Collins effect in semi-inclusive deeply inelastic scattering and in \(e^+e^-\)-annihilation

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The Collins fragmentation function is extracted from HERMES data on azimuthal single spin asymmetries in semi-inclusive deeply inelastic scattering, and BELLE data on azimuthal asymmetries in \(e^+e^-\)-annihilations. A Gaussian model is assumed for the distribution of transverse parton momenta and predictions are used from the chiral quark-soliton model for the transversity distribution function. We find that the HERMES and BELLE data yield a consistent picture of the Collins fragmentation function which is compatible with COMPASS data and the information previously obtained from an analysis of DELPHI data. Estimates for future experiments are made.

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I. INTRODUCTION

The chirally odd Collins fragmentation function \(H_1^\perp\) describes a possible left-right-asymmetry in the fragmentation of transversely polarized quarks [1–3], the effects of which can be observed in two ways. In \(e^+e^-\) annihilations it may give rise to specific azimuthal asymmetries [4, 5] where one can access \(H_1^\perp q H_1^\perp \bar{q}\) adequately weighted with electric or electro-weak charges. In semi-inclusive deeply inelastic scattering (SIDIS) with a transversely (or longitudinally) polarized target it may give rise to azimuthal single spin asymmetries (SSA) [6, 7] where it enters in combination with the transversity distribution function \(h_1^\perp(x)\) [8, 9] (or other chirally-odd distribution functions). For these observables transverse parton momenta play a crucial role requiring a careful generalization [10–13] of standard (“collinear”) factorization theorems [14].

The first evidence for the Collins fragmentation function was looked for in a study of SMC data from SIDIS with transversely polarized targets [15], and in a study of DELPHI data on charged hadron production in \(e^+e^-\) annihilations at the \(Z^0\)-pole [16]. More recently the HERMES Collaboration has published (and shown further preliminary) data on the Collins SSA in SIDIS from a proton target giving the first unambiguous evidence that \(H_1^\perp\) and \(h_1^\perp(x)\) are non-zero [17–19], while in the COMPASS experiment the Collins effect from a deuteron target was found compatible with zero within error bars [20]. Finally, the BELLE collaboration presented data on sizeable azimuthal asymmetries in \(e^+e^-\) annihilations at a center of mass energy of 60 MeV below the \(\Upsilon\)-resonance [21].

One question which immediately arises is: Are all these data from different SIDIS and \(e^+e^-\) experiments compatible, i.e. are they all indeed due to the same effect, namely the Collins effect?

In order to answer this question one may try to fit the HERMES \([18, 19]\), COMPASS \([20]\) and BELLE \([21]\) data simultaneously. While worthwhile trying [22] such a procedure has also disadvantages. When using SIDIS data the obtained fit for the Collins fragmentation function is inevitably biased by some model assumption on the transversity distribution, and the — presently not fully understood — scale dependence of \(H_1^\perp\) is neglected.

For these reasons we shall follow a different strategy and extract \(H_1^\perp\) separately from HERMES \([19]\) and BELLE \([21]\) data, and compare then the results to each other and to other experiments.

By focussing on adequately defined ratios of \(H_1^\perp\) to \(D_1\) (“analyzing powers”) we observe that the indications from DELPHI [16] and the data from HERMES \([17–19]\), COMPASS \([20]\) and BELLE \([21]\) are compatible with each other. Experience with spin observables where radiative and other corrections are known, see e.g. \([23, 24]\), suggests that such analyzing powers could be less scale-dependent than the respective absolute cross sections.

For the analysis of the HERMES data we use [25] predictions for \(h_1^\perp(x)\) from the chiral quark-soliton model [26] and assume a Gaussian distribution of transverse parton momenta. Our results are in agreement with the first extraction of \(H_1^\perp\) from the HERMES data [27], where different models were assumed.\(^1\) On the basis of a comparison to Ref. [27] we discuss which of the observations made here and in [27] are robust, and which are likely to be model-dependent.

We shall furthermore present some estimates for future experiments in SIDIS and at \(e^+e^-\) colliders. The comparison of these estimates to the outcome of the experiments will help to solidify our understanding of the Collins fragmentation function and the transversity distribution.

The SIDIS data from HERMES and COMPASS [17–20] also provide the first unambiguous evidence for the equally interesting Sivers effect [38–41]. We refer the reader to Refs. [27, 42–47] for studies of this effect.

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\(^1\) Previously, the preliminary result [15] and longitudinal target SSA [28–31] data were used to extract \(H_1^\perp\) from SIDIS [32, 33]. However, in these studies twist-3 effects [34] were not or only partially considered. Still earlier attempts to learn about the Collins effect from SSA in the hadronic processes \(pp^\perp\) or \(pp^\perp \to \pi X\) [35] were made in [36], but recent studies indicate that the Collins effect is not likely to be the dominant source of SSA in these processes [37] as it was assumed in [36].
II. COLLINS EFFECT IN SIDIS

For reasons which will become clear in the sequel we start our study with the Collins effect in SIDIS — although this case is more involved because here in addition to $H^\perp_1$ also the unknown transversity distribution $f^a_1(x)$ enters. For the latter we use here predictions from the chiral quark-soliton model [26] which provides a consistent and successful [48, 49] field theoretic description of the nucleon. The distribution functions $f^a_1(x)$ and $g^a_1(x)$ computed in this model at a low normalization point [50] agree to within (10-30)% with parameterizations [51]. One may assume the same accuracy for the model predictions for transversity, which are in between the popular working assumptions, namely at a low normalization point [50] agree to within (10-30)% with parameterizations [51]. One may assume the same

The way the HERMES [18, 19] (and BELLE [21]) data were analyzed requires to assume also a model for transverse parton momenta. We assume the distributions of transverse parton momenta in the transversity distribution and transverse hadron momenta in the Collins fragmentation function to be Gaussian, and take the respective widths $\langle p^2_{h1} \rangle$ and $\langle K^2_{H1} \rangle$ to be flavour and x- or z-independent

$$h^a_1(x, p^2_T) \equiv h^a_1(x) \exp(-p^2_T/(\langle p^2_{h1} \rangle)),$$

$$H^{\perp a}_1(z, K^2_T) \equiv H^{\perp a}_1(z) \exp(-K^2_T/(\langle K^2_{H1} \rangle))$$.

The Gauss Ansatz can be considered at best as a crude approximation to the true distribution of transverse momenta [53]. For example, it does not yield a 1/$P_{h\perp}$ power-like suppression of the Collins SSA expected at large transverse hadron momenta [2]. However, this Ansatz gives a satisfactory effective description in several hard reactions, provided the transverse momenta are much smaller than the hard scale of the process [54]. This is the situation in the HERMES experiment where the mean transverse hadron momenta $\langle P_{h\perp} \rangle \sim 0.4$ GeV $\ll \sqrt{Q^2} \sim 1.5$ GeV.

