Nucleon - Nucleon Interaction with Confined One Gluon
Exchange Potential and One Pion Exchange Potential

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Abstract

Confined One Gluon Exchange Potential (COGEP) along with the One Pion Exchange Potential (OPEP) has been used to obtain the singlet (1S0) and triplet (3S1) state Nucleon-Nucleon (N-N) Interaction potentials in the framework of a Relativistic Harmonic Model (RHM) using Resonating Group Method (RGM) in the Born Oppenheimer approximation. The full Hamiltonian used in the study consists of the kinetic energy, two body confinement potential, COGEP and OPEP. The contributions of COGEP and OPEP to the adiabatic potential of the N-N interaction has been discussed.

1 Introduction

Nucleon-Nucleon (N-N) Interaction has always remained a formidable and challenging problem in nuclear physics. With the advent of Yukawa’s meson exchange theory, N-N interaction is explained by the exchange of various mesons [1,2]. It is necessary to include the exchange of σ and π mesons to understand the bulk of N-N attraction. With the acceptance of Quantum Chromodynamics (QCD) as the theory of strong interaction, attempts have been made to explain N-N interaction using QCD based models. Based on such models, N-N interaction is completely determined by the underlying quark-gluon dynamics.

There are various relativistic and non relativistic quark models which try to explain the N-N interaction [3–14]. The existence of a short range repulsion between the nucleons which is crucial in understanding the stability of nuclei brings a difficulty to such models. The study of N-N interaction has implications in nuclear physics, particle physics and astrophysics. Even though there are theoretical and experimental explanations in this regard explaining the same, there is no proper understanding of short range repulsion as such. In a work by Vinh Mau et al., it is seen that the behaviour of N-N interaction at short distances is dependent on additional terms added for the intermediate and long range forces [15]. With the experimental data provided by the J-PARC [16], PANDA [17], NICA [18], and HIAF [19] projects and also the work by Naghdi [20], there is good progress in this regime.

In conventional quark models, the Hamiltonian consists of kinetic energy term, one gluon exchange potential (OGEP) term and confinement potential. In all these models, the exchange part of the color magnetic interaction is considered responsible for the short range repulsion [21,22]. The OGP is obtained from the QCD Lagrangian in the non relativistic limit by retaining terms to the order of $\frac{1}{c^2}$. The gluon propagators used to derive the OGP are similar to the free photon propagators in QED, which is used to obtain the Fermi-Breit interaction. Since confinement of color implies confinement of quarks and gluons, to determine the nature of N-N interaction, a decisive role should be played by the confined dynamics of the gluons. For the confinement of quarks we make use of the relativistic harmonic model (RHM) and for the confinement of gluons we make use of the current confinement model (CCM) [21]. The confined one gluon exchange potential (COGEP) used in the present work is derived in the CCM using confined gluon propagators (CGP) [21,22].

To obtain the partially conserved axial current, the OPEP must be included as it is important to know the contribution of OPEP to the N-N potential [23]. Pion is treated as an elementary field in non relativistic quark models (NRQM) which couples to the quarks with a strength which reproduces the experimental π-N coupling strength at zero momentum transfer [3]. It is seen that the OPEP provides state independent repulsion to the N-N interaction [3,24]. To take care of the effect of inner

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structure of pions at short range, a suitable form factor with a cutoff mass $\Lambda$ has been used in a different approach \[25\]. The long and medium range of the N-N potential were introduced phenomenologically in the earlier versions of quark cluster models \[26–28\]. Simple $q\bar{q}$ pair exchange potential dominates the long and intermediate range parts. The $q\bar{q}$ pair is a color singlet structure with pseudoscalar or vector meson quantum numbers. The obtained potentials have characteristics of one boson exchange potential (OBEP). Fujiwara and Hecht have incorporated the $q\bar{q}$ excitations in the non relativistic model of N-N interaction to include the mesonic degrees of freedom \[29\]. The coupling constant $f_{NN\pi}$ calculated in the work is one third of the observed $f_{NN\pi}$ value \[29\]. It is not possible to obtain simultaneously proper values for $f_\pi$ and the charge radius $\langle r^2\rangle$. If pion is treated as a single $q\bar{q}$ pair, $\langle r^2\rangle$ turns out to be very large to match $f_\pi$ \[30\]. Hence just the inclusion of a $q\bar{q}$ pair is not enough to explain the long range behaviour of N-N interaction. Inclusion of $(3q)(q\bar{q})^2$ components will lead to potentials that have characteristics of exchange of $\sigma$ mesons and gives an additional medium range attraction that binds the deuteron \[31\].

