Tetraquark Interpretation of the Charged Bottomonium-like states $Z_{b}^{±}(10610)$ and $Z_{b}^{±}(10650)$ and Implications

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We present a tetraquark interpretation of the charged bottomonium-like states $Z_{b}^{±}(10610)$ and $Z_{b}^{±}(10650)$, observed by the Belle collaboration in the $\pi^{±}Y(nS)$ ($n = 1, 2, 3$) and $\pi^{±}h_{b}(mP)$ ($m = 1, 2$) invariant mass spectra from the data taken near the peak of the $Y(5S)$. In this framework, the underlying processes involve the production and decays of a vector tetraquark $Y_{b}(10890)$, $e^{+}e^{-} \rightarrow Y_{b}(10890)$, followed by the decays $[Z_{b}^{±}(10610), Z_{b}^{±}(10650)] \rightarrow \pi^{±}Y(nS), \pi^{±}h_{b}(mP)$. Combining the contributions from the meson loops and an effective Hamiltonian, we are able to reproduce the observed masses of the $Z_{b}^{±}(10610)$ and $Z_{b}^{±}(10650)$. Our formalism implies mixing between the mass eigenstates and the tetraquark spin states. The analysis presented here is in agreement with the Belle data and provides crucial tests of the tetraquark hypothesis.

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Recently Belle $[1]$ (updating a previous publication $[2]$) reported the measurement of the $\pi^{±}Y(nS)$ ($n = 1, 2, 3$) and $\pi^{±}h_{b}(mP)$ ($m = 1, 2$) invariant mass spectra from the data taken near the peak of the $Y(5S)$ resonance in the processes $e^{+}e^{-} \rightarrow Y(nS)\pi^{+}\pi^{-}$ and $e^{+}e^{-} \rightarrow h_{b}(mP)\pi^{+}\pi^{-}$, in which two charged bottomonium-like states $Z_{b}^{±}(10610)$ and $Z_{b}^{±}(10650)$ are discovered. Hereafter, these states will be abbreviated to $Z_{b}$ and $Z_{b}^{′}$ respectively. The masses and decay widths averaged over the five different final states are in MeV $[1]$: $m_{Z_{b}^{±}} = 10607.2 \pm 2.0$, $m_{Z_{b}^{±}} = 10622.2 \pm 1.5$, $\Gamma_{Z_{b}^{±}} = 18.4 \pm 2.4$, $\Gamma_{Z_{b}^{±}} = 11.5 \pm 2.2$.

The angular distribution analysis indicates that the quantum numbers of both $Z_{b}^{±}$ and $Z_{b}^{±}$ are $I^{G}(J^{P}) = 1^{+}(1^{+})$. These states def a standard bottomonium assignment, as in the valence approximation they consist of four quarks $budd$ (and charge conjugates).

Due to the proximity of the $Z_{b}$ and $Z_{b}^{′}$ masses with the $BB^{*}$ and $B^{∗}B^{∗}$ thresholds $[3]$, it has been proposed that the former could be realized as $S$-wave $BB^{*}$ and $B^{∗}B^{∗}$ molecular states, respectively $[4][10]$. In this scenario, the heavy quark spin structure of the $Z_{b}$ and $Z_{b}^{′}$ is expected to mimic that of the corresponding meson pairs $|Z_{b}^{′}\rangle = (0_{bb}^{−} \otimes 1_{q\bar{q}}^{−} - 1_{bb}^{−} \otimes 0_{q\bar{q}}^{−})/\sqrt{2}$, $|Z_{b}\rangle = (0_{bb}^{−} \otimes 1_{q\bar{q}}^{+} + 1_{bb}^{−} \otimes 0_{q\bar{q}}^{+})/\sqrt{2}$, \(1\)

where $0^{−}$ and $1^{−}$ denotes the para and ortho- states with negative parity, respectively. One anticipates the mass splitting to follow $\Delta m_{Z_{b}} \equiv m_{Z_{b}^{′}} - m_{Z_{b}} = m_{B^{∗}} - m_{B} \approx 46$ MeV, in near agreement with the observed value $\Delta m_{Z_{b}} = (45 \pm 2.5)$ MeV $[11]$. Moreover, the structure in Eq. $[10]$ predicts that $Z_{b}$ and $Z_{b}^{′}$ should have the same decay width, which is approximately in agreement with the data.

