Structural and analytical mesomechanics of fracture

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Abstract. Developing methodology of structural-analytical theory of strength, this article outlines methods for constructing a variant of defining equations of structural-analytical mesomechanics of failure, based on a rational synthesis of the main achievements of continuum mechanics, physics of plasticity and strength of solids, and other related disciplines. Loaded material is considered as a multi-level system in which micro, meso and macro levels are organically interconnected. Each scale level is characterized by its mechanisms and laws of deformation, the evolution of structure and damaging.

A method for constructing a two-level fracture criterion is being developed, which takes into account the hierarchical nature of the development of damaging and mutual influence of structural concentrators zones (SCZ) and macro stress concentrators. In order to take into account the influence of SCZ at the mesoscale level, experimentally justified tensor parameters of the magnetomechanical effect are introduced: magnetic distortion tensor and tensor intensity of intrinsic magnetic field of dispersion that occurs in the SCZ of the loaded body. Possible directions for the development of fracture mesomechanics are discussed.

1. Introduction

It is known that the ability of material to resist nucleation and development of cracks strongly depends on the organization of defective material structure, the presence of SCZ, which necessitates the development of fracture mechanics in order to introduce parameters characterizing the structural state of the material based on technical diagnostics methods, which led to the emergence of a new scientific direction – mesomechanics of destruction. For the named direction, attempts to link the criterion of fracture mechanics with the parameters of meso and microstructure of the material are characteristic. In this case, there is an urgent task of identifying effective parameters that allow controlling the real state of structural concentrators, and interaction with macro concentrators in order to formulate the criterion of destruction reflecting the influence of the heterogeneity of the stressed state.

To solve the problem of establishing relationship between microstructure and macro characteristics with the goal of formulating the criteria for fracture, a very promising approach is that combining the method of orientation averaging and the method of continuous approximation [1–3]. This approach allows us to formulate the criteria for the destruction of representative volume characterizing the mechanical properties of material point at macroscale level, based on information on the micro and mesostructure of local region of the material in the process of loading.
It is fundamentally important that the method of orientational averaging allows one to take into account different spatial orientation of structural elements at micro and mesoscale levels [2], and to naturally model their evolution in the process of deformation up to destruction [1, 2]. The formal algorithm of the orientation averaging method involves the following procedure. Let the structural-mechanical properties of representatively macro-volume of solid body be characterized by response \{\Pi\} when certain physical field \{\Sigma\} is applied to it. It is assumed that this field acts uniformly on each structural element at micro and mesoscale level within the representative macro volume. The response \(\pi(\Omega)\) of each structural element is a function of the field, the structural and mechanical properties of micro and meso volumes, and such is determined by the orientation of the corresponding structural elements relative to active field \{\Sigma\}:

\[
\pi(\Omega) = \pi(\{\Sigma\}; \{a_i\}; \{\Omega\}).
\] (1)

Performing averaging the parameters \(\pi(\Omega)\) over all orientations (superposition is assumed), we obtain the desired functional dependence:

\[
\{\Pi\} = \int_{[\Omega]} f(\Omega)\pi(\{a_i\}; \{\Sigma\}; \{\Omega\})d^3\Omega.
\] (2)

where \(f(\Omega)\) is density distribution function according to the orientations of the representative volumes of structural elements. Function \(f(\Omega)\) must satisfy the natural normalization conditions. With this approach, it is possible to obtain analytical relationships that reflect adequate relationships between different types of deformation, damage to the structure, on the one hand, and the stresses, temperatures, radiation, electrical, magnetic fields, etc. that generate them, – with another.

Mathematical side of the problem is connected with the development of averaging methods and continuous approximation methods for hierarchically structured environments of various scale levels, formally represented by formulas (1) and (2). The physical aspect of the problem is related to the formulation of the relations of type (1) for the corresponding physical fields. The problem of specifying dependencies (1) is fundamental in this approach and should be solved taking into account the physical content of the problem, as well as reflecting the main achievements obtained in the mechanics of deformable solid. This aspect is considered in detail in [1, 3].

