In memory of Alexei Pokrovskii: 
his hysteresis effect on economics

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Abstract. This memorial note assesses the impact of Alexei Pokrovskii, a pioneer in the 
mathematical analysis of systems with hysteresis, on economic analysis. Apart from the seminal 
contributions made, Alexei was a source of inspiration and encouragement to others seeking to 
understand the imprints of hysteresis on economic behaviour.

I first came across Alexei’s name as the co-author of Systemy s Gisteresisom (Krasnosel’skii and 
Pokrovskii, 1983), having obtained a photocopy of this seminal work by way of the inter-library 
loan service. At the time, the mid-1980s, I was looking for a mathematical analysis of systems 
with hysteresis that was more interesting than the unit root special case of a linear difference 
equation that I had been using since the late 1970s in an attempt to understand economic 
fluctuations, particularly those in unemployment. Apart from the problems in applying unit 
root analysis to a bounded variable such as the rate of unemployment, the elephantine memory 
properties were implausible. My knowledge of the Russian language, however, was even more 
lacking than my knowledge of the mathematics.

It took a chance conversation with Marek Niezgodka in Warsaw in 1990, when I was 
working for the Polish National Bank, to set me on the path leading to Alexei. Marek had 
translated Systemy s Gisteresisom into Systems with Hysteresis (Krasnosel’skii and Pokrovskii, 
1989). This book paved the way for an economist such as myself to see hysteresis as a general 
systems property that could maybe be applied to economic systems. I wrote papers that used 
the brilliant Isaac Mayergoyz (1991) staircase partition simplification of the Krasnosel’skii - 
Pokrovskii analysis to show how economic systems could be haunted by a selective, erasable 
memory of demand shocks (see, for example, Cross, 1993). I sent the papers to Alexei who 
responded with his characteristic enthusiasm. He subsequently became not only my mentor on 
matters relating to hysteresis but also a good friend.

In this memoir I will attempt to sketch how Alexei came to make a valuable contribution 
to understanding how economic systems respond to shocks such as the financial bubble that 
imploded on the world stage in 2007. To fill in the background to this interdisciplinary aspect of 
Alexei’s work, it is useful to begin with the coining of the term ‘hysteresis’. I will then discuss 
the flaws in conventional economic analysis that hysteresis explanations attempt to redress. Alexei’s 
contributions are then considered in this context.
1. The Coining of the Term Hysteresis
The intellectual curiosity across a wide range of disciplines that was the hallmark of Alexei was evident in the coining of the term ‘hysteresis’. Many of the key players were from Scotland and Alexei took a keen interest in this background. The point of departure was James Clerk Maxwell’s formalisation of the concept of a field of force. Incredibly, Maxwell had become redundant when the University of Aberdeen merged its two colleges and decided that one of the two physics professors had to go! Various experimenters observed violations of Maxwell’s equations in magnetic fields involving ferric metals, the fields not returning to their previous states once a temporary magnetising force was removed (see Glazebrook, 1935 for this background).

James Alfred Ewing, originally from Dundee, was one of the researchers who observed violations of homeostasis and reversibility in such fields. He coined the term ‘hysteresis’ to describe such effects in a paper published in 1881: “...these curves exhibit, in a striking manner, a persistence of previous state... to this action... the author now gives the term Hysteresis.” (J A Ewing, 1881, p. 22). His assessor for the Royal Society of London paper was the University of Glasgow professor, Sir William Thomson (later Lord Kelvin). Thomson objected to the use of the mysterious term ‘hysteresis’, suggesting that the phrase ‘effects of retentiveness’ be used instead. Ewing stuck to his guns, arguing that ‘hysteresis’ was a generic phenomenon that would be observed in a wider range of contexts than electromagnetism (see A W Ewing, 1939).

Ewing’s prescience became evident in Alexei’s work on formalising hysteresis as a general systems property (Krasnosel’skii and Pokrovskii, 1983, 1989), and in applying the properties to a wide set of problems such as soil wetting and drying processes (Applebe et al, 2009) and trade cycles (McNamara and Pokrovskii, 2008).

2. Mainstream Economic Analysis
Mainstream economic analysis, which has encountered particularly searching criticism since the world financial crisis and subsequent recessions began in 2007, does not incorporate hysteresis effects. The analytical methods used in neoclassical economics involve conservation and reversibility properties. At the micro level this means that supply-side shocks, such as the burning of cotton fields during the US Civil War, have no lasting effects on the structure of demand, consumer tastes reverting to the status quo ante once the cotton fields are replanted, despite the plausible shifts in demand arising from learning-by-consuming non-cotton materials during the disturbance. And demand shifts, such as those arising form increased demand for air travel in the 1960s, have no lasting effects on the supply conditions, despite the learning-by-producing efficiency gains in aircraft production. At the macro level this means that recessions have no lasting deleterious effects on potential output and employment, despite the debilitating effects of recessions on bankrupted firms and unemployed workers. And booms in economic activity also have no lasting beneficial effects on potential output, despite the new capital investments made and learning-by-working of employees. Financial markets are also taken to be efficient, asset prices accurately reflecting all the relevant information, and not being haunted by waves of optimism or pessimism about the unknowable future.

