Beamstrahlung at the NLC

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The detection of beamstrahlung visible light, divided in its polarization components, effectively images the beam-beam collision (BBC). Monitoring and correction of drifts are reviewed. Monitoring of beam jitter is also possible. The properties of coherent beamstrahlung in the microwave part of the spectrum (and its usage) are introduced.

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I. INTRODUCTION.

There has been a lot of simulation and theoretical work about beamstrahlung at future linear colliders over the years, yet that work only scratches the surface of beamstrahlung phenomenology. Beamstrahlung is of interest to the particle physicist, who needs to know the energy distribution of colliding beam particles at collision time (dL/dE), and to the accelerator physicist who must make the beams collide and then steer the spent beams out of the Interaction Region.

In a series of recent papers we have made clear that recovering complete information on low energy beamstrahlung effectively recovers most of the available information about the BBC in $e^+e^-$ colliders[1−3]. A large-angle infrared beamstrahlung detector is being built for CESR[3].

As remarked in Ref.[2], there are seven transverse degrees of freedom (dof) in the BBC that may decrease luminosity and need to be monitored (Fig. 1). In the following the discussion is restricted, without loss of generality, to the four BBC presented in Fig. 2.

Without the accurate measurement and monitoring of so many dof, even the most accurate of BBC simulations is of limited usefulness. For example, if the bunches collide perfectly (Fig. 2a) the particles in the center of each beam will have maximal fractional luminosity and will produce zero beamstrahlung. If the two beams are offset by 1.5 sigma (Fig. 2b), the particles in the center of each beam will have close to maximal fractional luminosity and close to maximal beamstrahlung. The dL/dE curve is vastly different in the two cases, yet both BBC types contribute usable amounts of luminosity.

The idea underlying the usage of low energy beamstrahlung is actually a simple one. Given the four Maxwell equations,

$$\nabla \cdot E = 4\pi \rho, \quad \nabla \times B - \frac{1}{c} \frac{\partial E}{\partial t} = 4\pi J, \quad \nabla \times E + \frac{1}{c} \frac{\partial B}{\partial t} = 0, \quad \nabla \cdot B = 0,$$

the beams are the currents, and beamstrahlung is the emitted EM field. The equations describe the correlation between currents and fields. We know the correlations, and we measure the field to figure out the currents. The fields are vectors, and that is why is necessary to measure their components. Fig. 3 shows the beamstrahlung polarization components for each beam, corresponding to the four BBC sketched in Fig. 2.

In practice, it is difficult to measure the polarization of photons of energy higher than UV, and that is why we limit ourselves to the study of low energy beamstrahlung in the present paper. Coherent beamstrahlung is available for observation in the microwave region, however in that case the polarization information is not meaningful (see Section IV).

This paper is written with three goals in mind:

1. to make it clear that large angle incoherent beamstrahlung (IB) is a necessary feature, if polarization
information is to be had (Section II). Because large angle observation is also used for background suppression at CESR, background suppression at the NLC is also briefly discussed;

2. to discuss the possibility of measuring beam jitter at the NLC (Section III);

3. to introduce coherent beamstrahlung (CB), its properties and its potential (Section IV).

II. LOW ENERGY BEAMSTRahlung PHENOMENOLOGY.

Some points need to be addressed in regard to the usage of this technique at the NLC. We show that polarization information at the NLC is as pristine as at CESR. We also discuss separation of signal and backgrounds and point out that there are four proposed methods to improve the S/B ratio.

A single bunch crossing will generate approximately $10^{12}$ visible photons at the NLC, a good statistics to work with. Very low energy photons have a much larger angular spread than the usual $1/\gamma$ angle. The total beamstrahlung power at the NLC is of order 1MW, a rate at which any kind of instrumentation is unlikely to survive. The radiation is mostly confined to a spot of order 1 mrad.

The Maxwell equations point out the need to measure the polarization of beamstrahlung. In the classical description of synchrotron radiation (SR), and therefore of beamstrahlung, the polarization is mostly carried by high energy photons (of energy comparable to the critical energy)[4]. Even at CESR the polarization of $\approx$10-keV X-rays can not be measured. At the NLC, a precision measurement of the polarization of 100-1000 GeV gamma rays is probably out of the question.

