Research on the correction method of the boundary water level of the mountainsides in seepage analysis

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Abstract. The accuracy of the calculation results in seepage analysis is determined largely by the boundary conditions. The construction of underground caverns and reservoir impoundments will inevitably lead to a redistribution of the underground seepage fields and the change in groundwater level. However, commonly used finite-element simulation methods rarely consider the effects of changes in the boundary water level. This study examined the mechanism of the change in groundwater level caused by impoundment. The drainage of underground caverns on the boundary water level of the model on the mountainsides, and the boundary water-level correction method is proposed. By applying simplified engineering examples, the groundwater levels of the boundaries were calculated (impoundment condition). The results show that the distribution of the seepage field before and after the correction is different. Furthermore, the calculation results of the flow rate are exaggerated (drainage condition of the cavern), as the groundwater drawdown at the computation boundary is ignored. The proposed method can correct the water level at the boundary of the model and effectively improve the calculation accuracy of seepage analysis.

1. Introduction

Seepage movement is controlled by groundwater mass conservation and linear momentum conservation equations. However, the evolution process is restricted by the initial conditions, boundary conditions, and calculation parameters [1]. The main objectives of seepage analysis are as follows: to determine the basic physical quantities of the seepage field, such as the water head, seepage velocity distribution, seepage flow rate, and seepage gradient; to evaluate the safety and economic benefits of the project [2]; to select reasonable seepage control measures [3]. Fluid mechanics, hydraulics, flow nets, electric analog tests, and numerical-analysis methods are commonly used for seepage analysis. In particular, numerical-analysis methods have been developing rapidly in recent years owing to their high calculation accuracy, speed, and convenience, as well as their suitability for complex boundaries and solving large-scale problems. Currently, the finite-element method is used mainly for seepage control analyses.

The accuracy of the seepage calculation results is closely related to the selection of the boundary conditions of the seepage zone [4]. The boundary conditions in calculating the seepage field can be divided into three types [5]: the constant water head boundary, also called the Dirichlet condition; the constant-discharge boundary, also called the Neumann condition; the potential overflow boundary. The lateral boundary of the model on the mountainsides is the infiltration boundary of the seepage field. Many studies have analyzed and investigated the boundary conditions. One considers it the zero-discharge boundary, and the other regards it as a constant water head boundary. For example, while considering that the seepage field is affected only by the reservoir water, Hu et al. [6] regarded the...
boundary as the zero-discharge boundary. They did not consider the recharge of the natural groundwater of the mountain on both sides in the seepage calculation domain. Such boundary treatment cannot reflect the actual seepage field accurately, which is detrimental to engineering safety. Tang et al. [7] considered the boundary a constant water head boundary, and the water head value was determined using known geological survey data or inverse modeling of the initial seepage field. Such calculations are often based on natural hydrogeological data. The impact of the reservoir water level on the boundary water level after reservoir impoundment was not considered. The dynamic evolution characteristics of the mountain seepage field were ignored, leading to inaccurate boundary conditions. Similarly, it also affected engineering safety.

At present, many scholars have different opinions on how to apply the boundary and obtain the value, which will affect the calculation accuracy of the seepage field directly. However, no further research has been conducted on this topic. This research aims to determine the value of the boundary, explore the influence mechanism of reservoir impoundment and the drainage of underground caverns on the boundary water level, and examine the correction of the water level.

2. Correction Method for Boundary Water Level

2.1. Basic assumptions
(1) The slope of the reservoir bank is homogeneous and isotropic. (2) The natural groundwater level in the slope of the reservoir bank is high, and the groundwater level at the mountainside boundary of the seepage calculation domain is higher than the normal water level of the reservoir. (3) After the water level of the reservoir rises and stabilizes, groundwater flow becomes slow, and the slope of the phreatic surface decreases. The water head does not change with depth assuming that the phreatic surface is flat and the water head is vertical.

