Nonequilibrium transition induced by mass media in a model for social influence

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We study the effect of mass media, modeled as an applied external field, on a social system based on Axelrod’s model for the dissemination of culture. The numerical simulations show that the system undergoes a nonequilibrium phase transition between an ordered phase (homogeneous culture) specified by the mass media and a disordered (culturally fragmented) one. The critical boundary separating these phases is calculated on the parameter space of the system, given by the intensity of the mass media influence and the number of options per cultural attribute. Counterintuitively, mass media can induce cultural diversity when its intensity is above some threshold value. The nature of the phase transition changes from continuous to discontinuous at some critical value of the number of options.

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In recent years, the research on complex systems has extended to social science in order to understand how collective behaviors arise in social systems. Several mathematical models, inspired by analogies with physical systems, have been proposed to describe a variety of phenomena occurring in social dynamics\textsuperscript{[1, 2, 3, 4]}. Processes such as self-organization, cooperation, epidemic spreading, opinion formation, propagation of information, economic exchanges and evolution of social structures have been studied by means of discrete-time, discrete-space dynamical systems. In this context, there has been interest in the model introduced by Axelrod\textsuperscript{[5]} to investigate the dissemination of culture among interacting agents in a social system\textsuperscript{[6, 7, 8, 9, 10, 11, 12, 13]}. From the point of view of statistical physics, this model is appealing because it exhibits nontrivial out of equilibrium dynamics, as in other well studied systems with phase ordering properties\textsuperscript{[13]}. Studies on this model have mainly focused on the collective properties that result from the interactions between the elements representing endogenous social influences.

In this paper we investigate the effect of external cultural influences such as mass media on a social system. Our approach is based on the adaptive interaction dynamics of Axelrod’s model for the dissemination of culture. Agents can interact with their neighbors in the system and with the mass media according to the cultural similarities that they share, in each case. The concept of culture is intended here as a set of individual features or attributes that are subject to social or external influence. The numerical simulations show that, depending on the value of a parameter that represents the intensity of the mass media influence and on the number of options available per cultural attribute, the system displays a phase transition between an specific ordered phase (a homogeneous culture) imposed by the mass media and a disordered (culturally fragmented) phase. Surprisingly, mass media can induce cultural diversity when its intensity is above some threshold value. The nature of this transition changes from continuous to discontinuous when the number of options per cultural feature is increased.

The model consists of $N$ agents as the sites of a square lattice. The cultural state $c_i$ of agent $i$ is defined as a vector of $F$ components (cultural features) $c_i = (\sigma_{i1}, \sigma_{i2}, \ldots, \sigma_{iF})$. Each $\sigma_{ij}$ can take any of the $q$ values in the set $\{0, 1, \ldots, q-1\}$ (cultural traits), initially assigned randomly with a uniform distribution. There are $q^F$ possible cultural vectors. We define a mass media cultural message as a vector $M = (\mu_1, \mu_2, \ldots, \mu_F)$, where $\mu_f \in \{0, 1, \ldots, q-1\}$, that can interact with any of the agents in the system. We also define a parameter $B \in [0, 1]$ that measures the relative intensity of the mass media message with respect to the local interactions, or the probability that this message has to attract the attention of the agents in the system. The parameter $B$ represents enhancing factors of the transmitted message that can be varied externally, such as its amplitude, frequency, attractiveness, etc. It is assumed that $B$ is uniform, i.e., the mass media message reaches all the agents with the same intensity, as a uniform field. At any given time, we assume that any agent can either interact with the mass media message or with other agents in the system. Thus each agent in the network possesses a probability $B$ of interacting with the message and a probability $(1 - B)$ of interacting with its neighbors.

The discrete-time dynamics of the system subject to the mass media influence is defined by iterating the following steps:

1. Select at random an element $i$ in the lattice (active element) having a cultural state $c_i$. Its attention is drawn to interact with the message $M$ with probability $B$.

2. If the attention of element $i$ is drawn to the message, then

   (2) Calculate the cultural overlap (number of shared features) between the active element and the message $l(i, M) = \sum_{f=1}^{F} \delta_{\sigma_{if}, \mu_f}$.

3. If $0 < l(i, M) < F$, the element $i$ and the message interact with probability $l(i, M)/F$. In case of interaction, choose $h$ randomly such that $\sigma_{ih} \neq \mu_h$ and set $\sigma_{ih} = \mu_h$. 


