An Analysis of the Decay $B \to D^* X \ell \bar{\nu}_\ell$
with Predictions from Heavy Quark and Chiral Symmetry*

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ABSTRACT

This paper considers the implications of the heavy quark and chiral symmetries for the semi-leptonic decay $B \to D^* X \ell \bar{\nu}_\ell$. The general kinematic analysis for decays of the form pseudoscalar meson $\to$ vector meson + pseudoscalar meson + lepton + anti-lepton is presented. This formalism is applied to the above exclusive decay which allows the differential decay rate to be expressed in a form that is ideally suited for the experimental determination of the different form factors for the process through angular distribution measurements. Heavy quark and chiral symmetry predictions for the form factors are presented, and the differential decay rate is calculated in the kinematic region where chiral perturbation theory is valid.

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1. INTRODUCTION

When the mass of a quark is taken to infinity with its four-velocity held fixed, its strong interactions become independent of its mass and spin, and depend only on its velocity. This gives rise to a new SU(2\(_N\)) spin-flavor symmetry, for \(N\) heavy quark flavors, that is not manifest in the full theory of QCD. These symmetries are made explicit in a heavy quark effective field theory (HQEFT) which has been a powerful tool in understanding the strong dynamics of hadrons containing a heavy quark.[1]

Another symmetry of the strong interactions that has been known for some time is that in the limit where the mass of the light \(u,d,s\) quarks become massless, QCD has a SU(3)\(_L\) \(\times\) SU(3)\(_R\) chiral symmetry. This symmetry is spontaneously broken down to the SU(3)\(_V\) vector subgroup by the strong interactions, and the associated pseudo-Goldstone bosons are the light pseudoscalar octet mesons \(\pi, K, \eta\). Chiral Lagrangians which incorporate this symmetry have been used extensively to study low energy interactions involving the pseudoscalar mesons.

Recently, a synthesis of the above heavy quark and chiral symmetries has been achieved. This theory describes the low energy interactions of hadrons containing a single heavy quark (which will hereafter be referred to as a heavy hadron and excludes \(Q\bar{Q}\) quarkonium bound states of heavy quarks) with the pseudoscalar octet mesons. \(^{2-6}\) Reference [6] examined the constraints that these symmetries place on \(B_{\ell 4}\) and \(D_{\ell 4}\) decays (without a final state vector meson). In this paper, similar considerations are applied to the decay \(B \rightarrow D^* X \ell \bar{\nu}_\ell\), where \(X\) is a pseudo-Goldstone boson, and the development here parallels the one in that paper.

The heavy meson chiral Lagrangian density which describes the low momentum interactions of the ground state heavy mesons with the pseudo-Goldstone bosons is given by\(^2\)

\[
\mathcal{L} = \frac{f^2}{8} \text{Tr}(\partial^\mu \Sigma \partial_\mu \Sigma^\dagger) + \lambda_0 \text{Tr}(m_q \Sigma + m_q \Sigma^\dagger) - i \text{Tr} \bar{H}_a v_\mu \partial^\mu H_a \\
+ \frac{i}{2} \text{Tr} \bar{H}_a H_b v_\mu (\xi^\dagger \partial^\mu \xi + \xi \partial^\mu \xi^\dagger)_{ba} + \frac{i g}{2} \text{Tr} \bar{H}_a H_b \gamma_5 (\xi^\dagger \partial^\mu \xi - \xi \partial^\mu \xi^\dagger)_{ba} \\
+ \lambda_1 \text{Tr} \bar{H}_a H_b (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)_{ba} + \lambda_1' \text{Tr} \bar{H}_a H_a (m_q \Sigma + m_q \Sigma^\dagger)_{bb} \\
+ \frac{\lambda_2}{m_Q} \text{Tr} \bar{H}_a \sigma_{\mu \nu} H_a \sigma^{\mu \nu} + \ldots ,
\]

(1.1)

where the ellipsis denotes terms containing more derivatives, factors of the light quark mass matrix

\[
m_q = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_d & 0 \\
0 & 0 & m_s
\end{pmatrix}
\]

(1.2)

which explicitly violates SU(3)\(_L\) \(\times\) SU(3)\(_R\) chiral symmetry, or factors of \(1/m_Q\) (where \(m_Q\) is the mass of the heavy quark of flavor \(Q\)) associated with the breaking of the SU(2N) heavy
quark spin-flavor symmetry. The light quark flavor indices \( a, b \) run over \( 1, 2, 3 \) (corresponding to \( u, d, s \)) and repeated indices are implicitly summed.

The heavy pseudoscalar and vector meson fields, \( P_a \) and \( P_a^\star \), are degenerate in the heavy quark mass limit, and in the case where \( Q \) is the \( c \) quark,

\[
(P_1, P_2, P_3) = (D^0, D^+, D_s^+)
\]

and

\[
(P_1^\star, P_2^\star, P_3^\star) = (D^{*0}, D^{*+}, D_s^{*+}).
\]

In the above Lagrangian, these heavy meson fields are combined into the \( 4 \times 4 \) matrix

\[
H_a = \frac{1 + \frac{\gamma}{2}}{\gamma} (P_a^\mu \gamma^\mu - P_a \gamma_5),
\]

(1.3)

and

\[
\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0.
\]

(1.4)

The field \( H_a \) is a doublet under the heavy quark spin symmetry \( SU(2)_v \) and an anti-triplet under chiral \( SU(3)_V \):

\[
H_a \rightarrow S(HU^\dagger)_a,
\]

(1.5)

where \( S \in SU(2)_v \), which is the symmetry group for a single flavor of heavy quark at velocity \( v \), and \( U \) is defined below.

