Digital sorting of laser beams by radial number: degenerate and non-degenerate states

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Abstract. We have created and tested a computer sorting method of Laguerre-Gauss (LG) radial modes with permanent topological charge arising from the perturbation of both single LG beams and their composition using a diaphragm with different radius $R$. The method based on computer analysis of the intensity moments. We considered two types of perturbed beams: single LG beam and complex LG beams with different radial number included in an integral array. The diaphragm acts on LG beam in such a way that when the radius of the diaphragm decreases, a set of secondary LG modes with different radial indices, which are characterized by different mode spectra, arise. Reconstructed spectrum using digital processing allows one to reconstruct the real LG modes and calculate the measure of uncertainty arising under the action of a disturbance, using the concept of information entropy. The calculations performed showed that the correlation degree of the measured disturbed beams is about 0.94. It was also found that the perturbation of a complex beam leads to the appearance of a degenerate spectrum in amplitudes, since one spectral line corresponds to a whole set of modes with different radial indices. To detect the spectrum, it was required to know $M$ keys, which are the spectra of the amplitudes of nondegenerate perturbed beams in our experiment, degree of correlation was also 0.94.

1. Introduction

One of the highlighted problems of data information processing for optical communications and quantum key distributions for cryptography is the vortex modes sorting of a combined vortex beams by radial $n$ and azimuthal $m$ quantum numbers. Solutions to this problem are mainly based on two approaches. The first approach involves the use of diffraction optical elements in conjunction with interferometric devices and modulators (see e.g. [1-5]). The second one proposes to use digital mode sorting based on higher order intensity moments technique that allows to significantly simplify optical devices and extend their capabilities (see e.g. [6,7]). Such a digital approach was developed and implemented only for sorting vortices by their topological charge. The purpose of our communication is to consider the process of digital sorting of Laguerre-Gauss beams by radial numbers.

The LG mode sorting model is based on perturbation of a standard LG beams array via a conventional hard-edged aperture with a circular hole. The beam perturbation results in a broad spectrum of secondary LG modes, the wavefield of which can be represented for a single initial LG beam as
\[ \Psi_{m,n}(r,\phi,R) = r^{|l|}e^{in\phi}\exp\left(-r^2\right)\sum_{k=0}^{\infty} C_{m,n,k}(R) L_k^{|l|}(2r^2) , \] 

where \( r = \rho/\rho_w \), \( \rho \) and \( \phi \) are the polar coordinates, \( \rho_w \) stands for a beam, \( R \) is a normalized aperture radius while the beam amplitudes are described by the equation

\[ C_{m,n,k} = \frac{\int_0^R \Psi(r,\phi) \Psi^*(r,\phi) r \, dr}{\int_0^\infty \left| \Psi(r,\phi) \right|^2 r \, dr} \] 

and \( \Psi_{m,n}(r,\phi) \) denotes the complex amplitude of a single non-perturbed LG beam. The analysis and measurement of the intensity of a singular beam perturbed by a circular diaphragm \( \Im_{m,n}(r,\phi) = \left| \Psi_{m,n}(r,\phi) \right|^2 \) is carried out strictly in the plane of the focus of the lens, resorting to the analysis of the moments of intensity of higher orders using the moments of intensity presented in the form

\[ J_{p,q} = \int_{D} F_{p,q}(r,\phi) \Im_{m,n}(r) \, dS , \] 

where \( F_{p,q}(r,\phi) \) is a function of the moments of intensity, \( p,q = 0,1,2,\ldots \). Based on the fact that the intensity of the transverse beam distribution is described by an axisymmetric function, it can be reduced to the form \( F_{p,q}(r) \). The representation of the intensity of the moments in expression (3) can be represented in the form of a square matrix, the elements of which are the squares of the amplitudes \( C_{m,n,k}^2 \) and the cross terms \( 2C_{m,n,k}C_{m,n,s} \). The right side of equations (3) really depends on the squares of the coefficients, as well as the cross terms

\[ J_{p,q} = \int_0^\infty \int_0^\pi F_{p,q}(r,\phi) L_k^{|l|}(2r^2) L_j^{|l|}(2r^2)e^{-2r^2} \, r \, dr \] 

On the one hand, we can measure this matrix experimentally, and on the other hand, calculate it as a measurable value.

The number of squares of amplitudes is equal \( N \), and the number of cross-factors in the intensity distribution is found by a combination equal to the binomial coefficient \( N!!/(2!!(N-2)!!) \). It is worth noting that the column vector \( \mathbf{C} \) of mode amplitudes and the column vector \( \mathbf{J} \) of the intensity moments are determined among themselves using a linear relationship \( \mathbf{C} = \mathbf{J}^{-1}\mathbf{J} \).

