Heavy and Light Pentaquark Chiral Lagrangian

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Using the SU(3) flavor symmetry, we construct the chiral Lagrangians for the light and heavy pentaquarks. The correction from the nonzero quark is taken into account perturbatively. We derive the Gell-Mann–Okubo type relations for various pentaquark multiplet masses and Coleman-Glashow relations for anti-sextet heavy pentaquark magnetic moments. We study possible decays of pentaquarks into conventional hadrons. We also study the interactions between and within various pentaquark multiplets and derive their coupling constants in the symmetry limit. Possible kinematically allowed pionic decay modes are pointed out.

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I. INTRODUCTION

Since LEPS Collaboration announced the discovery of the exotic baryon with very narrow width Θ+(1540) [1], many other groups have claimed the observation of this state [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. NA49 observed a new pentaquark Ξ−(1862) [13], which needs confirmation from other groups [14]. Recently, H1 Collaboration claimed the discovery of a heavy pentaquark around 3099 MeV with the quark content udud ¯c [15]. It is interesting to note that several groups reported negative results [16, 17, 18].

There is preliminary evidence that the Θ+ pentaquark is an iso-scalar because no enhancement was observed in the pK+ invariant mass distribution [4, 6, 7, 12]. Most of the theoretical models assume that Θ+ is in SU(3)f ¯10 representation.

The parity of Θ pentaquark remains unknown. Theoretical approaches advocating positive parity include the chiral soliton model (CSM) [19], the diquark model [20], some quark models [21, 22, 23, 24, 25], a lattice calculation [26]. On the other hand, some other theoretical approaches tend to favor negative parity such as two lattice QCD simulations [27, 28], QCD sum rule approaches [29, 30], several quark model study [31, 32, 33], and proposing stable diamond structure for Θ+ [34].

The narrow width of Θ pentaquark is another puzzle. All the experiments can only determine the upper bound of the pentaquark widths up to the detector resolution. The reanalysis of previous pion kaon scattering data indicates the decay width of Θ+ should be one or two MeV or less [35], which makes the theoretical interpretation very difficult.

There have appeared several attempts to explain the narrow width. One possibility is the mismatch between the spin-flavor wave functions of the initial and final state when Θ pentaquark decays through the fall-apart mechanism [23, 36, 37, 38].

Another possible interpretation of the narrow width puzzle is the possible mismatch between the spatial wave functions of initial and final states [39]. The reason is simple. The Θ+ pentaquark with the stable diamond structure and bound by non-planar flux tubes is hard to decay into hadrons bound by planar flux tubes [34]. But this scheme has not been studied quantitatively.

In the chiral soliton model, the narrowness of Θ+ results from the cancellation of the coupling constants at different Nc orders [30]. It is suggested that one of two nearly degenerate pentaquarks sharing the same dominant decay mode can be arranged to decouple from the decay channel after diagonalizing the mixing mass matrix via kaon nucleon loop [40].

Recently heavy pentaquarks have received much attention [20, 21, 28, 11, 12, 33, 14, 35, 16, 17, 18, 19, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51]. In the heavy quark limit, the heavy anti-quark decouples and acts as a spectator. The pentaquark system simplifies significantly. In fact, the heavy pentaquark system can be used as a test-ground of the various models developed for the light pentaquarks.

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Model calculation has shown that the anti-decuplet and the even-parity pentaquark octet lie close to each other and ideal mixing occurs if quantum number allows \[20\]. The odd-parity pentaquark nonet is several hundred MeV lower than the anti-decuplet and even-parity octet. Strong transitions between different pentaquark multiplets may occur \[52\].

At present the underlying dynamics which binds four quarks and one anti-quark into a narrow resonance above threshold is still a mystery. We will explore the strong interactions between pentaquark multiplets using the \(SU(3)\) flavor symmetry as the guide. Chiral Lagrangians have been used to study the strong decay modes of pentaquarks \[49, 52, 53, 54, 55\].

In Section II, we will construct the chiral Lagrangian involving light and heavy pentaquark multiplets. Then we discuss the mass splitting from the current quark mass correction within the same multiplet. In Section IV, we derive the coupling constants of the pentaquark interactions and discuss possible strong decay modes. The final section is a short discussion.

### II. CHIRAL LAGRANGIAN

#### A. Notation

The approximate chiral symmetry and its spontaneous breaking have played an important role in hadron physics. Through the nonlinear realization of spontaneous chiral symmetry breaking, we may study the interaction between the chiral field and hadrons, which always involves the derivative of the chiral field. The nonzero current quark mass breaks the chiral symmetry explicitly. These corrections are taken into account perturbatively together with the chiral loop correction. Generally speaking, chiral symmetry provides a natural framework to organize the hadronic strong interaction associated with the light quarks.

