There’s Nothing “Black” about a Black Hole

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Quantum gravity suggests that the paradox recently put forward by Almheiri et. al. (AMPS) can be resolved if matter does not undergo continuous collapse to a singularity but condenses on the apparent horizon. One can then expect a quasi-static object to form even after the gravitational field has overcome any degeneracy pressure of the matter fields. We consider dust collapse. If the collapse terminates on the apparent horizon, the Misner-Sharp mass function of the dust ball is predicted and we construct static solutions with no tangential pressure that would represent such a compact object. The collapse wave functions indicate that there will be processes by which energy extraction from the center occurs. These leave behind a negative point mass at the center which contributes to the total energy of the system but has no effect on the the energy density of the dust ball. The solutions describe a compact object whose boundary lies outside its Schwarzschild radius and which is hardly distinguishable from a neutron star.

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I. INTRODUCTION

The final fate of gravitational collapse has long been a mystery. Classical collapse models suggest that a star that is massive enough to overcome all degeneracy pressures will undergo collapse beyond the apparent horizon eventually forming a naked or covered singularity of spacetime, depending on the initial conditions. But there is something deeply unsatisfying about this picture since it does not take into account quantum gravity, which is expected to be significant in the final stages of collapse.

If the collapse begins with initial data that lead to the formation of a naked singularity then a semi-classical treatment of the radiation (assuming the validity of effective field theory) from the singularity suggests that the final stages will be catastrophic. It is not known what the final fate of such a collapse is: either the collapsing star will dissipate entirely or a remnant will attempt to form a covered singularity. However, if the initial data are such as to lead to the formation of a covered singularity and an event horizon forms, then Hawking pointed out that the semi-classical theory would yield thermal radiation from the point of view of the observer who remains outside the black hole provided that the freely falling observer detects nothing unusual while crossing the horizon, as required by the equivalence principle.

The semi-classical analysis would seem to suggest that information is lost if the black hole evaporates completely, since what is left is a density matrix and not a wave function. But if the quantum theory is unitary then either (a) the evaporation is not in fact thermal and the Hawking radiation is pure or (b) the thermal evaporation process should, by an as yet unknown mechanism, leave behind a stable remnant that contains all the information that fell into the hole. The second option is difficult to imagine since a relatively small object would be required to possess a huge degeneracy while remaining stable. Moreover, it is ruled out if we assume that quantum gravity is CPT invariant.

This leaves just the first option, that the Hawking radiation is pure. In 1993, Susskind et. al., building on the work of ’t Hooft and Preskill, proposed that the unitarity of the Hawking radiation could be preserved if information is both emitted at the horizon and passes through it, so an observer outside would see it in the Hawking radiation and an observer who falls into the black hole would see it inside but no single observer would be able to confirm both pictures.

Although there is no precise mechanism by which it is can be said to occur, thought experiments that appear to support this picture of “Black Hole Complementarity” rely on three fundamental assumptions, viz., (a) the unitarity of the Hawking radiation, (b) the validity of effective field theory outside a “stretched” horizon and (c) the equivalence principle. Recently, however, Almheiri et. al. (AMPS) have argued that the three assumptions are logically inconsistent and would lead to a violation of the strong subadditivity of the entanglement entropy. To resolve the paradox the authors suggested giving up the third assumption, i.e., the equivalence principle.

But Hawking has proposed an intriguing alternative, suggesting that no event horizon would form in the first place if somehow the collapse did not continue beyond the apparent horizon. Since no event horizon is formed the entire discussion about information loss becomes moot. One is, however, left with the question of
what sort of object is left and how it evolves after the formation of the apparent horizon.

