Obstructions to dimensional reduction in hot QCD

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I describe results on screening masses in hot gauge theories. Wilsonian effective long distance theories called dimensionally reduced (DR) theories describe very well the longest screening length in pure gauge theories. In the presence of fermions, meson-like screening lengths dominate the long-distance physics for $3T_c/2 \leq T < 3T_c$, and thus obstruct perturbative DR. Extrapolation of our results indicates that a form of this obstruction may remain till temperatures of $10T_c$ or higher, and therefore affect the entire range of temperature expected to be reached even at the Large Hadron Collider.

1. Dimensional reduction

Finite temperature field theory in its Euclidean formulation exists on an infinite spatial volume but a finite (Euclidean) temporal extent of $1/T$. As a result, the Fourier modes of gluon fields have a countably infinite set of momentum components in this direction—$k_0 = 2\pi n T$ for $n = 0, \pm 1, \pm 2, \cdots$. One might be able to integrate over the non-zero modes to find a Wilsonian effective theory at distances larger than $1/T$ [1]. The matching of correlation functions in the two theories must be performed at a momentum scale $\Lambda_T \approx O(T)$ [2]. Consistency in applying perturbation theory then demands that $\alpha_S(\Lambda_T) \ll 1$. After matching correlation functions in the two theories, the low-energy effective theory must have exactly the same physics as the full 4-d theory—the same correlation functions, the same long distance screening behaviour, and so on [3].

This procedure works extremely well in scalar $\phi^4$ theories [3]. It works for the gauge-Higgs system that describes the dynamics of the electroweak theory near its finite temperature phase transition [4]. It has also been tested for SU(2) pure gauge theory at $T \geq 2T_c$. The spectrum of screening masses in SU(2) has been extracted from correlation functions of local operators built from gluon fields [3]. The perturbatively determined dimensionally reduced (DR) theory has also been simulated numerically and its screening masses have been determined [3]. The lowest screening mass (longest screening length) belongs to the thermal scalar sector (see [7] for the group theory). At $2T_c$ we find

$$\frac{\mu(0_+)}{T} = \begin{cases} 2.9 \pm 0.2 & (4\text{-d theory}), \\ 2.86 \pm 0.03 & (\text{DR theory}) \end{cases}$$

Thus, at length scales of about $1/3T$ and more, the DR theory gives a good description of the physics of the 4-d theory.

2. Limits of DR and puzzles

How does DR fare at distances less than $1/\mu(0_+)$? To answer this we look at the next higher screening masses in the two theories. Some notation will be useful—screening masses describe propagation from one two-dimensional slice of space to another [3]. As a result, they are labelled, not by the angular momentum $J$ of 3-d, but by its two dimensional analogue, $J_z$, the projection along the normal to the 2-d slices of space. For every even $J$, the $J_z = 0$ states are the thermal scalar $0_+$, and the $J_z = 0$ of odd $J$ are the thermal quasi-scalar $0_-$. The $J_z = \pm 1$ states make the real two-dimensional irrep, the
1. Similarly the $J_z = \pm 2$ make up the thermal 2. Measurements showed that $\mu(0_+) < \mu(0_-) < \mu(1) \approx \mu(2)$. So, the question is about the comparison of the higher screening masses in the 4-d and DR theories.

In Figure 1 a comparison of the 4-d and DR theories is shown. Notice the following points—

1. In the 4-d simulations, finite volume effects are under good control. The finite volume measurements extrapolate smoothly to the infinite volume limit.

2. Lattice spacing effects are under reasonable control, since the error ellipses for lattice spacings of $1/8T_c$ and $1/12T_c$ overlap significantly.

3. The 4-D theory lies many standard deviations away from the DR theory by this measure.

In the light of the previous discussion, this mismatch is not unexpected, since the 4-d and DR theories are matched at $\Lambda_T = 2\pi T_c$, and the higher screening masses all lie rather close to this scale. However, we have gained a quantitative bound to the length scale at which DR fails in the pure gauge theory— DR cannot describe physics at length scales of $1/\mu(0_-)$ or shorter.

This, in fact, is a puzzle. Recall that we have labelled states by the dimensionally reduced symmetry group. The very fact that this symmetry is obeyed even by the higher states shows that some version of DR occurs in the theory, although it cannot be obtained by perturbative matching.

3. Including fermions

Even if the weaker form of perturbative DR works, then inclusion of Fermions involves no new problems. Fermion modes are antisymmetric in the Euclidean time direction and hence have $k_0 = \pi(2n + 1)T$. There are no zero modes and Fermions can be entirely integrated out. As a result, there are no Fermion remnants in the long-distance theory— every correlation function involving Fermion field operators is integrated out of the DR theory. The only traces of Fermions are subtle: they influence the dimensionful couplings in the long-distance effective theory. A simulation of QCD with four flavours of dynamical fermions shows that this picture breaks down completely at $T \approx 1–3 T_c$.

