Hamiltionian Mechanic Systems on the
Standard Cliffordian Kähler Manifolds

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February 23, 2009

Abstract

This study introduces standard Cliffordian Kähler analogue of Hamiltonian mechanic systems. In the end, the some results related to standard Cliffordian Kähler dynamical systems are also discussed.

Keywords: Cliffordian Kähler Geometry, Hamiltonian Mechanic Systems.

PACS: 02.40.
1 Introduction

It is well-known that modern differential geometry explains explicitly the dynamics of Hamiltonians. Therefore, if $Q$ is an $m$-dimensional configuration manifold and $H : T^*Q \to \mathbb{R}$ is a regular Hamiltonian function, then there is a unique vector field $X$ on $T^*Q$ such that dynamic equations are given by

$$i_X \Phi = dH \quad (1)$$

where $\Phi$ indicates the symplectic form. The triple $(T^*Q, \Phi, X)$ are called Hamiltonian system on the cotangent bundle $T^*Q$.

Nowadays, there are a lot of studies about Hamiltonian mechanics, formalisms, systems and equations [1, 2] and there in. There are real, complex, paracomplex and other analogues. We say that in order to obtain different analogous in different spaces is possible.

Quaternions were invented by Sir William Rowan Hamilton as an extension to the complex numbers. Hamiltonian’s defining relation is most succinctly written as:

$$i^2 = j^2 = k^2 = ijk = -1 \quad (2)$$

If it is compared to the calculus of vectors, quaternions have slipped into the realm of obscurity. They do however still find use in the computation of rotations. A lot of physical laws in classical, relativistic, and quantum mechanics can be written pleasantly by means of quaternions. Some physicists hope they will find deeper understanding of the universe by restating basic principles in terms of quaternion algebra. It is well-known that quaternions are useful for representing rotations in both quantum and classical mechanics [3]. We say that Cliffordian manifold is quaternion manifold. Therefore, all properties defined on quaternion manifold of dimension $8n$
also is valid for Cliffordian manifold. Thus, it is possible to construct mechanical equations on Cliffordian Kähler manifold.

The paper is structured as follows. In second 2, we recall Cliffordian Kähler manifolds. In second 3 we introduce Hamiltonian equations related to mechanical systems on Cliffordian Kähler manifold. In conclusion, we discuss some geometrical and physical results about Hamiltonian equations and fields obtained on the base manifold.

2 Preliminaries

Hereafter, all mappings and manifolds are assumed to be smooth, i.e. infinitely differentiable and the sum is taken over repeated indices. By \( \mathcal{F}(M) \), \( \chi(M) \) and \( \Lambda^1(M) \) we denote the set of functions on \( M \), the set of vector fields on \( M \) and the set of 1-forms on \( M \), respectively.

2.1 Cliffordian Kähler Manifolds

Here, we recalled the main concepts and structures given in [4, 5]. Let \( M \) be a real smooth manifold of dimension \( m \). Suppose that there is a 6-dimensional vector bundle \( V \) consisting of \( F_i(i = 1, 2, ..., 6) \) tensors of type (1,1) over \( M \). Such a local basis \( \{F_1, F_2, ..., F_6\} \) is called a canonical local basis of the bundle \( V \) in a neighborhood \( U \) of \( M \). Then \( V \) is called an almost Cliffordian structure in \( M \). The pair \( (M, V) \) is named an almost Cliffordian manifold with \( V \). Hence, an almost Cliffordian manifold \( M \) is of dimension \( m = 8n \). If there exists on \( (M, V) \) a global basis \( \{F_1, F_2, ..., F_6\} \), then \( (M, V) \) is said to be an almost Cliffordian manifold; the basis \( \{F_1, F_2, ..., F_6\} \) is called a global basis for \( V \).