In this paper we will first justify Hawking’s conjecture in an exact quantization of dust collapse [13, 14]. Two kinds of solutions of the Wheeler-DeWitt equation may be given [13]. In one the dust shells coalesce onto the apparent horizon on both sides and the interior, outgoing waves are suppressed by the Boltzmann factor at the Hawking temperature of the collapsing shells. In the other matter moves away from the apparent horizon on both sides, but the exterior outgoing rays are suppressed by the Boltzmann factor at the Hawking temperature. We can think of these exterior, outgoing waves as Hawking radiation. Continued collapse can only be achieved by combining the two and requiring the net flux to vanish at the apparent horizon. However, the AMPS paradox is resolved if we retain the first solution and discard the second, so there is no Hawking radiation but all the shells coalesce onto the apparent horizon [16].

If the dust condenses on the apparent horizon, we expect to end up with a spherically symmetric, quasi-static configuration of finite extension and with a specific mass function as the end state of the collapse. Clearly no classical, static, extended dust configuration exists, but we will argue that strong quantum fluctuations in the interior during the collapse will create the conditions appropriate for a quasi-static configuration. In effect these quantum fluctuations generate a negative mass point source at the center of the star. The mass is then decreasingly negative as we move out from the center and approaches zero at the Schwarzschild radius. However, there are no horizons and the collapsed matter itself extends to twice this radius.

We allow for radial but no tangential pressure. This is in keeping with the mid-superspace quantization that informs our construction [13, 14]. Moreover, while quantum uncertainty may be expected to “push” shells outwards (as is evident from the presence of interior, outgoing waves) giving rise to an effective radial pressure, it is difficult to imagine how non-rotating, collision free dust will develop tangential pressure. With the addition of a constant vacuum energy and including radial pressure, static solutions exist.

In section II we summarize our arguments for terminating the collapse at the horizon. In section III we construct the static, spherically symmetric solutions described above and analyze the solutions. In section IV we estimate the size of the region in which the mass is negative. We conclude and summarize our results in section IV. We take $\hbar = c = 1$ in what follows.

## II. QUANTUM DUST COLLAPSE

Dust collapse in any dimension, with or without a cosmological constant, is described by the LeMaitre-Tolman-Bondi family of solutions [17]. In comoving and synchronous coordinates, $(t, r, \theta, \phi)$, one has

$$ds^2 = d\tau^2 - \frac{R'(\tau, \rho)^2}{1 + f(\rho)} d\rho^2 - R^2(\tau, \rho) d\Omega^2,$$

where the area radius, $R(\tau, \rho)$ obeys the Einstein equation

$$\dot{R}(\tau, \rho) = \sqrt{f(\rho) + \frac{2G \rho}{R(\tau, \rho)} + \frac{1}{3} \Lambda R^2(\tau, \rho)}$$

and the energy density is given by

$$\epsilon(\tau, \rho) = \frac{F'(\rho)}{R^2(\tau, \rho) R(\tau, \rho)}.$$  

$\Lambda$ is the cosmological constant. There are two integration functions, $F(\rho)$ and $f(\rho)$, that are interpreted as the twice the gravitational (Misner-Sharp) mass contained within a shell located at $\rho$ and the total energy contained within the same shell respectively. They are “mass” and “energy” functions of the collapse [13, 15].

By considering the expansion of an outgoing, radial null congruence, $\Theta = \frac{2 R'(\tau, \rho)}{R(t, \rho)} \left[ 1 - \sqrt{\frac{2G F(\rho)}{R(\tau, \rho)} + \frac{1}{3} \Lambda R^2(\tau, \rho)} \right]$, one sees that the condition for trapping is met when

$$\frac{2G F(\rho)}{R(\tau, \rho)} + \frac{1}{3} \Lambda R^2(\tau, \rho) = 1,$$

which condition can be used to determine the time of formation, $\tau_{\text{Ah}}(\rho)$, of the apparent horizon.

