Type Ia Supernovae

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Abstract. At its heart, a Type Ia supernova is a problem in turbulent nuclear combustion. There are four sub-problems, each of which has eluded solution for decades, but which can be addressed by large scale simulation. First is the ignition - where and how often the burning is ignited in the convective core of an exploding white dwarf star. The outcome is sensitive to the initial conditions near the star’s center, which may be chaotic. Second is the propagation of the flame. Until near the end, the flame is an unresolvably narrow sheet moved around by instabilities and the turbulence that its own motion produces, yet how fast it moves determines the strength and brightness of the explosion. Third is whether and how the subsonic burning makes a spontaneous transition to a detonation. Observations favor this outcome, but the physics of the transition is obscure. Fourth, is the radiation transport problem. Why does the supernova look the way it does and can its light curve be relied upon to do precision cosmology? Our Consortium has made genuine progress in each of these areas, as well as in planning, with observers, future observational strategies for SNAP/JDEM and LSST. A new generation of codes is being optimized for the four tasks.

1. Introduction
Occasionally, once every other century in our Galaxy, but perhaps once a second in the observable universe, a white dwarf star explodes with a brightness comparable to ten billion suns. This brilliant display, powered by the decay of radioactive elements made in the explosion, fades after a few weeks, but briefly rivals the luminosity of the galaxy in which it resides. The light curves of these Type Ia supernovae (SN Ia) are mostly regular from event to event, making them useful for cosmology. Even more useful is the fact that the small residual variations in their peak brightness are tightly correlated with how long they remain bright. There is a “width-luminosity relation” – broader equals brighter.
Astronomers are not fully agreed upon how the white dwarf initially reaches criticality, except that it must somehow grow to some critical mass by accreting from a binary companion. Based upon spectra and light curves, a class of models called "Chandrasekhar Mass Explosions" is greatly favored [1]. In this model, the regularity of the light curves is a consequence of a common starting point for all explosions - a 1.38 solar mass white dwarf star made out of carbon and oxygen with a central density of $3 \times 10^9$ g cm$^{-3}$. Despite the similarity in initial conditions, the outcome of the explosion is still variable because the ignition conditions are chaotic. The outcome is additionally difficult to calculate and potentially variable because the burning is subject to Rayleigh-Taylor and Kelvin-Hemholtz instabilities and to the turbulence that these instabilities create. As we shall see, modeling these explosions is a petascale project, if not more, because of the resolution of disparate length scales it requires. To first order, the brightness of the explosion is determined by the amount of radioactivity it produces, but an important goal of such studies must be the characterization and systematics of "second parameters" (and third and fourth, etc.) that affect the shape of the light curve and may contaminate cosmological measurements.

2. Ignition
The actual explosion of the white dwarf, which only takes a second or two, is preceded by several centuries of convection. During this time, carbon burning, which proceeds at a rate proportional to the temperature to the 23rd power, accelerates while the pressure, due to degenerate electrons, hardly rises. Too much heat is generated for conduction to carry it all away, so the fluid convects with increasing speed and efficiency. Eventually, convective speeds reach ~100 km s$^{-1}$ (about 1% sonic) and the convective region grows to include most of the star. In addition, because it has accumulated much of its mass from a centrifugally supported accretion disk, the white dwarf is probably rotating rapidly.

Within this context, the first “sparks” are ignited that rapidly become bounded aggregations of ash and fuel separated by a thin flame approximately $10^{-3}$ cm thick. At a central temperature of $7 \times 10^8$ K and density $3 \times 10^9$ g cm$^{-3}$, convection can still - barely - carry the away the nuclear energy produced by carbon burning, though the time scales for convective turn over and temperature rise have become comparable, both ~ 100 s [2]. The heat capacity of the degenerate gas is small and the reaction rate remains extremely temperature sensitive, so the runaway accelerates. By $1.1 \times 10^9$ K, material is burning in place and the temperature rises to its final value when all the fuel has burned of $9 \times 10^9$ K in a small fraction of a second. This large increase in temperature only causes a small decrease in density, ~20%, but because of the high gravitational acceleration, $g_{\text{eff}} \sim 10^9$ cm s$^{-2}$, the hot ash floats in the cold fuel very rapidly.

