Enhanced Weiss oscillations in graphene

A. Matulis$^{1,2}$ and F. M. Peeters$^1$

$^1$Departement Fysica, Universiteit Antwerpen
Groenenborgerlaan, B-2020 Antwerpen, Belgium
$^2$Semiconductor Physics Institute, Gostauto 11, LT-01108 Vilnius, Lithuania

(Dated: March 23, 2022)

The magneto-conductivity of a single graphene layer where the electrons are described by the Dirac Hamiltonian weakly modulated by a periodic potential is calculated. It is shown that Weiss oscillations periodic in the inverse magnetic field appear, that are more pronounced and less damped with the increment of temperature as compared with the same oscillations in a typical two-dimensional electron system with a standard parabolic energy spectrum.

PACS numbers: 72.20.My, 72.80.Rj, 73.50.Dn, 73.40.-c

I. INTRODUCTION

Recently the successful preparation of monolayer graphene films has generated a lot of activity in the physics of two-dimensional (2D) Dirac fermions. The massless energy spectrum and the specific density of states of electrons and holes described by the Dirac Hamiltonian enabled to study experimentally the chiral tunnelling and Klein paradox in graphene, and led to the discovery of the unconventional "half-integer quantum Hall effect". The presence of holes in graphene with 2D Dirac-like spectrum was confirmed by measurements of de Haas-van Alphen and Shubnikov-de Haas (SdH) oscillations. These magnetic oscillations appear due to the interplay of the quantum Landau levels with the Fermi energy in the metal, and serve as a powerful technique to investigate the Fermi surface and the spectrum of electron excitations.

Another technique which was successfully used to obtain information on the electron spectrum of 2D systems is based on the interaction of electrons with artificially created periodic potentials with periods in the submicron range. Such electrical modulation of the 2D system was created by two interfering laser beams, or by depositing an array of parallel metallic strips on the layer surface, and led to the discovery of Weiss oscillations in the magneto-resistance. These oscillations are a consequence of the commensurability of the electron cyclotron orbit radius at the Fermi energy and the period of the above electrical modulation. They were found to be periodic in the inverse magnetic field like the SdH oscillations, but have a different period versus electron density dependence. The period for Weiss oscillations varies with electron density ($n_e$) as $\sqrt{n_e}$, whereas that of the SdH ones as $n_e$. Theoretical calculations of these oscillations were presented in Refs. and it was shown that Weiss oscillations in the magneto-resistance for motion perpendicular to the oscillating potential can be understood as being a semiclassical effect.

The Klein paradox in graphene where Dirac electrons can penetrate through potential barriers with a rather high probability shows that electric control of Dirac electrons cannot be realized. Therefore, it is interesting to investigate the sensitivity of Dirac electrons on the electrical modulation of the layer. Thus, we subjected the system to a periodic potential that introduces a new length scale and a new energy scale into the problem. We found that such a periodic potential also leads to Weiss oscillations in graphene which are even more pronounced than in typically 2D electron gases with a parabolic electron spectrum.

The paper is organized as follows. In the next Sec. all necessary expressions for the magneto-conductivity calculation are given, and in Sec. III the obtained results for graphene layer are compared with results for the standard 2D electron system. In Sec. IV the asymptotic expression valid in the quasi-classical region is obtained. In Sec. V some simple classical explanation of obtained results are presented, and in the last Sec. the short conclusions are given.

II. ELECTRICAL MAGNETOTRANSPORT

We consider the graphene layer within the single electron approximation where the low energy excitations are described by the two-dimensional (2D) Dirac-like Hamiltonian:

$$H_0 = v_F \sigma \left(-i\hbar \nabla + \frac{e}{c} A\right).$$

Here $\sigma = \{\sigma_x, \sigma_y\}$ are the Pauli matrices, and the vector potential $A = \{0, Bx\}$ describing the magnetic field $B = \{0, 0, B\}$ perpendicular to the graphene layer is chosen in the Landau gauge. The parameter $v_F$ characterizes the electron velocity which is usually about 300 times smaller than the velocity of light. The total Hamiltonian consists of two parts

$$H = H_0 + U(x),$$

where the additional potential

$$U(x) = V_0 \cos(2\pi x/a_0)$$

describes the static electrical modulation of our 2D system in the $x$ direction.
The conductivity tensor $\sigma_{\mu\nu}(\omega)$ within the one-electron approximation was evaluated in Ref. 13. We shall restrict ourselves to the diffusion contribution (i.e., which is the dominant contribution) which stems from the diagonal part of the density operator. Following Ref. 13 it can be written as

$$\sigma_{yy} = \frac{\beta e^2}{L_x L_y} \sum_{\zeta} f(E_\zeta) \{1 - f(E_\zeta)\} \tau(E_\zeta) (v^\zeta_y)^2.$$  (4)

