

CONVECTION IN GALAXY-CLUSTER PLASMAS DRIVEN BY ACTIVE GALACTIC NUCLEI AND COSMIC-RAY BUOYANCY

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ABSTRACT

Turbulent heating may play an important role in galaxy-cluster plasmas, but if turbulent heating is to balance radiative cooling in a quasi-steady state, some mechanism must regulate the turbulent velocity so that it is neither too large nor too small. This paper explores one possible regulating mechanism associated with an active galactic nucleus at cluster center. A steady-state model for the intracluster medium is presented in which radiative cooling is balanced by a combination of turbulent heating and thermal conduction. The turbulence is generated by convection driven by the buoyancy of cosmic rays produced by a central radio source. The cosmic-ray luminosity is powered by the accretion of intracluster plasma onto a central black hole. The model makes the rather extreme assumption that the cosmic rays and thermal plasma are completely mixed. Although the intracluster medium is convectively unstable near cluster center in the model solutions, the specific entropy of the thermal plasma still inhibits cooling of intracluster plasma to temperatures below the thermal conductivity of conduction to balance cooling, where $\kappa_S$ is in the range $0.1 \kappa_S - 1.0 \kappa_S$, where $\kappa_S$ is the Spitzer thermal conductivity. However, models in which cooling is balanced entirely by conduction face some difficulties. Different clusters need different values of $\kappa_T$ to fit the observations, and it is not clear why $\kappa_T$ would vary in the required way. Relatively cool cluster plasmas can require values of $\kappa_T$ in excess of $\kappa_S$ for conduction to balance cooling, values which are difficult if not impossible to justify. Theoretical studies of heat transport in tangled intracluster magnetic fields find values of $\kappa_T$ toward the lower end of the range of required values ($\kappa_R$ and $\kappa_M$). Chadran & Maron 2004; Chandran & Cowley 1998; N. Bessho, J. Maron, & B. Chandran, in preparation; and references therein). In addition, pure conduction models are thermally unstable, although the implications for clusters are debated, since the instability growth time is long ($\sim 2-5$ Gyr for $\kappa_T \sim 0.2 \kappa_S-0.4 \kappa_S$) (Kim & Narayan 2003a; Soker 2003).

Other heating mechanisms include dissipation of turbulent motions (Loewenstein & Fabian 1990; Churazov et al. 2004; Dennis & Chandran 2004) and turbulent mixing (Cho et al. 2001; Voigt & Fabian 2004; Kim & Narayan 2003b; Dennis & Chandran 2004; Narayan & Kim 2004). Observational evidence of plasma motion in the Perseus cluster at roughly half the sound speed $c_s$ (Churazov et al. 2004) provides support for models invoking turbulent heating but also poses a theoretical challenge, for the following reason. As the rms turbulent velocity $u$ varies from 0.1$c_s$ to $c_s$, and as the dominant velocity length scale at radius $r$ varies from 0.1$r$ to $r$, the turbulent heating rate in a typical cluster varies from a value much smaller than the radiative cooling rate $R$ to a value much larger than $R$ (Dennis & Chandran 2004). If turbulent heating in fact balances radiative cooling, an explanation is needed for the fine-tuning of the turbulence amplitude to the required value.

A number of authors have considered plasma heating by an active galactic nucleus (AGN) at a cluster’s center. Various mechanisms have been considered for transferring AGN power to the ambient plasma, including buoyantly rising bubbles of cosmic rays or centrally heated gas (Tabor & Binney 1993; Churazov et al. 2000, 2001; Begelman 2001a, 2001b; Reynolds 2001; Brüggen et al. 2002; Fabian et al. 2003; Ruszkowski & Begelman 2002; Mathews et al. 2003; Reynolds et al. 2004), wave-mediated plasma heating by cosmic-ray electrons (Rosner & Tucker 1989) or cosmic-ray protons (Böhringer & Morfill 1988; Loewenstein et al. 1991), turbulence (Loewenstein & Fabian 1990; Churazov et al. 2004), Compton heating (Binney & Tabor 1995; Ciotti & Ostriker 1997, 2001), and jet mechanical luminosity and shocks (Binney & Tabor 1995). An attractive feature of AGN-heating models is that the heating rate increases with the mass accretion rate of intracluster plasma, which allows the models to produce globally stable equilibria. If the mass accretion rate rises above the equilibrium value, AGN heating increases, and radiative cooling of the intracluster medium (ICM) is less able to cause the net cooling that drives mass accretion onto the central supermassive black hole. If the mass accretion rate falls below the
equilibrium value, the net cooling rate rises, restoring the mass accretion rate to its equilibrium level. If the thermal conductivity is not dramatically reduced relative to the Spitzer value, small-scale thermal instabilities are also suppressed (Rosner & Tucker 1989; Ruszkowski & Begelman 2002; Zakamska & Narayan 2003). A powerful motivation for AGN-heating models is the observation that almost all clusters with strongly cooling cores possess active central radio sources (Eilek 2003).

