X-RAY SPECTRA FROM MAGNETOHYDRODYNAMIC SIMULATIONS OF ACCRETING BLACK HOLES

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ABSTRACT

We present the results of a new global radiation transport code coupled to a general relativistic magnetohydrodynamic simulation of an accreting, non-rotating black hole. For the first time, we are able to explain from first principles in a self-consistent way all the components seen in the X-ray spectra of stellar-mass black holes, including a thermal peak and all the features associated with strong hard X-ray emission: a power law extending to high energies, a Compton reflection hump, and a broad iron line. Varying only the mass accretion rate, we are able to reproduce a wide range of X-ray states seen in most galactic black hole sources. The temperature in the corona is $T_e \approx 10$ keV in a boundary layer near the disk and rises smoothly to $T_e \approx 100$ keV in low-density regions far above the disk. Even as the disk’s reflection edge varies from the horizon out to $\approx 6M$ as the accretion rate decreases, we find that the shape of the Fe Kα line is remarkably constant. This is because photons emitted from the plunging region are strongly beamed into the horizon and never reach the observer. We have also carried out a basic timing analysis of the spectra and find that the fractional variability increases with photon energy and viewer inclination angle, consistent with the coronal hot spot model for X-ray fluctuations.

Key words: accretion, accretion disks – black hole physics – X-rays: binaries

Online-only material: color figures

1. INTRODUCTION

Since the initial discovery of the magnetorotational instability (MRI) by Balbus & Hawley (1991) over two decades ago, tremendous progress has been made in simulating astrophysical accretion disks. Large-scale magnetohydrodynamic (MHD) simulations have steadily improved their resolution and physical accuracy, leading to greater understanding of the fundamental physics of accretion. Yet despite all this success, we are not much closer to reproducing X-ray observations of accreting black holes than we were after the original papers on the structure of their surrounding disks (Shakura & Sunyaev 1973; Novikov & Thorne 1973). From basic conservation laws, it is relatively easy to understand why the spectrum seen in some black holes can be ascribed to a multi-color disk model (Mitsuda et al. 1984). However, it was realized very early on that a great deal of the power of both stellar-mass black holes and active galactic nuclei (AGNs) is in the form of high-energy X-rays well above the thermal peak (Oda et al. 1971; Elvis et al. 1978).

Although it is now widely accepted that this hard flux comes from the inverse-Compton (IC) scattering of seed photons from the disk through a hot corona (Liang & Price 1978; Haardt & Maraschi 1993), we still know little or nothing about the origin or detailed properties of this corona. In classical disk theory there is no particular reason even to suppose such a corona exists. Consequently, almost all previous work has been performed phenomenological. Cartoon sketches are drawn to suggest the corona geometry, while parameterized models are used to divide the total dissipation between the disk and corona in an attempt to fit the data (Svensson & Zdziarski 1994; Done & Kubota 2006). Numerous papers have shown that for both galactic black holes and AGNs the hard spectra commonly observed can be explained only if the coronal heating is spatially localized and inhomogeneous (Haardt et al. 1994; Stern et al. 1995; Zdziarski et al. 1996; Poutanen et al. 1997), but there are many geometries in which this can happen, and most of those proposed are arbitrary and without firm grounding in dynamics. At best they have drawn on qualitative arguments and analogy with the solar corona (Galeev et al. 1979). One notable exception is Kawanaka et al. (2008), in which a similar method to our own is used to couple Monte Carlo radiative transfer to the global MHD simulations of Kato et al. (2004). However, their simulations are non-relativistic, assume that the corona is radiatively inefficient, and treat the corona as dynamically decoupled from the disk proper, so that the thermal seed photon input is determined without any connection to coronal properties.

Recent advances in numerical simulation methods now allow us to approach this problem from an approach founded directly on disk dynamics. Angular momentum transport, the central mechanism of accretion, can be calculated directly, as it arises from correlations induced by orbital shear in MHD turbulence stirred by the MRI (Balbus & Hawley 1998). Moreover, the same magnetic fields essential to creating internal stresses automatically rise buoyantly above the dense regions of the disk and dissipate, creating a hot corona. Thus, the mechanisms treated by MHD simulations lead directly to physical processes that promise to explain coronal phenomenology.

Until now, however, we have lacked the tools necessary for closing the loop and comparing the results of the simulations directly with the observations of coronal radiation. In this paper, by employing the radiative transfer code Pandurata (Schnittman & Krolik 2013) as a “post-processor” to simulation data generated by the general relativistic MHD code Harm3d (Noble et al. 2009), we make a critical step toward bridging the gap between theory and observation. In so doing, we will attempt to answer the question, “Can the coronae predicted by MHD dynamics produce hard X-rays with the observed luminosity and spectra?”
Our answer to that question will be founded on genuine disk physics. Whereas analytic disk models rely on dimensional analysis to describe the scaling of shear stress with pressure, direct calculation of the nonlinear development of the MRI allows us to compute quantitatively the rate of angular momentum transfer through the actual magnetic stresses. Thus, we can dispense with the greatest uncertainties of the traditional accretion model: the dependence of stress on local physical conditions; the spatial distribution of dissipation; and the inner boundary condition, which can now be moved inside the horizon, and thus made physically irrelevant because any numerical noise or physical information cannot propagate outward to the main simulation. The rate of energy dissipation, rather than being guessed via some parameterized relation to pressure, can be easily monitored with the flux-conservative code Harm3d, which contains an heuristic cooling function designed to generate thin disks (Noble et al. 2009). Especially important to the question we raise about coronal radiation, dissipation in the corona is the direct result of explicit dynamical calculation, not a scaling guess about the strength of the magnetic field. While Shakura & Sunyaev (1973) and subsequent phenomenological analyses have been restricted to steady-state, azimuthally and vertically averaged quantities, the MHD simulations provide dynamic, three-dimensional (3D) information about the fluid density, 4-velocity, magnetic pressure, gas pressure, and cooling at every point throughout the computational domain.

The first step on the path from simulation to observation is to convert the code variables to physical units by specifying the black hole mass and accretion rate. We can then distinguish between the optically thick disk body and the optically thin corona. In so doing, we divide the total energy released into a portion associated with the disk body and a portion associated with the corona. To predict the spectrum radiated from the disk body, we make an assumption similar to one made by Shakura & Sunyaev (1973) and Novikov & Thorne (1973), that energy dissipated in the disk is emitted as thermal radiation from the disk surface. To predict the spectrum radiated from the corona, we first demonstrate that other potentially relevant emission mechanisms (e.g., thermal bremsstrahlung, synchrotron) are negligible. We then balance local energy dissipation and IC up-scattering of disk seed photons in order to find the equilibrium electron temperature and radiation intensity as functions of position throughout the corona. After both these steps have been accomplished, we use the distribution of coronal radiation we have just found to predict the hard X-ray illumination of the disk surface and the Fe Kα fluorescence line generated by absorption of those hard X-rays.

As described in a companion paper (Schnittman & Krolak 2013), Pandurata is a fully relativistic Monte Carlo radiation transport code that integrates photon trajectories from the disk surface, accounts for scattering through the hot corona, and transports them to their ultimate destination, either a distant observer or the black hole horizon. For efficient transport, Pandurata tracks bundles of many photons along each geodesic path. These photon packets cover a wide range of energies, further increasing the efficiency in modeling the broad-band spectra expected from accreting black holes. While Pandurata includes polarization effects in all its scattering calculations, the results in this paper focus on spectral features alone.

By varying the mass accretion rate, we can reproduce many of the features that define the three main accretion states described in Remillard & McClintock (2006): hard, thermal, and steep power law (SPL). Although the results in this paper are based entirely on a single simulation whose structure most closely matches the classical predictions for a disk with $\dot{m} \approx 0.1–0.3$, there is qualitative agreement with observations spanning the entire range of $\dot{m} = 0.01–1.0$. We have also included a simple model for fluorescent line production and can reproduce Fe Kα features similar to those seen in many galactic black holes and AGNs (Miller et al. 2004, 2006a; Walton et al. 2012). For very low accretion rates, the vertically integrated optical depth falls below unity in the inner regions, so for Kα production the disk is effectively truncated around $r \approx 4–6M$. Interestingly, the iron line profile appears to be independent of the location of the reflection edge, as long as it is inside the innermost stable circular orbit (ISCO).

Last, we include some rudimentary variability analysis, finding results consistent with a large body of observations: the low-hard state is more variable than the disk-dominated state, and in both the thermal-dominant and SPL states the fractional rms amplitude increases with photon energy (Cui et al. 1999; Churazov et al. 2001; Gierlinski & Zdziarski 2005; Remillard & McClintock 2006). On short timescales, the amplitude of fluctuations increases with observer inclination angle, consistent with the coronal hot spot model of X-ray variability.

Although, as we have outlined, numerical simulation data offer many advantages for spectral predictions, the current state of the art in computational astrophysics imposes certain limitations. Two are of particular relevance here. The first is that practical and accurate algorithms for treating radiation forces simultaneously with MHD are still restricted to shearing-box models, and are not yet ready for application to global models (Hirose et al. 2006; Jiang et al. 2012; Davis et al. 2012). As a result, all 3D global simulations to date assume that the disk is supported by a combination of gas and magnetic pressure alone, even though radiation forces can be an important influence on disk structure, often dominating the disk’s support against the vertical component of gravity (Shakura & Sunyaev 1973). Although Pandurata provides a fully relativistic radiation transport post-processor, its calculation is not incorporated directly into the Harm3d simulation, and thus has no effect on the accretion dynamics. The second limitation is that because an adequate description of MHD turbulence requires a wide dynamic range in length scales (Hawley et al. 2011; Sorathia et al. 2012), the spatial resolution necessary to simulate disks as thin as some of those likely to occur in nature remains beyond our grasp. Thus, in some respects, our calculations represent an intermediate step toward drawing a complete connection between fundamental physics and output spectra. Nonetheless, as we will discuss in detail below, they offer new insights, and, for certain parameter values, are already good enough approximations to permit direct comparison with observations.

2. FROM SIMULATION DATA TO PHOTONS

2.1. Description of Harm3d

The data we analyze for this paper are drawn from the highest resolution simulation reported in Noble et al. (2010, 2011), designated “ThinHR” in those papers. Harm3d, the code used to generate the data, is an intrinsically conservative 3D MHD code in full general relativity; this particular simulation was computed in a Schwarzschild spacetime. Because it uses a coordinate system based on Kerr-Schild, Harm3d is able to place the inner boundary of the computation volume inside the black hole’s event horizon, thus obviating the need for
any guessed inner boundary conditions. The stress-energy conservation equation is modified to include a local cooling function; that is, we write \( \nabla \cdot T^\mu_\nu = -\mathcal{L}u^\mu, \) where \( T^\mu_\nu \) is the stress-energy tensor, \( u^\mu \) is the specific 4-momentum, and \( \mathcal{L} \) is non-zero only for gravitationally-bound gas, and only when the local temperature is greater than a target temperature \( T_e. \) When the temperature exceeds that threshold, the excess heat is radiated away on an orbital timescale. The target temperature \( T_e \) is chosen so as to keep the disk’s aspect ratio\(^5 \) \( H_{\text{gms}}/r \) close to a single pre-set value at all radii. In dimensionless code units, \( T_e \equiv (\pi/2)(R_c/r)(H_{\text{gms}}/r)^2, \) where \( R_c \) describes the correction to the vertical gravity due to relativistic effects (Noble et al. 2010). For the ThinHR simulation, the target scale height was \( H_{\text{gms}}/r = 0.06. \)

We took special pains to ensure the numerical quality of these simulations. Every 20\( M \) in time,\(^6 \) we measured the number of cells across the fastest growing MRI wavelength in both the vertical and the azimuthal directions (\( \lambda_z \) and \( \lambda_\phi \)). The minimum number to achieve the correct linear growth rate for vertical modes is 6 cells per \( \lambda_z \) (Sano et al. 2004); to describe nonlinear behavior, at least 20 cells per \( \lambda_z \) and at least 10 per \( \lambda_\phi \) are necessary (Hawley et al. 2011). The mass- and time-weighted values in ThinHR were 25 (vertical) and 18 (azimuthal). As discussed in Hawley et al. (2011), by this and several other measures, ThinHR is the best-resolved global thin-disk accretion simulation in the literature. By examining the time-dependent hydrodynamic and radiative properties of the fluid at several fiducial radii, Noble et al. (2010) determined that the final 5000\( M \) of the ThinHR simulation met the relevant criteria for inflow equilibrium in the inner disk. We therefore restrict our analysis of the simulation data to that period.

