Neutrino mass matrix with U(2) flavor symmetry
and neutrino oscillations

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ABSTRACT

The three neutrino mass matrices in the $SU(5) \times U(2)$ model are studied focusing
on the neutrino oscillation experiments. The atmospheric neutrino anomaly could be
explained by the large $\nu_\mu - \nu_\tau$ oscillation. The long baseline experiments are expected to
detect signatures of the neutrino oscillation even if the atmospheric neutrino anomaly is
not due to the neutrino oscillation. However, the model cannot solve the solar neutrino
deficit while it could be reconciled with the LSND data.

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The U(2) flavor symmetry \[1, 2\] is an interesting candidate for beyond SM. This flavor symmetry should be more precisely tested by future experiments \[3\]. The significant test will be possible in the neutrino sector as well as in the quark sector.

The extension of the U(2) model to neutrino masses and mixings has been presented by Carone and Hall \[4\]. They have found that a simple modification is required for the flavor symmetry breaking pattern if the light three neutrino masses are obtained by the see-saw mechanism. This fact suggests that the U(2) symmetry will be tested seriously in neutrino oscillations. The purpose of our paper is to present the systematic analyses in order to clarify the phenomenological implications of the $SU(5) \times U(2)$ model in neutrino oscillations.

In the U(2) flavor symmetry \[1\] the lighter two generations transform as a doublet and the third generation as a singlet of U(2). Only the third generation of the fermion can obtain a mass in the limit of unbroken symmetry limit. It is assumed that the quark and lepton mass matrices can be adequately described by VEV of flavons $\phi^a$, $S^{ab}$ and $A^{ab}$, in which $S^{ab}$ and $A^{ab}$ are symmetric and antisymmetric tensors, respectively. Furthermore, it is assumed $\langle \phi^2 \rangle/M = \epsilon$, $\langle S^{22} \rangle/M = \epsilon$ and $\langle A^{12} \rangle/M = \epsilon'$, where $M$ is the cutoff scale of a flavon effective theory. Other VEV’s are assumed to be zero. Thus, the U(2) symmetry breaks to U(1) with breaking parameter $\epsilon$ and U(1) breaks to nothing with $\epsilon'$.

The neutrino mass matrix has been discussed by Carone and Hall \[4\]. The right-handed Majorana mass matrix gives a zero Majorana mass due to the absence of the contribution from the antisymmetric flavon $A^{12}$. They proposed a simple solution which is to relax the assumptions: $\langle \phi^1 \rangle = 0$, $\langle S^{11} \rangle = 0$ and $\langle S^{12} \rangle = 0$. We present the systematic analyses of the modified neutrino mass matrices focusing on neutrino oscillations.

In the $SU(5) \times U(2)$ model, the modified Majorana mass matrix for the right-handed neutrinos is generated at leading order by the operators

$$\Lambda_R \left( \nu_3 \nu_3 + \frac{1}{M} \phi^a \nu_a \nu_3 + \frac{1}{M^2} \phi^a \phi^b \nu_a \nu_b + \frac{1}{M^3} S^{ab} \Sigma_Y \Sigma_Y \nu_a \nu_b \right), \quad (1)$$
where $\nu_3$ and $\nu_a$ are $SU(5)$ singlets, and $\Sigma_Y$ is a flavor singlet and a $24$ of $SU(5)$. On the other hand, the neutrino Dirac mass matrix is generated by the operators

$$\overline{F}_3 H \nu_3 + \frac{1}{M} (\phi^a \overline{F}_3 H \nu_a + \phi^a \overline{F}_a H \nu_3 + A^{ab} \overline{F}_a H \nu_b) + \frac{1}{M^2} (\phi^a \phi^b \overline{F}_a H \nu_b + S^{ab} \Sigma_Y F_a H \nu_b), \quad (2)$$

