Numerical study of thermal stress of a silicon mirror forming a beam of synchrotron radiation

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Abstract. Numerical simulation of heat transfer in a mirror for focusing a synchrotron radiation beam and its thermally stressed state has been carried out. The choice of the method for cooling the mirror through contact with the water-cooled plates, which provides the specified limitations on thermal deformation, has been substantiated. The modes of heat transfer, implemented under different conditions of heat transfer at the boundary of the mirror with water-cooled plates, are compared.

1. Introduction
At present, the construction of the Siberian Circular Photon Source, the SKIF synchrotron, is underway in Novosibirsk. The first stage involves the creation of six research workstations. The designs of workstations that provide studies of the structure of objects in materials science, biology, geology, and medicine are unique. The energy density in the beam on such devices (with normal incidence) can reach values of the order of 1000 mW/m² [1], which requires special approaches to temperature management when creating workstations [2-3].

The operating conditions of the focusing mirrors for the formation of powerful beams of ultraviolet and X-ray synchrotron radiation are characterized by a significant thermal load [4]. Controlling the shape of the mirrors, which requires reliable heat dissipation, is a critical task. The most effective method of mirror cooling is channel liquid cooling [5-7]. Its use is undesirable due to the requirements of the tightness of the vacuum volume; therefore, it should be resorted to only in the case of a proven impossibility to provide restrictions on the deformation of the mirror by other means. From the point of view of technical implementation contact cooling through tightly pressed plates is significantly simpler (see Fig. 1). The disadvantage of this cooling method is that when it is used, almost the entire silicon array in one way or another participates in the process of heat transfer from the heat release region to the cooling plates. Consequently, the temperature gradient that generates thermal deformation is present throughout the entire volume of the body.
The objective of this work is to conduct preliminary studies on the applicability of the method of cooling a mirror using cooled plates. Computational methods for studying heat transfer and thermal stress are used. The object of the study is the toroidal mirror of the projected workstation 1-6 of the Siberian Circular Photon Source [8]. According to the requirements for the quality of radiation focusing, the deviation of the mirror surface from the original shape should not exceed 2 μm. At this stage of the research, the issues of heat transfer in the plates and heat exchange of the plates with the cooling liquid are of secondary importance. The possibility or impossibility of using the discussed cooling method is determined first by the features of the temperature field in the mirror material. Consequently, the analysis can be limited to the area of the mirror material, while the presence of a difference between the temperature in the areas of contact with the plates and the cooling liquid temperature can be described by introducing an effective coefficient of heat transfer with the external environment.

2. Mathematical model
The heat transfer in the polycrystalline silicon mirror and the thermally stressed state of the mirror are studied. The governing equations of the mathematical model are the thermal conductivity equation and the thermoelastic equations.

The mirror shape (Fig. 1) is close to the rectangular hexagon with edge sizes AB = AE = 60 mm, AD = 400 mm. The reflecting face ABCD deviates from a plane by less than 0.5 mm and all other faces are plain. A beam of synchrotron radiation falls on the central region of the reflecting surface with a size of 344×10 mm and forms a complex distribution of the absorbed heat flux with an average value of 31.3 kW/m² (Fig. 2). The distribution of the heat flux is considered, according to the conditions of the
problem, to be specified, independent of temperature. It is symmetric with respect to the line IJ (points I, J are the centers of the edges AB, CD), therefore the problem is symmetric with respect to the plane of geometric symmetry of the body passing through IJ. In this regard, in the computational study, 1/2 of the body is considered. The corresponding boundary conditions are set on the plane of symmetry: the absence of normal components of the vectors of heat flux and displacement. The reflective gold layer on the surface of the mirror has a small thickness (300 Å) and its presence does not affect heat transfer.

On the ADHE face, which is the zone of contact with the cooling plate, the boundary condition for heat transfer is set

\[ \mathbf{qn} = \alpha(T - T_{cool}), \]  

where \( \mathbf{q} \) is the heat flux density vector, \( \mathbf{n} \) is the unit vector normal to the surface and directed outward, \( \alpha \) is the effective heat transfer coefficient, the value of which characterizes qualitatively the presence of heat transfer resistances associated with the heat transfer through the contact with the cooling plate, through the material of the cooling plate and between the cooling channel walls and the coolant; \( T_{cool} \) is the coolant temperature that is set equal to 285 K. On the faces ABFE, ADEH, DCGH, EFGH the conditions of radiation heat exchange with neighbor walls of vacuum volume are considered as boundary conditions and the temperature of the neighbor walls \( T_{ext} \) is set constant:

\[ q_n = \sigma_\varepsilon(T^4 - T_{ext}^4). \]

The parameters of the conditions are: \( T_{ext} = 285 \, \text{K}, \varepsilon = 0.9. \)

We will formulate the problem of the thermally stressed state in such a way as to consider the conditions most favorable for maintaining the initial shape of the mirror during its thermal deformation. By such conditions, we mean those under which some points of the mirror surface exactly retain their location regardless of thermal deformation. Let us clarify that, since the surface of the mirror is almost flat, the displacement of a point on the surface in a direction parallel to the surface practically does not change the shape of the mirror. Surface deformation is determined only by the \( y \)-component of the displacement vector (in the coordinate system shown in Fig. 1). Without imposing restrictions, the implementation of which will require a force acting on the body and taking into account the symmetry, we can assume that the \( y \)-components of the displacement vector are equal to zero no more than at two points of the mirror surface. We take points I, J as such points.

