Fracture analysis of magneto-electro-elastic smart materials: a brief review

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Abstract. The magneto-electro-elastic (MEE) materials are able to effectively realize the mutual transformation among the electric energy, magnetic energy and mechanical energy. They are widely used in many important areas (e.g., in aerospace, biomedicine and intelligent manufacturing) due to their excellent properties. However, owing to low fracture toughness and high defect sensitivity, MEE materials are prone to crack. In this paper, we give a brief summary on fracture studies of MEE smart materials, mainly from analytical and numerical perspectives.

1. Introduction
With the rapid development of modern science and technology, the demand for multi-functional and intelligent materials in high-tech industries has grown dramatically, and various sorts of smart materials have emerged. Typical smart materials include piezoelectric materials, piezomagnetic materials, shape memory alloys, electrorheological materials, magnetorheological materials, magneto-electro-elastic (MEE) materials, and so forth. As the representatives, the MEE materials are composed of piezoelectric and piezomagnetic materials in a certain way, which can realize the conversion among electrical energy, magnetic energy and mechanical energy. Simply speaking, when the mechanical loads cause the material to deform, the electric and magnetic field can be generated; conversely, applying the electric or magnetic field to the material will cause it to deform. This kind of material combines the advantages of piezoelectric and piezomagnetic material, and has more excellent electromagnetic coupling performances than a single piezoelectric or piezomagnetic material. Due to their excellent self-sensing, self-diagnosing, self-motivating and self-repairing properties, the MEE smart materials have been playing a significant role in many fields, e.g., magnetic field detection (see Figure 1(a)), sensors (see Figure 1(b)), actuators (see Figure 1(c)), electronic packaging, medical imaging and big data.

![Figure 1. Applications of MEE materials (pictures from internet)](image)

MEE materials are typical brittle materials with low fracture toughness and high defect sensitivity.
Defects such as dislocations, cracks, holes and inclusions are inevitable during their manufacture and service (see Figure 2 for crack in BiFeO$_3$), and these may further lead to the initiation and propagation of cracks under pure or coupled magnetic/electrical/mechanical field, which will directly affect the performances and functions of the components, and even cause the premature failure of the entire structure. Therefore, detailed and in-depth research on the fracture behavior of MEE smart materials has crucial theoretical value and application prospects. Herein, the research progress on this topic in recent years is briefly introduced from analytical and numerical aspects.

Figure 2. Crack in BiFeO$_3$[1]

