Quaternion-Based Discriminant Analysis Method for Color Face Recognition

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Abstract

Pattern recognition techniques have been used to automatically recognize the objects, personal identities, predict the function of protein, the category of the cancer, identify lesion, perform product inspection, and so on. In this paper we propose a novel quaternion-based discriminant method. This method represents and classifies color images in a simple and mathematically tractable way. The proposed method is suitable for a large variety of real-world applications such as color face recognition and classification of the ground target shown in multispectrum remote images. This method first uses the quaternion number to denote the pixel in the color image and exploits a quaternion vector to represent the color image. This method then uses the linear discriminant analysis algorithm to transform the quaternion vector into a lower-dimensional quaternion vector and classifies it in this space. The experimental results show that the proposed method can obtain a very high accuracy for color face recognition.

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Introduction

Color images can provide a large quantity of appearance information of the real-world objects and allow the objects to be more accurately described than the grey-scale image [1–3]. In the field of face recognition, many literatures have shown that color face recognition usually can obtain a higher accuracy than conventional face recognition using the gray image of the face. There are three kinds of color face recognition methods. The first kind usually first converts the 3-D color space into a new lower-dimensional space and then perform classification in the new space. For example, an optimum conversion is proposed by Neagoe to transform the 3-D color space into a 2-D color space [4]. It was showed that the obtained 2-D color space was better for face recognition. Jones and Abbott proposed to convert the original 3-D color space to 1-D space, using Karhunen-Loeve (KL) analysis, linear regression, and genetic algorithms [5]. Yang et al. proposed the optimal discriminant model of color face images [6]. The second kind focuses on transforming the original color space into a new color space for better classification result. For example, Kittler and Sadeghi proposed the IGr(R–G) color space for face verification [7]. This color space includes the following three color channels: an intensity (the mean of R, G, B channels), a chromaticity (normalized G) and an opponent chromaticity (normalized R–G) channel. Shih and Liu proposed the optimal color configuration YQCr for color face recognition, where Y and Q color components are from the YIQ color space and Cr is from the YCbCr color space [8]. Liu proposed the so-called uncorrelated color space (UGS), the independent color space (ICS), and the discriminating color space (DCS) for color face recognition [9]. By using these spaces, a very high face recognition accuracy can be obtained [9]. Wang et al. used a sparse tensor discriminant color space (STDCS) model to represent the color image as a third-order tensor [10]. This model is able to preserve the underlying spatial structure of color images and to enhance robustness. The third kind integrates color information and the texture information for face recognition. For instance, Liu et al. used a hybrid color and frequency feature (CFF) method to perform color face recognition [11]. Liu et al. also fused multiple global and local features derived from a hybrid color space RCrQ [12]. Choi et al. proposed color local Gabor wavelets (CLGWs) and color local binary pattern (CLBP) for face recognition [13]. The color local texture features proposed in [13] can use the discriminative information derived from spatiochromatic texture patterns of different spectral channels.

Color images require more storage space than grey-scale images. Moreover, the transmission of the color image also needs a larger bandwidth. The amount of the color image data such as a RGB, HIS, or YCbCr color image is usually three times of that of a grey-scale image with the same size. As a result, it is crucial to seek a way to effectively represent the color image in a low-dimensional space.

We note that classical image processing algorithm is not able to simultaneously mathematically deal with the three channels of the color image. Instead, when dealing with the color image, previous methods first separate the color image into three channels and then apply the traditional image processing algorithms to these three channels, respectively.

The quaternion can be used as a way to represent the color pixel consisting of three components [14–20]. Actually, the quaternion allows the three components of a color pixel to be
simply denoted by a denotation in the form of a “number”. Moreover, the color image can be straightforwardly represented by a quaternion matrix. Because a quaternion is composed of four real numbers, a simple means to represent the color pixel by a quaternion is to let three real numbers in the quaternion be respectively equal to the three color components of the color pixel and let the remaining real number in the quaternion be zero.

The quaternion representation of the color has been used in the context of color texture region segmentation [17]. Shi et al. also used a Hessian matrix defined on the basis of the quaternion to measure the curvature in color images [17]. Angulo [10] exploited the structure tensor of color quaternion image representations to perform feature extraction. Besides the color image, the quaternion was also used for greyscale images [10]. Moreover, we can also use a quaternion matrix to represent the multi-spectrum images such as three or four-channel remote images.

