3D numerical simulation of ribbon electron beam bending by relativistic proton bunch

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Abstract. Nowadays experiments in the field of high-energy physics require increasing the intensity and brightness of charged particle beams and developing new methods for their diagnosis. One of the noninvasive methods for diagnosing the parameters of an intense bunch of charged particles is based on the use of a low-energy ribbon electron beam. This paper presents a numerical model of the interaction process of the testing electron beam and the proton bunch under study. The model used to solve the problem consists of the equations of motion of the electron beam under the action of an electromagnetic field created by the relativistic proton bunch. The beam is modeled by calculating of electron trajectories passing through the 3D computational domain. The dependence of the electron beam deflection on its energy, the angle of inclination to the direction of the proton bunch, the magnitude of the space charge and the velocity of the proton bunch is investigated. It is shown that the amplitude of the electron beam deflection is proportional to the charge and gamma factor of the proton bunch and inversely proportional to the energy of the electron beam.

1. Introduction
Currently experiments in the field of high-energy physics for fundamental and applied research require a constant increase in the intensity and brightness of charged particle beams and developing new methods for their diagnosis [1]. Such an improvement can be achieved on the basis of methods for diagnosing charged particle beams that do not worsen the quality of the studied beam. A significant role is played by non-invasive diagnostic methods that can be continuously used with intense beams of accelerated particles. One of the possibilities for diagnosing and measuring the parameters of intense bunch of charged particles is based on the use of a low-energy electron beam- a beam sensor [2]-[3]. A beam sensor is a device based on the use of a low-energy electron beam as a tool for studying intense bunches of charged particles. The possibility of measuring the parameters of intense bunches by an electron beam is due to the deflection of the electrons of the testing beam by a certain angle in the fields of the studied bunches. The values of these deflection angles determine the distribution of the testing charged particles on the observed plane and the projection of the electron beam on the luminescent screen is sensitive to the shape of the proton bunch. The possibilities of this method of non-destructive diagnostics can be fully used only with a detailed study of the process of electron beam-proton bunch interaction. Since the problem of the motion of a beam of charged particles in electromagnetic fields cannot be solved analytically in general, for studying the structure of the beam, numerical modeling is a method that plays an essential role in experiments. The
numerical simulation are also of significant importance in the interpretation of images obtained on the device screen. The results of experiment and computer simulation to determine the density distributions of accelerated particles by an scanning electron pencil beam are presented in paper [4]. However, the electron pencil beam device used is large and complex and makes it difficult to use this technique for accelerated beam diagnostics. Instead of a scanning pencil electron beam, it was proposed in [5] to use a ribbon electron beam created by a strip cathode. The apparatus with the strip cathode has better accuracy of measurement and better time resolution.

In the paper we present a 3D numerical model and a results of computer simulation of the interaction of the testing ribbon electron beam and the relativistic proton bunch. As a results of simulation the dependence of the electron beam deflection on its energy, the angle of inclination to the direction of the proton bunch, the magnitude of the space charge and the velocity of the proton bunch was investigated. In the second section, the statement of the problems of using an electron beam to study the characteristics of relativistic proton bunches is presented. The third section of the paper is dedicated to describe a numerical model of ribbon electron beam deflection by relativistic proton bunch. The fourth section contained the description the results of computer simulation for parameters corresponding to requirements of laboratory experiments. The main conclusions are presented in the fifth section of the paper.

2. The statement of the problem
A numerical solution of the three-dimensional problem of the electron beam deflection in the electromagnetic field of an ultrarelativistic proton bunch is considered. The diagram for measuring the beam profile as well as simulation domain for the 3-dimensional model are shown in the Fig. 1. The calculation area has the form of a parallelepiped with size $-k < x < k$, $-l < y < l$, $-d < z < h$. A relativistic proton bunch with dimensions $a$, $b$, $p$ along the $x$, $y$, and $z$ axes, respectively, passes through the center of the domain and is directed along the $y$-axis. A ribbon electron beam emitted by cathode of width $L$ and zero thickness. The cathode is located in $x - y$ plane at a distance $z = -d$ from the center of the relativistic proton bunch. Electrons are launched from the cathode line with the velocity parallel to the $z$-axis: $v = (v_x, v_y, v_z)$,