2. A brief summary on fracture of MEE materials

2.1. Analytical work

So far, researchers have carried out extensive and effective analytical work on the fracture of MEE materials, using various techniques such as integral transform, singular integral equation and complex function theory. Feng et al.[2] adopted the integral transform technique to investigate the dynamic stress intensity factors of penny-shaped crack in MEE material. With the integral transform approach, Singh et al. [3] derived closed-form solutions for finite MEE layer with two collinear cracks. Zhou et al. [4] made use of the symplectic expansion to find the solutions of edge-cracked circular MEE media with general boundary conditions. Li et al.[5] tackled a penny-shaped interfacial crack between dissimilar MEE layers using the Hankel transform technique. Rekik[6] reported the effect of a center crack on the behavior of a functionally graded MEE material by virtue of Fourier transform and singular integral equations. Liu et al.[7] employed the generalized Almansi’s theorem and the Schmidt method to cope with 3D rectangular cracks in MEE materials. Yang and Li[8] focused on the problem of the scattering of the SH wave in an arbitrary direction by a crack in a semi-infinite MEE material bonded to a half-space of piezoelectric material by way of the Copson method. Using the Fourier and Laplace transform techniques, Rogowski[9] investigated the fracture behavior of MEE materials with a straight-line crack parallel to the poling direction. Viun et al.[10] determined the fracture parameters of periodic cracks in a MEE medium through the complex function theory. Liu and Li[11] discussed the fracture properties of MEE interlayer in multiferroic composites with the combined Green’s functions, dislocation theory and singular integral equations. By means of the Green’s functions, Fabrikan[12] studied crack problems in MEE semi-infinite space subjected to points loadings or dislocations. With the dislocation theory and integral transform method, Bagheri et al.[13] examined moving cracks in a functionally graded MEE strip. Kaczynski and Kaczynski[14] considered an anti-crack in a transversely isotropic MEE space with the singular integral equation. Through the singular integral equation, Zhang[15] explored the fracture behavior of layered multiferroic structures under concentrated force. Wu and Li[16] utilized the generalized potential theory to inspect an elliptical crack in a 3D infinite transversely isotropic MEE composite media. Xiao et al. probed the fracture characteristic of MEE materials with inclusions[17] and nanoscale defects[18] using analytical tools (e.g., conformal mapping technique). Tupholme[19] took a generalized dislocation layer method to analyze propagating shear cracks under non-uniform load in MEE body. With the complex potential theory and conformal mapping technique, Yang and Liu[20] analyzed anti-plane fracture of MEE materials with cracks emanating from hole in nanoscale.
2.2. Computational work
Wang and Mai[21] took advantage of the finite element method (FEM) to investigate the behaviors of a crack in a MEE medium. Based on the FEM, Rao and Kuna[22] proposed the domain-form interaction integral for the calculation of fracture parameters of MEE bodies. Sladek et al.[23] modeled cracked nano-sized MEE media with the FEM and the gradient theory. Garcia-Sanchez et al.[24] adopted the dual boundary element method (BEM) to look into the fracture of 2D linear MEE composite materials. Within the time domain, Wunsche et al.[25] studied the dynamic cracking of MEE solids through a hypersingular BEM. Lei et al.[26] applied the time-domain BEM to tackle dynamic interface cracks in MEE bi-materials. Considering exact crack face boundary conditions and with the BEM, Zhao et al.[27] put forward an iteration approach to investigate a cracked finite MEE solid.

3. Concluding remarks
In this paper, the widely used analytical and numerical tools in the fracture study of MEE materials are briefly introduced. Generally speaking, with the analytical methods, closed-form solutions can be obtained and the research cost is relatively low. However, these approaches are mainly applicable to problems with simple geometries/crack configurations/boundary conditions (e.g., single, symmetric or periodic stationary crack in infinite or semi-infinite domain under simple load, and crack propagation with preset path, etc.). For more complex cases (e.g., problems involving arbitrarily intersecting/branched crack and its growth, arbitrary form of load/geometry/boundary conditions), numerical approaches (e.g., the FEM, BEM, MLPG, XFEM and IGA) with stronger applicability and higher repeatability are more suitable. Among the widely used computational tools, the FEM is the most mature; yet, when solving fracture problems, meshes should be consistent with the crack geometry and therefore remeshing is required to capture the crack path. The BEM is able to simulate crack problems with lower cost by dimensionality reduction, but it is not convenient in discontinuity characterization, nonlinear analysis and fundamental solution acquisition. The MLPG and other meshless methods use a series of points to discretize domain and can model stationary cracks and also crack propagation problems with high accuracy and convenience; however, they still encounter difficulties in boundary condition treatment, crack description, numerical integration and solution efficiency. The meshes used in the XFEM and extended IGA do not need to be conforming to the crack faces, and the solution accuracy can be improved by introducing crack-tip enrichment functions into the field approximations; in addition, the crack propagation can be simulated without remeshing; nevertheless, there are also limitations in describing the jump of physical field across the crack surface for complex cracks (e.g., intersecting and branching cracks).

Although plenty of work has been carried out around the fracture of MEE smart materials, due to the complexity, there are still many issues to be further explored, e.g., in the development of (1) more universal fracture criteria; (2) non-linear fracture theory; (3) advanced numerical methods for 2D and 3D crack initiation, branch and propagation; (4) advanced test methods and equipment under multi-filed coupling; (5) advanced MEE materials with higher fracture toughness.
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