Determination of Neutron Flux Viewed from the Maxwell-Boltzmann’s Statistical Distribution Using Numerical Approach

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Abstract. This article aims to investigate the computational distribution of neutron flux using Maxwell-Boltzmann statistics. The reactor geometry used is a cylindrical shape. Identify the flux neutron only can when a neutron collides. Data analysis uses numerical analysis with the MATLAB 2013a program. The graph generated from a cylinder geometry is a histogram graph with the Poisson distribution by the Maxwell-Boltzmann distribution. There is a difference in the value of the neutron flux due to statistical uncertainty using a numerical approach. The purpose of writing this article is to determine the computational distribution of neutron fluxes in a cylindrical geometry reactor using Maxwell Boltzmann statistics. By writing this article it is hoped that it will become a reference material for further research or studies on computing the distribution of neutrons in other cases

Keywords: Maxwell-Boltzmann, Numerical, Statistics.

1. Introduction

Statistical mechanics deals with macroscopic systems. Macroscopic systems consist of many particles. The number of particles will move freely, so it is difficult to identify the physical quantities of the macroscopic system. But, it can be solved based on probability. Probability has a random principle. Moving particles have the same change that they are at any point in the system and are moving in any direction with momentum following the rules. One of the moving particles is a neutron. There has been much discussion about the Monte Carlo method in the nuclear field. The Monte Carlo N-Particle Code (MCNP) with ENDF-VII cross section data was used to simulate the thermal neutron flux distribution in the core. However, few discussed numerical or computational analysis in the field of nuclear radiation. Therefore, computational studies as the basis of statistical mechanics [1].

In statistical mechanics, there are many high-level mathematical formulations, so it is necessary to have a deeper understanding of mathematics before understanding statistical mechanics. Statistical mechanics discuss macroscopic systems consisting of very many particles that make up the system, so it is difficult to determine the exact physical quantities one by one particle [2]. In statistical mechanics itself, there are three types of distribution, namely Maxwell Boltzmann, Bose-Einstein, and Fermi Dirac. One characteristic of Maxwell Boltzmann's statistical distribution is distinguished while the Bose-Einstein and Fermi Dirac distributions use identical or indistinguishable particles [3].

Maxwell and Boltzmann assumed a normal distribution for the velocity of the atoms in the gas. As for nanoparticles like neutrons so far, their energy distribution and, consequently, the temperature distribution follows a normal distribution [4]. Neutrons as differentiated particles can be identified using...
the Maxwell Boltzmann statistical distribution. The neutron source is an accelerator or research reactor [5]. Nuclear reactors usually use fissile materials and fertilizers as fuel. The fissile material is Uranium, while the other of fertile material is Thorium [6].

The gap analysis is to find the flux using the Maxwell Boltzmann distribution with numerical methods. The purpose of writing this article is to determine the computational distribution of neutron fluxes in a cylindrical geometry reactor using Maxwell Boltzmann statistics. By writing this article it is hoped that it will become a reference material for further research or studies on computing the distribution of neutrons in other cases.

The reason for using the Maxwell Boltzmann distribution in this paper is because the neutron flux has a velocity so that the probability can be formulated by the Maxwell Boltzmann distribution. The Maxwell Boltzmann distribution also can form the kinetic theory of gases, so it is very suitable to be used to analyze the neutron flux velocity.

2. Neutron Flux Distribution

Based on previous research conducted by Purwoningsih [7], geometric modeling in the distribution of cylindrical neutron fluxes with output in the form of neutron distribution patterns at different depths. The resulting graph in the neutron flux distribution pattern follows the Poisson distribution pattern using Maxwell-Boltzmann statistics. The most basic neutron flux of only four processes at the atomic nucleus level in its evolution, such as 1) spontaneous neutron emission from unstable nuclei, 2) neutron scattering, 3) neutron-induced fission, and 4) neutron capture [8].

The technique of data collection used includes the following steps:

1. Determination of variables
2. Determination of the initial state of the variable
3. Determination of the existence or first position of a neutron
4. Selecting particles into cells
5. Calculation of physical thermodynamic quantities when neutrons collide. Particle collision in statistical mechanics is a link between microscopic behavior and macroscopic phenomena [9].

