Practical Solution to Efficient Flight Path Control for Hypersonic Vehicles*

Qixia WU,1) Mingwei SUN,1)† Zenghui WANG,2) and Zengqiang CHEN1)

1)College of Computer and Control Engineering, Nankai University, Tianjin 300350, China
2)Department of Electrical and Mining Engineering, University of South Africa, Florida 1710, South Africa

There is a severe lag between the pitch attitude and flight path angle of hypersonic vehicles, imposing extraordinary difficulty in achieving high-precision trajectory control. A comprehensive flight path regulation scheme is proposed to solve this problem. The inherent relationship between the pitch attitude and flight path angle is sufficiently used to establish a feed-forward control framework where the flight path reference is directly fed into the attitude loop in order to quickly obtain the major control component. To achieve efficient tracking, an attitude control method based on active disturbance rejection control (ADRC) is proposed. The new attitude control method utilizes an extended state observer (ESO) to estimate the unknown model dynamics and external disturbances. The remaining part of control is generated by ADRC acting on the flight path angle, which plays a complementary role. The stability margin tester is used to tune the attitude controller explicitly. Finally, simulation examples are provided to demonstrate the effectiveness of the proposed method.

Key Words: Hypersonic Vehicle, Flight Path, Active Disturbance Rejection Control (ADRC), Extended State Observer (ESO), Feed-forward Control, Tuning, Stability Margin Tester

Nomenclature

- \( V \): velocity (m/s)
- \( h \): altitude (m)
- \( T \): engine thrust (N)
- \( D \): drag (N)
- \( L \): lift (N)
- \( M_{yy} \): pitch moment (kg·m²)
- \( \alpha \): angle of attack (rad)
- \( \gamma \): flight path angle (rad)
- \( \theta \): pitch angle (rad)
- \( q \): pitch angular rate (rad/s)
- \( m \): mass (kg)
- \( g \): gravitational constant (kg/s²)
- \( I_{yy} \): moment of inertia (kg·m²)
- \( \rho \): atmospheric density (kg/m³)
- \( z_T \): thrust eccentric distance (m)
- \( S \): reference area (m²)
- \( \bar{c} \): mean aerodynamic chord (m)
- \( \delta_r \): rudder reflection (rad)
- \( \omega_n \): actuator natural frequency (rad/s)
- \( \xi \): actuator damping

1. Introduction

The flight control of hypersonic vehicles is a main topic of research recently.\(^1\)–\(^4\) In addition to the precise attitude control requirement for hypersonic vehicles, an efficient path control is also necessary. For the longitudinal dynamics, it is well known that there are correlations between the pitch angle and flight path angle. Therefore, common flight vehicles can regulate the flight path angle by adjusting the attitude in a short time. However, compared to conventional subsonic and supersonic flight vehicles, hypersonic aircraft demonstrate unique dynamics. It should be noted that there is a severe lag of about 30 s or more between the flight path angle and pitch attitude, which is determined by the flight speed and atmospheric density gradient of the flight environment,\(^5\)–\(^7\) and this is a particular phenomenon for hypersonic flights. High precision flight path control is difficult to achieve due to this large lag. Therefore, an appropriate solution is needed to resolve this issue.

In the past few years, a great deal of attention has been paid to advanced modern control methods in the realm of hypersonic flight. Among these advanced modern control methods, nonlinear control has become a hot topic. Xu et al.\(^8\) proposed a multi-input and multi-output adaptive sliding mode controller for controlling the speed and altitude of hypersonic vehicles. Fiorentini et al.\(^9\) utilized a nonlinear robust adaptive controller to control hypersonic vehicles. With the development of artificial intelligence, the neural-network\(^10\) and fuzzy-logic-based control\(^11\) approaches have also become popular. Moreover, robust control is an important control scheme for hypersonic flight. Wang and Stengel\(^12\) presented a robust flight control system with a nonlinear dynamic inversion structure. Buschek and Calise\(^13\) used the \( H_{\infty} \) approach to obtain a fixed-order controller. Despite the excellent performance exhibited in numerical simulations, these model-based control methods are difficult to implement in practice because of their complexities and incompatibilities with currently used control approaches. In fact, PID control is still widely used in traditional aircraft control systems and it is a preferred option for practitioners\(^14\) due to its simplicity and effectiveness. There is a practical significance to incorporate the modern control philosophy in the PID control framework as well as the empirical knowledge.
about flight mechanics. A comprehensive scheme including several mature techniques should be sought instead of a simple and solely complicated control algorithm.

As a unique kind of model-free control strategy, active disturbance rejection control (ADRC) was established by Han.15 The main philosophy or advantage of ADRC is treating the combination of internal dynamics, modeling inaccuracy and external disturbances as a virtual uncertainty variable, which is called “extended state.” This state can be estimated by a state observer in real-time, and then compensation is performed in order to transform the original plant into an integrator chain that can be easily controlled by a simple PD controller. In this process, an approximate differentiator can be realized by an integrator that can eliminate direct numerical differentiation, which is quite sensitive to measurement noises, as used in another model-free controller: time delay control (TDC).16,17 Another advantage of ADRC is that it does not require that all the states of a system be available for feedback, which is the foundation of the uncertainty and disturbance estimator (UDE)18,19 that is a developed version of TDC. Moreover, ADRC has already demonstrated its effectiveness in many fields.20-31

In this paper, the intrinsic characteristics of the flight mechanics are well utilized to design a novel feed-forward-feedback control framework such that high flight path control performance can be achieved for hypersonic flight. The feed-forward channel feeds the flight path reference straightforwardly to the pitch control loop where the major control component can be generated in an efficient way. To raise the servo precision of the pitch loop, a linear ADRC is utilized. The remaining tracking error of the flight path angle is also compensated by a low-gain ADRC. Moreover, a stability margin tester is used to tune the attitude controller.

The remaining parts of this paper are organized as follows. In Section 2, a longitudinal dynamic model of hypersonic flight is presented and the main problem is formulated. Then the proposed method is given in Section 3. The simulation examples are provided in Section 4. Section 5 contains the concluding remarks.

