Bound entanglement maximally violating Bell inequalities: quantum entanglement is not equivalent to quantum security

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It is shown that Smolin four-qubit bound entangled states [Phys. Rev. A, 63 032306 (2001)] can maximally violate two-setting Bell inequality similar to standard CHSH inequality. Surprisingly this entanglement does not allow for secure key distillation, so neither entanglement nor violation of Bell inequalities implies quantum security. It is also pointed out how that kind of bound entanglement can be useful in reducing communication complexity.

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Quantum entanglement is one of the most intriguing phenomenon within quantum physics. The pure state of composite quantum system is called to be entangled if it is impossible to describe its subsystems by pure states. Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} and Schrödinger\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. The so-called EPR paradox has started long debate whether local realistic theories can simulate quantum mechanics. The well-known Bell theorem\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} says that such a simulation is, in general, impossible. On the other hand, entanglement has been found as an important resource in quantum information theory\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} being an essential ingredient of coding, entanglement based cryptographic scheme\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}, quantum dense coding\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}, quantum teleportation\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} and quantum computing\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}.

To overcome the problem of noisy entanglement\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} the idea of entanglement distillation has been invented\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} which is useful in quantum privacy amplification\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. While any entangled two-qubit (or qubit-qutrit) state can be distilled to a singlet form\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} this is not true in general and this fact reflects the existence of so called bound entanglement phenomenon\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. This is very weak type of entanglement which can not serve in dense coding or teleportation. However, in multipartite case (see\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}) can be useful for remote quantum information concentration\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} (bipartite) activation\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} and (multipartite) superactivation\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}, classical cryptographic key distillation\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} or nonadditivity of quantum channels with multiple receivers\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. Remarkably bipartite BE states have not been reported to violate any Bell inequalities so far (see\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}). For multipartite case the seminal result has been obtained by Dür\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} who showed that some multiqubit BE states violate two-settings Bell inequalities. The quite interesting question has arose: what is the minimal size of quantum system (in terms of subsystems) that admits bound entanglement to violate Bell inequalities?.

It is important question since violation of Bell inequalities by entangled states seems to imply that the entanglement is useful for some classical tasks. In particular, it has been shown that violation of wide class of Bell inequalities is equivalent to possibility of reduction of communication complexity by the corresponding states\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. There is even a conjecture following cryptography analysis (see\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}) that violation of Bell inequalities is an indicator of usefulness of entanglement in general\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}.

The minimal number of bound entangled qubits violating the inequalities in Dür scheme was $N = 8$. This limit has been pushed down by Kaszlikowski et al. to $N \geq 7$ with help of three apparatus settings per site and further by Sen et al.\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} to $N \geq 6$ with help of so-called functional Bell inequalities\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. The relation of the results to bipartite distillability has been analyzed in details in\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}.

In the present paper we show that there are BE states that violate maximally standard Bell inequalities, i.e., with two setting per site\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} for $N = 4$. The inequality is very similar to standard CHSH inequality for two qubits\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. Moreover, the robustness of the considered bound entanglement is comparable to that of entangled two-qubit Werner states. Note that maximal violation of Bell inequalities by mixed states has already been known\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. However, this is the first time when such a violation is reported for bound entangled states.

Surprisingly, despite maximal violation, of the same kind as for pure GHZ state, the considered states do not allow for secure key distillation as it was for bipartite BE states from Ref.\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. This is a striking feature: knowing about Ekert protocol for bipartite states\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} one might expect that violation of Bell inequalities is always indicator of secure key distillation. A fundamental implication of our result is the conclusion that being a precondition for quantum cryptography\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)} entanglement is not sufficient for the latter.

The BE states we shall show to violate Bell inequalities are Smolin states\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}\footnote{Historically importance of quantum entanglement has been recognized by Einstein, Podolsky and Rosen (EPR)}. They have a special property: if the four particles are far apart no entanglement between any subsystems can be distilled. If however two particles are in the some location - it is possible to create maximal
entanglement between remaining two particles by means of LOCC operations. Detailed analysis has shown further interesting aspects: the entanglement cost of such states corresponds just to a singlet state \( \rho = |\psi_1\rangle \langle |\psi_1| |\psi_2\rangle \langle |\psi_2| \) that considered bound entanglement can serve to reduce communication complexity.

