$B \to X_S l^+ l^-$ in the minimal gauged $(B - L)$ supersymmetry

Tai-Fu Feng*, Jin-Lei Yang, Hai-Bin Zhang†, Shu-Min Zhao, Rong-Fei Zhu

Department of Physics, Hebei University, Baoding, 071002, China

Abstract

Applying the effective Hamilton for $b \to s l^+ l^-$, ($l = e, \mu$) in the framework of minimal supersymmetric extension of the standard model with local $B - L$ gauge symmetry, we investigate branching ratios and forward-backward asymmetries of rare decay $B \to X_S l^+ l^-$ in low and high $q^2$ regions, respectively. In addition we also study the $CP$ asymmetries depending on new $CP$ phases from soft breaking terms in low and high $q^2$ regions. With some assumptions on parameter space of the model, the numerical analyses of the supersymmetric contributions to the branching ratios, forward-backward and $CP$ asymmetries of $B \to X_S l^+ l^-$ are presented in low and high $q^2$ regions, respectively.

PACS numbers: 12.60.Jv, 14.60.St, 14.80.Cp

Keywords: supersymmetry, gauge symmetry, rare $B$ decay

* email:fengtf@hbu.edu.cn
† email:hbzhang@hbu.edu.cn
I. INTRODUCTION

The study on rare $B$ decays can detect new physics beyond the standard model (SM) since the theoretical evaluations of corresponding observations are not seriously affected by the uncertainties originating from unperturbative QCD effects. So far the Charmless semileptonic $B$ decays are studied extensively in the SM, the authors of Ref. [1–4] analyze the QCD corrections to the branching ratios of rare $B$-decay, and the authors of Ref. [5–7] present corrections from $c\bar{c}$ resonances to the branching ratio of $B \to X_s l^+ l^-$. In order to obtain the QCD evolution effects precisely in rare $B$-decay, the relevant two-loop QCD anomalous dimension matrix (ADM) for all the flavour-changing four-quark dimension-six operators is given in Ref. [8].

Considering those corrections mentioned above, one obtains the theoretical evaluations in the SM as

\[
BR(B \to X_s l^+ l^-)^{SM}_{q^2 \in [1,6]\text{GeV}^2} = (1.59 \pm 0.11) \times 10^{-6},
\]

\[
BR(B \to X_s l^+ l^-)^{SM}_{q^2 \in [14.4,25]\text{GeV}^2} = (2.3 \pm 0.7) \times 10^{-7}, (l = e, \mu). \tag{1}
\]

Only the contributions to the branching ratios in the low $q^2$ region with $1 \text{GeV}^2 \leq q^2 \leq 6 \text{GeV}^2$ and the high $q^2$ region with $14.4 \text{GeV}^2 \leq q^2 \leq 25 \text{GeV}^2$ are evaluated respectively. Here $q^2 = (p_+ + p_-)^2$ denotes the dilepton invariant mass squared. Averaging available experimental data from BaBar [12] and Belle [13], we obtain the experimental averages for the branching ratios in two regions as follows [14]

\[
BR(B \to X_s l^+ l^-)^{exp}_{q^2 \in [1,6]\text{GeV}^2} = (1.63 \pm 0.50) \times 10^{-6},
\]

\[
BR(B \to X_s l^+ l^-)^{exp}_{q^2 \in [14.4,25]\text{GeV}^2} = (4.3 \pm 1.2) \times 10^{-7}, (l = e, \mu). \tag{2}
\]

Obviously the SM theoretical evaluations on those branching ratios coincide with the experimental data in three standard deviations, and the coming precise measurements on the rare $B$-decay processes will set more strong constraints on the new physics beyond SM. The main purpose of investigation of $B$-decays is to search for traces of new physics and determine its parameter space. Besides the branching ratios, the forward-backward asymmetries in the process $B \to X_s l^+ l^- (l = e, \mu)$ are the physics quantities to detect new physics beyond the
SM. The updated experimental data from Belle Collaboration\cite{15} are
\[
A_{FB}(B \to X_s l^+ l^-)|^{exp}_{q^2 \in [1, 6] \text{ GeV}^2} = 0.30 \pm 0.24 \pm 0.03 ,
\]
\[
A_{FB}(B \to X_s l^+ l^-)|^{exp}_{q^2 \in [14.4, 25] \text{ GeV}^2} = 0.28 \pm 0.15 \pm 0.01 ,
\]
where the first uncertainty is statistical and the second uncertainty is systematic. The corresponding SM predictions are given in Ref.\cite{16} as:
\[
A_{FB}(B \to X_s l^+ l^-)|^{SM}_{q^2 \in [1, 6] \text{ GeV}^2} = -0.07 \pm 0.04 ,
\]
\[
A_{FB}(B \to X_s l^+ l^-)|^{SM}_{q^2 \in [14.4, 25] \text{ GeV}^2} = 0.40 \pm 0.04 .
\]
For the present experimental uncertainty is large, we cannot yet apply the experimental data of the physics quantity to test the SM precisely. Nevertheless, the future experimental data will constrain the parameter space of new physics strongly with accumulating of data sample.

Meanwhile the SM evaluation of the CP asymmetry in the entire $q^2$ region is very small\cite{17}, the theoretical prediction on the CP asymmetry may be enhanced significantly\cite{18}. The updated experimental data on the CP asymmetry from Babar Collaboration are\cite{19}
\[
A_{CP}(B \to X_s l^+ l^-)|^{exp}_{q^2 \in [1, 6] \text{ GeV}^2} = -0.06 \pm 0.22 \pm 0.01 ,
\]
\[
A_{CP}(B \to X_s l^+ l^-)|^{exp}_{q^2 \in [14.4, 25] \text{ GeV}^2} = 0.19^{+0.18}_{-0.17} \pm 0.01 .
\]
In supersymmetric extensions of the SM, new sources of flavor and CP violations may appear in those soft breaking terms\cite{20}. Actually the analyses of constraints on extensions of the SM are extensively discussed in literature. The calculation of the branching ratio of inclusive decay $B \to X_s \gamma$ is presented by authors of\cite{21–23} in the two-Higgs doublet model (2HDM). The supersymmetric effect on $B \to X_s \gamma$ is discussed in\cite{24–28} and the next-to-leading order (NLO) QCD corrections are given in\cite{29}. The transition $b \to s \gamma \gamma$ in the supersymmetric extensions of the SM is computed in\cite{30}. The hadronic $B$ decays\cite{31} and CP-violation in those processes\cite{32} have been discussed also. The authors of\cite{33} have discussed possibility to observe supersymmetric effects in rare decays $B \to X_s \gamma$ and $B \to X_s e^+ e^-$ at the $B$-factory. Studies on decays $B \to (K, K^*) \mu^+ \mu^-$ in the SM and supersymmetric models have been carried out in\cite{34}, a relevant review can be found in
Ref. [35] also. The theoretical analyses on oscillations of $B_0 - \bar{B}_0$ ($K_0 - \bar{K}_0$) have been done in the SM and 2HDM. In supersymmetric extensions of the SM, the calculation involving the gluino contributions should be re-studied carefully for gluino has a nonzero mass. At the NLO approximation, the QCD corrections to the $B_0 - \bar{B}_0$ mixing in the supersymmetry extensions have been discussed also. Adopting the mass-insertion approximation (MIA) method the authors of [36–38] estimate QCD corrections to the $B_0 - \bar{B}_0$ mixing, and later we have re-derived the formulation by including the contribution of gluinos [39].

The discovery of Higgs on the Large Hadron Collider (LHC) implies that the searching of particle spectrum predicted by the SM is finished now [40, 41]. One main target of particle physics is testing the SM precisely and searching for the new physics (NP) beyond it. The updated bound from ATLAS collaboration on the gluino mass is $m_{\tilde{g}} \geq 1460$ GeV, and the bound on the mass of scalar top is $m_{\tilde{t}} \geq 780$ GeV [42]. Additionally the LHCb experiment can measure the quantities of exclusive hadronic, semi-leptonic, and leptonic $B$ and $B_s$ decays at a high sensitivity [43]. The measurements on inclusive rare $B$ decay and decays with neutrino final states will be performed also in two next generation $B$ factories in near future [44, 45].

The discrete symmetry R-parity in supersymmetry is defined through $R = (-1)^{3(B-L)+2S}$, where $B$, $L$ and $S$ are baryon number, lepton number and spin respectively for a concerned field [46]. In the minimal supersymmetric extension of SM (MSSM) with local $U(1)_{B-L}$ symmetry, R-parity is spontaneously broken when left- and right-handed sneutrinos acquire nonzero vacuum expectation values (VEVs) [47–50]. Meanwhile, the nonzero VEVs of left- and right-handed sneutrinos induce the mixing between neutralinos (charginos) and neutrinos (charged leptons). Furthermore, the MSSM with local $U(1)_{B-L}$ symmetry naturally predicates two sterile neutrinos [51–53], which are favored by the Big-bang nucleosynthesis (BBN) in cosmology [54]. In other words, there are exotic sources to mediate flavor changing neutral current processes (FCNC) in this model.

Here we investigate some interesting physical quantities in the FCNC processes $B \to X_s l^+ l^-$, $(l = e, \mu)$ in the MSSM with local $U(1)_{B-L}$ symmetry, our presentation is organized as follows. In section [II] we briefly summarize the main ingredients of the MSSM with local $U(1)_{B-L}$ symmetry, then present effective Hamilton for $b \to s l^+ l^-$ in section [III]. The
formulæ of decay widths, forward-backward asymmetries, CP asymmetries at hadronic scale are given in \[\text{IV} \] respectively. The numerical analyses are given in section \[\text{V} \] and our conclusions are summarized in section \[\text{VI} \] finally.

II. THE MSSM WITH LOCAL $U(1)_{B-L}$ SYMMETRY

When $U(1)_{B-L}$ is a local gauge symmetry, one can enlarge the local gauge group of the SM to $SU(3)_C \otimes SU(2)_L \otimes U(1) \otimes U(1)_{(B-L)}$. In the model proposed in Ref.[47–50], the exotic superfields are three generation right-handed neutrinos $\hat{N}^c_i \sim (1, 1, 0, 1)$. Meanwhile, quantum numbers of the matter chiral superfields for quarks and leptons are given by

\begin{align*}
\hat{Q}_I &= \begin{pmatrix} \hat{U}_I \\ \hat{D}_I \end{pmatrix} \sim (3, 2, \frac{1}{3}, \frac{1}{3}) , \quad \hat{L}_I = \begin{pmatrix} \hat{\nu}_I \\ \hat{E}_I \end{pmatrix} \sim (1, 2, -1, -1) , \\
\hat{U}^c_i &\sim (3, 1, -\frac{4}{3}, -\frac{1}{3}) , \quad \hat{D}^c_i \sim (3, 1, \frac{2}{3}, -\frac{1}{3}) , \quad \hat{E}^c_i \sim (1, 1, 2, 1) ,
\end{align*}

with $I = 1, 2, 3$ denoting the index of generation. In addition, the quantum numbers of two Higgs doublets are assigned as

\begin{align*}
\hat{H}_u &= \begin{pmatrix} \hat{H}_u^+ \\ \hat{H}_u^0 \end{pmatrix} \sim (1, 2, 1, 0) , \quad \hat{H}_d &= \begin{pmatrix} \hat{H}_d^0 \\ \hat{H}_d^- \end{pmatrix} \sim (1, 2, -1, 0) .
\end{align*}

The superpotential of the MSSM with local $U(1)_{B-L}$ symmetry is written as

\[ W = W_{\text{MSSM}} + W_{(B-L)}^{(1)} . \]

Here $W_{\text{MSSM}}$ is superpotential of the MSSM, and

\[ W_{(B-L)}^{(1)} = \left(Y_N \right)_{ij} \hat{H}_u^T i\sigma_2 \hat{L}_i \hat{N}^c_j . \]

Correspondingly, the soft breaking terms for the MSSM with local $U(1)_{B-L}$ symmetry are generally given as

\[ L_{\text{soft}} = L_{\text{soft}}^{\text{MSSM}} + L_{\text{soft}}^{(1)} . \]

Here $L_{\text{soft}}^{\text{MSSM}}$ is soft breaking terms of the MSSM, and

\[ L_{\text{soft}}^{(1)} = -(m_{\tilde{N}^c}^2)_{ij} \hat{N}^{c*}_i \hat{N}^c_j - (m_{BL} \lambda_{BL} \lambda_{BL} + m_{1BL} \lambda_1 \lambda_{BL} + h.c.) \\
+ \left( \left( A_N \right)_{ij} \hat{H}_u^T i\sigma_2 \hat{L}_i \hat{N}^c_j + h.c. \right) , \]

5
with $\lambda_{BL}$ denoting the gaugino of $U(1)_{B-L}$, $m_{1BL}$ denoting the mixing mass parameter between the $U(1)_Y$ gaugino and $U(1)_{B-L}$ gaugino, respectively. After the $SU(2)_L$ doublets $H_u, H_d, \tilde{L}_i$ and $SU(2)_L$ singlets $\tilde{N}_i^c$ acquire the nonzero VEVs,

$$
H_u = \left( \begin{array}{c} H_u^+ \\ \frac{1}{\sqrt{2}} \left( v_u + H_u^0 + iP_u \right) \end{array} \right),
$$

$$
H_d = \left( \begin{array}{c} H_d^- \\ \frac{1}{\sqrt{2}} \left( v_d + H_d^0 + iP_d \right) \end{array} \right),
$$

$$
\tilde{L}_i = \left( \begin{array}{c} \frac{1}{\sqrt{2}} \left( v_{L_i} + \bar{\nu}_{L_i} + iP_{L_i} \right) \\ \tilde{L}_i^- \end{array} \right),
$$

$$
\tilde{N}_i^c = \frac{1}{\sqrt{2}} \left( v_{N_i} + \bar{\nu}_{R_i} + iP_{N_i} \right),
$$

the R-parity is broken spontaneously, and the local gauge symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_{(B-L)}$ is broken down to the electromagnetic symmetry $U(1)_e$, and the neutral and charged gauge bosons acquire the nonzero masses as

$$
m^2_Z = \frac{1}{4} (g_1^2 + g_2^2) \nu_{SM}^2,
$$

$$
m^2_W = \frac{1}{4} g_2^2 \nu_{EW}^2,
$$

$$
m^2_{ZBL} = g_{BL}^2 \left( \nu_N^2 + \nu_{EW}^2 - \nu_{SM}^2 \right).
$$

Where $\nu_{SM}^2 = \nu_u^2 + \nu_d^2$, $\nu_{EW}^2 = \nu_u^2 + \nu_d^2 + \sum_{a=1}^3 \nu_{L_a}^2$, $\nu_N^2 = \sum_{a=1}^3 \nu_{N_a}^2$, and $g_2, g_1, g_{BL}$ denote the gauge couplings of $SU(2)_L$, $U(1)_Y$ and $U(1)_{(B-L)}$, respectively.

