-1}\) respectively. Considering \( S_0 = 0.2 \) and 0.8 the frequency sweep rate \( df/d\tau \), (equation-7), is found to lie between 144 kHz/sec and 216 kHz/sec for first case, 59.9 kHz/sec and 89.86 kHz/sec for second case and the third case 31.14 kHz/sec and 46.7 kHz/sec, which is somewhat larger than the observed value of 11.01, 7.49 and 4.25 kHz/sec respectively. Using equation (8), we can compute the amplitude of the discrete VLF chorus emission, by putting \( \gamma_{\text{BWO}} = 20, 12.9, 9.3 \) s\(^{-1}\) for \( L = 4.62, 4.5, 4.56 \) as \( B_- = 1.39, 0.53 \) and 0.29 my for figure 2 (a,b,c) respectively. The computed parameters of observed discrete VLF chorus emissions derived from backward wave oscillator theory are given in Table 1. Although the theoretical estimate for the repetition period is rather rough but the observed period is well centered in the predicted interval. The average frequency sweep rate determine by the theory is consistent with the observations.

5. Conclusions:
In this paper, we have reported discrete VLF chorus emissions recorded at low latitude ground station Jammu. The generation mechanism for various temporal and spectral features of these events is presented on the basis of backward wave oscillator (BWO) generator operating in the magnetosphere. On the basis of BWO theory, various discrete VLF chorus emission parameters as well as some magnetospheric parameters are computed. These computed parameters are comparable to that of the observed values, which support the possibility for a wide use of discrete VLF chorus emissions data for the ground based diagnostics of the state of the magnetospheric plasma during sub storm period.

Further experimental and theoretical studies of discrete VLF chorus emissions at low latitude would definitely contribute to a more detailed understanding of this phenomenon.
Table 1: Observed and computed parameters of discrete VLF chorus emissions recorded at low latitude ground station Jammu on 4th May 1998 at 22:50 – 23:30 Hrs IST.

| Parameter   | Observed Set-I | Computed | Observed Set-II | Computed | Observed Set-III | Computed |
|-------------|----------------|----------|-----------------|----------|-----------------|----------|
| L<sub>Source</sub> | —             | 4.62     | —               | 4.5      | —               | 4.56     |
| F<sub>UB</sub> (kHz) | 4.45      | —        | 4.85            | —        | 4.66            | —        |
| f<sub>LB</sub> (kHz) | 2.98     | —        | 3.33            | —        | 3.43            | —        |
| T (Sec.)     | 0.2         | 0.10<T<1.3 | 0.31       | 0.09<T<1.17 | 0.43         | 0.10<T<1.13 |
| l (km)       | 2500       | 1220     | 4000            | 1098     | 1220            | 5000     |
|              |              | 15860    | 14274           | 1098     | 1220            | 13786   |
| df<sub>/df</sub> (kHz) | 11.01  | 144 (S=0.2) | 7.49   | 59.9 (S=0.2)  | 4.25     | 31.14 (S=0.2) |
|              |              | 216 (S=0.8) | 89.86 (S=0.8) | 46.70 (S=0.8) |
| γ<sub>w0</sub> (Sec<sup>-1</sup>) | —       | 20       | —               | 12.9     | —               | 9.3      |
| B<sub>L</sub> (my)       | —             | 3.15 × 10<sup>5</sup> | —       | 3.41 × 10<sup>5</sup> | —       | 3.28 × 10<sup>5</sup> |
| B<sub>W</sub> (my)       | —             | 1.39     | —               | 0.53     | —               | 0.29     |

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References:
[1] Helliwell R 1965 A Whistler an Related Ionospheric Phenomena (Stanford USA: Stanford Univ. Press)
[2] Sazhin S S and Hayakawa M 1992 Magnetospheric chorus emissions: a review Planet Spac. Sci. 40 681-97
[3] Helliwell R A 1967 Journal Geophys. Res. 72 4773-90
[4] Helliwell R A 1969 Low frequency waves in the magnetosphere Rev. Geophys. 7 281-303
[5] Smith A J, Freeman M P and Reeves G D 1996 Post midnight VLF chorus events, a substorm signature observed at the ground near L=4 Journal Geophys. Res. 101 24641-53
[6] Trakhtengerts V Y 1999 A generation mechanism for chorus emissions Ann. Geophys. 17 95-100
[7] Singh R, Patel R P, Singh R P and Lalmani 2000 An experimental study of hiss triggered chorus emissions at low latitude Earth Planet Space 52(1) 37-40
[8] Singh R P and Patel R P 2004 Hiss-triggered chorus emissions at Indian stations J. of Atmos. and Solar-Terr. Phys. 66 1027-33
[9] Singh A K and Ronnmark K 2004 A generation mechanism for VLF chorus emissions observed at low latitude ground station Annales Geophysicae 22 2067-72
[10] Santolik O 2008 New results of investigations of whistler-mode chorus emissions Non Linear Processes in Geophys 15 621-30
[11] Nunn D A 1974 Self-consistent theory of triggered VLF emissions Planet Spac. Sci. 22 349-78
[12] Trakhtengerts V Y 1995 Magnetosphere cyclotron maser: Backward wave oscillator generation regime Journal Geophys. Res. 100 17205-10
[13] Jhonson H R 1995 Backward wave oscillator *Proc. IEEE* **43** 684
[14] Omura M, Nunn D, Matsumoto H and Rycroft M J 1991 A review of observational, theoretical and numerical studies of VLF triggered emissions *J. of Atmos. and Solar-Terr. Phys.* **53** 351-68
[15] Tsurutani B T and Smith E J 1974 Postmidnight chorus: a substorm phenomenon. *Journal Geophys. Res.* **79** 118-27
[16] Burtis W J and Helliwell R A 1976 Magnetospheric Chorus: Occurrence pattern and normalized frequency *Planet space science* **24** 1007-24
[17] Bespalov P A and Trakhtengerts V Y 1986 The cyclotron instability of the Earth’s radiation belt *Reviews of Plasma Physics* edited by Leontovich M A **10** 155-292 Plenum, New York.
[18] Kennel C F and Petscheck H E 1966 Limit on stably trapped particle fluxes *Journal Geophys. Res.* **71** 1-28
[19] Lauben D S, Inan U S, Bell T F and Gurnett D A 2002 Source characteristic of ELF/VLF chorus *Journal of Geophy. Res.* **107** 1429-46
[20] Smirnova N A, Novkov Yu P, Kleimenova N G and Titova E E 1976 Some spectral peculiarities of VLF emissions registered on the Earth surface near the plasmapause projection *Journal of Atmospheric-Terrestrial Phys.* **38** 1247
[21] Smirnova N A 1984 A fine structure of the ground observed VHF chorus as an indicator of the wave particle interaction process in the magnetosphere *Planet Space Sci.* **32** 425
[22] Carpenter D L and Anderson R R 1992 An ISEE/Whistler model of equatorial density in the magnetosphere *Journal Geophys. Res.* **97** 1097-1108