Research and Realization of Seamless Splicing of Quasi-geoid

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Abstract Considering the differences in area, precision, datum and resolution existing among the national, provincial and urban quasi-geoids, some methods such as datum transformation and weighting according to precision are presented in this paper to normalize the quasi-geoid models with different areas, precisions, datums and resolutions. In this way, the seamless splicing of quasi-geoid is realized.

Keywords systematic difference; datum transformation; splicing of quasi-geoid

Introduction

The global navigation satellite system (GNSS) has brought about a profound revolution in the field of navigation and positioning, and has played an important role in the development of geospatial information. The height obtained by the global positioning technique is the ellipsoidal height $H$, while the elevation system adopted in China is the normal height $h$. The difference between them is called height anomaly which is presented by $\xi = H - h$, as shown in Fig.1.

In recent years, the precision of GPS-derived ellipsoidal height is quite high. If a quasi-geoid model $\xi$ with high precision is obtained, the transformation from GPS ellipsoidal height to the leveling height will be realized and the modernization of the elevation survey will be achieved\(^1\). In the 10\(^{th}\) Five-Year Plan of the National Fundamental Surveying and Mapping in China, the State Bureau of Surveying and Mapping definitely requires that terrestrial gravity data, GPS/leveling data, digital terrestrial elevation model data and satellite altimetry, airborne gravity data, and gravity gradient measurement techniques should be comprehensively utilized to improve the

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precision of quasi-geoid determination to ±5-10 cm for the eastern part of China, ±10-20 cm for the middle and western part, and ±20-30 cm for the western mountainous areas, so that in medium-small scale mapping, the burdensome leveling work is practically replaced by GPS altimetry.

Researchers from Wuhan University and some other research institutions have solved many difficult technical problems in local quasi-geoid determination after many years of intensive study. The determination of the local quasi-geoid for the state as well as some provinces and cities has been accomplished\(^{[2-5]}\). However, the quasi-geoid models determined by different departments in different periods are not the same in area, precision, datum and resolution. These differences bring about many problems to the application and popularization of the quasi-geoid\(^{[2]}\). It is of great significance to study the normalization of the quasi-geoid in different areas with different precisions. This paper utilizes the existing quasi-geoid refining results to form a new generation of the national quasi-geoid numerical model which has a uniform datum and resolution as well as a precision higher than CQG2000 by means of the splicing method presented in this paper, without adding massive field observation.

1 Status of local quasi-geoid determination in China

As early as the 1990s, China commenced the study on the determination of local quasi-geoid. Since then, China has established The Technical Manual for Provincial Quasi-geoid Determination and accomplished the determination of the local quasi-geoid in Hainan, Jiangsu, Hebei, Qinghai, Guangdong, Guangxi, Gansu, and Shanxi provinces and cities such as Wuxi, Qingdao, Changzhou, Changzhi, Shouzhou, Datong, Jinzhong, Songbei, Dongguan, Guangzhou, Shenyang, Putian, Zhenjiang, and Wuhan\(^{[6]}\). Some typical examples are as follows.

1) The new generation of Chinese quasi-geoid CQG2000. The resolution is 15′×15′. The accuracy is ±0.5 m in places north of 36°N and west of 108°E, ±0.3 m in places north of 36°N and east of 108°E, ±0.6 m in places south of 36°N and west of 108°E, ±0.3 m in places south of 36°N and east of 108°E.

2) Hainan Quasi-geoid. It is the first provincial quasi-geoid in China with a resolution of 2′30″×2′30″. The external accuracy is ±9.5 cm.

3) Guangxi Quasi-geoid. It was established in 2006 with a resolution of 2′30″×2′30″. The external accuracy is ±3.3 cm. It has been the most accurate provincial quasi-geoid in China so far.

4) Dongguan Quasi-geoid. It was established in 2005 with a resolution of 2′30″×2′30″. The external accuracy is ±0.6 cm. It has been the most accurate urban quasi-geoid in China so far.

