The X-rule: universal computation in a non-isotropic Life-like Cellular Automaton

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Abstract
We present a new Life-like cellular automaton (CA) capable of logic universality – the X-rule. The CA is 2D, binary, with a Moore neighborhood and λ parameter similar to the game-of-Life, but is not based on birth/survival and is non-isotropic. We outline the search method. Several glider types and stable structures emerge spontaneously within X-rule dynamics. We construct glider-guns based on periodic oscillations between stable barriers, and interactions to create logical gates.

keywords: universality, cellular automata, glider-gun, logical gates.

1 Introduction

Ever since the game-of-Life cellular automaton (CA) was created by Conway in 1970, and its first glider-gun constructed subsequently by Gosper[5, 2], the search has continued for alternative rules capable of universal computation[1, 4], where logical gates can be constructed from mobile and static configurations, and where a glider-gun, a logical information mechanism for generating gliders at regular intervals, is a key requirement.

This paper presents a new Life-like CA, the X-rule (figure 12) – a 2d binary CA with a Moore neighborhood and a λ parameter[6] similar to the game-of-Life, but which is not based on birth/survival and is non-isotropic, and where glider-guns based on periodic oscillations between stable

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Figure 2: Snapshots of glider-guns in other 2D CA: (a) Conway’s game-of-Life b3/s23 showing Gosper’s glider-gun\(^2\), (b) Eppstein b35/s236\(^4\), (c) Sapin\(^7\), (d) Adamatsky-Wuensche spiral-rule\(^13\).

barriers are constructed (figures\(^1\)\(^,\)\(^30\))\(^,\) and interactions between gliders and stable structures (eaters) are arranged to create the logical gates required for logic universality – implementing any logic circuit, and potentially universality in the Turing sense, though this is reserved for a later investigation.

Other 2D CA with aspects of universality have been presented in addition to the game-of-Life involving both glider-guns and stable structures (figure 2). Eppstein\(^4\) has come up with several birth/survival CA, notably b35/s236 with a constructed glider-gun. Sapin’s isotropic outer-totalistic rule\(^7\), not based on birth/survival, features a spontaneously emergent glider-gun. In 3-value systems there is the Adamatzky-Wuensche 2D hexagonal 7-neighbor totalistic “spiral” rule\(^13\) with spontaneously emergent glider-guns.

A remarkable quality of the X-rule is that it achieves logical universality, and potentially also in the Turing sense, by means of constructed glider-guns, analogous to Gosper’s (figure 2a) but different in that they are made from a kit of parts, gliders and reflectors, that can be put together in many combinations to produce periodic oscillators based on bouncing/reflecting behaviour – pairs of gliders bouncing against each other and trapped between reflectors from which other glider types are ejected at periodic intervals – this is achieved by introducing specific non-isotropic outputs (figure 14) within an isotropic\(^1\)\(^,\) precursor rule (figure 5). Increasing the gap between reflectors increases the glider-gun period and reduces glider ejection frequency. These properties provide the X-rule with flexible and versatile computational dynamics.

\(^1\)Isotropic in the sense we use it means that given a neighborhood pattern, any orientation of the pattern (spin, reflection, vertical flip) has the same output in the rule-table, so the resulting CA dynamics will be equivalent whatever the orientation of an initial state.
Succeeding sections describe the following:

2. The search of isotropic rule-space for emergent gliders coexisting with stable structures.
3. The pivotal section in the paper – the construction of glider-guns based on simpler periodic oscillators, and the derivation of the X-rule itself.
4. The X-rule universe, including its gliders, eaters and collisions.
5. A detailed description of two types of basic glider-guns.
6. Compound glider-guns, combining basic glider-guns to enhance the diversity of gun dynamics.
7. Examples of the logical gates, NOT, AND, OR and NAND, to achieve logic universality.

2 Searching rule-space for emergent gliders

![Figure 3: The scatter-plot of a sample of 93000+ rules, plotting min-max entropy variability against mean entropy. The left panel shows the location of the shortlist of about 70 rules, and the precursor to the X-rule.](image)

Definitions of Cellular Automata (CA) can be found from many sources, so will not be repeated here, other than to note that this paper is dealing with binary 2D classical synchronous CA, comparable to the Game-of-Life (GoL) with a Moore neighborhood but not based on birth/survival, and with periodic (or null) boundary conditions.

The Moore neighborhood has 3x3=9 neighbors giving a full lookup-table with $2^9=512$ outputs, a rule-space of $2^{512}$, but we started off with isotropic rules only – equal outputs for any neighborhood rotation, reflection, or vertical flip – where the number of effective outputs reduces to 102, thus rule-space $= 2^{102}$. 


Figure 4: The scatter-plot in Fig.3 showing rule frequency ($\log y$) of rules on a 256x256 grid, showing the area of exhaustive search and the short list of glider rules. Characteristic dynamical behaviour is found in different parts of the landscape. Rules in the high variability "complex" sector of the scatter plot were found unsuitable because of over-active dynamics, including unabated glider collisions and the lack of stable structures.

We searched in a sample of isotropic rule-space where look-up tables were selected at random but biased probabilistically to have a similar density of 1s as GoL ($\lambda$ parameter $= 0.273 \pm \approx 0.06$). This rule-space is more general than GoL – not based on birth/survival, and not totalistic. Because of the $\lambda$ bias, the isotropic rule-space searched was smaller than $2^{10^2}$ in [8]. In addition, the all-0 neighborhood was made to output 0.

