Robust output feedback stabilization for discrete-time systems with time-varying input delay

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In this paper, a robust output feedback stabilization method is proposed for discrete-time systems subject to time-varying input delay. By introducing an augmented state, the robust stabilization problem is converted into that of a delay-free system. Using Artstein’s reduction method and the scaled-bounded real lemma, sufficient conditions for robust stability are established via matrix inequalities, by which the output feedback controller can be derived. A tuning parameter is introduced to efficiently solve these matrix inequalities, therefore facilitating the improvement of control performance. An illustrative example is used to show the effectiveness and merit of the proposed method.

Keywords: time-varying delay; scaled-bounded real lemma; dynamic output feedback; matrix inequality

1. Introduction

Time delay is usually associated with industrial applications (Liu & Gao, 2012; Seborg, Edgar, & Mellichamp, 2004). Advanced control design for industrial processes with time delay has drawn a lot of attention in the past decades. For the convenience of control design, various delays in the system response have been mostly merged into the state delay or input delay for consideration in the literature. For the case of state delay, Xia, Liu, Shi, Rees, and Thomas (2007) investigated the stability of discrete-time systems with a constant delay by using a lifting method. Regarding time-varying state delay, a few stability criteria were proposed in the literature, for example, Wu (2003) and Wu and Grigoriadis (2001), for which the main concern was focused on reducing the control conservatism. By using a free-weighting matrix, a flexible output feedback control design for a discrete-time system with a time-varying state delay was presented by He, Wu, Liu, and She (2008). Based on the scaled small gain theorem, new stability criteria were proposed in the recent paper (Li & Gao, 2011) in terms of linear matrix inequalities in combination with an approximation on the state delay. For industrial batch processes with time-varying state delay, two-dimensional stability conditions were established for robust closed-loop iterative learning control (ILC) design (Liu & Gao, 2010). For the case of input delay, Yue (2004) addressed the problem of robust feedback stabilization for uncertain continuous-time input-delayed systems. For stable/unstable or minimum/non-minimum phase systems with long time delay, Albertos and Garcia (2009) proposed a robust control method based on a predicted undelayed output in frequency domain, and later improved the adjusting capability between the output performance and robust stability by introducing a tuning parameter (Garcia & Albertos, 2013). Robust stability analysis of the filtered Smith predictor control structure was addressed by Normey-Rico, Garcia, and Gonzalez (2012) for stable or unstable processes with time-varying input delay. An internal model control-based ILC method was proposed to cope with uncertain input delay for batch process operation (Liu, Gao, & Wang, 2010). By comparison, a predictor-based controller design was given by Gonzalez, Sala, and Sanchis (2013) based on the analysis of the Lyapunov–Krasovskii stability condition developed by Manitius and Olbrot (1979), which was then extended in the recent paper (Gonzalez, Sala, & Albertos, 2012) to allow for larger delay variation.

Recently, Najafi, Hosseinnia, Sheikholeslam, and Karimadini (2013) addressed the problem of state feedback control for continuous-time systems with known or time-invariant input delay by sequential sub-predictors, which was further extended to output feedback in the recent paper (Najafi, Sheikholeslam, Wang, & Hosseinnia, 2014). Based on using an interval observation technique, an output feedback stabilization method for time-varying input delay systems without model uncertainties was proposed by Polyakov, Efimov, Perruquetti, and Richard (2013). In discrete-time domain, robust stabilization of linear discrete-time systems with time-varying input delay was studied by Gonzalez (2013) using Artstein’s reduction...
method (Artstein, 1982) and the scaled-bounded real lemma (Apkarian & Gahinet, 1995), leading to superior control performance in comparison with previous methods.

Considering that output feedback is widely used in engineering practice and state measurement is not available in many industrial applications, this paper proposes an dynamic output feedback control method to stabilize discrete-time systems with input delay, based on the output measurement and past input information. Moreover, the input delay variation and plant uncertainties are also taken into account for robust control. By introducing an augmented state, such a system description is transformed into a delay-free model involved with uncertainties for robust stabilization. Sufficient conditions for robust stability are established in terms of bilinear matrix inequalities which can be solved efficiently by configuring a tuning parameter. For clarity, the paper is organized as follows. In Section 2, the control problem and some preliminary knowledge for analysis are presented. By using the scaled-bounded real lemma, stability analysis of the transformed systems is given in Section 3. In Section 4, the design of dynamic output feedback controller is detailed. An illustrative example is provided in Section 5 to demonstrate the effectiveness of the proposed method. Finally, some conclusions are given in Section 6.

