LINKAGE BETWEEN GOLD AND STOCK RETURN: EVIDENCE FROM NONLINEAR CAUSALITY AND ROLLING WINDOW VOLATILITY SPILLOVER

Abstract. This paper investigates nonlinear causality relationship between gold and stock return in the USA for the period of December 1969-November 2021. The nonlinearity is detected in both series, and nonlinear unit root tests are applied for each variable. It is seen that both return series are nonlinear stationary according to whole nonlinear unit root findings. There is unidirectional nonlinear Granger causality from stock to gold return in all lags, except lag three, whereas there is unidirectional nonlinear Granger causality from gold to stock return just in lag three. So, this finding reflects safe haven seeking of investors between gold and stock markets. Moreover, causality findings of volatility spillover display that causal link is from gold to stock return before 2000, and shift direction after 2000. Meanwhile, rolling window spillover causality is employed from beginning to ending period in 120 month fixed range of consecutive movement. It is attained that bidirectional reciprocal causality exists and mainly present at the fixed periods among December 1999-January 2015, mirror financial crisis impact.

Keywords: Gold, Stock, Return, Safe Haven, Nonlinear Causality, Volatility Spillover, Rolling Window.

JEL Classification: C14, E44, G11, G15, N20

1. Introduction

The relationship between gold and stock return is a discussion subject in economics and financial analysis. Economists also have long discussed whether gold submit a safe haven hallmark at the era of financial stress periods. But, one of the important point is about the structural dynamics of this relationship whether linear or not. If an asset is a safe haven for another one, then price of the safe haven assets compensates negative shocks originated from the other one (Baur and Lucey, 2006). The safe haven resembles the protection of a ship in a harbour at the time of destructive storm weather. So, a safe haven asset brings opportunity to protect wealth at the era of negative conditions (Baur and McDermott, 2010).

Baur and Lucey (2006) distinguish safe haven and hedge concepts from each other. If an asset serves as hedge for another one, it does not necessitate a safe haven situation for the same asset. Meanwhile, the reverse of this condition is valid.
as well. The difference between two lies on the behaviours of the assets holders. Hedge asset is negatively correlated or uncorrelated with other one based on average values, whereas safe haven asset is negatively correlated or uncorrelated with other at extreme market conditions, which can be stated as crisis periods.

Gold is a valuable commodity and used as money in many countries up to previous century. Investment in gold is seen as one of the instrument at the financial stress conditions. Investors think gold investment as more certain and reliable tool at the negative circumstances due to its store of value function. The idea has been discussed by Bagehot (1871) that accumulation of gold increases at the cautious and uninvesting period of industry. Meanwhile, historical experiences reflect that people mostly choose to haven gold port at the era of financial market slumps.

Smith (1776) declares that the demand of precious metals is originated from their utility and beauty. The worth of beauty rises with the further increment in scarcity. Gold can be evaluated more valuable than diamond, if the scarcity of gold is much more than the scarcity of diamond. Therefore, same ounce of gold can buy more goods and services with further increment in its scarcity. In accordance with this Smith (1776) exemplifies that the discovery of new precious metals in America decreased the value of gold and silver about a third of previous value in Europe at 16th century due to its abundancy. Gold has always been a player of the game up to now, even if sometimes on the bench, and sometimes on the field.

2. Literature Review

Choudhry and et al. (2015) state that non-causality displays the importance of gold in reducing portfolio risk at the era of stable economic conditions. They examine the nexus between gold and stock return in Japan, the UK, and the USA for the daily period of January 2000-March 2014. The behaviour of stock holders is mainly interrogated at the period and pre-period of financial crisis. As far as nonlinear causality findings, there exists weak evidence for the causality at the pre-crisis period that signifies save haven structure of gold for three countries. On the other hand, significant causality is more dominant at the financial stress era, which means that gold is not a safe haven at this period. Baek (2019) seek for the relationship gold, bond, and stock return in the USA for the monthly period of January 2009-December 2018. According to Johansen co-integration findings, there is no co-integration between gold and stock return. In respect to Granger causality findings, gold return is a Granger cause of stock return in the short run, and gold is decided as a better safe haven than bonds for market slumps in the USA.

