Little Higgses from an Antisymmetric Condensate

Ian Low $^a$, Witold Skiba $^b$, and David Smith $^b$

$^a$Jefferson Physical Laboratory, Harvard University, Cambridge, MA 02138
$^b$Center for Theoretical Physics, Massachusetts Institute of Technology, Cambridge, MA 02139

Abstract

We construct an $SU(6)/Sp(6)$ non-linear sigma model in which the Higgses arise as pseudo-Goldstone bosons. There are two Higgs doublets whose masses have no one-loop quadratic sensitivity to the cutoff of the effective theory, which can be at around 10 TeV. The Higgs potential is generated by gauge and Yukawa interactions, and is distinctly different from that of the minimal supersymmetric standard model. At the TeV scale, the new bosonic degrees of freedom are a single neutral complex scalar and a second copy of $SU(2) \times U(1)$ gauge bosons. Additional vector-like pairs of colored fermions are also present.
I. INTRODUCTION

There are many ways to embed the standard model in a more complete theory which is valid at energies beyond a few TeV. Those that remain weakly coupled at the TeV scale are preferred by precision electroweak data, and low-scale supersymmetry is certainly the best studied example in this class of theories. Another potentially weakly coupled extension of the standard model, featuring the Higgs as a pseudo-Goldstone boson, has been explored in far less detail. In order for the pseudo-Goldstone Higgs (PGH) to be weakly coupled in the TeV region, the effective theory describing the PGH must have a cutoff somewhat larger than a few TeV. Early attempts at constructing PGH models \[1, 2\] could achieve a high cutoff scale only by fine tuning of parameters, because the Higgs mass suffered from a quadratic sensitivity to the cutoff just as in the standard model. Motivated by deconstruction \[3, 4\], a PGH with a larger cutoff scale has been realized recently \[5\]. In the model of Ref. \[5\], the Higgs mass is quadratically sensitive to the cutoff only at two loops. Using the set of observations described in \[5, 6\] it would be possible to construct models where the sensitivity to the cutoff is suppressed up to even higher orders in perturbation theory \[5\], but it is not a high priority to do so since infrared-dominated contributions to the Higgs mass prevent raising the cutoff scale arbitrarily high. In any case, the cutoff can be pushed to around 10 TeV simply by eliminating one-loop quadratic divergences, and we will not probe beyond this scale anytime soon.

The interactions that contribute most to the quadratic divergence of the Higgs mass are the top Yukawa coupling, the gauge interactions, and the Higgs quartic coupling. How is it possible to eliminate the one-loop divergences arising from these interactions? The key is to arrange for a large approximate global symmetry which yields the desired pseudo-Goldstone bosons after the symmetry is spontaneously broken. Contained in this global symmetry is a product gauge group, such as \((SU(2) \times U(1))^2\) or \(SU(2) \times U(1) \times SU(3)^n\), which is broken to the electroweak gauge group when the global symmetry is spontaneously broken. Each individual interaction, for instance a single \(SU(2)\) group, explicitly breaks only part of the global symmetry. The remaining unbroken subgroup is sufficient to produce an exactly massless Higgs. Therefore, the Higgs mass can arise only from an interplay between two, or more, interactions and cannot become quadratically divergent at one loop. Similar observations apply to fermion interactions.

The first models to incorporate this strategy for suppressing dominant one-loop quadratic divergences \[3, 4, 7\] were based on a QCD-like pattern of symmetry breaking (“moose models”). The global symmetries of these models consist of multiple copies of \(SU(k) \times SU(k)\) spontaneously broken to \(SU(k)\), in analogy to chiral symmetry breaking in QCD with \(k\) flavors. Very recently, a model based on a simple global symmetry group was constructed using the breaking pattern \(SU(5) \to SO(5)\) \[8\]. This model has very few additional particles below the cutoff scale: a complex triplet of scalars, an extra copy of \(SU(2) \times U(1)\) gauge bosons, and a vector-like pair of colored Weyl fermions. The model of Ref. \[8\] also has several other interesting features such as the generation of the Higgs potential by the gauge interactions and the top Yukawa couplings. These theories, in which the lightness of the Higgs is due to its being a pseudo-Goldstone boson, have become known as “little Higgs” theories.

Moose models based on a QCD-like symmetry breaking pattern can be analyzed in a systematic manner \[10\] using notions of topology to predict the low-energy spectrum and interactions. A similar understanding of models with simple global symmetry groups is lack-
ing. Since these models offer interesting alternatives for TeV scale physics, it is important to understand their general features. One way to approach the problem is to study alternative models and check which properties appear to be generic and which properties are model dependent. In this article we study a model based on an $SU(6) \to Sp(6)$ symmetry breaking pattern.

