Contribution to time truncating reliability single sampling plan for weighted Rayleigh distribution

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Abstract
The Rayleigh distribution, a well-known one, is naturally applicable in life testing studies. This article is indeed an attempt made to study the Weighted Rayleigh Distribution as a model for a life time random variable. It discusses the concept of truncated single acceptance sampling plan at a pre-assigned time. With the given probability levels, different acceptance numbers and the values of the ratio of the specified test time to the specified mean life are obtained, further it is found that the minimum sample sizes with assured the specified mean life time are discussed. This paper also elaborates the operating characteristic values of the sampling plans and producer’s risk. Tables are given and illustrated with example.

Keywords: Weighted Rayleigh Distribution (WRD), truncated life test, producer's risk and consumer's risk

Introduction
Acceptance sampling is one of the most popularly used sampling technique in Quality Control. The purpose of the plan is to find an optimal plan parameters such as sample size and its acceptance number to save the cost and time of the experiment. This becomes particularly important if the quality of the product is defined by its lifetime. It is easy to understood that when the acceptance sampling procedure involved with its quality characteristics which follow the lifetime of products then it is called life testing studies. The life time data that are collected during the life test, if the time period in which a product operated successfully. Life time of the products can be measured in hours, miles, cycles to failure or any other metric with which the life or exposure of a product can be measured. All such type of products life time is expressed in the term ‘Life Data’. These requires certain specification of a probability model which follow the life time of the products then such a procedure is called Acceptance sampling plan based on life test. Thus this plan considered two types of risks in which the consumer decides to accept or reject a lot of products shipped by the producer based on the inspection. The consumer risk and producer risk are then the probabilities that a bad is accepted and a good lot is rejected.

The objective of these life time experiments is to set a lower confidence limit on the mean life. In this case, the aim of the life test is verifying that whether the mean life of the product exceeds the specified one, say $\sigma \geq \sigma_0$ this means that the true mean life of the products exceeds specified mean life then the lot is accepted. The decision criteria is based on the number of failures observed in the sample of size $n$ during the specified time period $t$. If the observed number of failures is greater than $c$ (acceptance number), then the lot is rejected otherwise accepted. Based on this criteria and the confidence limits, the minimum sample size $n$ is attained to save the time and cost.

Review of literature
Among the pioneers who have been working on the Truncated Acceptance Sampling plans for several years, Epstein [3] is the most prominent who introduced life test plans and designed the truncated life test for single acceptance sampling plans which follow exponential life time distribution. Similar to this, several authors have also developed various life test plans based on several distributions.
Weighted Rayleigh distribution
(Area Biased Rayleigh Distribution)

The probability density function of the Rayleigh Distribution is given by

\[ f(t; \sigma) = \frac{t}{\sigma^2} e^{-\frac{t^2}{2\sigma^2}}; \ t > 0, \ \sigma > 0 \]  \hspace{1cm} (1)

While the cumulative distribution function of the Rayleigh distribution

\[ F(t; \sigma) = 1 - e^{-\frac{t^2}{2\sigma^2}} ; \ t > 0, \ \sigma > 0 \]  \hspace{1cm} (2)

Suppose \( T \) is a non-negative random variable with its density function \( f(t; \sigma) \), \( \sigma \) is a parameter, then \( f_w(t; \sigma) \) distribution is a weighted version of \( f(t; \sigma) \) and is defined as:

\[ f_w(t; \sigma) = \frac{w(t)f(t; \sigma)}{E[w(t)]} \]  \hspace{1cm} (3)

Where \( w(t) \) is an arbitrary non-negative function.

For \( w(t) = t^\alpha \), when \( \alpha = 1 \), it is called size biased and \( \alpha = 2 \), it is called area biased distribution according to Shakila Bashir and Mujahid Rasul [17]. This paper made a attempt to study the Area biased Rayleigh Distribution under Industrial shop floor condition.

The probability density function of size biased Rayleigh Distribution is

\[ f(t; \sigma) = \frac{\alpha^2}{\sigma^2} \frac{t}{\sqrt{\pi}} e^{-\frac{t^2}{2\sigma^2}} ; \ t > 0, \ \sigma > 0 \]  \hspace{1cm} (4)

Using equation (3) and equation (1), the probability density function of the area biased Rayleigh distribution is given by

\[ f(t; \sigma) = \frac{\alpha^3}{2\sigma^3} e^{-\frac{t^2}{2\sigma^2}} ; \ t > 0, \ \alpha > 0 \]  \hspace{1cm} (5)

The cumulative distribution of the ARD is

\[ F(t; \sigma) = \gamma(2, t^\alpha) \), where \( t^\alpha = \frac{t^2}{2\sigma^2} \]  \hspace{1cm} (6)

Where \( \gamma(n, x) = \int_0^x t^{n-1}e^{-t} \, dt \),

It is a lower incomplete gamma function.
Truncated sampling plans

In some cases, the real data set does not follow any of the standard probability distribution. Henceforth the quality of the procedures used in statistical theory depends strongly on some assumed probability distributions. In this article, it is assumed that the product lifetime follows a new Weighted Rayleigh distribution it is also called as Area biased Rayleigh distribution.

