Numerical simulation of the mathematical model of treated Schistosomiasis spread

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Abstract. Schistosomiasis is one of infectious disease caused by a trematoda or blood worm, classified as Schistosoma genus. One species of Schistosoma genus that endemically exist in Napu valley, Besoa and Lindu highland Central Sulawesi Indonesia is Schistosoma Japonicum. This paper governs a mathematical model that represents the Schistosomiasis spread which consider praziquantel as a treatment that given to the infected human. The SI model is adapted to the human population, besides the Schistosoma Japonicum life cycle. The Oncomelania hupensis lindoensis growth is assumed to follow the logistic growth model. The stability of the governed model is analyzed by the Jacobian matrix at the critical points. There an unstable disease free and a stable endemic critical point. The stability is guaranteed by the existence requirement of the critical point that is the recruitment rate parameter. Another requirement parameter to be needed is the death rate of the mature worm. The research result also shows that the using of 60 mg doses of praziquantel, to the infected human subpopulation, indicates the reduction of the number of both infected human and the mature worm. We also come to the bifurcation point $B = \mu_S$ where the system gives a different characteristic.

1. Introduction

Schistosomiasis is an infectious disease caused by a blood worm (Trematode) from the genus Schistosoma as a parasite. Such species of schistosoma worms has a different intermediate hosts and could only be found in some endemic areas of Africa, South America, Caribbean, Brazil, Venezuela, Suriname, China, Indonesia, Philippines, Cambodia, Laos, and France [1]. In Indonesia, Schistosomiasis is only found in the Napu, Besoa and Lindu highlands, Central Sulawesi. This is related to the existence of an intermediate host of the worm, Schistosoma Japonicum, and the snail, Oncomelania hupensis lindoensis that endemically live in the focused area [2]. The spread of Schistosomiasis disease occurs when the water sources are contaminated by infected human or animals feces that contain the parasitic eggs that hatch in the water being larvae called mirasidium that swims to looking for an intermediate snail and penetrates to the snail's body and develops to a holding sporocyst. Later on the sporocyst leaves the snail's body as a cercariae that ready to infect humans [2].

To treat the uncontrolled growth of Schistosoma Japonicum in the infected human, as an important role in Schistosomiasis controlling, Praziquantel is to against all of Schistosoma species [3]. Mathematical modeling becomes an interesting approach tool to study the spread of Schistosomiasis. A mathematical review of the spread of Schistosomiasis was conducted. This paper studies a mathematical model that considers the human population in two groups, namely vulnerable human subpopulation (Susceptible) and infected human subpopulation (Infected). The spread begins when such susceptible infected human makes a direct contact to water resources contaminated by cercariae containing feces. The cercariae develop to be a adult worm and produces eggs in the human body that start to lives outside the body through the improper human feces. When the eggs that contained in feces hatch to be a mirasidium, it infects the intermediate snail and develops to be cercariae that ready to reinfect humans. The life cycle of Schistosoma Japonicum have to be pay attention in the effort to control the disease spread. The health ministry uses 60 mg doses of praziquantel given the infected human subpopulation. In this paper the intervention is consider as the parameter of the model.
The stability of the governed mathematical model is analyzed at the critical point that considered the stagnant condition of the model that represents the long term behavior of the model. The stability of the critical points is determined from the characteristic equation that derived from the Jacobian matrix of the linearized system of the model. Because of the complexity of the characteristic equation, the identification of the stability requires the Routh Hurwitz criteria. The stability analysis also appears the possibility of the bifurcation point.

2. The mathematical model

The mathematical model of controlling the spread of Schistosomiasis is derived from the diagram of the disease spread and some assumptions. The SI model is also adapted to represents the human population dynamic. The human population is divided into 2 compartments that determine vulnerable subpopulation ($S_W$) and infected subpopulation ($I_H$), while the Schistosoma Japonicum life cycle is into 4 compartments determine the adult worm subpopulation ($C_D$), worm eggs ($C_P$), mirasidium larvae ($C_M$), and cercariae ($C_3$). The Oncomelania hupensis lindoensis snail ($S_P$) is also considered and assumed to follow the logistic equation with the internal interaction of snails in food competing to survive. The compartment diagram of Fig. 1 shows the transition, interaction, growth, and death of each sub-population of the model, where the description of the involved variables and parameters are respectively shown in Table 1 and Table 2.

