Non leptonic heavy hadron decays and local duality

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Abstract

We discuss local duality for weak non leptonic $B$ and $\Lambda_b$ decays in the $m_b \to \infty$ limit under the hypothesis of factorization and in the Shifman-Voloshin limit. We show that, under these hypotheses, local duality holds for the first two terms in the $1/m_b$ expansion and at the order $\alpha_s$ in the perturbative expansion. The possible relevance of these results for the operator product expansion evaluation of the $b$-hadron lifetimes is discussed.
1 Introduction: inclusive heavy hadron decays and OPE

In the last few years there has been an increasing theoretical interest in inclusive heavy hadron weak decays, mainly motivated by the possibility to use the theoretical method of Operator Product Expansion (OPE) to get information on the decay widths in the infinite heavy quark mass limit: \( m \rightarrow \infty \) \( [1, 2] \). By employing OPE, a time ordered product of operators, in the present case the weak hamiltonian responsible for the decay, is expanded in terms of a sum of local operators of increasing dimension, multiplied by inverse powers of the heavy-quark mass. In the \( m \rightarrow \infty \) limit, one may truncate this expansion to the first few terms, that are calculable in terms of perturbative QCD and some non perturbative contributions that depend on some known matrix elements of higher dimension operators.

This procedure can be illustrated by taking the semileptonic inclusive decay width as an example. For a hadron \( H_b \) containing one heavy quark \( b \) (e.g. \( B \) or \( \Lambda_b \)) the inclusive width is given by

\[
\Gamma(H_b) = \frac{1}{2m_H} \text{disc}\langle H_b | T | H_b \rangle
\]

where

\[
T = i \int d^4x T(\mathcal{L}(x)\mathcal{L}(0))
\]

and \( \mathcal{L} \) is the effective lagrangian responsible for the transition. In the following we consider the transition \( b \rightarrow c \) only. For semileptonic inclusive decays one has:

\[
\mathcal{L}_{sl} = \frac{G_F}{\sqrt{2}} V_{cb} \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma_\mu (1 - \gamma_5) \nu_\ell = \frac{G_F}{\sqrt{2}} V_{cb} J^\mu_h J^\mu_\ell
\]

where \( J^\mu_h \) and \( J^\mu_\ell \) are the hadronic and leptonic currents respectively.

As we mentioned already, the operator product expansion, as applied to the product of operators appearing in \( \mathcal{L} \), gives rise to an expansion in terms of decreasing powers of the \( b \)-quark mass \( m_b \); for semileptonic decays the first terms of this expansion have the form \( [1] \):

\[
\Gamma^{OPE}_{sl}(H_b \rightarrow X_c) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left[ R(x) \left( 1 - \frac{K}{m_b^2} \right) + R'(x) \frac{G}{m_b^2} \right]
\]
where \( x = m_c^2/m_b^2 \) and

\[
R(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \log x
\]

\[
R'(x) = 3 - 8x + 24x^2 - 24x^3 + 5x^4 + 12x^2 \log x.
\]

The leading term in (4) corresponds to the leading \( \bar{b}b \) operator in the OPE, whereas the next-to-leading terms (of order \( O(1/m_b^2) \)) arise from the color magnetic moment operator and the \( b \)-quark kinetic energy; they are proportional to the hadronic matrix elements

\[
G = Z \langle H_b(v) | \bar{b}_v \frac{g \sigma^{\mu \nu} G_{\mu \nu}}{4} b_v | H_b(v) \rangle
\]

\[
K = -\langle H_b(v) | \bar{b}_v \left( \frac{(iD)^2}{2} \right) b_v | H_b(v) \rangle.
\]

Here \( b_v \) is the effective, velocity dependent heavy quark operator of the heavy quark effective theory (HQET), \( Z \) is a renormalization factor equal to one at the scale \( \mu = m_b \). In the \( m_b \to \infty \) limit, \( G \) and \( K \) are finite. Let us incidentally note that, for the leading term in (4), corresponding to the parton model result (i.e. (4) without the corrections of the order \( O(1/m_b^2) \)), also the \( O(\alpha_s) \) corrections are available [2], i.e.

\[
\Gamma_{sl}(b \to c \ell \nu_{\ell}) = \frac{G_F^2 m_b^5}{192 \pi^3} |V_{cb}|^2 \left[ R(m_c^2/m_b^2) + \frac{\alpha_s}{\pi} A_0(m_c^2/m_b^2) \right]
\]

where \( A_0(x) \) contains the perturbative corrections and can be found in [2].

