The Vortex Solution in the (2+1)-Dimensional Yang-Mills-Chern-Simons Theory at High Temperature

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Abstract

The vortex-like solution to the no-linear field equations in a two-dimensional SU(2) gauge theory with the Chern-Simons mass term is found at high temperature. It is derived from the effective Lagrangian including the leading order finite temperature corrections. The discovered field configuration possesses the finite energy and the quantized magnetic flux. At the centre of the vortex the point charge is located which is surrounded by the distributed charge of the opposite sign and the vortex is neutral as a whole. At high temperature the energy of the vortex is negative and it corresponds to the ground state. The derived solution is considered to be a result of heating the lattice vacuum structure formed at zero temperature.

1 Introduction

Over the last decade gauge theories in two spatial dimensions have been the object of considerable interest. The origins of this interest are manifold. First, these theories have a number of interesting features which make them quite different from theories in the conventional space-time. There is, for example, a possibility of introducing a mass term for the gauge field—the Chern-Simons (CS) topological mass—in a gauge invariant way [4]. In fact, if not added by hand, this mass is necessarily generated by the fermion loop corrections [2, 3]. Second, at high temperature field theories in a (3+1)-dimensional space-time effectively behave as theories in a 3-dimensional Euclidean space [4]. Third, low-dimensional theories are successfully applied to the condensed matter physics phenomena—quantum Hall effect, high-T superconductivity—which proved to have an essentially planar nature [3, 6].

It is well known that the non-Abelian gauge field vacuum becomes unstable in an external magnetic field due to a tachyonic mode present in the gluon spectrum [4, 5]. This holds for both three- and two-dimensional models. Searches for the true vacuum state of...
the non-abelian gauge fields have led to the discovery of gauge field condensation in an external magnetic field \[9–11\]. The mechanism of the structure generation included exciting of tachyonic modes in intense magnetic fields. Also, a scalar field should be present in order to stabilize the condensate. An essential feature of this vacuum is its lattice structure which results in a breakdown of vacuum homogeneity. This picture is similar to that of superconductivity \[12\], although the origin of tachyonic modes is different.

In Ref. \[13\] it has been shown that in 2D gauge theories there is a mechanism of the vacuum structure formation quite distinct from that in 3+1 space-time. It occurs due to the CS mass \(m\) presence, in the magnetic fields below the tachyonic threshold in the gluon spectrum, and does not involve the condensation of unstable modes. The vacuum state of the \(SU(2)\) theory in external magnetic field has been found to be a periodic electromagnetic structure; both magnetic and electric fields are present, and form a triangular lattice with a period \(\sim 1/m\). Each cell contains magnetic flux quantized in a manner similar to superconductivity. The configuration is a perturbative solution to the non-linear field equations. Generation of a pure magnetic structure is suppressed by the CS term, which mixes electric and magnetic components of the colour fields. As a result, the \(A_0\) component of the gauge potential plays an important role in the vacuum structure formation. As the threshold of the tachyonic mode appearance set by the topological mass is not reached, the derived state is stable. The lattice has been found to lower the energy of the applied homogeneous magnetic field. Moreover, when \(H = 0\), the energy density is negative and the condensate can be generated spontaneously without an external magnetic field. It is the CS mass that gives the origin to this new type of vacuum structure, which has no analogues among lattices studied in the other models.

The analysis carried out in Ref. \[13\] has a general character and allows extension to the case when the CS mass is generated at the one-loop level. Therefore, the noted mechanism could produce the gluon field vacuum structure in various external environments. This possibility has been examined in the finite temperature model \[14\]. The technique applied was essentially the one of zero temperature case. As it appeared, the periodic vacuum structure persists at finite temperature, and is generated due to the same mechanism as at \(T = 0\). The crucial factor for the lattice formation is again the topological mass presence. However, it has been found that the amplitude of the gauge field condensate increases as the temperature rises. As a consequence, at high temperature \(T \gg m\) the results obtained within a perturbative method should be treated with caution. It has been argued that the analysis of the non-linear equations is needed to describe the vacuum evolution at high temperature.

