Supersolidity in electron-hole bilayers with a large density imbalance

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Abstract – We consider an electron-hole bilayer in the limit of extreme density imbalance, where a single particle in one layer interacts attractively with a Fermi liquid in the other parallel layer. Using an appropriate variational wave function for the dressed exciton, we provide strong evidence for the existence of the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase in electron-hole bilayers with a large density imbalance. Furthermore, within this unusual limit of FFLO, we find that a dilute gas of minority particles forms excitons that condense into a two-dimensional “supersolid”.

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Introduction. – Pairing phenomena in two-component Fermi systems are a topic of fundamental interest, having relevance to a range of fields spanning superconductivity to QCD. For an attractive interspecies interaction, one can have Bardeen-Cooper-Schrieffer (BCS) pairing in the weak-coupling limit or a Bose-Einstein condensate (BEC) of tightly bound pairs in the strong-coupling limit.

Of particular interest is the case where the densities of the two fermionic species are imbalanced, so that the interspecies pairing is then frustrated. Here, one expects more exotic pairing scenarios such as the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) spatially modulated phase [1,2], where fermions pair at finite centre-of-mass momentum, and both gauge invariance and translational invariance are spontaneously broken. However, an unambiguous observation of the FFLO phase has remained elusive, despite being predicted more than four decades ago. The creation of spin-imbalanced Fermi gases in ultracold atomic gases [3,4] has recently revived the hope of realising the FFLO state, but, thus far, the atomic system has been dominated by phase separation between superfluid and normal phases, with FFLO only occupying a tiny sliver of the predicted phase diagram in the three-dimensional (3D) case [5,6].

On the other hand, electron-hole bilayers, where electrons and holes in a semiconductor are spatially separated into two closely spaced quantum wells, may provide a better route to achieving the FFLO state. Here, electrons and holes can pair to form excitons which can then in principle condense [7,8]. Such bilayers have already been successfully produced by optically pumping coupled GaAs quantum wells (for recent experiments, see [9,10]). More recently, independently contacted layers have been fabricated in GaAs [11,12], where electron and hole layers can be separately loaded by biasing and doping, thus allowing the densities in each layer to be controlled individually and providing a means for generating a density imbalance. The additional advantage of considering these structures over optically pumped coupled quantum wells lies in the extremely long lifetimes, where the exciton decay rate due to tunnelling recombination is essentially negligible and an equilibrium phase diagram can be regarded as accurate.

The reason why electron-hole bilayers provide the ideal conditions for realising the FFLO state is twofold. Firstly, the reduced dimensionality of the bilayer system favours the FFLO phase over the normal phase owing to the enhanced Fermi-surface “nesting” [13], and secondly, the intra-layer Coulomb repulsion acts to suppress any macroscopic phase separation that might compete with the FFLO phase, in contrast to the cold-atom system where...
phase separation dominates. However, recent theoretical work on density-imbalanced electron-hole bilayers [14–17] currently paints a rather uncertain picture of FFLO. While some studies do predict regions of FFLO in the phase diagram, they either rely on an artificial tight-binding Hamiltonian with contact interaction [17] or on approximations that neglect screening [15]. Indeed, as we will show, screening crucially affects the stability of the FFLO phase. Other studies [14,16] neglect the finite momentum of the excitons in the FFLO phase and find that FFLO is often out-competed by other phases where the excess particles simply coexist with the excitonic superfluid. Moreover, none of them consider competing phases such as charge-density wave (CDW) or Wigner crystal phases, which have been shown to be dramatically enhanced in mass-asymmetric electron-hole bilayers [18].

Clearly, further work is needed to ascertain the existence of FFLO.

In this letter, we provide strong evidence for the existence of the FFLO phase in an electron-hole bilayer with a large density imbalance. To address the problem in a more controlled manner, we consider the limit of extreme imbalance where we essentially have a single particle in one layer interacting attractively with a Fermi liquid in the other parallel layer. This allows us to rule out CDW or Wigner crystal phases induced by the presence of the other layer. It also enables us to include interactions between excitons that go beyond mean-field theory. From this analysis, we expose an unusual bosonic limit of FFLO, where a dilute gas of excitons forms a condensate with a 2D spatial modulation, a phase otherwise known as a supersolid. Our strong-coupling version of FFLO should also be of interest to the communities working on strongly correlated superconductors, where one often encounters spatial textures coexisting with superconductivity.

