Galactic Halo Cusp versus Core: Tidal Effects in Mergers

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Abstract. We show how the buildup of halos by merging satellites forces an inner cusp, with a density profile $\rho \propto r^{-\alpha}$ where $\alpha \to \alpha_a \gtrsim 1$. Our analysis is based on a new prescription for tidal stripping as a function of $\alpha(r)$, using a simple toy model which matches N-body simulations. In a core of $\alpha < 1$ there is tidal compression rather than stripping and the satellites sink towards the halo center, causing a rapid steepening of the profile to $\alpha > 1$. Where $\alpha > 1$, the stripping of each satellite shell is preceded by gradual puffing up, which makes the stripping more efficient at larger $\alpha$, causing flattening where $\alpha$ is large enough. Therefore, we can show using linear perturbation analysis that a sequence of mergers slowly leads to a fixed point $\alpha(r) = \alpha_a$. This result implies that a cusp is enforced as long as enough satellite material makes it into the inner halo and is deposited there. We conclude that in order to maintain a flat core, as indicated by observations, satellites must be disrupted outside the core, e.g., because of puffing up due to baryonic feedback effects.

1. Introduction

The ‘standard’ model of cosmology, CDM, which assumes hierarchical buildup of structure, is facing difficulties in explaining observed properties of galaxies, such as the number density of dwarfs (e.g., Klypin et al. 1999b), the angular-momentum crisis (e.g., Navarro & Steinmetz 2000), and the cusp/core problem. Our approach in addressing these problems within CDM is to isolate and model in simple physical terms the key relevant processes, as a guide for possible solutions. We first model the buildup of dark-matter (DM) halos in N-body simulations based on tidal effects, and then incorporate the inevitable baryonic feedback processes in an attempt to explain the apparent discrepancies. We address the angular momentum problem in Maller & Dekel (2002) and Dekel & Maller (2002), and summarize here our progress in the cusp/core problem (Dekel & Devor 2002; Dekel et al. 2002).

Cosmological N-body simulations have revealed that the density profiles of DM halos scatter about a universal shape, $\rho(r) = \rho_c (r/r_c)^{-\alpha} (1 + r/r_c)^{\alpha-3}$, with an inner cusp of slope $-\alpha$. Navarro, Frenk & White (1995, NFW) found this function, with $\alpha \simeq 1$, to be a good fit in the range $(0.01 - 1)R_{\text{vir}}$ for different hierarchical cosmological scenarios. High-resolution simulations of a few individual halos (Moore et al. 2001; Klypin et al. 2001) found that the cusp could be as steep as $\alpha \simeq 1.5$, though it may flatten towards $\alpha \simeq 1$ at $r < 0.01R_{\text{vir}}$ (private comm. with Navarro, Frenk, Springel & White). While the formation
of a cusp with $1 \leq \alpha \leq 1.5$ has been established in the simulations, a basic understanding of its origin is still lacking. An even more intriguing puzzle is introduced by observations of low surface-brightness galaxies, whose centers are dominated by their DM halos, which indicate flatter inner cores with $\alpha \simeq 0$ (de Block et al. 2001). This seems to introduce a challenge to the CDM paradigm.

In the first two sections we develop a toy model for tidal stripping and test it against an N-body simulation. In §2 we describe the compression and rapid steepening in a core. In §3 we address stripping where $\alpha > 1$. In §4 we analyze the convergence to an asymptotic profile. In §5 we discuss our results.

2. Core into Cusp due to Tidal Compression

We consider a fixed spherical halo of mean density profile $\bar{\rho}(r) \propto M(r)/r^3$. Denote $\alpha(r) \equiv -d\ln \bar{\rho}/d\ln r$ such that locally $\bar{\rho} \propto r^{-\alpha}$, with $\alpha(r)$ constant or increasing in the range $0 \leq \alpha \leq 3$. Consider a satellite at $r$, spiraling into the halo under gravity and dynamical friction. The maximum tidal force by the halo on a unit mass at satellite radius $\ell$ ($\ell \ll r$) is

$$F_{\text{tide}} = \frac{\mu(r)\ell}{r^3}, \quad \mu(r) = 2M(r) - r \frac{dM}{dr} = (\alpha - 1)M(r).$$

