Cosine Gear Planetary Transmission with Small Teeth Number Difference

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Abstract. This paper presents a model of cosine gear planetary transmission with small number teeth difference (CG_PTSTD). Firstly, the mathematical model of the CG_PTSTD is established based on the theory of engagement, including the equations of the conjugate internal gear and the meshing line. Then an example of the CG_PTSTD is developed. In this example, the 2D and 3D models of the CG_PTSTD are established, and the bending and contact stresses are analyzed by finite element software ABAQUS. The results show that the cosine gear can be successfully applied to the PTSTD without tooth tip interference and there are multi teeth simultaneously participating in the mesh.

1. Introduction

PTSTD is widely used in robot joints, aerospace, weapons and other fields due to its large transmission ratio, compact structure and high transmission accuracy [1]. The harmonic reducer [2] and RV reducer [3] are the most typical PTSTD which are widely used in robot joints. Compared with harmonic reducer, RV reducer has better stiffness and larger bearing capacity due to the absence of flexible components. Hence it is widely used in joints with high bearing capacity requirements such as waist and big arm of robot [4]. F. Litvin et al. [5] established the cycloid tooth profile equation and deduced the conditions for avoiding singularity of the cycloid curve. Shuting Li [6] developed the design software of cycloid gear reducer, and conducted contact analysis of this reducer based on the presented mechanical model and finite element software. Xuan Li et al. [7] comprehensively considered the influences of clearances, tooth profile modification and elastic deformation when establishing the load distribution model of cycloid gear reducer. Ta-shi Lai [8] proposed the geometric design flow of roller drives from the perspective of reducing processing cost and difficulty. Chiu-fan Hsieh [9] proposed to replace the pin tooth with an internal cycloid gear to overcome the output speed fluctuation and the uneven stress distribution. Bingkui Chen et al. [4] introduced a new cycloidal engaged meshing pair which is characterized that there has two contact points simultaneously in a specific meshing area. The above studies belong to cycloid type PTSTD. However, the cycloid tooth profile has a large pressure angle [10], leading to a large stress on the support bearing, and the cycloid pin tooth meshing pair is very sensitive to machining and assembly errors.

The involute PTSTD is also a commonly used PTSTD type [1]. Shuting Li [11] studied the contact problem of involute PTSTD. R. Maiti et al. [12] modified the involute tooth profile from the perspective of avoiding tooth tip interference. Wen-yi Lin et al. [13] optimize the involute tooth profile comprehensively considered the two important factors of pressure angle and contact ratio. However, as mentioned in the above literature, since the whole involute gear consists of several curves,
complex interferences are prone to occur in involute internal meshing, especially in the PDSTD. Only through complex and accurate calculations can the valid design parameters be obtained.

From the above literature, it can be concluded that gear tooth profile, as one of the vital factors affecting gear transmission performance, has not been studied enough in the PTSTD. Shanming Luo et al. [14] proposed a novel cosine gear and introduced a corresponding processing method [15]. In external gearing drive, the cosine gear drive has smaller sliding coefficient and larger bearing capacity but lower contact ratio compared with the involute gear and mainly used in external gear pumps. Since the cosine tooth profile is generated by the cosine curve and the shape is similar to "S", it is theoretically suitable for the internal gear transmission. However, to the best of our knowledge, no research has been done on the cosine internal gear drive, especially in the PTSTD. This paper establishes the CG_PTSTD mathematic model according to the meshing theory. An example of CG_PTSTD solid model is presented. The results indicate that the cosine gear can be successfully applied to the PTSTD without tooth interference and with multi-tooth elastic meshing.

