We present a new class of slowly rotating black hole solutions in \((n+1)\)-dimensional \((n \geq 3)\) Einstein-Maxwell-dilaton gravity in the presence of Liouville-type potential for the dilaton field and an arbitrary value of the dilaton coupling constant. Because of the presence of the dilaton field, the asymptotic behaviour of these solutions are neither flat nor (A)dS. In the absence of a dilaton field, our solution reduces to the \((n+1)\)-dimensional Kerr-Newman modification thereof for small rotation parameter \(\varepsilon\). We also compute the angular momentum and the gyromagnetic ratio of these rotating dilaton black holes.

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I. INTRODUCTION

The motivation for studying higher dimensional black holes originates from developments in string/M-theory, which is believed to be the most promising approach to quantum theory of gravity in higher dimensions. It was argued that black holes may play a crucial role in the analysis of dynamics in higher dimensions as well as in the compactification mechanisms. In particular to test novel predictions of string/M-theory, microscopic black holes may serve as good theoretical laboratories \[^{2,3}\]. Another motivation for studying higher dimensional black holes comes from the braneworld scenarios, as a new fundamental scale of quantum gravity. For a while it was thought that the extra spatial dimensions would be of the order of the Planck scale, making a geometric description unreliable, but it has recently been realized that there is a way to make the extra dimensions relatively large and still be unobservable. This is if we live on a three dimensional surface (brane) in a higher dimensional spacetime (bulk) \[^{4,5}\]. In such a scenario, all gravitational objects such as black holes are higher dimensional. One of the most interesting phenomena predicted within this scenario is the possibility of the formation in accelerators of higher dimensional black holes smaller than the size of extra dimensions \[^{6}\].

The pioneering study on higher dimensional black holes was done by Tangherlini several decades ago who found the analogues of Schwarzschild and Reissner-Nordstrom solutions in higher dimensions \[^{7}\]. These solutions are static with spherical topology. The rotating black hole solutions in higher dimensional Einstein gravity was found by Myers and Perry \[^{8}\]. These solutions \[^{8}\] have no electric charge and can be considered as a generalization of the four dimensional Kerr solutions. Besides, it was confirmed by recent investigations that the gravity in higher dimensions exhibits much richer dynamics than in four dimensions. For example, there exists a black ring solution in five dimensions with the horizon topology of \(S^2 \times S^1\) \[^{9}\] which can carry the same mass and angular momentum as the Myers-Perry solution, and consequently the uniqueness theorem fails in five dimensions. While the nonrotating black hole solution to the higher-dimensional Einstein-Maxwell gravity was found several decades ago \[^{7}\], the counterpart of the Kerr-Newman solution in higher dimensions, that is the charged generalization of the Myers-Perry solution in \((n+1)\)-dimensional Einstein-Maxwell gravity, still remains to be found analytically. Indeed, the case of charged rotating black holes in higher dimensions has been discussed in the framework of supergravity theories and string theory \[^{10,11,12,13,14,15}\]. Recently, charged rotating black hole solutions in \((n+1)\)-dimensional Einstein-Maxwell theory with a single rotation parameter in the limit of slow rotation has been constructed in \[^{1}\] (see also \[^{16,17}\]). More recently a class of charged slowly rotating black hole solutions in Gauss-Bonnet gravity has been presented in \[^{18}\].

It is also of great interest to generalize the study to the dilaton gravity. A dilaton is a kind of scalar field which appears in the low-energy effective action of string theory and can be coupled to gravity and gauge fields \[^{19}\]. When a dilaton is coupled to Einstein-Maxwell theory, it has profound consequences for the black hole solutions. This fact may be seen in the case of four dimensional rotating Einstein-Maxwell-dilaton (EMd) black holes which does not possess the gyromagnetic ratio \(g = 2\) of Kerr-Newman black hole \[^{20,21,22,23}\]. Therefore, the study on the rotating solutions of Einstein-Maxwell gravity in the presence of a dilaton field is well motivated. Of particular interest is to investigate the effect of the dilaton field on the physical properties of the solutions. The appearance of the dilaton field changes the asymptotic behavior of the solutions to be neither asymptotically flat nor (A)dS. A motivation to