An almost Cliffordian connection on the almost Cliffordian manifold \( (M, V) \) is a linear
connection $\nabla$ on $M$ which preserves by parallel transport the vector bundle $V$. This means that if $\Phi$ is a cross-section (local-global) of the bundle $V$, then $\nabla_X \Phi$ is also a cross-section (local-global, respectively) of $V$, $X$ being an arbitrary vector field of $M$.

If for any canonical basis $\{J_1, J_2, ..., J_6\}$ of $V$ in a coordinate neighborhood $U$, the identities

$$g(J_i X, J_i Y) = g(X, Y), \ \forall X, Y \in \chi(M), \ i = 1, 2, ..., 6,$$

hold, the triple $(M, g, V)$ is named an almost Cliffordian Hermitian manifold or metric Cliffordian manifold denoting by $V$ an almost Cliffordian structure $V$ and by $g$ a Riemannian metric and by $(g, V)$ an almost Cliffordian metric structure.

Since each $J_i (i = 1, 2, ..., 6)$ is almost Hermitian structure with respect to $g$, setting

$$\Phi_i(X, Y) = g(J_i X, Y), \ i = 1, 2, ..., 6,$$

for any vector fields $X$ and $Y$, we see that $\Phi_i$ are 6 local 2-forms.

If the Levi-Civita connection $\nabla = \nabla^g$ on $(M, g, V)$ preserves the vector bundle $V$ by parallel transport, then $(M, g, V)$ is called a Cliffordian Kähler manifold, and an almost Cliffordian structure $\Phi_i$ of $M$ is called a Cliffordian Kähler structure. A Clifford Kähler manifold is Riemannian manifold $(M^{8n}, g)$ For example, we say that $\mathbb{R}^{8n}$ is the simplest example of Clifford Kähler manifold. Suppose that let $\{x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}\}$, $i = 1, n$ be a real coordinate system on $\mathbb{R}^{8n}$. Then we define by $\left\{\frac{\partial}{\partial x_i}, \frac{\partial}{\partial x_{n+i}}, \frac{\partial}{\partial x_{2n+i}}, \frac{\partial}{\partial x_{3n+i}}, \frac{\partial}{\partial x_{4n+i}}, \frac{\partial}{\partial x_{5n+i}}, \frac{\partial}{\partial x_{6n+i}}, \frac{\partial}{\partial x_{7n+i}}\right\}$ and $\{dx_i, dx_{n+i}, dx_{2n+i}, dx_{3n+i}, dx_{4n+i}, dx_{5n+i}, dx_{6n+i}, dx_{7n+i}\}$ be natural bases over $\mathbb{R}$ of the tangent space $T(\mathbb{R}^{8n})$ and the cotangent space $T^*(\mathbb{R}^{8n})$ of $\mathbb{R}^{8n}$, respectively. By structures
$J_1, J_2, J_3$, the following expressions are obtained

\[
J_1 \left( \frac{\partial}{\partial x_i} \right) = \frac{\partial}{\partial x_{n+i}}, \quad J_1 \left( \frac{\partial}{\partial x_{n+i}} \right) = -\frac{\partial}{\partial x_i}, \quad J_1 \left( \frac{\partial}{\partial x_{2n+i}} \right) = \frac{\partial}{\partial x_{4n+i}}, \quad J_1 \left( \frac{\partial}{\partial x_{3n+i}} \right) = -\frac{\partial}{\partial x_{5n+i}},
\]

\[
J_1 \left( \frac{\partial}{\partial x_{4n+i}} \right) = -\frac{\partial}{\partial x_{2n+i}}, \quad J_1 \left( \frac{\partial}{\partial x_{5n+i}} \right) = -\frac{\partial}{\partial x_{3n+i}}, \quad J_1 \left( \frac{\partial}{\partial x_{6n+i}} \right) = \frac{\partial}{\partial x_{7n+i}}, \quad J_1 \left( \frac{\partial}{\partial x_{7n+i}} \right) = -\frac{\partial}{\partial x_{6n+i}},
\]