Whether one computes visible beamstrahlung by using the classical formulae[4] or by using the “short-magnet” formulae[5], the polarization content of the radiation, integrated over the solid angle, is virtually zero (Table I). Ref.6 discusses the validity of the two approaches (Ref.[4] versus Ref.[5]) in different regions of radiation phase space. By applying those formulae, we find that the “short magnet” approximation is most accurate for all visible beamstrahlung at the NLC, regardless of angle. It is used to produce the numbers in Table I.

Beamstrahlung yields suffer no significant quantum corrections at low energies, making classical formulae precise enough to be usable. The only quantum corrections arise from those beam particles which, having lost much of their energy through beamstrahlung, have special trajectories through the other beam. These corrections (Table I) are at the percent level and can be ignored throughout this paper.

FIG. 2: Three BBCs that lead to wasted luminosity; a) the beams overlap perfectly, no luminosity is wasted; b) a $y-$offset; c) $y-$bloating; and d) a beam-beam rotation. The “bad” beam is represented by the dashed ellipse.

FIG. 3: Beamstrahlung diagrams corresponding to the four pathologies of Figure 2. The dashed vectors in parts a) and b) are slightly displaced for display purposes.

However, Ref.[6] shows that, if an electron is subject to a transverse force, its large angle radiation is unpolarized as a whole but its azimuthal pattern at large angles
Table I: NLC nominal parameters and beamstrahlung yield for each bunch crossing[7].

| Parameter                        | Value               |
|----------------------------------|---------------------|
| Beam charge $N$                  | $7.5 \times 10^{10}$ e |
| Vertical beam width $\sigma_y$   | 3 nm                |
| Horizontal beam width $\sigma_x$| 243 mm              |
| Beam length $L$                  | 110 $\mu$m          |
| Beam energy                      | 500 GeV             |
| Beamstrahlung average energy loss| 5.4%                |
| Beamstrahlung yield,            |                     |
| $350 < \lambda < 700$ nm, $1 < \theta < 2$ mrad | $3 \times 10^8$ photons |
| Beamstrahlung polarization       |                     |
| $350 < \lambda < 700$ nm, $1 < \theta < 2$ mrad | 0. |
| Beamstrahlung yield, offset= $3 \sigma_y$ | $5 \times 10^8$ |
| $400 < \lambda < 500$ nm, $1 < \theta < 2$ mrad | $16 W$ |
| $\lambda > 100 \mu$m             |                     |

exhibits 100% linear polarization at 8 nodes (for an elementary derivation of these equations, see Ref[2]):

$$I_\perp(\theta, \phi, \omega) = I_0(\theta, \omega) \cos^2(2\phi), \quad (1)$$

$$I_\parallel(\theta, \phi, \omega) = I_0(\theta, \omega) \sin^2(2\phi), \quad (2)$$

where $I_\perp$ and $I_\parallel$ are the polarization components w.r.t. to the bending force. $\phi$ is the azimuthal angle with respect to the transverse force. At nodes equal to a multiple of $\pi/4$ the polarization is 100%, either parallel or perpendicular to the bending force.

In practice, the beams are 3-dimensional and flat (horizontal transverse size much larger than the vertical), with electric charge moving both horizontally and vertically as the BBC progresses. A fixed reference frame has to be chosen (naturally, one chooses the horizontal and vertical directions) and the spacetime charge distribution of the beams always generates some particle deflection along each of the axes. Eqs. (1-2) become

$$I_\perp(\theta, \phi, \omega) = I(\theta, \omega) \frac{U_x \cos^2(2\phi) + U_y \sin^2(2\phi)}{U_0}, \quad (3)$$

$$I_\parallel(\theta, \phi, \omega) = I(\theta, \omega) \frac{U_x \sin^2(2\phi) + U_y \cos^2(2\phi)}{U_0}. \quad (4)$$

The $U$ factors are form factors describing the integral over the beam charge distribution. The normalizing factor $U_0$ is introduced, which is the form factor (for either component) when the BBC is an exact overlap (Fig. 2a)) [2]. The same factors are the coordinates of the diagrams in Fig. 3.

