Testing Brane World Models with Ultra High Energy Cosmic Rays

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ABSTRACT: The arrival time coherence of particles in the Ultra High Energy Air Showers where the center of mass energy of the interaction is of the order of $10^{15}\,eV$, puts strict constraint on the propagation of particles in a hypothetical extra-dimension. We first argue that at such high energies bulk modes and massive KK-modes can be produced abundantly and in many models their phase space volume is larger than confined modes. Then, we study the minimum propagation time in one and two-brane models and show that a large part of the parameter space of these models are ruled out unless the confinement of fields is protected by symmetries up to energies not accessible even to the high energy tail of Ultra High Energy Cosmic Rays (UHECRs). As a by-product we confirm the result obtained in some previous works about the close relation between a small Cosmological Constant and the hierarchy problem.

KEYWORDS: Large Extra-Dimensions, Ultra High Energy Cosmic Rays
1. Introduction

Ever since the proposition by Th. Kaluza and O. Klein in 1920s to use a 5-dimensional space-time for unification of Gravity with Electromagnetism [1], space-times with more than 4 dimensions have been the hope of physicists to solve problems of High Energy Particle Physics. Last ideas in these series are suggestions by N. Arkani-Hamed, et al. [2] and by L. Randall and R. Sundrum [3] for using large extra dimensions and localized matter to solve mass hierarchy problem inspired by some previous works of V. Rubakov and M. Shaposhinkov [4] on domain walls in higher dimensional spaces and P. Horava and E. Witten [5] on M-theory models with Compactification in spaces with D-brane boundaries.

In the first proposals only gravity could propagate in the bulk. It has been however found that the total localization of all fields except graviton on 3-branes is not realistic. In fact brane solution are cosmologically unstable and at least one scalar bulk field (radion) [6] [7] is necessary to stabilize the distance between branes. In some brane models inflaton [8] also has to propagate to the bulk to make inflation with necessary properties. A deeper insight to the propagation of gravitational waves and massive particles with bulk modes in models with infinite bulk has illustrated that even the warping of the bulk can not stop their escape from branes [10] [11].
Most of localization mechanisms are evolutionary i.e. based on special configuration of matter fields with localized properties like topological defect solutions which can arise during phase transitions in the Early Universe [12]. In these models the real dimensionality of the space-time is larger than 4 but our Universe is confined to a domain wall (3-brane) where fields specially at low energies (with respect to quantum gravity scale) are geometrically or gravitationally localized. However, geometrical settings like a warped metric are not always enough to confine fields on the branes. It has been shown that in spaces with \( \geq 3 \) extra-dimensions gravity can not be geometrically confined to a 3-brane and p-form fields in the bulk must be added to stabilize the brane (defect) [13]. In 5-dim. models gravity and scalar fields can be localized on the brane with negative tension [14] (or the brane with smallest value of warp factor in 2-brane models of [15] [16]). Warp geometry can localize fermions only on the positive tension brane (which can not solve the hierarchy problem). Vector fields can not be geometrically confined. Localization of gauge fields and fermions on the negative tension brane is achievable through special particle physics setups [14] [17]. The localization scale however is considered to be not much higher than warping scale, otherwise a new hierarchy can appear [18]. At higher energies one expects that symmetry restoration (e.g. chiral symmetry of fermions) leads to escape of particles from the brane.

Even when a warped geometry is enough to confine fields on the brane, the wave function of the zero mode can penetrate to the bulk (but has an exponential maximum on the brane) [3]. In infinite bulk models KK continuum begins from \( m = 0 \) and this affects the long range behavior of gravity and other massless fields [10] [19] [20]. Massive fields if they have bulk modes can decay to the bulk with a life time which depends on the fundamental scale of gravity [19] [20]. For orbifoldized models the spectrum of KK modes is discrete. The long range effect of massive graviton modes is less important but the probability of decay of massive modes to the bulk is unchanged (see next section). Universal extra-dimension models in which all SM particles propagate to the compact dimensions are not ruled out for compactification radius of order \( TeV^{-1} \) or even lower [21].

At present brane world models have been constrained only based on the probability of direct observation of processes involving the production of gravitons and its Kaluza-Klein modes [22] [23] [24]. A detail investigation of observable signal of the RS type models in Tevatron and LHC are performed in [25] and KK-mode production in the early Universe in [26]. The existent and near future accelerators can constrain the scale of gravity (and thus the size of the extra-dimensions) up to \( \sim 30TeV \). The interaction of Ultra High Energy Cosmic Rays (UHECR) with protons in the terrestrial atmosphere has a CM energy close to 1000TeV and is the most energetic interaction of elementary particles we can study today. It can be used to constrain the fundamental scale of gravity and the compactification scale up to much higher energies.

The mass of KK modes detected by an observer on the brane is the result of smeared dimensions in the wave function. Classically however, it can be interpreted as a delay in the displacement of particles. For the observer on the brane if the delay slightly modifies the propagation of the particle in the detector, it is interpreted as a larger particle mass,
otherwise it is seen as an arrival delay, specially with respect to the particles which propagate only on the brane. In the case of an air shower, the time coherence of the showers will be destroyed.

In this work we calculate the minimum propagation time of particles ejected to extra-dimensions for a number of warped brane models and compare them with arrival time resolution of present Air Shower detectors. We restrict our study to $D = 5$ models. This is enough for understanding the general characteristics of the propagation in extra-dimension from the point of view of an observer on a $(3 + 1)$-brane and can be considered as a special configuration for models with higher dimensions. Our attention is mostly concentrated on the classical structure of the brane models because results are independent of the detail of quantum field contents and origin of the branes. However, before doing this we must assess the possibility and the probability that UHECRs’ interaction in the atmosphere can produce bulk modes i.e. escaping particles. Given the rarity of UHECRs, only models in which the probability of production of bulk modes is very high can be constrained by this method.

2. Production of Bulk Modes

The whole idea of constraining brane models with UHECRs depends on the possibility and probability that remnants of UHECR interaction in the atmosphere can penetrate into the extra-dimensions. In this section we review the localization of fields on the brane with a special attention on models which provide bulk modes i.e. not all fields are confined to the visible brane at all energies. We estimate the probability of producing these modes at energy scale of interaction between UHECRs and nucleons in the terrestrial atmosphere.

