Designing FDA Radars Robust to Contaminated Shared Spectra

WENKAI JIA
University of Electronic Science and Technology of China, Chengdu, China

ANDREAS JAKOBSSON, Senior Member, IEEE
Lund University, Lund, Sweden

WEN-QIN WANG, Senior Member, IEEE
University of Electronic Science and Technology of China, Chengdu, China

This article considers the problem of jointly designing the transmit waveforms and weights for a frequency diverse array (FDA) in a spectrally congested environment in which unintentional spectral interferences exist. Exploiting the properties of the interference signal induced by the processing of the multichannel mixing and low-pass filtering FDA receiver, the interference covariance matrix structure is derived. With this, the receive weights are formed using the minimum variance distortionless response method for interference cancellation. Owing to the fact that the resulting output signal-to-interference-plus-noise ratio (SINR) is a function of the transmit waveforms and weights, as well as due to the ever-greater competition for the finite available spectrum, a joint design scheme for the FDA transmit weights and the spectrally compatible waveforms is proposed to efficiently use the available spectrum while maintaining a sufficient receive SINR. The performance of the proposed technique is verified using numerical simulations in terms of the achievable SINR, spectral compatibility, as well as several aspects of the synthesized waveforms.

Manuscript received 11 April 2022; revised 11 August 2022; accepted 31 October 2022. Date of publication 10 November 2022; date of current version 9 June 2023.

DOI. No. 10.1109/TAES.2022.3221030

Refereeing of this contribution was handled by S. Bidon.

This work was supported in part by the National Natural Science Foundation of China under Grant 62171092, and in part by Swedish SRA ESSENCE under Grant 2020 6:2.

Authors’ addresses: Wenkai Jia and Wen-Qin Wang are with the School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu, 611731, China, E-mail: (wenkai.jia@matstat.lu.se; wqwang@uestc.edu.cn); Andreas Jakobsson is with the Division of Mathematical Statistics, Center for Mathematical Sciences, Lund University, SE-22100 Lund, Sweden, E-mail: (andreas.jakobsson@matstat.lu.se). (Corresponding author: Wen-Qin Wang.)

0018-9251 © 2022 IEEE
increment is larger than the bandwidth of the transmitted baseband signal [24], [25], [26]. At the same time, in the presence of radar interference, the ability of a communication receiver to recover the transmitted signal will be severely degraded [27]. Therefore, a better solution is to construct the systems to allow different services to operate within the same frequency bands with a tolerable level of distortion.

In addition to the aforementioned discussion, the statistical characteristics of interferences must be considered when analyzing and processing FDA output signals. In [19], a noise model is derived to allow for the fact that the FDA output noise does not obey the independence assumption after passing through the multichannel receiver, in contrast to that of a MIMO or a phased-array (PA). Likewise, the interference model needs to be modified to ensure that the appropriate statistical characteristics are provided to obtain a more accurate estimate of the interference-plus-noise covariance matrix. Meanwhile, modern digital technology enables the creation of precise radar waveforms with spectral nulls at particular frequency bands. This also allows for an optimal solution wherein the FDA waveforms may be designed specifically for the spectrally congested environment in which they will operate, taking into account the structure of the present interference signals.

There are many excellent reviews in the literature dealing with the design of PA or MIMO radar in a spectral congestion scenario (see, e.g., [28], [29], [30], and [31]). However, unlike PA, which transmits a single waveform [32], and MIMO, where the multiple waveforms will cause a dependence of the waveform spectral distribution on the spatial direction [31], FDA contains multiple waveforms with large frequency increments, resulting in an omnidirectional distribution of electromagnetic energy. To the best of our knowledge, no well working solution to this problem has been proposed and how one should design spectrally compatible waveforms for FDA is still a problem that merits further attention. Not limited to waveform design, in recent years, several approaches have been proposed to deal with the problem of spectrum congestion and to allow for a more efficient spectrum usage [21], [22], [33]. The advantage of the waveform design approach is that, relying on real-time spectrum occupancy sensing, the waveform can be dynamically selected based on changing conditions, thereby controlling its impact on other compatible systems. In addition, the underlying optimization process can also benefit from multiple design degrees of freedom to further enhance the radar performance.

Rather than assuming that no other radiators will occupy the same part of the spectrum as the current radar application, we will in this work take into account for the increasingly crowded spectrum, proposing an FDA system that allows for the presence of other applications in the same spectral band that is used by the radar. The main contributions and novelties of this work are summarized as follows.

1) Employing the multichannel mixing and low-pass filtering-based FDA receiver presented in [20], the transmit waveforms can be extracted from the signals at the receiver output end. This allows the FDA spectrum compatibility constraint and the interference-plus-noise covariance matrix to be derived. Since the requirements for FDA waveform separation in the receiver requires that a constant energy for each waveform must be guaranteed [20], the transmit weights are designed to achieve less mutual interference than fixed weights.

2) Aiming at minimizing the mutual interference induced by frequency overlaid systems, the joint design of the FDA transmit waveforms and weights is formulated as an optimization problem with multiple nonconvex constraints, using the output SINR as the objective function to measure the system performance. In order to solve the resulting nonconvex problem, an iterative algorithm based on the semidefinite relaxation (SDR) technique presented in [34] is introduced.

Various numerical simulations are carried out to demonstrate the performance of the proposed algorithm from the point of achievable SINR and spectral compatibility as well as with respect to several aspects of the radar environment. It is found that the design scheme based on waveform optimization not only maximizes the output SINR, but also enables finer control over the operating spectrum of the system.

The remainder of this article is organized as follows. In Section II, the assumed signal model and the statistical analysis of the potential interferences are presented. Next, we formulate the optimization problem maximizing the output SINR subject to the derived FDA spectrum compatibility constraint in Section III. In Section IV, an iterative method based on an SDR technique is developed to solve the resulting optimization problem. Simulation results are given in Section V, and finally, Section VI concludes this article.

II. SIGNAL MODEL

A. Receive Signal Model

Consider a uniform linear FDA, with \( N_T \) transmit antennas, each of which radiates a different waveform \( s_m(t) \), \( m = 1, 2, \ldots, N_T \). The transmitted radio frequency FDA signal may be expressed as

\[
x(t) = \sum_{m=1}^{N_T} w_m s_m(t) e^{j2\pi f_m t}
\]

where \( f_m = f_c + (m-1)\Delta f \), with \( \Delta f \) being the frequency increment, and \( w_m \) denoting the carrier frequency and the transmit weight of the \( m \)th transmit antenna, respectively. Thus, the synthesized signal from a given far-field point target location \((\theta, r)\), where \( r \) and \( \theta \) represent the slant range and azimuth angle, respectively, can be expressed as:

\[
x(t) = \sum_{m=1}^{N_T} w_m s_m(t) e^{j2\pi f_m t}
\]
as \[ y(t) = \sum_{m=1}^{N_T} w_m s_m(t - \frac{r_t - md_t \sin \theta_t}{c}) e^{j2\pi f_m (t - \frac{r_t - md_t \sin \theta_t}{c})} \]

\approx \sum_{m=1}^{N_T} w_m s_m(t - \frac{r_t}{c}) e^{j2\pi f_m (t - \frac{r_t}{c})} e^{j2\pi \frac{md_t \sin \theta_t}{c}}

