Consistent couplings between fields with a gauge freedom and deformations of the master equation

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Abstract

The antibracket in BRST theory is known to define a map
$H^p \times H^q \rightarrow H^{p+q+1}$ associating with two equivalence classes of BRST
invariant observables of respective ghost number $p$ and $q$ an equivalence class of BRST invariant
observables of ghost number $p+q+1$. It is shown that this map is trivial in the space of all functionals, i.e.,
that its image contains only the zeroth class. However it is generically non trivial in the space of local functionals.

Implications of this result for the problem of consistent interactions among fields with a gauge freedom are then drawn. It is shown
that the obstructions to constructing such interactions lie precisely in
the image of the antibracket map and are accordingly inexistent if one
does not insist on locality. However consistent local interactions are
severely constrained. The example of the Chern-Simons theory is con-
sidered. It is proved that the only consistent, local, Lorentz covariant
interactions for the abelian models are exhausted by the non-abelian
Chern-Simons extensions.

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1 introduction

The antifield formalism [1, 2] appears to be one of the most powerful and elegant methods for quantizing arbitrary gauge theories. Originally presented as a set of efficient working rules, its physical foundations have been gradually clarified by showing how gauge invariance is completely captured by BRST cohomology [3, 4, 5]. Some of its geometrical aspects (Schouten bracket, role of Stokes theorem in the proof of the gauge independence of the path integral) have been developed in [6] and more recently in [7, 8, 9, 10]. The somewhat magic importance of the antifield formalism in string field theory [11, 12, 13, 14, 15, 16] and its remarkable underlying algebraic structure [17, 18, 19, 20, 21] have attracted further considerable attention (see also [22]). It is fair to believe that more interesting results are still to come.

The purpose of this letter is to reanalyze the long-standing problem of constructing consistent interactions among fields with a gauge freedom in the light of the antibracket formalism. We point out that this problem can be economically reformulated as a deformation problem in the sense of deformation theory [23], namely that of deforming consistently the master equation. We then show, by using the properties of the antibracket, that there is no obstruction to constructing interactions that consistently preserve the gauge symmetries of the free theory if one allows the interactions to be non local. Obstructions arise only if one insists on locality. We provide a reformulation of the deformation of the master equation that takes locality into account, and illustrate the new features to which this leads by considering the three dimensional Chern-Simons theory. We show that the only local, Lorentz covariant, consistent interactions for free (abelian) Chern-Simons models are given by the non-abelian Chern-Simons theories. We also establish the rigidity of the non-abelian Chern-Simons models with a simple gauge group. A fuller account of our results will be reported elsewhere [24].

These results are in line with the light-front analysis of [24], as well as with the work of [25] where the role of the master equation is also strongly stressed.
2 The master equation and the antibracket map

We first recall some basic properties of the antifield formalism. The starting point is the action in Lagrangian form $S_0[\varphi^i]$, with gauge symmetries

$$\delta_\varepsilon \varphi^i = R^i_\alpha \varepsilon^\alpha. \quad (1)$$

Given $S_0[\varphi^i]$, one can, by introducing ghosts and antifields, construct the solution $S[\varphi^A, \varphi_A^*]$ of the master equation,

$$S = S_0 + \varphi_A^* R^i_\alpha C^\alpha + ... \quad (2)$$

$$S, S = 0 \quad (3)$$

where $\varphi^A \equiv (\varphi^i, C^\alpha, ...)$ denotes collectively the original fields, the ghosts and the ghosts of ghosts if necessary, while $\varphi_A^*$ stands for the antifields. The solution $S$ of the master equation captures all the information about the gauge structure of the theory. The existence of $S$ reflects the consistency of the gauge transformations. The Noether identities, the (on-shell) closure of the gauge transformations and the higher order gauge identities are contained in the master equation $(S, S) = 0$. The original gauge invariant action $S_0$ itself and the gauge transformations can be recovered from $S$ by setting the antifields equal to zero in $S$ or in $\delta S / \delta \varphi_A^*$,

$$S_0 = S[\varphi^A, \varphi_A^* = 0] \quad (4)$$

$$\delta_\varepsilon \varphi^i = \frac{\delta S}{\delta \varphi_i^*}[\varphi^A, \varphi_A^* = 0] (C^\alpha \rightarrow \varepsilon^\alpha). \quad (5)$$