2. Method for constructing criterion for the destruction of material in high gradient stress fields

When constructing fracture criterion, we will use the methods of structural-analytical mesomechanics [2] supplementing them with hypotheses reflecting the factor of substantial non-locality of structural-mechanical properties associated with the influence of inhomogeneity of macrostress concentrators [4]. Let us consider the method for constructing two-level criterion for destruction of material under a macrohomogeneous stress state.

The representative volume of macroscale level will be given by laboratory basis \(xyz\). In this basis, stress and strain tensors \(\sigma_{ik}\), deformation \(\varepsilon_{ik}\) and temperature \(T\) are identified [1–3]. The representative volume of the structural element will be specified by the local basis \(lmn\) using the geodetic coordinate system \{\Omega\} [1]. In the local bases of cut and separation, dyad \(n_i l_k\) on the plane of separation with the normal \(n\) is specified, and the direction of the formation of slit cracks is given by the unit ort \(l\), Figure 1. Diagram presented in figure 1 at \(r = r_m\) depicts macro point M, which is located at a distance \(r_m\) from the tip of macro concentrator, and its surroundings are characterized by structural elements in the form of local points of a sphere with a radius \(r_m\) which is considered to be structural characteristic of material (analogous to parameter \(\rho_0\) in the M.Ya. Leonov – K.N. Rusinko model).
In theory of strength within the framework of the Neuber-Novozhilov-type integral criterion, it is advisable to use orientation space shown in figure 1 at $r = r_m$. For the case of using the gradient approach, it is sufficient to represent the orientation space $\{\Omega\}$ as figure 1 at $r_m = 1$. As noted in [1-3], in the case of using the Cauchy stress tensor, because of its symmetry in the analytical representation of structural elements, instead of sphere, it is enough to consider hemisphere with radius $r = r_m$ or $r_m = 1$. It significantly, reduces the counting time in numerical simulation.

Figure 1. Diagram of orientation coordinate system $\{\Omega\}$.

Note that for the case of macro homogeneous net stretching, you can specify structural elements on one-fourth of the hemisphere. The presented variants of the analytical display of structural elements relate to tactical techniques.

Figure 2 presents the scheme of the orientational coordinate system $\{\Omega\}$, and also shows the matrix of direction cosines $\alpha_{ik}$, which translates laboratory basis $xyz$ into local coordinate system $lmn$.

Figure 2. The scheme of orientation coordinate system $\{\Omega\}$: $xyz$ is laboratory basis; $lmn$ is local basis: $0 \leq \alpha \leq 2\pi$; $0 \leq \beta \leq \pi/2$; $0 \leq \omega \leq 2\pi$. 
\[ \alpha_{ik} = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} \\ \alpha_{21} & \alpha_{22} & \alpha_{23} \\ \alpha_{31} & \alpha_{32} & \alpha_{33} \end{pmatrix} ; \quad \alpha_{ik} = \begin{pmatrix} -\sin \alpha \cos \omega - \sin \alpha \sin \omega - \cos \alpha \cos \beta \\ -\cos \alpha \sin \omega \sin \beta - \cos \alpha \sin \beta \cos \omega \\ \cos \alpha \cos \omega - \cos \alpha \sin \omega - \sin \alpha \cos \beta \\ -\sin \alpha \sin \beta \sin \omega - \sin \alpha \sin \beta \cos \omega \\ \cos \beta \sin \omega - \cos \beta \cos \omega - \sin \beta \\ \cos \beta \sin \omega - \cos \beta \cos \omega - \sin \beta \end{pmatrix}. \quad (3) \]

Small solid angle \( d^3 \Omega \) for considered orientational space \( \alpha, \beta, \omega \) is determined by the formula

\[ d^3 \Omega = \cos \beta \, d\alpha \, d\beta \, d\omega. \quad (4) \]

Note that in the laboratory basis \( xyz \), the parameters of the macroscale level are set, including the tensor of idealized \( \sigma_{ik} \) and effective stresses \( \sigma_{ik}^* \), and in the local basis \( lmn \), the corresponding parameters of the structural elements, including the effective stress tensor \( \tau_{ik}^* \).