= [w \odot a_T(\theta_t)]^T \tilde{s}(t - \frac{r_t}{c}) \tag{2}

where \( w = [w_1, w_2, \ldots, w_{N_T}]^T \) denotes the transmit weight vector, with \( T \) being the transpose operator, \( \tilde{s}(t) = s(t - \frac{r_t}{c}) \odot e(t) \), with

\[ s(t) = [s_1(t), s_2(t), \ldots, s_{N_T}(t)]^T \tag{3} \]

and

\[ e(t) = [e^{j2\pi f_1 t}, e^{j2\pi (f_1+\Delta f) t}, \ldots, e^{j2\pi (f_1+(N_T-1)\Delta f) t}]^T \tag{4} \]

being the transmit waveform vector and the carrier vector, respectively, and \( \lambda = \frac{c}{\Delta_1} \) the reference wavelength, with \( c \) denoting the speed of light.

\[ a_T(\theta_t) = [1, e^{j2\pi \frac{md_t \sin \theta_t}{c}}, \ldots, e^{j2\pi (N_T-1) \frac{md_t \sin \theta_t}{c}}]^T \tag{5} \]

with \( d_t = \frac{\lambda}{2} \) being the interelement spacing of the transmit array, and \( \odot \) is the Hadamard product operator. It is worth noting that the transmit power allocation is enabled by coupling of the transmit weight vector \( w \) with the transmit waveform vector \( s(t) \). Here, we assume that the \( N_R \) receive antennas are colocated with the transmit antennas, at half-wavelength intervals, \( d_r = \frac{\lambda}{2} \). Using the multichannel mixing and low-pass filtering-based FDA receiver [20], as shown in Fig. 1, where a reflected signal is first mixed by the multichannel mixers with local carrier frequencies \( \{f_m\}_{m=1}^{N_T} \), and subsequently, processed by the low-pass filter, the filtered output matrix may be obtained as

\[ \hat{R}(t) = [r_1(t), r_2(t), \ldots, r_{N_R}(t)] \]

\[ = \zeta(r_t, \theta_t) \text{diag} [w \odot a_T(r_t, \theta_t)] s(t - \frac{2r_t}{c}) b_R^T(\theta_t) \]

\[ = \zeta(r_t, \theta_t) \text{diag} [s(t - \frac{2r_t}{c}) \odot a_T(r_t, \theta_t)] w b_R^T(\theta_t) \tag{6} \]

where \( \zeta(r_t, \theta_t) \) is the target complex reflection coefficient, and \( \text{diag}[w \odot a_T(r_t, \theta_t)] \) denotes the diagonal matrix with entries formed by \( w \odot a_T(r_t, \theta_t) \) as

\[ b_R(\theta_t) = [1, e^{j2\pi \frac{md_t \sin \theta_t}{c}}, \ldots, e^{j2\pi (N_T-1) \frac{md_t \sin \theta_t}{c}}]^T \tag{7} \]

denotes the FDA angle-dependent receive steering vector, and \( r_n(t) \) the filtered output of the \( n \)th receive antenna, given by

\[ r_n(t) = \zeta(r_t, \theta_t) e^{j2\pi \frac{md_t \sin \theta_t - \lambda b_n}{c}} \text{diag} [w \odot a_T(r_t, \theta_t)] \tag{8} \]

with

\[ a_T(r_t, \theta_t) = [1, e^{j2\pi \frac{md_t \sin \theta_t - 2\lambda b_n}{c}}, \ldots, e^{j2\pi (N_T-1) \frac{md_t \sin \theta_t - 2\lambda b_n}{c}}]^T \tag{9} \]

being the FDA range-angle-dependent transmit steering vector. Let \( z_i(t) \) denote the \( i \)th undesired interference signal with \( i = 1, 2, \ldots, N_T \). For the \( n \)th receiver element, the filtered output of the its \( m \)th channel may then be expressed as

\[ \bar{z}_{m,n}(t) = e^{-j2\pi \frac{md_t \sin \theta_t - \lambda b_m}{c}} z_{m,i}(t) \tag{10} \]

for \( m' = 1, 2, \ldots, N_T \), where \( z_{m,i}(t) \) is

\[ z_{m,i}(t) = [e^{j2\pi f_0 t} z_i(t)] * h_m(t) \tag{11} \]

with \( h_m(t) \) denoting the low-pass filter function. Stacking all \( N_T \) channel outputs yields

\[ \bar{z}_{n,i}(t) = [\bar{z}_{n,1,i}(t), \bar{z}_{n,2,i}(t), \ldots, \bar{z}_{n,N_T,i}(t)]^T \]

\[ = e^{-j2\pi \frac{md_t \sin \theta_t - \lambda b_n}{c}} z_i(t) \tag{12} \]

where \( z_i(t) = [e^{j2\pi f_0 t} z_i(t)] * h(t) \) with \( \odot \) and \( * \) denoting the conjugate and convolution operators, respectively, and \( h(t) = [h_1(t), h_2(t), \ldots, h_{N_T}(t)]^T \). If introducing the presence of \( \Upsilon \) undesired interferences and as well as an additive Gaussian noise, the FDA receive signal matrix may be obtained as

\[ R_{\text{FDA}}(t) \]

\[ = \zeta(r_t, \theta_t) \text{diag} [w \odot a_T(r_t, \theta_t)] s(t - \frac{2r_t}{c}) b_R^T(\theta_t) \]

\[ + \sum_{i=1}^{\Upsilon} \{z_i(t) b_R^T(\theta_t)\} + N(t) \tag{13a} \]

\[ = \zeta(r_t, \theta_t) \text{diag} \{ s(t - \frac{2r_t}{c}) \odot a_T(r_t, \theta_t) \} w b_R^T(\theta_t) \]

\[ + \sum_{i=1}^{\Upsilon} \{z_i(t) b_R^T(\theta_t)\} + N(t) \tag{13b} \]

where \( N(t) \) represents the noise matrix, assuming it is both spatially and temporally white. These two equivalent expressions for \( R_{\text{FDA}}(t) \) are used throughout this article, i.e., the transmit waveform expression shown in (13a) and
the transmit weight expression shown in (13b). For convenience, we stack the columns of \( R_{\text{FDA}}(t) \) to yield the vector

\[
\mathbf{r}_{\text{FDA}}(t) = \text{vec} \{ R_{\text{FDA}}(t) \}
\]

\[
= \zeta (r_i, \theta_i) \left[ \mathbf{b}_R (\theta_i) \otimes \text{diag} \{ \mathbf{w} \otimes \mathbf{a}_T (r_i, \theta_i) \} \right] s \left( t - \frac{2r_i}{c} \right)
\]

\[
+ \sum_{i=1}^{\Upsilon} \left\{ \mathbf{b}_R (\theta_i) \otimes \mathbf{z}_i (t) \right\} + \mathbf{n}(t)
\]

(14)

where \( \mathbf{n}(t) = \text{vec} \{ \mathbf{N}(t) \} \), and vec\{\cdot\} and \( \otimes \) denote the vectorization operator and the Kronecker product, respectively.

