Finishing of micro-aspheric tungsten carbide mold using small viscoelastic tool

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Abstract
Tungsten carbide is widely used as the material of replication mold to produce small aspheric optics, and the polishing process determines the precision of these molds. However, for micro-aspheric tungsten carbide mold, the existing polishing methods are difficult to realize the form error modification during a polishing process due to the polishing tool is always larger than small mold. Therefore, a polishing tool using polyester fiber cloth to wrap small-size rigid ball is used in this paper. In order to predict the tool influence function (TIF) of this polishing tool, a series of theoretical analysis and experimental verification are carried out in this paper. Firstly, by analyzing the structural and viscoelastic characteristics of the fiber cloth, the pressure distribution in the polishing contact area is determined. And the polishing speed distribution is obtained by analyzing the kinematic movement of polishing tool; Then, combined with Preston equation, the TIF is derived; Afterward, through a series of single point polishing experiments, it is verified that the volume error between the theoretical removal model and the experimental removal is less than 6.9% when the polishing pressure is low; Finally, the TIF is applied to the form error corrective polishing of small size symmetric aspheric tungsten carbide mold. After a form error corrective polishing test, the PV value (peak to valley) of form error is decreased from 0.268 to 0.083 μm, which verifies the effectiveness of this polishing method for small size tungsten carbide mold in form error correction.

Keywords Tungsten carbide · Micro-aspheric mold · Viscoelastic tool · Deterministic polishing

1 Introduction
Aspheric lens has the advantages of reducing volume of optical system, improving image quality and avoiding image error of spherical lens. It is widely used in mobile phone, vehicle imaging system, and infrared system [1]. For the fabrication of aspheric lens, small aperture aspheric lens is mainly produced by glass aspheric molding for low cost. The aspherical lens is formed by replicate the surface profile of the aspheric mold. Thus, the form accuracy and surface roughness of the aspheric mold determine the accuracy of the aspheric lens [2].

In the glass aspherical molding technology, tungsten carbide, silicon carbide, and other cemented carbide materials with stable performance at molding temperature are usually used to manufacture aspherical molds [3]. This kind of cemented carbide needs to be ground for preliminary processing. However, the process of grinding will conduct damaged layer on the surface of the mold. To investigate the fundamental mechanisms of grinding, Zhang et al. studied the damage layer caused by single grain grinding at nanoscale depth of cut with high grinding speed [4–6], and developed a new grinding diamond wheel to obtain minimally damaged surface layer [7, 8]. They point out damage layer and grinding crack will reduce surface quality. Therefore, it is necessary to remove the damage layers caused by grinding through polishing process [9]. This process determines the final accuracy and surface roughness of aspheric mold [10]. Furthermore, an accuracy TIF is essential for deterministic polishing to reduce the form error of aspheric profile.

At present, aspheric polishing is mainly based on Preston equation [11], and TIF is always established by analyzing pressure distribution, polishing speed distribution of
polishing contact area, and removal constant that related to 

the workpiece material and polishing particle size.

There are mainly two methods to obtain a TIF. The first one is measuring it by experiment, and the other one is predicting it by analyzing the stress distribution and velocity distribution in polishing contact area. Due to an aspheric surface having different curvature, TIF will change with polishing position and time. Thus, a large amount of work needs to be done if measuring TIF by experiment. Therefore, predicting the TIF of aspheric surface is an efficient choice to save time and labor in industrial production. However, the change of TIF with time and polishing position and the relevancy of other process parameters make it difficult to be predicted precisely.

In the past decades, many scholars have taken a lot of studies on TIF of aspheric polishing. In the 1980s, Aspden et al., researched the removal mechanism of workpiece materials and proposed a polishing removal model based on pressure, polishing speed, and time [11]. Subsequently, Jones [12] proposed a dwell time solution model based on Aspden’s research. The model uses a convolution iteration method to calculate the dwell time of the polishing tool at each position of the workpiece, and achieves the purpose of reducing the form error of the workpiece by controlling the polishing dwell time. The research work of Aspden and Jones constructed a theoretical foundation for computer-controlled optical surface polishing. However, the studies of Aspden and Jones have a certain limit of application; thus, many other scholars improved the aspheric polishing theory and put forward new polishing methods under their research framework.

Different polishing methods were appeared in the field of aspheric cemented carbide mold polishing. In 1990s, Kordonsky et al. [13] invented a new polishing technology called magnetorheological polishing (MRF). This technology make magnetorheological fluid hardens in a short time and turns into viscoplastic status under a high-intensity gradient magnetic field. And the hardened fluid is used as polishing head. When the polishing head moves relative to the workpiece, this medium has shear force on the workpiece surface, which results in material removal. In order to adapt to the polishing of small diameter cemented carbide mold and avoid interference between polishing tool and concave mold, Yin [14] proposed inclined axis MRF technology to avoid interference by controlling the angle of polishing tool. Using this method, the tungsten carbide mold with 8-mm diameter can be polished. However, for concave mold with smaller curvature radius, inclined axis MRF method is also difficult to be executed. MRF technology could bring high surface quality to tungsten carbide molds, but its removal efficiency is too low and it is just available to mold that has large radius of curvature.

