Broadband coherent perfect absorption by cavity coupled to three-level atoms in linear and nonlinear regimes

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Abstract
A broadband coherent perfect absorption (CPA) scheme consisting of an optical resonator coupled with three-level atoms excited by single cavity mode is proposed and analyzed. We show the output light field from the system is completely suppressed under specific conditions when the system is excited in linear and nonlinear regimes by two identical light fields from two ends of optical cavity. An analytical broadband CPA criterion for central and sideband excitations of cavity quantum electrodynamics system is derived in linear regime. Moreover, we show the resonant excitation criterion for CPA is greatly extended in nonlinear regime. A new type of bistability behavior is found. The output field intensity and the bistability curve can be well tuned by dynamically adjusting system parameters. Our results demonstrate that the CPA is quite universal, and it should be useful in a variety of applications in optical logic and optical communication devices.

1. Introduction
In recent years, the phenomenon of coherent perfect absorption (CPA) [1–18] has attracted tremendous interest. CPA phenomenon is generally designed by illuminating of an artificially fabricated medium with two or more coherent incoming waves. Then the radiation is completely absorbed by the medium and transferred to other form of internal energy. There is no output wave emitted from the absorber [19]. CPA-based devices are usually required to have feedback interaction for incoming waves. The physical mechanism of CPA is a combination of coherence interference and energy dissipation, which allows trapping of incoming wave energy within the absorbing media of special structures. So CPA in optical frequency range can also be regarded as a time-reversed process of lasing at threshold point [1]. The CPA mechanism was first proposed in year of 2010 by Chong et al [20]. From then on, considerable theories and experiments about CPA are conducted. Now CPA phenomena have been demonstrated in different materials, including graphene [21, 22], high-contrast grating [23, 24], waveguide [25], photonic crystals [26], plasmonic array [27], etc [28]. Meanwhile, for extensive applications in biosensing [29], molecular detection [30], photovoltaics [31], photodetection [32, 33], optical communications [34] and quantum informations, etc, CPA has been studied in a very broad wave range from acoustics [35], to infrared [18, 36–39] and optical frequencies [40–45]. Recently, CPA of nonlinear matter waves has been realized experimentally in an atomic Bose–Einstein condensate system [46]. This progress opens new avenue to manipulate and control interaction between quantum matter and light fields. All of the above imply the important development and application prospects of CPA.

For many practical applications in optics and optoelectronics, it is important to absorb a large amount of light in a relatively small volume structure of absorbing material. However, it is usually not easy to obtain the actively tunable broadband resonance absorption in traditional absorbers. That is to say, the resonance absorption frequency is fixed once the physical structure is designed. The absorption intensity and input
frequency cannot be dynamically tuned in a broad spectrum range, which has greatly limited its wide
applications in practice. On the other hand, when one wants to remanufacture the CPA-device, there are
lots of technical challenges to maintain the equivalent coupling rate and dissipative rate, which make
integrated CPAs difficult. Inspired by this, we propose to use a three-level atom ensemble excited by single
cavity mode in an optical resonator to fabricate a CPA device. Different from the two-level atom-cavity
system, in which the CPA is invalid when the input field frequency is near the atom resonance frequency
[47]. The proposed CPA can be realized in a broadband and continuous input frequency range. Besides,
under the collective strong coupling condition, the CPA excited by three-level atoms in a cavity depends on
the system parameters, which makes the CPA criterion more easily to be met. The proposed CPA with these
properties has great potential for practical applications in modulators [48], signal processors [49], quantum
information [50], optical communications [51], all optical switching [52, 53], all optical logic [54], etc.

The article is organized as follows. In section 2, we describe our theoretical model of a three-level atom
ensemble excited by single cavity mode in optical resonator. In section 3, we give theoretical analysis about
the CPA in linear regime. The analytical CPA criterion is given and discussed. In section 4, we give
theoretical analysis about the CPA in nonlinear regime. The CPA properties influenced by system
parameters are discussed in detail. Finally, section 5 gives a conclusion.

