Collisional Evolution in the Vulcanoid Region: 
Implications for Present-Day Population Constraints

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15 Pages
06 Figures
01 Table
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Vulcanoid Collisional Evolution

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ABSTRACT

We explore the effects of collisional evolution on putative Vulcanoid ensembles in the region between 0.06 and 0.21 AU from the Sun, in order to constrain the probable population density and population structure of this region today. Dynamical studies have shown that the Vulcanoid Zone (VZ) could be populated. However, we find that the frequency and energetics of collisional evolution this close to the Sun, coupled with the efficient radiation transport of small debris out of this region, together conspire to create an active and highly intensive collisional environment which depletes any very significant population of rocky bodies placed in it, unless the bodies exhibit orbits that are circular to \( \sim 10^{-3} \) or less, or highly lossy mechanical properties that correspond to a fraction of impact energy significantly less than 10% being imparted to ejecta. The most favorable locale for residual bodies to survive in this region is in highly circular orbits near the outer edge of the dynamically stable Vulcanoid Zone (i.e., near 0.2 AU), where collisional evolution and radiation transport of small bodies and debris proceed most slowly. If the mean random orbital eccentricity in this region exceeds \( \sim 10^{-3} \), then our work suggests it is unlikely that more than a few hundred objects with radii larger than 1 km will be found in the entire VZ; assuming the largest objects have a radius of 30 km, then the total mass of bodies in the VZ down to 0.1 km radii is likely to be no more than \( \sim 10^{-6} M_\oplus \), \(< 10^{-3} \) the mass of the asteroid belt. A 0.01 AU wide ring near the outer stability boundary of the VZ at 0.2 AU would likely not contain over a few tens of objects with radii larger than 1 km. Despite the dynamical stability of large objects in this region (Evans & Tabachnik 1999), it is plausible that the entire region is virtually empty of km-scale and larger objects.
1. INTRODUCTION

Over the past 20 years, our understanding of the solar system has grown dramatically, as evidenced by the detection of a series of heretofore wholly or largely undetected populations of small bodies. These include the Kuiper Belt region beyond Neptune, the population of Centaurs orbiting in the giant planet region, and the near Earth asteroid (NEA) zone (for additional background on each, see articles in the review volume edited by Rettig & Hahn [1996]). These new populations are revealing valuable insights into both the architecture of our solar system (and by extension, others), and the nature and origin of small bodies, and with regard to impact hazards on Earth.

Among the few stable dynamical niches which remain largely unexplored today is the region interior to Mercury’s orbit, where a population of small, asteroid-like bodies called the Vulcanoids has long been hypothesized to reside (e.g., Perrine 1902; see Campins et al. 1996 for a recent review). This putative reservoir is of interest because it would plausibly contain a sample of condensed material from the early inner solar system, and because it would bear relevance to our understanding and the interpretation of Mercury’s cratering record, and thus Mercury’s surface chronology. Owing to the intense thermal conditions and comparatively high collision velocities characteristic of this region, the Vulcanoid population might also be expected to contain unique chemical (e.g., ultra-refractory) signatures not seen in more heliocentrically distant, small body reservoirs.

The “Vulcanoid Zone” (VZ) extends inward from a stability limit near 0.21 AU, set by orbital eccentricity excitations due to Mercury and the other planets (Leake et al. 1987, Evans & Tabachnik 1999; S. Brooks, priv. comm 1999). The VZ is likely to be effectively bounded on the inside by the combination of thermal conditions and dynamical transport effects (i.e., Poynting-Robertson (PR) drag and the Yarkovsky effect; e.g., Leake et al. 1987; Campins et al. 1996). Even pure Fe bodies with radius \( r < 50 \text{ km} \) would evaporate under solar insolation in 4.5 Gyr at or inside 0.06 AU, and pure Fe bodies with radius \( r < 1 \text{ km} \) would evaporate under solar insolation in 4.5 Gyr at or inside 0.07 AU (Lebofsky 1975; Campins et al. 1996). PR drag extends this limit outward somewhat because it can move a \( \rho = 4 \text{ gm cm}^{-3} \), 1 km radius object from 0.08 to 0.07 AU in 4 Gyr, where it would then be evaporated; the Yarkovsky effect may dominate over PR drag, thereby removing some 1 km-scale primordial objects from the zone from even greater distances. Based on these results, we adopt for what follows an effective inner boundary of the VZ at 0.06 AU, but point out that if a population of primordial objects were to exist inside 0.1 AU, a steep heliocentric depletion would be expected to manifest itself inside \( \approx 0.08 \text{ AU} \).

Unfortunately, despite the fact that the Vulcanoid region is a plausible dynamical reservoir for small bodies, any Vulcanoid population will be particularly hard to detect. This is because the small bodies believed to be there are close to the Sun (in angular terms),[1]

[1] From 1 AU, the VZ inner and outer limits correspond to maximum solar elongation
and comparatively faint (i.e., $9<V<13$) compared to the sky at twilight. The angular proximity of the VZ to the Sun itself constrains ground-based visible-wavelength searches to brief windows of difficult, twilight geometry, or alternatively, to total solar eclipses. As a result, few VZ searches have been carried out, and those that have (e.g., Campbell & Trumpler 1923; Courten 1976; see Campins et al. 1996) exhibited comparatively shallow limiting magnitudes. Still, owing to the strongly increased flux of the Sun on the Vulcanoid region, even these early studies were sensitive to objects with radii down to $\sim 50$ km. No objects were discovered.

