Factoring Third Order Ordinary Differential Operators over Spectral Curves

Sonia L. Rueda¹, Maria-Angeles Zurro² [sonialuisa.rueda@upm.es]

¹ Dpto. de Matemática Aplicada. Universidad Politécnica de Madrid. Spain
² Dpto. de Matemáticas. Universidad Autónoma de Madrid. Spain

We consider the factorization problem of a third order ordinary differential operator $L - \lambda$, for a spectral parameter $\lambda$ and an irreducible operator $L$, whose coefficients belong to a differential field $K$. It is assumed that $L$ is algebro-geometric over $K$, guaranteeing a nontrivial centralizer, which can be seen as the ring of an affine curve, the famous spectral curve $\Gamma$.

Based on the nature of $\Gamma$, we give a symbolic algorithm to factor $L - \lambda$ over the spectral curve using differential subresultants. In this context, the first explicit example of a non-planar spectral curve arises, as well as the factorization it provides for $L - \lambda$. As far as we know, it is the first factorization algorithm for third order irreducible operators over the field extension $K(\Gamma)$ of $K$. The coefficient field $K$ is extended to the field of rational functions on $\Gamma$ to obtain a right factor with coefficients in this field.

Factorizations over planar spectral curves have been presented in other articles, for instance for second order operators [1], or fourth order operators with rank 2, [3]. The present work is the natural continuation in a program dedicated to the factorization of rank 1 algebro-geometric differential operators, that was already successful in the order 2 case, [1]. Our ultimate goal is an effective approach to the direct spectral problem and the development of the appropriate spectral Picard-Vessiot fields containing all the solutions of the operator $L - \lambda$. Spectral Picard-Vessiot fields were studied for Schrödinger operators in [2].

Keywords
Factorization, ordinary differential operators, differential subresultant, spectral curve.

References
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