Renormalization of the antisymmetric tensor field propagator and dynamical generation of the $1^{+-}$ mesons in Resonance Chiral Theory\(^*$

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We discuss the renormalization of the $1^{--}$ vector meson propagator within Resonance chiral theory at one loop. Using the particular form of the interaction Lagrangian we show that additional poles of the renormalized propagator corresponding to $1^{+-}$ degrees of freedom can be generated. We give a concrete example of such an effect.

1. Introduction

Non-perturbative investigation of the QCD dynamics in the low energy region by means of the effective Lagrangian approach has made considerable progress recently. In the very low energy region ($E \ll \Lambda_H \sim 1 \text{ GeV}$) the chiral perturbation theory ($\chi$PT)\(^[123]\) based on the spontaneously broken chiral symmetry $SU(3)_L \times SU(3)_R$ grew into a very successful model-independent tool for description of the Green functions (GF) of quark currents and related low-energy phenomenology. $\chi$PT is organized as a rigorously defined simultaneous perturbative expansion in small momenta and light quark masses. Recent calculations are performed at the next-to-next-to-leading order $O(p^6)$\(^[4]\).

In the intermediate energy region ($\Lambda_H \leq E < 2 \text{ GeV}$), however, the situation is less satisfactory. The set of relevant degrees of freedom includes now the low lying resonances and because there is no mass gap existing in the spectrum, the effective theory in this region cannot be constructed as a straightforward extension of the $\chi$PT low energy expansion. On the other hand, the considerations based on the large $N_C$ expansions together with high-energy constraints derived from perturbative QCD and operator product expansion (OPE) allow to introduce another type of effective Lagrangian description, corresponding to the leading order in $1/N_C$ and reflecting the basic features of QCD in the $N_C \rightarrow \infty$ limit. Namely, the spectrum consisting of an infinite tower of free stable mesonic resonances exchanged in each channel requires infinite number of resonance fields in the $U(3)_L \times U(3)_R$ symmetric Lagrangian with interaction vertices suppressed by an appropriate power of $N_C^{-1/2}$ and (since the $1/N_C$ expansion is correlated with semiclassical expansion) only tree graphs have to be taken into account in the leading order. An approximation to this general picture consisting in limiting the number of resonance field to one in each channel and matching the resulting theory in the high energy region with OPE is known as Resonance Chiral Theory ($R\chi T$)\(^[56]\). Integrating out the resonance fields from the Lagrangian of $R\chi T$ in the low energy region and subsequent matching with $\chi$PT has become a very successful tool for estimates of the resonance contribution to the values of the $O(p^4)$\(^[5]\) and $O(p^6)$\(^[78]\) low energy constants (LEC) entering the $\chi$PT Lagrangian.

Though the usual chiral power counting fails within $R\chi T$ due to the presence of an additional heavy scale (the mass of the resonances) and the usual Weinberg formula\(^[1]\) cannot be generalized here (because of the lack of a scale playing the role

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\(^[1]\) Perturbative QCD and operator product expansion (OPE) allow to introduce another type of effective Lagrangian description, corresponding to the leading order in $1/N_C$ and reflecting the basic features of QCD in the $N_C \rightarrow \infty$ limit. Namely, the spectrum consisting of an infinite tower of free stable mesonic resonances exchanged in each channel requires infinite number of resonance fields in the $U(3)_L \times U(3)_R$ symmetric Lagrangian with interaction vertices suppressed by an appropriate power of $N_C^{-1/2}$ and (since the $1/N_C$ expansion is correlated with semiclassical expansion) only tree graphs have to be taken into account in the leading order. An approximation to this general picture consisting in limiting the number of resonance field to one in each channel and matching the resulting theory in the high energy region with OPE is known as Resonance Chiral Theory ($R\chi T$)\(^[56]\). Integrating out the resonance fields from the Lagrangian of $R\chi T$ in the low energy region and subsequent matching with $\chi$PT has become a very successful tool for estimates of the resonance contribution to the values of the $O(p^4)$\(^[5]\) and $O(p^6)$\(^[78]\) low energy constants (LEC) entering the $\chi$PT Lagrangian.

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analogous to $\Lambda_H$), it seems to be fully legitimate to go beyond the tree level $R\chi T$ and calculate the loops [9]. Being suppressed by one power of $1/N_C$, the loops allow to encompass such NLO effects in the $1/N_C$ expansion as resonance widths and final state interaction and to determine the NLO resonance contribution to LEC (and their running with the renormalization scale).

