Stochastic source seeking with forward and angular velocity regulation

Jinbiao Lin \(^a\), Shiji Song \(^{a,1}\), Keyou You \(^a\), Miroslav Krstic \(^b\)

\(^a\) Department of Automation and Tsinghua National Laboratory for Information Science and Technology, Tsinghua University, Beijing, 100084, China
\(^b\) Department of Mechanical and Aerospace Engineering, University of California, San Diego, La Jolla, CA 92093-0411, USA

Article history:
Received 6 April 2016
Received in revised form 30 March 2017
Accepted 11 April 2017
Available online 10 July 2017

Keywords:
Source localization
Nonholonomic unicycle
Extremum seeking
Stochastic averaging
Adaptive control

Abstract

This paper studies a stochastic extremum seeking method to steer a nonholonomic vehicle to the unknown source of a spatial field in a plane. The key challenge lies in the lack of vehicle’s position information and the distribution of the scalar field. Different from the existing stochastic strategy that keeps the forward velocity constant and controls only the angular velocity, we design a stochastic extremum seeking controller to regulate both forward and angular velocities simultaneously in this work. Thus, the vehicle decelerates near the source and stays within a small area as if it comes to a full stop, which solves the overshoot problem in the constant forward velocity case. We use the stochastic averaging theory to prove the local exponential convergence, both almost surely and in probability, to a small neighborhood near the source for elliptical level sets. Finally, simulations are included to illustrate the theoretical results.

1. Introduction

Source seeking is a problem of steering single or multiple autonomous agents to seek the source of an unknown scalar field, which may be thermal, electromagnetic, acoustic, or the concentration of a chemical agent. Source seeking is of interest in many areas, such as environmental studies, explosive detection, localizing the sources of hazardous chemicals leakage or pollutants, etc. There are a diversity of approaches to source seeking problem, such as bio-inspired methods (Li, Farrell, Pang, Arrieta, et al., 2006; Russell, 2004), mathematical programming methods (Khong, Tan, Manzie, & Nešić, 2014; Ogren, Fiorelli, & Leonard, 2004; Porat & Nehorai, 1996; Teel & Popović, 2001), and source-likelihood mapping methods (Jakuba, 2007; Pang & Farrell, 2006).

In this work, we consider steering a single nonholonomic vehicle to locate a static source which creates a continuous signal map in a plane. Recently there is a growing interest in the study of locating such a source without position information (Azuma, Sakar, & Pappas, 2012; Cochran & Krstic, 2009; Matveev, Teimoori, & Savkin, 2011). The lack of position information is taken account for vehicles operated in environments where their position information is unavailable or costly. Obviously, this constraint, along with the nonholonomic constraint of the vehicle kinematics, renders the guidance of the vehicle interesting and challenging.

Extremum seeking (ES) is a model free optimization method for dynamical systems with limited information (Krstić & Wang, 2000). It has been proved to an effective method for nonholonomic source seeking problems even without position information. In Cochran and Krstic (2009) and Zhang, Arnold, Ghods, Siraniosian, and Krstic (2007) ES was applied to tune the forward or angular velocity of the vehicle to locate the source. In Ghods and Krstic (2010) Ghods and Krstic regulated both velocities to control the vehicle to stop near the source. While the above works focus on the 2D vehicles, Lin, Song, You, and Wu (2016) considered the more complicated 3D case. Different from the above perturbation-based ES methods, a novel perturbation-free regulator is proposed in Durr, Krstic, Scheinker, and Ebenbauer (2017) and Scheinker and Krstic (2014).

Motivated by the chemotactic behavior of bacteria (Berg, 2008), Liu and Krstic proposed a stochastic ES method in Liu and Krstic (2010a) and applied it to the source seeking problem in Liu and Krstic (2010b). The seeker can successfully locate the source but with an unpredictable, “nearly random” trajectory. This feature would be useful when the seeker itself is pursued by another hostile pursuer. In their work the forward velocity of the vehicle...
is constant, resulting in complicated asymptotic behaviors. Particularly, the vehicle cannot settle when it approaches close to the source. Instead it exhibits certain overshoots and finally revolves around the source. A small constant forward velocity may improve the asymptotic performance, but decreases the convergence rate.

In order to improve the asymptotic performance of the vehicle, we apply the stochastic ES to tune both forward and angular velocities simultaneously, which is different from Liu and Krstic (2010b). While the deterministic case has been discussed in Ghods and Krstic (2010), this work focuses on designing a stochastic excitation to modulate the velocities. Under a tunable forward velocity, the vehicle can slow down around the source and converge closer to the source. In addition, the undesired overshoots are eliminated due to a tunable forward velocity. We adopt the stochastic averaging theory to establish local exponential convergence, both almost surely and in probability, to a small neighborhood near the source, for signal fields with elliptical level sets. Note that in Cochran and Krstic (2009), Ghods and Krstic (2010) and Liu and Krstic (2010b) only the stability for circular level sets was proved.

It should be mentioned that there are other methods to address stochastic source seeking problem without position information. In Manzie and Krstic (2009) Manzie and Krstic proposed a discrete-time stochastic ES. Azuma et al. (Azuma et al., 2012) adopted the stochastic approximation technique to solve this problem by sequentially generating source-oriented waypoints, which can work for a switching signal field. However, the controller is discontinuous and requires a clock to precisely decide when to take next action. Another representative method using a swarm of autonomous vehicles is proposed in Mesquita, Hespanha, and Åström (2008), which achieves high vehicle densities near the maximum. Compared with those methods, the advantage of the stochastic ES is that we can use a single vehicle under a simple continuous controller to locate the source. Moreover, the exponential convergence (in probability and almost surely) to a small attractor near the source can be established.

Overall, the advantages of the proposed stochastic ES method are as follows: (1) Only a single vehicle under a simple continuous controller is used to locate the source; (2) The forward velocity is tuned to achieve a better asymptotic behavior; (3) The local exponential convergence for elliptical signal map is established.

The rest of the paper is organized as follows. In Section 2 we describe the nonholonomic source seeking problem and propose the stochastic ES scheme. In Section 3 we prove the local exponential convergence for signal fields with elliptical level sets. We first derive an average system to approximate the original system, then we consider the local stability under a small bias forward velocity. After that we discuss the result for circular level sets as a special case. In Section 4 we include simulation results to illustrate the effectiveness of the control scheme.

