Influence of surface roughness on the mixed elastohydrodynamic lubrication performance of vehicle gears contact area

Wen Cao
Department of Vehicle Application, Army Academy of Armored Forces, No.1 Huayuan Road, Changchun, Jilin Province, China

Corresponding author and e-mail: Wen Cao, 58986717@qq.com

Abstract. Most engineered rough surfaces approximately follow non-Gaussian distributions, which play an important role in Vehicle gears lubrication performance in contract area. A three-dimensional elastohydrodynamic lubrication (EHL) model for involute Vehicle gears is developed by making the contact between surfaces during meshing tooth surfaces equivalent to an infinite line-contact model. By using the proposed model and non-Gaussian random roughness surfaces of given parameters generated by numerical simulation method based on FFT, the meshing process of spur and helical Vehicle gears is analyzed. The film thickness, contact area ratio and contact load ratio of contacting points of vehicle gears with different surface roughness characteristics are obtained as well.

1. Introduction
High precision, high reliability, large torque and long life are becoming the development trend of modern Vehicle gear transmission. Involute Vehicle gear transmission is the most widely used mechanical transmission method, and its reliability and life are closely related to the lubrication of the tooth surface. Therefore, in-depth study of the lubricating characteristics of Vehicle gears during meshing can provide key theoretical guidance for reducing the friction and wear of Vehicle gears and improving the overall performance of Vehicle gear transmission.

Most of the early numerical calculation models used for EHL in the gear meshing process of involute Vehicles assume that the two contact tooth surfaces are smooth surfaces [1-4]. Wang Youqiang [5] et al. used a rough surface simulated by sinusoids to obtain a complete numerical solution of the involute straight tooth transient microscopic thermoelastic fluid lubrication. However, its modeled approximate surface is not a real Vehicle gear tooth surface and cannot fully reflect the actual lubrication. Working condition. Ren et al. The tooth surface roughness is usually of the same order as the tooth surface lubrication film thickness. In fact, the distribution of most common roughness peaks on machined surfaces has a certain degree of skewness and kurtosis, which is approximately subject to non-Gaussian distribution. For example, electrolysis, turning and planing produce positively inclined rough surfaces, while grinding, honing and milling will produce rough surfaces with negative peak skewness. Therefore, setting the rough tooth surface of the Vehicle gear to
Gaussian distribution has a certain deviation from engineering practice. It is necessary to study the effect of the non-Gaussian rough surface on the contact lubrication characteristics of the vehicle gear.

2. Vehicle gear mesh analysis

2.1. Involute Vehicle gear contact parameters

When the gears of an involute vehicle are meshed, the radius of normal curvature and the speed of tangential movement of the two tooth surfaces at the meshing point continuously change. But at the same moment, the motion state of all the meshing points on the gear contact line of the straight vehicle is the same, while the motion state of each meshing point on the gear contact line of the oblique vehicle is different. The involute vehicle gear front end face meshing process is shown in Figure 1.

N1N2 is the meshing line, points A and B are the meshing and meshing points of the small vehicle gear, respectively. For the hypothetical vehicle gear front end surface, when the meshing point is B', the entire gear teeth mesh out (that is, the gear tooth front surface meshed after that, rotate the rear end face before engaging). The relationship between the gear coincidence of vehicles and the turning angle in the figure is:

\[
\varphi_0 = \frac{2\pi}{z_1} \\
\varepsilon_a = \varphi_a / \varphi_0 \\
\varepsilon_b = \varphi_b / \varphi_0
\]

(1)

Among them, \(z_1\) is the number of gear teeth of the small vehicle, one gear tooth of the small vehicle gear corresponds to the center angle, and \(\varepsilon_a\) and \(\varepsilon_b\) are the coincidence degrees of the end face and the normal face, respectively.

Figure 1. Meshing diagram of front end face.

Viewed from the front end, during the process of any gear teeth of the involute oblique vehicle gear from the initial engagement \((0)\) to the final engagement \((\theta)\), the contact line length changes as shown in Figure 2, where \(b\) is the tooth width and is the base circle Helix angle. After the gear teeth are engaged and before the next engagement, the length of the contact line is \(0\), so the relative rotation angle after the front face of the watch is engaged, then the length of the contact line of the gear teeth can be expressed by the function \(l(x)\) with a period of \(z_1\).