We have simulated QCD on lattices of various sizes at temperatures of $3T_c/2$, $2T_c$, and $3T_c$ and collected statistics over 1000–2000 mutually uncorrelated configurations. Simulation details can be found elsewhere. Over these configurations of thermal gauge fields, we have constructed correlators from quark-anti-quark operators. The corresponding screening masses we call meson-like. We have used operators corresponding to the $T = 0 \pi$ and $\sigma$ and the vector and pseudo-vector mesons. All of these give non-trivial correlations in the $0_+$ state. The $\pi$ and $\sigma$-like screening masses are equal within our statistics as are the vector and pseudo-vector $0_+$ screening masses. However, these two sets of $0_+$ screening masses are not equal to each other. The $\pi$-like screening mass, $\mu_\pi(0_+)$ is smaller. Measurements on various lattice sizes (Figure 1) show the lack of finite volume effects.
We also measured the $0_+$ screening mass in the glue sector of the theory, $\mu_g(0_+)$, and found that $\mu_\pi(0_+) < \mu_g(0_+)$ for $T < 3T_c$ (Figure 3). This is a direct obstacle to DR at $T < 3T_c$; since the DR theory does not admit any Fermion operators, it is ineffective in describing the true long-distance physics of the QCD plasma, which arises in the Fermion sector.

Actually the Fermionic roadblock to DR is even wider. In a free-Fermion theory, the meson-like screening correlators would decay at long distances with an effective mass of $2\pi T$. $O(a^2)$ lattice artifacts would change this numerical value at the lattice spacing, $a$, that we work at into the lower number indicated in Figure 3. This is the value $\mu_\pi(0_+)$ must have if perturbation theory were to be reliable at the scale of $\mu_\pi(0_+)$. Within perturbation theory unless $\mu_\pi(0_+) \approx 2\pi T$. Hence, this less obvious roadblock to DR.

We have tried various 2 parameter fits to our three data points to see at what $T$ we get $\mu_\pi(0_+) \approx 2\pi T$. The envelope of these fits are given by the two curves shown in Figure 3. This purely phenomenological approach tells us that the obstruction to DR would persist up to $T \approx 10T_c$ or greater. It would, of course, be best to simulate the theory at such temperatures. However, to avoid finite volume effects with $N_t = 4$ one would then have to take spatial sizes greater than $40^3$ lattice units. Such a computation with dynamical fermions is prohibitively difficult at present.

4. Other phenomena

Recently, screening masses have been computed in a DR theory obtained by perturbative matching to QCD with dynamical Fermions [10]. Knowing of the problems with this DR theory, it is nevertheless interesting to ask what it predicts for the screening masses in the glue sector, and how it compares with data from the 4-d theory. The temperatures in [10] have been specified in units of $\Lambda_{\overline{MS}}$ whereas those in [8] are in
units of $T_c$. The quantitative connection between these units is made possible by a recent determination \[ T_c/\Lambda_{\overline{\text{MS}}} = 1.07 \pm 0.05 \] for 4-flavour QCD at the quark masses used in \[ 5. \]

At $T = 2\Lambda_{\overline{\text{MS}}}$ we find

$$\frac{\mu_g(0_+)}{T} = \left\{ \begin{array}{l}
4.04 \pm 0.05 \quad (\text{4-d theory}), \\
4.87 \pm 0.07 \quad (\text{DR theory}).
\end{array} \right.$$ (2)

There is a statistically significant mismatch between these two numbers, as we might now expect. The number quoted above for the 4-d theory is at $T = 2T_c = (2.14 \pm 0.10)\Lambda_{\overline{\text{MS}}}$. Moving to $T = 2\Lambda_{\overline{\text{MS}}}$ would mean lowering the temperature. As seen in Figure 3 this would lower $\mu_g(0_+)$ slightly, making the discrepancy slightly worse.

The test can be pushed further in terms of the ratios of screening masses, as shown in Figure 4. In the DR theory all the masses have been extracted. In the 4-d theory a preliminary measurement of the glue sector $\mu(2)$ has been made, and the ratio $\mu(2)/\mu(0_+)$ is shown as the horizontal band. The mismatch between the DR and 4-d theories is obvious.

5. Conclusions

Dimensional reduction with perturbative matching of the DR theory to the full 4-d finite temperature theory seems to work extremely well in many cases. For 4-d $SU(2)$ pure gauge theory it works superbly at the scale of the longest screening length, but fails at the scale of the next screening length. This is expected if we consider DR as a Wilsonian effective theory at the scale of the longest screening length. However, for QCD with Fermions at $T \lesssim 3T_c$ it fails in all respects—it does not contain the Fermion composite states which give the longest correlation length, it gives wrong results for the longest gluon screening length, and it does not correctly reproduce the ratios of gluon screening lengths. At higher temperatures, up to about $10T_c$, arguments given here lead us to believe that it would be impossible to construct this effective theory in QCD through a perturbative matching of correlation functions.

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