However, we have to be ready for both technical and conceptual complications connected with renormalization of the effective theory for which no natural organization of the expansion (other than the $1/N_C$ counting) exists. Especially, because there is no natural analog of the Weinberg power counting in $R\chi T$, we can expect mixing of the naive chiral orders in the process of the renormalization (e.g. the loops renormalize the $O(p^2)$ LEC and also counterterms of unusually high chiral orders are needed). Also, lack of appropriate protective symmetry can bring about appearance of new poles in the GF corresponding to new degrees of freedom which are frozen at the tree level. The latter might be felt as a pathological artefact of the not carefully enough formulated theory, particularly because this extra poles might be negative norm ghost or tachyons [10]. On the other hand, however, we could also try to take advantage of this feature and adjust the poles in such a way that they correspond to the well established resonance states.

In the following we will illustrate these problems in more detail. As an explicit example we use the one-loop renormalization of the propagator corresponding to the antisymmetric tensor field which originally describes the $1^{--}$ vector resonance ($\rho$ meson) at the tree level. We will show that the loop corrections to the propagator could lead to the dynamical generation of various types of $1^{--}$ and $1^{+-}$ states and that the appropriate adjustment of coupling constants allows us to generate in this way the one which could be identified with the $b_1(1235)$ meson. The details of the calculations and further discussion will be provided in [11].

2. Antisymmetric tensor field propagator

The $1^{--}$ resonance part of the $R\chi T$ Lagrangian within the antisymmetric tensor formalism reads [50]

$$ \mathcal{L} = -\frac{1}{4} (D^{\mu} R_{\mu\nu} D_{\alpha} R^{\alpha\nu}) + \frac{1}{2} M^2 (R^{\mu\nu} R_{\mu\nu}) + \mathcal{L}_{\text{inter}} , $$

where $R_{\mu\nu} = R^{a}_{\mu\nu} T^a$ with normalized $U(3)$ generators $T^a$, $R^{a}_{\mu\nu}$ are antisymmetric tensor fields with appropriate quantum numbers and $D_\alpha$ is the usual chiral covariant derivative. $\mathcal{L}_{\text{int}}$ is the interaction Lagrangian which will be specified later. The full antisymmetric tensor field propagator $\delta^{ab}\Delta_{\mu\nu,\alpha\beta}(p) = -iF.T.(0|T R^{a}_{\mu\nu}(x) R^{b}_{\alpha\beta}(0)|0)$ has in general the following tensor structure

$$ \Delta_{\mu\nu,\alpha\beta}(p) = -2\Pi^{L,T}_{\mu\nu,\alpha\beta} \Delta^{L}(p^2) + 2\Pi^{T}_{\mu\nu,\alpha\beta} \Delta^{T}(p^2), $$

where $\Pi^{L,T}_{\mu\nu,\alpha\beta}$ are longitudinal and transverse projectors ($P^{\nu}_{\mu} = g_{\nu\mu} - p_\nu p_\mu/p^2$)

$$ \Pi^{T}_{\mu\nu,\alpha\beta} = \frac{1}{2} \left( p^{T}_{\mu\alpha} p^{T}_{\nu\beta} - p^{T}_{\nu\alpha} p^{T}_{\mu\beta} \right), $$

$$ \Pi^{L}_{\mu\nu,\alpha\beta} = \frac{1}{2} \left( g_{\mu\nu} g_{\alpha\beta} - g_{\alpha\nu} g_{\mu\beta} \right) - \Pi^{T}_{\mu\nu,\alpha\beta}. $$

Note that, for $p^2 = m^2 > 0$ we can express $\Pi^{L,T}_{\mu\nu,\alpha\beta}$ as the polarization sums

$$ -2\Pi^{L}_{\mu\nu,\alpha\beta} = \sum_{\lambda} u^{(\lambda)}_{\mu\nu}(p) u^{(\lambda)\ast}_{\alpha\beta}(p), $$

$$ 2\Pi^{T}_{\mu\nu,\alpha\beta} = \sum_{\lambda} w^{(\lambda)}_{\mu\nu}(p) w^{(\lambda)\ast}_{\alpha\beta}(p), $$

where

$$ u^{(\lambda)}_{\mu\nu}(p) = \frac{i}{m} \left( p_{\mu} \epsilon^{(\lambda)}_{\nu}(p) - p_{\nu} \epsilon^{(\lambda)}_{\mu}(p) \right), $$

$$ w^{(\lambda)}_{\mu\nu}(p) = \frac{1}{2} \epsilon^{(\lambda)}_{\mu\nu} \epsilon^{(\lambda)\ast}_{\alpha\beta}(p) $$

and $\epsilon^{(\lambda)}_{\nu}(p)$ are the usual spin-one polarization vectors with mass $m$. The possible poles $p^2 = m^2_{L,T} > 0$ of $\Delta^L(p^2)$ and $\Delta^T(p^2)$ correspond therefore both to the spin-one states which couple to the fields $\partial^a R^{a}_{\mu\nu}$ and $\epsilon^{\mu\nu,\alpha\beta}\partial^\nu R^{a}_{\alpha\beta}$ respectively and have therefore the same quantum numbers up to the parity.

