On computation of solid fuel regression rate in ramjet combustor

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Abstract. Development of a solid fuel ramjets requires mathematical modeling of the solid fuel regression inside a combustor with accounting of solid fuel gasification and combustion processes and a numerical method to calculate parameters of these processes. This report presents a quasi-one-dimensional model of processes inside the solid fuel ramjet combustor. The model allows to calculate the solid fuel regression rate and gas flow parameters at the combustor outlet while air flow parameters at the combustor inlet are fixed. The model is based on mass, energy, species and momentum conservation equations and deals with thermochemical processes inside the ramjet combustor. It considers gas flow inside the combustor, gasified fuel combustion, convective heat transfer, solid fuel pyrolysis kinetics. The model is verified by comparison of the numerical results with the experimental data available from other authors. The analysis of the numerical results shows a dependence of the flow structure and thermochemical parameters of a solid fuel employed on the regression rate.

1. Introduction
A solid fuel ramjet (SFR) is one of the most simple air breathing propulsion systems. Typical SFR, as can be seen on Figure 1, consists of an air intake, a combustion chamber with a solid fuel block inside and a nozzle Simplicity, high fuel density and high specific impulse at high flight speed makes SFR one of the most attractive designs for supersonic propulsion systems [1].

![Figure 1. Schematic illustration of a SFR.](image-url)
During the flight of an aircraft with SFR the incoming air is decelerating by shock waves at the air intake, yielding an increase of pressure and temperature. At the combustion chamber inlet, a diaphragm creates a sudden expansion of the incoming air, inducing a recirculation zone, which provides flame stabilization. Inside the combustion chamber the solid fuel is pyrolyzed by the hot air and by heat flux from diffusion flame caused by reactions between the gasified fuel and the oxygen. Combustion of gasified fuel increases gas flow internal energy which is converted to kinetic energy in the nozzle, yielding thrust. Polymers, solid hydrocarbons and their compositions with metal and boron particles are considered as solid fuels for ramjets [1].

SFR design optimization requires engineering models of its elements and computing algorithms for performance parameters evaluation. The aim of this work is to develop such model of SFR combustor and evaluate some of combustor performance parameters.

2. Model description

Parameters of the air flow at the combustor inlet, e.g. bulk temperature $T_0$, static pressure $P_0$ and air mass flux $(\rho V)_0$ are entry data of the model. Since polymers such as polyethylene (PE), hydroxyl-terminated polybutadiene (HTPB) and polymethylmethacrylate (PMMA) are considered as the main components of solid fuel for ramjets [1], properties of these materials are used in present calculations. The solid fuel block considered to be a hollow cylinder. It is assumed that solid fuel pyrolysis takes place in thin layer near the block surface consuming all the heat supplied from the gas phase and yielding monomer which then oxidizes in the only reaction with oxygen. Thus, four macro-species are considered to be the gas mixture components: $N_2$, $O_2$, gasified fuel and combustion products. Since solid fuel linear regression rates are much lower than an average flow velocity a quasi-steady approach is implemented. At each time step a steady problem is solved and solid fuel block radius distribution is modified for the next time step using calculated regression rate.

2.1. Flow parameters calculation

The flow is governed by mass, momentum, energy and species conservation equations. These equations are as follows:

\[
\begin{align*}
\frac{d(\rho VF)}{dx} &= U \rho_{fu} \Pi \\
\frac{d[F(\rho V^2 + P)]}{dx} &= -\Pi C_l \rho V^2 + P \frac{dF}{dx} \\
\frac{d[\rho VF(H + \frac{V^2}{2})]}{dx} &= U \rho_{fu} \Pi \Delta_f H_{fu} \\
\frac{d(\rho VFY)}{dx} &= Q_j,
\end{align*}
\]

In these equations $\rho$ – bulk gas density, $V$ – bulk gas flow velocity, $F$ – cross section surface, $x$ – longitude coordinate, $U$ – linear regression rate, $\rho_{fu}$ – solid fuel density, $\Pi$ – cross section channel perimeter, $P$ – static pressure, $C_l$ – friction coefficient, $H$ – bulk flow specific enthalpy, $\Delta_f H_{fu}$ – standard specific enthalpy of solid fuel formation, $Y$ – bulk mass fraction of $j$-th gas specie, $Q_j$ – mass generation rate of $j$-th gas specie in unit length. Bulk flow parameters space distributions at each time step are obtained by integrating the system conservation equations closed by the expression for specific enthalpy and the equation of state of the perfect gas:
\[
H = \sum_j Y_j \left( \int_{298K}^{T} c_p \Delta T + \Delta_i H_{fi} \right),
\]

\[
P = \sum_j \frac{Y_j}{M_j} \rho R_{\mu} T .
\]

2.2. Fuel regression rate

During the integration of the system (1) it is needed to obtain a solid fuel linear regression rate at each longitudinal step. It can be done by coupling block surface energy balance equation with kinetic law of fuel pyrolysis at the block surface. With assumptions made energy balance equation is

\[
\rho_{\mu} U [\Delta_i H_{g0} - \Delta_i H_{fu0} + c_e (T_S - 298K)] = q,
\]

where \( \Delta_i H_{g0} \) - standard enthalpy of formation of gasified fuel, \( T_S \) - block surface temperature. Left hand side of equation (2) describes heat, consumed by pyrolysis in unit time at unit block surface; right hand side describes local heat flux \( q \). If combustion takes place in the boundary layer local heat flux can be calculated by [2]:

\[
q = \alpha (Y_{ox} Q_c / (v_{ox} c_p) + T - T_S),
\]

where \( \alpha \) – convective heat transfer coefficient, \( Q_c \) – specific heat of gasified fuel combustion, \( v_{ox} \) – stochiometric ratio of gasified fuel combustion.

