On correlation between global polarization, angular momentum and flow in heavy-ion collisions

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Possible correlations of the global polarization of Λ hyperons with the angular momentum and transverse flow in the central region of colliding nuclei are studied based on refined estimate of the global polarization. Simulations of Au+Au collisions at collision energies $\sqrt{s_{NN}} = 6$–$40$ GeV are performed within the model of the three-fluid dynamics (3FD). Within the crossover and first-order-phase-transition scenarios this refined estimate quite satisfactorily reproduces the experimental STAR data. Hadronic scenario fails at high collision energies, $\sqrt{s_{NN}} > 10$ GeV, and even predicts opposite sign of the global polarization. It is found that the global polarization correlates with neither the angular momentum accumulated in the central region nor with directed and elliptic flow. At the same time we observed correlation between the angular momentum and directed flow in both their time and collision-energy dependence.

PACS numbers: 25.75.-q, 25.75.Nq, 24.10.Nz
Keywords: relativistic heavy-ion collisions, hydrodynamics, vorticity

I. INTRODUCTION

Experimental observation of the global hyperon polarization in heavy-ion collisions by the STAR Collaboration at the Relativistic Heavy Ion Collider (RHIC) [1–3] gave us the evidence for the creation of the most vortical fluid ever observed. Theoretical simulations within the hydrodynamic approaches [4–7] and transport models [8–13] based on thermal equilibration of the spin degrees of freedom [14–16] succeeded to describe the measured global hyperon polarization [11–13]. An alternative approach based on the axial vortical effect (AVE) [17–19] also reasonably reproduced the observed global polarization [20]. Although problems still persist, see recent review in Ref. [21], this gives us confidence that our current understanding of the heavy-ion dynamics and, in particular, the vortical motion is basically compatible with observed polarization.

This phenomenon of the global polarization is usually related to the Barnett effect [22], i.e. magnetization by rotation, where a fraction of the orbital angular momentum associated with the body rotation is transformed into the spin angular momentum. In the Barnett effect the magnetization, i.e. spin alignment, is proportional to the angular momentum. On the contrary, the global hyperon polarization decreases with collision energy rise, i.e. with increase of the total angular momentum [11–13]. This mismatch was explained by that the global polarization is measured in central region of the colliding system (near the midrapidity), while the angular momentum is mostly accumulated in peripheral regions at the freeze-out stage [6, 23, 24]. Then the question arises: whether the global polarization in central region correlates with the angular momentum accumulated in this region? In the present paper we study this question.

We start with a more accurate estimate of the global polarization than that made in the previous paper [6]. Then we compare collision-energy dependence and time evolution of the global polarization and the angular momentum accumulated in central region. We also compare the above quantities with those of directed and elliptic flow to test their possible correlation. The simulations are performed within the model of the three-fluid dynamics (3FD) [25] in the energy range of the Nuclotron based Ion Collider fAcility (NICA) in Dubna and the Beam Energy Scan (BES) program at RHIC.

II. POLARIZATION IN 3FD MODEL

High-energy heavy-ion collisions are characterized by a finite stopping power resulting in a counterstreaming regime of baryon-rich matter at early stage of the collision. Within the 3FD [25] this nonequilibrium regime is modeled by two interpenetrating baryon-rich fluids initially associated with constituent nucleons of the projectile (p) and target (t) nuclei. Newly produced particles, predominantly populating the midrapidity region, are attributed to a fireball (f) fluid. Each of these fluids is governed by conventional hydrodynamic equations coupled by friction terms in the right-hand sides of the Euler equations. The physical input of the present 3FD calculations is described in Ref. [26]. Three different equations of state (EoS’s) were used in simulations. These are a purely hadronic EoS [27] and two versions of the EoS with the deconfinement transition [28], i.e. a first-order phase transition (1PT) and a crossover one.

The global polarization of hyperons was measured in the midrapidity region, i.e. at pseudorapidity $|\eta| < 1$ [11–13]. Similarly to that in Ref. [6], we associate the
global midrapidity polarization with the polarization of \( \Lambda \) hyperons emitted from a central slab of Au+Au colliding system

\[
P_{\Lambda} \simeq \frac{\langle \varepsilon_{zz} \rangle}{2} \left( 1 + \frac{2}{3} \frac{\langle E_\Lambda \rangle - m_\Lambda}{m_\Lambda} \right),
\]

where \( m_\Lambda \) is the mass of \( \Lambda \) hyperon, \( \langle E_\Lambda \rangle \) is energy of the \( \Lambda \) hyperon averaged over the central slab, and \( \langle \varepsilon_{zz} \rangle \) is the \( zz \) component of the thermal vorticity averaged over the central slab with the weight of the energy density \( \varepsilon \)

\[
\langle \varepsilon_{\mu\nu}(t) \rangle = \int d^3 x \left[ \varepsilon_B^B(x,t) \varepsilon_B(x,t) + \varepsilon_f^f(x,t) \varepsilon_f(x,t) \right] / \langle \varepsilon \rangle(t).
\]

where

\[
\varepsilon_{\mu\nu} = \frac{1}{2} \left( \partial_\mu \hat{\beta}_\nu - \partial_\nu \hat{\beta}_\mu \right),
\]

