Connection of surface roughness to hysteresis loss in spine implants

Mohammad HODAEI* and Kambiz FARHANG*

*Southern Illinois University Carbondale
Carbondale, IL, 62901, USA
E-mail: mhodaei@siu.edu

Received 10 October 2014

Abstract
To investigate roughness effect in spine implant, a contact model is offered in this paper. The contact surfaces are modeled macroscopically as semi-spherical containing micro-scale roughness. A minimum mean surface separation between two rough surfaces is defined and related to contact force. This is accomplished through a statistical account of elastic and plastic micro contact at the surface roughness scale and the subsequent integration of micron level events to obtain macro level expectation of force as a function of mean surface separation. Contact force - minimum separation relation facilitates the derivation of hysteretic energy loss during a load-unload event. It is found that the surface high plasticity index yields more energy per cycle. Using the force-minimum surface separation in a dynamic interaction of the implant, approximate leads to the prediction of contact natural frequencies and damping ratio of the implant. These characteristics along with energy loss in implant affect implant’s performance and durability.

Key words : Contact mechanics, Spine implant, Roughness, Wear, Blood, Toxicity

1. Introduction
One of the most important structural components in the human body is spinal column. A schematic of human body spine consisting of thirty-three bony vertebrae is shown in Fig. 1. The sections forming the spine are called cervical, thoracic, lumbar, sacrum, and coccyx. The first three forms the movable components of each spine unit and the last two components are fixed. For the designing implants and performing experiments, it is important to use an identical vocabulary for anatomic directions and motions. This paper investigates bending sideways known as lateral bending shown in the figure 1(b).

Figure 1: Schematic of spine
The functionality of the human body is strongly influenced by the spine, so any injury to the spinal column can cause severe disability; any injury to the spinal cord above the C3, cervical 3, for example, can terminate breathing ability. The spine holds and protects the spinal cord, the only means of communication between the brain and the rest of the body. The brain regulates an individual’s reflex. When such function is impaired more reflex can cause the spasticity (Brockenbrough, 2009; Kmeter et al. 2004). It is found with neurologic lumber radiculopathy that a diagnosis of hip arthritis can be clarified by examining lumber 5, showing a connection between spine and hip and other parts of the body. The critical importance of the spine in a human body provides a high degree of motivation to understand its structural integrity and the effective replacement of its joints when necessary, through appropriate implant designs. Lee and Wong, 2002 showed that in the sagittal and horizontal planes, the movement patterns of the spine and hip were in phase, whereas in the coronal plane, the spine generally moved earlier than the hips. They concluded that clinical examination of the back should include kinematic measures of both the lumbar spine and hips.

The main application of implant or replacement discs are replacing injured intervertebral spine. With regards to some research, Spinal implant surgery is relatively new in the United States dating back to October 2001, while the spine surgery training has taken hold in Europe; European neurosurgical trainees during their residency, Boszczyk et al. 2009.

Even though spinal implants cannot function the same as the body’s natural spine, they are still very beneficial. The goal of the implants is to provide, in addition to structural integrity, as much as possible the needed mobility in several planes, providing bending and torsion. In addition, an implant must provide adequate mechanical and chemical surface contact stability so as to minimize gradual erosion or excessive wear, which can lead to the onset of toxicity and fatal danger to the patient. Consequently, a designer must probe all parameters such as wear, roughness, erosion, tribology, and materials when considering a spinal implant. Rohl et al. 2009 investigated the functionality of cervical spine with two methods. The first method involved curing by disc replacement and the second method used treatment by decompression and fusion. The results showed that both methods can work, however, using prostheses provides an improved total mobility. Additionally, their conclusion describes this result in tetraplegics. Sankar et al. 2010 evaluated the level of complications involved in spinal surgery for various spinal implant designs. They found that spinal surgery can be made prohibitively complicated by the nature of the spinal implant design.

