Universal Landauer conductance in chiral symmetric 2d systems

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We study transport properties of an arbitrarily shaped ultraclean graphene sheet, adiabatically connected to leads, composed by the same material. If the localized interactions do not destroy chiral symmetry, we show that the conductance is quantized, since it is dominated by the quasi one-dimensional leads. As an example, we show that smooth structural deformations of the graphene plane do not modify the conductance quantization.

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I. INTRODUCTION

In recent years an explosive amount of attention has been paid to the novel material graphene, a 2d carbon sheet. One of the important reasons for this great interest is the peculiar behavior of the associated transport properties.

Essentially, graphene is a semi-metal with a gap over almost all the 2d Brillouin zone, except for two symmetrical points where the low energy excitations are gapless fermions with a linear dispersion relation. The low energy description of graphene can be done in terms of four-component massless Dirac fermions. In this scenario, curious behaviors such as quantized conductivity, unconventional integer quantum Hall effect, and the Klein paradox (unimpeded penetration through high and wide potential barriers), can be discussed.

With regard to conductance, although current experiments observe a dissipative (Ohmic) regime, from a theoretical point of view, the study of transport properties of charged 2d massless Dirac modes is a fascinating problem, far away from its complete understanding.

In ref. 14 for a thin ultraclean graphene strip, a quantized conductance related with the zero modes at the edge of the strip was found; similarly to what happens in nanotubes, where evidence of conductance quantization was recently reported. The value of the conductance depends on whether the edges are cut with zig zag or armchair geometry. In that work, a one particle approximation was considered, disregarding the effect of any kind of impurities or interactions.

In this work, we claim that this behavior is not only correct, but it can be extended to more general situations. The conductance quantization is preserved if, in a localized region, the system suffers an adiabatic change with respect to the situation at the strip, preserving chiral symmetry.

We recall that, in planar models involving a four-component spinor ψ, two independent matrices γ 3, γ 4 that anticommute with the Dirac matrices can be defined, which play a similar role to the usual γ 5, in one and three-dimensional Dirac theories. At the classical level, a planar system of massless Dirac fermions, including (total) charge density and current interactions, are symmetric under the continuous chiral transformations, generated by γ 3, γ 4. Then, as examples of adiabatic change, we can consider an adiabatic widening of the 2d sample, or even effective current interactions coming, for instance, from a smooth deformation of the graphene plane. In the above mentioned conditions, chiral symmetry will be sufficient to show that Landauer conductance is dominated by the leads, as occurs in 1d systems such as quantum wires, adiabatically connected to Fermi liquid reservoirs. Then, following a line of reasoning similar to the one presented in refs. 18 and 19, a simple argument relying on general properties of the system will be given, in spite of the fact that the 2d sample contains complicated gapless modes, modeled as confined massless interacting Dirac fermions.

From a technical point of view, we will use the functional bosonization for 2d systems, where the current is mapped into a topological current, containing effective “electric” and “magnetic” fields for a vector potential Aaka, Aka, Aka (the bosonizing fields). In principle, a closed form for the dual bosonized action describing 2d Dirac fermions is only known in the large mass limit. However, on the leads, as the typical scale of one of the dimensions is small, we argue that results from 1d bosonization can be applied there. This amounts to approximating, on the leads, the 2d fermion determinant by considering only the contribution of fermion zero modes, and taking into account the decoupling of the other modes for small widths.

Another important ingredient in our derivation will be the universal character of the bosonization rule for fermion currents in a general 2d system. In some sense, we will see that bosonization implements the idea of an
electron wave guide, enabling the discussion of transport properties in terms of similar concepts associated with “electromagnetic” wave guides, such as geometric and material dispersion for the coupled modes. For instance, the decoupling of the zero modes on the quasi one-dimensional leads amounts to the suppression of the geometric dispersion, in wave guide language.

In section IV we describe the bosonization technique, stressing its physical meaning and showing the similarities and differences between 1d (IIA) and 2d systems (IIB). In particular, in IIB and III we explain how to construct the bosonized action for the general system (2d sample plus quasi one-dimensional leads), showing how the bosonization technique implements the idea of electron wave guide. In section IV we derive the main result of this paper, showing the conductance quantization for a wide class of ultraclean systems with interactions. Finally, we discuss our results in IV.

II. CHIRAL TRANSPORT AND BOSONIZATION

A. One dimensional massless fermions

In this section we give a brief summary of one dimensional bosonization. Although the procedure is very well established, we would like to emphasize the physical concepts involved that will lead us to new results, developed in the following sections.