2. Problem Formulation

2.1. Model of hypersonic vehicle

Here, a longitudinal dynamic model for a generic hypersonic vehicle is chosen as the baseline for this study. It can be described by a set of differential equations32 as

\[
\begin{align*}
v &= \frac{T \cos \alpha - D}{m} - g \sin \gamma \\
\dot{v} &= \frac{T \sin \alpha + L - mg \cos \gamma}{mV} \\
\dot{\gamma} &= \frac{M_{yy}}{I_{yy}} \\
\dot{\theta} &= q \\
\dot{h} &= V \sin \gamma \\
\alpha &= \theta - \gamma
\end{align*}
\]

and

\[
\begin{align*}
L &= \frac{1}{2} \rho V^2 S C_L(\alpha, \delta_e) \\
D &= \frac{1}{2} \rho V^2 S C_D(\alpha, \delta_e) \\
M_{yy} &= z_T T + \frac{1}{2} \rho V^2 S C_{M,\alpha}(\alpha, \delta_e) \\
T &= C^*_T \alpha^3 + C^*_T \alpha^2 + C^*_T \alpha + C^*_T
\end{align*}
\]

where \( C_L(\alpha, \delta_e) \) and \( C_D(\alpha, \delta_e) \) represent the lift and drag coefficients due to \( \alpha \) and \( \delta_e \), respectively; and \( C_{M,\alpha}(\alpha, \delta_e) \) and \( C_{M,\delta_e}(\alpha, \delta_e) \) denote the moment coefficients with respect to \( \alpha \) and \( \delta_e \), respectively. One has

\[
\begin{align*}
C_L &= C^*_L \alpha + C^*_L \delta_e + C^*_L \theta_e \\
C_D &= C^*_D \alpha^2 + C^*_D \alpha + C^*_D \delta_e^2 + C^*_D \theta_e + C^*_D \\
C_{M,\alpha} &= C^*_{M,\alpha} \alpha^2 + C^*_{M,\alpha} \alpha \\
C_{M,\delta_e} &= C^*_{M,\delta_e} \\
C_T &= C^*_{T,\alpha} \beta + C^*_{T,\alpha} \\
C_T &= C^*_{T,\alpha} \beta + C^*_{T,\alpha} \\
C_T &= C^*_{T,\alpha} \beta + C^*_{T,\alpha} \\
C_T &= C^*_{T,\alpha} \beta + C^*_{T,\alpha}
\end{align*}
\]

where \( \beta \) is the engine throttle.32

The actuator can be modeled by a second-order equation with the slew rate limit described as

\[
\begin{align*}
x_1 &= \delta_e, \quad x_2 = \dot{\delta}_e \\
\dot{x}_1 &= -\sigma, \quad x_2 < -\sigma \\
\dot{x}_2 &= x_2, \quad x_2 \in [-\sigma, \sigma] \\
\dot{x}_2 &= x_2, \quad x_2 > \sigma \\
\dot{x}_2 &= -2\xi\omega_s x_2 - \omega_s^2 x_1 + \omega_s^2 \delta_e
\end{align*}
\]

where \( \xi \) is the drive voltage for the elevator, and \( \sigma \) is the slew rate limit; and the block diagram of this actuator is shown in Fig. 1.

2.2. Main problem statement

For the longitudinal dynamics of hypersonic flight, there are three modes of motion: the short period mode and the phugoid, which are also the modes of conventional flights; and the height mode, which is the unique mode of hypersonic flight. The path-attitude consonance for conventional flights can be depicted by Eq. (5)

\[
\frac{\gamma(s)}{\theta(s)} \approx \frac{1}{T \sigma s + 1}
\]

Fig. 1. Block diagram of actuator with slew rate limit.
As a $\theta$ stabilizer. This scheme has the drawback that the control variable $\delta_e$ is completely produced by two feedbacks, and it has rather low efficiency. According to the natural cause-effect relationship between $\theta$ and $\gamma$, a novel feed-forward-feedback control scheme can be proposed and plotted as shown in Fig. 3, and it is possible to speed up the $\gamma$ response. Both the inner attitude controller $C_\theta$ and the outer flight path controller $C_\gamma$ are based on ADRC.

### 3.1 Attitude control design

Combining Eqs. (1), (2) and (3) yields

$$\dot{\theta} = \dot{q}$$

$$= \left[\tau T \left(C_\gamma^\alpha \alpha^3 + C_\gamma^\alpha \alpha^2 + C_\gamma^\alpha \alpha + C_\gamma^0\right) + \rho V^2 \bar{S} \left(C_{M,\alpha}^\alpha \alpha^2 + C_{M,\alpha}^\alpha \alpha + C_{M,\alpha}^0\right)\right]/I_{yy}$$

$$+ \rho V^2 \bar{S} \bar{c} e_c \delta_e/(2I_{yy})$$

where the first term is not directly related to $\delta_e$. Here we define

$$\begin{align*}
    x_\theta & = \dot{x}_\theta + Bu_	heta \\
    \dot{x}_\theta & = f_1 \\
    y_\theta & = x_\theta \\
    u_\theta & = \delta_e
\end{align*}$$

Equation (7) can be reformulated in a state-space form as

$$\begin{align*}
    \dot{x}_\theta & = x_\theta + Bu_	heta \\
    \dot{y}_\theta & = y_\theta
\end{align*}$$

Here, $x_\theta$ is the so-called extended state, which has no physical meaning; $f_1$ is the derivative of $x_\theta$, and can be seen as a representation of all the aerodynamic uncertainties and external disturbances. $f_1$ is a low-frequency bandwidth signal because the major aerodynamic and atmospheric uncertainties are generally slow time-varying. It should be noted that only one uncertainty exists in Eq. (9) (i.e., steady gain $B$), which can be obtained with high accuracy.