The main goal of the present paper is to investigate how the vacuum structure is affected by high temperature when the perturbation theory is not adequate. To do this we solve the non-linear equations of motion for the topologically massive gluon fields derived from the effective Lagrangian (EL). In contrast to the magnetic lattice case, when the CS mass is present an electric field is generated in a vacuum and, so, the \(A_0\) component is not decoupled. Therefore, we should not fail to take into account the Debye mass when constructing the \(\mathcal{L}_{\text{eff}}\). We consider the one-loop gluon effective Lagrangian of the two-dimensional gauge theory at high temperature. The thermal contribution is introduced via the one-loop gluon
polarization tensor in the static limit. We take into consideration the leading in $T$ term of the Debye mass, $\mu$, which is $\sim g^2 T$ in 2+1 dimensions. The one-loop induced CS coefficient in the EL is usually modified at finite temperature by the factor of $\tanh m_f / T$. We will discuss the implications of such temperature dependence below. With thermal effects thus included, the following results have been obtained.

An exact solution to the non-linear gluon field equations has been found. The solution is static and axially symmetric; the fields form a vortex in both coordinate and gauge spaces. At spatial boundary the gluon potentials constituting the solution become a pure gauge. Both electric and magnetic fields are present in the derived configuration. Due to the CS and the temperature masses presence the fields are short-range, and vanish exponentially at $r \to \infty$ and $T \to \infty$. The extent of the area in which the fields are localized is determined by the inverted effective mass $M = \sqrt{m^2 + \mu^2}$. So, at spatial infinity the configuration approaches the perturbative vacuum. The important feature of the found solution is finiteness of the total energy of the vortex. The sign of energy is not constant, it is determined by the sign of the combination $m^2 - \mu^2$. The electric field has a Coulomb-like behaviour at the origin signalling the presence of a point charge. It cancels exactly the density of the continuously distributed charge resulting in the neutral field configuration. Thus, the derived solution to the field equation describes the composite of a vortex, which possesses finite energy and magnetic flux, and a point charge.

The picture summarized above corresponds quite well to the lattice solution found in Refs. [13, 14]. In fact, the vortex can be considered as a result of heating the vacuum structure. Depending on the sign of the free energy, the vortex solution can describe either particle-like excitation over the periodic vacuum structure, or the vacuum state represented by the system of isolated vortices. We note once again that the discovered configuration exists because the equation of motion are derived from the effective Lagrangian, which includes the temperature mass contribution for $A_0$ component of the gluon field.

2 Vortex solution to the field equations

Two-dimensional $SU(2)$ gauge theory in the Minkowski space-time is governed by the following generating functional:

$$ Z = \int d\psi d\bar{\psi} dA \exp \left[ i \int d^3 x \left[ -\frac{1}{4} G_a^\mu G^a_{\mu\nu} + \bar{\psi} \gamma^\mu (i\partial_\mu + \frac{1}{2} g \sigma_a A_a^\mu + m_f) \psi \right] \right], \quad (1) $$

where $G_{\mu\nu}^a = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \varepsilon_{abc} A^b_\mu A^c_\nu$, $A^a_\mu$ are Yang-Mills (YM) potentials, $\psi, \bar{\psi}$ are the fermion fields, $\sigma_a$ are Pauli matrices, $g$ is the gauge coupling constant.

