ACCELERATION AND DECELERATION IN CURVATURE INDUCED PHANTOM MODEL OF THE LATE AND FUTURE UNIVERSE, COSMIC COLLAPSE AS WELL AS ITS QUANTUM ESCAPE

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Abstract

Here, cosmology of the late and future universe is obtained from $f(R)$–gravity with non-linear curvature terms $R^2$ and $R^3$ ($R$ being the Ricci scalar curvature). It is different from $f(R)$–dark energy models [6] where non-linear curvature terms are taken as gravitational alternative of dark energy. In the present model, neither linear nor non-linear curvature terms are taken as dark energy. Rather, dark energy terms are induced by curvature terms in the Friedmann equation derived from $f(R)$–gravitational equations. It has advantage over $f(R)$–dark energy models in the sense that the present model satisfies WMAP results and expands as $\sim t^{2/3}$ during matter dominance. So, it does not have problems due to which $f(R)$–dark energy models are criticised in [7]. Curvature-induced dark energy, obtained here, mimics phantom. Different phases of this model, including acceleration and deceleration during phantom phase, are investigated here. It is found
that expansion of the universe will stop at the age $(3.87t_0 + 694.4)$ kyr ($t_0$ being the present age of the universe) and, after this epoch, it will contract and collapse by the time $(336.87t_0 + 694.4)$ kyr. Further, it is shown that universe will escape predicted collapse (obtained using classical mechanics) on making quantum gravity corrections relevant near collapse time due to extremely high energy density and large curvature analogous to the state of very early universe. Interestingly, cosmological constant is also induced here, which is extremely small in classical domain, but very high in the quantum domain.

**Key Words**: $f(R)$—gravity cosmology, phantom dark energy, acceleration and deceleration in phantom universe, contraction of the universe, cosmic collapse in future and quantum escape of collapse.

### 1. Introduction

Astrophysical observations, made at the turn of the last century [1, 2] show conclusive evidence for acceleration in the late universe, which is still a challenge for cosmologists. Theoretically, it is found that dark energy (DE) violating *strong energy condition* (SEC) or *weak energy condition* (WEC) is responsible for it. So, in the recent past, many DE models violating one of these conditions, were proposed to explain the *late cosmic acceleration* taking some exotic matter being the possible DE source. A comprehensive review of these models is available in [3]. As the exotic matter is not known, different scalar fields with special attention to quintessence, phantom, K-essence and tachyons, were tried upon to probe actual nature of the matter responsible for required DE [3]. Apart from, field-theoretic models, some fluid dynamical models were also proposed. Chaplygin gas and generalized Chaplygin gas
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models of DE are prominent among these, as it has super-symmetric generalization and negative pressure needed for violation of SEC or WEC[3, 4].

As no satisfactory model came up, it was thought that curvature, being the source of gravitation, could also be a source of DE. The Einstein-Hilbert term \( R/16\pi G \) (\( R \) being the scalar curvature and \( G \) being the gravitational constant) yields the geometrical component in Einstein’s gravitational equation, so it was realized to use non-linear terms of curvature replacing matter lagrangian in the Einstein’s gravitational action. Motivated by this idea, S. Capozziello et al. and S.M. Carroll et al. proposed non-linear terms of curvature \( R^{-n} \) with \( n > 0 \) as a possible source of DE [5]. Although this model explained late cosmic acceleration, it exhibited instability and failed to satisfy solar system constraints. Further, this idea was taken up by Nojiri and Odintsov and models for gravitational alternative of DE were proposed taking different forms of \( f(R) \) other than \( R^{-n} \). These improved models satisfied solar system constraints exhibiting late cosmic acceleration for small curvature and early inflation for large curvature. Thus, in \( f(R) – dark\ energy\ models \), non-linear curvature terms were considered as an alternative for DE [6, for detailed review]. Recently, Amendola et al. have criticized \( f(R) – dark\ energy\ models \) and have shown that these models with dominating powers of \( R \) for large or small \( R \) can not yield viable cosmology as results contradict the standard model and do not satisfy Wilkinson Microwave Anisotropy Probe (WMAP) observations, though these models pass solar system constraints and explain late acceleration [7]. Moreover, it is also shown that the most popular model with \( f(R) = R + \alpha R^m + \beta R^{-n} \) (\( m > 0, n > 0 \)) considered in [6] is unable
to produce matter in the late universe prior to the beginning of late acceleration [7].

The recent developments against $f(R)$–dark energy models indicate the need to think for some alternative approach to get DE from curvature. Such an approach has been proposed in [8, 9, 10, 11, 12] and it is adapted here too. This approach yields a cosmology emerging from $f(R)$–gravity satisfying WMAP results as well as results of the standard cosmological model. On the contrary, these results (WMAP results as well as results of the standard cosmological model) are not satisfied by $f(R)$–dark energy models as well as it is noted in [7]. There is a crucial difference between the cosmology from $f(R)$–gravity, being addressed here as well as in [8, 9, 10, 11, 12] and $f(R)$–dark energy models. In the latter case, non-linear terms of curvature are taken as dark energy lagrangian a priori. On the contrary, in the former case, neither linear nor non-linear term is considered as dark energy. Rather, in the present cosmological model from $f(R)$–gravity, DE is induced by linear (Einstein-Hilbert term) as well as non-linear terms $R^2$ and $R^{(2+r)}$ in the action ( $r$ being a real number such that $(2 + r) > 0$). In $f(R)$–dark energy models, dark energy terms depend on $f(R)$ terms and its derivative $F = df/dR$. In the former case, induced DE terms depend on the scale factor $a(t)$ of the homogeneous and flat of Friedmann - Robertson - Walker (FRW) model of the universe.

In what follows, $f(R)$–gravity with $R^2$ and $R^3$ is considered. It is found that curvature terms induce dark energy, dark matter and cosmological constant in the Friedmann equation (FE) for the late universe, derived from $f(R)$–gravitational equations. It is interesting to
see that curvature-induced dark energy, obtained here, mimics phantom with equation of state (EOS) parameter $w = -5/4$. Moreover, FE contains phantom DE term as $\rho_{\text{de}}[1 - \rho_{\text{de}}/2\lambda]$. The correction term $-\rho_{\text{de}}^2/2\lambda$ with $\lambda$ being the cosmic tension [10, 12] is analogous to such term in RS-II model based FE [13] as well as loop quantum gravity correction [14]. Like references [10, 12], here also, this term is obtained from $f(R)$-gravity. Here, cosmic tension $\lambda$ is evaluated to be $5.77\rho_{\text{cr}}^0$ with $\rho_{\text{cr}}^0$ being the present critical density of the universe. Further, it is shown that universe, derived by curvature-induced dark matter, decelerated up to time $0.59t_0$. At this epoch and small red-shift $z = 0.36$, transition from deceleration to acceleration took place. Interestingly, it is noted that as phantom energy density increases, effect of the term $-\rho_{\text{de}}^2/2\lambda$ gradually increases. As a result, it is found that universe will super-accelerate (expansion with high acceleration) during the period $0.59t_0 < t < 2.42t_0$, it will accelerate (expansion with low acceleration) during the period $2.42t_0 < t < 3.44t_0$ and, during the period $3.44t_0 < t < 3.87t_0$, universe will decelerate even during the phantom phase. Phantom-dominance will end when $\rho = 2\lambda = 11.54\rho_{\text{cr}}^0$ at time $t = 3.87t_0$ causing re-dominance of dark matter with the re-beginning of decelerated expansion. It is natural to think that, even during this phase with deceleration (derived by dark matter), phantom DE will grow with expansion causing $\rho_{\text{de}}[1 - \rho_{\text{de}}/2\lambda]$ negative due to $\rho_{\text{de}} > 2\lambda$ during re-dominance of dark matter. In the beginning of this phase, this term can be ignored, but its effect will grow causing expansion to stop at time $t_m = 3.87t_0 + 694.4 \text{ kyr}$. As a consequence, universe will retrace back and contract. Further, it is found that by the time $i_{\text{col}} = 336.87t_0 + 694.4 \text{ kyr}$, universe will collapse and energy density will
diverge. It is argued that this state of the future universe is analogous to the state of very early universe, where energy density is extremely large and curvature is very high. So, like early universe, it is reasonable to think that quantum gravity effects will be dominant near the collapse time. Motivated by this idea, quantum corrections are made in FE in the small interval of time around the time of collapse and it is found that cosmic collapse, predicted by classical mechanics, can be avoided. Earlier too, Quantum gravity corrections were done in the future phantom universe to avoid finite time singularities [15, 16, 17].

