Characterization and Modeling of Supercapacitors Used for Hybrid Electric Energy Storage

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ABSTRACT
The electricity storage is a major issue due to the intermittency of the renewable energy sources. In general, lead-acid batteries are the most common and widely used electrochemical devices for energy storage. In order to extend the lifetime of these batteries, new and complementary materials were invented, such as supercapacitors, which in some cases led to the development of new electric energy storage systems. In this paper, we present a study that investigates characterization and modeling of the supercapacitors by using two models: RC-constructor model and multibranch model, under Matlab-Simulink environment. This study led us to predict the electrical behavior of the supercapacitors in electricity production facilities based on renewable energy.

General Terms
Super capacitors

Keywords
Electricity Storage; Supercapacitors; RC-constructor model; Multibranch model

1. INTRODUCTION
The electric energy storage devices are useful in many applications such as telecommunications, memory backups, electric and hybrid vehicles, and photovoltaic [1]. The main parameters considered in these devices are: the energy that is expressed in watt-hour (Wh) [2], the maximal power that is expressed in Watts (W), the weight, the sizing, the lifetime, and the initial cost. In general, the classifications reported to these devices are delicate and arbitrary. In fact, according to the energy density and the power density, there are the following electrochemical components categories [4]:

- the accumulators that characterized by high energy density and low power density;
- the classical capacitors possess high power density but lower energy density;
- the supercapacitors or electric double-layer capacitors that possess the characteristics of the accumulators and the classical capacitors.

The renewable energy sources, like wind and solar, require large devices to store the energy. Effectively, the batteries and the supercapacitors have been used in complementary manner to solve the problem of the energy storage. Considering the available instantaneous power and the quantity of the stored energy, the combination of these devices increases the electric energy storage systems. The supercapacitors have high instantaneous, respectively, power and energy than batteries and classical capacitors. Their lifetime is about 1 to 20 years, which is high compared to batteries. The electrochemical capacitors have very high power density but very lower specific energy. Generally, the capacitors are used for time constants less than a few hundreds of milliseconds. In contrary, the batteries have very low power density and high specific energy, and may used for longer times that attempt to one minute.

Indeed, the supercapacitors are particularly ideal for a wide range of renewable energy applications and for energy storage with high efficiency and in a short time (about 1 to 10 s). Furthermore, designing these materials taken into account the reliability and maintenance, where they are fundamentally considered as maintenance-free and do not require expensive testing compared to the batteries that, in particular, need a follow-up evaluation of their state of health (SOH) and charge (SOC). For these reasons, these materials are useful for installations based on renewable energy sources that, in general, installed outback.

To understand the behavior of a battery or a supercapacitor, it is necessary to create models able to predict their function through simulation. In fact, the study of electric storage systems modeling is very complex as confirmed by many researchers [5], [6]. In the literature, several models were developed to modeling batteries [7], [8]. These models are often represented by an equivalent electric circuit that consists of resistance, capacitance, and the other cells with fixed or varied parameterization such as charge state and temperature. On the other hand, there are several models were used for modeling supercapacitor function such as RC-constructor model and multibranch model with two or more branches.

This paper is organized as follow: after an introduction, we detailed the proposed models of supercapacitors in section 2. Then, in section 3, we presented and discussed the obtained results of simulation. Eventually, the conclusion is presented at the end of the manuscript.

2. SUPERCAPACITORS MODELING

2.1 Supercapacitors pack sizing
Supercapacitor pack is defined as a serial and/or parallel combination of the supercapacitors cells, which it is sized according to the required power and timespan allowed to achieve this power [9]. The supercapacitor pack sizing consists of:

- Fixing the nominal voltage and current levels;
- Determining the used total capacity of the supercapacitors pack;
- Determining the number of cells in series $N_s$ and in parallel $N_p$.

In this study, the supercapacitors pack (SC) equivalent model consists of equivalent capacity in series $C_{eq}$ with an equivalent
The capacity and the global resistance of the supercapacitor pack are evaluated by equations (1) and (2):

\[ C_{sc} = \frac{N_p}{N_s} C_{sc, elem} \]  

(1)

\[ R_{sc} = \frac{N_s}{N_p} R_{sc, elem} \]  

(2)

Where \( C_{sc, elem} \) and \( R_{sc, elem} \) are the capacity and the resistance of one supercapacitor cell, respectively.

