Hydrodynamic, Heat and Acoustic Processes Modelling in Transport of Rheologically Complex Viscous Media Technology in Pipelines

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Abstract. The paper describes the results of mathematical modelling of acoustic processes, hydrodynamics and heat exchange in case of oil products transportation in pipelines with constant and variable cross-section. The turbulence model features of RANS approach and intensification of heat exchange in substances with anomalous rheology are reviewed. It is shown that statistic second order models are appropriate to use for forecasting details of the pulsating flows. The paper states the numerical integration features of determining equations. The properties of vibratory effect influence are determined. Vortex and heat perturbations, rheological changes impact on resistance regularities and intensity of heat exchange are analyzed.

1. Introduction
Nowadays oil and gas, chemical, technological, heat-and-power engineering and space industries are facing problems of modern, efficient and safe equipment inculcation. State-of-the-art equipment is expected to function without failures in the intensive thermal and hydrodynamic effects regime, accompanied by the transitional phase, vortex and acoustic wave processes, heat and mass exchange and chemical reactions and etc. Due to that the problem of adequate description of momentum, heat and mass transfer in difficult shear flows of viscous media (oil products, drop and gaseous homogeneous and heterogeneous mixes) in channels and the pipelines with constant and variable cross-section is the most difficult one. The details of noted flows are demanded for making metal-consuming equipment that is conservative to high stresses. The optimization of its work during crude oil with abnormal rheological properties transportation is quite vital as well.

The references analysis of the abovementioned issues shows (for example, \cite{1-4}), that, despite Russian and foreign researchers’ wide experience of studying various aspects of flows with heat and mass exchange in internal systems, there is no agreement concerning a complex assessment of attendant factors defining complex substances flows such as influence of non-stationarity, acoustics, phase transitions, flow contamination, turbulization, relaminarity of an entering flow, heat exchange with environment. Usually researches of regularities in changes of dynamic flow structure, resistance and heat exchange are carried out simplifying real process assumptions. In addition, it should be mentioned that almost all real non-isothermal flows of viscous substances are turbulent in the pipelines functioning under high stresses. Their full direct numerical...
modelling forecast is linked with well-known problems of numerical character, restrictions of modern computers [4]. In these conditions RANS method looks more preferable, especially considering positive experience of statistical multiple parameter models of the second rank in tension and flows computing. Despite the great potential opportunities, adaptation of the mentioned models to calculation of technological processes in the oil and gas industry faces serious difficulties.

In this regard the objective of this paper is the creation of complex mathematical model of the physical processes accompanying flows and heat and mass transfer in pipelines; explanation of acoustic, vortex and thermal processes influence on resistance and heat return, generation of large-scale formations in carrying agent; establishment of energy intense zones’ sizes and rate of their intensity.

1.1. Acoustic-hydrodynamic interactions in the pipeline
The solution of heat and mass transfer intensification becomes seriously complicated in case of the attempt to account effects of interaction of acoustic, hydrodynamic and thermal processes in the carrying agent and a pipeline wall. Particularly, problems of acoustic processes in pipeline system are fully viewed in papers [5,6] where analytical model expressions of pulsations in carrying fluid have been received for the system with established fluctuations [5], also the mathematical model of vibroacoustic processes in pipeline systems of difficult spatial configuration, considering simultaneous impact on system of power irritation of carrying liquid and kinematic energizing of the attached objects.

According to the idea of [5], it is supposed in the analysis of pipeline wall reaction on dynamics of the internal processes to investigate effect of axis deflections of the pipeline with cross-section vibrations on the differential equation [1]:

\[ EI \frac{\partial^4 L_y}{\partial x^4} = F_P - F_N \frac{\partial^2 L_y}{\partial x^2} - 2 \frac{\partial F_N}{\partial x} \frac{\partial L_y}{\partial x} - L_y \frac{\partial^2 F_N}{\partial x^2}, \]

where \( E \) - the Young’s modulus, \( I \) – the inertia moment of section relative to neutral axis, \( L_y \) – a deflection of the pipeline, \( F_N \) - longitudinal compressing force, \( F_P \) – load of unit of length.

