Dynamic distribution of labor resources by region of investment

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Abstract. This article discusses the problem of the optimal distribution of labor resources by investment region, taking into account changing conditions. A deterministic model is considered, that is, both the investment region and the invested resources go into new states with a probability equal to one. Several economic agents jointly organize the process of investing labor resources in several investment regions.

1. Introduction
At the initial moment, many types of invested labor resources and investment regions were identified, the efficiency with which each investment resource in a particular investment region is also known. Each investment resource at any time can be in one of a finite number of states that differ in the efficiency of use of investment resources. It is necessary to invest investment resources in the investment region in an optimal way, that is, so that the total investment efficiency is maximum. By the beginning of the next period of time, depending on the decision made at the previous stage, and, possibly, for other reasons, many investment regions may change (some may appear), many labor resources invested, as well as the efficiency of a particular investment resource or another region of investment [1–5]. Consequently, a new situation arises in which it is necessary to solve a new problem of optimal appointment. There are many such situations. Thus, a dynamic problem arises. It can be solved by the method of dynamic programming, and as strategies at each step, various solutions to the optimal assignment problem are used.

2. Formal statement of the problem
We now turn to the formal statement of the problem. There are many $M$ labor resources investing. We number them by index $m = 1, 2, \ldots, |M|$. There are many $L$ regions of investment. We number them by index $l = 1, 2, \ldots, |L|$. Investing takes place over $T$ periods $t = 1, 2, \ldots, T$ [6–9]. We will evaluate the effectiveness of the resource in the investment region $l$ size $p(l)$. We introduce some system of numbers
\[ U : 0 = u_0 < u_1 > u_2 < \ldots < u_{|q|} = I. \] We will say that the investment region is in a state \( ak \), if the inequality \( u_k < p(l) \leq u_{k+1} \). If, before investing the invested resource, the investment region \( l \) was at the level of efficiency \( ak \), and after the period of investment resource investment \( m \) he is in a state \( a_{k_2} \), then under the efficiency of investing an investment resource \( m \) in the region of investing \( l \) we will understand the value \( E_{ml} = k_2 - k_1 \). Denote the set of conditions in which regions of investment may be located through \( A \) and number them by index \( a = 1,2,\ldots,|A| \). Investment regions can be combined into a group, the effectiveness of which will be determined by the state vector of individual investment regions \([10–13]\). The set of conditions in which investment resources may be located is denoted by \( B \) and we index \( b = 1,2,\ldots,|B| \). Functions defined \( F(m,b,l,a) \) transition region investment \( l \) from one state to another, if an investment resource is invested in a region \( m \), in condition \( b \), and functions \( \overline{F}(m,b,l,a) \) the transition of an investment resource from one state to another, if it was invested in the investment region \( l \), in condition \( b \) \([14–15]\). Now we can talk about the state of a certain system, determined by the conditions in which individual investment regions and individual investment resources are located, that is, the state of the system is determined by the vector 

\[
S = \{b_1^1, b_2^2, \ldots, b_{|q|}^{|q|}, a_1^1, a_2^2, \ldots, a_{|A|}^{|A|}\}.
\]

Denote by \( S \) many states of the system and number them by index \( s = 1,2,\ldots,|S| \). The number of system states is determined by the expression

\[
|S| = |A|^{|q|} \cdot |B|^{|q|}.
\] (1)

Denote the set of strategies by \( Q : q = 1,2,\ldots,|Q| \), using the transition functions \( F(m,b,l,a) \) and \( \overline{F}(m,b,l,a) \), can get functions \( \overline{F}(q,s) \) system transition from one state to another \([16]\).

When the system transitions from state \( S_t \) to state \( S_{t+1} \) income can be earned \( r_{s,t+1}(q) \), which can be calculated in various ways, for example,

\[
r_{s,t+1}(q) = \sum_{i=1}^{max} (a_i^{t+1} - a_i^t).
\] (2)

To calculate the optimal income from the functioning of the system during \( T \) time periods at finite \( T \) we will use the recurrence relations of dynamic programming. Consider them \([17–21]\).

Let be \( V^{T-t}(S_t) \) - maximum income from the functioning of the system during \( T-t \) time periods from state \( S_t \), where \( S_t = 0,1,2,\ldots,|S| \), but \( t = 0,1,2,\ldots,T \) under optimal policy. The maximum income of the system during one period of time is determined by the formula

\[
V^1(S_0) = \max_{q \in Q} (r_{s,t}^1(q)).
\] (3)

The maximum income from the functioning of the system for two periods of time is delivered by the expression

\[
V^2(S_0) = \max_{q \in Q} [r_{s,t}^1(q) + V^1(S_1)],
\] (4)

Where \( V^1(S_1) \) – maximum income for the functioning of the system in a one-step process. To calculate the optimal income, we have the following functional relation

\[
V^{T-t}(S_t) = \max_{q \in Q} [r_{s,t}^{T-t}(q) + V^{T-t-1}(S_{t+1})].
\] (5)

\( V^0(S_t) \) can be set to zero, which is natural. Applying relation (5) sequentially for \( t = T-1,T-2,\ldots,0 \), we calculate \( V^1(S_{T-1}), V^2(S_{T-2}), \ldots, V^{T-1}(S_1), V^T(S_0) \) and we will be able to
indicate the optimal distribution of investment labor resources by investment region at any given time [22–23]. The specified calculation method can be implemented using a computer.