Within these constraints, a sample of 93000+ rules was generated in DDLab[15,14] using Wuenche's input-entropy method[11,12], which creates a scatter-plot of min-max input-entropy variability against mean entropy, and which separates rule-space into fuzzy sectors of chaos, order, and complexity (figures [3] and [4]). To generate the scatter-plot we track how frequently the different entries in the lookup-table are actually looked up in a moving window of time-steps, started once the CA has settled into its typical behaviour. The Shannon entropy of this frequency distribution, the input-entropy $S$, at time-step $t$, for one time-step ($w=1$), is given by $S_t = -\sum_{i=0}^{L-1} (Q_i^t \times \log Q_i^t)$, where $Q_i^t$ is the lookup frequency of neighborhood $i$ at time $t$. $L$ is the rule-table size, and $n$ is the size of the CA.

A number of parameters need to be defined. For this experiment they were as follows. The measures were smoothed by being averaged over a moving window of $w = 10$ time-steps. The measures were started after 30 time-steps, and then
taken for a further 400 time-steps. The 2d CA was $40 \times 40$ – the lattice should be big enough but not too big for effective tracking of entropy variability\cite{12}. Each rule was run from 5 random initial states and average measures were plotted – course grained onto a $256 \times 256$ grid – the entropy variability ($x$-axis) against the mean entropy ($y$-axis). Variability was measured according to min-max, meaning the maximum increase in entropy from any dynamical minimum. The assembled sample was sorted by both decreasing $x$ and $y$, and the data plotted in figures\cite{3} and\cite{4}. The plot roughly classifies rule-space between chaos, order and complexity. Individual rules from the plot can be selected and visually scanned by efficient methods in DDLab\cite{12,13}.

2.1 Shortlist of glider rules in the ordered sector

Avoiding the densely populated chaotic sector, an exhaustive search was made of CA dynamics in the scatter-plot looking for spontaneously emerging gliders and stable structures. Although gliders were frequent in the complex sector – the sector with high entropy variability – this also turned out to be too unstable, although this is the sector were GoL, Eppstien’s and Sapin’s rule would occur.

The exhaustive search thus concentrated within the ordered sector of the scatter plot (figure\cite{5}), where a shortlist of about 70 rules with both gliders and stable structures were identified. From this list we selected just 5 rules with gliders travelling both orthogonally and diagonally.

3 Constructing glider-guns

3.1 Periodic oscillators – the X-rule precursor

Figure 5: The rule-table of the X-rule precursor, 512 neighborhood outputs are shown in descending order of neighborhood values\cite{9}, from left to right, then in successive rows from the top.

Having found gliders the next step was to build a glider-gun. To do this, the main idea was to build a periodic bouncing-colliding structure, a dynamical oscillator driving periodic collisions, which eventually, with some modification to the pattern or rule, might eject gliders.

With this in mind, from the final short-list of 5 isotropic rules, we selected a rule, the X-rule precursor, where bouncing from collisions was observed. This bouncing behaviour was promising because it could provide components for a periodic bouncing-colliding structure.

Two spontaneously emergent gliders\footnote{A third glider, Gb (figure\cite{10}) is also supported but less likely to emerge than Ga or Gc, because its phase patterns are more complex and Gb lacks a simple predecessor.} in the X-rule precursor are glider Ga.\[\]
free to move in all diagonal directions (fig 6) and glider \(Gc\) free to move in all orthogonal directions (fig 7). The glider speed of \(Ga = c/4\) and of \(Gc = c/2\), where \(c\) is the speed of light.

An example of gliders “bouncing” is when two \(Gc\) gliders collide head-on and reverse direction. The initial separation must be an even number including zero, as in figure 8 where the initial separation is 8 cells.

Also observed in the X-rule precursor were small stable emergent configurations (eaters \(Ea\)) with two \(Ea\) eaters we built a reflector \(Rc\) that “reflects” glider \(Gc\) back after a collision, where \(Rc\) itself remains stable (figure 9). This reflecting behaviour occurs when the initial gap between glider \(Gc\) and \(Rc\) is an even number including zero.

For clarity we will reserve the word “bounce” for glider-glider interactions as in figure 8 and “reflect” for interactions between a glider and any stable configuration such as \(Ea\) or \(Rc\), as in figure 9.

As the rule is isotropic, the mirror image of the reflecting interaction would also be is valid, so we could construct a periodic oscillator with one \(Gc\) glider reflecting back and forth between two \(Rc\) reflectors, called a “simple reflecting oscillator”, SRO (Figure 10).

Finally we obtained what we were looking for, a more complex periodic collision structure (figure 11) formed from two \(Gc\) gliders continually colliding and bouncing between two reflectors, which we call a “reflecting/bouncing oscillator” RBO. The RBO is an assembly of several components, so is flexible in that

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3 In game-of-Life terminology a structure moving diagonally is a “glider” – moving orthogonally its a “space-ship”. In this paper we use “glider” for both types.

4 A Moore neighborhood limits the maximum displacement of any pattern to one cell per time-step either orthogonally or diagonally. This is the CA’s speed of light, \(c\).
the distance between reflectors (figure 11) and the phase of the various collisions could be varied. We ran experiments to test these variations, hoping to obtain gliders, but without success. Although the RBO we constructed did not eject gliders, the rule itself seemed a promising candidate whereby mutations to its lookup-table might achieve a glider-gun, so we kept this rule as the precursor to the X-rule.