**Model.** – The basic Hamiltonian for a 2D electron-hole bilayer, \( H = H_0 + H_{\text{int}} \), consists of a kinetic part, \( H_0 = \sum_{k_\sigma} \epsilon_{k_\sigma} c_{k_\sigma}^\dagger c_{k_\sigma} \), and an interaction part (\( \Omega \) is the system area):

\[
H_{\text{int}} = \sum_{kkq} g_{q} c_{k_1,1}^\dagger c_{k_2,2}^\dagger c_{k+q,1} c_{k-q,1} + \frac{1}{2\Omega} \sum_{kk'q} U_{q} c_{k_\sigma,1}^\dagger c_{k_{\sigma}',1}^\dagger c_{k_{\sigma} q,1} c_{k_{\sigma} q,1}.
\]

Here, the labels \( \sigma = \{1, 2\} \) denote the different 2D layers, and we approximate the dispersions as quadratic, \( \epsilon_{k_\sigma} = \hbar^2 k^2/2m_\sigma \), which is reasonable for sufficiently small momenta in GaAs quantum wells. The bare Coulomb inter- and intra-layer interactions are, respectively, given by

\[
U_{q} = \frac{2\pi e^2}{\epsilon q}, \quad g_{q} = -U_{q} e^{-qd},
\]

where \( d \) is the bilayer distance, \( e \) is the electron charge, and \( \epsilon \) is the material dielectric constant. By introducing twice the reduced mass \( m = 2(1/m_1 + 1/m_2)^{-1} \), we can define the exciton Bohr radius \( a_0 = \epsilon h^2/m e^2 \) and the exciton Rydberg \( E_0 = e^2/\epsilon a_0 \). We also ignore the spin degrees of freedom and assume the layers are completely spin-polarised by a parallel magnetic field, but we shall return to this point later.

We focus on the problem of a single particle in the 2nd (\( \sigma = 2 \)) layer interacting attractively with a Fermi liquid in the 1st (\( \sigma = 1 \)) layer. This minority particle can either be a hole interacting with the electron layer or an electron interacting with the hole layer — the two cases are simply obtained by inverting the mass ratio \( \alpha \equiv m_2/m_1 \). For a fixed bilayer distance \( d/a_0 \), the only other relevant parameter is the dimensionless density \( r_s \equiv 2/k_Fa_0 \), where \( k_F = 2\sqrt{\pi n_1} \) is the Fermi wave vector of the filled 1st layer with density \( n_1 \). Note that this work focuses on establishing the equilibrium phase diagram of a fully imbalanced electron-hole bilayer, a problem somewhat different from the phenomenology of the X-ray-edge singularity [19], where a dynamical transition is caused by a sudden local perturbation, e.g., the emission or absorption of a photon. Such a dynamical transition would also require a term in the Hamiltonian that transfers electrons between layers, which is clearly absent in (1).

In order to determine the phase diagram of the fully imbalanced electron-hole bilayer, we need to establish the system ground state. To this end, we consider the following variational state for an excitonic quasi-particle (electron-hole pair):

\[
|\Psi(Q)\rangle = \sum_{k>k_F} \phi_{kQ} c_{k,1}^\dagger c_{k,2}^\dagger |FS\rangle,
\]

where \( |FS\rangle \) represents the Fermi sea of 1-particles filled up to wave vector \( k_F \), and we use the notation \( \sum_{k>k_F} \equiv \sum_k \delta_{k,k_F} \). According to its definition, the excitonic wave function \( \phi_{kQ} \) has relative momentum \( k \) and centre-of-mass momentum \( Q \). The spread in relative momentum \( k \) of \( \phi_{kQ} \) is set by the inverse of the size of the exciton bound state. Note that eq. (3) gives a good description of both low- and high-density limits: In the low-density limit, the state (3) coincides with the exact two-body (molecular excitonic) state in the vacuum limit \( (r_s \to \infty) \), while in the high-density limit, where the particles barely interact, it gives the non-interacting “unbound” state:

\[
|\Psi_0\rangle = c_{k_F,2}^\dagger c_{k_F,1}^\dagger |FS\rangle
\]

corresponding to \( \phi_{kQ} = \delta_{k,k_F} \delta_{Q,k_F} \). In this case, the minority particle is not bound to a majority particle and instead occupies a scattering state with well-defined (zero) momentum. We handle the regime between these two limits by considering screened interactions within the Random Phase Approximation (RPA). For the single-minority-particle case, the screened interactions \( U_{q}^{sc} \), \( g_{q}^{sc} \) have the simple expressions