Assuming solid fuel pyrolysis takes place in the thin surface layer of the block the law of pyrolysis takes form [3]:

\[
(\rho_{\mu} U)^2 = \frac{\lambda_{\mu} \rho R_{\mu} T_{\mu}^2 Z_S}{[\Delta_i H_{g0} - \Delta_i H_{fu0} + c_e (T_S - 298K)] E_A} \exp \left( - \frac{E_A}{R_{\mu} T_{\mu}} \right),
\]

Values of kinetic (preexponential factor \( Z_S \) and activation energy \( E_A \)) and thermal parameters of solid fuels pyrolysis were proposed by various experimental investigators, for example [4], [5]. Parameters used in present work are given in Table 1.

| Substance | Formula | \( Z_S \), s\(^{-1} \) | \( E_{\text{hu}} \), J/mole | \( \Delta_i H_{g0} \), J/kg | \( \Delta_i H_{fu0} \), J/kg | \( \rho_{\mu} \), kg/m\(^3 \) | \( \lambda_{\mu} \), W/(m·K) |
|-----------|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| PMMA \((C_5H_8O_2)n\) | 3.92 \(10^7\) [4] | 1.16 \(10^7\) [4] | -4.3 \(10^6\) [7] | -3.32 \(10^6\) | 1150 [4] | 0.185 [4] |
| PE \((C_2H_4)n\) | 4 \(10^{11}\) [6] | 1.99 \(10^7\) [6] | -1.92 \(10^6\) [7] | 1.87 \(10^6\) | 922 [1] | 0.293 [1] |
| HTPB \((C_4H_6)n\) | 2.63 \(10^{10}\) [5] | 2.04 \(10^7\) [5] | 0.02 \(10^7\) [5] | 2.08 \(10^4\) | 920 [5] | 0.151 [5] |

Solid fuel block surface temperature and linear regression rate at each space step are calculated through solving system consisting of equations (2) and (3) using iterative procedure. Channel radius change during the time step is calculated using:

\[
\Delta R = \int_{t_i}^{t_{i+\Delta t}} U \cos \beta \, dt,
\]

where \( \beta \) is the angle between the cross section plane and the local fuel block surface normal vector. With the distribution of solid fuel linear regression rate and the block form known solid fuel mass regression rate can be obtained from:

\[
G_{fu} = \int_0^L \Pi U \rho_{\mu} \, dx.
\]
3. Results and discussion

Figure 2 shows solid fuel regression rate distribution along a solid fuel block surface calculated using presented model and experimental data published in [1]. A flow behind step can be divided into three adjacent zones: recirculation zone, area of reattachment point in which convective heat transfer and fuel gasification are the most intensive and zone of a fully-developed boundary layer where regression rate gradually decreases since the boundary layer thickness rises and oxygen bulk mass fraction lowers. It can be seen from the Figure 2, that numerical data is in a good agreement with experimental results.

![Figure 2. Linear regression rate distribution along solid fuel block. Solid line – numerical results, points – experimental data.](image)

Figure 3 shows dependences of mass gasification rates for blocks of PMMA, PE and HTPB on initial air mass flux. In each case the solid fuel block is considered to have length $L = 1$ m, channel radius $R = 0.08$ m. Regime values of SFR are accounted by setting air flow temperature and static pressure at the combustor inlet: $T_0 = 1000$ K, $P_0 = 5$ atm. It can be seen that regression rates of solid fuels increases as air mass flux increases. Despite fuel consumption increases, outlet bulk temperature decreases as air mass flux rises as can be seen on Figure 4.

![Figure 3. Influence of air mass flux on mass regression rate of different solid fuels.](image)
The PMMA block which has the lowest pyrolysis kinetic parameters and heat of gasification in general regresses faster than blocks of PE and HTPB. HTPB has the highest combustion heat and yields temperature at the outlet close to that of PMMA. PE and HTPB have close combustion heats and kinetic parameters of pyrolysis law but heat of gasification of HTPB two times lower. Thus, PE regresses in general two times slower and yields the lowest bulk temperature at the combustor outlet.

Let us consider transient behavior of gasification parameters of same fuel blocks in an air flow with following parameters: $T_0 = 1000$ K, $P_0 = 5$ atm, $(\rho V)_0 = 30$ kg/(m$^2$s). During the operation of the SFR combustor, the channel of the solid fuel block expands leading to a decrease of the convective heat transfer coefficient. Thus, as the operation time passes solid fuel regression rate decreases (Figure 5). As can be seen on Figure 6 the temperature at the combustor outlet decreases as well.

**Figure 4.** Influence of air mass flux on bulk gas temperature at the combustor outlet for different fuels.

**Figure 5.** Solid fuel mass regression rate change during the combustor operation for different fuels.
4. Conclusion
A model of gasification and combustion of solid fuel block in a ramjet combustor is presented and validated using available published experimental data. The model allows to predict SFR combustor performance parameters. Dependences of air mass flux and combustor working time on solid fuel regression rate are obtained using the model. Features of gasification of different kinds of solid fuels in a ramjet combustor are shown.

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Figure 6. Change of the bulk gas temperature at the combustor outlet during the combustor operation for different fuels.