Finite-Time Stability and Stabilization of Impulsive Stochastic Delayed Neural Networks With Rous and Rons

TAO CHEN, SHIGUO PENG, YINGHAN HONG, AND GUIZHEN MAI

1School of Automation, Guangdong University of Technology, Guangzhou 510006, China
2School of Physics and Electronic Engineering, Hanshan Normal University, Chaozhou 521041, China
3School of Computer Science and Technology, Guangdong University of Technology, Guangzhou 510006, China

Corresponding author: Yinghan Hong (honyinghan@163.com)

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ABSTRACT

This paper mainly tends to investigate finite-time stability and stabilization of impulsive stochastic delayed neural networks with randomly occurring uncertainties (ROUs) and randomly occurring nonlinearities (RONs). Firstly, by constructing the proper Lyapunov-Krasovskii functional and employing the average impulsive interval method, several novel criteria for ensuring the finite-time stability of impulsive stochastic delayed neural networks are obtained by means of linear matrix inequalities (LMIs). Then, some conditions about the state feedback controller are derived to ensure the finite-time stabilization of impulsive stochastic delayed neural networks with ROUs and RONs. Finally, numerical examples are provided to demonstrate the effectiveness and feasibility of the proposed results.

INDEX TERMS

Finite-time stability and stabilization, impulsive stochastic neural networks, average impulsive interval, time-varying delay, ROUs, RONs.

I. INTRODUCTION

Neural networks (NNs) are usually considered as one of the simplified models of neural processing in human brain [1]–[5]. By simulating the nature of message passing between neurons, NNs have significant advantages in estimating data and learning algorithms compared with many traditional approaches. In the past few decades, NNs have been successfully applied in various fields, such as combinatorial optimization [6], pattern recognition [7], image processing [8] and so on. Besides, as an useful dynamical model, the dynamics of NNs is of great significant research subject.

Stability of complex networks, as the fundamental collective dynamical phenomenon, plays an important role in the analysis of other dynamical behaviors. Up to now, a large number of scholars have focused on the study of stability [9]–[11]. In fact, most of the studies were about whether the networks can tend towards stability in infinite time.

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For example, asymptotic stability, exponential stability, bounded-input-bounded-output stability and so on, all of these stability schemes make the networks stable in an infinite time interval. However, it is worth noticing that the networks are often required to reach stability within finite time interval in practical applications due to their finite service life and energy efficiency. As an efficient scheme, on the one hand, the finite-time stability has better robustness and disturbance rejection properties. On the other hand, in some instantaneous systems, the finite-time stability is more suitable than the infinite-time stability. Therefore, compared with infinite-time stability, the finite-time stability appeals to many researchers strongly and many results have been established to this concept [12]–[18]. In [14], global finite-time stabilization of uncertain time-varying nonlinear systems by means of time-varying state feedback. By combining the time-varying methods and finite-time control technique, the authors proposed a new nonadaptive design scheme to achieve the finite-time stabilization. In [16], a class of delayed non-linear Hamiltonian systems were investigated. Based on the Razu-mikhin method, authors designed the finite-time $H_\infty$ control
and proposed the finite-time stability criterion about the delayed non-linear systems.

It is widely known that transmission of information in the practical systems is not always instantaneous, which is frequently affected by the time delay. Similarly, time delay signals are inevitable during the application of neural networks, which frequently appear not only in the state of system but also in the derivatives of the state of system [19]–[25]. This is the main reasons for leading to NNs poor performance and even instability. Since the time delay signals in a dynamical system often leads to deterioration or instability of the system, the stability analysis of delayed NNs has important theoretical and practical significance. For instance, to the discrete-time neural networks with time-varying delay, authors in [24] proposed two improved delay-dependent stability rules by means of constructing a Lyapunov-Krasovskii functional with several augmented terms. In [25], based on the Lyapunov-Razumikhin techniques, the linear time-varying time-delay system could be ensured finite-time stability and finite-time contractive stability.

Furthermore, in real life, there exist many stochastic disturbances in physical systems and mechanical equipment, for instance, noise disturbances, randomly occurring uncertainties (ROUs) and randomly occurring nonlinearities (RONs) [26]. NNs, of course, are often subject to kinds of stochastic disturbances. Therefore, it is of great importance to research on the stochastic models of NNs. And in the meantime, most of the scholars are fascinated by the stochastic neural networks (SNNs) [26]–[31]. In [26], authors proposed the impulsive pinning control to investigate the exponential consensus of stochastic multi-agent systems with ROUs and RONs by means of the Lyapunov function and the Halanay differential inequality. In [31], by using the Razumikhin techniques, authors studied the robust exponential stability of impulsive SNNs with delayed impulses.