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investigate non-asymptotically flat, non-asymptotically (A)dS solutions of Einstein gravity is that these might lead to possible extensions of AdS/CFT correspondence. Indeed, it has been speculated that the linear dilaton spacetimes, which arise as near-horizon limits of dilatonic black holes, might exhibit holography [24]. Another motivation is that such solutions may be used to extend the range of validity of methods and tools originally developed for, and tested in the case of, asymptotically flat or asymptotically (A)dS black holes. It has been shown that the counterterm method inspired by the AdS/CFT correspondence can be applied successfully to the computation of the conserved quantities of non-asymptotically (A)dS black holes/branes (see e.g. [25, 26, 27, 28]). While exact static dilaton black hole/string solutions of EMd gravity have been constructed in [24, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38], exact rotating black holes solutions with curved horizons have been obtained only for some limited values of the dilaton coupling constant [21, 39, 40, 41]. For general dilaton coupling constant, the properties of charged rotating dilaton black holes only with infinitesimally small charge [42] or small angular momentum have been investigated [22, 23, 43, 44, 45]. When the horizons are flat, rotating solutions of EMd gravity with Liouville-type potential in four (25) and (n + 1)-dimensions have also been constructed [26]. These solutions ([25, 26]) are not black holes and describe charged rotating dilaton black strings/branes. It is worth noting that these rotating solutions ([25, 26]) are basically obtained by a Lorentz boost from corresponding static ones; they are equivalent to static ones locally, although not equivalent globally. So far, charged rotating dilaton black hole solutions with curved horizons, for an arbitrary value of the dilaton coupling constant in (n + 1)-dimensional EMd theory have not been constructed. In this paper, as a new step to shed some light on this issue for further investigation, we present a class of rotating solutions in (n + 1)-dimensional rotating solutions of the above field equations. For infinitesimal rotation, we can solve Eqs. (3)-(5) to first order in the angular momentum parameter $a$. Inspection of the (n + 1)-dimensional Kerr solutions shows that the only term in the metric changes to $O(a)$ is $g_{t\phi}$. Similarly, the dilaton field does not

II. BASIC EQUATIONS AND SOLUTIONS

The action of (n + 1)-dimensional ($n \geq 3$) Einstein-Maxwell gravity coupled to a dilaton field can be written

$$S = \frac{1}{16\pi} \int d^{n+1}x \sqrt{-g} \left( \mathcal{R} - \frac{4}{n-1}(\nabla\Phi)^2 - V(\Phi) - e^{-4\alpha\Phi/(n-1)}F_{\mu\nu}F^{\mu\nu} \right) - \frac{1}{8\pi} \int_{\partial M} d^n x \sqrt{-\gamma} \Theta(\gamma),$$

(1)

where $\mathcal{R}$ is the Ricci scalar curvature, $\Phi$ is the dilaton field and $V(\Phi)$ is a potential for $\Phi$. $\alpha$ is a constant determining the strength of coupling of the scalar and electromagnetic field, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor and $A_\mu$ is the electromagnetic potential. The last term in Eq. (1) is the Gibbons-Hawking boundary term which is chosen such that the variational principle is well-defined. The manifold $\mathcal{M}$ has metric $g_{\mu\nu}$ and covariant derivative $\nabla_\mu$. $\Theta$ is the trace of the extrinsic curvature $\Theta_{ab}$ of any boundary(ies) $\partial M$ of the manifold $\mathcal{M}$, with induced metric(s) $\gamma_{ab}$. In this paper, we consider the action (1) with a Liouville type potential,

$$V(\Phi) = 2\Lambda e^{2\zeta\Phi},$$

(2)

where $\Lambda$ and $\zeta$ are arbitrary constants. The equations of motion can be obtained by varying the action (1) with respect to the gravitational field $g_{\mu\nu}$, the dilaton field $\Phi$ and the gauge field $A_\mu$ which yields the following field equations

$$\mathcal{R}_{\mu\nu} = \frac{4}{n-1} \left( \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{4} g_{\mu\nu} V(\Phi) + 2e^{-4\alpha\Phi/(n-1)} F_{\mu\eta} F^{\rho\eta} - \frac{1}{2(n-1)} g_{\mu\nu} F_{\lambda\eta} F^{\lambda\eta} \right),$$

