CKM Matrix: Present and Future

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Abstract

We review the present status of the CKM matrix and we offer some visions of its future. After a brief presentation of the theoretical framework for weak decays we discuss the following topics: i) CKM matrix from tree level decays, ii) Standard analysis of the unitarity triangle, iii) CKM matrix from rare and CP violating K- and B-decays, iv) CKM matrix from CP violation in two-body B-decays. In particular we compare the CKM potentials of the standard analysis of the unitarity triangle, of the very clean $K \rightarrow \pi \nu \bar{\nu}$ decays and of CP asymmetries in B-decays.

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1 Introduction

1.1 CKM Matrix

An important target of particle physics is the determination of the unitary \(3 \times 3\) Cabibbo-Kobayashi-Maskawa matrix \(\hat{V}_{CKM}\), which parametrizes the charged current interactions of quarks. CP violation in the Standard Model is supposed to arise from a single phase in this matrix. The standard parametrization of the CKM matrix is given in terms of \(c_{ij} = \cos \theta_{ij}\), \(s_{ij} = \sin \theta_{ij}\) and the phase \(\delta\) with \(i\) and \(j\) being generation labels \((i, j = 1, 2, 3)\). \(s_{13}\) and \(s_{23}\) turn out to be small numbers: \(O(10^{-3})\) and \(O(10^{-2})\), respectively. Consequently to an excellent accuracy the four independent parameters to be determined experimentally are:

\[
s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta
\]  

On the other hand, in the phenomenological applications, it is customary these days to work with the CKM-matrix expressed in terms of four Wolfenstein parameters \((\lambda, A, \varrho, \eta)\) with \(\lambda = |V_{us}| = 0.22\) playing the role of an expansion parameter and \(\eta\) representing the CP violating phase:

\[
\hat{V}_{CKM} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & A\lambda^3(\varrho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \varrho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4) \tag{2}
\]

The Wolfenstein parametrization is certainly more transparent than the standard parametrization. However, if one requires sufficient level of accuracy, the higher order terms in \(\lambda\) have to be included in phenomenological applications.
An efficient and systematic way to achieve this without the lost of transparency is to define the parameters $(\lambda, A, \varrho, \eta)$ through

\[
s_{12} \equiv \lambda \quad s_{23} \equiv A\lambda^2 \quad s_{13} e^{-i\delta} \equiv A\lambda^3(\varrho - i\eta) \quad (3)
\]

Making this change of variables in the standard parametrization, we find the CKM matrix as a function of $(\lambda, A, \varrho, \eta)$ which, in contrast to (2) satisfies unitarity exactly! Expanding next in powers of $\lambda$ we recover the matrix in (2) and in addition find explicit corrections of $O(\lambda^4)$ and higher order terms.

The definition of $(\lambda, A, \varrho, \eta)$ given in (3) is useful because it allows to improve the accuracy of the original Wolfenstein parametrization in an elegant manner. In particular

\[
V_{us} = \lambda \quad V_{cb} = A\lambda^2 \\
V_{ub} = A\lambda^3(\varrho - i\eta) \quad V_{td} = A\lambda^3(1 - \tilde{\varrho} - i\tilde{\eta}) \\
V_{ts} = -A\lambda^2 + \frac{1}{2}A(1 - 2\varrho)\lambda^4 - i\eta A\lambda^4
\]

where

\[
\tilde{\varrho} = \varrho(1 - \frac{\lambda^2}{2}) \quad \tilde{\eta} = \eta(1 - \frac{\lambda^2}{2})
\]

turn out to be excellent approximations to the exact expressions.

The advantage of this generalization of the Wolfenstein parametrization over other generalizations found in the literature is the absence of relevant corrections to $V_{us}$, $V_{cb}$ and $V_{ub}$. Corrections to $V_{us}$ and $V_{cb}$ appear only at $O(\lambda^7)$ and $O(\lambda^8)$, respectively. $V_{ub}$ as given above is valid to all orders. Simultaneously the simple modification of $V_{td}$ relative to the one in (3) allows a simple generalization of the unitarity triangle beyond the leading order in $\lambda$.

![Figure 1: Unitarity Triangle.](image-url)
The unitarity triangle is obtained by using the unitarity relation
\[ V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \] (8)
rescaling it by \( |V_{cd}V_{cb}^*| = A\lambda^3 \) and depicting the result in the complex \((\bar{\rho}, \bar{\eta})\) plane as shown in fig. 1. The lengths CB, CA and BA are equal respectively to 1,
\[ R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = (1 - \frac{\lambda^2}{2}) \frac{1}{\lambda} |V_{ub}| \quad \text{and} \quad R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} |V_{td}|. \] (9)

The triangle in fig. 1, \(|V_{us}|\) and \(|V_{cb}|\) give the full description of the CKM matrix. Looking at the expressions for \(R_b\) and \(R_t\) we observe that within the Standard Model the measurements of four CP conserving decays sensitive to \(|V_{us}|, |V_{cb}|, |V_{ub}|\) and \(|V_{td}|\) can tell us whether CP violation \((\bar{\eta} \neq 0)\) is predicted in the Standard Model. This is a very remarkable property of the Kobayashi-Maskawa picture of CP violation: quark mixing and CP violation are closely related to each other. This property is often used to determine the angles of the unitarity triangle without the study of CP violating quantities.

There is of course the very important question whether the KM picture of CP violation is correct and more generally whether the Standard Model offers a correct description of weak decays of hadrons. In order to answer these important questions it is essential to calculate as many branching ratios as possible, measure them experimentally and check if they all can be described by the same set of the parameters \((\lambda, A, \bar{\rho}, \bar{\eta})\). In the language of the unitarity triangle this means that the various curves in the \((\bar{\rho}, \bar{\eta})\) plane extracted from different decays should cross each other at a single point as shown in fig. 2. Moreover the angles \((\alpha, \beta, \gamma)\) in the resulting triangle should agree with those extracted one day from CP-asymmetries in B-decays. More about this below.

Since the CKM matrix is only a parametrization of quark mixing and of CP violation and does not offer the explanation of these two very important phenomena, many physicists hope that a new physics while providing a dynamical origin of quark mixing and CP violation will also change the picture given in fig. 2. That is, the different curves based on the Standard Model expressions, will not cross each other at a single point and the angles \((\alpha, \beta, \gamma)\) extracted one day from CP-asymmetries in B-decays will disagree with the ones determined from rare K and B decays.

Clearly the plot in fig. 2 is highly idealized because in order to extract such nice curves from various decays one needs perfect experiments and perfect theory. One of the goals of this review is to identify those decays for which at least the theory is under control. For such decays, if they can be measured
with a sufficient precision, the curves in fig. 2 are not unrealistic. In order to understand this we have to discuss briefly the present theoretical framework.

1.2 Theoretical Framework

The basic problem in the extraction of CKM parameters from meson decays is related to strong interactions. Although due to the smallness of the effective QCD coupling at short distances, the gluonic contributions at scales $\mathcal{O}(M_W, M_Z, m_t)$ can be calculated within the perturbative framework, the fact that mesons are $q\bar{q}$ bound states forces us to consider QCD at long distances as well. Here we have to rely on existing non-perturbative methods, which are not yet very powerful at present.

The separation of the short and long distance contributions to a given amplitude is achieved by using the powerful method of the Operator Product Expansion (OPE) combined with the renormalization group approach. Explicitly the amplitude for an exclusive decay $M \to F$ is written as

$$A(M \to F) = \frac{G_F}{\sqrt{2}} V_{\text{CKM}} \sum_i C_i(\mu) \langle F \mid Q_i(\mu) \mid M \rangle$$  \hspace{1cm} (10)

where $V_{\text{CKM}}$ denotes the relevant CKM factor. The scale $\mu$ separates the physics contributions in the “short distance” contributions (corresponding to
scales higher than $\mu$) contained in the Wilson coefficients $C_i(\mu)$ and the “long distance” contributions (scales lower than $\mu$) contained in $\langle F | Q_i(\mu) | M \rangle$. Here $Q_i$ are local operators generated by QCD and electroweak interactions which govern “effectively” the decays in question. The $\mu$ dependence of $C_i(\mu)$ is governed by the renormalization group equations. It must be canceled by the one present in $\langle Q_i(\mu) \rangle$ so that the resulting physical amplitudes do not depend on $\mu$. Generally this cancellation involves many operators due to the operator mixing under renormalization. The list of the operators originating in the tree diagrams, in one-loop diagrams of fig. 3 and in QCD corrections to them, as well as technical details of the calculations of $C_i(\mu)$ can be found in [6].

