Because of a luminosity of up to five orders of magnitude larger than at LEP, electroweak precision measurements at the FCC-ee – the Future Circular Collider with electron-positron beams – would provide improvements by orders of magnitude over the present status and constitute a broad search for the existence of new, weakly interacting particles up to very high energy scales. The FCC-ee will address centre-of-mass energies ranging from below the $Z$ pole to the $t\bar{t}$ threshold and above. At collision energies around the $Z$ pole, the $Z$-boson mass and width can be measured to better than 100 keV each. Asymmetry measurements at the $Z$ pole allow improvements in the determination of the weak mixing angle by at least a factor 30 to $\delta \sin^2 \theta^{\text{eff}}_W \simeq 6 \times 10^{-6}$. An independent determination of the electromagnetic coupling constant at the $Z$ energy scale, $\alpha_{\text{QED}}(m_Z^2)$, to a relative precision of $3 \times 10^{-5}$ can be obtained via measurement of the forward-backward asymmetry of lepton pairs at two energy points $\pm 3.2$ GeV away from the $Z$ peak. At collision energies around the WW threshold, high-statistic cross section measurements can provide a determination of the $W$ mass to 300 keV. The key breakthrough advantage of the FCC-ee in these achievements, beside the large integrated luminosity, is the possibility of a continuous, precise determination of the beam energy by resonant depolarization at the $Z$ peak and at the WW threshold. Precise measurements of the hadronic branching fractions of the $Z$ and $W$ bosons allow for considerably improvements in the determination of the strong coupling constant down to a precision of $\delta \alpha_s(m_Z^2) \simeq 0.0001$. An energy scan around the 350 GeV $t\bar{t}$ threshold allows a 10 MeV measurement of the top-quark mass.
1. Introduction

The Higgs boson discovery [1, 2] in 2012 at the relatively low mass of 125 GeV has revived the interest in circular electron-positron colliders that can serve as, in particular, Higgs factories. The design study [3] of the Future Circular Collider (FCC) in a ~100-km tunnel in the Geneva area was initiated by CERN in early 2014. The ultimate goal of the FCC programme is a 100 TeV proton-proton collider. In the current plans, the first step of the FCC physics programme would exploit a high-luminosity $e^+e^-$ collider, called FCC-ee, able to address centre-of-mass energies ranging from below the Z pole to the $t\bar{t}$ production threshold and above. A first look at the physics case of the FCC-ee can be found in Refs. [4, 5]. The use of techniques inspired from b-factories, very strong focussing combined with full-energy top-up injection into separate $e^+$ and $e^-$ storage rings, allows the FCC-ee to achieve very high instantaneous luminosities of $8.4 \times 10^{36}$ cm$^{-2}$s$^{-1}$ at the Z pole, $1.5 \times 10^{36}$ cm$^{-2}$s$^{-1}$ at the WW threshold, $3.5 \times 10^{35}$ cm$^{-2}$s$^{-1}$ at 240 GeV for HZ production, and $8.4 \times 10^{34}$ cm$^{-2}$s$^{-1}$ at the $t\bar{t}$ threshold [6], where four interaction points have been assumed. Compared to linear collider projects, the advantages of the FCC-ee include the higher luminosity, the possibility to instrument several interaction points, and the possibility to measure the centre-of-mass energy with a precision of 100 keV. The luminosity advantage is about tenfold for the Higgs-strahlung process $e^+e^- \rightarrow ZH$ at $E_{CM} = 240$ GeV increasing steeply at lower energies. In summary, the FCC-ee programme aims at collecting $10^{12}$–$10^{13}$ Z decays in an energy scan around the Z mass, $2 \times 10^8$ W pairs at and above production threshold, $10^6$ Higgs decays at 240 GeV, and $10^6 t\bar{t}$ pairs at and just above threshold.

In this report, I discuss the prospects for precision electroweak measurements at the Z pole and at the WW production threshold. Top quark properties, mass and couplings, can be measured with high precision at the $t\bar{t}$ production threshold. Here, only the mass measurement is mentioned; a dedicated discussion of top-quark couplings can be found in Ref. [7] and in these proceedings [8]. Also in these proceedings can be found discussions of the prospects for the FCC-ee as a Higgs factory [9] and for high precision flavour physics measurements [10].