Despite these striking patterns, the fact that both $Z_{b}$ and $Z_{b}^{′}$ lie above their respective thresholds by about 2 MeV speaks against a molecular interpretation. In particular, a one-pion exchange potential which would produce such a bound state, does not support an $S$-wave $BB^{*}$ resonance above threshold in an effective field theory $[11]$. Also, the measured total decay widths appear much too large compared to the naively expected ones for loosely bound states, and this suggests that both $Z_{b}$ and $Z_{b}^{′}$ are compact hadrons. In addition, the measured cross sections in question are too big to be interpreted in terms of the decays $Y(5S) \rightarrow (Y(nS), h_{b}(mP))\pi^{±}\pi^{-}$.

In this Letter, we pursue a different ansatz in which the observed processes arise from the production and decays of a vector tetraquark $Y_{b}(10890)$ $[12][14]$, having a (Breit-Wigner) resonant mass of $[10888.4^{+2.7}_{−2.5}(stat) \pm 1.2(syst)]$ MeV and a width of $[30.7^{+8.3}_{−7.0}(stat) \pm 3.1(syst)]$ MeV $[15][16]$. The mass and, in particular, the decay width of $Y_{b}(10890)$ differ from the Particle Data Group entries assigned to the $Y(5S)$ $[3]$. We propose that the states $Z_{b}$ and $Z_{b}^{′}$ seen in the decays of $Y_{b}(10890)$ are themselves charged tetraquark candidates having the flavor configuration $[bu][d\bar{d}]$ (and charge conjugates) (see Refs. $[17][18]$ for earlier suggestions along these lines). Their neutral isospin counterparts with $I_{3} = 0$ have $J^{PC} = 1^{−}$ and their masses were calculated in the effective Hamiltonian approach in $[12]$. Ignoring the small isospin-breaking effects $[12][19]$, $Z_{b}$ and $Z_{b}^{′}$ have the same masses as those of their neutral counterparts. As shown below, these estimates yield a too large value for $\Delta m_{Z_{b}}$ compared to the Belle measurement. We recalculate the masses of $Z_{b}$ and $Z_{b}^{′}$ states by taking into account the meson loop contributions involving the Zweig-allowed two-body intermediate states $BB^{∗}, B^{∗}B^{∗}, h_{b}(mP)\pi, Y(nS)\pi$ and $h_{b}\rho$. Theoretical estimates presented here account for the observed masses; in particular, the precisely measured mass difference $\Delta m_{Z_{b}}$ is reproduced in terms of the partial decay widths of $Z_{b}$ and $Z_{b}^{′}$. In our approach, the mass eigenstates $Z_{b}$ and $Z_{b}^{′}$ are rotated with respect to the tetraquark spin states $\tilde{Z_{b}}$ and $\tilde{Z_{b}^{′}}$, and we determine this mixing angle.

We start with the classification of the $\tilde{Z_{b}}$ and $\tilde{Z_{b}^{′}}$ tetraquark states in terms of the spin and orbital an-
gular momentum of the constituent diquark and antidiquark. A diquark has positive parity and may be a scalar (spin-0, or “good” diquark) or an axial-vector (spin-1, or “bad” diquark) and is assumed to be a color antitriplet $\bar{\mathbf{3}}$. We shall be using a non-relativistic notation to characterize the tetraquark states in which a matrix representation of the interpolating operators is used in terms of the $2 \times 2$ Pauli matrices $\sigma_i$ ($i = 1, 2, 3$): $0_{[QQ]} \equiv Q^T \sigma_2 Q/\sqrt{2}$, $1_{[QQ]} \equiv Q^T \sigma_3 \epsilon Q/\sqrt{2}$ and $0_{Q\bar{Q}} \equiv Q \bar{Q}/\sqrt{2}$, $1_{Q\bar{Q}} \equiv Q\epsilon \bar{Q}/\sqrt{2}$, $Q$ being any quark. The two tetraquark spin states $\tilde{Z}_b$ and $\tilde{Z}_b'$ are represented as