Coronado and et al. (2018) scrutinize the relationship between gold, oil and stock prices in the USA for the daily period of January 1986-June 2017. First of all, unit root structure of return variables are reviewed with ADF and RALS tests, whilst the nonlinear structure is evaluated with BDS and Tsay tests. In regards to Diks and Panchenko (2006) nonlinear causality findings, there exists bidirectional causality between gold and stock returns. Besides, nonlinear causal linkage is
examined with rolling window method yearly from 1986 to 2017, which confirm findings.

Liu and et al. (2016) interrogate safe haven and hedge structure of gold against US dollar and stock with Capula model for the weekly period of January 2000-November 2013. It is claimed that gold is a hedge at normal epochs, while it is a safe haven at crisis periods. Gao and Zhang (2016) express that the movement from riskier asset to safer one called as “flight to quality”. Choudhry (2015) called this as “flight to safety” as well. Thus, Gao and Zhang (2016) focus the impact of economic policy uncertainty on the correlation between gold and stock with GARCH models in the UK for the monthly period of January 1997-March 2015. It is seen that gold demand increases due to flight to quality effect at the uncertain economic periods. Miyazaki and Hamori (2013) seek for the causality in mean and variance between gold and stock return in the USA for the daily period of January 2000-April 2011. There is unidirectional causality in mean from stock to gold, but no causality in variance at the entire period. While there is bidirectional causality in mean for the sample period of January 2000-August 2007, there is unidirectional causality from stock to gold at the sample period of August 2007-April 2011.

Smith (2001) examines the causality between gold and stock in the USA for the daily period of January 1991-October 2001. Gold and stock prices have unit root at level, but series are not co-integrated in the long-run. Moreover, there is unidirectional Granger-causality for the morning prices from stock to gold return, but not for the afternoon prices. Qin and et al. (2019) investigate the causal relationship between global economic policy uncertainty and gold prices with bootstrap rolling window causality method for the monthly period of January 1997-November 2018. It is stated that there is mutual interaction at the crisis periods, which reflects the hedging structure of the gold prices. Li and et al. (2015) search the causal link between economic policy uncertainty and stock return with bootstrap rolling window approach in China for the period of February 1995-February 2013. According to findings, there exists bidirectional causality in subsamples of rolling windows. Finally, Bampinas and Panagiotidis (2015) resort to nonlinear and time varying causality methods to peruse the relationship between crude oil and gold prices in the USA for the monthly period of January 2003-December 2012. According as findings, the intensity of causal linkage increases after the financial crisis based on time varying causality, and bidirectional causal link also exists after crisis in terms of nonlinear causality.

3. Data and Methodology

Data is employed for the bivariate relationship between gold and stock prices covering the monthly period spanning from December 1969 to November 2021. Monthly stock data is attained from MULTIPL website at https://www.multpl.com/s-p-500-historical-prices/table/by-month based on S&P 500 prices, whereas monthly gold data is retrieved from World Gold Council
website at https://www.gold.org/goldhub/data/gold-prices leaning on per ounce prices. Return series are achieved with respect to logarithmic difference of series \( \ln(p_t - p_{t-1}) \), where \( p_t \) is the price of gold and stock at \( t \) dimension. All calculations are realized after seasonal adjustment of each series with Census X-13.

