Nonspectator Effects and $B$ Meson Lifetimes

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We review the $B$ meson lifetime problems and nonspectator effects. The predictions of $B$ meson lifetime ratios depend on four unknown hadronic parameters $B_1$, $B_2$, $\epsilon_1$ and $\epsilon_2$, where $B_1$ and $B_2$ parametrize the matrix elements of color singlet-singlet four-quark operators and $\epsilon_1$ and $\epsilon_2$ the matrix elements of color octet-octet operators. To understand contributions of the nonspectator effects to the $B$ meson lifetime ratios, we derive the renormalization-group improved QCD sum rules for these parameters within the framework of heavy quark effective theory.

I. INTRODUCTION

A QCD-based operator-product-expansion (OPE) formulation for treatment of inclusive heavy hadron decays has been developed in past years [1]. The optical theorem tells us that the inclusive decay rates are related to the imaginary part of certain forward scattering amplitudes along the physical cut. Since, based on the hypothesis of quark-hadron duality, the final state effects which are nonperturbative in nature, are eliminated after adding up all of the states, the OPE approach thus can be employed for such the smeared or averaged physical quantities. In order to test the validity of (local) quark-hadron duality, it is very important to have a reliable estimate of the heavy hadron lifetimes within the OPE framework and compare them with experiment.

In the heavy quark limit, all bottom hadrons have the same lifetimes in the parton picture. With the advent of heavy quark effective theory, which gives a systematic way in expansion of the initial heavy hadron, and the OPE approach for the analysis of inclusive weak decays, it is realized that the first nonperturbative correction to bottom hadron lifetimes starts at order $1/m_b^2$. However, the $1/m_b^2$ corrections are small and essentially negligible in the lifetime ratios. The nonspectator effects such as $W$-exchange and Pauli interference due to four-quark interactions are of order $1/m_Q^2$, but their contributions can be potentially significant due to a phase-space enhancement by a factor of $16\pi^2$. As a result, the lifetime differences of heavy hadrons come mainly from the above-mentioned nonspectator effects.

II. DIFFICULTIES OF THE OPE APPROACH

The world average lifetime ratios of bottom hadrons are [2]:

$$\frac{\tau(B^-)}{\tau(B_d^0)} = 1.07 \pm 0.03,$$

$$\frac{\tau(B_s^0)}{\tau(B_d^0)} = 0.94 \pm 0.04,$$

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\[
\frac{\tau(\Lambda_b)}{\tau(B_d^0)} = 0.79 \pm 0.05 .
\] (2.1)

Since, to order \(1/m_b^2\), the OPE results for all of the above ratios are very close to unity [see Eq. (3.11) below], the conflict between theory and experiment for this lifetime ratio is quite striking \([3–6]\). One possible reason for the discrepancy is that (local) quark-hadron duality may not work in the study of nonleptonic inclusive decay widths. Another possibility is that some hadronic matrix elements of four-quark operators are probably larger than what naively expected so that the nonspectator effects of order \(16\pi^2/m_b^3\) may be large enough to explain the observed lifetime ratios. Therefore, one cannot conclude that (local) duality truly fails before a reliable calculation of the four-quark matrix elements is obtained \([4]\).

Conventionally, the hadronic matrix elements of four-quark operators are evaluated using the factorization approximation for mesons and the quark model for baryons. However, as we shall see, nonfactorizable effects absent in the factorization hypothesis can affect the 

\[B\text{approximation for mesons and the quark model for baryons. However, as we shall see, nonfactorizable effects absent in the factorization hypothesis can affect the}\text{B}\text{estimate of the hadronic parameters}\text{. Applying the QCD sum rule to calculate these unknown parameters.}\]

### III. THEORETICAL REVIEW

In this talk we will focus on the study of the four-quark matrix elements of the \(B\) meson. Before proceeding, let us briefly review the theory. Applying the optical theorem, the inclusive decay width of the hadron \(H_b\) containing a \(b\) quark can be expressed as

\[
\Gamma(H_b \to X) = \frac{1}{m_{H_b}} \text{Im} i \int \text{d}^4x \langle H_b | T \{ \mathcal{L}_{\text{eff}}(x), \mathcal{L}_{\text{eff}}(0) \} | H_b \rangle ,
\] (3.1)

where \(\mathcal{L}_{\text{eff}}\) is the relevant effective weak Lagrangian that contributes to the particular final state \(X\). When the energy release in a \(b\) quark decay is sufficiently large, it is possible to express the nonlocal operator product in Eq. (3.1) as a series of local operators in powers of \(1/m_b\) by using the OPE technique. In the OPE series, the only locally gauge invariant operator with dimension four, \(\bar{b}i\gamma_\mu Db\), can be reduced to \(m_b\bar{b}b\) by using the equation of motion. Therefore, the first nonperturbative correction to the inclusive \(B\) hadron decay width starts at order \(1/m_b^2\). As a result, the inclusive decay width of a hadron \(H_b\) can be expressed as \([1]\)

\[
\Gamma(H_b \to X) = \frac{G_F^2 m_b^5 |V_{\text{CKM}}|^2}{192\pi^3} \frac{1}{2m_{H_b}} \left\{ c_3^X \langle H_b | \bar{b}b | H_b \rangle + c_3^X \frac{\langle H_b | \bar{b}_5 \gamma_\mu \sigma \cdot G b | H_b \rangle}{m_b^2} \right\} + \sum_n c_6^{X(n)} \left( \frac{\langle H_b | O_6^{(n)} | H_b \rangle}{m_b^4} + O(1/m_b^4) \right) ,
\] (3.2)

where \(\sigma \cdot G = \sigma_{\mu\nu}G^{\mu\nu}\), \(V_{\text{CKM}}\) denotes some combination of the Cabibbo-Kobayashi-Maskawa parameters and \(c_i^X\) reflect short-distance dynamics and phase-space corrections. The matrix elements in Eq. (3.2) can be systematically expanded in powers of \(1/m_b\) in heavy quark effective theory (HQET), in which the \(b\)-quark field is represented by a four-velocity-dependent field denoted by \(h_v^{(b)}(x)\). To first order in \(1/m_b\), the \(b\)-quark field \(b(x)\) in QCD and the HQET-field \(h_v^{(b)}(x)\) are related via

\[
b(x) = e^{-m_b v \cdot x} \left[ 1 + i \frac{D}{2m_b} \right] h_v^{(b)}(x) .
\] (3.3)

