Magnetoquantum Oscillations in Mesoscopic Multi-Channel Rings

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Abstract

We obtain exact analytical expressions for the electronic transport through a multi-channel system, also with an applied magnetic field. The geometrical structure of the electrodes is found to cause a splitting of the conduction band into many subbands, depending on the number and the length of the chains and the conductance approaches zero when the chain number is sufficiently large, due to quantum interference. In the presence of a magnetic field a very complicated oscillatory behavior of the conductance is found with a very sensitive dependence on the number of chains and their lengths, in a remarkable distinction from the usual oscillations in two-channel Aharonov-Bohm (AB) rings. In the multi-channel system the obtained oscillation patterns and their periodicities depend on the partitioning of the magnetic flux in the areas enclosed by the electronic paths. The present study may provide a useful information for quantum dots with a special configuration.

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I. INTRODUCTION

Quantum transport through artificially fabricated nanostructures has been extensively studied both experimentally and theoretically during the past years. The miniaturization of quantum dots or wires has now reached a stage where devices can be fabricated at sizes smaller than the single particle electronic coherence length. In such mesoscopic systems, the wavefunction maintains its phase coherence so that the electron can travel coherently through the sample. The access to “coherent transport” is granted by the advances made in lithography techniques which have opened a very rich field of theoretical and experimental research concerning quantum wires and quantum dots. Electro-optical experiments in solid state devices could lead to new switches which use the quantum wave nature of the electron.

Many interesting quantum effects can be also found in coupled nanostructures where the electronic transport is drastically affected by the phenomenon of quantum interference. Moreover, the application of a magnetic field, which is often used to probe the properties of devices, can also induce characteristic changes in the phase coherence of the electronic wavefunctions which, in turn, give rise to particular interference effects for the electronic transport. In the pioneering work of Aharonov and Bohm, such an effect was demonstrated via a thought experiment and it was shown that the conductance of a ring should oscillate as a function of the magnetic flux threaded through it. Among the manifestations of the Aharonov Bohm (AB) effect usually are the periodic magnetoresistance oscillations in normal metal rings and in electrostatically defined heterojunction rings. The AB effect is a result of the relative phase shift between two electron beams enclosing a magnetic flux $\phi$, where the magnetic field causes a $2\pi \phi / \phi_0$ change of the phase difference between the two arms of the ring. In this system the magnetoresistance oscillations have period $\phi_0$, which allows tuning of the phase of a wavepacket with destructive and constructive interference in cycles.

Owing to the great variety of the possible configurations for quantum dots it is of great interest to investigate the change of the interference effect in AB rings when the channel number is greater than two. In this paper, we study the electron transport properties of such
a multi-chain structure with common leads attached at its ends. We show that it can provide many alternative options for tuning quantum interference effects in the electronic transport. In the multi-channel structure an initial wave splits up into complementary waves $\psi_1 \ldots \psi_N$, where $N$ is the total number of chains involved. These waves propagate independently in every chain and are finally recombined at the outgoing lead. Interference effects among the different waves can be observed from the behavior of the electrical resistance calculated between the two leads. We also show that if the number of chains involved in the system is large enough most of the states are reflected and only a few of them can propagate through the system. This kind of “blocking” or “localization” of the electron waves, which occurs via quantum interference effects caused by the geometrical structure, is observed despite the absence of any disorder in the system.

In the presence of a magnetic field an electron moving around a loop will experience a phase change determined by the flux threaded through the loop. In a multi-chain system the phase changes are not the same for different paths of propagation so that they can lead to particular interference phenomena accompanied by much more complicated conductance oscillations than in the ordinary two-channel AB ring. We find that the pattern of these magnetoquantum oscillations is very sensitive to the number and the length distribution of the chains involved in the structure. Moreover, the oscillation patterns and their periodicities are also very sensitive to the partitioning of the flux among the areas enclosed by the paths. It should be pointed out that the results obtained in this paper could be useful towards understanding quantum dots with a special configuration.