Figure 1 shows the adoption of Dupuit’s assumption in the slowly varying pressureless seepage on the horizontal impervious layer. The single-width flow rate and water depth at any distance $x$ can be expressed as follows [8]:

\[
q = \frac{k\left(h_1^2 - h_2^2\right)}{2x},
\]

\[
h = \left[h_2^2 + \left(h_1^2 - h_2^2\right)\left(\frac{x}{L}\right)^{0.5}\right],
\]

where $h_1$ and $h_2$ represent the water levels of the upstream and downstream faces, respectively; $h$ is the water head at any distance $x$, from the downstream; $q$ is the single-width flow rate; $k$ is the hydraulic conductivity.

![Figure 1. Dupuit’s hypothetical seepage water-depth change.](image-url)
2.2. Computational theory

The reservoir begins to store water. After the water level rises and stabilizes, the location of the seepage point becomes significantly higher than that of the initial seepage field, i.e., from \( A_0 \) to \( A_1 \), as shown in figure 2. In this case, the water level at the model boundary \( FG \) should be \( H_{B_1} \) (figure 2). The groundwater level around the caverns will decrease when there are underground caverns in the slope because of the drainage effect of the caverns. In this case, the groundwater level at the boundary \( FG \) should be \( H_D \) (figure 2). Therefore, more study is needed to examine the mechanism for the influence of these external factors on the boundary water level and determine the changed boundary value.

![Figure 2. Typical chart of the geological section.](image)

Typically, the seepage-free surface of a homogeneous slope follows the Dupuit seepage theory. Therefore, the seepage-free surface \( A_1B_1C \) of the reservoir bank slope satisfied the Dupuit curve equation as follows:

\[
z = \left[ H_1^2 + \left( H_{\text{max}}^2 - H_1^2 \right) \left( \frac{x}{L_1 + L_2} \right) \right]^{0.5},
\]

where \( H_1 \) is the normal water level of the reservoir; \( H_{\text{max}} \) is the groundwater level at the surface watershed on the slope of the reservoir bank, which is generally regarded as an underground watershed; \( L_1 \) is the horizontal distance from the model boundary to the intersection of the reservoir water surface and the bank slope; \( L_2 \) is the horizontal distance from the model boundary to the surface watershed.

The single-width flow rates, \( q \), of the cross-section passing points \( A_1 \) and \( C \) can be calculated as follows:

\[
q_{A_1} = k J_{A_1} H_1,
\]

\[
q_C = k J_C H_{\text{max}},
\]

where \( k \) is the hydraulic conductivity, and \( J_{A_1} \) and \( J_C \) are the hydraulic gradients at points \( A_1 \) and \( C \), respectively.

When \( x = L_1 \), the water head at \( B_1 \) is formulated as

\[
H_{B_1} = \sqrt{\frac{L_1 H_{\text{max}}^2 + L_2 H_1^2}{L_1 + L_2}},
\]

where \( H_{B_1} \) is the initial calculated water head value at the boundary \( FG \) on the mountainsides of the model. Furthermore,

\[
|H_{B_0} - H_{B_0-1}| \leq \varepsilon, \varepsilon \in (0.01, 0.1),
\]
where $H_{Bn}$ and $H_{Bn-1}$ are the calculated water head values of the $n^{th}$ and $n-1^{th}$ times at the boundary $FG$ on the mountainsides of the model, respectively; $n = 1, 2, 3…$

3. Engineering Applications

3.1. Boundary water level correction under impoundment

In this study, the left-bank slope of the Baihetan Project on the Jinsha River in Southwest China is taken as an example [9], as shown in figures 3 and 4. The left-bank slope watershed of the pivot area is far from the Jinsha River valley. A 2D finite-element model of a typical left-bank profile was established to analyze the distribution of the initial seepage field on the left bank and the water level change characteristics of the reservoir at the boundary on the mountainsides of the model after reservoir impoundment, as shown in figure 5.