Applying this relation, one can replace \(b\) by the effective field \(h_v^{(b)}\) in Eq. (3.2) to obtain

\[
\frac{\langle H_b | \bar{b}b | H_b \rangle}{2m_{H_b}} = 1 - \frac{K_{H_b}}{2m_b} + \frac{G_{H_b}}{2m_b} + O(1/m_b^3) ,
\]

\[
\frac{\langle H_b | \bar{b}_5 \gamma_\mu \sigma \cdot G b | H_b \rangle}{2m_{H_b}} = G_{H_b} + O(1/m_b) ,
\] (3.4)
where

\[ K_{H_b} \equiv - \frac{\langle H_b | h_c^{(b)} (iD_c)^2 h_c^{(b)} | H_b \rangle}{2m_{H_b}}, \quad G_{H_b} \equiv \frac{\langle H_b | h_c^{(b)} \bar{b} \to \bar{c} \gamma \sigma \cdot G h_c^{(b)} | H_b \rangle}{2m_{H_b}}. \]

Note that here we adopt the convention \( D^\alpha = \partial^\alpha - ig_s A^\alpha \). The inclusive nonleptonic and semileptonic decay rates of a bottom hadron to order \( 1/m_b^2 \) are given by [1]

\[
\Gamma_{NL}(H_b) = \frac{g^2 m_b^5}{192\pi^3} N_c |V_{cb}|^2 \frac{1}{2m_{H_b}} \left\{ \left( c_1^2 + c_2^2 + \frac{2c_1 c_2}{N_c} \right) \times \right. \\
\left. \left[ (\alpha I_0(x,0) + \beta I_0(x,x,0)) \langle H_b | \bar{b}b | H_b \rangle \
- \frac{1}{m_b} \left( I_1(x,0,0) + I_1(x,x,0) \right) \langle H_b | \bar{b}g_s \cdot G b | H_b \rangle \\
- \frac{4}{m_b^2} c_1 c_2 \left( I_2(x,0,0) + I_2(x,x,0) \right) \langle H_b | \bar{b}g_s \cdot G b | H_b \rangle \right\},
\]

(3.6)

where \( N_c \) is the number of colors, the parameters \( \alpha \) and \( \beta \) denote QCD radiative corrections to the processes \( b \to c\bar{u}d \) and \( b \to c\bar{c}s \), respectively [8], and

\[
\Gamma_{SL}(H_b) = \frac{g^2 m_b^5}{192\pi^3} N_c |V_{cb}|^2 \eta(x,\mu,0) \frac{1}{2m_{H_b}} \\
\times \left[ I_0(x,0,0) \langle H_b | \bar{b}b | H_b \rangle - \frac{1}{m_b} I_1(x,0,0) \langle H_b | \bar{b}g_s \cdot G b | H_b \rangle \right],
\]

(3.7)

where \( \eta(x,\mu,0) \) with \( x = (m_t/m_q)^2 \) is the QCD radiative correction to the semileptonic decay rate and its general analytic expression is given in [7]. In Eqs. (3.6) and (3.7), \( I_{0,1,2} \) are phase-space factors (see e.g. [5] for their explicit expressions): \( I_0(x,0,0) \) for \( b \to c\bar{u}d \) transition and \( I_1(x,x,0) \) for \( b \to c\bar{c}s \) transition. Note that the CKM parameter \( V_{ud} \) does not occur in \( \Gamma_{NL}(H_b) \) and \( \Gamma_{SL}(H_b) \) when summing over the Cabibbo-allowed and Cabibbo-suppressed contributions.

In Eq. (3.6) \( c_1 \) and \( c_2 \) are the Wilson coefficients in the effective Hamiltonian

\[
H_{\text{eff}}^{B=1} = \frac{G_F \sqrt{2}}{\Lambda} \left[ V_{cb} V_{us}^* \right] \left[ c_1(\mu) O_1^u(\mu) + c_2(\mu) O_2^u(\mu) \right] \\
+ V_{cb} V_{cs}^* \left[ c_1(\mu) O_1^s(\mu) + c_2(\mu) O_2^s(\mu) + \cdots \right] + \text{h.c.},
\]

(3.8)

where \( q = d, s \), and

\[
O_1^u = \bar{c} \gamma_\mu (1 - \gamma_5) b \bar{q} \gamma^\mu (1 - \gamma_5) u, \quad O_2^u = \bar{q} \gamma_\mu (1 - \gamma_5) b \bar{c} \gamma^\mu (1 - \gamma_5) u.
\]

(3.9)

The scale and scheme dependence of the Wilson coefficients \( c_{1,2}(\mu) \) are canceled out by the corresponding dependence in the matrix element of the four-quark operators \( O_{1,2} \). That is, the four-quark operators in the effective theory have to be renormalized at the same scale \( \mu \) and evaluated using the same renormalization scheme as that for the Wilson coefficients.

