QCD tools for the LHC

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This contribution provides a pedagogical introduction to and review of the current status and ongoing progress in the development of Monte Carlo tools for the calculation and simulation of high-$Q^2$ processes in hadronic collisions.

1 Introduction

In April 2007 the Large Hadron Collider (LHC) will start providing $pp$ collisions at $\sqrt{s} = 14$ TeV. The main goal [1] of this new enterprise is the exploration of a yet uncharted energy domain, and QCD will provide an essential tool for the analysis and the decoding of the immense set of data that will be collected by the three detectors currently being built [2]. The production rates for most SM particles and processes are mindboggling. Huge statistics of final states nowadays totally unaccessible at the current accelerators will be available, and will allow measurements of unprecedented accuracy and depth [3]. Table 1 gives few examples of cross-sections for some of the most relevant processes.

| Process                                      | $\sigma$ (nb) | $\equiv$ evts/s |
|----------------------------------------------|---------------|----------------|
| Jets, $E_T > 0.1$ (2) TeV                   | $10^{-9}$ ($10^{-7}$) |                |
| $W^{\pm} \rightarrow e\nu$                  | 20            |                |
| $Z \rightarrow e^+e^-$                       | 2             |                |
| Photons ($E_T > 60$GeV)                     | 20            |                |
| $e\bar{e}$, $b\bar{b}$                      | $8 \times 10^5$, $5 \times 10^5$ |         |
| $t\bar{t}$                                  | 0.8           |                |

Table 1. Benchmark cross-sections for few SM processes at the LHC, $L = 10^{33}$cm$^{-2}$s$^{-1}$. In the case of jets and photons, we assume a $|\eta| < 1$ rapidity cut.

In addition to direct manifestations of new phenomena, these measurements could ultimately lead to indirect evidence for physics beyond the SM itself. The huge lever arm in energy available through the measurement of high transverse-energy ($E_T$) jets [4] will allow to probe the smallest distance scales ever accessed. One year of high-luminosity running will give tens of events with jets with $E_T > 3$ TeV. Compared to the Tevatron, where jets up to 600 GeV will be observed, this is a factor of 5 increase in the scale at which the quark form factor can be explored.

The immense samples of EW gauge bosons will enable high-precision measurements of the $W$ mass ($\pm 15$ MeV) and of the gauge bosons self-couplings [5]. The study of Drell-Yan final states will be sensitive to several possible new phenomena. For example, possible contact interactions mixing light quarks with leptons will be probed up to scales of the order of 25-30 TeV, well in the region where new strongly interacting phenomena related to the EWSB may take place. New $U(1)$ gauge bosons with $Z$-like couplings will be observed up to masses of the order of 4-5 TeV. The large statistics will allow measurements of the $W$ and $Z$ total cross sections with accuracies significantly better than 1%, providing luminosity monitors which are only limited by the precision of the theoretical predictions for these rates (predictions currently estimated to be around 5%, dominated by uncertainties in the partonic densities of the proton [4]).

Top quarks will be produced in great abundance, at a rate of approximately 1 pair/s. Precise determinations of the top decay properties and the top mass with an uncertainty of about 1 GeV will be possible [6]. The study of flavour will be enriched by a thorough b-physics programme [7]. The tens of billions of bottom quarks produced [8], will allow to pin down with great accuracy and redundancy the CKM matrix elements, to probe in full detail the parameters of CP in the $B_q$ systems ($q = u, d, s$), and to study rare decays with branching ratios at the level of $10^{-9}$.

In all these cases, it is required that the best possible control on the QCD production and decay mechanisms be available. For this reason, in the recent years a great effort has been put into the development of tools enabling the description of the final states resulting from high-energy $pp$ collisions. These tools go under the common name of Monte Carlo (MC) codes, since the state-of-the-art knowledge about QCD is implemented using numerical MC techniques. MC development is a very technical topic. This review is intended to provide a pedagogic and qualitative introduction to the main issues, and to some of the most relevant ideas and topics which are driving the fast development in the field. More accurate discussions, and all the necessary details, will have to be looked for in the references.