2. Electroweak precision measurements in the Higgs era

Electroweak loops have the remarkable property of being sensitive to the existence of weakly-coupled particles, even if these cannot be directly produced or observed in current or future experiments. Historically, electroweak precision measurements have been instrumental in predicting and determining free parameters of the Standard Model. Now, with the Higgs boson discovered, all SM particles are known, and, within the SM, there are no free knobs left to turn. The precision offered by the FCC-ee on all electroweak observables will therefore allow potential new physics to be identified, and may be used towards either constraining or fitting the parameters of Beyond-Standard-Model theories.

3. Physics at the Z pole

One of the cornerstones of electroweak precision measurements is the determination of the mass and width of the Z boson. These parameters were determined at LEP via the line-shape scan to
precisions of 2 MeV each. Key to this measurement was a precise knowledge of the centre-of-mass energy, which saturated the systematic uncertainty on the Z mass. At LEP, a relative precision on the beam energy of $2 \times 10^{-5}$ was reached [11] with the technique of resonant depolarisation [12]. A detailed discussion of this technique for FCC-ee can be found in these proceedings [6].

It was found at LEP that the intrinsic precision of each individual beam energy measurement was 100 keV or better. Since, however, this measurement was performed only in dedicated polarisation runs, the final uncertainty was more than tenfold larger because the measurements had to be extrapolated to the conditions of the physics collision runs. At FCC-ee, it will be possible to exploit the full precision of the technique by continuously applying it to a few non-colliding bunches (out of several tens of thousands), leading to uncertainties better than 100 keV on both the Z mass and width.

The five orders of magnitude larger statistics than at LEP will allow also for much improved measurements of other Z-pole observables such as Z partial widths and numerous asymmetries sensitive to the weak mixing angle. The benefit of the increased statistics can, in particular, be reaped for the measurement of ratios, where systematic effects tend to cancel. Important examples are $R_\ell$, the ratio between the hadronic and leptonic decay widths, and $R_b$, the ratio between the b-quark and the total hadronic widths. For these two parameters, respectively, relative precision of $5 \times 10^{-5}$ and $3 \times 10^{-4}$ are realistic targets. This bears importance for the determination of the strong coupling constant discussed below.

At LEP, the determination of the effective weak mixing angle, $\sin^2 \theta_{\text{eff}}^W$, was based on a variety of measurements such as the leptonic and hadronic forward-backward asymmetries and the $\tau$ polarisation in $Z \rightarrow \tau \tau$ decays. The single most precise measurement, however, came from the SLD experiment from the inclusive left-right beam-polarisation asymmetry, $A_{LR}$. Studies are ongoing to understand whether FCC-ee would be able to operate with longitudinally polarised beams, and whether, in this case, one would be still able to maintain the required precision on the beam energy calibration. Even without longitudinally polarised beams, the increased statistics of FCC-ee will lead to a sizeable improvement compared to the LEP measurements, which were largely statistics limited. Detailed studies of the various asymmetry measurements are under way. A first study [13] of the forward-backward asymmetry for muon pairs, $A_{FB}^{\mu \mu}$, indicates that this channel alone could provide an improved precision on $\sin^2 \theta_{\text{eff}}^W$ by a factor 50 relative to LEP to $6 \times 10^{-6}$, where the dominant systematic uncertainty contribution from the centre-of-mass energy calibration exceeds the statistical uncertainty by a factor two.

The very large statistics accumulated at FCC-ee should allow a new range of searches for very rare phenomena and tests of conservation laws that remain to be investigated. An example, is the search for sterile, right-handed partners of neutrinos in Z decays [14, 5].

4. Physics at the WW production threshold

With $O(10^8)$ W pairs collected at and above the production threshold, FCC-ee would be able to provide precise measurements of W-boson properties, such as mass, width, and branching fractions.

For the determination of the W mass and width, operation at a few energy points within one or two GeV from production threshold is particularly interesting, since, in this narrow region, the WW cross section is maximally sensitive to the values of the W mass and width. Again, to perform
precise measurements, the accurate centre-of-mass energy calibration is necessary. Transverse beam polarisation builds up naturally in a storage ring by the Sokolov-Ternov effect [15]. To maintain polarisation at a useful level, particles have to avoid depolarising resonances spaced by 440.7 MeV in beam energy. In a storage ring, the beam-energy spread scales approximately as $\sigma_E \propto E^2/\sqrt{\rho}$, where $\rho$ is the bending radius. The rapid increase with energy of the beam-energy spread effectively imposes an upper limit on the energy where a useful polarisation level can be maintained. At LEP the maximum beam energy at which polarisation was observed was 60.6 GeV. With the larger bending radius of FCC-ee, beam polarisation should therefore be available at the WW threshold, allowing a measurement of the W mass to 300 keV.

