Back-reaction instabilities of relativistic cosmic rays

A K Nekrasov

Institute of Physics of the Earth, Russian Academy of Sciences, 123995 Moscow, Russia
E-mail: anekrasov@ifz.ru and nekrasov.anatoly@gmail.com

Received 9 December 2012, in final form 2 April 2013
Published 5 June 2013
Online at stacks.iop.org/PPCF/55/085007

Abstract
We explore streaming instabilities of an electron–ion plasma with relativistic and ultra-relativistic cosmic rays in the background magnetic field using the multi-fluid approach. Cosmic rays can be both electrons and ions. The drift speed of cosmic rays is directed along the magnetic field. In equilibrium, the return current of the background plasma is taken into account. One-dimensional perturbations parallel to the magnetic field are considered. The dispersion relations are derived for transverse and longitudinal perturbations. It is shown that the back-reaction of magnetized cosmic rays generates new instabilities one of which has a growth rate that can approach the growth rate of the Bell instability. These new instabilities can be stronger than the cyclotron resonance instability. For unmagnetized cosmic rays, the growth rate is analogous to the Bell one. We compare two models of the plasma return current in equilibrium with three and four charged components. Some differences between these models are demonstrated. For longitudinal perturbations, an instability is found in the case of ultra-relativistic cosmic rays. The results obtained can be applied to investigation of astrophysical objects such as the shocks by supernova remnants, galaxy clusters, intracluster medium and so on, where interaction of cosmic rays with turbulence of the electron–ion plasma produced by them is of great importance for cosmic-ray evolution.

1. Introduction

It has been known for a long time that a return current arises in a plasma penetrated by an external beam current [1]. A theory of this phenomenon for the laboratory plasma has been developed in a number of previous papers [1–5]. It was shown that the induced plasma current depends on the spatio-temporal shape of the imposed current and is transferred by plasma species. For external currents of cylindrical shape, it has been found that the return current is nearly equal to the imposed beam current and lies almost entirely within the beam channel [2–4]. In a bound magnetized plasma with a given nonstationary sheet current, the return current can change with time and be not equal to the external current [5]. However, inclusion of the surface current in the perfectly conducting walls results in the full compensation of both currents [5].

The return currents in astrophysics are considered for media where cosmic rays are present. It is assumed that in equilibrium the total current of cosmic rays and plasma is equal to zero. Models are explored, in which the equilibrium current is directed along [6–8] and across [9, 10] the background magnetic field. In the case of currents parallel to the magnetic field, one considers a three-component medium, consisting of the electrons, ions and cosmic rays, where each species has its own drift velocity [6], as well as a four-component one. In the last case, one assumes that the background plasma has no drift velocities while cosmic rays and an additional electron component (for the proton cosmic rays) drift together [7, 8, 11].

The kinetic consideration of cosmic rays drifting along the magnetic field has been provided by Achterberg [6], Zweibel [7], Bell [8] and Reville et al [12] also for perturbations parallel to the magnetic field. The well-known non-resonant Bell instability [8] has a large growth rate for perturbation wavelengths shorter than the mean Larmor radius of cosmic-ray protons defined by their longitudinal momentum. In this case, the contribution of cosmic rays to the kinetic dispersion relation is small [7, 8] and the instability is due to the electron return current. Thus, the back-reaction of cosmic rays is absent in unstable short-wavelength perturbations mentioned above. In the opposite case of long-wavelength perturbations, the perturbed currents of cosmic rays (protons) and of additional electrons compensate each other, if only the perturbed electric drift of particles is taken into account [7].
However, involving the Doppler-shifted polarizational current (back-reaction) of cosmic rays is also important for cosmic-ray streaming instabilities. This effect was not considered in [7, 8]. As we show here, the back-reaction of magnetized cosmic rays gives rise to new streaming instabilities, one of which has a growth rate of the order of that of the Bell instability [8] in the vicinity of the instability threshold and less far from it. However, in the long-wavelength spectral part, for example, these new instabilities can be more powerful in comparison with the cyclotron resonance instability.

In this paper, we investigate streaming instabilities of an electron–ion plasma in the background magnetic field with cosmic rays up to ultra-relativistic energies using the multi-fluid approach. We assume that cosmic rays, which can be both protons and electrons, drift along the magnetic field. One-dimensional perturbations also parallel to the magnetic field are treated. In this case, transverse and longitudinal movements are split. For generality, we take into account the thermal energy exchange between background electrons and ions and the electron thermal conductivity. We derive dispersion relations for the transverse and longitudinal perturbations. For the first case, two models with three and four components described above are used and the corresponding results are compared. (Analogous consideration of these models for shocks has been provided by Amato and Blasi [13].) New instabilities due to the back-reaction of relativistic cosmic rays are found.

The paper is organized as follows. Section 2 contains the fundamental equations for plasma, cosmic rays and electromagnetic fields. Equilibrium state is discussed in section 3. In section 4, the transverse perturbations with magnetized and unmagnetized cosmic rays are explored. We investigate longitudinal perturbations in section 5. In section 6, we discuss results obtained in the preceding sections. Concluding remarks are given in section 7.

2. Basic equations for plasma and cosmic rays

The fundamental equations for the plasma that we consider here are the following:

\[
\frac{\partial v_j}{\partial t} + v_j \cdot \nabla v_j = -\frac{\nabla p_j}{m_j n_j} + \frac{q_j}{m_j} E + \frac{q_j}{m_j c} v_j \times B + C_j,
\]

the equation of motion, \( \frac{\partial n_j}{\partial t} + \nabla \cdot n_j v_j = 0 \),

the continuity equation, \( \frac{\partial T_e}{\partial t} + v_j \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot v_j = v_{ei} (n_e, T_e) \) \((T_e - T_i)\)

and

\[
\frac{\partial T_e}{\partial t} + v_e \cdot \nabla T_e + (\gamma - 1) T_e \nabla \cdot v_e = - (\gamma - 1) \frac{1}{n_e} \nabla \cdot q_e - v_{ei} (n_i, T_e) \) \((T_e - T_i)\) \]

are the temperature equations for ions and electrons. In equations (1) and (2), the subscript \( j = i, e \) denotes ions and electrons, respectively. The notation in equations (1)–(4) is as follows: \( q_j \) and \( m_j \) are the charge and mass of species \( j = i, e \); \( u_j \) is the hydrodynamic velocity; \( n_j \) is the number density; the terms \( C_e = -v_{te}(v_e - v_i) \) and \( C_i = -v_{ti}(v_i - v_e) \) take into account the collisional momentum exchange between electrons and ions, where \( v_{te} (v_{ti}) \) is the electron (ion)–ion (electron) collision frequency; \( p_j = n_j T_j \) is the thermal pressure; \( T_j \) is the temperature; \( v_{ei} (n_e, T_e) \) is the frequency of the thermal energy exchange between ions (electrons) and electrons (ions) being \( v_{ei} (n_e, T_e) ) = 2n_{te} \) [14]; \( n_e v_e^2 (n_e, T_e) = n_i v_{ti}^2 (n_i, T_i) \); \( \gamma \) is the ratio of the specific heats; \( E \) and \( B \) are the electric and magnetic fields and \( c \) is the speed of light in vacuum. We include the thermal exchange between electrons and ions because \( v_{ei} (n_e, T_e) ) = 2n_{te} \) must be compared with the dynamical frequency. The value \( q_e \) in equation (4) is the electron heat flux [14]. In a weakly collisional plasma which is considered here, the electron Larmor radius is much smaller than the electron collisional mean free path. In this case, the electron thermal flux is mainly directed along the magnetic field,

\[
q_e = -\chi_e b (b \cdot \nabla) T_e,
\]

where \( \chi_e \) is the electron thermal conductivity coefficient and \( b = B/B \) is the unit vector along the magnetic field. We assume that the thermal flux in equilibrium is absent.