To satisfy present electroweak precision observations we assume the mass of neutral $U(1)_{(B-L)}$ gauge boson $m_{Z_{BL}} > 1$ TeV which implies $\nu_N > 1$ TeV when $g_{BL} < 1$, then we derive $\max((Y_N)_{ij}) \leq 10^{-6}$ and $\max(\nu_{L_i}) \leq 10^{-3}$ GeV to explain experimental data on neutrino oscillation. Considering the minimization conditions at one-loop level, we formulate the $3 \times 3$ mass-squared matrix for right-handed sneutrinos as

$$
m^2_{\tilde{N}_i^c} \approx 
\begin{pmatrix}
\Lambda_{N_1}^2 - \Lambda_{BL}^2 & 0 & -\frac{\nu_{N_1}}{\nu_{N_2}} \Lambda_{N_1}^2 \\
0 & \Lambda_{\tilde{N}_2}^2 - \Lambda_{BL}^2 & -\frac{\nu_{N_2}}{\nu_{N_3}} \Lambda_{\tilde{N}_2}^2 \\
-\frac{\nu_{N_1}}{\nu_{N_3}} \Lambda_{N_1}^2 & -\frac{\nu_{N_2}}{\nu_{N_3}} \Lambda_{\tilde{N}_2}^2 & \frac{\nu_{N_1}^2}{\nu_{N_3}^2} \Lambda_{N_1}^2 + \frac{\nu_{N_2}^2}{\nu_{N_3}^2} \Lambda_{\tilde{N}_2}^2 - \Lambda_{BL}^2
\end{pmatrix}
$$

(14)
with $\Lambda_{BL}^2 = m_{\nu_{BL}}^2 / 2 + \Delta T_8$. Where $\Delta T_8$ denotes one-loop radiative corrections to the mass matrix of right-handed sneutrinos from top, bottom, tau and their supersymmetric partners [53].

III. EFFECTIVE HAMILTON FOR $b \to s l^+ l^-$, $(l = e, \mu, \tau)$

The transition $b \to s l^+ l^-$ is attributed to the effective Hamilton at hadronic scale

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \left[ C_i \mathcal{O}_i + C_2 \mathcal{O}_2^ \cdot + \sum_{i=3}^{6} C_i \mathcal{O}_i + \sum_{i=7}^{10} C_i \mathcal{O}_i + \sum_{i=S,P} C_i \mathcal{O}_i \right]. \quad (15)$$

where $\mathcal{O}_i$ $(i = 1, 2, \cdots, 10, S, P)$ are defined as [55]

$$\begin{align*}
\mathcal{O}_1^a &= (\bar{s}_L \gamma^\mu T^a u_L)(\bar{u}_L \gamma^\mu b_L) , \quad \mathcal{O}_2^a = (\bar{s}_L \gamma^\mu u_L)(\bar{u}_L \gamma^\mu b_L) , \\
\mathcal{O}_3 &= (\bar{s}_L \gamma^\mu b_L) \sum_q (\bar{q} \gamma^\mu q) , \quad \mathcal{O}_4 = (\bar{s}_L \gamma^\mu T^a b_L) \sum_q (\bar{q} \gamma^\mu T^a q) , \\
\mathcal{O}_5 &= (\bar{s}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \sum_q (\bar{q} \gamma^\nu \gamma^\rho T^a q) , \quad \mathcal{O}_6 = (\bar{s}_L \gamma^\mu \gamma^\nu \gamma^\rho T^a b_L) \sum_q (\bar{q} \gamma^\nu \gamma^\rho T^a q) , \\
\mathcal{O}_7 &= \frac{e^2}{g_s^2} m_b (\bar{s}_L \sigma_{\mu\nu} b_R) F_{\mu\nu} , \quad \mathcal{O}_8 = \frac{1}{g_s^2} m_b (\bar{s}_L \sigma_{\mu\nu} T^a b_R) G_{a,\mu\nu} , \\
\mathcal{O}_9 &= \frac{e^2}{g_s^2} (\bar{s}_L \gamma^\mu b_L) \bar{t} \gamma^\mu l , \quad \mathcal{O}_{10} = \frac{e^2}{g_s^2} (\bar{s}_L \gamma^\mu b_L) \bar{t} \gamma^\mu \gamma_5 l , \\
\mathcal{O}_S &= \frac{e^2}{16\pi^2} m_b (\bar{s}_L b_R) \bar{t} \gamma_5 l , \quad \mathcal{O}_P = \frac{e^2}{16\pi^2} m_b (\bar{s}_L b_R) \bar{t} \gamma_5 l . \quad (16)
\end{align*}$$

Here we adopt the MIA to get the corrections to relevant Wilson coefficients from supersymmetric particles because the updated experiment data push the energy scale of supersymmetry far above the electroweak energy scale. Furthermore we can formulate the relevant Wilson coefficients depending on the flavor changing sources from scalar quark sectors transparently by the MIA method. At the electroweak energy scale $\mu_{\text{EW}}$, the Wilson coefficients $C_{9,NP}(\mu_{\text{EW}})$, $C_{S,NP}(\mu_{\text{EW}})$ from the new physics beyond SM can be found in our previous work [56], other relevant Wilson coefficients are split as following

$$\begin{align*}
C_{9,NP}(\mu_{\text{EW}}) &= C_{9,NP}^\gamma(\mu_{\text{EW}}) + C_{9,NP}^Z(\mu_{\text{EW}}) + C_{9,NP}^{Z_{BL}}(\mu_{\text{EW}}) + C_{9,NP}^{\text{box}}(\mu_{\text{EW}}) , \\
C_{10,NP}(\mu_{\text{EW}}) &= C_{10,NP}^\gamma(\mu_{\text{EW}}) + C_{10,NP}^Z(\mu_{\text{EW}}) + C_{10,NP}^{Z_{BL}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) , \\
C_{S,NP}(\mu_{\text{EW}}) &= \sum_{i=1}^2 C_{S,i,NP}(\mu_{\text{EW}}) + C_{S,NP}^{\text{box}}(\mu_{\text{EW}}) ,
\end{align*}$$
\begin{equation}
C_{\mu,NP}(\mu_{\text{EW}}) = C_{\mu,NP}^{A_0}(\mu_{\text{EW}}) + C_{\mu,NP}^{\text{box}}(\mu_{\text{EW}}),
\end{equation}
where the superscripts $\gamma$, $Z$, $Z_{BL}$, $H_{1}^{0}$, $A_{0}$, box denote that new physics corrections to relevant Wilson coefficients originate from $\gamma$, $Z$, $Z_{BL}$, $H_{1}^{0}$, $A_{0}$; penguins and box diagrams, respectively. In order to formulate the corrections transparently, we split those pieces further as

\begin{align*}
C_{9,NP}^{\gamma}(\mu_{\text{EW}}) &= C_{9,H_{1}^{0}}^{\gamma}(\mu_{\text{EW}}) + C_{9,NP}^{\gamma}(\mu_{\text{EW}}) + C_{9,H_{1}^{0}}^{\gamma}(\mu_{\text{EW}}) + C_{9,2^{BL}}^{\gamma}(\mu_{\text{EW}}), \\
C_{9,10,NP}^{Z}(\mu_{\text{EW}}) &= (4s_{W}^{2} - 1)C_{10,NP}^{Z}(\mu_{\text{EW}}), \\
C_{9,1,NP}^{Z}(\mu_{\text{EW}}) &= C_{10,H_{1}^{0}}^{Z}(\mu_{\text{EW}}) + C_{9,2}^{Z}(\mu_{\text{EW}}) + C_{9,0}^{Z}(\mu_{\text{EW}}) + C_{9,2^{BL}}^{Z}(\mu_{\text{EW}}), \\
C_{9,10,NP}^{H_{0}}(\mu_{\text{EW}}) &= C_{9,2}^{H_{0}}(\mu_{\text{EW}}) + C_{9,0}^{H_{0}}(\mu_{\text{EW}}) + C_{9,2^{BL}}^{H_{0}}(\mu_{\text{EW}}), \\
C_{9,1,NP}^{A_{0}}(\mu_{\text{EW}}) &= C_{10,NP}^{A_{0}}(\mu_{\text{EW}}) + C_{10,NP}^{A_{0}}(\mu_{\text{EW}}) + C_{10,NP}^{A_{0}}(\mu_{\text{EW}}) + C_{10,NP}^{A_{0}}(\mu_{\text{EW}}), \\
C_{9,1,NP}^{\text{box}}(\mu_{\text{EW}}) &= C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}), \\
C_{9,1,NP}^{\text{box}}(\mu_{\text{EW}}) &= C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}), \\
C_{9,1,NP}^{\text{box}}(\mu_{\text{EW}}) &= C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}) + C_{10,NP}^{\text{box}}(\mu_{\text{EW}}),
\end{align*}

where the concrete expressions for the corrections involving $U(1)_{B-L}$ interaction are presented in Eq. (17).

The Wilson coefficients in Eq. (17) are calculated at the matching scale $\mu_{\text{EW}}$, then evolved down to hadronic scale $\mu \sim m_u$ by the renormalization group equations. In order to obtain hadronic matrix elements conveniently, we define effective coefficients

\begin{align*}
C_{7}^{\text{eff}} &= \frac{4\pi}{\alpha_{s}}C_{7} - \frac{1}{3}C_{3} - \frac{4}{9}C_{4} - \frac{20}{3}C_{5} - \frac{80}{9}C_{6}, \\
C_{8}^{\text{eff}} &= \frac{4\pi}{\alpha_{s}}C_{8} + C_{3} - \frac{1}{6}C_{4} + 20C_{5} - \frac{10}{3}C_{6}, \\
C_{9}^{\text{eff}} &= C_{9}^{\text{eff,SM}} + \frac{4\pi}{\alpha_{s}}C_{9}^{NP}, \\
C_{10}^{\text{eff}} &= \frac{4\pi}{\alpha_{s}}C_{10}, \\
C_{7,8,9,10}^{\text{eff}} &= \frac{4\pi}{\alpha_{s}}C_{7,8,9,10}. 
\end{align*}

In the SM, the Wilson coefficient $C_{7}^{\text{eff,SM}}$ and $C_{10}^{\text{eff,SM}}$ are real. Nevertheless, the Wilson coefficient $C_{9}^{\text{eff,SM}}$ contains slightly complex $CP$ phase originating from the continuum part
of $u\bar{u}$ and $c\bar{c}$ loop which is proportional to $V_{ub} V^*_{ub}$:

$$C^{eff,SM}_{q} = \frac{4\pi}{\alpha_s} C^{SM}_{q} + \xi_1(q^2) + \frac{V_{ub} V^*_{ub}}{V_{tb} V^*_{ts}} \xi_2(q^2), \quad (20)$$

with

$$\xi_1(q^2) = 0.138\omega\left(\frac{q^2}{m^2_b}\right) + g\left(\frac{m_q}{m_b}, \frac{q^2}{m^2_b}\right)\left(\frac{4}{3} C_1(\mu_b) + C_2(\mu_b) + 3C_3(\mu_b) + 3C_5(\mu_b)\right)$$

$$+ \frac{2}{3}\left(C_3(\mu_b) + C_5(\mu_b)\right) - \frac{1}{2}g\left(\frac{m_q}{m_b}, \frac{q^2}{m^2_b}\right)\left(C_3(\mu_b) + \frac{4}{3} C_4(\mu_b)\right)$$

$$- \frac{1}{2}g\left(1, \frac{q^2}{m^2_b}\right)\left(4C_3(\mu_b) + \frac{4}{3} C_4(\mu_b) + 3C_5(\mu_b)\right),$$

$$\xi_2(q^2) = \left[g\left(\frac{m_q}{m_b}, \frac{q^2}{m^2_b}\right) - g\left(\frac{m_q}{m_b}, \frac{q^2}{m^2_b}\right)\right]^{\frac{4}{3} C_1(\mu_b) + C_2(\mu_b)}. \quad (21)$$

Where the concrete expressions for $\omega(z), g(x, y)$ are written respectively as [55]:

$$\omega(z) = -\frac{2}{9}\pi^2 - \frac{4}{3}Li_2(z) - \frac{2}{3}\ln z \ln(1-z) - \frac{5 + 4z}{3(1 + z)} \ln(1-z)$$

$$- \frac{2z(1 + z)(1 - 2z)}{3(1 - z)^2(1 + z)} \ln z + \frac{5 + 9z - 6z^2}{6(1 - z)(1 + z)},$$

$$g(x, y) = \frac{8}{27} - \frac{8}{9} \ln \frac{m_q}{\mu_b} - \frac{8}{9} \ln x + \frac{16x^2}{9y}$$

$$- \frac{4}{9}(1 + \frac{2x^2}{y})\sqrt{1 - \frac{4x^2}{y}} \begin{cases} \ln \left| \frac{\sqrt{y} - 4x^2 + \sqrt{y}}{\sqrt{y} - 4x^2 - \sqrt{y}} \right| - i\pi, & \text{if } y > 4x^2 \\ 2 \arctan \frac{\sqrt{y}}{\sqrt{4x^2-y}}, & \text{if } y < 4x^2 \end{cases}. \quad (22)$$

In the limit of $x = 0$, $g(0, y) = \frac{8}{27} - \frac{8}{9} \ln \frac{m_q}{\mu_b} - \frac{8}{9} \ln y + i\frac{4}{9}\pi$. In the following numerical analysis, we take $\mu_b = m_b$ for simplification.

