Model Transformations for Ranking Functions and Total Preorders

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Abstract
In the field of knowledge representation, the considered epistemic states are often based on propositional interpretations, also called worlds. E.g., epistemic states of agents can be modelled by ranking functions or total preorders on worlds. However, there are usually different ways of how to describe a real world situation in a propositional language; this can be seen as different points of view on the same situation. In this paper we introduce the concept of model transformations to convert an epistemic state from one point of view to another point of view, yielding a novel notion of equivalence of epistemic states. We show how the well-known advantages of syntax-splitting, originally developed for belief sets and later extended to representation of epistemic states and to nonmonotonic reasoning, can be exploited for belief revision via model transformation by uncovering splittings not being present before. Furthermore, we characterize situations where belief change operators commute with model transformations.

1 Introduction
In the field of knowledge representation, the considered objects are often based on propositional logic. A statement can be modelled as a logical formula directly; a conditional \((B|A)\) formalizes a defeasible rule “If \(A\) then usually \(B\)” for logical formulas \(A, B\). Other representations are based on propositional interpretations, also called (possible) worlds. Epistemic states of agents can be modelled, e.g., by a ranking function assigning a rank to each world, a total preorder on the set of worlds, or a belief set which can be represented by the set of its models. Common to these approaches is that they assume an underlying (propositional) signature on which the formulas are based and which determines the set of propositional interpretations occurring in the epistemic states. When choosing which part of a situation is described with which atomic sentence, there are often different ways to model the same subject.

Example 1. Two programs \(P_1\) and \(P_2\) are running on a computer. Usually either both or none of the programs has access to the internet, depending on whether the computer is connected to a network with an internet connection. But sometimes a weird firewall configuration causes the situation that one of the programs has internet access but not the other program. We could model this situation with two signature variables \(a, b\) where \(a\) is true if program \(P_1\) can access the internet and \(b\) is true if program \(P_2\) can access the internet. Another way of modelling would be to introduce two variables \(c, d\) where \(c\) is true if \(P_1\) has internet access and \(d\) is true if a weird firewall configuration is in place that allows exactly one program to access the internet. While the two ways of choosing are different, the four interpretations of each signature correspond to the same four elementary events. For example, the situation where \(P_1\) has internet access but \(P_2\) not is modelled by \(a\) and by \(c\) or respectively.

The different approaches to modelling in the example can be seen as different points of view on the same situation. In this paper, we introduce the concept of model transformations that allows transforming between these points of view by establishing a connection between the worlds induced by each signature. As epistemic states that can be transformed into each other by model transformations can be seen as different points of view on the same situation.

Epistemic states are often used in combination with operations realizing belief changes or inferences. If an operator uses only the semantic side of an epistemic state based on worlds, then applying this operator and a model transformation is equivalent to applying the model transformation first and then the operator. We formalize such operations as language independent.

One important property of an epistemic state is if it allows for syntax splittings. Parikh (1999) introduced the concept of syntax splittings to formulate the revision postulate (P) describing that only the relevant parts of the belief base should be changed by belief revision operators. Later the notion of syntax splitting was extended to ranking functions and total preorders on worlds (Kern-Isberner and Brewka 2017). As syntax splittings depend on the language used, applying a model transformation to an epistemic state might yield a new or a finer syntax splitting. In this paper, we generalize the syntax splitting postulate (P) to also consider syntax splittings that can be obtained by a model transformation.

To summarize, the main contributions of this paper are

\begin{itemize}
  \item the introduction of model transformations as transformations between different points of view for ranking functions and total preorders,
  \item the introduction of language independence as property for operators on epistemic states,
  \item a generalized syntax splitting postulate for belief sets that considers model transformations.
\end{itemize}
In Section 2 we briefly recall the required background on conditional logic. In Section 3 we introduce model transformations, and in Section 4 we consider language independent operations. We investigate syntax splitting in combination with model transformations in Section 5 before concluding and discussing future work in Section 6.