Fermion condensation is one possible origin for this symmetry breaking. Condensed fermions that transform under a complex representation of a strong gauge group could yield a non-linear sigma model that is based on the coset $SU(k) \times SU(k)/SU(k)$, where $k$ is the number of strongly interacting fermions. A breaking pattern of this type is employed in the “minimal moose” model of Ref. [7]. When the condensed fermions transform under a real representation of the strong gauge group, the condensate gives rise to a non-linear sigma model based on $SU(k)/SO(k)$, which is considered in Ref. [8]. The third possibility is that the strongly interacting fermions transform under a pseudoreal representation, which produces the $SU(k) \to Sp(k)$ breaking pattern that interests us here\(^1\). In this case the fermion condensate is antisymmetric in flavor.

However, there are a variety of possibilities for the underlying physics of our model, and the question of what dynamics generates the breaking of $SU(6)$ to $Sp(6)$ is not essential for our purposes. From the low-energy point of view, all available information about the model is encoded in the nonlinear sigma-model describing the coset space $SU(6)/Sp(6)$. If we made some assumptions about the physics at the cutoff we might hope to compute, perhaps by using lattice techniques, the coefficients of operators generated at the cutoff. We will simply use naive dimensional analysis\(^[9]\) to estimate the magnitudes of the coefficients of any such operators.

In our model, the spontaneous breaking of $SU(6)$ to $Sp(6)$ generates $14 = 35 - 21$ (pseudo)Goldstone bosons. Four of these become the longitudinal components of vector bosons, since the condensate breaks the gauge group of the model, $(SU(2) \times U(1))^2$, to the electroweak $SU(2)_W \times U(1)_Y$. Eight others form two doublets of $SU(2)_W$ and remain light because they are protected from one-loop quadratic divergences. The remaining two form a complex neutral singlet. This singlet is not protected from quadratic divergences and is heavier than the Higgs fields.

Therefore, at low energies our model is a two Higgs doublet model. At energies of order TeV there is a set of massive gauge bosons responsible for canceling one-loop divergences originating from the gauge sector, along with the neutral singlet responsible for softening the divergences arising form the Higgs quartic coupling. To eliminate the divergences associated with the top Yukawa we also add extra colored fermions in vector-like pairs. Under the electroweak symmetry these fermions transform as two doublets and two pairs of charged singlets. We will explain in detail why this particular assignment is selected. As in the model of Ref. [8], the gauge and Yukawa interactions are sufficient for generating a viable Higgs potential.

The outline of the rest of the paper is as follows: in Section [I], we present the symmetry breaking pattern, the embedding of the gauge group, and the effect of approximate symmetries on the masses of the pseudo-Goldstone bosons. We also describe the symmetry arguments which make keeping track of quadratic divergences transparent. Section [II] contains a discussion of the top Yukawa coupling and the Higgs potential. Section [IV] considers

\(^1\) Of course, at the level of the effective field theory, there is no need to restrict oneself to cosets that arise from fermion condensation.
the issue of vacuum stability in the context of various possible UV completions. Section V contains a brief discussion of experimental constraints and signatures.

II. THE GAUGE SECTOR

We consider a non-linear sigma model whose sigma field transforms under global $SU(6)$ transformations $V$ according to

$$\Sigma \rightarrow V \Sigma V^T. \quad (1)$$

As stressed in the introduction, we will be interested in this model as a low-energy effective field theory and will not specify the underlying physics that provides its UV completion. We assume a condensate of the form

$$\langle \Sigma \rangle = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad (2)$$

where $I$ is the $3 \times 3$ unit matrix. The global $SU(6)$ symmetry is thus spontaneously broken down to $Sp(6)$.

We can divide the $SU(6)$ generators into broken ($X_a$) and unbroken ($T_a$) ones. They satisfy

$$X_a \langle \Sigma \rangle - \langle \Sigma \rangle X_a^T = 0, \quad (3)$$

$$T_a \langle \Sigma \rangle + \langle \Sigma \rangle T_a^T = 0. \quad (4)$$

The fluctuations of the sigma field around its vacuum expectation value can then be parametrized by Goldstone boson fields $\Pi_a$ as in

$$\Sigma = e^{i\Pi_a X_a/(2f)} \langle \Sigma \rangle e^{i\Pi_a X_a^T/(2f)} = e^{i\Pi_a X_a/f} \langle \Sigma \rangle. \quad (5)$$

Using Eq. (3) we determine that the matrix of Goldstone bosons has the form

$$\Pi_a X_a \sim \begin{pmatrix} A & B \\ B^+ & A^* \end{pmatrix}, \quad (6)$$

where $A$ is a traceless and Hermitian $3 \times 3$ matrix and $B$ is an antisymmetric $3 \times 3$ matrix, giving a total of 14 real fields.

