Born-Infeld condensate as a possible origin of neutrino masses and dark energy

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We discuss the possibility that a Born-Infeld condensate coupled to neutrinos can generate both neutrino masses and an effective cosmological constant. In particular, an effective field theory is provided capable of dynamically realizing the neutrino superfluid phase firstly suggested by Ginzburg and Zharkov. In such a case, neutrinos acquire a mass gap inside the Born-Infeld ether forming a long-range Cooper pair. Phenomenological implications of the approach are also discussed.

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1. INTRODUCTION

The idea that the cosmological vacuum energy and neutrino masses have a common origin was suggested by several authors in the past without a final convincing implementation of it. It was stimulated by an intriguing coincidence of numbers: the vacuum energy is $\rho_{\text{vacuum}} \sim \Lambda M_{Pl}^2 \sim (10^{-3} \text{eV})^4$ while neutrino masses can be $m_\nu \sim 10^{-3} \text{eV}$. Naively, the cosmological constant and neutrino masses could appear as disconnected phenomena: their hierarchy is essentially given by

$$\frac{\Lambda}{m_\nu^2} \sim \frac{\lambda_\nu^2}{r_H^2} \sim 10^{-58} \quad (1)$$

where $r_H$ is the Hubble radius of the Universe and $\lambda_\nu$ the neutrino Compton wavelength. However, the vacuum energy is coupled with gravitational field by $G_N \sim M_{Pl}^{-2}$, so that $\rho_{\text{vacuum}} \sim m_\nu^4$ and $\Lambda = G_N \rho_{\text{vacuum}} \sim r_H^{-2}$. A part these numerology, the mechanism behind the generation of vacuum energy and neutrino masses could be framed under the standard of some effective theory. In view of this goal, neutrino masses could be generated by some condensate interacting with neutrinos and providing a source for dark energy. This approach has been pursued in some previous works where the idea of neutrino mixing condensate is capable of giving rise to a dynamically evolving dark energy density [1–4].

On the other hand, a condensate could be derived from the Born-Infeld theory. In particular, the idea of an invisible Born-Infeld condensate acting as dark energy density was first suggested in Ref.[5]. It is worth mentioning that also the standard visible electrodynamics can be extended à la Born-Infeld so that it could contribute, in some sense, to the vacuum energy [6,7].

Here, we want to show how an invisible Born-Infeld condensate can be related to the neutrino masses and give rise to the dark energy content of the Universe.

The old theory of a non-linear electrodynamics was suggested as a non-linear extension of Maxwell electrodynamics [10]. The main purpose of that model was to solve the insidious self-energy problem of point-like charged particles. This model has a certain attractiveness: it is invariant under Dirac electric-magnetic duality, it predicts the existence of solitonic objects and it has physical propagations without shock waves and dichroism [11]. Even if the original attempts were not successful for electron-self-energy, it turns out to be deeply connected to string theory. The Born-Infeld electrodynamics is the low-energy theory of open strings and it also re-appears as a contribution part of world-volume action of D3-branes as string solitons. So that it is well known, in string theory, how to regularize a Born-Infeld theory in the UV limit. Otherwise problems with quantization and unitarity would severely afflict the proposal. So that, in the contest of D-brane worlds or intersecting D-brane worlds cosmology, we can envisage an invisible electrodynamics confined on a D3-brane and gravitationally interacting in our D-brane world. This picture
has important implications in particles physics because of non-pertubative stringy effects related to exotic instantons.
For instance, exotic instantons can generate new effective operators in the low energy limit such as an effective
Majorana mass for the neutron besides neutrinos [12–17].

In this picture, the presence of open strings attached to the ordinary D-brane world and the dark D3-brane can
generate effective interactions among the Invisible Born-Infeld field and the Standard Model particles. In fact, they
develop a high tension, i.e. first levels of Kaluza-Klein or Regge modes acquire a very high mass and they can be
integrated out at low energy limit, generating new effective operators.

In this paper, we will argue how massless neutrinos can dynamically get an effective mass term in the propagation
inside the Born-Infeld ether. This inevitably leads to the formation neutrino-antineutrino Cooper pairs, leading to
superfluid neutrino phase. The superfluid neutrino phase was an old idea firstly suggested by Ginzburg and Zharkov [8].
This idea has phenomenological implications particularly interesting in cosmology as we are going to discuss.

