The thermal effect on the left-handedness of the mesoscopic composite right-Left handed transmission line

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Abstract

Starting from the quantum fluctuation of current in the mesoscopic composite right-left handed transmission line (CRLH-TL) in the thermal Fock state, we investigate the left-handedness dependent of the frequencies, intensity and quantum fluctuation-s of the current field in the CRLH-TL under different thermal environment. The results show that the intensity and quantum fluctuations of current field in lower frequency bands affect the left-handedness distinctly under different thermal environment. The thermal effect on the left-handedness in the mesoscopic CRLH-TL deserves further experimental investigation in its miniaturization application.

Keywords: Mesoscopic composite right-Left handed transmission line, Left-handedness, Thermal effect

1. Introduction

Left-handedness indicates materials exhibiting simultaneously negative effective permittivity \(\epsilon\) and permeability \(\mu\) in the same frequency window [1, 2, 3], which has been the center of significant interest in the physics and engineering communities since it was first realized at microwave frequencies [4] due to their exotic electromagnetic properties and their fascinating applications. However, one key obstacle in the applications of left-handed materials is absorption which is particularly important in the optical regime [5, 6, 7], and significant efforts have been spent to realize low-loss left-handed materials [8, 9, 10]. Meanwhile, the transmission lines (TLs) are reportedly low dissipation and wide frequency bands for left-handed materials [11, 12, 13, 14], i.e., the composite right-Left handed transmission lines (CRLH-TLs) [15]. Recently, an universal applications for CRLH-TLs are implemented by its left-handed phenomena. Such as in the acoustic wave frequency band, one-dimensional acoustic negative refractive index metamaterial is presented by the CRLH-TL [16], and the CRLH-TL is firstly reported to achieve the acoustic dispersive prism which has the capability of splitting a broadband acoustic wave.

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into its constituent Fourier components within the audible frequency range of 800 Hz-1300 Hz[17]. In the the antenna application, the fractional bandwidth can be enhanced from 0.31% to 12.5% in a resonant antenna with an asymmetric coplanar waveguide based on the CRLH-TL[18]. And a new class of miniaturized nonreciprocal leaky-wave antenna is proposed for miniaturization, nonreciprocal properties and wide-angle scanning at the same time[19]. With four unit cells of CRLH-TL a wide-band loop antenna is proposed in a compact size[20]. Nowadays, under the nanotechnology and microelectronics influence, the compact application has turned to a tendency for CRLH-TLs.

However, when the compact size of the CRLH-TL approaches the Fermi wavelength, the quantum effects on the CRLH-TL must be taken into account similarly to the mesoscopic circuits[21, 22, 23, 24, 25]. In our former work, we firstly deduced the quantum features of negative refraction index (NRI) of the lossless mesoscopic left-handed transmission line (LH-TL)[26]. And then the quantized lossy LH-TL[27] entered our the next work, we discussed the characteristics of NRI of the lossy LH-TL[27] in a displaced squeezed Fock state. And some novel quantum effects caused by the dissipation were revealed and were reputed significance for the miniaturization application of LH-TL.

Thus, from the point of against this miniaturization application challenge, this paper exploits the thermal effect on the left-handedness of the CRLH-TL in the thermal Fock state. And this paper is organized as follows. In Sec.2, we quantize the travelling current field in the unit-cell circuit of the CRLH-TL, and deduce the permittivity $\epsilon$ and permeability $\mu$ in the thermal Fock state. Then we evaluate the left-handedness in the thermal Fock state in Sec.3. Sec.4 presents our summary and conclusions.

