Non-commuting ETC corrections to $zt\bar{t}$ vertex

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Abstract

In this paper we calculate the corrections to $zt\bar{t}$ couplings induced by non-commuting ETC interactions. The extra parameters $\delta g_b^L$, $s^2$ and $\frac{1}{x}$ of non-commuting ETC models and the usual SM parameters will be assumed to be determined from a global fit to the LEP 1 EW data. We find that in the heavy (light) case $F^t_L$ is modified by at most 4% (2.8%) relative to its SM value provided $\delta R_b < .0088$. This implies that it will be very difficult to disentangle the ETC corrections from SM corrections to $zt\bar{t}$ vertex or to probe the effects of non-commuting ETC on $zt\bar{t}$ couplings with the projected NLC precision of measuring them.
I. Introduction

In ETC models the large mass of the top quark is presumably [1] due to a low enough ETC scale ($\approx$ 1 Tev). The sideways ETC interaction that gives rise to the large mass for the top quark also gives rise to a sizeable negative correction [2] to $R_b$. This result was considered to be disastrous for ETC models since the LEP value for $R_b$ at that time was already $2.2\sigma$ above the SM prediction. In order to resolve the $R_b$ anomaly in the context of ETC models Chivukula, Simmons and Terning (CST) proposed the non-commuting ETC models [3] in which the electroweak $SU(2)_h$ gauge group for the heavy third generation fermions is embedded in the ETC gauge group. The electroweak $SU(2)_l$ gauge interactions of the fermions belonging to the first two generations were however assumed to commute with the ETC interactions as usual. The same authors also showed that although these models contain three extra parameters ($\delta g^b_L$, $s^2$ and $\frac{1}{x}$), they provide a better fit [4] to the LEP 1 EW data than the SM particularly if $\alpha_s(M^2_Z) \approx .112$ as suggested by the low energy deep inelastic scattering experiments. LEP 1 data however does not probe the $zt\bar{t}$ couplings, the corrections to which can vary from one ETC model to another even for a fixed $\delta R_b$. Precision study of $zt\bar{t}$ coupling can therefore complement our understanding of third generation flavor physics derived from LEP 1 data on $zb\bar{b}$ coupling. In order to probe non-commuting ETC effects in the process $e^+e^- \rightarrow t\bar{t}$ far below the $Z_H$ resonance one has to depend on precision measurements of $zt\bar{t}$ couplings at NLC 500 or higher. The aim of this article is to calculate the corrections to $zt\bar{t}$ couplings for non-commuting ETC models assuming $R_b$ to lie around its present LEP 1 value ($R_b = .2178 \pm .0011$) [5] and to compare them with the projected NLC sensitivity for measuring them.

In non-commuting ETC models one has to go through a sequence of symmetry breakings- $G_{etc} \times SU(2)_l \times U(1)' \rightarrow G_{tc} \times SU(2)_h \times SU(2)_l \times U(1)_y \rightarrow G_{tc} \times SU(2)_L \times U(1)_y \rightarrow G_{tc} \times U(1)_Q$ in order to give masses to EW gauge bosons and ordinary fermions. To implement the symmetry breaking $SU(2)_h \times SU(2)_l \times U(1)_y \rightarrow SU(2)_L \times U(1)_y \rightarrow U(1)_Q$ one needs two order parameters $<\sigma>$ and $<\phi>$. $<\sigma>$ breaks $SU(2)_h \times SU(2)_l$
into $SU(2)_L$ and $\langle \phi \rangle$ breaks $SU(2)_L \times U(1)_Y$ into $U(1)_Q$. Two simplest possibilities of $SU(2)_h \times SU(2)_l$ transformation properties of $\langle \sigma \rangle$ and $\langle \phi \rangle$ that produce the correct mixing and breaking of the gauge groups are: a) $\langle \phi \rangle = (2, 1)_\frac{1}{2}$, $\langle \sigma \rangle = (2, 2)_0$ (heavy case) and b) $\langle \phi \rangle = (1, 2)_\frac{1}{2}$, $\langle \sigma \rangle = (2, 2)_0$ (light case). In the heavy (light) case $\langle \phi \rangle$ transforms non-trivially under $SU(2)_h(SU(2)_l)$. The heavy case corresponds to the situation where the symmetry breaking mechanism that gives mass to the top quark also provides the bulk of EW symmetry breaking as evidenced in W and Z boson masses. On the other hand in the light case the physics responsible for top quark mass does not provide the bulk of W and Z boson masses. Corrections to $z\bar{b}b$ and $zt\bar{t}$ couplings for non-commuting ETC models can arise from two sources: i) sideways ETC induced vertex correction and ii) $Z_1 - Z_2$ mass mixing.

