Maximum-Service Channel Assignment in Vehicular Radar-Communication

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This work was supported by the American University of Sharjah under Grants no. EN0281:SCRI18-07 and FRG20-M-E10. The work of Ahmed S. Ibrahim was supported in part by the National Science Foundation under Award CNS-1816112.

ABSTRACT Spectrum sharing between different technologies has become a necessity as the RF spectrum has become more congested than ever due to the increasing number of connected devices and applications all wishing to access the spectrum simultaneously. Vehicular networks are one of many scenarios of spectrum sharing as vehicles are expected to share the same spectrum for radar sensing and communication purposes. Channel assignment among radar sensors and communication transceivers in automotive systems is crucial for the success of future vehicular networks. This paper proposes an optimization framework for the channel assignment to multiple automotive radars and communication transceivers aiming at maximizing the number of served vehicles under hard quality of service requirements. The channel assignment problem is formulated as an integer linear optimization with binary variables. A heuristic solution that is based on sequential channel assignment (OSCA) is proposed and is shown to achieve at least 92% of the solution obtained from branch-and-cut based techniques with much less computational complexity and run-time; at most 2% of the run-time of branch-and-cut.

INDEX TERMS Channel assignment, quality of service, radar systems, vehicular networks.

I. INTRODUCTION

The congested Radio Frequency (RF) spectrum constitutes a real challenge to the future of wireless communication and its ability to support the increasing number of connected devices and the growing demand on wireless services. Spectrum sharing between radar and communication is one way to make the most of the available RF spectrum. Such sharing should be managed in a way that does not harm the performance of either technologies. Vehicular systems are one of the systems that could benefit from coexistence between automotive radars and communication transceivers on vehicles. Modern vehicles are equipped with multiple radar sensors with each sensor serving different purposes and having different specifications [1]. Automotive radars are generally categorized into Long Range (LR) radars for automatic cruise control and collision avoidance as well as Short Range (SR) and Very Short Range (VSR) radars for blind spot detection and lane change assist. The most common automotive radars use linear Frequency Modulated Continuous Wave (FMCW) and operate in the 77-81 GHz Millimeter-Wave (mmWave) band. This wide available spectrum could enable high communication data rates that cannot be supported by the narrow bandwidth currently available in the sub-6 GHz band.

Radar-communication spectrum sharing has two main forms; spectral coexistence and Dual Function Radar Communication (DFRC) [2], [3]. In coexistence scenarios, different radar and communication devices share the same spectrum, and consequently mutual interference should be managed either proactively or reactively. In DFRC scenarios, the two functions of radar sensing and communication are performed simultaneously using the same hardware and waveform. This can be done by using the communication waveform for ranging, by embedding communication data in the radar waveform, or by designing a waveform for joint ranging and data transmission. In this paper, we focus on spectral coexistence scenarios. For more information on the literature of DFRC, we refer the reader to [4], [5].
A. LITERATURE REVIEW

Different research works in the literature on radar-communication coexistence handled interference via different techniques including spectrum sensing and opportunistic spectrum access [6], [7], orthogonal time/frequency resource allocation [8], [9], optimizing transmission parameters [10]–[15], or interference estimation and removal [16], [17]. To elaborate, [6] considered a cognitive radar operating over multiple disjoint narrow bands that are not used for communication. The radar optimizes its transmission over the least noisy bands to maximize its performance. In [7], energy based spectrum sensing is used to locate idle chunks of spectrum, where radar signal and communication data are transmitted on disjoint sub-carriers in the detected idle spectrum. Orthogonal resource allocation was considered in [8], [9]. Reference [8] considered vehicles with communication transceivers and FMCW radars where radar sensing and communication took place over disjoint frequency bands in order to prevent mutual interference. Communication transceivers in [8] were also allocated orthogonal resources to eliminate interference on the communication band, and radars were scheduled in time to avoid mutual interference between them. The work in [9] considered vehicles with dual function radar-communication using stepped Orthogonal Frequency Division Multiplexing (OFDM). Vehicles were allotted orthogonal time-frequency resources to avoid inter-vehicle interference, and the objective was to maximize a weighted sum of communication rate and radar Cramer-Rao bound. Different transmission parameters of radars and/or communication systems were optimized in [10]–[15] to control interference levels. Specifically, [10] designed a radar waveform to maximize its Signal to Interference plus Noise Ratio (SINR) under constraints on the generated interference over bands shared with other telecommunication systems. References [11]–[13] optimized the transmit power in a coexistence scenario between a multi-carrier radar and a base station. Specifically, in [11], the total band was divided into an exclusive communication band, an exclusive radar band, and a shared band. The transmit powers of the radar and the base station were then optimized over different sub-carriers by alternatively maximizing the radar mutual information and the base-station capacity. On the other hand, in [12], full band sharing was allowed and power was allocated to maximize the base station capacity subject to constraint on the radar SINR. The authors in [13] considered spectrum sharing between a multi-carrier airborne radar and a ground base station, where the total transmission power of the radar was minimized under constraints on its mutual information and the capacity of the ground base station. Reference [14] considered the coexistence between a base station with a Multi-Input-Multi-Output (MIMO) radar, and it optimized the beamforming of the base station signal for two problems: 1) minimizing the transmit power under constraints on the SINR at communication receivers and the interference at the radar, 2) minimizing the interference at the radar under constraints on the transmission power and the SINR at the communication receivers. Reference [15] considered beamforming at a MIMO ship-borne radar to minimize the interference at a nearby cellular system with MIMO coordinated multi-point transmission. Reactive interference management through interference estimation and removal was considered in [16], [17]. Reference [16] proposed iterative algorithms at the communication receiver to jointly estimate the radar interference and demodulate the communication data. Reference [17] proposed an atomic norm based algorithm to estimate and cancel the interference of an automotive FMCW radar at an OFDM vehicular receiver.

