Study on electro-optic noise in crystalline coatings toward future gravitational wave detectors

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(Dated: October 18, 2022)

Thermal noise in high-reflectivity mirror coatings is a limiting factor in ground-based gravitational wave detectors. Reducing this coating thermal noise improves the sensitivity of detectors and enriches the scientific outcome of observing runs. Crystalline gallium arsenide and aluminum-alloyed gallium arsenide (referred to as AlGaAs) coatings are promising coating candidates for future upgrades of gravitational wave detectors because of their low coating thermal noise. However, AlGaAs-based crystalline coatings may be susceptible to an electro-optic noise induced by fluctuations in an electric field. We investigated the electro-optic effect in an AlGaAs coating by using a Fabry-Perot cavity, and concluded that the noise level is well below the sensitivity of current and planned gravitational-wave detectors.

I. INTRODUCTION

Direct detection of gravitational waves (GWs) by ground-based laser interferometric gravitational wave detectors (GWDs) has provided unique insight into the Universe [1–3]. In the current laser interferometric GWDs, km-scale Fabry-Perot arm cavities are used which employ test mass mirrors coated with high-reflectivity amorphous coatings [4, 5].

The sensitivity of current GWD such as advanced LIGO (aLIGO) is partially limited by thermal noise arising from amorphous silica and titania-doped tantala coatings at their most sensitive frequency band [6, 7]. Future GWDs are planned to employ low thermal noise coatings so that one can explore further into the Universe with improved sensitivity [8–11]. Therefore, development of low thermal noise mirror coatings plays an important role in the development of future GWDs.

Crystalline gallium arsenide (GaAs) and aluminum-alloyed gallium arsenide (AlxGa1−xAs) coatings (referred to as AlGaAs coatings), which have demonstrated low thermal noise, are one of the coating candidates for future GWDs [12, 13]. In addition to exhibiting low elastic losses, optical absorption and scatter in AlGaAs are also low [14, 15]. Therefore, AlGaAs coatings have a potential to improve the performance of GWDs, resulting in fruitful scientific outcomes. There is a coordinated research effort to realize AlGaAs coating mirrors in future upgraded GWDs [16–18].

While crystalline AlGaAs coatings can reduce thermal noise, they may also be susceptible to coupling from fluctuations in the electric field. Refractive indices of AlGaAs coatings vary in proportion to the electric field via the electro-optic (EO) effect [19, 20]. Fluctuations in the electric field couple to the cavity length fluctuations through the change in refractive indices of coatings, and can show up as noise in a GWD [21, 22].

In order to investigate the impact of the noise induced by the EO effect in AlGaAs coatings, we have developed an experimental setup using a Fabry-Perot cavity. In this study, we focused on the coupling between the electric field normal to the mirror surface and the cavity length. From this experiment, we estimated the noise level of the EO effect, which was well below the strain sensitivity of current and future proposed GWDs. We conclude that the EO noise in AlGaAs coating will not be a limiting noise source in these systems.

II. THEORY OF ELECTRO-OPTIC EFFECT

When an electric field is applied to certain materials, the refractive indices vary depending on this field. This effect is called the electro-optic (EO) effect. In this section, we briefly review the theory of the EO effect. More details can be seen in the references [19, 20].

Refractive indices of a crystal can be expressed in terms of its index ellipsoid as

\[ \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1, \]

where \( x, y, \) and \( z \) represent the coordinate axes, with the \( z \)-axis along the [100] axis as shown in Fig. 1. And \( n_x, n_y, \) and \( n_z \) are the three principal refractive indices with the crystallographic axes as the optical axes [20].

For the case of zincblende crystals such as GaAs and AlGaAs, these refractive indices are \( n_x = n_y = n_z = n_0 \).

When the electric field is applied to the zincblende crystal, the index ellipsoid becomes \[ \frac{x^2}{n_0^2} + \frac{y^2}{n_0^2} + \frac{z^2}{n_0^2} + 2r_{41}(Exyz + Eyzz + Ezxy) = 1, \]

where \( r_{41} \) is the electro-optic coefficient.