The structure of the paper with the exposition of our results is as follows: in section II we describe the studied structure and give analytic expressions for the electronic transport, in section III we demonstrate different kinds of transport induced by special quantum interference effects with and without a magnetic field. The obtained results are summarized and discussed in Section IV.
II. MODEL AND FORMULA

We consider a ring of many chains with two common leads at its ends, threaded by a magnetic field which produces a flux in every loop enclosed by two nearest-neighbor chains. In addition we suppose that the multi-chain system is embedded in an infinite perfectly conducting chain with a left and a right part serving as the two electrodes. The configuration is shown in Fig.1 and the transport properties for non-interacting electrons in this system are studied via the tight binding Hamiltonian

\[ H = -t_0 \sum_{\alpha=1}^{N} (c_{0,\alpha}^{\dagger} c_{\alpha,1} + e^{i\phi_\alpha} c_{\alpha,N_\alpha}^{\dagger} c_s + \sum_{i=1}^{N_{\alpha}-1} c_{\alpha,i}^{\dagger} c_{\alpha,i+1} + \text{h.c.}) , \]  

where \( c_{i,\alpha} \) (\( c_{i,\alpha}^{\dagger} \)) is the annihilation (creation) operator which annihilates (creates) an electron on the site \( i \) of chain \( \alpha \), \( N_\alpha \) is the number of sites in the \( \alpha \)th chain (excluding the two nodes), \( N \) is the total number of chains and the two lead node sites are labelled by 0 and \( s \). The first two terms in the sum of Eq. (1) describe hopping of the electrons between the ends of every chain and the two leads 0 and \( s \). Moreover, since the chains are connected to each other only at their ends we can make a convenient choice of the gauge for the vector potential to affect only the phase of the wave functions at the hopping bonds between the right ends of each chain (site \( N_\alpha, \alpha = 1, 2, ..., N \)) and the right node \( s \). In the Hamiltonian of Eq. (1) the magnetic field is expressed by the second term in the sum where the phase difference \( \phi_\alpha - \phi_{\alpha-1} \) is proportional to the flux \( HW_\alpha, \alpha = 2, 3, ..., N \), where \( H \) is the strength of the field and \( W_\alpha \) the area enclosed by the \( \alpha \) and the \( (\alpha - 1) \)th chains. The phase of the first chain is chosen to be zero \( \phi_1 = 0 \) and the hopping strength for all the bonds \( t_0 = 1 \) is the energy unit used throughout the paper.

Our picture of the electronic transport in the system consists of an electron wave incident from the source into the perfect chain, then ramified into the \( N \) chains also experiencing different phase increments and eventually recombined into one channel at the output lead. Thus, an electronic beam incident from the right should be partially transmitted and partially reflected by the multi-chain system. In the site representation the coefficients of the
wave function at the left and the right parts of the pure chain can be written as

\[ a_j = e^{-ikj}, \text{ for } j \leq 0, a_j = Ae^{-ik(j-s)} + Re^{ik(j-s)}, \text{ for } j \geq s, \] (2)

where \( k = \cos^{-1}(E/2) \) is the wave vector of a wave function with energy \( E \), \( A \) is the amplitude of the incident wave, \( R \) is the amplitude of the reflected wave and the wave function for all the bonds is normalized so that the transmitted wave amplitude is unity. The transmission coefficient which measures the transparency of the system is, subsequently, defined as \( |t|^2 = 1/|A|^2 \). The wave function coefficients in the \( \alpha \)th chain, by including the left node 0, can be expressed as a linear combination of the propagating and the reflected plane waves via

\[ a_{\alpha,j} = A_{\alpha}e^{ikj} + R_{\alpha}e^{-ikj}, \text{ for } 0 \leq j \leq N_{\alpha}, \] (3)

where \( a_{\alpha,j} \) is the coefficient at the \( j \)th site of the \( \alpha \)th chain. The coefficient at the left lead node \( j=0 \) from Eqs. (2) and (3) gives the relation

\[ A_{\alpha} + R_{\alpha} = 1. \] (4)

Similarly, we can calculate the coefficient at the right node \( j=s \), by including the flux-induced phase shift term, and a comparison with Eq. (2) gives a second relation

\[ (A_{\alpha}e^{ik(N_{\alpha}+1)} + R_{\alpha}e^{-ik(N_{\alpha}+1)})e^{-i\phi_{\alpha}} = A + R. \] (5)