A concise survey on matrices of quaternion entries is presented in [14]. Denis [16] studied the quaternionic Fourier spectrum and attempted to explain the color information contained in the new domain and how the different real and imaginary parts of the spectral quaternionic domain interact with the pure quaternion component chosen to encode colors in the spatial domain. Quaternion Fourier Transform was also proposed as a frequency analysis tool [20–23]. The quaternion has also been used to separate polarized waves and in block truncation coding [24,25]. Miron et al. [24] considered the problem of direction of arrival and polarization parameters estimation in the case of multiple polarized sources impinging on a vector-sensor array and devised a MUSIC-like algorithm to estimate direction of arrival of waves.

We also note that based on the quaternion algebra that was developed in recent years, we can also easily apply matrix operations to the quaternion matrix. This means that once we denote a color image by a quaternion matrix, we can directly exploit the quaternion algebra to implement some image processing techniques such as image denoising, image segmentation, edge detection and image transform [26–32].

In this paper, we aim at transforming the color image into lower-dimensional data for color face recognition. In order to do so, we extend the widely used linear discriminant analysis technique [33,34] to the quaternion matrix that represents the color image. Previous research has shown that dimensional reduction is usually helpful for classification, as dimensional reduction is able to extract the important and robust features from the image and to neglect the trivial and non-salient information of the image. The experimental results show that the proposed method can not only reduce dimensional quaternion array to describe the color image but also can achieve a high recognition accuracy. The proposed method has the following rationale: as it is a feature level fusion method, it can convey much richer information of the bimodal biometric traits than the matching score level fusion and decision level fusion methods [35–37].

We first denote each color pixel of a color image by quaternion matrix. Because a quaternion is composed of four real numbers, a simple means to represent the color pixel by a quaternion is to let three real numbers in the quaternion be respectively equal to the three color components of the color pixel and let the remaining real number in the quaternion be zero. The quaternion representation of the color has been used in the context of color texture region segmentation [17]. Shi et al. also used a Hessian matrix defined on the basis of the quaternion to measure the curvature in color images [17]. Angulo [10] exploited the structure tensor of color quaternion image representations to perform feature extraction. Besides the color image, the quaternion was also used for greyscale images [10]. Moreover, we can also use a quaternion matrix to represent the multi-spectrum images such as three or four-channel remote images.

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Algorithm of the quaternion-based discriminant method

We note that as $S$ is a quaternion matrix, it is hard to directly solve its eigenvalues and eigenvectors. However, we can solve this problem in the following way: first of all we construct an equivalent complex matrix of $S$. Then we solve the eigenvalues and eigenvectors of the equivalent complex matrix.

Corollary 1. If the quaternion matrix $Q = G + Hj$, then the equivalent complex matrix $E$ of $Q$ is defined as follows:

$$E = \begin{bmatrix} G & H \\ -H & G \end{bmatrix}.$$  \hfill (9)

If $Q$ is a $M \times M$ quaternion matrix, then $E$ is an $2M \times 2M$ complex matrix.

For example, if $Q$ is the $2 \times 2$ quaternion matrix

$$Q = \begin{bmatrix} a_0 + a_1 i + a_2 j + a_3 k & b_0 + b_1 i + b_2 j + b_3 k \\ c_0 + c_1 i + c_2 j + c_3 k & d_0 + d_1 i + d_2 j + d_3 k \end{bmatrix},$$

then we have

$$E = \begin{bmatrix} a_0 + a_1 i & b_0 + b_1 i \\ c_0 + c_1 i & d_0 + d_1 i \end{bmatrix} + \begin{bmatrix} b_2 + b_3 i & a_2 + a_3 i \\ d_2 + d_3 i & c_2 + c_3 i \end{bmatrix}j. \quad \hfill (10)

