Lattice Boltzmann modeling and simulation of forced-convection boiling on a cylinder

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When boiling occurs in a liquid flow field, the phenomenon is known as forced-convection boiling. We numerically investigate such a boiling system on a cylinder at a saturated condition. To deal with the complicated liquid-vapor phase-change phenomenon, we develop a numerical scheme based on the pseudopotential lattice Boltzmann method (LBM). The collision stage is performed in the space of central moments (CMs) to enhance numerical stability for high Reynolds numbers. The adopted forcing scheme, consistent with the CMs-based LBM, leads to a concise yet robust algorithm. Furthermore, additional terms required to tune thermodynamic consistency are derived in a CMs framework. The effectiveness of the present scheme is successfully tested against a series of boiling processes, including nucleation, growth, and departure of a vapor bubble for Reynolds numbers varying between 30 and 30000. Unlike the Navier–Stokes-based simulations, our CMs-based LBM can reproduce all the boiling regimes, i.e., nucleate boiling, transition boiling, and film boiling, without any artificial input such as initial vapor phase. We find that the typical boiling curve, also known as the Nukiyama curve, appears even though the focused system is not the pool boiling but the forced-convection system. Also, our simulations support experimental observations of intermittent direct solid-liquid contact even in the film-boiling regime. Finally, we provide quantitative comparison with the semi-empirical correlations for the forced-convection film boiling on a sphere or cylinder on the $Nu$-$Ja$ diagram.

Key words: Forced-convection boiling, Lattice Boltzmann method, Multiphase flow, Phase change, Boiling curve, Heat transfer coefficient

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1. Introduction

Since the pioneering work of Nukiyama (1934), boiling phenomena have long been investigated. By referring to the so-called boiling curve (figure 1), three regimes can be identified: nucleate boiling, transition boiling, and film boiling with respect to surface superheat (Koizumi et al. 2017). The passage from one regime to another is identified by salient characteristic points, i.e., ONB (onset of nucleate boiling), CHF (critical heat flux), and MHF (minimum heat flux). If the system has no condensation ability against the generated vapor, it is called saturated boiling. Otherwise, the system is regarded as subcooled boiling. Pool boiling arises when vapor bubbles and heated liquid move only due to gravity, whereas the presence of other forces promotes the so-called forced-convection boiling.

Investigating heat transfer coefficient (HTC) of forced-convection boiling on a sphere has been a critical topic, especially in the nuclear safety field. That is because the HTC between a high-temperature melt particle and liquid plays a key role in modeling fuel-coolant interactions (FCIs) during any accidental event of nuclear power plants (Kobayasi 1965; Dhir & Purohit 1978; Epstein & Hauser 1980; Liu & Theofanous 1994). A schematic diagram of the possible FCI scenario is shown in figure 2(a), where melt fragments are generated from the ejected melt jets penetrating the coolant. Since the interaction between a cluster of fragments and coolant would be quite difficult to be modeled, the one between a single melt particle and liquid, as in figure 2(b), is often considered in melt-coolability evaluation. To this end, the HTC correlation of the forced-convection boiling, which is generally modeled within the film-boiling regime, is required. One can find that the forced-convection boiling on a sphere or cylinder simulates the situation in figure 2(b). The choice of such HTC correlations would lead to large uncertainties of melt-particle coolability. However, there has been little discussion about selecting an appropriate HTC correlation.

Starting from Bromley’s study in chemical engineering (Bromley 1950), film-boiling heat transfer has attracted much attention. Bromley et al. (1953) experimentally and analytically studied forced-convection film boiling on a cylinder at saturation temperature. They also derived an appropriate theoretical analysis under steady-state assumptions.
Similar analyses for the cases of flow past a sphere were reported later by Kobayasi (1965) and Witte (1968). Ito et al. (1981) analyzed the phenomena with various liquid media as water, ethanol, and hexane, based on the two-phase boundary-layer theory. Dhir & Purohit (1978) studied the pool and forced-convection film boiling on metal spheres under subcooled conditions from 0 to 50 K. Epstein & Hauser (1980) derived a semi-empirical correlation by solving conservation equations for vapor and liquid boundary layers at the stagnation point.

Liu & Theofanous (1994) reviewed the existing studies of the forced-convection film boiling on a sphere or cylinder, in which they classified the film-boiling HTC correlations into two modes as Mode 1 and Mode 2. In the dimensionless space, the film-boiling HTC is expressed as the Nusselt number:

\[ Nu = \frac{hD}{\lambda_v}, \]  

(1.1)

and is described in terms of the liquid-vapor density ratio \( \gamma \), viscosity ratio \( \eta \), liquid Reynolds number \( Re \), Prandtl number \( Pr \), and Jacob number \( Ja \) as follows:

\[ \gamma = \frac{\rho_l}{\rho_v}, \]  

(1.2)

\[ \eta = \frac{\nu_l}{\nu_v}, \]  

(1.3)

\[ Re = \frac{u_lD}{\nu_l}, \]  

(1.4)

\[ Pr = \frac{\nu_v}{\alpha_v}, \]  

(1.5)

\[ Ja = \frac{c_{p,v} \Delta T}{h_{fg}}, \]  

(1.6)

where \( h \) is the heat transfer coefficient, \( D \) is the characteristic length (typically, diameter of sphere or cylinder), \( \lambda \) is the thermal conductivity, \( \rho \) is the density, \( \nu \) is the kinematic viscosity, \( u \) is the flow velocity, \( \alpha \) is the thermal diffusivity, \( c_p \) is the specific heat at constant pressure, \( \Delta T \) is the superheat degree, and \( h_{fg} \) is the latent heat of vaporization. The subscript \( l \) and \( v \) denote the liquid and the vapor phases, respectively. Now let \( C \) denote a given constant. With dimensionless form, Mode 1 correlation (Bromley et al.
Mode 2 correlation (Epstein & Hauser 1980; Ito et al. 1981; Liu & Theofanous 1994), on the other hand, can be written as

\[
Nu = C \gamma^{1/4} \eta^{1/2} Re^{1/2} Pr^{1/4} Ja^{-1/4}.
\]

Both types of correlations state that \( Nu \propto \eta^{1/2} Re^{1/2} \), but have different dependencies on \( \gamma, \eta, Pr \), and \( Ja \). Such correlations are usually validated with experimental data, but the experimental setup and conditions generally include unexpected uncertainties. Thus, numerical simulations would be an effective approach to further understanding the heat transfer characteristics of the phenomena.

Due to its important role in elucidating the mechanism and heat transfer characteristics, numerical simulations of boiling phenomena have been carried out since late 1990s. The first attempt was made by Son & Dhir (1997), who studied the evolution of the liquid-vapor interface during saturated film boiling with a level-set method. Juric & Tryggvason (1998) extended a front-tracking method to simulate horizontal film boiling by adding a source term to the continuity equation. Welch & Wilson (2000) proposed a volume-of-fluid based method to simulate horizontal film boiling. Since then, a lot of numerical studies have been conducted to investigate boiling phenomena [see e.g., Kunugi (2012); Dhir et al. (2013); Cheng et al. (2014); Kharangate & Mudawar (2017) for further details]. However, most of these methods assume an initial vapor phase as an artificial input. Therefore, they cannot simulate nucleation in boiling phenomena.

In recent years, as a different approach from the aforementioned Navier–Stokes-based method, several kinds of lattice Boltzmann methods (LBMs) for multiphase flows have been applied to simulating liquid-vapor phase-change phenomena. Multiphase LBMs can be divided into four categories, namely color-gradient (Gunstensen et al. 1991; Grunau et al. 1993), pseudopotential (Shan & Chen 1993, 1994), free-energy (Swift et al. 1995, 1996), and mean-field (He et al. 1999) models. This is not an exhaustive classification; for instance, the latter two model types are sometimes called phase-field models since the Cahn–Hilliard (or similar) interface tracking equations can be derived from them. The multiphase LBMs have been successfully applied to a variety of complex multiphase-flow problems, such as liquid drop (Kang et al. 2005) and bubble (Frank et al. 2005) dynamics, flow through porous media (Parmigiani et al. 2011), wetting-drying process (Liu & Zhou 2014), liquid-jet breakup (Saito et al. 2017), etc. Among the multiphase LBMs, the pseudopotential and the phase-field models are generally used in phase-change simulations (Li et al. 2016). In most of the phase-field models, an interface-tracking equation is solved to capture the liquid-vapor interface and a source term is incorporated into the continuity or the Cahn–Hilliard equations to define the phase-change rate. This implies that the rate of the liquid-vapor phase change is an artificial input.