The canonical dynamics of the collapsing dust shells is described by embedding the spherically symmetric ADM metric in the LTB spacetime of [18]. After a series of canonical transformations [13, 19, 20], they are described in a phase space consisting of the dust proper time, $\tau(t, r)$, the area radius, $R(t, r)$, the mass density, $\Gamma(r) = F'(r)$, and their conjugate momenta, $P_\tau(t, r)$, and $P_R(t, r)$ respectively by two constraints:

$$\mathcal{H}_\tau = \tau' P_\tau + R' P_R - \Gamma P'_\tau \approx 0$$

$$\mathcal{H} = P^2_\tau + \mathcal{F} P^2_R - \frac{\Gamma^2}{\mathcal{F}} \approx 0,$$

where the prime denotes a derivative with respect to the ADM radial label coordinate, $r$, and

$$\mathcal{F} \overset{\text{def}}{=} 1 - \frac{2G F}{R} - \frac{1}{3} \Lambda R^2.$$
The apparent horizon occurs when $F = 0$. In the absence of a cosmological constant, this says that on the apparent horizon the physical radius of each shell is given by

$$R(\tau, \rho) = 2GF(\rho). \quad (6)$$

Dirac’s quantization of the constraints in (3) yields a formal Wheeler-DeWitt equation whose solution may be given by making the following ansatz [13],

$$\Psi[\tau, R, \Gamma] = \exp \left[ -\frac{i}{2} \int d\tau \Gamma(r) W(\tau(r), R(r), \Gamma(r)) \right],$$

(7)

with a well defined continuum limit ($\sigma \to 0$), where $a_i = 1/\sqrt{1 + f_i}$ is related to the energy function and $\omega_i = \sigma \Gamma_i/2$. Diffeomorphism invariance also requires that both $a_i$ and $b_i$ depend on $r$ via the mass function, i.e., $a_i = a_i(F_i)$ and $b_i = b_i(F_i)$.

These solutions are defined everywhere except at the apparent horizon, where the Wheeler-DeWitt equation has an essential singularity. Thus there are “exterior” wave functions that must be matched to “interior” wave functions across the singular horizon. This can be accomplished by analytically continuing the solutions into the complex $R$ plane, taking $F_i = \epsilon \exp[i\varphi]$, with $\epsilon > 0$, and comparing them at $\varphi = \pi/2$. One then finds two sets of matched solutions, with support everywhere; the first is given by [13],

$$\Psi = \lim_{\sigma \to 0} \prod_i \psi_i(\tau_i, R_i, \Gamma_i) = \lim_{\sigma \to 0} \prod_i e^{i\omega_i b_i} \times \exp \left\{ -i\omega_i \left[ a_i \tau_i \pm \int_{R_i}^{R_i} dR_i \sqrt{1 - a_i^2 F_i} \right] \right\},$$

(8)

and the second by

$$\psi_{i, \text{col}}^{(2)}(\tau_i, R_i, F_i) = \left\{ \begin{array}{ll} e\sqrt{\frac{\omega_i \pm \pi}{2}} \times \exp \left\{ -i\omega_i \left[ a_i \tau_i - \int_{R_i}^{R_i} dR_i \sqrt{1 - a_i^2 F_i} \right] \right\} & \text{if } F_i > 0 \\
e\sqrt{\frac{\omega_i \pm \pi}{2}} \times \exp \left\{ i\omega_i \left[ a_i \tau_i - \int_{R_i}^{R_i} dR_i \sqrt{1 - a_i^2 F_i} \right] \right\} & \text{if } F_i < 0 \end{array} \right. \quad (9)$$

and

$$\psi_{i, \text{col}}^{(1)}(\tau_i, R_i, F_i) = \left\{ \begin{array}{ll} e\sqrt{\frac{\omega_i \pm \pi}{2}} \times \exp \left\{ i\omega_i \left[ a_i \tau_i + \int_{R_i}^{R_i} dR_i \sqrt{1 - a_i^2 F_i} \right] \right\} & \text{if } F_i > 0 \\
e\sqrt{\frac{\omega_i \pm \pi}{2}} \times \exp \left\{ -i\omega_i \left[ a_i \tau_i + \int_{R_i}^{R_i} dR_i \sqrt{1 - a_i^2 F_i} \right] \right\} & \text{if } F_i < 0 \end{array} \right. \quad (10)$$

where $g_{i, h} = \partial_\tau F(R)|_{R_i, h}/2$ is the surface gravity of the $i^{th}$ shell at the apparent horizon.

