The high-frequency conductivity of a thin semiconductor wire in the longitudinal magnetic field

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Abstract. The problem of high-frequency conductivity of a thin semiconductor cylindrical wire at arbitrary temperature in the longitudinal magnetic field is solved by the kinetic method. The charge carrier reflection from inner wire surface is assumed to be diffuse. The ratio between the mean free path of charge carriers and the wire radius is supposed to be arbitrary. The results obtained for limiting cases of degenerate and nondegenerate semiconductor were subjected to comparative analysis.

1. Introduction
The electrical properties of conductive materials with the linear size comparable to the mean free path of charge carriers differ substantially from the corresponding properties of bulk materials [1]. Particularly a nontrivial dependence of conductivity on ratio \(R/\lambda\) (\(R\) is the wire radius, \(\lambda\) is the mean free path of charge carriers) appears.

In many conventional semiconductors at room temperature the mean free path of charge carriers \(\lambda = 10 \div 1000\) nm, and de Broglie wavelength is \(\lambda_B \sim 10\) nm. In high-conductivity metals we have \(\lambda = 10 \div 100\) nm, and de Broglie wavelength is of the order of the interatomic distance: \(\lambda_B \approx 0.3\) nm [2, 3]. Modern technology allows one to prepare the materials with the size of the order of several dozen nanometers. Therefore the situation when we can disregard the quantum confinement effects and take into account the classical size effects is actually occurs. It should be noted that in recent years because of rapid development of micro- and nanoelectronics the problems of thin wire conductivity become much more actual.

In work [1] the problem about the influence of surface scattering on the conductivity and the magnetic properties of thin films and wires is considered for the case of constant electrical and magnetic fields. In some works [4 – 7] the problems of the conductivity of thin wire with a rectangular [4] and circular [5 – 7] cross-section are solved. A metal wire was considered in works [4 - 7] and a semiconductor wire was considered in work [7]. In [8 – 10] the problem of the conductivity of a thin metal wire in longitudinal magnetic field was studied.

In this paper the expression of high-frequency conductivity of a thin semiconductor wire with a circular cross section in a longitudinal magnetic field at arbitrary temperature is obtained by the kinetic method. Charge carriers are assumed to be reflected from the wire boundary diffusely.
2. Conductivity calculation

We consider a cylindrical semiconductor wire with radius $R$, which is much smaller than length $L$. This wire located in a magnetic field with induction $B$. An alternating electrical voltage with frequency $\omega$ is applied to its ends. The directions of electrical and magnetic fields coincides with the wire symmetry axis. A skin effect isn’t considered (we assume that $R < \delta$, where $\delta$ is the skin layer depth).

A homogeneous time-periodic electric field

$$E = E_0 \exp(-i \omega t)$$

(1)

affects on the charge carriers and induces the deviation of the distribution function of those $f_i$ from the equilibrium Fermi-Dirac’s distribution function $f_0$:

$$f(\mathbf{r}, \mathbf{v}, t) = f_0(\varepsilon) + f_1(\mathbf{r}, \mathbf{v}, t) = f_0(\varepsilon) + f_1(\mathbf{r}, \mathbf{v}) \exp(-i \omega t),$$

(2)

where $\varepsilon = \frac{mv^2}{2}$ is the kinetic energy of an electron (a hole) for the case of spherically symmetric energy band, $\mathbf{v}$ and $m$ are the velocity and the effective mass of an electron (a hole), $\mathbf{r}$ is the radius vector of an electron (a hole).

The distribution function $f_1$ is found from the solution of the Boltzmann kinetic equation in the relaxation time approximation and in the linear approximation in the external field [11]:

$$-i \omega f_1 + \mathbf{v} \frac{\partial f_1}{\partial \mathbf{r}} + e \mathbf{vE} \frac{\partial f_0}{\partial \varepsilon} + \frac{e}{m} [\mathbf{v}, \mathbf{B}] \frac{\partial f_1}{\partial \mathbf{v}} = -\frac{f_1}{\tau},$$

(3)

where $\tau$ is the relaxation time.

We assume the diffuse reflection of charge carriers from the wire inner surface as a boundary condition [11]:

$$f_1(\mathbf{r}_{\perp}, \mathbf{v}_{\perp}, \mathbf{v}_z) = 0 \quad \text{at} \quad \begin{cases} |\mathbf{r}| = R, \\ \mathbf{r}_{\perp} \cdot \mathbf{v}_{\perp} < 0, \end{cases}$$

(4)

where $\mathbf{r}_{\perp}$ and $\mathbf{v}_{\perp}$ are respectively the components of the electron (hole) radius vector $\mathbf{r}$ and velocity $\mathbf{v}$ in the plane perpendicular to the wire symmetry axis; $\mathbf{v}_z$ is the electron (hole) velocity component along the wire symmetry axis.