By definition in universal models [21] any field has bulk modes and propagates in all space-time dimensions. The interesting case for solving the hierarchy problem is when the size of the compactified dimensions are of the order of weak interaction or lower$^1$. This is $\sim 3$ orders of magnitude smaller than the CM energy of UHECRs’ interaction. Therefore in the case of universal models, UHECRs can produce lowest KK-modes abundantly. In [23] it is argued that UHECRs can not probe the physics at very high energies simply because their interactions is dominantly electromagnetic. It is true that probability of the exchange of a heavy particle e.g a massive boson related to symmetries beyond Standard Model is very small with respect to a low transverse momentum EM cascade. However, in universal models as SM fields have bulk modes at very high energies all dimensions are “seen” as to be the same and the parameter space of EM cascades with non-zero momentum component in the extra-dimensions is much larger than cascades restricted to the three infinite dimensions. In the language of KK-modes, with a good accuracy the lowest modes can be considered as massless and they can be produced with the same probability as zero modes. In the following subsections we argue that for non-universal models one expects that at the CM energy of

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$^1$In some of universal models the extra-dimensions are not warped. Here we only study the propagation in warped spaces. Nevertheless, when the compactification scale $L^{-1}$ is much larger than $\mu$ (see Sec. (1) for definition), the warp factor is very close to one and the results of following sections are applicable.
UHECRs interaction, most of localization mechanisms be no longer active and Standard Model particles can escape to the bulk.

2.1 Scalars and Spin-2 Fields

In warped models scalars and spin-2 fields can be confined geometrically. The zero mode of these fields has an exponentially decreasing wave function in the bulk [3] [27]. For models with infinite bulk the KK-spectrum begins from zero and therefore there is not a real confined zero mode. In addition, if the field has a non-zero 5-dim. mass, it has been shown [11] that 4-dim mass eigen modes on the brane are complex and decay to the bulk with a width:

\[ \Gamma / m_4 \propto (m_4 / \mu)^2 \]

\[ m_5^2 = m_4^2 / 2 \]  

(2.1)

where \( m_4 \) is the real part of the 4-dim. mass eigen value, \( m_5 \) is the 5-dim. mass of the field, and \( \mu \) is the warp scale in the static RS metric (Eq. (4.1) below). If the fundamental scale of Quantum Gravity is comparable to the weak interaction scale, at CM energy of UHECRs interaction it is expected that due to radiative corrections even massless particles like gravitons have an effective non-zero mass. For short distances relevant to the propagation of UHECRs in the terrestrial atmosphere, massive modes have a Yukawa type potential and their coupling is exponentially suppressed [11]. However, if the \( m_4 \) is much smaller than CM energy, the effect of exponential term in Yukawa potential is negligible. For most energetic UHECRs \( E_{CM} \sim 10^{15} \text{eV} \). This means that the coupling to modes as massive as \( 1 \text{TeV} \) is roughly the same as massless modes. The width in (2.1) depends on the effective 5-dim. mass of the field and the warping scale. We discuss their implication on the decay of massive modes to the bulk and on the test of brane models in Section 4.

The above argument is also true in the case of 2-brane models where the spectrum is discrete. We show briefly that in static/quasi-static models the zero mode of scalar/spin-2 fields with \( m_5 > 0 \) has an imaginary part i.e. it decays to the bulk (See also [28]). We determine the propagator of scalar/graviton using the Green function method discussed in detail in [27].

After changing variables \( y \) in metric (4.1) to:

\[ z \equiv \frac{1}{\mu} e^{\mu y} \]

(2.2)

\[ ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2) \]

(2.3)

we apply the boundary conditions to both branes. Without loss of generality we assume one of them is at \( y = 0 \) or \( z = 1 / \mu \equiv R \) and the other at \( y = L \) or \( z = 1 / \mu e^{\mu L} \equiv R' \). The Green function (2-point propagator) \( \Delta(x, z, x', z') \) is the solution of 4-dim. mass eigenstate

\[ ^2 \text{We are only interested in the case where bulk is static. Therefore implicitly it is assumed that these points correspond to the fixed points of the radion field.} \]

\[ ^2 \]
equation:
\[
(z^2 \partial_z^2 + z \partial_z + p^2 z^2 - d^2) \Delta_p(z, z') = \frac{R^3}{z} \delta(z - z')
\]  
\(2.4\)
\[
\Delta(x, z, x', z') = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')} \Delta_p(z, z')
\]  
\(2.5\)
\[
\Delta_p(z, z') \equiv \left(\frac{zz'}{R^2}\right)^2 \hat{\Delta}_p(z, z')
\]  
\(2.6\)
\[
d = \sqrt{4 + R^2 m_5^2}
\]  
\(2.7\)

The boundary and matching conditions for right and left propagators:
\[
\hat{\Delta}_< \equiv \hat{\Delta}_p(z, z') \quad z < z', \quad \hat{\Delta}_\geq \equiv \hat{\Delta}_p(z, z') \quad z > z'
\]  
\(2.8\)
are as follows (boundary conditions are deduced from \(2.8\) and matching condition from \(2.4\)):
\[
\partial_z (z^2 \hat{\Delta}_\geq) |_{z = R} = 0
\]  
\(2.9\)
\[
\partial_z (z^2 \hat{\Delta}_<) |_{z = R} = 0
\]  
\(2.10\)
\[
\hat{\Delta}_< |_{z = z'} = \hat{\Delta}_\geq |_{z = z'}
\]  
\(2.11\)
\[
\partial_z (\hat{\Delta}_< - \hat{\Delta}_\geq) |_{z = z'} = \frac{R^3}{z^6}
\]  
\(2.12\)

Solutions of \(2.4\) are linear combination of Bessel Functions:
\[
A(z', R, R') J_\nu(pz) + B(z', R, R') N_\nu(pz)
\]  
\(2.13\)

