Observation of negative absolute resistance in a Josephson junction

J. Nagel, D. Speer, T. Gaber, A. Sterck, R. Eichhorn, P. Reimann, K. Ilin, M. Siegel, D. Koelle, and R. Kleiner

Physikalisches Institut – Experimentalphysik II and Center for Collective Quantum Phenomena and their Applications, Universität Tübingen, Auf der Morgenstelle 14, D-72076 Tübingen, Germany

Institut für Mikro- und Nanoelektronische Systeme, Universität Karlsruhe (TH), Hertzstraße 16, D-76187 Karlsruhe, Germany

(Dated: February 9, 2022)

We experimentally demonstrate the occurrence of negative absolute resistance (NAR) up to about \(-1\,\Omega\) in response to an externally applied dc current for a shunted Nb-Al/O-AI-Nb Josephson junction, exposed to a microwave current at frequencies in the GHz range. The realization (or not) of NAR depends crucially on the amplitude of the applied microwave current. Theoretically, the system is described by means of the resistively and capacitively shunted junction model in terms of a moderately damped, classical Brownian particle dynamics in a one-dimensional potential. We find excellent agreement of the experimental results with numerical simulations of the model.

PACS numbers: 05.45.-a, 05.40.-a, 05.60.Cd, 74.50.+r

When a static force is applied to a system consisting of mobile particles, these particles usually move in the direction of the force, i.e., they show positive mobility, which leads to, e.g., a positive conductance or resistance in electrical systems. Also well known is the fact that such a system can exhibit regions of negative differential mobility/resistance. However, the absolute mobility/resistance usually remains positive. The opposite response, i.e., a motion against the static force is termed negative absolute mobility or negative absolute resistance (NAR). This is clearly a quite counter intuitive effect which, at first glance, might seem even to be in conflict with Newton’s laws and thermodynamic principles. Yet, nonlinear systems being driven far from equilibrium can indeed exhibit not only a negative differential resistance but also a NAR effect. Unambiguous and convincing experimental observations of NAR are still quite scarce, involving systems consisting of electrons in a sample of bulk GaAs, electrons in semiconductor heterostructures, electrons in low dimensional conductors, and charged Brownian particles in structured microfluidic devices. Apart from the low dimensional conductors, the system was always driven out of equilibrium by means of an ac driving force and then its response to an externally applied static perturbation was studied. On the theoretical side, a considerably larger literature is available, most notably on different types of semiconductors and semiconductor heterostructures. In all those cases (except) NAR is based on purely quantum mechanical effects which cannot be transferred into the realm of classical physics. For classical systems, a first theoretical demonstration of the effect was provided in the context of a spatially periodic and symmetric model system of interacting Brownian particles, subject to multiplicative white noise. While each of the different ingredients of the model is quite realistic in itself, their combined realization in an experimental system seems difficult. In particular, the main physical mechanism is based on collective effects of at least three interacting particles. An entirely different mechanism was later on suggested theoretically for a realistic, classical model dynamics of a single Brownian particle in a suitably tailored, two-dimensional potential landscape in Ref. and subsequently realized experimentally in Refs. As a first application of NAR, the separation of different particle species has been realized in Ref. While the underlying basic physical mechanism still requires at least two spatial dimensions, very recently, NAR has been analyzed and predicted theoretically to occur also in the simplest possible case of a single Brownian particle dynamics in one dimension. More precisely, two basically different physical mechanisms capable of generating NAR in such systems have been unraveled, namely a purely noise induced effect in Ref. and a transient chaos induced effect in Refs. In both cases, an experimental realization by means of a Josephson junction subjected to suitable dc and ac currents has been proposed. In this Letter we show that a moderately damped Josephson junction being driven by microwaves indeed shows NAR of the type predicted in Refs. A first hint along these lines can be found in Fig. 13 of, although without further explanation or discussion and no direct reference to the resistively and capacitively shunted junction model.

To model the Josephson junction we use the resistively and capacitively shunted junction model. It describes the equation of motion for the difference \(\Phi\) of the phases of the superconducting order parameter in the two electrodes