Symbols $L_x, L_y$ characterize the dimensions of the layer, and $\beta$ is the inverse temperature. This diagonal part is caused by the influence of the electrical modulation on the electron drift in crossed electric and magnetic fields and is in order of magnitude larger than the other diagonal component $\sigma_{xx}$ which appears due to scattering on imperfections. Symbol $\zeta$ denotes the quantum numbers of the electron eigenstates, and the velocities $v^\zeta_y$ can be calculated as derivatives of the energy eigenvalue over the corresponding electron momenta.

The resistivity tensor is given in terms of the inverse conductivity tensor, namely, $\rho_{xx} = \sigma_{yy} / (\sigma_{xx} \sigma_{yy} - \sigma_{xy} \sigma_{yx})$ what in our case of small conductivity reduces to $\rho_{xx} \approx \sigma_{yy} / \sigma_{xx}^2$, and its $\rho_{xx}$ component is actually proportional to the conductivity in the considered perpendicular direction.

The energy eigenvalues are defined through the solution of the stationary Schrödinger equation

$$\{H - E\} \Psi(r) = 0$$  (5)

with total Hamiltonian (3). We will follow Ref. 13 and assume that the electrical modulation is small enough and restrict to a lowest perturbation expansion in $V_0$. For this purpose we have to solve the zero order Schrödinger equation with the Hamiltonian $H_0$. Due to the system homogeneity along $y$ axis we substitute the eigenfunction as

$$\Psi(r) = \frac{1}{\sqrt{L_y}} e^{i k_y y} \begin{pmatrix} a(x) \\ b(x) \end{pmatrix},$$  (6)

and transform the zero order eigenvalue problem into the following two ordinary differential equation set for the wave function components:

$$-i\hbar v_F \left( \frac{\partial}{\partial x} - \frac{i}{\hbar} \frac{\partial}{\partial y} + \frac{x}{l^2} \right) b - Eb = 0,$$  (7a)

$$-i\hbar v_F \left( \frac{\partial}{\partial x} + \frac{i}{\hbar} \frac{\partial}{\partial y} - \frac{x}{l^2} \right) a - Ea = 0,$$  (7b)

where $l = \sqrt{\hbar e / e B}$ is the magnetic length. Solution of these equations can be easily obtained making use of the analogy with the harmonic oscillator eigenvalue problem. It reads

$$E_n^D = \frac{v_F h}{l} \sqrt{2n},$$  (8a)

$$\Phi_{n,k_y}(r) = \frac{e^{ik_y y}}{\sqrt{2 L_y}} \left(-i\Phi_{n-1}([x + x_0]/l) - \Phi_n([x + x_0]/l) \right).$$  (8b)

where

$$\Phi_n(x) = \frac{e^{-x^2/2}}{\sqrt{2^n n! \sqrt{\pi}}} H_n(x),$$  (9)

is expressed in the normalized Hermitian polynomials, and $x_0 = l^2 k_y$ indicates the localization of the electron in the $x$ direction.

For the first order correction one has to add to the energy eigenvalue the diagonal matrix element of potential (3) calculated with the above eigenfunctions:

$$\Delta E_{n,k_y} = \int_{-\infty}^{\infty} \int_0^{L_y} dy \Psi_{n,k_y}^* (r) U(x) \Phi_{n,k_y} (r)$$

$$= \frac{V_0}{2} \cos(K x_0) e^{-u/2} \{ L_n(u) + L_{n-1}(u) \},$$  (10)

where $K = 2\pi / a_0$, $u = K^2 l^2 / 2$, and $L_n(u)$ is a Laguerre polynomial. This energy correction makes the degenerate Landau levels $k_y$-dependent, expands them into bands, what finally leads to non zero velocities:

$$v^\zeta_y = v_n^{n,k_y} = \frac{1}{\hbar} \frac{\partial}{\partial k_y} \Delta E_{n,k_y}$$

$$= -\frac{V_0}{\hbar R} \cos(K x_0) e^{-u/2} \{ L_n(u) + L_{n-1}(u) \} \sin(K x_0).$$  (11)