The first of these solutions describes an infalling shell in the exterior together with an interior, outgoing shell, whose relative probability is suppressed by the Boltzmann factor at the Hawking temperature of the shell. It
represents a net flow toward the apparent horizon. The second solution describes an ingoing shell in the interior together with an outgoing shell, whose amplitude is once again suppressed by one half the Boltzmann factor, in the exterior. It represents a flow away from the horizon on both sides and thermal radiation in the exterior. This wave function is responsible for the Hawking radiation, but it requires continued collapse beyond the apparent horizon to a central singularity. It follows that if we take \( \mathcal{F}_i = 0 \) to represent the collapse there is no thermal radiation outside the apparent horizon and there is no continued collapse to a central singularity. The collapse terminates at the apparent horizon \( (\mathcal{F}_i = 0) \), which agrees with Hawking’s proposal \[14\].

### III. A QUASI-CLASSICAL CONFIGURATION

We now look for static, spherically symmetric solutions of Einstein’s equations without a cosmological constant, in which the source occupies a finite region and possesses an energy density that is characteristic of a dust cloud that has condensed onto its apparent horizon. In this region, we take the metric to be of the form

\[
ds^2 = e^{2A} dt^2 - e^{2B} dr^2 - r^2 d\Omega^2,
\]

where \( A = A(r) \), \( B = B(r) \) and \( r \) represents the physical radius. In this coordinate system, if we take the components of the stress-energy to be \( T_{\mu \nu} = \text{diag}(-\varepsilon(r), p_r(r), p_\theta(r), p_\phi(r)) \) but impose no equations of state, the field equations are

\[
1 - e^{2B} - 2rB' = -8\pi G r^2 e^{2B} \varepsilon
\]

\[
1 - e^{2B} + 2rA' = 8\pi G r^2 e^{2B} p_r
\]

\[
rA'^2 - B' + A'(1 - rB') + rA'' = 8\pi Gr e^{2B} p_\theta,
\]

where a prime indicates a derivative with respect to the radius, \( r \). The conservation of energy-momentum gives a constraint,

\[
\varepsilon A' + p_r' + p_r \left[ \frac{2}{r} + A' \right] - \frac{2}{r} p_\theta = 0,
\]

which represents the condition for static equilibrium. Two of the stress-energy components may be chosen arbitrarily and then the third is determined by either Einstein’s equations or by the conservation law. Below we will choose the energy density and set the tangential pressure to zero.

The first equation in \[12\] may be re-expressed as

\[
[r(1 - e^{-2B})]' = 8\pi Gr^2 \varepsilon,
\]

which is straightforwardly integrated to give

\[
r(1 - e^{-2B}) = 8\pi G \int_0^r dr' r'^2 \varepsilon(r') - r_0,
\]

where \( r_0 \) is an integration constant, which is usually set to zero in stellar models to avoid a central singularity. We will not do so here for reasons that will become clear in the following. The Misner-Sharp mass function of the dust is to be identified with the integral on the right,

\[
F(r) = 4\pi \int_0^r dr' r'^2 \varepsilon(r).
\]

Now, according to \[3\], the mass function that may be expected of a dust ball whose collapse has terminated at the apparent horizon is

\[
F(r) = \frac{r}{2G},
\]

for a total gravitational mass of \( M_{ms} = F(r_b) = r_b/2G \). It corresponds to an energy density of

\[
\varepsilon(r) = \frac{1}{8\pi Gr^2}
\]

and \[15\] gives

\[
e^{2B} = r/r_0.
\]

We see that the constant \( r_0 \) is essential and cannot be set to zero. Without it there do not exist real solutions for \( B \) with the desired mass function, even if pressure is included. Strictly it describes a negative mass point source the center. Such a negative mass source is actually predicted by our wave functions \[3\] to form during the collapse because every infalling shell of dust in the exterior is accompanied by the emission of a positive energy shell from the center with a probability equal to the Boltzmann factor at the Hawking temperature appropriate to that shell. This process of energy extraction from the center continues until the collapse terminates. It describes the effect of strong quantum fluctuations near the center.