A usual procedure in life testing is that it terminates the life test by a pre-fixed time \( t \) and observes the number of failures (assuming that a failure is well defined). One of the objectives of the life test is to set a lower confidence limit on the average life time. It is then desired to establish a specified mean life with a probability of at least \( p^* \).

The decision criteria are

- To accept the lot, the specified average life occurs if and only if the number of observed failures at the end of the fixed time \( t \) does not exceed a given acceptance number \( c \).
- To reject the lot when the number of failures exceeds \( c \), in which case the test may get terminated before the time \( t \) is reached.
- Thus the sampling plan consists of the triplet \((n, c, \frac{t}{\sigma_0})\)

Where

\( n \rightarrow \) Number of units on test
\( c \rightarrow \) An acceptance number

The ratio \( \frac{t}{\sigma_0} \rightarrow t \) is the maximum test duration and \( \sigma_0 \) is the specified average.

For this kind of truncated life test and its decision rule, it is aimed at finding the minimum sample size to fulfil the objective of the reduction of time and cost.

Consumer's risk

The condition required to obtain the minimum sample size is the probability of rejecting a bad lot to be at least \( p^* \). Obviously the consumer’s risk is the probability of accepting a bad lot not to exceed 1 - \( p^* \). A bad lot is the lot with true average life, which is below the specified average life \( \sigma_0 \). Here, it is to be assumed that the lot is infinitely large size so that binomial distribution can be applied. For the given \( p^* \), the acceptance number \( c \) and the ratio \( t/\sigma_0 \), one can obtain the smallest positive integer \( n \), which satisfies the inequality,

\[
\sum_{i=0}^{c} \binom{n}{i} p^i (1 - p)^{n-i} \leq 1 - p^* \quad \text{... (7)}
\]

Where \( p = F(t; \sigma_0) \) is given by (6). It indicates that the probability of observed failure before time \( t \) depends only on the ratio \( t/\sigma_0 \). This ratio is sufficient for designing lifetime experiment.

If the number of observed failures before time \( t \) when it is less than or equal to \( c \), from (7) one can obtain:

\[
F(t; \sigma) \leq F(t; \sigma_0) \iff \sigma \geq \sigma \quad \text{... (8)}
\]

The minimum sample size satisfies the inequality (7) for given \( p^* = 0.75, 0.90, 0.95 \) and \( 0.99 \) and \( \frac{t}{\sigma_0} = (0.7, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0) \) Hence it is obtained and presented in Table-1.

| Table 1: Minimum sample sizes for the given \( \sigma_0 \) with \( p^* \) for the acceptance number \( c \). |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \( p^* \) | \( c \) | \( 0.7 \) | \( 0.9 \) | \( 1 \) | \( 1.5 \) | \( 2 \) | \( 2.5 \) | \( 3 \) | \( 3.5 \) | \( 4 \) |
| 0.75 | 0 | 54 | 22 | 15 | 5 | 2 | 1 | 1 | 1 | 1 |
| 1 | 105 | 42 | 30 | 8 | 4 | 3 | 2 | 2 | 2 |
| 2 | 153 | 62 | 43 | 12 | 7 | 4 | 3 | 3 | 3 |
| 3 | 200 | 117 | 56 | 16 | 8 | 5 | 5 | 4 | 4 |
| 4 | 250 | 146 | 69 | 20 | 10 | 7 | 6 | 5 | 5 |
| 5 | 300 | 200 | 82 | 23 | 12 | 8 | 8 | 6 | 6 |
| 6 | 344 | 236 | 93 | 27 | 13 | 10 | 8 | 7 | 7 |
| 7 | 378 | 255 | 106 | 30 | 15 | 11 | 9 | 8 | 8 |
| 8 | 425 | 281 | 119 | 34 | 18 | 12 | 10 | 9 | 9 |
| 9 | 465 | 311 | 131 | 37 | 19 | 13 | 11 | 10 | 10 |
| 10 | 509 | 346 | 142 | 41 | 21 | 14 | 13 | 12 | 11 |
| 0.9 | 0 | 90 | 36 | 25 | 8 | 3 | 2 | 2 | 2 |
| 1 | 150 | 62 | 42 | 12 | 6 | 4 | 2 | 2 | 2 |
| 2 | 200 | 86 | 48 | 17 | 8 | 6 | 4 | 3 | 3 |
| 3 | 260 | 104 | 73 | 20 | 10 | 6 | 5 | 4 | 4 |
| 4 | 311 | 125 | 87 | 24 | 12 | 9 | 6 | 5 | 5 |
| 5 | 362 | 146 | 100 | 28 | 14 | 9 | 8 | 7 | 7 |
| 6 | 410 | 166 | 115 | 32 | 17 | 13 | 10 | 8 | 8 |
| 7 | 459 | 185 | 128 | 36 | 18 | 12 | 10 | 9 | 9 |
| 8 | 507 | 205 | 142 | 40 | 19 | 13 | 11 | 10 | 9 |
p-values for fixed value of c (say c=2) calculated and given in Table

