Effective power analysis of multi-tone excitation transmitting system

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Abstract: Inter-modulation would occur when the transmitting system is under multi-tone excitation; therefore, the non-linear response of power amplifier unit should be considered in the system design. Here, by establishing non-linear theory simulation model of the transmitting system, we introduce the channel consistency factor, put forward the concept of effective jamming power, analyse the effective power of the multi-channel array transmitting system under multi-tone excitation, and carry out relevant experiments for verification, thereby providing reference for the design of multi-signal transmitting system.

1 Introduction

Modern warfare has entered the era of information warfare, and the electromagnetic environment has become more complex than ever before. In modern battlefield, one-to-many and many-to-many system confrontation scenario has become the basic operational requirements. Therefore, the multi-target jamming capability of radar jamming system becomes an important factor to measure its performance, which also becomes a research focus in recent years. Reference [1] studies the allocation of jamming resources under multi-target jamming scenario, establishes an evaluation model of jamming effect, and proposes jamming resources allocation methods. However, these studies did not involve the effective power of the system under multi-tone excitation when conducting multi-target jamming. Reference [2] studies the experimental methods of multi-target jamming capability. In this study, the devices at all levels in transmitting feeder link are regarded as linear microwave device and it is assumed that non-linear frequency component would not be produced. However, non-linear microwave devices are widely used in jamming system, such as frequency mixer and power amplifier. Thus, when the signals pass through these non-linear microwave devices, the non-linear effect will be caused, which would then result in non-linear spectrum distortion. The jamming power spectrum distribution plays a crucial role in jamming effect. The spectrum distortion would result in reduction of jamming power of fundamental wave. The spectrum leakage can also be easily detected by radar system and corresponding counter-jamming measures would be adopted. Therefore, it is necessary to conduct quantitative analytical study for jamming signal power spectrum under multi-target jamming scenario.

2 Multi-tone excitation transmitting system

The current radar countermeasures systems widely adopt Rotman lens feed transmitting method to simultaneously confront multiple emitter targets at different azimuth and frequency. It can work at instantaneous bandwidth, cover wide airspace, scan agile beams, and make good use of jamming energy of multiple targets. The operating principle diagram is shown in Fig. 1.

N-channel jamming signals pass through the switch network route and then enter the lens from different beam ports. After phase assignment in the lens, equiphase difference is generated between adjacent output ports. The signals are then amplified in power amplifier array and output to M element antenna array to form beam in space and complete beam scanning.

3 How non-linear features of power amplifier affect the transmitting system

3.1 Mathematic model for non-linear features of power amplifier

As known from the analysis in the previous section, transmitting signals have passed through microwave switch network, lens, and power amplifier array before they arrive at antenna aperture. Among the devices, the microwave network and lens are passive microwave devices with lower non-linear degree, so only the effect of non-linear characteristic of power amplifier array is to be considered. In the transmitting system, the power amplifier often works at the compression point of about 1 dB, where non-linear characteristic is obvious.

Unlike the power amplifier in the communication system, the operation bandwidth of the power amplifier in electronic warfare can reach three octaves and the signal power is in W order of magnitude. The memory effect of power amplifier cannot be ignored. Volterra series model is often used to describe the non-linear features of such power amplifier [3, 4] (see (1)). \( x(t) \) refers to power amplifier excitation function; \( a_n(t_1, t_2, \ldots, t_n) \) refers to the nth-order time domain kernel function of non-linear power amplifier, which represents non-linear features of power amplifier; \( y(t) \) refers to non-linear response output of power amplifier. Formula (1) indicates that the time domain response of non-linear power amplifier is not instantaneous, but is convolutional with impact response of the system, which, considering the historical status and memory effect of the power amplifier, would cause inconsistency of frequency domain response and AM/AM and AM/PM conversion. Actually in real systems, only the first few of the Volterra series have a significant impact on the output of the system. Moreover, high-order non-linear distortion is usually beyond the system’s working bandwidth and cannot be taken into account. In the process of analysis, only the first 3 orders of the non-linear model of power amplifier are used, so that the approximation hardly affects the non-linear description of the
system, and it can also achieve a better computational convergence [5].