In studying simulations intended to represent statistically steady accretion, it is important to recognize that when there is only a finite amount of mass on the grid, some of it must move out in order to absorb the angular momentum removed from accreted material. Consequently, the radial range over which the disk can be said to be in inflow equilibrium is limited. For the simulations under consideration here, that range was typically \( r \lesssim 20M. \) In Noble et al. (2011), the disk beyond this radius was simply replaced with a standard relativistic Novikov–Thorne (N-T) thin disk (Novikov & Thorne 1973). That paper focused exclusively on thermal radiation, so a thin disk was an appropriate extrapolation of the simulation data beyond 20\( M. \) Here, we are primarily interested in the coronal properties of the accretion flow, for which there are no simple analytic solutions. Therefore, we include the entire body of simulation data out to \(~60M, \) beyond which the surface density of the gas begins to decrease rapidly, and the accretion disk is effectively truncated.

Despite the fact that the disk is not strictly in inflow equilibrium outside of \(~20M, \) the dissipation profile still roughly follows that expected for a classical disk. This can be seen in Figure 1, where we plot the radial shell-integrated dissipation profile for the \( \text{Harm3d} \) data, along with that given by N-T for the same accretion rate. Unlike Noble et al. (2011), here we make no attempt to normalize the dissipation profile by the radial mass accretion rate, which explains the somewhat larger deviation of \( \text{Harm3d} \) from N-T outside \(~10M \) shown in this figure than in

\[ \rho_{\text{cgs}} = \rho_{\text{code}} \frac{4\pi c^2}{\kappa GM} \frac{\dot{m}/\eta}{M_{\text{code}}}, \]

where \( \kappa = 0.4 \text{ cm}^2 \text{ g}^{-1} \) is the electron scattering opacity, and \( \dot{m} \) is the Eddington-scaled accretion rate assuming a radiative efficiency \( \eta = 0.06. \) For ThinHR, the mean accretion rate in code units was \( M_{\text{code}} = 3 \times 10^{-4}. \) Regardless of the exact value for \( \eta \) used in the conversion from code units to cgs, Pandurata

\[ \text{Figure 1. Luminosity profile } dL/d(\log r), \text{ integrated over } \theta \text{ and } \phi, \text{ and averaging over time. The solid curve is the Harm3d data, and the dashed curve is the Novikov–Thorne prediction. For both cases, the Eddington-normalized accretion rate is } \dot{m} = 0.1. \]

Figure 2 of Noble et al. (2011). Since the disk surface temperature scales like \( r^{-3/4} \) at large radius, and the corona temperature also decreases outward (see below, Figure 10), we do not expect significant contribution to the X-ray flux from outside of \( r = 60M. \)

### 2.2. Conversion from Code to Physical Units

When comparing the \( \text{Harm3d} \) predictions with real physical systems, the first step is to convert the fluid variables from dimensionless code units to physical cgs units. This conversion requires specifying the black hole mass \( M, \) which sets the natural length and timescales, and the accretion rate \( \dot{M}, \) which determines the scale for the gas density, cooling rate, and magnetic pressure. One technique for enacting this conversion is described in the Appendix of Noble et al. (2011), where the actual ray-tracing calculation is done in dimensionless units, and only the final observed spectrum is converted to physical units. That approach works best for optically thin systems where a single emission mechanism is used throughout the accretion flow and the photons do not interact with the matter.

In this work, however, we are interested in very different radiation processes in the disk and the corona. Therefore, before we even begin the Pandurata ray-tracing calculation, a photospheric boundary must be defined to distinguish between the cool, dense disk and the relatively hot, diffuse corona.\(^7 \) Since the location of this photosphere is a function of the gas density, it will be different for different values of the accretion rate.

For this reason, we must convert from code to physical units at the beginning of the calculation. There is only one consistent way to do this. Following Schnittman et al. (2006) and Noble et al. (2009), we relate the physical density to the code density by

\[ \rho_{\text{cgs}} = \rho_{\text{code}} \frac{4\pi c^2}{\kappa GM} \frac{\dot{m}/\eta}{M_{\text{code}}}, \]

where \( \kappa = 0.4 \text{ cm}^2 \text{ g}^{-1} \) is the electron scattering opacity, and \( \dot{m} \) is the Eddington-scaled accretion rate assuming a radiative efficiency \( \eta = 0.06. \) For ThinHR, the mean accretion rate in code units was \( M_{\text{code}} = 3 \times 10^{-4}. \) Regardless of the exact value for \( \eta \) used in the conversion from code units to cgs, Pandurata

\[ {^5} \] Throughout this paper we will consider multiple different scale heights. Here we refer to the gas density-weighted scale height \( H_{\text{gms}}. \)

\[ {^6} \] We set \( G = c = 1, \) so time has units of \((M/M_\odot) \cdot 4.9 \times 10^{-6} \) s, and distance has units of \((M/M_\odot) \cdot 1.5 \times 10^5 \) cm.

\[ {^7} \] Throughout this paper, we use the terms “photosphere” and “photospheric boundary” interchangeably. The term “corona” refers to the volume outside of the photosphere surface.
itself results in an independent value for the radiative efficiency, which is listed in Table 1. As shown there, it is never far from $\approx0.06$ when radiation from the outer disk is included.

Once the physical density is specified, the location of the photosphere at each point in the disk at any particular time is calculated by integrating the optical depth $d\tau = \kappa \rho(r, \theta, \phi) r d\theta$ at constant $(r, \phi)$ from the poles at $\theta = 0, \pi$ down toward the disk. The photosphere is then defined as the surface where the integrated optical depth reaches unity. For the top and bottom of the disk, the photospheric surfaces can be written as $\Theta_{\text{top}}(r, \phi)$ and $\Theta_{\text{bot}}(r, \phi)$ as in Schnittman & Krolik (2013):

$$\int_{\theta=0}^{\theta=\Theta_{\text{top}}} d\tau = \int_{\theta=\Theta_{\text{bot}}}^{\theta=\pi} d\tau = 1,$$

and the height of the photosphere is then simply given by $H_{\text{phot}} = r/|\cos \Theta|$.

With increasing $m$, the photosphere height increases, making the disk more like a bowl or inverted cone (imagine rotating the contours of Figure 2 around the $z$-axis). This shape increases the probability that photons scatter off other parts of the disk surface (the relativistic version of this effect is sometimes called “returning radiation”; see Cunningham 1976) and may subsequently be captured by the black hole. Thus, the radiative efficiency decreases steadily with larger $m$. This effect may be interpreted as the beginning of “super-Eddington photon trapping.”

For a Range of Mass Accretion Rates: the Bolometric Radiative Efficiency $\eta$, the Time-averaged Fraction of Total Luminosity in the Corona, the Radius of the Reflection Edge $R_{\text{refl}}$, the Disk-corona Transition Radius $R_{\text{trans}}$, and the Height $H_{\text{phot}}$ of the Scattering Photosphere (Averaged Over $r = 10-30M$)

| $m$ | $\eta$ | $L_{\text{cgs}}/L_{\text{tot}}$ | $R_{\text{refl}}/M$ | $R_{\text{trans}}/M$ | $H_{\text{phot}}/r$ |
|-----|-------|-----------------|-------------------|-------------------|-----------------|
| 0.01 | 0.056 | 0.40 | 6.1 | 8.8 | 0.11 |
| 0.03 | 0.052 | 0.29 | 4.4 | 7.4 | 0.19 |
| 0.1 | 0.051 | 0.19 | 2.1 | 6.4 | 0.31 |
| 0.3 | 0.048 | 0.13 | 2.0 | 5.7 | 0.43 |
| 1.0 | 0.042 | 0.09 | 2.0 | 5.1 | 0.55 |

Notes. The dependence of $\eta$ on $m$ is in part an artifact of our model, as explained in the text. Note also that emission outside $R = 60M$, ignored here, adds an additional $\approx0.012$ to the radiative efficiency.

2.3. Disk Structure

Figure 2 shows a snapshot of the gas density in the $(r, z = r \cos \theta; \phi = 0)$ plane for fiducial values of the black hole mass $M = 10 M_\odot$ and accretion rate $m = 0.1$. The solid contour lines show surfaces of constant optical depth. Note that while the density-weighted scale height of the disk $H_{\text{dens}}/r$ is the order of $0.06$, the photosphere is located at a height several times that above the midplane, with $H_{\text{phot}}/r \approx 0.3$ in the region of peak emission $r = 10-30M$ for this choice of accretion rate. This is to be expected; in stratified shearing box simulations with careful treatment of thermodynamics and radiation transfer, the scattering photosphere often lies 3–4 scale heights from the plane (Hirose et al. 2009).

For $m = 0.1$, the total optical depth of the disk ranges from order unity in the plunging region up to $r \approx 100-200$ in the disk body at $r > 10M$. Where the total optical depth is less than 2, we say that there is no disk, only corona (i.e., no solution exists for Equation (2)). We denote the radius of this transition by $R_{\text{crt}}$; in the language of Krolik & Hawley (2002), this is the radius of the “reflection edge.”

In Figures 3 and 4 we show the magnetic energy density and local cooling function, respectively. The Harma3d data correspond to the same time and the same slice in the $(r, z)$ plane as shown in Figure 2, for $M = 10 M_\odot$ and $m = 0.1$. Comparing the gas density and magnetic pressure, we see...
that both are concentrated in the disk, but the magnetic scale height is significantly greater than the gas scale height. This contrast naturally leads to a corona that is dominated by magnetic pressure, as seen in most shearing box and global MHD simulations. From Equation (1), we see that the physical density scales inversely with black hole mass, but the physical length scale is proportional to $M$, so the location of the photosphere—and thus the relative fraction of power from the disk and corona—is independent of $M$, and depends only on $m$.

The cooling profile appears to closely follow the magnetic field, consistent with earlier models that use magnetic stress as a proxy for heat dissipation (Beckwith et al. 2008), as well as stratified shearing box simulations in which the actual dissipation rate is computed (Hirose et al. 2006). As described above, Harm3d uses a local cooling function $\mathcal{L}$ to keep the disk relatively thin. This cooling can also be thought of as the local dissipation of heat, so we will often identify $\mathcal{L}$ as the emissivity of the gas. Because at any given time some of the fluid elements are actually below their target temperatures, the contours of $\mathcal{L}$ show numerous isolated patches with no emission (black in Figure 4). In fact, the coronal emission is extremely inhomogeneous, and the vast majority of it comes from a relatively small volume of space. Figure 5 shows the cumulative fraction of the coronal volume responsible for the cumulative fraction of the coronal luminosity: 50% of the corona volume generates only 1% of the luminosity, while 10% of the corona generates 90% of the luminosity, consistent with many earlier models that assume, without any dynamical justification, a highly inhomogeneous heating profile in the corona (Haardt et al. 1994; Stern et al. 1995; Zdziarski et al. 1996; Poutanen et al. 1997). We now understand that such strongly inhomogeneous dissipation arises naturally in the MHD simulations, and the simulation data give a quantitative description of its character.