The BRST differential $s$ in the algebra of the fields and the antifields is generated by $S$ through the antibracket,

$$sA \equiv (A, S). \quad (6)$$

The BRST cohomology is denoted by $H^*(s)$. It is easy to verify that the antibracket induces a well defined map in cohomology,

$$\langle \cdot, \cdot \rangle : H^p(s) \times H^q(s) \rightarrow H^{p+q+1}(s) \quad (7)$$

\footnote{We shall follow the presentation of the antifield formalism given in \textsuperscript{4} (chapters 15, 17 and 18). We refer the reader to that reference for more information.}
where \([A]\) denotes the cohomological class of the BRST-closed element \(A\).
We call (7) “the antibracket map”. If one takes \([A] = [B]\) in (8), one gets a map from \(H^p(s)\) to \(H^{2p+1}(s)\) sending \([A]\) on \([(A, A)]\).

It is sometimes useful to introduce auxiliary fields in a given theory, namely, fields that can be eliminated by means of their own equations of motion. This may, for instance, simplify the gauge structure and the geometric interpretation of the theory. One then has various equivalent formulations and a natural question to ask is: what is the relationship between the BRST cohomologies and the antibracket map of these equivalent formulations? Not surprisingly, one has

**Theorem 1** the BRST cohomologies \(H^*(s)\) and \(H^*(s')\) associated with two formulations of a theory differing in the auxiliary field content are isomorphic. Furthermore, the isomorphism \(i : H^*(s) \rightarrow H^*(s')\) commutes with the antibracket map.

*Proof*: the proof is direct and based on the explicit relationship between the solutions of the master equation of both formulations worked out in \([27]\). We leave it as an exercise to the reader.

Using theorem 1, one can now establish the crucial result that the antibracket map is trivial.

**Theorem 2** the antibracket map is trivial, i.e., the antibracket of two BRST-closed functionals is BRST-exact.

*Proof*: the proof consists in two steps: (i) One adds auxiliary fields and fixes the gauge in such a way that (a) the gauge fixed equations of motion are of first order in the time derivatives and can be solved for \(\dot{\varphi}^A\); and (b) the BRST variation of the fields depends only on the fields and not on their time derivatives or on the antifields. This can be done for instance by going to the Hamiltonian formalism, and, as we have seen, modifies neither the BRST cohomology nor the antibracket map. (ii) By expressing the fields
in terms of initial data on a Cauchy hypersurface, one proves the existence, in each BRST cohomological class, of a representative that does not involve the antifields. More precisely, let $A[\varphi^A, \varphi^*_A]$ be a solution of $sA = 0$ and let $\tilde{A}[\varphi^A]$ be the functional of the free initial data that coincides with $A[\varphi^A, \varphi^*_A = 0]$ on-shell. One easily verifies that $s\tilde{A} = 0$. Furthermore, $A$ and $\tilde{A}$ are in the same cohomological class due to general properties of the antifield formalism [7]. For representatives that do not involve the antifields, $(A, B)$ vanishes identically and not just in cohomology. This proves the theorem (a more detailed analysis will be given in [26]).

3 Higher order maps

The triviality of the antibracket map enables one to define higher order operations in cohomology. For example, if $[A] \in H^p(s)$, one can define a squared map $H^p(s) \to H^{3p+1}(s)$ as follows: the antibracket $(A, A)$ is a coboundary. Accordingly, there exist a functional $B$ of degree $2p$ such that $(A, A) = (B, S)$. The functional $B(\varphi, \varphi^*)$ is defined up to a cocycle. Now $(A, B)$ is easily verified to be BRST-closed and the cohomological class of $(A, B)$ does not depend on the ambiguity in $B$. Furthermore, $[(A, B)] = [(A', B')]$ if $[A] = [A']$. Hence, the application $H^p(s) \to H^{3p+1}(s)$ that maps $[A]$ on $[(A, B)]$ is well-defined. In our case, however, the squared map and all the other higher order maps that can be defined in a similar fashion are trivial since one can choose representatives in $H^p(s)$ such that $(A, A)$ and hence $B$ both strictly vanish.