2. Mathematical model of cosine internal gear pairs

Figure 1 shows the established coordinate systems. \( g_1 \) and \( g_2 \) respectively denote the cosine gear and its conjugate internal gear. The moving coordinate systems \( C_1(X_1, Y_1, Z_1) \) and \( C_2(X_2, Y_2, Z_2) \) (abbreviated as \( C_1 \) and \( C_2 \)) are respectively established at the centres of these two gears \( O_1 \) and \( O_2 \). The centre of the fixed coordinate system \( C_f(X_f, Y_f, Z_f) \) (abbreviated as \( C_f \)) is established on the pitch point \( P \). At the initial position, \( X_1 \), \( X_2 \) and \( X_f \) are horizontal. The pitch circle radii of the \( g_1 \) and \( g_2 \) are \( r_1 \) and \( r_2 \), the number of teeth is \( z_1 \) and \( z_2 \), respectively. The centre distance between the two gear centres is \( d \). Assuming the pivoted arm \( O_1O_2 \) is fixed. If the cosine gear rotates \( \theta_1 \) anticlockwise around the \( Z_1 \) axis, the conjugate internal gear will rotate \( \theta_2 \) anticlockwise around the \( Z_2 \) axis and \( \theta_1/\theta_2 = z_2/ z_1 \).

According to [19], as shown in Figure 2, the cosine gear in \( C_1 \) can be expressed as:

\[
\mathbf{G}^{(x)} = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}^T, \text{ where } x_1 \text{ and } y_1 \text{ represent the abscissa and ordinate of the cosine gear, respectively, which are equal to:}
\begin{align*}
x_1 &= [r_1 + h \cos(z_1 \varphi)] \sin \varphi \\
y_1 &= [r_1 + h \cos(z_1 \varphi)] \cos \varphi
\end{align*}
\]

where \( \varphi \) and \( h \) represent the inherent parameter and the tooth dedendum height of the cosine gear, respectively, and \( h = m \, h^* \) where \( m \) and \( h^* \) denote the gear modulus and tooth height coefficient, respectively.

According to the meshing principle [16], the locus of the contact point in \( C_2 \) is the conjugate tooth profile, which is determined by the following equation:
\[ G^{(2)} = M_{2i}G^{(1)} \]
\[ f(\varphi, \theta) = n - v_{i(2)}^{(2)} = 0 \]  \hspace{1cm} (2)

where \( M_{2i} \) represents the transformation matrix of the contact point from the \( C_1 \) to the \( C_2 \), which can be expressed as:

\[
M_{2i} = \begin{bmatrix}
\cos(\theta_i - \theta_j) & -\sin(\theta_i - \theta_j) & d \sin \theta_j \\
\sin(\theta_i - \theta_j) & \cos(\theta_i - \theta_j) & d \cos \theta_j \\
0 & 0 & 1
\end{bmatrix}  \hspace{1cm} (3)
\]

The function \( f(\varphi, \theta) > 0 \) represents the meshing equation of the cosine gear drive. It represents the angular displacement of a point on the cosine gear turning from its initial position to the position where it becomes a contact point. From \([14]\), it can be obtained as:

\[
\theta_i = \arcsin \left( \frac{1}{\mu} \left[ r_i + h \cos(z_i \varphi) \sin(\varphi + \psi) \right] \right) - \psi
\]  \hspace{1cm} (4)

where \( \psi \) denotes the angle from the positive direction of the \( X_1 \) axis to the tangent to the point on the cosine gear. From the equation (1), it can be obtained:

\[
\psi = \arctan \left( \frac{dy}{dx} \right) = \frac{d}{d\varphi} \sin(z_i \varphi) + \frac{r_i + h \cos(z_i \varphi)}{r_i + h \cos(z_i \varphi) - h z_i \sin(z_i \varphi) \tan \varphi}  \hspace{1cm} (5)
\]

Substituting equation (5) into equation (4), the relationship between \( \theta_i \) and the parameter \( \varphi \) can be obtained. Substitute equations (1), (3) and (4) into equation (2) to obtain the expression of conjugate internal gear in \( C_2 \) as follows:

\[
\begin{align*}
x_1 &= \left[ r_i + h \cos(z_i \varphi) \right] \sin[\varphi - (1 - \mu) \theta_j] + d \sin(\mu \theta_j) \\
y_1 &= \left[ r_i + h \cos(z_i \varphi) \right] \cos[\varphi - (1 - \mu) \theta_j] + d \cos(\mu \theta_j)
\end{align*}
\]  \hspace{1cm} (6)

where \( \mu \) is the reciprocal of the gear ratio, \( \mu = \theta_j / \theta_i \).

