Role of the singular factors in the standard fits for initial parton densities

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Total resummation of double- and single-logarithms of $x$ contributing to the spin-dependent structure function $g_1(x, Q^2)$ ensures its steep rise at small $x$. In the asymptotic limit $x \to 0$, the resummation leads to the Regge behavior of $g_1$ and allows to calculate the non-singlet and singlet intercepts of $g_1$. DGLAP lacks such a resummation but suggests special phenomenological fits for the initial parton densities such that the singular factors $x^{-\gamma}$ in the fits mimic the resummation and also provide $g_1$ with the steep (power-like) rise at the small-$x$ region. Accounting for the total resummation of logarithms of $x$ allows to drop the singular factors in the fits and leads to a remarkable simplification of the fits.

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I. INTRODUCTION

The standard theoretical instrument for investigating the DIS structure function $g_1(x, Q^2)$ is DGLAP\cite{1}. In this approach, $g_1^{DGLAP}$ is a convolution of the coefficient functions $C_{DGLAP}$ and evolved parton distributions which are also expressed as a convolution of the splitting functions $P_{DGLAP}$ and initial parton densities. The latter are found with fitting experimental data at $x$ close 1 and $Q^2 \sim 1$ GeV$^2$. There is an obvious asymmetry in treating $Q^2$- and $x$- contributions in DGLAP. Indeed, the leading $Q^2$- contributions, $\ln(Q^2)$, are accounted to all orders in $\alpha_s$ whereas $C_{DGLAP}(x)$ and $P_{DGLAP}(x)$ are known in first two orders of the perturbative QCD. The reason for such asymmetry is the fact that originally DGLAP was constructed for operating at large $Q^2$ and $x$ not so far from 1. In this region $\ln^k Q^2$ were large whereas $x$- contributions from higher loops were small and could be neglected. On the contrary, in the small-$x$ region the situation looks opposite: logarithms of $x$, namely double logarithmic (DL), i.e. the terms $(\alpha_s \ln^2(1/x))^k$, and single logarithms (SL), the terms $(\alpha_s \ln(1/x))^k$ with $k=1,2,...$, are becoming quite sizable and should be accounted to all orders in $\alpha_s$. When the total resummation of DL terms was done\cite{2}, it led to new expressions, $g_1^{DL}$, for $g_1$. In particular, Refs.\cite{2} demonstrated that the small-$x$ asymptotics of $g_1^{DL}$ was of the Regge (power-like) form and was much greater than the well-known small-$x$ asymptotics of $g_1^{DGLAP}$ (see Eq.\cite{3}) at $x \leq 0.01$. However, this result was strongly criticized (see e.g. Refs.\cite{3}) where it was shown that using the standard NLO DGLAP formulae together with appropriate fits for initial parton densities led to the opposite result: $g_1^{DL} \ll g_1^{DGLAP}$ at small $x$. After that the total resummation of logarithms of $x$ for $g_1$ was claimed irrelevant for available range of $x$.

Strictly speaking, comparison of results of Refs.\cite{2} and\cite{3} could not be done in a straightforward way because DGLAP -evolution equations have always used the running $\alpha_s$, with the parametrization

$$\alpha_s^{DGLAP} = \alpha_s(Q^2),$$

whereas Refs.\cite{2} operated with $\alpha_s$ fixed at an unknown scale. Trying to set a scale for $\alpha_s$ when the approach of Refs.\cite{2} is used, Refs.\cite{4} suggested that the argument of $\alpha_s$ should be $Q^2$ like in DGLAP. The parametrization $\alpha_s^{DGLAP} = \alpha_s(Q^2)$ appears when the argument of $\alpha_s$ in each of the ladder rungs of the involved Feynman graphs is $k_1^2$, with $k$ being the momentum of the upper parton (a quark or a gluon) of the rung. A deeper investigation of this matter\cite{2} led us to the conclusion that the DGLAP-parametrization of Eq.\cite{1} can be a good approximation at $x$ not far from 1 only. Instead of it, a new parametrization was suggested where the argument of $\alpha_s$ in every ladder rungs is the virtuality of the horizontal gluon (see Ref.\cite{2} for detail). This parametrization is universally good for both small $x$ and large $x$. It converges to the DGLAP-parametrization at large $x$ but differs from it at small $x$. Using this new parametrization allowed us to obtain in Refs.\cite{2} expressions for $g_1$ accounting for all-order resummmations of DL and
SL terms, including the running $\alpha_s$ effects. In the first place, it was used to obtain numerical values of the intercepts of the singlet and non-singlet $g_1$. It is worth to mention that these results were immediately confirmed by several independent groups who fitted available HERMES data and extrapolated them into the asymptotic region $x \to 0$.

Nevertheless, it is known that, despite DGLAP lacks the total resummation of $\ln x$, it successfully operates at $x \ll 1$. As a result, the common opinion was formed that not only the total resummation of DL contributions in Refs. [2] but also the much more accurate calculations performed in Refs. [7] should be out of use at available $x$. In Ref. [8] we argued against such a point of view and explained why DGLAP can be so successful at small $x$: in order to be able to describe the available experimental data, DGLAP uses the singular fits (see for example Refs. [10, 11]) for the initial parton densities. Singular factors (i.e. the factors which $\to \infty$ when $x \to 0$) in the fits mimic the total resummation of Refs. [7]. Using the results of Ref [8] allows to simplify the quite complicated structure of the standard DGLAP fits down to normalization constants at small $x$.