We take the equations for relativistic cosmic rays in the form [15]

\[
\frac{\partial (p_{cr} v_{cr})}{\partial t} + v_{cr} \cdot \nabla (p_{cr} v_{cr}) = - \nabla p_{cr} \frac{n_{cr}}{n_e} + q_{cr} \left( \frac{E}{c} + v_{cr} \times B \right),
\]

\[
\left( \frac{\partial}{\partial t} + v_{cr} \cdot \nabla \right) \left( \frac{p_{cr} T_{cr}^{\gamma_{cr}}}{n_{cr}} \right) = 0,
\]

where

\[
R_{cr} = 1 + \frac{\Gamma_{cr}}{\Gamma_{cr} - 1} \frac{T_c}{T_{cr}}.
\]

In these equations, \( p_{cr} = \gamma_{cr} m_{cr} v_{cr} \) is the momentum of a cosmic-ray particle having the rest mass \( m_{cr} \) and velocity \( v_{cr} \); \( q_{cr} \) is the charge; \( p_{cr} \gamma_{cr} = n_{cr} T_{cr} \) is the kinetic pressure; \( n_{cr} \) is the number density in the laboratory frame; \( \Gamma_{cr} \) is the adiabatic index; \( \gamma_{cr} = (1 - v_{cr}^2/c^2)^{-1/2} \) is the relativistic factor. The continuity equation is the same as equation (2) for \( j = cr \).

Equation (8) can be used for both cold non-relativistic, \( T_{cr} \ll m_{cr} c^2 \), and hot relativistic, \( T_{cr} \gg m_{cr} c^2 \), cosmic rays. In the first (second) case, we have \( \Gamma_{cr} = 5/3 \) (4/3) [15]. The general form of the value \( R_{cr} \) applied to any relation between \( T_{cr} \) and \( m_{cr} c^2 \) can be found, e.g., in [16, 17].

Equations (1)–(4), (6) and (7) are solved together with Maxwell’s equations

\[
\nabla \times E = -\frac{1}{c} \frac{\partial B}{\partial t},
\]

and

\[
\nabla \times B = \frac{4\pi}{c} j + \frac{1}{c} \frac{\partial E}{\partial t},
\]

where \( j = j_{pl} + j_{cr} = \sum_j q_j n_j v_j + j_{cr} \).
3. Equilibrium state

We will consider a uniform plasma embedded in the uniform magnetic field \( B_0 \) (subscript 0 here and below denotes background parameters) directed along the \( z \)-axis. We assume that in equilibrium the plasma is penetrated by a uniform beam of cosmic rays having a uniform streaming velocity \( v_{cr0} \) along the \( z \)-axis. The return plasma current along this axis compensating for the current of cosmic rays is provided by the streaming velocities of electrons, \( v_{e0} \), and ions, \( v_{i0} \). The quasi-neutrality is satisfied due to cosmic-ray charge neutralization from the background environment \([18]\). Thus, we have two equations in equilibrium:

\[
q_en_{e0}v_{e0} + q_in_{i0}v_{i0} + q_{cr}n_{cr0}v_{cr0} = 0
\]

and

\[
q_en_{e0} + q_{i0}n_{i0} + q_{cr}n_{cr0} = 0.
\]

Such a three-component model corresponds to the one considered by Achterberg \([6]\). In papers by Zweibel \([7]\) and Bell \([8]\), a four-component model has been explored, in which plasma species are immobile and the additional electrons (in the case of proton cosmic rays) have the cosmic-ray number density and drift with the cosmic-ray drift velocity. We show below that there are some differences between these two models.

4. Transverse perturbations

We will treat one-dimensional perturbations along the background magnetic field. From equations (9) and (10), it follows that in this case the transverse and longitudinal background magnetic field. From equations (9) and (10), we will treat one-dimensional perturbations along the \( B_0 \) magnetic field.

\[
\text{We will consider a uniform plasma embedded in the uniform background parameters) directed along the z-axis. The return plasma current along this axis compensating for the current of cosmic rays is provided by the streaming velocities of electrons, } v_{e0}, \text{ and ions, } v_{i0}. \text{ The quasi-neutrality is satisfied due to cosmic-ray charge neutralization from the background environment} [18]. \text{ Thus, we have two equations in equilibrium:}
\]

\[
q_en_{e0}v_{e0} + q_in_{i0}v_{i0} + q_{cr}n_{cr0}v_{cr0} = 0
\]

and

\[
q_en_{e0} + q_{i0}n_{i0} + q_{cr}n_{cr0} = 0.
\]

Such a three-component model corresponds to the one considered by Achterberg [6]. In papers by Zweibel [7] and Bell [8], a four-component model has been explored, in which plasma species are immobile and the additional electrons (in the case of proton cosmic rays) have the cosmic-ray number density and drift with the cosmic-ray drift velocity. We show below that there are some differences between these two models.

4.1. Magnetized species

We first consider equation (16) in the case in which all species are magnetized, i.e.

\[
\omega^2_{ij} \gg D^2_{ij},
\]

\[
\omega^2_{cr} \gg D^2_{cr},
\]

where \( \omega_{ij,cr} = q_{ij,cr}B_0/m_{ij,cr}c \) is the cyclotron frequency, \( D_{ij} = \gamma_{cr}R_{cr0}D_{cr} \), \( D_{ij,cr} = -i\omega + ik_zv_{ij,cr} \). Using conditions (17), we calculate the values \( \epsilon_{plex,y} \) and \( \epsilon_{crxx,y} \) given by equations (A16) and (B11), respectively, and substitute them into equation (15). Then from equation (16), we derive the following dispersion relation:

\[
k_z^2c^2 + \sum_j \frac{\omega_{pj}^2}{\omega_{cj}^2} \epsilon_{crxx} = \gamma_{cr0}R_{cr0} \omega^2_{per}/\omega^2_{cr}.
\]

In equation (18), we have neglected the contribution of the displacement current and small terms proportional to \( D_{ij,cr}^2/\omega_{crxx} \). According to equations (11) and (12), the right-hand side of equation (18) is equal to zero. Thus, we obtain \( \alpha_v (\omega - k_zv_{e0})^2 + \alpha_i (\omega - k_zv_{i0})^2 + \alpha_{cr} (\omega - k_zv_{cr0})^2 = k_z^2c^2 \),

\[
\text{where } \alpha_j = \omega_{pj}^2/\omega_{cj}^2 \text{ and } \alpha_{cr} = \gamma_{cr0}R_{cr0}\omega^2_{per}/\omega^2_{cr}.
\]

The solution of equation (19) is given by

\[
\omega = \frac{A_2}{A_1} k_z \pm \frac{1}{A_1} k_z \left( A_2^2 - A_1 A_3 \right)^{1/2},
\]

where

\[
A_1 = \alpha_v + \alpha_i + \alpha_{cr},
\]

\[
A_2 = \alpha_v v_{e0} + \alpha_i v_{i0} + \alpha_{cr} v_{cr0},
\]

\[
A_3 = \alpha_v v_{e0}^2 + \alpha_i v_{i0}^2 + \alpha_{cr} v_{cr0}^2 - c^2.
\]

Using equation (21), we find the expression for \( A_2^2 - A_1 A_3 \):

\[
A_2^2 - A_1 A_3 = A_1 c^2 - \alpha_v a_1 (v_{e0} - v_{i0})^2 - \alpha_i a_{cr} (v_{cr0} - v_{i0})^2 - \alpha_{cr} a_{cr} (v_{cr0} - v_{e0})^2.
\]

Equation (20) describes the streaming instability if \( A_2^2 - A_1 A_3 < 0 \).

