Pair production of the lightest chargino at $\gamma\gamma$-collider

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Chargino pair production via photon-photon collision is investigated in Minimal Supersymmetric Standard Model at a future linear collider. The process is computed using all the possible diagrams at the next-to-leading order level, including box, triangle, and self-energy diagrams. The numerical analysis is carried out for the production rates of the lightest chargino pairs in RNS, NS, mSUGRA, BB, NUHM2, and NUGM scenarios. These distinct benchmark models were introduced in the light of LHC results presented at $\sqrt{s} = 7 - 8$ TeV. Among these scenarios, Radiatively Driven Susy has the highest cross-section for the $\gamma \gamma \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$. The partonic cross-section reaches up to 4.15 pb at $\sqrt{s} = 250$ GeV. The convoluted cross-section in $e^+e^-$-collider is $\sim 950$ fb at $\sqrt{s} = 350$ GeV, which makes it accessible even at very low energies. Besides, the cross-section for each scenario is presented for the possible polarization configurations of the incoming photon beams. The results show that the opposite polarization of the photon beams enhances the cross-section. The total convoluted cross-section with the photon luminosity in a $e^+e^-$ machine is given as a function of center-of-mass energy, and up to $\sqrt{s} = 1$ TeV.

I. INTRODUCTION

The discovery of the Higgs boson at the LHC [? ? ?] is a strong confirmation of the electroweak symmetry breaking mechanism. Considering the measurements of the Standard Model (SM), there is substantial literature on a higher scale called the new physics beyond the standard model (BSM). Among all the proposals of BSM physics, supersymmetry is one of these theories which explains the gauge couplings of the strong and the electroweak interactions from Planck scale down to the weak scale. That is achieved by introducing superfields which combines the SM particles with their supersymmetric partners. That also helps to reduce the quadratic divergences arising from the self-interaction of the scalar fields to logarithmic ones. There are various models which include supersymmetry, and Minimal Supersymmetric Standard Model (MSSM)[? ] is the minimal extension which realizes the supersymmetry. According to the model, if a symmetry called the R-parity is conserved in the decay of superparticles, the lightest supersymmetric particle (LSP) becomes a natural candidate for a weakly-interacting dark matter. After the LSP, charginos are the next which attracts attention. They are solely the fermionic mass eigenstates of the supersymmetric partners of the $W^\pm$ and the charged Higgs $H^{\pm}_{1,2}$. Measuring their masses and production could give the possibility of determining the gaugino, and the higgsino couplings. In the light of the results presented by the LHC in pp collisions at $\sqrt{s} = 7, 8, 13$ TeV, strict limits on SUSY parameter space and constraints are set on the sparticles masses [? ? ? ?].

In the last decade, particle physics community is preparing itself for the next collider projects. It is certain that the next collider will be a lepton collider - Particularly for studying the properties of the Higgs boson - and there are several proposals. These are the Circular Electron-Positron Collider (CEPC, $\sqrt{s} = 240$ GeV) in China [? ? ? ?], the Future Circular Collider (FCC-ee, $\sqrt{s} = 350$ GeV) [? ] at CERN [? ?], and the International Linear Collider (ILC, $\sqrt{s} = 500$ GeV) in Japan [? ]. In all these proposals, the electron-positron collisions are planned. However, it is also possible to design a lepton collider which operates as a $\gamma\gamma$ collider by extracting Compton backscattered photons [? ?]. The $\gamma\gamma$-collider is considered as a future option with the integrated luminosity of the order of $100$ fb$^{-1}$ yearly [? ? ?]. The machine could be upgradeable to $\sqrt{s} = 1$ TeV with the total integrated luminosity up to $300$ fb$^{-1}$ yearly. Therefore, photon-photon collisions will provide a different way to produce the chargino pair which deserves a detailed study.

The neutralino and the chargino pairs in $e^+e^-$-colliders are studied before at the NLO-level accuracy with including the infrared divergences [? ? ? ?]. The chargino pair production rates in $\gamma\gamma$-collider is also inspected before [? ] without treating the infrared corrections. In this work, the numerical calculation for the chargino pair production with all the possible one-loop level Feynman diagrams in $\gamma\gamma$-collider including the radiative corrections, moving on from previously defined popular supersymmetric models to the new ones which might have interesting phenomenology, are presented. The numerical calculation is performed for the benchmark points defined in following scenarios which are proposed after the results obtained at the LHC at $\sqrt{s} = 7, 8$ TeV collision data [? ]:

i.) Radiatively driven natural SUSY (RNS),

ii.) Natural Susy and mSUGRA/CMSSM scenario,

iii.) Brummer Buchmuller (BB) benchmark point,

iv.) NUHM2 and NUGM scenarios.

These scenarios and benchmark points are outside of the limits presented by the LHC so far. Because the masses

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of the squarks and the sleptons are specifically arranged to be above the TeV. However, the lightest neutralino and the chargino are deliberately set in below TeV range, and that makes them accessible at the Future Lepton Colliders (FLC) particularly in $\gamma\gamma$-collision mode. These benchmark models show the potential of the FLC and $\gamma\gamma$-colliders concerning the supersymmetry searches. In this study, the total cross-section as a function of the center-of-mass (COM) energy of the lightest chargino pair is calculated. In addition to these, the distributions for the total cross-section integrated with the photon luminosity in a $e^+e^-$-collider are discussed.