B. CONTRIBUTION

In this paper, we consider channel assignment for vehicular radar-communication coexistence where vehicles wish to access the shared spectrum for different radar or communication purposes under different Quality of Service (QoS) (or performance) requirements. Managing channel assignment in such environment is challenging due to the inter-vehicle interference, the limited spectrum, and the diverse QoS requirements of different radars and communication transceivers. This is besides the different channel assignment patterns that should be considered for radars/communication transceivers. For example, linear FMCW should be assigned contiguous channels, while communication transceivers can be assigned contiguous or non-contiguous channels.

We consider coordinated channel assignment where vehicles send their channel assignment requests and their QoS requirements to a coordinator vehicle that announces which vehicles are granted access to the spectrum and what are their assigned channels. We consider hard QoS requirements, meaning that vehicles that are granted spectrum access (and consequently assigned channels) must have their QoS requirements satisfied, and for simplicity, we call these vehicles the served vehicles. While some vehicles are served, others may be blocked due to inter-vehicle interference over the shared spectrum. Obviously, to make the most of the shared spectrum, channel assignment should be optimized to maximize the number of served vehicles. This is equivalent to minimizing the number of blocked vehicles (i.e., vehicles that are denied channel assignment). Blocking is a metric that was originally considered in cellular planning where cells were assigned channels based on their incoming call traffic to minimize the call blocking probabilities under specific frequency reuse pattern [18], [19]. In cellular systems, an incoming call request is served by the base station when enough resources are available and is blocked otherwise.

We formulate the channel assignment problem as an Integer Linear program (ILP) and propose a heuristic algorithm that uses an Ordered Sequential Channel Assignment (OSCA). We compare the performance of OSCA with the solution obtained from branch-and-cut based techniques and we show that OSCA has much less computational complexity and run-time, which is necessary in vehicular scheduling, at the cost of slight performance degradation.
Despite the fact that the literature of radar-communication coexistence is rich with a lot of great research work, we believe that, in this paper, we approach the channel assignment problem with a quite different perspective so as to fill some gaps in the literature, which are explained as follows.

- We do not assume a pre-determined sharing criterion (e.g. orthogonal or non-orthogonal). On the contrary, we formulate and solve a general channel assignment problem that decides which channels should be assigned exclusively and which channels should be shared simultaneously. This decision depends on the relative positions of vehicles in the cluster and their required QoS.
- We optimize the channel assignment to maximize the number of served vehicles over the shared limited spectrum under hard QoS constraints which is challenging as we consider multiple interfering vehicles with different types of radars and communication applications with different bandwidth requirement and interference tolerance. Looking back at the discussed literature, QoS was usually considered in scenarios of spectrum sharing between a single communication transmitter and a single multi-carrier radar that can operate on non-contiguous channels, which is not the case for Linear FMCW automotive radars which we consider in this paper. Although multiple vehicles were considered in\cite{8,9}, time and/or frequency resources were allocated orthogonally without constraints on QoS requirements.

The rest of this paper is organized as follows: the system model and the radar-communication performance metrics are discussed in Section II and Section III, the channel assignment problem is formulated in Section IV and the heuristic solution, namely OSCA, is proposed in Section V. Numerical results are then presented in Section VI before the paper is finally concluded in Section VII.

II. SYSTEM MODEL

A. NETWORK MODEL

Consider a vehicular ad-hoc network, in which vehicles group themselves in clusters. Clustering of vehicles breaks the large challenging network into smaller organized groups of vehicles. Consequently, it enhances the scalability and stability of the network and facilitates its resource management. Each cluster has one or more cluster heads, which is an ordinary vehicle that works as a group coordinator and handles spectrum access of the vehicles in the cluster. A vehicle joins a cluster through some steps that include neighborhood discovery and cluster selection \cite{20}. In neighborhood discovery, the vehicle announces its presence over a control channel and collects information about nearby vehicles, including their positions, through passive sensing or active requests. Based on the collected data, the vehicle decides whether to form a new cluster or join an already existing cluster and announce itself as a cluster member. A cluster member vehicle periodically evaluates its link to the cluster head to decide whether to remain in the cluster or leave it. Similarly, a cluster head vehicle periodically checks its cluster members to keep track of who still belongs to the cluster. Vehicles in the same cluster exchange information periodically to establish, coordinate, and maintain the cluster. Cluster formation, association, and maintenance depend on several methods including channel monitoring, mobility prediction, machine learning, etc. In addition, clusters change their formation along the road due to vehicles mobility. The lifetime of a cluster depends on many factors including the clustering technique, the traffic conditions, the road structure, etc. In an ideal sense, clustering techniques should be reliable and robust to mobility and sudden changes in the network. Clustering techniques in vehicular networks has a rich literature \cite{20}; however, it is out of the scope of this paper.

We consider an arbitrary cluster of vehicles, denoted by $\mathcal{V}$, on a multi-lane road as shown in Fig. 1. The set cardinality (i.e., the number of elements in the set) is denoted by $|\cdot|$ such that $|\mathcal{V}|$ is the number of vehicles in the cluster. Radar sensors are mounted on the vehicles front-ends and back-ends and the vehicles are also equipped with communication transceivers for vehicle-to-vehicle (V2V) communication. Radar sensing and communication both take place on the same shared spectrum, which is divided into a set of channels $\mathcal{K}$, each of bandwidth $B$. Let $\mathcal{R} \subseteq \mathcal{V}$ and $\mathcal{C} \subseteq \mathcal{V}$ denote, respectively, the subsets of vehicles wishing to access the spectrum for radar sensing and for communication. Vehicles send their channel assignment requests along with their QoS requirements to the cluster head, which handles the received requests and announces the channel assignment to the vehicles of the cluster. We assume that the vehicles of $\mathcal{V}$ have negligible relative velocities. This is a simplifying assumption to have stable members and shape of the cluster. Although clusters change their formation along the road due to vehicles mobility, the assumption holds validity in some cases such as urban roads with low-speed limits where vehicles tend to travel with close enough velocities, and consequently clusters tend to have more stable formations. On the other hand, freeways with different speed limit per lane may experience more frequent changes in cluster shape and formation. The channel assignment needs to be updated whenever the cluster changes its formation. For example, channels occupied by members that left the cluster should be freed, while new members of the cluster should be assigned channels upon their request based on the current cluster and channel occupation status. The lifetime of a cluster, and hence the assumed stable formation, depends on many factors, which are out of the scope of our work \cite{20}.