The number density of cosmic rays is considerably smaller than the number density of the background plasma. Therefore, we can conclude from equation (11) that \( v_{cr0} \ll v_{e0} \). In this case, equation (22) can be written in the form

\[
A_2^2 - A_1 A_3 \simeq (\alpha_v + \alpha_{cr}) c^2 - \alpha_i a_{cr} v_{cr0}^2.
\]

The growth rate of instability \( \delta = \text{Im} \omega \) found from equation (20) in the case \( \alpha_i a_{cr} v_{cr0}^2 \gg (\alpha_v + \alpha_{cr}) c^2 \) is equal to

\[
\delta = \left( \frac{\alpha_i a_{cr}}{\alpha_v + \alpha_{cr}} \right)^{1/2} k_z v_{cr0}.
\]

This new instability arises due to the cosmic-ray back-reaction, i.e. due to the same dynamics of cosmic rays as that of the...
plasma connected with the polarizational drift of the species (see equation (18)).

For a four-component model consisting of the background ions and electrons without drift velocities, proton cosmic rays and additional electrons with the cosmic-ray number density and drift velocity [7], equation (18) has the solution
\[ \omega = -\frac{\alpha_{cr}}{\alpha_i + \alpha_{cr}} k_z v_{e0} \pm \frac{k_z}{\alpha_i + \alpha_{cr}} \left[ -\alpha_i \alpha_{cr} v_{e0}^2 + (\alpha_i + \alpha_{cr}) c^2 \right]^{1/2}. \]

We see that this solution gives the same growth rate as that given by expression (24) (see equation (23)). However, the real frequency (or the phase velocity) for a four-component model is different from that for a three-component one.

4.2. Unmagnetized cosmic rays

In this section, we assume that the cosmic rays are unmagnetized:
\[ D_{cr}^2 \gg \omega_{cr}^2. \] (25)

If \( \omega_{cr}^2 \gg (\omega - k_z v_{e0})^2 \), this condition can be satisfied for relativistic cosmic rays for which \( \gamma_{cr} R_{cr} \gg 1 \). Then, we obtain
\[ \epsilon_{crxx} = -\frac{\omega_{per}^2}{\gamma_{cr} R_{cr} \omega_{cr}^2}, \] (26)
\[ \epsilon_{crxy} = -\frac{\omega_{per}^2 \omega_{cr}}{\gamma_{cr} R_{cr} D_{cr} \omega_{cr}^2}. \]
The plasma ions and electrons stay magnetized. By substituting equation (26) and \( \epsilon_{p,xx,xy} \) into equation (16), we will have
\[ k_z^2 c^2 - \alpha \omega (\omega - k_z v_{e0})^2 = \pm \beta_{cr} (\omega - k_z v_{e0}), \] (27)
where \( \beta_{cr} = \omega_{per}^2 / \omega_{cr} \). For obtaining the right-hand side of this equation, we have used equations (11) and (12). We note that equation (27) does not contain the contribution of the cosmic-ray perturbed dynamics, which is small in comparison with the plasma current produced by the electric drift velocities of ions and electrons. The solution of equation (27) is given by
\[ \omega = \frac{1}{\alpha_i} \left( \alpha_i k_z v_{e0} + \alpha k_z v_{i0} + \frac{\beta_{cr}}{2} \right) \pm \frac{1}{\alpha_i} \left[ \pm \alpha_i \beta_{cr} k_z v_{e0} - \alpha \omega (\omega - k_z v_{e0})^2 \right]^{1/2}, \] (28)

From equations (11) and (12), it follows that \( v_{e0} - v_{i0} \simeq (q_{eR_{cr}}/q_{iR_{cr}}) v_{i0} / v_{e0} \) (\( q_i = -q_e \)). An estimation of the ratio of the second term in the squared brackets in equation (28) to the first one gives the value \((n_{eR_{cr}}/n_{iR_{cr}})(k_z v_{e0}/\omega_{cr})\), which is generally speaking much smaller than unity.

The solution of the dispersion relation for the four-component medium considered above is the following:
\[ \omega = \frac{\beta_{cr}}{2 \alpha_i} \pm \frac{1}{\alpha_i} \left[ \pm \alpha_i \beta_{cr} k_z v_{e0} - \alpha \frac{\beta_{cr}}{\alpha_{cr}} k_z^2 v_{e0}^2 \right]^{1/2}, \] (29)
where the sign \( \pm \) denotes an absolute value. We see some differences between equations (28) and (29). The growth rates are the same (neglecting the small terms), while the phase velocities are different for the two models.

Equation (29) applied to the proton cosmic rays coincides with equation (8) given in the paper by Zweibel and Everett [11], if we neglect the term proportional to \( v_i^2 \) (assuming that \( k_z v_{i0} \ll |\omega_{cr}| \) and take the lower sign (see also [7, 8]). This coincidence is due to the absence of the dynamical contribution of unmagnetized cosmic rays to the dispersion relation (27) as also in the case considered in [7, 8, 11]. However, conditions of unmagnetization are different in both cases. In our one-dimensional magnetohydrodynamic case, the transverse perturbations of cosmic rays do not contain the thermal pressure, and the condition of unmagnetization has the form (25). At the same time, the kinetic consideration shows that cosmic rays are also unmagnetized in perturbations with wavelengths much smaller than their Larmor radius defined by the thermal velocity along the magnetic field [7, 8]. Thus, in both limiting cases, the back-reaction of cosmic rays is negligible, which results in the same growth rates of instability due to the return plasma current.

We note that if we set \( v_{e0} = 0 \) in equation (28) (or on the left-hand side of equation (27)), we return to equation (29) without the term \( \propto v_{e0}^2 \).

5. Longitudinal perturbations

5.1. Dispersion relation

We now consider potential perturbations along the background magnetic field. The wave equation is the following (see equation (10)):
\[ 4\pi j_{l1} + \partial E_{l1} / \partial t = 0. \] (30)

In appendices A and B, the plasma, \( j_{pl1} \), and cosmic-ray, \( j_{cr1} \), perturbed currents (equations (A17) and (B12), respectively) are obtained. Substituting them into equation (30) and the Fourier transformation lead to the dispersion relation
\[ 0 = D \left[ \frac{\omega_{per}^2}{D_{te}} \left( L_{i1} D_{te} - L_{z2} D_{ti} q_{m_e} / q_{m_i} \right) \right] + \frac{\omega_{per}^2}{D_{ti}} \left( L_{i1} D_{te} - L_{z2} D_{ti} q_{m_i} / q_{m_e} \right) + \frac{\omega_{cr}^2}{L_{cr}} + 1, \] (31)
where \( \partial / \partial t = -i \omega \) and \( \partial / \partial z = ik_z \). This equation will be treated in the limiting cases.

5.2. Cold electrons and ions

We first consider the cold plasma species for which
\[ D_{te} (D_{te} + v_{e0}) \gg k_z^2 T_{0}, \] (32)
\[ D_{ti} (D_{ti} + v_{i0}) \gg k_z^2 T_{0}, \]
For cosmic rays, we assume here and below that the following condition is satisfied:
\[ D_{cr}^2 \gg \frac{\Gamma_{cr} T_{e0}}{\gamma_{cr} R_{cr} E_{cr0} m_{cr}} k_z^2, \] (33)
where \[ E_{cr0} = R_{cr0} - \frac{\Gamma_{cr} T_{cr0} v_{cr0}^2}{m_{cr} c^2}. \]

In this case, the first term on the right-hand side of equation (B13) is dominant. We note that the temperature of cosmic rays can be relativistic, i.e. \( T_{cr0} / m_{cr} c^2 \gg 1 \). Then, using equations (A6), (A8), (A10), (A12) and (B13) under conditions defined by equations (32) and (33), we obtain equation (31) in the form

\[
0 = \frac{1}{(D_{l} D_{e} + D_{t} v_{t e} D_{l} v_{t e})} \left( \frac{\alpha_{pi}^2}{D_{l} D_{e}} D_{ti} + \frac{\alpha_{pi}^2}{D_{l} D_{e}} D_{te} \right) + \frac{\omega_{pi}^2}{\gamma_{cr0} E_{cr0} D_{tcr}^{3}},
\]

(34)

where for simplicity we have neglected unity.

