Where are we in the theory of High Temperature Superconductors°?

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In this talk I will briefly review our present theoretical understanding of some of the important issues in the high Tc cuprates. In view of its success, at a qualitative level and some times quantitative level, the theory initiated by Anderson and developed further by him and collaborators will be discussed. Many issues that still challenges us will be pointed out.

I. INTRODUCTION

I feel honored to speak at this conference which commemorates the Birth Centenary of an eminent son of India, Sir K S Krishnan, the co-discoverer of Raman effect, and the Platinum Jubilee year of this Physics Department that has nurtured many excellent physicists over the years. Krishnan would have enjoyed seeing the development in the field of high Tc superconductivity, in which a two dimensional metallic character and quantum magnetism play fundamental role. Krishnan in late 30’s, pioneered the study of anisotropic transport and magnetic properties of graphite, an excellent example of two dimensional metal. He had deep insights in the magnetism of transition metal and rare earth ions through his innovative susceptibility measurements of families of magnetic systems such as CuSO₄·5H₂O, that are actually Mott insulators in the current parlance.

Nearly 12 years ago Bednorz and Muller discovered superconductivity in Ba doped LSCO and broke the barrier of the record Tc of 23 K exhibited by the A-15 family member Nb₃Ge. Soon many cuprates were synthesized and now the maximum Tc is nearly 160 K in some Tl/Hg based cuprate under pressure. From experiment point of view the quality of single crystals have improved considerably over years and we have a good set of several reproducible experimental results that begs quantitative explanation.

From theory point of view the Resonating Valence Bond (RVB) theory of Anderson that had a lead right from the beginning in view of its strong foundation on the available body of experimental facts, has made significant progress, considering the nature of the hard quantum many body problem that cuprates posed. This fertile theory, by its fairly penetrating character also initiated a resurgence in the theory of strongly correlated systems and quantum magnetism. This theory has remained the leading guidance in the sense of providing the right directions and emphasizing the crucial aspects of the problem, albeit with occasional changes compelled by new experimental results.

In this talk I will enumerate some of the important issues and point out our current understanding from the point of RVB theory. This talk will be sketchy and some details and many helpful hints for making further progress can be found in Anderson’s book.

II. ANDERSON’S ORIGINAL PROPOSAL AND INITIAL PROGRESS

Anderson’s original proposal presented at the Bangalore conference in January 87 identified the relevant interactions and presented it in a succinct form as a one band large U Hubbard model or the equivalent t-J model. The insulating parent compound LSCO was suggested to be a 2d Mott insulator in a disordered spin liquid or RVB state. The resonating spin singlets are neutral in the insulating state - they do not transport charges at low energies. On doping they start transporting charges leading to superconductivity. Anderson also suspected the presence of neutral spin half excitations (which was later named spinons) with their own pseudo fermi surface. RVB mean field theory that brought out the neutral spinons and their pseudo fermi surface in the insulating state and superconductivity in the doped case were discussed by Anderson and collaborators. Affleck, Martson and Kotliar brought out an energetically better mean field state namely the d-RVB or the flux state. Inspired by Anderson’s suggestions Kivelson and collaborators discussed short range RVB in some detail focusing on spinon and holon excitations. Slave boson theories and gauge theories followed suit and there were intense activities and speculations, including the possible parity violating superconducting states with connection to Laughlins quantum Hall state.

Looking back, the proposals of Anderson, with its emphasis on strong correlation, the ensuing non fermi liquid states, spin charge decoupling, spinon fermi surface, has remained robust and has given us a good way to think about this complex problem. In particular the ARPES, neutron scattering, NMR relaxation and transport properties can be qualitatively understood from the point of view of the above proposal. However, quantitative understanding is yet to be achieved.

The precise mechanism of superconductivity, particularly in the single layer materials, is some thing that has eluded a sharper theoretical understanding so far, even though it was one of the first issues that caught the attention of the condensed matter community. Very fundamental and new ideas have however emerged through the notion of inter layer pair tunneling, which we will...
discuss at the end. The electron kinetic energy gain as the origin of the superconducting condensation energy is also a novel aspect of the present system.