2. Problem description and control scheme

In this section we firstly describe the vehicle model and formulate the source seeking problem. Then we propose a stochastic ES scheme to adjust the forward and angular velocities of the vehicle.

2.1. Problem description

Similar to Cochran and Krstic (2009), we consider an autonomous vehicle modeled as a 2D nonholonomic unicycle, see Fig. 1 for illustration. The heading angle is defined by , and the position of the vehicle center is defined by . A sensor is mounted at the front end , a distance away from the vehicle center . The vehicle has actuators which are used to impart the forward velocity and the angular velocity . The kinematic equations of motion for the vehicle center and the sensor are

\[
\dot{r}_c = v e^{\psi}, \quad (1) \\
\dot{\psi} = \xi, \quad (2) \\
\dot{r}_s = r_c + R e^{\psi}, \quad (3)
\]

where and are written as complex variables.

The task of vehicle is to seek a static source in a plane. We denote the signal strength at the location by and the unknown source location by , both of which are constant versus time and non-random. We make the following assumption.

**Assumption 1.**

(1) The signal strength can be measured by the sensor, but the position is unavailable.

(2) decays away from the source and is twice continuously differentiable, satisfying that

\[
\nabla f(r^*) = 0, \quad \nabla^2 f(r^*) \quad \text{is negative definite,} \quad (4)
\]

where \(\nabla f(r^*)\) and \(\nabla^2 f(r^*)\) denote the gradient and Hessian of at respectively.

Note that the traditional gradient searching strategy is not suitable for the problem due to the lack of position information. By (4), we can approximate the signal distribution by a quadratic map when studying the local convergence. Without loss of generality, we assume the quadratic map takes the form

\[
J = f^* - q_x(x - x^*)^2 - q_y(y - y^*)^2, \quad (5)
\]

where \(r^*_s = [x^*_s, y^*_s]^T, r^* = [x^*, y^*]^T\), and \(q_x\) and \(q_y\) are unknown positive constants.

**2.2. Control scheme**

We employ the stochastic ES method to tune the angular velocity and the forward velocity indirectly. The control scheme is depicted in Fig. 2. The control laws are given by

\[
v = V_c + b \xi, \quad (6) \\
\psi = a \tilde{\eta} + c \xi \sin(\eta), \quad (7) \\
\xi = \frac{s}{s + h} |J|, \quad (8) \\
\eta = \frac{g \sqrt{E}}{\varepsilon s + 1} |W|, \quad (9)
\]

where the parameters , , , , , and are positive and will affect the performance of the approach. \(J\) is the sensor reading, and \(W(t)\) is a standard Brownian motion defined in a complete
probability space \((\mathcal{O}, \mathcal{F}, P)\) with the sample space \(\mathcal{O}\), the \(\sigma\)-field \(\mathcal{F}\), and the probability measure \(P\). Here the colored noise \(\eta\) is used as a stochastic perturbation in ES.

In our control scheme, the angular velocity \(\psi\) is tuned according to the idea of the stochastic ES tuning law \((\text{Liu & Krstic, 2010a})\). The perturbation term \(a\eta\) is added to persistently excite the system while the corresponding demodulation term \(\sin(\eta)\) is used to estimate the gradient of the nonlinear map \(f\). Different from the deterministic case which uses a sinusoidal perturbation \((\text{Cochran & Krstic, 2009}; \text{Ghods & Krstic, 2010})\), the stochastic perturbation results in a partly random trajectory. The forward velocity \(v\) is designed to be positively correlated to \(\xi\), since \(\xi\) describes the variation of the sensor reading \(J\) in some sense. As a result, the vehicle would speed up when approaching the source, and slow down when deviating from the source.

It is worthy mentioning that our control scheme is different from the one in \((\text{Liu & Krstic, 2010b})\), where a vehicle with a constant forward velocity is considered. Employing the basic stochastic ES method in \((\text{Liu & Krstic, 2010a})\), the vehicle with a constant forward velocity cannot settle even if it has reached the source. In addition, it easily overshoots the source and has to turn around. This process may repeat before the vehicle finally revolves around the source. In \((\text{Liu & Krstic, 2010b})\) a nonlinear damping item is added to tune the angular velocity to improve the performance and achieve exponential stability. In this work, we tune the forward velocity along with the angular velocity. Intuitively this is a better way to control the vehicle, as we are able to smartly adjust the vehicle to speed up or slow down depending on different circumstances. Thus we can apply the basic stochastic ES control law to tuning the angular velocity directly without employing the nonlinear damping.

### 3. Stability analysis

The dynamics of the closed-loop system is intricate on account of nonlinearities of the vehicle model and the signal map and the existence of the stochastic perturbation. We adopt the stochastic averaging theory in \((\text{Liu & Krstic, 2010a})\) to prove the local exponential convergence for elliptical level sets. In Section 3.1 we derive an average system to approximate the closed-loop system. In Section 3.2 we prove that the vehicle converges, almost surely and in probability, to an attractor near the source under a small bias forward velocity. In Section 3.3 we consider a special case where the signal distribution is circular. In this case, the local exponential convergence can be established no matter whether the bias forward velocity is small or large.

#### 3.1. Average system for elliptical level sets

We firstly rewrite the elliptical signal map (5) as

\[
J = f^* - (q_1 + 2q_p)(x_2 - x^*)^2 - (q_2 - 2q_p)(y_2 - y^*)^2
\]

where \(q_1\) and \(q_p\) are unknown and \(q_2 > 2|q_p| > 0\).

Before analyzing the stability of the closed-loop system, we define an output error variable \(e_\xi = \frac{1}{\sqrt{v+h}}[J - f^* - e_\xi]\) to express the output of the washout filter as

\[
\xi = \frac{s}{s+h}[J - f^* - e_\xi].
\]

Thus, we obtain \(\dot{e}_\xi = \frac{h}{s+h}e_\xi\).