### Table 1. Descriptive Statistics

|        | DLSP   | DLGP   |
|--------|--------|--------|
| Mean   | 0.006  | 0.006  |
| Median | 0.010  | 0.003  |
| Maximum| 0.111  | 0.268  |
| Minimum| -0.223 | -0.212 |
| Std. Deviation | 0.035  | 0.051  |
| Skewness | -1.430* | 0.251  |
| Kurtosis | 9.020* | 6.619* |
| Jarque-Bera | 1153.284* (0.000) | 346.522* (0.000) |

Note: DLSP and DLGP are the logarithmic differences of price values. Kurtosis > 3 and the values in parenthesis reflect probabilities.

Mean values of both gold and stock returns are approximately same, whereas maximum value of gold return is more than stock return. The leptokurtic values in both stock and gold, and skewness in stock bring the nonlinearity possibility into mind. Meanwhile, Jarque-Bera statistics reject the null hypothesis of normal distribution of errors in both of them.

In order to clarify causal relationship between gold and stock return, unit root and linearity structure of variables are discussed at the first step. Unit root structure of price and return values of gold and stock have been investigated with Augmented Dickey Fuller (ADF) and Phillips-Perron (PP) tests by considering raw, constant, and constant and trend structures, respectively (Baek, 2019). Then, RALS-ADF, and RALS-LM tests are applied to take the information of non-normal errors into consideration for a more powerful unit root findings. Nonlinearity of variables is checked with BDS statistics. The existence of linearity is examined at the null hypothesis against alternative nonlinearity.

\[
\Delta y_t = I_t \rho_1 [y_{t-1} - a_0] + [1 - I_t] \rho_2 [y_{t-1} - a_0] + \varepsilon_t, I_t = \begin{cases} 1 & \text{if } y_{t-1} \geq a_0 \\ 0 & \text{if } y_{t-1} < a_0 \end{cases} \tag{1}
\]

Enders and Granger (1998) introduce a nonlinear unit root test based on TAR model, which is expressed at Equation 1. The null hypothesis of unit root \( (\rho_1 = \rho_2 = 0) \) is tested against nonlinear stationarity \( (\rho_1 = \rho_2 \neq 0) \) alternative (Güriş, 2020). Sollis (2009), and Hu and Chen (2016) unit root tests, and single frequency Fourier ADF test of Enders and Lee (2012) are also applied to diversify findings. Meanwhile, nonlinear co-integration is analysed with Enders and Siklos (2001) methodology.
Univariate ARCH (1,1), GARCH (1,1), and E-GARCH (1,1) models are estimated to express and compare volatility between gold and stock return. The value of $\alpha$ represents the ARCH effect, and $\beta$ represents GARCH effect in variance equation ($\sigma_t^2 = \omega + \alpha \mu_{t-1}^2 + \beta \sigma_{t-1}^2$). Firstly, stability conditions of parameters are checked to ensure whether intercept $\omega \geq 0$, ARCH parameter $\beta \geq 0$, and GARCH parameter $\alpha + \beta < 1$ (Nazlioglu and et al., 2015). Moreover, E-GARCH model is added to evaluate asymmetric impact of shocks for each univariate model, and diagnostic checks are realized for heteroscedasticity, and autocorrelation.

3.1. Nonlinear Causality Methodology

Diks and Panchenko (2006) develop a nonlinear Granger causality model to cope with over-rejection bias problem on the basis of Hiemstra and Jones (1994) test based upon nonparametric correlation integrals. $X_{t+1}$ is conditionally independent for the finite past values of $X$ and $Y$. Statement of $Y_{t+1} \parallel (X_t^x, Y_t^y) \sim Y_{t+1} | Y_t^y$ represents conditional independence, where $X_t^x = (X_{t-1}^x, ..., X_t)$ and $Y_t^y = (Y_{t-1}, Y_{t-1}^y, ..., Y_t)$ in a strictly stationary bivariate series. Meanwhile, $W = (X, Y, Z)$ is stated as an invariant random continuous variable. Density function of joint probability and marginal values of $f_{X,Y,Z}(x, y, z)$ must meet the condition at Equation 2, and $X$ and $Z$ are independent conditionally on $Y = y$ for every fixed value of $y$ (Nazlioglu, 2011).