\[ p = F(t; \sigma) \]

\[ L(p) = \sum_{i=0}^{\infty} \binom{n}{i} p^i (1 - p)^{n-i} \]

\[ \frac{1}{\sigma_0} \]

The operating characteristic function of the truncated life sampling plan \((n, c, \frac{1}{\sigma_0})\) gives the probability that a lot can be accepted with \(L(p)\).

\[ L(p) = \sum_{i=0}^{\infty} \binom{n}{i} p^i (1 - p)^{n-i} \] \quad \ldots (9)

Where

\[ p = F(t; \sigma) \]

is a function of lot quality parameter \(\sigma\). It is to be noted that the operating characteristic function is an increasing function of \(\sigma\). For given \(p^*\) and \(\frac{1}{\sigma_0}\), the choice of \(c\) and \(n\) can be made on the basis of operating characteristics function. The OC values for fixed value of \(c\) (say \(c=2\)) calculated and given in Table 2.

**Table 2: Operating characteristic values of the sampling plan \((n, c, \frac{1}{\sigma_0})\) for given \(p^*\) and \(c=2\)**

| \(p^*\) | \(n\) | \(t/\sigma\) | 1    | 1.25  | 1.5    | 1.75   | 2   | 2.25  | 2.5    | 2.75   | 3    |
|-------|-------|------------|------|-------|--------|--------|-----|-------|--------|--------|-----|
| 0.75  | 153   | 0.7        | 0.248| 0.759 | 0.947  | 0.988  | 0.997| 0.999 | 1      | 1      | 1   |
| 0.75  | 61    | 0.9        | 0.254| 0.751 | 0.942  | 0.987  | 0.997| 0.999 | 1      | 1      | 1   |
| 0.75  | 43    | 1          | 0.243| 0.736 | 0.936  | 0.985  | 0.996| 0.999 | 1      | 1      | 1   |
| 0.75  | 12    | 1.5        | 0.229| 0.691 | 0.913  | 0.977  | 0.994| 0.998 | 0.999 | 1      | 1    |
| 0.75  | 7     | 2          | 0.102| 0.496 | 0.809  | 0.938  | 0.981| 0.994 | 0.998 | 0.999 | 1    |
| 0.75  | 4     | 2.5        | 0.153| 0.535 | 0.816  | 0.936  | 0.978| 0.992 | 0.997 | 0.999 | 1    |
| 0.9   | 207   | 0.7        | 0.1  | 0.597 | 0.892  | 0.974  | 0.994| 0.998 | 0.999 | 1      | 1    |
| 0.9   | 84    | 0.9        | 0.095| 0.574 | 0.879  | 0.97   | 0.992 | 0.998 | 0.999 | 1      | 1    |
| 0.9   | 58    | 1          | 0.096| 0.566 | 0.873  | 0.968  | 0.991 | 0.998 | 0.999 | 1      | 1    |
| 0.9   | 17    | 1.5        | 0.066| 0.461 | 0.806  | 0.942  | 0.983 | 0.995 | 0.998 | 0.999 | 1    |
| 0.9   | 8     | 2          | 0.054| 0.39  | 0.743  | 0.911  | 0.971 | 0.99   | 0.997 | 0.999 | 0.999 |
| 0.9   | 6     | 2.5        | 0.012| 0.188 | 0.535  | 0.795  | 0.919 | 0.969 | 0.988 | 0.995 | 0.998 |
| 0.95  | 244   | 0.7        | 0.05   | 0.492 | 0.847  | 0.961  | 0.99    | 0.999 | 0.999 | 1      | 1    |
| 0.95  | 98    | 0.9        | 0.05  | 0.473 | 0.832  | 0.955  | 0.988  | 0.997 | 0.999 | 1      | 1    |
| 0.95  | 68    | 1          | 0.049| 0.46  | 0.823  | 0.952  | 0.987  | 0.996 | 0.999 | 1      | 1    |
| 0.95  | 18    | 1.5        | 0.05  | 0.42  | 0.782  | 0.933  | 0.98   | 0.994 | 0.998 | 0.999 | 1    |
| 0.95  | 9     | 2          | 0.027| 0.301 | 0.675  | 0.881  | 0.959  | 0.986  | 0.995 | 0.998 | 0.999 |
| 0.95  | 5     | 2.5        | 0.045| 0.328 | 0.675  | 0.872  | 0.953  | 0.983  | 0.994  | 0.997 | 0.999 |
| 0.99  | 325   | 0.7        | 0.01  | 0.301 | 0.733  | 0.923  | 0.978  | 0.994  | 0.998 | 0.999 | 1    |
| 0.99  | 130   | 0.9        | 0.01  | 0.283 | 0.712  | 0.913  | 0.975  | 0.992  | 0.998 | 0.999 | 1    |
| 0.99  | 90    | 1          | 0.01  | 0.274 | 0.699  | 0.906  | 0.973  | 0.992  | 0.997 | 0.999 | 1    |
| 0.99  | 25    | 1.5        | 0.007| 0.203 | 0.605  | 0.856  | 0.952  | 0.984  | 0.995 | 0.998 | 0.999 |
| 0.99  | 11    | 2          | 0.007| 0.171 | 0.54   | 0.809  | 0.93   | 0.975  | 0.991 | 0.996 | 0.999 |
| 0.99  | 7     | 2.5        | 0.003| 0.102 | 0.411  | 0.71   | 0.878  | 0.951  | 0.981 | 0.992 | 0.997 |
Producer's risk

