A Clifford Dyadic Superfield from Bilateral Interactions of Geometric Multispin Dirac Theory

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Abstract. Multivector quantum mechanics utilizes wavefunctions which are Clifford aggregates (e.g. sum of scalar, vector, bivector). This is equivalent to multispinors constructed of Dirac matrices, with the representation independent form of the generators geometrically interpreted as the basis vectors of spacetime. Multiple generations of particles appear as left ideals of the algebra, coupled only by now-allowed right-side applied (dextral) operations. A generalized bilateral (two-sided operation) coupling is proposed which includes the above mentioned dextral field, and the spin-gauge interaction as particular cases. This leads to a new principle of poly-dimensional covariance, in which physical laws are invariant under the reshuffling of coordinate geometry. Such a multigeometric superfield equation is proposed, which is sourced by a bilateral current. In order to express the superfield in representation and coordinate free form, we introduce Eddington E-F double-frame numbers. Symmetric tensors can now be represented as 4D "dyads", which actually are elements of a global 8D Clifford algebra. As a restricted example, the dyadic field created by the Greider-Ross multivector current (of a Dirac electron) describes both electromagnetic and Morris-Greider gravitational interactions.

Key words: spin-gauge, multivector, clifford, dyadic

1. Introduction

Multivector physics is a grand scheme in which we attempt to describe all basic physical structure and phenomena by a single geometrically interpretable Algebra. A conservative approach recognizes the Dirac algebra as belonging to a Clifford manifold having both spin and coordinate aspects. The spin gauge theory approach to grand unification makes use of a spin Clifford algebra which necessarily commutes with coordinate geometry. We propose a direct projection from this abstract space into concrete coordinate geometric algebra. Ultimately we eliminate spin space entirely by using Clifford aggregates of coordinate geometry to replace ‘spinors’. Spin gauge
theory, an artifact of spin geometry therefore vanishes. However, we gain in having multiple generations of particles appear which are coupled by new dextrad (right-sided multiplication) gauge transformations.

To accommodate all the known couplings we must somehow recover the spin-gauge formalism. This requires transformations which literally reshuffle the geometry, i.e. the basis vectors for one observer might be the trivectors for another observer. This leads us to propose that the general physical laws are invariant under these transformations, a new principle called poly-dimensional covariance. We postulate a single multigeometric superfield equation, which will require two commuting coordinate Clifford algebras, analogous to Eddington’s E-F “double frame” numbers[7]. This dyadic Clifford algebra can be reinterpreted as a single 8D multigeometric space. Multivector Dirac theory expressed in this full algebra potentially has enough degrees of freedom to represent all the fermions of the standard model.

2. Geometric Algebras and Multi-Spinors

We present at first the standard view that abstract entities (e.g. spinors) exist outside of the realm of concrete coordinate geometry. Dirac algebra belongs to a Clifford manifold which has both spin and coordinate features. We propose a direct projection between spin space and coordinate geometry in eq. (2) below.

2.1. Spacetime and the Majorana Algebra

Factoring the second-order meta-harmonic Klein-Gordon equation to the first order meta-monogenic Dirac form requires four mutually anticommuting algebraic elements \( \{\gamma^1, \gamma^2, \gamma^3, \gamma^4\} \),

\[
(\Box^2 - m^2)\Phi(x) = (\Box - m)(\Box + m)\Phi(x), \quad (1a)
\]

\[
\Psi(x) = (\Box + m)\Phi(x) = (\gamma^\mu \nabla_\mu + m)\Phi(x), \quad (1b)
\]

\[
(\gamma^\mu \nabla_\mu - m)\Psi(x) = 0, \quad (1c)
\]

where \( \nabla_\mu = \partial_\mu \) in flat spacetime. Requiring the formulation to be Lorentz covariant imposes the defining condition of a Clifford algebra, \( \frac{1}{2}\{\gamma_\mu, \gamma_\nu\} = g_{\mu\nu} = e_\mu \cdot e_\nu \), where \( e_\mu \) are the coordinate basis vectors. If the use of the abstract \( i \) is excluded, the above factorization of eq. (1a) only works in the metric signature of \( (+ + + -) \). The lowest order matrix representation of the \( \{\gamma_\mu\} \) is \( \mathbb{R}(4) \), i.e. 4 by 4 real (i.e. no commuting \( i \) ) matrices, commonly known as the (16 dimensional) Majorana algebra. The explicit matrix form of the algebra generator \( \gamma_\mu^\alpha_\beta \) can be determined from the Riemann space metric \( g_{\mu\nu} \) up to a similarity spin transformation: \( \gamma'_\mu = S\gamma_\mu S^{-1} \).
2.2. Spin Space