$$f_{X,Y,Z}(x, y, z) = f_{X,Y}(x, y) f_{Y,Z}(y, z) f_{Y}(y)$$

Diks and Panchenko (2006) reorganize the null hypothesis of nonlinear Granger causality with $q$ statistics as equal to zero:

$$q = E[f_{X,Y,Z}(X, Y, Z)f_y(Y) - f_{X,Y}(X, Y)f_{Y,Z}(Y, Z)]$$

Indicator function of $q$ estimator is expressed based on the value of $(\epsilon)$ bandwidth as:

$$T_n(\epsilon) = (2\epsilon)^{-d_x - d_y - d_z} \sum_{i} \left[ \sum_{k, j \neq i} (I_{ik}^{XY} I_{ij}^{Y} - I_{ik}^{XY} I_{ij}^{Y}) \right]$$

Value $I(\cdot)$ is the indicator function, where $I_W^Y = I(\|W_i - W_j\| < \epsilon)$. Value $\|\|_\cdot$ reflects the maximum norm, whereas $\epsilon$ is the sample size depended on bandwidth (Coronado and et al., 2018).

$$f^*_W(W_i) = \frac{(2\epsilon)^{d_w}}{(n-1)} \sum_{j \neq i} I_{ij}^{W}$$

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Local density estimator is represented with \( f_{\omega} \) function of a \( d_{\omega} \)-variate random vector \( W \) at \( W_t \), and test statistics is expressed at Equation 6 (Diks and Panchenko, 2006).

\[
T_n(\epsilon) = \frac{(n-1)}{n(n-2)} \sum \left( \hat{f}_{X,Y,Z}(X_t,Y_t,Z_t)\hat{f}_Y(Y_t) - \hat{f}_{X,Y}(X_t,Y_t)\hat{f}_{Y,Z}(Y_t,Z_t) \right)
\]  \hspace{1cm} (6)

Providing that bandwidth depends on the sample size as \( Cn^{-\beta} \), and \( C > 0 \), then \( \beta \) lies between \( (1/4 < \beta < 1/3) \) for one lag \( \tau = 1 \).

\[
\sqrt{n} \left( T_n(\epsilon_n - q) \right) \overset{d}{\rightarrow} N(0,1)
\]  \hspace{1cm} (7)

Value of \( S_n \) is the estimator of asymptotic variance of \( T_n(. \) ), and test statistics of Equation 7 ensures asymptotically normal distribution for the independence of vectors of \( W_t \). The null hypothesis of no nonlinear Granger causality is accepted if statistic values are lower than 1.28 (Nazlioglu, 2011).

### 3.2. Volatility Spillover Methodology

Hafner and Herwartz (2006) introduce causality in variance test by applying LM (Lagrange Multiplier) method based on univariate GARCH residuals.

\[
H_0: \text{Var} \left( \epsilon_{it} | {\mathcal{F}}_{t-1}^{(j)} \right) = \text{Var} \left( \epsilon_{it} | {\mathcal{F}}_{t-1} \right), j = 1, ..., N \text{ and } i \neq j \]  \hspace{1cm} (8)

Causal hypothesis in variance (volatility spillover) is expressed between two series at Equation 8, where \( {\mathcal{F}}_t^{(i)} = {\mathcal{F}}_t / \sigma(\epsilon_{it}, \tau \leq t) \), and \( \epsilon_{it} \) is the estimated residual of univariate GARCH model (Nazlioglu and et al. 2015).

\[
\epsilon_{it} = \xi_{it} \sqrt{\sigma_{it}^2(1 + z'_{it} \pi)}, z_{jt} = (\epsilon_{jt-1}^2, \sigma_{jt-1}^2)' \]  \hspace{1cm} (9)