As can be seen from Figure 4, for $m = 0.1$, the majority of the total emission comes from within the disk body, with a sizable contribution from the corona in the innermost regions. Outside a transition radius $R_{\text{trans}} \approx 6M$, the shell-integrated luminosity is dominated by the disk, while inside of this radius the corona dominates. By increasing $m$ for the same simulation, the density scale increases (see Equation (1)), encompassing a greater fraction of the total luminosity within the optically thick disk. Conversely, for small values of $m$, the disk shrinks and the corona becomes more dominant. Table 1 shows how the relative contributions of the disk and corona change with $m$, as well as the locations of $R_{\text{eff}}$ and $R_{\text{trans}}$. The corona dominates throughout the plunging region for all sub-Eddington values of $m$. This can also be seen in Figure 6, where we have plotted the fraction of total dissipation (angle- and time-averaged) occurring in the corona at each radius for a range of $m$. This fraction can be directly compared to the $f$ parameter used in the coupled disk-corona model of Done & Kubota (2006), where the total dissipation at each radius is divided between disk and corona. Thus it should come as no surprise that our resulting spectra (see below in Section 3) are qualitatively similar to those that they predicted for comparable values of $f$.

### 2.4. Global Radiation–Matter Thermal Balance

All the radiation processes we treat are thermal at some level. However, defining the temperature in this context involves some subtlety, both inside and outside the disk. Given the density, internal energy, and an equation of state, it is possible to derive a gas temperature directly from the simulation data. However, because Harm3d does not include any radiation pressure in its equation of state, this inferred temperature would not be very meaningful. Inside the optically thick disk, the code temperature should be regarded as a measure of the total thermal (i.e., gas plus radiation) pressure per unit mass supporting the gas against the vertical component of gravity,
but this is not necessarily the temperature relevant for its radiated spectrum. The reason is that, for much of the interesting range of accretion rates and radii, the material’s support is in fact dominated by radiation pressure. Consequently, the actual gas temperature is considerably lower than the code temperature. In the corona, although radiation forces should be less important, the code temperature is once again unsuitable for spectral considerations because it is tied to the ad hoc target temperature used by the simulation in order to regulate the disk’s geometric thickness. Fortunately, though, it is possible to use other simulation data to self-consistently determine the corona temperature through consideration of explicit thermal balance. Moreover, unless the thermal balance temperature turns out to be considerably higher than the code temperature, the dynamics of the simulation remain self-consistent because support against gravity in the corona is primarily magnetic: the plasma β is in the range \(\sim 0.03–0.3\) almost everywhere in the corona. Thus, the gas temperature (whether code or Compton equilibrium) has little influence on the density structure in the corona. In the remainder of this subsection we describe how we estimate the genuine physical temperature both inside and outside the disk photosphere.

Inside the disk (i.e., between the two electron scattering photospheres), we assume all the emitted radiation is able to thermalize, and all the heat generated within the disk is radiated from the same \((r, \phi)\) where it is made. These two assumptions allow us to define an effective temperature at each point on the disk surface:

\[
\sigma T_{\text{eff}}^4(r, \phi) = \frac{1}{2} \int_{\theta_{\text{top}}}^{\theta_{\text{bot}}} \mathcal{L}(r, \theta, \phi) d\ell ,
\]

where \(\sigma\) is the Stefan–Boltzmann constant and the factor of 1/2 is due to the fact that half the radiation is emitted from the top and half from the bottom of the disk.

In the regions of the disk where most of the flux is generated, electron scattering opacity is always much greater than the opacity due to other processes such as free–free absorption (Shakura & Sunyaev 1973). We therefore assume the mean intensity in the fluid frame at the photosphere has the spectrum of a diluted blackbody:

\[
J_\nu = f_{\text{hard}}^4 B_\nu(f_{\text{hard}} T_{\text{eff}}) ,
\]

where \(B_\nu(T)\) is the Planckian blackbody function, and the hardening parameter \(f_{\text{hard}}\) is taken to be 1.8 (Shimura & Takahara 1995). As described in Schnittman & Krolik (2013), the disk flux also has an angularly dependent intensity (limb-darkening) and polarization (Chandrasekhar 1960).

In the corona, the picture is not so simple. Unlike in the disk, in the corona the radiation is not expected to be thermalized. We have considered the contributions from a number of different radiation mechanisms, including bremsstrahlung, cyclo/synchrotron, and IC, but, at least for the stellar-mass black holes of interest here, we find the coronal power is completely dominated by IC.

Bremsstrahlung and synchrotron emission are both fundamentally local processes, and depend only on the local electron density, temperature, and magnetic field. Since the density, magnetic field, and net emissivity are given by the Ham3d data, to solve for the temperature we could simply invert the following equation at each point in the corona if they were the only cooling agents:

\[
\mathcal{L} = P_{\text{brem}}(\rho, T_e) + P_{\text{synch}}(\rho, T_e, B) ,
\]

where \(P_{\text{brem}}\) and \(P_{\text{synch}}\) are the local bremsstrahlung and synchrotron power density (in erg s\(^{-1}\) cm\(^{-3}\)), respectively. But this approach is incomplete because the corresponding absorption opacity

\[
\alpha_\nu = \frac{j_\nu}{B_\nu(T_e)}
\]

can also be important. Here \(\alpha_\nu\) and \(j_\nu\) are the absorption and emission coefficients respectively for either bremsstrahlung or synchrotron. For typical coronal conditions of \(T_e \sim 10–1000\) keV, \(n_e \sim 10^{16–10^{18}}\) cm\(^{-3}\), and \(B \sim 10^6–10^7\) G, we find that free–free emission and absorption are both negligible, while synchrotron emission can actually contribute a significant fraction of the total cooling function. However, the typical cyclotron frequency for these parameters lies in the infrared, where self-absorption is strong. Since the corona is optically thick to synchrotron radiation, it does not end up contributing significantly to the total cooling: every photon that is emitted is almost instantly re-absorbed. We are thus left with IC as the dominant emission process in the corona.

Unlike bremsstrahlung or synchrotron, IC is a fundamentally non-local process because it requires a population of seed photons to be up-scattered by the hot electrons. Moreover, the IC seeds can come from distant parts of the accretion disk. Local treatments are therefore insufficient.

For a mono-energetic population of electrons with isotropic velocity \(v\), the IC power is (Rybicki & Lightman 2004)

\[
P_{\text{IC}} = \frac{4}{3} \sigma_T c \gamma^2 \beta^2 n_e U_{\text{ph}} .
\]

Here \(\sigma_T\) is the Thomson cross section, \(\beta = v/c\), \(\gamma = (1 - \beta^2)^{-1/2}\), \(n_e\) is the electron density, and \(U_{\text{ph}}\) is the energy density of the local photon distribution. This local photon density is not known a priori from the simulation data, so it must be solved for using radiation transport. Because the corona has an optical depth of order unity (i.e., neither optically thin nor optically thick approximations can be used) and its geometry is complex, we use Monte Carlo ray-tracing including scattering to model the transport in a global manner (Schnittman & Krolik 2013).

As described in Schnittman & Krolik (2013), at every step along its geodesic trajectory a photon packet has a small chance that it will scatter off an electron. When a scattering event occurs, the electron velocity is selected by choosing a particular velocity in the local fluid frame from the relativistic Maxwell–Boltzmann distribution for the local temperature. The direction of the scattered photon packet is chosen from the probability distribution associated with the Compton scattering partial cross section. The change in energy of the scattered photons is then calculated exactly in terms of relativistic kinematics. When a photon packet encounters the disk photosphere, its new direction is chosen from the probability distribution given by Chandrasekhar (1960). At the temperatures characteristic of the inner regions of accretion disks in black hole binaries, few medium-\(Z\) elements are unstripped, so the reflection albedo should be high (we treat Fe K photoionization separately; see Section 4). Nonetheless, the packet does lose energy by conventional Compton recoil because the electrons in the disk surface are “cold,” i.e., \(kT_e \ll m_e c^2\).

A somewhat similar method has been used by Kawashima et al. (2012) to predict the spectrum from black holes accreting at super-Eddington rates. It resembles ours in that they employ Monte Carlo ray-tracing including IC scattering to predict X-ray spectra. It differs from ours in that the underlying...
data is drawn from an axisymmetric MHD simulation; this restrictive symmetry assumption prevents the simulation from properly following the development of MHD turbulence. The simulation is pseudo-Newtonian rather than relativistic, and the ray-tracing likewise assumes a flat spacetime. In addition, their coronal electron temperature is not made self-consistent with the simulation’s dissipation rate. They assume that their coronal electron temperature is not made self-consistent.

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Figure 7. Instantaneous luminosity profile $dL/d(log r)$, integrated over $\theta$ and $\phi$, considering only coronal emission. The black curve is the Harm3d data, and the red curve is the Pandurata ray-traced reconstruction. The luminosity is $0.1 L_{\text{Edd}}$. The Monte Carlo calculation used $\approx 5 \times 10^7$ photon packets for this snapshot.

(1) Initially estimate the local value of the radiation density in terms of the thermal contribution at the surface of the disk: $U_{\text{ph}}(r, \theta, \phi) = \sigma_T T_{\text{disk}}^4(r, \phi)$.

2. Solve Equation (9) with $P_{\text{IC}} = L$ to get $\gamma(r, \theta, \phi)$ throughout the corona. Derive the electron temperature at each point from the relation $T_e = (2/3) (m_e c^2/k_B) (\gamma - 1)$ (the non-relativistic expression works well as an initial guess for the corona temperature).

3. Carry out a complete Monte Carlo ray-tracing calculation with Pandurata, using thermal seed photons from the disk photosphere surface, and propagate photons through the corona via Compton scattering.

4. For each volume element in the corona, determine the total amount of IC power generated in that zone by comparing the ingoing and outgoing energy of every photon packet that scatters within that zone.

5. Compare the coronal power from Pandurata with the cooling function $L(r, \theta, \phi)$ obtained by Harm3d. Where the coronal power from the ray-tracing calculation exceeds the cooling rate in the simulation, the initial guess for $U_{\text{ph}}$ was too low, giving a $T_e$ that is too high, and vice versa.

6. Revise the coronal temperature estimates up or down accordingly, and repeat the full ray-tracing calculation, getting a new 3D map of the cooling function, which is again compared with the target values from the simulation data.

7. Repeat until the global solution for $T_e$, $U_{\text{ph}}$, and $L$ is self-consistent throughout the corona.

This iterative procedure is similar to the technique briefly described in Kawanaka et al. (2008), where they attempt to balance electron cooling from IC with electron heating from Coulomb collisions with energetic ions.

Because the Monte Carlo technique is inherently noisy, the Pandurata calculation and the Harm3d target cooling rate for any individual fluid element are unlikely to agree very well. We typically use $\sim 10^7$–$10^8$ photon packets for a single snapshot, while the simulation volume contains roughly $10^6$ cells. Consequently, for the majority of the corona volume, where $\tau < 0.1$, a given cell might see an average of only one photon packet. It should therefore not be surprising that point-to-point Poisson fluctuations are quite large. Furthermore, as can be seen in Figure 4, even the target cooling function is highly non-uniform, characterized by large-amplitude fluctuations on a small spatial scale.