The content is organized as follows: In Sec. II, the chargino sector in MSSM is presented briefly. In Sec. III, the one loop Feynman diagrams, the radiative corrections and the convolution of the cross-section in a $e^+e^-$-machine are discussed. The stabilization of the infrared and ultraviolet divergences are also addressed. In Sec. IV, numerical results of the total cross-section for each benchmark point are delivered. At last, the conclusion is drawn in Sec. V.

II. EXPRESSIONS FOR THE CHARGINO SECTOR

Charginos are the mass eigenstates of the charged superpartners of the W-boson (wino) and the charged Higgs-boson (higgsinos). They are defined as a linear combination of the wino and the charged higgsinos. The relevant part of the MSSM Lagrangian that is responsible for the chargino masses is defined as following

$$\mathcal{L} = -\frac{1}{2}(\psi_i^+ \psi_i^-) \begin{pmatrix} 0 & \mathcal{M}^- \xi^\pm \\ \mathcal{M}^+ \xi^\pm & 0 \end{pmatrix} (\psi_i^+ \psi_i^-).$$

(1)

The mass eigenstates in Dirac notation $\tilde{\chi}^\pm_i$ are obtained employing the Weyl states $\tilde{\psi}_i^+ = (-i\lambda^+, \psi^0_H)$ and $\tilde{\psi}_i^- = (-i\lambda^-, \psi^0_H)$ with the following relations

$$\tilde{\chi}^\pm_i = \left(\begin{array}{c} \chi^\pm_i \\ \chi^\mp_i \end{array}\right), \quad \chi^+_i = V_{ij} \tilde{\psi}_j^+, \quad \chi^-_i = U_{ij} \tilde{\psi}_j^-.$$

(2)

Then, the mass matrix of the charginos is defined as

$$\mathcal{M}_{\tilde{\chi}^\pm} = \begin{pmatrix} M_{SU(2)} & \sqrt{2m_W} \cos \beta \\ \sqrt{2m_W} \sin \beta & \mu \end{pmatrix}.$$  

(3)

where $m_W$ is the mass of the W-boson, $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of two Higgs fields, $M_{SU(2)}$ represents the gaugino mass parameter associated with the $SU(2)$ symmetry group, and $\mu$ is the supersymmetric Higgs mass parameter. The diagonalization of the chargino mass matrix $\mathcal{M}_{\tilde{\chi}^\pm}$ is performed with two unitary matrices $U$ and $V$ defined below

$$diag(\tilde{\chi}^\pm_i, \tilde{\chi}^\mp_i) = U^* \mathcal{M}_{\tilde{\chi}^\pm} V^\dagger.$$  

(4)

In general, the mass parameters in the chargino mass matrix could be complex for CP non-invariant cases. As the CP violation is ignored in this study, all the parameters in the mass matrix are taken as real.

III. THE CALCULATION OF THE DIAGRAMS

In this section, the analytical expressions of the cross-section, and the convolution process with the photon luminosity in $e^+e^-$ collider for the chargino pair production are given. The process is denoted as

$$\gamma(k_1, \mu) + \gamma(k_2, \nu) \rightarrow \tilde{\chi}^+_i(k_3) + \tilde{\chi}^-_j(k_4) \quad (i, j = 1, 2),$$

where $k_a$ $(a = 1, \ldots, 4)$ are the four momenta of the incoming photons and outgoing charginos, respectively, in addition $\mu$ and $\nu$ represents the polarization vectors of the incoming photons.

A. The contributing Feynman diagrams

In Fig. 1, the tree level Feynman diagrams contributing to the process $\gamma\gamma \rightarrow \tilde{\chi}^+_i \tilde{\chi}^-_j$ are plotted. Compared to the neutralino pair production via $\gamma\gamma$ collisions [1], the chargino pair production is possible at the tree level as well. The diagrams are generated using FeynArts[2]. The amplitudes for each type of diagrams are constructed using FeynArts, the relevant part of the Lagrangian and the corresponding Feynman rules for the vertices are defined in [2] and the FeynArts implementation of these rules are given in [2]. After summing over the helicities of the charginos and averaging over the polarization vectors of the incoming photons, the cross-section of the chargino pair production via photon-photon fusion.

![Fig. 1. Tree level Feynman diagrams for the chargino pair production via photon-photon fusion.](image-url)

where $\lambda(\hat{s}, m_{\tilde{\chi}_i^\pm}^2, m_{\tilde{\chi}_j^-}^2) = \lambda(\hat{s}, m_{\tilde{\chi}_i^\pm}^2, m_{\tilde{\chi}_j^-}^2)$ and the factor $\frac{1}{4}$ is the average of the photons polarization vectors, $i$ and $j$ run over the flavor of the charginos at the final state, and $\mu, \nu$ is the polarization of the initial state photons.