The idea that OPE can be applied to heavy hadron decays is based on the notion of quark-hadron duality, a concept which is widely used, but remains nonetheless rather vague. The physical meaning of duality is intuitively clear: because of the large \( b \)-quark mass, the energy release to the final hadrons is large as compared to \( \Lambda_{QCD} \); therefore one expects that QCD can be applied and this would justify the approximation of identifying the hadronic decay width with the corresponding OPE expression.

However a closer scrutiny shows that this expectation is in general false and can be retained only if some further assumptions are made. The reason why the physical and the OPE widths cannot be identical is in the different structure of their singularities: in one case there are multiparticle hadronic
final states and, therefore, multihadron thresholds, while in the other case one has quark and gluon production with a different set of thresholds.

There are two possibilities for avoiding this negative conclusion. The first one consists in working far away from the physical thresholds; in this case OPE becomes necessarily true and one can therefore safely compute the hadronic quantities by their simpler OPE counterparts. However this local parton-hadron duality does not hold in semileptonic $H_b \to X_c \ell \nu_\ell$ weak decays (neither it occurs in other high energy processes, e.g. $e^+e^- \to \text{hadrons}$). The reason for this can be easily understood by looking at the differential decay rate $d\Gamma/dq^2dydv$ (here $q = p_\ell + p_\nu$, $y = E_\ell/m_b$, $p_b^\mu = m_b v^\mu$, $v = vz/m_b$). For $q^2$ and $y$ fixed, the differential decay rate, in the complex $v\bar{q}$ plane, has a cut, corresponding to multihadron production. For OPE to be valid, one should be away from the cut, which is not the case. The second possibility, that can be applied in semileptonic $H_b$ decays, consists in considering, instead of the hadronic and OPE expressions, some averages, i.e. their smearing over a region of the order $\Lambda_{QCD}$ \cite{3}. The use of smeared quantities is generally referred to as global duality and has been adopted in \cite{4} to prove the validity of OPE for semileptonic $B$ and $\Lambda_b$ decays in a particular kinematical region, the Shifman-Voloshin (SV) limit \cite{5} (see below). Considering the integrated semileptonic width, the authors in \cite{4} show that OPE is valid as well, even without explicit smearing. The matching between the hadronic and the OPE expressions for the widths is proved to two orders in the $1/m$ expansion and to first order in $\alpha_s$. As already suggested in \cite{3}, the validity of OPE is due to the fact that the integration contour can be deformed in such a way that the part which remains near the cut is of order $\Lambda_{QCD}/m$ and therefore is vanishingly small in the $m \to \infty$ limit.

The validity of OPE for non-leptonic decays is less clear. For non-leptonic transitions, neglecting penguin operators and four quarks $b\bar{c}\bar{c}s$ operators, we have the effective lagrangian:

$$\mathcal{L}_{nl} = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [c_1 O_1 + c_2 O_2]. \quad (10)$$

Here $c_1$ and $c_2$ are Wilson coefficients that in the leading-log approximation
are given by:
\[
c_1 = \frac{1}{2} \left( \left[ \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right]^{6/23} + \left[ \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right]^{-12/23} \right)
\]
\[
c_2 = \frac{1}{2} \left( \left[ \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right]^{6/23} - \left[ \frac{\alpha_s(m_W)}{\alpha_s(m_b)} \right]^{-12/23} \right)
\]
(11)

and the operators \(O_j\) are given by:
\[
O_1 = \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{d} \gamma_\mu (1 - \gamma_5) u
\]
\[
O_2 = \bar{c} \gamma_\mu (1 - \gamma_5) u \bar{d} \gamma_\mu (1 - \gamma_5) b .
\]
(12)