A detailed description of the central region is likely to depend very strongly on the intricate details of particular models of quantum gravity. However, regardless of the details, the size of this region is characterized by \( r_0 \), so we will take our quasi-classical solutions seriously only for \( r \geq r_0 \) and, in the following section, we will estimate \( r_0 \).

With \( B(r) \) given in \[19\] and no tangential pressure, we solve the Riccati equation in \[12\] for \( A(r) \) and find

\[
ds^2 = r^2 \left( 1 + \frac{\gamma}{r^{3/2}} \right)^2 dt^2 - \frac{r}{r_0} dr^2 - r^2 d\Omega^2,
\]

where \( \gamma \) is another integration constant.
There are curvature singularities at \( r = 0 \) and at \( r = (\gamma)^{2/3} \), but because our solutions are valid only when \( r \geq r_0 \), we will ignore the singularity at \( r = 0 \). To avoid the singularity at \( r = (\gamma)^{2/3} \), either \( \gamma \) must be positive or \( (\gamma)^{2/3} \) must lie outside the outer boundary of the collapsed star, where the solution no longer applies. We will soon show that the second condition cannot be met.

We determine the pressure directly from the second equation in (22)

\[
p_v(r) = \frac{1}{8\pi G r^2} \left[ 1 - \frac{3r_0}{r} \left( 1 + \frac{\gamma}{r^{3/2}} \right)^{-1} \right],
\]

so with \( \gamma \geq 0 \) our solutions are well behaved for \( r \geq r_0 \) and they obey the weak energy conditions.

If \( r_b \) denotes the outer boundary of the collapsed star, we want to match the interior geometry to an external vacuum, described by the Schwarzschild metric

\[
ds^2 = f(R) dt^2 - f^{-1}(R) dR^2 - R^2 d\Omega^2,
\]

where

\[
f(R) = \left( 1 - \frac{2GM_s}{R} \right)
\]

and \( M_s \) is the Schwarzschild mass of the dust ball. The junction conditions require that

\[
R_b = r_b, \quad T_b = \frac{e^{A(r_b)}}{\sqrt{f(r_b)}} t
\]

and therefore

\[
r_0 = r_b - r_s, \quad \gamma = 2r_s^{3/2} \left( 1 - \frac{3r_s}{2r_b} \right),
\]

where we have let \( r_s = 2GM_s \) be the Schwarzschild radius.

The first condition says that the physical radius of the boundary must lie outside its Schwarzschild radius. Therefore, as expected, the Schwarzschild mass of the star is less than the Misner-Sharp mass of the dust,

\[
M_s = \frac{r_s}{2G} = \frac{r_b - r_0}{2G} = M_{ms} + M_0,
\]

by precisely the negative central mass, \( M_0 \). If \( \gamma \geq 0 \), the second condition requires that \( r_b \geq 3r_s/2 \). But, for \( \gamma < 0 \) a regular solution is obtained only if we require the singularity to lie outside the boundary of the star. According to (22), this can happen if

\[
2 \left( \frac{3r_s}{2r_b} - 1 \right) > 1.
\]

But this would imply that \( r_s > r_b \). As this is not possible, the star will be singularity free only if \( r_b \geq 3r_s/2 \). This implies that \( r_b \leq 3r_0 \) and \( r_s \leq 2r_0 \).

**IV. ESTIMATING \( r_0 \)**

We can provide a simple estimate of the radius, \( r_0 \), in which the quantum fluctuations are expected to dominate, as follows. The energy extraction from the center occurs during the collapse because every collapsing shell is accompanied by an interior, outgoing wave, which will extract energy from the center. We want to estimate how much energy is extracted in this process. If \( r_b \) is the outer boundary of the star and the shell spacing is \( \sigma \), then we will have \( N = r_b/\sigma \) collapsing shells sharing the total energy, \( r_b/2G \) of the star. Thus the average energy, \( \omega_i \) of each shell will be roughly \( \sigma/2G \).