In the multi-target confrontation scenario, assuming input excitation is signal with \( N \) frequency points: \( x(t) = \sum_{j=1}^{N} A_j e^{j\omega_j t} \), \( A_j \) refers to the amplitude of the \( j \)th signal, \( \omega_j \) refers to the angular frequency of the \( j \)th signal. Substitute them into formula (1) for calculation, and it comes out the output expression of Volterra series. Then, substitute the \( n \) dimension Fourier transform of kernel function, and it comes out the first 3 orders output of power amplifier:

\[
y(t) = \sum_{j=1}^{N} A_j [H_1(\omega_j) e^{j\omega_j t} + H_2(-\omega_j) e^{-j\omega_j t}] + \sum_{j=1}^{N-1} \sum_{k=1}^{j} A_j A_k \frac{1}{4} [H_1(\omega_j + \omega_k) e^{j(\omega_j + \omega_k) t} + H_2(\omega_j - \omega_k) e^{-j(\omega_j - \omega_k) t} + H_1(-\omega_j - \omega_k) e^{-j(\omega_j + \omega_k) t} + H_2(-\omega_j + \omega_k) e^{j(\omega_j - \omega_k) t}]
\]

(3)

(see (4)). As seen from formulas (2), (3), and (4), multi-frequency excitation signals would generate plenty of non-linear spectrum components in power amplifier response, such as harmonic signal and intermodulation signal. When these spectrum components are within the system operation bandwidth, the effective jamming power of the system will be affected. Particularly, when \( N = 2 \), the above formulas degenerate to double-tone intermodulation model.

3.2 Non-linear response of the system

In the radar countermeasures system, the method of using parallel power amplifier to output signals to transmitting array to synthesise a signal in space is widely adopted. The non-linear features of power amplifier mainly affect ERP of the system. In order to describe the engineering implementation model more accurately, the channel gain consistency factor \( \beta \) of the power amplifier is introduced. Taking both non-linear features of power amplifier and channel gain difference effect of power amplifier into consideration, the ERP calculation formula of the system under multi-target jamming is:

\[
P_b = \sum_{i=1}^{N} \beta_i P G_i \quad i = 1, 2, \ldots, \infty
\]

(5)

where, \( P_b \) refers to the radiation ERP of the \( i \)th non-linear spectrum component; \( i \) refers to all spectrum components in ergodic power amplifier response; \( \beta_i \) refers to inter-channel consistency factor, indicating the gain difference between channels of power amplifier and changing with power amplifiers and corresponding spectrum components; \( P_i \) refers to power amplifier output power of the \( i \)th non-linear spectrum component; \( N \) refers to the number of power amplifier channels; \( \eta \) refers to combined efficiency; \( G_i \) refers to the antenna gain corresponding to the \( i \)th non-linear spectrum component, obtained via antenna simulation software. For antenna array, the gain is:

\[
G_i(\theta) = \frac{4\pi N d^2 \cos(\theta) G_e}{\lambda_i^2}
\]

(6)

where \( G_i(\theta) \) refers to the antenna gain corresponding to the \( i \)th non-linear spectrum component; \( G_e \) refers to the gain of element antenna; \( \theta \) refers to the scanning angle of beams; \( \lambda_i \) refers to the wave length of the \( i \)th spectrum component; \( d \) refers to the interval between antenna array elements. Ignore the phase difference introduced by power amplifier channel, analyse the time domain response of Volterra series, and we know that the radiation angle of fundamental wave jamming signal stays unchanged, but the spatial direction in synthetic pattern of other non-linear spectrum increment has changed.

The antenna gain is known. After the non-linear spectrum component of power amplifier is output, the effective jamming power ratio under power amplifier's non-linear operating status is:

\[
P_{sat} = \sum_{i=1}^{N} P_b = \sum_{i=1}^{N} (P_{li} + P_{hui})
\]

(7)

where, \( K_i(\omega_i, \omega_i, \cdots, \omega_i) \) refers to the effective jamming power ratio, it means the ratio of ERP of \( \omega_i \) frequency component to saturation output power of power amplifier when the jamming signal contains \( N \) frequency points under \( N \)-target confrontation scenario; \( P_{li} \) and \( P_{hui} \) respectively, refer to output power of fundamental wave and harmonic wave of power amplifier module under single-tone \( \omega_i \) excitation; \( P_{sat} \) refers to saturation output power of power amplifier. It can be seen that when the jamming system is in dual targets confrontation, the system ERP does not simply reduce by twice. On the contrary, it is related to the non-linear degree of power amplifier in a complicated way, and it follows the principle of conservation of energy.

