The Schneider-Vigneras functor
for principal series

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30th September 2014

Abstract

We compute the Schneider-Vigneras functor attaching a module
over the Iwasawa algebra $\Lambda(N_0)$ to a $B$-representation for irreducible
modulo $p$ principal series of the group $\text{GL}_n(F)$ for any finite field
extension $F|\mathbb{Q}_p$. It turns out to be a finitely generated module with
rank 1 over $\Lambda(N_0)/\pi\Lambda(N_0)$.

1 Introduction

Let $\mathbb{Q}_p$ be the field of $p$-adic numbers, $\overline{\mathbb{Q}}_p$ its algebraic closure, $F, K \leq \overline{\mathbb{Q}}_p$
finite extensions of $\mathbb{Q}_p$. Let $o_F$, respectively $o_K$ be the rings of integers in
$F$, respectively in $K$, $\pi_F \in o_F$ and $\pi_K \in o_K$ uniformizers, $\nu_F$ and $\nu_K$ the
standard valuations and $k_F = o_F/\pi_F o_F$, $k_K = o_K/\pi_K o_K$ the residue fields.

The Langlands philosophy predicts a natural one-to-one correspondence
between smooth admissible representations of $\text{GL}_n(F)$ over Banach $K$-vector
spaces and certain $n$-dimensional $K$-representations of the Galois-group
$\text{Gal}(\overline{\mathbb{Q}}_p|F)$.

Colmez proved the existence of such a correspondence in the case of
$\text{GL}_2(\mathbb{Q}_p)$, but for any other group even the conjectural picture is not de-
veloped yet. It turned out, that Fontaine’s theory of $(\varphi, \Gamma)$-modules is a
fundamental intermediary between the representations of $\text{Gal}(\overline{\mathbb{Q}}_p/\mathbb{Q}_p)$ and
the representations of $\text{GL}_2(\mathbb{Q}_p)$. Schneider and Vigneras managed to gener-
alize parts of Colmez’s work to reductive groups other than $\text{GL}_2(\mathbb{Q}_p)$.

Our aim is to understand the construction of Schneider and Vigneras, at-
taching a generalized $(\varphi, \Gamma)$-module to a smooth torsion $o_K$-representation of
$G$, for principal series representations $V$ in the case $G = \text{GL}_n(F)$. Originally this functor is defined only for $F = \mathbb{Q}_p$, but our considerations work for any finite extension $F|\mathbb{Q}_p$ and the analogous definitions.

In order to that, we need to understand the $B_+$-module structure of the principal series, where $B_+$ is a certain submodule of a Borel subgroup $B$ in $G$. The first step is to decompose $G$ to open $N_0$-invariant subsets (where $N_0$ is a maximal compact open subgroup in the unipotent radical of $B$), indexed by the Weyl group, which has a similar but finer partial order than the Bruhat decomposition.

With this it is possible to prove, that there exists a minimal generating $B_+$-subrepresentation $M_0$ of the principal series, generated as a $B_+$-module by $n!$ elements.

By looking at the filtration of $M_0$ containing the $B_+$-modules generated by the first elements of the above set of generators, it is shown that as a $\Lambda(N_0)$-module $D(V)$ is finitely generated and has rank 1 over $\Lambda(N_0)/\pi\Lambda(N_0)$, where $\Lambda(N_0)$ is the Iwasawa-algebra of $N_0$.

In the last section we point out some properties of $D(V)$, which makes the picture more difficult, compared to the case of $\text{GL}_2(\mathbb{Q}_p)$.

**Acknowledgments.** I gratefully acknowledge the financial support and hospitality of the Central European University, and the Alfréd Rényi Institute of Mathematics, both in Budapest. I would like to thank Gergely Zábrádi for introducing me to this field, and for his constant help, valuable comments and all the useful discussions. I am also grateful to Levente Nagy for reading through an earlier version of this paper.

## 2 Notations

Let $G$ be the $F$-points of a $\mathbb{Q}_p$-split connected reductive group over $\mathbb{Q}_p$.

As an $\mathfrak{o}_K$-representation of $G$ we mean a pair $V = (V, \rho)$, where $V$ is a torsion $\mathfrak{o}_K$-module, $\rho : G \to \text{GL}(V)$ is a group homomorphism. $V$ is smooth if $\rho$ is locally constant ($\forall v \in V \exists U \subset G$ open, such that $\forall u \in U : \rho(u)v = v$). $V$ is admissible if for any $U \leq G$ open subgroup, the vector space $k \otimes_{\mathfrak{o}_K} V^U$ is finite dimensional.

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For an $o_K$-representation $V$ let $V^* = \text{Hom}_{o_K}(V, K/o_K)$ be the Pontrjagin dual of $V$. Pontrjagin duality sets up an anti-equivalence between the category of $o_K$-representations and the category of all compact linear-topological $o_K$-modules.

Let $G_0 \leq G$ be a compact open subgroup and $\Lambda(G_0)$ denote the completed group ring of the profinite group $G_0$ over $o_K$. Any smooth $o_K$-representation $V$ is the union of its finite $G_0$-subrepresentations, therefore $V^*$ is a left $\Lambda(G_0)$-module (through the inversion map on $G_0$).

From now on fix $n \in \mathbb{N}$, and let $G = \text{GL}_n(F)$.

Let $B$ be the set of upper triangular matrices in $G$ - a fixed Borel subgroup, $T$ the set of diagonal matrices - the maximal torus in $B$, $N$ the set of upper unitriangular matrices - the unipotent radical of $B$. Let $N^-$ be the lower unipotent matrices - the opposite of $N$ - and $N_0$ be a maximal compact subgroup of $N$ - with elements in $o_K$, define the following submonoid of $T$:

$$T_+ = \{t \in T | tN_0t^{-1} \subset N_0\} = \{\text{diag}(x_1, x_2, \ldots, x_n) | i > j : \nu_F(x_i) \geq \nu_F(x_j)\}.$$  

We have the following partial order on $T_+$: $t \leq t'$ if there exists $t'' \in T_+$ such that $tt'' = t'$.