$$
\tilde{Z}_b = (0_{[bq]} \otimes 1_{[bq]} - 1_{[bq]} \otimes 0_{[bq]})/\sqrt{2},
$$

$$
\tilde{Z}_b' = 1_{[bq]} \otimes 1_{[bq]}.
$$

Performing a Fierz transformation, the flavor and spin content in the $b\bar{b} \otimes q\bar{q}$ and $b\bar{q} \otimes q\bar{b}$ product space can be made explicit:

$$
\tilde{Z}_b = (1_{b\bar{b}} \otimes 0_{q\bar{q}} + 0_{b\bar{q}} \otimes 1_{q\bar{b}})/\sqrt{2} = 1_{b\bar{q}} \otimes 1_{q\bar{b}},
$$

$$
\tilde{Z}_b' = (1_{b\bar{b}} \otimes 0_{q\bar{q}} + 0_{b\bar{q}} \otimes 1_{q\bar{b}})/\sqrt{2} = (1_{b\bar{q}} \otimes 0_{q\bar{b}} + 0_{b\bar{b}} \otimes 1_{q\bar{b}}). \quad (3)
$$

Eq. (3) shows that the $\tilde{Z}_b$ and $\tilde{Z}_b'$ have similar coupling strengths with different final states. The labels $0_{[bq]}$ and $1_{[bq]}$ in Eq. (3) can be viewed as $\bar{B}$ and $B^*$, respectively. It follows that $\tilde{Z}_b$ couples to $B^*B^*$ state while $\tilde{Z}_b'$ couples to $BB^*$. We stress that this identification is in contrast with the molecular interpretation, in which the transition $\tilde{Z}_b' \to BB^* + h.c.$ is forbidden by the spin symmetry since $\tilde{Z}_b'$ is assumed to be essentially a $B^*B^*$ molecule. This difference can be tested in the future and is of great importance in order to distinguish between the tetraquark and the hadronic molecule interpretations.

In the effective Hamiltonian approach, the $2 \times 2$ mass matrix for the $S$-wave $1^+$ tetraquarks $M$ is given by

$$
M = \left(2m_{[bq]} + \frac{3}{2}\Delta - \kappa_{q\bar{q}} - \kappa_{b\bar{q}}\right)I + \begin{pmatrix} -a & b \\ b & a \end{pmatrix}, \quad (4)
$$

where $I$ is a $2 \times 2$ unit matrix, $a = \Delta/2 + (\kappa_{bq})_3 - \kappa_{b\bar{q}}$ and $b = \kappa_{q\bar{q}} - \kappa_{b\bar{q}}$. In the above $(\kappa_{bq})_3$ accounts for the spin-spin interaction between the quarks inside the diquark and antidiquark, $\kappa_{q\bar{q}}$ and $\kappa_{b\bar{q}}$ are the couplings ranging from the quarks in the diquark to the antiquarks in the antidiquark, and $\Delta$ is the mass difference between the spin-1 and spin-0 diquarks. Using the default values of the parameters $[12, 13]$ (in units of MeV)

$$
\kappa_{q\bar{q}} = 79.5, \quad \kappa_{b\bar{q}} = 9, \quad \kappa_{bq} = 5.75, \quad (\kappa_{bq})_3 = 6, \quad (5)
$$

yields the diquark mass $m_{[bq]} \approx 5200$ MeV (from the $Y_{1s}(10890)$ mass). The value of $\Delta$ is uncertain, with $\Delta \approx 200$ MeV for the light quarks $[22]$. Reducing its value drastically for the $c$ and $b$ quarks will reduce the level spacing of the corresponding tetraquark states for which the experimental evidence is rather sparse. We adopt a range $\Delta = (120 \pm 30)$ MeV for our numerical calculations. These parameters yield the following values for the two charged tetraquark masses and the mass difference

$$
m_{Z_b} = (10443^{+352}_{-36})\text{MeV}, \quad m_{Z'_b} = (10628^{+352}_{-25})\text{MeV}, \quad \Delta m_{Z_b} = 2\sqrt{a^2 + b^2} = (185^{+352}_{-25})\text{MeV}. \quad (6)
$$

We note that the prediction for $\Delta m_{Z_b}$ given above is much larger than the experimental data, and there is no easy-fix for this mismatch at hand in terms of the parameters in the effective Hamiltonian. We argue that additional contributions to the mass matrix arise from meson loops.

