Graphs, Constraints, and Search for the Abstraction and Reasoning Corpus

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Abstract

The Abstraction and Reasoning Corpus (ARC) aims at benchmarking the performance of general artificial intelligence algorithms. The ARC’s focus on broad generalization and few-shot learning has made it difficult to solve using pure machine learning. A more promising approach has been to perform program synthesis within an appropriately designed Domain Specific Language (DSL). However, these too have seen limited success. We propose Abstract Reasoning with Graph Abstractions (ARGA), a new object-centric framework that first represents images using graphs and then performs a search for a correct program in a DSL that is based on the abstracted graph space. The complexity of this combinatorial search is tamed through the use of constraint acquisition, state hashing, and Tabu search. An extensive set of experiments demonstrates the promise of ARGA in tackling some of the complicated object-centric tasks of the ARC rather efficiently, producing programs that are correct and easy to understand.

Introduction

In an attempt to better measure the gap between machine and human learning, the Abstraction and Reasoning Corpus (ARC) was created by Chollet in 2019. The dataset is a collection of 1000 image-based reasoning tasks, where each task asks for an output image given an input. To “learn” a procedure that produces said output, each task comes with 2–5 input-output image pairs as training instances; these training inputs are different from the actual test input, but can be solved by the same (unknown) procedure. Some examples are shown in Figure 1. A competition with over 900 teams was hosted on Kaggle to solve the ARC (Kaggle 2020). Despite a massive effort, the solutions only achieved 20% accuracy on the hidden test set, at best. In fact, the first-place solution could not solve two of the three examples shown in Figure 1 despite their simplicity to a human.

Recognizing objects, actions performed on them, and relationships between them makes up a large portion of human cognition core systems (Spelke and Kinzler 2007). The ARC embodies this notion in its tasks. In fact, Acquaviva et al. (2021) found that when humans attempt to solve ARC tasks through language, half of the phrases they use relate to object detection. Therefore, an object-centric approach to solving the ARC is highly promising. Surprisingly, this key insight is yet to be leveraged.

Abstract Reasoning with Graph Abstractions

Toward this goal, we propose Abstract Reasoning with Graph Abstractions (ARGA), an object-centric framework for solving ARC tasks. Our design rationale is to build a computationally efficient, extensible, object-aware ARC solver through careful integration of the following:

– **Representation:** Enabling object awareness requires a move from treating the input as pixels towards a graph of objects with spatial or other relations. We design a variety of such graph abstractions to cater to the diversity of the ARC and its different definitions of objects.
**Structure:** Grounded in first-order logic, our graph-based DSL makes it possible to define complex but interpretable solution programs for tasks of the ARC. This is in contrast to pure neural network-type approaches that attempt to map input to output in an often black-box fashion.

**Search:** With the representation and DSL in place, we opt for a complete tree search algorithm. Given a task, the search seeks a program in the DSL that produces the correct outputs for each of the task’s training examples. Whenever a correct program for a task exists in our DSL, the search can find it given sufficient time.

**Constraints:** Leveraging the observation that (solved) training examples not only tell us what a correct program does but also what it should not do (e.g., in Fig. 1 (left), objects should not move), we use constraint acquisition to simplify our combinatorial search space. Constraints are expressed in the very same graph DSL and may be acquired by an arbitrary algorithm.

Fig. 2 illustrates the DSL, Search, and Constraints components of ARGA; Fig. 4 illustrates the Representation. With ARGA, we hope to provide AI researchers who are interested in the ARC and similar few-shot reasoning situations with the first such system upon which they can build and explore the capabilities of graph and search-based reasoning. Our implementation is available on GitHub.

Because object-oriented abstraction and reasoning are major failure modes of state-of-the-art ARC solvers, we define criteria to select a subset of object-oriented ARC tasks as a testbed for the evaluation of our methods in comparison to other top solvers. The 160 tasks in question span a wide range of challenging problems that can be categorized as object recoloring, object movement, and object augmentation. We show how ARGA’s design and performance are favorable in the following ways:

**Extensibility and Modularity:** Every component of ARGA can be extended almost independently to target additional ARC tasks or optimize performance: novel graph abstractions can be added, additional object filters and transformations can be appended to the DSL, new search strategies can be tested, and faster constraint acquisition algorithms may seamlessly replace ours.