In contrast, the pseudopotential model has no such a limitation. The most distinct feature of this model is that the phase separation is achieved via an interparticle potential. In the case considering temperature change, the liquid-vapor phase change is driven by the equation of state (EOS). Hence no artificial phase-change terms need to be added to the temperature equation (Li et al. 2016). After the first attempt by Zhang & Chen (2003), many researchers proposed the LBM models to deal with the phase-change phenomena (Albernaz et al. 2017; Qin et al. 2019), and succeeded in simulating boiling
process without artificial terms (Házi & Márkus 2009; Márkus & Házi 2012a,b; Biferale et al. 2012; Gong & Cheng 2013).

Most of the aforementioned models have the common feature that a thermal lattice Boltzmann equation (LBE) with a temperature distribution function is used to mimic the macroscopic energy equation. However, the forcing term included in the thermal LBEs introduces a spurious term into the macroscopic temperature equation (Li et al. 2015), and such a forcing term leads to significant errors in the simulation of thermal flows (Li & Luo 2014). Furthermore, in temperature-based thermal LBEs, another error term proportional to $\nabla \cdot (T \nabla \rho / \rho)$ appears in the macroscopic temperature equation, that can be seen in Refs. (Li et al. 2008; Chai & Zhao 2013). This term yields considerable errors in multiphase flows with density varying. According to Li et al. (2015), solving the temperature equation using the classical finite-difference method enables us to be free from the problems.

Beside, the original pseudopotential model with the BhatnagarGrossKrook (BGK) (Bhatnagar et al. 1954) approximation generally suffers from numerical instability under high-Re (low-viscosity) conditions. One way to overcome this issue is to modify the collision operator (Luo et al. 2011; Coreixas et al. 2019). To cope with this problem, Lycett-Brown & Luo (2014) introduced the cascaded scheme (Geier et al. 2006) into a pseudopotential model to enhance numerical stability. Later, Lycett-Brown & Luo (2016) updated their multiphase LBM model with their original forcing scheme (Lycett-Brown & Luo 2015). The cascaded collision operator relaxes the so-called central moments (CMs) (Geier et al. 2006, 2017; De Rosis 2016, 2017b; De Rosis et al. 2019), instead of raw moments used in the multiple-relaxation-time (MRT) schemes (d’Humières 1992; Lallemand & Luo 2000). Undoubtedly, the CMs-based LBM drastically outperforms both BGK and MRT in terms of stability for high-Re multiphase flow (Lycett-Brown & Luo 2014, 2016; Lycett-Brown et al. 2014; Saito et al. 2018; De Rosis et al. 2019).

In this paper, aiming at numerically investigating the forced-convection boiling on a cylinder up to high Reynolds numbers, we develop a numerical scheme based on the LBM. The pseudopotential model and its forcing are formulated in the framework of nonorthogonal CMs. The flow and energy equations are coupled through the EOS to represent the liquid-vapor phase change. The developed numerical scheme is applied to the forced-convection boiling on a cylindrical body.

The rest of the paper is organized as follows. §2 describes formulation of the CMs-based pseudopotential LBM model with the consistent forcing scheme, and derivation of the macroscopic energy equation to be solved with finite-difference method. §3 verifies the present scheme through three numerical tests, namely four rolls mill, Poiseuille flow, and static bubble. §4 presents simulations of forced-convection boiling on a cylindrical body at $Re = 30$ and $30000$. Obtained HTCs are compared with several semi-empirical correlations. Finally, §5 concludes this paper.

2. Methodology

2.1. Central-moments-based lattice Boltzmann equation

Let us consider an Eulerian basis $x = [x, y]$ and the D2Q9 velocity space (Succi 2001). The LBE predicts the space and time evolution of the particle distribution functions $|f_i| = [f_0, f_1, \ldots, f_8]^{\top}$ colliding and streaming on a fixed square grid along the generic link $i = 0 \ldots 8$ with lattice velocity $c_i = [c_{ix}, c_{iy}]$, where

\[
|c_{ix}| = [0, 1, -1, 0, 0, 1, -1, 1, -1]^{\top}, \\
|c_{iy}| = [0, 0, 0, 1, -1, 1, -1, -1, 1]^{\top}.
\]
Let us employ the symbols $|\cdot\rangle$ and $\top$ to denote a column vector and the transpose operator, respectively. The LBE with the forcing term can be generally expressed as (McCracken & Abraham 2005; Fei & Luo 2017)

$$
| f_i(x + \delta_t c_i, t + \delta_t) \rangle = | f_i(x, t) \rangle + \mathbf{A} [ | f^\text{eq}_i(x, t) \rangle - | f_i(x, t) \rangle ] + \delta_t (I - \mathbf{A}/2) | F_i(x, t) \rangle ,
$$

(2.2)

where $f^\text{eq}_i$ is the discrete local equilibrium, and the time step is set to $\delta_t = 1$. When the collision matrix $\mathbf{A}$ has only one relaxation frequency as $\omega I$, (2.2) describes the so-called BGK LBE. As usual, the LBE can be divided into two steps, i.e., collision

$$
| f^*_i(x, t) \rangle = | f_i(x, t) \rangle + \mathbf{A} [ | f^\text{eq}_i(x, t) \rangle - | f_i(x, t) \rangle ] + \delta_t (I - \mathbf{A}/2) | F_i(x, t) \rangle ,
$$

(2.3)

and streaming

$$
f_i(x + \delta_t c_i, t + \delta_t) = f^*_i(x, t),
$$

(2.4)

where the superscript $\star$ denotes post-collision quantities here and henceforth. The dependence on the space and the time will be implicitly assumed in the rest of this section. The term $F_i$ accounts for external body forces $\mathbf{F} = [F_x, F_y]$ and its role will be elucidated later. The fluid density $\rho$ and velocity $\mathbf{u} = [u_x, u_y]$ are computed as

$$
\rho = \sum_i f_i, \quad \rho \mathbf{u} = \sum_i f_i c_i + \frac{\mathbf{F}}{2} \delta_t ,
$$

(2.5)

respectively. Following the works by Malaspinas (2015) and Coreixas (Coreixas et al. 2017; Coreixas 2018), the equilibrium distribution can be expanded into a basis of Hermite polynomials $\mathcal{H}^{(n)}$ as

$$
f^\text{eq}_i = w_i \rho \left[ 1 + \frac{c_i \cdot \mathbf{u}}{c_s^2} + \frac{1}{2c_s^2} \mathcal{H}^{(2)}_{i} : \mathbf{u}\mathbf{u} 
+ \frac{1}{2c_s^4} \left( \mathcal{H}^{(3)}_{ixxy} u_x^2 u_y + \mathcal{H}^{(3)}_{ixxy} u_x u_y^2 
+ \frac{1}{4c_s^8} \mathcal{H}^{(4)}_{ixxyy} u_x^2 u_y^2 \right) \right] ,
$$

(2.6)

with $w_0 = 4/9, w_{1...4} = 1/9, w_{5...8} = 1/36$ and $c_s = 1/\sqrt{3}$ is the lattice sound speed. Notice that the maximum order of the expansion is equal to four in the D2Q9 space. Moreover, the model recovers the classical second-order truncated equilibrium when $\mathcal{H}^{(3)}$ and $\mathcal{H}^{(4)}$ are neglected.

