D1D5 systems and AdS/CFT correspondences with 16 supercharges

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Abstract: We study the spectra of BPS excitations of D1D5 bound states in a class of free orbifolds/orientifolds of type IIB theory and its dual descriptions in terms of chiral primaries of the corresponding AdS₃ supergravities.

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AdS/CFT correspondence [1] relates type IIB string theory on AdS₃ × S³ × M, with M = T⁴ or K3, to N = (4, 4) two-dimensional CFT’s describing the infrared dynamics of D1D5 bound state systems. Tests of this conjecture were performed in [2, 3, 4, 5], where multiplicities of BPS excitations of the CFT’s were shown to agree with those of chiral primary states in the underlying supergravities. Although the supergravity description is expected to be valid only for large values of the brane charges, the correspondence was shown to work for all N = Q₁Q₅, once a new additive quantum number, the degree d, is introduced on the supergravity side [4]. This is a non-negative integer that allows to cut-off multiparticle states and implement the exclusion principle [2]: one keeps only products of chiral primaries whose total degree is ≤ N.

In this paper we study the spectrum of chiral primary states and their descendants in CFT₂/AdS₃ supergravity pairs, arising from the D1D5 system in a class of freely acting Z₂ orbifolds/orientifolds of type IIB theory. Correspondingly, the near horizon geometries are certain freely acting Z₂ orbifolds of AdS₃ × S³ × T⁴.

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The associated boundary CFTs have been studied in great detail in [7]. The freely acting $Z_2$ group generators are defined by accompanying the orbifold and/or orientifold actions $\Omega$ (worldsheet parity), $I_4$ ($Z_2$ reflection of $T^4$), $\Omega I_4$ with a shift $\sigma_{p6}$ along a circle transverse to the D1/D5 system, with compact coordinate $X^6$. We refer to these theories as models I, II and III respectively.

In [7] the effective gauge theories associated to the type IIB orbifold/orientifold D1D5 systems were argued to flow in the infrared to CFTs locally equivalent to the one appearing for the D1D5 system in type IIB theory on $T^4 \times S^1$, but with additional $Z_2$ global identifications induced by the orbifold group actions [7]. The resulting target spaces in the three models are of the form:

$$\mathcal{M}_{\text{higgs}} = \left( R^3 \times S^1 \times T^4 \times (T^4)^N / S_N \right) / Z_2 \quad (0.1)$$

The $Z_2$'s are generated by $(-)^F I_4^{c.m.}$, $I_4^{c.m.} I_4^{sp}$ and $(-)^F I_4^{sp}$ for the models I, II and III respectively, with $(-)^F$ the left moving spacetime fermionic number, $I_4^{c.m.}$ the reflection of the first $T^4$ factor in (0.1) and $I_4^{sp}$ the diagonal $Z_2$ reflection of the $N$ copies of $T^4$ in the symmetric product part. Pure D1D5 states correspond to states in the $Z_2$-untwisted sectors of (0.1). The resulting CFTs are of type $\mathcal{N} = (4, 4)$ for models II and $\mathcal{N} = (4, 0)$ for models I and III, which involve the $\Omega$ world sheet parity projection\footnote{A closely related example of $\mathcal{N} = (4, 0)$ D1D5 system where the $Z_2$ acts as a reflection of the transverse $R^4$ accompanied with a longitudinal shift have been recently studied in [8]. It would be nice to apply the techniques developed in this paper to that system}.

The spectrum of charges and multiplicities of D1D5 BPS excitations (right moving ground states $N_R = 0$) was obtained from the elliptic genus

$$\sum_N p^N \left( Z_N^{(1)} + Z_N^{(g)} \right) \equiv \sum_N p^N \text{Tr}_{\mathcal{H}_N} \left( \frac{1+g}{2} \right) q^{L_0-c/24} y^{J_3} \bar{y}^{\bar{J}_3} \quad (0.2)$$

evaluated in the CFT Hilbert spaces $\mathcal{H}_N$ defined by (0.1). $q = e^{2\pi i \tau}$ describe the genus-one worldsheet modulus, $L_0$, $L_0$ are the Virasoro generators and $J_3$, $\bar{J}_3$ are Cartan generators of an $SU(2)_R \times SU(2)_L$ current algebra to which the sources $y$ and $\bar{y}$ couple respectively. The elliptic genera have been evaluated in [7] using generalizations of the DMVV symmetric product formulas [9].

