Quantum Hall effect in spin-degenerate Landau levels: Spin-orbit enhancement of the conductivity

D. G. Polyakov
Institut für Theoretische Physik, Universität zu Köln, Zülpicher Str. 77, 50937 Köln, Germany

M. E. Raikh
Department of Physics, University of Utah, Salt Lake City, Utah 84112

The quantum Hall regime in a smooth random potential is considered when two disorder-broadened Zeeman levels overlap strongly. Spin-orbit coupling is found to cause a drastic change in the percolation network which leads to a strong enhancement of the dissipative conductivity at finite temperature, provided the Fermi level \( E_F \) lies between the energies of two delocalized states \( E = \pm \Delta, 2\Delta \) being the Zeeman splitting. The conductivity is shown to exhibit a box-like behavior with changing magnetic field: \( \sigma_{xx} \) is \( \sim e^2/h \) at \( |E_F| < \Delta \) and exponentially small otherwise. Two peaks of \( \sigma_{xx} \) arising as \( T \to 0 \) are found to be strongly asymmetric.

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The conventional picture of the integer quantum Hall effect (QHE) implies that there is only one delocalized state in the middle of a disorder-broadened Landau level. Numerical simulations [1–3] support this concept: the localization length \( \xi \) is shown to diverge as \( |E|^{\gamma} \), \( \gamma \simeq 2.3 \), when the energy \( E \) approaches the Landau level center \( (E = 0) \). This conclusion can also be drawn from low-temperature measurements of the longitudinal conductivity \( \sigma_{xx} \) in GaAs heterostructures: the width of a peak of \( \sigma_{xx} \) shrinks with lowering \( T \) as \( T^\kappa \) down to 25 mK [4,5]. The localization-length exponent \( \gamma \simeq 2.3 \) was measured directly by analyzing how the peak width scales with the sample size in small Hall-bar geometries [6].

The above picture applies when the disorder-induced width of the Landau level \( \Gamma \) is smaller than the Zeeman splitting \( 2\Delta \). In the opposite case, \( \Gamma \gg \Delta \), one should expect the existence of two delocalized states within a single peak of the density of states, each corresponding to a different projection of spin. As a result, two \( \sigma_{xx} \)-peaks may correspond to one peak of the density of states. Whether the \( \sigma_{xx} \)-peak spin-splitting is observable is determined by both \( T \) and the strength of disorder: the peaks merge with increasing \( T \) at a characteristic temperature which is a growing function of the parameter \( \Delta/\Gamma \).

The splitting has been recently observed [6] by tilting the sample with respect to the magnetic field (this technique makes it possible to increase the effective \( g \)-factor). The analysis of the temperature behavior of \( \sigma_{xx} \) at different values of \( \Delta/\Gamma \) enabled the authors to conclude that the data for \( \sigma_{xx} \) as a function of the Fermi level position cannot be represented as a superposition of two single peaks not related to each other. Namely, confirming previous experimental results [1,7], spin-unresolved \( \sigma_{xx} \)-peaks were claimed to shrink with decreasing \( T \) anomalously slow - with the exponent \( \kappa \) approximately half that for a single Zeeman level. The anomaly in the behavior of spin-degenerate \( \sigma_{xx} \)-peaks has been recently reported also with respect to their microwave-frequency broadening [10] and broadening with current [11,12]. However, it was argued in subsequent discussions [13,14] that a wider range of experimental parameters is needed in order to make a conclusive statement about the value of \( \kappa \). What we would like to note in this connection is that when the sweeping of the Fermi level shows two close spin-split \( \sigma_{xx} \)-peaks, the width of each of them, though being characterized by the “normal” value of \( \kappa \), is much larger than that of well-separated Zeeman peaks at the same \( T \). In other words, if the width of the peak \( \Delta \nu \), \( \nu \) being the filling factor, is represented in the form \( \Delta \nu = (T/T_1)^\kappa \), the characteristic temperature \( T_1 \) for close spin-split peaks is observed to be much smaller than for nonoverlapping Zeeman levels. For example, the values of \( T_1 \) extracted from the data presented in Ref. [4] are as follows: \( T_1 \sim 30K \) for \( N = 1 \) spin-split peaks whereas it is \( \sim 600K \) for \( N = 0 \) \( \downarrow \) peak. This observation seems to be compelling experimental evidence that overlapping of Zeeman levels indeed can strongly impede the localization of electron states.