We could classify the one-loop diagrams into three distinct groups, namely as the box-type, the triangle- and the bubble-type s-channel diagrams, and the self-energy diagrams. They are depicted in Fig. 2-4 where F states the fermions in the propagator, $S$ for a scalar particle, and $V$ for a vector boson propagator. They could be the SM particles as well as their supersymmetric counterparts.

The corresponding Lorentz invariant matrix element for the one-loop level process is written as a sum over
the box-diagrams (Fig. 2), the triangle- and bubble-type diagrams (Fig. 3), and the self-energy (Fig. 4),

\[ \mathcal{M}_{\text{virt}} = \mathcal{M}_{\text{box}} + \mathcal{M}_{\text{tri}} + \mathcal{M}_{\text{self}}. \]

Accordingly, the one-loop virtual correction is calculated by the following formula.

\[ \hat{\sigma}_{\text{virt}}(\hat{s}) = \frac{\lambda(\hat{s}, m_{\tilde{\chi}_1}^2, m_{\tilde{\chi}_2}^2)}{16\pi^2} \frac{1}{4} \sum_{\text{hel}} 2\text{Re} [\mathcal{M}_{\text{tree}}^* \mathcal{M}_{\text{virt}}] \tag{7} \]

where \( \hat{s} \) represents the COM energy in the photon-photon collision frame.

**B. Ultraviolet and infrared divergences**

In the computation, the ultraviolet divergence arising in the calculation is cured by taking into account the renormalization constants. The calculation is performed in the \( \text{'t Hooft-Feynman} \) gauge where the gauge boson propagators are in simple form, hence the computation requires less computing power. We employed the constrained differential renormalization \([? ?]\), which is equivalent to the dimensional reduction \([? ?]\) at the one-loop level \([? ?]\). Besides, that also preserves supersymmetry \([? ?]\), and guarantees that the SUSY relations are kept intact. All the counterterms which are taken into account, as well as the vertices which are renormalized, are indicated by a cross sign in Fig. 5. After the renormalization procedure, the equation 7 becomes UV finite. Nevertheless, the UV finiteness, in the evaluation of scalar and tensor one-loop integrals, is checked numerically in \texttt{LoopTools} \([? ? ?]\) by varying the parameters \( \mu \) and \( \Delta \) on a large scale. These two parameters regularize the divergent integrals, and the cross-section is stable in the numerical precision.

Besides of the UV divergence, infrared (IR) divergence could also arise in the computation due to the massless particles propagating with very small energy in the diagrams. They lead to a singularity, and what is called the IR-divergence occurs. If the photon had a mass such as \( \lambda \), these divergent terms would be proportional to \( \log \lambda \). This problem is cured by the fact that in nature the experimental apparatus makes the measurement with a finite resolution and minimum energy. Therefore, if there are photons emitted with an energy of less than \( \Delta E \), It
The diagrams plotted in Fig. 6 have an additional photon at the final state. Consequently, adding these soft photon contributions cancels those IR-divergent ones appeared in the loop diagrams with an internal photon.

Fortunately, the soft bremsstrahlung correction is already implemented in FormCalc following the description given in [?]. The photons are considered soft if their energy is less than $\Delta E = \delta_s E = \delta_s \sqrt{s}/2$ which separates soft and hard photon radiation. Since the diagrams where IR divergence arises are involved with a photon propagating in the loops are regularized by a photon mass parameter $\lambda$. Adding the virtual ($\sigma_{\text{virt}}(\lambda)$) and the soft ($\sigma_{\text{soft}}(\lambda, \Delta E)$) contributions drops the dependence on the photon mass parameter $\lambda$. On the other hand, the hard photon radiation is also needed to be computed and summed for a complete picture of the process. Otherwise, the total cross-section will be dependent on the $\delta_s$, $\Delta E$. In the computation, the cancellation of these IR divergences and dependence on the parameter $\lambda$ is numerically tested. The sum of virtual and soft photon radiation is stable in modifying the parameter $\lambda$ on a large scale. That means, the dependence on the $\lambda$ dropped out. Next, the contribution coming from the hard photon emission is calculated as a function of $\delta_s$. It can be seen in Fig. 7 that all the contributions are plotted for the benchmark point $\text{RNS (NS)}$ at the top (bottom), respectively. The total NLO correction is around $-8.3\%$ for $\text{NS} (\sqrt{s} = 0.5 \text{ TeV})$ and $-2.6\%$ for $\text{RNS} (\sqrt{s} = 1 \text{ TeV})$. Most importantly, the sum labeled by the green line in each of the distribution is stable by varying the $\delta_s$ logarithmically. Stability is also seen at varying the COM energy.