In 2001, Walker et al. [15] proposed bonnet polishing technology. It is reported that the bonnet polishing technology can achieve a removal rate of 0.025–120 mm³/min and achieve a smoother surface quality [16]. Due to the high material removal rate, bonnet polishing technology is widely used in the polishing of optical element materials with brittle and hard property [17]. For bonnet polishing technology, an accurate TIF is also essential to the polishing accuracy. Therefore, Kim and Kim [18], Wang et al. [19], and Cheung et al. [20] established TIF with Hertz contact theory and geometric motion of bonnet polishing head from the perspective of macro. Cao and Cheung [21, 22] analyzed the removal mechanism of single free abrasive particles in bonnet polishing, and established a TIF by calculating the removal of all abrasive particles. Lin et al. [23] analyzed the influence of the surface morphology of polyurethane polishing pad on the TIF through simulation. In the above research about bonnet polishing, it is assumed that the bonnet is linearly elastic and its viscoelastic properties are ignored. Although bonnet polishing technology can effectively remove cemented carbide materials, it is limited by the size of bonnet tool and is easy to interfere with small concave mold. Bonnet polishing is also difficult to realize the from error modification during the polishing for small-diameter tungsten carbide mold.

The polishing methods mentioned above all have large size of polishing tools, which results in large polishing spots. Therefore, they are not suitable for form error corrective polishing of micro-aspheric tungsten carbide molds (especially <5 mm). As a result, this paper uses a rigid ball as a small-size polishing tool. The surface of the tool is wrapped with a layer of cloth which is made of polyurethane and nylon. In order to build TIF, a polishing pressure distribution function is established through analyzing the viscoelastic properties and geometric structure of the polishing cloth, and velocity distribution of the polishing contact area is obtained through the kinematic movement of the polishing tool. Finally, a series of experiments are carried out to verify the accuracy of TIF and the effectiveness of form error corrective polishing for small-size mold.

2 Material removal mechanism and model of TIF

In this paper, the relative position of the polishing tool and workpiece is shown as Fig. 1. According to Preston’s empirical formula, modeling the TIF of a small tool needs to obtain the pressure distribution of polishing contact area and the relative velocity distribution between workpiece surface and polishing tool. On one hand, the pressure distribution in contact area is determined by the viscoelastic properties and the structure of the polishing cloth. On the
other hand, the speed distribution is determined by the relative movement of the polishing tool and workpiece. In order to simplify the modeling of TIF, we make the following assumptions:

1. Before polishing starts, the polishing tool wrapped with cloth is regarded as a standard ball.
2. The polishing tool and the workpiece are regarded as rigid bodies.
3. The polishing slurry and particles are uniformly attached to the surface of the polishing cloth, and its final effect on the polishing removal is reflected as removal constant \( k_p \).
4. The height of high-frequency unevenness on the surface of the polishing cloth obeys normal distribution, and the height of intermediate-frequency unevenness on the surface of the polishing cloth obeys sinusoidal distribution.
5. The polishing cloth is a non-uniform viscoelastic medium. During the compression process of the cloth, the probability of the silk participating in the compression deformation obeys the exponential probability distribution.

As is well known, a TIF is mostly based on the Preston equation. During a polishing process, the removal depth of material is proportional to the pressure, the relative velocity, and the dwell time in contact area. The relationship between these factors is expressed as follows [24]:

\[
dh = k_p p v_T \, dt
\]

In formula (1), \( dh \) is the amount of material removal at the one point on workpiece surface, \( k_p \) is the removal constant, \( p \) is the pressure exerted by polishing tool at workpiece, \( v \) is the relative speed of polishing tool and workpiece on contact area, and \( dt \) is the dwell time of polishing tool on workpiece.

As we know, the accuracy of TIF is essential to the result of form error corrective polishing process. Thus, in order to achieve the deterministic removal of surface material in a polishing process, the TIF should be predicted accurately. Based on the Preston equation, a TIF can be expressed as follows [17]:

\[
R(x, y) = \frac{k_p}{T} \int_0^T p(x, y)v(x, y) \, dt
\]

In formula (2), \( k_p \) is the remove constant, \( T \) is the total polishing time, \( p(x, y) \) presents the pressure distribution function, and \( v(x, y) \) presents the relative velocity distribution function.

