Dynamical quantum phase transitions in non-Hermitian lattices

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In closed quantum systems, a dynamical phase transition is identified by nonanalytic behaviors of the return probability as a function of time. In this work, we study the nonunitary dynamics following quenches across exceptional points (EPs) in non-Hermitian lattice realized by optical resonators. Dynamical quantum phase transitions with topological signatures are found when an isolated exceptional point is crossed during the quench. A topological winding number defined by a real, noncyclic geometric phase is introduced, whose value features quantized jumps at critical times of these phase transitions and remains constant elsewhere, mimicking the plateau transitions in quantum Hall effects. This work provides a simple framework to study dynamical and topological responses in non-Hermitian systems.

Introduction.− Dynamical quantum phase transitions (DQPTs) are characterized by nonanalytic behavior of physical observables as functions of time [1, 2]. These transitions happen in general if the system is ramped through a quantum critical point. As a promising framework to classify quantum dynamics of nonequilibrium many-body systems, DQPTs have been studied intensively in recent years [3–19]. The generality and topological feature of DQPTs were demonstrated in both lattice and continuum systems [20], across different spatial dimensions [21–25], and under various dynamical protocols [26–28]. The defining features of DQPTs have also been observed in recent experiments [29–31].

Following the initial proposal, most studies on DQPTs focus on closed quantum systems undergoing unitary time evolution. Efforts have been made to generalize DQPTs to systems prepared in mixed states [32, 33]. However, DQPTs in systems with gain and loss, and therefore subject to nonunitary evolution are largely unexplored. One such class of open systems can be described by a non-Hermitian Hamiltonian. This type of system, realizable in various platforms like photonic lattice [34], phononic media [35], LRC circuits [36] and cold atoms [37, 38], has attracted great attention in recent years due to their nontrivial dynamical [39–48], topological [49–62] and transport properties [63–70]. Many of these features can be traced back to non-Hermitian degeneracy (i.e., exceptional) points mediating gap closing and reopening transitions on the complex plane [71–76]. In this work, we explore DQPTs in non-Hermitian systems, with a focus on topological signatures in nonunitary evolution following quenches across exceptional points (EPs).

Theory.– We start by summarizing the theoretical framework of DQPTs for systems described by non-Hermitian lattice Hamiltonians. The nonunitary time evolution of the system is governed by a Schrödinger equation $i \frac{d}{dt} |\Psi(t)\rangle = H |\Psi(t)\rangle$. For concreteness, we present the formalism with a one-dimensional two-band lattice model in mind, while the generalization to multiple-band systems is straightforward [77].

Consider a system described by Hamiltonian $H = H^\theta(t) + H^\prime(-t)$, where $\theta(t)$ is the step function. At time $t = 0$, the system undergoes a sudden quench with its Hamiltonian switched from $H^\prime$ to $H^\prime$. At time $t > 0$, the return amplitude of the system to its initial state $|\Psi(t)\rangle$ is given by $G(t) = \langle \Psi(t)|e^{-iH^\prime t}|\Psi(t)\rangle$. In our study, the post-quench Hamiltonian $H^\prime$ is non-Hermitian, and the evolution operator $e^{-iH^\prime t}$ is therefore nonunitary. Nontrivial dynamics is expected if $|\Psi(t)\rangle$ is not a right eigenvector of $H^\prime$. Specially, when the evolving state $|\Psi(t)\rangle = e^{-iH^\prime t}|\Psi(t)\rangle$ becomes orthogonal to the initial state at some critical time $t_c$, we have $G(t_c) = 0$. Then the rate function of the return amplitude $f(t) = \lim_{N \to \infty} \frac{1}{N} \ln G(t)$ becomes nonanalytic at $t = t_c$, where $N$ is the number of degrees of freedom of the system. Similar to DQPTs in unitary evolution, we identify the vanishing of $G(t)$ at critical times as signatures of DQPTs in non-Hermitian systems.