As we know, owing to the sudden changes of systems at a certain time, impulsive phenomenon appears widely in the world and it is called the impulsive effect. As an inevitable phenomenon, impulsive effect is adorned with a lot of advantages. Not only can impulsive effect enhance the data security but also it can reduce the cost of control. Besides the merits, impulsive effect has also many disadvantages. It can influence the stability of the systems, and what’s worse, it can destroy the performance of the systems. Therefore, the research of impulsive effect has become a hot spot in many scholars and many relevant results have been reported in [32]–[34]. For instance, based on certain average impulsive interval method, [32] investigated the finite-time synchronization of coupled networks with Markovian topology and impulsive effects, where the impulse could be either synchronizing impulses or desynchronizing impulses with the impulsive interval. In [33], a class of delay switched systems with delayed impulsive effects was studied. By the idea of Lyapunov stability theory of impulsive differential and the technique of inequalities, the finite-time stability criteria were established. For the NNs with impulsive effects and time-varying delay, authors in [34] demonstrated the finite-time stability by means of Lyapunov-Krasovskii functional and the average impulsive interval method whether it was the stabilizing impulses or the destabilizing impulses.

At present, although a great deal of stability or stabilization results of dynamic systems have been obtained in many existing papers [14]–[16], [19], [22]–[25], [35], finite-time stability and stabilization results of impulsive stochastic delayed neural networks (ISDNNs) with ROUs and RONs are still very few. Therefore, the study for finite-time stability and stabilization of ISDNNs with ROUs and RONs has aroused our great interest.

Motivated by the aforementioned discussions, the problem of finite-time stability and stabilization for ISDNNs with ROUs and RONs is considered in this paper. This note presents the following main novelties.

- This paper considers a class of ISDNNs with ROUs and RONs, which has just a little research. The network model is quite comprehensive, which is closer to the engineering practice. Since finite-time stability has better disturbance rejection properties, the results of this work are very important.
- By structuring proper Lyapunov-Krasovskii functional and by means of the average impulsive interval method, some new finite-time stability criteron in terms of LMI are derived for ISDNNs with ROUs and RONs.
- On the basic of the obtained results, a state feedback controller is proposed to ensure the same networks finite-time stabilization.

This paper is structured as follows. In Section II, some preliminaries and network formulation are presented. The finite-time stability of ISDNNs with ROUs and RONs is further analyzed in Section III. The state feedback controller is designed to ensure the same networks finite-time stabilization in Section IV. In Section V, we provide some examples to demonstrate the effectiveness of these analytical results. Finally, the conclusions are drawn in Section VI.

Notations: The notations used throughout this article are standard. I_n denotes the identity matrix. \( \mathbb{R} \) represents the sets of real numbers. \( \mathbb{R}^n \) is the n-dimensional Euclidean space, and \( \mathbb{R}^{n \times m} \) denotes the set of all \( n \times m \) real matrices. \( \mathbb{N} \) and \( \mathbb{N}_+ \) denote the set of natural numbers and positive integers, respectively. diag \{ \cdots \} stands for a block-diagonal matrix and \( \text{trace} \{ A \} \) denotes the trace of matrix A. \( A^T \) stands for its transpose. \( \| z \| = \sqrt{z^T z} \) stands for the Euclidean norm. \( \lambda_{\max} (A) \) and \( \lambda_{\min} (A) \) mean the largest and the smallest eigenvalue of matrix A, respectively. For a symmetric real matrix A, \( A \succ 0 \) (\( A \preceq 0 \)) means that the matrix A is positive (negative) definite. Let \( (\Omega, \mathcal{F}, \{ \mathcal{F}_t \}_{t \geq 0}, \mathcal{P}) \) be a complete probability space with filtration \( \{ \mathcal{F}_t \}_{t \geq 0} \) satisfying the usual conditions (i.e., the filtration contains all \( \mathcal{P} \)-null sets and is right continuous), and \( \mathbb{E} \{ \cdot \} \) stands for the mathematical expectation operator with respect to a given probability measure \( \mathcal{P} \).
II. PROBLEM STATEMENT AND PRELIMINARIES

In this section, some preliminaries about the model and necessary assumptions are given. Firstly, we briefly describe the problem formulation. Consider the following stochastic delayed NNs with ROUs and RONs:

\[
\begin{align*}
dz(t) &= \left[ C(t)z(t) + \beta_1(t)g(z(t)) + \beta_2(t) \right] dt \\
&\quad + \sigma \left( z(t), z(t - \tau(t)) \right) dw(t), \quad t > 0, \\
z_0(\ell) &= \varphi(\ell), \quad -\tau \leq \ell \leq 0,
\end{align*}
\]

