Different Types of Neutrosophic Chromatic Number

Henry Garrett
Independent Researcher
DrHenryGarrett@gmail.com
Twitter’s ID: @DrHenryGarrett | ©DrHenryGarrett.wordpress.com

Abstract

New setting is introduced to study chromatic number. Different types of chromatic numbers and neutrosophic chromatic number are proposed in this way, some results are obtained. Classes of neutrosophic graphs are used to obtains these numbers and the representatives of the colors. Using colors to assign to the vertices of neutrosophic graphs is applied. Some questions and problems are posed concerning ways to do further studies on this topic. Using different types of edges from connectedness in same neutrosophic graphs and in modified neutrosophic graphs to define the relation amid vertices which implies having different colors amid them and as consequences, choosing one vertex as a representative of each color to use them in a set of representatives and finally, using neutrosophic cardinality of this set to compute types of chromatic numbers. This specific relation amid edges is necessary to compute both types of chromatic number concerning the number of representative in the set of representatives and types of neutrosophic chromatic number concerning neutrosophic cardinality of set of representatives. If two vertices have no intended edge, then they can be assigned to same color even they’ve common edge. Basic familiarities with neutrosophic graph theory and graph theory are proposed for this article.

Keywords: Neutrosophic Connectedness, Neutrosophic Graphs, Chromatic Number

AMS Subject Classification: 05C17, 05C22, 05E45

1 Background

Fuzzy set in Ref. [15], neutrosophic set in Ref. [2], related definitions of other sets in Refs. [2,13,14], graphs and new notions on them in Refs. [5–11], neutrosophic graphs in Ref. [3], studies on neutrosophic graphs in Ref. [1], relevant definitions of other graphs based on fuzzy graphs in Ref. [12], related definitions of other graphs based on neutrosophic graphs in Ref. [4], are proposed.

In this section, I use two subsections to illustrate a perspective about the background of this study.

1.1 Motivation and Contributions

In this study, there’s an idea which could be considered as a motivation.
**Question 1.1.** Is it possible to use mixed versions of ideas concerning “connectedness”, “neutrosophic graphs” and “neutrosophic coloring” to define some notions which are applied to neutrosophic graphs?

It’s motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Connections amid two items have key roles to assign colors. Thus they’re used to define new ideas which conclude to the structure of coloring. The concept of having specific edge from connectedness inspires me to study the behavior of specific edge in the way that, both types of chromatic numbers and types of neutrosophic chromatic numbers are the cases of study.

The framework of this study is as follows. In the beginning, I introduced basic definitions to clarify about preliminaries. In section “New Ideas”, new notion of coloring is applied to the vertices of neutrosophic graphs. Specific edge from connectedness has the key role in this way. Classes of neutrosophic graphs are studied in the terms of different types of edges in section “New Results”. In section “Applications in Time Table and Scheduling”, one application is posed for neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications are featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions are formed.

### 1.2 Preliminaries

**Definition 1.2.** $G : (V,E)$ is called a **crisp graph** where $V$ is a set of objects and $E$ is a subset of $V \times V$ such that this subset is symmetric.

**Definition 1.3.** A crisp graph $G : (V,E)$ is called a **neutrosophic graph** $G : (\sigma,\mu)$ where $\sigma = (\sigma_1,\sigma_2,\sigma_3) : V \rightarrow [0,1]$ and $\mu = (\mu_1,\mu_2,\mu_3) : E \rightarrow [0,1]$ such that $\mu(xy) \leq \sigma(x) \land \sigma(y)$ for all $xy \in E$.

**Definition 1.4.** A neutrosophic graph is called **neutrosophic empty** if it has no edge. It’s also called **neutrosophic trivial**. A neutrosophic graph which isn’t neutrosophic empty, is called **neutrosophic nontrivial**.

**Definition 1.5.** A neutrosophic graph $G : (\sigma,\mu)$ is called a **neutrosophic complete** where it’s complete and $\mu(xy) = \sigma(x) \land \sigma(y)$ for all $xy \in E$.

**Definition 1.6.** A neutrosophic graph $G : (\sigma,\mu)$ is called a **neutrosophic strong** where $\mu(xy) = \sigma(x) \land \sigma(y)$ for all $xy \in E$.

**Definition 1.7.** A path $v_0,v_1,\cdots,v_n$ is called **neutrosophic path** where $\mu(v_iv_{i+1}) > 0$, $i = 0,1,\cdots,n-1$. $i$-path is a path with $i$ edges, it’s also called **length** of path.

**Definition 1.8.** A crisp cycle $v_0,v_1,\cdots,v_n,v_0$ is called **neutrosophic cycle** where there are two edges $xy$ and $uv$ such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0,1,\cdots,n-1} \mu(v_iv_{i+1})$.

**Definition 1.9.** A neutrosophic graph is called **neutrosophic t-partite** if $V$ is partitioned to $t$ parts, $V_1,V_2,\cdots,V_t$ and the edge $xy$ implies $x \in V_i$ and $y \in V_j$ where $i \neq j$. If it’s neutrosophic complete, then it’s denoted by $K_{\sigma_1,\sigma_2,\cdots,\sigma_t}$ where $\sigma_i$ is $\sigma$ on $V_i$ instead $V$ which mean $x \not\in V_i$ induces $\sigma_i(x) = 0$. If $t = 2$, then it’s called **neutrosophic complete bipartite** and it’s denoted by $K_{\sigma_1,\sigma_2}$ especially, if $|V_1| = 1$, then it’s called **neutrosophic star** and it’s denoted by $S_{1,\sigma_2}$. In this case, the vertex in $V_1$ is called
center and if a vertex joins to all vertices of neutrosophic cycle, it's called neutrosophic wheel and it's denoted by \( W_{1,\sigma} \).

**Definition 1.10.** Let \( G : (\sigma, \mu) \) be a neutrosophic graph. For any given subset \( N \) of \( V \), \( \sum_{n \in N} \sigma(n) \) is called neutrosophic cardinality of \( N \) and it’s denoted by \( |N|_n \).

**Definition 1.11.** Let \( G : (\sigma, \mu) \) be a neutrosophic graph. Neutrosophic cardinality of \( V \) is called neutrosophic order of \( G \) and it’s denoted by \( O_n(G) \).

**Definition 1.12.** Let \( G : (\sigma, \mu) \) be a neutrosophic graph. The number of vertices is denoted by \( n \) and the number of edges is denoted by \( m \).

