High Power Microwave Signal Detection Based On Second Order Multisynchrosqueezing Transform

Li Dengxin, Jia Hongyang, Ye Yuchen, Wang Chao and Yuan Naichang

State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System
National University of Defense Technology, Changsha Hunan, 410073, China
Email: lidengxin_123@163.com

Abstract—This paper puts forward a generalization of the multisynchrosqueezing transform (MSST) using the second order local estimate of instantaneous frequency, which could localize a signal with high precision both in time and frequency. Numerical signal simulation result verifies its advantage on processing high power microwave (HPM) signals in different environments, this time-frequency analysis method provides an idea for HPM signal detection and feature analysis.

1. INTRODUCTION
With the rapid development of studies on radar signal receiving and processing, conventional radar signals can be detected and identified quickly, so modern radar often carries more complicated signals with different waveforms and types to improve its ability to prevent detection. In recent decades, with the rapid development of HPM [1] technology, HPM radar [2][3] has gained much attention. Because the signal emitted by this type of radar is very different from the conventional radar signal, it is difficult to obtain the desired radar results using traditional processing methods, which inspired us to explore a more effective method for HPM signal detection and feature analysis.

Time frequency analysis (TFA) is an effective tool to analyze time-varying signal. The traditional TFA methods, such as short-time Fourier transform (STFT) and wavelet transform (WV), are often utilized to characterize signal in the time frequency (TF) plane. Restricted by Heisenberg uncertainty principle, TF representations generated via these methods are often blurry, which also limits the application occasion of these analysis methods. L. Cohen proposed Wigner-Ville method in[4] to improve the TF representation, which also introduced the interference of cross term that affected the performance of TFA. In order to further improve the performance of TFA, F. Auger proposed a rearrangement (RM) algorithm in[5]. The main idea of this method is to rearrange the energy of every point in the spectrogram generated by STFT, to let spectrum barycenter of every point replace its original point. RM algorithm has achieved better performance compared with the previous TFA methods, but lost the ability to reconstruct the signal. I. Daubechies proposed synchrosqueezed wavelet transform (SST) in[6], which was a post-processing method based on WV method. It not only sharpens the TF representation significantly, but also realized the reconstruction of the signal. Using the principle of wavelet-based SST, Thakur and WU proposed an extension of SST to the TF representation given by STFT (FSST) [7].
In recent years, many nonlinear TFA methods have been proposed to address non-stationary signals\cite{8}\cite{9}, these methods combined SST and the demodulated technique to show its effectiveness in generating highly energy-concentrated TF results. High order SST was developed to improve the accuracy of Instantaneous frequency(IF) estimation\cite{10}\cite{11}, but the increasing SST order cause heavy computational cost. Based on FSST, Gang Yu proposed a multisynchrosqueezing transform (MSST) which gradually concentrates the fuzzy time-frequency energy by using iterative redistribution method \cite{12}.

This paper aim to extend the MSST using the second order local estimate of IF which could obtain the more accurate estimates of each mode. The simulation result prove that this technique could get a more concentrated TF representation.

2. SECOND ORDER MULTISYNCHROSQUEEZING TRANSFORM

2.1 Multisynchrosqueezing Transform

Synchrosqueezing transform is a post-processing method based on traditional TFA, this paper will be based on the framework of short-time Fourier transform to begin our research.

For a given signal \( s(t) \in L^2(\mathbb{R}) \), the short-time Fourier of the signal is defined as

\[
V(t,\omega) = \int_{-\infty}^{\infty} g(u-t)s(u)e^{-i\omega(u-t)}du
\]

(1)

g(t) is the real window function, it compactly supports in \([-\Delta,\Delta]\), for mono-component signal model

\[
s(t) = A(t)e^{i\phi(t)}
\]

(2)

The signal (2) can be regarded as the pure harmonic signal in the short time \([t-\Delta, t+\Delta]\]. Expand \( A(u) \) and \( \phi(u) \) at the time t, and the related expressions can be rewritten as \( A(u) \approx A(t) \) and \( \phi(u) \approx \phi(t) + \phi'(t)(u-t) \) the signal (2) become

\[
s(u) = A(t)e^{i\phi(t)(t-u)}
\]

