Strongly Mismatched Regime of Nonlinear Laser–Plasma Acceleration: Optimization of Laser-to-Energetic Particle Efficiency

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Abstract—Laser electron accelerators utilize a bubble regime of nonlinear plasma waves driven as laser wakefields that, from theoretical considerations, require a matched laser spot size incident on plasma. A strongly mismatched regime of nonlinear laser–plasma acceleration in the bubble regime, favored by experiments, is introduced and modeled for optimization of laser-to-particle energy efficiency with application to the recently proposed laser positron accelerator. Strong mismatch, in contrast with the matched condition, arises from the incident laser spot size being much larger than that needed for equilibration of the laser ponderomotive and electron-ion charge-separation forces in the nonlinearly driven density structure of a plasma bubble. This is shown to be favorable for optimization of large self-injected electron charge and ultralow transverse emittance without precluding beam spectral shaping. It is shown that there are prominent signatures of the mismatched regime, strong optical-shock excitation, and bubble elongation, which are validated using multidimensional particle-in-cell simulations. This paper thus uncovers a generalized regime that apart from being used in many laser–plasma acceleration experiments also opens a novel pathway for a wide range of future applications.

Index Terms—Laser beams, particle beams, plasma accelerators, plasma simulations, positrons, plasma wave.

I. INTRODUCTION

LASER-PLASMA accelerators (LPA) [1] using laser-driven nonlinear electron density waves in the “bubble” regime [2] have enabled centimeter-scale acceleration [3], [4] of quasi-monoenergetic electron “beams.” This regime [2] has now inspired a worldwide effort on LPAs. These efforts on LPA [5]–[9] have continued to show enhancement in the electron beam properties.

The theoretical model of these bubble LPAs is based on a “matched” regime [2]. Theoretically, the maximum energy gain is considered to be only possible if the incident laser radial spot size is matched to the “bubble” size that equilibrates the electron-ion charge separation and the laser ponderomotive force. This initially matched laser spot-size condition is held to be exclusively optimal for properties of the acceleration structure and electron beam [10].

However, a wide range of well-known groundbreaking experimental results [3]–[8] that have been a key to establish these LPAs have favored a mismatched regime. In the strongly mismatched regime, modeled here for the first time, the laser focal spot size is significantly larger than the matched condition [10].

As opposed to the minimization of the relative energy spread of the accelerated electron beam that is the exclusive pursuit of almost all of the ongoing LPA efforts [3]–[9], [11], this paper optimizes the laser-to-energetic particle conversion efficiency without precluding beam spectral shaping. This laser-to-beam energy conversion optimization makes possible applications that require high electron charge at high energies in a micrometer-scale spot size. For example, this is the requirement of the feeder stage of a recently proposed laser positron accelerator [12]. Past experimental work has often used this regime because it has also been found to be more effective for certain beam characteristics over the matched regime. Despite the higher electron energies and other qualities that experimentally establish the profound importance of the mismatched regime, no earlier work has investigated its underlying physical mechanisms.

Physical processes underlying the mismatched regime are here shown to significantly differ from the matched regime. Two prominent signature processes of this regime—strong optical-shock excitation and bubble elongation—are elucidated here. The process of laser slicing uncovered here is significantly different from the well-known effect of laser etching in plasma. Second, the process of bubble elongation is quite distinct from the isotropic bubble expansion explored earlier. Thus, while this paper reveals novel laser–plasma dynamics, it also opens up an alternative to the matched regime.

Apart from merely the tendency of experiments to favor the mismatched regime, several important factors motivate its study. First, a larger vacuum focal spot size at a given laser power is known to produce higher “mode quality” in the far field (low beam-propagation factor or $\mathcal{M}^2$-number or TEM00 times diffraction-limited number). This is because a larger spot is less affected by various aberrations [13]. High mode quality is not equivalent to the maximization of intensity percentage within the focal spot, as characterized by the Strehl ratio [14]. This is because of the well-defined “no-TEM00 Gaussian” problem [13]. Second, self-injection mechanisms...
that rely on symmetry breaking processes promise beams with distinct phase-space properties and higher net charge due to the inherent mismatch [15]–[20].

Ideal laser focal-spot mode characteristics are assumed in the bubble regime theory of self-guided LPAs [2]. A laser with predominantly TEM_{00}-mode is self-guided in a homogeneous plasma with electron density, n_{0} over multiple Rayleigh lengths, \( Z_{R} \equiv \pi w_{0}^{2}/\lambda_{0} = \pi W_{0}^{2}/(\mathcal{M}^{2}\lambda_{0}) \) (\( \lambda_{0} \) is the central wavelength of the laser and \( w_{0} \) is the spot size of TEM_{00} mode and \( W_{0} \) is the measured spot size) if it drives electron cavitation [16], [17]. To self-guide the TEM_{00}-mode, the laser power \( P \) has to exceed a critical power \( P_{c} = 17.4 \times 10^{19} \ (\omega_{0}/\omega_{pe})^{2} \) W (\( \omega_{0} \) is laser frequency and \( \omega_{pe} = (4\pi n_{0} e^{2}/m_{e})^{1/2} \) is the plasma frequency). When \( P \geq P_{c} \), the plasma refractive index profile is shaped by relativistic quiver [16], [17] and ponderomotive channeling [18], which counter the diffraction of the focused laser.

Focused laser modal composition characteristics are constrained as the bubble plasma wave excitation demands the peak normalized laser vector potential, \( a_{0} \gg 1 \) [2] (where \( a_{0} = \text{max}[eA/m_{e}c^{2}] = 8.55 \times 10^{-10} n_{0}[\mu m] [\mu m(W/cm^{2})]^{1/2} \), \( \mathcal{A} \) is the laser vector potential, \( I_{0} \) is the peak intensity, and \( \lambda_{0} \) is the wavelength) which typically also satisfies \( P \geq P_{c} \). Importantly, the bubble spatial profile is dictated by the laser focal mode characteristics that, under the required tight focusing, suffer from optics-induced aberrations and distortions [13].

In the bubble regime, the electron–ion charge separation equilibrates with the laser ponderomotive force at the matched spot size [10]

\[
w_{0-m} = 2\sqrt{a_{0}} \frac{c}{\omega_{pe}} = R_{\text{bubble}} \tag{1}
\]

and thus the matched initial conditions enforce a vacuum laser spot size with \( w_{0-m} = R_{\text{bubble}} \).

