Dynamic boundary estimation of human heart within a complete cardiac cycle using electrical impedance tomography

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Abstract. This paper presents an EKF based boundary estimation algorithm to estimate the shape and size of human heart ventricle during a complete cardiac cycle. First-order kinematic model is used as a state evolution model. The boundary of the heart is expressed as coefficients of truncated Fourier series and the conductivity distribution inside the thorax region is assumed to be known \textit{a priori}. The proposed method is tested with the use of a realistic chest shape FEM mesh.

1. Introduction

Efficient and robust representations of cardiac morphology and morphodynamics are instrumental to conduct any large scale statistical analysis of regional heart shape and wall motion characteristics [1]. Electrical impedance tomography (EIT) is a non-invasive imaging modality which offers high temporal resolution making it a formidable technique to track cardiac related changes inside human body [2],[3],[4]. In EIT, a cross sectional image of the conductivity distribution inside human body can be obtained through the measurement of boundary potentials across the electrodes attached to the skin of the body. The poor spatial resolution of EIT makes it difficult to estimate the organ boundaries, thus undermining its anatomical significance. Key haemodynamic parameters related to heart physiology such as the cardiac output and ejection fraction can be better estimated if the boundary of the heart (ventricle) is estimated instead of its conductivity [4]. Dynamic algorithms such as linearized Kalman filter (LKF) or extended Kalman filter (EKF) can be used for this purpose [4],[5],[6],[7]. However, the estimation performance of these dynamic algorithms is still non-optimal due to the modeling uncertainty of the state evolution model, such as the \textit{random-walk} model, used by them [3],[4],[5]. Since the size of target in cardiac motion varies smoothly, the estimation performance can be further improved with the use of a more suitable state evolution model such as the kinematic model [6],[7]. This paper presents an EKF based boundary estimation algorithm to estimate the cross sectional shape and size of the left ventricle of human heart during a complete cardiac cycle. First-order kinematic model is used as a state evolution model. The organ boundaries are expressed as coefficients of truncated Fourier series and the conductivity distribution inside the thorax region is assumed to be known \textit{a priori} [8]. The proposed method is tested with the use of a realistic chest shape FEM mesh.
2. The boundary representation
The outer boundary $\partial \Omega$ of the body is assumed to be known. The organ boundary $C_i(s)$ is assumed to be sufficiently smooth to be expressed as coefficients of truncated Fourier series of order $N_\alpha$ [4],[8]

$$C_i(s) = \left( x_i(s), y_i(s) \right) = \sum_{n=1}^{N_\alpha} \left( \gamma_n^x \theta_n^x(s), \gamma_n^y \theta_n^y(s) \right)$$

where $\theta_n(s)$ is the periodic and smooth basis function of the form

$$\theta_n(s) = \begin{cases} 
\sin(2\pi n s) & n = 1, 2, ..., \\
\cos(2\pi n s) & n = 1, 2, ...
\end{cases}$$

Here, $s \in [0,1]$, $\alpha$ denotes $x$ or $y$ and $S$ is the number of boundaries. Expanding (1) and (2), the boundaries $C_i$ can be represented as $\gamma$ of the shape coefficients, i.e.,

$$\gamma = (\gamma_1^x, ..., \gamma_n^x, \gamma_1^y, ..., \gamma_n^y, ..., \gamma_1^x, ..., \gamma_n^x, ..., \gamma_1^y, ..., \gamma_n^y)$$

where $\gamma \in \mathbb{R}^{2Sn}$. It is assumed that the boundaries of the lungs, the backbone as well as the outer heart are known a priori, while an approximate knowledge of the right ventricle boundary is assumed. The inverse problem, therefore, is a simplified case of only estimating the left ventricle elliptic boundary, i.e., $C_i$ with $S=1$, and $N_{\alpha}=3$ has to be estimated

$$\gamma = (\gamma_1^x, ..., \gamma_S^x, \gamma_1^y, ..., \gamma_S^y)$$

3. Electrical impedance tomography: A Kalman filter approach
EIT is governed by a partial differential equation, with appropriate boundary condition such as the complete electrode model, establishing a relationship between voltage measurements and the conductivity distribution [3],[6]. The dynamic state estimation problem in EIT is based on a state space-space representation of the system, comprising of state evolution and observation models [5]

$$\gamma_{k+1} = f_k \gamma_k + w_k$$

$$U_k = g_k(\gamma_k) + v_k$$

where the subscript $k$ is the discrete time index, $f_k$ is the state transition model and $g_k$ is the non-linear observation model. $w_k$ and $v_k$ denote the process and measurement noise respectively and are assumed to be white Gaussian noise. First-order kinematic model has been used as a state evolution model to estimate the radii $(\gamma_x^e, \gamma_y^e)$ of the elliptic ventricle boundary, while the random-walk model is used for the rest of the parameters. The evolution model for the radius $r^\alpha$, $\alpha = x, y$, can be described as

$$\gamma_{k+1}^\alpha = \gamma_k^\alpha + \gamma_k^\alpha \Delta T + w_k$$

$$\gamma_{k+1}^\alpha = \gamma_k^\alpha + w_k$$

where $\gamma_k^\alpha$ is the velocity with which the boundary is expanding/contacting and $\Delta T$ denotes the time difference between two consecutive measurements $k$ and $k+1$.

