Fatigue behavior and predictive modeling of short fiber thermoplastic composites

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Abstract

Cyclic deformation and fatigue behavior of two short fiber thermoplastic composites (SFTCs) under a number of loading and environmental conditions are investigated. The considered environmental effects include those of low and elevated temperatures as well as moisture (or water absorption). Fatigue behavior is also explored under the action of non-zero mean stress (or $R$ ratio) in addition to fully-reversed ($R = -1$), as well as various cyclic loading frequencies. Material anisotropy and geometrical discontinuity effects (i.e. stress concentration) are other aspects considered in this study. Based on experimental observations and analysis, a number of analytical and empirical models are developed for predicting fatigue behavior under different conditions. Empirical equations are presented to characterize self-heating under cyclic loading. Tsai-Hill criterion is applied to account for the effect of fiber orientation on fatigue life. Mean stress effect is corrected with several mean stress parameters and a shift factor of Arrhenius type is defined to characterize the effect of temperature on fatigue life. Two methodologies are presented to estimate fatigue properties based on tensile properties. Estimation of notched fatigue behavior based on smooth fatigue behavior is also presented.

Keywords: Fatigue; Short Fiber; Thermoplastic Composites; Predictive Modeling

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1. Introduction

Application of short fiber thermoplastic composites (SFTCs) is increasingly growing due to their remarkable properties. Light weight, low manufacturing cost with a high volume production rate, and the capability to be molded in complex geometries are some characteristics of SFTCs. A wide range of effects related to microstructure, environment and load conditions are involved in fatigue design and application of SFTCs. However, a relatively small number of studies have been conducted on fatigue behavior characterization of SFTCs, although components made of these materials are typically subjected to cyclic loads. Due to the complexity as well as a large number of parameters influencing mechanical behavior of SFTCs, fatigue behavior has been mainly evaluated through experimental techniques, while less attention has been given to fatigue behavior modeling and predictions [1].

Under cyclic loading, a continuous softening is generally observed in SFTCs, which is typically due to initiation and growth of damage in the matrix, at fiber ends, and at fiber matrix interface [2, 3]. Thermoplastic materials exhibit time dependent properties and relatively low melting temperatures. As a result, significant effect of load frequency, including self-heating, is observed on fatigue behaviour of SFTCs [4].

Increased fatigue performance of SFTCs is a function of fiber reinforcement, its orientation and distribution. This in turn is associated with the geometry of fibers and the component, viscoelastic behavior of the matrix, and flow field during the injection molding process [5]. A shell-core morphology across the thickness of a molded part has frequently been reported for SFTCs, where higher degree of fiber alignment exists in two shell layers compared with the core layer [6]. Recent micro-tomography studies have been performed on SFTCs for fatigue their damage investigation [7]. However, the effect of fiber orientation effect has been commonly evaluated through conducting fatigue tests on samples of different thicknesses and with fibers in different directions with respect to the loading [6].

Effects of environment including temperature and moisture on fatigue behavior of SFTCs have been explored in several studies. A significant degradation of fatigue strength has been observed from temperatures below to above the glass transition temperature ($T_g$) [6]. A recent survey has been performed on high temperature fatigue behavior of SFTCs in [8]. The effect of water absorption is highly depends on the polymer type and fiber-matrix coupling agents [9].

A relatively small number of studies have been devoted to effects of mean stress and stress concentration on fatigue behavior of SFTCs. A significant effect of mean stress, which may be accompanied by cyclic creep or ratcheting is observed on fatigue behavior of SFTCs [10]. The reduction of fatigue strength due to mean stress has been observed to be less for notched specimens as compared with smooth specimens, due to the presence of stress gradient near the notch. Modified Goodman and Gerber mean stress equations have been used with the use of creep rupture strength, rather than ultimate tensile strength in the original forms of the equations, to correct for the mean stress effect [11, 12].