B. Interference Covariance Matrix and Interference Mitigation

For the interference signal \( z_{m'::i}(t) \), according to Parseval’s theorem

\[
z_{m'::i}(t) = \left[ e^{-j2\pi f t} \hat{z}_{m}(t) \right] \ast h_m(t)
\]

\[
= \int_{-\infty}^{\infty} Z_i (f + f_m) H_m(f) e^{j2\pi f t} df
\]

(15)

where \( H_m(f) \) and \( Z_i(f) \) are the Fourier transforms of \( h_m(t) \) and \( z_i(t) \), respectively. Thus

\[
\mathbb{E} \left\{ z_{m'::i}(t) z_{m'::i}^*(t) \right\} = \mathbb{E} \left\{ \int_{-\infty}^{\infty} Z_i (f + f_m) H_m(f) e^{j2\pi f t} df \int_{-\infty}^{\infty} Z_i^* (f + f_m') H_m^*(f') e^{-j2\pi f t} df' \right\}
\]

\[
= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R_{Z_i} (f, f') H_m (f - f_m') H_m^* (f - f_m) e^{j2\pi f t} df df'
\]

(16)

where \( \mathbb{E} \{ \cdot \} \) represents the statistical expectation. Without loss of generality, suppose that \( z_i(t) \) may be well modeled as a stationary circularly symmetric complex random process with zero mean. Following the results in [35]

\[
R_{Z_i} (f, f') = \mathbb{E} \left\{ Z_i(f) Z_i^* (f') \right\} = P_{Z_i}(f) \delta (f - f')
\]

(17)

Substituted into (16), this yields

\[
\mathbb{E} \left\{ z_{m'::i}(t) z_{m'::i}^*(t) \right\} = e^{j2\pi \left( f_{m'} - f_{m'}' \right) t} \int_{-\infty}^{\infty} P_{Z_i}(f) H_m (f - f_m) H_m^* (f - f_m) df.
\]

(18)

Since the multichannel mixing and low-pass filtering-based FDA receiver only works when the FDA transmit waveforms have nonoverlapping spectrum, i.e.,

\[
f_{m'} - f_{m'}' > \frac{B_{h_{m'}(t)} + B_{h_{m'}(t)}}{2}
\]

(19)

it holds that

\[
\mathbb{E} \left\{ z_{m'::i}(t) z_{m'::i}^*(t) \right\} = \begin{cases} P_{m'}, m' = m'' \\ 0, m' \neq m'' \end{cases}
\]

(20)

where \( P_{m'} = \int_{-\infty}^{\infty} P_{Z_i}(f) H_m (f + f_m)^2 df, \) which is a reasonable estimate for the interference power density (PSD) at \( f_{m'} \) when \( N_F \) is sufficiently large. From the derived results, it can be seen that the interference signal processed by the FDA receiver can be regarded as a spectral interference signal whose frequency is located at \( f_{m'} \). It is worth noting that this statistical property is different from conventional PA and MIMO radar systems. Finally, the interference covariance matrix of the interference output by the FDA receiver, \( \mathbf{Q} \), can be obtained as

\[
\mathbf{Q} = \mathbb{E} \left\{ \begin{bmatrix} \sum_{i=1}^{\Upsilon} \left\{ \mathbf{b}_R (\theta_i) \otimes \mathbf{z}_i (t) \right\} \end{bmatrix}^H \right\} = \sum_{i=1}^{\Upsilon} \left\{ \mathbf{Q}_i \right\}
\]

(21)

where \( ^H \) is the conjugate transpose operator, and

\[
\mathbf{Q}_i = \mathbf{P}_i \otimes \left\{ \mathbf{b}_R (\theta_i) \mathbf{b}_R^H (\theta_i) \right\}
\]

(22)

with \( \mathbf{P}_i = \mathbb{E} \left\{ \mathbf{z}_i (t) \mathbf{z}_i^H (t) \right\} \). Furthermore

\[
\mathbf{P}_i (l, k) = \begin{cases} P_k, l = k \\ 0, \text{ otherwise} \end{cases}
\]

(23)

resulting in \( \mathbf{P}_i = \sum_{k=1}^{N_T} P_k \mathbf{e}_k \mathbf{e}_k^T \), where \( \mathbf{e}_k \) indicates a vector with the \( k \)-th element being 1 and 0 otherwise. Considering (15), the FDA snapshot \( \mathbf{r}_{\text{FDA}} (l) \) after analog-to-digital conversion (ADC) can therefore be expressed as

\[
\mathbf{r}_{\text{FDA}} (l) = \zeta (r_i, \theta_i) \left[ \mathbf{b}_R (\theta_i) \otimes \text{diag} \{ \mathbf{w} \otimes \mathbf{a}_T (r_i, \theta_i) \} \right] s(l)
\]

\[
+ \sum_{i=1}^{\Upsilon} \left\{ \mathbf{b}_R (\theta_i) \otimes \mathbf{z}_i (t) \right\} + \mathbf{n}(l)
\]

with \( i = 1, 2, \ldots, L \), where \( L \) denotes the number of discrete time samples. Further, stacking all \( L \) outputs yields

\[
\hat{\mathbf{r}}_{\text{FDA}} = \text{vec} \left\{ \left[ \mathbf{r}_{\text{FDA}} (1), \mathbf{r}_{\text{FDA}} (2), \ldots, \mathbf{r}_{\text{FDA}} (L) \right] \right\}
\]

\[
= \zeta (r_i, \theta_i) \mathbf{A} (r_i, \theta_i; \mathbf{w}) \mathbf{s} + \sum_{i=1}^{\Upsilon} \left\{ \text{vec} \left\{ \mathbf{I} (\theta_i) \right\} \right\} + \mathbf{n}
\]

\[
= \zeta (r_i, \theta_i) \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) \mathbf{w} + \sum_{i=1}^{\Upsilon} \left\{ \text{vec} \left\{ \mathbf{I} (\theta_i) \right\} \right\} + \mathbf{n}
\]

(24)

where

\[
\mathbf{I} (\theta_i) = \left[ \mathbf{b}_R (\theta_i) \otimes \mathbf{z}_i (1), \ldots, \mathbf{b}_R (\theta_i) \otimes \mathbf{z}_i (L) \right]
\]

\[
\mathbf{A} (r_i, \theta_i; \mathbf{w}) = \mathbf{I}_L \otimes \left[ \mathbf{b}_R (\theta_i) \otimes \text{diag} \{ \mathbf{w} \otimes \mathbf{a}_T (r_i, \theta_i) \} \right]
\]

\[
\tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) = \begin{bmatrix} \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}(1)) \\ \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}(2)) \\ \vdots \\ \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}(L)) \end{bmatrix}
\]

(25a)

(25b)

(25c)

\[
\mathbf{n} = \text{vec} \left\{ \left[ \mathbf{n} (1), \mathbf{n} (1), \ldots, \mathbf{n} (1) \right] \right\}
\]

\[
\mathbf{s} = \text{vec} \left\{ \left[ \mathbf{s} (1), \mathbf{s} (2), \ldots, \mathbf{s} (L) \right] \right\}
\]

(25d)

(25e)

(25f)

where \( \mathbf{A} (r_i, \theta_i; \mathbf{w}) \in \mathbb{C}^{N_T N_F L \times N_t L} \) and \( \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) \in \mathbb{C}^{N_T N_F L \times N_t L} \), with \( \mathbb{C}^{N \times N} \) being the set of \( N \times N \) complex-valued matrices. Employing a receive weight
vector \( \mathbf{v} \in \mathbb{C}^{N_T N_H L \times 1} \) to synthesize the multichannel outputs yields

\[
\mathbf{x}_{\text{FDA}} = \zeta(r_i, \theta_i) \mathbf{v}^H \mathbf{A} (r_i, \theta_i; \mathbf{w}) \mathbf{s} + \sum_{i=1}^{Y} \{ \mathbf{v}^H \text{vec} \{ \mathbf{I}(\theta_i) \} \} + \mathbf{v}^H \mathbf{n} = \zeta(r_i, \theta_i) \mathbf{v}^H \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) \mathbf{w} + \sum_{i=1}^{Y} \{ \mathbf{v}^H \text{vec} \{ \mathbf{I}(\theta_i) \} \} + \mathbf{v}^H \mathbf{n}. \quad (26)
\]