5.2.1. Collisionless case. We now assume that \( D_{tcr} \gg v_{ei,cr0} \).

Then equation (34) takes the form

\[
\frac{\omega_{pe}^2}{(\omega - k_z v_{cr0})^2} + \frac{\omega_{pi}^2}{(\omega - k_z v_{cr0})^2} + \frac{\omega_{per}^2}{\gamma_{cr0} E_{cr0} (\omega - k_z v_{cr0})^2} = 0.
\]

(35)

In the vicinity of \( \omega \approx k_z v_{cr0} \), when the back-reaction of cosmic rays plays a role, the solution of equation (35) is the following:

\[
\omega = k_z v_{cr0} \left( 1 + i \frac{\gamma_{cr0}^{1/2} E_{cr0}^{1/2} \omega_{per}}{\omega_{pe}} \right).
\]

(36)

In the region \( \omega \approx k_z v_{cr0} \), equation (35) gives

\[
\omega = k_z v_{cr0} + i \left( \frac{m_{e}}{m_{i}} \right)^{1/2} k_z |v_{i0} - v_{e0}|,
\]

(37)

where \( v_{i0} - v_{e0} \approx - (n_{cr0} / n_{i0}) v_{cr0} \) (see (11)). The ratio of the growth rate defined by equation (37) to that of equation (36) is equal to \( \gamma_{cr0}^{1/2} E_{cr0} (m_{e} / m_{i})(n_{cr0} / n_{i0}) \). This value can be much less than unity even at \( \gamma_{cr0} \gg 1 \) and \( T_{cr0} \gg m_{cr} c^2 \).

5.2.2. Collisional case. In the collisional case \( v_{cr0} \gg D_{tcr} \),

we find from equation (34) the solution in the region \( \omega \approx k_z v_{cr0} \):

\[
\omega = k_z v_{cr0} + \left( \frac{-1 + i}{\sqrt{2}} \right) (v_{cr0} k_z v_{cr0})^{1/2} \gamma_{cr0}^{-3/2} E_{cr0}^{-1/2} \frac{\omega_{per}}{\omega_{pe}}.
\]

(38)

Thus, the back-reaction of relativistic cosmic rays can result in an instability of potential perturbations.

5.3. Hot electrons and cold/hot ions

Consideration shows that in the cases \( D_{tcr} (D_{tcr} + v_{cr0}) \ll (T_{0} / m_{e}) k_{e0}^2 \), \( D_{l} (D_{l} + v_{l e}) \gg (T_{0} / m_{i}) k_{i0}^2 \) and \( D_{l} (D_{l} + v_{l e}) \ll (T_{0} / m_{e}) k_{e0}^2 \), the frequency \( \omega \) is of the order of \( k_z v_{cr0} \) as that in equations (36) and (40). Equation (33) results in the condition \( \gamma_{cr0} \gg 1 \) when \( v_{cr0} \approx c \). Thus, the temperature of the background plasma should be relativistic. However, this contradicts the basic equations, where a plasma is a non-relativistic one. Therefore, conditions for hot plasma are invalid. Taking into account other terms in equation (B13) does not give an instability.

6. Discussion

We now discuss the growth rates and conditions of their derivation for transverse perturbations considered in section 4. For magnetized cosmic rays obeying condition (17), the growth rate is given by equation (24). Below, we assume that ions and cosmic rays are protons. Let us first consider the case in which \( \alpha_{e} \gg \alpha_{cr} \) or

\[
1 \gg \gamma_{cr0} R_{cr0} n_{cr0} n_{i0} / n_{e0}.
\]

(39)

Then, the condition of instability can be written in the ‘soft’ form

\[
\gamma_{cr0} R_{cr0} n_{cr0} n_{e0} / n_{i0} \lesssim c_{Al}^2 / v_{cr0}.
\]

(40)

where \( c_{Al} = (B_0^2 / 4 \pi n_{cr0} m_{e})^{1/2} \) is the ion Alfvén velocity (see equation (23)). The growth rate is equal to

\[
\delta = \left( \gamma_{cr0} R_{cr0} n_{cr0} n_{e0} / n_{i0} \right)^{1/2} k_z v_{cr0}.
\]

(41)

This growth rate increases with the wave number \( k_{\parallel} \). However, the value \( k_{\parallel} \) is limited from above by the condition of magnetization (17). For cosmic rays, this condition has the form

\[
\frac{\omega_{cr}^2}{\gamma_{cr0} R_{cr0} v_{cr0}} \gg k_{\parallel}^2.
\]

If we set, for estimation,

\[
k_{\parallel max} \sim \frac{\alpha_{ci}}{\gamma_{cr0} R_{cr0} v_{cr0}}.
\]

and substitute this value into expression (41), we obtain the maximal growth rate \( \delta_{max} \):

\[
\delta_{max} \sim \frac{\alpha_{ci}}{\gamma_{cr0} R_{cr0} v_{cr0}} \left( \frac{1}{n_{cr0} n_{i0}} \right)^{1/2}.
\]

(42)

We note that according to condition (40), \( \delta_{max} \lesssim \delta_{Bell} \) and \( k_{\parallel max} \lesssim k_{Bell} \), where

\[
\delta_{Bell} = \frac{1}{2} \alpha_{ci} \frac{n_{cr0} v_{cr0}}{n_{i0} c_{Al}}
\]

and

\[
k_{Bell} = \frac{1}{2} \alpha_{ci} n_{cr0} v_{cr0} / n_{i0} c_{Al}^2
\]

(43)
are the growth rate and the wave number of the fastest growing mode for the Bell instability [8, 11]. From equations (20) and (21), we see that \( \text{Re} \omega \lesssim \delta \).

The case \( \alpha \varepsilon \ll \alpha \), or

\[
\gamma \varepsilon \rho \varepsilon \frac{n_{\varepsilon 0}}{n_{00}} \gg 1
\]  

(42)

can be satisfied for ultra-relativistic cosmic rays for which \( \gamma \varepsilon \rho \varepsilon \gg 1 \) and/or \( R \varepsilon \rho \varepsilon \gg 1 \). In the last case, the temperature of the cosmic rays is relativistic, \( T \varepsilon \varepsilon \gg m_{\varepsilon}c^2 \). The ‘soft’ condition of instability has the form

\[
v_{\varepsilon \varepsilon 0}^2 \lesssim c_{\varepsilon 0}^2. \tag{43}
\]

The growth rate is equal to

\[
\delta = \left( \frac{1}{\gamma \varepsilon \rho \varepsilon \gamma \varepsilon \rho \varepsilon \frac{n_{\varepsilon 0}}{n_{00}}} \right)^{1/2} \frac{n_{00}}{n_{\varepsilon 0}} k \varepsilon \varepsilon \rho \varepsilon. \tag{44}
\]

In the case under consideration, we have \( \text{Re} \omega = k \varepsilon \varepsilon \rho \varepsilon \gg \delta \) (see equations (20) and (21)). Thus, we find from (17) the upper limit for \( k \varepsilon \varepsilon \rho \varepsilon \):

\[
\delta_{\max} = \frac{\alpha}{l_{\varepsilon 0}} \frac{1}{n_{\varepsilon 0}}. \tag{32}
\]

From conditions (42) and (43), it follows that \( \delta_{\max} \ll \delta_{\text{Bell}} \) and \( k_{\max} \ll k_{\text{Bell}} \).

