Resonantly enhanced coherence by laser-assisted tunneling

Hong-Xia Hao¹, Shiping Feng¹ and Shi-Jie Yang¹,²

¹ Department of Physics, Beijing Normal University, Beijing 100875, People’s Republic of China
² State Key Laboratory of Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, People’s Republic of China

E-mail: yangshijie@tsinghua.org.cn

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Abstract
We study quantum coherence of strongly interacting cold bosons in a double-well potential driven by a laser field. The system is initially in a Fock state and, either with or without a static tilting field, evolves into coherent states. The coherence is resonantly enhanced by photon-assisted tunneling. For the tilted wells, it reveals a two-branch pattern that corresponds to multiple photon absorption or emission.

Keywords: double-well, ultracold boson, coherence, photon-assisted tunneling

1. Introduction

Ultracold atoms confined in optical lattices are well-suited to explore many-body phenomena in condensed matter physics. Experimental and theoretical investigations regarding this field have experienced amazing progress. The phenomena of quantum tunneling [1–3], superfluid-Mott phase transition [4–6], and disorder effects [7–9] have been explored extensively. In a double-well, atomic Josephson oscillations take place in a Bose–Einstein condensate (BEC) when the initial population imbalance is below a critical value [10, 11] and the phenomenon of macroscopic quantum self-trapping, where the atoms essentially stay in one well, is observed in experiments [12, 13]. In recent years, the non-equilibrium phenomena in cold atom systems have attracted great interests [14]. The behavior of the collapse and revival of the matter wave field is demonstrated in the dynamic evolution of the interference pattern [15]. Research on periodic shaking lattices has invoked effects ranging from coherent destruction of tunneling, dynamic localization, field-induced barrier transparency, super Bloch oscillations, phase-jumps, and dynamics of bound pairs as well as artificial magnetic fields in many-body physics [16–26]. One of the most remarkable effects of periodically driving potential is photon-assisted tunneling. The ‘photons’ are time-dependent potential modulations in the kilohertz regime rather than real photons. This effect may induce a tunable superfluid–Mott insulator transition as the external driving frequency matches the interaction energy [27] and has been observed experimentally [28].

In this paper, we study the quantum dynamics of N bosons confined in a double-well under the influence of a laser field. By making a two-mode approximation, the Hamiltonian is written as [29]

\[
\hat{H}(t) = -\nu (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \frac{1}{2} U \sum_{i=1,2} \hat{n}_i (\hat{n}_i - 1) + \frac{1}{2} (\Delta + A \cos \Omega t) (\hat{n}_1 - \hat{n}_2),
\]

where \(\hat{a}_{1(2)}^\dagger\) and \(\hat{a}_{1(2)}\), respectively, denote the creation and annihilation operators for bosonic atoms localized in either side of the well. \(\nu\) is the hopping matrix element between the neighboring sites. \(\hat{n}_i\) is the number operator at the \(i\)th site. \(U\) is the Hubbard interaction between a pair of bosons occupying the same site, which can be adjusted by the Feshbach resonance technique. \(\Delta\) is the static energy difference of the tilted wells. \(A\) and \(\Omega\) are the amplitude and frequency of the driving laser field, respectively.

We focus on the strong interaction regimes \(U \gg \nu\), either with \((\Delta \neq 0)\) or without \((\Delta = 0)\) a static tilting field applied on the double-well. We investigate, with the help of both analytical arguments and numerical simulations, the dependence of the coherence on the driving field as the system evolves from the...
initial Fock state. As the driving frequency matches the sum of the interaction energy and the tilted energy difference, the system resonantly absorbs or emits energy, exhibiting a multiple photon process. The single particle tunneling is strengthened and the quantum coherence are greatly enhanced.

This paper is organized as follows. In section 2, we provide a theoretical description of the resonant coupling of the many-boson system with a driving field. In section 3, we numerically display the temporal evolution of the system for untilted wells by introducing a physical quantity to depict the coherence degree. Section 4 presents the coherence evolution for the tilted wells. A brief summary is provided in section 5.