To construct a TIF, the shape of the contact area, the pressure distribution, and the relative velocity distribution in contact area need to be predicted. Therefore, the process of predicting a TIF in this paper is shown as Fig. 2.

2.1 Structural simulation and viscoelastic properties of polishing cloth

2.1.1 Structural modeling of polishing cloth

In this paper, a small rigid ball which wrapped with a layer of cloth is used as polishing tool. The cloth is woven from polyurethane and nylon composite twisted core yarns, and has a mid-frequency and high-frequency profile in structure. A sketch of polishing tool and polishing cloth is shown as Fig. 3. The structure and viscoelastic properties of the polishing cloth determine the stress distribution under a certain deformation, so it is necessary to explore the structure of the polishing cloth. In order to simplify the mathematical model of the three-dimensional structure of polishing cloth, we assume that the structure of the polishing cloth is composed of a base layer and a rough layer. The cross-sectional morphology of the polishing cloth is
shown in Fig. 4. According to the structural characteristics of polishing cloth, the height deviation of the rough layer is superimposed by two types of periodic height that are intermediate frequency periodic height and high frequency periodic height. This paper assume that the height deviation of the intermediate frequency periodic height conforms to the sinusoidal distribution, and the initial phase of the sinusoidal distribution conforms to the normal distribution \( N(\mu_1, \sigma_1) \); the height deviation of the high-frequency height conforms to the normal distribution \( N(\mu_2, \sigma_2) \). Figure 4 shows the height distribution diagram of the YZ section.

In order to simulate the three-dimensional structure of the cloth, we extend two-dimensional structure diagram of the YZ section along the X direction. As is shown in Fig. 5a, it is a simulation process of the polishing cloth structure.

And Fig. 5b shows the experimental results measured by a stylus profilometer and simulation results of cloth surface. The height deviation of the rough layer is recorded as \( \delta(x, y) \) in this study.

### 2.1.2 Viscoelastic properties of polishing cloth

The silk of polishing cloth is made of polyurethane and nylon composite twist core, which emerge viscoelastic properties. Therefore, a four-parameter viscoelastic model [25] is used to describe the viscoelastic properties of the polishing cloth. As is shown in Fig. 6, it is a schematic diagram of the deformation and deformation recovery process of the polishing cloth under a certain pressure. In Fig. 6, \( \delta \) is the pressure, \( \eta_1 \) and \( \eta_2 \) are the viscous coefficients of viscous elements, respectively, \( E_1 \) and \( E_2 \) are the elastic coefficients of elastic elements, respectively, \( \varepsilon_1 \) is the initial deformation, \( \varepsilon_2 \) is maximum compression, and \( \varepsilon_3 \) is unrecoverable deformation. When we make a deformation to the cloth by a force, the cloth will appear an unrecoverable deformation after the force is withdrawn. The unrecoverable deformation is mainly produced by viscous element 1, which will cause the shape of the polished contact area to change from a circle to an ellipse.

In order to get the relationship between initial deformation and unrecoverable deformation, a compression experiment is designed, as is shown in Fig. 7a. Firstly, a compression \( \varepsilon_2 \) is form by exerting a force. Then, the force is withdrawn and the unrecoverable deformation \( \varepsilon_3 \) is measured. The experimental result is shown in Fig. 7b. The experimental results indicate a linear relationship between compression and unrecoverable deformation. The linear relationship between them can be expressed by a proportional coefficient \( k_c \).

If the same amount of compression is carried out again after the first compression, the unrecoverable deformation is almost the same. Therefore, the unrecoverable deformation are constant, and the viscoelastic model of the polishing cloth is simplified to a three-parameter viscoelastic model, as is shown in Fig. 8. According to the theory of viscoelasticity, the stress relaxation equation of the three-parameter model is expressed as follows [25]:

\[
\sigma(t) = E_1 \varepsilon + \frac{E_2 \varepsilon}{\eta_2} e^{-\frac{t}{\eta_2}}
\]

where \( \sigma \) is the total stress, \( t \) is the time, \( \varepsilon \) is the total strain, \( E_1 \) is the elastic modulus of elastic element 1, \( E_2 \) is the elastic modulus of elastic element 2, and \( \eta_2 \) is the viscosity coefficient of the viscous element.