In our numerical analyses, we evaluate the Wilson coefficients from the SM to next-to-next-to-logarithmic (NNLL) accuracy in Table I at hadronic energy scale. On the other hand, the corrections to the Wilson coefficients from new physics are only included to one-loop accuracy:

$$\hat{C}_{NP}^\dagger(\mu) = \hat{U}(\mu, \mu_0) \bar{C}_{NP}(\mu_0),$$

| $C^{eff,SM}_{q}$ | $C^{eff,SM}_{q}$ | $C^{eff,SM}_{q}$ | $C^{eff,SM}_{q}$ |
|------------------|------------------|------------------|------------------|
| -0.304           | -0.167           | 4.211            | -4.103           |

TABLE I: At hadronic scale $\mu = m_b \simeq 4.8$GeV, SM Wilson coefficients to NNLL accuracy.
\[ \bar{C}_{NP}^\theta (\mu) = \bar{U}(\mu, \mu_0) \bar{C}_{NP}^\theta (\mu_0) \]  

(23)

with

\[
\bar{C}_{NP}^\theta T = (C_{1, NP}, \ldots, C_{6, NP}, C_{8, NP}^{\text{eff}}, C_{9, NP}^{\text{eff}}, C_{10, NP}^{\text{eff}}, Y(q^2), C_{10, NP}^{\text{eff}}),
\]

\[
\bar{C}_{NP}^{\prime, T} = (C_{7, NP}^{\prime, \text{eff}}, C_{8, NP}^{\prime, \text{eff}}, C_{9, NP}^{\prime, \text{eff}}, C_{10, NP}^{\prime, \text{eff}}).
\]

(24)

Correspondingly the evolving matrices are approached as

\[ \bar{U}(\mu, \mu_0) \simeq 1 - \left[ \frac{1}{2\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right] \hat{\gamma}^{(0)T}, \]

\[ \bar{U}^{\prime}(\mu, \mu_0) \simeq 1 - \left[ \frac{1}{2\beta_0} \ln \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right] \hat{\gamma}^{\prime(0)T}, \]

(25)

where the anomalous dimension matrices can be read from Ref. [57] as

\[
\hat{\gamma}^{(0)} = 
\begin{pmatrix}
-4 & 8/3 & 0 & -2/9 & 0 & 0 & -208/243 & 173/102 & -2272/729 & 0 \\
12 & 0 & 0 & 4/3 & 0 & 0 & 416/81 & 70/27 & 1952/243 & 0 \\
0 & 0 & 0 & -52/3 & 0 & 2 & -176/81 & 14/27 & -6752/243 & 0 \\
0 & 0 & -40/9 & -100/9 & 4/9 & 5 & -152/243 & 587/162 & -2192/729 & 0 \\
0 & 0 & 0 & -256/3 & 0 & 20 & -6272/81 & 6596/27 & -84032/243 & 0 \\
0 & 0 & -256/9 & 56/9 & 40/9 & -2/3 & 4624/243 & 4772/81 & -37856/729 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 32/3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -32/9 & 28/3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

(26)

\[
\hat{\gamma}^{\prime(0)} = 
\begin{pmatrix}
32/3 & 0 & 0 & 0 \\
-32/9 & 28/3 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

In addition, the operators \( O_{S,P}^{(\prime)} \) do not mix with other operators and their Wilson coefficients are given by the corresponding coefficients at matching scale.
IV. DIFFERENTIAL DECAY BRANCHING RATIOS, FORWARD-BACKWARD AND CP ASYMMETRIES

Keeping full dependence on the lepton mass while neglecting the strange quark mass, we write the unnormalized double differential decay width for \( B \rightarrow X_s l^+ l^- \) \((l = e, \mu, \tau)\) as

\[
\frac{d^2\Gamma}{dq^2 d\cos \theta} = \frac{\alpha_{\text{EW}}^2 G_F^2 m_b^5 |V_{tb} V_{ts}^*|^2}{16\pi^2} \left(1 - \frac{q^2}{m_b^2}\right)^2 \sqrt{1 - \frac{4m_s^2m_b^2}{(q^2)^2}}
\times \left\{ 6 \left(1 + \frac{2m_s^2}{q^2}\right) \Re(C_{e_7}^{\text{eff}} C_{e_9}^{\text{eff}}(q^2)) + \frac{3m_s^2}{m_b^2} \left[|C_{e_9}^{\text{eff}}(q^2)|^2 - |C_{e_{10}}^{\text{eff}}|^2\right] 
+ 3 \left(1 + \frac{2m_s^2}{q^2}\right) |C_{e_7}^{\text{eff}}|^2 \frac{m_b^2}{q^2} (1 + \cos^2 \theta) + (1 - \cos^2 \theta) \right\}
\times \left\{ \frac{3}{4} \left[|C_{e_9}^{\text{eff}}(q^2)|^2 + |C_{e_{10}}^{\text{eff}}|^2\right] \left[(1 - \cos^2 \theta) + \frac{q^2 - m_s^2}{m_b^2} + \frac{m_s^2}{q^2}(1 + \cos^2 \theta)\right] 
+ \frac{9}{32} q^2 \left[|C_s|^2 + |C_p|^2\right] (1 + \cos^2 \theta) 
+ \frac{9}{4} m_s \Re(C_s C_{e_{10}}^{\text{eff}} + C_p C_{e_{10}}^{\text{eff}})(1 - \cos^2 \theta) 
- \frac{9q^2}{8m_b^2} \Re(C_{e_9}^{\text{eff}}(q^2) C_{e_{10}}^{\text{eff}}) \cos \theta - 6 \Re(C_{e_7}^{\text{eff}} C_{e_{10}}^{\text{eff}}) \cos \theta 
+ \frac{3m_s}{2} \Re((2C_s^{\text{eff}} + C_{e_9}^{\text{eff}}(q^2))(C_s^* + C_p^*)) \cos \theta \right\} ,
\] (27)

where \(\theta\) denotes the angle between the \(l^+\) and \(B\) meson three momenta in the di-lepton rest frame. In order to reduce the uncertainties originating from the bottom quark mass and CKM matrix elements, one generally normalize the observables by the semileptonic decay width of \(B\) meson:

\[
\Gamma(B \rightarrow X_c e \bar{\nu}_e) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 f(z) \kappa(z) .
\] (28)

Here \(f(z) = 1 - 8z^2 + 8z^6 - z^8 - 24z^4 \ln z\) is phase space factor, \(k(z) = 1 - 2\alpha_s/3\pi[(\pi^2 - 31/4)(1 - z)^2 + 3/2]\) is the QCD correction factor with \(z = m_e/m_b\), respectively. Using Eq.(27) and Eq.(28), we get the normalized differential decay branching ratio of \(B \rightarrow X_s l^+ l^- \) \((l = e, \mu, \tau)\) at hadronic scale as

\[
R(q^2) = \frac{1}{\Gamma(B \rightarrow X_c e \bar{\nu}_e)} \frac{d\Gamma(B \rightarrow X_c l^+ l^-)}{dq^2}
\]
\[
\frac{\alpha_{\text{EW}}^2}{4\pi^2} V_{tb} V_{ts}^* \left( 1 - \frac{q^2}{m_b^2} \right)^2 \left( 1 - \frac{4m^2_{l_b}m^2_b}{(q^2)^2} \right) \times \left\{ 4 \left( 1 + \frac{2m^2_{l_b}}{q^2} \right) \left[ 3 \Re (C_7^{\text{eff}} C_9^{\text{eff}*}(q^2)) + (1 + \frac{2m^2_{l_b}}{q^2}) |C_7^{\text{eff}}|^2 \right] \\
+ \left( 1 + \frac{2q^2}{m_b^2} + \frac{2m^2_{l_b}}{q^2} + \frac{4m^2_{l_b}}{m_b^2} \right) |C_9^{\text{eff}}(q^2)|^2 \\
+ \left( 1 + \frac{2q^2}{m_b^2} + \frac{2m^2_{l_b}}{q^2} - \frac{8m^2_{l_b}}{m_b^2} \right) |C_{10}^{\text{eff}}|^2 \\
+ \frac{3}{4} q^2 \left( 1 - \frac{4m^2_{l_b}}{q^2} \right) \left( |C_3|^2 + |C_5|^2 \right) \\
+ 3m_b \Re (C_5 C_{10}^{\text{eff}*} + C_3 C_{10}^{\text{eff}*}) \right\},
\]

where \( q^2 = (p_{l+} + p_{l-})^2 \) denotes the dilepton invariant mass squared. Meanwhile the unnormalized forward-backward asymmetry is formulated as

\[
\bar{A}_{FB}(q^2) = \frac{1}{\Gamma(B \to X_{e\nu})} \int^{1}_{-1} d\cos\theta \frac{d^2\Gamma(B \to X_{e\nu})}{d\cos\theta dq^2} \text{Sgn}(\cos\theta) \\
= -\frac{3\alpha_{\text{EW}}^2}{4\pi^2} \left| V_{tb} V_{ts}^* \right|^2 \frac{1}{f(z)k(z)} \left( 1 - \frac{q^2}{m_b^2} \right)^2 \left( 1 - \frac{4m^2_{l_b}m^2_b}{(q^2)^2} \right) \times \left\{ \frac{q^2}{m_b^2} \Re (C_7^{\text{eff}}(q^2) C_9^{\text{eff}*}) + 2\Re (C_7^{\text{eff}} C_9^{\text{eff}*}) \\
- \frac{m_b}{2} \Re ((2C_7^{\text{eff}} + C_9^{\text{eff}}(q^2))(C_3^{\text{eff}} + C_5^{\text{eff}})) \right\},
\]

and the normalized forward-backward asymmetry is

\[
A_{FB}(q^2) = \frac{1}{R(q^2)} \bar{A}_{FB}(q^2),
\]

respectively. The global forward-backward asymmetry in the region \( q^2 \in [a, b] \) GeV\(^2\) is defined through

\[
A_{FB} \big|_{q^2 \in [a, b] \text{ GeV}^2} = \frac{N(l^+_{++}) - N(l^+_{+-})}{N(l^+_{++}) + N(l^+_{+-})} \big|_{q^2 \in [a, b] \text{ GeV}^2} \bigg|_{q^2 \in [a, b] \text{ GeV}^2} = \frac{\int^b_a dq^2 \bar{A}_{FB}(q^2)}{\int^b_a dq^2 R(q^2)}.
\]

The direct \( CP \) asymmetry in \( B \to X_{l} l^+ l^- \) is defined by

\[
A_{CP}(q^2) = \frac{d\Gamma(B \to X_{l} l^+ l^-)/dq^2 - d\Gamma(\bar{B} \to \bar{X}_{l} l^+ l^-)/dq^2}{d\Gamma(B \to X_{l} l^+ l^-)/dq^2 + d\Gamma(\bar{B} \to \bar{X}_{l} l^+ l^-)/dq^2} = \frac{\Delta D(q^2)}{D(q^2)},
\]

where

\[
\text{for } a < q^2 < b, \quad B \to X_{l} l^+ l^- \rightarrow f^{+}f^{-}\ell^{+}\ell^{-}, \quad f^{+}f^{-}\ell^{+}\ell^{-} \rightarrow a_{1}^{+}a_{1}^{+}a_{1}^{-}a_{1}^{-},
\]
with
\[
\Delta D(q^2) = 2\left(1 + \frac{2m^2}{q^2}\right)\left\{3 \left(\frac{V_\mu V_\mu^*}{V_{tb} V_{ts}^*}\right) \left[2 \left(1 + \frac{2q^2}{m_b^2}\right) \Im(\xi_1 \xi_2^*) - 12C_{eff,SM}^*(\mu_b) \Im(\xi_2)\right] + 2\left(1 + \frac{2q^2}{m_b^2}\right) \left[\Re(\xi_1) \Im(\xi_2) \left(C_{eff,NP}(\mu_b)\right) + \Im(\xi_2) \Im\left(\frac{V_{ub} V_{ub}^*}{V_{tb} V_{ts}^*} C_{7f,NP}(\mu_b)\right)\right] + 12 \left[\Re(\xi_1) \Im\left(\frac{V_{ub} V_{ub}^*}{V_{tb} V_{ts}^*} C_{7f,NP}(\mu_b)\right)\right]\right\},
\]
\[
D(q^2) = \left(1 + \frac{2m^2}{q^2}\right) \left\{B_1 + 2C_{eff,NP}(\mu_b)\right\}^2 + 4\Re(\xi_1) \Im\left(C_{eff,NP}(\mu_b)\right) + 12\left(1 + \frac{2m^2}{q^2}\right) \left\{B_2 + 2C_{eff,SM}(\mu_b)\right\} \Im\left(C_{eff,NP}(\mu_b)\right) + 2\Re(\xi_1) \Re\left(C_{eff,SM}(\mu_b)\right) + 2\Re(\xi_2) \Re\left(C_{eff,NP}(\mu_b)\right) + 8\left(1 + \frac{2m^2}{q^2}\right) \left(1 + \frac{2q^2}{m_b^2}\right) C_{eff}\right\}^2 + 2\left(1 + \frac{2q^2}{m_b^2} + \frac{2m_1^2}{q^2} - \frac{8m_1^2}{m_b^2}\right) C_{eff}\right\^2 + \frac{3}{2} q^2 \left(1 - \frac{4m_1^2}{q^2}\right) \left(C_S^2 + |C_P|^2\right) + 6m_c \Re\left(C_S C_{10}^{eff} + C_P C_{10}^{eff}\right) \right\}.
\]

Here
\[
B_1 = 2\left\{\left|\xi_1\right|^2 + \left|\frac{V_{ub} V_{ub}^*}{V_{tb} V_{ts}^*} \xi_2\right|^2 + 2\Re\left(\frac{V_{ub} V_{ub}^*}{V_{tb} V_{ts}^*}\right) \Re(\xi_1 \xi_2)\right\},
\]
\[
B_2 = 2C_{eff,SM}(\mu_b) \left\{\Re(\xi_1) + \Re\left(\frac{V_{ub} V_{ub}^*}{V_{tb} V_{ts}^*}\right) \Re(\xi_2)\right\}.
\]

The global CP asymmetry in the region \(q^2 \in [a, b]\) GeV\(^2\) is correspondingly defined through
\[
A_{CP}\bigg|_{q^2 \in [a, b]} = \frac{\int_a^b dq^2 \Delta D(q^2)}{\int_a^b dq^2 D(q^2)}.
\]
theoretical predictions in this region are \[59\]:

\[
\text{Br}(B \rightarrow X_s e^+ e^-)_{q^2 \in [1, 6] \text{ GeV}^2}^{\text{SM}} = (1.64 \pm 0.11) \times 10^{-6},
\]

\[
\text{Br}(B \rightarrow X_s \mu^+ \mu^-)_{q^2 \in [1, 6] \text{ GeV}^2}^{\text{SM}} = (1.59 \pm 0.11) \times 10^{-6}.
\]

In the region \( q^2 \geq 14.4 \text{GeV}^2 \), the theoretical uncertainty is relatively larger than that in the low region \( 1 \text{GeV}^2 \leq q^2 \leq 6 \text{GeV}^2 \), and the SM theoretical evaluations are given as \[59\]:

\[
\text{Br}(B \rightarrow X_s e^+ e^-)_{q^2 \in [14.4, 25] \text{ GeV}^2}^{\text{SM}} = (0.21 \pm 0.07) \times 10^{-6},
\]

\[
\text{Br}(B \rightarrow X_s \mu^+ \mu^-)_{q^2 \in [14.4, 25] \text{ GeV}^2}^{\text{SM}} = (0.24 \pm 0.07) \times 10^{-6}.
\]

In our analysis, the lepton-flavor-averaged branching ratio for \( B \rightarrow X_s l^+ l^- \) is averages of the individual \( \text{Br}(B \rightarrow X_s e^+ e^-) \) and \( \text{Br}(B \rightarrow X_s \mu^+ \mu^-) \). Furthermore, the updated experimental data on the forward-backward and \( CP \) asymmetries constrain the parameter space of new physics concretely.