**Definition 1.13.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. It’s called neutrosophic connected if for every given couple of vertices, there’s at least one neutrosophic path amid them.

**Definition 1.14.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. Suppose a path \( P : v_0, v_1, \cdots, v_n-1, v_n \) from \( v_0 \) to \( v_n \), \( \min_{i=0,1,2,\cdots,n-1} \mu(v_iv_{i+1}) \) is called neutrosophic strength of \( P \) and it’s denoted by \( S_n(P) \).

**Definition 1.15.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. The number of maximum edges for a vertex, amid all vertices, is denoted by \( \Delta(N) \).

2 New ideas

New ideas are applied on this model to explore behaviors of these models in the mathematical perspective. Another ways to make sense about them, are used by relatively comparable results to conclude analysis. Having different colors when two vertices have common “connection”. Common connection can only be an edge. An edge with special attribute can be common “connection”. Using neutrosophic attributes are expected to make sense about the study in this framework. In what follows, some definitions are introduced to be in the form of common “connection”.

**Question 2.1.** What-if the common “connection” is beyond having one common edge?

The first step is the definition of common “connection”.

**Definition 2.2.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A neutrosophic edge \( xy \) is called type-I if value of \( xy \) is connectedness which is a maximum strength of paths amid them.

**Example 2.3.** Consider Figure (1).

(i) : From \( n_1 \) to \( n_2 \), there’s no edge which is type-I but \( n_2n_3 \).

(ii) : From \( n_2 \) to \( n_3 \), there’s no edge which is type-I but \( n_2n_3 \).

(iii) : From \( n_1 \) to \( n_3 \), there’s no edge which is type-I but \( n_1n_3 \).

There’s a curious question.

**Question 2.4.** Is there a neutrosophic graph whose edges are type-I?

Yes but only one class. Two upcoming Propositions give simple answers about a class of neutrosophic graphs. Other classes of neutrosophic graphs have at least one edge which isn’t type-I.

**Proposition 2.5.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is fixed-edge. Then all edges are type-I.
Figure 1. Two edges aren’t type-I.

**Proposition 2.6.** Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong fixed-vertex. Then $N = (\sigma, \mu)$ is fixed-edge.

**Proposition 2.7.** Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong fixed-vertex. Then all edges are type-I.

**Example 2.8.** Consider Figure (2). All edges are type-I.

Figure 2. Neutrosophic graph which is fixed-edge but not strong fixed-vertex.

**Definition 2.9.** Let $N = (\sigma, \mu)$ be a neutrosophic graph. A neutrosophic edge $xy$ is called **type-II** if value of $xy$ is lower than **connectedness** which is a maximum strength of paths amid them.

**Example 2.10.** The comparison amid the variant of edges which are either type-I or type-II, is possible when common neutrosophic graphs are studied.
(a) : Consider Figure (1).

(i) : From \( n_1 \) to \( n_2 \), there’s no edge which is type-II but \( n_1 n_2 \).

(ii) : From \( n_2 \) to \( n_3 \), there’s no edge which is type-II but \( n_1 n_2 \).

(iii) : From \( n_1 \) to \( n_3 \), there’s no edge which is type-II but \( n_1 n_2 \) and \( n_2 n_3 \).

(b) : Consider Figure (2). There’s no edge which is type-II.

**Definition 2.11.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A neutrosophic edge \( xy \) is called **type-III** if value of \( xy \) is the only value which is **connectedness** which is a maximum strength of paths amid them.

**Example 2.12.** The comparison amid the variant of edges which are either type-I or type-II or type-III, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (1).

(i) : From \( n_1 \) to \( n_2 \), there’s no edge which is type-III but \( n_2 n_3 \).

(ii) : From \( n_2 \) to \( n_3 \), there’s no edge which is type-III but \( n_2 n_3 \).

(iii) : From \( n_1 \) to \( n_3 \), there’s no edge which is type-III but \( n_1 n_3 \) and \( n_2 n_3 \).

(b) : Consider Figure (2). There’s no edge which is type-III.

**Definition 2.13.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A neutrosophic edge \( xy \) is called **type-IV** if value of \( xy \) is **connectedness** which is a maximum strength of paths amid them but in \( N = (\sigma, \mu) \) doesn’t have \( xy \).

**Example 2.14.** The comparison amid the variant of edges which are either type-I or...or type-IV, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (1).

(i) : From \( n_1 \) to \( n_2 \), there’s no edge which is type-IV.

(ii) : From \( n_2 \) to \( n_3 \), there’s no edge which is type-IV.

(iii) : From \( n_1 \) to \( n_3 \), there’s no edge which is type-IV.

(b) : Consider Figure (2). All edges are type-IV.

**Definition 2.15.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A neutrosophic edge \( xy \) is called **type-V** if value of \( xy \) is lower than **connectedness** which is a maximum strength of paths amid them but in \( N = (\sigma, \mu) \) doesn’t have \( xy \).

**Example 2.16.** The comparison amid the variant of edges which are either type-I or...or type-V, is possible when common neutrosophic graphs are studied.

(a) : Consider Figure (1).

(i) : From \( n_1 \) to \( n_2 \), edge \( n_1 n_2 \) is type-V.

(ii) : From \( n_2 \) to \( n_3 \), there’s no edge which is type-V.

(iii) : From \( n_1 \) to \( n_3 \), there’s no edge which is type-V.

(b) : Consider Figure (2). There’s no edge which is type-V.

**Definition 2.17.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A neutrosophic edge \( xy \) is called **type-VI** if value of \( xy \) is greater than **connectedness** which is a maximum strength of paths amid them but in \( N = (\sigma, \mu) \) doesn’t have \( xy \).
Example 2.18. The comparison amid the variant of edges which are either type-I or...or type-VI, is possible when common neutrosophic graphs are studied.

(a): Consider Figure (1).

(i): From \( n_1 \) to \( n_2 \), there’s no edge which is type-VI.

(ii): From \( n_2 \) to \( n_3 \), edges \( n_2n_3 \) and \( n_1n_3 \) are type-VI.

(iii): From \( n_1 \) to \( n_3 \), edges \( n_2n_3 \) and \( n_1n_3 \) are type-VI.

(b): Consider Figure (2). There’s no edge which is type-VI.