The extended state observer (ESO) for Eq. (9) can be designed as

$$\begin{align*}
    \dot{z}_\theta & = z_\delta + Bu_0 + \beta_0 (y_\theta - z_\delta) \\
    \dot{z}_\beta & = \beta_0 (y_\theta - z_\beta)
\end{align*}$$

where $z_\delta$ and $z_\beta$ provide estimations for $x_\theta$ and $x_\delta$, respectively; and $L_0 = [\beta_0, \beta_0]_\tau$ represents the observer gain vector. By placing the closed-loop poles of the observer at $-\omega_{o_0}$, the ESO can be parameterized as

$$L_0 = \left[2\omega_{o_0}, \omega_{o_0}^2\right]_\tau$$

where $\omega_{o_0}$ is the bandwidth of the observer.

With a well-tuned observer, one can obtain $z_\delta \approx x_\theta$. If the control signal is

$$u_0 = \frac{u_\theta - z_\delta}{B}$$

graphs and tables are not available in the natural text representation.
where $u_0$ is a virtual control variable, the original system (7) can be appropriately reformulated as

$$\dot{\theta} = u_0$$  \hspace{1cm} (13)

Since the regulated output is $\theta$, the classical PD controller for $u_0$ can be used, and can be designed as

$$u_0 = k_p(\theta_{ref} - \theta) - k_d q$$  \hspace{1cm} (14)

where $\theta_{ref}$ represents the reference of $\theta$. In all, the ADRC law for $\theta$ control is

$$u_0 = \frac{k_p(\theta_{ref} - \theta) - k_d q - z_{\theta}}{b}$$  \hspace{1cm} (15)

Since the controller is linear, it is called “linear active disturbance rejection control” (LADRC).33) LADRC can be applied to a wide range of nonlinear, time-varying, uncertain processes with very little model information because it can treat all nonlinear dynamics and uncertainties as estimated signals. From a signal point of view, they have no distinction at all. Note that this LADRC can be regarded as an extension of the PID controller.33)

### 3.2. Flight path control design

According to Fig. 3, the reference for the pitch angle is

$$\theta_{ref} = K\gamma_{ref} + u_\gamma$$  \hspace{1cm} (16)

where $K$ is the feed-forward coefficient; and $\gamma_{ref}$ and $u_\gamma$ represent the reference and the control variable, respectively, for the flight path angle.

Combining Eqs. (1), (2) and (3) gives rise to

$$\dot{\gamma} = \frac{1}{m V} \left[ C_0^\gamma \alpha^3 + C_1^\gamma \alpha^2 + C_2^\gamma \alpha + C_3^\gamma + \frac{1}{2} \rho V^2 S C_L^\gamma \right] \gamma$$

$$+ \frac{1}{2} \rho V^2 S (C_\delta^\gamma \delta_c + C_\theta^\gamma) - mg \cos \gamma$$

$$+ \frac{1}{m V} \left( C_0^\theta \alpha^3 + C_1^\theta \alpha^2 + C_2^\theta \alpha + C_3^\theta + \frac{1}{2} \rho V^2 S C_L^\theta \right) \theta$$  \hspace{1cm} (17)

Besides the effect of $\delta_c$, other components in the first term are all related to $\gamma$, a relatively slow variable compared to $\dot{\theta}$. On the other hand, $\delta_c$ plays a small role in changing $\gamma$, which is often used as a non-minimum phase benchmark for control design. In practice, this minimal effect can be neglected. Therefore, considering Eq. (16), Eq. (17) can be rewritten as

$$\dot{\gamma} = a + bu_\gamma$$  \hspace{1cm} (18)

where

$$a = \left[ C_0^\gamma \alpha^3 + C_1^\gamma \alpha^2 + C_2^\gamma \alpha + C_3^\gamma + \frac{1}{2} \rho V^2 S C_L^\gamma \right] (K\gamma_{ref} - \gamma) + \frac{1}{2} \rho V^2 S C_L^\gamma$$

$$b = \frac{1}{m V} \left( C_0^\theta \alpha^3 + C_1^\theta \alpha^2 + C_2^\theta \alpha + C_3^\theta + \frac{1}{2} \rho V^2 S C_L^\theta \right)$$  \hspace{1cm} (19)

Following the similar procedure illustrated in Sub-section 3.1, Eq. (17) can also be rewritten as

$$\begin{align*}
\dot{x}_1 &= x_2 + bu_\gamma \\
\dot{x}_2 &= f_2 \\
y_\gamma &= x_1
\end{align*}$$  \hspace{1cm} (20)

where $x_1 = \gamma$, $x_2 = a$, $x_3$ is the extended state, and $f_2$ is assumed to be the total disturbance.

Using the similar extended state observer as Eq. (10), the estimation of $x_3$ can be obtained as $z_\gamma$ by

$$\begin{align*}
\dot{z}_1 &= z_2 + bu_\gamma + \beta_1 (y_\gamma - z_\gamma) \\
\dot{z}_2 &= \beta_2 (y_\gamma - z_\gamma)
\end{align*}$$  \hspace{1cm} (21)

Similarly, $L_\gamma = [\beta_1, \beta_2]^T$ represents observer gain vector, and can also be parameterized as

$$L_\gamma = \begin{bmatrix} 2\omega_{\text{aoa}}, \omega_{\text{aoa}}^2 \end{bmatrix}^T$$  \hspace{1cm} (22)

It should be noted that $\omega_{\text{aoa}}$ should be smaller than $\omega_{\text{aoa}}$, due to the faster response speed of the inner loop than the outer loop. In general, the control law of $\gamma$ is

$$u_\gamma = \frac{K_p (\gamma_{ref} - \gamma) - z_{\gamma_2}}{b}$$  \hspace{1cm} (23)

It should be noted that the feed-forward component in Eq. (16), or $K\gamma_{ref}$, is also included in the extended state. However, due to the lag of the observer, the observer error caused by large $K$ is helpful to raise the closed-loop speediness of the flight path. This can be illustrated by the simulation examples in Section 4. This is the essential point in this paper to avoid high-gain flight path angle feedback.