Consequently, the output SINR \( \Pi_{\text{FDA}}(\mathbf{s}, \mathbf{w}, \mathbf{v}) \) can be written as

\[
\Pi_{\text{FDA}}(\mathbf{s}, \mathbf{w}, \mathbf{v}) = \frac{\mathop{\mathbb{E}} \left\{ \left[ \zeta(r_i, \theta_i) \mathbf{v}^H \mathbf{A} (r_i, \theta_i; \mathbf{w}) \mathbf{s} \right]^2 \right\}}{
\mathop{\mathbb{E}} \left\{ \sum_{i=1}^{Y} \{ \mathbf{v}^H \text{vec} \{ \mathbf{I}(\theta_i) \} \} + \mathop{\mathbb{E}} \left\{ \mathbf{v}^H \mathbf{n} \right\} \right\} = \frac{\text{SNR} \cdot \mathbf{v}^H \tilde{\mathbf{Q}}_{i+n} \mathbf{v}}{
\mathbf{v}^H \mathbf{Q}_{i+n} \mathbf{v}} \quad (27)
\]

where

\[
\tilde{\mathbf{Q}}_{i+n} = \frac{1}{\sigma^2} \sum_{i=1}^{Y} \left\{ \mathop{\mathbb{E}} \left\{ (\text{vec} \{ \mathbf{I}(\theta_i) \}) (\text{vec} \{ \mathbf{I}(\theta_i) \})^H \right\} \right\} + \mathbf{I}_{N_T N_H L} = \frac{1}{\sigma^2} \sum_{i=1}^{Y} \left\{ (\mathbf{I}_{L \times 1} \mathbf{1}^T_{L \times 1}) \otimes \mathbf{Q}_i \right\} + \mathbf{I}_{N_T N_H L} \quad (28)
\]

and where \( \text{SNR} = \frac{\mathbb{E} \{ |\mathbf{c}(r_i, \theta_i)|^2 \}}{\mathbb{E} \{ |\mathbf{n}|^2 \}} \), with \( \sigma^2 \) denoting the noise power. To suppress the present interferences, the adaptive minimum variance distortionless response (MVDR) receive weight vector \( \mathbf{v}_{\text{opt}} \) is employed, with

\[
\mathbf{v}_{\text{opt}} = \frac{\tilde{\mathbf{Q}}_{i+n}^{-1} \mathbf{A} (r_i, \theta_i; \mathbf{w}) \mathbf{s}}{\mathbf{s}^H \mathbf{A}^H (r_i, \theta_i; \mathbf{w}) \tilde{\mathbf{Q}}_{i+n}^{-1} \mathbf{A} (r_i, \theta_i; \mathbf{w}) \mathbf{s}} = \frac{\tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) \mathbf{w}}{\mathbf{w}^H \tilde{\mathbf{A}}^H (r_i, \theta_i; \mathbf{s}) \tilde{\mathbf{Q}}_{i+n}^{-1} \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) \mathbf{w}} \quad (29)
\]

The corresponding output SINR may then be expressed as

\[
\Pi_{\text{FDA}}(\mathbf{s}, \mathbf{w}) = \mathbf{s}^H \tilde{\Psi}(\mathbf{w}) \mathbf{s} = \mathbf{w}^H \tilde{\Psi}(\mathbf{s}) \mathbf{w} \quad (30)
\]

where

\[
\tilde{\Psi}(\mathbf{w}) = \mathbf{A}^H (r_i, \theta_i; \mathbf{w}) \tilde{\mathbf{Q}}_{i+n}^{-1} \mathbf{A} (r_i, \theta_i; \mathbf{w}) \quad (31a)
\]

\[
\tilde{\Psi}(\mathbf{s}) = \tilde{\mathbf{A}}^H (r_i, \theta_i; \mathbf{s}) \tilde{\mathbf{Q}}_{i+n}^{-1} \tilde{\mathbf{A}} (r_i, \theta_i; \mathbf{s}) \quad (31b)
\]

It is worth noting that the output SINR is a function of the transmit waveforms and the weights. We proceed to jointly designing the FDA transmit waveforms and weights with the aim of improving the output SINR, as well as limiting the waveform energy over the overlaid frequency bands.

III. PROBLEM FORMULATION

In this section, we derive the FDA spectrum compatibility constraint and formulate the proposed method for constructing the transmit waveforms and weights accounting for various practical requirements.

A. Constraints

1) Energy and Bandwidth Constraints: Both energy and bandwidth constraints must be imposed on the transmit waveforms to allow these to be separated by the receiver. According to the results in [20], the energy constraint can be expressed as

\[
s_T^H \Sigma_m s_T = \frac{1}{N_T} \quad (32)
\]

for \( m = 1, 2, \ldots, N_T \), where \( \Sigma_m \in \mathbb{C}^{N_T \times N_T} \) represents a block diagonal matrix, with its \( m \)-th diagonal block being an identity matrix \( \mathbf{I}_L \), and where \( s_T = \mathbf{T}(N_T \mathbf{L}) \mathbf{w} \), with \( \mathbf{T}(N_T \mathbf{L}) = \sum_{i=1}^{L} (\mathbf{e}_i^T \otimes \mathbf{I}_N \otimes \mathbf{e}_j) \) being the commutation matrix. The bandwidth constraint has the form

\[
s_T^H B_m s_T \geq \frac{\gamma_m}{N_T} \quad (33)
\]

for \( m = 1, 2, \ldots, N_T \), where \( \gamma_m \in (0, 1) \) is a user-defined scalar that defines the tolerance for in-band energy, a typical choice being \( \gamma_m = 0.91 \) [20], whereas \( B_m \) represents a block diagonal matrix with its \( m \)-th diagonal block being

\[
\int_0^{f_{m,lp}} \mathbf{e}_f^H \mathbf{f} \mathbf{d} \mathbf{f} \in \mathbb{C}^{N_T \times N_T} = \left\{ \begin{array}{ll}
f_{m,lp} \exp(j2\pi f_{m,lp} \nu - \nu^2 / 2 \sigma^2) & \text{if } p = q \\int_0^{f_{m,lp}} \mathbf{e}_f^H \mathbf{f} \mathbf{d} \mathbf{f} \end{array} \right. \quad (34)
\]

where \( f_{m,lp} \) denotes the normalized cut-off frequency of \( m \)-th low-pass filter and \( \mathbf{e}_f = [\exp(j2\pi f_1 \nu), \exp(j2\pi f_2 \nu), \ldots, \exp(j2\pi f_L \nu)]^T \).

2) Similarity Constraint: Implementing similarity constraint allows one to indirectly control desirable features of the transmit waveforms and weights. For the transmit waveforms,

\[
\| \mathbf{s}_T - \mathbf{s}_{\text{Ref}} \|^2 \leq \varepsilon^2 \quad (35)
\]

where \( \| \cdot \|^2 \) represents the square of the Euclidean norm, \( \mathbf{s}_{\text{Ref}} \) the reference waveform, and \( \varepsilon^2 \) a user-defined parameter to control the extent of the similarity. The similarity constraint for the transmit weights is similarly

\[
\| \mathbf{w}_T - \mathbf{w}_{\text{Ref}} \|^2 \leq \mu^2 \quad (36)
\]

where \( \mathbf{w}_{\text{Ref}} \) and \( \mu \) denote the reference weight and similarity level, respectively.

