Cosmology from Gamma Ray Bursts

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Abstract. In this study we propose to use Gamma Ray Bursts (GRBs) as standard candles in order to constrain the expansion history of the universe up to redshifts of $z \sim 6$. In particular, we utilize the 69 GRB dataset recently compiled by Cardone et al. (2009). Performing a joint likelihood analysis of the recent supernovae type Ia (SNIa) data and the GRBs we can put constraints on the main cosmological parameters ($\Omega_m, w$). However, the use of the current GRBs to trace the Hubble relation, as an alternative to the traditionally used SNIa, can not break the degeneracy between the $\Omega_m$ and the dark energy equation of state parameter.

1. Introduction
Several cosmological observations (Tegmark et al. 2004; Davis et al. 2007; Kowalski et al. 2008; Komatsu et al. 2009; Hicken et al. 2009 and references therein) have converged during the last decade towards a cosmic expansion history that involves a recent accelerating expansion of the universe. This expansion has been attributed to an energy component (dark energy) with negative pressure which dominates the universe at late times and causes the observed accelerating expansion. The simplest type of dark energy corresponds to the cosmological constant (Peebles & Ratra 2003 and references therein).

The geometrical probes (Lazkoz, Nesseris & Perivolaropoulos 2008) used to map the cosmic expansion history involve a combination of standard candles [Type Ia supernovae (SNIa) Davis et al. 2007] and standard rulers [clusters, CMB sound horizon detected through Baryon Acoustic Oscillations (BAO; Percival et al. 2007 ) and through the CMB perturbations angular power spectrum Komatsu et al. (2009)]. These observations probe the integral of the Hubble expansion rate $H(z)$ either up to redshifts of order $z \sim 1 – 2$ (SNIa, BAO, clusters) or up to the redshift of recombination ($z \sim 1089$). Alternatively, dynamical probes (Bertschinger 2006; Nesseris & Perivolaropoulos 2008) of the expansion history based on measures of the growth rate of cosmological perturbations are also confined to relatively low redshifts up to $z \sim 1$. It is therefore clear that the redshift range $2 – 1000$ is not directly probed by any of the above observations. Even though many models of dark energy predict a decelerating expansion in that redshift range due to matter domination, the possibility of non-trivial expansion properties at higher redshifts can not be excluded. In order to investigate this possibility we need a visible distance indicator at redshifts $z > 2$.

Gamma Ray Bursts (GRBs) are the most violent and bright explosions in the universe. They are produced by a highly relativistic bipolar jet outflow from a compact source (Rhoads 1999; Piran 2004). The fact that GRBs are detected up to very high redshifts (Kawai et al. 2006)
makes it tempting to try and use them as standard candles that could be used to constrain the cosmological expansion history in a similar way as SNIa. The problem is that GRBs appear to be anything but standard candles: they have a very wide range of isotropic equivalent luminosities and energy outputs. Several suggestions have been made to calibrate them as better standard candles by using correlations between various properties of the prompt emission, and in some cases also the afterglow emission. While there is good motivation for such cosmological applications of GRBs, there are many practical difficulties (Basilakos & Perivolaropoulos 2008).

Despite the latter, the potential benefits of obtaining even approximate standard candles at redshifts as high as \( z \sim 6 \) has prompted a significant activity towards both testing the usefulness of GRBs as standard candles (Amati et al. 2002; Ghirlanda, Ghisellini & Firmani 2006; Firmani et al. 2006; Li 2007; Butler et al. 2007; Zhang & Xie 2007; Hooper & Dodelson 2007; Basilakos & Perivolaropoulos 2008) and eagerly utilizing them to constrain cosmological parameters (Schaefer 2003; Zhang & Meszaros 2004; Dai, Liang & Xu 2004; Di Girolamo, et al. 2005; Schaefer 2007; Bertolami & Tavares Silva 2006; Wang & Dai 2006; Demianski, et al. 2006). This activity has lead to a debate about the usefulness of GRBs as standard candles with both discouraging (Li 2007; Butler et al. 2007) and encouraging (Amati et al. 2002; Ghirlanda et al. 2006; Firmani et al. 2006; Schaefer 2007; Zhang & Xie 2007; Hooper & Dodelson 2007; Capozziello & Izzo 2008; Izzo et al. 2009; Wang et al. 2009; Tsutsui et al. 2009a,b) results.

The aim of the present study is along the above lines. In particular, we use the 69 GRB dataset compiled by Cardone et al. (2009) and using a maximum likelihood method of \( \chi^2 \) we attempt to put constrains on the cosmological parameters. Finally, we perform a direct statistical comparison of the GRB results with those found using the SNIa data. The structure of the paper is as follows. The basic elements of the cosmological equations are presented in section 2. Section 3 outlines the statistical results of the current analysis. Finally, we draw our conclusions in section 4.