Once the flame is born, it will not die until the star has been completely disrupted, but just how much burning occurs and how bright the supernova later appears is sensitive to where the initial flame ignites and how fast it moves afterwards. Flames that ignite in a single point off-center give weak explosions, unless a detonation occurs; ignition at many points scattered isotropically throughout the central region give stronger explosions.

2.1. Recent models
The actual geometry of the ignition remains uncertain. Numerical studies by Kuhlen et al. [3] showed that the convective flow pattern for a non-rotating white dwarf approaching a runaway is a dipole, as anticipated by Chandrasekhar 40 years ago [4]. Fuel passes rapidly through the center of the star, flows out in a “jet” where the temperature is highest, then travels around the extremity of the star to return again from the other side. Kuhlen’s calculations, which showed off-center, one-sided ignition, could be criticized because the anelastic code used spectral coordinates, and thus had an empty “hole” in the middle to avoid a coordinate singularity. The calculations were also at too low a Reynolds number.
Figure 1. The distribution of entropy perturbations in a 3D anelastic simulation of the convection leading up to a Type Ia supernova explosion. Left is a non-rotating model; right, the model has slow rotation, about 2% of Keplerian at the surface. Red color indicates regions of high entropy and maximal excess temperature ($\delta T/T$). The total temperature, $T + \delta T$, is greatest at the base of the red region, and that is where ignition occurs.

Figure 2. Flow patterns in 2D slices of the 3D calculations shown in Fig. 1. The figures are color-coded by entropy-perturbation, yellow and red being highest. In the rotating model (right) the flow is more chaotic and the dipole flow not so apparent. The high temperature region overlaps the center.

We have now repeated Kuhlen’s study using a 3D Cartesian grid (but still anelastic hydrodynamics) that has no coordinate singularity [5,6]. Our new results (Figs. 1 and 2) agree with his earlier findings. The maximum temperature is still found in an outflow on one side, displaced from the center by ~100 km. Interestingly, calculations of the same star with the same code in 2D showed a very different ignition pattern. The dipole convective flow was still clearly present, but the hot spots developed in a torus around the jet where the flow stagnated. The 3D results, though low in resolution (Re ~ 1000), are regarded as superior because of the well known tendency of 2D turbulence to cascade to artificially large scales.
The calculation was also repeated with a mild amount of rotation, $\omega = 0.84 \, \text{rad} \, \text{s}^{-1}$, or about 2% of break up. Even this relatively small amount of rotation affected the flow pattern. The dipole was still present, but fractured and twisted by the Coriolis force. Ignition still occurred a little off center on one side, but the ignition region overlapped the center. Lower viscosity might change things.

To illustrate the dependence on Reynolds number, we carried out 2D anelastic calculations of convection between two cylinders at $\text{Re} \sim 10^6$, essentially 2D Rayleigh-Benard convection with a gravitational potential (Fig. 3). The non-rotating case again showed a dipole character, though with more structure. The dipole is completely missing from the rotating case, but is replaced by a structured cylindrically symmetric pattern, highly differential. In fact, the inner regions counter-rotate with respect to the outer ones. We intend to carry these calculations to higher Reynolds number (but more realistic Rossby number) to determine more precisely the conditions for “breaking” the dipole. It may be that the brightness of a SN Ia is related to the rotation rate of the presupernova star.

Figure 3. 2D anelastic simulations at high Reynolds number ($10^6$) of the flow between two cylinders in a central gravitational field that goes as $r^{-1}$. The left figure shows the dipolar pattern typical of non-rotating flow. The right hand side shows a similar calculation that included a small amount of rotation. The Ekman number (ratio of viscous forces to Coriolis force) was very low – $10^{-8}$, but much larger than in the actual star. The Rossby number (ratio of fluid speed to rotational speed) was about 0.01, whereas in the white dwarf it is $\sim 1$. Calculations at higher Rossby number and with a smaller inner boundary are in progress. These calculations took about 100,000 hours each at NERSC.