(13)

The OPE expression for the non leptonic decay width can be written in terms of the semileptonic \(\Gamma_{sl}^{OPE}\) as follows [7]:
\[
\Gamma_{nl}^{OPE} = N_c |V_{cb}|^2 \Gamma_{sl}^{OPE} \left\{ (c_1^2 + c_2^2) \left( 1 + \frac{\alpha_s}{\pi} \right) + \right. \\
+ \frac{2c_1c_2}{N_c} \left[ 1 - \frac{16 \alpha_s}{3 \pi} + \frac{16}{m_b^2} \left( 1 - \frac{m_c^2}{m_b^2} \right)^3 G \right] \right\} .
\]
(14)

In this case, however, we have \textit{a priori} no reason to expect that the contribution near the physical cut is small, because there is no external momentum \(q\) and no integration contour to deform. Therefore, for non leptonic decays, in order to justify OPE, one has to \textit{assume} local duality or, alternatively, to prove it under some additional hypothesis. Clearly this puts the validity of OPE for non leptonic heavy hadron decays on a less firm ground [6]; a signal of this might be the experimental result for the ratio of the total lifetimes of \(\Lambda_b\) and \(B\). Experimentally one has \(\tau(\Lambda_b)/\tau(B) = 0.76 \pm 0.06\) (see e.g. [8]), whereas, from OPE, one expects \(\tau(\Lambda_b)/\tau(B) \approx 1\) (including corrections to the third order in the \(1/m\) expansion [9]). As an explanation of this discrepancy, a possible failure of local duality for non leptonic heavy hadron decays has been suggested [10].

The aim of this paper is to investigate the validity of OPE for non leptonic \(B\) and \(\Lambda_b\) decays. We shall prove that local duality is indeed valid and that OPE can be proved under the following hypotheses:
1) one works in the SV limit:

\[ m_b, m_c >> \delta m = m_b - m_c >> \Lambda_{QCD} \]  

(15)

2) the weak non-leptonic amplitude is factorized in two parts, the first one corresponding to the transition \( b \rightarrow c \) and the second to a transition between light quarks. Since factorization has been proved only in the \( N_c \rightarrow \infty \) limit \((N_c = \text{number of colours})\), we shall implicitly assume this limit, even if we retain some of the \( \mathcal{O}(1/N_c) \) corrections in order to keep track of terms violating factorization and/or duality.

We shall show in the next section how these hypotheses are used to prove duality while in Section 3 we shall conclude this letter by a discussion of our results.

2 \( B \) and \( \Lambda_b \) non leptonic decays and duality

Before considering the problem of local duality for non-leptonic \( B \) and \( \Lambda_b \) decays, we shall briefly review the results obtained in [4] for the semileptonic case.

These authors work in SV limit (15). The OPE result for semileptonic decays, as expressed by (4) (with the corrections \( \mathcal{O}(\alpha_s) \) in (9)), becomes, in this limit:

\[ \Gamma_{sl}^{\text{OPE}}(H_b \rightarrow X_c) = \frac{G_F^2}{192\pi^3} |V_{cb}|^2 \left[ \left( 1 - \frac{\alpha_s}{\pi} \right) \left( \frac{64}{5} \delta m^5 - \frac{96}{5} \frac{\delta m^4}{m_b} \right) \right. \]

\[ + \left. 64 \frac{G\delta m^4}{m_b} + \ldots \right] \]  

(16)