Throughout this paper, the following notations are used: $\mathbb{R}^{n \times m}$ denotes an $n \times m$ real matrix space. For any matrix $P \in \mathbb{R}^{m \times m}$, $P > 0$ (or $P < 0$) means $P$ is a positive-(or negative-) definite symmetric matrix, in which the symmetric elements are indicated as *'. Denote by $P^T$ the transpose of $P$, and by $P^{-1}$ the inverse of $P$. The identity vector/matrix with appropriate dimension is denoted by $I$.

2. Problem description and preliminary knowledge

Consider a discrete-time system with time-varying input delay described by

$$x(t + 1) = (A + \Delta A(t))x(t) + (B + \Delta B(t))u(t - d(t)), \quad y(t) = Cx(t), \quad x(t) = \varphi(t), \quad t = -h_2, \ldots, 0, \quad h_1 \leq d(t) \leq h_2,$$

(1)

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, and $h_{12} = h_2 - h_1$. $x(t)$ is the state, $u(t)$ is the control input, and $\varphi(t) (t = -h_2, \ldots, 0)$ is a given initial condition sequence. Denote by $\Delta A(t)$ and $\Delta B(t)$ the model uncertainties which may be described by the following form:

$$\Delta A(t) \Delta B(t) = \delta F \Delta A(t) (E_A E_B), \quad (2)$$

where $F$, $E_A$, and $E_B$ are known matrices of appropriate dimensions; $\Delta(t)$ is a time-varying matrix with unknown elements, satisfying $\Delta(t)^T \Delta(t) \leq I$, and $\delta$ is a positive scalar reflecting the bound of uncertainties. Note that only the measured output and the past input information will be used to design the controller for the convenience of implementation.

Define the following state transformation in terms of Artstein’s reduction method (Artstein, 1982),

$$z(t) = x(t) + \phi^p_1(h_1) + \phi^p_2(h_2),$$

where $\phi^p_i(h) : L_2([0, \infty), \mathbb{R}^m) \rightarrow \mathbb{R}^n$ is an operator with respect to $u(t)$ defined by

$$\phi^p_i(h) = \sum_{i=0}^{h-1} A^{-i} B \frac{1}{2} u(t - h + i), \quad h \in \mathbb{N}^+.$$ 

(4)

For the ease of comprehension, the following lemma from the proof of Proposition 1 in Gonzalez (2013) is briefly presented as below, which will be used in the later analysis.

**Lemma 1** The system described by Equation (1) can be transformed into the following form using the state transformation in Equation (3),

$$z(t + 1) = Az(t) + \theta_B u(t) + \delta F \omega_z + \frac{h_{12} B}{2} \omega_d,$$

$$\tilde{y}(t) = Cz(t), \quad \sigma_z = E_A z(t) + \theta_E u(t - 1) + E_B u(t) + \sum_{i=1}^{2} \theta_i \omega_i,$$

(5)

where

$$\omega_z = \Delta(t) \sigma_z,$$

$$m_1 = \left\| \sum_{r=1}^{2} \frac{h_{12} B}{2} \omega_d \right\|_{\infty},$$

$$m_2 = \left\| \sum_{r=1}^{2} \frac{h_{12} B}{2} \omega_d \right\|_{\infty},$$

$$\theta_1 = -m_1 E_B, \quad \theta_2 = m_2 E_A, \quad \nu(t) = u(t) - u(t - 1),$$

$$\omega_d = \frac{h_{12}}{2} \left( u(t - d(t)) - \frac{1}{2} \left( u(t) + u(t - h_2) \right) \right).$$

Note that $\omega_d$ can be expressed as $\omega_d = \Delta_d \nu(t)$, $\Delta_d : \nu \rightarrow \omega_d$ is a time-varying delay operator satisfying $\|\Delta_d\|_{\infty} \leq 1$, $\omega_d = \Delta_d \nu(t), i = 1, 2$, and $\Delta_i : \nu \rightarrow \omega_i$ is an operator satisfying $\|\Delta_i\|_{\infty} = 1$ and
Based on the augmented system description in Equations (5) and (6) can be obtained (Jungers, Castelan, Moraes, & Moreno, 2013),

\[
\begin{align*}
\eta(t+1) &= \bar{A}\eta(t) + \bar{B}_d\omega_d(t) + \delta \bar{F}\omega_{\Sigma_i} + \bar{B}_d\omega_d, \\
\sigma_{\Sigma_i} &= \bar{C}_F\eta(t),
\end{align*}
\]

where

\[
\bar{A} = \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{B}_d = \begin{bmatrix} \theta_B \\ 0 \end{bmatrix}, \quad \bar{F} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad \bar{C}_F = \begin{bmatrix} C^T \\ 0 \end{bmatrix}.
\]