Here we use the effective Wilson coefficients \( c_1 \) which are scheme-independent [5].

\[
c_1 = 1.149, \quad c_2 = -0.325.
\]

(3.10)

Using \( m_b = 4.85 \) GeV, \( m_c = 1.45 \) GeV, \( |V_{cb}| = 0.039 \), \( G_B = 0.36 \) GeV\(^2\), \( G_L = 0 \), \( K_B \approx K_L \approx 0.4 \) GeV\(^2\) together with \( \alpha = 1.063 \) and \( \beta = 1.32 \) to the next-to-leading order [8], we find numerically

\[
\frac{\tau(B^-)}{\tau(B_d)} = 1 + O(1/m_b^3), \quad \frac{\tau(B_s)}{\tau(B_d)} = 1 + O(1/m_b^3), \quad \frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.99 + O(1/m_b^3).
\]

(3.11)
It is evident that the $1/m_\pi^2$ corrections are too small to explain the shorter lifetime of the $\Lambda_b$ relative to that of the $B_d$. To the order of $1/m_\pi^2$, the nonspectator effects due to Pauli interference and $W$-exchange parametrized in terms of the hadronic parameters [4]: $B_1$, $B_2$, $\epsilon_1$, $\epsilon_2$, $\hat{B}$, and $r$ (see below), may contribute significantly to lifetime ratios due to a phase-space enhancement by a factor of $16\pi^2$. The four-quark operators relevant to inclusive nonleptonic $B$ decays are

\begin{align*}
O_{V,A}^q &= \tilde{b}_L\gamma_\mu q_L q_L\gamma^\mu b_R, \\
O_{S,P}^q &= \tilde{b}_R q_L \tilde{q}_L b_R, \\
T_{V,A}^q &= \tilde{b}_L\gamma_\mu t^a q_L q_L\gamma^\mu t^a b_R, \\
T_{S,P}^q &= \tilde{b}_R t^a q_L \tilde{q}_L b_R, 
\end{align*}

(3.12)

where $q_{R,L} = \frac{1+\gamma_5}{2} q$. For the matrix elements of these four-quark operators between $B$ hadron states, following, [4] we adopt the definitions:

\begin{align*}
\frac{1}{2m_{B_q}} \langle \bar{B}_q | O_{V,A}^q | \bar{B}_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} B_1, \\
\frac{1}{2m_{B_q}} \langle \bar{B}_q | O_{S,P}^q | \bar{B}_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} B_2, \\
\frac{1}{2m_{B_q}} \langle \bar{B}_q | T_{V,A}^q | \bar{B}_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} \epsilon_1, \\
\frac{1}{2m_{B_q}} \langle \bar{B}_q | T_{S,P}^q | \bar{B}_q \rangle &= \frac{f_{B_q}^2 m_{B_q}}{8} \epsilon_2, \\
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | O_{V,A}^q | \Lambda_b \rangle &= -\frac{f_{\Lambda_b}^2 m_{\Lambda_b}}{48} r, \\
\frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | T_{S,P}^q | \Lambda_b \rangle &= -\frac{1}{2} (\hat{B} + \frac{1}{3}) \frac{1}{2m_{\Lambda_b}} \langle \Lambda_b | O_{V,A}^q | \Lambda_b \rangle.
\end{align*}

(3.13)

Under the factorization approximation, $B_i = 1$ and $\epsilon_i = 0$, and under the valence quark approximation $\hat{B} = 1$ [4].

The destructive Pauli interference in inclusive nonleptonic $B^-$ decay and the $W$-exchange contributions to $B^0_d$ and $B^0_s$ are [4] \footnote{The penguin-like nonspectator contributions to $B_s$ are considered in [10], but they are negligible compared to that from the current-current operators $O_1$ and $O_2$ introduced in Eq. (3.9).}

\begin{align*}
\Gamma^{\text{ann}}(B^0_d) &= -\Gamma_0|V_{ud}|^2 \eta_{\text{haspec}}(1-x)^2 \left\{ (1 + \frac{1}{2}x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_1 + 2c_1^2 \epsilon_1 \right] \\
&\quad - (1 + 2x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_2 + 2c_1^2 \epsilon_2 \right] \right\}, \\
\Gamma^{\text{int}}(B^-) &= \Gamma_0 \eta_{\text{haspec}}(1-x)^2 \left\{ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) (B_1 + 6\epsilon_1) + 6c_1 c_2 B_1 \right\}, \\
\Gamma^{\text{ann}}(B^0_s) &= -\Gamma_0|V_{cs}|^2 \eta_{\text{haspec}}(1-x)^2 \left\{ (1 + \frac{1}{2}x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_1 + 2c_1^2 \epsilon_1 \right] \\
&\quad - (1 + 2x) \left[ (\frac{1}{N_c} c_1^2 + 2c_1 c_2 + N_c c_2^2) B_2 + 2c_1^2 \epsilon_2 \right] \right\}.
\end{align*}
Likewise, the nonspectator effects in inclusive nonleptonic decays of the \( \Lambda \) with
\[
\begin{align*}
\Gamma_{\text{ann}}(A_b) &= \frac{1}{2} \Gamma_0 \eta_{\text{nspec}} r (1 - x)^2 \left( \hat{B} (c_1^2 + c_2^2) - 2 c_1 c_2 \right), \\
\Gamma_{\text{int}}(A_b) &= -\frac{1}{4} \Gamma_0 \eta_{\text{nspec}} r \left[ |V_{cd}|^2 (1 - x)^2 (1 + x) + |V_{ud}|^2 \sqrt{1 - 4x} \right] \left( \hat{B} c_1^2 - 2 c_1 c_2 - N_c c_2^2 \right).
\end{align*}
\]

Using the values of \( c_i \) in Eqs. (3.10), we obtain
\[
\begin{align*}
\Gamma_{\text{ann}}(B_d) &= \Gamma_0 \eta_{\text{nspec}} (-0.0087 B_1 + 0.0098 B_2 - 2.28 \epsilon_1 + 2.58 \epsilon_2), \\
\Gamma_{\text{int}}(B^-) &= \Gamma_0 \eta_{\text{nspec}} (-0.68 B_1 + 7.10 \epsilon_1), \\
\Gamma_{\text{ann}}(B_s) &= \Gamma_0 \eta_{\text{nspec}} (-0.0085 B_1 + 0.0096 B_2 - 2.22 \epsilon_1 + 2.50 \epsilon_2), \\
\Gamma_{\text{int}}(A_b) &= \Gamma_0 \eta_{\text{nspec}} r (0.59 \hat{B} + 0.31), \\
\Gamma_{\text{int}}(A_b) &= \Gamma_0 \eta_{\text{nspec}} r (-0.30 \hat{B} - 0.097).
\end{align*}
\]

