Single Charged Higgs production as a probe of CP violation at a Muon Collider

A.G. Akeroyd and S. Baek

KEK Theory Group, Tsukuba,
Ibaraki 305-0801, Japan

Abstract

We consider single charged Higgs ($H^\pm$) production in association with a $W^\pm$ boson at $\mu^+\mu^-$ colliders, in the context of the general CP violating Two Higgs Doublet Model (2HDM). We find that large cross-sections for the processes $\mu^+\mu^- \rightarrow H^+W^-, H^-W^+$ are possible, and offer an attractive way of producing $H^\pm$ at $\mu^+\mu^-$ colliders. The difference in the cross-sections for $H^+W^-$ and $H^-W^+$ may exceed 1000 fb, and this represents a novel way of probing CP violation in the Higgs sector.
1 Introduction

Charged Higgs bosons ($H^\pm$) are predicted in many extensions of the Standard Model (SM), in particular the Minimal Supersymmetric Standard Model (MSSM). Their phenomenology [1] has received much attention both at $e^+e^-$ colliders [2] and at hadron colliders [3], [4], [5]. At $e^+e^-$ colliders production proceeds via the mechanism $e^+e^- \rightarrow \gamma^*, Z^* \rightarrow H^+H^-$, with higher order corrections evaluated in [6], and detection is possible for $M_{H^\pm}$ up to approximately $\sqrt{s}/2$. The combined null–searches from all four LEP collaborations derive the lower limit $M_{H^\pm} \geq 77.3$ GeV (95\% c.l) [7].

In recent years an increasing amount of work has been dedicated to the physics possibilities of $\mu^+\mu^-$ colliders [8], [9]. Such colliders offer novel ways of producing Higgs bosons, and much attention has been given to the study of neutral Higgs bosons produced as resonances in the s-channel [10], [11]. The phenomenology of $H^\pm$ at $\mu^+\mu^-$ colliders has previously been considered to be more or less identical to that at $e^+e^-$ colliders. This is because the pair production processes of $e^+e^-, \mu^+\mu^- \rightarrow H^+H^-$ have been assumed to have the same rate at both colliders. This is the case in the MSSM, where the Higgs mediated s-channel diagrams have been shown to be negligible at a $\mu^+\mu^-$ collider [12]. The single production of $H^\pm$ via the process $e^+e^- \rightarrow H^\pm W^\mp$ [13], which proceeds dominantly via loops, has relatively small rates. An analysis in the context of the LHC has been covered in [14]. At a muon collider this process can have a much larger cross-section because the tree-level diagrams, which are suppressed by $m_e^2$ in the $e^+e^-$ case, are proportional to $m_\mu^2$, and become by far the dominant contribution.

The mechanism $\mu^+\mu^- \rightarrow H^\pm W^\mp$ was first considered in [13] and subsequently developed in [12]. It possesses several advantages over the conventional pair production process, $\mu^+\mu^- \rightarrow H^+H^-$. In particular, $H^\pm$ may be produced on-shell for $M_{H^\pm} \leq \sqrt{s} - M_W$, which compares favourably with the kinematic reach for pair production ($M_{H^\pm} \leq \sqrt{s}/2$). In addition, backgrounds are expected to be relatively small, since for $H^\pm \rightarrow t\bar{t}$ decays the main background would be from $\mu^+\mu^- \rightarrow t\bar{t}$ production which has a cross-section of 700 fb at $\sqrt{s} = 500$ GeV. In [12] an analysis in the context of the MSSM showed that sizeable cross-sections ($\geq 20$ fb) can be attained for $\tan\beta \geq 40$. In this paper we consider the general (non-SUSY) Two Higgs Doublet Model (2HDM), which has the added advantage of allowing CP-violation in the tree-level Higgs potential. In contrast to the MSSM, all the Higgs masses may be taken free parameters and so one would expect larger cross-sections, as well as CP asymmetries in the rates for $\mu^+\mu^- \rightarrow H^+W^-, H^-W^+$. We will show that such a production mechanism may provide a copious source of $H^\pm$ as well as offering a novel way of probing CP violation in the Higgs sector, the latter not being possible in the standard mechanism $\mu^+\mu^- \rightarrow H^+H^-$. Our work is organized as follows. In Section 2 we introduce the 2HDM potential, and section 3 derives explicit formulae for the cross-sections. In Section 4 we present our numerical analysis while section 5 contains our conclusions.