By inserting the control laws (6)–(9) into the system (1)–(3) and expressing \(\hat{\eta}\) as

\[
\hat{\eta} = \frac{g}{\sqrt{v}}\left[W - \frac{1}{\sqrt{v}}(\frac{g}{s+1}g + \frac{g}{v}W) = \frac{g}{\sqrt{v}}W - \frac{1}{\sqrt{v}}\eta,\right.
\]

the closed-loop system is written as

\[
\begin{align*}
\dot{c}_\xi &= (V_c + b\xi)e^{i\phi(\xi)}dt, \quad (10) \\
\dot{d}_\xi &= -\frac{a}{\sqrt{v}}\eta dt + c\xi \sin(\eta)dt + \frac{ag}{\sqrt{v}}dW, \quad (11) \\
d\xi &= \frac{1}{\sqrt{v}}\eta dt, \quad (12) \\
\dot{d}_\xi &= -\frac{1}{\sqrt{v}}\eta dt + \frac{g}{\sqrt{v}}dW, \quad (13) \\
\xi &= J - f^* - e_\xi, \quad (14) \\
r_\xi &= r_* + R e^{i\theta}. \quad (15)
\end{align*}
\]

To analyze the closed-loop system, we firstly re-express it by variable transformation. Then we redefine \(r_\xi\) in its polar coordinates for the convenience of the calculation of the equilibria. To this end, we start by defining shifted variables

\[
\dot{r}_\xi = r_* - r^* - \theta, \quad \dot{\theta} = \theta - a\xi.
\]

The dynamics of the shifted system is given by

\[
\begin{align*}
\dot{r}_\xi &= (V_c + b\xi)e^{i\hat{\phi}(\hat{\xi})}, \quad (10) \\
\dot{\theta} &= c\xi \sin(\eta)dt, \quad (11) \\
\dot{e}_\xi &= h\xi, \quad (12) \\
\hat{\xi} &= -\frac{1}{\sqrt{v}}\eta dt + \frac{g}{\sqrt{v}}dW, \quad (13) \\
\xi &= -\frac{1}{\sqrt{v}}\eta dt + \frac{g}{\sqrt{v}}dW. \quad (14)
\end{align*}
\]

By (13) and the definition of Its stochastic differential equation, we obtain \(\eta(t) = \eta(0) - \int_0^t \frac{1}{\sqrt{v+h}}e^{\int_0^\tau \frac{1}{\sqrt{v}}d\tau}dt + \int_0^t \frac{1}{\sqrt{v}}dW(\tau)\). Then we have \(\eta(\tau) = \eta(0) - \int_0^\tau \eta(\tau)dt + \int_0^\tau \frac{1}{\sqrt{v}}dW(\tau)\). By defining \(\chi(t) = \eta(\tau)\) and \(B(t) = \int_0^t W(\tau)dt\), we have \(d\chi(t) = -\chi(t)dt + dB(t)\), where \(B(t)\) is a stationary Brownian motion and the process \(\chi(t)\) is an Ornstein–Uhlenbeck (OU) process which is ergodic with invariant distribution \(\mu(dy) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{y^2}{2\sigma^2}}dy\).

We redefine \(\hat{r}_\xi\) by its polar coordinates

\[
\hat{r}_\xi = |\hat{r}_\xi|e^{i\hat{\theta}}, \quad \hat{r}_\xi = \hat{r}_\xi e^{i\hat{\theta}}, \quad \hat{\theta} = \arg(\hat{r}_\xi) = \arg(r_* - r_\xi),
\]

where \(\hat{r}_\xi\) is the distance between the vehicle center and the source, \(\hat{\theta}\) represents the heading angle from the vehicle center towards...
the source. We also define $\tilde{e}_x = e_x + qgR^2$ for convenience. Using these new definitions, $\xi$ is expressed as

$$\xi = -\tilde{e}_x - 2qgR^2 \cos(2\hat{\theta} + 2\alpha(t/e)) - \hat{r}_c^2 (q_r + 2qg \cos(2\theta^*)) + 2R \hat{q}_c \cos(\hat{\theta} - \theta^* + a\chi(t/e)) + 2qg \cos(\hat{\theta} + \theta^* + a\chi(t/e)).$$  

(16)

Now we obtain the following shifted error system

$$\frac{d\hat{r}_c}{dt} = -(V_c + b\xi) \cos(\hat{\theta} - \theta^* + a\chi(t/e)), \quad (17)$$

$$\frac{d\hat{\theta}}{dt} = -\frac{V_c + b\xi}{r_c} \sin(\hat{\theta} - \theta^* + a\chi(t/e)), \quad (18)$$

$$\frac{d\hat{\chi}}{dt} = c\xi \sin(\chi(t/e)), \quad (19)$$

$$\frac{d\hat{e}_x}{dt} = h\xi, \quad (20)$$

$$d\chi(t) = -\chi(t)dt + gdB(t). \quad (21)$$

According to the stochastic averaging theory (Liu & Krstic, 2010a), the error system can be approximated by an average system, which is given by

$$\frac{d\hat{e}_x^{av}}{dt} = (b\phi_0 - V_c) \cos(\hat{\theta}^{av} - \theta^{av}) \chi_1(a, g) + bqgR^2 \phi_1 - b\hat{r}_c^{av} R\phi_2, \quad (22)$$

$$\frac{d\theta^{av}}{dt} = \frac{b\phi_0 - V_c}{\hat{r}_c^{av}} \sin(\hat{\theta}^{av} - \theta^{av}) \chi_1(a, g) + bqgR^2 \phi_3 - bR\phi_4, \quad (23)$$

$$\frac{d\hat{\theta}^{av}}{dt} = 2cqgR^2 \sin(2\hat{\theta}^{av}) \chi_2(2a, g) - 2c\hat{r}_c^{av} R\phi_3 \chi_2(2a, g), \quad (24)$$

$$\frac{d\hat{e}_x^{av}}{dt} = -2hqgR^2 \cos(2\hat{\theta}^{av}) \chi_1(2a, g) - h\phi_0 + 2h\hat{r}_c^{av} R\phi_1(2a, g). \quad (25)$$

where $\chi_1(a, g) = \int_R \cos(ay) \mu(dy)$, $\chi_2(2a, g) = \int_R \sin(ay) \mu(dy)$,

$$\sin(\gamma) \mu(dy) = \frac{1}{2} \left[ e^{\frac{(a-i\gamma)^2}{4}} - e^{\frac{(a+i\gamma)^2}{4}} \right]$$

and

$$\phi_0 = \hat{e}_x^{av} + (\hat{r}_c^{av})^2 (q_r + 2qg \cos(2\theta^av)), \quad (26)$$

$$\phi_1 = \cos(3\hat{\theta}^{av} - \theta^{av}) \chi_1(3a, g) + \cos(\hat{\theta}^{av} + \theta^{av}) \chi_1(a, g), \quad (27)$$

$$\phi_2 = q_r \cos(2\hat{\theta}^{av} - 2\theta^{av}) \chi_1(2a, g) + 2qg \cos(2\theta^av) + 2qg \cos(2\theta^av) \chi_2(2a, g) + q_r \cos(3\theta^av) \chi_2(3a, g), \quad (28)$$

