Session-ocaml: a Session-based Library with Polariesties and Lenses

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Abstract. We propose \texttt{session-ocaml}, a novel library for session-typed concurrent/distributed programming in OCaml. Our technique solely relies on parametric polymorphism, which can encode core session type structures with strong static guarantees. Our key ideas are: (1) polarised session types, which give an alternative formulation of duality enabling OCaml to automatically infer an appropriate session type in a session with a reasonable notational overhead; and (2) a parameterised monad with a data structure called ‘slots’ manipulated with lenses, which can statically enforce session linearity and delegations. We show applications of \texttt{session-ocaml} including a travel agency usecase and an SMTP protocol.

1 Introduction

Session types [6], from their origins in the $\pi$-calculus [19], serve as rigorous specifications for coordinating link mobility in the sense that a communication link can move among participants, while ensuring type safety. In session type systems such mobility is called delegation. Once the ownership of a session is delegated (transferred) to another participant, it cannot be used anymore at the sender side. This property is called linearity of sessions and appears indispensable for all session type systems.

Linearity of session channels, however, is a major obstacle to adopt session type disciplines in mainstream languages, as it requires special syntax extensions for session communications [10], or depends on specific language features, such as type-level functions in Haskell [12,18,22,28], and affine types in Rust [14], or even falling back on run-time and dynamic checking [8,9,24,29]. For instance, a common way in Haskell implementations is to track linear channels using an extra symbol table which denotes types of each resource conveyed by a parameterised monad. A Haskell type for a session-typed function is roughly of the form:

\[ t_1 \rightarrow \cdots \rightarrow M \{ c_1 \mapsto s_1, c_2 \mapsto s_2, \cdots \} \{ c_1 \mapsto s'_1, c_2 \mapsto s'_2, \cdots \} \alpha \]

where $M$ is a monad type constructor of arity three, $\alpha$ is a result type and the two $\{ \cdots \}$ are symbol tables before (and after) evaluation which assign each channel $c_i$ to its session type $s_i$ (and $s'_i$ respectively). This symbol table is represented at the type level, hence the channel $c_i$ is not a value, but a type which reflects an identity of a channel. Since this static encoding is Haskell-specific using type-level functions, it is not directly extendable to other languages.
This paper proposes the *session-ocaml* library, which provides a fully static implementation of session types in OCaml without any extra mechanisms or tools (i.e., sessions are checked at compile-time). We extend the technique posted to the OCaml mailing list by Garrigue [4] where linear usage of resources is enforced solely by the parametric polymorphism mechanism. According to [4], the type of a file handle guarantees linear access to multiple resources using a symbol table in a monad-like structures. Adapting this technique to session types, in *session-ocaml*, multiple simultaneous sessions are statically encoded in a parameterised monad. More specifically, we extend the monad structure to a *slot monad* and the file handles to *lenses*. The slot monad is based on a type \((p, q, a)\) *monad* (hereafter we use postfix type constructor of OCaml) where \(p\) and \(q\) are called *slots* which act like a symbol table. Slots are represented as a sequence of types represented by nested pair types \(s_1 \ast (s_2 \ast \cdots)\). Lenses [16] are combinators that provide access to a particular element in nested tuples and are used to manipulate a symbol table in the slot monad. These mechanisms can provide an idiomatic way (i.e., code does not require interposing combinators to replace standard syntactic elements of functional languages) to declare session delegations and labelled session branching/selections with the static guarantee of type safety and linearity (unlike FuSe [24] which combines static and dynamic checking for linearity, see § 5).

To enable session-type inference solely by unification in OCaml, *session-ocaml* is equipped with polarised session types which give an alternative formulation of duality (binary relation over types which ensures reciprocal use of sessions). In a polarised session type \((p, q)\) *sess*, the polarity \(q\) is either *serv* (server) or *cli* (client). The usage of a session is prescribed in the protocol type \(p\) which provides an objective view of a communication based on a communication direction of *req* (request; client to server) and *resp* (response; server to client). For example, the protocol type for sending of a message type \(\text{'v'}\) from client to server is \([\text{'msg of req'} \ast \text{'v'} \ast \text{'s'}]\) and the opposite is \([\text{'msg of resp'} \ast \text{'v'} \ast \text{'s'}]\). Duality is not necessary for protocol types as it shows a protocol common to both ends of a session rather than biased by either end. Then the session type inference can be driven solely by type unification which checks whether a protocol matches its counterpart or not. For instance, the dual of \((p, \text{cli})\) *sess* is \((p, \text{serv})\) *sess* and vice versa. When a session is being initiated, polarities are assigned to each end of a session according to the primitives used, namely *cli* for the proactive peer and *serv* for the passive peer. The protocol types also provide a usual prefixing declaration of session types, which is more human-readable than FuSe types [24] (see § 5).

The rest of the paper is as follows. Section 2 outlines programming with *session-ocaml*. Section 3 shows the library design with the polarised session type and the slot monads. In Section 4, we present two examples, a travel agency use case and SMTP protocol implementations. Section 5 discusses comparisons with session type implementations in functional languages. Section 6 concludes and discusses further application of our technique. Technical report [11] includes the implementation of *session-ocaml* modules and additional examples. Appendix A shows the implementation of *session-ocaml* modules. For an additional example,
Appendix B shows an example of database server. Session-ocaml is available at https://github.com/keigoi/session-ocaml.

## 2 Programming with session-ocaml

In this section, we overview session-typed programming with session-ocaml and summarise communication primitives in the library.

**Send and receive primitives** Listing 1 shows a server and client which communicate boolean values. The module Session0\(^1\) introduces functions of session-ocaml in the scope. xor.ch (line 2) is a service channel (or shared channel) that is used to start communication by a client connecting (connect_) to the server waiting (accept_) at it.\(^2\) The server (lines 3–7) receives (recv) a pair of booleans, then calculates the exclusive-or of these values, transmits (send) back the resulting boolean, and finishes the session (close). These communication primitives communicate on an implicit session endpoint (or session channel) which is connected to the other endpoint. For inferring session types by OCaml, communication primitives are concatenated by the bind operations >>= and >>= of a parameterised monad \([\_\_]\) which conveys session endpoints. The syntax let pat = e in e binds the value returned by e to the pattern pat and executes e, which is shorthand for e >>= fun pat -> e (the % symbol indicates a syntax extension point in an OCaml program). The client (lines 8-12) sends a pair of boolean, receives from the server and finishes the session, as prescribed in the following type. These server and client behaviours are captured by the protocol type argument of the channel type inferred at xor.ch as follows:

\[
\text{['msg of req * (bool * bool) * ['msg of resp * bool * ['close]]] channel}
\]

The protocol type is the primary language of communication specification in session-ocaml. Here, \([\text{msg of } r * v * p]\) is a protocol that represents communication of a message of type v before continuing to p. r \(\in\{\text{req,resp}\}\) indicates a communication direction from client to server and vice versa, respectively. ['close] is the end of a session. Thus the above type indicates that by a session established at xor.ch, (1) the server receives a request of type bool * bool and then (2) sends a response of type bool back to the client.