Concerning the augmented system shown in Equation (7), a full-order dynamic output feedback controller is proposed as

\[
\begin{align*}
x_c(t+1) &= \hat{A}x_c(t) + B_c\hat{y}(t), \\
u(t) &= C_cx_c(t) + D_c\hat{y}(t),
\end{align*}
\]

where \(x_c(t)\) is the controller state, \(A_c, B_c, C_c, D_c\) are the controller matrices to be designed later.

**Remark 1** For the convenience of control design, the measured output is combined with the past input information to construct a delay-free predicted output shown in Equation (5).

### 3. Stability analysis

Based on the augmented system description in Equations (7) and (8), applying the controller shown in (11) results in the following closed-loop system,

\[
\begin{align*}
x_g(t+1) &= \hat{A}x_g(t) + \delta \hat{F}\omega_{\Sigma_i} + \bar{B}_d\omega_d, \\
\sigma_{\Sigma_i} &= \hat{C}_x(t) + \sum_{i=1}^{2} \theta_i \omega_i + \frac{h_{12}E_B}{2}\omega_d, \\
\end{align*}
\]

where

\[
\begin{align*}
x_g(t) &= \begin{bmatrix} \eta(t) \\ x_c(t) \end{bmatrix},
\end{align*}
\]

and

\[
\begin{align*}
\hat{A} &= \begin{bmatrix} A + \bar{B}_dD_c\bar{C} & \bar{B}_dC_c \\ B_c\bar{C} & A_c \end{bmatrix}, \quad \hat{F} = \begin{bmatrix} F \\ 0 \end{bmatrix}, \quad \bar{B}_d = \begin{bmatrix} B_d \\ 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} E_BD_c\bar{C} & E_BC_c \end{bmatrix}.
\end{align*}
\]

For stability analysis, the following theorem is given.

**Theorem 1** The system described by Equation (1) using the controller shown in Equation (11) is asymptotically stable if there exist matrices \(P > 0, S > 0\) and scalars \(\varepsilon_1 > 0, \varepsilon_2 > 0\) such that the following matrix inequality holds:

\[
\begin{bmatrix}
-P & A_s & B_s & 0 & 0 \\
*A & -P^{-1} & 0 & C_s^T & 0 \\
*B & 0 & -W_1 & W_1D_i^T & 0 \\
C & 0 & 0 & -W_2 \\
\end{bmatrix} < 0,
\]

where \(A_s = \hat{A}, B_s = \begin{bmatrix} \delta \hat{F} & \hat{B} & 0 & 0 \end{bmatrix}, P = \delta \hat{L}, L = \begin{bmatrix} L + D_sC & C_s \\ L \end{bmatrix}, W_1 = \text{diag}(\rho I_n, S, \varepsilon_1 I_m, \varepsilon_2 I_n), W_2 = \text{diag}(\rho I_n, S, \varepsilon_1 I_m, \varepsilon_2 I_n).

\]

Proof Using Equations (12), (13), and Lemma 1, we have

\[
\begin{bmatrix} x_g(t+1) \\ \sigma_{\Sigma_i} \end{bmatrix} = \begin{bmatrix} A_s & B_s \\ C_s & D_s \end{bmatrix} \begin{bmatrix} x_g(t) \\ \omega_i \end{bmatrix}, \quad \bar{\omega}_i = \Delta\sigma_{\Sigma_i},
\]

where

\[
\Delta = \text{diag}(\Delta_1, \Delta_2, \Delta_1, \Delta_2), \quad ||\Delta||_{\infty} \leq 1,
\]

\[
\bar{\omega}_i = \begin{bmatrix} \omega_i^T \omega_i^T \omega_i^T \end{bmatrix}, \quad \sigma_{\Sigma_i} = \begin{bmatrix} \sigma_{\Sigma_i}^T \sigma_{\Sigma_i}^T \sigma_{\Sigma_i}^T \end{bmatrix}.
\]

