Concordance:
In-Flight Calibration of X-ray Telescopes 
without 
Absolute References

Herman L. Marshall (MIT), Vinay Kashyap, Jeremy Drake, Pete Ratzlaff, Paul Plucinsky (SAO), Matteo Guainazzi (ESA)

Yang Chen (Harvard, UMich.), Xiao-Li Meng (Harvard), Xufei Wang (Harvard, Two Sigma), David van Dyk (ICL)
The Goal

• The problems
  • Discrepant results from X-ray observatories in orbit
  • Cluster temperatures and fluxes
  • Blazar fluxes from simultaneous observations
  • SNR line fluxes
  • Imperfect ground cal, performance changes in flight
  ‣ Instrument area priors $a_i$ differ from “true values” $A_i$
  • No absolute calibrators across all bands in flight: no “true” $F_j$

• Specific task: derive $\hat{A}_i$ for optimal agreement

$\Rightarrow$ Let flux $f_{ij} = c_{ij}/T_{ij}/a_i$
where $a_i = \text{prior on } A_i$
$c_{ij} = \text{observed counts}$
$T_{ij} = \text{known exposure time}$
Some Poor Methods

- Use the average flux as the ‘true’ flux: \( F_j = \langle f_{ij} \rangle \)
  - If statistical weighting, answer depends on \( T_{ij} \) and \( a_i \)
  - If no weighting, then “agnostic” but not stable
  - Problematic statistical inference: \( \hat{A}_i = \frac{c_{ij}}{T_{ij}F_j} \)
- Use one instrument as “given”: \( F_j = f_{Xj} \) for some \( X \)
  - Reference choice is subjective
  - Still problematic statistically

Let flux \( f_{ij} = \frac{c_{ij}}{T_{ij}/a_i} \)

where \( a_i = \text{prior on } A_i \)
\( c_{ij} = \text{observed counts} \)
\( T_{ij} = \text{known exposure time} \)
Better: Multiplicative Shrinkage

(Chen+ ’19)
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\[ y_{ij} = B_i + G_j - \frac{\sigma_i^2}{2} + e_{ij} , \quad y_{ij} \equiv \log(c_{ij}/T_{ij}) , \quad B_i \equiv \log A_i , \quad G_j \equiv \log F_j \]
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\[ \hat{B}_i = W_i(\bar{y}_i - \bar{G}_i) + (1 - W_i)b_i \quad \text{and} \quad \hat{G}_j = \bar{y}_j - \bar{B}_j \]

\[ W_i = \frac{M\sigma_i^{-2}}{\tau_i^{-2} + M\sigma_i^{-2}} \]
Better: Multiplicative Shrinkage
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\[ y_{ij} = B_i + G_j - \frac{\sigma_i^2}{2} + e_{ij} \quad , \quad y_{ij} \equiv \log(c_{ij}/T_{ij}) \quad , \quad B_i \equiv \log A_i \quad , \quad G_j \equiv \log F_j \]

\[ \hat{B}_i = W_i(\bar{y}_i. - \bar{G}_i) + (1 - W_i)b_i \quad \text{and} \quad \hat{G}_j = \bar{y}'_j - \bar{B}_j \]

EA prior uncertainties

\[ W_i = \frac{M\sigma_i^{-2}}{\tau_i^{-2} + M\sigma_i^{-2}} \]
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\[ y_{ij} = B_i + G_j - \frac{\sigma_i^2}{2} + e_{ij} \]

\[ y_{ij} \equiv \log(c_{ij}/T_{ij}) \]

\[ B_i \equiv \log A_i \]

\[ G_j \equiv \log F_j \]

\[
\hat{B}_i = W_i(\bar{y}_i' - \bar{G}_i) + (1 - W_i)b_i \\
\text{and} \\
\hat{G}_j = \bar{y}_j' - \bar{B}_j
\]

\[
W_i = \frac{M\sigma_i^{-2}}{\tau_i^{-2} + M\sigma_i^{-2}}
\]

EA prior uncertainties \hspace{1cm} Data uncertainties
Better: Multiplicative Shrinkage