In this study, a number of aspects related to fatigue behaviors of two short fiber thermoplastic composites (SFTCs) were experimentally investigated. Methodologies are presented to account for these aspects in fatigue life predictions. First the materials and specimen geometries used, as well as the experimental procedure are described. Then, experimental results for the different effects considered are presented and discussed, followed by the fatigue analysis model used to represent or
predict each effect. The considered effects include cyclic deformation, load frequency and self-heating, anisotropy or fiber orientation effect, moisture, temperature, mean stress, and stress concentration.

2. Material, specimen geometry, and experimental method

The two composite materials considered were a polybutylene terephthalate with 30 wt% short glass fiber (here referred to as PBT) and a polyamide-6 with about 10 wt% rubber and 35 wt% short glass fiber (here referred to as PA6). The glass transition temperature \( T_g \) of both materials was about 60 °C, as obtained from dynamic mechanical analysis, and the fiber aspect ratio was about 26 [13, 14].

Materials were injection molded in rectangular plaques with dimensions of 100 mm × 200 mm in 3 and 3.8 mm thicknesses. To study the effect of fiber orientation, rectangular strips were machined from molded plaques in four angles with respect to the injection mold flow direction, as shown in Figure 1(a). A dog-bone shape specimen with an optimized geometry was cut from the strips, as shown in Figure 1(b). For notched tests, a circular hole with diameter of 2 mm was drilled in the center of the specimen gage section.

![Figure 1. (a) Specimen cutting directions with respect to mold flow and, (b) specimen geometry designed for fatigue tests. For notched specimens a 2 mm diameter central hole was drilled in the middle of the gage section (all dimensions are in mm).](image)

Fatigue tests were conducted on a uniaxial servo-hydraulic testing machine and controlled by a digital controller. A mechanical extensimeter was used to measure strain and a thermal imaging camera was used to measure surface temperature rise in room temperature tests. For temperature effect study, an environmental chamber employing an electronic heating element and a liquid nitrogen cooling system was used. Specimen immersion at room temperature water for certain period of time prior to testing was considered for moisture or water absorption effect study.

Load-controlled unnotched fatigue tests were performed in a range of cycles to failure between \(10^3\) and \(10^6\) cycles on specimens machined in 0°, 18°, 45°, and 90° directions of mold flow. Fatigue tests were conducted at -40 °C, 23 °C, and 125 °C under the stress ratios of -1, 0.1, and 0.3. Notched fatigue tests were conducted using specimens in both longitudinal and transverse directions and under stress ratios of -1 and 0.1.
3. Experimental results, analysis, and models

3.1 Cyclic deformation behavior

Incremental step cyclic deformation tests were performed with a sinusoidal load-controlled wave form under fully-reversed ($R = -1$) condition at -40°, 23° and 125 °C and in both longitudinal and transverse directions. Test continued at each stress level until a relatively stabilized strain was reached. Test results were then used to obtain cyclic stress-strain curves and evaluate cyclic softening.

Progressive cyclic deformation of PBT in the transverse direction at a stress level corresponding to 65% of tensile strength is shown in Figure 2(a). The area under the hysteresis loops increases and the cyclic modulus decreases with continued cycling. Due to higher straining in tension than in compression, the strain amplitude and mean strain increase with continued cycling. This behaviour was observed for both materials, in both longitudinal and transverse directions, and at all test temperatures. The aforementioned changes in hysteresis loops were more pronounced at higher temperatures and at higher stress levels, such that at stress levels corresponding to fatigue life of $10^3$ cycles or shorter, no stabilized hysteresis loop was observed.

The stress-strain response of the materials under cyclic loading can be quite different from that under monotonic loading. Cyclic stresses and strains from relatively stabilized hysteresis loops were used to obtain the cyclic stress-strain curves and Ramberg-Osgood equation was used to mathematically represent the cyclic stress-strain behaviour, expressed as [15]:

$$\varepsilon'_a = \frac{\sigma'_a}{E'} + \left( \frac{\sigma'_u}{K'} \right)^{1/n'}$$

where $K'$ and $n'$ are cyclic strength coefficient and cyclic hardening exponent, respectively, and obtained from the fit of true stress amplitude versus true plastic strain amplitude.