(3)

Substitute signal (3) into STFT expression, we can have

\[
V(t,\omega) = \int_{-\infty}^{\infty} g(u-t)A(t)e^{i\phi(t)(t-u)}e^{-i\omega(u-t)}du
= A(t)e^{i\phi(t)}\int_{-\infty}^{\infty} g(u-t)e^{i\phi'(t)(u-t)}e^{-i\omega(u-t)}du
= A(t)e^{i\phi(t)}\hat{g}(\omega-\phi'(t))
\]

(4)

\( \hat{g} \) denotes the Fourier transform of window \( g(t) \), where \( A \) and \( \phi'(t) \) are constants, therefore, the instantaneous frequency of the signal \( s(t) \) can be calculated by the following formulation

\[
\partial_t V(t,\omega) = \hat{\partial}_t \left( A(t)e^{i\phi(t)}\hat{g}(\omega-\phi'(t)) \right)
= A(t)e^{i\phi(t)}\hat{g}(\omega-\phi'(t))\hat{\phi}'(t)
= V(t,\omega)\hat{\phi}'(t)
\]

(5)

According to the formulation (5), for the common signal \( x(t) \), we know the \((t,\omega)\) point in the \( V(t,\omega) \) time-frequency domain

\[
\hat{\phi}(t,\omega) = \frac{\partial_t V(t,\omega)}{iV(t,\omega)}
\]

(6)

\( \hat{\phi}(t,\omega) \) is called the phase transformation operator, SST based STFT is to redistributes the transform coefficient in the TF domain according to the mapping relationship of \((t,\omega) \rightarrow \hat{\phi}(t,\omega(t,\omega)) \), The mathematical expression is

\[
V(t,\eta) = \int_{-\infty}^{\infty} V(t,\omega)\delta(\eta-\hat{\phi}(t,\omega))d\omega
\]

(7)
Through the SST operator, the blurry energy obtained by STFT can be concentrated in a compact region around the instantaneous frequency trajectory. For multi-component signal models

$$s(t) = \sum_{k=1}^{K} s_k(t) = \sum_{k=1}^{K} A_k(t)e^{j\phi_k(t)}$$  \hfill (8)

Because the short-time Fourier transform is linear, for multi-component signals, the short-time Fourier transform is the linear superposition of the short-time Fourier transform of each mono-component signal, and does not produce cross-term interference. And according to its reversibility, for a given multi-component signal, when \( A_k(t) \) and \( \phi_k(t) \) meet certain conditions, the signal \( s_k(t) \) can be reconstructed from the transformation

$$s_k(t) \approx \left\{2\pi g(0)\right\}^{-1} \int_{-\infty}^{\infty} V_k(t,\omega)d\omega$$  \hfill (9)

ds is the SST reconstruction bandwidth.

Based on SST, Gang Yu proposed a new method called Multisynchrosqueezing transform. by iteratively applying multiple SST operations, the accuracy of signal instantaneous frequency estimation is further improved. Its mathematical expression is

$$V_{\delta}^{[2]}(t,\eta) = \int_{-\infty}^{\infty} V_{\delta}^{[1]}(t,\omega)\delta(\eta - \hat{\omega}(t,\omega))d\omega$$
$$V_{\delta}^{[3]}(t,\eta) = \int_{-\infty}^{\infty} V_{\delta}^{[2]}(t,\omega)\delta(\eta - \hat{\omega}(t,\omega))d\omega$$
$$\ldots$$
$$V_{\delta}^{[N]}(t,\eta) = \int_{-\infty}^{\infty} V_{\delta}^{[N-1]}(t,\omega)\delta(\eta - \hat{\omega}(t,\omega))d\omega$$  \hfill (10)

At this expression, \( V_\delta(t,\eta) \) in equation (7) is expressed here as \( V_\delta(t,\omega) \), \( \eta \) is the number of iterations, and \( N \geq 2 \). Through executing multiple iterations, TF representation appears more concentrated.