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\(^1\) The suffix 0 means that it only uses the slot 0 (see later in this section).

\(^2\) The suffixed underscore means that they run immediately instead of returning a monadic action (see later).
Listing 2  A logical operation server

|   | open Session0 | type binop = And | Or | Xor | Imp |
|---|---------------|-----------------|----|----|----|
| 3 | let log_ch = new_channel () | let eval_op = function |
| 5 | | | | | |
| 7 | | | | | |
| 9 | | | | | |

```ocaml
let rec logic_server () =
  match%branch0 () with
  | `bin -> let op = recv () in
  let x,y = recv () in
  send (eval_op op x y) >>= logic_server |
  | `fin -> close ();;
```

Branching and recursion  A combination of branching and recursion provides various useful idioms such as exception handling. As an example, Listing 2 shows a logical operation server. The protocol type inferred for log_ch is:

```
['branch of req * ['bin of ['msg of req * binop * ['msg of req * (bool * bool) * ['msg of resp * bool * 'a]]]]]
['fin of ['close]]] as 'a
```

This represents a protocol that branches to pi when label labi is communicated. Here r is a communication direction. t as 'a is an equi-recursive type [26] of OCaml that represents recursive structure of a session where 'a in t is instantiated by t as 'a. Lines 8-14 describe the body of the server. It receives one of the labels bin or fin, and branches to a different protocol. `match%branch0 () with ...` is the syntax for branching to the expression ei after label labi is received. Upon receipt of bin, the server receives requests for a logical operation from the client (type binop and bool * bool), sends back a response and returns to the branch (note that the server is recursively defined by let rec). In the case of fin, the session is terminated. [%select0 `lab`] is a syntax to select one of branches with a label lab.3

A client using selection is shown in lines 18-25: It selects the label bin, requests conjunction, and selects fin; then the session ends.

For the branching primitive on arbitrary labels, session-ocaml uses OCaml polymorphic variants and syntax extensions. By using equi-recursive types, recursive protocols are also directly encoded into OCaml types.

Link mobility with delegation  Link mobility with session delegation enables one to describe a protocol where the communication counterpart dynamically changes during a session. A typical pattern utilising delegation incorporates a main thread accepting a connection and worker threads doing the actual work to increase responsiveness of a service.

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3 Here the bracket is another form of a syntax extension point applied to an expression (see the OCaml manual).
In session-ocaml, a program using delegation handles multiple sessions simultaneously. We explicitly assign each session endpoint to a slot using slot specifiers \(_0, _1, \ldots\) which gives an idiomatic way to use linear channels. Listing 3 shows an example of a highly responsive server using delegation. The server receives repeated connection requests on channel \(\log\_ch\), consisting of the main thread and six worker threads. The module \texttt{SessionN} provides slot specifiers and accompanying communication primitives, where the suffix \(N\) means that it can handle on arbitrary number of sessions. The main thread (lines 3-7) accepts a connection from a client (\texttt{accept}) with \(\log\_ch\) and assigns the established session to slot 0 (\(-\texttt{bindto}:\_0\)). Next, it connects (\texttt{connect}) to a worker waiting for delegation at channel \(\texttt{worker\_ch}\) (line 2) and assigns the session to slot 1 (\(-\texttt{bindto}:\_1\)). Finally it delegates the session with the client to the worker (\texttt{deleg\_send}), then ends the session with the worker and accepts the next connection. The worker thread (lines 8-12) receives the delegated session from the main thread (\texttt{deleg\_recv}) and assigns the session to slot 0, then continues to \texttt{logic\_server} (Listing 2). Here, \texttt{Session0} module used by \texttt{logic\_server} implicitly allocates the session type to slot 0, hence can be used with \texttt{SessionN} module. Line 14 starts the main thread and workers. Here \texttt{run} is a function that executes \texttt{session-ocaml} threads.

The protocol type of \(\texttt{worker\_ch}\) is inferred as follows:

\[
\texttt{[deleg of req * (logic\_p, serv) sess * [\texttt{close}]]}
\]

Here \texttt{logic\_p} is the protocol type of \(\log\_ch\) and \texttt{[deleg of r * s * p]} is the delegation type. \(r\) is a communication direction, \(s\) is a polarised session type (a type with protocol and polarity which we explain next) for the delegated session and \(p\) is a continuation. By inferring the protocol types, session-ocaml can statically guarantee safety of higher-order protocols including delegations.

The polarised session types Communication safety is checked by matching each protocol type inferred at both ends. The polarised session type \((p, q)\) \(sess\) given to each endpoint plays a key role for protocol type inference. Here \(p\) is a protocol type, and \(q \in \{\texttt{serv,cli}\}\) is the polarity determined at session initiation. \texttt{serv} is assigned to the \texttt{accept} side and \texttt{cli} to the \texttt{connect} side. \texttt{serv} and \texttt{cli} are dual to each other.

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\[\texttt{~arg:e}\] is a labelled argument \(e\) for a named parameter \texttt{arg}.
The polarised session type gives a simple way to let the type checker infer a uniform protocol type according to a communication direction and a polarity assigned to the endpoint. For example, as we have seen, we deduce `resp` (response) from server transmission (`send`) and client reception (`recv`). Table 1 shows correspondences between polarities and communication directions.

|      | send | select | deleg_send | recv | branch | deleg_recv |
|------|------|--------|------------|------|--------|------------|
| `cli` | `req` | `req`  | `req`      | `resp`| `resp` | `resp`     |
| `serv`| `resp`| `resp` | `resp`     | `req` | `req`  | `req`      |

Table 1. Correspondence between polarities and communication directions

To track the entire session, a polarised session type changes in its protocol part as a session progresses. Fig. 1 shows changes of the session type in slot 0 of the xor server (here we use the `SessionN` module). The server first accepts a connection and assigns the session type to slot 0, where the type before acceptance is `empty`. After the subsequent reception of the pair of booleans and transmission of the xor values of those booleans, `req` and `resp` are consumed, and becomes `empty` again at the end of the session. Similar type changes occur on both `main` and `worker` and their types would be:

```
unit -> (empty * (empty * 'ss), 'tt, 'a) session
```

Here the type `(s, t, a)` session specifies that it expects slot sequence `s` at the beginning, and returns another slot sequence `t` and a value of type `a`. The type `empty * (empty * 'ss)` denotes that slot 0 and 1 are empty at the beginning, and since they never return the answer (i.e. the recursion runs infinitely), the rest of types `tt` and `a` are left as variables.

