Parity-odd Neutrino Torque Detection

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Cosmological observations are promising ways to improve our understanding of the neutrino mass properties. The upper bound on their sum of mass is given by the cosmic microwave background and large scale structures. These measurements are all parity-even. Here we show that, the presence of neutrino mass provides a unique contribution to the directions of the angular momentum of galaxies, which is the first parity-odd neutrino effect of galaxies or halos. This parity-odd observable is free of the contamination of linear perturbation theory, and can be cleanly separated from other non-gravitational effects. A complete 21-cm survey deep to redshift 1 can give a 5σ confidence level of detecting the neutrino torque effect if the sum of neutrino masses is 0.05 eV.

Introduction.— Neutrino mass is a long-standing physics problem. The flavour oscillation experiments [1] discovered the mass splittings of neutrinos and placed a lower bound of the sum of their mass $M_\nu \equiv \sum_{i=1}^{3} m_\nu \gtrsim 0.05$ eV [2]. The existence of neutrino mass has profound impacts on cosmic evolution, and the current cosmic microwave background observations provide an upper bound of $M_\nu \lesssim 0.23$ eV [3]. In the cosmic structure formation, neutrinos are non-relativistic, and contribute to the matter energy density $\Omega_m$. Unlike the majority of matter, the cold dark matter (CDM) and baryons, neutrinos maintain a high velocity dispersion, known as “free-streaming”, which reduces their gravitational collapse on small scales. A number of large scale structure (LSS) surveys [4, 5] will improve this upper bound using neutrino effects on LSS, however we usually encounter difficulties in disentangling neutrino effects from other parameters, and in understanding halo bias and cosmic variance. Recently, new nonlinear neutrino effects on LSS are proposed to give independent measurements on neutrino mass [6–8]. For these effects one needs to carefully separate and exclude the contaminations from non-gravitational contributions. The only clean, gravitational-only neutrino effect on LSS is the damping of weak gravitational lensing power spectrum [4]. However the amount of information and constraining power of the lensing is limited by the projection onto the 2-dimensional sky. Is there a 3-dimensional measure of spectral lines we determine the plus-minus sign of the spin, which is parity-odd. A parity-odd spin measurement cannot be contaminated by the linear perturbation theory. Only gravity affects spin statistics of halos or galaxies at large separations, since baryons do not travel to the distance of the free-streaming of neutrinos.

We present the neutrino torque effect, to modulate the spin of dark matter halos. Here we use halos to represent galaxies. The galaxy spin can be observed to a much larger radii than baryons do. This suggests that the spins of a galaxy and the whole halo are well correlated.

Theory.— In the picture of LSS formation, gravitational instability lets initial density fluctuations form dark matter halos, where galaxies are embedded. In these highly nonlinear structures, uncertainties of halo bias, halo merging history and baryonic mechanisms obstruct us from clearly understanding the statistics like number counts and morphologies. In comparison, the spins of galaxies/halos represent a local probe of gravity, especially contributed from the linear epoch of the structure formation.

The initial halo spin is written in Lagrangian space as $j_L \propto -\int V_L q \times \nabla \phi_c d^3q$, where $\phi_c$ is the gravitational potential of CDM, $q$ is the Lagrangian coordinates relative to the center of mass of the protohalo in volume $V_L$. In the tidal torque theory [10], it can be written as, $j_T \propto e I_q T_c$, where $I_q = (I_{ij}) = (\int V_L q_i q_j d^3q)$, $T_c = (T_{ij}) = (\partial_i \partial_j \phi_c)$ are the protogalactic inertia tensor and the local tidal shear tensor1, and $e = (\epsilon_{ijk})$ is

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1 $I_q$ and $T_c$ differ from textbook by trace, which does not contribute to $j_T$.  