\[
J_2 \left( \frac{\partial}{\partial x_{n+i}} \right) = \frac{\partial}{\partial x_{2n+i}}, \quad J_2 \left( \frac{\partial}{\partial x_{5n+i}} \right) = -\frac{\partial}{\partial x_{4n+i}}, \quad J_2 \left( \frac{\partial}{\partial x_{2n+i}} \right) = -\frac{\partial}{\partial x_i}, \quad J_2 \left( \frac{\partial}{\partial x_{3n+i}} \right) = -\frac{\partial}{\partial x_{5n+i}}.
\]

\[
J_3 \left( \frac{\partial}{\partial x_{n+i}} \right) = \frac{\partial}{\partial x_{3n+i}}, \quad J_3 \left( \frac{\partial}{\partial x_{5n+i}} \right) = -\frac{\partial}{\partial x_{2n+i}}, \quad J_3 \left( \frac{\partial}{\partial x_{2n+i}} \right) = \frac{\partial}{\partial x_{7n+i}}, \quad J_3 \left( \frac{\partial}{\partial x_{3n+i}} \right) = -\frac{\partial}{\partial x_{6n+i}},
\]

A canonical local basis $\{J_1^*, J_2^*, J_3^*\}$ of $V^*$ of the cotangent space $T^*(M)$ of manifold $M$ satisfies the condition as follows:

\[
J_1^{*2} = J_2^{*2} = J_3^{*2} = J_1^* J_2^* J_3^* J_2^* J_1^* = -I,
\]

defining by

\[
J_1^*(dx_i) = dx_{n+i}, \quad J_1^*(dx_{n+i}) = -dx_i, \quad J_1^*(dx_{2n+i}) = dx_{4n+i}, \quad J_1^*(dx_{3n+i}) = dx_{5n+i},
\]

\[
J_1^*(dx_{4n+i}) = -dx_{2n+i}, \quad J_1^*(dx_{5n+i}) = -dx_{3n+i}, \quad J_1^*(dx_{6n+i}) = dx_{7n+i}, \quad J_1^*(dx_{7n+i}) = -dx_{6n+i},
\]

\[
J_2^*(dx_i) = dx_{2n+i}, \quad J_2^*(dx_{n+i}) = -dx_{4n+i}, \quad J_2^*(dx_{2n+i}) = -dx_i, \quad J_2^*(dx_{3n+i}) = dx_{6n+i},
\]

\[
J_2^*(dx_{4n+i}) = dx_{n+i}, \quad J_2^*(dx_{5n+i}) = -dx_{7n+i}, \quad J_2^*(dx_{6n+i}) = -dx_{3n+i}, \quad J_2^*(dx_{7n+i}) = dx_{5n+i},
\]

\[
J_3^*(dx_i) = dx_{3n+i}, \quad J_3^*(dx_{n+i}) = -dx_{5n+i}, \quad J_3^*(dx_{2n+i}) = -dx_{6n+i}, \quad J_3^*(dx_{3n+i}) = -dx_i,
\]

\[
J_3^*(dx_{4n+i}) = dx_{7n+i}, \quad J_3^*(dx_{5n+i}) = dx_{n+i}, \quad J_3^*(dx_{6n+i}) = dx_{2n+i}, \quad J_3^*(dx_{7n+i}) = -dx_{4n+i}.
\]

3 Hamiltonian Mechanics

Here, we obtain Hamiltonian equations and Hamiltonian mechanical system for quantum and classical mechanics structured on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$. 
Firstly, let \((\mathbb{R}^{8n}, V)\) be a standard Cliffordian Kähler manifold. Assume that a component of almost Cliffordian structure \(V^*\), a Liouville form and a 1-form on the standard Cliffordian Kähler manifold \((\mathbb{R}^{8n}, V)\) are shown by \(J_1^*, \lambda_{J_1^*}\) and \(\omega_{J_1^*}\), respectively.