The treatment of spines is mostly done by interspinous devices. Ansetti et al. 2010 performed a test by using the interspinous devices in reducing segmental lordosis and in stabilizing motion by evaluation of the role of laces, and the size and position of devices. They recommend the application of proper device size and placement with a correct surgical operation to lead to a successful result. Even though disc replacement is one of the best ways for reducing pain, more attention should be directed to the phenomenon of adjacent-segment disease after cervical fusion. Lazaro et al. 2010 investigated three different types of disc implants to evaluate the effect of these given devices to maintain and restore normal segmental alignment. They found all three disc types to be acceptable so long as misalignment does not exceed six degrees during surgery. There are various investigations performed by finite element model of the spines, especially the lumbar spine. Because of some reasons such as examination of biomechanical behavior of the healthy spine, investigation of spine performance facing with disease, investigation of the effect of different instrumentations on spine behaviors and developing of new spinal implants finite element methods has been used in spine research by Fagan et al.2002.

The examination of simple and complex finite element model of single vertebrae, motion segments, multi-motion segments, and the whole lumber spine is investigated by (Liebschner et al. 2003; Pollikeit 2002; Pitzen et al. 2002; Naoailly et al., 2003; Zander et al., 2001; Kuntz et al., 2000; Kurutz, 2010) in order to check the effects of variation of material properties for different spinal components. The geometry of the majority of recently built 3D models of the spine has been derived from Computed tomography (CT) data by (Shirazi et al., 2000; Pintar et al., 1992; Polineit et al, 2003; Shirazi 1994).

The goal of this study is the prediction of the effect of roughness in spine joint implants. The objective in spine implant is not just curing pain but it is also providing joint functions in patients.

To address the issues related to wear and thereby potential toxicity effects of an implant it is important to have a mathematical tool to predict the effect of implant surface finish. The use of FEM is impractical in this case since such an effort would involve a prohibitively large number of elements amounting to more than one million elements for a mere one square millimeter nominal area. This paper develops a statistically-based contact mechanics model of spine joint taking into account the effect the surface finish property and surface roughness geometry of the implant. An elastic-plastic model of the spheres in contact representing the ball and socket implants is developed. The contact mechanics approach in this paper allows the development important predictive metrics for spine implants. The first is the per load-unload cycle energy loss due to rough surface elastic-plastic interactions of the spine implant, an indicator of plastic wear in the implant. The next two are the natural frequency and damping properties of the implant’s rough surface interaction. These two parameters can show potential discomfort in the spine implant due to normal activities and environmental exposure of the body of the implant recipient. For example, existence of environmental machinery noise/vibration can excite the implant frequencies during their operation.
2. Spine Contact Model

The schematic diagram of a spine joint is shown in Fig 2. The contact force between the spinal body and ball-and-socket will be increased due to force transfer to the spine. The macro geometry of the ball and the socket are approximated by semi spheres with radii of curvature $R_1$ and $R_2$, respectively.

![Figure 2: Schematic depiction of spine joint](image)

The contact between two semi spheres of radii of $R_1$ and $R_2$ is detailed in Fig. 3. For a contact model incorporated with roughness of surface, it is axiomatic that the load-carrying zone be defined by minimum separation with symmetrically distributed pressure around the minimum separation. The mean surface separation has influence on the number of contact points and respective pressure, so the development of the expression of mean separation as a function of minimum separation and geometries of two contact areas can be very imperative. There is a conformal contact between ball and socket as it is represented by the semi sphere contact in Fig. 3.

![Figure 3. Semi-spheres in contact](image)
With regards to the schematic drawing of semi spheres for ball and socket contacts, for the minimum separation of two contact surfaces $h_0$, the offset of two semi spheres centers, $\delta$, can be expressed in terms of $h_0$.

$$R_1 + h_0 + \delta = R_2 \rightarrow \delta = R_2 - R_1 - h_0 \tag{1}$$

$$R_1 + h = x \tag{2}$$

In the triangle shown in figure 2, mean plan separation, $h$, can be derived as a function of minimum separation, $h_0$, radii of the two semi spheres, $R_1$ and $R_2$, and angular location derived with respect to the ball, inner sphere.