$$v_z = \sqrt{2W/m}, \quad v_x = 0, \quad v_y = 0,$$

where $m$ is electron mass, $W$ is the electron beam energy. The electron beam intersects the proton bunch at an angle $\theta$ to the $x$-axis. The electrons are deflected by the electric field of the
space charge and by the magnetic field of the relativistic proton bunch and are received by a luminescent screen in the $x - y$ plane at a distance $z = h$ from the center of the proton bunch. The model used to solve the problem consists of the equations of motion of the electron beam under the action of an electromagnetic field created by a relativistic proton bunch. Emitted by cathode a ribbon electron beam passing through the computational domain is treated as a set of $N_p$ non-relativistic electron beamlets (macroparticles). The motion of each macroparticle is tracked by calculating the equation of motion. Each trajectory is computed independent of all the others. To determine the potential of the electric field created by the spatial charge of a relativistic proton beam, the Poisson equation is used. The electric field $E$ and the magnetic field $B$ are determined in accordance with the Lorentz transformation [6]. The runs were carried out for the size computational domain $k = 2$ cm, $l = 2$ cm, $d = 2$ cm, $h = 4$ cm, the electron beam size $L = 1$ cm, the angle between cathode and $x$-axis $100^\circ < \theta < 850^\circ$, the electron beam energy $W = 30 \div 200$ keV, $1 < \gamma < 250$, $0.01$ mm $< a < 2.0$ mm, $b = 60$ mm, $0.01$ mm $< p < 2.0$ mm, proton bunch charge $Q = N \times 1.6 \cdot 10^{-19}$ C, $N = 2 \cdot 10^{10} \div 2 \cdot 10^{11}$.

3. Set up of mathematical model

The equations of motion of electrons in the electromagnetic field created by a relativistic beam of protons have the following form (CGS units):

$$m \frac{dv_x}{dt} = e \gamma E_x + \frac{e}{c} (v_y B_z - v_z B_y),$$

$$m \frac{dv_y}{dt} = e E_y + \frac{e}{c} (v_z B_x - v_x B_z),$$

$$m \frac{dv_z}{dt} = e \gamma E_z + \frac{e}{c} (v_x B_y - v_y B_x),$$

$$\frac{dX}{dt} = v, \quad X = (x, y, z), \quad v = (v_x, v_y, v_z),$$

$$B_x = \beta \gamma E_y, \quad B_y = 0, \quad B_z = \beta \gamma E_x, \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}}.$$  

Here $v_x, v_y, v_z$ are the components of the electron velocity, $e$ is the electron charge, $c$ is a speed of light, $B_x, B_y, B_z$ and $E_x, E_y, E_z$ are the components of the electric field and magnetic field, respectively, created by a space charge of the proton bunch, $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$ is a relativistic factor of the proton beam $\beta = u/c = (1 - 1/\gamma^2)^{1/2}$ is the dimensionless velocity of the proton bunch $u = (0, u_y, 0)$. Taking into account the condition $v_x, v_y \ll v_z$, the equations of electron motion assume the form:

$$m \frac{dv_x}{dt} = e \gamma E_x,$$

$$m \frac{dv_y}{dt} = e E_y \left(1 + \gamma \frac{v_z \beta}{c}\right),$$

$$m \frac{dv_z}{dt} = e \gamma E_z,$$

$$\frac{dx}{dt} = v_x, \quad \frac{dy}{dt} = v_y, \quad \frac{dz}{dt} = v_z.$$  

The charge density of the relativistic proton bunch $\rho$ is determined by the equation:

$$\rho(x, y, z) = \frac{Q}{84abc\pi^{3/2}} \exp \left[- \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{C^2}\right)\right].$$
Velocity \( c = 3 \cdot 10^{10} \text{ cm/s} \) – speed of light

Spatial size \( L_0 = 1 \text{ cm} \)