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the Levi-Civita symbol to collect the asymmetric components generated by the misalignment between $I_q$ and $T_r$. Here, $I_q$ and $T_r$ are parity-even, $\epsilon$ is parity-odd, and thus $j^T_r$ is parity-odd. In linear perturbation theory, $\phi_r$ remains constant, so $T_r$ decays as $a^{-2}$ due to the cosmic expansion, where $a$ is the scale factor. Also, the protohalo shrinks in size and turns more spherical in shape. A decayed tidal field is hard to torque small and round objects, so the spin are expected to be contributed mostly in the linear regime. These concepts can be straightforwardly tested in numerical simulations. In Lagrangian space, CDM and baryonic matter are torqued by the same gravitational shear, so the spin direction is not affected by baryonic effects. If a halo has a merging history, it simply collects disconnected regions in Lagrangian space but conserves their total spin.

Massive neutrinos contribute sub-percent fraction of the matter ingredient, and their unique spatial distribution and evolution should contribute a unique torque to halos and galaxies. In particular, neutrino density field traces CDM on large scales while their small scale structures are smoothed out by their free-streaming. They contribute a predictable tidal tensor $T(\nu)$ depending on the their mass [11]. The interplay between neutrinos and dark matter should contribute a unique torque to the matter ingredient, and their unique spatial distribution in shape. A decayed tidal field is hard to torque small and round objects, so the spin are expected to be contributed mostly in the linear regime. These concepts can be straightforwardly tested in numerical simulations. In Lagrangian space, CDM and baryonic matter are torqued by the same gravitational shear, so the spin direction is not affected by baryonic effects. If a halo has a merging history, it simply collects disconnected regions in Lagrangian space but conserves their total spin.

Neutrino distribution shares the same Fourier phases of CDM from halo reconstruction [11, 12], but only differs by the ratio of their linear transfer function, depending on the neutrino masses. The reconstruction $T_r$ is reliable even if it is applied directly on halos [11], because they are relatively linear even at present epoch.

The reconstruction of $I_q$ relies on the fact that $I_q$ and $T_r$ are highly correlated in Lagrangian space [13, 14], which can be understood that $I_q$ is a collection of matter to be shell-crossed to form a halo, being parallel with $T_r$. The latter depends on a precise reconstruction of $I_q$ and $T_r$.

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Recent years, many emerging reconstruction methods have achieved unprecedented accuracies. For example, the isobaric halo reconstruction [15] can recover the initial conditions of the Universe from spatial distribution of halos. In the case of high halo numbers densities the reconstructed density field is correlated with the true initial density field on scales $k \lesssim 0.7h$/Mpc [12], close to the limit ($k \lesssim 1h$/Mpc) of isobaric reconstruction from using direct CDM density field [16], or the limit of reconstruction from the true displacement field [17]. ELUCID simulations are able to reconstruct the full evolution history of the real local Universe [18]. These reconstruction techniques enable us to study the tidal field from both CDM and neutrinos in an unprecedented precision, at different epochs of the cosmic evolution. We construct an equivalent inertia $I_R$ from $T_r$, in order to maximize the cross-correlation $\mu$ between $\epsilon I_R T_r$ and $j_{\nu0}^T$ (see Appendix A).

The final neutrino torque can be understood as an interaction between two scales – a small collapsing scale $T_r$, a large neutrino free-streaming scale $T_{\nu}$, and with an antisymmetric (parity-odd) operator $\epsilon$ collecting the antisymmetric (parity-odd) contributions from the multiplication of these two symmetric (parity-even) tensors.

Simulation. — These correlations and coefficients are tested across a set of high-resolution $N$-body simulations [19]. Given any halo formed in the simulation, all the belonging particles are mapped back to Lagrangian space. The status of this definite set of particles can be traced in a resimulation of the exact same initial conditions.