The paper is organized as follows.

In section 2, phantom-phase of the late and future universe is investigated. Transition from deceleration to acceleration is probed and cosmic tension is evaluated. Section 3 contains discussion on energy conditions during phantom era and its consequences. In section 4, re-dominance of dark matter, maximum expansion, contraction and possible cosmic collapse using the classical approach have been addressed. Avoidance of collapse, using quantum gravity effects, is demonstrated in sub-section 4(c). Important results are summarized in the last section.

Natural units ($k_B = \hbar = c = 1$) (where $k_B, \hbar, c$ have their usual meaning) are used here. GeV is used as a fundamental unit and we have $1\text{GeV}^{-1} = 6.58 \times 10^{-25} \text{sec}$.

2. Phantom phase of the late universe and future universe from $f(R)$-gravity

Here, the action for $f(R)$-gravity is taken as

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \alpha R^2 + \beta R^3 \right]. \quad (2.1)$$
where \( G = M_P^{-2} (M_P = 10^{19}\text{GeV} \) is the Planck mass), \( \alpha \) is a dimensionless coupling constant and \( \beta \) is a constant having dimension \((\text{mass})^{-2}\) (as \( R \) has mass dimension 2).

Action (1) yields field equations
\[
\frac{1}{16\pi G} (R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R) + \alpha (2\nabla_\mu \nabla_\nu R - 2g_{\mu\nu} \Box R - \frac{1}{2}g_{\mu\nu}R^2 + 2RR_{\mu\nu}) \\
+ \beta (3\nabla_\mu \nabla_\nu R^2 - 3g_{\mu\nu} \Box R^2 - \frac{1}{2}g_{\mu\nu}R^3 + 3R^2R_{\mu\nu}) = 0
\] (2.2a)
using the condition \( \delta S/\delta g_{\mu\nu} = 0 \). The operator \( \Box \) is defined as
\[
\Box = \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\mu} \left( \sqrt{-g} g^{\mu\nu} \frac{\partial}{\partial x^\nu} \right).
\] (2.2b)

Taking trace of (2.2a), it is obtained that
\[
- \frac{R}{16\pi G} - 6(\alpha + 3\beta R) \Box R - 18\beta \nabla^\mu R \nabla_\mu R + \beta R^3 = 0
\] (2.3)

In (2.3), \((\alpha + 3\beta R)\) emerges as a coefficient of \( \Box R \) due to presence of terms \( \alpha R^2 \) and \( \beta R^3 \) in the action (2.1). If \( \alpha = 0 \), effect of \( R^2 \) vanishes and effect of \( R^3 \) is switched off for \( \beta = 0 \). So, an effective scalar curvature \( \tilde{R} \) is defined as
\[
\gamma \tilde{R} = \alpha + 3\beta R,
\] (2.4)
where \( \gamma \) is a constant having dimension \((\text{mass})^{-2}\) being used for dimensional correction.

Connecting (2.3) and (2.4), it is obtained that
\[
- \Box \tilde{R} - \frac{1}{\tilde{R}} \nabla^\mu \tilde{R} \nabla_\mu \tilde{R} = \frac{1}{6\gamma} \left[ \frac{1}{16\pi G} - \frac{\alpha^2}{3\beta} \right] - \frac{1}{6\gamma} \left[ \frac{1}{16\pi G} - \frac{\alpha^2}{9\beta} \right] \frac{1}{\gamma \tilde{R}} \\
- \frac{\tilde{R}}{54\beta} [\gamma \tilde{R} - 3\alpha].
\] (2.5)

Experimental evidences [18] support spatially homogeneous flat model of the universe
\[
dS^2 = dt^2 - a^2(t)[dx^2 + dy^2 + dz^2]
\] (2.6)
with \( a(t) \) being the scale-factor.

In this space-time, (2.5) is obtained as

\[
-\dddot{R} - \frac{3}{a} \ddot{R} - \frac{\dot{R}^2}{R} = \frac{1}{6\gamma} \left[ \frac{1}{16\pi G} - \frac{\alpha^2}{3\beta} \right] - \frac{1}{6\gamma} \left[ \frac{1}{16\pi G} - \frac{\beta^2}{9\beta} \right] \frac{1}{\gamma R} - \frac{\dot{R}}{54\beta} [\gamma \dot{R} - 3\alpha] \tag{2.7}
\]

due to spatial homogeneity.

For \( a(t) \), being the power-law function of \( t \), \( \tilde{R} \sim a^{-n} \). For example, \( \tilde{R} \sim a^{-3} \) for matter-dominated model. So, there is no harm in taking

\[
\tilde{R} = \frac{A}{a^n}, \tag{2.8}
\]

where \( n > 0 \) is a real number and \( A \) is a constant with mass dimension 2.

Using (2.8) in (2.7), it is obtained that

\[
\frac{\dot{a}}{a} (\dot{a}) + (3 - 2n)(\dot{a})^2 = \frac{Ca^n}{nA} \left[ 1 - \frac{Da^n}{C} \right] - \frac{\gamma A}{54n\beta} \left[ \frac{1}{a^n} - \frac{3\alpha}{\gamma A} \right].
\]

In the late universe, \( a(t) \) is large, so this equation reduces to

\[
\frac{\dot{a}}{a} (\dot{a}) + (3 - 2n)(\dot{a})^2 = \frac{Ca^n}{nA} \left[ 1 - \frac{Da^n}{C} \right] + \frac{a_0}{18n\beta} \tag{2.9a}
\]

where

\[
C = \frac{1}{6\gamma} \left[ \frac{1}{16\pi G} - \frac{\alpha^2}{3\beta} \right] \tag{2.9b}
\]

and

\[
D = \frac{1}{6\gamma} \left[ \frac{1}{16\pi G} - \frac{\beta^2}{9\beta} \right]. \tag{2.9c}
\]

(2.9a) can be re-written as

\[
\ddot{a} + (2 - 2n)\frac{\dot{a}^2}{a} = \frac{Ca^n+1}{nA} \left[ 1 - \frac{Da^n}{C} \right] - \frac{A\alpha}{18n\beta}, \tag{2.10}
\]

which integrates to

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{E}{a^{6-4n}} + \frac{2C}{nA} \left[ \frac{a^n}{(6 - 3n)} - \frac{Da^{2n}}{C(6 - 2n)} \right] - \frac{3\alpha}{9n\beta(6 - 4n)}, \tag{2.11a}
\]

where \( E \) is an integration constant having dimension \((\text{mass})^2\).
(2.11a) is the modified Friedmann equation (MFE) giving cosmic dynamics. Terms, on r.h.s. of this equation, are imprints of curvature. The first term of these, proportional to \(a^{-(6-4n)}\) emerge spontaneously. It is interesting to see that this term (the first term on r.h.s. of (15a)) corresponds to matter density if \(n = 3/4\) i.e. for this value of \(n\), it reduces to \(Ea^{-3}\) and yields the density of non-baryonic matter being spontaneously induced by curvature. So, we recognize it as dark matter density.

Thus, for \(n = 3/4\), (2.11a) looks like

\[
\left(\frac{\dot{a}}{a}\right)^2 = \left[\frac{E}{a^3} + \frac{16\alpha}{729\beta}\right] + \frac{32Ca^{3/4}}{45A}\left[1 - \frac{5Da^{3/4}}{6C}\right].
\] (2.11b)

On r.h.s. of (11b), there are terms proportional to \(a^{3/4}\) and \(a^{3/2}\). If the density term \(\rho_{de} = 4Ca^{3/4}/15A\pi G\) is put in conservation equation

\[
\dot{\rho}_{de} + 3H(\rho_{de} + p_{de}) = 0
\] (2.11c)

with \(p_{de} = w\rho_{de}\), we obtain

\[
w = -\frac{5}{4} < -1.
\] (2.11d)

This result shows that \(\rho_{de} = 4Ca^{3/4}/15A\pi G\) behaves as phantom dark energy density being induced by \(f(R)\)– gravity.