The density of structural damages \( \pi(\Omega) \), oriented inside the solid angle \( d^3 \Omega \) in the direction given for normal tearing cracks \( n_i = n_i(\alpha, \beta) \), and for shear cracks by dyad \( n_i/\kappa = f(\alpha, \beta, \omega) \) will be characterized by parameter \( \pi(\Omega) \cdot d^3 \Omega \).

Fracture criterion in the zone of stress macroconcentrator, as in the case of a macrohomogeneous stress state [3], must take into account the necessary criterion conditions of kinetic and force character at the structural and macroscale levels. However, the specificity of the structural-mechanical state of material associated with sharp localization of macrostresses in the vicinity of a macro concentrator requires when building a fracture model to take into account the influence of the type of stress state, the degree and nature of localization of macrostresses in the stress concentrator zone [4].

Let us formulate the criterion of destruction, considering that destruction will occur when the maximum damage in the most loaded structural element reaches a critical value, i.e.

\[ \{\Pi\} = \int f(\Omega) \delta(\Omega - \Omega_0) \pi(\Omega) d^3 \Omega = \text{const} ; \quad \max \pi(\Omega_0) = 1. \quad (5) \]

Here, \( \delta(\Omega - \Omega_0) \) is Dirac delta function; \( \Omega_0 \) is coordinates of structural element in which damaging \( \pi(\Omega) \) reaches maximum. As argument of functional \( \pi(\Omega) \), you can use effective normal stress \( \tau_{nn}^*(\Omega) \), i.e.

\[ \pi(\Omega_0) = \tau_{nn}^*(\Omega_0) / \tau_0 = \tau_{33}^*(\Omega) / \tau_0, \quad (6) \]

where \( \tau_0 \) is critical stress by separation equals to ultimate strength of material under the net tension \( (\sigma_\theta) \) of standard sample. In this case, criterion (5), (6) for a complex stress state coincides in form with the criterion of maximum tensile stresses, formulated in terms of the effective stresses of the laboratory basis \( xyz \), i.e.

\[ \max \tau_{nn}^*(\Omega) = \sigma_i^*. \quad (7) \]
Note that, in (6) and (7), the kinetic aspect of preparing material structure for macrofracture is not taken into account.

3. Tensor characteristics of intrinsic dispersion of magnetic field

Important task is the development of experimental-theoretical models reflecting the basic laws of the evolution of structural stresses based on the registration of parameters of intrinsic magnetic field of scattering (SIMF). It is necessary to introduce appropriate conceptual representations and mathematical tools. In this case, by measuring magnetic field strength $H_i$, one can determine the distribution of $H_i$ in terms of coordinates of investigated article.

To test the existence of SIMF in the sample as vector object after the material was deformed, experiments were performed on thin steel plates. Plates were used without concentrators and with various macro concentrators in the form of central round and elliptical holes, side cuts of various configurations from steel 3, steel 45, steel 09G2S.

Serially manufactured specialized control device SIMF and a three-component fluxgate sensor developed by the company “Energodiagnostika” were used. In particular, a stress concentration meter magnetometric IKN-1M-4 was used, with a microprocessor-based recording device. $H_i$ measurements were performed at various points of the sample, and at each point the components of the vector were scanned, $H_i$ rotating the sensor at angles of $30^\circ$, $45^\circ$, $60^\circ$ and $90^\circ$. Each experiment was repeated three times. The measurement results confirmed the $H_i$ invariance, i.e. the law of conservation of the first rank tensor was fulfilled when the coordinate system was rotated

$$H'_i = \alpha_{ik} H_k, \quad (8)$$

where $\alpha_{ik}$ are guiding cosines.