In the case of jets and...
2 Overview of the available approaches

The starting point of all QCD analyses of high-$Q^2$ processes is summarized by the factorization theorem, most clearly expressed by the following relation:

$$\frac{d\sigma}{dX} = \sum_X \sum_{j,k} f_j(x_1, Q_i) f_k(x_2, Q_1) \frac{d\hat{\sigma}_{jk}(Q_i, Q_f)}{dX} F(\hat{X} \rightarrow X; Q_f),$$

(1)

where: $X$ represents a given hadronic final state (FS); $\hat{X}$ is an arbitrary partonic FS; $f_j(x, Q_i)$ is the number density of partons of type $j$ carrying the momentum fraction $x$ of the nucleon at a resolution scale (factorization scale) $Q_i$; $\hat{\sigma}_{jk}(Q_i, Q_f)$ is the partonic cross-section for the transition between the initial partonic state $jk$ and the final partonic state $\hat{X}$, considered at resolution scales $Q_i$ and $Q_f$ for initial and final state factorization, respectively; $F(\hat{X} \rightarrow X; Q_f)$ represents a transition function from the partonic final state to the given observable $X$. This may include fragmentation functions, hadronization effects, as well as the result of experimental cuts or jet definitions. The sum over final states $\hat{X}$ can be thought of as a sum over all possible histories leading to the same observable configuration. In a similar fashion, the parton density (PDF) $f_j(x, Q_i)$ can be thought of as the sum over all initial-state (IS) evolution histories leading to the initial-state parton $j$ with momentum fraction $x$. According to the factorization theorem, the possible IS and FS histories, and their relative probabilities, are independent of the hard process, and only depend on the flavours of the partons involved and on the resolution scales. Once an algorithm is developed to describe IS and FS evolution, it can therefore be applied to the partons arising from the calculation of an arbitrary hard process. Depending on the extent to which possible IS and FS histories affect the value of $X$, three different realizations of the factorization theorem are used: cross-section “evaluators”, parton-level event generators, and shower MC event generators. We shall now review these in more detail.

2.1 Cross-section evaluators

In this case only some component of the FS is singled out for the measurement, all the rest being ignored (i.e. integrated over). One example is the Drell-Yan (DY) process, where we typically look at the inclusive spectrum (mass, $p_T$, rapidity, etc.) of a leptonic FS. In this approach, no event needs to be generated, it is enough to define the variables relative to the considered object (the lepton(s), or a jet). Experimental selection criteria (e.g. a jet definition or a detector acceptance) are applied to parton-level quantities. Provided these are infrared and collinear finite, it does not matter what $F$ is, as we assume that all histories of the final partonic state lead to the same observable, and integrate to 1 because of unitarity: $\sum X F(\hat{X}, X) = 1$. Thanks to the inclusiveness of the result, it is conceptually “straightforward” to include higher-order corrections, as well as to resum classes of dominant and subdominant logarithms. Next-to-leading order (NLO) results are known [11] for most processes both within and beyond the Standard Model (SM). In addition, next-to-next-to-leading order (NNLO) cross-sections have been calculated for the DY-type processes [11] and for Higgs production [12]. For a review of progress in NNLO calculations for higher-multiplicity final states, see [13].