(Chen+ '19)

\[ y_{ij} = B_i + G_j - \frac{\sigma_i^2}{2} + e_{ij}, \quad y_{ij} \equiv \log(c_{ij}/T_{ij}), \quad B_i \equiv \log A_i, \quad G_j \equiv \log F_j \]

\[ \hat{B}_i = W_i(\tilde{y}_i' - \tilde{G}_i) + (1 - W_i)b_i \quad \text{and} \quad \hat{G}_j = \tilde{y}_j' - \tilde{B}_j \]

\[ \tilde{y}_i' = \bar{y}_i + 0.5\sigma_i^2, \quad \bar{y}_i = \frac{\sum_{j=1}^M \tilde{y}_{ij}\sigma_i^{-2}}{\sum_{j=1}^M \sigma_i^{-2}}, \quad \tilde{y}_j' = \frac{\sum_{i=1}^N \tilde{y}_{ij}\sigma_i^{-2}}{\sum_{i=1}^N \sigma_i^{-2}}, \quad \tilde{G}_i = \frac{\sum_{j=1}^M \hat{G}_j\sigma_i^{-2}}{\sum_{j=1}^M \sigma_i^{-2}}, \quad \tilde{B}_j = \frac{\sum_{i=1}^N \hat{B}_i\sigma_i^{-2}}{\sum_{i\in I_j} \sigma_i^{-2}} \]

\[ W_i = \frac{M\sigma_i^{-2}}{\tau_i^{-2} + M\sigma_i^{-2}} \quad \text{EA prior uncertainties} \]

\[ \text{Data uncertainties} \]
Input Data

- **Paper I**
  - 1E0102 with 13 instruments (N=13), O & Ne (M=2)
  - 2XMM catalog targets, N=3, M=41; soft, medium, hard
  - XCAL bright targets, N=3, M=94-108; soft, medium, hard

- **New paper (Marshall+, in prep.)**
  - Same 3 sets as in Paper I
  - Also Capella with Chandra gratings, N=8, M=15
  - Added correlations of XMM hard, medium, soft
  - Added correlations of O, Ne fluxes of 1E0102
  - Used heterogeneous tau values

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**Table 5. 2XMM Concordance Fluxes – Medium Band**

| Target          | $f_{ij}$ | $\sigma_{ij}$ | $f_{ij}$ | $\sigma_{ij}$ | $f_{ij}$ | $\sigma_{ij}$ |
|-----------------|----------|---------------|----------|---------------|----------|---------------|
| 1127-145        | 0.481    | 0.049         | 0.496    | 0.053         | 0.490    | 0.052         |
| 1E0919+515      | 0.053    | 0.053         | 0.069    | 0.066         | 0.068    | 0.065         |
| 4C06.41         | 0.131    | 0.015         | 0.142    | 0.017         | 0.143    | 0.018         |
| APM08279+5255   | 0.085    | 0.041         | 0.088    | 0.042         | 0.082    | 0.040         |
| CenX-4          | 0.088    | 0.035         | 0.089    | 0.022         | 0.091    | 0.023         |
| CoD-33 7795     | 0.275    | 0.136         | 0.287    | 0.143         | 0.276    | 0.136         |
| ESO323-G077     | 0.425    | 0.184         | 0.438    | 0.202         | 0.439    | 0.203         |
| GRB00944117     | 0.348    | 0.006         | 0.415    | 0.008         | 0.419    | 0.009         |
| Holmberg IX     | 0.514    | 0.083         | 0.517    | 0.084         | 0.556    | 0.090         |
| IRAS13317-1627  | 0.938    | 0.818         | 0.914    | 0.793         | 1.000    | 0.873         |
| LBQS1228+1116   | 0.154    | 0.009         | 0.156    | 0.010         | 0.162    | 0.010         |
| M31 NN1         | 0.173    | 0.005         | 0.196    | 0.007         | 0.195    | 0.007         |
| MS0205.7+3509   | 0.283    | 0.087         | 0.304    | 0.095         | 0.293    | 0.092         |
| MS1229.2+6430   | 0.326    | 0.086         | 0.356    | 0.092         | 0.355    | 0.101         |
| NGC 1313        | 0.200    | 0.021         | 0.212    | 0.023         | 0.215    | 0.023         |
| NGC 4278        | 0.281    | 0.032         | 0.291    | 0.035         | 0.307    | 0.037         |
| NGC 5204 X-1    | 0.140    | 0.032         | 0.140    | 0.033         | 0.148    | 0.036         |
| NGC 5204 X-1    | 0.192    | 0.034         | 0.195    | 0.035         | 0.196    | 0.036         |
| NGC 5252        | 0.326    | 0.092         | 0.327    | 0.095         | 0.328    | 0.091         |

