HIGH-ENERGY PARTICLE TRANSPORT IN THREE-DIMENSIONAL HYDRODYNAMIC MODELS OF COLLIDING-WIND BINARIES

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ABSTRACT

Massive stars in binary systems (such as WR 140, WR 147, or η Carinae) have long been regarded as potential sources of high-energy γ-rays. The emission is thought to arise in the region where the stellar winds collide and produce relativistic particles that subsequently might be able to emit γ-rays. Detailed numerical hydrodynamic simulations have already offered insight into the complex dynamics of the wind collision region (WCR), while independent analytical studies, albeit with simplified descriptions of the WCR, have shed light on the spectra of charged particles. In this paper, we describe a combination of these two approaches. We present a three-dimensional hydrodynamical model for colliding stellar winds and compute spectral energy distributions of relativistic particles for the resulting structure of the WCR. The hydrodynamic part of our model incorporates the line-driven acceleration of the winds, gravity, orbital motion, and the radiative cooling of the shocked plasma. In our treatment of charged particles, we consider diffusive shock acceleration in the WCR and the subsequent cooling via inverse Compton losses (including Klein–Nishina effects), bremsstrahlung, collisions, and other energy loss mechanisms.

Key words: acceleration of particles – binaries: general – gamma rays: stars – hydrodynamics – stars: winds, outflows

Online-only material: color figures

1. INTRODUCTION

In recent years, several models of a mostly analytical nature have addressed the question whether binary systems of massive stars without compact objects are liable to produce high-energy γ-ray emission (e.g., Reimer et al. 2006; Pittard et al. 2006; Benaglia & Romero 2003). These studies argue that such objects (i.e., systems containing a Wolf–Rayet (WR) star and an OB-type star) provide suitable environments for efficient particle acceleration and subsequent γ-ray emission. Electrons and protons are thought to be accelerated at the shock fronts tracing the edge of the regions where stellar winds collide with supersonic velocities. Several γ-ray emission mechanisms—inverse Compton (IC) scattering, relativistic bremsstrahlung, and π0 decay—can produce γ-rays at GeV and TeV energies with sufficient fluxes to allow for detection with instruments such as the Fermi Large Area Telescope (LAT) or perhaps even H.E.S.S., MAGIC, and VERITAS.

In contrast to expectations, no such detection of γ-ray emission linked to colliding-wind binaries (CWBS) has been reported so far, with one notable exception: the highly unusual object η Carinae is the only CWB system unambiguously linked to high-energy γ-ray emission (Reitberger et al. 2012). This system most likely consists of a WR star and a high-mass luminous blue variable (LBV). Both stars are enveloped in a huge dust and gas cloud, the Homunculus nebula, which originated in a massive outburst of the LBV in the year 1843. Although this source exhibits a number of unique characteristics, no explanation for its high γ-ray flux, compared to the nondetection of other CWBs, can be given so far. Dedicated observations of WR–OB systems such as WR 147 or WR 140 (for which models predicted fluxes above LAT detection thresholds) have yielded upper limits (Werner et al. 2013).

Detailed three-dimensional (3D) hydrodynamical (HD) simulations (Pittard 2009) have recently explored the highly dynamical nature of the wind collision region (WCR) in CWBs and its strong dependence on stellar and orbital parameters. The complex density, velocity, and temperature structure of the colliding winds have further been used to model the thermal radio and X-ray emission in such systems (Pittard 2010; Pittard & Parkin 2010).

This work aims at a numerical computation of the spectral energy distribution of charged particles within a numerical HD model of the WCR. By solving a transport equation including spatial convection, diffusive shock acceleration (DSA), and various cooling processes for electrons and protons at every grid point of the HD simulation, we simulate the time-dependent 3D spatial distribution of particles at different energies. As cooling processes we take IC emission, bremsstrahlung, nucleon–nucleon interaction, adiabatic cooling, and others into account. The particles are injected at the shock fronts of the WCR and subsequently gain energy by DSA.

We developed a code that by taking stellar, stellar wind, and orbital parameters of a given binary system as input, solves the 3D distribution of density, velocity, and temperature fields of the wind plasma as well as the energy spectra for electrons and protons for every point on a numerical grid. The resulting spatially varying particle spectra will serve as input for complementary studies, computing the components of ensuing γ-ray emission.

In Section 2, we introduce the numerical and HD setup that we use to simulate a 3D distribution of radiatively driven winds in a binary system. Our method of dealing with the spatial and energetic evolution of particle species via solving a transport equation is thoroughly discussed in Section 3. In Section 4, we present results obtained for a typical CWB system in terms of spatial and spectral distribution of high-energy electrons and protons. Section 5 provides a summary of our findings as well as an outlook on future developments.
2. HYDRODYNAMICS

2.1. Numerical Setup

To simulate the hydrodynamics of the stellar winds, we use the Cronos code (Kissmann et al. 2008; Kleimann et al. 2009), which is a finite-volume magnetohydrodynamic (MHD) code optimized for the simulation of compressible astrophysical plasmas. The code is second order accurate in space and allows for Cartesian, cylindrical, and spherical grid layouts. Cronos uses approximate Riemann solvers for the time integration of the HD and MHD equations, where the choice of Riemann solvers includes Harten–Lax–van Leer (hil, used in this study), hill-Contact (h1lc), and hill-Discontinuities (hhld). Time integration is done via a second- or third-order Runge–Kutta integrator, where the time discretization utilizes the semidiscrete approach.

In the present case, Cronos is used to solve the HD equations, with external forces,

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0
\]

(1)

\[
\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v} + \mathbf{P}) = \rho \mathbf{f}
\]

(2)

\[
\frac{\partial \epsilon}{\partial t} + \nabla \cdot [(\epsilon + P)\mathbf{v}] = \left(\frac{\rho}{m_H}\right)^2 \Lambda(T) + \rho \mathbf{f} \cdot \mathbf{v},
\]

(3)

where the left-hand sides of the equations are solved by Cronos intrinsically, whereas the right-hand sides (i.e., the force term \( \mathbf{f} \) and the radiative cooling term \( \Lambda(T) \)) are dealt with in dedicated additional modules as outlined below. Here \( \rho \) is the mass density, \( \mathbf{v} \) the velocity vector, \( P \) the scalar pressure, \( \mathbf{f} \) is the sum of all external forces, \( \epsilon = (\rho/2)v^2 + e \) is the total energy per volume, \( e \) is the internal energy per volume, and \( T \) is the temperature. We use the ideal gas equation of state \( P = (\gamma - 1)e \) with the adiabatic index for a monoatomic gas \( \gamma = 5/3 \).

2.2. The Force Term \( f \)

The force density term \( f \) consists of three components: gravity, radiative line acceleration due to the ions in the wind, and the radiative acceleration due to photons scattering off electrons. It can be written as

\[
f = \sum_{i=1}^{2} \left( -GM_{*i} \frac{r_i}{r_i^3} + g^L_{\text{rad,}i} \right) + g^e_{\text{rad,}i} \left( \frac{\sigma_e L_{*i}}{4\pi r_i^2 c} \right),
\]

(4)

where the index \( i \) indicates each star. The vector \( r_i \) is given relative to the star \( i \). We assume radiative acceleration to be directed radially from the stars. Therefore,

\[
g^L_{\text{rad,}i} = \frac{g^L_{\text{rad,}i}}{r_i} \quad \text{with} \quad g^e_{\text{rad,}i} = \sigma_e L_{*i} \frac{\sqrt{\pi}}{2}. \]

(5)

where \( \sigma_e \) is the specific electron opacity due to Thomson scattering and \( L_{*i} \) is the luminosity of star \( i \).