B. TRANSMITTED SIGNALS

We consider mono-static linear FMCW radars, where a radar $i \in \mathcal{R}$ transmits a sequence of chirps, each being a linearly frequency modulated signal as follows: \cite{8,21}

$$T_i(t) = \sqrt{P_i} \exp\left(j2\pi \left(f_i^* + \frac{B_f}{2T}\right) t\right), \quad (1)$$

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The binary indicator that takes the value 1 if vehicle $K_i$ is the speed of light. The second and third terms represent $G_{tx,i}$ and $G_{rx,i}$, respectively. In this case, the radar operates on the channels in the continuous frequency band $[f_1, f_1 + B_i]$. We consider multi-carrier communication where a vehicle establishes a one hop communication link with another vehicle. Hence, the transmitted communication signal of vehicle $j \in \mathcal{C}$ is given by

$$T_j(t) = \sum_{k \in \mathcal{K}_j} P_k^r \sqrt{\phi_k} \exp(j2\pi f_k t),$$

where $P_k^r$ is the transmit power per channel of vehicle $j \in \mathcal{C}$ and $\mathcal{K}_j$ is the set of channels assigned to vehicle $j$. Also, $s_{k,j}$ is the data symbol of vehicle $j$ transmitted over channel $k$ and $f_k$ is the frequency of channel $k$.

## C. RECEIVED SIGNALS

The received signal at the receiver of radar $i \in \mathcal{R}$ is given by

$$R_i(t) = \sum_{k \in \mathcal{K}_i} \sum_{v \in \mathcal{V}_i} x_{k,v} \sqrt{G_{rx,i}^r G_{tx,v}^r \lambda^2 / (4\pi d_{i,v}^2)} T_v(t - d_{i,v} / c)$$

$$+ \sum_{k \in \mathcal{K}_i, v \in \mathcal{V}_i} x_{k,v} \sqrt{G_{rx,i}^r G_{tx,v}^r \lambda^2 / (4\pi d_{i,v}^2)} T_v(t - d_{i,v} / c)$$

$$+ n_i(t).$$

The first term in $R_i(t)$ is the desired reflected signal, where $G_{rx,i}^r$ and $G_{tx,v}^r$ are the transmit and receive antenna gains and $\phi_i$ is the operating range, all of radar $i \in \mathcal{R}$, $\lambda$ is the wavelength, $\Omega$ is the radar cross section of the target, and $c$ is the speed of light. The second and third terms represent the interference from other vehicles in $\mathcal{R}$ and $\mathcal{C}$ that are assigned channels in the set assigned to vehicle $i \in \mathcal{R}$ (i.e., $\mathcal{K}_i$), respectively. In these two terms, $x_{k,v} \in \{0, 1\}$ is a binary indicator that takes the value 1 if vehicle $v \in \mathcal{V}$ is assigned channel $k \in \mathcal{K}_i$ and 0 otherwise.

### III. PERFORMANCE METRICS

#### A. RADAR PERFORMANCE

The performance of radars is measured by the probability of detection $\zeta$, the probability of false alarm $\eta$, and the error in parameter estimation over their operating ranges. These metrics are determined by the radar SINR and bandwidth. In particular, the probabilities of detection and false alarm of radar $i \in \mathcal{R}$ are related to its SINR as follows:

$$\zeta_i = \frac{1}{2} \text{erfc} \left( \text{erfc}^{-1}(2\eta_i) - \sqrt{\gamma^i} \right),$$

where $\text{erfc}(\cdot)$ is the complementary error function. $\gamma^i$ is the SINR at radar $i \in \mathcal{R}$ and is calculated from $R_i(t)$ as follows:

$$\gamma^i = \frac{P^r_x G_{tx,x}^r G_{tx,i}^r |\Omega|^2}{4\pi \phi_i^4 \sum_{k \in \mathcal{K}_i, v \in \mathcal{V}_i} x_{k,v} I_{i,v,}}.$$ 

The term $I_{i,v}$ in $\gamma^i$ is the interference from vehicle $v \in \mathcal{V}$ to radar $i \in \mathcal{R}$. It is calculated from $R_i(t)$ as follows:

$$I_{i,v} = \frac{G_{rx,i}^r |\Omega|^2}{4\pi \phi_i^4} \times \left\{ \frac{P^r_x G_{tx,i}^r G_{tx,v}^r}{P^r_x G_{tx,v}^r} \right\} \forall v \in \mathcal{R} \setminus \{i\}, \forall v \in \mathcal{C}.$$ 

Also, from [21], the mean squared error of range estimation of radar $i \in \mathcal{R}$ is lower bounded by the Cramer-Rao bound, $\epsilon_i$ as follows:

$$\epsilon_i = \frac{3}{2(2\pi)^2 \gamma^i} (\Delta r_i)^2.$$
where $\gamma_i^c$ is the SINR in (6), and $\Delta r_i$ is the range resolution given as

$$\Delta r_i = \frac{c}{2B_i^r}, \quad (9)$$

with $c$ being the speed of light, and $B_i^r$ being the chip bandwidth (i.e., the difference between the end and start frequencies of the chip) of radar $i \in \mathcal{R}$.