where \( z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T \in \mathbb{R}^n \) is the state vector associated with the neurons; \( C(t) \) stands for the weight matrix and can be further expressed as \( C(t) = C + c(t) \Delta C(t) \), \( \Delta C(t) = MY(t) U \) in which \( C, M \) and \( U \) can be seen as the real constant matrix with appropriate dimensions. Besides, \( Y(t) \) stands for the non-linear time-varying function satisfied that \( Y^T(t) Y(t) \leq I, g : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) denotes the activation functions of the neurons; \( u(t) \in \mathbb{R}^n \) is the control input, \( \tau(t) \) is an unknown but bounded time-varying delay satisfying \( 0 \leq \tau(t) \leq \tau \); \( \hat{\tau} \leq \ell \leq 0 \); \( \sigma : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m} \) is the noise intensity function. \( w(t) = [w_1(t), w_2(t), \ldots, w_m(t)]^T \) is an m-dimensional Wiener process defined on \( (\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, \mathbb{P}) \), \( w_i(t) \) is independent of \( w_j(t) \) for \( i \neq j \) and \( i = 1, 2, \ldots, m \). \( \varphi(\ell) \) is the continuous initial condition.

Remark 1: The term \( \Delta C(t) \) is used to stand for the phenomena of ROUs, and the terms \( \beta_1(t) g(\cdot), \beta_2(t) g(\cdot) \) are used to express as the phenomena of RONs. What’s more, the stochastic variables \( c(t), \beta_1(t) \) and \( \beta_2(t) \) are all subject to Bernoulli distributing white sequences where their values are either one or zero. And they meet the following assumptions:

\[
\begin{align*}
Pr \{ c(t) = 1 \} &= c, \quad Pr \{ c(t) = 0 \} = 1 - c, \\
Pr \{ \beta_1(t) = 1 \} &= \beta_1, \quad Pr \{ \beta_1(t) = 0 \} = 1 - \beta_1, \\
Pr \{ \beta_2(t) = 1 \} &= \beta_2, \quad Pr \{ \beta_2(t) = 0 \} = 1 - \beta_2,
\end{align*}
\]

where \( c, \beta_1, \beta_2 \in [0, 1] \) are known constants. Obviously, according to (2), we can get that \( \mathbb{E} \{ c(t) - c \} = 0, \mathbb{E} \{ \beta_1(t) - \beta_1 \} = 0, \mathbb{E} \{ \beta_2(t) - \beta_2 \} = 0 \). Furthermore, we suppose that the stochastic variables \( c(t), \beta_1(t), \beta_2(t) \) and \( w(t) \) are mutually independent.

Owing to the impulsive phenomenon everywhere, we should consider the impulsive effects on the finite-time stability, the ISDNs can be obtained as follows:

\[
\begin{align*}
dz(t) &= \left[ C(t)z(t) + \beta_1(t)g(z(t)) + \beta_2(t)g(z(t - \tau(t))) \right] dt \\
&\quad + \sigma \left( z(t), z(t - \tau(t)) \right) dw(t), \quad t \neq t_k, \\
z(t_k) &= Jz(t_k^-), \quad t = t_k, \quad k \in \mathbb{N},
\end{align*}
\]

where \( J \) is an impulse control gain matrix to be designed; \( t_k, k \in \mathbb{N} \) is the impulse instant satisfying \( 0 < t_0 < t_1 < t_2 < \cdots < t_k < \cdots \) with \( \lim_{k \rightarrow \infty} t_k = \infty \). Without loss of generality, we assume that \( z(t) \) is the right-hand continuous at \( t = t_k \), i.e., \( z(t_k) = z(t_k^+) \).

Remark 2: Compared with the system (1) in [34], we add \( \sigma (\cdot, \cdot, \cdot) dw(t) \) in (3), meanwhile consider ROUs and RONs. Besides, compared with system (2) in [29] without considering impulsive effects for finite-time stochastic synchronization, this paper considers comprehensively the stochastic NNs with impulsive effects.

The following definitions, lemmas and assumptions are required to obtain our results.

Inspired by [17] and [25], our aims are all the finite-time stability. But different from the above literatures, we focus on the finite-time stability of the ISDNs with ROUs and RONs. And we can derive the following finite-time stability definition.

Definition 1: The ISDNs with ROUs and RONs (3) is said to be finite-time stable with respect to \( (c_1, c_2, \hat{T}) \) with \( c_1 < c_2 \), where \( c_1 \) and \( c_2 \) are positive constants, if for the given any initial conditions satisfying:

\[
\mathbb{E} \left( \sup_{t \in [0, \hat{T}]} \| z(t) \|^2 \right) \leq c_1, \quad \forall t \in [0, \hat{T}],
\]

one has

\[
\mathbb{E} \left( \| z(t) \|^2 \right) \leq c_2, \quad \forall t \in [0, \hat{T}],
\]

where \( \hat{T} \) is a positive constant.