The paper is organized as follows: in Sec. 2, we describe the formalisms of the invisible Born-Infeld theory. Sec. 3
is devoted to the neutrino condensation mechanism and the dynamical generation of neutrino masses. The possibility
of a neutrino superfluid phase is discussed in Sec.4 In Sec. 5, we draw our conclusions and discuss phenomenological
implications.

2. A BORN-INFELD MODEL FOR DARK ENERGY

Let us consider a Born-Infeld model describing an Abelian theory of electrodynamics with non-linear interactions
$A_\mu$ formulated in 4 dimensions:

$$S_{BI} = -\lambda \int d^4 x \left\{ \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})} - \sqrt{-\det(g_{\mu\nu})} \right\},$$

where

$$\det(g_{\mu\nu} + F_{\mu\nu}) = (-g) \left[ 1 + \frac{1}{2} F_{\mu\nu} F^{\mu\nu} + \frac{1}{16g} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right],$$

with $g = \det g_{\mu\nu}$ and $\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$. The Lagrangian can be also rewritten as

$$\mathcal{L} = -\lambda \sqrt{1 - \frac{E^2 - B^2}{\lambda^2}} - \frac{(E \cdot B)^2}{\lambda^2} + \lambda,$$

where $E = F_{0i}$ and $B_{ij} = F_{ij}$ are the electric and magnetic fields; $\lambda$ is a dimensional parameter with dimensions
$[\lambda] = M^4$. Let us define $\lambda = \mathcal{M}^4$ for notational convenience for the following analysis. The associated energy-
momentum tensor is

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta S_{BI}}{\delta g_{\mu\nu}} = \frac{\lambda}{2} \left\{ \frac{g^{\mu\nu}(1 + \frac{1}{2} F_{\mu\nu} F^{\mu\nu}) - F^{\mu\nu}_{\mu\nu}}{\sqrt{1 + \frac{1}{2} F_{\sigma\rho} F^{\sigma\rho} + \frac{1}{16g} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2}} - g_{\mu\nu} \right\},$$

to be compared with the standard Maxwell energy-momentum tensor

$$T^0_{\mu\nu} = \frac{1}{4} g_{\mu\nu} F^{\rho\sigma} F_{\rho\sigma} - F_{\mu\rho} F^{\rho}_{\nu}.$$

The Born-Infeld energy-momentum tensor can be rewritten as

$$T_{\mu\nu} = \epsilon T^0_{\mu\nu} + \frac{1}{4} g_{\mu\nu} \Delta,$$

where

$$\epsilon = -\frac{\partial V_{eff}(P, S)}{\partial S}, \quad \Delta = T^{\mu}_{\mu} = -\mathcal{M} \frac{d\Phi_{eff}}{d\mathcal{M}},$$

$$V_{eff} = -\mathcal{S} + \mathcal{M}^4 \Phi_{eff} \left( \frac{S}{\mathcal{M}^2}, \frac{P}{\mathcal{M}^4} \right),$$

with

$$\mathcal{S} = -\mathcal{M}^4 \Phi_{eff} \left( \frac{S}{\mathcal{M}^2}, \frac{P}{\mathcal{M}^4} \right) + \frac{1}{2} g_{\mu\nu} \left( \frac{S}{\mathcal{M}^2}, \frac{P}{\mathcal{M}^4} \right) \frac{d\Phi_{eff}}{d\mathcal{M}}.$$
\( \Phi_{\text{eff}} = \frac{1}{M^4} S + (1 - \sqrt{1 + 2S/M^4 - P^2/M^8}) \) (10)

and

\( S = \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad P = \frac{1}{4} \tilde{F}_{\mu\nu} F^{\mu\nu} \). (11)

The explicit dependence of \( V_{\text{eff}} \) from \( S, P \) is

\[ V_{\text{eff}} = M^4(1 - \sqrt{1 + 2S/M^4 - P^2/M^8}) \] (12)