Consequently, the total one-loop corrections are decomposed into virtual, soft, and hard parts as the following:

$$\sigma_{\text{NLO}} = \sigma_{\text{virt}}(\lambda) + \sigma_{\text{soft}}(\lambda, \Delta E) + \sigma_{\text{hard}}(\Delta E).$$ 

![FIG. 6. The Feynman diagrams which contributes to the radiative corrections in the chargino pair production via photon-photon fusion. These diagrams show the process $\gamma\gamma \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$](image)
C. The convoluting the cross-section with the photon luminosities

This process could be studied in a future lepton collider where the photon beam could be obtained by the laser back-scattering technique, and these photon beams could make them collide. The big fraction of the center of mass energy of the electron beam could be transferred to the photon collisions. Then, the production of the chargino pairs could be taken as a subprocess in $e^+e^-$ collisions. The total cross-section of $e^+e^-\rightarrow \gamma\gamma \rightarrow \tilde{\chi}^+_i \tilde{\chi}^-_j$ could be computed by convoluting the cross-section $\hat{\sigma}_{\gamma\gamma \rightarrow \tilde{\chi}^+_i \tilde{\chi}^-_j}(\hat{s})$ with the photon luminosity in $e^+e^-$ collider.

The total integrated cross-section is defined as following:

$$\sigma(s) = \int_{x_{\text{min}}}^{x_{\text{max}}} \hat{\sigma}_{\gamma\gamma \rightarrow \tilde{\chi}^+_i \tilde{\chi}^-_j}(\hat{s}; s = z^2\hat{s}) \frac{dL_{\gamma\gamma}}{dz}, \quad (9)$$

where $s$ and $\hat{s}$ are the COM energy in $e^+e^-$ collisions and $\gamma\gamma$ subprocess, respectively. $x_{\text{min}}$ is the threshold energy to produce the chargino pair, and defined as $x_{\text{min}} = (m_{\tilde{\chi}^+_i} + m_{\tilde{\chi}^-_j})/\sqrt{s}$. The maximum fraction of the photon energy is taken as $x_{\text{max}} = 0.83$ [? ]. The distribution function of the photon luminosity is given by

$$\frac{dL_{\gamma\gamma}}{dz} = 2\sqrt{x} \int_{z^2/x_{\text{max}}}^{x_{\text{max}}} \frac{dx}{x} F_{\gamma/e}(x) F_{\gamma/e} \left(\frac{z^2}{x}\right), \quad (10)$$

where $F_{\gamma/e}(x)$ is the energy spectrum of the Compton back scattered photons from the initial unpolarized electrons, and it is defined as a function of the fraction $x$ of the longitudinal momentum of the electron beam [? ].

IV. NUMERICAL RESULTS AND DISCUSSION

In the analysis the following input parameters are taken from [? ] where $m_{W} = 80.399$ GeV, $m_{Z} = 91.1887$ GeV, $m_{t} = 173.21$ GeV, $s_{W} = 0.222897$, and $\alpha(m_{Z}) = 1/137.035999$. The prominent feature of the supersymmetry is that all three gauge-couplings are unified at the grand scale which is also predicted by GUTs and string theories. However, there are no any superpartners discovered yet at the weak scale. To accommodate this fact, it is assumed that the supersymmetry is slightly broken. Therefore, all the superpartners acquired mass higher than the electroweak scale. On the other hand, the breaking scale could not be at the order of ten TeV. Because the soft SUSY breaking parameters are intimately linked to the breakdown of electroweak symmetry. It is assumed that the masses of these sparticles are not far away from the electroweak scale. However, the results coming from the LHC at $\sqrt{s} = 7, 8, 13$ TeV made that lovely picture to fade away and questioned the simple weak scale SUSY picture. The exclusion of the mass difference between the sparticle masses and the weak scale increases the breaking scale. Unfortunately, that also resurrects the so-called little hierarchy problem [? ]. The benchmark points considered in this study are introduced to fit into this picture drawn by the LHC results. These benchmark points were chosen for having low contribution to the electroweak observables. A detailed discussion is delivered in [? ], according to that, to achieve small $\Delta_{EW}$, it is required that $m_{H_u}^2$, Higgs-doublet mixing parameter $\mu$, and the radiative contribution $\Sigma_u^2$ to be around $m_{Z}^2/2$ to within a factor of a few [? ].

The computation is presented for the following benchmark points in each subsection. They are introduced explicitly for a future lepton collider by the constraints set from the LHC results, and more information for each benchmark point could be found at [? ] and references therein. Just in case, the finite width scheme is introduced for the Higgs states. Therefore, the decay widths of the neutral Higgs bosons are obtained at the NLO-level accuracy employing FeynHiggs [? ]. The input parameters for FeynHiggs are taken from each of the benchmark points considered. In all these scenarios, the second chargino mass is above the TeV except for the RNS and the mSUGRA scenarios, for that reason, the production of $\tilde{\chi}^+_2 \tilde{\chi}^-_2$ requires much greater center-of-mass energy than the lightest chargino pair and $\sqrt{s} = 1$ TeV. On the other hand, the mass of the lightest chargino is quite low, and the production of $\tilde{\chi}^+_1 \tilde{\chi}^-_1$ could be accessed even at a collider whose center-of-mass energy is very low such as FCC-ee.