We assume that $\langle \Sigma \rangle$ breaks an $(SU(2) \times U(1))^2$ gauge group to the electroweak $SU(2)_W \times U(1)_Y$. In our model the $SU(2)^2$ is a subgroup of the global $SU(6)$ but the $U(1)^2$ is not, for reasons that will become clear later. The full gauge group is contained in a global $U(6)$, and could also be embedded in a larger group, such as $SU(8)$, in a way consistent with our goals. Let us label the two $SU(2)$ groups with a subscript $i = 1, 2$. The generators of $SU(2)_i$ are

$$Q^a_1 = \frac{1}{2} \begin{pmatrix} \sigma^a & 0 \\ 0 & 0 \end{pmatrix}, \quad \text{and} \quad Q^a_2 = -\frac{1}{2} \begin{pmatrix} 0 & \sigma^a \\ 0 & 0 \end{pmatrix}. \quad (7)$$
It is easy to check, using Eqs. (3) and (4), that the sum of $Q_1$ and $Q_2$ is unbroken, while the difference is broken. The unbroken linear combination generates $SU(2)_W$. The Goldstone bosons which coincide with the broken gauged generators become the longitudinal components of the massive gauge bosons of the broken $SU(2)$.

The gauging of $(SU(2) \times U(1))^2$ explicitly breaks all of the global $SU(6)$ symmetry, and the $\Pi_a$'s are not exact Goldstone bosons. It is useful to decompose the pseudo-Goldstone states into representations of $SU(2)_W$, giving

$$
\Pi_a X_a = \left( \begin{array}{cccc}
0 & \phi_1 & 0 & s \\
\phi_1^\dagger & 0 & -s^T & 0 \\
0 & -s^T & -\phi_2 & 0 \\
\phi_2^\dagger & 0 & \phi_1^T & 0 \\
\end{array} \right).
$$

Here, $\phi_1$ and $\phi_2$ are both $SU(2)_W$ doublets and $s$ is an $SU(2)_W$ singlet. We have set the eaten fields to zero, and anticipated that the breaking of the gauged $U(1)^2$ to $U(1)_Y$ will eat the remaining diagonal piece of the Goldstone boson matrix left over from the Higgsing of $SU(2)^2 \rightarrow SU(2)_W$. Of the 14 real fields identified earlier, four are eaten, four each are contained in $\phi_1$ and $\phi_2$, and two are contained in $s$.

The fields $\phi_1$, $\phi_2$, and $s$ all acquire masses through their gauge interactions, but the $\phi_i$'s do not obtain one-loop mass contributions quadratic in the cutoff scale. We will show that this is the case by analyzing the subgroup unbroken by each individual interaction, and finding that the breaking of the exact symmetry produces massless weak doublets. This type of reasoning \cite{8} will be central to the rest of the paper, and in particular will allow us to incorporate Yukawa couplings in the following section.

If either of the $SU(2)$ gauge couplings $g_1$ or $g_2$ vanish, the theory possesses an exact $SU(4)$ global symmetry. Suppose that $g_2$ is turned off, then $SU(4)$ acting on the indices (3456) is an exact symmetry; if $g_1$ is turned off, an $SU(4)$ acting on (1236) is exact. Both of these $SU(4)$'s generate two weak doublets under the breaking by $\langle \Sigma \rangle$. This indicates that neither $\phi_1$ nor $\phi_2$ receives a quadratically divergent mass squared at one loop. To generate a mass term for the doublets both gauge interactions have to participate, so a quadratically divergent contribution can only occur at two loops. On the other hand, the $s$ scalar is not protected by the $SU(4)$ global symmetries and should acquire a quadratically divergent mass squared at one loop.

The quadratically divergent gauge contribution to the Coleman-Weinberg potential is

$$
\frac{3}{32\pi^2} \lambda^2 \text{tr} \left[ M^2(\Sigma) \right],
$$

where $\lambda$ is the cutoff of the non-linear sigma model and $M^2(\Sigma)$ is the gauge boson mass squared matrix in the presence of a background value for $\Sigma$. This matrix can be deduced from the kinetic term of the sigma model Lagrangian

$$
- \frac{f^2}{4} \text{tr}[(D_\mu \Sigma)(D^\mu \Sigma)^*],
$$

where $f$ is the electroweak vacuum expectation value.
where
\[ D_\mu \Sigma = \partial_\mu \Sigma + ig_1 A_1^a (Q_1^a \Sigma + \Sigma Q_1^{aT}) + ig_2 A_2^a (Q_2^a \Sigma + \Sigma Q_2^{aT}). \]  \hfill (11)

For simplicity, we include only the $SU(2)^2$ gauge interactions for the time being. The presence of this quadratically divergent contribution tells us that we must include in the effective Lagrangian a potential of the form
\[ \frac{2}{3} f^4 \left( g_1^2 \text{tr}[(Q_1^a \Sigma)(Q_1^a \Sigma)^*] + g_2^2 \text{tr}[(Q_2^a \Sigma)(Q_2^a \Sigma)^*] \right), \]  \hfill (12)

where $c$ is a coefficient of order one (the factor of $2/3$ is included for later convenience). The minus sign in Eq. (10) is required by the antisymmetry of $\Sigma$, and has the consequence that the quadratically divergent gauge contribution to the Coleman-Weinberg potential is negative. Whether the corresponding operators included in the effective Lagrangian come with the same or opposite sign depends on the details of the UV completion of the effective theory.