Therefore, from the two Eqs. (4) and (5) we can express the wave function coefficient of Eq. (3) for all chains \( \alpha \) via

\[ A_{\alpha} = \frac{e^{-ik(N_{\alpha}+1)} - (A + R)e^{i\phi_{\alpha}}}{2i \sin(k(N_{\alpha} + 1))}, \] (6)

\[ R_{\alpha} = \frac{e^{ik(N_{\alpha}+1)} - (A + R)e^{i\phi_{\alpha}}}{2i \sin(k(N_{\alpha} + 1))}, \] (7)

in terms of \( A \) and \( R \).

On the other hand from the Schrödinger difference equations at the two lead nodes 0 and \( s \) we obtain
\[ E = e^{ik} + \sum_{\alpha} (A_\alpha e^{ik} + R_\alpha e^{-ik}), \]  
(8)

\[ E(A + R) = A e^{-ik} + R e^{ik} + \sum_{\alpha} e^{-i\phi_\alpha} (A_\alpha e^{ikN_\alpha} + R_\alpha e^{-ikN_\alpha}) \]  
(9)

and by a substitution of Eqs. (8), (7) into Eqs. (8), (9) eventually obtain two equations for the \( A \) and \( R \). Finally, the transmission coefficient can be calculated from their solution, which gives

\[ |t|^2 = \frac{4|f_0|^2 \sin^2 k}{|c_0 - e^{-ik}|^2 - |f_0|^2}, \]  
(10)

where

\[ f_0 = \sum_{\alpha} e^{i\phi_1} e^{i(\phi_\alpha - \phi_1)} \sin k \sin k(N_\alpha + 1), \]

and

\[ c_0 = \sum_{\alpha} \frac{\sin kN_\alpha}{\sin k(N_\alpha + 1)}, \]

for arbitrary chain numbers \( N_\alpha, \alpha = 1, 2, ..., N \).

Eq. (10) is the most important result of this paper, which presents the general analytical expression for the transmission coefficient in a multi-channel ring. This expression can be further simplified for special geometries, for example, if the chain lengths are equal to \( N_1 = N_2 = ... = N_N \equiv L \), and the two nearest neighbor chains enclose a fixed area \( \phi = \phi_\alpha - \phi_{\alpha-1} \), then \( f_0 \) and \( c_0 \) become

\[ f_0 = \frac{e^{i(N-1)\phi/2} \sin (N\phi/2) \sin k}{\sin(\phi/2) \sin k(L + 1)}, \]  
(11)

\[ c_0 = \frac{N \sin kL}{\sin k(L + 1)}, \]  
(12)

and a simpler exact expression for the transmission coefficient \(|t(E)|^2\) can be obtained. It can be seen from Eq. (10) that the magnetic field dependence is solely due to \(|f_0|\) so that if the factors \( \frac{1}{W_2}, \frac{1}{W_2+W_3}, ..., \frac{1}{\sum_{\alpha=2}^{N} W_\alpha} \) have common multiples the oscillations of \(|t(E)|^2\) as a function of a magnetic field have a period of \( \nu \phi_0 \), where \( \nu \) is the smallest common multiple.
with $\phi_0$ the flux quantum. The electronic conductance can be also directly computed from the transmission coefficient via the Landauer formula:

$$\sigma(E) = \frac{|t(E)|^2}{1 - |t(E)|^2},$$

(13)

at a Fermi energy $E$.

**III. QUANTUM OSCILLATIONS FOR VARIOUS MULTI-CHAIN CONFIGURATIONS**

In this section we show our results associated with quantum interference effects in multi-chain systems by application of Eq. (10). Our purpose is to illustrate the relationship between electronic wave transport in the multi-channel system and the geometric ring structure, also in the presence of an external magnetic field.