The data analysis stage uses numerical or computational analysis with the MATLAB 2013a program that includes coding equations used in the distribution of neutron fluxes. The neutron flux determined by using the relation of the number of neutrons (n) to the distribution of velocity (v) in equations:

\[ \Phi = n \cdot v \]  

(1)

2.1. Kinetic Energy of Particles

In statistical mechanics, particles do the Brownian motion or random motion. If a system has each mass, namely \( m_1, m_2, m_3 \) up to \( m_N \) while the velocity \( v_1, v_2, v_3 \) to \( v_N \), then the average energy of particles can calculate using their kinetic energy. The particle kinetic energy equation is as follows:

\[ E_k = \frac{1}{N} \left( \sum_{i=1}^{N} m_i v_i^2 \right) \]  

(2)

If the mass is the same, then:

\[ E_k = \frac{1}{2} m \left( \frac{1}{N} \sum_{i=1}^{N} v_i^2 \right) \]

\[ E_k = \frac{1}{2} m \overline{v_{rms}^2} \]  

(3)

\( \overline{v_{rms}^2} \) is the average square velocity described by:

\[ \overline{v_{rms}^2} = \frac{1}{N} \left( \sum_{i=1}^{N} v_i^2 \right) \]

\[ \overline{v_{rms}^2} = \frac{1}{N} (v_1^2 + v_2^2 + \cdots + v_N^2) \]  

(4)
Because the average kinetic energy of a particle is \( E_k = \frac{1}{2} m v_{rms}^2 \), so the relation of the average energy of a gas particle with absolute temperature it is written:
\[
\bar{E} = \frac{3}{2} k_B T
\]  
(5)

Where \( k_B \) is the Boltzmann constant and \( T \) is an absolute temperature. Thus to determine the average kinetic energy by entering equation (5) into equation (3) to obtain:
\[
v_{rms} = \sqrt{\frac{3k_B T}{m}}
\]  
(6)

2.2. Maxwell-Boltzmann Distribution

The shape of the speed distribution follows with the Maxwell-Boltzmann distribution. In this distribution, physical quantities determine with the most probability of a particle. The general form of the Maxwell Boltzmann distribution function :
\[
f(E) = Ae^{-E/k_BT}
\]  
(7)

The magnitude of the Boltzmann constant which is \( 1.38 \times 10^{-23} \) J/K. In the Maxwell-Boltzmann distribution the magnitude of the velocity of the greatest probability can be determined by :
\[
v_p = \sqrt{\frac{2k_B T}{m}}
\]  
(8)

The calculation of the distribution of neutron fluxes can also be by mixing diffusion equations. The reactor core geometry reviewed is cylindrical and will be shown in Figure 1.

![Figure 1. Geometry of a Cylindrical [10].](image)

Neutrons produce from \((\alpha, n)\) reaction. These neutrons are heated quickly through elastic scattering so that their speed can be determined using the Maxwell Boltzmann distribution [11]. In most reactors using thermal neutrons to support chain fission reactions. This thermal reactor contains a neutron moderator that can slow the speed of the neutrons towards the thermal equilibrium in the system \((E < 1 \text{ eV})\).

The neutron flux will consider the overall length of the path taken by the neutron every one cubic centimeter for one second. Mathematically, the neutron flux equation as in equation (1) [1].
Research conducted by Silva, Martinez, and Goncalves [12] has reconstructed the neutron flux in a slab reactor by verifying the analytical solution using numerical methods with particles of the same mass.

Supported by Fayyadh, et al [13] explains that by using numerical analysis it is easier to determine the distribution of a neutron. The results of neutron flux measurements conducted by Rabir [1] show that the thermal neutron flux peaks at the center of the nucleus and decreases gradually towards the outer side.