$$R_2^2 = \delta^2 + x^2 - 2x\delta \cos(\pi - \theta) \rightarrow R_2^2 = \delta^2 + x^2 - 2x\delta \cos(\pi - \theta) \tag{3}$$

$$x^2 + 2x\delta \cos \theta + \delta^2 - R_2^2 = 0 \rightarrow x = -\delta \cos \theta \pm \sqrt{\delta^2 \cos^2 \theta + R_2^2 - \delta^2} \tag{4}$$

In Eq. (4), the accepted solution is in positive range of $x$, so

$$x = -\delta \cos \theta + \sqrt{\delta^2 \cos^2 \theta + R_2^2 - \delta^2} \tag{5}$$

With substitution for $x$ from Eq. (1) to (5) it will be found

$$R_1 + h = x \rightarrow h = R_2 \left( -\frac{\delta}{R_2} \right) \cos \theta + \sqrt{1 - \left( \frac{\delta}{R_2} \right)^2 \sin^2 \theta} - R_1 \tag{6}$$

Substitute for $\delta$ from Eq. (1)

$$\delta = R_2 - R_1 - h_0 \tag{7}$$

Therefore,

$$h = R_2 \left( -\frac{R_2 - R_1 - h_0}{R_2} \right) \cos \theta + \sqrt{1 - \left( \frac{R_2 - R_1 - h_0}{R_2} \right)^2 \sin^2 \theta} - R_1 \tag{8}$$

With regard to mean surface separation found from Eq. (8), the contact force between elastic plastic interaction of the roughness of the ball and socket can be derived. The expression of force per unit area presented by Sepehri and Farhang, 2007 is as follows:

$$P(h) = P_e(h) + P_p(h) \tag{9}$$

Where, $P_e(h)$ is the elastic force given by the following equation,

$$P_e(h) = C \left[ \int_0^\infty (s - h)^\frac{3}{2} e^{-\frac{s^2}{2 \sigma^2}} ds - \int_0^{\omega_c} (s - h)^\frac{3}{2} e^{-\frac{s^2}{2 \sigma^2}} ds \right] \tag{10}$$

Where,

$$C = \frac{4}{3\sqrt{\pi}} E \eta \beta \sigma^2 \tag{11}$$

$s$ and $h$ are both dimensionless. $s$ is the ratio of an asperity height over the standard deviation of asperity summit distribution, $\sigma$, and $h$ is the ratio of the mean surface separation over $\sigma$. It is noteworthy to mention that it is adequate for the model to include micron-scale level of roughness since for the force ranges considered and the nature of the spine implant macro geometry the lower scales of roughness would not contribute significantly to the contact force. When the surfaces are pressed together, there may be locations within the contact zone where asperity interference results in the onset of plastic deformation. Greenwood and Williamson, 1966 defines asperity critical interference to be the onset of plastic deformation. In following the Chang et al.1987, the authors employed the definition of the critical interference to formulate the elastic-plastic model of contact. In Eq. (13) $\omega_c$ represents the critical interference. Greenwood and Williamson,1966 defines plasticity index for a surface as follows:

$$\phi = \frac{E}{H} \sqrt{\frac{R}{\eta}} \tag{12}$$

Where, $R$ is the average asperity summit radius of curvature, $E$ is the equivalent modulus and $H$ is the hardness of the softer material. They also define critical interference as

$$\omega_c = \left(\frac{\psi}{E} \right)^2 \frac{\sigma^2}{R} \tag{13}$$
Letting $w_c = \frac{\omega_c}{\sigma}$ be the dimensionless critical interference, the plasticity index, is related to the $w_c$ as follows:

$$\psi = \frac{1}{\sqrt{w_c}}$$

(14)

Equation (10) uses a constant $C$ and the dimensionless force expression in integral form for the elastic part of the surface interaction. $C$ is defined as given by Eq. (11), in which $E$ is the reduced modulus of elasticity of the two surfaces, $\beta$ is the dimensionless equivalent average asperity radius of curvature. The reduced modulus of elasticity is derived from the properties of the material used in the implant. It is given by the following equation

$$\frac{1}{\bar{E}} = \frac{1}{\bar{E}_1} + \frac{1}{\bar{E}_2}$$

(15)