In the case \( \alpha \varepsilon \sim \alpha \varepsilon \), solution (20) takes the form

\[
\omega = \frac{1}{2} (1 + i) k \varepsilon \varepsilon \rho \varepsilon, \tag{45}
\]

when \( v_{\varepsilon \varepsilon 0}^2 \lesssim 2c_{\varepsilon 0}^2 \). The upper limit for \( k \varepsilon \) for solution (45) is the same as for the case (39). Thus,

\[
\delta_{\max} = \frac{1}{2} \alpha \varepsilon \frac{n_{\varepsilon 0}}{n_{00}}. \tag{46}
\]

Let us compare, for example, solution (41) with the growth rate \( \delta_{\text{res}} \) of the cyclotron resonance instability of cosmic rays [29], which is thought to play a crucial role in the early stages of cosmic-ray acceleration in shocks (e.g. [30]). For the real frequency \( \omega = k_{\varepsilon}c_{\varepsilon 0} \), the growth rate \( \delta_{\text{res}} \lesssim k_{\varepsilon}c_{\varepsilon 0} \) for a particular distribution function [7] can be written in the form

\[
\delta_{\text{res}} = \frac{1}{2} \frac{\omega_{\varepsilon}}{n_{\varepsilon 0} n_{00}} \left( \frac{v_{\varepsilon 0}}{c_{\varepsilon 0}} - 1 \right) p_{1} \left( p_{1}/p_{0} \right)^{2}, \tag{47}
\]

where \( p_{1} = m_{\varepsilon} \alpha_{\varepsilon} \varepsilon c_{\varepsilon 0}/k_{\varepsilon} \) and \( p_{0} \) is a typical momentum of cosmic rays. This growth rate has a maximum of the order of the Bell growth rate at \( p_{1} = p_{0} \), when a wavelength of the perturbation is equal to a typical Larmor radius \( \rho_{\varepsilon} = p_{0}/m_{\varepsilon} \alpha_{\varepsilon} \varepsilon c_{\varepsilon 0} \) multiplied by \( 2\pi \). In the long-wavelength part of the spectrum, \( p_{1} \gg p_{0} \) or \( 1 \gg k_{\varepsilon} \rho_{\varepsilon} \), expression (46) becomes

\[
\delta_{\text{res}} = \frac{1}{2} \frac{\omega_{\varepsilon}}{n_{\varepsilon 0} n_{00}} \left( \frac{v_{\varepsilon 0}}{c_{\varepsilon 0}} - 1 \right) k_{\varepsilon} \rho_{\varepsilon}. \tag{48}
\]

The ratio of the growth rate (41) to that of (47) is equal to

\[
\gamma \varepsilon \rho \varepsilon \frac{n_{\varepsilon 0}}{n_{00}} \frac{1}{m_{\varepsilon} c_{\varepsilon 0}} \frac{1}{p_{0}}. \tag{49}
\]

The case \( \delta \gg \delta_{\text{res}} \) results in

\[
4 \gamma \varepsilon \rho \varepsilon \frac{n_{\varepsilon 0}}{n_{00}} \frac{1}{m_{\varepsilon} c_{\varepsilon 0}} \frac{p_{0}^{2}}{m_{\varepsilon}^{2} c_{\varepsilon 0}^{2}}. \tag{50}
\]

Condition (48) can be satisfied. An analogous consideration for solution (44) gives

\[
\delta = \left( \frac{1}{\gamma \varepsilon \rho \varepsilon \gamma \varepsilon \rho \varepsilon \frac{n_{\varepsilon 0}}{n_{00}}} \right)^{1/2} \frac{n_{00}}{n_{\varepsilon 0}} \frac{1}{m_{\varepsilon} c_{\varepsilon 0}} \frac{p_{0}^{2}}{m_{\varepsilon}^{2} c_{\varepsilon 0}^{2}}. \tag{51}
\]

In this case, it is also possible that \( \delta \gg \delta_{\text{res}} \).

For unmagnetized cosmic rays satisfying condition (25) and magnetized background electrons and ions, the solution of equation (27) is given by equation (28). In the case \( v_{\varepsilon \varepsilon 0}^2 > c_{\varepsilon 0}^2 \), to neglect the term \( \rho_{\varepsilon}^{2} \varepsilon \varepsilon \rho \varepsilon \), the growth rate has the form \( \delta_{\text{Bell}} \) (see above). The frequency \( \omega \) is smaller than \( k_{\text{Bell}}v_{\varepsilon \varepsilon 0} \). Thus, condition (25) takes the form

\[
\gamma \varepsilon \rho \varepsilon \frac{n_{\varepsilon 0}}{n_{00}} \frac{v_{\varepsilon 0}^{2}}{c_{\varepsilon 0}^{2}}, \tag{52}
\]

where we have inserted \( k_{\text{Bell}} \). We note that under condition (49) cosmic rays do not contribute to the dispersion relation, i.e. the cosmic-ray back-reactation is absent. In the kinetic consideration, we obtain an analogous result for hot cosmic rays \( p_{1} \varepsilon \varepsilon \gg m_{\varepsilon} \alpha_{\varepsilon} \varepsilon c_{\varepsilon 0}/k_{\varepsilon} \), where \( p_{1} \varepsilon \varepsilon \) is the average momentum along the magnetic field [7, 8]. Substituting the value of \( k_{\text{Bell}} \) into the last condition gives

\[
\frac{n_{\varepsilon 0}}{n_{00}} \frac{v_{\varepsilon 0}^{2}}{c_{\varepsilon 0}^{2}} \frac{p_{1} \varepsilon \varepsilon}{m_{\varepsilon} c_{\varepsilon 0}^{2}} \gg 1. \tag{53}
\]

Let us discuss longitudinal perturbations. In the case of a cold background plasma expressed by condition (32) and under condition (33) for cosmic rays, the solution of equation (34) in the collisionless case is given by equation (36). Condition (32) can be written as \( v_{\varepsilon \varepsilon 0}^{2} \gg T_{0}/m_{\varepsilon} \). For cosmic rays, condition (33) takes the form

\[
\frac{n_{\varepsilon 0}}{n_{00}} \frac{v_{\varepsilon 0}^{2}}{c_{\varepsilon 0}^{2}} \frac{T_{0}}{\gamma \varepsilon \rho \varepsilon m_{\varepsilon} c_{\varepsilon 0}^{2}}. \tag{54}
\]

Condition (50) can be satisfied for ultra-relativistic cosmic rays with \( \gamma \varepsilon \rho \varepsilon \gg 1 \) when \( v_{\varepsilon \varepsilon 0} \approx c \). The growth rate \( \delta \) is the following:

\[
\delta = \frac{1}{2} \frac{\omega_{\varepsilon}}{n_{\varepsilon 0} n_{00}} \left( \frac{v_{\varepsilon 0}}{c_{\varepsilon 0}} - 1 \right) k_{\varepsilon} v_{\varepsilon \varepsilon 0}. \tag{55}
\]

This growth rate is considerably smaller than that given, for example, by equation (41). The collisional growth rate
In our investigation, we have not included collisions in the momentum equation for the transverse perturbations. It can be shown that in the present case it is possible under the condition $\omega \gg \nu_{\text{e}} D_{\text{ci}}^2 / \sigma$ (e.g. [31, 32]) (see also (A15) and (A16)). In the temperature equations, we did not take into account the heating due to viscosity and the Joule heating. These effects can result, in particular, in damping of perturbations [14, 29]. For our model, the resistive damping $\delta_{\text{diss}}$ is equal to $\delta_{\text{diss}} = k_0^2 \rho^2 / 8 \pi \sigma$, where $\sigma = q^2 n_0 / \nu_e m_e$ is the electric conductivity, and the viscous damping is $\delta_{\text{visc}} = 0.6 k_0^2 T_{\text{ci}} v_{\text{ti}} / \omega_k^2 m_1$, where $v_{\text{ti}}$ is the ion–ion collision frequency [14, 29]. In [29], it has been shown that these dampings are negligible in comparison, for example, with the ion–neutral collision damping. We also assume here that the growth rates can exceed the dissipative effects. The presence of the background plasma current in equilibrium can also give rise to other specific instabilities (e.g. [33]). However, all these additional questions are outside the scope of this paper, which is devoted to the effect of back-reaction of streaming cosmic rays.