Visible-wavelength searches to date have covered most of the VZ inside 0.25 AU, but only reached $V \approx 8.5$. As shown in Figure 1, this corresponds to comparatively large objects with radii of 30 to 50 km at 0.20 AU; objects smaller than this would have escaped detection, even if present in great numbers. The most constraining search published to date worked in daylight conditions to detect the thermal-IR signature of Vulcanoids in the L (3.5 µm) band (Leake et al. 1987). This effort reached a magnitude limit of $L=5$, corresponding to objects with radii near 3.5 km at 0.21 AU, but covered only 5.8 deg$^2$ of sky, which is $<5\%$ of the available search area. Owing to the small area of this search, it is not possible to rule out populations containing a few objects with radii exceeding 25 km, and a some dozens of objects larger than 5 km in radius; the population of still smaller objects remains almost wholly unconstrained.\[^{[2]}\] Together these various observational results imply that the present-day VZ certainly cannot contain a large population of objects with radii in excess of 3.5 km, and likely contains zero (or perhaps only a handful) of objects with radii of $>25$ km.

This brief summary recapitulates much of what is known about the Vulcanoid Zone, and demonstrates that an ensemble of small bodies in the size range 1 km to a few 10s of km in radius could exist and remain undetected there. In this report we examine the effects of collisional evolution on a suite of hypothetical Vulcanoid populations, with the specific objective of further constraining the extent of any small-body population interior to Mercury’s orbit.

### 2. COLLISION ENERGETICS IN THE VULCANOID ZONE

The consequences of collisions in the Vulcanoid Zone depend on whether the collisional environment promotes net erosion or net accretion. As one intuitively expects, collisions in angles of only 4 deg to 12 deg.

\[^{[2]}\] Searches for IR emission from dust close to the Sun which might result from recent collisions among Vulcanoids (e.g., Hodapp et al. 1992, MacQueen et al. 1995) have also yielded negative results. A study of the zodiacal light using the photometer aboard the Helios spacecraft (Leinert et al. 1981) never penetrated the region inside 16 deg from the Sun where the Vulcanoids are expected to reside.
the VZ are highly energetic, owing to the high Keplerian orbital velocities close to the Sun. To illustrate this point, consider an orbiting swarm with mean random inclination $i$ and eccentricity $e$ in approximate statistical equilibrium, i.e., $\langle i \rangle = \frac{1}{2} \langle e \rangle$, the mean encounter speed at infinity, as a function of heliocentric semi-major axis $a$, is of scale:

$$V_{\text{enc}} = 180 \langle e \rangle \sqrt{\frac{0.1 \text{ AU}}{a}} \text{ km s}^{-1}. \quad (1)$$

Even higher encounter speeds would be achieved if $\langle i \rangle > \frac{1}{2} \langle e \rangle$, as is the case in the asteroid belt. Still, even for the case in Eqn. (1) and $\langle e \rangle = 0.01$, $V_{\text{enc}}$ is $\sim 2$ km s$^{-1}$, over an order of magnitude higher than the 0.1 km s$^{-1}$ escape speed from a 50 km radius body with density equal to the Earth’s iron core. As such, even for lossy collisions into mechanically strong objects in the VZ, one expects collisions to be highly erosive.

This result can be further quantified by adopting an analytical formalism which derives a critical collision velocity, or equivalently, a critical orbital eccentricity $e^*$, above which impacts eject more mass from the object than the mass of the impactor, and below which the target body gains mass and thereby grows (Stern 1995). This critical eccentricity is a function of several target parameters, including strength, size, and mass.

The results of a set of $e^*$ calculations for the Vulcanoid Zone are shown in Figure 2. This figure shows that, assuming $f_{KE} = 0.08$, objects across the VZ with radii of a few km or less will suffer erosion by impacts even if the orbital eccentricities of the colliding objects are as low as $5 \times 10^{-5}$ to $10^{-4}$, depending on their mechanical properties. For both weak and strong mechanical properties, $e^*$ is $\sim 2$ times lower at the 0.06 AU VZ inner-boundary than at the 0.21 AU VZ outer boundary. Larger objects are more resistive to erosion owing to their gravitational binding energy, which acts to return low-velocity ejecta. Still, however, their $e^*$ boundary also occurs at comparatively low orbital eccentricities, owing primarily to the high Kepler velocities, and therefore the high specific impact energies inherent in the VZ. Even the largest objects still marginally permitted by searches, i.e., those with radii near 50 km, will suffer erosion if their orbital eccentricities are as low as $10^{-3}$ to $5 \times 10^{-3}$, depending on their mechanical properties.