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where \( r_{41} \) represents the electro-optic coefficient. If the electric field is applied along the \( z \) axis, i.e., \( E_x = E_y = 0 \), Eq. (2) becomes
\[
\frac{x'^2}{n_0^2} + \frac{y'^2}{n_0^2} + \frac{z'^2}{n_0^2} + 2r_{41}E_zxy = 1. \tag{3}
\]

We define the new principal axes, \( x' \), \( y' \), and \( z' \), when the electric field is applied as
\[
\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & \sqrt{2} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{4}
\]

By using the new coordinate, Eq. (3) can be rewritten as
\[
\left( \frac{1}{n_0^2} - r_{41}E_z \right) x'^2 + \left( \frac{1}{n_0^2} + r_{41}E_z \right) y'^2 + \frac{z'^2}{n_0^2} = 1. \tag{5}
\]

Therefore, refractive indices of new principal axes, \( n_{x'} \) and \( n_{y'} \), become
\[
n_{x'} = \left[ \frac{1}{n_0^2} (1 - n_0^2 r_{41}E_z) \right]^{-1/2}, \tag{6}
\]
\[
n_{y'} = \left[ \frac{1}{n_0^2} (1 + n_0^2 r_{41}E_z) \right]^{-1/2}. \tag{7}
\]

Assuming that \( n_0^2 r_{41}E \ll 1 \), these can be rewritten as
\[
n_{x'} = n_0 + \frac{1}{2} n_0^2 r_{41}E_z, \tag{8}
\]
\[
n_{y'} = n_0 - \frac{1}{2} n_0^2 r_{41}E_z. \tag{9}
\]

Thus, an electric field changes the refractive indices of zincblende crystals such as GaAs and AlGaAs, hence AlGaAs coatings. When the polarization of beams are aligned to new principal axes, \( x' \) or \( y' \), of AlGaAs coatings, optical path lengths in the coatings can be perturbed by the EO effect, causing perturbations in the phase of the reflected beam. If the polarization is not aligned to \( x' \) or \( y' \) axes, the EO effect introduces birefringence.

It should be noted that \( x' \) and \( y' \) axes are 45 degree rotated with respect to the positive \( z \)-axis as shown in Fig. 1. For the case of GaAs and AlGaAs, \( x \) and \( y \) axes correspond to [010] and [001] directions, respectively. Similarly, \( x' \) and \( y' \) axes are along the [011] and [011] directions. Therefore, the changes of refractive indices due to the normal electric field are induced in principal axes of [011] and [011] directions.

Crystalline AlGaAs coatings may be susceptible not only to the EO effect, but also to the piezoelectric effect. However, this effect does not directly couple to the cavity length fluctuations when the electric field is normal to the surface. In this study, we only consider the EO effect that is much more dominant coupling source than the piezoelectric effect.

\[ E_z \] represents the electric field along to \( z \) axis.

### III. EXPERIMENT

#### A. Setup

In order to experimentally investigate the EO effect in AlGaAs coatings, we developed an optical setup using a Fabry-Perot cavity. Fig. 2 shows the schematic of the experimental setup. The Fabry-Perot cavity is composed of two high-reflectivity mirrors — an amorphous coating front mirror and AlGaAs coating end mirror. The AlGaAs coating is composed of 35.5 periods (71 layers) of alternating GaAs and Al_{0.92}Ga_{0.08}As, that have been
where $\Delta$ and $\nu$ are the cavity length fluctuation and the laser frequency. When the laser is locked to the cavity, fluctuations of the laser frequency, $\Delta \nu$ satisfies

$$\frac{\Delta \nu}{\nu} = - \frac{\Delta L}{L}, \quad (10)$$

where $\Delta L$ is the cavity length fluctuations, and $\nu$ is the laser frequency. The phase perturbation in AlGaAs coatings induced by the electric field is imprinted onto the cavity displacement, hence the PDH error signal. By probing the PDH feedback signal, the displacement due to the EO effect can be measured.

The input beam is linearly polarized, and its polarization can be aligned to the crystal axes of AlGaAs coatings by rotating a $\lambda/2$ plate in front of the Fabry-Perot cavity. The AlGaAs sample mirror is installed as shown in Fig. 3.

FIG. 3. Front view of actual mirror mount for AlGaAs mirror without electrodes. The AlGaAs mirror is clamped by a nylon screw with moderate torque. The green arrows indicate the fast or slow axis where the refractive index is disturbed by the EO effect. The visible defects near the edges of the coating are due to excessive handling and are not typical of AlGaAs coatings. These defects do not impact the EO effect nor any results of this study.

B. Axis identification

As described in the previous section, AlGaAs coatings have the fast and slow axes i.e., [011] and [011] orientations, whose refractive indices are perturbed by the EO effect. Prior to the measurements of the EO effect, we identified the fast and slow axes of the AlGaAs coating. In order to determine the fast and slow axes, we used the higher-reflectivity mirror as the input mirror instead of the one used for the EO measurement. With this configuration, the finesse of the cavity increased to about $10^3$.