4 Constructing consistent couplings as a deformation problem

We now turn to the problem of introducing consistent interactions for a “free” action $S_0^{(0)}[\varphi^i]$ with “free” gauge symmetries

$$\delta_\varepsilon \varphi^i = R^{(0)}_\alpha \varepsilon^\alpha, \quad (9)$$

$$\frac{\delta (0)}{\delta \varphi^i} R^{(0)}_\alpha = 0. \quad (10)$$
We want to modify $S_0$

\begin{equation}
S_0 \rightarrow S_0 = S_0 + g S_0 + g^2 S_0 + \ldots
\end{equation}

in such a way that one can consistently deform the original gauge symmetries,

\begin{equation}
R_\alpha^i \rightarrow R_\alpha^i = R_\alpha^i + g R_\alpha^i + g^2 R_\alpha^i + \ldots
\end{equation}

By “consistently”, we mean that the deformed gauge transformations $\delta_\varepsilon \varphi^i = R_\alpha^i \varepsilon^\alpha$ are indeed gauge symmetries of the full action (11),

\begin{equation}
\frac{\delta (S_0 + g S_0 + g^2 S_0 + \ldots)}{\delta \varphi} (R_\alpha^i + g R_\alpha^i + g^2 R_\alpha^i + \ldots) = 0.
\end{equation}

This implies automatically that the modified gauge transformations close on-shell for the interacting action (see [7], chapter 3). In the case where the original gauge transformations are reducible, one should also demand that (12) remain reducible. Interactions fulfilling these requirements are called “consistent”. [It may be necessary to add further consistency requirements, but this will not be considered here].

A trivial type of consistent interactions is obtained by making field redefinitions $\varphi^i \rightarrow \tilde{\varphi}^i = \varphi^i + g F^i + \ldots$. One gets

\begin{equation}
S_0 \rightarrow S_0 = S_0 + g \varphi^i + g^2 \varphi^i + \ldots = S_0 + g \delta \varphi F^i + \ldots
\end{equation}

Interactions that can be eliminated by field redefinitions are usually thought of as being no interactions. We shall say that a theory is rigid if the only consistent deformations are proportional to $S_0$ up to field redefinitions. In that case, the interactions can be summed as

\begin{equation}
S_0 \rightarrow S_0 = (1 + k_1 g + k_2 g^2 + \ldots) S_0
\end{equation}

and simply amount to a change of the coupling constant in front of the unperturbed action.

The problem of constructing consistent interactions is a complicated one because one must simultaneously modify $S_0$ and $R_\alpha^i$ in such a way that
is valid order by order in $g$. It has been studied for lower spins by many authors (see for instance [28, 29, 30] and references therein) and some aspects of the algebraic structure underlying the construction were clarified in [31]. One can reformulate more economically the problem in terms of the solution $S$ of the master equation. Indeed, if the interactions can be consistently constructed, then the solution $S$ of the master equation for the free theory can be deformed into the solution $(0)S$ of the master equation for the interacting theory

$$S \rightarrow S = (0)S + g (1)S + g^2 (2)S + ... \quad (16)$$

The master equation $(S, S) = 0$ guarantees that the consistency requirements on $S_0$ and $R_i^\alpha$ are fulfilled.

There is a definite advantage in reformulating the problem of consistent interactions as the problem of deforming the master equation. It is that one can bring in the cohomological techniques of deformation theory. The master equation for $S$ splits according to the deformation parameter $g$ as

$$2(0)S, (0)S = 0 \quad (18)$$
$$2(0)S, (1)S = 0 \quad (19)$$
$$2(0)S, (2)S + (1)S, (1)S = 0 \quad (20)$$

The first equation is satisfied by assumption, while the second implies that $(1)S$ is a cocycle for the free differential $(0)s \equiv (\cdot, (0)S)$. Suppose that $(1)S$ is a coboundary, $(1)S = (1)T, (0)S$. This corresponds to a trivial deformation because $(0)S_0$ is then modified as in (14)

$$S_0 \rightarrow S_0 - g \left( \frac{\delta (1)T}{\delta \varphi_i^*} \delta (0)S \right) \quad (21)$$

Deforming the master equation also appears in renormalization theory where (17) is replaced by the equation $(\Gamma, \Gamma) = 0$ for the generating function of proper vertices [32].
(the other modifications induced by $T_a$ affect the higher order structure functions which carry some intrinsic ambiguity \[3\]). Hence, non trivial deformations are determined by the zeroth cohomological space $H^0(\mathcal{O})$ of the undeformed theory. This space is generically non-empty: it is isomorphic to the space of observables \[3, 4, 7\].