V. NUMERICAL ANALYSES

In order to perform our numerical analyses, we present the relevant SM inputs from \[60\] in table\[II\]. The supersymmetric parameters involved here are soft breaking masses of the 2nd and 3rd generation squarks, \( m_{\tilde{Q}_2,3}^2 \), \( m_{\tilde{U}_{2,3}}^2 \), \( m_{\tilde{D}_{2,3}}^2 \), soft breaking masses of the 1st and 2nd generation sleptons \( m_{\tilde{L}_{1,2}} \), \( m_{\tilde{E}_{1,2}} \), \( m_{\tilde{N}_{1,2}} \), neutralino and chargino masses \( m_{\chi_0} \), \( m_{\chi^\pm} \), \((\alpha = 1, \cdots, 4, \beta = 1, 2)\) and their mixing matrices. Additionally the masses and mixing matrix of \( B - L \) gaugino/right-handed neutrinos are mainly determined from nonzero VEVs of right-handed sneutrinos, local \( B - L \) gauge coupling \( g_{BL} \) and soft gaugino mass parameters \( m_{1BL}, m_{1BL} \). The flavor conservation mixing between left- and right-handed of the third generation squarks \( (\delta_{u}^{LR})_{33} = m_{t_X}^2 / \Lambda_{NP}^2, (\delta_{d}^{LR})_{33} = m_{b_X}^2 / \Lambda_{NP}^2 \) are chosen to give the lightest Higgs mass in the range 124 – 126 GeV, where the concrete expressions of \( m_{t_X}^2 \), \( m_{b_X}^2 \) are presented in appendix \[A\]. The \( b \rightarrow s \) transitions are mediated by those flavor changing insertions \( (\delta_{U,D}^{LR})_{23}, (\delta_{U,D}^{LR})_{32}, (\delta_{U,D}^{RR})_{23} \), which are originated from flavour-violating scalar mass terms and trilinear scalar couplings in soft breaking terms.
The updated bound from ATLAS collaboration on the gluino mass is $m_{\tilde{g}} \geq 1460$ GeV, and the bound on the mass of scalar top is $m_{\tilde{t}} \geq 780$ GeV \cite{42}. To coincide with those experimental data, we always assume $\Lambda_{NP} = m_{\tilde{Q}_{2,3}} = m_{\tilde{U}_{2,3}} = m_{\tilde{D}_{2,3}} = 2$ TeV, $m_{\tilde{t}_{1,2}} = m_{\tilde{b}_{1,2}} = m_{\tilde{N}_{1,2}} = 1$ TeV, and $m_{\tilde{g}} \geq 1.5$ TeV in our numerical discussion unless specified.

Certainly the experimental data of 125 GeV constrain the parameter space of supersymmetric extension of the SM strongly. The radiative corrections to mass of the lightest Higgs subtly depend on the parameters $\tan\beta$, $\mu$, the squark masses of the third generation and relevant trilinear couplings $A_t$, $A_b$ in soft terms. Furthermore the observed average on branching ratio of $B^0_s \to \mu^+ \mu^-$ is \cite{60}

$$BR(B \to \mu^+ \mu^-)_{\text{exp}} \simeq \left(3.1 \pm 0.7\right) \times 10^{-9},$$ \hspace{2cm} (39)

which coincides with the SM evaluation:

$$BR(B \to \mu^+ \mu^-)_{\text{SM}} \simeq \left(3.23 \pm 0.27\right) \times 10^{-9}. \hspace{2cm} (40)$$

TABLE II: Input parameters \cite{60} of the SM used in the numerical analysis

| Input | Input |
|-------|-------|
| $m_B = 5.280$ GeV | $m_{K^*} = 0.896$ GeV |
| $m_{B_s} = 5.367$ GeV | $m_{\mu} = 0.106$ GeV |
| $m_w = 80.40$ GeV | $m_Z = 91.19$ GeV |
| $\tau_B = 2.307 \times 10^{12}$ GeV | $f_B = 0.190 \pm 0.004$ |
| $\alpha_s(m_Z) = 0.118 \pm 0.002$ | $\alpha_{em}(m_Z) = 1/128.9$ |
| $m_e(m_e) = 1.27 \pm 0.11$ GeV | $m_b(m_b) = 4.18 \pm 0.17$ GeV |
| $m_t^{pole} = 173.1 \pm 1.3$ GeV | |
| $\lambda_{CKM} = 0.225 \pm 0.001$ | $A_{CKM} = 0.811 \pm 0.022$ |
| $\tilde{\rho} = 0.131 \pm 0.026$ | $\tilde{\eta} = 0.345 \pm 0.014$ |

15
FIG. 1: Taking $(\delta_{D}^{LL})_{23} = (\delta_{D}^{RR})_{23} = (\delta_{D}^{LR})_{23} = 0.125$, we plot (a) $BR(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2} \times 10^6$, (b) $BR(B \to X_s l^+ l^-)_{q^2 \in [14.4,25.0] \text{GeV}^2} \times 10^6$ (c) $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2}$, (d) $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [14.4,25.0] \text{GeV}^2}$, varying with the gluino mass. Where the solid lines denote $\tan \beta = 5$, dashed lines denote $\tan \beta = 10$, dotted lines denote $\tan \beta = 30$, dashed-dotted lines denote $\tan \beta = 50$, respectively.

The average experimental data on the branching ratio of the inclusive $\bar{B} \to X_s \gamma$ reads

$$BR(\bar{B} \to X_s \gamma)^{\text{exp}} \simeq \left(3.40 \pm 0.21\right) \times 10^{-4}, \quad (41)$$

and the corresponding SM prediction at NNLO order is

$$BR(\bar{B} \to X_s \gamma)^{\text{SM}} \simeq \left(3.36 \pm 0.23\right) \times 10^{-4}. \quad (42)$$

The experimental data from $\bar{B} \to X_s \gamma$ and $B^0_s \to \mu^+ \mu^-$ also constrain on correlations between the flavor-changing parameters and energy scale of new physics strongly.

To obtain mass of the lightest Higgs in reasonable range, we further choose the mass of CP-odd Higgs $m_A = 1 \text{ TeV}$, and the following assumptions on the parameter space.
FIG. 2: Taking $(\delta_D^{LL})_{23} = (\delta_D^{RR})_{23} = (\delta_D^{LR})_{23} = 0.125$, we plot (a) $BR(B \rightarrow X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2} \times 10^6$, (b) $BR(B \rightarrow X_s l^+ l^-)_{q^2 \in [14.4,25.0] \text{GeV}^2} \times 10^6$ (c) $A_{FB}(B \rightarrow X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2}$, (d) $A_{FB}(B \rightarrow X_s l^+ l^-)_{q^2 \in [14.4,25.0] \text{GeV}^2}$, varying with the $CP$ phase $\theta_g$. Where the solid lines denote $\tan \beta = 5$, dashed lines denote $\tan \beta = 10$, dotted lines denote $\tan \beta = 30$, dashed-dotted lines denote $\tan \beta = 50$, respectively.

- Taking $\tan \beta = 5$, $A_t = 1.5 \text{ TeV}$, $A_b = 1 \text{ TeV}$, $\mu = -500 \text{ GeV}$, one gets $m_h \simeq 124.6 \text{ GeV}$ correspondingly.

- Taking $\tan \beta = 10$, $A_t = 1 \text{ TeV}$, $A_b = 1 \text{ TeV}$, $\mu = 500 \text{ GeV}$, one gets $m_h \simeq 125.3 \text{ GeV}$ correspondingly.

- Taking $\tan \beta = 30$, $A_t = 0.5 \text{ TeV}$, $A_b = 1 \text{ TeV}$, $\mu = 500 \text{ GeV}$, one gets $m_h \simeq 125.2 \text{ GeV}$ correspondingly.

- Taking $\tan \beta = 50$, $A_t = .5 \text{ TeV}$, $A_b = 0.5 \text{ TeV}$, $\mu = 500 \text{ GeV}$, one gets $m_h \simeq 125.3 \text{ GeV}$ correspondingly.
FIG. 3: Taking $(δ^{LL}_D)_{23} = (δ^{RR}_D)_{23} = (δ^{LR}_D)_{23} = 0.125$, we plot (a) $A_{CP}(B \rightarrow X_s l^+l^-)_{q^{2\in[1,6]GeV^2}} \times 10^2$, (B) $A_{CP}(B \rightarrow X_s l^+l^-)_{q^{2\in[14,425,0]GeV^2}} \times 10^2$, varying with the CP phase $θ_δ$. Where the solid lines denote $tan β = 5$, dashed lines denote $tan β = 10$, dotted lines denote $tan β = 30$, dashed-dotted lines denote $tan β = 50$, respectively.

For the gauge coupling of local $B - L$ symmetry and nonzero VEVs of right-handed sneutrinos, we take $g_{BL} = 0.7, \nu_N = (0, 0, 3)$ TeV here. This choice induces the mass of $U(1)_{B-L}$ gauge boson $m_{z_{BL}} = 2.1$ TeV. Through scanning the parameter space, we find that the theoretical evaluations depend on the $U(1)_{B-L} \times U(1)_Y$ gauginos masses $|m_1|$ and $|m_{BL}|$ mildly. In our numerical analysis below, we choose $m_1 = |m_{BL}| = 1$ TeV for simplification. Furthermore, the parameter $m_{1BL}$ only evokes the mixing between $U(1)_{B-L}$ and $U(1)_Y$ gauginos, and affects our numerical results gently. Not loss of generality, we also take $m_{1BL} = 0$ in our numerical analysis. Under the assumptions above, the numerical results are decided by the gaugino masses $|m_2|, |m_3|$, the CP violating phases $θ_δ, θ_2, θ_{BL}$, and the corresponding flavor-changing insertions $(δ^{LL}_{U,D})_{23}, (δ^{RR}_{U,D})_{23}, (δ^{LR}_{U,D})_{23}$.

Assuming $|m_2| = 600$ GeV, $θ_δ = θ_2 = θ_{BL} = 0$, we present $BR(B \rightarrow X_s l^+l^-)_{q^{2\in[1,6]GeV^2}} \times 10^6$ versus $|m_3|$ in Fig[1(a)], $BR(B \rightarrow X_s l^+l^-)_{q^{2\in[14,425,0]GeV^2}} \times 10^6$ versus $|m_3|$ in Fig[1(b)], $A_{FB}(B \rightarrow X_s l^+l^-)_{q^{2\in[1,6]GeV^2}}$ versus $|m_3|$ in Fig[1(c)], and $A_{FB}(B \rightarrow X_s l^+l^-)_{q^{2\in[14,425,0]GeV^2}}$ versus $|m_3|$ in Fig[1(d)], respectively. Since the gluino affects our numerical results through the down-type squark sector, we choose $(δ^{LL}_D)_{23} = (δ^{RR}_D)_{23} = (δ^{LR}_D)_{23} = 0.125$, $(δ^{LL}_U)_{23} = (δ^{RR}_U)_{23} = (δ^{LR}_U)_{23} = 0$ here. As $|m_3| < 1.8$ TeV, the numerical evaluations
FIG. 4: Taking $(\delta_U^{LL})_{23} = (\delta_U^{RR})_{23} = (\delta_U^{LR})_{23} = 0.125$, we plot (a) $BR(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2} \times 10^6$, (b) $BR(B \to X_s l^+ l^-)_{q^2 \in [14,4,25.0] \text{GeV}^2} \times 10^6$ (c) $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2}$, (d) $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [14,4,25.0] \text{GeV}^2}$, varying with the $SU(2)$ gaugino mass $|m_{\tilde{g}}|$. Where the solid lines denote tan $\beta = 5$, dashed lines denote tan $\beta = 10$, dotted lines denote tan $\beta = 30$, dashed-dotted lines denote tan $\beta = 50$, respectively.

of $BR(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2}$ exceed $2 \times 10^{-6}$. With increasing of $|m_{\tilde{g}}|$, the numerical evaluations of $BR(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2}$ decrease slowly. Meanwhile the corresponding numerical results depend on the parameter tan $\beta$ mildly because the main corrections originate from the operators $O_{7,9,10}$ in low $q^2$ region. Similarly the numerical evaluations of $BR(B \to X_s l^+ l^-)_{q^2 \in [14,4,25] \text{GeV}^2}$ exceed $0.5 \times 10^{-6}$ as $|m_{\tilde{g}}| < 2 \text{ TeV}$. With increasing of $|m_{\tilde{g}}|$, the numerical evaluations of $BR(B \to X_s l^+ l^-)_{q^2 \in [14,4,25] \text{GeV}^2}$ decrease mildly. The corresponding numerical results depend on the parameter tan $\beta$ subtly because the main corrections originate from the operators $O_{S,P}$ in high $q^2$ region. In addition the numerical evaluations on $A_{FB}$ in low $q^2$ region is lying in the range $-0.32 \leq A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [1,6] \text{GeV}^2} \leq -0.18$, the numerical evaluations on $A_{FB}$ in high $q^2$ region is lying in
the range 0.28 ≤ $A_{FB}(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.4, 25.0] \text{GeV}^2} ≤ -0.18$ as 1.6 ≤ $|m_\tilde{g}|$/TeV ≤ 4, which are all coincide with the experimental data within three standard deviations. Because the main corrections originate from the operators $O_{S,P}$ in high $q^2$ region, the numerical evaluations of $A_{FB}(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.4, 25.0] \text{GeV}^2}$ depend on the parameter tan $\beta$ subtly.