The use of the renormalization group is necessary in order to sum up large logarithms $\log M_W/\mu$ which appear for $\mu = O(1 − 2 \text{GeV})$. In the so-called leading logarithmic approximation (LO) terms $(\alpha_s \log M_W/\mu)^n$ are summed. The next-to-leading logarithmic correction (NLO) to this result involves summation of terms $(\alpha_s)^n(\log M_W/\mu)^{n-1}$ and so on. This hierarchic structure gives the renormalization group improved perturbation theory.

The rather formal expression for the decay amplitudes given in (10) can always be cast in the form:

$$A(M \rightarrow F) = \sum_i B_i V_{CKM}^i \eta_{QCD}^i F_i(m_t, m_c) \quad (11)$$
which is more useful for phenomenology. In writing (11) we have generalized
result from the evaluation of loop diagrams with internal top and charm ex-
changes (see fig. 3) and may also depend solely on \( m_t \) or \( m_c \). In the case of new
physics they depend on masses of new particles such as charginos, stops and
charged Higgs scalars as well as new couplings. The factors \( \eta_{\text{QCD}} \) summarize
the QCD corrections which can be calculated by formal methods mentioned
above. Finally \( B_i \) stand for nonperturbative factors related to the hadronic
matrix elements of the contributing operators: the main theoretical uncer-
tainty in the whole enterprise. In leptonic and semi-leptonic decays for which
only the matrix elements of weak currents are needed, the non-perturbative
\( B \)-factors can fortunately be determined from leading tree level decays redu-
ning or removing the non-perturbative uncertainty. In non-leptonic decays
this is generally not possible and we have to rely on existing non-perturbative
methods. A well known example of a \( B_i \)-factor is the renormalization group
invariant parameter \( B_K \) defined by

\[
B_K = B_K(\mu) [\alpha_s(\mu)]^{-2/9} \quad \langle \bar{K}^0 \mid (\bar{s}d)_{V-A}(\bar{s}d)_{V-A} \mid K^0 \rangle = \frac{8}{3} B_K(\mu) F_K^2 m_K^2
\]  

(12)

In order to achieve sufficient precision, \( C_i(\mu) \) or equivalently \( \eta_{\text{QCD}} \equiv \eta_i \)
have to include NLO corrections in the renormalization group improved per-
turbation theory. In particular, unphysical left-over \( \mu \)-dependences in decay
amplitudes, resulting from the truncation of the perturbative series, are con-
siderably reduced by including NLO corrections. These corrections are known
by now for the most important and interesting decays and are reviewed in [6].

In the case of decaying B-mesons, the approach discussed here can be
generalized to inclusive decays where it is known under the name of \textit{Heavy
Quark (1/}\( m_b \)) \textit{Expansions}. The leading term in these expansions corresponds
to the spectator model corrected for short distance QCD effects. HQE are
discussed by Thomas Mannel in these proceedings.

2 CKM Matrix from Tree Level Decays and Unitarity

2.1 The seven elements of the CKM Matrix

The seven elements of the CKM matrix which have been determined in tree
level decays without using unitarity are given as follows:

\[
|V_{ud}| = 0.9736 \pm 0.0010 \quad |V_{us}| = \lambda = 0.2205 \pm 0.0018
\]  

(13)
as given in [4] and

\[
|V_{ud}| = 0.9740 \pm 0.0005
\]  

(14)
obtained subsequently at the Chalk River Laboratory

\[ |V_{cb}| = 0.224 \pm 0.016 \quad |V_{cs}| = 1.01 \pm 0.18 \quad (15) \]

taken from \cite{3},

\[ |V_{cb}| = (40 \pm 3) \cdot 10^{-3} \quad |V_{ub}| = (3.2 \pm 0.8) \cdot 10^{-3} \quad (16) \]
discussed below and

\[ |V_{tb}| = 0.99 \pm 0.15 \quad (17) \]
as obtained from CDF \cite{9}.

We do not have any tree level results for \(|V_{ts}| and \(|V_{td}|. Because of very small branching ratios for such tree level top quark decays we will most likely be able to determine these elements only in loop induced decays discussed in subsequent sections.

2.2 Including Unitarity of CKM

Setting \( \lambda = 0.22 \), scanning \(|V_{cb}| and \(|V_{ub}| in the ranges \cite{10} and \cos \delta in the range \(-1 \leq \cos \delta \leq 1\), we find \cite{11}:

\[ 4.5 \cdot 10^{-3} \leq |V_{td}| \leq 13.7 \cdot 10^{-3}, \quad 0.0353 \leq |V_{ts}| \leq 0.0429 \quad (18) \]

and

\[ 0.9991 \leq |V_{tb}| \leq 0.9993, \quad 0.9736 \leq |V_{cs}| \leq 0.9750. \quad (19) \]

From \cite{11} we observe that the unitarity of the CKM matrix requires approximate equality of \(|V_{ts}| and \(|V_{cb}|: 0.954 \leq |V_{ts}|/|V_{cb}| \leq 0.997 which is evident if one compares \cite{3} with \cite{11}. The determination of \(|V_{td}| will be improved in the next section by using the constraints from \(B_d^0 - \bar{B}_d^0\)-mixing and CP violation in the \(K\)-meson system.

Using \cite{12}, \(|V_{us}| and \(|V_{ub}| one finds:

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9972 \pm 0.0013, \quad (20) \]

where the contribution of \(|V_{ub}|^2 is negligible. Thus the departure from the unitarity constraint is by at least two standard deviations. The simplest solution to this “unitarity problem” would be to double the error in \(|V_{ud}| or to increase its value. Since the neutron decay data give, in contrast to \(0^+ \rightarrow 0^+\) superallowed beta decays, values for the unitarity sum higher than unity \cite{11}, such a shift is certainly possible. Clearly the current status of the \(|V_{ud}| determinations, in spite of small errors quoted above, is unsatisfactory at present.
Next, as stressed by Ben Nefkens at this symposium, the experimental error in the determination of $|V_{us}|$ in (13) is underestimated and a new improved experiment with the aim of determining this very important element to a high accuracy is desirable. It is clear that further investigations of $|V_{ud}|$ and $|V_{us}|$ should be made before one could conclude that the failure to meet the unitarity constraint signals some physics beyond the Standard Model.

2.3 More on $|V_{cb}|$ and $|V_{ub}|$

$|V_{cb}|$ from Inclusive Decays

The determination of $|V_{cb}|$ from inclusive decays uses

$$\Gamma(B \to X_c l\bar{\nu}) = \frac{G_F^2 m_b^5}{192\pi^3} |V_{cb}|^2 f_1(m_b, m_c, \lambda_1, \lambda_2, \alpha_s)$$

(21)

where $\lambda_i$ are the well known non-perturbative parameters entering the $1/m_b^2$ corrections to the spectator model result.

The most recent analysis\(^\text{11}\) updates the 95 analysis of Shifman, Uraltsev and Vainshtein\(^\text{12}\) with the result:

$$|V_{cb}|^{incl} = 41.0 \left[ \frac{Br(B \to X_c l\nu)}{0.104} \right]^{1/2} \left[ \frac{1.6\text{ps}}{\tau_B} \right]^{1/2} \cdot 10^{-3} \cdot \kappa$$

(22)

where

$$\kappa = 1 \pm 0.015_{\text{pert}} \pm 0.01_{m_b} \pm 0.012_{\lambda_1} \pm 0.012_{\text{rest}}.$$ \hspace{1cm} (23)

The last error is an educated guess for the uncertainties coming from assuming hadron-quark duality as well as other small corrections not shown explicitly in (23). The main progress this year has been the reduction of the perturbative uncertainty by a factor of approximately three through the two-loop calculations of Czarnecki and Melnikov\(^\text{13}\). The rather small error due to the uncertainty in $m_b$ is related to the fact that the leading $m_b^5$ dependence in (21) is substantially reduced through the additional phase space effects collected in $f_1$. This uncertainty is probably somewhat underestimated.