The pivotal idea to design any CMs-based collision operator is to shift the lattice directions by the local fluid velocity (Geier et al. 2006). Therefore, it is possible to define $\tilde{c}_i = [|\tilde{c}_{ix}|, |\tilde{c}_{iy}|]$, where

$$
|\tilde{c}_{ix}| = |c_{ix} - u_x| , \\
|\tilde{c}_{iy}| = |c_{iy} - u_y| .
$$

(2.7)

Then, one must choose a suitable basis of moments. Let us adopt the non-orthogonal
basis (De Rosis 2016) as the following matrix form:

\[
T = \begin{bmatrix}
\langle c_i^0 \rangle \\
\langle \bar{c}_i x \rangle \\
\langle \bar{c}_{ij} \rangle \\
\langle \bar{c}_{ix} + \bar{c}_{ij}^2 \rangle \\
\langle \bar{c}_{ix} - \bar{c}_{ij}^2 \rangle \\
\langle \bar{c}_{ix} \bar{c}_{iy} \rangle \\
\langle \bar{c}_{ix}^2 \bar{c}_{iy} \rangle \\
\langle \bar{c}_{ix}^2 \bar{c}_{ij}^2 \rangle \\
\langle \bar{c}_{ix}^2 \bar{c}_{ij}^2 \rangle \\
\end{bmatrix},
\]

(2.8)

where \(\langle \cdot |\) denotes the raw vector. Using equation (2.8), the collision matrix can be written as \(\Lambda = T^{-1}KT\), where \(K\) is the relaxation matrix specified later. Let us collect pre-collision, equilibrium and post-collision CMs as

\[
|k_i\rangle = [k_0, \ldots, k_i, \ldots, k_8]^T,
\]

\[
|k_{eq}^i\rangle = [k_{eq0}, \ldots, k_{eqi}, \ldots, k_{eq8}]^T,
\]

\[
|k^*_i\rangle = [k^*_0, \ldots, k^*_i, \ldots, k^*_8]^T,
\]

respectively. The first two quantities are evaluated by applying the matrix \(T\) to the corresponding distribution, that is

\[
|k_i\rangle = T |f_i\rangle, \quad |k_{eq}^i\rangle = T |f_{eq}^i\rangle,
\]

(2.10)

where \(|f_{eq}^i\rangle = [f_{eq0}, \ldots, f_{eqi}, \ldots, f_{eq8}]^T\). By adopting \(n = 4\) in the Hermite polynomials, equilibrium CMs can be computed as

\[
k_{eq0} = \rho, \quad k_{eq3} = \frac{2}{3} \rho, \quad k_{eq8} = \frac{1}{9} \rho,
\]

(2.11)

with \(k_{eq1,2} = k_{eq4,7} = 0\). Notably, only three equilibrium CMs assume values different from zero. It is of interest to notice that the discrete equilibrium CMs have the same form of the continuous counterparts when the full set of Hermite polynomials is considered. The post-collision CMs can be written as

\[
|k^*_i\rangle = (I - K) T |f_i\rangle + KT |f_{eq}^i\rangle + \left( I - \frac{K}{2} \right) T |F_i\rangle,
\]

(2.12)

\[
= (I - K) |k_i\rangle + K |k_{eq}^i\rangle + \left( I - \frac{K}{2} \right) |R_i\rangle,
\]

where

\[
K = \text{diag}[1, 1, 1, 1, \omega, \omega, 1, 1, 1],
\]

(2.13)

is a \(9 \times 9\) relaxation matrix with \(\omega = \left(\frac{\nu}{c_s^2 \delta t} + \frac{1}{2}\right)^{-1}\), \(\nu\) being the fluid kinematic viscosity.

The CMs of the discrete force term \(|R_i\rangle = T |F_i\rangle\) are computed as follows.

Now, let us define the discrete forcing term \(F_i\). Specifically, we employ the expression adopted by Huang et al. (2018):

\[
F_i(u) = w_i \left( \frac{F}{c_s} \cdot \mathcal{H}^{(1)} + \frac{[Fu]}{2c_s^2} \cdot \mathcal{H}^{(2)} + \frac{[Fuu]}{6c_s^3} \cdot \mathcal{H}^{(3)}_{[xyz]} + \frac{[Fuuu]}{24c_s^4} \cdot \mathcal{H}^{(4)}_{[xxyy]} \right),
\]

(2.14)

where the square bracket in Hermite coefficient denotes permutations (e.g., \([Fuu] = \)
\textbf{Fu} + u\textbf{Fu} + uu\textbf{F}). Notice that the popular formula proposed by Guo et al. (2002) is recovered if $H(3)$ and $H(4)$ are neglected. The CMs of the present discrete force term can be computed as
\begin{equation}
|R_i\rangle = T |F_i\rangle ,
\end{equation}
and read as follows:
\begin{align}
R_1 &= F_x, \quad R_2 = F_y, \quad R_6 = \frac{1}{3} F_y, \quad R_7 = \frac{1}{3} F_x,
\end{align}
with $R_0 = R_{3...5} = R_8 = 0$.

Some considerations should be drawn regarding these results. The same expressions of $|R_i\rangle$ can be achieved when the continuous Maxwell–Boltzmann distribution are considered (Premnath & Banerjee 2009; Fei & Luo 2017). This is consistent with a recent work of De Rosis & Luo (2019), where it is argued that the CMs of the discrete distribution collapse into the continuous counterpart when the full set of Hermite polynomials is considered. Moreover, $|R_i\rangle$ can be achieved by disregarding the velocity-dependent terms in equation (15) of De Rosis (2017). Again, this is consistent with De Rosis & Luo (2019), where the velocity terms vanish when the maximum $H$ is adopted to construct CMs. Finally, the present findings are different from and simpler than those in Huang et al. (2018) [see equation (9)] due to the adoption of a different basis.

In the following, we report the post-collision CMs:
\begin{align}
k_0^* &= \rho, \quad k_1^* = \frac{1}{2} F_x, \quad k_2^* = \frac{1}{2} F_y, \quad k_3^* = \frac{2}{3} \rho, \\
k_4^* &= (1 - \omega) k_4, \quad k_5^* = (1 - \omega) k_5, \\
k_6^* &= \frac{1}{6} F_y, \quad k_7^* = \frac{1}{6} F_x, \quad k_8^* = \frac{1}{9} \rho. \tag{2.17}
\end{align}
One can immediately appreciate that the present scheme is highly intelligible and the resultant algorithm is very concise. Then, the post-collision populations are reconstructed as
\begin{equation}
|f^*_i\rangle = T^{-1} |k^*_i\rangle ,
\end{equation}
that are eventually streamed. For practical implementation, it is easier to replace the above “one-step” reconstruction by the “two-step” reconstruction. For more details, the interested readers are kindly directed to Fei & Luo (2017).

2.2. Pseudopotential and thermodynamic consistency

In the pseudopotential model, the interaction force to mimic the molecular interactions plays as important role in phase separation, which is given by (Chen et al. 2014; Lycett-Brown & Luo 2016)
\begin{equation}
F_m(x) = -\mathcal{G} c_s^2 \psi(x) \sum_{i=0}^N w(|c_i|^2) \psi(x + c_i) c_i ,
\end{equation}
where $\psi$ is the interaction potential, $\mathcal{G}$ is the interaction strength, and $w(|c_i|^2)$ are the weights. The number of discrete velocities $N$ used in the force calculation need not to be equal to the number of lattice velocities. We consider the case of $N = 24$ (pseudo D2Q25 lattice) for calculation of the interaction force. The weights can be defined as $w(0) = 247/140, w(1) = 4/21, w(2) = 4/45, w(4) = 1/60, w(5) = 2/315$, and $w(8) = 1/5040$ (Sbragaglia et al. 2007; Leclaire et al. 2011; Fei et al. 2018b). This treatment means that the interaction force is discretized with the 6th-order isotropic gradient (Leclaire et al. 2011).
To incorporate a non-ideal equation of state (EOS), $p_{\text{EOS}}$, into the pseudopotential model, the following potential form should be chosen as† (He & Doolen 2002)

$$
\psi(x) = \sqrt{\frac{2(p_{\text{EOS}} - \rho c_s^2)}{\mathcal{G} c_s^2}},
$$

(2.20)

where $\mathcal{G}$ is only required to guarantee the term inside the square root to be positive (Yuan & Schaefer 2006; Chen et al. 2014).