![Figure 1. Compartment diagram](image)

| Symbol | Parameter | Value | Unit | References |
|--------|-----------|-------|------|------------|
| $A$    | The birth rate of human population | 0.001 | Day | Governed  |
| $\alpha$ | The infected rate of people by Cercariae | 0.0476 | Day | [1]        |
| $\gamma$ | The transition rate of infected people being health | 0.5 | Day | Governed  |
| $\mu_{H1}$ | The natural death rate of vulnerable people | 0.000042 | Day | 1/life time |
| $\mu_{H2}$ | The death rate people because of infection | 0.000054 | Day | 1/life time |
| $\rho$ | The successes of Praziquantel to cure the infected people | 0.06 | Gram | [4]        |
| $\delta$ | The transition rate of Cercariae being adult worms | 0.14 | Day | [2]        |
| $q$ | The chance of Cercariae being adult worms | 0.5 |      | Governed  |
| $\mu_C$ | The natural death rate of adult worms | 0.000058 | Day | [5]        |
| $\mu_M$ | The natural death rate of mirasidium | 1 | Day | [6]        |
| $V$ | The recruitment rate of eggs worms | 0.05 | Day | Governed  |
| $\beta_1$ | The natural birth rate of adult eggs | 0.00016 | Day | Exist & stable require |
| $\beta_2$ | The transition rate of eggs being mirasidium | 0.025 | Day | [2]        |
| $p$ | The chance of eggs being mirasidium | 0.5 |      | Governed  |
| $\beta_3$ | The transition rate of mirasidium being sporocyst | 0.0167 | Day | [6]        |
| $B$ | The natural birth rate of Oncomelania hupensis lindoensis | 0.34 | Day | Exist & stable require |
|       |           | 0.000034 | Day | Exist & stable require |
The critical point of the system is determined by evaluating the DES (1) in a stagnant condition by determining:

\[
\frac{dx}{dt} = 0, \quad x = (S_H, I_H, C_D, C_T, C_M, C_S, S_p)^T
\]

As a result, there are a disease-free critical point \( T_1 = \left( \frac{A}{\mu_{H1}}, 0, 0, V, \frac{pV}{\mu_M}, 0, 0 \right) \) and an endemic critical point \( T_2 \):

\[
T_2 = \left( \frac{\mu_{H1} + \gamma p}{\beta_1}, \frac{\gamma p}{\delta}, \frac{\gamma p}{\delta}, \frac{\gamma p}{\delta}, \frac{\gamma p}{\delta}, \frac{\gamma p}{\delta}, \frac{\gamma p}{\delta}, \frac{\gamma p}{\delta} \right)
\]

The existence of \( T_2 \) requires \( \beta_1 < \frac{p + \mu_C}{pq} \) and \( B > \mu_S \).

3.2 The stability of critical points

The stability of the critical point is determined from the characteristic equation that determined by the eigenvalue of \( \lambda \) from the determinant of \( J - \lambda I \), where \( J \) is the Jacobian matrix evaluated at the critical point and \( I \) is the identity matrix. The critical point of \( T_1 \) gives six negative eigenvalues \( \lambda_1 = -\mu_{H1}, \lambda_2 = -\mu_{H2} - \gamma p, \lambda_3 = -\mu_{C} - \rho, \lambda_4 = -p\beta_2 - (1 - p)\beta_2, \lambda_5 = -\mu_{M}, \lambda_6 = -\delta, \lambda_7 = B - \)
\( \mu_5 \). The critical point \( T_3 \) is stable if \( B < \mu_5 \). Otherwise, when \( B > \mu_5 \), the Jacobian matrix of \( T_2 \) leads to a negative eigenvalue \( \lambda = -B + \mu_5 \) which and a fourth and second order characteristic equation:

\[
P_1(\lambda) = a_4 \lambda^4 + a_3 \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0
\]
and
\[
P_2(\lambda) = e_2 \lambda^2 + e_1 \lambda + e_0
\]