Determining the line acceleration requires an integration over the finite stellar disk (as a point-source-based approach would yield significantly erroneous results close to the star; see Lamers & Cassinelli 1999). This integral can be simplified by assuming azimuthal symmetry of the stellar disk. To approximate the contribution of the wide spectrum of optically thick and thin spectral lines of the ions in the wind, we rely on the standard Castor–Abbott–Klein (CAK) formalism first introduced by Castor et al. (1975) and later modified and improved by Pauldrach et al. (1986). This approach replaces the required summation over all lines by a simple parameterization with two parameters, \( \alpha \) and \( k \). Taking all this into account, the line acceleration term (as given in Gayley et al. 1997) can, with some modification, be expressed as follows:

\[
g^L_{\text{rad}} = \frac{2k\sigma_e^{1-a} L_{*i}}{c} \int_0^{\theta_{i,\ast}} \left( n_i \cdot \nabla (n_i \cdot \mathbf{v}) \right) \sin \theta \cos \theta \sin \theta \, d\theta
\]

(6)

with \( k \) and \( \alpha \) being the CAK parameters mentioned above. Here \( \rho \) is the mass density of the wind in the cell for which the acceleration is calculated, \( v_{\infty} \) is the thermal velocity of ionized hydrogen, and the angle \( \theta_{i,\ast} \) marks the edge of the stellar disk relative to the point for which we derive the acceleration. It is defined by \( \sin \theta_{i,\ast} = R_{*i}/r_i \). The projected velocity gradient \( n_i \cdot \nabla (n_i \cdot \mathbf{v}) \) depends on the unit vector toward a point on the stellar surface \( n_i \) depending again on \( \theta \). In Euclidean coordinates, it can be expressed as

\[
\mathbf{n} \cdot \nabla (\mathbf{n} \cdot \mathbf{v}) = n_x^2 \frac{\partial v_x}{\partial x} + n_y^2 \frac{\partial v_y}{\partial y} + n_z^2 \frac{\partial v_z}{\partial z} + n_x n_y \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} + n_x n_z \frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}.
\]

(7)

In evaluating the integral in Equation (6), we use \( n_i = \sin \theta P + \cos \theta N \), where \( N \) is the unit vector pointing from the grid point toward the stellar center. \( P \) is a perpendicular vector chosen such that it is nonzero and lies in a plane of the numerical grid. Integration is then performed numerically using a simple Simpson rule with five steps in the interval \([0, \cos \theta_{i,\ast}]\).

Because of the ionization of the plasma, line driving is set to zero in cells with temperature above \( 10^6 \) K.

2.3. Radiative Cooling

Because of high densities and temperatures in the WCR, radiative cooling becomes important and thus has to be considered (see Equation (3)). Here we use the cooling function \( \Lambda(T) \) from Schure et al. (2009), who present a new radiative cooling curve based on a contemporary plasma emission code providing detailed \( \log T - \log N \) tables. We use the tabulated data and interpolate between individual data points.

2.4. Geometrical Setup and Initial Conditions

We use a Euclidean coordinate system defined such that the \( x \)-axis coincides with the semimajor axis of the binary system, thus connecting periastron and apastron. The origin is at the center of mass. The \( z \)-axis is perpendicular to the orbital plane.

Initially, we place the stars along the \( x \)-axis (choosing between periastron and apastron passage) and initialize the stellar winds by a \( \beta \) law approximation:

\[
v_i(r) = v_{\infty,i} \left( 1 - \frac{\xi R_{*i}}{r} \right)^{\beta_i}\).
\]

(8)

From the continuity Equation (1), it follows that

\[
\rho_i(r) = \frac{M_i}{4\pi r^2 v_i(r)}
\]

(9)
for the individual stars. The temperature $T$ is initialized with the value $10^4$ K. It is not allowed to drop below this value, simulating the effects of photoionization heating (cf. Pittard 2009).

As it is computationally expensive to self-consistently simulate the wind up from the stellar surface, it has been common practice to prescribe a fixed solution in at least three cells above the stellar surface at every time step (e.g., Pittard 2009). A classical $\beta$ law solution ($\xi = 1$, only accurate in the point source limit) usually results in a nonphysical kink between fixed cells and those dynamically solved. Multiple trials with one-dimensional (1D) simulations clearly show that a smooth transition is best achieved by initiating the fixed cells with $\xi \sim 0.9983$, which is also the value suggested by Lamers & Cassinelli (1999). To achieve smoother wind velocity profiles even at lower spatial resolutions, we prescribe the wind with this modified $\beta$ law at every time step up to five cells above the stellar surface. For a typical resolution of $\sim 15.6 R_\odot$ per cell, this gives satisfactory velocity and density profiles.

For given stellar parameters, we determine the best-fit value of $\beta$, as well as the values of the CAK parameters $\alpha$ and $k$, by an iterative sequence of 1D simulations based on the known observables $R_\star, L_\star, M_\star$, the wind’s terminal velocity $v_\infty$, and the mass loss rate $\dot{M}$.

To simulate the orbital motion of the stars, we use the standard equations for a Keplerian orbit. The differential equation for the eccentric anomaly $\Psi(t)$ ($\dot{\omega}t = \Psi - e \sin \Psi$ with eccentricity $e$) is solved by a 10 step Newton–Raphson method. The wind velocity that is prescribed at each step above the stellar surface is modified by the stellar velocity due to its orbital motion.

3. PARTICLE SPECTRA

3.1. The Transport Equation

In injecting electrons and protons inside the WCR that then suffer from various energy loss mechanisms, we widely rely on the work by Reimer et al. (2006), in which an analytical approach for a simplified wind setup is carried out in great detail.

We distinguish between acceleration cells, which are located at the shock front of the WCR, and all other cells. In order to discriminate between these two regions, we use the temperature structure of the wind plasma. Because of the imposed lower limit of $10^4$ K, the wind is isothermal until it reaches the WCR. There, the temperature increases by three to four orders of magnitude within a few grid cells (see Figure 1(b)). In our approach, we declare a cell to be an acceleration cell if three conditions are met: (1) the divergence of the velocity vector is negative (the wind is effectively slowing down), (2) the temperature is higher than in the unshocked wind, and (3) the temperature of at least one of the six neighboring cells is at the value of the unshocked wind. This yields a one-cell-thick skin of acceleration cells that envelops the WCR (see Figure 1(d)).

The time-dependent transport equation, as it is solved for each grid point, reads

$$\frac{\partial N(E)}{\partial \tau} + \nabla \cdot [\nu N(E)] + \frac{\partial}{\partial E} [\dot{E} N(E)] + \frac{N(E)}{\tau} = Q_0 \delta(E - E_0),$$

where $N$ is the differential number density of particles at energy $E$ in a grid cell at position $r$. We now discuss each term in detail:

1. The spatial convection term (where $\nu$ is the velocity vector of the wind material) handles the transport of charged particles downstream along the WCR. It is used for all grid cells except acceleration cells, which are treated in a leaky-box approach analogous to the acceleration region in Reimer et al. (2006).

2. Term (2) in Equation (10) handles all energy gain and loss processes, which depend on the considered particle species and on whether the cell is located at the shock front or not. For acceleration cells the term $\dot{E}$ includes DSA (not active outside the shock front) and radiative losses. We also consider adiabatic cooling, which becomes important as the particles accelerate traveling downstream. For acceleration cells and all cells where $\nabla \cdot \nu < 0$ (true for cells just behind the shock front), the adiabatic cooling term has to be switched off as it would allow additional acceleration, which is explicitly already taken care of by DSA.

$$\dot{E} = \begin{cases} \dot{E}_{\text{DSA}} + \dot{E}_\text{radiative} & \text{for acceleration cells} \\ \dot{E}_\text{radiative} & \text{elsewhere if } \nabla \cdot \nu < 0 \\ \dot{E}_{\text{radiative}} + \dot{E}_\text{adiabatic} & \text{elsewhere if } \nabla \cdot \nu > 0. \end{cases}$$

(11)

The individual energy gain and energy loss terms are discussed in detail in Sections 3.2 and 3.3.

