Quantum oblivious transfer protocols based on EPR states

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We describe efficient protocols for quantum oblivious transfer and for one-out-of-two quantum oblivious transfer. These protocols, which can be implemented with present technology, are secure against general attacks as long as the cheater can not store the bit for an arbitrarily long period of time.

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In 1970, Wiesner \[1\] wrote a highly innovative paper about quantum cryptography \[2\], \[3\]. In his paper, he also introduced the concept of Multiplexing, which was later rediscovered by Rabin, \[4\] and is now usually called Oblivious Transfer (OT). Let us briefly describe the OT protocol:

1 - Alice knows one bit $\lambda$, where $\lambda$ is either 1 or $-1$ \[5\].
2 - Bob obtains bit $\lambda$ from Alice with probability 0.5.
3 - Bob knows whether or not he obtained bit $\lambda$.
4 - Alice does not learn whether or not Bob obtained bit $\lambda$.

In this letter, we propose a very efficient protocol for quantum oblivious transfer which is secure even against cheaters with unlimited computing power, if we assume that Bob can not store the bit for an arbitrarily long period of time. It is worth noting that none of the known non-quantum protocols for oblivious transfer are perfectly secure; they all allow one among Alice or Bob to cheat without risk of detection if he or she can break some unproved cryptographic assumptions.

Before proceeding, it is useful to review some elementary features of quantum mechanics. We consider an unstable source emitting pairs of entangled Einstein, Podolsky, Rosen (EPR) \[8\] particles. We take the $z$ axis along the direction of the flight of the particles, and the $x$ and the $y$ axes along any two directions perpendicular to the $z$ axis. For a pair of particles in the EPR state $|\phi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)$, the expected value of the product of the spin of the particles along two arbitrary axes $\vec{a}$ and $\vec{b}$ in the $xy$ plane is \[9\]

$$\langle \phi | \sigma^a_1 \sigma^b_2 | \phi \rangle = \cos (\theta_a + \theta_b) ,$$

where $\sigma^a_1$ is the spin of the first particle along axis $\vec{a}$, $\sigma^b_2$ is the spin of the second particle along axis $\vec{b}$, and $\theta_a$ ($\theta_b$) is the angle between axis $\vec{a}$ ($\vec{b}$) and the $x$ axis. Thus the spin of the first particle along the $x$ axis, $m^x_1$, is perfectly correlated with the spin of the second particle along the same axis, i.e., $m^x_1 m^x_2 = 1$. However, the spin of the first particle along the $y$ axis is perfectly anticorrelated with the spin of the second particle along the same axis, i.e, $m^y_1 m^y_2 = -1$. Similarly for EPR states \[10\]

$$|\phi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle - |\downarrow\downarrow\rangle), \quad \langle \phi' | \sigma^a_1 \sigma^b_2 | \phi' \rangle = -\cos (\theta_a + \theta_b) ;$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle), \quad \langle \psi | \sigma^a_1 \sigma^b_2 | \psi \rangle = \cos (\theta_a - \theta_b) ;$$

$$|\psi'\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle), \quad \langle \psi' | \sigma^a_1 \sigma^b_2 | \psi' \rangle = -\cos (\theta_a - \theta_b) ;$$

$$|\alpha\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle + i |\downarrow\downarrow\rangle), \quad \langle \alpha | \sigma^a_1 \sigma^b_2 | \alpha \rangle = \sin (\theta_a + \theta_b) .$$

(2)
\[ |\alpha'\rangle = \frac{1}{\sqrt{2}} (|\uparrow\uparrow\rangle - i |\downarrow\downarrow\rangle), \quad \langle \alpha' | \sigma^x \sigma^y | \alpha' \rangle = -\sin (\theta_a + \theta_b), \]

\[ |\beta\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle + i |\downarrow\uparrow\rangle), \quad \langle \beta | \sigma^x \sigma^y | \beta \rangle = \sin (\theta_a - \theta_b), \]

\[ |\beta'\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - i |\downarrow\uparrow\rangle), \quad \langle \beta' | \sigma^x \sigma^y | \beta' \rangle = -\sin (\theta_a - \theta_b). \]