The dots indicate several higher order corrections, i.e. terms \( \mathcal{O}(\delta m^7/m_b^2) \) or \( \mathcal{O}(G\delta m^5/m_b^2) \) or \( \mathcal{O}(K\delta m^5/m_b^2) \); they can be found in [4]. As observed in [4] a correction \( \mathcal{O}(G\alpha_s/m_b) \) could also be added but has not been computed yet (note that \( G = 0 \) when \( H_b = \Lambda_b \) because the light degrees of freedom have zero angular momentum). Computing now \( \Gamma_{sl}(B \rightarrow D, D^*l\nu) \) and \( \Gamma_{sl}(\Lambda_b \rightarrow \Lambda_c l\nu) \)
in the same kinematical regime [13], [4] one obtains the results [4]:

\[
\Gamma_{sl}^{\text{had}}(B \to D, D^*\ell\nu_\ell) = \frac{G_F^2}{192\pi^3}|V_{cb}|^2 \left[ \left(1 - \frac{\alpha_s}{\pi} \right) \left( \frac{64}{5} \delta M^5 - \frac{96}{5} \frac{\delta M^6}{M_B} \right) \right] \\
+ 64 \left(1 - \frac{4\alpha_s}{3\pi} \right) \frac{G\delta M^4}{M_B} + \ldots \tag{17}
\]

with

\[
\delta M = M_B - M_D = \delta m \left[1 - \frac{K_B + G}{m_b^2} \right] + O(1/m_b^3) \tag{18}
\]

and

\[
\Gamma_{sl}^{\text{had}}(\Lambda_b \to \Lambda_c\ell\nu_\ell) = \frac{G_F^2}{192\pi^3}|V_{cb}|^2 \left[ \left(1 - \frac{\alpha_s}{\pi} \right) \left( \frac{64}{5} \delta M^5 - \frac{96}{5} \frac{\delta M^6}{M_{\Lambda_b}} \right) \right] + \ldots \tag{19}
\]

with

\[
\delta M = M_{\Lambda_b} - M_{\Lambda_c} = \delta m \left[1 - \frac{K'_b}{m_b^2} \right] + O(1/m_b^3) \tag{20}
\]

\(K_B\) and \(K'_b\) in (18) and (20) are given by (8) with \(H_b = B\) and \(H_b = \Lambda_b\) respectively. The inclusive semileptonic width for \(B\) decay is saturated, in the SV limit, by decays into \(D\) and \(D^*\) and, for \(\Lambda_b\), by its semileptonic decay into \(\Lambda_c\).

Eqs. (16), (17) and (19) show that, for semileptonic inclusive decays, duality holds in the SV limit for the first two terms in 1/m_b expansion and at the first order in \(\alpha_s\); the only term for which duality has not been checked in [4] is the term proportional to \(\alpha_s G\delta M^4/M_b\) appearing in (17), which has no matching in (16) (as we stressed already, the calculation of this correction to the OPE leading term is still missing).

Let us show how the OPE inclusive non-leptonic decay rate (14), can be matched by the sum over exclusive non-leptonic decay channels. Similarly to the case of semileptonic decays, we shall work in the SV limit and, therefore, we shall take the results of [4], i.e. that duality is proved for the first two terms in the 1/m_b expansion and at the first order in \(\alpha_s\) for the semileptonic

\[1\]Under the SV kinematical conditions, the form factors can be expanded around the zero recoil point and corrections to this limit appear only at the order 1/M^2.

decays. As a second hypothesis, we shall assume factorization of the decay amplitudes. Because of our hypotheses, we expect that, also for non leptonic decays, duality may be proved only for the first two leading terms in $1/m_b$ expansion and up to order $\alpha_s$.

The full width for non-leptonic decays is given by

$$\Gamma_{nl} = \frac{1}{2M_{H_b}} \text{disc} \left\{ i \int d^4x \langle H_b(v)|T(\mathcal{L}_{nl}(x)\mathcal{L}_{nl}(0))|H_b(v)\rangle \right\}$$

$$= \frac{1}{2M_{H_b}} \sum_X (2\pi)^4 \delta^4(p - p_X) |\langle X|\mathcal{L}_{nl}(0)|H_b(v)\rangle|^2$$

(21)

where $\mathcal{L}_{nl}$ is the effective lagrangian for non leptonic decays given in (10).