In fact, the HERMES data on $\langle P_{h\perp} \rangle$ can be described satisfactorily assuming for $f^a_1(x, p^2_T)$ and $D^a_1(z, K^2_T)$ Ansätze analog to (1) with [45]

$$\langle p^2_{f1} \rangle = 0.33 \text{ GeV}^2 \quad \text{and} \quad \langle K^2_{D1} \rangle = 0.16 \text{ GeV}^2.$$  

Noteworthy these numbers are in good agreement with results [43] inferred from EMC data [55] on the so-called Cahn-effect [56]. All that can be said about the Gaussian widths of the transversity distribution and Collins function is that positivity conditions require [57]

$$|h^a_1(x, p^2_T)| \leq f^a_1(x, p^2_T) \Rightarrow \langle f^a_1 \rangle \leq \langle p^2_{h1} \rangle,$$

$$\frac{|K^2_T|}{2m_\pi} H^{\perp a}_1(z, K^2_T) \leq D^a_1(z, K^2_T) \Rightarrow \langle K^2_{H1} \rangle \leq \langle K^2_{D1} \rangle$$

which are necessary conditions (but not sufficient for $H^{\perp}_1$, see [45] for the discussion of the analog case of the Sivers function).

In the Gaussian model (1) the Collins SSA as measured by HERMES [18, 19] is given by (we neglect throughout the soft factors [11, 12])

$$A^\sin(\phi + \phi_S)_{UT} \equiv 2\sin(\phi + \phi_S)$$

$$= 2 \sum_a e^a_\pi e^a_\upsilon x h^a_1(x) B_{\text{Gauss}} H^{(1/2)a}_1(z) \sqrt{\sum_a e^a_\pi \sqrt{x f^a_1(x) D^a_1(z)}},$$

where $x = Q^2/(2P \cdot q)$, $z = P \cdot P_h/P \cdot q$, $Q^2 = -q^2$ and $P$ is the target momentum. Other momenta and azimuthal angles are defined in Fig. 1. The (1/2)-moment of the Collins function is defined as and satisfies the inequality

$$H^{(1/2)a}_1(z) = \int d^2K_T \frac{|K_T|}{2z m_\pi} H^{\perp a}_1(z, K_T) \leq \frac{1}{2} D^a_1(z),$$

which follows from (3), see [57]. In Eq. (4) the dependence on the Gaussian model is contained in the factor

$$B_{\text{Gauss}}(z) = \frac{1}{\sqrt{1 + z^2 \langle p^2_{h1} \rangle / \langle K^2_{H1} \rangle}}.$$  

Within the Gaussian model Eq. (4) is just one convenient way of writing the result. Equivalently one could write

$$B_{\text{Gauss}} H^{(1/2)a}_1(z) = a_{\text{Gauss}} H^{(1)a}_1(z), \quad a_{\text{Gauss}} = \frac{\sqrt{\pi} m_\pi}{2 \sqrt{\langle p^2_{h1} \rangle + \langle K^2_{H1} \rangle / z^2}}.$$
Here the (1)-transverse moment of the Collins function is defined as in Eq. (5) but with an additional power of $|K_T|/(z m_a)$ in the weight, and $a_{Gauss}$ is defined in complete analogy to the notation of Ref. [45].

Had the Collins SSA been analyzed as $\langle \sin(\phi + \phi_S) P_{h\perp}/(z m_a) \rangle$, i.e. with an additional power of transverse hadron momentum $P_{h\perp}$ in the weight, the result would be given by an expression analogous to Eq. (4) but with $B_{Gauss} \to 1$ and $H_{1}^{(1/2)}$ replaced by $H_{1}^{(1)}$ independently of any model of transverse parton momenta [7]. It was argued that adequately weighted SSA might be less sensitive to Sudakov suppression effects [58]. Preliminary HERMES data analyzed in this way were presented in [17].

By defining $A_\perp = A_{UT}^{\sin(\phi + \phi_S)} \sum_a c_a^2 f_{a\perp}^2 D_1^{a/\perp}$ and using HERMES data [18, 19] on $\pi^\pm$ we can rewrite (4) as

$$\left( \frac{A_\perp^+}{A_\perp^-} \right) = \left( \frac{\frac{2}{3} x h_1^u + \frac{1}{3} x h_1^d}{\frac{2}{3} x h_1^u + \frac{1}{3} x h_1^d} \right) \cdot \left( \frac{2 B_{Gauss} H_{1}^{(1/2) fav}}{2 B_{Gauss} H_{1}^{(1/2) unf}} \right),$$

which — with our chosen model for $h_1^a(x)$ — can be inverted to give unambiguous results for $B_{Gauss} H_{1}^{(1/2) fav}$. The favoured and unfavoured Collins fragmentation functions are defined as, schematically:

$$H_{1}^{fav} = H_{1}^{a/\perp} = H_{1}^{d/\perp} = H_{1}^{u/\perp} = H_{1}^{d/\perp},$$

$$H_{1}^{unf} = H_{1}^{a/\perp} = H_{1}^{d/\perp} = H_{1}^{u/\perp} = H_{1}^{d/\perp}. $$

Here — in the study of SIDIS data — we neglect the effects of strange and heavier flavours, which is justified at the present stage because the corresponding distribution functions are rather small.

As a first step we extract in this way from the HERMES preliminary data [19] on the $x$-dependence of the Collins SSA the quantity $\langle 2 B_{Gauss} H_{1}^{(1/2) fav} \rangle$ which is averaged over $z$ within the HERMES cuts $0.2 \leq z \leq 0.7$. Using for $h_1^a(x)$ the chiral quark soliton model [26] and for $f_{2n}^a(x)$ and $D_1^q(z)$ the LO-parameterizations [51, 59] at $(Q^2) = 2.5$ GeV$^2$ which is the average scale in the HERMES experiment, we obtain the results shown in Fig. 2. As the SSA for $\pi^+$ and $\pi^-$ are independent observables the statistical errors shown in Fig. 2 are not correlated.

The $\langle 2 B_{Gauss} H_{1}^{(1/2) fav} \rangle$ are expected to be $x$-independent — provided the used models, the Gaussian Ansatz (1) and the model [26] for $h_1^a(x)$, are appropriate. In fact, the extracted $\langle 2 B_{Gauss} H_{1}^{(1/2) fav} \rangle$ are compatible with this expectation and can be fitted respectively to constants with reasonable $\chi^2$ per degree of freedom ($\chi^2_{ dof}$)

$$\langle 2 B_{Gauss} H_{1}^{(1/2) fav} \rangle = (3.5 \pm 0.8)\% , \quad \chi^2_{ dof} = 0.6 ,$$

$$\langle 2 B_{Gauss} H_{1}^{(1/2) unf} \rangle = -(3.8 \pm 0.7)\% , \quad \chi^2_{ dof} = 1.3. $$

These constants are indicated in Fig. 2. The published HERMES data which have a lower statistics [18] yield similar values but with larger uncertainties and worse $\chi^2_{ dof}$.