**A. Equal chain system without a magnetic field**

In the absence of a magnetic field the replacement $\phi = 0$ for $f_0$, $c_0$ is made in Eqs. (11), (12). The obtained results in this case correspond to a similar model of a total number of $N$ thin wires joined together at their two ends as it was introduced by Wang et al. in their study of electronic transport through a quantum cavity. These authors have predicted that the total electron transmission can be simply expressed as a coherent sum of the transmission coefficients obtained from every chain. Our results can give an even more complicated transmission behavior due to the geometrical structure of the electrodes.

In Fig. 2 we plot the transmission coefficient versus the electronic energy for such a multi-chain system made of equal chains. The pattern shown exhibits an interesting bridge-arc shape whose curvature becomes larger and the blank region below the arc smaller if the chain number increases, with some states still having high values of the transmission coefficient. Thus, if the chain number is large enough most of the states are reflected and only very few states can propagate through the multi-channel system. It must be emphasised that
the blocking of the electron propagation at most energies is merely caused by quantum interference due to the geometrical structure involved. It seems, however, a puzzle why such a rather symmetric geometry can give rise to a very complicated behavior of the outgoing wave. This is probably due to the fact that the translation symmetry is broken at the two contacts, which leads to partial destructive interference of the electron waves. Wang et al. have also observed a partial blocking of electron waves in the propagation regime of a quantum-wave-filter consisting of field-induced nanoscale cavities and 1D wires by varying the electronic wavelength. Our results could account for the reported experimental behavior.

We have also investigated the relationship between the obtained features of the conducting spectrum and the chain number for an equal chain multi-chain system. In Fig. 3 the conduction band as a function of the number of chains is illustrated and we observe that by increasing the chain number the transmission pattern becomes more and more sparse and the conduction band splits into several subbands. If the number of chains becomes large enough we find that most of the states cannot propagate through the system, becoming “blocked” or “localized”. Thus, a quantum interference effect causes the conduction band to become discrete in the absence of disorder and/or interchain couplings, except at the connections of the system at the two end nodes. A relation between the conductance and the chain number can be extracted from Fig.4, where a monotonic drop of the conductance is seen when the number of chains is increased. This is another indication of the trend shown by the system for large chain numbers to become more “insulating”.

B. Magnetoconductance oscillations

In Fig. 5(a) we show the characteristics of the transmission coefficient obtained in the absence of a magnetic field, such as the bridge-arc shape already seen in Fig. 2(a), in order to compare with the cases with an applied magnetic field (Figs. 5(b), 5(c) and 5(d)). We find a remarkable change of the transmission in the latter case when the areas between neighboring chains enclose equal magnetic fluxes. In Figs. 5(b), 5(c) and 5(d) the arc structure is no
longer present and the transmission becomes more and more sparse due to the increase of the magnetic flux through the system.

In Fig. 6 we present the magnetic field dependence of the transmission coefficient for equal chains with the same nearest-neighbor path areas. In this case the curves show periodic quantum-magnetic oscillations governed by the field dependence which enters $f_0$ via Eqs. (11) and (12), finally leading to $(N - 1)\phi_0$. These findings share many similarities with the optical multi-slit interference patterns with main common feature the $N - 1$ minima and the $N - 2$ subsidiary maxima between every two consecutive principal maxima. However, the obtained electronic transmission is more complicated when compared to the analogous optical case due to the complexity of the denominator in our expression for $|t(E)|^2$. Moreover, from Fig.6 we can observe many points of zero transmission which imply a magnetic-field induced destructive interference effect.

Our results for a four-chain system with equal chains but non-equal areas enclosed by every two nearest-neighbor paths are shown in Figs. 7 and 8. It can be seen that the interference pattern and the period of the magnetos oscillation vary, depending on the distribution of the magnetic flux between the closed paths. In fig.7 we show the electronic conductance versus the magnetic flux for a four-chain system made of equal chains with magnetic flux periods (a) $3\phi_0$, (b) $5\phi_0$, (c) $4\phi_0$ and (d) $4\phi_0$. One can easily deduce the relation between the oscillation period and the distribution of the magnetic fluxes by noticing that the phase shift for every chain must be an integer times $2\pi$. If the magnetic flux distribution has a small deviation, the mageto-oscillation pattern and its period changes abruptly, as shown in Fig.8. Without deviation, the spectrum has strict period $3\pi$ and destructive interference occurs twice during this period. Introducing the deviation, the spectrum has no strict periodicity and the interference pattern changes aperiodically.