Using the scaled-bounded real lemma (Apkarian & Gahinet, 1995) and the scaling matrices \(W_1\) and \(W_2\) that satisfy \(W_1\Delta = \hat{A}W_2\), we are sure the system in Equation (18) is stable if there exist matrices \(P > 0\) and \(S > 0\) such that

\[
\begin{bmatrix}
-P & A_s & B_s & 0 & 0 \\
*A & -P^{-1} & 0 & C_s^T & 0 \\
*B & 0 & -W_1 & W_1D_i^T & 0 \\
C & 0 & 0 & -W_2 \\
\end{bmatrix} < 0.
\]

Moreover, the inequality in (21) implies that

\[
\begin{bmatrix}
-P & A_s \\
*A & -P^{-1} \end{bmatrix} < 0,
\]

which guarantees the internal stability of the system in (18). Pre- and post-multiplying (21) by \(\text{diag}(I, I, W_1, I)\), we obtain the matrix inequality in (17). The proof is completed.

It should be noted that the dimension of the identity matrices in \(W_1\) and \(W_2\) may be different if \(\Delta\) is not square. In fact, \(\text{diag}(I, I, W_1, I)\) is symmetric diagonal matrix due to \(W_1\) is symmetric.
4. Design of dynamic output feedback controller

Based on the above stability analysis, the output feedback controller can be derived as stated in the following theorem.

**Theorem 2** Given a time-varying delay \( d(t) \) satisfying \( h_1 \leq d(t) \leq h_2 \), there exists a dynamic output feedback controller in the form of Equation (11), such that the closed-loop system (12) and (13) is asymptotically stable if there exist matrices \( Q > 0, P, A_c, B_c, C_c, \) and \( D_c \) and scalars \( \varepsilon_1 > 0, \varepsilon_2 > 0, \rho \) satisfying the following matrix inequality:

\[
\begin{bmatrix}
-\bar{P} & J_1 & J_2 & 0 \\
* & J_3 & 0 & J_4 \\
* & * & -\bar{W}_1 & J_5 \\
* & * & * & -\bar{W}_2
\end{bmatrix} < 0, \tag{23}
\]

where

\[
\bar{P} = \begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix}, \quad J_5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ h_1 \varepsilon_2^2 \eta_2^2 & 0 & 0 & 0 \\ \varepsilon_1 \theta_1^2 & 0 & 0 & 0 \\ \varepsilon_2 \theta_2^2 & 0 & 0 & 0 \end{bmatrix},
\]

\[
J_1 = \begin{bmatrix} U_1^T A_c + \bar{B}_c \bar{C}_c \bar{A}_c & \bar{A}_c V_1 + \bar{\theta}_b \bar{C}_c \end{bmatrix},
\]

\[
J_2 = \begin{bmatrix} U_1^T F & U_1^T \bar{B}_d & 0 & 0 \\ \bar{F} & \bar{B}_d & 0 & 0 \end{bmatrix},
\]

\[
J_3 = \begin{bmatrix} P_{11} - U_1^T - U_1 & \rho P_{12} - I - M^T \\ * & P_{22} - U_1^T V_1 - V_1 \end{bmatrix},
\]

\[
J_4 = \begin{bmatrix} E^T + C^T \bar{D}_c \bar{E}_b & \Pi_1 & \Pi_1 & \Pi_1 \\ V_1^T E^T + C^T \bar{D}_c \bar{E}_b & \Pi_2 & \Pi_2 & \Pi_2 \end{bmatrix},
\]

\[
\Pi_1 = L^T + C^T \bar{D}_c, \quad \Pi_2 = V_1^T L^T + \bar{C}_c^T,
\]

\[
M = V_1^T U_1 + V_1^T U_2, \quad \bar{W}_1 = \text{diag}\{\rho I_1, S^{-1}, \varepsilon_1 I_2, \rho_2 I_2\}.
\]

\( V_2 \) and \( U_2 \) can be obtained by using a singular value decomposition on \( M - V_1^T U_1 \), corresponding to the controller parameters given by

\[
\begin{align*}
D_c &= \bar{D}_c, \\
C_c &= (\bar{C}_c - D_c \bar{V}_1) V_2^{-1}, \\
B_c &= U_2^T (\bar{B}_c - U_1^T \bar{\theta}_b D_c), \\
A_c &= U_2^T (\bar{A}_c - U_1^T \bar{A} V_1 - U_1^T \bar{\theta}_b D_c \bar{C} V_1 - U_2^T B_c \bar{C} V_1 - U_1^T \bar{\theta}_b C_c V_2) V_2^{-1}.
\end{align*}
\]

**Proof** The proof is based on a suitable congruence transformation and changes of matrix variables. We introduce a slack variable \( G \) which is a nonsingular matrix.