3) Spectrally Compatible Constraint: Aside from the aforementioned requirements, the FDA needs to control the energy radiated on the shared frequency bands, allowing for more efficient use of the spectrum for the coexistence of different services. Although it is a multiwaveform emission mechanism, the energy distribution of the FDA is actually direction-independent due to its nonoverlapping waveform spectrum.

An example of an FDA spectrum is shown in Fig. 2, with the used parameters being listed in Table I. As can be seen the allocated frequency bands for each FDA transmit waveform are different, which would not be the case for a MIMO or a PA radar, indicating that the representation of the energy spectral density (ESD)-based spectral compatibility constraint should be adjusted [32]. Supposing that the
expressed as (see Appendix)

$$E_b = w^H \tilde{I}_b(S) w = s_f^H \mathbf{H}_b(w) s_T \leq \eta_b$$  \hspace{1cm} (38)$$

where $\eta_b$ represents the acceptable level for the $b$th compatible systems, and $\tilde{I}_b(S)$ and $\mathbf{H}_b(w)$ are given in (55a) and (55e), respectively.

### B. Optimization Problem

Based on the aforementioned discussions, aiming to maximize the output SINR and to control the energy over the shared frequency bands, the joint design of transmit weights and waveforms for FDA can be formulated as

$$\max_{s_f, w} \quad \Pi_{\text{FDA}}(s, w)$$

subject to

$$\|s_T - s_{\text{Ref}}\|_2^2 \leq \varepsilon^2$$
$$s_f^H \sum_m s_T = \frac{1}{N_T}$$
$$s_f^H \mathbf{B}_m s_T \geq \frac{\gamma_m}{N_T}, \quad \gamma_m \in (0, 1]$$
$$\|w - w_{\text{Ref}}\|_2^2 \leq \mu^2$$
$$\|w\|_2^2 = N_T$$
$$E_b = w^H \tilde{I}_b(S) w = s_f^H \mathbf{H}_b(w) s_T \leq \eta_b$$  \hspace{1cm} (39)$$

for $m = 1, 2, \ldots, N_T$, where $\Pi_{\text{FDA}}(s, w)$ denotes the output SINR given in (32). It should be noted that optimizing the transmit waveforms with fixed transmit weights will achieve the FDA spectrally compatible design while still maximizing the output SINR. However, as compared to the joint optimization, there are obvious disadvantages in controlling the transmission energy in the overlaid frequency bands and maintaining a constant energy in the transmit elements, which will also be demonstrated in the following simulations.

Generally speaking, there is no closed-form solution to $P_1$, as the energy and bandwidth constraints on the waveforms are nonconvex, as are the energy constraint on the weights and the objective function. In the following, we will examine how $P_1$ may be reformulated to allow for an approximative solution using the proposed iterative algorithm.

### IV. PROPOSED ALGORITHM

In order to relax $P_1$, we separate the maximization, such that this is done alternatingly with respect to $w$ and $s_T$, keeping the other fixed. Each of the resulting problems may then be further relaxed. In the following, we will examine both of these problems separately.

#### A. Optimize $w$ With Fixed $s_T$

For a fixed transmit waveform vector $s_T$, the problem $P_1$ can be optimized with respect to $w$ as

$$\max_w \quad \mathcal{W}(s, w)$$

subject to

$$\|w\|_2^2 = N_T$$
$$\|w - w_{\text{Ref}}\|_2^2 \leq \mu^2$$
$$w^H \tilde{I}_b(S) w \leq \eta_b$$  \hspace{1cm} (40)$$

where $\mathcal{W}(s, w)$ is the objective function.

#### B. Optimization Problem

For a fixed transmit waveform vector $s_T$, the problem $P_1$ can be optimized with respect to $w$ as

$$\max_w \quad \Pi_{\text{FDA}}(s, w)$$

subject to

$$\|s_T - s_{\text{Ref}}\|_2^2 \leq \varepsilon^2$$
$$s_f^H \sum_m s_T = \frac{1}{N_T}$$
$$s_f^H \mathbf{B}_m s_T \geq \frac{\gamma_m}{N_T}, \quad \gamma_m \in (0, 1]$$
$$\|w - w_{\text{Ref}}\|_2^2 \leq \mu^2$$
$$\|w\|_2^2 = N_T$$
$$E_b = w^H \tilde{I}_b(S) w = s_f^H \mathbf{H}_b(w) s_T \leq \eta_b$$  \hspace{1cm} (39)$$

for $m = 1, 2, \ldots, N_T$, where $\Pi_{\text{FDA}}(s, w)$ denotes the output SINR given in (32). It should be noted that optimizing the transmit waveforms with fixed transmit weights will achieve the FDA spectrally compatible design while still maximizing the output SINR. However, as compared to the joint optimization, there are obvious disadvantages in controlling the transmission energy in the overlaid frequency bands and maintaining a constant energy in the transmit elements, which will also be demonstrated in the following simulations.

Generally speaking, there is no closed-form solution to $P_1$, as the energy and bandwidth constraints on the waveforms are nonconvex, as are the energy constraint on the weights and the objective function. In the following, we will examine how $P_1$ may be reformulated to allow for an approximative solution using the proposed iterative algorithm.

### IV. PROPOSED ALGORITHM

In order to relax $P_1$, we separate the maximization, such that this is done alternatingly with respect to $w$ and $s_T$, keeping the other fixed. Each of the resulting problems may then be further relaxed. In the following, we will examine both of these problems separately.

#### A. Optimize $w$ With Fixed $s_T$

For a fixed transmit waveform vector $s_T$, the problem $P_1$ can be optimized with respect to $w$ as

$$\max_w \quad \mathcal{W}(s, w)$$

subject to

$$\|w\|_2^2 = N_T$$
$$\|w - w_{\text{Ref}}\|_2^2 \leq \mu^2$$
$$w^H \tilde{I}_b(S) w \leq \eta_b$$  \hspace{1cm} (40)$$

where $\mathcal{W}(s, w)$ is the objective function.
Following the strategies in [31] and [36], this may be reformulated as
\[
\begin{align*}
P_3: \quad & \max_w \quad w^H \tilde{\Psi}(s) w \\
& \text{s.t.} \quad \|w_T\|_2^2 = N_T \\
& \quad w^H (I_{N_T} - w_{\text{Ref}} w_{\text{Ref}}^H) w \leq \mu^2 \\
& \quad w^H I_0(s) w \leq \eta_b.
\end{align*}
\] (41)

Introducing a new variable \( W = w w^H \) yields
\[
\begin{align*}
P_4: \quad & \max_W \quad \text{Tr} \{ \tilde{\Psi}(s) W \} \\
& \text{s.t.} \quad \text{Tr} \{ W \} = N_T \\
& \quad \text{Tr} \{ (I_{N_T} - w_{\text{Ref}} w_{\text{Ref}}^H) W \} \leq \mu^2 \\
& \quad \text{Tr} \{ I_0(s) W \} \leq \eta_b \\
& \quad W \succeq 0, \text{ Rank} \{ W \} = 1
\end{align*}
\] (42)

where \( W \succeq 0 \) means that \( W \) is positive semidefinite, and with \( \text{Rank} \{ W \} \) and \( \text{Tr} \{ W \} \) denoting the rank and trace of \( W \), respectively. Reformulating \( P_2 \) to \( P_4 \) shows that the only nonconvex part of the problem is the rank constraint. A natural relaxation is thus obtained by dropping this constraint, yielding the semidefinite program (SDP)
\[
\begin{align*}
P_5: \quad & \max_W \quad \text{Tr} \{ \tilde{\Psi}(s) W \} \\
& \text{s.t.} \quad \text{Tr} \{ W \} = N_T \\
& \quad \text{Tr} \{ (I_{N_T} - w_{\text{Ref}} w_{\text{Ref}}^H) W \} \leq \mu^2 \\
& \quad \text{Tr} \{ I_0(s) W \} \leq \eta_b \\
& \quad W \succeq 0.
\end{align*}
\] (43)