The description of a conserved charge in a 1d system, is usually simplified by introducing a bosonic field $\phi$ such that

$$\rho = \partial_x \phi, \quad j_x = -\partial_t \phi,$$

which automatically leads to the continuity equation,

$$\partial_t \rho + \partial_x j_x = 0$$

for this reason, the $\phi$-field expressions are called a “topological current”.

For instance, we can consider a gapless 1d noninteracting fermionic mode $\psi$ characterized by a linear dispersion $E(p) = vp$. In quantum wires, this model can be used to represent those modes in the 2d Fermi liquid in the leads that couple to a quasi-one-dimensional quantum wire (sample), in this case $E$, and $\rho$ refer to energy and momentum measured with respect to a Fermi surface (points), at low excitation energies. In this case the field mode can be either right or left moving ($\psi_R = \psi_R(x-vt)$ or $\psi_L = \psi_L(x+vt)$, respectively). Then, the current densities,

$$\rho = e(\rho_R + \rho_L), \quad j_x = ev(\rho_R - \rho_L),$$

where $\rho_R = \psi_R^* \psi_R$ and $\rho_L = \psi_L^* \psi_L$, also satisfy,

$$v^{-2} \partial_t j_x + \partial_x \rho = 0,$$
In other words, eq. (12) represents the nonconservation of the chiral current at the quantum level. A classical symmetry that is not realized at the quantum level is called an anomaly. From a physical point of view, the anomaly represents the creation of particles and holes out of the vacuum or Fermi sea, when an external field is applied; of course, these concepts only exist in the quantum world. As it is well known, this effect fixes the value of $\alpha$ as follows. Consider an homogeneous electric field $E$ and an initial Fermi sea. From the equation of motion $\dot{p} = eE$ all right moving particles will gain a momentum $eEt$. If the system size is $L$, then the number of right moving particles created above the Fermi sea, at $p_x = 0$, is the volume occupied in phase space, $L(eEt)$, divided by Planck’s constant $h$, so that the density of right moving particles is $eEt/h$. This equals the density of left moving holes (or antiparticles) created, to conserve the system’s charge equal to zero (with respect to the charge of the Fermi sea). This is measured by the quantum version of $\rho = e(\rho_R + \rho_L)$. Now, because of the relative minus sign in $\rho_A = e(-\rho_R + \rho_L)$, at the quantum level, it will measure the total density of particles (times the electric charge $e$): $\rho_A = (2e^2Et/h)$. Since the system is homogeneous, eq. (2) then gives,

$$\partial_t \rho_A + \partial_x j^A_x = 2e^2E/h, \quad (13)$$

and the value,

$$\alpha = (1/v) 2e^2/h \quad (14)$$

is obtained. Note that if $N$ dispersionless one-dimensional channels $\psi_i$, $i = 1, .., N$ were considered, each one represented by a field $\phi_i$, the anomaly for the total chiral currents in eq. (12) would be $(vN\alpha)E$, $\alpha = (1/v) 2e^2/h$. Equivalently, if a single field $\phi$ were used to describe the total currents, then the value $\alpha = (1/v) N 2e^2/h$, should be used for the associated field theory.

B. Two dimensional four-component massless fermions confined to a strip

Following the same reasoning of the previous section, let as consider a 2$d$ system with a conserved charge, and the associated continuity equation,

$$\partial_t \rho + \partial_x j_x + \partial_y j_y = 0. \quad (15)$$

Formally, this equation is a “three-divergence” equal to zero, so that we can represent $\rho$, $j_x$ and $j_y$ as a “curl” or topological current,

$$\rho = \partial_x A_y - \partial_y A_x$$

$$j_x = \partial_y A_t - \partial_t A_y$$

$$j_y = \partial_t A_x - \partial_x A_t, \quad (16)$$

which is identically conserved.

Considering that $A_t$, $A_x$, $A_y$ and $A_t + \partial_t \chi$, $A_x + \partial_x \chi$, $A_y + \partial_y \chi$ represent the same charge density and current distribution, the $A$-fields must be gauge fields, physically equivalent when the above mentioned transformation is performed. It is useful to think about the charge density and currents as effective “magnetic” and “electric fields”. Making the identifications, we have,

$$\rho = B$$

$$j_x = +E_y$$

$$j_y = -E_x. \quad (17)$$

Of course, $E$ and $B$ are not real electromagnetic fields, they are simply useful auxiliary fields to represent charge density and currents. In particular, charge conservation now looks like a 2$d$ Faraday-Lenz law,

$$\partial_x E_y - \partial_y E_x = -\partial_t B. \quad (18)$$

Thus, charge variation in a given region is associated with nonzero current flux through the boundary, as changes in the magnetic flux piercing a surface are associated with an induced electric field, in bosonized language.