The cross-section of the exploration line IX2 was used as a typical section, as shown in figure 3. The model had a total length, height, and slope ratio of 1400 m, 800 m, and 1:1, respectively. The centerline of the riverbed is taken as the right boundary of the model, extending 1400 m from the reservoir area as the left boundary. The elevation of the bottom of the model is 300 m. Therefore, the right boundary of the model is a constant water head boundary. The total water head under natural conditions was 591 m based on Chen’s [9] inverse analysis of the seepage field on the left bank; the total water head value at the right boundary of the model under normal operating conditions was 825 m. The boundary condition at the left side of the model is regarded as a constant water head boundary. The water level at the left boundary was determined using the correction method. Four-node elements (figure 5) are used to maintain the balance between the resolution and computational efficiency. The model of the slope was divided into 1848 elements and 1938 nodes, as shown in figure 5.

Figure 3. Location and landscape of the Baihetan dam site.

Figure 4. Geologic cross-section along the IX2 exploration line [9].
3.1.1. Analysis of corrected results

Seepage calculations were conducted on the bank slope under natural and normal operating conditions. The water level of the slope at the calculation boundary of the model was corrected under normal operating conditions. The calculated boundary $x = 1400$ m indicates that the left-bank model range is 1400 m, as shown in figure 5. The calculation results under the scope of the model are as follows.

Table 1. Calculation results of the water level and discharge.

| Calculation boundary $x = 1400$ m | Natural conditions | Normal operating conditions |
|----------------------------------|--------------------|---------------------------|
|                                  |                    | Uncorrected               |
| Water level (m)                  | 955.30             | 955.30                    |
| Discharge $Q$ (m$^3$/s)          | $5.898 \times 10^{-3}$ | $2.621 \times 10^{-3}$     |
|                                  |                    | Corrected                |
| Water level (m)                  |                     | 1020.14                   |
| Discharge $Q$ (m$^3$/s)          | $5.761 \times 10^{-3}$ |                         |

Figure 5. Finite-element mesh for the slope.

Figure 6. Distribution of the phreatic surface and water head isoline before and after the correction.

Figure 7. Comparison of the water head values before and after the correction.

Generally, the water level at the boundary on the mountainsides of the model was obtained by inverting the initial seepage field. In this study, the water level was calculated by finite-element analysis based on the inverted boundary conditions. Table 1 lists the calculation results. The water level at the calculated boundary under natural conditions was 955.30 m, and the discharge through the calculated boundary was $5.898 \times 10^{-3}$ m$^3$/s. Compared to natural conditions, the reservoir water level under normal operating conditions increased by 234 m (80.4 %). When the water level at the calculated boundary was not corrected, the water level at the boundary was also 955.30 m, but the discharge through the calculated boundary was $2.621 \times 10^{-3}$ m$^3$/s.
However, as the water level rises after the reservoir impoundment, it affects the water level at the calculated boundary. Furthermore, as the watershed is far from the reservoir, its groundwater level will not change with increasing reservoir water level. The water level at the boundary increases to 1020.14 m, and the discharge through the calculated boundary is $5.761 \times 10^{-3}$ m$^3$/s using the correction method in this study.

Figures 6 and 7 compare the seepage-free surface and water head values before and after correction. The figures show significant differences in the water head values at different positions before and after the correction. The closer the model calculation boundary is to $x = 1400$, the more significant the difference. The water level at the calculated boundary was increased by approximately 9.9% after reservoir impoundment. When the water level at the calculated boundary was uncorrected, the discharge for replenishment from outside the seepage calculation domain to inside the domain was $2.621 \times 10^{-3}$ m$^3$/s, whereas when corrected, the recharge flow from outside the seepage calculation domain to inside the domain was $5.761 \times 10^{-3}$ m$^3$/s. Moreover, the recharge flow was reduced by approximately 54.5% when the water level at the calculated boundary was uncorrected. Therefore, the water level and discharge at the calculated boundary after reservoir impoundment have a significant impact that cannot be neglected.