As the mass of gluino is relatively light, the $CP$ phase of $m_\tilde{g}$ also affects our final result strongly. Taking $|m_2| = 600$ GeV, $|m_\tilde{g}| = 1.6$ TeV, and $\theta_2 = \theta_{BL} = 0$, we plot $BR(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.6] \text{GeV}^2} \times 10^6$ varying with $\theta_9$ in Fig.2(a), $BR(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.4, 25.0] \text{GeV}^2} \times 10^6$ varying with $\theta_9$ in Fig.2(b), $A_{FB}(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.6] \text{GeV}^2}$ varying with $\theta_9$ in Fig.2(c), and $A_{FB}(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.4, 25.0] \text{GeV}^2}$ varying with $\theta_9$ in Fig.2(d), respectively. In low $q^2$ region, the theoretical prediction on $BR(B \rightarrow X_s l^+ l^-)_{q^2 \in [1.6] \text{GeV}^2}$ increases from $2.5 \times 10^{-6}$ to $4.5 \times 10^{-6}$.
10^{-6} as the CP phase \( \theta_g \) increases from 0 to \( \pi \), and the forward-backward asymmetry \( A_{FB}(B \to X_s l^+ l^-) \) changes from -0.2 to 0.3 when the CP phase \( \theta_g \) increases from 0 to \( \pi \), respectively. In high \( q^2 \) region, the branching ratio \( BR(B \to X_s l^+ l^-) \) varies mildly with increasing of the CP phase \( \theta_g \) because the corrections from the operators \( O_{S,P} \) are important in this region. Nevertheless \( A_{FB}(B \to X_s l^+ l^-) \) changes from -0.2 to 0.18 when the CP phase \( \theta_g \) increases from 0 to \( \pi \).

Using the inputs presented in Table. (III), one gets the SM predictions on the CP asymmetries as \( A_{CP}(B \to X_s l^+ l^-) \sim 10^{-3} \), \( A_{CP}(B \to X_s l^+ l^-) < 10^{-4} \), respectively. Taking \( (\delta_D^{LL})_{23} = (\delta_D^{RR})_{23} = (\delta_D^{LR})_{23} = 0.125 \), we plot \( A_{CP}(B \to X_s l^+ l^-) \times 10^2 \) versus \( \theta_g \) in Fig. 3(a), \( A_{CP}(B \to X_s l^+ l^-) \times 10^2 \) versus the CP phase \( \theta_g \) in Fig. 3(b). Where the solid lines denote \( \tan \beta = 5 \), dashed lines denote \( \tan \beta = 10 \), dotted lines denote \( \tan \beta = 30 \), dashed-dotted lines denote \( \tan \beta = 50 \), respectively. The CP asymmetry \( A_{CP}(B \to X_s l^+ l^-) \) reaches 1.4\% as \( \theta_g = \pi/2 \), the CP asymmetry \( A_{CP}(B \to X_s l^+ l^-) \) changes from -1.2\% to 1.8\% when the CP phase \( \theta_g \) varies from \( \pi/2 \) to \( 3\pi/2 \). We anticipate that the CP asymmetries exceeding 0.01 can be detected in near future.

FIG. 6: Taking \( (\delta_U^{LL})_{23} = (\delta_U^{RR})_{23} = (\delta_U^{LR})_{23} = 0.125 \), we plot (a) \( A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [1,6] GeV^2} \times 10^2 \), (B) \( A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4,25.0] GeV^2} \times 10^2 \), varying with the CP phase \( \theta_g \). Where the solid lines denote \( \tan \beta = 5 \), dashed lines denote \( \tan \beta = 10 \), dotted lines denote \( \tan \beta = 30 \), dashed-dotted lines denote \( \tan \beta = 50 \), respectively.
FIG. 7: Taking \((\delta_{D}^{LL})_{23} = (\delta_{D}^{RR})_{23} = (\delta_{D}^{LR})_{23} = 0.125\), we plot (a) \(BR(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [1,6] \text{GeV}^{2}} \times 10^{6}\), (b) \(BR(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [14.4,25.0] \text{GeV}^{2}} \times 10^{6}\) \(A_{FB}(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [1,6] \text{GeV}^{2}}\), (d) \(A_{FB}(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [14.4,25.0] \text{GeV}^{2}}\), varying with the CP phase \(\theta_{g}\). Where the solid lines denote \(\tan \beta = 5\), dashed lines denote \(\tan \beta = 10\), dotted lines denote \(\tan \beta = 30\), dashed-dotted lines denote \(\tan \beta = 50\), respectively.

To investigate the corrections from chargino sector, we choose \((\delta_{D}^{LL})_{23} = (\delta_{D}^{RR})_{23} = (\delta_{D}^{LR})_{23} = 0\), and \(|m_{\tilde{g}}| = 4 \text{ TeV}\) to suppress the contributions from gluinos and down-type squarks. So far the experimental data do not exclude relatively light neutralinos and charginos with several hundred GeV masses yet. Assuming \(\theta_{\tilde{g}} = \theta_{2} = \theta_{BL} = 0\), and the insertions \((\delta_{U}^{LL})_{23} = (\delta_{U}^{RR})_{23} = (\delta_{U}^{LR})_{23} = 0.125\), we present \(BR(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [1,6] \text{GeV}^{2}} \times 10^{6}\) versus the \(SU(2)\) gaugino mass \(|m_{2}|\) in Fig.8(a), \(BR(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [14.4,25.0] \text{GeV}^{2}} \times 10^{6}\) versus the \(SU(2)\) gaugino mass \(|m_{2}|\) in Fig.8(b), \(A_{FB}(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [1,6] \text{GeV}^{2}}\) versus the \(SU(2)\) gaugino mass \(|m_{2}|\) in Fig.8(c), and \(A_{FB}(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [14.4,25.0] \text{GeV}^{2}}\) versus the \(SU(2)\) gaugino mass \(|m_{2}|\) in Fig.8(d), respectively. As \(|m_{2}| > 2 \text{ TeV}\), \(BR(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [1,6] \text{GeV}^{2}}\) and \(BR(B \rightarrow X_{s} l^{+} l^{-})_{q^{2} \in [14.4,25.0] \text{GeV}^{2}}\) both coincide with the experimental data in three standard
FIG. 8: Taking $(\delta_D^L)_{23} = (\delta_D^{RR})_{23} = (\delta_D^{LR})_{23} = 0.125$, we plot (a) $A_{CP}(B \to X_s l^+ l^-)_{q^2\in[1,6] GeV^2} \times 10^2$, (B) $A_{CP}(B \to X_s l^+ l^-)_{q^2\in[14,4,25,0] GeV^2} \times 10^2$, varying with the $CP$ phase $\theta_{BL}$. Where the solid lines denote $\tan \beta = 5$, dashed lines denote $\tan \beta = 10$, dotted lines denote $\tan \beta = 30$, dashed-dotted lines denote $\tan \beta = 50$, respectively.

deviations. The resonance peaks around 1 TeV originate from the box diagrams involving sleptons/sneutrinos and neutralinos/charginos under our assumptions on parameter space. As $|m_2| = 1.5$ TeV, the numerical evaluation on $A_{FB}(B \to X_s l^+ l^-)_{q^2\in[1,6] GeV^2}$ is about $-0.4$, and increases slowly with increasing of $|m_2|$. In high $q^2$ region the corrections from the operators $O_{S,P}$ affect our numerical results heavily. As $|m_2| = 2.5$ TeV, the numerical result indicates $A_{FB}(B \to X_s l^+ l^-)_{q^2\in[14,4,25] GeV^2} \simeq -0.5$. Along with increasing of $|m_2|$, the numerical evaluations on $A_{FB}(B \to X_s l^+ l^-)_{q^2\in[14,4,25] GeV^2}$ of $\tan \beta = 50$ are faster than those of lower of $\tan \beta$.

Under our assumptions on the parameter space, there is a relatively light chargino with mass in the range 500 GeV $\leq m_{\tilde{\chi}_1^\pm} \leq 1$ TeV. Therefore the $CP$ phase of $m_2$ also affects our numerical results. Taking $|m_2| = 600$ GeV, $|m_\tilde{g}| = 4$ TeV, and $\theta_\tilde{s} = \theta_{BL} = 0$, we present $BR(B \to X_s l^+ l^-)_{q^2\in[1,6] GeV^2} \times 10^6$ versus $\theta_2$ in Fig 5(a), $BR(B \to X_s l^+ l^-)_{q^2\in[14,4,25,0] GeV^2} \times 10^6$ versus $\theta_2$ in Fig 5(b), $A_{FB}(B \to X_s l^+ l^-)_{q^2\in[1,6] GeV^2}$ versus $\theta_2$ in Fig 5(c), and $A_{FB}(B \to X_s l^+ l^-)_{q^2\in[14,4,25,0] GeV^2}$ versus $\theta_2$ in Fig 5(d), respectively. When $\tan \beta = 5$ and $\pi/2 \leq \theta_2 \leq 3\pi/2$, we get $0.4 \times 10^{-6} \leq BR(B \to X_s l^+ l^-)_{q^2\in[1,6] GeV^2} \leq 0.8 \times 10^{-6}$, $0.07 \times 10^{-6} \leq BR(B \to X_s l^+ l^-)_{q^2\in[14,4,25,0] GeV^2} \leq 0.12 \times 10^{-6}$ respectively, which
coincide with the experimental data in $3\sigma$ permissions. As $\tan \beta = 10$, one finds the numerical evaluations of $BR(B \to X_s l^+ l^-)_{q^2 \in [1.6]\text{GeV}^2}$ satisfying the experimental constraint within three standard deviations. However the choice of $\tan \beta = 10$ is excluded by experimental observations because $0.02 \times 10^{-6} \leq BR(B \to X_s l^+ l^-)_{q^2 \in [14.4,25]\text{GeV}^2} \leq 0.05 \times 10^{-6}$, which does not satisfy the experimental data in $3\sigma$ standard deviations. For large $\tan \beta = 30, 50$, the corrections from the operators $O_{S,P}$ are enhanced drastically. Correspondingly the theoretical predictions on the branching ratios in low and high $q^2$ regions all coincide with the experimental data, respectively. The forward-backward asymmetries both in low and high $q^2$ regions depend on the $CP$ phase $\theta_2$ smoothly, the absolute values of corresponding evaluations exceed $0.05$ which can be detected in future.

As mentioned above, the SM predictions on the $CP$ asymmetries are $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [1,2]\text{GeV}^2} \sim 10^{-3}$, $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4,25]\text{GeV}^2} < 10^{-4}$, which are difficult to detected in near future. Taking $(\delta_{U}^{LL})_{23} = (\delta_{U}^{RR})_{23} = (\delta_{U}^{LR})_{23} = 0.125$, we plot $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [1,6]\text{GeV}^2} \times 10^2$ varying with $\theta_2$ in Fig.7(a), $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4,25,0]\text{GeV}^2} \times 10^2$, varying with the $CP$ phase $\theta_2$ in Fig.7(b), respectively. Assuming that the $CP$ violation originates from CKM in the SM, one finds $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [1,6]\text{GeV}^2} \sim 0.01$, $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [1,4.25,0]\text{GeV}^2} \sim 0.002$, respectively. The theoretical predictions of $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [1,6]\text{GeV}^2}$ depends on $\theta_2$ gently. Nevertheless, the new $CP$ phase $\theta_2$ modifies numerical results of $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4,25,0]\text{GeV}^2}$ strongly.

When $(\delta_{U}^{LL})_{23} = (\delta_{U}^{RR})_{23} = (\delta_{U}^{LR})_{23} = 0$, and $(\delta_{D}^{LL})_{23} = (\delta_{D}^{RR})_{23} = (\delta_{D}^{LR})_{23} \neq 0$, the $SU(2)$ gaugino mass $m_2$ and $U(1)$ gaugino mass $m_1$ affect our numerical results through the mixing matrix of neutralinos. The numerical evaluations indicate that those physics quantities depend on $m_2$, $m_1$ mildly in the sectors of parameter space. Similarly the $U(1)_{B-L}$ gaugino mass $|m_{BL}|$ also affects our results gently.

Under our assumptions on the parameter space, it is interesting to investigate the effect of $CP$ phase $\theta_{BL}$ on our numerical analyses. Taking $(\delta_{U}^{LL})_{23} = (\delta_{U}^{RR})_{23} = (\delta_{U}^{LR})_{23} = 0.125$, $(\delta_{U}^{LL})_{23} = (\delta_{U}^{RR})_{23} = (\delta_{U}^{LR})_{23} = 0$, $|m_2| = 600$ GeV, $|m_{\tilde{g}}| = 4$ TeV, and $\theta_\tilde{g} = \theta_2 = 0$, we present $BR(B \to X_s l^+ l^-)_{q^2 \in [1,6]\text{GeV}^2} \times 10^6$ versus $\theta_{BL}$ in Fig.7(a), $BR(B \to X_s l^+ l^-)_{q^2 \in [14.4,25,0]\text{GeV}^2} \times 10^6$ versus $\theta_{BL}$ in Fig.7(b), $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [1,6]\text{GeV}^2}$ versus $\theta_2$ in Fig.7(c), and $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [14.4,25,0]\text{GeV}^2}$ versus $\theta_{BL}$ in Fig.7(d), respectively. Our theo-
retical evaluations depend on the $CP$ phase $\theta_{BL}$ slowly. The numerical results on branching ratios in low $q^2$ and high $q^2$ regions satisfy the experimental data simultaneously in three standard permissions. $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [1.6] GeV^2} \sim -0.2$ for all $\tan \beta$ chosen. As $\tan \beta = 5, 10$, $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2} \sim -0.4$, and $A_{FB}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2} \sim -0.2$ as $\tan \beta = 30, 50$. Additionally we present $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2} \times 10^2$ varying with $\theta_{BL}$ in Fig.8(a), $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2} \times 10^2$, varying with the $CP$ phase $\theta_{BL}$ in Fig.8(b), respectively. Assuming that the $CP$ violation originates from CKM matrix elements, one finds $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2} \sim 0.01$, $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2} \sim 0.002$, respectively. The theoretical predictions of $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2}$ depends on $\theta_{BL}$ gently. Nevertheless, the new $CP$ phase $\theta_{BL}$ modifies numerical results of $A_{CP}(B \to X_s l^+ l^-)_{q^2 \in [14.4, 25.0] GeV^2}$ strongly.

In our assumptions on parameter space, the theoretical predictions on $Br(\bar{B} \to X_s \gamma)$ and $Br(B_s \to \mu^+ \mu^-)$ all coincide with the experimental observations in three standard permissions. Obviously our numerical results on branching ratios, forward-backward asymmetries, and $CP$ asymmetries in $B \to X_s l^+ l^-$ depend on the mass insertions $(\delta_{LL} U, D)_{23}$, $(\delta_{RR} U, D)_{23}$, $(\delta_{LR} U, D)_{23}$ and corresponding $CP$ phases subtly. Here we do not present theoretical evaluations on above quantities versus mass insertions explicitly, because some similar analyses are given in our previous works [39, 56]. In addition we take a relatively small coupling of $U(1)_{B-L}$ as $g_{BL} \leq g_2$, this choice avoid the Landau pole of $g_{BL}$ below the energy scale of grand unified theories.