where $\overline{F}$ and $H$ are $\overline{5}$ and $5$ representations of matter and Higgs scalar, respectively. The $SU(5)$ representations of $\phi^a$, $S^{ab}$ and $A^{ab}$ are assigned to be $1$, $75$ and $1$, respectively, in order to reproduce the quark mass hierarchy. As far as VEV’s of $\phi^1$, $S^{11}$ and $S^{12}$ vanish, one of the right-handed Majorana neutrinos is massless as seen in eq.(1). Therefore, we study three cases of the non-zero VEV, which were considered by Carone and Hall [4]: (1) $\langle \phi^1 \rangle / M = \delta_1$, (2) $\langle S^{11} \rangle / M = \delta_2$ and (3) $\langle S^{12} \rangle / M = \delta_3$. The value of $\delta_i$ should be $O(\epsilon')$ in the standard $U(2)$ model because there is no mechanism to provide $\delta_i$ far below the $U(1)$ breaking parameter $\epsilon'$. However, we take $\delta_i$ as a free parameter in the model, so the value of $\delta_i$ is constrained by investigating quark masses and mixings. We do not address to the origin of $\delta_i$ far below $\epsilon'$ in this paper. We will find some patterns of neutrino mixings depending on the magnitude of $\delta_i$.

The modified neutrino mass matrices are generally written as follows:

$$M_{RR} = \Lambda_R \begin{pmatrix} \delta_1^2 + \tilde{\delta}_2 \rho & \epsilon \delta_1 + \tilde{\delta}_3 \rho & \delta_1 \\ \epsilon \delta_1 + \tilde{\delta}_3 \rho & \alpha \epsilon^2 & \epsilon \\ \delta_1 & \epsilon & 1 \end{pmatrix}, \quad M_{LR} = \langle H \rangle \begin{pmatrix} \delta_1^2 + \tilde{\delta}_2 & \epsilon' + \tilde{\delta}_3 & \delta_1 \\ -\epsilon' + \tilde{\delta}_3 & \epsilon^2 & \epsilon \\ \delta_1 & \epsilon & 1 \end{pmatrix}, \quad (3)$$

where $\tilde{\delta}_i \equiv \rho \delta_i$ with $\rho \equiv \langle \Sigma_Y \rangle / M$, and coefficients of $O(1)$ are omitted except for the coefficient $a$ in each entry. It is noted that those coefficients should be chosen to give the non-singular neutrino mass matrix.

Let us discuss following three cases with $a \neq 1$. The case with $a = 1$ will be discussed later. The light neutrino mass matrix $M_{LL}$ can be easily computed by the see-saw mechanism.

**case (1):** $\langle \phi^1 \rangle / M = \delta_1 \neq 0$
Taking $\delta_2 = 0$ and $\delta_3 = 0$ in eq. (3), we obtain

$$M_{LL} \simeq \frac{(H)^2}{\kappa_1^2} \Lambda R \begin{pmatrix} \lambda^2 \kappa_1^2 & \lambda \kappa_1 & \delta \kappa_1^2 \\ \lambda \kappa_1 & (a - 1) \kappa_1 & \delta \kappa_1^2 \\ \delta \kappa_1^2 & \delta \kappa_1^2 & \kappa_1^2 \end{pmatrix},$$

(4)

where we define

$$\lambda \equiv \frac{\epsilon'}{\epsilon} \simeq 0.22, \quad \kappa_1 \equiv \frac{\delta}{\epsilon'}.\quad (5)$$

Analyses of quark masses and mixings give the constraint for the magnitude of $\kappa_1$. The serious constraint follows from $|V_{ub}/V_{cb}|$, which is expressed in terms of quark masses as follows:

$$|V_{ub}/V_{cb}| \simeq \sqrt{\frac{m_u}{m_c}} (1 + C \kappa_1),$$

(6)

where $C$ is a complex coefficients of $O(1)$. Taking into consideration the experimental values $|V_{ub}/V_{cb}| = 0.08 \pm 0.02$ and $\sqrt{m_u/m_c} = 0.06 \pm 0.01$ $^5$, we find the safe constraint $\kappa_1 \equiv \delta/\epsilon' \leq 0.5$.

