Tunneling Analysis Under the Influences of Einstein-Gauss-Bonnet Black Holes Gravity Theory

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I. INTRODUCTION

In quantum theory, the BH radiation as a solution of Hawking [1] is one of the significant physical phenomena. During the study of radiation phenomenon, the investigators effort to combine the gravitation within quantum mechanics and BH thermodynamics [2, 3]. In order to analyze the BHs radiation phenomena, many methods have been suggested in the literature. Many researchers have observed these radiation for the different well-known BHs [4]-[11]. The generalized uncertainty principle (GUP) relation plays a critical role in understanding the BHs nature, while the quantum effect may be studied as critical effect around the BH horizon. By applying the tunneling phenomenon incorporating GUP relation the thermodynamical properties of BHs have been analyzed [12]. The tunneling radiation from quartic and cubic BHs through the GUP corrected fermion and scalar particles have been investigated [13] as well as the author’s studied that BHs temperature depend on the tunneling particles properties and also observed the fermion and scalar particles temperature which appear alike in both interactions.

Using Hamilton-Jacobi method, the significance of field equation has been studied by considering electromagnetic interactions [14]-[17]. The massive bosons tunneling in both electric and magnetic fields and Hawking temperatures have been computed. The Hawking radiation as a semi-classical tunneling from BHs and black ring have been investigated. In this paper, by using the WKB approximation, authors analyzed the Lagrangian field equation for charged bosons. The gravitational effects on BH radiation as well as the instability and stability of BH have been analyzed and concluded that the rotation parameter effects the tunneling radiation.

In recent years, the investigators put a great interest to study the BHs thermal properties, especially with a spacetime having charged and cosmological constant. The main aim of this paper is to analyze the tunneling radiation phenomena with GUP effects with charged and coupling constant BHs from higher dimensions. Firstly, we assume a D-dimensional BH with charged and coupling constant. Then by applying the Hamilton Jacobi technique, we compute the boson tunneling for spin-1 particles. Moreover, we have observed the modified Hawking temperature for spin-1 boson particles. Further, we assume the graphical analysis of quantum gravity effects on modified Hawking temperature for spin-1 particles for charged BHs. The Hawking radiation process for the various spin particles has been analyzed in literature [18]-[20]. Furthermore, it has been analyzed that the Hawking radiation for various spin of particles.

The singularity of spacetime is a causal contact with an outside observer and the relevant geometry can not be explained by semiclassical or classical theories. So, to have a logically predictive ability, the singularity must be causally unconnected from distance observer. This is indeed the type for the space Schwarzschild result, where the horizon ignore the singularity from the distance observer. Simply their are also general relativity solutions which may comprise naked singularity (singularity without the event horizon). The creation of such naked singularity in
the physical existence would invalidate applicability semiclassical and classical physics. Therefore, naked singularity is unsuitable feature of general relativity.

The paper is sketched as follows, Sec. II contains the line element information of D-dimensional BHs. Subsection II.A, II.B and II.C represents the study of boson tunneling in the BHs for 3, 4 and D-dimension spaces. In Sec. III, we study the graphical analysis of quantum gravity effects on Hawking temperature for spin-1 particles. In Sec. IV, we resume the conclusions of our work.

II. EINSTEIN-GAUSS-BONNET BLACK HOLE GRAVITY THEORY

The result of Einstein equation will not consist any naked singularity. This result is the generalization of the Reissner-Nordström BHs in general relativity. We analyze the charged solution of D-dimension BHs in Einstein-Gauss-Bonnet gravity theory. We study the Lagrangian equation with Einstein-Gauss-Bonnet (EGB) gravity theory, \[ L = \hat{\beta}(R^2 + R_{abcd}R^{abcd} - 4R_{ab}R^{ab}) + R. \]

The BH of spherically symmetric solution of this gravity theory in D-dimension spacetime is of the form [21]

\[ ds^2 = -G(r)dt^2 + \frac{1}{G(r)}dr^2 + r^2d\Omega_{d-2}^2, \]

with

\[ G(r) = \frac{r^2}{4\hat{\beta}}[1 - \sqrt{1 - \frac{8\hat{\beta}q^2}{r^{2d-4}} + \frac{16\hat{\beta}M}{r^{d-2}}} + 1, \]

where \( q \) and \( M \) are BH charge and mass, the constant \( \hat{\beta} \) is associated to the cupling constant as \((d-3)(d-4)\hat{\beta}/2.\)

