Solution of problems of parametric optimization and control of electric drives state based on information about operability area boundary

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Abstract. The paper presents methods and algorithms of optimal parametric synthesis and control of electric drive state. The basis of methods is made up of information on boundary of the operability area of the electric drive. It is proposed to use an operability margin of the electric drive as an optimality criterion. The proposed methods and algorithms are illustrated by examples of parametric synthesis and state evaluation of direct current electric drives.

1. Introduction

A design of electric drives (ED) at the stage of the parametric synthesis should be reduced to solving two main problems: the determining of nominal values of ED system internal parameters and determining allowable limits of their change. The problem of parametric synthesis is reduced to the problem of parametric optimization when choosing the optimal internal parameters by any criterion.

Internal parameters are parameters of ED elements which characterize the state and properties of the system itself. When designing an ED system, these parameters determine vector $X$ of controlled parameters. A mathematical model of an ED represents an algorithm for calculating the output parameters of vector $Y$ for given vectors of internal parameters $X$ and external parameters $V$. External parameters describe characteristics of an ED external environment and impact its functioning. Output parameters characterize ED properties, in which consumers are interested. They are functional parameters, i.e. functional dependences of phase variables and parameters of ED, which are boundary values of external variable ranges that retain an operability of an ED system. Output parameters in parametric synthesis include destination indicators, parametric reliability and economy [1]. Working capacity margin is an indicator of parametric reliability under conditions of limited statistical data about laws of distribution of ED internal parameters in time [1].

Operability area $G = P \cap M$ defines a set of admissible values of internal parameters, under which all requirements for ED output parameters are met. It is determined by operability conditions [1]:

$$Y_{j\min} \leq Y_j = F_j(X) \leq Y_{j\max}, \quad j = 1, m;$$
$$X_{i\min} \leq X_i \leq X_{i\max}, \quad i = 1, n$$

where $Y_{j\max}(X_{i\max}), \quad Y_{j\min}(X_{i\min})$ are the maximum and minimum allowable values of the $j$-th
output $Y_j$ ($i$-th internal $X_i$) parameter respectively; $F$ is an operator, establishing a link between internal and output parameters; $D$ and $P$ are tolerance areas defined by the first and the second inequalities respectively (1). Area $D$ in an internal parameters space corresponds to tolerance area $M$.

The main tasks of an ED exploitation are the task of determining a state of a drive at a given time and the task of predicting its condition for the coming time. Papers [1, 2] show that the tasks of designing and exploitation of dynamic systems should be considered from a unified position of parametric and structural control of a state of these systems. At the same time, the most important indicator of parametric synthesis and diagnosis of a state of dynamic systems is their operability margin.

This article, as applied to ED control systems, considers methods of solution of problems stated above, based on the use of information on working capacity area boundary. The methods for solving the above-stated problems are considered in this article in relation to ED control systems, based on the use of information on the boundary of the operability area.

2. Quality indicators and optimality criteria
Parametric synthesis of ED control systems is generally reduced to a setting of their regulators. At the same time, default settings are used for technical and symmetrical optimum. These settings have well known disadvantages. Only two dynamic factors are considered: the time of the transient process and the maximum overshoot. Optimum synthesis of gain factors and time constants of regulators is carried out by the criterion of the main indicator [3].

Increasingly wide application in practice is found by regulators with state observers (SOR). Their advantages are a flexible regulator structure and a need for only a single sensor of controlled object output coordinates. A modal synthesis method based on a standard distribution of roots of a characteristic polynomial is often used for a parametric optimization of such systems. Moreover, this method does not take into account the index of a parametric reliability and possible coordinate limitations inherent in an ED [4].

Studies [1, 5-7] rightly point out that usage of methods of parametric optimization allow one to take into account the entire set of requirements for automatic control of an ED. But there is a difficulty in solving the optimization problem: it is complicated to form criteria of management quality, most of which are indirect. Another difficulty is a definition of a search algorithm convergence of a global extremum of a selected criterion function. Synthesis of systems with SOR should not be done based upon indirect criteria of optimality, but based on requirements applicable to a quality of ED control, including destination indicators (operation speed indicator, accuracy and energy efficiency) and parametric reliability [1].

Thus, the problem of optimal parametric synthesis of EDs and their control systems is the task of a multicriterion or vector optimization. Methodological aspects of solving multicriterion optimization of dynamic systems are still of high relevance and in demand [9-11].