The solution of the Dirac eq. (1c) is usually taken to be a four component column bispinor $\Psi^\alpha$, belonging to the left linear space for which the endomorphism algebra is the Majorana matrices. This spin space is transcendental, i.e. the postulates of quantum mechanics ordain that some attributes (e.g. quantum phase) of the wavefunction cannot be directly observed. The principle of representation invariance states that tangible results should be invariant under a spin transformation: $\Psi^\alpha' = S^\alpha_\beta \Psi^\beta$. It should therefore be possible to express the theory in a form which eliminates any reference to a particular representation without sacrificing any “physics”. To this end we introduce the spinor basis $\xi^\alpha$ as carriers for the representation. A spin transformation can now be interpreted as a passive change in spinor basis, which leaves the spin vector $\Psi = \Psi^\alpha \xi^\alpha$ unchanged.

The dual spinor basis $\bar{\xi}^\alpha$ is defined such that $\bar{\xi}^\alpha \xi^\beta = \eta_{\alpha\beta}$, where the spin metric $\eta_{\alpha\beta}$ has the diagonal signature $(+ + - -)$ in the standard matrix representation. We propose to interpret the representation independent form,

$$ e_\mu = \xi^\alpha \gamma^\beta_\mu \xi^\beta, \hspace{1cm} (2a) $$

as the “observable” basis vector of coordinate space. Mathematically this can be viewed as a map or projection from the Clifford manifold to the coordinate manifold. Hence we get a Dirac equation completely independent of spin basis or matrix representation: $(\Box - m)\Psi = 0$, where $\Psi = \Psi^\alpha \xi^\alpha$ and $\Box = e^\mu \partial_\mu$ is now the coordinate gradient.

2.3. Geometric interpretations of Gauge Algebras

There is a long standing tradition which views $i$ as only “existing” in spin space, as the internal $U(1)$ generator of unobservable quantum phase. Factors of $i$ are included as needed to make operators Hermitian (e.g. $\gamma_4$) so that expectation values will never contain a non-observable “imaginary” number. The usual Dirac matrices are the complexified Majorana algebra: $\mathbb{C}(4) = \mathbb{C} \otimes \mathbb{R}(4)$. This can be geometrically reinterpreted as a 5D geometric (anti de-Sitter) space, where the unit pseudoscalar (5-volume) plays the role of the $i = \gamma^1 \gamma^2 \gamma^3 \gamma^4 \gamma^5$, only if the fifth basis vector has positive signature. The obvious question would be the physical interpretation of the new fifth dimension, and the identification of its associated coordinate variable and conjugate momenta (mass?). We will address this question briefly below.

To represent an isospin doublet of bispinors (e.g. u & d quark) requires a commuting isospin Pauli $\{\sigma_j\}$ algebra. The wavefunction can be expressed as a matrix set of components $\Psi^{\alpha\kappa}$ contracted on a product basis $\xi^\alpha \lambda_\kappa$. There are only two elements to the isospinor basis $\{\lambda_1, \lambda_2\}$ which necessarily commute with the spinor basis $\xi^\alpha$. The direct product of a commuting
Majorana “spin” algebra and a Pauli “isospin” algebra can be reinterpreted as a 7D geometric algebra with metric signature (+ + + − − − −). The column spinor for which the endomorphism algebra is \( \mathbb{C}(8) = \mathbb{C}(2) \otimes \mathbb{R}(4) \) would now have 8 components.

To represent two observers in real spacetime requires a pair of coordinates, each with their own Clifford algebras. The direct product of these two commuting algebras can be geometrically reinterpreted as a 8D space with a mother algebra \( \mathbb{R}(16) = \mathbb{R}(4) \otimes \mathbb{R}(4) \). This encompasses all the above algebras, where the ‘second frame’ \( \mathbb{R}(4) \) algebra commutes with that of the ‘first frame’. Hence the ‘second’ algebra is the ‘internal’ gauge algebra for the ‘first’ frame observer and visa versa.