The streaming instabilities driven by cosmic rays may play a significant role in such phenomena as the shocks caused by supernova remnants [8, 19–22], galaxy clusters [23, 24], intracluster medium [25–28] and so on, where weakly collisional plasma consists mainly of electrons and ions (protons) and where high-energy cosmic rays are present. Therefore, our model and results are applicable to these astrophysical objects. The main point of this investigation is the finding that the back-reaction of magnetized cosmic rays can give rise to instabilities, the growth rate of which can approach that obtained earlier [8]. Although the kinetic derivation of the dispersion relation in [7, 8] contains the dynamics of cosmic rays, the contribution of the latter to the dispersion relation is negligible in the hot regime. The same result is obtained for unmagnetized cosmic rays in the fluid approximation. In these cases, instabilities arise due to the return plasma current.

The exploration carried out in this paper is relevant to the problem of generation of magnetic fields and acceleration of high-energy cosmic rays. At present, it is assumed that acceleration of cosmic rays occurs in supernova remnant shocks due to their multiple crossing of the shock front. This mechanism is known as the first-order Fermi acceleration. The multiple crossing happens as a result of cosmic-ray diffusion on magnetic inhomogeneities in the upstream and downstream regions of the shock being generated by possible instabilities. Such a process as a whole is called diffusive shock acceleration [34–38]. One powerful streaming instability was found by Bell [8], where the unperturbed cosmic-ray current was directed along the magnetic field. In the perturbed state, cosmic rays have been considered as unmagnetized with the Larmor radius defined by the longitudinal velocity much larger than wavelengths of perturbations. In the nonlinear regime, this instability amplifies magnetic fields in the upstream medium of shocks by a factor up to $\sim 10$ larger than typically expected in the interstellar medium [39]. However, x-ray observations [40, 41] show that magnetic fields in the downstream medium are $\sim 100$ times larger than those in the interstellar medium. Therefore, the search for new instabilities has been continued. One possibility using the pre-amplified magnetic fields by the Bell instability has been discussed by Riquelme and Spitkovsky [9]. In this paper, it has been shown that a new instability can arise due to the background cosmic-ray current streaming across the background magnetic field. A growth rate of the same order of magnitude as for the Bell instability has been found. However, Riquelme and Spitkovsky [9] have not considered the back-reaction of cosmic rays in their analytical treatment. In the paper by Nekrasov and Shadmehri [10], we have included the back-reaction of cosmic rays in the multi-fluid approach for the model considered by Riquelme and Spitkovsky [9] and found a growth rate for the streaming instability considerably larger than that of Bell [8] and of Riquelme and Spitkovsky [9] amounting to the square root of the ratio of plasma to cosmic-ray number densities. Therefore, it was of interest to take into account this effect also for the model considered by Bell [8]. For magnetized cosmic rays, we have found new instabilities, one of which has a growth rate comparable to that of Bell in the vicinity of the threshold of instability and smaller far from it in the wavelength region $k_0 \lesssim k_{\text{Bell}}$. Another instability for ultra-relativistic cosmic rays is weaker than the Bell one and excites at $k_0 \ll k_{\text{Bell}}$. Thus, magnetized cosmic rays can also amplify magnetic fields, which results in their diffusion in astrophysical settings. In shock wave fronts, these additional magnetic perturbations will increase the diffusion of cosmic rays and accordingly the efficiency of their acceleration. We have shown in our model that electrostatic perturbations can also be excited by streaming cosmic rays.

The results obtained represent a contribution to the picture of cosmic-ray acceleration studied in previous investigations and of generation of magnetic fields in other astrophysical objects. Taking into account the cosmic-ray back-reaction can be accomplished by making use of the multi-fluid approach, in which all species have their own velocities.

7. Conclusion

Using the multi-fluid approach, we have investigated streaming instabilities of a magnetized electron–ion plasma with relativistic and ultra-relativistic cosmic rays. Cosmic rays are assumed to drift along the background magnetic field. The return current of the background plasma in equilibrium is taken into account. One-dimensional perturbations parallel to the magnetic field are considered. We have derived dispersion relations for the transverse and longitudinal perturbations, whose electric field is polarized across and along the magnetic field, respectively. We have shown that the back-reaction of magnetized cosmic rays in transverse perturbations can result in new instabilities, one of which has a growth rate of the order of that of the Bell instability [8] in the vicinity of the instability threshold and less far from it. However, in the long-wavelength spectral part, for example, these new instabilities can be more powerful in comparison with the cyclotron resonance instability. For unmagnetized cosmic rays, we have obtained a growth rate that is the same as the Bell one. For longitudinal
perturbations, we have found an instability in the case of ultra-relativistic cosmic rays. The corresponding growth rate is less than that for transverse perturbations.

The results obtained can be applied to investigation of astrophysical objects such as supernova remnant shocks, galaxy clusters, intracluster medium, and so on, where interaction of cosmic rays with turbulence of the electron–ion plasma produced by them is of great importance for cosmic-ray scattering and acceleration.

Acknowledgments

The author is grateful to both anonymous referees for their very constructive and useful comments which have helped considerably to improve this paper.

Appendix A.

A.1. Perturbed velocities of ions and electrons

Let us put in equation (1) \( v_j = v_{j0} + v_{j1} \), \( p_j = p_{j0} + p_{j1} \), \( E = E_{j0} + E_{j1} \), \( B = B_{j0} + B_{j1} \). We assume that the medium and background velocities of the species are uniform. Then for perturbations depending only on the \( z \)-coordinate and \( v_{j0} \parallel z \), where \( z \) is the unit vector along the \( z \)-axis, the linearized equation (1) takes the form

\[
D_{j1} v_{j1} = - \nabla T_{j1} \frac{m_j}{T_{j0}} \frac{T_{j0}n_{j1}}{m_jn_{j0}} + F_{j1} + q_{j} \frac{e}{m_j c} v_{j1} \times B_{0},
\]

where we have used that \( p_{j1} = n_{j0} T_{j1} + n_{j1} T_{j0} z \) and introduced the notation \( D_{j1} = \partial / \partial t + v_{j0} \partial / \partial z \) and

\[
F_{j1} = \frac{q_i}{m_i} E_1 + \frac{q_i}{m_i c} v_{i0} \times B_1 + \nu_{ei0} \frac{\partial v_{i1z}}{\partial z} (v_{i0} - v_{i0}),
\]

\[
- \nu_{v0} (v_{i1z} - v_{i1z}),
\]

\[
F_{e1} = \frac{q_e}{m_e} E_1 + \frac{q_e}{m_e c} v_{e0} \times B_1 + \nu_{ei0} \frac{\partial v_{e1z}}{\partial z} (v_{e0} - v_{e0}),
\]

\[
- \nu_{v0} (v_{e1z} - v_{e1z}).
\]

(A1)

We do not include collisions for transverse perturbations. The corresponding condition is given in section 6. For obtaining (A2), we have taken into account a perturbation of scattering and acceleration.

A.2. Perturbed temperatures of ions and electrons

From the linearized equations (3) and (4), we obtain equations for the perturbed temperatures of ions and electrons, \( T_{i,e1} \). We will assume that the background ion and electron temperatures are equal to each other, \( T_{i0} = T_{e0} = T_0 \). In this case, the terms connected with the perturbation of thermal energy exchange frequency in equations (3) and (4) will be absent. However, for convenience of calculations to follow the symmetric contribution of ions and electrons, we formally retain different notation for the ion and electron temperatures. Then, we will have

\[
D_i T_{i1} - \Omega_{ie} T_{e1} = - (\gamma - 1) T_{i0} \frac{\partial v_{i1z}}{\partial z},
\]

\[
D_e T_{e1} - \Omega_{ie} T_{i1} = - (\gamma - 1) T_{e0} \frac{\partial v_{e1z}}{\partial z}.
\]