![Figure 9: Glider Gc reflecting off the reflector Rc, time-steps shown.](image)

![Figure 10: Glider Gc reflecting continuously between two stable Rc reflectors, gap=20, period=70, time-steps shown. We call this periodic structure a "simple reflecting oscillator", SRO.](image)

![Figure 11: Two Gc gilders reflecting/bouncing between two reflectors, gap=20, period=30, time-steps shown. We call this periodic structure a "reflecting/bouncing oscillator", RBO. (right) a representation showing 2D time-steps.](image)

### 3.2 Creating glider-guns – the X-rule

Our strategy for creating glider-guns was to mutate outputs in the X-rule precursor’s rule-table with no restriction on preserving isotropy. These mutations had to fulfil two objectives: firstly, to preserve the essence of the periodic oscillating behaviour SRO and RBO, and the gliders Ga and Gc, from the X-rule precursor described in section 3.1 and secondly to produce ejected gliders from the periodic oscillators. The methodology for achieving this was as follows:
We identified the 36 neighborhood outputs that had no effect on the oscillating behaviour or on the gliders, giving $2^{36}$ possible combinatorial mutations. However, for ease of computation we considered just a random subset of $2^{17}$.

2. We automatically tested rules in the subset sample with a “reflecting/bouncing oscillator”, RBO, varying the gaps between the reflectors, looking for periodic glider ejection.

3. In these experiments we obtained two different glider-guns in a rule later named the X-rule. One glider-gun GGa ejected Ga gliders SouthWest and NorthWest, the same glider that had previously been observed as freely emergent in Section 3.1. The other glider-gun GGb ejected a new type of glider Gb South and North. These glider-guns are described in detail in section 5.1.

In the testing sequence before and after the X-rule, GGa (but not GGb) was also detected in rules that were close variations of the X-rule, but we decided to focus on the X-rule itself because it supported two glider-guns, reasoning that two glider-guns are better than one.

The X-rule differs from its precursor by 11 out of 512 neighborhood outputs. The mutations were all from 1 to 0 – their positions in the rule-table are shown in figure 13 and the actual neighborhoods in figure 14. Six of these mutations retain isotropy, 5 are non-isotropic and their spins/flips give 12 non-isotropic neighborhoods. The remaining 500 isotropic neighborhoods allow most but not all gliders to move in reflected and rotated directions.
4 Emergent structures in the X-rule universe

4.1 Gliders

The X-rule conserves two emergent glider types from its precursor, glider Ga able to move in all diagonal directions (figure 6), and glider Gc able to move in all orthogonal directions (figure 7). Both these gliders emerge easily from a random initial seed because the phase patterns of Ga are very simple, and Gc has a simple predecessor – the pattern Gc-p and its rotations.

The X-rule presents two further gliders Gd and Gb. Glider Gd, not present in the precursor, is an emergent orthogonal asymmetric glider moving only West and East with speed $c/2$, in 4 phases where the asymmetry alternates about a horizontal axis.

![Gd diagram](image)

Figure 15: Asymmetric glider Gd shown moving East. The reverse moves West.

In the X-rule, Gb is an orthogonal glider moving only North and South with speed $c/2$, in 4 phases. In the X-rule precursor, Gb also functions – in all directions, but was not initially observed because each of its phases has a moderately intricate pattern with a low probability of emerging and surviving from a random seed, and Gb’s phases also lack a simple predecessor.

In the X-rule glider Gb is not free to move East and West, only North and South. If Gb (phase 1 in figure 16) is pointed towards the West it transits into glider Gc moving West via Gc-p as in figure 17.

![Gb diagram](image)

Figure 16: Glider Gb shown moving South. The reverse moves North.

![Gc and Gc-p](image)

Figure 17: Starting with glider Gb (phase 1) pointing West, the pattern transits all of its 4 phases, then transforms to Gc-p which is the predecessor of glider Gc.

Things become more complicated when glider Gb is pointed East. If Gb phase 4 in figure 17 is reversed towards the East it will transit via Gc-p into
glider Gc moving East – a reversed scenario to figure 17. Gc-p pointing West or East provides the predecessors of Gc.

Figure 18: Starting with glider Gb (phase 4) pointing East, the pattern changes to Gc-p, then transits to glider Gc moving East.

However, if Gb phase 1 in figure 17 is turned towards the East, it will transit Gb phase 2 and 3 pointing East but not reach phase 4, instead it will transform in stages to the final result of a pair of Ga gliders moving SW and NW, and a pair of Gc gliders moving West and East.

Figure 19: Starting with glider Gb (phase 1) pointing East the pattern transits the first 3 Gb phases, then transforms according the central part of glider-gun GGa. For example steps 14 and 24 above are the same as the central part of steps 27 and 37 in figure 31. At step 24 above, a pair of Ga gliders move SW and NW, and a pair of Gc gliders move West and East. The remaining patterns in the central area will disappear in 2 more time-steps.

4.2 Combined gliders

Ga gliders can be joined together to created combined Ga gliders of arbitrary size with a one cell overlap between adjacent gliders, as illustrated in figure 20 for combined Ga moving SE. Combined Ga gliders move in any diagonal direction cycling through the usual 4 phases.