In other words, the energy-momentum tensor can assume the form

\[
T_{\mu\nu} = \left( -\frac{\partial V_{\text{eff}}}{\partial S} \right) (g_{\mu\nu} S - F_{\mu\lambda} F^{\lambda} - \left( V_{\text{eff}} - P \frac{\partial V_{\text{eff}}}{\partial P} - S \frac{\partial V_{\text{eff}}}{\partial S} \right)),
\]

with the identification

\[
\frac{1}{4} M \frac{\partial \Phi_{\text{eff}}}{\partial M} = \left( V_{\text{eff}} - P \frac{\partial V_{\text{eff}}}{\partial P} - S \frac{\partial V_{\text{eff}}}{\partial S} \right). \]

Now, we can notice that the trace-term \( \Delta \) can provide a cosmological-constant-like contribution, while \( \epsilon \) can be interpreted as a non-linear dielectric function. We can also rewrite Eqs.(8) as

\[
\epsilon = -\frac{\partial V_{\text{eff}}}{\partial S} = \frac{1}{1 + 2S/M^4 - P^2/M^8}, \]

and

\[
\Delta = -4M^4 \left( 1 - \sqrt{\frac{1 + 2S/M^4 + S^2/M^8}{1 + 2S/M^4 - P^2/M^8}} \right). \]

Finally the two functions \( \epsilon, \Delta \) are related as

\[ \Delta = 4M^4 \left[ \epsilon(1 + S/M^4) - 1 \right], \]

and the energy density is

\[ T^{00} = \frac{\epsilon}{2} (E^2 + B^2) + \frac{1}{4} \Delta. \]

Let us now comment on the meaning of this result. First of all, the non-linear terms of Born-Infeld action are contributing to the trace-part of the electromagnetic energy-momentum tensor. Strictly speaking, the theory is no more conformally invariant at classical level. This differentiates it with respect to the standard Maxwell theory which is explicitly invariant under conformal rescaling. The essence of this proposal is that the energy density generated in such a way, which gives rise to a dynamically scaling density, is a sort of cosmological constant. However, it is worth stressing that the cosmological constant generated in such a model is dynamically varying with time.

To be more precise, let us assume the initial conditions

\[ \langle F_{\mu\nu} F^{\mu\nu} \rangle_{t=t_0} = \langle (E^2 - B^2) \rangle_{t=t_0} = f_0 \neq 0, \]

\[ \langle F_{\mu\nu} \tilde{F}^{\mu\nu} \rangle_{t=t_0} = \langle (E \cdot B) \rangle_{t=t_0} = g_0 \neq 0, \]

for the Born-Infeld fields in a Friedmann-Roberston-Walker space-time, where the symbols \( \langle \cdot \cdot \cdot \rangle \) indicate quantum vacuum expectation values. Clearly, such terms can be seen as contributions for an effective cosmological constant. However, we expect that the non-linear electrodynamics leads to quantum instabilities of condensates. Strictly speaking, also isotropy is expected to be dynamically violated. As a simplifying ansatz, we can consider isotropic but time-varying condensates given by smoothly varying time functions

\[ \langle F_{\mu\nu} F^{\mu\nu} \rangle = \langle (E^2 - B^2) \rangle = f(t), \]
\[ \langle F_{\mu \nu} \tilde{F}^{\mu \nu} \rangle = \langle (E \cdot B) \rangle = g(t) \sqrt{-g}, \] (22)

where the Born-Infeld dynamics is expected to slowly vary with respect to the initial conditions. This means that we can assume apparently bad initial conditions \([19-20]\) as a physical ansatz.

Let us now parameterize the vacuum expectation value of the operators \(\langle S^n P^m \rangle, \langle E^2_i \rangle\) and \(\langle B^2_i \rangle\) as
\[ \langle S^n P^m \rangle = \alpha^n \beta^m, \] (23)
\[ \langle E^2_i \rangle = \langle B^2_i \rangle = \epsilon(t), \] (24)
following the equipartition principle. The condition \(23\) parameterizes the quantum condensate contribution, while \(24\) are the energy contributions of the classical fields. This implies the following density and pressure relations
\[ \rho_{BI} = \frac{1}{2} M^4 \left( \frac{1 + \frac{\alpha}{2} - \frac{\alpha_s}{4} - \frac{\alpha_t}{4} - \beta^2}{1 + \frac{\alpha}{2} - \frac{\beta^2}{2}} - 1 \right), \] (25)
\[ p_{BI} = -\frac{1}{2} M^4 \left( \frac{1 + \frac{\alpha}{2} - \frac{\alpha_s}{4} - \frac{\alpha_t}{4} - \beta^2}{1 + \frac{\alpha}{2} - \frac{\beta^2}{2}} - 1 \right), \] (26)
where
\[ \alpha_t = \alpha - 4 \epsilon, \quad \alpha_s = \alpha + \frac{4}{3} \epsilon. \] (27)