The probability for the emission of an interior, outgoing wave of frequency \( \omega_i \) from the center is given by the Boltzmann factor at the Hawking temperature of the shell, so the average energy of the outgoing shell is

\[
\langle \omega_i \rangle = \frac{\int_0^{\sigma/2G} d\omega_i \omega_i e^{-2\pi\omega_i r_i}}{\int_0^{\sigma/2G} d\omega_i e^{-2\pi\omega_i r_i}} \approx \frac{\sigma}{4G} - \frac{\pi \sigma}{6} \left( \frac{\sigma}{2G} \right)^2 + \ldots
\]

This is the energy that will be extracted from the center during the collapse of the \( i \)th shell. In the limit as \( \sigma \to 0 \), the total energy extracted during the collapse will be

\[
M_0 = \frac{N\sigma}{4G} = \frac{r_b}{4G} = \frac{1}{2} M_{ms},
\]

which implies that \( r_0 = r_b/2 \) and, by the matching conditions, it follows that \( r_0 = r_s \). Therefore, the region of strong quantum fluctuations occupies the *entire* Schwarzschild radius of the star. This is a surprisingly large length scale over which quantum gravitational effects should predominate.

The conclusion is similar to the “fuzzball” models of the black hole (23). However, there is also a significant difference between the two pictures of the interior, and this difference comes with testable consequences. In this model, the “fuzzy” region is a region of negative energy that extends to half the boundary radius of the collapsed object and is necessarily surrounded by a cloud of ordinary matter. There are no horizons. A photon, emitted near the boundary of this cloud, would experience a relatively tame redshift of

\[
z = \sqrt{\frac{r_b}{r_0}} - 1 = \sqrt{2} - 1 \approx 0.414,
\]

which is compatible with the gravitational redshift of neutron stars of low core densities (24), suggesting that, in a collapse of realistic matter, quantum gravity could “kick in” much before previously imagined, very near the time at which nuclear densities are achieved. This is consistent with the notion that in extreme conditions quantum gravity may be relevant on distance scales much larger than previously anticipated.
V. DISCUSSION

In this paper we have examined the consequences of a simple quantum model of dust collapse. We have argued that the AMPS paradox is avoided if continued collapse does not occur and all dust shells coalesce onto the apparent horizon. We showed that the collapse process is accompanied by energy extraction from the center of the cloud by the outgoing waves of positive energy within the apparent horizon that necessarily accompany the collapsing shells of dust. This leads to an effective negative point mass at the center.

Stable classical solutions, with the given mass function and including pressure were determined. The solutions are governed by two parameters, the Schwarzschild radius, $r_{s}$, of the dust ball, equivalently its mass as measured by a distant observer, and the boundary radius, $r_{b}$. The difference between the two is the radius, $r_{0}$, of a region in which the total energy is negative and where we expect quantum fluctuations to dominate. A description of this region is most likely to depend strongly on the details of the models of quantum gravity being applied, but our quasi-classical solutions should be reliable outside this region.

There are strong constraints on the parameters $r_{0}$, $r_{s}$ and $r_{b}$ if the interior geometry is required to be regular everywhere (except at the center). We have shown that the “quantum” region should extend to more than one half the Schwarzschild radius and more than one third the radius of the entire star, so it will occupy a significant fraction of the star. A more detailed analysis of the process by which energy is extracted from the center during collapse indicates, however, that $r_{0} = r_{s} = r_{b}/2$, so that the entire Schwarzschild region is one in which large quantum fluctuations may be expected.