Where, $E_1$ and $\nu_1$ are the modulus of elasticity and Poisson ratio of the ball implant material and $E_2$ and $\nu_2$ are those of the socket implant. The equivalent asperity radius is found using

$$\frac{1}{\bar{\beta}} = \frac{1}{\beta_1} + \frac{1}{\beta_2}$$

(16)

Where, and $\beta_1$ and $\beta_2$ are the average asperity radius of curvature of the ball and socket implants, respectively. $\eta$ is the asperity density per unit area. Table 1 lists typical properties of implant materials used in spine.

The plastic force per unit nominal area is, Chang et al. (1987),

$$P_p(h) = C \left( \sqrt{w_c} \int_0^{\infty} [2(s - h) - w_c]e^{-s^2/2} \pi ds \right)$$

(17)

Thus the total force is the combination of elastic and plastic interactions

$$P(h) = P_e(h) + P_p(h)$$

(18)

To obtain contact force along a particular direction, one must sum the force components along that direction due to infinitesimal contact forces that occur over an infinitesimal area. Sum the force components parallel to radial line of symmetry with respect to nominal contact area to find the normal force

$$F = \int_0^{2\pi} \int_{\pi/2}^{\pi/2} P(h) R^2 \sin \theta \cos \theta d\theta d\phi$$

(19)

Where $R$ is the equivalent radius of curvature of the ball and socket radii of curvatures defined as

$$\frac{1}{\bar{R}} = \frac{1}{\bar{R}_1} + \frac{1}{\bar{R}_2}$$

(20)

The integral in Eq.(19) can be reduced to the following

$$F = 4\pi \sigma^2 \int_0^{\pi} P(h) R^2 \sin \theta \cos \theta d\theta$$

(21)

It will prove beneficial to express Eq. (21) as an explicit function of minimum separation, $h_0$. Since $h$ is function of integration variable $\theta$, Eq. (8), the integral in Eq. (21) can only be found numerically. As a result, it is set out to find an approximate relation between contact forces $F$ and the minimum separation $h_0$. It will be shown in the next section that the contact force may be estimated using function of the form $ae^{-ch_0}$.

### 2.1. Dependence of Coefficients on Spine Radii

In this section, ball and socket radii are used as a parameter in the approximate expression relating contact force to minimum mean surface separation. It can be shown that the approximate equation is of the following form

$$F_{n\alpha}(h_0) = \mathcal{F}(\psi, R_1, R_2)e^{-c(\psi) h_0^{1.2}}$$

(22)

The values are generated for various ball radii, ranging from 10 mm to 16 mm by Moghadas et al. 2012, where the coefficients $\mathcal{F}$ and $c$ are expected to depend on the geometry of the ball radii and the plasticity index.

$$F_n(h_0, R_1, R_2) = 4\pi \sigma^2 \int_0^{\pi} P(h) R_1^2 \sin \theta \cos \theta d\theta$$

(23)

In obtaining the approximate equation, the ball radius is varied and the socket radius is assumed to be 0.015mm larger.
than the ball radius.

\[ \mathcal{F}(\psi, R_1) = a(\psi) R_1^2 \]  

(24)

The coefficient \( \mathcal{F} \) is found to be quadratic with respect to \( R_1 \) and coefficient \( a \) is functions of the radius and plasticity index \( \psi \).

