Optical ratchets with discrete cavity solitons

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We propose a setup to observe soliton ratchet effects using discrete cavity solitons in a one-dimensional array of coupled waveguide optical resonators. The net motion of solitons can be generated by an adiabatic shaking of the holding beam with zero average inclination angle. The resulting soliton velocity can be controlled by different parameters of the holding beam. © 2022 Optical Society of America

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Spatial solitons in optical nonlinear cavities have attracted a lot of attention during last decades, see, e.g., reviews in Refs. 1, 2 and references therein. They represent nondispersive localized structures supported by a flat holding beam (pump) and result from the interplay between nonlinearity and diffraction, as well as gain, losses and internal feedback of the system. Due to multiple reflections of light at the boundary mirrors, which form the cavity, light interaction with the nonlinear material inside the cavity is effectively increased. As a consequence, spatial cavity solitons (CSs) may be formed at essentially reduced input powers, as compared to conventional spatial solitons in waveguides. All the parameters of cavity solitons (including energy and phase) are completely determined by the parameters of the holding beam, which adds more flexibility to the control of the process of creation and annihilation of CSs as well as to their evolution. Together with a high robustness, usually inherent for solitons, the above properties turn CSs into good candidates for information storage units in all-optical devices. Cavity solitons have been experimentally observed in semiconductor microcavities and single-mirror feedback loops with nonlinear elements.

The presence of gain and losses in the system introduces several qualitatively new features to the soliton dynamics. In particular, a unidirectional soliton motion under the influence of AC zero-mean external forces can be realized, provided certain symmetries of the system are broken. Such a soliton ratchet effect has been successfully observed in experiments with an annular Josephson junction, revealing a unidirectional topological fluxon motion driven by biharmonic microwaves of zero average.

Up to now the soliton ratchet effect has been discussed only for solitons bearing a non-zero topological charge. In such a case the excitation can not be destroyed due to large energy barriers in the system. In this Letter we extend the concept of soliton ratchets to much more fragile optical cavity solitons having zero topological charge. We consider a nonlinear optical waveguide array with dielectric mirrors at the end facets (array of coupled zero-dimensional optical resonators), driven by an external field (pump), see Fig. 1. Recently it was demonstrated that discrete cavity solitons (DCSs) can be excited in such a system within a reasonably wide range of control parameters. The evolution of the transmitted field pattern at the output facet obeys a discrete nonlinear Schrödinger (DNLS) type equation:

\[
\left( i \frac{\partial}{\partial \tau} + \Delta + i + \alpha |A_n|^2 \right) A_n + C(A_{n+1} + A_{n-1} - 2A_n) = A_n^{in}, \tag{1}
\]

where \(A_n\) and \(A_n^{in}\) correspond to the renormalized amplitudes of transmitted and input fields in the nth waveguide, respectively. The time \(\tau = t/T_0\) is measured in terms of the photon lifetime inside the cavity \(T_0 = 2d/|v_g(1 - \rho^2)|\), which may be of the order of picoseconds or larger. Here \(d\) is the length of each waveguide, \(v_g\) is the group velocity of light inside the waveguides and \(\rho\) is the real-valued reflection coefficient of the mirrors, \((1 - \rho^2) \ll 1\). The effective damping coefficient is rescaled to unity in Eq. 1, while parameters \(C = C_0/(1 - \rho^2)\) and \(\Delta = \Delta_0/(1 - \rho^2) + 2C\) define the effective coupling between adjacent waveguides and detuning from the linear resonance, respectively \((\Delta_0 = 2\beta_0d\), where \(\beta_0\) is the wave number corresponding to the laser frequency \(\omega_0\)). Both parameters can vary in a wide range by adjusting the reflectivity of mirrors \(\rho\), as well as the frequency of incident light and the distance between adjacent waveguides (the latter determines the value of \(C_0\)). The nonlinear Kerr coefficient \(\alpha\) can be rescaled to \(\alpha = \pm 1\). Increasing the coupling parameter \(C\) the model in Eq. 1 asymptotically approaches the continuous Lugliato-Levefer model for a one-dimensional res-
step we perform a symmetry analysis. There exist only a unique attractor solution. As a first approximation, the soliton dynamics is locked to the beam phase, i.e., disregarding the initial conditions (i.e., the initial phase $\phi_{in}$), the holding beam is adiabatically slowly oscillating in time $\tau$ with the characteristic photon lifetime in the cavity, $T \gg 1$, the soliton dynamics is locked to the phase $\phi_{GS}$, which involves periodic oscillations and/or mirror reflections, leaves the dynamical equations invariant and, at the same time, changes the sign of the soliton velocity. It should be applied to any trajectory, it will again generate a trajectory. Since the attractor is unique, the symmetry operation applied to the attractor will map it onto itself, and the average velocity on this attractor will be exactly zero, so that the soliton will perform periodic oscillations in space and time. Therefore, breaking such symmetries of the underlying model equations is a necessary condition to observe a soliton ratchet effect.