The electron energy gain in this matched regime based on 3-D particle-in-cell (PIC) simulations (dephasing and laser-etching limited) scales as [10]

\[
\Delta E \ [m_{e} c^{2}] \approx \frac{2}{3} a_{0} \left( \frac{n_{0}}{n_{c}} \right) \tag{2}
\]

where \( a_{0} \) is the vacuum vector potential that is incident on the plasma. The value of \( a_{0} \) in plasma is known to significantly vary over the acceleration length due to several laser–plasma interactions effects. These include a localized variation of the laser wavelength profile, group velocity profile, and pump depletion of the laser pulse in addition to radial self-focusing. Thus, this equation best models a scenario where \( a_{0} \) is stable over the acceleration length, as is argued to be the case for matched laser spot size incident on plasma.

However, contrary to the matched regime theory, exemplary experimental evidence [3]–[8] has shown the optimality of mismatched regime. In these experiments, density scans with a fixed laser spot size have shown that the beam energy gain and other properties optimize where incident spot size (\( W_{0} \)) is much larger than the matched spot size (\( w_{0-m} \)). A mismatch factor, \( \Gamma \), is defined as

\[
\Gamma = W_{0}/w_{0-m} \gg 1. \tag{3}
\]

A large mismatch inflates the difference between (2) and experiments (Section III). Thus, the analysis here is based on GeV-scale energy gain data [21], [22] that have shown a maximum beam energy gain in a mismatched regime.

The first bubble wave [2] (wave of bubble-like oscillations)-based self-guided experiments used the mismatched regime to demonstrate laser–plasma acceleration of quasi-monoenergetic electron beams [3], [4]. In [3], the incident laser intensity was \( 2.5 \times 10^{18} \) W cm\(^{-2} \) (\( a_{0} = 1.1 \) at \( n_{0} = 2 \times 10^{19} \) cm\(^{-3} \), the matched spot size is \( w_{0-m} \approx 3 \) \( \mu m \), whereas the launched spot size was \( W_{0} \approx 12 \mu m \) [full-width-at-half-of-maximum (FWHM) \( \approx 20 \mu m \)], and a mismatch of \( \Gamma \approx 4 \). The predicted energy from (2) is 40 MeV but experiments obtained a spectral peak at 70 MeV. Similarly, in [4], the intensity was \( 3.2 \times 10^{18} \) W cm\(^{-2} \) (\( a_{0} = 1.3 \) at \( n_{0} = 6 \times 10^{18} \) cm\(^{-3} \), the matched spot size is \( w_{0-m} \approx 5 \) \( \mu m \), whereas the launched spot size of \( W_{0} = 12.5 \mu m \) (FWHM \( \approx 21 \mu m \)), and a mismatch of \( \Gamma \approx 2.5 \). The expected beam energy is 155 MeV but the spectral peak was at 175 MeV.

Moreover, the mismatched regime attains higher relevance as it has been applicable to many other ground-breaking experiments that have driven LPAs forward. Some prominent examples are, Austin-2 GeV data [5]: \( W_{0} = 275 \mu m, a_{0} = 0.6, n_{0} = 5 \times 10^{17} \) cm\(^{-3} \), \( w_{0-m} \approx 12 \mu m \), and \( \Gamma = 22 \); Nebraska-0.3 GeV data [6]: \( W_{0} = 17 \mu m, a_{0} = 2.2, n_{0} = 2.5 \times 10^{18} \) cm\(^{-3} \), \( w_{0-m} \approx 10 \mu m \), and \( \Gamma \approx 2 \); Gwangju-2 GeV data [7]: \( W_{0} = 25.5 \mu m, a_{0} = 5, n_{0} = 1.4 \times 10^{18} \) cm\(^{-3} \), \( w_{0-m} \approx 20 \mu m \), and \( \Gamma \approx 1.3 \); and Strathclyde-125 MeV data [8]: \( W_{0} = 20 \mu m, a_{0} = 1.5, n_{0} = 1 \times 10^{19} \) cm\(^{-3} \), \( w_{0-m} \approx 5 \mu m \), and \( \Gamma = 4 \).

Further experimental evidence in favor of laser focal spot size larger than the matched size also comes from the observation of the reduction in experimental artifacts that affect the plasma-wave quality. A nonuniform laser focal-spot (a large \( \mathcal{M}^{2} \)-number) is known to affect the transverse characteristics of the plasma wave [23] and leads to nonoptimal acceleration and focusing field profiles in the plasma. This quality degradation is in addition to a faster laser energy loss due to the tendency of higher order modes in a focused laser spot to diffract faster. The plasma wave quality has been indirectly inferred using a laser focal profile, such as the presence of multiple hot spots, at the plasma entrance [5] and exit [22].

In this paper, the essential dynamics due to the mismatch is analytically modeled in Section II with a nonlinear envelope equation of a self-guided laser derived for a bubble plasma wave. This equation shows that the oscillations of the spot size increasingly become asymmetric in response to an increasing degree of mismatch, with shorter (and tighter) “squeeze” phases and longer “relaxation” phases of the laser spot size. This behavior is similar in characteristics to a “cnoidal” wave, whose form is given by the Jacobi elliptic function, “cn” [24] (also solution of Korteweg-de-Vries equation [25]). Such envelope oscillation behavior is not modeled in earlier analyses of the evolution of a self-guided laser envelope [15]–[19].

The peak intensities reached in the squeeze phase are many times higher than that in the matched regime with the same laser energy. In Section III, a heuristically derived adjusted-\( a_{0} \) model shows that the highest electron energies in the
strongly mismatched regime exceed the predictions of (2). Furthermore, the rate of change of intensity in the shortened squeeze phase is unprecedented. Consequently, the laser–plasma interaction processes in the rapid spot size squeeze events dictate the physical mechanisms that underlie this regime.

By an analysis of the physical mechanisms that underlie this regime using multidimensional PIC simulations in Section IV, it is here shown that the laser spot-size mismatch leads to processes yet unexplored.

The general applicability of the strongly mismatched regime is here proved using 3-D and 2.5-D PIC simulations. The validation of methodology of computational modeling is proved using the equivalence of 3-D PIC simulations with boosted vacuum-2 × a₀ 2.5-D PIC simulations for experimental data in [3].

Using 3-D and 2.5-D PIC simulations in Section IV-A, the two signature processes of the mismatched regime are validated. A rapid rise in the intensity in the squeeze phase is shown in Section IV-B to lead to rising ponderomotive force that drives a sharply rising density perturbation ahead of the peak of the laser. The interplay of density build-up and relativistic reduction in plasma frequency in the region of increasing intensity modify a part of the laser that overlaps with the electron density build-up, in the front of the bubble. A rapid shift in the group velocity of a part of the laser pulse leads to the slicing of the laser longitudinal envelope. This slicing sets the pulse into a state of a strong “optical shock” [26] as shown in Section IV-B. Second, the slicing breaks the oscillatory envelope mode due to the coupling of the transverse-to-longitudinal envelope dynamics.