From (9), the chirp bandwidth required by radar $i \in \mathcal{R}$ to satisfy specific range resolution requirement ($\Delta r_i$) is

$$\hat{B}_i^r = \frac{c}{2\Delta r_i}. \quad (10)$$

Also, from (5) and (8), the SINR required by radar $i \in \mathcal{R}$ to satisfy specific probability of detection, probability of false alarm, and range estimation error, denoted respectively by $\tilde{\zeta}_i$, $\tilde{\eta}_i$, and $\tilde{\epsilon}_i$ is

$$\gamma_i^r = \max\left(\frac{3(\Delta r_i)^2}{2(2\pi)^2\tilde{\epsilon}_i}, \left(\text{erfc}^{-1}(2\tilde{\eta}_i) - \text{erfc}^{-1}(2\tilde{\epsilon}_i)\right)^2\right). \quad (11)$$

Equivalently, the required interference threshold $\hat{I}_i^r$ is calculated from (6) and (11) as follows

$$\hat{I}_i^r = \frac{P_i^r G_{rx,i}^c(\theta_{i,i})G_{rx,j}^c(\theta_{j,j})\Omega \lambda^2}{(4\pi)^3 \nu_i^r \nu_i^r} - \sigma_i^r, \quad (12)$$

### B. COMMUNICATION PERFORMANCE

Obviously, different vehicles have different bit-rate requirements depending on the applications they run. We assume that, unlike radars, communication can be aggregated over non-contiguous channels such that the total bit-rate of a vehicle is the aggregate bit-rate over its assigned channels. Hence, the total bit-rate of a communication link between vehicle $j \in \mathcal{C}$ and its receiver $i^*$ is

$$R_j^c = \sum_{k \in \mathcal{K}_j} R_{j^*,k}^c (1 - \text{BER}_{j^*,k}^c), \quad (13)$$

where $R_{j^*,k}^c$ and $\text{BER}_{j^*,k}^c$ are the bit-rate and bit-error-rate on channel $k \in \mathcal{K}_j$. The value of $R_{j^*,k}^c$ depends on the modulation order and channel bandwidth, and the value of $\text{BER}_{j^*,k}^c$ depends on the modulation type and the SINR at the receiver on channel $k$. From (4), the SINR at $j^*$ on channel $k$ is

$$\gamma_{j^*,k}^c = \frac{P_j^c G_{rx,j}^c(\theta_{j^*,j})G_{rx,j}^c(\theta_{j,j})\lambda^2}{(4\pi)^3 \nu_j^c \nu_j^c} \left(\sigma_{j^*} + \sum_{v \neq j} I_{j^*,v} x_{k,v}\right). \quad (14)$$

The term $I_{j^*,v}$ in (14) is the interference from vehicle $v$ to $j^*$ calculated from (4) as follows

$$I_{j^*,v} = \frac{G_{rx,j}^c(\theta_{j^*,j})\lambda^2}{(4\pi d_{j^*j})^2} \times \begin{cases} \frac{P_v^c}{\pi d_{j^*v}^2} G_{rx,v}^c(\theta_{j^*,v}) \quad \forall v \in \mathcal{R} \\ P_v^c G_{rx,v}^c(\theta_{j^*,v}) \quad \forall v \in \mathcal{C} \end{cases}. \quad (15)$$

Different modulation types can be used on different channels according to the changing channel conditions. However, to avoid unnecessary complexity, we assume that channels have similar conditions as the coherence bandwidth of mmWave is wide especially in case of line of sight [23], [24]. Channel assignment with adaptive modulation and coding can be considered for future work. We consider, without loss of generality, a binary phase shift keying modulation such that

$$R_{j^*,k}^c = 0.5B_j, \quad \text{BER}_{j^*,k}^c = Q\left(\sqrt{2\gamma_{j^*,k}^c}\right), \quad (16)$$

where $Q(\cdot)$ is the well-known $Q$ function. From (16), the SINR required by vehicle $j \in \mathcal{C}$ to achieve specific bit-error-rate $\text{BER}_{j^*,}^c$ per channel is

$$\gamma_{j^*,k}^c = \frac{1}{2} \left(Q^{-1}\left(\text{BER}_{j^*,}^c\right)\right)^2. \quad (17)$$

Equivalently, the required interference threshold $\hat{I}_{j^*}^c$ is calculated from (14) and (17) as follows

$$\hat{I}_{j^*}^c = \frac{P_j^c G_{rx,j}^c(\theta_{j^*,j})G_{rx,j}^c(\theta_{j,j})\lambda^2}{(4\pi d_{j^*,j})^2} \gamma_{j^*,k}^c - \sigma_{j^*}^c, \quad (18)$$

where $\gamma_{j^*,k}^c$ is the SINR threshold in (17).

From (18), the number of channels required to achieve total bit-rate $\hat{R}_{j^*}$ is

$$N_{j^*}^c = \frac{2\hat{R}_{j^*}^c}{B_1 - \text{BER}_{j^*,}^c} \approx \frac{2\hat{R}_{j^*}^c}{B_j}. \quad (19)$$

where $\approx$ is because the required bit-error-rate is usually very small (i.e., $\text{BER}_{j^*,}^c << 1$).