In Fig. 1 (all quantities are projected onto the plane of this letter), we select a very massive halo $(7.8 \times 10^{14} M_\odot)$ to maximize the clarity of the visualization of the halo properties. We confirm in simulation that these properties (cross-correlations) have only weak dependence on halo mass. The background LSS at redshift $z = 0$ has the thickness $2r_{200}$ with $r_{200}$ being the halo radius within which the mean halo density is 200 times the mean matter density of the Universe. The Lagrangian mapping of this halo is shown by the protohalo’s column density with the orange clouds. To visualize the tidal torque theory, we plot equivalent ellipsoids (solid curves) with
their moment of inertia equal to $I_q$ and $I_{200}$, where $I_{200}$ is the moment of inertia within $r_{200}$. The ellipsoids with dotted lines correspond to $T_e$ and $T_{e'}$, normalized such that their volumes are $V_q$ and $8V_q$ respectively. As expected, $I_q$ and $T_e$ are aligned with their primary axes in parallel with the collapsing direction, perpendicular to the filament containing the halo. Their minor misalignment yields the tidal torque $j_T$, which is the first order approximation of the true initial spin $j_L$ (all the spin arrows are normalized to have $15$ Mpc/h). They are, in general, highly correlated with the spin of the final halo $j_0$. In comparison, the neutrino tidal shear $T_{e'}$ torques $I_{e'}$ in an other less correlated direction $j_{e'}$.

The validity of the tidal torque formulation is tested by an ensemble average over all halos, across $3$ orders of magnitude in mass range ($> 2 \times 10^{12} M_{\odot}$), and over simulations with different resolutions. As the Universe evolves from the initial condition to $z = 0$, the cross-correlation coefficients $\langle j(z) \cdot j_L \rangle$ and $\langle j(z) \cdot j_T \rangle$ smoothly decrease from $1$ to $0.80$, and from $0.75$ to $0.69$, respectively.

For neutrinos, the first-order tidal torque approximation gives a near perfect (with cross-correlation $0.99$) representation of the actual neutrino torque, and it has generally $< 0.2$ cross-correlates with CDM torques. This is expected in that $T_e$ dominates locally whereas $T_{e'}$ is contributed beyond the neutrino free-streaming scale. These two species, however, have a highly correlated contribution in structure formation. When we consider the gravitational forces that the two species exerted to the protohalo, $F_{c/\nu} \propto \int \nabla \phi_{c/\nu} d^3q$, the cross-correlation between two species is as high as $0.86$.

From simulations, we estimate the magnitude of integrated neutrino torque $\langle |j_T^{\nu}/|j_0| \rangle \simeq 3 \times 10^{-4}$. In particular, the effect given by the smoother distribution for neutrinos relative to CDM accounts $0.03$, while the neutrino fraction $f_\nu = 3.5 \times 10^{-3}$ (for $M_\nu = 0.05$ eV) and the back-reaction factor from neutrinos to CDM $\sqrt{S}$ [20] contribute the rest.

Measured from simulations, the cross-correlation between $\epsilon \Delta \mu T_e$ and $\epsilon \Delta \mu j_T^{\nu}$ is about $\mu = 0.19$ (see Appendix A). We then need $5 \times 10^9$ halos to have a $5\sigma$ detection (see Appendix B). If we improve the reconstruction of $I_R$ from studying the cosmic evolution history [18], the lower limit of required halos is $2 \times 10^8$.

Discussion. — We found that the cross-correlation coefficients between $j_0$ and initial values $j_L$ and $j_T$ are prominently higher than our previous understandings (e.g. [13]). We carefully investigate the numerical errors that may affect the results. P3M (particle-particle particle-mesh) algorithms result in higher cross-correlation between initial and final spins, compared to PM (particle-mesh), where additional tangential forces in PM violate the angular momentum conservation. Higher mass halos in a given simulation generally have slightly higher cross-correlations between initial and final spins, however the correlation is enhanced as we use higher mass resolutions. All other cross-correlation measure-ments have only weak dependencies on the halo mass, even in a fixed simulation. With different box sizes, mass resolutions, force resolutions, we find that the results are consistent across these simulations. The estimation of number of halo spins needed to detect the neutrino torque depends on $\mu = 0.19$. This has a very weak dependence on halo mass and configuration of the simulation. An accurate study of the neutrino torque requires mass and force resolutions to cover the wide range of halo mass, a large box size ($> 600$ Mpc/h) to account the neutrino tides at distance, and neutrino particles/fluids, further studies of reconstruction of $I_R$ (Appendix A) to calculate more precise nonlinear neutrino effects on an evolving halo. These require future simulations with computing power comparable to that of TianNu [21].