For this reason, before comparing the results of the ray-tracing calculation with the target Harm3d emissivity, we apply a smoothing kernel to both data sets to remove the fluctuations described above. This smoothing is not only numerically helpful, but it is also strongly motivated from a physical point of view. For any radiation transport problem in a roughly steady state, the photon energy and momentum density cannot change significantly over length scales much shorter than the mean free path. Thus, when testing for convergence of $U_{\text{ph}}$, it is eminently reasonable to smooth the cooling function $L$ over the characteristic scattering length.

In fact, by smoothing over an even greater volume, we can significantly improve the efficiency of our iterative solver. For example, if the smoothing length is comparable to the coronal scale height, then instead of trying to sample $L$ in $10^6$ fluid elements, we are effectively only probing 10–100, and thus can use many fewer photon packets. After converging at low resolution, we repeat the calculation with more photons and progressively shorter smoothing lengths until Pandurata and Harm3d agree to high accuracy everywhere down to the gridscale.

One way to see this agreement at a quantitative level is to compare the radial distribution of coronal emission $dL/d(\log r)$ as derived from the two codes for a single snapshot, shown in Figure 7. After only three levels of iteration, we are clearly able to resolve coronal hot spots as small as $\Delta r/r \sim 0.2$. By plotting $dL/d(\cos \theta)$, we see that the vertical profile of the corona is also well matched (see Figure 8).

2.5. Coronal Temperature

The global solution for the electron temperature is shown in Figure 9. The temperature within each vertical slice of
the disk body is constant, and of the order of 0.2−1 keV for these parameters. The corona is much hotter, with $T_e$ ranging from $\sim 10$–$100$ keV for $\tau$ between 0.01 and 1. By comparing Figures 2 and 4, we see that the electron density falls off faster with increasing altitude from the disk than the dissipation ($H_{\text{diss}} \simeq 3H_{\text{dens}}$), leading to higher coronal temperatures as more power must be released by a smaller quantity of gas. The temperature map also shows large fluctuations over small spatial scales, yet not quite as large as those seen in $L$. This is because the regions of high dissipation are correlated with regions of high density, which has the effect of smoothing out the temperature gradients (see Equation (9)).

By changing the $m$ used in Equations (1) and (4), we can investigate the coronal properties of different accretion states. For a given point in the corona, $L$, $n_e$, and $U_{\text{ph}}$ all scale linearly with $m$, so from Equation (9) one can see that the term $\gamma^2 \beta^2$ should scale like $m^{-1}$. At low electron temperatures we have

$$\gamma^2 \beta^2 \approx \frac{v^2}{c^2} \approx \frac{k_B T_e}{m_e c^2},$$

while in the relativistic regime,

$$\gamma^2 \beta^2 \approx \gamma^2 \approx 12 \left(\frac{k_B T_e}{m_e c^2}\right)^2,$$

recovering the well-known scaling of IC power with temperature (Rybicki & Lightman 2004). Thus, at a fixed height above the disk, the electron temperature should scale like $T_e \sim m^{-1}$ for $k_B T_e \ll m_e c^2$, while at high temperatures, $T_e \sim m^{-1/2}$, independent of the black hole mass.

However, since we have fixed the total coronal optical depth at unity, the characteristic density near the photosphere is $n_e \simeq (\sigma_T H_{\text{dens}})^{-1}$, regardless of $m$. Therefore, at a fixed optical depth $\tau$ in the corona, Equation (9) becomes

$$\gamma^2 \beta^2 \sim \frac{3}{4} \frac{H_{\text{dens}} L}{c U_{\text{ph}}} \tau^{-1}.$$

A rough model for how the temperature scales with $m$ can then be derived if we treat both the density and the dissipation profiles as exponentials with vertical scale heights $H_{\text{dens}}$ and $H_{\text{diss}}$, respectively. Because both scale linearly with $m$ (see Equations (1) and (4)), these two quantities can be described by $L(z) = m L_0 e^{-z/H_{\text{diss}}}$ and $\rho(z) = m \rho_0 e^{-z/H_{\text{dens}}}$. From the simulation data we find that $H_{\text{diss}} \simeq 3H_{\text{dens}}$. It then follows that, at fixed optical depth $\tau$, $L$ will scale with $m$ like $L \sim m^{1-\eta/dm/H_{\text{diss}}} \sim m^{2/3}$ (this also explains the scaling of $L_{\text{cor}}/L_{\text{hot}} \sim m^{-1/3}$ seen in Table 1). On the other hand, $U_{\text{ph}}$ is nearly constant throughout the corona, but scales linearly with $m$, so Equation (12) becomes

$$\gamma^2 \beta^2 \sim \frac{L}{U_{\text{ph}}} \tau^{-1} \sim m^{-1/3} \tau^{-1}.$$

We therefore expect the temperature at a fixed optical depth to scale like $T_e \sim m^{-1/3}$ in the non-relativistic regime, and $T_e \sim m^{-1/6}$ at high temperature. Similarly, at fixed $m$, we expect $T_e \propto \tau^{-1}$ non-relativistically and $T_e \propto \tau^{-1/2}$ when the electron temperature is relativistic.

Thus, whether comparing regions of constant latitude or constant optical depth, we see a clear trend that is consistent with decades of observations: low-luminosity states are characterized by hard X-ray flux from a hot corona, while high-luminosity states lead to a much cooler corona and softer spectrum. In Figure 10 we plot the time-averaged coronal temperature as a function of radius for a range of different accretion rates. In the top panel the mean temperature is calculated by integrating over $\theta$ and $\phi$ and weighting by the local cooling rate $L$, while in the bottom panel the temperature is weighted by the electron density $n_e$. The $L$-weighting is more closely related to the emergent spectrum and naturally probes the upper corona, while the $n_e$-weighting speaks to conditions in the majority of the coronal mass and is sensitive to the conditions near the disk. In either case, the trend with $m$ is clear and, at the level of approximation expected, consistent with our earlier rough scaling argument. We also see that in the bulk of the corona, especially outside the ISCO, the temperature changes very little with radius.

The time-averaged radial and vertical temperature profiles of the corona can be seen in greater detail in Figure 11 for $m = 0.1$. At six different values of $r$, we plot the temperature as a function of optical depth through the corona, where $\tau = 0$ corresponds to the $z$-axis, and $\tau = 1$ the disk surface (see contours of $\tau$ in Figure 9). From the base of the corona at $r = 1$ outward to $\tau \sim 0.01$, the predicted $\tau^{-1}$ scaling describes the results well for...
Figure 10. Mean coronal temperature as a function of radius, weighted by local cooling rate \( \mathcal{L} \) (top) and electron density \( n_e \) (bottom), for a range of luminosities. (A color version of this figure is available in the online journal.)

Figure 11. Mean coronal temperature as a function of optical depth at a range of radii, for \( L = 0.1 L_{\text{Edd}} \). (A color version of this figure is available in the online journal.)

all radii outside the ISCO. Between \( \tau = 1 \) and \( \tau = 0.1 \), where most of the scattering events occur, and roughly 50% of the coronal cooling takes place, the temperature is always between 3 and 20 keV. This is a relatively low temperature for a disk corona, resembling more a warm atmosphere than a hot corona. Only in the upper corona, where the density and optical depth are least, does the temperature surpass 100 keV. Yet, because it accounts for the other 50% of the coronal cooling, even this small amount of hot gas is sufficient to contribute a hard power-law tail to the X-ray continuum.

Another interesting feature seen in Figure 11 is the turnover in temperature for \( \tau \lesssim 10^{-3} \). As can be seen from the temperature map in Figure 9, this region is very close to the funnel/jet region, where significant outflows are expected. Since Harm3d cools only bound matter, \( \mathcal{L} \) is set to zero for much of this region, leading to a decreased average temperature. This is, of course, an artifact of the simulation. Because the black hole in this simulation does not rotate, the jet power is very small and this artifact should be unimportant; when the black hole rotates and the jet power is greater, the dissipation rate in the jet could be significant. At the same time, the large relativistic bulk motion of gas in this region can still lead to interesting Comptonization effects, as will be described below in Section 5.

Finally, we comment on the relationship between our calculated electron temperature \( T_e \) and the nominal gas temperature \( T_i \) found in the simulation. In a fully self-consistent picture, we would expect \( T_e \) to be close to the ion temperature \( T_i \) because the coronal densities are generally high enough to make ion–electron collisional coupling reasonably rapid: just above the photosphere, the electron heating rate due to Coulomb collisions with hotter ions is

\[
\frac{d}{dt} \ln(3/2kT_e) \simeq 0.05 \frac{g(T_e)}{4} \left( \frac{H_{\text{dens}}/r}{0.06} \right)^{-1} \times \left( \frac{r}{10 M} \right)^{-1} (T_i/T_e - 1) M^{-1},
\]

where we have estimated the local electron density by \( \simeq (\sigma_T H_{\text{dens}})^{-1} \). When the ion temperature is \( \ll m_p c^2 \), the function \( g(T_e) \) falls sharply with increasing temperature below \( \approx 50 \text{ keV} \), but varies slowly when \( T_e \gtrsim 100 \text{ keV} \). Consequently, for most of the range of interest, \( g(T_e) \approx 4 \) (Stepney & Guilbert 1983). The estimate of Equation (14) then shows that the thermal equilibration time is at most comparable to, and in much of the corona considerably shorter than, the dynamical time \( \approx 52(\tau/10 M)^{3/2} M \), so treating it as a single-temperature fluid should be a good approximation.

In practice, we find that the ratio \( T_e/T_i \) is generally considerably less than unity. For example, if \( m = 1 \), it ranges from \( \sim 5 \times 10^{-3} \) (very near the photosphere) to \( \sim 0.2 \) (in the hottest locations at high altitude). For other values of \( m \), this ratio will scale like \( T_e \), because \( T_i \) is a fixed property of the simulation. The actual gas pressure in the corona is therefore likely overestimated in the simulation. Fortunately, however, it has little influence on the density structure of the corona. It is smaller than both the radiation pressure and the magnetic pressure by factors of \( \sim 3 \rightarrow 30 \). Replacing the code temperature with \( T_e \) would therefore make the gas pressure even less significant in coronal dynamics.

3. BROAD-BAND SPECTRA

Having converged on a self-consistent, global map of the coronal electron temperature, there is little left to do but “turn the crank” with Pandurata, ray-tracing as many photon packets as computationally reasonable. As described in Schnittman & Krolik (2013), the photon packets are emitted from the photosphere of the disk with a (diluted) thermal spectrum, and subsequently up-scattered via IC in the corona, eventually either getting captured by the black hole or reaching an observer at infinity. Those photons that escape are binned by their energy and observer coordinates \((\theta, \phi)\), making it trivial to generate simulated X-ray spectra as a function of viewing angle. Since the Harm3d data is fundamentally dynamic, it is also straightforward to simulate X-ray light curves and investigate timing features such as quasi-periodic oscillations (QPOs) and
time lags between hard and soft bands. An in-depth study of these topics will be the subject of a future paper, but in this work, we generally average the spectra over multiple snapshots.

Specifically, we use ThinHR simulation data from snapshots between 10,000$M$ and 15,000$M$, sampled every 500$M$ in time. Our photon packets cover the range of energy from $10^{-3}$ to $10^{5}$ keV, with logarithmic spacing and spectral resolution of $\Delta E/E = 0.016$. By using multi-energy photon packets, we are able to resolve the thermal continuum with high accuracy and efficiency. However, the Monte Carlo IC scattering kernel still introduces a significant amount of numerical noise at high energies (Schnittman & Kolić 2013). This noise can be reduced by increasing the number of rays traced, but in practice seems to converge only slowly.