### IV. PROBLEM FORMULATION

Vehicles that wish to access the spectrum send their channel assignment requests to the cluster head. Alongside these requests, the vehicles also send their QoS requirements represented by their required bandwidth and interference threshold. These values are calculated from (10), (12) for vehicles in $\mathcal{R}$ and from (19), (18) for vehicles in $\mathcal{C}$. The cluster head optimizes the channel assignment based on the received requests and announces which vehicles are granted access to the spectrum (i.e., served) and what channels are assigned to them. We consider hard QoS requirements such that vehicles are granted access to the spectrum only if their QoS requirements (i.e., bandwidth and interference threshold) can be satisfied. Consequently, a cluster head grants spectrum access to the group of vehicles whose QoS requirements can be satisfied simultaneously. We assume that the cluster head has knowledge of the mutual interference between the cluster members (i.e., the terms $I_{i,v}$, $\forall i, v \in \mathcal{V}$) which, from (7) and (15), depend mainly on the vehicles' relative positions. Since a cluster head periodically polls the cluster members, such information can be exchanged periodically between the cluster members. Fig. 2 visually illustrates the channel assignment to a cluster of vehicles where multiple vehicles are allowed to share the same channel as long as their interference thresholds are satisfied. Moreover, communication vehicles in $\mathcal{C}$ can be assigned contiguous or disjoint channels, while radars in $\mathcal{R}$
are assigned contiguous channels due to their linear FMCW signal. In the example shown in Fig. 2, five vehicles requested channel assignment. Four of the five requesting vehicles were served (i.e., assigned channels), while vehicle number four was blocked (i.e., was denied channel assignment). It means that, due to inter-vehicle interference, no channel assignment can satisfy the QoS requirements of all the five vehicles simultaneously.

Based on the above discussion, and in order to make the most of the shared spectrum, we optimize channel assignment to maximize the number of vehicles served over the shared spectrum (i.e., vehicles that are assigned channels) subject to constraints on the bandwidth and interference requirements of the vehicles as follows:

\[
\begin{align*}
\text{maximize} & \sum_{v \in V} z_v \\
\text{subject to} & \sum_{k \in K} x_{k,j} = \tilde{N}_j z_j, \\
& \sum_{b \in B_i} y_{b,i} = z_i, \quad B_i = \{1, 2, \ldots, |K| - \tilde{N}_i + 1\}, \\
& \sum_{v \in V, v \neq j} x_{k,v} I_{j,v} \leq x_{k,j} \tilde{I}_j + (1 - x_{k,j}) F, \\
& \sum_{k=b+\tilde{N}_j-1}^{b+N_j-1} \sum_{v \in V, v \neq i} x_{k,v} I_{i,v} \leq y_{b,i} \tilde{I}_i + (1 - y_{b,i}) F, \\
& z_v, x_{k,v}, y_{b,i} \in \{0, 1\},
\end{align*}
\]

(20a) (20b) (20c) (20d) (20e) (20f) (20g)

The objective in (20a) is to maximize the number of served vehicles where \( z_v \in \{0, 1\} \) takes the value 1 if vehicle \( v \in V \) is served and 0 otherwise, \( x_{k,v} \in \{0, 1\} \) takes the value 1 if channel \( k \in K \) is assigned to vehicle \( v \in V \) and 0 otherwise. Constraint (20b) ensures that a served vehicle \( j \in C \) gets its required number of channels \( \tilde{N}_j \), and constraint (20c) ensures that the interference at its receiving vehicle \( j^* \) does not exceed a threshold \( \tilde{I}_j \) where \( F \) is a parameter of high value to stress that interference at the receiver \( j^* \) only matters on the channels assigned to vehicle \( j \). Similarly, constraint (20d) ensures that a served radar \( i \in R \) gets a band of \( \tilde{N}_i = B_i / B \) contiguous channels where \( y_{b,i} \in \{0, 1\} \) indicates whether the band, which starts at channel \( b \in B_i \) is assigned to radar \( i \). It is worth mentioning that the set \( B_i \) comprises \( |K| - \tilde{N}_i + 1 \) overlapping bands each consisting of \( \tilde{N}_i \) contiguous channels. Similar to (20e), constraint (20f) ensures that for any served radar \( i \in R \), the interference on its assigned band does not exceed a required threshold \( \tilde{I}_i \). Finally, constraint (20g) transforms the binary band assignment vector \( y_i \) (of size \( |K| - \tilde{N}_i + 1 \times 1 \)) to the binary channel assignment vector \( x_i \) (of size \( |K| \times 1 \)). If radar \( i \in R \) is served, then \( y_i \) has a single non-zero entry that indicates the index of its assigned band. This is achieved through the linear transformation matrix \( M_i \) that takes the form

\[
M_i = \begin{bmatrix}
0 \times 0 & 1 \times \tilde{N}_i' & 0 \times |K| - \tilde{N}_i' \\
0 \times 1 & 1 \times \tilde{N}_i' & 0 \times |K| - \tilde{N}_i' - 1 \\
0 \times 2 & 1 \times \tilde{N}_i' & 0 \times |K| - \tilde{N}_i' - 2 \\
\vdots & \vdots & \vdots \\
0 \times |K| - \tilde{N}_i' & 1 \times \tilde{N}_i' & 0 \times 0
\end{bmatrix}
\]

(21)

where 0 and 1 are the all-zeros and all-ones vectors, respectively.

Although the problem in (20) assumes same priorities for channel assignment requests of radars and communications, radars can be prioritized by assigning high weights to \( z_v \) for \( v \in R \) than for \( v \in C \) in the objective (maximizing a weighted sum of the served vehicles). Radars can also be prioritized through sequential assignment where radar requests are handled first and then communication requests.

The problem in (20) is a 0-1 ILP as the optimization variables (i.e., decision variables) are binary, and the objective and constraints are linear [26]. The problem has \(|K| + 1 \times (|V| + |R|) - \sum_{i \in R} \tilde{N}_i \) binary optimization variables and it can be solved directly using branch-and-cut based techniques [27]. However, these techniques have exponential complexity in the worst case while their average complexity is usually unknown as they include branching, cut generation, linear relaxation, and various searches [28, 29]. It was suggested in [30] that, for some cases, the number of visited nodes in the search tree by branch-and-cut has the order \( O(|V|^2) \), where \( |V| \) is the depth of the search tree. Assuming a binary tree, then approximately \(|K|^2 |V|^2 \) nodes are visited, each node is a linear relaxation of the problem that is solved using interior point methods which, in turn, have \( O(|K|^3 |V|^3) \) complexity [31], and consequently the average complexity of branch-and-cut techniques is expected to be around \( O(|K|^5 |V|^5) \). This is in addition to the complexity of cut generation and search rules. Since we consider channel assignment for vehicles, computational complexity and runtime should be favored over optimal assignment and conse-
quently, in the next section, we propose a heuristic channel assignment algorithm that has less complexity and run-time than the classical branch-and-cut based approaches.