Taking the estimate (22) with $\tau_B = (1.60 \pm 0.03) ps$ and $Br(B \to X_c l\nu) = 10.4 \pm 0.4\%$ one arrives at:

$$|V_{cb}|^{incl} = \left( \frac{41.0}{38.8} \right) \left[ Br(B \to X_c l\nu) \right]^{1/2} \left[ \frac{1.6\text{ps}}{\tau_B} \right] \cdot 10^{-3} \cdot \kappa$$

(24)

where the second estimate is based on the 95-analysis of Ball, Benecke and Braun\(^\text{14}\) with appropriate update in $\tau_B$ and $Br(B \to X_c l\nu)$.\(^\text{11,14}\) agree on the size of the theoretical error.
\[ |V_{cb}| \textbf{from Exclusive Decays} \]

The exclusive determination of \(|V_{cb}|\) uses the recoil spectrum of \(D^*\) in \(B \rightarrow D^* l \bar{\nu}\):

\[
\frac{d\Gamma(B \rightarrow D^* l \bar{\nu})}{d\omega} = f_2(m_B, m_{D^*}, \omega)|V_{cb}|^2 \mathcal{F}^2(\omega) \tag{25}
\]

where \(f_2\) collects kinematical factors and \(\mathcal{F}(\omega)\) is the relevant formfactor which in the absence of power and short distance QCD corrections reduces to the Isgur-Wise function. \(\mathcal{F}(\omega)\) can be calculated reasonably at the no-recoil point:

\[
\mathcal{F}(1) = 1 + \mathcal{O}(\alpha_s) + \mathcal{O}\left(\frac{1}{m_c^2}\right) + \mathcal{O}\left(\frac{1}{m_c m_b}\right) + \mathcal{O}\left(\frac{1}{m_b^2}\right) \tag{26}
\]

The perturbative short distance corrections are fully under control after the complete two-loop analysis of Czarnecki \(^{15}\). On the other hand there is some dispute between Neubert \(^{16}\) and the authors in \(^{11}\) on the uncertainty due to \(\mathcal{O}(1/m^2)\) corrections:

\[
\mathcal{F}(1) = 0.91 \pm 0.01_{\text{pert}} \pm \left[ 0.025 \begin{array}{c} [N] \vspace{1mm} \\ 0.050 \end{array} \right]_{1/m^2} = 0.91 \pm \left[ 0.03 \begin{array}{c} [N] \\ 0.06 \end{array} \right]_{[BSUV]} \tag{27}
\]

Using Gibbons 97 - Analysis:

\[
\mathcal{F}(1)|V_{cb}| = (35.1 \pm 2.5) \cdot 10^{-3} \tag{28}
\]

one arrives at

\[
|V_{cb}|_{exc} = \left( 38.6 \pm \left[ 1.2 \begin{array}{c} [N] \\ 2.4 \end{array} \right]_{\text{th}} \pm 2.7_{\text{exp}} \right) \cdot 10^{-3} \tag{29}
\]

I have no intention to make an analysis which somehow combines the inclusive and exclusive determinations. However, looking at (24) and (25), it appears that \(|V_{cb}| = 0.040 \pm 0.003\) as given in \(^{16}\) is a good summary of the present value for \(|V_{cb}|\). Clearly values as \(|V_{cb}| = 0.039 \pm 0.003\) or \(|V_{cb}| = 0.039 \pm 0.004\) would also be fine. On the other hand an error on \(|V_{cb}|\) as low as 0.002 \(^{16}\) appears to me difficult to defend at present.

\[ |V_{ub}|/|V_{cb}| \]

The present status and future prospects for \(|V_{ub}|\) have been discussed by Ball \(^{17}\), Gibbons and Flynn at this symposium and consequently I will be only very brief on this topic. The main results at present come from the end-point lepton energy spectrum in \(B \rightarrow X_u e \bar{\nu}_e\) which are affected by some model dependence.
This model dependence can be somewhat reduced by using the exclusive decays $B \to \rho(\pi)l\nu$. Yet the final result using this method contains still a ±25% error as seen in (10) and in

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.08 \pm 0.02$$

With the improved data and improved formfactors coming from lattice calculations and light-cone sum rules, this error could be possibly decreased to ±10%.

On the other hand there is a hope that the study of the hadronic energy spectrum or hadron invariant mass spectrum in $B \to X_u l\bar{v}_e$ could allow a measurement of $|V_{ub}|$ with an error of ±10%. Furthermore as suggested by Uraltsev, the ultimate measurement of $|V_{ub}|$ could be obtained from the inclusive semileptonic $b \to u$ rate by means of:

$$|V_{ub}| = 0.00458 \left( \frac{Br(B \to X_u l\nu)}{0.002} \right)^{1/2} \left[ \frac{1.6 ps}{\tau_B} \right]^{1/2} \cdot [1\pm0.025_{\text{pert}}\pm0.03_{m_b}].$$

In the case of very good data this would allow a ±5% measurement of $|V_{ub}|$. The measurement of the inclusive semileptonic $b \to u$ rate is very difficult, however, and it will take some time before this idea could be efficiently realized. In addition the view that this way of measuring $|V_{ub}|$ is theoretically very clean is not shared by everybody.

Finally one should recall that the measurement of the leptonic branching ratio for $B^+ \to l^+\nu_l$ ($l = e, \mu, \tau$) determines the product $|V_{ub}|F_B$ with essentially no theoretical uncertainties. Thus provided such branching ratio can be measured with respectable accuracy and $F_B$ can be calculated with high precision one day, one would have another independent measurement of $|V_{ub}|$.

3 Standard Analysis of the Unitarity Triangle

3.1 Basic Formulae

The standard analysis of the unitarity triangle proceeds in five steps:

Step 1:
From $b \to c$ transition in inclusive and exclusive B meson decays one finds $|V_{cb}|$ and consequently the scale of UT:

$$|V_{cb}| \Rightarrow \lambda|V_{cb}| = \lambda^3 A$$

Step 2:
From $b \to u$ transition in inclusive and exclusive B meson decays one finds $|V_{ub}/V_{cb}|$ and consequently the side $CA = R_b$ of UT:

$$|V_{ub}/V_{cb}| \Rightarrow R_b = 4.44 \cdot \left| \frac{V_{ub}}{V_{cb}} \right|$$  \hspace{1cm} (33)

**Step 3:**

From the observed indirect CP violation in $K \to \pi\pi$ described experimentally by the parameter $\varepsilon_K$ and theoretically by the imaginary part of the relevant box diagram in fig. 3 one derives the constraint:

$$\bar{\eta} \left[ (1 - \bar{\rho}) A^2 \eta_2 S(x_t) + P_0(\varepsilon) \right] A^2 B_K = 0.226 \quad S(x_t) = 0.784 \cdot x_t^{0.76}.$$  \hspace{1cm} (34)

Equation (34) specifies a hyperbola in the $(\bar{\rho}, \bar{\eta})$ plane. Here $x_t = m_t^2 / M_W^2$, $\eta_2$ is the QCD factor in the box diagrams with two top quark exchanges and $P_0(\varepsilon) = 0.31 \pm 0.05$, dependent on QCD factors $\eta_1$ and $\eta_3$, summarizes the contributions of box diagrams with two charm quark exchanges and the mixed charm-top exchanges. The error in $P_0(\varepsilon)$ is dominated by the uncertainties in $\eta_3$ and $m_c$.

Since the $P_0(\varepsilon)$ term contributes only 25% to (34) these uncertainties constitute only to a few percent uncertainty in the constraint (34). The NLO values of the QCD factors are: $\eta_1 = 1.38 \pm 0.20$, $\eta_2 = 0.57 \pm 0.01$, and $\eta_3 = 0.47 \pm 0.04$. The quoted errors reflect the remaining theoretical uncertainties due to $\mu$-dependences and $\Lambda_{\overline{MS}}$.

$B_K$ is the non-perturbative parameter defined in (12). The review of its values in various non-perturbative approaches can be found in \cite{10}. I only mention here new results not included there. The most accurate lattice results for $B_K$ come from JLQCD $B_K = 0.87 \pm 0.06$ \cite{25}. Similar result has been published in \cite{26} this year. On the other hand a recent analysis in the chiral quark model gives surprisingly a value as high as $B_K = 1.3 \pm 0.2$ \cite{27}. In our numerical analysis presented below we will use, as in \cite{10}, $B_K = 0.75 \pm 0.15$ which is in the ballpark of various lattice estimates and $B_K = 0.70 \pm 0.10$ from the $1/N$ approach \cite{28}.

Since $m_t$ and the relevant QCD factors are rather precisely known, the main uncertainties in the constraint (34) reside in $B_K$ and to some extent in $A^4$ which multiplies the leading term.

**Step 4:**

From the observed $B^0_d - \bar{B}^0_d$ mixing described experimentally by the mass difference $\Delta M_d$ or by the mixing parameter $x_d = \Delta M_d / \Gamma_B$ and theoretically by the relevant box diagram of fig. 3 the side $BA = R_t$ of the UT can be determined:

$$R_t = \frac{1}{\lambda} \left| \frac{V_{td}}{|V_{cb}|} \right| = 1.0 \cdot \left| \frac{|V_{td}|}{8.8 \cdot 10^{-3}} \right| \left| \frac{0.040}{|V_{cb}|} \right|$$  \hspace{1cm} (35)
with

\[ |V_{td}| = 8.8 \cdot 10^{-3} \left[ \frac{200 \, \text{MeV}}{\sqrt{B_{B_d} F_{B_d}}} \right] \left[ \frac{170 \, \text{GeV}}{m_t(m_t)} \right]^{0.76} \left[ \frac{\Delta M_d}{0.50/\text{ps}} \right]^{0.5} \sqrt{\frac{0.55}{\eta_B}}. \]  

(36)

Here \( \eta_B \) is the QCD factor analogous to \( \eta_2 \) and given by \( \eta_B = 0.55 \pm 0.01 \).