$$\phi_3 = \sin(3\hat{\theta}^{av} - \theta^{av}) \chi_1(3a, g) - \sin(\hat{\theta}^{av} + \theta^{av}) \chi_1(a, g), \quad (29)$$

$$\phi_4 = q_r \sin(2\hat{\theta}^{av} - 2\theta^{av}) \chi_1(2a, g) - 2qg \sin(2\theta^av) + 2qg \sin(2\theta^av) \chi_2(2a, g), \quad (30)$$

$$\phi_5 = q_r \sin(\hat{\theta}^{av} + \theta^{av}) + 2qg \sin(\hat{\theta}^{av} + \theta^{av}), \quad (31)$$

$$\phi_6 = q_r \cos(\hat{\theta}^{av} + \theta^{av}) + 2qg \cos(\hat{\theta}^{av} + \theta^{av}). \quad (32)$$

The average error system has eight equilibria as follows:

$$\textbf{eq}_1 = \left[ \rho(q_p), 0, 0, e(q_p) \right], \quad (26)$$

$$\textbf{eq}_2 = \left[ \rho(q_p), \pi, \pi, e(q_p) \right], \quad (27)$$

$$\textbf{eq}_3 = \left[ \rho(-q_p), \pi/2, \pi/2, e(-q_p) \right], \quad (28)$$

$$\textbf{eq}_4 = \left[ \rho(-q_p), -\pi/2, -\pi/2, e(-q_p) \right], \quad (29)$$

$$\textbf{eq}_5 = \left[ -\rho(q_p), \pi, 0, e(q_p) \right], \quad (30)$$

$$\textbf{eq}_6 = \left[ -\rho(q_p), 0, \pi, e(q_p) \right], \quad (31)$$

$$\textbf{eq}_7 = \left[ -\rho(-q_p), -\pi/2, -\pi/2, e(-q_p) \right], \quad (32)$$

$$\textbf{eq}_8 = \left[ -\rho(-q_p), \pi/2, -\pi/2, e(-q_p) \right]. \quad (33)$$

where $\textbf{eq}_i$ is of the form $[\tilde{r}_c^{av}, \phi^{av}, \hat{\theta}^{av}, \tilde{e}_x^{av}]$ and

$$\rho(q_p) \triangleq -\frac{V_L I_1(a, g) + bqgR^2 \gamma_1}{bR(q_r + 2qg \gamma_1)}, \quad (34)$$

$$e(q_p) \triangleq 2Rq_r + 2qg \gamma_1(a, g) - (q_r + 2qg)^2 \gamma_1. \quad (35)$$

Note that with positive $a$ and $g$, we have $I_1(a, g) > 0, I_2(a, g) > 0, \gamma_1 > 0$ and $\gamma_2 > 0$.

Each of the equilibria (26–33) represents an attractor around the source. The value of $\hat{r}_c^{av}$ should be real and positive as it represents the average distance between the vehicle center and the source. Note the difference between $\tilde{\theta}^{av}$ and $\hat{\theta}^{av}$ is either 0 or $\pi$, which indicates the average heading of the vehicle points either directly towards or away from the source.

### 3.2. Stability for elliptical level sets

Before we prove the stability, define an index variable $\iota$ as

$$\iota = \begin{cases} 
1 & \text{if } q_\rho \gamma_3 < 0 \text{ and } \rho(q_p) > 0 \\
3 & \text{if } q_\rho \gamma_3 > 0 \text{ and } \rho(-q_p) > 0 \\
5 & \text{if } q_\rho \gamma_3 > 0 \text{ and } \rho(q_p) < 0 \\
7 & \text{if } q_\rho \gamma_3 > 0 \text{ and } \rho(-q_p) < 0,
\end{cases} \quad (36)$$

where

$$\gamma_3 = (I_1(3a, g) - I_1(a, g)) I_2(a, g) + (1 - I_1(2a, g)) I_2(2a, g). \quad (37)$$

**Theorem 2.** Consider the system (5), (10)–(15) with positive parameters $a$, $g$, $b$, $c$, $h$, and $\xi \in (0, \xi_0)$. The parameters $a$, $g$, $b$, $c$, $h$, $V_c$ are chosen such that either

$$q_\rho \gamma_3 < 0, \quad (38)$$

$$V_L'(q_p) < V_c < V_c''(q_p), \quad \text{and } V_c \neq V_a, \quad (39)$$

or

$$q_\rho \gamma_3 > 0, \quad (40)$$

$$V_L'(-q_p) < V_c < V_c''(-q_p), \quad \text{and } V_c \neq V_a, \quad (41)$$

where

$$V_L'(q_p) \triangleq -\frac{1}{2I_1^2(a, g)} \left( b\rho^2 \gamma_1 (1 + I_1(2a, g)) ight) \quad (42)$$

$$V_c''(q_p) \triangleq \frac{bR^2 q_r - 2bRq_r \gamma_2 I_1(a, g)}{2c(1 + a, g) I_2(2a, g)} \quad (43)$$

$$V_c''(q_p) \triangleq -\frac{V_L(q_p)}{\Xi(0) - \textbf{eq}_{i+1}} \quad (44)$$

If the initial conditions $r_c(0), \theta(0), e_c(0)$ satisfy that either $\Xi(0) - \textbf{eq}_i$ or $\Xi(0) - \textbf{eq}_{i+1}$ is sufficiently small, where

$$\Xi(t) = \left[ |r_c(t) - r^*|, \arg(r^* - r_c(t)), \theta(t), e_c(t) - q_r R^2 \right]. \quad (45)$$

---

2 We have implicitly assumed $\theta^{av} \in (-\pi, \pi)$ and $\hat{\theta}^{av} \in (-\pi, \pi)$ to exclude repetitive equilibria.
then there exist constants $C_0 > 0$, $\gamma_0 > 0$, $T(\varepsilon) : (0, \varepsilon_0) \to \mathbb{N}$ such that for any $\delta > 0$, the trajectory of the vehicle center $r_c(t)$ satisfies the following properties,

$$\liminf_{t \to 0} \left\{ \left| r_c(t) - r^* - \bar{r}_i \right| > C_0 e^{-\gamma_0 t + \delta} \right\} = \infty, \quad \text{a.s.,}$$

$$\lim_{t \to -0} P \left\{ \left| r_c(t) - r^* - \bar{r}_i \right| \leq C_0 e^{-\gamma_0 t + \delta} \right\} = 1$$

$$\forall t \in [0, T(\varepsilon)]$$

where $\bar{r}_i$ denotes the first element of $\mathbf{eq}_i$, the constant $C_0$ depends on the initial condition $(r_c(0), \theta(0), \varepsilon(0))$ and on the parameters $a, g, b, c, h, V_i, q_i, a_{14}$, and the constant $\gamma_0$ depends on the parameters $a, g, b, c, h, V_i, q_i, q_p$.