Noisy Smolin states .-- Let us consider the following four-qubit mixed state \( \varrho \) defined on space \( (\mathbb{C}^2)^{\otimes 4} \):

\[
\varrho_{ABCD}(p) = \varrho^S(p) = (1 - p) \frac{I^{\otimes 4}}{16} + p \rho^S, \tag{1}
\]

where \( I \) stands for identity on one-qubit space, while \( \rho^S \) is the four-qubit bound entangled state introduced by Smolin [25] and is defined through the relationship

\[
\rho^S_{ABCD} = \rho^S = \frac{1}{4} \sum_{i=1}^{4} |\psi_i\rangle \langle |\psi_i| |\psi_i\rangle \langle |\psi_i|, \tag{2}
\]

where two-particle states \( |\psi_i\rangle = \frac{1}{\sqrt{2}} (|00\rangle \pm |11\rangle) \), \( |\psi_3(4)\rangle = \frac{1}{\sqrt{2}} (|01\rangle \pm |10\rangle) \) form the so-called Bell basis. It is worth noticing that the states \( |\psi_i\rangle \) are fully permutationally invariant, since they can be written in the form

\[
\varrho^S(p) = \frac{1}{16} (I \otimes I \otimes I \otimes I + p \sum_{i=1}^{3} \sigma_i \otimes \sigma_i \otimes \sigma_i \otimes \sigma_i), \tag{3}
\]

with \( \sigma_i \) denoting the standard Pauli matrices. One can see that for \( p = 1/3 \) the state \( \varrho^S(p) \) is separable. Indeed, for such value of \( p \) we can rewrite \( \varrho^S \) as

\[
\varrho^S(1/3) = \frac{1}{6} \sum_{i=1}^{3} \sum_{s=+} \varrho_i^{(s)} \otimes \varrho_i^{(s)}, \tag{4}
\]

where \( \varrho_i^{(\pm)} \) stand for two-qubit separable mixed states (see [37]):

\[
\varrho_i^{(\pm)} = \frac{1}{2} (P_k^{(\pm)} \otimes P_k^{(\pm)} + P_k^{(-)} \otimes P_k^{(\mp)}), \quad k = 1, 2, 3, \tag{5}
\]

\( P_{k}^{(\pm)} \) denotes projectors corresponding to the eigenvectors of \( \sigma_k \) with eigenvalues \( \pm 1 \). Since by LOCC we can add some noise to the state, the above fact implies that for all \( p \leq 1/3 \) the state \( \varrho \) is separable. On the other hand for \( p > 1/3 \) the state is bound entangled. Indeed, it is easy to see that for this region the state violates PPT separability criterion [35] if we transpose indices corresponding to single qubit i.e. against any of the cuts: \( A|BCD \), \( B|ACD \) etc. Thus the state is entangled. It is bound entangled since the state maintains the property of original \( \rho^S \): it is separable against bipartite symmetric cuts like \( AB|CD \), \( BC|AD \), etc., which makes sure that no maximal entanglement between any subsystems can be distilled.

Below we shall prove that for \( p > 1/\sqrt{2} \) the state violates Bell inequalities introduced in Refs. [20, 23].

Violation of Bell inequalities ..-

In the corresponding scenario each of the \( N \) parties corresponding to index \( j \) \( (j = 1, 2, ..., N) \) can choose between two dichotomic observables \( (O_{kj}) \), \( k_j = 1, 2 \). The set of \( 2^4 \), \( N \) Bell inequalities is [20, 21]:

\[
\left| \sum_{s_1, ..., s_N} S(s_1, ..., s_N) \sum_{k_1, ..., k_N} s_{k_1}^{1,-} s_{k_N}^{N,-} E(k_1, ..., k_N) \right| \leq 2^N, \tag{6}
\]

where correlation function \( E \) is defined through relation (average over many runs of experiment)

\[
E(k_1, ..., k_N) = \langle \prod_{j=1}^{N} O_{kj}^{(j)} \rangle_{\text{avg}}. \tag{7}
\]

Trying to predict the above (experimental) average local hidden variables (LHV) theories offer its calculation as an integral over probabilistic measure on space of hidden parameters. The measure correspond to classical states. In quantum mechanical regime the observables depend on vector parameters

\[
O_{kj}^{(j)} = \hat{n}_{k_j} \sigma_j \tag{8}
\]

and the corresponding average for given quantum state \( \varrho \) is calculated as follows:

\[
E_{QM}(k_1, ..., k_N)(\varrho) = \text{Tr} \left[ \varrho O_{k_1}^{(1)} \otimes ... \otimes O_{k_N}^{(N)} \right]. \tag{9}
\]

One can see that for the following non-trivial sign function:

\[
S(+, +, -,-) = S(+, -,+,-) = S(-,-,+,+), \quad S(-,-,-,+) = -1, \tag{10}
\]

where \( \pm \) stands for \( \pm 1 \) (for other cases sign function equals to unity), and for \( N = 4 \) we can derive the following Bell-inequality:

\[
|E(1,1,1,1) + E(1,1,1,2) + E(2,2,2,1) - E(2,2,2,2)| \leq 2. \tag{11}
\]

This inequality can be also derived very easily using the same technique as in standard CHSH inequality [31]. We keep the above derivation for purposes of further analysis.