At LO in the $1/N_C$ expansion the Lagrangian [11] gives $\Delta^L_{LO}(p^2) = 1/(p^2 - M^2)$ and $\Delta^T_{LO}(p^2) = \frac{1}{4} (D^{\mu} R_{\mu\nu} D_{\alpha} R^{\alpha\nu}) + \frac{1}{2} M^2 (R^{\mu\nu} R_{\mu\nu}) + \mathcal{L}_{\text{inter}}.
where the self-energies \( \Sigma_{L,T}(p^2) \) are of the order \( 1/N_C \) at least. In the next section we present the results of the calculation of the renormalized self-energies \( \Sigma_{L,T}(p^2) \) in the chiral limit at NLO for a concrete form of the interaction Lagrangian \( \mathcal{L}_{\text{int}} \). A more systematic treatment will be given in \([12]\).

3. The one-loop self-energies within R\(\chi\)T

In what follows we limit ourselves to the interaction Lagrangian \( \mathcal{L}_{\text{int}} \) with at most two derivatives and up to two resonance fields. Writing explicitly only those terms that contribute to the one-loop self-energies we have \([5, 13]\)

\[
\mathcal{L}_{\text{int}} = \frac{iG_V}{2\sqrt{2}} (R^{\mu\nu}[u_\mu, u_\nu]) + 2d_1\{D_\beta u^\alpha (\tilde{\rho}_{\alpha\beta}, R^{\alpha\beta})
\]

\[+ 2d_3\{u^\lambda\{D_\rho R^{\mu\nu}, \tilde{\rho}_{\mu\lambda}\} + 2d_4\{u_\nu\{D^\rho R^{\mu\nu}, \tilde{\rho}_{\mu\rho}\}\},
\]

where \( \tilde{\rho}_{\alpha\beta} = \varepsilon_{\alpha\beta\mu\nu} R^{\mu\nu}/2 \). In the large \( N_C \) limit the couplings are \( G_V = O(\lambda^{1/2}) \) and \( d_i = O(1) \) and apparently the intrinsic parity odd part is of higher order. However, the trilinear vertices contributing to the one-loop self-energies are \( O(N_C^{-1/2}) \) in both cases due to the appropriate power of \( 1/F = O(N_C^{-1/2}) \) accompanying \( u_\alpha \).

Therefore the operators with two resonance fields cannot be eliminated using the large \( N_C \) arguments. Also nonzero \( d_i \) are required in order to satisfy the OPE constraints for VVP GF at the LO \([13]\).

In order to cancel the infinite part of the one-loop self-energies we have to introduce a set of counterterms. Because the interaction terms are \( O(p^2) \) we would expect (by the analogy with \( \chi\)PT power counting) these counterterms to have four derivatives at most. However, the nontrivial structure of the free resonance propagator (namely the presence of the \( \Delta_{LO}^4 \Pi_{\mu\nu\rho\beta}^\alpha \) part) results in the failure of this naive expectation. In fact we need counterterms with up to six derivatives, namely

\[
\mathcal{L}_{\text{ct}} = \mathcal{L}_{\text{ct}}^{(0)} + \mathcal{L}_{\text{ct}}^{(2)} + \mathcal{L}_{\text{ct}}^{(4)} + \mathcal{L}_{\text{ct}}^{(6)}.
\]

The complete list of the counterterms and their infinite parts is postponed to \([12]\). Let us only note that \( \mathcal{L}_{\text{ct}}^{(2)} \) contains a new type of kinetic term \( \frac{i}{\pi} (D_\alpha R^{\mu\nu} D^\rho R_{\mu\nu}) \). Provided such a term was included in the LO Lagrangian from the very beginning, the propagator would have an additional pole in \( \Delta_{LO} \). However, interpretation of such a pole as a \( 1^{-+} \) state would be problematic. According to the sign of \( Y \) this state would be either a tachyon or a negative norm ghost \([10]\).