Dissidents have argued that this neoclassical system does not explain how actual economic systems behave, and have attempted to inject realism into the complacency of the neoclassical framework. John Maynard Keynes was a notable dissident, arguing that pessimistic ‘animal spirits’, leading consumers and firms to cut back on consumption and investment, can allow economies to become trapped in low output, high unemployment equilibria. While it is not too difficult to formalise Keynes’ precepts in hysteretic terms, Keynes himself did not use the terminology. Keynes and Ewing, the coiners of the term ‘hysteresis’, were contemporaries at King’s College, Cambridge. Whilst there is evidence of conversations between the two great men on international affairs, there is no indication that hysteresis was discussed (A W Ewing,
1939). Alexei, as was his wont, was fascinated by this tale of what might have been. Ewing, after establishing Engineering as an academic discipline at the University of Cambridge, had headed the enemy communications intercept group at the Admiralty during the 1914-18 war, and gone on to become the Principal of the University of Edinburgh. Keynes also had developed a wide portfolio as a man of affairs, so there would have been a variety of other topics of common interest.

3. Allowing for Hysteresis in Economic Systems
Alexei’s initial role in applying hysteresis to economic systems came in the invaluable encouragement he provided to those who were engaged in this task. I first met Alexei in person when he arranged for me to give a talk at a SIAM conference at the University of Edinburgh. Giving a paper at a mathematics conference could have been a daunting experience for a non-mathematician such as myself, but Alexei made the occasion more of an enjoyable next step rather than an ordeal with his interventions during the discussion of the paper. The ordeal, in fact, proved to be one of attempting to explain to a large group of mathematicians the relative merits of the different beers on offer in a real ale bar in Leith. There were at least a dozen brews on offer!

Alexei would pose fundamental questions, such as whether it was plausible to suppose that rate independence characterised the responses of economic agents to input perturbations. And he would guide me through the development of the hysteresis literature, patiently explaining how the Preisach model had been developed, for example.

Around this time I came to supervise the Ph.D. work of Laura Piscitelli at the University of Strathclyde, jointly with Michael Grinfeld of the Department of Mathematics. This thesis, “Hysteresis in Economics” (Piscitelli, 1997) reflected the brilliance of Laura, the valuable supervision of Michael, and that of Harbir Lamba, then a postdoctoral fellow at Strathclyde. A key innovation in the thesis was to devise a means of testing for the existence of hysteresis in time series regressions designed to explain unemployment. Harbir gave important guidance in implementing the programme designed to detect the selective memory of demand and supply side shocks affecting unemployment. Alexei also played an important role in this work, suggesting how sensitivity tests of the Preisach weighting function might help get round the problem of the absence of experimental data regarding agent switching points. At Alexei’s instigation, an extension of this work was later published as Darby, Cross and Piscitelli (2006).

There followed from Alexei an invitation to give a seminar at University College Cork. At the time Alexei had a temporary position there. After the seminar I got talking to Michael Mortell who, at the time, was President of UCC. Mike made some typically searching comments on the paper I had presented, but then we started talking about Alexei’s importance in the academic world. Alexei must have subsequently heard a summary of this conversation from Mike. I must have said something along the following lines: “First there was Isaac Newton, then James Clerk Maxwell, next Albert Einstein, now there is Alexei Pokrovskii!”. The implication was that UCC should offer Alexei a permanent position forthwith! Alexei took great delight in this story. This was not because he had the heightened sense of self-importance that plagues some academics. He did not suffer from such an affliction, being down-to-earth in the manner of all the truly great academics I have known. Instead, Alexei’s delight in this story reflected his keen sense of humour and appreciation of the absurd.

There followed regular visits to Cork for the MURPHYS conferences organised by Alexei and his co-researchers. Alexei established a research team of truly international significance that provided a welcome to interdisciplinary researchers. The trips to the bars of Cork, and to the fishmongers for raw fish to ingest with the beer, Russian-style, played an important role in establishing an esprit de corps for the Cork group and the leading world experts who gravitated to this centre of excellence.
But these meetings were also a serious engine of discovery. To take but one example in which I was directly involved, prolonged discussion revealed that the Preisach model needed to be revised for application to economic systems by allowing agent switching points to vary over time. The outcome, for which Alexei and Sasha Krasnosel’skii generously offered me co-authorship, was Cross, Krasnosel’skii and Pokrovskii (2001). This was an example of Alexei’s genius in being able to modify the mathematical analysis to deal with the circumstances of the application.