Definition 2.19. Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A neutrosophic edge \( xy \) is called type-VII if value of \( xy \) is the only value which is connectedness which is a maximum strength of paths amid them but in \( N = (\sigma, \mu) \) doesn’t have \( xy \).

Example 2.20. The comparison amid the variant of edges which are either type-I or...or type-VII, is possible when common neutrosophic graphs are studied.

(a): Consider Figure (1).

(i): From \( n_1 \) to \( n_2 \), there’s no edge which is type-VII.

(ii): From \( n_2 \) to \( n_3 \), there’s no edge which is type-VII.

(iii): From \( n_1 \) to \( n_3 \), there’s no edge which is type-VII.

(b): Consider Figure (2). There’s no edge which is type-VII.

Common way to define the number, could be twofold. One is about the cardinality and another is about neutrosophic cardinality.

Definition 2.21. Let \( N = (\sigma, \mu) \) be a neutrosophic graph. A vertex which has common type edge with another vertex, has assigned different color from that vertex. The cardinality of the set of representatives of colors, is called type chromatic number and its neutrosophic cardinality concerning the set of representatives of colors is called \( n \)-type chromatic number.

Definition 2.22. It’s worthy to note that there are two types of definitions. One is about the comparison amid edges and connectedness. Another is about one edge when it’s deleted, new connectedness is compared to deleted edge. Thus in first type, all edges are compared to connectedness but in second type, for every edge, there’s a computation to have connectedness. So in first type, connectedness is unique and there’s one number for all edges as connectedness but in second type, for every edge, there’s a new connectedness to decide about the edge whether has intended attribute or not. To avoid confusion, chromatic number is computed with respect to \( n_1 \) and \( n_2 \) where second style is used and all edges are labelled even they’re not deleted edges so third type is introduced when deletion of one edge, is enough to label all edges. Also first order is used to have these concepts.

In following example, third type of definitions which are except from type-IV,V,VI,VII, are studied.

Example 2.23. The comparison amid the variant of numbers which are either type-I or...or type-VII, is possible when common neutrosophic graphs are studied. Chromatic number is computed with respect to \( n_1 \) and \( n_2 \). Also first order is used to have these concepts.

(a): Consider Figure (1).
(i) : The set of representatives of colors is \( \{n_1, n_2\} \). Thus type-I chromatic number is 2 and n-type-I chromatic number is 1.73.

(ii) : The set of representatives of colors is \( \{n_1, n_2\} \). Thus type-II chromatic number is 2 and n-type-II chromatic number is 1.73.

(iii) : The set of representatives of colors is \( \{n_2, n_3\} \). Thus type-III chromatic number is 2 and n-type-III chromatic number is 1.28.

(iv) : The set of representatives of colors is \( \{n_2, n_3\} \). Thus type-IV chromatic number is 2 and n-type-IV chromatic number is 1.28.

(v) : The set of representatives of colors is \( \{n_1, n_2\} \). Thus type-V chromatic number is 2 and n-type-V chromatic number is 1.73.

(vi) : The set of representatives of colors is \( \{n_2, n_3\} \). Thus type-VI chromatic number is 2 and n-type-VI chromatic number is 1.28.

(vii) : The set of representatives of colors is \( \{n_2, n_3\} \). Thus type-VII chromatic number is 2 and n-type-VII chromatic number is 1.28.

(b) : Consider Figure (2).

(i) : The set of representatives of colors is \( \{n_1, n_2, n_3\} \). Thus type-I chromatic number is 3 and n-type-I chromatic number is 3.01.

(ii) : The set of representatives of colors is \( \{\} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \( \{\} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors is \( \{\} \). Thus type-IV chromatic number is 0 and n-type-IV chromatic number is 0.

(v) : The set of representatives of colors is \( \{\} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) : The set of representatives of colors is \( \{\} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) : The set of representatives of colors is \( \{\} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

3 New Results

In this section, I introduce some results concerning new ideas and in this ways, the results make sense more about their impacts on different models.

**Proposition 3.1.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is complete. If it’s fixed-edge, then

(i) : The set of representatives of colors is \( \{v_1, v_2, \cdots, v_n\} \). Thus type-I chromatic number is \( n \) and n-type-I chromatic number is neutrosophic cardinality of \( V \).

(ii) : The set of representatives of colors is \( \{\} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \( \{\} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors is \( \{v_1, v_2, \cdots, v_n\} \). Thus type-IV chromatic number is \( n \) and n-type-IV chromatic number is neutrosophic cardinality of \( V \).
(v) : The set of representatives of colors is \( \{ \} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) : The set of representatives of colors is \( \{ \} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) : The set of representatives of colors is \( \{ \} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{ v_1, v_2, \cdots, v_n \} \). The type-I chromatic number is \( n \) and n-type-I chromatic number is neutrosophic cardinality of \( V \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{ \} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which are type-IV. Thus the set of representatives of colors is \( \{ v_1, v_2, \cdots, v_n \} \). The type-IV chromatic number is \( n \) and n-type-IV chromatic number is neutrosophic cardinality of \( V \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \( \{ \} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \( \{ \} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic complete, every vertex has \( n - 1 \) vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \( \{ \} \). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proposition 3.2. Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is complete. If it’s fixed-vertex, then

(i) : The set of representatives of colors is \( \{ v_1, v_2, \cdots, v_n \} \). Thus type-I chromatic number is \( n \) and n-type-I chromatic number is \( n\sigma(v_i) \).

(ii) : The set of representatives of colors is \( \{ \} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \( \{ \} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors is \( \{ v_1, v_2, \cdots, v_n \} \). Thus type-IV chromatic number is \( n \) and n-type-IV chromatic number is \( n\sigma(v_i) \).
(v) The set of representatives of colors is \( \{\} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is \( \{\} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is \( \{\} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{v_1, v_2, \cdots, v_n\} \). The type-I chromatic number is \( n \) and n-type-I chromatic number is \( n\sigma(v_i) \).

(ii). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{\} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which are type-IV. Thus the set of representatives of colors is \( \{v_1, v_2, \cdots, v_n\} \). The type-IV chromatic number is \( n \) and n-type-IV chromatic number is \( n\sigma(v_i) \).