2 2HDM Potential

The most general 2HDM potential which violates CP and only softly breaks (by dimension 2 terms) the discrete symmetry \( \Phi_i \rightarrow -\Phi_i \) contains 8 free parameters at tree-level \[10\]. We will follow the notation of \[17\]. The potential is given as follows.

\[
V(\Phi_1, \Phi_2) = V_{\text{symm}} + V_{\text{soft}}
\]  

where

\[
V_{\text{symm}} = -\mu_1^2 \Phi_1^\dagger \Phi_1 - \mu_2^2 \Phi_2^\dagger \Phi_2 + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \\
\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 + \frac{1}{2}[\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + h.c]
\]  

and

\[
V_{\text{soft}} = -\mu_{12}^2 \Phi_1^\dagger \Phi_2 + h.c
\]  

CP violation, either spontaneous or explicit, requires the presence of \( V_{\text{soft}} \) which breaks the discrete symmetry softly. If all the parameters are real, spontaneous CP violation can still occur provided that:

\[
\left| \frac{\mu_{12}^2}{\lambda_5 v_1 v_2} \right| \leq 1.
\]  

CP violation will be explicit if \( \text{Im}(\mu_{12}^4 \lambda_5 \neq 0) \). In the CP conserving case one finds two CP even neutral scalar eigenstates, \( h^0, H^0 \), and a CP odd eigenstate \( A^0 \). In the CP violating case, mixing is induced between the CP even and CP odd neutral scalar fields, resulting in three mass eigenstates \( H_1, H_2, H_3 \) with no definite CP quantum numbers. In the MSSM, such mixing may be induced when one considers the 1-loop effective scalar potential. This would also lead to a rate asymmetry in the processes \( \mu^+ \mu^- \rightarrow H^+ W^-, H^- W^+ \) and this will be addressed in \[18\]. The neutral scalar mass squared matrix \( \mathcal{M}_S^2 \) is diagonalized by the matrix \( O_{ij} \):

\[
O^T \mathcal{M}_S^2 O = \text{diag}(M_{H_1}^2, M_{H_2}^2, M_{H_3}^2)
\]  

We will parametrize the matrix \( O_{ij} \) by using three Euler angles as follows.

\[
O_{ij} = \begin{bmatrix}
    c_{12} c_{13} & s_{12} s_{13} & s_{13} \\
    -s_{12} c_{23} - c_{12} s_{23} s_{13} & c_{12} c_{23} - s_{12} s_{23} s_{13} & s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} c_{23} s_{13} & -c_{12} s_{23} - s_{12} c_{23} s_{13} & c_{23} c_{13}
\end{bmatrix}
\]  

The CP conserving limit is obtained by taking two of the Euler angles equal to zero, and so the eigenstates of \( \mathcal{M}_S^2 \) become pure CP even eigenstates, \( h^0, H^0 \) and \( A^0 \). This results in a potential with 6 free parameters, \( V_{\text{symm}} \). The condition for maximum CP violation was considered in \[19\].
3 $\mu^+\mu^- \to H^\pm W^\mp$

Single $H^\pm$ production may proceed via an $s$–channel resonance mediated by $H_i$, and by $t$-channel exchange of $\nu_\mu$ (see Fig. 1). We will present explicit formulae for the processes $\mu^+\mu^- \to H^+W^-$ and $\mu^+\mu^- \to H^-W^+$ by adapting the formulae presented in [12], to which we refer the reader for a detailed explanation of our notation. As explained in [12], model II type couplings are required for this production mechanism to have an observable rate.

The CP violation originates from the $s$-channel diagrams and the $st$ interference, and is caused by the elements of $O_{ij}$ which mix the pure CP even and CP odd scalar fields. In the $s$-channel diagrams the couplings at the vertices $(g_{H_iH^\pm W^\mp}, g_{H_i\nu_\mu})$, which are either purely real or purely imaginary in the CP conserving case, possess both a real and imaginary part. We will show that this induces a difference in the rates for $\mu^+\mu^- \to H^+W^-$ and $\mu^+\mu^- \to H^-W^+$. The CP violating couplings are as follows:

$$g_{H_iH^\pm W^\mp} : (O_{2i} \cos \beta - O_{1i} \sin \beta, O_{3i})$$
$$g_{H_i\nu_\mu} : (O_{1i}, O_{3i} \sin \beta), \quad (7)$$

where $i = 1, 2, 3.$

We now present the formulae for the matrix elements for for $H^+W^-$ and $H^-W^+$ production. The matrix element squared for $\mu^+\mu^- \to H^+W^-$ is as follows:

