Kinematics analysis and optimization of 6R manipulator

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Abstract. In order to solve the problem of 6R manipulator kinematics, this paper analyzes the inverse kinematics solution and optimizes the solution process of forward kinematics. Aimed to the structural characteristics of the manipulator, this paper first introduces the concept of the natural invariants, establishes the Denavit-Hartenberg (D-H) model based on the natural invariants and proposes a new iterative algorithm for manipulator forward kinematics. Then, the inverse kinematics based on the natural invariants is proposed, which is divided into the position inverse solution and the inverse attitude solution, and the analytical solution is obtained by analyzing the singularity of the solution. Finally, the visual simulation model is built with MFC and Coin4D. The simulation results show that the proposed forward and inverse kinematics method of the six-degree-of-freedom manipulator has small error and high efficiency, and the accuracy and real-time performance of the algorithm are verified.

1. Introduction

The kinematics of manipulators is a central problem in the automatic control of manipulator. The kinematics of the manipulator includes positive kinematics and inverse kinematics. The positioning process of the manipulator is to calculate the joint variables of the manipulator according to the desired posture, so that the computer can control the manipulator. Domestic and foreign scholars have done a lot of research for the kinematics of the manipulator[1-4], at present, the analytic method[5,6] and the iterative method are mainly used to solve the inverse kinematics of the manipulator. The advantage of using the analytical method to solve the inverse kinematics of the manipulator is very obvious, all the solutions can be obtained by analyzing the model of the manipulator, and the solution is accurate and fast. Jorge Angeles[7] used Euclidean norm to solve the problem of six degrees of freedom decoupling manipulator, and simplified the solution of position inverse kinematics into a quartic equation of one variable by analytic method. The process of solving multivariate equations by iterative method is relatively simple, but it is impossible to determine how many sets of solutions are involved because of the initial value selection problem, and it is difficult to get all the solutions, and there is a danger that the iteration does not converge. Kalra[8] et al proposed an inverse kinematics algorithm for a six degrees of freedom manipulator based on genetic algorithm.

In this paper, by introducing the concept of natural invariants, a kinematic D-H model based on Axis Invariant is established and solved. On this basis, a new forward kinematics iterative algorithm is proposed. The inverse kinematics is divided into position inverse solution and inverse pose solution by pose separation and solving process. The analytical solution is obtained by analysis. Finally, the algorithm is verified correctness and feasibility.
2. Basic theory
According to the topological characteristics between the link of manipulator, the concept of the natural invariants is introduced. The movement of the manipulator can be more conveniently understood by the natural invariants.

2.1. Natural Invariants
When the system is at initial state, the coordinate system of axis $l$ and axis $T$ is $F[l]$ and $F[T]$ respectively; the projection relationship between $F[l]$ and $F[T]$ can be considered as linear transformation. Since both $F[l]$ and $F[T]$ are Cartesian coordinate systems, an orthogonal base transformation matrix $\mathbf{Q}$ can be used to describe:

$$\mathbf{Q} = \begin{pmatrix} e_x & e_y & e_z \end{pmatrix}$$

Taking the eigenvector corresponding to the eigenvalue of 1, and the eigenvector represented by the coordinate system of $F[l]$ is $e_l$, which can be obtained:

$$\mathbf{Q} e_l = e_l$$

Note is expressed as $\bar{e}_l = \begin{pmatrix} x_l & y_l & z_l \end{pmatrix}^T$ in the $F[T]$ coordinate system. According to the linear transformation matrix $\mathbf{Q}$, obtain

$$\begin{pmatrix} x_T \\ y_T \\ z_T \end{pmatrix} = \mathbf{Q} \begin{pmatrix} x_l \\ y_l \\ z_l \end{pmatrix} \Rightarrow \bar{e}_l = \mathbf{Q} e_l$$

Then the coordinate vectors represented by $e_l$ in $F[l]$ and $F[T]$ coordinate systems are the same, namely:

$$\bar{e}_l = e_l$$

It can be seen from the above equation that $\bar{e}_l$ and $e_l$ do not depend on the respective reference coordinate systems $F[l]$ and $F[T]$, so $\bar{e}_l$ or $e_l$ is said to be a natural invariant.

