Aspects of the Particle Finite Element Method applied to contact problems

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The Particle Finite Element Method (PFEM) developed by [1] is designed to allow changes in the outer shape of a body while performing a Finite Element Analysis. This is achieved by repeated remeshing and shape detection of a particle cloud, which represents the body under consideration. The shape detection method creates new surfaces if formerly connected points on the surface move away from each other sufficiently. This method allows to model material separation, e.g. in metal cutting, see [2]. Details on advanced strategies to refine the detected shape are presented. Considering abrasive wear of surfaces, a mechanism-based approach aims to model such separation of surface particles on a mesoscale. The basis of this approach consists of a contact algorithm that captures the interaction of solid bodies. This contribution presents a contact domain method [3] which is implemented in PFEM. The approach uses the abilities of the shape detection method as a contact search method. Furthermore, strategies for solving large strain boundary value problems using this contact model are discussed. Numerical examples show the abilities of the approach in an exemplary large strain contact problem.

1 Introduction

PFEM is based on a standard Finite Element Analysis, whereby the elements of this analysis do not persist during the simulation but only the node points, which are considered as particles. These particles are repeatedly remeshed and the outer shape of the body described by the particle cloud is captured by an α-shape method, see [4]. The α-shape method can also be used to detect contact surfaces when bodies approach each other. During contact detection, a layer of triangular segments is created between the bodies, the size of which defines the area of the expected contact. These segments are also used to discretise the contact-domain and to enforce contact constraints. This approach has been presented in [3], where also a penalty contact model for these contact elements is proposed. It is based on an approximation of the current distance between the bodies using small strain theory. In this work the model is applied to contact problems with large deformations in PFEM. The deformation of a contact segment within one loadstep is assumed to be small. However, the solution of this framework is not trivial, since a number of contact elements can come into contact or release within one loadstep which has large impact on neighbouring contact elements.

2 Theory

Following [5], the considered point cloud is meshed with an unconstrained Delaunay triangulation. An α-test is then carried out for each element to check whether a test radius fits into the circumsphere of the triangle. If this is the case, the triangle is discarded. The test for emptiness, which usually is part of the α-shape technique, is obsolete due to the emptiness property of the Delaunay triangulation. A point density dependent α-radius is applied to allow for inhomogeneous point densities within the body. A disadvantage of the mesh density dependent α-value is a sensitivity to long triangles near corners of the mesh where only a few elements contribute to the local test radius-value. One solution is to refine the α-shape by deleting elements outside the shape of the previous triangulation based on a criterion, see [5]. In this work an improved criterion is used which deletes all elements where an outward normal vector of the old shape points into the element.

The creation of a contact segment mesh in the region where contact is likely to occur, again uses the α-shape method. Here a particle-class consisting only of boundary particles is triangulised and the α-shape is detected. To guarantee a minimum distance between the bodies, they are slightly shrunk, only for the meshing step, see [6].

A straightforward contact model for contact patches taken from [3] is used in this work. The gap between contacting bodies is penalised as soon as penetration occurs. The current gap \( g_N \) is approximated based on the initial normal gap length \( G_N \) in direction \( n \) using the following small strain approach,

\[
g_N = G_N + |G_N| n \cdot \varepsilon \cdot n , \quad \text{with} \quad \varepsilon = \nabla_{\text{sym}} u .
\]  

The penalty term is not added to the weak form of equilibrium in terms of variation of a penalty potential, but in analogy to an internal force term. Hence, the contact model is formulated in a material model-like manner based on the definition of Cauchy stresses \( \sigma \). Normal penetration is therefore linearly penalised with the parameter \( \varepsilon \) and tangential reaction forces are taken into account using Coulomb friction, combined with tangent hyperbolic regularisation near the origin. The complete model
reads
\[ f^A_{\text{cont}} = \int_{B_c} \nabla_x N_A \cdot \sigma \, d\alpha, \quad \sigma = \frac{g_N}{t} n \otimes n + \mu |p_N| \tanh(2t \cdot \varepsilon \cdot n) [t \otimes n]^{\text{sym}}. \]  

Solution of the proposed framework suffers from two main difficulties: to find an unique set of active contact elements and to determine an adequate penalty parameter value. An Augmented-Lagrange-type strategy is proposed here: the penalty parameters are initialised with low values and increased individually until both, the active set and a chosen penetration tolerance are fulfilled. Update of active set and penalty parameters are only performed for converged states, i.e. a given timestep has to be calculated repeatedly. Adaptive timestepping increases stability if necessary.

### 3 Numerical Examples

A benchmark problem taken from [6] shows the ability of the model also for large strain simulation. In the shallow ironing test, a rounded tool is indented into a soft block of \(4 \times 12\) mm and slid over the surface. The block is clamped at the lower edge and the tool is moved displacement driven. Both materials are modelled hyperelastic with \(E_{\text{Tool}} = 68960\) MPa, \(E_{\text{Block}} = 6896\) MPa, \(\nu = 0.32\). Figures 1 and 2 show the overall results of the benchmark, whereas Figure 3 reveals the exact resulting friction characteristica. Although the simulation is carried out with local friction coefficient \(\mu = 0\), the indentation creates structural resistance up to \(\mu_{\text{eff}} = 0.5\) depending on indentation depth, see Table 1.

![Fig. 1: Rounded tool indenting in soft block and sliding over the surface, resulting vertical Cauchy stresses.](image1)

![Fig. 2: Contact forces and horizontal stresses show pressure transmission and structural resistance due to indentation using a purely normal contact law.](image2)

![Fig. 3: Summed reaction forces quantify frictional behavior for purely normal contact model.](image3)

### Table 1: Effective friction coefficients \(\mu_{\text{eff}}\), cf. [7], for different indentation depths.

| Indentation depth [mm] | \(\mu_{\text{eff}} = \int F_{\text{tangential}} / F_{\text{normal}} \, dt\) |
|------------------------|--------------------------------------------------|
| 0.2                    | 0.3236                                           |
| 0.4                    | 0.3309                                           |
| 0.8                    | 0.5001                                           |

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