2. The quantized CRLH-TL in the thermal Fock state

![Schematic diagram of equivalent unit-cell circuit of the mesoscopic CRLH-TL.](image.png)
The equivalent unit-cell circuit model of the proposed CRLH-TL is shown in Fig. 1. It consists of the series capacitor $C_l$ and inductance $L_r$, shunt inductance $L_l$ and capacitor $C_r$ [28]. And the dimension $\Delta z$ of the equivalent unit-cell circuit is much less than the wavelength at operating frequency. The permittivity $\epsilon$, permeability $\mu$ for the unit-cell circuit in Fig. 1 read as $\epsilon = C_r - \frac{1}{\omega^2 L_l}, \mu = L_r - \frac{1}{\omega^2 C_l}$ [29], respectively. Hence, let us now consider Kirchhoff's voltage and current laws for the unit-cell circuit in Fig. 1, which respectively read

$$u(z, t) = i(z, t) \left[ \frac{1}{j \omega \Delta z} + j \omega L_r \Delta z \right] + u(z + \Delta z, t),$$

$$i(z, t) = \left[ \frac{1}{j \omega \Delta z} + j \omega C_r \Delta z \right] u(z + \Delta z, t) + i(z + \Delta z, t)$$

where $u(z, t)$ is the voltage, $i(z, t)$ is the current, and $\omega$ is the angle frequency. When $\Delta z \to 0$, the above equations lead to the following system:

$$\frac{\partial^2 u(z, t)}{\partial z^2} = -[\omega L_r - \frac{1}{\omega C_l}] \times [\omega C_r - \frac{1}{\omega L_l}] u(z, t),$$

$$\frac{\partial^2 i(z, t)}{\partial z^2} = -[\omega L_r - \frac{1}{\omega C_l}] \times [\omega C_r - \frac{1}{\omega L_l}] i(z, t)$$

The above equations, together with the auxiliary equations $i(z, t) = C \frac{\partial u(z, t)}{\partial t}$ and $C = C_r + C_l$ lead to the forward plane-wave solutions:

$$u(z, t) = A e^{-j(\omega t - \beta z)} + A^* e^{j(\omega t - \beta z)}$$

$$i(z, t) = j \omega C [A^* e^{j(\omega t - \beta z)} - A e^{-j(\omega t - \beta z)}]$$

with $\beta = \sqrt{\left(\frac{\omega}{\omega_r}\right)^2 + \left(\frac{\omega}{\omega_l}\right)^2 - k \omega^2}$, where $\omega_r = \frac{1}{\sqrt{L_r C_r}}, \omega_l = \frac{1}{\sqrt{L_l C_l}}, k = L_r C_l + L_l C_r$. In Eq. (1) and Eq. (2), $A^*$ is the complex conjugate of $A$ for the normalization purpose. To simplify Eq. (1) and Eq. (2), two functions $\xi(z, t)$ and $\eta(z, t)$ are introduced by

$$u(z, t) = \xi(z, t),$$

$$i(z, t) = \frac{\omega^2}{z_0} \eta(z, t)$$

where $z_0$ is the length of per unit circuit. We assume that $\omega$ is given and choose the unit length of circuit $z_0$ to be fixed. Then, $\eta(z, t)$ and $\zeta(z, t)$ differential $t$ are