Now, (2.11b) is re-written as

\[
H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left[\frac{8\pi G}{3}\rho_{dm} + \frac{16\alpha}{729\beta}\right] + \frac{32Ca^{3/4}}{45A}\left[1 - \frac{5Da^{3/4}}{6C}\right]
\] (2.11e)

with

\[
\rho_{dm} = \frac{3E}{8\pi Ga^3}.
\] (2.11f)

Using current value of \(\rho_{dm}\) as \(0.23\rho_{cr}^0\), (2.11f) yields

\[
\rho_{dm} = 0.23\rho_{cr}^0 \left(\frac{a_0}{a}\right)^3,
\] (2.12a)
where $a_0 = a(t_0)$, $3E/8\pi G = 0.23\rho^0_{\text{cr}}a_0^3$ and

$$\rho^0_{\text{cr}} = \frac{3H^2_0}{8\pi G}$$

with $H_0 = 100\text{km/Mpc sec} = 2.32 \times 10^{-42}\text{GeV h}$ being the current Hubble’s rate of expansion and $h = 0.68$. The present age of the universe is estimated to be $t_0 \simeq 13.7\text{Gyr} = 6.6 \times 10^{41}\text{GeV}^{-1}$ \cite{19}. So,

$$H^{-1}_0 = 0.96t_0. \quad (2.12b)$$

Further, $a_0$ is normalized as

$$a_0 = 1. \quad (2.12c)$$

Connecting (2.12a) and (2.12c), it is obtained

$$\rho_{\text{dm}} = \frac{0.23\rho^0_{\text{cr}}}{a_0^3} \quad (2.12d)$$

WMAP \cite{19} gives decoupling of matter from radiation at red-shift

$$Z_d = \frac{1}{a_d} - 1 = 1089. \quad (2.13)$$

So, it is supposed that dark matter begins to dominate cosmic dynamics when $a > a_d$.

Now, (2.11e) is re-written as

$$\left(\frac{\ddot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[\left\{\rho_{\text{dm}} + \frac{2\alpha}{243\beta\pi G}\right\} + \rho_{\text{de}}\left\{1 - \frac{\rho_{\text{de}}}{2\lambda}\right\}\right], \quad (2.14a)$$

where

$$\rho_{\text{de}} = \rho^0_{\text{de}}a^{3/4} \quad (2.14b)$$

with $\rho^0_{\text{de}} = 0.73\rho^0_{\text{cr}} = 4C/15A\pi G$ using $a_0 = 1$ and

$$\lambda = \frac{3C\rho^0_{\text{de}}}{5D}, \quad (2.14c)$$

where $C$ and $D$ are given by (2.9b) and (2.9c) respectively.
The Friedmann equation (FE) (2.14a) is obtained from $f(R)$-gravity. Comparing it with general relativity based FE

$$
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \cdot \rho + \frac{\Lambda}{3},
$$

it is obtained that the constant term in (2.14a) behaves as cosmological constant

$$
\Lambda = \frac{16\alpha}{243\beta} \quad (2.14d)
$$

with vacuum energy density

$$
\rho_\Lambda = \frac{\Lambda}{8\pi G} = \frac{2\alpha}{243\beta\pi G}. \quad (2.14e)
$$

It is interesting to see that (2.14c) contains a term $-(\rho_{de})^2/2\lambda$ analogous to brane-gravity correction to the Friedmann equation (FE) for negative brane-tension [13] and modifications in FE due to loop-quantum effects [14]. Here $\lambda$ is called *cosmic tension* [9, 10]. (2.14c) shows that *cosmic tension* $\lambda$ depends on coupling constants $\alpha$ and $\beta$ in the gravitational action (2.1). Moreover, positive cosmological constant, too, emerges from curvature.

From (2.12a) and (2.14b), it is found that $\rho_{dm} \sim a^{-3}$ and $\rho_{de} \sim a^{3/4}$. So $\rho_{dm}$ decreases and $\rho_{de}$ increases with expansion of the universe. So, it is natural to think for values of these to come closer and to be equal at a certain time $t_*$. At this particular time, we have

$$
0.23a_*^{-3} = 0.73a_*^{3/4}
$$

using (2.12a), (2.12c) and (2.14b) as well as $a_* = a(t_*)$. This equation yields

$$
a_* = \left(\frac{23}{73}\right)^{4/15}. \quad (2.15)
$$
It shows that, for $a < a_\ast = (23/73)^{4/15}$, $\rho_{\text{dm}} > \rho_{\text{de}} > \rho_{\text{de}}^2$. So, (2.14a) is approximated as
\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left[ \rho_{\text{dm}} + \frac{2\alpha}{243\beta \pi G} \right].
\] (2.16)

Connecting (2.12a) and (2.16), it is obtained that
\[
\left(\frac{\dot{a}}{a}\right)^2 = \frac{0.23H^2_0}{a^3} \left[ 1 + \frac{\alpha}{B} \right]
\] (2.17a)
with
\[B = \frac{\rho_{\text{dm}}^0}{\rho_\Lambda} = \frac{10.48\beta H^2_0}{\alpha},\] (2.17b)
obtained using (2.14e).

(2.17a) integrates to
\[a = B^{1/2} \sinh^{2/3} \left[ \frac{3H_0 \sqrt{0.23}}{2B^{3/2}} (t - t_d) + \sinh^{-1} (a_d/B^{1/3})^{3/2} \right]\] (2.18a)
This result is not consistent with the scale factor, obtained in standard model of cosmology during matter dominance. So, to have a viable cosmology, it needs to be approximated as
\[a = a_d \left[ 1 + \frac{3H_0 \sqrt{0.23}}{2a_d^{3/2}} (t - t_d) \right]^{2/3},\] (2.18b)
which is possible till
\[\frac{3H_0 \sqrt{0.23}}{2B^{3/2}} (t - t_d) + \sinh^{-1} (a_d/B)^{3/2} \lesssim 1\] (2.18c)
as $\sinh 1 = 1.18 \simeq 1$. Here $a_d$ is given by (2.13), which is the scale factor at time $t = t_d = 386$ kyr $= 2.8 \times 10^{-5}t_0$. Connecting (2.13), (2.17b), (2.18a) and (2.18c), it is obtained that
\[\frac{\alpha}{\beta H^2_0} \gtrsim 10^{-6}.\] (2.18d)

Connecting (2.16e) and $\alpha/\beta H^2_0 \simeq 10^{-6}$ from (2.18d), it is evaluated that
\[\rho_\Lambda = 2.19 \times 10^{-8} \rho_{\text{cr}}^0 = 5.48 \times 10^{-55}\text{GeV}^4.\] (2.18e)
The approximated form (2.17a) is obtained when \( a \ll a^* \), but, as discussed above, \( \rho_{\text{dm}} \simeq \rho_{\text{de}} \) when \( a \lesssim a_* \). So, in the narrow strip around \( a = a_* \) for \( a < a_* \), (2.14a) is approximated as
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \left[ \frac{2\rho_{\text{dm}} + \frac{2\alpha}{243\beta\pi G}}{2\rho_{\text{de}}} \right] \tag{2.19}
\]
using \( \rho_{\text{dm}} \simeq \rho_{\text{de}} > \rho_{\text{de}}^2 \) in (2.14a).

Connecting (2.12a) and (2.19), it is obtained that
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{0.46H_0^2}{a^3} \left[ 1 + \left( \frac{a}{a_*} \right)^3 \right] \tag{2.20}
\]
with \( a_*^3 = \alpha/167.67\beta H_0^2 \). It is because at \( a = a_* \), mater-dominated phase ends and dark energy dominance begins.

(2.20) integrates to
\[
a^{3/2} = a_*^{3/2} \sinh \left[ \frac{3H_0\sqrt{0.46}}{2a_*^{3/2}} (t - t_d) + \sinh^{-1}(a_d/a_*)^{3/2} \right],
\]
being approximated as
\[
a = a_d \left[ 1 + \frac{3H_0\sqrt{0.46}}{2a_d^{3/2}} (t - t_d) \right]^{2/3} \tag{2.21}
\]
like the above case.