Time \( t_0 = L_0/c = 0.33 \cdot 10^{10} \text{ s} \)

Electric field \( E_0 = \frac{mc}{et_0} = 1.7 \cdot 10^3 \text{ statvolt/cm} = 5.1 \cdot 10^5 \text{ v/cm} \)

Potential \( \phi_0 = \frac{mc}{\sqrt{\hbar}} = 1.7 \cdot 10^3 \text{ statvolt} = 5.1 \cdot 10^5 \text{ v} \)

Charge density \( \rho_0 = \frac{m}{4\pi et_0^2} = 1.36 \cdot 10^2 \text{ statcoulomb/cm}^3 = 0.045 \text{ Coulomb/m}^3 \)

| Table 1. The values used for normalization |
|-------------------------------------------|

To calculate the electric field created by the spatial charge of a proton beam, the Poisson equations are used:

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -4\pi \rho(x, y, z), \tag{4}
\]

\[
E_x = -\frac{\partial \phi}{\partial x}, \quad E_y = -\frac{\partial \phi}{\partial y}, \quad E_z = -\frac{\partial \phi}{\partial z}.
\]

The planes \( x = \pm k, \ y = \pm l, \ z = -d \) and \( z = h \) are conductive, so the boundary condition \( \phi = 0 \) is assumed. To solve the problem, we use the dimensionless form of all equations. The values used for normalization are listed in Table 1. Next, all formulas are given in dimensionless variables.

Then the system of equations (1)-(4) has the following dimensionless form:

\[
\frac{dv_x}{dt} = -\gamma E_x, \tag{5}
\]

\[
\frac{dv_y}{dt} = -E_y (1 + \gamma \beta v_z), \tag{6}
\]

\[
\frac{dv_z}{dt} = -\gamma E_z, \tag{7}
\]

\[
\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = -\rho. \tag{8}
\]

Note that in the case of a constant electric field \( E(x, y, z) = \text{const} \), the system of equations (5)-(7) has an analytical solution, which is used to solve the equations of motion of an electron beam in the electromagnetic field of a relativistic proton bunch. Let’s consider in detail the algorithm for solving the problem.

a) Algorithm of calculating charge density of proton bunch on the regular grid with nodes \((x_i, y_k, z_l)\) is

\[
x_i = -x_m + (i - 1)h_x, \quad i = 1, \ldots, i_m, \quad h_x = \frac{2x_m}{i_m - 1},
\]

\[
y_k = -y_m + (k - 1)h_y, \quad k = 1, \ldots, k_m, \quad h_y = \frac{2y_m}{k_m - 1},
\]

\[
z_l = -z_m' + (l - 1)h_z, \quad l = 1, \ldots, l_m, \quad h_z = \frac{z_m' + z_m''}{l_m - 1},
\]

\[
\tilde{\rho}_{i,k,l} = \rho(x_i, y_k, z_l) = \frac{Q}{abC} \exp \left[ -\left( \frac{x_i^2}{a^2} + \frac{y_k^2}{b^2} + \frac{z_l^2}{C^2} \right) \right].
\]
All grid functions \((\rho, \phi, E_x, E_y, E_z)\) have 3 indices \((i, k, l)\), corresponding to the position of a point on a three-dimensional grid. To make the algorithm more visual, when describing the algorithm, we will omit the indexes that coincide with \(i, k\) or \(l\). Then the system of linear equations for determining the grid potential \(\phi_{i,k,l}\) has the form shown below.

b) Calculating the potential and electric field are based on formula:

\[
\frac{\phi_{i+1} - 2\phi_i + \phi_{i-1}}{h_x^2} + \frac{\phi_{k+1} - 2\phi_k + \phi_{k-1}}{h_y^2} + \frac{\phi_{l+1} - 2\phi_l + \phi_{l-1}}{h_z^2} + \rho = 0,
\]

\(i = 2, \ldots, i_m - 1, \quad k = 2, \ldots, k_m - 1, \quad l = 2, \ldots, l_m - 1\)

with boundary conditions: \(\phi_{\text{boundary}} = 0\). Upper relaxation method to calculate the potential was used.