\[
U_{q}^{sc} = \frac{U_{q}}{1 - U_{q} \Pi_1(q)}, \quad g_{q}^{sc} = -U_{q}^{sc} e^{-qd},
\]
where the static polarisation operator $\Pi_1$ for the 1st layer is given by the Lindhard function:

$$\Pi_1(q) = \frac{N_e m_1}{2\pi \hbar^2} \left[ \sqrt{\frac{q^2 - 4E^2}{q}} \theta(q - 2k_F) - 1 \right],$$

with the number of particle flavours $N_e = 1$ for the spin-polarised case. Considering screened interactions within RPA corresponds to effectively “dressing” the particles in our wave function with density fluctuations (i.e., an infinite number of particle-hole pairs). In addition, RPA provides a good approximation in the long-wavelength limit, $q < 2k_F$, and therefore gives a good estimate of the unbinding transition: Here, as we will show later, the wave function $\varphi_{kQ}$ in (3) is strongly peaked at $k = k_F$ and so the momentum transfer is small. More generally, RPA should be reliable for sufficiently large $RPA$ for the $d$-wave pairing coupling $\lambda_{kQ}$, and so the momentum transfer is small. However, for long-range interactions and an interacting majority Fermi sea, this perturbative expansion is bound to fail, as it is never profitable to excite just one particle-hole excitation on top of the non-interacting approximation to the Fermi sea [FS] for both the molecular and polaron wave functions [21,22]. This simple treatment is possible for cold atoms because the majority Fermi sea is non-interacting and the interspecies interaction is short-range. However, for long-range interactions and an interacting majority Fermi sea, this perturbative expansion is bound to fail, as it is never profitable to excite just one particle-hole pair. Indeed, the singular nature of the Coulomb interaction will generate an infinite number of particle-hole excitations at the Fermi surface. Thus, instead of explicitly including a small number of particle-hole excitations in the wave functions, we implicitly include an infinite number of particle-hole excitations by replacing the Coulomb potentials in eq. (7) with screened potentials and using, implicitly, an interacting Fermi sea.

**Phase diagram.** – By minimising the expectation value $\langle \Psi(Q)| (H - E) |\Psi(Q) \rangle$ with respect to the amplitude $\varphi_{kQ}$, we obtain an eigenvalue equation for the exciton energy $E$:

$$E \varphi_{kQ} = \left( \epsilon_{Q-k,2} + \epsilon_{K,1} - \frac{1}{\Omega} \sum_{k' < k_F} U_{k-k'}^s \right) \varphi_{kQ} + \frac{1}{\Omega} \sum_{k' > k_F} g_{k-k'}^s \varphi_{kQ},$$

Similarly to what was done in ultracold polarised Fermi gases [21,22], we compare the energy of the excitonic molecular wave function $|\Psi(Q)\rangle$ with that of the unbound state $|\Psi_0\rangle$, i.e., the exciton is bound if the lowest eigenvalue of (7) is such that $E < \hbar^2 k_F^2 / 2m_1 - \frac{1}{\Omega} \sum_{k' < k_F} U_{k-k'}^s$.

It is useful to note that the eigenvalue equation (7) coincides, in the limit of full imbalance, with the mean-field gap equation employed to describe the BEC-BCS crossover in imbalanced electron-hole bilayers (see, e.g., refs. [14–16]). To see this, we neglect the intra-layer Coulomb repulsion and rewrite eq. (7) in terms of the “gap”, $\Delta_{kQ} = \frac{1}{\Omega} \sum_{k' > k_F} g_{k-k'}^s \varphi_{kQ}$, therefore obtaining

$$\Delta_{kQ} = \frac{\Omega}{\Delta_{kQ}} \sum_{k' > k_F} g_{k-k'}^s \varphi_{kQ}.$$ (8)

This corresponds to the linearised version of the mean-field gap equation of refs. [14–16], which is just the form one would expect in this limit since the gap $\Delta_{kQ}$ becomes macroscopically small as we approach full imbalance. To complete the correspondence between eq. (8) and the linearised mean-field gap equation, we require that the chemical potential of the minority particles be $\mu_2 = -\hbar^2 k_F^2 / 2m_1$, since the chemical potential of the majority particles is fixed to $\mu_1 = \hbar^2 k_F^2 / 2m_1$. Thus, the conditions for a bound state outlined above require that $\mu_2 < 0$, implying that $\mu_2$ gives the exciton binding energy in this limit, which also matches with the mean-field theory.

