Kinetic modelling of quantum effects in laser-beam interaction

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Abstract
We present the results of kinetic modelling of quantum effects in laser-beam interaction. In the developed numerical model, electron-positron pair production by hard photons, hard photon emission and the electromagnetic fields generated by the created charged particles are taken into account. Interaction of a relativistic electron beam with a strong laser pulse is analyzed. It is shown that the quantum effects can be important even for moderately intense laser pulses when the number of emitted photons by single electron is not large. Electron-positron pair plasma production in extremely-intense laser field via development of electromagnetic cascades is also studied. The simulation results confirm the prediction of strong laser field absorption in the self-generated electron-positron plasma. It is shown that the self-generated electron-positron plasma can be an efficient source of energetic gamma-quanta.

Electromagnetic cascades are one of the basic phenomena of strong-field physics. As opposed to classical physics, in quantum physics the decay of high-energy photon in strong electromagnetic fields is possible. The photon can decay with creation of electron-positron \((e^- e^+)\) pair. If the energy of the electron and the positron is high enough, they can emit new photons that can, by-turn, decay with creation of new \(e^- e^+\) pairs, etc. This cascade process is also called ‘electromagnetic (or QED) shower’.

Due to its importance, the cascades recently are intensively studied. For example, electromagnetic showers that develop during the pass of high-energy particle (electron, positron or photon) through constant homogeneous magnetic field are theoretically investigated in Ref. [1,2]. In this case the energy for creation of secondary particles is transferred from the initial energy of the seed particle. In such shower the mean energy of a particle decreases with the lapse of time. The dynamics of electromagnetic showers in strong fields of pulsars also attracts much attention [3]. Showers in inhomogeneous, varying electromagnetic fields (fields of pulsars, for example) can be self-sustained. This means that secondary particles in such shower are accelerated by strong external electric field and the mean energy of a particle in such cascade is constant or increases.

There are little experiments devoted to investigation of self-sustained cascades. However, the recent progress in laser technologies will allow to fill up this gap. The laser intensity can be very high in near future so that quantum electrodynamic (QED) effects will be essential [4,5,6,7,8]. An electron (positron) moving in strong laser field can be accelerated up to very high energy and can produce electromagnetic cascade. If the laser intensity is high enough, the number of produced pairs can be so great that self-generated electromagnetic fields of \(e^- e^+\) plasma can strongly affect the further cascade dynamics. Moreover, a large portion of the laser energy can be absorbed by self-generated \(e^- e^+\) plasma. This effect is especially important to determine the limitations on the intensity of high power lasers [6,8]. The estimations of intensity threshold for cascade development in circularly polarized standing electromagnetic wave are presented in Ref. [6]. However, in order to study the cascade development and to verify estimations the self-consistent numerical models is needed.

We develop the self-consistent two-dimensional numerical model based on particle-in-cell (PIC) and Monte Carlo (MC) methods [9]. The model uses the probability rates of photon emission and electron-positron pair production calculated in the framework of QED theory [10,11,12]. It exploits the strong difference between the photon energy of laser-plasma fields and the characteristic energy of hard photons emitted by relativistic charged particles in electromagnetic fields. Making of use standard PIC technique the hard photons, electrons and positrons are modeled as quasiparticles while the laser and plasma field is calculated by nu-
numerically solving of Maxwell’s equations. The model has been benchmarked to the simulations performed by other MC codes [13]. The PIC part of the model is two-dimensional version of the model used in Ref. [14].

One of methods to probe QED in laboratory conditions is to study the interaction of a relativistic electron beam with an intense laser pulse [15, 16]. In the rest frame of the relativistic particle the laser field is very intense and can be close to critical. So at this field strength quantum effects become important. In this case the energy of a photon emitted by a beam electron undergoing oscillations inside the laser pulse can be close to the electron energy.