Figure 20: Glider Ga2 (left) and combined Ga2 gliders of increasing size 2 – 6, moving SE in 4 phases at a speed of c/4. There is a one cell overlap between adjacent gliders in the combination. Ga1 combines with Ga3, and Ga2 with Ga4.
4.3 Gliders colliding with gliders

The outcome of glider collisions is highly sensitive to the collision phases, and the point and angle of impact. Gliders can self destruct, form a stable structure, transform and bounce off at different angles. Figures 21 and 22 give just a flavour of the diversity of behaviour.

Figure 21: Examples of glider Ga moving SW and colliding with Ga moving NW with impact points P1 to P6 resulting in the following at time-step 9:
P1: GaSW⇔GaNW → GaNE. P2: GaSW⇔GaNW → two Ea6. P3: GaSW⇔GaNW destroyed. P4: GaSW⇔GaNW → two Ea6. P5: GaSW⇔GaNW → Ea6. P6: GaSW⇔GaNW → Ea6.

Figure 22: Examples of glider Ga1 moving NW and colliding with Gb1 moving South. with impact points P1 to P6 resulting in the following at time-step 12:
P1: GaSW⇔GbS destroyed. P2,P3: GaSW⇔Gb South → Ea6. P4: GaSW⇔Gb South → Gc South. P5: GaSW⇔Gb South → Gc North. P6: GaSW⇔Gb South → Gc West.
4.4 Gliders colliding with eaters

Just like its precursor, the X-rule presents small stable emergent configurations Ea of 6 types, Ea1 – Ea6, which include all rotations 1 2 3 4 5 6 . They are known as eaters because in most cases they destroy colliding gliders, but eaters Ea may themselves be destroyed or transformed in the process, or instead of being destroyed a glider can be transformed and/or reflected – the exact behaviour is highly sensitive to the collision phase, point and angle of impact, and the Ea type. As in collisions between gliders in section 4.3, the examples in figures 23 – 27 give just a flavour of the diversity of behaviour.

Figure 23: Examples of glider Ga moving SW colliding into eater Ea1 with impact points P1 to P7 resulting in the following at time-step 44:
P1, P2: Ea1 → Ea6, Ga destroyed. P3: Ea1 destroyed, Ga destroyed. P4: Ea1 destroyed, Ga → combined ×2-Ga NW and Ga SE. P5, P6: Ea1 stable, Ga destroyed. P7: Ea1 destroyed, Ga → Gc East.

Figure 24: Examples of glider Ga moving SW colliding into eater Ea4 with impact points P1 to P7 resulting in the following at time-step 13:
P1: Ea4 destroyed, Ga → Gc North. P2: Ea4 → Ea3 and Ea5, Ga destroyed. P3: Ea4 destroyed, Ga → Gc North. P4 – P7: Ea4 conserved, Ga destroyed.
Figure 25: Examples of glider Ga moving SW colliding into eater Ea5 with impact points P1 to P7 resulting in the following at time-step 14:
P1: Ea5 destroyed, Ga conserved. P2: Ea5 → Ea6, Ga destroyed. P3, P4: Ea5 destroyed, Ga → Gc North. P5, P6: Ea5 conserved, Ga destroyed. P7: Ea destroyed, Ga destroyed.

Figure 26: Examples of glider Ga moving SW and colliding into eater Ea6 with impact points P1 to P5 resulting in the following at time-step 34:
P1: Ea6 destroyed, Ga destroyed. P2: Ea6 conserved, Ga destroyed. P3: Ea6 conserved, Ga destroyed. P4: Ea6 → Ea5, Ga → Gc West. P5: Ea6 → Ea5, Ga → Gc West.

Figure 27: Examples of glider Gb moving South colliding into eater Ea1 with impact points P1 to P6 resulting in the following at time-step 44:
P1: Ea1 → Ea6 and Ea1, Gb destroyed. P2: Ea1 → Ea6, Gb → Gc1 East. P3,P4: Ea1 destroyed, Gb destroyed. P5: Ea1 → Ea6, Gb → Gc3 East. P6: Ea1 → Ea6 and Ea1, Gb destroyed.
4.5 Combined eaters make reflectors

As in its precursor, the simple Ea eaters in the X-rule can be combined with a three cells gap to make a stable reflector Rc, which can either destroy or reflect back Gc gliders. As shown in figure 3.1 if the distance between an incoming glider Gc1 and the reflector Rc is an odd number, Gc1 is destroyed – for an even number including zero, Gc1 is reflected. This behaviour is preserved in the X-rule and occurs with many combinations of Rc as shown in figure 28.

![Gc1 heading South approaches variations of Rc at a distance of 2 cells.](image1)

![Gc bounces off Rc shown at time-step 16 with Gc1 heading North.](image2)

Figure 28: Examples of glider Gc1 bouncing off various combinations of reflector Rc. The behaviour is valid in any orthogonal direction provided that the distance between Gc1 and Rc is an even number, otherwise glider Gc is destroyed.

4.6 Simple reflecting oscillators

In the X-rule precursor, figure 10, we described a simple reflecting oscillator, SRO, made up of two opposing Rc reflectors with a Gc glider reflected back and forth in between. The SRO is conserved in the X-rule in any orthogonal orientation.