3. An example of solving a dynamic problem

To illustrate the described method for solving a dynamic problem, we give a simple example. Two economic agents simultaneously invest in labor resources in two investment regions. Each investment resource can be in one of two possible states. Each investment region can be in one of three possible economic conditions determined by a system of numbers $U : 0, 1/3, 2/3, 1$. Obviously, the system can be in one of 36 possible states defined by the following table 1.

| Table 1. Possible economic conditions. |
|----------------------------------------|
| $S$ | $b_{m_1}$ | $b_{m_2}$ | $a_{l_1}$ | $a_{l_2}$ |
|-----------------------------|-----------------------------|
| 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 2 |
| 3 | 1 | 1 | 1 | 3 |
| 4 | 1 | 1 | 3 | 1 |
| 5 | 1 | 1 | 2 | 2 |
| 6 | 1 | 1 | 2 | 3 |
| 7 | 1 | 1 | 3 | 1 |
| 8 | 1 | 1 | 3 | 2 |
| 9 | 1 | 1 | 3 | 3 |
| 10 | 1 | 2 | 1 | 1 |
| 11 | 1 | 2 | 1 | 2 |
| 12 | 1 | 2 | 1 | 3 |
| 13 | 1 | 2 | 2 | 1 |
| 14 | 1 | 2 | 2 | 2 |
| 15 | 1 | 2 | 2 | 3 |
| 16 | 1 | 2 | 3 | 1 |
| 17 | 1 | 2 | 3 | 2 |
| 18 | 1 | 2 | 3 | 3 |
| 19 | 2 | 1 | 1 | 1 |
| 20 | 2 | 1 | 1 | 2 |
| 21 | 2 | 1 | 1 | 3 |
| 22 | 2 | 1 | 2 | 1 |
| 23 | 2 | 1 | 2 | 2 |
| 24 | 2 | 1 | 2 | 3 |
| 25 | 2 | 1 | 3 | 1 |
| 26 | 2 | 1 | 3 | 2 |
| 27 | 2 | 1 | 3 | 3 |
| 28 | 2 | 2 | 1 | 1 |
| 29 | 2 | 2 | 1 | 2 |
| 30 | 2 | 2 | 1 | 3 |
| 31 | 2 | 2 | 2 | 1 |
| 32 | 2 | 2 | 2 | 2 |
| 33 | 2 | 2 | 2 | 3 |
| 34 | 2 | 2 | 3 | 1 |
| 35 | 2 | 2 | 3 | 2 |
| 36 | 2 | 2 | 3 | 3 |

Many strategies consist of two elements:

1) the first resource is invested in the first investment region, the second resource is invested in the second investment region;

2) the first resource is invested in the second investment region, the second resource is added to the first investment region.

The efficiency of labor resources investment in investment regions is determined by the following table 2.

| Table 2. The effectiveness of the investment of labor resources. |
|---------------------------------------------------------------|
| $m$ | $b$ | $E_{ml_1}$ | $E_{ml_2}$ |
|-----------------------------|-----------------------------|
| 1 | 1 | 1 | 2 |
| 1 | 2 | 0 | 1 |
| 2 | 1 | 0 | 2 |
| 2 | 2 | 1 | 1 |

Remark 1. 9th, 19th, 27th. The 36th standing systems are the final states of the investment process. Investing from these conditions is pointless.
Remark 2. The effectiveness of labor resources investment may also depend on the state in which the investment region is located.

Transitions of investment regions from one state to another are determined by the following table 3.

**Table 3. Transitions of investment regions from one state to another.**

| m | b | l | a | F |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 2 |
| 1 | 1 | 2 | 3 | 2 |
| 1 | 1 | 3 | 3 | 2 |
| 1 | 2 | 1 | 3 | 2 |
| 1 | 2 | 2 | 3 | 2 |
| 1 | 2 | 3 | 3 | 2 |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 1 | 2 | 2 | 2 |
| 2 | 1 | 3 | 3 | 2 |
| 2 | 2 | 1 | 2 | 2 |
| 2 | 2 | 2 | 3 | 2 |
| 2 | 2 | 3 | 3 | 2 |

Transitions of the labor resources of investing from one state to another are determined by the following table 4.