5. The top-quark mass

The FCC-ee will enable precise top-quark studies as well, with over $10^6$ $t\bar{t}$ pairs produced at and above production threshold in a clean experimental environment. An important achievement will be a sizeable improvement in the measurement of the top-quark mass, which is currently determined at hadron colliders to a precision of about 0.5%, dominated by the theoretical understanding. At hadron colliders, the mass is determined from the invariant mass of the decay products. Because of colour reconnection between final state hadrons these cannot be unambiguously related to a single top quark, and hence the method is affected by an inherent uncertainty of the order of $\Lambda_{QCD} \sim 500$ MeV in the theoretical interpretation of the experimental measurement.

From precise measurements of the $e^+e^-\rightarrow t\bar{t}$ production cross section in a narrow scan region around threshold, the top-quark mass can be determined with a statistical precision of the order of 10 MeV. The measurement benefits from the precisely defined centre-of-mass energy, which, unlike at linear colliders, is not affected by strong beamstrahlung effects. The threshold behaviour of the cross section has a sizeable dependence on the strong coupling constant, and hence the measurement will profit from the independent determination of $\alpha_s$ at the Z and WW running as explained below. The threshold scan method is characterised by well-controlled theory uncertainties, likely making it the ultimate measurement of the top-quark mass.

6. The strong coupling constant

At LEP, a precise measurement of $\alpha_s(m_Z^2)$ was derived from the Z decay width ratio $R_\ell = \Gamma_{\text{had}}/\Gamma_\ell$. By reinterpreting the LEP measurement in the light of i) new N$^3$LO calculations, ii) the improved knowledge of $m_{\text{top}}$, and iii) the knowledge of $m_{\text{Higgs}}$, the uncertainty can now be expressed as $\delta(\alpha_s(m_Z^2))_{\text{LEP}} = \pm 0.0038$ (exp) $\pm 0.0002$ (other), and it is thus completely dominated by experimental effects. At FCC-ee, with the much improved experimental precision on $R_\ell$, a precision on $\alpha_s(m_Z^2)$ of 0.00015 is a reasonable target.

In a similar manner, $\alpha_s(m_W^2)$ can be derived from a measurement of the hadronic branching fraction of the W boson, $B_{\text{had}} = (\Gamma_{\text{had}}/\Gamma_{\text{tot}})_W$. At LEP, with $4 \times 10^4$ W-pair events, this quantity was measured with a relative precision of 0.4%, not precise enough for an interesting constraint on $\alpha_s$. With a factor $O(10^4)$ more data at FCC-ee, an improvement of up to two orders of magnitude on $B_{\text{had}}$ can be foreseen, resulting in a absolute uncertainty on $\alpha_s(m_W^2)$ of $\pm 0.00015$, thus matching
the uncertainty derived from Z decays. From a combination of the Z and W measurements, a precision of 0.0001 seems within reach for $\alpha_s(m_Z^2)$.

A recent review of the current status and future prospects of precision $\alpha_s$ measurements, at FCC-ee and elsewhere, can be found in Ref. [16].

7. The Z invisible width and the number of neutrinos

At LEP, the measurement of the Z peak hadronic cross section led to the determination of the number of active neutrino species, $N_\nu = 2.984 \pm 0.008$, which can be observed to be two standard deviations below the SM value of 3. This measurement is of great interest since it constitutes a direct test of the unitarity of the PMNS matrix – or of the existence of right-handed neutrinos, as pointed out in Ref. [17]. The experimental conditions at FCC-ee will be adequate to improve the experimental uncertainty considerably, but, to make the measurement worthwhile, a commensurate effort would have to be invested in the theoretical calculation of the small-angle Bhabha-scattering cross section used for the normalisation. Indeed, the current measurement is limited by the theoretical understanding of this process. A desirable goal would be to reduce the uncertainty on $N_\nu$ down to 0.001, but it is not clear that this can be achieved from the Z peak measurement.

Potentially more promising is the use of radiative return processes at centre-of-mass energies above the Z pole, i.e. the process $e^+e^- \rightarrow Z\gamma$, leading to a very clean photon-tagged sample of on-shell Z bosons, with which Z properties can be measured. From the WW threshold scan alone, 10 million $Z\gamma$ events with $Z \rightarrow \nu\bar{\nu}$ and a photon inside the detector acceptance will be produced per experiment. With in addition the statistics from the 240 and 350 GeV running, the invisible width can be measured with a statistical precision corresponding to 0.001 on $N_\nu$. Recently, it has been suggested to search for the direct $s$-channel production of Higgs bosons in a dedicated high-luminosity run at $E_{CM} = 125$ GeV [9]. Such a run would be ideal for the the neutrino counting exercise and could reduce the statistical uncertainty by about a factor of two relative to the runs at higher energies.