Figure 12 shows the observed spectra from ThinHR for a range of $\dot{m}$, integrated over all viewing angles. The dominant features include a broad thermal peak around 1–3 keV, a power-law tail, and a Compton reflection hump above 10 keV. There is no evidence for a cutoff up to at least 1000 keV, but the Monte Carlo statistics are very poor at those high energies (we do not currently include reprocessing of energy lost by photons above $\sim$100 keV due to Compton recoil in the disk, although from Figure 12 this is clearly a small fraction of the total energy budget). Also visible is a broadened iron line feature around 5–7 keV, which will be discussed in greater detail in the following section.

The most important result to be seen in Figure 12 is that, for the first time, we have been able to use the genuine physics of global MHD simulations to reproduce the X-ray spectra observed in a wide variety of black hole binary states. We carry out a simple phenomenological fit of the spectra in Figure 12 using a fully relativistic multi-color disk, a Gaussian peak around 6 keV, and a power law component at higher energies.

The following best-fit parameters are summarized in Table 2, using the classifications defined by Remillard & McClintock (2006). The disk fraction $f^b$ is limited to the 2–20 keV band, as in Remillard & McClintock (2006). Note that $f^b$ is particularly for small $\dot{m}$) smaller than 1 – $L_{\text{cor}}/L_{\text{tot}}$ shown in Table 1 because it is the fraction only within the 2–20 keV band, and much of the disk power is radiated at lower energies. The power-law index $\Gamma$ is taken from the number flux of photons per unit energy $N(E) \propto E^{-\Gamma}$. While we do not claim to completely fit the spectra with only a thermal peak, Gaussian line, and a single power-law tail, $f^b$ and $\Gamma$ are still valuable parameters for spectral classification.

When scaling the simulations to $\dot{m} = 0.01$, we reproduce some of the features that characterize the low-hard state described in Remillard & McClintock (2006), with $\Gamma < 2.1$ and the 2–20 keV flux dominated by the corona: $f^b < 0.2$. At $\dot{m} = 0.1$ and above, the spectra closely resemble observations of the SPL state, with $\Gamma > 2.4$ and a disk contribution of $0.2 < f^b < 0.8$. At the highest luminosities, we are most closely aligned with the thermal state, defined by $f^b > 0.75$ and little variability (see below, Section 6).

Despite the remarkable success of reproducing such a wide range of spectral behavior with a single simulation, we should note that these spectra represent just a one-dimensional slice through the hardness–luminosity plane that is populated by stellar-mass black holes with a wide diversity of behaviors. For example, LMC X-3 alone has been observed with $\dot{m}$ anywhere from <0.03 up to >0.5 in the thermal state alone (Steiner et al. 2010). Furthermore, with our current techniques, we are not able to reproduce the very pure thermal spectra used for inferring spin with the continuum fitting technique (McClintock et al. 2006). A complete description of the spectral states of black hole binaries may also require additional parameters such as black hole spin and magnetic field topology. For example, a magnetic field that is primarily toroidal may curb coronal activity (Beckwith et al. 2008). Exploration of how accretion rate and these additional parameters interact will be the topic of future simulations.

We also note that the thermal disk component in the $\dot{m} = 0.03$ case is somewhat larger than that traditionally inferred in the low-hard state. In part, this soft component is due directly to the thermal disk, but much of it is also due to the disk photons that get upscattered in the warm, high density regions ($T_e \approx 10$ keV; $\tau \gtrsim 0.1$) of the corona. This effect is evident in the Wien tails of the thermal peaks in Figure 12, which are noticeably harder for smaller values of $\dot{m}$ due to the higher coronal temperatures in those cases. In fact, for some observations of the hard state, more recent analyses have shown clear evidence for an optically thick thermal disk (Miller et al. 2006a, 2006b; Reis et al. 2010;
Hiemstra et al. (2011). In those observations, the disk extends in
to the ISCO with a relatively cool temperature of $\sim 0.2–0.35$ keV,
slightly lower than we find for $\dot{m} \leq 0.03$. On the other hand,
there are also observations of the hard state that show evidence
of a disk truncated at large radius (Esin et al. 2001; Done et al.
2007). It is possible that the inner regions of these systems may
be dominated by a radiatively inefficient flow (e.g., Esin et al.
1997), and thus are not well-represented by this simulation.
As with the thermal state, we are not able to reproduce every
observed black hole spectrum simply by varying $\dot{m}$ within a
single simulation.

The key elements in our model that lead to the shift in spectral
shape with accretion rate are the assumptions that the gas density
scales linearly with $\dot{m}$ (discussed below in Section 7) and that
the boundary of the corona is defined by the disk’s scattering
photosphere. The latter assumption is physically reasonable:
as shown in our calculation, the coronal temperature declines
close to the disk surface; in addition, the quickly rising density
there leads to a much greater importance for cooling processes
like bremsstrahlung. For fixed $H_{\text{diss}}$, the natural consequence
of these assumptions is that as $\dot{m}$ increases, a larger fraction of
the dissipation takes place inside the thermal disk and a smaller
fraction in the corona. The disk appears weaker at low accretion
rates because the corona, although only marginally thick by
assumption, can give so much energy to the average photon
created thermally in the disk. In Section 7, we will discuss how
a more realistic disk picture, in which $H_{\text{diss}}$ changes with $\dot{m}$,
may affect our spectral predictions.

We have also investigated the effect of observer orientation
on the shape of the spectrum, but in most cases find only
weak dependence on inclination. After accounting for projection
effects, we do see that high-inclination (i.e., edge-on) systems
have a smaller thermal peak, and somewhat harder spectrum
between 1 and 10 keV, consistent with the results found in Noble
et al. (2011). Above $\sim 30$ keV, the spectra are virtually identical.
This is quite reasonable considering the results of the previous
section, where we showed the coronal temperature increasing
significantly with distance above the disk. The highest-energy
photons are mostly generated in a large, diffuse volume in the
upper corona, which subtends roughly the same solid angle
independent of viewer inclination. The only part of the spectrum
that seems to be strongly sensitive to the inclination is the broad
iron line, which will be described in the following section.

In Figure 13 we plot the broad-band spectra for $\dot{m} = 0.01,
0.1,$ and 1.0, showing the relative contributions from different
regions of the accretion disk. The spectra are sorted by the
emission radius of the seed photon. We see a few clear trends
across all accretion rates, none of which is very surprising:
the spectra grow systematically softer with increasing radius,
the thermal emission is dominated by flux originating from
$r/M > 15$, the coronal emission is dominated by the region
$6 < r/M < 15$, and the plunging region contributes very little
to either the thermal or power law parts of the spectrum.

The fact that the plunging region contributes so little to
the spectrum does not mean that the classical N-T disk is an
adequate model for accretion dynamics. As shown in Noble
et al. (2011) and Kulkarni et al. (2011), the radial emissivity
profile from MHD simulations leads to thermal spectra that
are systematically harder than N-T would predict for the same
spin parameter. In part, this comes from the small amount of
dissipation from inside the ISCO, but an even more important
cause is the emission profile immediately outside the ISCO,
which peaks at a smaller radius than predicted by N-T, and thus
the MHD thermal spectra look like they come from black holes.
with somewhat higher spins (Noble et al. 2011; Kulkarni et al. 2011).

One of the major underlying assumptions of this paper is that the disk photosphere is placed at the $r_{\text{phot}} = 1$ surface. We believe this is an eminently reasonable and physically-motivated assumption, but it is worth investigating how much our central results are sensitive to it. To this end, we have repeated the entire radiation-temperature iterative solution for $\dot{m} = 0.1$, setting $\tau_{\text{phot}} = 0.5$ and again with $\tau_{\text{phot}} = 2.0$. The results are shown in Figure 14. While the total luminosity is of course unchanged, the relative flux in the hard X-ray tail necessarily increases with $\tau_{\text{phot}}$. However, while the normalization of this hard tail changes, the shape appears to be invariant, indicative of an identical temperature profile in the upper corona, regardless of the exact location of the photosphere. This follows from the basic nature of IC radiation: Equation (9) shows that the electron temperature is set by the total radiation density, not the spectrum (at very high photon energies, pair creation and relativistic corrections are set by the total radiation density, not the spectrum (at very high photon energies, pair creation and relativistic corrections). So the energy balance in the upper corona is completely insensitive to the detailed radiative processes taking place in the disk and boundary layer.

4. IRON EMISSION LINES

Relativistically broadened Fe Kα lines have been detected in numerous AGNs (Tanaka et al. 1995; Nandra 2007; Bremneman & Reynolds 2009), galactic black holes (Miller et al. 2004; Reis et al. 2008, 2009), and galactic neutron stars (Cackett et al. 2010). The underlying emissivity profile is nearly always inferred (e.g., Reynolds & Nowak 2003) by fitting the observed line profile to a phenomenological model in which the emissivity is zero inside the ISCO, rises abruptly to a maximum at the ISCO, and then declines as a power law (sometimes a broken power law) toward larger radii. The energy with which fluorescence photons arrive at a distant observer depends on the radius from which they are emitted and the direction in which they are sent, as well as the character of the spacetime in which they travel. It would, of course, be highly desirable both to find functional forms for the Kα emissivity that are more closely connected to physical considerations and to be able to use observational data to constrain the disk dynamics responsible for generating these lines.

To do so requires solving problems both of physics and of procedure. Fluorescence line production begins with the illumination of gas by X-rays of energy greater than the threshold for K-shell ionization; we must determine its intensity as a function of radius. The fraction of those photons absorbed by such ionization events depends on the total optical depth of the gas and the ratio between the absorption opacity and other opacities (predominantly Compton scattering). The total optical depth depends on the specifics of angular momentum transport within the disk. The absorption opacity (as well as the fluorescence yield and the line energy) depend on the ionization state of the Fe atoms, and that in turn depends on both the temperature in the absorbing layer and the ratio between the ionization rate and the recombination rate. Although relativistic ray-tracing in vacuum has long been a solved problem (Carter 1968; Bardeen et al. 1972), a significant fraction of Kα photons traversing a marginally optically thick corona may also gain or lose energy by Compton scattering. A significant procedural problem is posed by the question of how to separate line photons from the continuum (Miller 2007). This is particularly problematic in the case of stellar-mass black holes, where the thermal peak and the power-law tail intersect right around the iron line, making it challenging to determine the precise form of the underlying continuum spectrum.

Our new ray-tracing analysis of the Harm3d simulations directly solves many of these problems. The radial profile of hard X-ray illumination is a direct product of our global solution for the radiation field. The total optical depth of the disk is automatically computed by the underlying general relativistic MHD simulation, subject only to scaling with our choice of $\dot{m}$. Compton scattering en route also follows naturally from our Monte Carlo transfer solution. Even the continuum contribution is also an automatic by-product, greatly improving our ability to uniquely fit the shape of the iron line.

The principal remaining uncertainty is calculation of the ionization state. In this paper we assume a fixed ionization state, but the data required for a genuine calculation of the ionization state as a function of position are also supplied by the other components of our method, so even this last problem can be solved within our framework, although it will involve a certain amount of additional labor.

As the disk seed photons are scattered through the corona, many eventually return, with higher energy, to the disk photosphere. For stellar-mass black holes with disk temperatures of $\sim 1$ keV, essentially all Fe atoms will be ionized to only a few remaining electrons. However, the ability to produce a K-shell photon as a result of K-shell photoionization disappears only when the Fe is completely stripped. From Saha equilibrium, we find that most of the photosphere is dominated by He-like Fe xxv from $\dot{m} \leq 0.1$, but a mix of He-like, H-like, and fully stripped Fe xxv exists for $\dot{m} \geq 0.3$.