If gapless parity preserving fermions $\psi$ with a linear dispersion $E(\mathbf{p}) = v|\mathbf{p}|$ (no “material” dispersion) are considered, we can guess that in 2$d$ we will have to face a difficult problem because of “geometrical” dispersion. The fermion modes are general combinations of waves $\exp i(\mathbf{k}\cdot\mathbf{r} \pm \omega t)$ propagating with speed $v = \omega/|\mathbf{k}|$, satisfying the wave equation derived from Dirac’s equation. But the charge density and currents are $\psi$-field bilinears, not satisfying any simple equation, because of the continuum of possible directions given by $\mathbf{k}$ (“geometrical” dispersion). That is, if we try to write other equations defining $\rho$, and $j_i$, or their effective electric and magnetic counterparts, they will certainly be highly nontrivial.

On the other hand, if gapless excitations were confined to a 2$d$ strip, we expect that the effects of geometrical dispersion will decrease, as the strip width is reduced. Then, we could use the $\phi$-language representation of $\mathbb{A}_{14}$ for these modes. However, we would like to consider a general situation where the strip could be coupled adiabatically with an extended 2$d$ region, where a continuum of gapless modes exist, with no definite direction of propagation, described by the gauge field $A$. For this reason, we will translate the dispersionless modes in the strip from $\phi$ to $A$-language.

If the strip is defined by a region of length $L$ along the $x$-axis, and width $W$, that is $y \in [-W/2, +W/2]$, the confinement condition of having no charge flux across the limits, that is $j_y = 0$ at $y = \pm W/2$, amounts to $E_x = 0$ at $y = \pm W/2$. In terms of the effective model this means that the the border acts as a perfect conductor or wave guide (see ref. 26).

Assuming that the current distribution on the strip is all along the $x$-axis, that is $j_y = 0$, while $\rho$ and $j_x$ are
$y$-independent, we have,
\[ \partial_x y - \partial_y x = 0 , \quad v^{-2} \partial_y j_y + \partial_x j_x = 0, \] (19)
or in “effective” language,
\[ \partial_x E_x + \partial_y E_y = 0 , \quad \partial_y B - v^{-2} \partial_x E_x = 0. \] (20)
Now, using the condition \[ \text{i} \], for dispersionless 1$d$ modes, we also have,
\[ \partial_x B + v^{-2} \partial_t E_y = 0. \] (21)
Eqs. \[ \text{(18)} \] and \[ \text{(20)} \] are Maxwell’s equations in 2$d$, while the current distribution considered in the strip corresponds to an effective dispersionless $TE_0$ mode coupled in a wave guide. These equations can be derived from 2$d$ Maxwell’s action,
\[ S_0[A] = \int dt d^2 x \frac{1}{2 \beta} \left[ v^{-2}(E_x^2 + E_y^2) - B^2 \right], \] (22)
with $E_x$, $E_y$ and $B$ defined through eqs. \[ \text{(13)} \] and \[ \text{(17)} \].
We also note that in the radiation gauge $\partial_x A_x + \partial_y A_y = 0$, $A_t = 0$, and for the $TE_0$ mode we can use $A_x = 0$, so that eq. \[ \text{(16)} \] reads,
\[ \rho = +\partial_x A_y, \quad j_x = -\partial_y A_y, \quad j_y = 0. \] (23)
Then, considering that $\rho$ in 1$d$ and 2$d$ correspond to charge by unit length and area, respectively, while $j_x$ has units of current and current by transverse length, respectively, we can identify $W \rho$ and $W j_x$ in 2$d$ with one dimensional quantities, that is, we can identify $\phi \equiv W A_y$. We would like to underline that the dispersionless modes have a formal “Lorentz” invariance associated with boosts with speed parameter $v$. Under this boost $A_y$ is invariant, so that, in the quasi one-dimensional lead, its identification with the scalar $\phi$ is a natural one.

Summarizing, if we evaluate Maxwell’s action on the $TE_0$ mode, we obtain the identification $\beta = \alpha / W$, $\alpha = (1/v) N 2 \pi \hbar$, for the description of the total currents associated with $N$ dispersionless channels on a 2$d$ strip to be equivalent to $N 1$d dispersionless quantum modes.