The type of `logic_server` in Listing 2 has a session type:

```
((logic_p, serv) sess * 'ss, empty * 'ss, unit) session
```

Here `logic_server` expects a session assigned at slot 0 before it is called, hence it expects the session type `(logic_p, serv) sess` in its pre-type. A difference from
Table 2. session-ocaml primitives and protocol types

| Primitive | Pre-type (at cli) | Post-type | Synopsis |
|-----------|------------------|-----------|----------|
| send \_n e | \[`msg of req v p\] | p | sending e v at slot n |
| let\% pat = recv \_n in \cdots | \[`msg of resp v p\] | p | Reception at slot n, binding to pattern pat v |
| \[%select \_n \`lab_i\] | \[branch of req \_n p_i\] | p_i | Select label lab_i at slot n |
| match\%branch \_n with | \[branch of resp \_n p_i\] | t | Branch at n with labels lab_i (protocol type p_i is that of pre-type of e_i; t is a post-type of all e_i) |
| \[\`lab_0 \rightarrow e_0\] | \[\`lab_0 of p_0\] | \[\`lab_m of p_m\] | m | Delegate session at m with type s along n |
| \[\`lab_m \rightarrow e_m\] | n_p | m_empty | m | Receptance of delegation along n and assign it to m |
| deleg_send \_n | n_p | m_empty | m | Close session at slot n |

s is a polarised session type, \_n and \_m are slot specifiers, e is an expression of a base type, ch is a service channel, \`lab is a polymorphic variant and pat is a binding pattern.

\*1: At serv, req and resp are exchanged; \*2: s is a session type (not a protocol type); \*3: Slot type changes to empty.

Table 2 shows the type and communication behaviour before and after the execution of each session-ocaml communication primitive. Each row has a pre-type (the type required before execution) and post-type (the type guaranteed after execution). The protocol type at serv is obtained by replacing req with resp and resp with req. For example, the session send \_n e has pre-type (\[`msg of req v p\], cli) sess at cli and (\[`msg of resp v p\], serv) sess at serv where \_n is a slot specifier, e is an expression, v is a value type and p is a protocol type. Selection \[%select \_n\] has open polymorphic variant type [\>\ldots] in pre-type to simulate subtyping of the labelled branches.

3 Design and implementation of session-ocaml

In this section, we first show the design of polarised session types associated with communication primitives (§ 3.1); then introduce the slot monad which conveys multiple session endpoints in a sequence of slots and constructs the whole session.
type for a session (§ 3.2). In § 3.3, we introduce the slot specifier to look up a particular slot in a slot sequence with lenses which are a polymorphic data manipulation technique known in functional programming languages. We present the syntax extension for branching and selection, and explain a restriction on the polarised session types. This section mainly explains the type signatures of the communication primitives. Implementation of the communication behaviours is left to [11].

### 3.1 Polarity polymorphism

The polarity polymorphism is accompanied within all session primitives in that the appropriate direction type is assigned according to the polarity. This resolves a trade-off of having two polarised session types for one transmission. For instance, a transmission of a value could have two candidates, \([\text{msg of req } \times \text{v } \times \text{s}]\) and \([\text{msg of resp } \times \text{v } \times \text{s}]\) but they are chosen according to the polarity from which the message is sent. In order to relate the polarities to the directions, `cli` and `serv` are defined by type aliases as follows:

```ocaml
type cli = req * resp
type serv = resp * req
```

For each communication primitive, we introduce fresh type variables `r_req` and `r_resp` representing the communication direction, and put `r_req * r_resp` as the polarity in its session type. When its polarity is `cli`, we put `r_req` for `req` and `r_resp` for `resp`, while when it is `serv`, we put `r_req` for `resp` and `r_resp` for `req`. For example, the pre-type of `send` is `([\text{msg of } r1 \times \text{v } \times \text{p}], r1 \times r2 \times \text{s}) session` and that of `recv` is `([\text{msg of } r2 \times \text{v } \times \text{p}], r1 \times \text{r2} \times \text{s}) session`. The same discipline applies to branching and delegation. The actual typing is deferred to the following subsections.

### 3.2 The slot monad carrying multiple sessions

The key factor to achieve linearity is to keep session endpoints securely inside a monad. In `session-ocaml`, multiple sessions are conveyed in slots using the slot monad of type

\[(s_0 \times (s_1 \times \cdots), t_0 \times (t_1 \times \cdots), \alpha) \text{ session}\]

which denotes a computation of a value of type `\alpha`, turning each pre-type `s_i` of slot `i` to post-type `t_i`. We refer to slots before and after computation as pre- and post-slots, respectively. The type signature of the slot monad is shown in Listing 4. The operators `>>=` and `>>` (lines 3–4) compose computation sequentially while propagating type changes on each slot by demanding the same type `\text{q}` in the post-slots on the left-hand side and the pre-slots on the right-hand side. Usually they construct compound session types via unification. For example, in `send And >> send (true, false)` (from Listing 2) the left-hand side (`send Add`) has the following type:

```ocaml
([\text{msg of } \text{req } \text{binop} \text{p1}], \text{cli}) \text{ sess } \times \text{ssl}, ('\text{p1, cli}) \text{ sess } \times \text{ssl, unit}) \text{ session}
```

While the type of the right-hand side (`send (true, false)`) is:
The slot monad

**Listing 4** The slot monad

```ocaml
type ('p,'q,'a) session and empty and all_empty = empty * 'a as 'a
val return : 'a -> ('p,'p,'a) session
val (>>=) : ('p,'q,'a) session -> ('a -> ('q,'r,'b) session) -> ('p,'r,'b) session
val (>>) : ('p,'q,'a) session -> ('q,'r,'b) session -> ('p,'r,'b) session
val run : (all_empty,all_empty,unit) session -> unit
```

| Specifier | Type |
|-----------|------|
| _0        | (\(\langle 'a, 'b, 'a * 'ss, 'b * 'ss \rangle \) slot |
| _1        | (\(\langle 'a, 'b, 's0 * (\langle 'a + 'ss \rangle), 's0 * (\langle 'b + 'ss \rangle) \rangle slot |
| _2        | (\(\langle 'a, 'b, 's0 * (\langle 's1 + ('a + 'ss) \rangle), 's0 * (\langle 's1 + ('b + 'ss) \rangle) \rangle slot |
| _n        | (\(\langle 'a, 'b, 's0*\cdots*\langle 's_{n-1}*(\langle 'a*ss)\rangle\cdots\rangle, 's0*\cdots*\langle 's_{n-1}*(\langle 'b*ss)\rangle\cdots\rangle) \rangle slot |