The K-edge threshold varies slowly with ionization state, from $\sim 7$ keV for neutral iron, up to 8.8 keV for Fe xxv and 9.3 keV for Fe xxvi (Kallman et al. 2004). At photon energies much above this threshold, the photoionization cross section decreases sharply with energy. In Figure 15 we show the radial profile of the absorbed K-edge photon flux, defined as the incident photon number flux in the 9–30 keV band times the fraction of the disk that is optically thick at that radius.\(^9\)

\(^9\) The reflection edge of the disk is not a sharp boundary; at a given radius, the optical depth as a function of azimuth and time can change by a factor of a few.
For comparison, at $\dot{m} = 0.1$, we also show (dashed line) the number flux of seed photons emitted from the disk $F_{\text{em}}$ with energy greater than $\sim 3$ keV, i.e., those most likely to get up-scattered to $> 9$ keV. Note that this plot of the outgoing flux is normalized for comparison purposes.

In the outer regions of the disk, the absorbed K-edge flux profile is similar in shape to the emitted flux of photons above 3 keV. However, in the plunging region the disk becomes optically thin when $\dot{m} < 0.1$ (see Table 1), suppressing the absorption there. For $\dot{m} > 0.1$, the optically thick disk extends all the way to the horizon, and the shape of the absorbed flux profile is essentially independent of accretion rate. Even as the disk emission falls off with smaller radius, the illumination profile continues to rise as $F_{\text{em}} \sim r^{-\alpha}$, with $\alpha \approx 3/2$, indicative of the increasing importance of coronal flux in the inner disk.

We see no evidence for a steepening of the radial illumination profile with decreasing radius, as suggested by some AGN observations (Vaughan & Fabian 2004; Miniutti et al. 2007; Wilkins & Fabian 2011; Ponti et al. 2010) and predicted by “lamp post” models (George & Fabian 1991), in which the hard flux comes from a concentrated region along the black hole axis. This is not very surprising, considering the density and luminosity maps in Figures 2 and 4, which show an evacuated funnel around $\theta = 0$, essentially the opposite of the lamp post geometry. Indeed, a stationary point source on the rotation axis seems rather unlikely dynamically: a centrifugal barrier prevents much matter from approaching close to the axis, and any matter with little enough angular momentum to enter that region must either fall rapidly into the black hole or be ejected; in both cases, there would be strong beaming of any photons emitted in the direction of travel. It is possible that non-zero black hole spin may lead to relatively greater coronal dissipation at small radii, removing this discrepancy with observations; future simulations can test this conjecture.

As mentioned above, the line profile is also sensitive to the iron ionization state as a function of radius. Even if the surface density of the disk remains large inside of the ISCO, a line can be produced only if the iron is not fully ionized (Reynolds & Begelman 1997). Reynolds & Fabian (2008) initiated the use of simulation data to predict Fe Kα profiles by placing a source of ionizing radiation on the rotation axis $6M$ above the disk, and then using density data from a pseudo-Newtonian MHD simulation to estimate the ionization parameter in the disk. Since we know the vertical density profile as well as the incident spectrum at each point in the disk, it should be possible to completely solve the ionization balance equations as in Garcia & Kallman (2011) and Garcia et al. (2011). Such a detailed treatment is beyond the scope of the present work, but we can substitute reasonable approximations to obtain useful first-order results.

When a photon packet hits the disk photosphere, some part is absorbed by the iron atoms, while the remainder is reflected by electron scattering (other processes, such as free–free absorption in the disk, are insignificant). For a single photon incident on the disk, the probability of absorbing the photon in Fe K-shell photoionization is

$$P(E) = \frac{N_{\text{scat}} \kappa_{K\alpha}(E)}{N_{\text{scat}} \kappa_{K\alpha}(E) + \kappa_T},$$

where $\kappa_{K\alpha}$ and $\kappa_T$ are the Fe Kα and Thomson scattering opacities. $N_{\text{scat}}$ is the median number of scattering events a photon experiences before emerging from the atmosphere. Thus, the typical photon gets $N_{\text{scat}}$ chances to excite a Kα transition before exiting the disk, thereby enhancing the yield on the line production (Kallman et al. 2004). For accretion disks with roughly solar abundances and dominated by He-like Fe, we take

$$\kappa_{K\alpha}(E < 8.8 \text{ keV}) = 0$$

and from Monte Carlo scattering experiments, we find an angle-averaged value of $N_{\text{scat}} = 3$. This crude approximation to the K-shell opacity is appropriate provided most Fe atoms retain at least one electron; when most Fe atoms are stripped, $\kappa_{K\alpha}$ is smaller than our estimate by the ratio of Fe xxvii ions to the total. Of all the photons absorbed by iron in the disk, only a fraction $f_{K\alpha}$ produce a fluorescent line, while the excitation energy deposited by the rest is lost to Auger transitions, or, in the case of H-like and He-like Fe, more energetic K series recombination lines (Kallman et al. 2004). In Panduranga’s current form, the energy absorbed by K-edge opacity is simply removed from the spectrum during the Monte Carlo solution, while the energy in Kα emission is added back later. The fluorescence yield $f_{K\alpha}$ depends on ionization state (Krolik & Kallman 1987), growing slowly from $\sim 0.34$ to $\sim 0.5$ from Fe i to Fe xxii. At higher ionization stages, it can be as little as 0.11 (Fe xxiii), but is generally larger (0.5–0.75). For all the results presented below, we take $f_{K\alpha} = 0.5$, corresponding to a highly ionized state.

For a photon packet incident on the disk with initial spectral intensity $I_{\nu,0}$ (units of erg s$^{-1}$ Hz$^{-1}$), the number of Fe Kα photons produced per second will be

$$N_{K\alpha} = f_{K\alpha} \int d\nu P(\nu) \frac{I_{\nu,0}}{h\nu}.$$  

In our simplified model, all of these photons are added back to the photon packet as a delta function in energy at $E = 6.7$ keV, corresponding to the Kα emission for Fe xxv. Including absorption, the outgoing spectrum can be written

$$I_{\nu,\text{out}} = I_{\nu,0}[1 - P(\nu)] + N_{K\alpha} \delta(h\nu - 6.7 \text{ keV}) \times (6.7 \text{ keV}),$$

where both $I_{\nu,\text{out}}$ and $I_{\nu,0}$ are measured in the local frame of the disk.
This outgoing photon packet then propagates toward the observer, getting up-scattered by coronal electrons, possibly returning to the disk, or captured by the black hole. The Fe Kα emission line and absorption edge are both broadened by relativistic effects as well as the IC scattering. To show the magnitude and shape of these spectral features, in Figure 16 we plot (black curve) the ratio of the total observed spectrum to what would be observed if no absorption or emission were included in the calculation. We also show the absorption and emission contributions separately with the red and blue curves, respectively.

When plotted as a ratio to the hypothetical no-line spectrum, the absorption appears to dominate over the emission. Two facts account for this effect. First, $f_{\text{Kα}} = 0.5$ means that twice as many photons get absorbed as emitted. Second, although in the rest-frame only photons with energy $>8.8$ keV can be absorbed and all line photons have energy exactly 6.7 keV, relativistic broadening can shift part of the absorption feature downward in energy and part of the emission line upward. Where they overlap, there is substantial cancellation. Blueward of the point where they exactly cancel, the spectrum shows a sharp absorption feature that looks like $1 - P(E)$. Of course, the ratios in Figure 16 could never be directly observed, since they require knowledge of some hypothetical spectrum that conveniently ignores fluorescent line physics, but they do provide valuable insight into the physical processes at work here.

In practice, we observe spectra like those shown in Figure 12 and then attempt to infer the shape of the emission line by fitting the continuum with phenomenological models, an approach that can introduce serious systematic errors (e.g., Miller 2007 and references therein). The great advantage of this global radiation transport calculation is that we can simultaneously fit the entire spectrum with a single model based on physical parameters—black hole mass, spin, and accretion rate, Fe abundance, and observer inclination angle, obviating the historical reliance on more phenomenological models. Ultimately, we hope to apply our global radiation transport techniques to a large body of MHD simulations, resulting in a comprehensive suite of tabulated, self-consistent spectra that can be incorporated into a standard X-ray spectra analysis package like XSPEC (Arnaud 1996). As mentioned above, more detailed ionization physics will be required before Pandurata can be used to fit real iron line data with high precision. Nonetheless, in the meantime we can still use the simulated spectra to gain important insights into the behavior of the inner accretion flow.

To focus on the relativistic effects of broad line physics, in Figure 17 we plot the shape of the iron line (ratio of “emission only” to “no line physics”) for a range of observer inclination angles for $m = 0.01$ (the thinner disk highlights the relativistic effects). Again, we emphasize that these line profiles cannot be directly observed, but only inferred after fitting multiple spectral components such as the thermal peak, power law, reflection hump, and smeared absorption edge (Miller et al. 2004, 2012). More appropriately, these line profiles can be compared with theoretical semi-analytic calculations well-known in the literature for over two decades (e.g., Laor 1991), which typically assume planar circular orbits, and a $K\alpha$ surface brightness with a power-law radial profile abruptly cut off at the ISCO. Because these simple models are the ones generally used in packages such as XSPEC, they define the range of line shapes to be fitted; contrasting them with our profiles, based on more physical models for the hard X-ray illumination, the disk mass profile, and the fluorescing material’s velocity, can indicate what might be achieved with future XSPEC models based more directly on disk physics.

One important feature that is not often discussed is the high-energy tail clearly seen at all inclinations (Petrucci et al. 2001). This is due to IC scattering in the corona, the very process that generates the ionizing flux in the first place. Since the coronal scattering is nearly isotropic, the amplitude of the emission line above $\sim 8$ keV is independent of viewer inclination. At the same time, the optical depth from the disk directly to a distant observer increases with inclination angle, thereby reducing the total number of line photons that don’t get up-scattered, as can be seen by comparing the integrated line flux below 8 keV. The diminished contrast between line and continuum associated with high inclination angles may make it systematically more difficult to detect $K\alpha$ lines from those directions. Again, the emission-only component of the high-energy tail would not be observable directly, but would be combined with the dominant power-law continuum. We present it here to focus on the underlying scattering physics that produces the line.
One of the most important potential applications of the iron line is to use it as a probe of where (or if) the disk truncates. If there is a truncation radius, and this radius can be quantitatively related to the ISCO, measurement of the truncation radius could lead to a measurement of the black hole spin. Indeed, many have attempted to measure spin assuming that such a sharp truncation takes place exactly at the ISCO (a sampling of these efforts may be found in Martoccia et al. 2002, Miller et al. 2002, Duro et al. 2011, Reis et al. 2011, 2012, and Fabian et al. 2012). Even for the single spin value \( a/M = 0 \) simulated in ThinHR, we have shown in Table 1 and Figure 15 that the reflection edge of the disk can be adjusted by modifying \( \dot{m} \). Therefore we might reasonably expect very different line profiles for \( \dot{m} = 0.01, 0.03, \) and 0.1, corresponding to average reflection edge radii \( R_{\text{refl}}/M = 6.1, 4.4, \) and 2.1, respectively. Gravitational redshift is especially strong in the plunging region, so sizable contrasts in the red portion of the profile might be expected. A sample of line profiles is shown in Figure 18 for a range of spin parameters, we will explore whether the emergent line profiles have sufficiently strong dependence on spin that this diagnostic can be successfully used. It will also be crucial to better understand how to deconvolve the emission line profile from the other features of the observed spectrum.