### 3.3. Stability margin tester tuning method

When the form of a control law is determined, the subsequent procedure is tuning the controller, which is a key step to achieve good robustness and dynamic performance. In fact, hundreds of PID tuning rules have emerged for practitioners in the past half century.34) A systematical tuning method is also indispensable for the proposed LADRC scheme. The completely linear form of LADRC leads to a possible analysis in the frequency domain, which is commonly acceptable in practice. The analysis of robustness is very crucial for control system design. Gain and phase margins (GPM) have always served as important measures of robustness.33,35) Moreover, it is known from classical control theory that the phase margin is related to the damping of the system and therefore can also serve as a performance measure. Hypersonic flight dynamics can be linearized around the trimming point based on the small perturbation assumption, which holds true for air-breathing engines because it can only operate with a small angle of attack. Therefore, a D-decomposition graphical tuning approach, which is called a “stability margin tester,”23,36) can be employed to tune the controller to make the system meet the specific GPM requirement.

For flight attitude control, the open-loop transfer function should be reformulated to a linear function of $G_o$ (i.e., the
open-loop plant) in order to carry out robustness analysis. This linear model can be obtained by linearizing the longitudinal dynamic model, Eq. (1), based on the small perturbation theory as

\[
\dot{y} = b_a \alpha + b_3 \delta_v + b_r \gamma
\]
\[
\dot{q} = a_\alpha + a_\delta \delta_v
\]
\[
\dot{\theta} = q
\]
\[
\alpha = \theta - \gamma
\]  
(24)

where \( b_a, b_3 \) and \( b_r \) are the derivatives due to \( \alpha, \delta_v \) and \( \gamma \), respectively. It should be noted that \( b_r \) is too small to have an observable effect on the dynamic performance of hypersonic vehicles, and it is reasonable to assume \( b_r = 0 \). \( a_\alpha \) and \( a_\delta \) are the derivatives with respect to \( \alpha \) and \( \delta_v \), respectively. \(^{23}\)

Equation (24) can be reformulated in the frequency domain as

\[
G_1(s) = \frac{q(s)}{\delta_v(s)} = \frac{a_\delta s + a_\delta b_a - a_\delta b_3}{s^2 + b_b s - a_\delta}
\]  
(25)

According to Eq. (4), the linearized dynamic of the elevator is

\[
G_2(s) = \frac{a_\delta^2}{s^2 + 2 \xi \omega_n s + \omega_n^2}
\]  
(26)

Combining Eqs. (25) and (26), the open-loop plant becomes

\[
G_p(s) = G_1(s) G_2(s)
\]  
(27)

By some straightforward mathematical manipulations based on Eq. (10), one can obtain

\[
z_{d} = \frac{a_\delta^2 (sq - B \delta_v)}{(s + \omega_n)^2}
\]  
(28)

which is a second-order filtered differentiator of \( q \) if the part produced directly by the elevator or just the extended state \( x_{d} \) is not considered. Substituting Eq. (28) for \( z_{d} \) in Eq. (15) yields

\[
u_0 = u_{PD} + \frac{a_\delta^2}{s + 2 \omega_n} \left( \frac{u_{PD}}{s} - \frac{q}{B} \right)
\]  
(29)

where

\[
u_{PD} = \frac{k_p (\theta_{ref} - \theta) - k_d q}{B}
\]  
(30)

Combining Eqs. (25) – (30), one can obtain the closed-loop transfer function for attitude control as

\[
G_\alpha(s) = \frac{k_p (s + \omega_n)^2 G_p(s)}{B s^2 (s + 2 \omega_n) + (s + \omega_n)^2 (k_p + k_d s) + a_\delta^2 s^2} G_p(s)
\]  
(31)

According to Eq. (31), the open-loop transfer function can be obtained as

\[
G_\alpha(s) = \frac{(s + \omega_n)^2 (k_p + k_d s) + a_\delta^2 s^2}{B s^2 (s + 2 \omega_n) + (s + \omega_n)^2 (k_p + k_d s) + a_\delta^2 s^2} G_p(s)
\]  
(32)

with non-unit feedback. Letting \( s = j \omega \) gives rise to

\[
G_\alpha(j \omega) = \left[ (\omega + \omega_n)^2 (k_p + j k_d \omega) - a_\delta^2 \omega^2 \right] G_p(j \omega)
\]
\[
- B \omega^2 (\omega + 2 \omega_n)
\]  
(33)

which can be represented by two parts, the real part and the imaginary part

\[
G_\alpha(j \omega) = \text{Re}[G_\alpha(j \omega)] + j \text{Im}[G_\alpha(j \omega)]
\]  
(34)

Equation (34) can also be expressed as

\[
G_\alpha(j \omega) = |G_\alpha(j \omega)| e^{j \phi}
\]  
(35)

From Eq. (33) to Eq. (35), one has

\[
D(j \omega) G_\alpha(j \omega) [e^{j \phi} - N(j \omega)] = 0
\]  
(36)

i.e.,

\[
D(j \omega) - \frac{1}{|G_\alpha(j \omega)|^{j \phi} N(j \omega)} = 0
\]  
(37)

Define

\[
A = 1/|G_\alpha(j \omega)|
\]  
(38)

and

\[
\Theta = \phi + \pi
\]  
(39)

Then, Eq. (37) becomes

\[
\frac{D(j \omega)}{1 + A e^{-j \Theta} N(j \omega)} = 0
\]  
(40)

Note that \( A \) is the gain margin of the system if \( \Theta = 0 \), and \( \Theta \) is the phase margin of the system if \( A = 1 \). The physical meaning of Eq. (40) is that the gain margin and phase margin of a system can be determined by the gain-phase margin tester \( A e^{-j \Theta} \). Equation (40) can also be rewritten as

\[
1 + A e^{-j \Theta} G_\alpha(j \omega) = 0
\]  
(41)

The gain-phase margin tester \( A e^{-j \Theta} \) can be described as

\[
A e^{-j \Theta} = A \cos \Theta - j A \sin \Theta = X - j Y
\]  
(42)

where \( X = A \cos \Theta \) and \( Y = A \sin \Theta \). Combining Eq. (40) with Eq. (42), one has

\[
F(j \omega) = D(j \omega) + (X - j Y) N(j \omega) = 0
\]  
(43)

which can be depicted as

\[
F(j \omega) = F_r(k_p, k_d, A, \Theta, \omega) + j F_i(k_p, k_d, A, \Theta, \omega) = 0
\]  
(44)