where
\[
a_4 = -\theta < 0
\]
\[
a_3 = -\beta_2 \theta + (-B + \mu_5)\beta_3 - \theta(\mu_c + \rho + \delta + \mu_m)
\]
\[
a_2 = ((-B + \mu_3)\beta_3 - \theta(\mu_c + \rho + \delta + \mu_m))\beta_2 - (\rho + \delta + \mu_c)\beta_2 - \theta((\mu_c + \rho + \mu_m)\delta \mu_m (\rho + \mu_c))
\]
\[
a_1 = (-\rho + \delta + \mu_c)(B - \mu_3)\beta_3 - \theta((\mu_c + \rho + \mu_m)\delta + \mu_m(\rho + \mu_c))\beta_2 - (\rho + \mu_c)\delta(B - \mu_3)\beta_3 + \theta \mu_m
\]
\[
a_0 = (pq\beta_1 - \mu_c - \rho)(B - \mu_3)\beta_3 - \theta \mu_m(\rho + \mu_c)\delta
\]
\[
e_2 = ((pq\beta_1 - \mu_c - \rho)(B - \mu_3)\beta_3 - \theta \mu_m(\rho + \mu_c))\beta_2
\]
\[
e_1 = ((pq\beta_1 - \mu_c - \rho)(B - \mu_3)\beta_3 - \theta \mu_m(\rho + \mu_c))\beta_1 + ((pq\beta_1 - \mu_c - \rho)(B - \mu_3)\beta_3 - \theta \mu_m(\rho + \mu_c))(\gamma + \mu_{H2})\delta - V\mu_{H2}(\rho + \mu_c)
\]
\[
e_0 = ((pq\beta_1 - \mu_c - \rho)(B - \mu_3)\beta_3 - \theta \mu_m(\rho + \mu_c))\mu_{H1}(\gamma + \mu_{H2})\delta - V\mu_{H2}\mu_{H2}(\rho + \mu_c)(B - \mu_3)
\]

Using the criteria of Routh Hurwitz. Equation (3) and (4) are could be stated in in Table (3) and (4)

| \( \lambda^4 \) | \( \lambda^3 \) | \( \lambda^2 \) | \( \lambda \) | \( \lambda^0 \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( a_4 \)       | \( a_3 \)       | \( a_2 \)       | \( a_1 \)       | \( a_0 \)       |
| \( \lambda^3 \) | \( b_2 = a_0 \) | \( b_2 = a_0 \) | \( b_2 = a_0 \) | \( b_2 = a_0 \) |
| \( \lambda^2 \) | \( c_1 \)       | \( c_1 \)       | \( c_1 \)       | \( c_1 \)       |
| \( \lambda^1 \) | \( d_1 = b_2 = a_0 \) | \( d_1 = b_2 = a_0 \) | \( d_1 = b_2 = a_0 \) | \( d_1 = b_2 = a_0 \) |

| \( \lambda^0 \) | \( f_1 = e_0 \) | \( f_1 = e_0 \) | \( f_1 = e_0 \) | \( f_1 = e_0 \) |

It is known that \( a_4 < 0 \) and \( e_2 < 0 \) is satisfied by the existence of \( T_2 \) and \( \beta_1 < \frac{\rho + \mu_c}{pq} \). The coefficients \( a_3 < 0 \), \( b_1 < 0 \), and \( c_1 < 0 \) are fulfilled, due to the existence of the critical point \( B > \mu_5 \). The coefficients \( a_3 < 0 \), \( b_1 < 0 \), and \( c_1 < 0 \) are fulfilled due to the existence of the critical point \( B > \mu_5 \), and \( d_1 < 0 \), \( e_1 < 0 \), and \( f_1 < 0 \) are fulfilled due to the existence of the critical point \( B > \mu_5 \) and \( \beta_1 < \frac{\rho + \mu_c}{pq} \). Based on the Routh Hurwitz criteria, the critical point \( T_2 \) will be stable if \( a_3 < 0 \), \( b_1 < 0 \), \( c_1 < 0 \), \( d_1 < 0 \), \( e_1 < 0 \), and \( f_1 < 0 \), such that it can be concluded that the \( T_2 \) critical point is stable.

