The Application of Probabilistic Hesitation Fuzzy Linguistic in Urban Livability

Xiuming Chen*, Chouxuan Wang, Chenchen Zhang
Fuyang Normal University, School of Computer and Information Engineering, Fuyang 236037, China
Email: cxm9001@163.com

Abstract. With the continuous improvement of residents' living conditions and the improvement of spiritual civilization construction, urban livability has become the focus of attention of the government and urban residents, as well as a hot topic of research by experts and scholars at home and abroad. How to choose a suitable city to live in has become a group decision-making (GDM) problem that we have to deal with. However, the changeable decision environment leads to the fact that the decision information of the above GDM problem cannot be expressed by traditional display values. Therefore, we choose the probabilistic hesitation fuzzy linguistic term set (PHFLTS) and then study the consensus among preference relations (PRs). This paper will study the group consensus model based on PHFLTS. At present, it has been studied in supply chain management, public opinion prediction and other fields. This paper first defines the concept of PHF LTS and distance measure, then defines the similarity measure and the consistency index (CI) based on the distance measure, constructs the group consensus level (GCI), can express the uncertain preference information given by the decision maker more comprehensively. Finally, specific example is given to rank the cities and the future research direction is prospected.

Keywords: Probabilistic hesitant fuzzy sets; Urban livability; Preference relation; Consensus level.

1. Introduction
From slash-and-burn in the primitive society to the interconnection of everything in the information society, human beings have put forward higher and higher requirements for the livability of their gathering areas. As a symbol of carrying modern civilization, cities have highly developed secondary and tertiary industries. The development of these industries not only promotes human progress, but also inevitably tests the carrying capacity of urban environment: the one-sided pursuit of rapid industrial development, the extreme exploitation of natural resources, the urban environment level gradually deteriorated; With the continuous progress of urbanization, resulting in chaos of urban order and insufficient public resources[1]. How to balance the contradiction between livability of cities and development pains is related to the effect of ecological civilization construction. A convenient and accurate assessment is needed to compare and evaluate the trend of sustainable performance of urban construction environment, so as to help people make decisions [2]. In group decision making, the personal preference and environmental factors such as the complex decision-making, decision makers often have different evaluation results, so according to the evaluation results of group decision makers are satisfied results of the ranking, need to consider whether or not the decision makers in a certain extent, reached the consensus level, so the consensus model was an important research field of group
decision. Probabilistic hesitant fuzzy preference relation (PHFPRs) improves the flexibility and richness of linguistic information expression. At present, many researches have focused on the consensus measures and consensus improvement methods of GDM, but some researches only consider the consensus measures and consider the consensus improvement methods. For example, Zhang et al. [3] proposed consensus measures based on allocation assessment without considering consensus improvement methods. In addition, most scholars divide GDM process into two processes [4]. In the process of reaching consensus, a few papers consider consistency criteria before measuring the degree of consensus of the panel. For example, the consensus reaching process proposed by Garcia et al. [5] follows two important criteria, without considering the consistency of preference relations. This paper taking four cities in Anhui province as examples and proposes a probabilistic hesitation based fuzzy linguistic evaluation method. This paper first defines the distance formula of the PHFPRs matrix, then defines the consensus measure of the PHFPRs matrix based on the distance formula, the consensus model is established and the livability of the four cities is ranked according to the score function, and finally gives a concrete example for verification.

2. Preliminaries
This section introduces the concept of PHFLTS and the concept of PHFPRs matrix.

Definition 1 [6]
Let \( H = \{< x, h(x) | x \in X > \} \), where \( h(x) \) is the membership degree of element, \( p_x \) represents the corresponding probability, and satisfies \( \sum_{i} p_x = 1 \). In addition, \( h(p) = \{h' | h' \in [0,1], l = 1,2, \ldots, |h(p)| \} \) is the probabilistic hesitant fuzzy element (PHFE), and satisfies \( \sum_{i} h' = 1 \).

Definition 2
Let \( A = \{< x, h_i^n (p_x) | x \in X > \} \), \( B = \{< x, h_j^n (p_x) | x \in X > \} \) is the PHFLTS of set X, and the distance denoted as \( D(A,B) \).

Definition 3 [7]
PHFPRs matrix \( H = \{h_j \}_{n \in \mathbb{N}} \) is defined as a finite set of predefined PHFLTS, where \( h_j (p_j) = \{h_{j}^{g} | g = 1,2, \ldots, l(h_j) \} \) is the PHFLTS, where \( l(h_j) \) the number of elements represented by \( h_j \), \( h_{j}^{g} \) the g element represented by \( h_j \), where \( i,j=1,2,3 \ldots n \).

3. Group Consensus Description of Probabilistic Hesitant Fuzzy Linguistic
In this section, several distance formulas of the term set of probabilistic hesitant fuzzy linguistic are introduced.