### 2.2 Second Order Multisynchrosqueezing

Although SST can effectively enhance the time-frequency representations of the signal, its application is restricted to multi-component signals composed of pure harmonic. Many studies have shown that when addressing Non-stationary signal \([7][8]\), SST will generate a blurry TF representation. To overcome this limitation, an improved synchrosqueezing operator is introduced to a more accurate IF estimate in \([9][10]\).

For a given signal \( s \in L^2(\mathbb{R}) \), the complex reassignment operators \( \tilde{\omega}(t,\eta) \) and \( \tilde{\tau}(t,\eta) \) are respectively defined for any \( (t,\eta) \) s.t. \( V_\delta(t,\eta) \neq 0 \) as

$$\tilde{\omega}(t,\eta) = \frac{\partial V(t,\eta)}{2\pi V(t,\eta)}$$
$$\tilde{\tau}(t,\eta) = t - \frac{\partial V(t,\eta)}{2\pi V(t,\eta)}$$  \hfill (11)

Then we define the second-order complex modulation operator

$$\tilde{\eta}_\omega(t,\eta) = \frac{\partial \tilde{\omega}(t,\eta)}{\partial \tilde{\tau}(t,\eta)} \text{ whenever } \tilde{\tau}(t,\eta) \neq 0$$  \hfill (12)

In that case, the second-order local complex IF estimate is defined as:

$$\tilde{\omega}^{[2]}(t,\eta) = \begin{cases} \tilde{\omega}(t,\eta) + \tilde{\eta}_\omega(t,\eta)(t - \tilde{\tau}(t,\eta)) & \text{if } \partial \tilde{\tau} \neq 0 \\ \tilde{\omega}(t,\eta) & \text{otherwise} \end{cases}$$  \hfill (13)

The real part of \( \tilde{\omega}^{[2]}(t,\eta) \) is the desired IF estimate. Then the second-order SST expression can replace \( \tilde{\omega}(t,\omega) \) in (7) with \( \tilde{\omega}^{[2]}(t,\omega) \), that is

$$T_{\omega}^{[2]}(t,\eta) = \int_{-\infty}^{\infty} V(t,\omega)\delta(\eta - \tilde{\omega}^{[2]}(t,\omega))d\omega$$  \hfill (14)
In order to get a more accurate estimate of the IF, we combine the second-order SST and MSST. The expression is as follows

\[ T_{\nu}^{(\nu)}(t,\eta) = \int_{-\infty}^{\infty} T_{\nu}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \]

\[ T_{\nu}^{(\nu)}(t,\eta) = \int_{-\infty}^{\infty} T_{\nu}^{(\nu)}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \]

\[ \ldots \]

\[ T_{\nu}^{(\nu)}(t,\eta) = \int_{-\infty}^{\infty} T_{\nu}^{(\nu-d)}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \]

We substitute \( T_{\nu}(t,\eta) \) into \( T_{\nu}^{(\nu)}(t,\eta) \), then the second order MSST (N=2) can be expressed as

\[ T_{\nu}^{(\nu)}(t,\eta) = \int_{-\infty}^{\infty} T_{\nu}(t,\xi)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\xi)\right)d\xi \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V_{\nu}(t,\omega)\delta\left(\xi - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \times \delta\left(\eta - \hat{\omega}^{(\nu)}(t,\xi)\right)d\xi \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V_{\nu}(t,\omega)\delta\left(\xi - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \times \delta\left(\eta - \hat{\omega}^{(\nu)}(t,\xi)\right)d\xi d\omega \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} V_{\nu}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\xi)\right)d\xi d\omega \]

In [7], Gang Yu further proved that with the number of iterations increases, the instantaneous frequency estimate error depends on the second-order derivative of instantaneous phase and the distance between frequency and true IF, second-order SST has more accurate frequency estimate for strong time-varying than SST, so the TF representation obtained by second order MSST is much closer to the ideal TF representation.