This pulse slicing effect significantly differs from the slow pulse etching [27] which sharpens the laser front. The process of etching in the matched regime is due to the red shift of the pulse front. Slicing, on the other hand, essentially detaches the head of the pulse from the rest of its body and excites a strong optical shock.

The effect of a strong optical shock on the acceleration mechanism is shown to be optimal only over a narrow range of densities (Fig. 5 in Section IV). At the lower end of this range, the radial squeeze is slow and a weak optical shock is reached close to the laser energy depletion length. At the higher end, the radial squeeze is too short to allow a significant interaction between the injected beam and the short-lived peak field. The envelope oscillation wavelength is also short and results in closely spaced successive laser slicing events. At higher densities, both these effects inhibit optimal acceleration.

In response to the sliced pulse that forms a strong optical shock, the bubble rapidly elongates and drives a novel self-injection mechanism. The properties of the beam injected during this elongation are modeled using the PIC-based analysis in Section IV-C.

II. NONLINEAR LASER ENVELOPE EQUATION—ASYMMETRIC PHASES OF EVOLUTION IN BUBBLE

In this section, an analytical model for the laser envelope evolution in the strongly mismatched regime is derived based on principles of self-guiding of lasers. This follows the analytical model of the evolution of the laser envelope size under laser-excited electron response using individual ray equations of geometric optics in a homogeneous plasma as was first modeled in [16]. An envelope equation of the variation of radial envelope size \( R_s(z) \) with \( z = ct \) was derived in this work (size was defined as the root mean square (rms) of the radial location of individual rays). It was shown to be in the form of the evolution equation of a coasting particle beam [19].

This equation that assumes radial symmetry was then simplified using the source-dependent expansion with Laguerre–Gaussian eigenfunctions, under the assumption, \( a_{m=0} \gg a_{m>0} \) (where \( m \) is the mode number and represents the order of the Laguerre polynomial and \( m = 0 \) corresponds to TEM₀₀-mode). The equation thus obtained was then further simplified to determine the self-focusing critical power for TEM₀₀-mode under the asymptotic approximation of a large value of \( R_s(z)/a_{inc}R_{inc} \), where \( R_{inc} \) is the incident rms envelope size and \( a_{inc} \) is the peak incident vector potential.

In [17], a quasi-static approximation of a driven wave equation in vector potential was used to show that the critical laser power corresponds to the condition of complete expulsion of all electrons from within the laser volume, referred to as electron cavitation.

Envelope behavior in self-guided regime is modeled using the following equation. In this equation, vacuum diffraction is balanced by a laser-driven refractive index profile [17]. However, this predicts that the envelope undergoes a catastrophic radial collapse for \( P > P_c \) and was thus not useful above the threshold

\[
\frac{a_{inc}R_{inc}}{R_s(z)} \rightarrow 0
\]

\[
\frac{d^2 R_s(z)}{dz^2} = \frac{R_{inc}^2}{a_{inc}^2 Z_R^2} \left[ \frac{1}{R_s(z)} \left( \frac{a_{inc}R_{inc}}{R_s(z)} \right)^2 \left( 1 - \frac{P}{P_c} \right) \right]. \tag{4}
\]

The above result was obtained using an insightful effective potential approach that modeled the behavior of the envelope by using a simplified “single-particle” model. The location of the particle is set to \( R_s \) and the right-hand side of the envelope equation (4) is written as a normalized effective potential \( V(R_s, P/P_c) \), to model the oscillations of this “particle” as in the following equation [16], [18]:

\[
\frac{d^2 R_s(z)}{dz^2} = -\frac{R_{inc}^2}{a_{inc}^2 Z_R} \left( \frac{\partial V_{eff}}{\partial R_s} \right)
\]

\[
V_{eff}(R_s, P/P_c) = \frac{1}{2} \left( \frac{a_{inc}R_{inc}}{R_s} \right)^2 \left[ 1 - \frac{P}{P_c} \right]. \tag{5}
\]

A different equation was derived in the opposite asymptotic limit of \( P/P_c > 1 \) and small value of \( R_s(z)/a_{inc}R_{inc} \) of the form in the following equation. In this limit, the laser envelope begins to diffract when the spot size satisfies the condition that \( R_s < a_{inc}R_{inc}/(16 P/P_c)^{1/3} \), the rate of change of envelope becomes too high for the relativistic effects to continually focus it down. Under this condition, the envelope either oscillates or diffracts away depending upon its initial rate of change. However, the following equation is not applicable to
the bubble regime:

\[
\begin{align*}
R_{s}(z) & \rightarrow 0 \\
\frac{d^{2}R_{s}(z)}{dz^{2}} & = \frac{R_{s}^{2}}{a_{\text{inc}}^{2}z_{R}^{2}} \\
& \times \frac{1}{R_{s}(z)} \left( \frac{a_{\text{inc}}R_{\text{inc}}}{R_{s}} \right)^{2} \\
& \cdots \frac{1}{1 - \frac{P}{P_{c}} \left( \frac{R_{s}}{a_{\text{inc}}R_{\text{inc}}} \right)^{3}}. \quad \text{(6)}
\end{align*}
\]

The effective potential on a particle approach assumes that the laser spot size is equal to the bubble radius, \( R_{s} \approx R_{B} \). PIC simulations validate this assumption nearly over the entire length of the evolution of the laser to within a small factor of the order of unity. The sheath that surrounds the bubble cavity is effectively the particle in this nonlinear “single-particle” model. In the bubble regime, the condition, \( P \gg P_{c} \) and \( a_{0} \gg 1 \), is generally satisfied, and therefore, the effective potential is not dominated by the interplay between the relativistic focusing and vacuum defraction. In consideration of this catastrophic collapse predicted for \( P > P_{c} \) in (5), these terms are retained only when \( P \leq P_{c} \).