2. Theoretical model

Figure 1 shows the schematic diagram for a two-sided cavity quantum electrodynamics (CQED) system that
consists of N three-level atoms inside in a single mode cavity. The three-level atoms are cooled and trapped
by a magneto-optical trap (MOT) setup [55–59]. The CQED system is excited by two classical input light
fields \( a_1' \) and \( a_1 \) from two ends of the cavity. The cavity mode couples the atomic transitions \( |1\rangle \rightarrow |2\rangle \) and
\( |1\rangle \rightarrow |3\rangle \) simultaneously. \( \nu_c \) is the cavity frequency. The frequency detuning of cavity mode is defined as
\( \Delta_c = \nu_c - \nu_{31} \), \( \Delta_{31} \) is the frequency separation between atomic states \( |2\rangle \) and \( |3\rangle \). Two input fields have the
same frequency \( \nu \) and are detuned from the atomic transition by \( \Delta_p = \nu - \nu_{31} \).

Under the condition of rotating wave approximation by dropping the energy nonconserving terms
[60, 61], the Hamiltonian of the coupled cavity-atom system is:

\[
H = \hbar \sum_{i=1}^{3} \omega_i \sigma_i + \hbar \omega_c \hat{a}^\dagger \hat{a} + \hbar \left( \sum_{m=1}^{N} \sum_{i=2}^{3} \tilde{g}_i \sigma_i^{(m)} + H.c. \right). 
\]  (1)

Here \( \hat{a} \) and \( \hat{a}^\dagger \) are the annihilation and creation operators of the cavity photons. \( \tilde{g}_i = \mu_i \sqrt{\omega_c / 2 \hbar \epsilon_0 V} \) is
the cavity-atom coupling coefficient for transition path \( |i\rangle \) to \( |i\rangle \) and is assumed uniform for the \( N \) identical
atoms inside the cavity. \( \sigma_i \) is a Hermitian operator, \( \sigma_i^{(m)} \) and \( \sigma_i^{(m)*} \) \( (i = 2, 3) \) are raising and lowering
operators for the \( m \)th atom. The first and second items of equation (1) describe atom and cavity field
energies respectively. The third item denotes the coupled interaction energy between the cavity and atom
ensemble. The equations of motion for the CQED system are given by \( \frac{d\rho}{dt} = \frac{i}{\hbar}[H, \rho] + \hat{L} \rho \) [60, 61]. \( \hat{L} \) is a
quantum superoperator. Ignoring the quantum fluctuation terms \([9, 47, 62–64]\), the resulting equations of
motion for the expectation values of \( \sigma_i^{(m)} = \sigma_i (i, j = 1–3) \) and the intracavity light field \( a \) are:

\[
\dot{\sigma}_{11} = \gamma_2 \sigma_{22} + \gamma_3 \sigma_{33} + \left( i \tilde{g}_1 a \sigma_{21} + i \tilde{g}_1 a^\dagger \sigma_{31} + H.c. \right), 
\]  (2)

\[
\dot{\sigma}_{22} = -\gamma_2 \sigma_{22} - i (g_1 a \sigma_{21} - g_1^\dagger a^\dagger \sigma_{12}), 
\]  (3)
\[ \dot{\sigma}_{33} = -\gamma_3 \sigma_{33} - i(g_2 a \sigma_{31} - g_3 a^\dagger \sigma_{13}), \]
\[ \dot{\sigma}_{12} = -\left( \frac{\gamma_2}{2} + i(\Delta_p + \Delta_{23}) \right)\sigma_{12} + ig_1 a (\sigma_{22} - \sigma_{11}) + ig_2 a \sigma_{32}, \]
\[ \dot{\sigma}_{13} = -\left( \frac{\gamma_3}{2} + i\Delta_p \right)\sigma_{13} + ig_3 a^\dagger (\sigma_{33} - \sigma_{11}) + ig_1 a \sigma_{23}, \]
\[ \dot{\sigma}_{23} = \left( \frac{\gamma_3 + \gamma_3}{2} - i\Delta_{23} \right)\sigma_{23} + ig_3 a^\dagger \sigma_{13} - ig_2 a \sigma_{31}, \]
\[ \dot{a} = -\left( \frac{\kappa_1 + \kappa_2}{2} - i(\Delta_c - \Delta_p) \right) a + ig_2 N \sigma_{21} + ig_3 N \sigma_{31} + \frac{\sqrt{\kappa_1}}{\tau} a^0 + \frac{\sqrt{\kappa_2}}{\tau} a^0, \]

where \( \gamma_2 \) and \( \gamma_3 \) are decay rates of atom from excited states \([2]\) and \([3]\) to ground state \([1]\) respectively. In most cases, it is safe to assume \( \gamma_2 = \gamma_3 = \Gamma \). The loss rate of intracavity field on cavity mirror \( i \) is defined as \( \kappa_i = T_i/\tau \) (here \( T_i \) is the transmission rate of cavity mirror and \( \tau \) is the round trip time of light field inside the cavity). The cavity can be treated as symmetric so that \( \kappa_1 = \kappa_2 = \kappa \). Assuming a closed atomic system so \( \sigma_{11} + \sigma_{22} + \sigma_{33} = 1 \).