3. Spin Covariant Dirac Theory

The special theory of relativity requires the Dirac equation to have the same form under Lorentz transformations: \( dx^\mu = a^\mu_\nu dx^\nu \). It is usually argued that the generators \( \gamma^\mu \) are invariant scalars, i.e. the same for all observers, at the cost of forcing the bispinor wavefunction to obey a compensating spin transformation: \( \psi^\alpha_s = S^{\alpha}_{\beta} \psi^\beta \), where \( S^{-1} \gamma^\mu S = a^\mu_\nu \gamma^\nu \).

3.1. Coordinate Covariant Dirac Theory

The general principle of covariance will require the spin transformation to be local (different at each point in spacetime). This introduces a spin connection \( \Omega_\mu \) to the derivative \( \nabla_\mu \) of the Dirac eq. (1c),

\[
\nabla_\mu = \partial_\mu + \Omega_\mu, \tag{3a}
\]

\[
\partial_\mu \xi_\alpha = \Omega_\mu \xi_\alpha = \xi_\beta \Omega^{\beta}_{\mu \alpha}, \tag{3b}
\]

\[
\Omega_\mu = \Omega^j_\mu E_j = \Omega^j_\mu \Gamma^{\beta\alpha}_{(j)} \xi_\beta \bar{\xi}_\alpha = \Omega^\beta_\mu \bar{\xi}_\alpha, \tag{3c}
\]

\[
\Omega'_\mu = S \Omega_\mu S^{-1} + S \partial_\mu S^{-1}. \tag{3d}
\]

One of the 16 basis elements \( E_j \) of the geometric Clifford algebra is given by the generalization of eq. (2a),

\[
E^{(j)} = \Gamma^{\alpha\beta}_{(j)} \xi_\alpha \bar{\xi}_\beta, \tag{2b}
\]

where \( \Gamma^{(j)} \) is the corresponding basis element of the Dirac matrix algebra. Under the general coordinate transformations required by the equivalence principle, one must replace \( \gamma^\mu \rightarrow \gamma^a h^a_\mu(x) \) where the tetrad (vierbein) field \( h^a_\mu(x) \) transforms as a vector. This is equivalent to introducing position dependent \( \gamma^\mu(x) \) which transform like basis vectors.
For this reason and others, we adopt the “nontraditional” view that both $e^\mu$ and $\gamma^\mu$ of eq. (2a) transform as vectors, while $\xi_\beta$ and $\overline{\xi}_\alpha$ transform as coordinate scalars[9]. With this definition of constant spin basis, the spin connection is everywhere zero, hence the generally covariant Dirac equation is simply eq. (1c) with Dirac matrices which are a function of position. However, when the coordinate space is curved, one cannot have the spin connection vanish everywhere. The geometric definition of eq. (2a) forces the following relations:

$$\partial_\nu \gamma_\mu = C_{\nu\mu}^\omega \gamma_\omega - \Omega^{(j)}_{\nu} \{ \Gamma_{(j)}, \gamma_\mu \},$$  \hspace{1cm} (4a)$$

$$[K_{\sigma\tau}, e_\mu] = R_{\omega\sigma\mu}^\nu e_\nu,$$  \hspace{1cm} (4b)$$

$$K_{\omega\sigma} = K_{(j)}^{(j)} E_{(j)} = [\nabla_\omega, \nabla_\sigma],$$  \hspace{1cm} (4c)$$

The coordinate connection coefficient $C_{\nu\mu}^\omega$ (Christoffel symbol) is directly related to the spin connection by eq. (4a). Restricting our discussion to real spacetime algebra (no commuting $i$), the spin curvature $K_{\sigma\tau}$ is forced by eq. (4b) to be a bivector. Clearly it must be nonzero if the coordinate space is curved, i.e. described by the Riemann curvature tensor: $R_{\omega\sigma\mu}^\nu e_\nu = [\partial_\omega, \partial_\sigma] e_\mu$. It follows from eq. (4c) that the spin connection (which appears in the spin covariant derivative $\nabla_\sigma$) must have a nontrivial bivector part, commonly called the Fock-Ivanenko coefficient[9].