A. Radiatively driven natural SUSY (RNS)

This model is motivated by minimizing $\Delta_{EW}$, and it also sustains the unification of the gauge couplings. If it is ensured that the Higgs-doublet mixing parameter $\mu$ is $\sim 100-300$ GeV, that causes a small negative value of $m_{H_u}^2$ at the weak scale and large mixing between top squarks. The mass spectrum is calculated for the parameters given in Tab. 1 using ISASUGRA-v7.88 [? ]. The masses of the lightest two neutralinos and the lightest chargino are around the electroweak scale in RNS. However, all the other sparticle masses are beyond the TeV [? ? ].

The distribution of the cross-section at the tree level, the total NLO corrections (virtual + real), and the sum of all are plotted in Fig. 8 for the lightest chargino pair as a function of the center-of-mass energy. The ratio $\sigma_{NLO}/\sigma_{LO}$ is plotted at the bottom of the figure. It can be seen that the correction at the NLO level is positive in $\sqrt{s} < 300$ GeV, and it falls quickly moving to the higher COM energies. It reaches up to -7.6% at $\sqrt{s} \sim 1$ TeV. In Fig. 8, the unpolarized cross-section goes up to 4.4 pb around $\sqrt{s} = 290$ GeV, and it falls rapidly to 0.90 pb at

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1 The SLHA files for these benchmark points could also be obtained from [http://flc.desy.de/ilcphysics/research/susy](http://flc.desy.de/ilcphysics/research/susy).
The ratio $\sigma_{NLO}/\sigma_{LO}$ is plotted as a function of the energy. The branching ratio of the chargino is also calculated through three channels. The sum of the branching ratios $BR(\tilde{\chi}_1^+ \to \tilde{\chi}_i^0 + u + \hat{s}) = 0.333, BR(\tilde{\chi}_1^+ \to \tilde{\chi}_i^0 + c + d) = 0.333, \text{and } BR(\tilde{\chi}_1^+ \to \tilde{\chi}_i^0 + l + \nu_l) = 0.333$. Accordingly, the pattern of the chargino decay at the collider would be a missing transverse energy due to the LSP and two light quark jets or a lepton.

The branching ratio of the chargino is also calculated using ISASUGRA, and the signal for the discovery of the lightest chargino is examined. In the Radiatively Driven Susy (RNS) scenario, the lightest chargino decays mainly through three channels. The sum of the branching ratios is $\sim 0.999$. These channels are $BR(\tilde{\chi}_1^+ \to \tilde{\chi}_0^0 + u + s) = 0.333, BR(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 + c + d) = 0.333, \text{and } BR(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 + l + \nu_l) = 0.333$.}

TABLE I. The input parameters and the mass spectrum for all the benchmark points considered in this study. All masses are in TeV. The mass spectrum and the electroweak scale parameters are obtained with ISASUGRA.

| Benchmark Point | $m_0(1,2)$, $m_0(3)$ | $m_{1/2}$, $M_{1,2,3}$ | $A_0$ | $\tan\beta$ | $\mu$ | $m_{H^0/A^0/H^\pm}$ | $m_{\tilde{\chi}_{1/2}^\pm}$ |
|-----------------|----------------------|----------------------|-------|-------------|------|------------------|------------------|
| RNS             | 5                    | 0.7                  | -8.3  | 10          | 0.11 | 1                | (0.101, 0.61)    |
| NS              | 13.35, 0.76          | 1.38                 | -167  | 23          | 0.225| $\sim 1.550$    | (0.224, 1.18)    |
| mSUGRA          | 10                   | 0.5                  | -5.45 | 15          | 0.234| $\sim 9.700$    | (0.248, 0.7)     |
| BB              | $\sim 5.3, \sim 9.5$ | -                    | 48    | 0.160       | $\sim 4.050$ | (0.167, 9.52) |
| NUHM2           | 10                   | 0.8                  | -16   | 7           | 0.280| $\sim 0.280$    | (0.237, 5.9)     |
| NUGM            | 3                    | 0.7                  | -6.0  | 25          | 2.36 | $\sim 3.300$    | (0.131, 2.36)    |