\[
I = \frac{\Phi_0}{2\pi} C \delta + \frac{\Phi_0}{2\pi R} \dot{\delta} + I_0 \sin \delta + I_N.
\]
Here, $C$, $R$, and $I_0$ denote the junction capacitance, resistance and maximum Josephson current, respectively, dots indicate time-derivatives, $\Phi_0$ is the magnetic flux quantum, and $I = I_{dc} + I_{ac}\sin(\omega t)$ is the total current applied to the junction, consisting of a dc and a high frequency ac component. The first term on the right hand side of Eq. (1) describes the displacement current $CU$, where $U$ is the voltage across the junction, and has been rewritten in terms of $\delta$ using the Josephson relation $\delta = 2\pi U/\Phi_0$. The second term describes the current through the resistor $R$, the third term the Josephson current, and the last term the noise current arising from Nyquist noise in the resistor. Its spectral power density is assumed to be white with $S_{\eta}(f) = 4k_B T/R$, where $T$ is the temperature and $k_B$ Boltzmann's constant. The model (1) implicitly assumes that magnetic fields created by circulating supercurrents can be neglected (short junction limit). This holds when the lateral junction dimensions are below about $4\lambda_J$, where $\lambda_J = (\Phi_0/(4\pi\mu_0 j_0 \lambda_J))^{1/2}$ is the Josephson length in terms of the critical current density $j_0$, the London penetration depth $\lambda_L$, and the magnetic permeability $\mu_0$.

By integrating Eq. (1) one obtains $\delta$ and, by time averaging, the dc voltage $V = \Phi_0(\delta)/2\pi$ across the junction. This is the main observable of our present work, which is measured when recording $V(I_{dc})$, the current voltage characteristics (IVC). For numerical simulations, (1) can be rewritten in dimensionless units by normalizing currents to $I_0$, voltages to $I_0 R$, times to $t_c = \Phi_0/(2\pi I_0 R)$, and hence frequencies to $f_c = I_0 R/\Phi_0$, yielding

$$i = \beta_c \delta + \delta + \sin \delta + i_N,$$

where $i = i_{dc} + i_{ac}\sin(\tau f/f_c)$ is the normalized applied current, $\tau = t/t_c$ the normalized time, $\beta_c = (f_c/f_{pl})^2 = 2\pi I_0 R^2 C/\Phi_0$ the Stewart-McCumber parameter, $f_{pl} = (I_0/(2\pi\Phi_0 C))^{1/2}$ the Josephson plasma frequency, and $i_N$ the normalized noise current with spectral density $S_{\eta}(f/f_c) = 4\Gamma$ and noise parameter $\Gamma = 2 k_B T/\Phi_0$.

In a nutshell, the basic ingredients of NAR as predicted in [1] are as follows. The unperturbed deterministic dynamics (Eq. 2) with $i_{dc} = 0$ and $i_N = 0$ exhibits two symmetric attractors, carrying currents of opposite signs (zero crossing Shapiro steps). When an external perturbation in the form of a static bias $i_{dc}$ is applied, a subtle interplay of this bias force and the dissipation leads to a destabilization of that attractor, whose current points into the same direction as the applied bias. Its remnant is a strange repeller, exhibiting transient chaos, hence the name “transient chaos induced NAR” coined in [1]. The actual realization of NAR along these lines requires a careful choice of model parameters in (2) within the general regime of frequencies $f$ comparable to $f_{pl}$ and values of $\beta_c$ roughly between 1 and 100. To obtain precise quantitative results, we have solved Eq. (2) numerically for various such parameter values by integrating and averaging over typically $5 \cdot 10^3$ periods of the ac current.

For experiments, which were performed at $T = 4.2$ K, we used circular Nb-Al/AlO$_x$-Nb Josephson junctions with an area of $200 \mu m^2$, cf. upper left inset in Fig. 1(a). The junctions were shunted by a AuPd strip with resistance $R = 1.27 \Omega$ and integrated in a coplanar waveguide. We denote the critical current $I_c$ as the maximum dc current for which $V = 0$. In general, $I_c$ is a function of $I_{ac}$ and fluctuations. By measuring $I_c(I_{ac} = 0)$ and matching it with simulations we determined $I_0 = 197 \mu A$, yielding $I_0 R = 250 \mu V$, $f_c = 121 \text{GHz}$ and $\Gamma = 9 \cdot 10^{-4}$. The Josephson length is about $40 \mu m$, i. e., well above the $16 \mu m$ diameter of our junctions assuring the short junction limit. The design value of the capacitance was $8.24 \text{pF}$, yielding $f_{pl} = 43 \text{GHz}$, and $\beta_c = 7.9$. The actual value used in the simulations shown below is somewhat smaller, namely $\beta_c = 7.7$, reproducing particularly well the hysteretic IVC in the absence of microwaves. The transport measurements have been performed with a standard four terminal method, using filtered leads. Microwaves between 8 and 35 GHz, with variable output power $P_m$, were applied through a semirigid cable that was capacitively coupled to the 50 $\Omega$ coplanar waveguide. The samples were electromagnetically shielded and surrounded by a cryopem shield, to reduce static magnetic fields.