Let us remark that at small $x$ a deviation from the expected behaviour $\langle 2 B_{Gauss} H_{1}^{(1/2) fav} \rangle = constant$ would not be surprising for two reasons. First, the description of distribution functions in the small-$x$ region $x \lesssim 0.05$ is beyond the range of applicability of the chiral quark-soliton model [50]. Second, in the lowest $x$-bin of the HERMES experiment $(Q^2) = 1.3$ GeV$^2$ [31] could be at the edge of the applicability of the factorization approach.

However, the lack of a noticeable $x$-dependence of $\langle 2 B_{Gauss} H_{1}^{(1/2) fav} \rangle$ indicated in Fig. 2 gives certain confidence that the model [26] for $h_1^a(x)$ used here is — considering the accuracy of the data [18, 19] — reasonable. The results in Eqs. (10, 11) are, of course, model-dependent but Fig. 2 indicates that the systematic uncertainty due to model-dependence is less dominant than the statistical error in (10, 11).

We also remark that the results in Fig. 2 and Eqs. (10, 11) are not specific to the Gaussian Ansatz: Any analysis of the data [19] in which the factorized Ansatz $h_1^a(x, p_T^2) = h_1^a(x) G(p_T^2)$ is assumed with $G(p_T^2)$ some function of $p_T^2$ (and $h_1^a(x)$ is taken from [26]) would yield the same result (10, 11). (Then, however, this z-averaged quantity would be given by some different theoretical expressions.)

The results (10, 11) are unexpected from the point of view of studies [33, 60-63] of longitudinal SSA [28, 29]. In order to understand these data it was sufficient to assume favoured flavour fragmentation only and to neglect $H_{1}^{unf}$ completely, see [63] for a review. Later it became clear that these SSA are dominated by subleading twist effects [31], and are theoretically more difficult to describe [34]. Moreover, we now learn that the unfavoured fragmentation

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**Fig. 2.** The quantity $2 B_{Gauss} H_{1}^{(1/2) fav}$ averaged over $z$, i.e. practically the weight of $h_1^a(x)$ in the Collins SSA $A_{UT}^{\sin(\phi + \phi_S)}(x)$ in Eq. (4), vs. $x$ as extracted from the preliminary HERMES data [19]. This quantity does not show any significant $x$-dependence — as expected, see text.
The Collins SSA $A_{UT}^{\sin(\phi+\phi_2)}(x)$ as function of $x$. The preliminary HERMES data are from [19], the COMPASS data are from [20]. The theoretical curves are based on the fit in Eqs. (10, 11) and predictions for the transversity distribution from the chiral quark-soliton model [26]. Notice that different sign conventions are used in [19, 20]: $A_{UT}^{\sin(\phi_i+C)}(x) = -A_{UT}^{\sin(\phi_i+\phi_2)}(x)$.

In the Collins function cannot be neglected. Instead, in order to explain the HERMES data [18, 19] the favoured and unfavoured Collins fragmentation functions must be of similar magnitude and opposite sign. The string fragmentation picture provides a qualitative understanding of this behaviour [64] as do quark-hadron-duality motivated phenomenological considerations [27].

The different behaviour of the Collins function compared to the unfavoured fragmentation function becomes more evident by considering the analyzing powers defined as

$$\frac{\langle 2B_{Gauss}H_1^{(1/2)\text{fav}} \rangle}{\langle D_1^{(1/2)} \rangle}_{\text{HERMES}} = (7.2 \pm 1.7)\%,$$

$$\frac{\langle 2B_{Gauss}H_1^{(1/2)\text{unfav}} \rangle}{\langle D_1^{(1/2)} \rangle}_{\text{HERMES}} = -(14.2 \pm 2.7)\%.$$  \hspace{1cm} (12)

The positivity bound (5) and the fact that $B_{Gauss} < 1$, see Eq. (6), require the absolute values of the above numbers to be smaller than unity which is the case. Thus, the extracted results satisfy positivity.

Figs. 3a and b shows how the fit (10, 11) describes the preliminary HERMES data [19] on the Collins SSA from the proton target. As can be seen from Figs. 3c and d this fit is also in agreement with the COMPASS data which show a compatible with zero Collins effect from a deuteron target [20].

In principle, one could extract in an analog way also the $z$-dependence of the product $B_{Gauss}H_1^{(1/2)\alpha}$. However, we refrain from doing this here for the following reasons. First, due to the appearance of the unknown Gaussian widths in $B_{Gauss}$, see Eq. (6), we could not conclude anything on the $z$-dependence of $H_1^{(1/2)\alpha}$. Second, the study of the $z$-dependence requires to average over $x$ within the HERMES cuts $0.023 \leq x \leq 0.4$ which includes small $x$ where the model [26] is not applicable. We postpone the study of the $z$-dependence of the Collins SSA for Sec. IV.

### III. COLLINS EFFECT IN $e^+e^-$ ANNIHILATION AT BELLE

In the process $e^+e^- \rightarrow h_1h_2X$ where the two observed hadrons belong to opposite jets the Collins effect gives rise to a specific azimuthal distribution of hadron $h_1$ along the axis defined by hadron $h_2$ in the $e^+e^-$ center of mass frame [4, 5], see Fig. 4. Assuming the Gaussian model this azimuthal distribution is given by

$$\frac{d^4\sigma(e^+e^-\rightarrow h_1h_2X)}{d\cos\theta_2dz_1dz_2d\phi_1} = \frac{3\alpha^2}{Q^2} \left[ (1 + \cos^2\theta_2) \sum_a e_a^2 D_1^a(z_1)D_1^a(z_2) \
+ \cos(2\phi_1) \sin^2\theta_2 C_{Gauss} \sum_a e_a^2 H_1^{(1/2)\alpha}(z_1)H_1^{(1/2)\alpha}(z_2) \right]$$  \hspace{1cm} (13)

where $Q^2$ is the center of mass energy square of the lepton pair and $z_i = 2E_{h_i}/Q$ while $C_{Gauss}$ is defined as

$$C_{Gauss}(z_1, z_2) = \frac{16}{\pi} \frac{z_1z_2}{z_1^2 + z_2^2}.$$  \hspace{1cm} (14)

In principle, one can rewrite the result in (13) within the Gauss model also in terms of $H_1^+$ or $H_1^{(1)}$ with some different functions instead of $C_{Gauss}$.

Experimentally it is convenient to normalize the expression (13) with respect to its $\phi_1$-average and to define the following observable [21]

$$A_1 = 1 + \cos(2\phi_1) \frac{\langle \sin^2\theta_2 \rangle}{\langle 1 + \cos^2\theta_2 \rangle} \sum_a e_a^2 C_{Gauss} H_1^{(1/2)\alpha}(z_1)H_1^{(1/2)\alpha}(z_2) \sum_a e_a^2 D_1^a(z_1)D_1^a(z_2) \left[ \right]$$  \hspace{1cm} (15)

![Kinematics of the process $e^+e^- \rightarrow h_1h_2X$ and the definitions of azimuthal angles in the $e^+e^-$ rest frame.](image-url)
Here the average over $\theta_2$ is understood in the range of acceptance of the BELLE detector [21].

