Top quark electromagnetic dipole moments

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Abstract. The magnetic and electric dipole moments of the top quark are constrained indirectly by the $\text{Br}(B \to X_s\gamma)$ and the $A_{CP}(B \to X_s\gamma)$ measurements. They can also be tested by top quark production and decay processes. The recent measurement of $t \bar{t}\gamma$ production by CDF are used to set direct constraints. The $B \to X_s\gamma$ measurements by themselves define an allowed parameter region that sets up stringent constraints on both dipole moments. The measurement by CDF has a $\sim 37\%$ error that is too large to set any competitive bounds, for which a much lower $5\%$ error would be required. For the LHC it is found that with its higher energy the same measurement could indeed further constrain the allowed parameter region given by the $B \to X_s\gamma$ measurement [1]. In addition, the proposed LHeC experiment (electron-proton) could provide even more stringent constraints than the LHC via the $t \bar{t}$ photoproduction channel [2].

1. Introduction
Currently, the anomalous magnetic and electric dipole moments (MDM and EDM) of the top quark get the most stringent constraints from the $\text{Br}(B \to X_s\gamma)$. We make an evaluation of these constraints, where in addition to the $\text{Br}(B \to X_s\gamma)$ we consider a CP asymmetry for this process that indeed sets the strongest bounds on the EDM of the top quark. Our bounds are more stringent than reported previously [3]. The CDF collaboration has reported a measurement of $t \bar{t}\gamma$ production with $6 fb^{-1}$ of data [4]. (Some preliminary study has also been done for the LHC [5].) This process has been considered as a probe of the dipole moments of the top quark by Baur et. al [6]. Their overall conclusion was that at the energy of the Tevatron it would not be possible to set bounds as stringent as those from the $B \to X_s\gamma$ measurements. However, at the energy of the LHC the bounds would become as stringent. The reason for this is that since the dipole coupling is proportional to the momentum of the photon there is more relative contribution (compared to the QED coupling) as the energy of the collider increases. In this work, we take the experimental result by [4] and make an estimate of the bounds on the MDM and EDM, where indeed we corroborate that $t \bar{t}\gamma$ at the Tevatron is far from competing with $B \to X_s\gamma$. But on the other hand, we also find that the LHC could in principle set significant direct bounds that would further improve what we already have from the indirect bounds from $B \to X_s\gamma$. 
Figure 1. Feynman diagrams for $tt\gamma$ production at the Tevatron. In diagrams (a) the photon is radiated along with the on-shell top quarks, in diagrams (b) the photon is radiated by the decay products of one of the top quarks.

2. The MDM and EDM of the Top quark.

Following [7], we define the effective $t\bar{t}\gamma$ Lagrangian

$$\mathcal{L}_{t\bar{t}\gamma} = e\bar{t} \left( Q_t \gamma_{\mu} A^{\mu} + \frac{1}{4m_t} \sigma_{\mu\nu} F^{\mu\nu} (\kappa + i\bar{\kappa}\gamma_5) \right) t, \quad (1)$$

where the CP even $\kappa$ and CP odd $\bar{\kappa}$ terms are related to the anomalous MDM and EDM of the top quark, respectively. This Lagrangian is also defined in Refs. [6] and [3]; comparing with their effective Lagrangian (notice a relative minus sign in the charge term) we obtain the following relations:

$$\kappa = -F_{2V}^3 = \frac{2m_t}{e} \mu_t = Q_ta_t, \quad \bar{\kappa} = F_{2A}^3 = \frac{2m_t}{e} d_t. \quad (2)$$

Where $a_t = (g_t - 2)/2$ is the anomalous MDM in terms of the gyromagnetic factor $g_t$. The factors $F_{2V}^3$ and $F_{2A}^3$ are used in [6]. The factors $\mu_t$ and $d_t$ are used in [3]. The SM prediction for $a_t$ is $a_t^{SM} = 0.02$ [8], which translates to $\kappa^{SM} = 0.013$. The bounds for $\kappa$ that we will obtain will be about two orders of magnitude greater, therefore the SM prediction will not be considered in our calculations. On the other hand, the CP violating EDM factor $d_t$ is strongly suppressed in the SM: $d_t^{SM} < 10^{-30} e\text{ cm}$ ($\bar{\kappa} < 1.75 \times 10^{-14}$) [9]. The EDM is thus a very good probe of new physics. There are models with vector like multiplets that predict values as high as $10^{-19} e\text{ cm}$ ($\bar{\kappa} < 1.75 \times 10^{-3}$) [10].