Firstly, we assume the inequality (23) holds. Define

\[
G = \begin{bmatrix} V_1 & \bullet \\ V_2 & \bullet \end{bmatrix}, \quad G^{-1} = \begin{bmatrix} U_1 & \bullet \\ U_2 & \bullet \end{bmatrix}, \quad \Omega = \begin{bmatrix} U_1 & I \\ U_2 & 0 \end{bmatrix},
\]

where ‘\( \bullet \)’ denotes the elements that are uniquely determined from equations \( GG^{-1} = G^{-1} G = I \). Note that the matrix \( U_2 \) included in \( G^{-1} \) is assumed to be invertible.

It is easy to verify

\[
J_1 = \Omega^T A_c G \Omega, \quad J_2 = \Omega^T B_c W_1,
\]

\[
J_3 = \Omega^T (P - G^T - G) \Omega, \quad J_4 = \Omega^T G^T C_c^T,
\]

where the following changes of variables are used:

\[
\bar{P} = \Omega^T P \Omega, \quad \bar{D}_c = D_c, \\
\bar{C}_c = D_c \bar{V}_1 + C_c V_2, \quad \bar{B}_c = U_1^T \bar{\theta}_b D_c + U_2^T B_c, \\
\hat{A}_c = U_1^T \bar{A} V_1 + U_1^T \bar{\theta}_b D_c \bar{C} V_1 + U_2^T B_c \bar{C} V_1 + U_1^T \bar{\theta}_b C_c V_2 + U_2^T A_c V_2.
\]

Thus, inequality (23) can be rewritten as

\[
\begin{bmatrix} -\Omega^T P \Omega & \Omega^T A_c G \Omega & \Omega^T B_c W_1 & 0 \\ * & \Omega^T (P - G^T - G) \Omega & -W_1 & W_1 D_c^T \\ * & * & -W_2 & 0 \\ * & * & * & -W_2 \end{bmatrix} < 0. \tag{24}
\]

Since \( U_2 \) is invertible, matrix \( \Omega \) should be invertible. Performing a congruent transformation to Equation (24) by pre- and post-multiplying \( \text{diag}(\Omega^{-T}, \Omega^{-T}, I, I) \) and \( \text{diag}(\Omega^{-1}, \Omega^{-1}, I, I) \), respectively, we obtain

\[
\begin{bmatrix} -P & A_c G & B_c W_1 & 0 \\ * & * & -W_1 & W_1 D_c^T \\ * & * & * & -W_2 \end{bmatrix} < 0. \tag{25}
\]

Using the well-known inequality,

\[
(G^T - P) P^{-1} (G - P) \geq 0. \tag{26}
\]

we have

\[
-G^T P^{-1} G \leq P - G^T - G. \tag{27}
\]

Combining Equations (25) and (27) leads to the inequality (17) in Theorem 1. This completes the proof.

**Remark 2** Note that the sufficient condition in (23) is not a strict linear matrix inequality due to that \( S^{-1} \) enters in \( W_1 \) in a nonlinear manner. It is therefore suggested to let the matrix variable \( S \) be an identity matrix when using the LMIs toolbox to find a feasible solution, but in exchange for conservativeness. For a single-input system, it can be easily verified that the positive-definite matrix \( S \) is exactly a positive scalar. Hence, we can introduce a tuning parameter.
β > 0 instead of $S$ and monotonically decrease or increase it to solve inequality (23), owing to that $β$ enters inequality (23) linearly. For a multiple-input system, we can solve inequality (23) by letting $S = βI$ and adjusting $β$ to obtain a feasible solution in the same way, or using a cone complementarity linearization algorithm proposed by Ghaoui, Oustry, and AitRami (1997).

It should be noted that the robust performance against model uncertainties with a bounded delay interval $h_{12}$ can be evaluated by using the following optimization procedure,

$$\min \rho \quad \text{s.t.} \quad (23)$$

where the robust performance level against model uncertainties is indicated by $δ = ρ^{-1}$.