3.1. Extended Kalman Filter
The EIT observation equation (6) is nonlinear in nature. Suboptimal estimates can be obtained by using linear approximations. The most common approach for this purpose is the extended Kalman filtering in which linearization is obtained with respect to the previous state estimate [5-6]. The complete set of equations for the extended Kalman filter looks like

$$\gamma_{k+1} = f_k \gamma_k$$

$$C_{k+1} = f_k C_k f_k^T + Q$$

$$K_{k+1} = C_{k+1} J_{k+1}^T (U_{k+1} C_{k+1} J_{k+1}^T + R)^{-1}$$
\[ C_{k+\delta t+1} = (I_k - K_{k+1} J_{k+1}^{\varepsilon}) C_{k+\delta t} \]  
\[ \gamma_{k+\delta t+1} = \gamma_{k+\delta t} + K_{k+1} \{ \overline{U}_{k+1} - J_{k+1}^{\varepsilon} \gamma_{k+\delta t} \} \]  
\[ \overline{U}_k = U_k (\gamma_{d-1}) + J_k (\gamma_t - \gamma_{d-1}) + v_k \]  
\[ (12) \]
\[ (13) \]
\[ (14) \]

where \( J_k = \frac{\partial y_k}{\partial \gamma} \in \mathbb{R}^{L \times N} \) is Jacobian obtained after linearizing the observation equation about the prior state estimate i.e., \( \gamma = \gamma_{d-1} \), \( \overline{U}_k \in \mathbb{R}^{L \times P} \) is the linearized approximation of \( U_k \) in (6), \( C_{k} \in \mathbb{R}^{N \times N} \) is the error covariance matrix, \( Q \in \mathbb{R}^{N \times N} \) and \( R \in \mathbb{R}^{L \times L} \) are process noise and measurement noise covariance respectively. \( K_k \in \mathbb{R}^{N \times L} \) is Kalman gain and \( I_N \in \mathbb{R}^{N \times N} \) is an identity matrix. \( N \) is the number FEM mesh nodes, \( L \) is the number of electrodes and \( P \) is the number of current patterns. For details refer to [6].

4. Results and discussions

A realistic cross sectional chest shape phantom is considered to carry out the numerical simulations to verify the performance of the proposed method. Inhomogeneous FEM mesh structures, concentrated in the region of the heart, with 5768 and 5626 triangular elements have been used to solve the forward and inverse problems respectively. The dynamic boundary of the left ventricle of the heart has been recovered during complete cardiac cycle, while the boundaries of the lungs and the backbone are fixed and assumed to be known. The simulated scenario depicts the situation in which a patient is told to hold his breadth during EIT measurements. More accurate imaging techniques such as MRI can be used to recover the static boundaries. Clinically correct conductivities are assigned to the region, i.e., the conductivities of the heart muscle, blood, lungs, backbone and the background are 0.45, 0.75, 0.08, 0.1 and 0.2 S/m respectively [9]. A normal heart rate of 70 beats/min is assumed.

![Figure 1](image)

**Figure 1.** End diastolic (Left) and end systolic (Right) boundary. The broken lines represent the estimated boundary and the approximately known boundary for the left and right ventricles respectfully. The boundaries of the outside heart, lungs and the backbone are known *a priori*.

The first four modes of (cosine and sine) trigonometric current patterns are injected into 16 electrodes attached to the chest boundary, considering that they are more sensitive compared to the other modes. The complete electrode model with effective electrode contact impedance of 0.005 \( \Omega \) cm\(^2\) is used. In EIT, the voltage measurements are often noisy in nature. Therefore, zero-mean Gaussian noise with 0.5\% standard deviation relative to the corresponding measured voltages has been added to the simulated voltage data. Using this methodology, 25 measurement frames (16 frames for diastole and nine frames for systole) are obtained for each cardiac cycle. The simulations have been carried out for three consecutive cardiac cycles to verify the robustness of the method. The state matrix for the kinematic models consists of six variables corresponding to the Fourier coefficients and two variables corresponding to the evolution velocities of the radii of the elliptic boundary. The state noise covariance is chosen as \( Q = \text{diag}[10^{-4}, 4 \times 10^{-9}, 10^{-4}, 10^{-4}, 4 \times 10^{-5}, 4 \times 10^{-5}, 4 \times 10^{-4}] \) while the coefficients of the error covariance and observation noise covariance are initialized to \( 10^4 \) and \( 8 \times 10^2 \) respectively.
Figure 2. Evolution of the Fourier coefficients $\gamma_2$ and $\gamma_3$ which account for the radii of the elliptic ventricle in the x and y directions respectively (left), with their respective evolution velocities at each stage of the cardiac cycle (right). The solid lines are for the true values, whereas the dotted and broken lines represent the evolution of estimated parameters.

End diastolic and end systolic boundaries, as estimated in the third cycle, are shown in figure 1, while the evolution of the Fourier coefficients $\gamma_2$ and $\gamma_3$, which account for the radii of the elliptic ventricle in the x and y directions respectively, has been shown in the figure 2. The results demonstrate reasonable estimation performance of the proposed methodology. A key assumption used while applying the kinematic model is the approximate knowledge of the diastolic and systolic cycle durations. This information can be easily obtained by using any other heart beat detection technique.

5. Conclusion

An EKF based 2D EIT measurement technique has been used to recover the boundary of the left heart ventricle during complete cardiac cycle, expressed as truncated Fourier series coefficients. Assuming a smooth expansion and contraction of the heart, first-order kinematic model is used as a state evolution model. Zero-mean Gaussian-distributed measurement noise has been added to simulate voltage measurement errors. Results obtained from a three-cycle, normal heart-rate simulation demonstrate the robustness of the method. Future work includes the extension of the method to 3D EIT for in vivo measurement and the simultaneous reconstruction of organ boundaries and their conductivities.

Acknowledgments

This work is supported by the grant from Korea Science and Engineering Foundation (KOSEF) funded by the Korea government (MEST) (No. R01-2007-000-20155-0). Part of the researchers participating in this study have been supported by the grant from the 2nd phase BK21 project.

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