3. Limits from $t\bar{t}\gamma$ at the Tevatron

The CDF collaboration has reported a cross–section measurement of top–quark pair production with an additional photon that carries at least 10 GeV of transverse energy, $\sigma_{t\bar{t}\gamma} = 0.18 \pm 0.08$
Figure 2. The allowed parameter space for the anomalous magnetic and electric dipole moments of the top quark. Gray solid lines: region allowed by the experimental results for the branching ratio for \( B \rightarrow X_s \gamma \). Gray dashed lines: region allowed by the experimental results for the CP asymmetry for \( B \rightarrow X_s \gamma \). Black solid line: region allowed by the CDF measurement of \( \hat{R} \) for \( t\bar{t}\gamma \) production at \( \sqrt{s} = 2 \) TeV. Black dashed lines: regions allowed by the values of \( \hat{R} \) indicated in the figure.

in which systematic uncertainties are eliminated. The experimental result (3) is in excellent agreement with the SM prediction \( R_{SM} = 0.024 \pm 0.005 \) [4]. Since the main result by CDF is given in terms of \( \sigma_{t\bar{t}\gamma}/\sigma_{t\bar{t}} \), we shall consider that ratio as a function of the MDM \( \kappa \) and the EDM \( \tilde{\kappa} \) and then use the CDF measurement to set bounds on those parameters. In the radiative production process two modes are predominant: (1) \( t\bar{t} \) produced along with the radiated photon followed by the decay of the top pair, which is indeed \( t\bar{t}\gamma \) production proper (see Figure 1 (a)), and (2) \( t\bar{t} \) produced on-shell with one of them decaying radiatively (see Figure 1 (b)). The first mode may involve initial–state radiation if the initial partons are charged. The second mode may involve final–state radiation from the \( b \) jets, the intermediate \( W \) boson or the \( W \) decay products.

In order to quantify the impact of the top–quark MDM and EDM on the cross section, we focus our attention on a normalized ratio \( \hat{R} \equiv R/R_{SM} = \sigma_{t\bar{t}\gamma}/\sigma_{t\bar{t}} \). In this way, the CDF result (3) can be translated to \( \hat{R}^{\exp} = 1 \pm 0.375 \). We compute the cross sections for \( pp \rightarrow t\bar{t} \rightarrow bW^+\bar{b}W^-\gamma \rightarrow \text{FS} \) at leading order at the Tevatron energy, and the same processes with \( pp \) initial state at LHC energies. We choose semileptonic final states, as done in the CDF measurement, but consider also a simplified process with final state \( b\bar{b}WW \) as a cross check of our results.

The expression for \( \hat{R} \) must have in general the quadratic form \( \hat{R} = 1 + a_1\kappa + a_2\kappa^2 + b_2\tilde{\kappa}^2 \). By computing \( \hat{R} \) for several values of \( \kappa \) and \( \tilde{\kappa} \) we can obtain the coefficients \( a_i \) in \( \hat{R} \) at the desired energy. Then we use a relation of the form

\[
\hat{R}_1 \leq \hat{R} = 1 + a_1\kappa + a_2\kappa^2 + a_3\tilde{\kappa}^2 \leq \hat{R}_2
\]
to find the allowed parameter region for $(\kappa, \tilde{\kappa})$ at that energy. In the case of the CDF measurement (3), we set $\hat{R}_{1,2} = 1 \pm 0.375$ to define the allowed region at the 1σ level. As for the numerical coefficients we obtain $a_1 = -0.002, -0.008, -0.009$ for the Tevatron, LHC (7 TeV) and LHC (14 TeV). Similarly, $a_2 = 0.011, 0.055, 0.088$ and $a_3 = 0.011, 0.055, 0.089$. The measurement of $\hat{R}$ at the Tevatron by the CDF collaboration sets limits on $(\kappa, \tilde{\kappa})$ through (4).