C. General chiral symmetric fermions confined to a 2$d$ sheet

For a general fermionic 2$d$ model with interactions $I[\rho, j_x, j_y]$, only involving densities and currents, the general quantum equivalence with the $A$-language description is given by,
\[ Z = \int \mathcal{D} \psi \mathcal{D} \bar{\psi} \exp i K_F[\psi] + i I[\rho, j_x, j_y] \]
\[ = \int \mathcal{D} A \exp i K_B[A] + i I[B, E_y, -E_x], \] (24)
where $K_F[\psi]$ is the free fermionic action. The bosonizing action $K_B[A]$ is gauge invariant, and again is given by a “transverse” Fourier functional transform of the partition function associated with $K_F$.

The low energy description of graphene can be written in terms of a four component spinor $\psi$, which can be organized as a pair $\psi_K$, $\psi_{-K}$, where the label is associated with excitations in the two opposite valleys centered at the corners of the Brillouin zone, with wave vector $\pm \mathbf{K}$ (Dirac points). The fields $\psi_{\pm K}$ are two-component Dirac spinors, each component is associated with the amplitude of the wave function on the A and B sublattices of the honeycomb lattice.

We underline that while the effective action for two-component massless fermions contains a Chern-Simons term, in the case of four component fermions, each element of the pair must contribute with an opposite sign, in order to conserve parity. Then, as the model for graphene is based on a four-component $\psi$, $K_B$ contains no Chern-Simons term, and being gauge invariant it must be necessarily of the form, $K_B = K_B [B, E_x, E_y]$.

In general, if $K_F$ were a gapped (massive) theory, it would be possible to obtain a low energy expansion for $K_B$. However, in graphene the excitations are associated with massless Dirac fermions, and in an extended 2$d$ region $K_B[A]$ an unknown complicated functional, as there is no parameter to organize a perturbative expansion.

On the other hand, from the previous discussion, on a thin 2$d$ strip, $K_B[A]$ is expected to have the simple Maxwell form of eq. \[ \text{(22)} \], as the problem can be considered as quasi one-dimensional. To be more precise, a massless theory confined to a thin strip will have zero modes plus other modes that decouple in the small $W$ limit (see ref. \[ \text{[20]} \]). Thus, the partition function for $K_F$ on the strip is expected to be dominated by these zero modes; these are precisely the relevant modes to discuss transport in ultraclean graphene strips (see ref. \[ \text{[14]} \]).

Then, in the more general case, where a two dimensional system is adiabatically coupled to quasi one-dimensional leads, the mapping \[ \text{(21)} \] can be used, if corrections to the simple Maxwell form in the leads are incorporated in the bulk expression for $K_B$.

III. CONDUCTANCE OF A GENERAL GRAPHENE STRIP/SAMPLE/STRIP SYSTEM

From the previous discussion, if an extended 2$d$ region with ultraclean graphene (gapless Dirac fermions) is adiabatically coupled to two quasi one-dimensional strips, formed with the same material, the system’s bosonized action is expected to have the form,
\[ S_B = K_B + I , \quad K_B = S_0 + R, \] (25)
where $S_0$ is given by eq. \[ \text{(22)} \], $R$ represents deviations from the simple Maxwell form valid on the leads, localized in the sample’s bulk. The $I$-term represents charge
density and current interactions also localized at the bulk
In the present bosonized language, these interactions and
the deviations $R$ play a similar role. In addition, the
“electric” field $E_x$, $E_y$ must be orthogonal to the sam-
ple’s boundary (no charge flux across the boundaries).

The important point is that because of the adia-

tic condition, $K_B$ is defined by a single functional
of $B$, $E_x$ and $E_y$, whose form depends on the system’s
(lead/sample/lead) point under consideration. Although
the corrections in the bulk are very difficult to compute,
we will see that the above mentioned general structure
for $K_B$ is all we need to derive the conductance quanti-