We want to build the one loop effective Lagrangian of gauge fields and, to do this, integrate over the fermion degrees of freedom. As a result, two additional terms appear in $\mathcal{L}_{eff}$. The first is contributed by the parity-even part of gluon polarization tensor $\Pi^{ab}_{\mu\nu}$; the second is parity-violating one intrinsic to odd-dimensional space-time, the well known CS term [4 8]:

$$ \mathcal{L}_{CS} = \frac{1}{4} m \varepsilon^{\alpha\beta\gamma} \left( G^a_{\alpha\beta} A^\gamma_a - \frac{1}{3} g \varepsilon_{abc} A^a_\alpha A^b_\beta A^c_\gamma \right), \quad (2) $$

where $m$ is real. This term renders the gauge field excitations massive, with the mass $m$. For the theory to remain invariant under large gauge transformations, the CS coefficient must
be quantized. To be precise, the value of \( m \) should satisfy the condition

\[
\frac{m}{g^2} = \frac{n}{4\pi}.
\]

where \( n \) is an integer. We note that the coupling \( g \) has dimension of \( m^{1/2} \) in 2+1 space-time.

We are interested in obtaining the gluon effective Lagrangian at finite temperature. The polarization tensor \( \Pi_{\mu\nu}^{ab} = \Pi_{\mu\nu}\delta^{ab} \) contribution has been obtained by the authors earlier \[14\] in high-temperature approximation. The following expression for the \( \Pi_{00} \) component has been calculated

\[
\Pi_{00}(T, k_0 = 0, \vec{k} \to 0) \equiv \mu^2(T) = g^2 T \left( \frac{\ln 2}{2\pi} - \frac{1}{16\pi} \frac{m_f^2}{T^2} + O\left(\frac{m_f^4}{T^4}\right) \right),
\]

(4)

here \( m_f \) is the fermion mass and by \( \mu \) we denoted the Debye mass. All the other components of the polarization tensor \( \Pi_{\mu\nu} \) vanish in the zero-momentum limit \( k_0 = 0, \vec{k} \to 0 \).

Calculations of the CS term at finite temperature has drawn much attention recently. In a number of papers (see, for instance \[3, 15, 16\]) there has been obtained by applying various perturbative methods, that the one-loop expression for the CS mass is given at finite temperature by

\[
m(T) \sim m(T = 0) \tanh \frac{m_f T}{T}.
\]

(5)

Such a smooth dependence on the temperature is, of course, in conflict with the gauge invariance. As has been argued in Refs. \[17, 18\], since the CS mass has to take discrete values, it should not be modified by finite temperature corrections. Recently, there has been significant progress in resolving this puzzle. It has been found \[19–21\] that, when large gauge transformations are taken into account, the full effective action is gauge invariant, while the perturbative expansion is not. Thus, the apparent gauge non-invariance of the finite temperature Chern-Simons coefficient is only a result of considering the first term in the expansion of the effective action.

Therefore, we can write the expression for the high-temperature one-loop effective Lagrangian

\[
L_{\text{eff}} = - \frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + m \epsilon^{\alpha\beta\gamma} \left( G^a_{\alpha\beta} A^a_{\gamma} - \frac{g}{3} \epsilon_{abc} A^a_{\alpha} A^b_{\beta} A^c_{\gamma} \right) + \frac{1}{2} A_0^a \Pi_{00}(T) \delta^{ab} A_0^b,
\]

(6)

where \( \Pi_{00} \) is given by the leading term in (4), \( \Pi_{00} = g^2 T \frac{\ln 2}{2\pi} \). The non-linear equations of motion derived from (6) are

\[
\partial_\nu G^{\nu\mu}_a + g \epsilon^{abc} G_b^{\mu\nu} A^c_\nu + m \epsilon^{\mu\alpha\beta} G^{a}_{\alpha\beta} + g \mu \Pi_{00} A_0^a = 0.
\]

(7)

Our aim is to find an exact solution to these equations. To this end let us introduce the following ansatz for the gauge fields:\footnote{We will see below, that an effective mass appears in our model, \( \sim m^2 + gT \), and the influence of CS mass temperature dependence of the form \( \ln \) would be inessential at high \( T \).}

\[
A^a_\mu = \hat{\phi}^a \hat{\phi}_1(\vec{r}) + \delta^a_3 \phi_1 A(\vec{r}); \quad A_0^a = \hat{\phi}^a \Psi_2(\vec{r}).
\]

(8)