We represent the metric by \( d\Omega_{d-2}^2 \) for the unit \( d = 3 \) sphere of area \( A_1. \) The just nonzero element of the vector electromagnetic potential has the form

\[ A_t(r) = -\frac{q}{r^{d-3}}. \]

Here, \( q \) and \( M \) are associated to the BH Arnowitt-Deser-Misner charge \((\hat{q})\) and mass \( \hat{M} \) as

\[ q^2 = \frac{2(d-3)}{d-2} \hat{q}^2, \quad M = \frac{8\pi}{(d-2)A_{d-2}} \hat{M}. \]

A. 3-Dimensional Charged Black Hole

The spherically symmetric solution of BH in 3-dimensional spacetime is of the form

\[ ds^2 = -Gdt^2 + \frac{1}{G}dr^2 + r^2d\theta^2. \]

We focus on the effect of the quantum gravity on the boson tunneling from the BHs in EGB gravity theory. Firstly, we apply the GUP-corrected Lagrangian field equation for the massive charged vector field given by [7]

\[ \partial_{\mu}(\sqrt{-g}\psi^{\mu}) + \sqrt{-g}\frac{m^2}{\hbar^2}\psi^{\mu} + \sqrt{-g}\frac{i}{\hbar}A_{\mu}\psi^{\mu} + \sqrt{-g}\frac{i}{\hbar}eF^{\nu\mu}\psi_{\mu} + \alpha\hbar^2\partial_{\mu}\partial_{\nu}(\sqrt{-g}g^{\nu0}\psi^{\nu}) \]

\[ -\alpha\hbar^2\partial_{\mu}\partial_{\nu}(\sqrt{-g}g^{\nu\nu}\psi^{\mu}) = 0, \]

where \( \psi^{\mu\nu}, \ m \) and \( g \) are the anti-symmetric tensor, particle mass and coefficient matrix determinant, respectively. The \( \psi^{\mu\nu} \) can be defined as

\[ \psi_{\nu\mu} = (1 - \alpha\hbar^2\partial_0^2)\partial_0\psi_\mu - (1 - \alpha\hbar^2\partial_0^2)\partial_\mu\psi_0 + (1 - \alpha\hbar^2\partial_0^2)\frac{i}{\hbar}eA_\mu\psi_\nu - (1 - \alpha\hbar^2\partial_0^2)\frac{i}{\hbar}eA_\nu\psi_\mu. \]
Here, $A_\mu$, $e$ and $\alpha$ are denoted as the BH potential, charged of boson particle and gravity parameter, respectively. The $\psi$ components can be computed as

$$
\psi^0 = -\frac{1}{G} \psi_0, \quad \psi^1 = G \psi_1, \quad \psi^2 = \frac{1}{r^2} \psi_2, \quad \psi^{01} = -\psi_{01}, \quad \psi^{02} = -\frac{1}{Gr^2} \psi_{02}, \quad \psi^{12} = \frac{G}{r^2} \psi_{12}.
$$

The $\psi_\nu$ function of the boson particle is defined as [9]

$$
\psi_\nu = c_\nu \exp \left[ \frac{i}{\hbar} I_0(t, r, \theta) + \sum_{i=1}^{n} h^i I_i(t, r, \theta) \right],
$$

(4)

where $c$ is a constant and $I_0(t, r, \theta)$ is the classical action of particle. Substituting the $\psi_\nu$ field function from Eq. (4) into the field Eq. (3) for the lowest order of $\hbar$, we obtain the corrected Hamilton-Jacobi equations as follows:

$$
c_1(\partial_0 I_0)(\partial_1 I_0) + ac_1(\partial_0 I_0)^3(\partial_1 I_0) - c_0(\partial_1 I_0)^2 - ac_0(\partial_1 I_0)^4 + c_1eA_0(\partial_1 I_0) + c_1aeA_0(\partial_0 I_0)^2(\partial_1 I_0) + \frac{1}{Gr^2}
$$

$$
[c_2(\partial_0 I_0)(\partial_2 I_0) + ac_2(\partial_0 I_0)^3(\partial_2 I_0) - c_0(\partial_2 I_0)^2 - ac_0(\partial_2 I_0)^4 + c_2eA_0(\partial_2 I_0) + c_2aeA_0(\partial_0 I_0)^2(\partial_2 I_0)] - \frac{m^2c_0}{G} = 0,
$$