$$|\mathcal{M}|^2(\mu^+\mu^- \to H^+W^-) = \frac{sg^4m_\mu^2}{32M_W^4} \left[ \lambda(s, M_{H^\pm}^2, M_W^2) \sum_{i,j} g_{H_iH^\pm W^\mp}^* g_{H_jH^\pm W^\mp} S_{H_i} S_{H_j}^* \Re\{g_{H_i\nu_\mu}^* g_{H_j\nu_\mu}\} + 2 \tan^2 \beta S_F^2(t)(2M_W^2p_T^2 + t^2) + \tan \frac{\beta}{\cos \beta} S_F(t)(M_{H^\pm}^2 M_W^2 - s^2 - t^2) \sum_i \left\{ g_{H_iH^\pm W^\mp} g_{H_i\nu_\mu} S_{H_i} + c.c. \right\} \right] \quad (8)$$

Where $p_T^2 = \lambda(s, M_{H^\pm}^2, M_W^2) \sin^2 \theta/4s$, $S_F(t) = 1/t$, and the propagators $S_{H_i}$ are given by:

$$S_{H_i} = \frac{1}{s - M_{H_i}^2 + iM_{H_i} \Gamma_{H_i}} \quad (9)$$
The matrix element squared for $\mu^+\mu^- \to H^-W^+$:

$$ |\mathcal{M}|^2(\mu^+\mu^- \to H^+W^-) = \frac{sg^4 m_{\mu}^2}{32 M_W^4} \left( \frac{\lambda(s, M_{H^\pm}, M_W^2)}{\cos^2 \beta} \sum_{i,j} g_{H^i H^\pm W^\mp W^\pm}^* g_{H^j H^\pm W^\pm W^\mp} S_{H^i} S_{H^j}^* \Re \left\{ g_{H^i H^\pm W^\mp}^* g_{H^j H^\pm W^\pm} \right\} ight. $$

$$ + 2 \tan^2 \beta S_F^2(t)(2M_W^2p_T^2 + t^2) $$

$$ \left. + \frac{\tan \beta}{\cos \beta} S_F(t)(M_{H^\pm}^2 M_W^2 - sp^2_T - t^2) \sum_i \left\{ g_{h_i H^\pm W^\pm}^* g_{H^i H^\pm W^\pm} S_{H^i} + c.c. \right\} \right) $$

(10)

The origin of the CP violation is the interference between the weak phases (phases in the $g_{H_i H^\pm W^\mp}$ and $g_{H_i \bar{H}^\pm \mu \mu}$) and absorptive phases (phases in the $S_{H_i}$), as can be seen in (8) and (10).

The differential cross-section for $\sigma(\mu^+\mu^- \to H^\pm W^\mp)$ may be written as follows:

$$ \frac{d\sigma}{d\Omega} = \frac{\lambda^2(s, M_{H^\pm}, M_W^2)}{64\pi^2 s^2} |\mathcal{M}|^2 $$

(11)

The total cross-section, $\sigma_{tot}$, is defined by:

$$ \sigma_{tot} = \sigma(\mu^+\mu^- \to H^+W^-) + \sigma(\mu^+\mu^- \to H^-W^+) $$

(12)

In the CP conserving case the $g_{H_i H^\pm W^\mp}$ are $g_{H_i \bar{H}^\pm \mu \mu}$ are either purely real or purely imaginary and the two rates are the same. In the CP violating case one can define a rate asymmetry as follows:

$$ \frac{\sigma(\mu^+\mu^- \to H^+W^-) - \sigma(\mu^+\mu^- \to H^-W^+)}{\sigma(\mu^+\mu^- \to H^+W^+) + \sigma(\mu^+\mu^- \to H^-W^+)} $$

(13)

Although this is a measure of the magnitude of the CP violation, analogous to the direct CP asymmetry in the partial widths of B hadron decays, the difference in the rates ($\sigma_{diff}$) is of more use experimentally:

$$ \sigma_{diff} = \sigma(\mu^+\mu^- \to H^+W^-) - \sigma(\mu^+\mu^- \to H^-W^+). $$

(14)

## 4 Numerical results

We will present results for the CP violating 2HDM. For the CP conserving 2HDM, $\sigma_{tot}$ is usually very close in value to that of the CP violating case (for the same choice of Higgs masses and $\tan \beta$), and so we do not explicitly show results. The mass splittings of the Higgs bosons contribute to the $\rho$ parameter at the 1-loop level, and these extra contributions are constrained by $-0.0017 \leq \Delta \rho \leq 0.0027$ [20]. Therefore in our numerical analysis we impose the formulae for $\Delta \rho$ in [21], which are valid for the CP violating 2HDM. We will assume integrated luminosities of the order 50 fb$^{-1}$ per year.
Measurements of $b \rightarrow s\gamma$ strongly restricts the allowed values of $M_{H^\pm}$ in the 2HDM with Model II type couplings. Recent measurement suggest $M_{H^\pm} \geq 200$ GeV for $\tan \beta \geq 1$ \cite{22}.