At CESR, the detector has been placed at an angle of 10.4 mrad (or $\sim 100/\gamma$) [2]. Above angles $\sim 10/\gamma$, Eqs. (1-4) hold precisely, so that the polarization components are disentangled by extracting lights at large angle and appropriate azimuthal locations.

In the process, the large angle provides virtually all of the background suppression, using the fact that a "short magnet" (the beam) will produce a radiation cone far wider (in angle) than a "long magnet" (the various magnets of CESR) [1,2]. In short, at CESR the large angle does two things for the experimenter: make the polarization observable, and separate signal and background.

At the NLC [7], a beamstrahlung power of order 1MW imposes a stay-clear cone of 1 mrad (or $\sim 1000/\gamma$). Clearly at such a large angle the 8-fold pattern of Eqs. (1-4) will be available, however optically one can no longer hope to disentangle signal and background (assuming a diffraction-limited optical resolution of 1 mrad, as in the CESR case). Background will be rejected by other methods.

The visible beamstrahlung rates at the NLC for each bunch within the train are given in Table I, assuming full azimuthal acceptance. They are certainly abundant and capable of providing subpercent precision in the measurement of the beamstrahlung diagram.

The principal issue, as usual, is whether the backgrounds can be controlled. Three background rejection methods have already been suggested.

The first uses the fact that the beamstrahlung pulse is shorter than the coincident, SR background pulse by a factor of $2\sqrt{2}$. A streak chamber could disentangle the two components, with a possible background rejection of order $10^{-2}$.

A second method uses the fact that SR background tends to be strongly (90%) polarized radially. By extracting only tangential components, one could reduce backgrounds by one order of magnitude.

A much more powerful method than the previous two was the focus of Ref. [5]. An elliptical grating is the primary mirror, placed so that the Interaction Point (IP) is located at one of the ellipse foci, and the main collimator at the other focus. Such a device has extremely shallow field depth ($\sim 100 \mu$m at 10 meters distance). The background rejection is roughly equal to the number of lines in the grating ($\sim 10^3$). It has, however, also a very narrow frequency acceptance ($\sim 10^{-3}$), which may prove to be too large a signal reduction at the NLC.

Recently, I. Avrutsky [5] has systematically researched all possible methods of optical background rejection. He has found that whole-azimuth imaging, by a means of a ring-like mirror, offers at the same time a field depth of order one meter and a diffraction-limit of order 0.1 mrad. This method should allow background rejection at the $10^{-3}$ level, without any signal bandwidth loss.

III. MEASURING JITTER AT THE NLC.

The algorithm to make use of these diagrams was worked out in Ref. [2], and is summarized here. A set of four asymmetries, obtained directly from the diagrams of Fig. 3, is defined and ranked, and the feedback system acts when anyone of the asymmetries becomes significantly different from zero. The wasted luminosity is then expressed as a function of certain partial derivatives. Tuning of a single corrector magnet (dipole, quadrupole,
or sextupole) will correct any of the “pathologies” shown in Fig. 3. If more than one “pathology” exists, it was also proven that minimization of the asymmetries the order of their multipole ranking (dipole first) always converges to the proper overall correction.

The meaning of the diagram is that it can correct the various ways in which a beam can drift away from its nominal working point over time. The diagram both diagnoses and quantifies drift. It identifies which beam is going bad, which corrector magnet needs to be tuned, and by how much it needs to be tuned. If a machine drifts only (that is, if there is negligible beam jitter) it is important to notice that after a correction is applied, a new diagram is observed. That is why a 4-dimensional diagram monitors a 6-dimensional parameter space. The missing dof was identified with the smallest of the two $\sigma_y$, which can be measured by scanning one beam across the other (in the process, however, purely passive monitoring is lost).

When measuring jitter, control on the time evolution is lost. It is clear that the diagram will still work as a monitoring tool. For example, if the beam is oscillating between the BBC of Fig. 2a) and that of Fig. 2b) the diagram will be oscillating between those of Fig. 3a) and Fig. 3b). The diagram will still be able to pick out jitter components that other methods can not measure, and the diagram time evolution will be able to provide information about the frequency, waveform, and amplitude of the jitter.

