Simulations of coherent nonlinear neutrino flavor transformation in the core collapse supernova environment

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Abstract. We describe a set of codes developed under the auspices of the Terascale Supernova Initiative to simulate the coherent, nonlinear evolution of the neutrino and antineutrino fields in the core collapse supernova environment. Ours are the first simulations to include quantum entanglement of neutrino flavor evolution on different neutrino trajectories. We find that neutrinos and antineutrinos can undergo coherent, collective transformations of their flavors which could affect supernova dynamics and nucleosynthesis.

1. Introduction

Macroscopic quantum coherence is a very rare phenomenon in nature. Our codes reveal, however, that the neutrinos and antineutrinos streaming away from a hot, post-core-collapse neutron star exhibit just that. This is not an idle curiosity either, since most of the energy available in stellar core collapse (eventually $\sim 10^{53}$ erg, or 10% of the core rest mass) resides in seas of neutrinos and antineutrinos of all flavors ($\nu_e$, $\bar{\nu}_e$, $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_\tau$ and $\bar{\nu}_\tau$), and energy/entropy transport in this environment is dominated by neutrinos and neutrino interactions with matter. Energy/entropy deposition and the ratio of neutrons to protons in this environment principally involves $\nu_e$ and $\bar{\nu}_e$ through the processes $\nu_e + n \rightarrow e^- + p$ and $\bar{\nu}_e + p \rightarrow e^+ + n$. Furthermore, it may be that the energy spectra and/or the fluxes of $\nu_e$, $\bar{\nu}_e$, and $\nu_\mu\bar{\nu}_\mu\nu_\tau\bar{\nu}_\tau$ neutrinos differ. If they do, then the transformation of neutrino flavors $\nu_e \leftrightarrow \nu_\mu, \nu_\tau$ and $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu, \bar{\nu}_\tau$ could make a significant difference in how the supernova evolves and in what elements it creates. In turn, the macroscopic quantum coherence of the neutrino and antineutrino fields dictates how neutrino flavor transformation proceeds.

Neutrinos diffuse with short mean free paths inside the proto-neutron star, but decouple at a “neutrino sphere” at the location of steep matter density gradient at the star’s surface. Subsequently, above the proto-neutron star, neutrinos more or less freely stream at the speed of light. Neutrino emission from the neutrino sphere is incoherent and thermal in nature with the result that all neutrino species initially have nearly thermal, black body, Fermi-Dirac energy spectra. However, a neutrino’s flavor state $|\Psi(t)\rangle$ evolves nearly coherently along its world line (parametrized by $t$) as it propagates away from the neutrino sphere.

We have employ a mean-field Schrödinger-like equation to model flavor development along a
given neutrino trajectory:

\[ i \frac{\partial}{\partial t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle, \]

where, e.g., \( \langle \nu_e |\Psi(t)\rangle \) and \( \langle \nu_\tau |\Psi(t)\rangle \) are the amplitudes for the neutrino to be a \( \nu_e \) and \( \nu_\tau \), respectively, at a location along a neutrino’s world line given by parameter \( t \). A similar equation describes antineutrino flavor evolution. Here \( \hat{H} \) is the effective neutrino propagation Hamiltonian. It consists of a vacuum term dependent on neutrino mass-squared differences and on potentials arising from neutrino-electron and neutrino-neutrino forward scattering. This last potential renders neutrino flavor evolution non-linear in the sense that the flavor development of the neutrino field at any location depends on the flavor states of the neutrinos there [1, 2, 3].

Neutrino-neutrino forward scattering also can cause flavor evolution histories on intersecting trajectories to be coupled. This quantum entanglement is graphically illustrated in Figure 1.

2. FLAT: a versatile numerical code for neutrino oscillation problems

We have developed two independent sets of numerical codes using two different computer languages. We used them to provide cross checks to obtain consistent results. Here we highlight one of them, which is called FLAT.

FLAT is written in C++ and has employed many advanced features of the Object-Oriented Programming (OOP) paradigm. It is composed of a collection of modules (or “Classes” in C++). The internal implementation of each module is hidden behind an abstract interface (the names and arguments of the member functions of a class). The modules with the same interface are completely interchangeable. For example, in our problem, the physics and the numerical algorithms are fairly independent of the specific details of the Hamiltonian operator \( \hat{H} \) and the neutrino state \( |\Psi(t)\rangle \), which we implement in the NeuBin modules. Most of the time the remainder of the program will ask NeuBin to do only three things: (1) compute \( \hat{H} \) with some physical input (\( Y_e, E_\nu \), etc.); (2) compute \(|\Psi(t + dt)\rangle\) from \(|\Psi(t)\rangle\) with a given \( \hat{H} \); and (3) compute the probability that \(|\Psi(t)\rangle\) is in some flavor state. We can write two modules, NeuBin_F2C and NeuBin_F3C, with exactly the same interface performing the three tasks listed above, but implementing two-active neutrino flavor mixing and three-active neutrino flavor mixing, respectively. The advantage of this approach is obvious. Once we have solved the neutrino flavor transformation problem with two-active flavor mixing, we can tackle the three-flavor mixing problem without revamping the numerical code. The only thing required is to replace NeuBin_F2C with NeuBin_F3C and rerun the code.

