Closed string radiation from the interacting fractional-dressed Dp-branes with the transverse motions

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Abstract We determine the boundary states, associated with a fractional Dp-brane in the presence of the Kalb–Ramond field and a $U(1)$ gauge potential. The background spacetime is the non-compact orbifold $\mathbb{R}^{1,21} \otimes \mathbb{R}^{4}/\mathbb{Z}_2$. Besides, the brane has a transverse velocity. We compute the radiation amplitude of a massless closed string from the interaction of two fractional Dp-branes with the previous background fields. The branes have been dressed by different fields and have transverse motions. The total radiation amplitude will be investigated for large inter-brane separation. Finally, the graviton, Kalb–Ramond (axion) and dilaton emissions will be distinctly studied. Our calculations are in the context of the bosonic string theory.

1 Introduction

The D-branes play a crucial role in the explanation of string theory [1–3]. Some of the fundamental techniques in the string theory have been found by investigating the interactions of the D-branes. The boundary state formalism is an adequate technique for studying the interaction of the branes in the closed string channel [4–16]. By adding dynamics, various background and internal fields to the brane, the most general boundary state, corresponding to a Dp-brane has been obtained. The boundary state formalism has revealed that the dynamical branes with nonzero background and internal fields have some interesting properties [17–36].

Among the various configurations of the D-branes, the fractional D-branes exhibit attractive behaviors [37–52]. For instance, by using a specific system of the fractional branes, the gauge/gravity correspondence has been investigated [46–50]. Additionally, the fractional branes provide a deep insight into the Matrix theory [52–54].

In fact, the D-branes are sources of closed strings. Hence, they can generate closed strings in a wide range of configurations. One of them is the production of closed strings from a solitary unstable Dp-brane [55–59]. This kind of production has been investigated in the presence of various background fields [58,59], and its supersymmetric version has also been studied [60]. The closed string radiation from the interacting branes is another important configuration. This form of radiation has been studied only in some special configurations [61–64].

The background fields, the transverse motions of the branes and the fractionality of the branes, motivated and encouraged us to investigate the effects of these factors on the radiation of a massless closed string. In our setup, the massless string is radiated from the interacting parallel Dp-branes with background fields in a partially non-compact orbifoldized spacetime $\mathbb{R}^{1,21} \otimes \mathbb{R}^{4}/\mathbb{Z}_2$. Thus, in the context of the bosonic string theory, we shall use the boundary state formalism to compute the radiation amplitude. Hence, by inserting an appropriate vertex operator into the worldsheet of the closed string, exchanged between the branes, we obtain the radiation amplitude. We shall use the eikonal approximation in which the recoil of the branes, due to the string radiation, can be ignored. Finally, we shall acquire the radiation amplitude for the case that the distance of the branes is very large. We observe that only one of the following radiations can potentially occur: radiations from each of the interacting branes and one radiation between the branes.

This paper is organized as follows. In Sect. 2, we shall introduce the boundary states, corresponding to the untwisted and twisted sectors of a fractional-dressed Dp-brane with a transverse velocity. In Sect. 3.1, the total radiation amplitude for a massless closed string, resulting from the interaction of such branes, will be computed. In Sect. 3.2, the partition functions and various correlators in this amplitude will be explicitly calculated. In Sect. 3.3, the previous amplitude will
be deformed to represent the closed string radiation from the distant branes. In Sect. 4, for each of the graviton, Kalb–Ramond and dilaton states, we shall explicitly calculate the total emission amplitude. In Sect. 5, the conclusions will be given.

2 The boundary states

In this section, we obtain the boundary states which are corresponding to a fractional-dressed Dp-brane. The brane lives in the 26-dimensional orbifoldy spacetime \( \mathbb{R}^{1,21} \times \mathbb{R}^4/\mathbb{Z}_2 \) and has a transverse motion. The coordinates of the \( \mathbb{R}^4 \) are denoted by \( \{x^a|a = 22, 23, 24, 25\} \), and they are influenced by the action of the \( \mathbb{Z}_2 \)-group. This group has the cyclic structure \( [e, h]|h^2 = e \). Under the action of the element \( h \) we have the identification \( x^a \equiv -x^a \). The Dp-brane has been located at the fixed points of the orbifold, i.e. \( x^a = 0 \), which is a 22-dimensional hyperplane. Thus, the dimension of a moving brane is restricted by the upper bound \( p \leq 20 \).

In order to determine the boundary state, associated with a fractional Dp-brane with background fields, we consider the following closed string action

\[
S = -\frac{1}{4\pi\alpha'} \int d^2\sigma \left( \sqrt{-\hbar^{AB}G_{\mu\nu} + \varepsilon A^2 B_{\mu\nu}} \right) \partial_A X^\mu \partial_B X^\nu
+ \frac{1}{2\pi a'} \int d\sigma A_\sigma \partial_\sigma X^a.
\] (2.1)

In this action, we use the following notations: \( \mu \) and \( \nu \) are the spacetime indices and \( h^{AB} \), with \( A, B \in \{\tau, \sigma\} \), denotes the metric of the string worldsheet, while \( G_{\mu\nu} \) indicates the spacetime metric and \( B_{\mu\nu} \) represents the Kalb–Ramond field. Besides, \( A_\sigma \) is the gauge potential. The set \( \{X^a|a = 0, 1, \ldots, p\} \) shows the directions along the Dp-brane worldvolume. We restrict ourselves to the flat spacetime \( G_{\mu\nu} = \eta_{\mu\nu} = \delta_{\mu\nu} \), \( \delta_{\mu\nu} \) being the Kronecker delta, flat worldsheet and a constant Kalb–Ramond field. In addition, we adopt the usual gauge \( A_\sigma = -\frac{1}{2} F_{\alpha\beta} X^\beta \) with the constant field strength \( F_{\alpha\beta} \).

Apart from the equation of motion, the following boundary state equations are also acquired by vanishing the variation of the action (2.1),

\[
\left( \partial_\tau X^a + \mathcal{F}^a_{\beta} \partial_\sigma X^\beta \right)_{\tau = 0} |B_k \rangle = 0,
\] (2.2)

\[
\left( X^i - y^i \right)_{\tau = 0} |B_k \rangle = 0,
\] (2.3)

where \( \mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta} \) is the total field strength. The set \( \{x^I|I = p + 1, \ldots, 21\} \) belongs to the perpendicular directions of the Dp-brane worldvolume. Hence, the parameters \( y^I \) indicate the brane’s position.

For the non-orbifoldy directions, the solution of the equation of motion takes the form

\[
X^\rho(\tau, \sigma) = x^\rho + 2\alpha' \rho^\rho \tau
+ \frac{1}{2} \sqrt{2\alpha'} \sum_{m\neq 0} \frac{1}{m} \left( \alpha_0^m e^{-2im(\tau - \sigma)} + \bar{\alpha}_m^0 e^{-2im(\tau + \sigma)} \right).
\] (2.4)

Thus, for the twisted sector \((T)\) we have \( \rho \in \{a, i\} \), where the set \( \{x^I|I = p + 1, \ldots, 21\} \) represents the non-orbifoldy transverse directions to the brane. Since the Dp-brane locates on the fixed points of the non-compact orbifold \( \mathbb{R}^4/\mathbb{Z}_2 \), the orbifoldy coordinates of the closed string possess the following form

\[
X^a(\sigma, \tau) = \frac{i}{2} \sqrt{2\alpha'} \sum_{r \in \mathbb{Z} + \frac{1}{2}} \frac{1}{\tau} \left( \alpha_r^a e^{-2ir(\tau - \sigma)} + \bar{\alpha}_r^a e^{-2ir(\tau + \sigma)} \right).
\] (2.5)

In the twisted sector, Eq. (2.3) is split into the following equations

\[
\left( X^i - y^i \right)_{\tau = 0} |B_k \rangle^U = 0,
\]

\[
\left( X^a \right)_{\tau = 0} |B_k \rangle^T = 0.
\] (2.6)

Now we impose a transverse velocity to the brane. Since the brane has stuck at the orbifold fixed-points, it cannot move along the orbifold directions. Therefore, the boost direction should be a member of the set \( \{x^{p+1}, \ldots, x^{21}\} \), which we call it \( x^i \). Hence, Eqs. (2.2), (2.3) and (2.6), under the boost effect, take the forms

\[
\left[ \partial_\tau (X^0 - v X^i) + \mathcal{F}^0_{\alpha} \partial_\sigma X^\alpha \right]_{\tau = 0} |B_k \rangle^U = 0,
\]

\[
\left[ \partial_\tau \bar{X}^\alpha + \gamma^2 \mathcal{F}^\alpha_{\beta} \partial_\sigma (X^0 - v X^i) + \mathcal{F}^\alpha_{\beta} \partial_\sigma X^\beta \right]_{\tau = 0} |B_k \rangle^U = 0,
\]

\[
\left( X^i - y^i \right)_{\tau = 0} |B_k \rangle^U = 0,
\]

\[
\left( X^a \right)_{\tau = 0} |B_k \rangle^T = 0,
\] (2.7)

where \( \gamma = 1/\sqrt{1 - v^2} \) and \( \bar{\alpha} = (\alpha) - \{0\} \).