Under the steady-state condition, the left sides of equations (2)–(8) are equal to zero. \( \sigma_0 \) can be obtained by solving this set of equations. The steady-state solutions of equations (2)–(7) are obtained as:

\[ \sigma_{21} = ig_\chi (X_{21} \sigma_{11} + Y_{21} \sigma_{33} + Z_{21}) = a a_{21}' \]
\[ \sigma_{13} = ig_\chi (X_{13} \sigma_{11} + Y_{13} \sigma_{33} + Z_{13}) = a a_{13}' \]

and

\[ \sigma_{11} = \frac{2g^2 \sigma_{12} Re(Z_{13}) \left[ 2g^2 \sigma_{22} Re(Y_{21}) + \Gamma \right] - \left[ 2g^2 \sigma_{12} Re(Z_{21}) - \Gamma \right] \left[ 2g^2 \sigma_{22} Re(Y_{21}) + \Gamma \right]}{\left[ 2g^2 \sigma_{22} Re(Y_{13}) + \Gamma \right] - 2g^2 \sigma_{22} Re(X_{13}) + \left[ 2g^2 \sigma_{22} Re(Y_{21}) + \Gamma \right]}, \]
\[ \sigma_{33} = \frac{2g^2 \sigma_{12} Re(Z_{13}) \left[ 2g^2 \sigma_{22} Re(Y_{21}) + \Gamma \right] - \left[ 2g^2 \sigma_{12} Re(Z_{21}) - \Gamma \right] \left[ 2g^2 \sigma_{22} Re(Y_{21}) + \Gamma \right]}{\left[ 2g^2 \sigma_{22} Re(Y_{13}) + \Gamma \right] - 2g^2 \sigma_{22} Re(X_{13}) + \left[ 2g^2 \sigma_{22} Re(Y_{21}) + \Gamma \right]}, \]

where \( X_{21} = \frac{2d_2 d_3 x + 2a^2 d_3^2}{M}; Y_{21} = \frac{-d_2 y + 2d_3 d_1 + 2a^2 d_1 d_3}{M}; Z_{21} = \frac{2a^2 d_1 d_3}{M}; X_{13} = \frac{2a^2 d_1 d_3}{M}; Y_{13} = \frac{d_2 d_1 + 2a^2 d_3}{M}; \)
\( 2d_2 d_1 d_3 + g^2 a^2 d_1 + g^2 a^2 d_3; d_1, d_2 \) and \( d_3 \) are defined as \( d_2 = \gamma_2/2 - i(\Delta_p + \Delta_{23}) \)
\( d_3 = (\gamma_2 + \gamma_3)/2 - i\Delta_{13} \) and \( d_3 = \gamma_2/2 + i\Delta_{23} \) respectively.

The intracavity light field of equation (8) in steady-state can be re-written as:

\[ a = \frac{\sqrt{\kappa/\tau} (a_{1\text{in}}^0 + a_{2\text{in}}^0)}{\kappa - i(\Delta_p - \Delta_c) - i\Gamma N (\sigma_{21}' + \sigma_{13}')} \]

Assuming two input fields are identical \( a_{\text{2in}}^0 = a_{\text{1in}}^0 \), the relation between intracavity light field intensity \( I_{\text{in}}(\nu) \) and input light field intensity \( I_{\text{in}}(\nu) \) is:

\[ I(\nu) = \left| \frac{2\sqrt{\kappa/\tau}}{\kappa - i(\Delta_p - \Delta_c) - i\Gamma N (\sigma_{21}' + \sigma_{13}')} \right|^2 I_{\text{in}}(\nu) \]

Here \( I(\nu) = |a|^2 \nu \) and \( I_{\text{in}}(\nu) = |a_{\text{in}}|^2 \nu \). The optical pumping rate \( P_{\text{in}} \) of atom for transition \([1]\) \( \rightarrow \) \([1]\) can be depicted as \( P_{\text{in}} = \sigma_{11} I(\nu)/\nu \). Under the steady-state condition, it is easy to obtain that the populations of atoms in upper states \([2]\) and \([3]\) are \( N_2 = \frac{\nu}{2} \left( \frac{P_{\text{in}}}{\pi \nu} + 1 \right) \) and \( N_3 = \frac{\nu}{2} \left( \frac{P_{\text{in}}}{\pi \nu} + 1 \right) \), respectively [65].