Definition 2: [36] An impulsive sequence \( \xi = \{ t_1, t_2, \ldots \} \) is said to have average impulsive interval \( T_a \) if there exist positive integer \( N_0 \) and positive constant \( T_a \) such that

\[
\frac{\hat{T} - t}{T_a} - N_0 \leq N_\xi \left( \hat{T}, t \right) \leq N_\xi \left( \hat{T}, t \right) + \frac{\hat{T} - t}{T_a},
\]

for any \( \hat{T} > t \geq 0 \), where \( N_\xi \left( \hat{T}, t \right) \) denotes the number of impulsive times of the impulsive sequence \( \xi = \{ t_1, t_2, \ldots \} \) on the interval \( (t, \hat{T}] \).

Since we are interested in the dynamic characteristics of the system (3) within a finite time interval \( [0, \hat{T}] \). Therefore, it is supposed that there exists a scalar \( N_\xi \left( \hat{T}, 0 \right) \in \mathbb{N}_+ \) such that

\[
0 < t_1 < \cdots < t_{N_\xi \left( \hat{T}, 0 \right)} \leq \hat{T}.
\]

Assumption 1: There exist scalars \( \rho_1 > 0 \) and \( \rho_2 > 0 \) such that for any \( p, q \in \mathbb{R}^n \)

\[
trace \left[ \sigma^T(t, p, q) \sigma(t, p, q) \right] \leq \rho_1 p^T p + \rho_2 q^T q.
\]

Assumption 2: The vector \( g_i(\cdot) (i = 1, 2, \ldots, n) \) are assumed to be bounded and there exist scalars \( \tilde{k}_i, \tilde{k}_i \), such that for any \( a, b \in \mathbb{R}, a \neq b \),

\[
\tilde{k}_i \leq \frac{g_i(a) - g_i(b)}{a - b} \leq \tilde{k}_i.
\]
\( \omega : [0, \rho] \rightarrow \mathbb{R}^n \) such that the integrations concerned are well defined, then
\[
\rho \int_0^\rho \omega^T(s) W \omega(s) \, ds \geq \left( \int_0^\rho \omega(s) \, ds \right)^T W \left( \int_0^\rho \omega(s) \, ds \right).
\]  
(9)

**Lemma 3 (Gronwall inequality):** Let constants \( \nu \in \mathbb{R} \) and nonnegative constant \( \gamma \in \mathbb{R}_+ \). If there exists the function \( y(\cdot) \) satisfying
\[
y(t) \leq \nu + \int_a^t \gamma y(s) \, ds, \quad a \leq t \leq b,
\]
then one has
\[
y(t) \leq \nu e^{\gamma(t-a)}.
\]

In the following, we will assess the finite-time stability of (3) by establishing the sufficient conditions in forms of the main theorem of this paper.

### III. Finite-Time Stability Analysis

In this section, without considering the controller, namely, \( u(t) = 0 \), we shall focus on analyzing finite-time stability of ISDNNs with ROUs and RONs. In order to do so, we present the following Theorem 1 firstly.

**Theorem 1:** Suppose that the Assumptions 1 and 2 hold. The average impulsive interval of the impulsive sequence \( \zeta = \{ t_1, t_2, \ldots \} \) is \( T_{\zeta} \), given three positive constants, \( t \), \( c_1 \) and \( c_2 \) with \( c_1 < c_2 \), if there exist constants \( \varepsilon_1, \rho_1, \rho_2, \eta, N_0, \lambda > 0 \) and \( \delta \geq 1 \), symmetric positive definite matrices \( P > 0, Q > 0, R > 0 \), and diagonal matrix \( W_j > 0, (j = 1, 2) \) such that the following inequalities hold:
\[
\Sigma = \begin{bmatrix}
A_{11} & 0 & A_{13} & A_{14} & 0 & A_{16} \\
* & A_{22} & 0 & A_{24} & 0 & 0 \\
* & * & A_{33} & 0 & 0 & 0 \\
* & * & * & A_{44} & 0 & 0 \\
* & * & * & * & A_{55} & 0 \\
* & * & * & * & * & A_{66}
\end{bmatrix} \leq 0,
\]  
(10)

\[
J^T P \leq \delta P,
\]  
(11)

\[
P \leq \eta I_n,
\]  
(12)

\[
\frac{c_1}{\lambda_{\min}(P)} \delta N_0 e^{\left( \frac{\lambda_{\max}(P)}{2} \right) t} \times \left[ \lambda_{\max}(P) \right]
\]  
\[
+ \tau e^{\lambda_{\max}(P)} (Q) + \tau^2 e^{\lambda_{\max}(P)} (R) < c_2,
\]  
(13)

\[\begin{align*}
2z^T(t) & \left( PC + C^T P \right) z(t) \\
& = 2z^T(t) P (C + c(t) MY(t) U) z(t) \\
& = z^T(t) \left( PC + C^T P \right) z(t) \\
& + 2z(t) (t) PMY(t) Uz(t) \\
& + 2(c(t) - c) z^T(t) PMY(t) Uz(t) \\
& \leq z^T(t) \left[ PC + C^T P \right] z(t) \\
& + 2(c(t) - c) z^T(t) PMY(t) Uz(t),
\end{align*}\]  
(18)