4. Alexei and Economics
An international forum that Alexei introduced me to was the bi-annual Hysteresis and Micromagnetics conference. He arranged an invitation for me to give a paper at the Washington DC conference in 2001. Problems with Alexei’s visa meant that he arrived late, but just in time to hear my presentation. This applied the Preisach model to the Economic and Monetary Union project initiated in Europe in 1999 (Cross, 2000). One of the implications was that interest rates would be too low in booming economies such as Ireland, with inevitable dire consequences once the ensuing property bubble burst. As with many implications of economic models, this was treated with suspicion— and took until 2007 to be borne out! Again, this was Alexei opening doors for others, allowing me to meet other greats of the hysteresis profession such as Isaac Mayergoyz.

The Micromagnetics and Hysteresis conference in Naples in 2007 honoured Alexei with a presidential address. It was at about this time that Alexei had started to expand the economics component of his research portfolio. Maybe it was the growing signs of major turbulence in the world economy that stimulated his heightened interest. Alexei chose to unveil some new economic models in his presidential address (Cross, Pokrovskii and Raschinskii, 2008, and Cross, McNamara and Pokrovskii, 2008). The models were based on a modification of Darcy’s Law, with output flows responding to differences between potential and actual demand. Hysteresis in output flows arose from differences between behaviour in booms - the ‘wetting’ phase - and in recessions - the ‘drying’ phase. Alexei showed that these models followed the ‘hydraulic’ tradition in economic modelling, surprising many in the audience with photographs of working models that had been built by the economists Irving Fisher and Bill Phillips to illustrate the determination of relative prices, and of aggregate output. The difference was that Alexei’s equations involved non-reversible paths for the flows within economic systems rather than the reversible flows involved in the Fisher and Phillips’ models.

Alexei’s supervision of Hugh McNamara’s Ph.D thesis at Cork involved extending these models. By this time recessions had come to afflict many economies with a vengeance after the implosion of the financial bubble in 2007. Ireland, for example, experienced the largest cumulative drop in output, in percentage terms, suffered by an advanced economy since the Great Depression of the early 1930s. Mainstream economic models assume that such recessions have only temporary effects, with output subsequently recovering to its previous growth path. The models that Alexei designed do not have this felicitous property but instead have recessions bringing lasting curses by displacing potential output levels downwards. The evidence from the last three years suggests that Alexei was right. One implication of this analysis is that fiscal retrenchment in recessions can be self-defeating. In a recent paper, ‘Memory of Recessions’, (Cross, McNamara and Pokrovskii, 2010), we chronicled the evidence on how the recessions that usually follow financial crises displace potential output paths downwards, and showed how hysteretic models, in which the rate of change of output responds to demand gradients, can mimic the key features of this evidence. Alexei’s work may well play an important role in the reformulation, with Keynesian properties, of economic models of macroeconomic fluctuations.

The last time I saw Alexei in person was when he attended an Institute of Advanced Studies workshop in Glasgow in the summer of 2009. This “Limits to Rationality in Financial Markets”
workshop was co-organised with Michael Grinfeld and Harbir Lamba. Alexei was brimming with ideas as to how hysteresis models could throw light on how the financial crisis had emerged, and on its aftermath. Along with Gary Friedman, we had begun to work on the application of return-point memory models to interbank interest rates and their effects on bank lending.

Alexei had a most distinctive manner of speaking. Instead of using full stops, he would punctuate his sentences with a variety of smiles, ranging from the rueful to the exuberant. When we talked on the telephone I could always picture the type of smile on his face. I will miss those conversations.

In good biblical tradition, Alexei saved some of the best until the last. The day before he died he sent me the last draft of a paper entitled ‘Periodic Sequences of Arbitrage: A Tale of Four Currencies’ (2010) which Victor Kozyakin, Brian O’Callaghan, Alexei’s son Alexey Pokrovskiy, myself and Alexei had been working on over the summer. Mainstream theories in economics and finance assume that all profitable arbitrage opportunities are exploited so that the ‘law of one price’ always holds. The evidence suggests significant and persistent deviations of actual goods and asset prices from this law. This paper uses combinatorial analysis to show that arbitrage sequences can be periodic in nature, rather than involving a smooth convergence towards a law of ‘one price’. The implications of these results for equilibrium theorising in economics and finance are profound. The mathematical techniques for analysing asynchronous systems used in this paper could well prove to have a wide set of applications to other problems in economics and finance.

The peer review jury is still out, but one eminent US economist has described the results as ‘novel’ and ‘beautiful’. These adjectives provide a fitting description of the contributions Alexei has made to the difficult task of understanding how economic systems behave. Others will continue Alexei’s work. We will be sadder for his absence, but much wiser for the legacy he has bequeathed. Alexei’s ship may have passed beyond the horizon but, in a very real sense, the ship still sails on.

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