Following the general philosophy of Maldacena AdS/CFT correspondences one can associated to these D1D5 CFT's a dual description in terms of near horizon $AdS_3 \times S^3$ supergravities. If we consider the radius $R_6$ of the circle along which the shift is performed, very large, in such a way that the space transverse to the D1D5 system is effectively $R^4$, then the near horizon geometry will still be...
AdS\(3 \times S^3 \times T^4\). However, in doing the KK reduction the various modes will come with non-trivial \(Z_2\) phases due to the orbifold group actions (\(\Omega, I_4\) and \(\Omega I_4\) according to the model) in the way we will specify below.

The relevant \(Z_2\)-eigenvalues for 6-dimensional massless fields of \(\mathcal{N} = (2,2)\) supergravity, together with their transformation properties under the little group \(SO(4)\), are displayed in the following table:

| I  | II  | III |
|----|-----|-----|
| \(\Omega\) | \(I_4\) | \(\Omega I_4\) |
| \(+\) | \(+\) | \(+\) |
| \(+\) | \(+\) | \(+\) |
| \(-\) | \(-\) | \(-\) |
| \(-\) | \(-\) | \(-\) |

| bosons | fermions |
|--------|----------|
| \((2,2)+(0,2)+(2,0)+17(0,0)\) | \(2(1,2)+10(1,0)\) |
| \(4(0,2)+4(2,0)+8(0,0)\) | \(2(1,2)+10(1,0)\) |
| \(8(1,1)\) | \(2(2,1)+10(0,1)\) |
| \(8(1,1)\) | \(2(2,1)+10(0,1)\) |

Table 2: \(SO(4)\) field content with \(Z_2\) eigenvalues

If we denote by \(G^i_{-\frac{1}{2}}, \tilde{G}^i_{-\frac{1}{2}}\) with \(i = 1, 2\) the left and right moving lowering supersymmetry charges in the AdS supergroup associated to the \(AdS_3 \times S^3 \times T^4\) vacuum one can easily see that while \(\tilde{G}^i_{-\frac{1}{2}}\) moves the states within each row, \(G^i_{-\frac{1}{2}}\) moves vertically between first two rows as well as the last two rows.

The spectrum of KK harmonics on \(S^3\) can be determined essentially by group theory \([10, 11, 3]\). The end result can be written as \([7]\)

\[
\mathcal{H}_{\text{single particle}}^A = \bigoplus_{m \geq 0} h_{r,s} (m + r, m + s) \epsilon_{A(r,s)}^{m+1}
\]

where \(h_{r,s}\) denotes the Hodge numbers of the \(T^4\) torus and

\[
(m, m') = \sum_{i,j=0}^2 \binom{i}{2} \binom{j}{2} (m - i, m' - j),
\]

collects states sitting in \(\mathcal{N} = (4,4)\) supermultiplets of the unorbifolded theory. Supermultiplets \((m, m')_d\) are labeled by the \(SO(4)\) quantum numbers of the highest weight primary \((m = 2j, m' = 2j')\) and are constructed by acting on this state with the lowering operators \(L_- , L_-, J_0^-, \bar{J}_0^-, G^i_{-\frac{1}{2}}, \tilde{G}^i_{-\frac{1}{2}}\). The prime denotes the omission of the term \(m = r = s = 0\) in the sum. The indices \(A = I, II, III\) label the model while the subscript \(m + 1\) denotes the degree. Finally \(\epsilon_A(r,s)\) are the \(Z_2\) eigenvalues of the highest weight state inside the supermultiplet and are given by:

\[
\epsilon^I(r, s) = (-1)^s, \quad \epsilon^{II}(r, s) = (-1)^{r+s}, \quad \epsilon^{III}(r, s) = (-1)^r
\]
For models I and III, although we have grouped the states in terms of the original (4,4) multiplets, we have to remember that $G_{-\frac{1}{2}}$ anticommutes with the $Z_2$‘s, and therefore the descendants that involve odd numbers of $G_{-\frac{1}{2}}$‘s will appear with an extra minus sign under the $Z_2$ action.