As remarked in Ref.[5], the parameter \(\alpha\) has a quantum origin. It gives the parameterization of the quantum vacuum expectation value of the correlator \(23\). In fact, since the electric field is perpendicular to the magnetic field, \(\langle F \tilde{F} \rangle = \langle E \cdot B \rangle = 0\) is expected at purely classical level, i.e. \(\alpha = 0\) in the expression \(23\).

Now, the energy-momentum tensor of the Born-Infeld field has to be conserved according to the Bianchi identities, that is:
\[ \nabla_{\mu} T_{BI}^{\mu \nu} = 0 \rightarrow \nabla_{\mu} \left[ -\frac{\lambda}{2} \left\{ \frac{g^{\mu \nu}(1 + \frac{1}{2} f_{\mu \nu} f^{\mu \nu}) - f_{\rho \nu} f_{\mu \rho}}{\sqrt{1 + \frac{1}{2} f_{\rho \sigma} f^{\rho \sigma}} + \frac{1}{16g} (f_{\mu \nu} \tilde{F}^{\mu \nu})^2} - g_{\mu \nu} \right\} \right] = 0. \] (28)

Under the assumption of the Friedmann-Robertson-Walker background, Eq.(28) reduces to the simple form
\[ \dot{\rho}_{BI} + 3H (\rho_{BI} + p_{BI}) = 0, \] (29)
where \(H = \dot{a}/a\). From Eqs.(25), (26), and expanding up to the first order the \(\epsilon\) parameter, we obtain
\[ w = -1 + \frac{128 \epsilon}{96 \epsilon + 33}. \] (30)

This assumption has physical meaning in the limit of \(\alpha \gg \epsilon\), that means that the quantum condensate dominates on the classical field contribution. In other words, the equation of state \(w(t)\) will evolve in time in the limit of quantum condensate dominance.

### 3. Neutrino Condensation in Born-Infeld Electrodynamics

The above considerations can be straightforwardly related to the phenomenon of neutrino condensation. Let us suppose that neutrinos interact with the invisible electrodynamics according to the following Lagrangian
\[ \mathcal{L}_\nu = i \bar{\nu} \gamma_\mu \partial^\mu \nu + g_\nu A_\mu \bar{\nu} \gamma^\mu \nu, \] (31)
or, alternatively,
\[ \mathcal{L}_\nu = i \bar{\nu} L \gamma_\mu \partial^\mu \nu_L + g_\nu A_\mu \bar{\nu} L \gamma_\mu \nu_L. \] (32)
Figure 1: a) Photon interaction with the background Born-Infeld condensate through the Euler-Heisenberg interaction term. b) Neutrino interaction with the superfluid condensate through an effective four-fermion interaction induced by the Born-Infeld interaction.

Two neutrino particles with opposite chirality with an attractive interaction between them form a Cooper pair. At tree level, neutrinos and antineutrinos exchange a Born-Infeld dark photon with an effective attractive potential, which induces neutrino condensation.

The condensation process can only happen in non-relativistic regime, with the four-neutrinos effective interaction given by the Hamiltonian

$$H_{int} = - \sum_{k,\sigma} V_{kk'} c_{k',\sigma}^\dagger c_{k,\sigma}^\dagger c_{k,\sigma} c_{k',\sigma},$$

(33)

$$V_{kk} = -\frac{4\pi^2 g_{\nu}^2}{q^2 + V_{eff}^2},$$

On the other hand, the propagating dark Born-Infeld photon in the constant background gets an effective mass which, for small momentum transfers $q << M$, is

$$m_{\gamma}^{eff} \simeq M + O(q^2).$$

This because of the higher-derivative Euler-Heisenberg interactions among propagating photons and background field photons. So that,

$$V_{\gamma}^{eff} \simeq \frac{2\pi^2 g_{\nu}^2}{2M^2},$$

(34)

As a consequence, an effective four fermion effective interaction is generated with an effective Fermi-like coupling $\bar{G}_F \simeq -\frac{g_{\nu}}{2M^2}$. In the quantum field theory formalism, the corresponding interaction term has the form $\bar{G}_F \nu^{\gamma}_{\mu} \nu^{\gamma}_{\mu} \nu \bar{\nu}$. On the other hand, a neutrino propagating in the Born-Infeld background gets the effective mass term

$$m_{\nu}^{eff} \simeq \frac{g_{\nu}}{M^2}.$$

However, there is also an extra effective mass contribution for neutrino coming from the effective occupation number of cold neutrino, which is parametrized by a chemical potential $\rho$ and which we will discuss later.