Here, as for the excitonic wave function $\varphi_{kQ}$, the order parameter $\Delta_{kQ}$ describes a pair with centre-of-mass momentum $Q$. Therefore, in this context, it is natural to identify an exciton with minimum energy at non-zero momentum $Q$ with the FFLO phase in the large-imbalance limit —this will be justified further when we consider the case of a dilute gas of minority particles. We emphasise that in this work, similarly to ref. [15], and contrary to refs. [14,16], where only the limit $Q \to 0$ has been considered, we allow the pair centre-of-mass momentum to be finite and minimise the energy with respect to $Q$.

The phase diagram for the fully imbalanced bilayer system is obtained by converting the eigenvalue equation (7) into a matrix equation and numerically solving for the lowest energy eigenvalue. We fix the electron-hole mass ratio to 4, which is approximately its value in GaAs experiments, so that the relevant mass ratios are $\alpha = 0.25, 4$. In the low-density limit, $r_s \to \infty$, we recover the two-body limit, and thus we expect a bound exciton with $Q = 0$ (which we refer to as SF) for sufficiently large $r_s$, as shown in fig. 1. In the opposite limit, where $r_s$ is small, we see that the screened interactions cause the exciton to eventually unbind and enter the “normal” (N) phase, as expected. However, the crucial point is that in a significant region of the phase diagram at intermediate densities, the system ground state is a bound exciton with finite momentum $Q$, which we label as FFLO, as previously explained. The size of the FFLO region is greatest if the minority particle is an electron ($\alpha = 0.25$) rather than a hole ($\alpha = 4$). Indeed, the FFLO region is generally enhanced (and shifted to larger $r_s$) when the minority particle is lighter, as demonstrated in the bottom panel of fig. 1. The
In the top two panels, the bilayer distance $d/a_0$ varies and the mass ratio $\alpha = m_2/m_1$ is fixed to typical values in GaAs ($\alpha = 0.25, 4$), while the bottom panel has $\alpha$ varying and $d/a_0 = 1$. In all cases, the inter- and intra-layer interactions have been screened using RPA. The superfluid (SF) region corresponds to excitons with centre-of-mass momentum $Q = 0$, while the “FFLO” excitons have their lowest energy when $Q \neq 0$. The normal (N) region is where there are no bound excitons. Refer to fig. 2 for the behaviour of $Q$ within the FFLO region for $d/a_0 = 1$.

reason for this is simple: an exciton with $Q = 0$ requires the minority particle to sit above the Fermi sea, but a small $\alpha$ increases the kinetic energy cost for this, thus favouring the formation of an FFLO exciton, where the minority particle can sit below the Fermi surface.

The FFLO region is also enlarged by increasing the bilayer distance $d$ (fig. 1). For large $d$, large momentum scattering $|k - k'| > 1/d$ is suppressed in $g_{k-k'}$ and this tends to favour FFLO, where the wave function $\phi_{kQ}$ is peaked in the direction of $Q$, over SF. However, larger $d$ also demands larger $r_s$ to achieve FFLO, and so we eventually expect Wigner crystallisation in the 1st layer to destroy FFLO. According to estimates from Quantum Monte Carlo (QMC) calculations [23,24], Wigner crystallization occurs once $r_s \gtrsim 70\alpha/(1 + \alpha)$. Thus, the distance $d$ required to see FFLO sensitively depends on $\alpha$, e.g. for $\alpha = 0.25$, we ideally want $d/a_0 \lesssim 1$.

Finally, we note that both the SF-FFLO and FFLO-N transitions are second order, with the exciton momentum $Q$ varying continuously (fig. 2). Also, we always find that $Q = k_F$ at the FFLO-N transition.

Interestingly enough, we find that the size of the FFLO region actually strongly depends on the screening of the interaction. In order to understand this, we consider the gap-equation form of the eigenvalue equation (8) for the case of unscreened interactions. After converting sums into integrals, and rescaling momenta by $a_0^{-1}$ and energies by $E_0$, we obtain

$$\Delta k Q = \int_{k' > -2\pi} \frac{d^2 k' e^{-d|k-k'|}}{2\pi \pi |k-k'|} \Delta k Q = \delta_{k,k'} \delta_{Q,k-k'}.$$  \hspace{1cm} (9)

Now, at the unbinding transition ($E = 4\alpha/r_s^2(1 + \alpha)$ in rescaled units), we see that that the integral is logarithmically divergent for $Q = k' = k = 2k/r_s$ and so we must take $r_s = 0$ for the equation to be satisfied. This implies that, for the bare Coulomb inter-layer interaction $g_{q_0}$, the exciton with momentum $Q = 2/r_s \equiv k_F$ for $r_s \ll 1$ is always bound, a point which we have also confirmed numerically. In other words, FFLO will extend all the way down to $r_s = 0$ in the phase diagram, which is contrary to what was found in [15]. Note that this is not the case when the interaction is screened, since this removes the singularity at $k' = k$, thus leaving an integrable singularity at $Q = k' = 2k/r_s$.