Even in this simple model the picture that emerges is very different from the traditional view of a black hole. The collapsed object bores to twice its Schwarzschild radius, there is no horizon and the spacetime geometry is regular. Observationally, what is being encountered has more in common with a neutron star, except that what holds the system up is not matter degeneracy pressure but vacuum energy. What is traditionally viewed as the radius of the hole (the Schwarzschild radius) is in fact surrounded by a matter cloud. Radiation from the boundary of this cloud should not suffer a redshift much larger than from neutron stars of relatively low core densities.

[1] S.W. Hawking and R. Penrose, Proc. Phys. Soc. London, Sect. A 300 (1967) 182; ibid Proc. Phys. Soc. London, Sect. A 314 (1970) 529;
[2] S.W. Hawking and G.F.R. Ellis, The Large Scale Structure of Spacetime, Cambridge University Press, Cambridge (1973).
[3] P.S. Joshi, Global Aspects in Gravitation and Cosmology, Clarendon Press, Oxford (1993).
[4] C. Vaz, L. Witten, Phys. Lett. B325 (1994) 27; ibid Class. Quant. Grav. 13 (1996) L59; ibid Nucl. Phys. B 487 (1997) 409;
[5] S. Barve, T.P. Singh, C. Vaz and L. Witten, Nucl. Phys. B 532 (1998) 361; ibid Phys. Rev. D 58 (1998) 104018.
[6] S.W. Hawking, Comm. Math. Phys. 43 (1975) 199; ibid Phys. Rev. D 14 (1976) 2460.
[7] L. Susskind, L. Thorlacius, J. Uglum, Phys. Rev. D 48 (1993) 3743.
[8] G. ’t Hooft, Nucl. Phys. B 256 (1985) 727; ibid Nucl. Phys. B 335 (1990) 138.
[9] J. Preskill, arXiv:hep-th/9209058.
[10] A. Almheiri, D. Marolf, J. Polchinski, J. Sully, arXiv:1207.3123; A. Almheiri, D. Marolf, J. Polchinski, D. Stanford, J. Sully, arXiv:1304.6483.
[11] A precursor to the idea of a firewall was proposed by S. Braunstein, S. Pirandola, K. Życzkowski, arXiv:0907.1190 and published in Phys. Rev. Lett. 110, (2013) 101301.
[12] S. W. Hawking. arXiv:1401.5761
[13] C. Vaz, T.P. Singh and L. Witten, Phys. Rev. D 63 (2001) 104020.
[14] C. Vaz, L. Witten, T.P. Singh, Phys. Rev. D 69 (2004) 104029; C. Vaz, R. Tibrewala, T.P. Singh, Phys. Rev. D 78 (2008) 024019.
[15] C. Vaz and L.C.R. Wijewardhana, Phys. Rev. D 82 (2010) 084018; C. Vaz and K. Lochan, Phys. Rev. D 87 (2013) 024045.
[16] C. Vaz, Int. J. Mod. Phys. D 23 (2014) 1441002.
[17] G. LeMaitre, Annales de la Societe Scientifique de Bruxelles A 53, 51 (1933); for an English translation see Gen. Rel. Grav. 29 641 (1997); R. Tolman, Proc. Natl. Acad. Sci. USA 20 (1934) 410; H. Bondi, Mon. Not. Astron. Soc. 107 (1947) 343.
[18] J. Plebansky and A. Krasinski, An Introduction to General Relativity and Cosmology, Cambridge University Press, Cambridge (2006).
[19] K. Kuchař, Phys. Rev. D 50, (1994) 3961.
[20] J. Brown and K. Kuchař, Phys. Rev. D 51 (1995) 5600.
[21] C. Vaz, T.P. Singh, L. Witten, Phys. Rev. D 69 (2004) 104029.
[22] Claus Kiefer, J. Mueller-Hill and C. Vaz, Phys. Rev. D 73 (2006) 044025.
[23] S.D. Mathur, Fortsch. Phys. 53 (2005) 793; ibid Class. Quant. Grav. 26 (2009) 224001.
[24] L. Lindblom, Ap. J. 278 (1984) 364.