Although the potential form of equation (2.20) is consistent with the EOS, the problem of thermodynamic inconsistency still persists (Li et al. 2016). To approximately restore the thermodynamic consistency, we here derive the corrected forcing term $\left| R'_i \right|$ according to the idea of Li et al. (2012, 2013). To do this, the macroscopic velocity $u$ in equation (2.14) is replaced by the following modified velocity:

$$
u' = \begin{cases} 
\frac{\sigma F_m}{(\omega^{-1} - 1/2)\delta t \psi^2} \quad (i = 4, 5), \\
\frac{\sigma F_m}{(1 - 1/2)\delta t \psi^2} \quad \text{(otherwise)},
\end{cases}
$$

(2.21)

where $\sigma$ is a constant to tune thermodynamic consistency. By substituting equation (2.21) into (2.14) and transforming into central-moment space (e.g., calculating $T |F_i(u')|$), we can obtain the corrected forcing term. Actually, we have found that $R_{3..8}$ are non-zero, but for simplicity and consistency with the derived macroscopic equations (see Appendix A for the analysis), we can modify only $R'_3$ and $R'_8$ as follows:

$$
R'_3 = 4\alpha |F_m|^2, \quad R'_8 = \frac{4}{3} \alpha |F_m|^2,
$$

(2.22)

where $\alpha = \sigma / (\psi^2 \delta t)$ and $|F_m|^2 = F_{mx}^2 + F_{my}^2$. The other elements are kept to be the original ones (e.g., $R'_i = R_i$ for $i = 0...2$ and $i = 4...7$). Compared with the original forcing term in equation (2.16), the terms for $i = 3...8$ have been modified. When we choose $\sigma = 0$, equation (2.22) reduces to the original forcing term of equation (2.16). Due to this modification, we can rewrite the post-collision CMs as follows:

$$
k'_0 = \rho, \quad k'_1 = \frac{1}{2} F_x, \quad k'_2 = \frac{1}{2} F_y, \quad k'_3 = \frac{2}{3} \rho + 2\alpha |F_m|^2, \\
k'_4 = (1 - \omega) k_4, \quad k'_5 = (1 - \omega) k_5, \\
k'_6 = \frac{1}{6} F_y, \quad k'_7 = \frac{1}{6} F_x, \quad k'_8 = \frac{1}{9} \rho + \frac{2}{3} \alpha |F_m|^2.
$$

(2.23)

Comparing to equation (2.17), the underlined terms have appeared in equation (2.23) to tune the thermodynamic consistency.

### 2.3. Energy equation and non-ideal EOS

Now let us derive the energy equation in terms of temperature $T$. If we assume that the thermal diffusion is governed by the Fourier law, the energy equation in terms of the internal energy $e$ can be generally given by

$$
\rho \frac{De}{Dt} = \nabla \cdot (\lambda \nabla T) - p (\nabla \cdot u) + \Phi,
$$

(2.24)

† Note that one can choose the potential form of $\psi(x) = \sqrt{2(p_{\text{EOS}} - \rho c_s^2)/\mathcal{G} c_s^2}$. In this case, the pressure tensor in the recovered macroscopic equation will be slightly modified.
where $\lambda$ is the thermal conductivity, $\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \mathbf{u} \cdot \nabla$ is the material derivative, and $\Phi = \nabla : \nabla \mathbf{u}$ is the viscous heat dissipation (Landau & Lifshitz 1987), which is neglected in the following. The second term in equation (2.24) describes the pressure work. Meanwhile, for the internal energy and temperature, we have the following thermodynamic relation:

$$
d e = c_v dT + T \left( \frac{\partial p}{\partial T} \right)_v - p \, dv, \tag{2.25}$$

where $c_v$ is the specific heat at constant volume and $v = 1/\rho$ is the specific volume. Dividing equation (2.25) by $dt$ and replacing $d/dt \rightarrow D/Dt$ yields

$$
\frac{D e}{D t} = \rho c_v \frac{D T}{D t} + \rho \left[ T \left( \frac{\partial p}{\partial T} \right)_v - p \right] \frac{D v}{D t} \\
= \rho c_v \frac{D T}{D t} + T \left( \frac{\partial p}{\partial T} \right)_\rho - p \left( \nabla \cdot \mathbf{u} \right), \tag{2.26}
$$

where we use the continuity equation:

$$
\frac{D \rho}{D t} = -\rho \left( \nabla \cdot \mathbf{u} \right), \quad \frac{D v}{D t} = \frac{1}{\rho} \left( \nabla \cdot \mathbf{u} \right). \tag{2.27}
$$

Note that we replaced the subscript $v \rightarrow \rho$ in equation (2.26). Substituting equation (2.26) into (2.24) results in

$$
\rho c_v \frac{D T}{D t} = \nabla \cdot (\lambda \nabla T) - T \left( \frac{\partial p}{\partial T} \right)_\rho \nabla \cdot \mathbf{u}. \tag{2.28}
$$

Finally, the energy equation based on temperature is given by

$$
\frac{\partial T}{\partial t} + \mathbf{u} \cdot \nabla T = \frac{1}{\rho c_v} \nabla \cdot (\lambda \nabla T) - T \left( \frac{\partial p_{\text{EOS}}}{\partial T} \right)_\rho \nabla \cdot \mathbf{u}, \tag{2.29}
$$

where we replaced $p \rightarrow p_{\text{EOS}}$. Although the derivation in this paper is based on the internal energy, equation (2.29) is identical with the one obtained by Li et al. (2015), who began with the local entropy balance. On the other hand, equation (2.29) is different from that of Gong et al. (2018). See Li et al. (2018b) for the discussion of the energy equation for phase-change LBMs.

To solve equation (2.29), we use the second-order Runge-Kutta scheme for time and the third-order upwind scheme for the convection term. The first-order derivative and the Laplacian are computed by the isotropic central difference (Shan 2006; Sbragaglia et al. 2007) with the same stencils as the interparticle force in equation (2.19).

As can be seen in equations (2.19) and (2.29), the coupling between the pseudopotential model and the finite-difference scheme is established via a non-ideal EOS. The realistic EOS widely used in the multiphase lattice Boltzmann simulations includes van der Waals, Carnahan-Starling, Peng-Robinson, etc. We adopt the Peng-Robinson EOS in this simulations, which is given by (Yuan & Schaefer 2006)

$$
p_{\text{EOS}} = \frac{\rho RT}{1 - b \rho} - \frac{a \varphi(T) \rho^2}{1 + 2b \rho - b^2 \rho^2}, \tag{2.30}
$$

where $\varphi(T) = [1 + (0.37464 + 1.54226 \omega - 0.26992 \omega^2)(1 - \sqrt{T/T_c})^2$ with the acentric factor $\omega$. The critical properties can be obtained as follows: $a = 0.4572 R^2 T_c^2 / p_c$ and

† We stress that some upwind-type schemes should be adopted for the convection term. Otherwise, the non-physical spurious oscillations will appear for convection-dominant cases.
In this paper, we set $a = 3/49$, $b = 2/21$, $R = 1$, and the acentric factor $\omega = 0.344$ (for water). Then, the critical density, temperature, and pressure can be calculated as $\rho_c = 2.657$, $T_c = 0.1094$, and $p_c = 0.08936$, respectively.

3. Numerical tests

In this section, we test the numerical properties of the above-outlined methodology.

3.1. Driving forces: four rolls mill

Let us consider a square periodic box of size $N \times N$, where the fluid is initially at rest. Let us apply a constant force field, that is

$$F(x) = \phi [\sin(x) \sin(y), \cos(x) \cos(y)].$$

with $\phi = 2\nu u_0 \psi^2$. Then, the pressure and velocity fields must converge to a steady state that reads as follows

$$p(x) = p_0 \left[ 1 - \frac{u_0^2}{4cs^2} (\cos(2\psi x) - \cos(2\psi y)) \right],$$

$$u(x) = u_0 [\sin(\psi x) \sin(\psi y), \cos(\psi x) \cos(\psi y)],$$

(3.2)

where $u_0 = 10^{-3}$, $\psi = 2\pi/N$ and $p_0 = \rho_0 c_s^2$. This test is known as four rolls mill and is a modification of the canonical Taylor–Green vortex (Taylor & Green 1937). Simulations characterized by different grid sizes are carried out, i.e. $N = [8, 16, 32, 64, 128]$, at a Reynolds number $Re = u_0 N / \nu$ equal to 100. The performance of our scheme is elucidated by computing the relative discrepancy between analytical predictions and numerical findings. For this purpose, the vectors $r_{an}$ and $r_{num}$ are introduced, storing the values of the velocity field from equation (3.2) and those provided by our numerical experiments, respectively. Then, the relative error is computed as

$$err = \frac{\|r_{an} - r_{num}\|}{\|r_{an}\|}$$

(3.3)

and is depicted in figure 3 as a function of the grid dimension. An excellent convergence rate equal to 1.998 is found, which is totally consistent with the second-order nature of the LBE.