In our case, the split frequency of these two eigenmodes was about 500 kHz. However, as described in the next section, two orthogonal polarization eigenmodes separated by about 500 kHz were observed when we replaced the input mirror to the one used for the EO measurement. With this configuration, the finesse of the cavity increased to about $4.5 \times 10^3$, and the FWHM line-width was about 300 kHz.

FIG. 4 shows the response of the reflected beam power when the laser frequency is scanned. The laser frequency was swept by actuating the laser PZT with a triangle wave at 100 Hz. Then we adjusted the $\lambda/2$ plate to maximize or minimize the amount of the split peak. When the beam polarization was aligned to the fast or slow axis, only single eigenmode was observed as shown by the blue curve. Thus, we determined the angle of $\lambda/2$ plate which can align the input beam polarization to the fast or slow axis. By tilting the $\lambda/2$ plate, the distinct split peak appeared as indicated by the orange line. In our case, the split frequency of these two eigenmodes was about 500 kHz.

The AlGaAs coated mirror and two electrodes are housed in the same mirror mount made of machinable glass, MACOR. Each electrode has a hole with 3 mm diameter to pass the beam through. The distances between the mirror surface and front electrode and back electrode are 0.39 mm and 0.20 mm, respectively. Source voltage is amplified to the front electrode by a HVA up to 2 kV peak to peak. On the other hand, the back electrode is grounded, which introduces an electric field normal to the AlGaAs sample mirror surface.

| Symbol | Description                  | Value  |
|--------|------------------------------|--------|
| $\lambda$ | Laser wavelength | 1064 nm |
| $L$     | Cavity length               | 0.105 m |
| $x$     | Aluminum alloying fraction  | 0.92   |
| $d_H$   | Thickness of GaAs           | 76.43 nm |
| $n_H$   | Refractive index of GaAs    | 3.48   |
| $d_L$   | Thickness of Al$_{0.92}$Ga$_{0.08}$As | 89.35 nm |
| $n_L$   | Refractive index of Al$_{0.92}$Ga$_{0.08}$As | 2.98   |

TABLE I. Parameters of experimental setup.
FIG. 4. Response of the reflected beam power when the laser frequency is scanned. As long as the input beam polarization is aligned to the fast or slow axis, the cavity shows single eigenmode as shown in blue curve. On the other hand, when the polarization is misaligned from the fast or slow axis, two separated eigenmodes are observed due to the birefringence in the AlGaAs coating (orange curve).

After identifying the fast and slow axes, we switched the input mirror to what we originally used. The reason why we employed the lower-reflectivity input mirror is because the lock to the cavity was more stable and the cavity-pole was much higher than the frequency region where we measured the EO effect. We then tuned the $\lambda/2$ plate so that the laser polarization was aligned to the fast or slow axis where the EO effect can be observed. As the birefringence in amorphous coatings are so small that the impact of replacing the input mirror is negligible [28].

C. Measurement scheme

FIG. 5. Block diagram of measurement scheme. Transfer function from source signal $V_{in}$ to PDH feedback signal $V_{out}$ is measured by using a SR785. $v_{laser}$, $v_S$, and $v_F$ denote the noises of the laser, RFPD, and FSS, respectively.

Voltage applied to the electrode is converted into an electric field which penetrates the AlGaAs sample mirror. This conversion efficiency, $E [(V/m)/V]$, is computed by...
an effective 3D static solution based on the geometry of the optics. Fig. 7 shows the computed electric field when the unit voltage is applied to the front electrode i.e., the conversion efficiency, $E [(V/m)/V]$. The electric field close to the mirror center where the beam hits is 42 V/m. In our setup, the beam spot size on the AlGaAs mirror is about 100 µm, and the electric field within the beam spot on the AlGaAs mirror can be treated as uniform. Therefore, we apply the conversion efficiency as $E = 42 (V/m)/V$ and assume it is constant within the frequency region of interest.

3. **PZT response**

The internal PZT of the NPRO laser is used to actuate the laser frequency. Its actuation efficiency is measured by scanning the laser frequency by a triangle wave. Generally, the NPRO’s laser PZT response has frequency dependence. However, we cannot drive enough voltage to scan the laser frequency above a few kHz due to the low-pass filter of the HVA connected to the laser PZT. On the other hand, the laser PZT response can be regarded as constant between $1 - 100$ kHz [29]. Therefore, we measured the actuation efficiency with 1 kHz triangle wave and assume that observed laser PZT efficiency is flat between $1 - 100$ kHz.