((\['msg of req*+ (bool+bool) * p2, cli] sess * s'ss2, (p2, cli) sess * s'ss2, unit) session

By unifying the post-type in the preceding monad with the pre-type in the following monad (and the rest of slots \(s'ss1\) with \(s'ss2\)), the bind operation produces a chain of protocol type in the pre-slots as follows:

```
((\['msg of req*+ binop* \['msg of req* (bool+bool) * p2], cli) sess * s'ss2,
 (p2, cli) sess * s'ss2, unit) session
```

In line 5, `run` executes the slot monad and requires all slots being `empty` before and after execution, thus it precludes use of unallocated slots, and mandates that all sessions are finally closed (which corresponds to the absence of contraction in linear type systems). The type `all_empty` (line 1) is a type alias for OCaml equi-recursive type `empty * 'a as 'a`, enabling use of arbitrarily many slots.

### 3.3 Lenses focusing on linear channels

In order to provide access to session endpoints conveyed inside a slot monad, we apply lenses [16] to to slot specifiers \(_0, _1, \cdots\) which are combinators to manipulate a polymorphic data structure. The following shows the type of a slot specifier which modifies slot \(n\) of a slot sequence:

```ocaml
type ('a, 'b, 's0*\cdots*\langle 's_{n-1}*(\langle 'a*ss)\rangle\cdots\rangle, 's0*\cdots*\langle 's_{n-1}*(\langle 'b*ss)\rangle\cdots\rangle) \rangle slot
```

The type says that it replaces the type \('a\) of slot \(n\) in the slot sequence \('s0*\cdots*\langle 's_{n-1}*(\langle 'a*ss)\rangle\cdots\rangle\) with \('b\) and the resulting sequence type becomes to \('s0*\cdots*\langle 's_{n-1}*(\langle 'b*ss)\rangle\cdots\rangle\). The type of each slot specifier \(_0, _1, \cdots\) is shown in Table 3.

Listing 5 exhibits type signatures of `accept`, `connect`, `close`, `send`, `recv`, `deleg_send` and `deleg.recv` which are compiled from lenses, the polarised session types (§ 3.1),

\[5\text{ In order to have such a type, we compile the code with the -rectypes option. If we chose types for slots using objects or polymorphic variants, there is no need to use this option.}\]
Listings 5 Signatures for communication primitives in session-oaml

```ocaml
val accept : 'p channel -> bindto:(empty, ('p, serv) sess, 'pre, 'post) slot
-> ('pre, 'post, unit) session
val connect : 'p channel -> bindto:(empty, ('p, cli) sess, 'pre, 'post) slot
-> ('pre, 'post, unit) session
val close : ([`close], 'r1*'r2) sess, empty, 'pre, 'post) slot
-> ('pre, 'post, unit) session
val send : ([`msg of 'r1*'v*'p], 'r1*'r2) sess, ('p,'r1*'r2) sess, 'pre, 'post) slot
-> 'v -> ('pre, 'post, unit) session
val recv : ([`msg of 'r2*'v*'p], 'r1*'r2) sess, ('p,'r1*'r2) sess, 'pre, 'post) slot
-> ('pre, 'post, 'v) session
val deleg_send :
  ([`deleg of 'r1*'s*'p], 'r1*'r2) sess, ('p,'r1*'r2) sess, 'pre, 'mid) slot
-> release:('s, empty, 'mid, 'post) slot -> ('pre, 'post, 'v) session
val deleg_recv :
  ([`deleg of 'r2*'s*'p], 'r1*'r2) sess, ('p,'r1*'r2) sess, 'pre, 'mid) slot
-> bindto:(empty, 's, 'mid, 'post) slot -> ('pre, 'post, 'v) session
val select_left : ([`branch of 'r1 * [>left of 's1]], 'r1*'r2) sess,
  ('s1,'r1*'r2) sess, 'pre, 'post) slot -> ('pre, 'post, unit) session
val select_right : ([`branch of 'r1 * [>right of 's2]], 'r1*'r2) sess,
  ('s2,'r1*'r2) sess, 'pre, 'post) slot -> ('pre, 'post, unit) session
val branch2 :
  ([`branch of 'r2 * [>left of 's1 | >right of 's2]], 'r1*'r2) sess,
  ('s1,'r1*'r2) sess, 'pre, 'mid1) slot * (unit -> ('mid1, 'post, 'a) session)
  -> ([`branch of 'r2 * [>left of 's1 | >right of 's2]], 'r1*'r2) sess,
  ('s2,'r1*'r2) sess, 'pre, 'mid2) slot * (unit -> ('mid2, 'post, 'a) session)
  -> ('pre, 'post, 'a) session
```

slot monads (§ 3.2), and pre- and post-types in Table 2 (§ 2). Note that `bindto:`
and `release:` are named parameters of a primitive.

`accept` and `connect` (lines 1–4) assign a new session channel to the `empty`
slot, whereas `close` (lines 5–6) finishes the session and leave the slot `empty`
again. `send` and `recv` (lines 7–10) proceed the protocol type by removing a `msg`
prefix.

`deleg_send` and `deleg_recv` (lines 11–16) update a pair of slots; one is for
the transmission/reception and the other is for the delegated session. To update
the slots twice, they take a pair of slot specifiers which share an intermediate
slot sequence `mid`. They embody an aspect of linearity: `deleg_send`
releases the ownership of the delegated session by replacing the slot type to `empty`,
while `deleg_recv` allocates another `empty` slot to the acquired session.

The primitives for binary selection `select_left`, `select_right` and branching
`branch2` (lines 17–25) communicate left and right labels. `branch2` takes a pair of
continuations as well as a pair of slot specifiers. According to the received label,
one of the continuations is invoked after the pre-type of the invoked continuation
is assigned to the corresponding slot.

Finally, we present how to embed slot type changes into a pair of slot sequences
in a slot monad where the position of the slot is specified by applying a slot
specifier. In each type signature, the first and second type arguments of type
`slot` prescribes how a slot type changes. The third and fourth arguments do not
specify a slot in the slot sequence conveyed by the slot monad. For example,
the type of function application `close _1` is given by the following type substitution:
A note on delegation and slot assignment The delegation `$deleg of r * s * p$ distinguishes polarity in the delegated session $s$. This results in a situation where two sessions exhibiting the same communicating behaviour cannot be delegated at a single point in a protocol, if they have different polarities from each other. It is illustrated by the following (untypeable) example.

```ocaml
if b then connect ch1 ~bindto:_1 >> deleg_send _0 ~release:_1
else accept ch2 ~bindto:_1 >> deleg_send _0 ~release:_1
```

Recall that `connect` yields a `CLI` endpoint while `accept` gives a `SERV`. Due to the different polarities in the delegated session types, the types of `then` and `else` clause conflict with each other, even if they have the identical behaviour. In [34], where polarity is not a type but a syntactic construct, such a restriction does not exist. A similar restriction exists in GV [32] which has polarity in `end (end ! and end?)`.