In addition, the voltage and the current of the SC pack were computed using the equations (3) and (4):

\[ V_{sc} = N_s V_{sc, elem} \]  

(3)

\[ I_{sc} = N_p I_{sc, elem} \]  

(4)

Where \( V_{sc, elem} \) and \( I_{sc, elem} \) are the supercapacitor pack voltage and current, respectively. \( I_{sc, elem} \) is the current of one supercapacitor cell, and \( V_{sc, elem} \) is its voltage.

Further, the stored energy in a supercapacitor was computed using the equation (5):

\[ W_{sc, elem} = \frac{1}{2} C_{sc, elem} V_{sc, elem}^2 \]  

(5)

In this study, two constraints were considered [10]:

- The voltage \( V_{sc} \) at the supercapacitor terminals should be less than the maximal voltage \( V_{sc, max} \) in order to not deteriorate and limit the lifetime of the supercapacitor,

- The all stored energy \( W_{sc, elem} \) is not usable. In fact, if the all energy will used, the voltage at the supercapacitor terminals should be decreased from \( V_{sc, max} \) to 0 V. Consequently, a minimal voltage \( V_{sc, min} \) should be determined in order to not consume the all stored energy.

Therefore, the usable energy \( W_{sc, elem} \) by a supercapacitor is:

\[ W_{sc, elem} = \frac{1}{2} C_{sc, elem} \left( V_{sc, elem, max}^2 - V_{sc, elem, min}^2 \right) \]  

(6)

So, the available usable energy by a supercapacitor pack that consists of \( N_c \) cells is:

\[ W_{pack} = N_c W_{sc, elem} = N_c N_p W_{sc, elem} \]  

(7)

Where, \( N_s \) and \( N_p \) are the number of the combined cells in series and parallel, respectively.

The used supercapacitor pack expects to provide the high power demand \( P_{charge} \) during a period of \( \Delta t = 3 \) min, thus, inducing a requested power of about \( P_{charge} = 240 \) W. Where the extracted energy from the supercapacitor pack is expressed as follow:

\[ P_{N, charge} \Delta t = \eta_s W_{pack} = \frac{1}{2} \eta_s \frac{N_p}{N_s} C_{sc, elem} \left( V_{sc, max}^2 - V_{sc, min}^2 \right) \]  

(8)

Where, \( \eta_s \) is the supercapacitor pack efficiency, which consider the internal losses \( \eta_s \) of is about 0.9 [11].

The equation (8) allows us to use 75% of the stored energy in the SC pack if we discharge it from its maximal voltage \( V_{sc, max} \) to half. According to Choi et al [12] the maximal voltage should equal to 80% of the DC bus voltage. In this study, \( V_{bus} = 24 \) V which mean \( V_{sc, max} = 20 \) V.

Commonly, the maximal voltage of one supercapacitor cell is \( V_{sc, max} = 2.8 \) V. If we considered \( N_p = 1 \), the number of the combined supercapacitor in series, \( N_s \), is given by the equation in follow:

\[ N_s = \frac{V_{sc, max}}{2.8} = \frac{20}{2.8} = 7.14 \]  

(9)

If we maximize \( N_s \) value to \( N_s = 8 \), then from equation (8), the capacity of one SC pack cell will be as shown in following equation:

\[ C_{sc, elem} = \frac{P_{N, charge} \Delta t}{\frac{1}{2} \eta_s \frac{N_p}{N_s} \left( V_{sc, max}^2 - V_{sc, min}^2 \right)} = 2600 \ F \]  

(10)

### 2.2 Supercapacitors modeling

#### 2.2.1 RC-constructor model

To modeling the supercapacitors, the constructors use a model of base called "RC-constructor". In this case, each supercapacitor cell consists of a capacitor in series \( C_{sc, cell} \) with a resistance \( R_{sc, cell} \) (Figure 1).

The RC-constructor model is usually used in the context of theenergetical systems functional modeling. Also, it is used to establish the technical sheets of the supercapacitors [13],[14].

The resistance \( R_{sc, cell} \) and the capacity \( C_{sc, cell} \) values can be determined through charge/discharge cycles in succession. The figure 2 shows the voltage \( V_{sc, cell} \) curve during one cycle of charge/discharge.