Fig. 8 shows the response of the PDH error signal scanned by 1 kHz triangle wave. We calculated the actuation efficiency by fitting the error signal. From the fitting result, the PZT efficiency is estimated as 1.7 MHz/V.
IV. RESULTS

![Calibrated transfer functions](image)

**FIG. 9.** Calibrated transfer functions. The black dashed line shows the fitted result between 20 – 40 kHz.

From Eq. (14) and the obtained calibration data, one can evaluate the coupling between the electric field and cavity length, \( C \). Fig. 9 shows the calibrated results of the coupling. Here we used Eq. (10) to convert the unit from \([\text{Hz}/(\text{V/m})]\) to \([\text{m}/(\text{V/m})]\). Measured coupling levels at both axes are almost the same at the order of \(10^{-16} \text{ m}/(\text{V/m})\).

The coupling, \( C \), can be decomposed to mechanical coupling, \( C_m \), and the EO effect, \( C_{\text{EO}} \), as \( C = C_m + C_{\text{EO}} \). As described later, the signs of phase perturbation due to the EO effect are opposite between fast and slow axes. This can be expressed as

\[
C_{\text{EO,slow}} = |C_{\text{EO}}| e^{i\psi},
\]

\[
C_{\text{EO,fast}} = -|C_{\text{EO}}| e^{i\psi} \left( = |C_{\text{EO}}| e^{i(\psi+\pi)} \right),
\]

where \( \psi \) is the phase offset. Assuming that the mechanical coupling is common to both fast and slow axes at this frequency region, differential between these two transfer functions becomes

\[
\text{Diff.} = C_{\text{slow}} - C_{\text{fast}} = (C_m, \text{slow} + C_{\text{EO,slow}}) - (C_m, \text{fast} + C_{\text{EO,fast}}) = 2|C_{\text{EO}}| e^{i\psi}.
\]

Therefore, the magnitude of differential between the TFs of fast and slow axes ideally becomes twice the magnitude of the EO effect in AlGaAs coatings.

The green dashed line shown if Fig. 9 is the differential between transfer functions of fast and slow axes, \( 2|C_{\text{EO}}| e^{i\psi} \). Mechanical couplings from the mirror mount and resonances of mirror itself disturb the cavity length below \( \sim 10 \) kHz and around \( \sim 50 – 80 \) kHz. Above \( \sim 40 \) kHz, the differential shows the frequency dependence. One possibility of this behavior is that the frequency dispersion of the electro-optic coefficients of GaAs and AlGaAs [30]. Further studies may be needed to fully understand the behavior at those higher frequency region. On the other hand, for the case of current terrestrial GWDs, the important frequency region is between \( \sim 10 \) Hz and several kHz. As shown in the previous work, the electro-optic coefficient tend to show the flat response below a few tens of kHz [30]. We focus on the frequency region 20 – 40 kHz where the impacts of mechanical couplings can be considered small and the differential has flat response, and assume that the EO effect in AlGaAs coatings is frequency independent below 40 kHz. From the above assumptions, we obtain \( 2|C_{\text{EO}}| = 2.2 \times 10^{-17} \text{ m}/(\text{V/m}) \) by fitting the result. Therefore, the coupling level of the EO effect is estimated as \( |C_{\text{EO}}| = 1.1 \times 10^{-17} \text{ m}/(\text{V/m}) \).

V. DISCUSSIONS

A. Comparison to theoretical estimation

The level of EO effect can be numerically computed by using a transfer matrix calculation of the coating multi-layer. The perturbation of the reflected field induced by \( k \)-th coating layer can be described as [31, 32]

\[
\frac{\partial \phi_k}{\partial \phi_k} = \Im \left( \frac{1}{M_{21}} \frac{\partial M_{21}}{\partial \phi_k} - \frac{1}{M_{22}} \frac{\partial M_{22}}{\partial \phi_k} \right),
\]

where \( M_{ij} \) are elements of the transfer matrix of coatings, \( M \), and \( \Im \) denotes the imaginary part. The transfer matrix of the total coating can be given by

\[
M = Q_N D_N \cdots Q_k D_k \cdots Q_1 D_1 Q_0,
\]

where \( Q_0 \) is the transition between vacuum and 1st layer, and \( Q_k \) is the transition matrix from \( k \)-th layer to \((k+1)\)-th layer defined as