**Computational Efficiency:** Our DSL contains a number of object-based selection filters as well as transformations (e.g., recoloring, moving, etc.). Because these can be composed together to form a candidate program for an ARC task, the resulting search space is combinatorially large. Nonetheless, through experiments on 160 object-based ARC tasks, we show that when ARGA finds a solution, it does so by exploring a minute number of possible solutions, effectively three orders of magnitude fewer than the winner of the Kaggle competition.

**Effectiveness:** Our current DSL includes only 4 base filters and 11 transformations. Yet, we solve 57 of 160 tasks, only slightly behind the Kaggle winner’s 64 of 160. The latter includes a much larger body of transformations that were obtained by examining many more ARC tasks.

**System Overview**

We propose a two-stage framework that takes an object-centric approach to solving an ARC task. First, the graph abstraction stage inspired by work on Go (Graepel et al. 2001), where the 2D grid images are mapped to (multiple) undirected graph representations that capture information about the objects in the images at a higher abstracted level. Second, the solution synthesis stage, where a constraint-guided search is used to formulate the series of operations to be applied to the abstracted graphs that will lead to a solution. The space of possible operations is defined by an ARGA-specific relational Domain Specific Language (DSL).

Since the DSL defines operations on the abstracted graphs, we will first describe the graph abstraction stage and formally define the structure of the abstracted graphs. Then, the DSL will be defined in detail. Finally, the solution synthesis stage will be discussed.
Figure 3: Example solution generated by ARGA. The input image is first abstracted into a graph in which each node represents a set of adjacent pixels that are not black. Two nodes share an edge iff there is at least one cell in each node with the same coordinate value along either axis. The solution here first colors in blue all nodes not containing exactly six pixels, then colors in red all nodes with exactly six pixels. The number of pixels contained in a node is defined as its “size”, a node attribute that ARGA can use in its search for a correct program.

### Table 1: Object Types in ARGA.

| Object Type Set | Object Type Description |
|-----------------|-------------------------|
| $i \in Image$  | A 2D grid image         |
| $p \in Pixel$   | A pixel on an image     |
| $g \in Graph$   | An abstracted graph     |
| $n \in Node$    | A node in an abstracted graph |
| $e \in Edge$    | An edge in an abstracted graph |
| $c \in Color$   | Color (including background) |
| $s \in Size$    | Size of a node (# pixels) |
| $d \in Direction$ | Directions within the 2D image |
| $pa \in Pattern$ | A pattern found on the image |
| $t \in Type$    | Generic Type (any above) |

### Table 2: Example Object Relations in ARGA.

| Typed Object Relations | Description |
|------------------------|-------------|
| hasNode(Graph, Node)   | Graph contains Node |
| hasPixel(Node, Pixel)  | Node contains Pixel |
| neighbor(Node, Node)   | Nodes are connected by edge |
| color(Node, Color)     | color of Node |
| size(Node, Size)       | size of Node |
| Rel(Type, Type)        | Generic Relation (any above) |

**Graph Abstraction**

Graph abstraction allows us to search for a solution at a macroscopic level. In other words, we are modifying groups of pixels at once, instead of modifying each individual pixel separately. As a result, this approach has a smaller search space than its non-abstracted, raw image counterpart.

We now formally introduce terminology that aids in defining our abstracted graphs (such as those shown in Figure 3) that will be leveraged by the DSL of the next section. The language we use builds on first-order logic which provides a flexible and expressive language for describing typed objects and relations. Object types in our DSL are shown in Table 1 and can be used as unary predicates, e.g., $Node(n)$ is true iff $n \in Node$. Some example relations between objects are shown in Table 2 and the full set of relations can be found in Appendix Table 6.

Let $i$ be any input or output 2D grid image from an ARC task. $i$ can be completely specified by its set of pixels $p$. Let $g$ be an abstracted graph with sets of abstracted nodes $n$. The relations that hold between these types are shown in Table 2. Each Node $n$ represents an object that is detected in the original image $i$ based on the rules of the abstraction (e.g., one graph abstraction is “non-black neighboring pixels of the same color form a node”) and relations between the nodes represent relationships between these objects.

Therefore, the graph abstraction process executes a mapping that generates some abstracted graph $G$ for image $I$. We note that there are multiple ways in which this mapping can be defined. Different graph abstractions can be used to identify objects in the image using different definitions of what an object is. Since the resulting abstracted graphs from

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**Figure 4:** Visualization of graph abstractions. Applying two different graph abstractions to an image. Left: non-background single-color connected pixels. Right: non-background single-color vertically-connected pixels.
overlapping structure, we are able to expand the solution space significantly without modifying the DSL.