When a light field transmits through an absorbing medium, the width and profile of spectral line are generally influenced by the Doppler broadening, collisional broadening, transit-time broadening, etc [65]. For our case of magneto-optical trapped cold atoms in cavity, the Doppler broadening effect and collisional broadening (also called pressure broadening) are greatly suppressed since the atom velocity \( v_{\text{a}} \) \( (v_{\text{a}} = \sqrt{2kT/m} \propto \sqrt{\gamma_T}; T_\gamma \) is the temperature of atom. \( k \) is Boltzmann constant and \( m \) is atom mass) decreases more than 10^3 times compared to it in normal atmospheric temperature (298.15 K). The collision effects between atoms, and atoms with their surrounding buffer gases, also increase the velocities of some atoms so that they would run out of the MOT, but these effects mainly influence the lifetime of atom in MOT. The population and density of trapping atoms also depend on the trapping rate of MOT. In a confocal cavity, the incident light field is precisely focused to the center position of cavity by lens, so transit-time broadening can be neglected since it is mainly due to the width of laser field.
3. CPA in linear regime

Assuming the intensity of input field $I_{\text{in}}$ ($\nu$) is weak, the populations of atoms in upper states $|2\rangle$ and $|3\rangle$ are $N_2 \approx 0$ and $N_3 \approx 0$. The optical pumping effects for atoms from ground state $|1\rangle$ to excited states $|2\rangle$ and $|3\rangle$ are neglected. Then $\sigma_{22} = \sigma_{33} = 0$ and $\sigma_{11} \neq 0$. The CQED system is excited in linear regime [47].

Solving equations (2)–(8) in steady-state case, the solution of intracavity light field is then simplified as:

$$a = \frac{\sqrt{\nu}}{\kappa - i(\Delta_\kappa - \Delta_\nu)} (a_{\text{in}}^\dagger + a_{\text{in}}),$$

where $T$ is the transmission coefficient of two cavity mirrors. Combining equations (15)–(17), the steady-state solutions of the output light fields from two sides of the CQED system are:

$$a^\dagger = \sqrt{T} a - a_{\text{in}}^\dagger,$$
$$a^\prime = \sqrt{T} a - a_{\text{in}}^\dagger,$$

respectively. Here $\zeta = \Delta_\nu + \Delta_{23}, \xi = \Delta_\kappa - \Delta_\nu$. Assuming the collective-coupling coefficients $g_i \sqrt{N}$ ($i = 2, 3$) of the cavity mode to two transition paths are identical, i.e. $g_2 \sqrt{N} = g_3 \sqrt{N}$. Under this condition, it is easy to find that two output fields at two ends of CQED system are equal, $a^\dagger = a^\prime = a_{\text{out}}$. The CPA occurs when the output light field is zero ($a_{\text{out}} = 0$). From equations (18) and (19), the required criterion for CPA can be derived as:

$$\kappa B - A \xi \Gamma = 2g^2 N, \quad (20)$$
$$A \kappa \Gamma + B \xi = A g^2 N, \quad (21)$$

where $A = \zeta + \Delta_\nu = 2\Delta_\nu + \Delta_{23}$, $B = \Gamma^2 - \zeta \Delta_\nu = \Gamma^2 - (\Delta_\nu + \Delta_{23}) \Delta_\nu$.

When equations (20) and (21) are satisfied, there is no output light emitted from the CQED system, but the intracavity light intensity $|a|^2$ is not zero. The incident light fields can be regarded as perfectly trapped in the CQED system, and the cavity-atom structure behaves as a perfect photon absorber.