The null hypothesis of no causality \((H_0: \pi = 0)\) is tested against alternative one \((H_0: \pi \neq 0)\), where \( \xi_{it} \) is the standardized residual, \( \sigma_{it}^2 \) is conditional volatility, \( \epsilon_{jt-1}^2 \) is the disturbance term in square, and \( \sigma_{jt-1}^2 \) is the conditional variance. LM test is used with average values of univariate GARCH (Nazlioglu and et al., 2015).

\[
\lambda_{LM} = \frac{1}{4T} \left( \sum_{t=1}^{T} (\xi_{jt}^2 - 1)z_{jt}' \right) V(\theta_i)^{-1} \left( \sum_{t=1}^{T} (\xi_{jt}^2 - 1)z_{jt}' \right) \overset{d}{\rightarrow} \chi^2(2)
\]  \hspace{1cm} (10)

where,

\[
V(\theta_i) = \frac{K}{4T} \left( \sum_{t=1}^{T} z_{jt}z_{jt}' - \sum_{t=1}^{T} z_{jt}x_{it}' \left( \sum_{t=1}^{T} x_{it}x_{it}' \right)^{-1} \sum_{t=1}^{T} x_{it}z_{jt}' \right)
\]  \hspace{1cm} (11)
Asymptotic distribution of $\lambda_{LM}$ lies on the misspecification indicators number in $z_{jt}$. Two misspecifications indicators induce two degrees of freedom in chi-square asymptotic distribution at Equation 10, and $K$ equals $\frac{1}{T} \sum_{t=1}^{T} (\xi_{jt}^2 - 1)^2$ at Equation 11 (Hafner and Herwartz, 2006). The rejection of null hypothesis means that there exists volatility spillover from $j$ to $i$ series (Nazlioglu and et al., 2015).

4. Empirical Findings

Firstly, unit root structure is tested for each series. Unit root findings of series are summarized in Table 2 leaning on ADF, PP and RALS tests in raw, demeaned and detrended models.

### Table 2. Findings of Unit Root Tests

| Tests       | Raw          | Demeaned    | Detrended   |
|-------------|--------------|-------------|-------------|
|             | LSP | LGP | LSP | LGP       | LSP | LGP |
| ADF         | 3.644 | 2.688 | 0.268 | -2.254 | -2.097 | -2.320 |
| PP          | 3.482 | 2.098 | 0.259 | -2.133 | -2.269 | -2.443 |
| RALS-ADF    | 0.672 | -1.909 | -2.633 | -3.178 |
| RALS-LM     | -2.435 | -2.354 |

First Difference (*)

| Tests       | Raw          | Demeaned    | Detrended   |
|-------------|--------------|-------------|-------------|
|             | LSP | LGP | LSP | LGP       | LSP | LGP |
| ADF         | -19.681 | -24.335 | -20.211 | -24.685 | -20.217 | -24.733 |
| PP          | -20.409 | -25.235 | -20.381 | -25.181 | -20.384 | -25.181 |
| RALS-ADF    | -24.016 | -28.128 | -24.009 | -28.040 |
| RALS-LM     | -19.924 | -13.176 |

Note: * refers the significance at the 0.1 level, and (*) implies the significance of all first difference values at 0.01 level. Schwarts criterion is used for all tests.

It is detected that the null hypothesis of unit root is accepted in both level values of stock and gold, and transformed into stationarity at difference values. So, there is unit root at price series, but series are stationary at return values. But, the nonlinear structure of variables was entreated to reach a better modelling.