How do these critical eccentricity results compare to expected eccentricities in the region? One worthwhile comparison is obtained by noting that a population of 10 km radius bodies would mutually excite orbital eccentricities to levels of $10^{-4}$ in the VZ if there is no substantial population of still smaller bodies causing dynamical drag. Mean random eccentricities of the $10^{-4}$ level could also have been excited by a former population of objects in Mercury’s feeding zone with masses of 10% to 20% of Mercury. Still larger eccentricities could have been excited either by large interlopers in the region, or by sweeping secular

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[3] i.e., Neglecting mutual gravitational acceleration due to the binding energy of the impactor and target.
resonances, including those generated by solar spin-down (Ward et al. 1976). Given these considerations, and the characteristic eccentricity levels seen in the terrestrial planet zone, we consider it plausible that the mean random eccentricity of VZ orbits could be as low as a $10^{-4}$ or as high as several times $10^{-1}$.

Given the low critical eccentricity required for erosion in the VZ, and the plethora of dynamical processes which could have excited the region to random orbital eccentricities above $e^*$, we find it highly unlikely that the $e^*$ boundary has not been exceeded; therefore, present-day collisions in the VZ are likely to be erosional.

This is not a surprising conclusion. However, it does imply the interesting corollary that the conditions necessary to accrete objects in the primordial VZ would have required extremely low eccentricities, or $f_{KE}$ significantly below 8%, or both; low eccentricities would in turn imply either the virtual absence of large perturbers inside 0.4 AU, or the aid of some random velocity damping mechanism such as nebular gas-drag to prevent self-stirring by a population of growing embryos, or both. Whether the conditions necessary for accretion in this region ever obtained is not clear, but the absence of a planet in this region suggests that either accretion never proceeded very far in the VZ, or that any large (i.e., $r>50$ km) objects which formed were subsequently dynamically removed (a process which would likely have contributed to further dynamical excitation and clearing of the region).

Despite our pessimism that accretion may have ever been able to proceed in the VZ, for the remainder of this paper we posit that accumulation did take place for a sufficient period some 4.5 Gyr ago to allow objects up to size scales with radii near 50 km (the observational detection upper limit). We then examine the constraints that collisional models can place on the number of such objects that persist to the present.

### 3. POPULATION CONSTRAINTS FROM PRESENT-DAY CATASTROPHIC DISRUPTION TIMESCALES

A scale for the collisionality of the VZ region can be achieved from a simple particle-in-a-box (PIB) estimate in which the collision time $t_{\text{coll}}$ on a target of cross section $\sigma$ depends only on $\sigma$, the projectile number density $n$, and the mean random speed of the projectiles $v$. That is,

$$t_{\text{coll}} = (n \sigma v)^{-1}. \quad (2)$$

For the assumption of just $10^4$ radius $r>0.1$ km projectiles in a VZ extending from 0.09 AU to 0.21 AU with a mean random eccentricity of 10%, we find that the mean time between collisions on any given target with radius 3 km is $\sim 350$ Myr; for a target with radius 30 km the collision time is $\sim 3.5$ Myr. Although these are purely just collision time estimates, given the erosive affects of such collisions (see §2 above), one can conclude that if there is
a substantial population of objects larger than 3 km in the VZ, their fates will be strongly affected by collisional evolution.

Although the simple collisionality estimate given above demonstrates the merit of collisional considerations with respect to the Vulcanoid Zone, a far better first-order assessment of the present-day collisional environment of the VZ can be derived using a static, multi-zone collision rate model (CRM) which assesses the frequency of collisions in any specified population. This CRM code (Stern 1995; Durda & Stern 2000) is based around a simple but robust statistical PIB formalism, and computes orbit-averaged collision rates (and thus collision timescales) for objects crossing heliocentric zones using accurate Kepler time-of-flight calculations for the fractional time the target spends at each heliocentric zone it crosses, depending on its orbital semi-major axis and eccentricity. We used this model to explore various plausible VZ population distributions as a function of heliocentric distance and assumed mean random eccentricity (again, with \( \langle i \rangle = \frac{1}{2} \langle e \rangle \)). Our objective is to assess the range of collisional timescales onto targets which observational constraints allow to exist in the VZ today.

Figure 3 depicts results of model runs for six plausible VZ cases spanning a wide range of target strengths, \( \langle e \rangle \)’s, and VZ population. In each of these cases we assumed that the heliocentric surface mass density in the VZ declines like \( R^{-2} \), where \( R \) is heliocentric distance, and assumed the canonical -2.5 cumulative power law index Dohnanyi collisional equilibrium population size distribution (e.g., Williams & Wetherill 1994). The three CRM runs shown in Figure 3 assume populations of \( 10^2 \), \( 10^3 \), and \( 10^4 \) objects with radius \( r > 1 \) km in the VZ, respectively; the largest object in these three simulations (a direct result of this population constraint and the Dohnanyi power law) is 4.0 km, 10.2 km, and 25.8 km in radius, respectively. Populations with significantly larger numbers of bodies with radii larger than 1 km cannot exist in collisional equilibrium without violating observational constraints. The total mass of the three population ensembles, down to sizes of 0.1 km radius was \( 1.9 \times 10^{-9} \) M\(_\oplus\), \( 2.3 \times 10^{-8} \) M\(_\oplus\), and \( 3.8 \times 10^{-7} \) M\(_\oplus\), respectively.[4]

Consider now the catastrophic collisional disruption timescale results shown in Figure 3. The smallest projectile capable of disrupting and dispersing the largest object in each given VZ population shown in Figure 3 is indicated along the collision timescale curves by either a filled or open circle. Filled circles are for the case of strong objects (both projectiles and targets) in the VZ, and open circles are for the case of weak objects. Here, ‘strong’ and ‘weak’ are defined from the strongest and weakest of the published scaling laws in the literature. Specifically, the strongest scaling law is from Benz & Asphaug (1999), and the weakest is from Durda et al. (1998). The assumed specific disruption energies, \( Q_D^* \), for these cases are summarized in Table 1.