From the reference, we know real eigenvalues of $E$ always appear as a pair and so do the complex eigenvectors of $E$. In other words, $E$ has the following eigenvalues and eigenvectors: $\lambda_1, \lambda_1^*, \lambda_2, \lambda_2^*, ..., \lambda_M, \lambda_M^*$. $\lambda_i, \lambda_i^*$ is the composite of $\lambda_i$ and $\lambda_i^*$ is the associated eigenvector of $\lambda_i$. It is clear that $\lambda_i = \lambda_i^*$. From the above context, we know that $S = S_1 + S_3j$, then $S$ has the following equivalent complex matrix $S_E = \begin{bmatrix} S_1 & S_2 \\ -S_2 & S_1 \end{bmatrix}$. We also note that eigenvalues and eigenvectors of the quaternion matrix and its equivalent complex matrix have the following relationship [14]: the complex eigenvalues of a quaternion matrix are the same as the eigenvalues of its equivalent complex matrix. In addition, if $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ is the eigenvector with respect to the eigenvalue $\lambda$ of equivalent complex matrix $E$, then $\bar{u} = [u_1 - u_2j]$ is also an eigenvector with respect to the eigenvalue $\lambda$ of quaternion matrix $Q$. Hereafter $\bar{\cdot}$ denotes the complex conjugate.

The main steps of the algorithm to implement the discriminant method are as follows.

Step 1. Represent each pix of every color image by a quaternion number and denote a color image by a quaternion vector.

Step 2. Use Eq.(7) to calculate the covariance matrix $S$ of the class mean of the quaternion arrays corresponding to the training samples. Then construct the equivalent complex matrix of the covariance matrix.

Step 3. Compute the eigenvalues $\lambda_1, \lambda_1^*, \lambda_2, \lambda_2^*, ..., \lambda_M, \lambda_M^*$ and eigenvectors $v_1, v_1^*, v_2, v_2^*, ..., v_M, v_M^*$ of the equivalent complex matrix. Suppose that the eigenvalues have the relationship $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_M = \lambda_M^*$. We exploit $v_1, v_2, \ldots, v_d$ to construct a $2M \times d$ complex transform matrix $P = [v_1, v_2, \ldots, v_d]$.

Step 4. Convert the quaternion array corresponding to each color image into its equivalent complex matrix. For a quaternion array $q = q_1 + q_2j$, its equivalent complex matrix is

$$\begin{bmatrix} \frac{q_1}{\sqrt{q_1^2 + q_2^2}} + \frac{q_2}{\sqrt{q_1^2 + q_2^2}} \left( \frac{1}{\sqrt{q_1^2 + q_2^2}} \right)^2 & \frac{-q_2}{\sqrt{q_1^2 + q_2^2}} + \frac{q_1}{\sqrt{q_1^2 + q_2^2}} \left( \frac{1}{\sqrt{q_1^2 + q_2^2}} \right)^2 \\ \frac{-q_1}{\sqrt{q_1^2 + q_2^2}} + \frac{q_2}{\sqrt{q_1^2 + q_2^2}} \left( \frac{1}{\sqrt{q_1^2 + q_2^2}} \right)^2 & \frac{q_2}{\sqrt{q_1^2 + q_2^2}} + \frac{q_1}{\sqrt{q_1^2 + q_2^2}} \left( \frac{1}{\sqrt{q_1^2 + q_2^2}} \right)^2 \end{bmatrix}.$$
of the PCA feature extraction results of the three channels for the ultimate face recognition. Fusion LDA does similarly except that the performed feature extraction procedure is LDA rather than PCA. From Figure 2, we see that our method, quaternion LDA, obtains a much higher classification accuracy than quaternion PCA, fusion LDA and fusion PCA. Figure 2 shows classification accuracies of LDA using a single color channel on the GTFB database.

Conclusion

This paper, for the first time, proposes the quaternion-based discriminant analysis method. This method can represent the color images in a simple and tractable way. Since the proposed method is feature level fusion method, it is able to convey richer information of the color image than the score level and decision level fusion methods. Moreover, when the method transforms the color image into a very low-dimensional space, it can also well represent the image. The experiments show that the proposed method can perform very well in color face recognition.

Author Contributions

Conceived and designed the experiments: YX. Performed the experiments: YX. Analyzed the data: YX. Contributed reagents/materials/analysis tools: YX. Wrote the paper: YX. YX.