2.2. D-H Frame based on natural invariants
The D-H Frames plays a very important role in the kinematics calculation of the manipulator. The D-H system of each link is established based on the natural invariant. The D-H Frames of the link $T$ and link $l$ is represented by $F[l]$ and $F[T]$ respectively, as shown in figure 1.

(i)Assume that $z_T$ and $z_l$ pass the natural invariants $\bar{e}_T$ and $\bar{e}_l$ respectively.

(ii)Draw the common perpendicular of axis $z_T$ and axis $z_l$. Mark the points intersecting with $z_T$ and $z_l$ as $O_T$ and $O_l$ respectively, wherein $O_T$ is the origin of the Middle Frame $F[l]$. $O_l$ is the origin of Frame $F[l]$. Define the direction from axis $z_T$ to axis $z_l$ as axis $x_l$. If $z_T \parallel z_l$, select one common perpendicular line to be axis $x_l$.

The D-H parameter is determined:

$$c_T = r_T - \text{The Axial Offset Distance from the point } O_T \text{ to the point } O_T.$$
\[ \phi_\tau = \phi^E_\alpha \ - \text{Rotational Angle from the axis } \overline{r_x} \text{ to axis } l_x. \]

\[ a_\tau = r^T_1 \ - \text{The Axis Distance from the point } O_x \text{ to the point } O_1. \]

\[ \alpha_\tau = \phi^E_\beta \ - \text{The Torsional Angle from the axis } \overline{l_1} \text{ to axis } l_i. \]

\[ \phi^0_\tau \ - \text{The initial angle between two adjacent } x \text{-axes.} \]

From the above modeling process, the general formula of the transformation matrix of the adjacent rod coordinate system is: (Note: \( C() = \cos() \), \( S() = \sin() \))

\[
\begin{align*}
^T Q_i = \begin{bmatrix}
C(\phi_\tau) & -\lambda_\tau \cdot S(\phi_\tau) & \mu_\tau \cdot S(\phi_\tau) \\
S(\phi_\tau) & \lambda_\tau \cdot C(\phi_\tau) & -\mu_\tau \cdot C(\phi_\tau) \\
0 & \mu_\tau & \lambda_\tau
\end{bmatrix}
\end{align*}
\]

\[ (5) \]

Wherein \( \lambda_i = C(\alpha_i) \), \( \mu_i = S(\alpha_i) \)

\[ ^1 Q_\tau = ^1 Q_n^T \cdot ^1 Q_\tau = ^1 Q_\tau^{-1} = ^1 Q_\tau^{-T} \]

\[ (6) \]

The rotational transformation matrix \( ^T Q_\tau \) of the D-H system is a unit orthogonal matrix, and its inverse matrix \( ^1 Q_\tau \) is equal to its own transpose. The characteristic of \( ^T Q_\tau \) provides great convenience for solving the inverse kinematics of manipulator.

3. Kinematics analysis of the manipulator

3.1. Forward kinematics of the manipulator

The schematic diagram of the industrial robot arm is shown in figure 2. From bottom to top, it consists of joints such as base, waist, shoulder, elbow, and wrist.

The kinematics model of the manipulator is obtained by obtaining the position and attitude of the end of the manipulator relative to the base of the manipulator. D-H Frame was established according to the kinematics model of the manipulator, D-H parameters are shown in table 1.

According to the D-H Frame of manipulator, the positive equation of motion is:

\[
\begin{align*}
^1 r_7 &= ^1 r_1 + ^1 Q_1 \cdot ^2 r_2 + ^1 Q_1 \cdot ^2 Q_2 \cdot ^3 r_3 + \ldots \\
^1 Q_1 \cdot ^2 Q_1 \cdot ^3 r_2 + ^1 Q_1 \cdot ^2 Q_2 \cdot ^3 Q_2 \cdot ^4 r_4 + ^1 Q_2 \cdot ^2 Q_3 \cdot ^4 Q_3 \cdot ^5 r_5 + ^1 Q_3 \cdot ^2 Q_4 \cdot ^3 Q_4 \cdot ^6 r_6
\end{align*}
\]

\[ (7) \]
Figure 2. Schematic diagram of manipulator model.