The multigroup neutron diffusion equation is:

\[
-\vec{\nabla}.D_g(\vec{r})\vec{\nabla}\varphi_g(\vec{r}) + \sum_{r,g} (\vec{r})\varphi_g(\vec{r}) = \frac{X_g}{k_{eff}}\sum_{g'=1}^{G} v \sum_{f,g} (\vec{r})\varphi_{g'}(\vec{r}) + \sum_{g'=1}^{G} \sum_{s,g\rightarrow g} (\vec{r})\varphi_{g'}(\vec{r})
\]  

(9)

where the indices are below \( g \) and \( g' \) are neutron group number; \( X_g \) is the chance that fission neutrons have energy in the group \( g \) region; \( D \) is the diffusion constant; \( \varphi \) is a neutron flux; \( \sum_r \) and \( \sum_f \) each is a cross-section of macroscopic removal and macroscopic fission; \( k_{eff} \) is an effective multiplication factor; \( \sum_{s,g\rightarrow g} \) is probability of neutrons from group \( g' \) scattered to group \( g \) [10].

The magnitude of the neutron flux depends on the cross-section of the removal and the scattering. The greater the cross-sectional removal value so make smaller the scale flux in the region. That is because cross-sectional removal is a chance that neutrons transfer to the energy group \( g \) due to collisions. That way, the higher the cross-sectional makes the neutrons move and vice versa. However, the appearance of scattering shows the number of neutrons scattered without being absorbed. The higher the value of the scattering latitude, the lower the scale flux and vice versa [14].

3. Analysis of Neutron Flux Distribution

In using the numerical analysis of the Jacobi method, iterative calculations are used:

\[
\varphi_I^{m+1} = -\sum_{j=1}^{n} \frac{(a_{ij})}{(a_{ii})} \varphi_j^m + \frac{(s_i)}{(a_{ii})} \leq i \leq n, \ m \geq 0
\]

(10)

Equation (10) is called the total-step iterative method [15]. The graphs generated from the MATLAB computational program on cylindrical geometry form the shape of a histogram. Equation (10) is called the total-step iterative method [15]. The graphs generated from the MATLAB computational program on cylindrical geometry form the shape of a histogram.

3.1. Analytical Solution of Neutron Flux Distribution

The neutron flux distribution pattern observed by Purwoningsih [6] shows the distribution of Poisson because it does not follow a normal model distribution (Gaussian).
From Figure 2 can be seen that the pattern of the neutron flux distribution curve is the resulting pattern of a Poisson distribution. It can be viewed when in bin 3, the curve will increase sharply and decrease when it is in bin 6 to zero. Thus, the magnitude of the neutron flux will never reach zero because the kinetic energy of the neutrons will not be zero due to the speed of the nucleus. While of the analytics approach, the thermal neutron flux graph produced is:

The graph to verify an analytical solution. This solution can reproduce the results of the numerical method, which is the finite reference method. Some simplified cases present to compare analytical results with numerical ones. It shows that when it is possible to obtain an analytical solution for a simplified geometry, the results obtained by the analytical solution are close to the numerical results. The analytical solution needs to reconstruct the flux
point-by-point that comes from the numerical method is the eigen-value $k_{\text{eff}}$. That is an information intrinsic of the reactor due to its geometry and nuclear parameters and the initial flux $\varphi_{g0}$ that is obtained by the numerical solution [12].

This study has research limitations, such as this research cannot study the neutron flux in terms of Maxwell Boltzman's expectations so that it can be developed with other theories. The results of this study are expected to help and become a reference to carry out further research in the future. However, this research has several implications for improvement in academics and practitioners. For the academic community, this research contributes to expanding the knowledge of statistical mechanics concerning the study of the neutron flux. For the practitioners, it is possible to develop findings regarding the distribution of the neutron flux that are not limited to one design but can be related to another.

**Conclusion**

The distribution of the neutron flux can be determined using computation with numerical analysis. The numerical analysis uses the MATLAB program by calculating the iterative Jacobi method. In a cylindrical geometric reactor, the neutron flux distribution produces a Poisson distribution histogram curve pattern according to Maxwell-Boltzmann Statistics. The neutron flux differences are due to statistical uncertainty in the numerical approximation. Therefore, further research is needed to minimize the difference in neutron flux using a numerical approach. Also, it is necessary to vary the geometric shape of the cylinder side reactor.

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