In the nonlinear bubble regime, the effective potential is dictated by interplay between the ponderomotive potential and the ion electrostatic potential. Thus, for \( P \geq P_{c} \), the effective potential is the sum of the ponderomotive potential of the laser pulse in the following equation:

\[
\phi_{p}(P, R_{s}) = \Phi_{p}(P, R_{s})/(m_{e}c^{2}) = \gamma_{\perp}(P, R_{s}) - 1 = \sqrt{1 + a^{2}(P, R_{s})} - 1 \quad \text{(7)}
\]

where \( a \) is the normalized vector potential, and the ion cavity potential in the following equation on the “particle” represents the electron sheath that surrounds the laser pulse:

\[
\phi_{\text{cav}}(R_{s}) = \Phi_{\text{cav}}(R_{s})/(m_{e}c^{2}) = k_{p e}^{2}R_{s}^{2}/4 \quad \text{(8)}
\]

where \( k_{p e} = \omega_{p e}c^{-1} \). Thus, the total effective potential is as in (10) and the corresponding nonlinear envelope equation in (10). \( \mathcal{H} \) is the Heaviside step function

\[
\begin{align*}
V_{\text{eff}} \left( R_{s}, \frac{P}{P_{c}} \right) & = \frac{1}{2} \left( \frac{a_{\text{inc}}R_{\text{inc}}}{R_{s}} \right)^{2} \left[ 1 - \frac{P}{P_{c}} \right] \mathcal{H}(P_{c} - P) \\
& \cdots + \left( \phi_{p}(P, R_{s}) + \phi_{\text{cav}}(R_{s}) \right) \mathcal{H}(P - P_{c}) \\
= \frac{1}{2} \left( \frac{a_{\text{inc}}R_{\text{inc}}}{R_{s}} \right)^{2} \left[ 1 - \frac{P}{P_{c}} \right] \mathcal{H}(P_{c} - P) \\
& \cdots + \left( \sqrt{1 + a^{2}(P, R_{s})} - 1 + \frac{k_{p e}^{2}R_{s}^{2}}{4} \right) \\
& \times \mathcal{H}(P - P_{c}) \quad \text{(9)}
\end{align*}
\]

\[
\begin{align*}
\frac{d^{2}R_{s}(z)}{dz^{2}} & = \frac{R_{s}^{2}}{a_{\text{inc}}^{2}z_{R}^{2}} \times \cdots \frac{1}{R_{s}(z)} \left( \frac{a_{\text{inc}}R_{\text{inc}}}{R_{s}} \right)^{2} \left[ 1 - \frac{P}{P_{c}} \right] \mathcal{H}(P_{c} - P) \\
& + \cdots \frac{1}{2} \left[ \frac{a^{2}(P, R_{s})}{\sqrt{1 + a^{2}(P, R_{s})}} - k_{p e}^{2}R_{s} \right] \mathcal{H}(P - P_{c}) \right) \\
& \times \mathcal{H}(P - P_{c}). \quad \text{(10)}
\end{align*}
\]

Note that the radial mode of the laser spot over its evolution is assumed to remain \( \text{TEM}_{00} \), and thus,

\[
\begin{align*}
a^{2}(r, z) = \frac{a_{\text{inc}}^{2}R_{\text{inc}}^{2}}{R_{s}^{2}(z)} e^{-2r^2/R_{s}(z)^{2}} \\
& \times \frac{\partial^{2}a^{2}(r, z)}{\partial r^{2}} \big|_{r=R_{s}} = - \frac{1}{4R_{s}^{2}} a^{2}(r = R_{s}, z). \quad \text{(11)}
\end{align*}
\]

From (10), the matched spot size \( R_{\text{inc}}^{m} \) is inferred to be a critical point. It is the critical incident spot size where the ponderomotive force that pushes the envelope out equals the electrostatic ionic potential that pulls it in and is found to be the bubble radius matching condition in (1) [under the approximation, \( a(r = R_{s}) \approx a_{\text{inc}} \) and \( 1 + a^{2}P_{c}^{1/2} \approx a_{\text{inc}} \)]

\[
R_{\text{inc}}^{m} = w_{0} \approx 2 = \sqrt{a_{\text{inc}}k_{p e}^{1} \equiv R_{\text{bubble}}. \quad \text{(12)}
\]

There is an initial radial “velocity” of the envelope for any value of \( R_{\text{inc}} \neq R_{\text{inc}}^{m} \). For \( R_{\text{inc}} > R_{\text{inc}}^{m} \), the ion electrostatic force is dominant and the envelope initially develops a negative radial velocity. On the other hand, in the opposite limit, \( R_{\text{inc}} < R_{\text{inc}}^{m} \), ponderomotive force dominates and the envelope gains a positive initial radial velocity. Here, the negative initial velocity condition is studied due to its experimentally established optimality.

Note that the local changes in group velocity and wavelength within the laser envelope due to local electron density variations within the frame of the laser pulse are not accounted for in this model. Similarly, laser energy depletion is not accounted. Using PIC simulations of the strongly mismatched regime, these effects are shown to become quite important to the envelope behavior.

Numerical solutions in (10) are presented in Fig. 1 for laser energy of \( E_{L} = 10 \) J and intensity FWHM pulse length \( \tau_{p} = 49 \) fs at electron density, \( n_{0} = 2 \times 10^{18} \) cm\(^{-3}\). The plasma is initialized with a 500-\( \mu \)m rising plasma density ramp from vacuum to \( n_{0} \) to model experimental conditions and for consistency with PIC simulations. Two different incident spot sizes, \( R_{\text{inc}} = w_{0} = 16.4 \) \( \mu \)m and 37.4 \( \mu \)m (with experimentally relevant intensity FWHM spot sizes of 19.25 and 44 \( \mu \)m, respectively [21], [22]), and corresponding \( a_{0} \sim 4.6 \) and 2.0 are, respectively, compared. In Fig. 2, the initial focal spot size is fixed at \( R_{\text{inc}} = w_{0} = 37.4 \) \( \mu \)m and three different densities are compared.

In Fig. 1(a), spot-size evolution is compared for \( R_{\text{inc}} = 16.4 \) \( \mu \)m, matched at \( n_{0} = 2 \times 10^{18} \) cm\(^{-3}\) for \( E_{L} = 10 \) J (shown as dashed line), and strongly mismatched (by a factor of 4) \( R_{\text{inc}} = 37.4 \) \( \mu \)m. It is evident that with a high degree of mismatch at \( R_{\text{inc}} = 37.4 \) \( \mu \)m, the envelope oscillations become asymmetric, whereas they have a small amplitude sinusoidal evolution at \( R_{\text{inc}} = 16.4 \) \( \mu \)m. It is also seen that in the strongly mismatched regime, \( a_{0} \) in Fig. 1(b) sharply rises to many times its incident value in the asymmetric radial squeeze phase.
This is a critical result, as it implies rapid variation in the laser-driven plasma wave properties. Note that $R_{\text{inc}} = 37.4 \, \mu m$ is matched at $n_0 = 1.5 \times 10^{17} \, \text{cm}^{-3}$.