2 Background: Logic, OCFs, and TPOs

A (propositional) signature is a finite set \( \Sigma \) of identifiers; we denote the propositional language over \( \Sigma \) by \( \mathcal{L}_\Sigma \). Usually, we denote elements of the signatures with lowercase letters \( a, b, c, \ldots \) and formulas with uppercase letters \( A, B, C, \ldots \). We may denote \( A \land B \) by \( AB \) and \( \neg A \) by \( \overline{A} \) for brevity of notation. The set of interpretations over \( \Sigma \) is denoted as \( \Omega_\Sigma \). Interpretations are also called worlds and \( \Omega_\Sigma \) is called the universe. An interpretation \( \omega \in \Omega_\Sigma \) is a model of a formula \( A \in \mathcal{L}_\Sigma \) if \( A \) holds in \( \omega \), denoted as \( \omega \models A \). The set of models of a formula over \( \Sigma \) is \( \text{Mod}_\Sigma (A) = \{ \omega \in \Omega_\Sigma \mid \omega \models A \} \). A formula \( A \) entails a formula \( B \) if \( \text{Mod}_\Sigma (A) \subseteq \text{Mod}_\Sigma (B) \), denoted as \( A \models B \).

The deductive closure of a set \( S \) of formulas is \( \text{Cu}(S) = \{ F \in \mathcal{L}_\Sigma \mid S \models F \} \); for formulas \( A, B, \ldots \) we abbreviate \( \text{Cu}(\{ A, B, \ldots \}) \). A belief set \( K \) is a deductively closed set of formulas, i.e., \( \text{Cu}(K) = K \). The theory for a set of interpretations \( I \subseteq \Omega_\Sigma \) is \( \text{Th}(I) = \{ F \in \mathcal{L}_\Sigma \mid \omega \models F \) for every \( \omega \in I \} \). For sets \( S, T \) of formulas we define \( S + T = \text{Cu}(S \cup T) \).

A conditional \( (B | A) \) connects two formulas \( A, B \) and represents the rule “If \( A \) then usually \( B \)”. The formula \( A \) is called the antecedent and the formula \( B \) the consequent of the conditional. A finite set of conditionals is called a conditional belief base. We use a three-valued semantics for conditionals in this paper (de Finetti 1937). For a world \( \omega \) a conditional \( (B | A) \) is either verified by \( \omega \) if \( \omega \models AB \), falsified by \( \omega \) if \( \omega \models \overline{AB} \), or not applicable to \( \omega \) if \( \omega \models \overline{A} \).

Two popular semantics for conditionals and conditional knowledge bases are ranking functions and total preorders.

A ranking function (Spohn 1988), also called ordinal conditional function (OCF), is a function \( \kappa : \Omega_\Sigma \to N_0 \cup \{ \infty \} \) such that \( \kappa^{-1}(0) \neq \emptyset \). The intuition of an OCF is that the rank of a world is lower if the world is more plausible. Ranking functions are extended to formulas by \( \kappa(A) = \min_{\omega \in \text{Mod}(A)} \kappa(\omega) \) with \( \min_\emptyset(\ldots) = \infty \). An OCF \( \kappa \) models a conditional \( (B | A) \), denoted as \( \kappa \models (B | A) \) if \( \kappa(AB) \leq \kappa(\overline{A}) \), i.e., if the verification of the conditional is strictly more plausible than its falsification. An OCF \( \kappa \) models a conditional belief set \( \Delta \), denoted as \( \kappa \models \Delta \) if \( \kappa \models r \) for every \( r \in \Delta \).

A total preorder (TPO) is a total, reflexive, and transitive binary relation. The meaning of a total preorder \( \preceq \) on \( \Omega_\Sigma \) as model for an epistemic state is that \( \omega_1 \) is at least as plausible as \( \omega_2 \) iff \( \omega_1 \preceq \omega_2 \) for \( \omega_1, \omega_2 \in \Omega_\Sigma \). Total preorders on worlds are extended to formulas by \( A \preceq B \) if \( \min(\text{Mod} \Sigma (A), \preceq) \preceq \min(\text{Mod} \Sigma (B), \preceq) \). A total preorder \( \preceq \) models a conditional \( (B | A) \), denoted as \( \preceq \models (B | A) \) if \( AB \preceq \overline{AB} \), i.e., if the verification of the conditional is strictly more plausible than its falsification. A total preorder \( \preceq \) models a conditional belief set \( \Delta \), denoted as \( \preceq \models \Delta \), if \( \preceq \models r \) for every \( r \in \Delta \).