Expanding Eq. (12) up to terms quadratic in $s$ and quartic in the doublets yields the potential
\[ cf^2 \left( g_1^2 \left| s + \frac{i}{2f} \tilde{\phi}_2 \phi_1 \right|^2 + g_2^2 \left| s - \frac{i}{2f} \tilde{\phi}_2 \phi_1 \right|^2 \right), \]  \hfill (13)

where $\tilde{\phi}_2 = i\sigma_2 \phi_2^*$. If these are the only contributions to the potential with coefficients of this magnitude, the stability of the vacuum of Eq. (2) requires $c > 0$, which we assume to be true. Only $s$ is massive at this level, and it can be integrated out to yield a low-energy quartic term for the light Higgses,
\[ \frac{c g_1^2 g_2^2}{g_1^2 + g_2^2} \left| \tilde{\phi}_2 \phi_1 \right|^2. \]  \hfill (14)

An order one quartic coupling is generated by integrating out the singlet, just as it is generated by integrating out the triplet in the model of Ref. [3].

For this quartic term to be suitable for stabilizing the Higgs condensates, $\phi_1$ and $\phi_2$ must have opposite hypercharge, or else the quartic term vanishes in the direction that preserves the $U(1)$ of electromagnetism. Moreover, the two $U(1)$’s we gauge must not induce one-loop quadratically divergent corrections to the Higgs masses. We choose the $U(1)$’s generated by $Y_1 = \text{Diag}(0, 0, 1, 0, 0, 0)$ and $Y_2 = \text{Diag}(0, 0, 0, 0, 0, -1)$, which are broken by $\langle \Sigma \rangle$ to ordinary hypercharge $Y = \text{Diag}(0, 0, 1, 0, 0, -1)$. This choice gives $\phi_1$ and $\phi_2$ opposite hypercharges as desired, and makes the singlet $s$ neutral. Neither $Y_1$ nor $Y_2$ respect the global $SU(4)$ approximate symmetries that protect the Higgs masses from quadratically divergent contributions from $SU(2)$ gauge interactions. However, they each respect an $SU(5)$ global symmetry, and these $SU(5)$’s also prevent one-loop quadratic divergences. In fact, in the limit of either $SU(5)$ being preserved, $\phi_1, \phi_2, \text{and } s$ become exact Goldstone bosons, so there are no quadratically divergent contributions to the one-loop potential proportional to the $U(1)$ coupling constants.

The form of the potential in Eq. (13) could have been deduced using the global symmetry transformation properties of $\phi_1, \phi_2$, and $s$ [3]. $SU(4)_1$ preserves global symmetries that act on these fields as
\[ \phi_1 \rightarrow \phi_1 + \epsilon_1 + \cdots, \]
\[ \phi_2 \to \phi_2 + \epsilon_2 + \cdots, \]
\[ s \to s - \frac{i}{2f}(c^\dagger \phi_1 + \tilde{c}^\dagger \phi_2 \epsilon_1) + \cdots, \]
while \( SU(4)_2 \) preserves global symmetries that act as
\[ \phi_1 \to \phi_1 + \eta_1 + \cdots, \]
\[ \phi_2 \to \phi_2 + \eta_2 + \cdots, \]
\[ s \to s + \frac{i}{2f}(\eta_2^\dagger \phi_1 + \tilde{\eta}_2^\dagger \phi_2 \eta_1) + \cdots, \]
where \( SU(4)_i \) is the symmetry preserved by \( SU(2)_i \). Given these transformation laws, the quadratically divergent contributions to the one loop potential are forced to be proportional to Eq. (13).

III. THE TOP YUKAWA COUPLING

The structure of our model guarantees that the Higgses are protected from one-loop quadratic divergences coming from gauge interactions and self couplings. The challenge remains to couple \( \Sigma \) to fermions in a way that gives rise to a large top Yukawa coupling without inducing quadratic divergences from these additional interactions. For this purpose we consider the following fermion content: the standard model fermions are taken to be singlets under \( SU(2)_1 \) and \( Y_1 \), and have the usual assignments under \( SU(2)_2 \) and \( Y_2 \). Thus the third generation quarks have the charge assignments \( b^r(1_0, 1_{1/3}) \), \( t^r(1_0, 1_{-2/3}) \), and \( Q(1_0, 2_{1/6}) \). To the standard model fermions we add a colored pair of vector-like doublets and two colored pairs of vector-like singlets: \( Q'(2_{1/6}, 1_0) + Q'^c(2_{-1/6}, 1_0) \), \( \psi_1(1_{1/2}, 1_{1/6}) + \tilde{\psi}_1(1_{-1/2}, 1_{-1/6}) \), and \( \psi_2(1_0, 1_{-1/3}) + \tilde{\psi}_2(1_0, 1_{1/3}) \). We denote in parenthesis transformation properties under the gauge symmetries.