Similarly, the locus of the contact point in the \( C_i \) is the meshing line. Therefore, by transforming the coordinates of the contact point from the \( C_1 \) to the \( C_c \), the meshing line equation can be obtained as follows:

\[
\begin{align*}
G^{(c)} = M_{ci}G^{(1)} \quad &\text{and} \\
f(\varphi, \theta) = n - v_{i(2)}^{(1)} = 0
\end{align*}
\]  \hspace{1cm} (7)

where \( M_{ci} \) is the transformation matrix of the above coordinate transformation, which can be expressed as:

\[
M_{ci} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 \\
\sin \theta_i & \cos \theta_i & -r_i \\
0 & 0 & 1
\end{bmatrix}  \hspace{1cm} (8)
\]

Substitute equations (1), (4), and (8) into equation (7), the meshing line equation is:

\[
\begin{align*}
x_i &= \left[ r_i + h \cos(z_i \varphi) \right] \sin(\varphi - \theta_i) \\
y_i &= \left[ r_i + h \cos(z_i \varphi) \right] \cos(\varphi - \theta_i) - r_i
\end{align*}
\]  \hspace{1cm} (9)

3. Design example and evaluation

3.1. Solid modeling

Assuming that the design parameters are \( m=1 \), \( h^* = 1.2 \), \( z_1 = 100 \), \( z_2 = 105 \). The tooth number difference is \( \Delta = z_2 - z_1 \). First, 2D model of the CG_PTSSTD is obtained in Wolfram Mathematica 8.0, as shown in Figure 3. According to \([17]\), the contact ratio of this drive under the given parameters is 1.17, meeting the requirement that the contact ratio should be greater than or equal to 1 to ensure the continuity and stationarity of the transmission. It is assumed that \( r_{a1} \) as well as \( r_{a2} \) and \( r_{a2} \) represent the addendum
and dedendum circles’ radii of the cosine gear and its conjugate internal gear, respectively. They can be obtained in the calculation process of the contact ratio. It can be obtained that \( r_{a1} = 50.84 \text{ mm} \), \( r_{f1} = 48.80 \text{ mm} \), \( r_{a2} = 51.65 \text{ mm} \) and \( r_{f2} = 53.70 \text{ mm} \). According to the above parameters, the 2D model is imported into NX 10.0 to establish the 3D model of the CG_PTSTD, as shown in Figure 4. It can be seen that there is no tooth tip interference between the two engaged gears. Therefore, the cosine gear can be successfully applied in the PTSTD without tooth interference.

3.2. Evaluation

The analysis of bending and contact stresses of the CG_PTSTD are carried out by the FEA program ABAQUS. The hexahedral solid elements are used to construct the FEA model. The model is partitioned and the mesh density of the contact area was increased. The minimum mesh width is 0.2mm. The properties of materials are: \( \mu = 0.3 \), \( E = 200\text{GPa} \), density=7.8e3kg/m3. It is assumed that the sliding coefficient is 0.01. The internal gear was fixed. Except for the rotation about the \( Z_1 \) axis, other freedoms of the cosine gear were fixed. The torque applied to the cosine gear was 180 Nm. Figure 5 shows the simulation analysis results. Figure 5 (a) shows that the maximum contact stress is about 199.11 MPa, which is distributed near the tip of the cosine tooth profile. The maximum bending stress is about 164.35 MPa, distributed near the root of the cosine tooth profile. Compared with the applied torque, the stresses are relatively small. Therefore, the cosine gear PTSTD has greater carrying capacity. Figure 5(b) shows that the number of teeth that are simultaneously engaged are 4. However, as mentioned before, the theoretical contact ratio of this gear drive is 1.17. Therefore, there are multi-tooth elastic meshing in the cosine gear PTSTD.
4. Conclusion
The model of the CG_PTSTD is proposed in this paper. First, the equations of the conjugate internal gear and the meshing line are deduced based on the meshing theory and the equation of the cosine gear. Then a solid model example of the CG_PTSTD is established. The analysis of the bending and contact stresses of this CG_PTSTD are carried out by FEA program. The results show that the cosine gear can be successfully applied to the PTSTD without tooth tip interference and there are multi teeth simultaneously participating in the mesh.

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