The present paper builds on previous studies of turbulent heating and AGN feedback in clusters and explores one possible explanation for how the turbulent velocity could achieve the value needed to balance radiative cooling. A steady-state model of the ICM is presented in which radiative cooling causes inflow of intracluster plasma and accretion onto a central supermassive black hole, leading to cosmic-ray production by a central radio source. The cosmic-ray luminosity $L_{\text{cr}}$ is taken to be proportional to the mass accretion rate $\dot{M}$, and it is assumed that the cosmic rays mix completely into the thermal intracluster plasma. Since the cosmic rays provide pressure without noticeably increasing the density of the ICM, they increase the buoyancy of the plasma, eventually leading to convection. Convection heats the plasma in three ways: through the viscous dissipation of turbulent motions, by mixing hot plasma from the outer regions of a cluster in toward cluster center, and by providing a vehicle for cosmic-ray pressure to do work on the thermal plasma. Convection is treated with a two-fluid (thermal plasma and cosmic ray) mixing length theory developed in §2. The turbulent velocity and turbulent heating rate increase with increasing $\dot{M}$ and $L_{\text{cr}}$. In equilibrium, $\dot{M}$ and $L_{\text{cr}}$ attain those values for which turbulent heating and thermal conduction balance radiative cooling. This equilibrium is expected to be stable for the same reasons as other AGN-feedback/thermal-conduction models.

Once the outer temperature and density are specified, the model can be used to calculate the density, temperature, and cosmic-ray pressure throughout the cluster. The model density profiles, however, are not as steep as observed profiles within the central $\sim$50 kpc, where the ICM is convective. The reason for this discrepancy is that the cosmic-ray pressure has to be comparable to or greater than the thermal pressure within the convective region in order for cosmic-ray buoyancy to make the ICM convectively unstable. The large cosmic ray pressure gradient provides much of the support of the ICM against gravity, reducing the thermal pressure gradient and decreasing the central plasma density. A similar discrepancy was found in an MHD-wave–mediated cosmic ray heating model (Loewenstein et al. 1991), which also involved significant nonthermal pressure. The present model also provides a self-consistent calculation of the profiles of the turbulent velocity and turbulent heating rates in the ICM. The resulting velocities are subsonic, implying that convective regions are fairly close to marginal convective stability. However, because the cosmic-ray pressure modifies the stability criterion, the specific entropy of thermal plasma still increases outward.

The discrepancy between the model and observations may be linked to the assumption that cosmic rays are completely mixed into the thermal plasma. This assumption, which is made to simplify the analysis, is inconsistent with the "X-ray cavities" (depressions in X-ray emission often associated with enhanced synchrotron emission) observed in roughly one-fourth of the clusters in the Chandra archive (Birzan et al. 2004). When mixing is incomplete, cosmic-ray pressure is less able to support the thermal plasma, the thermal pressure gradient has to be larger, and therefore the central plasma density has to be larger. A model that accounts for incomplete mixing may have greater success.

The remainder of the paper is organized as follows. In §2, the model is described in detail and example solutions are compared to observations of Abell 478. Section 3 summarizes the results of the paper.

2. THE TWO-FLUID CONVECTION MODEL

In the cooling flow model of intracluster plasmas (Fabian 1994), entropy-generating heat sources are neglected, and radiative losses are balanced by inward advection of plasma internal energy as well as gravitational and $pdV$ work. The resulting average radial velocity corresponds to mass accretion rates above $1000 M_{\odot}$ yr$^{-1}$ in some clusters. In contrast, in the present model, radiative cooling is balanced by heating from turbulence and thermal conduction, leading to much smaller radial velocities and mass accretion rates. The average fluid velocity is neglected in the fluid equations, and the mass accretion rate is set equal to the Bondi (1952) rate corresponding to the equilibrium plasma parameters at cluster center,

$$\dot{M} = \pi G^2 M_{\text{bh}}^2 \rho(0) \frac{a_{\text{ad}}(0)}{c_{\text{rs}}(0)}$$

where $M_{\text{bh}}$ is the mass of a supermassive black hole at cluster center, $\rho(0)$ is the central density, and $c_{\text{rs}}(0)$ is the adiabatic sound speed at cluster center.$^2$

The inferred mass accretion rate is used to determine the cosmic-ray luminosity through the equation

$$L_{\text{cr}} = \eta \dot{M} c^2$$

where $\eta$ is a dimensionless efficiency factor. The spatial distribution of cosmic-ray injection into the ICM is a major uncertainty in the model. Some clues are provided by radio observations, which show that cluster-center radio sources (CCRS) differ morphologically from radio sources in other environments. As discussed by Eilek (2003), roughly half of the CCRS in a sample of 250 sources studied by Owen & Ledlow (1997) are "amorphous," or quasi-isotropic, presumably because of jet disruption by the comparatively high-pressure cluster-core plasma. With the exception of Hydra A, the CCRS in the Owen & Ledlow (1997) study are smaller than non-cluster-center sources, with most extending less than 50 kpc from cluster center (Eilek 2003). Complicating matters is the possibility that cosmic-ray bubbles rise buoyantly away from their acceleration site before mixing into the thermal plasma, effectively distributing cosmic-ray injection over a larger volume. In this paper, the cosmic-ray energy introduced into the intracluster medium per unit volume per unit time is taken to be

$$S(r) = S_0 e^{-r^2/r_s^2},$$

where the constant $r_s$ is a free parameter and the constant $S_0$ is determined from the equation $L_{\text{cr}} = 4\pi \int_0^\infty dr \ r^2 S(r)$.