[4] A set of 3 similar runs differing only in that we assumed that the heliocentric surface mass density in the VZ declines like \( R^{-1} \), produced the same qualitative results.
| Target Radius | $Q_\text{D}^*$ (strong) | $Q_\text{D}^*$ (weak) |
|--------------|-------------------------|------------------------|
| 04 km        | $1 \times 10^8$ ergs g$^{-1}$ | $1 \times 10^6$ ergs g$^{-1}$ |
| 10 km        | $2 \times 10^8$ ergs g$^{-1}$ | $7 \times 10^6$ ergs g$^{-1}$ |
| 25 km        | $5 \times 10^8$ ergs g$^{-1}$ | $6 \times 10^7$ ergs g$^{-1}$ |

Even in the smallest of these three VZ population scenarios (upper panel), which has just 100 objects with radii larger than 1 km in the entire VZ, the catastrophic collisional disruption timescale of the largest object in the swarm, a 4 km radius body, is less than the age of the solar system; this result obtains over the full range of $\langle e \rangle$ explored, throughout the VZ if the target is mechanically weak. If the target is mechanically strong, this result obtains out to a heliocentric distance of 0.17 AU. Strong objects at larger heliocentric distance survive longer owing to a combination of lower collision rates and lower collision velocities (thus requiring progressively larger impactors to cause disruption); the latter factor dominates this progression. Since collision timescales increase with target radius squared in this size regime, smaller objects have collisional disruption timescales that are longer than those shown here for the 4 km target by the ratio $(4 \text{ km}/r)^2$, implying that there is a significant region of strength-heliocentric distance parameter space for objects of 0.1 km to 1 km scale to survive against both collisions and PR drag.$[^5]$ Qualitatively similar behaviors are seen for the two larger hypothetical VZ population runs, which have their results depicted in the lower two panels of Figure 3, respectively.

From Figure 3 we conclude that few if any objects with radii of $\approx 1$–25 kilometers are likely to survive against collisions for the age of the solar system in standard population structures like the ones we explored, a result in accord with the observational absence of objects in the 10 km to 50 km size range. This result is not unexpected, of course, because the volume of space in the VZ is so small and the orbit speeds are so high.

From the results shown in Figure 3, one concludes that virtually no primordial objects with $1 \text{ km}<r<25 \text{ km}$ could have survived to the present in the low-mass VZ models we have considered here. Higher mass models with the same population structure would be even more collisional.

One could imagine scenarios, however, in which larger objects formed or were transported into the region, and then subsequently suffered collisional erosion owing to the growth of orbital eccentricities. To model such populations it is necessary to use time-dependent collisional evolution simulations. We discuss such simulations next.

$[^5]$ Recall that a 0.1 km objects of density $4 \text{ g cm}^{-3}$ will spiral from the outer limit of the VZ at 0.21 AU to its evaporation limit near 0.07 AU in 4.5 Gyr.
4. TIME-DEPENDENT COLLISIONAL EVOLUTION SIMULATIONS

The time-dependent model we use to investigate collisional evolution in the Vulcanoid Zone was adapted from the Kuiper Belt code described by Stern & Colwell (1997), with its target mechanical properties changed to reflect the range of likely object types in the Vulcanoid Zone. Very briefly, this model uses a “moving bin” (i.e., Lagrangian) approach to size bins first described by Wetherill (1990). This technique has the advantage of being particularly straightforward, and particularly accurate in its mass accounting. The radius bins we used were separated by factors of $2^{1/3}$; we ran bin radii from 1 m to 100 km for the initial VZ simulations. Three-body, Keplerian shear-limited, gravitational collision cross sections were computed following Ward’s (1996) prescription. For each collision pair of mass $m_k < m_l$ colliding at relative velocity $v_{kl}$, the specific impact energy is computed according to the standard definition, $Q^* = \frac{1}{2} m_k v_{kl}^2 / m_l$ (e.g., Housen & Holsapple 1990), and then compared to a threshold value for catastrophic disruption, $Q_D^*$. We used a strain-rate scaling model (Housen & Holsapple 1990; HH90); this results in objects slightly stronger than the strong cases in §3, above.