(5)

$$
c_1(\partial_0 I_0)^2 + c_1(\partial_0 I_0)^4 - c_0(\partial_0 I_0)(\partial_1 I_0) - ac_0(\partial_0 I_0)(\partial_1 I_0)^3 + eA_0(\partial_1 I_0) + c_1aeA_0(\partial_0 I_0)(\partial_1 I_0)^2 - \frac{G}{r^2}
$$

$$
[c_2(\partial_1 I_0)(\partial_1 I_0) + ac_2(\partial_1 I_0)^3(\partial_1 I_0) - c_1(\partial_2 I_0)^2 - ac_1(\partial_2 I_0)^4] + eA_0[c_1(\partial_1 I_0) + c_1(\partial_0 I_0)^3 - c_0(\partial_1 I_0) - ac_0(\partial_1 I_0)^3 + eA_0c_1 + c_1aeA_0(\partial_0 I_0)^2] + m^2Gc_1 = 0,
$$

(6)

$$
\frac{1}{Gr^2} \left[ c_2(\partial_1 I_0)^2 + ac_2(\partial_1 I_0)^4 - c_0(\partial_1 I_0)(\partial_2 I_0) - ac_0(\partial_0 I_0)(\partial_2 I_0)^3 + c_2eA_0(\partial_0 I_0) + c_2aeA_0(\partial_0 I_0)^3 + \frac{G}{r^2} 
$$

$$
[c_2(\partial_1 I_0)^2 + ac_2(\partial_1 I_0)^4 - c_1(\partial_1 I_0)(\partial_2 I_0) - ac_1(\partial_1 I_0)(\partial_2 I_0)^3] + \frac{m^2c_2}{r^2} + \frac{eA_0}{Gr^2}[c_2(\partial_0 I_0) + ac_2(\partial_0 I_0)^3 - c_0(\partial_1 I_0) - ac_0(\partial_1 I_0)^3 + c_2eA_0 + c_2aeA_0(\partial_0 I_0)^2] = 0.
$$

(7)

We can take the classical action of particle in the following form

$$
I_0 = -(E - j\omega t) + W(r) + K\theta,
$$

(8)

where $\hat{E} = E - j\omega$, $E$, $j$ and $\omega$ shoe the energy of particle, angular momentum and angular velocity, respectively. From Eqs. (5)-(7), the field equation can be expressed as

$$
U(c_0, c_1, c_2)^T = 0,
$$

(9)

where $U$ is a $3 \times 3$ ordered of matrix and its elements are given below

$$
U_{00} = -[\hat{W}^2 + a\hat{W}^4] - \frac{1}{Gr^2}[K^2 + aK^4] - \frac{m^2}{G},
$$

$$
U_{01} = -[\hat{E} + a\hat{E}^3 - eA_0 - aeA_0\hat{E}^2]\hat{W},
$$

$$
U_{02} = -\frac{1}{Gr^2}\hat{E} + a\hat{E}^3 - eA_0 - aeA_0\hat{E}^2]K,
$$

$$
U_{10} = -[\hat{W} + a\hat{W}^3]\hat{E} + eA_0[\hat{W} + a\hat{W}^3],
$$

$$
U_{11} = -[\hat{E}^2 + a\hat{E}^4 - 2eA_0\hat{E} - aeA_0\hat{E}^3 - \big{(}eA_0\big{)}^2 + (eA_0)^2\hat{W}^2] - \frac{G}{r^2}[K^2 + aK^4] - m^2G,
$$

$$
U_{12} = \frac{G}{r^2}[\hat{W} + a\hat{W}^3]K,
$$

$$
U_{20} = \frac{1}{Gr^2}[K + aK^3]\hat{E} + \frac{eA_0}{Gr^2}[\hat{K} + aK^3],
$$

$$
U_{21} = -\frac{G}{r^2}[K + aK^3]\hat{W},
$$

$$
U_{22} = -\frac{1}{Gr^2}[\hat{E}^2 + a\hat{E}^4 - 2eA_0\hat{E} - 2aeA_0\hat{E}^3 + (eA_0)^2 + (eA_0)^2a\hat{E}^2] + \frac{G}{r^2}[\hat{W}^2 + a\hat{W}^4] - \frac{m^2}{r^2},
$$

