Kaons in Nuclear Matter

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Abstract

The kaon energy in a nuclear medium and its dependence on kaon-nucleon and nucleon-nucleon correlations is discussed. The transition from the Lenz potential at low densities to the Hartree potential at high densities can be calculated analytically by making a Wigner-Seitz cell approximation and employing a square well potential. As the Hartree potential is less attractive than the Lenz one, kaon condensation inside cores of neutron stars appears to be less likely than previously estimated.

1 Introduction

Kaon condensation in dense matter was suggested by Kaplan and Nelson [1], and has been discussed in many recent publications [2, 3]. Due to the attraction between $K^{-}$ and nucleons its energy decreases with increasing density, and eventually if it drops below the electron chemical potential in neutron star matter in $\beta$-equilibrium, a Bose condensate of $K^{-}$ will appear. It is found that $K^{-}$'s condense at densities above $\sim 3 - 4\rho_0$, where $\rho_0 = 0.16$ fm$^{-3}$ is normal nuclear matter density. This is to be compared to the central density of $\sim 4\rho_0$ for a neutron star of mass $1.4M_\odot$ according to the estimates of Wiringa, Fiks and Fabrocini [4] using realistic models of nuclear forces. The condensate could change the structure and affect maximum masses and cooling rates of neutron stars significantly.

Recently, [5, 7] we have found that the kaon-nucleon and nucleon-nucleon correlations conspire to reduce the $K^{-}N$ attraction significantly already at rather low densities when the interparticle distance is comparable to the range of the $KN$ interaction. We have calculated the kaon energy as function of density in a simple model where also the low and high density limits and the dependence on the range of the interaction can be extracted.
2 Low Density Limit: Lenz Potential

In neutron matter at low densities when the interparticle spacing is much larger than the range of the interaction, \( r_0 \gg R \), the kaon interacts strongly many times with the same nucleon before it encounters and interacts with another nucleon. Thus one can use the scattering length as the “effective” kaon-nucleon interaction, \( a_{K-n} \simeq -0.41 \text{fm} \). The kaon energy deviates from its rest mass by the Lenz potential

\[
\omega_{\text{Lenz}} = m_K + \frac{2\pi}{m_R} a_{K-n} \rho ,
\]

which is the optical potential obtained in the impulse approximation.

3 High Density Limit: Hartree Potential

At high densities when the interparticle spacing is much less than the range of the interaction, \( r_0 \ll R \), the kaon will interact with many nucleons on a distance scale much less the range of the interaction. The kaon thus experiences the field from many nucleons and the kaon energy deviates from its rest mass by the Hartree potential:

\[
\omega_{\text{Hartree}} = m_K + \rho \int V_{K-n}(r) d^3r ,
\]

As shown in [5], the Hartree potential is considerably less attractive than the Lenz potential. This is also evident from Fig. (1).

4 General Case

To demonstrate this transition from the low density Lenz potential to the high density Hartree potential we solve the Klein-Gordon equation for kaons in neutron matter in the Wigner-Seitz cell approximation. With the simplified square well potential this can in fact be done analytically.

4.1 The Recoil Corrected Klein-Gordon Equation

We choose to describe the kaon-nucleon interaction by a vector potential \( V \) dominated by the Weinberg-Tomozawa term. In the analysis of Ref. [6] the \( K^+N \) interaction was also found to be dominated by \( \omega \) and \( \rho \) vector mesons.
The energy of the kaon-nucleon center-of-mass system with respect to the nucleon mass is then

\[ \omega = \sqrt{k^2 + m_N^2 + V + \frac{k^2}{2m_N}}, \]  

(3)

where \( k \) is the kaon momentum (we use units such that \( \hbar = c = 1 \)), in c.m. frame. We have included the recoil kinetic energy of the nucleon assuming that terms of order \( k^4/8m_N^3 \) and higher can be neglected. For a relativistic description of the kaon in a vector potential we employ the following recoil corrected Klein-Gordon equation (RCKG) obtained by quantizing Eq. (3)

\[ \left\{ (\omega - V)^2 + \frac{m_N + \omega - V}{m_N} \nabla^2 - m_K^2 \right\} \phi = 0. \]  

