Improvement of Calculation of Stresses in the Earth Bed and Layers of Road Clothes from Granulated Materials. Part 1. Analysis of Decisions and a New Method

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Abstract. The bases of road clothes made of discrete materials have become widespread in the practice of building roads throughout the world. Experimental studies of various loads have shown that an adequate calculation of stresses by formulas in the mechanics of a continuous medium is practically impossible. The article presents a method for modifying stress calculation models, the application of which allows supplementing the solutions of granular medium mechanics and engineering methods with dependencies for calculating the minimum main stress $\sigma_3$. Following this method, some solutions have been modified, and modified models are given. These models make it possible to calculate the main stresses in a layer of finite thickness in the section on the axis of symmetry of the load distributed along the circular platform. In accordance with this method, in the section along the axis of symmetry of the load at the point $z=0$ compression pressure occurs, that is $\varepsilon_2=\varepsilon_3=0$. Within the limits of depth change $0<z<\infty$ the soil operates under conditions of triaxial compression, experiencing lateral expansion strains $\varepsilon_2=\varepsilon_3<0$. At the point $z=\infty$ the ground undergoes uniaxial compression $\sigma_2=\sigma_3=0$ and $\varepsilon_2=\varepsilon_3=-\mu\varepsilon_1$.

1. Introduction.
In practice, the construction of bases of road pavements is widespread discrete materials. Such materials in comparison with materials processed by binders, have lower levels of strength and deformability, but have a high maintainability. For example, cold recycling allows re-using discrete materials by processing them astringent. In this regard, abroad in new construction base road pavements arrange from discrete materials, and the overhaul of their strengthen the various astringent with a deep recycling. If you adopt such a strategy for the construction and repair of roads, it becomes evident that adequate experimental data, the calculation of the stress strain state (SSS) of discrete materials will allow you to properly select the materials of the layers and calculate their thickness. The solution to this problem will allow to achieve compliance with the design and actual lifetimes of the structure.

The problem of construction of bases of road clothes from discrete materials includes three tasks. The first problem is material authority. It is associated with the selection of the compositions of macadam-sandy (CSSM) and sand-gravel mixtures (ASG), as well as with the development...
requirements. Developing requirements for the discrete materials they have in the dynamic triaxle compression devices, which allow to apply repeated loads. Limiting residual deformation and providing a fading nature of plastic deformation, experts are regulated by compositions of the CSSM and ASG. In these studies there is an influence on the mechanism of deformation of the content of silty-clay particles, moisture content, drainage conditions of the sample, coefficient of consolidation and others [1-4].

The second task must be given to technology aimed at building bases with the required properties of materials in the layer. For this the optimal modes of preparation and compaction of the mixtures are set.

The third task is aimed at improving methods of calculation of road designs. This direction of the research selected by the authors. The essence of the work is to attempt the improvement of soils and calculation of discrete materials on the shear strength. In addition, improved formulas for calculating the principal stresses can be used to calculate the deformations accumulated in granular materials under the action of a repeated load.

We note that at the present time a large number of logarithmic, degree and exponential functions are known that describe the regularity in the accumulation of residual deformation by the granular material from the number of loads [1-7].

The calculation of the structural layer of the pavement and soil subgrade on shear strength is performed by checking conditions

\[ \tau_{tr} \leq \frac{c_{N} \cdot k_{d} + \gamma \cdot z_{d} \cdot \tan \phi_{st}}{K_{str}}, \]  

where \( \tau_{tr} \) – active shear stress from traffic load, MPa; \( c_{N} \) – adhesion after exposure to N-th design load, MPa; \( k_{d} \) – coefficient taking into account features of the design; \( \gamma \) – weighted average specific gravity of the structural layers, check the layers located above, MN/cm\(^3\); \( z_{d} \) – the depth of surface layer subject to shear resistance, cm; \( \phi_{n} \) – value of angle of internal friction in a single static long-term load, \(^{0}\).