2. Theoretical description

We divide the time-dependent Hamiltonian (1) into two parts: the hopping part \( H_1 \) and the interaction \( H_0(t) \),

\[
\hat{H}_1 = -\nu (\hat{a}_1^+ \hat{a}_2 + \hat{a}_2^+ \hat{a}_1),
\]

\[
\hat{H}_0(t) = \frac{1}{2} U \sum_{i=1,2} \hat{n}_i (\hat{n}_i - 1) + \frac{1}{2} (\Delta + A \cos \Omega t) (\hat{n}_1 - \hat{n}_2). \tag{3}
\]

The operators (2) and (3) are represented with the complete set of Fock bases \( \{\lfloor N, 0\rfloor, \lfloor N-1, 1\rfloor, \ldots, \lfloor 0, N\rfloor\} \) as

\[
\langle k|\hat{H}_1|j\rangle = -(\delta_{k,j-1} + \delta_{k,j+1}) \nu \sqrt{N-k} \sqrt{k+1}, \tag{4}
\]

\[
\langle k|\hat{H}_0(t)|j\rangle = \delta_{k,j} \left[ \frac{U}{2} (N^2 - 2Nk - N + 2k^2) + \frac{1}{2} (\Delta + A \cos \Omega t) (N - 2k) \right]. \tag{5}
\]

where \( |k\rangle = |k, N-k\rangle \) with \( k = 0, \ldots, N \). The many-body state is expressed as

\[
|\psi(t)\rangle = \sum_{k=0}^{N} c_k(t) |N-k, k\rangle. \tag{6}
\]

By making the transformation to the interaction picture through [33, 34]

\[
c_k(t) = a_k(t) e^{-i \int_0^t \langle k|\hat{H}_0|k\rangle dt}, \tag{7}
\]

inserting the state into the Heisenberg equation of motion, one obtains

\[
i\dot{a}_k(t) = \langle k|\hat{H}_1|k+1\rangle h_{k+1}(t) a_{k+1}(t) + \langle k|\hat{H}_1|k-1\rangle h_{k-1}(t) a_{k-1}(t), \tag{8}
\]

where the phase factor reads

\[
h_{k+1}(t) = e^{-i U/2 (N+2k+1) - i/2 \Delta + A \sin \Omega t}. \tag{9}
\]

The phase factor can be computed by employing the expansion 
\( e^{i \cos \Omega t} = \sum_{n=-\infty}^{\infty} J_n(z) e^{i n \Omega t} \), where \( J_n(z) \) is the \( n \)-th order Bessel function of the first kind, yielding

\[
h_{k+1}(t) = \sum_{n=-\infty}^{\infty} J_n(A / \Omega) e^{i n \Omega t - \Delta n}, \tag{10}
\]

for the initial Fock state \( |\psi(0)\rangle = |N/2, N/2\rangle \).

In the limit of strong interactions \( U/\nu \gg 1 \), the phase factor \( h_k(t) \) contains rapidly oscillating terms, which are averaged over the time period \( T = 2\pi/\Omega \). With
\[
\frac{1}{T} \int_{0}^{T} e^{i(n\Omega-U)t} = \delta(n\Omega - U),
\]

it follows that
\[
h_{k+1} = \sum_{n=-\infty}^{\infty} J_n(\Lambda/\Omega) \delta(n\Omega - U - \Delta),
\]
\[
h_{k-1} = \sum_{n=-\infty}^{\infty} J_n(\Lambda/\Omega) \delta(n\Omega - U + \Delta).
\]

As indicated in equation (8), the lower frequency parts of the motion are contributed by the hopping terms, which can be taken as quasi-static when we take the time-average over the high frequency parts.

The resonance occurs at
\[
n\Omega = U \pm \Delta.
\]

Under this circumstance, the equation of motion (8) can be reduced to
\[
\begin{align*}
\dot{a}_k(t) &= \langle k | \hat{a}_k \rangle \hat{H} | k + 1 \rangle + \langle k | \hat{a}_{k+1} \rangle \hat{H} | k \rangle \\
&= \delta_{k,0} + \sum_{n=1}^{\infty} J_n(\Lambda/\Omega) \delta(n\Omega - U - \Delta),
\end{align*}
\]

which constitutes a series of coupled equations of free oscillators with a renormalized tunneling parameter as
\[
\tilde{\alpha}_n = \alpha_n(\Lambda/\Omega).
\]

3. Untilted wells

We obtained the exact results by numerically solving the Heisenberg equation of motion based on the Hamiltonian (1). To characterize the coherence of the system, we adopted a quantity of coherence degree introduced by one of the authors [5].

\[
\alpha(t) = \frac{|\lambda_1 - \lambda_2|}{\lambda_1 + \lambda_2}
\]

where \(\lambda_1\) and \(\lambda_2\) are the eigenvalues of the single-particle density \(\rho_{\nu}\) \(\langle \Psi(t) | \hat{a}_{\nu}^\dagger \hat{a}_{\nu} \Psi(t) \rangle (\mu, \nu = 1, 2)\) [30, 31]. As \(\alpha(t) \rightarrow 1\), the system is in the coherent (quasi-coherent) state since, in this case, there is only one large eigenvalue of matrix \(\rho_{\nu}\). \(\alpha(t) \rightarrow 0\) indicates the system is in the decoherent or fragmented state because there are two densely populated natural orbits. At the weak interaction or strong tunneling limit \((U/\hbar \ll 1)\), each atom is in a coherent superposition of the left-well and right-well states. At the strong interaction or weak tunneling limit \((U/\hbar \gg 1)\), the tunneling term is negligible. The state is a product of the number operators for the left and right wells. This regime is analogous to the Mott insulator (MI) phase in optical lattices.