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**Figure 1: Supercapacitor pack model**

The voltage and the current of the SC pack were computed using the equations (10) and (11):

\[ V_{sc} = \frac{N_p}{N_s} V_{sc, elem} \]  

(10)

\[ I_{sc} = \frac{N_s}{N_p} I_{sc, elem} \]  

(11)
Indeed, during charge phase, the capacity \( C_{sc,\text{elem, char}} \) is given as follow:

\[
C_{sc,\text{elem, char}} = I_{sc,\text{char}} \frac{\Delta V_{sc,\text{char}}}{\Delta t_{\text{char}}} \tag{11}
\]

In contrast, during same charge phase, the resistance \( R_{sc,\text{elem, char}} \) is determined through the observed voltage drop at time of the current discharge application [15] according to the following equation:

\[
R_{sc,\text{elem, char}} = \frac{U_3 - U_4}{I_{sc,\text{char}}} \tag{12}
\]

Furthermore, during the discharge phase, the capacity \( C_{sc,\text{elem, déchar}} \) and the resistance \( R_{sc,\text{elem, déchar}} \) of the used model are given as shown in following equations:

\[
C_{sc,\text{elem, déchar}} = I_{sc,\text{déchar}} \frac{\Delta V_{sc,\text{déchar}}}{\Delta t_{\text{déchar}}} \tag{13}
\]

\[
R_{sc,\text{elem, déchar}} = \frac{U_2 - U_1}{I_{sc,\text{déchar}}} \tag{14}
\]

In this study, we searching to find the voltage \( V_{sc,\text{cel}} \) in function of resistance \( R_{sc,\text{cel}} \) and capacity \( C_{sc,\text{cel}} \) of the RC-constructor model through simulate it in Matlab-Simulink environment (Figure 3).

In fact, we can compute the voltage \( V_{sc,\text{cel}} \) by using the equation (15),

\[
V_{sc,\text{cel}} = R_{sc,\text{cel}} I_{sc} + \frac{1}{C_{sc,\text{cel}}} \int_0^t I_{sc,\text{cel}} \, dt \tag{15}
\]

To establish the multibranch model in any software’s of electric circuit simulation, the transmission line should decomposed into an access resistance \( R_t \) and a finite number of network of \( n \) identic branches \( R_{sc,\text{cell}} \) et \( C_i = C_{LT} \) for \( i > 1 \)[16].

### Parameters model identification

In following lines, we describe the employed techniques to determine the parameters of the multibranch model, including:

- The access resistance to the transmission line;
- The total capacity \( C_{LT} \) of the transmission line;
- The total resistance \( R_{LT} \) of the transmission line;
- The capacity \( C_l \) and the resistance \( R_l \) of the different complementary branches.

**a) Transmission line parameters**

- Access resistance \( R_t \)

Figure 5 presents the variation of the voltage \( V_{sc} \) at the supercapacitors terminals, allowing us to determine the access resistance \( R_t \) value at the transmission line. Which due to the voltage variation \( \Delta V_R \) during a charge (current) application as expressed in equation (16).
\[ R_L = \frac{\Delta V_{R1}}{\Delta I} \]  

- Total capacity of the transmission line

If the complementary branches are inactive, it can be concluded the total capacity \( C_{LT} \) of the transmission line in function of voltage \( V_{sc,elem} \) through few partial charge tests [10]. Practically, this capacity can be determined from slope of the \( V_{sc,elem}(t) \) curve.

The capacity values \( C_{LT}(V_{sc,elem} = 0 \text{ V}) \) and \( C_{LT}(V_{sc,elem} = 0.5 \text{ V}) \) can be computed using the equations (17) and (18), respectively, as shown in figures 6 and 7.

\[
C_{CL}(V_{sc,elem} = 0 \text{ V}) = \frac{I_{sc}\Delta V_{sc,elem}}{\Delta V(V_{sc,elem} = 0 \text{ V})} (17)
\]

\[
C_{CL}(V_{sc,elem} = 0.5 \text{ V}) = \frac{I_{sc}\Delta V(V_{sc,elem} = 0.5 \text{ V})}{\Delta V(V_{sc,elem} = 0.5 \text{ V})} (18)
\]

Then, it will be easy to find the expression of the transmission line capacity \( C_{LT} \) by increasing the voltage from 0.5 V until it reaches the nominal voltage of the supercapacitor.