Where \( ^{\mathcal{T}} r_i \) is the coordinate of the origin of the coordinate system \( i \) in the \( \mathcal{T} \) coordinate system.

### Table 1. D-H parameters of manipulator.

|   | \( a_i \) (m) | \( c_i \) (m) | \( \alpha_i \) (deg) | \( \phi_i \) (deg) |
|---|--------------|--------------|--------------------|-------------------|
| 1 | 0.16         | 0.43         | 90                 | 90                |
| 2 | 0.58         | 0            | 0                  | 90                |
| 3 | 0.125        | 0            | 90                 | 0                 |
| 4 | 0            | 0.65         | 90                 | 180               |
| 5 | 0            | 0            | 90                 | -90               |
| 6 | 0            | 0.16         | 0                  | 0                 |

Since positive kinematics involves a lot of matrix operations, it is very time consuming. In order to improve the speed of the positive movement of the manipulator, an iterative algorithm is proposed in this paper. The specific form is shown in (8).

\[
^{1}r_7 = ^{1}r_2 + ^{1}Q_2 \cdot \left( ^{2}Q_3 + ^{3}Q_3 \cdot (^{3}r_4 + ^{4}Q_3 \cdot (^{4}r_5 + ^{5}Q_4 \cdot (^{5}r_6 + ^{6}Q_4 \cdot ^{6}r_7)) \right) \tag{8}
\]

The iterative algorithm established in this paper avoids the matrix multiplication operation by calculating the product of the inner layer matrix and the three-dimensional vector. Compared with the positive motion equation established by (7), the calculation amount can be greatly reduced, as shown in table 2.

### Table 2. Computational complexity of kinematic equations.

|                        | Addition calculation | Multiplication calculation |
|------------------------|----------------------|---------------------------|
| General positive kinematics equation | 117                  | 153                       |
| Iterative positive kinematics equation | 45                   | 45                        |

3.2. Position inverse solution

The position of the wrist of the orthogonal decoupling manipulator is only related to the first three joint angles, and the desired wrist position is \( ^{3}_{d}r_{IC} \).

\[
^{3}_{d}r_{IC} = ^{1}r_2 + ^{1}Q_2 \cdot \left( ^{2}r_3 + ^{2}Q_3 \cdot (^{3}r_4 + ^{3}Q_4 \cdot ^{3}r_6) \right) \tag{9}
\]
Let \( \mathbf{r}_l = [a_l \quad \mu_l \quad c_l]^T \), then
\[
\mathbf{r}_l = \mathbf{Q} \mathbf{r}_l' \tag{10}
\]
Where \( \mathbf{r}_l' \) represents the projection of position vector \( \mathbf{r}_l \) in the \( l \) coordinate system.