Using (2.21), it is obtained that
\[
a_*^{3/2} \simeq a_d^{3/2} + \frac{3H_0\sqrt{0.46}}{2} (t_* - t_d).
\]
This result yields
\[
t_* \simeq 0.59t_0 \tag{2.22}
\]
using values of \( a_* \) (from (2.15)), \( H_0^{-1} \) (from (2.12b)) and \( t_d \).

It is discussed above that for \( a > a_* \), \( \rho_{\text{dm}} < \rho_{\text{de}} \). In this case, (2.14a) is approximated as
\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho_{\text{de}} \left\{ 1 - \frac{\rho_{\text{dm}}}{2\lambda} \right\}. \tag{2.23}
\]
Thus, in the late universe, a phantom model is obtained from curvature without using any source of exotic matter. But this model contains a correction term $-4\pi G\rho_{de}^2/3\lambda$ due to curvature-induced cosmic tension $\lambda$ being evaluated below.

Connecting (14b) and (2.23), it is obtained that
\[ H^2 = \left(\frac{\dot{a}}{a}\right)^2 = 0.73H_0^2a^{3/2}\left[a^{-3/4} - \frac{0.73\rho_{cr}^0}{2\lambda}\right]. \] (2.24)

(2.24) integrates to
\[ a(t) = \left[\frac{0.73\rho_{cr}^0}{2\lambda} + \left\{\sqrt{1.26 - \frac{0.73\rho_{cr}^0}{2\lambda}} \right\} - \frac{3}{8}H_0\sqrt{0.73(t - t^*)}\right]^{-4/3}. \] (2.25a)
as $a_*^{-3/4} = 1.26$. (25a) shows that phantom model, obtained here, is singularity-free.

From (2.25a), it is obtained that
\[ \ddot{a} = 0.27H_0^2a^{3/2}\left[\frac{1.7\rho_{cr}^0}{\lambda} - \frac{11}{3}a^{-3/4}\right]. \] (2.25b)
This shows $\ddot{a} > 0$, when
\[ \frac{1.7\rho_{cr}^0}{\lambda} > \frac{11}{3}a^{-3/4}. \] (2.25c)

Further, (2.25a) yields
\[ 1 = a_0 = \left[\frac{0.73\rho_{cr}^0}{2\lambda_{ph}} + \left\{\sqrt{1.26 - \frac{0.73\rho_{cr}^0}{2\lambda_{ph}}} \right\}^{-4/3} - \frac{3}{8}H_0\sqrt{0.73(t_0 - t^*)}\right]. \] (2.26)

Using (2.22) for $t^*$ in (2.26), $\lambda$ is evaluated as
\[ \lambda = 5.77\rho_{cr}^0. \] (2.28)

(2.25a) exhibits accelerating universe when $t > t_*$. Thus, a transition from declaration to acceleration takes place at
\[ t = t_* = 0.59t_0 \] (2.29a)
and red-shift

\[ z_\ast = \frac{1}{a_\ast} - 1 = \left( \frac{73}{23} \right)^{4/15} - 1 = 0.36, \] (2.29b)

which is within the range 0.33 \leq Z_\ast \leq 0.59 given by 16 Type supernova observations [2]. (2.25a) shows that universe expands till \( \rho_{de} \) becomes equal to 2\( \lambda \) as it grows with expansion. It happens till \( a(t) \) increases to \( a_{pe} \) satisfying

\[ a_{pe}^{3/4} = \frac{2\lambda}{0.73\rho_{cr}}. \] (2.30)

Thus, expansion (2.25a) stops at time

\[ t_{pe} = t_\ast + \frac{8}{3H_0\sqrt{0.73}} \sqrt{1.26 - \frac{0.73\rho_{cr}}{2\lambda}}. \] (2.31)

3. Cosmic energy conditions as well as acceleration and deceleration during phantom era

In what follows, it is found that correction term, \(-4\pi G \rho_{de}^2/3\lambda\) due to curvature-induced cosmic tension \( \lambda \) in (23), change the bahviour of the phantom model drastically.

From (2.11c) and (2.23), it is obtained that

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left[ 3(\rho + p) \left[ 1 - \frac{\rho}{\lambda} \right] - 2\rho \left[ 1 - \frac{\rho}{2\lambda} \right] \right], \] (3.1)

The correction term, in this equation, is caused due to curvature-induced cosmic tension \( \lambda \) in (2.23). This type of equation was obtained earlier in [16, 17] in the context of RS-II model in brane-gravity.

In the GR-based theory, (3.1) looks like

\[ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3} [\rho + 3P]. \] (3.2)

Comparing (2.5) and (2.6), the effective pressure density \( P \) is given as

\[ \rho + 3P = 3(\rho + p) \left[ 1 - \frac{\rho}{\lambda} \right] - 2\rho \left[ 1 - \frac{\rho}{2\lambda} \right]. \] (3.3)
Using (2.11d), (3.3) yields the effective pressure density in the curvature induced phantom model as

\[ P = -\frac{5}{4}\rho + \frac{7\rho^2}{12\lambda}. \]  

(3.4)

(3.4) yields

\[ \rho + P = -\frac{\rho}{4} + \frac{7\rho^2}{12\lambda}. \]  

(3.5)

This equation shows that WEC is violated when \( \rho < 3/7\lambda = 2.47\rho_{\text{cr}}^0 \) with \( \lambda \) given by (2.28). Moreover, \( \rho + P = 0 \) for \( \rho = 2.47\rho_{\text{cr}}^0 \) and \( \rho + P > 0 \) for \( \rho > 3/7\lambda = 2.47\rho_{\text{cr}}^0 \).

From (2.14b), it is found that energy density \( \rho \) for phantom fluid increases with increasing scale factor \( a(t) \). It is interesting to note from (3.5) that curvature-induced phantom fluid, obtained here, behaves effectively as phantom violating WEC till \( \rho < 2.47\rho_{\text{cr}}^0 \), but phantom characteristic to violate WEC is suppressed by cosmic tension when \( \rho \) increases and obeys the inequality \( \rho > 2.47\rho_{\text{cr}}^0 \).

Further, using (2.11d) in (3.3), it is also obtained that

\[ \rho + 3P = -\frac{11\rho}{4\lambda} - \frac{7\rho^2}{4\lambda}. \]  

(3.6)

Connecting (2.28) and (3.3), it is obtained that SEC is violated when \( \rho < 11\lambda/7 = 9.07\rho_{\text{cr}}^0 \). Also, it is found that \( \rho + 3P = 0 \) for \( \rho = 9.07\rho_{\text{cr}}^0 \) and \( \rho + 3P > 0 \) for \( \rho > 9.07\rho_{\text{cr}}^0 \).

Thus, it is obtained that (i) WEC is violated for \( \rho < 2.47\rho_{\text{cr}}^0 \), (ii) for \( 2.47\rho_{\text{cr}}^0 \leq \rho < 9.07\rho_{\text{cr}}^0 \) WEC is not violated, but SEC is violated and (iii) for \( \rho > 9.07\rho_{\text{cr}}^0 \) neither of the two conditions is violated. Also it is interesting to note that these corrections cause effective phantom divide at

\[ \rho = 2.47\rho_{\text{cr}}^0. \]  

(3.7)
Moreover, these results suggest that a transition from violation of SEC to non-violation of SEC will take place at

\[ \rho = 9.07 \rho_{cr}^0. \]  

(3.8)

Also, universe will super-accelerate till \( \rho_* < \rho < 2.47 \rho_{cr}^0 \), accelerate when \( 2.47 \rho_{cr}^0 < \rho < 9.07 \rho_{cr}^0 \) and decelerate when \( 9.07 \rho_{cr}^0 < \rho < 11.54 \rho_{cr}^0 \) as expansion of phantom phase of the universe will stop at \( \rho = 11.54 \rho_{cr}^0 \).