We couple \( \Psi \) to \( \Sigma \) through two terms, both of which preserve a subgroup of the global \( SU(6) \) that protects the Higgs masses. Consider the Lagrangian terms
\[ \lambda_1 f \left( Q'^T \right)^T \begin{pmatrix} \psi_1 & (i\sigma_2 Q)^T \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ t^c & 0 & 0 & 0 \end{pmatrix} \Sigma^* \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_2 f \left( Q'^T \right)^T \begin{pmatrix} i\sigma_2 Q'^c \\ \psi_1^c \\ 0 \\ \psi_2^c \end{pmatrix} \Sigma \begin{pmatrix} i\sigma_2 Q'^c \\ \psi_1^c \\ 0 \\ \psi_2^c \end{pmatrix} + \text{h.c.} \] (17)

Neither of these terms induces one-loop quadratic divergent Higgs masses: the first term respects the same global \( SU(5) \) as does \( Y_2 \) and the second respects the global \( SU(4)_2 \). These terms are gauge invariant given the charge assignments listed above. We also introduce Dirac masses linking the vector-like fermion pairs:
\[ \lambda_3 f Q'^T i\sigma_2 Q'^c + \lambda_4 f \psi_1^c \psi_1 + \lambda_5 \psi_2^c \psi_2. \] (18)

Expanding the two terms in Eq. (17) gives
\[ -\lambda_1 [f t^c \psi_1 - it^c (\phi_1^\dagger Q' + \phi_2^\dagger Q)] + \cdots + \text{h.c.} \] (19)
and
\[ \lambda_2 [f Q'^T i\sigma_2 Q'^c + iQ'^T (\phi_1^c \psi_1^c + \phi_2^c \psi_2^c)] + \cdots + \text{h.c.} \] (20)
where $\tilde{\phi}_2 = i\sigma_2 \phi_2^*$. In the limit of unbroken electroweak symmetry, the first terms in these expansions combined with Dirac masses of Eq. (18) yield the massless linear combinations

$$
t^c_0 = \frac{\lambda_1 \psi_1^c + \lambda_4 t^c}{\sqrt{\lambda_1^2 + \lambda_4^2}} \quad (21)
$$

$$
Q_0 = \frac{\lambda_2 Q' - \lambda_3 Q}{\sqrt{\lambda_2^2 + \lambda_3^2}}. \quad (22)
$$

These light states are the ordinary right-handed top quark and left-handed doublet. The second term in Eq. (19) contains the Yukawa couplings for the top quark,

$$
i \frac{\lambda_1}{\sqrt{(\lambda_1^2 + \lambda_4^2)(\lambda_2^2 + \lambda_3^2)}} t^c_0 \left[ \lambda_2(\lambda_4 - \lambda_3)\phi_1 - \lambda_3\lambda_4\tilde{\phi}_2 \right]^\dagger Q_0. \quad (23)
$$

Therefore, an order one top Yukawa coupling is easily obtained if the various $\lambda$’s are order one. It is important that a linear combination of $\phi_1$ and $\tilde{\phi}_2$ appears in the Yukawa coupling rather than $\phi_1$ or $\tilde{\phi}_2$ alone, which is why the term in Eq. (17) proportional to $\lambda_2$ is necessary. In its absence, the couplings of Eqs. (18) and (19) respect a Peccei-Quinn symmetry that would forbid the mass term $(\phi_1^\dagger \phi_2 + h.c.)$. In this case, the Higgs potential either would preserve electroweak symmetry or would possess a runaway direction.

Only $\lambda_2$, and not $\lambda_1$, contributes to the quadratically divergent piece of the Coleman-Weinberg potential, giving

$$
\frac{3}{8\pi^2}\Lambda^2 \left| s - \frac{i}{2f} \tilde{\phi}_2^\dagger \phi_1 \right|^2, \quad (24)
$$

Since $\lambda_2$ preserves $SU(4)_2$, its contribution is proportional to the second term of Eq. (13). The presence of this additional quadratic divergence requires that the potential of Eq. (13) is modified to

$$
cg^2 f^2 \left| s + \frac{i}{2f} \tilde{\phi}_2^\dagger \phi_1 \right|^2 + (cg_2^2 + c'\lambda_2^2) f^2 \left| s - \frac{i}{2f} \tilde{\phi}_2^\dagger \phi_1 \right|^2, \quad (25)
$$

where $c'$ is another order one coefficient. The mass squared of $s$ is positive provided that $c'\lambda_2^2 + c(g_1^2 + g_2^2) > 0$. In this case, integrating out $s$ gives the low energy quartic coupling $\lambda|\tilde{\phi}_2^\dagger \phi_1|^2$, with

$$
\lambda = c \frac{g_2^2[1 + (c'/c)\lambda_2^2]}{g_1^2 + g_2^2 + (c'/c)\lambda_2^2}. \quad (26)
$$

Note that $\lambda > 0$ requires $c > 0$.