At the Tevatron energy $\sqrt{s} = 2$ TeV, the production of $t\bar{t}$ and $t\bar{t}\gamma$ receives its dominant contribution from $u\bar{u}$ initial states, but we took into account also the smaller contributions from initial $d\bar{d}$ and $gg$. For the numerical computation we considered the semileptonic process $p\bar{p} \rightarrow t\bar{t} \rightarrow b\bar{b}qq'(\nu_{\ell}\gamma)$ with three lepton flavors, where the final photon can originate from any initial, intermediate or final charged particle. In analogy with the measurement reported by CDF [4], we applied cuts in the transverse energy of the photon, missing transverse energy and pseudorapidity of the final particles given by

$$E_T^\gamma > 10 \text{GeV}, \ E_T > 20 \text{GeV}, |\eta_\gamma| < 3.6, |\eta_b| < 2, |\eta_\ell| < 1, |\eta_\gamma| < 1.$$  (5)

With those cuts we obtain a SM cross section $\sigma_{t\bar{t}\gamma}^{SM} = 0.07261 \text{ pb}$ at $\sqrt{s} = 2$ TeV, in agreement with the leading-order result reported in [4]. In order to increase the sensitivity of the process to the dipole moments of the intermediate top quark it is necessary to reduce the background from photons originating in final–state charged particles. For that purpose we impose a lower bound on the distance from the photon to the charged particles in the $\eta-\phi$ plane, $\Delta = \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$, $\Delta_{\gamma,ch} > 0.4$, which plays the same role as the analogous cuts introduced in the actual measurement [4]. With the cut on $\Delta$ the SM cross section at 2 TeV is $\sigma_{t\bar{t}\gamma}^{SM} = 0.0193 \text{ pb}$.

4. Limits from $B \rightarrow X_s \gamma$

We separate the SM value $C_7^{SM}(m_b) = -0.31$ from the NP contributions:

$$C_7(m_b) = -0.31 + 0.67 \delta C_7(\mu_b) + 0.09 \delta C_8(\mu_b) + \cdots,$$  (6)

We set $\delta C_8(\mu_b) = 0$ and keep $\delta C_7(\mu_b)$ which is given in terms of $\kappa$ and $\tilde{\kappa}$ in Ref. [7]. A numerical expression for the branching ratio $B(B \rightarrow X_s \gamma)$ in terms of the coefficients $C_7(\mu_b)$:

$$\delta B(B \rightarrow X_s \gamma) \equiv B(B \rightarrow X_s \gamma) - B^{SM}(B \rightarrow X_s \gamma) = 10^{-4} \times$$

$$\times \left( \text{Re}(-7.184 \delta C_7 - 2.225 \delta C_8 + 2.454 \delta C_7 \delta C_8^* + 4.743 |\delta C_7|^2 + 0.789 |\delta C_8|^2) \right),$$

Using the predicted value for $B^{SM}(B \rightarrow X_s \gamma)$ and a measured value for $B(B \rightarrow X_s \gamma)$ we get the relation:

$$\delta B(B \rightarrow X_s \gamma) = (3.43 \pm 0.22) - (3.15 \pm 0.23) = -0.234\kappa + 0.005\kappa^2 + 0.039\tilde{\kappa}^2,$$

The CP asymmetry

$$A_{CP}(B \rightarrow X_s \gamma) = \frac{\Gamma(\bar{B} \rightarrow X_\gamma) - \Gamma(B \rightarrow X_{\bar{\gamma}})}{\Gamma(\bar{B} \rightarrow X_\gamma) + \Gamma(B \rightarrow X_{\bar{\gamma}})}$$  (7)

was first proposed in [15]. Its latest experimental value is quoted in [13] as $A_{CP}^{exp}(B \rightarrow X_\gamma) = (-0.8 \pm 2.9)\%$. Concerning the SM prediction, the most recent study is given in Ref. [16]. With the definition $C_7(m_b)/C_7^{SM}(m_b) = r_7 e^{i\theta_7}$ we get

$$A_{CP}[^\%] = a_7 \frac{\sin(\theta_7)}{r_7} + 1.07 \frac{\cos(\theta_7)}{r_7} + \frac{0.03}{r_7^2},$$
Figure 3. Gray lines as in previous figure. Black solid and dashed lines delimit the regions allowed by hypothetical measurements of $\hat{R}$ for $t\bar{t}\gamma$ production at the LHC. In (a) $\sqrt{s} = 7$ TeV. In (b) $\sqrt{s} = 14$ TeV.