In momentum space, the pre-/post-quench Hamiltonian $H^\prime/\pi$ can be expressed as $H^\prime/\pi = \sum_k \mathcal{H}^\prime/\pi_k |k\rangle\langle k|$, where $k \in [0, 2\pi)$ is the quasimomentum and $\mathcal{H}^\prime/\pi_k = d^\dagger_{x,k}(k)\sigma_x + d^\dagger_{y,k}(k)\sigma_y + d^\dagger_{z,k}(k)\sigma_z$ is the Bloch Hamiltonian, with $\sigma_{x,y,z}$ being Pauli matrices in their usual representation. The right eigenvectors $|\psi_{x,y,z}^\pi/\pi_k\rangle$ of $\mathcal{H}^\prime/\pi_k$ satisfy $\mathcal{H}^\prime/\pi_k|\psi_{x,y,z}^\pi/\pi_k\rangle = \pm d^{\dagger}_{x,y,z}(k)\psi_{x,y,z}^\pi/\pi_k$, with $d^{\dagger}_{x,y,z}(k) = \sqrt{|d^\dagger_{x,k}(k)|^2 + |d^\dagger_{y,k}(k)|^2 + |d^\dagger_{z,k}(k)|^2}$. Here we focus on the case in which $\mathcal{H}^\prime_i$ is Hermitian with a gapped spectrum, and the initial state is a Slater determinant of lower band eigenstates $|\psi_{x,y,z}^\pi\rangle$ [78]. The translational symmetry allows us to treat the dynamics of each state $|\psi_{x,y,z}^\pi\rangle$ separately. The return amplitude is then given by $G^\prime(t) = \prod_k \mathcal{G}_{x,y,z}^\prime(t)$, where $\mathcal{G}_{x,y,z}^\prime(t) = \cos (d^\dagger_{x,y,z}(k)t) - i \sin (d^\dagger_{x,y,z}(k)t) \frac{\partial}{\partial t}|\psi_{x,y,z}^\pi\rangle$. In the thermodynamic limit, the rate function of return probability reads

\[
F(-t) = F(t) = -\frac{1}{2}\ln |\mathcal{G}_{x,y,z}^\prime(t)|^2
\]
\[ g^-(t) = -\frac{1}{2\pi} \int_0^{2\pi} dk \ln \left| \mathcal{G}^-(t) \right|^2. \] (1)

A DQPT is expected when a line of Lee-Yang-Fisher (LYF) zeros \([79-81]\), defined as
\[ z_{n,k}^- = \frac{i}{2} \left( n + \frac{1}{2} \right) + \frac{1}{d_k} \arctan \left( \frac{\mathcal{H}_k^f}{\mathcal{H}_k^d} |\psi_k^i| \right) \quad n \in \mathbb{Z}, \] (2)
crosses the imaginary time axis at a critical momentum \( k_c \), yielding a critical time \( t_{c,n} = -iz_{n,k_c}^- \) at which \( \mathcal{G}_k^-(t_c) = 0 \). These conclusions hold for both Hermitian and non-Hermitian systems.

To capture topological signatures of DQPTs in non-Hermitian systems, we study the time dependence of the winding number
\[ \nu_D(t) = \frac{1}{2\pi} \int_0^{2\pi} dk \left[ \partial_t \phi_k^G(t) \right], \] (3)
where the geometric phase of the return amplitude is given by \( \phi_k^G(t) \equiv \phi_k(t) - \phi_k^{\text{dyn}}(t) \), with the total phase \( \phi_k(t) = -i \ln \left[ \frac{\psi_k(t)}{\psi_k(0)} \right] \) and the dynamical phase
\[ \phi_k^{\text{dyn}}(t) = -\int_0^t ds \frac{\langle \psi_k^i(s) | \mathcal{H}_k^f | \psi_k^i(s) \rangle}{\langle \psi_k^i(s) | \psi_k^i(0) \rangle}, \] \[ + \frac{i}{2} \ln \left[ \frac{\langle \psi_k(t) | \psi_k^i(t) \rangle}{\langle \psi_k^i(0) | \psi_k^i(0) \rangle} \right]. \] (4)

It can be shown that the geometric phase thus defined is real and gauge invariant \([82]\). These are the critical properties for us to introduce the real-valued, quantized topological winding number \( \nu_D(t) \) for non-Hermitian systems. Furthermore, the winding number of \( \phi_k^G(t) \) in \( k \)-space will experience a quantized jump at every critical time \( t_c \) when the system passes through a DQPT point \([83]\). As will be shown, the value of \( \nu_D(t) \) will change monotonically in time if the initial state is topologically trivial and if the quench crosses an isolated EP.

**Model.** To demonstrate our theory explicitly, we study a non-Hermitian lattice model introduced in Ref. \([61]\), which may be realized in optical resonators. In momentum representation, the model Hamiltonian is \( \hat{H} = \sum_k \mathcal{H}_k |k \rangle \langle k | \), where
\[ \mathcal{H}_k = h_x(k) \sigma_x + \left[ h_z(k) + \frac{i\gamma}{2} \right] \sigma_z, \] (5)
with \( h_x(k) = \mu + r \cos k \) and \( h_z(k) = r \sin k \). When the non-Hermitian hopping amplitude \( \gamma \) satisfies \( \mu - r < \frac{\gamma}{2} < \mu + r \) or \( \mu - r < -\frac{\gamma}{2} < \mu + r \), the trajectory of vector \( \mathbf{h}(k) = [h_x(k), h_z(k)] \) versus quasimomentum \( k \) will encircle one of the two EPs at \( \mathbf{h}(k) = (\pm \frac{\gamma}{2}, 0) \). In this case, the system is topologically nontrivial and characterized by a winding number defined in a \( 4\pi \) Brillouin zone \([61]\).