The quasi-geoid models were established by different departments (including the Bureaus of Surveying and Mapping, Planning Bureaus and Land Resources Bureaus) in different periods, and there are differences in terms of area, precision, datum and resolution. The differences are listed in Table 1.

| Local quasi-geoid          | Area (km²) | Precision (m) | Datum          | Resolution       |
|---------------------------|------------|---------------|----------------|------------------|
| National quasi-geoid      | 9600000    | ±0.3−±0.6     | ITRF93         | 15′×15′          |
| Provincial quasi-geoid    | 50000-500000 | ±0.05−±0.08   | ITRF93, ITRF97 | 2′30″×2′30″      |
| Urban quasi-geoid         | 2000-10000 | ±0.01−±0.02   | ITRF97, ITRF2000 | 2′30″×2′30″, 2′×2′ |

2 Splicing method of quasi-geoid models

The result of a quasi-geoid model is the height anomaly at a grid point within a certain range with a certain resolution which is presented as \((B, L, \xi)\). The height anomaly out of the given range is usually denoted as 9999.000. The splicing of quasi-geoid mod-
els is actually the normalization of height anomalies at regular distributed grid points.

To splice quasi-geoid models with different areas, precisions, datums and resolutions, the crucial problems are presented as follows.

2.1 Normalization of the geodetic height datums

In order to replace order leveling with GPS ellipsoidal height, the core problem is to define a local quasi-geoid model, that is, to determine the height anomaly with high precision. Considering \( \xi = H - h \), the datum of height anomaly is determined by the geodetic and leveling height datum. The height anomaly at a certain point varies with respect to different geodetic height datums. Therefore, in order to splice quasi-geoid models, different geodetic height datums should be normalized first.

At present, spatial datums of quasi-geoid models applied in provinces and cities are mainly the ITRF93, ITRF97, and ITRF2000 framework at a certain reference epoch. In fact, the coordinate difference under the ITRF93, ITRF97, and ITRF2000 framework referring to a certain epoch shows up as a kind of systematic difference. The process to eliminate this kind of systematic difference is to normalize the spatial datums. It is realized by the following 7-parameter transformation method:

\[
\begin{bmatrix}
X_2 \\
Y_2 \\
Z_2
\end{bmatrix} = (1 + m)
\begin{bmatrix}
1 & e_x & -e_y \\
-e_y & 1 & e_x \\
-e_x & -e_y & 1
\end{bmatrix}
\begin{bmatrix}
X_1 \\
Y_1 \\
Z_1
\end{bmatrix} +
\begin{bmatrix}
\Delta X_0 \\
\Delta Y_0 \\
\Delta Z_0
\end{bmatrix}
\]

(1)

where \( \Delta X_0, \Delta Y_0, \Delta Z_0 \) are the three translation parameters; \( e_x, e_y, e_z \) are the three rotation parameters; \( m \) is the scale parameter; \( X_1, Y_1, Z_1 \) and \( X_2, Y_2, Z_2 \) denote the rectangular spatial coordinates in two different spatial datums.

After computing the transformation parameters \( \Delta X_o, \Delta Y_o, \Delta Z_o, e_x, e_y, e_z, m \) by at least three common points, the grid values \( (B, L, \xi) \) in the quasi-geoid grid model file are fixed as \( (B, L, 0) \). The grid values \( (B, L, 0) \) should be converted to rectangular spatial coordinates \( (X, Y, Z) \) first. Then the rectangular spatial coordinates \( (X', Y', Z') \) in the normalized datum are calculated by applying the transformation parameters computed beforehand. In the next step, the rectangular spatial coordinates \( (X', Y', Z') \) should be converted to their corresponding geodetic coordinates \( (B', L', \Delta H) \). A point’s plane coordinates in different frames and at different reference epochs have only slight difference of which the effect on height anomaly can be ignored. Therefore, \( \Delta H \) can be seen as the change of the quasi-geoid grid model caused by the normalization of the geodetic height datums and it is expressed as \( \xi' = \xi + \Delta H \).