Equation (44) is equivalent to

\[
\begin{cases}
F_r(k_p, k_d, A, \Theta, \omega) = 0 \\
F_i(k_p, k_d, A, \Theta, \omega) = 0
\end{cases}
\]  
(45)

Equation (45) is a set of linear functions of \( k_p \) and \( k_d \) as

\[
\begin{cases}
F_r(k_p, k_d, A, \Theta, \omega) = B_1 k_p + C_1 k_d + D_1 = 0 \\
F_i(k_p, k_d, A, \Theta, \omega) = B_2 k_p + C_2 k_d + D_2 = 0
\end{cases}
\]  
(46)

where

\[
B_1 = (-\omega^2 X + \omega^2_n X + 2 \omega_n \omega Y) \text{Re} G_p(j \omega)
\]
\[
+ (-2 \omega_n \omega X - \omega^2 Y + \omega^2_n Y) \text{Im} G_p(j \omega)
\]  
(47)

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wherein there are two requirements: the feedbacks with the

Table 1. Constant parameters.

| Parameter | Value | Parameter | Value |
|-----------|-------|-----------|-------|
| $C^1_2$ (rad$^{-1}$) | 4.6773 | $C^1_0$ (rad$^{-1}$) | 2.7699 $\times 10^{-4}$ |
| $C^2_2$ (rad$^{-1}$) | 7.6224 $\times 10^{-1}$ | $C^2_0$ (rad$^{-1}$) | 1.0131 $\times 10^{-2}$ |
| $C^3_2$ (rad$^{-1}$) | $-1.8714 \times 10^{-2}$ | $C^3_0$ (rad$^{-2}$) | 6.2926 |
| $C^4_2$ (rad$^{-2}$) | 5.8224 | $C^4_0$ (rad$^{-1}$) | 2.1335 |
| $C^5_2$ (rad$^{-1}$) | $-4.5315 \times 10^{-2}$ | $C^5_0$ (rad$^{-2}$) | 8.1993 $\times 10^{-1}$ |
| $c_s$ (rad$^{-1}$) | $-1.2897$ | $\zeta_r$ (m) | 2.5481 |
| $C^1_2$, $\omega_1$ | $-3.7693 \times 10^5$ | $C^1_0$, $\omega_1$ | $-3.7225 \times 10^4$ |
| $C^2_2$, $\omega_1$ | $2.6814 \times 10^4$ | $C^2_0$, $\omega_1$ | $-1.7277 \times 10^4$ |
| $C^3_2$, $\omega_1$ | $3.5542 \times 10^4$ | $C^3_0$, $\omega_1$ | $-2.4216 \times 10^5$ |
| $C^4_2$, $\omega_1$ | $6.3785 \times 10^4$ | $C^4_0$, $\omega_1$ | $-1.0090 \times 10^5$ |
| $T_r$ (kg·m$^2$) | 1.1524 $\times 10^7$ | $\rho$ (kg·m$^{-3}$) | $3.4800 \times 10^{-2}$ |
| $S$ (m$^2$) | 26.849 | $\omega_a$ (rad$^{-1}$) | 150 |
| $\xi$ | 0.7 | $m$ (kg) | 7.4429 $\times 10^4$ |

Fig. 4. Feasible phase margin curves.

where $k_p$ and $k_d$ can be selected in the given intersection region by appropri-ately specifying $A$ and $\Theta$, such that a certain robustness can be ensured.

3.4. Overall performance and stability analysis

The system stability and control performance are the two major concerns for this proposed flight path control scheme, and they need to be investigated. As can be seen, this strategy is composed of LADRCs and a feed-forward term to achieve good control performance. However, LADRC is not a Lyapunov-based stability constructive approach as can be seen from its control structure and only using measurement outputs. Additionally the closed-loop stability analysis containing ADRC is rather difficult, or even an impossible task, as illustrated in Huang and Xue$^{37}$ and Guo and Zhao,$^{38}$ wherein there are two requirements: the feedbacks with the number of full-state and sufficiently high bandwidth of
However, these two requirements are unable to coincide with practical applications. Therefore, we attempt to seek other ways to investigate these two concerns. Due to the angle of attack limitation from the requirements of air-breathing engines, the linearized model is sufficient to represent the essential dynamics of a hypersonic vehicle. The dynamic performance amelioration will be interpreted in Appendix A. Based on a Lyapunov indirect method, the system closed-loop stability is analyzed around its sole equilibrium point, which is obtained using the numerical optimization method explained in Appendix B. It should be noted that to simplify the investigation process without loss of generality, reduced-order observers for Eq. (9) and Eq. (20) are respectively employed instead of the original second-order observers.

4. Simulation Examples

In this section, several simulation examples are presented to demonstrate the effectiveness of the proposed control scheme.

The characteristic parameters of Eqs. (2) and (3) are listed in Table 1. The model is valid for $-4^\circ \leq \alpha \leq 4^\circ$. The initial conditions of the simulation are consistently set with $\alpha = 0$ rad, $\theta = 0$ rad, $\gamma = 0$ rad, $q = 0$ rad/s, $V = 4590.288$ m/s and $h = 20000$ m. Here, the slew rate limit $\sigma$ of the actuator is chosen as 100 deg/s.

At first, the pitch attitude controller needs to be tuned. The observer bandwidth is set as $\omega_0 = 20$ rad/s according to the empirical knowledge and $B$ is set as its nominal value of $B = -5.55$. The stability margin testers are shown in Fig. 4. There are three boundaries: $\Lambda = 1$ and $\Theta = 0$ for
the nominal stability; \( A = 2 \) and \( \theta = 0 \) for the 6 dB gain margin; and \( A = 1 \) and \( \theta = 45 \) deg for the 45 deg phase margin. According to Fig. 4, we can select \( k_p = 50 \) and \( k_d = 20 \). The corresponding step responses are shown in Fig. 5. From Fig. 5(a) and Fig. 5(b), we can find that the ESO can achieve accurate state estimation. According to Fig. 5(c), it can be observed that satisfactory control performance for the pitch angle can be achieved. Meanwhile, there is a considerable lag of about 25 s between the flight path angle and pitch angle.