With the above in mind, we now proceed to describe the following protocol:

1. Alice and Bob agree that the bit \( \lambda \) is encoded in the product of the spin of the first and the second particles along the \( x \) axis (\( m_1^x m_2^x \) or \( -m_1^x m_2^x \)), or the spin of the first and the second particles along the \( y \) axis (\( m_1^y m_2^y \) or \( -m_1^y m_2^y \)), or the spin of the first particle along the \( y \) axis and the spin of the second particle along the \( x \) axis (\( m_1^y m_2^x \) or \( -m_1^y m_2^x \)), or the spin of the first particle along the \( -x \) axis and the spin of the second particle along the \( y \) axis \( (m_1^{-x} m_2^y \text{ or } -m_1^{-x} m_2^y) \). They also agree on a set \( A \) defined as \( A = \{m_1^x m_2^x, m_1^y m_2^y\} \), and on a set \( B \) defined as \( B = \{m_1^y m_2^x, m_1^{-x} m_2^y\} \).

2. Alice encodes the bit to be obliviously transferred in \( m_1^x m_2^x \), or \( -m_1^x m_2^x \), or \( m_1^y m_2^y \), or \( -m_1^y m_2^y \), or \( m_1^y m_2^x \), or \( -m_1^y m_2^x \), or \( m_1^{-x} m_2^y \), or \( -m_1^{-x} m_2^y \), chosen randomly by her. She randomly chooses an appropriate state (as shown below, there are two appropriate states for any of her choices). She then prepares a pair of particles in that state and sends both particles to Bob.

3. Bob measures randomly either \( m_1^x m_2^x \) or \( m_1^y m_2^y \), or \( m_1^y m_2^x \), or \( m_1^{-x} m_2^y \).

4. Alice asks Bob if his measurements have been successful. If he says no, then she goes to step 2. If he says yes, then she considers the following two cases:

   (I) Alice asks Bob if his measurement belongs to set \( A \). If he says no, then she goes to (II). If he says yes (but of course he does not tell her along which axes he performed his measurements), and if she has chosen one of the four state \( |\alpha\rangle \), or \( |\alpha'\rangle \), or \( |\beta\rangle \), or \( |\beta'\rangle \), then she tells him that the protocol has not been successful, and she goes to step (2). But if she has chosen one of the four states \( |\phi\rangle \), or \( |\phi'\rangle \), or \( |\psi\rangle \), or \( |\psi'\rangle \), then she tells him only one of the following four alternatives:

   (i) \( \lambda \) is encoded in \( m_1^x m_2^x \),
   (ii) \( \lambda \) is encoded in \( -m_1^x m_2^x \),
   (iii) \( \lambda \) is encoded in \( m_1^y m_2^y \),
   (iv) \( \lambda \) is encoded in \( -m_1^y m_2^y \).

(II) Alice asks Bob if his measurement belongs to set \( B \). If he says yes, and if she has chosen one of the four state \( |\phi\rangle \), or \( |\phi'\rangle \), or \( |\psi\rangle \), or \( |\psi'\rangle \), then she tells him the protocol has not been successful, and she goes to step (2). But if she has chosen one of the four states \( |\alpha\rangle \), or \( |\alpha'\rangle \), or \( |\beta\rangle \), or \( |\beta'\rangle \), then she tells him only one of the following four alternatives:
\( (i) \) \( \lambda \) is encoded in \( m^y_1 m^z_2 \),
\( (ii) \) \( \lambda \) is encoded in \( -m^y_1 m^z_2 \),
\( (iii) \) \( \lambda \) is encoded in \( m^{-x} m^y_2 \),
\( (iv) \) \( \lambda \) is encoded in \( -m^{-x} m^y_2 \).

**Theorem:** Assuming that Bob can not store the bit until step 4 (when Alice asks him whether his measurement belongs to set \( A \) or \( B \)), the above oblivious transfer protocol is secure even against cheater with unlimited computing power.