Therefore, to the order of \( 1/m_b^3 \), the \( B \)-hadron lifetime ratios are given by
\[
\begin{align*}
&\frac{\tau(B^-)}{\tau(B_d^0)} = 1 + \left( \frac{f_B}{185 \text{ MeV}} \right)^2 (0.043 B_1 + 0.0006 B_2 - 0.61 \epsilon_1 + 0.17 \epsilon_2), \\
&\frac{\tau(B_s)}{\tau(B_d^0)} = 1 + \left( \frac{f_B}{185 \text{ MeV}} \right)^2 (-1.7 \times 10^{-5} B_1 + 1.9 \times 10^{-5} B_2 - 0.0044 \epsilon_1 + 0.0050 \epsilon_2), \\
&\frac{\tau(A_b)}{\tau(B_d^0)} = 0.99 + \left( \frac{f_B}{185 \text{ MeV}} \right)^2 \left[ -0.0006 B_1 + 0.0006 B_2 \\
& -0.15 \epsilon_1 + 0.17 \epsilon_2 - (0.014 + 0.019 \hat{B}) r \right].
\end{align*}
\]

We see that the coefficients of the color singlet–singlet operators are one to two orders of magnitude smaller than those of the color octet–octet operators. This implies that even a small deviation from the factorization approximation \( \epsilon_i = 0 \) can have a sizable impact on the lifetime ratios. It was argued in [4] that the unknown nonfactorizable contributions render it impossible to make reliable estimates on the magnitude of the lifetime ratios and even the sign of corrections. That is, the theoretical prediction for \( \tau(B^-)/\tau(B_d) \) is not necessarily larger than unity. In the next section we will apply the QCD sum rule method to estimate the aforementioned hadronic parameters, especially \( \epsilon_i \).
IV. THE QCD SUM RULE CALCULATION

In HQET where the $b$ quark is treated as a static quark, we can use the renormalization group equation to express them in terms of the operators renormalized at a scale $\Lambda_{\text{QCD}} \ll \mu \ll m_b$. Their renormalization-group evolution is determined by the “hybrid” anomalous dimensions [11] in HQET. The operators $O_{V-A}^i$ and $T_{V-A}^i$, and similarly $O_{S-P}^i$ and $T_{S-P}^i$, mix under renormalization. In the leading logarithmic approximation, the renormalization-group equation of the operator pair $(O, T)$ reads

$$\frac{d}{dt} \begin{pmatrix} O \\ T \end{pmatrix} = \frac{3\alpha_s}{2\pi} \begin{pmatrix} C_F & -1 \\ -C_F & 1 \end{pmatrix} \begin{pmatrix} O \\ T \end{pmatrix},$$

where $t = \frac{1}{2} \ln(Q^2/\mu^2)$, $C_F = (N_c^2 - 1)/2N_c$, and effects of penguin operators induced from evolution have been neglected.

The solution to the evolution equation Eq. (4.1) has the form

$$\begin{pmatrix} O \\ T \end{pmatrix}_Q = \begin{pmatrix} \frac{8}{5} \\ \frac{2}{3} \end{pmatrix} \begin{pmatrix} \frac{8}{27} \\ \frac{8}{9} \end{pmatrix} \left( L_Q^{9/(2\beta_0)} \right) \begin{pmatrix} 0 \\ 1 \end{pmatrix} D_\mu,$$

where

$$D_\mu = \begin{pmatrix} D_1 \\ D_2 \end{pmatrix}_\mu = \begin{pmatrix} O - \frac{3}{2}T \\ \frac{1}{2}O + T \end{pmatrix}_\mu,$$

$$L_Q = \alpha_s(\mu)/\alpha_s(Q)$$ and $\beta_0 = \frac{11}{3} N_c - \frac{2}{3} n_f$ is the leading-order expression of the $\beta$-function with $n_f$ being the number of light quark flavors. The subscript $\mu$ in Eq. (4.3) and in what follows denotes the renormalization point of the operators. Given the evolution equation (4.2) for the four-quark operators, we see that the hadronic parameters $B_i$ and $\epsilon_i$ normalized at the scale $m_b$ are related to that at $\mu = 1$ GeV by

$$B_i(m_b) \simeq 1.54 B_i(\mu) - 0.41 \epsilon_i(\mu),$$

$$\epsilon_i(m_b) \simeq -0.090 B_i(\mu) + 1.07 \epsilon_i(\mu),$$

with $\mu = 1$ GeV, where uses have been made of $\alpha_s(m_Z) = 0.118$, $\Lambda_{\text{QCD}}^{(4)} = 333$ MeV, $m_b = 4.85$ GeV, $m_c = 1.45$ GeV. The above results (4.4) indicate that renormalization effects are quite significant.

It is easily seen from Eqs. (4.2) and (4.3) that the normalized operator $D_1$ (or $D_2$) is simply multiplied by $L_Q^{9/(2\beta_0)}$ (or 1) when it evolves from a renormalization point $\mu$ to another point $Q$. In what follows, we will apply this property to derive the renormalization-group improved QCD sum rules for $D_j$ at the typical scale $\mu = 1$ GeV. We define the new four-quark matrix elements as follows

$$\frac{1}{2m_{B_q}} \langle \bar{B}_q | D_j^{(i)}(\mu) | \bar{B}_q \rangle \equiv \frac{f_{B_q}^2 m_{B_q}}{8} d_j^{(i)}(\mu),$$

where the superscript $(i)$ denotes $(V - A)$ four-quark operators for $i = 1$ and $(S - P)$ operators for $i = 2$, and $d_j^{(i)}$ satisfy

$$\begin{pmatrix} d_1^{(i)} \\ d_2^{(i)} \end{pmatrix}_\mu = \begin{pmatrix} B_i - \frac{3}{4} \epsilon_i \\ \frac{1}{6} B_i + \epsilon_i \end{pmatrix}_\mu.$$