The cyclic stress-strain data, corresponding Ramberg-Osgood curves, as well as and monotonic tension curves were superimposed in Figure 2(b) for PBT in the longitudinal direction and at temperatures of -40 °C, 23 °C, and 125 °C. Significant cyclic softening is observed at room temperature, while small softening is observed at -40 °C and 125 °C. Similar behaviour was observed for the transverse direction, as well as for PA6.

Figure 2. (a) Progressive deformation of PBT in the transverse direction with hysteresis loops shown from initial cycles to and near fracture cycles and, (b) Cyclic stress-strain data and the corresponding Ramberg-Osgood (dashed) and monotonic tension (solid) curves for PBT in the longitudinal direction.
3.2 Temperature rise effect and modelling

The effect of frequency on $R = -1$ fatigue life of PA6 in the longitudinal direction is shown in Figure 3(a). Two stress amplitudes corresponding to 40% and 34% of ultimate tensile strength ($S_u$) were used. Considerable effect of frequency on fatigue life was observed by changing the frequency by a factor of 4 for the higher stress amplitude test (from 0.25 Hz to 1 Hz) and by a factor of 2 for the lower stress amplitude test (from 2 Hz to 4 Hz).

Displacement amplitude versus applied cycles for these tests are shown in Figure 3(b), along with the increase in measured surface temperature. As can be seen from this figure, the temperature increase was more than 13 °C at 1 Hz for the higher stress amplitude test and more than 35 °C at 4 Hz for the lower stress amplitude test. The degree of cyclic softening, as reflected by the increase in displacement amplitude, is directly related to the amount of increase in temperature.

Regardless of the chosen stress amplitude and test frequency, when the temperature rise exceeded 10 °C, the displacement amplitude rapidly increased and thermal failure occurred. Therefore, low test frequencies were chosen to limit the temperature rise to a maximum of 10 °C in fatigue tests in order to prevent significant self-heating. A relatively higher sensitivity to frequency was observed for PA6 as compared with PBT, due to a higher dissipation of energy in PA6.

![Figure 3. (a) Effect of testing frequency on fatigue life and, (b) Displacement amplitude versus applied cycles for PA6 in the longitudinal direction at room temperature under $R = -1$ condition for stress amplitudes of 40% and 34% of $S_u$.](image)

To characterize the effects of stress level and frequency on temperature rise, incremental step tests with increasing frequency at each step were performed. At each stress level, cycles were applied under a constant frequency until the surface temperature of specimen stabilized. Then the test was stopped for a period of time, until the surface temperature returned to the room temperature.

Transient temperature rise curves with applied cycles at different frequencies are shown in Figure 4(a) for a longitudinal sample of PBT under stress amplitude of 32% of $S_u$. The stabilized temperature is higher at higher frequency, as expected. At a critical frequency, the temperature rise was more than 10 °C and surface temperature did not stabilize.
At all stress levels of both materials and in both longitudinal and transverse directions of mold flow, a linear relationship between stabilized temperature rise and cycling frequency was obtained, as seen in Figure 4(b). As the stress level is increased, a higher rate of temperature rise is observed with increased frequency.

Energy-based models were applied to the incremental step frequency data to generalize correlation of temperature rise as a function of cycling frequency and stress amplitude for each material. A linear relationship between the temperature rise and dissipated energy per unit volume and time was obtained, as follows [4]:

\[ \Delta T = B \omega f \]  

where \( f \) is test frequency, \( \omega \) is the area inside the hysteresis loop, \( \Delta T \) is temperature rise, and \( B \) is a material parameter. A linear correlation of temperature rise data presented in Figure 4(b) with parameter \( (\omega f) \) in Equation 2 is shown in Figure 4(c).

Another model based on a constant energy approach was also applied to the data. This model is based on a linear one-dimensional conductive heat transfer and assumes negligible energy storage in the test specimen, expressed as [16]:

\[ \Delta T = C \sigma_a \varepsilon_a^2 f \]  

where \( C \) is a material parameter and obtained by a linear fit of temperature rise data on Equation 3. Linearity of the relationship between temperature rise and \( (\sigma_a \varepsilon_a^2 f) \) parameter is observed in Figure 4(c).