We simulate the interaction between a relativistic electron bunch and a laser pulse by the means of our numerical model. The laser pulse is linearly polarized, has the Gaussian envelope $E_y = B_z = a_0 \exp[-\gamma^2/\sigma_s^2]$, where $E$ and $B$ are the electric and magnetic field envelopes, respectively, $a_0 = |eE_0/(mc\omega)|$, $E_0$ is the maximal field strength of the laser pulses, $\omega = 2\pi c/\lambda$, $\lambda$ is the laser wavelength, $m$ and $e$ are the electron mass and charge, respectively, $c$ is the speed of light, $\sigma_s$ is the pulse duration, $\sigma_y$ is the pulse width. From here on coordinates are normalized to $\lambda$ and time is normalized to $2\pi/\omega$. The parameters of the simulation are the following. The wavelength $\lambda = 0.8 \, \mu m$, $\sigma_s = 32$, $a_0 = 5$, the laser spot size $\sigma_y$ is assumed to be much large then the bunch width. The initial gamma-factor of the bunch electrons is $\gamma_0 = 2000$. For these parameters $\chi \approx 0.03 \ll 1$ and the photon emission regime should be classical. The energy distribution of the electron bunch after passing through laser pulse is shown in Fig. [1]. The mean gamma-factor of the electron after interaction is $\gamma = 1568$. The task about the interaction between an electron and a laser pulse has exact theoretical solution in the framework of Landau – Lifshitz representation of radiation reaction force [17]. This solution yields the following gamma-factor of the electron passing the laser pulse with Gaussian envelope:

$$\gamma = \frac{\gamma_0}{1 + \mu e^2 \omega^2 / (3mc^3)}.$$

where $\mu = 2e^2 \omega^2 / (3mc^3)$. For the given parameters this formula yields $\gamma = 1700$ that is in good agreement with the simulation result.

It is interesting to note that quantum nature of photon emission can reveal itself even in classical regime ($\chi \ll 1$). It is seen from Fig. [1] that the energy spread of the electron bunch is large in contrast to the classical theory. The reason is that each bunch electron emits a small number of photons. The characteristic time be-
between consecutive photon emissions (photon emission time) can be estimated as the ratio of the characteristic energy of emitted photon to the radiation power. In the classical limit the photon energy is \( \hbar \omega_{cm} \sim a_0 \gamma^2 \hbar \omega \) and in the quantum limit \( \hbar \omega_{cm} \sim \gamma mc^2 \) \([11]\). Making use of the expression for the radiation power in the classical and quantum limits \([12, 11]\) we can find the photon emission time in the classical and quantum limits

\[
t_{rad} \approx \frac{hc}{e^2} \left( 1 + \chi^{1/3} \right) t_f, \tag{2}
\]

where \( ct_f \) is so-called radiation formation length, which is equal to length of the electron trajectory path, over which the particle is deflected by angle \( 1/\gamma \) \([11]\) \([12, 13]\). The radiation formation length for an electron colliding with ultra-high intense laser pulse can be estimated as follows: \( t_f = 1/(2\pi a_0) \). For the given parameters \( \sigma_c/(ct_{rad}) \approx 6 \) that is not much larger than unity. Therefore the number of photons emitted by a bunch electron passing through the laser pulse is not large. Because of quantum nature of photon emission the spread in the number of the photons and energy of the photons emitted by bunch electrons can be significant.

The electron energy distribution of the electron bunch after interaction with shorter and more intense laser pulse (\( \sigma_x = 1.6, a_0 = 20 \)) is shown in Fig. 4. The initial electron gamma-factor is \( \gamma_0 = 2000 \). For these parameters \( \chi \approx 0.1 \) and \( t_{rad} \approx 1.1 \). As \( \sigma_x/t_{rad} \approx 1 \) a bunch electron emits approximately one photon in the average. As the electron distribution function peaks at
In this work we also present the results of 2D numerical simulations of QED cascade in the field of two colliding linearly polarized laser pulses. Laser pulses have the Gaussian envelopes at initial instant \( t = 0 \) and propagate along \( x \)-axis towards each other. At initial instant of time the distance between laser pulses is equal to \( 2\sigma_x \) and coordinates of the pulse centers are \( x = -\sigma_x \) and \( y = 0 \). The cascade was initiated by \( 5 \times 10^8 \) GeV photons situated near \( x = -\sigma_x, y = 0 \). The mean photon number emitted by the laser pulse in quantum regime is classical and the total loss of the bunch energy due to the photon emission can be calculated in the classical approach the distribution function of the electron bunch after interaction cannot be described in the framework of classical electrodynamics if the mean photon number emitted by the bunch electron is small.