**Sample Data (Marshall+ in prep.)**
Complications I: Flux Measurements

Concordance: find $A_i$ where $C_{ij} = T_{ij}A_i F_j$, $A(E) = A_i \alpha_i(E)$

- Fluxes in band $(E_1, E_2)$ derived by an inversion process
- Input: observation $c_{ijk}$ for counts in channel $k$

Then fit to model $C'_{ijk} = t_{ij} a_i f_{ij} \frac{\int_{E_1}^{E_2} \alpha_i(E) q_j(E) \Phi_k(E) dE}{\int_{E_1}^{E_2} q_j(E) dE} = T_{ijk} a_i f_{ij}$

where $f_{ij} = \int_{E_1}^{E_2} n_E(\Theta_{ij}) dE = n_{ij} \int_{E_1}^{E_2} q_j(E) dE$ and $\tilde{A}(E) = a_i \alpha_i(E)$ define shape functions $q_j(E)$ and $\alpha_i(E)$, the detector response is $\Phi_k(E)$, and $\sum_k \Phi_k(E) = 1$

Now, $C_{ij} = \sum_k C_{ijk}$, $T_{ij} = \sum_k T_{ijk}$
Complications II: Eff. Area Correlations

- Assume we have EA parameters $\vec{\xi}$ giving $\log \tilde{A}(E; \vec{\xi}) = \tilde{B}(E; \vec{\xi})$ with $p(\vec{\xi})$

- Then $\hat{B}(E) = \int \tilde{B}(E; \vec{\xi}) p(\vec{\xi}) d\vec{\xi}$ is the best (prior) estimate of B and $\tau^2(E) = \int [\tilde{B}(E; \vec{\xi}) - \hat{B}(E)]^2 p(\vec{\xi}) d\vec{\xi}$ should be the prior’s variance

- Consider two energies, $E_i$ and $E_{i'}$, then the correlation between these is $\rho_{i,i'} = \frac{1}{\tau(E_i)\tau(E_{i'})} \int [\tilde{B}(E_i; \vec{\xi}) - \hat{B}(E_i)][\tilde{B}(E_{i'}; \vec{\xi}) - \hat{B}(E_{i'})] p(\vec{\xi}) d\vec{\xi}$

- In reality, a Monte Carlo method is used to compute the correlations…

Table 8. Correlation matrix for 2XMM and XCAL Analyses

| Band          | Soft band | Medium band | Hard band |
|---------------|-----------|-------------|-----------|
| Soft band     | 1         | 0.60        | 0.13      |
| Medium band   | 0.60      | 1           | 0.53      |
| Hard band     | 0.13      | 0.53        | 1         |
Complications III: Assessing Priors

- Collecting **prior** (fractional) uncertainties on effective areas
- Cal scientists assessed their instruments