(4)

4.2 The Wigner-Seitz Cell Approximation

The Wigner-Seitz cell approximation simplifies band structure calculations enormously. Though it is a poor approximation for solids it is better for liquids. As we only consider qualitative effects we shall assume the Wigner-Seitz cell approximation for the strongly correlated nuclear liquid because the periodic boundary condition is a computational convenience. It contains the important scale for nucleon-nucleon correlations given by the interparticle spacing and, as we shall see, it naturally gives the correct low density (Lenz) and high density (Hartree) limits. We only consider neutrons since the \( \sim 10\% \) protons expected in neutron stars do not change results by much.

4.3 Square Well Potential

Since the kaon-nucleon potential is not known in detail and we are only interested in qualitative effects, we will for simplicity approximate it by a simple square well potential

\[ V(r) = -V_0 \Theta(R - r). \]  

(5)

The range of the interaction \( R \) and the potential depth \( V_0 \) are related through the s-wave scattering length \( a = R - \tan(\kappa_0 R)/\kappa_0 \), where \( \kappa_0^2 = (2m_K V_0 + V_0^2) \cdot m_N/(m_N + m_K + V_0) \). If \( V_0 \ll m_K \) this reduces to the standard result \( \kappa_0^2 = 2m_R V_0 \), where \( m_R = m_K m_N/(m_K + m_N) \) is the kaon-nucleon reduced mass. However, for small ranges of interaction the potential becomes significant as
compared to the kaon mass which necessitates a relativistic treatment with the Klein-Gordon equation instead of the Schrödinger equation.

The above arguments suggest that the range $R = 0.4 - 1.0 \text{fm}$ covers most realistic possibilities above. For a kaon-neutron scattering length of $a_{K^{-}n} = -0.41 \text{ fm}$ we find for typical ranges $R = 0.4 - 1.0 \text{ fm}$ kaon-neutron potentials ranging from $V_0 = 463 \text{ MeV}$ down to $V_0 = 49 \text{ MeV}$. 

![Fig. 1. Kaon energy as function of neutron density. Our calculation (Eq. (6), full curves) are shown for $R = 0.4 \text{ fm}$, $R = 0.7 \text{ fm}$ and $R = 1.0 \text{ fm}$. At low densities they approach the Lenz result (Eq. (1), dotted curve) and at high densities they approach the Hartree result (Eq.(2), dashed curves).]

5 Results

As shown in [7] the RCKG equation can now be solved for s-waves and the kaon wave-function found. Applying the periodic boundary condition, $\phi'(r_0) = 0$, as required by the Wigner-Seitz cell approximation we arrive at the closed equation

$$\frac{k}{\kappa} \tan(\kappa R) = \frac{e^{2k(R-r_0)} - (1 - kr_0)/(1 + kr_0)}{e^{2k(R-r_0)} + (1 - kr_0)/(1 + kr_0)}, \quad (6)$$
which determines $k$ and thus the kaon energy. Here, $k^2 = (m_K^2 - \omega^2) m_N/(m_N + \omega)$ and $\kappa^2 = ((\omega + V_0)^2 - m_K^2) m_N/(m_N + \omega + V_0)$. The resulting kaon energy is shown in Fig. (1). By expanding (6) at low densities the kaon energy becomes the Lenz potential whereas at high densities it becomes the Hartree potential.

6 Summary

Kaon-nucleon correlations reduce the $K^-N$ interaction significantly when its range is comparable to or larger than the nucleon-nucleon interparticle spacing. The transition from the Lenz potential at low densities to the Hartree potential at high densities begins to occur already well below nuclear matter densities. For the measured $K^-n$ scattering lengths and reasonable ranges of interactions the attraction is reduced by about a factor of 2-3 in cores of neutron stars. Relativistic effects further reduce the attraction at high densities. Consequently, a kaon condensate is less likely in neutron stars.

Coulomb energies have not been included in the above analysis. They may, as discussed in [8, 9], lead to a mixed phase of nuclear matter with and without a kaon condensate. However, the Coulomb energies are small as compared to kaon masses and therefore the reduction in the kaon energy will also be minor.

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