### Table 3. Findings of BDS Statistics

| m | Residual DLSP | Residual DLGP | DLSP | DLGP |
|---|---------------|---------------|------|------|
| 2 | 0.017 [0.000] | 0.024 [0.000] | 0.020 [0.000] | 0.025 [0.000] |
| 3 | 0.031 [0.000] | 0.045 [0.000] | 0.037 [0.000] | 0.046 [0.000] |
| 4 | 0.043 [0.000] | 0.057 [0.000] | 0.050 [0.000] | 0.059 [0.000] |
| 5 | 0.046 [0.000] | 0.063 [0.000] | 0.054 [0.000] | 0.063 [0.000] |
| 6 | 0.044 [0.000] | 0.061 [0.000] | 0.052 [0.000] | 0.063 [0.000] |
| 7 | 0.041 [0.000] | 0.058 [0.000] | 0.050 [0.000] | 0.059 [0.000] |
| 8 | 0.038 [0.000] | 0.053 [0.000] | 0.046 [0.000] | 0.054 [0.000] |
| 9 | 0.033 [0.000] | 0.050 [0.000] | 0.040 [0.000] | 0.050 [0.000] |
| 10| 0.029 [0.000] | 0.046 [0.000] | 0.036 [0.000] | 0.046 [0.000] |
Nonlinear distribution is interrogated at the return series to find out proper causal modelling. Nonlinearity findings of return series are displayed in Table 3. It is noticed that linearity is rejected in both series and their VAR residuals.

Table 4. Findings of Nonlinear Unit Root

| Tests       | Raw     | Demeaned | Detrended |
|-------------|---------|----------|-----------|
|             | LSP     | LGP      | LSP       | LGP       |
| E-G         | 6.770c  | 4.336b   | 0.023     | 4.066b    | 2.122     | 4.031     |
| Sollis      | 6.627c  | 4.658b   | 3.544     | 3.105     | 4.546     | 3.393     |
| Hu-Chen     | 13.748b | 16.696c  | 8.144     | 15.206b   | 7.381     | 6.636     |
| Fourier ADF | 0.325   | -1.853   | -2.731    | -2.135    |
| E-S         | 0.231   | 0.210    | 0.202     |

First Difference (c)

| Tests       | Raw     | Demeaned | Detrended |
|-------------|---------|----------|-----------|
|             | LSP     | LGP      | LSP       | LGP       |
| E-G         | 122.967 | 144.913  | 130.603   | 149.319   | 130.746   | 150.354   |
| Sollis      | 8.403   | 11.369   | 10.912    | 11.479    | 10.882    | 11.275    |
| Hu-Chen     | 400.207 | 634.146  | 410.849   | 638.129   | 411.687   | 639.671   |
| Fourier ADF | -20.397 | -25.123  | -20.406   | -25.141   |

Note: a, b, c indicate significance at the 0.1, 0.05 and 0.01 levels, respectively. (c) implies the significance of all first difference values at 0.01 level, and BIC criterion is used for all test statistics up to maximum 13 lags.

Table 4 demonstrates nonlinear unit root findings propped up Enders and Granger (1998), Sollis (2009), Hu and Chen (2016), and Enders and Lee (2012) tests. The null hypothesis of linear unit root is rejected at return series in all models, which reflect nonlinear stationarity distribution. Nonlinear co-movement of price series is analyzed with Enders and Siklos (2001) test. It is seen that there is no nonlinear co-integration. So, series are not moving together in the long-run.

Table 5. Findings of Diks and Panchenko Nonlinear Causality I

| DLSP ≠ DLGP |
|-------------|-----------|-----------|-----------|-----------|
| m | RAW" | VARc | GARCHd | EGARCHe |
|----|-------|------|------|--------|
| 2  | 1.454a | 1.435a | 1.493a | 1.528a |
| 3  | 1.510a | 1.488a | 1.501a | 1.491a |
| 4  | 1.218  | 0.953 | 1.076 | 1.072 |
| 5  | 1.339a | 1.428a | 1.618a | 1.644a |
| 6  | 1.803b | 1.712b | 1.780b | 1.817b |
| 7  | 1.786b | 1.622a | 1.753b | 1.771b |
| 8  | 1.310b | 1.456a | 1.473b | 1.532a |
| 9  | 1.485a | 1.545a | 1.622a | 1.672b |
| 10 | 2.052b | 2.007b | 2.016b | 2.070b |
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Note: a, b indicate significance at the 0.1, and 0.05 levels, respectively. w raw data based on logarithmic values of seasonally adjusted gold and stock returns, x is the VAR residuals attained from logarithmic values of seasonally adjusted gold and stock returns, y is the residuals of GARCH(1,1), z is the residuals of EGARCH(1,1) models. m is the embedding dimension, and bandwidth is determined as 1.5 according to observation number.