**Proof:** Construct the following Lyapunov-Krasovskii functional
\[
V(t) = \sum_{i=1}^{3} V_i(t),
\]  
(14)

where
\[
V_1(t) = z^T(t) Pz(t),
\]
\[
V_2(t) = \int_{-\tau(t)}^{t} e^{\lambda(t-s)} z^T(s) Qz(s) \, ds,
\]
\[
V_3(t) = \int_{-\tau(t)}^{t} \int_{t+\theta}^{t} e^{\lambda(t-s)} z^T(s) Rz(s) \, ds \, d\theta.
\]

We use \( LV(t) \) to denote the infinitesimal operator of \( V(t) \). For \( t \in [t_k, t_{k+1}), k \in \mathbb{N} \), differentiating \( V(t) \) along the solution of (3) gives that
\[
L V_1 = 2z^T(t) P \left[ C(t)z(t) + \beta_1(t)g(z(t)) + \beta_2(t)g(z(t) - \tau(t)) \right]
\]
\[
+ \text{trace} \left[ \sigma^T(t, z(t), z(t) - \tau(t)) \right] \times P \sigma(t, z(t), z(t) - \tau(t)) - z^T(t) \lambda P z(t) + \lambda V_1,
\]  
(15)

\[
L V_2 = \int_{-\tau(t)}^{t} e^{\lambda(t-s)} z^T(s) Qz(s) \, ds + z^T(t) Q z(t)
\]
\[
- (1 - \check{\tau}(t)) e^{\lambda(t-t)} z^T(t - \tau(t)) Q z(t - \tau(t)) \leq z^T(t) Q z(t) - (1 - \mu) e^{\lambda(t-t)} z^T(t - \tau(t)) Q \times z(t - \tau(t)) + \lambda V_2,
\]
\[
\leq z^T(t) Q z(t) - \mu e^{\lambda(t-t)} z^T(t - \tau(t)) Q \times z(t - \tau(t)) + \lambda V_2,
\]  
(16)

\[
L V_3 = \tau z^T(t) R z(t) - \int_{t-\tau(t)}^{t} e^{\lambda(t-s)} z^T(s) R z(s) \, ds
\]
\[
+ \lambda \int_{t-\tau(t)}^{t} z^T(s) R z(s) \, d\theta \leq \tau z^T(t) R z(t) - \int_{t-\tau(t)}^{t} z^T(s) R z(s) \, ds + \lambda V_3.
\]  
(17)

According to Lemma 1, it yields that
\[
2z^T(t) P C(t) z(t) \]
\[
= 2z^T(t) P (C + c(t) MY(t) U) z(t) \]
\[
= z^T(t) \left( PC + C^T P \right) z(t) \\
+ 2z(t) (t) PMY(t) Uz(t) \\
+ 2(c(t) - c) z^T(t) PMY(t) Uz(t) \leq z^T(t) \left[ PC + C^T P \right] z(t) \\
+ 2(c(t) - c) z^T(t) PMY(t) Uz(t).
\]  
(18)

\[
2\beta_1(t) z^T(t) P g(z(t)) \]
\[
= 2\beta_1 z^T(t) P g(z(t)) \\
+ 2(c(t) - c) z^T(t) P g(z(t)) + 2(c(t) - \check{c}) z^T(t) P g(z(t)).
\]  
(19)
\[ 2\beta_2 (t) z^T (t) Pg (z(t - \tau (t))) \\
= 2\beta_2 z^T (t) Pg (z(t - \tau (t))) \\
+ 2 (\beta_1 (t) - \beta_1) z^T (t) Pg (z(t - \tau (t))). \quad (20) \]

Note that the Assumption 1 and \( P \leq \eta I \), it yields that

\[ \text{trace} \left[ \sigma^2 \left( t, z(t) , z(t - \tau (t)) \right) P \sigma \left( t, z(t) , z(t - \tau (t)) \right) \right] \leq \eta \rho_1 z^T (t) z(t) + \rho_2 z^T (t - \tau (t)) z(t - \tau (t)). \quad (21) \]

Based on applying Lemma 2, one obtains

\[ - \int_{t-\tau(t)}^t z^T (t) R z (s) ds \leq - \frac{1}{\tau} \left[ \int_{t-\tau(t)}^t z^T (s) ds \right]^T R \left[ \int_{t-\tau(t)}^t z (s) ds \right]. \quad (22) \]