(v). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \( \{\} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \( \{\} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). By it’s fixed-vertex and it’s neutrosophic complete, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic complete, every vertex has \( n-1 \) vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \( \{\} \). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

\[
\text{Proposition 3.3. Let } N = (\sigma, \mu) \text{ be a neutrosophic graph which is strong. If it’s fixed-edge, then}
\]

(i) The set of representatives of colors is \( \{v_1, v_2, \cdots, v_t\} \) where \( t = \Delta(N) \). Thus type-I chromatic number is \( t \) and n-type-I chromatic number is neutrosophic cardinality of \( \{v_1, v_2, \cdots, v_t\} \).

(ii) The set of representatives of colors is \( \{\} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.
(iii): The set of representatives of colors is \( \{ \} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv): The set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \) where \( t = \Delta(N) \). Thus type-IV chromatic number is \( t \) and n-type-IV chromatic number is neutrosophic cardinality of \( \{ v_1, v_2, \ldots, v_t \} \).

(v): The set of representatives of colors is \( \{ \} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi): The set of representatives of colors is \( \{ \} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii): The set of representatives of colors is \( \{ \} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

**Proof.** (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \). The type-I chromatic number is \( t \) and n-type-I chromatic number is neutrosophic cardinality of \( \{ v_1, v_2, \ldots, v_t \} \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{ \} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which are type-IV. Thus the set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \). The type-IV chromatic number is \( t \) and n-type-IV chromatic number is neutrosophic cardinality of \( \{ v_1, v_2, \ldots, v_t \} \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \( \{ \} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \( \{ \} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \( \{ \} \). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

\[ \square \]

**Proposition 3.4.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is strong. If it’s fixed-vertex, then
(i) The set of representatives of colors is \( \{v_1, v_2, \cdots, v_t\} \) where \( t = \Delta(N) \). Thus type-I chromatic number is \( t \) and n-type-I chromatic number is \( t\sigma(v_i) \).

(ii) The set of representatives of colors is \( \{\} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) The set of representatives of colors is \( \{\} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) The set of representatives of colors is \( \{v_1, v_2, \cdots, v_t\} \) where \( t = \Delta(N) \). Thus type-IV chromatic number is \( t \) and n-type-IV chromatic number is \( t\sigma(v_i) \).

(v) The set of representatives of colors is \( \{\} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is \( \{\} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is \( \{\} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{v_1, v_2, \cdots, v_t\} \). The type-I chromatic number is \( t \) and n-type-I chromatic number is \( t\sigma(v_i) \).

(ii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{\} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which are type-IV. Thus the set of representatives of colors is \( \{v_1, v_2, \cdots, v_t\} \). The type-IV chromatic number is \( t \) and n-type-IV chromatic number is \( t\sigma(v_i) \).

(v). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \( \{\} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \( \{\} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic strong, there’s a vertex has \( t = \Delta(N) \) vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \( \{\} \). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

\( \square \)
Proposition 3.5. Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong and path. If it’s fixed-edge, then

(i) : The set of representatives of colors is $\{v_i, v_j\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $\sigma(v_i) + \sigma(v_j)$.

(ii) : The set of representatives of colors is $\emptyset$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is $\emptyset$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren’t defined.

(v) : The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren’t defined.

(vi) : The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren’t defined.

(vii) : The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren’t defined.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j\}$. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j\}$.

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is $\emptyset$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is $\emptyset$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-IV. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

Proposition 3.6. Let $N = (\sigma, \mu)$ be a neutrosophic graph which is strong and path. If it’s fixed-vertex, then

(i) : The set of representatives of colors is $\{v_i, v_j\}$. Thus type-I chromatic number is 2 and n-type-I chromatic number is $2\sigma(v_i)$. 


(ii) : The set of representatives of colors is \{\}. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \{\}. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren’t defined.

(v) : The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren’t defined.

(vi) : The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren’t defined.

(vii) : The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren’t defined.

**Proof.** (i). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \{vi, vj\}. The type-I chromatic number is 2 and n-type-I chromatic number is \(2\sigma(v_i)\).

(ii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \{\}. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \{\}. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-IV. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(v). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vi). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

**Proposition 3.7.** Let \(N = (\sigma, \mu)\) be an even cycle. If it’s fixed-edge, then

(i) : The set of representatives of colors is \{vi, vj\}. Thus type-I chromatic number is 2 and n-type-I chromatic number is \(\sigma(v_i) + \sigma(v_j)\).

(ii) : The set of representatives of colors is \{\}. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \{\}. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv) : The set of representatives of colors is \( \{v_i, v_j\} \). Thus type-IV chromatic number is 2 and n-type-IV chromatic number is \( \sigma(v_i) + \sigma(v_j) \).

(v) : The set of representatives of colors is \( \{\} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) : The set of representatives of colors is \( \{\} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) : The set of representatives of colors is \( \{\} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{v_i, v_j\} \). The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of \( \{v_i, v_j\} \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{\} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s cycle, all vertices have 2 vertices which have common edges which are type-IV. By deletion of one edge, it’s possible to compute connectedness. Thus the set of representatives of colors is \( \{v_i, v_j\} \). The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of \( \{v_i, v_j\} \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

Proposition 3.8. Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is strong and even cycle. If it’s fixed-vertex, then

(i) : The set of representatives of colors is \( \{v_i, v_j\} \). Thus type-I chromatic number is 2 and n-type-I chromatic number is \( 2\sigma(v_i) \).

(ii) : The set of representatives of colors is \( \{\} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \( \{\} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv) The set of representatives of colors is \( \{v_i, v_j\} \). Thus type-IV chromatic number is 2 and n-type-IV chromatic number is \( 2\sigma(v_i) \).

(v) The set of representatives of colors is \( \{\} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is \( \{\} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is \( \{\} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

**Proof.** (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By its cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{v_i, v_j\} \). The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of \( \{v_i, v_j\} \) which is \( 2\sigma(v_i) \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{\} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By its cycle, all vertices have 2 vertices which have common edges which are type-IV. By deletion of one edge, it’s possible to compute connectedness. Thus the set of representatives of colors is \( \{v_i, v_j\} \). The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of \( \{v_i, v_j\} \) which is \( 2\sigma(v_i) \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{\} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

Proposition 3.9. Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is an odd cycle. If it’s fixed-edge, then

(i) The set of representatives of colors is \( \{v_i, v_j, v_k\} \). Thus type-I chromatic number is 2 and n-type-I chromatic number is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) \).