Now, we can obtain the mixing matrix $V$, which corresponds to the mixing matrix obtained in neutrino oscillation experiments, by calculating $U_{E\nu}^\dagger U_{\nu}$. The unitary matrix $U_{\nu}$ is obtained by diagonalizing the matrix in eq.(4), while the unitary matrix $U_{E}$ which diagonalizes the charged lepton mass matrix $M_E$, is given as follows:

$$U_{E} \simeq \begin{pmatrix} 1 & \sqrt{\frac{m_\mu}{m_\tau}} & 0 \\ -\frac{m_\mu}{\sqrt{3} m_\tau} \sqrt{\frac{m_\mu}{m_\mu}} & 1 & \frac{1}{3} m_\mu \\ \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\mu}} & -\frac{1}{3} m_\mu & 1 \end{pmatrix},$$

(7)

where the $CP$ violating phase is neglected.

The neutrino mass ratio is obtained approximately as follows:

$$m_3 : m_2 : m_1 \simeq 1 : \kappa_1^2 : \lambda^2 \kappa_1^2,$$

(8)

and the mixing matrix $V$ is given as:

$$V \simeq \begin{pmatrix} 1 & \lambda + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\mu}} & \lambda \kappa_1 - \sqrt{\frac{m_\mu}{m_\mu}} \\ \lambda \kappa_1 + \sqrt{\frac{m_\mu}{m_\mu}} & -\kappa_1 - \frac{1}{3} m_\mu & 1 \\ -\lambda & 1 & \kappa_1 + \frac{1}{3} m_\mu \end{pmatrix}.\quad (9)$$
The heaviest neutrino is the \( \mu \)-like one and the maximal mixing which is consistent with the atmospheric neutrino anomaly is not obtained. The mixing \( V_{e2} \) cannot solve the solar neutrino deficit because \( V_{e2} \propto \lambda \propto 0.22 \) is too large for the small angle solution in the resonant MSW transitions: \( \Delta m^2 = (3 \sim 12) \times 10^{-6} \text{eV}^2 \) with \( \sin^2 2\theta = (4 \sim 12) \times 10^{-3} \).

Taking account of the formula of neutrino oscillations in the case of \( m_3 \gg m_2 \gg m_1 \):

\[
P(\nu_\mu \to \nu_\alpha) \simeq 4V_{\alpha 3}V_{\mu 3}^2 \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right), \quad (\alpha = e, \, \tau),
\]

we can discuss the possibility to find neutrino oscillations in the accelerator and reactor neutrino experiments. The constraint for the heaviest neutrino mass \( m_3 \) depends on the mixing \( |V_{e3}| \simeq |\lambda \kappa_1 - 0.07| \). As far as \( \kappa_1 = 0.20 \sim 0.44 \) in eq. (5), the experimental upper bound of \( P(\nu_\mu \to \nu_e) \) allows the neutrino mass of \( m_3 \geq 1 \text{eV} \), which is consistent with the hot dark matter (HDM). Moreover, this mixing matrix is also consistent with the LSND data, which suggest typical parameters such as \( \Delta m^2 \sim 2 \text{eV}^2 \) with \( \sin^2 2\theta_{\text{LSND}} \simeq 2 \times 10^{-3} \) [10]. Then, in the CHORUS and NOMAD experiments, we predict \( P(\nu_\mu \to \nu_\tau) = (1 \sim 4) \times 10^{-4} \), which may be a detectable magnitude.

LBL accelerator experiments are planned to operate in the near future [12, 13]. The first experiment will begin in K2K (250 Km). For LBL experiments, the relevant formula of neutrino oscillations are

\[
P(\nu_\mu \to \nu_\alpha) \simeq -4V_{\alpha 1}V_{\mu 1}V_{\mu 2}V_{\mu 3}^2 \sin^2 \left( \frac{\Delta m^2_{31} L}{4E_\nu} \right) + 2V_{\alpha 3}^2V_{\mu 3}^2, \quad (\alpha = e, \tau),
\]

where we assume \( \Delta m^2_{31} \gg \Delta m^2_{21} \). Let us predict the neutrino oscillation in K2K, where the average energy of the \( \nu_\mu \) beam is taken as \( E_\nu = 1.3 \text{GeV} \) with \( L = 250 \text{Km} \). If we take \( \kappa_1 = 0.2 \sim 0.4 \) and \( \Delta m^2_{21} = 0.01 \text{eV}^2 \) tentatively, we predict \( P(\nu_\mu \to \nu_e) = 0.01 \sim 0.03 \) and \( P(\nu_\mu \to \nu_\tau) = 0.1 \sim 0.4 \). Thus, the K2K experiment is expected to detect signatures of neutrino oscillations in this model.