In sight of Assumption 2 and \( g_i (0) = 0 \), it is obvious that

\[ \left( g_i (z(t)) - \hat{k}_i z(t) \right) \left( \hat{k}_i z(t) - g_i (z(t)) \right) \geq 0. \quad (23) \]

And for any positive diagonal matrix \( W_i = \text{diag} \{ w_{i1}, w_{i2}, \ldots, w_{im} \}, i = 1, 2, \) combining with (23) which leads to

\[ 2z^T (t) W_1 K_1 g(z(t)) - g^T (z(t)) W_1 g(z(t)) - z^T (t) W_1 K_2 z(t) \geq 0, \quad (24) \]

\[ 2z^T (t - \tau (t)) W_2 K_1 g(z(t - \tau(t))) - g^T (z(t - \tau(t))) W_2 g(z(t - \tau(t))) - z^T (t - \tau (t)) W_2 K_2 z(t - \tau(t)) \geq 0, \quad (25) \]

where \( K_1 = \text{diag} \{ \hat{k}_1 \tilde{k}_1, \hat{k}_2 \tilde{k}_2, \ldots, \hat{k}_m \tilde{k}_m \}, \quad K_2 = \text{diag} \{ \tilde{k}_1 \hat{k}_1, \tilde{k}_2 \hat{k}_2, \ldots, \tilde{k}_m \hat{k}_m \}. \]

Then in view of (18)-(25), we can get

\[ \mathcal{L} V (t) = \mathcal{L} V_1 + \mathcal{L} V_2 + \mathcal{L} V_3 \]

\[ \leq z^T (t) [P C + C^T P + \varepsilon_1^{-1} e^2 PMMT P + \varepsilon_1 UU^T \ln (t) + 2 (c(t) - c) z^T (t) P MMT P \times U_z (t) + 2 \beta_1 z^T (t) Pg (z(t)) + 2 (\beta_1 (t) - \beta_1) z^T (t) Pg (z(t)) + 2 \beta_2 z^T (t) Pg (z(t - \tau (t))) + 2 (\beta_2 (t) - \beta_2) z^T (t) Pg (z(t - \tau (t))) \times \hat{\eta} z^T (t) P z (t + \tau (t)) + z^T (t - \tau (t)) Q z (t - \tau (t)) - (1 - \mu) z^T (t - \tau (t)) Q z (t - \tau (t)) + \tau z^T (t) R z (t)] \\
\]

\[ - \frac{1}{\tau} \left[ \int_{t-\tau(t)}^t z^T (s) ds \right]^T R \left[ \int_{t-\tau(t)}^t z (s) ds \right] + 2z^T (t) W_1 K_1 g(z(t)) - g^T (z(t)) W_1 g(z(t)) - z^T (t) W_1 K_2 z(t) + 2z^T (t - \tau (t)) W_2 K_1 \\
\times g(z(t - \tau(t))) - g^T (z(t - \tau(t))) W_2 g(z(t - \tau(t))) W_2 + g(z(t - \tau(t))) - z^T (t - \tau (t)) W_2 K_2 \\
x (t - \tau (t)) + \lambda V_1 + \lambda V_2 + \lambda V_3 \leq \phi^T (t) \Sigma \phi (t) + 2 (c (t) - c) z^T (t) PMY^T (t) U \times z (t) + 2 (\beta_1 (t) - \beta_1) z^T (t) Pg (z(t)) + 2 (\beta_2 (t) - \beta_2) z^T (t) Pg (z(t - \tau (t))) + \lambda V \leq \lambda V + 2 (c(t) - c) z^T (t) PMY^T (t) U z (t) + 2 (\beta_1 (t) - \beta_1) z^T (t) Pg (z(t)) + 2 (\beta_2 (t) - \beta_2) z^T (t) Pg (z(t - \tau (t))) \], \]
On the other hand, from the definition of $V(t)$, one can easily obtain the following inequalities

$$
\mathbb{E} V(t_0) \leq \lambda_{\max} (P) \mathbb{E} \left[ \sup_{t \in [t_0, T]} \| z(t) \|^2 \right],
$$
(35)

$$
\mathbb{E} V(t_0) \leq \mathbb{E} V(t) + \lambda_{\max} (Q) \mathbb{E} \left[ \sup_{t \in [t_0, T]} \| z(t) \|^2 \right],
$$
(36)

$$
\mathbb{E} V(t_0) \leq \mathbb{E} V(t) + \lambda_{\max} (Q) \mathbb{E} \left[ \sup_{t \in [t_0, T]} \| z(t) \|^2 \right].
$$
(37)

Thus, from (35),(36) and (37), one obtains

$$
\mathbb{E} V(t) \leq \lambda_{\max} (P) + \lambda_{\max} (Q) + \lambda_{\max} (R) \mathbb{E} \left[ \sup_{t \in [t_0, T]} \| z(t) \|^2 \right].
$$
(38)