Applying the conditions \(2.9\)-\(2.12\) to this solution leads to an equation which determines the KK mass spectrum:
\[
\frac{p R' J_\nu(p R') + (1 - \nu) J_{\nu+1}(p R')}{p R' N_\nu(p R') + (1 - \nu) N_{\nu+1}(p R')} = \frac{p R J_\nu(p R) + (1 - \nu) J_{\nu+1}(p R)}{p R N_\nu(p R) + (1 - \nu) N_{\nu+1}(p R)}
\]  
\(2.14\)

with \(\nu \equiv d - 1\). Finding the exact solution of \(2.14\) is not trivial. To consider only a simple case we assume that 4-dim. mass of the scalar field \(|p| \ll \mu = \frac{1}{R}\), i.e. \(p R \ll 1\). Regarding the Standard Model, this can be applied to a confined Higgs when the scale of compactification is much higher than Higgs mass or to a light axion like scalar or to graviton with a small mass due to radiative corrections. We keep only lowest powers of \(p R\) in the expansion of \(J_\nu\). For solving hierarchy problem \(R' \gg R\). Using the asymptotic expansion of Bessel functions, \(2.14\) reduces to:
\[
\eta' \eta^{-(\nu+1)} \sqrt{\frac{2}{\pi \eta'}} \left(\cos(\eta' - \frac{\pi \nu}{2} - \frac{\pi}{4}) - \frac{1 - \nu}{\eta'} \sin(\eta' - \frac{\pi \nu}{2} - \frac{\pi}{4})\right)
\]
\[
\frac{\eta^2 (1 + \frac{\eta}{2(\nu-1)}) + (\nu - 1)(2\nu + \eta)}{\nu 2^{-\nu} \sin(\nu \pi) \Gamma(-\nu)} = 0
\]  
\(2.15\)
where η ≡ pR and η′ ≡ pR′. For ν ≥ 1 the solutions of (2.15) lead to the following mass spectrum:

\[ \begin{align*}
    ip_0 &= m_4 - i\Gamma \\
    m_4 &\approx \mu \sqrt{2\nu(\nu - 1)} \approx \frac{m_5}{\sqrt{2}}, \quad \Gamma \approx m_5^2/8\mu \\
    |p_\nu'| &\approx \mu e^{-\mu L(n\pi + \frac{\pi\nu}{2} + \frac{3\pi}{4})} \quad \text{For large } n.
\end{align*} \] (2.16)

(2.17)

Continuity properties of \( J_\nu \) guarantees that (2.13) is also valid when \( m_5 \to 0 \) or equivalently \( \nu \to 1 \). In this case \( \eta = 0 \) or \( p_0 = 0 \). For \( pR \ll 1 \) the mass difference between KK-modes \( p_n \) is \( \Delta p \propto 1/R' \ll 1/R = \mu \). Due to special properties of Bessel Function with integer index, the zero mode of massless fields is protected from decay even when the spectrum of KK-modes for massive particles begins roughly from zero. Tunneling probability depends on 5-dim. mass of the field and on warping scale \( \mu \). If \( \mu \) is large, the probability of zero-mode decay to the bulk can be small. However, except for very light particles \( \mu \) as large as \( 1 TeV \) provides an enough large width (\( \Gamma > 10^{-10}eV \)) for decay to the bulk during propagation in the terrestrial atmosphere if \( m_5 \) is in the mass range of SM particles.

To see the effect of mass on the coupling we can investigate the mass dependence of the propagator on the branes. Using (2.8) and (2.9)-(2.12), one can determine the integration coefficients \( A(z', R, R') \) and \( B(z', R, R') \) in (2.13) and right and left propagators:

\[ \begin{align*}
    \hat{\Delta}_<(z, z') &= \frac{\pi R^3}{z^2(\Delta_0\Delta_3 - \Delta_1\Delta_2)} \left( (\Delta_0 J_{\nu+1}(p_{z'}) - \Delta_1 N_{\nu+1}(p_{z'})) - (\Delta_2 J_{\nu+1}(p_{z}) - \Delta_3 N_{\nu+1}(p_{z})) \right) \\
    \hat{\Delta}_>(z, z') &= \frac{\pi R^3}{z^2(\Delta_0\Delta_3 - \Delta_1\Delta_2)} \left( (\Delta_2 J_{\nu+1}(p_{z'}) - \Delta_3 N_{\nu+1}(p_{z'})) - (\Delta_0 J_{\nu+1}(p_{z}) - \Delta_1 N_{\nu+1}(p_{z})) \right)
\end{align*} \] (2.19)

(2.20)

\[ \begin{align*}
    \Delta_0 &\equiv pR'N_{\nu}(pR') + (1 - \nu)N_{\nu+1}(pR') \\
    \Delta_1 &\equiv pR'J_{\nu}(pR') + (1 - \nu)J_{\nu+1}(pR') \\
    \Delta_2 &\equiv pRN_{\nu}(pR) + (1 - \nu)N_{\nu+1}(pR) \\
    \Delta_3 &\equiv pRJ_{\nu}(pR) + (1 - \nu)J_{\nu+1}(pR)
\end{align*} \] (2.21)

(2.22)

(2.23)

(2.24)

Restricting these equations to the branes gives the 2-point propagators:

\[ \begin{align*}
    \Delta(x, R, x', R) &= \frac{1}{(2\pi)^4} \int dp^4 e^{ip(x-x')} \frac{p^{-1}(\Delta_0 J_{\nu+1}(pR) - \Delta_1 N_{\nu+1}(pR))}{(\Delta_0\Delta_3 - \Delta_1\Delta_2)} \\
    \Delta(x, R', x', R') &= \frac{R'^2}{R^2} \frac{1}{(2\pi)^4} \int dp^4 e^{ip(x-x')} \frac{p^{-1}(\Delta_2 J_{\nu+1}(pR') - \Delta_3 N_{\nu+1}(pR'))}{(\Delta_0\Delta_3 - \Delta_1\Delta_2)}
\end{align*} \] (2.25)

(2.26)

The term \( R'^2/R^2 \) in (2.26) reflects the difference between metric on the branes. Consistently, the roots of dominator in (2.23) and (2.26), are the same as (2.14) and correspond to KK-modes. Near each mass mode the propagators can be written as a Yukawa propagator with
complex mass and a coupling equal to the residue of the integrand. Applying this procedure to (2.26) one finds that on the brane at $R'$ for $\nu > 1$:

$$
\frac{g_0^2}{g_n^2} \sim \frac{|p_0/\mu|^{2\nu}}{|p'_n/\mu|^{2(\nu-1)}}
$$

(2.27)

In our approximation $|p_0/\mu| \propto m_5/\mu \ll 1$, $|p'_n/\mu| > |p_0/\mu|$ and it can be even larger than 1. Therefore coupling to KK-modes is larger than to zero-mode. The probability of production of zero-mode with respect to $n^{th}$ KK-mode is:

$$
V \sim \frac{|p_0|^{2(\nu-1)}|p'_n|^{2}}{|p'_n/\mu|^{2}}
$$

(2.28)

For KK-modes with $|p'_n| \sim \mu$ the branching ratio $V < 1$ and therefore the probability of production of these modes is larger than zero-mode.

In conclusion, radiative corrections that can induce an effective mass for scalar/spin-2 fields weakens their confinement on the brane. In warped 2-brane models even when $m_5 = 0$ the mass difference between the zero mode and massive KK-modes on the brane with smallest warp factor is small and they can be abundantly produced in high energy interactions.