**Proof:** First we consider Alice’s strategy. We assume that Alice has chosen one of the four state \( |\phi\rangle \), or \( |\phi'\rangle \), or \( |\psi\rangle \), or \( |\psi'\rangle \), and Bob’s measurement belongs to set \( A \), i.e., he has measured \( m^y_1 m^z_2 \) or \( m^y_1 m^y_2 \). Alice should consider the following four cases:

1. First assume \( \lambda = 1(-1) \), and Alice decides to encode \( \lambda \) in \( m^y_1 m^z_2 \). In this case, Alice should choose either \( |\phi\rangle \) or \( |\psi\rangle \), since for both of these states \( m^y_1 m^z_2 = 1 \). If Bob measures the spin of the two particles along the \( x \)-axis, then he learns the value of \( \lambda \). However, if he measures the spins of the particles along the \( y \)-axis, then he does not learn any information about \( \lambda \), since \( \langle \phi | \sigma^y_1 \sigma^y_2 | \phi \rangle = -1 \), but \( \langle \psi | \sigma^y_1 \sigma^y_2 | \psi \rangle = 1 \).

2. Next assume \( \lambda = -1(1) \), and Alice decides to encode \( \lambda \) in \( m^y_1 m^z_2 \). In this case, Alice should choose either \( |\phi'\rangle \) or \( |\psi'\rangle \), since for both of these states \( m^y_1 m^z_2 = -1 \). If Bob measures the spin of the two particles along the \( x \)-axis, then he learns the value of \( \lambda \). However, if he measures the spins of the particles along the \( y \)-axis, then he does not learn any information about \( \lambda \), since \( \langle \phi' | \sigma^y_1 \sigma^y_2 | \phi' \rangle = 1 \), but \( \langle \psi' | \sigma^y_1 \sigma^y_2 | \psi' \rangle = -1 \).

3. Next assume \( \lambda = 1(-1) \), and Alice decides to encode \( \lambda \) in \( m^y_1 m^y_2 \). In this case, Alice should choose either \( |\phi'\rangle \) or \( |\psi'\rangle \), since for both of these states \( m^y_1 m^y_2 = 1 \). If Bob measures the spin of both particles along the \( y \)-axis, then he learns the value of \( \lambda \). However, if he measures the spins of the particles along the \( x \)-axis, then does not learn any information about the value of \( \lambda \), since \( \langle \phi' | \sigma^x_1 \sigma^x_2 | \phi' \rangle = -1 \), but \( \langle \psi' | \sigma^x_1 \sigma^x_2 | \psi' \rangle = 1 \).

4. Finally assume \( \lambda = -1(1) \), and Alice decides to encode \( \lambda \) in \( m^y_1 m^y_2 \). In this case, Alice should choose either \( |\phi\rangle \) or \( |\psi'\rangle \), since for both of these states \( m^y_1 m^y_2 = -1 \). If Bob measures the spin of both particles along the \( y \)-axis, then he learns the value of \( \lambda \). However, if he measures the spins of the particles along the \( x \)-axis, then does not learn any information about the value of \( \lambda \), since \( \langle \phi | \sigma^x_1 \sigma^x_2 | \phi \rangle = 1 \), but \( \langle \psi' | \sigma^x_1 \sigma^x_2 | \psi' \rangle = -1 \).

We now consider Bob’s strategy. If Bob is honest, then the oblivious transfer protocol can succeed without any difficulty (see above). Consider now a cheating Bob who measures the spin of the first particle along axis \( \vec{a} \) and measures the spin of the second particle along axis \( \vec{b} \), i.e., \( m^a_1 m^b_2 \).