i) Effect of sideways gauge boson exchange: In non-commuting ETC models the sideways ETC gauge boson exchange induces the following coupling [4] between the Z boson and the LH t-b doublet.

$$\delta L_{sf}^S \approx \frac{g}{c_w} \frac{\xi^2 f^2}{2 f^2} \bar{\Psi}_L \gamma^\mu \Psi_L Z_\mu. \quad (1)$$

This implies that $\delta g_{bs}^L = -\frac{\xi^2 f^2}{2 f^2}$ and $\delta g_{ts}^L = -\frac{\xi^2 f^2}{2 f^2}$ where we have divided out by a common factor of $-\frac{g}{c_w}$. In the above $f = 2 \frac{M_{etc}}{g_{etc}}$ is the scale at which the ETC gauge group breaks down. In the weak perturbative realization of ETC gauge interactions $M_{etc}$ is the mass of the ETC gauge boson and $g_{etc}$ is the ETC gauge coupling.

ii) Effect of $Z_1 - Z_2$ mass mixing: In non-commuting ETC models the effect of $Z_1 - Z_2$ mass mixing on $\delta g_{bs}^L$ and $\delta g_{ts}^L$ depend on whether the symmetry breaking pattern corresponds to heavy case or light case. We shall discuss them separately in the following.

a) Heavy case: In the heavy case the $Z_1 - Z_2$ mass mixing gives rise to the following [4] mass eigenstates: $Z_1 \approx Z_L - \frac{c_s}{x_{cw}} Z_H$ and $Z_2 \approx Z_H + \frac{c_s}{x_{cw}} Z_L$. Here $Z_L$ and $Z_H$ are mass eigenstates. $Z_L$ will be identified with the light neutral Z boson of the SM. $\phi$ is the angle that characterizes the mixing between $SU(2)_h$ and $SU(2)_l$. So in the heavy case the
mixing effect gives rise to the following coupling between $Z_L$ and the LH (t-b) doublet:

$$\delta L^m = \frac{g}{c_w} s^4 \bar{\Psi}_L \gamma^\mu T_{3h} \Psi_L Z^\mu_L.$$  (2)

It then follows that $\delta g_L^{bm} = \frac{s^4}{2x}$ and $\delta g_L^{tm} = -\frac{s^4}{2x}$. The overall correction to the $zb\bar{b}$ and $zt\bar{t}$ couplings in the heavy case are given by $\delta g_L^b = -\frac{\xi^2 f_Q^2}{2f^2} + \frac{s^4}{2x}$ and $\delta g_L^t = -\frac{\xi^2 f_Q^2}{2f^2} - \frac{s^4}{2x}$. We find that in the heavy case the effects due to mixing and sideways gauge boson induced vertex correction interfere constructively (destructively) in LH $zt\bar{t}$ ($zb\bar{b}$) coupling. This suggests that in the heavy case the $zt\bar{t}$ vertex correction can be large even if $zb\bar{b}$ vertex correction is small provided the mixing and vertex correction contributions are individually large and nearly equal.