**Proof.** The Jacobians of the equilibria (26)–(33) are as follows,

$$A^{eq_1} = A^{eq_2} = J_1(q_p), \quad A^{eq_3} = A^{eq_4} = J_1(-q_p),$$

$$A^{eq_5} = A^{eq_6} = J_2(q_p), \quad A^{eq_7} = A^{eq_8} = J_2(-q_p),$$

where $J_1(q_p)$ and $J_2(q_p)$ are defined as

$$J_1(q_p) \triangleq \begin{bmatrix} a_{11}(q_p) & 0 & 0 & a_{14} \\ 0 & a_{22}(q_p) & a_{23}(q_p) & 0 \\ a_{41}(q_p) & 0 & 0 & -h \\ a_{41}(q_p) & 0 & 0 & -a_{14} \end{bmatrix},$$

$$J_2(q_p) \triangleq \begin{bmatrix} a_{11}(q_p) & 0 & 0 & -a_{14} \\ 0 & a_{22}(q_p) & a_{23}(q_p) & 0 \\ 0 & a_{22}(q_p) & a_{23}(q_p) & 0 \\ -a_{41}(q_p) & 0 & 0 & -h \end{bmatrix},$$

and

$$a_{11}(q_p) = 2b \left( q_p + 2q_p \right) \rho(q_p) l_1(a, g) - bR \left( q_p + 2q_p \right) \left( 1 + l_1(a, g, q) \right),$$

$$a_{14} = bh l_1(a, g),$$

$$a_{22}(q_p) = -bR(q_p - 2q_p) \left( 1 - l_1(a, g, q) \right),$$

$$a_{23}(q_p) = 2bR(q_p + 2q_p) \left( l_2(a, g) - l_1(a, g, q) + \frac{1}{\rho(q_p)} \right),$$

$$\times \left( 3l_1(3a, g) - l_1(a, g, q) - 2l_1(a, g, q) l_1(a, g) \right),$$

$$\times b q_p R^2 I_1 \left( a, g \right),$$

$$a_{32}(q_p) = 2c R \left( q_p - 2q_p \right) \rho(q_p) l_2(a, g),$$

$$a_{33}(q_p) = 4c q_p R^2 I_2(2a, g) - 2c R \left( q_p + 2q_p \right) \rho(q_p) l_2(a, g),$$

$$a_{41}(q_p) = 2bR \left( q_p + 2q_p \right) l_1(a, g, q) - 2h \left( q_p + 2q_p \right) \rho(q_p).$$

The Jacobians $A^{eq_1}, A^{eq_2}, A^{eq_3}$ and $A^{eq_6}$ have the same characteristic equation, which is given by

$$\det \left[ \lambda^2 + \left( h - a_{11}(q_p) \right) \lambda - a_{11}(q_p) h - a_{14}(q_p) a_{41}(q_p) \right] \times \det \left[ \lambda^2 - \left( a_{22}(q_p) + a_{33}(q_p) \right) \lambda + a_{22}(q_p) a_{33}(q_p) \right] - a_{23}(q_p) a_{32}(q_p) = 0.$$  (38)

To guarantee that all roots of characteristic equation (38) have negative real parts, we need

$$a_{11}(q_p) - h < 0,$$

$$a_{11}(q_p) h - a_{14}(q_p) a_{41}(q_p) < 0,$$

$$a_{23}(q_p) a_{32}(q_p) - a_{22}(q_p) a_{33}(q_p) < 0.$$  (39)

All the above requirements are satisfied under conditions (34) and (35), and the Jacobians $A^{eq_1}, A^{eq_2}, A^{eq_3}$ and $A^{eq_6}$ are Hurwitz. Hence, equilibria $\mathbf{eq}_1, \mathbf{eq}_2, \mathbf{eq}_3$ and $\mathbf{eq}_6$ are exponentially stable. Similarly we can prove that equilibria $\mathbf{eq}_4, \mathbf{eq}_5, \mathbf{eq}_7$ and $\mathbf{eq}_8$ are exponentially stable under conditions (36) and (37). By Theorem 2 in Liu and Krstic (2010a), there exist constants $C_0 > 0$, $r_0 > 0$, $\gamma_0 > 0$ and a function $T(\varepsilon) : (0, \varepsilon_0) \to \mathbb{N}$, such that for any $\delta > 0$ and any initial condition $|A^{eq_1}(0)| < r_0^0$,

$$\liminf_{t \to 0} \left\{ \left| \varepsilon^{eq_1}(t) \right| > C_0 e^{-\gamma_0 t + \delta} \right\} = \infty, \quad \text{a.s.,}$$

$$\lim_{t \to -0} P \left\{ \left| \varepsilon^{eq_1}(t) \right| \leq C_0 e^{-\gamma_0 t + \delta} \right\} = 1$$

$$\forall t \in [0, T(\varepsilon)]$$

where $\varepsilon^{eq_1}(t) = |\varepsilon(t) - \mathbf{eq}_1|$. With the fact $|\bar{r}_i(t) - \bar{r}_i| < |\varepsilon^{eq_1}(t)|$, we obtain

$$\liminf_{t \to 0} \left\{ \left| \bar{r}_i(t) - \bar{r}_i \right| > C_0 e^{-\gamma_0 t + \delta} \right\} = \infty, \quad \text{a.s.,}$$

$$\lim_{t \to -0} P \left\{ \left| \bar{r}_i(t) - \bar{r}_i \right| \leq C_0 e^{-\gamma_0 t + \delta} \right\} = 1$$

$$\forall t \in [0, T(\varepsilon)]$$

where $C_0 = C_0 \varepsilon^{eq_1}(0)$. The proof is completed. □

**Theorem 2** implies the vehicle can locate a source in an elliptical signal map under small $V_i$. The vehicle finally points either directly towards or away from the source on the average. In fact the averaging heading of the vehicle is finally aligned with one of the coordinate axes. In other words, the vehicle converges to one point at the major or minor axis of the elliptical map. Note that the stability for the elliptical map under a constant forward velocity remains outstanding (Cochran & Krstic, 2009; Liu & Krstic, 2010b), since in that case there does not exist a stable equilibrium to analyze (in the polar coordinates). For the case under large $V_i$, we cannot find an analytic equilibrium due to the complexity of the average error system (22)–(25), though simulation in Section 4 indicates the vehicle can also approach the source in this case. In Section 3.3 we shall study the result in a circular signal map, and compare it against the result in Liu and Krstic (2010b).