Assume (without loss of generality) that Bob obtains \( m^a_1 m^b_2 = 1 \). Bob then asks the following question: Given that \( m^a_1 m^b_2 = 1 \), what is the probability
that \( m_1^x m_2^x = 1 \), i.e., what is \( p ( m_1^x m_2^x = 1 \mid m_1^a m_2^b = 1 ) \)? To answer this question, he notes that only states \( | \phi \rangle \) and \( | \psi \rangle \) can contribute to \( m_1^x m_2^x = 1 \). Thus

\[
p ( m_1^x m_2^x = 1 \mid m_1^a m_2^b = 1 ) = p ( | \phi \rangle \mid m_1^a m_2^b = 1 ) + p ( | \psi \rangle \mid m_1^a m_2^b = 1 ). \tag{3}
\]

To calculate \( p ( | \phi \rangle \mid m_1^a m_2^b = 1 ) \), note that

\[
p ( | \phi \rangle , m_1^a m_2^b = 1 ) = p ( | \phi \rangle ) p ( m_1^a m_2^b = 1 \mid | \phi \rangle ),
\]

\[
= p ( m_1^a m_2^b = 1 ) p ( | \phi \rangle \mid m_1^a m_2^b = 1 ). \tag{4}
\]

But

\[
p ( | \phi \rangle ) = \frac{1}{4}, \tag{5}
\]

\[
p ( m_1^a m_2^b = 1 ) = \frac{1}{2}, \tag{6}
\]

\[
p ( m_1^a m_2^b = 1 \mid | \phi \rangle ) = \cos^2 \left( \frac{\theta_a + \theta_b}{2} \right), \tag{7}
\]

where (5) follows from the fact that Alice chooses any state with probability \( \frac{1}{4} \), (6) follows from the symmetry of the problem, and (7) follows from the standard rules of quantum mechanics.

Substituting the above formulas in Eq. (4), we obtain

\[
p ( | \phi \rangle \mid m_1^a m_2^b = 1 ) = \frac{1}{2} \cos^2 \left( \frac{\theta_a + \theta_b}{2} \right). \tag{8}
\]

Similar argument shows that

\[
p ( | \psi \rangle \mid m_1^a m_2^b = 1 ) = \frac{1}{2} \cos^2 \left( \frac{\theta_a - \theta_b}{2} \right). \tag{9}
\]

Thus

\[
p ( m_1^x m_2^x = 1 \mid m_1^a m_2^b = 1 ) = \frac{1}{2} \cos^2 \left( \frac{\theta_a + \theta_b}{2} \right) + \frac{1}{2} \cos^2 \left( \frac{\theta_a - \theta_b}{2} \right). \tag{10}
\]

Bob now asks the following question: Given that \( m_1^a m_2^b = 1 \), what is the probability that \( m_1^y m_2^y = 1 \), i.e., what is \( p ( m_1^y m_2^y = 1 \mid m_1^a m_2^b = 1 ) \)? To
answer this question, he notes that only states $|\phi'\rangle$ and $|\psi\rangle$ can contribute to $m_1^y m_2^y = 1$. Thus

$$p \left( m_1^y m_2^y = 1 \mid m_1^a m_2^b = 1 \right) = p \left( |\phi'\rangle \mid m_1^a m_2^b = 1 \right) + p \left( |\psi\rangle \mid m_1^a m_2^b = 1 \right). \quad (11)$$

Similar argument as before shows that

$$p \left( m_1^y m_2^y = 1 \mid m_1^a m_2^b = 1 \right) = \frac{1}{2} \sin^2 \left( \frac{\theta_a + \theta_b}{2} \right) + \frac{1}{2} \cos^2 \left( \frac{\theta_a - \theta_b}{2} \right). \quad (12)$$

Now the probability that Bob learns the value of $\lambda$ is $\frac{1}{2} \left[ p \left( m_1^y m_2^y = 1 \mid m_1^a m_2^b = 1 \right) \right] + p \left( m_1^a m_2^b = 1 \mid m_1^a m_2^b = 1 \right)$. From Eqs. (12) and (14), we have

$$\frac{1}{2} \left[ p \left( m_1^y m_2^y = 1 \mid m_1^a m_2^b = 1 \right) \right] + p \left( m_1^a m_2^b = 1 \mid m_1^a m_2^b = 1 \right) = \frac{1}{4} + \frac{1}{2} \cos^2 \left( \frac{\theta_a - \theta_b}{2} \right). \quad (13)$$

Note that the maximum value of the RHS of (13) is $\frac{3}{4}$. Thus the best strategy for Bob is to measure the spins of both particles either along the same axis. In particular, if Bob does not cheat and measures the spins of both particles either along the $x$-axis or along the $y$-axis, then he obtains maximum information about the value of the OT bit $\lambda$.