### 2.2 Fermions and Gauge Fields

Fermions can not be localized to the $TeV$ scale brane (i.e. brane at $R'$) gravitationally but a chiral symmetry breaking can confine them [14]. Their escape to the bulk when their $m_5 > 0$ has been studied in detail for one brane models in [11] and [29]. The detail of the formalism is very similar to the case of a scalar field except that the mass eigenstate equation includes a chiral mass term due to interaction of fermion modes with a bulk scalar responsible for the symmetry breaking. The width of the zero mode depends on the coupling between fermions and the inferred scalar field and without detail knowledge of underlying particle physics it is difficult to assess the probability of decay to the bulk. For 2-brane models the situation should not be very different and general conclusions of Sec.2.1 must be applicable.

At present the particle physics models don’t fix the scale of the symmetry breaking. For not creating a new hierarchy however it can not be much larger than compactification scale [18] or fundamental scale of gravity $M_5$. Therefore, at energy scale of UHECRs interaction in the atmosphere, not only it is possible to produce KK-modes, it is very probable that at such energies the restoration of chiral symmetry completely removes the confinement of fermions and open the extra-dimension even to fermionic zero modes (presumably SM matter).

As for gauge bosons, the most successful scenario for their confinement on the branes is based on adding an induced kinetic term to the action of bulk gauge fields on the brane. It appears due to the interaction of these fields with confined charged scalar or fermions on the branes [17]. Other suggestions are mostly equivalent to this scenario [30]. Once the charged
fields become able to escape to the bulk, they drag their interaction vertex with gauge fields to the bulk and release them from confinement. It has been shown [31] that the coupling of gauge boson KK-modes to fermions on the brane is stronger than coupling of their corresponding zero-mode (similar to the self coupling of scalars $2.27$).

The general conclusion of this section is that regarding:

- Very high CM energy of interaction of most energetic Cosmic Rays in the atmosphere which is much higher than natural confinement scale of Standard Model particles on the brane and natural fundamental scale of gravity to solve the hierarchy problem;

- The fact that confinement of SM fields is not intrinsic but the result of either a broken symmetry (for fermions) or interactions (for bosons);

- That the particle physics in 5-dim can not be completely massless and at least part of the particle spectrum must acquire mass as it is the case in observable 4-dim Universe. In fact as radion is in fact the scalar component of 5-dim. metric perturbations [9], it couples to all bulk fields and radiatively induces a small mass term. Consequently, zero-modes (presumably SM particles) are not stable and in a finite time decay to the bulk unless another phenomenon like symmetry breaking prevent it;

the phase space of production of bulk modes at high energy tail of UHECRs spectra seems to be higher than confined modes.

Until now more than 100 coherent air showers have been observed with $E_{CM} \gtrsim 300 TeV$ (assuming interaction with nucleons in the atmosphere), 17 between them have energies more than 450 TeV and one has a CM energy close to 1000 TeV [32]. Assuming that UHECRs interaction with $E_{CM} \gtrsim 300 TeV$ produces bulk modes abundantly, the mere observation of coherent showers up to energies close to $10^{21} eV$ constraints the parameter space of brane models.

As the assessment of cross-sections and other details are model dependent, in the rest of this work we simplify the problem of constraining brane models and consider only the classical propagation of the bulk modes. This method has been already used by other authors to study some of cosmological consequences of brane models [33] [10].

3. Propagation

The geodesic path of particles in the bulk has been already studied in a number of previous works [33] [10] [11] [34]. However, most of them are concerned with the possible acausality of paths for observer on a brane and their purpose is to see if it can solve the cosmological horizon problem in the early universe. Here we are concerned with the present evolution of the Universe and simplifying assumptions will be based on its present very slowly changing state. The metric of the 5-dim brane models can be written as the following:

$$ds^2 = n^2(t,y)dt^2 - a^2(t,y)\delta_{ij}dx^i dx^j - b^2(t,y)dy^2.$$
For a static bulk \( b^2(t, y) \) is constant and we can normalize coordinates such that \( b = 1 \). The geodesic of a particle is defined by:

\[
\frac{du^0}{d\tau} + \frac{n}{n} u^0 u^0 + 2n' u^4 u^0 + \frac{a}{n^2} u^i u^i \delta_{ij} = 0
\]

(3.2)

\[
\frac{du^i}{d\tau} + \frac{2a'}{a} u^i u^4 + 2a u^i u^0 = 0
\]

(3.3)

\[
\frac{du^4}{d\tau} + n n' u^0 u^0 - a a' u^i u^j \delta_{ij} = 0
\]

(3.4)

\[
u^0 = \frac{dt}{d\tau}, \quad u^i = \frac{dx^i}{d\tau}, \quad u^4 = \frac{dy}{d\tau}
\]

(3.5)

\( \tau \) is the proper time parameter along the particle world line. It is easy to see that (3.3) is integrable and:

\[
u^i = \frac{\theta^i}{a^2}
\]

(3.6)

where \( \theta^i \) is an integration constant. As we are only interested in the minimum delay in the arrival of particles due to the propagation in the extra-dimensions, we put \( \theta^i = 0 \). Later we try to estimate qualitatively the effect of a non-zero \( \theta^i \). Even after this simplification the system of equations (3.2)-(3.4) is highly non-linear and coupled. In the following we calculate an analytical solution for the case \( \dot{n}/n \approx 0 \). This approximation is justified when we are interested in the propagation of particles in an extremely short period of time with respect to the expansion rate of the bulk or the brane. In fact from the solution of the Einstein equations [35], \( n(t, y) \) can be normalized such that:

\[
n(t, y) = \frac{\dot{a}(t, y)}{\dot{a}_0(t)}
\]

(3.7)

where \( a_0(t) = a(t, y = 0) \) assuming that one of the branes is at \( y = 0 \). At present, both \( \dot{a}(t, y) \) and \( \dot{a}_0(t) \) are very slowly varying quantities. Therefore \( n(t, y) \sim n(t_0, y) \) where \( t_0 \) is the present time, and \( \dot{n}(t, y) \sim 0 \).