In principle, it is possible to automatically assign numbers to slot specifiers `locally` in a function instead of writing them explicitly. However, since sequential composition of the session monad requires each post- and pre-type to match with each other, the `global` assignment of slot specifiers would require a considerable amount of work and can be hard to predict its behaviour. As shown in Listing 3, one can handle two sessions by just using two slot specifiers.

Syntax extension for arbitrarily labelled branch Since the OCaml type system does not allow to parameterise type labels (polymorphic variants), we provide macros for arbitrarily-labelled branching. Listing 6 provides helper functions for the macros. For selection, the macro `$select ...$ is expanded to `$select ... (fun x -> `lab(x))`, where the helper function `$select` transmits label $lab_i$ on the slot $n$. `match` is expanded to:

```ocaml
.branch_start ... ((function |`lab1(p1),q -> .branch ... (p1,q) (e1) | ... 
|  labk(pk),q -> .branch ... (pk,q) (ek))
: ['lab1 of 'p1|...|'labk of 'pk]*'x->'y)
```

The helper functions `.branch_start` and `.branch` have the type shown in Listing 6. The anonymous function will have type

```ocaml
['lab1 of p1 | ... | 'labk of pk]*q -> (pre, post, v) session
```

where $q$ is the polarity and $p_i$ is the protocol type in the pre-type at slot $n$ in $e_i$. When a label $lab_i$ ($i \in [1...k]$) is received, `.branch_start ... f` passes a pair of witness `$lab_i(p_i)$ and $q$ of a polarised session type $(p_i,q) sess$ to the function $f$. The anonymous function extracts the witness and by `.branch` it rebuilds the session type (`p_i',q') sess and passes the session to the continuation $e_i$ as the
Listing 6 The helper functions for branching/selection with arbitrary labels

```
val _select : 
  ((([`branch of 'r2 * 'br],[`r1*'r2]) sess, ('p,'r1*'r2) sess, 'pre, 'post) slot
  -> ('p -> 'br) -> ('pre, 'post, unit) session
val _branch_start : 
  ((([`branch of 'r1 * 'br],[`r1*'r2]) sess, 'x, 'pre, 'dummy) slot
  -> ('br * ('r1*'r2) -> ('pre, 'post,'v) session) -> ('pre, 'post, 'v) session
val _branch : 
  ((([`branch of 'r1 * 'br],[`r1*'r2]) sess, ('p,'r1*'r2) sess, 'pre, 'mid) slot
  -> 'p * ('r1*'r2) -> ('mid,'post,'v) session -> ('pre, 'post, 'v) session
```

Listing 7 Travel agency

```
let customer cst_ch = 
  connect cst_ch ~bindto:_0 >> 
  [%select _0 `quote] >> 
  send _0 "London to Paris" >> 
  let% cost = recv _0 in 
  if cost > 100. then 
    [%select _0 `reject] >> 
    close _0 
  else 
    [%select _0 `agree] >> 
    close _1 
  let% d : date = recv _0 in 
  send _0 (Address("London")) >> 
  let service svc_ch = 
    accept svc_ch ~bindto:_1 >> 
    deleg_recv _1 ~bindto:_0 >> 
    let rec loop () =
      let% (Address(addr)) = recv _1 in 
    send _0 (now()) >> 
  let agency cst.ch svc.ch = 
    accept cst.ch ~bindto:_0 >> 
    deleg_send _1 ~release:_0 >> 
    close _1
```

pre-type. The type annotation `[lab, of 'p1|···|labk of 'pk]*'x->'y erases the row type variable [<···] generated by the anonymous function. The annotation is necessary because the row type variable turns into a useless monomorphic row type variable [<···] in the inferred protocol type. This may cause a problem while compiling since the compiler requires monomorphic type variables to not escape from compilation units.

4 Applications

4.1 Travel agency

We demonstrate programming in session-ocaml using the Travel agency scenario from [10], which consists of typical patterns found in business and financial protocols. The scenario is played by three participants: customer, agency and service (Listing 7). customer and service initially do not know each other, and agency mediates a deal between them by session delegation.
customer begins an order session with agency and binds it to their own slot 0 (each process has a separate slot sequence). Then customer requests and receives the price for the desired journey after sending the quote label. In our scenario, customer requests “London to Paris” and agency replies with a fixed price 80.0.

Then customer might send the agree label to proceed the transaction with the current price. Or if customer does not agree with the price, customer can cancel the transaction by sending the reject label. Or, customer can send quote again and this will be repeated an arbitrary number of times for different journeys (we omit this branch from the code). In our program, customer agrees with agency at a price less than 100.0, or otherwise rejects it and terminates the transaction.

Next, if customer agrees with the price, agency opens the session with service and binds it to slot 1. Then it delegates to service, through slot 1, the interactions with customer remaining for slot 0. customer then sends the billing address (unaware that he/she is now talking to service), and service replies with the dispatch date (now()) for the purchased tickets. The transaction is complete.

The protocol type between customer and agency is inferred as:

```
| branch of req *
| quote of [msg of req * string * [msg of resp * float * 'a]]
| reject of ['close]
| agree of [msg of req * addr * [msg of resp * date * ['close]]] as 'a
```

Delegation from agency to service is inferred in the channel of service as:

```
| deleg of req *
| ([msg of 'r1 * addr * [msg of 'r2 * date * ['close]]], 'r1' 'r2) sess * ['close]
```

The delegated type is polymorphic on the polarity and communication directions (§ three.prop / one.prop), hence the service can handle both polarities. It reflects the part after agree in the protocol above where ‘r1 is req and ‘r2 is resp. Thus delegation with the polarised session types and slots effectively gives a way to coordinate higher order communication incurred by link mobility.

Static checking of delegation makes it easier to find errors otherwise hard to analyse due to the indirect nature of delegation. Consider a case that service changes its behaviour to receive addr * paymeth. Now the inferred protocol type at service would be:

```
| deleg of req *
| ([msg of 'r1 * (addr * paymeth) * [msg of 'r2 * date * ['close]]], 'r1' 'r2) sess * ['close]
```

Whereas that of agency remains same as before, it results in a type error at the moment when a service channel is passed. Without static typing, the run-time error would be deferred until the beginning of actual client-service communication.