In the sum rule calculation, the factorization scale $\mu$ cannot be chosen too small, otherwise the strong coupling constant $\alpha_s$ would be so large that Wilson coefficients cannot be perturbatively calculated.
Since the terms linear in four-quark matrix elements are already of order $1/m_b^3$, we only need the relation between the full QCD field $b(x)$ and the HQET field $h_v^{(b)}(x)$ to the zeroth order in $1/m_b$: $b(x) = e^{-im_b v \cdot x} \{h_v^{(b)}(x) + \mathcal{O}(1/m_b)\}$. In the following, within the framework of HQET, we apply the method of QCD sum rules to obtain the value of the matrix elements of four-quark operators. We consider the three-point correlation function

$$
\Pi^{D_j^{(i)}}_{\alpha,\beta}(\omega, \omega') = i^2 \int dx dy e^{i x v \cdot x - i \omega' y} \langle 0 | \{ \bar{q}(x) \Gamma_\alpha h_v^{(b)}(x) \} D_j^{(i)}(0) \{ \bar{q}(y) \Gamma_\beta h_v^{(b)}(y) \}^\dagger | 0 \rangle, \tag{4.7}
$$

where the operator $D_j^{(i)}$ is defined in Eq. (4.3) but with $b \to h_v^{(b)}$ and $\Gamma_\alpha$ is chosen to be $v_\alpha \gamma_5$ (some further discussions can be found in [12]).

The correlation function can be written in the double dispersion relation form

$$
\Pi^{D_j^{(i)}}_{\alpha,\beta}(\omega, \omega') = \int \int \frac{ds}{s - \omega} \frac{ds'}{s' - \omega'} \rho^{D_j^{(i)}}. \tag{4.8}
$$

The results of the QCD sum rules are obtained in the following way. On the phenomenological side, which is the sum of the relevant hadron states, this correlation function can be written as

$$
\Pi^{PS}_{D_j^{(i)}}(\omega, \omega') = \frac{F^2(m_b) F^2(\mu) \rho^{D_j^{(i)}}}{16(\Lambda - \omega)(\Lambda - \omega')} + \cdots, \tag{4.9}
$$

where $\Lambda$ is the binding energy of the heavy meson in the heavy quark limit and ellipses denote resonance contributions. The heavy-flavor-independent decay constant $F$ defined in the heavy quark limit is given by

$$
\langle 0 | \bar{q}\gamma^\mu \gamma_5 h_v^{(b)} | \bar{B}(v) \rangle = i F(\mu) v^\mu. \tag{4.10}
$$

The decay constant $F(\mu)$ depends on the scale $\mu$ at which the effective current operator is renormalized and it is related to the scale-independent decay constant $f_B$ of the $B$ meson by

$$
F(m_b) = f_B \sqrt{m_B}. \tag{4.11}
$$

On the theoretical side, the correlation function can be alternatively calculated in terms of quarks and gluons using the standard OPE technique. Then we equate the results on the phenomenological side with that on the theoretical side. However, since we are only interested in the properties of the ground state at hand, e.g., the $B$ meson, we shall assume that contributions from excited states (on the phenomenological side) are approximated by the spectral density on the theoretical side of the sum rule, which starts from some thresholds (say, $\omega_{i, j}$ in this study). To further improve the final result under consideration, we apply the Borel transform to both external variables $\omega$ and $\omega'$. After the Borel transform [13],

$$
\mathcal{B}[\Pi^{D_j^{(i)}}_{\alpha,\beta}(\omega, \omega')] = \lim_{m_{\omega, \omega'} \to \infty} \lim_{m'_{\omega, \omega'} \to \infty} \frac{1}{n!m!}(\omega')^m(\omega)^n \frac{d}{d\omega'} m(\omega)^{n+1} \frac{d}{d\omega'} n \Pi^{D_j^{(i)}}_{\alpha,\beta}(\omega, \omega'), \tag{4.12}
$$

the sum rule gives

$$
\frac{F^2(m_b) F^2(\mu)}{16} e^{-\bar{\Lambda}/t_1} e^{-\bar{\Lambda}/t_2} d_j^{(i)} = \int_{0}^{\omega_{i, j}} ds \int_{0}^{\omega_{i, j}} ds' e^{-(s/t_1 + s'/t_2)} \rho^{QCD}, \tag{4.13}
$$

where $\omega_{i, j}$ is the threshold of the excited states and $\rho^{QCD}$ is the spectral density on the theoretical side of the sum rule. Because the Borel windows are symmetric in variables $t_1$ and $t_2$, it is natural to choose $t_1 = t_2$. However, unlike the case of the normalization of the Isgur-Wise function at zero recoil, where the Borel mass is approximately twice as large as that in the corresponding two-point sum rule [14], in the present case of the three-point sum rule at hand, we find that the working Borel windows can be chosen as the same as that in the two-point sum rule since in our analysis the output results depend weakly on the Borel mass. Therefore,
we choose \(t_1 = t_2 = t\). By the renormalization group technique, the logarithmic dependence \(\alpha_s \ln(2t/\mu)\) can be summed over to produce a factor like \([\alpha_s(\mu)/\alpha_s(2t)]^\gamma\). After some manipulation we obtain the sum rule results:

\[
\frac{F^2(m_\ell) F^2(\mu)}{16} e^{-2 \Lambda/\mu} \begin{pmatrix}
\frac{\delta_1^{(i)}}{\alpha_s(2t)} \\
\frac{\delta_2^{(i)}}{\alpha_s(\mu)}
\end{pmatrix} \left( \begin{array}{cc}
\frac{\alpha_s(2t)}{\alpha_s(\mu)} & \frac{1}{2} \\
0 & \frac{1}{2}
\end{array} \right) \left( \begin{array}{c}
\text{OPE}_{B_{i,1}} - \frac{3}{4} \text{OPE}_{\epsilon_{i,1}} \\
\frac{1}{6} \text{OPE}_{B_{i,2}} + \text{OPE}_{\epsilon_{i,2}}
\end{array} \right),
\tag{4.14}
\]