**case (2):** \( \langle S^{11} \rangle/M = \delta_2 \neq 0 \)

Taking \( \delta_1 = 0 \) and \( \delta_3 = 0 \) in eq. (3), we obtain the light neutrino mass matrix:
\[ M_{LL} \simeq \frac{\langle H \rangle^2}{(a-1)\rho \Lambda R} \begin{pmatrix} \rho \lambda^2 & -(a-1)\epsilon' & \rho \lambda \\ -(a-1)\epsilon' & (a-1)\epsilon'^2/\delta_2 & \epsilon \rho \\ \epsilon \rho & \epsilon' \rho \delta_3 & \rho \delta_3 \end{pmatrix}. \]

(12)

We get the mass ratio as:

\[ m_3 : m_2 : m_1 \simeq 1 : \tilde{\kappa}_2/\lambda : \lambda \tilde{\kappa}_2, \]

(13)

where \( \tilde{\kappa}_2 \equiv \delta_2/\epsilon' \) is defined. The constraint for \( \tilde{\kappa}_2 \) is given by \( V_{us} \), which is expressed in terms of quark masses as follows:

\[ |V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \left( 1 + \frac{\kappa_2}{\lambda} \right)^{-\frac{1}{2}} - \sqrt{\frac{m_u}{m_c}} \left( 1 + C_\kappa \frac{\kappa_2}{\lambda} \right)^{-\frac{1}{2}}. \]

(14)

Taking into consideration the accuracy of 5% for \( |V_{us}| = \sqrt{m_d/m_s} \), we obtain a tight constraint \( \kappa_2 \leq 0.1\lambda \simeq 0.02 \), which gives \( \tilde{\kappa}_2 \leq 0.02\rho \simeq 0.0004 \). The mixing matrix \( V \) is given as

\[ V \simeq \begin{pmatrix} 1 & \lambda + \frac{1}{3} \frac{m_u}{m_r} \sqrt{\frac{m_u}{m_\mu}} & -\tilde{\kappa}_2 - \sqrt{\frac{m_\mu}{m_u}} \\ \tilde{\kappa}_2 + \sqrt{\frac{m_\mu}{m_u}} & \lambda \tilde{\kappa}_2 - \frac{1}{3} \frac{m_\mu}{m_r} & 1 \\ -\lambda & 1 & -\tilde{\kappa}_2^* + \frac{1}{3} \frac{m_\mu}{m_r} \end{pmatrix}. \]

(15)

In this case, the heaviest neutrino is also the \( \mu \)-like one and the large mixing which is consistent with the atmospheric neutrino anomaly cannot be realized. Since \( \tilde{\kappa}_2 \) is very small, we cannot find new interesting phenomena. This mixing matrix also cannot solve the solar neutrino problem due to \( V_{e2} \simeq \lambda \simeq 0.22 \).

If the heaviest mass \( m_3 \) is constrained to be \( m_3 \leq 0.8 \text{ eV} \), the LSND data is consistent with our obtained mixing \( |V_{e3}| \simeq 0.07 \). However, this mass value is not consistent with HDM. In the LBL experiment at K2K, we can expect \( P(\nu_\mu \rightarrow \nu_e) \simeq 0.01 \) and \( P(\nu_\mu \rightarrow \nu_\tau) \simeq 0.01 \) as far as \( m_3 \geq 0.1 \text{ eV} \) is satisfied.