2.2. Problems and outlook
The results of the previous section highlight a computational conundrum. Two-dimensional studies at high Reynolds number, though nowhere near the actual astronomical value, show a different structure than 3D at low resolution. Clearly one wants high-resolution 3D studies. But the Cartesian version of the anelastic code is not very efficient on large parallel machines and the spectral version is – currently – singular at the origin. We are thus engaged in constructing a new code, MAESTRO [7-9] that is based on a low Mach number formulation of the equations for full-star, reacting flow with a time-evolving stratified background state. MAESTRO runs efficiently on massively parallel platforms, and is designed specifically for modelling the convection and ignition of SNe Ia with realistic equation of state and nuclear physics. For Mach numbers of a few percent or less, MAESTRO is orders of magnitude faster than compressible hydrodynamics codes like FLASH [10]. A paper by Almgren et al. in these proceedings provides further details and scaling studies for MAESTRO.
3. Propagation
Once the flame is born, the computation enters a new phase where fluid instabilities and turbulence are the dominant effects and challenges and the Mach number is \(\sim30\%\) instead of \(\sim1\%\). The hot ash is buoyant with respect to the cold fuel in which it is immersed. As it rises, shear gives rise to turbulence which cascades to smaller length scales where it affects the motion of the flame. Our previous studies using a low Mach number code, SNe [11-13], showed that the large scale anisotropic motions (floating plumes) drive isotropic turbulence with a Kolmogorov spectrum after cascading downwards approximately one decade in length scale. For plumes of characteristic size hundreds of kilometers this sets the integral length scale for turbulence to be about 10 km. Any physical description of the flame must either very finely resolve this length scale, or use a subgrid model for the turbulence – or both. Since the initial radius of the star is 1800 km, and growing rapidly as it explodes, sub-km resolution in 3D is not yet feasible (though it is a lot easier than trying to resolve the flame itself which is only \(10^{-4} - 10^{-1}\) cm thick). We thus employ a subgrid model.

Our subgrid scale turbulence model has been discussed extensively in the astrophysical literature [14 -15] and is based on an approach that has been successfully applied in the combustion community [16]. A balance equation keeps track of the budget of subgrid-scale turbulent energy where the production term is modelled according to a similarity assumption between the smallest resolved and the largest unresolved scales. Due to the localized treatment of the turbulence energy production rate the model is applicable to situations without homogeneity and stationarity on the large scales which is the case in combustion in SN Ia. It will be important to test this subgrid model in studies where the integral length scale is highly resolved. This will require 3D studies with \(~10^{10}\) zones, even with adaptive mesh (i.e., < 1 km resolution for a 5000 km star), or a well-calibrated subgrid model for turbulence.

3.1. Recent results for off-center ignition
Last year we reported results for single point off-center ignition [17] calculated in 2D and 3D. Those calculations have now been published in the refereed literature [18]. The initial conditions were similar to those of Plewa et al [19], and our study, at that time, was the first extension of the Chicago model for “gravitationally confined detonation” (GCD) to 3D. The conclusion then was that models that looked promising in 2D showed a much lower likelihood of late time detonation in 3D. Since that time, Lamb et al (this volume) have also carried out 3D studies, and claim a more robust detonation than Röpke et al. We believe our previous results and conclusions are correct. At issue is the degree to which the white dwarf expands before the first plume breaks out. More expansion gives a weaker collision on the far side that is not favorable for detonation. Essentially, the disagreement is an argument about the role of turbulence in spreading the burning during the first (“rising plume”) stage of the model. The Chicago study does not include a subgrid model for turbulence and – we suspect – fails to sufficiently resolve at least one decade of Kolmogorov turbulence over the entire flame surface. In any case, numerical disagreements ultimately have numerical answers, which the next year or two of computations should provide.

3.2. A 2D and 3D survey
If a transition to detonation does occur, by whatever means, it should be possible to parameterize it in terms of e.g., ignition conditions and a density and location where the transition occurs. While continuing to pursue “first principles’ models”, we are also carrying out a survey in order to provide models for spectroscopic and photometric diagnostics. Such a multi-dimensional survey can be parameterized. The radiation transport can also be calculated in 2D and 3D (2D at present) with various approximations concerning the level populations of the ions (Sec. 5).