Under this approximation:

\[
\frac{dn}{d\tau} \approx n' u^4
\]

(3.8)

and:

\[
\frac{du^0}{d\tau} + \frac{2 u^0}{n} \frac{dn}{d\tau} = 0
\]

(3.9)

\[
\frac{du^4}{d\tau} + \frac{n(u^0)^2}{u^4} \frac{dn}{d\tau} = 0
\]

(3.10)

The solution of (3.9) and (3.10) is straightforward:

\[
u^0 = \theta n^{-2}, \quad u^4 = \pm \left( \frac{\theta^2}{n^2} \right)^{\frac{1}{2}}
\]

(3.11)
The parameters $\theta$ and $\eta$ are integration constants and must be determined from the initial conditions. The $\pm$ sign defines the direction of the propagation. In the rest of this letter we neglect the direction and consider only the absolute value of $u^4$.

For a particle leaving the visible brane placed at $y = y_b$ at time $t = t_0$:

$$\theta = u^0(t_0, y_b) n^2(t_0, y_b), \quad \eta = \frac{\theta^2}{n^2(t_0, y_b)} - (u^4(t_0, y_b))^2 = \begin{cases} 1 & \text{Massive particles,} \\ 0 & \text{Massless particles.} \end{cases}$$

(3.12)

After eliminating the proper time from $u^0$ and $u^4$, one obtains the equation of motion in the bulk (For simplicity we assume that $Dy/d\tau \approx dy/d\tau$ and $Dt/d\tau \approx dt/d\tau$):

$$\frac{dy}{dt} = \frac{n^2(t, y)}{\theta} \sqrt{\frac{\theta^2}{n^2(t, y)} - \varepsilon} \quad \quad (3.13)$$

$$\varepsilon \equiv \begin{cases} 1 & \text{Massive particles,} \\ 0 & \text{Massless particles.} \end{cases}$$

(3.14)

Our approximations are valid only when $dy/dt$ is real. This puts limits on the testable part of the parameter space of the models (see below).

Einstein equations give the solution for $a(t, y)$ and $n(t, y)$ [35]. For a flat visible brane:

$$a^2(t, y) = A(t) \cosh(\mu y) + B(t) \sinh(\mu y) + C(t), \quad (3.15)$$

$$\dot{a}^2(t, y) = n^2(t, y)a_0^2(t) = \left(\dot{A}(t) \cosh(\mu y) + \dot{B}(t) \sinh(\mu y) + \dot{C}(t)\right)^2, \quad (3.16)$$

$$A(t) = a_0^2(t) - \dot{C}(t), \quad (3.17)$$

$$B(t) = -\dot{\rho}_B a_0^2(t), \quad (3.18)$$

$$C(t) = -\frac{2\dot{a}_0^2(t)}{\mu^2}, \quad (3.19)$$

$$\mu \equiv \sqrt{\frac{2\kappa^2}{3} |\rho_B|} \quad (3.20)$$

For any density $\rho$, $\rho' \equiv \rho/\Lambda_{RS}$, $\Lambda_{RS} \equiv 3\mu/\kappa^2$. The densities $\rho'_{b0}$ and $\rho_B$ are effective total energy density of the brane at $y = 0$ and the bulk respectively. We consider only AdS bulk models with $\rho_B < 0$. The constant $\kappa^2 = 8\pi/M_5^3$ is the gravitational coupling in the 5-dim. space-time. The model dependent details like how $\rho'_{b0}$ and $\rho_B$ are related to the field contents in the bulk and on the brane and how they evolve are irrelevant for us as long as we assume a quasi-static model. The solution (3.15) is valid both for one brane and multi-brane models. The only difference between them is in the application of Israel junction conditions [15] [16]. Equation (3.14) is non-linear and its integration non-trivial. We use again the quasi-static properties of the present Universe and its low energy density to simplify the integration. Matter density on the branes at late time is much smaller than the brane tension or induced
tension by scalar fields \[38\]. Therefore, it is not unreasonable to neglect time dependence of densities in (3.17)-(3.19) and to consider only cosmological constant type energy-momentum densities. This simplification is even more justified in our case where we have to deal only with very short duration of the propagation in the extra-dimensions. This approximation and (3.17)-(3.19) lead to:

\[
\frac{\dot{C}}{\mu^2 a_0(t)} = -\frac{4\dot{a}_0^3(t)}{\mu^2 a_0(t)}, \quad (3.21)
\]

\[
\dot{A} = 2a_0(t)\dot{a}_0(t) - \dot{C}(t), \quad (3.22)
\]

\[
\dot{B} = -2\rho'_{b_0} a_0(t)\dot{a}_0(t). \quad (3.23)
\]

After changing variable \(y\) to \(z = e^{\mu y}\) and using (3.7):

\[
\dot{a}^2(t, z) = -\frac{\mu^2 C(t)}{2z} D(t, z), \quad (3.24)
\]

\[
D \equiv \frac{1}{2} \left[ (1 - \rho'_{b_0} - \frac{C(t)}{a_0^2}) z^2 + \frac{2C(t)}{a_0^2} z + (1 + \rho'_{b_0} - \frac{C(t)}{a_0^2}) \right], \quad (3.25)
\]

\[
\frac{dz}{dt} = \mu \sqrt{D(z - \frac{\epsilon}{\mu^2} D)}. \quad (3.26)
\]

If an ejected particle to the bulk comes back to the brane, \(u^4\) must go to zero at some point in the bulk before the particle arrives to the bulk horizon (if it is present). The roots of (3.26) correspond to these turning points and determine the propagation time in the bulk. In the next simplifying step we use again the fact that the typical propagation time we are interested in is very much shorter than the age of the Universe and therefore \(A, B, C\) and \(\dot{A}, \dot{B}, \dot{C}\) during propagation are roughly constant, the right hand side of (3.26) depends only on \(z\) and is easily integrable:

\[
\Delta t_{\text{propag}} \equiv 2(t_{\text{stop}} - t_0) = \int_{z_0}^{z_{\text{stop}}} \frac{2dz}{\mu \sqrt{D(z - \frac{\epsilon}{\mu^2} D)}} \quad (3.27)
\]

In (3.27), \(t_0\) is the initial time of propagation in the extra-dimension and \(t_{\text{stop}}\) is the time when the particle’s velocity changes its direction, i.e. when \(dz/dt = 0\). The integral in (3.27) is related to the elliptical integrals of the first type \(F(\omega, \nu)\) where \(\omega\) and \(\nu\) are analytical functions of the denominator roots in (3.27) and \(z_0\) \[39\]. Note that \(z_{\text{stop}}\) corresponds to the closest root to \(z_0\).