\footnote{This ansatz has been introduced in Ref. \[22\] in context of the search for an exact solution to Yang-Mills equations at zero temperature. The solutions found in that paper have infinite energy, and are time-dependent, \( A_\mu^a \sim \exp(-t) \).}
Here and below we use the unit vectors of the cylindrical coordinate system \( \hat{\phi}_i = \frac{\epsilon_{ij} x^j}{r} \), \( \hat{\varrho}_i = \frac{x_i}{r} \). Upon substituting the potentials (8) into the system (7), after rather lengthy but straightforward transformations the equations of motion take the form

\[
- \partial^2_r \Psi_2 - \frac{1}{r} \partial_r \Psi_2 - \frac{1}{r^2} \partial^2_\phi \Psi_2 - \frac{1}{r} \partial_\varrho \partial_\phi \Psi_1 + m \left( \partial_r \Psi_1 + \frac{1}{r} \Psi_1 \right) + \mu^2 \Psi_2 = 0,
\]

\[
\partial^2_\varrho \Psi_1 - \partial^2_\phi \Psi_1 - \frac{2}{r^2} \partial^2_\phi \Psi_1 + \frac{1}{r^2} \Psi_1 - \frac{1}{r} \partial_\varrho \partial_\phi \Psi_2 + m \partial_r \Psi_2 = 0,
\]

\[
- \partial_\varrho \partial_r \Psi_2 - \frac{1}{r} \partial_\varrho \partial_\phi \Psi_1 - \frac{1}{r^2} \partial^2_\phi \Psi_1 + m \left( \partial_\varrho \Psi_1 - \frac{1}{r} \partial_\phi \Psi_2 \right) = 0.
\]

Here we have also set \( A(r) = \frac{1}{gr} \). These equations are now linear, but still rather complicated for exhaustive analysis. Further simplification can be achieved if we set the ansatz functions \( \Psi_1, \Psi_2 \) to be time independent and axially symmetric. Under these assumptions our ansatz describes a static solution to non-linear equations of motion that has the form of a vortex in both coordinate and internal spaces. Then the third equation in (9) is satisfied identically, while the other two reduce to

\[
- \partial^2_r \Psi_2 - \frac{1}{r} \partial_r \Psi_2 + m \left( \partial_r \Psi_1 + \frac{1}{r} \Psi_1 \right) + \mu^2 \Psi_2 = 0,
\]

\[
- \partial^2_\varrho \Psi_1 - \frac{1}{r} \partial_\varrho \Psi_1 + \frac{1}{r^2} \Psi_1 + m \partial_r \Psi_2 = 0.
\]

Now one is able to solve the system exactly. Multiplying the second equation by \( m \) and taking the derivative of the first one, we add the resultant expressions together to obtain

\[
\Psi_2'' + \frac{1}{r} \Psi_2'' - \left( \frac{1}{r^2} + M^2 \right) \Psi_2' = 0,
\]

here we denoted differentiation with respect to \( r \) by prime, and introduced the effective mass squared

\[
M^2 \equiv m^2 + \mu^2 = m^2 + g^2 T \ln \frac{2}{2\pi}.
\]

We see that both squared masses, Chern-Simons one and temperature generated, enter the equations with the same sign. Note that \( M \) increases linearly as the temperature rises. We recognize (11) as the modified Bessel equation for \( \Psi_2' \) with the general solution:

\[
\Psi_2'(r) = a K_1(Mr) + b I_1(Mr).
\]

Integrating (13) and leaving out the part of the solution that diverges exponentially at spatial infinity, we obtain

\[
\Psi_2(r) = - \frac{a}{M} K_0(Mr)
\]

and, solving equation for \( \Psi_1(r) \),

\[
\Psi_1(r) = \frac{am}{M^2} K_1(Mr) + \frac{c}{r},
\]

where \( a, c \) are integration constants.
Finally, the expressions for the gauge potentials that solve the non-linear equation of motion (7) read

\[ A^a_i = \hat{\phi}^a \hat{\phi}_i \left( \frac{am}{M^2} K_1(Mr) + \frac{c}{r} \right) + \delta^a_3 \phi_i \frac{1}{gr} , \]
\[ A^a_0 = -\hat{\phi}^a \frac{a}{M} K_0(Mr). \]  

(16)

The properties of this vortex configuration will be discussed in the next section.