Within assumptions of one-dimensional character of a viscous fluid flow, this equation is simplified to the kind, that is ready for parametrical analysis:

\[ \frac{\partial^4 L_y}{\partial x^4} + a_{10}(x,t) \frac{\partial^2 L_y}{\partial x^2} + a_{20}(x,t) \frac{\partial^2 L_y}{\partial x^2} + a_{30}(x,t) \frac{\partial^2 L_y}{\partial x^2} + \]

\[ + a_{40}(x,t) \frac{\partial^2 L_y}{\partial x^2} + a_{50}(x,t) L_y + a_{30}(x,t) = 0, \]

where \( a_{i0} \) – known parametrical functions [5].

With the help of (2) the Bubnov-Galyorkin method [7], we will receive the system consisting of \( n \) the differential equations:

\[ \sum_{i=1}^{n} A_{ik} f''_k + B_{ik} f'_k + C_{ik} f_k - D_{ik} = 0; n = 1, 2, ..., n, \]

where factors \( A_{ik}, B_{ik}, C_{ik}, D_{ik} \) have a certain expression, and \( f_k \) are time functions which are received with approach of \( L_{yy}(x,t) = \sum_{i=1}^{n} f_i(t) \varphi_i(x) \). It is necessary to notice that the analytical calculation of such system is rather difficult.

In this paper the generalization (2) of the spatial character of velocity field change is described. It should be noticed that the change of the velocity field is received from the solution of a multidimensional hydrodynamic task, intensified by the difficult shift flow of the hydrocarbonic
fluids in the pipeline. In turbulent mode the flow structure was predicted on the basis of two-parametrical models by G. S. Glushko and D. Wilkox. It should be noted that the defining equations system of acoustic and hydrodynamic processes interaction is based on the method of final differences with the approximation of derivatives on spatial co-ordinates with the second order of accuracy. In the definition of the pressure field L.M. Simuni’s generalised algorithm is used [1], because that algorithm is well-proved in forecasts of internal flows. The results of calculation show that in case of the developing flows convective and diffusive mechanisms of velocity field change cause an intensification of acoustic processes in the pipeline. Some ideas of advantages of model and complex flow calculation method can be received in extract 2 of the paper.

2. Hydrodynamics and heat exchange in constant non-Newtonian oil products flow conditions

As it was mentioned above, most of hydrocarbon media flows are related to the complex rheology flows. Due to modern computer technology development an interest in its mathematical modelling is growing [2,4,8].

This paper discusses both laminar and turbulent isothermal flow in pipelines, to illustrate the mathematical model of the flow and heat and mass transfer equations are given below describing the axisymmetric case flow.

The mathematical model of nonisothermal viscous-inertial vortex flow of hydrocarbon fluids is described by equations representing conservation laws: mass (continuity equation), momentum (Navier-Stokes equation), energy (for the carrier media) and thermal conductivity for the walls.

Mathematical formulation meets the conditions of the flow with the axial symmetry. It is assumed that the nature of the movement throughout the range of variation of perturbation corresponds to the stable laminar and turbulent transport processes, complicated conjugate heat transfer. Below, for simplicity, the vector and index form in generally accepted notation is used in constitutive equations [1]:

\[
\begin{align*}
\text{div} \tau &= 0; \\
\frac{D \tau}{Dt} &= \text{Div} \mathbf{\tau}; \\
\frac{DT}{Dt} &= \text{div} (a \text{grad} T) + S_T; \\
\frac{\partial T_w}{\partial t} &= \text{div} (a_w \text{grad} T_w);
\end{align*}
\]

In this case the velocity vector \( \mathbf{v} \) and its components are as following: \( u_1 \) - axial velocity, \( u_2 \) - radial velocity; \( \frac{D}{Dt} = \frac{\partial}{\partial t} + (\mathbf{v} \cdot \nabla) \) \( \equiv \frac{\partial}{\partial t} + u_j \frac{\partial}{\partial x_j} \) - total derivative; \( T, T_w \) - temperature of transported media and pipeline wall as accordingly; \( a, a_w \) - media and wall thermal diffusivity coefficients; \( S_T \) - volumetric heat source; \( \mathbf{\tau} \) - stresses tensor, which is described by three-constant equation of nonlinear-viscoplastic liquids in the terms of abnormal rheology model:

\[
\begin{align*}
\tau_{ik} &= -p \delta_{ik} + 2 \left[ \frac{\tau_0(T)}{h} + \kappa(T) \frac{1}{h^{m-1}} \right] \cdot S_{ik}, \\
S_{ik} &= \frac{1}{2} \left( \frac{\partial u_i}{\partial x_k} + \frac{\partial u_k}{\partial x_i} \right).
\end{align*}
\]