The usefulness of having multiple definitions of an object can be observed in the example shown in Figure 1 (Middle). Upon first inspection, one may think that objects are defined as connected pixels with the same color. However, upon further inspection, we realize that the connected red pixels in different columns are in fact different objects as they do not share the same modification in the output images. Therefore, defining multiple abstraction processes improves ARGA’s ability to correctly capture object information from the input images. The two different abstractions mentioned in the example are further discussed in the following:

different graph abstraction definitions share the same underlying structure, we are able to expand the solution space significantly without modifying the DSL.

The usefulness of having multiple definitions of an object can be observed in the example shown in Figure 1 (Middle). Upon first inspection, one may think that objects are defined as connected pixels with the same color. However, upon further inspection, we realize that the connected red pixels in different columns are in fact different objects as they do not share the same modification in the output images. Therefore, defining multiple abstraction processes improves ARGA’s ability to correctly capture object information from the input images. The two different abstractions mentioned in the example are further discussed in the following:

**non-background single-color connected pixels**: In this abstraction, an object (or node) is defined as a set of connected pixels sharing the same color. The pseudocode for the abstraction algorithm is shown in Appendix Algorithm 1 and an illustration of this abstraction is in Figure 4.

**non-background single-color vertically-connected pixels**: In this graph abstraction, an object is defined as a set of vertically connected pixels that are not the background color. An illustration of this abstraction is in Figure 4.

**Overlapping Objects**

Note that our representation allows for a pixel to be associated with multiple nodes in the graph, as objects are modified. This can be intuitively understood by observing that objects may overlap with one another on the grid as one applies a sequence of transformations to solve a given task. Our graph abstraction ensures that although some objects may be partially obscured, they are still kept track of and considered to be a whole object. This allows the system to have the object persistence knowledge prior.

**A Graph DSL for the ARC**

We now introduce a lifted relational DSL for ARGA built upon the objects and relations defined in the previous section. The DSL is used to formally describe the filter language used to match node patterns, determine graph transformation parameters, and carry out transformations on abstracted graphs as described in the following. An example solution expressed using the DSL is shown in Figure 3.

| Transformation | Description |
|----------------|-------------|
| updateColor(Node, Color) | Update color of Node to Color |
| move(Node, Direction) | Update pixels of Node to move in Direction |
| rotate(Node) | Update pixels of N to rotate it clockwise |
| extend(Node, Direction) | Add additional pixels to Node in Direction |
| transform(Node, v₁, . . ., vₖ) | Generic transformation with k parameter values (v₁, . . ., vₖ) |

Table 3: Example Transformations.
While this is generally undesirable behavior, we note that we deterministically order and return the first matching information (it is considered a parameter binding). We remark that this dynamic parameter grammar includes inherently in the training examples.

We implement a greedy best-first search. Suppose ARC task \( t \) has \( m \) training instances, with input-output images \( \{input_i, output_i\} \) for \( i \in \{1, \ldots, m\} \). Each node in our search tree contains a set of graphs \( \{g_{input,i}\} \) for \( i \in \{1, \ldots, m\} \). \( g_{input,i} \) represents \( input_i \) after the abstraction process and the application of a sequence of operations \( (o_1, o_2, \ldots, o_k) \) for \( j \in \{1, \ldots, k\} \), where each \( o_j \) is a full operation as defined previously. The special case of \( k = 0 \) corresponds to no operations applied, i.e., the root node of the search tree. To expand a node with abstracted graphs \( \{g_{input,i}\} \), we first identify the set of all valid full operations \( O \). Then, for each \( o \in O \), we apply it to \( g_{input,i} \) and obtain updated abstracted graphs \( \{g'_i\} \) for \( i \in \{1, \ldots, m\} \). We add the new abstracted graphs \( \{g'_i\} \) into the search tree as a new node and update the sequence of operations that led to it by appending operation \( o \) to obtain \( (o_1, \ldots, o_k, o) \).

### Heuristic Function

To determine the node to be expanded in each iteration of the search, our primary metric measures how close the node is to the target training output. For each node, we reconstruct the corresponding 2D image for each of the abstracted graphs \( \{g_{input,i}\} \). We then compare the reconstructed images with the training outputs \( \{output_i\} \) and calculate a penalty score based on pixel-wise accuracy, as detailed in Appendix Table 9. Large mismatch in pixels between the “predicted” and the actual output results in a large penalty. The node in the search tree with the lowest score is selected for expansion.