The temperature dependence of the thermal conductivity coefficient of polycrystalline silicon was set based on the data from [9]. The rational function describing the temperature dependence of the thermal conductivity is chosen, which approximates the experimental data [9] within the temperature interval from 150 to 350 K with accuracy 0.4%. The data for the thermal expansion coefficient depending on the temperature are taken from [10]. Linear interpolation of the experimental data is used. The Poisson coefficient value \( \mu = 0.22 \) is taken according to [11].

3. Calculation results

The calculated distributions of temperature and the \( y \)-components of the displacement vector along the segment IJ are shown in Fig. 3. The inverse of the effective heat transfer coefficient in the boundary condition (1) on the ADHE face, measured in m\(^2\)K/kW, is indicated in the notation of the curves. In what follows, this value will be referred to as the coefficient of thermal resistance (CTR). The peculiarities of the temperature profile – two maximum points – are a consequence of the complex distribution of the heat flux density on the surface (see Fig. 2). First of all, it should be noted that in all the cases considered, the maximum deviation from the initial position of the reflecting surface does not exceed 0.8 μm, which is almost 3 times lower than the permissible limit. With an increase in the CTR, the deformation of the mirror increases. Note that this is not a direct consequence of an increase in temperature, since, according to the accepted conditions of the problem, under conditions of uniform heating, the \( y \)-component of the displacement on the mirror surface is strictly zero. However, at low CTR, the region of heat flux propagation in the body or, therefore, the region of the presence of a temperature gradient that generates deformation is limited to the upper part of the body, while at large
CTR this region extends to the entire body. This can be seen from the distribution of the heat flux on the heat-transfer surface – the ADHE face (Fig. 4).

![Figure 3](image1.png)

Figure 3. The distributions of temperature and y-component of the displacement vector along the IJ segment. Curves are noted by the value of CTR in m²K/kW.

![Figure 4](image2.png)

Figure 4. Distribution of the heat flux on the surface of contact with the cooling plate, W/m², for different values of CTR in m²K/kW. The negative values correspond to the outgoing heat flux.

The character of the thermal deformation of the mirror that is presented in Fig. 3 is quite simple. The heating of its upper part leads to the expansion of the material in this area, as a result of which an upward bulge appears and the center point of the mirror rises relative to the edges. At low values of CTR, even small changes in this parameter change the value of deformation significantly, while at the largest of the considered CTR values, the effect of this parameter weakens. So, approximately the same increase in deformation occurs when the CTR changes from 0 to 0.05 and from 0.2 to 1 m²K/kW (two first and two last of the considered values). The reason for this behavior is that as the CTR grows, the temperature field undergoes a rearrangement: at CTR=0 the condition of constant temperature is implemented on the ADHE face, whereas at CTR→∞ the condition of homogeneous heat flux is implemented on the same face. The calculated cases with the largest of the considered values of CTR indeed demonstrate a close to uniform distribution of the heat flux on the heat-transfer surface (Fig. 4). An increase in CTR, which is already large, leads to an almost uniform increase in temperature throughout the body, slightly changing its drop; in accordance with the accepted conditions of the problem, a uniform increase in temperature does not contribute to deformation.
Conclusion
The study has shown the presence of a non-trivial relationship between the temperature field and thermal deformation of the mirror for focusing synchrotron radiation. The distribution of the energy flux of synchrotron radiation in the cross-section of the beam, as a rule, has a complex shape. The calculated results confirmed the possibility of fulfilling the restrictions on the thermal deformation of the toroidal mirror of the projected station 1-6 of the Siberian Circular Photon Source when using mirror cooling by contact heat removal to the water-cooled plates: a method that is much easier to implement and operate in comparison with direct liquid cooling of the mirror material. The calculation results show that it is important to organize the mirror mounting correctly. If the mounting of the mirror ensures the absence of the normal component of displacement of two points of its surface, which does not impose a load on the material of the mirror, then the homogeneous component of heating does not lead to the displacement of other points of the surface. As a result, in particular, the presence of contact thermal resistance at the boundary of the mirror material with the plates does not prevent the fulfillment of the restrictions on thermal deformation of the mirror. In further studies, the objects of research should include the cooling plates, the fluid flow in their channels, and the area of contact of the mirror material with the plates, which creates contact resistance. In the extended formulation of the problem, the issue of optimizing the shape of the plates and their channels, the flow regimes of the cooling coolant can be solved to minimize the deformation of the mirror.

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