Diproton correlation in a proton-rich Borromean nucleus $^{17}$Ne

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We carry out three-body-model calculations for a proton-rich Borromean nucleus $^{17}$Ne by assuming a $^{15}$O + $p$ + $p$ structure. To this end, we use a density-dependent contact interaction between the valence protons, explicitly treating also the Coulomb interaction. We find that the two-particle density distribution for $^{17}$Ne is similar to that for $^{16}$C, which has two valence neutrons outside the $N = 8$ core. That is, the two protons take a spatially compact configuration, while the Coulomb repulsion plays a minor role. This indicates that there is a strong diproton correlation in the ground state of the $^{17}$Ne nucleus. We also show that the Coulomb interaction reduces the expectation value of the proton-proton interaction by about 14%.

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I. INTRODUCTION

Physics of unstable nuclei have attracted much attention both experimentally and theoretically $^{[1,3]}$. These nuclei are located far from the $\beta$-stability line, having a large asymmetry in the proton-to-neutron ratio. In the neutron-rich side, a few exotic features have been observed. These include i) a large extension of density distribution, referred to as halo or skin structure $^{[4]}$, ii) a narrow momentum distribution $^{[5]}$, and iii) a large concentration of the dipole strength distribution at low energies $^{[6-8]}$. For $^{11}$Li and $^6$He nuclei, the so called Borromean structure has also been discussed extensively $^{[9,10]}$. The Borromean is defined as a three-body bound system in which any two-body subsystem does not bound. The pairing interaction between the valence neutrons plays an essential role in stabilizing these nuclei $^{[9]}$.

One of the topics of current interest in physics of unstable nuclei is dineutron correlations in neutron-rich nuclei. Although two neutrons do not form a bound state in vacuum, theoretical calculations have shown that they take a spatially compact configuration in finite nuclei $^{[3,9,15]}$. This feature is enhanced significantly when the binding of $^{17}$Ne is dineutron correlations in neutron-rich nuclei.

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Experimental observation of the strong low-lying dipole strength $^{[6-8]}$. For $^{11}$Li and $^6$He nuclei, the so called Borromean structure has also been discussed extensively $^{[9,10]}$. The Borromean is defined as a three-body bound system in which any two-body subsystem does not bound. The pairing interaction between the valence neutrons plays an essential role in stabilizing these nuclei $^{[9]}$.

We describe the ground state of $^{17}$Ne by assuming an inert core nucleus $^{15}$O and two valence protons. We consider the following three-body Hamiltonian for this system:

$$H(r_1, r_2) = h^{(1)} + h^{(2)} + \frac{P_{1} \cdot P_{2}}{A_{c} m} + V_{PP}(r_1, r_2),$$

where $m$ and $A_c$ are the nucleon mass and the mass number of the inert core nucleus, respectively. $h$ is the single-particle (s.p.) Hamiltonian for a valence proton, in which $V_{PC}$ is the potential between the proton and the core nucleus. $V_{PP}$ is the interaction between the valence protons. The diagonal component of the recoil kinetic energy of the core nucleus is included in the s.p. Hamiltonian $h$ through the reduced mass $\mu = A_{c} m / (A_{c} + 1)$, whereas

$$h^{(i)} = \frac{P_{c}^{2}}{2\mu} + V_{PC}(r_i),$$

where $m$ and $A_c$ are the nucleon mass and the mass number of the inert core nucleus, respectively.
the off-diagonal part is taken into account in the last term in Eq. (11). For the single-particle potential $V_{pc}$, we use a Woods-Saxon potential together with the Coulomb interaction between a valence proton and the core nucleus. That is,

$$V_{pc}(r) = \left[ V_0 + V_s r_0^2 (\ell \cdot s) \frac{1}{r} \right] f(r) + V_C(r),$$