As a consequence, neutrinos which are cooled enough by the background ($E << M$) are described by the non-relativistic Lagrangian

$$\mathcal{L} = \sum_{\alpha=\uparrow,\downarrow} \bar{\nu}_\alpha^\dagger \left( i \frac{\partial}{\partial t} - \frac{\nabla^2}{2m_{\nu,\alpha}^{eff}} + \mu \right) \nu_\alpha - \bar{\nu}_\alpha^\dagger \nu_\alpha\nu_\alpha + h.c.,$$

(35)

which leads to the expectation value (in the Born-Infeld medium)

$$\langle \bar{\nu}\nu \rangle = \mu/\bar{G}_F = \mu M^2/(2\pi^2 g_{\nu}^2),$$

(36)

It is convenient to work in the matrix notation $\Psi = (\nu_{\uparrow}^\dagger, \nu_{\downarrow}^\dagger)^T$, $\Psi^\dagger = (\nu_{\uparrow}^\dagger, \nu_{\downarrow})$, with a partition function $Z = \int D\Psi^\dagger D\Psi e^{i S_{\Psi}}$. Considering a scalar field $\Phi$, we can rewrite the four-fermion interaction term as

$$\bar{G}_F |\nu_\downarrow^\dagger \nu_\uparrow|^{2} = \frac{1}{\bar{G}_F} |\Phi - \bar{G}_F \nu_\downarrow^\dagger \nu_\uparrow|^{2} - \frac{1}{\bar{G}_F} |\Phi|^{2} + \bar{\Phi}^{\dagger} \nu_\downarrow \nu_\uparrow + h.c.,$$

(37)

However, we are interested to the combination $\bar{G}_F \nu_\uparrow^\dagger \nu_\downarrow^\dagger$, which renders null the first term. The partition function can be rewritten in $\Phi, \Phi^\dagger$ variables as

$$Z = \int D\Phi^\dagger D\Phi e^{i S[\Psi, \Psi^\dagger, \Phi, \Phi^\dagger]},$$

(38)
\[ S[\Psi, \bar{\Psi}^\dagger, \Phi, \Phi^\dagger] = \int d^4x \left[ \bar{\Psi} \left( \frac{i\partial_t - \frac{\nabla^2}{2m_{\nu eff}} + \mu}{-\Phi^\dagger} - i\partial_t - \frac{\nabla^2}{2m_{\nu eff}} - \mu \right) \Psi + \frac{1}{G_F} |\Phi|^2 \right]. \] (39)

Now, using the Grassmanian integral
\[ \int \mathcal{D}\Psi \mathcal{D}\bar{\Psi} e^{-\bar{\Psi}^\dagger M \Psi} = \det M, \] (40)

the partition function can be rewritten as
\[ Z = \int \mathcal{D}\bar{\Phi} \mathcal{D}\Phi e^{-\frac{\delta S}{\mathcal{O}} \int d^4x |\Phi|^2}, \] (41)

and
\[ \mathcal{O} = \left( \begin{array}{cc} i\partial_t + \frac{\nabla^2}{2m_{\nu eff}} + \mu & -\Phi \\ -\Phi^\dagger & i\partial_t - \frac{\nabla^2}{2m_{\nu eff}} - \mu \end{array} \right), \]

which is equivalent to
\[ Z = \int \mathcal{D}\bar{\Phi} \mathcal{D}\Phi e^{iS[\Phi, \Phi^\dagger]}, \] (42)

\[ S[\Phi, \Phi^\dagger] = \int d^4x \left[ \frac{i}{G_F} |\Phi|^2 + \int \frac{d^4k}{(2\pi)^4} \text{tr} \left( k^0 - \frac{|k|^2}{2m_{\nu eff}} + \mu \frac{\Phi}{-\Phi^\dagger} + \Phi^\dagger k^0 + \frac{|k|^2}{2m_{\nu eff}} - \mu \right) \right]. \]

