Compact Storage Ring for an X-Ray Source

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Abstract. We propose a new design of a compact storage ring for a source of X-ray radiation on the basis of reverse Thomson scattering of laser radiation by electrons with the energy of 35–50 MeV, which has small number of optical elements and a significant clear space for the placement of a beam injection-extraction system and a RF cavity. The original laser cavity layout has been considered. The ring dynamic aperture after correction of chromaticity and a second-order dispersion function is sufficient for the injection and stable circulation of an electron bunch in the ring.

1. Introduction

This work continues a cycle of the previously conducted studies of an X-ray radiation source — the laser-electron generator (LEG) based on Thomson scattering of laser radiation by electrons [1–5]. The new design of the storage ring, which is a part of LEG, considered in the present work is intended to retain electron bunches with the energy adjustable within the range of 35 to 50 MeV with the maximum charge of 1 nC and a root-mean-square normalized emittance of about 4 mm mrad at the moment of injection. Intrabeam scattering of particles and their interaction with laser radiation at such a low electron’s energy and a large charge of the bunches leads to the growth of transverse emittance and a momentum spread, which requires replacement of the bunches by means of a fast beam injection-extraction system with the period of 20 ms, which is much smaller than the time constant of radiation damping due to synchrotron radiation.

Our previously considered [5] version of the ring “C” was taken as the basis. We have conducted an optimization of the ring lattice, minimization of chromatic aberrations, performed calculations of dynamic aperture and the growth of emittance due to intrabeam scattering. The calculations were performed with the help of the MAD-X [6] and OPA [7] codes. Consideration of the optimal RF cavity operating frequency has been conducted. Since the laser resonator is an integral part of the LEG ring, we have considered an option for its placement, which does not lead to an increase of the ring size and allows to obtain a small value of the $\beta$-function at the interaction point, with its values in other parts of the ring being moderate.
Figure 1. LEG storage ring lattice. $Q_1$, $Q_2$, $Q_3$, $Q_4$, $Q_5$, $Q_6$ — quadrupoles, $S_x$, $S_y$ — sextupoles, $M_1$, $M_2$, $M_3$ — dipoles, IP — interaction point.

2. The ring lattice

The ring lattice (Fig. 1) is formed by two achromatic 180° turns composed of three 60° dipole magnets: $M_1$, $M_2$, $M_3$, providing the beam focusing in horizontal and vertical planes. The beam focusing at the interaction point is achieved with the help of two pairs of six quadrupoles $Q_1$ – $Q_6$. The ring lattice has a mirror symmetry with regard to the plane passing through the interaction point, IP.

| Parameter                                      | Value  |
|------------------------------------------------|--------|
| RF cavity operating frequency, $f_{RF}$, MHz   | 2856   |
| Bunch revolution frequency, $f_{rev}$, MHz     | 30     |
| Orbit length, $L$, m                           | 9.762  |
| Momentum compaction factor, $\alpha$           | 0.136  |
| Voltage across the RF cavity gap, $V_{RF}$, kV  | 300    |
| Momentum of the synchronous particle, $p_s$, MeV/c | 50.5  |
| Number of bunches at orbit, $M$                | 1      |
| Average beam current, $I_b$, mA                | 35.7   |
| Synchrotron tune, $\nu_z$                      | 0.05   |
| Betatron tunes, $\nu_x/\nu_y$                  | 2.7/1.8|
| Betatron functions at the interaction point, $\beta_x/\beta_y$, m | 0.05/0.05 |
| Relative RMS momentum spread at the injection, $\Delta p_{rms}/p_s$ | 0.001  |

The ring lattice was optimized taking into account the following correction of chromatic aberrations and the second-order dispersion function, achieving large dynamic aperture, and a feasibility of the laser cavity placement. The betatron functions and the dispersion function behavior are shown in Fig. 2, and the ring parameters are given in Tab. 1. An important feature of the ring is the absence of quadrupoles in most part of orbit except for the section containing
the IP. At that, the maximum value of the betatron functions does not exceed 4 m, which corresponds to the maximum RMS beam radius equals to about 0.4 mm. At the IP, the value of the betatron function equals 5 cm, and respectively, the RMS beam radius equals 45 μm. The other feature of the ring is a large value of the momentum compaction factor, which requires a high value of the operating RF frequency; the discussion of this point is given below.

3. Intrabeam scattering and interaction with laser radiation

In the course of the bunch revolution in the ring, the growth of both the RMS transverse beam emittance in the bending plane and the relative RMS momentum spread occurs as a result of intrabeam scattering. The reason for the emittance growth in the bending plane is a change of longitudinal momentum within a dispersion path due to particles scattering within a bunch, which leads to violation of the achromatic conditions for these particles. The growth of relative momentum spread is related to the transverse momentum transfer into the longitudinal plane; at that in the laboratory reference frame the change of longitudinal momentum grows proportionally to the relative energy of particles [8–11]. Time constant for the emittance and the momentum spread growth depends on the ring lattice functions and is inversely proportional to the charge density in the phase volume occupied by the beam. In this connection, as the emittance and the momentum spread grow, the growth rate is slowing down. It should also be noted that the time constant for the emittance growth contains an addend, which is inversely

![Image of Figure 2, Betatron and dispersion functions of the ring. Q₁ – Q₆ — quadrupoles, Sₓ, Sᵧ — sextupoles, M₁, M₂, M₃ — dipoles, IP — interaction point.](image-url)
proportional to the fourth power of the relative beam energy. In this regard, the given problem is especially significant for the rings with low beam energies.