The quadratic terms in the one-loop Higgs potential are logarithmically divergent and take the form

$$
m_1^2|\phi_1|^2 + m_2^2|\phi_2|^2 + b^2(\phi_1^\dagger \tilde{\phi}_2 + h.c.). \quad (27)
$$

The quartic term in the Higgs potential features flat directions that are stabilized only if both $m_1^2$ and $m_2^2$ are positive. In this case the potential is minimized for vacuum expectation

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2 Because we have assumed for simplicity that the coefficients in Eqs. (17) and (18) are real, $b^2$ will be real, but phase redefinitions of the Higgs doublets could have absorbed the phase of $b^2$ were it complex.
values that may be chosen to have the form
\[ \phi_1 = \begin{pmatrix} 0 \\ v_1 \end{pmatrix} \quad \text{and} \quad \phi_2 = \begin{pmatrix} v_2 \\ 0 \end{pmatrix}, \] (28)
where \( v_1 \) and \( v_2 \) are both real, with the same sign if \( b^2 \) is positive and opposite sign if \( b^2 \) is negative. Electroweak symmetry breaking will occur if and only if the determinant of the Higgs mass matrix is negative,
\[ m_1^2 m_2^2 - b^4 < 0. \] (29)

Fermion loops induce logarithmically divergent contributions to \( m_1^2, m_2^2 \) and \( b^2 \). Assuming the heavy fermion masses, \( M' = f \sqrt{\lambda_1^2 + \lambda_3^2} \), \( M_Q' = f \sqrt{\lambda_2^2 + \lambda_3^2} \), and \( M_{\psi_2} = f \lambda_5 \), to be the same order of magnitude and, to an excellent approximation, taking all the logarithms to a common value, \( \log(\Lambda/M) \), these contributions are
\[ m_{1f}^2 = \frac{3f^2}{8\pi^2} (\lambda_1^2 - \lambda_3^2)(\lambda_3^2 - \lambda_1^2) \log \frac{\Lambda^2}{M^2} \] (30)
\[ m_{2f}^2 = \frac{3f^2}{8\pi^2} (\lambda_1^2 \lambda_2^2 + \lambda_3^2 \lambda_5^2 - \lambda_2^2 \lambda_3^2 - \lambda_1^2 \lambda_5^2) \log \frac{\Lambda^2}{M^2} \] (31)
\[ b_f^2 = \frac{3f^2}{8\pi^2} \lambda_1^2 \lambda_2 (\lambda_3 - \lambda_4) \log \frac{\Lambda^2}{M^2}. \] (32)

Gauge and scalar loops contribute only to \( m_1^2 \) and \( m_2^2 \), with
\[ m_{1g}^2 = m_{2g}^2 = \frac{3}{64\pi^2} \left( 3g_2^2 m_2^2 \log \frac{\Lambda^2}{M_2^2} + g_1^2 M_{\nu_2}^2 \log \frac{\Lambda^2}{M_{\nu_2}^2} \right), \] (33)
\[ m_{1s}^2 = m_{2s}^2 = \frac{\lambda}{16\pi^2} M_s^2 \log \frac{\Lambda^2}{M_s^2}, \] (34)
where \( M_g = f \sqrt{(g_1^2 + g_2^2)/2} \), \( M_{\nu_2} = f \sqrt{(g_1^2 + g_2^2)/2} \), and \( M_s = f \sqrt{c(g_1^2 + g_2^2) + c'd^2} \) are the masses of the heavy \( SU(2) \) gauge boson, the heavy \( U(1) \) gauge boson, and \( s \), respectively. The crucial ingredient in the above formulae is the logarithmically divergent contribution to \( b^2 \) coming from fermion loops. Because all the entries of the Higgs mass matrix have similar logarithmic enhancement, there is a broad range of parameters for which Eq. (24) is satisfied and electroweak symmetry is broken.

There may be other terms in the effective Lagrangian, involving \( \Sigma \) alone, that contribute to the Higgs potential. In this case, these extra terms may be used to break the Peccei-Quinn symmetry, for example through the gauge invariant operator
\[ af^4 \epsilon_{ij} \Sigma_{\alpha i} \Sigma_{\beta j} \Sigma^*_{\gamma x} \Sigma^*_{\delta x} + \text{h.c.} = -(af^2 \phi_2^\dagger \phi_1 + \text{h.c.}) + \cdots, \] (35)
where \( i, j = 1, 2 \) and \( x = 4, 5 \), and \( a \) is an order one coefficient. It is natural for \( a \) to be small \( \ll 16\pi^2 \) because this term breaks all of the global \( SU(4) \)'s and \( SU(5) \)'s identified earlier. If the Peccei-Quinn symmetry is broken in this way, simpler possibilities arise for the fermion sector. For instance, one could remove \( \psi_2 \) and \( \psi_5^* \) from the previous model and do away with the term in Eq. (17) proportional to \( \lambda_2 \). In this case the additional fermionic matter content beyond the standard model is one vector-like pair of singlets and one vector-like pair of doublets.
IV. VACUUM STABILITY AND UV COMPLETIONS

In this section we would like to address the issue of the stability of the vacuum we have chosen, Eq. (2), and possible UV completions of the non-linear sigma model. As mentioned in the introduction, one way to UV complete the effective theory is by using strong gauge dynamics at the cutoff scale. The desired symmetry breaking pattern could arise from a condensate of six Weyl fermions transforming under a pseudoreal representation of a strong gauge group, for example the fundamental representation of $Sp(6)$. If the strong group confines and breaks chiral symmetry then the global $SU(6)$ symmetry associated with the six flavors is spontaneously broken down to the $Sp(6)$ subgroup.