Belief sets, OCFs, and TPOs can each be used to model the epistemic state of an agent. In an evolving world, an agent needs to update her beliefs to account for new information. The process of including new beliefs into the current epistemic state and resolving possible inconsistencies is called belief revision. Such belief revisions can be formalized by a belief revision operator \( * \) mapping the epistemic state before the revision and the incoming information to the new epistemic state; \( K * A \) denotes the result of revising epistemic state \( K \) with the information \( A \) (e.g., (Alchourrón, Gärdenfors, and Makinson 1985; Darwiche and Pearl 1997; Parikh 1999)). General belief changes are denoted as \( K \tau A \).

To draw inferences from conditional beliefs inductive inference operators can be used. Inductive inference operators (Kern-Isberner, Beierle, and Brewka 2020) formalize the inductive completion of a conditional belief base according to an inference method; they are defined as a mapping \( C : \mathcal{R} \to \mathcal{R} \) that maps each belief base to an inference relation such that direct inference (DI) and trivial vacuity (TV) are fulfilled, i.e., if \( (B | A) \in \Delta \) implies \( A \tau B \) and if \( \Delta = \emptyset \) and \( A \tau B \) imply \( A \models B \).

3 Model Transformations

In this paper, we want to formalize changes as illustrated in Example 1. The two approaches to model the situation in the example resulted in different descriptions of the same real-world situations. For the general case, we define model transformations as bijections between two universes over possibly different signatures.

Definition 1 (model transformation). Let \( \Sigma_1, \Sigma_2 \) be signatures of the same size. A model transformation is a bijective mapping \( \phi : \Omega_{\Sigma_1} \to \Omega_{\Sigma_2} \).

Model transformations \( \phi \) can be lifted to OCFs and TPOs.

Definition 2 (model transformations for OCFs and TPOs). For \( \kappa \) over \( \Sigma_1 \) we define \( \phi(\kappa) = \kappa' \) where \( \kappa' \) is an OCF over \( \Sigma_2 \) such that \( \kappa'(\omega) = \kappa(\phi(\omega)) \) for any \( \omega \in \Omega_\Sigma \).

For \( \preceq \) over \( \Sigma_1 \) we define \( \phi(\preceq) = \preceq' \) where \( \preceq' \) is a TPO over \( \Sigma_2 \) such that \( \omega \preceq' \omega' \) iff \( \phi^{-1}(\omega) \preceq \phi^{-1}(\omega') \) for any \( \omega, \omega' \in \Omega_\Sigma \).

This definition implies \( \kappa(\omega) = \kappa'(\phi(\omega)) \) for every \( \omega \in \Omega_\Sigma \) and \( \omega \preceq \omega' \) iff \( \phi(\omega) \preceq' \phi(\omega') \) for \( \omega, \omega' \in \Omega_\Sigma \).

Note that model transformations go far beyond renamings of the underlying signature as in (Beierle and Haldimann 2022). While each bijection \( \sigma : \Sigma_1 \to \Sigma_2 \) induces a model transformation \( \phi_\sigma : \Omega_{\Sigma_1} \to \Omega_{\Sigma_2} \) by \( \phi_\sigma(\omega) = \sigma(\omega) \), in general, model transformations cannot be obtained from signature renamings.

Example 2. Consider the signatures \( \Sigma_{abc} = \{ a, b, c \} \) and \( \Sigma_{xyz} = \{ x, y, z \} \). The function \( \phi : \Omega_{\Sigma_{abc}} \to \Omega_{\Sigma_{xyz}} \) is a model transformation. We have \( \phi(\kappa_{abc}) = \kappa_{xyz} \) where \( \kappa_{abc} \) and \( \kappa_{xyz} \) are the OCFs displayed in Figure 1.

Applying model transformations to formulas is more complex. If we consider the syntactic structure of a formula,
While some operators depend on the valuation of signature without non-trivial syntax splitting, we cannot apply model transformations directly. But if OCF \( \kappa \) and trivial update (Parikh 1999) for belief sets.

\[ \phi(\omega_1, \ldots, \omega_n) \]

To work with formulas in this paper, from now on we assume that every formula is in CDNF. \( \phi \) is lifted to conditionals by \( \phi((B|A)) = (\phi(B)|\phi(A)) \) and to belief sets by \( \phi(K) = \text{Th}(\phi(\text{Mod}(K))) \).