By substituting Eqs. (2.4) and (2.5) into Eqs. (2.7), we get the boundary state equations in terms of the oscillators and zero modes. For the oscillating part of both sectors we obtain

\[
\left[ \alpha_m^0 - v \alpha_m^i - \mathcal{F}^0_{\alpha} \bar{\alpha}_m^\alpha + \alpha_0^m - v \alpha_{-m}^i - \mathcal{F}^0_{\alpha} \bar{\alpha}_{-m}^\alpha \right] |B_k \rangle^U_\text{osc} = 0,
\]

\[
\left[ \alpha_m^i - v \bar{\alpha}_{-m}^i - v (\alpha_m^0 - \bar{\alpha}_{-m}^0) \right] |B_k \rangle^U_\text{osc} = 0,
\]

\[
\left[ \alpha_m^0 - v \alpha_m^0 - (\alpha_0^m - \alpha_m^0) \right] |B_k \rangle^U_\text{osc} = 0,
\]

\[
\left[ \alpha_m^i - v \bar{\alpha}_{-m}^i - v (\alpha_m^0 - \bar{\alpha}_{-m}^0) \right] |B_k \rangle^U_\text{osc} = 0,
\]
For the zero-mode part, we have
\[
(\nu^0 - v\nu^i) |B_x\rangle_{0/T}^U = 0, \\
\nu^\hat{a} |B_x\rangle_{0/T}^U = 0, \\
(x^i - u\lambda^0 - y^i) |B_x\rangle_{0/T}^U = 0, \\
(x^i - y^i) |B_x\rangle_{0/T}^U = 0, \quad i \neq i_v, \\
(x^i - y^i) |B_x\rangle_{0/T}^U = 0, \quad I \neq i_v. \tag{2.9}
\]

By using the coherent state approach, the oscillating parts of the boundary states find the following features
\[
|B\rangle_{osc}^T = \sqrt{-\det M} \exp \left[ - \sum_{m=1}^{\infty} \frac{1}{m} \alpha_{m-}^\nu S_{\nu\nu^0} \tilde{g}_{\nu\nu^0}^m \right] \\
\times \exp \left[ \sum_{r=1/2}^{\infty} \frac{1}{r} \alpha_{r-}^\nu \delta_{\nu\nu^0} \tilde{g}_{\nu\nu^0}^r \right] |0\rangle, \tag{2.10}
\]
\[
|B\rangle_{osc}^U = \sqrt{-\det M} \exp \left[ - \sum_{m=1}^{\infty} \frac{1}{m} \alpha_{m-}^\nu \tilde{S}_{\nu\nu^0} \tilde{g}_{\nu\nu^0}^m \right] |0\rangle. \tag{2.11}
\]

The matrices $S$ and $\tilde{S}$ are defined by
\[
S_{\rho\rho'} \equiv (Q_{\lambda\lambda'}, -\delta_{ij}), \quad i, j \neq i_v, \tag{2.12}
\]
\[
\tilde{S}_{\mu\nu} \equiv (Q_{\lambda\lambda'}, -\delta_{ij}, -\delta_{ab}), \quad i, j \neq i_v, \tag{2.13}
\]
where $Q_{\lambda\lambda'} \equiv (M^{-1}N)_{\lambda\lambda'}$ and $\lambda, \lambda' \in \{\alpha, i_v\}$. The matrices $M$ and $N$ are given by
\[
M^0_{\lambda} \equiv \gamma \left( \delta^0_{\lambda} - v\delta^i_{\lambda} - F^\alpha_{\nu\nu^0} \tilde{g}_{\nu\nu^0}^\alpha \right), \\
N^0_{\lambda} \equiv \gamma \left( \delta^0_{\lambda} - v\delta^i_{\lambda} + F^\alpha_{\nu\nu^0} \tilde{g}_{\nu\nu^0}^\alpha \right), \tag{2.14}
\]

The normalization factor in Eqs. (2.10) and (2.11) are derived from the disk partition function [4,11].

Applying the quantum mechanical technics, the zero-mode components of the boundary states find the following solutions
\[
|B_x\rangle_{0}^U = \delta(x^i - v\lambda^0 - y^i) |p^\nu = 0\rangle \\
\times \prod_{i=p+1}^{21} \left[ \delta(x^i - y^i) |p^i = 0\rangle \right] \prod_{\alpha=0}^{p} |p^\alpha = 0\rangle, \tag{2.15}
\]
\[
|B_x\rangle_{0}^U = \delta(x^i - u\xi^0 - y^i) |p^\nu = 0\rangle \\
\times \prod_{i=p+1}^{25} \left[ \delta(x^i - y^i) |p^i = 0\rangle \right] \prod_{\alpha=0}^{p} |p^\alpha = 0\rangle. \tag{2.16}
\]

For the next purposes, we employ the integral versions of the Dirac $\delta$-functions, which yield
\[
|B_x\rangle_{0}^T = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} \exp \left( -ip\gamma_i \right) \prod_{\rho} |p^\rho \rangle, \tag{2.17}
\]
\[
|B_x\rangle_{0}^U = \int_{-\infty}^{+\infty} \frac{dp}{2\pi} \exp \left( -ip y_i \right) \prod_{\mu} |p^\mu \rangle. \tag{2.18}
\]

According to Eq. (2.9) we have $\nu^0 = v\nu^i, \nu^\hat{a} = 0$, and the remaining components of the momentum in each sector are nonzero.

The following direct product represents the total boundary state, associated with the $Dp$-brane
\[
|B\rangle_{tot}^U = \frac{T_p}{2} |B_x\rangle_{osc}^T \otimes |B_x\rangle_{0}^U \otimes |B\rangle_g, \tag{2.19}
\]
where $|B\rangle_g$ is the boundary state of the conformal ghosts
\[
|B\rangle_g = \exp \left[ \sum_{m=1}^{\infty} \left( c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m} \right) \right] \times \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle \otimes |\tilde{q} = 1\rangle. \tag{2.20}
\]

Since the ghost fields have no interaction with the matter fields, their contribution to the boundary states is not affected by the background fields, the motion of the brane and the orbifold projection.

### 3 Radiation of a massless closed string

In this section, we compute the massless closed string radiation from the interaction of two parallel $Dp$-branes with the background fields and transverse motions in the spacetime $\mathbb{R}^{1,21} \otimes \mathbb{R}^4 / \mathbb{Z}_2$. To generalize our computations, let us suppose that the fields and velocities of the two $Dp$-branes are different. For showing this difference, we shall use the subscripts 1 and 2.
3.1 The total radiation amplitude

Using the relevant vertex operator, we can obtain the radiation of a closed string state. From the mathematical point of view, one should compute the amplitudes

\[
A^{U/T} = \int_0^\infty \int_0^\infty \int_0^\pi d\sigma \int_0^\pi d\tau \ U_{tot}^{U/T}(B_1|e^{-iH^{U/T}}V(\tau, \sigma)|B_2)^{U/T},
\]

where \( V(\tau, \sigma) \) is the vertex operator of the radiated massless state, \( H^T \) and \( H^U \) are the closed string Hamiltonians in the twisted and untwisted sectors, respectively, and are

\[
H^T = H_g + a^\prime \rho p^\rho p_\rho + 2 \sum_{n=1}^\infty (\alpha^\mu_n \alpha_{n\rho} + \tilde{\alpha}^\nu_n \tilde{\alpha}_{n\nu})
+ 2 \sum_{r=1/2}^\infty (\alpha^\alpha_r \alpha_{r\alpha} + \tilde{\alpha}^\beta_r \tilde{\alpha}_{r\beta}) - 3,
\]

(3.2)

\[
H^U = H_g + a' \rho'^\mu p_\mu + 2 \sum_{n=1}^\infty (\alpha^\mu_n \alpha_{n\rho} + \tilde{\alpha}^\nu_n \tilde{\alpha}_{n\nu}) - 4.
\]

(3.3)

Note that, in Eq. (3.1) we employed the integrated version of the vertex operator, which is independent of the conformal ghosts [1, 65]. It has been demonstrated that for calculating the bosonic emission amplitude, insertion of either the integrated form of the closed string vertex operator or its unintegrated form gives rise to the same result [1, 65]. However, this equivalence will fail for the supersymmetric configurations with certain compactifications [66-69]. Our setup does not belong to this set of the configurations. Hence, we have this equivalence, and the conformal ghosts do not appear in the integrated vertex operator. Their contributions only appear in the boundary state \(|B_1\rangle \) and the Hamiltonian \( H_g \).