The variation of envelope oscillations over $n_0 = 0.9, 2, 5 \times 10^{18} \, \text{cm}^{-3}$ for a fixed incident spot size is shown in Fig. 2(a). The oscillation wavelength and the minima of the spot size in the squeeze phase of the oscillation become smaller at higher densities (higher degree of mismatch) while the asymmetry in the oscillations increases.

The nonlinear envelope equation compares well with the PIC simulations, as shown in Fig. 2(b). PIC simulations are described in Section IV. From simulations at $n_0 = 2 \times 10^{18} \, \text{cm}^{-3}$ with an incident spot size of $w_0 = 37.4 \, \mu m$, the first radial squeeze minima is at around 3 mm, as shown in the numerical solutions of (10). The model also correctly predicts the trend of wavelength of envelope evolution and its minima over a range of densities. However, the envelope behavior changes after the first radial squeeze as seen in the PIC simulation results. Note that PIC snapshots are 250 fs apart.

Asymmetric envelope oscillation behavior has been reported earlier in [28] but for subcritical laser power ($P < P_t$) in a channel-guided mismatched regime, where the incident spot size is not equal to the matched spot size for channel guiding.

It is important to point out that as the radial envelope squeeze phases become shortened and the change in the laser envelope radius and $a_0$ becomes more rapid, and in simulations, it is important to more carefully resolve the radial dimension. In comparison, the matched-regime simulations set a weaker constraint on the resolution of the radial dimensions.

In Fig. 1(b), at $n_0 = 2 \times 10^{18} \, \text{cm}^{-3}$, the value of $a_0$ varies around its peak over distances that are of the order of 100–200 $\mu m$. Similarly, in Fig. 2(b), the change in radial size is over a few plasma wavelengths.

In consideration of this important change in the radial envelope dynamics, fully resolved 2.5-D PIC simulations are used instead of transversely underresolved 3-D simulations. Comparison against a well-resolved 3-D PIC simulation is used to validate the methodology used in the 2.5-D PIC simulations.

This also opens up the case for an “optical plasma lens.” If the plasma-based focal spot squeezing process is experimentally confirmed to result in a higher focal-spot quality with lesser aberrations compared to vacuum optics, such a lens is quite attractive. From PIC simulations, it is observed that the energy loss of the laser over the first squeeze phase is relatively small. This allows the possibility of a mode quality and energy tradeoff. This is quite similar but operates based on different physical mechanisms compared to a “beam plasma lens” [29].

### III. ADJUSTED-$a_0$ HEURISTIC MODEL

An “adjusted-$a_0$ model” is here introduced to account for the mismatch in (2) and provides a good agreement with
experimental observations. This model assumes that the entire laser pulse energy launched at the entrance of the plasma is coupled into the plasma and is then squeezed down to the matched spot size corresponding to the launched $a_0$. This will, therefore, increase $a_0$ by the factor $\Gamma = W_0/w_{0-m}$ upon the culmination of the squeezing process for a radially symmetric focal spot. Using these heuristic arguments, energy gain in the adjusted-$a_0$ model is in (13), $F$ is the optical $F$-number of the focal spot, $F = (\pi w_0/2a_0)$. It is related to the focusing optics $F$-number ($F = f/D$, $f$ is the focal length and $D$ is the aperture of the focusing parabola)

$$\Delta E_{\text{adj}}[m_ec^2] = \frac{2\pi}{3} \sqrt{a_0-\text{inc}} \left( \frac{W_0}{\omega_{\text{inc}}} \right) \sqrt{\frac{n_c}{n_0}} \sqrt{\frac{n_c}{n_0}} F$$

circ.: $a_0(\text{adj.}) = a_0-\text{inc} \Gamma = a_0-\text{inc} \frac{W_0}{w_{0-m}}$

ellip.: $a_0(\text{adj.}) = a_0-\text{inc} \sqrt{\left( \frac{W_0 - 1}{w_{0-m}} \right) \left( \frac{W_0 - 2}{w_{0-m}} \right)}$. (13)

Using (13), the results of many ground-breaking experiments are much better explained where significantly higher energies were obtained in comparison with the predictions of the matched regime model (2). For the experiment in [3], (13) predicts a peak energy gain of 155 MeV, whereas the observed spectral peak at 70 MeV. In stark comparison from 3-D PIC simulations, energies as high as 200 MeV are obtained (see the Supplementary Material). Similarly, for the experiments in [4], (13) predicts a peak energy of 324 MeV, whereas the observed spectral peak was at 175 MeV. Note that the lower than theoretically predicted accelerated beam energies are expected in experiments due to experimental factors such as inferior mode quality, $M^2 \gg 1$, of the laser focal spot.

This paper focusses on GeV-scale acceleration due to its relevance to current experimental frontiers of the field of LPAs. Experimental data in [21] ($\approx$ 2 GeV beam) and [22] ($\approx$ 1 GeV beam) are used to further detail the mismatched regime.

The predicted accelerated beam energies from (13) in comparison with (2) for laser and plasma parameters of the experiments in [21] (in Table I) are shown in Fig. 3. The peak energies predicted by (2) for $n_0 = 1.5 - 3 \times 10^{18} \text{cm}^{-3}$ are $\lesssim 1$ GeV, whereas the experiments obtained energy $> 2$ GeV. Thus, at lower and higher intensities, the disagreement between the predictions of (2), from the 3-D simulation-based matched the regime model in [10] and the experiments grow.

This significant disagreement between the energies predicted in (2) and the experimentally obtained energies is because the laser-plasma acceleration process in [21] corresponds to a "strongly mismatched regime." This is evident from (1), the matched $w_0$ for this interaction at the incident $a_0 = 10.3 \mu$m and for incident laser energy is 16.4 $\mu$m at $n_0 = 2 \times 10^{18} \text{cm}^{-3}$. In contrast, the launched elliptical laser spot size has its minor-axis waist size of 37.4 $\mu$m (a factor of 4 mismatch).

It is important to note that in [21], the matched spot size is 16.4 $\mu$m for a laser energy of 10 J. In addition, the experiments reported at this matched spot size in [22] with 10-J laser energy only observed 100-MeV energy gain at $n_0 < 5 \times 10^{18} \text{cm}^{-3}$.