There are, however, two limitations that arise in the case of jitter. The first and most obvious one is that each singular BBC needs to be recorded. We have seen in Table I that each BBC provides large statistics, and this is not expected to be a problem (for comparison, at CESR rates are integrated over 1 sec, or 17 millions BBCs). The second limitation is due to lack of control on the diagram evolution. The diagram jitters uncontrollably and the space it covers is equal to its own dimensionality, or four. Three dof are folded in without possibility of detection.

IV. COHERENT BEAMSTRahlUNG AT THE NLC.

If one wants to monitor the luminosity of a machine, any of several low-$Q^2$ QED processes can be used. For the purpose of discussion, let us consider $e^+e^- \rightarrow e^+e^-\gamma$. Most of these events (above a minimum angle) consist of one fermion and one photon at low angle in the same emisphere, balanced in $p_T$, while the other fermion continues down the beam pipe. One can reasonably speak of a radiating beam, the beam in the same hemisphere as the photon. The event rate in each emisphere, $R_{1,2}$, is proportional to the luminosity, and therefore to the product of the two beam populations

$$R_{1,2} \propto L \propto N_1N_2.$$  

When IB is considered, SR formulae are used. In SR theory, the power is proportional to the beam charge and proportional to the square of the bending force. This readily translates into a photon rate

$$R_1 \propto N_1N_2^2.$$  

When CB is used, the beam moves coherently under the influence of the EM field of the other beam. Radiation is proportional to the square of the emitting charge, so that

$$R_{1,2} \propto N_1^2N_2.$$  

A brilliant description of the coherent and incoherent limits for SR can be found in Ref. [10], which concisely derives the $N_1$ and $N_2^2$ factors in Eqs. (6-7).

Equations (5-7) show at a glance why beamstrahlung is preferable to quantum processes - the $N$ factors are huge numbers which make for more abundant, more precisely measured rates. As we will find in this Section, CB has other unique properties.

We have already noted at CESR that the overall $N^3$ dependence of IB does not favor the early development of the detector. Weak beams (a factor of ten below nominal) will result in a signal a thousand times weaker than nominal (at CESR, such a factor is enough to lower the signal down to the observed background rate). At the NLC there will be an extensive initial phase of machine development, with weak, relatively broad beams. CB provides the large enhancement needed to observe precisely such weak beams. This is a first, important property of coherent beamstrahlung.

The relativistically invariant coherence condition is

$$\int d^4x \rho(x)e^{ik\cdot x} \sim 1,$$

where $k$ is the observed photon 4-momentum, and $\rho(x)$ is the electron probability distribution in space-time (normalized to 1). Given that both the beam and the emitted photons are extremely longitudinal, the coherence condition becomes simply

$$R = \frac{\lambda}{\sigma_z} \sim 1.$$

If the wavelength is of order of the beam length, the radiation emitted will be coherent, otherwise it will be incoherent. At CESR, this translates to wavelengths greater than 1cm. At the NLC, the wavelengths of interest are those in excess of 0.1mm. The expected enhancement is also huge, of order $N_{1,2} \sim 10^{10}$. A second important property of CB is that one can reasonably expect it to be background-free. The SR from the magnets will be incoherent, and therefore much weaker, because the magnets are much longer than $\lambda$.

It becomes immediately clear that at CESR observation is hampered by having a beam pipe whose diameter is comparable to the wavelength (3cm), resulting in the
well-known absorption of EM waves as they travel down the beam pipe. At the NLC, however, 0.1 mm waves are a factor of 25 shorter than the beam pipe diameter and will be able to travel long distances. Detection of signals is also much easier in the 0.1 mm range than in the 1cm range. A third important property is that at the NLC the experimental conditions are much more favorable than at current storage rings (basically due to much shorter beams).

Another feature of coherent beamstrahlung radiation can be inferred directly from Fig. 2. It is clear that, for radiation to become coherent, the whole beam has to move in a certain direction coherently. In Figs. 2c and 2d), different parts of the beams move in opposite directions and interfere destructively. It is only in the case of a beam-beam offset (Fig. 2b)) that the beam as a whole moves vertically. Therefore coherent radiation will only appear in the presence of a non-zero offset and will primarily measure an offset. It is also immediately evident that (since as the offset along the \(x\)–axis is not significant) only the \(y\)–component of the polarization will be coherent, as the coherent motion is purely along that direction. Thus coherent radiation will isolate and amplify two single components (one for each beam) of the diagrams of Fig. 3.