For all impacts, we initially add the mass of the impactor to the target, and then remove the appropriate amount of debris, based on the target and impactor properties and the collision energetics. The result is net accretion if the mass of the escaping ejecta is less than the impactor mass, and net erosion if the ejected mass exceeds the mass of the impactor. Our catastrophic fragmentation model is the same as that used in Colwell & Esposito (1993). If $Q^* > Q_D^*$, then the mass fraction with escape velocity from the colliding pair is given by $f( > v_{esc}) = 1/2(v_{esc}/v_{med})^{-3/2}$, where $v_{med} = \sqrt{2 f_{KE} Q^*}$ is the median fragment velocity, $f_{KE}$ is the fraction of impact energy partitioned into fragment kinetic energy, and $v_{esc}$ is the escape velocity. Following experimental results (see Fujiwara et al. 1989), we have set $f_{KE} = 0.10$.\[^{[6]}\]

In our model the total mass of escaping debris is distributed to smaller mass bins following a standard, two-component power-law size distribution, with slopes computed based on laboratory experiments (e.g., Davis & Ryan 1990). The result of any given collision can range from complete accretion (no debris achieves escape velocity from the colliding pair), to complete erosion (in which the object is destroyed because greater than half the target mass has escape velocity). Cratering impacts ($Q^* < Q_D^*$) are handled similarly: following the literature, the debris size distribution is a single-valued power-law ($n( > m) \propto m^{-5/6}$); the fragment velocity distribution uses a power law exponent of $-1.2$ (for weak target\[^{[6]}\] Since, particularly for larger objects, lower values of $f_{KE}$ might be more appropriate, we note that any significant lowering of $f_{KE}$ would lengthen collisional erosion and catastrophic disruption timescales. We have found that this lengthening scales somewhere between $1/f_{KE}$ and $1/\sqrt{f_{KE}}$, depending on target size and bulk mechanical properties.)

\[^{[6]}\]
runs) and −2.0 (for our hard target runs). Debris smaller than the smallest discrete bin (1 meter in the initial runs presented below) is placed into a “dust” bin. PR drag operates to remove small debris from the simulation, based on their size- and \(a, e\)-dependent PR drag lifetime. Runs of the model typically conserve mass to a few parts in \(10^{-15}\) over \(10^{10}\) years.

Figure 4 presents the first of several sets of VZ collisional evolution simulations we performed using this model. For all of the runs presented below we assumed uniform density and impact strengths of 4 \(\text{gm cm}^{-3}\) and \(3\times10^6 \text{ergs gm}^{-1}\), respectively, for all objects in the simulation. This combination of density and strength corresponds to competent basalt, i.e., somewhat stronger objects than in HH90. Figure 4 presents various results for objects with semi-major axis \(a=0.20\) AU. Each simulation was evolved until it achieved an end state with no objects larger than 1 m among the population with \(a=0.20\) AU. Note, however, that the plots and timescales in Figure 4 refer to the time at which the given population evolves to have no objects with \(r>1\) km; the loss of smaller objects proceeds rapidly after this point. The purpose of these collisional evolution simulations was to explore the evolution of various populations that fit with the available VZ observational constraints reviewed in §1.

The set of simulations shown in the upper two panels of Figure 4 were started with 300 objects with \(r>1\) km in the VZ (i.e., between 0.06 and 0.21 AU); smaller debris down to \(r=1\) m in radius was extrapolated from the large object population using a Dohnanyi cumulative power-law population index of -2.5. The simulations shown in the lower two panels started with \(10^4\) objects with \(r>1\) km in the VZ. The two left-hand panels in Figure 4 refer to an assumed, constant mean random eccentricity \(\langle e\rangle=0.0032\); the two right-hand panels assume \(\langle e\rangle=0.1024\). Owing to the high Kepler velocities at 0.20 AU, even for the strong objects assumed here, both population cases are erosive across all of the populated size bins.

As stated above, the two cases shown in the upper panels were started with only 300 objects in the entire VZ larger than 1 km radius; this corresponds to a starting condition with just 8 objects larger than 1 km radius (largest object \(r=2.05\) km) in our 0.01 AU wide bin centered at \(a=0.20\) AU. While the \(\langle e\rangle=0.1024\) case took only 1.2 Gyr yrs to eliminate all objects with \(r>1\) km, the \(\langle e\rangle=0.0032\) case took 6.3 Gyr. Examining the intermediate-time population structures in each of these cases reveals differing population structure erosion styles. In the case with \(\langle e\rangle=0.1024\), high energy collisions by the numerous small bodies quickly destroyed the largest objects (i.e., \(r>100\) m) through catastrophic collisions. The case with \(\langle e\rangle=0.0032\), though still erosive, was not sufficiently energetic to induce rapid catastrophic impacts on the largest objects, and resulted in a more gradual erosion of the population structure throughout the run.

Now consider the two cases shown in the lower panels of Figure 4, i.e., the runs with \(10^4\)
objects in the VZ larger than 1 km radius at the simulation start. This corresponds to
a starting condition with 272 objects larger than 1 km radius (largest object r=8.2 km)
in our 0.01 AU wide bin centered at a=0.20 AU. In the case with ⟨e⟩=0.1024, the system
population number density was so high and the collisions so energetic that in just 1.6
Myr it evolved to a state with no objects with radius larger than 1 km. In the case with
⟨e⟩=0.0032, however, this evolution did not obtain in the 10 Gyr length of the simulation.
However, after 4.5 Gyr this run contained only 9 objects with r>1 km. As shown in Figure
5, additional runs with both 300 and 10^4 objects initially in the VZ with r>1 km, but
started with intermediate ⟨e⟩’s of 0.012 to 0.025, yielded timescales of 1 Gyr to 3 Gyr to
erode down to populations with no objects with r>1 km and a=0.20±0.005 AU.