Let us now write

$$|X\rangle = |X_uX_c\rangle$$

(22)

where $X_u$ is a set of light particles, and $X_c$ is a set of hadronic charmed states. Let us now consider the matrix element

$$\langle X_uX_c|\mathcal{L}_{nl}(0)|H_b(v)\rangle \ .$$

(23)

Assuming factorization, i.e. inserting the vacuum in all possible ways, we obtain, by using the Fierz identities,

$$\langle X_uX_c|\mathcal{L}_{nl}(0)|H_b(v)\rangle = \left( c_1 + \frac{c_2}{N_c} \right) \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* \langle X_u|\bar{d}\gamma^\mu(1 - \gamma_5)u|0\rangle \times$$

$$\langle X_c|\bar{c}\gamma^\mu(1 - \gamma_5)b|H_b(v)\rangle.$$  

(24)

A comment on the factorization procedure we have adopted is now in order. From eqs.(10),(12) and (13), we see that, using the Fierz theorem, one can write for $\mathcal{L}_{nl}(0)$ the following expressions:

$$\mathcal{L}_{nl}(0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [(c_1 + \frac{c_2}{N_c})O_1 + \tilde{O}_1]$$

(25)

or

$$\mathcal{L}_{nl}(0) = \frac{G_F}{\sqrt{2}} V_{cb} V_{ud}^* [(c_2 + \frac{c_1}{N_c})O_2 + \tilde{O}_2] \ .$$

(26)

$\tilde{O}_1$ and $\tilde{O}_2$ are written as products of two coloured currents; in the factorization approximation they give no contribution because the matrix elements
of a coloured current between physical hadronic states vanish. In exclusive
decays, \((25)\) and \((26)\) produce respectively the so-called class I and class II
decays. We shall now show that, within our hypotheses, we can always
choose the form \((25)\) for \(L_{nl}\). In order to be definite let us consider
\(H_b = B^0\) decays. If \(\langle X_u X_c \rangle\) contains \(D^+\) or \(D^{*+}\) states, we can assume eq.\((25)\) and
apply factorization as in \((24)\), with \(\langle X_c \rangle = \langle D^+ \rangle\) or \(\langle D^{*+} \rangle\) and neglecting \(\tilde{O}_1\).
If \(\langle X_u X_c \rangle\) contains \(D^0\) or \(D^{*0}\) states, we can evaluate the matrix element in
two ways.

1) We can write \(L_{nl}(0)\) according to \((25)\) and neglect \(\tilde{O}_1\). In this case one
shall factorize the amplitude as follows (considering only \(D^0\) for simplicity):

\[
\langle X_u D^0|L_{nl}(0)|\bar{B}^0 \rangle = \left( c_1 + \frac{c_2}{N_c} \right) \frac{G}{\sqrt{2}} V_{cb} V_{ud}^* \langle X^- | \bar{d} \gamma^\mu (1 - \gamma_5) u | 0 \rangle \times
\langle X^+ D^0 | \bar{c} \gamma^\mu (1 - \gamma_5) b | H_b(v) \rangle. \tag{27}
\]

where \(X^+, X^-\) are multiparticle states containing only light hadrons (the
superscript \(\pm\) indicates the total charge). On the basis of the results for
semileptonic decays contained in \([4]\) and \([5]\), \((27)\) represents in the SV limit
a higher order correction (of the order of \(O(\delta M^2/M_b^2)\)) because \(B^0\) decays
semileptonically only to \(D^+, D^{*+}\) in this limit.

2) The second way to evaluate the contribution of these states is by
making use of \((26)\) and neglecting, in the factorization approximation, \(\tilde{O}_2\).
However one can see immediately that, also by this choice, the matrix element
is a higher order correction and should be neglected. As a matter of fact, since
we work in the \(N_c \to \infty\) limit, where factorization can be proved to be valid,
we should evaluate the resulting expression in this limit (for discussion on this
point see \([11]\), \([12]\)). But in the \(N_c \to \infty\) limit these states would contribute
to the width a term \(O(c_2 + c_1/N_c)^2 = O(c_2)^2 = O(\alpha_s^2)\); in the approximation
when one neglects higher order \(O(\alpha_s^2)\) perturbative contributions, this term
should be neglected as well.