In this section, we consider the untitled double-well case \((\Delta = 0)\). We take the units of \(\hbar = 1\) and fix the Hubbard energy \(U = 100\) and the particle number \(N = 10\) as examples. The initial state is a symmetric Fock state \(|\Psi(0)\rangle = |N/2, N/2\rangle\). In absence of the driving field \((A = 0)\), the single particle tunneling is damped and coherence is suppressed. The driving field may assist the particle tunneling between the wells through exchanging energy with the bosons and enhancing the coherence. Figure 1 shows the real-time evolution of the coherence degree \(\alpha\) for various driving frequencies at fixed ratio \(A/\Omega = 3.2\). In figures 1(a), (d) and (e), where \(\Omega = 0.03U, 2U/3, 2U\), respectively, the coherence \(\alpha(t)\) remains very low, which indicates that single particle tunneling is still suppressed. The bosons can nearly exchange the energy with the driving field. In contrast, in figures 1(b), (c) and (f), which respectively corresponds to the driving frequencies \(\Omega = U/3, U/2, U\), the coherence is greatly increased in a simple oscillating mode which, implies single particle tunneling is strengthened through exchanging energy with the driving field. This photon-assisted tunneling occurs resonantly as the driving frequency satisfies \(\Omega = U/n\) (with \(n\) integer), analogous to the \(n\)-photon absorption process.

To provide a more explicit description, we investigate the maximum value of the coherence degree \(\alpha(t)\). Figure 2 illustrates \(\alpha_{\text{max}}\) versus the ratio \(A/\Omega\) for various driving frequencies. The three lower curves are for \(\Omega = 0.03U, 2U/3, 2U\), respectively. In comparison, the coherence is greatly enhanced for frequencies \(\Omega = U/3, U/2, U\), and the coherence is greatly increased in a simple oscillating mode which, implies single particle tunneling is strengthened through exchanging energy with the driving field. This photon-assisted tunneling occurs resonantly as the driving frequency satisfies \(\Omega = U/n\) (with \(n\) integer), analogous to the \(n\)-photon absorption process.

Figure 3 displays the topographical graph of the \(\alpha_{\text{max}}\) versus \(U/\Omega\) and \(A/\Omega\). The horizontal ridges, indicated by the integer numbers \((n = 1, 2, 3, 4, 5)\), clearly reveal the resonant tunneling, which is assisted by multiple photon absorption. The resonances occur as the driving frequency is an integer fraction of the interaction strength \(U/\Omega = n\), as claimed in equation (13). In this process, the system absorbs \(n\)-photons.
The ridge is broken at dips that correspond to the correlated destructive tunneling as \( \frac{A}{\Omega} \) takes the zeros of the Bessel functions.

4. Tilted wells

The phenomenon becomes more intriguing as the system is tilted, which induces a finite potential energy difference \( \Delta \neq 0 \) between the two wells. This static tilting can be realized experimentally by a constant acceleration or a gradient magnetic field. The evolution from the initial state \( |\Psi(0)\rangle = |N/2, N/2\rangle \) to \( |N/2 + 1, N/2 − 1\rangle \) or \( |N/2 − 1, N/2 + 1\rangle \) through one-particle hopping induces an energy difference of either \( U + \Delta \) or \( U − \Delta \), which takes place as it matches multiple photon absorption or emission energy. The competition between Hubbard energy and tilting potential leads to reducing or raising effective particle tunneling [32].

The ridge is broken at dips that correspond to the correlated destructive tunneling as \( \frac{A}{\Omega} \) takes the zeros of the Bessel functions.

**Figure 3.** Topographical map of \( \alpha_{\text{max}} \) versus \( A/\Omega \) and \( U/\Omega \) in the untitled double-well. The horizontal ridges indicate resonant enhancement of the coherence. The integer numbers specify the orders of the resonant absorption of multiple photons.