1 See Ensslin (2003) for a detailed discussion of the escape of cosmic rays from radio-galaxy cocoons.

2 See Quataert & Narayan (2000), Nulsen (2003), and Böhringer et al. (2003) for further discussions of Bondi accretion in clusters.
The magnetic pressure is taken to be much less than the thermal pressure. Magnetic terms are then neglected in the fluid equations, and the effects of the magnetic field on convection are ignored. Although the magnetic field does not affect the bulk fluid velocity, for any plausible field strength the field plays an important role in heat transport by constraining charged particles to move primarily along field lines. Since the magnetic field in clusters is disordered and probably turbulent (Kronberg 1994; Taylor et al. 2001, 2002), chaotic field-line trajectories inhibit large radial particle excursions, suppressing radial heat transport to some degree. In this paper, the thermal conductivity is set equal to the Spitzer (1962) value for a nonmagnetized plasma, $\kappa_S$, multiplied by a suppression factor $\theta$,

$$\kappa_T = \theta \kappa_S,$$

where

$$\frac{\kappa_S}{ni_e k_B} = 9.2 \times 10^{30} \left( \frac{k_B T}{5 \text{ keV}} \right)^{5/2} \left( \frac{10^{-2} \text{ cm}^{-3}}{n_e} \right) \left( \frac{37}{\ln \Lambda_c} \right) \text{cm}^2 \text{s}^{-1}.$$  

$n_e$ is the electron density, $T$ is the temperature, $k_B$ is the Boltzmann constant, and $\ln \Lambda_c$ is the Coulomb logarithm. Recent theoretical studies suggest that $\theta \sim 0.2$ (Narayan & Medvedev 2001; Chandran & Maron 2004; Maron et al. 2004). The heating rate per unit volume from thermal conduction is

$$H_{\text{he}} = \nabla \cdot (\kappa_T \nabla T).$$

Following Tozzi & Norman (2001) and Ruszkowski & Begelman (2002), the radiative cooling per unit volume per unit time for free-free and line emission is taken to be

$$R = n_i n_e \left[ 0.0086 \left( \frac{k_B T}{1 \text{ keV}} \right)^{-1.7} + 0.058 \left( \frac{k_B T}{1 \text{ keV}} \right)^{0.5} + 0.063 \right] \times 10^{-22} \text{ ergs cm}^{-3} \text{s}^{-1},$$

where $n_i$ is the ion density and the numerical constants correspond to 30\% solar metallicity. Radiative cooling of cosmic rays is ignored, which is reasonable if protons make the dominant contribution to the cosmic-ray pressure. Cosmic rays are assumed to diffuse relative to the thermal plasma, but the value of the cosmic ray diffusion coefficient $D_{\text{cr}}$ in clusters is not known. In the model,

$$D_{\text{cr}} = \sqrt{D_0^2 + v_d^2 r^2},$$

where $D_0$ and $v_d$ are constants.

The thermal plasma and cosmic rays are treated as coextensive fluids that interact only in limited ways. Wave pitch-angle scattering causes the two fluids to move with the same bulk fluid velocity $v$, although the finite efficiency of scattering allows for cosmic-ray diffusion as already discussed. Collisional and microphysical collisionless energy exchange between the two fluids is neglected, but the cosmic rays can heat the plasma, and vice versa, through $pdV$ work. The thermal plasma is treated as a fluid of adiabatic index $\gamma$,

$$p = (\gamma - 1) \rho \epsilon,$$

where $p$ is the plasma pressure, $\rho$ is the plasma density, and $\epsilon$ is the plasma internal energy per unit mass. The cosmic rays are treated as a fluid of adiabatic index $\gamma_{\text{cr}}$,

$$p_{\text{cr}} = (\gamma_{\text{cr}} - 1) \rho_{\text{cr}} \epsilon_{\text{cr}},$$

where $p_{\text{cr}}$ is the cosmic-ray pressure, $\rho_{\text{cr}}$ is the cosmic-ray density, and $\epsilon_{\text{cr}}$ is the cosmic-ray internal energy per unit mass. Both $\gamma$ and $\gamma_{\text{cr}}$ are treated as constants (although the adiabatic index of a fluid should really vary with temperature from 5/3 in the nonrelativistic limit to 4/3 in the ultrarelativistic limit).

The first law of thermodynamics can be written

$$\frac{d \rho}{dt} = H + \frac{p}{\rho^2} \frac{dp}{dt},$$

where $d/dt \equiv (\partial/\partial t + v \cdot \nabla)$, and $H$ is the net heating per unit mass per unit time. Equations (9) and (11) together give

$$\frac{1}{\gamma - 1} \left( \frac{d \rho}{dt} - \gamma \frac{dp}{dt} \right) = \rho H.$$

The equation of continuity is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = S_M,$$

where $S_M$ is the rate at which mass is introduced (by, e.g., stellar winds) per unit time per unit volume. Equations (12) and (13) give

$$\frac{1}{\gamma - 1} \left[ \frac{d \rho}{dt} + \nabla \cdot (\rho v) \right] = -p \nabla \cdot v + \rho H + \frac{\gamma \rho S_M}{(\gamma - 1) \rho}.$$  