Subsequently, for the state given by [22] we choose the following observables:

\[
\hat{n}_i^{(1)} = \hat{x}, \quad \hat{n}_i^{(2)} = \hat{y}, \quad i = 1, 2, 3
\]

\[
\hat{n}_1^{(4)} = \hat{x} + \hat{y}, \quad \hat{n}_2^{(4)} = \hat{x} - \hat{y}. \tag{12}
\]

In virtue of the above equations, one gets the violation of the Bell inequality [11]:

\[
E_{QM}(1,1,1,1)(\varrho^S) + E_{QM}(1,1,1,2)(\varrho^S) + E_{QM}(2,2,2,1)(\varrho^S) - E_{QM}(2,2,2,2)(\varrho^S) = 2\sqrt{2}p. \tag{13}
\]
where $S'[g] = g/|g| = \pm 1$ is the sign function of $g$ and $f \in \{-1, 1\}$. The success is achieved when all parties get the correct value of $f$. Their joint task is to maximize the probability of success.

Following the broad class of quantum protocols

- After receiving $x_1 = 1$ ($x_i = 2$), the $i$-th party set its apparatus to measurement of a dichotomic observable $O^{(i)}_1$ ($O^{(i)}_2$). Values obtained in such measurements are elements of the set $\{-1, 1\}$ and will be denoted by $a_i$. Each party distribute one bit of classical information $e_i = a_i \cdot y_i$.

- Finally, all parties compute the product $y_1 \cdot \ldots \cdot y_N \cdot a_1 \cdot \ldots \cdot a_N$ and put it as a value of function $f$ it is proven in [24] that probability of success in quantum case is higher than in classical one iff for given entangled state one of the following Bell inequality for correlation function

$$
\sum_{x_1, \ldots, x_N = 1}^2 g(x_1, \ldots, x_N) E(x_1, \ldots, x_N) \leq B(N)
$$

is violated. In particular, the above class contains Bell inequalities given by [15] with $B(N) = 2^N$ and

$$
g(x_1, \ldots, x_N) = \sum_{s_1, \ldots, s_N}^{\pm 1} s_1 s_{x_1-1} \ldots s_{x_N-1}.
$$

It is worth noticing that the probability $P$ of success in case of classical protocols is estimated as follows (for proof see [24])

$$
P \leq \frac{1}{2} \left( 1 + \frac{B(N)}{\sum |g(x_1, \ldots, x_N)|} \right).
$$

On the other hand the above inequalities are equivalent to Bell inequalities [13], i.e., are violated iff the inequalities [15] are violated. Therefore, one can see that violation of Bell inequality implies violation of respective inequality [20] and can result in higher probability of success in case of quantum entanglement.

To show that Smolin state is useful to reduce communication complexity it is sufficient to consider the function $g$ given by

$$
g(x_1, \ldots, x_N) = 4\sqrt{2} \cos{(2(x_1 - x_2)\pi)} \sin{(2(1-x_4)} - x_3)\pi
$$

and to put $B(4) = 16$. Note that function [21] is equivalent to the function obtained after substitution of (10) to (14) for $x_1 \in \{1, 2\}$.

By virtue of [20], we infer that maximal probability achievable in any classical protocol is $P^C_{\text{max}} = 3/4$, whereas in quantum protocol is $P^Q_{\text{max}} = \frac{1}{2}(1 + \frac{1}{\sqrt{2}})$.  

which gives violation for any $p \in (\frac{1}{2}, 1]$. Below we shall show that for $p = 1$, i.e., for original Smolin states $g^{\text{S}}(p) = \frac{1}{2}$ the above violation is maximal, i.e., there is no other quantum state that can make RSH of the above equation greater than $2\sqrt{2}$. To this aim we use the method [10] applied for CHSH inequality, namely, we shall calculate the second power of the Bell operator:

$$
B = O^{(1)}_1 \otimes O^{(2)}_1 \otimes O^{(3)}_1 \otimes O^{(4)}_1
+nO^{(1)}_1 \otimes O^{(2)}_2 \otimes O^{(3)}_1 \otimes O^{(4)}_2
+O^{(1)}_2 \otimes O^{(2)}_1 \otimes O^{(3)}_2 \otimes O^{(4)}_1
-nO^{(1)}_2 \otimes O^{(2)}_2 \otimes O^{(3)}_2 \otimes O^{(4)}_2.
$$