The pseudo-Goldstone boson fields are incorporated into the Lagrangian of eq. (1.1) via

\[
\Sigma = \xi^2
\]

(1.6)

where

\[
\xi = \exp(iM/f_\pi)
\]

(1.7)

with

\[
M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+

-\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^- & K^0

K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta
\end{pmatrix}
\]

(1.8)

and \( f_\pi \approx 132 \text{ MeV} \) is the pion decay constant. Under chiral \( SU(3)_L \times SU(3)_R \) transformations

\[
\Sigma \rightarrow L \Sigma R^\dagger
\]

(1.9a)

and

\[
\xi \rightarrow L \xi U^\dagger = U \xi R^\dagger,
\]

(1.9b)
where \( L \in SU(3)_L \), \( R \in SU(3)_R \), and \( U \) is a unitary matrix which depends on \( L, R \) and is a function of space-time through its non-linear dependence on the pseudo-Goldstone boson fields.

Equation (1.1) is the most general Lagrangian invariant under both the heavy quark and chiral symmetries to first order in the Goldstone boson momenta, \( m_q \) and \( 1/m_Q \). It is remarkable that these symmetries combine to constrain the the Lagrangian so that at leading order there is only one unknown coupling \( g \) independent of the heavy quark flavor and spin. This same coupling enters into the decay width for \( D^* \to D \pi \) (where \( Q = c \)):

\[
\Gamma(D^{**} \to D^0 \pi^+) = \frac{g^2}{6\pi} \frac{|P_\pi|^3}{f_\pi^2} \tag{1.10}
\]

A recent experimental measurement of this width gave the limit \( \Gamma(D^{**} \to D^0 \pi^+) < 72 \text{ keV} \)\(^7\) which translates to \( g^2 < 0.4 \). There is no phase space for the corresponding decay when \( Q = b, B^* \to B \pi \). However, the exclusive semi-leptonic decay \( B \to D^* X \ell \bar{\nu}_\ell \) could be used to probe the heavy flavor dependence of \( g \).

The kinematical formalism for treating an exclusive decay of the form under consideration is developed in Sect. 2. In Sect. 3, the heavy quark and chiral symmetry predictions for the form factors that appear in \( B \to D^* X \ell \bar{\nu}_\ell \) decay are calculated and used to determine the rate for this process. Concluding remarks are made in Sect. 4.

2. KINEMATICAL ANALYSIS

In this section, the general kinematic analysis for decays of the form

\[
\text{pseudoscalar meson} \to \text{vector meson + pseudoscalar meson + lepton + anti-lepton} \quad (2.1)
\]

is presented. For definiteness, we consider the decay \( B \to D^* X \ell \bar{\nu}_\ell \); however, this formalism is more generally applicable to any decay of the form given by eq. (2.1). If \( p_B, p_{D^*}, p_X, p_\ell, p_\bar{\nu} \) are the four-momenta of the \( B, D^* \) (which also has a polarization vector \( \varepsilon \)), \( X, \ell, \bar{\nu}_\ell \), respectively, then the kinematics of the decay can be more conveniently expressed in terms of quantities involving the following combinations of these four-momenta.

\[
P = p_{D^*} + p_X, \tag{2.2a}
\]

\[
Q = p_{D^*} - p_X \tag{2.2b}
\]

\[
L = p_\ell + p_\bar{\nu} \tag{2.2c}
\]

\[
N = p_\ell - p_\bar{\nu} \tag{2.2d}
\]

Apart from spin, four-body decay is kinematically parameterized by five variables. By choosing these variables appropriately, one can express the distribution for the decay in a form...
where the dependence of the angular distribution on the hadronic and leptonic currents factorizes. This can be achieved by the choice

i. \( s_H = P^2 \), the effective mass of the hadron pair, \( D^* \) and \( X \);

ii. \( s_L = L^2 \), the effective mass of the lepton pair, \( \ell \) and \( \nu_\ell \);

iii. \( \theta_H \), the angle between the \( D^* \) three-momentum in the \( D^*X \) rest frame and the line of flight of the \( D^*X \) in the rest frame of the \( B \);

iv. \( \theta_L \), the angle between the \( \ell \) three-momentum in the \( \ell\nu_\ell \) rest frame and the line of flight of the \( \ell\nu_\ell \) in the rest frame of the \( B \);

v. \( \phi \), the angle from the normal of the plane formed by the hadron pair to the normal of the plane formed by the lepton pair.