\[
Q_k = \frac{1}{2n_{k+1}} \left( n_{k+1} + n_k \right) \left( n_{k+1} n_k - n_k \right).
\]
$D_k$ is the propagator through the $k$-th coating layer expressed as
\[
D_k = \begin{pmatrix} e^{-i\phi_k/2} & 0 \\ 0 & e^{i\phi_k/2} \end{pmatrix},
\] (21)
where $\phi_k = 4\pi n_k d_k/\lambda$ is round trip phase change. From Eqs. (19) - (21), partial derivative of transfer matrix can be calculated as
\[
\frac{\partial M}{\partial \phi_k} = Q_N D_N \cdots Q_k D_k \begin{pmatrix} -i/2 & 0 \\ 0 & i/2 \end{pmatrix} Q_{k-1} D_{k-1} \cdots Q_1 D_1 Q_0.
\] (22)
From the definition of round trip phase change, $\phi$, $\partial \phi_k/\partial E$ becomes
\[
\frac{\partial \phi_k}{\partial E} = \frac{4\pi n_k}{\lambda} \frac{\partial n_k}{\partial E} = \pm \frac{2\pi}{\lambda} n^3_k d_k r_{41,k},
\] (23)
where the signs depend on the AlGaAs axes. By using the chain rule, the phase perturbation induced by the electro-optic effect can be expressed as
\[
\frac{\partial \phi_c}{\partial E} = \frac{\partial \phi_c}{\partial \phi_k} \frac{\partial \phi_k}{\partial E}.
\] (24)
Here we assume the EO coefficients of GaAs and Al$_x$Ga$_{1-x}$As as $r_{41,GaAs} = -1.33 \times 10^{-12} \text{ m/V}$, and $r_{41,AlGaAs} = -(1.33 - 0.45z) \times 10^{-12} \text{ m/V}$, respectively \[33, 34\]. As a result, one can compute the phase perturbation induced by the electro-optic effect as
\[
\left| \frac{\partial \phi_c}{\partial E} \right| = \left| \frac{\partial \phi_c}{\partial \phi_k} \frac{\partial \phi_k}{\partial E} \right| = 3.9 \times 10^{-11} \text{ rad/(V/m)}. \tag{25}
\]
This phase perturbation can be converted to the Fabry-Perot cavity displacement, $\partial L/\partial E$. Round trip phase of a Fabry-Perot cavity, $\phi$, satisfies the relationship as
\[
\phi = \frac{2 L \omega}{c} = \frac{4\pi L}{\lambda},
\] (26)
where $L$, $\omega$, $c$, and $\lambda$ are the cavity length, angular frequency, the speed of light, and the wavelength of laser, respectively. From Eq. (26), one can obtain
\[
\frac{\partial \phi}{\partial E} = \frac{4 \pi \partial L}{\lambda \partial E}.
\] (27)
Consequently, the coupling of EO effect to cavity length can be calculated as
\[
\left| \frac{\partial L}{\partial E} \right| = \frac{\lambda}{4\pi} \left| \frac{\partial \phi_c}{\partial E} \right| = 3.3 \times 10^{-18} \text{ m/(V/m)}. \tag{28}
\]
This value is about one-third of the measured value.

**B. Implications for gravitational wave detectors**

We evaluate the impacts of noise induced by the EO effect on future GWDs. In GWDs such as aLIGO, horizontally polarized (P-polarized) beam is employed for laser interferometry \[35\]. The impacts of the EO effect on GWDs depend on the alignment between the beam polarization and AlGaAs axes.

Firstly, we consider the case that the polarization of the beam is aligned to the AlGaAs \[110\] or \[110\] axes ($x'$ or $y'$) where the reflected optical phase is perturbed by the EO effect. The measured fluctuations in the electric field next to the test mass in aLIGO is $3 \times 10^{-6} \text{ (V/m)/\sqrt{Hz} at 100 Hz}$ \[36\]. Assuming that the fluctuations in the electric fields next to each of the four test masses are the same level, and uncorrelated with each other, the strain noise due to the EO effect at 100 Hz can be calculated as
\[
\sqrt{4 \times 1.1 \times 10^{-18} \text{ m/(V/m) x 3 \times 10^{-6} (V/m)/\sqrt{Hz}}} = 1.6 \times 10^{-26} \text{ 1/\sqrt{Hz}}.
\] (29)
Here we assumed that the EO effect has flat response and the arm cavity length is 4 km. The target sensitivity of A+, future upgrade plan of aLIGO, is about $2 \times 10^{-24} \text{ 1/\sqrt{Hz} at 100 Hz}$ \[37\]. Therefore, the noise level of EO effect is about two orders of magnitude smaller than the sensitivity of A+. As long as fluctuations in the electric field are kept below $\sim 2 \times 10^{-5} \text{ (V/m)/\sqrt{Hz}}$ at 100 Hz, the noise level of the EO effect is below $10^{-25} \text{ 1/\sqrt{Hz}}$, and will not affect the sensitivity of GWDs.