The kinetic equation (3) is solved by the method of characteristics [12]:

$$f_1(t') = -\frac{e(\mathbf{vE})}{v} \frac{\partial f_0}{\partial \varepsilon} \left[1 - \exp(-vt')\right],$$

(5)

where $V = \tau^{-1} - i \omega$ is the complex scattering frequency, $t' = \phi \delta / v_{\perp}$ is the electron (hole) movement time from the boundary, in which reflection happens, to the current point. The trajectory of charge carrier movement in the magnetic field represents a spiral line. This trajectory may not cross the wire boundary (in this case $t' \to \infty$). The projection of the trajectory in the plane perpendicular to wire symmetry axis looks as the arc of a circle. The arc radius is $\delta = mv_{\perp} / eB$, $\phi$ is the central angle respecting to this arc.

Find the wire integral conductivity by the mean similar to the work [8] for the case of a metal wire:
\[ G = G_0 P(x_0, y_0, \mu_0), \quad G_0 = \frac{3ne^2R^3}{mv_L L}, \quad (6) \]

\[
P(x_0, y_0, \mu_0) = \frac{1}{3I_0z_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \xi \sqrt{U_z} \frac{\exp(U_{\parallel} + U_{\perp} - U_\mu)}{\exp(U_{\parallel} + U_{\perp} - U_\mu) + 1} \left[ 1 - \exp \left( -\frac{\varphi_{z_0}}{\mu_0} \right) \right] d\xi d\mu dU_{\perp} dU_{\parallel} d\xi, \]

\[ \varphi = 2 \arctan \left[ \frac{2 \xi \sqrt{U_{\perp}} \cos \alpha - \sqrt{D}}{\mu_0 (\xi^2 - 1)} \right], \quad (8) \]

\[ D = 4 \frac{U_{\parallel}}{V_1} (1 - \xi^2 \sin^2 \alpha) - 4 \xi \mu_0 \sqrt{U_{\perp}} \sin \alpha (1 - \xi^2) - \mu_0^2 (1 - \xi^2)^2. \]

Nondimensional parameters \( x_0, y_0, z_0, \xi, I_0, U_{\perp}, U_{\parallel}, U_\mu \), also parameters \( \nu_1, \tilde{V}_1 \) are defined in the work [7]. \( \mu_0 = eRB/mv_L \) is the nondimensional magnetic field induction, \( \alpha \) is the angle in the velocity space.

3. Limited cases

3.1. The case of degenerate semiconductor

Let us consider the case of degenerate semiconductor \( (U_\mu \gg 1) \) that is equal to the case of a metal wire. The expression of the dimensionless integral conductivity will have the form:

\[
P(x_0, y_0, \mu_0) = \frac{1}{3I_0z_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \xi \rho \sqrt{1 - \rho^2} \left[ 1 - \exp \left( -\frac{\varphi_{z_0}}{\mu_0} \right) \right] d\rho d\xi d\rho d\xi; \quad (7) \]

\[ \varphi = 2 \arctan \left[ \frac{2 \xi \rho \cos \alpha - \sqrt{4 \rho^2 (1 - \xi^2 \sin^2 \alpha) - 4 \xi \mu_0 \rho \sin \alpha (1 - \xi^2) - \mu_0^2 (1 - \xi^2)^2}}{\mu_0 (\xi^2 - 1)} \right]; \quad (8) \]

where the designation \( \rho = \nu_{\parallel} / \nu_F \) is entered, \( \nu_F \) is Fermi velocity.

3.2. The case of nondegenerate semiconductor

Let us consider the case of degenerate semiconductor \( (U_\mu \to -\infty) \). The expression of the dimensionless integral conductivity will have the form:

\[
P(x_0, y_0, \mu_0) = \frac{5}{3I_0z_0} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \xi \rho \exp \left( -\frac{5}{2} \rho^2 \right) \left[ 1 - \exp \left( -\frac{\varphi_{z_0}}{\mu_0} \right) \right] d\rho d\xi d\rho d\xi. \quad (9) \]

Here the angle \( \varphi \) is determined by the expression (8), if \( \rho \) is implied by the following expression \( \rho = \nu_{\parallel} / \nu_T \). \( \nu_T = \sqrt{3k_BT/m} \) is the average thermal velocity.
4. Analysis of obtained results