5. BULK COMPTONIZATION

In addition to the thermal IC processes described above, the corona can also transfer energy to the seed photons through “bulk Comptonization” when the fluid velocity of the corona is large relative to the disk. Some authors have used this process to explain the hard tail seen in some thermal state observations (Zhu et al. 2012), or the SPL state when the bulk flow is convergent (Titarchuk & Shrader 2002; Turolla et al. 2002) or turbulent (Socrates et al. 2004; Socrates 2010). To quantify this effect in the \( \text{Harm}3d \) simulations, we simply set the electron temperature everywhere in the corona to zero while maintaining the turbulent motion above the disk and the convergent flow in the plunging region.

As before, we calculate the total Compton power in each fluid element by subtracting the energy in the incoming photon packet (as measured at infinity) from that of the outgoing ray. For an electron at rest, the photon will always transfer energy to the electron, giving negative IC power (hence the distinction between Compton scattering and IC scattering). In Figure 19 we plot the bulk coronal power in terms of \( dL/d(\log r) \), normalized by the total disk luminosity, for a range of accretion states. The \( y \)-axis is logarithmic and signed, so we set \( 10^{-4} = 0 = -10^{-4} \) for improved visualization. Where \( dL/d(\log r) > 0 \), the bulk velocity of the gas transfers energy to the radiation field. Where \( dL/d(\log r) < 0 \), the typical energy of a seed photon is greater than the bulk kinetic energy of the coronal electrons, and the radiation field loses energy to the corona.

Three conclusions may be drawn from Figure 19: (1) bulk Comptonization plays a very minor role in the overall energetics of thin accretion disks; (2) bulk Comptonization is most significant for high accretion rates; and (3) for all values of \( \dot{m} \), \( dL/dr > 0 \) in the inner regions and \( dL/dr < 0 \) in the outer disk. The explanations of these effects are straightforward: (1) turbulent velocities in the corona are simply not very large. In the plunging region, there are either not enough seeds (low \( \dot{m} \)) or the optical depth is high (large \( \dot{m} \)), so the seeds are advected into the black hole without being able to sample a wide range of converging velocities. Not surprisingly, the observed spectra for the bulk Comptonization runs are nearly indistinguishable from pure thermal disks. (2) Large \( \dot{m} \) corresponds to large \( H_{\text{phot}}/r \),

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Figure 18. Iron line profile as a function of luminosity, for observer inclination \( i = 30' \). The red wings of the lines are remarkably similar, despite differences in \( R_{\text{refl}} \) and illumination profiles.

(A color version of this figure is available in the online journal.)

Figure 19. Net Compton power \( dL/d(\log r) \) in the corona when \( T_e = 0 \), normalized to the total seed luminosity from the thermal disk. Note the unusual labeling of the \( y \)-axis, with logarithmic scaling above and below 0. A color version of this figure is available in the online journal.)
and we find that the turbulent velocities generally increase with scale height above the disk, so higher luminosity systems are sampling more turbulent regions of the corona. (3) For disks with constant $H_{\text{phot}}/r$, turbulent velocity should scale like the orbital velocity $v_{\text{bulk}} \sim v_{\text{orb}} \sim r^{-1/2}$, so the turbulent kinetic energy scales like $r^{-3/4}$. The seed photon energy, on the other hand, scales like $r^{-3/2}$ in the outer disk, and actually begins to decrease in the inner region as the disk becomes optically thin.

In Figure 20 we show the average kinetic energy in the corona as a function of radius (solid curves), along with the seed photon energy (dashed curves). Here, the specific kinetic energy is defined as $1/2 \sigma_{\phi}^2(r)$, where $\sigma_{\phi}(r)$ is the variance of the 3-velocity $u_\phi$, sampled over all $\phi, t$, and optical depth $\tau = 0.1-1$ at each radius. Outside of $r \approx 10 M$, the photon energy is higher, and thus transfers energy into the corona, giving $dL/dr < 0$. Note that, for $m = 0.01$, the nearly laminar flow at the midplane is considered part of the corona, not the disk, thus explaining the turnover in turbulent kinetic energy inside of $\sim 4 M$.

6. X-RAY VARIABILITY

Up to this point, all the discussion in this paper has focused on steady-state behavior of the simulated spectra. As mentioned in Section 2, the results are based on snapshots of the ThinHR simulation, spaced every 500 M between 10,000 M and 15,000 M, roughly the period of inflow equilibrium. With these 11 snapshots, we are also able to carry out some very coarse timing analysis. Figure 21 shows simulated light curves in four different energy bands, for $m = 0.1$ and observer inclination $i = 60^\circ$. Over the period shown, the bolometric luminosity of the simulation changes by about 20%, quite typical of global MHD simulations. To focus on the intrinsic variability, we have normalized all light curves by a single linear trend over this period. In Figure 21, each individual light curve has also been normalized by its mean value, to show the relative amplitude of fluctuations.

In Table 3 we list the rms variation in the flux in different energy bands for different values of $m$, again normalized by the linear trend in the bolometric luminosity. At low energies, corresponding to the thermal peak, we see a clear anti-correlation between accretion rate and variability, due to the fact that at low $m$, the inner disk is moving in and out, changing the thermal seed flux. The low variability of the case with $m = 1.0$ further strengthens its classification in the thermal state (Remillard & McClintock 2006).

**Figure 20.** Seed photon energy (dashed curves) and specific turbulent kinetic energy of the fluid in the corona (solid curves) for $L/L_{\text{Edd}} = 0.01, 0.1, 1.0$. (A color version of this figure is available in the online journal.)

**Figure 21.** X-ray light curves for $m = 0.1$ and viewer inclination of $i = 60^\circ$, taken from simulation snapshots sampled every 500 M in time during the period of inflow equilibrium. In each energy band, the light curve is normalized to the mean flux in that band, and divided by the linear trend of the bolometric flux. (A color version of this figure is available in the online journal.)

**Table 3**

| $L/L_{\text{Edd}}$ | 0.1–3 keV (%) | 3–10 keV (%) | 10–30 keV (%) | >30 keV (%) |
|---------------------|--------------|--------------|---------------|------------|
| 0.01                | 15           | 24           | 35            | 54         |
| 0.03                | 14           | 26           | 40            | 56         |
| 0.1                 | 12           | 24           | 32            | 58         |
| 0.3                 | 8.4          | 21           | 26            | 54         |
| 1.0                 | 6.8          | 19           | 28            | 51         |

**Notes.** To remove the secular trend, the light curves in each energy band have been normalized by dividing out a linear fit to the bolometric luminosity.

Our findings show an opposite trend in variability with $m$ than found in Noble & Krolik (2009). They measured the bolometric flux from dissipation in the corona only. Since they neglected the seed flux from the thick disk and ignored all IC scattering/heating physics in the corona, their only source of variability was the intrinsic dynamic variability of coronal turbulence and dissipation. Thus their trend of increasing variability with increasing $m$ suggests larger intrinsic fluctuations at higher altitudes above the disk, consistent with increased turbulent velocities, as shown above in Figure 20.

At all $m$, there is a clear increase in variability with photon energy, as seen in observations of the thermal dominant and SPL states of some black holes (Cui et al. 1999; Churazov et al. 2001; Gierlinski & Zdziarski 2005). When there is evidence for a thermal disk in the hard state, this same energy–rms relation is seen (Wilkinson & Uttley 2009). On the other hand, for hard states with no clear evidence for a disk, the rms appears to be constant or even decrease with energy (Nowak et al. 1999a; Gierlinski & Zdziarski 2005).

One possible explanation for the different rms–energy scaling found in different states is that for softer states, the seed photons generally have higher energies and thus can get scattered up to 50 keV after only a few scatterings, which is possible to do within a single coronal hot spot. For the hard state, the lower-energy seed photons will typically need more scatterings to reach the same energy, and thus sample a larger volume of the corona, averaging over many hot spots. This explanation is consistent with the rms variability seen in AGNs (Vaughan et al. 2003), which increases with energy at low energy (≤1 keV; few
that the variability is coming from the corona and not from the disk. Any fluctuations in the seed photons would be smoothed out when propagating through a uniform corona, just as pulses from a lighthouse are dispersed in fog, and would give lower variability at high energy because larger numbers of scatterings are required for the seeds to reach high energy (Schnittman 2005). Combined, these results are highly suggestive of a coronal hot-spot model for high-frequency X-ray variability in black hole binaries.

7. COMPARISON WITH CLASSICAL DISK THEORY

The spectral states of galactic black hole binaries are roughly correlated with their bolometric luminosities in the sense that low luminosity states generally have hard spectra, while higher luminosities permit a broader range of spectral states but exhibit a preference for softer states (Fender et al. 2004; Remillard & McClintock 2006). Our model is able to reproduce this observed correlation, yet does so in a fashion that differs in several respects from classical disk theory. In our model, increased accretion rate leads to a proportionately larger surface density, but leaves the scale height $H_{\text{dens}}$ unchanged. The systematic shift in spectral shape with accretion rate is due to a change in how the corona and the thermal disk share the dissipation: as the accretion rate increases, the photosphere moves to larger multiples of $H_{\text{dens}}$, so that more of the dissipation occurs within the thermal disk. By contrast, in classical disk theory, both the surface density $\Sigma$ and $H_{\text{dens}}$ are functions of accretion rate (Shakura & Sunyaev 1973). As we will show in a moment, most of our parameter space lies in the radiation-dominated regime, in which classical disk theory predicts $\Sigma \propto \dot{m}^{-1}$ and $H_{\text{dens}} \propto \dot{m}$. Moreover, for the accretion rates we consider, classical disk theory assumes that all the dissipation takes place inside the disk, leaving no room for a corona at all, and identifies the photosphere precisely with $H_{\text{dens}}$. These contrasts raise two questions: Is there any $\dot{m}$ at which our model and classical theory overlap? And should the trends with $\dot{m}$ predicted by the single-simulation model presented here be expected to carry over to global MHD simulations with different values of $H_{\text{dens}}$, corresponding to different values of $\dot{m}$?

As shown in Figure 13, most of the light is produced in the range $r \approx 6-30M$ for all the accretion rates we studied. Our account of the predictions of conventional disk theory therefore centers on that range. According to this theory, radiation pressure exceeds gas pressure inside disks when the accretion rate is greater than

$$\dot{m}_g \approx 0.02\alpha_{55}^{-1/8}(M/M_\odot)^{-1/8}(r/10M)^{21/16} \times (R_g/0.2)^{-9/8}R_T^{1/8}R_5^{5/8},$$

(19)

where $\alpha_{55}$ is the usual ratio of vertically integrated fluid-frame stress to vertically integrated pressure, $R_g \lesssim 1$ is a function of radius that adjusts the vertically integrated fluid-frame dissipation rate for both the net angular momentum flux through the disk and relativistic corrections, $R_T \simeq R_g$ is a similar correction factor applied to the vertically integrated fluid-frame stress, and $R_5$ (usually slightly greater than unity) introduces relativistic corrections into the vertical component of gravity (notation as in Krolik 1999). Because $R_g$ increases outward in this range of radii, $\dot{m}_g$ rises only gradually with radius. Thus, almost the entire span of accretion rates we consider falls into the radiation-dominated regime.

Conventional analytic disk theory estimates disk thickness in the radiation-dominated limit by suggesting that radiation
provides all the disk’s support against the vertical component of gravity and that all dissipated energy is conveyed outward by radiation flux (Shakura & Sunyaev 1973; Novikov & Thorne 1973). This pair of assumptions combined with the condition of hydrostatic equilibrium leads to the conclusion that all of the dissipation within the disk must be accomplished within a distance

\[ H_{\text{rad}} = \left( \frac{3}{2} \right) \left( \frac{m}{\eta n} \right) \left( \frac{G M}{c^2} \right) R_R \]

(20)
of the disk midplane. Thus, for radiation-dominated disks, scale height and accretion rate are directly proportional, and one can be used as a proxy for the other. Conventional theory also assumes that the density is constant for \( |z| < H_{\text{rad}} \) and zero outside \( H_{\text{rad}} \) (Shakura & Sunyaev 1973). Because this theory has no explicit place for coronae, it is unclear what luminosity it predicts for them, but presumably any corona begins outside \( H_{\text{rad}} \).