**Table 4. The transitions of the labor resources of investing from one state to another.**

| l | a | m | b | F |
|---|---|---|---|---|
| 1 | 1 | 1 | 2 | 2 |
| 1 | 1 | 2 | 1 | 2 |
| 1 | 2 | 1 | 1 | 2 |
| 1 | 2 | 2 | 2 | 2 |
| 2 | 1 | 1 | 1 | 2 |
| 2 | 1 | 2 | 2 | 2 |
| 2 | 2 | 1 | 1 | 2 |
| 2 | 2 | 2 | 2 | 2 |
| 3 | 1 | 1 | 1 | 2 |
| 3 | 1 | 2 | 3 | 2 |
| 3 | 2 | 1 | 1 | 2 |
| 3 | 2 | 2 | 2 | 2 |

Using the tables above, you can determine the function of the transition of the system from one state to another. Table 5 below shows the table definition of the function \( \tilde{F}(q,s) \).

**Table 5. Table task function \( \tilde{F}(q,s) \).**

| q | S | F | r | q | S | F | r | q | S | F | r | q | S | F | r |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 33 | 3 | 1 | 19 | 12 | 2 | 2 | 21 | 3 | 2 | 19 | 19 | 0 |
| 1 | 2 | 15 | 2 | 1 | 20 | 3 | 1 | 2 | 21 | 2 | 2 | 20 | 6 | 1 |
| 1 | 3 | 15 | 1 | 1 | 21 | 3 | 0 | 2 | 21 | 3 | 0 | 2 | 21 | 3 | 0 |
| 1 | 4 | 36 | 3 | 1 | 22 | 33 | 2 | 2 | 24 | 2 | 2 | 22 | 23 | 1 |
In table 5, in addition to the values of the function of the transition of the system from one state to another, the values of the income received are indicated. The amount of income was calculated by the formula (5). The determination of the optimal strategy in a one-step process is carried out by simple comparisons of the incomes obtained for different strategies and the choice of the strategy for which this value will be greater. Determining the optimal strategy in a two-step process is as follows:

1) for each strategy, the amount of income received during the transition and the state into which the system will go with this strategy are determined;
2) determine the amount of income received in one-step processes from the states to which the system can go when using various strategies;
3) incomes are summarized according to (5) and compared with each other.

The calculation results for this example are summarized in table 6, in which, for each state of the system, the optimal strategies and the values of the received income are shown in the one-step, two-step, and three-step investment processes, respectively. If the column contains 0, then this means that investing does not make sense, since the state is final. If - is indicated, then this indicates the pointlessness of investing, since the final state is achieved in fewer steps.

Table 6. The results of the calculations.

| T-1 | 1 | 2 | 3 | T-1 | 1 | 2 | 3 |
|-----|---|---|---|-----|---|---|---|
| S   | q | V | q | V | q | V | q | V |
|-----|---|---|---|---|---|---|---|---|
| 1   | 1 | 3 | 1 | 4 | - | 19 | 1 | 2 | 1 | 3 | 1 | 4 |
| 2   | 1 | 2 | 2 | 3 | - | 20 | 1 | 1 | 2 | 2 | - |   |
| 3   | 1 | 1 | 1 | 2 | - | 21 | 1 | 0 | 1 | 1 | 1 | 2 |
| 4   | 1 | 3 | - | - | 22 | 1 | 2 | 1 | 3 | - |   |
| 5   | 1 | 2 | - | - | 23 | 2 | 1 | 2 | 2 | - |   |
| 6   | 1 | 1 | - | - | 24 | 2 | 0 | 2 | 1 | - |   |
| 7   | 1 | 2 | - | - | 25 | 1 | 2 | - | - |   |
| 8   | 1 | 1 | - | - | 26 | 1 | 1 | - | - |   |
| 9   | 0 | 0 | 0 | 0 | 27 | 0 | 0 | 0 | 0 |   |   |
| 10  | 2 | 3 | 2 | 4 | - | 28 | 2 | 2 | 2 | 4 | - |   |
| 11  | 1 | 2 | 2 | 3 | - | 29 | 2 | 2 | 2 | 3 | - |   |
| 12  | 2 | 1 | 2 | 2 | - | 30 | 2 | 1 | 2 | 2 | - |   |
| 13  | 1 | 3 | - | - | 31 | 2 | 2 | 1 | 3 | - |   |
| 14  | 1 | 2 | - | - | 32 | 2 | 2 | - | - |   |   |
| 15  | 2 | 1 | - | - | 33 | 2 | 1 | - | - |   |   |
| 16  | 2 | 2 | - | - | 34 | 1 | 1 | 1 | 2 | - |   |
4. Conclusion
Table 6 contains the solution to the problem of optimal investment. It is seen from it that the problem can be solved in no more than three time periods, and if the initial state is any, except for the 19th and 21st, then the problem can be solved in no more than two time periods. Thus, for any situation (see table 1 of the example), you can specify the time for which it is possible to solve the problem of optimal investment and optimal strategies at any time.