Similarly,

$$\frac{1}{\gamma_{\text{cr}} - 1} \left[ \frac{d \rho_{\text{cr}}}{dt} + \nabla \cdot (\rho_{\text{cr}} v_{\text{cr}}) \right] = -p_{\text{cr}} \nabla \cdot v + \rho_{\text{cr}} H_{\text{cr}} + \frac{\gamma_{\text{cr}} p_{\text{cr}} S_{\text{M, cr}}}{(\gamma_{\text{cr}} - 1) \rho_{\text{cr}}},$$

where $H_{\text{cr}}$ and $S_{\text{M, cr}}$ are, respectively, the cosmic-ray heating and mass-injection rates. For the thermal plasma,

$$\rho H = H_{\text{diss}} + H_{\text{he}} - R,$$

where $H_{\text{diss}}$ is the rate of heating from viscous dissipation of turbulent motions, quantified below in equation (50). For simplicity, it is assumed that

$$S_M = 0.$$  

The right-hand side of equation (15) is simplified by assuming that

$$\rho_{\text{cr}} H_{\text{cr}} + \frac{\gamma_{\text{cr}} p_{\text{cr}} S_{\text{M, cr}}}{(\gamma_{\text{cr}} - 1) \rho_{\text{cr}}} = \nabla \cdot (D_{\text{cr}} \nabla p_{\text{cr}}) + S(r),$$

with $D_{\text{cr}}$ given by equation (8) and $S(r)$ given by equation (3). The first term on the right-hand side of equation (18) is a
simplified representation of diffusive energy transport, and the second term models the energy injection from the central radio source.

Convection and convective stability are treated using the following simple two-fluid mixing-length theory. Consider a parcel of plasma and cosmic rays initially at a distance \( r = r_0 \) from cluster center. The parcel is displaced radially by an amount \( l \) (positive or negative) to \( r = r_1 = r_0 + l \), where \(|l|\) is the mixing length. For simplicity, thermal conduction and cosmic-ray diffusion into and out of the parcel are neglected at this stage, so that the parcel expands adiabatically. The parcel is then mixed with its surroundings. The values of \( \rho, p, \rho_\text{cr} \), and \( p_\text{cr} \) in the parcel are initially the same as the average values at \( r = r_0 \), denoted \( \rho_0, p_0, \rho_\text{cr,0}, \) and \( p_\text{cr,0} \). After the parcel is displaced radially outward, the new fluid quantities within the parcel are denoted \( \rho', p', \rho'_\text{cr} \), and \( p'_\text{cr} \). The average fluid quantities at \( r = r_1 \) are denoted \( \rho_1, p_1, \rho_\text{cr,1}, \) and \( p_\text{cr,1} \). The difference between the thermal-plasma density in the displaced parcel and its surroundings at \( r = r_1 \) is denoted

\[ \Delta \rho = \rho' - \rho_1. \]  

Similarly, \( \Delta p = p' - p_1 \), etc. Since the parcel expands adiabatically,

\[ p' = p_0 \left( \frac{\rho'}{\rho_0} \right)^\gamma, \]  

\[ p'_\text{cr} = p_{\text{cr,0}} \left( \frac{\rho'_\text{cr}}{\rho_{\text{cr,0}}} \right)^{\gamma_{\text{cr}}} \]  

The volume occupied by the thermal plasma and the volume occupied by the cosmic rays expand by the same amount, which implies that

\[ \frac{\rho'}{\rho_0} = \frac{\rho'_{\text{cr}}}{\rho_{\text{cr,0}}} \equiv 1 + \delta. \]

It is assumed that the turbulent velocities are subsonic, so that the total pressure

\[ p_{\text{tot}} = p + p_{\text{cr}} \]

inside the parcel remains approximately the same as the total pressure outside the parcel. It is now assumed that \(|l| \ll r_0 \), which implies that \( \delta \ll 1 \). To lowest order in \(|l|/r_0\), equations (20)–(23) give

\[ \delta = \frac{l}{\gamma p + \gamma_{\text{cr}} p_{\text{cr}}} \frac{dp_{\text{tot}}}{dr}, \]

\[ \Delta \rho = \frac{l \rho}{\gamma p + \gamma_{\text{cr}} p_{\text{cr}}} \frac{dp_{\text{tot}}}{dr} - l \frac{d \rho}{dr}, \]

\[ \Delta p = \frac{l \gamma p}{\gamma p + \gamma_{\text{cr}} p_{\text{cr}}} \frac{dp_{\text{tot}}}{dr} - l \frac{d \rho}{dr}, \]

\[ \Delta p_{\text{cr}} = -\Delta p. \]

Since only the lowest order terms have been kept, it is not necessary to specify in equations (24)–(26) whether \( \rho, p, \) etc., are evaluated at \( r_0 \) or \( r_1 \), and so the subscripts on \( \rho, p, \) etc., have been dropped.

Since the displacement \( l \) is a signed quantity, the criterion for convective stability is

\[ \frac{\Delta \rho}{l} > 0. \]  

If \( \rho \gg p_{\text{cr}} \), equation (28) is equivalent to the Schwarzchild criterion

\[ \frac{d}{dr} \ln \left( \frac{p}{\rho^\gamma} \right) > 0. \]  

More generally, the ICM can be convectively unstable even when the specific entropy of the thermal plasma increases outward, provided that \( dp_{\text{cr}}/dr \) is negative and \( |dp_{\text{cr}}/dr| \) is sufficiently large.