According to estimate of $(H_i H_i)^{1/2}$ invariant, the measurement error did not exceed $(1-3)\%$, and for individual components, spread did not exceed $5\%$. The obtained results on the distribution of vector field $H_i$ allowed to introduce into consideration the model of the SIMF distortion tensor as a gradient of intensity vector $H_i$ according to the formula

$$H_{ik} = \nabla_i H_k, \quad (9)$$

where $\nabla_i$ is the Nabla operator [1].

Processing the experimental data using formula (2) allowed us to obtain experimentally grounded mathematical object that characterizes the heterogeneity of the SIMF. The next step was to experimentally check the parameters of $H_{ik}$ (9) in order to preserve $H_{ik}$ as a second-rank tensor when coordinate system is rotated by the angles of: $30^\circ$, $45^\circ$, $60^\circ$ and $90^\circ$. The results of processing the data obtained showed satisfactory compliance with the law of conservation:

$$H'_{ik} = \alpha_{ip} \alpha_{kq} H_{pq}. \quad (10)$$

Scatter in the values of the first, second, and third $H_{ik}$ invariants was within $15\%$, which confirmed the tensor nature of the parameters $H_{ik}$. It should be noted that these experimental studies required the creation of original equipment, each experiment lasted for 4-5 hours. Within three years, a large number of experiments were carried out, which confirmed the tensor character of the introduced parameter $H_{ik}$, called the distortion of the intrinsic magnetic field of scatter.

The next step was the introduction of symmetric and antisymmetric tensors on the basis of the $H_{ik}$ distortion tensor:
\[ H_{ik}^s = \frac{1}{2} \left( \nabla_i H_k + \nabla_k H_i \right), \quad H_{ik}^d = \frac{1}{2} \left( \nabla_i H_k - \nabla_k H_i \right). \]  (11)

Introduction of \( H_{ik}^s \) allows the development of symmetric structural mechanics in accordance with the traditional symmetric mechanics of deformable solid. Introduction of parameter \( H_{ik}^d \) opens up the possibility of developing structural models that take into account moment structural stresses and corresponding twist-bending.

4. Effective stresses in high gradient fields of macrostresses

When analyzing the strength of material in high gradient force fields, the question of choosing an equation for effective stress is of particular importance. The effective stress must take into account the influence of structural concentrators and their evolution under load. This circumstance can be taken into account using vector-invariant intensity of intrinsic magnetic field of scattering \( S_i \) [3], measured directly in the vicinity of the macroconcentrator.

Another important aspect is the need to take into account the influence of type of stress state, the nature and degree of heterogeneity of the stress state in the zone of stress concentrator on the structural and mechanical properties of material. These circumstances can be reflected in the fracture criteria using the achievements obtained in the development of gradient methods [4].

Critical analysis of the main results obtained in gradient approach, taking into account the structural and mechanical parameters of metal magnetic memory method, made it possible, based on the methods of structural and analytical mesomechanics [1–3], to formulate the following equation for effective stresses \( \tau_{ik}^* \) acting in the structural elements experiencing the opening of cracks by separation and cut:

\[ \tau_{ik}^*(\Omega) = \delta_{i3}\delta_{k3}\tau_{33}^*(\Omega) + (\delta_{i1}\delta_{k3} + \delta_{i3}\delta_{k1})\tau_{31}^*(\Omega). \]  (12)

taking into account criteria (5)

\[ \max \tau_{33}^*(\Omega_0) = \sigma_1^*; \quad \max \tau_{31}^*(\Omega_0) = \tau_{\text{max}}^*. \]  (13)