Diks and Panchenko (2006) nonlinear Granger causality findings from stock to gold return is displayed in Table 5. It is detected that there is nonlinear Granger causality in all embedding dimensions, except 4. Gold renders a safe stop for risk aversion, and uncertain conditions in stock market. So, unidirectional causality from stock to gold return revises “flight to quality/safety” condition.

Table 6.Findings of Diks and Panchenko Nonlinear Causality II

| m   | RAW" | VAR' | GARCH' | EGARCH' |
|-----|------|------|--------|---------|
| 2   | 1.351a | 1.087 | 1.205   | 1.206   |
| 3   | 0.674    | 0.868 | 0.808   | 0.785   |
| 4   | 1.283a   | 1.309a | 1.325a  | 1.280   |
| 5   | 0.798    | 0.847 | 0.665   | 0.629   |
| 6   | 0.540    | 0.540 | 0.535   | 0.511   |
| 7   | 0.561    | 0.359 | 0.371   | 0.370   |
| 8   | 0.459    | 0.390 | 0.263   | 0.269   |
| 9   | 0.219    | -0.123 | -0.182 | -0.173 |
| 10  | -0.319   | -0.413 | -0.424 | -0.401 |

Note: a indicate significance at 0.1 level. w raw data based on logarithmic values of seasonally adjusted gold and stock returns, x is the VAR residuals attained from logarithmic values of seasonally adjusted gold and stock returns, y is the residuals of GARCH(1,1), z is the residuals of EGARCH(1,1) models. m is the embedding dimension, and bandwidth is determined as 1.5 according to observation number.

Nonlinear Granger causality is displayed in Table 6 from gold to stock return. It is seen that there is significant causality at the embedding dimension 4. Unidirectional causal link from gold to stock might represent “flight to quality/safety” for investment purposes at the third lag. So, it corresponds to investment attempts from gold to stock market.

Univariate ARCH (1,1), GARCH (1,1), and EGARCH (1,1) models are expressed in Appendix 1. ARCH and GARCH stability conditions are valid in both gold and stock returns. While the sum of ($\alpha + \beta < 1$) ARCH and GARCH parameters are less than 1, it is very close to 1 in stock return model. Therefore, it reflects the persistency of volatility shocks in stock market. Moreover, EGARCH
parameter ($\theta$) is negative in stock return, whereas it is positive in gold return. So, negative shocks have larger impact than positive shocks on the volatility of stock return, whilst vice versa is valid in gold return. Static volatility forecasting of ARCH models are taken place in Appendix 2. Both gold and stock volatility lies within standard error bands, but intense volatility can be foreseeable in stock return after 2008 Global Financial Crisis. Besides, stock return exhibits high intensity after 2020 due to destructive impacts of Covid 19 in all over the world.

| Causality in variance for | $DLSP \not\Rightarrow DLGP$ | $DLGP \not\Rightarrow DLSP$ |
|--------------------------|-------------------------------|-------------------------------|
| All period               | 5.811$^a$                     | 1.923                         |
| 1969m12-1979m11          | 0.474                         | 0.659                         |
| 1979m12-1989m11          | 0.918                         | 4.611$^a$                     |
| 1989m12-1999m11          | 0.317                         | 4.921$^a$                     |
| 1999m12-2009m11          | 6.298$^b$                     | 4.003                         |
| 2009m12-2019m11          | 4.790$^a$                     | 2.685                         |

Note: $^a$, $^b$ indicate significance at the 0.1 and 0.05 levels for LM statistics.