4. Test of Brane Models

In this section we apply the formalism discussed in the previous section to most popular brane models and determine the propagation time of high energy particles in the fifth dimension. Note that the calculation of propagation time in these models under our approximations is valid only for durations very smaller than the age of the Universe and if in the following figures in part of the parameter space the propagation time can be larger, this part of the figure should not be considered.
4.1 2-Brane Models

It has been shown in [15] [16] that by imposing constraints on the visible brane to obtain the observed value of cosmological constant \( \Lambda \) and Newton coupling constant \( G \) and to solve the hierarchy problem, all parameters of this class of models i.e. \( \rho_0, \rho_L \) and \( \Lambda_{RS} \), can be determined as a function of \( \mu L \) where \( L \) is the distance between two branes. It is not however possible to find an exact analytical form for the solutions. Moreover, the analytical solution in [16] has been obtained for a special setup which decouples hidden and visible branes. Here we free some of the constraints, first to be able to find analytical solutions, and second to extend this study to a larger number of models.

4.1.1 Geodesics in Static RS Models

In the original RS model with static metric:

\[
ds^2 = e^{-\mu y} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2. \tag{4.1}
\]

the main constraint on the model is the cancellation of the cosmological constant on the visible brane which leads to the equal and opposite sign tensions \( \rho_0 = -\rho_L = \Lambda_{RS} \). The solution of the hierarchy problem limits the range of the parameter \( \mu L \). It has been shown [10] that in the fine-tuned RS model photons leave the brane and never return. Assuming the visibility of the all dimensions of the space-time at very high energies, in this model we could never observe the ultra high energy particles and therefore it is automatically ruled out. Nonetheless, to test the formalism of the previous section we apply it to this model.

For a very small cosmological constant (as it is assumed in the process of fine-tuning) \( C(t_0) \approx 0 \) and \( D(t_0, z) \approx 1 \). For massive particles, the denominator in (3.27) has only one root: \( z_1 = 1/\theta \) and:

\[
\Delta t_{propag} = \frac{2e^{\mu L}}{\mu} \sqrt{1 - \frac{e^{\mu L}}{(u^0_L(t_0))^2}} \quad \theta = e^{-\mu L} u^0_L(t_0). \tag{4.2}
\]

Fig. 1 shows \( \Delta t_{propag} \) as a function of \( \mu L \) and \( M_5 \) for massive relativistic particles. With present air shower detectors time resolution of order \( 10^{-6} \) sec, only when \( M_5 \gtrsim 10^{18} \text{eV} \), the model is compatible with the observed time coherence of the UHE showers. For fine-tuned RS model \( \mu \approx G/\kappa^2 \) [3] i.e. \( \mu = GM_5^3 \sim 10^{-3} \text{eV} \) for \( M_5 \sim 10^{18} \text{eV} \) [3]. Due to smallness of \( \mu \) and consequently lightness of KK-modes for SM particles even this model with large \( M_5 \)
has already been ruled out [31] unless a conserved quantum number prevents the production of KK-modes [21].

For massless particles:
\[
z(t) - z_0 = \mu (t - t_0)^2 \quad (4.3)
\]
In (4.3), \(z(t)\) is monotonically increasing and there is no stopping point. With our approximations there is no horizon in the bulk because \(a(t, y)\) is roughly constant. Therefore (4.3) means that massless particles simply continue their path to the hidden brane and their fate depends on what happens to them there. At very high CM energy of UHECR interactions if charged particles can escape to the bulk photons are also dragged to the bulk and never come back. However, we must remember also that due to small \(\mu\) in this model the life-time of zero-modes of light SM particles (2.17) can be larger than propagation time in the atmosphere. Therefore it is meaningless to exclude the model based on the propagation time in the bulk.

### 4.1.2 General Solution

Numerical solution of constrained 2-brane models in [15] [16] shows that for \(\mu L \gtrsim 5\) the tension on both branes is positive and very close to \(\Lambda_{RS}\). We can use constraints on the Cosmological Constant and hierarchy to find \(\rho_0'\) and \(\rho_L'\). We redefine them as \(\rho_0' = 1 + \Delta \rho_0'\) and \(\rho_L' = 1 + \Delta \rho_L'\). To solve hierarchy problem (See equations 29-31 in [16]):
\[
\frac{M_0^2}{M_{pl}^2} \sim N^2 \equiv \frac{n_0^2}{n_0^2} = \frac{\rho_{\Lambda_0}'(1 - \cosh(\mu L)) + \sinh(\mu L)}{\rho_{\Lambda_0}'(1 - \cosh(\mu L)) + \sinh(\mu L)} \ll 1 \quad (4.4)
\]
This leads to:
\[
\Delta \rho_0' = \frac{N^2 \left( 1 - e^{-\mu L} + \Delta \rho_L' (1 - \cosh(\mu L)) \right)}{1 - \cosh(\mu L)} - \frac{1 - e^{-\mu L}}{1 - \cosh(\mu L)} \quad (4.5)
\]
For a very small \(N^2\) and \(\Delta \rho_L' \lesssim 1\), the first term in (4.5) is \(O(N^2)\) and:
\[
\Delta \rho_0' \approx -\frac{1 - e^{-\mu L}}{1 - \cosh(\mu L)} \approx -\frac{1}{1 - \cosh(\mu L)} \approx 2e^{-\mu L} \quad (4.6)
\]
Using \(\dot{a}_L^2/a_L^2 = H^2\) where \(H\) is the Hubble Constant on the visible brane [16]:
\[
\Delta \rho_L' = \frac{1}{2N^2 \sinh(\mu L)} \left[ (1 - \cosh(\mu L)) \frac{2H^2}{\mu^2} + e^{-\mu L} - 1 \pm \sqrt{((1 - \cosh(\mu L)) \frac{2H^2}{\mu^2} + e^{-\mu L} - 1)^2 + N^2 \sinh(\mu L)(\frac{2H^2}{\mu^2} + 2e^{-\mu L})} \right] \quad (4.7)
\]
In (4.7) the solution with plus sign gives \(\Delta \rho_L' \approx -2\) which deviates from our first assumption \(|\Delta \rho_L'| < 1\) and leads to a negative tension on the visible brane like static RS model. The solution with negative sign is:
\[
\Delta \rho_L' \approx 2e^{-\mu L} \quad (4.8)
\]
and both branes have positive tension close to $\Lambda_{RS}$.

When the matter densities on the branes and in the bulk are negligible [16], $\dot{a}_0^2/a_0^2 = \dot{a}_L^2/a_L^2 = H^2$ and:

$$\frac{C(t)}{a_0^2} \equiv C' = -\frac{2H^2}{\mu^2}$$

(4.9)

It is easy to see that $D$ and $a^2(t, y)$ have the same roots. Models with a horizon i.e. a point $y_h$ such that $a^2(t, y_h) = 0$ are pathological (because no particles/brane behind it is observable).