**case (3):** \( \langle S^{12} \rangle/M = \delta_3 \neq 0 \)

Taking \( \delta_1 = 0 \) and \( \delta_2 = 0 \) in eq. (3), the light neutrino mass matrix is given as:

\[ M_{LL} \simeq \frac{\langle H \rangle^2}{\rho^2 \delta_3^2 \Lambda R} \begin{pmatrix} (1 - a)\epsilon e^2 \delta_3^2 & (a - 1)\epsilon e' \delta_3 & \rho e \delta_3^2 \\ (a - 1)\epsilon e' \delta_3 & (1 - a)\epsilon^2 e'^2 & \epsilon e' \rho \delta_3 \\ \rho e \delta_3^2 & \epsilon e' \rho \delta_3 & \rho^2 \delta_3^2 \end{pmatrix}. \]

(16)
The hierarchy of this matrix is somewhat complicated, but using the relation $\epsilon \simeq \rho$ we obtain approximately the mass ratio:

$$m_3 : m_2 : m_1 \simeq 1 : \frac{a}{1-a} \bar{\kappa}_3^2 : \frac{1}{a(1-a)} \bar{\kappa}_3^2,$$  \hspace{1cm} (17)\]

where $\bar{\kappa}_3$ is constrained in the quark sector. Since the $|V_{us}|$ is expressed in terms of quark masses as follows:

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}}(1 + \kappa_3) - \sqrt{\frac{m_u}{m_c}}(1 + \kappa_3),$$  \hspace{1cm} (18)\]

we obtain a tight constraint $\kappa_3 \leq 0.05$, which leads to $\bar{\kappa}_3 \leq 0.05 \rho \simeq 0.001$. Therefore, we predict the huge mass hierarchy $\mathcal{O}(10^6)$ in eq.(17).

The mixing matrix $V$ is obtained as:

$$V \simeq \begin{pmatrix}
\bar{\kappa}_3 + \sqrt{\frac{m_\mu}{m_\tau}} \alpha + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} & \alpha \bar{\kappa}_3 + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} - \bar{\kappa}_3 - \frac{m_\mu}{m_\tau} & 1 \\
-\frac{m_\mu}{m_\tau} \alpha \bar{\kappa}_3 + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} & \lambda + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} & -1 \\
\lambda - \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} & 1 & -1
\end{pmatrix},$$  \hspace{1cm} (19)\]

where $\alpha$ is a coefficients of $\mathcal{O}(\leq \lambda)$. This mixing matrix may solve the solar neutrino problem since $V_{e2} \simeq \alpha$ could be small. However, $m_3$ should be larger than $10^3$ eV as seen in eq.(17) if the solar neutrino mass scale $\Delta m^2_{21} \simeq 10^{-6}$ eV$^2$ is fixed. This large mass $m_3$ is not consistent with the present experimental bound of E776 [9] because of $|V_{e3}| \simeq 0.07$. In conclusion, the solar neutrino problem is not explained as well as the atmospheric neutrino deficit in this case. On the other hand, for the LBL experiment at K2K, we predict $P(\nu_\mu \rightarrow \nu_e) \simeq 0.01$ and $P(\nu_\mu \rightarrow \nu_\tau) \simeq 0.01$.

Let us consider a possibility of the maximal mixing which is reconciled with the atmospheric neutrino anomaly. If $a = 1$ is satisfied exactly with $\kappa_3^2 \ll 1$ in eq.(4) of case (1) or with $\tilde{\delta}_3 \ll 1$ in eq.(16) of case (3), this matrix gives the maximal mixing because the diagonal entries of the matrix are suppressed. Then, the mixing matrix with the maximal mixing is given for both cases as follows:

$$V \simeq \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2} \lambda - \sqrt{\frac{m_\mu}{m_\tau}} + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} & \lambda + \sqrt{\frac{m_\mu}{m_\tau}} + \frac{1}{3} m_\mu \sqrt{\frac{m_\mu}{m_\tau}} & 1 \\
\sqrt{2} \sqrt{\frac{m_\mu}{m_\tau}} & 1 & -1 \\
-\sqrt{2} \lambda & 1 & 1
\end{pmatrix}. \hspace{1cm} (20)\]
The mass ratio is approximately given for case (1) (case (3)):

\[ m_3 : m_2 : m_1 \simeq 1 : 1 : \lambda^2 \kappa_1 (\lambda^2 \tilde{\kappa}_3). \]  

Actually, for the case (1), the choice of \( \kappa_1 \simeq 0.01 \) reproduces the maximal \( \nu_\mu - \nu_\tau \) mixing with \[ \Delta m_{32}^2 = (0.3 \sim 3) \times 10^{-2} \text{ eV}^2. \]  

Therefore, the LBL experiment at K2K will find the considerably large \( \nu_\mu - \nu_\tau \) oscillation probability. For the \( \nu_\mu - \nu_e \) oscillation, \( P(\nu_\mu \rightarrow \nu_e) \simeq 0.03 \) is expected.