To produce quantitative results, the beam-beam simulation program of Ref. was developed further to include coherent radiation scoring. This program is one of many cloud-in-cell programs existing on the market, and since it was developed for CESR, beamstrahlung energy loss by the beam particles is not included. This is a small deficiency of the program that does not affect the main results produced below - at the NLC (Table I) the typical beamstrahlung loss is of order a few to several percent (small corrections like these can be introduced at a later stage).

When scoring incoherent beamstrahlung, under the assumption discussed above that one can recover 100% linear polarization, one makes use of the following formulae to find the force exerted by all cells (index \(i\)) in beam 2 on a cell in the beam 1 (index \(j\)) is

\[
\Delta r_{1j} = \sum_{i} P_{2i} B_{ij} / b_{ij},
\]

\[
F_{1j} = \gamma mc^2 / 2 \Delta z \Delta r_{1j}.
\]

\(\gamma\) is the relativistic factor, \(m\) the electron mass, \(c\) the speed of light, \(\Delta z\) the step along the beam collision axis, and \(\Delta r_{1j}\) the (transverse) deflection during such a step. \(F_{1j}\) is the force exerted on one particle of beam 1 by the whole beam 2.

The \(P\) are the fractional charge population in each cell, and \(b\) is the transverse impact parameter between the centers of the two cells. The energy vector \(U_1\) for beam 1 is computed by summing

\[
U_{1x} = \sum_{i} \Delta U_{1ij} = \frac{2 N r_x \Delta z \gamma^2}{3mc^2} \sum_{j} P_{j} F_{1jx}^2,
\]

\[
U_{1y} = \sum_{i} \Delta U_{1ij} = \frac{2 N r_y \Delta z \gamma^2}{3mc^2} \sum_{j} P_{j} F_{1jy}^2.
\]

The \(U\) quantities are the low energy power emitted by the beams (appropriately scaled by the perfect collision power \(U_0\), they form the diagrams of Fig. 3).

When scoring coherent beamstrahlung, the formulae become

\[
W_{1x} = \frac{2 N^2 r_x \Delta z \gamma^2}{3mc^2} \left( \sum_{j} P_{j} e^{ikx} F_{1jx} \right)^2,
\]

\[
W_{1y} = \frac{2 N^2 r_y \Delta z \gamma^2}{3mc^2} \left( \sum_{j} P_{j} e^{ikx} F_{1jy} \right)^2,
\]

and we apply the normalization condition \(W_0 = U_0\). The limitation of the method is that there exists a transition region between incoherent and coherent beamstrahlung, where the program will not work. The two sets of formulae do coincide, in the limit of very large statistics and short wavelength, and in a way that is consistent with Ref. However the cell population \(C\) inside each beam is finite (typically \(C \sim 3 \times 10^4\)), and we found that the CB program would be numerically stable only if the coherent enhancement was greater than the number of cells

\[
C < \frac{U_{CB}}{U_{IB}}.
\]

The total power emitted in the microwave region may exceed 10W when full strength NLC beams are offset by a few \(\sigma_y\) (Table I). The main simulation results are shown in Figs. 4-6 for “weak” beams. In Fig. 4, the microwave power (in units of IB power) is shown as a function of the beam-beam vertical offset (in beam width units). The curves show the dependence of the coherent yield for various \(R\) ratios.

Fig. 5 shows the side-to-side power ratio, for beams which have different beam lengths \((\sigma_{y2}/\sigma_{y1}) = 0.8\). The ordinate in this plot is the ratio of the powers emitted by the beams. The shorter beam will obtain coherence at a lower wavelength than the other one, resulting in substantially more power. From the ratio, and its dependence on wavelength, one measures accurately the two beam lengths (with a precision which is probably dominated by uncertainties in the wavelength in use).