Fig. 1 (a, b) shows the dependences of the dimensionless integral conductivity module $|P|$ on the dimensionless induction of the external magnetic field $\mu_0$ for the case of stationary electric field ($y_0 = 0$) for the limited cases of a metal wire (7) and of a semiconductor wire made of nondegenerate semiconductor (9). The values of the dimensionless reciprocal mean free path of charge carriers are 0,1 (Fig. 1(a)) and 1 (Fig. 1(b)). In these figures the contributions to the total integral conductivity from the charge carriers that do not collide with the wire boundary (the volume part of the total dimensionless integral conductivity) (curves 2, 5), and from the charge carriers colliding with the wire boundary (the surface part of the total dimensionless integral conductivity) (curves 3, 6) are considered. The behaviour of this curves is explained by the fact that for the small values of dimensionless induction of the external magnetic field $\mu_0 << 1$ the charge carriers colliding with the wire boundary contribute generally to the total integral conductivity. With increasing of magnetic field the amount of charge carriers moving along a trajectory not crossing with the wire boundary increases, and the contribution from those to the total integral conductivity grows. Therefore the curves 1 and 2 (for the case of a metal wire) and the curves 4 and 5 (for the case of a semiconductor wire) converge asymptotically each other, and the curves 3 and 6 smoothly converge to zero. The converging velocity of the curves 1, 4 and 2, 5 and the velocity of the dropping to zero of the curves 3 and 6 for the case of the semiconductor wire are greater than those for the case of the metal wire. For sufficiently great values of the dimensionless reciprocal mean free path of charge carriers (Fig. 1(b)) the dimensionless integral conductivity discontinues to depend on the dimensionless induction of the magnetic field because of the small contribution from charge carrier surface scattering to the conductivity.

In Fig. 2 (a, b) the dependences of the dimensionless integral conductivity module $|P|$ (a) and argument $\arg(P)$ (b) on the dimensionless induction of the external magnetic field $\mu_0$ for the case of the metal (7) and the semiconductor (9) wire for the dimensionless electric field frequence $y_0 = 1$ and for different values of the dimensionless reciprocal mean free path of charge carriers are built. In Fig. 2(a) we see that the dimensionless integral conductivity module has a minimum more pronounced for the case of the metal wire. For the wire with the determined radius with charge carrier mean free path increasing the wire conductivity grows. With dimensionless induction of the external magnetic field increasing the dimensionless integral conductivity argument (Fig. 2(b)) monotonically grows and converges to $\pi/2$. Thus the conductivity argument value for the case of the semiconductor wire is greater than that for the case of the metal wire. This difference increases with charge carrier dimensionless reciprocal mean free path growing.

In Fig. 3 (a, b) the dependences of the dimensionless integral conductivity module $|P|$ (a) and argument $\arg(P)$ (b) for the case of the metal (7) and the semiconductor (9) wire on the dimensionless electric field frequence $y_0$ for the dimensionless external magnetic field induction value $\mu_0 = 1$. The conductivity module value for the case of the semiconductor wire is greater than for the case of the metal wire. At that with charge carrier mean free path increasing this difference increases. For great values of $y_0$ the dimensionless integral conductivity module decreases and all curves merge. This is due to the fact that the free charge carriers system not keeping up with electric field intensity oscillating behaves as bound charge collection not contributed to the conduction current. With electric field frequence increasing the conductivity argument grows and converges to $\pi/2$ for large frequencies, that is the wire conductivity becomes the purely imaginary value.
**Figure 1.** The dependence of the dimensionless integral conductivity module $|P|$ on the external magnetic field induction $\mu_0$ at $x_0 = 0.1$ (a), $x_0 = 1$ (b) and $y_0 = 0$ for the cases of a metal wire (solid curves 1–3) and a semiconductor wire (dashed curves 4–6): 1, 4 are the total dimensionless integral conductivity module; 2, 5 are volume integral conductivity module; 3, 6 are surface integral conductivity module.

**Figure 2.** The dependence of a) module $|P|$ and b) argument $\arg(P)$ of the wire dimensionless integral conductivity on the external magnetic field induction $\mu_0$ at $y_0 = 1$ for the cases of a metal wire (solid curves 1–3) and a semiconductor wire (dashed curves 4–6): 1, 4 – $x_0 = 0.1$; 2, 5 – $x_0 = 0.3$; 3, 6 – $x_0 = 0.5$. 
Figure 3. The dependence of a) module $|P|$ and b) argument $\arg(P)$ of the wire dimensionless integral conductivity on the dimensionless electric field frequency $y_0$ at $\mu_0 = 1$ for the cases of a metal wire (solid curves 1 – 3) and a semiconductor wire (dashed curves 4 – 6): 1, 4 – $x_0 = 0.2$; 2, 5 – $x_0 = 0.5$; 3, 6 – $x_0 = 1$.

5. Conclusion
In this work the results obtained demonstrate that magnetic field influences substantially on a thin wire conductivity (for example the conductivity minima were discovered for determined dimensionless external magnetic field induction values). Also we showed the differences in the behaviour of the metal and semiconductor wire conductivity. Thus for specified parameter values (the electric field frequency, the charge carrier mean free path) the conductivity module and argument for the case of the semiconductor wire are greater than those for the case of the metal wire.

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