In Fig.9 we present results for a four-chain system of both different chain lengths and non-equal areas enclosed by two nearest-neighbor paths. From the realizations of Eq. (10) we find that the conductance changes when the chain length varies because of variations in both the numerator and the denominator of Eq. (10). For a certain length distribution we
observe a quasi-periodic pattern close to about $1.5\phi_0$, but its real period is $3\phi_0$ as in Fig. 9(d). Thus, we may conclude that even a small variation of the chain lengths causes abrupt changes in the conductance oscillation patterns. It is, perhaps, worth mentioning that the sensitivity found could provide an opportunity for the application of the studied multi-chain structure to the electronic device engineering.

**IV. DISCUSSION**

Quantum interference plays a central role in the quantum physics of mesoscopic systems. We have shown that for a multi-chain system an incident wave splits into several chain beams at the entrance and recombines at the exit. Thus, the conduction band becomes discrete and the electronic transport properties are drastically modified by the introduction of a “localization” effect, despite the absence of any disorder. Moreover, in the presence of a magnetic flux we obtain magneto-oscillations, which are much more complicated than these known in the usual AB rings. In the AB effect a magnetic field is threaded through the center of a ring so that the electrons passing via each of the two chains experience different phase shifts. If the magnetic field is varied one can modulate the phase and produce conductance oscillations in the wave transport from one terminal to the other.

The magnetic field dependence of the electrical conductance also shows an oscillating behavior very different from the AB ring effect, since the multi-chain system exhibits more complicated interference effects determined by the phase shifts in the various propagation paths. Each phase shift is caused by both the electronic momentum and the magnetic flux, so that momentum variations and changes in the chain lengths as well as variations in the distribution of the magnetic fluxes can modify the interference pattern. Electron wave propagation through a multi-chain system pierced by a magnetic field has also an interesting analogy with optical interference phenomena by many slits. In both phenomena $N-2$ subsidiary maxima and $N-1$ minima between two consecutive principal maxima occur. Of course, between each pair of minima a subsidiary maximum exists$^4$, as it is confirmed
by our numerical calculations. It must be pointed that our results are also relevant to the case of Andreev scattering which occurs in normal-superconductor interfaces. It turns out that if the right hand side periodic chain attached to the dot structure is replaced by a clean superconducting wire our obtained results of the transmission coefficient for the normal-dot-normal can be also used for finding the transmission through the normal-dot-superconductor system. This can be achieved via Beenaker’s formula for the conductance

\[ G = \frac{(2e^2/h)\frac{2|t|^4}{(2-|t|^2)^2}} \] expressed from the transmission of the non-superconducting dot part only. If we use the obtained $|t|^2$ from Eq. (10) extra doubling of the periodicities should occur for the dot-superconductor interface.

In summary, we have systematically studied the electronic properties of a multi-chain system connected at its two ends. A recursion method was employed and an exact analytic expression for the electronic conductance was presented. Many interesting features in the transmission coefficient and the magnetoconductance were also shown for various configurations: 1) The geometrical structure of the electrodes is found to cause a discreteness of the conduction band, which eventually affects remarkably the transport properties leading to a kind of “localization” in the absence of disorder. 2) We find various magneto-oscillation periodicities and interference patterns, by varying the distribution of the relative magnetic flux through the structure, and also abrupt changes in the plot of the conductance versus the magnetic-flux if the length distribution of the system is modulated, which is useful to distinguish even slight chain length variations. 3) The studied system can be also used to probe the distribution of the magnetic field since the obtained interference patterns are very sensitive to the distribution of the magnetic flux among neighbouring closed paths.
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FIGURES

Figure 1. The considered multi-chain system with the left and right nodes indicated by 0 and s, respectively. The total number of chains is \( N = 5 \) and the number of sites in the \( \alpha \)th chain is \( N_\alpha, \alpha = 1, 2, ..., N \) without counting of the nodes 0 and s.