**Figure 4.** Resonant enhancement of coherence versus the static tilting \( \Delta \) for various driving frequencies. (a)–(f) \( \Omega = 0.03U, U/3, U/2, 2U/3, U, \) and \( 2U \), respectively. The solid curves are calculated for \( A/\Omega = 3.2 \). The dashed curves in (b), (c), and (e) are calculated respectively as \( A/\Omega = 3.8, A/\Omega = 5.1, \) and \( A/\Omega = 6.3 \), which are zeros of the Bessel functions \( J_1(z), J_2(z), \) and \( J_3(z) \). In these cases, the first resonant peak is damped by the destructive coherent tunneling.
driving field renormalizes the particle tunneling under specific circumstance.

Figure 4 displays the numerical results of $\alpha_{\text{max}}$ versus the static tilting $\Delta$ at fixed $A/\Omega = 3.2$. From $(a)$–$(f)$, the driving frequency takes $\Omega = 0.03U$, $U/3$, $U/2$, $2U/3$, $U$, and $2U$, respectively. Evidently, the equidistant resonant peaks occur as the parameters satisfy the following relation, \( n_\pm \Omega = U \pm \Delta, \) (17)

where $n_\pm$ are integers that respectively indicate two coinciding sets of resonant peaks as $\Delta \neq 0$. The $\pm$ sign on the right-hand side of (17) corresponds to a particle hop from the lower well to the higher well and vice versa. A special case occurs at the untilting point $\Delta = 0$, where only one set of resonant peaks occurs. These peaks can be damped by taking the values of $A/\Omega$ as the zeros of the corresponding Bessel functions, as shown in figures 4$(b)$, $(c)$ and $(e)$ where the dashed curves correspond respectively to the 3rd, 2nd, and 1st orders of Bessel functions. All other peaks for $\Delta \neq 0$ are robust, regardless of the values of the driving parameters.

Figure 5 shows the $\alpha_{\text{max}}$ for tilting $\Delta = U$. The horizontal ridges indicated by $n_+ = 1, 2, 3, \ldots$ correspond to the resonant tunneling to the state $|N/2 + 1, N/2 - 1\rangle$. In comparison to figure 3, the multiple photon-assisted enhancements are observed at $U/\Omega = 0.5, 1.5, 2.5, \ldots$, which satisfies the relation of $n_+ \Omega = U + \Delta$. The $n_-$ series are absent because $U - \Delta = 0$. The vertical valleys are from the zero points of the zeroth order of Bessel functions, $A/\Omega = 0.5, 1.5, 2.5, \ldots$, and so on. Figure 6 uses the same parameters but with $\Delta = U/5$, as in figure 5. In this case, two branches of resonant enhancement $n_+$ and $n_-$ are clearly present. The ridges for $n_+ = n_-$ have the same dips because they belong to the same order of Bessel functions and have the same effective tunneling parameter.

Finally, in figure 7, we illustrate $\alpha_{\text{max}}$ dependence on the ratios of $\Delta/U$ and $\Omega/U$ in the tilted double-well system by fixing $A/\Omega = 3.2$. The resonant tunneling to the state $|N/2 + 1, N/2 - 1\rangle$ exhibits straight ridges with positive slope, which are marked in the figure with $n_- = 0, 1, 2, 3, \ldots$. For the other branch, $n_+ \Omega = U - \Delta$, the resonant enhancement displays a symmetrical fan-shape structure starting from $n_- = 0$ $(\Delta/U=1)$ at the center to $n_- = \pm 1, \pm 2, \ldots$ successively. The ridges with positive value of $n_-$ lie on the left-side of the fan, which indicates absorption of photons, while the ridges with negative value of $n_-$ lie on the right-side of the fan, which indicates emission of photons. The numerical results agree with the theoretical analysis very well.
exhibits straight lines with positive slopes. The other resonant branch for \( n \) exhibits a symmetrical fan structure with \( n = 0 \) at the center to \( n = \pm 1, \pm 2, \ldots \) successively (not specified in the figure).

5. Summary

In summary, we have investigated the effects of photon-assisted tunneling on the quantum coherence in a periodically driven double-well, both untitled and tilted. In the strongly interacting regime \( U/\hbar \gg 1 \), the correlated tunneling coefficient is renormalized. The resonant enhancement of coherence takes place as the driving frequency matches the energy difference of the quantum transition \( n\Omega = U \pm \Delta \), which involves multiple photon absorption or emission processes.

Acknowledgments

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Figure 7. Topographical map of \( \alpha_{\text{max}} \) versus \( \Delta U \) and \( U/\Delta U \) in the tilted double-well at fixed \( A/\Omega = 3.2 \). One of the resonant branches for \( n \) exhibits a symmetrical fan structure from \( n = 0 \) at the center to \( n = \pm 1, \pm 2, \ldots \) successively (not specified in the figure).