Let us define \( z = \sigma + i \tau \) and \( \theta = \theta \). The vertex operator in Eq. (3.1) for a general massless closed string has the form

\[
V(z, \bar{z}) = \epsilon_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu e^{ip \cdot X},
\]

(3.4)

where \( \epsilon_{\mu\nu} \) represents the polarization tensor, and “\( p^2 \)” (with \( p^2 = 0 \)) shows the momentum of the radiated massless closed string. Besides, we shall apply the momenta \( k_1 \) and \( k_2 \) for the emitted closed string from the first brane and the absorbed one by the second brane, respectively. According to the decomposition \( X^\mu = X^\mu_0 + X^\mu_{osc} \), we have

\[
\bar{\partial} X^\mu \partial X^\nu e^{ip \cdot X} = \left[ \partial X^\mu_0 \partial X^\nu_0 + \partial X^\mu_0 \partial X^\nu_{osc} + \partial X^\mu_{osc} \partial X^\nu_0 + \partial X^\mu_{osc} \partial X^\nu_{osc} \right] e^{ip \cdot X_0} e^{ip \cdot X_{osc}}.
\]

(3.5)

Hence, the four terms should be sandwiched between the boundary states. The terms in the vertex operator may be expressed in a generic form \( A(X_0) e^{ip \cdot X_0} \times B(X_{osc}) e^{ip \cdot X_{osc}} \), where \( A(X_0) \in \{ 1, \tilde{\partial} X_0 \partial X_0, \partial X_0, \tilde{\partial} X_0 \} \) and \( B(X_{osc}) \in \{ 1, \tilde{\partial} X_{osc} \partial X_{osc}, \partial X_{osc}, \tilde{\partial} X_{osc} \} \). With this notation we obtain

\[
\begin{align*}
&U_{tot}^{U/T}(B_1|e^{-iH^{U/T}}V(\tau, \sigma)|B_2)^{U/T} \\
&= T_0^2 \delta \left[ \int_0^\infty \int_0^\infty e^{3t} \prod_{i=p+1}^{21} \frac{dk^1_i}{2\pi} \frac{dk^2_i}{2\pi} e^{ik^1_{i\nu} e^{-ik^2_{i\nu}}} \right] \times e^{-\tau' k^1_{i\nu} e^{-\tau' k^2_{i\nu}}} \prod_{i=p}^p \delta(k^1_i + (k_2 + p))^\rho.
\end{align*}
\]

(3.6)

In the twisted sector, first consider \( A^T(X_0) = 1 \),

\[
T_0^T(B_1|e^{-iH^{T}(X_0)} e^{ip \cdot X_0^T}|B_2)^{T}
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{3t} \prod_{i=p+1}^{21} \frac{dk^1_i}{2\pi} \frac{dk^2_i}{2\pi} e^{ik^1_{i\nu} e^{-ik^2_{i\nu}}} \times e^{-\tau' k^1_{i\nu} e^{-\tau' k^2_{i\nu}}} \prod_{i=p}^p \delta(k^1_i + (k_2 + p))^\rho.
(3.7)
\]

Using \( \delta(\alpha x) = \frac{1}{|\alpha|} \delta(x) \), we find

\[
\prod_{i=p+1}^{21} \delta(k^1_i + (k_2 + p))^\rho,
\]

(3.8)

Therefore, using the Wick’s rotation \( \tau \rightarrow -i \tau \), Eq. (3.7) takes the feature

\[
T_0^T(B_1|e^{-iH^{T}(X_0)} e^{ip \cdot X_0^T}|B_2)^{T}
= \frac{e^{3t}}{(2\pi)^{20-2p} \ |v_1 - v_2|} \prod_{\alpha=1}^p \delta(p^\alpha) \times \int_{-\infty}^{\infty} \prod_{i\neq \alpha}^{21} \frac{dk^1_i}{2\pi} e^{ik^1_{i\nu} e^{-\tau' k^2_{i\nu}} e^{-\tau' k^2_{i\nu}}}.
\]

(3.9)

in which, after the Wick’s rotation, we introduced another proper time \( \tau' = t - \tau \). The transverse vector \( b = y_1 - y_2 \) represents the impact parameter.

The remaining components of \( A^T(X_0) \) can be simply written as

\[
\begin{align*}
&\prod_{i=p+1}^{21} \delta(k^1_i + (k_2 + p))^\rho,
\end{align*}
\]

where

\[
\begin{align*}
&A(X_0) \in \{ 1, \tilde{\partial} X_0 \partial X_0, \partial X_0, \tilde{\partial} X_0 \} \text{ and } B(X_{osc}) \in \{ 1, \tilde{\partial} X_{osc} \partial X_{osc}, \partial X_{osc}, \tilde{\partial} X_{osc} \}. \end{align*}
\]
where identities

Furthermore, in both sectors, we shall utilize the following

Now we consider the oscillating portion and define the following general correlator

where

is the oscillating and ghost parts of the partition function. Furthermore, in both sectors, we shall utilize the following identities

Note that the related indices of the corresponding sector should be exerted.

The new proper time enables us to modify the integrations as

The variables $t'$ and $\tau$ represent the proper times of the radiated closed string from the first and second brane, respectively. Thus, $t' = 0$ denotes the radiation from the first brane, while $\tau = 0$ implies the radiation from the second brane. When the radiation occurs from the inter-brane region, the condition is $\tau, t' > 0$.

Finally, the general amplitudes (3.1) are rewritten by inserting all combinations of Eq. (3.5). For the untwisted sector we acquire

where

The additional indices $\gamma$ and $\eta$ belong to the spacetime directions.

For the twisted sector we find

in which

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After decomposing the indices $\gamma$ and $\eta$ to the orbifold and non-orbifold directions, one can easily deduce that the mixed-indices correlators vanish.

The total amplitude for radiating a massless closed string is given by

$$A_{\text{tot}} = A^T + A^U.$$  \hfill (3.21)

The presence of the $\delta$-function in both sectors obviously clarifies that the closed string is radiated perpendicular to the branes.

3.2 The partition functions and correlators

Now we compute the partition functions and correlators that appeared in Eqs. (3.17)–(3.20). Since we employed the Wick’s rotation to calculate the zero-mode parts, the oscillating portion should also be obtained in this frame.

According to the Hamiltonians (3.2) and (3.3), the partition functions in both sectors become

$$Z_{\text{osc-}T} = \sqrt{\text{det}(M_1 M_2)} \prod_{n=1}^{\infty} \left[ \left( 1 - q^{2n} \right) \right]^{-1} \left( 1 - q^{2n-1} \right)^2,$$

$$Z_{\text{osc-}U} = \sqrt{\text{det}(M_1 M_2)} \prod_{n=1}^{\infty} \left[ \left( 1 - q^{2n} \right) \right]^{-1} \left( 1 - q^{2n-1} \right)^{-1},$$

where $q \equiv e^{-2\tau}$ and $Q \equiv Q^T_1 Q_2$.

For the correlators with a single derivative, we obtain the following expressions

$$\langle \partial X^\alpha X^\beta \rangle_{\text{osc-}U}^T = i \alpha' \sum_{n=0}^{\infty} \left\{ \text{Tr}_{26 \times 26} \left( \eta^{\alpha\beta} + Q_1 \left( 1 - Q \right) q^{2(n+1)} \right) - Q_2 \partial^U \partial' \right\},$$

$$\langle \partial X^I X^J \rangle_{\text{osc-}U}^T = i \alpha' \delta^{IJ} \sum_{n=0}^{\infty} \left\{ \text{Tr}_{26 \times 26} \left( 1 + (1 - Q) q^{2(n+1)} \right) - \partial^U \partial' \right\},$$

for the untwisted sector, and

$$\langle \partial X^\alpha X^\beta \rangle_{\text{osc-}T}^U = i \alpha' \sum_{n=0}^{\infty} \left\{ \text{Tr}_{26 \times 26} \left( \eta^{\alpha\beta} + Q_1 \left( 1 - Q \right) q^{2(n+1)} \right) - Q_2 \partial^T \partial' \right\},$$

$$\langle \partial X^I X^J \rangle_{\text{osc-}T}^U = i \alpha' \delta^{IJ} \sum_{n=0}^{\infty} \left\{ \text{Tr}_{26 \times 26} \left( 1 + (1 - Q) q^{2(n+1)} \right) - \partial^T \partial' \right\},$$