\( \hat{\beta}_\mu = \hbar \beta_\mu, \beta_\mu = u_\mu/T, u_\mu \) is local four-velocity of a fluid, and \( T \) is local temperature. Here \( B \) and \( f \) label quantities related to unified baryonic (\( p \) and \( t \)) fluid and the f-fluid, respectively, and

\[
\varepsilon \simeq \varepsilon_B + \varepsilon_f.
\]

In Eq. 1 all quantities are taken at the freeze-out instant. Expression 4 is a good approximation because of unification of the baryon-rich fluids and small relative (between baryon-rich and fireball fluids) velocities at the later stages of the collision [29].

The above equations need certain comments. Eq. 1 (without averaging over the slab) was derived for polarization vector averaged over the momentum direction of emitted hyperons [10]. In Ref. 6, where a narrow central slab was used, we neglected the longitudinal motion of the \( \Lambda \) hyperon in that slab and therefore approximated \( \langle E_\Lambda \rangle \) by the mean midrapidity transverse mass, \( \langle m_\Lambda^2 \rangle_{\text{midrap}} \). As we consider a wider slab in the present calculation (see discussion below), we compute \( \langle E_\Lambda \rangle \) with account of the longitudinal motion. We consider the proper-energy-density weighted vorticity 4 which allows us to suppress contributions of regions of low-density matter. It is appropriate because abundant production of hyperons takes place in highly excited regions of the system.

A simplified version of the the freeze-out was used in Ref. 6. The freeze-out instant was associated with time, when the energy density (\( \varepsilon(t) \)) averaged over the central slab reached the value of freeze-out energy density \( \varepsilon_{\text{frz}} \) = 0.4 GeV/fm\(^3\). This parameter is the same for all EoS’s and all collision energies.

In actual calculations of observables a differential, i.e. cell-by-cell, freeze-out is implemented in the 3FD [30]. The freeze-out procedure starts when the local energy density drops down to the freeze-out value \( \varepsilon_{\text{frz}} \). The freeze-out criterion is checked in the analyzed cell and in eight cells surrounding this cell. If the freeze-out criterion is met in all cells and if the analyzed cell is adjacent to the vacuum (i.e. if at least one of the surrounding cells is “empty”\(^1\)), then this considered cell is counted as frozen out. The latter condition prevents formation of bubbles of frozen-out matter inside the dense matter still hydrodynamically evolving. This results in the actual energy density of frozen-out cell \( \varepsilon_{\text{frz}} \) being lower than \( \varepsilon_{\text{frz}} \). Thus, \( \varepsilon_{\text{frz}} \) has a meaning of a “trigger” that indicates possibility of the freeze-out. The physical pattern behind this freeze-out resembles the process of expansion of a compressed and heated classical fluid into vacuum, mechanisms of which were studied both experimentally and theoretically, see discussion in Ref. 31. The freeze-out is associated with evaporation from the surface of the expanding fluid.

The actual value \( \varepsilon_{\text{frz}} \) depends on dynamics of expansion and consequently on the collision energy, EoS and impact parameter (\( b \)). This actual freeze-out energy density, averaged over frozen-out system, is illustrated in Fig. 1.

![Fig. 1](image_url)

FIG. 1: Average actual freeze-out energy density versus collision energy \( \sqrt{s_{\text{NN}}} \) in Au+Au collisions at impact parameters \( b=2 \) and \( 8 \) fm calculated with different EoS’s. Pale colors are used for \( b=2 \) fm.