The rms turbulent velocity \( u \) is assumed to satisfy

\[ \rho u^2 \approx \left\{ \begin{array}{ll} \frac{\Delta \rho g l}{8} & \text{if } l^{-1} \Delta \rho < 0 \\ 0 & \text{if } l^{-1} \Delta \rho > 0, \end{array} \right. \]  

where \( g \) is the gravitational acceleration. That is, the bulk-flow kinetic energy of a moving parcel in a convective region is approximately the buoyancy force on the fully displaced parcel times the mixing length times the standard numerical coefficient used in mixing-length theory (Cox & Giuliani 1968). Equation (30) is then modified slightly so that \( du/dr \) varies continuously to zero as \( \Delta \rho \) increases through zero:

\[ u^2 = \left\{ \begin{array}{ll} u_0^2 + q g l / 8 & \text{if } l^{-1} \Delta \rho < 0 \\ u_0^2 e^{-eq} & \text{if } l^{-1} \Delta \rho > 0, \end{array} \right. \]

where

\[ q = \frac{\Delta \rho}{\rho}, \]

\[ \sigma = -\frac{qg l}{8u_0^2}, \]

\[ u_0 = u_0' r, \]

and \( u_0' \) is a constant chosen so that \( u_0 \) remains much smaller than the sound speed throughout the cluster. The value of \( \sigma \) in equation (33) is chosen so that \( \partial u^2 / \partial q \) is continuous at \( q = 0 \).

The approximate (signed) value of \( \nabla \cdot \mathbf{v} \), denoted \( \text{div } \mathbf{v} \), is given by the fractional change in the volume of a parcel as it rises a distance \( l \), [(\( \rho' \))\(^{-1} \) \( - \rho_0^{-1} \)] / \( \rho_0^{-1} \)), divided by the time for the parcel to rise, \(|l|/u|\):

\[ \text{div } \mathbf{v} = -\frac{\text{sgn}(l) u}{\gamma p + \gamma_{\text{cr}} p_{\text{cr}}} \frac{dp_{\text{tot}}}{dr}, \]

where \( \text{sgn}(l) = l/|l| \).

From equation (14), the steady-state thermal-plasma energy equation is

\[ \frac{1}{\gamma - 1} \nabla \cdot (\rho \mathbf{v}) = -p \nabla \cdot \mathbf{v} + \dot{H}_{\text{diss}} + \dot{H}_{\text{ic}} - R. \]  

(36)
The steady-state cosmic ray energy equation, from equations (15) and (18) is
\[ \frac{1}{\gamma_{cr} - 1} \nabla \cdot (\rho v_{cr}) = -p_{cr} \nabla \cdot v + \nabla \cdot (D_{crt} \nabla p_{cr}) + S. \tag{37} \]

To obtain averaged equations, each fluid quantity is written as an average value plus a turbulent fluctuation:
\[ v = \langle v \rangle + \delta v, \tag{38} \]
\[ p = \langle p \rangle + \delta p, \tag{39} \]
etc. As mentioned above, \( \langle v \rangle \) is set equal to zero. It is assumed that averaged quantities depend only on the radial coordinate \( r \). The average \( \langle \delta v \delta p \rangle \), which is \( \gamma - 1 \) times the thermal-plasma internal-energy flux, is estimated to be \( c_{\text{mix}} \langle u \text{ sgn}(l) \rangle \Delta p \), where \( c_{\text{mix}} \) is a constant of order unity, giving
\[ \langle \delta v \delta p \rangle = \hat{r} DQ, \tag{40} \]
where
\[ D = c_{\text{mix}} u |l| \tag{41} \]
is the eddy diffusivity, and
\[ Q = \frac{\Delta p}{l} = \frac{\langle p \rangle}{\gamma + \gamma_{cr} p_{cr}} \frac{d p_{\text{tot}}}{d r} - \frac{\rho}{d r}. \tag{42} \]
The average \( \langle \delta v \delta p_{cr} \rangle \) is estimated to be \( c_{\text{mix}} \langle u \text{ sgn}(l) \rangle \Delta p_{cr} \), which yields
\[ \langle \delta v \delta p_{cr} \rangle = -\langle \delta v \delta p \rangle. \tag{43} \]
The average \( \langle \delta p \nabla \cdot \delta v \rangle \) is taken to be \( c_{\text{mix}} \Delta p (\text{div} v) \), or
\[ \langle \delta p \nabla \cdot \delta v \rangle = -DQ \frac{d p_{\text{tot}}}{\gamma + \gamma_{cr} p_{cr}}. \tag{44} \]
Similarly, \( \langle \delta p_{cr} \nabla \cdot \delta v \rangle \) is taken to be \( c_{\text{mix}} \Delta p_{cr} (\text{div} v) \), giving
\[ \langle \delta p_{cr} \nabla \cdot \delta v \rangle = -\langle \delta p \nabla \cdot \delta v \rangle. \tag{45} \]

Equations (43) and (45) reflect the assumption that the total-pressure fluctuation in a displaced fluid parcel vanishes. The mixing length is set equal to
\[ |l| = \alpha r, \tag{46} \]
where \( \alpha \) is a constant. Although \( |l| \ll r \) was previously assumed, it is now assumed that \( \alpha \) is of order unity, an inconsistency that also characterizes standard mixing-length theory.

Upon averaging equations (36) and (37), discarding terms proportional to the average velocity, and dropping the angle brackets \( \langle \ldots \rangle \) around averaged quantities, one obtains
\[ \frac{1}{(\gamma - 1)r^2} \frac{d}{d r} (r^2 DQ) = \frac{DQ}{\gamma + \gamma_{cr} p_{cr}} \frac{d p_{\text{tot}}}{d r} + H_{\text{tot}} + H_{\text{ec}} - R \tag{47} \]
and
\[ \frac{1}{(\gamma - 1)r^2} \frac{d}{d r} (r^2 DQ) = \frac{DQ}{\gamma + \gamma_{cr} p_{cr}} \frac{d p_{\text{tot}}}{d r} + \frac{1}{r^2} \frac{d}{d r} \left( r^2 D_{crt} \frac{d p_{cr}}{d r} \right) + S. \tag{48} \]