- Total resistance \( R_{LT} \) of the transmission line

In the literature, some authors reported that we consider impedance approximation index to compute the resistance \( R_{LT} \) of the transmission line [16], which it is a response to current of line amplitude \( I_{sc} \) for very short times. From [16], this impedance can be expressed:

\[ Z_{ind}(t) = \frac{I}{C_{LT}} \left[ 1 + 2\pi \sum_{n=1}^{\infty} \text{erf}c \left( \frac{\pi}{t} n \right) \right] \]

(19)

With:

\[ \lim_{n \to \infty} 2\pi \sum_{n=1}^{\infty} \text{erf}c \left( \frac{\pi}{t} n \right) = 0 \]

(20)

Thus, the behavior of the impedance index lead us to find a law in function of \( \sqrt{t} \) as expressed in equation (21):

\[ Z_{ind}(t) = \frac{4R_{LT}}{\pi C_{LT}} \sqrt{t} \]

(21)

So, the voltage response can be written as:

\[ V_{sc,elem}(t) = R_{sc}I_{sc} + Z_{sc,elem}(t)I_{sc} \]

(22)

Where:

\[ V_{sc,elem}(t) = R_{sc}I_{sc} + \sqrt{\frac{4R_{LT}}{\pi C_{LT}}} \sqrt{t} \]

(23)

Considering the form of a line equation as:

\[ V_{sc,elem}(t) = A \sqrt{t} + b \]

(24)

Thus, \( A = \frac{4R_{LT}}{\pi C_{LT}} \) et \( B = R_{sc}I_{sc} \)

Further, for a voltage between 0 V and 0.5 V, we can determine the resistance \( R_{LT} \) by measuring the slope of the voltage response in function of \( \sqrt{t} \) as shown in figure 8. It can be computed by the following equation:

\[ R_{LT} = \frac{\pi C_{LT}}{4H_{sc}^2} \]

(25)

Figure 6: Portion of the curve \( V_{sc}(t) \) identifying the capacity \( C_{LT} \) of the transmission line for \( V_{sc,elem} = 0 \text{ V} \)

Figure 7: Portion of the curve \( V_{sc}(t) \) identifying the capacity \( C_{LT} \) of the transmission line for \( V_{sc,elem} = 0.5 \text{ V} \)

- Complementary branch parameters

It should be noted that, in this study, we use one complementary branch.

- Capacity \( C_{comp} \)

The quantity of charge \( Q_{sc,comp} \) stored in a supercapacitor during charge phase (Figure 9) can be computed using the following formulation:
Eventually, the resistance $R_{comp}$ can be expressed as follow:

$$R_{comp} = \frac{t_{reds}}{5 \tau_{comp}}$$ (34)

### 3. SIMULATION RESULTS

In this section, we present the simulation results achieved by both models: RC-.constructor and multibranch. Table 1 lists the electric characteristics of Maxwell supercapacitor type BCAP0010 that have a capacity of 2600 F [13,14] used in this study.

Table 1: Electric characteristics of the supercapacitor BCAP0010

| Specifications                        | Value          |
|---------------------------------------|----------------|
| Capacitance (F)                       | 2600 (en DC et 25°C) |
| Capacitance tolerance                 | ±20 %          |
| Voltages (V):                         |                |
| nominal                               | 2.5            |
| maximal                               | 2.8            |
| Resistance inches (mΩ)                | 0.7 (en DC et 25°C) |
| Specific power density (Wkg⁻¹)        | 4300 (±2.5 V)  |
| maximal current (A)                   | 600            |
| Temperature (°C):                     |                |
| of operation                          | -35 à +65      |
| of storage                            | -35 à +65      |
| Lifetime (year)                       | 10 (à 25°C)    |
| Cyclability                            | 5000000 (at 25°C and $I = 20$ A) |

Table 2 lists the values of the relative parameters of the RC- constructor during charge and discharge obtained for current $I_{sc,elem}$ = 30 A after several simulations.

Table 2: Parameters values of the RC- constructor model

| Current (A) | $R_{sc,elec}$ (mΩ) | $C_{sc,elec}$ (F) | $R_{sc,elec}$ (mΩ) | $C_{sc,elec}$ (F) |
|-------------|---------------------|-------------------|---------------------|-------------------|
| 50          | 38.015              | 2523              | 32.700              | 2502              |

For multibranch model, we obtained the following parameters values: the access resistance $R_1 = 3.2$ mΩ, the total resistance of the transmission line $R_{LT} = 22.11$ mΩ, and the total capacity values of the transmission line $C_{LT}$, which is summarized in table 3.