What happens for the accelerator and reactor neutrino experiments in this case? Since \( m_3 \simeq m_2 \gg m_1 \) is satisfied, the relevant formula of neutrino oscillations are given as:

\[ P(\nu_\mu \rightarrow \nu_\alpha) \simeq 4 V_{\alpha 1}^2 V_{\mu 1}^2 \sin^2 \left( \frac{\Delta m_{31}^2 L}{4 E_\nu} \right), \quad (\alpha = e, \tau). \]  

The accelerator and reactor neutrino experiment of \( \nu_\mu \rightarrow \nu_e \) strongly constrains the value of the heaviest neutrino mass \( m_3 \) since \( V_{\mu 1} \simeq 0.07 \) is considerably large as already discussed by Carone and Hall. In this case, one obtains \( m_3 \leq 0.8 \text{ eV} \), which is not consistent with HDM. If the LSND data is taken seriously, \( m_3 = 0.5 \sim 0.8 \text{ eV} \) is obtained. Then, in the CHORUS and NOMAD experiments, we predict \( P(\nu_\mu \rightarrow \nu_\tau) \simeq 4 \times 10^{-7} \) and \( P(\nu_\mu \rightarrow \nu_e) \simeq 8 \times 10^{-5} \), which are out of their experimental sensitivity.

For the case (3), the choice of \( \tilde{\kappa}_3 \simeq 0.001 \) explains easily the atmospheric neutrino deficit by reproducing \( \Delta m_{32}^2 \simeq 10^{-2} \text{ eV}^2 \) with the \( \nu_\mu - \nu_\tau \) oscillation. Predictions of neutrino oscillations are same ones as the ones in case (1).

Since above results are very interesting, it may be important to give following comments. The very small value of \( \kappa_1 \) and \( \tilde{\kappa}_3 \) cannot be understood solely in terms of a \( U(2) \) symmetry breaking pattern. Moreover, the \( U(2) \) flavor symmetry does not guarantee the value of \( a = 1 \).

For above all cases, we summarize expected signatures of neutrino oscillation experiments in Table 1. The model cannot solve the solar neutrino deficit while it could be
reconciled with the LSND data by taking the adjustable $m_3$. In the special case of $a = 1$ in the right-handed Majorana mass matrix, the maximal mixing of $\nu_\mu - \nu_\tau$ is derived. Therefore, the atmospheric neutrino anomaly is explained by the large $\nu_\mu - \nu_\tau$ oscillations in those cases. Thus, we have clarified the phenomenological implications of the $SU(5) \times U(2)$ model in the neutrino sector. The model will be tested seriously in coming neutrino oscillation experiments.

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\[ \langle \phi^1 \rangle / M = \delta_1 \quad a \neq 1 \quad a = 1 \]
\[ \langle S^{11} \rangle / M = \delta_2 \quad a \neq 1 \quad a = 1 \]
\[ \langle S^{12} \rangle / M = \delta_3 \quad a \neq 1 \quad a = 1 \]

| Cases           | $\langle \phi^1 \rangle / M = \delta_1$ | $\langle S^{11} \rangle / M = \delta_2$ | $\langle S^{12} \rangle / M = \delta_3$ |
|-----------------|---------------------------------------|---------------------------------------|---------------------------------------|
| Solar           | No                                    | No                                    | No                                    |
| Atmospheric     | No                                    | Yes                                   | No                                    |
| LSND            | Yes                                   | Yes                                   | Yes                                   |
| HDM             | Yes                                   | No                                    | No                                    |
| CHORUS/NOMAD    | Yes                                   | No                                    | No                                    |
| LBL(K2K)        | Yes                                   | Yes                                   | Yes                                   |

Table 1: Expected experimental signatures of neutrino oscillations in the model. "Yes" denotes that signatures are detectable if the relevant neutrino masses are taken. On the other hand, "No" denotes that the model cannot explain the phenomenon.