### 2.2. Dependence of Coefficients on Plasticity Index \( \Psi \)

In the previous section it was shown that coefficient \( F \) is a quadratic function of Spine radius with coefficient \( a \) being functions of the radius and plasticity index, \( \psi \). In this section, approximate functional relationship between \( a \) and plasticity index are established for plasticity index ranging 0.6 to 1.3. Keep in mind that for surfaces characterized by \( \psi < 0.6 \) the surface is considered predominantly, while for \( 0.6 < \psi < 1 \) the surface is viewed as elastic plastic.

\[ a(\psi) = a_3 \psi^3 + a_2 \psi^2 + a_1 \psi + a_0 \]  

(25)

\[ a_3 = -9.204 \times 10^{-5}, \quad a_2 = 2.511 \times 10^{-4}, \quad a_1 = 1.96 \times 10^{-4}, \quad a_0 = 7.841 \times 10^{-5} \]  

(26)

Likewise, the fitted function for \( c \) is

\[ c(\psi) = c_3 \psi^3 + c_2 \psi^2 + c_1 \psi + c_0 \]  

(27)

As

\[ c_3 = 0.4545, \quad c_2 = -1.474, \quad c_1 = 1.537, \quad c_0 = 1.447 \]  

(28)

Finally, plasticity function with low percent error for \( a \) and \( c \) is

\[ F_{na}(h_0) = \left[ a(\psi) R_1^2 \right] e^{-c(\psi) h_0^{1.21}} \]  

(29)

In the Fig 4, it is shown that the max error between fitted plasticity function and original function for elastic-plastic contact is less than 6% over the entire range of parameters considered. For part of recovery, no plastic, the coefficients and equations are:

\[ a(\psi) = a_3 \psi^3 + a_2 \psi^2 + a_1 \psi + a_0 \]  

(30)

\[ a_3 = 1.156 \times 10^{-4}, \quad a_2 = -3.215 \times 10^{-4}, \quad a_1 = 2.477 \times 10^{-4}, \quad a_0 = -2.638 \times 10^{-5} \]  

(31)

Likewise, the fitted function for \( c \) is

\[ c(\psi) = c_3 \psi^3 + c_2 \psi^2 + c_1 \psi + c_0 \]  

(32)

As

\[ c_3 = 1.245, \quad c_2 = -3.859, \quad c_1 = 3.028, \quad c_0 = 1.245 \]  

(33)

Finally, plasticity function with low percent error for \( a \) and \( c \) is

\[ F_{na}(h_0) = \left[ a(\psi) R_1^2 \right] e^{-c(\psi) h_0^{1.2}} \]  

(34)

Figure 4. (a) Exact and approximate normal contact force and (b) the max error (percent) between the two

\[ a_3 = 1.156 \times 10^{-4}, a_2 = -3.215 \times 10^{-4}, a_1 = 2.477 \times 10^{-4}, a_0 = -2.638 \times 10^{-5} \]  

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As

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\[ F_{na}(h_0) = \left[ a(\psi) R_1^2 \right] e^{-c(\psi) h_0^{1.2}} \]  

(34)
3. Energy Loss in Spine Implant

The contact between the ball and socket implant surfaces consists of asperities experiencing elastic and plastic deformation. A close look at the loading and unloading process reveals that both damping and elastic recovery are involved in the process. During the increase in contact load both elastic and plastic deformations can occur at asperity deformation level. However, during unloading asperities undergo only elastic recovery. Therefore, the load and unload process will follow different paths, resulting in hysteresis type energy loss in the hip joint contact.

It can be employed the approximate equations for elastic-plastic contact and purely elastic contact to represent the loading and unloading process mathematically. The force during loading is denoted $F_{nL} = \alpha_1 L e^{a_2 L h_0 a_3 L}$, and that during unloading, $F_{nU} = \alpha_1 U e^{a_2 U h_0 a_3 U}$. Based on the results of the previous section, the respective coefficients of contact force during loading and unloading are as follows:

$$\alpha_{1L} = a(\psi) R_1^2$$

$$\alpha_{2L} = -c(\psi)$$

$$\alpha_{3L} = \alpha_{3U} = 1.21$$

$$\alpha_{1U} = a(\psi) R_1^2$$

$$\alpha_{2U} = 1.2$$

To study energy loss and storage in a spine joint, an equilibrium contact force is considered. For example this may correspond to an individual standing still and a contact force equal to the equilibrium force exists between the ball and socket. The equilibrium contact force is associated with an equilibrium minimum plane separation, $h_0$. A disturbance from equilibrium is denoted as $x$. Therefore, to study the behavior of the contact near an equilibrium state, the contact force equations above can be used. Depending on the nature of the disturbance, the load may increase from equilibrium or decrease from it. If the load is increasing from equilibrium then both elastic and plastic contacts must be included in the calculation of the contact force. If the load is decreasing from the equilibrium state, then only elastic contacts contribute, since this is a load recovery process. The following expressions will be adequate to account for either load change scenarios.