For effective stresses \( \tau_{ik}^*(\Omega) \) in the framework of criterion (5), we formulate the equations for \( \sigma_1^* \) and \( \tau_{\text{max}}^* \). Under non-uniform stress state in the SCZ, to formulate the criterion of brittle fracture by separation, taking into account the equivalence principle [3], we will compare with the ultimate strength of the material \( \sigma_0 \) the maximum effective stress \( \sigma_1^* \). Taking into account the achievements obtained in the gradient approach [4], the following equation is formulated for the calculation \( \sigma_1^* \):

\[ \sigma_1^* = \sigma_1 \left( 1 - \beta + \sqrt{\beta^2 + L_1 q_2} \right); \quad \beta = \frac{L}{d}, \]  (14)

\[ q_1 = \frac{S_i^{\text{max}}}{S_i^{\text{inf}}}, \]  (15)

\[ q_2 = (\text{grad} \sigma_1 \cdot \text{Sign} \text{ grad} \sigma_1) / \sigma_1, \]  (16)

where \( q_1 \) is relative vector intensity invariant \( S_i \) [3] of the intrinsic magnetic field of scattering,
characterizes at the structural level the damageability of metal at hardening stage; $q_2$ is relative gradient of the first principal stress, is found from the elastic solution of corresponding boundary value problem [4]; $L$ is parameter, has the dimension of length and characterizes the effective size of structural stress concentrators The designations $\text{max}$ and $cp$ in (15) indicate the maximum and average values $S_1$ in the vicinity of macro concentrator; $d$ is the characteristic size of macro stress concentrator.

In the case of development of transverse or longitudinal slides in the stress concentration zone, to formulate the criterion of shear failure, based on the equivalence principle, we will compare with the shear strength limit $\tau_S$ maximum effective stress value $\tau_{\text{max}}^*$ that is less than maximum shear stress $\tau_{\text{max}}$ (taken as equivalent to shear failure criterion):

\[
\tau_{\text{max}}^* = \tau_{\text{max}} \left[ 1 - \beta + \sqrt{\beta^2 + Lq_1q_3} \right]
\]

\[
q_3 = (\text{grad} \tau_{\text{max}} \cdot \text{sign} \text{grad} \tau_{\text{max}})(\tau_{\text{max}} \cdot \text{sign} \tau_{\text{max}}),
\]

where $q_3$ is the relative gradient of the maximum shear stress $\tau_{\text{max}}$.

It should be noted that introduction of product of parameters $L \cdot q_1 \cdot q_2$ and $L \cdot q_1 q_3$ into relations (14) – (18) reflects the influence of interaction of structural stresses arising in SCZ and gradient of macroscopic stresses in the vicinity of macro concentrator on the structural and mechanical properties of the material in high gradient stress fields.

5. Two-level structural and analytical failure criterion
We formulate a variant of two-level fracture criterion, suggesting that the formation and development of crack in the vicinity of a macroconcentrator requires the fulfillment of fracture criterion at both the macroscale and structural (micro and mesoscale) levels [1–3].

Let us first consider the case of destruction of article with a macroconcentrator by mechanism of separation. We assume that separation by breaking in the neighborhood of the local macro-point will occur when two conditions are fulfilled: the achievement of the ultimate strength $\sigma_\text{ult}$ by effective stresses $\sigma_1^*$ and the mandatory fulfillment of the fracture criterion at the structural level, i.e. when the parameter $q_1$ reaches its value $q_{\text{np}}$.

The latter means, according to [1–3], the emergence in this structural element of critical defect structure characterized by the limiting state of the zones of structural concentrators. Then, taking into account the condition that the initial propagation of macrocrack will occur at the site of the maximum normal stress $\tau_{\text{nn}}$, i.e. of the first main stress and will occur only when the two conditions mentioned are fulfilled, we formulate the final macroscopic fracture criterion by breaking $\Pi_0^M$ in the form:

\[
\Pi_0^M = X(\sigma_1^* - \sigma_\text{ult})X(q_1 - q_{\text{np}}) = 1,
\]

where $X(x)$ is the Heaviside function.