Scalar mesons have been difficult since scalar resonances are difficult to resolve because some of them have large decay widths which cause a strong overlap between resonances and background. One expects non $q\bar{q}$ scalar objects, like glueballs and multiquark states in the mass range below 2 GeV. A possible candidate for the $\sigma$ meson that has been listed by the PDG is the scalar meson $f_0(500)$ with $I^G(0^+)$ and $J^{PC}(0^{++})$. Details of scalar mesons below 2 GeV is given in the PDG \[32\]. $q\bar{q}$ excitations also lead to an attraction in the region between 0.8 fm to 1.5 fm but this attraction is too weak to bind the Deuteron and reproduce the low energy S wave scattering parameters \[29\].

The current work aims to make a detailed study of the contribution of COGEP and OPEP to the $1S_0$ and $3S_1$ of the adiabatic N-N potential in the framework of RHM. The adiabatic N-N potentials have been computed using the Born-Oppenheimer approximation.

The paper is organised as follows: in section 2.1, we review the RHM, in section 2.2, the resonating group method (RGM) is briefly described. The results and discussions are given in section 3 and the conclusions are given in section 4.

# 2 The Model

## 2.1 Relativistic Harmonic Model

In the RHM \[33\] the quarks in a Nucleon are considered to be confined by a Lorentz scalar plus vector harmonic oscillator potential,

$$\frac{1}{2}(1 + \gamma_0)\alpha^2 r^2 + M$$

where $\gamma_0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is the Dirac’s matrix, $M$ is a constant mass and $\alpha^2$ is the confinement strength parameter. In RHM the quark wave function $\psi$ is given by,

$$\psi = N \begin{bmatrix} \phi \\ \sigma_p \phi \end{bmatrix}$$

where

$$N = \sqrt{\frac{2(E + M)}{3E + M}}$$

Here $E$ is an eigen value of the single particle Dirac equation that is used for quarks in the RHM. The lower component of $\psi$ is eliminated to obtain a harmonic oscillator wave equation in $\phi$

$$(\frac{p^2}{E + M} + \alpha^2 r^2)\phi = (E - M)\phi$$

The full Hamiltonian used in this work is

$$H = K + V_{int} + V_{conf} - K_{CM}$$

where $K$ is the Kinetic energy, $V_{int}$ is the interaction potential, $V_{conf}$ is the confinement potential and $K_{CM}$ is the kinetic energy of the centre of mass.

$$K = \sum_{i=1}^{6} \frac{p_i^2}{E + M}$$
\[ K_{CM} = \frac{P^2}{6(E + M)} \]

where \( \frac{(E+M)^2}{2} \) is the dynamic effective mass of the quarks, \( p_i \) is momentum of the \( i^{th} \) quark and \( P \) is the momentum of the centre of mass. The confinement potential is given by,

\[ V_{conf} = -\sum_{i<j} \alpha^2 r_{ij}^2 \lambda_i \lambda_j \]

where \( r_{ij} \) is the distance between the \( i^{th} \) and \( j^{th} \) quarks, \( \alpha^2 \) is the confinement strength and \( \lambda_i \) and \( \lambda_j \) are the generators of the color \( SU(3) \) group for the \( i^{th} \) and the \( j^{th} \) quarks. The interaction potential is given by,

\[ V_{int} = V_{COGEP} + V_{OPEP} \]

where

\[ V_{COGEP} = \frac{\alpha_s}{4} N^4 D_0(r) + \frac{1}{(E + M)^2} (4\pi \delta^3(r) - c^4 r^2 D_0(r))(1 - \frac{2}{3} \sigma_i \sigma_j) \lambda_i \lambda_j \]

In the above equation, \( \sigma_i \) and \( \sigma_j \) are the Pauli spin operators of the \( i^{th} \) and the \( j^{th} \) quarks and \( \alpha_s \) is the strong coupling constant.

The OPEP [3] is given by

\[ V_{OPEP} = \frac{f^2}{3} \sum_{i<j} e^{-mr_{ij}} \sigma_i \sigma_j \tau_i \tau_j \]

where \( \tau_i \) and \( \tau_j \) are the isospins of the \( i^{th} \) and the \( j^{th} \) quarks. \( f_q \) is the OPEP strength parameter and is related to the pion-nucleon coupling constant by the relation \( f_q^2 = \frac{f_{\pi NN}^2}{4\pi} \) [23].