Let $W \simeq N_G(T)/C_G(T)$ be the Weyl group of $G$, by the abuse of notation $w \in W$ be the permutation matrices - representatives of $W$ in $G$ (with $w_{ij} = 1 \iff w(j) = i$), and also let $w$ denote the corresponding permutation of the set $\{1, 2, \ldots, n\}$. For $w \in W$ denote length of $w$ - the length of the shortest word representing $w$ in the terms of the standard generators of $W$ - by $l(w)$.

Let $G_0 = \text{GL}_n(o_F)$, a compact open subgroup in $G$. Let the kernel of the projection $pr : G_0 \to \text{GL}_n(k_F)$ be $U^{(1)}$. This is a compact open pro-$p$ normal subgroup of $G_0$. We have $G = G_0B$ and $U^{(1)} \subset (N^- \cap U^{(1)})B$.

$k_F$ has canonical (multiplicative) injection to $o_F \subset F$, hence any subgroup of $H(k_F) \leq G(k_F)$ is mapped injectively to $G$ (however this is not a group homomorphism). We denote this subset of $G$ by $\overline{H}(k_F)$.

Let $C^\infty(G)$ denote the set of locally constant, $G \to k_K$ functions, with the group $G$ acting by left multiplication (by the inverses). Let

$$\chi = \chi_1 \otimes \chi_2 \otimes \cdots \otimes \chi_n : T \to k_K^*.$$
be a locally constant character of $T$ with $\chi_i : F^* \to k_K^*$, multiplicative. Note that then for all $i$ $\chi_i(1 + \pi_F o_F) = 1$ and $\chi_i(o_F^*) \subset k_F^* \cap k_K^* \leq \mathbb{F}_p^*$. Since $T = B/[B,B]$, also denote the corresponding $B \to k_K^*$ character by $\chi$. Let

$$V = \text{Ind}_B^G(\chi) = \{ f \in C^\infty(G) | \forall g \in G, b \in B : f(gb) = \chi^{-1}(b) f(g) \}$$

$V$ is called a principal series representation of $G$. $V$ is irreducible exactly when for all $i$ we have $\chi_i \neq \chi_{i+1}$ ([4], theorem 4).

We can understand the structure of $V$ better (see [6], section 4.), by the Bruhat decomposition $G = \bigcup_{w \in W} B w B$. Fix a total order $\prec_T$ refining the Bruhat order $\prec_B$ of $W$, and let

$$w_1 = \text{id}_W \prec_T w_2 \prec_T w_3 \prec_T \cdots \prec_T w_n! = w_0.$$ 

Let us denote by $G_m = \bigcup_{l \leq m} B w_l B$ - a closed subset of $G$. We obtain a descending $B$-invariant filtration of $V$ by

$$V_m = \text{Ind}_B^G(\chi) = \{ F \in \text{Ind}_B^G(\chi) | F \mid_{G \setminus G_w} \equiv 0 \} \quad (0 \leq m \leq n!),$$

with quotients $V_{m-1}/V_m$ via $f \mapsto f(\cdot w_m)$ isomorphic to $V(w_m, \chi) = C_c^\infty(N/N_{w_m}' \cap N)$, where $N_{w_m}' = N \cap w_m N w_m^{-1}$, with $N$ acting by left translations and $T$ acting via

$$(t \phi)(n) = \chi(w_m^{-1} t w_m) \phi(t^{-1} n t).$$

For any $w \in W$ put

$$N_w = \{ n \in N | \forall i < j, w^{-1}(i) < w^{-1}(j) : n_{ij} = 0 \} = N \cap wNw^{-1} \leq N,$$

and $N_{0,w} = N_0 \cap N_w$. Then we have the following form of the Bruhat decomposition $G = \bigsqcup_{w \in W} N_w w B$.

### 3 The action of $B_+$ on $G$

The first goal is to find a "nice" set of representatives of the double cosets $U^{(1)} \setminus G/B$. With these we can better understand the $B_+$-action on $V$, which is needed for the following parts.

**Definition** Let for any $w \in W$ $r_w : N^- \cap N_0 \to G(k_F), n^- \mapsto pr(wn^- w^{-1})$, $R_w = wr_w^{-1}(N_0(k_F))$, $R = \bigcup_{w \in W} R_w$ and $U_w = R_w B$. 

We have that
\[
R_w = \begin{cases} 
(a_{ij}) \in G | \forall i, j : a_{ij} 
= 1, & \text{if } w^{-1}(i) = j \\
= 0, & \text{if } w^{-1}(i) < j \\
\in o_F, & \text{if } w^{-1}(i) > j \text{ and } w(j) > i \\
\in \pi_F o_F, & \text{if } w^{-1}(i) > j \text{ and } w(j) < i
\end{cases}
\]

For \( n = 3 \) in details (with \( o = o_F \) and \( \pi = \pi_F \)):

| \( w \) | \( R_w \) | \( w \) | \( R_w \) |
|--------|--------|--------|--------|
| \( \text{id} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) | \( \begin{pmatrix} 1 & 0 & 0 \\ \pi o & 1 & 0 \\ \pi o & \pi o & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ \pi o & \pi o & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 1 & 0 \\ \pi o & o & 1 \\ \pi o & 1 & 0 \end{pmatrix} \) |
| \( (12) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \) | \( \begin{pmatrix} o & 1 & 0 \\ 1 & 0 & 0 \\ \pi o & \pi o & 1 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \) | \( \begin{pmatrix} o & o & 1 \\ \pi o & 1 & 0 \end{pmatrix} \) |
| \( (132) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} o & 1 & 0 \\ \pi o & 1 & 0 \end{pmatrix} \) | \( \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \) | \( \begin{pmatrix} o & o & 1 \\ \pi o & 1 & 0 \end{pmatrix} \) |

**Proposition 3.1** A set of double coset representatives of \( U^{(1)} \setminus G/B \) is \( \cup_{w \in W} \overline{N}_w(k_F)w \). Every element of \( G \) can be written uniquely in the form \( rb \) with \( r \in R \) and \( b \in B \).

