Characteristic profile of analytical thinking in mathematics problem solving

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Abstract. Analytical thinking is a thinking ability to help individuals in solving problems of mathematics. It is important for understanding the parts of situation, the ability to scrutinize and breakdown facts. However, there is a differentiation or variation in the way to solve these problems. The differentiation or variation is described as the characteristics of analytical thinking. The characteristics consisted of pre-analytical, partial-analytical, semi-analytical, and analytical. This study aims at describing analytical thinking characteristic profile of high school students in problem solving using two dimension materials. This is a qualitative study. The participants of this study are two high ability students at the eleventh grade of Public Senior High School 1 Kedungwaru Tulungagung. Think Aloud Method is applied to collect the data. The findings showed that the high-ability students have pre-analytical thinking characteristic when they are understanding the problems, planning the steps, and checking back the answer, and they have semi-analytical and pre-analytical thinking characteristic when they are implementing the plan. As the conclusion, the high-ability students have two out of four analytical thinking characteristics, namely pre-analytic and semi-analytic.

1. Introduction
In the 21st century, competition in the field of education is very strict. Education is one of the ways to enhance the degree of human life in this dynamic era. Government increases the quality of education by improving the curriculum in the various fields. National Education Standards Agency accommodates four characteristics of 21st century learning called 4C. These characteristics are creativity, critical thinking, collaboration, and communication skills [1]. One area of education that cannot be separated from the improvement is mathematics education.

Mathematics education is a basic science that has an important role in the development of science and technology since it covers a way to solve problems. As stated by Pimta, Tayruakham, and Nuangchaleri, they view that problem solving is considered as the heart of mathematics learning because it emphasizes on the development of thinking ability method instead of study the subjects [2]. In accordance with the purpose of learning mathematics related to the problem solving, each student will certainly encounter problems in mathematics learning. Although problem solving requires high-level thinking, but the problem-solving ability can actually be trained [3].

In Indonesia, students' mathematics ability in understanding and absorbing information is still relatively slow. Based on a survey of the Program of International Students Assessment in 2015 with the evaluated materials is measuring the ability of science, reading and mathematics, the results showed that Indonesia ranked 63 out of 69 countries on mathematics ability. Another International survey,
**Trends International Mathematics and Science** revealed that Indonesia ranked 36 out of 49 countries. Another research on mathematics ability is also conducted by Atmojo. The results revealed that the low-ability students are incomplete in understanding the problem because of the imperfection in the assimilation thinking process [3].

The results of the survey indicated that the Indonesian students’ achievement is not good enough. The students are lack of understanding and they are also lack of the way to solve mathematics problems. Another factor that influences students’ achievement is the differentiation or variation in solving the problems. The students encounter various difficulties depending on how they use the thinking ability that have been learned during the learning process.

Thinking ability is one of the important components in the 21st century. Analytical thinking is a way of thinking ability to overcome mathematics problems. According to Amer, "Analytical thinking is a powerful thinking tool for understanding the parts of the situation, is the ability to scrutinize and break down facts and thoughts into their strengths and weaknesses" [4]. In accordance with the explanation, Angela proposes that analytical thinking can be characterized by a virtue of a systematic approach [5]. Moreover, analytical thinking implies logical connection by encoding reality into abstract symbol, words, or numbers [6].

Analytical thinking is very crucial in the truth of completion. According to Sujadi, he states that analytical cognition explains the truth of a statement because it becomes evidence that analytical cognition is a non-intuitive cognition [7]. Furthermore, Syam states that critical thinking is analytical thinking, deliberate and involving original thinking [8]. This is supported by Kahneman in Rusou. He states that analytical mode can be started by monitoring, validating, correcting, or overriding or ignoring initial intuitive responses [9]. However, this is not entirely contrary to intuitive, for example reading is a complex ability, but reading is something that we usually do without a lot of analytical thinking [10]. In coincidence with the experts’ opinion, analytical thinking is essential to help students in solving mathematics problems.

Moreover, there are other researches in the field of analytical thinking, such as Parta’s research. The results of Parta’s research revealed that analytical thinking has characteristics, namely pre-analytical, analytical-partial, semi analytical, and analytical (complete analytics). Each characteristic has indicators. The indicator of pre-analytical is the use of standard procedures in the form of pictures. The partial-analytic is characterized by the breakdown of one part of analytical procedures and other analytical procedures. The indicator of semi-analytical is characterized by "bending" procedures and "vague" or duplicate procedures. Analytical (complete analytics) is marked by the clarity of the algorithm, the demands of reasoning, and the existence of the essential statement that underlies the process [11].