with

$$f(r) = \frac{1}{1 + \exp[(r - R_{\text{core}})/a_{\text{core}}]},$$

where $R_{\text{core}} = r_0 A_n^{1/3}$ is the radius of the core nucleus and $V_C$ is the Coulomb potential for a uniform charge distribution. For the parameters, we use $r_0 = 1.22$ fm, $a_{\text{core}} = 0.65$ fm, $V_0 = -53.73$ MeV, and $V_s = 15.01$ MeV. This parameter set yields an $s_{1/2}$ and a $d_{5/2}$ resonances at 0.675 MeV and 1.129 MeV, respectively. These are spin-averaged energies of the observed $0^-$ resonance at 0.535 MeV and $1^-$ resonance at 0.722 MeV, and $2^-$ at 0.951 MeV and $3^-$ at 1.257 MeV in $^{16}$F, respectively (see Fig. 1).

We solve the three-body Hamiltonian, Eq. (1), by expanding the ground state wave function with the single-particle wave functions, $\phi_{nljm}(r)$ [9, 11, 27],

$$\Psi_{g.s.}(r_1, r_2) = \sum_{n \leq n'} \sum_{l, j} a_{nm'lj} \tilde{\psi}_{nm'lj}(r_1, r_2),$$

$$\tilde{\psi}_{nm'lj}(r_1, r_2) = \frac{1}{\sqrt{2(1 + \delta_{nm'})}} \sum_m \langle j, m; j, -m | 0, 0 \rangle \times \left[ \phi_{nljm}(r_1) \phi_{nljm}(r_2) + \phi_{nljm}(r_1) \phi_{nljm}(r_2) \right],$$

where we have assumed that the valence protons form a $0^+$ configuration. The continuum s.p. states are discretized in a large box with the size of $R_{\text{max}} = 30$ fm. We include the s.p. angular momentum $l$ up to 12. In the expansion, we explicitly exclude the $1s_{1/2}$, $1p_{3/2}$, and $1p_{1/2}$ states, which are occupied by the protons in the core nucleus.

In this work, we employ a density-dependent contact interaction for the nuclear part of the proton-proton interaction [9, 11, 27, 30], while the Coulomb potential is also explicitly included. That is,

$$V_{pp}(r_1, r_2) = V_{pp}^{(N)}(r_1, r_2) + V_{pp}^{(C)}(r_1, r_2),$$

with

$$V_{pp}^{(N)}(r_1, r_2) = \delta(r_1 - r_2) \left[ v_0 + \frac{v_p}{1 + \exp[(r_1 - R_p)/a_p]} \right],$$

and

$$V_{pp}^{(C)}(r_1, r_2) = \frac{e^2}{r_1 - r_2},$$

As is well known, a contact interaction has to be handled in a truncated space defined with a cutoff energy $E_{\text{cut}}$. We take $E_{\text{cut}} = 60$ MeV, so that the s.p. space is truncated as $27$,

$$\epsilon_{nlj} + \epsilon_{n'm'l} \leq \frac{A_n + 1}{A_c} E_{\text{cut}},$$

where $\epsilon_{nlj}$ is a s.p. energy. For the Coulomb interaction, we also use the same cutoff energy. We have confirmed that our results do not change significantly even if we use a larger value of $E_{\text{cut}}$ for the Coulomb interaction. For a given cutoff, the strength of the pairing interaction, $v_0$, is determined so as to reproduce an empirical scattering length $a_{27}$, that is,

$$v_0 = \frac{2\pi^2\hbar^2}{m} \frac{2a_{27}}{\pi - 2ak_{\text{cut}}},$$

where $k_{\text{cut}} = \sqrt{mE_{\text{cut}}/\hbar^2}$. The empirical scattering length for a neutron-neutron scattering is $a_{nn} = -18.5$ fm [31], while that for a proton-proton scattering is $a_{pp} = -7.8063$ fm [32]. The difference between $a_{nn}$ and $a_{pp}$ is mainly due to the Coulomb repulsion in a two-proton system. Since we explicitly include the Coulomb interaction in our calculations, we use the neutron-neutron scattering length $a_{nn}$ to determine the strength of the pairing interaction, Eq. (11), assuming charge independence of nuclear force. Once $v_0$ is determined in this way, the parameters for the density-dependent term, $v_p, R_p$, and $a_p$ are adjusted so as to reproduce the ground state energy of $^{17}$Ne, $E_{g.s.} = -0.944$ MeV, measured from the threshold of two-proton emission. See the first row in Table I for the values of the parameters.