\[
\bar{\phi} = \frac{\phi_{n+1}^i + \phi_{n-1}^i + \phi_{n+1}^k + \phi_{n-1}^k + \phi_{n+1}^l + \phi_{n-1}^l}{h_x^2 + h_y^2 + h_z^2} + \rho \left( \frac{2}{h_x^2} + \frac{2}{h_y^2} + \frac{2}{h_z^2} \right)^{-1},
\]

\(\phi^{n+1} = (1 - \omega)\phi^n + \omega\bar{\phi}\),

\(n\) – iteration number, \(\omega\) – iteration parameter \((\omega = 1.93)\). Iterations are performed until the condition is met

\[
|\phi_{i,k,l}^{n+1} - \phi_{i,k,l}^n| < \varepsilon, \quad \varepsilon = 10^{-10},
\]

where \(\varepsilon\) is a calculation accuracy. The obtained potential values are used to find the components of the electric field:

\[
E_{x,i-1/2} = -\frac{\phi_i - \phi_{i-1}}{h_x},
\]

\[
E_{y,k-1/2} = -\frac{\phi_k - \phi_{k-1}}{h_y},
\]

\[
E_{z,l-1/2} = -\frac{\phi_l - \phi_{l-1}}{h_z}.
\]

c) The initial coordinates of the electron beam particles on the plane \((x, y, z = z_{\text{min}})\) are determined as follows:

\[
x_n = A_x n + B_x, \quad y_n = A_y n + B_y, \quad z_n = z_{\text{min}}.
\]

\[
A_x = \frac{x_2 - x_1}{N_p - 1}, \quad B_x = x_1 - A_x,
\]

\[
A_y = \frac{y_2 - y_1}{N_p - 1}, \quad B_y = y_1 - A_y,
\]

where \(N_p\) is a number of trajectories, \(n\) is a trajectory number, \(x_1 = -\frac{L}{2}\cos\theta, y_1 = -\frac{L}{2}\sin\theta, x_2 = \frac{L}{2}\cos\theta, y_2 = -\frac{L}{2}\sin\theta\). Initial velocities of electron beam at the time \(t = 0\) are equal to \((v_x, v_y, v_z) = (0, 0, \sqrt{2W})\).

d) Algorithm to solve the particles motion equations includes next steps:

• interpolation of the electric field components to the particle location \([7]\):

\[
E(x^m, y^m, z^m) \equiv \{E_x, E_y, E_z\};
\]
• calculation of the electron beam velocity based on the use of analytical solutions of the equations of motion obtained under the condition that the electric field is constant at each time step \( \tau \):

\[
\begin{align*}
    v_{x}^{m+1/2} &= v_{x}^{m-1/2} - \gamma E_{x} \tau, \\
    v_{y}^{m+1/2} &= v_{y}^{m-1/2} - E_{y} \left(1 + \gamma \beta v_{z}^{m-1/2}\right) \tau + \frac{1}{2} \gamma^{2} \beta E_{y} E_{z} \tau^{2}, \\
    v_{z}^{m+1/2} &= v_{z}^{m-1/2} - \gamma E_{z} \tau;
\end{align*}
\]

• calculation of new coordinates and time

\[
\begin{align*}
    x^{m+1} &= x^{m} + \tau v_{x}^{m+1/2}, \\
    y^{m+1} &= y^{m} + \tau v_{y}^{m+1/2}, \\
    z^{m+1} &= z^{m} + \tau v_{z}^{m+1/2}, \\
    t^{m+1} &= t^{m} + \tau,
\end{align*}
\]

where \( \tau \) is the time step, \( m \) is the number of the time step. When the particle reaches the boundary of the calculation area \( (z^{m+1} > z_{\text{max}}) \) the coordinates and time of the intersection of the trajectory with the \( z = z_{\text{max}} \) calculated \( t = t_{\text{end}} \):

\[
\begin{align*}
    s &= \frac{h_{z} - z^{m}}{z^{m+1} - z^{m}}, \\
    x_{\text{end}} &= x^{m} + s(x^{m+1} - x^{m}), \\
    y_{\text{end}} &= y^{m} + s(y^{m+1} - y^{m}), \\
    t_{\text{end}} &= t^{m} + \tau s.
\end{align*}
\]

We developed this method following the co-design approach [8],[9]. The algorithm of solving the particle motion equations has a high potential for vectorization on modern Intel and AMD processors with enough data size. At this moment, our tests from section 4 were done on a personal workstation, but our future work will be focused on parallel CPU code with deep vectorization.