Here, the following notation is introduced:

\[
D_i = D_{i0} + \Omega_{ie}, \quad D_e = D_{e0} + \Omega_{ie} + \Omega_{ei},
\]

\[
\Omega_{ie} = \nu_{ei}^0 (n_{e0}, T_{e0}), \quad \Omega_{ei} = \nu_{ei}^0 (n_{i0}, T_{e0}),
\]

\[
\Omega_x = - (\gamma - 1) \frac{x_0}{\nu_{ei0}} \frac{\partial^2}{\partial z^2},
\]

where we have used equation (5) for obtaining \( \Omega_x \). Solutions of equation (A5) for \( T_{i,e1} \) are given by

\[
DT_{i1} = - D_e (\gamma - 1) T_{0} \frac{\partial v_{i1z}}{\partial z} - \Omega_{ie} (\gamma - 1) T_{0} \frac{\partial v_{e1z}}{\partial z},
\]

\[
DT_{e1} = - D_i (\gamma - 1) T_{0} \frac{\partial v_{e1z}}{\partial z} - \Omega_{ei} (\gamma - 1) T_{0} \frac{\partial v_{i1z}}{\partial z}.
\]

(A7)

A.3. Equations for longitudinal velocities \( v_{i1z} \) and \( v_{e1z} \)

Let us substitute equation (A7) into equation (A4) written for the ions and electrons. Then, we obtain

\[
L_{i1} v_{i1z} + L_{i2} v_{e1z} = DD_{i0} \frac{q_i}{m_i} E_{1z},
\]

\[
L_{e1} v_{e1z} + L_{e2} v_{i1z} = DD_{e0} \frac{q_e}{m_e} E_{1z}.
\]

(A9)

Here, we have introduced notation

\[
L_{i1} = DD_{i0} (D_{i0} + \nu_{ei0}) - T_{i0} \frac{\partial^2}{\partial z^2} - (\gamma - 1) T_{0} \frac{\partial v_{i1z}}{\partial z},
\]

\[
L_{i2} = DD_{i0} (D_{i0} + \nu_{ei0}) - T_{0} \frac{\partial^2}{\partial z^2} - (\gamma - 1) T_{0} \frac{\partial v_{i1z}}{\partial z},
\]

\[
L_{e1} = DD_{e0} (D_{e0} + \nu_{ei0}) - T_{e0} \frac{\partial^2}{\partial z^2} + \nu_{ei0} D_{i0},
\]

\[
L_{e2} = DD_{e0} (D_{e0} + \nu_{ei0}) - T_{0} \frac{\partial^2}{\partial z^2} + \nu_{ei0} D_{i0}.
\]

(A10)
Solutions of equation (A9) are the following:

\[ v_{r1z} = \frac{D}{L} \left( L_1 D_{n1} \frac{q_1}{m_1} - L_2 D_{e1} \frac{q_e}{m_e} \right) E_{1z}, \]  

\[ v_{e1z} = \frac{D}{L} \left( L_1 D_{e1} \frac{q_e}{m_e} - L_2 D_{i1} \frac{q_i}{m_i} \right) E_{1z}, \]  

(A11)

where

\[ L = L_{li} L_{ie} - L_{2i} L_{2e}. \]  

(A12)

A.4. Expressions for perturbed transverse velocities via \( E_1 \)

Using equation (9), we can find the components of \( F_{j1} \) given by equation (A2). In the case under consideration, we obtain

\[ F_{j1x,y} = \frac{d_j}{m_j} D_{lj} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1x,y}. \]  

(A13)

Substitution of equation (A13) into equation (A3) gives

\[ (D_{jj}^2 + \omega_{cj}^2) v_{j1x} = \frac{d_j}{m_j} \omega_{cj} D_{lj} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1y} + \frac{d_j}{m_j} D_{lj} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1x}, \]

\[ (D_{jj}^2 + \omega_{cj}^2) v_{j1y} = -\frac{d_j}{m_j} D_{lj} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1x} + \frac{d_j}{m_j} \omega_{cj} D_{lj} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1y}. \]  

(A14)

A.5. Perturbed plasma current

The components of the transverse and perturbed plasma current \( j_{p1x,y} = \sum_j q_j n_j v_{j1x,y} \) are found by using equation (A14). The expressions for \( 4\pi (\partial / \partial t)^{-1} j_{p1x,y} \) can be given in the form

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{p1x} = \varepsilon_{pxx} E_{1x} + \varepsilon_{pxy} E_{1y}, \]  

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{p1y} = -\varepsilon_{pxx} E_{1x} + \varepsilon_{pxy} E_{1y}, \]  

(A15)

where \( \omega_{pj} = (4\pi n_j q_j^2 / m_j)^{1/2} \) is the plasma frequency. The following notation is introduced into equation (A15):

\[ \varepsilon_{pxx} = \sum_j \frac{\omega_{pj}^2 D_{jj}^2}{(D_{jj}^2 + \omega_{cj}^2)} \left( \frac{\partial}{\partial t} \right)^{-2}, \]  

\[ \varepsilon_{pxy} = \sum_j \frac{\omega_{pj} \omega_{cj} D_{lj}}{(D_{lj}^2 + \omega_{cj}^2)} \left( \frac{\partial}{\partial t} \right)^{-2}. \]  

(A16)

The longitudinal perturbed plasma current \( j_{pl1z} = \sum_j q_j n_j v_{j1z} \) is found using equations (2) and (A11):

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{pl1z} = \frac{D_{pl}}{D_{ll}} \left( L_{1e} D_{n1} - L_{2i} D_{e1} \frac{q_e m_e}{q_i m_i} \right) E_{1z} + \frac{2 \omega_{pe} D_{pl}}{D_{ll} L} \left( L_{1i} D_{ie} - L_{2e} D_{i1} \frac{q_i m_e}{q_i m_i} \right) E_{1z}. \]  

(A17)

Appendix B.

B.1. Perturbed velocity of cosmic rays

The linearized version of equation (6) for \( p_{cr1} = p_{cr} - p_{cr0} \) and \( v_{cr0} \parallel z \) has the form

\[ R_{cr0} D_{cr} p_{cr1} + p_{cr0} D_{cr} R_{cr1} = -\nabla p_{cr1} + m_{cr} F_{cr1} + \frac{q_{cr}}{c} v_{cr1} \times B_0, \]  

(B1)

where \( D_{cr} = \partial / \partial t + v_{cr0} \partial / \partial z \) and

\[ m_{cr} F_{cr1} = q_{cr} E_1 + \frac{q_{cr}}{c} v_{cr0} \times B_1. \]  

(B2)

For the perturbed transverse velocities of cosmic rays \( v_{cr1x,y} \), we find from equation (B1) the following solutions:

\[ (D_{cr}^2 + \omega_{c}^2) v_{cr1x} = \frac{q_{cr}}{m_{cr}} \omega_{c} D_{cr} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1y} + \frac{q_{cr}}{m_{cr}} D_{cr} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1x}, \]

\[ (D_{cr}^2 + \omega_{c}^2) v_{cr1y} = -\frac{q_{cr}}{m_{cr}} \omega_{c} D_{cr} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1x} + \frac{q_{cr}}{m_{cr}} D_{cr} \left( \frac{\partial}{\partial t} \right)^{-1} E_{1y}. \]  

(B3)

where \( D_{cr} = \gamma_{cr0} R_{cr0} D_{cr} \). When obtaining equation (B3), we have expressed \( F_{cr1x,y} \) through \( E_{1x,y} \) using equation (A13) for \( j = cr \) (see equation (B2)). The \( z \)-component of equation (B1) is given by

\[ D_{cr} v_{cr1z} + v_{cr0} R_{cr0} D_{cr} \gamma_{cr1} + \gamma_{cr0} v_{cr0} D_{cr} R_{cr1} = -\frac{1}{m_{cr} n_{cr0}} \frac{\partial p_{cr1}}{\partial z} + F_{cr1z}, \]  

(B4)

where \( \gamma_{cr1} = \gamma_{cr0}^3 v_{cr0} v_{cr1} / c^2 \).