Clearly, the anomalous behavior of \( \Delta \nu \) for two close Zeeman levels can be accounted only for the spin-orbit (SO) interaction. The question is, How can the weak SO-interaction manifest itself strongly in the conductivity? The purpose of the paper is to give answer to the question by considering a quasiclassical model of electron motion in a long-range random potential. We will show that in this case the SO-interaction can lead to a drastic change in the percolation network which causes a strong enhancement of the conductivity in the QHE regime.

To study the role of the SO-interaction comprehensively, understanding of the nature of the localization in the QHE regime should have been a starting point. However, by now an analytical theory of the quantum, disorder-induced, localization of the Landau level states is missing. There exists a completely classical approach to the localization [13]. It pertains to the case of a smooth random potential \( V(\mathbf{\rho}) \) with a correlation radius \( d \) much
larger than the magnetic length $\lambda$. Electrons move along the equipotential lines $V(\rho) = E$, so that their trajectories are closed. The exception is one of the equipotentials $V(\rho) = 0$ which penetrates through the entire system. In the classical picture, only electrons on this percolating trajectory contribute to $\sigma_{xx}$ at $T = 0$. As it was first pointed out by Chalker and Coddington \cite{12}, the picture has the defect that tunneling through saddle-points of $V(\rho)$ is ignored. Meanwhile this tunneling becomes crucial in an energy band of the width $\Delta t$ around the level $E = 0$ causing the coupling of electron states in adjacent cells of the percolation network.

Since $\Delta t \ll \Gamma$ in the smooth potential, a plausible situation is that the Zeeman splitting \cite{13}, being much smaller than $\Gamma$, is still much larger than $\Delta t$ ($\Delta t \ll \Gamma$). In other words, the Zeeman levels may overlap while the tunneling through the saddle-points may be neglected. It is this case that is considered in the Letter (the opposite case, $\Delta \lesssim \Delta t$, has been considered numerically in Refs. \cite{13}). The absence of the tunneling makes it reasonable to start with the classical picture [15].

The trajectories corresponding to the same energy, close to that of a saddle-point, in the presence of spin-splitting. The saddle-point separates two trajectories for spin-down but does not split up the trajectory for spin-up. Arrows on the trajectories indicate direction of motion. Due to the spin-orbit interaction, an effective coupling between the spin-down states is provided by the spin-up trajectory without any tunneling through the saddle-point.

We characterize the strength of SO-coupling by the length $L_{so}$. Its physical meaning is that an electron wave packet which is initially on, say, a spin-up trajectory will be equally distributed among the spin-up and spin-down trajectories typically after traveling the length $L_{so}$. According to the picture above, the crucial parameter is the ratio $L_{so}/L(\Delta)$. Evaluation of $L_{so}$ in the case of a smooth random potential will be published elsewhere.

The question we now turn to is, Suppose $L_{so} \lesssim L(\Delta)$, how then will a classical electron travel over the system? Let us demonstrate that if $|E| < \Delta$, the electron, following the classical trajectories, can percolate. On the contrary, if $E$ is outside the band ($-\Delta, \Delta$), its motion is restricted to a finite area. First of all, note that the trajectories with opposite spins at a given $E$ may be obtained as the equipotentials $V(\rho) = E \pm \Delta$. Therefore, if one considers a point in the space between two neighboring trajectories with opposite spins, it will belong to an equipotential whose energy is somewhere in the interval $(E - \Delta, E + \Delta)$ and vice versa: all equipotentials with energies lying in this interval are confined between the spin-up and spin-down trajectories with the energy $E$. Now let us color in the area between these trajectories. It is important that at $|E| < \Delta$ the dashed regions form an infinite network [Fig. 2(a)]. To prove this statement, notice that the infinite equipotential $V(\rho) = 0$ goes at $|E| < \Delta$ exclusively inside the dashed space. Therefore, an electron can travel throughout the entire system fol-
lowing the boundaries of the dashed regions. Treating $R(\Delta)$ as an elementary step we may view the electron motion as a random walk process. Consider now the case $|E| > \Delta$. Then there is no percolation at all. Indeed, the dashed area contains now equipotentials either with only positive or negative energies [Fig. 2(b)]. In either case, the infinite equipotential corresponding to zero energy lies outside the dashed space. Hence, passage of an electron through the sample is inavoidably associated with tunneling between the finite clusters the characteristic distance between which is of order $\lambda[(|E| - \Delta)/\Delta]^{1/2}$. In other words, if $|E|$ exceeds $\Delta$ by the small energy $\Delta$, the transport is exponentially suppressed. We thus conclude that the SO-enhancement of the classical transport occurs in the energy interval $|E| < \Delta$.

