Spin-wave modes of elliptical skyrmions in magnetic nanodots

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Abstract
Magnetic skyrmions, whose shapes are ellipse due to the presence of anisotropic Dzyaloshinskii–Moriya interaction (DMI), have already been discovered in experiments recently. By using micromagnetic simulations, we discuss the ground state and the spin-wave modes of a single elliptical skyrmion in a confined nanodot. It is found that the shapes of skyrmion are stretched into a horizontal ellipse, vertical ellipse, or stripe shape under different strengths of anisotropic DMI. When elliptical skyrmions are excited by in-plane ac magnetic fields, the spin-wave mode contains a counterclockwise rotation mode at high frequencies and a clockwise (CW) rotation mode at low frequencies, and the CW mode depends on the strength of anisotropic DMI. When elliptical skyrmions are excited by out-of-plane ac magnetic fields, the spin-wave mode is split from a simple breathing mode into two complex breathing modes, including a mixed mode of CW rotation and breathing, and another anisotropic breathing mode. Our results provide an understanding of the rich spin-wave modes for skyrmions, which may contribute to the applications in magnonics.

1. Introduction
Magnetic skyrmions, as chiral spin textures carrying a topological number, have attracted a lot of attention because of their topological properties and potential applications in spintronic devices [1–5]. In most cases, the Dzyaloshinskii–Moriya interaction (DMI) stabilizes magnetic skyrmions and determines the chiral structure of the skyrmions, typically, Bloch-type skyrmions are discovered in bulk non-centrosymmetric magnetic lattices due to the presence of bulk DMI [9]; Néel-type skyrmions are observed in ultrathin ferromagnetic/heavy-metal multilayered films lacking inversion symmetry due to the presence of interfacial DMI [10]. Recently, several works report that the shape of skyrmions is elliptical rather than circular when there is a spatial anisotropy in the DMI. For example, the elliptical skyrmions are investigated in quasi-two-dimensional systems [11] and the Heisenberg triangular antiferromagnet [12] by using Monte Carlo simulations; current-induced motion of elliptical skyrmions is performed by using micromagnetic simulations [13]; Néel-type elliptical skyrmions can be realized in an out-of-plane magnetized epitaxial Co/W(100) stack [14] and a laterally asymmetric magnetic multilayer [15]. Moreover, the shape of elliptical skyrmions can be controlled by an in-plane magnetic field [16], an out-of-plane magnetic field [17], an in-plane magnetic anisotropy [18], the temperature [19], or even confined nanostructures [20, 21] in addition to the anisotropic DMI.

In addition to the potential applications of skyrmions in spintronic devices, ac magnetic field-induced spin excitations of skyrmions are promising for the potential application in the field of magnonics. Three typical spin-wave modes have been reported both in the skyrmion crystal phase [22] and a single skyrmion in confined-geometry systems [23–25], i.e., two gyrotropic modes including the clockwise (CW) and counterclockwise (CCW) rotation modes are found for in-plane ac magnetic fields; a breathing mode is
excited for an out-of-plane ac magnetic field. In the CW or CCW modes, the skyrmion rotates CW or CCW around the center position periodically; in the breathing mode, the area of skyrmion extends and shrinks periodically. These modes have been experimentally verified by using microwave transmittance spectroscopy [26], all-electrical broadband spectroscopy [27], and all-optical spin wave spectroscopy [28]. Up to now, most researches have focused on the spin-wave modes of circular skyrmions with the isotropic DMI, while the spin-wave modes of elliptical skyrmions with the anisotropic DMI have not been reported. In this work, through using micromagnetic simulations, we started with the phase diagram of the ground state of elliptical skyrmions by changing the anisotropic DMI. Then we calculated the power spectrum and the corresponding spin-wave modes of the elliptical skyrmion. At last, we investigated the time evolutions of elliptical skyrmion excitations, which help to better understand each spin-wave mode.