In writing down the pentaquark chiral Lagrangians, we follow the standard notation in the chiral perturbation theory. First the eight Goldstone bosons are introduced exponentially. We use the short-hand notation \(\pi\) to denote them.

\[
\Sigma \equiv \xi^2 = \exp\left(\frac{2i\pi}{F_\pi}\right), \tag{1}
\]

\[
\pi = \begin{pmatrix}
\frac{\pi^0}{\sqrt{2}} + \frac{\eta_0}{\sqrt{6}} & \pi^+ & K^+
\frac{-\pi^0}{\sqrt{2}} + \frac{\eta_0}{\sqrt{6}} & K^0 & -\frac{2\eta_0}{\sqrt{6}}
\end{pmatrix}, \tag{2}
\]

where \(F_\pi = 92.4\) MeV is the pion decay constant.

Under the \(SU(3)_L \times SU(3)_R\) chiral transformation, \(\Sigma(x)\) and \(\xi(x)\) transform as

\[
\Sigma(x) \rightarrow L\Sigma(x)R^\dagger, \\
\xi(x) \rightarrow L\xi(x)U^\dagger(x) = U(x)\xi(x)R^\dagger \tag{3}
\]

where \(L \in SU(3)_L, R \in SU(3)_R, U(x)\) is a non-linear function of \(\pi(x)\) and \(L, R\).

The chiral connection \(V_\mu\) and the axial vector field \(A_\mu\) are defined as

\[
V_\mu = \frac{1}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger), \\
A_\mu = \frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger). \tag{4}
\]

The vector \(V_\mu\) and axial vector \(A_\mu\) transform under chiral \(SU(3)\) as

\[
V_\mu \rightarrow UV_\mu U^\dagger + U\partial_\mu U^\dagger, \\
A_\mu \rightarrow UA_\mu U^\dagger. \tag{5}
\]

With the chiral connection, we can construct the chirally covariant derivative \(D_\mu\). For the matter field \(\phi\) which is in the fundamental representation, we have

\[
D_\mu \phi = (\partial_\mu + V_\mu) \phi, \\
D_\mu \rightarrow UD_\mu U^\dagger. \tag{6}
\]
For the matter field in the adjoint representation like the nucleon octet $B$, we have

$$\mathcal{D}_\mu B = \partial_\mu B + [V_\mu, B]$$

(7)

where the octet baryon field reads

$$(B_j^i)' = \begin{pmatrix} \Sigma^0 & \Delta^+ & p \\ \Sigma^- & -\Sigma^0 & n \\ \Xi^- & -\Xi^0 & \Xi^0 \end{pmatrix}.$$  

(8)

For the $\Delta^{++}$ decuplet, the chirally covariant derivative reads

$$\mathcal{D}_\mu D^{ijk} = \partial_\mu D^{ijk} + V_{\mu, a} D^{ajk} + V_{\mu, a} D^{iak} + V_{\mu, a} D^{jia}.$$  

(9)

B. Matter fields

In Jaffe and Wilczek's diquark model [20], the color wave function of the two diquarks within the pentaquark must be antisymmetric $3_c$. In order to get an exotic anti-decuplet, the two scalar diquarks combine into the symmetric $SU(3)_F$ 6 representation: $[ud]^2$, $[ud]|ds|_{+}$, $[su]^2$, $[su]|ds|_{+}$, $[ds]^2$, and $|ds|ud$. Bose statistics demands symmetric total wave function of the diquark-diquark system, which leads to the antisymmetric spatial wave function with one orbital excitation. The resulting anti-decuplet $P_{ij}$ and octet pentaquarks $O_{ij}^1$ have $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^+$. We use $J^P = \frac{1}{2}^-$ case to illustrate the formalism.

Our discussion makes use of the flavor symmetry only. So the results are valid for both $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^+$. Later, we pointed out [52] that lighter pentaquarks can be formed if the two scalar diquarks are in the antisymmetric $SU(3)_F$ 3 representation: $[ud]|su|_-$, $[ud]|ds|_-$, and $[su]|ds|_-$, where $[q_1q_2][q_3q_4]$ = $\sqrt{2}([q_1q_2][q_3q_4] - [q_3q_4][q_1q_2])$. No orbital excitation is needed to ensure the symmetric total wave function of two diquarks since the spin-flavor-color part is symmetric. The total angular momentum of these pentaquarks is $\frac{1}{2}$ and the parity is negative. There is no accompanying $J = \frac{3}{2}$ multiplet. The two diquarks combine with the antiquark to form a $SU(3)_F$ pentaquark octet $O_{ij}^1$, and singlet $N_1$.