\[
l(x) =
\begin{cases}
x / \varepsilon_a \cdot b / \cos \beta_b & (0 < x \leq \varepsilon_a) \\
b / \cos \beta_b & (\varepsilon_a < x \leq \varepsilon_a + \varepsilon_b) \\
(\varepsilon_a + \varepsilon_b - x) / \varepsilon_a \cdot b / \cos \beta_b & (\varepsilon_a < x \leq \varepsilon_a + \varepsilon_b) \\
0 & (\varepsilon_a + \varepsilon_b < x \leq z_1)
\end{cases}
\]

(2)
The contact line lengths of the first, second, ... \( z_1 \)-th gear teeth before the gear teeth are \( l(x+1) \), \( l(x+2) \), ... \( l(x+ z_1 -1) \), so the entire oblique armor The total length of the contact line of the vehicle gear pair is:

\[
L(x) = l(x) + l(x+1) + \cdots + l(x+ z_1 -1)
\]

Obviously \( L(x) \) is a function with a period of 1. In this paper, the percentage of contact line method is used to consider the load distribution of oblique Vehicle gears. Assuming that the load is evenly distributed along the contact line, the line load in the contact area is \( F(x) = \frac{F_n}{L(x)} \), \( F_n \) is the normal force of the oblique Vehicle gear, \( T \) is the input torque, and \( r_b \) and \( \alpha \) are the base wheel radius and helix angle of the driving wheel, respectively.

![Figure 2. Relationship between contact line length and relative rotation angle.](image)

At the front end face, let the contact point at a certain moment be \( C \). At this time, the contact point of the tooth surface from the center of the Vehicle gear is. Define the tooth profile parameters as follows.

\[
\xi_c = \sqrt{r_c^2 - r_i^2} : z/(2\pi b)
\]

Represents the ratio of the radius of curvature of the involute Vehicle gear at the contact point \( C \) to the base node \([10]\). For involute straight Vehicle gears, the tooth profile parameter at the point \( A \) of the front end face engagement point is set to exist at any contact point \( C \). The oblique Vehicle gear is cut perpendicularly to the axial direction into countless slices, which can be regarded as composed of countless straight Vehicle gears with the same end face parameters and different phases. For each sliced straight Vehicle gear, there are corresponding characterizing contact points \( s \) position. Each contact point with the axial distance \( l \) of the gear contact line of the oblique Vehicle from the front end face has tooth profile parameters as follows:

\[
\xi_{cc} = \xi_c - \varepsilon_s l / b
\]

Among them, the value of \( l \) changes with the end surface. At that time, the tooth of the driving wheel was axially close to the part of the tooth profile near the root of the front face. At that time, the entire gear tooth was engaged in the meshing along the axial direction. At that time, the gear tooth was close to \( A \) part of the tooth profile of the rear end face contacts near the tooth crest.

2.2. Movement state of gear mesh point of Vehicle

For straight Vehicle gear contact point \( C \), the normal curvature radius and tangential movement speed of the two tooth surfaces are as follows.

\[
\begin{align*}
\rho_1 &= \xi_c p_b \\
\rho_2 &= N_1 N_2 - \xi_c p_b \\
u_1 &= \omega_c \xi_c p_b \\
u_2 &= \omega_c (N_1 N_2 - \xi_c p_b)
\end{align*}
\]
For oblique Vehicle gears, when the contact point of the front end section is C, the normal curvature radius and tangential movement speed of the two tooth surfaces at the other contact points of the gear tooth are as follows.

\[
\begin{align*}
\rho_1 &= \frac{\xi_{cc} \rho_b}{\cos \beta_b} \\
\rho_2 &= (N_1 N_2 - \xi_{cc} \rho_b) / \cos \beta_b \\
u_1 &= \omega_b \xi_{cc} \rho_b \\
u_2 &= \omega_b (N_1 N_2 - \xi_{cc} \rho_b)
\end{align*}
\] (7)

The suction speed at the tooth surface contact is generated by the tooth surface tangential velocity component. For straight Vehicle gears or oblique Vehicle gears, the tooth surface suction speed and sliding speed at any meshing point are as follows.

\[
\begin{align*}
u_s &= \frac{u_1 + u_2}{2} \\
u_r &= u_1 - u_2
\end{align*}
\] (8)

### 3. Non-Gaussian distribution rough surface

For a Gaussian rough surface determined by the autocorrelation function, its characteristics are described by two parameters—the standard deviation $R_{rms}$ (or) and the correlation length $\beta$ ($\beta_x$ and $\beta_y$ for anisotropic surfaces); for non-Gaussian rough surfaces, two other parameter descriptions are needed, skewness (Sk) and kurtosis (Ku). The Gaussian distribution of the rough surface has a skewness of 0 and a kurtosis of 3. If $z(x, y)$ is used to represent the height of the rough surface, the standard deviation, skewness and kurtosis can be expressed by the following formula.