Now substituting the velocities in Eq. (4) and specifying the summation over quantum numbers as

$$\sum_{\zeta} = \frac{L_y}{2\pi} \int_0^{L_x/l^2} dk_y \sum_{n=0}^{\infty}$$

we obtain the final expression for the diffusion contribution for the dc conductivity under the consideration

$$\sigma_{yy} = A \Phi,$$  (13)

where

$$A = 8\pi^2 e^2 V_0^2 \tau \beta / h,$$  (14)

and the function

$$\Phi = \frac{1}{4} u e^{-u} \sum_{n=0}^{\infty} \frac{f(E_n)}{[f(E_n) + 1]^2} [L_n(u) + L_{n-1}(u)]^2$$  (15)

will be considered as a dimensionless conductivity. In order to simplify this expression we introduced the following exponential function $f(E) = \exp \{ \beta (E - E_F) \}$ where $E_F$ is the Fermi energy.

### III. COMPARISON WITH THE USUAL WEISS OSCILLATIONS

It is interesting to compare the obtained expression for dc conductivity with the same conductivity calculated in
Ref.\textsuperscript{12} for the case of a 2D electron system localized at the interface between two semiconductors and having a parabolic energy dependence on its momentum. Comparing velocity expression (11) with the equivalent one in Ref.\textsuperscript{13} (see Eq. (4)) we notice that the diffusive conductivity for the system of Dirac electrons differs from the one for the standard interface case by replacement of the Laguerre polynomial $L_n(u)$ by the average of two successive polynomials $[L_n(u) + L_{n-1}(u)]/2$. And of course, two different expressions for the Landau level energies (Eq. (5a)) for Dirac electrons and

$$E_n^P = \hbar \omega_c(n + 1/2), \quad \omega_c = \frac{eB}{mc}$$

for the parabolic electron spectrum case) has to be used.

These differences in the expressions, however, lead to essentially different results for the dimensionless conductivity as shown in Fig. 1. The results are shown as function of the inverse magnetic field for the temperature $T = 6$ K, electron density $n_e = 3 \cdot 10^{13}$ cm$^{-2}$, and the period of electric modulation $a_0 = 350$ nm. The dimensionless magnetic field $b = B/B_0$ is introduced using the characteristic magnetic field $B_0 = e\hbar c/ea_0^2$ corresponding to the magnetic length equal to the modulation period $a_0$, which in the above case is equal to $B_0 = 1$ T.

![Image of conductivity versus inverse magnetic field](image)

**FIG. 1:** The dimensionless conductivity versus inverse magnetic field: 1 – Dirac electrons, 2 – electrons with the parabolic energy spectrum.

We see that in graphene (curve 1) the Weiss oscillations are more pronounced as compared with the system of electrons with the standard parabolic energy spectrum (curve 2). Furthermore, the Weiss oscillations in graphene are much more robust with respect to temperature damping in the quasi-classical region of small magnetic fields. The physical reasons for these differences lay in different Fermi velocities of Dirac and standard electrons. In order to confirm the above statement we shall consider the asymptotic behavior of Weiss oscillations in the quasi-classical region which according to Refs.\textsuperscript{3,14} describes the main features of the above oscillations and which also allow for explicit analytical expressions.