III. 2D QUANTUM ANTIFERROMAGNET AND UNDER DOPED REGIME

A good understanding of the Mott insulator should help one to understand the doped Mott insulator better. In this insulating state, kinetic or super exchange dominates and only spin degrees of freedom governed by Heisenberg Hamiltonian are present at low energies. An emerging local gauge symmetry in the insulating state was found and formalized by Anderson and the present author as a gauge theory. It was later discovered that this gauge field captures the physics of chiral fluctuations among the interacting spins in the low doped regime. The d-RVB state or the Affleck-Marston-Kotliar phase can be thought of as a uniform RVB state in which \( \pi \) fluxes are condensed at low temperatures.

There are good theoretical indications that the 2d Heisenberg model has a long range antiferromagnetic order. Several static and quasi static phenomenon are well explained by the spin wave theory inspired non linear sigma model analysis. However, the dominant correlations in the ground state is that of a d-RVB state; as suggested by Hsu it is meaningful to think of the ordered state as a spinon density wave in a spin liquid state. The antiferromagnetic order is fragile and disappears at about 1.5% of doping. Recent ARPES study in under doped and insulating layered cuprates point out that the d-RVB with its massless Dirac like spinon spectrum is a good reference state to describe the Mott insulating and the under doped state. This questions the real relevance of the non linear sigma model in its present form, in the doped quantum melted region.

The physics in the under doped regime is complicated by disorder, long range coulomb interaction, charge localization and micro phase separation effects. This is the origin of the stripe phase. Some theoretical work is going on in this regime. A recent work by Fisher and collaborators, addresses the issue of how an insulator to superconductor transition takes place in the ground state by their theory of nodal fermi liquids. Their reference state is a d-wave superconductor, where quantum fluctuations induced by coulomb correlations drive a metal(insulator) superconductor transition. This theory captures some of the physics of the t-J model and over emphasizes pair fluctuation of charges. There are also some fundamental questions whether we can have a boson metal ground state for low doping.

IV. ‘SOLVING’ THE T-J MODEL

Several experimental results indicated the validity of the t-J model for the conducting cuprates. From theory point of view the derivation of the t-J model has been shown rather satisfactorily through sharpening of the Zhang-Rice singlet arguments through detailed cluster calculations. The t-J model however remains unsolved in a satisfactory fashion, in view of the on site constraint involved. In the absence of an exact solution or a good many body theory, a natural way to solve the t-J model is to look for the next level of effective theories, guided by experiments and some time theoretical arguments, that will point to the correct final solution.

In constructing the next level of effective theories there has been considerable work using slave boson mean field theories and the related gauge field theories. This approach, though presents itself with many new possibilities that often agree with the experimental trends, involves uncontrolled approximations and is far from satisfactory as a quantitatively correct many body theory. These theories are in a sense reaction against the conventional restrictive fermi liquid type of perturbative many body theories that fail in these class of correlated conductors.

In the numerical front there has been many efforts by several groups. But they have not been very helpful in our understanding the rich low energy physics offered by the t-J model - they most often capture some high energy features and face serious difficulties when it comes to low energy physics.

In the analytical front there is hope however, in the sense explained below, arising from Anderson’s proposal of failure of fermi liquid theory in 2d Hubbard model for arbitrarily small repulsion and the associated notion of 2d tomographic Luttinger liquid. This means that \( U^* = 0 \) is an unstable fermi liquid fixed point as in the 1d Hubbard model; and in principle the strong coupling non fermi liquid fixed point \( U^* = \infty \) can be understood by a careful study of small \( U \).

Anderson, through a phase shift analysis of two particles on the fermi surface in the \( 2k_F \) and singlet channel argues for the presence of a singular Landau parameter that leads to a different spin charge velocities on the fermi surface and anomalous exponent for the electron propagator. This point has remained controversial and the present author has given some supporting arguments for Anderson’s proposal. The present author has also brought out a related mechanism involving zero sound that could destabilize fermi liquid state in 2 and 3 dimensions.