![FIG. 10. Schematic of the electric field of horizontally polarized beam, $\vec{E}$, and tilted AlGaAs axes.](image)

Secondly, we consider the case when the $x'$ and $y'$ axes are tilted $\theta$ degrees from the beam polarization as shown in Fig. 10. By defining the field of the beam as $(E_0 \ 0)^T$, its projection onto $[011]$ and $[111]$ axes can be expressed as
\[
\begin{pmatrix} E_{[011]} \\ E_{[111]} \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} E_0 \\ 0 \end{pmatrix} = E_0 \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}.
\] (30)
We denote the optical phase perturbation induced by the EO effect as $\phi_{EO}$. Then the field perturbed by the EO
effect becomes
\[
\left( \frac{\tilde{E}_{(011)}}{E_{(011)}} \right) = E_0 \begin{pmatrix} e^{i\phi_{EO}} & 0 \\ 0 & e^{-i\phi_{EO}} \end{pmatrix} \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}.
\] (31)

By converting the coordinates from AlGaAs axes to beam polarization axes, one can get
\[
\left( \frac{E_P}{E_S} \right) = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \left( \frac{\tilde{E}_{(011)}}{E_{(011)}} \right),
\]
\[
= E_0 \begin{pmatrix} e^{i\phi_{EO}} \cos^2 \theta + e^{-i\phi_{EO}} \sin^2 \theta \\ -e^{i\phi_{EO}} \cos \theta \sin \theta + e^{-i\phi_{EO}} \cos \theta \sin \theta \end{pmatrix}.
\] (32)

Assuming $|\phi_{EO}| \ll 1$, Eq. (32) can be approximated as
\[
\left( \frac{E_P}{E_S} \right) \approx E_0 \begin{pmatrix} 1 + i\phi_{EO}(\cos^2 \theta - \sin^2 \theta) \\ -2i\phi_{EO} \cos \theta \sin \theta \end{pmatrix}.
\] (33)

When the polarization of the beam is aligned to the [100] axis, $\theta = 45$ deg, Eq. (33) becomes
\[
\left( \frac{E_P}{E_S} \right) \approx E_0 \begin{pmatrix} 1 \\ -i\phi_{EO} \end{pmatrix}.
\] (34)

Therefore, the EO effect in AlGaAs coatings induces birefringence, and a P-polarized beam is generated by this effect, leading to elliptical polarization. However, the amplitude of the P-polarized beam converted from S-polarization is the order of $|\phi_{EO}| \sim 10^{-16}$, and it can be negligible. Moreover, the reflected phase of the S-polarized beam is not disturbed by the EO effect. As a result, in ideal cases, the impacts of the EO effect can be mitigated when the polarization is aligned to the [001] family of axes. However, those axes can show the static birefringence in AlGaAs coatings. Our study helps pave a path for utilizing AlGaAs mirror coatings in future upgraded GWDs. Further studies will lead to the large-area substrate transferred crystalline test mass coatings.

\section{VI. CONCLUSION}

Crystalline AlGaAs coatings, with their lower coating thermal noise, have the potential to dramatically improve the sensitivity and detection rate of GWDs and greatly bolster the new field of GW astrophysics. We investigate the noise induced by the EO effect in AlGaAs coating caused by the fluctuations in the electric field. This study yields that the EO effect will not be a limiting noise source in future upgraded GWDs. Our study helps pave a path for utilizing AlGaAs mirror coatings in future upgraded GWDs. Further studies will lead to the large-area substrate transferred crystalline test mass coatings.

\section{ACKNOWLEDGMENTS}

This work was supported with funding from the National Science Foundation grants: PHY-1707863, PHY-1912699, PHY-2011688, and PHY-2011723. A portion of this work was performed in the UCSB Nanofabrication Facility, an open access laboratory. This paper has LIGO Document number LIGO-P2200244.
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