### 4.2 An SMTP protocol

This section shows an SMTP client implementation by session-ocaml. Listing 8 and Listing 9 show the protocol type of SMTP and message types representing SMTP commands and replies; and Listing 10 shows the client implementation. Line 2 in Listing 10 generates a service channel for connecting to the SMTP server. Here smtp_adapter is an adapter that converts a sequence of session messages to a
TCP stream. Its definition is shown in Listing 11 and built using the combinators shown in Listing 12. The functions req and resp accept a function to convert between a message of type ’v and a command string and construct an adapter. bra and sel are branching and selection respectively, and cls is the end of the session.

The function with the same name as the message type is a function for converting to a string (or vice versa) and is responsible for actual stream processing. In OCaml, f @@ g means function composition fun x -> f (g x), and begin e end means (e). Since OCaml evaluates eagerly, each function is η-expanded with a parameter ch so that it does not recurse infinitely.

The adapter for branch bra is asymmetric in its parameters [20]. bra has a parser of type string -> ’v option on the left side since the adapter determines a continuation in a branch according to the parsed result of a received string. The adapter chooses left if the parser succeeds (returns Some(x)), and right if it fails (None). By nesting bra, any nesting of branch can be constructed.

Comparing to the existing Haskell implementation in [12], an advantage is that our OCaml version enjoys equi-recursive session types, so we avoid the manual annotation of repeated unwind operations needed to unfold iso-recursive
Listing 10 An implementation of SMTP client

```ocaml
open Session0

let ch = TcpSession.new_channel smtp_adapter "smtp.example.com:25"
let smtp_client () = connect_ ch begin fun () ->
  let R200 s = recv () in
  send (EHLO("me.example.com")) >>
  select_left () >> (* enter into the main loop *)
  send (MAIL("alice@example.com")) >>
  select_left () >> (* enter into recipient loop *)
  send (RCPT("bob@example.com")) >>
  branch2 (fun () ->
    select_right () >> (* proceed to sending the mail body *)
    send DATA >>
    select_right () >> send QUIT >> close ()) end

(fun () ->
  let R200 s = recv () in
  select_right () >>
  send (escape mailbody) >>
  select_right () >> send QUIT >> close ()) end
```

Listing 11 The TCP adapter for a SMTP client

```ocaml
let rec smtp_adapter ch =
  (resp r200 @@ req ehlo @@ resp r200 @@ ml_p) ch
and ml_p ch =
  sel ~left:(req mail @@ resp r200 @@ rp_p) ~right:(req quit @@ cls) ch
and rp_p ch =
  sel ~left:(req rcpt @@ bra ~left:(r200, rp_p) ~right:((req r500 @@ req quit @@ cls))
  ~right:bd_p ch
and bd_p ch =
  (req data @@ resp r354 @@ req string_list @@ resp r200 @@ ml_p) ch
```

types in Haskell. A shortcoming of the OCaml version is the explicit nature of adapter. However, since the adapter and the protocol type have the same structure, it can be generated semi-automatically from the type declaration in Listing 8 when OCaml gains ad hoc polymorphism such as type classes. We expect this to be possible with modular-implicits [33], which will be introduced in a future version of OCaml. On the other hand, it is also possible to omit the protocol type declaration in Listing 8 by inferring the type of the adapter.

5 Related work

We discuss related work focusing on the functional programming languages. For other related work, see § 1.

Implementations in Haskell The first work done by Neubauer and Thiemann [20] implements the first-order single-channel session types with recursions. Using parameterised monads, Pucella and Tov [28] provide multiple sessions, but manual reordering of symbol tables is required. Imai et al. [12] extend [28] with delegation, handling multiple sessions in a user-friendly manner by using type-level functions. Orchard and Yoshida [22] use an embedding of effect systems in
Haskell via graded monads based on a formal encoding of session-typed π-calculus into PCF with an effect system. Lindley and Morris [18] provide an embedding of the GV session-typed functional calculus [32] into Haskell, building on a linear λ-calculus embedding by Polakow [27]. Duality inference is mostly represented by a multi-parameter type class with functional dependencies [1]; For instance, class Dual t t' | t -> t', t' -> t declares that t can be inferred from its dual t' and vice versa. However, all of the above works depend on type-level features in Haskell, hence they are not directly applicable to other programming languages including OCaml. See [23] for a detailed survey. session-ocaml generalises the authors’ previous work in Haskell [2] by replacing type-level functions with lenses, leading to wider applicability to other programming languages.

Implementations in OCaml Padovani [24] introduces FuSe, which implements multiple sessions with dynamic linearity checking and its single-session version with static checking in OCaml. Our session-ocaml achieves static typing for multiple sessions with delegation by introducing session manipulations based on lenses; and provides an idiomatic way to declare branching with arbitrary labels; while FuSe combines static and dynamic approach to achieve them.

The following example shows that session-ocaml can avoid linearity violation, while FuSe dynamically checks it at the runtime.

```ocaml
let rec loop () = let s = send "*" s in
  match branch s with `stop s -> close s |
  `cont _ -> loop ()
loop sends "*" repeatedly until it receives label stop. Although the endpoint s
should be used linearly, the condition is violated at the beginning of the second
iteration since the endpoint is disposed by using the wildcard _ at the end of
the loop. In FuSe 0.7, loop is well-typed and terminates in error InvalidEndpoint
at runtime. In session-ocaml, this error inherently does not occur since each
endpoint is implicit inside the monad and indirectly accessed by lenses.

[24] gives a micro-benchmark which measures run-time performance between
the static and dynamic versions of FuSe. Based on the benchmark, it is shown
that the overhead incurred by dynamic checking is negligible when implemented
with a fast communication library such as Core [13], and concludes that the static
version of FuSe performs well enough in spite of numerous closure creations in
a monad. The FuSe implementation has been recently extended to context free session types [31] by adding an endpoint attribute to session types [25].

On duality inference, a simple approach in OCaml is firstly introduced by Pucella and Tov [28]. The idea in [28] is to keep a pair of the current session and its dual at every step; therefore the notational size of a session type is twice as big as that in [6]. FuSe [24] reduces its size by almost half using the encoding technique in [3] by modelling binary session types as a chain of linear channel types as follows. A session type in FuSe (‘a, ‘b) t prescribes input (‘a) and output (‘b) capabilities. A transmission and a reception of a value ‘v followed by a session (‘a,’b) t are represented as (_0,’v* (‘a,’b) t) t and (_’v* (‘a,’b) t, _0) t respectively, where _0 means “no message”; then the dual of a session type is obtained by swapping the top pair of the type. A drawback of these FuSe notations is it becomes less readable when multiple nestings are present. For example, in a simplified variant of the logic operation server in Listing 2 with no recursion nor branch, the protocol type of log_ch becomes:

```
[‘msg of req*binop*[‘msg of req*(bool*bool)*[‘msg of resp*bool*[‘close]]]]
```

In FuSe, at server’s side, the channel should be inferred as:

```
(binop* (bool*bool)*(_0,bool*(_0,_0) t) t,_0) t
```

Due to a sequence of flipping capability pairs, more effort is needed to understand the protocol. To recover the readability, FuSe supplies the translator Rosetta which compiles FuSe types into session type notation with the prefixing style and vice versa. Our polarised session types are directly represented in a prefixing manner with the slight restriction shown in § 3.3.

6 Conclusion

We have shown session-ocaml, a library for session-typed communication which supports multiple simultaneous sessions with delegation in OCaml. The contributions of this paper are summarised as follows. (1) Based on lenses and the slot monad, we achieved a fully static checking of session types by the OCaml type system without adding any substantial extension to the language. Previously, a few implementations were known for a single session [24,28], but the one that allows statically-checked multiple sessions is new and shown to be useful. To the authors’ knowledge, this is the first implementation which combines lenses and a parameterised monad. (2) On top of (1), we proposed macros for arbitrarily labelled branches. The macros “patch up” only the branching and selection parts where linear variables are inevitably exposed due to limitation on polymorphic variants. (3) We proposed a session type inference framework solely based on the OCaml built-in type unification. Communication safety is guaranteed by checking equivalence of protocol types inferred at both ends with different polarities.

Type inference plays a key role in using lenses without the burden of writing any type annotations. Functional programming languages such as Standard ML, F# and Haskell have a nearly complete type inference, hence it is relatively easy to apply the method presented in this paper. On the other hand, languages
such as Scala, Java and C# have a limited type inference system. However, by a recent extension with Lambda expressions in Java 8, lenses became available without type annotations in many cases (see a proof-of-concept at https://github.com/keigoi/slotjava). The main difficulty for implementing session types is selection primitives since they require type annotations for non-selected branches. Development of such techniques is future work.

Our approach which uses slots for simultaneous multiple sessions resembles parameterised session types [2,21], and it is smoothly extendable to the multiparty session type framework [7]. We plan to investigate code generations from Scribble [30] (a protocol description language for the multiparty session types) along the line of [8,9] integrating with parameterised features [2,21].

Acknowledgments We thank Raymond Hu and Dominic Orchard for their comments on an early version of the paper. The third author thanks the JSPS bilateral research with NFSC for fruitful discussion. This work is partially supported by EPSRC projects EP/K034413/1, EP/K011715/1, EP/L00058X/1, EP/N027833/1 and EP/N028201/1; by EU FP7 612985 (UPSCALE), and COST Action IC1405 (RC); by JSPS International Fellowships (S15051), and KAKENHI JP17KI2662, JP25280023 and JP17H01722 from JSPS, Japan.

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Listing 13 Basic operations on slot monad

(* The slot monad *)
type ('ss,'tt,'a) session = 'ss -> 'tt * 'a
let return a = fun ss -> ss, a
let ( >>= ) m f = fun ss -> let tt, a = m ss in f a tt

type empty = Empty
let rec all_empty = Empty * all_empty
let run m = m all_empty

(* Slot specifiers *)
type ('a,'b,'ss,'tt) slot = ('ss -> 'a) * ('ss -> 'b -> 'tt)
let _0 = (fun (a,_) -> a), (fun (_,ss) b -> (b,ss))
let _1 = (fun (_,(a,_)) -> a), (fun (s0,(_,ss)) b -> (s0,(b,ss)))
let _2 = (fun (_,(_,(a,_))) -> a), (fun (s0,(s1,(_,ss))) b -> (s0,(s1,(b,ss))))

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A Implementing slot monads

This section explains the implementation details of slot monads shown in Section 3.

A.1 Bind, return and slots

Listing 13 shows an implementation of basic operations for a slot monad. A slot monad is defined as a varying-type state monad ‘ss -> ‘tt * ‘a (line 2) where ‘ss and ‘tt are types of initial and final states, respectively, and ‘a is the result type of a monadic action. return (line 3) is an operation that takes a value and returns a “pure” action without any effects on the state. >>= (line 4) returns a composed action which firstly takes a state and run the action on the left-hand
Listing 14 Buffer channel module

```ocaml
module Channel : sig
  type 'a t
  val create : unit -> 'a t
  val send : 'a t -> 'a -> unit
  val receive : 'a t -> 'a
end
```

Listing 15 An implementation of polarised session types

```ocaml
(* Communication direction and polarity *)
type req = Req and resp = Resp

(* The polarised session type and message wrapper *)

let ('p, 'r) sess = 'p wrap Channel.t * 'r
let ('p wrap =
  Msg : ('v * 'p wrap Channel.t) -> ['msg of 'r * 'v * 'p] wrap
| BranchL : 'p1 wrap Channel.t -> ['branch of 'r * ['left of 'p1]] wrap
| BranchR : 'p2 wrap Channel.t -> ['branch of 'r * ['right of 'p2]] wrap
| Chan : (('pp, 'rr) sess * 'p wrap Channel.t) -> ['deleg of 'r * ('pp, 'rr) sess * 'p] wrap