Calculation of shear stress \( \tau_{tr} \) carries out from the plasticity condition Mohr – Coulomb. We made the analysis of this condition in the first part of the article [8]. In this work, we modified this criterion. The essence of the modification consists in entering of the third material parameter \( d \) in the original criterion of Coulomb – Mohr. In accordance with the modified condition of plasticity the shear stress \( \tau_{n} \) is determined by the formula:

\[ \tau_{n} = \frac{1}{2} \cdot \left( \sigma_{1} \cdot \left( \frac{1 - \sin\phi_{N}}{1 + \sin\phi_{N}} \right)^{d} - \left( \frac{1 + \sin\phi_{N}}{1 - \sin\phi_{N}} \right)^{d} \cdot \sigma_{3} \right), \]  

where \( \phi_{n} \) – is the angle of internal friction after exposure to N-th design loads, hail, \( d \) – is the material parameter depending on the deformation of soil as the limit value in the triaxial test.

Analyzing the dependence (2) note that when \( d=0 \) and \( d=0,5 \) it takes the form:

\[ \tau_{tr} = \frac{\sigma_{1} - \sigma_{3}}{2}. \]  

\[ \tau_{tr} = \frac{1}{2} \cdot \left( \sigma_{1} \cdot \frac{1 - \sin\phi_{N}}{1 + \sin\phi_{N}} - \sigma_{3} \cdot \frac{1 + \sin\phi_{N}}{1 - \sin\phi_{N}} \right), \]  

The expression (3) is a well-known formula of mechanics that allows you to calculate the maximum shear stress used in the third theory of strength. The dependence (4) is a well-known
formula of soil mechanics, designed for determination of the tangential stresses on the Mohr – Coulomb’s condition written in one of known forms [9]. Thus, the proposed dependence (2) when decreasing the parameter d from 0.5 to 0 describes the increase in shear stress from the value corresponding to the Mohr – Coulomb’s condition, up to a maximum value corresponding to the third theory of strength. From the analysis of dependences (2) – (4) follows, that for calculation of shear stresses it is necessary to determine the principal stresses.

The methods of calculating of stresses in the layers of the pavement and the subgrade can be divided into three groups:

– methods of continuum mechanics;
– methods of the granular medium;
– engineering methods of calculation.

In figure 1 shows the block diagram used to discrete the materials, to solve the problem of SSS methods of continuum mechanics.

\[
\begin{align*}
\sigma_1 &= p \left(1 - \left[1 + \left(\frac{R}{z}\right)^2\right]^{-1.5}\right),
\end{align*}
\]

where \( p \) – pressure in half-space, MPa; \( R \) is the radius of the platform, m; \( z \) – distance from the surface to the point belonging to axis of symmetry of the load, which is calculated from the voltage, m.
\[ \sigma_2 = \sigma_3 = p \left( \frac{1+2 \cdot \mu}{2} - \frac{1+\mu}{1+\left(\frac{R}{z}\right)^2} - \frac{1}{2 \cdot \left[1+\left(\frac{R}{z}\right)^2\right]^{1.5}} \right) \]  \tag{6}

where \( \mu \) is the Poisson's ratio.

Dependences (5) and (6) are used in the calculation of the stress state from the transport and loading are given in the thesis [14, 15]. The results of calculation by formulas (5) and (6) coincide with the exact solution tabulated [10, 11]. In the RF solution (5) and (6) was extended by taking into account the components of the principal stresses from the self weight of the soil above the point under consideration. The advantage of the formulas of continuum mechanics is the possibility of determining all three principal stresses, whereby it is possible to solve the problem of precipitation of the array and the resistance of material to shear. These formula shave draw backs related to the fact that:

– in the upper part of the soil half-space the value of the minimum principal stress is more than the amount by which materials and soils have compression. Compression is characterized by the absence of deformations the lateral expansion, i.e. \( \varepsilon_2=\varepsilon_3=0 \). These experiments show that under the Central part of the bent flexible boards materials and soils have compression [16].

– at some depth the magnitude of the stresses \( \sigma_2=\sigma_3 \) takes a negative value, it means that they turn from compressive to tensile. This is contrary to the design scheme figure 1, according to which at any point in the discrete environment, these stresses are compressive. Note that in the solutions of problems on SSS when exposed to other stresses [12, 17], for example distributed along the endless flexible band, such is no shortage of voltages \( \sigma_3 \) is positive at all points in the half-space.