Fig. 6 shows the same plot as Fig. 4, for beams which have different beam widths \((\sigma_{y2}/\sigma_{y1}) = 3\). The slower turn-on of the coherence curve of the wider beam is noted. Clearly CB measures two distinct degrees of freedom, which are, roughly speaking but not exactly, the ratio of the beam-beam offset and the vertical width of each beam.

The alert reader will notice that in general the coherent enhancements are greater than \(N\) at large offsets,
FIG. 4: CB yield as a function of the beam-beam offset. The simulations were done with NLC nominal conditions (Table I), but weaker beams ($N_1 = N_2 = 0.3 \times 10^{10}$, $\sigma_{y1} = \sigma_{y2} = 19$nm). Plots are shown for four different wavelength-beam length ratios. The LINX working points are discussed in the main text.

As beamstrahlung will evolve from CESR to the NLC, it will be required to do more to monitor the quality and shape of the BBC. At CESR, like at the NLC, there will be a slow machine drift that incoherent beamstrahlung (IB) can monitor almost completely by itself (six out of seven $dof$, with the seventh one being measurable by scanning beams).

At NLC, there will be also substantial beam jitter (also potentially a 7-dimensional phenomenon). IB will be able to monitor only four of these seven $dof$. In part to counter this limitation, we have introduced the idea of measuring the coherent, microwave part of beamstrahlung. This part of the spectrum provides two extra, independent $dof$, bringing the total back to six and effectively providing almost complete monitoring. CB will also provide precision measurements of the beam length, and will work initially with very weak beams.

Like the beamstrahlung diagrams of Fig. 3, the CB plots presented here are semi-universal. What that means is that, up to small corrections related to the beams disruption during the BBC, Figs. (4-6) are universal. That is why both axes are scaled variables, with each curve depending on a third, scaled parameter.

By introducing two extra, independent measurements with CB, one may wonder whether the BBC is now fully monitored in a purely passive way, both for drift and for jitter. The short answer is no. Refs.[1, 2] discuss how the total IB power is insensitive to $\sigma_y$. Consider now a situation where the BBC is jittering between Fig. 2a) and

V. CONCLUSIONS.

At the NLC, incoherent beamstrahlung (IB) should retain its usefulness as a near-complete BBC monitor. The expected light signals are large, and there are, on paper, methods to reduce machine backgrounds.

FIG. 5: CB ratio of yields (beam 1 versus beam 2) as a function of the beam-beam offset. The simulations conditions are described in Fig. 4, but $\sigma_{z2} = 88\mu m$. 
Experimental issues will be discussed in a future paper. Once the visible SR backgrounds at the IP will be available, one or more of the four background rejection methods will be implemented in a final design.

In the case of CB, the rates are truly enormous (Table I), which allows the usage of anything sensitive to microwaves (including microantennas inside the beam pipe). CB will probably be measured above a certain threshold in the beam-beam offset, of order $0.1\sigma_y$. We note that the experimental issues related to CB are entirely technical, within including how to avoid burning the microwave detector, how to ensure a very large dynamic range, and how to read the microwave signal for each bunch (with a time separation of $1.4$ nsec).

Finally, one needs to point out that there are some significant differences between the low energy beamstrahlung method and the beam-beam deflection method (see for example[12]). This method is purely passive, and it is sensitive, in a passive way, to pathologies other than offsets. It also measures the ratio of the two $\sigma_y$ in a purely passive way, so that one beam’s detuning is diagnosed instantly. When the beams are scanned through one another, this method measures both $\sigma_y$ (as opposed to the quadratic sum of the two). This method will not be affected by different bunch lengths, and will provide a positive signal when the beams are colliding properly, whereas the beam-beam deflection provides zero signal. Finally, beamstrahlung is sensitive to vertical jitter (expressed in units of $\sigma_y$) far smaller than the beam-beam deflection method.

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Fig. 2c). IB will track that jitter, but will only provide the time evolution of a quantity which is the ratio of the two $\sigma_y$. There will be no CB. The problem will be identified, and the jitter of the ratio well measured. The absolute size of the smaller of the two $\sigma_y$ will have to be determined by scanning one beam through the other. Every other of the seven $dof$ is, however, accounted for and measurable passively.

![Coherent BS power](image)

**FIG. 6:** Same as Fig. 4, but $\sigma_{y_1} = 57$nm.

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