The propagation of pulses of a special shape in an inhomogeneous anisotropic medium with dispersion and torsion of the optical axis

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Abstract. An oblique incidence of polarized light pulses with different envelope shapes is considered: triangular, Gaussian, rectangular and in the form of giant laser pulses onto a flat inhomogeneous anisotropic layer with torsion of the optical axis and optical dispersion. The wavelength of an optical pulse carrier is close to the resonance wavelength of an inhomogeneous anisotropic layer. The problem of the propagation of s- and p-polarization waves through a layer, the reflection of impulses of cross-polarized components, the dependence of modulation coefficients during medium torsion and frequency detuning is considered.

1. Introduction
Currently, the interest of researchers to optical materials with anisotropy inhomogeneity and dispersion has increased significantly. The use of optical technologies in telecommunications systems is associated with effective scaling of integrated circuits and processing devices, as well as with increasing frequency and throughput. Optical properties of materials determine the direction of propagation, birefringence, polarization, phase of a light wave in a medium [1].

Matrix methods are traditionally used to calculate electromagnetic waves in planar structures [2]. They appeared in the 40s of the 20-th century and were improved as new challenges and technologies developed. For an anisotropic layer, the solution has the form of a 4 × 4 matrix [3].

Heterogeneous structure was represented as a set of layers with homogeneous parameters. A change in the direction of the optical axis affects the transmission of light by an inhomogeneous optical medium [4]. It finds application in various optical devices, for example, polarizers, light modulators [5]. The Berreman method was developed for inhomogeneous uniaxial anisotropic media. For calculations, a model of a layered medium is used. The transfer matrix of the inhomogeneous layer is obtained by multiplying the transfer matrices of the layers. For ease of calculation, the Berreman method was later reduced to the 2 × 2 method [6] and adapted for the calculation of liquid crystal displays [7].

Most programs for modeling liquid crystal devices use simplified mathematical algorithms. The book [8] presents a rigorous approach to the optical modeling of LCD devices. The authors consider the anisotropic properties of liquid crystals, the dependence of these properties on external influences, classify theoretically rigorous and efficient methods and determine how these methods are best used to develop new applications of optics and photonics. Currently, there is a growing interest in photonic crystal waveguides, which can direct or prohibit the propagation
of light at any wavelength, including the visible range. In liquid crystals, the heterogeneity of the photonic crystal structure can be provided by external fields [9].

Using external influences, you can change the direction of the optical axis of an anisotropic liquid crystal and, thus, determine the transmission spectrum of the cell or the mode of propagation of a waveguide wave. This paper is devoted to the influence of the dispersion and torsion of an anisotropic medium on the envelope shape of optical signals of various shapes: a triangular, square wave, Gaussian pulse and a signal in the form of a giant laser pulse.

2. Formulation of the problem

Consider an oblique incidence of an optical pulse on a flat anisotropic layer with torsion of the optical axis and dispersion. The dielectric constant of the medium depends on the coordinate z and the frequency ω. The drop of a wave on a layer is shown in Figure 1. The change in the direction of the optical axis is shown with a distance from the interface z = 0. Vectors $k_i$, $k_r$, $k_t$ are wave vectors of the incident, reflected and transmitted waves.

The dielectric constant of the medium depends on the z coordinate and on the frequency ω:

$$\epsilon = \begin{pmatrix}
\epsilon_o(\omega) \cos^2 \chi(z) + \epsilon_e(\omega) \sin^2 \chi(z) & \sin 2\chi(z) \cdot \frac{\epsilon_e(\omega) - \epsilon_o(\omega)}{2} & 0 \\
\sin 2\chi(z) \cdot \frac{\epsilon_e(\omega) - \epsilon_o(\omega)}{2} & \epsilon_o(\omega) \sin^2 \chi(z) + \epsilon_e(\omega) \cos^2 \chi(z) & 0 \\
0 & 0 & \epsilon_o(\omega)
\end{pmatrix}. \quad (1)$$

The dependence of the dielectric constant of the anisotropic medium on the radiation frequency is:

$$\epsilon_o,\epsilon_e = \epsilon_{\perp,\parallel} + \frac{F_{\perp,\parallel} \eta^2}{\omega^2 - \omega^2 - i\omega \gamma_{\perp,\parallel}}. \quad (2)$$

For a liquid crystal MBBA: $\lambda_0 = 9.07 \mu m$, $\gamma_\perp = 8.03E - 3 \cdot \omega_0$, $\gamma_\parallel = 4.35E - 3 \cdot \omega_0$ [10], $\epsilon_o = 5.14$, $\epsilon_e = 4.53$ [11]. The thickness of the layer of material is $d = 15\lambda_0$. We introduce the average value for $\gamma_\parallel$ and $\gamma_\perp$: $\gamma_{1,2} = \frac{\gamma_\perp + \gamma_\parallel}{2}$.