In figure 29, as well as glider Gc, we observe the small pattern Gc-p, rotations of predecessors of Gc, which are able to oscillate between two Rc reflectors with a minimal gap of 3 or 4 cells, thereafter Gc-p still appears in the oscillation sequence with the next gap of 6 big enough to accommodate a complete Gc glider, and thereafter any even number gap can continue to increase provided Gc or Gc-a is started at an appropriate position.
5 Glider-guns

5.1 Two basic glider-guns

Figure 30: Snapshots of the two basic glider-guns and glider evolution. (left) Diagonal glider-gun GGa shoots Ga gliders NW and SW with speed= $c/4$, time-step 37 in figure 31. (right) Orthogonal glider-gun GGb shoots gliders Gb North and South with speed= $c/2$, time-step 14 in figure 32.

Snapshots of the X-rule’s two types of glider-gun, GGa and GGb, constructed in section 3.2 are shown in figure 30, with greater detail in figures 31 and 32 and in the context of attractors in figure 33. A summary of glider-gun properties are as follows, where $c$ is the speed of light.

Glider-gun GGa shoots Ga gliders diagonally SW and NW with speed= $c/4$. The GGa minimum gap between reflectors (as shown) is 24 cells with an oscil-
Gliding and glider ejection period of 38 time-steps, with a pair of gliders shot for each period.

Glider-gun GGb shoots Gb gliders orthogonally South and North with speed=$c/2$. The GGB minimum gap between reflectors (as shown) is 23 cells and its oscillation period is 110 time-steps, but this is divided into two sub-periods – pairs of gliders are shot every 55 time-steps, alternating between the center and offset by 1 cell left of center.

Both glider-guns GGa and GGb are periodic machines developed from the periodic oscillator of the X-rule precursor in figure 11. Of the X-rule’s 512 outputs just 12 are non-isotropic but this results in spans of continuous time-steps where the periodic oscillator is either symmetric or asymmetric. Glider-gun GGa starts to eject Ga gliders towards the end of its asymmetric span, whereas glider-gun GGb starts to eject Gb gliders at the transition from symmetry to asymmetry. Both glider-guns can be seen as attractors on a finite periodic (toroidal) lattice where colliding gliders self-destruct (figure 33). Both continue to work for larger gaps between reflectors, in steps of +4, giving correspondingly larger periods described in section 5.2.

Although Ga gliders can move in any diagonal direction, the glider direction from GGa is restricted to SW and NW. However, by combining glider-guns and creating periodic collisions, any direction can be achieved by the resulting compound glider-guns (section 6).

Note that the glider-guns in the game-of-Life (figure 2a) and in the X-rule are both artificially constructed – their spontaneous emergence would be highly improbable. In contrast glider-guns in Sapin’s rule[7], (figure 2c), and the 3-value spiral rule[13] (figure 2d) emerge spontaneously.

Figure 31: The basic diagonal glider-gun GGa has a minimum reflector gap of 24 cells, an oscillator period 38 time-steps. The snapshots (on a 30×21 lattice) show spans of symmetry (sym) 1–26, and asymmetry (asym) 27–38 of the oscillating structures. Gliders Ga detach at time-steps 36 in the asymmetric span, moving NW and SW.
Figure 32: The basic orthogonal glider-gun GGb has a minimum reflector gap of 23 cells, an oscillator period 110 time-steps, consisting of 2 sub-periods of 55 time-steps. The snapshots (on a 29×29 lattice) show the span of each sub-period 1–55 and 56–110, and spans of symmetry (sym), displaced symmetry (dsym) and asymmetry (asym) of the oscillating structures. In sub-period 1, symmetric structures and gliders lie centrally relative to the gap, at cell 12. Sub-period 2 is a modified copy of sub-period 1, where the axis of symmetric structures and gliders is initially displaced to the left, at cell 10 relative to the gap. Sub-period 2 has another span of displaced symmetry (85–98) where the axis is displaced to the right of center at cell 14 relative to the gap. Gliders Ga detach completely at time-steps 14 and 69 moving North and South, but they become apparent about 3 time-steps earlier. Note that in the first sym/dsym span of phase 1 and 2 there is an isolated time-step, 9 and 64, with the same structure (right) which has a minor asymmetry.
(above) GGa as in figure 31 showing all 38 attractor states – time-step 38 is due East. Symmetric/asymmetric spans are indicated. Gliders eject at time-step 36.

(above) GGb as in figure 32 showing part of the attractor including time-steps 110, 1–21, and 22. Time-step 110 is due East. Symmetric/asymmetric spans are indicated. Gliders eject at time-step 14.

Figure 33: Glider-guns can be seen as periodic attractors on a finite toroidal lattice where colliding gliders self-destruct. (top left) glider-gun GGa complete attractor. (bottom right) glider-gun GGb partial attractor. The direction of time is clockwise.
5.2 Variable gap between Glider-gun reflectors

X-rule glider-guns have a special property in that the gap between reflectors can be enlarged from the minimum – 24 for GGa and 23 for GGb. Only increments of +4 are valid in each case to preserve the glider-gun, which increases the oscillation period and thus reduces the frequency of the resulting glider-stream. From experiment a regular pattern of gaps and periods emerge that extend indefinitely – listed in table 5.2 and example are shown figure 5.2.

| Glider-gun GGa | +step | Reflector gap | Oscillator period |
|----------------|-------|---------------|------------------|
|                |       | 24            | 38               |
|                |       | 28            | 46               |
|                |       | 32            | 54               |
|                |       | 36            | 62               |
|                |       | 40            | 70               |
|                |       | 44            | 78               |
|                |       | 64            | 118              |

| Glider-gun GGb | +step | Reflector gap | Oscillator period |
|----------------|-------|---------------|------------------|
|                |       | 23            | 110              |
|                |       | 27            | 126              |
|                |       | 31            | 142              |
|                |       | 35            | 158              |
|                |       | 39            | 174              |
|                |       | 43            | 190              |
|                |       | 63            | 270              |

Table 1: GGa and GGB reflector gaps and oscillator periods.