b) Light case: In the light case $Z_1 - Z_2$ mixing gives rise to the following [4] mass eigenstates: $Z_1 \approx Z_L - \frac{se^3}{xc_w} Z_H$ and $Z_2 \approx Z_H + \frac{se^3}{xc_w} Z_L$. Hence the neutral gauge boson mixing produces the following corrections to $Z_L$ couplings

$$\delta g_L^{fm} = -\frac{c^3 s}{x} (\frac{c}{s} T_{3l} - \frac{s}{c} T_{3h}).$$  (3)

In the light case the overall corrections to the LH $zb\bar{b}$ and $zt\bar{t}$ couplings are given by $\delta g_L^b = -\frac{\xi^2 f_Q^2}{2f^2} - \frac{c^2 s^2}{2x}$ and $\delta g_L^t = -\frac{\xi^2 f_Q^2}{2f^2} + \frac{c^2 s^2}{2x}$. We therefore find that in the light case the effects due to mixing and sideways ETC induced vertex correction interfere constructively (destructively) in $zb\bar{b}$ and $zt\bar{t}$ coupling.

II. $zt\bar{t}$ vertex correction in the heavy case

The gauge boson mixing both in the charged and neutral sector modifies not only the $zt\bar{t}$ and $zb\bar{b}$ couplings but also the SM prediction to many other EW observables that are accessible at LEP 1. The two additional parameters $s^2$ and $\frac{1}{x}$ that determine the the gauge boson mixing can be determined along with the usual standard model parameters from a global fit to the precision EW data. In the heavy case for $s^2 = .97$ and $\alpha_s(M_Z^2) \approx .115$ the best fit values of $\frac{1}{x}$ and $\delta g_L^b$ obtained by CST [4] are given by $\frac{1}{x} = \approx .0027 \pm .0093$ and $\delta g_L^b \approx -0.0064 \pm 0.0074$. We shall assume the above values for $s^2$ and $\frac{1}{x}$, but treat $\delta g_L^b$
as a relatively free parameter since the LEP 1 value for $R_b$ has been undergoing frequent changes. More precisely we shall let $\delta R_b$ to assume the values .0022, .0044 .0066 and .0088 which correspond to 1%, 2% 3% and 4% deviations relative to the SM prediction for $R_b$. It can be shown that in the heavy case the ETC correction to $R_b$ is given by

$$\frac{\delta R_b}{R_b} \approx .8973\xi^2 \frac{m_t}{4\pi f_Q} - .0047$$

which can be used to find the value of $\xi^2 \frac{m_t}{4\pi f_Q}$ for a given $\delta R_b$. We find that for $\delta R_b = .0022, .0044, .0066 and .0088$, $\frac{\delta F_{L,sm}^t}{F_{L,sm}^t}$ is given by -.0159, -.0240, -.0324 and -.0405 respectively. Here $\delta F_{L}^t$ is the ETC induced correction to the LH form factor for $zt\bar{t}$ vertex and $F_{L,sm}^t = \frac{1}{2} - \frac{2}{7}s_w^2$ is its value in the SM. Note that $\delta F_R^t = 0$ both in heavy and light scenarios since: i) $Z_2$ does not couple to $t_R$ and therefore $Z_1 - Z_2$ mixing does not renormalize the $Z_Lt_R\bar{t}_R$ coupling and ii) the sideways ETC induced four fermion term $t_R\gamma^\mu t_R\bar{t}_R\gamma^\nu t_R$ after Fierz rearrangement does not contain any isospin triplet technifermion current.