3.2. Poiseuille flow

A simple yet effective test is represented by the Poiseuille flow. Let us consider a fluid initially at rest with density set to $\rho(x, 0) = \rho_0 = 1$ everywhere. A laminar flow develops in the $x$ direction within a domain of dimensions $L_x \times L_y$, that are respectively aligned to the $x, y$ Cartesian reference axis. No-slip boundary condition is enforced at two $y$-normal planes placed at the top and bottom sections of the domain, while the domain is periodic at the other two sides. By applying a constant uniform horizontal rightward force $F_x = 8\nu U_0 / L_y^2$, the flow field must converge to the analytical predictions

$$u_x(y) = -\frac{4U_0 y}{L_y} \left( 1 - \frac{y}{L_y} \right).$$

(3.4)

The peak velocity of the imposed velocity profile is set to $u_0 = 0.001$ and the Reynolds number is $Re = u_0 L_y / \nu = 100$. The value of $L_y$ varies in order to achieve different grid resolutions, i.e. $L_y \in [5 : 1025]$. Here, the simulations stop when the steady state is reached. In order to evaluate the accuracy and convergence properties of our algorithm,
Figure 3. Four rolls mill: slope of the line fitting our results (triangles) indicates a convergence rate of 1.998.

Figure 4. Poiseuille flow: (a) convergence analysis showing an optimal convergence rate equal to 2.04 and (b) profile of the horizontal component of the velocity, $u_x$, normalized with respect to $u_0$ from the analytical predictions (continuous line) and our numerical scheme (circles).

we define two vectors, $\mathbf{r}_{\text{an}}$ and $\mathbf{r}_{\text{num}}$ collect the analytical and numerical solutions, respectively. In figure 4(a), results obtained by a convergence analysis are reported. It is possible to appreciate that our algorithm shows an optimal convergence rate (i.e. the slope of the line fitting the values) equal to 2.04, that is consistent with the second-order accuracy of the LBM. For $L_y = 1015$, the profile of the horizontal component of the velocity, $u_x/u_0$, from our numerical simulation is reported in figure 4(b), together with the analytical predictions. It is possible to appreciate that the two solutions are overlapped.

3.3. Static bubble

To check the thermodynamic consistency of the present LBM scheme, static bubble tests are conducted. In the computational domain discretized into $200 \times 200$, periodic boundary conditions are set everywhere. Following Li et al. (2013), we set the initial bubble radius to $R = 50$. The analytical solution is obtained via the Maxwell equal area rule, i.e., by numerical integration of the EOS.

Figure 5 shows the comparison of simulation results with analytical solution of Maxwell construction. As reference, the case with the original forcing scheme (Fei & Luo 2017)
is shown in the same figure. From the figure 5, we can see that the present model well describes the analytical solution of Maxwell construction by setting \( \sigma = 0.38 \), whereas the case with the original forcing scheme (\( \sigma = 0 \)) does not agree with the analytical solution. Based on such observations, we confirmed that this model can approximately perform thermodynamic-consistent phase-change simulations.

To validate the surface tension obtained by the present LBM scheme, additional static-bubble tests are conducted. We use the same computational setup as the last tests, and the parameter \( \sigma \) is set to 0.38. A stationary bubble with various radius \( R \) is initially placed in the domain. From the Laplace law for two-dimensional case, the pressure difference between the inside and the outside of a bubble can be given by

\[
\Delta p = \frac{\gamma}{R},
\]

where \( \gamma \) and \( R \) are the surface tension and the bubble radius, respectively. At the equilibrium state, we measure the pressure difference for three reduced temperature values, \( T_r = 0.86, 0.90, \) and 0.95. Figure 6 shows the results. As can be seen, our results well fit the Laplace law, that is, the pressure difference is proportional to \( 1/R \).

4. Simulations of forced-convection boiling

In this section, we aim at simulating the melt particle settling in a coolant as described in figure 2\((b)\).

4.1. Setup and initial parameters

The computational setup of the present simulations is described in figure 7. A constant velocity \( u_l = 0.05 \) is prescribed at the inlet section (Zou & He 1997), while convective outlet is imposed at the outflow boundary (Lou et al. 2013). A cylindrical body is modeled by the interpolated bounce-back scheme (Bouzidi et al. 2001; De Rosis et al. 2014), which is a similar approach to Hatani et al.’s condensation simulation (Hatani et al. 2015). In the cylindrical region with a diameter \( D = 30 \), we set superheat temperature \( T_w \), which
is higher than the saturated temperature $T_{sat}$. The computational domain, filled with saturated liquid, is discretized into $W \times H = 20D \times 45D = 600 \times 1350$. The center of the cylinder is located at $(0.5W, 0.2H)$. The initial liquid velocity is set to be $T_{sat}$ and $u_l$. For the initial 3000 time steps, we did not solve the energy equation and forcing term.

The liquid Reynolds number is an important parameter to investigate the effect of forced convection. In the following simulations, two cases are considered: $Re = 30$ and 30000. The corresponding kinematic viscosity are 0.05 and 0.00005, respectively. According to Bromley et al. (1953), the flow regime can be regarded as forced-convection boiling when $Fr^{1/2} > 2$, where $Fr^{1/2} = u_l/(gD)^{1/2}$ is the Froude number. To set $Fr^{1/2} = 4$, the gravitational acceleration is chosen as $g = 5.21 \times 10^{-6}$.

In this paper, the reduced temperature is set to $T_r = T_{sat}/T_c = 0.86$. In this case, the saturation temperature corresponds to 556 K in actual physical units and the system pressure to $\sim 7$ MPa. The liquid-vapor density ratio results in $\rho_l/\rho_v \sim 17$. The physical properties required for the simulations (kinematic-viscosity ratio, Prandtl number, thermal conductivity ratio, specific heat, etc.) under the reduced temperature are taken from the steam table for the saturation condition (JSME 1983), which gives

Figure 6. Laplace’s law for several reduced temperatures: $T_r = 0.86$, $T_r = 0.90$, and $T_r = 0.95$.

Figure 7. Boundary conditions for the boiling simulations.
the Prandtl number for vapor phase equal to $Pr = 1.5$. We have assumed that specific heat ratio for each phase is unity, and we set to $c_{p,v} = 1$. The specific latent heat, $h_{fg}$, is determined from the EOS with the same procedure in Gong & Cheng (2013). Our parameter choice of $a$ and $b$ in equation (2.30) leads to $h_{fg} = 0.572$. The wall superheat $\Delta T$ is varied from 0.005 to 0.065.

### 4.2. Boiling regimes, heat flux, and boiling curve

Simulation results for $Re = 30$ are shown in figure 8. Density fields around a cylinder with different superheat degrees are cut out and magnified. All the simulations started with being fully immersed by liquid phase. After a certain time elapses, nucleation naturally arises around the superheated cylindrical domain, confirming the capability of the present approach to capture this feature. The appeared vapor phase grows up and finally breaks up in different forms, depending on the superheat degree. For the low superheat case [figure 8(a)], it seems that only the upper part of the cylinder is in contact with the vapor phase. In contrast, the high superheat cases [figure 8(c)] implies that most of the cylinder’s surface is covered with vapor phase. This may be recognized as film-boiling regime.

The above observations of boiling regimes are qualitative. In order to perform a more quantitative investigation, we calculate the average heat flux on the cylinder surface, that is

$$\bar{q}(t) = \frac{1}{W} \int_0^W q(x) ds,$$  \hspace{1cm} (4.1)

where $W$ is the circumference length, and the local heat flux is given by

$$q(x) = -\lambda(x) \frac{\partial T}{\partial n}|_{\text{wall}},$$  \hspace{1cm} (4.2)
Figure 9. Time histories of heat flux on a cylindrical body at $Re = 30$ and $Pr = 1.5$: (a) $\Delta T = 0.010$, (b) $\Delta T = 0.023$, and (c) $\Delta T = 0.060$.

where $\lambda$ is the thermal conductivity and $n$ is the normal direction. Figure 9 represents the time history of the calculated heat flux. Simulation conditions are the same as figure 8. For low superheat case [figure 9(a)], the heat flux data seems to converge to a certain value. As the superheat is increased, the heat flux starts to intensely fluctuate. This is due to intermittent contact of superheated wall and liquid. When it is in contact with the liquid, higher heat flux is observed; when it is in contact with the vapor, lower heat flux is observed. We find that the intervals of this intermittent fluctuation shortened with the increase of superheat.