for the twisted sector.
enough time, via the massless closed strings. Thus, after a sufficiently long
from now on, we study the closed string radiation from the
3.3 Large distance branes

for the untwisted sector, and

\[
\langle e^{ip \cdot X_{\text{osc}}^T} \rangle^T_U = \prod_{n=0}^{\infty} \left[ \det_{(p+1) \times (p+1)} \left( 1 - Q q^{2(n+1)} e^{-4\tau} \right) \right]^{-\alpha_{\text{ut}}^T} (S_{S_1})^{\alpha_{\text{ut}}^T} \times \left[ \frac{1}{(1 - q^{2n} e^{-4\tau})^{21(p-\delta)}} \right]^{\alpha_{\text{ut}}^T} \times \det_{(p+1) \times (p+1)} \left( 1 - Q q^{2n} e^{-4\tau} \right)^{-\alpha_{\text{ut}}^T} (S_{S_1})^{\alpha_{\text{ut}}^T} \times \det_{22 \times 22} \left[ \exp \left( \frac{\alpha_{\text{ut}}^T p \cdot p \cdot S_{S_1}}{2(n+1)} (1 - S q^{2n} e^{-4\tau})^{-1} \right) \right] \times \det_{22 \times 22} \left[ \exp \left( \frac{\alpha_{\text{ut}}^T p \cdot p \cdot S_{S_1}^{\alpha_{\text{ut}}^T}}{2(n+1)} (1 - S q^{2n} e^{-4\tau})^{-1} \right) \right].
\]

\[\text{(3.31)}\]

for the twisted sector.

Inserting all correlators into the untwisted and twisted parts of the amplitude \((3.21)\), one receives the radiation amplitude of the massless closed string. Because of the long length of the resultant amplitude, we do not explicitly write it.

In fact, the integrands of the amplitudes \((3.17)\) and \((3.19)\) are independent of the worldsheet coordinate \(\sigma\). They merely depend on the coordinates \(\tau\) and \(\tau'\) (see Eqs. \((3.22)-(3.32)\)). Thus, the integration over \(\sigma\) is trivial. One can always multiply the amplitudes by the factor \(\pi\) to restore the effect of the integration over \(\sigma\).

3.3 Large distance branes

From now on, we study the closed string radiation from the branes which are located far from each other. For two D-branes with large distance, the interaction effectively occurs via the massless closed strings. Thus, after a sufficiently long enough time, \(t \to \infty\), which is equivalent to large inter-brane distance, only the graviton, Kalb–Ramond and dilaton are effectively exchanged. We should note that the limit \(t \to \infty\) must be applied only to the oscillating portion of the amplitude, namely the correlators and partition functions.

For simplification, we impose the extra conditions \(S = \hat{S} = 1\) to our formulation, which put constraints on the parameters of our configuration. These conditions enable us to compare our results with the results of Refs. \([61,62]\). When the large distance limit is applied, the partition functions in both sectors find the form

\[
\mathcal{Z}^U_{\text{osc}} | \hat{S} = 1 = \mathcal{Z}^T_{\text{osc}} | \hat{S} = 1 = \frac{\sqrt{\det(M_1 M_2)}}{\mathcal{Z}^U_{\text{osc}} | \hat{S} = 1}. \tag{3.33}
\]

Besides, the correlators of each sector are given by

\[
\langle \partial X^a X^b U \rangle_{\text{osc}} | \hat{S} = 1 = -i \alpha' \left( \frac{13}{2} \eta^{ab} - Q_2^{ab} T_U^{\partial} - Q_1 T_2^{\partial} T_U^{\partial} \right), \tag{3.34}\]

\[
\langle \partial X^a X^b T \rangle_{\text{osc}} | \hat{S} = 1 = \alpha' \left( \frac{11}{2} \eta^{ab} - Q_2^{ab} T_T^{\partial} - Q_1 T_2^{\partial} T_T^{\partial} \right), \tag{3.35}\]

\[
\langle \partial X^a X^b T \rangle_{\text{osc}} | \hat{S} = 1 = i \alpha' \delta^{ij} \left( 11 + T_T^{\partial} + T_T^{\partial} \right), \tag{3.36}\]

\[
\langle \partial X^a X^b T \rangle_{\text{osc}} | \hat{S} = 1 = \langle \partial X^a X^b U \rangle_{\text{osc}} | \hat{S} = 1 = -4i \alpha' \delta^{ab} \left[ f(\tau) - f(\tau') \right], \tag{3.38}\]

where we used these definitions

\[
T_T^{\partial} = \text{Tr} \frac{e^{-4\tau}}{1 - e^{-4\tau}} \left[ 1 + \frac{1}{1 - e^{-4\tau}} \right]; \tag{3.39}\]

\[
T_T^{\partial} = -\text{Tr} \frac{e^{-4\tau}}{1 - e^{-4\tau}} \left[ 1 + \frac{1}{1 - e^{-4\tau}} \right]; \tag{3.40}\]

\[
f(t', \tau) = \frac{e^{-4(t' - \tau)}}{1 - e^{-4(t' - \tau)}}. \tag{3.41}\]

In the right-hand side we have \(\text{Tr} U^{26}, \text{Tr} T^{22} \to 22\) and \(1 \to 1\).

In this limit, the exponential correlators are deformed as

\[
\langle e^{ip \cdot X_{\text{osc}}^U} | \hat{S} = 1 \rangle_U \to \infty \]

\[
= \left[ 1 - e^{-4\tau} \right] p \cdot p_1 \tilde{S}_1^{\mu \nu} \left( 1 - e^{-4\tau} \right) p \cdot p_2 \tilde{S}_2^{\mu \nu} \right]^{-13 \alpha'} \times \exp \left[ 13 \alpha' p \cdot p_1 \left( \tilde{S}_1^{\mu \nu} - \frac{\tilde{S}_1^{\mu \nu}}{1 - e^{-4\tau}} + \frac{\tilde{S}_2^{\mu \nu}}{1 - e^{-4\tau}} \right) \right] + \left( \tilde{S}_1 + \tilde{S}_2 \right)^{\mu \nu} (\gamma_{EM} - 1) \right] \tag{3.42}\]
for the untwisted sector, and
\[
\langle e^{ip \cdot X_{\text{osc}}} \rangle_{\text{osc}} \overset{S=1}{=} \left[ (1-e^{-4i})^{22p_{\rho} p_{\sigma} S_{\rho}^\sigma - 4p_{\rho} p_{\sigma} S_{\rho}^\sigma - 4p_{\rho} p_{\sigma} S_{\rho}^\sigma} \right]^{-a'/2} \\
\times \exp \left\{ 11a' p_{\rho} p_{\sigma} \left( \frac{S_{\rho}^\sigma}{1-e^{-4i}} + \frac{S_{\rho}^\sigma}{1-e^{-4i}} \right) \right. \\
+ (S_{1} + S_{2}) \rho_{\rho} (\gamma_{EM} - 1) \right\} \tag{3.43}
\]

for the twisted sector. The Euler-Mascheroni number $\gamma_{EM} = 0.577 \ldots$ was entered through a regularization scheme.

We can rewrite the two-derivative term as a combination of one-derivative terms. The contribution of the two-derivative term to the untwisted part of the amplitude is given by
\[
A_{U} \overset{S=1}{\rightarrow} \frac{T_{p}^{2}}{8(2\pi)^{23-2p} |v_{1} - v_{2}|} \prod_{\alpha=1}^{p} \delta(p_{\alpha}^{2}) \\
\times \int_{0}^{\infty} dt' \int_{0}^{\infty} dt \int_{-\infty}^{+\infty} \int_{l \neq \alpha}^{25} d \kappa_{1}^{l} e \kappa_{1}^{l} b_{l} \epsilon_{\mu \nu} \times \gamma^{-t_{0}^{l}} \gamma^{k_{1}^{l} \mu} \gamma^{k_{1}^{l} \nu} \\
e^{-t_{0}^{l} \kappa_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l} e^{(t_{0}^{l} + t)}} \gamma^{U_{\text{osc}, g}} \overset{S=1}{\rightarrow} \gamma^{e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times \left( p_{\rho} p_{\sigma} \langle e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} \right) + \frac{i}{2} \gamma^{k_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l}} \right) \\
+ \text{Non-integral term.} \tag{3.44}
\]

With the help of integration by part, one finds
\[
A_{U} \overset{S=1}{\rightarrow} \frac{T_{p}^{2}}{8(2\pi)^{23-2p} |v_{1} - v_{2}|} \\
\times \prod_{\alpha=1}^{p} \delta(p_{\alpha}^{2}) \int_{0}^{\infty} dt' \int_{0}^{\infty} dt \int_{-\infty}^{+\infty} \int_{l \neq \alpha}^{25} d \kappa_{1}^{l} e \kappa_{1}^{l} b_{l} \epsilon_{\mu \nu} \\
\times \gamma^{-t_{0}^{l} \kappa_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l} e^{(t_{0}^{l} + t)}} \gamma^{U_{\text{osc}, g}} \overset{S=1}{\rightarrow} \gamma^{e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times \left( p_{\rho} p_{\sigma} \langle e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} \right) + \frac{i}{2} \gamma^{k_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l}} \right) \\
+ \text{Non-integral term.} \tag{3.45}
\]

From the physical point of view, we should exclude the non-integral terms. Consequently, omission of the surface term at $t, T = 0$ implies that the quantities $p_{\mu} p_{\rho} S_{\mu}^{\rho}$ and $p_{\mu} p_{\rho} S_{\mu}^{\rho}$ should be negative.