The condition to have no real root i.e no horizon in the bulk is:

$$-2 + \frac{C'^2}{2} \leq \Delta \rho'_{0} + C' \leq -\frac{C'^2}{2}$$

(4.10)

For massive particles, the denominator of the integrand in (3.27) can have two roots:

$$z_{\pm} = \frac{C' - \theta^2 \pm \sqrt{(C' - \theta^2)^2 + (2 + \Delta \rho'_{0} + C')(\Delta \rho'_{0} + C')}}{\Delta \rho'_{0} + C'}$$

(4.11)

The model is consistent only if $D(z - D/\theta^2) > 0$ in the range of integration. Therefore:

$$z_+ \leq z_0 = e^{\mu L} \leq z_-.$$  

(4.12)

The matter on the brane is confined only if $z_+ > 1$ and $z_+ \to z_0$ when $u^4 \to 0$. To first order in $e^{-\mu L}$ and $N^2$ this leads to the following relation between parameters of the model:

$$-C' = \Delta \rho'_{0} + \frac{2}{z_0}(N^2 + \Delta \rho'_{0})$$

(4.13)

This condition is not an addition to the model described in [16]. It is in fact the result of solutions (4.6) and (5.8) under the approximations considered here. It is not evident whether such a constraint appear in the full theory.

For $\mu L \gtrsim 5$ the right hand side of (4.13) is positive. Therefore $-C' \propto H^2$ can not be zero.

This relation between a small but non-zero value of the Hubble Constant or equivalently Cosmological Constant on the visible brane and the smallness of $N^2$ and $\mu$ which is related to the strength of the induced gravitational coupling on the brane, confirms the same observations in [16] for an analytical solution of 2-brane models with some approximations and in [37] for the exact solution of some special models.

Finally the propagation time in the bulk is given by:

$$\Delta t_{propag} = \frac{4}{\mu(8|\Delta \rho'_{0} + C'|)^{\frac{1}{4}}} \mathcal{F}(\alpha, Q).$$

(4.14)

$$\alpha = 2 \arctan \sqrt{\frac{q(z_+ - z_0)}{p(z_0 - z_+)}},$$

(4.15)

$$Q = \frac{1}{2} \sqrt{2 + \frac{2}{pq} \left[ \frac{C'(C' - \theta^2) + 4(2 + \Delta \rho'_{0} + C')(\Delta \rho'_{0} + C')}{(\Delta \rho'_{0} + C')^2} \right].}$$

(4.16)
\[
\begin{align*}
p^2 &\equiv \left(\frac{C'}{\Delta \rho_0 + C'} - z_-\right)^2 + r^2, \\
q^2 &\equiv \left(\frac{C'}{\Delta \rho_0 + C'} - z_+\right)^2 + r^2, \\
r^2 &\equiv -\left(\frac{C'}{\Delta \rho_0 + C'}\right)^2 - \left(\frac{2 + \Delta \rho_0 + C'}{\Delta \rho_0 + C'}\right).
\end{align*}
\] (4.17)

Fig. 2 shows the propagation time for models which satisfy simultaneously (4.6), (4.8) and (4.13). In equation (4.13) up to first order, \(C'\) depends only on \(\mu L\) and thus in (4.9) the value of \(\mu\) is independent of \(M_5\). Only models with large \(\mu L \gtrsim 150\) are not ruled out. This is due to (4.14) and smallness of the observed Hubble Constant \(H\). The same conditions make \(\mu > 1eV\) which is much higher than the \(\mu\) for the fine-tuned RS model. With \(\mu \lesssim m_{\text{radion}}\) [6] [9] these class of brane models are also consistent with constraint on the fifth-force measurements [41]. From (2.17) the corresponding life-time for SM fields with \(m_5 > 0\) is shorter than \(10^{-16}\) sec, much shorter than propagation time in the atmosphere and also much shorter than propagation time in the bulk. This justifies the classical treatment of propagation.

We have also applied the formalism described here to the fine tuned model of [16]. In this model the equation of state on the hidden brane is fine-tuned to neutralize its effect on the visible brane. This results to a unique definition of \(G\) and the only free parameter in the model is \(M_5\). Although this model has been obtained just by phenomenological arguments, a field theory model suggested by Arkani-Hamed et al. [36] to solve the Cosmological Constant problem has the same form for \(G\) if \(\mu L \sim \kappa^2 \phi(0)\) where \(\phi(0)\) is the vev of radion on the visible brane (The Arkani-Hamed et al. model has only one brane but it includes a horizon in the bulk which limits the accessible size of the extra-dimension and makes it similar to a two brane model).
Fig. 3 shows the propagation time and $\mu$ as a function of $M_5$. Unfortunately despite physical interest of this model it is only compatible with very high $M_5 \gtrsim 10^{19} \text{eV}$ unless the decay to the bulk is prevented up to such high energies. The corresponding $\mu$ however is consistent with present constraint on the Fifth force [41] and the life-time of zero-modes of massive 5-dim. fields for such high $M_5$ is enough short to permit particles to decay to the bulk during their propagation in the atmosphere. The value for $M_5$ is some 6 orders of magnitude larger than $10^{17} \text{eV}$, presumably the Electroweak interaction scale and it would be a matter of speculation to consider this model as having no hierarchy problem. Another problem in testing these models with UHECRs is that as the natural scale of gravity $M_5$ is very high, it is possible that the symmetry breaking scale which is necessary for the localization of fermions (and indirectly gauge bosons) is also much higher than CM energy of UHECRs interaction. In this case only very weakly interacting particles like gravitons can decay to the bulk. As the total production cross-section for them can be tiny, the number of observed UHECRs event can be not enough to constrain such models. We have also tested the general 2-brane

models without taking into account (4.13). Roughly speaking, it is equivalent to having a comparable matter density and tension on the hidden brane. The value of $\mu$ becomes a free parameter. The result is shown in Fig. 4 for 3 different values of $\mu$. Models with $\mu \gtrsim 10^4 \text{eV}$ and $\mu L \lesssim 70$ are compatible with the present observation of UHECRs. The lower limit for $\mu$ from this test is higher than the constraint obtained from Fifth force experiments [41].