### Dilute gas of minority particles

To understand the implications of our single-minority-particle phase diagram for a dilute gas of minority particles (and, thus, the density-imbalanced electron-hole bilayer), we must determine the interaction between minority particles. Firstly, in the normal phase, the effective interaction $V^{22}_{q}$ between two unbound minority particles within RPA
satisfies $V_{q}^{22} = U_{q} + g_{q} \Pi_{1} \Psi_{q}^{*} [25]$ and therefore is given by

$$V_{q}^{22} = U_{q} + \frac{g_{q}^{2} \Pi_{1}}{1 - \Pi_{1} U_{q}}. \quad (10)$$

In the long-wavelength limit ($q \rightarrow 0$), this interaction is repulsive and dipolar ($V^{22}(r) \sim 1/r^{2}$), and thus we expect a dilute gas of minority particles (with density $n_{2} \ll 1/d^{2}$ when $d \gg 1$) to form a weakly interacting Fermi liquid. This is similar to what was found in ref. [26] for the minority spins in a strongly polarised 2D electron gas.

The interaction between well-separated excitons is also easily calculated by treating the excitons as static dipoles and summing up the inter- and intra-layer contributions:

$$V_{q}^{ex} = 2g_{q}^{ex} + U_{q}^{ex} + V_{q}^{22} \sim 4\pi d(1 - qd). \quad (11)$$

The exact-exciton interaction in the vacuum is clearly repulsive and dipolar at large distances, and we see from eq. (11) that this remains the case in the presence of a Fermi sea. This suggests that FFLO excitons in the regime of large density imbalance will be thermodynamically stable against phase separation, unlike in the cold-atom case. However, note that our approximation for $V_{q}^{ex}$ is no longer valid when the exciton size becomes comparable to the distance between minority particles, which is the case near the FFLO-N transition. There is also the possibility that the short-range exciton-exciton interactions are attractive, in which case there will be biexciton formation. However, QMC calculations for the two-exciton problem show that biexcitons cannot form when $d \gtrsim 0.25$ for mass ratio $a = 4$ [27]. Indeed, biexciton formation will be even more suppressed in our spin-polarised case due to Pauli exclusion.

Now that we have an estimate for the exciton-exciton interaction, we can determine the structure of the FFLO phase at large density imbalance. For a low density of minority particles, we can treat the excitons as simple bosons and define the mean-field complex order parameter $\psi(r)$, where $|\psi(r)|^{2}$ corresponds to the exciton density. Fluctuations will, of course, prevent true long-range order at finite temperature in 2D, but our excitonic phases can still possess quasi-long-range order.

The effective low-energy thermodynamic potential $F[\psi]$ is similar to that used in weak crystallisation theory [28]:

$$F[\psi] = \int d^{2}r \left[ -\mu |\psi|^{2} + \gamma |(\nabla^{2} + Q_{min}^{2})\psi|^{2} + \frac{\lambda}{2} |\psi|^{4} \right].$$

Here, $\mu$ is the chemical potential for the excitons, where $\mu < 0$ describes an empty 2nd layer, while the term $\gamma > 0$ sets the exciton’s dispersion minimum at momentum $Q_{min}$. The repulsive interactions are taken to be zero-ranged, with $\lambda = V_{0}^{ex} - \lambda_{sm}^{ex}$ —the momentum dependence of $V_{q}^{ex}$ should not affect our solution below provided $Q_{min} \ll 1$, i.e. the typical length scale of the spatial modulations is large. Indeed, this condition is always satisfied within the FFLO regions of fig. 1. For the FFLO phase, we consider solutions of the form $\psi(r) = \sum_{n} a_{n} e^{i q_{n} \cdot r}$, with $|q_{n}| = Q_{min}$. Substituting this into $F[\psi]$ we get