On the basis of this discussion we conclude that, in the SV limit, and
taking into account only the first two terms in the \(1/m_b\) expansion and at
order \(\alpha_s\) in the perturbative series, we can limit ourselves to consider only
class I decays, as expressed by \((24)\).
From (21) and (24) one obtains
\[
\Gamma_{nl}(H_b) = \left(1 + \frac{c_2}{N_c}\right)^2 \int \frac{d^4q}{(2\pi)^3} W^{\mu\nu}_{H_b}(q, v) T_{\mu\nu}(q),
\]
(28)
where \(W^{\mu\nu}_{H_b}\) is the same hadronic tensor appearing in the semileptonic \(b \to c\) decays
\[
W^{\mu\nu}_{H_b} = (2\pi)^3 \sum_{X_u} \delta^4(p_{H_b} - q - p_{X_u}) \times
\langle H_b | \bar{b} \gamma^\mu (1 - \gamma_5) c | X_u \rangle \langle X_u | \bar{c} \gamma^\nu (1 - \gamma_5) b | H_b \rangle
\]
(29)
while \(T_{\mu\nu}\) is given by:
\[
T_{\mu\nu}(q) = (2\pi)^4 \sum_{X_u} \delta^4(q - q_{X_u}) \times
\langle 0 | \bar{u} \gamma_\mu (1 - \gamma_5) d | X_u \rangle \langle X_u | \bar{d} \gamma_\nu (1 - \gamma_5) u | 0 \rangle.
\]
(30)

The vector part of the tensor \(T_{\mu\nu}\) is theoretically known for large values of \(q^2\) up to the order \(\alpha_s^3\) from \(e^+e^-\) into hadrons: here we need only the expression to the first order in \(\alpha_s\). We shall write
\[
T^{\mu\nu}(q) = T^{\mu\nu}_R(q) + T^{\mu\nu}_{QCD}(q),
\]
(31)
where \(T^{\mu\nu}_{QCD}(q)\) is the \(QCD\) contribution to \(T^{\mu\nu}(q)\); neglecting the \(u\) and \(d\) quark masses, it is given by:
\[
T^{\mu\nu}_{QCD}(q) = 2N_c \frac{1}{6\pi} \left(1 + \frac{\alpha_s}{\pi}\right) (q^\mu q^\nu - q^2 g^{\mu\nu}) \theta(q^2 - q_0^2).
\]
(32)
The factor 2 with respect to the result for \(R_{e^+e^-}\) is due to the contribution of both vector and axial currents in (31); \(q_0^2 \sim 1 - 2 \text{GeV}^2\) is the onset of the \(QCD\) behaviour. \(T^{\mu\nu}_R(q)\) takes into account the contribution of the low lying \((m_R^2 \leq q_0^2)\) particles: \(\pi, a_1, \rho, \rho'\). It can be easily checked that, in the SV limit, these resonances contribute to the non-leptonic rate with terms of the order \(G_F^2 \delta m_3 f_R^2\), where \(f_R\) is a mass parameter, of order \(\Lambda_{QCD}\), characteristic of the resonance. As we shall discuss below, these terms represent corrections \(\mathcal{O}(\Lambda_{QCD}/\delta m)^2\) to the leading contribution arising from \(T^{\mu\nu}_{QCD}\) and will be
neglected because, in the SV limit, $\delta m >> \Lambda_{QCD}$ (see eq. (13)). We shall therefore put
\[ T_{\mu\nu}(q) \approx T_{\mu\nu}^{QCD}(q) \] (33)