VI. SUMMARY

Rare $B$-meson decays are very sensitive to new physics beyond the SM since the theoretical evaluations on corresponding physical quantities are not seriously affected by the uncertainties originating from unperturbative QCD effects. Considering the constraint from the observed Higgs signal at the LHC, we study the supersymmetric corrections to the branching ratios $BR(B \to X_s l^+ l^-)$, ($l = e, \mu$) in the MSSM with local $U(1)_{B-L}$ symmetry [47, 50] with nonuniversal soft breaking terms. After obtaining the Wilson coefficients at matching scale, we evolve the Wilson coefficients from the SM down to hadronic
scale at NNLL accuracy, and evolve that from new physics down to hadronic scale at LL accuracy, respectively. The lightest neutral Higgs with mass around 125 GeV constrains the correlation between $\tan \beta$ and the soft Yukawa coupling $A_t$, $A_b$ strongly, nevertheless constrains neutral flavor changing mass insertions weakly. Under our assumptions on parameters of the considered model, the numerical analyses indicate that the branching ratios, and forward-backward asymmetries depend on the gaugino masses $m_{\tilde{g}}$, $m_{\tilde{2}}$ strongly, new possible $CP$ phases can enhance the $CP$ asymmetries exceed 1%, which can be detected in near future.

Acknowledgments

The work has been supported by the National Natural Science Foundation of China (NNSFC) with Grant No. 11275036, and No. 11535002, the Open Project Program of State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, China(No.Y5KF131CJ1), the Natural Science Foundation of Hebei province with Grant No. A2013201277, No. A2016201010, No. A2016201069, and Natural Science Foundation of Hebei University with Grant No. 2011JQ05, No. 2012-242.

Appendix A: The mass squared matrices for squarks

With the minimal flavor violation assumption, the $2 \times 2$ mass squared matrix for scalar tops is given as

$$
Z_t^\dagger \begin{pmatrix} m_{t_L}^2 & m_{t_X}^2 \\ m_{t_X}^2 & m_{t_R}^2 \end{pmatrix} Z_t = \text{diag}(m_{t_1}^2, m_{t_2}^2),
$$

(A1)

with

$$
m_{t_L}^2 = \frac{(g_1^2 + g_2^2) v_{EW}^2}{24} \left( 1 - 2 \cos^2 \beta \right) \left( 1 - 4 c_W^2 \right) + \frac{g_{3 L}^2}{6} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) + m_t^2 + m_{Q_3}^2,
$$
\[ m_{t_R}^2 = -\frac{g_1^2 v_{EW}^2}{6} \left( 1 - 2 \cos^2 \beta \right) \]
\[ -\frac{g_2^2}{6} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) + m_{t_3}^2 + m_{\tilde{U}_3}^2, \]
\[ m_{t_X}^2 = -\frac{v_u}{\sqrt{2}} A_t Y_t + \frac{\mu v_d}{\sqrt{2}} Y_t. \]  

(A2)

Here \( Y_t, A_t \) denote Yukawa coupling and trilinear soft-breaking parameters in top quark sector, respectively. In a similar way, the mass-squared matrix for scalar bottoms is

\[
\mathcal{Z}_b^\dagger \begin{pmatrix}
m_{b_L}^2 & m_{b_X}^2 \\
m_{b_X}^2 & m_{b_R}^2
\end{pmatrix} \mathcal{Z}_b = \text{diag}(m_{b_1}^2, m_{b_2}^2),
\]

(A3)

with

\[
m_{b_L}^2 = \frac{(g_1^2 + g_2^2) v_{EW}^2}{24} \left( 1 - 2 \cos^2 \beta \right) \left( 1 + 2 c_2^W \right) \]
\[ + \frac{g_2^2}{6} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) + m_b^2 + m_{Q_3}^2, \]
\[
m_{b_R}^2 = \frac{g_1^2 v_{EW}^2}{12} \left( 1 - 2 \cos^2 \beta \right) \]
\[ - \frac{g_2^2}{6} \left( v_N^2 - v_{EW}^2 + v_{SM}^2 \right) + m_b^2 + m_{D_3}^2, \]
\[
m_{b_X}^2 = \frac{v_d}{\sqrt{2}} A_b Y_b - \frac{\mu v_u}{\sqrt{2}} Y_b, \]  

(A4)

here \( Y_b, A_b \) denote Yukawa couplings and trilinear soft-breaking parameters in \( B \) quark sector, respectively.

**Appendix B: The Wilson coefficients from \( U(1)_{B-L} \) interaction at electroweak scale**

Adopting mass insertion approximation, we present the corrections to those Wilson coefficients from \( U(1)_{B-L} \) interaction here

\[
C_{9,2_{BL}}^7 (\mu_{EW}) = \frac{Q_d \alpha_s(\mu_{EW}) \alpha_{BL} \Lambda_{NP}^2}{324 \pi \Lambda_{NP}^2} \delta^2 m_{LL} \left( \delta^2 m_{LR} \right)_{23} \]
\[ \times T_3(x_{2_{BL}}, x_{b_L}, x_{s_L}) \]
\[ - \frac{Q_d \alpha_s(\mu_{EW}) \alpha_{BL} \Lambda_{NP}^2}{324 \pi \Lambda_{NP}^2} \delta^2 m_{DR} \left( \delta^2 m_{LR}^* \right)_{33} \]
\[ \times D_3(x_{2_{BL}}, x_{b_L}, x_{b_R}, x_{s_L}), \]