With this premise, the renormalized masses can be obtained by computing the two-point functions. At the one-loop level, the self-energy corrections to the unperturbed propagator $\Sigma(p^2) g_{\mu\nu}$, depicted in Fig. 1, are written as

$$
-i(g^{\mu\alpha} - p^\mu p^\alpha/p^2)i\Sigma(p^2) g_{\alpha\beta} -i(g^{\beta\nu} - p^\beta p^\nu/p^2)/p^2 - M^2, \quad (7)
$$

Taking the $h_0\pi$ state as an example, we find

$$
\Sigma(s) = \frac{g_{Z_b}^{(i)} h_0^\nu h_\pi^\mu}{(4\pi)^2} \int_0^1 dx s x \Lambda \left[1 - \log \left(\frac{\Lambda}{\mu^2}\right)\right], \quad (8)
$$

where $\Lambda = x^2 s - xs + x m_{h_0}^2 + (1 - x) m_{h_b}^2 - i\epsilon$, and the coupling constants appearing above are defined through the hadronic interaction

$$
\mathcal{L} = \epsilon_{\mu\nu\alpha\beta} g_{Z_b}^{(i)\nu} h_\pi^\mu \partial^\alpha \bar{Z}_b^\nu \partial^\beta \bar{h}_b^\pi + h.c.. \quad (9)
$$

In deriving $\Sigma(s)$, the $\text{MS}$ scheme has been used to remove the UV divergence with the scale $\mu \sim m_{Z_b}$. We recall that the real parts of $\Sigma(s)$ contribute to the mass matrix, while the imaginary parts of $\Sigma(s)$ are related to the decay widths of $Z_b$ and $Z_b'$. In particular, the transitions $Z_b \to (\Upsilon(nS)\pi, h_0(mP)\pi, h_\eta(nS)\rho) \to Z_b'$ contribute to the off-diagonal terms in the $2 \times 2$ mass matrix and provide significant effects on the mixing of the two tetraquark-spin eigenstates.

The (genuine) real part of the loop contributions $\text{Re}\Sigma_{\text{gen}}(s)$ can be obtained by a subtraction procedure at some suitable mass scale $s_0$ [24].

$$
\text{Re}\Sigma_{\text{gen}}(s) = \text{Re}\Sigma(s) - \text{Re}\Sigma(s_0). \quad (10)
$$

![FIG. 1. Two-body meson-loop corrections to the function $\Sigma(s)$ defined in Eq. (8). The intermediate states $B^{(*)}B^*$ contribute only to the diagonal terms in the mass matrix while $\Upsilon(nS)\pi, h_0(mP)\pi$ and $h_\eta(nS)\rho$ contribute to both the diagonal and non-diagonal terms.](image-url)
Including the loop corrections, we now have the following structure for the $2 \times 2$ mass matrix

$$M = \hat{M} + \sum_i c_i \left( \frac{\Gamma_i^{Z_b}}{\Gamma_i^B - \Gamma_i^{Z_b}} - \sqrt{\frac{\Gamma_i^{Z_b}}{\Gamma_i^B}} \right),$$

where $i$ runs over the two-body channels shown in Fig. 4; the coefficients $c_i$ are defined as $c_i \equiv -\text{Re} \Sigma(s)/\text{Im} \Sigma(s)/2$, in which $s$ is taken as the physical mass squared from the data. The sign in the $\Upsilon(nS)$ contribution to the off-diagonal terms is reversed due to the spin symmetry as shown in Eq. (3). In the case of open bottom mesons, the $BB^*$ loop impacts on $M_{22}$ while $B^*B^*$ modifies $M_{11}$. Note, that via the optical theorem the imaginary parts are directly related to the decay widths, and our parametrization in Eq. (11) makes this manifest.