(ii) The set of representatives of colors is \( \{\} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) The set of representatives of colors is \( \{\} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv) : The set of representatives of colors is \( \{v_i, v_j, v_k\} \). Thus type-IV chromatic number is 2 and n-type-IV chromatic number is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) \).

(v) : The set of representatives of colors is \( \{} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) : The set of representatives of colors is \( \{} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) : The set of representatives of colors is \( \{} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{v_i, v_j\} \). The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of \( \{v_i, v_j, v_k\} \) which is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is \( \{v_i, v_j\} \). The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of \( \{v_i, v_j, v_k\} \) which is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \( \{} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \( \{} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \( \{} \). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proposition 3.10. Let \( \sigma, \mu \) be a neutrosophic graph which is strong and odd cycle. If it’s fixed-vertex, then

(i) : The set of representatives of colors is \( \{v_i, v_j, v_k\} \). Thus type-I chromatic number is 2 and n-type-I chromatic number is \( 3\sigma(v_i) \).

(ii) : The set of representatives of colors is \( \{} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \( \{} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.
(iv) : The set of representatives of colors is \{v_i, v_j, v_k\}. Thus type-IV chromatic number is 2 and n-type-IV chromatic number is 3σ(v_i).

(v) : The set of representatives of colors is \{\}. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) : The set of representatives of colors is \{\}. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) : The set of representatives of colors is \{\}. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By its cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \{v_i, v_j\}. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of \{v_i, v_j, v_k\} which is 3σ(v_i).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \{\}. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \{\}. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is \{v_i, v_j\}. The type-IV chromatic number is 2 and n-type-IV chromatic number is neutrosophic cardinality of \{v_i, v_j, v_k\} which is 3σ(v_i).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \{\}. The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \{\}. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \{\}. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proposition 3.11. Let \(N = (\sigma, \mu)\) be an even wheel. If it’s fixed-edge, then

(i) : The set of representatives of colors is \{v_i, v_j, v_k\}. Thus type-I chromatic number is 2 and n-type-I chromatic number is \(\sigma(v_i) + \sigma(v_j) + \sigma(v_k)\).

(ii) : The set of representatives of colors is \{\}. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \{\}. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors is \{v_i, v_j, v_k\}. Thus type-IV chromatic number is 3 and n-type-IV chromatic number is \(\sigma(v_i) + \sigma(v_j) + \sigma(v_k)\).
(v) The set of representatives of colors is \( \{ \} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is \( \{ \} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is \( \{ \} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By its cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{ v_i, v_j, v_k \} \). The type-I chromatic number is 3 and n-type-I chromatic number is neutrosophic cardinality of \( \{ v_i, v_j, v_k \} \) which is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{ \} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By its cycle, all vertices have 2 vertices which have common edges which are type-IV. By deletion of one edge, it’s possible to compute connectedness. Thus the set of representatives of colors is \( \{ v_i, v_j, v_k \} \). The type-IV chromatic number is 3 and n-type-IV chromatic number is neutrosophic cardinality of \( \{ v_i, v_j, v_k \} \) which is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By its cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

Proposition 3.12. Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is strong and even wheel. If it’s fixed-vertex, then

(i) The set of representatives of colors is \( \{ v_i, v_j, v_k \} \). Thus type-I chromatic number is 3 and n-type-I chromatic number is \( 3\sigma(v_i) \).

(ii) The set of representatives of colors is \( \{ \} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) The set of representatives of colors is \( \{ \} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) The set of representatives of colors is \( \{ v_i, v_j, v_k \} \). Thus type-IV chromatic number is 3 and n-type-IV chromatic number is \( 3\sigma(v_i) \).
(v) The set of representatives of colors is \( \{ \} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is \( \{ \} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is \( \{ \} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{ v_i, v_j, v_k \} \). The type-I chromatic number is 3 and n-type-I chromatic number is neutrosophic cardinality of \( \{ v_i, v_j, v_k \} \) which is \( 3\sigma(v_i) \).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{ \} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s cycle, all vertices have 3 vertices which have common edges which are type-IV. By deletion of one edge, it’s possible to compute connectedness. Thus the set of representatives of colors is \( \{ v_i, v_j, v_k \} \). The type-IV chromatic number is 3 and n-type-IV chromatic number is neutrosophic cardinality of \( \{ v_i, v_j, v_k \} \) which is \( 3\sigma(v_i) \).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

Proposition 3.13. Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is an odd wheel. If it’s fixed-edge, then

(i) The set of representatives of colors is \( \{ v_i, v_j, v_k, v_s \} \). Thus type-I chromatic number is 4 and n-type-I chromatic number is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s) \).

(ii) The set of representatives of colors is \( \{ \} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) The set of representatives of colors is \( \{ \} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) The set of representatives of colors is \( \{ v_i, v_j, v_k, v_s \} \). Thus type-IV chromatic number is 2 and n-type-IV chromatic number is \( \sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s) \).
(v) The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i) All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. The type-I chromatic number is 4 and n-type-I chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k, v_s\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s)$.

(ii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. The type-IV chromatic number is 4 and n-type-IV chromatic number is neutrosophic cardinality of $\{v_i, v_j, v_k, v_s\}$ which is $\sigma(v_i) + \sigma(v_j) + \sigma(v_k) + \sigma(v_s)$.

(v) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proposition 3.14. Let $N=(\sigma, \mu)$ be a neutrosophic graph which is strong and odd wheel. If it’s fixed-vertex, then

(i) The set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. Thus type-I chromatic number is 4 and n-type-I chromatic number is $4\sigma(v_i)$.

(ii) The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) The set of representatives of colors is $\{v_i, v_j, v_k, v_s\}$. Thus type-IV chromatic number is 4 and n-type-IV chromatic number is $4\sigma(v_i)$.
(v) : The set of representatives of colors is \{\}. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) : The set of representatives of colors is \{\}. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) : The set of representatives of colors is \{\}. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s cycle, all vertices have 2 vertices which have common edges which are type-I. Thus the set of representatives of colors is \{v_i, v_j, v_k, v_s\}. The type-I chromatic number is 4 and n-type-I chromatic number is neutrosophic cardinality of \{v_i, v_j, v_k, v_s\} which is 4\sigma(v_i).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \{\}. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \{\}. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s cycle, all vertices have 2 vertices which have common edges which are type-IV. Thus the set of representatives of colors is \{v_i, v_j, v_k, v_s\}. The type-IV chromatic number is 4 and n-type-IV chromatic number is neutrosophic cardinality of \{v_i, v_j, v_k, v_s\} which is 4\sigma(v_i).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic strong, there’s a vertex has 2 vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \{\}. The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \{\}. The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s cycle, all vertices have 2 vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \{\}. The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proposition 3.15. Let \(N = (\sigma, \mu)\) be a neutrosophic graph which is complete \(t\)-partite. If it’s fixed-edge, then

(i) : The set of representatives of colors is \(\{v_1, v_2, \cdots, v_t\}\). Thus type-I chromatic number is \(t\) and n-type-I chromatic number is \(\sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t)\).