We show in Fig. 2a and 2b $\sigma_{\text{tot}}$ and $\sigma_{\text{diff}}$ as a function of $\sqrt{s}$, for $\tan \beta = 4, 20, 50$. We have fixed the Euler angles such that the values $O_{21} = O_{22} = O_{23} = 1/\sqrt{3}$ are reproduced, and the masses of $H_1, H_2, H_3$ are fixed at 100, 400, 700 GeV respectively; we also take $M_{H^\pm} = 200$ GeV. In Fig. 2a one can clearly see the large rises in $\sigma_{\text{tot}}$ when $\sqrt{s} \approx M_{H_i}$, which corresponds to the familiar resonance effect. Such an enhancement is never possible in the MSSM case \cite{12} since $M_A \approx M_H \approx M_{H^\pm}$, and so the conditions for on-shell production ($\sqrt{s} \geq M_{H^\pm} + M_W$), and the resonance condition ($\sqrt{s} = M_{H_i}$) can never simultaneously be satisfied. Fig. 2a shows that $\sigma_{\text{tot}}$ is maximized at the resonance ($\sqrt{s} \approx M_{H_i}$) and large $\tan \beta$. In such cases $\sigma_{\text{tot}} \geq 1000$ fb is possible, and represents a copious source of $H^\pm$.

In Fig. 1b we can see that $\sigma_{\text{diff}}$ is maximized with the same conditions that maximized $\sigma_{\text{tot}}$, and is always negative for the input parameters considered. Values of $\sigma_{\text{diff}}$ up to 150 fb are possible for large $\tan \beta$. With the expected luminosities of order 50 fb$^{-1}$, even $\sigma_{\text{diff}} \geq 2$ fb would lead to a mismatch of $\geq 100$ events in the rates for $H^+W^- \text{ and } H^-W^+$. In Fig. 3 we fix two Higgs masses almost equal ($M_{H_2} = 400$ GeV, $M_{H_3} = 410$ GeV), and show the dependence of $\sigma_{\text{tot}}$ and $\sigma_{\text{diff}}$ on $\sqrt{s}$. We take $M_{H_1} = 280$ GeV, $\tan \beta = 50$ and the Euler angles are the same as in Fig.2. In this case the cross-sections are strongly peaked at the resonance, where $\sigma_{\text{tot}} \approx 4700$ fb and $\sigma_{\text{diff}} \approx 1200$ fb, and this corresponds to an asymmetry (eq.(13)) of $\approx -26\%$. Away from resonance the cross-sections fall sharply with $\sqrt{s}$, in contrast to the case in Fig. 2a and 2b where sizeable values for $\sigma_{\text{tot}}$ and $\sigma_{\text{diff}}$ were possible over a wide range of $\sqrt{s}$.

In Fig. 4a we plot $\sigma_{\text{diff}}$ as a function of $O_{ij}$, for the same Higgs mass input parameters as used in Fig. 2. We fix $\tan \beta = 50$ and $\sqrt{s} = 400$ GeV, and vary 2 Euler angles in order to explicitly show the dependence of $\sigma_{\text{diff}}$ on $O_{ij}$. One can see that the maximum value of $\sigma_{\text{diff}}$ arises when $O_{ij} = 1/\sqrt{3}$, which is the maximum CP violation condition applied in Fig. 2. Note that $\sigma_{\text{diff}}$ may be both positive and negative. The inner dots are eliminated by the $\rho$ parameter constraint, while the thicker dots survive. We note that the latter points include the points that maximally violate CP. The $\rho$ parameter constraint has a strong effect on the magnitude of $\sigma_{\text{tot}}$, and rules out a sizeable parameter space where $\sigma_{\text{tot}}$ exceeds 3000 fb. This is shown is Fig. 4b, where one can see that the points which correspond to the largest values of $\sigma_{\text{tot}}$ are eliminated by the $\rho$ parameter constraint.