$$F[\psi] = -\mu A + \frac{\lambda}{2} \left( 2A^{2} - \sum_{n} |a_{n}|^{4} + B^{*} B \right) - \sum_{n} |a_{n}|^{2} |a_{n-1}|^{2},$$

where $A = \sum_{n} |a_{n}|^{2}$ and $B = \sum_{n} a_{n} a_{n-1}$. Minimising with respect to the amplitude $a_{n}$ we find that the lowest energy solution ($F[\psi]/\Omega = -\mu B/3\lambda$) requires that $B^{*} a_{1} = |B| a_{-1}^{*}, B^{*} a_{2} = -|B| a_{-2}^{*}, |a_{1}| = |a_{2}| = \sqrt{\mu/6\lambda}$, while $a_{n} = 0$ for $n > 2$. Such a state is thus composed of four $q_{n}, i.e. \pm q_{1}$ and $\pm q_{2}$, so that we have

$$\psi_{0}(r) = e^{i\theta} \sqrt{\frac{2\mu}{3\lambda}} \left[ \cos(q_{1} \cdot r) - i \cos(q_{2} \cdot r + \theta_{12}) \right]. \quad (12)$$

Note that the phases $\theta$, $\theta_{12}$ and the directions $q_{1}$, $q_{2}$ are randomly chosen. Equation (12) corresponds to an exciton condensate with a 2D spatial modulation—a supersolid. Contrast this with the opposite limit of weak polarization in the BCS regime, where the favored state is believed to be a single cosine [2].

Discussion. – The supersolid is expected to enjoy a sizeable region of existence away from Wigner crystallisation, but one possible issue in the large imbalance limit is the formation of three-body states, or trions. These are known to exist for all mass ratios in the limits $d \rightarrow 0$ and $k_{F} \rightarrow 0$ [29]. However, we expect trions to disappear with increasing $d$, like in the bieexciton case, since the intralayer repulsion will eventually dominate over the interlayer attraction. Moreover, we find that the spin-polarised trion is barely bound at $d = 0$ and so we expect trions to exist only for $d \ll 1$ in fig. 1. We speculate that a spin-polarised electron-hole bilayer (as considered in this work) may be better for achieving the FFLO phase than an unpolarised one, since spin polarisation both suppresses trion formation and enhances the inter-layer attraction, due to the reduction in screening in eq. (6).

Thus far, our calculations have been restricted to zero temperature and we have ignored the effects of thermal fluctuations. However, the exciton binding energy $E_{B}$ provides a temperature scale below which the exciton should be robust against thermal fluctuations. Assuming that the minority particles are electrons ($a \approx 0.25$) and using the parameters in GaAs ($a_{0} \approx 7 \text{nm}$ and $E_{0} \approx 17 \text{meV}$), we find that $E_{B}$ for FFLO excitons at $d/a_{0} \approx 0.5$ is of the order of 5 K near the SF-FFLO transition. Thus, FFLO excitons should be experimentally accessible.

In order to access the FFLO phase itself, we require a sufficiently large exciton density, i.e. a sufficiently large minority-particle density $n_{2}$, since we expect the critical temperature to scale with $n_{2}$. Our predictions should be valid at finite $n_{2}$ provided the exciton size is smaller than the spacing between excitons, i.e. we must have
For an ordinary excitonic superfluid in 2D, there will be a Berezinskii-Kosterlitz-Thouless transition [30,31] to the superfluid state, with transition temperature given by

\[ T_{BKT} = \frac{\alpha}{(1 + \alpha)^2} \frac{E_0}{k_B r_s^2} \frac{n_2}{n_1}. \]  

(13)

It is unclear whether or not this is also true for the FFLO phase, since translational as well as gauge invariance has been spontaneously broken. However, on general grounds, we expect the FFLO transition temperature to have the same scaling as \( T_{BKT} \), so we can use it to obtain an estimate. For the typical parameters \( r_s \sim 10 \) and \( n_2/n_1 \sim 0.2 \), we get an FFLO transition temperature of order 100 mK, which is smaller than the exciton binding energy (as expected), but still within reach experimentally.

The FFLO phase we predict can be observed experimentally via light scattering off of the spatial modulations. In addition, if the electrons and holes are allowed to recombine, then a signature of the finite momentum pairing will appear in the angular emission of the photons [33], where we expect peaks in the exciton photoluminescence to occur at large angles with respect to the plane normal, corresponding to \( \pm q_1 \) and \( \pm q_2 \).

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