(ii) : The set of representatives of colors is \{\}. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \{\}. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors is \(\{v_1, v_2, \cdots, v_t\}\). Thus type-IV chromatic number is \(t\) and n-type-IV chromatic number is \(\sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t)\).
(v) The set of representatives of colors is \( \{ \} \). Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) The set of representatives of colors is \( \{ \} \). Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) The set of representatives of colors is \( \{ \} \). Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i) All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By its neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which are type-I. Thus the set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \). The type-I chromatic number is \( t \) and n-type-I chromatic number is neutrosophic cardinality of \( \{ v_1, v_2, \ldots, v_t \} \) which is \( \sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t) \).

(ii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \( \{ \} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which are type-IV. Thus the set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \). The type-IV chromatic number is \( t \) and n-type-IV chromatic number is neutrosophic cardinality of \( \{ v_1, v_2, \ldots, v_t \} \) which is \( \sigma(v_1) + \sigma(v_2) + \cdots + \sigma(v_t) \).

(v) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \( \{ \} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \( \{ \} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic complete, there’s a vertex has \( t - 1 \) vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \( \{ \} \). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proposition 3.16. Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is complete \( t \)-partite. If it’s fixed-vertex, then

(i) The set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \). Thus type-I chromatic number is \( t \) and n-type-I chromatic number is \( t\sigma(v_1) \).

(ii) The set of representatives of colors is \( \{ \} \). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) The set of representatives of colors is \( \{ \} \). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) The set of representatives of colors is \( \{ v_1, v_2, \ldots, v_t \} \). Thus type-IV chromatic number is \( t \) and n-type-IV chromatic number is \( t\sigma(v_1) \).


t: The set of representatives of colors is \{\}. Thus type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi): The set of representatives of colors is \{\}. Thus type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii): The set of representatives of colors is \{\}. Thus type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which are type-I. Thus the set of representatives of colors is \(\{v_1, v_2, \cdots, v_l\}\). The type-I chromatic number is \(t\) and n-type-I chromatic number is neutrosophic cardinality of \(\{v_1, v_2, \cdots, v_l\}\), which is \(t\sigma(v_i)\).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \(\{\}\). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \(\{\}\). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which are type-IV. Thus the set of representatives of colors is \(\{v_1, v_2, \cdots, v_l\}\). The type-IV chromatic number is \(t\) and n-type-IV chromatic number is neutrosophic \(c\{v_1, v_2, \cdots, v_l\}\) which is \(t\sigma(v_i)\).

(v). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is \(\{\}\). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is \(\{\}\). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic complete, there’s a vertex has \(t - 1\) vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is \(\{\}\). The type-VII chromatic number is 0 and n-type-VII chromatic number is 0.

Corollary 3.17. Let \(N = (\sigma, \mu)\) be a neutrosophic graph which is complete bipartite. If it’s fixed-edge, then

(i): The set of representatives of colors is \(\{v_1, v_2\}\). Thus type-I chromatic number is 2 and n-type-I chromatic number is \(\sigma(v_1) + \sigma(v_2)\).

(ii): The set of representatives of colors is \(\{\}\). Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii): The set of representatives of colors is \(\{\}\). Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv): The set of representatives of colors is \(\{v_1, v_2\}\). Thus type-IV chromatic number is 2 and n-type-IV chromatic number is \(\sigma(v_1) + \sigma(v_2)\).
(v): The set of representatives of colors is \( \{ \} \). Thus type-V chromatic number is 0 and 
n-type-V chromatic number is 0.

(vi): The set of representatives of colors is \( \{ \} \). Thus type-VI chromatic number is 0 and 
n-type-VI chromatic number is 0.

(vii): The set of representatives of colors is \( \{ \} \). Thus type-VII chromatic number is 0 
and n-type-VII chromatic number is 0.

Proof. (i). All edges have same amount so the connectedness amid two given edges is 
the same. All edges are type-I. By it’s neutrosophic complete, there’s a vertex has 1 
which have common edges which are type-I. Thus the set of representatives of colors is 
\( \{ v_1, v_2 \} \). The type-I chromatic number is 2 and n-type-I chromatic number is 
neutrosophic cardinality of \( \{ v_1, v_2 \} \) which is \( \sigma(v_1) + \sigma(v_2) \).

(ii). All edges have same amount so the connectedness amid two given edges is the 
same. All edges aren’t type-II. By it’s neutrosophic complete, there’s a vertex has 1 
vertices which have common edges which aren’t type-II. Thus the set of representatives 
of colors is \( \{ \} \). The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the 
same. All edges aren’t type-III. By it’s neutrosophic complete, there’s a vertex has 1 
vertices which have common edges which aren’t type-III. Thus the set of representatives 
of colors is \( \{ \} \). The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). All edges have same amount so the connectedness amid two given edges is the 
same. All edges are type-IV. By it’s neutrosophic complete, there’s a vertex has 1 
vertices which have common edges which are type-IV. Thus the set of representatives 
of colors is \( \{ v_1, v_2 \} \). The type-IV chromatic number is 2 and n-type-IV chromatic number 
is neutrosophic cardinality of \( \{ v_1, v_2 \} \) which is \( \sigma(v_1) + \sigma(v_2) \).

(v). All edges have same amount so the connectedness amid two given edges is the 
same. All edges aren’t type-V. By it’s neutrosophic complete, there’s a vertex has 1 
vertices which have common edges which aren’t type-V. Thus the set of representatives 
of colors is \( \{ \} \). The type-V chromatic number is 0 and n-type-V chromatic number is 0.

(vi). All edges have same amount so the connectedness amid two given edges is the 
same. All edges aren’t type-VI. By it’s neutrosophic complete, there’s a vertex has 1 
vertices which have common edges which aren’t type-VI. Thus the set of representatives 
of colors is \( \{ \} \). The type-VI chromatic number is 0 and n-type-VI chromatic number is 0.