### III. RESULTS

We now numerically solve the three-body Hamiltonian and discuss the ground state properties of the $^{17}$Ne nucleus. The results of our calculation are summarized in
Table I

As we mentioned in the previous section, these results were obtained with \( a = a_{\text{nn}} = -18.5 \) fm. On the other hand, Refs. [11, 27] used a somewhat different value, \( a = -15 \) fm, in order to take into account finite momentum effects. We have checked that the results are almost the same even if we use \( a = -15 \) fm, together with the parameter set for the pairing interaction shown in Table II.

In Table II, \( \langle V_{pp}^{(N)} \rangle \) and \( \langle V_{pp}^{(C)} \rangle \) are expectation values of the nuclear and the Coulomb interactions, respectively. We find that the ratio of these quantities is about \(-0.14\), that is, the Coulomb repulsion reduces the pairing energy by about 14%. This value is similar to what has been found [28] using a non-empirical pairing energy density functional for proton pairing gaps \( P([s_{1/2}]^2), P([d_{5/2}]^2), \) and \( P([d_{3/2}]^2) \) are the probabilities of the \( s_{1/2}, d_{5/2}, \) and \( d_{3/2} \) configurations in the ground state wave function, respectively. One sees that the \( d_{5/2} \) configuration dominates in the ground state wave function. \( \sqrt{\langle r_{NN}^2 \rangle} \equiv \sqrt{\langle \Psi_{g.s.}|(r_1 - r_2)^2|\Psi_{g.s.}\rangle} \) and \( \sqrt{\langle r_{C-2N}^2 \rangle} \equiv \sqrt{\langle \Psi_{g.s.}|(r_1 + r_2)^2|\Psi_{g.s.}\rangle} \) are the root-mean-square (rms) distances between the valence protons and between the center of mass of the protons and the core nucleus, respectively. With these rms distances, the increment of the radius of \( ^{17}\text{Ne} \) due to the valence protons is calculated as [27, 30],

\[
\delta \langle r^2 \rangle = \frac{A_c}{A_c + 2} \langle r^2 \rangle_{A_c + 2} - \frac{A_c}{A_c + 2} \langle r^2 \rangle_{A_c},
\]

\[
= \frac{2A_c}{(A_c + 2)^2} \langle r_{C-2N}^2 \rangle + \frac{1}{2(A_c + 2)} \langle r_{NN}^2 \rangle.
\]

Once \( \delta \langle r^2 \rangle \) is evaluated in this way, the matter radius for \( ^{17}\text{Ne} \) can be estimated with Eq. (12) to be 2.62 fm. To this end, we used an empirical rms matter radius of the core nucleus, \( r_{\text{rms}}(^{15}\text{O}) \) =2.44 fm [38]. The s-wave probability, \( P([s_{1/2}]^2) \), in our calculation is somewhat smaller than that in other calculations, e.g., Ref. [23], and the rms radius for \( ^{17}\text{Ne} \) is slightly underestimated as compared to the experimental value, 2.75 fm [38].