Choosing the subtraction point as $s_0 = [(10.385 \pm 0.05)\text{GeV}^2]$, which corresponds to the mass of the lowest ($0^{++}$) tetraquark state, we estimate the following values for the coefficients $c_i$ (ignoring errors on the smaller $c_i$s):

| $c_{b_i(2P)\pi}$ | $c_{b_i(2S)\pi}$ | $c_{b_i(3S)\pi}$ | $c_{b_i(1S)\pi}$ | $c_{b_i(1P)\pi}$ | $c_{b_iB^*}$ |
|----------------|----------------|----------------|----------------|----------------|--------------|
| $45^{+11}_{-10}$ | $-0.01$ | $-0.1$ | $-1.3$ | $-1.1$ | $3 \pm 1$ | $-1.1$ |

In these estimates the Lagrangian $\mathcal{L} = g \sqrt{Z_b^{(*)}} V_{i}^{\nu} Z_{b_i}^{(*)} \pi$ with $V = Y_b, \Upsilon(nS)$ has been adopted. Instead using the form inspired by the chiral symmetry

$$\mathcal{L} = g \sqrt{Z_b^{(*)}} V_{i}^{\nu} \theta^{\nu} \sqrt{Z_{b_i}^{(*)}} \pi,$$

we obtain larger values for the coefficients $c_{\Upsilon(nS)\pi}$:

$$c_{\Upsilon(1S)\pi} = -0.7, \quad c_{\Upsilon(2S)\pi} = -2.1, \quad c_{\Upsilon(3S)\pi} = -6.5.$$

The main reason for the dominance of the coefficient $c_{b_i(2P)\pi}$ is that the limited phase space and the p-wave decay character of $Z_b^{(*)} \rightarrow h_b(2P)\pi$ results in a small value for the partial decay width and also the imaginary part of $\Sigma(s)$. The mass difference of the $1^+$ tetraquarks is approximately given as $\Delta m_{Z_b} = 2\sqrt{a'^2 + b'^2}$, where $a' = a - c_i (\Gamma_i^{Z_b} - \Gamma_i^{Z_b^{(*)}})/2$, $b' = b - c_i \sqrt{\Gamma_i^{Z_b} \Gamma_i^{Z_b^{(*)}}}$ and $i$ denotes $h_b(2P)\pi$, as we keep only the dominant contribution.

The corresponding mass eigenstates are

$$|Z_b^i\rangle = \cos \theta_{Z_b}|\tilde{Z}_b^i\rangle - \sin \theta_{Z_b}|\tilde{Z}_b^i\rangle,$$

$$|Z_b^{(*)}^i\rangle = \sin \theta_{Z_b}|\tilde{Z}_b^i\rangle + \cos \theta_{Z_b}|\tilde{Z}_b^i\rangle,$$

with $\theta_{Z_b} = \tan^{-1}[b'/\sqrt{(a'^2 + \Delta m_{Z_b}/2)}].$

In Fig. 2, we show the constrained partial decay widths from the masses observed by Belle. The left panel shows the constraints on the widths of the tetraquark mass eigenstates $\tilde{Z}_b^i$ for the default values of $\Delta$ and $c_{b_i(2P)\pi}$. In the spin-symmetry limit, $\Gamma_i^{Z_b}$ and $\Gamma_i^{Z_b^{(*)}}$ are equal. As seen in this panel, the resulting contours intersect at two regions, the lower one of which implies large symmetry breaking effects and hence is not entertained any further.

The upper region in which the two couplings differ by approximately 40% is further analyzed. In the right-hand panel, the corresponding constraints on the $Z_b$ and $Z_b^{(*)}$ partial decay widths are depicted. The black region denotes the default values of $\Delta$ and $c_i$ given above, while the extended (red) region is obtained by varying these two parameters, as stated in the text.