A compression test was performed on the polishing cloth by exerting a constant deformation to the cloth and measure the stress release with time. The elastic modulus \( E_1 \) and \( E_2 \) and viscosity coefficient \( \eta_2 \) can be obtained by fitting the test data with formula (3). The compression test parameters and results are shown in Table 1 and Fig. 9.
During a polishing process, the deformation of the polishing cloth at different points will change with the rotation of the polishing tool, as is shown in Fig. 10. Since the cloth is the sparse and non-uniform viscoelastic medium that composed of silk threads, it is assumed that the proportion of silk threads participating in the compression conforms to exponential probability distribution. According to Boltzmann superposition principle [25] and the probability and statistics theory, the relationship between stress and strain for a point at the cloth in contact area can be expressed as follows [25]:

\[ \text{Stress} = \text{Strain} \times \text{Modulus} \]

where Modulus is a function of the exponential probability distribution of the proportion of silk threads.
\[
\sigma(t) = D(t) \ast d\epsilon(t) \times F(\epsilon(t)) \\
= \frac{1}{E_0}(1 + \beta(1 - e^{-\lambda t})) \ast d\epsilon(t) \times (1 - e^{-\mu t})
\]
(4)

\[
\beta = \frac{E_1}{E_2}
\]
(5)

**Table 1** Experimental parameters and results

| Experimental parameters | Value     |
|-------------------------|-----------|
| Compressive strain \(\epsilon\) (mm) | 0.02948   |
| Relaxation time \(t\) (S)               | 0.9       |
| Elastic modulus \(E_1\) (MPa)           | 60.11     |
| Elastic modulus \(E_2\) (MPa)           | 6.511     |
| Coefficient of viscosity \(\eta_2\)     | 1.759     |

\(\omega\)

**Fig. 6** Four-parameter model of polishing cloth

**Fig. 7** Compression test diagram of polishing cloth. a Sketch of polishing cloth compression, b Compression test results

**Fig. 8** Three-parameter model

**Fig. 9** Stress relaxation test

**Fig. 10** Sketch of the change in compression
where $\sigma$ is the stress, $D$ is the relaxation modulus of the cloth, $\varepsilon$ is the deformation of the cloth, $F$ is the probability distribution relate to $\varepsilon$, $t$ is the deformation time of the cloth, $d\varepsilon$ is the derivative of the cloth deformation, $E_1$ is the elastic modulus of the elastic element 1, and $E_2$ is the elastic modulus of the elastic element 2, $\eta_2$ is the viscosity coefficient of the viscous element, $\mu$ is the probability related constant, and $*$ is the convolution operator.

In order to determine the value of the probability-related constant $\mu$, a compression test of the cloth was performed. The stress–strain curve was fitted with formula (4) to obtain the value of the probability-related constant. The test results are shown in Fig. 11. Through the results, it is known that the stress–strain relationship of the cloth is nonlinear.

2.2 Pressure distribution modeling

In the previous studies on TIF, most of the analysis of pressure is based on the Hertzian contact theory, and the applicable object of the Hertzian contact theory is a uniform elastic medium with a smooth surface. In this paper, a rigid ball wrapped with cloth is used as the polishing tool, and the polishing cloth is not a uniform elastic medium with a smooth surface. Therefore, the Hertzian contact theory is not applicable to the analysis of the pressure distribution in the polished contact area. Thus, the pressure distribution of the polishing contact area is calculated by the compressive deformation of cloth in the polishing contact area.

The contact between the polishing tool and the surface of the workpiece can be divided into two stages, as is shown in Fig. 12. In the first stage, it is assumed that the polishing tool is a standard ball, and the contact area between the ball and the surface of the workpiece is a circle. And in the second stage, when the polishing tool rotates once, the contact area between the polishing tool and the surface of the workpiece becomes elliptical due to the irreversible deformation of the polishing cloth after it is compressed, since most of the polishing time is in the second stage where the contact area between the polishing tool and the surface of the workpiece is elliptical. Thus, this paper only analyzes the pressure distribution in the second stage.

After the irreversible deformation occurring, the compression distribution of the cloth in the polishing area is expressed as follows:

$$z(x, y) = \sqrt{r^2 - x^2} - (r - d) - k_\omega \times \sqrt{r^2 - y^2} - (r - d) + \delta(x, y) \times (1 - k_\omega)$$  \hspace{1cm} (8)

In formula (8), $r$ is the radius of the cloth-covered ball head, $d$ is the maximum compression, $k_\omega$ is the proportional constant between the compression and the irreversible deformation mentioned earlier, and $\delta$ is the height difference of the polishing cloth surface mentioned earlier. Combining
formulas (4) and (8), the pressure distribution of the polishing contact area can be expressed as follows:

\[
P(x, y) = \sigma(x, y, t) = D(x, y, t) * \text{de}(x, y) * F(\epsilon(x, y)) = \frac{1}{E_0} \left(1 + \beta(1 - e^{-\mu \epsilon(x, y)}) \right) * dz(x, y, t) \times (1 - e^{-\mu \epsilon(x, y)})
\]  

(9)