To obtain the non leptonic decay width $\Gamma_{nl}(H_b)$ from (28) we observe that for the semileptonic inclusive $b \rightarrow c$ decay, the following formula holds

\[ \Gamma_{sl}(H_b) = \frac{G_F^2 |V_{cb}|^2}{\pi} \int \frac{d^4q}{(2\pi)^3} W_{H_b}^{\mu\nu}(q, v) T_{\mu\nu}^l(q) \] (34)

where, for $q^2 > q_0^2$, the leptonic tensor $T_{\mu\nu}^l$ differs from (32) only by the overall factor $N_c(1 + \alpha_s/\pi)$:
\[ T_{\mu\nu}^l(q) = \frac{2}{6\pi} \left( q_\mu q_\nu - q^2 g_{\mu\nu} \right) \] (35)

From the previous formulae one obtains an expression for the non leptonic width, computed in the factorization approximation, as a function of the semileptonic one:
\[ \Gamma_{nl}^{had}(H_b) = N_c \left( 1 + \frac{\alpha_s}{\pi} \right) |V_{cb}|^2 \left( c_1 + \frac{c_2}{N_c} \right)^2 \Gamma_{sl}^{had}(H_b) + ... \] (36)

where the omitted terms are $\mathcal{O}(\Lambda_{QCD}^2/\delta m^2)$ or $\mathcal{O}(\delta m^2/m_b^2)$. In the SV limit we can substitute in this equation expressions (17) and (19) for $B$ and $\Lambda_b$ respectively.

To compare this result with (14), we develop the Wilson coefficients $c_1$ and $c_2$ to the order $\alpha_s$:
\[ c_1(\mu) = 1 - \frac{3}{N_c} \frac{\alpha_s(M_W)}{4\pi} \log(\mu^2/M_W^2) \]
\[ c_2(\mu) = \frac{3}{4\pi} \frac{\alpha_s(M_W)}{4\pi} \log(\mu^2/M_W^2) \] (37)

We notice that the combination $c_1 + c_2/N_c$ in (36), does not contain first order $\alpha_s$ corrections. By developing (14) in the SV limit, we obtain, at the

The presence of $q_0^2 \neq 0$ in (32) introduces corrections $\mathcal{O}(\Lambda_{QCD}^2/\delta m^2)$, as it can be easily verified. Similarly to our discussion above, we neglect this correction in the SV limit.
first order in $\alpha_s$:

$$
\Gamma_{nl}^{OPE} = \frac{G_F^2 N_c}{192\pi^3} |V_{cb}|^2 \left[ \frac{64}{5} \delta m^5 - \frac{96}{5} \frac{\delta m^6}{m_b} + 64 \frac{G \delta m^4}{m_b} + \ldots \right] 
\times \left( 1 + \frac{20c_1c_2 G}{N_c \delta m^2} \right) .
$$

(38)

On the other hand, from (36), after substitution of (17) or (19) we have the expression

$$
\Gamma_{nl}^{had}(H_b) = \frac{G_F^2 N_c}{192\pi^3} |V_{cb}|^2 \left[ \frac{64}{5} \delta M^5 - \frac{96}{5} \frac{\delta M^6}{M_{Hb}} + 64 \frac{G \delta M^4}{M_{Hb}} + \ldots \right]
$$

(39)

which is valid in the same limit. Comparing these two equations, we see that, a part from the term $20c_1c_2 G/(N_c \delta m^2)$, which is present in (38) and has no counterpart in (39), there is matching between the two expressions. We notice that, in this term, $G$ is of the order $\Lambda_{QCD}^2$; therefore it represents a correction of the order $\alpha_s \Lambda_{QCD}^2 \delta m^3$ and therefore belongs to a class of corrections that we have neglected (for example those coming from the low-lying resonances)\footnote{In this approximation one should neglect the term $64G \delta m^4/m_b$ as well. It can be noted, however, that this term has an exact counterpart in the hadronic expression.}. Moreover it represents a correction $O(1/N_c)$ and should be neglected, in the approximation we are considering.