Now we construct the multiparticle Hilbert space $H_{\text{multiparticle}}$ and identify the finite $N$ CFT Hilbert space with the subset of states in $H_{\text{multiparticle}}$ that have degree less than or equal to $N$. To this end it is convenient to introduce a parameter $p$ that keeps track of the degree $d = \sum_i (m_i + 1)$ of multi-particle states. Carrying out the trace over all (chiral,chiral) primaries above one is left with the result

$$Z^g(p, q, y, \tilde{y}) = \prod_{m=0}^{\infty} \prod_{r,s=0}^{2} (1 - \epsilon^A(r, s)p^{m+1}y^{m+r}\tilde{y}^{m+s})^{-\frac{1}{2}h_{r,s} + O(q)} \quad (0.6)$$

The factor $\frac{1}{1-p}$ ($m = r = s = 0$) previously missing in (0.3) takes into account the fact that finite $N$ CFT Hilbert space is identified with states in $H_{\text{multiparticle}}$ that have degree up to $N$ and not just $N$. It is easy to see that the elliptic genera (0.6) exactly reproduce the CFT results (formulas (5.13) in [6]) for the three models after spectral flow from Ramond to NS sector [7].

One can now, following de Boer, check the correspondence beyond the (c,c) primaries. The idea is to construct the finite $N$ elliptic genus which is obtained by taking the trace over states of the form chiral on the right-moving sector and any state on the left-moving sector, and setting $\tilde{y} = \tilde{q}^{-1/2}$. The comparison of the states can, of course, only be made for dimensions much less than $N$, since otherwise gravity approximation would break down. In [3] de Boer showed that for the K3 case the matching of the states goes all the way up to left dimension equal to $(N+1)/4$. This is exactly the bound at which black hole is expected to form. In the right-moving sector arbitrary chiral states are allowed, and they have a bound on the dimension which is of order $N/2$.

We will restrict our elliptic genus computation to models II and III since the elliptic genus of model I vanishes for $\tilde{y} = \tilde{q}^{-1/2}$. Here one can repeat the analysis of [5] and take 2 derivatives with respect to $\tilde{y}$ before setting it to $\tilde{q}^{-1/2}$. In this case however we do not get any information; in fact for states satisfying the bound $h \leq (N+1)/4$ only the ground state contributes as shown in [5].

The elliptic genus for the multi-particle states is then

$$Z^A_{\text{multiparticle}}(p, q, y) = \prod_{n,m,\ell} \left[ \frac{1 + p^n q^m y^\ell}{1 - p^n q^m y^\ell} \right] c_{\text{csg}}^{A_{\text{c}}} (n,m,\ell). \quad (0.7)$$
where $c_{sgr}^{A^+}(n,m,\ell)$'s are the coefficients in the expansion of

$$Z^A(p,q,y) = \frac{1}{2} \sum_{m,r,s} \sum_{t=0}^{\min(2,m+s)} \sum_{k=0}^{\infty} d(r)d(s)d^A(t)\epsilon^A(r,s)p^{m+1}q^{\frac{m+s+t}{2}+k}$$

$$\times \sum_{j=0}^{m+s-t} y^{m+s-t-2j}$$

(0.8)