![FIG. 2. Schematic illustration of the percolation network in the presence of the SO-coupling at (a) $|E| < \Delta$ and (b) $|E| > \Delta$. The bold line denotes the infinite equipotential. Dashed is the space between trajectories with opposite spins (dotted and dashed lines) and the same energy $E$.](image)

The classical treatment above is valid if the phase-breaking length $L_\phi$, associated with inelastic scattering at finite $T$, is much shorter than $\xi(0)$, where $\xi(0)$ is the quantum localization length at $E = 0$. We thus predict that at $L_\phi \lesssim \xi(0)$, $T \ll \Delta$, and $L_{so} \lesssim L(\Delta)$ the dissipative conductivity $\sigma_{xx}$ as a function of the Fermi energy $E_F$ exhibits a box-like behavior [Fig. 3]: $\sigma_{xx} \sim e^2/h$ inside the band $|E_F| < \Delta$ and is exponentially small otherwise. The point is that for $|E_F| > \Delta$ the conductivity is only due to activation to the nearest percolation threshold, while a metallic band with well-pronounced boundaries exists between the percolation thresholds. The easiest way to see that $\sigma_{xx} \sim e^2/h$ at $|E_F| < \Delta$ is as follows. The diffusion coefficient $D$ provided by the SO-coupling of neighboring clusters is of order $R^2(\Delta)/v_\perp L(\Delta)$, where $v_\perp$ is the typical drift velocity. On the other hand, the density of electron states on the loop of the length $L(\Delta)$ is $g \sim 1/[R^2(\Delta)]L(\Delta)/hv_\perp$. Thus, if the SO-coupling is strong enough, i.e. $L_{so} \lesssim L(\Delta)$, we have $\sigma_{xx} = e^2gD \sim e^2/h$. In the absence of the SO-coupling, the wave functions decay on the scale of $\lambda$ from the equipotentials and so the conductivity inside the band $|E_F| < \Delta$ would be dominated by activation as long as $T \ll \Delta$. Let us stress that both conditions $L_\phi \lesssim \xi(0)$ and $T \ll \Delta$ are necessary for the box-like behavior of $\sigma_{xx}$. In this Letter we do not specify whether $L_\phi$ is limited by electron-electron or electron-phonon scattering and keep $L_\phi$ as a phenomenological length. We wish to note, however, that in any case the conditions can be fulfilled simultaneously at small enough Zeeman splitting as $\xi(0)$ grows rapidly with decreasing $\Delta$ (see below).

![FIG. 3. Shown schematically is the dissipative conductivity as a function of the Fermi level position in overlapping disorder-broadened Zeeman levels separated by the energy $2\Delta$ with (solid line) and without (dashed line) spin-orbit coupling. Both pictures correspond to the same temperature.](image)

Now consider what happens when $L_\phi$ exceeds $\xi(0)$ with lowering $T$. Obviously, then a drop of $\sigma_{xx}$ occurs between the centers of the Zeeman levels and two well-pronounced $\sigma_{xx}$-peaks appear. To estimate $\sigma_{xx}$ between the peaks, note that owing to the SO-coupling the wave functions of the localized states at $|E| < \Delta$ are built up from pieces of the classical trajectories, so that the overlap integral of two states does not involve tunneling through the saddle-points. As a result, the wave functions overlap strongly. Accordingly, $\sigma_{xx}$ in the middle between the peaks is due to hopping between the states separated by $R(\Delta)$, the hopping rate being $\sim (v_\perp/L(\Delta))(\xi(0)/L_\phi)$, i.e. the diffusion coefficient is $L_\phi/\xi(0)$ times smaller than in the classical case considered above. It follows that $\sigma_{xx} \sim (e^2/h)(\xi(0)/L_\phi)$ [19]. Thus two peaks of $\sigma_{xx}$ as a function of $E_F$ should be strongly asymmetric: $\sigma_{xx}$ falls off rapidly at $|E_F| > \Delta$ (activation) and slowly at $|E_F| < \Delta$ (hopping).