A suite of runs just like those in the upper two panels of Figure 4 but with ⟨e⟩=0.0004 also
produced erosion. Though the timescale to fully deplete the population of 1 km objects at
0.20 AU exceeded the age of the solar system, only 14 objects with r>1 km remained from
a starting Dohnanyi population with 10^4 r>1 km objects after 4.5 Gyr. Figure 5 presents
a similar set of run results for cases with ⟨e⟩=0.0124 and ⟨e⟩=0.0256, respectively.

Together these various results at 0.20 AU indicate that, except in the case where ⟨e⟩ can
be maintained significantly below 4×10^{-4} (i.e., below the e^* boundary), any substantial
VZ population near 0.20 AU extending up to objects with radii of a few tens of km must
be collisionally eroded by the present 4.5 Gyr age of the solar system to a point where r=1
km and larger objects are either rare or non-existent.\[7\]

One of course expects more rapid evolution at smaller heliocentric distance, owing to
number density enhancements and increased collision energetics. Figure 6 shows a set of
simulations identical to those in Figure 4, but at a=0.10 AU. The resulting evolutions in
population size structure are qualitatively similar, but with the timescales accelerated by
factors of 7 to 10.

Weaker mechanical properties, steeper initial power-law population ensembles, higher mean
random eccentricities, higher inclinations with respect to eccentricity, and the inclusion of
a bombarding flux from the asteroid belt and cometary reservoirs would each shorten the
VZ erosion timescales quoted above.\[8\] The Yarkovsky effect increases the rate of small

\[7\] We also conducted a set of simulations identical to those in Figure 4, but removed
all objects with r<1 km from the starting populations. Though evolution proceeded more
slowly at first owing to the need to build up the population of small projectiles from
collisions among km-sized and larger bodies, in all 4 cases run, we again found that the
population of objects with r>1 km was reduced to 10 or less objects remaining at a=0.20
AU over the age of the solar system.

\[8\] Regarding collisions with objects on heliocentric orbits outside the VZ, we find that
such collisions are rare, and that collisional lifetimes exceed the age of the solar system.
More specifically, based on Levison et al.’s (2000) cometary impact rates on Mercury, we
debris transport and removal over PR drag alone (e.g., Farinella et al. 1998, Vokrouhlický 1999), thereby reducing the impact flux on larger objects in the VZ. Therefore, this effect, though not modelled here owing to its wide range of free parameter choices, will tend to moderately increase the estimated lifetimes of bodies with diameters of several km and larger, but may actually allow objects as large as 1 km in diameter to be dragged into the Sun, even from 0.2 AU over 4.5 Gyr (W. Bottke, pers. comm. 1999).

Our results demonstrate that unless $f_{KE}$ is substantially below 10%, or the VZ has been maintained below the $e^*$ boundary, i.e., $\langle e \rangle$ below $\sim 10^{-4.5}$, population structures that fit under the present-day observational constraint boundaries would self destruct owing to collisions, resulting in a VZ which today is so thinly populated that collisions are rare. This population constraint implies that a few tens, and quite likely a very much smaller number of objects in the 1 km to 50 km size range are extant today inside 0.20 AU.

Figure 7 presents some results from one additional set of simulations we performed at 0.20 AU. In this set of cases we examined the evolution of much more massive starting populations with objects as large as $r=330$ km. Although observations trivially rule out such a massive VZ population today, it is instructive to examine what the evolution of such swarms would be, so as to determine what if any signatures of such a primordial population might exist today. In these simulations the total mass of the VZ down to our cutoff radius (10 m in this case) was $9 \times 10^{-4} M_\oplus$, i.e., somewhat in excess of the present-day mass of the asteroid belt ($7 \times 10^{-4} M_\oplus$).

Specifically, the lefthand panel in Figure 7 shows the gentlest and slowest evolving of the four runs we performed in this scenario. In this case we assumed $\langle e \rangle = 0.0032$; this is a low enough eccentricity to actually allow the largest object in the starting population to grow. The end result of this run was that after 22 Myr, two thirds of the starting mass in this zone had been removed from the simulation owing to grinding and subsequent PR drag loss. Further, after 22 Myr, no objects remained in our standard 0.01 AU wide model zone at 0.20 AU with $r>1$ km, except a single, largest body that had grown to $r=371$ km. After 45 Myr, no objects remained with $r>0.1$ km, except the large object which was stranded in the population but which could not grow appreciably because there was so little mass left in the population of small debris. Of course, this simulation is not fully self-consistent in that we simply began with large bodies up to 330 km in radius at the start. As the results in Figure 2 show, achieving growth from km-scale and smaller bodies to this stage, requires maintaining $\langle e \rangle$’s an order of magnitude or more lower than in this run until objects with $r \sim 10$ km are grown.