(* The service channel *)
let 'p channel = 'p wrap Channel.t Channel.t
```

side, then applies the result value and intermediate state to the continuation on
the right-hand side. run (line 9) takes the empty slots of type all_empty (line 8)
as the initial state to run the whole sessions.

A slot specifier (line 12) is a pair of getter and setter for accessing on a
sequence of slots. Slot specifiers _0, _1, · · · (lines 14-16) are defined as getters and
setters for specific positions of (nested) pair types.

### A.2 Implementing communication primitives

Since session types allow a channel to send and receive messages with heterogeneous types, they cannot be implemented directly by OCaml types. A resolution is
to use untyped channels as an underlying communication medium as Padovani [24]
did in his FuSe work, which we do not explain here because it is relatively easy
to extend the technique of [24] by slots.

Instead, we build session types based on the encoding from session types to
linear types by Kobayashi et al [17]. This implementation is inherently safe since
it does not use dangerous operations on untyped channels. A shortcoming is
that the encoding incurs some overhead to generate a channel each time when a
message is sent or received.

Listing 15 shows the definitions of service channels (line 15) and session
channels (line 6) where the signature of underlying Channel module are given in
Listing 14. Hereafter channels from Channel module are called plain channels. A session channel (`p, `r) sess (line 6) is a pair of a plain channel `p wrap Channel.t and a polarity `r. Here, wrap (line 7) is the type of the wrapper of the message payload (if any) and a plain channel for subsequent communications. The wrapper uses GADT [5] to associate session messages with the three kinds of protocol types. The constructor Msg (lines 8) is communicated by send and recv and has the protocol type of [`msg ..] and the payload is the OCaml’s value type (`v).

Branch{L, R} (lines 9-10) is for branching (branch2, select_left and select_right) and the protocol type is [`branch of [`left .. | `right ..]]. The wrapper labels have no payload since each constructor represents a branching label itself. Chan (lines 11-12) is for delegation (deleg_send and deleg_recv) and the protocol type is [`deleg ..]. It has a delegated session channel (`pp, `rr) sess as a payload.

Listing 16 shows the implementation of communication primitives. new_channel (line 1) creates a new channel and returns it. The primitives connect (lines 3-9) and accept (lines 11-13) create a new session channel and share the session over a service channel. Primitives for session communications send (lines 15-23), recv (lines 25-28), select_left (lines 30-32) and select_right (lines 34-36) (1) take the session channel from the slot string with the lenses; and (2) communicate using the wrapper, storing the new session channel in the slot. Branching branch2 (lines 38-42) matches on the received message and branches to the appropriate continuations. Delegation deleg_send (lines 44-49) transmits the session channel stored in the second slot and empties it. deleg_recv (lines 51-55) receives the delegated channel and stores it in the second slot. Finally, close just throws away the session channel and fills the slot with Empty.

B Further application: a database server

As a more practical application with delegation, we show an implementation of a database server. This database processes queries with a thread pool in order to handle a number of queries and authenticate client connections to the server by the main thread. At the same time, in order to respond to the clients quickly, a session established in the main thread is delegated to the worker thread.

The database server shown in Listing 17 behaves as follows.

1. A client starts a session with the server on db.ch and sends the authentication information (cred: credential) (line 8).
2. The main thread accepts or rejects the client according to the authentication result. If it rejects, the session ends (lines 10-11).
3. The main thread connects to a worker thread waiting on worker.ch and delegates the session to it (line 15).
4. The database session enters a loop which receives a command and returns a result. The client either terminates the session (line 25) or sends another query (query: query) to the worker thread (line 27). The worker thread returns the query result (res: result) and repeats the loop (lines 29-30).
The protocol type of the database channel (db.ch) is shown in Listing 18. Lines 2-5 are for receiving credential and branching where the label \texttt{left} is for rejection and \texttt{right} is for acceptance. Lines 7-10 corresponds to loop part in the Listing 17. The \texttt{left} branch finishes the protocol. \texttt{right} branch communicates a query and its result, then recurses to the second branch again.

Delegation appears in the type of \texttt{worker.ch} as follows.

```ocaml
val worker_ch :
  ['deleg of req * (dbprotocol, serv) sess * ['close]]
```

Here, the delegated session has polarity \texttt{serv}, as the delegated session is originally established by \texttt{accept}. 

Listing 16 Implementation of communication in the slot monad

```ocaml
let new_channel = Channel.create

let connect ch ~bindto:(_,set) ss =
  (* Generates a session channel *)
  let ch' = Channel.create () in
  (* Establish a connection and share the session channel with the server *)
  Channel.send ch ch';
  (* Then put it in a slot and return *)
  let ss (ch',(Resp,Req)), () =

let accept ch ~bindto:(_,set) ss =
  let ch' = Channel.receive ch in
  set ss (ch',(Req,Resp)), ()

let send (get,set) v ss =
  (* Extract the session channel (q is polarity) *)
  let ch,q = get ss
  (* Generate a session channel for subsequent communication *)
  and ch' = Channel.create () in
  (* wrap the message and the above channel in Msg and transmit it *)
  Channel.send ch (Msg(v,ch'));
  (* Then put it in a slot and return *)
  set ss (ch',q), ()

let recv (get,set) ss =
  let ch,q = get ss in
  let Msg(v,ch') = Channel.receive ch in
  set ss (ch',q), v

let select_left (get,set) ss =
  let ch,q = get ss and ch' = Channel.create () in
  Channel.send ch (BranchL(ch')); set ss (ch',q), ()

let select_right (get,set) ss =
  let ch,q = get ss and ch' = Channel.create () in
  Channel.send ch (BranchR(ch')); set ss (ch',q), ()

let branch2 ((get1,set1),f1) ((get2,set2),f2) ss =
  let (ch1,p) = get1 ss in
  match Channel.receive ch1 with
  | BranchL(ch1') -> f1 () (set1 ss (ch1',p))
  | BranchR(ch2') -> f2 () (set2 ss (ch2',p))

let deleg_send (get0,set0) ~release:(get1,set1) ss =
  let ch0,q1 = get0 ss and ch0' = Channel.create () in
  let tt = set0 ss (ch0',q1) in
  let ch1,q2 = get1 tt in
  Channel.send ch0 (Chan((ch1,q2),ch0'));
  set1 tt Empty, ()

let deleg_recv (get0,set0) ~bindto:(get1,set1) ss =
  let ch0,q0 = get0 ss in
  let Chan((ch1,q1),ch0') = Channel.receive ch0 in
  let tt = set0 ss (ch0',q0) in
  set1 tt (ch1,q1), ()

let close (get,set) ss = set ss Empty, ()
```

Listing 17 The implementation of a database server

```ocaml
open SessionN

let db_ch = new_channel ()
and worker_ch = new_channel ()

let rec main () =
    accept db_ch ~into:_0 >>
    let cred = recv _0 in
    if bad_credential cred then
        send_left _0 >>
        close _0
    else
        send_right _0 >>
        connect worker_ch ~into:_1 >>
        deleg_send _1 ~release:_0 >>
        close _1 >>=
        main

let rec worker () =
    accept worker_ch ~into:_0 >>
    deleg_recv _0 ~into:_1 >>
    close _0 >>
    let rec loop () =
        branch2 _1
        (fun () -> close _1)
        (fun () ->
            let query = recv _1 in
            let res = do_query query in
            send _1 res in
            loop ()
        )
in loop () >>= worker
```

Listing 18 The protocol of a database server

```ocaml
type dbprotocol =
    ['msg of req * credential *
    | 'branch of resp *
      [ 'left of ['close],
        | 'right of query_loop ]]
and query_loop =
    ['branch of req *
     [ 'left of ['close]
       | 'right of ['msg of req * query *
         [ 'msg of resp * result * query_loop ]]]
```