5 Conclusions

We have considered the mechanism $\mu^+\mu^- \rightarrow H^\pm W^\mp$ in the context of the CP violating 2HDM, which proceeds via Higgs mediated $s$-channel diagrams and $\nu_\mu$ exchange in the $t$-channel. We showed that large values are possible for both the total cross-section ($\sigma_{\text{tot}}$) and the difference in the cross-sections ($\sigma_{\text{diff}}$) for $H^+W^- \text{ and } H^-W^+$. The latter represents a novel way of probing CP violating effects in the Higgs sectors. The CP violation originates
from the interference between the weak phases in the vertices \((H, H^\pm W^\mp, H_i\bar{\mu}\mu)\) and the strong phases in the propagators. We showed that both \(\sigma_{\text{tot}}\) and \(\sigma_{\text{diff}}\) are maximized for large \(\tan\beta\) and for \(\sqrt{s} \approx M_{H_i}\), the latter corresponding to the familiar resonance effect. Values of \(\sigma_{\text{tot}} \geq 4000\) fb are possible at resonance for \(\tan\beta = 50\), and this provides a copious source of \(H^\pm\). Large values of \(\sigma_{\text{diff}}\) provide a clear way of observing CP violation, and we found that \(\sigma_{\text{diff}} \geq 1000\) fb is possible. Even \(\sigma_{\text{diff}} \geq 2\) fb would correspond to a mismatch of \(\geq 100\) in the number of \(H^+W^-\) and \(H^-W^+\) events, which should be readily observable.

Acknowledgements

A.G. Akeroyd was supported by the Japan Society for Promotion of Science (JSPS). S. Baek was supported by Korea Science and Engineering Foundation (KOSEF). We thank C. Dove for reading the manuscript.

References

[1] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter’s Guide* (Addison–Wesley, Reading, 1990).

[2] S. Komamiya, Phys. Rev. D38 (1988) 2158; A. Sopczak, Z.Phys. C65 (1995) 449; S. Moretti and K. Odagiri, J. Phys. G23 (1997) 537.

[3] E. Eichten, I. Hinchliffe, K. Lane and C. Quigg, Rev. Mod. Phys. 56 (1984) 579; J. Gunion, H.E. Haber, F.E. Paige, W.K. Tung and S.S.D. Willenbrock, Nucl. Phys. B294 (1987) 621; R.M. Barnett, H.E. Haber and D.E. Soper, B306 (1988) 697; D.A. Dicus, J.L. Hewett, C. Kao, and T.G. Rizzo, Phys. Rev. D40 (1989) 787; V. Barger, R.J.N. Philips and D.P. Roy, Phys. lett. B324 (1994) 236; J.L. Diaz–Cruz and O.A. Sampayo, Phys. Rev. D50 (1994) 6828.

[4] Jiang Yi, Ma Wen-Gan, Han Liang, Han Meng and Yu Zeng-hui; J. Phys. G24 (1998) 83; J. Phys. G23 (1997)385; A. Krause, T. Plehn, M. Spira and P.W. Zerwas, Nucl. Phys. B519 (1998) 85; S. Moretti and K. Odagiri, Phys. Rev. D55 (1997) 5627; Li Gang Jin, Chong Sheng Li, R.J. Oakes and Shou Hua Zhu, Eur. Phys. J. C14 (2000) 91; A.A. Barrientos Bendezu and B.A. Kniehl, Nucl. Phys. B568 (2000) 305; O. Brein and W. Hollik, Eur. Phys. J. C13 (2000) 175.

[5] K. Odagiri. Phys. Lett. B452 (1999) 327; K. Odagiri, hep-ph/9901332; D.P. Roy, Phys. Lett. B459 (1999) 607; F. Borzumati, J.L. Kneur and N. Polonsky, Phys. Rev. D60 (1999) 115011; D.J. Miller, S. Moretti, D.P. Roy and W.J. Stirling, Phys. Rev. D61 (2000) 055011; S. Moretti and D.P. Roy, Phys. Lett. B470 (1999) 209.
[6] A. Arhrib, M. Capdequi Peyranère and G. Moultaka, Phys. Lett. B341 (1995) 313; M.A. Diaz and Tonnis A. ter Veldhuis, hep–ph/9501313; A. Arhrib and G. Moultaka, Nucl. Phys. B558 (1999) 3.

[7] Combined Experimental Limits; ALEPH 99-081 CONF 99-052; DELPHI 99-142 CONF 327; L3 Note 2442; OPAL Technical Note TN–614.