Far away from the soliton center the field distribution $A_n^{GS}(\tau) \approx a^{GS}(\tau) \exp(i\phi^{GS}n)$ and its intensity $I^{GS}(\tau) = |a^{GS}(\tau)|^2$ are obtained from

$$
[\Delta - 4C \sin^2(\phi_{in}) + i + \alpha |a^{GS}|^2] a^{GS} = a .
$$

Then, we can define the position of the center of soliton $X(\tau)$ and its velocity $V(\tau)$

$$
X(\tau) = \frac{\sum_n |A_n(\tau)|^2 - I^{GS}(\tau)}{\sum_n |A_n(\tau)|^2 - I^{GS}(\tau)} , V(\tau) = \dot{X}(\tau) .
$$

Now we note, that any single-harmonic variation of $\phi_{in}(\tau) = \phi_M \sin[\omega(\tau - \tau_0)]$ possesses the time-shift symmetry

$$
\phi_{in}(\tau + T/2) = -\phi_{in}(\tau) .
$$

Provided this symmetry holds, the dynamical equation remains invariant under the operation $\hat{S}_a$

$$
\hat{S}_a : \tau \to \tau + T/2, n \to -n.
$$

At the same time, this operation changes the sign of the soliton velocity. Therefore, no rectification of a single-harmonic AC force is possible with cavity solitons. However, the above symmetry can be broken e.g. by choosing a biharmonic variation of the incidence angle:

$$
\phi_{in}(\tau) = \phi_a \sin(\omega \tau) + \phi_b \sin(2\omega \tau + \theta) .
$$

Yet another possibility is to use two holding beams, which oscillate at the first and second harmonic frequency, respectively:

$$
A_n^{in} = a \exp \left[ i\phi_{in}^{(1)}(\tau) n \right] + b \exp \left[ i\phi_{in}^{(2)}(\tau) n \right] ,
$$

$$
\phi_{in}^{(1)} = \phi_a \sin(\omega \tau) \quad \phi_{in}^{(2)} = \phi_b \sin(2\omega \tau + \theta) n .
$$

In both cases a unidirectional propagation of DCSs can be observed, despite the fact that the averaged value of the phase (and thus that of the force acting on the DCS) is zero, see Fig. 3. The resulting average velocity of the soliton strongly depends on the system parameters and parameters of shaking beam(s) (cf. maximum velocities of dark and bright DCSs for different holding beams and coupling parameters in Fig. 4). One of the possible ways to control the DCS motion is to adjust the relative phase $\theta$ in Fig. 5, see Fig. 6.
holding beams with different frequencies. The velocity inclination angle \( \theta \) or by a superposition of two shaking done e.g. by bi-harmonic variation of the holding beam \( S \). This soliton ratchet effect is the violation of the symmetry \( \theta \). The necessary condition for the observation of means of shaking holding beams with zero average inclination. The velocity inclination. The necessary condition for the observation of generating a rectified motion of discrete cavity solitons by \( \theta \) can be taken arbitrary small. 

We note, that in the continuum limit of Eq. (1) the soliton velocity \( \mathcal{V}(\phi_{in}) \) is a linear function of the incidence angle.\(^{10}\) Thus, for any choice of periodic functions \( \phi_{in} \) with zero mean the resulting average velocity \( \mathcal{V}(\phi_{in}) \) zero in the adiabatic limit. The ratchet effect for slowly varying incidence angle(s) \( \phi_{in} \) can appear only for nonlinear functions \( \mathcal{V}(\phi_{in}) \). This nonlinearity is induced by the discreteness of the system due to Peierls-Nabarro potential.\(^{11}\) In that case the resulting soliton net motion does not depend on the actual choice of the frequency \( \omega \) which can be taken arbitrary small.

To conclude, we have demonstrated the possibility to generate a rectified motion of discrete cavity solitons by means of shaking holding beams with zero average inclination. The necessary condition for the observation of this soliton ratchet effect is the violation of the symmetry \( S_{\phi} \) of the underlying model equations. This can be done e.g. by bi-harmonic variation of the holding beam inclination angle \( \theta \) or by a superposition of two shaking holding beams with different frequencies. The velocity of the resulting soliton net motion can be adjusted by the parameters of the holding beams \( \omega, \phi_{a}, \phi_{b}, \theta \). This opens a new prominent perspective for soliton steering and various all-optical switching schemes. Our results are also instructive for the general problem of nontopological soliton ratchets in spatially extended discrete and continuous systems.

References

1. W.J. Firth and G.K. Harkness, in Spatial Solitons, S. Trillo and W. Torruellas, eds. (Springer, 2001), p. 343.
2. U. Peschel, D. Michaelis, and C.O. Weiss, IEEE J. Quantum Electron. 39, 51 (2003).
3. S. Barland, J.R. Tredicce, M. Brambilla, L.A. Luglato, S. Balle, M. Giudici, T. Maggipinto, L. Spinelli, G. Tissoni, T. Knoll, M. Miller, and R. Jager, Nature 419, 699 (2002).
4. B. Schäpers, M. Feldmann, T. Ackermann, and W. Lange, Phys. Rev. Lett. 85, 748 (2000).
5. S. Flach, Y. Zolotaryuk, A.E. Miroshnichenko, and M.F. Fistul, Phys. Rev. Lett. 88, 184101 (2002).
6. M. Salerno and Y. Zolotaryuk, Phys. Rev. E 65, 056603 (2003).
7. A.V. Ustinov, C. Coqui, A. Kemp, Y. Zolotaryuk, and M. Salerno, Phys. Rev. Lett. 93, 087001 (2004).
8. U. Peschel, O. Egorov, and F. Lederer, Opt. Lett. 29, 1909 (2004).
9. L.A. Luglato, and R. Lefever, Phys. Rev. Lett. 58, 2209 (1987).
10. S. Fedorov, D. Michaelis, U. Peschel, C. Etrich, D.V. Skryabin, N. Rosanov, and F. Lederer, Phys. Rev. E 64, 036610 (2001).
11. O. Egorov, U. Peschel, and F. Lederer, Phys. Rev. E 72, 066603 (2005).