Note that in general case, the maximum values of effective stress $\sigma_1^*$ and the structural parameter $q_1$ can be reached at the point that does not coincide with the point of the maximum value of the first main stress. Therefore, when calculating the strength using criterion (14) - (19), it is necessary to check criterion (19) not only at the apex of concentrator, but also in other points of the body.
In accordance with equation (19), if \( \Pi_0^M = 1 \), then the body will collapse, and it will not be destroyed if \( \Pi_0^M = 0 \). Thus, macroscopic destruction of body with macro stress concentrators will occur when the critical level is reached and effective stresses \( \sigma_1^* \) (in this case, the force nature of the fracture is reflected) and the parameter that characterizes the kinetic nature of the fracture. Note that parameter \( q_1 \) is a constant of material and characterizes the degree of structural heterogeneity and the maximum deformation capacity of metal in the zones of structural stress concentrators.

Now suppose that in vicinity of macroconcentrator the stress-strain and structural-kinetic state correspond to the fracture variant by the development of transverse or longitudinal shear cracks. For the formulation of the fracture criterion, as in formulation of criterion in the event of breakdown by separation, we apply the hypothesis that macroscopic destruction in the stress concentration zone will occur if fracture criteria are met at both structural \( (q_1 = q_{np}) \) and macroscopic levels, i.e. when the effective shear stress \( \tau_{\text{max}}^* \) reaches value equal to shear strength \( \tau_S \). Then, the final criterion of destruction by cut \( \Pi_C^M \) at macroscale level will look like:

\[
\Pi_C^M = X\left(\tau_{\text{max}}^* \text{sign} \tau_{\text{max}}^* - \tau_S\right)X(q_1 - q_{np}) = 1
\]  

(20)

Note that, as in case of fracture criterion with normal break (19), the maximum values of effective stress \( \tau_{\text{max}}^* \) and structural parameter \( q_1 \) in the fracture criterion by slice (20) can be reached at the point not coinciding with the point of the maximum value of idealized tangential stress; therefore, it is necessary to check criterion (20), both at the top of the concentrator and at other points in the body. Criteria of destruction (19) and (20), reflecting various cases of limiting state, can be combined and write generalized criterion of destruction in the form:

\[
\Pi_\Sigma^M = X(q_1 - q_{np})X(\sigma_1^* - \sigma_B) + X(\tau_{\text{max}}^* \text{sign} \tau_{\text{max}}^* - \tau_S) \geq 1.
\]  

(21)

The formulated version of structural-analytical criterion of destruction is fundamentally different from existing criteria of destruction. The main difference is that structural-analytical criterion contains two necessary conditions for final destruction, namely, the kinetic criterion of destruction at the structural level, i.e. requirement for the parameter \( q_1 \) to reach the limiting value \( q_{np} \), which reflects the creation of critical defect structure, and the force criterion at macroscopic level by tear-off or shear mechanism.

An important point in this approach is the natural combination of methods of fracture mechanics and methods of technical diagnostics of the structural-mechanical state of the material, which allows you to create models that take into account the mutual influence of internal structural stresses in the SCZ and macrostresses in the vicinity of the macroconcentrator on the strength properties of materials. It is characteristic that to describe the strength properties of a material around macroconcentrators, it is necessary to introduce two complex parameters: structural-mechanical \( (Lq_1q_2) \) and structural-geometric \( (\beta = L/d) \) content. The introduced complexes reflect the mutual influence of structural concentrators and stress macroconcentrators on the strength properties of material.

In the structural-mechanical complex \( Lq_1q_2 \), one of parameters is a characteristic of the representative sizes of the structural heterogeneity of materials \( (L) \), the second reflects the influence of structural stress gradients \( (q_1) \), and the third parameter characterizes the influence of macrostress gradients \( (q_2) \). The structural-geometric complex \( \beta \) is the ratio of the characteristic size of the structural heterogeneity \( L \) to the representative size of the macro concentrator \( d \). An important element of the formulated criterion is
also the fact that strength characteristics, namely, tensile strength under uniaxial tension \( \sigma_u \), shear strength \( \tau_s \) and yield strength \( \sigma_y \), as well as the invariant structural parameter \( q_{np} \).