**Proof** By the Bruhat decomposition of \( G(k_F) \) a set of double coset representatives of \( U^{(1)} \setminus G_0/B \) is the set as above. Since \( G = G_0B \), we have the first part of proposition.

Let \( g = unwb \in G \) with \( u \in U^{(1)} \), \( w \in W \), \( n \in \overline{N}_w(k) \) and \( b \in B \). Then \( g = w(w^{-1}nw)u'b \) with \( u' = w^{-1}n^-1uw \in U^{(1)} \). But then there exist \( n' \in N^- \cap U^{(1)} \) and \( b' \in B \) such that \( u' = n'b' \). Then \( g = w(w^{-1}nwn')b'b' \), where \( w^{-1}nwn' \in r^{-1}_w(N_0(k_F)) \) because of the definition of \( N_w \).

It is clear that for any \( w \in W U^{(1)}N_w(k)B = R_wB \). Hence the uniqueness follows: if \( rb = r'b' \) then there exists \( w \in W \) such that \( r, r' \in R_w \) and \( b'b^{-1} = (r^{-1}w^{-1})(wr) \in B \cap N^- = \{1\} \).

**Corollary 3.2** For any \( w \in W \) let \( U_w = U^{(1)}\overline{N}_w(k_F)wB \). This way we partitioned \( G \) into open subsets indexed by the Weyl group. We obviously have \( U_w = R_wB \).
Proof Let $n' \in N_0$ and $x = unwb \in U(1)\overline{N}_w(k_F)kB$. We have $N_0 = N_{0,w}(N'_w \cap N_0)$, thus $n'n = mm'$ for some $m \in N_{0,w}$ and $m' \in N'_w \cap N_0$, moreover we can write $m = m_1m_0 \in (N_w \cap U(1))\overline{N}(k_F)$. By the definition of $N'$

$$n'x = (n'un'^{-1}m_1)m_0w(w^{-1}m'wb) \in U(1)\overline{N}(k_F)B,$$

meaning that $U_w$ is $N_0$-invariant.

$\text{Ind}_B^{U_{w_0}}(\chi)$ is the minimal generating $B_+$-subrepresentation of the Steinberg representation $\text{Ind}_B^{B_{w_0}B}(\chi)$.

Remark For $n \leq 3$ it is true that if $x \in U_w, t_+ \in T_+, n_0 \in N$ and $t_+^{-1}n_0^{-1}x \in U_w$, then $w' \preceq_B w$. For $n = 4$ it is not true: (again with $\pi = \pi_F$)

$$\begin{pmatrix} \pi^{-1} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -\pi^{-1} & 0 & \pi^{-1} \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -\pi & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

but $(12)(34) \not\in_B (1345)$.

Later it will be useful to know better the partial order induced by the $B_+$-action, so we need this:

Definition Let $w, w' \in W$. We say $w' < w$ if there exist transpositions $w_1, w_2, \ldots, w_i \in W$ such that $w' = w_1w_2\ldots w_i$ and $l(w) > l(w_1w) > l(ww_1w_2) > \cdots > l(ww_1w_2\ldots w_i)$.

In the Bruhat order the $w_j$-s had to correspond to simple roots, but we could multiply from the left as well. But on $W$ multiplying from the left with a simple transposition can be written as a multiplication from the right with a (not necessarily simple) transposition. So $<$ is a finer ordering than Bruhat, but less finer than the length.

Proposition 3.3 Let $y \in R_wB$, $nt \in B_+ = N_0T_+$, and $x = t^{-1}n^{-1}y \in R_{w'}B$. Then $w' \preceq_B w$. 
Proof Let $y = rb$ with $r \in R_w$ and $b \in B$. By the previous proposition we might assume that $n = \text{id}$. If $t = \text{diag}(t_1, t_2, \ldots, t_n) \in U^{(1)}$, then
\[
x = w(w^{-1}t^{-1}w(w^{-1}r)w^{-1}tw)(w^{-1}t^{-1}wb),
\]
where $w^{-1}t^{-1}w(w^{-1}r)w^{-1}tw \in \tau^{-1}(N_0(k_F))$, because modulo $\pi_F$ it is $w^{-1}r$ and it is in $N^-$. $T_+$ as a monoid is generated by $T \cap U^{(1)}$, $Z(G)$ and the elements with the form $(\pi_F, \pi_F, \ldots, \pi_F, 1, 1, \ldots, 1)$, hence it is enough to prove the proposition for such $t$-s.

So fix $t = (t_1 = \pi_F, t_2 = \pi_F, \ldots, t_i = \pi_F, t_{i+1} = 1, t_{i+2} = 1, \ldots, t_n = 1), r = (r_{ij})$ and try to write $x$ in the form as in Proposition 3.1. For all $j = 0, 1, 2, \ldots, n$ we construct inductively a decomposition $x = (t^{(j)})^{-1}r^{(j)}b^{(j)}$ together with $w^{(j)} \in W$, where

- with $w^{(j)} \preceq w^{(j+1)}$ for $j < n$ and such that the first $j$ columns of $w^{(j)}$ is the same as the first $j$ columns of $w^{(j+1)}$,
- $t^{(j)} = \text{diag}(t^{(j)}_i) \in T$ with
  \[
t^{(j)}_i = \begin{cases} 
1, & \text{if } (w^{(j)})^{-1}(i) \leq j \\
t_i, & \text{if } (w^{(j)})^{-1}(i) > j
\end{cases}
\]
- $r^{(j)} \in R^{w(j)}$, and if we change the first $j$ columns of $r^{(j)}$ to the first $j$ columns of $(t^{(j)})^{-1}r^{(j)}$ it is still in $R^{w(j)}$ (by definition of $t^{(j)}$, it might be in $R^{w(j)}$, moreover it is enough to verify this second condition),
- $b^{(j)} \in B$.