The next simulation is to evaluate the capability of the proposed feed-forward-feedback scheme to track a specified flight path reference. The observer bandwidth of \( C_v \) is chosen as \( \omega_{o2} = 10 \) rad/s, which is smaller than \( \omega_{o1} = 20 \) rad/s and \( b \) is set as five times its nominal value of 0.135. We select \( K_p = 0.8 \). The effects of different feed-forward gains on flight path tracking are shown in Fig. 6. It is found that increasing \( K \) is an effective measure for increasing the response speed of the flight path without using large feedback gains, which is a primary concern in practice. The cost for this increased speed is a larger angle of attack requirement. With the proposed comprehensive scheme, excellent tracking performance for the flight path angle can be achieved. It is obvious that the lag between the flight path angle and pitch angle has been reduced greatly. From Fig. 6(c) and Fig. 6(d), it can be seen that the estimation accuracy is acceptable. When \( \omega_{o2} = 0 \), the outer LADRC law degenerates into a simple proportional controller and it is unable to achieve satisfactory performance as shown in Fig. 6(e). When \( \omega_{o2} = 0 \), the speed of the inner attitude response cannot match the requirement from the outer flight path angle, and this mismatch can cause significant attitude oscillation. Therefore, we can conclude that the two LADRCs and the feed-forward term are all indispensable components of this comprehensive flight path control scheme.

Finally, this scheme is used for the altitude control of a hypersonic vehicle. In this case, \( \gamma_{ref} \) is to make the hypersonic vehicle climb from the current cruising altitude of 20,000 m to the altitude of \( h_{ref} = 22,000 \) m. We design \( \gamma_{ref} \) as

\[
\gamma_{ref} = \begin{cases} 
    k_1(t - t_0)/57.3 & t_0 \leq t \leq t_s \\
    k_2(h_{ref} - h)/57.3 & t > t_s
\end{cases} \tag{55}
\]

where \( k_1 = 0.210 \) and \( k_2 = 0.002 \), \( t_0 \) represents the initial time to climb, \( t \) is the time variable, and \( t_s \) represents the switching instant when the value of the first part of Eq. (55) is equal to the value of the second part of Eq. (55). Equation (55) is a combination of the open-loop and closed-loop altitude references. At first, the first equation of Eq. (55), which is a time series, takes effect to trigger an upward motion with sufficient speed when the altitude error is large, such that command saturation can be avoided. Then, the altitude closed-loop control starts to ensure the control precision when the altitude error falls within a small range. This is also a practical altitude control mechanism. The corresponding simulation results are shown in Fig. 7. Figure 7(a) shows that the angle of attack is within its limit; and the angular behaviors are excellent, as shown in Fig. 7(b). According to Fig. 7(c), it can be demonstrated that it takes only 30 s for the hypersonic vehicle to climb 2,000 m. The effectiveness of this method in altitude control is evident.

5. Conclusion

A comprehensive flight path control scheme for hypersonic vehicles was proposed to attenuate the severe lag between the flight path angle and pitch angle, where the feed-forward channel feeds the flight path reference into the attitude loop directly in order to speed up response while maintaining attitude stability. A linear active disturbance rejection controller, an extension of the PID controller, was utilized for both attitude and flight path angles. The stability margin tester was used to tune the attitude controller. Finally, the effectiveness
of the proposed method was demonstrated through numerical simulations. The proposed scheme can be used as a guideline for practitioners.