where

\[
\begin{align*}
\text{OPE}_{B_{i,j}} & \simeq \frac{1}{4} (\text{OPE})_{2pt;}^{2pt}, \\
\text{OPE}_{\epsilon_{i,j}} & \simeq -\frac{1}{16} \left[ \frac{\langle \bar{q} g_s \sigma \cdot G q \rangle}{8 \pi^2} (1 - e^{-\omega_{i,j}/t}) + \frac{\langle \alpha_s G^2 \rangle}{16 \pi^3} t^2 (1 - e^{-\omega_{i,j}/t})^2 \right], \\
\text{OPE}_{\epsilon_{2,j}} & \simeq O(\alpha_s),
\end{align*}
\tag{4.15}
\]

with

\[
\langle \text{OPE} \rangle_{2pt;}^{2pt} = \frac{1}{2} \left\{ \int_0^{\omega_{i,j}} ds \ s^2 e^{-s/t} \frac{3}{\pi^2} \left[ 1 + \alpha_s \left( \frac{17}{3} + \frac{4 \pi^2}{9} - 2 \ln \frac{s}{t} \right) \right] - \left( 1 + \frac{2 \alpha_s}{\pi} \right) \langle \bar{q} q \rangle + \frac{\langle \bar{q} g_s \sigma \cdot G q \rangle}{16 \pi^2} \right\}.
\tag{4.16}
\]

For reason of consistency, in the following numerical analysis we will neglect the finite part of radiative one loop corrections in \(\text{OPE}_{B_{i,j}}\) and \(\text{OPE}_{\epsilon_{i,j}}\) (and in Eq. (4.19)). The parameter \(\delta\) in (4.14) is some combination of the \(\beta\) functions and anomalous dimensions (see Eq. (4.2) of [15]) and is numerically equal to \(-0.23\). The relevant parameters normalized at the scale \(t\) are related to those at \(\mu\) by [15,13]

\[
\begin{align*}
F(2t) &= F(\mu) \left( \frac{\alpha_s(2t)}{\alpha_s(\mu)} \right)^{-2/\beta_0} \left( 1 - \frac{\delta \alpha_s(\mu)}{\pi} \right), \\
\langle \bar{q} q \rangle_{2t} &= \langle \bar{q} q \rangle_\mu \left( \frac{\alpha_s(2t)}{\alpha_s(\mu)} \right)^{-4/\beta_0}, \\
\langle g_s \bar{q} \sigma \cdot G q \rangle_{2t} &= \langle g_s \bar{q} \sigma \cdot G q \rangle_\mu \left( \frac{\alpha_s(2t)}{\alpha_s(\mu)} \right)^{2/(3 \beta_0)}, \\
\langle \alpha_s G^2 \rangle_{2t} &= \langle \alpha_s G^2 \rangle_\mu,
\end{align*}
\tag{4.17}
\]

where \(\langle \cdots \rangle\) stands for \(\langle 0 | \cdots | 0 \rangle\) and [13]

\[
\begin{align*}
\langle \bar{q} q \rangle_{\mu=1 \text{ GeV}} & = -(240 \text{ MeV})^3, \\
\langle \alpha_s G^2 \rangle_{\mu=1 \text{ GeV}} & = 0.0377 \text{ GeV}^4, \\
\langle \bar{q} g_s \sigma_{\mu \nu} G^\nu \mu q \rangle_{\mu=1 \text{ GeV}} & = (0.8 \text{ GeV}^2) \times \langle \bar{q} q \rangle_{\mu=1 \text{ GeV}}.
\end{align*}
\tag{4.18}
\]

Some remarks are in order. First, in Eqs. (4.14) and (4.15). \(\text{OPE}_{B_i}\) is obtained by substituting \(D_{ij}^{(i)}\) by \(O\) and it can be approximately factorized as the product of \(\text{OPE}_{2pt;i,j}\) with itself, which is the same as the theoretical part in the two-point \(F(\mu)\) sum rule [14,15]. In the series of \(\langle \text{OPE} \rangle_{2pt;i,j}\), we have neglected the contribution proportional to \(\langle \bar{q} q \rangle^2\). (More precisely, it is equal to \(\alpha_s \langle \bar{q} q \rangle^2 \pi/324\); see Ref. [14].) Nevertheless, the result of \(\text{OPE}_{B_i}\) in Eq. (4.15) is reliable up to dimension six, as the contributions from the \(\langle \bar{q} q \rangle^2\) terms in \(\langle \text{OPE} \rangle_{2pt;i,j}\) are much smaller than the term \((1 + \alpha_s/\pi)^2 \langle \bar{q} q \rangle^2 /16\) that we have kept [see Eq. (4.16)]. Second, in \(\text{OPE}_{B_i}\), the contribution involving the gluon condensate is proportional to the light quark
mass and hence can be neglected. Third, \( \text{OPE}_\epsilon \) is the theoretical side of the sum rule, and it is obtained by substituting \( D_j^{(t)} \) by \( T^\nu \). Here we have neglected the dimension-6 four-quark condensate of the type \( \langle q\Gamma\lambda^\nu q\rangle \). Its contribution is much less than that from dimension-five or dimension-four condensates and hence unimportant (see [16] for similar discussions). It should be emphasized that nonfactorizable contributions to the parameters \( B_i \) arise mainly from the \( O^\nu - T^\nu \) operator mixing.