As the resulting problem is convex, a solution may be obtained in polynomial time using standard optimization tools, such as the convex optimization toolbox CVX [37]. If the rank of the globally optimal solution \( \mathbf{W}^* \) obtained for the problem \( P_3 \) is one, then \( \mathbf{W}^* = w^* w^{*H} \), with \( w^* \) being the globally optimal solution to the problem \( P_2 \). Otherwise, a fundamental issue that one must address is how to convert the solution \( \mathbf{W}^* \) into a feasible solution for the problem \( P_2 \). To this end, a randomization method will here be used, as described in Algorithm 1. In essence, the key concept is to construct random vectors that share \( \mathbf{W}^* \) as a covariance matrix. Each random vector is then rescaled appropriately to ensure feasibility.

B. Optimize \( s_T \) With Fixed \( w \)

Proceeding, for a fixed transmit weight vector, \( w \), the optimal transmit waveform vector \( s_T \) can be obtained by rewriting \( P_1 \) as
\[
\begin{align*}
P_6: \quad & \max_{s_T} \quad s_T^H T(N_T, L) \Psi(w) T(L, N_T) s_T \\
& \text{s.t.} \quad \|s_T - s_{\text{Ref}}\|_2 \leq \varepsilon^2 \\
& \quad s_T^H \Sigma_m s_T = \frac{1}{N_T} \\
& \quad s_T^H B_m s_T \geq \frac{\gamma_m}{N_T}, \gamma_m \in (0, 1] \\
& \quad s_T^H H_b(w) s_T \leq \eta_b
\end{align*}
\] (44)

The resulting maximization is convex and the globally optimal solution \( \mathbf{S}^* \) of problem \( P_2 \) may thus be obtained using standard optimization tools. As aforementioned, a similar randomization technique may be employed to find an approximate solution to problem \( P_6 \). The proposed iterative algorithm is summarized in Algorithm 2.

V. NUMERICAL SIMULATIONS

In this section, various numerical simulations are provided to assess the performance of the proposed FDA design scheme. We first consider the problem of the FDA spectrally compatible design with different design schemes, namely, designing only the transmit waveforms, designing only transmit weights, and joint design of the transmit waveforms and weights. We compare the output SINR
and achieved ESD for these approaches over the overlaid frequency bands. Next, we examine the ACF of the designed waveforms. Finally, the behavior of the FDA receive beam-pattern is illustrated. In all simulations, it is assumed that the target of interest is located at \((r_t, \theta_t) = (15 \text{ km}, 40^\circ)\) with the input SNR being 0 dB. Furthermore, the signal is assumed to be corrupted by these interference sources, whose details are listed in Table II. We also assume \(\Omega = 2\) compliant systems operating on the normalized frequency bands \(F_1 = (0.073, 0.200)\) and \(F_2 = (0.556, 0.884)\) that overlap with FDA frequency band. The assumed system parameters are listed in Table III.

### A. FDA Spectrally Compatible Design

1) Designing Only the Transmit Waveforms: Usually, the performance compatibility is evaluated by the amount of energy produced over the shared frequency bands [29], [32], [36]. For PA and MIMO systems, the compatibility requirements can be met by designing only the transmit waveform. However, for FDA with its nonoverlapping spectra, it is difficult to achieve the same requirement under a constant energy constraint. In particular, for the second shared system, the normalized frequency band \(F_2\) covers the spectrum of the transmit waveform 4, 5, and 6, resulting in an achievable energy greater than \(1/N_T\), which is not applicable in practice. Here, we set the energy attenuation of the first frequency band \(F_1\) to 10 dB, that is, \(\eta_1 = 2/N_T \cdot \frac{1}{10} = \frac{1}{50}\). At the same time, the allowable energy in the second band \(F_2\) is set to \(\eta_2 = \frac{2}{N_T} \cdot \frac{1}{10} + \frac{1}{N_T} = \frac{101}{300}\). Fig. 3 shows the ESD of the synthesized waveforms with a similarity value \(\epsilon^2 = 6\). Therein, the shared frequency bands are shaded in heavy gray, and the green straight lines mark the frequency bands of the different transmit waveforms. The result reveals that, for FDA, the scheme is capable of limiting the radiated energy on the shared bands, as required by the imposed spectrally compatible constraints. But the achieved compatibility \(\eta_2 = \frac{101}{300}\) does not perform well on the second frequency band due to the constant energy limitation.

2) Designing Only the Transmit Weights: Proceeding, the energy attenuation on \(F_1 = (0.073, 0.200)\) and \(F_2 = \)
obtained ESD for different transmit waveform similarity values with a fixed transmit weight similarity level $\mu^2 = 15$.

![Fig. 6](image)

**Fig. 6.** Obtained ESD for different transmit waveform similarity values with a fixed transmit weight similarity level $\mu^2 = 15$.

| Performance Comparison |
|-------------------------|
| **Type**                  | **Spectral compatibility** | **Output SINR [dB]** |
| Transmit waveform design  | $(\eta_1 = \frac{3}{10}, \eta_2 = \frac{16}{10^3}) \leftrightarrow (10 \, dB, 1.72 \, dB)$ | 5.99 |
| Transmit weight design    | $(\eta_1 = \frac{2}{10}, \eta_2 = \frac{1}{10^2}) \leftrightarrow (10 \, dB, 20 \, dB)$ | 2.36 |
| Joint design of transmit waveforms and weights | $(\eta_1 = \frac{1}{30}, \eta_2 = \frac{1}{200}) \leftrightarrow (10 \, dB, 20 \, dB)$ | 6.02 |

![Table IV](image)

**Table IV**—Performance Comparison

(0.556, 0.884) are set to $\eta_1 = \frac{1}{10}$, $\frac{1}{10}$, and $\eta_2 = \frac{3}{10}$, $\frac{1}{20}$, respectively. In Fig. 4, the spectral distributions after designing the transmit weights with a similarity value $\mu^2 = 15$ is shown. Benefiting from the nonoverlapping spectrum, with flexible transmit power allocation, FDA can properly control the transmit power at the shared frequencies without adjusting the waveform. Fig. 5 compares the designed weights and the reference weights. It can be seen that the designed weights are well matched to the frequency bands of the compatible systems. However, in practice, the severely nonconstant energy exhibited will reduce the power efficiency of the amplifier and broaden the transmitted spectrum. At the same time, the SINR performance, as is further shown below, is greatly degraded.

3) Jointly Designing the Transmit Waveforms and Weights: With the same energy attenuation settings as in Fig. 4, Fig. 6 illustrates the spectral behavior with a fixed transmit weight similarity level $\mu^2 = 15$ but with different transmit waveform similarity values $\epsilon^2 = 1, 3, \text{and } 6$. As expected, by jointly adjusting both the transmit waveforms and weights, a better energy distribution is achievable, as well as less variation in energy allocated to each transmit element, as shown in Fig. 7.

