Effect of three-body forces on response functions in infinite neutron matter

D. Davesne, J. W. Holt, A. Pastore, and J. Navarro

1Université de Lyon, F-69003 Lyon, France; Université Lyon 1, 43 Bd. du 11 Novembre 1918, F-69622 Villeurbanne cedex, France
CNRS-IN2P3, UMR 5822, Institut de Physique Nucléaire de Lyon
2Department of Physics, University of Washington, Seattle, WA 98195, USA
3Institut d’Astronomie et d’Astrophysique, CP 226, Université Libre de Bruxelles, B-1050 Bruxelles, Belgium
4IFIC (CSIC-Universidad de Valencia), Apartado Postal 22085, E-46.071-Valencia, Spain

(Dated: November 13, 2014)

We study the impact of three-body forces on the response functions of cold neutron matter. These response functions are determined in the random phase approximation (RPA) from a residual interaction expressed in terms of Landau parameters. Special attention is paid to the non-central part, including all terms allowed by the relevant symmetries. Using Landau parameters derived from realistic nuclear two- and three-body forces grounded in chiral effective field theory, we find that the three-body term has a strong impact on the excited states of the system and in the static and long-wavelength limit of the response functions for which a new exact formula is established.

PACS numbers: 21.30.Fe 21.60.Jz 21.65.-f 21.65.Mn

I. INTRODUCTION

The low-energy, long-wavelength dynamics of neutron matter are encoded in a set of response functions characterizing the coupling of the strongly-interacting medium to probes of various symmetries. Of particular interest in astrophysical applications related to neutron star evolution are the vector and axial-vector response functions governing neutrino and anti-neutrino propagation in dense matter [1–5]. Although three-nucleon forces (3NF) are expected to play a significant role in determining the neutron matter equation of state for densities \( \rho \geq 0.25\rho_0 \) [6–10], their effect on the response functions of neutron matter have received relatively little attention [11].

Linear response theory has already been used to calculate neutron matter response functions using various models of the quasiparticle interaction, including in particular the one deduced from Landau’s theory of Fermi liquids [12]. Landau theory is a powerful effective theory for describing strongly interacting Fermi systems at low temperatures and has been successfully applied to various systems such as liquid \(^3\)He, nuclear matter and finite nuclei. The key quantity arising in the theory is the quasiparticle interaction, which can be conveniently parametrized with a set of Fermi liquid parameters obtained either from phenomenology or microscopic many-body theory. Keeping the most important Landau parameters, the neutron matter response function has been calculated [1–4, 13] in the limit of zero momentum transfer \( q \) but finite \( \omega/q \), with \( \omega \) the transferred energy. This limit has been relaxed in Ref. [12], showing that in a large interval of values of \( q \) and \( \omega \), a rapid convergence is obtained with at most four central and three tensor Landau parameters. In the present work we generalize this formalism to include the effect of additional noncentral components of the quasiparticle interaction depending on the two-particle center-of-mass momentum. These terms are of the same magnitude as the normal exchange tensor contribution and result in strong cancellations in select spin channels. Their effects are therefore qualitatively important in the description of the neutron matter response function.

In the present work we compute the Fermi liquid parameters over a broad range of densities from realistic nuclear two- and three-body forces grounded in chiral effective field theory (\( \chi \)-EFT). The \( \chi \)-EFT approach provides a systematic framework for constructing nuclear forces [10] by exploiting the separation of energy scales in the meson spectrum and by incorporating dynamical constraints from the symmetries and symmetry-breaking pattern of QCD. To better understand the dependence of the response functions on the resolution scale at which nuclear dynamics is resolved and to estimate the theoretical uncertainty in our calculation, we employ chiral low-momentum nuclear interactions [14, 15] with momentum-space cutoffs ranging from 414 – 500 MeV. These potentials have been shown to produce realistic equations of state for both neutron and nuclear matter when treated in perturbation theory [14, 15], which motivates a consistent treatment of the neutron matter response functions via microscopic Fermi liquid theory.

The article is organized as follows: in Sec. [II] we present a description of the microscopic approach to Landau Fermi liquid theory based on high-precision nuclear potentials, and in Sec. [III] we discuss how these results can be used to

*Electronic address: davesne@ipnl.in2p3.fr
construct the relevant response functions in the RPA framework. Finally in Sec. IV we present our conclusions.