(1)

The maximum is obtained along the line connecting the halo centers, but while the radial component is smaller in all other directions, it is of the same sign. The familiar case is of a point mass, $\alpha = 3$, for which the tidal force is pulling outwards from the satellite. For flatter halo slopes it becomes weaker $\propto (\alpha - 1)$. An important feature that is often overlooked is that when the halo density profile is flat enough, $\alpha < 1$, the tidal force reverses direction into compression, resulting in accretion rather than stripping. Note that the critical slope of unity coincides with the cusp of the NFW profile, and that in a core, $\alpha \sim 0$, the tides induce strong compression.

This effect can be demonstrated using an N-body simulation of a merger (following Mihos & Hernquist 1996). We use a total of $10^5$ DM particles, the mass ratio is 1:10, the satellite spirals in on a quasi-circular orbit, and the initial profile is a truncated isothermal sphere with a core, where $\alpha$ ranges in practice from 0.6 to 2.8. In Fig. 1 (left), the stripping of each satellite shell, marked by the onset of a steep rise, can be identified with a halo radius $r$, with $\alpha(r)$ decreasing from 3 downwards as one moves from outer shells inwards. However, as the stripping point approaches $\alpha \sim 1$, inwards to the 30% satellite mass shell, the stripping stops and some shrinking can be seen instead.

The compression at $\alpha < 1$ implies that any part of the satellite which makes it intact into the halo core would sink towards the center without further stripping. This should cause a rapid steepening of the profile to $\alpha > 1$, as seen in Fig. 1 (right). The NFW inner slope of $\alpha = 1$ is thus a robust lower bound, as we know from cosmological simulations; a flatter density core cannot survive as long as satellites deposit enough mass in the inner halo.
3. Tidal Puffing-Up and Stripping at $\alpha > 1$

At $\alpha > 1$, the effects are more subtle. Let $\ell$ mark shell radii within the unperturbed satellite, whose mean-density profile is $\bar{\sigma}(\ell) \propto m(\ell)/\ell^3$. Assume that when it is at halo radius $r$, mass is lost beyond a momentary stripping radius $\ell(r)$ and is added to the halo at $r$ (on average). We wish to determine the correspondence between $\ell$ and $r$ at stripping. Define $\psi(r, \ell) \equiv \bar{\rho}(r)/\bar{\sigma}(\ell)$. Traditionally, the stripping radius is assumed to be determined by the resonance condition $\psi(r, \ell) = 1$, but this ignores the earlier effects of tides on the satellite structure. A key new feature in our analysis is that, as $r$ decreases, the tides stretch the satellite orbits, which can be modeled as an effective puffing up of the relevant shells before they are being torn away. We define for every shell $\ell$ when the satellite is at $r$ a momentary puffing factor by $p(r, \ell) \equiv \ell_p/\ell$, where $\ell_p(r)$ is the momentary shell radius. One can then show that the resonance condition, for $r$ and $\ell$ at stripping, becomes

$$\psi = \alpha^{-1} p^{-3}. \tag{2}$$

In the regime where $\alpha > 1$, we expect puffing, $p > 1$, so the corrected resonance condition implies $\psi < 1$, and for large $\alpha$ even $\psi \ll 1$. This means more efficient stripping compared to the old condition ignoring puffing. To obtain an explicit stripping condition we wish to express $\psi$ as a function of $\alpha$, so we need to estimate how $p(\ell)$ evolves as the satellite falls into smaller $r$ positions. By applying an adiabatic invariant, we obtain a puffing equation for any shell $\ell$ when it is at $r$:

$$p - (\alpha - 1)\psi p^4 = 1. \tag{3}$$

In the two equations above we have omitted geometrical factors of order unity, to be calibrated later using simulations (see Dekel & Devor 2002). When we
combine the above equations we obtain a new stripping condition:

$$\frac{\bar{\rho}(r)}{\bar{\sigma}(\ell)} = \psi[\alpha(r)] = \begin{cases} \alpha^{-4} & 1 < \alpha < \alpha_c \sim 1.4 \\ 0.1/(\alpha - 1) & \alpha > \alpha_c \end{cases}.$$