3. By considering the acceleration cells similar to a leaky-box model (see, e.g., Protheroe & Staney 1999), the escape time $\tau$ describes the rate at which particles are lost by diffusing out of the system. The escape time can be approximated via the diffusion coefficient and the wind velocity perpendicular to the shock $v_{\text{Shock}}$. For WCR cells outside the shock front, the diffusional leakage out of the system is set to zero by choosing an infinite escape time (see Martin & Dubus 2013).

$$\tau = \begin{cases} \frac{c_r D}{v_{\text{Shock}}} & \text{for acceleration cells} \\ \infty, & \text{elsewhere} \end{cases}$$

(12)

with the compression ratio $c_r$ and the energy-independent diffusion coefficient $D$, which is an approximation for the sum of the upstream and downstream components of the diffusion coefficients at the shock ($D \approx D_1 + c_r D_2$; e.g., Schure et al. 2010). Kirk et al. (1998) provide a comparison to the alternative assumption of an energy-dependent diffusion coefficient. In the present work, we choose the energy-independent approach in order to allow direct comparison with Reimer et al. (2006). Thus, $D$ is constant throughout this work. Note that an energy-independent diffusion coefficient demands additional consideration of the Bohm limit, at which further acceleration of particles is inhibited as their gyroradii become comparable to the characteristic size of the shock. We include this in our simulations by setting $\tau = 0$ as soon as the Bohm diffusion coefficient ($\infty E$) exceeds the chosen energy-independent diffusion coefficient $D$:

$$\tau = 0, \quad \text{for } E > E_{\text{Bohm}} = 3eBD,$$

(13)

where the magnetic field strength $B$ is approximated as described below.

4. Particles are injected into the system at an energy $E_0$ (usually chosen to be 1 MeV). The choice of the injection rate $Q_0$ is limited to the constraints of particle number and energy conservation. A given grid cell in the shock region
cannot inject more particles than it initially carries; neither can it deposit more energy in the injected particles than it has. Following Martin & Dubus (2013), we approximate the number of injected particles by a constant fraction \( \eta \) of the mass density.

Assuming a wind that predominantly consists of ionized hydrogen, partly ionized helium, and electrons, it follows that

\[
n_e = \frac{\rho}{m_H} \frac{1 + I_{\text{He}} \zeta_{\text{He}}}{1 + 4 \zeta_{\text{He}}} \quad \text{and} \quad n_p = \frac{\rho}{m_H (1 + 4 \zeta_{\text{He}})}
\]

with \( n_p \) and \( n_e \) being the number densities of free electrons and protons in a given cell, \( \rho \) being the local wind density, \( \zeta_{\text{He}} = n_{\text{He}} / n_H \), and \( I_{\text{He}} \) being the number of electrons provided per helium nucleus. Hydrogen is assumed to be completely ionized. (We assume \( \zeta_{\text{He}} = 0.1 \) and \( I_{\text{He}} = 2 \) for wind temperatures higher than 30,000 K and \( \zeta_{\text{He}} = 1 \) below.) This imposes a limit on the maximum number density for electrons \( Q_0^e \) and protons \( Q_0^p \) that can be accelerated per time step \( dt \), of which we take the fractions \( \eta_e \) and \( \eta_p \).

Other particle species liable for significant acceleration in the WCR (such as He ions) are not considered in the present study:

\[
Q_0 = \begin{cases} 
\frac{\eta_e \rho}{m_H} \frac{1 + I_{\text{He}} \zeta_{\text{He}}}{1 + 4 \zeta_{\text{He}}} & \text{or} \\
\frac{\eta_p \rho}{m_H} \frac{1 + I_{\text{He}} \zeta_{\text{He}}}{1 + 4 \zeta_{\text{He}}} \quad & \text{for acceleration cells} \\
0 & \text{electrons, protons).} \\
& \text{elsewhere}
\end{cases}
\]

The injection fractions are used as free parameters. Typically, we take \( \eta_p = 10^{-3} \) and \( \eta_e = 10^{-5} \). The ensuing mass density decrease in the wind plasma is small enough to be negligible. As the energy deposited in the shock is also proportional to \( \eta_{e,p} \) (and thus very small), there is no significant reduction of the kinetic energy of the wind. Energy conservation therefore holds. We do not consider alternative sources of particles entering the acceleration process (e.g., via \( \gamma-\gamma \) pair production.)
3.2. Particle Acceleration

Of the various acceleration mechanisms for nonthermal particles that are discussed in the literature, DSA (such as a variant of the first-order Fermi acceleration principle in which particles gain energy by repeatedly traversing a shock) appears to be the most feasible process to take into account for our models. Alternative mechanisms such as various turbulent processes (e.g., the second-order Fermi processes) or magnetic reconnection are assumed to be either less efficient or incapable of supplying particles that reach energies sufficient for $\gamma$-ray emission (see Pittard & Dougherty 2006). Thus, we restrict ourselves to considering DSA the sole acceleration process of electrons and protons for the remainder of this work. The average rate of momentum gain by DSA (see, e.g., Schure et al. 2010) is

$$\dot{E}_{\text{DSA}} = \left( c_r - \frac{1}{3} \right) \frac{V_{\text{Shock}}^2}{D} E,$$  \hspace{1cm} (16)

where $V_{\text{Shock}}$ is the shock velocity, $c_r$ is the compression ratio, and $D$ is the energy-independent spatial diffusion coefficient as motivated above.

We define the shock velocity $V_{\text{Shock}}$ as the upstream velocity normal to the shock and determine it by computing the velocity component of the plasma perpendicular to the WCR. As a tracer for the orientation of the collision region, we take the gradient of the temperature field $\nabla T$ and compute

$$V_{\text{Shock}} = \frac{\mathbf{v} \cdot \nabla T}{|\nabla T|}.$$  \hspace{1cm} (17)

The compression ratio $c_r$ at the shock is also directly determined from the wind structure by computing the ratio of postshock mass density and preshock mass density. The former is determined by interpolating the mass density at a distance of three cell widths along the shock normal toward the WCR. The choice of three cell widths is motivated by the fact that the shock front in the simulations is, in general, three cells wide. A choice of four or more cell widths probes the mass density too far inside the WCR. As the mass density minimum of the wind plasma is, in general, three cells wide. A choice of three cell widths is motivated by the fact that the shock front is determined by interpolating the mass density at a distance of three cell widths probes the mass density too far inside the WCR. As the mass density minimum of the wind plasma prior to reaching the WCR is generally located at the shock position for which the compression ratio is computed, we set the postshock mass density to the local value.

After being accelerated, the electrons and protons are subjected to several loss mechanisms, which we discuss below.

