Mathematics pre-service teachers’ thinking process in solving modeling task in differential calculus course

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Abstract. This study aims to both designing a modeling task for differential calculus course with a contextual approach and analyzing the results after the task implementation. Interviews were also conducted to six mathematics pre-service teachers. The researchers initially have suspected that the modeling process which occurs would follow the ideal mathematical modeling cycle but this study found that the modeling process did not necessarily follow sequential development according to model construction as described in various mathematical modeling cycles.

1. Introduction

Chain rule is one of the key concepts in differential calculus [1]. Many studies have shown numerous problems that were encountered by students when learning about this topic. One of the main reasons is that the chain rule is connected with other various concepts in mathematics [2,3]. Another reason notes students’ unawareness when applying the chain rule. It happens since the instruction itself rarely discuss the core meaning of this rule and the approach is merely about the trick of algebra [1,4,5]. Despite this condition, very few attention is given to find alternative ways to facilitate students’ inquiry to make sense of the chain rule.

Many researchers argue that one of the ways to learn the meaning of the chain rule is by explaining the link among changes of quantities in the real world situation [6,7]. It is because the key aspect of the chain rule is about the combined ratio among variables, hence it will be easier if it is contextualized into the real world situation. However, the studies had been conducted were only giving understanding to students on how to use the chain rule in the daily life, while strategy to facilitate the students' inquiry is yet to be done. Meanwhile, other researchers try to utilize modeling activity to find the solution [4,8].

Modeling activity can encourage students to explore the meaning of mathematics concepts or procedure in connection with the real world problems [9,10]. The chain rule is considered to be an excellent mathematical concept to explore the use of modeling because calculus acts as the basis for modeling and problem solving. Along with promoting 21th century skill through mathematics, it is important to explore deeper into the thinking process of mathematics student candidate in doing mathematical modeling. This is where our research takes on a role.
2. Method
This research is a qualitative descriptive study. The researchers develop a modeling task for the differential calculus course, especially the chain rule topic. The task is made based on the stages in ideal mathematics modeling [11] presented in Figure 1.

![Figure 1. Modeling cycle adapted from Blum and Leiß.](image)

Figure 1 shows that there are two conditions for mathematics modeling cycle, namely real-world context and mathematics context. The cycle includes six stages of model construction (a-f) and is connected to six transitions (1-6). Indicators that were used in this research to explain each modeling stage are referred to the previous studies [12].

Giving a task is one to see how one thinks process. Referring to the modeling cycle, we constructed a modeling task that can be seen in Figure 2.

![Figure 2. Balloon depreciation task.](image)

There is a balloon that has initial largest cross-sectional area 154 centimeters square. Due to leaking, the balloon’s cross-sectional area shrinks 5 centimeters square per minute. Sub-task 1: define the conditions that represent the problem and how the formula works. Sub-task 2: determine the instantaneous rate of change of the balloon’s length diameter after 10 minutes and explain why your answer makes sense. Conjecture the general formula to find the instantaneous rate of change of the balloon’s length diameter. Sub-task 3: verify whether the formula you set (in sub-task 2) can be applied to find the instantaneous rate of change of the balloon’s length diameter after 13 minutes and 15 minutes.

From modeling task in Figure 2, students were asked to determine the instantaneous rate of change of the balloon’s length diameter by linking the three components of the change in cross-sectional area (A), change of length of diameter (d), and change of time (t). The goal is divided into three sub-tasks. In sub-task 1, students were asked about formulating a mathematical model that represents the real state given. The model formed is a modeling of the change in cross-sectional area (A). Based on the stages of mathematical modeling [11], sub-task 1 is aimed at directing students in the constructing stage, i.e. forming the initial idea of what problem was asked and in the simplifying stage, i.e. determine the conditions that represent the problem. Furthermore, in sub-task 2, students were asked to determine the instantaneous rate of change from the length of diameter in the 10th minute and make guesses to the general formula. In this sub-task, at least lead to three modeling stages. First, the simplifying stage, in which the student is required to make assumptions about the shape of the balloon section, declare the
variables and make an alignment for the diameter, time, and cross-sectional area, introduce the external knowledge, e.g. the area of the circle if it assumes that the shape of the cross-section is a circle. Second, the matematizing stage, where students are required to connect between changes in cross-sectional area (A) formulated in sub-task 1, change of diameter length (d), and change of time (t). Third, the working mathematically stage, i.e. the student performs an algebraic operation to determine the instantaneous rate of change of the diameter length in the 10th minute and shows the logic of the given answer. In this sub-task, it involves two stages of modeling. First, the interpreting stage, in which students perform the recontextualization of mathematical results for the 13th and 15th minutes. Second, the validating stage, where the student validates the formula submitted on sub-task 2, checks the rationality of the answer, and checks for extreme cases.

The modeling task that have been made was implemented to explore the students’ thinking process of mathematics students in the modeling process. This research involves 21 students of mathematics education at Universitas Islam Darul Ulum, Indonesia in 2018 who studied the material of chain rules on differential calculus courses to complete the modeling tasks. Based on the similarity of the answer, the researchers classify the results of student work into 3 groups of answers. From each group of answers, two students were selected to represent their group for an in-depth analysis of how their thinking processes in doing the modeling work by interviews.

3. Results and discussion

In this study, we investigated the students’ thinking processes in completing the modeling task of chain rules that have been prepared. The discussion was done for each sub-task. But for sub-task 3, we conveyed the discussion in other articles. The following is presented a discussion for each sub-task.