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References

1) Petersen, C., Baldwin, M., and Kolmanovsky, I.: Model Predictive Control Guidance with Extended Command Governor Inner-Loop Flight Control for Hypersonic Vehicles, AIAA Guidance, Navigation, and Control Conference, Boston, MA, 2013, pp. 1–20.
2) Sun, H., Yang, Z., and Zeng, J.: New Tracking-Control Strategy for Airbreathing Hypersonic Vehicles, J. Guid., Control Dynam., 36 (2013), pp. 846–859.
3) Sridharan, S., Rodriguez, A., Dickeson, J., and Soloway, D.: Constraint Enforcement and Robust Tube-Based Control for Scramjet-Powered Hypersonic Vehicles with Significant Uncertainties, American Control Conference, Montreal, QC, 2012, pp. 4619–4624.
4) Huang, Y., Sun, C., Qian, C., Zhang, J., and Wang, L.: Polytopic LPV Modeling and Gain-Scheduled Switching Control for a Flexible Air-Breathing Hypersonic Vehicle, J. Syst. Eng. Electronics, 24 (2013), pp. 118–127.
5) Sachs, G.: Path-Attitude Decoupling and Flying Qualities Implications in Hypersonic Flight, Aerospace Sci. Technol., 2 (1998), pp. 49–59.
6) Sachs, G.: Increase and Limit of Tg in Super- and Hypersonic Flight, J. Guid. Control Dynam., 22 (1998), pp. 181–183.
7) Sachs, G.: Longitudinal Long-Term Modes in Super- and Hypersonic Flight, J. Guid. Control Dynam., 28 (2005), pp. 539–541.
8) Xu, H. J., Mirrirmani, M. D., and Ioannou, P. A.: Adaptive Sliding Mode Control Design for a Hypersonic Flight Vehicle, J. Guid. Control Dynam., 27 (2004), pp. 829–838.
9) Fiorentini, L., Serrani, A., Bolender, M. A., and Doman, D. B.: Non-Linear Robust Adaptive Control of Flexible Air-breathing Hypersonic Vehicles, J. Guid. Control Dynam., 32 (2009), pp. 401–416.
10) Xu, B., Gao, D. X., and Wang, S. X.: Adaptive Neural Control Based on HGO for Hypersonic Flight Vehicles, Science in China-Information Science, 54 (2011), pp. 511–520.
11) Gao, D. X. and Sun, Z. Q.: Fuzzy Tracking Control Design for Hypersonic Vehicles via T-S Model, Science in China-Information Science, 54 (2011), pp. 521–528.
12) Wang, Q. and Stengel, R. F.: Robust Nonlinear Control of a Hypersonic Aircraft, J. Guid. Control Dynam., 23 (2000), pp. 577–585.
13) Buschek, H. and Calise, A. J.: Uncertainty Modeling and Fixed-Order Controller Design for a Hypersonic Vehicle Model, J. Guid. Control Dynam., 20 (1997), pp. 42–48.
14) Brett, D. R. and McFarland, M. B.: Tailoring Theory to Practice in Tactical Missile Control, IEEE Control Syst. Magazine, 19 (1999), pp. 49–55.
15) Han, J. Q.: From PID to Active Disturbance Rejection Control, IEEE Trans. Ind. Electronics, 56 (2009), pp. 900–906.
16) Youcef-Toumi, K. and Wu, S. T.: Input/Output Linearization Using Time Delay Control, J. Dynam. Syst. Meas. Control, 114 (1992), pp. 10–19.
17) Lee, C. H., Kim, T. H., and Tahk, M. J.: Agile Missile Autopilot Design Using Nonlinear Backstepping Control with Time-Delay Adaptation, T. Jpn. Soc. Aeronaut. Space Sci., 57 (2014), pp. 1–8.
18) Zhong, Q. C. and David, R.: Control of Uncertain LTI Systems Based on an Uncertainty and Disturbance Estimator, ASME J. Dynam. Syst. Meas. Control, 126 (2004), pp. 905–910.
19) Talole, S. E. and Phadke, S. B.: Model Following Sliding Mode Control Based on Uncertainty and Disturbance Estimator, ASME, J. Dynam. Syst. Meas. Control, 130 (2008), 034501.
20) Talole, S. E., Godbole, A. A., and Kolhe, J. P.: Robust Roll Autopilot Design for Tactical Missiles, J. Guid. Control Dynam., 34 (2011), pp. 107–117.
21) Godbole, A. A., Libin, T. R., and Talole, S. E.: Extended State Observer-Based Robust Pitch Autopilot Design for Tactical Missiles, Proc. IMechE Part G: J. Aerospace Eng., 226 (2012), pp. 1482–1501.
22) Zhu, E., Pang, J., Sun, N., Gao, H., Sun, L., and Chen, Z.: Airship Horizontal Trajectory Tracking Control Based on Active Disturbance Rejection Control (ADRC), Nonlinear Dynamics, 75 (2014), pp. 725–734.
23) Sun, M., Wang, Z., and Chen, Z.: Practical Solution to Attitude Control within Wide Envelope, Aircraft Eng. Aerospace Technol.: Int. J., 86 (2014), pp. 117–128.
24) Wang, Y., Sun, M., Wang, Z., Liu, Z., and Chen, Z.: A Novel Disturbance-Observer Based Friction Compensation Scheme for Ball and Plate System, ISA Trans., 53 (2014), pp. 671–678.
25) Huang, C. E., Li, D. H., and Xue, Y. L.: Active Disturbance Rejection Control for the ALSTOM Gasifier Benchmark Problem, Control Eng. Practice, 21 (2013), pp. 556–564.
26) Sun, M., Wang, Z., Wang, Y., and Chen, Z.: On Low-Velocity Compensation of Brushless DC Servo in the Absence of Friction Model, IEEE Trans. Ind. Electronics, 60 (2013), pp. 3897–3905.
27) Przybyla, M., Kordasz, M., Mado, R., Herman, P., and Sauer, P.: Active Disturbance Rejection Control of a 2DOF Manipulator with Significant Modeling Uncertainty, Bull. Polish Acad. Sci. Tech. Sci., 60 (2012), pp. 509–520.
28) Goforth, F. J., Zheng, Q., and Gao, Z.: A Novel Practical Control Approach for Rate Independent Hysteretic Systems, ISA Trans., 51 (2012), pp. 477–484.
29) Li, S., Xia, C., and Zhou, X.: Disturbance Rejection Control Method for Permanent Magnet Synchronous Motor Speed-Regulation System, Mechatronics, 22 (2012), pp. 706–714.
30) Zheng, Q. and Gao, Z.: Predictive Active Disturbance Rejection Control for Processes with Time Delay, ISA Trans., 53 (2014), pp. 873–881.
31) Liu, H. and Li, S.: Speed Control for PMSM Servo System Using Predictive Control and Extended State Observer, IEEE Trans. Ind. Electronics, 59 (2012), pp. 1171–1183.
32) Parker, J. T., Serrani, A., Yurkovich, S., Bolender, M. A., and Donman, D. B.: Control-Oriented Modeling of an Air-Breathing Hypersonic Vehicle, J. Guid. Control Dynam., 30 (2007), pp. 856–869.
33) Gao, Z.: Scaling and Bandwidth-Parameterization Based Controller Tuning, Proceedings of the America Control Conference, Denver, CO, USA, 2003, pp. 4989–4996.
34) Lee, C. H.: A Survey of PID Controller Design Based on Gain and Phase Margins, Int. J. Comput. Cognition, 2 (2004), pp. 63–100.
35) Sun, M., Jiao, G., Yang, R., and Chen, Z.: ADRC for Stability Unstable Longitudinal Flight Dynamics, Proceedings of the 30th Chinese Control Conference, Yantai, China, 2011, pp. 6274–6280.
36) Chang, C. H. and Han, K. W.: Gain Margins and Phase Margins for Control Systems with Adjustable Parameters, J. Guid. Control Dynam., 13 (1990), pp. 404–408.
37) Huang, Y. and Xue, W. C.: Active Disturbance Rejection Control: Methodological and Theoretical Analysis, ISA Trans., 4 (2014), pp. 963–976.
38) Gao, B. and Zhao, L.: On Convergence of the Nonlinear Active Disturbance Rejection Control for MIMO Systems, SIAM J. Control Optimization, 2 (2013), pp. 1727–1757.
39) Khalil, H. K.: Nonlinear Systems, Prentice Hall, New Jersey, 1996.