In the following, we compare our analysis with the similar QCD sum rule studies in [16] and [6]. First, Chernyak [16] used the chiral interpolating current for the \( B \) meson, so that all light quark fields in his correlators are purely left-handed. As a result, there are no quark-gluon mixed condensates as these require the presence of both left- and right-handed light quark fields. Instead, the gluon condensate contribution enters into the \( \epsilon_1 \) sum rule with an additional factor of 4 in comparison with ours; thus their \( \text{OPE}_\epsilon \) is in rough agreement with ours. Second, our results for \( \text{OPE}_\epsilon \) are very different from that obtained by Baek et al. [6]. The reason is that their results are mixed with the \( 1^+ \) to \( 1^+ \) transitions. Also a subtraction of the contribution from excited states is not carried out in [6] for the three-point correlation function, though it is justified to do so for two-point correlation functions. Indeed, in the following analysis, one will find that after subtracting the contribution from excited states, the contributions of \( \text{OPE}_\epsilon \) are largely suppressed. Furthermore, as in the study of the \( B \) meson decay constant [14], we find that the renormalization-group effects are very important in the sum rule analysis. Moreover, \( \epsilon_i \) at \( \mu = m_b \) are largely enhanced by renormalization-group effects.

The value of \( F \) in Eq. (4.14) can be substituted by

\[
F^2(\epsilon) = \left[ \frac{\alpha_s(2t)}{\alpha_s(\mu)} \right]^2 \left[ 1 - \frac{2\delta\alpha_s(2t)}{s} \right] \left\{ \int_0^{\omega_0} ds s^2 e^{-s/t} \frac{3}{\pi} \left[ 1 + \frac{\alpha_s(2t)}{\pi} \left( \frac{17}{3} + \frac{4\pi^2}{9} - 2 \ln \frac{s}{t} \right) \right] - \left( 1 + \frac{2\alpha_s(2t)}{\pi} \right) \langle \bar{q}q \rangle_{2t} + \frac{\langle \bar{q}G_qG_q\rangle_{2t}}{16t^2} \right\},
\]

which is from the two-point sum rule approach [15]. Next, to determine the thresholds \( \omega_{i,j} \) we employ the \( B \) meson decay constant \( f_B = (185 \pm 25 \pm 17) \text{ MeV} \) obtained from a recent lattice-QCD calculation [17] and the relation [18]

\[
f_B = \frac{F(m_b)}{\sqrt{m_B}} \left( 1 - \frac{2\alpha_s(m_b)}{3\pi} \right) \left( 1 - \frac{(0.8 \sim 1.1) \text{ GeV}}{m_b} \right),
\]

that takes into account QCD and \( 1/m_b \) corrections. Using the relation between \( F(m_b) \) and \( F(\mu) \) given by Eq. (4.17) and \( m_b = (4.85 \pm 0.25) \text{ GeV} \), we obtain

\[
F(\mu = 1 \text{ GeV}) \cong (0.34 \sim 0.52) \text{ GeV}^{3/2}.
\]

Since the \( \bar{\Lambda} \) parameter in Eq. (4.19) can be replaced by the \( \bar{\Lambda} \) sum rule obtained by applying the differential operator \( t^2 \frac{\partial}{\partial t} \ln \frac{\partial}{\partial t} \) to both sides of Eq. (4.19), the \( F(\mu) \) sum rule can be rewritten as

\[
F^2(\mu) = \left( \text{right hand side of Eq. (4.19))} \times \exp \left[ t \frac{\partial}{\partial t} \ln \right. \text{(right hand side of Eq. (4.19))} \right],
\]

which is \( \bar{\Lambda} \)-free. Then using the result (4.21) as input, the threshold \( \omega_0 \) in the \( F(\mu) \) sum rule, Eq. (4.22), is determined. The result for \( \omega_0 \) is \( 1.25 - 1.65 \text{ GeV} \). A larger \( F(\mu = 1 \text{ GeV}) \) corresponds to a larger \( \omega_0 \). The working Borel window lies in the region \( 0.6 \text{ GeV} < t < 1 \text{ GeV} \), which turns out to be a reasonable choice. Substituting the value of \( \omega_0 \) back into the \( \bar{\Lambda} \) sum rule, we obtain \( \bar{\Lambda} = 0.48 - 0.76 \text{ GeV} \) in the Borel window \( 0.6 \text{ GeV} < t < 1 \text{ GeV} \). This result is consistent with the choice \( m_b = (4.85 \pm 0.25) \text{ GeV} \), recalling that in the heavy quark limit, \( \bar{\Lambda} = m_B - m_b \). To extract the \( d_j^{(t)} \) sum rules, one can take the ratio of Eq. (4.19) and Eq. (4.14) to eliminate the contribution of \( F^2/\exp(\bar{\Lambda}/t) \). This means one has chosen the same \( \bar{\Lambda} \) both in Eq. (4.19) and Eq. (4.14). Since quark-hadron duality is the basic assumption in the QCD sum rule approach, we expect that the same result of \( \bar{\Lambda} \) also can be obtained using the \( \bar{\Lambda} \) sum rules derived from Eq. (4.14) (see [13] for a further discussion). This property can help us to determine consistently the
threshold in 3-point sum rule, Eq. (4.14). Therefore, we can apply the differential operator \( t^2 \partial \ln / \partial t \) to both sides of Eq. (4.14), the \( d^{(i)} \) sum rule, to obtain new \( \Lambda \) sum rules. The requirement of producing a reasonable value for \( \Lambda \), say 0.48 – 0.76 GeV, provides severe constraints on the choices of \( \omega_{i,j} \). With a careful study, we find that the best choice in our analysis is

\[
\omega_{1,1} = -0.02 \text{ GeV} + \omega_0, \quad \omega_{1,2} = -0.5 \text{ GeV} + \omega_0, \quad \omega_{2,2} = -0.22 \text{ GeV} + \omega_0.
\] (4.23)