Producer's risk is the probability of rejecting a good lot. When the true average life is greater than the specified average life, the lot can be classified as a good lot. Therefore producer's risk is obtained as a function \( \frac{\sigma}{\sigma_0} \) by using the binomial distribution. For the given value of producer's risk i.e. 0.05, one may be interested to obtain the minimum value of \( \frac{\sigma}{\sigma_0} \) that will ensure the producer's risk less than or equal to 0.05. Hence the probability function is obtained as

\[
p = F\left( \frac{L}{\frac{\sigma_0}{\sigma}} \right)
\]  

... (10)

The value \( \frac{\sigma}{\sigma_0} \) is the minimum ratio for which the following inequality holds:

\[
\sum_{i=0}^{n} \binom{n}{i} p^i(1 - p)^{n-i} \geq 0.95
\]  

... (11)

For the specified confidence level \( p^* \), Minimum values of \( \frac{\sigma}{\sigma_0} \) satisfying the inequality (11) are given in Table 3.

Table 3: Minimum ratio of \( \frac{\sigma}{\sigma_0} \) for the acceptability of a lot with producer’s risk of 0.05

| \( p^* \) | 0.7  | 0.9  | 1.5 | 2   | 2.5  | 3   | 3.5  | 4   |
|----------|------|------|-----|-----|------|-----|------|-----|
| 0.75     | 0.6  | 0.7  | 1.2 | 1.5 | 2    | 3   | 3.5  | 4   |
| 0.75     | 1.0  | 1.7  | 2.5 | 3.0 | 3.5  | 4   | 4.5  | 5.2 |
| 0.75     | 1.5  | 1.5  | 1.5 | 1.8 | 2    | 2.2 | 2.6  | 3.0 |
| 0.75     | 2.0  | 1.4  | 1.4 | 1.5 | 1.6  | 1.6 | 1.9  | 2.4 |
| 0.75     | 2.5  | 1.3  | 1.3 | 1.4 | 1.5  | 1.5 | 1.8  | 2.1 |
| 0.90     | 2.0  | 1.2  | 1.2 | 1.3 | 1.4  | 1.4 | 1.6  | 1.7 |
| 0.90     | 2.5  | 1.1  | 1.1 | 1.2 | 1.3  | 1.3 | 1.5  | 1.6 |
| 0.90     | 3.0  | 1.0  | 1.0 | 1.1 | 1.2  | 1.2 | 1.4  | 1.5 |
| 0.90     | 3.5  | 0.9  | 0.9 | 1.0 | 1.1  | 1.1 | 1.3  | 1.4 |
| 0.90     | 4.0  | 0.8  | 0.8 | 0.9 | 1.0  | 1.0 | 1.2  | 1.3 |
| 0.95     | 2.5  | 1.5  | 1.5 | 1.6 | 1.7  | 1.7 | 1.9  | 2.0 |
| 0.95     | 3.0  | 1.4  | 1.4 | 1.5 | 1.6  | 1.6 | 1.8  | 2.0 |
| 0.95     | 3.5  | 1.3  | 1.3 | 1.4 | 1.5  | 1.5 | 1.7  | 2.0 |
| 0.95     | 4.0  | 1.2  | 1.2 | 1.3 | 1.4  | 1.4 | 1.6  | 2.0 |
| 0.95     | 4.5  | 1.1  | 1.1 | 1.2 | 1.3  | 1.3 | 1.5  | 2.1 |
| 0.95     | 5.0  | 1.0  | 1.0 | 1.1 | 1.2  | 1.2 | 1.4  | 2.2 |