The adjusted-$a_0$ model in (13) is used to calculate the expected electron energies for parameters of the experiments in [21] (in Table I). In these experiments, a linearly polarized laser with $a_0 \approx 1.9$ and spot size $W_0 = 37.4 \mu$m is incident on a plasma with density $n_0 = 2 \times 10^{18} \text{cm}^{-3}$. The electron beam energy expected from (2) is $< 1$ GeV, whereas from experiments energy gain $\Delta E = 2.2$ GeV. Using the adjusted-$a_0$ model (13), the predicted energy gain is $\Delta E [a_0(\text{adj.})] = 7.4$ GeV.

For the parameters given in Table I, the value of average accelerating plasma field in [10] is $E_{\text{acc}}(a_0) = 1.9 = 0.5(a_0)^{1/2}m_eC_0a_0e^{-1} = 93.7 \text{ GV}^{-1}$ but with the adjusted-$a_0$ model, it is $E_{\text{acc}}(a_0[\text{adj.}] = 7.4)) = 185 \text{ GV}^{-1}$. Note that, at $2 \times 10^{18} \text{cm}^{-3}$, the wave-breaking field is $E_{\text{wb}} = m_eC_0a_0e^{-1} = 136 \text{ GV}^{-1}$. The adjusted-$a_0$ model accurately predicts even in this case, because in the concerned experiments, the peak beam energy of 2.2 GeV is gained over a total of 13 mm (length up to the injection point in not accounted). This gives an average acceleration gradient of $\approx 170 \text{ GV}^{-1}$. It is found that that the peak plasma fields in this regime are of the order of $a_0(\text{adj.}) \times m_eC_0a_0e^{-1} = 1006 \text{ GV}^{-1}$. Peak fields of 800 $\text{GV}^{-1}$ are observed in simulations [see Fig. 4(b)] where data are only available in increments of 250 fs.

Not surprisingly, an agreement with the energy-gain predictions of the adjusted-$a_0$ model is also obtained for experiments reported in [22]. At $5.5 \times 10^{18} \text{cm}^{-3}$ with $a_0 = 3.9$, the matched $w_0$ is 8.95 $\mu$m, whereas the launched
in [21], only with 2.5-D simulations. The experiments in [3] and [21] with entirely physical mechanisms in two different experiments that use a model is established by the proof of the equivalence of plasma-interaction dynamics. It also shows that these mechanisms significantly differ from the matched spot size.

However, not all the processes that play a role in electron acceleration to high energies are fully explained with the adjusted-\(a_0\) model. The important observations of optimal densities range \(n_0 = 1.5 - 3 \times 10^{18} \text{ cm}^{-3}\) for the experiments in [21] and a similar optimal range \(n_0 = 5 - 7 \times 10^{18} \text{ cm}^{-3}\) in [22] raises many questions. It is clear from above that the adjusted-\(a_0\) model is not applicable over a broad range of densities.

IV. MULTIDIMENSIONAL PIC SIMULATIONS

In this section, 3-D and 2.5-D PIC (two spatial and three velocity dimensions) simulations with parameters from accessible experimental data are presented using the EPOCH code [30]. An analysis of the simulations reveals the processes that underlie the acceleration mechanism in the strongly mismatched regime and shows the complex laser–plasma interaction dynamics. It also shows that these mechanisms significantly differ from the matched regime.

The general applicability of the strongly mismatched regime model is established by the proof of the equivalence of physical mechanisms in two different experiments that use the strongly mismatched regime in [3] and [21] with entirely different laser and plasma parameters. The experiments in [3] are modeled with 3-D and 2.5-D PIC simulations, whereas in [21], only with 2.5-D simulations.

The validity of the methodology of using \(2 \times a_0\) in 2.5-D simulations for equivalence to 3-D PIC simulations and thus to the experiments is proved by the comparison of this simulation in [3]. From the movies in the Supplementary Materials that compare the evolution of electron density, laser field, and plasma field, an excellent agreement is found between \(2 \times a_0\)-2.5-D and 3-D PIC simulations. A good agreement is also found in the evolution of the beam energy spectra for 2.5-D and 3-D simulations (movies provided). Further validation of the equivalence of the 2.5-D and 3-D PIC simulations is established by a good agreement between the evolution of the bubble size and the laser pulse length also in the Supplementary Materials. In \(2 \times a_0 - 2.5\)D PIC simulation, the initially boosted vacuum-\(a_0\) primarily accounts for the squeeze-phase peak plasma-\(a_0\) expected from the nonlinear envelope equation.

The two signature processes of the strongly mismatched regime, strong optical shock and bubble elongation, are clearly evident in 3-D and 2.5-D simulations of the experiments in [3] and [21]. These are shown to agree with the location of the squeeze phases predicted from the nonlinear envelope equation in (10) (for [3] shown in the Supplementary Material). The correspondence of the slicing of the laser pulse that drives the optical shock in 3-D and 2.5-D PIC simulations in [3] is shown using Wigner–Ville transform snapshots in the Supplementary Materials. The elongation of the bubble is also evident in both these simulations from the movies.

The simulations are set up in a moving simulation box that tracks the laser pulse at its unperturbed group velocity. A linearly polarized laser with a Gaussian envelope is initialized such that it entirely enters the box before the box starts moving. Absorbing boundary conditions are used for both the fields and particles. The laser pulse is incident from the left boundary (using a laser boundary condition) and propagates in 50 \(\mu\)m of free-space before it focusses onto the plasma with a spot size equal to the minor axis of the elliptical focal spot before the box starts to move.

In 2.5-D simulations, a Cartesian grid is used with 25 cells per laser wavelength (\(\lambda_0\)) in the longitudinal direction and 15 cells per laser wavelength in the transverse. In 3-D simulations, the longitudinal direction is resolved with 22.5 cells per laser wavelength and the two transverse directions with 7.5 cells per laser wavelength. The 2.5-D simulations are initialized with 4 particles per cell and 3-D with 1 particle per cell.

A gas jet is simulated with a linear density gradient of 50 \(\mu\)m before the homogeneous plasma, whereas the gas cell has a 500-\(\mu\)m linear density gradient to mimic experimental profiles.

A. Novel Dynamics of Laser–Plasma Interaction

The laser energy evolution does not exhibit any correlation with the electron beam energy or possess any specific signature of the laser–plasma interaction process in the mismatched regime. Interestingly, the energy loss over the first “squeeze” phase is relatively small, which allows this regime to be useful as an efficient optical plasma lens and the adjusted-\(a_0\) model to be valid.

An analysis of the evolution of the laser–plasma dimensions and fields as shown in Fig. 4 is much more instructive. Fig. 4(a) shows the evolution of dimensions and Fig. 4(b) shows the evolution of fields for \(n_0 = 2 \times 10^{18} \text{ cm}^{-3}\). Enumerated in the following are several laser–plasma effects of interest that are inferred by the study of this evolution.