In the terms of formulation and independent variables \{t, x, r\} described in the paper, components of stress tensor deviator \( \left( \mathbf{\tau} \right) \) quasi-Newtonian viscosity (\( \mu^* \)) in summarized rheological equation [3] are formulated as:

\[
\begin{align*}
\tau_{x\bar{x}}^{(d)} &= 2 \mu^* \cdot \frac{\partial u_x}{\partial x}, \\
\tau_{r\bar{r}}^{(d)} &= 2 \mu^* \cdot \frac{\partial u_r}{\partial r}, \\
\tau_{\bar{x}\bar{r}}^{(d)} &= \mu^* \left( \frac{\partial u_r}{\partial x} + \frac{\partial u_x}{\partial r} \right), \\
\mu^* &= \frac{\tau_0(T)}{h} + \kappa(T) h^{1-m}.
\end{align*}
\]
The rheology of Newtonian liquid is determined by a known law:

$$\tau_{ik} = -p\delta_{ik} + 2\mu \dot{S}_{ik} + (\zeta + \mu S) \frac{\partial u_i}{\partial x_j} \delta_{ik}, \quad (10)$$

where $\mu, \zeta, \mu S$ - dynamic, volume and second viscosity coefficients accordingly, $p$ - hydrodynamic pressure.

It is also expected that $S_T$ defines the value of inner heat sources taking dissipation processes into account:

$$S_T = \mu^* \cdot h, \quad h = 2 \left[ \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial r} \right)^2 + \left( \frac{\partial u}{\partial r} \right)^2 + \left( \frac{\partial v}{\partial \gamma} \right)^2 \right]. \quad (11)$$

In the terms of forecasting of the turbulent flow and heat exchange with the known values $(u, v, p, T)$ the system (4) - (11) uses its average analogue (time Reynolds averaging). To define additional unknown values in this case - Reynolds turbulent stresses $(\dot{u}'u'_j)$, specific turbulent heat fluxes $(\dot{u}'t')$ modern second order statistical models that use RANS method are engaged. The models use original and rarely used in practice base of applied caculations of turbulence kinetic energy $(k)$ and pulsation time $(\tau)$ equations, originally developed by C. Speziale and A.F. Kurbatskiy [9,10] and common to the class of internal flows.

General view of defining equations of RSS-TF-$k\tau$ turbulent model in index notation is presented below [1]:

$$\frac{D}{Dt} \left[ (\nu + c_\mu f_\mu \dot{u}_i \dot{u}_j \tau) \frac{\partial u_i}{\partial \alpha} \right] = \frac{\partial}{\partial \alpha} \left[ (\nu + \frac{\nu}{\nu + 2} + \frac{c_{\mu \theta} f_\mu \theta \dot{u}_i \dot{u}_j \tau}) \frac{\partial \dot{u}_i \dot{u}_j}{\partial \alpha} \right] - \frac{2}{\tau} \frac{\partial \dot{u}_i \dot{u}_j}{\partial \alpha} \frac{\partial u_i \dot{u}_j}{\partial \alpha}, \quad (12)$$

$$\frac{D}{Dt} \left[ \frac{\partial \dot{u}_i}{\partial \alpha} \right] = \frac{\partial}{\partial \alpha} \left[ (\nu + \frac{\nu}{\nu + 2} + c_\mu f_\mu \dot{u}_i \dot{u}_j \tau) \frac{\partial \dot{u}_i}{\partial \alpha} \right] - \frac{c_3 (\nu + \frac{\nu}{\nu + 2}) \dot{u}'t'}{\dot{u}'t'} - \frac{\partial \dot{u}_i}{\partial \alpha} \frac{\partial \dot{u}_i}{\partial \alpha} \frac{\partial \dot{u}_i}{\partial \alpha}, \quad (13)$$

$$\frac{Dk}{Dt} = \frac{\partial}{\partial \alpha} \left[ \left( \nu + c_\mu f_\mu \dot{u}_i \dot{u}_j \tau \right) \frac{\partial k}{\partial x_i} \right] - \frac{c_{\mu 2} f_\mu \dot{u}_i \dot{u}_j \tau \partial U_i}{\partial \alpha} - \frac{k}{\tau}, \quad (14)$$

$$\frac{D\tau}{Dt} = \frac{\partial}{\partial \alpha} \left[ \left( \nu + c_\mu f_\mu \dot{u}_i \dot{u}_j \tau \right) \frac{\partial \tau}{\partial x_i} \right] - \frac{2}{\tau} \left( \nu + c_\mu f_\mu \dot{u}_i \dot{u}_j \tau \right) \frac{\partial \tau}{\partial x_i} + \left( c_{\mu 2} f_\mu - 1 \right) \frac{\partial \dot{u}_i \dot{u}_j \partial U_i}{\partial \alpha} + \frac{2}{k} \left( \nu + c_\mu f_\mu \dot{u}_i \dot{u}_j \tau \right) \frac{\partial k \partial \tau}{\partial x_i} \frac{\partial \tau}{\partial x_i} \frac{\partial \tau}{\partial x_i}, \quad (15)$$