### Constraint-Guided Search

We illustrate this concept with an example. All objects in Figure 1 (Left) should not change in position. We can therefore define the constraint \( positionUnchanged \), which is satisfied when a node and the updated version of that node share
the same set of pixels, thus making sure that the node’s position on the image remains unchanged through the transformation. All transformations that modify a node’s pixels can therefore be pruned by this constraint in the search tree. A visualization is shown in Figure 2.

The constraints can be defined using the same language we’ve introduced earlier. For instance, \(positionUnchanged\) is defined as:

\[
positionUnchanged(n : Node, n' : Node) \equiv \\
\forall p \in P \text{ containsPixel}(n, p) \equiv \text{containsPixel}(n', p)
\]

which holds if for all pixels \(p \in P\), \(\text{containsPixel}(n, p)\) and \(\text{containsPixel}(n', p)\) return the same value. Constraints defined for ARGA can be found in Appendix Table 10.

Given a set of constraints \(C\) that must be satisfied and a node in the search tree with a set of graphs \(\{g_{\text{input}_i}\}\) for \(i \in \{1, \ldots, m\}\), the search space is pruned as follows. Suppose we have a full operation \(o\) that selects \(n\) from \(g_{\text{input}_i}\) with filter operation \(f\) and transforms it with operation \(t\) to produce updated node \(n'\). If \(\exists c \in C \land c(n, n') = \text{False}\), then the branch in the search tree created by applying \(o\) to \(\{g_{\text{input}_i}\}\) is pruned, as it does not satisfy constraint \(c\).

**Constraint Acquisition**

To obtain a set of constraints to prune the search space, we introduce a simple constraint acquisition algorithm inspired by the ModelSeeker (Beldiceanu and Simonis 2012) and Inductive Logic Programming (Lallouet et al. 2010).

We have the generic constraint \(constraint(n : Node, n' : Node)\) where \(n, n'\) can be understood as a node before and after modification by a transformation. To determine the constraints that must hold for a particular ARC task from the set of all possible constraints, we compare the training output images to the corresponding input images.

While expanding a node in the search tree, we apply the same abstraction process for the output images \(\{g_{\text{output}_i}\}\) as the input images to obtain \(\{g_{\text{input}_i}\}\) for \(i \in \{1, \ldots, m\}\). Then, for each full operation \(o \in O\), we apply its filter operation \(f\) to \(g_{\text{input}_i}\) and \(g_{\text{output}_i}\) to obtain pairs of \(n_{in}\) and \(n_{out}\). For each constraint \(c\), if \(c(n_{in}, n_{out})\) evaluates to True for all pairs found by \(f\), we say that constraint \(c\) must be satisfied for all nodes selected by filter \(f\). Therefore, all full operations \(o\) with filter \(f\) and transformation \(t\) that violate constraint \(c\) can be pruned.

**Hashing**

It is highly likely that different transformations or sequences of transformations result in the same abstracted graph. To avoid duplicate search efforts, we hash each node in the search tree so that equivalent nodes are only explored once. The search tree therefore has the structure of a Directed Acyclic Graph. An example of this is shown in Figure 2.

**Tabu List**

In our current implementation, abstracted graphs from different abstractions share the same search tree. It is therefore possible that greedy best-first search will get stuck in unpromising local solutions. To avoid this, we implement a simple Tabu List, which keeps tracks of the performance of each abstraction. If an abstraction is generating increasingly worse results, we temporarily place it on the Tabu List so that no nodes with this abstraction will be explored.

**Experiments**

Chollet (2019) states that the ARC aims to evaluate “Developer-aware generalization”, and all ARC tasks are unique and do not assume any knowledge other than the core priors. Therefore, implementing and evaluating ARGA on a subset of ARC tasks should provide useful insight into the effectiveness of our method without the need for extensive development of transformation functions, which are not the focus of our contribution.

We focus on a subset of 160 object-centric tasks from the ARC and categorize them into three groups: (1) **Object Re-coloring** tasks, which change colors of some objects in the input image; (2) **Object Movement** tasks, which change the position of some objects in the input image; (3) **Object Augmentation** tasks, which expand or add certain patterns to objects from the input images. An example task from each of the three sub-categories is shown in Figure 1.