Model transformations are compatible with the models relation and with logical entailment.

**Proposition 1.** Let \( \phi : \Omega_{\Sigma_1} \to \Omega_{\Sigma_2} \) be a model transformation. Let \( \omega \in \Omega_{\Sigma_1} \) and \( A, B \in \mathcal{L}_{\Sigma_1} \). Then \( \omega \models A \iff \phi(\omega) \models \phi(A) \); and \( A \models B \iff \phi(A) \models \phi(B) \).

Let \( \kappa \) be an OCF over \( \Sigma_1 \) and \( \preceq \) be a TPO over \( \Sigma_1 \). Let \( (B|A) \) be a conditional over \( \Sigma_1 \). Then \( (B|A) \models \kappa \iff \phi((B|A)) \models \phi(\kappa) \); and \( (B|A) \models \preceq \iff \phi((B|A)) \models \phi(\preceq) \).

Proposition 1 ensures that \( A \models K \iff \phi(A) \models \phi(K) \) for any formula \( A \), belief set \( K \), and model transformation \( \phi \).

### 4 Language Independent Operations

While some operators depend on the valuation of signature variables in each world, many operators for belief change only consider worlds as atomic objects. With model transformations we can formalize belief revision operators that do not depend on syntax at all. These operators are independent of the application of a model transformation.

**Definition 3** (language independent belief change operators). A belief change operator \( \circ \) is called language independent if \( \phi(X) \circ \phi(Y) = \phi(X \circ Y) \) for each model transformation \( \phi \).

Many belief change operators in the literature are language independent; they focus only on the semantic side of epistemic states and formulas.

**Proposition 2.** The following belief change operators are language independent:

- **moderate, natural, and lexicographic contraction** (Ramachandran, Nayak, and Orgun 2012) for TPOs
- **natural revision** (Boutilier 1996) and simple lexicographic revision (Nayak, Pagnucco, and Peppas 2003) for TPOs
- **expansion +** (Alchourrón, Gärdenfors, and Makinson 1985) and trivial update (Parikh 1999) for belief sets.

Dalal’s revision operator (Dalal 1988) for belief sets is not language independent.

**Proof sketch.** This can be verified by considering the definitions of these operations. The moderate, natural, and lexicographic contraction as well as the natural and simple lexicographic revision can be defined in a way that only considers the position of each world in the relation before the belief change and whether the world is a model of the input formula. Expansion and trivial update can be also defined in a way that only considers if the worlds are a model of the initial belief set and if they are a model of the input formula. Dalal’s revision is based on a TPO on worlds that compares the number of differently valued variables in different worlds. \( \square \)

We can see that language independence is a property that occurs naturally in many revision operators, but not every revision is language independent.

We can define language independence for inductive inference operators as well.

**Definition 4** (language independent inference operators). An inductive inference operator \( C : \mathcal{R} \to \sim_{\mathcal{R}} \) is called language independent if, for every model transformation \( \phi \), it holds that \( A \sim_{\mathcal{R}} B \iff \phi(A) \sim_{\phi(\mathcal{R})} \phi(B) \).

There are many examples of language independent inductive inference operators in the literature.

**Proposition 3.** \( p \)-entailment (Adams 1965), system Z (Pearl 1990) and lexicographic inference (Lehmann 1995) are language independent inductive inference operators.

Similar to the belief change operators in Proposition 2, the inference operators in Proposition 3 are defined in a way that only considers which conditionals in the belief base are verified and which are falsified by each world. Hence, they are language independent.

### 5 Transformations and Syntax Splitting

An important property of an epistemic state is whether it has a syntax splitting. A syntax splitting is a partition of the signature describing that a belief set, a total preorder, or a rankinfunktion, respectively, consists of independent information on different parts of the signature partitioning (Parikh 1999; Kern-Isberner and Brewka 2017). There are belief revision postulates describing that only the relevant part of the epistemic state must be revised. Respecting syntax splittings in belief revision leads to more intuitive revision operators and can also reduce the computational complexity of the belief revision by allowing to process several small parts of an epistemic state independently.

While syntax splittings are a highly desirable property they do depend on the underlying signature. Interesting about model transformations is that they can uncover new syntax splittings not being present before the transformation.