**IV. ASYMPTOTIC EXPRESSIONS**

The asymptotic expression for the conductivity (15) is obtained following the approach of Ref.\textsuperscript{15} for case of standard electrons. That approach is applicable when many Landau levels are filled, and it is based on the following asymptotic expression for the Laguerre polynomials:

$$e^{-u/2}L_n(u) \rightarrow \frac{1}{\sqrt{\pi \sqrt{n}u}} \cos\left(2\sqrt{nu} - \pi/4\right).$$

Taking the continuum limit

$$n \rightarrow \frac{1}{2} \left(\frac{lE}{v_F \hbar}\right)^2, \quad \sum_{n=0}^{\infty} \left(\frac{l}{v_F \hbar}\right)^2 \int_0^\infty EdE,$$

and having in mind that $u = 2\pi^2/b$, we transform Eq. (15) into the following integral:

$$\Phi = \frac{\sqrt{u}}{\pi} \left(\frac{l}{v_F \hbar}\right)^2 \int_0^\infty \frac{f(E)EdE}{\left[\frac{f(E)}{Ea_0}\right]^2 + \frac{1}{4}}\left(\frac{\pi v_F \hbar}{Ea_0}\right)^2 \cos^2\left(\frac{\pi v_F \hbar}{Ea_0}\right)$$

$$\times \cos^2\left(\frac{2\pi a_0E}{v_F \hbar} - \frac{\pi}{4}\right).$$

Now assuming that the temperature is low ($\beta^{-1} \ll E_F$) and replacing $E = E_F + s\beta^{-1}$ we rewrite the above integral as

$$\Phi = \frac{2a_0}{v_F \hbar \beta} \cos^2\left(\frac{\pi}{p}\right) \int_{-\infty}^\infty \frac{ds e^s}{(e^s + 1)^2} \times \cos^2\left(\frac{2\pi p}{b} - \frac{\pi}{4} + \frac{2\pi p}{b\beta}\right),$$

where symbol

$$p = \frac{E_F a_0}{v_F \hbar} = k_F a_0 = \sqrt{2\pi n_e a_0}$$

stands for the dimensionless Fermi momentum of the electron. Note in Eq. (20) we replaced all energies $E$ by the Fermi energy $E_F$ except that one which is included in the last cosine function, where the small energy correction can influence the damping of the Weiss oscillations.

The obtained expression for the dimensionless conductivity can be calculated using the standard integral

$$\int_0^\infty \frac{\cos(ax)}{\cosh(\beta x)} dx = \frac{\pi}{2\beta \sinh(\beta\pi/2\beta)},$$

as...
and the result can be presented as

\[
\Phi(T) = \frac{T}{4\pi^2 T_D} \cos^2 \left( \frac{\pi}{p} \right) \left\{ 1 - A \left( \frac{T}{T_D} \right) \right\} + 2A \left( \frac{T}{T_D} \right) \cos^2 \left[ 2\pi \left( \frac{p}{b} - \frac{1}{8} \right) \right],
\]

where

\[
A(x) = \lim_{x \to \infty} \frac{x}{\sinh(x)} = 2xe^{-x},
\]

and the symbol \( T_D \) defined as

\[
k_B T_D = \frac{\hbar}{4\pi^2 a_0} v_F
\]

\((k_B \text{ is the Boltzmann constant})\) gives the temperature scale for damping of the Weiss oscillations.

The validity of the asymptotic expression is seen in Fig. 2 where the Weiss oscillations for the Dirac electrons (curve 1) are shown together with their asymptotic expression (curve 2) for the same parameter values as in Fig. 1 in the region of strong magnetic fields where deviations of the asymptotic expression to the exact expression are largest. We see that the coincidence of the exact

result and its asymptotic expression is rather good everywhere except the region of very strong magnetic field where SdH oscillations become superimposed on top of the Weiss oscillations.

We compare now the obtained result for the conductivity for the Dirac electron system with the similar asymptotic result for the system of electrons with the standard parabolic energy spectrum taken from Ref.18 which can be presented as follows:

\[
\Phi(T) = \frac{T}{4\pi^2 T_P} \left\{ 1 - A \left( \frac{T}{T_P} \right) \right\} + 2A \left( \frac{T}{T_P} \right) \cos^2 \left[ 2\pi \left( \frac{p}{b} - \frac{1}{8} \right) \right],
\]

where the critical temperature reads

\[
k_B T_P = \frac{bp\hbar^2}{4\pi^2 m a_0^2}.
\]

Having in mind that

\[
\frac{p}{m} = \frac{k_F a_0}{m} = \frac{a_0}{h} v_F,
\]

where \( v_F \) is the velocity of the standard electron on the Fermi surface, the critical temperature can be presented as

\[
k_B T_P = \frac{bh}{4\pi^2 a_0} v_F.
\]

For parameters used in Fig. 1 plot \( k_F \sim 1.4\cdot10^6, p \sim 50, \) and \( \cos(\pi/p) \sim 1. \) Thus the asymptotic behavior of the dimensionless conductivity for Dirac electrons and standard electrons with parabolic energy spectrum differ mostly due to the very different critical temperatures.