If the attention of element $i$ is not caught by the message, then

(4) Select at random a site $j$ in the nearest neighborhood of site $i$.

(5) Calculate the cultural overlap $l(i,j) = \sum_{i=1}^{F} \delta_{\sigma_{i},\sigma_{j}}$.

(6) If $0 < l(i,j) < F$, sites $i$ and $j$ interact with probability $l(i,j)/F$. In case of interaction, choose $h$ randomly such that $\sigma_{ih} \neq \sigma_{jh}$ and set $\sigma_{ih} = \sigma_{jh}$.

In this model, the fixed mass media cultural message $M$ can interact with any element in the system with the same intensity or probability $B$, but the effect of that interaction may be different on each element, depending on the specific cultural overlap between the element and the message. For simplicity, we are assuming a fixed mass media cultural message acting uniformly over the system, as for example in a global broadcasting; however several variations of this condition can be implemented by extending this basic algorithm. The case $B = 0$ corresponds to the original Axelrod's model.

In any finite network the dynamics settles into an absorbing or frozen state, characterized by either $l(i,j) = 0$ or $l(i,j) = F$, $\forall i,j$. Homogeneous or multicultural states correspond to $l(i,j) = F$, $\forall i,j$, and obviously there are $q^F$ possible configurations of this state. Inhomogeneous or multicultural states consist of two or more homogeneous domains interconnected by elements with zero overlap. A domain is a set of contiguous sites with identical cultural traits. In the absence of external influences it has been shown that the system reaches ordered, multicultural states for $q < q_o$, and disordered, multicultural states for $q > q_o$, where $q_o$ is a critical value that depends on $F$ [55]. This order-disorder phase transition becomes better defined as $N$ increases and it has been argued that it is of first order in two-dimensional systems [6]. While this transition is of second order in one-dimensional systems [42]. The critical value of the number of traits for $F = 10$ has been numerically estimated at $q_o \approx 55$ in two dimensions [6].

When the mass media cultural influence is applied to the system, the order-disorder phase transition persists, but the critical value $q_o$ for which the transition takes place decreases as the intensity of the message $B$ is increased. Figure 1 shows the spatial configurations of the final states of the system subjected to mass media, with $F = 10$ and $q = 35 < q_o$. In the absence of external message, i.e. $B = 0$, the system settles into any of the possible $q^F$ multicultural states. When the intensity of the message is increased, the system is driven towards a multicultural state equal to the state of the mass media, i.e., $c_f = M$, $\forall i$. However, there is a critical value of the intensity $B_c \approx 0.05$ above which the system no longer converges to the state of the message $M$ but reaches a multicultural state consisting of an increasing number of domains as $B$ is increased. Domains having a cultural state equal to the mass media survive above the threshold intensity $B_c$, but they become smaller in size as $B$ is increased. Thus, we find the counterintuitive result that, above some threshold value of intensity, mass media actually promotes cultural diversity in the system.

FIG. 1: Spatial patterns for different values of the intensity $B$, for $F = 10$, $q = 35$, and $N = 50 \times 50$. The color code of the mass media state $M$ is white. Top left: $B = 0$; top right: $B = 0.005$; bottom left: $B = 0.1$; bottom right: $B = 0.9$.

In order to characterize the transition from the monocultural state imposed by the message $M$ to a multicultural state when the intensity $B$ is varied, we consider as an order parameter the average fraction of cultural domains $g = \langle N_g \rangle / N$, where $N_g$ is the number of domains formed in the final state for a given realization of initial conditions [12]. Figure 2 shows $g$ as a function of $B$ for different values of the number of traits $q$. For values of $q > q_o$, the system always reaches a multicultural state (with $\langle N_g \rangle > 1$), independently of the intensity $B$ of the message. On the other hand, for each value of $q < q_o$, we observe a transition at a critical value $B_c$, from the homogeneous cultural state imposed by the message, characterized by values $q < 1$, to a multicultural state for which $q$ increases with $B$. The critical value $B_c$ decreases with increasing $q$ for $q < q_o$. The variation of the order parameter $g$ near the value $B_c$ can be characterized by a critical exponent $\beta(q)$ as the scaling relation $g \sim (B - B_c)^{\beta(q)}$. Figure 2 also shows log-log plots of $g$ vs. $(B - B_c)$ for different values of $q < q_o$. The exponent $\beta$ as a function of $q$ can be calculated from the slope of each curve.