$$F_{nL}(h_0, x) = \alpha_{1L} e^{a_2 L (h_0 - x) a_3 L}$$

$$F_{nU}(h_0, x) = \alpha_{1U} e^{a_2 U (h_0 - x) a_3 U}$$

Here $F_{nL}$ denotes the normal contact load due to both elastic and plastic interaction of surface roughness, and $F_{nU}$ is the normal contact force due to only elastic interaction of the roughness. When the disturbance is small, the above force equations can be written in linear form using truncated Taylor series expansion of $F_{nL}$ and $F_{nU}$ about the equilibrium minimum separation.

$$F_{nL}(h_0, x) = -\left(\frac{h_0 a_3 L^{-1} a_{1L} a_{2L} a_{3L} e^{h_0 a_3 L a_2 L}}{1!}\right) x + \alpha_{1L} e^{a_2 L h_0 a_3 L}$$

$$F_{nU}(h_0, x) = -\left(\frac{h_0 a_3 U^{-1} a_{1U} a_{2U} a_{3U} e^{h_0 a_3 U a_2 U}}{1!}\right) x + \alpha_{1U} e^{a_2 U h_0 a_3 U}$$

Based on the dissipated energy formula, the plot of two terms loading and unloading forces can show the damped energy of the system. In Fig. 5, the dissipated energy per cycle is depicted as the area between both load and unload forces can explain the damping ratio of spine implant when plasticity index is 0.6. The area between both unloads and load forces will be expanded with increasing plasticity index as it is shown in the Fig. 6.
With regard to two different types of plasticity i.e., 0.6 and 1.3, it can be seen that for high plasticity the dissipative energy is more. To obtain the energy loss in a single cycle, integration of force over displacement in the load and unload phases is performed. For amplitude of oscillation of $x_a$ from equilibrium, the energy loss per cycle is expressed as follows

$$E = \int_{-x_a}^{x_a} F_n L dx - \int_{-x_a}^{x_a} F_n U dx$$  \hspace{1cm} (44)$$

That can be simplified by using the linear approximation of each load function in Eq. (42) and (43). It will be

$$E_L = 2Cx_a(\alpha_{1L} e^{a_{2L} h_o a_{3L}} - \alpha_{1U} e^{a_{2U} h_o a_{3U}})$$  \hspace{1cm} (45)$$

$E_L$ is the energy loss per cycle. The energy per cycle can be expressed in dimensionless form by dividing Eq. (45) by $Cx_a$. So the dimensionless energy loss per cycle is

$$E_{L} = 2(\alpha_{1L} e^{a_{2L} h_o a_{3L}} - \alpha_{1U} e^{a_{2U} h_o a_{3U}})$$  \hspace{1cm} (46)$$

Figure 7 illustrates dimensionless energy per cycle and plasticity index as functions of dimensionless critical interference. When critical interference is low (high plasticity index), the interference enters the plastic regime for less contact load.
Therefore, energy loss per cycle is higher for low critical interference. As critical interference increases, the number of asperities experiencing plastic interference decreases, thereby, reducing the energy per cycle.

![Figure 7: Dimensionless energy loss per cycle and surface plasticity index versus critical asperity interference](image)

A similar plot, which directly relates energy loss per cycle to surface roughness, is shown in Fig. 8. In this case the abscissa represents the dimensionless average radius of curvature. It is observed that as dimensionless asperity summit radius of curvature is increased (surface is made more smooth) the energy loss per cycle decreases.