(vii). All edges have same amount so the connectedness amid two given edges is the 
same. All edges aren’t type-VII. By it’s neutrosophic complete, there’s a vertex has 1 
vertices which have common edges which aren’t type-VII. Thus the set of representatives 
of colors is \( \{ \} \). The type-VII chromatic number is 0 and n-type-VII 
chromatic number is 0.

\[ \square \]

**Corollary 3.18.** Let \( N = (\sigma, \mu) \) be a neutrosophic graph which is complete bipartite. If 
it’s fixed-vertex, then

(i): The set of representatives of colors is \( \{ v_1, v_2 \} \). Thus type-I chromatic number is 2 
and n-type-I chromatic number is \( 2\sigma(v_i) \).

(ii): The set of representatives of colors is \( \{ \} \). Thus type-II chromatic number is 0 and 
n-type-II chromatic number is 0.

(iii): The set of representatives of colors is \( \{ \} \). Thus type-III chromatic number is 0 and 
n-type-III chromatic number is 0.

(iv): The set of representatives of colors is \( \{ v_1, v_2 \} \). Thus type-IV chromatic number is 
t and n-type-IV chromatic number is \( 2\sigma(v_i) \).
(v) The set of representatives of colors is $\{\}$. Thus type-V chromatic number is 0 and $n$-type-V chromatic number is 0.

(vi) The set of representatives of colors is $\{\}$. Thus type-VI chromatic number is 0 and $n$-type-VI chromatic number is 0.

(vii) The set of representatives of colors is $\{\}$. Thus type-VII chromatic number is 0 and $n$-type-VII chromatic number is 0.

Proof. (i) All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-I chromatic number is 2 and $n$-type-I chromatic number is neutrosophic cardinality of $\{v_1, v_2\}$ which is $2\sigma(v_i)$.

(ii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is $\{\}$. The type-II chromatic number is 0 and $n$-type-II chromatic number is 0.

(iii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is $\{\}$. The type-III chromatic number is 0 and $n$-type-III chromatic number is 0.

(iv) All edges have same amount so the connectedness amid two given edges is the same. All edges are type-IV. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which are type-IV. Thus the set of representatives of colors is $\{v_1, v_2\}$. The type-IV chromatic number is 2 and $n$-type-IV chromatic number is neutrosophic $\{v_1, v_2\}$ which is $2\sigma(v_i)$.

(v) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-V. Thus the set of representatives of colors is $\{\}$. The type-V chromatic number is 0 and $n$-type-V chromatic number is 0.

(vi) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-VI. Thus the set of representatives of colors is $\{\}$. The type-VI chromatic number is 0 and $n$-type-VI chromatic number is 0.

(vii) All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-VII. Thus the set of representatives of colors is $\{\}$. The type-VII chromatic number is 0 and $n$-type-VII chromatic number is 0.

Corollary 3.19. Let $N = (\sigma, \mu)$ be a neutrosophic graph which is star. If it’s fixed-edge, then

(i) The set of representatives of colors is $\{c, v_2\}$. Thus type-I chromatic number is 2 and $n$-type-I chromatic number is $\sigma(c) + \sigma(v_2)$.

(ii) The set of representatives of colors is $\{\}$. Thus type-II chromatic number is 0 and $n$-type-II chromatic number is 0.

(iii) The set of representatives of colors is $\{\}$. Thus type-III chromatic number is 0 and $n$-type-III chromatic number is 0.

(iv) The set of representatives of colors, type-IV chromatic number and $n$-type-IV chromatic number aren’t defined.
Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic complete, there’s a vertex has 1 which have common edges which are type-I. Thus the set of representatives of colors is \{v_1, v_2\}. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of \{v_1, v_2\} which is \(\sigma(v_1) + \sigma(v_2)\).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \{\}. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \{\}. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-IV. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(v). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vi). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

\[\square\]

Corollary 3.20. Let \(N = (\sigma, \mu)\) be a neutrosophic graph which is star. If it’s fixed-vertex, then

(i) : The set of representatives of colors is \{v_1, c\}. Thus type-I chromatic number is 2 and n-type-I chromatic number is \(2\sigma(c)\).

(ii) : The set of representatives of colors is \{\}. Thus type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii) : The set of representatives of colors is \{\}. Thus type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv) : The set of representatives of colors, type-IV chromatic number and n-type-IV chromatic number aren’t defined.

(v) : The set of representatives of colors, type-V chromatic number and n-type-V chromatic number aren’t defined.

(vi) : The set of representatives of colors, type-VI chromatic number and n-type-VI chromatic number aren’t defined.
(vii) : The set of representatives of colors, type-VII chromatic number and n-type-VII chromatic number aren’t defined.

Proof. (i). All edges have same amount so the connectedness amid two given edges is the same. All edges are type-I. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which are type-I. Thus the set of representatives of colors is \{c, v_2\}. The type-I chromatic number is 2 and n-type-I chromatic number is neutrosophic cardinality of \{c, v_2\}. which is 2\sigma(c).

(ii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-II. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-II. Thus the set of representatives of colors is \{\}. The type-II chromatic number is 0 and n-type-II chromatic number is 0.

(iii). All edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-III. By it’s neutrosophic complete, there’s a vertex has 1 vertices which have common edges which aren’t type-III. Thus the set of representatives of colors is \{\}. The type-III chromatic number is 0 and n-type-III chromatic number is 0.

(iv). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-IV. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(v). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-V. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vi). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VI. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

(vii). By it’s fixed-vertex and it’s neutrosophic strong, all edges have same amount so the connectedness amid two given edges is the same. All edges aren’t type-VII. Since it’s impossible to define when there’s no cycle in neutrosophic graph.

4 Applications in Time Table and Scheduling

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style.

Separating the duration of work which are consecutive, is the matter and it has important to avoid mixing up.

Step 1. (Definition) Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

Step 2. (Issue) Scheduling of program has faced with difficulties to differ amid consecutive section. Beyond that, sometimes sections are not the same.

Step 3. (Model) As Figure (3), the situation is designed as a model. The model uses data to assign every section and to assign to relation amid section, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There’s one restriction in that, the numbers amid two sections are at least the number of the relation amid them. Table (1), clarifies about the assigned numbers to these situation.