Comparing Eqs. (25) and (29) we see that

\[
\frac{T_P}{T_D} = \frac{v_F^P}{v_F},
\]

or the ratio of critical temperatures for the standard and Dirac electrons is equal to the ratio of the corresponding velocities on the Fermi surface. It can be estimated as

\[
\frac{T_P}{T_D} = \frac{\hbar\sqrt{2\pi n_e}}{\sqrt{v_F ma_0}}
\]

and for typical cases is less than unity. For instance, in the case of the parameters used in Fig. 1 it is \( T_P/T_D \sim 0.24 \) what explains the different slope and damping of the Weiss oscillations.

V. QUASI-CLASSICAL EXPLANATION

The obtained results can be understood from a simple physical picture. In order to estimate the oscillation period we write down the momentum of the electrons on the Fermi surface:

\[
p_F = m\omega_c R_e
\]

which is identical in both cases. Then it follows that the radius of electron orbit in the magnetic field is

\[
R_e = \frac{p_F}{m\omega_c} = \frac{\hbar k_F c}{eB} = \frac{p_F^2}{a_0}.
\]
The physical reason for the appearance of the Weiss oscillations is the commensurability of the electron orbit radius with the period of the electrical modulation, consequently, the argument of the cosine function has to be proportional to

\[ \frac{R_e}{a_0} = p \frac{l^2}{a_0^2} = \frac{p}{b}, \quad (34) \]

which we can see in both expressions (22) and (28).

The damping factor of the oscillations can be estimated as follows. Due to the finite temperature there are electron orbits with various radii. The effective damping can be estimated as the ratio of the above broadening of the orbit to the period of the modulation:

\[ \delta = \frac{1}{a_0} \frac{\partial R_e}{\partial E_F} b^{-1}. \quad (35) \]

In the case of standard electrons (when \( E_F = p_F^2/2m \)) it reads

\[ \delta_p = \frac{m}{a_0} \frac{\partial R_e}{\partial p_F} = \frac{m}{a_0} \beta k_F m \omega_c \]

\[ = \frac{k_B T}{p \hbar \omega_c} = \frac{m a^2}{\hbar^2} k_B T, \quad (36) \]

in agreement with the definition of the critical temperature for standard electron (27).

In the case of Dirac electrons (when \( E_F = v_F p_F \)) the above damping parameter can be estimated as

\[ \delta_D = \frac{1}{\beta a_0 v_F} \frac{\partial R_e}{\partial p_F} = \frac{c k_B T}{a_0 \hbar v_F} = \frac{a_0}{\hbar \hbar v_F} k_B T, \quad (37) \]

what coincides with the obtained earlier result for the critical temperature up to the same constant \( 4\pi^2 \). Now dividing Eq. (36) by Eq. (37) we obtain the same critical temperature ratio dependence on the ratio of the Fermi velocities (30) what confirms the quasi-classical nature of Weiss oscillations.

VI. CONCLUSIONS

In conclusion, we studied the Weiss oscillations in electrically modulated single layer of graphene. It was shown that the static conductivity oscillations are periodic in \( 1/B \) with period varying with electron density as \( \sqrt{n_e} \) like in the 2D electron system with standard parabolic energy spectrum. Due to the larger Fermi velocity the conductivity oscillations of the Dirac electron system are more pronounced and less damped compared with the 2D system of electrons with parabolic energy spectrum in case of analogous parameters. The found Dirac electron sensitivity to the electric perturbation does not contradict the Klein paradox, because in contrary to electron tunneling through barriers where both the electron and the hole nature of the excitation plays a role, only the electrons at the Fermi energy are responsible for the conductivity and for the studied Weiss oscillations. Thus, their behavior in the electric field is not overshadowed by the admixture of the hole states.

Acknowledgments

Part of this work is supported by the Flemish science foundation (FWO-Vl) and the Belgian Science policy (PAI).