[8] Proceedings of the Workshop on Physics at the First Muon Collider and front end of a Muon Collider, Fermilab, November 6-9, 1997; \( \mu^+\mu^- \text{ Collider: a Feasibility Study} \), BNL–52503, Fermilab–Conf-96/092, LBNL–38946, July 1996; Phys. Rep. 286 (1997) 1.

[9] J. Gunion, hep–ph/9802258, V. Barger, hep–ph/9803480.

[10] R. Casalbuoni, A. Deandrea, S. De Curtis, D. Dominici, R. Gatto and J.F. Gunion, Phys. Rev. Lett. 83 (1999) 1525; V. Barger, M.S. Berger, J.F. Gunion and T. Han, Phys. Rev. Lett. 75 (1995) 1462.

[11] E. Asakawa, A. Sugamoto and I. Watanabe, hep–ph/0004005; E. Asakawa, S.Y. Choi and J.S. Lee, hep–ph/0005113.

[12] A.G. Akeroyd, A. Arhrib and C. Dove, Phys. Rev. D61 (2000) 071702.

[13] A. Arhrib, M. Capdequi Peyranere, W. Hollik, G. Moultaka, Nucl. Phys. B581 (2000) 34; S. Kanemura, hep–ph/9911541 (to appear in Eur.Phys.J.C); Shou Hua Zhou, hep–ph/9901221.

[14] A.A. Barrientos Bendezu and B.A. Kniehl, Phys. Rev. D59 (1999) 015009; Phys. Rev. D61 (2000) 097701; hep–ph/0007226; S. Moretti and K. Odagiri, Phys. Rev. D59 (1999) 055008.

[15] R.A. Alanakyan, hep–ph/9804247.

[16] T.D. Lee, Phys. Rev. D8 (1973) 1226; G.C. Branco, Phys. Rev. D22 (1980) 2901.

[17] B. Grzadkowski, J.F. Gunion and J. Kalinowski, Phys. Rev. D60 (1999) 075011; Phys. Lett. B480 (2000) 287.

[18] A.G. Akeroyd and S. Baek, work in progress.

[19] A. Mendez and A. Pomeral, Phys. Lett. B272 (1991) 313.

[20] J. Erler and P. Langacker, talk given at the 5th International Wein Symposium (WEIN 98), Santa Fe, Jun 1998 (hep–ph/9809352).

[21] C.D. Froggatt, R.G. Moorhouse and I.G. Knowles, Phys. Rev. D45 (1992) 2471; A. Pomarol and R. Vega, Nucl. Phys. B413 (1994) 3; G.C. Joshi, M. Matsuda and M. Tanimoto, Phys. Lett. B341 (1994) 53;
[22] F.M. Borzumati and C. Greub, PM/98/23, hep-ph/9810240, Phys. Rev. D\textbf{58} 074004 (1998); Phys. Rev. D\textbf{59} 057501 (1999).
Figure Captions

Fig. 2a $\sigma_{\text{tot}}$ as a function of $\sqrt{s}$ for various values of $\tan\beta$. We take $M_{H_1} = 100$ GeV, $M_{H_2} = 400$ GeV, $M_{H_3} = 700$ GeV.

Fig. 2b $\sigma_{\text{diff}}$ as a function of $\sqrt{s}$ for various values of $\tan\beta$. For $M_{H_i}$ we use the values in Fig. 2a.

Fig. 3 $\sigma_{\text{tot}}$ and $\sigma_{\text{diff}}$ as a function of $\sqrt{s}$. We take $\tan\beta = 50$ and $M_{H_i}$ as displayed in the figure.

Fig. 4a $\sigma_{\text{diff}}$ as a function of $O_{23}$. We fix $\tan\beta = 50$ and $M_{H_i}$ are the same as in Fig. 2a. The thin dots violate the $\rho$ parameter constraint.

Fig. 4b Same as Fig. 4a but for $\sigma_{\text{tot}}$. 
\[ \sigma_{\text{tot}} = \sigma(H^+ W^-) + \sigma(H^- W^+) \]

\[ \tan \beta = 50 \]

\[ \tan \beta = 20 \]

\[ \tan \beta = 4 \]
$\sigma_{\text{diff}} = \sigma(H^+W^-) - \sigma(H^-W^+)$

Figure 2b
Figure 3

$\sigma_{\text{tot}}$

$\sigma_{\text{diff}}$

$M_{H1} = 280$ GeV

$M_{H2} = 400$ GeV

$M_{H3} = 410$ GeV

$\tan \beta = 50$