Having demonstrated that if Alice chooses one of the four states $|\phi\rangle$, or $|\phi'\rangle$, or $|\psi\rangle$, or $|\psi'\rangle$, and if Bob’s measurement belongs to set $A$, then the OT protocol can be implemented successfully, we now consider the other alternative. We assume that Alice has chosen one of the four state $|\alpha\rangle$, or $|\alpha'\rangle$, or $|\beta\rangle$, or $|\beta'\rangle$, and Bob’s measurement belongs to set $B$, i.e., he has measured $m_1^y m_2^y$, or $m_1^x m_2^y$. Again Alice should consider the following four cases:

1. First assume $\lambda = 1(-1)$, and Alice encodes $\lambda$ in $m_1^y m_2^y (-m_1^y m_2^y)$. In this case, Alice should choose either $|\alpha\rangle$ or $|\beta\rangle$, since for both of these states $m_1^y m_2^y = 1$. If Bob measures the spin of the first particle along the $y$ axis, and spin of the second particle along the $x$ axis, then he learns the value of $\lambda$. However, if he measures the spins of the first particle along the $-x$ axis, and the spin of the second particle along $y$ axis, then he does not learn any information about $\lambda$, since $\langle \alpha \mid \sigma_{-x}^{\alpha} \sigma_{y}^{\alpha} \mid \alpha \rangle = -1$, but $\langle \beta \mid \sigma_{-x}^{\beta} \sigma_{y}^{\beta} \mid \beta \rangle = 1$.

2. Next assume $\lambda = -1(1)$, and Alice encodes $\lambda$ in $m_1^y m_2^y (-m_1^y m_2^y)$. In this case, Alice should choose either $|\alpha'\rangle$ or $|\beta'\rangle$, since for both of these states
\(m^y_1 m^z_2 = -1\). If Bob measures the spin of the first particle along the \(y\) axis, and spin of the second particle along the \(x\) axis, then he learns the value of \(\lambda\). However, if he measures the spins of the first particle along the \(-x\) axis, and the spin of the second particle along \(y\) axis, then he does not learn any information about \(\lambda\), since \(\langle \alpha' \mid \sigma^{-x}_1 \sigma^y_2 \mid \alpha' \rangle = 1\), but \(\langle \beta' \mid \sigma^{-x}_1 \sigma^y_2 \mid \beta' \rangle = -1\).

(3) Next assume \(\lambda = 1(-1)\), and Alice encodes \(\lambda\) in \(m^{-x}_1 m^y_2 \left( -m^{-x}_1 m^y_2 \right)\). In this case, Alice should choose either \(\mid \alpha \rangle\) or \(\mid \beta \rangle\), since for both of these states \(m^{-x}_1 m^y_2 = 1\) If Bob measures the spin of the first particle along \(-x\) axis and spin of the second particle along the \(y\)-axis, then he learns the value of \(\lambda\). However, if he measures the spin of of the first particle along the \(y\) axis and spin of the second particle along the \(x\) axis, then he does not learn any information about the value of \(\lambda\), since \(\langle \alpha' \mid \sigma^y_1 \sigma^z_2 \mid \alpha' \rangle = -1\), but \(\langle \beta \mid \sigma^y_1 \sigma^z_2 \mid \beta \rangle = 1\).