In Fig. 3 we show the resulting graph of $\beta$ vs. $q$, for $q < q_o$ corresponding to $F = 10$. The dependence of the exponent $\beta$ with $q$ is well accounted by the linear relation $\beta(q) \propto (q_o - q)$. As $q$ increases towards the value $q_o$, the exponent $\beta$ becomes smaller and the corresponding phase transition from the monocultural state induced by the message $M$ to multicultural state gets more abrupt. The character of the phase transition changes from higher order ($\beta > 1$) to second order ($0 < \beta < 1$) at $q \approx 30$. At the value $q = q_o$ for which the exponent $\beta$ vanishes, the character of the transition should change from second
order to first order. Figure 3 shows the extrapolation of the straight line $\beta(q)$ until its intersection with the $q$ axis, predicting a critical value $q_o = 54$ for the occurrence of a first order transition.

The critical values of the intensity of the message $B_c$ as a function of $q$, calculated from Fig. 2, are shown in Fig. 4. The points in Fig. 4 are well fitted by the relation $B_c \propto (e^{-k_1 q} - e^{-k_2 q_o})$, which yields $B_c = 0$ for $q = q_o = 54$ as indicated in that figure. Thus, in absence of mass media, the critical value of the number of traits for the first order phase transition from cultural homogeneity to a multicultural state predicted by our model is $q_o = 54$. This agrees with the approximated critical value obtained numerically in Ref. [8] for the order-disorder transition in the original Axelrod model with $F = 10$. The critical curve $B_c$ vs. $q$ in Fig. 4 separates the multicultural region from the region where the monocultural state induced by the mass media occurs on the space of parameters $(B, q)$. This critical curve reflects the competition between the tendency towards the specific ordering imposed by the mass media and the disorder present in the initial configuration of the system, represented by the number of options per feature $q$. Figure 4 reveals that the ability of the mass media message to transmit its state on the entire system decreases as the number of options per cultural feature increases. Above this threshold curve, the mass media is not longer able to drive the system towards homogeneity, and a multicultural state sets in. The above results suggest that the order parameter $g$ behaves, near the critical boundary, as $g(B, q) \sim [B - B_c (e^{-k_1 q} - e^{-k_2 q_o})]^{k_3 (q_o - q)}$, where $k_1$, $k_2$ and $k_3$ are constants.

In the presence of the mass media message, the system reaches frozen configurations of each phase (homogeneous or multicultural) much faster than in the original Axelrod model having only local interactions. In Fig. 5 we show a log-log plot of the average time $\tau$ to reach a frozen configuration as a function of the intensity $B$, for several values of $q < q_o$. For $B < B_c$, the time $\tau$ decays according $\propto \tau \sim B^{-0.95}$, independently of $q$ for $q < q_o$ (where the exponent is determined from the slopes of the curves in Fig. 5). The times for convergence to the monocultural state induced by the message are much longer than the corresponding times to reach a multicultural state beyond the critical boundary in Fig. 4. Therefore, for values of intensity $B < B_c$ the mass media message is able to transmit its cultural state to all the elements in the system, but the process takes longer times than the partial convergence to the message state achieved when $B > B_c$. 

FIG. 2: Order parameter $g$ as a function of the intensity $B$ for different values of $q$, with $F = 10$. Error bars indicate typical standard deviations about mean values obtained over 10 realizations for each value of $B$. The inset shows a log-log plot of $g$ vs. $(B - B_c)$ for different values of $q$. Size $N = 50 \times 50$. Values of $q$ are 20 (squares), 30 (diamonds), 45 (circles), and 60 (triangles).

FIG. 3: Critical exponent $\beta$ for the phase transition induced by mass media, as a function of $q$.

FIG. 4: Critical boundary $B_c$ vs. $q$ separating the region of the monocultural phase induced by mass media from that of multicultural states, as indicated. The dashed line corresponds to the fitting of the points. Fixed $F = 10$. 