Next \( F_{B_d} \) is the B-meson decay constant and \( B_{B_d} \) denotes a non-perturbative parameter analogous to \( B_K \).

There is a vast literature on the lattice calculations of \( F_{B_d} \) and \( B_{B_d} \). The world averages given by Flynn and Bernard can be summarized by:

\[ F_{B_d} = 175 \pm 25 \, \text{MeV} \quad \text{and} \quad B_{B_d} = 1.31 \pm 0.03. \]

This result for \( F_{B_d} \) is compatible with the results obtained using QCD sum rules. In our numerical analysis we will use \( F_{B_d} \) given in table 1.

Step 5:

The measurement of \( B_0^0 - \bar{B}_0^0 \) mixing parametrized by \( \Delta M_s \) together with \( \Delta M_d \) allows to determine \( R_t \) in a different way. Setting \( \Delta M_s^{max} = 0.482/\text{ps} \) and \( |V_{ts}/V_{cb}|^{max} = 0.993 \) (see table 1) I find a useful formula:

\[ (R_t)_{max} = 1.0 \cdot \xi \sqrt{\frac{10.2/\text{ps}}{(\Delta M)_s}} \quad \xi = \frac{F_{B_s} \sqrt{B_{B_s}}}{F_{B_d} \sqrt{B_{B_d}}}. \]  

(37)

where \( \xi = 1 \) in the SU(3)–flavour limit. Note that \( m_t \) and \( |V_{cb}| \) dependences have been eliminated this way and that \( \xi \) should in principle contain much smaller theoretical uncertainties than the hadronic matrix elements in \( \Delta M_d \) and \( \Delta M_s \) separately.

The most recent values relevant for (37) are:

\[ \Delta M_s > 10.2/\text{ps} \quad (95\% \text{ C.L.}) \quad \xi = 1.15 \pm 0.05 \]  

(38)

The first number is the improved lower bound from ALEPH. The second number comes from quenched lattice calculations summarized by Flynn and Bernard here. A similar result has been obtained using QCD sum rules.

The fate of the usefulness of the bound (37) depends clearly on both \( \Delta M_s \) and \( \xi \). For \( \xi = 1.2 \) the lower bound on \( \Delta M_s \) in (38) implies \( R_t \leq 1.20 \) which,
as we will see, has a moderate impact on the unitarity triangle obtained on
the basis of the first four steps alone.

Finally, I would like to point out that whereas step 5 can give, in contrast
to step 4, the value for $R_t$ free of the $|V_{cb}|$ uncertainty, it does not provide
at present a more accurate value of $|V_{td}|$. The point is, that having $R_t$, one
determines $|V_{td}|$ by means of the relation (35) which, in contrast to (36), de-
pends on $|V_{cb}|$. In fact as we will see below, the inclusion of step 5 has, with
$\xi = 1.2$, a visible impact on $R_t$ without essentially any impact on the range of
$|V_{td}|$ obtained on the basis of the first four steps alone.

3.2 Numerical Results

Input Parameters

The input parameters needed to perform the standard analysis using the first
four steps alone are given in table 1. We list here the "present" errors based on
what we have discussed above, as well as the "future" errors. The latter are a
mere guess, but as we will see in the subsequent sections, these are the errors
one should aim at, in order that the standard analysis could be competitive in
the CKM determination with the cleanest rare decays and the CP asymmetries
in B-decays.

$m_t$ in table 1 refers to the running current top quark mass normalized at
$\mu = m_t$: $\overline{m}_t(m_t)$. Its value given there corresponds to $m_t^{pole} = 175 \pm 6 \text{ GeV}$
from CDF and D0.

Table 1: Collection of input parameters.

| Quantity        | Central | Present  | Future  |
|-----------------|---------|----------|---------|
| $|V_{cb}|$        | 0.040   | $\pm 0.003$ | $\pm 0.001$ |
| $|V_{ub}/V_{cb}|$ | 0.080   | $\pm 0.020$ | $\pm 0.005$ |
| $B_K$           | 0.75    | $\pm 0.15$ | $\pm 0.05$ |
| $\sqrt{B_d F_{B_d}}$ | 200 MeV | $\pm 40 \text{ MeV}$ | $\pm 10 \text{ MeV}$ |
| $m_t$           | 167 GeV | $\pm 6 \text{ GeV}$ | $\pm 3 \text{ GeV}$ |
| $\Delta M_d$    | 0.464 ps$^{-1}$ | $\pm 0.018 \text{ ps}^{-1}$ | $\pm 0.006 \text{ ps}^{-1}$ |

Output of a Standard Analysis

The output of the standard analysis depends to some extent on the error
analysis. This should be always remembered in view of the fact that different
authors use different procedures. In order to illustrate this I show in tables 2
("present") and \(\bar{3}\) ("future") the results for various quantities of interest using two types of error analyses:

- **Scanning**: Both the experimentally measured numbers and the theoretical input parameters are scanned independently within the errors given in Table 1.
- **Gaussian**: The experimentally measured numbers and the theoretical input parameters are used with Gaussian errors.

Clearly the "scanning" method is a bit conservative. On the other hand using Gaussian distributions for theoretical input parameters can be certainly questioned. I think that at present the conservative "scanning" method should be preferred, although one certainly would like to have a better method. An interesting new method is discussed by Schune in these proceedings. The analysis discussed here has been done in collaboration with Matthias Janin and Markus Lautenbacher.

Table 2: Present output of the Standard Analysis. \(\lambda_t = V_{ts}^* V_{td}\).

| Quantity                  | Scanning       | Gaussian       |
|----------------------------|----------------|----------------|
| \(|V_{td}| / 10^{-3}\)     | 6.9 – 11.3     | 8.6 ± 1.1      |
| \(|V_{ts}/V_{cb}|\)      | 0.959 – 0.993  | 0.976 ± 0.010  |
| \(|V_{td}/V_{ts}|\)      | 0.16 – 0.31    | 0.213 ± 0.034  |
| \(\sin(2\beta)\)        | 0.36 – 0.80    | 0.66 ± 0.13    |
| \(\sin(2\alpha)\)       | −0.76 – 1.0    | 0.11 ± 0.55    |
| \(\sin(\gamma)\)        | 0.66 – 1.0     | 0.88 ± 0.10    |
| \(\text{Im}\lambda_t / 10^{-4}\) | 0.86 – 1.71 | 1.29 ± 0.22   |

In figs. 4 and 5 we show the ranges for the upper corner A of the UT in the case of the "present" input and "future" input respectively. The circles correspond to \(R_t^{max}\) from (37) using \(\xi = 1.20\) and \(\Delta M_s = 10/\text{ps}, 15/\text{ps} \text{ and } 25/\text{ps}\), respectively. The present bound (38) is represented by the first of these circles. This bound has not been used in obtaining the results in Tables 2 and 3. Its impact will be analysed separately below. The circles from \(B^0_d - \bar{B}^0_d\) mixing are not shown explicitly for reasons to be explained below. The impact of \(\Delta M_d\) can however be easily seen by comparing the shaded area with the area one would find by using the lower \(\varepsilon\)-hyperbola and the \(R_0\)-circles alone.

The allowed region has a typical "banana" shape which can be found in many other analyses (33). The size of the banana and its position...
depends on the assumed input parameters and on the error analysis which varies from paper to paper. The results in figs. 4 and 5 correspond to a simple independent scanning of all parameters within one standard deviation.

As seen in fig. 4 our present knowledge of the unitarity triangle is still rather poor. Fig. 5 demonstrates very clearly that this situation may change dramatically in the future provided the errors in the input parameters will be decreased as anticipated in our "future" scenario.

Comparing the results for \(|V_{td}|\) given in table 2 with the ones obtained on the basis of unitarity alone (18) we observe that the inclusion of the constraints from \(\varepsilon\) and \(\Delta M_d\) had a considerable impact on the allowed range for this CKM matrix element. This impact will be amplified in the future as seen in table 3. An inspection shows that with our input parameters the lower bound on \(|V_{td}|\) is governed by \(\varepsilon_K\), whereas the upper bound by \(\Delta M_d\).