\[
\begin{align*}
R_{rms} &= \sqrt{\frac{1}{A} \int z^2(x,y)dxdy} = \sigma \\
Sk &= \frac{1}{\sigma^3} \cdot \frac{1}{A} \int z(x,y)dxdy \\
Ku &= \frac{1}{\sigma^4} \cdot \frac{1}{A} \int z^4(x,y)dxdy
\end{align*}
\] (9)

Where $A$ is the rough surface area. The rough surface can be regarded as a random process, using the following autocorrelation function.

\[
R(x, y) = \sigma^2 \exp(-2.3 \sqrt{k^2 / \beta_x^2 + y^2 / \beta_y^2})
\] (10)

Where $\beta_x$ and $\beta_y$ are the autocorrelation lengths along the x and y directions, respectively, and they are equal for an isotropic rough surface.

The probability density function of the rough surface can be obtained from the autocorrelation function through FFT:

\[
P_{ij} = \frac{1}{mn} \sum_{m=1}^{m-1} \sum_{n=1}^{n-1} R_{ij} \exp[-2\pi(kr/m + ls/n)i] = \frac{1}{mn} \text{FFT}(R_{ij})
\] (11)

$R_{ij}$, $s$ and $P_{ij}$ denote grid discrete values of autocorrelation function and probability density function, respectively. The non-Gaussian rough surface can be represented by amplitude and angular components as follows.

\[
z_{p,q} = \sum_{k=0}^{m-1} \sum_{l=0}^{n-1} P_{ij} \exp[i\phi_{ij} + i2\pi(kp/m + lq/n)] = \text{IFFT}(\sqrt{P_{ij}} \exp(i\phi_{ij}))
\] (12)

In the formula, IFFT means two-dimensional inverse Fourier transform, which is helpful to accelerate the calculation and generate rough surface. The probability density function can be obtained
through the autocorrelation function, so the key to generating a non-Gaussian rough surface is to find the phase angle component.

4. Mixed lubrication model and its numerical calculation

4.1. Mixed lubrication model

Regarding the gear contact model of Vehicles, the length of the contact area is much larger than the width of the contact area, and the contact area can be regarded as infinite. For three-dimensional line contact, this paper uses a lubrication-contact method based on the unified Reynolds equation [14-16] to solve. This method has proved to be effective in both the lubricating film contact area and the rough peak contact area, and can solve the problem of three-dimensional hybrid elastic hydrodynamic lubrication under extreme working conditions. The unified Reynolds equation for controlling the oil film pressure distribution in the three-dimensional infinite-line contact isothermal transient elastohydrodynamic lubrication model is:

\[
\frac{\partial}{\partial x} \left( \frac{\rho h^3}{12 \eta^*} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\rho h^3}{12 \eta^*} \frac{\partial p}{\partial y} \right) = u_x \frac{\partial (\rho h)}{\partial x} + v_x \frac{\partial (\rho h)}{\partial t} \tag{13}
\]

In the formula, \( p \) is the fluid pressure or contact pressure, \( h \) is the thickness of the oil film, \( \eta^* \) is the equivalent viscosity of the lubricant, \( \rho \) is the density of the lubricant, \( u \) is the suction speed, and \( t \) is the time. Considering the non-Newtonian characteristics such as shear thinning of the lubricating oil, the equivalent viscosity \( \eta^* \) in equation (13) is expressed as:

\[
\frac{1}{\eta^*} = \frac{1}{\eta} \left[ \frac{\tau_0}{\tau_1} \text{Sinh} \left( \frac{\tau_1}{\tau_0} \right) \right] \tag{14}
\]

Instantaneous oil film thickness equation:

\[
h = h_0(t) + \frac{x^2}{2R} + v(x, y, t) + \delta_1(x, y, t) + \delta_2(x, y, t) \tag{15}
\]

In the formula, \( h_0 \) is the initial center film thickness, \( x/2R \) is the original geometric gap of the contact surface, and \( v \) is the surface elastic deformation amount, which can be calculated by Boussinesq integration, as follows:

\[
v(x, y, t) = \frac{2}{\pi E'} \iint_{\Omega} \frac{p(\xi, \eta, t)}{\sqrt{(x-\xi)^2 + (y-\eta)^2}} d\xi d\eta \tag{16}
\]

\( \delta_1 \) and \( \delta_2 \) are the rough peak heights of the two contact rough surfaces. For theoretically smooth surface lubrication, \( \delta_1 \) and \( \delta_2 \) can be directly set to 0. In this paper, the non-Gaussian distributed rough surface is considered, so the roughness value generated by the numerical simulation method is used as \( \delta_1 \) and \( \delta_2 \) to participate in the lubrication calculation process, so that the three-dimensional infinite long-line contact isothermal transient elastohydrodynamic lubrication model (equation (13)) can be solved arbitrarily Surface contact problems.