III. $zt\bar{t}$ vertex correction in the light case.

In the light case for $s^2 = .97$ and $\alpha_s(M_Z) = .115$, CST [4] found that the best fit value for $\frac{1}{x}$ lies in the unphysical region ($\frac{1}{x} = -.17 \pm .75$). However since the fit is fairly insensitive to the value of $\frac{1}{x}$, there is a substantial range of values of $\frac{1}{x}$ which provide a good fit to the data. Following CST we shall choose $\frac{1}{x} = .055$ in the light case. Putting the mixing and vertex corrections together we find that

$$\frac{\delta R_b}{R_b} = .8973\xi^2 \frac{m_t}{4\pi f_Q} + .7844(2.2879s^2 + 2.3426c^2)\frac{c^2}{x},$$

which can be used to find the value of $\xi^2 \frac{m_t}{4\pi f_Q}$ for a given $\delta R_b$. We find that for $\delta R_b = .0022, .0044, .0066 and .0088$, $\frac{\delta F_{L,sm}^t}{F_{L,sm}^t}$ is given by -.0035, -.0119, -.0200 and -.0281.

IV. Discussion of results

We note the following features in the non-commuting ETC induced correction to $zt\bar{t}$ couplings:

i) $\delta F_{L}^t$ is always very small in non-commuting ETC models. For $\delta R_b < .0088$ the correction to $F_{L}^t$ is at most 4.1%(2.8%) in the heavy (light) case. Since this is of the same order as the usual SM corrections to $F_{L}^t$ it will be very hard to disentangle one from the other at NLC. It should be noted that the calculation of $\delta F_{L}^t$ does not depend on the value
ii) With increasing $\delta R_b$, $\delta F^t_L$ increases in magnitude but the increase is very slow (almost inappreciable) in both light and heavy cases.

iii) Non-commuting ETC interactions renormalize the $z t \bar{t}$ vertex in such a way that the LH weak charge of the top quark always decreases in magnitude.

v) For $\delta R_b = .0044$ we find that $\frac{f_Q}{\xi^2} = 497 \text{Gev}$ in the heavy case and 718 Gev in the light case. But $f_Q$ in the light case is expected to be less than its value in the heavy case. This can happen only if $\xi^2$ in the light case is smaller than its value in the heavy case for non-commuting ETC models.

The $z t \bar{t}$ couplings will be probed with good precision at NLC. By including the $l^\pm + \text{jets}$ mode and making use of the angular distribution of different polarization states of $t \bar{t}$, Ladinsky and Yuan [6] found that $F^t_L$ can be determined at NLC 500 to within about 3% (8%) at 68% (90%) confidence limit. While $F^t_R$ can be known to within roughly 5% (18%). The corrections to $F^t_L$ both in the heavy and light cases being always less than or barely equal to the projected NLC sensitivity of measuring it such corrections cannot be probed at NLC unless there is a drastic improvement in its sensitivity. It is interesting to note that for $\delta R_b = .0044$, $\delta F^t_v = -\delta F^t_a \approx -.0434$ for non-commuting ETC scenario. Whereas for $m_s^2 = m_d^2$ and the same value for $\delta R_b$ $\delta F^t_v \approx -.179$ and $\delta F^t_a \approx -.068$ for diagonal ETC scenarios. Non-commuting ETC scenarios are therefore expected to be much less constrained by $z t \bar{t}$ vertex correction than diagonal ETC scenarios. A 1 Tev machine can do better than a 500 Gev machine in determining the corrections to $z t \bar{t}$ vertex because firstly the RR and LL events are suppressed relative to LR and RL events and secondly the top quark is boosted more which makes the determination of its momentum direction more accurate.

**Conclusion**

In this article we have calculated the corrections to $z t \bar{t}$ couplings due to non-commuting ETC interactions. We find that the for $\delta R_b < .0088$ $\delta F^t_L$ is always less than 4% (2.8%) in
the heavy case (light case). Whereas $\delta F^i_R = 0$ to the order to which we have worked. The non-commuting ETC induced corrections to $zt\bar{t}$ couplings are therefore too small to be probed with the projected NLC sensitivity. Further since these corrections are of the same order as usual SM corrections it will be very hard to disentangle one effect from the other. We therefore conclude that it will be very hard to probe non-commuting ETC effects by precision measurements of $zt\bar{t}$ couplings.

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