2. Micromagnetic simulation details

Our micromagnetic simulation results are performed by using the Object Oriented MicroMagnetic Framework (OOMMF) public code [29], which contains the additional modules for the anisotropic DMI [13]. The magnetization dynamics is described by numerically solving the Landau–Lifshitz–Gilbert (LLG) equation, as follow:

$$\frac{dm}{dt} = -\gamma m \times H_{\text{eff}} + \alpha m \times \frac{dm}{dt},$$  

where $m$ is the unit vector of the local magnetization, $\gamma$ is the gyromagnetic ratio, the Gilbert damping $\alpha$ is set to 0.5 and 0.01 for the equilibrium state and the resonant excitation procedure, respectively.  

$$H_{\text{eff}} = -\frac{\partial}{\partial m \times \mu_0 \chi m}$$  

is the effective field derived from the free energy density $E$, which includes the exchange, the anisotropy, the demagnetization field, and the anisotropic DMI terms. The anisotropic DMI energy is given by [30]:

$$E_{\text{DM}} = D_x \left( m_x \frac{\partial m_z}{\partial x} - m_z \frac{\partial m_x}{\partial x} \right) + D_y \left( m_y \frac{\partial m_z}{\partial y} - m_z \frac{\partial m_y}{\partial y} \right),$$  

where $D_x$ and $D_y$ are the DMI energy parameters, as shown in figures 1(a) and (b). It can be seen that the interfacial anisotropic DMI is used in this work. As discussed in the introduction, the shape of the skyrmion can be manipulated in many methods, but when only considering the effect of the anisotropic DMI interaction on the shape of the elliptical skyrmion, we emphasize that the DMI can be anisotropic in the laterally asymmetric magnetic multilayer Ta/Pt/Co/Ta/Pt due to the presence of an interfacial anisotropic strain introduced by the oblique-angle deposition [15]. Moreover, in the out-of-plane magnetized Au/Co/W(100) with a $C_{6v}$ symmetry [14], the DMI strength along the bcc[110] is 2–3 times larger than that of along the bcc[001] direction, which can stabilize an isolated elliptical skyrmion in a magnetic thin film. To eliminate the influence of the boundary anisotropy on the shape of skyrmions, a circular nanodot system is considered in our simulations. The diameter and the thickness of the nanodot are respectively set to 100 nm and 0.4 nm with the mesh size of 1 nm.

In this work, we mainly focus on the spin-wave modes of a single elliptical skyrmion in a nanodot. To this end, the microwave magnetic field along the $x$ or $z$ directions is exploited to excite the spin-wave modes with different symmetries. The excitation field is a sinc-function type magnetic field, $h(t) = h_0 \text{sinc}(2\pi ft) = h_0 \sin(2\pi ft)/(2\pi ft)$ ($i = x$ or $z$), where the amplitude $h_0 = 10 \text{ mT}$ and the cut-off frequency $f = 50 \text{ GHz}$. The time evolution of the $x$ or $z$ component of the magnetization in each mesh is sampled every 2 ps with the total time of transient dynamics over 10 ns. After applying the excitation field, both the exciting magnetic field $h(t)$ and the magnetization distribution $m_i$ are transformed from the time domain to the frequency domain by using the fast Fourier transform (FFT) method, then we can obtain the resonance frequencies, i.e., the imaginary part of susceptibility ($\text{Im } \chi$), and the spatial distribution of the FFT power and phase of each magnetic component at different resonance frequencies. Correspondingly, depending on
3. Results and discussion