Knowing the constant \( B \) in Equation 2 or the constant \( C \) in Equation 3, temperature rise as a function of the loading (stress and strain) and the applied frequency can be estimated. These constants for each material and mold flow direction can be obtained from a small number of tests.

Figure 4. (a) Transient surface temperature rise curves at the stress amplitude of 32% \( S_u \), (b) Stable surface temperature rise as a function of cycling frequency at different stress amplitudes and, (c) Fits of stable surface temperature rise versus parameters introduced in Equations 2 and 3 [17].
3.3 Anisotropic fatigue behavior and modeling

Figure 5 shows the effect of fiber orientation on fatigue behavior of PA6 at 23 °C under $R = 0.1$ condition. Specimens in 3 mm and 3.8 mm thicknesses were machined in the longitudinal and transverse directions of mold flow. For samples in the longitudinal directions, a comparison was also made between the fatigue strength in edge and middle of the plaque geometry shown in Figure 1(a).

Fatigue lives of the middle specimens were reduced by about a factor of two, compared to the edge specimens, as seen in Figure 5(a). This is thought to be due to the relatively thinner core layer in the edge samples, as compared to the middle samples. No effect of thickness is observed in the longitudinal direction, while in the transverse direction fatigue lives of 3 mm samples were reduced by about a factor of 5 for PA6, as compared to the fatigue lives of 3.8 mm samples. The effect of thickness in the longitudinal direction of PBT was also negligible, but a factor of 4 reduction in fatigue life was observed from 3.8 mm to 3 mm thickness.

The effect of thickness is due to a shell-core morphology observed across the thickness of the short fiber composites. Due to the gradient of velocity of mold during the injection molding process, fibers in the core layer are mainly oriented perpendicular to the injection molding direction, while they are highly in line with the mold flow direction in the two shell layers near the walls, as seen from Figures 5(b) and 5(c). The core layer comprises about 0.5 mm and 0.2 mm of the specimen thickness for 3.8 mm and 3 mm thickness samples, respectively. The effect of thickness on fatigue strength of such materials can, therefore, be related to the thicknesses of the core and shell layers.

![Figure 5](image.png)
The effect of mold flow direction was evaluated in 0°, 18°, 45°, and 90° directions relative to the injection molding direction. Due to the presence of fiber reinforcement, fatigue performance was highly dependent on the mold flow direction. With increasing the specimen angle with respect to the mold flow direction, fatigue strength increases such that longitudinal specimens have about 40% higher fatigue limit (defined as fatigue strength at $10^6$ cycles) than transverse specimens, as seen in Figure 6(a) for PBT at $R = -1$ condition. A higher degree of anisotropy is observed at 125 °C, compared with 23 °C and -40 °C, such that fatigue limit increased about 60% from the transverse direction to the longitudinal direction of injection mold flow.

The Tsai-Hill [18] criterion was utilized to predict the off-axis fatigue strength, which is commonly used for orthotropic laminate composites, expressed as:

$$
\sigma_{fat}(\theta) = \left[ \frac{\cos^2(\theta) \sigma_{L,fat}^2(N)}{\sigma_{L,fat}^2(N)} + \frac{\sin^2(\theta) \sigma_{T,fat}^2(N)}{\sigma_{T,fat}^2(N)} + \frac{\cos^2(\theta) \tau_{LT,fat}^2(N)}{\tau_{LT,fat}^2(N)} \right]^{1/2}
$$

where $\sigma_{L,fat}(N)$, $\sigma_{T,fat}(N)$, and $\tau_{LT,fat}(N)$ are the experimental fatigue strengths for a specimen life of $N$ cycles. $S-N$ curves generated in 0°, 45°, and 90° directions were used to determine the directional properties of Tsai-Hill equation to estimate the $S-N$ curves for the 18° direction tests. The Tsai-Hill criterion reasonably captures the effect of mold flow direction on fatigue behavior. Comparison of the experimental fatigue strength and Tsai-Hill criterion correlations as a function of fiber orientation angle with respect to the load direction for different fatigue lives can be observed in Figure 6(b).