Table 3: Total capacity values $C_{LT}$ of the transmission line

| Voltage (V) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 |
|-------------|---|-----|---|-----|---|-----|
| Capacity $C_{LT}$ (F) | 1960.8 | 2354.2 | 2400.0 | 2767.8 | 3054.1 | 3054.1 |

Using the values listed in table 3 and based on a tool implemented in Matlab software, we computed the interpolation polynomials coefficients of the order 2 and 3 that reflecting the dependency of capacity $C_{LT}$ with voltage $V_{sc,elec}$. The polynomials functions are given by the equations (35) and (36):

$$C_{LT} = -71.8 V_{sc,elem}^2 + 632.8 V_{sc,elem} + 1972$$ (35)

$$C_{LT} = -67 V_{sc,elem}^3 + 179.3 V_{sc,elem}^2 + 403.5 V_{sc,elem} + 1997.1$$ (36)

In order to compare $C_{LT}$ values listed in table 3 and those may computed by using the equation (35) and (36), we presented the curve of the polynomials functions of $C_{LT}$ in function of
the voltage $V_{sc,elem}$ of the supercapacitor (Figure 10).

![Figure 10: Polynomials variations of $C_{LT}$ in function of $V_{sc,elem}$](image1)

In general, the complementary branches representing charge redistribution are characterized by times constant relatively very high of that of the transmission line. Note of that, in this study, contrary to what commonly used in literature [16], we used one complementary branch.

The obtained parameters values are: the complementary branch resistance $R_{comp} = 67.6 \, \text{m} \Omega$, and the complementary capacity values $C_{comp}$ are listed in Table 4.

| Voltage (V) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 |
|-------------|---|-----|---|-----|---|-----|
| $C_{comp}$ (F) | 50.78 | 60.97 | 62.16 | 71.68 | 88.62 | 88.62 |

![Figure 11: Complementary capacity $C_{comp}$ variation in function of $V_{sc,elem}$](image2)

We used a tool implemented in Matlab software to generating the interpolation polynomials coefficients based on parameters values listed in Table 4. The equations (37) and (38), given the polynomials functions of the order 2 and 3, respectively, which represent the evolution of the complementary capacity $C_{LT}$ in function of voltage $V_{sc,elem}$ (Figure 11).

$$C_{comp} = 0.8598 \, V_{sc,elem}^2 + 13.945 \, V_{sc,elem} + 51.07 \quad (37)$$

$$C_{comp} = -3.1431 \, V_{sc,elem}^3 + 12.6463 \, V_{sc,elem}^2 + 3.18 \, V_{sc,elem} + 52.25 \quad (38)$$

Branch used in multibranch model are 14 that gives best results.

![Figure 12: Evolution of $V_{sc,elem}$ and $I_{sc,elem}$ of the supercapacitor BCAP0010](image3)

In fact, there are similarity between the simulation results of both models with lower difference noted between $t = 220 \, \text{s}$ and $t = 280 \, \text{s}$ (Figure 12).

| Time (s) | $V_{sc,elem}$ (V) | RC-constructor model | Multibranch model |
|----------|------------------|----------------------|-------------------|
| 50       | 0.68             | 0.63                 |
| 100      | 1.29             | 1.23                 |
| 150      | 1.86             | 1.82                 |
| 200      | 2.38             | 2.34                 |
| 250      | 2.65             | 2.55                 |
| 300      | 2.39             | 2.35                 |
| 350      | 1.91             | 1.86                 |
| 400      | 1.37             | 1.24                 |
| 450      | 0.66             | 0.64                 |
| 500      | 0                | 0                    |

4. CONCLUSION

In this paper, we have presented the characterization and modeling of supercapacitors. These devices can be used for the storage of electrical energy in a hybrid or multi-source renewable energy system and can be used in complementary ways to batteries to extend their lifetime. For this, we used
two models, namely: the "RC-Constructor" model and the "Multi-branch" model. The obtained results show that there is similarity between the simulation results of the two models. This will make it possible to predict the electrical behavior of supercapacitors based on these studied models. The perspective of this work is to validate this simulation by experimental measurements.

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