The optimal way to solve the problem of optimal investment can be implemented on a computer using simple programs, although it should be noted that the amount of required memory increases significantly with an increase in the number of investment regions, the number of investment labor resources and the number of their conditions.

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References
[1] Bondarenko L A, Zubov A V, Orlov V B, Petrova V A and Ugegov N S 2016 Application in practice and optimization of industrial information systems Journal of Theoretical and Applied Information Technology Vol. 85 3
[2] Bondarenko L A, Zubov A V, Zubova A F, Zubov S V and Orlov V B 2015 Stability of quasilinear dynamic systems with after effect Biosciences Biotechnology Research Asia Vol. 12 1
[3] Dikusar V V, Zubov A V and Zubov N V 2010 Structural minimization of stationary control and observation systems Journal of Computer and Systems Sciences International Vol. 49 4
[4] Malafeev O 1995 On the existence of nash equilibria in a noncooperative n-person game with measures as coefficients Communications in Applied Mathematics and Computational Science 5 (4) 689–701
[5] Malafeev O and Nemnyugin S 1996 Generalized dynamic model of a system moving in an external field with stochastic components Theoretical and Mathematical Physics 10 (3) 770
[6] Malafeyev O, Zaitseva I, Onishenko V, Zubov A, Bondarenko L, Orlov V, Petrova V and Kirjanen A 2019 Optimal location problem in the transportation network as an investment project: A numerical method AIP Conference Proceedings 2116 450058
[7] Murashko A Y, Orlov V B, Zubov A V, Bondarenko L A and Petrova V A 2019 Qualitative analysis of the behavior of one mechanical system International Journal of Innovative Technology and Exploring Engineering 8 (7)
[8] Neverova E G, Malafeyev O A, Alferov G V and Smirnova T E 2015 Model of interaction between anticorruption authorities and corruption groups” in International Conference on Stability and Control Processes in Memory of V.I. Zubov (SCP, Proceedings, SPb) 488–490
[9] Pichugin Y A, Malafeyev O A, Rylov D and Zaitseva I 2018 A statistical method for corrupt agents detection AIP Conference Proceedings https://doi.org/10.1063/1.5043758
[10] Sychev S 2015 Technologies for fast economical construction of residential buildings ARPN Journal of Engineering and Applied Sciences 10 (17)
[11] Sychev S and Badjin G An interactive construction project for method of statement based on BIM technologies for high-speed modular building Architecture and Engineering Vol. 1
[12] Sychev S and Sharipova D 2015 Monitoring and logistics of erection of prefabricated modular buildings Indian Journal of Science and Technology 8 (29)
[13] Vlasov M A, Glebov V V, Malafeyev O A and Novichkov D N 1986 Experimental study of an electron beam in drift space Soviet journal of communications technology & electronics 31 (3) 145–149
[14] Zaitseva I 2019 Numerical method of distribution of labor resources by game-theoretic model
Zaitseva I, Malafeyev O, Dolgopolova A, Zhukova V and Vorokhobina Y 2019 Numerical method for computing equilibria in economic system models with labor force AIP Conference Proceedings 2116 450057

Zaitseva I, Malafeyev O, Marenchuk Y, Kolesov D and Bogdanova S 2019 Competitive Mechanism for the Distribution of Labor Resources in the Transport Objective Journal of Physics: Conference Series

Zaitseva I, Malafeyev O, Poddubnaya N, Vanina A and Novikova E 2019 Solving a dynamic assignment problem in the socio-economic system Journal of Physics

Zaitseva I, Malafeyev O, Strekopytov S, Bondarenko G and Lovyannikov D 2018 Mathematic model of regional economy development by the final result of labor resources AIP Conference Proceedings

Zubov A V, Dikusar V V and Zubov N V 2010 Controllability criterion for stationary systems Doklady Mathematics Vol. 81 1

Zubov A V, Murashko A Y, Kolyada L G, Volkova E A and Zubova O A 2016 Fidelity issue of engineering analysis and computer aided calculations in sign models of dynamic systems Global Journal of Pure and Applied Mathematics. Vol. 12 5

Zubov A V, Orlov V B, Petrova V A, Bondarenko L A and Pupysheva G I 2017 Engineering and Computer Aided Calculations upon Determination of Quality Characteristics of Dynamic Systems International Journal of Pure and Applied Mathematics Vol. 117 22

Zubov A V 2007 Stabilization of program motion and kinematic trajectories in dynamic systems in case of systems of direct and indirect control Automation and Remote Control Vol. 68 3

Zubov I V and Zubov A V 2009 The stability of motion of dynamic systems Doklady Mathematics Vol. 79 1