Heat transfer in PCM based closed radiator heated from a source of variable power

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Abstract. In the present study, the numerical problem of heat and mass transfer in a closed copper radiator filled with n-octadecane heated from an energy source with an unsteady volumetric heat generation was considered. Governing equations in dimensionless non-primitive variables “stream function – vorticity – temperature” describing conductive and convective heat and mass transfer processes taking into account the melting phenomenon were given. The evolution of temperature fields and the integral heat exchange coefficient at different heat release regimes was considered.

1. Introduction
Phase change materials are widely used in modern applications. One of the developing areas is the use of materials in the cooling of electronic equipment. The use of PCMs in the conditions of extreme heat fluxes or periodic heat generation modes allows reducing temperature peaks and improving the operating conditions of the device [1]. Materials such as paraffins have high latent heat, and a suitable melting point range. PCM embedded in the heat sink increases its performance when the melting point is exceeded. To increase the thermal conductivity inside the PCM heat sink, various heat transfer enhancers are used, such as metal foams, nano-sized particles or finning [2,3].

Nowadays, some experimental studies of PCM behavior in different regions were performed. Thus, an experimental analysis of heat transfer in PCM heat sink with external fins was carried out in [4]. To enhance the thermal conductivity inside the volume filled with PCM, graphite nanoplatelets were used. The use of nanoparticles significantly improves the performance of heat sink: thermal conductivity of the material increases by 537% with the addition of 15% wt nanoplatelets, the time of heating to 55°C increases from 33 minutes to 58 minutes. In [5], the effect of horizontal fins on temperature distributions in an aluminum case filled with PCM was considered. It was shown that natural convection plays a significant role in heat transfer processes. Changing the profile design or material properties affects the performance of the entire system, so the study of melting processes in radiators is a topical problem of heat and mass transfer and it requires detailed numerical analysis.

2. Physical and mathematical model
This paper focuses on numerical study of the heat and mass transfer in a closed radiator filled with phase change material under the influence of element with time-dependent volumetric heat generation. The copper profile with internal edges and aspect ratio \( L/H = 2 \) was located above the rectangular silicon heater. The lateral and upper boundaries were cooled by natural convection at a constant ambient temperature. At the moment of source switching at \( t = 0 \), its temperature and temperature of the entire system coincided with the ambient temperature. The melt motion inside the radiator was
considered as a laminar motion of a viscous Newtonian fluid, satisfying the Boussinesq approximation.

![Considered cavity.](image)

The equations describing the motion of the melt in the primitive variables "velocity – pressure – temperature" are of the form:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}
\]

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\mu}{\rho} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right). \tag{2}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \frac{\mu}{\rho} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho \beta (T - T_m). \tag{3}
\]

The energy equations for paraffin were initially written in the enthalpy formulation for liquid (4) and solid phases (5):

\[
\frac{\partial h}{\partial t} + u \frac{\partial h}{\partial x} + v \frac{\partial h}{\partial y} = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{4}
\]

\[
\frac{\partial h}{\partial t} = k_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \tag{5}
\]

The equations of heat conduction were solved inside the solid bodies of the profile and energy source. To describe the unsteady heat generation mode, the time dependent function was used \( Q(t) = Q_0 (1 - \sin(t/f)) \):

\[
\frac{\partial h}{\partial t} = k_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) \tag{6}
\]

\[
(\rho c)_s \frac{\partial T}{\partial t} = k_s \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q(t) \tag{7}
\]
Equations (1) - (7) are solved in dimensionless non-primitive variables: temperature $\Theta$, stream function $\Psi$ ($U = \partial \Psi / \partial Y$, $V = -\partial \Psi / \partial X$), and vorticity $\Omega$ ($\Omega = \partial V / \partial X - \partial U / \partial Y$) ($\Psi = \psi / (V_0 H)$, $\Omega = \omega H / V_0$):

$$\frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\Omega,$$

$$\frac{\partial \Omega}{\partial \tau} + U \frac{\partial \Omega}{\partial X} + V \frac{\partial \Omega}{\partial Y} = \frac{\mu_m}{\mu_f} \frac{\rho_m}{\rho_f} \sqrt{Pr} \nabla^2 \Omega + \frac{(\rho \beta_{\text{m}} / (\rho \beta_{\text{f}}))}{Pr} \frac{\partial \Theta}{\partial X}$$

$$\zeta(\varphi)[\frac{\partial \Theta}{\partial \tau} + U \frac{\partial \Theta}{\partial X} + V \frac{\partial \Theta}{\partial Y}] + \frac{\rho_m \ell_{\text{m}} \text{Ste}}{\rho \ell_f} \left[ \frac{\partial \varphi}{\partial \tau} + U \frac{\partial \varphi}{\partial X} + V \frac{\partial \varphi}{\partial Y} \right] + \frac{\xi(\varphi)}{\sqrt{Ra \cdot Pr}} \nabla^2 \Theta.$$
The obtained differential equations (8) – (12) were solved using the finite difference method on a uniform orthogonal mesh of $481 \times 201$ with a time step of 0.0007. The algorithm and testing on experimental data are presented in more detail in [6, 7]. It should be noted that the hydrodynamic equations were solved in the region of the melt, the boundaries of which were determined as the isotherm $\Theta = \eta$. The transition from the enthalpy formulation to the temperature formulation made it possible to solve the unified energy equation in PCM without highlighting the phase transition boundary.