These results are also supported by (2.25b) as (2.25b) and (2.14b) yield

\[ \ddot{a} = 0.27 H_0^2 a^{5/2} \left[ 0.29 - \frac{2.68 \rho_{cr}^0}{\rho} \right]. \]  

(3.9)

Connecting (2.14b), (2.25a), (2.28) and (2.29a), it is obtained that

\[ \rho = 0.73 \rho_{cr}^0 [0.06 + \{1.094 - 0.32 H_0 (t - 0.59 t_0)\}^2]^{-1}. \]  

(3.10)

(3.7) and (3.10) yield that effective phantom divide is obtained at time

\[ t \simeq 2.42 t_0. \]  

(3.11)

(3.8) and (3.10) yield that transition time for violation of SEC to non-violation of SEC will take place at

\[ t \simeq 3.44 t_0. \]  

(3.12)

These results imply super-acceleration during the time interval \( 0.59 t_0 < t < 2.42 t_0 \), acceleration during the time interval \( 2.42 t_0 < t < 3.44 t_0 \) and deceleration during the time interval \( 3.44 t_0 < t < 3.87 t_0 \). Expansion, driven by phantom, will stop at time \( t = 3.87 t_0 \) as \( \rho_{de} \) will acquire the value \( 2 \lambda \) by this time.

When \( t > 3.87 t_0 \), deceleration, driven by matter, will resume and Freidmann equation reduces to (2.17).
4. Cosmic collapse using classical approach and its quantum escape

4(a). Re-appearance of matter-dominance and cosmic collapse

As mentioned above, dark energy terms are switched off, in (2.14a), at $\rho_{de} = 2\lambda = 11.54\rho_{cr}^0$ (obtained from (2.28)), $a = a_{pe}$ given by (2.30) when $t = t_{pe} = 3.87t_0$ given by (2.31). So, for $t > t_{pe} = 3.87t_0$. (2.14a) will look like

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left\{ \rho_{dm}^{pe} \left(\frac{a_{pe}}{a}\right)^3 \left\{ 1 + \tilde{\Lambda}^{-1} \left(\frac{a}{a_{pe}}\right)^3 \right\} \right\}, \quad (4.1a)$$

where

$$\rho_{dm}^{pe} = 3.68 \times 10^{-6} \rho_{cr}^0, \quad (4.1b)$$

$$\tilde{\Lambda} = \frac{\rho_{dm}^{pe}}{\rho_{cr}^{pe}} = \frac{243\pi G\beta \rho_{dm}^{pe}}{2\alpha} = 168, \quad (4.1c)$$

which is evaluated using (2.18d), (2.30) and (4.1b). and

$$\rho_{pe}^{pe} \left(\frac{a}{a_{pe}}\right)^{3/4} \left\{ 1 - \left(\frac{a}{a_{pe}}\right)^{3/4} \right\} = 0 \quad (4.1d)$$

as

$$\rho_{de}^{pe} = 2\lambda = 11.54\rho_{cr}^0. \quad (4.1e)$$

(4.1a) integrates to

$$a(t) = \tilde{\Lambda}^{1/3} a_{pe} sinh^{2/3} \left[ sinh^{-1} \tilde{\Lambda}^{-1/2} + 2.22 \times 10^{-4} H_0(t - t_{pe}) \right] \quad (4.2a)$$

yielding

$$\frac{\ddot{a}}{a} = -\frac{2}{9} [2.22 \times 10^{-4} H_0^2] [cosech^2 \theta(t) - 2] \quad (4.2b)$$

with $\theta(t) = sinh^{-1} \tilde{\Lambda}^{-1/2} + 2.22 \times 10^{-4} H_0(t - t_{pe})$. (4.2b) shows decelerated expansion caused by matter dominance till

$$sinh \theta(t) < 1/\sqrt{2} = 0.707. \quad (4.2c)$$

The result (4.2a) is obtained when expansion is driven by the term

$$\rho_{dm}^{pe} \left(\frac{a_{pe}}{a}\right)^3 \left\{ 1 + \tilde{\Lambda}^{-1} \left(\frac{a}{a_{pe}}\right)^3 \right\}$$
Moreover, though at $a = a_{pe}$,

$$
\rho^{pe}_{de} \left( \frac{a}{a_{pe}} \right)^{3/4} \left\{ 1 - \left( \frac{a}{a_{pe}} \right)^{3/4} \right\}
$$

vanishes, it will be negative for $a > a_{pe}$. So for $a > a_{pe}$, Friedmann equation (2.14a) is obtained as

$$
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi G}{3} \left[ \{ \rho_{dm}^{pe} \left( \frac{a_{pe}}{a} \right) \} \left\{ 1 + \bar{\Lambda}^{-1} \left( \frac{a}{a_{pe}} \right)^3 \right\} - \rho_{de}^{pe} \left( \frac{a}{a_{pe}} \right)^{3/4} \left\{ \left( \frac{a}{a_{pe}} \right)^{3/4} - 1 \right\} \right].
$$

(4.3)

Obviously, the negative terms in (4.3) will try to stop expansion and, on sufficient growth of $a(t)$ up to $a_m$, expansion will reach its maximum such that $\dot{a} = 0$ and $a_m$ satisfies the equation

$$
\rho^{pe}_{dm} \left\{ \left( \frac{a_{pe}}{a_m} \right)^3 + \bar{\Lambda}^{-1} \right\} = 2 \lambda \left( \frac{a_m}{a_{pe}} \right)^{3/4} \left\{ \left( \frac{a_m}{a_{pe}} \right)^{3/4} - 1 \right\}
$$

(4.4a)

being approximated as

$$
\rho^{pe}_{dm} \bar{\Lambda}^{-1} \approx 2 \lambda \left( \frac{a_{pe}}{a_{dm}} \right)^{3/4} \left\{ \left( \frac{a_{pe}}{a_{dm}} \right)^{3/4} - 1 \right\}.
$$

(4.4b)

(4.4b) yields the solution

$$
\left( \frac{a_m}{a_{pe}} \right) = \left\{ \frac{1}{2} \left[ 1 + \sqrt{1 + 2 \rho^{pe}_{dm}/\lambda} \right] \right\}^{4/3} = 1 + 4.25 \times 10^{-7}.
$$

(4.4c)

The negative sign ($-$) is ignored here as it yields $a_m < a_{pe}$, which is not possible. Using (4.4c), it is obtained that

$$
\rho_{dm}^m = \rho_{dm}^{pe} \left[ 1 - 1.28 \times 10^{-6} \right].
$$

(4.4d)

Using (4.4c) in (4.2a), It is obtained that the time $t = t_m$ corresponding to $a = a_m$ is given by

$$
\left[ sinh^{-1} \bar{\Lambda}^{-1/2} + 2.22 \times 10^{-4} H_0 (t_m - t_{pe}) \right]
$$

$$
= sinh^{-1} \left[ \frac{1}{4 \sqrt{\bar{\Lambda}}} \left[ 1 + \sqrt{1 + 2 \rho^{pe}_{dm}/\lambda} \right]^2 \right]
$$

$$
= sinh^{-1} (0.18123) = 0.18123.
$$
(4.4e) confirms deceleration during time period $t_{pe} < t < t_m$ as it satisfies the condition (4.2c).

So, for $t_{pe} < t < t_m$, (4.2a) is obtained as

$$a(t) = a_{pe} \left[ 1 + 2.22 \times 10^{-4} H_0 \tilde{\Lambda}^{1/3} a_{pe}^{-3/2}(t - t_{pe}) \right]^{2/3}. \quad (4.5)$$

Connecting (2.12b), (4.1c) and (4.4e), $t_m$ is evaluated as

$$t_m - t_{pe} = 5.32 \times 10^{-4} t_0 = 3.51 \times 10^{38} \text{GeV}^{-1} = 694.4 \text{kyr}. \quad (4.6)$$

Further, it is interesting to note that the curve $a = a(t)$ will be continuous at $t = t_m$, but the direction of tangent to this curve (pointed at $a = a_m$) will change because it will attain its maximum at $t = t_m$ yielding $\dot{a} < 0$ for $t > t_m$, which used to be positive for $t < t_m$. It means that universe will retrace back at $t = t_m$ and will begin to contract. During the contraction phase, term proportional to $a^{-3}$ will dominate over terms proportional to $a^{3/4}$ and (4.3) will yield

$$\frac{\dot{a}}{a} \simeq -\left[ \frac{8\pi G}{3} \rho_m \left( \frac{a_m}{a} \right)^3 \{ 1 + \tilde{\Lambda}^{-1} \left( \frac{a}{a_m} \right)^3 \} \right]^{1/2}. \quad (4.7)$$

On integrating (4.7), it is obtained that

$$a(t) = \tilde{\Lambda}^{1/3} a_m \left\{ \sinh \left[ 2.22 \times 10^{-4} H_0 (t_{col} - t) \right] \right\}^{2/3}, \quad (4.8a)$$

where

$$t_{col} = t_m + \left[ 2.22 \times 10^{-4} H_0 \right]^{-1} \sinh^{-1} \tilde{\Lambda}^{-1/2} = t_m + 3.33 \times 10^2 t_0. \quad (4.8b)$$

(4.8a) shows that $a(t) = 0$ at $t = t_{col}$ and energy density of the universe

$$\rho = \left[ \rho_{dm} \left( \frac{a_m}{a} \right)^3 + \rho_{de} \left( \frac{a}{a_{pe}} \right)^{3/4} \right] \simeq \rho_{dm} \left( \frac{a_m}{a} \right)^3 \quad (4.9)$$
will diverge. It means that universe will collapse at $t = t_{\text{col}}$.