Put (10) into (9), obtain:
\[
2\mathbf{Q}_3 \mathbf{r}_3' + 3\mathbf{Q}_4 \mathbf{r}_4' + 3\mathbf{Q}_5 \mathbf{r}_5'(c) = 2\mathbf{Q}_3(\mathbf{r}_C - \mathbf{r}_2) \tag{11}
\]
Take the Euclidean norm on both sides of the (11):
\[
A \cdot C(\phi_1) + B \cdot S(\phi_2) + C \cdot C(\phi_3) + B \cdot S(\phi_3) = -E \tag{12}
\]
wherein:
\[
A = 2a_1 \cdot x_c + 2a_1 \cdot y_c, \\
C = 2a_2 \cdot a_2 - 2c_1 \cdot c_2 \cdot \mu_2 \cdot \mu_2, \\
D = 2a_3 \cdot c_2 \cdot \mu_2 + 2c_4 \cdot a_2 \cdot \mu_3 \\
E = a_4^2 + c_2^2 + a_2^2 + c_2^2 + c_4^2 \\
\text{\( \lambda_1 \) + 2c_2 \cdot c_2 \cdot \lambda_2 + 2c_4 \cdot c_3 \cdot \lambda_3 + 2c_4 \cdot c_2 \cdot \lambda_3 \cdot \lambda_3} \\
\text{\( \lambda_4 \) + 2c_4 \cdot z_c - x_c^2 - y_c^2 - z_c^2 - a_1^2 - c_1^2}
\]
Since the third component of (11) is equal:
\[
F \cdot C(\phi_1) + G \cdot S(\phi_2) + H \cdot C(\phi_3) + I \cdot S(\phi_3) = -J \tag{13}
\]
Wherein:
\[
F = y_c \cdot \mu_1, G = -x_c \cdot \mu_1, H = -c_4 \cdot \mu_2 \cdot \mu_3, I = a_3 \cdot \mu_2 \\
J = c_2 + c_1 \cdot \lambda_1 + c_4 \cdot \lambda_2 \cdot \lambda_3 + c_3 - z_c \cdot \lambda_3
\]
Express \( \phi_1 \) by \( \phi_3 \)
\[
S(\phi_1) = \frac{1}{\Delta_{11}} \begin{bmatrix} F \left( -E \cdot D \cdot S(\phi_3) - C \cdot C(\phi_3) \right) \\ -A \left( -J - I \cdot S(\phi_3) - H \cdot C(\phi_3) \right) \end{bmatrix} \tag{14}
\]
\[
C(\phi_1) = \frac{1}{\Delta_{11}} \begin{bmatrix} G \left( -E \cdot D \cdot S(\phi_3) - C \cdot C(\phi_3) \right) \\ -B \left( -J - I \cdot S(\phi_3) - H \cdot C(\phi_3) \right) \end{bmatrix}
\]
Wherein \( \Delta_{11} = -2a_1 \cdot (x_c^2 + y_c^2) \cdot \mu_1 \cdot \mu_1 \)
\( \Delta_{11} \) vanishes if and only if any of the factors \( a_1 \cdot x_c^2 + y_c^2 \) and \( \mu_1 \) does. (12) or (13) contains only the unknown variable \( \phi_3 \), and (15) will degenerate into a quadratic form. If \( x_c^2 + y_c^2 = 0 \), the wrist center is on the axis 1, and if \( a_1 \cdot \mu_1 \) is simultaneously zero, axis 1 and axis 2 is coaxial and the configuration is wrong.
The squares of the two sides of (14) are respectively squared. And use the universal formula to replace
\[ C(\phi_3), S(\phi_3) \] with
\[ \tan \left( \frac{\phi_3}{2} \right) \] where
\[ R \cdot t^4 + S \cdot t^3 + T \cdot t^2 + Ut + V = 0 \] (15)

Wherein:
\[ R = 4a_1^2 \left( H - J \right)^2 + \mu_1^2 \left( C - E \right)^2 - 4a_1^2 \mu_1^2 \rho^2 \]
\[ S = 16a_1^2 I \left( J - H \right) + 8\mu_1^2 D \left( E - C \right) \]
\[ T = 8a_1^2 \left( J^2 - 2I^2 - H^2 \right) + 2\mu_1^2 \left( E^2 + 2D^2 - C^2 \right) - 8a_1^2 \mu_1^2 \rho^2 \]
\[ U = 16a_1^2 I \left( J + H \right) + 8\mu_1^2 D \left( E + C \right) \]
\[ V = 4a_1^2 \left( H + J \right)^2 + \mu_1^2 \left( C + E \right)^2 - 4a_1^2 \mu_1^2 \rho^2 \]

The four sets of solutions obtained by (15) are solved, and \( \phi_3 \) is further obtained.
\[ \phi_3 = A \tan 2(\sin(\phi_3), \cos(\phi_3)) \] (16)