FLAT has a hierarchical structure of four levels. This is illustrated in Figure 2. At the lowest level NeuBin implements the Hamiltonian \( \hat{H} \) and the neutrino wave function \(|\Psi(t)\rangle\). At the second level NeuBeam employs an array of \(|\Psi(t)\rangle\) with some energy binning mechanism. At the third level NBGroup is an umbrella of three submodules: NBGroup_PM describes the physical and geometrical environment of the problem; NBGroup_PM implements the numerical scheme such
as error control and step size adjustment; and NBGroup\_IO handles data input and output. At the top level Driver processes any command-line options, initializes NBGroup and calls appropriate functions to solve the problem.

FLAT can be made parallel in three different ways. We have implemented two instances of NBGroup\_PM, i.e., NBGroup\_MM and NBGroup\_MM\_SMP, both of which use the same basic algorithm. NBGroup\_MM is designed to work on an array of NeuBeam in sequence, while NBGroup\_MM\_SMP creates a set of POSIX threads each of which works on a subset of the array simultaneously. Employing multiple threads, FLAT can run efficiently on a computer node having multiple CPU’s and/or cores and/or with Hyper-Threading technology enabled. Another way to make FLAT parallel is to compile it with flag \texttt{-USE\_MPI} and then run the executable binary on multiple computer nodes using Message Passing Interface (MPI). It is also possible to run FLAT in the hybrid mode, i.e., with both POSIX threads and MPI enabled, which is ideal for supercomputers composed of Symmetric MultiProcessing (SMP) computing nodes.

3. Numerical Results

We have used our codes to compute the flavor evolution histories of neutrinos in the hot bubble epoch with various parameters \cite{4}. We have found that both neutrinos and antineutrinos can undergo flavor transformation over broad ranges of energy and on all trajectories simultaneously. This behavior sets in earlier than would be expected with ordinary Mikheyev-Smirnov-Wolfenstein (MSW) \cite{5, 6} evolution alone. As a result, neutrino flavor transformation may have a more significant impact on r-process nucleosynthesis than previously thought. We find that the collective neutrino/antineutrino flavor transformation can take place in either the synchronized \cite{7} or bi-polar \cite{8} mode. This transformation is driven largely by the flavor off-diagonal potential and this is consistent with predictions in Ref. \cite{9}.

We have also found that the energy spectra of supernova neutrinos can be significantly modified by collective neutrino flavor transformation. In Figure 3 we plot the neutrino survival probability as a function of both neutrino energy $E_\nu$ and emission angle $\vartheta_0$ at $r = 250$ km, where $\vartheta_0$ is defined as the angle from the normal at the neutron star surface. In this calculation we have employed an atmospheric mass scale mass-squared difference ($\delta m^2 \simeq 3 \times 10^{-3}$ eV$^2$) and a vacuum mixing angle $\theta = 0.1$. We have taken the entropy per baryon in the hot bubble to be $S = 140k_B$ and neutrino luminosity to be $L_\nu = 10^{51}$ erg/s for each flavor. In the normal mass hierarchy scenario ($\delta m^2 > 0$), neutrinos of a given flavor with energies below some critical value mostly are transformed into the other flavor, while those with energies above the critical value mostly survive. The neutrino survival probability in the inverted mass hierarchy scenario ($\delta m^2 < 0$) is nearly a mirror image of that in the normal mass hierarchy case.

A detailed description of our results can be found in Ref. \cite{4}.
Figure 3. Neutrino survival probabilities as functions of emission angle $\vartheta_0$ and neutrino energy $E_\nu$ at $r = 250$ km. The left panel is for the normal mass hierarchy, and the right panel is for the inverted mass hierarchy.

4. Outlook
Our calculations are the first to solve the nonlinear, coherent neutrino flavor transformation problem with coupled flavor histories on all neutrino trajectories. This is a rich problem and we intend to employ our codes in modeling the shock reheating [2] and the $r$-process [10, 3] epochs of supernova evolution.

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