The previous analysis shows that the inclusive non leptonic $B$ and $\Lambda_b$ decay widths, computed in the Shifman-Voloshin limit (15) by factorization, are equal to their OPE expressions including the leading term $O(\delta m^5)$ and the next-to-leading term $O(\delta m^6/m_b)$ in the OPE, at the order $\alpha_s$ in the perturbative expansion and up to terms $O(\Lambda_{QCD}^2/\delta m^2)$ that are negligible in the SV limit. Within this limitations our analysis shows that local duality is indeed satisfied by non leptonic heavy hadron decays.

## 3 Conclusions

Let us summarize and discuss our results. We have proved local duality for $B$ and $\Lambda_b$ non leptonic decays, i.e. the equalities

$$
\Gamma_{nl}^{OPE}(H_b) = \Gamma_{nl}^{had}(\Lambda_b) = \Gamma_{nl}^{had}(B) .
$$

(40)
for the first two terms in the $1/m_b$ expansion and at the order $\alpha_s$ in the perturbative expansion. These results have been proved in the Shifman-Voloshin limit and assuming factorization of the weak amplitudes.

One may wonder on the experimental significance of SV limit; for example, in this kinematical regime, the $B$ meson would decay semileptonically only to $D$ and $D^*$. This prediction however, does not agree with the data, since experimentally one has

$$\frac{\Gamma(B \to D, D^* \ell \nu_\ell)}{\Gamma(B \to X \ell \nu_\ell)} = 0.64 \pm 0.08$$

which is obtained considering both $B^0$ and $B^\pm$ semileptonic decays \[13\]. This discrepancy is expected since the assumption \[15\] $m_c >\! > \! \delta m$ is clearly violated.

Secondly, our results are based on the assumption of factorization of the hadronic final state into two parts, one containing a single charmed particle ($D, D^*$ or $\Lambda_c$) and the other containing only light hadrons. Factorization holds in the $N_c \to \infty$ limit, and can therefore be violated by terms $\mathcal{O}(1/N_c)$. Moreover, there are arguments that can justify factorization in the $m_b \to \infty$ limit \[14\], but it is not clear to which order in the $1/m_b$ expansion it holds. In \[15\], an estimate is given of the nonfactorizable part of the exclusive process $B^0 \to D^+ \pi^-$: they found a term proportional to the matrix element $G$ (see eq. (7)) and color-suppressed with respect to the leading one; moreover these authors find that in the SV limit such a term is not suppressed by powers of $1/m_b$. This result agrees with our eq. (38), where similar terms are present (those proportional to $20c_1c_2G/(N_c\delta m^2)$). As discussed above, besides being colour suppressed, in the SV limit these contributions are negligible because are of order $\Lambda_{QCD}^2/\delta m^2$; it is quite possible that, beyond the SV limit, nonfactorizable terms arise that are not suppressed by $1/m_b^2$ in the infinite heavy quark mass limit. In \[16\] the problem of factorization in inclusive non leptonic $B$ decays is discussed assuming a standpoint complementary to the one taken in the present paper: these authors assume local duality from the outset and discuss the limits of factorization. As far as our approach is comparable with theirs, the results of the present paper agree with those of \[16\].
Finally let us note that, because of the assumed SV limit, the non leptonic widths depend on $\delta M$ and not on $M_{H_b}$. Because of (18) and (20), both for $B$ and $\Lambda_b$ decays $\delta M = \delta m + \mathcal{O}(1/m_b^2)$ and there is no way for a $\mathcal{O}(1/m_b)$ term to appear in the ratio $\tau(\Lambda_b)/\tau(B)$, contrarily to the possibility suggested in [10]. This result, however, might be heavily dependent on the kinematics of the SV limit and the possibility remains that, beyond this limit, $\mathcal{O}(1/m_b)$ corrections to the non leptonic $B$ and $\Lambda_b$ lifetimes appear.

Acknowledgements
This work was carried out under the program Human Capital and Mobility, contract nr. ERBCHRXCT940579, OFES nr.950200. A.D. acknowledges the support of a TMR research fellowship of the European Commission under contract nr. ERB4001GT955869. We thank F. Feruglio for reading of the manuscript.

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