Runs starting with the large bodies up to $r=330$ km but with higher $\langle e \rangle$ produced much lower collision rates and thus longer estimated lifetimes for objects down to 1 km in radius. In the center of the VZ, the catastrophic collision lifetime for objects down to 1 km in radius is in excess of $10^{10}$ years; for 10 km radius targets in the center of the VZ the estimated catastrophic collision lifetime exceeds $4 \times 10^{11}$ years.
greater quantities of debris owing to the more energetic collisions, and therefore evolved much faster. For example, the righthand panel in Figure 7 shows a run with $\langle e \rangle = 0.0128$. In 18 Myr this ensemble ground away 80% of its mass, and contained only 33 objects with $r > 1$ km, the largest of which had $r = 34$ km. As noted above, these runs were performed at 0.20 AU heliocentric distance; we found that evolution proceeds about an order of magnitude faster still at 0.10 AU heliocentric distance.

The more massive VZ scenarios just described demonstrate that even if the VZ was able to create a Vulcanoid belt of similar scale to the asteroid belt early in the history of the solar system, it would by today either have been eroded away (if $\langle e \rangle$ exceeded $e^*$ for as little as 1% the age of the solar system), or (if $\langle e \rangle$ remained well below $e^*$) it would have grown a small number of larger objects which are not seen today. Had that latter condition occurred, dynamical stability results (e.g., Evans & Tabachnik 1999) imply that one of more of these objects would remain and have been detected.

5. CONCLUSIONS

We have examined the role of collisional evolution in the Vulcanoid Zone (VZ), where searches for a population of small bodies have been conducted several times. The Vulcanoid Zone, owing to its shorter dynamical times and smaller volume is far “older” collisionally (and dynamically) than the asteroid belt. Unless our $f_{KE} = 0.1$ is a gross overestimate, or the Vulcanoids are far denser or stronger than our adopted values, then:

- If the mean random orbital eccentricity exceeds a critical value, $e^*$ (a function of target mass, mechanical properties, and heliocentric distance), efficient collisional grinding and erosion must take place. Given that the largest objects which observations allow to exist in the VZ today has a radius near 30 km, this implies that $e^*$ is today less than a few times $10^{-3}$, and could be an order of magnitude smaller if the largest bodies in the VZ are only a few km in radius.

- Collisional grinding and the subsequent radiation transport of debris out of the VZ dramatically depletes starting populations that are consistent with the existing observational constraints (i.e., VZ masses $\sim 10^{-6} M_\oplus$, largest objects with $r \approx 25$ km). This obtains whether one starts the evolution with or without a Dohnanyi-like debris tail of objects. This evolution results in populations which, unless eccentricities are below $\sim 10^{-3}$, cannot contain more than a few hundred objects with radii exceeding 1 km.

- Even allowing for ancient VZ ensembles with collisional equilibrium power-law population structures and embedded objects up to 330 km in radius (i.e., leading to a mass somewhat in excess of the asteroid belt), collisional evolution is so fast and collision energies are so high, that populations with mean random orbital eccentricities above $\sim 3 \times 10^{-3}$ will “self-destruct” down to levels with only a residuum of widely spaced
(and therefore collisionally non-interacting bodies) in \( \sim 1\% \) the age of the solar system. In our simulations, this residuum contained only a few hundred objects across the entire VZ with \( r < 1 \) km for orbits with \( \langle e \rangle \) near \( 10^{-2} \).

- Collisional evolution will proceed most quickly at smaller heliocentric distances; this, combined with PR drag and the Yarkovsky effect will cause any former or present-day Vulcanoid Zone population to be depleted by collisional grinding from the “inside out” over time.

- The characteristic erosion timescale for the VZ can range from \( 10^7 \) yrs to \( 10^{10} \) years, depending on \( \langle e \rangle, f_{KE} \), and the initial population density. Therefore, a wide range of VZ erosion timescales may exhibit themselves in solar systems with architectures like our own. The observational signatures of VZ erosion, i.e., thermal emission at \( \sim 2-5 \) \( \mu m \) and photospheric pollution with silicate-iron signatures may someday be detected in other planetary systems.

- These considerations suggest it is unlikely that, unless we have grossly overestimate \( f_{KE} \), more than a few hundred objects with radii larger than 1 km will be found in the VZ. The most favorable location to search for such bodies is in highly circular orbits near the outer edge of the dynamically stable VZ (i.e., near 0.2 AU), where collisional evolution and radiation transport of small bodies and debris proceed most slowly.

Although our exploration of parameter space is not fully complete (e.g., we did not examine scenarios with 1000 km radius and larger bodies in the starting population), we do believe that the work discussed here shows that the present-day VZ is likely to be either depleted or almost depleted of km-scale and larger objects. If any such objects are found, then collisional evolution arguments imply it is highly likely that their number density will be so low, and their spacings so great, that they will form a thin, collisionally-decoupled population remnant from an ancient era.

In conclusion, our work suggests that large numbers of objects with radii of km scale or larger are unlikely to be found unless the VZ region of the solar system has never been dynamically excited to orbital eccentricities above \( \sim 10^{-3} \), which seems unlikely. Nevertheless, the detection of any such population, regardless of how low, would shed valuable light on the dynamical, and possibly the accretional/erosional, history of this end-member region of our solar system, and would no doubt bear on our understanding of extra-solar planetary systems as well.
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FIGURE CAPTIONS

Figure 1. Observational constraints on possible Vulcanoid sizes as a function of heliocentric distance and assumed surface geometric albedo. For the magnitude-limited searches, radii have been calculated assuming a visual magnitude $V=8.5$, observations at quadrature (phase angle 90 deg), a phase function like that of Mercury (Veverka et al. 1988), and three different albedos: $p=0.05$ (dark asteroidal), $p=0.14$ (Mercury), and $p=0.30$ (bright asteroidal).