II. DIFFERENCE BETWEEN DGLAP AND OUR APPROACH

In DGLAP, $g_1$ is expressed through convolutions of the coefficient functions and evolved parton distributions. As convolutions look simpler in terms of integral transforms, it is convenient to represent $g_1$ in the form of the Mellin integral. For example, the non-singlet component of $g_1$ can be represented as follows:

$$g_{1,DGLAP}^{NS}(x, Q^2) = (e_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{(1/x)^{\omega}}{Q^2} C_{DGLAP}(\omega) \delta q(\omega) \exp \left[ \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \gamma_{DGLAP}(\omega, \alpha_s(k^2)) \right]$$  \hfill (2)

with $C_{DGLAP}(\omega)$ being the non-singlet coefficient functions, $\gamma_{DGLAP}(\omega, \alpha_s)$ the non-singlet anomalous dimensions and $\delta q(\omega)$ the initial non-singlet quark densities in the Mellin (momentum) space. The expression for the singlet $g_1$ is similar, though more involved. Both $\gamma_{DGLAP}$ and $C_{DGLAP}$ are known in first two orders of the perturbative QCD. Technically, it is simpler to calculate them at integer values of $\omega = n$. In this case, the integrand of Eq. (2) is called the $n$-th momentum of $g_1^{NS}$. When the moments for different $n$ are known, $g_1^{NS}$ at arbitrary values of $\omega$ is obtained with interpolation of the moments. Expressions for the initial quark densities are defined from phenomenological consideration, with fitting experimental data at $x \sim 1$. Eq. (2) shows that $\gamma_{DGLAP}$ govern the $Q^2$- evolution whereas $C_{DGLAP}$ evolve $\delta q(\omega)$ in the $x$-space from $x \sim 1$ into the small $x$ region. When, at the $x$-space, the initial parton distributions $\delta q(x)$ are regular in $x$, i.e. do not $\to \infty$ when $x \to 0$, the small-$x$ asymptotics of $g_{1,DGLAP}$ is given by the well-known expression:

$$g_{1,DGLAP}^{NS}, g_{1,DGLAP}^S \sim \exp \left[ \sqrt{\ln(1/x) \ln \left( \ln(Q^2/\mu^2)/\ln(\mu^2/\Lambda_{QCD}^2) \right)} \right].$$  \hfill (3)

On the contrary, when the total resummation of the double-logarithms (DL) and single-logarithms of $x$ is done, the Mellin representation for $g_{1,NS}^{DS}$ is

$$g_{1,NS}^{DS}(x, Q^2) = (e_q^2/2) \int_{-\infty}^{\infty} \frac{d\omega}{2\pi i} \frac{(1/x)^{\omega}}{Q^2} C_{NS}(\omega) \delta q(\omega) \exp \left( H_{NS}(\omega) \ln(Q^2/\mu^2) \right),$$  \hfill (4)

with new coefficient functions $C_{NS}$,

$$C_{NS}(\omega) = \frac{\omega}{\omega - H_{NS}(\omega)},$$  \hfill (5)

and anomalous dimensions $H_{NS}$,

$$H_{NS} = (1/2) \left[ \omega - \sqrt{\omega^2 - B(\omega)} \right].$$  \hfill (6)

where

$$B(\omega) = (4\pi C_F(1 + \omega/2)A(\omega) + D(\omega))/(2\pi^2).$$  \hfill (7)

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1 The parametrization of see Ref. [2] was used later in Refs. [2] for studying the small-$x$ contribution to the Bjorken sum rule.
Equations for the non-singlets are written in the x-space as convolutions of splitting functions $P_{qg}$ with evolved parton distributions $\Delta q$ and the latter are written as another convolution:

$$\Delta q(x) = C_q(x, y) \otimes \delta q(y),$$

with $C_q$ being the coefficient function. Written in this way, $\Delta q$ is sometimes believed to be less singular than $\delta q$ because of the evolution. However applying the Mellin transform to Eq. (14) immediately disproves it.
IV. CONCLUSION

Comparison of Eqs. (3) and (13) shows explicitly that the singular factor $x^{-\alpha}$ in the Eq. (11) for the initial quark density converts the exponential DGLAP-asymptotics into the Regge one. On the other hand, comparison of Eqs. (10) and (13) demonstrates that the singular factors in the DGLAP fits mimic the total resummation of logarithms of $x$. These factors can be dropped when the total resummation of logarithms of $x$ performed in Ref. [7] is taken into account. The remaining, regular $x$-terms of the DGLAP fits (the terms in squared brackets in Eq. (11)) can obviously be simplified or even dropped at small $x$ so that the rather complicated DGLAP fits can be replaced by constants. It immediately leads to an interesting conclusion: the DGLAP fits for $\delta q$ have been commonly believed to represent non-perturbative QCD effects but they actually mimic the contributions of the perturbative QCD, so the whole impact of the non-perturbative QCD on $g_1$ at small $x$ is not large and can be approximated by normalization constants.

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