~12~
Illustration of tables

Assume that the life distribution is followed by a Area biased Rayleigh distribution and the manufacturer wants to determine the parameters of truncated acceptance life sampling plan for assuring the average life of the electronic devices. The manufacturer is interested in proving that the true unknown average life of the electronic devices is at least 1000 hours. Assume that the consumer’s risk is set to be $1 - p^* = 0.25$. It is desired to stop the experiment at $t=1000$ hours. Then for an acceptance number $c = 2$, $\frac{t}{\sigma_0} = 1.0$ and $p^* = 0.75$, the minimum sample size which is obtained from Table-1 is $n=43$. 

Thus, 43 units is taken to be tested. If during 1000 hours, no more than two failures out of these 43 units are observed, then the manufacturer can assert with the confidence level of $p^* =0.75$ that the average life is at least 1000 hours. The operating characteristic values for the sampling plan $(n=43, c=2, \frac{t}{\sigma_0}=1.0)$ from the Table-2 are

| $\sigma/\sigma_0$ | 1.0 | 1.25 | 1.50 | 1.75 | 2.0 | 2.25 | 2.50 | 3 |
|------------------|-----|------|------|------|-----|------|------|---|
| L(p)             | 0.243 | 0.736 | 0.936 | 0.985 | 0.996 | 0.999 | 1.0 | 1 |

From the above operating characteristic value, it is concluded that if the true mean life is twice the specified mean life (i.e., $\frac{\sigma}{\sigma_0} = 2$) the producer’s risk is approximately 0.004.

From the Table 3, it is obtained the minimum ratio of true average life to the specified average life (i.e., $\frac{\sigma}{\sigma_0}$) for various choices of $c$ and $\frac{t}{\sigma_0}$, in order that the producer’s risk may not exceed 5%. Thus for the acceptance sampling plan $(43, 2, 1.0)$ the value of $\frac{\sigma}{\sigma_0}$ obtained from the table is 1.54. It shows that the product should have an average life of 1.54 times the specified average life 1000 hours in order that the product is accepted with the probability of at least 0.95.

![OC curve](image-url)

**Fig 1:** Shows the OC curve for the above given values

Real life example

Consider the example of life time (in hours) of the type of surface-mounted electrolytic capacitors. Three tests each with eight units, were conducted at different elevated voltage levels of 80,100 and 120V respectively and the failure is considered when the capacitance drifts more than 25%. Here we consider the life times of the voltage level 100 as a weight. (Guangbin Yang [6], p260) The failure times of eight observations are in the order of 1090, 1907, 2147, 2645, 2903, 3357, 4135, 4381.

Let the specified mean life be 1000 hours and life test time be 1500 hours and it leads to the ratio $\frac{t}{\sigma_0} = 1.5$ such as the corresponding $n$ and $c$ as 8, 1 from Table 1 for the confidence level $p^* = 0.75$. Thus the truncated life acceptance sampling plan is $(n=8, c=1, \frac{t}{\sigma_0} = 1.5)$. Based on this plan, the product is accepted only if the number of failures after 1500 hours is less than or equal to 1. It is revealed from the above sample of 8 failures that there is only one failure at 1090 hours before 1500 hours, hence the product is accepted.

Conclusion

In this article, A new truncated life acceptance sampling plan has been developed when life time of the test items follow a Area-Biased Rayleigh distribution. Tables have been presented for the minimum sample size, operating characteristic values and the minimum ratio based on the proposed sampling plan. The real time application of the proposed sampling plan is given using the electronic product which can be used effectively in analyzing the data. This proposed distribution might attracts wider sets of application in reliability based studies and it is very much interesting to study for various other sampling plans available in the literature.

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