3. A choice of a criterion function
In work [8], it is proposed to use expanded quality criteria, including normalized operation speed indicator $\bar{Y}_1$, accuracy $\bar{Y}_2$, parametric robustness $\bar{Y}_3$ and energy costs of control $\bar{Y}_4$:

$$\bar{Y}_m = 1 - (\prod_{i=1}^{m} \bar{Y}_i)^{1/m},$$

where $m = 4$ - the number of quality control indicators in the composition of the criterion. The rise time of transient characteristic $t_r$ is assumed here as operation speed indicator $\bar{Y}_1$. As accuracy indicator $\bar{Y}_2$ - relative deviation average module of a system output coordinate in the time interval from $t_r$ to $t$, is taken:
where $T_r = t_r / T_0$ and $T_t = t_t / T_0$ are the relative rise time and the transient time, $T_0$ is a sampling period.

It is proposed to use the average deviation modulus of the transient characteristic of the system when the parameters of the control object are varied relative to the designed values during the transient time as an indicator of robustness $\bar{Y}_3$.

$\bar{Y}_3 = \frac{1}{N} \sum_{j=1}^{N} \left| \frac{Y_j - Y_j^*}{Y_j} \right| \cdot 100\%$,

where $Y_j$, $Y_j^*$ are values of output coordinates in case of designed and changed parameters of the system.

A peak current of an electric motor, which is limited by overload capacity of the power part of the system is proposed to use as the indicator of the energy cost of control $\bar{Y}_4$.

The main disadvantage of the proposed criterion is its multiplicative form of the task, which is subjective and has certain limitations [1].

The analysis of literature sources shows that for the majority of EDs, the requirement of high reliability is put first. Providing a parametric reliability is especially relevant for EDs, which are characterized by a parametric instability. With regard to the problem being solved, this means that as the optimization parameter it is advisable to choose an ED operability margin, which should be maximized. The rationale and peculiarities of the choice of the proposed criterion function for solving problems of the synthesis of electro-technical systems are considered in [1].

The operability margin is understood as the degree of approximation of the vector of the actual state of the system to its maximum permissible value.

Metric $l$ is introduced in the $R^m$ space internal parameters. Metric $l$ is a function of coordinates of any two points in this space, such as points $A$ and $B$. In this case:

$l = \left[ \sum_{i=1}^{n} \mu_i (X_i(A) - X_i(B))^2 \right]^{1/2}$,

where $X_i(A)$, $X_i(B)$ are coordinates of points of $A$ and $B$ vectors, respectively; $\mu_i$ is a normalizing factor for the $i$-th coordinate of the $X$ parameter. If one of the points, for example, point $A$, is a boundary point of the operability area, and point $B$ is located within this area and its coordinates characterize the state of an ED at a given time, then the given metric will determine the operability margin and serve as a criterion of coordinate search of the optimal point of the system.

4. Methods and algorithms of the parametric synthesis according to the criterion of the operability margin

A choice of a problem solving method is determined by the dimension of space of parameter $X$, as well as the presence or absence of information on the boundary of the operability area. In case such information is available and it is defined as an array of boundary points, analytical and search methods of the parametric optimization are developed, using, respectively, mechanical and electrical analogy. A simpler analytical method can be used only in the case of convex area of the operability. This method supposes a direct calculation of optimal point coordinates by the known coordinates of boundary points. The search method can be used with any configuration of the operability area. In addition, as shown in paper [1], the criterion function is as follows:
\[
F = \frac{1}{N} \sum_{k=1}^{N} \left( \sum_{i=1}^{n} (R_i - X_{ik})^2 \right)^{-1},
\]

where \( N \) is the total number of given boundary points of the operability area; \( n \) is number of internal parameters \( X \); \( R_i \) and \( X_{ik} \) are, respectively, coordinates of internal point and boundary points.

In the absence of information on the boundary of the operability area, as well as for the large dimension of the space of internal parameters, a method of convergent areas has been developed that allows one to obtain a unique solution for the most general case. The method assumes a consistent narrowing of the operability area until an optimal solution is obtained. The essence of the method is as follows.