### 2.3 Modeling the velocity distribution in the contact area of the small tool

Figure 13 shows the positional relationship between tool and contact area of the workpiece. In a polishing process, the tool head rotates around the axis \( \vec{Z}_t \), the angular velocity of rotation is \( \omega_p \), and the angular velocity vector is in a form of \( \vec{w} \). \( \vec{Z}_t \) is a unit vector, and \( \vec{w} \) is expressed as follows:

\[
\vec{w} = \omega_p \vec{Z}_t = \omega_p(0, \sin \sigma y, \cos \sigma z)
\]  

(10)

In formula (10), \( \omega_p = |\omega| \). In Fig. 13, point \( P(x_p, y_p, 0) \) is a point in the polishing area, and \( \vec{t} \) is the velocity vector from \( O_t = (0, 0, R_p) \) to point \( P \) that is expressed as follows:

\[
\vec{t} = \vec{O}_t \vec{P} = (x_p, y_p, R_p)
\]  

(11)

In formula (11), \( R_p = R - d \), where \( R \) is the radius of polishing tool and \( d \) is the maximum compression of the polishing cloth. Since the linear velocity that is generated by the rotation of the polishing tool in the contact area is \( v_x \), it can be expressed as the cross product of the angular velocity vector \( \vec{w} \) and the vector \( \vec{t} \) like that:

\[
v_x = \vec{w} \times \vec{t} = \omega_p \left[ \begin{array}{c} -R_p \sin \sigma - \cos \sigma y_p \\ \cos \sigma x_p \\ -\sin \sigma x_p \end{array} \right]
\]  

(12)

In the polishing process, the amount of material removal is only related to the moving linear velocity in \( X \) and \( Y \) directions, and it is not related to the velocity of the component in \( Z \) direction. Therefore, it just needs to consider the \( X \) and \( Y \) components of the velocity. The combined velocity of a point \( p(x_p, y_p, z_p) \) in a contact area can be expressed as the following formula:

\[
v(x_p, y_p) = \sqrt{(v_x^p)^2 + (v_y^p)^2}
\]  

(13)

### 2.4 Modeling of TIF

After the above analysis, combined with formulas (2), (8), (9), and (13), the TIF can be expressed as follows:

\[
R(x, y) = \frac{k_p}{T} \int_0^T p(x, y)v(x, y)dt = \frac{k_p}{T} \int_0^T \frac{1}{E_0} (1 + \beta(1 - e^{-\mu \epsilon(x, y)}) \right) * dz(x, y, t) \times (1 - e^{-\mu \epsilon(x, y)}) \times v(x, y)dt
\]  

(14)

From the TIF formula (14), it is clear that the polishing removal rate of the material is proportional to the compression of the polishing cloth, and proportional to the relative speed distribution of the contact area.

![Sketch of polishing equipment](image)

(a) Sketch of polishing equipment. (b) Sketch of the movement axis of the polishing equipment.
In order to verify the effectiveness of the theoretical TIF, a series of single-point polishing experiments were carried out. In the experiments, the tungsten carbide plane was used as the polished workpiece, and the diamond with a diameter of 1um was used as the polishing abrasive grain, the cloth was used as the polishing cloth, and the white light interferometer produced by Bruker was used to measure the three-dimensional removal topography. The polishing equipment used in experiment is shown as Fig. 14, and the characteristics of the polishing spots are explored by setting the experimental parameters in Table 2. These processing parameters that affect a TIF include polishing dwell time, polishing tool rotation speed, and polishing pressure. The image of single-point polishing results for one group is shown in Fig. 15. It is known through the results that the polishing spot occurs intermediate frequency trace due to the surface characteristic of polishing cloth.

### 3.2 The influence of processing parameters on the removal rate

In order to obtain the relationship between polishing time and removal volume, experiments in Table 2 with serial numbers 1 to 5 were carried out. The experimental results are shown in Fig. 16. A linear fit was performed on the removal volume and the dwell time, and the goodness of fit $R^2$ was 0.9945. It can be seen that the removal volume has a linear relationship with the dwell time.

To study the influence of polishing tool rotation speed on polishing removal rate, experiments with serial numbers 6 to 10 in Table 2 were carried out. The experimental results are shown in Fig. 17. The polishing removal rate is linearly fitted to the rotation speed of the polishing tool, and the fitting $R^2$ is 0.9921. It can be seen that the rotation speed of the polishing tool has a linear relationship with the polishing removal rate.
To explore the influence of polishing pressure on polishing removal rate, experiments with serial numbers 11–15 in Table 2 were carried out. The experimental results are shown in Fig. 18. When the polishing pressure is low, the polishing removal rate has a linear relationship with the polishing pressure. However, when the pressure increases to a certain level, it starts to deviate from the linear relationship. This is because part of the abrasive particles is embedded in the polishing fiber cloth and part is embedded in the workpiece. When the force exerting on the polishing cloth increases, the depth of the abrasive particles pressing into the surface of the workpiece also begins to increase. But when the force increases to a certain extent, more increased cloth fiber contact directly on the surface of the workpiece without abrasive particle between them. Therefore, it has little impact on the depth of the abrasive particles pressed into the surface of the workpiece. In order to simplify the analysis, this paper only discusses the linear relationship between the polishing pressure and removal rate.