B.2. Perturbed temperature and pressure of cosmic rays

We now find \( R_{cr1} \) and \( p_{cr1} \). From equation (8), we see that

\[ R_{cr1} = \frac{\Gamma_{cr}}{\Gamma_{cr} - 1} \frac{T_{cr1}}{T_{cr0} - 1} \frac{m_{cr}^2}{e^2}. \]  

(B5)

The perturbation of the temperature \( T_{cr1} \) found from the equation \( T_{cr} = \gamma_{cr} p_{cr} / n_{cr} \) is equal to

\[ T_{cr1} = T_{cr0} \left( \frac{p_{cr1}}{p_{cr0}} - \frac{n_{cr1}}{n_{cr0}} + \frac{\gamma_{cr1}}{\gamma_{cr0}} \right). \]  

(B6)

From equation (7), we can find the pressure perturbation \( p_{cr1} \):

\[ p_{cr1} = p_{cr0} \Gamma_{cr} \left( \frac{n_{cr0}}{n_{cr1}} - \frac{\gamma_{cr1}}{\gamma_{cr0}} \right). \]  

(B7)
where

\[ n_{cr1} = -n_{cr0} \frac{\partial \psi_{cr1z}}{D_{cr} \partial z} \]  \hspace{1cm} (B8)

### B.3. Equation for \( \psi_{cr1z} \)

Substituting equations (B5)–(B8) into equation (B4) and using equation (A13) for cosmic rays, we obtain

\[ \gamma_{cr}^3 \left( R_{cr0} = \frac{\Gamma_{cr} T_{cr0} v_{cr0}^2}{m_{cr}^2 c^2} \right) D_{cr}^2 \psi_{cr1z} \]

\[ -2\gamma_{cr0} v_{cr0} \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} D_{cr} \frac{\partial \psi_{cr1z}}{\partial z} - \frac{\Gamma_{cr} T_{cr0}}{\gamma_{cr0} m_{cr}} \frac{\partial^2 \psi_{cr1z}}{\partial z^2} \]

\[ = D_{cr} \frac{q_{cr}}{m_{cr}} E_{1z}. \]  \hspace{1cm} (B9)

### B.4. Perturbed cosmic-ray current

The perturbed transverse cosmic-ray currents \( j_{cr1x,y} = q_{cr} n_{cr0} \psi_{cr1x,y} \) are found using equation (B3). For the values \( 4\pi (\partial / \partial t) \psi_{cr1x} \), we will have

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{cr1x} = \varepsilon_{crxx} E_{1x} + \varepsilon_{crxy} E_{1y}, \]

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{cr1y} = -\varepsilon_{crxy} E_{1x} + \varepsilon_{crxx} E_{1y}. \]  \hspace{1cm} (B10)

Here

\[ \varepsilon_{crxx} = \frac{\omega_{cr}^2 D_{cr} D_{cr}}{\left( \frac{\partial}{\partial t} \right)^2}, \]

\[ \varepsilon_{crxy} = \frac{\omega_{cr}^2 \omega_{cr} D_{cr} \omega_{cr}}{\left( \frac{\partial}{\partial t} \right)^2}, \]  \hspace{1cm} (B11)

where \( \omega_{cr} = \sqrt{4\pi n_{cr0} q_{cr}^2 / m_{cr}} \) is the cosmic-ray plasma frequency. The longitudinal perturbed cosmic-ray current is equal to \( j_{crz} = q_{cr} n_{cr0} \psi_{cr1z} \). Making use of the linearized continuity equation for cosmic rays and equation (B9), we find

\[ 4\pi \left( \frac{\partial}{\partial t} \right)^{-1} j_{crz} = \frac{q_{cr} \omega_{cr}^2}{L_{cr}} \psi_{cr1z}, \hspace{1cm} (B12) \]

where

\[ L_{cr} = \gamma_{cr0}^3 \left( R_{cr0} = \frac{\Gamma_{cr} T_{cr0} v_{cr0}^2}{m_{cr}^2 c^2} \right) D_{cr}^2 \]

\[ -2\gamma_{cr0} v_{cr0} \frac{\Gamma_{cr} T_{cr0}}{m_{cr} c^2} D_{cr} \frac{\partial}{\partial z} - \frac{\Gamma_{cr} T_{cr0}}{\gamma_{cr0} m_{cr}} \frac{\partial^2}{\partial z^2}. \]  \hspace{1cm} (B13)

### References

[1] Roberts T G and Bennett W H 1968 *Plasma Phys.* 10 381
[2] Cox J L Jr and Bennett W H 1970 *Phys. Fluids* 13 182
[3] Hammer D A and Rostoker N 1970 *Phys. Fluids* 13 1831
[4] Lee R and Sudan R N 1971 *Phys. Fluids* 14 1213
[5] Berk H L and Pearlstein L D 1976 *Phys. Fluids* 19 1831
[6] Achterberg A 1983 *Astron. Astrophys.* 119 274
[7] Zweibel E G 2003 *Astrophys. J.* 587 L653
[8] Bell A R 2004 *Mon. Not. R. Astron. Soc.* 353 550
[9] Riquelme M A and Spitkovsky A 2010 *Astrophys. J.* 717 1054
[10] Nekrasov A K and Shadmehri M 2012 *Astrophys. J.* 756 77
[11] Zweibel E G and Everett J E 2010 *Astrophys. J.* 709 1412
[12] Reville B, Kirk J G, Duffy P and O’Sullivan S 2007 *Astrophys. J.* 475 435
[13] Amato E and Blasi P 2009 *Mon. Not. R. Astron. Soc.* 392 1591
[14] Braginskii S I 1965 *Rev. Plasma Phys.* 1 205
[15] Lontano M, Bulanov S and Koga J 2001 *Phys. Plasmas* 8 5113
[16] Toepfer A J 1971 *Phys. Rev. A* 3 1444
[17] Dzhavakhishvili D I and Tsintsadze N L 1973 *Sov. Phys.—JETP* 37 666
[18] Alfvin H 1939 *Phys. Rev.* 55 425
[19] Koyama K, Petre R, Gottheil E V, Hwang U, Matsuura M, Ozaki M and Holt S S 1995 *Nature* 378 255
[20] Allen G E et al 1997 *Astrophys. J.* 487 L97
[21] Tanimore T et al 1998 *Astrophys. J.* 497 L25
[22] Vink J and Laming J M 2003 *Astrophys. J.* 584 758
[23] Brunetti G, Setti G, Feretti L and Giovannini G 2001 *Mon. Not. R. Astron. Soc.* 320 365
[24] Pfrommer C and Enßlin T A 2004 *Astrophys. J.* 413 17
[25] Enßlin T A 2003 *ASTRO2000 Science Missions Report AAS 399 409
[26] Guo F and Oh S P 2008 *Mon. Not. R. Astron. Soc.* 384 251
[27] Sharma P, Chandran B D G, Quataert E and Parrish I J 2009 *Astrophys. J.* 699 348
[28] Sharma P, Parrish I J and Quataert E 2010 *Astrophys. J.* 720 652
[29] Kulsrud R and Pearce W P 1969 *Astrophys. J.* 156 445
[30] Gargate L and Spitkovsky A 2012 *Astrophys. J.* 744 67
[31] Nekrasov A K and Shadmehri M 2010 *Astrophys. J.* 724 1165
[32] Nekrasov A K 2011 *Astrophys. J.* 739 88
[33] Heyvaerts J 1974 *Astrophys. J.* 197 567
[34] Krymski G F 1977 *Sov. Phys.—Dokl.* 22 327
[35] Axford W I, Leer E and Skadron G 1973 *Proc. 15th Int. Cosmic Ray Conf. (Plovdiv)* vol 11 (Budapest: Central Research Institute for Physics) p 132
[36] Bell A R 1978 *Mon. Not. R. Astron. Soc.* 182 147
[37] Blandford R D and Ostriker J P 1978 *Astrophys. J.* 221 L29
[38] Drury L O’C 1983 *Rep. Prog. Phys.* 46 973
[39] Riquelme M A and Spitkovsky A 2009 *Astrophys. J.* 694 626
[40] Ballet J 2006 *Adv. Space Res.* 37 1902
[41] Uchiyama Y, Aharonian F A, Tanaka T, Takahashi T and Maca Y 2007 *Nature* 449 576