In solving problems, the students use different ways. There are students who use routine steps, and the other one use non-routine steps. Students tend to use a conscious mental process in the form of analytical thinking process to solve problems. They also use non-routine steps by employing mental activities with formal cognition in solving problems. Chrysostomou states that there are many factors affected students’ ability in solving problems, such as intelligence, logical thinking skill, creativity and cognitive style, personality, values, attitudes and interests [12]. Another idea proposed by Macchi, he describes analytical thinking as the thinking process in information using different ways, depending on the characteristics used by the subject [13]. However, cognitive researcher usually views mistakes as unwanted but unavoidable feature of the human mind [14].

In coincidence with the phenomenon above, it can be indicated that there is a differentiation in thinking activities that involves analysis which is used by the students. As a result, the researcher wants to dig more deeply related to the students' thinking activities used in solving mathematics problems. If the students who have high mathematics thinking ability make many mistakes in solving mathematics problems, it is also possible for students with moderate and low mathematics thinking ability produce similar mistakes. Therefore, the researcher is interested in investigating students' thinking activities in Geometry materials for two dimension.
2. Method
The data obtained from this study are written and verbal data, so this study employed qualitative. The objective of this study is to describe analytical thinking characteristic profile of high school students who have high mathematics ability in problem solving using two dimension materials. Two high ability students at the eleventh grade of Public Senior High School 1 Kedungwaru Tulungagung involved as the subjects of this study. The researcher selected the participants through purposive sampling by choosing students who have good ability in communicating their ideas both in writing and oral. Think Aloud Method is applied to collect the data. The data are obtained by asking students to solve the mathematics problems accompanied by verbal expression about the thoughtful ideas [15]. Vygotsky describes language as an analytical thinking tool and the mind is not only expressed in the form of words but also appears through words [16].

In this study, there are two types of instruments used to collect the data. The first is the researcher itself as the main instrument and the second is supporting instruments in the form of worksheets, and unstructured interview guidelines. The data are analyzed based on written test and interview. The worksheet consists of a set of questions as follow: 1) there are 4 isosceles trapezoid combined on the hypotenuses will form an outer and inner square. The outer square has an area of 100 cm$^2$. The height of trapezoid is 3 cm. Calculate the area of the inner square!; 2) there are 6 equally large trapezoid having a 90-degree angle will form rectangle. The rectangle has a length of 15 cm and an area of 9 cm. Calculate the area of one trapezoid!

The researcher took the data twice in order to get the valid data. The valid data are compared with the indicators of analytical thinking ability between two subjects. The collected data is analyzed based on Miles and Huberman’s theory by the following steps: 1) Data reduction consists of selection process, simplification process, and then extracting and transforming the raw data; 2) Data display by classifying and identifying the data until the data are organized; 3) Conclusion based on the results. After that, the researcher applied method triangulation to get the valid data.

3. Results and discussion
In the answer sheets given, the subjects answer the questions related to the two dimension problem and explain the idea verbally. Here are some descriptions of written data and verbal transcripts of subject S1 in figure 1 and figure 2, whereas the subject S2 in figure 3 and figure 4.

![Figure 1](image1.png)
![Figure 2](image2.png)

**Figure 1.** The answer of Subject 1 on the first question.  **Figure 2.** The answer of Subject 1 on the second question.

Figure 1 shows that the subject can solve the problem and explain the steps. The subject writes unnecessary answer for the next step. The following is a quote of verbal data related to the analytical thinking solving in two dimension material. The subject says, "From this calculation, the area of all trapezids is 84. It means a is equal to 4, so that the area of inner square is 4 times by 4 is equal to 16."
The result of the inner square area is 16". Based on the explanation of the subject, subject 1 uses the steps marked by a turning procedure or a vague procedure.

Figure 2 shows that the subject solves the problem as it has been done on the first question. The subject writes unnecessary answers for the next step. When the subject executes the plan, he writes down the area of two trapezoids. However, the subject does not use it in the next step.

The subject reveals his idea when solving a given problem. The following is a quote of verbal data related to analytical thinking solving in two dimension materials. The subject says, "On the area of two trapezoids is 45. So to determine the trapezoid area by the formula of rectangle which is divided into two. At the length of a plus b is 9 and the height is 5. Then, 9 times by 5 is 45, and 45 divided by 2 is 22.5. The result of 1 trapezoid having a 90-degree angle area is 22.5 cm²".

The subject explanation related to the way in determining the area of 1 trapezoid, the previous step is not used for the next step. The result of the subject, subject 1 uses the steps marked by a turning procedure and a vague procedure. There are steps that should be unused or unnecessary to be applied become the whole part in solving the problem. Based on the test results data and verbal data, they are analyzed and triangulated to get the valid data. So it is found that the subject 1 in solving the problem on the first and the second question uses the steps marked by a turning procedure or a vague procedure.