(3)

$$\nabla^2 \Phi = \frac{n-1}{8} \frac{\partial V}{\partial \Phi} - \frac{\alpha}{2} e^{-4\alpha\Phi/(n-1)} F^{\lambda\eta} F_{\lambda\eta},$$

(4)

$$\nabla_\mu \left( e^{-4\alpha\Phi/(n-1)} F^{\mu\nu} \right) = 0.$$  

(5)

We would like to find (n + 1)-dimensional rotating solutions of the above field equations. For infinitesimal rotation, we can solve Eqs. (3)-(5) to first order in the angular momentum parameter $a$. Inspection of the (n + 1)-dimensional Kerr solutions shows that the only term in the metric changes to $O(a)$ is $g_{t\phi}$. Similarly, the dilaton field does not
change to \(O(a)\) and \(A_\phi\) is the only component of the vector potential that changes. Therefore, for infinitesimal angular momentum up to \(O(a)\), we can take the following form of the metric

\[
ds^2 = -U(r)dt^2 + \frac{dr^2}{U(r)} - 2af(r)\sin^2 \theta dt d\phi + r^2 R^2(r) \left( d\theta^2 + \sin^2 \theta d\phi^2 + \cos^2 \theta d\Omega^2_{n-3} \right),
\]

where \(a\) is a parameter associated with its angular momentum, and \(d\Omega^2_{n-3}\) denotes the metric of an unit \((n-3)\) sphere. The functions \(U(r)\), \(R(r)\) and \(f(r)\) should be determined. In the particular case \(a = 0\), this metric reduces to the static and spherically symmetric cases. For small \(a\), we can expect to have solutions with \(U(r)\) still a function of \(r\) alone. The \(t\) component of the Maxwell equations can be integrated immediately to give

\[
F_{tr} = \frac{q e^{4\alpha \Phi/(n-1)}}{(r R)^{n-1}},
\]

where \(q\) is an integration constant related to the electric charge of the solutions. Defining the electric charge via

\[
Q = \frac{q \omega_{n-1}}{4\pi},
\]

where \(\omega_{n-1}\) represents the area of the unit \((n-1)\)-sphere. In general, in the presence of rotation, there is also a vector potential in the form

\[
A_\phi = a q h(r) \sin^2 \theta.
\]

Notice that for infinitesimal rotation parameter, the electric field \(17\) does not change from the static case. In order to solve the system of equations \([3]-[5]\) for four unknown functions \(f(r), R(r), \Phi(r)\) and \(h(r)\), we take the following ansatz

\[
R(r) = e^{2\alpha \Phi/(n-1)}.
\]

Inserting \([10]\), the Maxwell fields \([17]\) and \([19]\), and the metric \([20]\), into the field equations \([3]-[5]\), one can show that these equations have the following solutions

\[
U(r) = -(n-2) (\alpha^2 + 1)^2 b^{-2\gamma} r^{2\gamma} - \frac{m}{r^{(n-1)(1-\gamma)-1}} + \frac{2q^2 (\alpha^2 + 1)^2 b^{-2(n-2)\gamma} r^{2(n-2)(\gamma-1)}},
\]

\[
f(r) = \frac{m (\alpha^2 + n - 2) b^{(n-3)\gamma}}{\alpha^2 + 1} r^{(n-1)(\gamma-1)+1} - \frac{2q^2 (\alpha^2 + 1) b^{(1-n)\gamma}}{n-1} r^{2(n-2)(\gamma-1)},
\]

\[
\Phi(r) = \frac{(n-1) \alpha}{2(\alpha^2 + 1)} \ln \left( \frac{b}{r} \right),
\]

\[
h(r) = r^{(n-3)(\gamma-1)-1}.
\]

Here \(b\) is an arbitrary constant and \(\gamma = \alpha^2/(\alpha^2 + 1)\). In the above expressions, \(m\) appears as an integration constant and is related to the ADM (Arnowitt-Deser-Misner) mass of the black hole. According to the definition of mass due to Abbott and Deser \([16]\), the mass of the solution is \([38]\)

\[
M = \frac{b^{(n-1)\gamma} (n-1) \omega_{n-1}}{16\pi (\alpha^2 + 1)} m.
\]