In the following analysis, one finds that over much of the available phase space including the region where chiral perturbation theory is valid, terms that depend on the mass of the lepton are suppressed, \( m_\ell / s_L \ll 1 \), so that the lepton mass may be neglected. With \( m_\ell = 0 \),

\[
Q^2 = 2(m_{D^*}^2 + m_X^2) - s_H = (\chi^2 - U^2)s_H, \tag{2.3a}
\]

\[
N^2 = -s_L, \tag{2.3b}
\]

\[
P \cdot L = V = \frac{m_B^2 - s_H - s_L}{2}, \tag{2.3c}
\]

\[
P \cdot Q = m_{D^*}^2 - m_X^2 = \chi s_H, \tag{2.3d}
\]

\[
P \cdot N = W \cos \theta_L, \tag{2.3e}
\]

\[
L \cdot N = 0, \tag{2.3f}
\]

\[
Q \cdot L = \chi V + UW \cos \theta_H, \tag{2.3g}
\]

\[
Q \cdot N = (\chi W + UV \cos \theta_H) \cos \theta_L - U \sqrt{s_H s_L} \sin \theta_H \sin \theta_L \cos \phi, \tag{2.3h}
\]

\[\epsilon_{\mu\nu\rho\sigma} P^\mu Q^\nu L^\rho N^\sigma = -UW \sqrt{s_H s_L} \sin \theta_H \sin \theta_L \sin \phi, \tag{2.3i}\]

In eqs.(2.3)

\[\chi = \frac{m_{D^*}^2 - m_X^2}{s_H}, \tag{2.4a}\]

\[U \] is the magnitude of the \( D^* \) three-momentum in the \( D^*X \) rest frame,

\[U = (s_H^2 + m_{D^*}^4 + m_X^4 - 2s_H m_{D^*}^2 - 2s_H m_X^2 - 2m_{D^*}^2 m_X^2)^{1/2} / s_H, \tag{2.4b}\]

and

\[W = (V^2 - s_H s_L)^{1/2}. \tag{2.4c}\]

The invariant transition amplitude for the decay \( B \to D^*X \ell\nu_\ell \) is given by

\[
\mathcal{M}_{fi} = \frac{G_F}{\sqrt{2}} \lambda_{chi}(X(p_X)D^*(p_{D^*}, \varepsilon)|\bar{c}\gamma_\mu(1 - \gamma_5)b|B(p_B)) \bar{u}(p_\ell)\gamma^\mu(1 - \gamma_5)v(p_\nu), \tag{2.5}\]
where $G_F$ is the Fermi constant and $V_{cb}^*$ is the Cabibbo-Kobayashi-Maskawa matrix element for $b \to c$ transitions. The hadronic matrix element can be expressed in terms of fifteen form factors:

$$
\langle X(p_X) D^*(p_{D^*}, \varepsilon) | \bar{c} \gamma_{\mu} (1 - \gamma_5) b | B(p_B) \rangle =
\left[ i(a_+ \varepsilon^* \cdot P + b_+ \varepsilon^* \cdot p_B) + \frac{w_+}{2} \epsilon_{\alpha \beta \gamma \delta} L^\alpha P^\beta Q^\gamma \varepsilon^{* \delta} \right] p_\mu
+ \left[ i(a_- \varepsilon^* \cdot P + b_- \varepsilon^* \cdot p_B) + \frac{w_-}{2} \epsilon_{\alpha \beta \gamma \delta} L^\alpha P^\beta Q^\gamma \varepsilon^{* \delta} \right] Q_\mu
+ \left[ i(c \varepsilon^* \cdot P + d \varepsilon^* \cdot p_B) + \frac{w}{2} \epsilon_{\alpha \beta \gamma \delta} L^\alpha P^\beta Q^\gamma \varepsilon^{* \delta} \right] L_\mu
+ i f \varepsilon^* _\mu
+ g_+ \epsilon_{\mu \alpha \beta \gamma} L^\alpha P^\beta \varepsilon^{* \gamma} + g_- \epsilon_{\mu \alpha \beta \gamma} L^\alpha Q^\beta \varepsilon^{* \gamma} + r \epsilon_{\mu \alpha \beta \gamma} P^\alpha Q^\beta \varepsilon^{* \gamma}
+ (u_1 \varepsilon^* \cdot P + u_2 \varepsilon^* \cdot p_B) \epsilon_{\mu \alpha \beta \gamma} L^\alpha P^\beta Q^\gamma,
$$

where the form factors $a_\pm, b_\pm, c, d, f, g_\pm, r, u_1, u_2, w,$ and $w_\pm$ are functions of $s_H, s_L,$ and $\theta_H$. The absolute value of the transition amplitude squared when summed over the vector meson and lepton polarizations is then

$$
\sum_{\text{spins}} |\mathcal{M}_{fi}|^2 = \frac{G_F^2}{2} |V_{cb}|^2 H_{\mu \nu} L^\mu L^\nu,
$$

where

$$
H_{\mu \nu} = \langle X(p_X) D^*(p_{D^*}, \varepsilon) | \bar{c} \gamma_{\mu} (1 - \gamma_5) b | B(p_B) \rangle \times \langle X(p_X) D^*(p_{D^*}, \varepsilon) | \bar{c} \gamma_{\nu} (1 - \gamma_5) b | B(p_B) \rangle^*,
$$

$$
L^\mu \nu = 4 (L^\mu L^\nu - N^\mu N^\nu - s_L g_\mu \nu - i \epsilon^{\alpha \mu \beta \nu} L_\alpha N_\beta).
$$