By averaging the heat flux during the quasi-steady state, we can calculate the time-averaged heat flux. When the heat flux is plotted against the superheat, the obtained curve describes the boiling curve, which is also known as the Nukiyama curve. Figure 10 shows the boiling curve for $Re = 30$. The error bar indicates the standard deviation of the time-averaged heat flux. Typical snapshots at $\Delta T = 0.010$, $\Delta T = 0.023$, and $\Delta T = 0.060$ are displayed in figure 10. The tendency of the heat flux against superheat degree agrees with that in figure 1. At low superheat conditions, the heat flux monotonously increases and reaches the maximal point. Although the present system is the convection boiling, the boiling regime in this low superheat region can be regarded as nucleate boiling. The maximal point would be the CHF. After the CHF point, the heat flux gradually decreases and reaches the minimal point, MHF. The boiling regime between the CHF and MHF would correspond to transition boiling. Finally, the heat flux is found to increase with the superheat degree over the MHF. This boiling regime would be the film boiling. We stress here that the present simulation can deal with a series of boiling processes, i.e., vapor bubbles nucleation, growth, and departure. Since all the boiling regimes can be reproduced, the results are not limited to the film-boiling regime without any artificial input unlike Navier–Stokes-based modeling (Son & Dhir 1997; Juric & Tryggvason 1998; Welch & Wilson 2000; Yuan et al. 2008; Phan et al. 2018).

Figure 11 shows the boiling curve for $Re = 30000$ with three snapshots of boiling behavior. The averaged heat-flux values indicate that there can be found the maximal (CHF) and minimal (MHF) points in figure 11. It should be noted that our CMs-based LBM approach enables to stably perform forced-convection boiling simulations up to very high $Re$ with the order of $O(10^4)$. The error bars of heat flux beyond the CHF point are much larger than the case for $Re = 30$ in figure 10, since the inertia of the liquid phase increases. As mentioned above, this is due to the intermittent contact with vapor and liquid. This behavior promotes an unstable interfacial deformation as can be seen in the snapshot at $\Delta T = 0.060$ within figure 11. From the boiling curve characteristics, we can recognize that the film boiling begins when $\Delta T$ is around 0.03. However, the
present simulations indicate that the cylindrical body intermittently contacts with liquid. Such behavior was also slightly seen even at $Re = 30$. In case of $Re = 30000$, the solid-liquid contact was promoted mainly because the inertial force of forced convection was larger than the force driven by vaporization around the cylinder. Destabilization of the interface would also help the intermittent direct solid-liquid contacts. Actually, experimental observations show that the local and intermittent solid-liquid contact occurs even in the film-boiling regime (Bradfield 1966; Yao & Henry 1978; Kikuchi et al. 1992). Such solid-liquid direct contacts were actually confirmed from the present simulations. Our results support the experimental considerations of solid-liquid direct contact even in the film-boiling regime.
Figure 12. Comparison of simulation result and conceptual illustration of several film-boiling regimes reproduced from Liu & Theofanous (1994). (a) Snapshot of simulation result for $Re = 30000$, (b) Conceptual illustration of film-boiling regime, including saturated pool film boiling, subcooled pool film boiling, completely saturated film boiling in forced convection, and slightly subcooled film boiling in forced convection.

We then compare the present simulation results with the conceptual illustration of several film-boiling regimes reported by Liu & Theofanous (1994). They schematically illustrated characteristic film-boiling regimes, including saturated and subcooled pool film boiling and completely saturated and slightly subcooled film boiling in forced convection. Figure 12 shows one of the typical simulation snapshots of film boiling for $Re = 30000$ and Liu–Theofanous’ conceptual illustration of film-boiling regimes. From the simulation result [figure 12(a)], we can see that the vapor is stretched into downstream region due to the vaporization and forced convection. Most of the cylindrical body is covered with vapor phase. Globally, the present simulation seems to capture the characteristics of the type of “completely saturated” film boiling in forced convection as in figure 12(b), although a slight difference in the interfacial shape still remains.

4.3. Comparison with film boiling heat transfer correlations

Focusing on the heat-flux data of the film-boiling regime, one can compare the present simulation with the existing film boiling HTC correlations (Bromley et al. 1953; Witte 1968; Epstein & Hauser 1980; Ito et al. 1981; Liu & Theofanous 1994; Dhir & Purohit 1978). All the correlations considered here have been reduced for the saturation condition, i.e., the term related subcooling degree has been dropped. The functional form of them are summarized in Appendix B.

The Nusselt number, $Nu$, calculated from the simulation results is shown in figure 13 for $Re = 30$ and $Re = 30000$. For both low- and high-$Re$ data, $Nu$ decreases as $Ja$ increases. In figure 13, six kinds of correlations are also shown together with the simulation data. These can be classified as the Mode 1 (Bromley et al. 1953; Witte 1968), the Mode 2 (Epstein & Hauser 1980; Liu & Theofanous 1994; Ito et al. 1981), and the other (Dhir & Purohit 1978) correlations. On the $Nu$ and $Ja$ numbers, the Mode 1 correlations state that $Nu \propto Ja^{-1/2}$, while the Mode 2 and the Dhir–Purohit correlations show $Nu \propto Ja^{-1/4}$. Let us first focus on the low-$Re$ case [figure 13(a)]. When $Ja < 1.0$, the simulation results are located between Bromley et al. and Epstein–Hauser correlations. In the region of $Ja > 1.0$, the Epstein–Hauser correlation might be in better agreement with the simulation results. Then we turn to the high $Re$ case [figure 13(b)]. All the correlations overestimate the $Nu$ obtained from the present simulations, but the slope of the simulation data was close to several correlations except for Bromley
et al. and Epstein–Hauser ones. This means that adjusting the coefficient $C$ in the correlations would corroborate the simulation results. The coefficient $C$ was actually used as a fitting parameter in past experiments. Eventually, the present comparison of simulations and correlations implies that one should carefully choose the correlation depending on system’s Reynolds number, and it would be difficult to identify a unique (optimal) solution for different flow conditions.

Although our proposed numerical method enables the stable simulation at an extremely high Reynolds number ($Re = 30000$), the obtained heat flux is lower than the one given by all the afore-mentioned correlations. One of the reasons should be found in the decrease of thermal boundary layer when the Reynolds number is high. In our simulations, the Prandtl number $Pr$ is fixed at constant value regardless of the Reynolds numbers. The expected situation is schematically illustrated in figure 14 as a one-dimensional relation between the exact and discrete solutions on lattice grids. Let us assume the grid resolution is the same and the Prandtl number is fixed at a same value. For low-$Re$ case [figure 14(a)], the thermal boundary layer can be easily resolved with the discrete grid. In contrast, the high $Re$ [figure 14(b)] would have lead to very thin thermal boundary layer. The fact makes it difficult to accurately capture the temperature gradient near the wall, and the evaluated value of temperature gradient will underestimate the exact one.

To examine this assumption, we conducted additional simulations with extremely low Prandtl number ($Pr = 1.5/1000$), while keeping the high Reynolds number $Re = 30000$. By setting the virtually low Prandtl number, we aim to artificially increase the thickness of thermal boundary layer. Figure 15 again plots the simulation results and correlations on the $Nu-Re$ diagram. It is clearly shown that the Dhir–Purohit correlation does not capture the effect of lowering the Prandtl number, but Mode 1 and Mode 2 correlations follow the lowered $Pr$. Unlike the previous simulations with high $Re$ [figure 13(b)], the present results are in good agreement with several correlations: the Mode 1 correlations of Bromley et al. and Witte. The Mode 2 correlations except for the Epstein–Hauser one also give fair predictions to the simulation results. Virtually lowering $Pr$ is considered to have improved the heat-flux calculation around the cylindrical wall. All in all, the present examination proves the importance of the thickness of thermal boundary layer in heat flux calculation. As a result of improving the thermal boundary layer thickness,
Figure 14. One-dimensional concept of the relation between exact and discrete solutions: (a) low Reynolds number (b) high Reynolds number.

Figure 15. Comparison of the low-Prandtl-number simulation results (open symbols) with the Mode 1 (Bromley et al. 1953; Witte 1968), Mode 2 (Epstein & Hauser 1980; Ito et al. 1981; Liu & Theofanous 1994), and the other (Dhir & Purohit 1978) correlations.

we found that Mode 1 correlation represented the best results for the high Re. This fact suggests that under the high-Re condition the Nusselt number \( Nu \) is determined by \( \eta \), \( Pr \), \( Re \), and \( Ja \) from equation (1.7).

5. Concluding remarks

In this paper, we developed a numerical approach based on the pseudopotential LBM to simulate forced-convection boiling. To enhance numerical stability even at high Reynolds numbers, a CMs-based formulation was constructed. The forcing scheme, consistent with the CMs-based LBM, led to a concise yet robust algorithm. Furthermore, additional terms required to ensure thermodynamic consistency were derived in a CMs framework. Four types of numerical tests verified the accuracy and convergence of the present approach.