**Table 1. Effective Area Uncertainty Priors (τ)**

| Instrument   | 0.15-0.33 | 0.33-0.54 | 0.54-0.8 | 0.8-1.2 | 1.2-1.8 | 1.8-2.2 | 2.2-3.5 | 3.5-5.5 | 5.5-10 |
|--------------|-----------|-----------|----------|---------|---------|---------|---------|---------|--------|
| Astrosat SXT | ⋮         | 15        | 15       | 10      | 10      | 10      | 10      | 10      | 10     |
| Chandra ACIS | 3         | 3         | 3        | 2.6     | 3.3     | 3.3     | 4.9     | 5       |        |
| Chandra HETGS| ⋮         | 10        | 5        | 4       | 4       | 4       | 5       | 7       |        |
| Chandra LETGS| 5         | 7         | 7        | 7       | 7       | 10      | 10      |         |        |
| ROSAT PSPC   | 10        | 10        | 10       | 10      | 10      |         |         |         |        |
| Suzaku XIS1  | ⋮         | 20        | 15       | 10      | 10      | 15      | 5       | 5       | 5      |
| Suzaku XIS0,2,3 | ⋮   | 15        | 10       | 10      | 10      | 15      | 5       | 5       | 5      |
| Swift PC/WT  | ⋮         | 15        | 7.5      | 7.5     | 10      | 5       | 5       | 5       |        |
| XMM MOS1,2   | 20        | 10        | 6        | 6       | 6       | 6       | 6       | 10      |        |
| XMM pn       | 2         | 2         | 2        | 2       | 2       | 2       | 2       | 2       | 3      |
| XMM RGS      | ⋮         | 8         | 5        | 5       | 5       |         |         |         |        |

*The τ values are given as percentages. The ellipses indicate bandpasses where the instrument has an insignificant effective area.*

**Table 2. Effective Area Uncertainty Priors (τ)**

| Instrument   | 2.2-3.5 | 3.5-5.5 | 5.5-10 | 15-25 | 25-50 | 50-100 | 100-300 |
|--------------|---------|---------|--------|-------|-------|--------|---------|
| Astrosat CZTI | ⋮       | ⋮       | ⋮      | 20    | 20    | 20     | 25      |
| Astrosat LAXPC| ⋮       | 15      | 15     | 15    | 15    | 20     | ⋮       |
| INTEGRAL IBIS| ⋮       | ⋮       | ⋮      | 8     | 15    | 20     | ⋮       |
| INTEGRAL SPI | ⋮       | ⋮       | ⋮      | 5     | 5     | 5      | 5       |
| NuSTAR       | ⋮       | 4       | 3      | 3     | 15    | 20     | ⋮       |
| RXTE PCA     | 5       | 10      | 3      | 3     | 10    | 50     | ⋮       |
| RXTE HEXTE   | ⋮       | ⋮       | ⋮      | 5     | 5     | 5      | ⋮       |
| Suzaku HXD   | ⋮       | ⋮       | ⋮      | 20    | 20    | 20     | 20      |
| Swift BAT    | ⋮       | ⋮       | ⋮      | 15    | 4     | 4      | 12      |

*The τ values are given as percentages.*
Concordance 1: 1E0102

O7 & O8

Large $\tau$ for Suzaku

HETG and S3 corr’d w/ Ne

Instruments

ACIS-I3
ACIS-S3
ACIS/HETG
Suzaku/XIS0
Suzaku/XIS1
Suzaku/XIS2
Suzaku/XIS3
Swift/XRT-PC
Swift/XRT-WT
XMM/MOS1
XMM/MOS2
XMM/pn
XMM/RGS1

Tau
- Correlated EAs
- Heterogeneous
- $\tau = 0.025$
- $\tau = 0.05$
Concordance 2: 2XMM

- Based on 42 sources from the 2XMM catalog
- Unaffected by pileup
- Fixed $\tau$: no EA change required
- Result (hetero. $\tau$): 1% for pn indicated, 5-7% for MOS
Concordance 3: XMM Blazars

- 117 bright XMM sources from Matteo Guainazzi
- PSF clipped to reduce effect of pileup
- Result (fixed $\tau$): 5% adjustment to pn indicated, 1-2% for MOS
- Result (hetero. $\tau$): 1% for pn indicated, 5-7% for MOS
Concordance 4: Capella

- Lines from Chandra grating spectra
  - Ne x, Fe xxvii (15 Å), Fe xxvii (17 Å), O VIII
- 5 sets of adjacent observations compared
- Not all instruments used each time
- Result: ±1 generally consistent, LETGS are low of HETGS

Marshall+ in prep.
Conclusions

• We can bring observations into Concordance

• Simple situations give reasonable answers: consistent with other analyses

• More complex situations:
  • Outliers handled with t distribution
  • Fluxes in bands are related globally, not independent
  • Instrument areas are time-dependent