(4) Finally assume \(\lambda = -1(1)\), and Alice encodes \(\lambda\) in \(m^{-x}_1 m^y_2 \left( -m^{-x}_1 m^y_2 \right)\). In this case, Alice should choose either \(\mid \alpha \rangle\) or \(\mid \beta' \rangle\), since for both of these states \(m^{-x}_1 m^y_2 = -1\) If Bob measures the spin of both particles along the \(y\)-axis, then he learns the value of \(\lambda\). However, if he measures the spin of of the first particle along the \(y\) axis and spin of the second particle along the \(x\) axis, then he does not learn any information about the value of \(\lambda\), since \(\langle \alpha \mid \sigma^y_1 \sigma^z_2 \mid \alpha \rangle = -1\), but \(\langle \beta \mid \sigma^y_1 \sigma^z_2 \mid \beta \rangle = 1\).

If Bob is honest, then the OT protocol can succeed without any difficulty (see above). However, if Bob is dishonest, the same argument as before shows that he does not gain any additional information by cheating. Thus the OT protocol is secure even against cheaters with unlimited computing power.

There is another flavor of OT which is called one-out-of-two oblivious transfer. The goal of this protocol is:

1 - Alice has two bits \(\lambda_1\) and \(\lambda_2\) where \(\lambda_1\) (or \(\lambda_2\)) is either 1 or -1.
2 - Bob chooses to obtain either bit \(\lambda_1\) or \(\lambda_2\).
3 - Bob knows whether or not he has obtained the bit.
4 - Alice does not learn which bit Bob has chosen.

Less formally, Alice has two bits. Bob can get only one of them, and Alice does not learn which bit Bob obtained.

This protocol can be implemented by

(1) Alice and Bob agree that \(\lambda_1\) is encoded in \(m^x_1 m^z_2\), or \(-m^x_1 m^z_2\), or \(m^y_1 m^y_2\), or \(-m^y_1 m^y_2\), and \(\lambda_2\) is encoded in \(m^y_1 m^z_2\), or \(-m^y_1 m^z_2\), or \(m^y_1 m^y_2\), or \(-m^y_1 m^y_2\). They also agree on a set \(A\) defined as \(A = \{m^x_1 m^z_2, m^y_1 m^y_2 \}\), and on a set \(B\) defined as \(B = \{m^y_1 m^z_2, m^{-x}_1 m^y_2 \}\).

(2) Alice encodes \(\lambda_1\) in \(m^x_1 m^z_2\), or \(-m^x_1 m^z_2\), or \(m^y_1 m^z_2\), or \(-m^y_1 m^z_2\), and \(\lambda_2\) in \(m^y_1 m^y_2\), or \(-m^y_1 m^y_2\), or \(m^{-x}_1 m^y_2\), or \(-m^{-x}_1 m^y_2\), chosen randomly by her. She randomly chooses an appropriate state (as shown below, there are two appropriate states for any of her choices). She then prepares a pair of particles in that state and sends both particles to Bob.
(3) Bob measures randomly either $m_1^x m_2^x$ or $m_1^y m_2^y$, or $m_1^y m_2^x$, or $m_1^x m_2^y$.

(4) Alice asks Bob if his measurements have been successful. If he says no, then she goes to step 2. If he says yes, then she considers the following two cases:

(I) Alice asks Bob if his measurement belongs to set $A$. If he says no, then Alice goes to step (II). If he says yes (but of course he does not tell her along which axes he performed his measurements), and if she has chosen one of the four states $|\alpha\rangle$, or $|\alpha'\rangle$, or $|\beta\rangle$, or $|\beta'\rangle$, then she tells him that the protocol has not been successful, and she goes to step (2). But if Alice has chosen one of the four states $|\phi\rangle$, or $|\phi'\rangle$, or $|\psi\rangle$, or $|\psi'\rangle$, then she tells him that $\lambda_1$ is encoded in $m_1^x m_2^x$ or $-m_1^y m_2^x$, and $\lambda_2$ is encoded in $m_1^y m_2^y$ or $-m_1^y m_2^x$.

(II) Alice asks Bob if his measurement belongs to set $B$. If he says yes, and if she has chosen one of the four states $|\phi\rangle$, or $|\phi'\rangle$, or $|\psi\rangle$, or $|\psi'\rangle$, then she tells him that the protocol has not been successful, and she goes to step (2). But if Alice has chosen one of the four states $|\alpha\rangle$, or $|\alpha'\rangle$, or $|\beta\rangle$, or $|\beta'\rangle$, then she tells him that $\lambda_1$ is encoded in $m_1^y m_2^x$ or $-m_1^y m_2^x$, and $\lambda_2$ is encoded in $m_1^x m_2^y$ or $-m_1^x m_2^y$.