![Fig. 7](image)

**Fig. 7.** Comparison between the reference weights and the designed weights.

![Fig. 8](image)

**Fig. 8.** Output SINR versus the iteration number for different $\epsilon^2$ with $\mu^2 = 15$.

Fig. 8 shows the output SINR for different transmit waveform similarity levels. It is worth noting that the
similarity value has a greater impact on the output SINR than the number of iterations. This result may arise due to, for the proposed algorithm, the transmit weights and waveforms play similar roles, making it is difficult to improve the output SINR by increasing the number of iterations.

Table IV summarizes the output SINRs for the different designs. It may be noted that the scheme that only optimizes the transmit waveforms achieves the worst spectral compatibility. Meanwhile, the output SINR of the transmit weight design shows the worst performance due to the transmit degrees of freedom (DoF) of the FDA is being sacrificed in order to fulfill the spectral compatibility constraint. Compared with the weight design, the joint design can obtain the same spectral compatibility, and compared with the waveform design, the joint design can obtain a slightly larger output SINR.

B. ACF Property

The ACF profiles of the waveforms with different similarity values obtained by the joint design are compared with that of the reference LFM in Fig. 9. The results show that the peak sidelobe level (PSL), or range resolution, gets better as the required similarity level decreases, again because of the reduced remaining DoF available to design the waveforms. This fact suggests a compromise between the SINR and the resolution or PSL characteristics. It can also be observed that the fluctuations of the designed waveform becomes more pronounced as the similarity value increases. The reason is that for the larger similarity values, the more DoFs are available in the optimization problem. It should be noted that the waveform of the 5th transmitter has a higher PSL and greater range resolution than the other waveforms. This may be because the second shared band completely occupies its spectrum.
C. Receive Beampattern

As can be seen in Table II, interference 2 enters through the receiver mainlobe pointing to 40° and others enter through the sidelobe. Fig. 10 shows the power spectrum of the interferences distributed in the frequency-angle domain, \( P(f_m, \theta) \), calculated by

\[
P(f_m, \theta) = [e_m \otimes b_R(\theta)]^H \Theta_{v+}^{-1} [e_m \otimes b_R(\theta)].
\]  

The resulting receive beampattern \( P_R(\theta) \) after applying the joint design is displayed in Fig. 11. The beampattern is computed as

\[
P_R(\theta) = \left| v^H_{opt} \left[ L \times b_R(\theta) \otimes \text{diag} \left[ w^* \otimes a_T(r, \theta) \right] \right] \right| s^2. \]

Although the beampattern is irregular, it can be clearly noticed that nulls are formed at the location of the interference entering through the side lobes, and an energy peak appears at the target location. Fig. 12 compares the output SINR of FDA and MIMO, where orthogonal LFM is chosen as the transmit waveform for FDA. The high output SINR of FDA demonstrates the effect of interference suppression on the mainlobe. This advantage is attributed to the additional controllable DoF of FDA in the range dimension.

VI. CONCLUSION

This article has addressed the joint design of transmit waveforms and weights for FDA in a spectrally crowded environment. Employing the multichannel mixing and lowpass filtering-based FDA receiver, the transmit waveforms are extracted from the signals at the receiver output end. Then, considering that the interference exhibits different statistical properties after passing through the FDA receiver, the interference-plus-noise covariance matrix is derived and the multichannel outputs are synthesized by the MVDR receive weights to suppress the interferences. In order to maximize the resulting waveform-weight-dependent output SINR and to control the energy distribution over the shared frequency bands, an optimization problem involving a set of energy, bandwidth, and similarity constraints is, therefore, formulated. As the resulting problem is nonconvex, we develop an iterative SDR algorithm to solve it. Via simulation, we show that the joint design can not only achieve the expected spectral compatibility, but also obtain the optimal output SINR. More importantly, the proposed design does not affect the efficient utilization of the controllable DoF of FDA in the range dimension.

APPENDIX

To prove (37), we divide the constraint into three separate cases on the basis of the frequency band positions occupied by any compatible systems.

1) For the case where \( f^b_i \) and \( f^b_h \) are in the same frequency band as the transmit waveform, corresponding to the \( f^b_i \) and \( f^b_h \) in Fig. 2, i.e., \( p_b = p_b = 0 \), the transmitted energy \( E_b \) of FDA in the \( b \)th frequency band can be calculated as

\[
E_b = \int_{f^b_i - p_b, \Delta f}^{f^b_i - p_b, \Delta f} \left| w_{p_b+1} \sum_{l=1}^{L} S(p_b, l) e^{i2\pi fl} \right|^2 df
\]

where \( S = [s(1), s(2), \ldots, s(L)] \) is the transmit waveform matrix, with \( S(p_b, l) \) indicating its \( p_b \)th row and \( l \)th column element.

2) Similarly, for the case \( p_b = p_b = 1 \), corresponding to the \( f^b_i \) and \( f^b_h \) in Fig. 2, \( E_b \) is given as

\[
E_b = \int_{f^b_i - p_b, \Delta f}^{f^b_i - p_b, \Delta f} \left| w_{p_b+1} \sum_{l=1}^{L} S(p_b, l) e^{i2\pi fl} \right|^2 df + \int_{0}^{f^b_i - p_b, \Delta f} \left| w_{p_b+1} \sum_{l=1}^{L} S(p_b, l) e^{i2\pi fl} \right|^2 df.
\]

3) Finally, in all other cases, \( E_b \) is computed as

\[
E_b = \int_{f^b_i - p_b, \Delta f}^{f^b_i - p_b, \Delta f} \left| w_{p_b+1} \sum_{l=1}^{L} S(p_b, l) e^{i2\pi fl} \right|^2 df + \int_{0}^{f^b_i - p_b, \Delta f} \left| w_{p_b+1} \sum_{l=1}^{L} S(p_b, l) e^{i2\pi fl} \right|^2 df + \frac{1}{N_T} \sum_{m=p_b+2}^{p_b} |w_m|^2.
\]

It is worth noting that (48)–(50) can be expressed in a unified and concise form; taking the third case as an example, (50) may be rewritten as

\[
E_b = w_{p_b+1}^2 S(p_b) K_{f_j} S^H(p_b) + w_{p_b+1}^2 S(p_b) K_{f_j} S^H(p_b) + \frac{1}{N_T} \sum_{m=p_b+2}^{p_b} |w_m|^2 = w^H \sum_{m=p_b+2}^{p_b} (S) w = s_{f_j}^H \sum_{m=p_b+2}^{p_b} (S) w
\]
where

$$
\mathbf{I}_{p_b,\tilde{p}_b} (p, q; S) = \begin{cases} 
S(p_b) K_f p^{H} (p_b), & p = q = p_b + 1 \\
S(p_b) K_f p^{H} (p_b), & p = q = \tilde{p}_b + 1 \\
\frac{1}{N'}, & p = q = m, m = p_b + 2, \ldots, \tilde{p}_b \\
0, & \text{otherwise}
\end{cases}
$$

(52a)

and

$$
\mathbf{H}_{p_b,\tilde{p}_b} (p, q; w) = \begin{cases} 
\frac{w_{p_b+1}^2}{K_f} p, & p = q = p_b + 1 \\
\frac{w_{\tilde{p}_b+1}^2}{K_f} p, & p = q = \tilde{p}_b + 1 \\
\frac{w_{p_b+1}^2}{K_f} I_{p, q} = \frac{w_{\tilde{p}_b+1}^2}{K_f} I_{p, q} = \frac{w_{p_b+1}^2}{K_f} 0_L, & p = q \in \{p_b + 2, \ldots, \tilde{p}_b\}
\end{cases}
$$

(52b)

Therefore, the FDA spectrally compatibly constraint can be expressed as

$$
E_b = w^H \mathbf{I}_b (S) w = s_b^H \mathbf{H}_b (w) s_T \leq \eta_b
$$

(54)

which completes the proof.