If we were to use strong dynamics to UV complete our model, we could assign two of the six flavors to transform as a doublet of $SU(2)_1$, another two flavors as a doublet of $SU(2)_2$, and the remaining two flavors as $SU(2)_i$ singlets and carry charges $(1,0)$ and $(0,-1)$ under $U(1)_1 \times U(1)_2$, respectively. This assignment gives the same charges for the nonlinear sigma model as described in Sect. II. The fermions charged under the $U(1)$ symmetries induce mixed anomalies with the strong group that can be canceled by the Green-Schwarz mechanism. Alternatively, one could enlarge the theory, for example to one based the $SU(8)/Sp(8)$ coset space, to accommodate traceless $U(1)$ generators.

An important question is whether the vacuum we have chosen, Eq. (2), is stable in a model with strong dynamics. For the time being, let us concentrate on the operators induced by the gauge interactions Eq. (12). A generalization of the standard argument, drawing on the observation that $\pi^+ - \pi^0$ mass difference is positive in QCD, would imply that the coefficient $c$ is negative for the $SU(6)/Sp(6)$ model. In QCD-like theories, the true vacuum is aligned such that the sum of the gauge boson masses squared is minimized, which selects not the vacuum of Eq. (2), but instead $\langle \Sigma \rangle = \sigma^2_2 \otimes \sigma^6_2 \otimes \sigma^{15}_2$, which preserves both $SU(2)$’s. The superscripts on the $\sigma_2$ matrices indicate the indices the $SU(6)$ indices.

However, the dynamics of the $Sp(N)$ gauge theories could be very different from that of the QCD-like theories, in which case a true determination of the sign of $c$ would require a lattice calculation. If the sign of $c$ is indeed negative, there are two options for obtaining a viable UV completion. One possibility is to consider different dynamics responsible for breaking the global symmetry. It is likely, for instance, that either sign of $c$ is possible in a supersymmetric UV completion. Another possibility is that other interactions overwhelm the gauge interactions and yield a stable vacuum.

For concreteness, we discuss the second possibility in detail. For example, it is possible for the fermion contributions to stabilize the potential by introducing another vector-like pair of quarks $Q''(1,0,2_{1/6}) + Q''c(1,0,2_{-1/6})$ and adding to Eq. (17) the coupling

$$\lambda_2 f \left( Q''^T \begin{pmatrix} 0 \\ \psi_1^c \\ i\sigma_2 Q''c \\ \psi_2^c \end{pmatrix} \right) + \text{h.c.} \quad (36)$$

This term preserves an $SU(4)_1$ global symmetry, whereas the $\lambda_2$ term in Eq. (17) preserves a different $SU(4)_2$ symmetry. We also include in the Lagrangian all possible gauge-invariant mass terms involving the quarks. Then a linear combination of $Q$, $Q'$, and $Q''$ will be massless before electroweak symmetry breaking, and the top Yukawa coupling comes from
the $\lambda_1$ term in Eq. (17) as before. Now the potential generated at the cutoff has the form

$$\left( c g_1^2 + c' \lambda_1^2 \right) f^2 \left| s + \frac{i}{2f} \tilde{\phi}_1^\dagger \phi_1 \right|^2 + \left( c g_2^2 + c' \lambda_2^2 \right) f^2 \left| s - \frac{i}{2f} \tilde{\phi}_2^\dagger \phi_1 \right|^2 .$$

(37)

To achieve the electroweak symmetry breaking we need both $c g_1^2 + c' \lambda_1^2$ and $c g_2^2 + c' \lambda_2^2$ to be positive. The stability of the vacuum follows.

Alternatively, the vacuum might be stabilized by operators involving $\Sigma$ alone. We call these “plaquette operators” in analogy with Ref. [5], even though our operators have no apparent lattice interpretation. For example, suppose that in the effective Lagrangian we include

$$a_1 f^4 \Sigma_{ij}^* \Sigma_{ji} + a_2 f^4 \Sigma_{xy}^* \Sigma_{yx},$$

(38)

with $i, j$ summed over 1, 2 and $x, y$ summed over 4, 5. These operators yield a contribution to the potential

$$2 f^2 a_1 \left| s + \frac{i}{2f} \tilde{\phi}_1^\dagger \phi_1 \right|^2 + 2 f^2 a_2 \left| s - \frac{i}{2f} \tilde{\phi}_2^\dagger \phi_1 \right|^2 .$$

(39)

If $a_1$ and $a_2$ are larger than $c$ times the gauge couplings squared, these terms can stabilize the vacuum and achieve the electroweak symmetry breaking at the same time.

Note that the stability is accomplished by operators that either contain two fermions and one power of the $\Sigma$ field, or two powers of $\Sigma$. Both can be generated by four-fermion interactions and could easily come from an analog of extended technicolor.