$$\frac{\partial \xi(z, t)}{\partial t} = \frac{\omega^2}{C z_0} \eta(z, t),$$

$$\frac{\partial \eta(z, t)}{\partial t} = -C z_0 \xi(z, t)$$
It is straightforward to find the following relation,
\[
\frac{\partial}{\partial \xi}(z, t) \left( \frac{\partial \xi(z, t)}{\partial t} \right) + \frac{\partial}{\partial \eta}(z, t) \left( \frac{\partial \eta(z, t)}{\partial t} \right) = 0 \tag{5}
\]
So, \( \eta(z, t) \) and \( \zeta(z, t) \) are the canonically conjugate variables and obey the hamiltonian canonical equations,
\[
\frac{\partial \xi(z, t)}{\partial t} = \frac{\partial H}{\partial \eta(z, t)}, \tag{6}
\]
\[
\frac{\partial \eta(z, t)}{\partial t} = -\frac{\partial H}{\partial \xi(z, t)} \tag{7}
\]
The integrals of Eq.(6) and Eq.(7) can deduce the hamiltonian for the electromagnetic wave with the ignored integral initial constant as follows,
\[
H = \frac{\omega^2}{2Cz_0} \eta^2(z, t) + \frac{1}{2}Cz_0 \xi^2(z, t) \tag{8}
\]
If we associate hermitian operators of \( \hat{\eta}(z, t) \) and \( \hat{\xi}(z, t) \) and require that they satisfy the commutation relation,
\[
[\hat{\xi}, \hat{\eta}] = j\frac{Cz_0}{\omega} [A e^{-j(\omega t - \beta z)} + A^* e^{j(\omega t - \beta z)}, A^* e^{j(\omega t - \beta z)} - A e^{-j(\omega t - \beta z)}] = j\hbar \tag{9}
\]
then, we achieve the quantized unit-cell circuit with the definitions,
\[
\hat{A} = \hat{a} \sqrt{\frac{\hbar \omega}{2Cz_0}}, \\
\hat{A}^* = \hat{a}^* \sqrt{\frac{\hbar \omega}{2Cz_0}}
\]
Then the two operators \( \hat{\eta}(z, t) \) and \( \hat{\xi}(z, t) \) are,
\[
\hat{\xi}(z, t) = \sqrt{\frac{\hbar \omega}{2Cz_0}} [\hat{a} e^{-j(\omega t - \beta z)} + \hat{a}^* e^{j(\omega t - \beta z)}], \tag{9}
\]
\[
\hat{\eta}(z, t) = j \sqrt{\frac{\hbar \omega}{2Cz_0}} [\hat{a}^* e^{j(\omega t - \beta z)} - \hat{a} e^{-j(\omega t - \beta z)}] \tag{10}
\]
and the quantum Hamiltonian of the unit-cell circuit can be written as \( \hat{H} = \hbar \omega (\hat{a}^\dagger \hat{a} + \frac{1}{2}) \) which is analogous to the oscillator’s Hamiltonian operator in Schrödinger representation.
As for the equilibrium situation, the so-called thermo field dynamics (TFD) extends the usual quantum field theory to a finite temperature\[30\], in which the tilde space accompanies with the Hilbert space. The creation and annihilation operators $\hat{a}^\dagger$, $\hat{a}$ associate with their tilde operators $\tilde{\hat{a}}^\dagger$, $\tilde{\hat{a}}$ according the rules[31]:

\[
[\tilde{\hat{a}}, \tilde{\hat{a}}^\dagger] = 1, [\tilde{\hat{a}}^\dagger, \tilde{\hat{a}}] = [\hat{a}, \hat{a}^\dagger] = 0.
\]

The number operators in the Hilbert space and tilde space are read as $\hat{n} = \hat{a}^\dagger \hat{a}$, $\tilde{\hat{n}} = \tilde{\hat{a}}^\dagger \tilde{\hat{a}}$. In this direct product space, the thermal Fock state at finite temperature $|\hat{n}\tilde{n}\rangle_T$ can be built by the thermal Bogoliubov transformation[31] through the Fock state at zero temperature $|\hat{n}\tilde{n}\rangle_0$:

\[
|\hat{n}\tilde{n}\rangle_T = \hat{T}(\theta)|\hat{n}\tilde{n}\rangle,
\]

where $\hat{T}(\theta)$ is a thermal unitary operator which is defined as

\[
\hat{T}(\theta) = \exp[-\theta(\hat{a}\tilde{\hat{a}} - \hat{a}^\dagger \tilde{\hat{a}}^\dagger)]
\]

the parameter $\theta$ is the thermal parameter relating the thermal photos $n_0$ in the thermal vacuum state: $n_0 = \sinh \theta$. The thermal photos $n_0$ and temperature $T$ are ruled by the Boltzmann distribution $n_0 = [\exp(\hbar \omega/k_B T) - 1]^{-1}$, in which $k_B$ is the Boltzmann constant.