Definition 4
\( H_1 = \{h_{j,1} (p_{j,1}) \}_{i=1}^{n} \) and \( H_2 = \{h_{j,2} (p_{j,2}) \}_{i=1}^{n} \) represents two arbitrary PHFPRs matrices, where \( h_{j,1} (p_{j,1}) = \{h_{j,1}^{g} | g = 1,2, \ldots, l(h_{j,1}) \} \) and \( h_{j,2} (p_{j,2}) = \{h_{j,2}^{g} | g = 1,2, \ldots, l(h_{j,2}) \} \) is expressed as a set of PHFLTS, then the generalized distance is defined as:

\[
D(H_1, H_2) = \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{1}{l(h_{j,1})} \sum_{k=1}^{l(h_{j,1})} \left| l(h_{j,2}^{k}) - l(h_{j,2}^{k}) \right|
\]
The similarity distance is defined as

\[
\frac{1}{2} \sum_{g=1}^{m} \sum_{p=1}^{n} p_{g,1}^{q(x)} p_{g,2}^{q(x)} \left( \sum_{g=1}^{m} \sum_{p=1}^{n} p_{g,1}^{q(x)} p_{g,2}^{q(x)} \right)^{-1/2}.
\]

(1)

Where, \( I(h_{g,1}) \) the number of elements represented by \( h_{g,1} \), \( h_{g,1}^{q(x)} \) and \( h_{g,2}^{q(x)} \) the g element represented by \( h_{g,1} \) and \( h_{g,2} \). \( I(h_{g,1}^{q(x)}) \) is denoted by subscript.

When \( \lambda = 1 \) is denoted by the Hamming distance formula, and when \( \lambda = 2 \) is denoted by the Euclidean distance formula.

After completing the probabilistic information, another important consideration should be taken into account: given any two PHFE's, in most cases they tend to be of different lengths. In other words, two PHFE have different numbers of members, which may cause some difficulty in the calculation. This method does not change the properties of primary PHFE by using the model [8] and recording the probability of newly added fuzzy hesitating elements as 0.

**Definition 5**

\[
S D( H_1, H_2) = m a \{ -1 \ (d \ H_1, H_2) \}
\]

which \( C I (H_1, H_2) = S D(H_1, H_2) \in [0,1] \).

**Definition 6**

\[
G C I (H_1, \cdots, H_m) = \frac{2}{m(m-1)} \sum_{p=1}^{m-1} \sum_{q=p+1}^{m} S D(H_p, H_q).
\]

which \( G C I (H_1, \cdots, H_m) \in [0,1] \).

Based on formula (1)(2)(3), it can be further rewritten to:

\[
G C I (H_1, \cdots, H_m) = \frac{2}{m(m-1)} \sum_{p=1}^{m-1} \sum_{q=p+1}^{m} \max \left\{ 1 - \frac{1}{n} \left[ \frac{1}{n} \sum_{g=1}^{m} \sum_{p=1}^{n} p_{g,p}^{q(x)} \right] \right\} \left( \frac{n}{1} \right)^{-1/2}.
\]

(4)

### 4. Example Analysis

#### 4.1. Example Description

With the continuous improvement of residents' living conditions and the improvement of spiritual civilization construction, urban livability has become the focus of attention of the government and urban residents, as well as a hot topic of research by experts and scholars at home and abroad. Based on the availability of references and data, this paper selects four indicators \( c_j (j = 1,2,3,4) \): economic affluence( \( c_1 \) ), urban environment beauty( \( c_2 \) ), regional resource carrying capacity( \( c_3 \) ) and living cost( \( c_4 \) ). We use \( x_i (i = 1,2,3,4) \) alternatives to represent four cities in Anhui: Hefei, Anqing, Fuyang and Wuhu. Then three decision making organizations were invited to evaluate four \( x_1, x_2, x_3, x_4 \)
projects. The threshold value $GCI = 0.90$ is given. The consensus model proposed in this paper is used to measure and adjust the consensus of the four projects. The three decision organization choices are based on the following set $\mathcal{L} = \{ s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \} = \{ \text{particularly bad}, \text{bad}, \text{slightly bad}, \text{bad}, \text{medium}, \text{slightly nice}, \text{nice}, \text{very nice} \}$. The three decision organizations conducted an evaluation to convert it to PHFLPRs $H_1, H_2$ and $H_3$. And the results are as follows:

$$H_1 = \left\{ \begin{array}{c}
\{ s_1(1) \} & \{ s_0(0.6), s_1(0.4) \} & \{ s_0(0.4), s_1(0.6) \} & \{ s_2(0.3), s_1(0.4), s_1(0.3) \} \\
\{ s_1(0.3), s_1(0.7) \} & \{ s_1(1) \} & \{ s_1(1) \} & \{ s_1(1) \} \\
\{ s_1(0.7), s_1(0.3) \} & \{ s_1(1) \} & \{ s_1(1) \} & \{ s_1(0.4), s_1(0.2), s_1(0.4) \} \\
\{ s_1(0.4), s_1(0.3), s_1(0.2) \} & \{ s_1(1) \} & \{ s_1(0.2), s_1(0.2), s_1(0.6) \} & \{ s_1(1) \} \\
\end{array} \right\},$$