To discuss transport, we include in eq. (24) a sim-
ple coupling with external electric and magnetic fields
$\{A_0, A_x, A_y\}$ to probe the system, that is, we consider
the replacement,

$$I \rightarrow I + I_e, \quad I_e = \int d^3x (-\rho A_0 + j_x A_x + j_y A_y), \quad (26)$$

or in $A$-language,

$$I \rightarrow I + \int d^3x (E_y A_x - E_x A_y - B A_0). \quad (27)$$

The associated saddle point equations are obtained by
taking functional derivatives with respect to $A_t$, $A_x$ and
$A_y$, respectively,

$$\partial_x \frac{\delta S_B}{\delta E_x} + \partial_y \frac{\delta S_B}{\delta E_y} = B$$
$$\partial_y \frac{\delta S_B}{\delta B} + \partial_x \frac{\delta S_B}{\delta E_x} = -\mathcal{E}_y$$
$$\partial_x \frac{\delta S_B}{\delta B} - \partial_y \frac{\delta S_B}{\delta E_y} = -\mathcal{E}_x \quad (28)$$

where we have defined the system’s bosonized action,

$$S_B = K_B + I. \quad (29)$$

Now, we can follow a reasoning very similar to the
one of Maslov and Stone used to derive the universal
behavior of conductance in quantum wires (see also ref.
[18].)

Let us consider an electric field that is switched on
to attain a stationary value $\mathcal{E}_x(x, y)$, $\mathcal{E}_y(x, y)$. At late
times a uniform current $I$ is expected to be settled in the
system, so that on the strips we can consider an ansatz
$A_t = 0$, $A_x = 0$ and $A_y = f(x) - k t$, $I/W = j_x = -\partial_y A_y = k, \quad \text{so that } I = k W.$

Because of causality, $A_y$ on the left (right) lead must
be given by a perturbation propagating to the left (right),
with speed $v$. Then, considering symmetry under $x$-
reflection (to simplify the argument), $f(x)$ must have the
form $-(k/v)x$ on the left lead and $+(k/v)x$ on the right
lead. This ansatz can be extended to a solution on the
whole system, and considering the last two equations in

$$\partial_y \left[-(1/\beta)B + \frac{\delta (R + I)}{\delta B}\right] = -\mathcal{E}_y$$
$$\partial_x \left[-(1/\beta)B + \frac{\delta (R + I)}{\delta B}\right] = -\mathcal{E}_x. \quad (30)$$

Integrating on a curve $C$ going from the left to the right
lead,

$$-(1/\beta)[B|_{\text{right}} - B|_{\text{left}}] = -\int_C (d x \mathcal{E}_x + d y \mathcal{E}_y) = \Delta V, \quad (31)$$

and using eqs. (13) and (17), and the ansatz above,

$$B|_{\text{right}} = (\partial_x A_y - \partial_y A_x)|_{\text{right}} = +\frac{K}{v}$$
$$B|_{\text{left}} = (\partial_x A_y - \partial_y A_x)|_{\text{left}} = -\frac{K}{v}. \quad (32)$$

Therefore, we obtain,

$$I = -(W v \beta/2) \Delta V = -N e^2/h \Delta V, \quad (33)$$

where $N$ is the number of zero fermion modes on the
strips.

IV. SUMMARY AND DISCUSSION

We have analyzed the problem of linear transport in
an arbitrarily shaped ultraclean graphene sheet, adia-
batically connected to quasi one-dimensional leads, formed
with the same material. This means that a smooth
widening of the strips is permitted at the sample’s bulk,
where effective interactions are localized, which depend
on the total charge density and currents associated with
both Dirac points. Under this circumstances, we have shown
that the conductance is quantized, as it is domi-
nated by the ultraclean quasi one-dimensional leads.

The situation is similar to what happens in quantum
wires, where conductance is quantized because of the
dominance of the Fermi liquid reservoirs.

There is an interesting realization of the scenario dis-
cussed here, recently studied in the literature, quite rele-
ants when one tries to compare models with actual exper-
iments. In ref. [14] Castro Neto and Kim analyzed the ef-
cfect of the curvature of the (otherwise plane) carbon sheet
on the electronic structure of the sample. They showed
that the net effect of smooth curvature fluctuations is
that fermions become minimally coupled with an effec-
tive “elastic” gauge field. Permanent deformations could
evitably break time reversal symmetry at one point.
However the effect on the other Dirac point $-K$ restores
this symmetry. In our formalism, that means that the
bosonized action $K_B$ has no induced Chern-Simons term,
due to the cancellation between both species of fermions.
On the other hand, the elastic field couples symmetrically
the two Dirac points, meaning that smooth fluctuations of this gauge field will induce total charge and current interactions of the class considered in this paper, so that our quantization argument can be applied in this case.

In general, our argument works whenever the chiral symmetry displayed by 2d four-component massless fermions is preserved, so that the obtained perfect conductance can be associated with effective gapless excitations defined along the whole system. Bosonization gives a nice interpretation for this phenomenon, looking to the sample as an "electromagnetic" wave guide, without backscattering, guiding the $TE_0$ modes at the leads.

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