3.1. Ground state of elliptical skyrmions

We first investigated the ground states of the magnetic nanodot, where a single skyrmion is set in the center of the nanodot as the initial state. Figure 2 shows the different magnetization distributions of the nanodot, where $D_y$ is fixed to 3.5 mJ m$^{-2}$ and $D_x$ varies from 1.1 mJ m$^{-2}$ to 4.9 mJ m$^{-2}$. For $D_x \leq 1.1$ mJ m$^{-2}$, the skyrmion disappears from the nanodot although $D_y$ is large, i.e., the ground state is a ferromagnetic uniform state (FM), as shown in figure 2(a). For 1.1 mJ m$^{-2} < D_x < 3.5$ mJ m$^{-2}$, the skyrmion stably exists in the nanodot with the form of a horizontal elliptical skyrmion (H-SKY) due to $D_x < D_y$, as shown in figure 2(b). For $D_x = 3.5$ mJ m$^{-2}$, the skyrmion shape is circular due to $D_x = D_y$, as shown in figure 2(c). For $3.5$ mJ m$^{-2} < D_x < 4.9$ mJ m$^{-2}$, the skyrmion is stretched in the vertical direction, which is called vertical elliptical skyrmion (V-SKYR), as shown in figure 2(d). For $D_x \geq 4.9$ mJ m$^{-2}$, the skyrmion is further stretched into a SD, as shown in figure 2(e).
3.2. Map of power spectral density of elliptical skyrmions
Considering that we investigate the spin-wave modes of elliptical skyrmions, in order to guarantee the existence of skyrmion in the nanodot in the following simulations, the power spectral density phase diagram of the elliptical skyrmion is obtained by varying the value of $D_x$ in the range of 2.3 to 4.6 mJ m$^{-2}$ with $D_y$ fixed to 3.5 mJ m$^{-2}$, as shown in figure 3, which can help us better understand the effect of changes in the shape of the skyrmion on the resonant frequency. Figure 3(a) shows the in-plane power spectral density phase diagram of the elliptical skyrmion. One strong resonance mode 1 first decreases from 49.6 to 25 GHz and then remains unchanged with $D_x$ increasing linearly from 2.3 to 4.6 mJ m$^{-2}$. In addition, a weak resonance mode 2 with the lowest frequency appears when $D_x$ is larger than 3.6 mJ m$^{-2}$, and its resonance frequency linearly increases from 0.5 to 2.7 GHz with the increase in $D_x$. We later demonstrate that mode 1 and mode 2 are CCW and CW rotation modes, respectively. The out-of-plane power spectral density phase diagram of the elliptical skyrmion is seen in figure 3(b). When $D_x = D_y$, there is a single strong resonance mode at 7.8 GHz, which is the breathing mode of the circular skyrmion (see the appendix B). When $D_x \neq D_y$, the shape of the skyrmion is stretched into an ellipse, the rotational symmetry of the skyrmion texture is broken, the breathing mode is therefore split into two complex resonance modes. One split resonance mode 3 varies slightly with $D_x$, i.e., the resonance frequency first decreases from 13.2 to 7.5 GHz and then increases to 11.5 GHz with $D_x$ increasing linearly from 2.3 to 4.6 mJ m$^{-2}$. The other split resonance mode 4 decreases from 69.7 to 2.7 GHz as $D_x$ increases from 2.3 to 4.1 mJ m$^{-2}$.

3.3. In-plane and out-of-plane spin-wave modes of an elliptical skyrmion
Then we take the two in-plane resonance frequencies (0.5 and 29.3 GHz) and the two out-of-plane resonance frequencies (3.2 and 7.5 GHz) marked by the gray circles in figure 3 as an example to investigate the spin-wave modes of the elliptical skyrmion at each resonance frequencies in detail, as shown in figure 4. Let us first look at the two in-plane spin-wave modes of the elliptical skyrmion. For $f = 0.5$ GHz, the resonance amplitude of the $\delta m_z$ magnetization component is the largest with the spatial distribution of an elliptical ring region, whose phase changes continuously from $-\pi$ to $\pi$. For $f = 29.3$ GHz, the spatial distribution of resonance amplitude is the same as the case of 0.5 GHz, but the phase of the $\delta m_z$ changes continuously from $\pi$ to $-\pi$, which is opposite to the case of 0.5 GHz. Following, we investigate the two out-of-plane spin-wave modes. The spatial distribution of the resonance amplitude of the $\delta m_z$ is a double crescent shape and an elliptical ring shape at 3.2 and 7.5 GHz, respectively. The corresponding phase has a $4\pi$ jump change at 3.2 GHz and has a slight change at 7.5 GHz.