2.2 Parton-level event generators

Here parton-level (PL) configurations (i.e. states with quarks and gluons) are generated, with probabilities proportional to the relative perturbative matrix element (ME). The transition function between a FS parton and the observed object (jet, missing energy, lepton, etc.) is unity, so there is no need to model the exclusive realization of the histories associated to the PDF and to $F$, as they all lead to the same observable. Experimentally, this is equivalent to assuming a smart jet algorithm (that would associate a jet to each hard parton) and linear detector response (the energy and direction of a measured jet will not depend on its inner structure). The advantage over the cross-section evaluators is that, with the explicit representation of the kinematics of all hard objects in the event, more refined detector analyses can be performed, implementing complicated cuts and correlations which are otherwise hard to simulate with the inclusive approach. PL event generators are typically used to describe final states with several hard jets. Due to the complexity of the ME evaluation for these many-body configurations, calculations are normally available only for leading-order (LO) cross-sections. In this case, several tools [14]-[23] have recently become available, covering all of the necessary processes for signal and background LHC studies, with jet multiplicities all the way up to 4, 5 or 6, depending on the specific process: $(W \rightarrow f\bar{f})Q\bar{Q}^{+}J$ jets (Q being a heavy quark, and $f = \ell, q$); $(Z/\gamma^{*} \rightarrow f\bar{f})Q\bar{Q}^{+}J$ jets ($f = \ell, \nu$); $(W \rightarrow f\bar{f})^{+}J$ charm + $N$ jets ($f = \ell, q$); $(W \rightarrow f\bar{f})^{+}J$ jets and $(Z/\gamma^{*} \rightarrow f\bar{f})^{+}J$ jets; $nW + mZ + H + J$ets; $Q\bar{Q}^{+}$ jets; $Q\bar{Q}^{*}Q \bar{Q}^{+}$ jets, with $Q$ and $Q'$ heavy quarks; $HQQ$ jets; $N$ jets; $N\gamma$+ jets. Most of these processes have been evaluated by more
than one group, providing additional robustness to the calculations and their numerical implementation. NLO PL event generators are also available for several low-jet-multiplicity final states, as discussed in Sect. 4.1.

2.3 Shower MC event generators

Shower MC generators [25] provide the most complete description of the FS. Their goal is to generate events consisting of physical, measurable hadrons, with a correct description of their multiplicity, kinematics and flavour composition. These final states can therefore be processed through a complete detector simulation, providing the closest possible emulation of a real event. Shower MC codes such as HERWIG [20], PYTHIA [27], ISAJET [28] or ARIADNE [29] have been known and used for several years now, and new tools, SHERPA, are becoming available [30].

After the generation of a given PL configuration (typically using a LO ME for $2 \rightarrow 1$ or $2 \rightarrow 2$ processes), all possible IS and FS histories (“showers”) are generated, with probabilities defined by the shower algorithm. By algorithm, we mean a numerical, Markov-like evolution, which implements within a given approximation scheme the QCD dynamics. This includes the probabilities for parton radiation (gluon emission, or $g \rightarrow q\bar{q}$ splitting), an infrared cutoff scheme, and a hadronization model. The radiation probabilities are defined as exclusive quantities, unitarized by the inclusion of Sudakov form factors. This means that the shower evolution itself does not alter the overall cross-section, as estimated from the ME evaluation for the initial hard process. Therefore a shower MC based on LO matrix elements cannot provide an estimate of the $K$ factor. Radiation probabilities implement leading soft and collinear logarithms, plus some subleading classes of logs. Quantum-mechanical correlations between different emissions are negligible in the case of collinear emission, since in this case gauges can be defined where the interference between different diagrams is numerically suppressed; in the case of soft emission at large angle, the association to a specific emitter is ambiguous, as the interference between different emission diagrams is large. Fortunately, soft emission at large angle is heavily suppressed, as the interference effects between different diagrams lead to destructive interference. Quantum coherence can then be implemented in a branching evolution via, for example, an angular ordering prescription [31]: if $\theta_{ij}$ is the angle between two colour-connected partons, the soft-gluon emission probability from the pair $i, j$ is given by the sum of two independent emission terms, constrained within angular regions close to the two emitters:

$$P(\theta_i)\Theta(\theta_{ij} - \theta_i) + P(\theta_j)\Theta(\theta_{ij} - \theta_j)$$

(2)

where $\theta_i, \theta_j$ are the angles between the soft gluon and the colour connected partons emitting it, and $P(\theta)$ is a positive definite probability.

3 Complementarity of the three approaches

Each of the above three approaches has its virtues and shortcomings. Until recently, we had to select the approach that was closest to our needs, and live with the fact that it had drawbacks. We first outline a comparison between the merits of the three approaches, and then review the recent progress in merging them.