Interaction of laser radiation with electrons leads to radiation damping of transverse oscillations and decrease of transverse emittance of the beam, which in our case is accompanied by significant growth of RMS momentum spread, as compared to the spread at the injection [12,13]. However, the time constant of this process for the ring with our parameters is much greater than the period of bunch replacement, and interaction with laser radiation leads primarily to the growth of RMS momentum spread in accordance with the following expression [14]:

$$\frac{\delta p_{\text{rms}}(t)}{p_s} = \sqrt{\left\langle \frac{E_{\gamma}}{2E_s} \right\rangle^2 \frac{N_\gamma}{N_e} t},$$

where $\langle E_{\gamma} \rangle$ — average energy of X-ray radiation, $E_s$ — ring synchronous energy, $N_\gamma$ — number of X-ray photons emitted in unit of time, $N_e$ — number of electrons in a bunch. In our case $\langle E_{\gamma} \rangle \approx 20$ keV, for $E_s = 50$ MeV, $N_e \approx 6.25 \times 10^9$, immediately after injection $N_\gamma \approx 5 \times 10^{13}$ s$^{-1}$, thus, over the operating cycle of the ring due to interaction with laser radiation $\delta p_{\text{rms}}(20 \text{ ms})/p_s \approx 0.25\%$. It should be reminded that at the moment of injection $\delta p_{\text{rms}}(0)/p_s \approx 0.12\%$.

![Figure 3](image_url)

Figure 3. (a) — RMS emittance, (b) — RMS momentum spread growth due to intrabeam scattering and interaction with laser radiation.

Calculation results for RMS emittance and RMS momentum spread growth for the ring lattice under study, performed with the help of the MAD-X code [6], are given in Fig. 3. Initial values of the RMS normalized emittance in the bending plane and the momentum spread were 3.8 mm mrad and 0.1% respectively, the bunch charge was 1 nC. The calculations accounted for the growth of momentum spread described by expression (1).

As one can see from the given graphs, over the time of 20 ms the beam emittance grew 10.5 times (from 3.7 to 14.2 mm mrad), whereas the momentum spread grew 2.14 times (from 0.1 to 0.257%). It needs to be taken into account that at the moment of injection from a linear accelerator, the phase extent of the bunch is much less than the equilibrium value in the ring for the given momentum spread, and during about few thousand revolutions it approaches the equilibrium one.
4. Correction of chromaticities and second-order dispersion function

![Diagram](image)

**Figure 4.** Position of operating point at the tunes diagram (momentum spread ±1.5%). (a) — without correction, (b) — with chromaticity correction. Point color on the diagram means the momentum deviation value, yellow color $Δp/p = 0$, blue $Δp/p = -1.5\%$, pink $Δp/p = +1.5\%$.

Values of natural chromaticities of the considered lattice are $ξ_x = -7$, $ξ_y = -4.5$. Fig. 4 shows position of the operating point at the ring tunes diagram, which was optimized using the OPA code [7]. Momentum spread, including that caused by intrabeam scattering and interaction with laser radiation, leads to an intersection with the strong first and third order resonances (Fig. 4 (a)), which requires the chromaticity correction. Correction of natural chromaticities was carried out with the help of two families of sextupoles ($S_x, S_y$, Fig. 1), located at dispersion paths. Chromaticity values after the correction are $ξ_{x, corr} = -3$, $ξ_{y, corr} = -3$. These chromaticity values, driven also by the need to correct the second-order dispersion function (see below), allow one to avoid intersections with strong resonances (Fig. 4 (b)).

The achromat structure we have selected allows one to carry out chromaticity correction using weak sextupole fields not resulting in strong nonlinear effects, which could limit the dynamic aperture value. Calculation results for the dynamic aperture at the injection point performed for 1000 revs with the help of the MAD-X code [6] for various relative pulse spreads are shown in Fig. 5. Note that further increase in the number of revs will insignificantly change the dynamic aperture dimensions. As one can see, the dynamic aperture almost completely covers geometric dimensions of the vacuum chamber shown with a solid line.