On the other side, if we calculate the second-order MSST according to equation (15), we need to execute multiple SST operations. But according to [12]’s conclusion, the SST operator is a little bit time consuming. We keep on substitute \( T_{\nu}(t,\eta) \) into \( T_{\nu}^{(\nu)}(t,\eta) \) more iterations, we can calculate the IF estimate of second-order MSST (N=3) is \( \hat{\omega}^{(3)}(t,\omega) = \hat{\omega}^{(3)}(t,\hat{\omega}^{(3)}(t,\omega))) \). Therefore, we let \( \hat{\omega}^{(2)}(t,\omega) \) denote the IF estimate of second-order MSST, and rewrite equation (15) into

\[ T_{\nu}^{(\nu)}(t,\eta) = \int_{-\infty}^{\infty} V_{\nu}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \]

According to (17), we can construct the IF estimate \( \hat{\omega}^{(2)}(t,\omega) \) of second-order MSST by using the function iteration, and then execute the SST operation only once to calculate the result. This can decrease the time consuming significantly.

Similarly, we can prove that the signal addressing by the second-order MSST can still be perfectly reconstructed.

\[ \int_{-\infty}^{\infty} T_{\nu}^{(\nu)}(t,\eta)d\eta \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T_{\nu}^{(\nu)}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega d\eta \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T_{\nu}^{(\nu)}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega d\eta \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} T_{\nu}^{(\nu)}(t,\omega)\delta\left(\eta - \hat{\omega}^{(\nu)}(t,\omega)\right)d\omega \]

\[ \ldots \]

\[ \int_{-\infty}^{\infty} T_{\nu}(t,\xi)d\xi \]

\[ = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(u-t)\delta(u)e^{-i\omega(u-t)}d\mu d\xi \]

\[ = 2\pi \int_{-\infty}^{\infty} g(u-t)\delta(u)d\mu \]

\[ = 2\pi g(0)r(t) \]
So, the original signal can be recovered by

\[ s(t) = (2\pi g(0))^{-1} \int_{-\infty}^{\infty} T_s(t, \omega) d\omega \]  

(19)

3. ANALYSIS AND MODELING OF HPM SIGNALS

High power microwave refers to electromagnetic waves with peak power above 100MW and frequency between 1-300GHz. It has the characteristics of short pulse duration, high peak power and wide spectrum. In order to improve anti-jamming capability, modern radar combines HPM with short pulse to be a radar transmission signal. The stationary signal model is no longer applicable when we analyze the HPM short pulse signal.

Wang Lang collected the HPM signal through an oscilloscope to analyze and reconstruct it in [13], summarized the following characteristics of HPM signal: irregular envelope, large phase jitter, and non-stationary. According to these characteristics, we employ the following model to research the HPM signal.

\[ s(n) = g(n)e^{j\phi(n)} \]  

(20)

According to Wang Lang’s description in [1], the envelope of the HPM signal is very close to the Gaussian curve, so this paper uses a Gaussian model to represent the signal envelope.

\[ g(x) = ae^{-\frac{(x-b)^2}{2c^2}} \]  

(21)

In [1], the author expresses the phase as \( \phi(n) + A\cos(n\omega_0) \), \( A\cos(n\omega_0) \) is the phase jitter factor. Considering that modern radar will do more complex modulation on signal, this paper uses other frequency modulation methods.

4. NUMERICAL SIMULATIONS AND VALIDATION

For HPM signal with a very short duration, it is difficult to detect it from multiple radar signals only from the time domain or frequency domain. But TFA methods can solve this problem by mode decomposition. In this section, we compare the TF concentration between the second-order MSST and other advanced TFA methods on the HPM signal in different environment.

4.1 There is only HPM signal

Let’s consider an ideal electromagnetic environment first, there is no noise and other jamming signal. The HPM signal is a triangle frequency-modulated signal with Gaussian envelope. The time domain waveform is shown below.

Fig.1 time domain waveform of HPM

The TF representations generated by different methods are shown in Fig.2(a)-(f).
Fig. 2 (a) STFT result, (b) PWVD result, (c) RM result, (d) SST result, (e) MSST result, (f) 2nd MSST result

It can be seen that STFT result is extremely blurry in fig.2(a), the accurate frequency information of the signal is difficult to obtain. PWVD can roughly describe the TF characteristics of the signal, but it will be affected by the unexpected cross terms.