For \((\alpha = 0 = \gamma)\) the mass of the black hole reduces to

\[
M = \frac{(n-1) \omega_{n-1}}{16\pi} m.
\]

In order to fully satisfy the system of equations, we must have

\[
\zeta = \frac{2}{\alpha(n-1)}, \quad \Lambda = \frac{(n-1)(n-2)\alpha^2}{2b^2(\alpha^2 - 1)}.
\]
One may also note that in the absence of a non-trivial dilaton ($\alpha = \gamma = 0$), the above solutions reduce to

\[ U(r) = 1 - \frac{m}{r^{\alpha-2}} + \frac{2q^2}{(n-1)(n-2)r^{2(n-2)}}, \]

where $\alpha$ is the mass parameter and $m > 0$.

\[ f(r) = \frac{m(n-2)}{r^{\alpha-2}} - \frac{2q^2}{(n-1)r^{2(n-2)}}, \]

\[ h(r) = r^{2-n}, \]

which describe an $(n+1)$-dimensional Kerr-Newman black holes in the limit of slow rotation \cite{1}. The metric corresponding to \cite{11,13} is neither asymptotically flat nor (anti)-de Sitter. In order to study the physical properties of these solutions, we first look for the curvature singularities. In the presence of a dilaton field, the Kretschmann scalar $R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa}$ diverges at $r = 0$, it is finite for $r \neq 0$ and goes to zero as $r \to \infty$. Thus, there is an essential singularity located at $r = 0$. As one can see from \cite{11}, the solution is ill-defined for the string case where $\alpha = 1$.

The cases with $\alpha > 1$ and $\alpha < 1$ should be considered separately. For $\alpha > 1$, we have cosmological horizons, while there is no cosmological horizons if $\alpha < 1$ (see fig. 1). In fact, in the latter case ($\alpha < 1$) the spacetimes exhibit a variety of possible causal structures depending on the values of the metric parameters (see figs. 2,3). One can obtain the causal structure by finding the roots of $f(r) = 0$. Unfortunately, because of the nature of the exponent in \cite{11}, it is not possible to find analytically the location of the horizons. To get better understanding on the nature of the horizons, we plot in figures 4 and 5 the mass parameter $m$ as a function of the horizon radius for different values of dilaton coupling constant $\alpha$. It is easy to show that the mass parameter $m$ of the dilaton black hole can be expressed in terms of the horizon radius $r_h$ as

\[ m(r_h) = \frac{(n-2)(\alpha^2 + 2b-2\gamma)(3-n)}{(\alpha^2 - 1)(n + \alpha^2 - 2)} r_h^{n-2+\gamma(3-n)} + \frac{2q^2(\alpha^2 + 1)^2 b^{-2\gamma(n-2)}}{(n-1)(n + \alpha^2 - 2)} \frac{1}{r_h^{(n-3)(\gamma-1)-1}}. \]

These figures show that for a given value of $\alpha$ or $q$, the number of horizons depend on the choice of the value of the mass parameter $m$. We see that, up to a certain value of the mass parameter $m$, there are two horizons, and as we decrease the $m$ further, the two horizons meet. In this case we get an extremal black hole with mass $m_{\text{ext}}$. The entropy of the black hole typically satisfies the so called area law of the entropy which states that the entropy of the black hole is a quarter of the event horizon area \cite{47}. This near universal law applies to almost all kinds of black holes, including dilaton black holes, in Einstein gravity \cite{15}. Since the surface gravity and the area of the event horizon do not change up to the linear order of the rotating parameter $a$, we can easily show that the Hawking temperature and the entropy of dilaton black hole on the outer event horizon $r_+$ can be written as

\[ T_+ = \frac{\kappa}{2\pi} = \frac{f'(r_+)}{4\pi}, \quad S = \frac{A}{4}, \]

where $\kappa$ is the surface gravity and $A$ is the horizon area. Then, one can get

\[ T_+ = -\frac{(\alpha^2 + n - 2)m}{4\pi (\alpha^2 + 1)} r_+^{(n-1)(\gamma-1)} + \frac{b^{-2\gamma(n-2)}(\alpha^2 + 1)}{2\pi (1 - \alpha^2)} r_+^{2\gamma-1}, \]