For comparison, we evaluated the Kaggle Challenge’s first-place model (top quarks 2020) on the same subset of tasks. The model was executed without the time limitation enforced by the competition and the highest-scored candidate produced by the model was used to generate the final prediction.

**Results**

The performance of ARGA and the Kaggle competition’s first-place solution are shown in Table 4. With the exception of Object Movement tasks, our model performed slightly worse than the Kaggle winner in terms of accuracy. This is likely due to the solution space spanned by our DSL not being expressive enough, as it was developed using only a subset of the 160 tasks. On the other hand, the DSL used in the Kaggle solution was developed by first manually solving 200 tasks from the ARC (top quarks 2020).

Despite lower accuracy, ARGA achieves much better efficiency in search as we are able to reach the solution with 3 order magnitude fewer nodes explored. This suggests that with a more expressive DSL and a more efficient implementation, ARGA should be able to solve more tasks with much less search effort (ARGA is currently implemented in Python while the Kaggle solution is implemented in C++).

Furthermore, the gap between the number of tasks for which all training instances are solved (# Training Correct) and the number of tasks for which the single test instance is solved (# Testing Correct) is much smaller for ARGA. This suggests that ARGA is better at finding solutions which generalize correctly while the Kaggle solution often overfits to the training instances.

**Ablation Study**

Table 5 shows the performance of different variations of ARGA; the accuracies are reported on all 160 tasks. We see that the use of constraint acquisition is very effective in reducing the search space, resulting in 38% lower average nodes explored before reaching the solution.
There have been many attempts at solving the ARC. Most of those have shown some success, leveraging a DSL within the program synthesis paradigm (Kaggle 2020). It has been shown that humans are able to compose a set of natural language instructions that are expressive enough to solve most of the ARC tasks, which suggests that the ARC is solvable with a powerful enough DSL and an efficient program synthesis algorithm (Johnson et al. 2021). Indeed, this is the approach suggested by Chollet (2019) when introducing the dataset: “A hypothetical ARC solver may take the form of a program synthesis engine” that “generate candidates that transform input grids into output grids.”

Solutions using this approach include the winner of the Kaggle challenge, where the DSL was created by manually solving ARC tasks and the program synthesis algorithm is a search that utilizes directed acyclic graphs (DAG). Each node in the DAG is an image, and edges between the nodes are transformations (top quarks 2020). The second-place solution introduces a preprocessing stage before following a similar approach (de Miquel Bleier 2020). Many other Kaggle top performers share this approach (Golubev 2020; Liukis 2020; Penrose 2020). Fischer et al. (2020) propose a Grammatical Evolution algorithm to generate solutions within their DSL. Alford et al. (2021) utilize an existing program synthesis system called DreamCoder (Ellis et al. 2020) to create abstractions from a simple DSL through the process of compression. The program then composes the solution for new tasks using neural-guided synthesis.

Other approaches to solving the ARC include the Neural Abstract Reasoner, which is a deep learning method that succeeds in a subset of the ARC’s problems (Kolev, Georgiev, and Penkov 2020). Assouel et al. (2022) developed a compositional imagination approach which generates unseen tasks for better generalization. Ferré (2021) develops an approach based on descriptive grids. However, these approaches have not achieved state-of-the-art results.

### Constraint Acquisition (CA)

CA is a field that aims to generate Constraint Programming (CP) models from examples (De Raedt, Passerini, and Teso 2018). State-of-the-art CA algorithms may be active, requiring interaction from the user (Bessiere et al. 2013; Arcangioli, Bessiere, and Lazaar 2016), or passive, requiring only initial examples (Bessiere et al. 2005).

The passive CA algorithm used for ARGA was influenced by ModelSeeker (Beldiceanu and Simonis 2012), which finds relevant constraints from the global constraint catalog (Beldiceanu, Carlsson, and Rampon 2005) as well as the system developed by Lalouet et al. (2010) which uses Inductive Logic Programming (ILP) and formulates constraints from logical interpretations.

### Conclusion

We proposed Abstract Reasoning with Graph Abstractions (ARGA), an object-centric framework that solves ARC tasks by first generating graph abstractions and then performing a constraint-guided search. We evaluated our framework on an object-centric subset of the ARC dataset and obtained promising results. Notably, the efficiency in reaching the solution within the search space shows that with further development of the DSL, our method has the potential to solve far more complicated problems than state-of-the-art methods.
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