### 3.2. Spin Gauge Theory

The principle of local matrix representation invariance or equivalently a principle of spin basis covariance is invoked to induce via minimal coupling a non-trivial spin connection[4]. This is a gauge theory where the generators $\Gamma_{(j)}$ of the general spin transformation are usually restricted to be Dirac bar-negative in order to preserve the spin norm $\bar{\Psi} \Psi$ (i.e. the spin metric $\xi_\alpha \xi_\beta = \eta_{\alpha\beta}$ is invariant). The standard (5D) Dirac algebra which has the bar negative pseudoscalar $i$, would contain the 16 element group structure $U(2,2)$. Electromagnetism is associated with $i$, which by itself would force the space curvature to be zero. It is tempting to interpret the 10 bivectors (of 5D) with group structure $SO(4,1)$ as the gauge fields which cause gravitational curvature through eq. (4b).

Grand unification is approached by Chisholm and Farwell[1] by resorting to higher dimensions (e.g. 11D) to introduce more fields. They only consider spin transformations of the form: $\gamma^\mu = \gamma^a h_a^\mu(x)$, generated by bivectors or the pseudoscalar $i$. They avoid those bivectors which would rotate spacetime into a higher dimension (e.g. $\gamma^5 \gamma^1$). The remaining bivectors which operate on spacetime form the 6 element Lorentz group $SL(2,C)$, potentially insufficient to accommodate a full description of gravitation.
3.3. Local Automorphism Invariance

Alternatively, the entire automorphism group $U(2,2)$ of the Dirac algebras is allowed by Crawford[2]. Previously, the non-bivector generators were excluded by equations (4a) & (4b). These constraints are relaxed because Crawford does not require the geometric interpretation of eq. (2ab). This allows him to consider generalized spin transformations of the form: $\Gamma^{(j)} = \Gamma^a \Delta_a^{(j)}(x)$, where $\Gamma^{(j)}$ is a basis element of the full “spin” (Dirac) Clifford algebra. The dreibein fields $\Delta_a^{(j)}(x)$ (“spin-legs”) reshuffle multivector rank in the Clifford spin manifold (e.g. vector $\leftrightarrow$ bivector) without doing the same to the “observable” coordinate geometry.

A Lagrangian formulation can show that the field equation is,

$$K^{(i)} = K_{\mu\nu}^{(i)} \Delta^{(j)} e^{\mu} \wedge e^{\nu} \Gamma^{(i)}.$$  

The current $j_\mu$ is similar to the spin gauge connection $\Omega_\mu$ in being a coordinate vector while also a Clifford aggregate over the spin algebra $\Gamma^{(i)}$. The spin curvature $K$ can be geometrically interpreted as a dyad of a coordinate geometric bivector and a spin algebra Clifford aggregate.

$$K_{\mu\nu}^{(i)} \Delta^{(j)} e^{\mu} \wedge e^{\nu} \Gamma^{(i)}.$$  

Elements of the coordinate geometry $E^{(i)}$ commute with the spin algebra $\Gamma^{(i)}$ because Crawford does not postulate the geometric connection of eq. (2ab). Note that the bivector part of the spin curvature is no longer constrained by eq. (4b) to be related to the space curvature.

4. Multivector Gauge Theory

The basic difference from standard theory is the replacement of column spinors by algebraic wavefunctions, i.e. Clifford aggregates of Dirac matrices[6]. Most authors only consider restricted combinations called minimal ideals, which have the same degrees of freedom as a single column spinor. In our approach, the form of the multivector wavefunction is unrestricted, having the same number of degrees of freedom as the elements of the Clifford group. The complete solution can be interpreted as a geometric multispinor: $\Psi = \Psi^{(i)} E^{(i)} = \Psi^{\alpha\beta} \xi_\alpha \lambda_\beta$. Here the $\xi_\alpha$ is no longer a basis spinor, but an element of a left ideal, hence eq. (3b) is no longer valid. The isospin element is part of the same algebra: $\lambda_\beta = \xi_\beta$, which does not commute with $\xi_\alpha$ whereas as it did in standard formulation. In 4D spacetime algebra (no commuting $i$) the geometric multispinor has been shown[11] to be an isospin doublet of Dirac bispinors, where the role of $i$ is played by right-side applied (i.e.
*dextrad multiplication* time basis vector \( e_4 \). In 5D (standard Dirac algebra) one has enough degrees of freedom to represent four quarks (i.e. u,d,s,c), where the (u,d) and (s,c) isospin doublets are uncoupled.