6. Comparison of experimental studies with theoretical calculations

This section provides analysis of the adequacy of fracture criteria for calculation of limit state of thin steel plates with stress concentrators. The advantage of structural-analytical gradient fracture criterion is substantiated. For convenience of analysis, the results of calculating limiting state of plates with macroconcentrators according to gradient fracture criterion and experimental data are presented in Table 1. Note that the experimental data were obtained from the results of testing at least 3 samples for each type of concentrator and Table 1 shows the average values.

Table 1. Comparison of calculation results for structural-analytical gradient fracture criterion with experimental data.

| Concentrator type | Calcula- tion method | Theoretical concentration coefficient \( \alpha_T \) | Average value of stress \( \sigma \), MPa | Relative difference by \( \sigma \) |
|-------------------|----------------------|---------------------------------|----------------------------|------------------|
| Circle \( D=12 \) mm | Theory | 2.4 | 460.2 | 4.8% |
| Circle \( D=18 \) mm | ANSYS | 2.508 | 482.5 | 0.16% |
| Circle \( D=18 \) mm | Theory | 2.1 | 368 | 8% |
| Circle \( D=18 \) mm | ANSYS | 2.31 | 437.4 | 8.5% |
| Ellipsis \( l=12 \) mm | Theory | 3.8 | 480 | 1% |
| Ellipsis \( l=18 \) mm | ANSYS | 3.16 | 434.8 | 8.4% |
| U-shaped \( l=12 \) mm | Theory | 4.4 | 368.2 | 3.9% |
| U-shaped \( l=18 \) mm | ANSYS | 3.37 | 388.5 | 1.3% |
| U-shaped \( l=18 \) mm | ANSYS | 5 | 284.6 | 24.1% |
| V-shaped \( l=12 \) mm | Theory | 5.9 | 336.7 | 10.2% |
| V-shaped \( l=18 \) mm | ANSYS | 3.79 | 301 | 11.9% |
| V-shaped \( l=18 \) mm | ANSYS | 3.53 | 234.6 | 0.05% |

Prediction made by the structural-analytical gradient fracture criterion is in good consistent with experimental data. The deviation of the calculation results from the data obtained during tests is in the range from 0.5% to 10%, which practically coincides with variation in experimental studies [6].

7. Conclusion

It should be noted that the pronounced specificity of structural concentrators along grain boundaries takes place in materials with submicro and nanostructure. The introduced tensor characteristics of SIMF make it possible to monitor the change in the grain size during plastic deformation up to submicro and nanostructured states [6].
The developed approach takes into account multi-level and multi-stage nature of destruction. The emergence of microcracks is necessarily preceded by the formation of zones of structural concentrators with critical defective structure, which is characterized by diagnostic parameters of the magnetic memory method in the form of invariant (12) and integral criterion, which together with the field of effective stresses $\tau_{ik}^*$ determine the formation of structural (macro and meso) cracks. Cracks create structural damage that can grow under the right conditions. When such property is acquired by large number of structural cracks, prerequisites appear for macroscopic destruction, which will occur only if the body is sufficiently damaged.

References
[1] Likhachev V A and Malinin V G 1993 Structural analytic theory of strength (Saint-Petersburg: “Science” Publ.) p 471
[2] Malinin V G and Malinina N A 2005 Structural analytic mezomekhanics deformed solid body Phisics mezomekhanics 23 31–45
[3] Malinin V V 2011 Structural-mechanical approach in fracture mesomechanics: Appendix to Engineering Journal Reference Publ. in Engineering journal: Spravochnik 6 52–56
[4] Legan M A 1994 Determination breaking load places and directions of development by means of gradient approach PMTF 5/35 117–124
[5] Dubov A A and Dubova A A 2006 Method of magnetic memory and control devices (Moscow: “Tisso” Publ) p 332
[6] Golenkov V A, Malinin V G and Malinina N A 2009 Structural and analytic mezomekhanics and its applications (Moscow: “Mechanical engineering” Publ.) p 635