**Example 3.** The OCF \( \kappa_{abc} \) from Example 2 does not have a non-trivial syntax splitting. The OCF \( \phi(\kappa_{abc}) = \kappa_{xyz} \) has the syntax splitting \( \{ \{x\} \cup \{y, z\} \} \).

To capture syntax splittings that exist only after application of a model transformation we introduce the following generalized notion of syntax splitting.
Definition 5 (syntax splitting with respect to model transformations). Let $\Sigma$ be a signature. Let $X$ be a belief set, a TPO, or an OCF over $\Sigma$. A syntax splitting for $X$ with respect to model transformations is a pair $(P, \phi)$ consisting of a partitioning $P$ of $\Sigma$ and a model transformation $\phi : \Omega_\Sigma \rightarrow \Omega_\Sigma$ such that $P$ is a syntax splitting for $\phi(X)$.

Note that the restriction to model transformations from $\Sigma$ to $\Sigma$ does not limit the kind of partitions that occur in the syntax splittings. If we have a belief set, a TPO, or an OCF $X$ and there is a model transformation $\phi' : \Omega_\Sigma \rightarrow \Omega_\Sigma$ such that $P$ is a syntax splitting for $\phi'(X)$, then we can concatenate $\phi'$ with the model transformation $\phi_\sigma$ induced by a bijection $\sigma : \Sigma' \rightarrow \Sigma$ on the signatures to obtain a model transformation $\phi = \phi_\sigma \circ \phi'$ such that $\phi : \Omega_\Sigma \rightarrow \Omega_\Sigma$ and $(\phi, (P), \phi)$ is a syntax splitting with respect to model transformations for $X$.

Syntax splitting with respect to model transformations is a generalization of syntax splitting.

Proposition 4. If a belief set, a TPO, or an OCF $X$ has a syntax splitting $P$, then $(P, \text{id})$ is a syntax splitting for $X$ with respect to model transformations with the identity $\text{id}$.

Example 4. Consider again Example 2. Then $((\{a\}, \{b, c\}, \psi), \sigma)$ is a syntax splitting with respect to model transformations for the OCF $\kappa_{abc}$ with $\psi = \sigma \circ \phi$ and $\sigma : \Sigma_{xyz} \rightarrow \Sigma_{abc}; x \mapsto a, y \mapsto b, z \mapsto c$.

For belief sets, i.e., deductively closed sets of propositional formulas, Parikh introduced the postulate (P) to describe that only the information about the relevant sub-signatures in the syntax splitting should be changed.

Postulate (P), see (Parikh 1999). Let $K$ be a belief set and $A$ a formula. If there is a syntax splitting $\{\Sigma_1, \Sigma_2\}$ for $K$, i.e., if there are $C \in L_{\Sigma_1}, D \in L_{\Sigma_2}$ such that $K = \text{Cn}(C, D)$, and $A \in L_{\Sigma_1}$, then $K \ast A = (\text{Cn}(C) \ast A) + D$.

The postulate (P) not only ensures a more sensible outcome of belief revision operators, it is also useful for the computation of belief changes. Assume that we want to revise a belief set $K = \text{Cn}(C, D)$ with a syntax splitting $\{\Sigma_1, \Sigma_2\}$ and $C \in L_{\Sigma_1}, D \in L_{\Sigma_2}$ with a formula $A \in L_{\Sigma_1}$. If we use a revision operator that fulfills (P), we only have to calculate $\text{Cn}(C) \ast A$ and add $D$ unchanged to obtain $K \ast A$.

We adapt the syntax splitting postulate to the notion of syntax splitting with respect to model transformations.

Postulate (Language Independent P). Let $K$ be a belief set and $A$ a formula. If $K$ has a syntax splitting with respect to a model transformation $\{\Sigma_1, \Sigma_2\}, \phi$ and $\phi(A) \in L_{\Sigma_1}$, then $K \ast A = \phi^{-1}((\phi(K) \cap L_{\Sigma_1}) \ast \phi(A) + (\phi(K) \cap L_{\Sigma_2}))$.

The intuition of (Language Independent P) is that if the belief base has a syntax splitting with respect to model transformations, then we should be able to conduct the revision from this point of view and respect the syntax splitting.