3 The properties of the vortex solution

Now let us investigate the physical characteristics of the obtained field configuration. Electric field is along the radius in the coordinate space, while in the internal space it has only the angular component

\[ E^a_i = G^a_{i0} = \hat{\phi}^a \hat{\phi}_i \frac{\partial \Psi_2}{\partial r} = \hat{\phi}^a \hat{\phi}_i a K_1(Mr). \]  

(17)

Magnetic field is

\[ H^a = G^a_{12} = \frac{1}{2} \varepsilon_{ij} G^a_{ij} = -\hat{\phi}^a \left( \frac{\partial \Psi_1}{\partial r} + \frac{1}{r} \Psi_1 \right) = \hat{\phi}^a \frac{am}{M} K_0(Mr). \]  

(18)

One can see that both electric and magnetic fields are screened by the mass \( M \), and vanish exponentially at spatial infinity. This is what should be expected, for the Yang-Mills potentials (16) asymptotically behave as

\[ A^a_i \xrightarrow{r \to \infty} \hat{\phi}^a \hat{\phi}_i \frac{c}{r} + \delta^a_3 \phi_i \frac{1}{gr} , \quad A^a_0 \xrightarrow{r \to \infty} 0 , \]

(19)

which is a pure gauge. Magnetic flux through the two-dimensional space is finite,

\[ \Phi^a = \int d^2 x H^a = \phi^a \frac{2\pi am}{M^3} . \]  

(20)

Now we turn to the total energy of the vortex determined as the integral of the Hamiltonian density

\[ \mathcal{E} = \int d^2 x \mathcal{H} = \int d^2 x \left( A^a_{\sigma,0} \frac{\partial \mathcal{L}}{\partial A^a_{\sigma,0}} - \mathcal{L} \right) \]
\[ = 2\pi \int dr r \left[ \frac{1}{2} \left( \partial_r \Psi_2 \right)^2 + \frac{1}{2} \left( \partial_r \Psi_1 + \frac{1}{r} \Psi_1 \right)^2 + \frac{m}{2} \left( \Psi_2 \partial_r \Psi_1 - \Psi_1 \partial_r \Psi_2 + \frac{1}{r} \Psi_1 \Psi_2 \right) - \frac{\mu^2}{2} \Psi_2^2 \right] . \]

The energy density \( \mathcal{H} \) in the above expression is divergent at the origin \( r \to 0 \)

\[ \mathcal{H}|_{r \to 0} = \frac{1}{r^2} \left( \frac{a^2 \mu^2}{2M^2} - \frac{acm}{2M} \right) + O(\ln r) . \]  

(22)

However, with appropriately chosen constants \( a, c \) we can cancel the term \( \sim r^{-2} \) and make this singularity integrable. Then the integrand in (21) will read

\[ \mathcal{H} = \frac{a^2 \mu^2}{2M^2} K_1^2(Mr) - \frac{a^2 \mu^2}{2M^2 r} K_1(Mr) + \frac{a^2 (2m^2 - \mu^2)}{2M^2} K_1^2(Mr) , \]

(23)
and performing integration we get the following finite result for the full energy of the vortex field configuration
\[
E = \frac{a^2 \pi (m^2 - \mu^2)}{(m^2 + \mu^2)^2}.
\]
(24)

Thus, we have found the static solution to the Yang-Mills equations that possesses finite energy, i.e. a two-dimensional soliton. Since asymptotic values of the fields correspond to the perturbative vacuum solution, this solitonic configuration does not have a topological nature. We note that the finiteness of energy became possible due to the Debye mass being taken into account. If we set \( \mu = 0 \), the only choice of the constants leading to cancellation of \( 1/r^2 \) divergence in \( \mathcal{H} \) would be \( a = c = 0 \). This would leave us with a pure gauge solution equivalent to \( A_\mu^a = 0 \). One can see as well, that the full energy changes sign: it is positive if \( m > \mu \), and negative otherwise. Implications of this fact we will discuss below.

