Feasibility study to measure the muon bremsstrahlung cross section with the energy loss profile using neutrino telescopes

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Abstract. Muons are the dominant event signature for neutrino telescopes like IceCube and they are the main background for neutrino searches. Furthermore, they are used to investigate extended air showers. In both cases, the stochasticity of the muon propagation plays a key role in the data extraction step and an accurate understanding, even of the edge cases, is crucial. The main process driving stochastic losses for TeV scale muons is bremsstrahlung.

In this paper, a feasibility study is presented to measure the cross section of stochastic losses using neutrino-induced muons. The simulation study is based on the propagation of muons using the Monte-Carlo library PROPOSAL. For different reconstruction methods and resolutions, the energy loss distribution for different muon energies is used to estimate the sensitivity to measure the bremsstrahlung cross section. Two further systematic parameters, the detection efficiency, which scales the amount of detected light, and the spectral index are also estimated to analyze their correlation to the estimated bremsstrahlung normalization. The statistics of the simulated dataset correspond to 10 years of up-going muon neutrino data in IceCube.

1. Introduction

For neutrino telescopes and other deep underground experiments, muons are the main background, dominating the event rate by several orders of magnitude. Due to their stochastic behavior during propagation, muons may remain undetected by the veto mechanism (e.g. [1]) while producing a single large energy loss inside the detector volume, mimicking neutrino-like signal events. Therefore, a precise description of the muon propagation in the Monte-Carlo simulation is crucial for an accurate background estimation, especially for the edge cases like the highest energetic losses. This importance is even more pronounced for searches of extremely rare events such as tau neutrinos [2] or Glashow resonances [1].

The large, stochastic energy losses are mainly driven by bremsstrahlung interactions, cf. figure 2. The bremsstrahlung cross section has an overall uncertainty of a few percent [3], while recent calculations increase the accuracy below a percent [4]. However, both calculations assume that the outgoing particles are highly boosted in the forward direction, which does not hold for the highest energy losses resulting in increased uncertainties in these regions.
To answer the question whether neutrino telescopes are sensitive to these uncertainties, this simulation study was developed. For three different detector resolutions, the sensitivity to measure the normalization of the bremsstrahlung cross section is calculated using the energy loss distribution of muons propagated through the detector.

Measurements of the muon cross sections have been made up to the TeV region [5], where ionization is still dominating the average energy loss. At higher energies, the radiative processes pair production, bremsstrahlung and inelastic interaction with nuclei dominate the energy loss. A measurement around 10 TeV would be the first measurement at these energies. Therefore, the muon energy range is chosen between 1 TeV and 100 TeV with the upper limit chosen due to the limited event statistic.

Although atmospheric muons are abundant, their usage is limited for this measurement as it requires a sample of single muons. Atmospheric muons predominantly enter the detector as bundles which are hard to distinguish from single muons. Therefore, a neutrino-induced muon sample is used which ensures a selection of single muons. The event statistic in this simulation study is loosely based on the Northern track sample of the IceCube detector from the diffuse $\nu_\mu$ analysis [6], which contains almost 10 years of data. According to the event spectrum of the aforementioned selection, there are around 245,000 events between the neutrino energies of a TeV and 100 TeV with a spectral index of 1.63. The resulting secondary muons have a steeper spectrum due to propagation losses. Their distribution is further shifted to lower energies because a part of the neutrino energy is transferred to the hadronic cascade at the vertex. The muon spectrum assumed here is a rough estimation, but nevertheless sufficient for this feasibility study.

2. Simulation and Reconstruction

The Monte-Carlo simulation of the muons is performed using the lepton propagator PROPOSAL [7,8]. Muons with a flat energy spectrum between 100 GeV and 1 PeV are propagated through ice. To take into account the different propagation lengths inside the detector, the maximum propagation distance is varied between 100 m and 1 km. The produced secondaries through interactions or decays can be stacked to an energy loss distribution shown in figure 2. The further simulation steps, especially the photon propagation, require a large amount of resources. Acceptance corrections, trigger simulation and reconstruction methods are detector specific and also depend on the specifics of the event selection. In order to reduce resource requirements and to ensure applicability to neutrino telescopes in all generality, a simplified photon and
detector simulation is applied. In this simplified approach, the secondaries produced in the muon simulation are smeared out to produce realistic and measurable energy loss segments along the muon track.