### 4.1. Dextral Gauge Theory

The generally covariant multivector Dirac equation \( (e^\mu \partial_\mu - m)\Psi^{(i)} E_{(i)} = 0 \), where \( e_\mu(x) \) are the local coordinate basis vectors, is manifestly matrix representation independent. We have in fact completely eliminated spin space, specifically spin basis \( \xi_\alpha \) and spin algebra \( \gamma^\alpha_{\beta} \) in favor of there being only the geometrically interpretable coordinate Clifford algebra \( E_{(i)} \). Hence, spin gauge theory, an artifact of spin space, is now inaccessible!

The multiple particle generations in the multivector wavefunction can be coupled by now-allowed right-side applied *dextrad gauge transformation*\(^3\). The new gauge fields enter as a *dextrad connection*: \( D_\mu = D_\mu^{(i)} E_{(i)} \),

\[
\nabla_\mu(\Psi) = \partial_\mu \Psi + \Psi D_\mu,
\]

\( (6a) \)

coupling to the multivector parts of Greider’s current\(^6\),

\[
  j_\mu^{(i)} = Tr(E^{(i)} \bar{\Psi} e_\mu \Psi) = Tr(\Psi E^{(i)} \bar{\Psi} e_\mu).
\]

\( (6b) \)

A Lagrangian formulation\(^11\) will require the geometric generators \( E_{(i)} \) of the dextrad connection \( D_\mu \) to be bar negative. In 4D spacetime, the subset which is also unitary generates the electroweak group: \( U(1) \otimes SU(2) \), where isospin rotations are generated by spacelike bivectors and the role of \( i \) played by right-sided (dextrad) multiplication of the time basis element \( e_4 \).

### 4.2. Poly-Dimensional Covariance

The spin gauge formalism can be recovered by proposing that the automorphism transformations operate on the very real, concretely observable spacetime coordinate Clifford algebra: \( E_{(i)} = E_{(j)} \Delta_{(i)}^{(j)}(x) \). The *geobein* fields \( \Delta(x) \) (“geometry-legs”) are completely analogous to Crawford’s *drehbeins*\(^2\) except that now we are reshuffling observable geometry. We are tautologically committed to propose a new principle of *local poly-dimensional covariance*. By this we mean that the basis vectors of a coordinate frame displaced from the origin may be “rotated” in dimension, e.g. be a multivector that is part vector plus part bivector relative to the reference geometry.

The generalized *poly-dimensional connection* \( \Lambda_{(i)}^{(j)} \) is defined,

\[
\Box E_{(i)} = \Lambda_{(i)}^{(j)} E_{(j)}.
\]

\( (7a) \)
The right side of this equation is recognized as a linear transformation on the full Clifford algebra $\mathbb{R}(4)$. In general $\Lambda_{(i)}^{(j)}$ belongs to the endomorphism algebra $\text{End } \mathbb{R}(4) \cong \mathbb{R}(4) \otimes \mathbb{R}(4)$, hence it is an element of the mother algebra $\mathbb{R}(16)$ [14]. This leads to a new generalized poly-dimensional covariant Dirac equation,

$$[(\Box - m)\Psi^{(j)} + \Psi^{(i)}\Lambda_{(i)}^{(j)}]E_{(j)} = 0,$$

(7b)

where the coordinate gradient in eq. (7b) is understood now NOT to operate on $E_{(j)}$. This is not a particularly useful form, as it is expressed in terms of the multivector basis $E_{(j)}$ instead of an ideal basis which would more closely resemble standard spinor form. The main annoying feature is that each multivector piece of the wavefunction couples to a different connection coefficient. Further, the poly-dimensional connection cannot itself be expressed as a multivector within the $\mathbb{R}(4)$ spacetime algebra.