– there is no possibility of accounting indicators of the mechanical properties of the material.

To account for the properties of bases from soils and discrete materials in dependence (5) was introduced the parameter \( n \), proposed by O. C. Frohlich to the classic solution of Boussinesq. As a result, the dependence (5) has the form [15]

\[ \sigma_1 = p \left[ 1 - \left( \sqrt{1+\left(\frac{R}{z}\right)^2} \right)^{-n} \right] \]  \tag{7}

where \( n \) is the Frolich's parameter.

Another known form of writing the expression (7), which has the form [16]

\[ \sigma_1 = p \left[ 1 - \left( \frac{z}{R} \right)^n \cdot \left[ 1+\left(\frac{z}{R}\right)^2 \right]^{-\frac{n}{2}} \right] \]  \tag{8}

Applying the analogy and entering the Frohlich’s parameter in dependence (6), we obtain the formula

\[ \sigma_3 = p \left( \frac{1+2 \cdot \mu}{2} - (1+\mu) \left[ 1+\left(\frac{R}{z}\right)^2 \right]^{-\frac{n}{3}} + \frac{1}{2} \left[ 1+\left(\frac{R}{z}\right)^2 \right]^{-\frac{n}{2}} \right) \]  \tag{9}
The advantage of dependences (7) – (9) is that the selection of the Frolich’s parameter for various materials and soils. Therefore, in these formulas there is an opportunity to consider differences in the structure of soils and materials, as well as its influence on the magnitude of the stresses. However, the minimum principal stress, calculated by the formula (9) at a certain depth change the sign, keeping the lack of expression (6).

Disadvantages of dependencies (6) and (9) has led to the fact that when solving problems only began to apply formula (5), (7) or (8), thereby replacing the triaxial uniaxial simple. In this case, the value of the minimum principal stress is conventionally assumed equal to zero, which leads to an overestimation of the values of the stress deviator \( \sigma_d = \sigma_1 - \sigma_3 \) at any point of the considered cross-section.

The development of methods of mechanics of granular medium originates in the works of Soviet scientists: G. I. Pokrovsky, M. N. Goldstein, I. I., Kandaurova, R. A. Muller. Abroad the ideas of I. I. Kandaurova was used by M. Harr [18]. Mechanics of granular medium is based on mathematical statistics that fundamentally sets it apart from continuum mechanics. In our model the cross-section voltage \( \sigma_1 = \sigma_z \) are defined in formula I. Kandaurov

\[
\sigma_1 = \sigma_z = p \cdot \left( 1 - \exp \left( -\frac{4 \cdot v_R \cdot R^2}{z^2} \right) \right). \tag{10}
\]

where \( v_R \) – coefficient of proportionality, characterizing the distribution capacity of the environment.

M. Harr [18, 19] proposes an alternative according to I. I. Kandaurova formula

\[
\sigma_1 = \sigma_z = p \cdot \left( 1 - \exp \left( -\frac{R^2}{2 \cdot v_p \cdot z^2} \right) \right). \tag{11}
\]

The disadvantage of the formulas (11) and (13) is the impossibility of calculating the minimum principal stress.

Engineering models and formulas of the granular medium provide the ability to count only the value of the maximum principal stress. This is their advantage.

The formulation of goals and objectives.
Considering the review, the authors establish the goal: To develop a modified model to calculate the principal stresses in the cross section passing through the axis of symmetry of the load distributed over a circular area, in the half-space and layer of finite thickness of a discrete material.

The goal requires the sequential solution of a series of tasks:
1. To develop a method of calculating of the minimum principal stress \( \sigma_3 \) in the considered cross-section of the half-space, in accordance with which a function changes \( \sigma_3 \) must be waning and unbreakable, which allows to calculate the voltage at all points of the considered cross-section. Additionally, this function must satisfy the following conditions:
   - In the upper half-space of the point located on the axis of symmetry of the load, the material have to experience the compression, that is, not to test the minimum of the main deformations \( \varepsilon_2 = \varepsilon_3 = 0 \). This requirement satisfies the experimental data of work [16, 20]. In point located at the axis of symmetry of the load and having an ordinate equal to infinity, the soil should test uniaxial compression, and the minimum principal stresses take a value of zero, that is, \( \sigma_2 = \sigma_3 = 0 \). This restriction corresponds to the hypothesis of Fedorovsky – Buzvolev [20].
- At other points of the considered cross-section should have an axial triaxial $\sigma_1>\sigma_2=\sigma_3$ when $\sigma_1>0$ and $\sigma_2=\sigma_3>0$. Voltage $\sigma_2$ and $\sigma_3$ depth needs to fade more rapidly than $\sigma_1$, whereby the lack of lateral deformation $\varepsilon_2=\varepsilon_3$ should decrease with depth. This statement fits into the generally accepted ideas of continuum mechanics.