As the syntax splitting exists only in the transformed belief set, we have to apply the model transformation of the syntax splitting to the belief set to separate the two parts. Using that $\text{Cn}(C, D) \cap L_{\Sigma_1} = \text{Cn}(C)$ for $C \in L_{\Sigma_1}, D \in L_{\Sigma_2}$ and $\{\Sigma_1, \Sigma_2\}$ is a partition of $\Sigma$, the part of the belief set containing the information about $\Sigma_i$ after the model transformation is $\phi(K) \cap L_{\Sigma_i}$ for $i \in \{1, 2\}$.

For revision operators that behave especially well with respect to model transformations, (Language Independent P) can already be inferred from (P).

Proposition 5. A language independent revision $\ast$ fulfills (Language Independent P) iff it fulfills (P).

Proof. To see that (Language Independent P) implies (P) consider the syntax splitting $\{\{\Sigma_1, \Sigma_2\}, \text{id}\}$.

For the other direction, let $\ast$ be a language independent revision operator that fulfills (P). Let $K$ be a belief set such that $\{\{\Sigma_1, \Sigma_2\}, \phi\}$ is a syntax splitting with respect to model transformations for $K$ and let $A$ be a formula such that $\phi(A) \in L_{\Sigma_1}$. Then we have $K \ast A = \phi^{-1}(\phi(K \ast A)) = \phi^{-1}(\phi(K) \ast \phi(A)) = \phi^{-1}((\phi(K) \cap L_{\Sigma_1}) \ast \phi(A) + (\phi(K) \cap L_{\Sigma_2}))$. \(\square\)

(Language Independent P) can be applied in strictly more situations than (P), implying that model transformations can uncover syntax splittings not being present before.

Proposition 6. There are belief sets that fulfill the prerequisites for (Language Independent P) but not for (P).

Proof. Assume we have the belief set $K = \text{Cn}(ab \lor \overline{ab})$ over the signature $\Sigma = \{a, b\}$ from Example 1, i.e., we believe that exactly one of the two programs has internet access. Now we want to revise $K$ with $A = ab \lor \overline{ab}$, i.e., we learn that we are actually in the usual situation that both or no programs have internet access. Even if we chose a revision operator fulfilling (P), we would have to consider the complete signature for this revision as $K$ does not have a syntax splitting. However, $K$ does have the syntax splitting with respect to model transformations $\{\{a\}, \{b\}\}, \phi$ with $\phi = \{ab \rightarrow \overline{cd}, a \overline{b} \rightarrow \overline{cd}, \overline{a} \overline{b} \rightarrow \overline{cd}\}$. Divergent from Definition 5 we use, just as in Example 1, the different signature $\{c, d\}$ for the transformed formulas to enhance readability, where $c$ is true if P1 has internet access and $d$ is true if a weird firewall configuration is in place that allows exactly one program to access the internet. In $\phi(K)$ the information about these two things are independent. If we know that our revision operator fulfills (Language Independent P), we can calculate $K \ast A$ by calculating $(\phi(K) \cap L_{\Sigma_1}) \ast \phi(A) = \text{Cn}(d) \ast \overline{d}$ on a smaller signature, combining it with $\phi(K) \cap L_{\Sigma_2} = \overline{c}$, and transforming it back with $\phi^{-1}$. \(\square\)

In the proof of Proposition 6 we see how (Language Independent P) ensures that syntax splitting with respect to model transformations is respected in a situation where (P) is not applicable. Additionally, knowing that $\ast$ fulfills (Language Independent P) allows to calculate the revision on only a part of the signature. In applications with larger signatures where only a small part of the belief set is relevant for a revision utilizing syntax splittings with respect to model transformations can be of advantage for the computation.

6 Conclusion and Further work

In this short paper we introduced the notion of model transformations. We outlined several applications of this notion, among them the definition of equivalence with respect to
model transformations, the definition of language independence as property of belief change and inference operators, and a generalized version of Parikh’s postulate (P).

In our current work, we want to further investigate syntax splittings postulates in the context of model transformations. Especially, we want to transfer the idea of (Language Independent P) to syntax splittings on OCFs and TPOs. Another open question is if there are other properties of a belief base, OCF, or TPO besides syntax splitting that can be improved by applying model transformations.

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