Boundary conditions for the integration of the complete system of constitutive equations describing the flow and heat transfer in a carrier media and the wall are: inlet - homogeneous distribution of desired characteristics (or their known experimental field); output - "firm" conditions; on the inner wall surface of the pipe - for the dynamic slip condition parameters conjugation - a field temperature; on the axis of symmetry - symmetry conditions required parameters; on the outer wall of the pipe (for the heat equation) - the boundary conditions of the 1st or 2nd kind, determining the environmental impact.
3. Results and discussion

The research of the flows and the heat exchange in pipelines was conducted in wide range of defining parameters changes, particularly: \( L/D \sim 10 \div 1000, \ D/d \sim 1 \div 2, \ T_c/T_w \sim 1 \div 2, \ Pr_c/Pr_w \sim 0.25 \div 6, \ Re = 10 \div 10^4 \); heat transfer agents were oil and water; the wall was manufactured from Steel 14ХТС.

The conclusion about steadiness, accuracy and reliability of the algorithm and features of closure correlations for RSS-TF-\( k\tau \) model were acquired using data of numerous comparisons of the process local and integral parameters calculations with available experimental data in wide range of input characteristics on laminar and turbulent flows class [12].

On figures [1][2] below individual results of dynamic heat disturbance from the entrance and the wall on the structure of transported Newtonian media with the variable heat and physical properties are depicted \([a = a(T)] \) according to Reynolds-Philonov law.

Thus, figure [1] represents features of evolution of recirculating zones of vortex flow and its intensity in the pipeline including sudden enlargement places/sections enlargement. Tendencies typical for the developed flow are observed in the near-wall zones \( (X^+ \leq 0.1) \). The experience shows that in such conditions the structure of the turbulent eddy becomes significantly heterogeneous, tendencies to anisotropic mechanisms in processes of heat and momentum transfer are growing. In these situations it is unlikely that the forecasting of resistance and heat exchange patterns would be satisfactory with inclusion of simple turbulence models that use scalar formulation of molar viscosity and thermal diffusivity coefficients. That is why it is quite valid that the RSS-TF models of turbulence and evolution data of "firm" flow parameters in energy-strained zones may be useful.

It is necessary to point out that regarding heat processes, as it seen from figure 2, the impact from heat and physical properties’ variability in the working media and pipeline wall on heat exchange intensity \( (Nu/Nu_c) \) is to be expected in \( X < 0.1 \) zones [both during heating and cooling of the flow by the wall: \( Pr_c/Pr_w = 0.25 \) (line 1); 0.5(2); 2(3); 4(4)]. \( Nu_c \) is a characteristic value of Nusselt number that corresponds to the flow conditions in “working media - wall” under variable heat and physical properties. Note that the comparative analysis of transfer process intensity in high-viscous systems have shown a rapid attenuation of dynamic disturbances. In case of oil or water transportation the impact of properties variability on friction and heat exchange is expressed in pipeline proximal zone where the disturbances are form the dynamic and heat
pattern \( (X \leq 0.01) \). The calculations show that one variability of heat and physical properties in the media may intensify up to 10-15% the exchange processes in heat and momentum transfer. However, models with constant properties are quite successful to use in zones with \( X > 0.1 \). 

The evaluation of perspectives of multiparametric turbulence models usage has stated that the model is very sensitive to disturbance character coming from the walls and entrance and requires a precise definition of full spectrum of pulsar values in working zone entrance. In addition, these models may be a rational compromise from the point of expenditures calculation and its precision in comparison with less complex (but quite reduced in the forecasting of flow pattern and emergency regimes of pipeline functioning) algebraic models. This fact approves the advantages of RANS approach and the second order statistical models that include transport equations for stresses and flows of velocity and the temperature pulsar fields in relation to the forecasting of the complex processes of crude oil transport.

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