It is clear that the quantum localization length $\xi(0)$ resulting from the SO-coupling is of order $R(\Delta)$ at $L_{so} \sim L(\Delta)$. The question is whether the ratio $\xi(0)/R(\Delta)$ remains finite as $L_{so}/L(\Delta) \rightarrow 0$. The problem can be mapped on that considered numerically in Ref. [2] if the percolation network is replaced by a square lattice with a lattice constant $R(\Delta)$. To reconcile the model [2] with our picture, the amplitudes to go to the left or to the right at the nodes of the lattice should be chosen equal to either 0 or 1 depending on the spin-orientation. According to the numerical simulation [2], the localization length in this case is about $3 \times 10^2$ lattice spacings in the limit of complete spin-mixing (i.e. it is much larger numerically than would be anticipated on the basis of
the scaling arguments, which may be important from the experimental point of view). Apart from the large numerical coefficient, we are now in a position to find the energy dependence of the localization length $\xi(E)$ at $L_{so} \lesssim L(\Delta)$. As $E$ approaches either of the percolation thresholds, the characteristic radius of the critical cluster corresponding to one of the spin projections diverges as $R(\Delta - |E|) \sim R(\Delta)|\Delta/(\Delta - |E|)|^{1/3}$ while that corresponding to the other remains equal to $R(\Delta)$. If we consider only these two scales and neglect all others, the two-scale conducting network can be modeled similarly to Ref. [12]. Coupling of large cells with each other is then provided by cells of smaller size. Clearly, the quantum effect shows up in a well-defined range of energies lying between the centers of the Zeeman levels. The picture corresponding to one of the spin projections diverges $\sim L(\Delta)$. As $E$ approaches either of the percolation thresholds, $|E| \lesssim L(\Delta)$, the quantum localization length, which describes gradual (as compared to the case with no SO-coupling) fall-off of the wave function, scales as the classical percolation radius $L(\Delta)$.

In conclusion, we demonstrated that, for electrons in a smooth random potential in the QHE regime, the SO-coupling of two disorder-broadened Zeeman levels can result in a strong suppression of the localization. The effect shows up in a well-defined range of energies lying between the centers of the Zeeman levels. The picture suggested does not include effects of electron-electron interaction, such as screening of the random potential or exchange interactions. This issue warrants further study.

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* Also at A. F. Ioffe Physico-Technical Institute, 194021 St. Petersburg, Russia.

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[15] To estimate the width of the band, we note that, at a given $E$, the saddle-points that separate critical percolation-clusters have heights $\sim |E|$. According to H. A. Fertig and B. I. Halperin, Phys. Rev. B 36, 7969 (1987), the tunneling transmittivity of a saddle-point with a height $E$ and curvatures along the principal directions $V''_x$ and $V''_y$ is $[1 + \exp(2\pi E/V''/\lambda^2)]^{-1}$, where $V'' = (V''_xV''_y)^{1/2}$. In the random potential, the characteristic value of $V''$ is $\Gamma(d^2)$. So the classical picture breaks within the band $|E| \lesssim \Gamma/\lambda$ for the tunneling exponent of the critical saddle-points is of order unity.

[16] The SO-coupling leads to the appearance of the effective $g$-factor which is not zero even if the “bare” $g$-factor is neglected. We mean by $2\Delta$ the “net” energy distance between the centers of the disorder-broadened Zeeman levels, which is determined by the SO-interaction as well.

[17] For a review, see M. B. Isichenko, Rev. Mod. Phys. 64, 961 (1992), and references therein.

[18] It is worth noting that other saddle-points with heights $\sim |\Delta|$ which they meet on their way to the critical one do not change the estimate. Though their number, $\sim L(\Delta)/d^2 \sim R(\Delta)/d$, is large, typically they just connect up loops of size $d$. The probability density of the loop perimeter falls off as a power-law function whose exponent, $11/7$, can be found from the condition that there is only one loop of the maximum length $L(\Delta)$. Since the average distance between the saddle-points along the trajectories, $\sim d^2/\delta$, is much larger than $d$, only a small fraction of them form multiply connected figures.

[19] This law is violated with decreasing $T$ only when $T$ becomes so low that the typical energy spacing between neighboring states exceeds $T$, then the mechanism of transport changes to the variable-range-hopping.

[20] The law $\xi(E) \sim R(\Delta - |E|)$ is valid between the percolation thresholds as long as $\Delta - |E| \gtrsim \Delta$. Within the energy bands of the width $\Delta$, around the thresholds, tunneling between the critical clusters of size $R(\Delta)$ comes into play, so that the presence of larger clusters can be ignored and the characteristic size of the conducting-network cell, $R(\Delta_c)$, is independent of $E$. 