3.1 Final-state structure

Shower MC’s provide the most complete description. The full information about the event is available, in terms of physical hadrons. So this is the only tool that allows realistic detector simulations, crucial when it comes to measuring with precision some quantity, such as a particle mass or a coupling strength. In the case of PL MC’s only the information on the hard partons is given; this is good enough for studies with naive detector simulations based on pure geometry and exact energy resolution. This is an excellent tool to quickly study systematics such as the PDF dependence of an acceptance, but cannot be used, for example, as a tool to reconstruct the top quark mass from real $W+4$ jet events. Cross-section evaluators typically don’t provide much information on the structure of the final state.

3.2 Higher-order corrections: real, hard emissions

In the shower MC’s, emissions are treated within the collinear or soft approximation. In particular, the angular-ordering prescription, introduced to implement quantum coherence effects for large-angle gluon radiation, will suppress hard large-angle emissions as well. Since large-angle hard emissions are not constrained by angular ordering, the shower approximation will typically underestimate the rate of multijet final states. On the contrary, PL MC’s have all diagram interference terms among multiple hard partons taken into account, and provide the best tool to describe multijet final states.

3.3 Higher-order corrections: virtual effects

Virtual diagrams are included in the soft approximation in the shower MC’s. They are needed to cancel
the infrared (IR) singularities due to the real emission of soft gluons, and appear through the Sudakov form factors to enforce the unitarity of the shower evolution. Virtual effects are exactly included in all NLO cross-section evaluators. In addition to the exact treatment of real emission, NLO (or NNLO) cross-section evaluators provide the most accurate determination of inclusive rates. In order to implement them in ME event generators, however, one must deal with the problem of negative-weight events. These arise to enforce the cancellation of the positive-weight infinities present in the soft and collinear real emission diagrams, with negative-weight infinities associated to the interference of the virtual and tree-level diagrams.

3.4 Resummations

Resummation [32, 4, 10] of leading and subleading logarithms appearing at all orders of perturbation theory when largely different energy scales appear in the event kinematics (for example small-$p_T$ production of heavy objects, with $M \gg p_T$, or large-$p_T$ production of light objects, $p_T \gg m$), is possible in the context of cross-section evaluators. The resummation of logarithms corresponds to the integration over multiple-emission and multi-loop diagrams, and is therefore, by its nature, an inclusive calculation. As a result, so far no ME generator includes them, although new analytical [33, 34] and semi-numerical [35] approaches have been proposed to provide resummation corrections to arbitrary multi-parton final states. Shower MC’s incorporate resummations via the multiple emissions taking place in the shower, and the corresponding Sudakov form factors, which account for the multi-loops. However, due to the unitarity of the evolution, these resummations cannot affect the overall rate of a process. The resummation of threshold-like logarithms (such as those entering in the determination of the $t\bar{t}$ cross-section [36]) affect the rates, and so far are only included in cross-section evaluator codes. Resummation of small-$x$ effects [37], finally, is implemented in some shower MC’s [38, 39], mostly developed for HERA studies, and is not yet a standard component of LHC tools.

4 A Monte-Carlo of Everything

The efforts of the past few years have improved considerably the flexibility of the above tools, and have allowed the construction of MC codes which merge the merits of different approaches. The main lines of development are summarised, in a very simplified way, in this section.

4.1 Inclusion of NLO corrections in ME generators

At NLO one needs to account for both real-emission diagrams and virtual corrections. Virtual corrections to an N-body FS give still an N-body FS, while real emissions lead to an (N+1)-body FS. An NLO event generator must therefore generate both N-body and (N+1)-body FS’s. The two however cannot be treated separately, since the contribution of the virtual part to the cross-section is equal to minus infinity, while the real part has divergencies for collinear and soft configurations. The structure of the cross-section in these regions can be parameterized in an idealized form as follows:

$$\frac{d\sigma}{dx} = \frac{f(x)}{1-x^+},$$

where by definition of $(x^+)$:

$$\int_0^1 \frac{dx}{1-x} \frac{d\sigma}{dx} = \int_0^1 \frac{f(x) - f(1)}{(1-x)}$$