Then
\[
\omega_{J_1^*} = \frac{1}{2}(x_i dx_i + x_{n+i} dx_{n+i} + x_{2n+i} dx_{2n+i} + x_{3n+i} dx_{3n+i} + x_{4n+i} dx_{4n+i} + x_{5n+i} dx_{5n+i} + x_{6n+i} dx_{6n+i} + x_{7n+i} dx_{7n+i})
\]

and
\[
\lambda_{J_1^*} = J_1^*(\omega_{J_1^*}) = \frac{1}{2}(x_i dx_{n+i} - x_{n+i} dx_i + x_{2n+i} dx_{4n+i} + x_{3n+i} dx_{5n+i} - x_{4n+i} dx_{2n+i} - x_{5n+i} dx_{3n+i} + x_{6n+i} dx_{7n+i} - x_{7n+i} dx_{6n+i})
\]

It is well-known that if \(\Phi_{J_1^*}\) is a closed Kähler form on the standard Cliffordian Kähler manifold \((\mathbb{R}^{8n}, V)\), then \(\Phi_{J_1^*}\) is also a symplectic structure on Cliffordian Kähler manifold \((\mathbb{R}^{8n}, V)\).

Consider that Hamilton vector field \(X\) associated with Hamiltonian energy \(H\) is given by
\[
X = X^i \frac{\partial}{\partial x_i} + X^{n+i} \frac{\partial}{\partial x_{n+i}} + X^{2n+i} \frac{\partial}{\partial x_{2n+i}} + X^{3n+i} \frac{\partial}{\partial x_{3n+i}} + X^{4n+i} \frac{\partial}{\partial x_{4n+i}} + X^{5n+i} \frac{\partial}{\partial x_{5n+i}} + X^{6n+i} \frac{\partial}{\partial x_{6n+i}} + X^{7n+i} \frac{\partial}{\partial x_{7n+i}}
\]

Then
\[
\Phi_{J_1^*} = -d\lambda_{J_1^*} = dx_{n+i} \wedge dx_i + dx_{4n+i} \wedge dx_{2n+i} + dx_{5n+i} \wedge dx_{3n+i} + dx_{7n+i} \wedge dx_{6n+i}
\]

and
\[
i_X \Phi_{J_1^*} = \Phi_{J_1^*}(X) = X^{n+i} dx_i - X^i dx_{n+i} + X^{4n+i} dx_{2n+i} - X^{2n+i} dx_{4n+i} + X^{5n+i} dx_{3n+i} - X^{3n+i} dx_{5n+i} + X^{7n+i} dx_{6n+i} - X^{6n+i} dx_{7n+i}
\]

Moreover, the differential of Hamiltonian energy is obtained as follows:
\[
dH = \frac{\partial H}{\partial x_i} dx_i + \frac{\partial H}{\partial x_{n+i}} dx_{n+i} + \frac{\partial H}{\partial x_{2n+i}} dx_{2n+i} + \frac{\partial H}{\partial x_{3n+i}} dx_{3n+i} + \frac{\partial H}{\partial x_{4n+i}} dx_{4n+i} + \frac{\partial H}{\partial x_{5n+i}} dx_{5n+i} + \frac{\partial H}{\partial x_{6n+i}} dx_{6n+i} + \frac{\partial H}{\partial x_{7n+i}} dx_{7n+i}
\]
According to Eq. (11), if equaled Eq. (10) and Eq. (11), the Hamiltonian vector field is found as follows:

\begin{equation}
X = -\frac{\partial H}{\partial x_{n+i}} \frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{n+i}} \frac{\partial}{\partial x_{n+i}} - \frac{\partial H}{\partial x_{2n+i}} \frac{\partial}{\partial x_{2n+i}} - \frac{\partial H}{\partial x_{5n+i}} \frac{\partial}{\partial x_{5n+i}} + \frac{\partial H}{\partial x_{2n+i}} \frac{\partial}{\partial x_{2n+i}} + \frac{\partial H}{\partial x_{5n+i}} \frac{\partial}{\partial x_{5n+i}}
\end{equation}