3.4. Gyrotropic modes and breathing modes of an elliptical skyrmion under a microwave magnetic field
To further identify each spin-wave mode shown in figure 4, we trace the magnetization component of the elliptical skyrmion at different times by applying a sin-function oscillation magnetic field with different
resonant frequencies, as shown in figure 5. The sin-function oscillation magnetic field is set by the form
\[ B_i(t) = B_i \sin(2\pi ft) \] (i = x or z), where \( B_i = 10 \) mT, \( f \) is the resonance frequency of each in-plane and out-of-plane spin-wave modes. For the purpose of better observing the evolution of the elliptical skyrmion under the sin-function microwave magnetic field, we use \( \delta m_z(t) \) to present the snapshots at different times, where \( \delta m_z(t) \) is the difference between the time-varying \( z \)-direction magnetization component and the initial state \( z \)-direction magnetization component, i.e., \( \delta m_z(t) = m_z(t) - m_z(0) \).

Figures 5(a) and (b) show that the in-plane spin-wave modes of the elliptical skyrmion at 0.5 GHz and 29.3 GHz are CW and CCW rotation modes, respectively. In order to compare the difference between the spin-wave modes of an elliptical skyrmion and a circular skyrmion, we give the in-plane spin-wave modes of a circular skyrmion with different \( D \) in appendix A. For the case where \( D \) is small, such as \( D = 3.5 \) mJ m\(^{-2}\), only the CCW mode exists; when the DMI increases to 3.8 mJ m\(^{-2}\), a CW mode appears at the low resonance frequency. The interaction between the skyrmion and the nanodot boundary is responsible for exciting the CW mode, which is similar to the interaction between the skyrmions in the skyrmion lattice phase [32]. We also calculate the out-of-plane spin-wave modes of a circular skyrmion and find that there is only one breathing mode (see appendix A), which is due to the rotational symmetry of
both the nanodot structure and the spin texture. While for the elliptical skyrmion with the anisotropic DMI, there are two out-of-plane spin-wave modes, as shown in figures 5(c) and (d). Figure 5(c) shows the out-of-plane spin-wave mode of the elliptical skyrmion at a resonant frequency of 3.2 GHz. From five snapshots taken at different times in one period, it can be seen that the shape of the \( \delta m_z \) alternately changes to a circle and an ellipse with CW rotation, which means that the out-of-plane spin-wave mode at 3.2 GHz of the elliptical skyrmion is a mixed mode of CW rotation and breathing. Figure 5(d) shows that the out-of-plane spin-wave mode of the elliptical skyrmion at the resonance frequency of 7.5 GHz is also a type of breathing mode, whose magnitude of the contraction and expansion of the elliptical skyrmion is spatially anisotropic, i.e., the shape of the elliptical skyrmion is circular when expanding to its maximum and elliptical when contracting to its minimum, this breathing mode is therefore called an anisotropic breathing mode. Compared with the breathing mode of the circular skyrmion in appendix B, although the shape of the nanodot still has rotational symmetry, the rotational symmetry of the spin texture of the elliptical skyrmion is broken, which is the reason why the breathing mode is split into two complex breathing modes.