Now we consider the charge density of the system. The time component of the relevant Noether current
\[
gI_0^a = \partial^i E_i^a + mH^a - \mu^2 A_0^a = \hat{\phi}^a (\partial_r^2 \Psi_2 + \frac{1}{r} \partial_r \Psi_2 - \mu^2 \Psi_2 - m \partial_r \Psi_1 - \frac{m}{r} \Psi_1) = 0
\]
vanishes on the vortex solution (16). We see that for all the calculated quantities their non-Abelian structure factorizes as \( \hat{\phi}^a \). This is true for the gauge potentials as well, up to the pure gauge contribution. So, in a sense, the introduced ansatz can be thought of as an Abelian one. Further, we calculate the electric charge density as \( \text{div} \vec{E} \), and integrate it over the whole space to obtain
\[
Q^a = \int d^2 x \partial_i E_i^a = -2 \pi \phi^a \int dr \ r \ aMK_0(Mr) = -\phi^a \frac{2\pi a}{M}.
\]
(26)

So, the charge of the vortex appears to have the finite value. On the other hand the electric field flux through the spatial boundary vanishes due to the mass screening. These facts can be reconciled if we recall that the electric field behaves as \( 1/r \) at \( r \to 0 \)
\[
E_i^a \mid_{r \to 0} = \hat{\phi}^a \hat{\partial}_i \left( \frac{a}{Mr} + O(r \ln r) \right),
\]
(27)
which is the 2-dimensional Coulomb law. Thus a point charge is present at the centre of the vortex, of the value
\[
Q_{\text{point}} = \frac{2 \pi a}{M},
\]
(28)
and the total electric charge of the vortex \( Q + Q_{\text{point}} \) is zero. It well corresponds to the picture described in [13], where it has been found that each lattice cell has a positive core surrounded by negative charge distribution, in such a way that the total charge vanishes.

4 Discussion of the results

In the present paper, the exact solution to the non-linear equations of motion for the topologically massive gauge field is derived at high temperature. To treat the problem consistently,
the additional term $\sim A_0^2 \Pi_{00}(T)$ has been introduced to the effective Lagrangian. This is the key point of our analysis that has the general character at finite temperature in theories with the Chern-Simons mass. In this respect the situation is quite different as compared to the superconductivity and electroweak theory. In both those models there is no mixing between the electric and the magnetic sectors. The temperature influences the vortices through the mass of the condensed scalar field. In the considered model the vortex is formed from the short-range colour magnetic and electric fields. The full energy of the configuration is finite. As the fields are screened by the effective mass $M$ (12), at spatial infinity the perturbative vacuum solution is realized. The total charge of the vortex is zero; the magnetic flux through the plane is finite.

Let us discuss the most important features of the discovered field configuration. If we set $T = 0$, our solution transforms into the divergent-energy solution of Ref. [22]. We stress once again that it is the addition of the Debye mass into the effective Lagrangian that makes the full energy finite. In fact, at high temperature it alone determines the effective mass parameter $M \sim \mu$, and the role of the CS mass is restricted to introducing the ansatz solutions to the non-linear field equations. It is worth noting that obtained configuration is composed from the gauge fields only. The vortex solutions in Chern-Simons theories obtained earlier [23] have been derived from the Lagrangians including Higgs fields.

The physical contents of the derived field configuration varies. As we see, the energy of the vortex (24) is positive when $m^2 > \mu^2 \sim g^2 T$. Thus, if this condition holds the derived solution describes the two-dimensional non-topological temperature soliton. Since in this case the perturbative approach used in Ref. [14] is consistent, we have to conclude that when $T \leq m$ the ground state of the theory is the gluon condensate lattice with the period $\sim 1/m$ determined by the CS mass. The vortex solution, then, corresponds to the particle-like field excitation over the lattice vacuum structure.

