Probabilistic temperature forecasting: a comparison of four spread-regression models

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Abstract

Spread regression is an extension of linear regression that allows for the inclusion of a predictor that contains information about the variance. It can be used to take the information from a weather forecast ensemble and produce a probabilistic prediction of future temperatures. There are a number of ways that spread regression can be formulated in detail. We perform an empirical comparison of four of the most obvious methods applied to the calibration of a year of ECMWF temperature forecasts for London Heathrow.

1 Introduction

There is considerable demand within industry for probabilistic forecasts of temperature, particularly from industries that routinely use probabilistic analysis such as insurance, finance and energy. However there is considerable disagreement among meteorologists about how such forecasts should be produced and at present no adequately calibrated probabilistic forecasts are available commercially. Those who need to use probabilistic forecasts have to make them themselves.

How, then, should probabilistic forecasts of temperature be made? A number of very different methods have been suggested in the literature such as those described in Mylne et al. (2002), Roulston and Smith (2003) and Raftery et al. (2003). However it seems that all three of these methods, although complex, suffer from the shortcoming that they don’t calibrate the amplitude of variations in the ensemble spread but rather leave the amplitude to be determined as a by-product of the calibration of the mean.

We take a very different, and simpler, approach to the development of probabilistic forecasts than the authors cited above. Our approach is based on the following philosophy:

- The baseline for comparison for all probabilistic temperature forecasts should be a distribution derived very simply by using linear regression around a single forecast or an ensemble mean.
- More complex methods can then be tested against this baseline. Before anything more complex than linear regression is adopted on an operational basis it should be shown to clearly beat linear regression in out of sample tests. Unfortunately none of the studies cited above compared the methods they proposed with linear regression, and, given that they seem not to calibrate the ensemble spread correctly, it would seem possible that they might not perform as well.

We have followed this philosophy and, based on our analysis of one particular dataset of past forecasts and past observations we have shown that:

- Moving from constant-parameter linear regression to seasonal parameter linear regression gives a huge improvement in forecast skill for forecasts of both the mean temperature and the distribution of temperatures (Jewson, 2004a).
- Adding spread as a predictor gives only a very small improvement (Jewson et al., 2003, Jewson (2003b)).
- Generalising to allow for non-normality gives no improvement at all (Jewson, 2003a).

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All these results are summarised and discussed in Jewson (2004c).

In this article we focus on the second of these conclusions: that using the spread as an extra predictor brings only a very small improvement to forecast skill. This is somewhat disappointing given that it had been hoped by some that use of the ensemble spread would turn out to be an important factor in the creation of probabilistic forecasts. We are trying to get a better understanding of why the ensemble spread brings so little benefit in the tests we have performed. In Jewson (2004b) we concluded that this is because of:

1. The scoring system we use.

   We calibrate and score probabilistic forecasts using the likelihood of classical statistics (Fisher (1912), Jewson (2003c)). Likelihood, as we have used it, is a measure that considers the ability of the forecast to predict the whole distribution of future temperatures. Much of the mass in the distribution of temperature is near the mean and so the likelihood naturally tends to emphasize the importance of the mean rather than the spread. If we were to use a score that puts more weight onto the tails of the distribution then the spread might prove more important (although such a score would not then reflect our main interest, which is in the prediction of the whole distribution).

2. The low values of the coefficient of variation of spread (COVS).

   Once we have calibrated our ensemble forecast data we find that the uncertainty does not vary very much relative to the mean level of the uncertainty (i.e. the COVS is low). Thus if we approximate the uncertainty with a constant this does not degrade the forecast to any great extent, and we have not been able to detect a significant impact of the spread in out of sample testing. That the variations in the calibrated uncertainty are small could be either because the actual uncertainty does not vary very much or because the ensemble spread is not a good predictor for the actual uncertainty. In fact it is likely to be a combination of these two effects.

3. The low values of the spread mean variability ratio (SMVR).

   We have also found that the amplitude of the variations in the uncertainty in the calibrated forecast is small relative to the amplitude of the variations in the mean temperature (i.e. the SMVR is low). As a result accurate prediction of the (small) variations in the uncertainty is not very important relative to accurate prediction of the (large) variations in the mean temperature.

However in addition to these reasons it is also possible that we have been using the ensemble spread wrongly in our predictions. The model we have been using represents the unknown uncertainty $\sigma$ as a linear function of the ensemble spread (Jewson et al, 2003):

$$\sigma = \hat{\sigma} + \text{noise}$$

$$= \delta + \gamma s + \text{noise}$$

But this model is entirely ad-hoc. Why a linear function? We chose linear because it is the simplest way to calibrate both the mean uncertainty and the amplitude of the variability of the uncertainty, and not on the basis of any theory or analysis of the empirical spread-skill relationship. This suggests it is very important to test other models to see if they perform any better.