Next we observe that whereas the angle \(\beta\) is rather constrained, the uncertainties in \(\alpha\) and \(\gamma\) are huge:

\[
35^\circ \leq \alpha \leq 115^\circ \quad 11^\circ \leq \beta \leq 27^\circ \quad 41^\circ \leq \gamma \leq 134^\circ
\] (39)

The situation will improve when the "future" scenario will be realized:

\[
70^\circ \leq \alpha \leq 93^\circ \quad 19^\circ \leq \beta \leq 22^\circ \quad 65^\circ \leq \gamma \leq 90^\circ
\] (40)

Impact of \(\Delta M_s\)

The impact of the present lower bound on \(\Delta M_s\) is still very small except for the upper limits for \(|V_{td}|/|V_{ts}|\) and \(\gamma\) which are lowered in the "scanning" version to 0.27 and 129° respectively.

| Quantity | Scanning | Gaussian |
|----------|----------|----------|
| \(|V_{td}|/10^{-3}\) | 8.1 – 9.2 | 8.6 ± 0.3 |
| \(|V_{ts}/V_{cb}|\) | 0.969 – 0.983 | 0.976 ± 0.004 |
| \(|V_{td}/V_{ts}|\) | 0.20 – 0.24 | 0.215 ± 0.010 |
| \(\sin(2\beta)\) | 0.61 – 0.70 | 0.67 ± 0.03 |
| \(\sin(2\alpha)\) | -0.11 – -0.66.0 | 0.21 ± 0.21 |
| \(\sin(\gamma)\) | 0.90 – 1.0 | 0.96 ± 0.03 |
| \(\text{Im}\lambda_t/10^{-4}\) | 1.21 – 1.41 | 1.29 ± 0.06 |
Correlation between $\varepsilon_K$ and $\Delta M_d$

Now, why did we omit the explicit circles from $B^0_d - \bar{B}^0_d$ mixing in the plots of unitarity triangles above? I have to answer this question because some of my colleagues suspected that a plot similar to the one in fig. 4 and shown already at the Rochester conference in Warsaw was wrong. At first one would expect that the left border of the allowed area coming from $B^0_d - \bar{B}^0_d$ mixing should have a shape similar to the circles coming from $\Delta M_d/\Delta M_s$ and shown in the figures above. This expectation is correct at fixed values of $m_t$ and $|V_{cb}|$. Yet once these two parameters are varied in the allowed ranges, this is no longer true. In fact one can easily convince oneself that the uncertainties coming from $m_t$ and $|V_{cb}|$ in the analyses of $\varepsilon_K$ and $\Delta M_d$ cannot be represented simultaneously in the $(\bar{\rho}, \bar{\eta})$ plane in terms of nice hyperbolas and nice circles. This is simply related to the correlation between $\varepsilon_K$ and $\Delta M_d$ due to $m_t$ and $|V_{cb}|$. Neglecting this correlation one finds for instance that the most negative value of $\bar{\rho}$ corresponds to the maximal values of $(m_t, |V_{cb}|)$ in the case of $\varepsilon_K$ and to the minimal values of $(m_t, |V_{cb}|)$ in the case of $B^0_d - \bar{B}^0_d$ which is of course inconsistent. In figs. 4 and 5 we have decided to show the $\varepsilon_K$-hyperbolas. Consequently the impact of $B^0_d - \bar{B}^0_d$ mixing had to be found numerically and as seen it is not described by a circle. Since $m_t$ is already very well known,
this discussion mainly applies to the $|V_{cb}|$ dependence. Finally it should be stressed that similar correlations have to be taken into account in the future when various rare decays discussed in the next section will enter the game of the determination of the unitarity triangle. Needless to say, the radius $R^{\text{max}}_t$ determined through $(37)$ and shown in the UT plots, being independent of $(m_t, |V_{cb}|)$, is not subject to the correlation in question.

4 CKM from Rare Decays

4.1 Preliminary Remarks

Let us change the gears and move to rare K- and B-decays. Doing this we actually move into the future, a very interesting one as we will see. Not all rare decays are suitable, at least at present, for precise determinations of the CKM parameters. Several of them suffer from large hadronic uncertainties and only dramatic progress in non-perturbative techniques could change them to useful means for CKM determinations. On the other hand there is a small number of decays with small or even negligible theoretical uncertainties. These decays are particularly suitable for the determination of the CKM matrix. Here the main difficulty are very small branching ratios and the fate of the usefulness
of these decays after the completion of NLO-QCD corrections\textsuperscript{4} lies mainly in the hands of experimentalists.

We will begin our discussion with decays which have large theoretical uncertainties. Subsequently we will gradually move towards cleaner decays, ending our discussion with the \textit{gold plated decay} \( K_L \rightarrow \pi^0 \nu \bar{\nu} \) and its \textit{silver plated sister} \( K^+ \rightarrow \pi^+ \nu \bar{\nu} \).

\subsection{4.2 \( \varepsilon'/\varepsilon \), \( K_L \rightarrow \mu \bar{\mu} \), \( B \rightarrow K^+(q)\gamma \) and \( K_L \rightarrow \pi^0 e^+ e^- \)}

\( \varepsilon'/\varepsilon \) suffers from large uncertainties in the relevant hadronic matrix elements of QCD-penguin and electroweak penguin operators\textsuperscript{10}. The situation is further complicated by the strong cancellations between these contributions. This precludes a useful extraction of \( \text{Im} \lambda_t \) from future data unless some dramatic progress in non-perturbative calculations will take place. In spite of this a measurement of \( \varepsilon'/\varepsilon \) at the \( 10^{-4} \) level remains as one of the important targets of contemporary particle physics. A non-vanishing value of this ratio would give the first signal for the direct CP violation ruling out the superweak models\textsuperscript{38}. The clarification of this important issue should come soon from FNAL, CERN and at a later stage from DAΦNE.

The branching ratio for the decay \( K_L \rightarrow \mu \bar{\mu} \) has been already measured\textsuperscript{39}. Unfortunately, it is dominated by long distance contributions. The extraction of the well calculable short distance contribution (sensitive to \( \bar{\phi} \)) from these data remains a very important challenge. A recent discussion of this issue and the relevant references can be found in\textsuperscript{40}.

The exclusive decays \( B \rightarrow K^+(q)\gamma \) are known experimentally from CLEO measurements. Taken together they provide an upper bound on \( |V_{td}| \), which is, however, very weak. The strong model dependence in the relevant formfactors precludes a useful determination of \( |V_{td}| \) from these decays. This is at least my opinion. There are others who are more optimistic in this respect. Clearly a considerable progress in lattice calculations could improve this situation.

The decay \( K_L \rightarrow \pi^0 e^+ e^- \) receives three contributions: CP conserving, \textit{indirectly} CP violating and \textit{directly} CP violating. The directly CP violating part can be calculated very reliably: \( Br(K_L \rightarrow \pi^0 e^+ e^-)_{\text{dir}} = (4.5 \pm 2.6) \cdot 10^{-12} \), where the error comes dominantly from the uncertainties in the CKM parameters. Extracting this part from future data would give a clean determination of \( \text{Im} \lambda_t \). Unfortunately the other two components, although likely smaller, are not fully negligible. A better assessment of the importance of the indirect CP violation in \( K_L \rightarrow \pi^0 e^+ e^- \) will become possible after the measurement of \( Br(K_S \rightarrow \pi^0 e^+ e^-) \) at DAΦNE. On the other hand the prospects of the accurate estimate of the CP conserving part are less clear. The present ex-
perimentional bound: $Br(K_L \to \pi^0 e^+ e^-) < 4.3 \cdot 10^{-9}$ should be considerably improved in the coming years at FNAL. More details on this interesting decay can be found in [42, 44].

4.3 $B \to X_{d,s} \gamma$ and $B \to X_{d,s} l^+ l^-$

These decays are covered by Ali [43] and Hewett in these proceedings. Let me still make a few comments on them.

The decay $B \to X_s \gamma$ has been a subject of intensive efforts by experimentalists and theorists during the last 5-10 years. Experimentally its branching ratio is found to be:

$$Br(B \to X_s \gamma) = (2.32 \pm 0.57 \pm 0.35) \times 10^{-4}, \quad (41)$$

and a very preliminary result from ALEPH reads $(3.38 \pm 0.74 \pm 0.85) \times 10^{-4}$. Here the first errors are statistical and the second systematic. On the other hand the Standard Model NLO analyses [45] give

$$Br(B \to X_s \gamma) = (3.48 \pm 0.13 \text{ (scale)} \pm 0.28 \text{ (par)}) \times 10^{-4} = (3.48 \pm 0.31) \times 10^{-4} \quad (42)$$

where this particular estimate comes from the paper by Kwiatkowski, Pott and myself. We show separately scale and parametric uncertainties.