Notice that hard gluon radiation also gives rise to a $\cos(2\phi_1)$-dependence as do detector dependent effects. These effects, however, are flavour independent and — as long as the coefficient of the $\cos(2\phi_1)$-modulation is not large — one can get rid of them in the following way [21]. One considers the double ratio of $A_1^U$, where both hadrons $h_1 h_2$ are pions of unlike sign, to $A_1^L$, where both pions are of like sign, i.e.

$$A_1^U = 1 + \cos(2\phi_1) P_1(z_1, z_2).$$

(16)

The observable $P_1(z_1, z_2)$ is given by

$$P_1(z_1, z_2) \equiv \frac{\sin^2(\theta_2)}{1 + \cos^2(\theta_2)} C_{\text{Gauss}} \left[ \frac{5 H_1^{(1/2)\text{fav}}(z_1) H_1^{(1/2)\text{fav}}(z_2) + 7 H_1^{(1/2)\text{unf}}(z_1) H_1^{(1/2)\text{unf}}(z_2)}{5 D_1^{\text{fav}}(z_1) D_1^{\text{fav}}(z_2) + 7 D_1^{\text{unf}}(z_1) D_1^{\text{unf}}(z_2)} \right].$$

(17)

up to higher order terms in the Collins function. The systematic error of the double ratio method was estimated to be small [21].

In Eq. (17) it is assumed that the Collins fragmentation of $s$- and $\bar{s}$-quarks into pions is equal to the unfavoured fragmentation function defined in Eq. (9). At the high energies of the BELLE experiment just below the threshold of $b$-quark production this is a reasonable and commonly used assumption for $D_1^r$ [65]. We assume it here to be valid also for the Collins function. ($D_1^{\text{fav}}$ and $D_1^{\text{unf}}$ in (17) are defined analogously to Eq. (9).) Charm contribution does not need to be considered in Eq. (17) since the BELLE data are corrected for it [21].

In order to obtain a fit to the BELLE data we adopt the LO-parameterization [65] for $D_1^r(z)$ at $Q^2 = (10.52 \, \text{GeV})^2$ and choose the following simple Ansatz

$$H_1^{(1/2)a}(z) = C_a \, z \, D_1^{r}(z).$$

(18)

The two free parameters $C_\text{fav}$ and $C_\text{unf}$ introduced in (18) can be well fitted to the BELLE data [21]. We explored also other Ansätze proportional, for example, to $z^2 D_1^{r}(z)$, $D_1^{r}(z)(1 - z)$ or $z D_1^{r}(z)(1 - z)$ but none of them gave satisfactory solutions. The best fit has a $\chi^2_{\text{NDF}} = 0.6$ and is demonstrated in Fig. 5a in the $C_\text{fav}$-$C_\text{unf}$-plane. Two different, equivalent, best fit solutions exist. The reason for this is that the expression for $P_1(z_1, z_2)$ in Eq. (17) is symmetric with respect to the exchange $C_\text{fav} \leftrightarrow C_\text{unf}$ in our Ansatz, and manifests itself in Fig. 5a where the two solutions are mirror images of each other with respect to the axis defined by $C_\text{fav} = C_\text{unf}$.

What can unambiguously be concluded from Fig. 5a is that the BELLE data require the Collins favoured and unfavoured fragmentation functions to have opposite sign — as in the HERMES experiment. On the basis our study of the Collins effect in SIDIS we are tempted to select the solution with $C_\text{fav} > 0$ and $C_\text{unf} < 0$ in Fig. 5a as the appropriate one. Our result is thus

$$C_\text{fav} = 0.15, \quad C_\text{unf} = -0.45$$

(19)

and the resulting best fits and their 1-$\sigma$ regions are shown in Fig. 5b for $z > 0.2$ which is the low-$z$ cut in the BELLE experiment. The results satisfy the positivity condition (5). Notice that the errors of the favoured and unfavoured Collins functions are correlated.

In Fig. 6 the BELLE data [21] are compared to the theoretical result for $P_1(z_1, z_2)$ obtained on the basis of the best fit shown in Fig. 5b. Here the error-correlation of the fit results for the favoured and unfavoured Collins function is
taken into account and the resulting 1-σ error band is more narrow than in Fig. 5b. The description of the BELLE data is satisfactory — as can be seen from Fig. 6. (Notice that \( P_{1}(z_{1}, z_{2}) \) is symmetric with respect to the exchange \( z_{1} \leftrightarrow z_{2} \) such that the bins with \( z_{1} \neq z_{2} \) can be combined — as was done in the BELLE analysis [21] and in our fit procedure. Here, for a better overview, these bins are presented separately — whereby we disregard that strictly speaking the statistical error bars for bins with \( z_{1} \neq z_{2} \) should then be multiplied by \( \sqrt{2} \).

IV. ARE BELLE AND HERMES DATA COMPATIBLE?

In order to compare the Collins effect in SIDIS at HERMES [18, 19] and in \( e^{+}e^{-} \)-annihilation at BELLE [21] it is, strictly speaking, necessary to take into account the evolution properties of \( H_{T}^{1} \). However, in order to get a first rough idea — which is sufficient at the present stage — we instead consider ratios of \( H_{T}^{1} \) to \( D_{T}^{1} \), which may be expected to be less scale dependent. For example, integrating the BELLE fit result in Fig. 5b over the range of HERMES \( z \)-cuts 0.2 < \( z < 0.7 \), we obtain the following analyzing powers

\[
\frac{(2H_{T}^{1/(2)\text{fav}})}{(D_{T}^{1\text{fav}})}_{\text{BELLE}} = (5.3 \ldots 20.4)\% , \quad \frac{(2H_{T}^{1/(2)\text{undf}})}{(D_{T}^{1\text{undf}})}_{\text{BELLE}} = -(3.7 \ldots 41.4)\% .
\]

Comparing the above numbers (again the errors are correlated) to the result in Eq. (12) we see that the effects at HERMES and at BELLE — as quantified in Eqs. (12, 20) — are of comparable magnitude. The central values of the BELLE analyzing powers seem to be systematically larger than the HERMES ones. This could partly be attributed to evolution effects. However, notice that in the HERMES result (12) in addition the factor \( B_{\text{Gauss}} < 1 \) enters, which tends to decrease the result. Thus, the HERMES [18, 19] and BELLE [21] data seem in good agreement.