In this paper we will compare the original spread-regression model with 3 other spread-regression models. The four models we compare all have four parameters and so can be compared in-sample. This is important because the signals we are looking for are weak and obtaining long stationary series of past forecasts is more or less impossible at this point in time. At some point the numerical modellers will hopefully start providing long (i.e. multiyear) back-test time series from their models. This will allow more thorough out of sample testing of calibration schemes such as the spread-regression model and will facilitate the comparison of models with different numbers of parameters: meanwhile we do what we can with the limited data available.

## 2 Four spread regression models

The four spread-regression models that we will test are all based on linear regression between anomalies of the temperature and anomalies of the ensemble mean:

$$T_i \sim N(\alpha + \beta m_i, \hat{\sigma})$$

The difference between the models is in the representation of $\hat{\sigma}$. 
The original standard-deviation-based spread regression model is:

$$\hat{\sigma}_i = \gamma + \delta s_i$$

(4)

The variance-based model is:

$$\hat{\sigma}^2_i = \gamma^2 + \delta^2 s^2_i$$

(5)

The inverse-standard-deviation-based model is:

$$\frac{1}{\hat{\sigma}_i} = \gamma + \frac{\delta}{s_i}$$

(6)

and the inverse-variance-based-model is:

$$\frac{1}{\hat{\sigma}^2_i} = \gamma^2 + \frac{\delta^2}{s^2_i}$$

(7)

Following Jewson (2004a) the parameters $\alpha, \beta, \gamma, \delta$ all vary seasonally using a single sinusoid. We fit each model by finding the parameters that maximise the likelihood (using numerical methods).

We note that for very small variations in $s$ all these models can be linearised and end up the same as the linear-in-standard-deviation model given in equation 4.

3 Results

The first and most important test is to see which of the models achieves the greatest log-likelihood at the maximum. The results from this test are shown in figure 1 (actually in terms of negative log-likelihood so that smaller is better). In each case the spread-regression results (dashed lines) are shown relative to results for a constant-variance model (solid line). What we see is that the four models achieve roughly the same decrease in the negative log-likelihood and that in none of the cases is the decrease very large compared with the change in the log-likelihood from one lead time to the next. These changes are also small compared with the change in the log-likelihood that was achieved by making the bias correction vary seasonally (Jewson, 2004a).

Figure 2 shows the same data as is shown in figure 1 but as differences between the spread-regression models and the constant-variance model. Again we see that there is little to choose between the models.

Figure 3 shows a fifty-day sample of the calibrated mean temperature from the constant-variance model with the spread-regression calibrated temperatures overlaid. The differences are very small indeed and can only really be seen when they are plotted explicitly in figure 4.

Figure 5 shows the calibrated spread from the constant-variance model and the calibrated spread from the four spread-regression models. The uncertainty prediction from the constant variance model varies slowly from one season to the next and has a kink because of the presence of missing values in the forecast data. We now see rather significant differences between the four spread regression models. The size of these differences suggests that the variations in $s$ are not so small that the four spread regression models are equivalent to the linear-in-standard-deviation model.

4 Conclusions

How to produce good probabilistic temperature forecasts from ensemble forecasts remains a contentious issue. This is mainly because of disagreement about how to use the information in the ensemble spread. We have compared 4 simple parametric models that convert the spread into an estimate for the forecast uncertainty. All the models allow for an offset and a term that scales the amplitude of the variability of the uncertainty. Although the four models lead to visible differences in the calibrated spread we have found only tiny differences between the impact of these four models on the log-likelihood achieved. Also none of the models clearly dominates the others.

These results lead us to conclude that:

- the variations in $s$ are not so small that the calibration of the spread can be linearised, which would make all four models equivalent
- but the changes in the calibrated uncertainty are small enough that they do not have a great impact on the maximum likelihood achieved in any of the models
- implying that there is simply not very much information in the variations in the spread
It is possible that the models are overfitted to a certain extent. This is unavoidable given that we only have one year of data for fitting these multiparameter models. That none of the models dominates is rather curious: perhaps all the models are equally bad and none of them come close to modelling the relationship between spread and skill in a reasonable way. This raises the possibility that better results could perhaps be achieved by using other parametrisations. It is difficult to see how to make further progress on these questions until longer series of stationary back-test data is made available by the numerical modellers. Meanwhile it seems that a pragmatic approach to producing probabilistic forecasts would be to stick with the constant variance model since more complex models have shown only a small benefit in in-sample testing, and do not show a significant benefit in out-of-sample testing.

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Figure 1: The negative log-likelihood scores achieved by a linear regression (solid line) and four spread-regression models (dotted lines).
Figure 2: As for figure 1, but showing the differences between all models and linear regression on a much finer vertical scale.
Figure 3: The calibrated mean temperature from linear regression (solid line) and four spread-regression models (dotted lines). The dotted lines cannot be distinguished because they are so close to the solid lines.
Figure 4: As for Figure 4 but showing the differences between all models and linear regression.
Figure 5: The calibrated uncertainty from linear regression (solid line) and four spread-regression models (dotted lines).