\[ C_{9,2_{BL}}^7 (\mu_{EW}) \]
\[
C_{10,BL}^{Z}(\mu_{EW}) = \frac{\alpha_s(\mu_{EW})\alpha_{BL}}{144\pi\alpha_{EW}(\mu_{EW})} \left(1 - \frac{2}{3}s_w^2\right) A_{N_{LP}}^2 V_{tb} V_{ts} (T_B - \frac{\partial \theta_{1,1}}{\partial x_t} - \frac{\partial \theta_{2,1}}{\partial x_L}) \left(x_{z,BL}, x_{b,1}, x_{z,1}\right),
\]
\[
- \frac{\alpha_s(\mu_{EW})\alpha_{BL}}{144\pi\alpha_{EW}(\mu_{EW})} \left(\delta^2 m_{DR}^{LR}\right)_{23}^{(2)} (\delta^2 m_{DL}^{LR})^{*}_{33} \left[1 - \frac{2}{3}s_w^2\right] (D_B - \frac{\partial \theta_{2,1}}{\partial x_L} - \frac{\partial \theta_{1,1}}{\partial x_t}) \left(x_{z,BL}, x_{b,1}, x_{z,1}\right),
\]
\[
- \frac{2}{3}s_w^2 \frac{\partial \theta_{1,1}}{\partial x_t} \left(x_{z,1}, x_{b,1}, x_{z,1}\right),
\]
\[
C_{9_{H^Z}}^{Z}(\mu_{EW}) = \frac{\alpha_s(\mu_{EW})\alpha_{BL}}{24\pi\alpha_{EW}(\mu_{EW})} \left(\frac{\delta^2 m_{LL}}{\delta^2 m_{UL}}\right)_{23}^{(2)} (U_{+})_{i1}^* (U_{+})_{i1} \left(x_{w,2} \frac{x_w}{x_{z,BL}} (T_B - \frac{1}{2} T_{BL}) \left(x_{z,1}, x_{b,1}, x_{z,1}\right) \right.
\]
\[
+ \frac{x_w}{x_{w,s_{\beta}}^{1/2}} \sum_{i=1}^{2} \left(Z_{H^Z}\right)_{i1}^* \left(Z_{H^Z}\right)_{i1} \left[- 2 \theta_{1,1} - \frac{2}{3} \partial \theta_{1,1} + \frac{2}{3} \partial \theta_{1,1} \right] \left(x_{t,1} \right),
\]
\[
- \frac{\alpha_s(\mu_{EW})\alpha_{BL}}{24\pi\alpha_{EW}(\mu_{EW})} \left(\frac{\delta^2 m_{LL}}{\delta^2 m_{UL}}\right)_{23}^{(2)} (U_{+})_{i1}^* (U_{+})_{i1} \left(x_{w,2} \frac{x_w}{x_{z,BL}} (T_B - 2 T_{BL}) \left(x_{z,1}, x_{b,1}, x_{z,1}\right) \right.
\]
\[
+ \frac{x_w}{x_{w,s_{\beta}}^{1/2}} \sum_{i=1}^{2} \left(Z_{H^Z}\right)_{i1}^* \left(Z_{H^Z}\right)_{i1} \left[- 2 \theta_{1,1} - \frac{2}{3} \partial \theta_{1,1} + \frac{2}{3} \partial \theta_{1,1} \right] \left(x_{t,1} \right),
\]
\[
- \frac{\alpha_s(\mu_{EW})\alpha_{BL}}{24\pi\alpha_{EW}(\mu_{EW})} \left(\frac{\delta^2 m_{LL}}{\delta^2 m_{UL}}\right)_{23}^{(2)} (U_{+})_{i1}^* (U_{+})_{i1} \left(x_{w,2} \frac{x_w}{x_{z,BL}} (T_B - 2 T_{BL}) \left(x_{z,1}, x_{b,1}, x_{z,1}\right) \right.
\]
\[
+ \frac{x_w}{x_{w,s_{\beta}}^{1/2}} \sum_{i=1}^{2} \left(Z_{H^Z}\right)_{i1}^* \left(Z_{H^Z}\right)_{i1} \left[- 2 \theta_{1,1} - \frac{2}{3} \partial \theta_{1,1} + \frac{2}{3} \partial \theta_{1,1} \right] \left(x_{t,1} \right),
\]
\[
\times (D_B - \frac{1}{2} D_B) (\frac{1}{2}, x_{b_R}, x_{b_L}, x_{s_L})
\]
\[
+ \frac{\alpha_s(\mu_{EW}) \alpha_{BL} m_b}{24 \pi \alpha_{EW}(\mu_{EW}) m_w c_w c_\beta} \frac{(\delta^2 m_{LL})_{23} (\delta^2 m_{LR})_{33}}{\Lambda^4_N \nu_{\nu}^{-1} \nu_t^*} (U_N)_{3i} \mathcal{N}^i \frac{x_W}{x_{ZBL}}
\]
\[
\times (D_B - \frac{1}{2} D_B) (\frac{1}{2}, x_{b_R}, x_{b_L}, x_{s_L})
\]
\[
+ \frac{\alpha_s(\mu_{EW}) \alpha_{BL} m_s}{24 \pi \alpha_{EW}(\mu_{EW}) m_w c_w c_\beta} \frac{(\delta^2 m_{RR})_{23} (\delta^2 m_{LR})_{33}}{\Lambda^4_N \nu_{\nu}^{-1} \nu_t^*} (U_N)^* \mathcal{N}^i \frac{x_W}{x_{ZBL}}
\]
\[
C^Z_{9, 8 BL} (\mu_{EW}) = - \frac{\alpha_s(\mu_{EW}) \alpha_{BL} s^2_w}{9 \pi \alpha^2_{EW}(\mu_{EW})} \frac{\delta^2 m_{LL}}{\delta^2 m_{LR}}_{23} \frac{\delta^2 m_{LR}}{\delta^2 m_{LR}}_{33} (T_B - T_{BL}) (x_{g}, x_{b_L}, x_{s_L})
\]
\[
+ \frac{\alpha_s(\mu_{EW}) \alpha_{BL} s^2_w}{9 \pi \alpha^2_{EW}(\mu_{EW})} \frac{\delta^2 m_{LL}}{\delta^2 m_{LR}}_{23} \frac{\delta^2 m_{LR}}{\delta^2 m_{LR}}_{33} \frac{x_W}{x_{ZBL}} \frac{1}{D_B - D_{BL}} (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
C^Z_{9, 2 BL} (\mu_{EW}) = - \frac{\alpha_s(\mu_{EW}) \alpha_{BL} s^2_w}{5 \pi \alpha^2_{EW}(\mu_{EW})} \frac{\delta^2 m_{LL}}{\delta^2 m_{LR}}_{23} \frac{\delta^2 m_{LR}}{\delta^2 m_{LR}}_{33} \frac{x_W}{x_{ZBL}} \frac{1}{D_B - D_{BL}} (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
C^{H^0}_{9, 2 BL} (\mu_{EW}) = - \frac{\alpha_s(\mu_{EW}) \alpha_{BL} s^2_w}{18 \alpha_{EW}(\mu_{EW}) m_w^2 c^2_\beta} \frac{\delta^2 m_{LL}}{\delta^2 m_{LR}}_{23} \frac{\delta^2 m_{LR}}{\delta^2 m_{LR}}_{33} (Z_H)_{2k} \frac{x_W}{x_{ZBL}} \frac{1}{D_B} (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
+ \frac{\alpha_s(\mu_{EW}) \alpha_{BL} m_s}{18 \alpha_{EW}(\mu_{EW}) m_w c_w c_\beta} \frac{\delta^2 m_{LL}}{\delta^2 m_{LR}}_{23} \frac{\delta^2 m_{LR}}{\delta^2 m_{LR}}_{33} (Z_H)_{2k} \frac{x_W}{x_{ZBL}} \frac{1}{D_B} (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
\times \left[ (\zeta_{LL})_k \frac{\partial \theta_{1, 1}}{\partial x_{s_L}} + \frac{2}{3} s^2_w (\zeta_{RR})_k \frac{\partial \theta_{1, 1}}{\partial x_{b_L}} \right] (x_{g}, x_{b_L}, x_{s_L})
\]
\[
+ \frac{2 \alpha_{BL} m_s A_{\nu w}^e i_{BL} (\delta^2 m_{LL})_{23} (\delta^2 m_{LR})_{33}}{9 \alpha_{EW}(\mu_{EW}) m_w^3 c^2_\beta} \frac{Z_H}{2k} \frac{x_W}{x_{ZBL}} \frac{1}{D_B} (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
\times \left[ (\zeta_{LL})_k \frac{\partial \theta_{1, 1}}{\partial x_{s_L}} + \frac{2}{3} s^2_w (\zeta_{RR})_k \frac{\partial \theta_{1, 1}}{\partial x_{b_L}} \right] (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
- \frac{\alpha_s(\mu_{EW}) \alpha_{BL} m_s s^2_w}{9 \alpha_{EW}(\mu_{EW}) m_w c_w c_\beta} \frac{\delta^2 m_{LL}}{\delta^2 m_{LR}}_{23} \frac{\delta^2 m_{LR}}{\delta^2 m_{LR}}_{33} (Z_H)_{2k} \frac{x_W}{x_{ZBL}} \frac{1}{D_B} (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
\times \left[ (\zeta_{LL})_k \frac{\partial \theta_{1, 1}}{\partial x_{s_L}} + \frac{2}{3} s^2_w (\zeta_{RR})_k \frac{\partial \theta_{1, 1}}{\partial x_{b_L}} \right] (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
\times \left[ (\zeta_{LL})_k \frac{\partial \theta_{1, 1}}{\partial x_{s_L}} + \frac{2}{3} s^2_w (\zeta_{RR})_k \frac{\partial \theta_{1, 1}}{\partial x_{b_L}} \right] (x_{g}, x_{b_L}, x_{b_R}, x_{s_L})
\]
\[
C_{A_k}^{a L, \tilde{\tau}_{BL}} (\mu_{EW}) = \frac{\alpha_{BL} m_{\mu} e^{i\theta_{BL}}}{18 \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{RR}}{\delta^2 m_{D}^{RR}} \right)_{33} \frac{\mathbb{R}[A_{b\mu}(\delta^{2} m_{D}^{LR})]_{33}}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{bR}} \times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{bL}} \times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{sL}} \times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{sL}} \\
+ \frac{2 \alpha_{BL} m_{\mu} e^{i\theta_{BL}}}{9 \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{LL}}{\delta^2 m_{D}^{LL}} \right)_{23} \frac{\mathbb{R}[P_{b\mu}(\delta^{2} m_{D}^{LR})]_{33}}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{bR}} \times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{bL}} \times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{sL}} \times \frac{\partial q_{2, 1}^{L}(x_{\beta_{BL}}, x_{bR}, x_{bL}, x_{sL})}{\partial x_{sL}} \\
C_{9, \tilde{\tau}_{BL}}^{box} (\mu_{EW}) = -\frac{\alpha_{s}(\mu_{EW}) \alpha_{BL}^{2} m_{W}^{2}}{36 \pi \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{LL}}{\delta^2 m_{D}^{LL}} \right)_{23} \frac{\mathbb{R}[\mathcal{C}_{d}^{*} \mathcal{C}_{d}^{*}]}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \left[ \frac{\partial q_{2, 1}^{L}}{\partial x_{\beta_{BL}}} \left( x_{\beta_{BL}}, x_{sL}, x_{sL}, x_{sL} \right) \right] \\
+ \frac{\alpha_{s}(\mu_{EW}) \alpha_{BL}^{2} m_{W}^{2}}{36 \pi \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{LL}}{\delta^2 m_{D}^{LL}} \right)_{23} \frac{\mathbb{R}[\mathcal{C}_{d}^{*} \mathcal{C}_{d}^{*}]}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \left[ \frac{\partial q_{2, 1}^{L}}{\partial x_{\beta_{BL}}} \left( x_{\beta_{BL}}, x_{sL}, x_{sL}, x_{sL} \right) \right] \\
C_{9, \tilde{\tau}_{BL}}^{box} (\mu_{EW}) = -\frac{\alpha_{s}(\mu_{EW}) \alpha_{BL}^{2} m_{W}^{2}}{24 \pi \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{LL}}{\delta^2 m_{D}^{LL}} \right)_{23} \frac{\mathbb{R}[\mathcal{C}_{d}^{*} \mathcal{C}_{d}^{*}]}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \left[ \frac{\partial q_{2, 1}^{L}}{\partial x_{\beta_{BL}}} \left( x_{\beta_{BL}}, x_{sL}, x_{sL}, x_{sL} \right) \right] \\
C_{9, \tilde{\tau}_{BL}}^{box} (\mu_{EW}) = -\frac{\alpha_{s}(\mu_{EW}) \alpha_{BL}^{2} m_{W}^{2}}{48 \pi \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{LL}}{\delta^2 m_{D}^{LL}} \right)_{23} \frac{\mathbb{R}[\mathcal{C}_{d}^{*} \mathcal{C}_{d}^{*}]}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \left[ \frac{\partial q_{2, 1}^{L}}{\partial x_{\beta_{BL}}} \left( x_{\beta_{BL}}, x_{sL}, x_{sL}, x_{sL} \right) \right] \\
C_{9, \tilde{\tau}_{BL}}^{box} (\mu_{EW}) = -\frac{\alpha_{s}(\mu_{EW}) \alpha_{BL}^{2} m_{W}^{2}}{48 \pi \alpha_{EW}(\mu_{EW}) m_{W}^{2} c_{\beta}} \left( \frac{\delta^{2} m_{\phi}^{LL}}{\delta^2 m_{D}^{LL}} \right)_{23} \frac{\mathbb{R}[\mathcal{C}_{d}^{*} \mathcal{C}_{d}^{*}]}{2} \frac{(Z_{\beta})_{2k}}{x_{bR}^{A_{0}} x_{bL}^{A_{0}}} \\
\times \left[ \frac{\partial q_{2, 1}^{L}}{\partial x_{\beta_{BL}}} \left( x_{\beta_{BL}}, x_{sL}, x_{sL}, x_{sL} \right) \right]
\]
\[-4m_w s_w e^{iθ_{BL}} (U_N)_{1i}(U_N)_{3i}(x^1_i, x^2_{BL})^{1/2} \partial_{2,1}\] 
\[-\frac{α_s(μ_{EW})α_{BL}}{24πα_{EW}(μ_{EW})m_w c_3^2} \Lambda_{NP}^2 V_tv_t^s x_w \left[(U_N)^*_{3i} N^i_1 q_{2,1}\right] \] 
\[-4s_w e^{iθ_{BL}} (U_N)_{3i}^*(x^1_i, x^2_{BL})^{1/2} \partial_{2,1}\] 
\[+ \frac{α_s(μ_{EW})α_{BL}}{24πα_{EW}(μ_{EW})m_w c_3^2} \Lambda_{NP}^2 V_tv_t^s x_w \left[(e^{iθ_{BL}} N^i_1 N^i_1)^* q_{2,1}\right] \] 
\[-4s_w \mathcal{R} \left((U_N)_{1i}\right) N^i_1 (x^1_i, x^2_{BL})^{1/2} \partial_{2,1}\] 
\[+ \frac{α_s(μ_{EW})α_{BL} m_m m_s}{48πα_{EW}(μ_{EW})m_w c_3^2} \Lambda_{NP}^2 V_tv_t^s x_w \left[(U_N)_{3i}^* (U_N)_{3i} q_{2,1}\right] \] 
\[-4s_w \mathcal{R} \left((U_N)_{3i}\right) N^i_1 (x^1_i, x^2_{BL})^{1/2} \partial_{2,1}\] 
\[+ \frac{α_s(μ_{EW})α_{BL}}{24πα_{EW}(μ_{EW})m_w c_3^2} \Lambda_{NP}^2 V_tv_t^s x_w \left[(m_m N^i_1)^* (U_N)_{3i} q_{2,1}\right] \] 
\[= \frac{C_{10,2_{BL}}^{box}(μ_{EW})}{36πα_{EW}(μ_{EW})^2} x_w \left[\frac{\partial q_{2,1}}{\partial x^2_{BL}} + 2x^2_{BL} \frac{\partial q_{2,1}}{\partial x^2_{BL}}\right] (x^1_i, x^2_{BL}, x^3, x^4, x^5, x^6, x^7, x^8) \] 
\[-\frac{α_s(μ_{EW})α_{BL}}{36πα_{EW}(μ_{EW})^2} \Lambda_{NP}^2 V_tv_t^s x_w \left[(U_N)_{3i}^* (U_N)_{3i} q_{2,1}\right] \] 
\[= \frac{C_{10,8_{BL}}^{box}(μ_{EW})}{24πα_{EW}(μ_{EW})^2} x_w \left[\frac{\partial q_{2,1}}{\partial x^2_{BL}} + 2x^2_{BL} \frac{\partial q_{2,1}}{\partial x^2_{BL}}\right] (x^1_i, x^2_{BL}, x^3, x^4, x^5, x^6, x^7, x^8) \]
\[ C_{S,\alpha_2^{\text{BL}}} (\mu_{\text{EW}}) = \frac{\alpha_{\text{BL}} m_{\mu}}{6 c_{\text{EW}} (\mu_{\text{EW}}) m_{W}^2 c_{\beta} \Lambda_{\text{NP}}^2 V_{tb} V_{ts}^* x_{\text{W}} [(U_N)^{\ast}_{3i}(U_N)_{3i}] \frac{\theta_{2,1}}{m_{W} c_{\beta}} - \frac{\alpha_{\text{BL}} m_{\mu}}{18 c_{\text{EW}} (\mu_{\text{EW}}) m_{W}^2 c_{\beta} \Lambda_{\text{NP}}^2 V_{tb} V_{ts}^* x_{\text{W}} [2 s_{\text{W}} (U_N)^{\ast}_{3i}(U_N)_{3i}] \frac{\theta_{2,1}}{m_{W} c_{\beta}} + 3 c_{\text{W}} e^{i \theta_{BL}} N_{d}^{\ast} (U_N)^{\ast}_{3i} (x_{\text{BL}}^{\ast})^{1/2} \frac{\theta_{2,1}}{m_{W} c_{\beta}} \]
\[ -2s_{W} e^{\theta_{BL}} (U_{N})^* (x_{i_0} x_{\tilde{Z}_{BL}}^{*})^{1/2} \theta_{1,1} \] \( x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*}, x_{\tilde{b}_{L}}^{*}, x_{\tilde{b}_{R}}^{*}, x_{\tilde{l}_{L}}^{*}, x_{\tilde{l}_{R}}^{*} \)

\[ + \frac{\alpha_{BL} m_{\mu}}{6 \alpha_{EW}(\mu_{EW}) m_{w}^{2} c_{\beta}^{2}} \left( \delta^{2} m_{D}^{LR} \right)_{23}^{33} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \] \( (U_{N})_{3i}(U_{N})_{3i} \theta_{2,1} \)

\[ - c_{w} (U_{N})_{3i}(U_{N})_{3i}(x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*})^{1/2} \theta_{1,1} \] \( x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*}, x_{\tilde{b}_{L}}^{*}, x_{\tilde{b}_{R}}^{*}, x_{\tilde{l}_{L}}^{*}, x_{\tilde{l}_{R}}^{*} \)

\[ + \frac{\alpha_{BL} m_{\mu}}{6 \alpha_{EW}(\mu_{EW}) m_{w}^{2} c_{\beta}^{2}} \left( \delta^{2} m_{D}^{LR} \right)_{23}^{33} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \] \( (U_{N})_{3i}(U_{N})_{3i} x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*} \)^{1/2}

\[ \times \theta_{1,1} (x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*}, x_{\tilde{b}_{L}}^{*}, x_{\tilde{b}_{R}}^{*}, x_{\tilde{l}_{L}}^{*}, x_{\tilde{l}_{R}}^{*}) \]

\[ + \frac{\alpha_{BL} m_{\mu}}{18 \alpha_{EW}(\mu_{EW}) m_{w} m_{c_{W}} c_{\beta}} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \] \( (U_{N})_{3i}(U_{N})_{3i} \theta_{2,1} \)

\[ \times \left[ 2s_{W} (U_{N})_{3i}(U_{N})_{3i} \theta_{1,1} - 3 c_{w} e^{\theta_{BL}} N_{d}^{*} (U_{N})_{3i}(x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*})^{1/2} \theta_{1,1} \right] \]

\[ + \frac{\alpha_{BL} m_{\mu}}{6 \alpha_{EW}(\mu_{EW}) m_{w}^{2} c_{\beta}^{2}} \left( \delta^{2} m_{D}^{LR} \right)_{23}^{33} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \] \( (U_{N})_{3i}(U_{N})_{3i} x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*} \)^{1/2}

\[ \times \theta_{1,1} (x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*}, x_{\tilde{b}_{L}}^{*}, x_{\tilde{b}_{R}}^{*}, x_{\tilde{l}_{L}}^{*}, x_{\tilde{l}_{R}}^{*}) \]

\[ + \frac{\alpha_{BL} m_{\mu}}{18 \alpha_{EW}(\mu_{EW}) m_{w} m_{c_{W}} c_{\beta}} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \] \( (U_{N})_{3i}(U_{N})_{3i} \theta_{2,1} \)

\[ \times \left[ 2s_{W} (U_{N})_{3i}(U_{N})_{3i} \theta_{1,1} - 3 c_{w} e^{\theta_{BL}} N_{d}^{*} (U_{N})_{3i}(x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*})^{1/2} \theta_{1,1} \right] \]

\[ + \frac{\alpha_{BL} m_{\mu}}{6 \alpha_{EW}(\mu_{EW}) m_{w}^{2} c_{\beta}^{2}} \left( \delta^{2} m_{D}^{LR} \right)_{23}^{33} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \] \( (U_{N})_{3i}(U_{N})_{3i} x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*} \)^{1/2}

\[ \times \theta_{1,1} (x_{\chi_i}^{0} x_{\tilde{Z}_{BL}}^{*}, x_{\tilde{b}_{L}}^{*}, x_{\tilde{b}_{R}}^{*}, x_{\tilde{l}_{L}}^{*}, x_{\tilde{l}_{R}}^{*}) \]

\[ - \frac{\alpha_{BL} m_{\mu}}{18 \alpha_{EW}(\mu_{EW}) m_{w} m_{c_{W}} c_{\beta}} \Lambda_{NP}^{4} V_{tb} V_{ts}^{*} x_{W} \]
\begin{align}
&\times [2s_w(U_N)_{3k}(U_N)_{11}^*\varrho_{2,1} - 3c_w e^{i\theta_{BL}} N_d^{i*}(U_N)_{3k}(x_{\chi_i^0} x_{\bar{Z}_{BL}})^{1/2} \varphi_{1,1} \\
&+ 2s_w e^{i\theta_{BL}} (U_N)_{3k}(U_N)^*_{11}(x_{\chi_i^0} x_{\bar{Z}_{BL}})^{1/2} \varphi_{1,1}(x_{\chi_i^0}, x_{\bar{Z}_{BL}}, x_{\bar{e}_L}, x_{\bar{e}_L}, x_{\bar{e}_R})] \tag{B1}
\end{align}