II. LANDAU PARAMETERS FROM CHIRAL EFFECTIVE FIELD THEORY

In the present section we discuss how to derive from chiral effective field theory the quasiparticle interaction and the associated Landau parameters in homogeneous neutron matter, with a special focus on the noncentral terms. Within Landau’s theory of Fermi liquids, a strongly interacting system is described in terms of weakly interacting quasiparticles \[16,18\]. The quasiparticle interaction is weak in the sense that low-energy probes excite relatively few quasiparticles, and the theory is then formulated as an expansion in terms of the quasiparticle densities. Although the contribution to the excitation due to explicit three-quasiparticle interactions is thus expected to be small \[17,19\], genuine three-nucleon forces contribute to the two-quasiparticle interaction in the form of a density- or medium-dependent interaction \[20,21\]. Here we describe the microscopic approach based on high-precision two- and three-body chiral nuclear forces, which has the advantage of consistency with constraints from nuclear few-body systems, including nucleon-nucleon elastic scattering phase shifts and deuteron properties.

Due to momentum conservation, a general two-body interaction in the momentum representation depends at most on three momenta. For the particle-hole (ph) case we define the initial and final momenta of the hole to be \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \), and the external momentum transfer in the direct channel is denoted by \( \mathbf{q} \). In the Landau-Migdal approximation \[17\], it is assumed that the low-energy excitations of the system are described by putting the interacting particles and holes on the Fermi surface, that is \( |\mathbf{k}_1| = k_F = |\mathbf{k}_2| \), and \( q = 0 \). In this case the residual interaction between quasiparticles only depends on the relative angle \( \theta_{12} \) between momenta \( \mathbf{k}_1 \) and \( \mathbf{k}_2 \). In pure neutron matter it has the most general form

\[
V_{ph} = f(\theta_{12}) + g(\theta_{12}) (\sigma_1 \cdot \sigma_2) + h(\theta_{12}) S_{12}(\mathbf{k}_{12}) + k(\theta_{12}) S_{12}(\mathbf{P}_{12}) + l(\theta_{12}) A_{12}(\mathbf{k}_{12}, \mathbf{P}_{12}),
\]

(1)

where \( \mathbf{k}_{12} = \mathbf{k}_1 - \mathbf{k}_2 \) is the momentum transfer in the exchange channel and \( \mathbf{P}_{12} = \mathbf{k}_1 + \mathbf{k}_2 \) is the center of mass momentum. We have also defined the usual tensor operator

\[
S_{12}(\mathbf{k}_{12}) = 3(\mathbf{k}_{12} \cdot \sigma_1)(\mathbf{k}_{12} \cdot \sigma_2) - (\sigma_1 \cdot \sigma_2),
\]

(2)

\[
A_{12}(\mathbf{k}_{12}, \mathbf{P}_{12}) = (\sigma_1 \cdot \mathbf{P}_{12})(\sigma_2 \cdot \mathbf{k}_{12}) - (\sigma_1 \cdot \mathbf{k}_{12})(\sigma_2 \cdot \mathbf{P}_{12}),
\]

(3)

which are the center-of-mass tensor and cross-vector interactions. The latter one arises at second-order in perturbation theory from the coupling of spin-orbit terms in the free-space interaction with any other non-spin-orbit term \[21\].

The Landau parameters \( f_\ell, g_\ell, \ldots \) are defined as usual as the coefficients of the expansion of the corresponding \( f(\theta), g(\theta), \ldots \) functions in terms of Legendre polynomials:

\[
f = \sum_\ell f_\ell P_\ell(\mathbf{k}_1 \cdot \mathbf{k}_2), \ldots
\]

(5)

where \( P_\ell \) is the \( \ell \)th Legendre polynomial. It is convenient to use dimensionless parameters, defined as \( F_\ell = N_0 f_\ell, \ldots \), where \( N_0 = n_d k_F m^*/(2\pi^2) \) is the density of quasiparticle states at the Fermi surface \[17,18\] and \( n_d = 2 \) is the spin degeneracy factor. Natural units \((\hbar = c = 1)\) are used throughout this article.