2. Taking into account the final thickness of the base of the pavement and the stiffness of the material layers located below the considered road element of the design.

3. Modification of known models for calculating of the stresses, the originals of which are derived using methods of mechanics of continuous and granular medium, as well as engineering methods of calculation.

4. Conformity assessment of the results of the calculation of principal stresses for the modified models and experimental data obtained from testing of pavement moving loads.

2. Methods
To calculate the minimum principal stress, we propose to modify the well-known formula of soil mechanics, by a separate multiplier of the function of depth

$$\sigma_2 = \sigma_3 = \alpha \cdot \xi \cdot \sigma_1,$$

where $\alpha$ – coefficient as a function of depth; $\xi$ – coefficient of lateral pressure.

The product of the coefficients $\alpha$ and $\xi$ can be interpreted as a variable with depth the coefficient of lateral pressure, what differentiates (12) from the traditional formula.

To determine the coefficient $\alpha$ compare two formulas used to calculate deformation $\varepsilon_1$. The first formula is traditional, it is the deformation defined as the ratio of the product of the coefficient of lateral compression $\beta$ and maximum principal stress $\sigma_1$ module de formation. The second formula is an expression of Hooke’s law, written with the use according to (15). Thus, the considered formulas have the form:

$$\varepsilon_1 = \frac{\beta \cdot \sigma_1}{E_{\text{def}}}; \quad \varepsilon_1 = \frac{\sigma_1}{E_{\text{def}}} \left[ 1 - \frac{2 \cdot \alpha \cdot \mu^2}{1 - \mu} \right].$$

In the formula (13) is applicable the idea of Fedorovsky – Buzvolev, according to which $\beta$ is determined by the formula [20]

$$\beta = \beta_c + \sqrt{1 - K^2} \cdot (\beta_{\text{inf}} - \beta_c),$$

where $\beta_c$ – the coefficient considering the absence of transverse expansion of the soil in the compressing device, which according to [20] takes place at the point with ordinate $z=0$ (on surface); K – attenuation coefficient of vertical normal stress from the uniform load under the axis of its symmetry; $\beta_{\text{inf}}$ – factor at infinity (at the point with ordinate $z=\infty$).

Putting equality between the dependencies (13) and taking into account in this identity the expression (14) discover the mathematical relationship of the coefficients $\beta$ and $\alpha$, as a result of transformation where the solution reduces to the formula

$$\alpha = \alpha_c - \sqrt{1 - K^2} \cdot (\alpha_c - \alpha_{\text{inf}}).$$

Where $\alpha_c$ and $\alpha_{\text{inf}}$ – value functions $\alpha$ at points located on the surface and at infinity of the considered cross-section.

Taking for the surface of half-space assumption on its material in a state of compression, that is $\varepsilon_2=\varepsilon_3=0$, we find $\alpha_c=1$. Similarly, setting at infinity the condition for uniaxial compression, i.e. $\varepsilon_2=\varepsilon_1=-\mu \cdot \varepsilon_1$, get $\alpha_{\text{inf}}=0$. Substituting these values in formula (18) transforms it to the form
\[
\alpha = 1 - \sqrt{1 - K^2},
\]  
(16)

Thus, the ratio \( \alpha \) is a function of depth. Since the point on the surface of this section \( K=1 \), then \( \alpha=1 \) the result is that the material at this point, the material experiences compression. At the point at infinity \( K=0 \), hence \( \alpha=0 \), this means that the material is experiencing uniaxial compression.