4. Results of computer simulation

Let us consider the results of computer simulation of the interaction of the testing ribbon electron beam and the relativistic proton bunch.

The trajectories of deflection electron probe beam by proton bunch with \( \gamma = 100 \) and number of proton \( N = 2 \cdot 10^{11}, Q = 3.2 \cdot 10^{-8} C \) is shown in Fig. 2. The proton bunch has the dimensions: \( a = 0.1 \text{ cm}, b = 6.0 \text{ cm}, p = 0.05 \text{ cm} \). The parameters of electron beam are the next: \( W = 50 \text{ keV}, \theta = 45^o \) - (a); \( W = 100 \text{ keV}, \theta = 45^o \) - (b); \( W = 100 \text{ keV}, \theta = 20^o \) - (c). The number of injected electron beamlets is \( N_{p} = 140 \), number of presented trajectories is \( N_{tr} = 30 \). The projections of the electron beam on the cathode plane \( (z = -2 \text{ cm}) \) and the plane of the
Figure 2. 3D trajectories of electron beam.
luminescent screen ($z = 4$ cm) are represented by blue lines. These graphs show that with an increase in the energy of the electron beam $W$, there is a decrease in the amplitude of its deviation $\Delta x$ by the relativistic proton bunch (Fig. 2 a, b). From the comparison of Fig. 2 a and Fig. 2 c, it is seen that a decrease in the angle of inclination of the electron beam to the $x$-axis is accompanied by a decrease in the amplitude of its deviation $\Delta x$ in the electromagnetic field created by a relativistic proton beam. Tracks of deflected electron beam on the luminescent screen $z = 4$ cm is shown in Fig. 3.

In the considered cases, the energy of the electron beams is equal to 100 keV, the curves indicated by the numbers 1, 2, 3, 4, 5 correspond to the angle of inclination $\theta = 20^\circ, 30^\circ, 45^\circ, 60^\circ, 70^\circ$, accordingly. The dependence of the electron beam deflection size $\Delta y$ on the angle of inclination $\theta$ is shown in Fig. 4. In this way, with an increase in the angle of inclination of the cathode, the resolution of measuring the small transverse dimensions of the proton bunch can be significantly improved. The results of computer simulation have shown that the amplitude of deviation $\Delta x$ is proportional to the charge of proton bunch $Q$ and the $\gamma$-factor of proton bunch. For example, when $Q = 1.6 \cdot 10^{-8}C$, $3.2 \cdot 10^{-8}C$, $4.8 \cdot 10^{-8}C$, the
amplitudes of deviation $\Delta x$ are 0.15 cm, 0.29 cm, 3.1 cm, respectively. The dependence of the deviation $\Delta x$ on the $\gamma$-factor of proton bunch is shown in Fig. 5 ($Q = 3.2 \cdot 10^{-8} C$). Thus, based on the obtained results of numerical modeling, it is possible to determine the parameters of the proton bunch by the displacement of the test electron beam on a screen.

5. Conclusion

3D numerical model of the interaction process of the testing electron beam and the proton bunch based on PIC method is created. The dependence of the electron beam deflection in the electromagnetic field of a relativistic proton beam on the electron energy and the angle of inclination to the direction of the proton bunch is investigated. It is shown that the amplitude of the electron beam deflection is proportional to the charge and gamma factor of the proton bunch and inversely proportional to the energy of the electron beam. The obtained characteristics of the electron projection on the screen plane can be used to determine the main parameters of the relativistic proton bunch.

Acknowledgments

The work was carried out under the state contract with ICMMG SB RAS 0251-2021-0005.

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