Evaluating the one-loop Feynman graphs and adding the polynomial contributions from the counterterms \([4]\) we get the \( \chi\)PT minimally subtracted self-energies \( \Sigma_{LO}(p^2) \). The equation for the poles of \( \Delta_{LO}^2(p^2) \) then has an approximate perturbative solution \( p^2 = M^2 + \Sigma_{LO}(M^2) \) corresponding to the original \( 1^{-+} \) vector resonance with LO mass \( M^2 \), which develops a mass correction and a finite width of the order \( O(1/N_C) \) due to the loops and which we identify with the \( \rho \) meson. This allows to re-parameterize perturbatively \( \Sigma_{LO}(s) \) in terms of \( M_\rho \) and \( \Gamma_\rho \) and requiring further \( M^2 = M_\rho^2 \) we get for \( \sigma_\rho^L(x) = \Sigma_{LO}^L(xM_\rho^2)/M_\rho^2 \)[12]

\[
\sigma_\rho^L(x) = \frac{1}{\pi} \left[ 1 - x^2 \tilde{B}(x) + \frac{a_i(x_i - 1)}{\pi M_\rho} \right] - \frac{40}{9} \left( \frac{M_\rho}{4\pi F_\pi} \right)^2 d_3^2 (x^2 - 1)^2 \tilde{J}(x)
\]

\[
\sigma_\rho^T(x) = \frac{1}{\pi} \left[ 1 + 6\gamma + \gamma^2 \right] x + 2\gamma^2 x^2 \left( x - 1 \right)^2 \tilde{J}(x).
\]

Here we put for further numerical estimates \( F_\pi, \gamma = d_4/d_3 \sim O(1) \) (for \( d_3 \) we take the value from \([13]\) and we have introduced the re-scaled free parameters \( a_i \) and \( b_i \) with natural size \( O(1) \) in the large \( N_C \) expansion. These can be expressed in terms of renormalization scale independent combinations of the renormalized counterterms couplings and \( \chi\)logs. The loop functions
$\hat{B}(x)$ and $\hat{J}(x)$ are given on the first sheet as

\[
\hat{B}(x) = 1 - \ln(-x), \\
\hat{J}(x) = x^{-1} \left[ 1 - (1 - x^{-1}) \ln(1 - x) \right],
\]

(5)

where we take the principal branch of the logarithm ($-\pi < \text{Im} \ln x \leq \pi$) with a cut for $x < 0$. On the second sheet we have then $\hat{B}^{II}(x - i0) = \hat{B}^{I}(x + i0) = \hat{B}^{I}(x - i0) + 2\pi i$ and similarly for $\hat{J}^{II}(x)$.

The equation for the poles of $\Delta^{T}(p^2)$ has only non-perturbative solutions of the order $O(C_N)$. The $1^{+-}$ states corresponding to them therefore decouple in the $N_C \rightarrow \infty$ limit, however, for physical values of $M_\rho$, $\Gamma_\rho$ and $F_{\pi}$ (and for reasonable $O(1)$ values of the parameters $b_i$ and $\gamma$), the position of poles can lie well within the intermediate energy region we are interested in. The nature of the corresponding states, which is also controlled by the free parameters $b_i$ and $\gamma$, is rich and covers bound states $0 < p^2 < M_\rho^2$, (which also might be negative norm ghosts), tachyonic poles $p^2 < 0$, resonance poles in the lower complex half-plane on the second sheet or even complex conjugated pair of Lee-Wick poles on the first sheet. It is not straightforward to formulate general conditions for $b_i$ and $\gamma$ under which there is no pole in $\Delta^{T}(p^2)$ at all, on the other hand we can rather easily arrange them to obtain the pole corresponding e.g. to the $b_1(1235)$ meson. This can be achieved in many ways e.g. for the choice $b_0 = -b_1 = -3.52$, $b_2 = -3.07$, $b_3 = 1$ and $\gamma = -0.52$. The plot of the denominator of $\Delta^{T}(p^2)$ for this particular choice for the section $p^2/M_\rho^2 = x - iM_\rho\Gamma b_i/M_\rho^2$ on the second sheet and the shape of $|\Delta_{L}(p^2)|^2$ on the first sheet for $p^2/M_\rho^2$ real are depicted in Fig 1.

4. Conclusion

We have illustrated the problems connected with loop calculations within R\x{T} using the one loop renormalization of the propagator of antisymmetric tensor field which describes $1^{--}$ resonance multiplet at the leading order of the large $N_C$ expansion as a concrete example. We have found that new $1^{++}$ states of various nature (including pathological ones like negative norm

![Figure 1. The real (full line) and imaginary part (dots) of the function $1/(M_\rho^2\Delta^{T}(p^2))$ for the section $p^2/M_\rho^2 = x - iM_\rho\Gamma b_1/M_\rho^2$ on the second sheet. The $b_1(1235)$ pole corresponds to zero at $x_{b_1} = 2.55 - i0.29$. The shape of $|\Delta_{L}(p^2)|^2$ on the first sheet for $p^2$ real is in the small frame.](image)

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