According to the authors, the advantage of this method is the ability to modify a model granular medium mechanics and engineering methods of calculation of stresses, supplementing them with formulas that can calculate the minimum principal stress \( \sigma_3 \). In addition, to determine the coefficient of lateral pressure, you can use any of the known formulas. For example, calculating \( \xi \) through the angle of internal friction is sufficient to use the work of M.D. Bolton [21] or J. Jaky [22], in generalizing these dependences.

Note that for the calculation of the minimum principal stresses arising at the axis of symmetry of half-space load, it is sufficient in dependence (12) to substitute expressions (5) (7), (8), (10) or (11).

For a layer of finite thickness, which includes the substrate or additional layers of the bases of road clothes of a discrete material, a direct substitution of these formulas into the expression (15) cannot be done. This is due to the fact that (5) (7), (8), (10) and (11) do not take into account the stiffness of the layers of the road structure located below the discrete base.

Solving the second objective, the authors use the method of N. Odemark [23], according to which rigidity is given by the ordinate of the point in the array half-space \( z \) to the ordinate of the point in the layer of the pavement \( z_L \) is determined by the formula:

\[
z = z_L \cdot \frac{E_L}{E_b},
\]  
(17)

where \( z_L \) – ordinate point located on the axis of symmetry of the load layer thickness \( h \), that is, \( 0 \leq z_L \leq h \), \( m \); \( E_L \) the modulus of elasticity of the material of the layer, MPa; \( E_b \) – the overall modulus of elasticity on the surface of a homogeneous or layered half-space, we expect the underlying layer, MPa.

Substitution according to (17) in the formula (5), (7), (8), (10) or (11) allows to modify them to calculate the maximum stresses occurring in a layer of finite thickness.

We give an incremental algorithm of modification of models.

1. Any known formula for calculating the value of \( \sigma_1 \) or \( \sigma_2 \) needs to represent the product of the pressure transmitted to the layer and the attenuation factor of the voltage that is \( \sigma_1 = \sigma_2 = p \cdot K \). Effect of equality \( \sigma_1 = \sigma_2 = p \cdot K \) occurs only in the section along the axis of symmetry of the load, the attenuation coefficients of these stresses are equal and are recorded with the same expressions.

2. In expression of attenuation factor it is necessary to use formula (17) that allows you to move from calculating the maximum principal stresses in the half space for their computations in a layer of finite thickness.

3. Generalizing dependence (12) is to mind

\[
\sigma_2 = \sigma_3 = p \cdot \xi \cdot K \cdot \left( 1 - \sqrt{1 - K^2} \right).
\]  
(18)

4. Dependence in (18) are substituted in the formula for calculating the \( K \) coefficients obtained after step 2.

This algorithm allows to modify any model, using a method independent of the mathematical apparatus, used in the derivation of the formula for calculating \( \sigma_1 \).

In table 1 shows a modified model for the calculation of stresses in a layer of finite thickness, and their name is given by the authors who received the original decision or has carried out this work prior to any modification.
Table 1. The formulas of modified models of calculation the principal stresses from the load distributed over a circular area in cross-section along the symmetry axis of the load.

| The authors of the original decision and any subsequent modifications | Formulas to calculate principal stresses |
|---|---|
| Modified models of continuum mechanics |  |
|  \[ \sigma_1 = p \cdot K; \quad K = 1 - \frac{1}{2} \left( 1 + \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right) \]  |  |
| R.G. Ahlvin, C.R. Foster, H.H. Ulery. |  |
|  \[ \sigma_2 = \sigma_3 = p \cdot \zeta \cdot \left( 1 - \left( 1 + \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right) \right)^{\frac{3}{2}} \]  |  |
|  \[ \times \left( 1 - \left( 1 + \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right) \right)^{\frac{2}{3}} \]  |  |
| The same model, but with a coefficient O.K. Fröhlich. |  |
|  \[ \sigma_1 = p \cdot K; \quad K = 1 - \frac{1}{2} \left[ 1 + \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right] \]  |  |
|  \[ \times \left( 1 - \left( 1 + \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right) \right)^{\frac{2}{3}} \]  |  |
| M.Haar |  |
|  \[ \sigma_2 = \sigma_3 = p \cdot \zeta \cdot \left( 1 - \exp \left[ - \frac{1}{2 \cdot \sqrt{R}} \cdot \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right] \right) \]  |  |
|  \[ \times \left( 1 - \exp \left[ - \frac{1}{2 \cdot \sqrt{R}} \cdot \left( \frac{R}{z_L} \cdot \frac{E_b}{E_L} \right)^2 \right] \right) \]  |  |
3. Results and Discussion