At the first stage, the boundary of the \( G \) area is defined by known methods [1]. This boundary consists of a finite number of hypersurfaces \( \Phi_j \), each of which can be described with a given error with equations \( \Phi_j(X) = 0 \), where \( \Phi_j(X) = Y_{j,\text{max}} - F_j(X) \) or \( \Phi_j(X) = F_j(X) - Y_{j,\text{min}} \) are constraint functions in the system of inequalities (1). For this purpose, it is convenient to use methods of experiment planning [1]. In space \( R^n \) of internal parameters, metric \( l \) is introduced, which is a function of coordinates of any two points in this space, such as points \( A \) and \( B \). If one of points, such as point \( A \) - the boundary point of area \( G \) - and point \( B \), which is inside this area and its coordinates characterize the state of ED at a considered moment of time, then this metric will determine the margin of operability area \( \lambda \) of electric drive and will serve as a criterion for narrowing source area \( G^{(0)} \) to determine the coordinate of the optimal point. Further, the \( G^{(0)} \) area is narrowed by criterion \( l = \lambda \). For that purpose, the criterion is changed by the \( \Delta l \) amount, that is \( l^{(1)} = l^{(0)} + \Delta l \), further it obtains an analytical description of the area \( G^{(1)} \) boundary and determines the required set of its boundary points. Then equations \( \Phi^{(1)}_j \) are formed, which describe the \( G^{(1)} \) area [1].

On the basis of the use of logical R-functions [1] and equations obtained \( \Phi^{(1)}_j \), one equation is written which, with a specified methodical error, analytically describes tolerance area \( G^{(1)} \) at the first step of the search:

\[
\begin{align*}
G^{(1)}_1 &= 0.5 \left( G^{(1)}_{2(m+n)-1} + \Phi^{(1)}_{2(m+n)} - G^{(1)}_{2(m+n)-1} - \Phi^{(1)}_{2(m+n)} \right); \\
...G^{(1)}_1 &= 0.5 \left( G^{(1)}_{j+1} + \Phi^{(1)}_j - G^{(1)}_{j+1} - \Phi^{(1)}_j \right); \\
...G^{(1)}_1 &= \Phi^{(1)}_1
\end{align*}
\]

Then, in a similar way, tolerance area \( G^{(2)} \in G^{(1)} \) is defined and the process of narrowing original area \( G^{(0)} \) is cyclically repeated until an optimal solution is achieved under \( N=1 \).

The moment of the search process stop is determined under such internal parameters values for which the \( G^{(1)} \) area in accordance with a specified error will degenerate into a point. Values of \( X_{\text{opt}} \) parameters in this point determine the optimum (maximum) value of criterion \( l \), which corresponds to condition \( N=1 \).

Algorithms of solving the problem and their application fields are considered in studies [1, 2].

5. Methods and algorithms of an ED state evaluation
The task of evaluation of the technical state of the ED is to recognize the belonging of vector \( Y(t) \) to area \( D \) or the belonging of vector \( X(t) \), to area \( G \), and also the determination of the operability margin of the system, in the case when \( Y(t) \subset D \). The difficulty in solving the control problem is due to the necessity of constructing the boundary of operability area, which can have a very complex
configuration, and the need to calculate quantities \( t_1 \) and \( l_0 \). In addition, to determine vector \( \mathbf{X}(t) = \{X_1(t), X_2(t), \ldots, X_n(t)\} \), it is required to control all \( n \) parameters of \( \mathbf{X} \), which is very problematic for a large dimension of the \( R^n \) space. To simplify the task of recognizing the operability area, it is usually approximated by inscribed or circumscribed hyper parallelepiped of the greatest volume. Thus, the permissible limits of the change in the primary parameters are established independently of each other. The analysis has shown [2] that such approach leads to a large methodological error, which increases nonlinearly in a function of a number of controlled parameters. Besides, well known methods do not have an unambiguous solution for non-simply-connected operability areas. Thereby, the task of developing a method of ED control, which is providing a simplicity and high veracity, is crucial.

To solve the problem, let us establish a connection between primary parameters \( \mathbf{X} \) and controlled (measurable) parameters \( \mathbf{Z} \). The essence of the suggested approach is as follows. It is known that any dynamical system with a given error can be approximated by the second order system and solve the task of system identification using transient characteristic \( h(t) \), or frequency transmission function \( W(i\omega) \). In this case, there is matching \( \mathbf{Z} = \psi(\mathbf{X}) \) between parameters \( \mathbf{X} \) and \( \mathbf{Z} \), and tolerable value area \( F \) corresponds to the \( G \) area in space \( R^h \) of parameters \( \mathbf{Z} \) [2].