We performed 3FD simulations of Au+Au collisions at fixed impact parameters \( b=8 \) fm. This \( b \) was taken to roughly comply with the STAR centrality selection of 20-50% [1]. Glauber simulations of Ref. 31 were used to relate the experimental centrality and the mean impact parameter. In the present calculation we also apply the global freeze-out in the central slab but at the actual freeze-out energy density as it is displayed in Fig. 1. Detailed discussion of \( \varepsilon_{\text{frz}} \) dependence versus collision energy is presented in Ref. 32. In particular, it is

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\(^1\) Frozen-out cells are removed from the hydrodynamical evolution.
The above described improvements of the polarization calculation increase the polarization as compared to that reported in Ref. [6]. The results of this refined estimate are presented in Fig. 2. In order to get impression of effect of rapidity window \( \Delta y_h \) on the resulting polarization, we present calculations with two choices of the central slab: Eqs. (8) and (9). The polarization increases when the rapidity window expands because the polarization is higher at non-central rapidities. While the global polarization predicted by the crossover and 1PT EoS is very similar, this is not the case for the hadronic EoS. At high collision energies the hadronic-EoS results even in the negative polarization, which looks counter-intuitive from the point of view of spin alignment along the angular momentum. This will be analyzed in more detail in the next section.

Overall agreement of the present estimate with the STAR data [4] on the \( \Lambda \) polarization is quite reasonable. The \( \bar{\Lambda} \) polarization is very close to the \( \Lambda \) one, therefore we do not present it here. Note that feed-down contribution to \( \Lambda \) polarization due to decays of higher mass hyperons is not taken into account in the present estimate. This feed-down results in about 10–15% decrease of the resulting polarization, as demonstrated in Refs. [4, 16, 21, 34].

FIG. 2: Rapidity width \( \Delta y_h \) \( (|y_h| < \Delta y_h/2) \) of the central slab at the freeze-out instant versus collision energy \( \sqrt{s_{NN}} \) in Au+Au collisions at impact parameters \( b = 8 \) fm calculated with various EoS’s for two different prescriptions of the spatial slab width, Eqs. (8) and (9).

FIG. 3: Global polarization of \( \Lambda \) hyperons in Au+Au collisions at \( b = 8 \) fm as function of collision energy \( \sqrt{s_{NN}} \). Shaded bands for the crossover EoS and hadronic EoS indicate polarization sensitivity to choice of the central slab: the upper borders of these bands correspond to choice (8) and the lower borders - to choice (9). STAR data on global \( \Lambda \) and \( \bar{\Lambda} \) polarization [4] are also displayed.

III. CORRELATIONS BETWEEN POLARIZATION, ANGULAR MOMENTUM AND FLOW

The method described above is not optimal for calculating the polarization, which can then be compared...
with experimental data. At the same time it provides a definite advantage for study of correlation between the polarization and the angular momentum. In this method we can compare the global polarization with the angular momentum accumulated in the same space region. The angular momentum accumulated in the slab region is defined as

\[ J = \int_{\text{slab}} d^3x \sum_{\alpha=p,t,f} (z T_{10}^\alpha - x T_{30}^\alpha), \]  

where \( T_{\mu\nu}^\alpha \) is the energy-momentum tensor of the \( \alpha(=p,t,f) \) fluid and has the conventional hydrodynamical form, \( z \) is the beam axis, \( (x, z) \) is the reaction plane of the colliding nuclei. In view of further comparison of the polarization with slope of the directed flow in the center of colliding system, we consider a narrower \( \left| y_h \right| \leq 0.5 \) slab, see Eq. \( \text{(8)} \), in this section. We also limit our further consideration to crossover and 1PT scenarios as the most relevant to the experimental data, see Fig. 3.

The directed flow of the matter is calculated as

\[ v_1(y_h, t) = \frac{u^1}{\sqrt{(u^1)^2 + (u^2)^2}} \]  

in terms of the hydrodynamical 4-velocities \( u^\alpha(x) \), \( \langle \ldots \rangle \) means averaging over \( xy \) plane with the weight of the proper energy density and summation over fluids similarly to Eq. \( \text{(6)} \). The corresponding rapidity \( y_h \) is defined by Eq. \( \text{(5)} \) in terms of the same hydrodynamical 4-velocities. The slope of \( v_1 \) in the center of colliding system, i.e. at “midrapidity” \( y_h = 0 \), is calculated as

\[ \frac{dv_1(t)}{dy_h} = \frac{v_1(y_{\text{right}}, t) - v_1(y_{\text{left}}, t)}{y_{\text{right}}(t) - y_{\text{left}}(t)}, \]  

where \( y_{\text{right}}(t) \) and \( y_{\text{left}}(t) \) are \( y_h \) rapidities at the right and left borders of the slab, respectively, cf. Eq. \( \text{(7)} \). Elliptic flow of the matter at “midrapidity” \( y_h = 0 \) is defined similarly

\[ v_2(t) = \frac{\langle (u^1)^2 - (u^2)^2 \rangle}{\langle (u^1)^2 + (u^2)^2 \rangle}. \]  

Here the \( \langle \ldots \rangle \) averaging is done over \( z = 0 \) plane. The above defined quantities at the freeze-out instant are presented in Figs. 4 (for the crossover EoS) and 5 (for the 1PT EoS) as functions of the collision energy.