For the twisted sector, the two-derivative term possesses the following contribution to the amplitude
\[
A_{T} \overset{S=1}{\rightarrow} \frac{T_{p}^{2}}{8(2\pi)^{19-2p} |v_{1} - v_{2}|} \\
\times \prod_{\alpha=1}^{p} \delta(p_{\alpha}^{2}) \int_{0}^{\infty} dt' \int_{0}^{\infty} dt \int_{-\infty}^{+\infty} \int_{l \neq \alpha}^{21} d \kappa_{1}^{l} e \kappa_{1}^{l} b_{l} \\
\times \gamma^{-t_{0}^{l} \kappa_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l} e^{(t_{0}^{l} + t)}} \gamma^{X_{\text{osc}, g}} \overset{S=1}{\rightarrow} \gamma^{e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times \left\{ \gamma^{e_{\rho \sigma} \langle \gamma^{X_{\rho \sigma} X_{\rho \sigma}} U_{l} \rightarrow \infty} \right. \\
+ \frac{i}{2} \gamma^{k_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l}} \right) \right\}, \tag{3.46}
\]

where the additional constraints are
\[
(p_{\rho} p_{\rho} S_{\rho}^{\rho} < 0, \quad p_{\rho} p_{\rho} S_{\rho}^{\rho} > 0, \\
(21 - p) p_{\rho} p_{\rho} S_{\rho}^{\rho} > 4 p_{\rho} p_{\rho} a_{\rho} \\
(21 - p) p_{\rho} p_{\rho} S_{\rho}^{\rho} > 4 p_{\rho} p_{\rho} a_{\rho}. \tag{3.47}
\]

### 4 The particles emission

In this section, we shall calculate the graviton, Kalb–Ramond and dilaton emissions from the interacting branes with the large distance. Instead of writing three different amplitudes for the three massless states, at first for all massless states we write a general emission amplitude, which includes several variables. Then, we explicitly write these variables for each of the graviton, Kalb–Ramond and dilaton to separate the previous general amplitude to three distinct amplitudes. For each sector, the amplitude is given by
\[
A_{U} \overset{S=1}{\rightarrow} \frac{T_{p}^{2}}{8(2\pi)^{19-2p} |v_{1} - v_{2}|} \\
\times \prod_{\alpha=1}^{p} \delta(p_{\alpha}^{2}) \int_{0}^{\infty} dt' \int_{0}^{\infty} dt \int_{-\infty}^{+\infty} \int_{l \neq \alpha}^{21} d \kappa_{1}^{l} e \kappa_{1}^{l} b_{l} \\
\times \gamma^{-t_{0}^{l} \kappa_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l} e^{(t_{0}^{l} + t)}} \gamma^{X_{\text{osc}, g}} \overset{S=1}{\rightarrow} \gamma^{e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times \left\{ \gamma^{e_{\rho \sigma} \langle \gamma^{X_{\rho \sigma} X_{\rho \sigma}} U_{l} \rightarrow \infty} \right. \\
+ \frac{i}{2} \gamma^{k_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l}} \right) \right\}, \tag{4.1}
\]

for the untwisted sector, and
\[
T_{U} \overset{S=1}{\rightarrow} \frac{T_{p}^{2}}{8(2\pi)^{19-2p} |v_{1} - v_{2}|} \\
\times \prod_{\alpha=1}^{p} \delta(p_{\alpha}^{2}) \int_{0}^{\infty} dt' \int_{0}^{\infty} dt \int_{-\infty}^{+\infty} \int_{l \neq \alpha}^{21} d \kappa_{1}^{l} e \kappa_{1}^{l} b_{l} \\
\times \gamma^{-t_{0}^{l} \kappa_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l} e^{(t_{0}^{l} + t)}} \gamma^{X_{\text{osc}, g}} \overset{S=1}{\rightarrow} \gamma^{e^{p \cdot X_{\text{osc}}} U_{l} \rightarrow \infty} (\gamma^{X_{U}^{T} X_{U}^{T}} U_{l} \rightarrow \infty) \\
\times \left\{ \gamma^{e_{\rho \sigma} \langle \gamma^{X_{\rho \sigma} X_{\rho \sigma}} U_{l} \rightarrow \infty} \right. \\
+ \frac{i}{2} \gamma^{k_{1}^{l} k_{1}^{l} k_{2}^{l} k_{2}^{l}} \right) \right\}, \tag{4.2}
\]
for the twisted sector. Using the integration by part on the proper times, the following equivalence relations are obtained

\[
T_T^\alpha = \frac{\alpha' k^\alpha_0 k_2}{2 \alpha' p_\mu p_\sigma S^\alpha_1}, \quad T_T^\mu = \frac{\alpha' k^\mu_0 k_1}{2 \alpha' p_\mu p_\sigma S^\mu_2},
\]

\[
T_U^\alpha = \frac{\alpha' k^\mu_0 k_2}{2 \alpha' p_\mu p_\sigma S^\mu_1}, \quad T_U^\mu = \frac{\alpha' k^\mu_0 k_1}{2 \alpha' p_\mu p_\sigma S^\mu_2},
\]

\[
f(t) = \frac{-\alpha' k^\mu_1 k_1}{2 \alpha' p_\mu p_\rho}, \quad f'(t) = \frac{-\alpha' k^\mu_2 k_2}{2 \alpha' p_\mu p_\rho}. \tag{4.5}
\]

For example, for the untwisted sector see the Refs. \cite{61–63}. In the rest of this paper we shall explicitly calculate and concentrate on the variables \(A^U/T, B^U/T, C^U/T, D^U/T, E^U/T, F\) and \(G\) for the graviton, Kalb–Ramond and dilaton states.

Now we compute the integrals over the proper times in the eikonal approximation. For the twisted sector, the result is

\[
\int_0^\infty dt \int_0^\infty d\tau e^{-t' \alpha' k^\mu_1 k_1} e^{-t \alpha' k^\mu_2 k_2} e^{3(t' + \tau)} (e^{\mu' X_{\text{osc}}})_{T}^{S=1} = \mathcal{J}_1 \mathcal{J}_2, \tag{4.6}
\]

\[
\mathcal{J}_{1,2} = \frac{1}{4} \exp \left[11 \alpha' p_\rho p_\sigma S^{\rho\sigma}_1 (\gamma_{\text{YM}} - 1)\right]
\times \left\{ \frac{\left(\alpha' k^\rho_2 k_2 - 3\right)}{4} \left[ \Gamma \left(\frac{1}{2} \right) - \alpha' \left[ 11 p_\rho p_\sigma S^{\rho\sigma}_1 - 2 p_\rho p_\sigma \right] \right] \right\}
\times \left\{ \frac{\left(\alpha' k^\rho_2 k_2 - 3\right)}{4} \left[ \Gamma \left(\frac{1}{2} \right) - \alpha' \left[ 11 p_\rho p_\sigma S^{\rho\sigma}_1 - 2 p_\rho p_\sigma \right] \right] \right\}. \tag{4.7}
\]

For the untwisted sector, we obtain

\[
\int_0^\infty dt' \int_0^\infty d\tau e^{-t' \alpha' k^\mu_1 k_1} e^{-t \alpha' k^\mu_2 k_2} e^{3(t' + \tau)} (e^{\mu' X_{\text{osc}}})_{U}^{S=1} = \mathcal{J}_1 \mathcal{J}_2 \tag{4.8}
\]

\[
\mathcal{J}_{1,2} = \frac{1}{4} \exp \left[13 \alpha' p_\mu p_\nu S^{\mu\nu}_{1,2} (\gamma_{\text{YM}} - 1)\right]
\times \left\{ \frac{\left(\alpha' k^\mu_2 k_2 - 4\right)}{4} \left[ \Gamma \left(\frac{1}{2} \right) - 13 \alpha' p_\mu p_\nu S^{\mu\nu}_{1,2} \right] \right\}
\times \left\{ \frac{\left(\alpha' k^\mu_2 k_2 - 4\right)}{4} \left[ \Gamma \left(\frac{1}{2} \right) - 13 \alpha' p_\mu p_\nu S^{\mu\nu}_{1,2} \right] \right\}.
\]

These are the typical factors, which emerge in the two-point functions on the worldsheets with the disk topology (see e.g. \cite{70–73}).