where the couplings in the expression are

\begin{align}
N_d^i &= \frac{1}{3}(U_N)_{11}s_w - (U_N)_{2i}c_w , \\
N_i^j &= (U_N)_{1i}s_w + (U_N)_{2i}c_w , \\
C_{ij}^L &= 2(c_w^2 - s_w^2)\delta_{ij} + (U_-)^{1i}_1(U_-)^*_{1j} , \\
C_{ij}^R &= 2(c_w^2 - s_w^2)\delta_{ij} + (U_+)^{1i}_1(U_+)^*_{1j} , \\
B_{ij}^k &= \cos \beta (Z_{H^0})_{1k} - \sin \beta (Z_{H^0})_{2k} , \\
(\zeta_{LL}^i)_{k} &= (1 - \frac{4}{3}s_w^2)B_{H}^k + \frac{2m_u^2 c_w^2}{m_w^2 s_w} (Z_{H^0})_{1k} , \\
(\zeta_{RR}^i)_{k} &= B_{H}^k - \frac{3m_u^2 c_w^2}{2m_w^2 s_w^2 s_\beta} (Z_{H^0})_{1k} , \\
(\zeta_{LL}^d)_{k} &= (1 - \frac{2}{3}s_w^2)B_{H}^k - \frac{2m_u^2 c_w^2}{m_w^2 c_\beta} (Z_{H^0})_{2k} , \\
(\zeta_{RR}^d)_{k} &= B_{H}^k - \frac{3m_u^2 c_w^2}{m_w^2 s_w^2 c_\beta} (Z_{H^0})_{2k} , \\
(\xi_{k}^{\pm})_{ij} &= (Z_{H^0})_{1k}(U_-)^{1i}_1(U_+)^*_{2j} + (Z_{H^0})_{2k}(U_-)^{2i}_2(U_+)^*_{1j} , \\
(\xi_{k}^{-})_{ij} &= [(Z_{H^0})_{1k}(U_N)_{3j} - (Z_{H^0})_{2k}(U_N)_{4j}] [(U_N)_{11}s_w - (U_N)_{2i}c_w] , \\
(\eta_{k}^{-})_{ij} &= (Z_{H^0})_{1k}(U_+)^{1i}_1(U_-)^{2i}_2 + (Z_{H^0})_{2k}(U_-)^{2i}_2(U_+)^*_{1j} , \\
(\eta_{k}^{-})_{ij} &= [(Z_{H^0})_{1k}(U_N)_{3j} - (Z_{H^0})_{2k}(U_N)_{4j}] [(U_N)_{11}s_w - (U_N)_{2i}c_w] , \\
A_{\mu_{ij}}^{k} &= (Z_{H^0})_{1k}(Z_{H^0})_{1i} - (Z_{H^0})_{2k}(Z_{H^0})_{2i} , \\
A_{\mu_{ij}}^{k} &= A_{\mu_{ij}}^{k}(Z_{H^0})_{1k} + \mu^*(Z_{H^0})_{2k} , \\
A_{\mu_{ij}}^{k} &= A_{\mu_{ij}}^{k}(Z_{H^0})_{2k} + \mu^*(Z_{H^0})_{1k} , \\
P_{\mu_{ij}}^{k} &= A_{\mu_{ij}}^{k}(Z_{H^0})_{1k} - \mu^*(Z_{H^0})_{2k} , \\
P_{\mu_{ij}}^{k} &= A_{\mu_{ij}}^{k}(Z_{H^0})_{2k} - \mu^*(Z_{H^0})_{1k} . \tag{B2}
\end{align}
Here $U_N$, $U_\pm$ are the mixing matrices of neutralinos and charginos in the MSSM, respectively. $Z_H$ is the $2 \times 2$ mixing matrix of CP-even Higgs, and

$$
Z_H = \begin{pmatrix} c_\beta & -s_\beta \\ s_\beta & c_\beta \end{pmatrix} \tag{B3}
$$

is the mixing matrix between charged Higgs and Goldstone. Furthermore, we adopt the shorten-cutting notations as $c_\beta = \cos \beta$, $s_\beta = \sin \beta$, $c_w = \cos \theta_w$, $s_w = \sin \theta_w$.

Appendix C: The functions

The functions in the wilson coefficients of $\gamma-$ and $g-$ penguin operators are

$$
T_B(x, y, z) = \left[ 2\theta_{1,1} - \frac{\partial \theta_{2,1}}{\partial x} \right](x, y, z),
$$

$$
T_{BL}(x, y, z) = \frac{\partial \theta_{2,1}}{\partial y} + \frac{\partial \theta_{2,1}}{\partial z},
$$

$$
T_1(x, y, z) = \left[ \frac{\partial \theta_{1,1}}{\partial y} + \frac{\partial \theta_{1,1}}{\partial z} \right](x, y, z),
$$

$$
T_2(x, y, z) = \left[ \frac{\partial^2 \theta_{2,1}}{\partial y^2} + 2 \frac{\partial^2 \theta_{2,1}}{\partial y \partial z} + \frac{\partial^2 \theta_{2,1}}{\partial z^2} \right](x, y, z),
$$

$$
T_3(x, y, z) = \left[ \frac{\partial^3 \theta_{3,1}}{\partial y^3} + 3 \frac{\partial^3 \theta_{3,1}}{\partial y^2 \partial z} + 3 \frac{\partial^3 \theta_{3,1}}{\partial y \partial z^2} + \frac{\partial^3 \theta_{3,1}}{\partial z^3} \right](x, y, z),
$$

where $\theta_{1,1}$, $\theta_{2,1}$, $\theta_{3,1}$ are the mixing angles.

(C1)

and

$$
D_B(x, y, z) = \left[ 2\theta_{1,1} - \frac{\partial \theta_{2,1}}{\partial x} \right](x, y, z, u),
$$

$$
D_{BL}(x, y, z, u) = \frac{\partial \theta_{2,1}}{\partial y} + \frac{\partial \theta_{2,1}}{\partial z} + \frac{\partial \theta_{2,1}}{\partial u},
$$

$$
D_1(x, y, z, u) = \left[ \frac{\partial \theta_{1,1}}{\partial y} + \frac{\partial \theta_{1,1}}{\partial z} + \frac{\partial \theta_{1,1}}{\partial u} \right](x, y, z, u),
$$

$$
D_2(x, y, z, u) = \left[ \frac{\partial^2 \theta_{2,1}}{\partial y^2} + 2 \frac{\partial^2 \theta_{2,1}}{\partial y \partial z} + \frac{\partial^2 \theta_{2,1}}{\partial z^2} + 2 \frac{\partial^2 \theta_{2,1}}{\partial y \partial u} + \frac{\partial^2 \theta_{2,1}}{\partial z \partial u} \right](x, y, z, u),
$$

$$
D_3(x, y, z, u) = \left[ \frac{\partial^3 \theta_{3,1}}{\partial y^3} + 3 \frac{\partial^3 \theta_{3,1}}{\partial y^2 \partial z} + 3 \frac{\partial^3 \theta_{3,1}}{\partial y \partial z^2} + 3 \frac{\partial^3 \theta_{3,1}}{\partial y^2 \partial u} + 6 \frac{\partial^3 \theta_{3,1}}{\partial y \partial z \partial u} + 3 \frac{\partial^3 \theta_{3,1}}{\partial z^3} \right](x, y, z, u).
$$

(C2)
with
\[ g_{m,n}(x_1, x_2, \cdots, x_N) = \sum_{i=1}^{N} \frac{x_i^m \ln^n x_i}{\prod_{j \neq i} (x_i - x_j)}. \] (C3)

[1] G. Buchalla, A. J. Buras, Nucl. Phys. B400(1993)225.
[2] M. Misiak, Nucl. Phys. B393(1993)23[Erratum-ibid. B439(1995)461].
[3] A. J. Buras, M. Münz, Phys. Rev. D52(1995)186.
[4] G. Buchalla, A. J. Buras, M. E. Lautenbacher, Rev. Mod. Phys. 68(1996)1125.
[5] N. G. Deshpande, J. Trampetic, K. Panose, Phys. Rev. D39(1989)1461.
[6] C. S. Lim, T. Morozumi, A. I. Sanda, Phys. Lett. B218(1989)343.
[7] P. J. O’Donnell, H.K.K. Tung, Phys. Rev. D43(1991)2067.
[8] A. Buras, M. Misiak and J. Urban, Nucl. Phys. B586(2000)397;
[9] C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B574(2000)291; H. H. Asatrian, H. M. Asatryan, C. Greub and M. Walker, Phys. Lett. B507(2001)162, Phys. Rev. D65(2002)074004; C. Bother, P. Gambino, M. Gorbahn and U. Haisch, JHEP0404(2004)071; A. Ghincuulov, T. Hurth, G. Isidori and Y. Yao, Nucl. Phys. B685(2004)351; Z. Ligeti, F. J. Tackmann, Phys. Lett. B653(2007)404; C. Greub, V. Pilipp and C. Schupbach, JHEP0812(2008)040.
[10] T. Hurth, G. Isidori, J. F. Kamenik and F. Mescia, Nucl. Phys. B808(2009)326.
[11] T. Huber, E. Lunghi, M. Misiak and D. Wyler, Nucl. Phys. B740(2006)105.
[12] B. Aubert et al., BABAR collaboration, Phys. Rev. Lett.93(2004)081802; J. Lees et al., BABAR collaboration, Phys. Rev. D86(2012)032012.
[13] M. Iwasaki et al., BELLE collaboration, Phys. Rev. D72(2005)092005; J. Wei et al., BELLE collaboration, Phys. Rev. Lett.103(2009)171801.
[14] D. Asner et al., Heavy Flavor Average Group collaboration, Averages of B-hadron, c-hadron and τ-lepton properties, arXiv:1010.1589.
[15] Y. Sato et al., BELLE collaboration, Phys. Rev. D93(2016)032008; ibid.93(2016)059901.
[16] S. Fukae, C. S. Kim, T. Morozumi and T. Yoshikawa, Phys. Rev. D59(1999)074013; A. Ali, E. Lunghi, C. Greub, and G. Hiller, Phys. Rev. D66(2002)034002.
[17] A. Ali, G. Hiller, Eur. Phys. J. C8(1999)619.

[18] A. Soni, A.K. Alok, A. Giri, R. Mohanta, S. Nandi, Phys. Rev. D82(2010)033009; A. K. Alok, A. Dighe and S. Ray, Phys. Rev. D79(2009)034017.

[19] J. Lees et al., BABAR collaboration, Phys. Rev. Lett.112(2014)211802.

[20] E. Gabrielli et al., Phys. Lett. B394(1996)80.

[21] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B527(1998)21.

[22] P. Ciafaloni, A. Romanino and A. Strumia, Nucl. Phys. B524(1998)361.

[23] F.M. Borzumati, C. Greub, Phys. Rev. D58(1998)074004.

[24] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B353(1991)591.

[25] R. Barbieri and G. F. Giudice, Phys. Lett. B309(1993)86.

[26] F. Borzumati and C. Greub, T. Hurth and D. Wyler, Phys. Rev. D62(2000)075005.

[27] M. Causse and J. Orloff, Eur. Phys. J. C23(2002)749.

[28] S. Prelovsek and D. Wyler, Phys. Lett. B500(2001)304.

[29] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B534(1998)3.

[30] S. Bertolini and J. Matias, Phys. Rev. D57(1998)4197.

[31] W. N. Cottingham, H. Mehrban and I. B. Whittingham, Phys. Rev. D60(1999)114029.

[32] G. Barenboim and M. Raidal, Phys. Rev. D55(1997)105.

[33] JoAnne L. Hewett, J. D. Wells, Phys. Rev. D55(1997)5549.

[34] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D61(2000)074024.

[35] A. Masiero and L. Silvestrini, *Honolulu 1997, B physics and CP violation*, p.172

[hep-ph/9709244]; *Erice 1997, Highlights of subnuclear physics*, p.172

[hep-ph/9711401].

[36] M. Ciuchini et al., JHEP9810(1998)008.

[37] R. Contion, I. Scimemi, Eur. Phys. J. C10(1999)347.

[38] F. Krauss, G. Soff, Nucl. Phys. B633(2002)237.

[39] T.-F. Feng, X.-Q. Li, W.-G. Ma and F. Zhang, Phys. Rev. D63(2001)015013.

[40] CMS Collaboration, Phys. Lett. B716(2012)30.

[41] ATLAS Collaboration, Phys. Lett. B716(2012)1.

[42] M. Aaboud et al., ATLAS Collaboration, Phys. Rev. D94(2016)052009.

[43] B. Adeva et al., LHCb Collaboration, *Roadmap for selected key measurements of LHCb*,
[44] T. Aushev et al., *Physics at Super B Factory*, arXiv:1002.5012.

[45] B. O’Leary et al., *SuperB Collaboration, SuperB Progress Reports*, arXiv:1008.1541.

[46] R. Barbier et al., Phys. Rep. 420(2005)1; C.-H. Chang, T.-F. Feng, Eur. Phys. J. C12(2000)137.

[47] P. Fileviez Perez and S. Spinner, Phys. Lett. B673(2009)251.

[48] V. Barger, P. Fileviez Perez, and S. Spinner, Phys. Rev. Lett.102(2009)181802.

[49] P. Fileviez Perez and S. Spinner, Phys. Rev. D80(2009)015004.

[50] P. Fileviez Perez and S. Spinner, JHEP1204(2012)118.

[51] V. Barger, P. Fileviez Perez and S. Spinner, Phys. Lett. B696(2011)509.

[52] D. K. Ghosh, G. Senjanovic, Y. Zhang, Phys. Lett. B698(2011)420.

[53] C.-H. Chang, T.-F. Feng, Y.-L. Yan, H.-B. Zhang, S.-M. Zhao, Phys. Rev. D90(2014)035013.

[54] J. Hamann, S. Hannestad, G.G. Raffelt, I. Tamborra, and Y. Y. Y. Wong, Phys. Rev. Lett.105(2010)181301.

[55] W. Altmannshofer, P. Ball, A. Bharucha, A. J. Buras, D. M. Straub, and M. Wick, JHEP0901(2009)019.

[56] T.-F. Feng, Y.-L. Yan, H.-B. Zhang, and S.-M. Zhao, Phys. Rev. D92(2015)055024.

[57] P. Gambino, M. Gorbahn and U. Haisch, Nucl. Phys. B673(2003)238.

[58] G. Buchalla, A. J. Buras, M. E. Lautenbacher, Rev. Mod. Phys.68(1996)1125; N. Cabibbo, L. Manani, Phys. Lett. B79(1978)109; C. S. Kim, A. D. Martin, Phys. Lett. B225(1989)186.

[59] T. Huber, T. Hurth, and E. Lunghi, Nucl. Phys. B802(2008)40.

[60] K. A. Olive et al.(Particle Data Group), Chin. Phys. C,38(2014)090001.