B. Natural Susy (NS) and mSUGRA/CMSSM scenarios

The Natural Susy (NS) models are introduced and characterized by $[? ? ? ?]$. The lightest chargino masses for this benchmark points is $m_{\tilde{\chi}_{1/2}^\pm} \approx 224$ GeV, and the second one is greater than TeV. Besides of the third generation squarks ($\tilde{t}_{1,2}$ and $\tilde{b}_1$), all the other sparticles have a mass greater than TeV. The energy dependence of the total cross-section of $\hat{s}(\gamma\gamma \to \tilde{\chi}_1^0 \tilde{\chi}_1^0)$ is plotted in Fig. 10 (left). The cross-section reaches up to 0.89 pb at $\sqrt{s} = 630$ GeV at the leading-order. The total virtual corrections correction is positive and reaches up to $+10\%$ for less than 500 GeV, then it falls quickly and becomes negative beyond the $\sqrt{s} = 500$ GeV. Overall, the virtual corrections lower the total cross-section. Moving to the mSUGRA/CMSSM scenario, the LHC8 ruled out most of the region in the parameter space by direct searches of gluino and squarks. However, there still exists some space for the dark matter (LSP) and the mSUGRA scenario. This benchmark point employs the parameters at the GUT scale given in Tab. I, and the electroweak parameters calculated by ISASUGRA. Since the Higgs-doublet...
mixing parameter $\mu \sim 235$ GeV, the neutralino and the chargino masses (electroweakinos) are around the electroweak scale. Similarly, all the other sparticles are beyond TeV. Therefore, this point is beyond the reach of LHC. The chargino pair production $\hat{\sigma}(\gamma\gamma \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^-)$ for the unpolarized photon beam is given in Fig. 10 (right) as a function of COM energy. The cross-section has the same distribution with the NS scenario with a very small difference. The production of the second chargino pairs requires a higher COM energy, and it is reachable at a collider with $\sqrt{s} = 1$ TeV.

Comparing the NS and the mSUGRA scenarios show that the total distributions of the cross-sections for the same chargino pairs are very similar to each other, the virtual corrections also have a similar trend as a function of the energy. Therefore, the cross-section could not be enough to distinguish these two scenarios from each other. On the other hand, polarizing the incoming photon beams does not have much impact on the cross-section, and the figure is not given for this scenario. The distribution is almost identical given in the Fig. 9 at $\sqrt{s} = 1$ TeV, and at most the ratio $\sigma(P_\gamma,P_\gamma)/\sigma_{UU}$ is $\sim 1.08$ and $\sim 1.06$ for NS and mSUGRA, respectively.

In the NS and mSUGRA scenarios, $\tilde{\chi}_1^0$ decays mainly through the same channels as the RNS scenario, and the sum of these branching ratios is again $\sim 0.99$. However, only the flavor of the light quarks is different. The decay channels are $BR(\tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 + u + \bar{d}) = 0.333$, $BR(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 + c + \bar{s}) = 0.333$, and $BR(\tilde{\chi}_1^0 \rightarrow \tilde{\chi}_1^0 + l + \nu) = 0.333$. Therefore, the pattern at the detector would be the same with the RNS scenario.

**C. Brummer-Buchmuller benchmark (BB)**

Inspired by GUT-scale string compactifications, Brummer and Buchmuller proposed this scenario [22,23] where the Fermi scale emerges as a focus point. In this scenario, gauge-mediated soft terms are characterized by discrete numbers, and they are called the messenger indices. For certain models, the messenger indices are aligned such that the contributions to the mass of the Z boson cancel between the various soft terms. Besides, the Higgs-doublet mixing parameter $\mu$ arises from the gravitational interactions. It is predicted that the graviton mass and $\mu$ are in the same order ($\mu \simeq m_{3/2} \simeq 150 - 200$ GeV).

In this study, the messenger indices are $(N_1,N_2,N_3) = (46,46,20)$, the gauge-mediated soft mass per messenger pair is $m_{GM} = 250$ GeV, the ratio of Higgs vacuum expectation value $\tan\beta = 48$ is taken, the Higgs-doublet mixing parameter is $\mu = 167$ GeV, and $m_A = 4050$ GeV.
More detailed information is presented in [? ? ]. The results for the lightest chargino pair production in $e^+e^-$ collider with detector simulation at the ILC is presented in [? ]. Accordingly, this benchmark scenario specifically adopted for the future lepton collider studies.

The lightest chargino mass is around $m_{\tilde{\chi}_1^\pm} \sim 167$ GeV, but the second chargino mass is at the order of ten TeV. Consequently, only the lightest chargino pair is accessible in a $\gamma\gamma$ collider with $\sqrt{s} \leq 1$ TeV. The energy dependence of the total cross-section of $\sigma(\gamma\gamma \to \tilde{\chi}_1^0 \tilde{\chi}_1^\pm)$ is given in Fig. 11 (left). The total cross-section reaches up to 1.6 pb at $\sqrt{s} = 470$ GeV. The virtual corrections is positive below 400 GeV, and the ratio $\sigma_{NLO}/\sigma_LO$ goes up to +5.3% for low COM energies. Then, it slowly falls at higher energies. The mass split between the electroweakinos $(m_{\tilde{\chi}_1^\pm} - m_{\tilde{\chi}_1^0})$ is $\approx 770$ MeV, and the decay channels of the lightest charginos in a collider at the ILD are studied in [? ? ]. Accordingly, $BR(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 + e^+\nu_e) = 0.15$, $BR(\tilde{\chi}_1^+ \to \tilde{\chi}_1^0 + \mu^+\nu_\mu) = 0.137$, and $BR(\tilde{\chi}_1^- \to \tilde{\chi}_1^0 + \pi^-) = 0.604$. At last, polarizing the photon beams coming to the collision has the same distribution given in Fig. 9, and the ratio $\sigma(P_+P_+)/\sigma_{UU}$ reaches up to 1.2 in BB scenario.