4 Numerical simulation and experiments

Taking the multi-channel power amplifier array system as an example, this paper uses the above method to conduct multi-tone excitation simulation and effective power analysis. As per formulas (2), (3), and (4), non-linear increment of the output spectrum under the excitation of eight frequency points can be obtained through simulation calculation. Then, 13 frequency points will be output within the system working bandwidth. The frequency components

\[
y(t) = y_1(t) + \int_{-\infty}^{t} h_1(\tau) x(t - \tau) d\tau
\]

\[+ \int_{-\infty}^{t} \int_{-\infty}^{t} h_2(\tau_1, \tau_2) x(t - \tau_1) x(t - \tau_2) d\tau_1 d\tau_2
\]

\[+ \cdots + \int_{-\infty}^{t} \cdots \int_{-\infty}^{t} h_n(\tau_1, \tau_2, \cdots, \tau_n) x(t - \tau_1) x(t - \tau_2) \cdots x(t - \tau_n) d\tau_1 \cdots d\tau_n + \cdots
\]

\[
y_2(t) = \sum_{k=1}^{N} \sum_{j=1}^{N} \sum_{i=1}^{N} A_k A_j A_l \frac{1}{4} [H_1(\omega_k + \omega_j + \omega_l) e^{j(\omega_k + \omega_j + \omega_l) t} + H_2(\omega_k - \omega_j - \omega_l) e^{-j(\omega_k - \omega_j - \omega_l) t} + H_1(-\omega_k - \omega_j + \omega_l) e^{-j(\omega_k + \omega_j - \omega_l) t} + H_2(\cdots) + \cdots]
\]

(4)
of the system can be analysed through the principle of antenna array, and this paper takes two frequency components as examples for analysis. As shown in Fig. 2, the beam direction of two fundamental wave signals does not change, but all beam directions of non-linear spectrum increment are different from that of fundamental wave beam.

Therefore, when the jamming system is working, it will transmit more spectrum signals in different directions, leading to the waste of power and spectrum leakage.

During analysing the impact of non-linear features of power amplifier on system, the most important step is to calculate its frequency domain kernel function. The non-linear model of power amplifier can be established through testing data. The low-order kernel function can be calculated after determining the high-order kernel function. This paper uses dual-tone injection model to verify the method of this paper, and test system is as shown in Fig. 3a.

\[ K_{i}(\omega_1, \omega_2, \cdots, \omega_N) \] can be calculated based on the existing non-linear model of power amplifier.

According to the method of this paper, the ratio of effective jamming to power can be −1.78 dBc if \( K_{i}(\omega_1, \omega_2) \) or −6.24 if \( K_{i}(\omega_1, \omega_2) \), and the results of outfield test are −2.67 dBc and −5.8 dBc, respectively. Given that the composition of transmitting system is complex, with allowable error and good conformity, the effective jamming power of transmitting system declines under multi-tone excitation. Besides, non-linear frequency point increment coincides with theoretical analysis.

Through theoretical analysis and tests, this paper finds that transmitting system will generate plenty of non-linear spectrum increment under multi-tone excitation, leading to an obvious decline of the power of fundamental wave. Hence, the excitation waveform design should be paid sufficient attention to during system design, realising time-sharing excitation at the micro-level, so as to improve the effective power of transmitting system.

5 Conclusion
As for the non-linear response of transmitting system under multi-tone excitation, this paper analyses the effective power of transmitting system and puts forward a systematic and analytical method. Specifically, this paper uses Volterra series to establish the non-linear model of power amplifier and calculates the frequency domain response of array transmitting system under multi-tone excitation. By establishing the model of the impact of non-linear features of power amplifier on system, this paper analyses the pattern and effective power ratio of all spectrum components. Simulation calculation and tests are also designed to verify the effectiveness of the above method. It turns out that the method of this paper is in good accordance with the testing results, and such method can provide reference for design of multi-signal transmitting system.

6 References
[1] Libin, P., Chunsheng, L.: ‘Research on multi-target jamming resource allocation in wide-band array’, Electron. Inf. Warf. Technol., 2012, 02-0046-04
[2] Yan, M.: ‘Method for testing the jammer's multi-target jamming capability’, Shipboard Electron. Countermeas., 2001
[3] Peng, Z.K., Lang, Z.Q., Chu, F.L.: ‘On the nonlinear effects introduced by crack using nonlinear output frequency response functions’, Comput. Struct., 2008, 86, pp. 1809–1818
[4] Jing, X.J., Lang, Z.Q., Billings, S.A.: ‘Output frequency properties of nonlinear systems’, Int. J. Nonlinear Mech., 2010, 45, (7), pp. 681–690
[5] Aibo, L. et al.: ‘Wideband power amplifier predistortion principle’ (Science Press, Beijing, 2011), pp. 19–22

Fig. 2 Dual-tone stimulation response and multi-spectrum component power response

Fig. 3 Schematic diagram of (a) Infield test, (b) Outfield test