The presence of singular forward scattering, Anderson argues, leads to the so called tomographic Luttinger liquid (TLL) state, a non fermi liquid state exhibiting spin charge decoupling as well as branch point singularities for electron propagator on the fermi surface. The TLL the-
ory of Anderson is essentially a Landau’s fermi liquid theory, but with a singular forward scattering. I view it as a natural generalization of Landau’s fermi liquid theory in the following sense. In Landau’s fermi liquid theory occupied low energy quasi particle states influence each other pairwise only in a mean field fashion irrespective of their relative momenta. In the tomographic Luttinger liquid, those electrons that have vanishingly small relative momenta (that is, belonging to a given tomograph) influence each other pairwise, in a non mean mean field fashion, leading to a finite phase shift in the relative momentum channel. While electrons belonging to different tomographs do not influence each other. This has profound consequences, as argued by Anderson.

In view of its simple form, TLL theory should lend itself to more detailed analysis and comparison with experimental results, particularly of the normal state. Through Anderson’s proposal, which has a good phenomenological support, we have a possibility of analyzing the t-J model, a strong coupling limit, through a Landau type of theory. The parameters of this non fermi liquid theory can be determined from experiments.

This scenario should work well for the optimal doping regime and beyond. However, it becomes a difficult problem with new possibilities when we go to the under doped situation. The physics near this region has the spin gap phenomenon and it requires some fresh thinking, or to go back to old RVB ideas to make further progress. As mentioned earlier, disorder and long range interactions and the corresponding charge localization effects cloud the real issue that we are after.

V. NORMAL STATE AS A TOMOGRAPHIC LUTTINGER LIQUID

The normal state of cuprates is generally accepted as anomalous and non fermi liquid like, thanks to a variety of experimental results - frequency and temperature dependent conductivity, Hall effect, NMR relaxation, thermal conductivity, the non fermi liquid spectral functions seen in ARPES measurements and so on. While the semi phenomenological theory of Anderson’s tomographic Luttinger liquid suggests anomalous exponents, a satisfactory derivation of exponents and other details awaits further theoretical developments.

At another level, the two scattering rates on the fermi surface, one corresponding to the longitudinal resistance that scales as $T$ and the other rate measured from Hall angle scaling as $T^2$ needs to be formalized. Anderson’s suggestions and heuristics remain as fundamental proposals that calls for a satisfactory derivation to make quantitative progress.

The development of spin gap at low temperatures becomes prominent when we go to the under doped situation. In terms of the old RVB idea this has a natural explanation in terms of the development of neutral spinon pair condensation. However, the importance of interlayer pair tunneling and interlayer super exchange in providing the spin gap phenomenon has also been suggested by Anderson and collaborators. The real origin of the spin gap, identifying the correct in plane and inter plane contributions, needs to be sharpened further, in view of the contrast between the strong interlayer correlations in YBCO, compared to the equally high Tc compound Tl-2201 with weak interlayer interaction.

Charged stripes in the normal states are interesting phenomenon of localization of the heavy holes that also suppress superconductivity. It is becoming clear that they are not providing any obvious mechanism for superconductivity. However, there are some intriguing experimental suggestions at low doping of very low energy stripe and perhaps antiferromagnetic long range order in an incommensurate peak position in the superconducting states. These are likely to be complications that are not very essential for our understanding of the underlying robust physics of superconductivity and anomalous normal state. However, we need a good theoretical understanding of them.

VI. CONFINEMENT AND INTER LAYER PAIR TUNNELING

The original proposal of Anderson relied on the neutral singlets of the insulating RVB state getting charged on doping and leading to superconductivity. This suggestion was however challenged by the inter layer regularity of the Tc in various cuprates. So it was felt that perhaps the quantum fluctuations arising from the strong correlations in a single plane are strong enough to suppress superconductivity by enhanced gauge field or phase fluctuations.