\( \langle \theta_{12} \rangle \) in Table II is the opening angle between the valence protons. The value of \( \langle \theta_{12} \rangle \) = 76.64 degrees is in a good agreement with that in Ref. [33]. In Ref. [33], the opening angle was estimated using the total \( B(E1) \) value. We therefore compute also the sum rule value for the \( E1 \) transition (see Table I),

\[
B_{\text{SR}}(E1) = \frac{3}{\pi} \frac{g_{eff}^2}{\epsilon^2} \langle r_{2N-1}^2 \rangle,
\]

where

\[
Z_{\text{eff}} = \frac{A_c - Z_c}{A_c + 2},
\]

for a core+2p system, \( Z_c \) being the proton number of the core nucleus (notice that \( Z_{\text{eff}} = Z_c/(A_c + 2) \) for a core+2n system). This factor is \( Z_{\text{eff}} = 0.41 \) for \( ^{17}\text{Ne} \). We remind the readers that the actual value for the total \( B(E1) \) strength is somewhat smaller than the sum rule value due to the Pauli forbidden transitions [34].

It is instructive to compare our results with those obtained by mocking up the Coulomb interaction effectively in the density dependent contact interaction, i.e., by readjusting the parameters for the contact interaction without treating the Coulomb interaction explicitly. For this purpose, we use the proton-proton scattering length, \( a_{pp} = -7.81 \) fm, to determine the strength of the pairing interaction \( v_0 \). The other parameters are adjusted to reproduce the ground state energy. See the last row in Table II for the values of the parameters. The results are summarized in the second column in Table II. One sees that the results are almost the same between the first and the second columns in Table II. This indicates that the effect of Coulomb interaction on the pairing properties can be well simulated by adjusting the parameters for the pairing interaction.

We also compare our results for \( ^{17}\text{Ne} \) with those for \( ^{16}\text{C} \). Both nuclei have two valence nucleons outside the \( N \) or \( Z = 8 \) core, and one may expect some similari-

|  \( V_{pp}^{(N)} \) (MeV) | \( V_{pp}^{(C)} \) (MeV) | \( P([s_{1/2}]^2) \) (%) | \( P([d_{5/2}]^2) \) (%) | \( P([d_{3/2}]^2) \) (%) | \( \langle r_{C-2N}^2 \rangle^{1/2} \) (fm) | \( \delta \langle r^2 \rangle^{1/2} \) (fm) | \( \langle \theta_{12} \rangle \) (deg) | \( B(E1)_{\text{SR}} \) (e²fm²) | \( r(A^4Z) \) (fm) |
|---|---|---|---|---|---|---|---|---|---|
| \( -3.26 \) | 0.448 | 15.16 | 75.19 | 3.83 | 6.63 | 4.688 | 1.267 | 6.74 | 2.62 |
| \( 0.67 \) | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 1.273 | 1.273 | 1.273 | 2.62 |
| \( 0.67 \) | 0.67 | 0.67 | 0.67 | 0.67 | 0.67 | 1.273 | 1.273 | 1.273 | 2.62 |

Table II: Ground state properties for \( ^{17}\text{Ne} \) and \( ^{16}\text{C} \) obtained with a three-body model. See the text for the definition for each quantity.
ties between the two nuclei. The three-body model calculations for \(^{16}\)C have been already performed in Refs. 35, 37. We here repeat the same calculation as in Ref. 35, but using the density-dependent contact interaction. The results for \(^{16}\)C are summarized in the last column in Table [11]. In order to calculate the matter radius, we use \(r_{\text{rms}} = 2.30\) fm [38] for the radius of the core nucleus \(^{14}\)C. Interestingly, the ground state properties are almost the same between \(^{17}\)Ne and \(^{16}\)C, despite that the ground state energy for \(^{16}\)C (\(E_{\text{g.s.}} = -5.47\) MeV) is significantly different from that for \(^{17}\)Ne (\(E_{\text{g.s.}} = -0.94\) MeV). Both are \(d\)-wave dominant, and the opening angle for the valence nucleons is about 75 degrees. This similarity may be naturally understood if the Coulomb interaction between the valence protons and the core nucleus in \(^{17}\)Ne mainly perturbs the ground state energy without affecting significantly the ground state wave function.