Since the circular aperture does not produce secondary modes with a set of different topological charges \( m \), the orbital angular moment of the perturbed beam remains constant. However multiple secondary modes of perturbed LG beam give rise to uncertainty of the wavefield state, measure of which is the information entropy (Shannon entropy) [8,9]. The information entropy can be represented in terms of squared amplitudes as

\[ H_{m,n} = -\sum_{k=0}^{\infty} C_{m,n,k}^2(R,k) \log_2 C_{m,n,k}^2(R,k) > 0 . \] 

Experimental measurements of mode coefficients and, directly, the digital sorting of LG modes itself were carried out on a similar experimental setup, the diagram of which is shown in Fig. 1 of our previous article [10]. Typical spectra of mode amplitudes \( C_{m,n,k} \) and entropy distributions \( H_{m,n} \) of a single perturbed beam are shown in Fig. 1. From Fig.1a we see that the amplitude spectrum \( C_{m,n}(k) \) is limited in radial mode numbers \( k \), that enabled us to use a limited number of terms in Eq. (1) in the experiment. In addition, the alternation of the mode spectrum indicates the need to measure not only the squared amplitudes, but also the cross-mode coefficients.
Oscillating distribution of the mode amplitude in Fig.1b illustrates the contribution of the mode with radial number \( k = 0 \) and \( k = 6 \) to the amplitude mode with initial radial index \( n = 0 \). The measure of uncertainty \( H_{m,n=0}(R) \) of the initial state \( n = 0 \) increases as the radius of the aperture (Fig.1c) decreases due to the energy transfer into higher order modes, while the entropy \( H_{m,n=6}(R) \) of the state \( n = 6 \) in Fig.1d rapidly oscillates due to the energy redistribution between modes. The perturbed broad spectrum of the single LG beam can make fundamental changes to the spectrum of the perturbed beam array [11] in a data compression channel.

\[ C_{m,n,k} \]

\[ C_{m,n,k}^2 \]

\[ H \]

\[ H \]

**Figure 1.** Distribution of amplitude spectra \( C_{m,n} \) as a function of the radial number \( k \),(b) diaphragm radii \( R \); Shannon Entropy \( H_{m,n}(R) \) for states (a) \( m=3, n=0 \) and (b) \( m=3, n=6 \); lines – theory, circles – experiment.

2. LG mode sorting

We carried out series of experimental and computer simulation analysis of the mode spectra and mode sorting processes of perturbed beam arrays containing several modes LG with different set of radial numbers \( n \), but constant topological charge \( m \) and revealed that each mode in the spectrum is degenerate while a number of degeneracies is equal to a modes number in the initial beam array.

Indeed, every mode in the array of \( M \) beams experiences the same perturbation from the hard-edged aperture forming its own broad mode spectrum. As a result, each \( k \)-th mode in the perturbed spectrum contains a superposition of the \( M \) modes of the array. The spectrum becomes degenerate and a degeneracy number of each mode is \( M \). Figure 2a shows a typical spectrum of a beam array with topological charge \( m = 3 \), containing three modes with radial indexes \( n = 0 \), \( n = 3 \) and \( n = 6 \) perturbed by diaphragm with radii \( R = 2 \). Every mode in the spectrum is three-fold degenerated. The original modes of the array are highlighted in red. The mode degeneration affects the smoothing of the entropy distribution in Fig.1b (compare to Fig.1c,d).
Since the modes with different radial indices are independent, then to read the spectrum shown in Fig. 2a it is necessary to know $M = 3$ keys. In this case, the keys are the spectra of mode amplitudes $C_{m\times n}(R)$ separately for each perturbed LG beam for the aperture radius $R$.

Fig. 2c, d shows two intensity distributions of complex beams with different radial numbers: (c) before sorting and (d) complex beam, after digital sorting of modes for given $m = 3$, $n = 0$, $n = 3$ and $n = 6$, $R = 2$. The correlation of the presented beams exceeded the critical value $\eta = 0.90$ and was $\eta = 0.92$, while the value of the degree of correlation of a single sorted LG beam is approximately $\eta = 0.92$.

![Figure 2.](image)

**Figure 2.** (a) Degenerate amplitude spectrum $C_{m,n}(k)$, (b) entropy distribution $H_m(R)$, (c) intensity distribution of the initial LG beam and (d) intensity distribution of the LG beam restored of the sorted modes.

### 3. Conclusion
This paper demonstrates the possibility of digital measurement of the spectrum of radial Laguerre-Gauss modes. The presented technique makes it possible to simplify experimental installations for sorting by radial modes, since it excludes complex and expensive elements from the installation. The possibility of using not only a single LG mode, but also degenerate beams, which simultaneously carry several different radial modes, is shown. The degree of correlation between the obtained beams in the experiment and sorted using the technique exceeds the critical value $\eta = 0.90$.

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