with $\hat{E}$ and $\hat{W}$ denoting the energy and angular momentum of boson particle, respectively.
where $\dot{W} = \partial_t I_0$ and $K = \partial_\theta I_0$. For the result of non-trivial solution, we take $U = 0$ and get
\[
\text{Im}W^\pm = \pm \int \frac{\sqrt{\dot{E} - eA_0}^2 + X_1[1 + \alpha \Xi^2]}{G} \, dr, = \pm i\pi \frac{\dot{E} - eA_\psi}{2\kappa(r_+)}[1 + a\Xi],
\]
where $W^-$ and $W^+$ represent to the incoming and outgoing trajectories of boson particle, respectively, and the values of $X_1$ and $X_1$ are given as
\[
X_1 = -2eA_0\dot{E} + Gm^2, \quad X_2 = \dot{E}^4 - 2eA_0\dot{E}^3 + (eA_0)^2\dot{E}^2 - GW^4.
\]
where $\kappa(r_+)$ is the standard BH surface gravity and its explicit form is
\[
\kappa(r_+) = \frac{4\beta q^2 - 8\beta M + r\sqrt{8\beta(2M - q^2)}}{4\beta r^2 + 16\beta M - 8\beta q^2}.
\]
Hence, the boson particle tunneling can be obtained as
\[
\Gamma = \exp \left[ -2\pi \frac{4\beta \sqrt{r^2 + 16\beta M - 8\beta q^2}(\dot{E} - eA_0)(1 + a\Xi)}{4\beta q^2 - 8\beta M + r\sqrt{8\beta(2M - q^2)} - r^2} \right]. \tag{12}
\]
The gravitational Hawking temperature of the boson particle for the BH can be written as
\[
T_H = \frac{4\beta q^2 - 8\beta M + r_+\sqrt{8\beta(2M - q^2)} - r_+^2}{8\pi\beta \sqrt{r_+^2 + 16\beta M - 8\beta q^2}}[1 + a\Xi]. \tag{13}
\]
The boson particle tunneling depends on BH potential, BH charged, coupling constant, BH radius, BH mass, particle velocity, particles energy and particle momentum as well as quantum gravity. While, the temperature depends on properties of BH and quantum gravity parameter. This result shows that the corrected temperature of the boson charged particle is higher than the absolute temperature. Moreover, it indicates that the modified temperature in Eq. (13) depends upon not only the BH properties but also the quantum gravity parameter.

## B. 4-Dimensional Charged Black Hole

The spherically symmetric solution of BH in 4-dimensional spacetime is of the form
\[
ds^2 = -Gdt^2 + \frac{1}{G}dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\psi.
\]
For the requiring component of the electromagnetic vector potential($A_0$) and quantum gravity, the explicit form of the tunneling probability can be computed by the above way:
\[
\Gamma = \exp \left[ -2\pi \frac{4\beta \sqrt{r^4 + 16\beta Mr - 8\beta Q^2} - 4\beta M - r^3}{r^3 \sqrt{r^4 + 16\beta Mr - 8\beta Q^2} - 4\beta M - r^3} \right]. \tag{14}
\]
Furthermore, observing the quantum gravity terms, we calculate the Hawking temperature of the 4-dimensional charged BH in EGB gravity theory as follows
\[
T_H = \frac{r_+^3 \sqrt{r_+^4 + 16\beta Mr_+ - 8\beta Q^2} - 4\beta M - r_+^3}{8\pi\beta \sqrt{r_+^4 - 8\beta Q^2 + 16\beta r_+}}[1 + a\Xi]. \tag{15}
\]
This solution indicates that the temperature with gravity of the charged boson particle is higher than the temperature without gravity and depends on the BH properties. Also, it is similar from that of the fermion and scalar particles.
C. D-Dimensional Charged Black Hole

The spherically symmetric solution of BH in D-dimension spacetime is of the form

\[
\text{ds}^2 = -G(r)dt^2 + \frac{1}{G(r)}dr^2 + r^2d\Omega^2_{D-2}.
\]  

(16)

The tunneling rate of the D-dimension charged BH in EGB gravity theory can be calculated as follows

\[
\Gamma = \exp\left([-2\pi \frac{2\beta M(D-1)r^{2-D} + (4 - 2D)Q^2r^{5-2D}}{2\sqrt{1 - 8\beta Q^2r^{4-2D} + 16\beta Mr^{1-D}}} + r(1 - \sqrt{1 - 8\beta Q^2r^{4-2D} + 16\beta Mr^{1-D}})\right)
\]

\[
\left(\hat{E} - eA_0\right)(1 + a\xi)].
\]

(17)

The Hawking temperature of the D-dimension charged BH in EGB gravity theory can be calculated as

\[
T_H = \frac{2\beta M(D-1)r^{2-D} + (4 - 2D)Q^2r^{5-2D}}{4\pi \sqrt{1 - 8\beta Q^2r^{4-2D} + 16\beta Mr^{1-D}}} + \frac{r(1 - \sqrt{1 - 8\beta Q^2r^{4-2D} + 16\beta Mr^{1-D}})}{8\pi\hat{\beta}}\left[1 + a\xi\right].
\]

(18)

This solution indicates that the boson particle temperature depends on the BH dimension and properties. Also, it is different from the three and four dimensional BHs temperature of the boson particle.