![Figure 8: Dimensionless energy loss per cycle and surface plasticity index versus dimensionless average asperity summit radius of curvature](image)
4. Effect of Ball Radius on Energy Loss

In this section the effect of ball radius on energy loss is investigated. Fig. 9 shows that discs with ball radius of 10 to 11 micrometer offer more promise under higher loads and have lower energy loss in comparison with ball radius of more than 11 to 16 micrometers.

Figure 9: Energy per cycle versus radius of curvatures and plasticity index

5. Contact Frequency and Damping

It is possible to define contact frequency and damping if one is able to define an appropriate equivalent lumped mass near the interface of the ball head and the socket. Designating the lumped mass as $m_0$, Eq. (42) and (43) can be used in a nonlinear dynamic model of the contact to obtain contact frequency and damping ratio as follows:

$$\omega_n = \sqrt{-\frac{h_0 a_3 L^{-1} \alpha_1 \alpha_2 a_3 L \alpha_0 a_3 L \alpha_2 L}{\sigma m_0}}$$ (47)

$$\zeta = \frac{a_3 L \alpha_0 a_3 L \alpha_1 \alpha_2 L \alpha_0 a_3 L \alpha_2 L}{\pi a_0 \sigma m_0}$$ (48)

For an assumed $m_0 = 0.1$ kg, Fig. 10 depicts the resulting natural frequency and damping ratio versus the minimum dimensionless mean surface separation. Figure 11 shows the contour plots of the same. One should note that the dimensionless minimum separation corresponds to a certain contact load. Lower values of $h_0$ correspond to higher contact force and vice versa. It is interesting to note that natural frequency primarily changes with contact load (or $h_0$) and not the surface plasticity index. This is consistent with the fact that contact presents a nonlinear overall stiffness and therefore stiffness is increased with increase in contact load. Higher stiffness in turn means larger contact frequency. Figure 10 and 11(b) show contact damping ratio.
Figure 10: Contact natural frequency and damping ratio vs. minimum surface separation
Figure 11: Contour plot of natural frequency and damping ratio vs. minimum surface separation and plasticity index

It is clear that damping ratio depends on both applied contact force and surface plasticity index. It is noted here that the sensitivity of contact damping ratio to plasticity index increases for higher contact force (lower $h_0$).
Table 1. Material and surface properties used

| Parts | Materials             | Young’s Modulus | Poisson’s Ratio | Sphere Radius | Hardness |
|-------|-----------------------|-----------------|----------------|--------------|----------|
| Ball  | Cobalt-Chromium-Alloy | 110 GPa         | 0.33           | 10-16 mm     | 700 MPa  |
| Socket| Polyethylene          | 0.63 GPa        | 0.4            | 10.015-16.015mm | 120 MPa  |

**Surface Properties**

|       |                                                                 |                |
|-------|----------------------------------------------------------------|----------------|
| η     | The asperity density per unit area                              | $11 \times 10^{10}\text{ l/m}^2$ |
| σ     | Standard Deviation                                              | $1 \times 10^{-6}$ |
| β     | Average asperity radius of curvature                            | 200            |
| E     | Equivalent modulus of elasticity                                | $7.455 \times 10^8 \text{ GPa}$ |

6. Closing Remarks

An elastic-plastic contact model of Spine joint implant is developed in this paper. The vertebral body of spine implant, separated by a disc, is modeled using semi-spherical solids in internal conformal contact and accounting for the effect of roughness of both surfaces. Statistical integration of contact pressure over contact region of effective interaction between two semi-spherical rough surfaces finds an equation relating force to minimum mean surface separation. The approximate equation is used to find closed-form equation for contact energy loss per cycle. It is found that the surface of the weaker implant material is responsible for more energy loss. Contact natural frequency and damping ratio are derived using the approximate function lump mass representation of the implant hemispheres. The estimation of spine implant contact stiffness and damping ratio has a potential usefulness for investigation of noise and vibration generation and fatigue wear of implant surfaces.

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