2.2 Resonating Group Method

The N-N interaction exists only when there is exchange of quarks between the nucleons. If the quarks are not exchanged between the nucleons, there can be no N-N interaction arising from the quark-quark (qq) interaction. This can be seen from the fact that the matrix element of \( \lambda_i \lambda_j \) vanishes when \( i^{th} \) quark is in one nucleon and \( j^{th} \) is in the other nucleon in accordance with the Wigner-Eckart theorem. Hence the only way N-N interaction can arise from a qq interaction is by constructing a totally antisymmetric wave function for the six quark system in which case the exchange terms arising solely out of antisymmetrization. The RGM employs a totally antisymmetric wave function and treats the motion of the centre of mass correctly. Using the RGM technique we solve the equation [4],

\[ \langle \psi | (H - E) A | \psi \rangle = 0 \]

to get the energy \( (E) \) of the interacting nucleons. Here, \( H \) is the Hamiltonian, \( \psi \) is the wave function of the nucleons and \( A \) is the anti-symmetrization operator given by,

\[ A = \frac{1}{10} (1 - 9 P_{36}^{O^{STC}}) \]

where \( P_{36}^{O^{STC}} \) is the permutation operator for the quarks 3 and 6 and \( O^{STC} \) stands for orbital, spin, isospin and color. Thus \( P_{36}^{O^{STC}} \) operator exchanges the orbital, spin, isospin and color quantum numbers of the quarks 3 and 6.

The anti-symmetrization operator splits each term in the Hamiltonian into two parts: direct part and the exchange part. The direct part corresponds to the interaction without exchange of quarks and the exchange part corresponds to the interaction with exchange of one quark between the nucleons. At asymptotic distances, the exchange part of the interaction vanishes since the overlap of wave functions is absent.

The harmonic oscillator wave function used here is,

\[ \phi(r_i) = \frac{1}{(\pi b^2)^{3/4}} \exp\left(-\frac{1}{2b^2}(r_i - s_I)^2\right) \]

where \( b \) is the oscillator size parameter and \( s_I \) is the generator coordinate.
Using equation (8), the following kernels are computed: (a) the normalization kernel,

\[ \langle \psi | A | \psi \rangle \]  

(b) the kinetic energy kernel

\[ \langle \psi | K A | \psi \rangle \]  

and (c) the potential energy kernel.

\[ \langle \psi | (V_{\text{int}} + V_{\text{conf}}) A | \psi \rangle \]  

The energy is then given by,

\[ E = \frac{\langle \psi | H A | \psi \rangle}{\langle \psi | A | \psi \rangle} \]  

where the subscript \( l \) indicates that the quantities have been projected to the angular momentum value \( l \).

Here, each nucleon is considered as a cluster of three quarks and the two nucleon system is considered as cluster \( A \) and cluster \( B \). In RGM, the total wave function is expressed as an anti-symmetric product of the single particle wave functions. The total wave function of the six quark system is,

\[ \psi_{\text{TOT}}(\xi_A, \xi_B, R_{AB}) = A[\phi_A(\xi_A)\phi_B(\xi_B)\chi(R_{AB})] \]  

where \( \phi_A \) and \( \phi_B \) are the internal wave functions of the individual clusters \( A \) and \( B \) respectively and \( \chi \) is the relative wave function between the two clusters and \( A \) is the total anti-symmetric operator of the six quark system. To separate the total wave function given in equation (11), the following choice of coordinate is made,

\[ \xi_1 = r_1 - r_2, \xi_2 = r_3 - \frac{r_1 + r_2}{2}, R_A = \frac{1}{3}(r_1 + r_2 + r_3), \]

\[ \xi_3 = r_4 - r_5, \xi_4 = r_6 - \frac{r_4 + r_5}{2}, R_B = \frac{1}{3}(r_4 + r_5 + r_6), \]

\[ R_{AB} = R_A - R_B, R_G = \frac{1}{2}(R_A + R_B). \]

Here \( r_i \) is the coordinate of the \( i^{th} \) quark, the coordinates \( \xi_A = (\xi_1, \xi_2) \) and \( \xi_A = (\xi_3, \xi_4) \) are the internal coordinates of the two clusters \( A \) and \( B \) respectively, \( R_{AB} \) is the relative coordinate between the two clusters and \( R_G \) is the centre of mass coordinate of the total system.

Since the Hamiltonian is translationally invariant, \( L_{ij}^I \) can be written as,

\[ L_{ij}^I = \int [\phi_A^{+SM}(r_1, r_2, r_3; s_{ij} \frac{s_I}{2})\phi_B^{+SM}(r_4, r_5, r_6; -s_{ij} \frac{s_I}{2})Y_{lm}^*(s_I)][H - E]A \]

\[ \times \left[ \phi_A^{S}(r_1, r_2, r_3; s_{ij} \frac{s_I}{2})\phi_B^{S}(r_4, r_5, r_6; -s_{ij} \frac{s_I}{2})Y_{lm}(s_J) \right] \prod_{k=1}^{6}d^3r_kd^3\hat{s}_1d^3\hat{s}_J \]

To take into account all possible interactions between the quarks, we have to consider seven different types of operators for the potential \( V_{ij} \) in the Hamiltonian. They are \( V_{12DR}, V_{36DR}, V_{12EX}, V_{13EX}, V_{16EX}, V_{14EX} \) and \( V_{36EX} \), where \( DR \) stands for direct part of the quark interaction between quarks \( i \) and \( j \) and \( EX \) stands for the corresponding exchange part.