For a three-level atom ensemble in cavity, due to the collective strong coupling of atoms and cavity field $g_i^2 N > \Gamma \kappa$, two upper states can be excited simultaneously by a single cavity mode under a proper cavity detuning [59, 67]. The resulting three spectral peaks represent three polariton states which come from the superposition of the atomic excitation and photonic excitation. Meanwhile, locations of three spectral peaks denote resonant frequencies of three polariton states. Consider the case when the cavity frequency is tuned to the middle of two upper excited states $|2\rangle$ and $|3\rangle$, i.e. $\Delta_\kappa = - \frac{\Delta_\nu}{3}$, a symmetrical spectrum is thus obtained. Inserting equation (1) and the system wave-function into the Schrödinger equation, it is easy to derive that the central peak of three polariton excitations is located at $\Delta_\nu = \Delta_\kappa = - \frac{\Delta_\nu}{3}$, and two sideband peaks are located at $\Delta_\nu = - \frac{\Delta_\nu}{2} \pm \sqrt{\frac{\Delta_\nu^2}{4} + 2g^2 N}$. First, we consider the situation that the input field is tuned to resonate with the central polariton, i.e. $\Delta_\nu = - \frac{\Delta_\nu}{3}$. In this case, we have $A = \xi = 0, \zeta = \frac{2\Delta_\nu}{3}$, the CPA criterion of equations (20) and (21) can be simplified as:

$$\kappa B = 2g^2 N \Gamma.$$  

With $\kappa = \Gamma$, we get $g \sqrt{N} = \sqrt{B/2}$ from equation (22). In this case, $g^2 N > \Gamma \kappa$, the collective strong coupling condition is satisfied. Figure 2 shows the output light intensity $|a_{\text{out}}|^2$ and intracavity photonic excitation $|a|^2$ versus the input field detuning $\Delta_\nu$. In figure 2(a), CPA occurs at the central polariton when the CPA criterion of equation (22) is satisfied. However, the output light intensities at two sideband polaritons are not zero. The CPA is sensitive to the system parameters $\kappa$ and $\Gamma$. Figure 2(b) plots the intracavity field intensity $|a|^2$ versus $\Delta_\nu$. The intracavity field intensity is not zero and reaches the maximum value at central polariton when the CPA criterion is satisfied. It means that the input light radiation is completely absorbed by the CQED system and transferred to internal excitation energy at central polariton excitation of the system.
When \( g \) increases, the output light intensity is near zero at \( |\Delta| \approx \sqrt{2} \), which means the CPA occurs at \( |\Delta| \approx \sqrt{2} \). In figure 4(a), the CPAs occur at two sideband polaritons \( \Delta_\pm = \pm \frac{\Delta_0}{2} + g\sqrt{N} \) with \( g\sqrt{N} = 8\Gamma \). Figure 3(b) shows that the intracavity light intensities are not zero and reach the maximum values at two sideband peaks when the sideband CPAs occur.

Figure 4 shows the normalized output field intensity \( |a_{\text{out}}|^2 \) versus \( g\sqrt{N} \) for different input field detuning \( \Delta_p \). In figure 4(a), the CPA of central polariton excitation only occurs at \( g\sqrt{N} = \sqrt{2}/2 \) and \( \Delta_p = \pm \frac{\Delta_0}{2} \). The output light intensity firstly decreases to zero, and later it increases quickly as \( g\sqrt{N} \) increases. The output light intensity is near zero at \( g\sqrt{N} > 16\Gamma \), which means the absorption of the system is saturated. In figure 4(b), the sideband polariton excitation of the system strengthens as \( g\sqrt{N} \) increases. As a result, the output light intensity decreases quickly. The output light intensity is near zero at \( g\sqrt{N} > 8\Gamma \), which means the CPA occurs at \( \Delta_p = \pm \frac{\Delta_0}{2} + g\sqrt{2N} \) and \( \Delta_p \leq \pm \frac{\Delta_0}{2} - g\sqrt{2N} \). We note that the splitting

\[
\kappa B - A\xi\Gamma = \kappa \left[ \Gamma^2 + 2g^2N + \frac{\Delta_0^2}{4} \right],
\]

(23)

\[
2(\kappa\Delta\Gamma + B\xi)/A = 2 \left[ \kappa\Gamma - \frac{\Gamma^2}{2} + g^2N \right].
\]

(24)
of two upper atomic energy-levels can also be created by an external magnetic field, thus the frequency separation $\Delta_{23}$ of two upper states can be tuned easily [68]. Comparing with cases in which the cavity mode only couples one atom transition path [47, 62–64, 66], we show that the proposed CPA scheme can be realized in a continuous input frequency range without dead zone from above discussion. The demonstrated central and sideband CPAs show a broadband-tunable property in linear regime of CQED system.