Around the same time the notion of confinement was introduced by Anderson and Zou by looking at the large anisotropy of the normal state resistivity - the ratio of the c-axis resistivity to ab-plane resistivity was too large compared to band effective mass anisotropy. It was then argued that the spin charge decoupling in the anomalous normal state strongly suppresses the electron spectral weight close to the fermi surface leading to absence of coherent one electron transport between neighboring planes. This is called confinement. One electron kinetic energy between two conducting planes is frustrated. This frustration leading to incoherent transport is in a way more subtle than the way one electron kinetic energy is frustrated in a Mott insulator. There is excellent experimental support for confinement phenomenon from optical sum rule measurements.

A pair of electrons on the fermi surface in a spin singlet state with zero center of mass momentum, however, retains its identity without any quantum number fractionalization. Hence it can tunnel coherently between two planes. The selective suppression of one electron coherent tunneling, and not of two electrons, is the origin of
inter layer pair tunneling mechanism of superconductivity of Wheatley, Hsu and Anderson [29,4]. The frustrated one electron kinetic energy loss is gained by pair delocalization between two planes.

Anderson formalized the above through a BCS type of effective Hamiltonian that expressed the major source of pair condensation energy as a pair tunneling terms between two fermi liquid planes:

\[
H_{\text{pair}} = -\sum_{k} T_{J}(k)c_{k1}^\dagger c_{-k1} c_{-k2} c_{k2}^\dagger
\]

and a small intra plane BCS type of scattering term non local in k-space. Notice that in the above term, the individual electron momentum is conserved, making it a resonant tunneling between to planes. It is this resonant character or local character in k-space that leads to a large Tc proportional to the pair tunneling matrix element \(T_J\) - a non-BCS dependence of Tc on the pairing interactions.

At the level of models, formalizing Anderson’s BCS type of formalism is a very important issue. Early derivation by Muthukumar [31] in terms of slave boson variables, can be modified to bring out the electron momentum conservation. However, a satisfactory derivation, that also addresses the issue of one electron incoherence [32,31] and two electron coherence between planes or chains is still needed. The absence of bilayer splitting in the ARPES spectral functions is a good indication for low energy one electron electron incoherence between two layers.

The interlayer coherence phenomenon and possibility of application to the closely related organic conductors has been studied by some authors [32]. The present author has argued that certain extreme sensitivity of the Tc of organic superconductors to off chain or off plane disorder points towards the origin of inter layer pairing mechanism of superconductivity in these systems and an apparent violation of Anderson’s theorem for c-axis disorder.

**VII. IS INTER LAYER PAIR TUNNELING THE ONLY MECHANISM OF SUPERCONDUCTIVITY IN CUPRATES?**

The pair tunneling effective Hamiltonian has been successfully used by Anderson and collaborators to understand the origin of large \(T_c\) and also certain features of the gap function in k-space. In this context Anderson also proposed an important test for the interlayer pair tunneling mechanism: since the superconducting condensation energy arises primarily from pair tunneling between planes, the c-axis Josephson plasma energy should be the same as the pair condensation energy. This remarkable prediction was verified for the case of bilayer materials such as YBCO and the one layer LSCO. However, the one layer Tl and Hg cuprates have remained an exception and do not seem to follow the interlayer pair tunneling mechanism. The present author made a suggestion that part of this could be accounted for through the particle-hole pair tunneling mechanism. This still does not solve the problem completely. There are also other suggestions. This has become a challenge and perhaps a revision of part of Anderson’s Central dogma to the effect that ‘a single layer of cuprate may be superconducting’ may be called for.

Thus we have to go back to the original one layer RVB mechanism and see how it can explain the large \(T_c\) of one layer materials. RVB gauge theory ideas have been pursued a lot along these direction [33], including some instanton ideas and an idea of quantum tunneling of RVB \(\pi\) flux. It is becoming clear that an in plane kinetic mechanism is also operating in addition to the inter plane kinetic mechanism. It is also found [34] that the inter layer and intralayer mechanisms do not help each other and also dominate in different regime of doping.