The torsion angle of the optical axis increases with distance from the interface:

$$\chi(z) = \chi_0 + \frac{\Delta_\chi}{d} z. \quad (3)$$

The vector of the electric field $E^{(i)}(r,t)$ of the incident wave depends on the coordinates and time: $E^{(i)}(r,t) = E^{(i)}(r)A_i(t) \cdot \exp(i\omega t)$. The direction and length of the vector $E(r)$ in the layer is determined by its properties (1). The polarization of the incident wave is generally elliptical. Let us choose as a basis for the description of the polarization of the s- and p-wave.

$$E^{(s)}(r,t) = \begin{pmatrix} E_x^{(s)}(t) \\ E_y^{(s)}(t) \end{pmatrix} = E^{(i)}(r)A_i(t) \cdot \exp(i\omega t). \quad (4)$$

An optical signal of a special form is incident on a layer with torsion of the optical axis and dispersion. We consider cases where the signal envelope has the form of a) a laser giant pulse, b) a Gaussian pulse, c) a triangular pulse, and d) a rectangular pulse. Each of the envelope functions $A(t)$ has its own spectrum $S(\omega)$ The medium has a dispersion, so for different values of the conditions for the reflection of light will be different for the same angle $\theta_i$. The optical properties of the medium depend on $\omega$ according to (1) therefore the coefficients of the reflection matrix will also be functions of frequency:

$$R(\omega, \theta_i) = \begin{pmatrix} R_{pp}(\omega, \theta_i) & R_{ps}(\omega, \theta_i) \\ R_{sp}(\omega, \theta_i) & R_{ss}(\omega, \theta_i) \end{pmatrix}. \quad (5)$$
Figure 1. Oblique incidence of light on a layer with torsion of the optical axis.

As a result, the reflected impulse will have a modified form compared to the incident one. There is a relationship between the spectra of the incident and reflected pulses:

\[
\begin{pmatrix}
E_p^{(r)} \\
E_s^{(r)}
\end{pmatrix} = \begin{pmatrix}
R_{pp}(\omega, \theta_i) & R_{ps}(\omega, \theta_i) \\
R_{sp}(\omega, \theta_i) & R_{ss}(\omega, \theta_i)
\end{pmatrix} \begin{pmatrix}
E_p^{(i)} \\
E_s^{(i)}
\end{pmatrix}
\]  

(6)

The amplitude of the reflected p-polarization wave consists of two components:

\[
E_p^{(r)} = S(\omega) \cdot R_{pp}(\omega, \theta_i)E_p^{(i)} + S(\omega) \cdot R_{ps}(\omega, \theta_i)E_s^{(i)}
\]  

(7)

Similarly for s-wave:

\[
E_s^{(r)} = S(\omega) \cdot R_{sp}(\omega, \theta_i)E_p^{(i)} + S(\omega) \cdot R_{ss}(\omega, \theta_i)E_s^{(i)}
\]  

(8)

The term \(S(\omega) \cdot R_{ps}(\omega, \theta_i)E_s^{(i)}\) in (7) describes the spectrum of the p-polarization signal, which appears when a s-wave is incident and conversely, the term \(S(\omega) \cdot R_{sp}(\omega, \theta_i)E_p^{(i)}\) in (8) describes signal with polarization s arising from the reflection of p-polarization. The terms \(S(\omega) \cdot R_{pp}(\omega, \theta_i)E_p^{(i)}\) in (7) and \(R_{ss}(\omega, \theta_i)E_s^{(i)}\) in (8) are the spectra of the signals that retain their polarization. The dependences of these four projections of the electric field vector on time were calculated using the inverse Fourier transform.

The torsion of the optic axis in the layer is \(\Delta \chi = 90^\circ\).

The reflection matrix (5) of an inhomogeneous anisotropic layer was calculated using the Cauchy matrix

\[
\hat{N}(z, 0) = \hat{Y}(z)\hat{Y}^{-1}(0),
\]  

(9)

found in the framework of the Wentzel-Kramers-Brillouin method [12]. In (9) \(\hat{Y}(z)\) is the fundamental matrix for solving (FMS) a system of ordinary differential equations for projections of field vectors in an inhomogeneous anisotropic medium. To find the matrix \(\hat{Y}(z)\) used methods for solving differential equations, described in [13].

The calculations of the reflected signals with the envelope in the form of a meander, triangular, Gaussian and giant pulses for the case of oblique incidence are performed. The carrier frequency of the signal is \(\omega_c = \omega_0\).

Figure 2 through Figure 9 show the reflected signals calculated using Fourier transforms. Forms envelope of the original signals are shown by dashed thin lines.
**Figure 2.** The reflected pulse, in the form of a giant laser, at different angles of incidence for the wave s-polarization.

**Figure 3.** The reflected pulse, in the form of a giant laser, at different angles of incidence for the wave s-polarization.

**Figure 4.** The reflected pulse of the Gaussian form at different angles of incidence for the wave p-polarization.
Figure 5. The reflected pulse of the Gaussian form at different angles of incidence for the wave s-polarization.

Figure 6. The reflected pulse of a triangular shape at different angles of incidence for the wave p-polarization.

Figure 7. The reflected pulse of a triangular shape at different angles of incidence for the wave s-polarization.
3. Conclusions
Calculations show that the amplitude and phase of the signals depend on the angle of incidence. The envelopes of rectangular and triangular signals change shape when reflected from an anisotropic layer with a dispersion.

4. References
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