The linear response formalism adopted in our previous calculations \[12\] requires a slightly different definition of the non-central components of the quasiparticle interaction, which are distinguished from the previous ones with a tilde \[24,25\].

\[
V_{ph} = f(\theta_{12}) + g(\theta_{12}) \sigma_1 \cdot \sigma_2 + \tilde{h}(\theta_{12}) \frac{\mathbf{k}_{12}^2}{k_F^2} S_{12}(\mathbf{k}_{12}) + \frac{\tilde{k}(\theta_{12})}{k_F^2} \frac{\mathbf{P}_{12}^2}{2} S_{12}(\mathbf{P}_{12}) + \tilde{l}(\theta_{12}) \frac{\mathbf{k}_{12} \cdot \mathbf{P}_{12}}{k_F^2} A_{12}(\mathbf{k}_{12}, \mathbf{P}_{12}).
\]

(6)

Although both sets of functions \( h, k, l \) and \( \tilde{h}, \tilde{k}, \tilde{l} \) contain the same physical information, their expansions in Legendre polynomials (see Eq. (5)) have different convergence properties, the former converging faster. We shall employ
the Landau parameters with tildes in our calculations, because the factors \( k_{12}^2 P_{12}^2 \) and \( k_{12} \cdot P_{12} \) entering the latter definition are more adapted for our method of obtaining the response function. Therefore, we calculate the Landau parameters according to Eq. (1), but calculate the response functions according to Eq. (6). Landau parameters with and without tildes are related by:

\[
H_\ell = (2\ell + 1) \sum \tilde{H}_{\ell'} \int_{-1}^{1} dx (1-x) P_\ell(x) P_{\ell'}(x)
\]

\[
K_\ell = (2\ell + 1) \sum \tilde{K}_{\ell'} \int_{-1}^{1} dx (1+x) P_\ell(x) P_{\ell'}(x)
\]

\[
L_\ell = (2\ell + 1) \sum \tilde{L}_{\ell'} \int_{-1}^{1} dx \sqrt{1 - x^2} P_\ell(x) P_{\ell'}(x),
\]

where \( x = \cos \theta \). The sum over the right-hand side is infinite. Thus, to switch from one definition to the other, we have to truncate it. In this way, we introduce a small error in the Landau parameters with tildes. In the above equations we have done calculations up to \( \ell_{\text{max}} = 10 \) for both indices \( \ell \) and \( \ell' \), and we have checked that the resulting errors on the first (\( \ell = 0-3 \)) Landau parameters with tildes are negligible.

In Refs. [20, 26] it was found that a microscopic calculation of the Landau parameters including the first- and second-order perturbative contributions from two-body forces as well as the leading-order term from the chiral three-nucleon force led to a good description of the bulk equilibrium properties of symmetric nuclear matter, including the nuclear compression modulus and symmetry energy. In the present study, we employ such an approach for pure neutron matter and study the effect on the response functions due to all terms in the quasiparticle interaction.

It is worth emphasizing that the Landau parameters derived from chiral nuclear potentials depend on the choice of the momentum-space regularization cutoff. In the present article, we focus on the results for the case \( \Lambda = 450 \text{ MeV} \), but other choices are possible. In Table I we give the explicit values of the Landau parameters for three different values of the cutoff: \( \Lambda = 414, 450, 500 \text{ MeV} \). From these values alone, it is difficult to evaluate the importance of the cutoff dependence on the results. Although specific details of the response functions vary with the cutoff, we have checked that the qualitative features (e.g., the position of the maximum) are independent of the choice of cutoff. In Fig. 1 we present the dimensionless Landau parameters for the lowest values of \( \ell \) as a function of the neutron matter density. We observe that the parameters \( \tilde{H}_\ell, \tilde{L}_\ell, \tilde{K}_\ell \) are of the same order of magnitude as the central terms, and it is therefore not possible to discard them based on their absolute magnitude alone.
FIG. 1: (Color online) Left panel: central Landau parameters $F_l$ and $G_l$ associated with the chiral nuclear force at the cutoff scale $\Lambda = 450$ MeV as a function of the density of the system. Right panel: same but for the noncentral Landau parameters $\tilde{H}_l$, $\tilde{K}_l$ and $\tilde{L}_l$. Black circles, red squares and blue diamonds correspond to results obtained when only two-body interactions are included, while solid lines and symbols refer to the full two- and three-body interactions. See text for details.