The error in the theoretical estimate is dominated by parametric uncertainties of which the quark masses and the used branching ratio $Br(B \to X_s e \bar{\nu}_e)$ are most important ones. I believe that these parametric uncertainties will be reduced in the future by at least a factor of two, so that a prediction for $Br(B \to X_s \gamma)$ with an uncertainty of 5 – 10% will be available within the next, say, five years. Provided the experimental branching ratio can be measured with a similar accuracy this would allow a 5% measurement of the ratio $|V_{ts}/V_{cb}|$. Even if such a measurement would not improve our knowledge of this ratio within the Standard Model, its considerable departure from 0.975 would signal the physics beyond the Standard Model. To achieve the accuracy of 5% in the experimental branching ratio is of course a very difficult but not a fully unrealistic task. From the existing data Ali extracts $|V_{ts}| = 0.033 \pm 0.007$.

The issue of $B \to X_d \gamma$, sensitive to $|V_{td}|$, is a different story. It is much harder to measure than $B \to X_s \gamma$ and the analysis is not as simple as in the latter case because the CKM non-leading contributions, which are negligible in $B \to X_s \gamma$, have to be considered now [46]. Even if there are different opinions about this, many would agree with me that this decay cannot compete with the measurement of $|V_{td}|$ through $K^+ \to \pi^+ \nu \bar{\nu}$ or even through $B^0_d - \bar{B}_d$ mixing (provided $\sqrt{B_d F_{B_d}}$ will be improved). Yet the efforts should be made
to measure it because as in the case of $B \to X_s \gamma$ its branching ratio is sensitive to the physics beyond the Standard Model.

Similar comments apply to $B \to X_s, d, l^{-} l^{+}$ decays except that here one is faced with an additional obstacle: the long distance contributions, due mainly to $J/\psi$ and $\psi'$ resonances. An interesting new study of this issue can be found in $^{46}$. These difficulties can be avoided to some extent by studying special regions of the invariant dilepton mass spectrum or asymmetries of various sorts. These issues are discussed by Ali and Hewett in these proceedings. The extraction of the CKM parameters from these decays has been discussed recently in $^{47}$.

4.4 $B_{d,s} \to \mu \bar{\mu}$ and $B \to X_{d,s} \nu \bar{\nu}$

$B_{d,s} \to \mu \bar{\mu}$ and $B \to X_{d,s} \nu \bar{\nu}$ are the theoretically cleanest decays in the field of rare $B$-decays. They are dominated by the $Z^0$-penguin and box diagrams involving top quark exchanges. After the calculation of NLO-QCD corrections $^{48}$, the scale uncertainties in these decays are negligible. The same applies to long distance contributions $^{46}$. One has then:

$$Br(B_s \to \mu \bar{\mu}) = 3.5 \cdot 10^{-9} \left[ \frac{F_{B_s}}{210 \text{ MeV}} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{3.12} \left[ \frac{|V_{ts}|}{0.040} \right]^2 \left[ \frac{\tau_{B_s}}{1.6 \text{ ps}} \right]$$

(43)

and

$$Br(B \to X_s \nu \bar{\nu}) = 3.7 \cdot 10^{-9} \left[ \frac{|V_{ts}|}{|V_{cb}|} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{2.3} \left[ \frac{0.54}{f(z)} \right] \left[ \frac{Br(B \to X_c e\bar{\nu}_e)}{0.104} \right]$$

(44)

where $f(z)$ is a phase space factor with $z = m_c/m_b$.

The short distance character of these decays allows a clean determination of $|V_{td}|/|V_{ts}|$ by measuring the ratios $Br(B_d \to \mu \bar{\mu})/Br(B_s \to \mu \bar{\mu})$ and $Br(B \to X_{d,s} \nu \bar{\nu})/Br(B \to X_s \nu \bar{\nu})$. In particular the latter determination is very clean as the uncertainties related to $Br(B \to X_c e\bar{\nu}_e)$ and $f(z)$ cancel in the ratio. The corresponding determination using $B_{d,s} \to \mu \bar{\mu}$ suffers from the uncertainty in the ratio $F_{B_d}/F_{B_s}$ which however should be removed to a large extent by future lattice calculations.

Experimentally there exists an indirect upper bound (90% C.L.): $Br(B \to X_s \nu \bar{\nu}) < 7.7 \cdot 10^{-4}$ obtained by ALEPH in 1996. Yet it is fair to say that the actual measurements of this branching ratio and in particular of $Br(B \to X_{d,s} \nu \bar{\nu})$ are extremely difficult. The measurement of $B_{s,d} \to \mu \bar{\mu}$ is first of all difficult because of the expected tiny branching ratio. Still one should hope that our experimental colleagues and those who provide financial support will
4.5 $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$

$K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ are the theoretically cleanest decays in the field of rare K-decays. $K_L \to \pi^0 \nu \bar{\nu}$ is dominated by Z-penguins and box diagrams involving the top quark. $K^+ \to \pi^+ \nu \bar{\nu}$ receives additional sizable contributions from internal charm exchanges. The great virtue of $K_L \to \pi^0 \nu \bar{\nu}$ is that it proceeds almost exclusively through direct CP violation and as such is the cleanest decay to measure this important phenomenon. It also offers a clean determination of the Wolfenstein parameter $\eta$ and in particular as we will stress below offers the cleanest measurement of Im$\lambda_t = \text{Im}V_{ts}^*V_{td}$. $K^+ \to \pi^+ \nu \bar{\nu}$ is CP conserving and offers a clean determination of $|V_{td}|$. Due to the presence of the charm contribution it has a non-negligible scale uncertainty and an uncertainty in $m_c$ both absent in $K_L \to \pi^0 \nu \bar{\nu}$.

After the NLO-QCD calculations, the remaining scale uncertainty in the extraction of $|V_{td}|$ from $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ amounts to $\pm 4\%$. The remaining scale uncertainty in the determination of $\eta$ or Im$\lambda_t$ from $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ is below $\pm 1\%$ which is safely negligible. Since the relevant hadronic matrix elements of the weak currents entering $K \to \pi \nu \bar{\nu}$ can be related using isospin symmetry to the leading decay $K^+ \to \pi^0 e^+ \nu$, the resulting theoretical expressions for $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ and $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ do not contain any hadronic uncertainties. The isospin braking corrections have been calculated in $\cite{52}$ and found to be safely negligible. The long distance contributions to $K_L \to \pi^0 \nu \bar{\nu}$ are negligible as well. Finally the two-loop ($O(G_F^2m_t^4)$ at the amplitude level) electroweak corrections to $K_L \to \pi^0 \nu \bar{\nu}$ have been computed and found to be about 2% which is well below the experimental sensitivity in the foreseeable future.

The explicit expressions for $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ and $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ can be found in $\cite{6,10}$. In particular:

$$\text{Br}(K_L \to \pi^0 \nu \bar{\nu}) = 3.0 \cdot 10^{-11} \left[ \frac{\eta}{0.39} \right]^2 \left[ \frac{m_t(m_t)}{170 \text{ GeV}} \right]^{2.3} \left[ \frac{|V_{cb}|}{0.040} \right]^4.$$  \hspace{1cm} (45)

Scanning the "present" input parameters of table 1 one finds $\cite{36}$:

$$\text{Br}(K^+ \to \pi^+ \nu \bar{\nu}) = (9.1 \pm 3.8) \cdot 10^{-11}, \quad \text{Br}(K_L \to \pi^0 \nu \bar{\nu}) = (2.8 \pm 1.7) \cdot 10^{-11}.$$  \hspace{1cm} (46)
where the errors are dominated by the present uncertainties in $|V_{cb}|$, $|V_{td}|$ and $\eta$. These errors are decreased by roughly a factor of three if the "future" input parameters are used.

One of the highlights of August 97 was the observation by BNL787\footnote{BNL787} of one event consistent with the signature expected for this decay. The branching ratio:

$$Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (4.2 + 9.7 - 3.5) \cdot 10^{-10}$$

has the central value by a factor of 4 above the Standard Model expectation but in view of large errors the result is compatible with the Standard Model. This new result implies that $|V_{td}|$ lies in the range $0.006 < |V_{td}| < 0.06$ which is substantially larger than the range from the standard analysis of section 3. The analysis of additional data on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ present on tape at BNL787 should narrow this range in the near future considerably. It is also hoped that FNAL will contribute to the measurement of this branching ratio\footnote{FNAL}.

The most recent upper bound on $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ from FNAL-E799 is $1.8 \cdot 10^{-6}$. This experiment expects to reach the accuracy $O(10^{-8})$ and a very interesting new proposal AGS2000\footnote{AGS2000} expects to reach the single event sensitivity $2 \cdot 10^{-12}$ allowing a 10% measurement of the expected branching ratio. It is hoped that also JNAF, FNAL and KEK will make efforts to measure this gold-plated decay.