Thus, the modified model includes two formulas. The first formula allows to calculate the amount $\sigma_1$, it has the feel of the original decision or previous modifications, but the coefficient $K$ is determined based on the formula of N. Odemark (17). The second dependence, allowing to calculate $\sigma_3$, is new and additional to formulas to the granular medium mechanics and engineering methods. In the modification of the solutions of continuum mechanics dependencies for the calculation of the minimum principal stress, proposed by the authors replace the original formula of the solution. This eliminates the drawback of the original decision, related to a change at a certain depth of the mark $\sigma_3$.

This modification leads to the fact that across the layer depth, up to $z=\infty$, values $\sigma_3$ is positive, that is, they are compressive, which corresponds to the conventional design scheme Figure 1.

In figure 2 and figure 3 shows the diagrams of the shearing stresses, illustrating the calculation results by the proposed formula (2) with $d=0.4$ and different angles of internal friction. The calculation of the principal stresses in equation (2) is performed on the modified model of Love – Frölich – Olson in $E_L/E_B=1$ и $n=1$ и $n=3$ (see tab. 1).

![Figure 2](image1)

**Figure 2.** The dependence of the relative magnitude of the tangential stresses $\tau/R$ calculated according to the formula (2) with $d=0.4$ and calculate the principal stresses in the modified model, Love – Frölich – Olson (tab. 1) when $n=1$, from the relative depth $z/R$, and the angle of the internal friction $\phi$: 1 – 6 when the angle of the internal friction $\phi$ 0; 10; 20; 30; 40 and about 50; 7 – line of location of the most dangerous points.

![Figure 3](image2)

**Figure 3.** The dependence of the relative magnitude of the tangential stresses $\tau/R$ calculated according to the formula (2) with $d=0.4$ and calculate the principal stresses in the modified model, Love – Frölich – Olson (tab. 1) when $n=3$, from the relative depth $z/R$, and the angle of the internal friction $\phi$: 1 – 6 when the angle of the internal friction $\phi$ 0; 10; 20; 30; 40 and about 50; 7 – line of location of the most dangerous points.
From the analysis of the data figure 2 and 3 show that the extreme shear stress for all values $\varphi$, $d$ and $n$ occurs at a certain depth from the surface of the half-space or layer of the finite thickness. The point with the highest tangential stress is the most dangerous point for which you want to perform the calculation and check the condition of shear resistance (1). A similar plot of shearing stresses, but with quantitative differences, obtained by calculating of the principal stresses at any of the proposed models.

The adequacy of the calculation of the tangential stresses in the most dangerous point is largely driven by the reliability computing the principal stresses arising from exposure to moving load. Therefore, the results of the calculation of the principal stresses on a modified models are presented in table 1, it is necessary to compare with experimental data.

4. Conclusions
1. Modified model for the calculation of the principal stresses in the half space (12) and the layer of finite thickness (see table 1) satisfy the following conditions:
   - In cross-section along the axis of symmetry of the load on surface of half-space or layer of finite thickness occurs compression, that is, when $z=0$ main deformation $e_2=e_3=0$, and the principal stresses $\sigma_2 = \sigma_3 = \mu \sigma_1 / (1 - \mu)$.
   - When you change the depth corresponding to the thickness of the layer, in the range $0 < z < \infty$ layer material operates under conditions of triaxial compression $\sigma_2 = \sigma_3 < \mu \sigma_1 / (1 - \mu)$, strain testing lateral extensions $e_2 = e_3 < 0$.
   - On the axis of symmetry of the load at the point $z = e_2 = \infty$ the discrete material experiences a uniaxial compression $\sigma_2 = \sigma_3 = 0$ and $e_2 = e_3 = -1$.