2.2 Normalization of the leveling height datums

Because of historical reasons, the vertical systems adopted in China are mainly the Huanghai vertical datum of 1956 and the national vertical datum of 1985. In addition, there are some vertical systems defined by local mean sea level, such as the Wusong vertical datum. After the normalization of the geodetic height datums by 7-parameter transformation, the leveling height datums also need to be normalized. The difference among vertical systems is mainly caused by the inconsistency of the mean sea levels used to define the vertical systems. This difference is a kind of systematic difference which is expressed by \( h' \). Here \( h' \) is seen as the change of the quasi-geoid grid model caused by the normalization of the leveling height datums. After the normalization of the geodetic and leveling height datums, the new quasi-geoid grid model is expressed as \( \xi'' = \xi + \Delta H + \Delta h \).

2.3 Normalization of the resolutions of quasi-geoid models

For quasi-geoid with different areas, the resolutions may be different. For example, the resolution of the national quasi-geoid is 15’×15’, while the resolution of the provincial and urban quasi-geoid is 2’30”×2’30” or 2’×2’. The normalization of resolutions should be accomplished before splicing quasi-geoid models with different resolutions. The bilinear interpolation method is applied to interpolate for grid models with lower resolution to form a new grid model that has the same resolution with high resolution grid models.
2.4 Splicing of quasi-geoid models in the same area with different precisions

After the normalization of the geodetic height datums, leveling height datums and resolutions, the splicing of quasi-geoid models in the same area but with different precisions is accomplished by means of weighting according to precision.

The nominal precision of a quasi-geoid model is directly used to decide weight. The additional checking points are set to do external checks and compute the external accuracy, which is applied to decide weight. Finally, the quasi-geoid models with different precisions are spliced by the weighted average method.

Assuming that at a specific grid point, the height anomaly in quasi-geoid model A is $\xi_A$ with a precision of $\sigma_A$, whereas the height anomaly in quasi-geoid model B is $\xi_B$ with a precision of $\sigma_B$. Then the weighted height anomaly at this point is $\xi_{AB} = \frac{\xi_A \cdot \frac{1}{\sigma_A^2} + \xi_B \cdot \frac{1}{\sigma_B^2}}{\frac{1}{\sigma_A^2} + \frac{1}{\sigma_B^2}}$.

2.5 Splicing of quasi-geoid models in different areas

With the geodetic height datums, leveling height datums and resolutions normalized, in order to splice quasi-geoids in different areas, the problem considered mostly is the overlapping area in splicing. If there are grid model values in the overlapping area, the quasi-geoid models are directly spliced by means of weighting according to precision. If there is no grid model value in the overlapping area, that is, blank areas in the overlapping part exist, the blank areas should be patched by collocation algorithm first. Then the quasi-geoid models are spliced by weighting according to precision.

To splice the quasi-geoid models in different areas, the specific constraining conditions are applied such as the quasi-geoid values in the overlapping area should be the same. The local quasi-geoid models are adjusted by forcing the boundary to satisfy the constraining conditions. This process should be repeated to extend the splicing area and finally the complete splicing of quasi-geoid models will be achieved.

3 Conclusion

In this paper, the practical significance in seamless splicing of quasi-geoid in different areas with different precisions is presented based on the situation that the quasi-geoid models established by different departments in different periods have differences in area, precision, datum and resolution. A systematic research is made on the crucial problems for the splicing of quasi-geoid models including the normalization of the geodetic height datums, leveling height datums and resolutions, the splicing of quasi-geoid models in the same area but with different precisions, and the splicing of quasi-geoid models in different areas. A practical method of seamless splicing of quasi-geoid models is formed and it may have great significance in the popularization and application of the quasi-geoid achievements.

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