Dried specimens for PBT (at 120 °C for 6 hours) and PA6 (at 80 °C for 6 hours) in the longitudinal direction of injection mold flow were immersed in room temperature water. The percentage of water absorption was periodically measured by weighting the specimen. Water absorption variation with square root of exposure time ($t^{1/2}$) is shown in Figure 7(a) for longitudinal samples of PA6. As seen, percentage of water absorption by weight linearly increases until it reaches a plateau with a maximum percentage of water absorption of 5.2 wt%. This behavior follows the Fick’s law commonly used for modeling the kinetics of moisture absorption process, expressed as [19]:

$$
\frac{M}{M_m} = 1 - \frac{8}{\pi^2} \exp \left(-\left(\frac{D_t}{h^2}\right)\pi^2 \right)
$$
where $D$ is diffusion coefficient, $h$ is specimen thickness, $t$ is exposure time, $M_t$ is absorbed water, and $M_m$ is the maximum capacity of water absorption. The diffusion coefficient was estimated by fitting Equation 5 to absorption data at short times, shown in Figure 7(a). Diffusion coefficient was estimated at $5.5 \times 10^{-13} \text{m}^2/\text{s}$. Using the calculated diffusion coefficient and thickness of specimen, along with the maximum water absorption capacity, Equation 5 can be used to predict the absorbed water with exposed time.

Figure 7(b) indicates variations of tensile strength and elastic modulus with percentage of absorbed water for longitudinal samples of PA6. Exponentially decaying fits are observed for these properties. The degraded properties of samples with maximum water absorption (5.2 wt%) recovered by only 50% after 30 hours of hot drying, while it was fully recovered by hot vacuum drying after 12 hours. It should also be mentioned that the rate of water absorption for PBT was significantly lower than for PA6, such that after four days of immersion less than 0.1 wt% water absorption and no degradation of tensile properties were observed.

Fatigue behavior was studied in dry condition as well as wet condition with four days of immersion at room temperature water. Figure 7(c) shows the $R = 0.1$ S-N curves of PA6 in both longitudinal and transverse directions. Fatigue life reduced by more than an order of magnitude in both LCF and HCF life regimes and in both the longitudinal and transverse directions.

At the same stress amplitude levels, significantly larger stress-displacement loops at midlife were observed in wet samples, as compared with the dried samples. Stress-life data in dry and wet conditions became closer when plotted in terms of the area inside the stress-displacement loops at midlife. The ratio of fatigue strength at $10^6$ cycles to tensile strength was about 0.25 for both longitudinal and transverse directions in both wet and dry conditions.

Figure 7. (a) Kinetics of water absorption at room temperature water (the dashed line corresponds to Fick’s law), (b) Variations of tensile strength and elastic modulus with water absorption for longitudinal samples of PA6, and (c) S-N curves at room temperature under $R = 0.1$ condition showing the effect of moisture in both longitudinal and transverse directions of PA6 [20].
3.5 Test temperature effect and modeling

A significant effect of temperature was observed in both longitudinal and transverse directions. Fatigue strength at 125 °C significantly decreased compared to the 23 °C, and increased at -40 °C, as seen in Figure 8(a).

S-N fatigue data were correlated by shifting the fatigue life at various temperatures to a reference temperature \((T_0)\). A shift factor \((a_{T_0})\) is defined in order to generate a master curve for a particular material, direction, and stress ratio as:

\[
a_{T_0}(T) = \frac{N_f'}{N_f} \quad (6)
\]

where \(N_f'\) is the reduced cycles to failure due to the effect of temperature. The 23 °C data were selected as references and the 125 °C and -40 °C data were shifted to the right and left sides of 23 °C data, respectively, until sufficiently high data correlations were obtained for all the data at various temperatures. Master curves in the two mold flow directions of PA6 under \(R = -1\) condition are shown in Figure 8(b).