III. LINEAR RESPONSE

The method we are using to obtain the response function has been detailed in Ref. [12] for the case of a $ph$ interaction of the form given in Eq. (6), but without the center-of-mass and cross-vector interactions, which we consider here. The excitations of an infinite homogenous neutron system are characterized by the spin quantum numbers $\alpha = (S, M)$, where $M$ refers to the projection of the spin $S$ onto the $z$-axis. Once the matrix elements $V_{ph}^{(\alpha, \alpha')}\left(\mathbf{q}, \omega\right)$ of the $ph$ interaction are calculated, the response function of the system $\chi^{(\alpha)}(\mathbf{q}, \omega)$ is obtained through the analytical solution of the Bethe-Salpeter equations [27]. Here $\mathbf{q}$ and $\omega$ are the transferred momentum and energy, and for convenience $\mathbf{q}$ is chosen along the $z$-axis.

The presence of the center-of-mass and cross-vector terms in this formalism modifies the matrix elements of the $ph$ interaction. Both new tensor terms act only in the $S = 1$ channel, and to Eq. (2) of Ref. [12] we have simply to add the terms

\[
V_{ph}^{(\alpha, \alpha')/n_d} = \delta(S, 1) \sum_{\ell} \left( \tilde{k}_\ell P_{\ell} (\mathbf{k}_1 \cdot \mathbf{k}_2) K^{M, M'}_{T} (\mathbf{k}_1, \mathbf{k}_2) + \tilde{l}_\ell P_{\ell} (\mathbf{k}_1 \cdot \mathbf{k}_2) A^{M, M'}_{T} (\mathbf{k}_1, \mathbf{k}_2) \right),
\]

where the subscript $T$ emphasizes the tensor nature of the interactions. We have defined

\[
K^{M, M'}_{T} (\mathbf{k}_1, \mathbf{k}_2) = 3(-)^{M} (K^{(1)}_{12})_{-M}^{(1)} (K^{(1)}_{12})_{M'} - 2 \left[ 1 + (\mathbf{k}_1 \cdot \mathbf{k}_2) \right] \delta(M, M')
\]

and

\[
A^{M, M'}_{T} (\mathbf{k}_1, \mathbf{k}_2) = \frac{8\pi}{3} \left[ Y_{1, M}^* (\mathbf{k}_1) Y_{1, M'} (\mathbf{k}_2) - Y_{1, M} (\mathbf{k}_1) Y_{1, M'}^* (\mathbf{k}_2) \right]
\]

and $(K^{(1)}_{12})_{M} = \sqrt{\frac{4\pi}{3}} \left[ Y_{1, M} (\mathbf{k}_1) + Y_{1, M} (\mathbf{k}_2) \right]$ is a rank-1 tensor, and $Y_{1, M}$ is a spherical harmonic. As usual the product $\delta_{SS'}$ is implicit. We notice that this is a specific feature of the particle-hole angular momentum coupling scheme employed in the calculation of the RPA diagram. Within the alternative coupling scheme used in Ref. [21], one can see from Tab. I of that article that the operator $A_{12} (\mathbf{k}_{12}, \mathbf{P}_{12})$ mixes the $S=0$ and $S=1$ channels. The derivation of the response function follows closely the calculations performed in Ref. [12], to which we refer the reader for more details concerning the adopted numerical scheme.

Before considering the contributions of the extra noncentral terms to the response function, it is very instructive to consider first the static limit, i.e. taking $q \to 0$ in the $ph$ propagator and also $\nu = \omega m^*/(qk_F) = 0$. In this way we can derive the static susceptibility of the system, extending the results of Ref. [28], with the inclusion of the terms...
$K_\ell$ and $L_\ell$. These terms do not modify the well-known $S=0$ result, which reads

\[
\frac{\chi_{HF}(0)}{\chi_{RPA}(S=0)} = 1 + F_0,
\]

which is related to the compressibility of the system. We recall that in the static limit $\chi_{HF}(0) = N_0/n_d$. After some lengthy manipulations, we obtain for the static spin susceptibility the expression

\[
\frac{\chi_{HF}(0)}{\chi_{RPA}(S=1)} = 1 + \frac{T_1}{T_2},
\]

where

\[
T_1 = -2 \left( \tilde{H}_0 - \frac{2}{3} \tilde{H}_1 + \frac{1}{5} \tilde{H}_2 + \tilde{K}_0 + \frac{2}{3} \tilde{K}_1 + \frac{1}{5} \tilde{K}_2 \right)^2,
\]

\[
T_2 = 1 + \frac{1}{5} G_2 - \frac{7}{15} \tilde{H}_1 + \frac{2}{5} \tilde{H}_2 - \frac{3}{35} \tilde{H}_3 + \frac{7}{15} \tilde{K}_1 + \frac{2}{5} \tilde{K}_2 + \frac{3}{35} \tilde{K}_3 + \frac{2}{5} L_1 - \frac{6}{35} \tilde{L}_3.
\]