In the averages of \( H_{\text{ec}} \) and \( R \) the density and temperature are simply replaced by their average values. The average and turbulent velocities are neglected in the momentum equation, giving
\[ \frac{d p_{\text{tot}}}{d r} = -\rho \frac{d \Phi}{d r}, \tag{49} \]
where \( \Phi \) is the gravitational potential, which is assumed to be dominated by a fixed dark matter distribution. The average value of \( H_{\text{dis}} \) is set equal to
\[ H_{\text{dis}} = \frac{c_{\text{dis}} \rho u^3}{l}, \tag{50} \]
where \( c_{\text{dis}} \) is a dimensionless constant of order unity. Equations (47)–(50) describe both convectively stable and convectively unstable regions, but \( D \) and \( H_{\text{dis}} \) are effectively zero in stable regions that are not very close to marginal stability.

A few words on the relationship between the two-fluid convection model and standard mixing length theory are in order. The left-hand side of equation (47) is reminiscent of the divergence of the convective heat flux that appears in standard mixing length theory and discussions of forced turbulent mixing in clusters. However, the convective internal-energy flux \( \langle \delta v \delta p \rangle / (\gamma - 1) \) on the left-hand side of equation (47) is not proportional to the specific-entropy gradient. In fact, \( \langle \delta p \delta v \rangle \) vanishes as \( p_{cr} \rightarrow 0 \) because the total pressure perturbation in a fluid element is assumed to vanish. However, the approach taken in this paper would be similar to standard mixing length theory in the \( p_{cr} \rightarrow 0 \) limit if the average velocity were retained. In a convectively unstable region, downward falling fluid elements are denser on average than upwardly rising fluid elements. The random velocities are thus associated with an inward mass flux. In standard mixing length theory, the total mass flux vanishes because a steady state without mass sources or sinks is assumed (Cox & Giuli 1968). This requires a nonvanishing outward average radial velocity,
\[ \langle u \rangle = -\frac{\langle \delta p \delta v \rangle}{\langle p \rangle}. \tag{51} \]

If we add equation (36) to the dot product of \( v \) with the steady state momentum equation, employ \( \nabla \cdot (\rho v) = 0 \), and take the limit \( p_{cr} \rightarrow 0 \), we obtain the total-energy equation
\[ 0 = -\nabla \cdot \left( \frac{\gamma \rho v^2}{\gamma - 1} + \rho v^2 + \rho v \Phi + F_{v} \right) + H_{\text{cc}} - R, \tag{52} \]
where \( \gamma \rho v / (\gamma - 1) \) is the enthalpy flux, and \( F_{v} \) is the viscous energy flux (see, e.g., Rudiger 1989, eq. [8.40]). If one sets
\[ \langle \delta p \delta v \rangle = c_{\text{mix}} \Delta \rho \text{ sgn}(l) u, \tag{53} \]
the enthalpy flux associated with the average radial velocity of equation (51) is

$$\gamma(p) \langle v_r \rangle = -D\rho T \frac{ds}{dr}, \quad (54)$$

where the angle brackets around average quantities have been dropped on the right-hand side, $s = C_V \ln (p/\rho)$ is the specific entropy, and $C_V = \epsilon/T$ is the specific heat at constant volume. The enthalpy flux in equation (54) is the same as the heat flux in standard mixing length theory.

Although $\langle v_r \rangle$ must be included to recover standard mixing length theory in the pressure equation when the model is applied to clusters. In the limit of vanishing $\dot{m}$, the $\langle v_r \rangle$ terms in the average of equation (36) are smaller than the $(\delta v_r \delta p)$ terms, which can be seen as follows. Using equations (51) and (53), one can write

$$\frac{\langle v_r \rangle}{\langle \delta v_r \delta p \rangle} = \frac{\Delta \rho}{\rho} \frac{\Delta p}{p} \left( \frac{\rho}{p} \right)^{-1}. \quad (55)$$

This ratio is small for the following reasons. Convective regions must be close to marginal convective stability to produce the subsonic turbulent velocities required to balance cooling, which implies $|\Delta \rho| \ll |\rho/T|$. This condition, coupled with equations (25) and (26), gives

$$\Delta \rho \approx -\frac{\rho}{C_V} \frac{ds}{dr}. \quad (56)$$

Equation (56) implies that $|\Delta \rho| \approx \rho/|s|$, as $ds/dr$ is observed to be $-C_V \rho/s$ in clusters. Thus, $|\Delta \rho|/\rho |\Delta p|/p |< 1$. It is true that if $M$ is sufficiently large, then the $\langle v_r \rangle$ terms in the energy equation are comparable to the radiative-cooling term, as in the cooling flow model. However, in the model solutions to be presented below, $M$ is a factor $10^3$–$10^4$ smaller than in the cooling-flow model, so that the neglect of $\langle v_r \rangle$ is acceptable.\(^5\)

Equation (42) can be rewritten

$$Q = -\rho \frac{d\chi}{dr} + \frac{\gamma (\gamma - \gamma_c) \chi \rho_{cr}}{\gamma \rho_c + \rho_{c,cr}} \frac{d \rho_{tot}}{dr}, \quad (57)$$

where

$$\chi = \frac{\rho}{\rho_{tot}}. \quad (58)$$

One expects $d\chi/dr > 0$ throughout most of a cluster since cosmic rays contribute a larger fraction of the pressure nearer the central radio source. Since $\gamma \geq \gamma_c$, one expects $Q$ to be negative in clusters. Thus, if a fluid parcel is displaced inward ($\dot{t} < 0$), the thermal pressure inside the displaced parcel is larger than in the surrounding medium ($\dot{Q} > 0$), essentially because the total pressure is the same inside and outside the parcel and $\chi$ is larger inside. Since the inwardly displaced parcel gets compressed, the first term on the right-hand side of equation (47) ($-\langle \delta p \nabla \cdot \delta v_r \rangle$) is positive and acts as an energy source for the thermal plasma. This source term can be interpreted as in part due to $pdV$ work on the thermal plasma by the cosmic-ray pressure, which helps compress the displaced parcel.