where we have used the fact that $(-1)^{r+s}h_{r,s} = d(r)d(s)$ with $d(0) = d(2) = 1$ and $d(1) = -2$. The sum over $m, r, s$ takes into account all $(c,c)$ primaries $(m + r, m + s)$. We have also included here the $m = r = s = 0$, which is not a $(c,c)$ primary in supergravity, to take into account the redefined $Z$'s including a shift by $p/2$. The sum over $k$ takes into account the descendents coming from applying $L^{-1}$ and the primed sum over $k$ means that for $m + s = 0$ there is only one term in the sum, namely $k = 0$, and for $m + s \neq 0$ the sum is over all the non-negative integers. This is due to the fact that for $m + s = 0$ we have the left ground state and $L^{-1}$ annihilates this state. The sum over $j$ takes care of the descendents coming from applying $J^{-1}$ and finally sum over $t$ takes into account the descendents coming from applying $G^{-1}$. The upper bound on this sum means that if $m + s$ is less than 2, then we can only apply a maximum of $m + s$ $G^{-1}$'s to the chiral primary. $d^A(t)$ takes into account the multiplicities of these descendents, together with the $Z_2$ actions. Since in model $II$, the $Z_2$ commutes with $(4,4)$ supersymmetry, $d^{II}(t) = d(t)$. On the other hand for models $I$ and $III$, the $Z_2$'s anticommute with $G^{-1}$'s and therefore $d^{I}(t) = d^{III}(t) = d(t)$ for $t$ even and $d^{I}(1) = d^{III}(1) = -d(1) = 2$. Note that since $\epsilon^I(r,s) = (-1)^s$ the sum over $r$ yields zero on the right hand side as expected for model $I$. For models $II$ and $III$, since $\epsilon(r,s)$ contains $(-1)^r$, the summation over $r$ gives a factor of 4.

The same applies also to the CFT side. but now with $c_{cft}^{A^+}(m,\ell)$ given by the expansion coefficients of the partition function $Z_{N=1}^g$ over the $(4,4)$ CFT with target space $T^4$ in zero momentum and winding sectors. The $Z_2$ orbifold actions are given by $g^{II} = I_4$ and $g^{III} = (-1)^{F_L}I_4$. It follows that

$$\sum_{m,\ell} c_{cft}^{II+}(m,\ell)q^m y^\ell = 8 \left[ \frac{\theta_2(q,z)}{\theta_2(q,0)} \right]^2$$

$$\sum_{m,\ell} c_{cft}^{III+}(m,\ell)q^m y^\ell = -8 \left[ \frac{\theta_1(q,z)}{\theta_2(q,0)} \right]^2$$

(0.9)

where $y = e^{2\pi iz}$.

The matching of CFT states and supergravity states for dimensions less than $N/4$
implies the following relations
\[
\sum_n c_{\text{cft}}^A (4mn - n^2 - \ell^2) = \sum_n c_{\text{sgf}}^A (m, n, \ell)
\]
\[
\sum_n nc_{\text{cft}}^A (4mn - n^2 - \ell^2) = \sum_n nc_{\text{sgf}}^A (m, n, \ell)
\]

(0.10)

between the supergravity and CFT expansion coefficients. A detailed evaluation of the quantities entering in both sides of (0.10) for model II and III can be found in [7]. The results show a complete agreement between supergravity and CFT computations for model II in the expected range of validity. For model III however, we find a discrepancy for states with \(\ell = 0\) and \(m > 0\). This mismatch at the 3-charge level have been already observed in [6], when CFT counting formulas were tested against U-duality. More precisely, consistency with U-duality requires that counting formula should be invariant under the simultaneous exchanged of model II and III and the \(D_1\) and \(p_1\) charges. This implies that the elliptic genus for model III should be the same as model II with \(p\) and \(q\) exchanged. One can wonder whether the supergravity result in the model III agrees with the CFT counting formula in model II after exchanging \(p\) and \(q\). This turns out to be the case (see [7] for details), allowing us to test the \(\mathcal{N} = (4, 4)\) CFT in the regimes of small and large conformal dimensions by two different supergravity duals.

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