2. The basic Cosmological Equations

For homogeneous, isotropic and flat cosmologies, driven by non relativistic matter and an exotic fluid with equation of state, \( p_Q = w(\alpha)\rho_Q \) with \( p_Q < 0 \), the Einstein field equations can be given by:

\[
\left( \frac{\dot{\alpha}}{\alpha} \right)^2 = \frac{8\pi G}{3}(\rho_m + \rho_Q)
\]

and

\[
\ddot{\alpha} = -4\pi G \left[ \left( w(\alpha) + \frac{1}{3} \right) \rho_Q + \frac{1}{3} \rho_m \right],
\]

where \( \alpha(t) \) is the scale factor, \( \rho_m = \rho_{m0}\alpha^{-3} \) is the background matter density and \( \rho_Q = \rho_{Q0} f(\alpha) \) is the dark energy density, with:

\[
f(\alpha) = \exp \left[ 3 \int_{\alpha}^{1} \left( 1 + \frac{w(u)}{u} \right) \frac{du}{u} \right].
\]

Thus, one can prove that the Hubble parameter \( H \equiv \dot{\alpha}/\alpha \) is given by

\[
H(\alpha) = H_0 E(\alpha) \quad E(\alpha) = \left[ \Omega_m \alpha^{-3} + \Omega_Q f(\alpha) \right]^{1/2}
\]

where \( H_0 \) is the Hubble constant \(^1\). In this context, \( \Omega_m = 8\pi G \rho_{m0}/3H_0^2 \) is the matter density parameter and \( \Omega_Q = 8\pi G \rho_{Q0}/3H_0^2 \) is the dark energy parameter at the present time with \( \Omega_m + \Omega_Q = 1 \).

\(^1\) Note that we always marginalize with respect to the internally derived Hubble constant using the SNIa data. This is \( H_0 = 65.4 \text{Km/s/Mpc} \).
Figure 1. Likelihood contours (1σ, 2σ and 3σ) in the (Ω_m, w) plane (GRBs-red and SNIa-black). The contours are plotted where −2lnL/L_max is equal to 2.30, 6.16 and 11.83.

In addition, to Ω_m(α) also Ω_Q(α) could evolve with the scale factor as

$$\Omega_m(\alpha) = \frac{\Omega_m \alpha^{-3}}{E^2(\alpha)} \quad \text{and} \quad \Omega_Q(\alpha) = \frac{\Omega_Q f(\alpha)}{E^2(\alpha)}.$$  \hspace{1cm} (5)

Of course, in order to address the negative pressure term it is essential to define the functional form of the equation parameter \( w = w(\alpha) \). In this work, we consider the well known Chevalier-Polarski-Linder (CPL)\cite{Chevallier & Polarski 2001; Linder 2003} parametrization for which

$$w(\alpha) = w_0 + w_1(1 - \alpha)$$  \hspace{1cm} (6)

where \( w_0 \) and \( w_1 \) are constants. Therefore, inserting eq.(6) into eq.(3) we simply derive

$$f(\alpha) = \alpha^{-3(1+w_0+w_1)} e^{3w_1(\alpha-1)}.$$  \hspace{1cm} (7)

Obviously, if \( w_1 = 0 \) the current models reduce to the quintessence cosmological models \([f(\alpha) = \alpha^{-3(1+w)}]\).

3. Likelihood Analysis

In this section, we present the statistical results using the Hubble relation provided by the GRB and SNIa data respectively.

- **GRB distance modulii:** We first use \( N_{GRB} = 69 \) GRBs provided by Cardone et al. (2009) up to 6.6 redshift.

- **SNIa distance modulii:** We additionally utilize the 'Constitution set' of 397 type Ia supernovae of Hicken et al. (2009). In order to avoid possible problems related with the local bulk flow, we use a sub-sample of the overall sample in which we select those SNIa with \( z > 0.023 \). This sub-sample contains \( N_{SNIa} = 351 \) entries.
Figure 2. Likelihood contours in the \( (w_0, w_1) \) plane after marginalizing with respect to \( \Omega_m = 0.28 \).

The corresponding \( \chi^2 \) function is:

\[
\chi^2_j(p) = \sum_{i=1}^{N_j} \left( \frac{\mu_{\text{th}}(a_i, p) - \mu_{\text{obs}}(a_i)}{\sigma_i} \right)^2 j = \text{GRB, SN Ia}
\]

where \( a_i = (1 + z_i)^{-1} \) is the observed scale factor of the Universe, \( z_i \) is the observed redshift, \( \mu \) is the distance modulus corresponding to flat space:

\[
\mu = m - M = 5 \log d_L(a) + 25
\]

and \( d_L(a, p) \) is the luminosity distance

\[
d_L(\alpha, p) = \frac{c}{\alpha} \int_1^a \frac{dy}{y^2 H(y)},
\]

Note, that \( c \) is the speed of light. We can combine the above cosmologically tests, using a joint likelihood analysis, in order to put even more stringent constraints on the free-parameter space, according to:

\[
\mathcal{L}_{\text{tot}}(p) = \mathcal{L}_{\text{GRB}} \times \mathcal{L}_{\text{SN Ia}}
\]

or

\[
\chi^2_{\text{tot}}(p) = \chi^2_{\text{GRB}} + \chi^2_{\text{SN Ia}},
\]

with the likelihood estimator defined as \( \mathcal{L}_j \propto \exp[-\chi^2_j/2] \). Note that the overall number of data points used is \( N_{\text{tot}} = 420 \) and the degrees of freedom: \( \text{dof} = N_{\text{tot}} - n_{\text{fit}} \), with \( n_{\text{fit}} \) the number of fitted parameters, which vary for the different models.