By analyzing the experimental results, it is known that the polishing tool speed and polishing pressure have a linear relationship with the polishing removal rate within a certain range. From formula (14), the value of removal constant can be obtained by the following formula:

$$k_p = \frac{R(x, y)}{p(x, y)v(x, y)} = \frac{\int R(x, y)dxdy}{\int p(x, y)v(x, y)dxdy} = \frac{V}{Fv}$$  \hspace{1cm} (15)

Table 3: Experimental parameters of single-point polishing

| Experimental parameters       | Symbol | Value     | 1 | 2 | 3 |
|------------------------------|--------|----------|---|---|---|
| Elastic Modulus (MPa)        | E1     | 60.11    | 60.11 | 60.11 |
| Elastic Modulus (MPa)        | E2     | 6.511    | 6.511 | 6.511 |
| Coefficient of Viscosity     | η2     | 1.759    | 1.759 | 1.759 |
| Maximum compression (mm)     | d      | 0.07     | 0.07  | 0.096 |
| Diamond abrasive grain size  (μm) | D      | 1        | 1    | 1   |
| Polishing time (S)           | T      | 25       | 100  | 100 |
| Polishing shaft speed (rpm)  | ωp     | 180      | 120  | 280 |
| Polishing ball radius (mm)   | r      | 1.5      | 1.5  | 1.5 |
| Removal constant             | k_p    | 0.0131   | 0.0131 | 0.0131 |
| Polishing cloth thickness (mm) | h     | 0.35     | 0.35  | 0.35 |
| Probability correlation constant | μ    | 0.03276  | 0.03276 | 0.03276 |
| Irrecoverable deformation proportional constant | k_a | 0.75    | 0.75  | 0.75 |
where \( V \) is the removal volume per unit time, \( F \) is the pressure of the polishing tool on the workpiece, and \( v \) is the average speed of the contact area.

### 3.3 Single-point polishing removal prediction

In order to confirm the TIF obtained above, three experiments and simulations are carried out. The parameters and results of the experiments and simulations are shown in Table 3 and Figs. 19, 20, and 21. Among them, Figs. 19a to 21a are a three-dimensional diagram drawn by MATLAB. The morphology of polishing spot was measured by the white light interferometer produced by Bruker. Figures 19b to 21b are three-dimensional structure diagram of the TIF calculated by above theory, and Figs. 19c to 21c are the cross-sectional contour of the three-dimensional diagram on the YZ section. Figs. 19d to 21d are the cross-sectional contour of the three-dimensional figure on the XZ section. From the figure, it is obviously that the theoretically predicted removal model is consistent well with the actual processed three-dimensional removal amount when the removal volume is small, which proves the effectiveness of the TIF. However, when the removal volume becomes larger, the deviation will become larger too.

In order to verify the applicable scope of the removal model, the polishing processing parameters of spot 1 to spot 13 in Table 2 are used to predict the polishing removal volume. The result is shown in Fig. 22. Figure 22 is a comparison chart of actual polishing removal volume and predicted volume. The maximum removal error rate is 6.9%, and the variance of the
removal error rate is 3.1%. It can be seen that the theoretical removal model can greatly match the actual processing effect when the pressure is low.

### 4 Form error corrective polishing experiment

#### 4.1 Solving polishing dwell time

On the basis of TIF obtained above, a small-diameter aspheric tungsten carbide mold is subjected to form error corrective polishing. The aspheric profile formula is shown as Eq. (16).

\[
g(x, y) = \frac{(x^2 + y^2) \cdot c}{1 + \sqrt{1 - c \cdot (k + 1)(x^2 + y^2)}} + \sum_{j=1}^{10} A_j \times (x^2 + y^2)^j
\]

In the formula (16), \( c \) is the curvature of the apex of the aspheric surface, \( k \) is the quadratic constant, and \( A_j \) is the coefficient of the higher-order term of the aspheric surface.