Then the bosonic operators in TFD can relate each other by the thermal Bogoliubov transformation as following,

\[
\begin{align*}
\hat{T}^\dagger(\theta) \hat{a} \hat{T}(\theta) &= \mu \hat{a} + \tau \tilde{\hat{a}}^\dagger, \\
\hat{T}^\dagger(\theta) \hat{a}^\dagger \hat{T}(\theta) &= \mu \hat{a}^\dagger + \tau \tilde{\hat{a}}
\end{align*}
\]

where $\mu = \cosh \theta$, $\tau = \sinh \theta$. With the quantum fluctuation of the current $\langle (\Delta i)^2 \rangle = \langle \langle \hat{i}^2 \rangle - \langle \hat{i} \rangle^2 \rangle$ in the thermal Fock state in Heisenberg picture and the definitions of effective permittivity $\epsilon$, permeability $\mu$ for the unit-cell circuit [29], we deduce the permittivity $\epsilon$, permeability $\mu$ in the thermal Fock state as follows,

\[
\begin{align*}
\epsilon &= C_r - 2^{-\frac{2}{5}} \frac{\hbar (1 + 2n) \coth(\frac{\hbar \omega}{k_B T})}{C_{r0}^2 (\langle (\Delta i)^2 \rangle L_i^{3/2})^{2/5}}, \\
\mu &= L_r - 2^{-\frac{2}{5}} \frac{\hbar (1 + 2n) \coth(\frac{\hbar \omega}{k_B T})}{C_{r0}^2 (\langle (\Delta i)^2 \rangle C_i^{5/2})^{2/5}}
\end{align*}
\]

where $n$ is the numbers of field photon corresponding to the number operator $\hat{n}$ in the Hilbert space.

3. Numerical simulations and discussions

Now we investigate the left-handedness of the mesoscopic CRLH-TL. The parameters used in our simulation are listed in Table 1. The order of magnitudes of these parameters are referenced from Ref.[32]. We consider one unit length of the mesoscopic CRLH-TL and set the quantum fluctuation of the current $\langle (\Delta i)^2 \rangle = 25$, the field photon $n=5$ in Fig.2. As mentioned before, left-handedness means the simultaneously negative effective permittivity $\epsilon$ and permeability $\mu$ in the same frequency window. As shown in Fig.2, the effective permeability $\mu$ is negative in all
Table 1: Parameters for the equivalent unit-cell circuit of the mesoscopic CRLH-TL.

|     | $C_l$ | $L_l$ | $C_r$ | $L_r$ | $\omega$ |
|-----|-------|-------|-------|-------|----------|
| Fig.2 | 148 $\mu$F | 595 $\mu$H | 35 $m$F | 480 $p$H |           |
| Fig.3 | 250 $\mu$F | 6.50 $m$H | 9.0 $m$F | 350 $p$H | 2.9GHz    |
| Fig.4 | 550 $\mu$F | 1.00 $m$H | 45 $m$F | 600 $p$H | 2.9GHz    |

frequency bands, as a result the mesoscopic CRLH-TL shows left-handedness in the frequency bands where the permittivity $\epsilon$ is negative, and shows right-handedness where the permittivity $\epsilon$ is positive.

It’s noted that the frequency ranges for negative permittivity $\epsilon$ increase when the temperatures enhance from 5K to 150K in Fig.2. And in the lower frequency bands, the negative permittivity $\epsilon$ are larger than those in the higher frequency band. What’s more, the width of frequency bands for negative effective permittivity $\epsilon$ increases accompanying the temperature increase. We also notice that the left-handedness, i.e., the simultaneous values of negative effective permittivity $\epsilon$ and permeability $\mu$ can achieve surpassing the microwave frequency band, i.e. $\geq 3\text{GHz}$ when the temperature is higher than 85K. Fig.2 shows the left-handedness in the unit-cell circuit of the mesoscopic CRLH-TL succeeding in the lower frequency bands under a higher temperature.