(12)

Suppose that a curve

\[ \alpha : \mathbb{R} \rightarrow \mathbb{R}^{8n} \]

(13)

be an integral curve of the Hamiltonian vector field \( X \), i.e.,

\[ X(\alpha(t)) = \dot{\alpha}, \quad t \in \mathbb{R}. \]

(14)

In the local coordinates, it is obtained that

\[ \alpha(t) = (x_i, x_{n+i}, x_{2n+i}, x_{3n+i}, x_{4n+i}, x_{5n+i}, x_{6n+i}, x_{7n+i}) \]

(15)

and

\[ \dot{\alpha}(t) = \frac{dx_i}{dt} \frac{\partial}{\partial x_i} + \frac{dx_{n+i}}{dt} \frac{\partial}{\partial x_{n+i}} + \frac{dx_{2n+i}}{dt} \frac{\partial}{\partial x_{2n+i}} + \frac{dx_{3n+i}}{dt} \frac{\partial}{\partial x_{3n+i}} + \frac{dx_{4n+i}}{dt} \frac{\partial}{\partial x_{4n+i}} + \frac{dx_{5n+i}}{dt} \frac{\partial}{\partial x_{5n+i}} + \frac{dx_{6n+i}}{dt} \frac{\partial}{\partial x_{6n+i}} + \frac{dx_{7n+i}}{dt} \frac{\partial}{\partial x_{7n+i}}. \]

(16)

Considering Eq. (14), if equaled Eq. (12) and Eq. (16), it follows

\[ \frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \quad \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_{n+i}}, \quad \frac{dx_{2n+i}}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \quad \frac{dx_{3n+i}}{dt} = \frac{\partial H}{\partial x_{3n+i}}, \quad \frac{dx_{5n+i}}{dt} = -\frac{\partial H}{\partial x_{5n+i}}, \quad \frac{dx_{6n+i}}{dt} = \frac{\partial H}{\partial x_{6n+i}}, \quad \frac{dx_{7n+i}}{dt} = \frac{\partial H}{\partial x_{7n+i}}. \]

(17)

Thus, the equations obtained in Eq. (17) are seen to be Hamiltonian equations with respect to component \( J^*_1 \) of almost Cliffordian structure \( V^* \) on Cliffordian Kähler manifold \((\mathbb{R}^{8n}, V)\), and then the triple \((\mathbb{R}^{8n}, \Phi, J^*_1, X)\) is seen to be a Hamiltonian mechanical system on Cliffordian Kähler manifold \((\mathbb{R}^{8n}, V)\).

Secondly, let \((\mathbb{R}^{8n}, V)\) be a Cliffordian Kähler manifold. Suppose that an element of almost Cliffordian structure \( V^* \), a Liouville form and a 1-form on Cliffordian Kähler manifold \((\mathbb{R}^{8n}, V)\) are denoted by \( J^*_2, \lambda, \omega \), respectively.
Putting

\[
\omega_J^* = \frac{1}{2}(x_i dx_i + x_{n+i} dx_{n+i} + x_{2n+i} dx_{2n+i} + x_{3n+i} dx_{3n+i})
+ x_{4n+i} dx_{4n+i} + x_{5n+i} dx_{5n+i} + x_{6n+i} dx_{6n+i} + x_{7n+i} dx_{7n+i})
\]

we have

\[
\lambda_J^* = J^*_2(\omega_J^*) = \frac{1}{2}(x_i dx_{2n+i} - x_{n+i} dx_{4n+i} - x_{2n+i} dx_i + x_{3n+i} dx_{6n+i})
+ x_{4n+i} dx_{n+i} - x_{5n+i} dx_{7n+i} - x_{6n+i} dx_{3n+i} + x_{7n+i} dx_{5n+i}).
\]

Assume that \(X\) is a Hamiltonian vector field related to Hamiltonian energy \(H\) and given by Eq. (8).