The aspheric profile of formula (16) is a rotationally symmetrical aspherical surface. And the sketch of TIF on a rotationally symmetrical aspherical mold is shown as Fig. 23. In Fig. 23, \( O_0X_0Y_0Z_0 \) is the coordinate system of the aspheric mold profile, and \( O_iX_iY_iZ_i \) is the coordinate system of the TIF when the polishing tool dwells at points \( P(x, y, z) \) on aspheric...
Fig. 21 Comparison of single-point polishing experiment and simulation results. a Sketch of measured polishing three-dimensional removal. b Sketch of predictive polishing three-dimensional removal. c Sketch of Y–Z section profile. d Sketch of X–Z section profile

Fig. 22 Measured removal volume and predicted removal volume

Fig. 23 Sketch of removing function residency
surface. The $Z$ axis of the TIF coordinate system coincides with the normal direction of point $P$.

During a polishing process, the polishing tool is fixed. The position of polishing dwell point is controlled by controlling the rotation and translation of aspheric mold. As is shown in Fig. 24a and b, the equidistant concentric circle path is adopted as the polishing path of the rotationally symmetric aspheric mold, and the polishing tool is fed along the concentric circle path through the rotation of the mold.

When the polishing tool tracks along the circular arc whose radius is $r_i$, the mold needs to rotate around the $Y$ axis from the initial position and translate the aspheric mold along $XZ$ section. As is shown in Fig. 24c, point $p(x_i, z_i)$ is an intersection point of aspherical meridian and polishing arc trajectory. During a polishing, it is necessary to move the point $p(x_i, z_i)$ to the origin $O$ after rotation, as is shown in Fig. 24d. After rotation and translation of the mold, the coordinate system $O_iX_iY_iZ_i$ and $O_0X_0Y_0Z_0$ are coincidence. The coordinate change of any point on the aspherical mold after rotation and translation has the following relationship:

$$
\begin{align*}
    x' &= z \sin \alpha_i + (x - x_i) \cos \alpha_i \\
    y' &= y \\
    z' &= (z - z_i) \cos \alpha_i - (x - x_i) \sin \alpha_i
\end{align*}
$$

(17)

Fig. 24 Polishing sketch. a Sketch of polishing tool feed. b Sketch of polishing contact area. c The initial position of the aspherical cross-section profile. d Polishing position of aspheric cross-section profile
where \( x_i \) and \( z_i \) are the coordinates of point \( P \) in the coordinate system \( O_0X_0Y_0Z_0 \), \( (x', y', z') \) are the coordinate values of the aspherical surface in the \( O_iX_iY_iZ_i \) coordinate system after the rotation and translation of mold, and \((x, y, z)\) are the coordinate values of the aspherical surface in the \( O_0X_0Y_0Z_0 \) coordinate system.

Combining formulas (16) and (17), the height of the aspherical mold after rotation and translation in the \( Z \) axis direction of the polishing contact area is:

\[
z' = g(x', y') = \sqrt{r^2 - x'^2 - y'^2} - (r - d) + g'(x, y) - k_a \times (\sqrt{r^2 - y'^2} - (r - d) + g'(0, y)) + \delta(x, y) \times (1 - k_a)
\]

Combining formulas (14) and (19), the TIF at the dwell point \( p(x_i, 0, z_i) \) is:

\[
\begin{align*}
R(x, y) &= \frac{k}{T} \int_0^T p(x, y) v(x', y') \, dt \\
&= \frac{k}{T} \int_0^T \frac{1}{L_x} \left( 1 + \beta(1 - e^{-\beta \gamma}) \right) \times dz' \times (1 - e^{-\beta \gamma}) \times v(x', y') \, dt \\
x' &= g(x, y) \times \sin \alpha_i + (x - x_i) \cos \alpha_i \\
y' &= y
\end{align*}
\]

Due to aspheric surface that has the characteristics of rotational symmetry, this paper only analyzes the removal of the cross-sectional profile on the meridian of the aspheric mold. When the TIF of formula (20) feed along the arc trajectory of radius \( r_i \), the removal amount on the meridian is:

\[
R_i = \sum R(x, y) \times dt = \int R(x, y) \frac{ds}{\alpha \rho} 
\]

In formula (18), \( x_i \) and \( z_i \) are the coordinates of the dwell point \( P \), and \( \alpha_i \) is the tangent angle of dwell point \( P \) on the meridian. Combining formulas (18) and (8), the compression of the polishing cloth in the polishing area is:

\[
\text{Table 4 Aspheric profile parameters}
\]

| Parameter | Value       |
|-----------|-------------|
| \( c \)   | 0.1297      |
| \( k \)   | -0.6727051  |
| \( A_2 \) | -2.3476874E-04 |
| \( A_4 \) | 1.4470198E-0  |
| \( A_6 \) | 3.3733336E-06 |
| \( A_8 \) | 2.9646086E-07 |
| \( A_{10} \) | -1.7194687E-08 |
| \( A_{12} \) | 3.5428031E-10 |
| \( A_{14} \) | 0            |
| \( A_{16} \) | 0            |
| \( A_{18} \) | 0            |
| \( A_{20} \) | 0            |

| Table 5 Form error corrective polishing experiment parameters |
|------------------------------------------------------------|
| Experimental parameters | Value       |
| Maximum compression of polishing cloth (mm) | 0.07         |
| Diamond abrasive grain size (μm) | 1           |
| Polishing tool speed (rpm) | 180          |
| Polishing ball radius (mm) | 1.5          |
| Total polishing time (min) | 7.9          |
| Polishing cloth thickness (mm) | 0.35         |
In formula (21), ρ is the distance that from aspherical meridian point to the axis of symmetry, dt is time, \( R(x, y) \) is the removal depth per unit time, ds is arc differential, \( \omega \) is the angular velocity of the aspherical mold, and L is the circular path.