V. ORDERED SEQUENTIAL CHANNEL ASSIGNMENT

In this section, we propose a heuristic algorithm that uses ordered sequential channel assignment, namely OSCA in Algorithm 1. The algorithm uses an intuitive sequential approach to handle the heavily constrained problem in (20). The main idea of the algorithm is handling the vehicles requests in an ordered sequential way such that each vehicle is assigned the channels that satisfy its QoS requirements if possible, and is blocked (i.e., denied assignment) otherwise. We also present its idea as a flowchart in Fig. 3.

The inputs to the algorithm are the sets $\mathcal{R, C}$, the number of required channels and interference threshold ($N_v$ and $I_v$) besides the inter-vehicle interference $I_{v,l}$ $\forall v,l \in \mathcal{V}$. The outputs are the sets of channels assigned to each vehicle, denoted by $K_v \forall v \in \mathcal{R} \cup \mathcal{C}$. The set $\mathcal{R}$ in the initialization step denotes the set of assigned radars, and the sets $\mathcal{R}_k$ and $\hat{\mathcal{C}}_k$ denote, respectively, the sets of assigned vehicles in $\mathcal{R}$ and $\mathcal{C}$ on channel $k$. Step 1 in the algorithm determines the assignment order as vehicles are sorted in descending order according to their ratios of ingoing to outgoing interference (i.e., $\sum_{l \neq v} I_{v,l} / \sum_{m \neq v} I_{m,v}$ for all $v \in \mathcal{R} \cup \mathcal{C}$). In the algorithm, $v^*$ denotes the destination vehicle (i.e., receiver) of $v \in \mathcal{C}$. If $v \in \mathcal{R}$, then $v^* = v$ as we consider mono-static automotive radars. The rationale behind this sorting is that channel assignment to vehicles with high outgoing interference should be delayed so as not to hinder possible assignment of other vehicles. This is in contrast to assignment to vehicles with high ingoing interference, which should be early to consider their vulnerability when assigning other vehicles. For each vehicle $v$, Steps 3-5 determine the receiver of the vehicle, and Step 6 starts with assuming that all channels are available for use. Then, OSCA disables channel $k$ for vehicle $v$ if using it would cause high interference to previously assigned vehicles (Steps 9-19) or if vehicle $v$ receives high interference over it (Steps 20-21). In these steps, $\text{ToI}_{c,k}$ is the interference tolerance of $c \in \hat{\mathcal{C}}_k$ over channel $k$ and $\text{ToI}_{r}$ is the interference tolerance of $r \in \mathcal{R}_k$. The counter $i_v$ in Step 16 shows how many channels in the band assigned to a radar $r$ can be reused by vehicle $v$. Algorithm 2 namely Communication Channels Assignment (CCA), and Algorithm 3 namely Radar Bands Assignment (RBA), are called in Steps 24 and 26 to assign channels to vehicles in $\mathcal{C}$ and $\mathcal{R}$, respectively. Steps 30-31 reduce the interference tolerance of previously assigned vehicles by the amount of newly generated interference by $v$ over its assigned channels in $K_v$. Finally, Steps 32-38 add $v$ to the set of assigned vehicles and calculate its interference tolerance in order to be considered when assigning new vehicles. In the next two subsections, we explain the details of the CCA and RBA algorithms.

Algorithm 1 OSCA Procedure

Input: $\mathcal{C, R, v, I_v, N_v, \hat{I}_v, \forall v \in \mathcal{V}$
Output: $K_v, \forall v \in \mathcal{R} \cup \mathcal{C}$
Initialize $\mathcal{R} = \emptyset, \mathcal{R}_k, \hat{\mathcal{C}}_k = \emptyset, \forall k \in \mathcal{C}$
1: Form a queue $Q$ of all $v \in \mathcal{R} \cup \mathcal{C}$ sorted in descending order according to $\sum_{l \neq v} I_{v,l} / \sum_{m \neq v} I_{m,v}$.
2: for $v \in Q$ do
3: if $v \in \mathcal{R}$ then $v^* = v$
4: else $v^* = \text{destination}(v)$
5: end if
6: Set $\text{Valid}_k = 1 \ \forall k \in \mathcal{K}$
7: for $k \in \mathcal{K}$ do
8: $I_{v^*,k} = 0$
9: for $c \in \hat{\mathcal{C}}_k$ do
10: $I_{v^*,k} = I_{v^*,k} + I_{v^*,c}$
11: if $\text{ToI}_{c,k} < I_{v^*,c}$ then $\text{Valid}_k = 0$
12: end if
13: end for
14: for $r \in \mathcal{R}_k$ do
15: $I_{v^*,k} = I_{v^*,k} + I_{v^*,r}$
16: $i_v = \floor{\text{ToI}_{v^*,k} / I_{v^*,k}}$
17: if $i_v < 1$ then $\text{Valid}_k = 0$
18: end if
19: end for
20: if $I_{v^*,k} > \hat{I}_v$ then $\text{Valid}_k = 0$
21: end if
22: end for
23: if $v \in \mathcal{C}$ then
24: $K_v = \text{Call CCA procedure in Algorithm 2}$
25: else
26: $K_v = \text{Call RBA procedure in Algorithm 3}$
27: $\text{ToI}_v = \hat{I}_v$
28: end if
29: for $k \in K_v$ do
30: Update $\text{ToI}_{c,k} = \text{ToI}_{c,k} - I_{c^*,v}, \forall c \in \hat{\mathcal{C}}_k$
31: Update $\text{ToI}_r = \text{ToI}_r - I_{r,v}, \forall r \in \mathcal{R}_k$
32: if $v \in \mathcal{C}$ then
33: Update $\hat{\mathcal{C}}_k = \hat{\mathcal{C}}_k \cup \{v\}$
34: Set $\text{ToI}_{v,k} = \hat{I}_v - I_{v^*,k}$
35: else
36: Update $\mathcal{R}_k = \mathcal{R}_k \cup \{v\}$
37: Set $\text{ToI}_v = \text{ToI}_v - I_{v^*,k}$
38: end if
39: end for
40: end for
A. COMMUNICATION CHANNELS ASSIGNMENT (CCA)
The CCA procedure is detailed in Algorithm 2 and it is also represented as a flowchart in Fig. 4. The algorithm starts by sorting the valid channels in a descending order according to a calculated reward (Steps 1-2). The reward assigned to channel $k$ is the total number of previously assigned vehicles on $k$. The rationale behind this choice is that it is better to reuse channels as much as possible so as to save free channels for future vehicles. Steps 4-7 then assign a channel $k$ to the vehicle if it does not violate the reuse counters $i_r$ for all $r \in \mathcal{R}_k$. The algorithm ends in two cases: if $v$ is assigned its required number of channels or if the valid channels are not enough to schedule $v$, in this case, $v$ is not served.