The next equation (19) implies that $\mathcal{S}$ should be such that $\left(\mathcal{S}, \mathcal{S}\right)$ is trivial in $H^1(\mathcal{O})$. But we have seen that the map $H^0(\mathcal{O}) \to H^1(\mathcal{O})$ induced by the antibracket is trivial and so, this requirement is automatically satisfied. Similarly, the higher order maps $H^0(\mathcal{O}) \to H^1(\mathcal{O})$ are also trivial, which guarantees that the next terms $\mathcal{S}, \mathcal{S}, ...$ exist. Thus given an initial element $\mathcal{S}$ of $H^0(\mathcal{O})$, there is no obstruction in continuing the construction to get the complete $\mathcal{S}$. The next terms $\mathcal{S}, \mathcal{S}, ...$ are determined up to an element of $H^0(\mathcal{O})$, i.e., up to a gauge invariant function. At each order in $g$ there is the freedom of adding to the interaction an arbitrary element of $H^0(\mathcal{O})$.

We can thus conclude that in the absence of particular requirements on the form of the interactions such as spacetime locality or manifest Lorentz covariance, there is no obstruction to constructing interactions that preserve the initial gauge symmetries as in (13). In other words, there is no “no-go theorem”.

## 5 Spacetime locality of the deformation - The example of free abelian Chern-Simons models

The above construction does not yield, in general, a local action and is somewhat formal. In practice, it is usually demanded that the deformation be local in spacetime, i.e., that $\mathcal{S}, \mathcal{S}, ...$ be local functionals. This leads to interesting developments.

In order to implement locality in the above analysis, we recall that if $A$ is a local functional which vanishes for all allowed field configurations, $A = \int a = 0$, then, the $n$-form $a$ is a “total derivative”, $a = dj$, where $d$ is the
spacetime exterior derivative and \( j \) is such that \( \mathbf{j} = 0 \) (see e.g. [7] chapter 12). That is, one can “desintegrate” equalities involving local functionals but the integrands are determined up to \( d \)-exact terms.

Let \( \hat{S} = \int \hat{\mathcal{L}} \) where \( \hat{\mathcal{L}} \) is a \( k \)-form depending on the variables and a finite number of their derivatives, and let \( \{a, b\} \) be the antibracket for such \( n \)-forms, i.e.,

\[
(A, B) = \int \{a, b\}
\]

if \( A = \int a \) and \( B = \int b \). [Because \( (A, B) \) is a local functional, there exists \( \{a, b\} \) such that (22) holds, but \( \{a, b\} \) is defined only up to \( d \)-exact terms. This ambiguity plays no role in the subsequent developments]. The equations (18-20) for \( \hat{S} \) read

\[
\begin{align*}
2 \, s^{(0)(1)} \mathcal{L} & = d^{(1)} j \\
\mathcal{L}^{(1)} + \{(\mathcal{L}, \mathcal{L})\} = d^{(2)} j \\
\vdots
\end{align*}
\]

in terms of the integrands \( \mathcal{L} \). The equation (23) expresses that \( \mathcal{L} \) should be BRST closed modulo \( d \) and again, it is easy to see that a BRST-exact term modulo \( d \) corresponds to trivial deformations. Non trivial local deformations of the master equation are thus determined by \( H^0(\mathcal{L} \mid d) \). [Note that an element of \( H^0(\mathcal{L} \mid d) \) yields upon integration an element of \( H^0(\mathcal{L}) \) only if appropriate surface terms vanish. We shall not investigate this question here and work with all the elements of \( H^0(\mathcal{L} \mid d) \)].