5. Illustration

Consider the example studied by Zhang, Xu, and Zou (2008),

$$x(t + 1) = \begin{bmatrix} 1.0078 & 0.0301 \\ 0.5202 & 1.0078 \end{bmatrix} x(t) + \begin{bmatrix} -0.0001 \\ -0.0053 \end{bmatrix} u(t).$$

(28)

The state response of the system with no input ($u(t) = 0$) and initial condition $x_0 = (0.1, 0.1)^T$ is shown in Figure 1, indicating that the system is unstable. For illustration, here we assume the system output $y(t) = [10 \quad 1] x(t)$, which is used for feedback control. The corresponding full-order controller parameters can be obtained by solving the stability condition in inequality (23) with a fixed tuning parameter $β = 500$ for the system without model uncertainties,

$$A_c = 10^3 \begin{bmatrix} -0.0007 & 0.0000 & 0.0000 \\ -4.3312 & 0.0020 & 0.0002 \\ -0.0245 & 0.0000 & -0.0000 \end{bmatrix},$$

$$B_c = 10^3 \begin{bmatrix} -0.0006 \\ -1.5446 \\ -0.0087 \end{bmatrix},$$

$$C_c = [33.3430 \quad -0.0155 \quad -0.0014], \quad D_c = 11.8905.$$

The following state feedback control law was given in Zhang et al. (2008),

$$u(t) = \begin{bmatrix} 110.6827 & 34.6980 \end{bmatrix} x(t) - d(t),$$

(29)

which could stabilize the system in Equation (28), where $d(t)$ is a time-varying delay and satisfies $1 \leq d(t) \leq 4$. The simulation results for the system without model uncertainties are shown in Figures 2 and 3.

It is seen from Figure 2 that the state response of the closed-loop system by the proposed method recovers to zero obviously faster than that of Zhang et al. (2008). Figure 3 shows that an apparently smaller amount of control effort is needed by the proposed method in comparison with that of Zhang et al. (2008).
Then, assume that besides the delay variation, there exist model uncertainties with $F = (0.1, 0)^T$, $E_A = (0.1, 0.2)$, $E_B = 0.01$ as assumed by Gonzalez (2013). A robust state feedback control law can be obtained as $K = (204.5466, 52.5988, 0.0046)$ by solving the LMI conditions in the cited reference, which allows for a model uncertainty bound of $\delta = 0.454$.

Using the proposed method yields the following full-order output controller matrices by fixing $\beta = 450$ in solving the stability condition (23),

$$
A_c = 10^3 \times \begin{bmatrix}
-0.0026 & 5.1686 & 0.0013 \\
-0.0000 & 0.0034 & 0.0000 \\
-0.0000 & 0.0000 & -0.0000
\end{bmatrix},
$$

$$
B_c = 10^3 \times \begin{bmatrix}
-5.4921 \\
-0.0027 \\
-0.0000
\end{bmatrix},
$$

$$
C_c = \begin{bmatrix} 0.0123 & -24.2497 & -0.0062 \end{bmatrix}, \quad D_c = 25.7674.
$$

which allows for the maximum model uncertainty bound of $\delta = 0.3735$.

With $\Delta(t) = 0.5$ and the initial state condition of $x_0 = (0.1, 0.1)^T$, the resulting state response of the closed-loop system is shown in Figure 4, and the control signal is plotted in Figure 5. It is seen that the proposed method holds the control system robust stability well against model uncertainties and, in contrast, the closed-loop system becomes unstable by using the state feedback control method given by Zhang et al. (2008). Note that the proposed output feedback control method gives a similar control performance with that of Gonzalez (2013) which was based on using state feedback. Moreover, it can be verified that the maximum input delay variation of $1 \leq d(t) \leq 9$ is allowed by the proposed method as well as that of Gonzalez (2013), based on only using the output measurement.

### 6. Conclusion

Robust stabilization of discrete-time systems with time-varying input delay and model uncertainties has been addressed in this paper. Correspondingly, an output feedback control method has been proposed in contrast with a recently developed method (Gonzalez, 2013) depending on state measurement. By transforming such a process model into a delay-free one via introducing an augmented state, the scaled-bounded real lemma is adopted to establish sufficient stabilization conditions in terms of matrix inequality, which can be simply solved by configuring a tuning parameter. An illustrative example from the literature has been used to show the effectiveness and superior performance of the proposed output feedback control method.

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