Using eqs.(2.3a–i), the differential decay rate can then be written in the form

$$
d^5 \Gamma = \frac{G_F^2 |V_{cb}|^2}{(4\pi)^6 m_{D^*}^3} U W I(s_H, s_L, \theta_H, \theta_L, \phi) ds_H ds_L d \cos \theta_H d \cos \theta_L d \phi,
$$

with

$$
I = I_1 + I_2 \cos 2\theta_L + I_3 \sin^2 \theta_L \cos 2\phi + I_4 \sin 2\theta_L \cos \phi + I_5 \sin \theta_L \cos \phi
+ I_6 \cos \theta_L + I_7 \sin \theta_L \sin \phi + I_8 \sin 2\theta_L \sin \phi + I_9 \sin^2 \theta_L \sin 2\phi
$$

where $I_j, 1 \leq j \leq 9,$ are functions of $s_H, s_L, \theta_H$ only. As was alluded to earlier, the separation of the variables $s_H, s_L, \theta_H$ from $\theta_L, \phi$ in eq. (2.10) is a direct consequence this particular choice for the five variables parameterizing four-body decay. The distribution functions $I_j$ can be
written in a compact form by introducing the following combinations of kinematic factors and form factors.

\[
G_1 = \frac{1}{2m_{D^*}} \left\{ \lambda s_H [W a_+ + (\chi W + UV \cos \theta_H) a_-] + \left( \frac{m_B^2 + s_H - s_L}{2} \lambda + UW \cos \theta_H \right) [W b_+ + (\chi W + UV \cos \theta_H) b_-] + (\lambda W + UV \cos \theta_H) f \right\} 
\]

\[
G_2 = \frac{U \sqrt{s_H s_L}}{2m_{D^*}} \left\{ (\lambda s_H) a_- + \left( \frac{m_B^2 + s_H - s_L}{2} \lambda + UW \cos \theta_H \right) b_- + f \right\} 
\]

\[
G_3 = \sqrt{s_H} \left\{ [W a_+ + (\chi W + UV \cos \theta_H) a_-] + \left( \frac{m_B^2 + s_H - s_L}{2s_H} [W b_+ + (\chi W + UV \cos \theta_H) b_-] + \frac{W}{s_H} f \right) \right\} 
\]

\[
G_4 = U s_H \sqrt{s_L} \left( a_- + \left( \frac{m_B^2 + s_H - s_L}{2s_H} b_- \right) \right) 
\]

\[
G_5 = \frac{1}{\sqrt{s_H}} [W^2 b_+ + W (\chi W + UV \cos \theta_H) b_- + V f] 
\]

\[
G_6 = UW \sqrt{s_L} b_- 
\]

\[
G_7 = \sqrt{s_L} f 
\]

\[
G_8 = \frac{U W \sqrt{s_H s_L}}{2m_{D^*}} \left[ g_- - g_+ + (\lambda s_H) u_1 + \left( \frac{m_B^2 + s_H - s_L}{2} \lambda + UW \cos \theta_H \right) u_2 \right] 
\]

\[
G_9 = \sqrt{s_L} [W g_+ + (\chi W + UV \cos \theta_H) g_- + (Us_H \cos \theta_H) v] 
\]

\[
G_{10} = U \sqrt{s_H} (s_L g_- + V r) 
\]

\[
G_{11} = UW \sqrt{s_L} \left[ g_- - \left( s_H u_1 + \left( \frac{m_B^2 + s_H - s_L}{2} u_2 \right) \right) \right] 
\]

\[
G_{12} = UV \sqrt{s_L} (g_- - V u_2) 
\]

\[
G_{13} = U \sqrt{s_H} s_L (g_- - V u_2) 
\]

\[
G_{14} = UV \sqrt{s_H} (r + s_L u_2) 
\]

\[
G_{15} = Us_H \sqrt{s_L} (r + s_L u_2) 
\]

\[
G_{16} = UW \sqrt{s_H} [W w_+ + (\chi W + UV \cos \theta_H) w_-] 
\]

\[
G_{17} = U^2 W s_H \sqrt{s_L} w_- 
\]

In these equations, \( \lambda = 1 + \chi \).
Then

\[
I_1 = \frac{1}{2} (|G_1|^2 - |G_3|^2 + |G_5|^2) + \frac{3}{2} |G_7|^2 + \frac{3}{4} |G_9|^2
\]

\[
+ \frac{3}{4} (|G_2|^2 - |G_4|^2 + |G_6|^2 + |G_8|^2 + |G_{10}|^2
- |G_{11}|^2 + |G_{12}|^2 - |G_{13}|^2 - |G_{14}|^2 + |G_{15}|^2) \sin^2 \theta_H
\]

\[
+ \frac{1}{2} |G_{10} + G_{16}|^2 \sin^2 \theta_H + \frac{3}{4} |G_9 - G_{17}|^2 \sin^2 \theta_H|^2
\]

(2.12a)

\[
I_2 = -\frac{1}{2} (|G_1|^2 - |G_3|^2 + |G_5|^2) + \frac{1}{2} |G_7|^2 + \frac{1}{4} |G_9|^2
\]

\[
+ \frac{1}{4} (|G_2|^2 - |G_4|^2 + |G_6|^2 + |G_8|^2 + |G_{10}|^2
- |G_{11}|^2 + |G_{12}|^2 - |G_{13}|^2 - |G_{14}|^2 + |G_{15}|^2) \sin^2 \theta_H
\]

\[
- \frac{1}{2} |G_{10} + G_{16}|^2 \sin^2 \theta_H + \frac{1}{4} |G_9 - G_{17}|^2 \sin^2 \theta_H|^2
\]