8. Direct measurement of $\alpha_{\text{QED}}$ at the Z-mass scale

The exceptional experimental precision of the Z, W, and top-quark measurements will require a matching precision on the determination of the electromagnetic coupling constant at the Z-mass scale relevant for electroweak interactions, $\alpha_{\text{QED}}(m_Z^2)$, in order to exploit the full potential of the FCC-ee to unravel signs of new physics. The current determination of $\alpha_{\text{QED}}(m_Z^2)$ is based on the measurement of the fine structure constant at zero momentum transfer, $\alpha_{\text{QED}}(0)$, followed by an extrapolation to the Z-mass scale. The uncertainty on this extrapolation, involving photon self-energy diagrams, is dominated by the hadronic part, which is obtained by a dispersion relation over measured low energy hadronic cross sections in $e^+e^-$ annihilations. Current calculations result in a relative uncertainty on $\alpha_{\text{QED}}(m_Z^2)$ of $1.1 \times 10^{-4}$ [18]. An improvement by at least a factor of five is called for to match the FCC-ee experimental precision.

It has been recently demonstrated [19, 20] that the formidable statistics of the FCC-ee could allow a direct measurement of $\alpha_{\text{QED}}$ close to the Z-mass scale, eliminating the need for extrapolation from zero momentum transfer. The study exploits the $e^+e^- \rightarrow Z/\gamma^* \rightarrow \ell^+\ell^-$ process. At
Figure 1: Relative statistical uncertainty on $\alpha_{\text{QED}}$ versus centre-of-mass energy. One year of running at any energy point is assumed.

the peak of the Z resonance, this process is completely dominated by Z exchange and has no sensitivity to $\alpha_{\text{QED}}$. Away from the peak, however, $\gamma^*$ exchange gradually takes over, and the sensitivity to $\alpha_{\text{QED}}$ increases. Two observables have been investigated: The total production cross section, $\sigma_{\mu\mu} = \sigma(e^+e^- \rightarrow \mu^+\mu^-)$ and the forward-backward asymmetry, $A_{\text{FB}}^{\ell\ell}$, with $\ell = \mu, \tau$. Figure 1 summaries the study in terms of the achievable statistical precision on $\alpha_{\text{QED}}$ after one year of running at any given energy point in a wide region around the Z pole. Whereas the statistical uncertainty from the $\sigma_{\mu\mu}$ measurement continues to decrease away from the Z pole, the uncertainty from the $A_{\text{FB}}^{\ell\ell}$ measurement shows a more interesting pattern. In fact, the best relative precision on $\alpha_{\text{QED}}$ of $2 \times 10^{-5}$ appears at centre-of-mass energies of 87.9 and 94.3 GeV, i.e. only $\pm 3.2$ GeV away from the Z peak, energies which could be easily accommodated in the course of the Z-resonance scan. A comprehensive list of sources of experimental, parametric, and theoretical systematic uncertainties has been investigated. It is important to note, that most sources of parametric and theoretical uncertainties influence the measurement at the 87.9 and 94.3 GeV energy points in opposite directions, and thus their effects largely cancel in the overall measurement. The knowledge of the centre-of-mass energy turns out to be the dominant systematic uncertainty contribution, smaller than the statistical uncertainty, though, by a factor two.

9. Summary

The proposed FCC-ee, a large, circular $e^+e^-$ collider delivering very high luminosities at centre-of-mass energies from 90 to 350 GeV and above, will allow measurements of electroweak observables at an unrivalled level of precision. It would provide measurements of the Z-boson mass and width to better than 100 keV each, the W-boson mass to 300 keV, and the top-quark mass to 10 MeV. These and other selected electroweak precision measurements at the FCC-ee are summarised in Table 1, where the quoted systematic uncertainties are indicative and will be revisited in the course of the ongoing design study. If these goals are achieved, the contour line in the $(m_{\text{top}}, m_W)$-plane could evolve from today to FCC-ee as indicated in Figure 2, where both the results of the direct
Table 1: Selected set of precision measurements at FCC-ee. The systematic uncertainties are indicative and will be revisited in the course of the design study.

mass measurements and the indirect constraints from the precision Z pole measurements are indicated. With all of the Standard Model parameters precisely known, the prediction of a number of observables sensitive to electroweak radiative corrections become absolute, and any deviation between measurements would be a demonstration of the existence of new, weakly interacting particles. With the dramatic increase in precision, sensitivity to new physics up to energy scales of the order of 100 TeV can be envisioned as shown in the recent paper [21].