![Figure 2](image-url)

Figure 2: (Color online) The effective permittivity $\epsilon$ and permeability $\mu$ of the mesoscopic CRLH-TL as a function of the frequencies via different temperatures: $T = 5K, T = 35K, T = 85K, T = 150K$.

The role of the intensity of current field in the mesoscopic CRLH-TL is one major factor for left-handedness. On account of the quantum Hamiltonian of the unit-cell circuit is $\hat{H}=\hbar\omega(\hat{n} + \frac{1}{2})$, the intensity of the current field can be described by the numbers of the field photon. Profiting from Fig.2, the mesoscopic CRLH-TL operates in the microwave frequency band ($\omega = 2.9\text{GHz}$), and the quantum fluctuation of the current is set as $\langle (\Delta i)^2 \rangle = 1$ in Fig.3. It is noted that the effective permittivity $\epsilon$ is negative when the temperature surpasses 170K with the weak current field ($n=1$). While the intensity of the current field increases gradually, the left-handedness can acquire at a lower temperature. The effective permittivity $\epsilon$ can be negative if the
temperature surpasses 40K with a stronger current field (n=10). Fig.3 shows the fact that the mesoscopic CRLH-TL can be the left-handed material at a higher temperature when the weak current field travels through it, and at lower temperature with a more intense current field.

The most distinctive quantum feature is the fluctuation of the current field in the mesoscopic CRLH-TL. The effect of the quantum fluctuation at different temperatures on the left-handedness is plotted in Fig.4. The effective permittivity $\varepsilon$ and permeability $\mu$ dependent the temperatures are shown by different the quantum fluctuations with the field photon $n=1$ in Fig.4. As shown in Fig.4, the effective permeability $\mu$ is negative in all temperature ranges, the left-handedness depends on the temperature ranges where the permittivity $\varepsilon$ is negative. When the quantum fluctuation is weak $\langle (\Delta i)^2 \rangle = 1.0$, a more wider temperature range for left-handedness is obtained. In other word, the left-handedness can arise from a low temperature to the room temperature when the fluctuation is weak in the mesoscopic CRLH-TL. However, the left-handedness can be achieved by the large number of the quantum fluctuation $\langle (\Delta i)^2 \rangle = 4.5$ in a higher temperature environment.

![Figure 3: (Color online) The effective permittivity $\varepsilon$ and permeability $\mu$ of the mesoscopic CRLH-TL as a function of the temperatures under different numbers of the microwave field photon $n$: 1, 3, 6, 10.](image1)

![Figure 4: (Color online) The effective permittivity $\varepsilon$ and permeability $\mu$ of the mesoscopic CRLH-TL as a function of the temperatures under different the quantum fluctuation of the current $\langle (\Delta i)^2 \rangle$: 1.0, 1.5, 3.0, 4.5.](image2)
Before concluding this paper we would like to point out that the frequency, intensity and fluctuations of the current field with different temperature in the mesoscopic CRLH-TL show the distinct characteristics in the left-handedness. We also remark that the discussion about the left-handedness doesn’t relate to the quantum effect of the electronic components in the mesoscopic CRLH-TL, such as $C_l$, $L_r$, $L_l$ and $C_r$. How to develop our present studies involving the quantum effect of the electronic components is to be considered in our forthcoming investigations.

4. Conclusion

In the present paper, we investigated the thermal effect on the left-handedness of the mesoscopic CRLH-TL, and the quantum thermal effect on the left-handedness can be summarized as: When the weak current field with a lower frequency and large quantum fluctuations travels through the mesoscopic CRLH-TL, the left-handedness can distinctly be achieved in the higher temperature environment. While the intense current field with lower frequency and weak fluctuations facilitates left-handedness at a lower temperature. For these reasons, we think the thermal effect on the left-handedness of the mesoscopic CRLH-TL deserves further experimental investigation in its miniaturization application.

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