The limit $x \to 1$ corresponds to kinematical configurations where the (N+1)-body FS degenerates (via soft or collinear emission) to an N-body FS. In the case of collinear emission, for example, $x = \cos \theta$. For heavy quark ($Q$) pair production, $x = m_{Q+\bar{Q}}^2/s \to 1$ corresponds to the soft-emission limit. $f(x)$ is continuous in $x = 1$, and therefore any integral over an arbitrary range of $x$, including possibly $x = 1$, is finite. This is the essence of the virtual/real cancellation of infinities. If we formally remove the integration from eq. [4] we can “define” a non-singular differential cross-section as:

$$\frac{d\sigma}{dx} = \frac{f(x) - f(1)}{(1-x)}.$$  

One can interpret this relation as follows: for any given (N+1)-body kinematical configuration $C(x)$ ($x \neq 1$), whose weight is given by $f(x)/1-x$, we can associate an N-body virtual configuration $C(1)$ with weight $-f(1)/1-x$, whose kinematics is obtained by the $x \to 1$ limit of $C(x)$. These virtual configurations are typically called “counter-events”. One can construct an event generator by adding to each (N+1)-body final state its corresponding N-body counter-event. Both events will have a finite weight, although the counter-event’s is negative. Since their kinematics is different, event and counter-event will typically populate different bins of our histograms. For bin sizes sufficiently large (namely for observables sufficiently inclusive), summing events over the full phase-space will nevertheless lead to positive rates in each bin.
not guaranteed to happen when the bins are too small. For example, let us consider the integral of the cross-section in a bin covering the range $1 - \epsilon < x < 1$, with $\epsilon$ small:

$$\int_{1-\epsilon}^{1} d\sigma = \int_{1-\epsilon}^{1} dx \frac{f(x)}{(1-x)} - \int_{1-\epsilon}^{1} dx \frac{f(1)}{(1-x)} . \quad (6)$$

The first contribution is from the $N+1$-body FS's, the second from the counter-events which all accumulate at $x = 1$. Simple algebra leads to the following result:

$$\int_{1-\epsilon}^{1} d\sigma = C + f(1) \log \epsilon , \quad (7)$$

where $C$ is a finite constant when $\epsilon \to 0$. Since $f(1)$ is positive, when the bin-size $\epsilon$ is small enough the integral becomes negative. This is an indication that radiative corrections in this small corner of phase-space are large. In other words, if we try to push the calculation in a region of low-inclusivity, probing the final-state structure with very fine resolution, the fixed-order perturbative approximation breaks down. Higher-order corrections become very large, and have to be included to restore the positivity of the cross-section in that bin. The smaller the bin, the larger the number of orders required.

In concrete applications life is complicated by the identification of the functions $f(x)$, the interplay of soft and colliner singularities, and the description of the phase-space in terms of suitable variables; however the above simplified description captures the main features of the technique. It has been used, in various different implementations, for the development of NLO ME event generators covering several of the interesting LHC processes. Examples include: 2-jets \[42\] and 3-jets \[41\], heavy quarks \[42\], and vector boson \[43\] production. For a detailed list of available tools, see \[\] \[44\]. The extension of these techniques to NNLO calculations has yet to be formulated. Work is in progress\[\] \[44\], but the difficulty is immense, and it will still be some time before we can implement the already known NNLO matrix elements for DY and Higgs production into an event generator.

### 4.2 NLO corrections in shower MC’s

The necessity to include NLO corrections in ME generators is twofold. On one side, only shower MC’s provide a representation of the final state complete enough to allow realistic detector simulations. Inclusion of the NLO matrix elements for the hard process, which will provide cross-sections with full NLO accuracy, is a natural improvement of these essential tools. On the other hand, as mentioned in the previous subsection, the inclusion of NLO effects in fixed-order ME MC’s leads to distributions which are not positive definite, thus calling for a tool where these large (and possibly negative) logarithmic effects which arise at any fixed order in some corners of phase-space can be properly resummed. This goal can be achieved via the inclusion of the NLO ME’s in the shower MC. A priori one may expect this task to be ill defined, as shower MC’s already incorporate part of the NLO effects: they have real emissions, as well as virtual effects included in the Sudakov form factors. The naive introduction of NLO ME’s would then lead to double counting. This is what kept people skeptical for many years about the viability of a NLO shower MC. Brilliant work by Frixione and Webber \[15\] recently showed how this merging can be done very effectively in what they called a MC@NLO. One starts by identifying the analytic form of the approximation used by the shower MC to describe real emission and the leading-order virtual correction contained in the Sudakov form factor. One can then subtract these expressions from the NLO matrix elements; since the shower approximation has the correct residue for all singular contributions, the subtracted NLO matrix elements are finite. In the simple formalism used in the previous section, one can represent the subtraction from the real emission term as follows:

$$\frac{d\sigma}{dx} = \frac{f(x) - f_{MC}(x)}{(1-x)} , \quad (8)$$

where $f_{MC}(x)$ is the approximate MC expression for the real emission matrix element, with the condition that $f_{MC}(1) = f(1)$. In this way the $x \to 1$ singularity is not removed by the merging with the virtual correction, but by letting the shower algorithm handle it and absorb it into the Sudakov form factor. As for the virtual part, the singular contribution is all contained in the shower approximation, and what is left for the NLO correction to describe is just a finite term with the N-body, Born-like, kinematics. One is still left with positive and negative weight events, since the difference between the exact non-singular terms from the full NLO calculation and those used in the shower can have either sign. However, since the residual positive and negative weights are bounded, one can define an unweighting procedure whereby positive-weight events are unweighted against the maximum positive weight, and negative-weight events are unweighted against the minimum negative weight. This procedure has been successfully implemented in MC@NLO codes \[15\] \[40\] describing heavy-quark pair, Higgs, DY and gauge boson pair production. There is no obstacle to extending it to all remaining processes known at NLO. Other approaches have also been developed into alternative codes for DY \[47\] and vector boson pair \[48\] production.
The inclusion of NLO corrections in the shower MC guarantees that total cross-sections generated by the MC reproduce those of the NLO ME calculation, thereby properly including the K factors and reducing the systematic uncertainties induced by renormalization and factorization scale variations. At the same time, however, the presence of the higher-order corrections generated by the shower will improve the description of the NLO distributions, leading to departures from the parton-level NLO result. This is shown, for example, in fig. 1 which shows the $p_T$ spectrum of a $t\bar{t}$ pair resulting from the pure NLO calculation, from the LO shower, and from the MC@NLO improvement. At large $p_T$, a region dominated by the NLO effects, MC@NLO faithfully reproduces the hard, large-angle emission distribution given by the NLO matrix elements. At small $p_T$, a region dominated by multiple radiation and higher-order effects, the MC@NLO departs significantly from the NLO result, while properly incorporating the Sudakov resummation effects only available via the IS shower evolution.

![Figure 1](image-url) Transverse momentum distribution of top quark pairs using three different approaches: the LO HERWIG MC, the parton-level NLO MC, and the merging of the two into MC@NLO. Figure from [46].