3.3. Energy Losses

Inverses Compton emission (electrons only). IC cooling occurs as relativistic electrons scatter on stellar radiation fields within. It is a major energy loss mechanism for nonthermal electrons. Here we use the full Klein–Nishina cross section resulting in the loss term:

$$\dot{E}_{\text{IC}} = -b_{\text{IC}} E^2 f_{\text{KN}}(E)$$  \hspace{1cm} (18)

with

$$b_{\text{IC}} = \frac{4}{3m_e c^5} \sigma_{\text{Th}} u_{\text{ph}},$$  \hspace{1cm} (19)

where $\sigma_{\text{Th}}$ is the Thomson cross section and $f_{\text{KN}}(E)$ is the correction factor for the Klein–Nishina regime (following Moderski et al. 2005), which is

$$f_{\text{KN}}(\tilde{b}) = \frac{9}{\tilde{b}^3} \left[ \left( \frac{1}{2} \tilde{b} + 6 + \frac{6}{\tilde{b}} \right) \ln(1 + \tilde{b}) - \frac{11}{12} \tilde{b}^3 + 6\tilde{b}^2 + 9\tilde{b} + 4 \right] \frac{1}{(1 + \tilde{b})^2} - 2 + 2\text{Li}_2(-\tilde{b})$$  \hspace{1cm} (20)

with the dilogarithm function $\text{Li}_2$ and the dimensionless variable $\tilde{b} = 4\epsilon_\gamma (E/m_e c^2)$, with $\epsilon_\gamma$ being the energy of the target photon. For $\tilde{b} < 10^{-3}$ it is safe to set $f_{\text{KN}} = 1$.

The radiation energy density $u_{\text{ph}}$ (taking into account both stars) is

$$u_{\text{ph}} = \frac{1}{4\pi c} \left( \frac{L_{*1}}{r_1^2} + \frac{L_{*2}}{r_2^2} \right),$$  \hspace{1cm} (21)

where $L_{*j}$ are the stellar luminosities and $r_j$ is the distance to the stars. At present, we do not consider additional radiation fields (e.g., photons from synchrotron emission) as targets for IC scattering.

Synchrotron emission (electrons only). The loss term for synchrotron emission is

$$\dot{E}_{\text{syn}} = -b_{\text{syn}} E^2$$  \hspace{1cm} (22)

with

$$b_{\text{syn}} = \frac{4}{3m_e c^3} \sigma_{\text{Th}} u_B,$$  \hspace{1cm} (23)

where the magnetic energy density $u_B$ is defined as

$$u_B = \frac{B^2}{2\mu_0}.$$  \hspace{1cm} (24)

Because of the lack of knowledge of the magnetic field strength $B$ in most CWBs, we rely here on the approximations in Usov & Melrose (1992), who describe the magnetic field as either a classical dipole field, a radially dominated field, or a toroidally dominated field, depending on the distance from the star:

$$B \approx \begin{cases} B_\star \left( \frac{r}{r_A} \right)^3, & \text{for } r < r_A \\ B_\star \left( \frac{r}{r_A} \right)^{1.7}, & \text{for } r > r_A \text{ and } r < R_\star \frac{v_{\text{rot}}}{v_{\text{rot}}^\star} \\ B_\star \left( \frac{v_{\text{rot}} R_\star}{v_{\text{rot}}^\star R_\star} \right)^{1/4}, & \text{for } r > r_A \text{ and } r > R_\star \frac{v_{\text{rot}}}{v_{\text{rot}}^\star}, \end{cases}$$  \hspace{1cm} (25)

where $B_\star$ is the magnetic field at the stellar surface and $v_{\text{rot}}$ is the surface rotation velocity of the star, typically approximated by $v_{\text{rot}} \sim 0.1 v_\infty$. For the Alfvén radius $r_A$ we take

$$r_A \approx \begin{cases} R_\star (1 + \xi), & \text{for } \xi \ll 1 \\ R_\star \xi^{1/4}, & \text{for } \xi \gg 1 \end{cases}$$  \hspace{1cm} (26)

with $\xi = B_\star^2 R_\star^2 / M v_\infty$. As a reasonable value for the surface magnetic field $B_\star = 0.01$ T is assumed throughout this work for both stars (see, e.g., Reimer et al. 2006).

Thus, we have an estimate for the magnetic field depending on the distance from each star. Since we merely approximate the absolute value of the magnetic field vector without having knowledge of its individual components, we cannot compute its vector sum for both stars. As an approximation, we merely consider the dominant component at a given grid cell to calculate $u_B$ and $b_{\text{syn}}$. 

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In order to preserve consistency with Reimer et al. (2006), we do not acknowledge the compression of the magnetic field in proportion to the compression of the wind inside the WCR. Thus, the field strength $B$ is likely to be underestimated by a factor of $\sim 4$, which has no effect at lower energies but can influence the maximum particle energy due to higher synchrotron losses and a higher Bohm energy. In future work, a full MHD description of the wind will allow greater precision in the treatment of the magnetic field.

**Losses by thermal bremsstrahlung (electrons only).** As charged particles interact with the Coulomb fields of ions in the wind plasma, bremsstrahlung emission occurs. To account for the ensuing energy loss, we use

$$\dot{E}_{\text{br}} = -b_{\text{br}} E$$

with

$$b_{\text{br}} = \frac{2}{\pi} \alpha c \sigma_{\text{Th}} N_{\text{H}}.$$  

(28)

where $\alpha$ is the fine-structure constant and $N_{\text{H}}$ is the number density of the thermal ions in the wind, which we approximate from the mass density in a given grid cell, dividing it by $m_{\text{H}}(1 + 4 \zeta_{\text{He}})$:

$$N_{\text{H}} \approx \frac{\rho}{m_{\text{H}}(1 + 4 \zeta_{\text{He}})}.$$  

(29)

**Coulomb losses (electrons).** As a loss term for the electrons we consider the energy-independent Coulomb losses given by

$$\dot{E}_{\text{coul}} = -b_{\text{coul}} = -55.725 \alpha c \sigma_{\text{Th}} N_{\text{H}} m_{\text{e}} c^2.$$  

(30)

**Coulomb losses (protons).** In the dense winds, Coulomb losses can also become significant for protons. They are expressed by

$$\dot{E}_{\text{coul}} = -\frac{3c \sigma_{\text{Th}} m_{\text{e}} c^2 Z^2 \ln \lambda}{2} N_{\text{H}} \frac{\beta^2}{x_m^3 + \beta^3}$$

(31)

with

$$\beta = \sqrt{\frac{E(E + 2m_{\text{p}} c^2)}{E + m_{\text{p}} c^2}}.$$  

(32)

and

$$x_m = 0.2 \sqrt[3]{\frac{T_e}{10^8 \text{ K}}}.$$  

(33)

The term $\ln \lambda$ is the Coulomb logarithm, which we set to have a value of 20. The electron temperature is assumed to be $T_e \sim 10^8 \text{ K}$.

**Nucleon–nucleon interaction (protons only).** For nucleon–nucleon interactions the energy loss rate is

$$\dot{E}_{\text{pp}} = -b_{\text{pp}} E$$

(34)

with

$$b_{\text{pp}} = 1.3 \times 3c N_{\text{H}} \sigma_{\text{pp}} \frac{m_{\pi}}{m_{\text{p}}}$$

(35)

valid above the threshold for pion production at $E_{\text{thr}} \simeq 0.28 \text{ GeV}$. The cross section for nucleon–nucleon collision is $\sigma_{\text{pp}} = 3 \times 10^{-28} \text{ cm}^2$, and the factor 1.3 takes into account the element abundance ratio between H and He (9:1) assumed throughout this work.

**Adiabatic cooling.** For all grid cells outside the shock front with $\nabla \cdot v > 0$ we include the adiabatic cooling term

$$\dot{E}_{\text{adiab}} = \frac{E}{3} \nabla \cdot v.$$  

(36)

The term becomes obsolete for $\nabla \cdot v < 0$ as it would then yield additional acceleration that is already physically taken care of by the DSA term.