The deposited energies of the secondaries are smeared out along the track with a Gaussian distribution centered at each interaction point with a certain vertex resolution. The muon track is divided into segments with a certain length, accumulating the smeared out parts of losses inside these segments. In addition to the stochastic energy losses, the continuous losses are considered according to their length in each segment. The hits of the DOMs in each segment are simulated by sampling from a Poissonian distribution centered at the energy in MeV. To simulate the finite energy resolution, the sampled hits in each track segment are then smeared in log-space according to a Gaussian distribution with an energy dependent width. This energy dependent width is taken from [9] and scaled by a certain energy resolution factor. Finally, the limited detector sensitivity is taken into account by discarding the segments with an energy below a certain energy cutoff.

In addition to the energy losses per track segment, the propagation length is smeared independently by a Gaussian distribution with a certain propagation length resolution. Based on the track segments, the energy is reconstructed using two independent methods.

One method called truncated energy estimates the energy analogous to the truncated mean method from [9] using the correlation between the average energy loss and the muon energy. Thereby, the 10% of track segments with the largest losses are discarded as the stochastic losses are only minimally correlated to the energy. The average energy loss per distance is based on the remaining segments with the benefit, that this method is robust against small changes of the bremsstrahlung cross section, which just dominates the high energy losses.

The other method to estimate the muon energy uses a neuronal network. This second approach is employed as a cross-check to the truncated energy method. Recent advances in reconstruction techniques have shown the capabilities of neural network-based reconstruction methods [10]. In principle it can learn the same truncated mean method as described above while being able to use further information e.g. by using the stochastic losses as a lower limit. A convolutional network is trained on the track segments resulting in a similar performance.

Furthermore, the same network architecture is used to estimate an uncertainty on the estimated energy. This estimate is used for both energy reconstruction methods to discard events with a high energy uncertainty.

| Resolution and cuts               | High | Medium | Low |
|-----------------------------------|------|--------|-----|
| Vertex res. / m                   | 2    | 5      | 10  |
| Track Segment Length / m          | 5    | 15     | 30  |
| Energy res. factor                | 0.5  | 1.0    | 1.5 |
| Energy cutoff / MeV               | 10   | 50     | 100 |
| Propagation Length res. / m       | 10   | 50     | 100 |
| Energy Uncertainty cut            | 0.2  | 0.4    | 0.6 |
| $R^2$ cut                         | 0.9  | 0.95   | 0.995 |

Muons are simulated and reconstructed for three different resolution settings: high, medium and low, as described in table 1. The medium resolution settings are chosen to be similar to the resolution of the IceCube detector. The high resolution settings are more comparable to the DeepCore detector, a low energy extension inside of IceCube, and the low resolutions may be
expected for a neutrino detector with a string spacing as planned for IceCube Gen2. But for all three configurations, the assumed size of the detector remains the same in this study as the propagation length distribution is not adapted.

3. Measurement
An event selection is performed to discard mis-reconstructed events. First, events traveling less than 100 m are filtered out to avoid muons skimming the detector volume. The second cut is performed on the energy uncertainty and it depends on the resolution (listed in table 1). It is chosen to strike a balance between selecting well reconstructed events and retaining enough statistics. Furthermore, only muon energies between 1 TeV and 100 TeV are selected in five ranges. Lower energetic muons are dominated by ionization losses and at higher energies there is not enough statistics. The selected muon range is divided into three equally sized bins in log10-space between 1 TeV and 10 TeV and two bins between 10 TeV and 100 TeV.