Appendix

A. Analysis of the effect of feed-forward

Here, whether or not the dynamic performance of the flight path angle when using the feed-forward term can be improved is to be revealed.

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The reduced-order observer for Eq. (9) is chosen as

\[
\begin{align*}
\dot{z}_\text{r} &= -\omega_{\text{r}}(Bu_\text{r} + z_\text{r}) \\
\dot{z}_\text{o} &= z_\text{r} + \omega_{\text{r}} q
\end{align*}
\]  
(A.1)

and the one for Eq. (20) is designed as

\[
\begin{align*}
\dot{z}_\text{r} &= -\omega_{\text{r}}(Bu_\text{r} + z_\text{r}) \\
\dot{z}_\text{o} &= z_\text{r} + \omega_{\text{r}} \gamma
\end{align*}
\]  
(A.2)

where \( z_\text{r} \) and \( z_\text{o} \) are the intermediate variables. By some straightforward mathematical manipulations based on Eq. (A.1) and Eq. (A.2), one can obtain

\[
\dot{z}_\text{o} = \frac{\omega_{\text{r}}(sq - B\delta_\text{r})}{s + \omega_{\text{r}}} 
\]  
(A.3)

and

\[
\dot{z}_\text{r} = \frac{\omega_{\text{r}}(s\gamma - Bu_\text{r})}{s + \omega_{\text{r}}} 
\]  
(A.4)

Note that both (A.3) and (A.4) can be regarded as the first-order filtered differentiators of the respective outputs if we do not consider the parts produced directly by the corresponding inputs. The original second-order observers can also be reformulated as second-order filtered differentiators. Substituting Eq. (A.3) and Eq. (A.4) into Eq. (15) and Eq. (23), respectively, yields

\[
\begin{align*}
\delta_x &= u_\theta = \frac{k_p(\theta_r - \theta) - k_d q}{B} + \omega_{\text{r}} \left[ \frac{k_p(\theta_r - \theta) - k_d q}{B_s} - \frac{q}{B} \right] \\
u_\gamma &= \frac{K_p(\gamma_r - \gamma)}{b} + \omega_{\text{r}} \left[ \frac{K_p(\gamma_r - \gamma)}{B_s} - \frac{\gamma}{b} \right]
\end{align*}
\]  
(A.5)

Combining Eqs. (16), (A.5) and the linearized plant of Eq. (24), the closed-loop transfer function of the overall system can be obtained as

\[
\begin{align*}
\frac{\gamma}{\gamma_r} &= \frac{N_c(s)}{D_c(s)}
\end{align*}
\]  
(A.6)

where

\[
\begin{align*}
N_c(s) &= k_p(s + \omega_{\text{r}}) [(Kb + Kp)s + \omega_{\text{r}} Kp] G_p \\
D_c(s) &= bs \left[ \frac{1}{b_s} + 1 \right] \left[ \frac{ks + k_p(s + \omega_{\text{r}})}{b_s} \right] G_p + k_p(s + \omega_{\text{r}}) [Kp_s + \omega_{\text{r}} Kp + \omega_{\text{r}} s] G_p
\end{align*}
\]  
(A.7)

According to \( N_c(s) \), it can be found that the feed-forward term results in a phase lead by increasing \( Kp_s \) to \((Kb + Kp)s\). A faster response for the flight path angle can thus be expected.

**B. Analysis of overall stability**

On the basis of Appendix A, the overall closed-loop stability will be analyzed for the original nonlinear system.

Substituting Eq. (A.5) and Eq. (16) into Eq. (1), and only considering the short period variables (except \( V \) and \( h \)), the closed-loop system is

\[
\begin{align*}
\dot{\gamma} &= \frac{1}{m} \left[ \left( C^q_1(\theta - \gamma)^3 + C^q_2(\theta - \gamma)^2 + C^q_3(\theta - \gamma) + C^q_4 \right) \sin(\theta - \gamma) + \frac{1}{2} \rho V^2 S(C^q_1(\theta - \gamma) + C^q_2 \delta_e + C^q_3) \\
&\quad - mg \cos \gamma \right] \\
\dot{\delta}_e &= q \\
\dot{x} &= \frac{1}{b_s} \left[ (k_p Kb + k_p Kp) \gamma_r + (k_p \omega_{\text{r}} Kp + \omega_{\text{r}} k_p Kb \right. \\
&\quad + \omega_{\text{r}} k_p Kp) \gamma_r + (k_p Kp - k_p \omega_{\text{r}}) \dot{\gamma} + (- k_p \omega_{\text{r}} Kp \\
&\quad - \omega_{\text{r}} k_p Kp - \omega_{\text{r}} k_p \omega_{\text{r}}) \gamma_r - \omega_{\text{r}} k_p b \dot{\theta} - k_p \dot{\theta} \\
&\quad + (- k_d b - \omega_{\text{r}} b) \dot{q} - \omega_{\text{r}} k_d \dot{b} \dot{q} \\
&\quad + \omega_{\text{r}} k_p \omega_{\text{r}} Kp(\gamma_r - \gamma) \right]
\end{align*}
\]  
(B.1)

According to Eq. (B.1), the sole equilibrium point can be obtained as \( [\gamma_r, 0, \theta_0, \delta_e, 0]^T \) where the values of \( \theta_0, \delta_e \) can be determined with respect to each specific \( \gamma_r \) using the nonlinear optimization function \( \text{fsolve} \) in Matlab.

Then, the Jacobian matrix of Eq. (B.1) can be obtained around its sole equilibrium point. The eigenvalues of the Jacobian matrix are calculated with respect to each sampling \( \gamma_r \) in \([0, 6 \, \text{deg}]\) as shown in Fig. B1. It can be observed that the real parts of all eigenvalues are negative. Therefore according to the Lyapunov indirect method, the original nonlinear closed-loop system is asymptotically stable.

Y. Ochi

Associate Editor