Detailed stratified shearing box simulations of radiation-dominated disk segments (Hirose et al. 2009; Blaes et al. 2011) have shown that \( H_{\text{rad}} \) does give an order of magnitude estimator of the vertical scale height of such disks, but, unsurprisingly, the density distribution is crudely exponential. Because both gas and magnetic pressure can contribute to vertical support and some vertical energy transport is by radiation advection rather than diffusive flux, these simulations also find that the dissipation profile is more extended than indicated by conventional analytic theory. In particular, only about half the dissipation takes place within a distance \( H_{\text{rad}} \) of the midplane, and 90% of the dissipation is accomplished inside \( \lesssim 2 H_{\text{rad}} \). The photosphere is generally found at \( 3 \lesssim H_{\text{rad}} \). These simulations were conducted with surface densities \( \sim 10^4 - 10^5 \) g cm\(^{-2}\), a range relevant to the larger radius and smaller accretion rate portion of our parameter space, so the specific numbers quoted might require adjustment for smaller radii and larger accretion rates.

The parallel set of facts about our simulated disk is that its density scale height is always \( H_{\text{dens}} \approx 0.06 r_{\text{in}} \), but the height of its photosphere is a multiple of \( H_{\text{dens}} \) that increases with \( m \), ranging from \( \lesssim 2 \) to \( \lesssim 9 \) as \( m \) increases from 0.01 to 1.0 (see Table 1). This is, of course, why the corona’s share of the luminosity \( f_{\text{cor}} \) decreases with increasing \( m \) in our model, as explained above in Section 2.5.

Having placed the two pictures side-by-side, we can now locate the parameters for which they resemble each other. Because \( H_{\text{rad}} \propto R_R \) and \( R_{\text{K}} \) scales with \( r \) somewhat more slowly than linearly for Schwarzschild spacetimes between \( r = 10M \) and \( r = 30M \), while the simulation has \( H_{\text{dens}} \propto r \), it is possible to match the simulation photosphere to the photosphere predicted by classical disk theory across this range of radii. We find approximate agreement when \( m \approx 0.2 \).

We now turn to the question of how large the x-ray emission of accreting coronae on black holes is, and how it depends on the accretion rate. The most important scaling issue for the corona is likewise the fraction of the dissipation it receives. At the very crudest level, a global model for the corona would yield a luminosity \( \propto f_{\text{cor}} m \), where the fraction \( f_{\text{cor}} \) of the dissipation going into the corona can be a function of \( m \) (e.g., in our picture, a decreasing one with \( f_{\text{cor}} \sim m^{-1/3} \)). Our predictions for how the hard X-ray luminosity scales with \( m \) therefore stand or fall on the relative scaling of \( L_{\text{phot}} \) and \( L_{\text{diss}} \) just as our predictions for the thermal portion do.

The shape of the coronal spectrum depends in general on the mean temperature and optical depth, but the distribution of dissipation and density also play a significant role. To estimate how they might change in new simulations with a dependence of \( H_{\text{dens}} \) on \( m \) mimicking the one predicted for radiation-dominated disks, we use the scaling argument of Equation (12) because it is usually quite general. Although it assumes that the corona’s total vertical optical depth is always unity, the way the coronal temperature falls as the photosphere is approached suggests that this is a reasonably good approximation, and one unlikely to be much affected by changes in disk parameters. Its detailed development, leading to Equation (13), however, depends on an overall density scale that is \( \propto m \), and this can change with parameters. Nonetheless, for any relation between density and \( m \), within the corona one still finds that \( \gamma^2 \beta^2 \propto (L_{\text{phot}}/U_{\text{ph}}) \tau^{-1} \).

The question is how the ratio of dissipation to photon density at the photosphere depends on \( m \). This ratio is, of course, simply \( f_{\text{cor}} \).

As we emphasized earlier, the spectral shape is also influenced by the spatial distribution of heating, in the sense that greater inhomogeneity tends to yield harder spectra. The range of temperatures seen in the corona is primarily determined by the range in the local heating rate per unit mass. In our simulation, \( L/\rho \) increases outward through the corona. It seems plausible that this trend would be reproduced in simulations with different scale heights because it largely reflects magnetic buoyancy, an effect that should be universal in magnetized accretion flows. However, the robustness of this trend can be checked when additional simulations with other scale heights are performed. Similarly, we know of no reason why the statistics of inhomogeneity should change, but future simulations will reveal whether this is so.

8. DISCUSSION

Using the new radiation transport code Pandurata, we have analyzed data from a high-resolution, 3D MHD simulation of accretion onto a Schwarzschild black hole. Because the MHD code Harma3d is energy conserving, we are able to employ a cooling function that tracks the local dissipation of energy throughout the simulation volume. By combining the results from Pandurata and Harma3d, we have for the first time been able to produce a global, self-consistent solution for the radiation field around an accreting black hole, and predicted——on the basis of real physics—the coronal luminosity. The major results from this work can be summarized as follows:

1. We have shown—for the first time—that MHD turbulence in an accretion disk can lead to dissipation outside the disk’s photosphere strong enough to power hard X-ray emission comparable in luminosity to the disk’s thermal luminosity. This is a result long expected, but never before demonstrated directly.

2. For different values of the Eddington-normalized accretion rate \( m \), the location of the photosphere changes, in turn
varying the fraction of radiative power in the disk (thermal) and the corona (IC). The coronal temperature ranges from about 10 keV near the disk surface up to $\sim$100–300 keV in the upper, low-density corona. At a fixed optical depth above the photosphere, we find the corona temperature increases slowly with decreasing accretion rate.

3. By varying $m$ from 0.01 to 1, we naturally reproduce X-ray spectra consistent with those observed in the hard, SPL, and thermal states of galactic black hole binaries. The spectra are characterized by a thermal peak around 1 keV and a high-energy power-law tail extending above 100 keV. Although the fraction of the total heat released by the corona is never greater than $\simeq$40%, the highly inhomogeneous dissipation predicted by the MHD simulation provides the local concentration of heating necessary to create output spectra as hard as observed.

In most cases there is evidence for a Compton reflection hump between 30 keV and 100 keV.

4. The Fe Kα illumination profile of the disk follows the classical $r^{-\delta}$ scaling at large radius, then flattens to $r^{-\delta/2}$ in the inner disk. At lower values of $m$, the disk begins to disappear inside the ISCO, and the line production is correspondingly reduced. The iron line profiles consist of both an absorption edge above 8.8 keV, and a broad emission line around 6.7 keV with a strong red-shifted tail. The shape of the line is dependent on observer inclination, and in all cases has a significant tail above 8 keV due to up-scattering of the line photons in the corona. The broad iron line profile appears to be only weakly dependent on the location of the disk inner edge, as most photons generated in the plunging region never reach the observer. Thus iron lines may in fact be better at measuring the location of the ISCO than the disk’s reflection edge.

5. Bulk Comptonization plays a very minor role in the photon energetics, typically $\lesssim$1% of the total seed luminosity. In the outer region of the disk ($r \gtrsim 10M$), the thermal seed photons carry more energy than the bulk kinetic energy in the coronal electrons.

6. We have carried out some initial timing analysis of the simulated X-ray spectra, and find a number of trends that are consistent with observations: the fractional rms amplitude increases with decreasing luminosity, and for all accretion rates, the rms amplitude increases with photon energy. On short timescales, the variability increases with observer inclination and photon energy, as expected for a coronal hot-spot model of X-ray variability.

Although the progress made to date has been significant, this work is just the tip of the iceberg. We are currently in the process of analyzing new $\text{Hzm3d}$ simulations, carried out with resolution comparable to that of ThinHR, for a wide range of black hole spin parameters. This will allow us to explore both potential dependence of the disk/corona continuum spectrum on spin and greatly improve our understanding of the iron line as a probe of black hole spin, disk dynamics near the ISCO, and the nature of the plunging region. New simulations with high spin will also allow us to probe the properties of the relativistic jet, frequently seen in observations of the hard state, and most clearly present in simulations of spinning black holes (McKinney & Gammie 2004; Hawley & Krolik 2006).

In other future work, simulations of disks with different $H_{\text{obs}}(r)$ profiles will expand the applicability of our models to a wider range of X-ray states and accretion rates. We will explore what parameters other than $m$ and $H_{\text{obs}}(r)$ determine the state of a given black hole. Improving the physics of the fluorescent line, including ionization balance and more detailed excitation cross sections (Garcia & Kallman 2011; Garcia et al. 2011) will make predictions of its strength and profile more reliable. Including the energy lost to photoionization and Compton recoil in the disk surface will also permit a better treatment of hard X-ray reprocessing and that energy’s reemergence in disk continuum, processes relevant to AGNs.

We also plan on extending our preliminary variability analysis to the entire set of simulation data within ThinHR’s statistically steady epoch (over 5000 snapshots in time), allowing for a more detailed study of high-frequency fluctuations and the possible identification of QPOs, sometimes seen in galactic black holes in the SPL state (Remillard & McClintock 2006). In addition to QPOs, we should be able to characterize time lags between different energy bands as a function of frequency, and compare with a large body of observational results (e.g., Nowak et al. 1999b). These lags appear to scale like the light-crossing time for fluctuations in the thermal seed flux to propagate through the corona, and thus could be a powerful probe of the coronal geometry (Uttley et al. 2011). We can also investigate the effects of finite light-travel time through the simulation volume, which was found to suppress variability in Noble & Krolik (2009).

Pandurata was originally developed to study X-ray polarization (Schnittman & Krolik 2009, 2010), so that information comes along for free with all the calculations described in this paper. As techniques for high-sensitivity X-ray polarimetry continue to improve (Black et al. 2010), polarization predictions will become observationally testable; it will be interesting to compare predictions made from MHD simulations with the toy coronal models presented in Schnittman & Krolik (2010). Because our analysis includes broad-band spectra, line profiles, timing, and polarization information in a single self-consistent calculation, it is the ideal tool for integrating these complementary techniques for measuring black hole spin and probing the physical properties of the accretion flow.

Despite the remarkable progress we have made in bridging the gap between simulation and observation, there still exist numerous challenges and caveats. The state-of-the-art MHD simulations still do not include adequate thermodynamics or internal radiation transport coupled directly with the fluid dynamics. While much progress has been made in shearing-box simulations (Hirose et al. 2006, 2009), there remain serious conceptual and computational obstacles to incorporating these advances into global simulations. In the short-run, $H(r)$ profiles designed to mimic the mean effects of radiation pressure support can help, but in the long-run it will be necessary to include radiation forces explicitly in order to explore what effect they have on both temporal and spatial fluctuations in the structure of the corona.

The ray-tracing tools described here are also lacking in certain regards. Because of the photon-packet methodology used in Pandurata, we have been forced to use energy-independent scattering cross sections, which certainly breaks down at high photon energy. Similarly, the angular distribution of Fe Kα lines emerging from the disk is assumed to be identical with the angular distribution of photons in the same packet reflected by electron scattering as given by Chandrasekhar (1960). These shortcomings can be improved with relatively little effort, but at a cost to computational efficiency.

Without a doubt, the most important next step is the direct comparison of our Pandurata spectra with real X-ray data. To this end, we are fortunately blessed with a mass of archival...