Then $w' = w^{(n)} \preceq w^{(n-1)} \preceq w^{(n-2)} \preceq \cdots \preceq w^{(1)} = w$, hence we have the proposition.

For $j = 0$ we have $t^{(0)} = t, r^{(0)} = wr, b^{(0)} = b$ and $w^{(0)} = w$. From $j$ to $j + 1$:

- If $w^{(j)}(j+1) \leq l$, then let $w^{(j+1)} = w^{(j)}$, so $t^{(j+1)} = e^{(j+1)}_w$ and $r^{(j+1)} = e^{(j+1)}_{w(j+1)}$, where $e_l$ is the diagonal matrix with $\pi_F$ in the $l$. row and 1 everywhere else. We can choose $r^{(j+1)} = e^{(j+1)}_{w(j+1)}r^{(j)}e_{j+1}$, and $b^{(j+1)} = e^{(j+1)}_{j+1}b^{(j)}$.

Then the first $j$ column of $(t^{(j+1)})^{-1}r^{(j+1)}$ is equal of those of $(t^{(j)})^{-1}r^{(j)}$, and the not $w^{(j+1)}$, elements of the $j + 1$. column are multiplied by $\pi_F$. Because of the conditions for $r^{(j)}$, this is in $R^{w(j+1)}$. The other conditions for $w^{(j+1)}, t^{(j+1)}, r^{(j+1)}$ and $w^{(j+1)}$ obviously hold.
- If $w^{(j)}(j + 1) > l$ and for all $i \leq l$ $\nu_F(r_{i, j+1}) \geq 1$, then it suffices to choose $w^{(j+1)} = w^{(j)}$, $r^{(j+1)} = r^{(j)}$ and $b^{(j+1)} = b^{(j)}$.

- If $w^{(j)}(j + 1) > l$ and exists $i \leq l$ such that $\nu_F(r_{i, j+1}) = 0$. Let $i_0$ be the maximal such $i$. Then we must have $w^{(j+1)}(j + 1) = i_0$, and $t^{(j+1)} = e_{i_0}^{-1}t^{(j)}$.

Let $r' = e_{i_0}^{-1}r^{(j)}((r_{i_0, j+1})^{-1}, e_{j+1})$. Then $(t^{(j+1)})^{-1}r'$ is not in $GL_n(o)$ - for example $\nu_F(r'_{i_0, (w^{(j)})^{-1}(i_0)}) = -1$, and there might be some other elements of $r'$ in the $i_0$, row and columns between the $j + 2$, and $j' = (w^{(j)})^{-1}(i_0)$. However it satisfies the condition for the first $j + 1$ columns of $r^{(j+1)}$. So we want to find $r^{(j+1)} = r'b'$ with $b' \in B$ such that the first $j + 1$ columns of $b'$ is those of the identity matrix, and $(t^{(j+1)})^{-1}r^{(j+1)} \in S_{w^{(j+1)}}$ with $w^{(j)} \leq w^{(j+1)}$.

Let $j_0 = j + 1$, and if $j_i < j'$ then

$$j_{i+1} = \min\{h| j + 1 < h, r_{i_0, h} \notin o_F, w^{(j)}(j_i) > w^{(j)}(h)\},$$

and $s$ such that $j_{s} = j'$. We claim that $r^{(j+1)}$ will be in $R_{w^{(j)}}$ with $w^{(j+1)} = w^{(j)}(j_1, j_2, j_3) \ldots (j_{s-1}, j_{s})$ (and then the conditions for $w^{(j+1)}$ trivially hold).

Let $r_0 = r'$, and if $i \leq s$ and then define $r_{i+1} = r_i b^{(i)}$ with

$$b^{(i)}_{i, j} = \begin{cases} \delta_{i, j}, & \text{if } I \neq j_i \text{ and } J \neq j_{i+1}, \\ 0, & \text{if } I = j_i \text{ and } (J - j_i)(j_{i+1} - J) < 0, \\ -(r_i)_{j_i, j}, & \text{if } I = j_i \text{ and } j_i \leq J < j_{i+1}, \\ 0, & \text{if } J = j_{i+1} \text{ and } 2i' \leq i + 1 : I = j_{i'}, \\ (-1)^{i-i'}(\prod_{i'=1}^{i-1}(r_i)_{w^{(j)}(j_{i'}, j_{i'})}^{-1})^{-1}, & \text{if } J = j_{i+1} \text{ and } \exists i' < i + 1 : I = j_{i'}, \\ (-1)^i(\prod_{i'=1}^{i-1}(r_i)_{w^{(j)}(j_{i'}, j_{i'})}^{-1}), & \text{if } J = j_{i+1} \text{ and } I = j_{i+1}, \end{cases}$$

where $\delta_{i, j}$ is 1 if $I = J$ and 0 otherwise. (when multiplying $r_i$ with $b^{(i)}$, the first $j_i$ column is fixed, and in the mean time the multiplication of $w^{(j)}$ with $(j_i, j_{i+1})$ is done)

Now we can choose $b' = b^{(s-1)}b^{(s-2)} \ldots b^{(0)}$, $r^{(j+1)} = r'b'$ and $b^{(j+1)} = b'^{-1}e_{j_{i+1}}^{-1}b^{(j)}$. Then every condition is satisfied.