We are going to study DQPTs following non-Hermitian quenches in this lattice model. Specifically, we let \( \mathcal{H}_k^f = \mathcal{H}_k^0 + i\frac{\gamma}{2} \sigma_z \), where \( \mathcal{H}_k^0 = h_x(k) \sigma_x + h_z(k) \sigma_z \) is Hermitian. The initial state \( |\psi_k^i \rangle \) at each \( k \) is chosen to be the lower eigenstate of the prequench Hamiltonian \( \mathcal{H}_k^0 \) with energy \( -d_k^0 = -\sqrt{h_x^2(k) + h_z^2(k)} \). Dynamics following a quench in which the vector \( \mathbf{h}(k) \) encircles zero and one EP for both topologically trivial and nontrivial initial states will be studied below \([84]\).

**Results.** Fig. 1 gives a schematic of quench cases studied in this section. In cases 1, 2 and 3, the trajectory of \( \mathbf{h}(k) \) does not encircle the zero of \( h_x-h_z \) plane, implying that the initial state is topologically trivial \([85]\). In case 4, the trajectory of \( \mathbf{h}(k) \) encircles the zero of \( h_x-h_z \) plane, and the initial state is thus topologically nontrivial, characterized by a quantized winding number. We now study the line of LYF zeros \( z_{n,k}^- \), rate function of return probability \( g^-(t) \), geometric phase \( \phi_k^G(t) \) and winding number \( \nu_D(t) \) for nonunitary dynamics following the four quench cases. These quantities provide with us key information of DQPTs in the non-Hermitian lattice.

![FIG. 1. (color online) A sketch of the four quench cases. The hollow triangle denotes the zero of \( h_x-h_z \) plane. The two solid squares represent EPs of the Hamiltonian \( \mathcal{H}^0 \). The dashed circle represents the vector \( \mathbf{h}(k) = (h_x(k), h_z(k)) \). (a) Case 1: the initial state is topologically trivial, \( \mathbf{h}(k) \) encircles the EP \((\frac{\gamma}{2}, 0)\). (b) Case 2: the initial state is topologically trivial, \( \mathbf{h}(k) \) encircles the EP \((-\frac{\gamma}{2}, 0)\). (c) Case 3: the initial state is topologically trivial, \( \mathbf{h}(k) \) does not encircle any EPs. (d) Case 4: the initial state is topologically nontrivial, \( \mathbf{h}(k) \) encircles the EP \((\frac{\gamma}{2}, 0)\).](image)
First, the value of $\nu_D(t)$ is always quantized and changes monotonically in time if the initial state is topologically trivial and the postquench Hamiltonian vector $h(k)$ encircles one of the EP. This provides us with a clear window to look into the topological signature of an isolated EP in system’s dynamics. Following the classification scheme in unitary evolution, we refer to DQPTs with quantized and monotonically changing winding numbers as topologically non-trivial [14]. Second, the quench across the EP in case 1 induces a positive and monotonic increasing $\nu_D(t)$ in time, while the quench across the EP in case 2 induces a negative and monotonic decreasing $\nu_D(t)$. This difference originates from the fact that the trajectory of vector $h(k)$ in cases 1 and 2 crosses the branch cut connecting the two EPs along opposite directions. We therefore find a way to distinguish the two EPs from their topological response to a simple quench in non-Hermitian systems, complementing other existing approaches focusing on quasi-adiabatic evolution [41, 42].

In Fig. 4, we show results of $z_{n,k}^-$ and $g^-(t)$ for quench case 3. In this case, no EPs are encircled by the vector $h(k)$, and lines of LYF zeros have no crossings on the imaginary time axis as shown in Fig. 4(a). This indicates that there is no DQPTs in this case and the winding number remains zero in the nonunitary evolution, as also supported by results reported in Fig. 4(b) and calculations of $\nu_D(t)$. More generally, for a vector $h(k)$ encircling no EPs, each line of LYF zeros could form a closed trajectory, crossing over the imaginary time axis an even number of times [87]. This results in DQPTs with a winding number oscillating in time, and thus can also be regarded as topologically trivial [14].