### 3.4. Maximum Energies

The delicate balance of DSA and the above-mentioned loss terms is decisive for the shape and maximum energy of the electron and (to a lesser degree) the proton spectra. The final expression of $\dot{E}$ entering the transport equation for all acceleration cells is of the form

$$\dot{E} = \dot{E}_{\text{DSA}} - (b_{\text{IC}} f_{\text{KN}}(E) + b_{\text{syn}}) E^2 - b_{\text{br}} - b_{\text{coul}}$$  

(37)

for the electrons and

$$\dot{E} = \dot{E}_{\text{DSA}} - b_{\text{pp}} E - \frac{3c \sigma_{\text{Th}} m_{\text{e}} c^2 Z^2 \ln \lambda}{2} N_{\text{H}} \frac{\beta^2}{x_m^3 + \beta^3}$$  

(38)

for protons.

In the case of electrons, the approximately dependence of IC and synchrotron losses leads to a cutoff in the GeV range for an average WR–OB binary system. The specific energy of the cutoff and the question of whether IC or synchrotron losses are mainly responsible for it depend on the energy densities of radiation and magnetic field. For some configurations where radiative losses are small, we see a cutoff due to the Bohm diffusion limit (as described in Section 3.1, term (3)).

For the protons, neither Coulomb losses nor losses by nucleon–nucleon interaction suffice to produce a cutoff. Rather, it is the Bohm diffusion limit (and thus the gyroradii overcoming the characteristic size of the shock, as described above) that produces a cutoff, usually at a fewTeV for an average WR–OB system. At low energies, however, Coulomb losses can inhibit particle acceleration at its early stage and lead to notable spectral features, even for protons.

### 3.5. Implementation

Solving the transport Equation (10) is handled directly within the framework of the Cronos code, where a semi-Lagrangian solver (following Crouseilles et al. 2010) has been implemented as an additional module applied at each grid point and time step. To fully incorporate the particle spectra into the code, additional scalar fields are created, one for each energy bin and particle species. Instead of having merely 5 fields ($\rho$, $v_x$, $v_y$, $v_z$, $T$), we use 205, meaning that we add 100 logarithmically equally spaced energy bins $[E_i, E_{i+1}]$ with $E_i \in [1 \text{ MeV}, 10 \text{ TeV}] \ (i = 1, \ldots, 100)$ for both electrons and protons. By treating those fields as advected scalars (similar to $\rho$ in Equation (1)), Cronos intrinsically handles spatial convection (term (1) in Equation (10)) via its HD solver.

The aforementioned semi-Lagrangian solver then takes care of the remaining equation

$$\frac{\partial N(E)}{\partial t} + \frac{\partial}{\partial E} [\dot{E} N(E)] + \frac{N}{\tau} = Q_0 \delta(E - E_0).$$  

(39)

where the inhomogeneity on the right-hand side can be treated as a simple boundary condition at the lowest energy bin of the spectrum where $N(E_0) = Q_0 / \dot{E}(E_0)$.

Equation (39) is solved at each step after the application of the HD solver. It usually uses the same time step as the HD part of the solver, but it also does subcycling in the case where $\dot{E}$ is very large and thus the convection velocity in momentum space becomes too large (i.e., the distribution is shifted by more than one energy bin). This is most relevant when electrons leave the shock; $\dot{E}_{\text{DSA}}$ becomes zero, and the remaining losses are very high. However, we find that subcycling is rarely needed as the
HD time step is small enough. For the example we discuss in Section 4, the applied HD time step is typically of the order of \( \sim 10^7 \) \( \text{s} \). For the chosen stellar and stellar wind parameters, the spectra in the shock at the apex of WCR typically need \( \sim 10^6 \) \( \text{s} \) to build up until they reach convergence. If the acceleration is switched off and just the loss terms remain, the width of the bins at high energies is (due to the logarithmic scaling) still large enough to prevent the shift from being larger than one bin per time step. Even if it were otherwise, the implicit solver would yield a sufficiently accurate result.

The term \( N/\tau \) in Equation (39) is considered for acceleration cells only. As these are treated similar to a leaky box where spatial convection from one cell to the other has been turned off, additional care must be taken concerning the transport from one acceleration cell to the next in the direction of the inner WCR. We deal with this problem by assuming that those particles vanishing from a single acceleration cell due to the diffusion term \( N/\tau \) are not lost from the system but enter the next cell downstream of the shock. Further along, convection quickly becomes the dominant process for particle transport.

For acceleration cells the index of the resulting spectrum is highly dependent on the compression ratio of the wind. It can be easily shown that Equation (39) (for \( \dot{E} = \dot{E}_{\text{DSA}} \) and the definitions above) has a solution \( N(E) \sim E^{-p} \) with \( p = (c_\text{r} + 2)/(c_\text{r} - 1) \). For a typical strong shock of \( c_\text{r} = 4 \) we obtain \( p = 2 \). If there is no compression at all and \( c_\text{r} = 1 \) (no shock), the index approaches infinity as the spectrum disappears.

### 4. RESULTS

#### 4.1. Models Investigated

Now, we want to illustrate the capability of our code and show first results on how the distribution functions of high-energy electrons and protons evolve during a CWB’s orbit. To allow for direct comparison and consistency checks, we choose a system identical to the one used in the parameter studies in Reimer et al. (2006). Table 1 lists stellar and stellar wind parameters of the studied CWB with a B star and a WR star.

| Star | \( M_* \) | \( R_* \) | \( T_* \) | \( L_* \) | \( M_{\infty} \) | \( \alpha \) | \( k \) | \( \beta \) |
|------|----------|----------|----------|----------|----------|----------|----------|----------|
| B    | 30       | 20       | 23000    | 10\(^7\) | 10\(^{-6}\) | 4000     | 0.643    | 0.5      | 0.83     |
| WR   | 10       | 40       | 40000    | 2.3 \times 10\(^5\) | 4000     | 0.678    | 0.7      | 0.83     |

**Notes.** The given CAK parameters \( \alpha \) and \( k \) yield a stellar wind characterized by \( M \) and \( v_\infty \) and a \( \beta \)-function-type velocity law with the given \( \beta \).

Figure 1(a) shows the density structure obtained for the parameters given above. Without orbital motion the simulation converges quickly to the depicted state. Number densities of wind particles as high as \( \sim 10^{14} \text{ m}^{-3} \) are reached at the apex of the WCR. The WR star’s higher mass loss rate and higher wind momentum compared to its companion lead to a significant curvature of the WCR. Also note that the WCR is generally more dense on the side facing the WR star.

Figure 1(b) shows the system’s temperature distribution. As mentioned above, the wind is kept at a constant temperature of \( 10^4 \text{ K} \) outside the WR. It then heats up rapidly, reaching temperatures \( > 10^6 \text{ K} \) at the apex. Figure 1(b) suggests that temperature structure can be used as a tracer for the location of the shock fronts on the edge of the WCR.

Absolute wind velocity is shown in Figure 1(a). The plot reveals how winds are slowed down to zero at the apex of WCR and then accelerate farther downstream. Note that the regions where the wind of the B star reaches velocities exceeding its terminal velocity of 4000 km s\(^{-1}\) can be explained by the combined radiative acceleration effect of both stars. It does not have any direct effect on WCR velocities as radiative acceleration is turned off for temperatures above \( 10^6 \text{ K} \) because line driving does not affect a fully ionized plasma. However, the higher preshock velocities influence the postshock velocities, which are highest at the wings (the outer part of the arms of the WCR) facing the B star’s wind.

Figure 1(d) shows the shock velocity, which is the component of the wind velocity perpendicular to the shock. As it is undefined outside the shock, it also traces the acceleration region determined as outlined above. This is where DSA generates a population of relativistic particles that are then injected into the WCR, where they advect downstream and lose energy.

#### 4.2. Consistency Checks

Before applying the numerical solver for the transport equation to our HD simulations, we demonstrate consistency with previous studies: our treatment of the acceleration cells (see Section 3.1) can be directly compared with the treatment of the acceleration region in Reimer et al. (2006). For similar conditions with respect to shock velocity, density, and magnetic and radiation fields, our code reproduces spectra as shown in Figure 2.