**Corollary 3.4** For any $m_0 \leq n!$ we have that

$$\bigcup_{m \geq m_0} U_{w_m} \subset G \setminus G_{m_0+1} = \bigcup_{m \geq m_0} B w_m B.$$
Remark We can achieve the results of this section not only for GL\(_n\), but different groups: let \(G'\) be such that

- \(G'\) is isomorphic to a closed subgroup in \(G\) which we also denote by \(G'\),
- In \(G'\) a maximal torus is \(T' = T \cap G'\), a Borel subgroup \(B' = B \cap G'\) with unipotent radical \(N' = N \cap G'\), such that \(N_{G'}(T') = N_G(T) \cap G'\) and hence \(W' \leq W\) with \(w_0 \in W'\), with representatives \(w'\) of \(W'\) in \(G'\) can be written in the form \(w = w't\) such that \(t \in T \cap G_0\).
- \(G_0' = G_0 \cap G'\) with \(G' = G_0'B'\) and
- \(U'(1) = U(1) \cap G'\) such that \(U'(1) \subset (N' \cap U'(1))B'\) for \(N' = w_0N'w_0\).

For example these conditions are satisfied for the group \(SL_n\).

The proof of the first proposition works for such \(G'\), and from a decomposition \(x = r'b \in R'_wB' \subset G'\) we get some \(r \in R_w\) and \(b \in B\) such that \(x = rb \in G\). Hence the restriction of \(\prec\) to \(W'\) is an appropriate ordering for the \(B'_+\)-action on \(G'\).

4 Generating \(B_+\)-subrepresentations

For any torsion \(o_K\)-module \(X\) denote the (partially ordered) set of generating \(B_+\)-subrepresentations of \(X\) (those \(M \subset X\) \(B_+\)-submodules, for which \(BM = X\)) by \(\mathcal{B}_+(X)\).

Proposition 4.1 Let \(X\) be a smooth admissible and irreducible torsion \(o_K\)-representation, which is also a \(k_K\)-vector space. Then \(M_0 = B_+X^{U(1)}\) is a generating \(B_+\) subrepresentation of \(X\). For any \(M \in \mathcal{B}_+(X)\) there exists a \(t_+ \in T_+\) such that \(t_+M_0 \subset M\).

Proof Clearly \(M_0\) is a \(B_+\)-subrepresentation, and also a \(G_0\)-subrepresentation (because \(U(1) \vartriangleleft G_0\), with \(G\)-compatible action. \(G_0B = BG_0 = G\), so \(BM_0\) is a \(G\)-subrepresentation of \(X\). \(M_0\) is not \(\{0\}\), since \(U(1)\) is pro-\(p\) and since \(X\) is irreducible \(BM = X\), hence \(M\) is generating.

\(X\) is admissible, hence \(X^{U(1)}\) has a finite generating set, say \(R\). Let \(M\) as in the proposition. For any \(r \in R\) \(\exists t_r \in T_+\) such that \(t_r r \in M\) ([3], lemma 2.1). The number of \(r\)-s is finite, hence exists \(t_+ \in T_+\) such that \(\forall r : t_r^{-1} t_+ \in T_+\), and then \(t_+ M_0 \subset M\).
From now on let \( V = \text{Ind}_G^M(\chi) \) as before and \( M_0 = B_+ V^{U(1)} \). Then \( V^{U(1)} \) (as a vector space) is generated by

\[
f_r : \left\{ \begin{array}{ll}
\text{urb} & \mapsto \chi^{-1}(b) \\
y \neq \text{urb} & \mapsto 0
\end{array} \right.
\quad \left( r \in U^{(1)} \setminus G/B = \bigcup_{w \in W} N(k_F)w \right).
\]

For \( G = \text{GL}_n \) and \( V \) irreducible, \( V^{U(1)} = \text{Soc}_G(V) \) - the socle of \( V \) as a \( G_0 \)-representation.

If we denote the coset \( U^{(1)}wB \) also with \( w \), then \( V^{U(1)} \) is generated by \( \{f_w | w \in W\} \) as an \( N_0 \)-module. Hence any \( f \in M_0 \) can be written in the form \( \sum_{i=1}^a \lambda_i n_i t_i f_w \) for some \( \lambda_i \in k_K, n_i \in N_0, t_i \in T_+ \) and \( w_i \in W \).

**Proposition 4.2** \( M_0 \) is minimal in \( B_+(V) \).

**Proof** By the previous proposition, it is enough to show, that for any \( t' \in T_+ \) we have \( M_0 \subset B_+ t'M_0 \).

If \( t' \in U^{(1)} \), then \( t'ntf_w = (t'nt^{t^{-1}})tt'f_w = (t'nt^{t^{-1}})tf_w \), since \( f_w \) is \( U^{(1)} \)-invariant. \( t'nt^{t^{-1}} \in N_0 \), and we have \( t'M_0 = M_0 \). The same is true for \( t \in Z(G) \). So it is enough to prove for \( t' = (\pi_F, \pi_F, \ldots, \pi_F, 1, 1, \ldots, 1) \), that \( M_0 \subset B_+ t'M_0 \).

Let \( j_0 \in \mathbb{N} \) be such that \( t'_{j_0} = \pi_F \) and \( t'_{j_0+1} = 1 \). We need to show, that for all \( w \in W \) \( f_w \in B_+ t'M_0 \). We prove it by induction on \( w \) with respect to \( \prec \).

Let us denote \( N_{j_0}^{U(1)} = \{n \in N \cap U^{(1)} | \forall i < j, (j_0 - i)(j - j_0) < 0 : n_{ij} = 0\} \), \( N_{w,j_0} = N_w \cap N_{j_0}^{U(1)} \) and

\[
\Theta_{w,j_0} = \{\text{a set of representatives of } N_{w,j_0}/t'N_{w,j_0}t^{-1} \} \subset N_0 \cap U^{(1)}.
\]

It is enough to prove the following:

**Lemma 4.3** Let \( g = \sum_{m \in \Theta_{w,j_0}} mt'f_w \). Then \( \chi(w^{-1}t'w)f_w - g \) is in \( \sum_{w',w<j_0} N_0f_{w'} \).