Equations (47)–(49) form a system of two second-order equations and one first-order equation for $\rho, T$, and $p_{cr}$. Five boundary conditions are required to specify a solution. Two boundary conditions are obtained by imposing a density $\rho_{outer}$ and temperature $T_{outer}$ at radius $r_{outer}$. Two additional boundary conditions are obtained by taking $dT/dr$ and $dp_{cr}/dr$ to vanish at the origin. The fifth condition is obtained by assuming that $p_{cr} \rightarrow 0$ as $r \rightarrow \infty$. This condition is translated into an approximate condition on $p_{cr}$ at $r_{outer}$ as follows. The value of $r_{outer}$ is chosen to be much greater than $r_c$, and $D_0/\rho_{tot}$, so that for $r > r_{outer}$, $S \approx v_t r$. Equation (48) then implies that $p_{cr} \approx r_c \rho_C r^{-2}$ for $r > r_{outer}$, assuming the ICM is convectively stable and far from marginal stability. Since $p_{cr} \rightarrow 0$ as $r \rightarrow \infty$, $c_1 = 0$. The fifth boundary condition is then taken to be $dp_{cr}/dr = -2p_{cr}/r$ at $r_{outer}$. Numerical solutions are obtained using a shooting method. Values are guessed for $\rho, T$, and $p_{cr}$ at $r = 0$, and the equations are integrated from 0 to $r_{outer}$. The guesses are then updated using Newton’s method until the three boundary conditions at $r_{outer}$ are met.

Figure 1 shows an example solution for the following parameters: $M_{tot} = 10^{10} M_\odot$, $\eta = 0.003$, $\gamma = 5/3$, $\gamma_c = 4/3$, $\alpha = 1$, $u_0 = 0.1 \text{ km s}^{-1} \text{ kpc}^{-1}$, $r_c = 25 \text{ kpc}$, $v_{tot} = 18 \text{ km s}^{-1}$, $D_0 = 10^{28} \text{ cm}^2 \text{ s}^{-1}$, and $\theta = 1/3$. The values $c_{min} = 0.42$ and $c_{max} = 0.11$ are adopted based on a number of previous studies, as discussed by Dennis & Chandran (2004). The gravitational potential is taken to be

$$\Phi = \frac{v_1^2}{2} \ln \left[ 1 + \left( \frac{r}{r_c} \right)^2 \right] + \frac{v_q^2}{2} \ln \left[ 1 + \left( \frac{r}{r_q} \right)^2 \right]. \quad (59)$$

with $v_1 = 1150 \text{ km s}^{-1}$, $r_c = 40 \text{ kpc}$, $v_q = 400 \text{ km s}^{-1}$, and $r_q = 1 \text{ kpc}$. The term proportional to $v_q^2$ ($v_q^2$) represents the cluster (central-galaxy) potential. A mean molecular weight per electron $\rho/\rho_{mH}$ of 1.15 is assumed, where $m_H$ is the mass of a hydrogen atom, and the ratio $n_e/n_0$ is set equal to 0.9.\(^6\)

The values $\rho_{outer} = 5.38 \times 10^{-3} \text{ cm}^{-3}$ and $T_{outer} = 7.91 \text{ keV}$ at $r_{outer} = 200 \text{ kpc}$ are determined by linear interpolation between data points for Abell 478. Density and temperature data for Abell 478 from Chandran observations, provided by S. Allen, are also plotted in Figure 1. Abell 478 is a highly relaxed, X-ray luminous cluster, with a cooling-flow–model mass accretion rate of $\sim 1000 M_\odot \text{ yr}^{-1}$ and a comparatively weak central radio source (Allen et al. 1993; Sun et al. 2003).

It can be seen from Figure 1 that the model density is too small for $r \leq 50 \text{ kpc}$. This discrepancy arises because the specific entropy of the thermal plasma increases outward, and thus the cosmic ray pressure gradient must be large in order for the ICM to be convectively unstable and for turbulent heating to help balance radiative cooling. When cosmic rays contribute a large fraction of the total pressure support, the thermal pressure gradient does not have to be as large, and thus the

\(^4\) If one takes $p \rho \sim T/\rho$ and $\nabla \cdot (\delta v \delta p) \sim (\delta v_r \delta p)/r$, then $H_{diff}/\nabla \cdot (\delta v \delta p) \sim (\Delta p/\rho) \Delta (\delta \rho/\delta p)^{-1}$, which is again small because convective regions are near marginal convective stability. Thus, dissipation of turbulent motions is less important than turbulent mixing in the model of this paper, as is borne out by numerical solutions to eqs. (47)–(49). On the other hand, if an external source of turbulent motions is invoked, as in, e.g., El-Zant et al. (2004), then viscous dissipation may be more important in comparison to turbulent mixing than in the present model.

\(^5\) Very close to cluster center, $\langle v_r \rangle$ becomes large even for small $M$, and the model is inaccurate.