Encouraged by this observation let us see whether we can describes the HERMES data on \( A_{LT}^{\text{sin}(\phi^{+}+\phi^{s})}(z) \) on the basis of the \( z \)-dependence of \( H_{T}^{1} \) concluded from the BELLE data [21]. For that let us assume a weak scale-dependence not only for \( z \)-averaged ratios — as we did above — but also

\[
\frac{H_{1}^{1/(2)a}(z)}{D_{T}^{1}(z)}_{\text{BELLE \ scale}} \approx \frac{H_{1}^{1/(2)a}(z)}{D_{T}^{1}(z)}_{\text{HERMES \ scale}}.
\]

Nothing is known about the Gaussian widths of the transversity distribution and the Collins function which enter the factor \( B_{\text{Gauss}} \) in Eq. (6). Let us therefore assume their ratio to be similar — let us say to within a factor of two — to the corresponding ratio of the Gaussian widths of the unpolarized \( f_{1}^{T}(x,p_{T}^{2}) \) and \( D_{T}^{1}(z,K_{T}^{2}) \) in Eq. (2), i.e.

\[
1 \lesssim \frac{(p_{T}^{2})_{H_{T}^{1}}}{(K_{T}^{2})_{f_{1}^{T}}} \lesssim 4 .
\]
V. ARE BELLE DATA AND DELPHI RESULT COMPATIBLE?

Another interesting question is whether the BELLE and HERMES data are compatible with the results obtained from an analysis of DELPHI data [16] where azimuthal asymmetries in $e^+e^- \rightarrow h'h'X$ at the $Z_0$-peak were studied. At DELPHI $h, h'$ were leading charged hadrons in opposite jets. Also in this process the Collins effect gives rise to a $\cos(2\phi_1)$-asymmetry analog to (13) but with electric charges replaced by electro-weak ones and with modifications due to the finite mass and width of $Z_0$ [4, 5]. The cross-section differential in $\phi_1$ can be written as [4, 5]

$$\frac{d\sigma(e^+e^- \rightarrow h'h'X)}{d\phi_1} = P_0 \left( 1 + \cos(2\phi_1) \right) P_2$$

with

$$P_2 = \frac{\langle \sin^2 \theta_2 \rangle}{\langle 1 + \cos^2 \theta_2 \rangle} \int dz_1 \int dz_2 \sum_{h,h'} \sum_{a,b,c} C_{Gauss(a)} c_h^a H_1^{(1/2)a/h}(z_1) H_1^{(1/2)b/h}(z_2)$$

$$\int dz_1 \int dz_2 \sum_{h,h'} \sum_{a,b,c} c_h^a D_1^{a/h}(z_1) D_1^{b/h}(z_2).$$

The $c_h^a$ are defined as (the “+” sign refers to $i = 1$, the “−” sign to $i = 2$)

$$c_{1,2} = (T_a - 2e^{-9} \sin \Theta_W)^2 \pm (T_3)^2$$

with the weak-isospin $T_a^u = T_u^u, T_3^d = T_3^d = -T_3^s = -\frac{1}{2}$. The result obtained from the analysis of the DELPHI data reads [16]

$$P_{2,\text{DELPHI}} = -(0.26 \pm 0.18)\%.$$  

In order to have an idea whether the result (26) is compatible with BELLE let us make the following rough estimate. We assume a weak-scale dependence similarly to (21) and take the charged hadrons observed at DELPHI to be pions and kaons (which were, in fact, the most prolific particles). Then we obtain from the best fit to DELPHI data with the unpolarized fragmentation functions at $Q^2 = M_Z^2$ from [65]

$$P_2, \text{from BELLE} \approx -(0.052 + 0.007 + 0.004 + 0.103 + 0.099 + 0.024)\% = -(0.06 \ldots 0.29)\%.$$  

The different contributions are explained as follows. The contribution “$u, d, s \rightarrow \pi, \pi^\pm$" is due to Collins fragmentation of light quarks into pions. This is the only contribution which is determined by the best fit to the BELLE data without additional assumptions. Since pions and kaons are considered as Goldstone bosons of spontaneous chiral symmetry breaking one is lead to the conclusion that in the chiral limit [61]

$$\lim_{M_Z \rightarrow 0} H_1^{(1/2)a/\pi}(z) = \lim_{M_K \rightarrow 0} H_1^{(1/2)a/K}(z).$$

Notice that the analog relations with $H_1^+$ or $H_1^{(1)}$ would contain explicit factors of hadron masses. In the real world the unpolarized fragmentation functions for pions and kaons are found to be far off the analog chiral limit relation. Thus, one may expect the real-world Collins functions to be similarly far off the chiral limit relation (28). However, if the way off the chiral limit to the real world proceeded in a spin-independent way, one still could expect the relations (18, 19) between the Collins and unpolarized fragmentation functions for pions to be valid approximately also for kaons. This is what we did for the sake of our rough estimate. Future SIDIS and $e^+e^-$ data on kaon asymmetries will reveal to which extent these assumptions are justified.\(^2\)

Finally, to estimate the contribution due to the Collins

\(^2\)The relations (18, 19) cannot hold for all hadrons. Otherwise it would be impossible to satisfy the Schäfer-Teryaev sum rule [73].
fragmentation of heavier quarks into pions (and kaons) we assume the relations (18) (and their kaon-analogs) to hold also for charm and bottom with $C_c = C_b = C_{\text{unt}}$ from (19). These latter assumptions seem natural — at the $Z^0$-peak, where differences in the fragmentation of non-valence quark-flavours $q$ into a specific hadron might be expected to be of $O(m_q/M_Z)$. In fact, one finds for pions $D_1^{\text{unt}} \equiv D_1^0 \approx D_1^{\pi/\pi} \gtrsim D_1^{b/\pi}$ at $Q^2 = M_Z^2$ and qualitatively similar for kaons [65]. But the above assumptions need not to be correct and, for example, the Collins fragmentation of heavier flavours could be simply zero. This gives rise to the uncertainty in the final result in Eq. (27).

The estimate (27) — even if we consider only the Collins effect of light quarks — is in reasonable agreement with the result from DELPHI in Eq. (26). We refrain from quoting in Eq. (27) the error to due to the 1-$\sigma$ uncertainty of the BELLE fit as our result has presumably even larger theoretical uncertainties. Also it is important to keep in mind that the DELPHI result (26) has unestimated (and presumably sizeable) systematic uncertainties. What is important at this point, however, is that both numbers (26, 27) are of comparable order of magnitude — indicating that the DELPHI result [16] and the BELLE data [21] could be well due to the same effect.

VI. REMARK ON THE PRELIMINARY SMC RESULT

A preliminary result for the Collins SSA $A_N \equiv -A_{UT}^{\text{coll}}$ was obtained for charged hadrons from an analysis of SMC data on SIDIS from transversely polarized proton and deuteron targets at $Q^2 \sim 5$ GeV$^2$ and $\langle x \rangle \sim 0.08$ with $\langle z \rangle \sim 0.45$ and $\langle p_{t\perp} \rangle \sim (0.5 - 0.8)$ GeV [15]. A non-zero Collins SSA for positive hadrons from a proton target was reported, the other SSA were found compatible with zero within error bars, see Fig. 8.