The log shift factor obtained from experimental data is plotted as a function of the reciprocal of test temperature in Figure 8(c). The equation of line fits follows the Arrhenius equation form, expressed as:

\[
Log a_{T_0} = \frac{E_a}{8.314} \left( \frac{1}{T} - \frac{1}{T_0} \right) \quad (7)
\]

where \(T_0\) is reference temperature and \(E_a\) is the activation energy which is different for the temperature ranges above and below the glass transition temperature \((T_g)\).

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**Figure 8.** (a) Effect of temperature on fatigue behaviour, (b) Construction of master curves of stress amplitude for fully-reversed data of PA6 in both longitudinal and transverse directions and, (c) Variation of log shift factor with the reciprocal of temperature for PBT and PA6 under various test conditions [14].
Knowing $E_a$ and $T_0$ in the Arrhenius equation for a particular material, the shift factor at any temperature can be calculated from Equation 7. Using the master curve, the fatigue life at that temperature for a particular stress level can then be estimated. Since the Arrhenius equation form is identical for each material at both stress ratios ($R = -1$ and $R = 0.1$) and in both mold flow directions (L and T), only one mold flow direction, one stress ratio, and two temperatures above and two temperatures below $T_g$ are needed to obtain this equation.

3.6 Mean stress or R ratio effect and modeling

Figures 9(a) shows the effect of mean stress on fatigue behaviour of PA6 in the transverse direction at 23 °C, -40 °C, and 125 °C. A significant decrease of fatigue strength is observed under $R = 0.1$ loading condition, as compared to $R = -1$, at all test temperatures. The effect of tensile mean stress was more pronounced in the LCF regime, as compared to the HCF regime. Smaller or no difference in fatigue lives is observed between $R = 0.1$ and $R = 0.3$ conditions in the HCF regime, as compared with in the LCF regime.

Many mean stress parameters have been applied to predict the effect of mean stress. The Walker equation and a general fatigue life prediction model showed more accurate correlations of mean stress data for different test conditions considered in this study. The Walker equation can be expressed as [21]:

$$S_{N_f} = (S_a + S_m)^{-\gamma} (S_u)^{\gamma} \tag{8}$$

where $S_a$, $S_m$, and $S_{N_f}$ are stress amplitude, mean stress, and fully-reversed stress amplitude, respectively, and $\gamma$ is the mean stress parameter. The value of $\gamma$ was determined by the best fits obtained for $R = 0.1$ and 0.3 data to the fully-reversed data, for each material, mold flow direction, and temperature. A low $\gamma$ value indicates a higher mean stress sensitivity and a value of $\gamma = 1$ indicates no mean stress sensitivity.

The fits of experimental data for PA6 in the transverse direction based on the Walker equation are shown in Figure 9(b), indicating reasonable mean stress correction for all the temperatures considered. For both materials at 23 °C and -40 °C, the $\gamma$ value is nearly constant at about 0.47, with a range between 0.4 and 0.55, while at 125 °C, $\gamma$ values indicated the following relationship with the tensile strength in the corresponding direction and temperature:

$$\gamma = 0.0043S_u + 0.364 \tag{9}$$

A general fatigue life prediction model based on a strength degradation concept under constant amplitude loading was also applied to the experimental data. The simplified form of the equation arranged in an equivalent stress form is expressed as [22]:

$$S_{eq} = A \left( \frac{S_u - \frac{\Delta S}{1 - R}}{\alpha S_u^{R \sin \theta - 0.6}} + 1 \right)^{B / \beta} \tag{10}$$
where \( A \) and \( B \) are the intercept and slope of the fully-reversed \( S-N \) line, respectively, \( \Delta S \) is the stress range, \( R \) is the stress ratio, \( \theta \) is fiber orientation angle, and \( \alpha \) and \( \beta \) are the model parameters. Good correlations of equivalent stress values versus fatigue life for different stress ratios of PA6 in the transverse direction and at different temperatures are shown in Figure 9(c).