1) The laser pulse intensity-FWHM waist size [in blue in 4(a)] launched at 44 \(\mu\)m is squeezed down to a minimum spot size of \(\sim 10 \mu\)m in 10 ps (\(\sim 3\)mm) in a good agreement with (10) and Fig. 1. This process of the initial focal spot nearly squeezing down to the incident-intensity matched spot size occurs over a wide range of densities.

2) The laser pulse radial envelope oscillates due to the strong initial mismatch. However, the spot size remains close to the matched spot size that corresponds to the squeezing down of the laser energy to around 15 \(\mu\)m on average. The maximum radial excursions are less than half the launched spot size. More importantly, these are all precursors to the successive triggering of a state of “strong optical shock,” which inhibits free radial
expansion predicted by the nonlinear envelope equation (10). This radial confinement explains the agreement of the experiments in the strongly mismatched regime to the adjusted-\(a_0\) model, which is based on the laser energy squeezing to the matched spot size.

3) The laser pulse longitudinal or temporal envelope undergoes events of “catastrophic collapses.” This is inferred from the evolution of field-FWHM time duration of the laser pulse (in red) over time. There are about four such events shown in Fig. 4(a) around 4.2, 6.3, 8.7, and 12 mm. A rapid collapse of the laser time-FWHM indicates the triggering of a strong “optical shock” due to slicing of the laser. This leads to a sharp laser-front edge.

4) This laser slicing effect is observed to correspond with a rapid increase in the bubble length. The length of the bubble is initially equal to its radius. However, as the laser radial envelope squeezes the bubble length rapidly increases while its radius remains almost constant, as seen from the comparison of the blue and black curves [in 4(a)]. This is due to the optical shock-driven rapid elongation of the bubble.

5) The triggering of optical shock state and the excitation of rapid bubble elongation directly correspond to the injection of electrons in the back of the lengthened yet radially stable bubble.

The laser–plasma interaction effects that underlie the acceleration mechanism are also reflected in the evolution of the laser and plasma fields, as shown in Fig. 4(b). It is observed that the peak plasma field occurs when the laser pulse temporal field-FWHM starts to undergo a sudden collapse. The triggering of a strong optical shock drives a rapid bubble elongation with a peak plasma field of \(-800\) GV/m at around 6.3 mm. The highest energy bunches are injected as the bubble rapidly elongates in response to the rapid increase in longitudinal ponderomotive force from the steep rise in the intensity at the head of the optical shock

\[
E_{\text{equiv}}(x, r) \propto I(x, r) \lambda_0^2(x, r)
\]

Spherical bubble: \(\nabla^2_{\|} E_{\text{equiv}}(x, r) \simeq \nabla r E_{\text{equiv}}^2(x, r)\)

Elongated bubble: \(\nabla^2_{\|} E_{\text{equiv}}(x, r) \gg \nabla r E_{\text{equiv}}^2(x, r)\). (14)

This is a novel injection mechanism due to the imbalance between the longitudinal and radial ponderomotive forces, a condition represented in (14), where \(E_{\text{equiv}}(x, r)\) is the energy of plasma electron quiver motion in the laser field. The ponderomotively driven electrons have different longitudinal and radial oscillation periods. The injection occurs due to the lower radial momentum of the electrons pushed by the low-intensity sliced part of the pulse ahead of the shock. In particular, as these electrons return to the bubble axis much ahead of the electrons driven by the optical shock, they experience the shock-driven bubble fields.
At 6.3 mm (around 24 ps) as shown in Figs. 4(b) and 6, the optical shock is excited by slicing the laser close to the peak laser field, and the laser energy is still high enough in its evolution to cause the strongest disbalance between the radial and longitudinal forces (ponderomotive force evolution is in the Supplementary material).

A clear insight is also developed into the reasons behind an optimal density range in the mismatched regime for the highest energy gain using Fig. 5. In Fig. 5(a), the laser wavelength at the maximum laser field is shown for different densities with the optimum at \( n_0 = 2 \times 10^{18} \text{ cm}^{-3} \). It is observed that at the lower end of the optimum density range (\( n_0 = 9 \times 10^{17} \text{ cm}^{-3} \), in red), the wavelength change is slow and a jump in wavelength that corresponds with a shock occurs only around 12.5 mm, where the laser has significantly depleted. On the other hand, at the higher end of the optimum density range (\( n_0 = 5 \times 10^{18} \text{ cm}^{-3} \), in green), the triggering of shock occurs multiple times and thus rapidly depletes the laser pulse.

This is further demonstrated by the evolution of peak-\( a_0 \) in Fig. 5(b) for different densities. At the lower density end of optimum, the value of \( a_0 \) increases too slowly (from the initial \( a_0 = 1.9 \)), and for \( n_0 = 9 \times 10^{17} \text{ cm}^{-3} \), its average value over 20 mm is \( \langle a_0 \rangle = 5.4 \). At the higher density end of optimum, the value of \( a_0 \) initially increases too rapidly and falls to well below its initial value before 10 mm, with the average over 20 mm being \( \langle a_0 \rangle = 3.6 \). For the optimum density at \( n_0 = 2 \times 10^{18} \text{ cm}^{-3} \), the average value of \( \langle a_0 \rangle = 6.7 \), which is quite comparable to the value arrived at in the adjusted-\( a_0 \) model of 7.4.

In the matched regime that is simulated here with \( a_0 \sim 5 \) and \( \omega_0 = 16.4 \mu \text{m} \) [for matched parameters used in nonlinear envelope equation (10), minimum envelope oscillations are expected], the rate of wavelength change is much slower than in the mismatched regime. The average value of \( a_0 \) in the matched regime over 20 mm is much lower in the matched regime at \( \langle a_0 \rangle = 5.1 \) compared to the optimum for the mismatched regime at \( \langle a_0 \rangle = 6.7 \).

Here, the wavelength is calculated from the equation:

\[
\beta_g = \beta_{\text{g-\text{las}}}^{-1} \approx \beta_{\text{g0}} \left[ 1 + \frac{1}{2} \left( \frac{\langle a_\perp \rangle^2 - \langle \delta n \rangle}{n_0} \right) \right]
\]

\[
\beta_{\text{g0}} = \sqrt{1 - \frac{\omega_{\text{pe}}^2}{c^2}}, \quad \gamma_{\text{g0}} = \frac{\omega_0}{\omega_{\text{pe}}}.
\]

The third-order perturbative expansion-based relation for the laser pulse group velocity [31] in a quasi-static plasma wake with local parameters of plasma (\( \delta n(\xi, r)/n_0 \)) and laser (\( \alpha(\xi, r) \)) dictates the local group velocity as given in (15) (\( \xi = c_\beta g_0 t - z \) is a coordinate that copropagates with the laser). This equation is used to treat spatially localized laser–plasma interaction because it handles group velocity \( \beta_g(\xi, r) \) at each point in space in the co-moving coordinate.