$$H_2 = \left\{ \begin{array}{c}
\{ s_1(1) \} & \{ s_1(0.3), s_1(0.4), s_1(0.3) \} & \{ s_1(1) \} & \{ s_1(0.2), s_1(0.8) \} \\
\{ s_1(0.3), s_1(0.3), s_1(0.4) \} & \{ s_1(1) \} & \{ s_1(1) \} & \{ s_1(0.3), s_1(0.7) \} \\
\{ s_1(1) \} & \{ s_1(1) \} & \{ s_1(1) \} & \{ s_1(0.4), s_1(0.4), s_1(0.2) \} \\
\{ s_1(0.6), s_1(0.4) \} & \{ s_1(0.6), s_1(0.4) \} & \{ s_1(0.4), s_1(0.3), s_1(0.3) \} & \{ s_1(1) \} \\
\end{array} \right\},$$

$$H_3 = \left\{ \begin{array}{c}
\{ s_1(1) \} & \{ s_1(0.3), s_1(0.4), s_1(0.3) \} & \{ s_1(0.7), s_1(0.3) \} & \{ s_1(1) \} \\
\{ s_1(0.4), s_1(0.4), s_1(0.2) \} & \{ s_1(1) \} & \{ s_1(0.1), s_1(0.5), s_1(0.4) \} & \{ s_1(0.3), s_1(0.7) \} \\
\{ s_1(0.5), s_1(0.5) \} & \{ s_1(0.3), s_1(0.4), s_1(0.3) \} & \{ s_1(1) \} & \{ s_1(0.6), s_1(0.4) \} \\
\{ s_1(1) \} & \{ s_1(0.4), s_1(0.6) \} & \{ s_1(0.3), s_1(0.7) \} & \{ s_1(1) \} \\
\end{array} \right\}.$$

The following algorithm steps are proposed.

**Step 1:** Construct PHFLPRs matrix $H = (h_{y,x})_{n \times m}$.

**Step 2:** Use model [8][7] And the probability of the new fuzzy hesitant element is recorded as 0, and the normalization probability hesitant fuzzy preference relation (NPHFLPRs) matrix $\tilde{H}_p (p = 1, 2, 3)$ is obtained.

**Step 3:** Calculate CI and GCI by using formula (1)(2)(3). If the calculated $GCI$ is less than the $GCI$, then perform step 4, and end otherwise.

**Step 4:** Use the preset consensus model [8] adjusts the matrix and performs the step 3 after adjustment.

**Step 5:** According to reference [8], $H_c = (h_{y,x})_{n \times m}$ is aggregate into PHFLPRs $H_m = (h_{y,x})_{n \times m} (m = 1, 2, \ldots, n)$.

**Step 6:** Compute the score value $SV(h_{y,x})$, where

$$SV(h_{y,x}) = \sum I(h_{y,x}^{(j,k)}) \cdot p_{y,x}^{(j,k)} .$$

(5)

**Step 7:** Rank the alternatives base on $SV(h_{y,x})$. The bigger the score function, the better the alternative.

By the step 2 of the above algorithm, we can get the results of the NPHFLPRs matrix.
Let $\lambda = 2$, through the step 3 can be obtained, $SD(H_1, H_2) = 0.86374$ \text{ and } $SD(H_1, H_3) = 0.84386$.

$SD(H_2, H_3) = 0.83844$ \text{ and } $GCI(H_1, H_2, H_3) = 0.84868$. Therefore, Perform Step 4.

Through the step 3 calculation can be obtained, $GCI(H_1, H_2, H_3) = 0.955527$, satisfied $GCI(H_1, H_2, \cdots, H_m) \in [0,1]$ and $GCI(H_1, H_2, H_3) \geq GCI = 0.90$.