Figure 2. Critical collision eccentricities separating the erosive vs. accumulation regimes (see §2) are computed here as a function of target radius for both strong and weak target mechanical properties at 0.06 and 0.21 AU in the Vulcanoid Zone. Strong target parameters: density $\rho=4$ gm cm$^{-3}$ and the material shattering strength $Q^*_S=3\times10^6$ ergs gm$^{-1}$; weak target parameters: density $\rho=1$ gm cm$^{-3}$ and material shattering strength $Q^*_S=3\times10^4$ ergs gm$^{-1}$. Note: For bodies with non-negligible binding energy (i.e., mass), the shattering strength $Q^*_S$ is less than the disruption strength $Q^*_D$ because to disrupt such a body also requires removing half its mass to infinity.

Figure 3. Collisional timescale results, as a function of projectile radius and heliocentric distance, using the static, multi-zone collision rate model, for the three population cases described in §3 of the text. The horizontal dashed line is a timescale of 4.5 Gyr. The sloping collision timescale lines are shown at $R=0.09$ AU and $a=0.21$ AU. The dotted line cases assume $\langle e\rangle=0.2048$, and the solid lines assume $\langle e\rangle=0.0256$; together these two cases span a wide range of potential VZ eccentricities. The circles on each of these collision rate curves represent the boundary between cratering and catastrophic collisions; open circles correspond to the assumption of weak targets and filled circles correspond to the assumption of strong targets (see text).

Figure 4. Vulcanoid Zone collisional evolution simulations for objects with $a=0.20$ AU. The simulations shown in the upper two panels began with 300 objects with $r>1$ km in the entire 0.06–0.21 AU VZ. The simulations shown in the lower two panels began with $10^4$ objects with $r>1$ km in the entire 0.06–0.21 AU VZ. The two cases on the left assume $\langle e\rangle=0.0032$; the two cases on the right assume $\langle e\rangle=0.1024$. The dotted line is the initial population. The successively thicker, solid lines represent the population at 3%, 10%, 30%, and 100% of the run time shown, where the run time is the simulation time required to reach a state with no objects with radius $>1$ km in the 0.25 AU subzone. See text for additional simulation details.

Figure 5. Same as Figure 4, but for $\langle e\rangle=0.0124$ and $\langle e\rangle=0.0256$.

Figure 6. Same as Figure 4, but for $a=0.10$ AU.

Figure 7. Collisional evolution runs at 0.20 AU for massive VZ scenarios (total mass
$9 \times 10^{-4} M_\oplus$). At the simulation start, the population (as shown by the dotted line) contains objects up to 330 km in radius, and as small as 20 m in radius, connected by a Dohnanyi power-law ensemble of intermediate objects. Left panel: $\langle e \rangle = 0.0032$; right panel: $\langle e \rangle = 0.0128$. See text for discussion.
10^2 objects with $r > 1$ km
$r = 4$ km target

10^3 objects with $r > 1$ km
$r = 10$ km target

10^4 objects with $r > 1$ km
$r = 25$ km target
Best Constraint: \( V = 8.5 \) searches

Campbell and Trumper (1923)

Courten (1976)
CRITICAL ECCENTRICITIES IN THE VULCANOID REGION

CRITICAL ECCENTRICITY $e^*$

STRONG (0.21 AU)
STRONG (0.06 AU)
WEAK (0.21 AU)
WEAK (0.06 AU)

TARGET RADIUS (km)
\[<e> = 0.012 \quad 4.72 \times 10^9 \text{ yr} \]
\[a = 0.20 \text{ AU} \quad M_0 = 1.4 \times 10^{-9} \text{ M}_\odot \]

\[<e> = 0.025 \quad 4.85 \times 10^9 \text{ yr} \]
\[a = 0.20 \text{ AU} \quad M_0 = 1.4 \times 10^{-9} \text{ M}_\odot \]

\[<e> = 0.012 \quad 5.99 \times 10^9 \text{ yr} \]
\[a = 0.20 \text{ AU} \quad M_0 = 9.6 \times 10^{-8} \text{ M}_\odot \]

\[<e> = 0.025 \quad 7.89 \times 10^8 \text{ yr} \]
\[a = 0.20 \text{ AU} \quad M_0 = 9.6 \times 10^{-8} \text{ M}_\odot \]

TA001411
\[ \langle e \rangle = 0.003 \quad M_0 = 0.00090 \, M_\oplus \]
\[ a = 0.20 \, \text{AU} \quad t_f = 4.46 \times 10^7 \, \text{yr} \]

\[ \langle e \rangle = 0.012 \quad M_0 = 0.00089 \, M_\oplus \]
\[ a = 0.20 \, \text{AU} \quad t_f = 1.83 \times 10^7 \, \text{yr} \]