The CPA of single cavity mode coupled to three-level atoms in the CQED system can also be controlled by the relative phase $\Delta \phi$ between two input fields $a_{\text{in}}$ and $a_{\text{in}}'$. When the CPA criterion of equations (20) and (21) are satisfied, figure 5 shows that the CPA depends on the relative phase $\Delta \phi$ of two input fields. Here $a_{\text{in}} = a_{\text{in}} e^{i\phi}$ and $a_{\text{in}}' = a_{\text{in}} e^{i\phi'}$, $\Delta \phi = \phi_1 - \phi_2$. For a symmetric excitation $\Delta c = -\Delta p$, the input light is resonant with the central polariton excitation at $\Delta p = -\Delta c$ and $g\sqrt{N} = \sqrt{B/2}$. Figures 5(a) and (b) plot the output light intensity $|a_{\text{out}}|^2$ and intracavity light intensity $|a|^2$ versus $\Delta \phi$. By adjusting $\Delta \phi$ from 0 to $\pi$, the output field intensity $|a_{\text{out}}|^2$ increases from original value zero to the maximum value 1. Correspondingly, the intracavity light intensity $|a|^2$ changes from the original maximum value 1 to the minimum value zero.

Figures 5(c) and (d) show similar behaviors when the input field is tuned to the sideband polariton resonance. In this case, $\Delta p = -\Delta c = \pm \Delta_{23} + 2g^2N$ and $g\sqrt{N} = 8\Gamma$, the output light intensity is changed from the original minimum value zero to the maximum value 1, while the intracavity field intensity $|a|^2$ has an opposite behavior. Based on this, an ultrafast multichannel all-optical switch can be constructed by manipulating phase, intensity and frequency of input field from one side of cavity to control the other [69, 70]. Specifically, two output field intensities from two ends of the system are different from each other except at $\Delta \phi = 0$ or $\pi$ in figure 5(c), which exactly exhibit an asymmetric input–output behavior [71, 72]. This can be used for optical logic [54] and optical communication devices [51].

### 4. CPA in nonlinear regime

When the input field intensity $I_{\text{in}}(\nu)$ is strong enough, the optical pumping effect cannot be neglected so that $\sigma_{22}$ and $\sigma_{33}$ are not equal to 0. The populations of atoms in upper states [2] and [3] are $N_2 \neq 0$ and $N_3 \neq 0$. In this case, the CQED system is excited in nonlinear regime [63, 64, 66]. The denominator of the first item on the right side of equation (13) contains some high-order terms of the intracavity field $a$, so it shows a nonlinear dependent relation between the intracavity field $a$ and the input field $a_{\text{in}}$. Under the steady-state condition, output light fields from the right and left sides of the cavity are given by:

$$a' = \frac{\kappa (a_{\text{in}}' + a_{\text{in}}^d)}{\kappa - i(\Delta_c - \Delta p) - igN(\sigma_{21} + \sigma_{31})} - a_{\text{in}}^d, \quad (25)$$

and

$$a' = \frac{\kappa (a_{\text{in}}' + a_{\text{in}}^d)}{\kappa - i(\Delta_c - \Delta p) - igN(\sigma_{21} + \sigma_{31})} - a_{\text{in}}^d, \quad (26)$$

respectively.
Here the light intensity is equivalent to the photon flux, i.e. $|a_{r,l}^\text{in}|^2 = n_{r,l}^\text{in}$, and $|a_{r,l}^\text{in}|^2 = n_{r,l}$ [64]. Equations (25) and (26) show the nonlinear dependence of the output field intensity on the input field intensity.