The charged hadrons at SMC were mainly $\pi^\pm$ [15]. Neglecting the contributions from other hadrons we obtain the results shown as open circles in Fig. 8. Our results are not in conflict with the preliminary SMC results except for the only case where indications for a non-zero SSA were seen, namely for $\pi^+$ from a proton target. Is this a discrepancy and if so, how could one explain it?

Could, for example, the Collins effect of hadrons other than pions be of importance in the SMC kinematics? On the basis of Schäfer-Teryaev sum rule [73] one may, in fact, expect cancellations between contributions of different hadrons in SIDIS where the Collins effect is linear in $H_1^\perp$. (In $e^+e^-$-annihilations the effect is proportional to $H_1^\perp H_1^{\perp \bar{q}}$ and different signs of the Collins functions for different hadrons cancel.)

It is difficult to address these questions — in particular because the results of Ref. [15] retain a preliminary status and their systematic uncertainties are not known. Future data on the Collins effect of other hadrons may clarify the situation.

VII. COMPARISON TO THE EXTRACTION OF REF. [27]

In Ref. [27] $H_1^{\perp (1/2)}(z)$ was extracted from the HERMES data [19] in a similar way as in Sec. II but with some different model assumptions. It is worth to inspect these differences in some more detail. This may shed some light on the model dependence of the two approaches. (Notice the different notation $\delta \tilde{b}^{1/2}/(z) \equiv -2H_1^{1/2}(z)$ in [27].)

There are two main differences between the approach of [27] and ours. First, in [27] it was assumed that the transverse momentum of the hadrons produced in the SIDIS process is solely due to the fragmentation process, i.e. the possibility was disregarded that partons are emitted from the target with non-zero intrinsic transverse momenta. This corresponds to setting $h_1(x, p_{T\perp}) = h_1^{\text{Soff}}(x, p_{T\perp})$ and is equivalent to our Gaussian Ansatz (1) in the limiting case $p_{T\perp} \to 0$ in which $B_{\text{Gauss}} \to 1$ in Eq. (4). Second, for the transversity distribution the saturation of the Soffer sum rule [73] one may, in fact, expect cancellations between contributions.

The extracted analysing power in SIDIS is valid in any model approach where the factorized Ansatz $h_1^{\text{Soff}}(x, p_{T\perp}) = h_1^{\text{Soff}}(x, p_{T\perp}) G(p_{T\perp})$ is assumed (see Sec. II). This was done in [27] and therefore, it is possible to compare our results (12) directly to the fits of Ref. [27] which yield

$$\frac{\langle 2H_1^{\perp (1/2)} \rangle_{\text{fit}}}{\langle D_1^{\text{avr}} \rangle_{\text{Ref. [27]}}} = \begin{cases} (6.1 \pm 0.8)\% & \text{for set I} \\ (6.1 \pm 0.4)\% & \text{for set II,} \end{cases}$$

$$\frac{\langle 2H_1^{\perp (1/2)} \rangle_{\text{fit}}}{\langle D_1^{\text{avr}} \rangle_{\text{Ref. [27]}}} = \begin{cases} -(11.0 \pm 1.3)\% & \text{for set I} \\ -(11.5 \pm 1.4)\% & \text{for set II.} \end{cases}$$

Notice the different convention used in Ref. [15] where the azimuthal distribution of hadrons was analyzed as function of the “Collins angle” $\phi_C = \phi_h + \phi_S - \pi$. Hence, the opposite sign. The same sign convention is used at COMPASS, see caption of Fig. 3.
We stress that (12) and (29) show the same quantity but in different models of transverse hadron momenta. Sets I, II refer to different Ansätze in [27] for the Collins fragmentation function. We observe a good agreement with (12).

We remark that the Ansätze of [27] are of the type \( H_1^{(1/2)}(z) \propto z(1 - z)D_1(z) \). The factor \((1 - z)\) in this Ansatz is based on general considerations in QCD [2] and is “needed” in the approach of [27] to reproduce the tendency of the Collins SSA to decrease (or, at least, not to grow) with increasing \( z \) at HERMES [18, 19]. Interestingly we find that such an Ansatz is not suited to describe the BELLE [21] data which extend to higher \( z \) than the HERMES data [18, 19]. In our Gaussian model for transverse parton momenta, where the mean transverse momentum of the produced hadrons is taken \( z \)-independent, the “needed” decrease of the Collins SSA at larger \( z \) is provided by the factor \( B_{\text{Gauss}} \) (6). Thus, although both approaches describe the \( z \)-dependence of the Collins SSA comparably well, the Collins fragmentation functions extracted here and in [27] differ in particular at large-\( z \). This point nicely illustrates the model dependence both of our results and those of Ref. [27]. Both approaches are not meant to give a fully realistic account of transverse parton momentum dependent process — but are suggested as effective descriptions which are useful at the present stage and will have to be improved upon the impact of future, more precise data.

Notice that the good agreement of the analyzing powers (12) and (29) indicates that there is little sensitivity to the choice of the model for the transversity distribution function. We shall come back to this point below in Sec. IX.

**VIII. COMPARISON TO MODEL CALCULATIONS OF THE COLLINS FUNCTION**

The appearance of any SSA requires the presence of interfering amplitudes with different relative phases [66]. The Collins function is loosely speaking the imaginary part of such interfering amplitudes and this was explored in model calculations in Ref. [2] and further in [67–71]. The common spirit of these calculations is to generate the required imaginary part by a one-loop diagram — using effective quark degrees of freedom, and pions or (abelian) gluons. Such perturbative calculations do not aim at providing a realistic model for the dynamics of the non-perturbative fragmentation process. Rather their importance is to indicate the existence of the mechanism [74]. It is therefore not surprising to observe that the models [67–71] have difficulties to reproduce the quantitative properties and in particular the flavour dependence of \( H_1^d \), though this could possibly be achieved by exploring the freedom in the choice of model parameters. Further studies in this direction are required.

Presumably better suited to catch features of the non-perturbative fragmentation process is the approach of [64] based on the string fragmentation picture. Though quantitatively plagued by uncertainties in the choice and tuning of parameters this approach \( \text{a priori} \) predicted the favoured and unfavoured Collins fragmentation functions to be of similar magnitude and opposite sign. Explanations of this observation have been given in [27] on the basis quark hadron duality considerations.

**IX. HOW MUCH DO WE KNOW ABOUT THE TRANSVERSITY DISTRIBUTION?**

After the discussion of the picture of the Collins function emerging on the basis of experimental information [16–21], it is worthwhile asking how much do we really know about the transversity distribution. In other words, to what extent do HERMES and COMPASS data [17–20] on the Collins SSA in SIDIS constrain \( h_1^q(x) \)?