We then applied our CMs-based LBM to the system of convection-boiling heat transfer on a cylinder with the Reynolds number at 30 and 30000. Our forced-convection boiling simulations were not limited to the film boiling; a series of boiling processes, \textit{i.e.}, nucle-
ation, growth, departure of vapor bubbles, was also observed from simulations without any artificial input for phase change. We demonstrated that unlike the Navier–Stokes-based schemes, the present scheme can reproduce all the boiling regimes. Moreover, the present scheme was able to well reproduce the characteristic behavior of the boiling curve even though the focused system was not the pool boiling but the forced-convection boiling system. The simulation results supported the previous experimental observations of intermittent direct solid-liquid contact even in the film-boiling regime. The obtained heat flux was evaluated on the $Nu$-$Ja$ diagram to compare with the semi-empirical correlations for the film boiling around a sphere or cylinder. We found that our simulation results fall in between some correlations, but it was difficult to identify the best and unified correlations for different flow conditions. As a result of improving the thermal boundary layer thickness, we found that Mode 1 correlation represented the best results for the high-$Re$ regime. This fact suggests that under the high-$Re$ condition the Nusselt number $Nu$ is determined by $\eta$, $Pr$, $Re$, and $Ja$.

The authors believe that the present CMs-based LBM will be applicable to a wide range of the high-Reynolds-number multiphase flows with phase change phenomena.

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Appendix A. Theoretical analysis

The Chapman–Enskog analysis for the present lattice Boltzmann model is provided here. We focus on the second-order analysis. For convenience of the analysis, we now rewrite the LBE in a general multiple-relaxation-time (GMRT) framework (Fei & Luo 2017; Fei et al. 2018a). To this end, we first decompose the matrix $T$ into the transformation matrix $M$ and the shift matrix $N$ (Fei & Luo 2017). The transformation matrix $M$ can be computed as

$$
M = \begin{bmatrix}
\langle |c_1| \rangle \\
\langle c_{ix} \rangle \\
\langle c_{iy} \rangle \\
\langle c_{ix}^2 + c_{iy}^2 \rangle \\
\langle c_{ix}^2 - c_{iy}^2 \rangle \\
\langle c_{ix}c_{iy} \rangle \\
\langle c_{ix}^2 c_{iy} \rangle \\
\langle c_{ix} c_{iy}^2 \rangle \\
\langle c_{ix}^2 c_{iy}^2 \rangle \\
\end{bmatrix}
= \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 1 & -1 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 1 & -1 & 1 & -1 & -1 & 1 \\
0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\
0 & 1 & 1 & -1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
.$$ (A 1)
The shift matrix can be given through the relation $T = NM$:

$$N = TM^{-1}$$  \hspace{1cm} (A 2)

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-u_x & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-u_y & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-u_x^2 + u_y^2 & -2u_x & -2u_y & 1 & 0 & 0 & 0 & 0 & 0 \\
-u_x^2 - u_y^2 & -2u_x & 2u_y & 0 & 1 & 0 & 0 & 0 & 0 \\
u_xu_y & -u_y & -u_x & 0 & 0 & 1 & 0 & 0 & 0 \\
-u_x^2u_y & 2u_xu_y & u_x^2 & -u_y/2 & -u_y/2 & -2u_x & 1 & 0 & 0 \\
-u_x^2u_y^2 & -2u_xu_y^2 & -2u_yu_x & (u_x^2 + u_y^2)/2 & -(u_x^2 - u_y^2)/2 & 4u_xu_y & -2u_y & -2u_x & 1 \\
\end{bmatrix}
$$  \hspace{1cm} (A 3)

The transformation matrix $M$ transforms the distribution functions into the raw moments. The shift matrix $N$ transforms the raw moments into the central moments, and is a lower-triangular matrix.

Using equations (A 1) and (A 3), we rewrite the LBE with forcing term as

$$f_i(x + c_i \delta t, t + \delta t) = f_i(x, t) - \Lambda_{ij}[f_j(x, t) - f_j^{eq}(x, t)] + \delta t \frac{\delta t}{2} [F_i(x, t) + F_i(x + c_i \delta t, t + \delta t)],$$  \hspace{1cm} (A 4)

where $\Lambda = MN^{-1}KNM$. Note that when we introduce the relation $f_i = f_i - \delta t F_i/2$, the implicitness of equation (A 4) can be eliminated, and equation (2.2) in the manuscript can be recovered. For generality, we set the relaxation matrix as $K = \text{diag}[s_0, s_1, \ldots, s_8]$ in this analysis.

The Taylor-series expansion of equation (A 4) at $(x, t)$ yields:

$$\delta_t (\partial_t + c_i \cdot \nabla) f_i + \frac{\delta t^2}{2} (\partial_t + c_i \cdot \nabla)^2 f_i = -\Lambda_{ij}[f_j - f_j^{eq}] + \delta t F_i + \frac{\delta t^2}{2} (\partial_t + c_i \cdot \nabla) F_i,$$  \hspace{1cm} (A 5)

where we have neglected the $O(\delta t^3)$ terms. Multiplying equation (A 5) by $M$ leads to

$$(I\partial_t + D)m + \frac{\delta t^2}{2}(I\partial_t + D)^2 m = -\frac{N^{-1}KN}{\delta t}(m - m^{eq}) + S + \frac{\delta t^2}{2}(I\partial_t + D)S,$$  \hspace{1cm} (A 6)

where $m = M[f_i], m^{eq} = M[f_i^{eq}], S = M[F_i], D = M[(c_i \cdot \nabla)f]M^{-1}$. To perform the Chapman–Enskog analysis, the following multiscale expansions are introduced:

$$\partial_t = \varepsilon \partial_{t_1} + \varepsilon^2 \partial_{t_2}, \quad \nabla = \varepsilon \nabla_1, \quad f_i = f_i^{eq} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)}, \quad F = \varepsilon F^{(1)},$$  \hspace{1cm} (A 7)

which indicates that

$$D = \varepsilon D_1, \quad m = m^{eq} + \varepsilon m^{(1)} + \varepsilon^2 m^{(2)}, \quad S = \varepsilon S^{(1)},$$  \hspace{1cm} (A 8)

We can obtain:

$$O(\varepsilon) : (I\partial_{t_1} + D_1)m^{eq} = -\frac{N^{-1}KN}{\delta t}m^{(1)} + S^{(1)},$$  \hspace{1cm} (A 9)

$$O(\varepsilon^2) : \partial_{t_2} m^{eq} + (I\partial_{t_1} + D_1) \left( I - \frac{N^{-1}KN}{2} \right) m^{(1)} = -\frac{N^{-1}KN}{\delta t}m^{(2)}.$$  \hspace{1cm} (A 10)

Here, we require the explicit expressions for $D$, and $N^{-1}KN$ to proceed the analysis. It is interesting to find that the Chapman–Enskog analysis is basically the same with the one for non-orthogonal MRT models [e.g., Li et al. (2018a); Fei et al. (2019)], because $N^{-1}KN$ itself is a lower-triangular matrix.
From equation (A 9), we have the continuity and momentum equations at $O(\varepsilon)$ level:

\begin{align}
\partial_t \rho + \partial_x (\rho u_x) + \partial_y (\rho u_y) &= 0, \quad (A 11) \\
\partial_t (\rho u_x) + \partial_x \left( \frac{1}{3} \rho + \rho u_x^2 \right) + \partial_y (\rho u_x u_y) &= F_x, \quad (A 12) \\
\partial_t (\rho u_y) + \partial_x (\rho u_x u_y) + \partial_y \left( \frac{1}{3} \rho + \rho u_y^2 \right) &= F_y. \quad (A 13)
\end{align}

Analogously, we have the ones at $O(\varepsilon^2)$ level:

\begin{align}
\partial_t \rho = 0, \quad (A 14) \\
\partial_t (\rho u_x) + \frac{1}{2} \partial_x \left[ \left( 1 - \frac{s_3}{2} \right) m_3^{(1)} + \left( 1 - \frac{s_4}{2} \right) m_4^{(1)} \right] + \partial_y \left[ \left( 1 - \frac{s_3}{2} \right) m_3^{(1)} \right] = 0, \quad (A 15) \\
\partial_t (\rho u_y) + \partial_x \left[ \left( 1 - \frac{s_5}{2} \right) m_5^{(1)} \right] + \frac{1}{2} \partial_y \left[ \left( 1 - \frac{s_3}{2} \right) m_3^{(1)} - \left( 1 - \frac{s_4}{2} \right) m_4^{(1)} \right] = 0. \quad (A 16)
\end{align}