Due to the field interference, the output light intensity $|a_{\text{out}}|^2$ becomes different by varying the relative phase $\Delta \phi$ of two input fields. In the following, we focus on the case that two input fields are of same amplitude and phase, i.e. $|a_{\text{r}}^\text{in}|^2 = |a_{\text{l}}^\text{in}|^2$ and $\Delta \phi = 0$. In figure 6, we show the influence of the input field detuning and collective-coupling coefficient $g\sqrt{N}$ on the optical transmission in nonlinear regime with a symmetric excitation $\Delta c = -\Delta_{23}^2$. It is clear to see that CPA occurs at $\Delta p = -\Delta_{23}^2$ and $g\sqrt{N} = \sqrt{B}/2$ in figure 6(a). In this case, the input field is resonant with the central polariton and the linear CPA criterion equations (20) and (21) are also satisfied. It means that the CPA criterion in linear regime of central excitation is also valid for nonlinear regime. However, as we increase the collective-coupling coefficient $g\sqrt{N}$, the CPA also appears beyond equations (20) and (21), which is shown in figure 6(b). This means the CPA criterion is much broader compared with the case of linear regime. In figure 6(c), both the threshold (the input light intensity corresponding to the switching point of the output field intensity) and corresponding output field intensity increase as the input field detuning increases. At the same time, CPA in the CQED system disappears. This is because the increasing of input field detuning leads to the decreasing of absorption of the system. As seen in figure 6(d), the threshold (the lower one) of the bistability continues to increase as the input light detuning $\Delta p$ increases. Moreover, the input–output relation exhibits a near linear style except for the bistability region. Compared with the nonlinear CPA of two-level atoms excited by a single cavity mode [63], the CPA threshold of the proposed scheme is lower, which means lower light power cost for future applications [48–54].

To investigate how the performance of the CQED system varies with the collective-coupling coefficient $g\sqrt{N}$ at central polariton excitation, we plot the input and output light intensities versus $g\sqrt{N}$ in figure 7(a). As $g\sqrt{N}$ increases, the threshold $|a_{\text{in}}|^2$ also increases, but the output light intensity $|a_{\text{out}}|^2$ keeps zero under the case of $\Delta c = \Delta p = -\Delta_{23}^2$. It means that the CPA always exists with the increasing of $g\sqrt{N}$, which is very different from the CPA in linear regime. The reason of this difference can be explained by the
Figure 6. The output field intensity $|a_{\text{out}}|^2$ versus the input field intensity $|a_{\text{in}}|^2$. The red lines label the thresholds. (a) $\Delta_p = -\frac{\Delta_{23}}{2}$, $g\sqrt{N} = \sqrt{B/2}$; (b) $\Delta_p = -\frac{\Delta_{23}}{2}$, $g\sqrt{N} = 5\Gamma$; (c) $\Delta_p = -\frac{\Delta_{23}}{2} + 2\Gamma$, $g\sqrt{N} = \sqrt{B/2}$; (d) $\Delta_p = -\frac{\Delta_{23}}{2} + 4\Gamma$, $g\sqrt{N} = \sqrt{B/2}$. Other parameters are $a_{\text{in}} = a_{\text{out}}$, $\Delta_c = -\frac{\Delta_{23}}{2}$, $\Delta_{23} = 5\Gamma$ and $\kappa = \Gamma$.

Figure 7. The variation of threshold, output, absorption and dispersion properties of the CQED system in nonlinear regime with $\Delta_c = \Delta_p = -\frac{\Delta_{23}}{2}$. (a) The threshold $|a_{\text{in}}|^2$ and corresponding output light intensity $|a_{\text{out}}|^2$ versus the collective-coupling coefficient $g\sqrt{N}/\Gamma$. Blue curve: the threshold $|a_{\text{in}}|^2$; red dashed line: the corresponding output light intensity $|a_{\text{out}}|^2$; (b) the absorption and dispersion versus the input field intensity $|a_{\text{in}}|^2$ with $g\sqrt{N} = \sqrt{B/2}$. The blue curve and red dashed line correspond to absorption and dispersion, respectively. Other parameters are $\Delta_{23} = 5\Gamma$, $\kappa = \Gamma$.

Next, we investigate the CPA at the sideband excitation of the CQED system in nonlinear regime. For $\Delta_c = -\frac{\Delta_{23}}{2}$, two sideband peaks are excited symmetrically. It means the input–output curves are same for $\Delta_p = -\frac{\Delta_{23}}{2} + \sqrt{\Delta_{23}^2 + 2g^2N}$ and $\Delta_p = -\frac{\Delta_{23}}{2} - \sqrt{\Delta_{23}^2 + 2g^2N}$. Figure 8(a) plots the sideband excitation
Figure 8. The output field intensity $|a_{out}|^2$ versus the input field intensity $|a_{in}|^2$. The red lines label the thresholds. (a) $g\sqrt{N} = 8\Gamma$; (b) $g\sqrt{N} = 12\Gamma$. Other parameters are $\Delta_c = -2\Delta_2$, $\Delta_p = -2\Delta_2 \pm \sqrt{\Delta_2^2 + 2g^2N}$, $\Delta_{23} = 5\Gamma$ and $\kappa = \Gamma$. (Inset) The magnified input–output relation to show the CPA and threshold clearly.