Given $I_0$, $R$, $C$, $T$ and $I_{dc}$, all relevant model parameters are fixed, with the exception of the (frequency dependent) coupling factor between the microwave amplitude $\sqrt{P_m}$ applied from the source and the amplitude $I_{ac}$ of the ac current induced across the junction. We have fixed this factor by comparing the measured dependence of $I_c(\sqrt{P_m})$ with the calculated curve $I_c(I_{ac})$, as shown in the right inset of Fig. 1 for a microwave frequency of $19 \text{GHz}$ ($f/f_c \approx 0.16$). The experimental and theoretical curves are in good agreement. In particular, the main side maxima can be found, both in experiment and simulation. By adjusting the position of these maxima, we obtain a coupling factor $I_{ac}/\sqrt{P_m}(19 \text{GHz}) = 1.0 \text{mA}/\sqrt{\text{mW}}$.

Figure 1(a) shows IVCs under $f = 19 \text{GHz}$ microwave irradiation at three values of $I_{ac}$. In the absence of microwaves (black line) the IVC is hysteretic, exhibiting a critical current of $195 \mu A$ and a return current of $100 \mu A$ (black arrows). When the microwave field is applied, the hysteresis decreases with increasing $I_{ac}$, and step-like features appear on the IVC. At $P_m = 194 \text{mW}$ ($I_{ac} = 435 \mu A$; magenta line), we observe NAR with a resistance of $1.07 \Omega$, occurring in an interval $|I_{dc}| \leq 20 \mu A$ (i. e., approximately $10\%$ of $I_0$). When $I_{ac}$ is increased to $P_m = 253 \text{mW}$ ($I_{ac} = 497 \mu A$; green line) the NAR has disappeared. However, centered on a voltage which corresponds to the first Shapiro step ($V_1 = \Phi_0 f_c \approx 39 \mu V$), regions of negative differential resistance appear. In Fig. 1(b) simulated and measured IVCs for the two microwave amplitudes $435 \mu A$ and $497 \mu A$ are compared. For the former case, which is recorded at the microwave amplitude where the maximum NAR has been observed, the agreement between the experimental and the theoretical curve is nearly perfect. For the latter case some small differences can be seen, although the agreement is
still very good. To demonstrate the origin of the NAR, the grey curve in Fig. 1(b) shows a simulated IVC for $i_{ac} = 2.242\,\mu A$ and $\Gamma = 0$, i.e., for the noise-free case. The curve shows $n = -1$ Shapiro steps to be the cause of NAR, clearly revealing its nature to be of the type discussed in [18].

Figure 2 compares in more detail the measured and calculated dependence of V on $I_{dc}$ and on $I_{ac}$, for two frequencies (8 GHz and 19 GHz). For $f = 8\,\text{GHz}$, the comparison between measured $I_c(\sqrt{I_{m}})$ and simulated $I_c(I_{ac})$ curves yields a coupling factor $I_{ac}/\sqrt{I_{m}}(8\,\text{GHz}) = 0.33\,\text{mA/}\sqrt{\text{mW}}$. In the graphs, V is normalized to $\Phi_0 f$, yielding an integer value $n$ for the $n$-th Shapiro step. Again, the agreement between theory and experiment is very good. There are at least five $I_{ac}$ intervals where NAR appears at $f = 8\,\text{GHz}$, and three such intervals at $f = 19\,\text{GHz}$. Within those regimes, the resistance at $I_{dc} = 0$ reaches values up to about $-1\,\Omega$. In the case of $f = 8\,\text{GHz}$, the NAR persists up to values of $|I_{dc}| \approx 10\,\mu A$, for all values of $I_{ac}$ for which NAR shows up. In contrast, for $f = 19\,\text{GHz}$, the $I_{dc}$ interval for NAR decreases with increasing $I_{ac}$. When we increased the frequency further to 35 GHz, hysteretic Shapiro steps appeared on the IVC, crossing the voltage axis ($I_{dc} = 0$). As a consequence, NAR ceases to exist both in the experiment and the simulations.

In a second series of experiments we applied a magnetic field $B$ parallel to the junction plane in order to tune (decrease) its Josephson current $I_0$, making it a $B$-dependent function $I_0(B)$ [22]. Thus all $I_0$-dependent parameters entering the normalized equation [2] acquire a $B$-dependence, in particular, $\beta_c$, $f/f_c$, and $\Gamma$. Figure 3 shows a comparison of the measured and calculated dependence of the resistance upon $I_{ac}$ and $I_0(B)$. Again, we find excellent agreement between measurement and theory. Blue regions indicate NAR. Their most remarkable feature is that the values of $I_{ac}$, for which NAR appears, practically do not depend on $I_0$. Furthermore, we find that the NAR value can be tuned by $I_0$ via an applied magnetic field. For our junction parameters we find a maximum NAR at $I_0 \approx (0.4 \ldots 0.6) I_0(B = 0)$, which is increasing with $I_{ac}$.