Next we give a brief discussion on the parameter selection for Theorem 2. Without loss of generality, we assume $q_p > 0$. We also assume $a \in (0, 3)$ and $g \in (0, 3)$ to limit the strength of the stochastic perturbation.

Using the fact that $2g_1 \left( 1 + l_1(2a, g, q) \right) > g_2 l_1(a, g, q)$ and $q_p > 2q_i$, one can easily derive that $V_i^{eq}(\pm q_p) = \frac{-b \rho(q_p)}{2l_1(2a, g)} < 0$. We also have $V_i^{eq}(\text{sgn}(\varepsilon) q_p) > 0$ under the condition

$$b > \frac{2c R \text{sgn}(\varepsilon) q_p \gamma_1}{(q_p + 2 \text{sgn}(\varepsilon) q_p) \gamma_1}.$$  (39)

Thus under condition (39), we can always find an appropriate $V_i$ for Theorem 2 by choosing $V_i$ small enough.

The sign of $V_i - V_{io}$ decides the average heading of the vehicle around the equilibria. The average heading would point inward when $V_i < V_{io}$ and outward when $V_i > V_{io}$. In addition, we have $V_i^{eq}(q_p) < 0 < V_{io}$ when $\gamma_1 < 0$ and $V_i^{eq}(-q_p) < V_{io} < 0$ when $\gamma_1 > 0$. Observing that $\gamma_1 > 0$ if $a \in (0, 1)$ and $\gamma_1 < 0$ if $a \in (1, 3)$, we obtain the following corollary by summarizing the above analysis.

**Corollary 3.** Consider the system in Theorem 2 with $q_p > 0$, $a \in (0, 1) \cup (1, 3)$ and $g \in (0, 3)$, assume conditions in Theorem 2 and (39) are satisfied.

(a) When $a \in (0, 1)$, the vehicle center converges to a point at the major axis of the elliptical level sets. Specially, the vehicle eventually points away from the source on the average under a small positive $V_i$. 
3.3. Stability for circular level sets

One can easily derive the stability for circular level sets under a small $V_c$ by setting $q_{\rho} = 0$ in Theorem 2. Due to the special structure of the circular level sets, we can also prove the stability under a large $V_c$. We write the signal distribution as

$$J = f(r_c) = f^* - q_{\rho} |r_c - r^*|^2,$$

and rewrite the expression of $\xi$ as

$$\xi = -q_{\rho} \left( \hat{r}_c^2 - 2R \hat{r}_c \cos \hat{\theta} - \theta^* + a \chi(t/\epsilon) \right) - \hat{\xi}_c.$$

Observing the expression of $\xi$ and the shifted error system (17)–(21), the system order can be reduced by defining $\hat{\theta} = \theta - \theta^*$, which results in the following reduced shifted error system

$$\frac{d\hat{r}_c}{dt} = - (V_c + b \xi) \cos \left( \hat{\theta} + a \chi \left( \frac{t}{\epsilon} \right) \right),$$

$$\frac{d\hat{\theta}}{dt} = c \xi \sin \left( \chi \left( \frac{t}{\epsilon} \right) \right) + \frac{(V_c + b \xi)}{\hat{r}_c} \sin \left( \hat{\theta} + a \chi \left( \frac{t}{\epsilon} \right) \right),$$

$$\frac{d\hat{\xi}_c}{dt} = h \hat{\xi}_c,$$

$$\xi = -q_{\rho} \hat{r}_c^2 - \hat{\xi}_c + 2q_c R \hat{r}_c \cos \left( \hat{\theta} + a \chi \left( \frac{t}{\epsilon} \right) \right),$$

$$d\chi(t) = - (\chi(t) + g \theta_B(t)).$$

The corresponding average error system is

$$\frac{d\hat{r}_c}{dt} = (b q_{\rho} \hat{r}_c \cos \theta - V_c) \cos \left( \hat{\theta} + a \chi \left( \frac{t}{\epsilon} \right) \right),$$

$$\frac{d\hat{\theta}}{dt} = c \xi \sin \left( \chi \left( \frac{t}{\epsilon} \right) \right) + \frac{(V_c + b \xi)}{\hat{r}_c} \sin \left( \hat{\theta} + a \chi \left( \frac{t}{\epsilon} \right) \right),$$

$$\frac{d\hat{\xi}_c}{dt} = h \hat{\xi}_c,$$

$$\xi = -q_{\rho} \hat{r}_c^2 - \hat{\xi}_c + 2q_c R \hat{r}_c \cos \left( \hat{\theta} + a \chi \left( \frac{t}{\epsilon} \right) \right),$$

$$d\chi(t) = - (\chi(t) + g \theta_B(t)).$$

Different from the elliptical case, we can find analytical equilibria of the reduced-order system even under a large $V_c$. The average system has four equilibria defined by

$$\frac{\hat{r}_c}{\hat{r}_c^2} = \frac{\hat{\theta}}{\hat{\theta}^2} = \frac{\hat{\xi}_c}{\hat{\xi}_c^2} = [\rho_1, \pi, e_1],$$

$$\frac{\hat{r}_c}{\hat{r}_c^2} = \frac{\hat{\theta}}{\hat{\theta}^2} = \frac{\hat{\xi}_c}{\hat{\xi}_c^2} = [-\rho_1, 0, e_1],$$

$$\frac{\hat{r}_c}{\hat{r}_c^2} = \frac{\hat{\theta}}{\hat{\theta}^2} = \frac{\hat{\xi}_c}{\hat{\xi}_c^2} = [\rho_2, \alpha, e_2],$$

$$\frac{\hat{r}_c}{\hat{r}_c^2} = \frac{\hat{\theta}}{\hat{\theta}^2} = \frac{\hat{\xi}_c}{\hat{\xi}_c^2} = [-\rho_2, -\alpha, e_2],$$

where

$$\rho_1 = \frac{V_c I_1(a, g)}{b q_R \gamma_1},$$

$$\rho_2 = \frac{\sqrt{c V_c I_1(a, g) I_2(a, g) + b^2 q_R \gamma_y b}}{2c I_2^2(a, g) q_R R}.\]
4. Simulation

In this section we present simulation results to illustrate the behaviors of the vehicle in a signal map with circular or elliptical level sets. We also consider locating a source in a non-quadratic signal field. In all simulations we use band-limited white noise to approximate the white noise.