### 3 Results and Discussions

To analyse the contributions of the various components of the Hamiltonian, we have plotted the diagonal elements of the various kernels of the singlet and triplet N-N potentials as a function of the relative distance between the nucleons \( (s_I) \). We subtract the total energy in the asymptotic limit from the total

| \( M \) (MeV) | \( \alpha^2 \) (MeV \( f m^2 \)) | \( b \) (fm) | \( \alpha_s \) | \( c \) (fm\(^{-1}\)) | \( m_\pi \) (MeV) | \( f_\pi^2 \) |
|---|---|---|---|---|---|---|
| 160.6 | 200.01 | 0.6 | 6.5 | 1.7324 | 140.0 | 12.6 |

Table 1: List of parameters.
The adiabatic potential is calculated using the Born-Oppenheimer approximation given by,

\[ V_{12}^{Ad} = \langle \psi_1(s_f) | H | \psi_2(s_f) \rangle - \langle \psi_1(\infty) | H | \psi_2(\infty) \rangle \]

There are seven parameters in our model - the masses of the quarks \( M \), the confinement strength \( (\alpha_c^2) \), the harmonic oscillator size parameter \( b \), the mass of the pion \( m_{\pi} \) and the quark-pion coupling constant \( f_{q\pi}^2 \). The coupling constant \( \alpha_c \) is fixed by the N-\( \Delta \) mass splitting which comes from the color magnetic term of COGEP and \( f_{q\pi}^2 \) is fixed from the Goldberger-Treiman relation [23]. We have chosen the value of oscillator size parameter to be 0.6 fm which is consistent with the experimental results of the charge distribution of the nucleons and the axial charge distribution [3]. The parameters used are listed in Table 1.

Fig. 1 gives the plot of adiabatic N-N potential without OPEP and with OPEP. The intermediate range attraction for \( ^1S_0 \) state is reduced MeV in the presence of OPEP. The short range repulsion for \( ^1S_0 \) state is larger than that of \( ^3S_1 \) state. The color magnetic part of the COGEP can be considered as the reason for this difference.

At short distances, the exchange kernels of \( \delta^3(r) \) is dominant over the exchange kernels of \( c^4r^2D_0(r) \) and hence provide the short range repulsion. The short range repulsion is larger for \( ^1S_0 \) state than the \( ^3S_1 \) state. In the intermediate and long ranges, the exchange kernels of \( c^4r^2D_0(r) \) dominates over the exchange kernels of \( \delta^3(r) \) and hence provide the intermediate and long range attraction. The intermediate range attraction is larger for the \( ^3S_1 \) state than the \( ^1S_0 \).

Fig. 2 is a plot of the direct and exchange parts of the Hamiltonian and the exchange part of the color magnetic interaction in the adiabatic limit. The exchange part of the potentials of \( ^1S_0 \) and \( ^3S_1 \) states show repulsion in the short range. The \( ^1S_0 \) state exchange potential is completely repulsive and that of the \( ^3S_1 \) state shows a small attraction in the intermediate range. There is consistency with established results in the case of repulsive contribution to the adiabatic potential at short range, both to \( ^1S_0 \) and \( ^3S_1 \) states [6][11][14]. There is no contribution from the color electric term to the N-N interaction. Since the energy difference between \( 2(0s)^3 \) and \( (0s)^6 \) configuration must come from the expectation value of \( \lambda_i, \lambda_j \), the radial matrix elements are same for \( 2(0s)^3 \) and \( (0s)^6 \) configuration. The expectation value of \( \lambda_i, \lambda_j \), \( \sigma_i, \sigma_j \) in the color magnetic part of the COGEP for the \( 2(0s)^3 \) and \( (0s)^6 \) configuration does not vanish and thus, the color magnetic part provides short range repulsion [3].

4 Conclusions

In this work, we have investigated the role played by the COGEP and OPEP on \( ^1S_0 \) and \( ^3S_1 \) N-N adiabatic potentials using the Born-Oppenheimer approximation without evoking \( \sigma \) meson in our Hamiltonian. The COGEP with OPEP interactions explain the \( ^1S_0 \) and \( ^3S_1 \) adiabatic potentials.

The OPEP provides the state independent repulsion. The calculation clearly demonstrates the importance of the confinement of gluons on \( ^1S_0 \) and \( ^3S_1 \) state N-N potentials.
Figure 2: Direct and exchange components of NN potential (left) and color magnetic exchange part of COGEP (right).

To conclude, we have obtained \(^1S_0\) and \(^3S_1\) N-N adiabatic potentials starting from the dynamics of quarks and gluons including confinement of color and OPEP as a phenomenological input.

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