![FIG. 2. Constrained partial decay widths from the $Z_b$ and $Z_b^{(*)}$ masses measured by Belle. The left-hand panel shows the constraint on the partial decay widths of the tetraquark eigenstates $\tilde{Z}_b^i$ and $\tilde{Z}_b^{(*)i}$. The circular (green) contour is obtained by the mass difference $\Delta m_{Z_b} = 45 \pm 2.5$ MeV, while the slanted vertical (blue) band results from the averaged mass $m_{Z_b} + m_{Z_b^{(*)}}/2 = 10629.7 \pm 2.5$ MeV for the default values $\Delta = 120$ MeV and $c_{b_i(2P)\pi} = 45$. In the right-hand panel, the corresponding constraints on $Z_b$ and $Z_b^{(*)}$ partial decay widths are depicted. The solid (black) region results from default values, while the extended (red) region is obtained by varying these two parameters, as stated in the text.](image)
The structure of $A_{mZ_b}$ was worked out in the tetraquark picture in great detail in Refs. [13, 14]. As opposed to the amplitudes involved in typical diquark heavy Quarkonia transitions, such as $Y(4S) \rightarrow Y(1S)\pi^+\pi^-$, which are modeled after the Zweig-suppressed QCD multipole expansion [24], the amplitudes for the decays $Y_b(10890) \rightarrow Y(nS)\pi^+\pi^-$ are not Zweig-forbidden, and hence they are significant. In addition, they lead to a resonant structure in the $\pi\pi$ invariant mass spectrum. This is most marked in the $Y(nS)\pi^+\pi^-$ mode in the form of the resonances $f_0(980)$ and $f_2(1270)$. The measured dipionic invariant mass spectra by Belle in these final states is in conformity with the predictions [13, 14]. On the other hand, the amplitudes $A_{mZ_b}$ in the decays $Y_b(10890) \rightarrow h_b(mP)\pi^+\pi^-$ are expected to be neither resonant nor numerically significant. Only the transition $Y_b(10890) \rightarrow h_b(1P)f_0(980)$ is marginally allowed, heavily suppressed by the phase space and the $P$-wave decay character. The state $f_0(600)$ (or $\sigma(600)$) contributes, in principle. However, as this is a very broad resonance, the higher mass part is again suppressed by the phase space and hence the contribution of the $f_0(600)$ in the decay $Y_b(10890) \rightarrow h_b(1P)f_0(600)$ is both small and difficult to discern. This feature is also in accord with the Belle data [1]. Finally, we note that the absence of any anomalous production of the states $Y(nS)\pi^+\pi^-$ and $h_b(mP)\pi^+\pi^-$ in the decays of the bottomonium state $Y(11020)$ [16] is anticipated in the tetraquark picture, as opposed to the hadronic molecular interpretation of the Z_b and Z'_b for which the decays $Y(11020) \rightarrow Z_b^{(\pm)}\pi^\mp \rightarrow Y(nS)\pi^+\pi^-$, $h_b(mP)\pi^+\pi^-$ are expected to be enhanced by the larger phase space compared to the corresponding decays from the $Y(5S)$.

In summary, we have presented a tetraquark interpretation of the two observed states $Z_b^{(\pm)}(10610)$ and $Z'_b(10650)$. Combining the effective diquark-antidiquark Hamiltonian with the meson-loop induced effects, we are able to account for the observed masses in terms of the decay widths for the dominant channel $Z_b^{(\pm)} \rightarrow h_b(2P)\pi^\pm$, obtaining a ratio for the relative decay amplitudes in the decays $Z_b^{(\pm)} \rightarrow h_b(mP)\pi^\pm$ which agrees with the Belle data. Together with the resonant $\pi\pi$ structure in the decay modes $Y_b(10890) \rightarrow Y(nS)\pi^+\pi^-$, first worked out in [13, 14], this Letter provides additional support to the tetraquark hypothesis involving the states $Y_b(10890)$, $Z_b^{(\pm)}(10610)$ and $Z'_b(10650)$.

Precise spectroscopic measurements foreseen at the Super-B factories and at the LHC will provide definitive answers to several issues raised here and will help resolve the current and long-standing puzzles in the exotic bottomonium sector.

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