4.1. Signal maps with circular level sets

In this part, we examine the performance of the vehicle in a circular map. The map parameters are set as \( f^* = 0, r^* = (0, 0) \) and \( q_r = 2 \) and \( q_p = 0.5 \) and the initial conditions of the vehicle are set as \( r_c(0) = (1, 1) \), and \( \theta(0) = -\pi/2 \). The distance between the sensor and the vehicle center is set as \( R = 0.1 \). The controller parameters are chosen as \( a = 2, g = 1, \varepsilon = 0.01, b = 2, c = 500 \) and \( h = 2 \).

Fig. 3 illustrates the behavior of the vehicle dictated by Theorem 4 under small positive \( V_c \). The bias forward velocity is chosen as \( V_c = 0.0005 \). As shown in Fig. 3(a), the vehicle center converges to a small neighborhood very close to the source with its heading points away from the source on the average. The trajectory of the vehicle center is partly random due to the using of the stochastic perturbation. Fig. 3(b) shows the sensor reading and the forward velocity of the vehicle.

In Fig. 3 the vehicle finally moves in a small area near the source as if it comes to a full stop. This is quite different from the result of Liu and Krstic (2010b), where the vehicle finally drifts in an annulus around the source. In addition, the attractor in Fig. 3 is very close to the source under small \( V_c \) since \( \tilde{r}_{av \, eq} \) is positively correlated to \( |V_c| \). Note that the vehicle does not strictly stop at the source, although it seems evident from Fig. 3(a). We can see it keeps moving in the attractor with a small forward velocity from Fig. 3(b).

Fig. 4 illustrates the behavior of the vehicle dictated by Theorem 4 under large \( V_c \). The bias forward velocity is chosen as \( V_c = 0.01 \). The vehicle converges to an annular attractor and revolves around the source, which is similar to the result in Liu and Krstic (2010b). The average heading is more outward than inward, which coincides with the theoretical result.

4.2. Signal maps with elliptical level sets

In this part, we examine the performance of the vehicle in an elliptical map. The map parameters are set as \( f^* = 0, r^* = (0, 0) \), and \( q_r = 2 \) and \( q_p = 0.5 \) and the initial conditions of the vehicle are set as \( r_c(0) = (1, 1) \), and \( \theta(0) = -\pi/2 \). The distance between the sensor and the vehicle center is set as \( R = 0.1 \). The controller parameters are chosen as \( a = 2, g = 1, \varepsilon = 0.01, b = 2, c = 500 \) and \( h = 2 \).

Figs. 5 and 6 illustrate the behavior of the vehicle dictated by Theorem 2. In Fig. 5 and Fig. 6(a), we chose \( a = 2, g = 1.5 \) and \( V_c = -0.015 \). In Fig. 6(b), we chose \( a = 2, g = 1.5 \) and \( V_c = 0.015 \). In Fig. 6(c), we chose \( a = 0.5, g = 2 \) and \( V_c = -0.01 \). In Fig. 6(d), we chose \( a = 0.5, g = 2 \) and \( V_c = 0.001 \). As depicted in Figs. 5 and 6, the vehicle converges to a small area near the source under small \( V_c \). Fig. 6 illustrates the convergence to different equilibria under different parameters in the same signal map.

Fig. 7 illustrates the behavior of the vehicle under large \( V_c \). The parameters are chosen as \( a = 2, g = 1, \) and \( V_c = 0.01 \). In this case, the vehicle can also approach the source. As shown in Fig. 7, it overshoots the source, turns back and overshoots the source again, and so on.

4.3. Non-quadratic signal maps

Our control scheme also exhibits abilities to seek the sources of signal fields with non-quadratic maps. In Fig. 8 we assume the
Fig. 5. Vehicle trajectory for elliptical level sets under small positive $V_c$.

(a) $a = 2$, $g = 1.5$, $v_c = -0.015$.

(b) $a = 2$, $g = 1.5$, $v_c = 0.015$.

(c) $a = 0.5$, $g = 2$, $v_c = -0.01$.

(d) $a = 0.5$, $g = 2$, $v_c = -0.01$.

Fig. 6. Vehicle trajectories for elliptical level sets under different parameters.

Fig. 7. Vehicle trajectory for elliptical level sets under large $V_c$.

signal distribution is a Rosenbrock function, which takes the form $J = -x_1^2 - (y_1 - x_1^2)^2$. The Rosenbrock function has an isolated maximum at $(0, 0)$ and its Hessian at $(0, 0)$ is negative definite. The initial conditions of the vehicle and controller parameters are chosen to be the same as those in Fig. 3 except $V_c = -0.0005$. As depicted in Fig. 8, the vehicle can also well approach the source.

5. Conclusion

We have studied the nonholonomic source seeking problem in a plane. In our control scheme, both forward and angular velocities are tuned according to the stochastic extremum seeking method (Liu & Krstic, 2010a). As a result, the vehicle well approaches the source with a partly random trajectory. We adopted the stochastic averaging theory for nonlinear continuous-time systems to prove the local stability for static signal fields with elliptical level sets. We have established the local exponential convergence, both almost surely and in probability, to attractors in an annulus around the source. Under a small bias forward velocity the vehicle may virtually “stop” at the source without sacrificing the convergence rate.

References

Azuma, Shun-ichi, Sakar, Mahmut Selman, & Pappas, George J. (2012). Stochastic source seeking by mobile robots. *IEEE Transactions on Automatic Control*, 57(9), 2308–2321.

Berg, Howard C. (2008). *E. coli in motion*. Springer Science & Business Media.

Cochran, Jennie, & Krstic, Miroslav (2009). Nonholonomic source seeking with tuning of angular velocity. *IEEE Transactions on Automatic Control*, 54(4), 717–731.

Durr, Hans-Bernd, Krstic, Miroslav, Scheinker, Alexander, & Ebenbauer, Christian (2017). Extremum seeking for dynamic maps using Lie brackets and singular perturbations. *Automatica*, 83, 91–99.