In Fig. 6 one can see the behavior of the first and second order dispersion functions. The second order dispersion function at the IP was -1.3 m prior to correction (Fig. 6 (a)). As shown above, relative RMS momentum spread reaches 0.257% due to intrabeam scattering and interaction with laser radiation. In our case this results in an increase of RMS beam size at the IP due to second order dispersion by 81 $\mu$m, which is significantly more that the design value of 45 $\mu$m. We have corrected the second order dispersion with the help of sextupole lenses placed at the nonzero dispersion paths. Fig. 6 (b) shows dispersion functions after the correction: the second order dispersion function value was decreased to 0.08 m, which resulted in an increase of the RMS beam size at the IP by only 8 $\mu$m.
(a) $\frac{\delta p}{p} = -1\%$
(b) $\frac{\delta p}{p} = +1\%$
(c) $\frac{\delta p}{p} = 0$

Figure 5. Dynamic aperture of the LEG ring. Calculation for 1000 revs for three relative momentum spread values.

(a) \( \eta_1 \)
(b) \( \eta_2 \)

Figure 6. First and second order dispersion functions, (a) — prior to chromaticity and dispersion correction, (b) — after chromaticity and dispersion correction.
5. Optical cavity placement
An optical cavity in which laser radiation pulse with the energy of about $W_{ph} = 20$ mJ circulates with a bunch revolution frequency $f_{rep} = 30$ MHz, which corresponds to the average accumulated radiation power of over 600 kW, must be integrated into the ring design.

![Optical cavity layout](image)

**Figure 7.** Positional relationship of the six-mirror optical cavity laser beams layout and quadrupoles before the interaction point, $Q_1 - Q_6$ — quadrupoles, MIR$_1$ — MIR$_3$ — mirrors.

Known optical cavity placement methods are not suitable for the ring lattice we selected. Therefore, we have considered an option of six-mirror cavity with laser beams passing within the aperture of central quadrupoles ($Q_4$, $Q_5$, $Q_6$). Fig. 7 shows a scheme of the optical cavity ensuring an intersection of electron bunches and a laser pulse at the angle of $2\degree$. Placement of the optical cavity is feasible at the lens aperture of not less than 70 mm and the shape of the pole pieces allowing for placement of the cavity’s vacuum tubes. The problem is complicated by the fact that in order to achieve a small value of the beta-function at the interaction point, the quadrupoles gradient value must be not less than 22 T/m, and the multipole component contribution must be minimal.

The quadrupoles geometry found as a result of optimization is shown in Fig. 8 (a), and the distribution of magnetic field for the design value of the gradient is shown in Fig. 8 (b). The main problem for the achieving of high gradient is presented by saturation of the pole pieces material. In this connection we have considered two options for the material: super-permendur and vanadium-permendur. Fig. 8 (c) shows gradient as a function of current in the coils. The pole saturation effect begins to appear at the gradient’s value above 22–24 T/m. Note that the lens aperture value, which is greater than the beam size by at least two orders of magnitude, guarantees small influence of multipole components on the beam dynamics in the ring.

6. RF cavity frequency selection. Estimate of X-ray photon yield
As was noted above, the feature of the considered ring is a large value of the momentum compaction factor $\alpha = 0.136$, which determines, along with other parameters, the RMS bunch length (two standard deviations) in equilibrium conditions:

$$\Delta l_{rms} = 2 \frac{\Delta p_{rms}}{p_s} \sqrt{\frac{cL_0E_s}{2\pi f_{RF}V_{RF} \cos \varphi_s}},$$

(2)

where $c$ — speed of light, $\varphi_s$ — equilibrium phase, other notations are explained in Tab. 1.
When an electron bunch collides with a laser pulse at a nonzero angle, as it is in our case, the yield of X-ray photons drops significantly with the increase of the bunch length, which is true when the orbit expansion factor increases. To reduce bunch length, along with decreasing the orbit length, we have selected a rather high operating frequency of RF cavity of the ring, $f_{RF} = 2856$ MHz.

Thus, for RMS relative momentum spread at the injection of $\Delta p_{rms}/p_s = 0.001$, and for the ring parameters given in Tab. 1, the RMS length of the bunch will be $\Delta l_{rms} \approx 3.86$ mm (the RMS duration and phase length are about 13 ps and 13°, respectively). Based on the above data, data from Tab. 1 and optical cavity parameters, one can estimate that at the laser photon energy of $E_{ph} = 1.03$ eV, and electron energy of $E_e = 50$ MeV, total yield per unit of time

Figure 8. (a) — quadrupoles geometry. (b) — magnetic field distribution, (c) — dependence of quadrupole gradient on current in the coils: red plot — super permendur magnetic material, black plot — vanadium permendur.
of scattered photons with threshold energy of $E_\gamma \approx 40$ keV just after injection will amount to $dN_\gamma / dt \approx 5 \times 10^{13}$ phot/s.

7. Beam injection-extraction system and RF cavity placement

The beam injection-extraction system, which includes two septum magnets and a kicker (Fig. 1) is placed at a free path with zero dispersion. These elements are described in a separate paper, as well as the RF cavity, which is also placed within that path.

8. Conclusion

In this work we offer an option of a compact storage ring for an X-ray radiation generator, which has a large dynamic aperture allowing one to place an optical and a RF cavities, as well as a beam injection-extraction system.

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