From the requirement of \( B > \mu_5 \), we could draw the parameter stability plan of \((B, \mu_5)\) in Fig. 2 such that in the upper region could be found a stable point of \( T_2 \) and unstable point of \( T_1 \) and in the lower region could be found a stable point of \( T_1 \) and unstable point of \( T_2 \). We could also have a bifurcation diagram on Fig.3, by varying the value of \( B \), such that we just have stable critical point \( T_1 \) in case of \( B < \mu_5 \). Moving to the \( B > \mu_5 \) region, we have an unstable critical point \( T_1 \) and a stable critical point \( T_2 \), such that we have the bifurcation point of \( B = \mu_5 \).
3.3 Numerical simulation

The simulation of the mathematical model of the treatment spread of Schistosomiasis disease will be done by giving the required parameter values that meet the existence and stability of the critical point. Table 1 gives parameter values used to simulate. Simulation of the model is also carried out by giving the initial values of variables on Table 2.

3.3.1 The simulation of $T_1$

The simulation of disease-free conditions of the $T_1$ critical point in Fig. 4 represents the unstable conditions where the vulnerable human sub-populations is growing with respect to time. This is due to the decreasing of the infected mature worm sub-population in the human body that causes the human sub-population being infected. The existent of the infected human sub-populations is because the existent of infectious snail population such that the cercariae sub-population is also exists. This condition is satisfied in case of the birth rate of the infectious snail is greater than the natural death rate of the infectious snail, $B > \mu_S$.

The simulation of disease-free conditions of the $T_1$ critical point in Fig. 5 represents the stable conditions that show the growth of vulnerable human sub-populations and infected human subpopulations that continue to decline with respect to time. It is caused by $B < \mu_S$ where the decreasing of mature worm subpopulation and the transmitting of the decreasing of the snail population due to the death rate of infectious snails that greater than the rate of infectious snail transmitters. As a result, the subpopulation of cercariae is also declined.
3.3.2 The simulation $T_2$

The simulation of endemic conditions of $T_2$ critical point in Fig. 6 shows the growth of vulnerable human subpopulations that declining in between the first to the fourth day and increasing up to 21 individual in 206 days and stagnant in between 300 days. Infected humans increases in between the first to fourth day and decreases until 2 populations left on the day of 240 then it will be stagnant until the day of 300. The mature worm decreases in between the first to tenth day and then will run out in 102 days. The number of eggs worm will be decreasing to 2 individual on the day of 163 and stagnant until the day of 300. The eggs worm in the human body will not run out because there are worm eggs trapped in the human body that causes the population of worm eggs still to exist. Mirasidium has decreased and then run out on the day of 7. The population of cercariae has been decreased and will run out on 55 days. The snail population Oncomelaniahupensislindoensis decreased to the number 3 and will be stagnant until the day of 300.

![Figure 6. The dynamic of the spread of Schistosomiasis of $T_2$](image)

4. Concluding remark

Analysis of the stability of the critical point of the mathematical model of controlling Schistosomiasis disease in infected humans gives a disease-free critical point $T_1$ is a stable critical point with the stability requirement $B < \mu_S$ and the endemic critical point $T_2$ is a stable critical point with the condition of the existence of a critical point that ensures stability, that is $B > \mu_S$ and $\beta_1 < \frac{\rho + \mu_C}{pq}$. The simulation results show that controlling with 60 mg of Praziquantel drug provides effective results in minimizing the number of infected humans and reducing the number of worms in the infected human body. The disease-free critical point obtained is a stable critical point with stability condition $B < \mu_S$ so as to provide an opportunity to further examine the control of snail Oncomelaniahupensislindoensis in the spread of Schistosomiasis disease in infected humans.

5. Concluding remark critical

The stability analysis of the points of the mathematical model of Schistosomiasis disease controlling give a stable disease-free critical point $T_1$ where $B < \mu_S$ and an endemic critical point $T_2$ is a stable critical point with the condition of the existence of a critical point if $B$ satisfy $B > \mu_S$ and $\beta_1 < \frac{\rho + \mu_C}{pq}$. The simulation results show that the 60 mg of Praziquantel provides an effective result in minimizing the number of infected people and reducing the number of worms in the infected human body. The disease-free critical point $T_1$ is a stable critical point with stability requirement $B < \mu_S$ provided an opportunity to examine the control of Oncomelania hupensis lindoensis. We also come to the bifurcation point $B = \mu_S$ where the system gives a different characteristic.
Acknowledgement
The author would like to thank the Research and Development Center P2B2 Donggala and the Mathematics Research and Creativity Center (PRKM) Mathematics study program for collaboration during the research.

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