This result, suggested by the classical mechanics, is unphysical due to divergence of cosmic energy density. So, to have a viable physics around the epoch $t = t_{\text{col}}$, the diverging component of the density of the universe, which is proportional to $a^{-3}$, needs to be finite. As this unphysical situation is predicted by classical mechanics, we have no other alternative other than to resort to quantum gravity. In what follows, we proceed in this direction, which is justified due to extremely high energy density and large curvature in the small interval of time near this particular epoch $t_{\text{col}}$. This situation is analogous to very early universe, where quantum gravity effects are dominant. Earlier also, quantum gravity corrections were made in the equations of future universe under such circumstances to avoid finite time singularities [15, 16, 17].

4(b). Further analysis of contracting universe through classical theory approach and conformal scalars

(4.9) indicates that energy density of the contracting future universe will increase very high with increasing time. As a consequence, stellar equilibrium will break and, on sufficient rise in cosmic energy density, all matter in the universe, including celestial bodies, will get smashed to highly-relativistic baryonic as well as non-baryonic elementary particles. Moreover, binding energy of celestial objects will be released as radiation with high temperature being in the high energy states (though, here, universe is driven by dark energy and dark matter as these constitute dominant components of the cosmic energy density and emerge from gravitational sector on which model is based, but presence of non-dominant baryonic component is not ignored). Energy
distribution of these elementary particles (bosons and fermions) is given by
\[ \rho_r = \rho_b + \rho_f = \frac{\pi^2}{30} \left[ g_b + \frac{7}{8} g_f \right] T^4 \] (4.10)
in natural units used in this paper. Here \( \rho_b, \rho_f \) is the energy density and \( g_b, g_f \) is the helicity for bosons (fermions). \( T \) is the temperature. Moreover, pressure density for these particles, is obtained as
\[ p_r = p_b + p_f = \frac{1}{3} \left[ \rho_b + \rho_f \right] = \frac{1}{3} \rho. \] (4.11)

It shows that, on growth of energy density in the universe upto sufficiently high level, energy distribution will also change causing a change in the EOS of dominating fluid from \( p = 0 \) (for dark matter addressed in 4(a)) to EOS given by (4.11). Incorporating this change, cosmic energy density is obtained from conservation equation
\[ \dot{\rho}_r + 3H(\rho_r + p_r) = 0 \]
as
\[ \rho_r = \rho_{ch} \left( \frac{a_{ch}}{a} \right)^4, \] (4.12)
where \( \rho_{ch} \) and \( a_{ch} \) are energy density and scale factor at the epoch \( t = t_{ch} \) (when highly relativistic particles and radiation are produced due to smash of cosmic matter at the epoch).

EOS \( \rho_r = 3p_r \) for an ideal fluid is obtained when trace of the energy-momentum tensor components vanish. At the classical level, this is true for conformal scalars \( \phi \) also. So, it is reasonable to think of existence of conformal scalars \( \phi \) too along with other fields representing bosons and fermions giving energy density like (4.12). The action for conformal \( \phi(x,t) \) is given as
\[ S_\phi = \int d^4x \sqrt{-g} \frac{1}{2} \left[ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{6} R \phi^2 \right]. \] (4.13)
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According to above arguments, for \( t \geq t_{\text{ch}} \), cosmic matter will break to elementary particles having different energy distribution (4.12), so energy distribution (4.9) will be replaced by (4.12) in FE as source of \( \rho_{\text{dm}} \sim a^{-3} \) will vanish. So, for \( t \geq t_{\text{ch}} \), (4.7) and (4.8a) will not be valid. Now, in this stage, (4.3) is modified as

\[
\left( \frac{\dot{a}}{a} \right)^2 \simeq \left[ \frac{8\pi G}{3} \rho_{\text{ch}} \left( \frac{a_{\text{ch}}}{a} \right)^4 \left\{ 1 + \tilde{\Lambda}_{\text{ch}}^{-1} \left( \frac{a}{a_{\text{ch}}} \right)^4 \right\} \right].
\]

(4.14a)

with

\[
\tilde{\Lambda}_{\text{ch}} = \rho_{\text{ch}} / \rho_{\Lambda}.
\]

(4.14b)

(4.14a) yields

\[
\frac{\dot{a}}{a} \simeq - \left[ \frac{8\pi G}{3} \rho_{\text{ch}} \left( \frac{a_{\text{ch}}}{a} \right)^4 \left\{ 1 + \tilde{\Lambda}_{\text{ch}}^{-1} \left( \frac{a}{a_{\text{ch}}} \right)^4 \right\} \right]^{1/2}
\]

(4.14c)

replacing (4.7).

On integrating (4.14c), it is obtained that

\[
a(t) = \tilde{\Lambda}_{\text{ch}}^{1/2} a_{\text{ch}} \left\{ \sinh \left[ \sqrt{\frac{8\pi G \rho_{\text{ch}}}{3}} (t_{\text{ch}} - t) \right] \right\}^{1/2},
\]

(4.15a)

where

\[
t_{\text{ch}} = t_{\text{ch}} + \left[ \sqrt{\frac{3}{32\pi G \rho_{\text{ch}}}} \right] \sinh^{-1} \tilde{\Lambda}_{\text{ch}}^{-1/2}.
\]

(4.15b)

Like (4.8a), (4.12) and (4.15a) also exhibits cosmic collapse in future as \( a(t) \to 0 \) and \( \rho \to \infty \) when \( t \to t_{\text{ch}}^{\text{col}} \). Thus, in the changed circumstances too, classical theory predicts cosmic collapse in future.

All these results indicate that contracting phase of the future universe will undergo a revival of states of the early universe, where elementary particles in thermal equilibrium will dominate. According to the standard model of cosmology, thermal equilibrium of these particles are maintained at and above \( 10^{-3} \text{GeV} \). So, using it in (4.10), it is obtained that

\[
\rho_{\nu}^{\text{ch}} = \frac{11\pi^2}{60} T^4 = 1.8 \times 10^{-12} \text{GeV}^4
\]

(4.15c)
4(c). Quantum escape of collapse

In what follows, a possibility to escape cosmic collapse, obtained by classical approach, is probed using quantum corrections.

Quantum field theory suggests that anomalous terms arise in energy-momentum tensor components of conformal scalar $\phi$ [16, 17, 20] and its trace does not vanish contrary to the results obtained through the classical approach (as mentioned above, trace of the energy-momentum tensor vanishes for conformal scalars according to the classical theory). These terms are advantageous being free from ultra-violet and infra-red divergences with respect to energy modes. So, quantum gravity yields anomalous terms as renormalized energy density $\rho_A$ and pressure density $p_A$ as

$$\rho_A = \frac{N}{180(4\pi)^2}[3H^4 + 6H\dot{H} + 18H^2\dot{H} - 3\dot{H}^2]$$

and

$$p_A = -\frac{N}{360(4\pi)^2}(6H^4 + 8H^2\dot{H}) - \frac{N}{180(4\pi)^2}[2\ddot{H} + 12H\dot{H} + 18H^2\dot{H} + 9\dot{H}^2],$$

with finite positive integers $N$ being the number of conformal scalars.

On making quantum corrections, (4.14a) is obtained as

$$\frac{3}{8\pi G}H^2 = (\rho_r + \rho_A) + \rho^q_\Lambda$$

using (4.14b) and $\rho^q_\Lambda$ as $\rho_A$ after quantum corrections.