D. NUHM2 and NUGM scenarios

At last, the computation for the lightest chargino pair production is also carried out for the NUHM2 and the NUGM scenarios. The two-parameter non-universal Higgs model (NUHM2) is inspired by GUT models where $H_u$ and $H_d$ belong to different multiplets. Accordingly, the soft-breaking scalar masses of the two Higgs doublets $m_{H_u}^2$ and $m_{H_d}^2$ are taken as a free parameter. Since there is no driving force to assume the Higgs fields and sfermion fields unify at the GUT scale, the universal soft-breaking scalar mass terms are assumed different for the sfermions and the two Higgs doublet fields. It is argued that in [? ], the GUT scale masses $m_{H_u}^2$ and $m_{H_d}^2$ could be trade-off for the weak scale parameters $\mu$ and $m_A$. The SLHA files are calculated using ISASUGRA, and setting the GUT scale parameters $\mu = 6$ TeV and $m_A = 275$ GeV, given in Tab. I. Accordingly, the masses of all the Higgses and the electroweakinos become light at the order of $100 - 300$ GeV, whereas the rest of the sparticles are beyond the TeV, and consequently beyond the reach of LHC. The distribution of the cross-section is given in Fig. 11 (center), and it reaches to a maximum $\sigma_{\tilde{\chi}_1^\pm\tilde{\chi}_1^\pm} = 0.79$ pb at $\sqrt{s} = 0.67$ TeV. The sum of the virtual and the real corrections shares the same trend as the other scenarios, and it falls down to $-15.3\%$.

The non-universal gaugino masses (NUGM) scenario is also motivated by the GUT models where the universality of the gaugino masses are loosened at $M_{GUT}$, and besides, that also settles the little hierarchy problem [? ]. This model is beyond the reach of the LHC due to the mass spectrum which is at the order of TeV. However, the masses of the charginos and the lightest two neutralinos are around $100 - 250$ GeV. The mass spectrum and the relevant parameters at the weak scale are calculated with ISASUGRA using the input parameters given in Tab. I. The production cross-section is $\sigma_{\tilde{\chi}_1^\pm\tilde{\chi}_1^\pm} = 2.59$ pb at $\sqrt{s} = 0.37$ TeV, which is the second highest cross-section after RNS. At $\sqrt{s} > 0.33$ TeV the sum of the virtual and real corrections becomes negative, and it lowers the cross-section up to $-15.6\%$. This model along with the RNS has a rather large production cross-section, and they could easily be studied in a $\gamma\gamma$ collider.

In these two scenarios, the decays of the lightest chargino differ from the previous scenarios, and it mainly decays via the lightest neutralino and a W-boson with $BR(\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 + W^\pm = 0.999)$. Detecting large missing energy and two W bosons at the detector is enough for extracting the chargino pairs from backgrounds. Therefore, taking into account the production-cross-section and easy way to distinguish the signal from the background make it a rather promising channel to study in a $\gamma\gamma$ collider. The production of $\tilde{\chi}_1^\pm\tilde{\chi}_2^\pm$ and $\tilde{\chi}_1^\pm\tilde{\chi}_2^\pm$ is kinematically not possible at $\sqrt{s} = 1$ TeV for the NUHM2 and the NUGM scenarios.

E. The convoluted cross-section in $e^+e^-$-collider

The total cross-section convoluted by the photon luminosity in an $e^+e^-$-collider for the benchmark points considered in this study are presented in Fig. 12. It is clear in the figure that the convoluted cross-section for the BB, the NS, and the NUHM2 scenarios have a similar distribution due to the similar partonic distributions (Fig. 10 and Fig. 11(center)). The minimal energy required for a detection is $\sqrt{s} \sim 0.6$ TeV. The cross-section rises with the COM energy as expected, and at $\sqrt{s} = 1$ TeV they get in the range of 264 – 337 fb. On the other hand, the highest convoluted cross-section is obtained for the RNS scenario, because the partonic cross-section is also the highest. The process becomes accessible even as low as $\sqrt{s} \sim 0.30$ TeV with $\sigma_{\tilde{\chi}_1^\pm\tilde{\chi}_1^\pm} = 0.4$ pb, and the cross-section rises quickly above 2.3 pb for $\sqrt{s} \geq 0.7$ TeV. At higher COM energies, due to the $1/s$ suppression of the partonic cross-section, it reaches to saturation, then falls very slowly. Next, the NUGM scenario have a cross-section $\sim 0.75$ pb at $\sqrt{s} = 0.5$ TeV. The production of the lightest chargino pair is accessible in the NUGM and the RNS scenarios even at the FCC-ee with $\sqrt{s} = 0.35$ TeV about 440 fb and 70 fb, respectively. At last, the BB scenario has a reduced cross-section, and it could be observed at the ILC with $\sigma(e^+e^- \to \gamma\gamma \to \tilde{\chi}_1^\pm\tilde{\chi}_1^\pm) = 162$ fb at $\sqrt{s} = 0.5$ TeV.
V. CONCLUSION