FIG. 5: Log-log plot of the average time $\tau$ to reach a frozen configuration as a function of the intensity $B$, for several values of $q < q_o$. For $B < B_c$, the time $\tau$ decays according $\propto \tau \sim B^{-0.95}$, independently of $q$. (where the exponent is determined from the slopes of the curves in Fig. 5).
The interaction with the mass media message $M$ accelerates the convergence of those vectors $c_i$ in the system that possess an overlap $0 < l(i, M) < F$ towards the cultural state prescribed by $M$. The dynamics of the interaction with the mass media is essentially a coarsening process of such domains. For values of $B$ and $q$ below the critical boundary, this process takes longer times and allows enough local interactions to take place and spread the cultural state of the message, leading to the growth and coalescence of domains with the state $M$ into a single domain of size comparable to that of the whole system. For values $(B, q)$ above the critical boundary, the process of convergence to the message and the formation of domains with this cultural vector occurs faster; this contributes to the early distinction of the domains with state $M$ from the other domains and to limit their growth. Thus, the rapid convergence towards the mass media message when its intensity is above a threshold value ends up inducing cultural diversity in the system. The transition between the two phases in Fig. 4 can also be seen as separating two different dynamical regimes; one having a fast decay time for values $(B, q)$ above the critical boundary and another characterized by a slow decay for values $(B, q)$ below that boundary.

In summary, we have presented a model for the influence of mass media on a social system as an extension of Axelrod’s model for the dissemination of culture. The mass media cultural message has been assumed as a fixed vector acting uniformly over the system. We have found the nontrivial result that mass media can actually induce cultural diversity in conditions for which it was absent, as for values $q < q_o$ in the original Axelrod’s model. We have verified that this effect persists in lattices with periodic boundary conditions or with different local geometries.

We have calculated the critical boundary in the space of parameters $(B, q)$ that separates the monocultural phase induced by the mass media message from the multicultural phase. These two phases correspond to different dynamical regimes of the system as it evolves towards its frozen final state; the first having a slow decay and the second characterized by a fast decay. The character of the phase transition changes progressively from continuous to discontinuous as the number of traits per feature is increased. The critical value $q_o$ for the onset of the first order phase transition was predicted from the linear dependence observed in the critical exponent $\beta$ vs. $q$, as well as from the curve fitting the critical boundary. The predicted value $q_o$ agrees with the value estimated previously by numerical simulations in the original Axelrod’s model. The scaling behavior of the order parameter $g$ characterizing the transition near the critical boundary was also found numerically. Future extensions of this basic model should include the consideration of mass media messages varying in time and/or space, competing messages, noise, and more complex networks of interactions.

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[1] W. Weidlich, Phys. Rep. 204, 1 (1991).
[2] S. M. de Oliveira, P. M. C. de Oliveira, and D. Stauffer, Nontraditional Applications of Computational Statistical Physics (B. G. Teubner, Stuttgart, 1999).
[3] P.W. Anderson, K. Arrow, and D. Pines, The Economy as an Evolving Complex System (Addison-Wesley, Redwood, 1998).
[4] M. San Miguel, V. Eguiluz, R. Toral, and K. Klemm, Computing in Science and Engineering 7, 67 (2005).
[5] R. Axelrod, J. Conflict Res. 41, 203 (1997).
[6] C. Castellano, M. Marsili, and A. Vespignani, Phys. Rev. Lett. 85, 3536 (2000).
[7] D. Vilone, A. Vespignani, and C. Castellano, Eur. Phys. J. B 30, 299 (2002).
[8] K. Klemm, V. M. Eguiluz, R. Toral, and M. San Miguel, Phys. Rev. E 67, 026120 (2003).
[9] K. Klemm, V. M. Eguiluz, R. Toral, and M. San Miguel, Phys. Rev. E 67, 045101(R) (2003).
[10] Y. Shibanai, S. Yasuno, and I. Ishiguro, J. Conflict Res. 45, 80 (2001).
[11] K. Klemm, V. Eguiluz, R. Toral, M. San Miguel, Physica A 327, 1 (2003).
[12] The order parameter $g$ measures the degree of multiculturality as defined by Axelrod (number of cultural domains) in $\mathbb{R}^d$. A related order parameter used in $\mathbb{R}^d$ is the normalized average size of the largest domain, $\langle S_{\max} \rangle / N$. Homogeneity corresponds to $\langle S_{\max} \rangle / N \approx 1$, while complete disorder is given by $\langle S_{\max} \rangle / N \ll 1$, for large $N$.
[13] I. Dornic, H. Chaté, J. Chave, and H. Hinrichsen, Phys.
Rev. Lett. 87, 045701 (2001).