Figure 9. The threshold $|a_{in}|^2$ and the corresponding output light intensity $|a_{out}|^2$ versus $g\sqrt{N}$. (a) The threshold $|a_{in}|^2$ versus $g\sqrt{N}$; (b) the corresponding output light intensity $|a_{out}|^2$ versus $g\sqrt{N}$. Two input fields are of the same amplitude and $\Delta\phi = 0$. Other parameters are $\Delta_c = -\Delta_2$, $\Delta_p = -\Delta_2 \pm \sqrt{\Delta_2^2 + 2g^2N}$, $\Delta_{23} = 5\Gamma$ and $\kappa = \Gamma$.

with $g\sqrt{N} = 8\Gamma$. CPA occurs at $|a_{in}|^2 = 860$, and the corresponding $|a_{out}|^2$ is near zero. Figure 8(b) plots the sideband excitation with $g\sqrt{N} = 12\Gamma$. The threshold of bistability curve appears at $|a_{in}|^2 = 4355$ while the corresponding $|a_{out}|^2$ is not zero. Specifically, the input–output relation at sideband excitation of the CQED shows a bistability curve in figure 8. However, before and after the threshold points of bistability range, the input–output relation exactly exhibits linear correlation. To my knowledge, this type of bistability is not reported before. It can be used as a linear and broadband-controllable all-optical switch [73, 74].

In figure 9, we plot the performance of the CQED system versus the collective-coupling coefficient $g\sqrt{N}$ at sideband excitation. Both the threshold $|a_{in}|^2$ and the corresponding output light intensity $|a_{out}|^2$ increase as $g\sqrt{N}$ increases. For $g\sqrt{N} \leq 8\Gamma$, the output light intensity $|a_{out}|^2$ is near zero, which means the near CPA occurs with $g\sqrt{N} \leq 8\Gamma$. Then the threshold of bistability increases quickly as $g\sqrt{N}$ increases.

5. Conclusion

In summary, we demonstrate a tunable broadband CPA scheme in a cavity-atom structure with single cavity mode coupled to three-level atoms. The CPA properties in linear and nonlinear regimes are analyzed carefully. By altering system parameters, the coherent absorption intensity, threshold and frequency can be tuned continuously in a broadband range. Our study demonstrates that the CPA does exist in multi-level coupling cavity-atom system. Thus the concept and application of CPA are further extended. As the input signal field can be effectively controlled and adjusted by a counterpart field from the other side of cavity, this three-level excited CQED scheme is useful for future quantum phase gates [75], special quantum state generation [76], quantum manipulation and quantum coherent control [77–79].
The specific parameters used in our calculation $g \sqrt{N} = \sqrt{B/2} \approx 2 \Gamma$ and $\kappa = \Gamma$ can be easily obtained in lots of CQED experiments [55, 56] with $^{85}$Rb atoms. The cavity decay rate $\kappa$ and the decay rate $\Gamma$ of $^{85}$Rb atom approximate to 6 MHz. The frequency separation of two upper atomic states $|2P_{3/2}, F = 1\rangle$ and $|2P_{3/2}, F = 2\rangle$ is $5\Gamma$. The ground state is $|2S_{1/2}, F = 2\rangle$. Most recently, a single cavity mode coupling simultaneously with two separate transitions of three-level atoms [67] and three separate transitions of four-level atoms [59] are realized respectively in experiment. Therefore, the experimental realization of CPA in the CQED system with three-level atoms excited by a single cavity mode is anticipated.

Our model can be applied to one-sided cavity [62], atom-optomechanical system [80–82], etc, easily. The experimental realization of CPA in CQED is meaningful for fabricating all-optical switching [70], interferometers [1], and logic elements [69] in several physical relevant frameworks, particularly in nonlinear optics [22, 61], photonic lattice networks [83] and optical multiplexing [19].

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Conflict of interest

The authors declare that there are no conflicts of interest related to this article.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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