It is easy to verify that for \( r \in R_w \) we have

\[
t'f_w(r) = \left\{ \begin{array}{ll}
\chi(w^{-1}t'w), & \text{if } \forall i \leq j_0 < j, w^{-1}(i) > w^{-1}(j) : r_{ij} \in \pi_F^2 o_F, \\
0, & \text{otherwise}.
\end{array} \right.
\]

and therefore \( \chi(w^{-1}t'w)f_w|_{U_w} = \sum_{m \in \Theta_{w,j_0}} mt'f_w|_{U_w} \). Hence by the induction hypothesis and Proposition 3.3 it suffices to prove that \( g \) is \( U^{(1)} \)-invariant.
To do that, first notice that since $f_w$ is $U^{(1)}$-invariant, we have that $t'f_w$ is $t'U^{(1)}t'^{-1}$-invariant. Moreover, since for all $m \in \Theta_{w,j_0}$ we have $m \in N_0 \cap U^{(1)} = t'N_0t'^{-1}$, $m$ normalizes $t'U^{(1)}t'^{-1}$, $mt'f_w$ is also $t'U^{(1)}t'^{-1}$-invariant, and so is $g$.

On the other hand, we can write

$$g = \sum_{m \in \Theta_{w,j_0}} mt'f_w = \sum_{m \in \Theta_{w,j_0}} t'(t'^{-1}mt')f_w = t' \left( \sum_{n \in t'^{-1}N_{w,j_0}t'/N_{w,j_0}} nf_w \right),$$

where the sum in the bracket on the right hand side is obviously $t'^{-1}N_{w,j_0}t'$-invariant, hence $g$ is $N_{w,j_0}$-invariant.

Denote $N'_{w,j_0} = N'_{w} \cap N^{(1)}_{j_0}$. Then $N_{w,j_0}$ centralizes $t'^{-1}N'_{w,j_0}t'$; let $n_0 = I + m_0 \in t'^{-1}N'_{w,j_0}t'$, $n \in N_{w,j_0}$,

$$(n^{-1}n_0n - n_0)_{xy} = (n^{-1}m_0n - m_0)_{xy} = \sum_{x \leq s \leq t \leq y} (n^{-1})_{xs}(m_0)_{st}n_{ty} - (m_0)_{xy},$$

and by the definition $N^{(1)}_{j_0}$, $(m_0)_{st}$ is 0, unless $s \leq j_0 < t$ and hence $(n^{-1})_{xs}n_{st}n_{ty} = 0$, unless $x = s$ and $y = t$.

By the definition of $N'_{w,j_0}$ we have $w^{-1}N'_{w,j_0}w \subset B$, so for any $u \in U^{(1)}$ and $n_0 \in t'^{-1}N'_{w,j_0}t' \subset G_0$ we have $n_0uw = (n_0un_0^{-1})w(w^{-1}n_0w) \in U^{(1)}wB$, and hence $f_w$ is $t'^{-1}N'_{w,j_0}t'$-invariant.

Altogether for any representative $n \in \Theta_{w,j_0}$

$$nf_w(n_0x) = f_w(n^{-1}n_0x) = f_w(n_0n^{-1}x) = f_w(n^{-1}x) = nf_w(x),$$

meaning that $nf_w$ is $t'^{-1}N'_{w,j_0}t'$-invariant, and $t'nf_w$ is $N'_{w,j_0}$-invariant. So $g$ is also $N'_{w,j_0}$-invariant.

$U^{(1)}$ is contained in $\langle t'U^{(1)}t'^{-1}, N_{w,j_0}, N'_{w,j_0} \rangle$, so $g$ is $U^{(1)}$-invariant, and we are done.

**Corollary 4.4** For any $f \in M_0$ there exists $t \in T_+$ such that $f$ can be written in form $\sum_{i=1}^{s} \lambda_i n_itf_{w_i}$ for some $\lambda_i \in k_K$, $n_i \in N_0$ and $w_i \in W$.

It is obvious from the proof.

**Remark** $V$ is the modulo $\pi_K$ reduction of the $p$-adic principal series representation. This can be done with any $l \in \mathbb{N}$ for the modulo $\pi_K^l$ reduction. Then the $\pi_K$-torsion part of the minimal generating $B_+$-representation is exactly $M_0$.
Remark This can be carried out in the same way for groups $G'$ as in the previous section satisfying moreover $N_0 \subset G'$. For example $G' = \text{SL}_n$ has this property.

5 The Schneider-Vigneras functor

Following Schneider and Vigneras ([5], section 2) we introduce the functor $D$ from torsion $o_K$-modules to modules over the Iwasawa algebra of $N_0$.

Let us denote the completed group ring of $N_0$ over $o_K$ by $\Lambda(N_0)$, and define

$$D(X) = \varinjlim_{M \in B_+(X)} M^*,$$

as an $\Lambda(N_0)$-module (equipped with a natural $T_+^{-1}$-action).

On $D(V)$ the action of $\pi_K$ is 0, hence we can view it as a $\Omega(N_0) = \Lambda(N_0)/\pi_K \Lambda(N_0)$-module.

**Theorem 5.1** The $\Omega(N_0)$-module $D(V)$ is finitely generated and has rank 1.

**Proof** The minimality of $M_0 \in B_+(V)$ gives $D(V) = M_0^*$.

Define $M_{0,m} = \sum_{m' > m} B_+ f_{w_{m'}} \leq \text{Ind}_{B}^{G_n}(\chi)$. We obtain a descending filtration $M_0 = M_{0,0} \geq M_{0,1} \geq \cdots \geq M_{0,n!} = 0$.

The dual gives a series of maps

$$M_0^* \twoheadrightarrow M_{0,1}^* \twoheadrightarrow M_{0,2}^* \twoheadrightarrow \cdots \twoheadrightarrow M_{0,n!-1}^* \twoheadrightarrow 0,$$

where by the exactness of the (modulo $\pi_K$) Pontrjagin dual we have the following exact sequences:

$$0 \rightarrow (M_{0,m-1}/M_{0,m})^* \rightarrow M_{0,m-1}^* \rightarrow M_{0,m}^* \rightarrow 0.$$

Since $\Lambda(N_0)$ is noetherian, it suffices to prove, the following:

**Lemma 5.2** We have that $(M_{0,m-1}/M_{0,m})^*$ is a finitely generated $\Omega(N_0)$-module, which is torsion if $m < n! - 1$ and has rank 1 for $m = n! - 1$.