3.1. Constant equation of state - QP model
In this case the equation of state parameter is constant, \( w(a) = w \), (see Peebles & Ratra 2003). Such dark energy models do not have much physical motivation. In particular, a constant equation of state parameter requires a fine tuning of the potential and kinetic energies of the
Figure 3. Insert Panel: SNIa distance moduli as a function of redshift. Distance moduli comparison difference between the $\Lambda$-model ($\Omega_m = 0.28$ solid line) and the GRB data.

real scalar field. Despite the latter problem, these dark energy models have been used in the literature to their simplicity. Notice, that dark energy models with a canonical kinetic term have $-1 < w < -1/3$. On the other hand, models of phantom dark energy ($w < -1$) require exotic nature, such as a scalar field with a negative kinetic energy. Comparing the QP-models with the observational data (we sample $\Omega_m \in [0.1, 1]$ and $w \in [-1.4, -0.6]$ in steps of 0.01) we find that the best fit values are: a) using the GRBs only we find $\Omega_m = 0.40^{+0.03}_{-0.08}$ (while $w$ remains unconstraint due to large errors) and b) for the joint analysis the likelihood function peaks at $\Omega_m = 0.30^{+0.10}_{-0.02}$ and $w = -1.04^{+0.10}_{-0.30}$ with $\chi^2_{\text{tot}}(\Omega_m, w)/dof \simeq 540/418$. In figure 1 we present the 1$\sigma$, 2$\sigma$ and 3$\sigma$ confidence levels (corresponding to where $-2\ln\mathcal{L}/\mathcal{L}_{\text{max}}$ equals 2.30, 6.16 and 11.83) in the ($\Omega_m, w$) plane. We would like to stress that our results are in good agreement with the 5 years WMAP data (Komatsu et al. 2009). Also Davis et al. (2007) using the Essence-SNIa+BAO+CMB and a Bayesian statistics found $\Omega_m = 0.27 \pm 0.04$, while Kowalski et al. (2008) utilizing the Union08-SNIa+BAO+CMB obtained $\Omega_m = 0.285^{+0.02}_{-0.01}$. Obviously, our results coincide within 1$\sigma$ errors. It is worth noting that the concordance $\Lambda$-cosmology can be described by QP models with $w$ strictly equal to -1. In this case we find: $\Omega_m = 0.30 \pm 0.30$ with $\chi^2_{\text{tot}}(\Omega_m)/dof \simeq 540/419$.

3.2. The parametric Dark Energy model - CPL model
In the CPL (Chevallier & Polarski 2001; Linder 2003) parametrization the dark energy equation of state parameter is defined as a first order Taylor expansion around the present epoch (see eq.6). We sample the unknown parameters as follows: $w_0 \in [-2, -0.4]$ and $w_1 \in [-2.6, 2.6]$ and we find that for the prior of $\Omega_m = 0.28$ the overall likelihood function peaks at $w_0 = 1.14^{+0.22}_{-0.16}$ and $w_1 = 1.12^{+0.52}_{-1.54}$ while the corresponding $\chi^2_{\text{tot}}(w_0, w_1)/dof$ is 539/418. In figure 2 we show the GRB constraints in the ($w_0, w_1$) plane by marginalizing our solution over $\Omega_m = 0.28$. For comparison reasons we also present red contours, which where based on the SNIa data. Obviously, the different shape between the contours is due to the fact that the two data sets cover different redshift ranges. This is a hint that the use of alternative tracers for the Hubble relation at large redshifts ($z \gtrsim 2$), could be a useful avenue for providing further constraints of the cosmological parameters (Plionis et al. 2009). However, in order to use the GRBs properly
as standard candles to constrain the cosmological parameters, we have to significantly reduce the scatter in the distance modulus diagram (see figure 3). This implies that we need a much better calibration of the observed correlations required in order to measure the correct distance modulus (Basilakos & Perivolaropoulos 2008). In fact, a Monte-Carlo analysis have shown that the best strategy to decrease the uncertainties of the cosmological parameters based on the Hubble relation, is to build a sample of $\sim 80$ standard candles that trace the redshift range $2 \lesssim z \lesssim 6$ as well as to reduce the scatter in the resulting luminosity and thus in the distance modulus diagram by a factor of 1/2.

4. Conclusions
We have utilized the recent GRB data (Cardone et al. 2009) in order to constraint the main cosmological parameters. We find that within the context of flat cosmological models and under of specific conditions the GRB data can place constrains on the main cosmological parameters. However, due to the small number statistics as well as due to the large uncertainties involved in the statistical analysis, the next generation of the GRB catalogs have to (a) contain at least $\sim 80$ entries at large redshifts ($2 \lesssim z \lesssim 6$) and (b) to reduce the scatter in the diagram of the distance modulus by a factor of 1/2.

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