**TABLE I. RENYI ENTROPY BY SEVERAL TFA METHODS**

| TFA  | STFT | PWVD | RM  | SST  | MSST | 2nd MSST |
|------|------|------|-----|------|------|----------|
| Renyi Entropy | 4.9119 | 3.8465 | 1.1121 | 1.1021 | 0.8713 | 0.8132 |

Fig. 3 (a) STFT result, (b) PWVD result, (c) RM result, (d) SST result, (e) MSST result, (f) 2nd MSST result
RM, SST, MSST and second-order MSST all get an approximate ideal TF representation. In order to evaluate the energy concentration of different TFA methods, we introduced the concept of Renyi entropy to compare them quantitatively. A lower Renyi entropy value means a more concentrated TF representation, the results calculated in Table I shows that the second order MSST has the best TF concentration.

Then we add noise to the signal, where the SNR (signal to noise ratio) is equivalent 5 db. The TF representations are shown in Fig 3. It can be seen that in the case of low SNR, the second order MSST still get a most concentrated TF representation, which also prove this technique has a better noise robustness.

4.2 Multi-radar signals without overlap in frequency domain

The previous section proves that when there are only HPM signal, the TF representation generated by second order MSST method has a better performance. In order to further verify the effectiveness of the method, we set up four kinds of radar signals with different modulation methods, They are non-linearly modulated HPM signals, the binary phase-coded and linear frequency modulation signal, chirp signals and Sine frequency modulation continuous wave. Then we compare the TF energy concentration and TF resolution under the SNR is equivalent 10 db.

According to Fig. 4 (a), we can see that STFT can roughly obtain the TF information of each component. The PWVD method is already impossible to get any useful information because of cross terms and noise. The remaining four methods have a good TF representation. In order to compare their TF resolution more clearly, we zoom the HPM signal IF trajectory locally and display it on the right side. Obvious, compared with the other method, MSST has a better TF resolution From Fig. 4 (c)-(f).
We add the true IF trajectory of the HPM signal to compare the accuracy of IF estimation in Fig.5, and the red curve is the true IF trajectory of the signal. It can be seen that the second-order MSST result is around the true IF trajectory, the TF representation generated by MSST slightly deviates from the real IF trajectory.

We still list the Renyi entropies of these TF results in Table II, obviously, the second order MSST provided the most concentrated result among these TFA methods.

| TFA  | STFT | PWVD | RM  | SST | MSST | 2nd MSST |
|------|------|------|-----|-----|------|----------|
| Renyi Entropy | 8.4065 | 9.8453 | 4.1520 | 4.3275 | 2.6415 | 2.6130 |

Fig.5 (a) MSST result and it’s local zoom, (b) second order MSST result and it’s local zoom

4.3 Multi-radar signal with overlap in frequency domain

When the radar signal does not overlap in the frequency domain, second order MSST performs well. However, the HPM signal that have a wide spectrum is very common to overlap with other signals in complex electromagnetic environment, so we further analyze the performance of different TFA methods in this occasion.
The TF representation generated via STFT and Wigner-Ville method is blurry, so we will not discuss it too much here. In order to facilitate the comparison of the performance between different TF methods, the true IF trajectory of the HPM signal is added in the TF representation, the overlapped area are amplified and display it on the right side.

It can be seen from that the TF representation of the HPM signal obtained by different TFA methods deviates from the true IF trajectory Fig.6(a)-(d). The second-order MSST result appears to be more concentrated, however, the result also deviates seriously from the true IF trajectory. In addition, the IF trajectory of the HPM signal and the chirp signal are mixed together, so we cannot separate the IF ridge from the TF representation, even if the SNR is increased, the result is still not improved, this also means that we cannot reconstruct the HPM signal.

5. CONCLUSION
This paper puts forward a generalization of the MSST method using a second order estimate of instantaneous frequency, numerical simulation results prove that the second-order MSST has a more concentrated TF representation and a better TF resolution. Even under low SNR condition, it is still superior to other TFA methods. But when there is an overlap in the frequency domain, the IF trajectories obtained by these advanced TF methods deviate from the true IF trajectory of the signal, this is also the problem we need to study and solve next.

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