Applying the above relations with \( \omega_0 = (1.25 \sim 1.65) \text{ GeV} \) and substituting \( F(\mu) \) in Eq. (4.14) by (4.19), we study numerically the \( d^{(i)} \) sum rules. In Fig. 1, we plot \( B_i^v \) and \( \epsilon_i^v \) as a function \( t \), where \( B_i^v = 8d_1^{v(i)}/9 + 2d_2^{v(i)}/3 \), and \( \epsilon_i^v = -4d_1^{v(i)}/27 + 8d_2^{v(i)}/9 \). The dashed and solid curves stand for \( B_i^v \) and \( \epsilon_i^v \), respectively, where we have used \( \omega_0 = 1.4 \text{ GeV} \) (the corresponding decay constant is \( f_B = 175 \sim 195 \text{ MeV} \) or \( F(\mu) = 0.405 \pm 0.005 \text{ GeV}^{3/2} \)). The final results for the hadronic parameters \( B_i \) and \( \epsilon_i \) are (see Fig. 2)

\[
B_i^v(\mu = 1 \text{ GeV}) = 0.60 \pm 0.02, \quad B_2^v(\mu = 1 \text{ GeV}) = 0.61 \pm 0.01, \\
\epsilon_1^v(\mu = 1 \text{ GeV}) = -0.08 \pm 0.01, \quad \epsilon_2^v(\mu = 1 \text{ GeV}) = -0.024 \pm 0.006.
\] (4.24)

The numerical errors come mainly from the uncertainty of \( \omega_0 = 1.25 \sim 1.65 \text{ GeV} \). Some intrinsic errors of the sum rule approach, say quark-hadron duality or \( \alpha_s \) corrections, will not be considered here.

Substituting the above results into Eq. (4.4) yields

\[
B_1(m_b) = 0.96 \pm 0.04 + O(1/m_b), \quad B_2(m_b) = 0.95 \pm 0.02 + O(1/m_b), \\
\epsilon_1(m_b) = -0.14 \pm 0.01 + O(1/m_b), \quad \epsilon_2(m_b) = -0.08 \pm 0.01 + O(1/m_b).
\] (4.25)

It follows from Eq. (3.19) that

\[
\frac{\tau(B^-)}{\tau(B_d)} = 1.11 \pm 0.02, \\
\frac{\tau(B_s)}{\tau(B_d)} \approx 1, \\
\frac{\tau(A_b)}{\tau(B_d)} = 0.99 - \left( \frac{f_B}{185 \text{ MeV}} \right)^2 (0.007 + 0.020 \bar{B}) r,
\] (4.26)

to the order of \( 1/m_b^3 \). Note that we have neglected the corrections of \( \text{SU}(3) \) symmetry breaking to the nonspectator effects in \( \tau(B_s)/\tau(B_d) \). We see that the prediction for \( \tau(B^-)/\tau(B_d) \) is in agreement with
the current world average: $\tau(B^-)/\tau(B_d) = 1.07 \pm 0.03$ [2], whereas the heavy-quark-expansion-based result for $\tau(B_s)/\tau(B_d)$ deviates somewhat from the central value of the world average: $0.94 \pm 0.04$. Thus it is urgent to carry out more precise measurements of the $B_s$ lifetime. Using the existing sum rule estimate for the parameter $r$ [20] together with $\hat{B} = 1$ gives $\tau(\Lambda_b)/\tau(B_d) \geq 0.98$. Therefore, the $1/m_b^3$ nonspectator corrections are not responsible for the observed lifetime difference between the $\Lambda_b$ and $B_d$.

V. CONCLUSIONS

The nonspectator effects can be parametrized in terms of the hadronic parameters $B_1$, $B_2$, $\epsilon_1$ and $\epsilon_2$ [21], where $B_1$ and $B_2$ characterize the matrix elements of color singlet-singlet four-quark operators and $\epsilon_1$ and $\epsilon_2$ the matrix elements of color octet-octet operators. In OPE language, the prediction of $B$ meson lifetime ratios depends on the nonspectator effects of order $16\pi^2/m_b^3$ in the heavy quark expansion. Obviously, the shorter lifetime of the $\Lambda_b$ relative to that of the $B_d$ meson and/or the lifetime ratio $\tau(B_s)/\tau(B_d)$ cannot be explained by the theory so far. It is very likely that local quark-hadron duality is violated in nonleptonic decays.

As emphasized in [5], one should not be contented with the agreement between theory and experiment for the lifetime ratio $\tau(B^-)/\tau(B_d)$. In order to test the OPE approach for inclusive nonleptonic decay, it is even more important to calculate the absolute decay widths of the $B$ mesons and compare them with the data. From (3.6), (3.7), (4.25) and considering the contributions of the nonspectator effects, we obtain

$$
\Gamma_{\text{tot}}(B_d) = (3.61^{+1.04}_{-0.84}) \times 10^{-13} \text{GeV},
$$

$$
\Gamma_{\text{tot}}(B^-) = (3.34^{+1.04}_{-0.84}) \times 10^{-13} \text{GeV},
$$

noting that the next-to-leading QCD radiative correction to the inclusive decay width has been included. The absolute decay widths strongly depend on the value of the $b$ quark mass. The problem with the absolute decay width $\Gamma(B)$ is intimately related to the $B$ meson semileptonic branching ratio $B_{\text{SL}}$. Unlike the semileptonic decays, the heavy quark expansion in inclusive nonleptonic decay is a priori not justified due to the absence of an analytic continuation into the complex plane and hence local duality has to be invoked in order to apply the OPE directly in the physical region.

To conclude, we have derived in heavy quark effective theory the renormalization-group improved sum rules for the hadronic parameters $B_1$, $B_2$, $\epsilon_1$, and $\epsilon_2$ appearing in the matrix element of four-quark operators. The results are $B_1(m_b) = 0.96 \pm 0.04$, $B_2(m_b) = 0.95 \pm 0.02$, $\epsilon_1(m_b) = -0.14 \pm 0.01$ and $\epsilon_2(m_b) = -0.08 \pm 0.01$ to the zeroth order in $1/m_b$. The resultant $B$-meson lifetime ratios are $\tau(B^-)/\tau(B_d) = 1.11 \pm 0.02$ and $\tau(B_s)/\tau(B_d) \approx 1$.

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