Based on step 5 can obtained collective HFLPR $H_c$, shown as follows:

$$H_c = \begin{bmatrix}
\{s_1(1)\} & \{s_{1666} (0.2), s_{1667} (0.467), s_{1667} (0.333)\} \\
\{s_{1666} (0.233), s_{1666} (0.333), s_{1666} (0.433)\} & \{s_1 (1)\} \\
\{s_{1666} (0.133), s_{1666} (0.333), s_{1666} (0.533)\} & \{s_1 (0.1), s_1 (0.133333), s_1 (0.766667)\} \\
\{s_{1666} (0.033333), s_1 (0.166667), s_1 (0.8)\} & \{s_{1666} (0.0), s_{1666} (0.2), s_1 (0.8)\} \\
\{s_1 (0.3), s_{1666} (0.166667), s_{1666} (0.533333)\} & \{s_1 (1)\}
\end{bmatrix}$$

Based on step 6, we get the $SV(h_{i,c}) (i = 1, 2, \cdots, n)$ as follows: $SV(h_{1,c}) = 2.714583$, $SV(h_{2,c}) = 2.540278$, $SV(h_{3,c}) = 1.76875$, $SV(h_{4,c}) = 1.917361$. Based on step 7, Because $SV(h_{1,c}) > SV(h_{2,c}) > SV(h_{3,c}) > SV(h_{4,c})$. So, the order of the four cities is: $x_1 > x_2 > x_3 > x_4$.

In order to prove the influence of $\lambda$ on the level of $GCI$, formula (4) is used to calculate the PHFLPRs matrix after normalization. Let $\lambda = 2, 3, 4, 5, 6$, data can be obtained as shown in table 1. According to Figure, when $\lambda \geq 2$, the level of group consensus decreases with the $\lambda$ increase.

| $\lambda$ | GCI      |
|-----------|----------|
| 1         | 0.834423611 |
| 2         | 0.848679794 |
| 3         | 0.84748564  |
| 4         | 0.844590554 |
| 5         | 0.841728849 |
| 6         | 0.839150196 |

**Table 1.** The value of $A$ varies with the value of $B$.

**Figure 1.** GCI Graphical graph of variation of parameters $\lambda$ values.
4.2. Comparative Analysis

Compared with literature [1], this paper cites probabilistic hesitation fuzzy linguistic. The method proposed in literature [9, 10] only takes into account the consensus of HFLPRs and ignores the process of consistency check, which is improved in this paper by taking consistency check into consideration. Compared with literature [7] [8], this paper adds probability value for each membership degree on its basis, which can reflect expert preference and prevent information loss.

5. Conclusion

It can be concluded from the case that Hefei city of Anhui Province has the best livability. In this paper, the group consistency problem of decision matrix in probabilistic hesitation fuzzy linguistic is studied. Based on the newly defined distance measure, the group consensus measure, similarity measure and consistency index formula of probabilistic hesitation fuzzy language decision matrix are proposed, and these formulas are applied to group decision problems. We apply this algorithm to the actual GDM problems, and help people to choose suitable cities to live in. Finally, the influence of each parameter on the GCI is analyzed. In the following research, the concept of multi-granularity can be introduced into the model to explore more reasonable methods.

Acknowledgements

The authors would like to thank the anonymous reviewers for their valuable suggestions as to how to improve this paper. This work was supported by Natural Science Foundation of the Anhui Educational Committee of China (Grant Nos. KJ2019A0541, KJ2019A0533), and China University Industry Research Innovation Fund - new generation information technology innovation project (Grant Nos. 2019ITA01037), and Provincial Quality Engineering of colleges of Anhui Province - major online teaching reform research project (Grant Nos. 2020zdxsjg256).

References

[1] Li W W and Yi P T et al. 2020 Assessment of coordinated development between social economy and ecological environment: Case study of resource-based cities in Northeastern China Sustainable Cities and Society 59:102208
[2] Deng W and Peng Z and Tang Y T 2019 A quick assessment method to evaluate sustainability of urban built environment: Case studies of four large-sized Chinese cities Cities 89: 57-69
[3] Zhang G and Dong Y and Xu Y 2014 Consistency and consensus measures for linguistic preference relations based on distribution assessments Information Fusion 17: 46-55
[4] Pérez I J and Cabrerizo F J, et al. 2011 Group decision making problems in a linguistic and dynamic context Expert Systems with Applications 38(3): 1675-1688
[5] Garcia J M T and Moral M J D and Martinez M A, et al. 2012 A consensus model for group decision making problems with linguistic interval fuzzy preference relations Expert Systems with Applications 39(11): 10022-10030
[6] Zhu B and Xu Z S 2017 Probability-hesitant fuzzy sets and the representation of preference relations Technological & Economic Development of Economy 24(3)
[7] Zhang Z and Wu C 2014 On the use of multiplicative consistency in hesitant fuzzy linguistic preference relations Knowledge Based Systems 72: 13-27
[8] Zhang Z and Chen S M 2020 Group decision making based on acceptable multiplicative consistency and consensus of hesitant fuzzy linguistic preference relations Information Sciences 541: 531-550
[9] Wu Z B and Xu J 2016 Managing consistency and consensus in group decision making with hesitant fuzzy linguistic preference relations Omega 65: 28-40
[10] Wu P and Zhu J and Zhou L, et al. 2019 Local feedback mechanism based on consistency-derived for consensus building in group decision making with hesitant fuzzy linguistic preference relations Computers & Industrial Engineering 137:106001.1-106001.12