One can identify the influence of various energy loss mechanisms. As the density of the wind plasma decreases steadily from case 1 to case 5, the effects of Coulomb losses (visible for \( E < 10 \text{ MeV} \)) decrease accordingly (for electrons and protons). For electrons, the cutoff of the spectra at higher energies is either caused by synchrotron losses (cases 2 and 3), by IC losses (cases 1 and 2), or by the diffusion approaching the Bohm limit (cases 4 and 5). The gentle slope visible for case 2 results from the curious condition that the IC loss rate almost reaches the acceleration rate at around 1 GeV but fails to overcome it because of the Klein–Nishina effect. The final cutoff at higher energies is due to synchrotron losses. For protons the energy losses from nucleon–nucleon interaction and Coulomb losses do not suffice to overcome the acceleration. All the cutoffs are caused by the Bohm limit.

We find quantitative agreement between our results and those of (Reimer et al. 2006; i.e., Figures 3 and 7). Minor differences of slope and cutoff shape in cases 1 and 2 can be understood by our usage of the full Klein–Nishina cross section.
4.3. Properties of Accelerating Cells

The same procedure as described above is now performed for all acceleration cells in our numerical grid. Shock velocity and number density of the plasma are directly determined from the HD variables. The magnetic and the radiation energy density are derived as outlined in Section 3.3. At this point, we are still neglecting the effects of orbital motion.

We explore the varying conditions along the shock front that determine how many particles are accelerated and how

Figure 2. (a) Differential number density of electrons for an acceleration cell after convergence. The five depicted cases (1, solid line; 2, dotted line; 3, dashed line; 4, dash-dotted line; and 5, double-dot-dashed line) represent different values of stellar separation $D \approx 720, 1440, 2880, 7190, \text{and} 14,380 \, R_\odot$, magnetic field $B \approx 1.3, 0.5, 0.3, 0.1, \text{and} 0.05 \, \text{G}$, number density of the wind plasma $N_H \approx (21, 5.2, 1.3, 0.2, \text{and} 0.05) \times 10^{13} \, \text{m}^{-3}$, and shock velocity $V_s \approx 1768, 1884, 1942, 1977, \text{and} 1988 \, \text{km s}^{-1}$. (b) The same for protons.

Figure 3. Important properties of the acceleration region along the shock as a function of the distance from the apex of the WCR: (a) shock velocity, (b) compression ratio (black), (c) electron injection rate, and (d) maximum energy of electrons and protons. The solid lines represent the shock front facing the WR wind; the dotted lines represent the shock facing the B wind. Concerning (c), the proton injection rate is the same as for electrons multiplied by $(\eta_p/\eta_e)(1 + I_{\text{He}}\zeta_{\text{He}}) \approx 100$. In (d), the maximum energies are shown for protons (red) and electrons (black).

(A color version of this figure is available in the online journal.)
high their energies are. Figure 3 shows various properties that are significant for the acceleration process as they vary with growing distance from the apex along the thin surface of acceleration cells. A notable difference between the shock front facing the B star (in black) and the one facing the WR star (in red) becomes apparent. We will discuss each side in turn.

The WR side of the shock. Looking at the values for the shock velocity $V_{\text{shock}}$ in Figure 3(a), we see a slow and steady decrease with the distance from the apex. This decrease is caused by the shape of the WCR. As it curves around the B star, the velocity component along the shock decreases (even if the absolute wind velocities still increases). For distances from the apex greater than $\sim 1200 R_\odot$ the curvature of the shock is less pronounced. Thus, the shock velocity decreases more slowly until it reaches zero at infinity.

Figure 3(b) shows the compression ratio $c_r$. As expected for a strong shock, it remains fairly constant, ranging from 3.8 to 4. For compression ratios sufficiently above 1, the DSA rate ($E_{\text{DSA}} \sim \left((c_r - 1)/c_r\right)V_{\text{shock}}^2$) mainly resembles the behavior of the shock velocity. The maximum is located at the apex. Acceleration then continuously decreases until far out in the wings.

As the injection rate of new particles is assumed to be proportional to the density in the acceleration cells, it decreases quickly with growing distance from the apex. This is shown in Figure 3(c).

The maximum energy attained by the electrons depends on the various loss terms. IC scattering on photons in the radiation field of the B star is the dominant loss mechanism, but it quickly loses relevance farther away from the star. As the losses decrease faster than the DSA rate, Figure 3(d) shows increasing maximum electron energies for increasing distances from the apex. This trend is broken at $\sim 800 R_\odot$ when the electrons reach the Bohm limit and thus cannot be accelerated any further. The slight decrease in maximum energy for even larger distances is due to the limit’s proportionality to the magnetic field, which decreases farther out. Also shown (in red) is the maximum energy of the protons, which do not suffer any significant losses and attain their maximum energies close to the apex.

The B side of the shock. As the curvature of the WCR is larger toward the B star’s wind close to the star, the shock velocity drops more quickly than on the WR side. Also, it is considerably smaller at the apex due to the lower terminal velocity of the B star’s wind. As the shock front flattens out earlier than on the WR side, a lower rate of decrease in $V_{\text{shock}}$ is reached already at $\sim 800 R_\odot$.

Again, the compression ratio remains fairly constant, ranging from 3.4 to 4.2. Minor fluctuations are expected to disappear with more sophisticated methods to determine the postshock mass density and higher spatial resolutions. In general, the low level of variation in the compression ratio along both shock fronts serves as confirmation that the generic assumption of fixing $c_r$ to 4 is warranted for the kind of CWB system investigated.

Figure 3(c) shows that the injection rate for the B side of the shock close to the apex is even larger than for the shock toward the high-mass-loss WR wind. This is due to higher densities of the B wind in the proximity of the star. However, the steeper decrease in density soon overcomes the initially higher value. Farther out, the injection ratio in the shock toward the B wind is up to an order of magnitude lower compared to the WR side of the shock.

Figure 3(d) reveals that the lower acceleration rate (due to lower $V_{\text{shock}}$ and $c_r$) and higher loss rates (due to the proximity of the B star) prevent the electrons on the B side of the shock from reaching energies up to the Bohm limit until far out in the wings. The protons are again not affected by any significant losses.

4.4. Simulation Results on Particle Spectra

Having discussed the varying conditions along the shock, we now explore the resulting distribution functions of high-energy particles throughout the computational domain. We show particle distribution functions in the orbital plane for various energies as well as spectra for several selected positions.

4.4.1. Electrons

The electron distribution function for six different energies from 1 MeV to 1 TeV in the orbital plane is shown in Figure 4.

As expected, number densities drop quickly toward higher energies. It is interesting to observe that the highest energies are to be found in the wings of the WCR, close to the shock toward the WR star. This can be understood by considering that this is a region where radiative losses are low and the shock is strong, as the component of the wind velocity perpendicular to the WR side of the shock is still significant. Note that electrons at 10 GeV and higher can no longer penetrate into the center regions of the WCR because of severe radiation losses. They are confined to the proximity of the shock fronts. This behavior closely resembles the schematic diagrams of the relativistic electron distribution in Pittard et al. (2006).