As is shown in Fig. 25, the polishing tool has different TIF on different arc trajectories, which are discrete and superimposed. Formula (21) can discretize to removal control point matrix as follows:

\[
R_i = [0, 0, \cdots, h_{\rho 1}, h_{\rho 2}, \cdots, h_{\rho m}, 0, \cdots, 0]
\]  

(22)

The discretized TIF on different arc trajectories can be superimposed to form the overall TIF matrix on the entire meridian:

\[
\bar{R} = \begin{bmatrix} R_1^T, R_2^T, \cdots, R_n^T \end{bmatrix}
\]

(23)

The dwell time matrix of the polishing tool at each ring belt position is:

\[
\bar{T} = [T_1 \cdots T_i \cdots T_n]^T
\]

(24)

The form error on this meridian is H, and the dwell time matrix satisfying the following formula can be obtained by using the non-negative least square method [26]:

\[
\min \left\| \bar{R}T - H \right\|_2 \geq 0
\]

(25)

4.2 Surface form error corrective polishing experiment

In order to verify the form error corrective polishing effect of the mold, a small-diameter mold shown as Fig. 26 was polished. The aspheric profile coefficient of the mold is shown in Table 4. According to the polishing parameters in Table 5, TIF on the aspheric meridian section is calculated, and combined with the formula (25) and the surface error data of the mold, the dwell time is calculated by least square method.

Use Taylor profiler PGI2440 to measure the meridian of the mold after form error corrective polishing. The measurement result is shown in Fig. 27. Before polishing, the PV value of the mold is 0.268 μm in Y direction. After one cycle polishing, the PV value of the mold is reduced to 0.083 μm in Y direction. And the surface roughness measured by white light interferometer is shown as Fig. 28. Observed from the polishing results, the surface accuracy of the experimental mold has been improved.

![Fig. 27](image1.png)

Sketch of comparison before and after polishing

![Fig. 28](image2.png)

Surface roughness after polishing
5 Conclusion

For the polishing of small-size aspheric tungsten carbide molds, a small-size rigid ball wrapped with polyester fiber cloth is used as a polishing tool to adapt to the curvature of the aspheric mold. The main findings of this paper are summarized as follows:

1. In order to predict the pressure distribution of the polishing base area and the contact area, the investigations of polyester fiber cloth structure and viscoelastic property were conducted. From the investigations, it can be seen that the structure of the polyester fiber cloth causes the fluctuation of the polishing pressure; the irreversible deformation of the polyester fiber cloth causes the polishing contact area to be approximately elliptical; and the relationship between the polishing contact pressure and the deformation is nonlinear.

2. Combined with pressure distribution and velocity distribution that were obtained from fiber cloth property and the movement of polishing tool respectively, the TIF of polishing process is obtained.

3. In this study, the polishing dwell time, the relative speed of the workpiece, and polishing tool are directly proportional to the material removal rate, and the polishing pressure is proportional to the material removal rate when the polishing pressure is low. TIF is simulated according to the experimental parameters, and the maximum error of removal volume between simulation and actual polishing TIF is less than 6.9% when the polishing pressure is small. The accuracy of the theoretical prediction of the TIF is verified.

4. On the basis of obtaining TIF, a form error corrective polishing test for rotationally symmetrical aspheric mold was conducted. After the polishing test, the PV value of form error was reduced from 0.268 μm to 0.083 μm, which verified the effectiveness of TIF for micro-aspheric mold in this study.

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Data availability The datasets supporting the results of this article are included within the article and its additional files.

Declarations

Ethics approval Not applicable.

Consent for publication We would like to submit the manuscript entitled “Finishing of micro-aspheric tungsten carbide mold using small viscoelastic tool” for your consideration for publication in the International Journal of Advanced Manufacturing Technology. No conflict of interest exists in the submission of this manuscript, and the manuscript is approved by all authors for publication. On behalf of the co-authors, we declare that the work described was original review work that has not been published previously, and not under consideration for publication elsewhere, in whole or in part.

Competing interests The authors declare no competing interests.

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