Step 4. (Solution) As Figure (3) shows, neutrosophic model, proposes to use different types of chromatic number which is incomputable for types IV,V,VI,VII in the case which is titled \(T'\). In this case, \(i_1\) and \(c_1\) aren’t representative of these two colors and different types of chromatic number is incomputable for types.
Table 1. Scheduling concerns its Subjects and its Connections as a Neutrosophic Graph in a Model.

| Sections of $T$ | $s_1$ | $s_2$ | $s_3$ | $s_4$ | $s_5$ | $s_6$ | $s_7$ | $s_8$ | $s_9, s_{10}$ |
|-----------------|-------|-------|-------|-------|-------|-------|-------|-------|---------------|
| Values          | 0.1   | 0.8   | 0.7   | 0.8   | 0.1   | 0.3   | 0.6   | 0.5   | 0.2           |
| Connections of $T$ | $s_1s_2$ | $s_2s_3$ | $s_3s_4$ | $s_4s_5$ | $s_5s_6$ | $s_6s_7$ | $s_7s_8$ | $s_8s_9$ | $s_9s_{10}$ |
| Values          | 0.1   | 0.6   | 0.4   | 0.1   | 0.1   | 0.2   | 0.4   | 0.2   | 0.1           |

IV,V,VI,VI,II. The set $\{i_1,c_1\}$ doesn’t contain representatives of colors which pose different types of chromatic number and different types of chromatic number for types IV,V,VI,II,II. Thus the decision amid choosing the subject $c_1$ an $c_2$ isn’t concluded to choose $c_1$ for types IV,V,VI,II. To get brief overview, neutrosophic model uses one number for every array so 0.9 means (0.9,0.9,0.9). In Figure (3), the neutrosophic model $T$ introduces the common situation. The representatives of colors are $i_2$ and $c_1$. Thus different types of chromatic numbers is two for types I and IV and different types of neutrosophic chromatic number is 1.4 for types I and IV. Thus suspicion about choosing $i_1$ and $i_2$ is determined to be $i_2$. The sets of representative for colors are $\{i_2,c_1\}$ for types I and IV. Thus the comparative studies based on different types of chromatic number and neutrosophic chromatic number are concluded.

5 Open Problems

The two notions of coloring of vertices concerning different types of chromatic number and different types of neutrosophic chromatic number are defined on neutrosophic graphs when connectedness and as its consequences, different types of edges have key role to have these notions. Thus

Question 5.1. Is it possible to use other types edges via connectedness to define different types of chromatic number and different types of neutrosophic chromatic number?

Question 5.2. Are existed some connections amid the coloring from connectedness inside this concept and external connections with other types of coloring from other notions?

Question 5.3. Is it possible to construct some classes neutrosophic graphs which have “nice” behavior?
**Question 5.4.** Which applications do make an independent study to apply different types of chromatic number and different types of neutrosophic chromatic number?

**Problem 5.5.** Which parameters are related to this parameter?

**Problem 5.6.** Which approaches do work to construct applications to create independent study?

**Problem 5.7.** Which approaches do work to construct definitions which use all three arrays and the relations amid them instead of one array of three arrays to create independent study?

### 6 Conclusion and Closing Remarks

This study uses mixed combinations of different types of chromatic number and different types of neutrosophic chromatic number to study on neutrosophic graphs. The connections of vertices which are clarified by special edges and different edges from connectedness, differ them from each other and and put them in different categories to represent one representative for each color. Further studies could be about changes in the settings to compare this notion amid different settings of graph theory. One way is finding some relations amid array of vertices to make sensible definitions. In Table (??), some limitations and advantages of this study is pointed out.

**Table 2.** A Brief Overview about Advantages and Limitations of this study

| Advantages                                      | Limitations                      |
|------------------------------------------------|----------------------------------|
| 1. Using connectedness for labelling edges     | 1. General Results               |
| 2. Using neutrosophic cardinality              | 2. Connections with parameters   |
| 3. Using cardinality                           |                                  |
| 4. Applying Different Types of Edges           | 3. Connections of Results        |
| 5. Different Types of Chromatic Notions        |                                  |

### References

1. M. Akram, and G. Shahzadi, *Operations on Single-Valued Neutrosophic Graphs*, Journal of uncertain systems 11 (1) (2017) 1-26.

2. K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets Syst. 20 (1986) 87-96.

3. S. Broumi, M. Talea, A. Bakali and F. Smarandache, *Single-valued neutrosophic graphs*, Journal of New Theory 10 (2016) 86-101.

4. N. Shah, and A. Hussain, *Neutrosophic soft graphs*, Neutrosophic Set and Systems 11 (2016) 31-44.

5. Henry Garrett, *Big Sets Of Vertices*, Preprints 2021, 2021060189 (doi: 10.20944/preprints202106.0189.v1).

6. Henry Garrett, *Locating And Location Number*, Preprints 2021, 2021060206 (doi: 10.20944/preprints202106.0206.v1).
7. Henry Garrett, *Metric Dimensions Of Graphs*, Preprints 2021, 2021060392 (doi: 10.20944/preprints202106.0392.v1).

8. Henry Garrett, *New Graph Of Graph*, Preprints 2021, 2021060323 (doi: 10.20944/preprints202106.0323.v1).

9. Henry Garrett, *Numbers Based On Edges*, Preprints 2021, 2021060315 (doi: 10.20944/preprints202106.0315.v1).

10. Henry Garrett, *Matroid And Its Outlines*, Preprints 2021, 2021060146 (doi: 10.20944/preprints202106.0146.v1).

11. Henry Garrett, *Matroid And Its Relations*, Preprints 2021, 2021060080 (doi: 10.20944/preprints202106.0080.v1).

12. A. Shannon and K.T. Atanassov, *A first step to a theory of the intuitionistic fuzzy graphs*, Proceeding of FUBEST (Lakov, D., Ed.) Sofia (1994) 59-61.

13. F. Smarandache, *A Unifying field in logics neutrosophy: Neutrosophic probability, set and logic*, Rehoboth: American Research Press (1998).

14. H. Wang, F. Smarandache, Y. Zhang, and R. Sunderraman, *Single-valued neutrosophic sets*, Multispace and Multistructure 4 (2010) 410-413.

15. L. A. Zadeh, *Fuzzy sets*, Information and Control 8 (1965) 338-353.