The value of \( \alpha \) was nearly independent of stress ratio, but varied with temperature and mold flow direction. In the longitudinal direction of both PBT and PA6 and at all test temperatures, \( \alpha = 0.135 \) was suggested. In the transverse direction of both PBT and PA6, \( \alpha = 0.074 \) was suggested for both 23 °C and -40 °C tests and \( \alpha = 0.1 \) was suggested for 125 °C tests.

It should be mentioned that although the data and mean stress correlations are shown for PA6 in the transverse direction, similar behaviours and correlations were obtained for the longitudinal direction, as well as for PBT.

### 3.7 Estimation of fatigue strength from tensile strength

Fully-reversed (\( R = -1 \)) fatigue strengths at fatigue lives of \( 10^3 \) and \( 10^6 \) cycles linearly correlated well with the corresponding tensile strength at different temperatures and in different mold flow directions. These correlations between these fatigue strengths and tensile strength were observed to be relatively independent of material, temperature, and mold flow direction. Therefore, fully-reversed \( S-N \) fatigue line was estimated based on the extrapolation of fatigue strengths at \( 10^3 \) and \( 10^6 \) cycles, as:

\[
\frac{S_u}{S_{uf}} = 1.08 (N_f)^{-0.085}
\]  

(11)
Correlation of fatigue strength data normalized by tensile strength is shown in Figure 10(a). Therefore, in the absence of fatigue data, this relation may be used to roughly estimate fatigue life at a given stress amplitude for a particular material, temperature, and mold flow direction, based on the tensile strength for the material and conditions.

The general fatigue life prediction model introduced in Equation 10 can be also be used in a simpler form to correlate the fully-reversed fatigue data at of the two materials at different temperatures and in different mold flow directions, expressed as:

$$S_{eq} = 96.3 \left( \frac{S_u - \frac{\Delta S}{2}}{\alpha S_u - 0.6 (\Delta S)^{1.6}} + 1 \right)^{-0.16}$$  \hspace{1cm} (12)

This equation has only one material parameter $\alpha$, which is dependent on fiber orientation and can be approximated as $\alpha = -0.0005 \theta + 0.118$, where $\theta$ is the specimen angle with respect to the mold flow direction in degrees. A better correlation of data was obtained using Equation 12, as compared to Equation 11, as can be seen in Figure 10(b).

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3.8 Stress concentration effect and analysis

A significant decrease of fatigue life was observed in notched specimens, as compared with smooth (i.e. unnotched) specimens, as seen in Figure 11(a) for PBT in the transverse direction under $R = -1$ loading condition. Nominal stress amplitude for notched specimens plotted in this figure was obtained by dividing the applied load amplitude by the net cross section area of the specimen.

Fatigue strength sensitivity of a material to a notch can be characterized by the notch sensitivity factor, expressed as:

$$q = \frac{K_t - 1}{K_f - 1}$$  \hspace{1cm} (13)

where $K_t$ is the elastic stress concentration factor and $K_f$ is the fatigue notch factor (defined as unnotched fatigue strength divided by notched fatigue strength). Notch sensitivity factor takes values between zero (no notch sensitivity) and one (full notch sensitivity).
This factor was obtained from experimental results for fully-reversed fatigue strength at $10^6$ cycles to be about 0.5 in both longitudinal and transverse directions. In LCF regime a lower notch sensitivity was observed in both mold flow directions, such that smooth and notch specimen $S$-$N$ lines nearly converged at one cycle. This is due to notch plastic deformation in the LCF regime reducing notch sensitivity and is similar to the behaviour typically observed for ductile metallic materials [15].

The local strain or stress approach can be utilized for notch behaviour predictions by using finite element analysis (FEA) results, or by using a notch deformation rule such as the commonly used Neuber rule for metallic materials. To consider the effect of stress gradient, $K_f$ rather than $K_t$ is often used in Neuber rule for life predictions, and an approach such as the theory of critical distance (TCD) can be used in conjunction with the FEA results. These approaches were used for the notched fatigue data in this study, as detailed in [23].