At this point, we can try the extremal solution of the action in the ground state of \( \Phi \), i.e. \( \Phi \to \langle \Phi \rangle \) and \( \delta S[\langle \Phi \rangle, \langle \Phi^\dagger \rangle] = 0 \). One can show that the action is minimized by the condition
\[ 0 = \frac{i}{G_F} \langle \Phi \rangle + \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^0)^2 - |k/2m_{\nu eff} - \mu|^2 - |\langle \Phi \rangle|^2 + i\epsilon} \text{tr} \mathcal{O}_2, \] (43)

where
\[ \mathcal{O}_2 = \left( \begin{array}{cc} k^0 - |k|^2/2m_{\nu eff} + \mu & 0 \\ 0 & 0 \end{array} \right). \] (44)

Eq.(44) explicitly provides the fermion propagator with poles
\[ E_{\pm} = \pm \sqrt{|k|^2/2m_{\nu eff} - \mu|^2 + |\langle \Phi \rangle|^2}, \] (45)

In the Hamiltonian approach, the Hamiltonian of Cooper pairs reads as
\[ H = 2 \sum_k \epsilon_k b_k^\dagger b_k - \sum_{k \neq k'} V_{kk'} b_k^\dagger b_{k'}, \] (46)

where \( \epsilon_k = k^2/2m_{\nu eff} - \mu + V_{kk'}/2 \) and pairing operators \( b, b^\dagger \) create and destroy Cooper pairs of neutrinos on the ground state \( |\Psi_0\rangle \):
\[ b_k^\dagger = c_k^\dagger c_{-k}, \quad b_k = c_{-k} c_k. \]

The formation of a Cooper pair condensate \( \langle \nu_R \nu_L \rangle \) breaks the chiral symmetry as
\[ SU(2)_L \times SU(2)_R \to SU(2)_V \]

generating the scalar Nambu-Goldstone bosons. While neutrinos with energies \( E < M \) will undergo to condensation, neutrinos with \( E >> M \) are practically unbounded, but they get a mass term as
\[ -G_N \langle \bar{\nu}_L \nu_R \rangle \bar{\nu}_L \nu_R \to \mu \bar{\nu}_L \nu_R. \] (47)
The $\mu$ parameter can be interpreted as the finite density parameter of the condensate or the Fermi energy level, which is $\mu \sim M$.

The propagator of neutrinos in the Born-Infeld medium is defined as

$$S_F(x, y) = \langle T \{ \tilde{\nu} \nu \} \rangle,$$

where $\langle \ldots \rangle$ denotes the correlator in the non-trivial Born-Infeld vacuum. As argued in Ref.\[6,7\], the effective formula for fermion propagator inside the invisible Born-Infeld ether is

$$g_\nu M \frac{\partial V_{eff}(x, M)}{\partial M} = ig_\nu M \lim_{\epsilon \to 0} \text{tr}[\Delta S(\epsilon, x)],$$

where

$$\Delta S(\epsilon, x) = S_F(x + \epsilon, x - \epsilon, M) - S^0_F,$$

and $S^0_F$ is the free-field propagator. In Eq.(49), the non-linear correction to the free-propagator are contained in the term $M \partial V_{eff}(x, M) / \partial M$. Eq.(49) implies

$$g_\nu M(iS_F(x, x) - iS^0_F(x, x)) = g_\nu M \frac{\partial V_{eff}(x, M)}{\partial M} \to -g_\nu M \tilde{\nu} \nu.$$

Finally, let us comment on the possible Pontecorvo matrix necessary for neutrino oscillations. We can easily generalize the one-neutrino structure in Lagrangian (32) with

$$\mathcal{L}_\nu = i\tilde{\nu} f_\nu A_\mu \gamma^\mu \nu f',$$

where $f = 1, 2, 3$ is the flavor index of neutrinos and $G_{f f', \nu}$ is a flavor mixing matrix. From this Lagrangian we can generate a Pontecorvo matrix $M_{f f'} = G_{f f', \nu}(\tilde{\nu} \nu)$.