To allow a deeper understanding of the electron distribution function, Figure 5 shows a selection of spectra for different positions. Figure 5(a) shows spectra along the apex of the WCR along the x axis from right to left. The spectra at the shock fronts (solid and double-dot-dashed lines) are marked in red. As was shown above, the compression ratio at the WR side of the shock is close to 4, which yields a power-law index of $-2$. Therefore, in $E^2N$ scaling, we obtain a nearly flat spectrum with a cutoff where IC and synchrotron losses overcome the acceleration (red solid line). Moving farther into the WCR along the line connecting both stars, we find higher electron densities than at the shock (black dashed line). This is caused by a pileup of particles in the area behind the shock front that occurs because of very low fluid velocities in this region. As the spatial convection flux increases with number density, the cells behind the shock reach equilibrium between incoming and outgoing particles only at a certain number density, which is generally higher than in the shock itself. Closer to the center of the WCR (black dotted line), losses from cooling (which are larger at higher energies) become important. The spectra now show two components, the one stemming from the B side of the shock with energies up to $\sim 1$ GeV and the other stemming from the WR side with energies up to $\sim 10$ GeV. Still closer to the B shock (black dash-dotted line), the latter component is further subdue as cooling and advection in both downstream directions remove more and more particles stemming from the WR side of the shock. At the B shock (red double-dot-dashed line), a lower compression ratio ($c_r \sim 3.8$) yields a slightly softer spectral index of $-2.1$. Radiative losses cause the change of slope and the eventual cutoff that is at a lower energy than on the other side of the WCR because of the increased proximity of the B star.

Figure 5(b) shows spectra for several regions along the WR side of the shock front. In contrast to the spectra along the apex,
Figure 4. Differential number density of electrons (in MeV$^{-1}$ m$^{-3}$) as a function of kinetic particle energy. The color maps show the x-y plane of a 256 x 256 x 64 simulation at $z = 0$.

(A color version of this figure is available in the online journal.)

Figure 5. Electron spectra for various positions within the WCR. (a) Spectra along the connecting line of the stars, where distance from the shock front facing the WR wind (solid line) is $\sim 60$ (dashed line), $\sim 110$ (dotted line), $\sim 160$ (dash-dotted line), and $\sim 190 R_\odot$ (double-dot-dashed line). Spectra within the acceleration region are in red. Regions (b) along the WR shock, (c) along the B shock, and (d) along the center of the WCR. Here the distance to the corresponding region at the apex (solid line) is $\sim 500$ (dashed line), $\sim 1000$ (dotted line), $\sim 1500$ (dash-dotted line), and $\sim 2000 R_\odot$ (double-dot-dashed line).

(A color version of this figure is available in the online journal.)
it shows dramatic changes in maximum energy as loss rates decrease significantly with growing distance. Whereas it is still the IC losses that produce the cutoff close to the apex (solid and dashed lines), the highest energies of the spectra farther out are determined by the Bohm limit. However, the influence of the radiative losses can still be seen in the change of slope (dash-dotted and double-dot-dashed lines), very similar to the cases discussed in Section 4.2. A varying compression ratio leads to slight variations of the spectral index. Note that none of the spectra reach energies >1 TeV.

The spectra corresponding to the B side of the shock are shown in Figure 5(c). We again see the expected increase in maximum energy as one moves toward the wings of the WCR. As radiative losses have a stronger impact on the B side of the shock, the Bohm limit is not reached except for a distance of \( \sim 2000 R_\odot \) from the apex. Additional features relate to slight variations in the compression ratio and to the decreasing injection rate as the wind plasma density decreases outward.

Finally, Figure 5(d) explores the spectra along the line that is equidistant to the two shock fronts. It shows a mixture of previously noticed effects. The absence of electrons above \( \sim 1 \) GeV is because particles at those energies are produced at the shock front only at considerable distance from the apex. They do not have sufficient time to propagate into the center of the WCR before leaving the simulated region. Thus, the majority of the particles in this center region stem from close to the apex of the WCR.

### 4.4.2. Protons

Figures 6 and 7 are analogous to Figures 4 and 5 for the case of protons. Radiative losses are hardly significant, and thus the number density of protons depends mostly on the injection rate (which is proportional to the local wind plasma density), rather than on the distance to the stars. As radiative losses are of lesser importance, protons can reach energies of several TeV. Figure 6 illustrates that in contrast to electrons, the highest-energy protons are found near the shock toward the B star where a stronger magnetic field strength shifts the Bohm cutoff toward higher energies. Further details can be seen from the spectra in Figures 7(a)–(d). Along the apex, we see again a pileup behind the shock, as an equilibrium between the particle transport downstream by spatial convection and the flow of new particles from the shock by the diffusion term is not reached until high number densities are attained. Farther away from the WR shock toward the center of the WCR, particles are increasingly transported downstream. The influence of the second shock front toward the B star emerges as the spectral index changes accordingly. There, a lower compression ratio yields a softer spectrum.

Comparing spectra along the both shock fronts (Figures 7(b) and (c)), one can see various effects: the softening of the spectra with decreasing compression ratio, the decrease of the injection rate with decreasing mass density of the wind, and the slight decrease of the Bohm limit with increasing distance from the apex. The latter explains why protons above 1 TeV are found near the center but not in the wings of the WCR facing the WR star. Finally, Figure 7(d) shows spectra along the center of the WCR, which again are determined by the particle densities close to the apex.

### 4.5. Effects of Orbital Motion

In addition to the previous findings discussed above, our simulations show that the orbital motion of a binary system has the potential to significantly alter the distribution of high-energy particles. As was described in Section 2.4, our code is capable of moving the stars on a Keplerian orbit. In the present study, we merely investigate the circular case. By including orbital motion in our simulation and letting the system evolve for \( \sim 1.3 \) orbital periods, we obtain results as in Figure 8. For the given stellar masses and their distance, a Keplerian orbit has a period of \( \sim 820 \) days. The orbital velocity of the stars is \( \sim 45 \) km s\(^{-1}\), which is two orders of magnitude below the terminal velocity of the winds.

With orbital motion, the two arms of the WCR develop considerable differences. The effect becomes most notable at 10 GeV, where we see a contrast in differential number densities of about two orders of magnitude between the leading and trailing arm on the B side of the WCR for electrons. This is due to the early cutoff of the trailing arm, which has an energy of \( \sim 10 \) GeV. Only in the leading arm are energies above 100 GeV reached for the electrons along the shock toward the B star. For the WR side of the shock, maximum electron energies are reached in the trailing arm.

These differences stem for the greatest part from the deformation of the WCR due to orbital motion. It is no longer symmetric with respect to the apex. Figure 9(a) illustrates the effect of these geometrical differences. The fraction of shock velocity versus absolute wind velocity is shown as a fraction of the distance from the apex along the shock front on the B side of the shock. The wind velocity component perpendicular to the shock is significantly larger along the forward arm. The ensuing difference in shock velocity has a significant impact as the acceleration rate is proportional to \( V_{\text{Shock}}^2 \). Figure 9(b) shows the electron spectra for two characteristic positions in the forward arm and in the trailing arm of the WCR close to the shock toward the B star. The lower shock velocity in the case of the former causes a cutoff due to IC losses at \( \sim 10 \) GeV. In the case of the forward arm, the acceleration is able to compete with the losses up to higher energies.

Comparatively larger effects are expected for elliptical orbits in which changing stellar separation causes more severe contrasts in a large number of relevant properties, such as plasma density, radiation energy density, shock velocity, etc. In this work, we do not yet explore the effects of varying stellar separation. However, the significant differences between the forward and trailing arm clearly indicate that orbital motion can have a strong influence in a model of the WCR and the particle transport within.

### 5. DISCUSSION

We have demonstrated the feasibility of numerically solving the time-dependent transport equation of high-energy particles within a 3D hydrodynamic model of the stellar winds and their collision region for a massive star binary system. This approach provides a more realistic description of the energetic particle spectra compared to analytical models with inherent simplifications regarding the complex structure of the WCR and the dynamics of the wind interaction.