4. Transition to detonation
The observed properties of SN Ia show that a transition from a deflagration to a detonation is desirable in the late stages of the explosion. Deflagrations alone tend to give supernovae that agree with only the
faintest events [20]. On the other hand, premature detonation tends to make only the brightest. One wants a detonation that is “just right” and perhaps variable in when and where it occurs. Three possibilities are pulsational detonation [21], gravitationally confined detonation [GCD, 19], and spontaneous detonation in the distributed regime [22]. Our current efforts are focused on the third alternative – that as the flame passes through a density that is low enough, turbulence may tear the flame and mix hot ash and cold fuel in what is called the “distributed regime” of combustion. If a sufficiently large region becomes mixed, and if the burning in that region becomes, temporarily, supersonic, a detonation could spontaneously ignite. This is sometimes called DDT for “deflagration-detonation transition”.

![Image](image_url)

**Figure 4.** Distribution of turbulent energy in a 3D simulation of a deflagration. Blue shows regions where the turbulent velocity on an integral length scale of 10 km exceeds 100 km s\(^{-1}\). The greatest turbulent speeds are developed at the bases of rising plumes. The red dots are locations where the turbulent speed approaches 1000 km s\(^{-1}\) and the green arrow points to the greatest value. On the right, the probability density function of the turbulence shows an exponential fall off above 100 km s\(^{-1}\) with just a few points reaching 1000 km s\(^{-1}\).

4.1. Analytic

Woosley [23] has recently shown that the conditions for spontaneous transition to detonation may exist for a fuel density in a narrow range around \(10^7\) g cm\(^{-3}\) provided that the energy in turbulence becomes very large, \(~1000\) km s\(^{-1}\) on a length scale of 10 km. In related SciDAC supported work, Röpke [24] has shown that 3D models (but not 2D) do supply the necessary amount of turbulence (Fig. 4). Essentially, in the distributed regime, the flame is turbulently broadened into a nested set of layers. As the density declines and turbulent energy increases, the number of layers becomes fewer, the thickness of each greater, and the speed of an individual flame sheet, faster. For turbulent energies approaching one-quarter sonic and turbulent flame sheets as thick as the integral length scale (10 km), the expected large fluctuations in flame speed may carry it over the sonic threshold. Challenging numerical simulations will need to be carried out to check the assumptions and scaling relations predicted by the analytic mode, but for the first time, there are predictions to check.

4.2. Multi-dimensional simulations

Modeling burning in the distributed regime is a challenge, both for us and the chemical combustion community. We are taking two approaches. The direct approach involves maintaining Kolmogorov turbulence on a 3D adaptive grid and watching the interplay with the flame at various densities. Our first results [25] are shown in Fig. 5. These computations, with the SNe code, used several hundred thousand CPU hours on ATLAS and demonstrated the tearing of the flame by the turbulence. The grid was only \(~10\) cm though and the transition to detonation would have to happen on a much larger scale.
An alternative approach is to use a technique developed by Kerstein [26], called ODT, that captures many of the features of 3D turbulence, especially the folding and mixing, on a 1D grid. This can be run, in parallel, for a large dynamic range of length scales. We believe that the two approaches will be synergistic with direct simulation serving to calibrate ODT for the astrophysical environment.

Figure 5. – (left and center) Results from a first 3D study [25] on ATLAS at LLNL of turbulent flame mixing in the distributed regime of a Type Ia supernova. At a density of 2.35 x 10^7 g cm^{-3}, turbulence with intensity 0.6 km s^{-1} on a length scale of 1 cm, tears the fusion flame, mixing fuel and ash. The left frame shows the distribution of energy generation and the center frame shows the broadening of the flame (time increases to the lower right). The red line shows the locus of a laminar flame at the same density. (right) Mixing on larger scales can be calculated using a technique called “One Dimensional Turbulence [26]. On a scale of 200 m at the same density, turbulence tears the flame into corrugated sheets [27]. Detonation may occur in the mixture for still larger length scales and turbulent energies.

5. Light curves and spectra
Part of the motivation for finding better models for SN Ia is the more reliable application of observed light curves and spectra to cosmology. Our work in this area has three themes: a better understanding of the Phillips Relation itself, libraries of SN Ia model spectra and light curves using state-of-the-art radiation transport, and efficient interfaces between those models and the supernova observers.