4.1 The graviton emission

For obtaining a generalized formulation, we do not impose extra assumptions to the polarization tensor. The polarization tensors should satisfy the condition \(p^\mu e_{\mu,\nu} = 0\). The polarization tensor of the graviton is symmetric and traceless, i.e., \(e_{\mu,\nu} = e_{\nu,\mu}\) and \(e_{\mu,\mu} = 0\). Therefore, the untwisted part of the radiation amplitude of the graviton is given by Eq. (4.1) with the following variables

\[
A^U = e_\alpha^g \left[p_\mu p_\nu \left(Q_1^{\alpha\beta} Q_1^{\beta\gamma} - Q_1^{\alpha\gamma} Q_1^{\beta\rho}\right) + 2 p_\mu p_\nu T_{1,2}^{\alpha\gamma} + T_{1,2}^{\alpha\rho} Q_1^{\beta\rho} + e_\alpha^g \delta_{IJ} \left(p_\mu p_\nu Q_1^{\alpha\gamma} + p_\mu^2 p_\nu \right) - p_\mu^2 p_\nu \right],
\]

\[
B^U = e_\alpha^g \left[p_\mu p_\nu \left(Q_2^{\alpha\beta} Q_2^{\beta\gamma} - Q_2^{\alpha\gamma} Q_2^{\beta\rho}\right) + 2 p_\mu p_\nu Q_2^{\alpha\gamma} + p_\mu^2 p_\nu Q_2^{\beta\rho} + e_\alpha^g \delta_{IJ} \left(p_\mu p_\nu Q_2^{\alpha\gamma} + p_\mu^2 p_\nu \right) - p_\mu^2 p_\nu \right],
\]

\[
C^U = e_\alpha^g \left[26 \left(p^\mu + \frac{1}{13} k_1^\mu\right) \left(p^\rho + p_\sigma Q_1^{\rho\sigma}\right) - \frac{1}{2} \left(k_1^\mu k_1^\nu - k_2^\mu k_2^\nu\right) \right] - \frac{1}{2} e_\alpha^g \left[26 p^\mu p_\nu Q_2^{\rho\nu} + 26 p^\mu p_\nu + k_1^\mu k_1^\nu - k_2^\mu k_2^\nu \right] + e_\alpha^g \delta_{IJ} \left(2 k_1^\mu p^\nu - 26 p^\mu p_\nu \right),
\]

\[
D^U = e_\alpha^g \left[26 \left(p^\mu + \frac{1}{13} k_1^\mu\right) \left(p^\rho + p_\sigma Q_2^{\rho\sigma}\right) - \frac{1}{2} \left(k_2^\mu k_1^\nu - k_2^\mu k_2^\nu\right) \right] - \frac{1}{2} e_\alpha^g \left[26 p^\mu p_\nu Q_2^{\rho\nu} + 26 p^\mu p_\nu + k_1^\mu k_1^\nu - k_2^\mu k_2^\nu \right] + e_\alpha^g \delta_{IJ} \left(2 k_1^\mu p^\nu - 26 p^\mu p_\nu \right),
\]

\[
E^U = e_\alpha^g \left[13 \delta_{IJ} \eta_{\alpha\beta} \left(k_1^\mu k_1^\nu - k_2^\mu k_2^\nu\right) \right] + (26 p - k_1)^\alpha k_1^\beta - 169 p^\alpha p^\beta + 13 e_\alpha^g \left[(k_1 - 13 p)^J \left(p - \frac{1}{13} k_1\right)^I + \frac{1}{2} \delta_{IJ} \right].
\]
For the twisted sector, the graviton emission amplitude is described by Eq. (4.2) with the following variables

\[ A^T = e_{a\beta}^g \left[ p_\xi p_\phi \left( Q_1^{\alpha\beta} Q_2^{\xi\phi} - Q_1^{Ta\xi} Q_1^{T\beta\phi} \right) + 2p_\xi p_\phi Q_2^{a\beta} + p^i p_i Q_1^{\alpha\beta} \right] + e_{ij}^g \left[ \left( p_\sigma p_\rho Q_1^{\alpha\beta} + p^k p_k \right) - p^i p^j \right] + 2e_{a\alpha}^g p^a p_\beta Q_1^{\alpha\beta}, \]

(4.15)

\[ B^T = e_{a\beta}^g \left[ p_\xi p_\phi \left( Q_2^{a\beta} Q_2^{\xi\phi} - Q_2^{a\xi} Q_2^{\beta\phi} \right) + 2p_\xi p_\phi Q_2^{a\beta} + p^i p_i Q_2^{a\beta} \right] + e_{ij}^g \left[ \delta_{ij} \left( p_\sigma p_\rho Q_2^{a\beta} + p^k p_k \right) - p^i p^j \right] + 2e_{a\alpha}^g p^a p_\beta Q_2^{a\beta}. \]

(4.16)

\[ C^T = 22e_{a\beta}^g \left[ \left( \rho^a + \frac{1}{11} k_1^a \right) \left( p^\beta + p_\phi Q_1^{T\beta\phi} \right) - \frac{1}{44} Q_1^{a\beta} \left( k_1^a k_{1\rho} - k_2^a k_{2\rho} \right) \right] + e_{\alpha}^g \left( p_\sigma p_\rho Q_1^{\alpha\beta} + p^i p_i \right) - 22e_{a\alpha}^g p^a \left( p + \frac{1}{11} k_1 \right)^\alpha + e_{ij}^g \left( 2k_1^a p^i - 22p^i p^j \right) - e_{i}^{g\alpha} \left[ 21p^i p_j + (k_1^a k_{1\rho} - k_2^a k_{2\rho}) \right] + 11p_\alpha p_\beta \left( Q_1^{\phi\beta} + \eta^{\alpha\beta} \right) - p_\alpha p_\beta Q_1^{\alpha\beta} \right], \]

(4.17)

\[ D^T = 22e_{a\beta}^g \left[ \left( \rho^a + \frac{1}{11} k_1^a \right) \left( p^\beta + p_\phi Q_2^{\alpha\beta} \right) - \frac{1}{44} Q_2^{a\beta} \left( k_1^a k_{1\rho} - k_2^a k_{2\rho} \right) \right] + e_{\alpha}^g \left( p_\sigma p_\rho Q_2^{a\beta} + p^i p_i \right) - 22e_{a\alpha}^g p^a \left( p + \frac{1}{11} k_1 \right)^\alpha + e_{ij}^g \left( 2k_1^a p^i - 22p^i p^j \right) - e_{i}^{g\alpha} \left[ 21p^i p_j + (k_1^a k_{1\rho} - k_2^a k_{2\rho}) \right] + 11p_\alpha p_\beta \left( Q_2^{\phi\beta} + \eta^{\alpha\beta} \right) - p_\alpha p_\beta Q_2^{a\beta} \right], \]

(4.18)

\[ E^T = e_{a\beta}^g \left[ \frac{11}{2} \eta^{\alpha\beta} \left( k_1^a k_{1\rho} - k_2^a k_{2\rho} \right) + (22p - k_1)^a \left( k_1 - \frac{121}{2} p^a p^\beta \right) \right] + 11e_{ij}^g \left[ (k_1 - 11p_i) \left( p - \frac{1}{11} k_1 \right)^j + \frac{1}{2} \delta_{ij} \right] + 11e_{a\alpha}^g \left( p - \frac{1}{11} k_1 \right)^\alpha \left( k_1 + 11p \right)^i, \]

(4.19)

\[ F = e_{a\alpha}^g \left( \delta_{a\alpha} - p_\beta p_\rho - p^a p^\alpha \right), \]

\[ G = -\frac{1}{2} e_{a}^{g\alpha} \left( k_1^a k_{1\rho} - k_2^a k_{2\rho} \right). \]

(4.20)

4.2 The Kalb–Ramond emission

The Kalb–Ramond (axion) polarization tensor is anti-symmetric \( \epsilon_{\mu\nu} = -\epsilon_{\nu\mu}. \) According to the right-hand sides of Eqs. (3.35), (3.37) and (3.38), which are symmetric, the amplitude of the axion emission should be independent of the matrix elements \( \epsilon_{ij}, \epsilon_{ij}^\prime \) and \( \epsilon_{ab}. \) Hence, for the untwisted sector, we acquire

\[ A^U = -e_{a\beta}^g \left[ p_\xi p_\phi \left( Q_1^{a\beta} Q_1^{\xi\phi} + Q_1^{Ta\xi} Q_1^{T\beta\phi} \right) + p^i p_i Q_1^{a\beta} \right], \]