![Example snapshots of glider-guns with increasing gaps between reflectors and thus lower glider frequency.](above)
GGa with gaps 24, 28, 32.

![Example snapshots of glider-guns with increasing gaps between reflectors and thus lower glider frequency.](left)
GGb with gaps 23, 27, 31.
6 Compound glider-guns

In the X-rule, two types of ejected glider-streams have not been achieved with a single glider-gun; firstly, gliders Gc  in any orthogonal heading, although they are readily emergent, and secondly, gliders Ga heading NE  and SE  , although Ga’s are readily emergent in all diagonal directions and the glider-gun GGa sends Ga’s NW and SW.

To enhance the diversity of X-rule dynamics, these glider-streams can be created with compound glider-guns (CGGx) constructed from two or more basic glider-guns and eaters/reflectors, positioned and synchronised precisely, making self-contained and sustainable multiple oscillating colliding compound structures. In the following examples, which are not necessarily unique solutions or the simplest, we demonstrate CGGc shooting Gc gilders South (and North), where CGGcS is also a component within CGGaNE shooting Ga gilders NE (and SE), then we combine CGGcS with another GGa glider-gun to make CGGcW shooting Gc gilders West (and East).

6.1 Compound glider-guns CGGcS and CGGaNE

To shoot Ga gliders NE, we combined GGa glider-guns and employed various collision/reflection properties to create a compound glider-gun CGGaNE. In the process we created a compound glider-gun CGGcS shooting Gc gliders South. Ga gliders moving NE are generated by bouncing glider GcS off eater Ea4 which survives the collision. Figure 35 shows a space-time pattern snapshot of CGGaNE which contains CGGcS. A vertical flip of this pattern gives the equivalent CGGaSE and CGGcN.

To send a stream of Gc gliders South, two GGa glider-guns are lined up below each other separated by a minimum of 28 vertical cells. This compound glider-gun CGGcS will work if the initial state of the lower (GGa-14) is offset by +14 time-steps from the upper (GGa-0). In the attractor cycle presentation in figure 33 the snapshot has GGa-0 at time-step 38 (the last in the cycle) and GGa-14 at time-step 14. GGa-0 shoots gliders Ga SW and GGa-14 shoots gliders Ga NW, resulting in the a Gc  glider heading South every alternate collision. One collision leaves an eater Ea5 and both Ga gliders are destroyed. The next collision between GaNW and Ea5 produces a Ge  glider heading South, Ea5 is destroyed, GaSW continues and is destroyed by an eater, as are other extraneous gliders.

A continuous stream of GcS  gliders heads South at 38 cell intervals. Each GcS glider collides with a appropriately positioned Ea4 eater (which survives), and the Gc glider is transformed (reflects) to become a Ga  glider heading NE at 19 cell intervals, the closer intervals result from the difference in speed, Gc speed=\(c/2\), Ga speed=\(c/4\).
Figure 35: Snapshot (93×85) of the compound glider-gun CGGcS shooting Gc gliders towards the South, which bounce of eater Ea to send Ga gliders NE, the combined system is the compound glider-gun CGGaNE. A vertical flip of this pattern gives the equivalent CGGaSE and CGGcN. The inset (bottom right) shows the same snapshot with gliders leaving white trails whose length is proportional to speed – the grey background represents cells that have not changed for the last 20 time-steps.

6.2 Compound glider-gun CGGcW

To shoot Gc gliders West we combined CGGcS from section 6.1 shooting Gc gliders South with a GGaNE basic glider-gun shooting Ga NW. Each alternate collision produces a Gc glider heading West and leaves behind an Ea5 eater. The next GaNW destroys Ea5 but reflects to become another Gc gliders heading South which is destroyed by a stable Ea3 eater. The result is compound glider-gun CGGcW made from three out of phase GGa’s, shooting GcW West at 38 cell intervals.
Figure 36: Snapshot of the compound glider-gun CGGcW shooting Gc gliders towards the West made from a CGGcS compound glider-gun shooting Gc glider South, and a GGaNW glider-gun positioned below shooting Ga gliders NW, so three GGa glider-guns in all. GaNW gliders collide with alternate GcS gliders to create a GcW glider-stream. Alternate CGcS gliders that break through are destroyed by an eater. The inset (top left) shows the same snapshot with gliders leaving white trails whose length is proportional to speed – the grey background represents cells that have not changed for the last 20 time-steps.

Using similar constructions to those in figures 35 and 36, figure 6.2 gives examples of CGGc shooting Gc gliders North, East, and an alternative towards the West, demonstrating that Gc glider-streams can be projected in any orthogonal direction, though there may be different or simpler arrangements to achieve the same results.
Figure 37: Snapshots of CGGc glider-guns shooting Gc gliders North and East, and an alternative towards the West with a wider interval of 56 cells. Gliders (and other mobile patterns) are shown with green dynamic time-trails of 20 time-steps.
7 Logical gates – logic universality

To demonstrate that the X-rule is universal in the logic sense, that it can implement any logic circuit, we follow the game-of-Life method using glider-guns as “pulse generators” to construct logical gates\[2\], starting with the simplest gate, NOT, followed by AND and OR, and finally the functionally complete NAND gate – a combination of just NAND gates can implement any logic circuit.