3.2. Boundary water level correction under underground caverns

To analyze the influence of the drainage of underground caverns on the boundary water level, a simplified model of the slope with cavern and drainage system was established using finite-element software, as shown in figure 8. Tetrahedral elements (figure 9) were used in this study. The slope model was divided into 830078 elements and 522187 nodes, as shown in figure 9. The cavern had a width, height, length, and buried depth of 25.40, 50.27, 157.01, and 1200 m, respectively (figure 10). The diameter, spacing, and distance of the drainage hole array around the cavern were 5 cm, 5 m, and 20 m, respectively, and the drainage hole array on the top was distributed diagonally upward at 30°. The drainage holes in the simulation model were simplified to line elements [10, 11]. The groundwater level was considered to be 50 m below the ground surface. The hydraulic conductivity of the homogeneous layer was $10^{-7}$ m/s.

The boundary conditions on the upstream, downstream, and left and right sides of the model were constant head boundaries. The inner wall of the cavern and the drainage hole were the potential seepage boundaries. The upstream and downstream of the model were extended symmetrically for 200 m to examine the changes in the groundwater level at the boundary. Therefore, 11 numerical simulation experiments with different model ranges were carried out, and steady seepage analysis was carried out simultaneously.

Figure 8. Simulation model of the slope with an underground cavern, underground cavern, and drainage system [11].
3.2.1. Variation of groundwater level in calculation boundary

The results show that the initial distance simulation model (1400 m between the upstream and downstream) in figure 11 overstates the value of $Q$ because the groundwater level in the upstream and downstream boundary is constant. Indeed, the boundary groundwater level is influenced by the drainage effect of the cavern, and an increase in the model range can impair the influence. Thus, from another perspective, the boundary groundwater level can be reduced to avert the drainage effect of the cavern without increasing the model.

Figure 10 shows the results of downstream boundary groundwater level of the initial distance simulation model in figure 8 with increasing $R$. As shown in figure 10, the boundary groundwater level decreased remarkably with increasing $R$ and tended to be steady, which is similar to the change in $Q$ with increasing $R$. Replacing the boundary groundwater level of the initial distance simulation model in figure 8 with the results in figure 10 resulted in the data in figure 11, which presents the results of $Q$ confirmed by increasing $R$ and decreasing boundary groundwater level, respectively. The results of $Q$ in figure 11 confirmed by decreasing boundary groundwater level was very close, which was confirmed by the increasing model range. Thus, decreasing the boundary groundwater level can effectively eliminate the drainage effect of the cavern, but the mechanism of the relationship between the boundary groundwater level and engineering characteristics was unclear; more relevant studies are needed. The boundary water level can be corrected to eliminate the influence of cavern drainage for steady-state seepage analysis. However, for transient seepage analysis, boundary values that vary with time need to be used.

![Figure 9. Finite-element mesh for the slope.](image)

![Figure 10. Change in the downstream boundary groundwater level of the initial model in figure 8 with increasing $R$.](image)
4. Conclusions
Determining the boundary water level on mountainsides is key to seepage calculations. In this paper, two situations in engineering that can lead to changes in the boundary conditions were summarized. Subsequently, a method to correct the boundary water level was proposed considering the influence of reservoir impoundment and the drainage of underground caverns on the groundwater level at the boundary. Finally, they were applied to numerical simulation experiments.

The boundary types were complex, and the boundary values were difficult to determine, which is generally the case in the seepage analysis of reservoir-dam systems. Reservoir impoundment and the drainage of underground caverns cause dynamic changes and redistribution of the seepage field in the slope, leading to an inappropriate boundary value problem in the seepage calculation.

Based on Dupuit’s theory and combined with finite-element software, a method was proposed to correct the groundwater level of the boundary on the mountainsides. The distribution of the seepage field before and after the correction was significantly different, and the results for $Q$ and the groundwater level were affected significantly using inappropriate boundary values. In addition, the boundary water level was corrected using the proposed method, which can counteract the influence of change in groundwater level based on the condition of reservoir impoundment and the drainage to the underground caverns and eliminate the seepage analysis error caused by inappropriate boundary values.

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