Due to correction terms in (4.17), $a(t)$ (being the solution of this equation) will be different from (4.8a) or (4.15a). But, the required solution is not possible unless $\rho_A$ is obtained as a suitable function of $a(t)$. Under these circumstances, there is an alternative approach to assume a solution and find conditions to make this solution capable to yield physically viable results and satisfy (4.17). (4.8a) and (4.15a)
show that divergence is caused by dominating component of cosmic energy density when $a \to 0$ as $t \to t_{col}^{ch}$. Now, if quantum corrections remove this divergence, $a(t)$ should be finite at $t = t_{col}^{ch}$. In cosmology, mostly $a(t)$ is found either as a power-law solution or as an exponential solution of FE. According to investigations here, $t = t_{col}^{rmch}$ is the epoch where this universe is expected to end. So, if $a(t) \sim (t_{col}^{ch} - t)^n$ for the contracting universe for $t > t_{ch}$, divergence cannot be removed. It can be checked from (4.16a) that $\rho_A$ itself will diverge for this form of $a(t)$. So, these arguments suggest to choose an exponential form of $a(t)$ during the quantum regime as it is possible for this form to yield finite value $a_{col} = a(t_{col}^{ch})$.

With this motivation, for $t > t_c$, solution of (4.17) is taken as

$$a^4(t) = a_{col} \exp\left[\{D(t_{col}^{ch} - t)\} + C_1\{D(t_{col}^{ch} - t)\}^\gamma\right] \quad (4.18)$$

with $\gamma$ being a positive real number, $C$ being an arbitrary dimensionless constant and $D$ being a constant of mass dimension used for dimensional correction. Here $t = t_c$ is the epoch near $t = t_{col}^{rmch}$, when quantum gravity effects begin to dominate. In what follows, it is shown that (4.18) is physically valid satisfying (4.17) and numerical values $C_1, D, \gamma$ and $a_{col}$ are determined.

From (4.18), it is obtained that

$$H = -D - \gamma C_1 D [D(t_{col}^{ch} - t)]^{(\gamma - 1)},$$
$$\dot{H} = \gamma(\gamma - 1) C_1 D^2 [D(t_{col}^{ch} - t)]^{(\gamma - 2)},$$
$$\ddot{H} = -\gamma(\gamma - 1)(\gamma - 2) C_1 D^3 [D(t_{col}^{ch} - t)]^{(\gamma - 3)},$$
$$\dddot{H} = \gamma(\gamma - 1)(\gamma - 2)(\gamma - 3) C_1 D^4 [D(t_{col}^{ch} - t)]^{(\gamma - 4)}. \quad (4.19a,b,c,d)$$

From (4.16a) and (4.16b), we find that terms containing $H$ and $\dot{H}$ are dominant in $\rho_A$ and $p_A$. So, minimum possible value for $\gamma$ is taken
as
\[ \gamma = 2, \]  
(4.20)
because \( H \) and \( \dot{H} \) do not vanish even at \( t = t_{\text{col}}^{\text{ch}} \) for this value of \( \gamma \). But
\[ \ddot{H} = 0 = \dddot{H}. \]  
(4.21a, b)
Also, at \( t = t_{\text{col}}^{\text{ch}} \)
\[ H = -D \quad \text{and} \quad \dot{H} = 2C_1D^2, \]  
(4.22a, b)
which is evaluated using
\[ \lim_{\epsilon \to 0}[D(t_{\text{col}}^{\text{ch}} - [t_{\text{col}}^{\text{ch}} - \epsilon])]^{(\gamma-2)} = \lim_{\epsilon \to 0}[D(t_{\text{col}}^{\text{ch}} - [t_{\text{col}}^{\text{ch}} + \epsilon])]^{(\gamma-2)} = 1 \]  
(4.23)
for \( \gamma = 2 \).

Now, it is important to check whether \( a^q(t) \), given by (4.18), satisfies (4.17) with \( \rho_A \) from (4.16a). Connecting (4.16a), (4.17), (4.18) and (4.19a)-(4.19d), it is obtained that
\[ \frac{3}{8\pi}M_P^2D^2[1 + 4C_1 D(t_{\text{col}}^{\text{ch}} - t) + 4C_1^2 D(t_{\text{col}}^{\text{ch}} - t)]^2 = \rho_{dm}[1 - 4\{D(t_{\text{col}}^{\text{ch}} - t)\}] + 4(2 + C_1)\{D(t_{\text{col}}^{\text{ch}} - t)\}^2 + \rho_A + \frac{ND^4}{60(4\pi)^2}[1 - 4C_1^2] + 8C_1\{D(t_{\text{col}}^{\text{ch}} - t)\} + 24C_1^2\{D(t_{\text{col}}^{\text{ch}} - t)\}^2 \]  
(4.24)
approximating up to second order only as \( |D(t_{\text{col}}^{\text{ch}} - t)| < 1 \), because, according to above arguments, quantum gravity is effective near the epoch \( t = t_{\text{col}}^{\text{ch}} \). Comparing constant terms and coefficients of \( D(t_{\text{col}}^{\text{ch}} - t) \) as well as \( D(t_{\text{col}}^{\text{ch}} - t)^2 \) in (4.24), it is obtained that
\[ \frac{3}{8\pi}M_P^2D^2 = \rho_r|_{t=t_{\text{col}}^{\text{ch}}} + \rho_A + \frac{ND^4}{60(4\pi)^2}(1 - 4C_1^2), \]
\[ \frac{3}{2\pi}C_1M_P^2D^2 = -4\rho_r|_{t=t_{\text{col}}^{\text{ch}}} + \frac{8ND^4}{60(4\pi)^2}C_1, \]
\[ \frac{3}{2\pi}C_1^2M_P^2D^2 = 4(2 + C_1)\rho_r|_{t=t_{\text{col}}^{\text{ch}}} + \frac{24NM_P^4}{60(4\pi)^2}C_1^2. \]
Incorporating (4.16a) and (4.25a,b,c) in (4.18), it is obtained that
\[
\rho_r + \rho_A^2 = \frac{3}{8\pi G} H^2 - \rho_A \\
= \frac{3}{8\pi} M_P^2 D^2 + \frac{ND^4}{60(4\pi)^2} (1 - 4C_1^2). 
\]
(4.25d)
yielding \((\rho_r + \rho_A^2)\) finite at \(t = t_{\text{col}}^{\text{ch}}\).

Planck scale is the fundamental scale. It suggests the largest energy mass scale as Planck mass \(M_P\). At this scale, energy density is obtained \(\sim 10^{75}\) GeV\(^4\). As, at \(t = t_{\text{col}}^{\text{ch}}\), finite \(\rho_r\) is obtained through quantum corrections. As, at \(t = t_{\text{col}}^{\text{ch}}\), we obtain finite \(\rho_r\) due to quantum corrections, so

\[
\rho_r|_{t=t_{\text{col}}^{\text{ch}}} = \frac{1}{r} M_P^4 
\]
(4.26)
where \(r > 1\) is the real number.

(4.25b) yields
\[
\frac{C_1}{2\pi} M_P^2 D^2 \left[3 - \frac{N}{60\pi}\right] = -4\rho_r|_{t=t_{\text{col}}^{\text{ch}}} . 
\]
(4.27)

From (4.26) and (4.27), it is obtained that
\[
\frac{1}{2\pi} \left[3 - \frac{N}{60\pi}\right] C_1 = -\frac{4}{r}. 
\]
(4.28a)
and
\[
D = M_P 
\]
(4.28b)
as \(D\) has mass dimension and \(C\) is dimensionless.