Alternatively, the linear transformation can be written entirely within the smaller original $\mathbb{R}(4)$ spacetime algebra using two-sided multiplication[13]. We re-express eq. (7a) in terms of a new bilateral connection $\Omega^{(jk)}$,

$$\Box E_{(i)} = \Omega^{(jk)}E_{(j)}E_{(i)}E_{(k)}.$$

(8a)

The advantage of eq. (8a) over eq. (7a) is that the connection is now completely form independent of the operand element $E_{(i)}$. This allows us to rewrite the interaction term of the Dirac equation in terms of the full multivector wavefunction $\Psi$ instead of having to consider each multivector component $\psi^{(j)}$ separately as was done in eq. (7b). The resulting Dirac equation has the bilateral interaction term which was proposed earlier to empirically fit known mesonic couplings[12],

$$(\Box - m)\Psi = -E_{(i)}\Psi E_{(j)}\Omega^{(ij)},$$

(8b)

where again it is understood that the gradient does not operate on the multivector basis (as that has already been included on the right side of the equation). From a multivector Lagrangian formulation[12] it can be shown that the gauge connection $\Omega^{(ij)}$ of eq. (8a) couples to the bilateral current,

$$j^{(ij)} = \frac{1}{4}Tr(\bar{\Psi}E^{(i)}\Psi E^{(j)}) = \frac{1}{4}Tr(\Psi E^{(j)}\bar{\Psi}E^{(i)}),$$

(8c)

where $E^{(i)}$ and $E^{(j)}$ must both be bar-positive or both bar-negative. The dextral interactions of eq. (6a) are the special case where the sinistrad[12] (left-side applied) interaction element of eq. (8b) is the set of basis vectors: $E_{(ij)} = e_{\mu}$. When the dextrad (right-side applied) element $E_{(ij)}$ of eq. (8b) is either 1 or $i$, the interactions are of the same form proposed by Crawford[2].

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4.3. **Multivector Field Theory**

In order to have a fully geometric description of symmetric tensors, Greider[8] introduced a second commuting Clifford algebra \( F_{(k)} \) in analogy with Eddington’s E-F double-frame numbers[7]. A product of two elements \( E_{(j)} F_{(k)} \) is a geometric dyad which is an element of a global 8D mother[14] algebra: \( R(16) = R(4) \otimes R(4) \). Potentially this allows us to write a single superfield equation which is completely coordinate and poly-dimensional covariant in form. In the particular case of dextrad connection of eq. (6a), the superfield equation can be written in a sourced monogenic form,

\[
\square \mathcal{F} = \mathcal{J}, \tag{9a}
\]

where \( \mathcal{J} = j^\mu (j) f^\nu (j) \) is the vector-multivector supercurrent made from eq. (6b). The coordinate derivative is in the \( F_{(j)} \) algebra vector basis: \( \square = f^\mu \partial_\mu \). The superfield \( \mathcal{F} = F^{\mu \nu (j)} f^\mu (j) \wedge f^\nu (j) \) is a bivector in the “first-frame coordinate algebra” \( F_{(k)} \), while a Clifford aggregate in the “second-frame charge algebra” \( E_{(j)} \).

The Morris-Greider[8] theory of gravitation was based upon the particular case where \( E_{(j)} \) is limited to be a vector, the supercurrent then being a vector-vector dyad. The case where \( E_{(j)} \) is a trivector was explored by Differ[5]. It appears that the spin-gauge field eq. (5a) can also be written in this general form, where the commutator term is built into the equation if assumptions are made about the generalized connection coefficient of eq. (7a). The field equation for the general bilateral interaction of eq. (8b) has yet to be fully formulated.

5. **Summary**

Our development was based upon an underlying theme of using only algebra that is based on concrete spacetime geometry. This has led us to eliminate spin space, the principle of local spin covariance and ultimately spin gauge theory. In its place we propose the more grand scheme of local poly-dimensional covariance. While the results are promising for Dirac, gauge and classical field theory, it remains to be seen if its domain can be extended to classical mechanics. Further, the interpretation of the 8D geometry needed is not completely clear, although it appears to be connected with the classical symmetric tensor objects of 4D which are needed for formulations of gravitation.

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