REFERENCES

[1] P. Antonik, M. Wicks, H. Griffiths, and C. Baker, “Frequency diverse array radars,” in Proc. IEEE Conf. Radar, 2006, pp. 215–217.

[2] W.-Q. Wang, H. C. So, and A. Farina, “An overview on time/frequency modulated array processing,” IEEE J. Sel. Topics Signal Process., vol. 11, no. 2, pp. 228–246, Mar. 2017.

[3] P. Antonik, M. C. Wicks, H. D. Griffiths, and C. J. Baker, “Range-dependent beamforming using element level waveform diversity,” in Proc. Int. Waveform Diversity Des. Conf., 2006, pp. 1–6.

[4] W.-Q. Wang, “Phased-MIMO radar with frequency diversity for range-dependent beamforming,” IEEE Sensors J., vol. 13, no. 4, pp. 1320–1328, Apr. 2013.

[5] Y. Xu, X. Shi, W. Li, J. Xu, and L. Huang, “Low-sidelobe range-angle beamforming with FDA using multiple parameter optimization,” IEEE Trans. Aerosp. Electron. Syst., vol. 55, no. 5, pp. 2214–2225, Oct. 2019.

[6] W.-Q. Wang, “Range-angle dependent transmit beampattern synthesis for linear frequency diverse arrays,” IEEE Trans. Antennas Propag., vol. 61, no. 8, pp. 4073–4081, Aug. 2013.

[7] J. Xu, S. Zhu, and G. Liao, “Range ambiguous clutter suppression for airborne FDA-STATAP radar,” IEEE J. Sel. Topics Signal Process., vol. 9, no. 8, pp. 1620–1631, Aug. 2015.

[8] L. Lan, J. Xu, G. Liao, Y. Zhang, F. Fionanelli, and H. C. So, “Suppression of mainbeam deceptive jammer with FDA-MIMO radar,” IEEE Trans. Veh. Technol., vol. 69, no. 10, pp. 11584–11598, Oct. 2020.

[9] L. Wang, W.-Q. Wang, H. Guan, and S. Zhang, “LPI property of FDA transmitted signal,” IEEE Trans. Aerosp. Electron. Syst., vol. 57, no. 6, pp. 3905–3915, Jun. 2021.

[10] W.-Q. Wang and H. Shao, “Range-angle localization of targets by a double-pulse frequency diverse array radar,” IEEE J. Sel. Topics Signal Process., vol. 8, no. 1, pp. 106–114, Jan. 2014.

[11] W.-Q. Wang and H. C. So, “Transmit subaperturing for range and angle estimation in frequency diverse array radar,” IEEE Trans. Signal Process., vol. 62, no. 8, pp. 2000–2011, Aug. 2014.

[12] W.-Q. Wang, “Adaptive RF stealth beamforming for frequency diverse array radar,” in Proc. 23rd Eur. Signal Process. Conf., 2015, pp. 1158–1161.

[13] W.-Q. Wang, “Moving-target tracking by cognitive RF stealth radar using frequency diverse array antenna,” IEEE Trans. Geosci. Remote Sens., vol. 54, no. 7, pp. 3764–3773, Jul. 2016.

[14] C. Wang, J. Xu, G. Liao, X. Xu, and Y. Zhang, “A range ambiguity resolution approach for high-resolution and wide-swath SAR imaging using frequency diverse array,” IEEE J. Sel. Topics Signal Process., vol. 11, no. 2, pp. 336–346, Feb. 2017.

[15] R. Gui, W.-Q. Wang, A. Farina, and H. C. So, “FDA radar with doppler-spreading consideration: Mainlobe clutter suppression for blind-doppler target detection,” Signal Process., vol. 179, 2021, Art. no. 107773.

[16] R. Gui, Z. Zheng, and W.-Q. Wang, “Cognitive FDA radar transmit power allocation for target tracking in spectrally dense scenario,” Signal Process., vol. 183, 2021, Art. no. 108606.

[17] M. Seccmen, S. Demir, A. Hizal, and T. Eker, “Frequency diverse array antenna with periodic time modulated pattern in range and angle,” in Proc. IEEE Radar Conf., 2007, pp. 427–430.

[18] W. Khan and I. M. Qureshi, “Frequency diverse array radar with time-dependent frequency offset,” IEEE Antennas Wireless Propag. Lett., vol. 13, pp. 758–761, 2014.
Wen-Qin Wang (Senior Member, IEEE) received the B.E. degree in electrical engineering from Shandong University, Shandong, China, in 2002, and the M.E. and Ph.D. degrees in information and communication engineering from the University of Electronic Science and Technology of China (UESTC), Chengdu, China, in 2005 and 2010, respectively. From March 2005 to 2007, he was with the National Key Laboratory of Microwave Imaging Technology, Chinese Academy of Sciences, Beijing, China. Since September 2007, he has been with the School of Information and Communication Engineering, UESTC, where he is currently a Professor and Director. From June 2011 to May 2012, he was a Visiting Scholar with the Stevens Institute of Technology, Hoboken, NJ, USA. From December 2012 to December 2013, he was a Hong Kong Scholar with the City University of Hong Kong, Hong Kong. From January 2014 to January 2016, he was a Marie Curie Fellow with the Imperial College London, London, U.K. His research interests include the area of array signal processing and circuit systems for radar, communications, and microwave remote sensing.

Andreas Jakobsson (Senior Member, IEEE) received the M.Sc. degree in computer science from the Lund Institute of Technology, Lund, Sweden, in 1993, and the Ph.D. degree in signal processing from Upsala University, Upsala, Sweden, in 2000. He has held positions with Global IP Sound AB, the Swedish Royal Institute of Technology, King’s College London, and Karlstad University, as well as held an Honorary Research Fellowship with Cardiff University and a guest professorship with Harbin Engineering University. He has been a visiting Researcher with King’s College London, Brigham Young University, Stanford University, KU Leuven, and University of California, San Diego, as well as acted as an expert for the International Atomic Energy Agency. He is currently a Professor of mathematical statistics with Lund University, Lund. He has authored and co-authored more than 250 refereed journal and conference papers in his research findings, and has filed six patents. He has also authored a book on time series analysis (Studentlitteratur), and co-authored (together with M. G. Christensen) a book on multi-pitch estimation (Morgan & Claypool). His research interests include statistical and array signal processing, detection, and estimation theory, and related application in remote sensing, telecommunication, and biomedicine.

Dr. Jakobsson is a Member of The Royal Swedish Physiographic Society, as well as an Associate Editor for Elsevier Signal Processing. He has previously also been a Senior Associate Editor for IEEE TRANSACTIONS ON SIGNAL PROCESSING (2018–2022), a Member of the EURASIP Special Area Team on Signal Processing for Multisensor Systems (2015–2021), a Member of the IEEE Sensor Array and Multichannel (SAM) Signal Processing Technical Committee (2008–2013), an Associate Editor for the IEEE TRANSACTIONS ON SIGNAL PROCESSING (2006–2010), the IEEE SIGNAL PROCESSING LETTERS (2007–2011), the Research Letters in Signal Processing (2007–2009), and the Journal of Electrical and Computer Engineering (2009–2014).

JIA ET AL.: DESIGNING FDA RADARS ROBUST TO CONTAMINATED SHARED SPECTRA

Authorized licensed use limited to the terms of the applicable license agreement with IEEE. Restrictions apply.