As a result of the solution, the distributions of hydrodynamic characteristics and temperatures throughout the system at fixed points in time, as well as integral parameters within the system, which characterize the intensity of heat transfer, were obtained. The solutions for the following dimensionless parameters were obtained: the Prandtl number $Pr = 48.36 \ (Pr = \nu \rho c_p / \lambda)$, Rayleigh number $Ra = 4.03 \cdot 10^6 \ (Ra = g \beta \Delta T^3 / (\nu \alpha))$, Stefan number $Ste = 1.84 \ (Ste = L_f / (\alpha c))$, Ostrogradsky number $Os = 0.845 \ (Os = qH^2 / (\lambda \Delta T))$ and Biot number $Bi = 1.27 \ (Bi = \alpha H / \lambda)$. The maximum power source is determined by the dimensionless parameter Ostrogradsky number. In this study, the maximum power of the source was set $Q = 10 \ W/cm$, at the temperature and linear scales $T = 60 ^\circ C$ and $H = 1.5 \ cm$. The parameter $f$ was set to 0 (constant heat generation), 50 and 200. The influence of the frequency of the source power change on the convective melting conditions in paraffin and the source temperature was analyzed.

3. Results and discussions

As the PCM, n-octadecane with the following properties was considered: $T_m = 28.05 ^\circ C$, $L_f = 2.41 \cdot 10^5 \ J/kg$, $\lambda_t = 0.157 \ W/(m \cdot K)$, $\rho_t = 775 \ kg/m^3$, $\rho_t = 814 \ kg/m^3$, $\beta = 8.5 \cdot 10^{-4} \ K^{-1}$, $c_t = 2200 \ J/(kg \cdot K)$, $c_s = 1900 \ J/(kg \cdot K)$. For copper and silicon, the following values of thermal conductivity and thermal diffusivity were used: $\lambda_1 = 401 \ W/(m \cdot K)$, $\alpha_1 = 1.17 \cdot 10^{-4} \ m^2/s$, $\lambda_2 = 148 \ W/(m \cdot K)$, $\alpha_2 = 8.8 \cdot 10^{-5} \ m^2/s$.

As already noted, an essential role in the heat transfer inside the molten paraffin is played by thermogravitational convection. Due to the low thermal conductivity of paraffins, convective flows determine the intensity of melting and the movement of the interface with large volumes of melt. Figure 2 shows the isotherms and isolines of the stream function for $f = 50$ and $f = 200$. The thermal conductivity of the source material and the profile is much higher than that of paraffin, so throughout the melting process a rapid heating of the entire radiator is seen; while in the PCM in the near-wall zones a thickening of the isotherms is observed. At the time moment $\tau = 277.12$, for both values of the parameter $f$, the power of the source is close to the maximum value. At the initial stage of melting, uniform melting is observed due to the heat-conducting mode, but an increase in the melt region leads to the appearance of convective cells in the entire height of the radiator compartments. With the development of the melt region, heated streams ascending along the radiator fins facilitate the heating of the upper layers of the PCM. The descending flows, falling along the central part of the compartments, carry cold melt, as a result of which unmelted material remains in the center of the compartments. With the complete melting of the material, the temperature stratification of the liquid is observed, which also occurs due to two symmetric convective vortices. As a result, in the lower part of the region there is a colder melt, which contributes to the cooling of the energy source. For $f = 200$ at the time moment $\tau = 692.8$, a power close to maximum is characteristic. It is seen that the paraffin temperature is lower than the radiator temperature, in contrast to the case $f = 50$, where the source power decreased to an average value. It can be seen that paraffin is heated more intensively at this phase.

Figure 3 shows the change in the integral Nusselt number on the inner surface of the radiator with time. It should be noted that with such high loads, paraffin melts quite quickly and when establishing a periodic mode, natural convection without phase transformations is considered. The maximum values of heat transfer are observed during the melting period, when the convective mode of heat transfer is established inside the region. Depending on the period of change in power, the amplitude of $Nu$
oscillations varies. The maximum deviations of the heat transfer coefficient from the case with a constant heat generation are observed with the greatest amplitude, which is associated with a longer heating period. During the time of increased intensity of heat release in the melt, convective heat transfer is intensified and the temperature of the source increases, and therefore at large amplitudes the heat transfer coefficient is much higher.

**Figure 2.** Temperature fields (a,c) and streamlines (b,d) inside the considered area for different $f$. (a–b) – $f = 50$; (c–d) – $f = 200$.

**Figure 3.** The Nusselt number time curve.
Conclusions
In this paper, the influence of the time-dependent heat generation on the modes of material melting inside a high thermal conductivity radiator was numerically analyzed. It was shown that the effect of natural convection has a significant role on the melting regimes of paraffin. An increase in temperature at the source with an increase in heat generation leads to intensification of the convective regime in the melt and an increase in the Nusselt number on the inner surface of the radiator. With full melting of the material, the heat transfer intensity decreases, and the maximum heat transfer values are observed in the case of a long period of change in the source power.

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