5.1. The Phillips relation
A breakthrough in our understanding of the width-luminosity relation, broader equals brighter, came this year with the realization, based on numerical models, that it is largely a color effect, most pronounced in the blue-band [28]. Multi-color light curves and spectra were calculated for a set of 130 one-dimensional models [29] with variable yields of radioactivity, stable iron, and intermediate mass elements. The results (Fig. 6) showed emphatically that SN Ia are not necessarily a single parameter family. Explosions with the same amount of 56Ni made supernovae of comparable brightness, but the width of the light curve varied greatly depending on the explosion energy and masses of other elements ejected. For cosmology, this means that simply using the width of the light curve or some measure of decline rate is not enough to uniquely specify peak brightness. Fortunately, there are other observational diagnostics – the actual shape of the light curve in different colors, spectral features, etc. – but models will be very helpful in sorting this out and seeing if there might be evolutionary effects lurking in the additional parameters.
Figure 6. Width-luminosity relations for a large set of 1D models. Peak brightness is plotted vs. decline rate. Only points in the grey band agree with the properties of the most commonly observed supernovae. (left) Supernovae that make the same amount of $^{56}$Ni (points with the same color) tend to have the same brightness at peak, but the decline rate is affected by other parameters. Only those models with a limited range of explosion energies, mixing, and nucleosynthesis agree with observations (right). An important goal of multi-dimensional models will be to understand why nature makes these choices, and the extent to which residual sensitivity to additional parameters will affect the use of SN Ia for cosmology.

Figure 7. Light curves and peak light spectrum for a 2D model in which delayed detonation occurred at a prescribed density. The explosion produced 0.7 solar masses of radioactive $^{56}$Ni (red in the leftmost figure) and a kinetic energy of $1.2 \times 10^{51}$ erg. The comparison of the multi-color light curves and the spectrum of a typical Type Ia supernova, SN 1994D, is excellent.

5.2. New models
The 1D survey [29] provided interesting constraints on the models, favoring, for example, the class of “delayed detonations”. Our current project is a similar 2D study (Fig. 7), with a 3D survey soon to follow. The Monte Carlo radiation transport code, SEDONA, is 1D, 2D, or 3D, so the effects of deformation in the models can be studied. More results from the 2D study are given by Kasen et al. elsewhere in this volume. Ultimately, we would like for the radiation to be handled in 3D and to drop the assumption of local thermodynamic equilibrium in calculating the level densities. This has not be done before and will be computationally challenging.
5.3. Automated comparison of models and observations
So far, the search for spectral diagnostics of supernova brightness and what constitutes a “good fit” between a model and an observed data set has been largely left to experienced observers. However, with the coming advent of hundreds of models and hundreds of thousands of supernovae, an automated process for comparison needs to be developed. A greater understanding of the models, and especially the “extra parameters” beyond the abundance of $^{56}$Ni synthesized, will also aid in planning observational strategy.

Figure 8. The left panel shows the results of an automated fit (red line) to the near peak-light spectrum of SN 0506-006 discovered by the Supernova Factory in 2007 using SYN-APPS. Applying this to a spectral time history such as that on the right can be used to determine the composition of the supernova as a function of velocity.

We are developing a code, SYN-APPS, that is a combination of SYNOW [30] and APPSPACK [31]. SYN-APPS automatically scours a parameter space of over 100 variables (temperature, density, velocity, and abundances for over 30 ions) to determine the composition of an expanding supernova. It can be run in parallel and has been run effectively on over 2000 CPU. SYN-APPS will be run in two modes. In one mode, it will use all available spectroscopic data for a few well studied, nearby supernova (Fig. 8) to do “supernova tomography”, that is the reconstruction in velocity space of the ionization and composition structure of the ejecta. In its other mode, SYN-APPS will be used to search for the best representation in the space of existing models for a given set of observations. This will not only help determine the kinds of models most frequently realized in nature, but also search automatically for observable correlations with peak luminosity.