\textbf{4.6 $|V_{td}|$, Im$\lambda_t$, sin$2\beta$ and UT from $K \rightarrow \pi \nu \bar{\nu}$}

|$V_{td}$| from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Once $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \equiv Br(K^+)$ is measured, $|V_{td}|$ can be extracted subject to various uncertainties:

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = \pm 0.04_{\text{scale}} \pm \frac{\sigma(|V_{cb}|)}{|V_{cb}|} \pm 0.7 \frac{\sigma(\bar{m}_c)}{\bar{m}_c} \pm 0.65 \frac{\sigma(Br(K^+))}{Br(K^+)}$$  \hspace{1cm} (48)

Taking $\sigma(|V_{cb}|) = 0.002$, $\sigma(\bar{m}_c) = 100$ MeV and $\sigma(Br(K^+)) = 10\%$ and adding the errors in quadrature we find that $|V_{td}|$ can be determined with an accuracy of $\pm 10\%$ in the present example. This number is increased to $\pm 11\%$ once the uncertainties due to $m_t$, $\alpha_s$ and $|V_{ub}|/|V_{cb}|$ are taken into account. Clearly this measurement can be improved although a determination of $|V_{td}|$ with an accuracy better than $\pm 5\%$ seems rather unrealistic.
Im$\lambda_t$ and $|V_{cb}|$ from $K_L \to \pi^0 \nu \bar{\nu}$

The basic formulae here are (49):

$$\text{Im} \lambda_t = 1.36 \cdot 10^{-4} \left( \frac{170 \text{ GeV}}{m_t(m_t)} \right)^{1.15} \left[ \frac{Br(K_L \to \pi^0 \nu \bar{\nu})}{3 \cdot 10^{-11}} \right]^{1/2}.$$  \hspace{1cm} (49)

and

$$|V_{cb}| = 40.0 \cdot 10^{-3} \sqrt{\frac{0.39}{\eta}} \left( \frac{170 \text{ GeV}}{m_t(m_t)} \right)^{0.575} \left[ \frac{Br(K_L \to \pi^0 \nu \bar{\nu})}{3 \cdot 10^{-11}} \right]^{1/4}.$$  \hspace{1cm} (50)

(49) offers the cleanest method to measure Im$\lambda_t$; even better than the CP asymmetries in $B$ decays discussed in the following section. On the other hand (50) determines $|V_{cb}|$ without any hadronic uncertainties.\[\text{With } \eta \text{ determined one day from CP asymmetries in B-decays and } m_t \text{ measured very precisely at LHC and NLC, a measurement of } Br(K_L \to \pi^0 \nu \bar{\nu}) \text{ with an accuracy of 10% would determine } |V_{cb}| \text{ with an error of } \pm 0.001. \] A comparison of this determination of $|V_{cb}|$ with the usual one in tree level B-decays would offer an excellent test of the Standard Model and in the case of discrepancy would signal physics beyond it.

![Figure 6: Unitarity triangle from $K \to \pi \nu \bar{\nu}$](image)

**Unitarity Triangle and $\sin 2 \beta$**

The measurements of $Br(K^+ \to \pi^+ \nu \bar{\nu})$ and $Br(K_L \to \pi^0 \nu \bar{\nu})$ can determine the unitarity triangle completely, (see fig. 6), provided $m_t$ and $V_{cb}$ are known.\[\text{Of particular interest is the determination of } \sin 2 \beta \text{ which is independent of } |V_{cb}| \text{ and } m_t:\]

$$\sin 2 \beta = \frac{2r_s}{1 + r_s^2} \quad r_s = \frac{\sqrt{B_1 - B_2 - P_c}}{\sqrt{B_2}}.$$ \hspace{1cm} (51)
Table 4: Illustrative example of the determination of CKM parameters from $K \rightarrow \pi \nu \bar{\nu}$ and from the standard analysis of the unitarity triangle.

| Parameter          | Present          | Future          |
|--------------------|------------------|-----------------|
| $\sigma(|V_{cb}|)$ | $\pm 0.002$      | $\pm 0.001$     |
| $\sigma(\bar{\rho})$ | $\pm 0.16$      | $\pm 0.12$     |
| $\sigma(\bar{\eta})$ | $\pm 0.04$      | $\pm 0.03$     |
| $\sigma(\sin 2\beta)$ | $\pm 0.05$     | $\pm 0.05$     |
| $\sigma(\text{Im} \lambda_t)$ | $\pm 5\%$     | $\pm 5\%$     |

where

$$B_1 = \frac{Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{4.11 \cdot 10^{-11}}, \quad B_2 = \frac{Br(K_L \rightarrow \pi^0 \nu \bar{\nu})}{1.80 \cdot 10^{-10}}.$$  \hspace{1cm} (52)$$

and $P_c = 0.40 \pm 0.06$ represents the charm contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$.

An extensive numerical analysis of the unitarity triangle from $K \rightarrow \pi \nu \bar{\nu}$ can be found in [58,56]. Here we only briefly compare this determination with the one by means of the standard analysis of the unitarity triangle. We assume that $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ and $Br(K_L \rightarrow \pi^0 \nu \bar{\nu})$ are known to within $\pm 10\%$, $\sigma(m_c) = \pm 50\text{ MeV}$ and $\sigma(m_t) = \pm 3\text{ GeV}$. Then for two choices of the uncertainty in $|V_{cb}|$ one finds the results given in the second and the third column of table 4. In the fourth and fifth column the corresponding results of the standard analysis of the unitarity triangle are shown. We observe that very respectable determinations of all considered quantities except for $\bar{\rho}$ can be obtained from $K \rightarrow \pi \nu \bar{\nu}$. However even in this case there is a considerable progress when compared with the present analysis of the unitarity triangle. Of particular interest are the accurate determinations of $\sin 2\beta$ and $\text{Im} \lambda_t$. The comparison of this determination of $\sin 2\beta$ with the one through the CP asymmetry in $B_d \rightarrow \psi K_S$ is particularly suited for tests of CP violation in the Standard Model and offers a powerful tool to probe the physics beyond it. The discussion of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ beyond the Standard Model can be found in [59,5].

5 \hspace{1cm} $\alpha$, $\beta$ and $\gamma$ from two-body B-Decays

5.1 \hspace{1cm} CP-Asymmetries in B-Decays

CP violation in B decays is certainly one of the most important targets of B factories and of dedicated B experiments at hadron facilities. It is well known that CP violating effects are expected to occur in a large number of channels at a level attainable at forthcoming experiments. Moreover there exist channels
which offer the determination of CKM phases essentially without any hadronic uncertainties. Since CP violation in B decays has been already reviewed in two special talks by Fleischer and Sanda at this symposium and since extensive reviews can be found in the literature 60, 61, 10, let me concentrate only on the most important points.

The classic determination of $\alpha$ by means of the time dependent CP asymmetry in the decay $B_d^0 \rightarrow \pi^+\pi^-$ is affected by the "QCD penguin pollution" which has to be taken care of in order to extract $\alpha$. The recent CLEO results for penguin dominated decays indicate that this pollution could be substantial. The most popular strategy to deal with this "penguin problem" is the isospin analysis of Gronau and London 62. It requires however the measurement of $Br(B^0_d \rightarrow \pi^0\pi^0)$ which is expected to be below $10^{-6}$: a very difficult experimental task. For this reason several, rather involved, strategies have been proposed which avoid the use of $B_d \rightarrow \pi^0\pi^0$ in conjunction with $a_{CP}(\pi^+\pi^-,t)$. They are reviewed in 10. It is to be seen which of these methods will eventually allow us to measure $\alpha$ with a respectable precision. It is however clear that the determination of this angle is a real challenge for both theorists and experimentalists.

The CP-asymmetry in the decay $B_d \rightarrow \psi K_S$ allows in the Standard Model a direct measurement of the angle $\beta$ in the unitarity triangle without any theoretical uncertainties 64. Of considerable interest 61, 65 is also the pure penguin decay $B_d \rightarrow \phi K_S$, which is expected to be sensitive to physics beyond the Standard Model. Comparison of $\beta$ extracted from $B_d \rightarrow \phi K_S$ with the one from $B_d \rightarrow \psi K_S$ should be important in this respect.

The two theoretically cleanest methods for the determination of $\gamma$ are: i) the full time dependent analysis of $B_s \rightarrow D_s^+ K^-$ and $\bar{B}_s \rightarrow \bar{D}_s^- K^+$ and ii) the known triangle construction due to Gronau and Wyler 67 which uses six decay rates $B^\pm \rightarrow D_{CP}^0 K^\pm$, $B^+ \rightarrow D^0 K^+$, $\bar{D}^0 K^+$ and $B^- \rightarrow D^0 K^-$, $\bar{D}^0 K^-$. Both methods are unaffected by penguin contributions. The first method is experimentally very challenging because of the expected large $B_s^0 - \bar{B}_s^0$ mixing. The second method is problematic because of the small branching ratios of the colour supressed channels $B^\pm \rightarrow D^0 K^\pm$ giving a rather squashed triangle and thereby making the extraction of $\gamma$ difficult. Variants of the latter method which could be more promising have been proposed in 68, 69. It appears that these methods will give useful results at later stages of CP-B investigations. In particular the first method will be feasible only at LHC-B.