(2.12b)

\[
I_3 = \frac{1}{2} (-|G_2|^2 + |G_4|^2 - |G_6|^2 + |G_8|^2 + |G_{10}|^2
- |G_{11}|^2 + |G_{12}|^2 - |G_{13}|^2 - |G_{14}|^2 + |G_{15}|^2) \sin^2 \theta_H
\]

\[
- \frac{1}{2} |G_{17}|^2 \sin^4 \theta_H
\]

(2.12c)

\[
I_4 = \text{Re}(G_1 G_2^* - G_5 G_4^* + G_5 G_6^* - G_9 G_{10}^*) \sin \theta_H
\]

\[
+ \text{Re}(G_{16} G_{17}^*) \sin^3 \theta_H
\]

(2.12d)

\[
I_5 = 2 \text{Re}[G_1 G_8^* + G_3 G_{11}^* - G_5 (G_{12} + G_{15})^* - G_7 (G_{10} + G_{16})^*] \sin \theta_H
\]

(2.12e)

\[
I_6 = 2 \text{Re}\{[G_2 G_8^* + G_4 G_{11}^* - G_6 (G_{12} + G_{15})^*] \sin^2 \theta_H + 2 G_7 G_9^*
\]

\[
- G_7 G_{17}^* \sin^2 \theta_H\}
\]

(2.12f)

\[
I_7 = 2 \text{Im}(G_1 G_2^* - G_3 G_4^* + G_5 G_6^* + G_9 G_{10}^*) \sin \theta_H
\]

(2.12g)

\[
I_8 = \text{Im}[G_1 G_8 + G_3 G_{11}^* - G_5 (G_{12} + G_{15})^* + G_7 (G_{13} + G_{14})^*] \sin \theta_H
\]

(2.12h)

\[
I_9 = - \text{Im}[G_2 G_8 + G_4 G_{11}^* - G_6 (G_{12} + G_{15})^*] \sin^2 \theta_H
\]

(2.12i)

Eqs.(2.12) indicates that the partial wave expansions for the $G_i$ in eqs.(2.11) are of the form

\[
G_i(s_H, s_L, \cos \theta_H) = \sum_{l=0}^{\infty} \tilde{G}_{i,l}(s_H, s_L) P_l(\cos \theta_H), \tag{2.13a}
\]

for $i = 1, 3, 5, 7, 9,$

\[
G_i(s_H, s_L, \cos \theta_H) = \sum_{l=1}^{\infty} \tilde{G}_{i,l}(s_H, s_L) \frac{d}{\sqrt{l(l+1)}} \frac{d}{d \cos \theta_H} P_l(\cos \theta_H), \tag{2.13b}
\]

for $i = 2, 4, 6, 8, 10, 11, 12, 13, 14, 15, 16,$ and

\[
G_{17}(s_H, s_L, \cos \theta_H) = \sum_{l=1}^{\infty} \tilde{G}_{17,l}(s_H, s_L) \frac{d^2}{\sqrt{(l-1)(l+1)(l+2)}} \frac{d^2}{d \cos \theta_H} P_l(\cos \theta_H). \tag{2.13c}
\]
The form of the distribution given by eq.(2.9-2.13), where the dependence on the lepton angles \((\theta_L, \phi)\) is explicit, is ideally suited for the determination of the \(I_j\)'s and hence the form factors from angular distribution measurements.

Implementing eqs.(2.10,2.12,2.13) in eq. (2.9) and integrating over the angles yields

\[
d^2\Gamma = \frac{G_F^2 |V_{cb}|^2}{3(4\pi)^3 m_B^3} UW \left[ \sum_l \frac{2}{2l + 1} \left| \tilde{G}_{1,l} \right|^2 + \left| \tilde{G}_{5,l} \right|^2 + 2 \left| \tilde{G}_{7,l} \right|^2 + \left| \tilde{G}_{9,l} \right|^2 + \left| \tilde{G}_{11,l} \right| - \left| \tilde{G}_{17,l} \right|^2 \\
+ \left| \tilde{G}_{2,l} \right|^2 + \left| \tilde{G}_{4,l} \right|^2 + \left| \tilde{G}_{6,l} \right|^2 + \left| \tilde{G}_{8,l} \right|^2 + \left| \tilde{G}_{10,l} \right|^2 - \left| \tilde{G}_{11,l} \right|^2 \\
+ \left| \tilde{G}_{12,l} \right|^2 - \left| \tilde{G}_{13,l} \right|^2 - \left| \tilde{G}_{14,l} \right|^2 + \left| \tilde{G}_{15,l} \right|^2 + \left| \tilde{G}_{16,l} \right|^2 \right]; \\
\text{(2.14)}
\]

and the total decay rate is

\[
\Gamma = \int_{m_B^2}^{m_B^2} \left[ \int_0^{(m_B - \sqrt{s_H})^2} \left( \frac{d^2\Gamma}{ds_H ds_L} \right) ds_L \right] ds_H.
\text{(2.15)}
\]

The simplicity of the limits in the integration over phase space in eq. (2.15) is another advantage of our choice of the five variables describing four body decay.