Also, we have

$$
\mathbb{E} V(t) \geq \lambda_{\min} (P) \| z(t) \|^2.
$$
(39)

Hence, from (34), we have

$$
\| z(t) \|^2 \leq \frac{1}{\lambda_{\min} (P)} \mathbb{E} V(t)
$$

$$
\leq \frac{1}{\lambda_{\min} (P)} \mathbb{E} V(t_0)
$$

$$
\leq \frac{1}{\lambda_{\min} (P)} \mathbb{E} V(t) + \lambda_{\max} (P) \mathbb{E} \left[ \sup_{t \in [t_0, T]} \| z(t) \|^2 \right]
$$

$$
\times \left[ \lambda_{\max} (P) + \lambda_{\max} (Q) + \lambda_{\max} (R) \right].
$$
(40)

Therefore, under the conditions in Theorem 1, for all $t \in [0, T]$, $\| z(t) \|^2 \leq c_2$. According to the Definition 1, the ISDNNs with ROUs and RONs is finite-time stable. This completes the proof.

**IV. FINITE-TIME STABILIZATION ANALYSIS**

In this section, we will design a state feedback controller for the ISDNNs to guarantee the finite-time stabilization. Give the following state feedback control law

$$
u(t) = Fz(t),
$$
(41)

where $F$ is the state feedback controller gain matrix.

Therefore, the dynamics of ISDNNs via feedback controller (41) is expressed as follows

$$
\begin{aligned}
dz(t) &= \{C(t)z(t) + B_1(t)g(z(t)) + B_2(t)g(z(t - \tau(t))) + Fz(t)\}dt \\
+ \sigma(t, z(t), z(t - \tau(t)))dw(t), \quad t \neq t_k, \\
\end{aligned}
$$
(42)

$$
z(t_k) = J\zeta(t_k),
$$

$$
t = t_k, \ k \in \mathbb{N}_+.
$$

Next, we present a theoretical result to guarantee that the ISDNNs with ROUs and RONs can realize finite-time stabilization via feedback controller (41).

**Theorem 2:** Suppose that the Assumptions 1 and 2 hold. The average impulsive interval of the impulsive sequence $\zeta = \{t_1, t_2, \ldots\}$ is $T_d$, given three positive constants, $\bar{t}$, $c_1$ and $c_2$ with $c_1 < c_2$, if there exists constants $\epsilon_1, \rho_1, \rho_2, \eta, N_0, \lambda > 0$ and $\delta \geq 1$, symmetric positive definite matrices $Z > 0, G > 0, \hat{Q} > 0, \hat{R} > 0$, and diagonal matrix $\hat{W}_i > 0, (i = 1, 2)$ such that the following inequalities hold:

$$
\begin{bmatrix}
\hat{Q} & \hat{R} \n
\end{bmatrix} < c_2,
$$
(45)

where $\theta_{11} = CZ + ZC^T + G + G^T + \epsilon_1 c_2 M M^T + \hat{Q} + \hat{R} - \hat{W}_1 K_2 - \hat{W}_1 K_2 - \hat{L} Z, \theta_{12} = 2\beta_1 Z + 2W_1 K_1, \theta_{13} = 2\beta_2 Z, \theta_{14} = \beta_2 Z, \theta_{15} = \sqrt{\lambda_{\min} (Z)}, \theta_{16} = \sqrt{\lambda_{\min} (Z)}$, $\theta_{17} = \sqrt{\lambda_{\min} (Z)}$, $\theta_{18} = \sqrt{\lambda_{\min} (Z)}$, $\theta_{22} = -\hat{W}_2 K_2 - \hat{W}_2 K_2 - (\lambda_{\max} (Z))$, $\theta_{24} = -\hat{W}_1 K_1, \theta_{33} = -\hat{W}_1, \theta_{44} = -\hat{W}_4, \theta_{55} = -\hat{W}_5, \theta_{66} = -\hat{W}_6, \theta_{88} = -\hat{W}_8$. Other parameters are the same as the Theorem 1. Then system (42) is finite-time stable with respect to $(c_1, c_2, \bar{t})$.

**Proof:** Similar to the proof of Theorem 1, the same Lyapunov functional candidate is considered. According to (42), replacing the $C$ in (10) by $C + F$, it is not hard to find the following inequity:

$$
\begin{bmatrix}
\hat{Q} & \hat{R} \n
\end{bmatrix} < c_2,
$$
(46)

where $\hat{Q} = P(C + F) + (C + F)^T P + \epsilon_1 c_2 M M^T + \epsilon_1 U U^T + \eta_1 I_n + Q + \tau R - \hat{W}_1 K_2 - \hat{L} P, \hat{Q} = \eta_1 I_n - \hat{W}_2 K_2 - (\lambda_{\max} (Z)) Q$. Other parameters are the same as the Theorem 1.