Figure 2. The transmission coefficient as a function of the electronic energy for a \( N \)-chain system of equal chain lengths (a) \( N_\alpha = 100, \alpha = 1, 2, ..., N \) and (b) \( N_\alpha = 1000, \alpha = 1, 2, ..., N \). The chain numbers involved in each case are: (1) \( N = 2 \), (2) \( N = 4 \), (3) \( N = 10 \), (4) \( N = 40 \) and (5) \( N = 80 \).

Figure 3. The conduction band vs. the chain number \( N \) for a multi-chain system with chain lengths \( N_\alpha = 5000, \alpha = 1, 2, ..., N \). A conduction band is defined as being non-zero at the energy values where the corresponding transmission coefficient is higher than 0.1.

Figure 4. The conductance \( \sigma(E) \) as a function of the number of chains \( N \) for a system with equal chains at a Fermi energy \( E = 1.0 \), in the absence of a magnetic field.

Figure 5. A comparison of the transmission coefficient with and without a magnetic field. The structure consists of \( N = 4 \) channels of lengths \( N_\alpha = 2000, \alpha = 1, 2, 3, 4 \) and the magnetic flux threaded in the system is: (a) 0, (b) 0.1, (c) 0.5 and (d) 2.0.

Figure 6. The electronic transmission vs. the magnetic flux for a multi-chain system with equal chain lengths \( N_\alpha = 2000, \alpha = 1, 2, ..., N \) and fixed electron energy \( E = 1.1 \). The unit of the magnetic flux is the flux quantum \( \phi_0 = 1 \) and the chain numbers are: (a) \( N = 2 \), (b) \( N = 3 \), (c) \( N = 4 \), (d) \( N = 5 \) and (e) \( N = 9 \).

Figure 7. The electronic conductance vs. the magnetic flux for a multi-chain system \( (N = 4) \) with equal chain lengths \( N_\alpha = 2000, \alpha = 1, 2, 3, 4 \) and electron energy fixed at \( E = 1.1 \), with the magnetic flux quantum \( \phi_0 = 1 \). (a) \( \phi_2 - \phi_1 = \phi_3 - \phi_2 = \phi_4 - \phi_3 \), (b) \( \phi_2 - \phi_1 = \phi_3 - \phi_2 = \frac{1}{2}(\phi_4 - \phi_3) \), (c) \( \phi_2 - \phi_1 = 3\phi_3 - \phi_2 \) and \( \phi_4 - \phi_3 = 0 \), (d) \( \phi_2 - \phi_1 = 3\phi_3 - \phi_2 = 2(\phi_4 - \phi_3) \).
Figure 8. The electronic conductance vs. the magnetic flux for a multi-chain system $(N = 4)$ with equal chain lengths $N_\alpha = 2000, \alpha = 1, 2, 3, 4$ and electron energy fixed at $E = 1.1$, with the magnetic flux quantum $\phi_0 = 1$: (a) $\phi_2 - \phi_1 = \phi_3 - \phi_2 = \phi_4 - \phi_3$, (b) $0.99(\phi_2 - \phi_1) = \phi_3 - \phi_2 = 1.01(\phi_4 - \phi_3)$, (c) $0.98(\phi_2 - \phi_1) = \phi_3 - \phi_2 = 1.01(\phi_4 - \phi_3)$.

Figure 9. The electronic conductance vs. the magnetic flux for a multi-chain system $(N = 4)$ with almost equal chain lengths $N_\alpha, \alpha = 1, 2, 3, 4$ and electron energy fixed at $E = 1.1$, with the magnetic flux quantum $\phi_0 = 1$: (a) the chain lengths are (a) $N_\alpha = 2000, \alpha = 1, 2, 3, 4$, (b) $N_1 = 2000, N_2 = 2002, N_3 = 2004, N_4 = 2006$, (c) $N_1 = 2000, N_2 = 2010, N_3 = 2020, N_4 = 2030$, (d) $N_1 = 2000, N_2 = 2100, N_3 = 2200, N_4 = 2300$.
Fig. 2(a)
Fig. 2(b)
Fig. 5
Fig. 9