Results of $z_{n,k}^-$, $g^-(t)$, $\phi_k^O(t)$ and $\nu_D(t)$ for quench case 4 with a topologically nontrivial initial state are presented in Fig. 5. Different from the first three cases of trivial initial states, each line of LYF zeros now features two types of crossing points on the imaginary time axis. One type of them appears once for each line of LYF zeros regularly, while the other type contains a pair of critical times separated by a vanishingly small time window. DQPTs corresponding to the later type of critical times may then be regarded as originating from the memory of an evolving state about its initial topology. Notably, this memory decays fast and becomes quickly indistinguish-
able with the progress of time, leaving only signatures of topologically nontrivial DQPTs following the quench across an EP. These arguments are further supported by the time evolution of winding number shown in Fig. 5(d), where a pair of accidental DQPTs caused by the initial state topology appear at $t \approx 10.6$. The insensitivity of DQPTs in nonunitary evolution to the initial state topology has two implications. First, by choosing simple initial states, DQPTs due to quenching across an EP can be demonstrated analytically in a transparent manner [88]. Furthermore, the non-Hermitian quench may provide with us a useful strategy to generate topologically nontrivial dynamical phases of matter from easy-to-prepare initial states.

In a photonic setup, the non-Hermitian quench may be engineered by introducing gain or loss uniformly to half of the system along the propagation direction of the wave. Then the signature of DQPTs may be identified by measuring and collecting phase factors of each $k$-mode in space. The quantum walk setup reported in Ref. [53] could also be a candidate to experimentally study DQPTs in non-Hermitian systems.

**Conclusion and discussion.** In summary, we have discovered DQPTs in non-Hermitian lattice systems by quenching across EPs. The return probability vanishes when the nonunitary evolution of the initial state reaches a critical time, where the topological winding number displays a quantized jump. Two types of DQPTs were observed and distinguished by the parity of critical times on each line of LYF zeros. Topologically nontrivial DQPTs happen when the postquench vector $\mathbf{h}(k)$ encircles an isolated EP. These transitions are characterized by an odd number of critical times along each line of the LYF zeros, and accompanied by a monotonically changing topological winding number in time. Initial states with nontrivial topology can also leave transient signatures in postquench dynamics, resulting in topologically trivial accidental DQPTs. Whether the qualitative content of these findings hold for more general dynamical protocols like linear ramp and periodic driving, and in higher physical dimensions deserves further studies.

The concept of DQPT is initially introduced as an organizing principle to study dynamics in closed nonequilibrium many-body systems. Discoveries made in this work could serve as a starting point to generalize this concept to open quantum systems, whose dynamical evolutions are usually nonunitary. The geometric phase and winding number introduced here, together with their generalizations to density matrix formalism [32, 33, 88, 89], may also be useful tools to decode geometric and topological order in the nonequilibrium dynamics of many-body open quantum systems.

In some studies concerning full counting statistics phase transitions, quenching protocols with initial states prepared as right eigenvectors of non-Hermitian spin chains are considered [9]. The postquench Hamiltonian there is Hermitian and the dynamics following the quench is unitary, which is opposite to the situation considered in this paper. Also, DQPTs observed there are mainly related to the degeneracy point of a Hermitian system. In another study [49], the time evolution of fidelity in a non-Hermitian topological model is considered. There the focus is on coherence protection in finite-size systems. Interestingly, zeros in the fidelity as a function of certain time-dependent system parameters are also observed. The relationship between these zeros and DQPTs may deserve further explorations.
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[77] See supplementary note 1 for a sketch of the theory applicable to multiple band non-Hermitian lattice models.

[78] See supplementary note 4 for examples under more general prequench Hamiltonians.

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[82] See supplementary note 2 for a proof.

[83] See Ref. [14] for an elaboration on this argument.

[84] Examples under more general conditions will be presented in supplementary notes 3 and 4. Analytic results for another type of initial states will be presented in supplementary note 5.

[85] For a Hermitian $\mathcal{H}_k$, the winding number is given by $\nu = \frac{1}{2\pi} \int_0^{2\pi} dk \left[ \hat{h}(k) \times \partial_k \hat{h}(k) \right]$, where $\hat{h}(k) = h(k)/|h(k)|$ is a vector of unit length. In the topologically nontrivial (trivial) case, the trajectory of vector $\hat{h}(k)$ encircles (does not encircle) the zero of $h_x - h_z$ plane. Then the initial state, given by a Slater determinant of lower eigenstates $|\psi_i\rangle$ of $\mathcal{H}_k$, is topologically nontrivial (trivial) if winding number $\nu = 1 \ (0)$.

[86] Here to show the nonanalytic behavior of $g^-(t)$ around DQPT points more clearly, we computed a renormalized rate function $g(t) \equiv -\frac{1}{2\pi} \int_0^{2\pi} dk \ln[|G_k(t)|^2/|e^{-iH^T_k(t)}|\langle \psi_i^-|\hat{h}_k^-\rangle|^2]$. This renormalized definition does not affect the description of qualitatively behaviors of the system around the DQPT point.

[87] See supplementary note 3 for more examples of DQPTs for non-Hermitian quenches to Hermitian pre-quench Hamiltonians.

[88] See supplementary note 5 for more details.

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