In order to gain some insight into this question let us make the following exercise. We repeat the extraction of the analyzing power from the preliminary HERMES data [19] and use for \( h_1^q(x) \) the prediction from the chiral quark soliton model [26] but for \( h_1^q(x) \) we use instead the model

\[
\hat{h}_1^q(x) = r \ h_1^q(x) \biggr|^\text{Ref. [26],}
\]

The extracted result for \( \langle 2B_{\text{Gauss}} H_1^d \rangle \) is then a function of the parameter \( r \) and its sensitivity to \( r \) is shown in Fig. 9. Our actual result based on the model [26] is given for \( r = 1 \) and its 1-\( \sigma \) regions are indicated in Fig. 9 by dashed lines. We make the remarkable observation that the extracted results for the favoured and unfavoured \( \langle 2B_{\text{Gauss}} H_1^d \rangle \) are nearly insensitive to \( r \). Varying \( |h_1^q(x)| \) between zero and four (and more) times the modulus of \( |h_1^q(x)| \) as predicted by the chiral quark soliton model [26] yields the same results for \( \langle 2B_{\text{Gauss}} H_1^d \rangle \) within 1-\( \sigma \) uncertainty. This explains also why our results and those of Ref. [27] agree, as discussed in Sec. VII, in spite of the different models in particular for the \( d \)-quark transversity: \( h_1^d(x) < 0 \) in our approach [26] vs. \( h_1^d(x) > 0 \) in [27] from saturating the Soffer bound and choosing the positive sign. Sensitivity to the smaller (in the model [26]) transversity antiquarks is far weaker.

Notice that the Soffer bound severely constrains the region of \( r \) in Fig. 9. This means that the HERMES data [18, 19] leave \( h_1^d(x) \) practically unconstrained. The Soffer bound provides basically all we know at present about the \( d \)-quark transversity, see Fig. 10.
The situation is different for the $u$-quark transversity. Varying $h_1^u(x)$ within 30% of the chiral quark soliton model prediction (which is within the model accuracy) would result in a 30% variation of both the favoured and unfavoured $\langle 2B_{\text{Gauss}}H_1^u \rangle$. Variations much larger than that would spoil the good agreement with the BELLE data in Sec. IV. Thus, the model prediction [26] for $h_1^u(x)$ seems to fit — within its theoretical uncertainty and within the uncertainties of our study — precisely in what is needed to explain simultaneously the HERMES [18, 19] and BELLE [21] data with a compatible Collins fragmentation function. This gives rise to the picture for $h_1^u(x)$ shown in Fig. 10.

To conclude the HERMES data [18, 19] on the proton Collins SSA provide constraints for $h_1^u(x)$ but leave $h_1^d(x)$ practically unconstrained. This is not unexpected considering the $u$-quark dominance in the proton. In order to learn more about $h_1^d(x)$ and eventually face the issue of a flavour separation data from different targets are required. The Collins SSA from a deuteron target is smaller than from a proton target and more difficult to measure as was shown by COMPASS [20]. It might be promising to explore a $^3\text{He}$ target, see below Sec. X.

Further data on transverse target SSA in SIDIS is expected from COMPASS, HERMES and the JLab experiments. COMPASS plans to measure with a proton target, and HERMES with a deuteron target. Thus, the two experiments will supplement the results [17–20] in the respectively different kinematical regions. The best fit results to the data [17–20] in Figs. 3 and 7 show basically the predictions from our study for these experiments for charged pions.

The Collins SSA for neutral pions is an interesting observable. Due to isospin symmetry this observable provides information which is already contained in the charged pions SSAs. However, being an independent experimental information it provides a valuable cross check for our understanding of the Collins function. If our present picture of the Collins function is correct, one may expect the $\pi^0$ Collins SSA to be rather small. Our prediction for the $\pi^0$ Collins effect on a proton target is shown in Fig. 11a. (On a deuteron target the effect is even smaller.) Preliminary HERMES data for the $\pi^0$ Collins SSA were shown in [17] and indicate an effect compatible with zero within large uncertainties.

Rather interesting would be data on kaon Collins SSA. These would not only allow to test whether pions and kaons exhibit as Goldstone bosons of chiral symmetry breaking a similar Collins effect, see Eq. (28) and the subsequent discussion. The understanding of kaon Collins fragmentation is required to test the Schäfer-Teryaev sum rule [73].

The Collins SSA for pions can also be measured in the CLAS and HALL-A experiments at JLab on different targets in a somehow different kinematics at $Q^2 \sim 2\text{ GeV}^2$ and for $0.15 < x < 0.5$ which would provide further important information [75, 76]. For example, the HALL-A collaboration will measure with a transversely polarized $^3\text{He}$ target which — after nuclear corrections — will provide data on the neutron Collins (and Sivers) effect. On the basis of the numbers in Eqs. (10, 11) we estimate for the pion Collins SSA on a “neutron target” the results shown in Figs. 11b and 11c as shaded regions. The estimates of the SSA could be even more optimistic because in the HALL-A experiment larger $z = (0.4 – 0.6)$ will be probed than at HERMES, and the effect tends to increase at larger $z$, see Fig. 7. The displayed “error bars” in Fig. 11b are projections for 24 days of beam time from Ref. [76].

The $\pi^\pm$ Collins SSA on neutron is of similar magnitude as the $\pi^\mp$ Collins SSA on proton. However, the pion Collins SSAs on neutron are far more sensitive to $h_1^d(x)$. In order to illustrate this point we present in Figs. 11b and 11c (dotted lines) the result which follows from assuming that $h_1^d(x)$ has opposite sign to the chiral quark soliton model prediction [26] (in which $h_1^d(x)$ is negative). Clearly, on the basis of the HALL-A data it will be possible to discriminate the different scenarios, and to constrain $h_1^d(x)$ more strongly. HALL-A will also be able to measure the Sivers effect for kaons [76].

Before discussing the predictions for azimuthal asymmetries in $e^+e^-$ annihilation (via $\gamma^*$) let us first establish how many different observables exist in the fragmentation into pions. There are free independent observables, namely

$$A_1^\pi(z_1, z_2) = \pi_1^+ \cdot \pi_2^- = \pi_2^- \cdot \pi_1^+ = \left\{ \pi_1^+ \cdot \pi_2^- + \pi_1^- \cdot \pi_2^+ \right\}$$  \hspace{1cm} (31)

$$A_2^\pi(z_1, z_2) = \pi_{1/2}^+ \cdot \pi_{1/2}^- = \pi_1^- \cdot \pi_2^- = \left\{ \pi_1^+ \cdot \pi_2^- + \pi_1^- \cdot \pi_2^+ \right\}$$  \hspace{1cm} (32)

$$A_1^\pi(z_1, z_2) = \pi_1^+ \cdot \pi_2^0 = \pi_1^- \cdot \pi_2^0 = \left\{ \pi_1^+ \cdot \pi_2^0 + \pi_1^- \cdot \pi_2^0 \right\}$$  \hspace{1cm} (33)

X. PREDICTIONS FOR ONGOING AND FUTURE EXPERIMENTS

FIG. 10: $xh_1^u(x)$ vs. $x$ in the kinematic range of the HERMES experiment. The dashed/dotted lines are predictions for $h_1^u/h_1^d$ from the chiral quark soliton model [26]. The shaded range for the $u$-quark is bound from above by the Soffer bound and from below by the estimated theoretical uncertainty of the model result [26]. The shaded range for the $d$-quark is due to the Soffer bound. The Figure demonstrates roughly the emerging picture of the transversity distribution.
A measured) [21], see Sect. III. The third asymmetry obtained by summing up all charged pion events — as indicated in the second line of Eq. (33). Thus, this contains the same information. Noteworthy, thanks to isospin symmetry the same information is contained in the fragmented quarks. (We discard the less convenient possibility of combining the events for charged and neutral pions.)