For a typical CWB system of a WR and a B star where the WCR is much closer to the latter (e.g., \( \eta = 0.1 \)), we simulated the radiatively driven wind plasma and its collision region. On the basis of the HD description of the wind and the geometry of the WCR and following the propagation of accelerating electrons and protons in the relativistic domain, we derived relevant quantities related to particle acceleration such as shock velocity, compression ratio, maximum possible particle...
Figure 6. Same as Figure 4 for the case of protons.
(A color version of this figure is available in the online journal.)

Figure 7. Proton spectra for various positions within the WCR. (a) Spectra along the connecting line of the stars, where distance from the shock front facing the WR wind (solid line) is \( \sim 60 \) (dashed line), \( \sim 110 \) (dotted line), \( \sim 160 \) (dash-dotted line), and \( \sim 190 \) \( R_\odot \) (double-dot-dashed line). Spectra within the acceleration region are in red. Regions (b) along the WR shock, (c) along the B shock, and (d) along the center of the WCR. Here the distance to the corresponding region at the apex (solid line) is \( \sim 500 \) (dashed line), \( \sim 1000 \) (dotted line), \( \sim 1500 \) (dash-dotted line), and \( \sim 2000 \) \( R_\odot \) (double-dot-dashed line).
(A color version of this figure is available in the online journal.)
injection rate, and maximum attainable energies along the two shock fronts. All of these exemplify notable differences between the B side and WR side of the shock and also between the inner (apex) and outer part (wings) of the WCR. The geometrical structure of the WCR critically influences the shock velocity and, consequently, the maximum energy of electrons accelerated in the shock. Varying wind densities along the shocks affect the highest possible rate of particles injected at the shock. We find a decrease in the shock velocity with growing distance from the apex that differs corresponding to the dissimilar curvature and wind velocities at both sides of the WCR. By determining the compression ratio at the shock fronts directly from the mass densities provided by the HD simulation, we obtain values of $\sim 4$. This corresponds well to the generic assumption of $c_r = 4$ for strong shocks. Particle injection rates are largest close to the apex toward the B star because of its proximity. Farther away from the apex, the denser WR wind produces a higher possible injection rate.

By solving a transport equation within the entire computational domain, we obtained spectra of high-energy particles throughout the WCR. For electrons, we have shown that maximum energies are attained in the wings of the WCR where losses by IC scattering and synchrotron emission are significantly lower than at the apex because of the distance from the stars. Typically, spectra with indices between 1.9 and 2.2 are obtained. Since protons are less affected by energy loss mechanisms, their spectra show a cutoff determined by the Bohm limit. Varying indices and injection rates similarly relate to the wind properties and the geometry of the WCR. By studying spectra of high-energy particles in between the two shock fronts, we also provided insight into the evolution of particles after they leave the shock front on their journey downstream along the WCR.

**Figure 8.** Same as Figure 4, but with orbital motion. The stars move counterclockwise on a circular orbit.

(A color version of this figure is available in the online journal.)

**Figure 9.** (a) Ratio of the shock normal velocity to overall wind velocity as a function of distance from the apex along the B side of the WCR shock front. Negative distance traverses the shock along the forward arm, and positive distance traverses the shock along the trailing arm of the WCR. (b) Electron spectra for two characteristic positions in the forward arm (black solid line) and in the trailing arm (red dashed line) of the WCR close to the shock toward the B star.

(A color version of this figure is available in the online journal.)
In addition, we demonstrated the importance of orbital motion, which has considerable impact on the high-energy particle spectra. Even slight changes in the geometry of the WCR can cause contrasts in the number density of particles and critically influence their maximum energies.

6. OUTLOOK

On the basis of the approach we presented, several applications and future developments become feasible. Concerning CWBs, the next consequent step is to derive the photon emission along given lines of sight on the basis of our electron and nucleon spectra. The calculation of (anisotropic) IC, bremsstrahlung, and neutral $\pi$-decay components of the total nonthermal photon flux can be done directly from the simulated particle population and will be presented in a subsequent paper. In addition, we will study the integrated flux value as a function of the stellar separation along the orbital period and as a function of time. The application of this code to specific binary systems (e.g., WR 140) that remain undetected at $\gamma$-ray energies can provide limits on the fraction of injected electrons (which is an important free parameter in respective models). Dependencies of the $\gamma$-ray emission on other parameters, such as the magnetic field, can also be studied in detail. Future developments of the presented code include the consideration of additional physical processes, e.g., $\gamma-\gamma$ absorption close to the WCR. The transition from a HD to an MHD description of the wind plasma holds promise for providing a self-consistent magnetic field model at the WCR that only depends on the magnetic field models of the stars.

The consideration of an increasing variety of aspects might eventually provide an explanation of why all CWBs except $\eta$ Carinae have remained undetected at high-energy $\gamma$-rays so far. One critical factor that is now accessible to simulations is the dependence of particle evolution on the dynamics of the WCR, as it dramatically changes during the orbital period of high-eccentricity binary systems. Close to the periastron passage, especially strong cooling and strong velocity gradients produce very unstable conditions (see, e.g., Parkin et al. 2011; Madura et al. 2013) that could have a strong influence on the resulting particle distributions and nonthermal emission processes. This will be studied by a dedicated description of high-energy transport on the basis of 3D hydrodynamic models of CWBs.

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REFERENCES

Benaglia, P., & Romero, G. E. 2003, A&A, 399, 1121
Castor, J. I., Abbott, D. C., & Klein, R. I. 1975, ApJ, 195, 157
Crouseilles, N., Mehnenberger, M., & Sonnendrücker, E. 2010, JCoPh, 229, 1927
Gayley, K. G., Owocki, S. P., & Cranmer, S. R. 1997, ApJ, 475, 786
Kirk, J. G., Rieger, F. M., & Mastichiadis, A. 1998, A&A, 333, 452
Kissmann, R., Kleimann, J., Fichtner, H., & Grauer, R. 2008, MNRAS, 391, 1577
Kleimann, J., Kopp, A., Fichtner, H., & Grauer, R. 2009, AnGeo, 27, 989
Lamers, H. J. G. L. M., & Cassinelli, J. P. 1999, Introduction to Stellar Winds (Cambridge: Cambridge Univ. Press)
Madura, T. I., Gull, T. R., Okazaki, A. T., et al. 2013, MNRAS, 436, 3820
Martin, P., & Dubus, G. 2013, A&A, 551, A37
Moderski, R., Sikora, M., Coppi, P. S., & Aharonian, F. 2005, MNRAS, 363, 954
Parkin, E. R., Pittard, J. M., Corcoran, M. F., & Hamaguchi, K. 2011, ApJ, 726, 105
Pauldrach, A., Puls, J., & Kudritzki, R. P. 1986, A&A, 164, 86
Pittard, J. M. 2009, MNRAS, 396, 1743
Pittard, J. M. 2010, MNRAS, 403, 1633
Pittard, J. M., & Dougherty, S. M. 2006, MNRAS, 372, 801
Pittard, J. M., Dougherty, S. M., Coker, R. F., O’Connor, E., & Bolingbroke, N. J. 2006, A&A, 446, 1001
Pittard, J. M., & Parkin, E. R. 2010, MNRAS, 403, 1657
Protheroe, R. J., & Staney, T. 1999, ApJ, 10, 185
Reimer, A., Pohl, M., & Reimer, O. 2006, ApJ, 644, 1118
Reitberger, K., Reimer, O., Reimer, A., et al. 2012, A&A, 544, A98
Schure, K. M., Ackerberg, A., Keppens, R., & Vink, J. 2010, MNRAS, 406, 2633
Schure, K. M., Kosenko, D., Kaastra, J. S., Keppens, R., & Vink, J. 2009, A&A, 508, 751
Usov, V. V., & Melrose, D. B. 1992, ApJ, 395, 575
Werner, M., Reimer, O., Reimer, A., & Egberts, K. 2013, A&A, 555, A102