Now, while \( (S, \mathcal{L}) \) is always cohomologically trivial, it is not true, in general, that it is the BRST variation of a local functional. Hence, \( (\mathcal{L}, \mathcal{L}) \) may not be BRST-exact modulo \( d \), and the map

\[
H^p(\mathcal{L} \mid d) \times H^q(\mathcal{L} \mid d) \longrightarrow H^{p+q+1}(\mathcal{L} \mid d)
\]

defined by the antibracket appears to possess a lot of structure. Furthermore, even when the image of \( \{\mathcal{L}, \mathcal{L}\} \) is trivial in \( H^0(\mathcal{L} \mid d) \), so that the squared
map $H^0(s \mid d) \rightarrow H^1(s \mid d)$ can be defined, there is no guarantee that this squared map is trivial. For this reason, the construction of local, consistent interactions is a problem that is quite constrained.

To illustrate this point, we shall analyze the case of the abelian Chern-Simons models in three dimensions.

The action is given by
\[
S_0 = \int d^3 x \frac{1}{2} \varepsilon^{ijk} k_{ab} A_i^a F_{jk}^b
\]
where $k_{ab}$ is a non-degenerate, symmetric and constant matrix. The equations of motion imply $F_{ij}^a = 0$. An irreducible set of gauge transformations can be taken to be
\[
\delta_\varepsilon A_i^a = \partial_i \varepsilon^a.
\]
The minimal solution to the classical master equation is
\[
S = S_0 + \int d^3 x A_i^a \partial_i C^a
\]
and the local version of the BRST symmetry is then
\[
\varepsilon^{ijk} F_{jk}^b k_{ab} \partial^i A_i^a + \partial_i A_i^a - \partial_i A_i^a \frac{\partial}{\partial C^a} A_i^a + \partial_i \frac{\partial}{\partial A_i^a}
\]
with $[\varepsilon, \partial_i] = 0$ and $\varepsilon \partial + d \varepsilon = 0$. As we have pointed out, the perturbation $L$ should obey
\[
L + da[2] = 0
\]
i.e., should define an element of $H^0(s \mid d)$. The equation $(34)$ can be analyzed along lines familiar from the algebraic study of anomalies. Indeed, one gets from $(34)$ a set of “descent equations”
\[
\varepsilon \partial + d \varepsilon = 0
\]
To solve (30), one needs to find the most general element at the bottom of the ladder that can be lifted all the way up to yield an element of $H^{(0)}|d)$. This is the procedure followed in [35]. Now, the last element of a descent belongs to $H^{(0)}$, and must be a polynomial in the ghosts $C^a$. [Because the equations of motion imply $F_{ij} = 0$, $F_{ij}$ is trivial in cohomology]. Thus

$$a_{[0]} = f_{abc} C^a C^b C^c$$  \hspace{1cm} (35)

where $f_{abc}$ is completely antisymmetric. This implies

$$a_{[1]} = 3 f_{abc} A^a C^b C^c + m_{ab} C^a C^b$$  \hspace{1cm} (36)

where $m_{ab} C^a C^b$ belongs to $H^{(0)}$ and $m_{ab}$ is a constant 1-form. By Lorentz covariance, this term must be zero. This leads then to

$$a_{[2]} = -\frac{3}{2} f_{a b c}^* A_a^* C^b C^c + 3 f_{a b c} A_a^* \wedge A^b C^c$$  \hspace{1cm} (37)

and finally to

$$(^{(1)}\mathcal{L} = \frac{1}{6} f_{a b c}^* C^a C^b C^c + f_{a b c}^* 3 A_a^* \wedge A^b C^c + f_{a b c} A_a^* \wedge A^b \wedge A^c).$$  \hspace{1cm} (38)

It should be noted that $S = \int^{(1)} \mathcal{L}$ is $s$-trivial in the space of all functionals. Indeed, assuming that the fields decrease at infinity, one can decompose $A_a^a$ and $A_a^a$ as

$$A_a^a = \partial_i \varphi^a + A_i^a, \quad A_a^a = \partial_j \varphi^a + \varepsilon^{ijk} \partial_j \mu^a_k$$  \hspace{1cm} (39)