![Figure 3. The answer of Subject 2 on the first question.](image1)

![Figure 4. The answer of Subject 2 on the second question.](image2)

Figure 3 shows that the subject can solve the problem and explain the steps. The subject solves the problem briefly. When the subject executes the plan, the subject reveals his idea with a key problem in order to be resolved quickly. The following is a quote of verbal data by subject 2 related to the analytical thinking solving in two dimension materials. The subject says, "In essence, the trapezoid's height is 3 cm just like this image, so that the inner square sides is 10 reduced by the isosceles trapezoid’s height and reduced by the isosceles trapezoid’s height in the front. Then we get the sides of inner square is 4. So that the area of inner square is 4 times by 4, namely 16". The subject description of Subject 2 in solving the problem depends on what the subject has been described.

The results of test and interview in the first and second process is analyzed and triangulated in order to get the valid data. And then the valid data used to identify students’ characteristic of analytical thinking. The explanation is presented in the table below.

Figure 4 shows that the subject solves the problem as it has been done in the first problem. The subject solves the problem briefly. The following is a quote of verbal data by subject 2 related to the analytical thinking solving in two dimension material. The subject says, "Then, from this figure, first, determine the area of the second square which is smaller than the first square, then the initial area is divided by 3. Thus, 135 is divided by 3 equal to 45. In this figure, the area of the rectangle is divided by 2 trapezoids having a 90-degree angle. So, the area of trapezoid obtained from the area of a small
rectangle divided by 2 equal to 22.5°. The description of Subject 2 depends on what has been described by the subject when solving the problem.

Based on the test results data and verbal data, they are analyzed and triangulated to get the valid data. Thus, it is found that subject 2 in solving the problem on the first and second questions uses the steps depends on what is being described.

Based on the valid data, then it is used to know the characteristics of students’ thinking ability. The valid data can be described in the table 1 as follows:

| Subject 1 | Subject 2 |
|-----------|-----------|
| The student is able to make plan through steps in the form of pictures. | The student is able to make plans through steps in the form of pictures. |
| The student is able to deliver information obtained from the mathematics problems given by the researcher. | The student is able to deliver information in the form of pictures obtained from the mathematics problems given by the researcher. |
| The student is able to mention and explain what is being known and what is being asked in the mathematics problems. | The student is able to mention and explain what is being known and what is being asked in the mathematics problems. |
| The student is able to answer the mathematics problems correctly. However, he used inappropriate steps in solving problems. | The student is able to answer the mathematics problems correctly. He tended to focus on using basic information accompanied with pictures. |
| The student is able to correct the answer by explaining the strategy in the form of pictures to solve the problems. | The student is able to correct the answer by looking the steps that have been done using the pictures that was created in the beginning of solving problems. |

Based on the table 1, then the researcher compared the results with the indicators of analytical thinking ability which is explained by Parta [11]. The results are presented as follow.

- In understanding the problems, the subject 1 and the subject 2 have pre-analytical characteristic because they presented what is being known and what is being asked using information in the form of pictures.
- In making plans, the subject 1 and the subject 2 have pre-analytical characteristic because they deliver the plans based on the intended pictures. It can be seen from the written work and verbal explanation of subjek 1 and subjek 2 during the process of solving problems. The subject 1 said that the steps in solving problems are drawing rectangle and dividing it into 6 parts in the form of trapezoid having a 90-degree angle. Meanwhile, the subject 2 said that he drew six trapezoids having a 90-degree angle to form a rectangle.
- In doing the plans, there is a differentiation between the subject 1 and the subject 2. They have different characteristic of analytical thinking. The subject 1 has semi-analytical characteristic, whereas the subject 2 has pre-analytical characteristic. It can be seen from the written work and verbal explanation of subjek 1 and subjek 2 during the process of solving problems. The subject 1 used inappropriate steps in solving problems, while the subject 2 focused on using basic information accompanied with the pictures.

In correcting the answer, the subject 1 and the subject 2 have pre-analytical characteristic because they check back the answer using the pictures that was created in the beginning of solving problems.

4. Conclusion

Based on the analysis and explanation, it can be inferred that analytical thinking characteristic of high-ability students is pre-analytical in understanding the problems, planning the steps, and checking back the answer. However, there is a differentiation between the subject 1 and the subject 2 in doing a plan. The subject 1 has semi-analytical characteristic, whereas the subject 2 has pre-analytical characteristic.
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