Let $w = w_{m-1}$. We have that $B_+ f_w \twoheadrightarrow M_{0,m-1}/M_{0,m}$, hence by Lemma 4.3

$$\tilde{M}_w = \lim_{t \in T_+} k_k[N_0] tf_w \twoheadrightarrow M_{0,m-1}/M_{0,m}.$$
The dual is

$$\left( M_{0,m-1}/M_{0,m} \right)^* \leq \tilde{M}_w^* = \lim_{t \in T_+} (k_K[tf_w]^*).$$

For each $t \in T_+$ as a $k_K$-vector space $(k_K[tf_w]^*)$ is finite dimensional and hence has finite cardinality, since both $k_K$ and $N_0/t(N_0 \cap U(1))t^{-1}$ are finite and $ntf_w = n'tf_w$ already in $M_0$ if $n^{-1}n' \in t(N_0 \cap U(1))t^{-1}$. And $tf_w$ generates it as a $\Omega(N_0)$-module. The projective limit of finite modules, which are generated by one element is also generated by one element, say $\mathbb{1}_w$. Hence $(M_{0,m-1}/M_{0,m})^*$ has rank at most 1.

Finally, to show that it is torsion, it is enough to find an element $L \neq 0$ in $(M_{0,m-1}/M_{0,m})^*$ which is torsion. For $m < n! - 1$ we have $\{0\} \neq N'_w - 1 \subset \Omega(N_0)$. Let $L : f \mapsto f(w)$ the evaluation in $w$, then it is trivial that $(N'_w - 1)L = 0$.

For $m = n! - 1$, we have that $M_{n!-1} = M(w_{n!-1}, \chi) = C^\infty(N_0)$, so $M_{n!-1}^* = C^\infty(N_0)^* = \Omega(N_0)$.

**Remark** $D(V)$ is obviously not an étale $\Lambda(N_0)$-module (in the sense of [5]), because it has a non-trivial torsion submodule. However, we have that $D(V)/D(V) - \text{tors} = \Omega(N_0) = D(V_{n!-1})$, which is étale.

By [7] section 4, $D^0(V_{n!-1}) = D(V_{n!-1})$ and for $i > 0$ $D^i(V_{n!-1}) = 0$.

## 6 GL\(_n\)(\(\mathbb{Q}_p\)) with \(n > 2\)

In this section we point out some properties of $M_0$, which makes the picture more difficult than the well known case of $n = 2$. Some of these require a lot of computation, thus some of the proofs is quite sketchy here.

For $n = 2$ we have $M_0 \cap V_{n!-1} = M_{0,n!-1}$. For $n > 2$ that is not true:

**Proposition 6.1** Let $n = 3$, $F = \mathbb{Q}_p$, then $M_0 \cap V_{n!-1} \supseteq M_{0,n!-1}$.

**Proof** Assume that $\chi = \chi_1 \otimes \chi_2 \otimes \chi_3 : T \to k_K^*$ is a character, such that neither $\chi_1/\chi_2$, nor $\chi_2/\chi_3$ is trivial on $\mathfrak{o}_K^*$. Similar construction can be carried out in the other cases.

Let $\prec_T$ be the following total order of the Weyl group of $G$ refining the Bruhat order:

$$w_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \prec_T w_2 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \prec_T w_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \prec_T$$
\( \langle T \mathbf{w}_4 \rangle = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \), \( \langle T \mathbf{w}_5 \rangle = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \), \( \langle T \mathbf{w}_6 \rangle = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \mathbf{w}_0. \)

And let

\[
 h = \sum_{a=0}^{p^2-1} p^2 b = (a b) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} f_{w_2} \in M_0,
\]

\[
 f = h - \frac{1}{\chi_3(p^2)} \sum_{a=0}^{p^3-1} p^3 b = (a b 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} f_{w_5}.
\]

Then it is easy to verify that \( f \in M_0 \cap V_5 \), and that \( f(z) \neq 0 \) for

\[
 z = \begin{pmatrix} p^2 & 0 & 1 \\ 1 & 0 & 0 \\ p & 1 & 0 \end{pmatrix} \in Bw_0B \setminus N_0w_0B.
\]

However, if \( f \in M_0 \cap V_5 \) then \( \text{supp}(f) \) is contained in \( Bw_0B \cap \bigcup_{i>3} R_iB \): A straightforward computation shows that for any \( n \in N_0, t \in T_+, w \in W \) and

- for any \( r \in R_{w_1} \) we have \( nt f_w(r) = nt f_w(w_1) \). Let \( r' = w_1 \in G_5 \),

- for any \( r \in R_{w_2} \) we have \( nt f_w(r) = nt f_w(r') \) for

\[
 r' = \begin{pmatrix} \alpha & 1 & 0 \\ 1 & 0 & 0 \\ \beta' & 0 & 1 \end{pmatrix} \in G_5, \text{ where } r = \begin{pmatrix} \alpha & 1 & 0 \\ 1 & 0 & 0 \\ \beta' & \gamma & 1 \end{pmatrix},
\]

- for any \( r \in R_{w_3} \) we have \( nt f_w(r) = nt f_w(r') \) for

\[
 r' = \begin{pmatrix} 1 & \alpha' - \beta \gamma & 0 \\ 0 & \gamma & 1 \\ 0 & 1 & 0 \end{pmatrix} \in G_5, \text{ where } r = \begin{pmatrix} 1 & 0 & 0 \\ \alpha' & \gamma & 1 \\ \beta' & 1 & 0 \end{pmatrix}.
\]

Thus if \( i<4 \) and \( r \in R_{w_1} \), then since \( r' \notin Bw_0B \), \( f(r) = f(r') = 0. \)
Now we compare the dual filtration of $D(V)$ with the $D(V_{m-1}/V_m)$, where the $V_m$-s are the sub-$B$-modules of the Bruhat-filtration. Recall (§12) that $V_{m-1}/V_m \simeq V(w_m, \chi)$, which has a minimal generating $B_+$-subrepresentation

$$M(w_m, \chi) = C^\infty(N_0/N_{w_m} \cap N_0) \in B_+(V(w_m, \chi)).$$

**Proposition 6.2** The quotients $M_{0,m-1}/M_{0,m-1} \cap V_m$ via $f \mapsto f(w_m)$ are isomorphic to $M(w_m, \chi)$.