All the dependence of the tensor interaction is included in $T_1$ and $T_2$. Putting $\tilde{L}_\ell = \tilde{K}_\ell = 0$ we recover the results given in Ref. [28]. Using Eqs. (7) and (8) it is possible to write the combinations of $\tilde{H}_\ell, \tilde{K}_\ell$ entering $T_1$ and $T_2$ in terms of $H_\ell, K_\ell$, with the results.

\[
T_1 = -\frac{1}{8} \left( H_0 - H_1 + K_0 + K_1 \right)^2
\]

\[
T_2 = 1 + \frac{1}{5} G_2 - \frac{1}{4} H_0 - \frac{1}{4} H_1 + \frac{1}{10} H_2 - \frac{1}{4} K_0 + \frac{1}{4} K_1 + \frac{1}{10} K_2 + \frac{2}{5} L_1 - \frac{6}{35} \tilde{L}_3
\]

We have checked that these results are in agreement with the static susceptibility which can be deduced from Ref. [22]. However, we keep $\tilde{L}_1, \tilde{L}_3$ instead of their corresponding infinite sum in terms of $L_\ell$.

In Fig. 2(a), we have plotted the static susceptibility as a function of the density, as obtained from Eqs. [13, 14] for the complete chiral NN potential with $\Lambda = 450$ MeV. To see the effect of the tensor parameters on the $S=1$ case, we have also represented the results by dropping them or keeping only $\tilde{H}_\ell$ in Eqs. [15, 16]. We notice in this case that the contribution of the ratio $T_1/T_2$ is negligible compared to the contribution of the $G_0$ term. Actually we have checked that in the explored density range there is an approximate cancellation between the combination of parameters $\tilde{H}_\ell$ and $\tilde{K}_\ell$ entering (15), which results in $T_1 \approx 0$. It follows that for this set of Landau parameters the non-central terms play almost no role in the static susceptibility. In Fig. 2(b) we show the same quantity, but this time we switch off the 3NF contribution in the Landau terms. The most important difference among these results is the modifications in the $S = 0$ channel (note that we have reduced this term by a factor 1/4 in the right plot) introduced by the three body term. The compressibility without the 3NF is four times larger than in the complete case leading to a static deformation of the Fermi surface. The $S = 1$ channel is less affected by the presence (absence) of 3NF, but we notice a difference in the results of $\approx 20\%$ at $\rho = 0.24$ fm$^{-3}$. 
FIG. 2: (Color online) Static susceptibility from the chiral nuclear potential for $\Lambda = 450$ MeV as a function of the density. In the left panel the complete interaction is considered, while in the right panel the contributions arising from 3NF has been removed. In the $S = 0$ channel the ratio of the interacting and noninteracting static susceptibilities is multiplied by $1/4$ in the right panel.

We turn now our attention to the dynamic case, where we remove the restriction $q = \nu = 0$ in the $ph$-propagators. In Fig. 3 we show the response function of the system in the two channels $(1,0)$ and $(1,1)$, where we investigate the role of the extra tensor terms. As discussed in Ref. [12], we limit the number of Landau parameters to $\ell_{\text{max}} = 3$. As a benchmark, on the same figure we also show the Hartree-Fock (HF) case, i.e. when we switch off the residual interaction ($V_{\text{ph}}(\alpha,\alpha') = 0$) but keep the modification introduced to the ground state through the effective mass.

![Graph showing static susceptibility](image1)

FIG. 3: (Color online) Nuclear response function at $k_F = 1.68$ fm$^{-3}$ and $q/k_F = 0.5$ in the $S = 1$ channel calculated at $l_{\text{max}} = 3$ with and without the different contributions of the tensor terms. The chiral nuclear potential with a cutoff of $\Lambda = 450$ MeV is employed. See text for additional details.