3.1. Sub task 1

Table 1 shows a mathematical model compiled in sub-task 1. Through this model, we analyze the constructing and simplifying stages in mathematical modeling.

| Model         | Analogy                              |
|---------------|--------------------------------------|
| Student A     | \( L_{at} = L_0 - a.s \)             |
| Student B     | \( U_n = a + (n-1)b \) the analogy of the relationship between area and time |
| Student C     | \( L(t) = 154 - 5t \)                |

Table 1 shows that student A formulates the mathematical model of the given real state. By assuming the initial cross-sectional area as \( L_0 \), the cross-sectional area in the t-minute as \( L_{at} \) and time as \( s \), he arranges the \( L_{at} = L_0 - a.s \) relationship with \( an \) as the depreciation of the area. In this case, we can see that he succeeded beyond the constructing stage of understanding the known elements of the task and knowing what problems need to be resolved. Similarly, the simplifying stage which can be shown from the result in arranging the relationship between the known elements into the \( L_{at} = L_0 - a.s \). It’s just that he failed to interpret the shrinkage as a unit area (cm²), and instead think of it as a unit of length (cm).

Furthermore, student B attempts to use an analogy approach to formulate a mathematical model. Student B does not do an alignment to see the relationship between variables. In fact, student B does not show explicitly that the elements that exist in the task require variables that need to be exemplified. However, he is aware of changes in cross-sectional area as time changes. In formulating the conditions that represent the problem, he chose to use an arithmetic approach. It should be appreciated that even without a complete intersection, student B succeeds beyond the simplifying stage, without going through the stage of constructing.

Student C has gone beyond constructing and simplifying stage. Similarly, in Student A, student C does the same thing, i.e, performs an assignment for each element, except that it is the cross-sectional area of time as \( L(t) \) and time in minutes as \( t \).

In sub-task 1 we show that in the modeling task not every stage must be passed to go to the next modeling stage. This can be seen from the thinking process of student B, although not through the
constructing stage, further modeling process can still be continued. These findings indicate that in the modeling cycle, it is possible not to go through all the modeling steps. The existence of stepping stage modeling is possible because of the role of previous experience. A person’s prior experience has an important role in modeling activity [13]. This previous experience will form the link between mathematical modeling and meaningful knowledge [14].

3.2. Sub task 2
Table 2 shows students’ simplifying, mathematizing and working mathematically process in sub-task 2. In this sub-task, students are asked to determine the instantaneous rate of change of the diameter length in the 10th minute and conjecture the general formula.

Table 2. Student’ simplifying, mathematizing and working mathematically process in sub-task 2.

| Simplifying | Mathematizing | Working mathematically |
|-------------|---------------|------------------------|
| Student A   | Make the assumption that the cross section is a circle | $L = \pi r^2$ | $La(10) = \pi r^2$ |
|             | $d = 2r$      |                        |
| Student B   | Bring up the concept of arithmetic series | $U_n = a + (n-1)b$ | $U10 = 154 - (10 - 1)5$ |
| Student C   | Make the assumption that the cross section is a circle | $L = \frac{1}{4} \pi d^2$ | $d\left(\frac{\sqrt{4(154 - 5t)}}{\pi}\right) = 5$ |

Table 2 shows that student A and student C make the assumption that the balloon section is in the form of a circle, so using the broad circle formula approach to determine the diameter. Student A uses a wide circle of relations with the radius, whereas student C uses a wide circle of relations with diameter. However, student A is limited to determining the length of diameter at minute 10, regardless of the instantaneous rate of change in diameter. While student C uses the approach of diameter change over time, which leads to the inquiry of chain rules. This requires students C to pay attention to validate the answers obtained at the end of each step. This indicates that the validation stage should not be performed in the final stages only after interpreting and the task ends, but validation can be done at each stage if necessary. Modeling processes may be better described as “haphazard jumps between different stages and activities” [15,16]. In the ideal modeling cycle, the proposed stage shows a coherent and continuous unity [11,17]. But in this study, we show that the possibility of modeling process should not be coherent and can go back and forth between steps.

Unlike the case with student B which is limited to Un formula development to determine the length of diameter at minute 10. Regardless of whether or not students enter into the inquiry of chain rules, the process of approaching by students can vary greatly. Although in this case found the student barriers in mathematical modeling as found by other researchers [18,19].

Another finding in this study is mathematics teacher candidates consistently perform modeling tasks according to the analogy they constructed from the beginning. Impressively, this study shows that the analogy becomes the key in determining the thinking process used in subsequent modeling tasks as shown in Table 2. It appears that the thinking processes undertaken by each student follow the pathway they wake up from scratch. For example, student A, which is consistent with the assumption built in the simplifying stage that the balloon section is in the form of a circle, then in the mathematizing and working mathematically stage is also consistently terbantuk of the assumption. Similarly, in student B, which uses the arithmetic approximation concept, the modeling process also uses an approach to a consistent concept. Nevertheless, the built assumptions can lead to different work outcomes as shown in students A and C.
4. Conclusions
We recognize that developing theories do not necessarily fall off with new findings. In the discussion, we have shown that in modeling assignments, not every step has to be traversed to go to the next modeling stage. The researcher initially suspect that the modeling process that occurs would follow the ideal mathematical modeling cycle but this study found that the modeling process did not necessarily follow sequential development according to model construction as described in various mathematical modeling cycles. Unexpectedly, however, the thinking process undertaken by students is consistent as the analogy is built.

Acknowledgments
This research was supported by Direktorat Riset dan Pengabdian Masyarakat, Kemenristekdikti.

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