(4.21)

\[ B^U = e_{a\beta}^g \left[ p_\xi p_\phi \left( Q_2^{a\beta} Q_2^{\xi\phi} - Q_2^{a\xi} Q_2^{\beta\phi} \right) + p^i p_i Q_2^{a\beta} \right]. \]

(4.22)

\[ C^U = D^U = E^U = F = G = 0. \]

(4.23)

4.3 The dilaton emission

In contrast to the graviton’s and axion’s polarization tensors, the explicit form of the dilaton polarization tensor is available

\[ \epsilon_{\mu\nu}^\phi = \frac{1}{\sqrt{24}} \left( \eta_{\mu\nu} - p_\mu \tilde{p}_\nu - p_\nu \tilde{p}_\mu \right), \]

(4.27)

where \( \tilde{p}^\mu \) is a light-like vector, i.e. \( \tilde{p}^\mu \tilde{p}_\mu = 0, \) such that \( p^\mu \tilde{p}_\mu = 1. \) Thus, the untwisted sector of the dilaton emission possesses the following variables

\[ A^U = \frac{1}{\sqrt{24}} \left[ p_\xi p_\phi \left( Q_1^{a\alpha} Q_1^{\xi\phi} - Q_1^{Ta\xi} Q_1^{T\beta\phi} \right) + 2p_\alpha p_\beta Q_1^{a\beta} + p^i p_i Q_1^{a\beta} \right] + (p_\sigma \tilde{p}_\rho - p_\rho \tilde{p}_\sigma) \left[ p_\xi p_\phi \left( Q_1^{a\alpha} Q_1^{\xi\phi} + Q_1^{Ta\xi} Q_1^{T\beta\phi} \right) - 2p_\alpha p_\beta Q_1^{a\beta} \right] + \left[ 25p - p_\alpha p_\beta Q_1^{a\beta} + p^i p_i Q_1^{a\beta} \right] - 2p_\alpha p_\beta Q_1^{a\beta}. \]

(4.28)

\[ B^U = \frac{1}{\sqrt{24}} \left[ p_\xi p_\phi \left( Q_2^{a\alpha} Q_2^{\xi\phi} - Q_2^{a\xi} Q_2^{\beta\phi} \right) + 2p_\alpha p_\beta Q_2^{a\beta} + p^i p_i Q_2^{a\beta} \right] + 2p_\alpha p_\beta Q_2^{a\beta} + p^i p_i Q_2^{a\beta}. \]
\[ -(p_\alpha \bar{p}_\beta - p_\beta \bar{p}_\alpha) \left[ p_\xi p_\theta \left( Q_{1}^{a} Q_{1}^{\xi} - Q_{2}^{a} Q_{2}^{\xi} \right) \right. \]
\[ + 2 p_\xi p_\theta Q_{1}^{a} Q_{1}^{\xi} + p^i p^I Q_{1}^{a} Q_{1}^{\xi} \]
\[ + \left( 25 - p \right) \left( p_\alpha p_\beta Q_{2}^{a} + p^i p^I - p^i p^I \right) \]
\[ - 2 p^i p^I p_\alpha p_\beta Q_{2}^{a} \quad \cdots \quad \cdots \quad (4.29) \]
\[ \mathbf{c}_U = \frac{1}{\sqrt{24}} \left\{ 26 \left( p_\beta Q_{1}^{a} + p_\alpha \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ - \frac{1}{2} Q_{1}^{a} \left( k_{1\mu} - k_{2\mu} \right) \]
\[ - \frac{1}{2} Q_{2}^{a} \left( k_{1\mu} - k_{2\mu} \right) \]
\[ - \frac{1}{2} \left( p_\alpha \bar{p}_\beta - p_\beta \bar{p}_\alpha \right) \left[ 26 \left( p_\xi Q_{1}^{a} Q_{2}^{\xi} + p^i p^I \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ + \frac{1}{2} p^i p^I \left( 25 - 2 p I p^I \right) \left( 1 - 2 \bar{p}_J p^J \right) \]
\[ - (p + 1) \left( p_\alpha p_\beta Q_{1}^{a} + p^i p^I \right) \]
\[ + 2 p_\alpha \bar{p}_\beta \left( p_\xi p_\theta Q_{1}^{a} + p^i p^I \right) \]
\[ + \left( 25 - 2 p I p^I \right) \left[ 26 p^i p^J + \frac{1}{2} \left( k_{1\mu} - k_{2\mu} \right) \right. \]
\[ + 12 p_\alpha p_\beta \left( Q_{1}^{a} + \eta Q_{2}^{a} \right) \left\} \right. \]
\[ (4.30) \]
\[ \mathbf{d}_U = \frac{1}{\sqrt{24}} \left\{ 26 \left( p_\beta Q_{1}^{a} + p_\alpha \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ - \frac{1}{2} Q_{2}^{a} \left( k_{1\mu} - k_{2\mu} \right) \]
\[ - \frac{1}{2} \left( p_\alpha \bar{p}_\beta - p_\beta \bar{p}_\alpha \right) \left[ 26 \left( p_\xi Q_{1}^{a} Q_{2}^{\xi} + p^i p^I \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ + \frac{1}{2} p^i p^I \left( 25 - 2 p I p^I \right) \left( 1 - 2 \bar{p}_J p^J \right) \]
\[ - (p + 1) \left( p_\alpha p_\beta Q_{1}^{a} + p^i p^I \right) \]
\[ + 2 p_\alpha \bar{p}_\beta \left( p_\xi p_\theta Q_{1}^{a} + p^i p^I \right) \]
\[ + \left( 25 - 2 p I p^I \right) \left[ 26 p^i p^J + \frac{1}{2} \left( k_{1\mu} - k_{2\mu} \right) \right. \]
\[ + 12 p_\alpha p_\beta \left( Q_{1}^{a} + \eta Q_{2}^{a} \right) \left\} \right. \]
\[ (4.31) \]
\[ \mathbf{e}_U = \frac{1}{\sqrt{24}} \left\{ \frac{13}{2} \left( k_{1\mu} - k_{2\mu} \right) \right. \]
\[ + \left( 26 p + k_{1\mu} k_{1\alpha} + 169 p^a p_a \right) \]
\[ - 2 p_\alpha \bar{p}_\beta \left\{ \frac{13}{2} \eta Q_{1}^{a} \left( k_{1\mu} - k_{2\mu} \right) \right. \]
\[ + \left( 26 p + k_{1\mu} k_{1\alpha} + 169 p^a p^b \right) \]
\[ + \left. 13 \left( 25 - \frac{1}{2} \left( k_{1\mu} - k_{2\mu} \right) \right) \left( p - \frac{1}{13} k_1 \right) \right\} \]
\[ + \left( p_\alpha \bar{p}_\beta \right) \left( k_{1\mu} - k_{2\mu} \right) \]
\[ + \left[ 26 p_\alpha \bar{p}_\beta p^a \right. \]
\[ (4.32) \]
For the twisted sector we have
\[ \mathbf{A}_T = \frac{1}{\sqrt{24}} \left\{ p_\xi p_\theta \left( Q_{1}^{a} Q_{1}^{\xi} - Q_{1}^{a} Q_{1}^{\xi} \right) \right. \]
\[ + 2 p_\alpha p_\beta Q_{2}^{a} + p^i p^I Q_{1}^{a} \]
\[ + \left( p_\alpha \bar{p}_\beta - p_\beta \bar{p}_\alpha \right) \left[ p_\xi p_\theta \left( Q_{1}^{a} Q_{1}^{\xi} Q_{1}^{a} \right. \right. \]
\[ + \left( 21 - p \right) \left( p_\alpha p_\beta Q_{1}^{a} + p^i p^I \right) \]
\[ - 2 p_\xi \left( p_\alpha \bar{p}_\beta + p_\alpha \bar{p}_\beta \right) \left( Q_{1}^{a} \right) \left. \right. \]
\[ (4.33) \]
\[ \mathbf{B}_T = \frac{1}{\sqrt{24}} \left\{ p_\xi p_\theta \left( Q_{1}^{a} Q_{1}^{\xi} - Q_{1}^{a} Q_{1}^{\xi} \right) \right. \]
\[ + \left( 21 - p \right) \left( p_\alpha p_\beta Q_{1}^{a} + p^i p^I \right) \]
\[ - 2 p_\xi \left( p_\alpha \bar{p}_\beta + p_\alpha \bar{p}_\beta \right) \left( Q_{1}^{a} \right) \left. \right. \]
\[ (4.34) \]
\[ \mathbf{C}_T = \frac{1}{\sqrt{24}} \left\{ 22 \left( p_\xi Q_{1}^{a} Q_{2}^{\xi} + p_\alpha \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ - \frac{1}{2} Q_{1}^{a} \left( k_{1\mu} k_{1\alpha} + k_{2\mu} k_{2\mu} \right) \]
\[ - \left( p_\alpha \bar{p}_\beta - p_\beta \bar{p}_\alpha \right) \left[ 22 \left( p_\xi Q_{1}^{a} Q_{2}^{\xi} + p^i p^I \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ + \left. \left. \frac{1}{2} Q_{1}^{a} \left( k_{1\mu} k_{1\alpha} - k_{2\mu} k_{2\mu} \right) \right) \right\} \left. \right. \]
\[ (4.35) \]
\[ \mathbf{D}_T = \frac{1}{\sqrt{24}} \left\{ 22 \left( p_\xi Q_{2}^{a} + p_\alpha \right) \left( p + \frac{1}{13} k_1 \right)^2 \right. \]
\[ - \frac{1}{2} Q_{1}^{a} \left( k_{1\mu} k_{1\alpha} - k_{2\mu} k_{2\mu} \right) \]
\[ - \left( p_\alpha \bar{p}_\beta - p_\beta \bar{p}_\alpha \right) \left. \right. \]
\[
\begin{align*}
\mathcal{I}_1 & \approx \frac{1}{\alpha' k_1^\rho k_1^\rho} - \frac{3}{4}, \\
\mathcal{J}_2 & \approx \frac{1}{\alpha' k_1^\rho k_1^\rho} - \frac{3}{4}, \\
\mathcal{J}_1 & \approx \frac{1}{\alpha' k_2^\rho k_2^\rho} - \frac{1}{4}.
\end{align*}
\]