Considering

\[
\Phi_J^* = -d\lambda_J^* = dx_{n+i} \land dx_{4n+i} + dx_{2n+i} \land dx_i + dx_{5n+i} \land dx_{7n+i} + dx_{6n+i} \land dx_{3n+i}, \quad (18)
\]

then we calculate

\[
i_X \Phi_J^* = \Phi_J^* (X) = X^{n+i} dx_{n+i} - X^{4n+i} dx_{4n+i} + X^{3n+i} dx_i - X^i dx_{2n+i}
+ X^{5n+i} dx_{7n+i} - X^{7n+i} dx_{5n+i} + X^{6n+i} dx_{3n+i} - X^{3n+i} dx_{6n+i}.
\]

According to Eq. (11), if we equal Eq. (11) and Eq. (19), it follows

\[
X = -\frac{\partial H}{\partial x_{2n+i}} \frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{4n+i}} \frac{\partial}{\partial x_{n+i}} + \frac{\partial H}{\partial x_{2n+i}} \frac{\partial}{\partial x_{2n+i}} - \frac{\partial H}{\partial x_{6n+i}} \frac{\partial}{\partial x_{3n+i}}
- \frac{\partial H}{\partial x_{n+i}} \frac{\partial}{\partial x_{4n+i}} + \frac{\partial H}{\partial x_{7n+i}} \frac{\partial}{\partial x_{5n+i}} + \frac{\partial H}{\partial x_{3n+i}} \frac{\partial}{\partial x_{6n+i}} - \frac{\partial H}{\partial x_{5n+i}} \frac{\partial}{\partial x_{7n+i}}.
\]

Considering Eq. (14), Eq. (16) and Eq. (20) are equal, we find equations

\[
\frac{dx_i}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \quad \frac{dx_{n+i}}{dt} = -\frac{\partial H}{\partial x_{4n+i}}, \quad \frac{dx_{2n+i}}{dt} = -\frac{\partial H}{\partial x_{2n+i}}, \quad \frac{dx_{3n+i}}{dt} = -\frac{\partial H}{\partial x_{6n+i}},
\]

\[
\frac{dx_{4n+i}}{dt} = -\frac{\partial H}{\partial x_{n+i}}, \quad \frac{dx_{5n+i}}{dt} = -\frac{\partial H}{\partial x_{7n+i}}, \quad \frac{dx_{6n+i}}{dt} = -\frac{\partial H}{\partial x_{3n+i}}, \quad \frac{dx_{7n+i}}{dt} = -\frac{\partial H}{\partial x_{5n+i}}.
\]
In the end, the equations obtained in Eq. (21) are known to be Hamiltonian equations with respect to component $J_2^*$ of standard almost Cliffordian structure $V^*$ on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$, and then the triple $(\mathbb{R}^{8n}, \Phi_{J_3^*}, X)$ is a Hamiltonian mechanical system on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$.

Thirdly, let $(\mathbb{R}^{8n}, V)$ be a standard Cliffordian Kähler manifold. By $J_3^* \lambda_{J_2^*}$ and $\omega_{J_2^*}$, we denote a component of almost Cliffordian structure $V^*$, a Liouville form and a 1-form on Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$, respectively.