Let us next discuss the density distribution of the valence nucleons,

\[
\rho(r_1, r_2, \theta_{12}) = \left| \Psi_{\text{g.s.}}(r_1, r_2, \theta_{12}) \right|^2,
\]

which are normalized as

\[
\int_0^\infty 4\pi r_1^2 dr_1 \int_0^\infty r_2^2 dr_2 \int_0^\pi 2\pi \sin \theta_{12} d\theta_{12} \times \rho(r_1, r_2, \theta_{12}) = 1.
\]

See Refs. 11, 12 for its explicit form. Figure 2 shows the density distribution for \(^{17}\)Ne (the upper panel) and \(^{16}\)C (the lower panel). For a presentation purpose, we set the radius of the valence nucleons to be the same (i.e., \(r_1 = r_2 = r\)) and multiply a weight factor of \(4\pi r^2 \cdot 2\pi r^2 \sin \theta_{12}\). One clearly sees that the density distributions are similar to each other for both nuclei. That is, the density distribution is largely concentrated at a small opening angle \(\theta_{12} \sim 20\) (deg) (notice that the density looks zero in the figure at \(\theta_{12} = 0\) because of the weight factor). This implies that the Coulomb repulsion between the valence protons plays a minor role in the density distribution, causing a strong diproton correlation in the ground state of \(^{17}\)Ne, as has been conjectured in Ref. 39 using a quasi-2D system.

The three-peak-shaped density distribution is due to the \([d_{5/2}]^2\) configuration, that dominates in the ground state wave function. Figure 3 shows the density distribution for the pure \([d_{5/2}]^2\) configuration for \(^{17}\)Ne. This density distribution has symmetric three peaks, which is given by \(\rho(\theta) \propto (5/4) \cdot \cos^2 \theta - 1/2 \cdot \cos^2 \theta + 3/20\) [39] (the formula in Ref. 39 contains a small error, which is corrected here). With the pairing interaction, several single-particle levels with opposite parity are mixed up in the wave function, leading to the asymmetric distribution shown in Fig. 2.

We demonstrate the strong diproton correlation in \(^{17}\)Ne also in another way. Figure 4 shows the density distribution of the second valence nucleon when the first nucleon is located along the \(z\) axis at the mean separation distance between the core and the center of mass of the two valence nucleons, that is, \(r_1 = \sqrt{r_{C-2N}^2}\). It is plotted as a function of the coordinate of the second nucleon, \(z_2 = r_2 \cos \theta_{12}\) and \(x_2 = r_2 \sin \theta_{12}\). As has been shown in Fig. 2, the second nucleon is well localized in the vicinity of the first nucleon, both for \(^{17}\)Ne and \(^{16}\)C nuclei. Evidently, the strong diproton correlation exists in the ground state of \(^{17}\)Ne in spite of the Coulomb repulsion between the valence protons.
this end, we used a density-dependent contact pairing interaction between the valence protons. In addition, we explicitly treated the Coulomb interaction as well. We found that the Coulomb repulsion leads to an about 14% reduction of the pairing energy. The density distribution for the valence protons resembles the two-neutron density distribution in $^{16}$C, showing a clear and strong diproton correlation in the ground state of $^{17}$Ne. That is, the two valence protons take a spatially compact configuration despite of the Coulomb repulsion. This suggests that the Coulomb interaction plays a minor role in the pairing correlation. In fact, we found that the effect of the Coulomb interaction can be well simulated by readjusting the parameters for the pairing interaction.

The present study suggests that there is a strong diproton correlation in light proton-rich nuclei. It would be an interesting future work to study whether this is the case also for medium-heavy and heavy proton-rich nuclei. The effect of the Coulomb interaction is expected to be stronger in the mean-field potential for these nuclei, but it is an open question whether the Coulomb effect on the diproton correlation is stronger or not. Another future work would be to study the role of diproton correlation in two-proton radioactivities. We will report on it in a separate paper.

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