The dashed lines indicate tentatively the effects of a different scenario for $H_1^Q(x)$, see text. (c) The same as (b) but for $\pi^+$.  

The arrows indicate the opposite jets to which the pions $\pi_i$ belong, which carry the momentum fractions $z_i$ of the fragmented quarks. (We discard the less convenient possibility of combining the events for charged and neutral pions.) These asymmetries are given by

$$
A_C^U = 1 + \cos(2\phi_1) f_0 \left\{ 5H_2^{f_{42}} H_3^{f_{42}} + 5H_2^{\text{unf}} H_3^{\text{unf}} + 2H_1^s H_2^s \right\} / \{H^a \to D_1^a\}
$$

$$
A_L^L = 1 + \cos(2\phi_1) f_0 \left\{ 5H_2^{f_{42}} H_3^{f_{42}} + 5H_2^{\text{unf}} H_3^{\text{unf}} + 2H_1^s H_2^s \right\} / \{H^a \to D_1^a\}
$$

$$
A_C^C = 1 + \cos(2\phi_1) f_0 \left\{ 5H_2^{f_{42}} + H_2^{\text{unf}} \right\} \left\{ H_2^{f_{42}} + H_2^{\text{unf}} + 4H_1^s H_2^s \right\} / \{H^a \to D_1^a\}
$$

where $f_0 = \sin^2\theta_2/(1 + \cos^2\theta_2)$ and $H_2^a = C_{\text{Gauss}} H_1^{(1/2)a}(z)$ in our approach.

The unlike- and like-sign asymmetries $A_C^U$ and $A_C^C$ were measured at BELLE (more precisely, the ratio $A_C^U / A_C^L$ was measured) [21], see Sect. III. The third asymmetry $A_C^C$ can be accessed by observing a neutral and a charged pion as shown in the first line of Eq. (33). Actually, a linear combination of the observables in the first line of Eq. (33) contains the same information. Noteworthy, thanks to isospin symmetry the same information is contained in the asymmetry obtained by summing up all charged pion events — as indicated in the second line of Eq. (33). Thus, this alternative way allows to access the information content of $A_C^C$ possibly more easily — considering the more difficult $\pi^0$ detection.

To get rid of hard gluon and detector effects, see Sect. III, one may consider the double ratio

$$
\frac{A_C}{A_L} = 1 + \cos(2\phi_0) P_c(z_1, z_2), \quad (35)
$$

where $P_c(z_1, z_2)$ is given to leading order in the analyzing power by

$$
P_c(z_1, z_2) = f_0 \left[ \frac{5(H_2^{f_{42}} + H_2^{\text{unf}})(H_3^{f_{42}} + H_3^{\text{unf}}) + 4H_1^s H_2^s}{\{H^a \to D_1^a\}} \right.

\quad \left. - \frac{5H_2^{f_{42}} H_3^{f_{42}} + 5H_2^{\text{unf}} H_3^{\text{unf}} + 2H_1^s H_2^s}{\{H^a \to D_1^a\}} \right] \quad (36)
$$

in the notation introduced in the context of Eq. (34).

The predictions for $P_c(z_1, z_2)$ made on the basis of our study of the BELLE data [21] in Sect. III are shown in Fig. 12. We observe a qualitatively similar picture to that in Fig. 6 for the observable $P_5(z_1, z_2)$ defined in Eqs. (16, 17). Thus, $P_c(z_1, z_2)$ could be seen in the BELLE experiment in a similarly clear way as $P_5(z_1, z_2)$. We stress that $P_c(z_1, z_2)$ contains independent experimental information which would provide valuable constraints allowing to better pin down the Collins function and to discriminate between $H_1^{f_{42}}$ and $H_1^{\text{unf}}$ using less constrained Ansätze than e.g. (18).
XI. CONCLUSIONS

We presented a study of data on azimuthal asymmetries due to the Collins effect in SIDIS and in $e^+e^-$ annihilation. Our investigation bears considerable theoretical uncertainties due to the neglect of soft factors, Sudakov effects, scale dependence and choice of models for transverse parton momenta and the transversity distribution. We tried to take into account these uncertainties and to minimize them wherever possible. For example in order to confront results from experiments performed at different scales we preferred to compare appropriately defined ratios of spin-dependent to spin-independent quantities which might be expected to be less scale dependent than the respective absolute numbers.

We observed that — within the uncertainties of our study — the SIDIS data from HERMES [17–19] and COMPASS [20] on the Collins SSA from proton and deuteron targets are in agreement with each other and with BELLE data [21] on the azimuthal asymmetry in $e^+e^-$ annihilation, and seem compatible with the indications from DELPHI [16] for an azimuthal asymmetry in $e^+e^-$ annihilation at the $Z^0$-peak. The emerging picture for the so far unknown functions is as follows.

- The Collins fragmentation functions show the behaviour $H_1^{C(1/2)} \propto zD_1(z)$ at small $z$ as found here and in [27]. Within our model for transverse momenta this relation remains valid also at large $z$, but not in the approach of [27]. The favoured and unfavoured Collins fragmentation functions are of comparable magnitude but have opposite sign. SIDIS data require the favoured Collins fragmentation to be positive in the convention [77].

- The $u$-quark transversity distribution is positive and close (within 30%) to saturating the Soffer bound. In contrast, $h_1^q(x)$ and antiquark transversities are entirely unconstrained by present data. At the present stage the chiral quark soliton model [26] provides — within its theoretical uncertainty — a useful estimate for $h_1^q(x)$.

Future data from SIDIS and $e^+e^-$ colliders will help to refine and improve this first picture. Useful for that will be the confrontation of these data with the estimates for observables in SIDIS and $e^+e^-$ annihilation we presented which can be measured in running and planned experiments at HERMES, COMPASS, CLASS, HALL-A and BELLE.

Particularly helpful would be data on double transverse spin asymmetries in Drell-Yan lepton pair production from RHIC [78] and the planned PAX experiment at GSI [79] which could provide direct information on $h_2^q(x)$. In particular, the PAX experiment — making benefit from a polarized antiproton beam — could provide valuable constraints on the $u$-quark transversity distribution [80]. Only the combined analysis of SIDIS, $e^+e^-$ and Drell-Yan data will provide the full picture of the Collins fragmentation and transversity distribution function.

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