### 3. \(B \rightarrow D^* X \ell \bar{\nu}_\ell\) DECAY

Weak \(b \rightarrow c\) transitions like the decay \(B \rightarrow D^* X \ell \bar{\nu}_\ell\) are effected by the current \(\bar{c}\gamma_\mu(1 - \gamma_5)b\). Since this operator is a singlet under SU(3)\(_L\) × SU(3)\(_R\) chiral transformations, matching its matrix elements onto those in the heavy meson chiral theory gives \(^2\)

\[
\bar{c}\gamma_\mu(1 - \gamma_5)b = -C_{cb}\beta(v \cdot v') \text{Tr}[\bar{H}^{(c)}(v') \gamma_\mu(1 - \gamma_5)H^{(b)}(v)] + \ldots,
\text{(3.1)}
\]

where the ellipsis denotes terms with derivatives, insertions of the light quark mass matrix \(m_q\), or factors of \(1/m_Q\). The factor \(C_{cb}\) contains the calculable perturbative QCD corrections to the heavy quark current, while \(\beta\) is the universal Isgur-Wise meson form factor which accounts for the unknown non-perturbative effects in the current due to interactions with the light degrees of freedom. The function \(\beta\) is normalized at zero recoil:\(^9\)

\[
\beta(v \cdot v' = 1) = 1
\text{(3.2)}
\]

At leading order in the heavy meson chiral perturbation theory, \(\pi, K, \eta\) fields are absent in the operator of eq. (3.1), and hence the matrix element for \(B \rightarrow D^* X \ell \bar{\nu}_\ell\) decay is dominated by the tree-level pole-type Feynman graphs in Fig. 1. The Feynman rules for these
diagrams are obtained by expanding out eq. (1.1) and (3.1) in powers of the pseudo-Goldstone boson fields and the heavy meson fields $P_a$ and $P^*_{a\mu}$.

Calculating the Feynman diagrams for the case $X = \pi^\pm$ gives the following predictions for the form factors.

$$a_+ = A\left(\frac{1}{2m_{D^*}} + \frac{1}{m_B}\right)\left(\frac{1}{v' \cdot p_\pi + \Delta_D} - \frac{1}{v \cdot p_\pi + \Delta_B}\right)$$

(3.3a)

$$a_- = \frac{A}{2m_{D^*}}\left(\frac{1}{v' \cdot p_\pi + \Delta_D} - \frac{1}{v \cdot p_\pi + \Delta_B}\right)$$

(3.3b)

$$b_+ = \frac{A}{2m_B}\left[\frac{1}{m_{D^*}}\left(1 + \frac{v \cdot p_\pi}{v' \cdot p_\pi + \Delta_B}\right) + \left(\frac{1}{v' \cdot p_\pi + \Delta_D} - \frac{1}{v \cdot p_\pi + \Delta_B}\right)\right]$$

(3.3c)

$$b_- = \frac{A}{2m_B}\left[\frac{1}{m_{D^*}}\left(1 + \frac{v \cdot p_\pi}{v' \cdot p_\pi + \Delta_B}\right) - \left(\frac{1}{v' \cdot p_\pi + \Delta_D} - \frac{1}{v \cdot p_\pi + \Delta_B}\right)\right]$$

(3.3d)

$$c = \frac{A}{m_B}\left(\frac{1}{v' \cdot p_\pi + \Delta_D} - \frac{1}{v \cdot p_\pi + \Delta_B}\right)$$

(3.3e)

$$d = 0$$

(3.3f)

$$f = A\left[\frac{v' \cdot p_\pi - (v \cdot v')(v \cdot p_\pi)}{v \cdot p_\pi + \Delta_B} + \frac{v \cdot p_\pi}{v' \cdot p_\pi - v \cdot v'}\right]$$

(3.3g)

$$g_+ = -\frac{A}{2m_B}\left[\frac{1}{m_{D^*}}\left(\frac{v \cdot p_\pi}{v' \cdot p_\pi + \Delta_B}\right) + \frac{1}{v \cdot p_\pi + \Delta_B}\right]$$

(3.3h)

$$g_- = -\frac{A}{2m_B}\left[\frac{1}{m_{D^*}}\left(\frac{v \cdot p_\pi}{v' \cdot p_\pi + \Delta_B}\right) - \frac{1}{v \cdot p_\pi + \Delta_B}\right]$$

(3.3i)

$$r = -\frac{A}{2}\left\{\frac{1}{m_B}\left[\frac{1}{m_{D^*}}\left(\frac{v \cdot p_\pi}{v' \cdot p_\pi + \Delta_B}\right) - \frac{1}{v \cdot p_\pi + \Delta_B}\right]\right.\right.$$

$$+ \left.\frac{1}{m_{D^*}}\left(1 + v' \cdot p_\pi\right) - \frac{1}{v \cdot p_\pi + \Delta_B}\right\}$$

(3.3j)

$$u_1 = 0$$

(3.3k)

$$u_2 = 0$$

(3.3l)

$$w = 0$$

(3.3m)

$$w_+ = w_- = \frac{A}{2m_B m_{D^*} v' \cdot p_\pi}$$

(3.3n)

In these equations,

$$A = \sqrt{m_{D^*} m_B} g C_{\alpha \beta} \beta(v \cdot v')/f_\pi$$

(3.4a)

$$\Delta_D = m_{D^*} - m_D \approx 142 \text{ MeV},$$

(3.4b)

$$\Delta_B = m_{B^*} - m_B \approx 46 \text{ MeV}.$$
The above results are generally applicable when $X$ is any of the pseudo-Goldstone bosons with appropriate modifications to take into account isospin factors. However, the large masses of the kaon and eta compared to the chiral symmetry breaking scale ($\Lambda_{\chi} \sim 1$ GeV) may render leading order chiral perturbation theory inadequate, so in the remainder of this analysis we will continue to take $X$ to be a pion.