B. RADAR BANDS ASSIGNMENT (RBA)
RBA procedure is detailed in Algorithm 3 and it is represented as a flowchart in Fig. 5. RBA iterates over the possible contiguous bands for vehicle $v$ (Steps 1-12). In Steps 1-2, the algorithm decides the start and end channels of the current band. Step 3 checks if all channels in the band are valid for use and that the received interference over the band is acceptable. Steps 4-7 calculate the overlap between the current band and the bands of previously assigned radars and checks if the overlap is less than the overlap counter $i_r$ for all $r \in \mathcal{R}$.

A band that satisfies these conditions is added to the set of valid bands (Step 8) with a reward (Step 9), which is the total number of previous channel assignments in the band. In Steps 13-18, if the set of valid bands is non-empty, valid bands are sorted in descending order according to their rewards (Step 14). Similar to CCA, the rationale is to save spectrum by reusing the already occupied bands as much as possible. Finally, the vehicle is assigned the band with highest reward (Step 15-16), and is added to the set of assigned radars $\mathcal{R}$ (Step 17).

C. COMPLEXITY ANALYSIS
The complexity of step 1 in OSCA is the complexity of sorting an array of $|V|$ elements. We assume, for simplicity, a selection sort with complexity $O(|V|^2)$ [32]. Based on the number of nested loops and iterations, the complexities of Steps 7-22 and Steps 29-39 in OSCA are upper bounded by $O(|K||V|)$ [33]. Consequently, the complexity of OSCA is upper bounded by $O\left( (|K||V|^2 + |V| \max \left( O(CCA), O(RBA) \right)) \right)$. Following the same approach, the complexity of CCA is upper bounded by $O\left( |K||\mathcal{R}| + |K|^2 \right)$. As for RBA,
the complexity of Steps 1-12 is upper bounded by $O\left(\max_{i \in \mathcal{R}} \left( \left| \mathcal{R} \right| + \hat{N}_v + 1 \right) \right)$ and the complexity of Steps 13-18 is upper bounded by $O\left(\max_{i \in \mathcal{R}} \left( \left| \mathcal{K} \right| - \hat{N}_v + 1 \right) \right)^2$. Consequently, the complexity of RBA is $< O\left(\left| \mathcal{R} \right| \left| \mathcal{K} \right| + \left| \mathcal{K} \right|^2 \right)$. Combining the complexities of the algorithms, the worst case complexity of OSCA is found to be upper bounded by $O(\left| \mathcal{R} \right| \left| \mathcal{V} \right|^2 + \left| \mathcal{K} \right|^2 \left| \mathcal{V} \right|) < O(\left| \mathcal{K} \right|^2 \left| \mathcal{V} \right|^2)$, which is much lower than the average complexity of branch-and-cut based techniques.

**Algorithm 2 Communication Channels Assignment (CCA)**

Initialize $\text{Temp}\mathcal{K}_v, \mathcal{K}_v = \emptyset$

1. Calculate $r_w(k) = \left| \hat{\mathcal{R}}_k \right| + |\hat{\mathcal{C}}_k|, \forall k \in \mathcal{K}$ with $\text{Valid}_k = 1$
2. Form a queue $Q_K$ with all valid channels sorted in descending order according to $r_w(k)$
3. for $k \in Q_K$ do
4. if all $i_r - 1 \geq 0, \forall r \in \hat{\mathcal{R}}_k$ then
5. Update $\text{Temp}\mathcal{K}_v = \text{Temp}\mathcal{K}_v \cup \{k\}$
6. Update $i_r = i_r - 1, \forall r \in \hat{\mathcal{R}}_k$
7. end if
8. if $|\text{Temp}\mathcal{K}_v| = \hat{N}_v$ then
9. $\mathcal{K}_v = \text{Temp}\mathcal{K}_v$
10. Jump to \ref{Algorithm 3}
11. end if
12. end for
13. Exit