In conclusion, we have observed negative absolute resistance (NAR) of up to about $-15\,\Omega$ in a shunted Nb-Al/AlO$_x$-Nb Josephson junction device subjected to microwaves. To clearly see the effect, a careful choice of parameters is required, but still the range of suitable
FIG. 3: (a) Contour plot of the experimentally measured resistance $R := V/(I = 5 \mu A)/5 \mu A$ as a function of the Josephson current $I_0(B)$ and of the microwave amplitude $I_{ac}$. $I_0$ has been varied by applying a magnetic field to the junction. Graph (b) shows the corresponding simulated plot. For both graphs parameters at $B = 0$ are the same as in Fig. 2(a), (b). The similarity between our Fig. 3 and Fig. 2 in [9] suggests that with respect to NAR, purely quantum mechanical band structure and energy quantization effects may be imitated by inertia effects in a purely classical, one dimensional noisy dynamics. Moreover, our Fig. 3 exhibits many features which are quite similar to the corresponding plots in [18], while the intuitive explanation of the almost vertical stripe-pattern in Fig. 3 remains as an open problem. As an application, our present work opens the intriguing perspective of a new resistor-type electronic element which is tunable between positive and negative resistance via an easily accessible external control parameter, e. g., the amplitude of an ac driving or an externally applied magnetic field in the mT range.

This work was supported by the Deutsche Forschungsgemeinschaft (Grants No. KO 1303/7-1, RE 1344/5-1, and SFB 613).

[1] S. R. White and M. Barma, J. Phys. A 17, 2995 (1984).
[2] G. A. Griess and P. Serwer, Biopolymers 29, 1863 (1990).
[3] V. Balakrishnan and C. Van den Broeck, Physica A 217, 1 (1995).
[4] G. A. Cecchi and M. O. Magnasco, Phys. Rev. Lett. 76, 1968 (1996).
[5] G. W. Slater, H. L. Guo, and G. I. Nixon, Phys. Rev. Lett. 78, 1170 (1997).
[6] R. K. P. Zia, E. L. Praestgaard, and O. G. Mouritsen, Am. J. Phys. 70, 384 (2002).
[7] For a review see R. Eichhorn, P. Reimann, B. Cleuren, and C. Van den Broeck, Chaos 15, 026113 (2005).
[8] T. J. Banys, I. V. Parshleyunas, and Y. K. Pozhela, Sov. Phys. Semicond. 5, 1727 (1972).
[9] B. J. Keay, S. Zeuner, Jr. S. J. Allen, K. D. Maranowski, A. C. Gossard, U. Bhattacharya, and M. J. W. Rodwell, Phys. Rev. Lett. 75, 4102 (1995).
[10] H. S. J. van der Zant, E. Slot, S. V. Zaitsev-Zotov, and S. N. Artemenko, Phys. Rev. Lett. 87, 126401 (2001).
[11] A. Ros, R. Eichhorn, J. Regtmeier, T. T. Duong, P. Reimann, and D. Anselmetti, Nature 436, 928 (2005).
[12] P. Reimann, R. Kawai, C. Van den Broeck, and P. Hänggi, Europhys. Lett. 45, 545 (1999).
[13] C. Van den Broeck, B. Cleuren, R. Kawai, and M. Kam‐
bon, Int. J. Mod. Phys. C 13, 1195 (2002).
[14] R. Eichhorn, P. Reimann, and P. Hänggi, Phys. Rev. Lett. 88, 190601 (2002); Phys. Rev. E 66, 066132 (2002); Physica A 325.
[15] R. Eichhorn, A. Ros, J. Regtmeier, T. T. Duong, P. Reimann, and D. Anselmetti, Eur. Phys. J. Spec. Top. 143, 159 (2007); J. Regtmeier, S. Grauwin, R. Eichhorn, P. Reimann, D. Anselmetti, and R. Ros, J. Sep. Sci. 30, 1461 (2007).
[16] J. Regtmeier, R. Eichhorn, T. T. Duong, P. Reimann, D. Anselmetti, and R. Ros, Eur. Phys. J. E 22, 335 (2007).
[17] L. Machura, M. Kostur, P. Talkner, J. Luczka, and P. Hänggi, Phys. Rev. Lett. 98, 040601 (2007).
[18] D. Speer, R. Eichhorn, and P. Reimann, Europhys. Lett. 79, 10005 (2007); Phys. Rev. E 76, 051110 (2007).
[19] N. F. Pedersen, O. H. Soerensen, B. Dueholm, J. Mygind, J. Low. Temp. Phys. 38, 1 (1980).
[20] W. C. Stewart, Appl. Phys. Lett 12, 277 (1968).
[21] D. E. McCumber, J. Appl. Phys. 39, 3113 (1968).
[22] A. Barone and G. Paterno, Physics and Application of the Josephson Effect, John Wiley and Sons, New York, 1982.