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Apollonian ball packings and stacked polytopes. (English) Zbl 1351.52019
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The relation between Apollonian packings of \(d\)-dimensional balls, their tangency graphs, and stacked polytopes is studied. For \(d = 2\) it is quite easy to prove that the graph \(G\) is a tangent graph of an Apollonian circle packing if and only if it is the 1-skeleton of some 3-dimensional stacked polytope.

For \(d = 3\) it is proved that the 1-skeleton of a stacked 4-polytope is a tangency graph of an Apollonian 3-balls packing if and only if it does not contain six 4-cliques sharing a 3-clique.

On the other hand, for \(d = 3\) the tangency graph of an Apollonian 3-balls may not be the 1-skeleton of any stacked 4-polytope. But it is proved that this situation never occurs if \(d \geq 4\).

Reviewer: Anton Shutov (Vladimir)

MSC:
52C17 Packing and covering in \(n\) dimensions (aspects of discrete geometry)
52B11 \(n\)-dimensional polytopes
20F55 Reflection and Coxeter groups (group-theoretic aspects)

Keywords:
Apollonian ball packing; stacked polytope; \(k\)-tree; forbidden subgraph

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