Substituting \( \phi_3 \) into (14) yields \( \phi_1 \), \( \phi_2 \) is calculated from the first two scalar equations of (11).

\[ C(\phi_2) = \frac{1}{\Delta_{22}} \begin{bmatrix} X \cdot \left( x_c C(\phi_1) + y_c S(\phi_1) - a_1 \right) \\ - Y \cdot \left( -\lambda x_c S(\phi_1) + y_c \lambda C(\phi_1) \right) + \mu_1 \cdot (z_c - c_1) \end{bmatrix} \]
\[ S(\phi_2) = \frac{1}{\Delta_{22}} \begin{bmatrix} Y \cdot \left( x_c C(\phi_1) + y_c S(\phi_1) - a_1 \right) \\ + X \cdot \left( -\lambda x_c S(\phi_1) + y_c \lambda C(\phi_1) \right) + \mu_1 \cdot (z_c - c_1) \end{bmatrix} \] (17)

Wherein:
\[ \Delta_{22} = X^2 + Y^2 \]
\[ X = a_2 + c_1 \mu_3 \cdot S(\phi_3) + a_3 \cdot C(\phi_3) \]
\[ Y = -a_3 \lambda \cdot S(\phi_3) + c_1 \lambda \mu_3 \cdot C(\phi_3) + c_4 \mu_2 \lambda + c_1 \mu_2 \]

If \( \Delta_{22} = 0 \) (second singularity), then \( X, Y \) is zero at the same time, then the wrist is located on the axis of the second axis of rotation, then \( \phi_2 \) can take any value, for convenience we can take a value of zero.

3.3. Attitude inverse solution

In this paper, the normal unit vector \( \mathbf{N} \) and the radial unit vector \( \mathbf{R} \) at the end of the manipulator are used to describe the attitude of the end of manipulator.

\[ \mathbf{N} = \mathbf{Q}_0 \cdot \mathbf{N} \]
\[ \mathbf{R} = \mathbf{Q}_0 \cdot \mathbf{R} \] (18)
Wherein $\mathbf{N} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$, $\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$.

The last three axis of the orthogonally decoupled 6 DOF manipulator intersect at one point. The normal direction to the end of the manipulator is controlled by $\phi_4$ and $\phi_5$ to align with the end of desired normal direction, and the radial direction end of the manipulator is radially aligned with the end of desired radial direction by $\phi_6$.

Given the normal direction $\mathbf{N}$ of the desired end of the manipulator, obtain:

$$\mathbf{Q}_4 \cdot \mathbf{N} = \mathbf{Q}_4^{-1} \cdot \mathbf{N}$$

(19)

Put $\mathbf{N} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$ into (19), obtain

$$\begin{bmatrix} C(\phi_4) \cdot C(\phi_5) \\ S(\phi_4) \cdot C(\phi_5) \\ -C(\phi_5) \end{bmatrix} = \mathbf{N}$$

(20)

Wherein $\mathbf{N} = \mathbf{Q}_4^{-1} \cdot \mathbf{N}$

$$\begin{cases} \phi_4 = A \tan(\mathbf{N}^{[2]} \mathbf{N}^{[1]}) \\ \phi_5 = A \cos(\mathbf{N}^{[3]}) \end{cases}$$

or

$$\begin{cases} \phi_4' = -\phi_4' \\ \phi_5' = \phi_5 + \pi \end{cases}$$

(21)

After determining the joint angle $\phi_4$ and $\phi_5$, it is necessary to rotate the joint angle $\phi_6$ to ensure that the radial direction of the end of the manipulator is aligned with the radial direction of the desired radial direction. Then

$$\phi_6 = a \cos\left(\mathbf{Q}_6 \cdot \mathbf{R}ight)^T \cdot \mathbf{R}$$

(22)

It can be seen from (21) and (22) that there are at most 4 sets of attitude inverse solutions, and combined with the position inverse solution of the first three-axis, so the inverse solution of the 6 DOF manipulator has up to 16 groups.