Since the masses of the heavy mesons are so much greater than that of the pseudo-Goldstone bosons, it is appropriate to make the dependence on the heavy masses manifest and to neglect terms that are suppressed by factors of $m_{\pi}/m_B$ and $m_{\pi}/m_{D^*}$. The pertinent formulae in Sect.2 can be written in this form by expressing the pion’s four-momentum in terms of its four-velocity $v_\pi^\mu = p_\pi^\mu/m_\pi$ and by changing variables from $s_H$ and $s_L$ to $v \cdot v_\pi$ and $v' \cdot v_\pi$ so that the integration measure in eq. (2.9) becomes

$$ds_H \, ds_L \approx 4 \, m_B \, m_{D^*}^2 \, m_\pi d(v \cdot v') \, d(v' \cdot v_\pi).$$

Now we introduce the dimensionless quantities $\hat{G}_j$ which are defined in terms of the $G_j$ by

$$G_j = \frac{m_\pi^{3/2} \, m_{D^*} \, g \, C_{cb} \, \beta(v \cdot v')}{{\hat{f}}_{\pi} \, \hat{G}_j}$$

into eq. (2.9). Substituting

$$U \approx \frac{2 m_\pi}{m_{D^*}} (v' \cdot v_\pi)^2 - 1)^{1/2},$$

$$W \approx m_B \, m_{D^*} \, (v' \cdot v_\pi)^2 - 1)^{1/2},$$

and performing the integrations over $\theta_L$ and $\phi$ in the differential decay rate in eq. (2.9) yields

$$d\Gamma = \frac{8 G_1^2 m_B^2 m_{D^*}^2 |V_{cb}|^2}{3 (4\pi)^5} \left( \frac{m_\pi}{f_\pi} \right)^2 g^2 C_{cb}^2 \beta(v \cdot v')^2 [(v' \cdot v_\pi)^2 - 1]^{1/2} [(v' \cdot v_\pi)^2 - 1]^{1/2} \left[ (|\hat{G}_1|^2 - |\hat{G}_3|^2 + |\hat{G}_5|^2 + 2 |\hat{G}_7|^2 + |\hat{G}_9|^2 + |\hat{G}_9 - \hat{G}_{17} \sin^2 \theta_H|^2) + (|\hat{G}_2|^2 - |\hat{G}_4|^2 + |\hat{G}_6|^2 + |\hat{G}_8|^2 + |\hat{G}_{10}|^2 - |\hat{G}_{11}|^2 - |\hat{G}_{12}|^2 + |\hat{G}_{13}|^2 - |\hat{G}_{14}|^2 + |\hat{G}_{15}|^2 + |\hat{G}_{16}|^2) \sin^2 \theta_H \right] d(v \cdot v') \, d(v' \cdot v_\pi) \, d \cos \theta_H,$

where

$$v \cdot v_\pi = (v \cdot v') (v' \cdot v_\pi) - [(v \cdot v')^2 - 1]^{1/2} [(v' \cdot v_\pi)^2 - 1]^{1/2} \cos \theta_H.$$

A source of uncertainty in eq. (3.7) is the Isgur-Wise function $\beta(v \cdot v')$ since its value is only known at the zero recoil point given by eq. (3.2). However, the quantity $v \cdot v'$ is unconstrained, so this dependence on $\beta$ can be removed by normalizing this decay rate to
that for the corresponding semi-leptonic transition without the emission of pseudo-Goldstone bosons:

\[ B \to D^* \ell \bar{\nu}_\ell \]  

(3.9)

This transition is mediated by the current in eq. (3.1) and the hadronic matrix element is

\[
\langle D^*(v', \epsilon)|\bar{c}\gamma_\mu(1 - \gamma_5)b|B(v)\rangle
= \sqrt{m_Bm_{D^*}}C_{cb}\beta(v \cdot v')[-(1 + v \cdot v')\epsilon^*_\mu + (\epsilon^* \cdot v)v'^\mu + i\epsilon_{\mu\alpha\beta\gamma}\epsilon^{*\alpha}v'^\beta v^\gamma].
\]

(3.10)

Then the rate could be studied away from the zero recoil point.