**Algorithm 3 Radar Band Assignment (RBA)**

Initialize $\text{Temp}\mathcal{B}_v, \mathcal{K}_v = \emptyset$

1. for $b_{\text{Start}} = 1 : |\mathcal{K}| - \hat{N}_v + 1$ do
2. $b_{\text{End}} = b_{\text{Start}} + \hat{N}_v - 1$
3. if $\sum_{n = b_{\text{Start}}}^{b_{\text{End}}} \text{Valid}(b) = \hat{N}_v$ and $\sum_{b = b_{\text{Start}}}^{b_{\text{End}}} I_v(b) < \hat{I}_v$ then
4. for $r \in \hat{\mathcal{R}}$ do
5. $\text{BO}_r = \min (\mathcal{K}_r(1), b_{\text{End}}) - \max (\mathcal{K}_r(1), b_{\text{Start}}) + 1$
6. end for
7. if all $\text{BO}_r \leq i_r$ then
8. $\text{Temp}\mathcal{B}_v = \text{Temp}\mathcal{B}_v + \{b\}$
9. $r_w(b_{\text{Start}}) = \sum_{b = b_{\text{Start}}}^{b_{\text{End}}} |\hat{\mathcal{R}}_b| + |\hat{\mathcal{C}}_b|$
10. end if
11. end if
12. end for
13. if $\text{Temp}\mathcal{B}_v \neq \emptyset$ then
14. Sort the valid bands in $\text{Temp}\mathcal{B}_v$ in descending order according to $r_w$
15. $b^* = \text{Temp}\mathcal{B}_v(1)$
16. $\mathcal{K}_v = \{b^*: b^* + \hat{N}_v - 1\}$
17. Update $\hat{\mathcal{R}} = \hat{\mathcal{R}} \cup \{v\}$
18. end if

**VI. NUMERICAL RESULTS**

This section compares the performance of OSCA with the branch-and-cut solution, obtained using the CVX solver [27], using numerical Monte-Carlo simulations. We consider, without loss of generality, a cluster that includes all vehicles on a 50 m road segment such that the number of vehicles in the cluster is the number of lanes × vehicle density per lane (in m⁻¹) × 50. The locations of vehicles within the cluster are random and so are the types of their channel assignment requests. A typical vehicle can be in one of the following states: active front (or back) communication transmitter, active front (or back) communication receiver, active front (or back) radar, where radars can be LR, SR, or VSR. The number of channels required by a communication transmitter, $N_c$ is taken to be a uniform random variable over the set {1, 2, 3, ..., 10}. We consider, without loss of generality, a total of 40 channels, each of 100 MHz bandwidth in the automotive radar spectrum 77-81 GHz. The detailed simulation parameters are given in Table [1][1]. All vehicles are assumed to have the following antenna gain in dBi [1]

$$G_{tx} = G_{rx}(\theta) = \begin{cases} G_0 - 12u^2, & 0 \leq u \leq 1.152, \\ G_0 - 15 - 15\log(u), & 1.152 \leq u, \end{cases}$$

(22)

where $u = \theta/\Theta$, and $\Theta$ is the 3-dB beamwidth in degrees. We ignore the noise power since interference is the major concern in the model.

The performance of the considered algorithms (i.e., OSCA and the branch-and-cut) is represented by two metrics. The first is the percentage of served vehicles, defined as the
number of served (i.e., assigned) vehicles divided by the total number of vehicles that request channel assignment × 100%. The second is the run-time of the algorithm, which we take as a numerical measure of its complexity. Both quantities are averaged over different vehicles’ locations and assignment requirements.

Figs. 6 and 7 show the average percentage of served vehicles and the average run-time versus the vehicle density per lane. As seen in Fig. 6, the percentage of served vehicles decreases as the competition over spectrum increases (higher vehicle density or more lanes). Generally, OSCA serves fewer vehicles compared to branch-and-cut, however OSCA has much shorter run-time while the run-time of branch-and-cut increases considerably with the number of vehicles as seen in Fig. 7 where simulations are performed on a core i7-4790 machine. The performance of branch-and-cut degrades when the number of vehicles increases (due to large increase in the number of optimization variables and constraints) and sometimes it cannot even find a good solution within a maximum run-time of 15 minutes per instance according to our experiments.

The results in Figs. 6 and 7 are generated assuming non attenuated same-lane interference, which is the interference generated by other vehicles on the same lane. However, vehicles on the same lane are usually aligned such that only the interference from the nearest vehicle, ahead or behind depending on the transmission direction, is significant as the body of that vehicle attenuates, or ideally blocks, interference from farther vehicles on the same lane. We show in Figs. 8 and 9 how setting the same-lane interference to zero, except from the nearest vehicle, affects the performance of OSCA and branch-and-cut algorithms. As expected, the average number of served vehicles increases in the case without same-lane interference as vehicles on the same lane can share the same channels simultaneously without harming each other. The two cases (i.e., with and without same-lane interference) serve as lower and upper bounds to the performance of OSCA and branch-and-cut in practical environments where the same-lane interference is partially blocked. Interestingly, the absence of same-lane interference affects the average run-time of OSCA and branch-and-cut differently as shown in Fig. 8. In case of branch-and-cut, zero same-lane interference relaxes the constraints in (20e) and (20f), and consequently reduces the average run-time of the algorithm, especially in cases of crowded lanes (i.e., high vehicle density per lane). On the other hand, the average run-time of OSCA increases slightly in the case without same-lane interference. This is because less interference means that more vehicles can share same channels simultaneously, and consequently, OSCA needs to choose from a bigger pool of available channels. However, it is noticeable that the run-time of OSCA is still much less than the run-time of branch-and-cut.

VII. CONCLUSION

This paper proposed a fast heuristic channel assignment, namely OSCA, that maximizes the number of served vehicles on a shared spectrum between automotive radars and communication transceivers. OSCA has low computational
complexity and fast run-time compared to branch-and-cut based assignment techniques at the cost of a slight performance degradation. The work in this paper can be extended to include joint optimization of channel assignment and power allocation with adaptive modulation order.

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