At high temperature $\mu > m$ the situation is quite different. The vortex field configuration possesses negative energy and, therefore, one has to treat it as the ground state of the model. It is natural to consider this vacuum as the result of the evolution of the lattice vacuum structure of gluon condensate created at zero temperature, each cell developing into a vortex. Indeed, comparing the single lattice cell with the vortex puts forward a number of similarities. In both cases we have gluon condensate formed from the magnetic, as well as radial electric field. The magnetic field is maximal at the centre of the configuration, falling off towards the boundary; the magnetic flux is finite. The charge density distribution is similar as well: a core in the centre encircled by the charge of the opposite sign. Thus, one can conjure up the following qualitative picture of the vacuum affected by the temperature. As the vacuum structure is heated, the period of the lattice, which is determined by the CS mass value, grows $d \sim 1/m \sim T$, as it follows from equation (1). Eventually, the distance between flux ‘tubes’ exceeds the characteristic scale $\sim 1/M$, the lattice cells decouple and at high temperature each of them develops into the vortex field configuration. The ground state thus becomes a plane covered with scattered non-interacting gauge field vortices. The stability of the derived field configuration is ensured by the magnetic flux conservation. Therefore, at high temperature the vacuum of the considered model is represented as the system of isolated vortices formed from colour electric and magnetic fields.
References

[1] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) 140, 372 (1982).
[2] A. N. Redlich, Phys. Rev. D29, 2366 (1984).
[3] A. Niemi and G. Semenoff, Phys. Rev. Lett. 51, 2077 (1983).
[4] D. J. Gross, R. D. Pisarski, and L. G. Yaffe, Rev. Mod. Phys 53, 43 (1981).
[5] I. V. Krive and A. S. Rozhavsky, Sov. Phys. Usp. 30, 370 (1987).
[6] S. Randjbar-Daemi, A. Salam, and J. Strathdee, Nucl. Phys. B340, 403 (1990).
[7] V. V. Skalozub, Sov. J. Nucl. Phys. 28, 113 (1978).
[8] N. K. Nielsen and P. Olesen, Nucl. Phys. B144, 376 (1978).
[9] V. V. Skalozub, Sov. J. Nucl. Phys. 43, 665 (1986).
[10] J. Ambjørn and P. Olesen, Nucl. Phys. B330, 193 (1990).
[11] S. W. MacDowell and O. Törnkvist, Phys. Rev. D45, 3833 (1992).
[12] A. A. Abrikosov, Sov. Phys. JETP 5, 1174 (1957).
[13] V. V. Skalozub, S. A. Vilensky, and A. Yu. Zaslavsky, Phys. Rev. D50, 5300 (1994).
[14] V. V. Skalozub and A. Yu. Zaslavsky, Vestn. Dniepropetr. Univ 3, 46 (1998).
[15] E. R. Poppitz, Phys. Lett. B252, 417 (1990).
[16] I. J. R. Aitchison, C. D. Fosco, and J. A. Zuk, Phys. Rev. D48, 5895 (1993).
[17] R. D. Pisarski, Phys. Rev. D35, 664 (1987).
[18] S. Deser, L. Griguolo, and D. Seminara, Phys. Rev. Lett. 79, 1976 (1997), hep-th/9705052.
[19] G. Dunne, K. Lee, and C. Lu, Phys. Rev. Lett. 78, 3434 (1997), hep-th/9612194.
[20] C. D. Fosco, G. L. Rossini, and F. A. Schaposnik, Phys. Rev. D56, 6547 (1997), hep-th/9707199.
[21] I. J. R. Aitchison and C. D. Fosco, Phys. Rev. D57, 1171 (1998), hep-th/9709035.
[22] R. Teh, J. Phys. G16, 175 (1990).
[23] A. Khare, Fortsch. Phys. 38, 507 (1990).