Figure 1: The function $f(r)$ versus $r$ for $q = 1$, $b = 1$, $m = 2$ and $n = 4$. $\alpha = 0.5$ (bold line), $\alpha = 1.3$ (continuous line).
Equation (22) shows that for $\alpha > 1$ the temperature is negative. As we argued above in this case we encounter cosmological horizons, and therefore the cosmological horizons have negative temperature. Numerical calculations show that the temperature of the event horizon goes to zero as the black hole approaches the extreme case. It is a matter of calculation to show that the mass parameter of the extremal black hole can be written $(\alpha < 1)$

$$m_{\text{ext}} = \frac{2(n-2)(\alpha^2 + 1)^2 b^{-2\gamma}}{(1-\alpha^2)(\alpha^2 + n - 2)} r_+^{(2-n)(\gamma-1)+\gamma},$$

(24)

In summary, the metric of Eqs. (6) and (11)-(13) can represent a black hole with inner and outer event horizons located at $r_-$ and $r_+$, provided $m > m_{\text{ext}}$, an extreme black hole in the case of $m = m_{\text{ext}}$, and a naked singularity if $m < m_{\text{ext}}$. It is worth noting that in the absence of a non-trivial dilaton field ($\alpha = \gamma = 0$), expressions (22) and (24) reduce to that of an $(n+1)$-dimensional asymptotically flat black holes.

Finally, we calculate the angular momentum and the gyromagnetic ratio of these rotating dilaton black holes which appear in the limit of slow rotation parameter. The angular momentum of the dilaton black hole can be calculated through the use of the quasi-local formalism of the Brown and York [49]. According to the quasilocal formalism, the quantities can be constructed from the information that exists on the boundary of a gravitating system alone. Such quasilocal quantities will represent information about the spacetime contained within the system boundary, just like the Gauss’s law. In our case the finite stress-energy tensor can be written as

$$T^{ab} = \frac{1}{8\pi} (\Theta^{ab} - \Theta_{\gamma}^{ab}),$$

(25)
which is obtained by variation of the action $I$ with respect to the boundary metric $\gamma_{ab}$. To compute the angular momentum of the spacetime, one should choose a spacelike surface $B$ in $\partial M$ with metric $\sigma_{ij}$ and write the boundary metric in ADM form

$$\gamma_{ab}dx^adx^a = -N^2dt^2 + \sigma_{ij}(d\varphi^i + V^i dt)(d\varphi^j + V^j dt),$$

where the coordinates $\varphi^i$ are the angular variables parameterizing the hypersurface of constant $r$ around the origin, and $N$ and $V^i$ are the lapse and shift functions respectively. When there is a Killing vector field $\xi$ on the boundary, then the quasilocal conserved quantities associated with the stress tensors of Eq. (25) can be written as

$$Q(\xi) = \int_B d^{n-1}\varphi \sqrt{\sigma} n^a \xi^b,$$

(26)

where $\sigma$ is the determinant of the metric $\sigma_{ij}$, $\xi$ and $n^a$ are the Killing vector field and the unit normal vector on the boundary $B$. For boundaries with rotational ($\zeta = \partial/\partial \varphi$) Killing vector field, one obtains the quasilocal angular momentum

$$J = \int_B d^{n-1}\varphi \sqrt{\sigma} n^a \xi^b,$$

(27)

provided the surface $B$ contains the orbits of $\zeta$. Finally, the angular momentum of the black holes can be calculated through the use of Eq. (27). We find

$$J = \frac{(n - \alpha^2)(\alpha^2 + n - 2)b^{2(n-2)}/\omega_{n-1}}{8\pi n(n - 2)(\alpha^2 + 1)^2} ma.$$