For the five muon energy ranges, an energy loss histogram is created based on the reconstructed muon track segments while accounting for the simulation weights for the chosen energy spectrum. Multiple simulation datasets are created, each with a different scaling of the bremsstrahlung cross section, or bremsstrahlung multiplier, varied by ±10 %, to interpolate the differences in the energy loss histograms. Therefore, the histograms are normalized by the propagated length.

$$\text{Normalized Energy Loss Histogram} = \frac{\sum L_i w_i}{\sum d_i w_i}$$ (1)

with the energy loss histogram $L_i$, the energy weight $w_i$ and the propagated length $d_i$ per event $i$.

Two further systematic parameters are considered, the DOM efficiency and the spectral index of the muon energy spectrum as they both affect the energy loss distribution. The DOM efficiency scales the energy losses per track segment, representing the uncertainty in the efficiency of the photomultiplier. It is varied by ±10 %. The spectral index specifies the power law of the assumed muon energy spectrum, which is varied between 1.5 and 1.9 with 1.7 as the baseline value. Their effects on the energy loss profile are also interpolated, resulting in a three dimensional interpolation function for each energy loss bin. The coefficient of determination

$$R^2 = 1 - \frac{\sum (y - f(x))^2}{\sum (y - \bar{y})^2},$$ (2)

with $f(x)$ representing the interpolation function and $y$ a simulated data point, is calculated for each bin. With a resolution dependent $R^2$ cut (see table 1), energy loss bins are discarded in which the fluctuations are too high.

With these interpolations, a Poisson Likelihood is defined to describe the bin contents of the energy loss distributions. An MCMC sampling is then applied to estimate the three parameters including their correlations, shown exemplary in figure 2.

To estimate the performance of the bremsstrahlung multiplier measurement for the different resolutions, simulation sets with random bremsstrahlung multipliers and fixed systematic parameters are produced. The correlations of the MCMC results are shown in figure 3.

For the high resolution, the bremsstrahlung multiplier can be estimated with ±2 % and for the medium resolution, this increases to ±4 %. For the low resolution, it is not feasible to estimate the bremsstrahlung multiplier. There is no notable difference in the performance between the truncated energy and the neural network approach to estimate the muon energy.

4. Conclusion
In this feasibility study, the ability to measure the bremsstrahlung cross section with neutrino telescopes is analyzed. To this end, a toy Monte-Carlo simulation is employed to obtain realistic
Figure 2. Corner plot of an exemplary MCMC sampling for the high resolution setting visualizing the correlation between the Spectral index, the DOM efficiency and the bremsstrahlung multiplier. The muon energy is reconstructed using the truncated energy method.

Figure 3. Results of the MCMC samplings with the high resolution settings for the Spectral index, the DOM efficiency and the bremsstrahlung multiplier. The muon energy is reconstructed using the truncated energy method. The region below 0.95 and above 1.05 is neglected for the performance to avoid boundary effects during the MCMC sampling at the edge of the allowed interpolation region.
energy losses per track segment based on the secondaries calculated by PROPOSAL. Using the reconstructed energy loss distributions for muon energies between 1 TeV and 100 TeV, the bremsstrahlung is estimated along with the quantum efficiency of the photomultiplier and the spectral index of the event spectrum as further systematic parameters. A detector with an IceCube-like resolution can measure the normalization of the bremsstrahlung cross section with an uncertainty of ±4%.

As the bremsstrahlung uncertainties have detectable effects, they could be considered as a further systematic parameter in event selections which depend on the muon stochasticity. In addition, the uncertainties of the other muon interactions can be included in the systematic treatments. With the recently developed SnowStorm simulation procedure [11], where the variations of all possible systematics are considered in one dataset, this can easily be included without creating new simulation sets.

Apart from the neutrino-induced muons, atmospheric muon samples can also be used to analyze the muon physics experimentally. By applying a selection of incoming muons stopping inside the detector, which is dominated by single muons, ionization and DOM efficiency can be further investigated for lower energies. Furthermore, leading muon events, where one muon contains most of the muon bundle energy, can be used to analyze the stochasticity of muons. A combined analysis of these three approaches can further constrain the uncertainties on the muon cross section measurement.

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