Notched fatigue life data correlations based on the local stress approach using FEA, Neuber rule, and TCD method for PBT are shown in Figure 11(b). As can be observed from this figure, FEA notch stress results in overly conservative life predictions by several orders of magnitude. This is because the effect of stress gradient at the notch is also an important factor in controlling notched fatigue behaviour. Better correlations are obtained by using Neuber rule or TCD.

![Figure 11. (a) Effect of notch on fully-reversed fatigue behaviour of PBT in the transverse direction under $R = -1$ condition and, (b) Smooth and notched fatigue data correlations based on local stress approach and FEA, Neuber rule, and TCD methods [23].](image)

4. Summary and conclusions

Application of short fiber thermoplastic composites (SFTCs) has seen significant growth in recent years. A wide range of aspects related to microstructure, environment and load conditions affect fatigue behavior of these materials. A number of these effects were experimentally investigated by conducting uniaxial constant amplitude load-controlled tests for two short fiber thermoplastic composites in this study. The considered effects included cyclic deformation, load frequency and self-heating, anisotropy or fiber orientation, moisture, temperature, mean stress or $R$ ratio, and stress concentration. Fatigue analysis models for representing or predicting each effect were also presented.

Based on the experimental observations and the analyses conducted, the following conclusions can be made:

(a) 
(b)
1. Cyclic deformation behaviour indicated progressive modulus reduction and unsymmetrical straining in tension and compression. Cyclic softening was observed at room temperature, while little or no softening was observed at -40 ºC and 125 ºC. The cyclic stress-strain curves for different conditions could be represented by the Ramberg-Osgood equation.

2. Cycling frequency resulting in self-heating can lead to cyclic softening and a significant detrimental effect on fatigue behaviour. Energy-based models were used to characterize the temperature rise as a function of loading and frequency. Such models can be used to estimate the amount of temperature rise when conducting fatigue tests.

3. An effect of thickness on fatigue behaviour was observed in the transverse specimens, resulting from a core–shell morphology produced during the injection molding process. The effect of fiber orientation with respect to the loading on fatigue strength was significant. Tsai–Hill criterion could represent the effect of mold flow direction on fatigue strength reasonably well.

4. A significant detrimental effect of moisture was found on fatigue strength of PA6, while the effect was negligible for PBT. Water absorption by weight linearly increased with square root of time, until it reached a plateau of 5.2 wt% for PA6. The effect of moisture on fatigue strength reduction in both LCF and HCF and in both longitudinal and transverse directions of PA6 was nearly identical. The degraded properties can be recovered, at least partially, by drying.

5. A significant effect of temperature on fatigue behavior was observed in both longitudinal and transverse directions of both materials. Cold temperature had a beneficial effect and elevated temperature had a detrimental effect on fatigue life, as compared to at room temperature. Fatigue data at different temperatures were correlated by a shift factor represented by Arrhenius equation, the form of which was independent of stress ratio and mold flow direction.

6. A significant decrease of fatigue strength was observed under tensile mean stress for both materials, all mold flow directions, and at all test temperatures. The effect of tensile mean stress was more pronounced in the LCF regime, as compared to the HCF regime. The Walker equation and a more general fatigue life prediction model correlated the mean stress data at all temperatures and in both mold flow directions reasonably well.

7. Two methods were evaluated for estimating fatigue strength of SFTCs for different conditions (i.e. material, fiber orientation direction, temperature) as a function of cycles to failure based on ultimate tensile strength for the corresponding condition. These methods may be used as a reasonable first estimate of fatigue strength, in the absence of fatigue data.

8. A significant decrease of fatigue life was observed in notched specimens, as compared with smooth (i.e. unnotched) specimens. Lower notch sensitivity was observed in LCF regime, as compared to HCF regime, due to significant notch plastic deformation in LCF. Notched fatigue life predictions based on the local stress approach using FEA results were overly conservative, while the use of Neuber rule or TCD provided relatively accurate predictions.

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