A proof of universality in the Turing sense is a harder proposition, and will involve the construction of memory registers, auxiliary storage and other components. Although we believe that the ingredients for these structures exist in the diverse dynamics of the X-rule, we will leave this for a later investigation.

Under the game-of-Life approach, in the glider-stream, the presence of a glider represents the value 1, and the absence of a glider – a gap in the glider-stream – represents the value 0. When two suitably synchronised glider-streams approach each other at some angle (in these examples at 90°), gliders will either collide and self-destruct leaving a gap, or a glider will pass through a gap and survive. By combining perfectly synchronised input streams intersecting glider-streams generated by one or more glider-guns, the logical gates can be implemented. In the following examples, for simplicity the input/output strings consist of 4 bits, but in principle they could have arbitrary length. Note that extraneous glider-streams are stopped by strategically positioned eaters.

In the following sections 7.1 – 7.2 we gives examples of NOT, AND, OR and NAND gates built with basic GGa diagonal glider-guns, firstly towards the West employing basic GGa glider-guns, then towards the East which requires compound glider-guns and a longer glider interval.

Taken together, these gates demonstrate that the X-rule is logic universal in all diagonal directions. Gates in orthogonal directions, possibly with intersections at 45° including GGb and GGc glider-guns, are work in progress.

7.1 NOT, AND, OR and NAND gates towards the West

Gates towards the West are built with basic GGa diagonal glider-guns, shooting Ga gliders with a frequency of 38 time-steps. Because Ga glider speed=\(c/4\), the glider interval is 9 cells. Each of the following examples show two snapshots separated in time. The initial setup state including inputs and a glider-gun or guns are on the right (East) of a curved line – the output at some time-steps later is on the left (West).
To demonstrate the NOT-A gate (figure 38) a NW glider-stream is generated by a GGa glider-gun. A SW sequence of 4 gliders/gaps representing the input string A (1101), with the correct spacing and phases, is positioned to intersect the NW glider-stream. Gliders that collide self-destruct making a gap (value 0) in the output, whereas gliders that pass through a gap in the input continue NW representing value 1 in the output. If the first interaction with input A is at time zero, 152 time-steps later \((38 \times 4)\), the complete output of gliders/gaps has been built, moving NW away the intersection point, representing NOT-A (0010).

Figure 38: An example of the NOT gate showing two snapshots separated in time. Ghosts of absent gliders colored gray are gaps in the glider sequence representing 0s, real gliders coloured black represent 1s. The 4 bit input string A (1101) is transformed into the output string NOT-A (0010) after 152 time-steps from the first interaction, indicated by the red arrows, between input A and the glider-stream from the GGaNW glider-gun.
7.1.2 AND gate West

The A-AND-B gate (figure 39), builds on the initial interaction from the NOT-A gate. Gliders that pass through a gap in input A are able to intersect input string B (0101) which has been positioned 38 cells above string A. After about 214 time-steps from the first interaction with input A, the complete output string A-AND-B (0101) has emerged moving SW.

Figure 39: An example of the AND gate showing two snapshots separated in time. Two 4 bit input strings A (1101) and B (0101) are represented by SW gliders+gaps. Input A intersects a NW glider-stream generated by a GGaNW glider-gun. Gilder collisions between GaNW and A self-destruct, allowing a glider in input B to continue. A GaNW gilder meeting a gap in A passes through and would self-destruct if colliding with a glider in input B. This second interaction with input B is indicated by the red arrows. After about 214 time-steps from the first interaction with input A, the complete output string A-AND-B (0101) emerges moving SW. In the figure, A-AND-B has moved further SW and is shown after about 345 time-steps.
7.1.3 OR gate West

The A-OR-B gate (figure 40) includes procedures from the AND gate combined with a second GGaSW glider-gun – commentary is in the caption.

Figure 40: An example of the OR gate showing two snapshots separated in time. Two 4 bit input strings A (1101) and B (0101) are represented by SW gliders+gaps. Input A intersects a NW glider-stream generated by a GGaNW glider-gun. Gilder collisions between GaNW and A self-destruct, allowing a glider in input B to continue. A GaNW gilder meeting a gap in A passes through and would self-destruct if colliding with a glider in input B. NW gliders that have survived the transit of both A and B intersect a SW glider-stream from a second GGaSW glider-gun 38 cells above B. This last interaction occurs at a point indicated by the red arrows. After about 305 time-steps from the first interaction with input A, the complete output string A-AND-B (0101) is cut off and continues SW. This is the A-OR-B output (1101). The glider-streams from the two glider-guns heading NW and SW continue to self destruct. In the figure, A-OR-B has moved further SW and is shown after about 415 time-steps.
7.1.4 NAND gate West

The A-NAND-B gate (figure 41) is a combination of the AND and NOT gates, and includes two identical GGaNW glider-guns – commentary is in the caption.