Given the slope profile $\alpha(r)$, it relates every satellite shell $\ell$ to the position $r$ where it should be stripped. The function $\psi(\alpha)$ is thus predicted to decrease monotonically as a function of $\alpha$, towards values of order 0.1-0.2 at $\alpha \sim 2$ and below 0.1 at $\alpha = 3$, Fig. 2 (left). This implies that the stripping process is more efficient for steeper halo profiles. We also implement in the deposit prescription additional effects near and below $\alpha = 1$, due to the finite size of the satellite and the tidal compression in the inner halo. These effects may cause deposit of satellite material before the stripping condition is fulfilled.

The puffing before stripping can be seen in the merger simulation, e.g., for the outer and intermediate shells in Fig. 1 (left). The magnitude of the puffing is 30-50%, as expected, corresponding to a factor of 2-3 in density. A similar effect has been qualitatively noticed in simulations before (e.g. Klypin et al. 1999a, Fig. 6). In the merger simulation, we measure the deposit radius $r(\ell)$ from the final distribution of stripped satellite mass about the halo center (can be read from Fig. ?? Fig. 1 (left)). The corresponding values of $\psi(r, \ell)$ are plotted against $\alpha(r)$ in Fig. 2 (left). The qualitative agreement between the simulation result and the model predictions indicates that despite the crude approximations
made, our very simplified model mimics the main features of the tidal stripping and deposit process. When puffing is ignored, $\psi = 1$, the model clearly fails.

4. Halo Asymptotic Profile

If the stripping is described by a condition similar to eq. ??, with $\psi(\alpha)$ a decreasing function, the profile evolves slowly towards an asymptotic stable power law with $\alpha_a \gtrsim 1$. We assume that the halo and satellite are drawn from a cosmological distribution; they are homologous, with their characteristic radii and densities scaling like $\ell_c/r_c \propto m^{(1+\nu)/3}$ and $\sigma_c/\rho_c \propto m^{-\nu}$, where $\nu \simeq 0.33$ for $\Lambda$CDM. Fig. 2 (right) helps understanding the origin of an asymptotic slope. We write $\bar{\rho}_{\text{final}}(r) = \bar{\rho}(r) + \bar{\sigma}(\ell)/r^3$, and obtain for the change of $\alpha$ in a merger

$$\Delta \alpha(r) \propto \frac{d}{dr} \left( \frac{\bar{\sigma}(\ell) \ell^3}{\bar{\rho}(r) r^3} \right).$$

(5)

One can see that every power law is a self-similar solution, $\Delta \alpha(r) = 0$, but not necessarily a stable one. When $\alpha$ is increasing with $r$, $\ell/r$ is decreasing with $r$. Thus, when puffing is ignored, $\psi = \text{const.}$, one has continuous steepening, $\Delta \alpha(r) > 0$. With realistic puffing, $1/\psi$ is increasing with $r$, which can produce a stable fixed point at a certain asymptotic value $\alpha_a$, where $\Delta \alpha = 0$ and the second derivative is negative. A rigorous linear perturbation analysis determines the rate of convergence to $\alpha_a$ and yields an equation for its value for a sequence of mergers with the same mass ratio:

$$\Delta \alpha \propto \alpha(\alpha - 3) \psi'(\alpha)/\psi(\alpha) + 3 \ln[(m/M)^{-\nu} \psi(\alpha)] = 0.$$

The solutions are typically in the range $1 < \alpha_a \leq 1.5$.

In order to test the linear analysis, we perform toy simulations of the profile buildup by mergers, where we implement the stripping recipe of §3 (replacing the crude stripping recipe used in earlier work, e.g., Syre & White 1998). For given halo and satellite profiles, we solve the stripping equation for $\ell(r)$ and add the stripped satellite mass to the halo accordingly. Among other numerical complications, we implement a smoothing scheme to ensure that $\alpha(r)$ remains monotonic. We then follow a sequence of cosmological mergers and study the evolution towards an asymptotic slope. Fig. 3 (left) shows the convergence of $\alpha$ at a fixed $r$ to the asymptotic value. Fig. 3 (right) shows how the profile evolves through momentary profiles which are probably more relevant for comparison with real halos at different times during their buildup process. These figures are for certain given values of the geometrical factors, the mass ratio and $\nu$. In Dekel et al. (2002) we address a sequence of mergers with a cosmological distribution of mass ratios, and the robustness to the cosmological model.