Because $A_i^a = 0$ by the equations of motion, one finds that $S_0$ vanishes on-shell,

$$S_0 \approx \int 2 \partial_i \varphi^a \partial_j \phi^b \partial_k \phi^c \varepsilon^{ijk} f_{a b c} d^3 x = 0.$$  \hspace{1cm} (40)

This implies that $S$ is BRST-exact ([4]) and indeed

$$S = (F, S)$$  \hspace{1cm} (41)

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5Different boundary conditions or a non trivial spacetime topology would require a more sophisticated treatment.
with
\[ F = (0) \int d^3 x f_{bc}^a \left( \frac{1}{6} \mu^*_{ia} A^T_j A^T_k + \frac{1}{2} \mu^*_{ia} \partial_j \varphi^b \partial_k \varphi^c + \partial_i \mu^*_{ja} A^T_k \varphi^c \right) \varepsilon^{ijk} - \Box^{-1} \partial^a C^*_a A^T_i A^c - \Box^{-1} \partial^a C^*_a \partial_j \varphi^b C^c \right) . \] (42)

However, \( F \) is a non-local functional of the fields and thus, \( (1) S \) cannot be eliminated by local redefinitions. One then computes \( (1) S, (1) S \). One finds
\[ (1) S, (1) S = 8 \int d^3 x \left\{ (f_{abc} \varepsilon^{ijk} A^b_j A^c_k + f_{acb} A^b_i C^c) + f_{bc}^a (A^b_i A^c_i + C^*_a C^b) \left( \frac{1}{2} f_{de}^c C^d C^e \right) \right\} . \] (43)

The integrand of this expression is a \( (0) s \)-cocycle modulo \( d \) because the Jacobi identity for the local antibracket holds modulo \( d \). In order to construct a non-trivial local interaction, this cocycle must be trivial in \( H((0) s | d) \). Because the image of \( (0) s \) and \( d \) contains no terms without derivatives, a necessary and sufficient condition for this cocycle to be \( (0) s \)-trivial modulo \( d \) is that it vanishes. This is the case if and only if the constants \( f_{bc}^a \) verify the Jacobi identity, even though \( (1) S, (1) S \) is BRST-exact in the space of all functionals for arbitrary choices of \( f_{bc}^a \) thanks to (41). This implies that \( (0) S + (1) S \) is a solution to our deformation problem which corresponds of course to the well-known non-abelian Chern-Simons theories:
\[ (0) S_0 + (1) S_0 = \int d^3 x \left( \frac{1}{2} \varepsilon^{ijk} k_{ab} A^a_i F^b_j A^c_k + \frac{2}{3} \varepsilon^{ijk} f_{abc} A^a_i A^b_j A^c_k \right) \] (44)

Accordingly, the only consistent, Lorentz covariant couplings of abelian Chern-Simons models are the non-abelian extensions.

6 Rigidity of non-abelian Chern-Simons theory

We close this letter by proving the rigidity of the Chern-Simons theory with a simple gauge group. The descent equations for \( L \) are identical to (31)-(34), but this time, the 0-form \( a_{[0]} \) of ghost number 3 should be closed for
the non-abelian BRST differential. The only non trivial element of $H^3(s)$ is the primitive form $\alpha trC^3$, where $\alpha$ is a priori an invariant polynomial in the non-abelian field strength $F$, which, however can be set equal to zero, since the equations of motion are $F_{ij} = 0$. The primitive form $\alpha trC^3$ can be lifted as in the familiar Yang-Mills case to yield $\alpha$ times the Chern-Simons action. Since there is no cohomology in ghost degree 2, 1 or 0 (apart from the irrelevant constants), there is no other element that can be lifted to yield another solution from a shorter descent. This proves the rigidity of the Chern-Simons action.

7 Conclusion

Reformulating the problem of consistent interactions in terms of deformations of the master equation allows the use of powerful BRST cohomological techniques. The triviality of the antibracket map in cohomology in the space of all functionals allows to built consistent interactions from any gauge invariant functionals of the undeformed theory. However, these interactions may be non local and obstructions on consistent local couplings do exist in practice. The study of these obstructions require additional tools familiar from the study of anomalies. The analysis has been illustrated for the Chern-Simons models.

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