Let $\omega_{J_3^*}$ be given by

$$
\omega_{J_3^*} = \frac{1}{2}(x_i dx_i + x_{n+i} dx_{n+i} + x_{2n+i} dx_{2n+i} + x_{3n+i} dx_{3n+i} + x_{4n+i} dx_{4n+i} + x_{5n+i} dx_{5n+i} + x_{6n+i} dx_{6n+i} + x_{7n+i} dx_{7n+i})
$$

Then it holds

$$
\lambda_{J_2^*} = J_2^*(\omega_{J_2^*}) = \frac{1}{2}(x_i dx_{3n+i} - x_{n+i} dx_{5n+i} - x_{2n+i} dx_{6n+i} - x_{3n+i} dx_i + x_{4n+i} dx_{7n+i} + x_{5n+i} dx_{n+i} + x_{6n+i} dx_{2n+i} - x_{7n+i} dx_{4n+i}).
$$

It is well-known that if $\Phi_{J_3^*}$ is a closed Kähler form on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$, then $\Phi_{J_3^*}$ is also a symplectic structure on Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$.

Consider $X$. It is Hamiltonian vector field connected with Hamiltonian energy $H$ and given by Eq. (8).

Taking into

$$
\Phi_{J_3^*} = -d\lambda_{J_3^*} = dx_{3n+i} \wedge dx_i + dx_{n+i} \wedge dx_{5n+i} + dx_{2n+i} \wedge dx_{6n+i} + dx_{7n+i} \wedge dx_{4n+i}, \quad (22)
$$
we find

\begin{equation}
    i_X \Phi_{J_3} = \Phi_{J_3}(X) = X^{3n+i} dx_i - X^n dx_{n+i} + X^n dx_{5n+i} - X^n dx_{5n+i} \tag{23}
    \end{equation}

\begin{equation}
    + X^n dx_{6n+i} - X^n dx_{2n+i} + X^n dx_{4n+i} - X^n dx_{7n+i}.
    \end{equation}

According to Eq. (1), Eq. (11) and Eq. (23) are equaled, we obtain a Hamiltonian vector field given by

\begin{equation}
    X = - \frac{\partial H}{\partial x_{3n+i}} \frac{\partial}{\partial x_i} + \frac{\partial H}{\partial x_{5n+i}} \frac{\partial}{\partial x_{n+i}} + \frac{\partial H}{\partial x_{6n+i}} \frac{\partial}{\partial x_{2n+i}} + \frac{\partial H}{\partial x_{7n+i}} \frac{\partial}{\partial x_{4n+i}} \tag{24}
    \end{equation}

Taking into Eq. (14), we equal Eq. (16) and Eq. (24), it yields

\begin{equation}
    \frac{dx_i}{dt} = - \frac{\partial H}{\partial x_{3n+i}}, \quad \frac{dx_{n+i}}{dt} = \frac{\partial H}{\partial x_{5n+i}}, \quad \frac{dx_{2n+i}}{dt} = \frac{\partial H}{\partial x_{6n+i}}, \quad \frac{dx_{4n+i}}{dt} = \frac{\partial H}{\partial x_{7n+i}}, \tag{25}
    \end{equation}

Finally, the equations obtained in Eq. (25) are obtained to be Hamiltonian equations with respect to component $J_3^*$ of almost Cliffordian structure $V^*$ on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$, and then the triple $(\mathbb{R}^{8n}, \Phi_{J_3}, X)$ is a Hamiltonian mechanical system on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$.

4 Conclusion

Formalism of Hamiltonian mechanics has intrinsically been described with taking into account the basis $\{J_1^*, J_2^*, J_3^*\}$ of almost Cliffordian structure $V^*$ on the standard Cliffordian Kähler manifold $(\mathbb{R}^{8n}, V)$.

Hamiltonian models arise to be a very important tool since they present a simple method to describe the model for mechanical systems. In solving problems in classical mechanics, the rotational mechanical system will then be easily usable model.
Since physical phenomena, as well-known, do not take place all over the space, a new model for dynamic systems on subspaces is needed. Therefore, equations (17), (21) and (25) are only considered to be a first step to realize how Cliffordian geometry has been used in solving problems in different physical area.

For further research, the Hamiltonian vector fields derived here are suggested to deal with problems in electrical, magnetical and gravitational fields of quantum and classical mechanics of physics.

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