### 4. NEUTRINO SUPERFLUID PHASE

We will now discuss the intriguing possibility of a superfluid phase induced by Cooper pairs of neutrinos. This fact could have interesting cosmological consequences due to the cosmic neutrino background. As in the standard superfluidity theory, the problem can be treated from two perspectives: considering the Ginzburg-Landau model as an effective theory describing the Cooper pair as a scalar field [18]; considering the Bogolubov approach as an effective theory of interacting fermions (neutrinos) [19].

In the Ginzburg-Landau approach, one consider a non-relativistic effective field theory of Cooper scalar fields in a finite density $\rho$. The effective Lagrangian is

$$\mathcal{L} = i\Phi^\dagger \partial_\mu \Phi - \frac{1}{2m_{eff}} \partial_\mu \Phi^\dagger \Phi - g^2(\Phi^\dagger \Phi - \rho)^2,$$

where $m_{eff} = g_\nu M$ and $g$ is the Ginzburg-Landau adimensional effective coupling. Confronting Eq.(53) with Eq.(36), the $\mu$ parameter is equal to the finite density parameter $\rho$. In polar variables, $\Phi = \sqrt{\rho} e^{i\theta}$, where $\sqrt{\rho} = \sqrt{\rho} + a$. The Lagrangian (53) becomes

$$\mathcal{L} = -\tilde{\rho} \partial_\mu \theta - \frac{1}{m_{eff}} \left[ \frac{1}{\rho}(\partial_\mu \tilde{\rho})^2 + \tilde{\rho}(\partial_\mu \theta)^2 \right] - g^2(\tilde{\rho} - \rho)^2,$$

that is, by developing and expanding,

$$\mathcal{L} = -2\sqrt{\rho} a \theta - \frac{\rho}{2m_{eff}} (\partial_\mu \theta)^2 - \frac{1}{2m_{eff}} (\partial_\mu a)^2 - 4g^2 \rho a^2 + \ldots$$

Integrating out $a$, we obtain

$$\mathcal{L} = \rho \hat{\theta} \frac{1}{4g^2 \rho - (1/2m_{eff})^2} \hat{\theta} - \frac{\rho}{2m_{eff}} (\partial_\mu \theta)^2.$$

where $M$ is the free-field propagator. In Eq.(49), the non-linear correction to the free-propagator are contained in the term $M \partial V_{eff}(x, M) / \partial M$. Eq.(49) implies
This implies the presence of gapless Bogolubov modes propagating with a dispersion relation of the form

$$\omega^2 = \left( \frac{2g^2 \rho}{m_{\nu}^{\text{eff}}} \right) k_*^2.$$  \hfill (57)

Of course the low energy limit of this theory is simply reduced to the Lagrangian of a Nambu-Goldstone boson of the form

$$\mathcal{L} = \frac{1}{4g^2} \partial_\mu \theta \partial^\mu \theta.$$  \hfill (58)

Now let us consider the Bogolubov approach. The effective Hamiltonian is

$$\mathcal{H} = \int d^3x \left[ \nu_\alpha^\dagger(r,t) \left( -\frac{\mathbf{1}}{2m_{\nu}^{\text{eff}}} \nabla^2 - E_F \right) \nu_\alpha(r,t) + O(r,t)\nu_\gamma(r,t)\nu_\delta(r,t)^\dagger + h.c. \right],$$  \hfill (59)

where the Fermi energy $E_F$ is the chemical potential of the system. $O(r,t)$ is the order field, which reads

$$O_{\alpha\beta}(r,t) = \bar{G}_N \langle \nu_\alpha \bar{\nu}_\beta \rangle.$$  \hfill (60)

Comparing Eq.59 and Eq.42, the order field is a mean field of the quantum field operator $\Phi$, while the Fermi energy scale $E_F$ coincides with $\mu$. $\nu(r,t)$ is the non-relativistic neutrino wave function. The equations of motion are given by the standard Bogolubov-de Gennes matrix equation:

$$i \partial_t \begin{pmatrix} \nu_\uparrow \\ \nu_\downarrow \end{pmatrix} = \begin{pmatrix} -\frac{1}{2m_{\nu}^{\text{eff}}} \nabla^2 - E_F \\ O(r,t) \end{pmatrix} \begin{pmatrix} \frac{1}{2m_{\nu}^{\text{eff}}} \nabla^2 + E_F \end{pmatrix} \begin{pmatrix} \nu_\uparrow \\ \nu_\downarrow \end{pmatrix}.$$  \hfill (61)