This relation in (15) is used to estimate a locally zero group velocity condition given in the following equation:

\[
\beta_g(\xi, r) = 0 \quad \frac{1}{2} \left( \frac{\delta n}{n_0} - \langle a_\perp \rangle^2 \right) \approx \frac{n_c}{n_0}.
\]

Although the zero local envelope velocity condition is a mathematical construct because, in this work, the typical initial plasma density is around \( n_c/n_0 \approx 30 \), it does essentially demonstrate that the group velocity of a pulse in different parts of the wake significantly differs. It is quite evident that if \( \delta n(\xi, r) \rightarrow n_c \) when \( \langle a_\perp(\xi, r) \rangle^2 \rightarrow 0 \), then the local group
velocity, $\beta_g(\xi, r) \rightarrow 0$. This implies that a part of the envelope gets slowed down much more in comparison to the rest of the pulse, and thus, this part of the envelope is lost.

This leads to “slicing of the laser” into two distinct pulses under the conditions mentioned above. The laser, by defocusing, loses a $c/\omega_{pe}$ of its head while undergoing compression but here the process is longitudinal and related to density spike and near-zero laser group velocity. It should also be noted that $\beta_g(\xi, r) \rightarrow 0$ implies $\beta_{\phi-\text{las}}(\xi, r) \rightarrow \infty$, which means $\lambda_{\text{las}}(\xi, r) \rightarrow \infty$. Therefore, an increase in the wavelength in a local region implies a local reduction in the group velocity.

The time evolution of the on-axis laser field from PIC simulations is shown for one such event in Fig. 6, which corresponds to the triggering of an optical shock at 21 ps and its formation based on the completion of slicing at 25 ps. A rapid increase in the laser wavelength for $n_0 = 2 \times 10^{18} \text{ cm}^{-3}$ is shown with the average wavelength plotted in Fig. 5(a).

Here, around 21 ps, the laser wavelength has rapidly jumped to 1.2 $\mu$m, from the initially launched value of 0.8 $\mu$m.

The laser–plasma interaction dynamics that underlies “laser slicing” is also shown in parameters other than the on-axis dynamics in Fig. 6. These parameters are the radial intensity-FWHM in Fig. 4(a), the laser field in Fig. 4(b), and the laser wavelength in Fig. 5(a).

Fig. 6 shows the laser–plasma interaction dynamics in the front of the bubble. It is shown that, at 17 ps, the wavelength in the front of the wake, a region collocated with max-$\delta n/n_0$ (where the longitudinal ponderomotive force is the highest), begins to increase. This time also corresponds to a rapid surge in the laser electric field, and thus, the ponderomotive force rapidly increases. This also leads to an increase in the max-$\delta n/n_0$ at the laser head. At 21 ps, in the region of max-$\delta n/n_0$, the wavelength has significantly stretched. This corresponds to a rapid reduction in the local group velocity. At 25 ps, the laser envelope is broken into two distinct regions separated by a long wavelength, low-group velocity cycle. These laser cycles of long-wavelength and low group velocity lead to the detachment of the head of the laser pulse from it and the triggering of optical shock state. The duration of the persistence of sliced laser is also the time where the laser longitudinal ponderomotive force becomes largely imbalanced with the radial ponderomotive force.

The large imbalance between the laser longitudinal and radial ponderomotive forces is seen to have a direct effect on the length and the radial envelope of the bubble. The bubble length is seen to grow much more than the bubble radius. The bubble elongation driven by the large longitudinal ponderomotive force has a direct effect on the peak longitudinal plasma field that increases to around $-800$ GV/m at 25 ps, as shown in Fig. 4(b).

The bubble elongation that follows an optical shock excitation also drives the self-injection of a large amount of charge on to the bubble axis. Because the injection of charge occurs when the laser is in the state of an optical shock, the injected charge experiences much higher peak plasma field and accelerates to peak energies in less than a centimeter. The elongated bubble also has a longer dephasing length while it lasts in the elongated state.
The estimated effective geometrical emittance for the high-energy component of the beam is 
\[ \varepsilon_p = \text{rms-}y_p \times \text{rms-}\theta_p \cong 10^{-3} \text{ mm-mrad} \text{. This corresponds with normalized emittance } \varepsilon_{p-n} = \gamma_p \times \varepsilon_p = 0.04 \text{ mm-mrad.} \]

This observation of distinct properties of the particles at the peak energy in comparison to all the particles above 50 MeV points toward the adiabatic damping effect of the geometrical emittance of particles as they undergo acceleration in the plasma. It is also of interest to note the conservation of the emittance of the highest energy particles of the beam, which is seen in the anticorrelation between \( \text{rms-}y_p \) and \( \text{rms-}\theta_p \) shown in Fig. 8(a) and (b). Thus, the multi-GeV component of the laser–plasma-accelerated beam in the strongly mismatched regime behaves like a high-quality conventional particle beam.

**V. Conclusion**

A strongly mismatched regime that underlies many groundbreaking self-guided LPA experiments is modeled for the first time using the theoretical and large-scale computational analysis. The physical mechanisms that underpin laser evolution, electron beam injection, and acceleration in this regime are shown to significantly differ from the well-established perfectly matched regime model.

Two new signature physical processes of this regime, optical shock excitation and bubble elongation, have been investigated. The excitation of a strong optical shock, distinct from the well-known etching mechanism, occurs in the shortened laser spot-size squeeze phase due to the rapid density build up in the front of the bubble. It is also shown to lead to the second signature process of rapid bubble elongation. This is due to the longitudinal component of the ponderomotive force of the optical shock state of the laser far exceeding the radial component of the ponderomotive force. A novel self-injection method due to bubble elongation is shown to inject a high-quality beam on-axis with unique properties.

Therefore, launching larger focal-spot laser pulses in the strongly mismatched self-guiding regime based upon the underlying acceleration mechanisms uncovered here is a novel approach to produce high-energy beams with a large net self-injected charge of high transverse quality and higher overall laser-to-beam efficiency.

Electron beams of this type will be useful for future work on laser positron acceleration [12] and other applications that do not always require a quasi-monoenergetic electron beam.

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