Figure 41: An example of the NAND gate showing two snapshots separated in time. Gliders (and other mobile patterns) are shown with green dynamic time-trails of 20 time-steps. Two 4 bit input strings A (1101) and B (0101) are represented by SW gliders+gaps. The first set of interactions with a NW glider-stream generated by a GGaNW glider-gun follows the AND gate procedure similar to figure 39. The output A-AND-B then intersects a second NW glider-stream at a point indicated by the red arrows, generated from a second GGaNW glider-gun. This implements the NOT gate procedure similar to figure 38. The final A-NAND-B output (1010) heads NW, shown after about 374 time-steps from the first interaction and 163 from the second.

7.2 NOT, AND, OR and NAND gates towards the East

Gates towards the East are built with CGGa compound diagonal glider-guns, shooting Ga gliders with a frequency one every 76 time-steps. Because Ga glider speed=\(c/4\), the glider interval is 19 cells, wider than the basic GGa glider interval of 9 cells. Each of the following examples show two snapshots separated
in time. The initial setup state including inputs and glider-guns are on the left (West) of a curved line – the output at some time-steps later is on the right (East).

7.2.1 NOT gate East

To demonstrate the NOT-A gate towards the East (figure 42) we harness the CGGaNE compound glider-gun shooting gliders NE (figure 35), and construct the gate following similar interactions as in figure 38.

Figure 42: An example of the NOT towards the East showing two snapshots separated in time. Input A (1101) moving SE interacts with a NE glider-stream generated by the compound glider-gun CGGaNE, and transforms the glider stream into the output NOT-A (0010) which continues NE, show about 261 time-steps from the first interaction.
7.2.2 AND gate East

To demonstrate the A-AND-B gate towards the East (figure 43) we harness the CGGaNE compound glider-gun shooting gliders NE (figure 35), and construct the gate following similar interactions as in figure 39.

Figure 43: An example of the AND gate towards the East showing two snapshots separated in time. Inputs A (1101) and B (0111) moving SW interact in turn with a NE glider-stream generated by the compound glider-gun CGGaNE. The surviving glider-stream, perturbed by A, then interacts with B at the point indicated by the red arrows, and B is transformed into the output A-AND-B (0101) which continues SE, and is shown about 520 time-steps from the first interaction.
7.2.3 OR gate East

To demonstrate the A-OR-B gate towards the East (figure 44) we harness two compound glider-guns, CGGaNE and CGGaSE (figure 35), and construct the OR gate following similar interactions as in figure 40.

Figure 44: An example of the OR gate towards the East showing two snapshots separated in time. Inputs A (1010) and B (0111) moving SE interact in turn with a NE glider-stream from compound glider-gun CGGaNE. The surviving glider-stream then intersects a second glider-stream from the compound glider-gun CGGaSE at the point indicated by the red arrows. After about 700 time-steps from the first interaction the complete output string A-OR-B (1111) is cut off and continues SE.
7.2.4 NAND gate East

To demonstrate the A-NAND-B gate towards the East (figure 45) we use two compound glider-guns CGGaNE (figure 35), and construct the NAND gate following similar interactions as in figure 41.

Figure 45: An example of the NAND gate towards the East showing two snapshots separated in time. A glider-stream from the upper GGaNE compound glider-gun intersects input strings A (1101) and B (0111) in turn, following the AND gate procedure. The output A-AND-B then intersects a second NE glider-stream at a point indicated by the red arrows. This implements the NOT gate procedure giving the final A-NAND-B output (1010) heading NE, shown after about 600 time-steps from the first interaction.
8 Concluding remarks

The X-rule is a novel 2D binary CA with a diversity of emergent structures – gliders and eaters/reflectors – from which glider-guns and logical gates have been constructed and demonstrated, showing that the X-rule is universal in the logic sense in that it can implement any logic circuit. The structures, experiments and results made so far and documented in this paper are an initial exploration of what can be done, which suggests to us that memory functions required for universality in the Turing sense could be also feasible.

The key component in a rule that has the potential for universality is the periodic glider-gun, either naturally emergent or constructed. Whereas Gosper’s glider-gun in the game-of-Life is an asymmetric periodic structure within an isometric rule, the glider-guns in the X-rule have an underlying symmetry within a marginally non-isotropic rule. Just a few outputs, selected by an automatic method, were changed in the isotropic precursor’s rule-table, with the effect that periodic oscillators based on reflecting/bouncing behaviour temporarily break symmetry to eject gliders.

The X-rule’s glider-guns and compound gilder-guns, shooting a variety of glider types at various frequencies and in various directions, are constructed from a kit of parts that can be put together in many combinations, but just a few of the available ingredients have been included. A next step would be to complete a full catalogue of basic glider/glider and glider/eater collisions.

The search method for X-rule precursors suggests there are many binary isotropic rules in rule-space with periodic reflecting/bouncing properties, and that these rules would be susceptible to analogous non-isotropic adjustments to make glider-guns. Alternatively, glider-guns based on non-symmetric periodic oscillators in isotropic rules could be designed in ways analogous to Gosper’s glider-gun, and we intend to try this approach for the X-rule precursor.

In the scatter plot of entropy variability against mean entropy, the X-rule is found in a different place from both the game-of-Life and the Sapin rule. So logic universality occurs in unexpected places – we do not yet know the diversity of condition for logic universality.

Like the game-of-Life, the X-rule is open-ended. As in nature, and given a quasi-infinite space-time, it would be impossible to pin down a complete description of behaviour.

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