6. Conclusions
Understanding SN Ia requires computations on the largest, fastest machines available. The simulation of ignition conditions (Sec. 2) requires fine resolution to reach well into the chaotic regime of turbulence where the flow is predicted [4] to have a different character. Current calculations on JAGUAR at ORNL are, so far, barely turbulent. Flame propagation in full star models currently rely on subgrid models for turbulence (Sec. 3). To resolve at least one decade of Kolmogorov turbulence all over the flame will probably require two orders of magnitude more zones and finer time steps, yet current studies saturate the available resources. Determining the physics of the putative transition to detonation will require much larger grids than in Sec. 4, Fig. 5, yet those calculations are already using hundreds of thousands of hours on ATLAS at LLNL. But faster computers alone will not “solve the supernova problem”. It is important to use the right tools for the job. A compressible shock code is a
poor tool for a problem in tight hydrostatic equilibrium whose fastest speed is 1% sonic. Efficient, flexible visualization will be essential to digging the real physics out of the huge data sets, and just managing those data sets will be difficult. And the codes must operate efficiently on many CPU. This is why SciDAC 2’s philosophy of embedding computer scientists and applied mathematicians in the science application itself is so important.

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References
[1] Hillebrandt, W., and Niemeyer, J. 2000, Ann. Rev. Astron. Astrophys., 38, 191 - 230.
[2] Woosley, S. E., Wunsch, S., and Kuhlen, M. 2004, Astrophys. J., 607, 921 – 930.
[3] Kuhlen, M., Woosley, S. E., and Glatzmaier, G. A. 2006, Astrophys. J., 640, 407 – 416.
[4] Chandrasekar, S. 1961, Hydrodynamic and Hydromagnetic Stability, Dover Publ., p. 220ff.
[5] Evonuk, M., and Glatzmaier, G. A. 2006, Icarus, 181, 458 – 464.
[6] Ma, H., Woosley, S. E., Glatzmeier, G. A., Evonuk, M. and Kuhlen, M. Q. 2007, in prep. for Astrophys. J.
[7] Almgren, A.S., Bell, J.B., Rendleman, C.A., and Zingale, M. 2006, Astrophys. J., 649, 927-938.
[8] Almgren, A.S., Bell, J.B., Rendleman, C.A., and Zingale, M. 2006, Astrophys. J., 608, 883 - 906.
[9] Zingale, M., Woosley, S. E., Bell, J. B., Day, M. S., and Rendleman, C. A. 2004, Astrophys. J., 632, 1021 - 1034.
[10] Schmidt, W., Niemeyer, J. C., and Hillebrandt, W., Röpke, F. K. 2006, Astron. Ap., 450, 283 – 294.
[11] Schmidt, W., Niemeyer, J. C., Hillebrandt, W. 2006, Astron. Ap., 50, 265 – 281.
[12] Peters, N. 2000, Turbulent Combustion, Cambridge Univ. Press.
[13] Röpke, F. K., and Woosley, S. E. 2006, Journ. Phys. Conf. Ser., 41, 413 - 417.
[14] Röpke, F. K., Woosley, S. E., and Hillebrandt, W. 2007, Astrophys. J., 660, 1344 – 1356.
[15] Plewa, T., Calder, A. C., and Lamb, D. Q. 2004, Astrophys. J. Lettr., 612, L37 - 41 .
[16] Blinnikov, S. I. et al. 2006, Astron and Astrophys., 453, 229 – 240.
[17] Arnett, W. D., and Livne, E. 1994, Astrophys. J., 427, 330 – 341.
[18] Niemeyer, J. C., and Woosley, S. E. 1997, Astrophys. J., 475, 740 – 753.
[19] Woosley, S. E. 2007, Astrophys. J, in press.
[20] Röpke, F. K. 2007, Astrophys. J, in press.
[21] Aspden, A., Bell, J., Day, M., and Lijewski, M. 2007, unpublished.
[22] Kerstein, A. R. 1999, J. Fluid Mech., 392, 277 – 334.
[23] Woosley, S. E., Kerstein, A., Sankaran, V., and Röpke, F. 2007, unpublished.
[24] Kasen, D., and Woosley, S. E. 2007, Astrophys. J., 656, 661 – 665.
[25] Woosley, S. E., Kasen, D., Blinnikov, S., and Sorokina, E. 2007, Astrophys. J., 662, 487 – 503.
[26] See http://www.nhn.ou.edu/~parrent/synow.html
[27] See http://software.sandia.gov/appspack/version5.0/index.html