Load balance equation:

\[
w(t) = \iint_{\Omega} p(x, y, t) dxdy \tag{17}
\]

Pressure viscosity equation:

\[
\eta = \eta_0 \exp \left( \ln \eta_0 + 9.67 \right) \left[ \left( 1 + 5.1 \times 10^{-3} p \right)^{-1} \right] \tag{18}
\]

The density equation is used:
\[ \rho = \rho_0 \left( 1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p} \right) \]  

(19)

4.2. Numeral calculations

Suppose the two main curvature directions of the instantaneous contact area are the x and y directions, respectively, and the center of the nominal contact area is at \( x=0 \) and \( y=0 \). In order to ensure that the lubrication state is oil-rich lubrication, the boundary of the calculated area is \( x_s=-2.5a, x_e=1.5a; y_s=-1.5a, y_e=1.5a \), where \( a \) is the Hertz contact of the meshing area when the gear tooth is under maximum load Half width. The boundary conditions when solving the Reynolds equation are as follows.

\[ \frac{\partial p}{\partial y} = 0, \quad p(x,0) = p(x,0) = 0 \]

5. Calculation results and discussion

Figure 3 shows the relationship between the contact area ratio of the tooth surface and the load ratio of the contact area with the kurtosis value of the rough surface under different working conditions (Rms=0.16). The roughness kurtosis value has a more consistent effect on the mixed lubrication of Vehicle gears. From a large contact area, poor lubrication to a small contact area, and good lubrication, \( \text{Ac} \) and \( \text{Wc} \) values vary with the contact roughness kurtosis value increase and decrease. Also note that the values of \( \text{Wc} \) and \( \text{Ac} \) at \( \text{Sk}=-0.5 \) in Figure 3(a) are smaller than the corresponding values at \( \text{Sk}=0.5 \); the values of \( \text{Ac} \) and \( \text{Wc} \) at \( \text{Sk}=-0.5 \) in Figure 3(b) are the same as \( \text{Sk}=0.5 \) The corresponding values at time are similar; the \( \text{Wc} \) and \( \text{Ac} \) values of \( \text{Sk}=-0.5 \) in Figure 3(c) are larger than the corresponding values at \( \text{Sk}=0.5 \), and the influence of these skewness values on \( \text{Wc} \) and \( \text{Ac} \) is the same as the previous conclusion Consistent.

Figure 3. \( Wc \) and \( Ac \) against Kurtosis for different operating conditions.

6. Conclusions

In this paper, a complete three-dimensional infinite long-line mixed hydroelastic lubrication analysis model for straight Vehicle gears and oblique Vehicle gears is established. Using this model to analyze the meshing process of the gears of straight Vehicles and gears of oblique Vehicles, the thickness of
the oil film, the load of the contact zone and the ratio of the contact zone on the tooth surface at each meshing point can be solved. Combined with the non-Gaussian distribution rough surface generated by the numerical simulation method based on fast Fourier transform, the difference in the lubrication condition of the gear contact area of Vehicles under different skewness, kurtosis and standard deviation was studied. The main conclusions are as follows:

1) For non-Gaussian rough surfaces with equal standard deviations, the influence of the skewness value on the lubrication of Vehicle gears is closely related to the working conditions and changes with the changes in the operating conditions. Under the condition of good lubrication, the smaller the skewness value, the better the lubrication condition; under the poor lubrication condition, the larger the skewness value, the better the lubrication condition; when the lubrication is in the intermediate condition, the influence of the skewness on the lubrication condition is not obvious.

2) Under various working conditions, the influence of the kurtosis value on the lubrication condition of Vehicle gears shows that the greater the kurtosis value, the better the lubrication condition.

3) The larger the standard deviation of the contact rough surface, the greater the contact area ratio and the contact area load ratio, and the worse the lubrication condition; the change of the standard deviation will not change the law of the influence of kurtosis and skewness on lubrication.

4) Oblique Vehicle gears are similar to straight Vehicle gears, for which the rough surface roughness peak distribution has the same effect on the lubrication characteristics of the two.

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