### 4.3 Merging multijet ME generators and shower evolution

The inclusion of NLO ME’s in the shower MC’s guarantees the correct description of the emission of one extra hard parton (ultimately a jet) from the Born, LO process. In the case of NLO corrections to the dijet final states, for example, this means that all topologies with up to three jets will be accurately described. To go beyond this in a NLO framework, however, requires the knowledge of NLO ME’s which are not available today, and won’t be for a longtime to come in the case of the highest multijet FS’s which are of interest for several LHC studies. As mentioned earlier, the description of multijets obtained from the shower evolution is inaccurate, since hard radiation at large angle is suppressed by the angular ordering prescription. One therefore needs an approach in which multi-parton events generated using the exact LO ME generator can be consistently evolved into multi-jet FS’s via a shower MC. As in the case of the inclusion of NLO corrections, the main problem to be dealt with is that of double counting. I shall describe this problem by discussing a specific example [10]. Consider inclusive production of 3-jet events, with jets defined by a cone of size $R_j$ in $\eta - \phi$ space and transverse energy larger than $E_T^{min}$. We can generate these events by generating parton-level configurations with partons separated by $\Delta R > R_j$, and let the shower evolve them into jets. By and large, there will be a one-to-one correspondence between the generated hard partons and the jets, and the angular distributions of the three jets will include correctly all interference effects among the various diagrams. There will be configurations, however, where the correspondence is not guaranteed. Take for example events where two jets have $E_T$ larger than the third, and the third itself is well above the $E_T^{min}$ threshold: $E_{T1} \sim E_{T2} > E_{T3} \gg E_{T3}^{min}$. These events can be generated in two independent ways. On one side we can start from three partons with the kinematics of the three jets. After evolution, the partons will generate the desired jets. On the other, we can start from configurations where the two leading partons have energies of the order of $E_{T1}$ and $E_{T2}$, but the third can be as soft as $E_{T3}^{min}$. These events will typically not have a third jet with $E_T \sim E_{T3}$, since the generated parton will be softer. However, hard radiation by one of the two leading jets may lead, with probability $\alpha_s$, to the generation of an extra jet with the desired energy. In other words, the shower evolution in these cases could produce jets with transverse energy larger than that of partons already present in the hard event. While the probability of this happening is parametrically of order $\alpha_s$ relative to the LO process, and could therefore be considered a higher-order correction compatible with the LO approximation of the ME calculation, configurations with two hard partons and a much softer one are enhanced by large logarithms. This is the result of the large phase-space available for the emission of a soft parton in a hard event. One can estimate that this enhancement is of order $\log(E_{T3}/E_{T2}^{min})$. As a result the overall probability that the third jet be emitted from the shower becomes a number of order $\alpha_s \log(E_{T3}/E_{T2}^{min})$. On one side this could be numerically of order 1, therefore turning an NLO effect into a LO one. On the other,
this dependence on $E_T^{\min}$ of the cross-section for 3-jet events well above the $E_T^{\min}$ threshold is a totally unphysical result! This paradox is caused by the double counting of equivalent configurations which this approach gives rise to. To solve the paradox, and remove the $E_T^{\min}$ dependence, one has to carefully check that a given phase-space configuration is generated only once: either directly by the parton-level event, or by the shower evolution.

Some approaches to this problem, aiming at different levels of accuracy, have been introduced recently. The first \[51, 29\] is generically known as “matrix-element correction” technique (MEC), as it corrects the approximate ME for the emission of the hardest gluon in a given process by using the exact LO ME. The second is known as CKKW \[51\]; its goal is to implement multi-jet ME corrections at the leading (LL), or next-to-leading (NLL) logarithmic level. In the MEC, one starts by identifying analytically the phase-space region $\Omega$ covered by the shower algorithm. In the case of $e^+e^- \to gq\bar{q}$, for example, this is given by a subset of the full phase-space domain $\Delta$ defined by

$$\Delta = \{1 \leq x_1 + x_2 \leq 2 \} \cap \{x_i \leq 1\},$$

where $x_i = 2E_i/\sqrt{s}$. $\Omega$ contains the singular regions corresponding to soft ($x_i = 0$) and collinear ($x_i = 1$) gluon emission. The integral $\sigma_\Omega$ of the 3-body cross-section over the complement of $\Omega$, $\Omega^0 = \Delta - \Omega$, is therefore free of singularities and finite. The integral over $\Omega$, $\sigma_{\Omega}$, is also finite, once the virtual corrections at the edge of phase-space are included. The MEC MC generation then works by deciding on an event-by-event basis whether to generate the event in $\Omega^0$ or in $\Omega$, based on the relative value of the respective cross-sections. If the event falls in $\Omega$ one generates a LO $e^+e^- \to gq\bar{q}$ event. If it falls in $\Omega^0$, one generates a $e^+e^- \to q\bar{q}g$ event. In both cases, the events are then evolved through the shower. Small adjustments should then be made in the first case to ensure a proper continuity across the boundary between the two domains. This technique has been applied also to DY production \[52\], and to top decays \[53\]. Its extension to more complicated processes, however, is made particularly difficult by the need to provide an analytic description of $\Omega$. When there are more than 3 coloured partons in the process (as e.g. in dijet or heavy quark pair production at the LHC), this becomes very hard and impractical. The CKKW approach circumvents this problem by limiting its precision goal to a LL accuracy (NLL for $e^+e^- \to$ multijets). In this approach the double counting is removed not by exactly separating a priori the domains $\Omega$ and $\Omega^0$, but by a probabilistic rejection procedure applied to events falling in the overlap, so as to ensure that a given phase-space configuration is only counted once. One starts by generating samples of multi-parton events of different multiplicities, using the exact LO ME. The generation is carried out in a phase-space domain defined by a Durham-like jet algorithm suitable for hadronic collisions \[54\]: a resolution variable for two partons is defined by:

$$k_{ij} = \min(E_{T,i}, E_{T,j})R_{ij},$$

if both $i$ and $j$ are in the FS, or by $k_{ij} = E_{T,i}$ if $j$ is in the IS. $R_{ij}$ is a measure of separation in the transverse plane, for example the standard $\sqrt{\Delta x^2 + \Delta y^2}$ measure. An N-parton FS is then classified as an N-jet event if $k_{ab} > k_{cut}$ for all possible parton pairings. $k_{cut}$ is a resolution threshold, introduced as a parameter necessary to separate the generation of the events in samples of different jet multiplicity; the final cross-section, however, should be independent of its specific values. Events are extracted from the different N-jet samples with probability proportional to the sample cross-section. The jet algorithm can then be used to define a tree structure for the event. The two partons $i, j$ with the smallest $k_{ij}$ are clustered into a single virtual parton $\ell$, provided the parton types and flavours of $i$ and $j$ can be merged. The procedure is repeated after removing $i, j$ from and adding $\ell$ to the list of partons. The clustering continues until one gets a $2 \to 1$ or $2 \to 2$ process. The resulting tree is then interpreted as a shower configuration, but with a weight given by the exact ME. To be used as an exclusive FS, and to allow the successive evolution via the shower, the event weight needs to be corrected with the inclusion of Sudakov form factors for each vertex, and by a rescaling of the values of $\alpha_s$ reflecting the choice of scale for $\alpha_s$ made by the shower. This reweighting factor, which can be constructed so as to always be smaller than 1, is then used as a probability to keep or reject the event. The events kept are then showered, and shower splittings at a scale larger than $k_{cut}$ are vetoed, to avoid the duplication of configurations which will otherwise be present in ME samples corresponding to higher $N$. In the case of $e^+e^-$ collisions \[51, 55\] one can prove that this algorithm correctly reproduces the weight of an event to NLL accuracy, and to rates independent of $k_{cut}$. In hadron collisions \[56\] such a proof is still missing, but this framework provides a very good starting point for further developments. The results of the first studies in the case of $W$+jets production can be found in \[57\].

An additional improvement in the description of multijet final states may soon come from the development of new shower algorithms, being developed in the context of the new generation of C++ codes, such as HERWIG++ \[55\] and SHERPA \[50\]. Preliminary results for $e^+e^-$ collisions, comparisons with LEP/SLC data and examples of the improved description of multijet final states, can be found in \[58\]. Similar work is under way for the C++ version of PYTHIA \[60\].
5 Conclusions

I hope to have succeeded in giving a flavour of the big progress and flourishing of new ideas which is currently taking place in the field of MC development for hadronic collisions. Tools only dreamt of few years ago are now available, encouraging and justifying greater ambitions. Progress is also taking place in other directions which I have not had time to cover, such as the progress in the evaluation of NNLO matrix elements \cite{13}, in the description of power corrections \cite{61}, underlying event and multiple interactions \cite{62}, hadronization \cite{63}, as well as on the uniformization of input/output formats for the merging of PL and shower generators \cite{64}. The validation of MC tools against Tevatron data is also an area of active and successful research \cite{65,66}. Finally, great progress has been achieved in the determination of parton densities and their systematic uncertainties \cite{67}, an essential input for all event simulations. Hoping that the progress will continue, and that all of these tools will receive proper validation using the results available from the Tevatron, I look forward to their testing at the LHC.

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