Clearly if $O = 0$, the system of equations of motion decouples and the dispersion relations are reduced to particles-antiparticles being $E_{\pm} = \pm(p^2/2m_{\nu}^{\text{eff}} - E_F)$. If $O \neq 0$, quasiparticles get gap as $E^2 = E_{\pm}^2 + |O|^2$. and quasiparticles wave functions are

$$\begin{pmatrix} \phi_p \\ \phi_h \end{pmatrix} = C_1 e^{-iE_{\uparrow}^+ t + ip_+ \cdot r} + C_2 e^{-iE_{\downarrow}^- t + ip_- \cdot r},$$  \hfill (62)

where

$$C_1 = c_1 \begin{pmatrix} a_+ e^{i\delta} \\ a_- e^{-i\delta} \end{pmatrix},$$

$$C_2 = c_2 \begin{pmatrix} a_- e^{i\delta} \\ a_+ e^{-i\delta} \end{pmatrix},$$

$$a_\pm = \sqrt{\frac{1}{2}(1 \pm \frac{1}{E_{\pm}^2} \sqrt{E_{\pm}^2 - |O|^2})},$$

$$p_{\pm}^2/2m_{\nu}^{\text{eff}} = E_F \pm \sqrt{E_{\pm}^2 - |O|^2},$$

where $c_{1,2}$ are constants and $\delta$ is the phase of the complex order operator; $\phi_{p,h}$ can be seen as particle and hole excitations.

5. CONCLUSIONS AND OUTLOOKS

In this paper, we explored a mechanism generating neutrino masses and dark energy in the unified framework given by the Born-Infeld theory. In particular, we showed how an invisible non-linear electrodynamics coupled to neutrinos
can provide both a Dirac neutrino mass term and a dark energy contribution. This is also related to an exotic neutrino superfluid state firstly conjectured in [8].

We find similar phenomenological implications of the model recently suggested by Dvali and Fucik [20], even if our approach is different in origin and formulation. They suggested that the neutrino mass is somehow generated by a gravitational $\theta$-term condensate, by a gravitational condensation mechanism. Inspired by their results, we can resume the following implications for our model:

- In the minimal version of this model, the neutrino is a Dirac not a Majorana particle. Of course the model can be extended with Majorana-mass generating mechanisms [21–24], having in mind see-saw mechanisms and leptogenesis [25].

- The cosmological neutrino mass bound vanishes: neutrinos are massless until the phase transition at redshift $z \sim 1$ corresponds to $T \sim \text{meV}$.

- The flavor ratio of neutrinos could be constrained by IceCube data [26]: KATRIN experiment [27] could detect an overdense cosmic neutrino fluid in our Galaxy.

- In conclusion, neutrinos and antineutrinos annihilates into Nambu-Goldstone bosons leaving part of relic cosmological neutrinos while contributing to at least a part of cold-dark matter. As argued, they are converted from hot dark matter to cold dark matter during the phase transition.

We also would like to point out another possible connection with a model proposed by Berezhiani and Khoury where a superfluid dark matter can have Bogolubov modes interacting with the gravitational field and modifying the Newtonian potential in our galaxy. This can lead to a hybrid MOND/CDM model, with interesting consequences for phenomenology in astrophysics and cosmology to be explore in future.

In our model, we also derived that the neutrino masses are slowly running with the Invisible Born-Infeld field. We think that phenomenological implications of this prediction will deserve future investigations which go beyond the purposes of this paper.

A further open issue remains the stabilization of the cosmological term with respect to the radiative corrections. This problem could be related to quantum gravity and then addressed in some future unified theory.

An alternative model to our approach has been recently suggested: the dark energy could be generated by an invisible Yang-Mills or Invisible QCD condensates [30–32]. In this case, the generation of a neutrino mass gap is similar possible but it seems ruled out by observations of unconfined neutrinos. As final remark, it is worth saying that nonlinear electrodynamics could contain the whole budget of non-gravitational background of the Universe comprising neutrinos, radiation and dark energy in the form of a condensate. Further theoretical investigations and experimental evidences are necessary in this perspective.

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