We model the interaction between electron bunch and the laser pulse in quantum regime. The parameters of the laser pulse and the electron bunch are \( \sigma_x = 1.6 \), \( a_0 = 100 \) and \( \gamma_0 = 2 \times 10^4 \). For these parameters \( \chi \approx 6 \) and \( t_{\text{rad}} \approx 0.2 \). The electron distribution function after interaction with the laser pulse is shown in Fig. 8. It is seen from Fig. 8 that the bunch lost most of the initial energy. Moreover, some of the emitted photons decay and produce electron-positron pair. The similar effects are observed in the experiments [15] and discussed, for example, in Ref. [16].

In this work we also present the results of 2D numerical simulations of QED cascade in the field of two colliding linearly polarized laser pulses. Laser pulses have the Gaussian envelopes at initial instant \( t = 0 \) and propagate along \( x \)-axis towards each other. At initial instant of time the distance between laser pulses is equal to \( 2\sigma_x \) and coordinates of the pulse centers are \( x = -\sigma_x \) and \( y = 0 \). The values of simulation parameters are the following: \( \lambda = 1.24 \mu m, a_0 = 1.5 \cdot 10^3, \sigma_x = 19, \sigma_y = 8 \).
The spatial distributions of normalized electron and photon densities and laser intensity at three instants of time are shown at Figs. 4, 5, and 6. At initial stage of cascade development (Fig. 4) \( e^- e^+ \) plasma density rapidly reaches the value of the relativistic critical density \( n_{e\text{cr}} \), where \( n_{e\text{cr}} = m_e \omega_e^2/(8 \pi e^2) \) is the nonrelativistic critical density for \( e^- e^+ \) plasma. The \( x \)-scale of created plasma is about the laser wavelength. Creation of dense plasma leads to substantial absorption of laser pulses and to the decrease of laser fields in the region occupied by the plasma. At later stages of the cascade development (Figs. 5, 6) \( e^- e^+ \) plasma expands.

Strong absorption of the laser pulses starts at \( t \approx 24 \) when the plasma density becomes close to \( n_{e\text{cr}} \). Due to the asymmetry in the initial position of the seed particles one of the laser pulses partially has passed the region occupied by the plasma at \( t \approx 24 \), while only small part of the front side of the other pulse has passed this region. Then the laser pulse that starts at \( x = \sigma_x \) and the rear part of the other laser pulse are mostly absorbed by created \( e^- e^+ \) plasma (see Fig. 5(c)). The asymmetry in the initial conditions also leads to asymmetry in the distribution of hard photons emitted by the electrons and the positrons. Normalized electron and photon densities and the distribution of laser intensity averaged over the laser wavelength at \( x \)-axis are shown on Figs. 7, 8. The asymmetry of the laser pulses absorption and hard photon emission can be also seen in Figs. 7, 8. The steeping of the right edge of the electron distribution caused by momentum transmission from the absorbed laser pulse to \( e^- e^+ \) plasma can be seen in Fig. 5 (line 1).

In conclusion we present the simulation results obtained by the developed two-dimensional hybrid PIC/MC model. The model allows us to simulate QED processes in laser plasmas. First we analyze the QED processes during interaction of the relativistic electron beam with the intense laser pulse. It is shown that the QED effects can be important even for moderately intense laser pulses when the number of emitted photons by single electron is not large. Then avalanche-like cascade development in the field of two colliding linearly polarized laser pulses is simulated. We show that overdense \( (e^- e^+) \) plasma is produced in the region where laser pulses overlap. A significant portion of the laser energy is absorbed as a result of production and heating of the plasma. The anisotropic emission of hard photons is observed that can be used for development of bright radiation sources of gamma-quanta. The obtained results demonstrate that QED effects can be observed in future laser facilities like ELI [20] and HiPER [21], since the laser intensity \( (I \approx 2 \cdot 10^{22} \text{ W/cm}^2) \) can be achievable in such laser systems.

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