Then, pre- and post-multiplying the inequality (46) by $P^{-1}$, $F = GZ^{-1}, \hat{Q} = ZQZ, \hat{R} = ZRZ, W_1 = ZW_1 X$ and $W_2 = ZW_2 X$, according to the Schur complement Lemma, the inequality (43) holds.

Similarly, pre- and post-multiplying the inequality (11) by $P^{-1}$, we derive that inequality (44) holds.

And other proof procedures are similar as the proof of Theorem 1, so we omitted here for saving space. Based on Definition 1, the system (42) is finite-time stabilization. The proof is completed.

**Remark 3:** In this paper, we propose a feedback controller to ensure that the system is finite-time stabilization. But the
finite-time stability means that the system is subject to external interference and can still maintain a stable state within finite time.

V. NUMERICAL EXAMPLES

In this section, numerical simulations are given to show that our obtained theoretical results described in the previous section are effectiveness.

Example 1: Consider the three dimensional ISDNNs with ROUs and RONs. The corresponding parameters are described as follows:

\[ C = \begin{bmatrix} -0.16 & 0.5 & 0.3 \\ 0.45 & -0.25 & 0.2 \\ 0.5 & 0.4 & -0.1 \end{bmatrix}, \quad M = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & -0.3 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \]

\[ U = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & -0.3 \end{bmatrix}, \]

\[ W_1 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 0.5 \end{bmatrix}. \]

neuron of the activation function is described as \( g(z(t)) = \tanh(z(t)) \), and the delayed activation function is \( g_2(z(t)) = \tanh(z(t - \tau(t))) \), where \( \tau(t) \) is stood for the time-varying delay expressed as

\[ \tau(t) = \frac{e^t}{e^t + 1}, \]

and the intensity function of noise is taken as

\[ \sigma(t, z(t), z(t - \tau(t))) = 0.5z(t) + 0.5z(t - \tau(t)), \]

We assume that the state of neuron is \( x \in \mathbb{R}^3 \), The Bernoulli-distributed stochastic variables are assumed as \( c = 0.7, \beta_1 = 0.82, \beta_2 = 0.58 \). Thus, with the help of the MATLAB LMI toolbox, the LMIs in Theorem 1 have feasible solutions as follows

\[ P = \begin{bmatrix} 0.2857 & -0.0144 & 0.0154 \\ -0.0144 & 0.3136 & 0.0195 \\ 0.0154 & 0.0195 & 0.2617 \end{bmatrix}, \]

\[ Q = \begin{bmatrix} 0.3456 & -0.0082 & 0.0032 \\ -0.0082 & 0.5397 & 0.0014 \\ 0.0032 & 0.0014 & 0.7794 \end{bmatrix}, \]

\[ R = \begin{bmatrix} 0.2332 & -0.0211 & 0.0131 \\ -0.0211 & 0.2415 & 0.0100 \\ 0.0131 & 0.0100 & 0.2332 \end{bmatrix}. \]

Set the initial condition of Example 1 with \( z_1(\ell) = 0.1, z_2(\ell) = 0.05, z_3(\ell) = 0.2 \), where \( \ell \in [-1, 0] \). Letting \( c_1 = 0.25, c_2 = 20, \ell = 3 \). In addition, the condition (13) in Theorem 1 implies that \( T_a > 0.15 \) should be satisfied. So choose \( T_a = 0.3 \), step-length is 0.01. FIGURE 1 is shown as an impulsive sequence with \( T_a = 0.2, N_0 = 10 \). Obviously, it is easy to see that the system (3) is finite-time stability with respect to (0.25, 20, 3), which is depicted in FIGURE 2.

Remark 4: Generally, as we know, system stability has a tendency to converge a certain value such as Lyapunov stability. According to FIGURE 2, system is finite-time stability with respect to (0.25, 20, 3). But it is not always stable inside the structure of Lyapunov stability and this finite-time stability is different from the Lyapunov stability.

Remark 5: In FIGURE 3, we can see that the trajectories of \( z(t) \) with \( T_a = 0.12, N_0 = 10 \) are not the finite-time stability.
with average impulsive interval method, sufficient conditions for finite-time stability are presented. In addition, the state feedback controller is designed such that the ISDNNs is finite-time stable.

In the real world, we realize that fixed-time stability of ISDNNs is better and more challenging than the finite-time stability. Therefore, we would further investigate how to stabilize the NNs in fixed time.

VI. CONCLUSION

In this paper, we studied the finite-time stability of ISDNNs with ROUs and RONs. By employing the new criterion and structuring the proper Lyapunov-Krasovskii function coupled

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