\(^6\) These values are appropriate for a hydrogen fraction $X = 0.7$ and a helium fraction $Y = 0.28$, with fully ionized hydrogen and helium (Zakamska & Narayan 2003).
central plasma density does not have to be as large. A variety of model parameters have been investigated with similar results. The turbulent velocity in Figure 1 is also surprisingly small. This is in part because the model underestimates the plasma density in the central 50 kpc, which weakens radiative cooling and the need for turbulent heating. Figure 2 shows a numerical solution for the parameters $v_c = 1500 \text{ km s}^{-1}/C_0$, $x_c = 10 \text{ kpc}$, $v_d = 0$, $\theta = 0.37$, $v_d = 20 \text{ km s}^{-1}$, and $D_0 = 3 \times 10^{28} \text{ cm}^2 \text{ s}^{-1}$, with other parameters the same as in Figure 1. The enhanced gravitational acceleration increases the density and radiative cooling, which results in the need for greater turbulent heating and a larger $u(r)$. These parameters, however, are unrealistic since they imply a velocity dispersion of $\sim 1000 \text{ km s}^{-1}$ within the central galaxy at $r = 10 \text{ kpc}$, which is much larger than observed velocity dispersions in giant elliptical galaxies.

A further difficulty with the model is that a critical point is encountered for sufficiently large $v_d$ and/or $D_0$, or for sufficiently small $\theta$. It can be shown that a sufficient condition for avoiding a critical point is that $(\gamma - 1)D_0/(\gamma - 1)$ is everywhere less than $\kappa_T/n_B$, the diffusion coefficient of heat-carrying electrons, and the models plotted in Figures 1 and 2 satisfy this inequality. The potential for a critical point raises a number of interesting questions, but these lie beyond the scope of this paper.

For reference, in the numerical solution plotted in Figure 1, the Bondi accretion rate is $1.2 \, M_\odot \text{ yr}^{-1}$, $L_{\text{cr}} = 2.1 \times 10^{44} \text{ ergs s}^{-1}$, the radiative luminosity out to 200 kpc is $1.9 \times 10^{45} \text{ ergs s}^{-1}$, and the radiative luminosity out to 600 kpc is $4.9 \times 10^{45} \text{ ergs s}^{-1}$. For the model plotted in Figure 2, the Bondi accretion rate is $5.0 \, M_\odot \text{ yr}^{-1}$, $L_{\text{cr}} = 8.5 \times 10^{44} \text{ ergs s}^{-1}$, the radiative luminosity out to 200 kpc is $2.2 \times 10^{45} \text{ ergs s}^{-1}$, and the radiative luminosity out to 600 kpc is $4.5 \times 10^{45} \text{ ergs s}^{-1}$. (The luminosity out to 600 kpc is smaller in the model of Figure 2 because the larger gravitational acceleration causes the density to drop off faster at large $r$.)

3. SUMMARY

AGN feedback is a promising explanation for the heating of intracluster plasmas on both observational and theoretical
grounds. There is an active radio source at the center of almost every strongly cooling cluster (Eilek 2003), and AGN feedback in combination with thermal conduction can lead to globally stable equilibria (Rosner & Tucker 1989; Ruszkowski & Begelman 2002). The recent detection of moderately subsonic plasma motions in the Perseus cluster (Churazov et al. 2004) further suggests that turbulent mixing and/or turbulent dissipation plays an important role in heating the intracluster medium. This observation also poses a theoretical challenge. For velocities in the range of those observed, turbulent heating can either overwhelm radiative cooling or be too small to offset cooling, depending on the precise value of the rms velocity \( u \) and the velocity length scale. If turbulent heating indeed balances radiative cooling, an explanation is needed for the fine-tuning of \( u \) to the required value.

The model presented in this paper connects AGN feedback to turbulent heating and seeks to explain the fine-tuning of \( u \). In the model, radiative cooling of intracluster plasma is balanced by a combination of turbulent heating and thermal conduction. The turbulence is generated by convection, which in turn is produced by the buoyancy of cosmic rays generated by a central radio source. Convection is a natural way for a central relativistic jet to generate turbulence because of the jet’s small momentum flux to energy flux ratio: rather than a relativistic outflow stirring intracluster plasma like an oar in water, the radio source inflates the central region, allowing gravity to generate plasma momentum after parcels of plasma and cosmic rays in the central region become buoyant. A two-fluid (plasma and cosmic ray) mixing length theory is developed to treat convection in the ICM. By linking the turbulence amplitude to the mass accretion rate and AGN feedback luminosity, the model provides an explanation for how the turbulent velocity achieves the value required for heating to balance cooling. Although stability is not investigated in this paper, it is expected that the model equilibria are stable for the same reasons as other AGN-feedback/thermal-conduction models.

The model provides equilibrium solutions for \( n_e(r), T(r), p_{cr}(r), \) and \( u(r) \) once the plasma temperature and density are specified at some suitably large outer radius. For typical cluster parameters, the ICM stays fairly close to marginal convective stability in convective regions, implying subsonic turbulent velocities. The principal shortcoming of the model is that the
model density is too small within the central ~50 kpc, where the model ICM is convective. The reason is that the cosmic ray pressure gradient must be large for cosmic-ray buoyancy to generate convection. This in turn reduces the thermal pressure gradient in the central region and decreases the central plasma density. A more realistic model of convection in the ICM in which relativistic and thermal plasmas are only partially mixed may have greater success.

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