We can write $r_7 e^{i\theta_7} = C_7(m_b)/C_7^{SM}(m_b) = 1 - 0.0705\kappa - i 0.1962\tilde{\kappa}$. Then,

$$A_{CP}[^\%] = \frac{1.1 - 0.075\kappa - 3.08\tilde{\kappa}}{(1 - 0.07\kappa)^2 + 0.0385\tilde{\kappa}^2}.$$

5. **Allowed parameter space for $\kappa$ and $\tilde{\kappa}$**

We can now show the allowed region in the $\kappa$ vs. $\tilde{\kappa}$ plane. For the asymmetry we require $|A_{CP}(X_s\gamma)| < 4\%$, to be consistent with the uncertainty of the SM prediction as well as the experimental error. The region of allowed values for $(\kappa, \tilde{\kappa})$ is shown in figures 2 and 3 by the gray dashed lines. For the branching ratio, the region allowed at the 1$\sigma$ level is $-0.038 < \delta B(B \to X_s\gamma) < 0.598$. That region is delimited in figures 2 and 3 by gray solid lines. Roughly speaking the MDM is bounded to be $-2 < \kappa < 1$ which translated to the $m_{t\mu_t} = \kappa e/2 = 0.15\kappa$ term used in [3] means that $-0.3 < m_{t\mu_t} < 0.15$. Our limits are significantly more stringent than reported by [3].
At the 1σ level the allowed region for \((\kappa, \tilde{\kappa})\) is bounded by the inequalities \(0.625 < \hat{R} < 1.375\). The lower value turns out to be unattainable, so it does not set any bound. The region delimited by \(\hat{R} = 1.375\) is shown in figure 2 by the black solid line. The black dashed lines in that figure show the regions that would be delimited by hypothetical measurements \(\hat{R} = 1 \pm 0.1\) and \(1 \pm 0.05\). We see from the figure that, as expected from the analysis in [6], the bounds set by the Tevatron measurement of \(\hat{R}\) are much less constraining than those arising from the asymmetry and branching ratio for \(B \to X_s\gamma\). This is so even in the hypothetical case of an experimental result \(\hat{R} = 1 \pm 0.1\) with a 10% measurement error. Only a 5% measurement uncertainty could yield bounds of the same order of magnitude at most.

We have also performed the same computation for hypothetical measurements of \(\hat{R}\) for \(tt\gamma\) production in \(pp\) collisions at the LHC, both at \(\sqrt{s} = 7\) TeV and \(\sqrt{s} = 14\) TeV. In this case the dominant contribution to the production process comes from \(gg\) initial states, but we also took into account the smaller contributions from the initial states \(uu\) and \(dd\). The results are shown in figure 3 (a) for the lower LHC energy and in figure 3 (b) for the higher one. As seen in the figure, the hypothetical experimental results at the LHC would remove significant portions of the region of the \((\kappa, \tilde{\kappa})\) plane allowed by the measurements of the branching ratio and \(CP\) asymmetry of \(B \to X_s\gamma\). Whereas this is true already at \(\sqrt{s} = 7\) TeV, the constraints set by a measurement of \(\hat{R}\) at \(\sqrt{s} = 14\) TeV with an experimental uncertainty smaller than, say, 30% would lead to strikingly tighter bounds on \((\kappa, \tilde{\kappa})\) than those currently available.

Our overall conclusions agree with those given by Baur et. al [6] even though their analysis was not based on the ratio \(R\) reported by CDF but on the change in the \(p_T\) distribution of the photon.

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