**Theorem:** Assuming that Bob can not store the bit until step 4 (when Alice asks him whether his measurement belongs to set $A$ or $B$), the above one-out-of-two oblivious transfer protocol is secure against cheater with unlimited computing power.

**Proof:** First Assume that Bob performed his measurement in set $A$, and Alice has chosen one of the four states $|\phi\rangle$, or $|\phi'\rangle$, or $|\psi\rangle$, or $|\psi'\rangle$, (similar argument also applies if Alice has chosen one of the four states $|\alpha\rangle$, or $|\alpha'\rangle$, or $|\beta\rangle$, or $|\beta'\rangle$). She should consider the following four cases:

(i) First assume $\lambda_1 = 1$, $\lambda_2 = 1$, and Alice decides to encode $\lambda_1$ in $m_1^x m_2^x$, and $\lambda_2$ in $m_1^y m_2^y$, or $\lambda_1 = -1$, $\lambda_2 = -1$, and Alice decides to encode $\lambda_1$ in $-m_1^x m_2^x$, and $\lambda_2$ in $-m_1^y m_2^y$. In this case, Alice should choose state $|\psi\rangle$, since for this state $m_1^x m_2^x = 1$, and $m_1^y m_2^y = 1$.

(ii) Next assume $\lambda_1 = 1$, $\lambda_2 = -1$, and Alice decides to encode $\lambda_1$ in $m_1^x m_2^x$, and $\lambda_2$ in $m_1^y m_2^y$, or $\lambda_1 = -1$, $\lambda_2 = 1$, and Alice decides to encode $\lambda_1$ in $-m_1^x m_2^x$, and $\lambda_2$ in $-m_1^y m_2^y$. In this case, Alice should choose state $|\phi\rangle$, since for this state $m_1^x m_2^x = 1$, and $m_1^y m_2^y = -1$.

(iii) Next assume $\lambda_1 = -1$, $\lambda_2 = 1$, and Alice decides to encode $\lambda_1$ in $m_1^x m_2^x$, and $\lambda_2$ in $m_1^y m_2^y$, or $\lambda_1 = 1$, $\lambda_2 = -1$, and Alice decides to encode $\lambda_1$ in $-m_1^x m_2^x$, and $\lambda_2$ in $-m_1^y m_2^y$. In this case, Alice should choose state $|\phi'\rangle$, since for this state $m_1^x m_2^x = -1$, and $m_1^y m_2^y = 1$.

(iv) Finally assume $\lambda_1 = -1$, $\lambda_2 = -1$, and Alice decides to encode $\lambda_1$ in $m_1^x m_2^x$, and $\lambda_2$ in $m_1^y m_2^y$, or $\lambda_1 = 1$, $\lambda_2 = 1$, and Alice decides to encode $\lambda_1$ in $-m_1^x m_2^x$, and $\lambda_2$ in $-m_1^y m_2^y$. In this case, Alice should choose state $|\psi'\rangle$, since for this state $m_1^x m_2^x = -1$, and $m_1^y m_2^y = -1$.

The same argument that was used for quantum OT can be used to prove
that Bob does not gain any additional information by setting his polarizer at other angles. Thus the one-out-of-two OT protocol is secure even against cheaters with unlimited computing power.
References

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[10] Note that the states $|\phi\rangle$, $|\phi'\rangle$, $|\psi\rangle$, $|\psi'\rangle$, are orthonormal. Thus if we only consider these 4 states, then Bob can (at least in principle, although infeasible with present technology) determine which state was sent to him. If Bob does not have the technology to determine the state that was sent to him, then Alice should only consider the above 4 states. In this case, the one-out-of-two OT is secure even against cheaters with the ability to store the bit for an arbitrarily long period of time.