(28)
For $a = 0$, the angular momentum vanishes, and therefore $a$ is the rotational parameter of the dilaton black hole. For $(\alpha = \gamma = 0)$, the angular momentum reduces to the angular momentum of the $(n+1)$-dimensional Kerr black holes

$$J = \frac{ma\omega_{n-1}}{8\pi}. \quad (29)$$

Combining Eq. (15) with Eq. (29) we get

$$J = \frac{2Ma}{n-1}. \quad (30)$$

Next we calculate the gyromagnetic ratio of this rotating dilaton black holes. The magnetic dipole moment for this slowly rotating dilaton black hole is

$$\mu = Qa. \quad (31)$$

Therefore, the gyromagnetic ratio is given by

$$g = \frac{2\mu M}{QJ} = \frac{n(n-1)(n-2)(\alpha^2+1)}{(n-\alpha^2)(\alpha^2+n-2)b^{(n-3)\gamma}}. \quad (32)$$

It was argued in \cite{22,23} that the dilaton field can modify the gyromagnetic ratio of the asymptotically flat and asymptotically (A)dS four dimensional black holes. Our result here confirm their arguments. However, in contrast to the gyromagnetic ratio of the asymptotically flat or (A)dS four dimensional dilatonic black holes which is turned out to be $g \leq 2$, in our case in which the solutions are neither asymptotically flat nor (A)dS, we get $g \geq 2$ in four dimension. We have shown the behaviour of the gyromagnetic ratio $g$ of the dilatonic black hole ($\alpha < 1$) versus $\alpha$ in figures 6 and 7. From these figures we find out that for small $b$ the gyromagnetic ratio increases with increasing $\alpha$, while for large $b$ and $n \geq 4$ the gyromagnetic ratio decreases with increasing $\alpha$. In the absence of a non-trivial dilaton ($\alpha = \gamma = 0$), the gyromagnetic ratio reduces to

$$g = n - 1, \quad (33)$$

which is the gyromagnetic ratio of the $(n+1)$-dimensional Kerr-Newman black holes (see e.g. \cite{1}).

III. CONCLUSION

Though the nonrotating black hole solution to the higher-dimensional Einstein-Maxwell gravity was found several decades ago \cite{7}, the counterpart of the Kerr-Newman solution in higher dimensions, that is the charged generalization of the Myers-Perry solution \cite{8} in $(n+1)$-dimensional Einstein-Maxwell gravity, still remains to be found analytically. Recently, charged rotating black hole solutions in $(n+1)$-dimensional Einstein-Maxwell theory has been constructed with a single rotation parameter in the limit of slow rotation parameter \cite{1}. In this letter, as a next step to shed some light on this issue for further investigation, we further generalized these slowly rotating black hole solutions
(11) by including a dilaton field and a Liouville-type potential for the dilaton field in the action. These solutions describe charged rotating dilaton black holes with an arbitrary dilaton coupling constant in the limit of slow rotation parameter. In contrast to the rotating black holes in the Einstein-Maxwell theory, which are asymptotically flat, the charged rotating dilaton black holes we found here, are neither asymptotically flat nor (A)dS. Our strategy for constructing these solutions was the perturbative technique suggested in [22]. We first studied charged black hole solutions in \((n+1)\)-dimensional Einstein-Maxwell-dilaton gravity. Then, we considered the effect of adding a small amount of rotation parameter \(a\) to the black hole. We discarded any terms involving \(a^2\) or higher power in \(a\). Inspection of the Kerr-Newman solutions shows that the only term in the metric changes to \(O(a)\) is \(g_{t\phi}\). Similarly, the dilaton does not change to \(O(a)\) and \(A_\phi\) is the only component of the vector potential that change to \(O(a)\). We showed that in the absence of dilaton field (\(\alpha = 0 = \gamma\)), our solutions reduce to the \((n+1)\)-dimensional Kerr-Newman modification thereof for small rotation parameter \(a\). We computed temperature and entropy of black holes, which did not change to \(O(a)\) from the static case. We also obtained the angular momentum and the gyromagnetic ratio of these rotating dilaton black holes. Interestingly enough, we found that the dilaton field can modify the gyromagnetic ratio of the \((n+1)\)-dimensional rotating dilaton black holes. This is in agreement with the arguments in [22, 23].

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