In order to verify findings, volatility spillover causality test of Hafner and Herwartz (2006) is analysed as well. Findings are expressed in Table 7. It is detected that there is one-way causality from stock to gold return in whole period. Meanwhile, volatility spillover is tested for each 10 year interval starting from December 1969. It is seen that volatility spillover is from gold to stock return between periods 1980-2000, whilst interaction is reverse after 2000. Thus, findings reflect one-way interaction up to this point, which propound safe haven structure.
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Balcilar and et al. (2010) signify that rolling window method endure on movement of a fixed length consecutively from the beginning to the ending period. Rolling window causality findings are demonstrated at Figure 1. The fixed length of rolling window regression is determined as 120, and started from the date of December 1969 to ending November 2021 with 505 causality findings for each bivariate model. While the number of significant volatility spillover is 70 for the causal relationship from stock to gold return, 22 of them coincide with the fixed periods among December 1999-January 2015. So, it includes the Global Financial Crisis period of December 2007-June 2009 at the sub-samples of rolling window, which reflects risk aversion behaving of asset holders at this period.

The number of significant volatility spillover is 120 out of 505 for the causal nexus from gold to stock return, whereas 33 of them coincide with the fixed periods among December 1999-January 2015. There are 18 bidirectional causalities and 11 of them present among fixed periods of December 1999-January 2015.

5. Conclusions

The nexus between stock and gold return have been investigated in the USA for the period spanning from December 1969-November 2021. First of all, unit root structure and co-integration situation of variables have been discussed with nonlinear models due to nonlinear distribution of series. It is determined that return series are nonlinear stationary. Thereby, nonlinear causal relationship between return series have been examined with Diks and Panchenko (2006) methodology. It is recognized that there is nonlinear Granger causality running from stock to gold return in all lag values, except lag three. On the other hand, there is nonlinear Granger causality running from gold to stock return just in lag three. These findings reflect the safe haven structure of gold for stock market. So, nonlinear causality from stock to gold market is seen more dominant for this period, and gold compensates negative shocks originated from stock market.

Univariate ARCH (1,1), GARCH (1,1), and EGARCH (1,1) models were expressed to clarify volatility structure of variables. It is detected that negative information has larger impact on stock volatility, whereas reverse is valid for gold volatility. Moreover, Haffner and Herwartz (2006) volatility spillover causality is applied to compare findings with nonlinear causality. It is seen that there is unidirectional causality between stock and gold return, which is mainly verified with rolling window spillover findings as well. Meanwhile, it is observed that significant spillover causality is mostly from gold to stock return before 2000, but it is from stock to gold return at the entire period. As a result, there exists predominantly unidirectional interaction between gold and stock return, which reflect safety seeking behavior of investors. However, 11 numbers rolling window causality out of 18 corresponds to periods among December 1999-January 2015. The increment in reciprocal interaction reverberate mutual hedging pursuit of gold and stock at the slump periods of financial market. Gold is one of the preferred
investment tools, and serves its safe haven and hedging features for stock market needs. On the other hand, acceptance of non-causality between two reflect more stable conditions in financial markets. Non-causal links mainly coincide with non-crisis including periods at the rolling window analysis.

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### Appendix 1. Findings of Variance Equations

| Variables | ARCH (1,1) | GARCH (1,1) | EGARCH (1,1) |
|-----------|------------|-------------|-------------|
|           | coef | p-value | coef | p-value | coef | p-value |
| C         | 0.006 | 0.000 | 0.007 | 0.000 | 0.007 | 0.000 |
| Lag value | 0.234 | 0.000 | 0.192 | 0.000 | 0.192 | 0.000 |
Appendix 2. Static Volatility Forecasting ARCH models for Stock and Gold

Note: Autocorrelation check leans on correlogram of standardized residuals squared. White test includes cross terms.