5. Discussion

Our analysis demonstrates that the way to maintain a flat core is by disrupting satellites outside the core. This may be achieved if the cores of satellite halos are puffed up due to gas processes. As an example, the following speculative
Figure 3. \textit{Left}: Toy-simulation evolution of slope $\alpha$ at $r = 0.1r_c$ due to a sequence of mergers $n = 1,600$ with mass ratio $m/M = 0.3$. The initial profile is a generalized NFW with $\alpha$ either zero or 2. When $\alpha < 1$, the slope steepens rapidly to $\alpha > 1$ within a few mergers ($\S 2$), and then it converges slowly from either side towards an asymptotic value ($\S 4$). \textit{Right}: Corresponding evolution of slope profile $\alpha(r)$, starting with $\alpha = 1$ at $r \ll r_c$. A power-law region develops below the radius where $\Delta \alpha = 0$ (near $r_c$), with a slope that grows slowly from unity to the asymptotic value.

scenario is based on enhanced feedback due to tidal compression. Consider a satellite made of DM and $\sim 10\%$ baryons passing through the halo core towards a turn-around on the other side. Assume that the baryons have already cooled and contracted into the satellite center. The satellite loses its outer DM layers in the outer halo such that when it enters the halo core it is baryon-rich. In the core, the tides compress the satellite, creating shocks and an efficient burst of star formation. Before turn-around on the other side, there is time for the resulting supernovae to blow out the satellite gas. If the remaining satellite loses half its mass in this blow-out, its density drops by a factor between 8 to infinity, depending on whether the gas expulsion is adiabatic or impulsive. Thus, the remaining satellite becomes much more susceptible to tidal stripping, which disrupts it completely before it re-enters the halo core.

In our recent work, we address the different problems within the successful cosmological framework of CDM by appealing to inevitable feedback effects. In Maller & Dekel (2002) we address the angular-momentum catastrophe, where simulations including gas produce disks significantly smaller than the galactic disks observed, and with a different internal distribution of angular momentum. We first construct a toy model for the angular-momentum buildup by mergers based on tidal stripping and dynamical friction, which helps us understand the origin of the spin problem as a result of over-cooling in satellites. We then incorporate a simple model of feedback, motivated by Dekel & Silk (1986), and find that it can remedy the discrepancies, and in particular explain the low baryon fraction and angular-momentum profiles in dwarf disk galaxies. Feedback effects may also provide the cure to the missing dwarf problem, where the predicted number of dwarf halos in CDM is much larger than the observed number of dwarf galaxies (Bullock, Kravtsov & Weinberg 2000).
Another approach (e.g., Hogan & Dalcanton 2000) is to appeal to a Warm Dark Matter (WDM) scenario, despite the fact that it requires fine-tuning of the particle mass to \( \simeq 1 \text{ keV} \). The main feature of WDM is the partial suppression of small halos, which should help maintaining a core, but simulations of halos in WDM seem to still show inner cusps (Bullock, Kravtsov & Colin 2001). While the explicit merger picture modeled above may be invalid in this case, the gravitational processes involved in the halo buildup still mimic a similar behavior. We note that the tidal compression in the core may amplify density perturbations and make them behave like merging satellites.

The suppression of small halos in WDM may remedy the spin catastrophe by avoiding over-cooling (Sommer-Larsen & Dolgov 2001), but it is harder to see how it would explain the angular-momentum profile in galaxies. Furthermore, while the number of dwarfs is already suppressed in WDM, the addition of minimum feedback effects is likely to cause an overkill, where the number of dwarfs is predicted to be much lower than observed (J. Bullock, private comm.).

The success of our toy model in matching several independent observations indicates that it indeed captures the relevant elements of the complex processes involved, and in particular that feedback effects may indeed provide the cure to all three problems of galaxy formation in CDM.

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