In the process of parametric synthesis [1], area \( G \) is broken into sub-areas \( G_i \), each of which determines the margin of operability area \( \lambda_i \). Based on transformation \( \Phi_Z : G \rightarrow F \) areas, \( F_i \) will correspond to areas \( G_i \) in the space of \( \mathbf{Z} \) parameters.

The equation of area \( F \) should be obtained. For this purpose, for each hypersurface \( f_j \) of the \( G \) area boundary, sum total \( N_q \) of boundary points equal to the number of a significant coefficient of the unknown equations should be formed. By these points, hypersurface equations can be obtained that generate the \( F \) area. In this case [1]:

\[
X_i^{(1)} = X_i^{(0)} + \left( \frac{\partial f_j^{(0)}(\mathbf{X})}{\partial X_i} \right) \Delta t_i (\text{grad} f_j^{(0)}(\mathbf{X}))^{-1}.
\]

Similarly, the sub-areas \( F_i \) boundaries are defined. Each sub-area \( G_i(F_i) \) has its own working capacity margin of ED. As follows, for sub-area \( G_1(F_1) \): \( \lambda_1 \in [0;l_1/l_0] \), for \( G_2(F_2) \): \( \lambda_2 \in [l_1/l_0;l_2/l_0] \).

Estimation of the state of ED is reduced to the recognition in space \( R^c \) of parameters \( \mathbf{Z} \) of belonging to the current state vector of the ED of a given sub-area, \( F_\beta \), \( \beta = 1,S \), where \( S \) is the number of sub-areas, for each of which operability areas margin \( \lambda_\beta \) is determined.

The task of recognition is solved as follows. If values \( Z_q \), \( q = 1,k \) resulting from the control will be substituted in the equation above will give condition \( F < 0 \), then ED is in working state and the validity of satisfaction of inequalities \( F_\beta < 0, \beta = 1,2,\ldots(S+1) \) is verified. If \( F_\beta \leq 0 \) and \( F_{\beta+1} > 0 \), then the current state vector of the ED belongs to sub-area \( F_\beta \) and the operability areas margin is \( \lambda_\beta \). If \( F > 0 \), then an ED is inoperative. In this case, the parametric correction of ED configurable options is performed [1].

6. Conclusion
The considered approach to the solution of the problem of assessing the state of ED is characterized by high veracity and convenience of practical use. In the article by the example of control systems for DC drives, the methods and algorithms for solving the problem of parametric synthesis and monitoring of ED condition are considered.
References

[1] Saushev A V 2013 *Parametric Synthesis of Electrical Systems* (St. Petersburg: Admiral Makarov SUMIS) p 315

[2] Saushev A V 2015 *Informat. Technol. and Comput. Syst.* 3 65-73

[3] Anisimov A A, Kotov D G, Tararykin S V and Tyutikov V V 2011 *J. of Comp. and Syst. Scienc. Intern.* 50 698-719

[4] Anisimov A A and Tararykin S V 2012 *J. of Comp. and Syst. Scienc. Intern.* 51 617-627

[5] Afanasyev K S and Vashchenko K V 2017 *Scient. Bullet. of the Novosibirsk St. Technic. Univers* 1(66) 7-14

[6] Igumnov I V and Kutsyi N N 2017 *Mechatron., Autom., Contr.* 18-4 227-232

[7] Zaporozhets D Yu, Xalov A M and Pshenokova I A 2016 *Inform., Comp. Scien. and Engin. Educ.* 1(25) 14-26

[8] Anisimov A A and Tararykin S V 2009 *Mechatron., Autom., Contr.* 10 36-41

[9] Nyrkov A, Shnurenko A, Sokolov S, Chernyi S and Korotkov V 2017 *International Journal of Electrical and Computer Engineering (IJECE)* 7(6) 3578

[10] Nyrkov A, Shnurenko A, Sokolov S, Chernyi S and Korotkov V 2017 *Procedia Engineering.* 178 543-550

[11] Nyrkov A, Zhilenkov A, Korotkov A, Sokolov S and Chernyi S 2017 *Journal of Physics: Conference Series* 803 012108