2. According to the authors, the prospect of further application of modified models table 1 is the ability to improve the calculation of the pavement layers of discrete materials on the shear resistance. Within this calculation, there is an opportunity to calculate the first critical load of the proposed plasticity condition (2), in applying this modified model table 1.

References

[1] Aleksandrov A S 2013 Plastic deformation granodiorite gravel and sand and gravel when exposed to cyclic loading triaxial Magazine of Civil Engineering № 4 pp 22-34.
[2] Belt J, Ryynanen T and Ehrola E 1997 Mechanical properties of unbound base course Proceedings of the 8th International Conference on Asphalt Pavements vol 1 pp 771-781
[3] Brown S F 1974 Repeated load testing of a granular material Journal of Geotechnical Engineering Division, ASCE vol 100 № 7 pp 825-841
[4] Aleksandrov A S, Semenova T V and Aleksandrova N P 2016 Analysis of permanent deformations in granular materials of road structures Road and Bridges vol 15 pp 263-276
[5] Werkmeister S, Dawson A and Wellner F 2004 Pavement design model for unbound granular materials Journal of Transportation Engineering, ASCE vol 130 № 5 pp 665-674
[6] Werkmeister S, Dawson A and Wellner F 2005 Permanent Deformation Behavior of Granular Materials Road Materials and Pavement Design vol 6 № 1 pp 31-51
[7] Aleksandrov A S, Dolgih G V and Kalinin A L 2017 Analysis and Modeling of Process of Residual Deformations Accumulation in Soils and Granular Materials IOP Conf. Ser.: Mater. Sci. Eng. vol 262
[8] Aleksandrov A S and Kalinin A L 2015 Improvement of shear strength design of a road structure Part 1 Deformations in the Mohr – Coulomb plasticity condition Engineering and construction magazine № 7(59) pp 4–17
[9] Craig R F 2004 Soil Mechanics - Seventh edition. Department of Civil Engineering 447 p
[10] Foster C R and Ahlvin R G 1954 Stresses and deflections induced by a uniform circular load Proc. Highway Research Board vol 33 pp 236-246
[11] Ahlvin R G and Ulery H H 1962 Tabulated Values for Determining the Complete Pattern of
Stresses, Strains and Deflections Beneath a Uniform Load on a Homogeneous Half Space Bull 342 Highway Research Record pp 1-13
[12] Das B M 2008 Advanced soil mechanics. Third Edition New York, Taylor & Francis 567 p
[13] Al-Tayer T H 1995 A prototype simple shear and compaction apparatus with application to asphaltic concrete Ph.D. thesis, University of Arizona 229 p
[14] Appea A K 2003 Validation of FWD Testing Results at the Virginia Smart Road: Theoretically and by Instrument Responses Ph.D. thesis, Virginia Polytechnic Institute and State University 279 p
[15] Steven B D 2005 The development and verification of a pavement response and performance model for unbound granular pavements A thesis submitted in partial fulfillment of the requirements for the Degree of Doctor of Philosophy in the University of Canterbury 291 pp
[16] O’Kelly B C and Naughton P J 2009 Study of the yielding of sand under generalized stress conditions using a versatile hollow cylinder torsional apparatus Mechanics of materials vol 41 pp 187-198
[17] Olson R E 2003 Stress distribution Advanced Soil Mechanics
[18] Harr M E 1977 Mechanics of Particulate Media McGraw-Hill, New York 543 p
[19] Ullidtz P, Askegaard V and Sjolin F O 1996 Normal Stresses in a Granular Material under Falling Weight Deflectometer Loading Transportation Research Record 1540, National Research Council, Washington pp 24-29
[20] Fedorovsky V G and Bezvolev S G 2000 Calculation of the sediments of the foundations of shallow deep and the choice of the model of the base for the calculation of slabs Grounds, foundations and mechanics of soils № 4 pp 10-18
[21] Bolton M D 1991 A Guide to Soil Mechanics MD & K Bolton
[22] Jaky J 1944 The Coefficient of Earth Pressure at Rest Journal for Society of Hungarian Architects and Engineers pp 355-358
[23] Odemark N 1949 Investigations as to the Elastic Properties of Soils and Design of Pavements according to the Theory of Elasticity Ph.D. thesis. Statens Väginstitut, Mitteilung № 77