Absence of correlation between the global polarization and the angular momentum was found already in Ref. 1. However, there the authors considered the angular momentum accumulated in the whole participant region which steadily rises with the collision energy.

In fact, the absence of correlation between the global polarization and the angular momentum is not surprising. The polarization is intrinsically related to vorticity while the rotation of the fluid can be vortex-free. In such vortex-free rotation the vorticity is present only in close vicinity of the axis of the rotation. Thus, the angular momentum can be arbitrary large while the global polarization is generated only in the narrow region around the rotation axis. Moreover, there could be local islands, where the matter vortically rotates in the opposite direction to the global rotation. Then the angular momentum and the global polarization can have opposite signs, as it is the case in the Au+Au collisions at \( b = 8 \) fm, see Figs. 4 and 5. Such an island structure in the \( xz \) plane was observed in many simulations of nuclear collisions 5 [10, 11, 23, 24, 33, 34].

Figs. 4 and 5 also demonstrate collective flow. Scales
of the slopes of the directed flow \( (dv_1/dy) \) and elliptic flow \( (v_2) \) in Figs. 4 and 5 noticeably exceed those observed in experiment. This is because the considered quantities characterize the medium rather than observed particles. The flow of observed particles is considerably smeared out by thermal spread and resonance decays \[37,38\]. The angular momentum characterizes the medium. Therefore, we compare it with the flow of the medium rather than specific particles.

As seen, the slope of the directed flow \( (dv_1/dy) \) does not correlate with the polarization but does correlate with the slab angular momentum. The slope and the angular momentum even simultaneously change their signs. This indicates that tilting the central fireball, which courses the anti-flow \[39,40\], is accompanied by a change in its angular momentum. At the same time the elliptic flow \( (v_2) \) correlate with neither the polarization nor the angular momentum.

In order to check whether this flow–angular-momentum correlation in their \( \sqrt{s_{NN}} \) dependence is accidental or not, we also consider their time dependence at various energies. Examples of such time dependence are presented in Figs. 6 and 7. The time dependence indicates that indeed there is a correlation between the \( v_1 \) slope and angular momentum, which is less spectacular at lower collision energies, see Fig. 7. Apparently, this is because the chosen width of the central region, \( |y_0| \leq 0.5 \), is too large at lower collision energies in view of comments below. A correlation between \( v_1 \) flow and polarization is also absent in their time dependence.

Note that the slope of the directed flow \( (dv_1/dy) \) in Figs. 6 and 7 at the freeze-out instant is slightly different from that presented in Fig. 4. In Figs. 6 and 7, the directed flow is calculated at the Lagrangian stage of the code \[25\] in terms of test particles and with smaller step in \( y_0 \) rapidity than in \( 12 \), while that in Fig. 4 see Eq. \( 12 \), is computed on a fixed grid (so called Euler step of the scheme). The Lagrangian calculation is more accurate but more time consuming than the Euler one. Therefore, we performed this Lagrangian calculation with larger time step than the Euler one. To accurately fix the freeze-out instant (as in Fig. 4), we need a finer time step.

### IV. SUMMARY

Possible correlations of the global polarization of Λ hyperons with the angular momentum and transverse flow in the central region of colliding nuclei are studied based on refined estimate of the global polarization within the 3FP model. In the present approach the global polarization is associated with the Λ polarization in the central region of colliding nuclei. Within the crossover and first-order-phase-transition scenarios this estimate quite satisfactorily reproduces the experimental STAR data \[1\], especially its collision-energy dependence. The purely hadronic scenario fails at high collision energies, \( \sqrt{s_{NN}} > 10 \) GeV, and even predicts the opposite sign of the global polarization.

It is found that the global polarization correlates with neither the angular momentum accumulated in the central region nor with directed and elliptic flow. Contrary to the polarization, the angular momentum accumulated in the central region even changes its sign at later stages of nuclear collisions at high collision energies. At the same time we detected correlation between the angular momentum and directed flow. The midrapidity slope of the directed flow and the angular momentum even almost simultaneously change their signs.

### Acknowledgments

Fruitful discussions with O.V. Teryaev and V.D. Toneev are gratefully acknowledged. This work was carried out using computing resources of the federal collective usage center “Complex for simulation and data processing for mega-science facilities” at NRC “Kurcha-
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Science of the Russian Federation within the Academic Excellence Project of the NRNU MEPhI under contract No. 02.A03.21.0005.