Ghods, Nima, & Krstic, Miroslav (2010). Speed regulation in steering-based source seeking. *Automatica*, 46(2), 452–459.

Jakuba, Michael V. (2007). Stochastic mapping for chemical plume source localization with application to autonomous hydrothermal vent discovery. Massachusetts Institute of Technology and Woods Hole Oceanographic Institution.

Khong, Sei Zhen, Tan, Ying, Manzie, Chris, & Nešić, Dragan (2014). Multi-agent source seeking via discrete-time extremum seeking control. *Automatica*, 50(9), 2312–2320.

Krstić, Miroslav, & Wang, Hsin-Hsien (2000). Stability of extremum seeking feedback for general nonlinear dynamic systems. *Automatica*, 36(4), 595–601.

Li, Wei, Farrell, Jay, Pang, Shuo, Arrieta, Richard M., et al. (2006). Moth-inspired chemical plume tracing on an autonomous underwater vehicle. *IEEE Transactions on Robotics*, 22(2), 292–307.
Lin, Jinbiao, Song, Shiji, You, Keyou, & Krstic, Miroslav (2016). Stochastic source seeking with forward and angular velocity regulation. https://arxiv.org/abs/1611.04762.
Lin, Jinbiao, Song, Shiji, You, & Wu, Cheng (2016). 3-D velocity regulation for nonholonomic source seeking without position measurement. IEEE Transactions on Control Systems Technology, 24(2), 711–718.

Shiji Song was born in 1965. He received the B.S. in Mathematics from Harbin Normal University, China in 1986, and M.S. and Ph.D. degrees in Mathematics from Harbin Institute of Technology, China in 1989 and 1996, respectively. Since 2000, he has been with the Department of Automation, Tsinghua University, China, where he is currently a Professor. His research and teaching interests include system identification, fuzzy systems, stochastic neural network, and visual information system for ocean mineral resource and its application.

Liu, Shu-Jun, & Krstic, Miroslav (2010a). Stochastic averaging in continuous time and its applications to extremum seeking. IEEE Transactions on Automatic Control, 55(30), 2235–2250.

Liu, Shu-Jun, & Krstic, Miroslav (2010b). Stochastic source seeking for nonholonomic unicycle. Automatica, 46(9), 1443–1453.

Manzie, Chris, & Krstic, Miroslav (2009). Extremum seeking with stochastic perturbations. IEEE Transactions on Automatic Control, 54(3), 580–585.

Matveev, Alexey S., Teimoori, Hamid, & Savkin, Andrey V. (2011). Navigation of a unicycle-like mobile robot for environmental extremum seeking. Automatica, 47(1), 85–91.

Mesquita, Alexandre R., Hespanha, João P., & Åström, Karl (2008). Optimotaxis: A stochastic multi-agent optimization procedure with point measurements. In International workshop on hybrid systems: Computation and control (pp. 358–371). Springer.

Ogren, Petter, Fiorelli, Edward, & Leonard, Naomi Ehrich (2004). Cooperative control of mobile sensor networks: adaptive gradient climbing in a distributed environment. IEEE Transactions on Automatic Control, 49(8), 1292–1302.
Pang, Shuo, & Farrell, Jay A. (2006). Chemical plume source localization. IEEE Transactions on Systems, Man and Cybernetics, Part B (Cybernetics), 36(5), 1088–1080.

Porat, Boaz, & Nehorai, Arye (1996). Localizing vapor-emitting sources by moving sensors. IEEE Transactions on Signal Processing, 44(4), 1018–1021.

Russell, B. & Andrew (2004). Robotic location of underground chemical sources. Robotic, 22(01), 109–115.

Scheinker, Alexander, & Krstic, Miroslav (2014). Extremum seeking with bounded update rates. Systems & Control Letters, 63, 25–31.

Teel, Andrew R., & Popović, Dobrivoje (2001). Solving smooth and nonsmooth multivariable extremum seeking problems by the methods of nonlinear programming. In American control conference, 2001. Proceedings of the 2001, volume 3 (pp. 2394–2399). IEEE.

Zhang, Chunlei, Arnold, Daniel, Ghods, Nima, Siranosian, Antranik, & Krstic, Miroslav (2007). Source seeking with non-holonomic unicycle without position measurement and with tuning of forward velocity. Systems & Control Letters, 56(3), 245–252.

Keyou You received the B.S. degree in Statistical Science from Sun Yat-sen University, Guangzhou, China, in 2007 and the Ph.D. degree in Electrical and Electronic Engineering from Nanyang Technological University (NTU), Singapore, in 2012. After briefly working as a Research Fellow at NTU, he joined Tsinghua University in Beijing, where he is now an Associate Professor in the Department of Automation. He held visiting positions at Politecnico di Torino, Hong Kong University of Science and Technology, University of Melbourne, etc. His current research interests include networked control systems, parallel networked algorithms, and their applications.

Dr. You received the Guan Zhaozhi award at the 29th Chinese Control Conference in 2010, and a CSC-IBM China Faculty Award in 2014. He was selected to the national “1000-Youth Talent Program” of China in 2014.

Miroslav Krstic holds the Alspach endowed chair and is the founding director of the Cymer Center for Control Systems and Dynamics at UC San Diego. He also serves as Associate Vice Chancellor for Research at UCSD. As a graduate student, Krstic won the UC Santa Barbara best dissertation award and student best paper awards at CDC and ACC. Krstic is Fellow of IEEE, IFAC, ASME, SIAM, and IET (UK), Associate Fellow of AIAA, and foreign member of the Academy of Engineering of Serbia. He has received ASME Oldenburger Medal, ASME Nyquist Lecture Prize, ASME Paynter Outstanding Investigator Award, the PECASE, NSF Career, and ONR Young Investigator awards, the Axelby and Schuck paper prizes, the Chestnut textbook prize, and the first UCSD Research Award given to an engineer. Krstic has also been awarded the Springer Visiting Professorship at UC Berkeley, the Distinguished Visiting Fellowship of the Royal Academy of Engineering, the Invitation Fellowship of the Japan Society for the Promotion of Science, and honorary professorships from four universities in China. He serves as Senior Editor in IEEE Transactions on Automatic Control and Automatica, as editor of two Springer book series, and has served as Vice President for Technical Activities of the IEEE Control Systems Society and as chair of the IEEE CSS Fellow Committee. Krstic has coauthored twelve books on adaptive, nonlinear, and stochastic control, extremum seeking, control of PDE systems including turbulent flows, and control of delay systems.