V. PHENOMENOLOGY

In addition to the matter and gauge fields of the standard model, our model contains two light Higgs doublets. The other heavy states, with masses below $\Lambda \approx 10$ TeV, are an $SU(2)_W \times U(1)_Y$-neutral complex scalar, a copy of $SU(2) \times U(1)$ gauge bosons, a vector-like pair of $SU(2)_W$-doublet colored fermions, and two vector-like pairs of $SU(2)_W$-singlet colored fermions, all of which have masses around a few TeV. The ordering of the spectrum of TeV-scale particles is parameter dependent. Our consideration of these heavy states’ effects on the precision electroweak measurements parallels the analogous discussion in [8]. The number of electroweak doublet fermions which get mass through Yukawa couplings is tightly constrained by the $S$ parameter [12, 13, 14]. Fortunately, the additional fermions in our model are all vector-like, which was shown in Refs. [15, 16, 17] to have small contributions to both the $S$ and $T$ parameters. The effects of these heavy, weakly interacting particles decouple from the low energy physics and are suppressed by factors of $M_{\text{heavy}}^2/M_W^2$. Thus, their impact on the precision electroweak measurements should be similar to 2-loop standard model corrections and smaller than the current experimental errors. The scalar $s$ in our model is an $SU(2)_W$ singlet and integrating it out at the tree level does not induce custodial symmetry violating operators, as would be the case were it a triplet scalar [8]. Therefore, the contribution to the $T$ (or $\rho$) parameter is minimal. Finally, the heavy $SU(2) \times U(1)$ gauge bosons couple directly to standard model fermions, and induce tree-level contributions to muon decay, for instance. This places a lower bound on $M_{\rho}$ of roughly 3 TeV [18] if the heavy gauge bosons couple to light fermions with the same strength as the light gauge bosons, but if $g_1 \neq g_2$ this need not be the case and the bound can be weakened.
We are required to introduce a vector-like pair of $SU(2)_W$ doublet quarks to cancel the quadratically divergent contributions to the Higgs masses coming from the top Yukawa coupling, so we predict that at the TeV scale both charge 2/3 quarks and charge 1/3 quarks will be present. These can decay to ordinary top and bottom quarks by emitting $Z$, $W$, or Higgs particles. The $s$ scalar mixes with the Higgs doublets with mixing angle approximately $v/f$, and decays predominantly into pairs of Higgs bosons. It is unfortunately unlikely to be detected at the LHC because it is neutral under $SU(2)_W \times U(1)_Y$, but more work is needed to study possible production and decay channels of the singlet scalar.

The two light Higgs doublets yield two CP-even neutral scalars $h^0$, $H^0$, one CP-odd neutral scalar $A^0$, and two charged scalars $H^+$ and $H^-$, with masses

$$M^2_{h^0}, M^2_{H^0} = \frac{1}{2} \left( M^2_{A^0} \pm \sqrt{M^4_{A^0} - 4M^2_{H^\pm}(M^2_{A^0} - M^2_{H^\pm}) \sin^2(2\beta)} \right),$$

where $v^2 \sim 250$ GeV, and the ratio of vacuum expectation values, $\tan \beta \equiv v_2/v_1$, turns out to be equal to $\sqrt{m^2_1/m^2_2}$. It is interesting to note that the CP-odd scalar $A^0$ is heavier than the charged scalars $H^\pm$. This is contrary to the MSSM, a difference that arises from the fact that the quartic potential of our model has a different form than that of the MSSM. The lighter CP-even Higgs mass $M_{h^0}$ is maximized when $\sin^2(2\beta) = 1$ ($m^2 = m^2_2$), in which case the CP-even scalar masses are at tree level equal to $M^2_{H^\pm}$ and $M^2_{A^0} - M^2_{H^\pm}$.

The light fermions must be coupled to the two Higgs doublets in such a way that tree level flavor-changing neutral currents (FCNCs) are avoided [20], which can be accomplished either by coupling all the fermions to a single Higgs doublet, or by coupling up-type and down-type quarks to separate Higgs doublets. For instance, the light fermion masses might come from

$$y^u_{ij} f \left( 0 0 (i\sigma_2 Q_i)^T 0 \right) \Sigma^* \left( \begin{array}{c} 0 \\ 0 \\ 0 \\ u^c_j \end{array} \right) + y^d_{ij} f \left( 0 0 Q_i^T 0 \right) \Sigma \left( \begin{array}{c} 0 \\ 0 \\ 0 \\ d^c_j \end{array} \right),$$

in which case both up quarks and down quarks couple to $\phi_2$. These couplings induce one-loop quadratic divergences in the Higgs potential, but they are harmless because of the smallness of the light quark Yukawa couplings. The only tree-level FCNCs induced by Higgs exchange involve the top sector and so are not problematic.

Going beyond the low-energy effective theory, there could also be new, potentially dangerous sources of FCNCs in some UV completions, for instance if the fermion couplings to $\Sigma$ arise from four-fermi interactions as in extended technicolor [21]. Possible ways of suppressing these FCNCs are discussed in Ref. [8, 21].

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