In this study, the lightest chargino pair production in the photon-photon collisions is investigated, and the results are presented in detail. The sum of all the virtual, the soft and the hard QED corrections are included. Then, the numerical analysis is based on the various benchmark points and scenarios. The presented numerics show that the RNS has the highest cross-section compared to all the other scenarios considered in this paper. The partonic level cross-section goes over 4 pb. Besides of the RNS, the NUGM and the BB scenarios have elevated cross-section. The other scenarios such as NS, NUHM2, and mSUGRA have identical cross-section distributions at tree-level and small variations at the NLO-level. The sum of the virtual and the real photon corrections is also similar, and they reach a maximum of $\sim -8\%$ at $\sqrt{s} = 1$ TeV for NS and mSUGRA scenarios, whereas the NLO correction is more dramatic in the NUHM2 scenario and it reaches down to $\sim -16\%$. Among all the scenarios, NUHM2 has the highest $\frac{\sigma_{NLO}}{\sigma_{LO}}$ ratio which means that the one-loop corrections become important for this scenario. The second chargino is at the TeV region for all the scenario except the RNS and the mSUGRA. The production of $\tilde{\chi}_1^{\pm} \tilde{\chi}_2^{0}$ pair is not possible for a collider with COM energy $\sqrt{s} = 1$ TeV. However, the production of $\tilde{\chi}_1^{0} \tilde{\chi}_2^{0}$ is possible in theory, and they require collision energy greater than $\sim 0.711$ TeV. The calculation is carried for this channel, but the production cross-section is at the order of 10 ab. Therefore, these results are not presented in this paper.

Polarizing the incoming photon beams enhances or suppresses the contribution coming from the various Feynman diagrams given in Fig. 1 - 6. Therefore, it has a potential to increase the cross-section. Indeed, comparing each scenario shows that the cross-section is enhanced up to 40% in RNS for left-handed polarized photon beam and right-handed polarized photon beam at $\sqrt{s} = 1$ TeV. The distributions are identical in all the scenarios for the given COM energy. The enhancement is around 30% in NUGM and 20% in BB scenario. However, polarization does not have a significant impact on the mSUGRA, the NS, and the NUHM2 scenarios at $\sqrt{s} = 1$ TeV. On the other hand, inspecting various topological Feynman diagrams show that overall excluding only the box diagrams raises the virtual contribution for all COM energies and it is positive, whereas the total virtual contribution is negative if only the self or only the vertex diagrams are excluded.

The total cross-section in a $e^+e^-$-collider is calculated by convoluting the $\gamma\gamma \rightarrow \tilde{\chi}_1^{\pm} \tilde{\chi}_1^{0}$ with the photon luminosities. The convoluted cross-sections at the FLCs with varying $\sqrt{s} = 0.3 - 1.0$ TeV are plotted in Fig. 12. The RNS scenario has the highest cross-section of all the other scenarios as expected. It reaches up to 1.55 pb at $\sqrt{s} = 0.5$ TeV, the NUGM scenario is around 0.75 pb, and for the BB scenario it is 0.16 pb. These scenarios could be accessed at the ILC with the $\sqrt{s} = 0.5$ TeV.
However, the NS, MSUGRA, and NUHM2 scenarios becomes accessible at $\sqrt{s} > 0.6$ TeV.

In a detector, the lightest chargino follows three-body decay for the RNS, NS, mSUGRA scenarios, and the topology of the signal at the detector is large missing energy from the dark matter candidate (the lightest neutralino) + two jets or a lepton for each chargino. Regarding the lepton efficiency of a detector, it could be much easier to extract the signal of chargino pairs from the events having a large missing energy + 2 leptons. Besides, large missing energy + two light jets is also signal which could not be undermined. Moreover, the decay of the lightest chargino in the NUHM2 and the NUGM scenarios differs from the previous scenarios, and the chargino mainly decays via the lightest neutralino and a W-boson with $BR(\tilde{\chi}_1^\pm \to \tilde{\chi}_1^0 + W^\pm \approx 0.999)$. Then, the signal at the detector would be large missing energy + two $W$ bosons. In the BB scenario, the $\tilde{\chi}_1^\pm$ are mainly decayed through $\tilde{\chi}_1^0$ and a pion. Considering all the possible final states, more studies at the detector level including the instrumentation and the efficiency of a detector is required for better assessment of the signal.

If the LHC could not find any hint for the supersymmetry, a great portion of the parameter space would be excluded. However, there will be some region in the parameter space where the supersymmetry would be left out. Therefore, the future lepton-lepton or photon-photon colliders could rule out these parameter spaces and put the supersymmetry at rest or discover the supersymmetric particles. The results manifest the potential of the $\tilde{\chi}_1^+ \tilde{\chi}_1^-$ pair production in a $\gamma\gamma$-collider which could be used for possible optimization in accelerator and detector design. The FLC in $\gamma\gamma$ collision mode will have minimal background coming from various sources, and the new physics could be more easily extracted from the backgrounds. The FLC could be an ideal laboratory to study the supersymmetry. It could be stated that a photon collider could lead to discoveries which reveal the secrets of the universe.

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