**VIII. SHARP RESONANCE IN NEUTRON SCATTERING AND QUASI PARTICLE PEAK IN ARPES**

Another outstanding experimental result is the 41 meV resonance. This sharp 41 meV resonance, limited only by the instrumental resolution, in neutron scattering in YBCO in the superconducting state has a natural explanation in terms of pair tunneling mechanism as proposed by Anderson and collaborators. The peak corresponds to a transition between the bonding and antibonding state of an electron pair between the bilayers, induced by the spin flip scattering of the neutron within a layer. This resonance is strongly pinned to \((\pi, \pi)\) - this point has no satisfactory explanation so far in my opinion.

Similarly, in the superconducting state, one sees a rather sharp Bogoliubov quasi particle around \((\pi,0)\) at a finite energy of about 20 meV. These quasi particles are rather heavy and do not disperse in k-space. A satisfactory explanation for this phenomenon, including why this Bogoliubov quasi particle peak is confined to the Brillouin zone boundary, away from normal state fermi surface is not available so far.

**IX. SYMMETRY OF THE GAP FUNCTION AND MAGNETIC FIELD EFFECTS**

As it has been emphasized by Anderson and coworkers, the symmetry of the superconducting order parameter is not strongly dictated by the kinetic pairing mechanism. It view of its strong locality in k-space this mechanism determines only the magnitude of the gap. The detailed symmetry of the gap function is determined by the in plane short range repulsion effects, favoring a d-wave. That is the local on site constraint \(n_{1\uparrow} + n_{1\downarrow} \neq 2\) leads to
a global constraint on the pair amplitude in k-space:

\[ \langle c_k^+ c_i^\dagger \rangle = 0 \rightarrow \sum_k \langle c_k^+ c_{-k_i}^\dagger \rangle = 0 \]

The above global constraint in k-space is easily satisfied if the pair amplitude has a d-symmetry.

In the case of conventional s-wave superconductors, small magnetic field does not modify the symmetry of the gap function or does not collapse the gap. In the case of the cuprate with d-node even small magnetic field does not modify the symmetry of the gap function or does not collapse the gap. In the case of the cuprate with d-node even small magnetic field strongly modifies the nature of the superconducting state, as recently discovered by Krishana and Ong [45] in YBCO. Their observation of a magnetic field induced removal of the d-node has Krishana and Ong [45] in YBCO has revived the possibility of \( d_{x^2-y^2} + id_{xy} \) state at low temperatures and low magnetic fields. Laughlin, Wilczek and others proposal of anyonic superconducting state with spontaneous P and T violation is perhaps realized now, however, with a small help from a magnetic field. The physics at Dirac nodes of a d-wave superconductor seems to be filled with rich possibilities [46].

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X. OTHER APPROACHES TO STUDY THE T-J MODEL - SPIN FLUCTUATION, SPIN BAG, SO(5) SYMMETRY ETC.

The spin fluctuation theories [47] work on a fermi liquid basis and suggest pairing mediated by the exchange of spin fluctuation quanta. There are deep issues [48] related to the incompatibility of real space super exchange and the fermi liquid background apart from the fact that normal state anomalies and inter layer regularities in Tc are not explained satisfactorily. Spin bag theory [49] is essentially a spin fluctuation theory with a real space tinge to it. It also suffers from similar criticism.

In the SO(5) front Zhang and collaborators [50] view the zero temperature superconducting order of the doped 2d cuprates as one that is obtained by a rotation of the antiferromagnetic order. An alleged \( \pi \) operator is the generator of this rotation. Serious criticisms [51] starting from technical points to points of fundamental principles have been raised.

XI. CONCLUSION

While one thought that the mist is getting cleared, one sees some further mist that challenges us in the cuprate game. However, the direction provided by RVB related ideas has been a constant source of real understanding of these systems. And with some more concerted effort a satisfactory picture should emerge soon.

XII. ACKNOWLEDGEMENT

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