**Proof** It is obvious, that $f(w_m) = 0$ implies $f|_{G_m \setminus G_{m-1}} = 0$ and $f \in M_{0,m-1} \cap V_m$. Hence the map $M_{0,m-1}/M_{0,m-1} \cap V_m \to M(w_m, \chi)$, $f \mapsto f(w_m)$ is injective.

Let $t_0 = \text{diag}(\pi_F^{-1}, \pi_F^{-2}, \ldots, \pi_F, 1) \in T_+$, and for any $l \in \mathbb{N}$ let $U^{(l)} = \text{Ker}(G_0 \to G(\alpha_F/\pi_F^l \alpha_F))$. For $x = rb \in R_{w_m} B$ we have

$$\sum_{n \in (N_0 \cap U^{(l)})/N_{0}^{(l)}} nt_0^{l} f_{w_m}(rb) = \begin{cases} \chi^{-1}(b), & \text{if } r \in U^{(l)} w_m, \\ 0, & \text{if not.} \end{cases}$$

The image of these generate $M(w_m, \chi)$ as an $N_0$-module, so $f \mapsto f(w_m)$ is surjective.

Since $M_{0,m} \leq V_m$, $M(w_m, \chi)$ is naturally a quotient of $M_{0,m-1}/M_{0,m}$ and $D(V_{m-1}/V_m) \leq (M_{0,m-1}/M_{0,m})^*$. 

**Proposition 6.3** For $m = 1$ and $m = n! - n + 1, n! - n + 2, \ldots, n!$ $(M_{0,m-1}/M_{0,m})^* = D(V_{m-1}/V_m)$. For other $m$-s it is not true, for example if $n = 3$, $F = \mathbb{Q}_p$ and $m = 2, 3$.

**Proof** By the previous proposition it is enough to show that $M_{0,m} = M_{0,m-1} \cap V_m$ for $m = 1$ and $m > n! - n$.

For $m = 1$ the quotient is obviously $k_K$, for $m > n! - n$ we have $w < w_m$ implies $w = w_n!$, so if $f \in B_{+} f_w m \cap V_m = B_{+} f_w m \cap V_n$, then $\text{supp}(f) \subset U^{(1)} R_{w_{n!-1}}^{(1)} B$. But

$$M_{0,m!-1} \simeq C^\infty(N_0) \simeq \{ f \in V_{n!-1} | \text{supp}(f) \subset U^{(1)} R_{w_{n!-1}}^{(1)} B \}.$$ 

The function $f$ constructed in the beginning of this section is in $M_{0,1} \cap V_2 \setminus M_{0,2}$. The same can be done for $m = 3$. 

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Proposition 6.4 For \( n = 3, m = 2, 3 \) we even have that \( D(V_{m-1}/V_m) \) has a global annihilator, but \((M_{0,m-1}/M_{0,m})^* \) has not.

**Proof** \( D(V_{m-1}/V_m) \simeq (C^\infty(N_0/N'_w \cap N_0))^* \), where the right hand side is generated by one element: the evaluation at \( I \in N_0 \). It is easy to verify that this is annihilated by \( Z(N_0) - 1 \), hence \( (Z(N_0) - 1)D(V_{m-1}/V_m) = 0 \).

Let \( w = w_{m-1} \). After choosing a special basis of \( kK[N_0]tf_w \) there can be found an explicit generator \( 1_w \in \tilde{M}_w^* \) which is not annihilated by the \( Z(N_0) - 1 \):

For each \( t \in T_+ \) the elements of the set

\[
S_{w,t} = \{ nt f_w | n \in N_w / t(N_w \cap U^{(1)})t^{-1} \} \in M_{0,m-1}
\]

are \( kK \)-independent in \( M_{0,m-1}/M_{0,m} \), because for such \( n \)-s \( \text{supp}(nt f_w) \cap R_w B \) are disjoint, so even modulo \( M_{0,m} \) they are \( kK \)-independent. Fix a basis \( B_{w,t} \) of \( kK[N_0]tf_w \) complementing \( S_{w,t} \).

We can show that \( (1/\chi(w^{-1}tw)(tf_w)^*)_t \), where \( (tf_w)^* \in B_{w,t}^* \), constitute a compatible system in \( \tilde{M}_w^* \), and then it trivially generates the whole \( (M_{0,m-1}/M_{0,m})^* \). Thus let \( 1_w = (1/\chi(w^{-1}tw)(tf_w)^*)_t \).

Then it is easy to verify that

\[
\left(
\begin{pmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
- 1
\right)1_w \neq 0.
\]

**Remark** It is true that if \( (N_\alpha \cap N_0 - 1)1_w = 0 \), where \( \alpha \) is the simple root such that \( N_\alpha \subset N'_w \). Both this and the compatibility \( (1/\chi(w^{-1}tw)(tf_w)^*)_t \), are consequences of the following: for \( n \in N'_w \cap N_0 \) we have that \( nt f_w - tf_w \in M_{0,m} \).

This is not true in general, only for \( m = 2, 3, \ldots, n \).

However, in general I hope that it is possible to choose the basis \( B_{w,t} \) conveniently - meaning that \( (1/\chi(w^{-1}tw)(tf_w)^*)_t \) is compatible, and annihilated by \( N_\alpha \cap N_0 - 1 \) for any \( \alpha \) simple root such that \( N_\alpha \subset N'_w \).
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