For the time being the most promising for the extraction of $\gamma$ appears to be the method of Fleischer 70 and of Fleischer and Mannel 71 which uses the...
rates for $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$. This method implies the bound:

$$\sin^2 \gamma \leq \frac{Br(B_d \to \pi^\mp K^\pm)}{Br(B^\pm \to \pi^\mp K)} \equiv R$$ \hspace{1cm} (53)

The Fleischer-Mannel bound is of particular interest because the most recent CLEO data give $R = 0.65 \pm 0.40$. The firm conclusion cannot be reached at present because of substantial experimental error. However if improved data will continue to give $R < 1$, the FR bound would exclude the region around $\bar{\rho} = 0$ in the $(\bar{\rho}, \bar{\eta})$ space putting the "$\gamma = 90^\circ$ club" into serious difficulties.

It should be stressed that excluding the region around $\bar{\rho} = 0$ would have a profound impact on the unitarity triangle dividing the allowed region for its apex into well separated regions with $\bar{\rho} < 0$ and $\bar{\rho} > 0$. The former could then probably be eliminated by improving the lower bound on $\Delta M_s$ leaving only a small allowed area with $\bar{\rho} > 0$. The crucial question then is, whether $R$ is indeed smaller than unity? Hopefully CLEO will answer this question in the coming years. More details on the implications of the FR bound can be found in [71, 74, 75].

5.2 UT from CP-B and $K \to \pi \nu \bar{\nu}$

In what follows let us assume that $\alpha$, $\beta$ and $\gamma$ have been measured to certain accuracy and let us ask what such measurement will imply for $|V_{td}|$, $\bar{\rho}$, $\bar{\eta}$ and $\text{Im} \lambda_t$. To this end let us assume:

$$\sin 2\alpha = 0.40 \pm 0.10 \hspace{1cm} \sin 2\beta = 0.70 \pm 0.06 \hspace{1cm} (\text{scenario I}) \hspace{1cm} (54)$$

$$\sin 2\alpha = 0.40 \pm 0.04 \hspace{1cm} \sin 2\beta = 0.70 \pm 0.02 \hspace{1cm} (\text{scenario II}) \hspace{1cm} (55)$$

Scenario I corresponds to the accuracy being aimed for at $B$-factories, HERA-B and FNAL prior to the LHC era. An improved precision can be anticipated from LHC experiments, which we illustrate with our choice of scenario II. The assumed accuracy on $\alpha$ in the scenario II is probably unrealistic in view of the comments made above but let us try it anyway. In general the calculation of $\bar{\rho}$ and $\bar{\eta}$ from $\sin 2\alpha$ and $\sin 2\beta$ involves discrete ambiguities. We will assume that they can be removed by looking at other decays.

In table 5 we compare this way of determination of CKM parameters with the one achieved through $K \to \pi \nu \bar{\nu}$ and presented already in table 4. We set $|V_{cb}| = 0.040 \pm 0.002(0.001)$. As can be seen in Table 5, the CKM determination using $K \to \pi \nu \bar{\nu}$ is competitive with the one based on CP violation in $B$ decays in scenario I, except for $\bar{\rho}$ which is less constrained by the rare kaon processes. On the other hand as advertised previously $\text{Im} \lambda_t$ is better
Table 5: Illustrative example of the determination of CKM parameters from $K \to \pi \nu \bar{\nu}$ and B-decays.

|                  | $K \to \pi \nu \bar{\nu}$ | Scenario I       | Scenario II       |
|------------------|-----------------------------|------------------|------------------|
| $\sigma(|V_{td}|)$ | ±10%(9%)                    | ±5.5%(3.5%)      | ±5.0%(2.5%)      |
| $\sigma(\hat{q})$   | ±0.16(0.12)                 | ±0.03            | ±0.01            |
| $\sigma(\hat{\eta})$ | ±0.04(0.03)                | ±0.04            | ±0.01            |
| $\sigma(\sin 2\beta)$ | ±0.05                      | ±0.06            | ±0.02            |
| $\sigma(\text{Im} \lambda_t)$ | ±5%                        | ±14%(11%)        | ±10%(6%)         |

determined in $K \to \pi \nu \bar{\nu}$ even if scenario II is considered. The virtue of the comparison of the determinations of various parameters using CP-B asymmetries with the determinations in very clean decays $K \to \pi \nu \bar{\nu}$ is that any substantial deviations from these two determinations would signal new physics beyond the Standard Model.

6 Final Messages

The reduction of various scale uncertainties through the calculation of short distance NLO-QCD corrections to a large class of weak decays and a progress in non-perturbative methods like HQET, HQE, lattice calculations, chiral perturbation theory, 1/N approach and QCD sum rules of various sorts, decreased considerably theoretical uncertainties in the determination of the CKM matrix in the present and future experiments. In spite of this, further progress in non-perturbative methods is clearly desirable.

The improved data for semi-leptonic tree level B-decays combined with HQE and HQET allowed considerable improvement in the determination of the elements $|V_{cb}|$ and $|V_{ub}|$:

$$
\sigma(|V_{cb}|)_{97} \approx \frac{1}{3}\sigma(|V_{cb}|)_{90}
$$

At present these elements are known with an accuracy of ±7% and ±25% respectively but the hope that they will be measured one day in tree level decays with an accuracy of ±4% is not fully unrealistic.

The discovery of the top quark together with the theoretical progress mentioned above provided an improved determination of $|V_{td}|$. Yet the error on this element is still sizable: roughly ±25%.

Standard analysis of the unitarity triangle can give very useful results provided the experimental knowledge of $\Delta M_s$, $|V_{ub}/V_{cb}|$ and $|V_{cb}|$ and the
theoretical knowledge of $\sqrt{B_B F_B}$, $\xi$ and $B_K$ can be considerably improved. A numerical analysis of this issue has been presented in section 3.

Useful information about $|V_{ts}|$ and $|V_{td}|$ at the level of $10 - 15\%$ accuracy should come from $B \to X_{d,s} \gamma$ and $B \to X_{d,s} l^+ l^-$. Moreover these decays can be efficiently used to study the physics beyond the Standard Model.

$K^+ \to \pi^+ \nu \bar{\nu}$ should offer a measurement of $|V_{td}|$ with an accuracy of $5 - 10\%$ provided its branching ratio can be measured with an accuracy of at least $10\%$. $K_L \to \pi^0 \nu \bar{\nu}$ appears to be the cleanest decay to study direct CP violation and to measure $\text{Im}\lambda_t$. With the newly proposed experiment at BNL the present error on $\text{Im}\lambda_t$ of roughly $30\%$ should be reduced to $5\%$ which would be an important progress. Both decays taken simultaneously can offer a very clean measurement of $\sin 2\beta$ with an error of $\pm 0.05$.

The cleanest measurement of the ratio $|V_{td}/V_{ts}|$ can be obtained from $B \to X_{d,s} \nu \bar{\nu}$ and to a lesser extent from $B \to X_{d,s} l^+ l^-$. However, both decays are extremely challenging for different reasons from experimental point of view.

Clearly the future measurements of CP violation in two-body B-decays at SLAC, KEK, Cornell, HERA-B, FNAL and LHC will open a new chapter in the physics of weak decays and particle physics in general irrespective of whether $\alpha$, $\beta$ and $\gamma$ can be measured very precisely. A very accurate measurement of $\beta$ in $B \to \psi K_S$ and in $B \to \phi K_S$ appears to be very realistic. Precise measurements of $\alpha$ and $\gamma$ seem to be much harder. Yet one should emphasize the obvious fact that $\alpha$ and/or $\gamma$ have to be well measured in addition to $\beta$ in order to construct the unitarity triangle on the basis of two-body B-decays alone. In the next few years the only results here will come from CLEO.

The most interesting will be the comparison of CKM determinations from CP-B, $K \to \pi \nu \bar{\nu}$, rare B-decays and the standard analysis of the unitarity triangle as expressed in the plot of fig. 2. Such a comparison should give us some hints about the physics beyond the Standard Model. The possibility of having such a comparison within the next 5 - 10 years is very exciting.

The physics around the CKM matrix has a very bright future at least for the next 5-10 years irrespective of whether the KM scenario for CP violation will be confirmed or proved false in the coming experiments. By making continuous experimental and theoretical efforts in this field we will either achieve a high accuracy for the CKM parameters within the Standard Model or find some hints how to generalize it.

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