4.2 One-Brane Models

The solution of Einstein equations for symmetric one-brane models is the same as two-brane
Figure 4: Propagation time for relativistic particles in 2-brane models with $\mu$ as a free parameter. Top left: $\mu = 10^{-7} \text{eV}$; Top right: $\mu = 10^4 \text{eV}$; Bottom: $\mu = 10^7 \text{eV}$. Description of the curves is the same as Fig.1.

ones [35]. Due to existence of only one boundary however the bulk and brane tensions are not related. In addition, the cosmological evolution on the brane:

$$\frac{\dot{a}_0^2}{a_0^2} = \frac{\tilde{\kappa}^2}{6} \rho_B + \frac{\tilde{\kappa}^4}{36} (\rho_b + \rho_m(t))^2 + \frac{C(t)}{a_0^4}. \quad (4.18)$$

includes an arbitrary function $C(t)$ which is related to the bulk tension and matter [40]. Here we test two popular models studied in [35] and [40].

In the first model [35] $C(t) = 0$ and the brane tension is fine-tuned to cancel the effect of
quadratic term at late times:

\[ \frac{\dot{\kappa}^2}{6} \rho_B + \frac{\dot{\kappa}^4}{36} \rho_b^2 = 0 \]  

(4.19)

and:

\[ 8\pi G = \frac{\dot{\kappa}^4 \rho_b}{6} \]  

(4.20)

The only free parameter in the model is \( M_5 \). In the second model [40] \( C(t) \) (or equivalently \( T^5_5 \) component of the energy-momentum tensor) is adjusted such that the conventional evolution equation be obtained. At late times when the brane tension is much larger than time dependent matter terms these two models are roughly the same.

Equations (4.19) and (4.20) determine \( \rho_b \) and \( \mu \). It is easy to see that \( \rho_b = \Lambda_{RS} \). The definition of \( C' \) and roots are the same with \( \Delta \rho_0 = \Delta \rho_b = 0 \) and \( z_0 = 1 \). Fig. 5 shows the propagation time for these models.

![Figure 5: Left: Propagation time for one-brane models of [35]. Right: Parameter \( \mu \) as a function of \( \mu L \).]

4.2.1 Effect of \( \theta^i \neq 0 \)

When \( \theta^i \neq 0 \) equations (3.9) and (3.10) can be written as:

\[ \frac{du^0}{d\tau} + 2u^0 \frac{dn}{d\tau} + \frac{\dot{a} \theta^i \theta^j \delta_{ij}}{n^2 a^3} = 0 \]  

(4.21)

\[ u^4 \frac{du^4}{d\tau} - \frac{a' \theta^i \theta^j \delta_{ij}}{a^3} u^4 + (u^0)^2 n \frac{dn}{d\tau} = 0 \]  

(4.22)

At least formally Eq. (4.21) can be solved analytically:

\[ u^0 = \frac{\theta}{n^2} + \frac{\theta^i \theta^j \delta_{ij} G(t, y)}{n^2} \]  

(4.23)
\[ G(t, y) = \int d\tau \left( -\frac{\dot{a}}{a^3} \right) \]  

(4.24)

We don’t need to solve (4.22) directly. Knowing \( u^0 \) and \( u^i \) we can use the definition of velocity vector to determine \( u^4 \):

\[
n^2(u^0)^2 - a^2 u^i u^j \delta_{ij} -(u^4)^2 = \varepsilon
\]

(4.25)

The definition of \( \varepsilon \) is the same as in (3.14). After elimination of \( d\tau \) we obtain the formal description of the equation of motion in the bulk:

\[
dy/dt = \left[ \frac{\dot{a}^2}{n^2} - \varepsilon + \theta^i \theta^j \delta_{ij} \left( \frac{G^2 \theta^i \theta^j \delta_{ij}}{n^2} + \frac{2G \theta^0 \delta_{ij}}{n^2} - \frac{1}{a^2} \right) \right]^{\frac{1}{2}}\theta^i \theta^j \delta_{ij}
\]

(4.26)

We can use (3.6) to determine \( d\tau \) (As before for simplicity we assume that \( Dx^i/d\tau \approx dx^i/d\tau \)):

\[
d\tau = \frac{a^2 \delta_{ij} \theta^j dx^i}{\theta^i \theta^j \delta_{ij}}
\]

(4.27)

and:

\[
\theta^i \theta^j \delta_{ij} G(t(\tau), y(\tau)) = -\int \frac{\dot{a}}{a} \delta_{ij} \theta^j dx^i
\]

(4.28)

In (4.28) \( \dot{a}/a \) is independent of \( x^i \). The rest of right hand side of (4.28) i.e. \( \int \delta_{ij} \theta^j dx^i \) is the projection of the particles world line on the brane. The value of Hubble constant \( \dot{a}/a(t, y) \approx H^2 \) (from (3.17-3.19) and (3.21-3.23)) and thus \( \theta^i \theta^j \delta_{ij} G \) is very small when the projection distance traversed by the particle is small with respect to the Hubble radius. Therefore we presume that conclusions of the previous section will not be extremely modified when full propagation is considered.

5. Conclusion

The calculation in this work is mainly based on two assumptions:

- At interaction energy scale of most energetic Cosmic Rays the physics is high dimensional either because all symmetry based confinements are no longer at work or because there is a large cross-section and/or phase space volume which permits the production of massive KK-modes.

- In the time scale of the propagation of a particle in the extra-dimension, the bulk and the branes are quasi-static.

If these assumptions are valid the time coherence of Ultra High Energy Air Showers rules out a large part of the parameter space for a number of brane models unless some micro-physics phenomena confine particles to the brane at energies much higher than Electroweak scale. This makes a new hierarchy inconsistent with the spirit of brane models.
For most 2-brane models the acceptable range of $\mu$ is $\mu \gtrsim 1eV$ except for original RS model which needs $\mu \gtrsim 10^{-2} eV$. This lower limit is much lower than what can be obtained from non-observation of KK-mode production in accelerators [31]. However, at present accelerator energies it is always arguable that presence of some quantum conservations prevent the production of KK-modes. Presence of such conservations at the CM energy of UHECR interaction seems much less natural and constraints on $M_5$ more are robust.

The upper limit of $L$ is also a universal value for models with different range of $\mu L$: $L \lesssim 10^{-3} eV^{-1} \sim 10^{-8} cm$. It is much smaller than the upper limits obtained from gravity experiments [42] [41]. Again for static RS model the upper limit is $L \lesssim 10^{5}cm$, much larger than one obtained for other models. One brane models with interesting range of $M_5$ are ruled out.

This study has an additional interesting conclusion: The close relation between a very small but non-zero Cosmological Constant and the smallness of the Newton coupling constant (i.e. the hierarchy problem). In fact without the fine-tuning of these apparently independent physical quantities, the brane models are not consistent.

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