4. Conclusions

In summary, we first investigate the magnetic structure of a single elliptical skyrmion in a confined nanodot with the anisotropic DMI of different strengths. Following, we particularly focus on the in-plane and out-of-plane spin-waves modes of elliptical skyrmions. For in-plane spin-wave mode, the elliptical skyrmions exhibit a CCW rotation mode at the high resonance frequency, and exhibit a CW rotation mode at the low resonance frequency only when the anisotropy DMI is large enough. For out-of-plane spin-wave mode, the circular skyrmion presents a breathing mode, and as the skyrmion shape is stretched into an ellipse, the breaking of the rotational symmetry of the spin texture causes the simple breathing mode to split into two complex breathing modes, i.e., a mixed mode of CW rotation and breathing, and an anisotropic breathing mode. These results may provide guidance for manipulating elliptical skyrmions by using spin-wave resonance.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

Appendix A. In-plane spin-wave modes of circular skyrmions

In comparison to the in-plane spin-wave modes of the elliptical skyrmion \((D_y = 3.5 \text{ mJ/m}^2\) and \(D_x = 3.8 \text{ mJ/m}^2\)), we have investigated the in-plane spin-wave modes of two circular skyrmions with \(D = 3.5\) and \(3.8 \text{ mJ/m}^2\), respectively. Figure A1(a) shows that the spectrum of two circular skyrmions with different \(D\) when a sinc-function type magnetic field is applied along the \(x\) direction. For \(D = 3.5 \text{ mJ/m}^2\), there is only one resonant peak with a resonant frequency of 36.4 GHz, which corresponds to a CCW rotation mode of skyrmions. For \(D = 3.8 \text{ mJ/m}^2\), in addition to a resonance peak at 27.2 GHz, an additional resonance peak with a resonance frequency of 0.9 GHz appears at low frequencies, the spin wave-modes at 0.9 and 27.2 GHz are respectively a CW and CCW rotation modes of the skyrmion, which can be demonstrated in figures A1(b) and (c). Figure A1(b) shows the in-plane spin-wave modes of the circular skyrmion at different resonance frequencies with different DMI strengths. The spatial distributions of the resonance amplitude of the \(\delta m_x\) are all a ring shape at three resonance frequencies. However, when \(f = 36.4\) and 27.2 GHz, the phase changes continuously from \(\pi\) to \(-\pi\). When \(f = 0.9\) GHz, the phase changes continuously from \(-\pi\) to \(\pi\), which indicates that the skyrmion has different gyrotropic modes. Figure A1(c)
Figure A1. In-plane spin-wave modes of two circular skyrmions. (a) Imaginary part of the in-plane susceptibility spectrum with $D = 3.5$ and $3.8 \text{ mJ/m}^2$. (b) Spatial distribution of the $x$, $y$, and $z$ components of the FFT power and phase at different in-plane resonance frequencies with different DMI strengths. (c) Spin dynamics of the circular skyrmions corresponding to the three spin-wave modes in figure A1(b).

Appendix B. Out-of-plane spin-wave mode of a circular skyrmion

We have also calculated the out-of-plane spin-wave mode of a circular skyrmion, as shown in figure B1. Figure B1(a) shows the spectrum of a circular skyrmion with $D = 3.5 \text{ mJ/m}^2$ when a sinc-function type magnetic field is applied along the $z$ direction. There is only one resonant peak with the value of 7.8 GHz corresponding to a breathing mode of the circular skyrmion. Figure B1(b) shows the spin-wave mode of the circular skyrmion at this resonance frequencies in detail, the spatial distribution of the resonance amplitude of $\delta m_z$ is a ring shape and the phase has a constant value. By applying a 7.8 GHz sin-function oscillation magnetic field along the $z$ direction, the out-of-plane spin-wave mode proved to be a breathing mode, as shown in figure B1(c).
Figure B1. Out-of-plane spin-wave mode of a circular skyrmion. (a) Imaginary part of the out-of-plane susceptibility spectrum with $D = 3.5 \text{ mJ/m}^2$. (b) Spatial distribution of the $x$, $y$, and $z$ components of the magnetization, FFT power, and phase at the out-of-plane resonance frequency of 7.8 GHz. (c) Spin dynamics of the circular skyrmion corresponding to the spin-wave mode in figure B1(b).

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