From Fig. 3 we notice that in the dynamic case the role of the $K_\ell$ and $L_\ell$ terms is negligible, while the $\tilde{H}_\ell$ terms modify the position of the maximum of the response. As already discussed in Ref. [12], the effect of the tensor terms in a realistic potential is much smaller than the one originating from phenomenological interactions, e.g. Skyrme.
To quantify the effect of the 3NF, we show in Fig. 4 the response function of the system at $\rho = 0.16 \text{ fm}^{-3}$ for $\Lambda = 450 \text{ MeV}$ in the $(q, \omega)$ plane. Confirming the results for the static case, we observe that the presence of the three-body term strongly affects the response of the system. The pure two-body contribution gives a strongly attractive response function, with the possible presence of instabilities along the $\omega = 0$ axis, while the complete response function including both two- and three-body terms is more repulsive and free from instabilities.
IV. CONCLUSIONS

In the present article we have presented a derivation of the Landau parameters from realistic nuclear forces, containing explicit two- and three-body contributions, derived within chiral effective field theory. The presence of a filled Fermi sea introduces a preferred frame of reference in the homogeneous medium, and we have therefore considered the effect of two additional noncentral couplings related to the center-of-mass momentum [22]. We have shown that the magnitude of these extra contributions is the same as the standard exchange tensor term and thus cannot be discarded a-priori. We have generalized the RPA response function formalism presented in Ref. [12] to include these extra contributions and have shown that in the static limit the sum of the exchange and center-of-mass tensor terms gives a negligible contribution, resulting in a qualitatively different description compared to calculations including the exchange tensor contribution alone. In the dynamic case the noncentral interactions are important, although their effect is smaller relative to similar calculations performed with phenomenological interactions. Finally, we have studied in detail the effect of three-nucleon force contributions to the Landau parameters: in both the static and dynamic case the 3NF plays a crucial role, especially in the $S = 0$ channel, and without their contribution the system is unstable. Detailed results for the $S = 1$ channel are also presented.

Acknowledgments

This work has been supported by Mineco (Spain), grant FIS2011-28617-C02-02 and by the US DOE grant DE-FG02-97ER-41014.

[1] R. F. Sawyer, Phys. Rev. D 11, 2740 (1975).
[2] N. Iwamoto and C. J. Pethick, Phys. Rev. D 25, 313 (1982).
[3] S. Reddy, M. Prakash, and J. M. Lattimer, Phys. Rev. D 58, 013009 (1998).
[4] G. Shen, S. Gandolfi, S. Reddy, and J. Carlson, Phys.Rev. C 87, 025802 (2013).
[5] A. Pastore, M. Martini, V. Buridon, D. Davesne, K. Bennaceur, and J. Meyer, Phys. Rev. C 86, 044308 (2012).
[6] A. Akmal, V. R. Pandharipande, and D. G. Ravenhall, Phys. Rev. C 58, 1804 (1998).
[7] H. M"uther and A. Polls, Progress in Particle and Nuclear Physics 45, 243 (2000), ISSN 0146-6410.
[8] K. Hebeler and A. Schwenk, Phys. Rev. C 82, 014314 (2010).
[9] G. Wlazlowski, J. W. Holt, S. Moroz, A. Bulgac, and K. J. Roche, Phys. Rev. Lett. 113, 182503 (2014).
[10] E. Epelbaum, H.-W. Hammer, and U.-G. Mei$"{n}$ner, Rev. Mod. Phys. 81, 1773 (2009).
[11] C. Shen, U. Lombardo, N. V. Giai, and W. Zuo, Phys. Rev. C 68, 055802 (2003).
[12] A. Pastore, D. Davesne, and J. Navarro, J.Phys. G41, 055103 (2014), 1312.4696.
[13] O. Benhar, A. Cipollone, and A. Loreti, Phys. Rev. C 87, 014601 (2013).
[14] L. Coraggio, J. W. Holt, N. Itaco, R. Machleidt, and F. Sammarruca, Phys. Rev. C 87, 014322 (2013).
[15] L. Coraggio, J. W. Holt, N. Itaco, R. Machleidt, L. Marcucci, and F. Sammarruca, Phys. Rev. C 89, 044321 (2014).
[16] L. D. Landau, Sov. Phys. JETP 3, 920 (1957).
[17] A. B. Migdal, The Theory of Finite Fermi Systems (Wiley, New York, 1967).
[18] B. Friman and A. Schwenk, in From Nuclei to Stars: Festschrift in Honor of Gerald E. Brown (edited by S. Lee, World Scientific, Singapore, 2011).