Figure 2: The 68% c.l. contour in the ($m_W$, $m_{t_{top}}$)-plane as expected for the FCC-ee (indicated as TLEP in the figure) and other accelerators. The blue line indicates the expected contour from direct W and top-quark mass measurements, while the red line gives the expected precision from a fit to the Z-pole observables.

Acknowledgements

I would like to thank Alain Blondel, Patrick Janot, and Roberto Tenchini for their help in preparing the talk. For his careful reading of the manuscript, Patrick Janot deserves another merci.
References

[1] ATLAS Collaboration, Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC, Phys.Lett. B716 (2012) 1–29. arXiv:1207.7214, doi:10.1016/j.physletb.2012.08.020.

[2] CMS Collaboration, Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC, Phys.Lett. B716 (2012) 30–61. arXiv:1207.7235, doi:10.1016/j.physletb.2012.08.021.

[3] The FCC design study, http://cern.ch/fcc

[4] M. Bicer, et al., First Look at the Physics Case of TLEP, JHEP 1401 (2014) 164. arXiv:1308.6176. doi:10.1007/JHEP01(2014)164.

[5] A. Blondel, Physics at the FCC-ee, poster presented at EPS-HEP2015, Vienna, 22-29 July 2015, these proceedings.

[6] M. Koratzinos, FCC-ee energy and luminosity, poster presented at EPS-HEP2015, Vienna, 22-29 July 2015, these proceedings.

[7] P. Janot, Top-quark electroweak couplings at the FCC-ee, JHEP 04 (2015) 182.

[8] P. Janot, Top couplings at the FCC, talk presented at EPS-HEP2015, Vienna, 22-29 July 2015, these proceedings.

[9] M. Klute, Higgs physics at the FCC, talk presented at EPS-HEP2015, Vienna, 22-29 July 2015, these proceedings.

[10] S. Monteil, Flavours at the FCC-ee, talk presented at EPS-HEP2015, Vienna, 22-29 July 2015, these proceedings.

[11] The ALEPH, DELPHI, L3, OPAL, SLD Collaborations, the LEP Electroweak Working Group, the SLD Electroweak and Heavy Flavour Groups, Precision Electroweak Measurements on the Z Resonance, Phys. Rep. 427 (2006) 257–454. arXiv:hep-ex/0509008, doi:10.1016/j.physrep.2005.12.006

[12] L. Arnaudon et al., Accurate determination of the LEP beam energy with resonant depolarization, Z.Phys. C66 (1995) 45–62. doi:10.1007/BF01496579.

[13] A. Blondel, Talk given at the FCC-ee Physics Meeting, 2 March 2015, paper in preparation.

[14] A. Blondel, E. Graverini, N. Serra, M. Shaposhnikon, Search for Heavy Right Handed Neutrinos at the FCC-ee, arXiv:1411.5230.

[15] A. A. Sokolov, I. M. Ternov, On Polarization and spin effects in the theory of synchrotron radiation, Sov. Phys. Dokl. 8, 1203 (1964).

[16] David d’Enterria, Peter Z. Skands, et al., High-precision αs measurements from LHC to FCC-ee, arXiv:1512.05194 [hep-ph].

[17] C. Jarlskog, Neutrino counting at the Z-peak and right-handed neutrinos, Phys. Lett. B241, 579–583 (1990), http://dx.doi.org/10.1016/0370-2693(90)91873-A.

[18] J. Erler and A. Freitas, Electroweak Medial and Constraints on New Physics, in K. A. Olive et al. (PDG), Chin. Phys. C38, 090001 (2014), http://pdg.lbl.gov.

[19] P. Janot, Determination of α_{QED}(M_Z) at FCC-ee, presentation at FCC-ee Physics Meeting, 29 June 2015, https://indico.cern.ch/event/401698/
[20] P. Janot, *Direct measurement of $\alpha_{\text{QED}}(M_Z^2)$ at the FCC-ee*,
http://arxiv.org/abs/1512.05544. (submitted to JHEP).

[21] J. Ellis and T. You, *Sensitivities of Prospective Future $e^+e^-$ Colliders to Decoupled New Physics*,
arXiv:1510.04561