4. Algorithm verification

In this paper, a set of visual robotic kinematics simulation tools is written by MFC and Coin4D, and the correctness of the inverse kinematics solution is verified by the method of forward and inverse kinematics mutual proof. Given the desired pose of the manipulator:

$$\mathbf{r}_7 = \begin{bmatrix} 0 & -0.215122 & 1.490120 \end{bmatrix}^T$$

$$\mathbf{N} = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}$$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

(23)

Deriving the desired wrist center coordinates according to the given position and posture, obtained by (24).

$$\mathbf{r}_{7c} = \mathbf{r}_7 - 0.16 \cdot \mathbf{N}$$

(24)

The joint angles are solved by the simulation software written in this paper. The results are shown in table 3. The joint angles of the four sets are obtained by the inverse kinematics of manipulator (corresponding position inverse solution), the motion of the manipulator is shown in figure 3.
Table 3. Inverse kinematics results of the manipulator.

|   | $\phi_1$ (°) | $\phi_2$ (°) | $\phi_3$ (°) | $\phi_4$ (°) | $\phi_5$ (°) | $\phi_6$ (°) |
|---|-------------|-------------|-------------|-------------|-------------|-------------|
| 1 | 0           | 45.000317   | 44.999490   | 0           | 0.000193    | 0           |
| 2 | 0           | 45.000317   | 44.999490   | 180         | 179.9998    | 180         |
| 3 | 0           | 8.566680    | 113.229455  | 180.000002  | -148.203    | 180         |
| 4 | 0           | 8.566680    | 113.229455  | 0.000002    | -31.7961    | 0           |
| 5 | -180        | 20.158965   | 20.127770   | -0.000003   | -130.028    | -180        |
| 6 | -180        | 20.158965   | 20.127770   | 179.999997  | -49.7132    | 0           |
| 7 | -180        | -43.100641  | 138.101175  | 0           | 174.9994    | -180        |
| 8 | -180        | -43.100641  | 138.101175  | -180        | 5.000534    | 0           |

Note: The solution of the inverse kinematics algorithm does not consider the two values of $\phi_6$, so only 8 sets of solutions are obtained.

In order to verify the correctness and accuracy of the inverse kinematics of the manipulator, the output of the inverse kinematics is input into the forward kinematics, and the forward kinematics results are compared with the given expected poses to obtain the calculation error. As shown in table 4. The obtained solution of inverse kinematics is substituted into forward kinematics, and the error is less than $10^{-7}$m, which proves the accuracy of inverse kinematics.

Table 4. Error of inverse kinematics results.

| Position | error(mm) | normal vector error(mm) | radial vector error(mm) |
|----------|-----------|-------------------------|-------------------------|
| 1        | 7.95e-8   | 0                       | 1.00e-10                |
| 2        | 7.95e-8   | 0                       | 0                       |
| 3        | 5.53e-8   | 2.30e-8                 | 2.71e-8                 |
| 4        | 5.52e-8   | 0                       | 7.70e-9                 |
| 5        | 5.33e-8   | 1.33e-7                 | 0                       |
| 6        | 5.33e-8   | 1.33e-7                 | 2.39e-8                 |
| 7        | 4.05e-8   | 1.33e-7                 | 0                       |
| 8        | 4.05e-8   | 1.33e-7                 | 3.80e-9                 |

Note: All error terms are the square root of the difference between the forward kinematic output and the given value. The zero mean that the error less than 1e-10.

5. Conclusion
In this paper, the motion model of the manipulator based on the natural invariants is established to solve the kinematics problem of the 6 DOF manipulator. A forward kinematic iterative algorithm is proposed, which is faster and more efficient than the traditional method of positive kinematics. At the same time, the method of pose separation is used to divide the solution process into position inverse solution and inverse pose solution, which reduces the complexity of the solution. Then the singular phenomena appearing in the solution process are analyzed and solved, and finally the analytical solution is obtained. Finally, the visual simulation of forward and inverse kinematics is carried out, which proves that the method is accurate, fast and efficient, and has important influence on improving the position accuracy and working efficiency of the manipulator.

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