Since the above rate involves the ratio \(m_\pi/f_\pi\) which is close to unity, and is not suppressed by heavy quark masses, the rate for the decay as given by eq. (3.7) is appreciable in the region of phase space where chiral perturbation theory is valid. To show this, we introduce a scaled decay rate \(d^3\Gamma\) defined by

\[
d^3\Gamma = \frac{G_F^2m_B^5}{192\pi^3}|V_{cb}|^2g^2C_{cb}^2\beta(v \cdot v')^2d^3\hat{\Gamma}.
\]

(3.11)

The differential rate \(d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)\) is calculated for various values of \(v \cdot v'\) and \(v' \cdot v_\pi\) in Table 1. In Fig. 2, this rate is plotted as a function of \(v \cdot v'\) and \(v' \cdot v_\pi\) in the kinematic region where chiral perturbation theory is expected to be valid. Additional plots of the differential decay rate as a function of the various kinematic variables, which may be of relevance to experimental analyses, are not presented because in practice such analyses are typically performed by doing Monte Carlo simulations of the fully differential decay rate given by eq. (2.9).

Table 1 shows that the differential rate for \(B \to D^*\pi \ell \bar{\nu}_\ell\) decay is smaller than the corresponding rate for \(B \to D\pi \ell \bar{\nu}_\ell\) decay given in Table 1 of Ref.[6]. This enhancement for \(B \to D\pi \ell \bar{\nu}_\ell\) can be attributed in part to the \(D^*\) propagator, in Fig.2 of Ref.6, which becomes on-shell as its pole is approached. However, the presence of the \(D^*\) in the decay \(B \to D^*\pi \ell \bar{\nu}_\ell\) allows this process to be selected experimentally with much better signal to background (because of the small amount of phase space available for \(D^* \to D\pi\) decay) as compared to the decay mode \(B \to D\pi \ell \bar{\nu}_\ell\). Moreover, the decay rate for the former channel increases much more rapidly with \(v \cdot v'\) than in the latter channel. So an experimental study of \(B \to D^*\pi \ell \bar{\nu}_\ell\) decay would complement a similar study of \(B \to D\pi \ell \bar{\nu}_\ell\). A measurement of this decay rate could be used to test heavy quark flavor symmetry: if this symmetry were violated, there would be different couplings \(g_c\) and \(g_b\) for the \(D^*D\pi\) and \(B^*B\pi\) vertices in Fig.1 which would result in different expressions for the form factors in eqs.(3.3) and hence in a different decay rate.

The value that the differential decay rate takes is determined by the contributions coming from the pole-type graphs in Fig.1. In order for these pole diagrams to be the
dominant contribution to the perturbative chiral expansion, the pseudo-Goldstone boson must be emitted with low momentum. Or equivalently, the chiral expansion parameters $v \cdot p_\pi / \Lambda_\chi$ and $v' \cdot p_\pi / \Lambda_\chi$ should be small — with $v \cdot p_\pi$ and $v' \cdot p_\pi$ on the order of a few hundred MeV. An attempt to estimate the regime where chiral perturbation theory is valid for the decay $B \to D\pi \ell \bar{\nu}_\ell$ was made in Ref.[6]; in this analysis it was found that predictions of next-to-leading order effects in chiral perturbation theory could not be made because there were too many higher dimension operators with unknown coefficients. A similar study here yields the same result, but the predictions made in this paper on the basis of leading order chiral perturbation theory may well be valid over a kinematic range much larger than that exhibited in Table 1. An experiment would ultimately establish the region of phase space where our results are valid.

4. CONCLUDING REMARKS

In this paper, a complete kinematical analysis for $B \to D^*X\ell \bar{\nu}_\ell$ decay is presented. The constraints that the heavy quark and chiral symmetries impose on this decay are found to considerably simplify the dynamics and are used to determine the decay rate for this process. A number of extensions to this work can be pursued. For instance, it is interesting to determine how large symmetry-breaking effects are by calculating sub-leading $\Lambda_{QCD}/m_c$ corrections. Decays in which more than one pseudo-Goldstone boson is emitted can also be considered.

Note added. After the completion of this work, a short paper by H.-Y. Cheng [preprint IP-ASTP-18-92] appeared which also considers the decay analyzed here.

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Table 1
Differential decay rate $d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$ at different values of $v \cdot v'$ and $v' \cdot v_\pi$.

| $v \cdot v'$ | $v' \cdot v_\pi$ | $d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$ |
|---------------|-------------------|-----------------------------------------------|
| 1.2           | 1.1               | 0.0022                                         |
| 1.4           | 1.1               | 0.0037                                         |
| 1.2           | 1.2               | 0.0033                                         |
| 1.4           | 1.2               | 0.0057                                         |
| 1.2           | 1.3               | 0.0043                                         |
| 1.4           | 1.3               | 0.0074                                         |
| 1.1           | 1.4               | 0.0022                                         |
| 1.2           | 1.4               | 0.0052                                         |
| 1.4           | 1.4               | 0.0090                                         |
| 1.2           | 1.5               | 0.0061                                         |
| 1.4           | 1.5               | 0.011                                          |
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Figure Captions

Figure 1. Leading order Feynman diagrams for $B \rightarrow D^*X\ell\bar{\nu}_\ell$ decay. The shaded circle represents an interaction term coming from the heavy meson chiral Lagrangian of eq. (1.1), and the shaded box denotes an insertion of the weak current given by eq. (3.1).

Figure 2. The scaled differential decay rate $d^2\hat{\Gamma}/d(v \cdot v')d(v' \cdot v_\pi)$ for $B \rightarrow D^*\pi\ell\bar{\nu}_\ell$ decay plotted as a function of $v \cdot v'$ and $v' \cdot v_\pi$. 