III. Graphical Stability Analysis

The stability of the different-dimensional charged BHs in EGB gravity theory can be studied by the tunneling radiation. If the Hawking temperature is positive, then the BHs is stable or else it is unstable. For this purpose, to study the stability of the different-dimensional charged BHs in EGB gravity theory, we firstly compute its modified temperature. The temperature graphical analysis at constant charged(Q), constant arbitrary parameter(ξ), mass BH constant, coupling constant BH radius constant and vary the quantum gravity can be obtained by applying the following relations (13, 15, 18).

From Fig. 1, we see the approximation region \(0 < r_+ < 5\), it is stable. Since, we can say that the 3-dimensional charged BHs in EGB gravity theory become unstable to the presence of the GUP effect. However, if \(a = 100 - 300\) then corrected temperature is positive. Hence, the 3-dimensions charged BHs in EGB gravity theory undergoes a initial to stable. We can say that the BH will be undergo to unstable in the presence of large the quantum gravity and also temperature will be decrease.

In Fig. 2, our results show that the temperature of the 3-dimensions charged BHs in EGB gravity theory depend on the BH properties as well as BH mass and boson tunneling particles in the presence of the quantum gravity. It is worth to observe that the boson particle temperatures calculated through boson tunneling phenomenon. The modified temperature are lower than the BH stable and the corrected temperature are higher than the BH unstable. The corrected temperature are lower than the small quantum gravity and the corrected temperature are higher than the large quantum gravity.

In Fig. 3, the absence of the GUP parameter, i.e. \(a = 0\), the corrected temperature are obtained to the standard temperature. in the presence of the quantum gravity parameter, if the quantum gravity of the D-dimensions charged BHs in EGB gravity theory is positive, the BH becomes stable in the approximation range \(0 < r_+ < 5\) but, in the quantum gravity absence, the stability property of the BH does not change and the BH unstable in the approximation range \(5 < r_+ < \infty\). When we ignore quantum gravity parameter then BH is more stable. Our results shows that quantum gravity effects leave the remnants on the boson tunneling radiation implies non-thermal.

Finally, we observe that the quantum gravity effect on 3, 4 and \(D\) dimensions charged BHs in EGB gravity theory of temperature decreases and increases with increasing horizon \(r_+\) and also observe the physical significance of this temperature to see the under the more and lower influence of quantum gravity on \(T_H\) by observing the BH totally instable and may be stable(i.e., quantum gravity inversely proportional to stability).
IV. CONCLUSIONS

In this paper, we have analyzed the quantum gravity effect on the temperature of the 3, 4 and D dimensional charged BHs in EGB gravity theory in the context of the boson tunneling phenomenon of the massive charged spin-1 particle. Our results show that the modified temperature of the 3, 4 and D dimensional charged BHs in EGB gravity theory depend on the BH geometry, but also the quantum tunneling gravity parameter. It is deserving to mention
that the modified temperatures computed through boson particle tunneling are completely similar from each other particles such as scalar and fermion. The corrected temperatures are higher than the absolute temperatures in all different 3, 4 and D-dimensions charged BHs. In the quantum gravity effect absence, i.e. $\alpha = 0$, the corrected temperatures are obtained to the absolute temperatures in all cases.

In this study, we observe the stability property of the BHs by using the corrected temperature of the charged massive boson particle. The 3-dimensional charged BH undergoes an initial to stable and BH will be undergo to unstable when large quantum gravity and also temperature will be decrease(Fig. 1). The temperature are lower than the 4 and D dimensional BH stable and the temperature are higher than the BH unstable. The temperatures are lower than the small quantum gravity and the temperatures are higher than the large quantum gravity (figures 2 and 3). In the quantum gravity absence then the BHs are more stable. The boson particle temperature depends on the BH dimensions and properties. Also, it is different from the three dimension or higher dimensions BHs temperature of the boson particle.

Finally, we can analyze that the quantum gravity of the boson tunneling particles play a significant role in understanding the BHs evolution such that it can study radiation on the BHs final stage. From graphs, we see that the quantum gravity effect on 3, 4 and D dimensions charged BHs in EGB gravity theory of temperature decreases and increases with increasing horizon $r_+$. 

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