Surveys like the Hubble Sphere Hydrogen Survey (HSHS) [22] or a modified 21cm Cosmic Vision [23] are able to observe order of $10^9$ HI galaxies ($> 10^{12} M_{\odot}$), in a cosmic volume $(4 \text{Gpc}/h)^3$ below redshift $z \simeq 1$ [24]. Under the standard model of the Universe with $M_\nu = 0.05$ eV, a $5\sigma$ detection will be reachable. Beyond standard models, e.g., the neutrino mass could be generated by a gravitational $\theta$-term condensation as the source of dark energy [25], the tidal torque history will be different, and we need different numbers of galaxies to differentiate between these models.

Conclusion. — Enormous efforts has been contributed to the neutrino mass properties. Radio astronomic and cosmological surveys are promising low-cost experiments implemented in the Universe to probe basic physical mysteries. The angular momentum is a 3-dimensional, gravity-driven, parity-odd measure, and is readily observable, well modeled by the tidal torque theory, and well conserved over the cosmic evolution. It contains comparable amount of information as density field but poorly appreciated in application. Recent developed reconstruction techniques enable us to visit an-order-of-magnitude more precise initial conditions of the Universe in Lagrangian space, to accurately reconstruct the neutrino torque effects, and to unveil the neutrino mass properties in upcoming galaxy surveys.

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Appendix A: Reconstruction of $I_R$

In Lagrangian space, $I_q$ and $T_e$ are highly correlated. In linear, intermediate epochs, $I_q$ is firstly reshaped according to the dominating $T_e$, thus the linear evolved $T_e(\tau)$ will act on an evolved $I(\tau)$. Further, $I(\tau)$ will be affected by nonlinear effects, which are more difficult to predict, however halos are relatively small and their spin directions can hardly be changed dramatically. A deep study of the latter steps above is beyond the scope of this letter. Here we show that, even simplified to the
first point above, the total neutrino torque can still be reconstructed.

We construct an equivalent inertia $I_R$ from $T_c$, in order to maximum the cross-correlation coefficient $\mu$ between $\epsilon I_R T_\nu$ and $j'_0$. In the primary coordinate of $T_c$, $T_c$ can be eigen-decomposed as $T_c = \sum_{i=1}^{3} T_{i c}$ and we find $(\alpha_1, \alpha_2, \alpha_3) = (-0.7, 0.7, 0.08)$ (normalized such that $\sum \alpha_i^2 = 1$) and $I_R \propto \sum_{i=1}^{3} \alpha_i T_{i c}$ optimize $\mu$ to be 0.19.

Appendix B: Errors

Consider $N$ halos with their unit spin vector randomly distributed on a 2-dimensional sphere, i.e., $|j| = 1$ and $\langle j \rangle = 0$. Adding an additional vector $\epsilon \hat{x}$ ($\epsilon \ll 1$) to $j$ and normalize, $j' = (j + \epsilon \hat{x})/|j + \epsilon \hat{x}|$, then project $j'$ onto $\hat{x}$ and we get $p = j' \cdot \hat{x}$. Since $\langle p \rangle = 2\epsilon/3$ and $\sigma(p) = 1/\sqrt{3N}$, an $n\sigma$ detection requires $N = 3n^2/4\epsilon^2$.

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