(14)

Since the observables are dichotomic (i.e., one has $(O^{(i)}_k)^2 = I$) we get that

$$
B^2 = 4I \otimes I \otimes I \otimes I + (O^{(1)}_1 O^{(2)}_2 \otimes O^{(3)}_1 O^{(4)}_2)
-O^{(1)}_2 (O^{(2)}_1 \otimes O^{(3)}_2 \otimes O^{(4)}_1) \otimes (O^{(4)}_1 O^{(4)}_2),
$$

(15)

where the last term stands for commutator. Again, by virtue of dichotomic character of the observables involved, this can be written as $B^2 = 4I + (X - X') \otimes (Y - Y')$ with all $X, X', Y, Y'$ having operator norm not greater than one. This gives the estimate $|B^2| \leq 4 + (||X|| + ||X'||)(||Y|| + ||Y'||) \leq 8$ and thus the spectrum of $B$ lies within the interval $[-2\sqrt{2}, 2\sqrt{2}]$. This immediately implies that RSH of (13) achieves its maximum in $2\sqrt{2}$.

Remarkable that both separability and violation of Bell inequalities [11] is in the same regime as in two-qubit Werner states [9] if we write them in the form $g^{W}(p) = (1-p)\frac{1}{4}I \otimes I + p|\Phi_{-}\rangle\langle \Phi_{-}|$ and consider CHSH inequality.

Indeed, $g^{W}(p)$ is known to (i) be entangled for $p \in (\frac{1}{2}, 1]$ [13, 38] and (ii) to violate Bell–CHSH inequality for $p \in [\frac{1}{2}, 1]$ [39].

Communication complexity . In this section we analyze the Smolin state in context of communication complexity problems. To aim this let us consider the general class of problems, proposed by Brukner et al. [24]:

- The $i$-th party receives a two-bit input string $(x_i, y_i)$ ($x_i \in \{1, 2\}$, $y_i \in \{-1, 1\}$). Values of $y_i$ are chosen randomly, whereas values of $x_i$ are chosen according to probability distribution $Q(x_1, \ldots, x_N)$ for which one can find real–valued function $g(x_1, \ldots, x_N)$ such that

$$
Q(x_1, \ldots, x_i) = \frac{g(x_1, \ldots, x_N)}{\sum_{x_1, \ldots, x_N = 1}^2 |g(x_1, \ldots, x_N)|},
$$

(16)

- Each party is allowed to distribute one classical bit of information.

- The main goal of each party is to find correct value of the function

$$
f = y_1 \cdot \ldots \cdot y_N S[g(x_1, \ldots, x_N)],
$$

(17)
Discussion and conclusions. - Bound entanglement is quite unique type of entanglement that does not possess property of distillability. Nevertheless it can be useful to perform some quantum tasks. It is located in a sense in between usual free „strong”, entanglement and separability. As such it represents the region of quantumness where natural limits of classical theories can be tested. One of the fundamental questions are limits of local hidden variables theories - so far it has been known that they are excluded for BE states with number of qubits not less that six. We show that violation of hidden variable model test is possible by four–qubit system. In particular four–qubit Smolin states can violate CHSH-like Bell inequalities maximally. This is the first time when such violation is proved for BE states. Moreover, the result pushes down the number of qubits needed for BE to violate Bell inequalities: from 6 to 4. If we consider the family of two settings inequalities (which are most easy to implement) Bell inequalities the present offers significant progress from $N = 8$ to $N = 4$.

Even more striking are implications of the present result as far as quantum security is concerned: for the first time we have example showing that neither Bell inequalities violation nor entanglement itself implies quantum security. Indeed, the states are separable under any symmetric bipartite cut, which means (following analysis of Ref. 23) that no secure correlations can be distilled between any two groups of two people in the scheme. By full permutational symmetry of the state this means that no secure key between four people can be distilled. Moreover, from the result of the present paper it follows that possibilities of performing two important quantum information tasks are not equivalent. Indeed, possibility of generation of quantum secure key is not equivalent to possibility of reducing of communication complexity. The additional surprise is that at the same time the states violate the considered Bell inequality in the same way as (maximally entangled) pure GHZ state.

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