Here we should specify $m_3^{(1)}$, $m_4^{(1)}$, $m_5^{(1)}$. By using the $O(\varepsilon)$ equation again, we have

\begin{align}
\partial_t m_3^{eq} + \partial_x (m_1^{eq} + m_7^{eq}) + \partial_y (m_2^{eq} + m_6^{eq}) &= -s_3 \frac{m_3^{(1)}}{\delta t} + S_3^{(1)}, \quad (A 17) \\
\partial_t m_4^{eq} + \partial_x (m_1^{eq} - m_7^{eq}) - \partial_y (m_2^{eq} - m_6^{eq}) &= -s_4 \frac{m_4^{(1)}}{\delta t} + S_4^{(1)}, \quad (A 18) \\
\partial_t m_5^{eq} + \partial_x m_6^{eq} + \partial_y m_7^{eq} &= -s_5 \frac{m_5^{(1)}}{\delta t} + S_5^{(1)}. \quad (A 19)
\end{align}

Equations (A 17)–(A 19) can be rewritten as

\begin{align}
m_3^{(1)} &= -\frac{2\delta t}{s_3} \rho c_s^2 (\partial_x u_x + \partial_y u_y) + \frac{2\sigma |F_m|^2}{(1 - s_3/2)\psi^2} + (u_x^2 \partial_x \rho + u_y^2 \partial_y \rho + 3\rho u_x^2 \partial_x u_x + 3\rho u_y^2 \partial_y u_y), \quad (A 20) \\
m_4^{(1)} &= -\frac{2\delta t}{s_4} \rho c_s^2 (\partial_x u_x - \partial_y u_y) + (u_x^3 \partial_x \rho - u_y^3 \partial_y \rho + 3\rho u_x^2 \partial_x u_x - 3\rho u_y^2 \partial_y u_y), \quad (A 21) \\
m_5^{(1)} &= -\frac{\delta t}{s_5} \rho c_s^2 (\partial_y u_y + \partial_y u_x), \quad (A 22)
\end{align}

where we set $c_s^2 = 1/3$. Under low Mach-number assumptions, the underlined terms in equations (A 20) and (A 21) may be neglected. Recently, Huang et al. (2018) derived correction terms to eliminate such third-order terms. The double underlined term in equation (A 20) have appeared due to the modification described in § 2. Then, equations (A 15) and (A 16) becomes

\begin{align}
\partial_t (\rho u_x) &= \partial_x \left[ \rho (2\partial_x u_x) + \rho (\nu_b - \nu) (\nabla \cdot u) - \frac{\sigma |F_m|^2}{\psi^2} \right] + \partial_y \left[ \rho (2\partial_x u_y + \partial_y u_x) \right], \quad (A 23) \\
\partial_t (\rho u_y) &= \partial_x \left[ \rho \nu (\partial_x u_y + \partial_y u_x) \right] + \partial_y \left[ \rho (2\partial_y u_y + \rho (\nu_b - \nu) (\nabla \cdot u) - \frac{\sigma |F_m|^2}{\psi^2} \right], \quad (A 24)
\end{align}

where $\nu_b = c_s^2 \delta t (\omega_b^{-1} - 1/2)$ and $\nu = c_s^2 \delta t (\omega^{-1} - 1/2)$ with $s_3 = \omega_b$ and $s_4 = s_5 = \omega$, respectively.

Combining equations (A 11) with (A 14) through $\partial_t = \partial_{t_1} + \varepsilon \partial_{t_2}$, the continuity
equation can be obtained
\[
\partial_t \rho + \nabla \cdot (\rho u) = 0. \tag{A 25}
\]
Similarly, combining equations (A 12)–(A 13) with equations (A 23)–(A 24), we can obtain the Navier–Stokes equation as follows:
\[
\partial_t (\rho u) + \nabla \cdot (\rho uu) = -\nabla \cdot (\rho c_s^2 + \sigma G^2 c_s^4 |\nabla \psi|^2) I + \nabla \cdot [\rho (\nabla u + \nabla u^\top) + \rho (\nu_b - \nu) (\nabla \cdot u)] + F. \tag{A 26}
\]
We can find that the pressure tensor has been modified to be thermodynamically consistent, which is similar to the scheme proposed for the orthogonal MRT model (Li et al. 2013).

Appendix B. Film boiling HTC correlations

B.1. Mode 1 correlation

The original equation derived by Bromley et al. (1953) is inconvenient because the derivative appears non-linearly. After a certain amount of ad hoc approximation, the equation results in a simple form. At high flow regime \( Fr^{1/2} > 2.0 \), Bromley et al.’s correlation is given by
\[
Nu = C \eta^{1/2} Re^{1/2} Pr^{1/2} \left( \frac{c_{p,v} \Delta T}{h_{fg}'} \right)^{-1/2}, \tag{B 1}
\]
where \( C = 2.7 \) and \( h_{fg}' = h_{fg}(1 + 0.4c_{p,v} \Delta T/h_{fg})^2 \) is the effective latent heat of vaporization (Bromley 1952).

The correlation of Witte (1968) can be expressed by the same form as equation (B 1) with \( C = 0.698 \) for sphere and 0.636 for cylinder. A slight difference appears in the definition of \( h_{fg}' \) as \( h_{fg}' = h_{fg} + 0.68c_{p,v} \Delta T \), which is Rohsenow’s latent heat of vaporization (Rohsenow 1956). We used \( C = 0.636 \) in this paper.

B.2. Mode 2 correlation

Epstein & Hauser (1980) originally derived the HTC correlation for subcooled forced-convection film boiling. When the subcooling degree is assumed to be zero, their correlation degrades into
\[
Nu = C \gamma^{1/4} \eta^{1/2} Re^{1/2} Pr^{1/4} \left( \frac{c_{p,v} \Delta T}{h_{fg}'} \right)^{-1/4}, \tag{B 2}
\]
where \( C = 0.553 \) for sphere and 0.537 for cylinder from theoretical analysis. Note that they finally concluded that \( C = 1.13 \), used in this paper, provided a reasonable correlation of observed HTC for the experimental data of subcooled forced-convection film boiling from spheres or cylinders (Bromley 1950; Bromley et al. 1953; Motte & Bromley 1957; Dhir & Purohit 1978).

Ito et al. (1981) analyzed the forced-convection film boiling heat transfer from a horizontal cylinder to saturated liquid based on the two-phase boundary layer theory. For the conditions of predominant forced convection, they obtained the equation identical to equation (B 2). They obtained the constants as \( C = 0.46, 0.48, \) and 0.51 for water, ethanol, and hexane, respectively; we used \( C = 0.46 \) in this paper.

The correlation of Liu & Theofanous (1994) also has a similar form to Epstein–Hauser’s one [equation (B 2)]:
\[
Nu = C \gamma^{1/4} \eta^{1/2} Re^{1/2} Pr^{1/4} \left( \frac{c_{p,v} \Delta T}{h_{fg}'} \right)^{-1/4}, \tag{B 3}
\]
with $h'_{fg} = h_{fg} + 0.5c_{p,v} \Delta T$ and $C = 0.5$. Note that the terms related to subcooling was dropped different from the original Liu–Theofanous correlation.

B.3. Other correlation

Dhir & Purohit (1978) correlated their film boiling heat transfer data of a stainless steel sphere by the following empirical correlation for the saturated condition:

$$Nu = Nu_0 + 0.8Re^{1/2},$$

where $Nu_0$ is the Nusselt number based on the saturated film boiling HTC averaged over the sphere, defined as

$$Nu_0 = 0.8 \frac{g \rho_v (\rho_l - \rho_v) h_{fg} D^3}{\mu_v \lambda_v \Delta T}^{1/4},$$

$$= 0.8 Ar^{1/4} Pr^{1/4} Ja^{-1/4}.\quad (B\ 5)$$

$Ar = g(\rho_l - \rho_v) D^3/\rho_v \nu_v^2$ is the Archimedes number, expressing the ratio of buoyancy and inertia. If the second term in equation (B 4) is neglected, the equation is reduced to the correlation for saturated pool boiling. One can consider the Dhir–Purohit correlation took the forced-convection effect into account their pool boiling HTC.

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