Using (4.26), (4.28b) in (4.25c), we obtain
\[
\frac{3}{2\pi} \left[\frac{N}{160\pi} - 1\right] C_1^2 + \frac{4}{r} C_1 + \frac{8}{r} = 0. 
\]
(4.29)
(4.29) yields the real value of \( C \), when \( N < 60\pi \). So, we can take

\[
N = 188. \tag{4.30}
\]

Connecting (4.28a) and (4.30), \( C \) is obtained as

\[
C_1 = -\frac{3.99\pi}{r}. \tag{4.31}
\]

(4.29) and (4.31) yield

\[
r = 2\pi. \tag{4.32}
\]

and

\[
C_1 = -1.995. \tag{4.33}
\]

Connecting (4.25a), (4.26), (4.28b), (4.30), (4.32) and (4.33), we obtain

\[
\rho_\Lambda^q = 2.56 \times 10^{75}\text{GeV}^4. \tag{4.34}
\]

Thus, it is found that vacuum energy density \( \rho_\Lambda^q \), given by (4.34), is extremely high and rises by \( 10^{129} \) compared to \( \rho_\Lambda \) given by (2.18e) on making quantum correction. It means that quantum gravity feeds cosmological constant so well that it gets extremely fattened. Thus, it is interesting to see that, during quantum regime, cosmological constant is extremely heavy, but it is almost negligible in classical domain.

As \( a^q(t) \), given by (4.18), is valid during quantum dominance and it will have its maximum at \( t = t_c \), if quantum corrections begin to dominate at this epoch. So, from (4.19a), (4.20) (4.28b) and (4.33), we obtain

\[
H = -M_P + 3.99M_P[M_P(t_{ch}^{col} - t_c)] = 0 \tag{4.35}
\]

yielding

\[
t_c = t_{col} - 0.25t_P \tag{4.36}
\]
with $t_P = M_P^{-1}$ being the Planck time. Thus, quantum gravity corrections will be effective for

$$|M_P(t_{\text{col}} - t_c)| < 0.25. \quad (4.37)$$

(4.18), (4.20), (4.34b) and (4.36) yield

$$a_c = 1.13 a_{\text{col}}. \quad (4.38)$$

(2.30),(4.1b), (4.4d) and (4.15a) yield

$$a_{\text{ch}} = 1.468 \times 10^{-12}. \quad (4.39)$$

From (4.1c), (4.15a), (4.15c),(4.36) and (4.39), we obtain

$$a_c = 1.875 \times 10^{-34}. \quad (4.40)$$

Further, (4.38) and (4.40) imply

$$a_{\text{col}} = 1.659 \times 10^{-34}. \quad (4.41)$$

Thus, solution of FE (4.17) with quantum correction is obtained as

$$a^q(t) = 1.659 \times 10^{-34} \exp[|\{M_P(t_{\text{col}}^{\text{ch}} - t)\} - 1.995\{M_P(t_{\text{col}}^{\text{ch}} - t)\}^2|] \quad (4.42)$$

(4.41) yields $H < 0$ for $t < t_{\text{col}}$ and $H = M_P[1 - 3.99\{M_P(t - t_{\text{col}}^{\text{ch}})\}] > 0$ for $t > t_{\text{col}}$. It shows contracting universe for $0.25t_P \leq t < t_{\text{col}}^{\text{ch}}$ and expanding universe for $t_{\text{col}}^{\text{ch}} < t \leq 0.26t_P$ during quantum regime. It means that future universe will pass through the narrow quantum bridge from pre-collapse to post-collapse state avoiding cosmic collapse suggested by classical mechanics.

5. Summary

Here, cosmology of the late and future universe is obtained from $f(R)$-gravity, obtained by adding higher-order curvature terms $R^2$
and $R^3$ to the Einstein-Hilbert linear in scalar curvature $R$. It is explained in the first section on introduction that $f(R)$—gravity cosmology is different from $f(R)$—dark energy models [6]. Moreover, problems of $f(R)$—dark energy models, pointed out in [7], do not appear in $f(R)$—gravity cosmology [8, 9, 10, 11, 12] as well as in the present model. Here, curvature scalar contributes to both geometrical and physical components of the theory. Thus, it plays dual role as a geometrical as well as physical fields, which was obtained earlier in [21].

Here, it is found that, in the late universe, for the red-shift $z < 1089$, dark matter term emerge spontaneously and phantom dark energy emerge as imprints of linear and non-linear terms of curvature. It is found that, during $0.36 < z < 1089$, dark matter dominated and universe expanded with deceleration as $t^{2/3}$. A transition from deceleration to acceleration took place at $z = 0.36$ and at time $t = 0.59t_0$ ($t_0$ being the present age of the universe). This transition is caused by dominance of curvature-induced phantom dark energy over curvature-induced dark matter. Dark energy gives anti-gravity effect and phantom dark energy exhibits this effect more strongly due to violation of WEC. So, phantom energy puts high jerk causing super-acceleration.

$f(R)$—gravity inspired Friedmann equation, obtained here, contains two terms (i) $8\pi G\rho_{dm}/3$ (with $\rho_{dm}$ being the dark matter density) and (i) $(8\pi G\rho_{de}/3)[1 - \rho_{de}/2\lambda]$ (with $\rho_{de}$ being the phantom dark energy density). Here $\lambda = 5.777\rho^0_{cr}$ is called the cosmic tension. It is interesting to see that Friedmann equation, obtained here, contains a correction term $-4\pi G\rho^2_{de}/3\lambda$ analogous to such term in Friedmann equations from RS-II model of brane-gravity [13] and loop quantum cosmology[14]. This correction is not effective in the present universe as
\( \rho_{de}^0 \ll 2\lambda \) as well as it is ineffective till \( \rho_{de} \ll 2\lambda \). But, as \( \rho_{de} \) will increase in future with the growing scale factor \( a(t) \), effect of this term will increase. It is found that, on sufficient growth of \( \rho_{de} \), the effective EOS does not violate WEC (which characterizes phantom), but violates SEC. On further increase in \( \rho_{de} \), even SEC is not violated. Thus, in the beginning of the phantom phase, universe will super-accelerate during the period \( 0.59t_0 < t < 2.42t_0 \), it will accelerate during the period \( 2.42t_0 < t < 3.44t_0 \) and, during the period \( 3.44t_0 < t < 3.87t_0 \), universe will decelerate even during the phantom phase. Phantom-dominance will end when \( \rho = 2\lambda = 11.54\rho_{cr}^0 \) at time \( t = 3.87t_0 \). As a consequence, re-dominance of dark matter will begin giving decelerated expansion. But, as universe will expand, growth of \( \rho_{de} \sim a^{3/4} \) will also continue giving \( \rho_{de} > 2\lambda \). It causes the term \( (8\pi G \rho_{de}/3)[1 - \rho_{de}/2\lambda] \) to switch over from positive to negative. Growth of this negative term will try to slow down expansion more rapidly. As a result, universe will reach a state, when expansion will stop and scale factor will acquire its maximum value in finite time \( t_m = 3.87t_0 + 694.4\text{kyr} \). When \( t > t_m \), universe will retrace back and contract. Results, obtained here, show that contraction will continue for sufficiently long period \( 333t_0 \) and will collapse at time \( t_{col} = 336.87t_0 + 694.4\text{kyr} \), where energy density of the universe will diverge and scale factor will vanish. These results are obtained using prescriptions of classical mechanics. In this paper, it is probed further whether quantum gravity corrections can save the universe from the menace of collapse.

It is argued, here, that as in the very small time period close to \( t = t_{col} \), energy density is extremely high and curvature is very large, quantum gravity effects will be dominant. It is analogous to the state of very
early universe, where energy density is extremely high and curvature is very large. So, quantum gravity has a crucial role. Motivated by these arguments, quantum corrections are used in the future universe near time of collapse and it is found here that expected collapse of the future universe can be avoided. Moreover, results show that contracting universe, in pre-collapse era will expand with super-acceleration in the post-collapse era.

Interestingly, energy density due to cosmological constant $\rho_\Lambda$, during quantum regime, is found $10^{129}$ times $\rho_\Lambda$ during regime of classical mechanics. It means that quantum gravity nourishes the cosmological constant well and makes it very strong, otherwise it is very weak under the rule of classical mechanics. So, when $\rho_\Lambda$ is out of quantum domain, it falls down immediately by 129 order.

The state of the universe near the epoch $t = t_{col}$ can be realized as revival of the state of early universe in future due to high energy density, large curvature and dominance of quantum gravity effects. If it is true, study of this state of the future universe can unveil many hidden knowledge of the early universe. For example, here, results of the cosmological constant near collapse time explains largeness of $\rho_\Lambda$ in the very early universe and its extremely small value in the present universe. It is important to see that this result is obtained without any fine-tuning.

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