Now let us apply the constant shifts \(3/\alpha'\) and \(4/\alpha'\) to the squared momenta in the twisted and untwisted sectors, respectively. Adding all these together gives the following amplitudes for the low energy limit case

\[
\mathcal{A}_\text{ELL}^{U,I}(\ell_i \to 0) = \frac{T^2}{8(2\pi)^{23-2p}} \left| \frac{1}{|v_1 - v_2|} \prod_{\tilde{a}=1}^{p} \delta(p_{\tilde{a}}) \right| \times \int_{\tilde{a}=1}^{p} d k_i^\rho e^{i k_i^\rho b_i} \times \left\{ \begin{array}{l} 
\mathcal{E}_{\text{shift}}^{U} \left( k_1^\rho k_1^\rho k_2^\rho k_2^\rho \right) - \mathcal{L}_{\text{shift}}^{U} \left( k_1^\rho k_1^\rho k_2^\rho k_2^\rho \right) \end{array} \right\}
\]

for the untwisted sector, with

\[
\begin{align*}
\mathcal{L}_{\text{shift}}^{U} & = \frac{1}{2p_\mu p_\sigma S_1} \left( C_{\text{shift}}^{U} - \frac{k_1^\rho k_2^\rho}{2p_\mu p_\sigma S_1} \mathcal{A}_{\text{shift}}^{U} \right), \\
\mathcal{Y}_{\text{shift}}^{U} & = \frac{1}{2p_\mu p_\sigma S_1} \left( D_{\text{shift}}^{U} - \frac{k_1^\rho k_2^\rho}{2p_\mu p_\sigma S_1} \mathcal{B}_{\text{shift}}^{U} \right).
\end{align*}
\]

For the twisted sector, we find

\[
\begin{align*}
\mathcal{A}_\text{ELL}^{T,I}(\ell_i \to 0) & = \frac{T^2}{8(2\pi)^{23-2p}} \left| \frac{1}{|v_1 - v_2|} \prod_{\tilde{a}=1}^{p} \delta(p_{\tilde{a}}) \right| \times \int_{\tilde{a}=1}^{p} d k_i^\rho e^{i k_i^\rho b_i} \\
& \times \left\{ \begin{array}{l} 
\mathcal{E}_{\text{shift}}^{T} \left( k_1^\rho k_1^\rho k_2^\rho k_2^\rho \right) - \mathcal{L}_{\text{shift}}^{T} \left( k_1^\rho k_1^\rho k_2^\rho k_2^\rho \right) \end{array} \right\},
\end{align*}
\]

where

\[
\begin{align*}
\mathcal{L}_{\text{shift}}^{T} & = \frac{1}{2p_\mu p_\sigma S_1} \left( C_{\text{shift}}^{T} - \frac{k_1^\rho k_2^\rho}{2p_\mu p_\sigma S_1} \mathcal{A}_{\text{shift}}^{T} \right) + \frac{1}{2p_\mu p_\sigma S_1} \left( D_{\text{shift}}^{T} - \frac{k_1^\rho k_2^\rho}{2p_\mu p_\sigma S_1} \mathcal{B}_{\text{shift}}^{T} \right), \\
\mathcal{Y}_{\text{shift}}^{T} & = \frac{1}{2p_\mu p_\sigma S_1} \left( D_{\text{shift}}^{T} - \frac{k_1^\rho k_1^\rho}{2p_\mu p_\sigma S_1} \mathcal{B}_{\text{shift}}^{T} \right).
\end{align*}
\]

The summation of the amplitudes (4.41) and (4.43) represents the low energy emission of one of the graviton, Kalb–Ramond and dilaton states. Our physical results are comparable with Refs. [61, 62].

The denominators \(k_1^\rho k_1^\rho\) and \(k_2^\rho k_2^\rho\) (and similarly \(k_1^\rho k_1^\rho\) and \(k_2^\rho k_2^\rho\)) correspond to the particle’s propagator. Besides, the terms with \(\mathcal{X}_{\text{shift}}^{U/T}\) and \(\mathcal{Y}_{\text{shift}}^{U/T}\) originate from a single-pole process. We observe that the quantities \(\mathcal{X}_{\text{shift}}^{U/T}\) and \(\mathcal{Y}_{\text{shift}}^{U/T}\)
and \( \mathcal{S}_{\text{shift}} \) completely depend on the velocity and fields of the first (the second) brane. Thus, the \( \mathcal{S}^{T/U}_{\text{shift}} \) -term (\( \mathcal{S}^{T/U}_{\text{shift}} \) -term) elaborates that a massless closed string is emitted by the second (the first) brane, then is absorbed by the first (the second) brane, then after traveling as an excited state, it redescents by emitting a massless particle on the first (second) brane.

The \( E^{U} \text{shift} \) -terms denote the double-pole mechanism. We saw that the quantities \( E^{U} \text{shift} \) and \( E^{T} \text{shift} \) are independent of the velocities and fields of both branes. Hence, these terms clarify that the radiation occurs from the exchanged closed string between the branes. The latter closed string causes the interaction of the branes.

Since all quantities in Eqs. (4.10)–(4.20) and in Eqs. (4.28)–(4.39) are nonzero, we conclude that each of the graviton and dilaton radiations, while the axion radiation cannot occur in the middle region of the branes. In other words, the axion emission can take place only on the surface of one of the branes.

5 Conclusions

We computed the boundary states, which correspond to a fractional Dp-brane, in the untwisted and twisted sectors of the bosonic string theory. The fractional brane has been located at the fixed-points of the non-compact orbifold \( \mathbb{R}^{4}/\mathbb{Z}_{2} \), and has been dressed by the Kalb–Ramond field and a \( U(1) \) gauge potential. Besides, it has a transverse motion. The orbifold part of the spacetime, the internal and background fields and dynamics of the brane drastically affected the boundary states.

In the closed string channel, the interaction of the D-branes happens via the exchange of a closed string. We inserted the general form of an integrated vertex operator, associated with a massless closed string, into the interaction amplitude of the branes. This defines the amplitude of the closed string radiation from the branes with an arbitrary distance. The presence of various parameters in the setup prominently generalized the radiation amplitude. By adjusting the parameters, the value of this amplitude can be adjusted to any suitable configurations.

We obtained the radiation amplitude for the case that the distance of the interacting branes is very large. To compare our results with the Refs. [61,62], the assumptions \( S = \hat{S} = 1 \) were imposed, which yield some relations between the parameters of our setup. Then, we explicitly calculated the amplitude of the graviton, Kalb–Ramond and dilaton emissions. We observed that only one of the following radiations could potentially take place: one radiation between the interacting branes and two radiations from the branes. All of these three possible processes can occur for the dilaton and graviton radiations, while the axion radiation cannot occur in the middle region of the branes.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: This paper is a theoretical work. No experimental data were used.]

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