Replacing the light anti-quark by one anti-charm or anti-bottom quark in Jaffe and Wilczek's model leads to one even parity anti-sextet $S_{ij}$ [20, 47] and one odd parity triplet $T^i$ [40, 48]. In Karliner and Lipkin's diquark tri-quark model, there is an additional even parity heavy pentaquark triplet [45]. In the following discussion, we only make use of the flavor symmetry to write down the chiral Lagrangian. Hence the results are not limited to Jaffe and Wilczek's model only. The heavy pentaquark multiplets are

$$(S_{ij}^c) = \begin{pmatrix} \Xi_{-5c}^- & -\frac{1}{\sqrt{2}} \Xi_{5c}^- & \frac{1}{\sqrt{2}} \Xi_{3c}^- \\ -\frac{1}{\sqrt{2}} \Xi_{-5c}^- & \Xi_{5c}^- & \Xi_{3c}^- \\ \Xi_{-5c}^- & \Xi_{5c}^- & -\Xi_{3c}^- \end{pmatrix},$$

(10)

$$(S_{ij}^b) = \begin{pmatrix} \Xi_{-5b}^- & -\frac{1}{\sqrt{2}} \Xi_{5b}^- & \frac{1}{\sqrt{2}} \Xi_{3b}^- \\ -\frac{1}{\sqrt{2}} \Xi_{-5b}^- & \Xi_{5b}^- & \Xi_{3b}^- \\ \Xi_{-5b}^- & \Xi_{5b}^- & -\Xi_{3b}^- \end{pmatrix},$$

$$(T^c_1) = \begin{pmatrix} \Xi_{2b}^0 \\ \Xi_{2b}^- \\ \Xi_{2b}^0 \end{pmatrix},$$

(11)

$$(T^b_1) = \begin{pmatrix} \Xi_{2b}^0 \\ \Xi_{2b}^- \\ \Xi_{2b}^0 \end{pmatrix}. $$

In writing down the chiral Lagrangians, we need the pseudoscalar heavy meson triplet $\bar{Q}'$ in the fundamental representation:

$$(Q_i) = (Q \bar{u}, Q \bar{d}, Q \bar{s}) .$$

(12)
Under chiral transformation, the matter fields transform as

\[ B^i_j \rightarrow U^i_a B^a_j U^b_j, \]
\[ D^{ijk} \rightarrow U^i_a D^a b c D^{abc}, \]
\[ O^{ij}_i \rightarrow U^i_a O^{ia}_b U^b_j, \]
\[ O^{ij}_2 \rightarrow U^i_a O^{ia}_b U^{ab}_2, \]
\[ \Lambda_1 \rightarrow \Lambda_1, \]
\[ P^{ijk} \rightarrow P^{ia}_a U^{ib}_j U^{ic}_k, \]
\[ \bar{Q}^i \rightarrow U^i_a Q^a, \]
\[ S^{ij} \rightarrow S^{ia}_a U^{ib}_j, \]
\[ T^i \rightarrow U^i_a T^a. \]

The chirally covariant derivatives of these matter fields have the same transformation as the matter fields. They are

\[ \mathcal{D}_\mu B^i_j = \partial_\mu B^i_j + V^i_{\mu a} B^a_j - B^i_a V^a_{\mu j}, \]
\[ \mathcal{D}_\mu O^{ij}_i = \partial_\mu O^{ia}_j + V^i_{\mu a} O^{ia}_j - O^{ia}_a V^a_{\mu j}, \]
\[ \mathcal{D}_\mu O^{ij}_2 = \partial_\mu O^{ia}_2 + V^i_{\mu a} O^{ia}_2 - O^{ia}_2 V^a_{\mu j}, \]
\[ \mathcal{D}_\mu P^{ijk} = \partial_\mu P^{ia}_a V^a_{\mu k} + P^{ia}_a V^a_{\mu j} + P^{ia}_a V^a_{\mu i}, \]
\[ \mathcal{D}_\mu \bar{Q}^i = \partial_\mu \bar{Q}^i + V^i_{\mu j} \bar{Q}^j, \]
\[ \mathcal{D}_\mu S^{ij} = \partial_\mu S^{ia}_a V^a_{\mu j} + S^{ia}_a V^a_{\mu i}, \]
\[ \mathcal{D}_\mu T^i = \partial_\mu T^i + V^i_{\mu j} T^j. \]

The current quark mass matrix \( m = \text{diag}(\hat{m}, \hat{m}, m_s) \) transforms as \( m \rightarrow L m R^\dagger = R m L^\dagger \) under \( SU(3)_L \times SU(3)_R \) chiral transformation, where we have ignored the isospin breaking effect and adopt \( m_u = m_d = \hat{m} \). Hence, the following combination of \( m \) and \( \xi \) transforms as the matter field:

\[ (\xi m \xi^\dagger + \xi^\dagger m \xi) \rightarrow U(\xi m \xi^\dagger + \xi^\dagger m \xi) U^\dagger. \]

C. Mass, kinetic term and interaction with the chiral field

With these matter fields and their corresponding transformations under chiral transformation, we first write down the chiral Lagrangian involving mass term, kinematic terms, and interaction terms between the matter field and the chiral field.

\[ \mathcal{L} = \mathcal{L}_\Sigma + \mathcal{L}_B + \mathcal{L}_P + \mathcal{L}_O_1 + \mathcal{L}_O_2 + \mathcal{L}_\Lambda_1 + \mathcal{L}_Q + \mathcal{L}_S + \mathcal{L}_T + \mathcal{L}_{\text{int}}, \]
where

\[ \mathcal{L}_\Sigma = \frac{F^2_\pi}{4} \text{Tr} \left[ \partial_\mu \Sigma \partial^\mu \Sigma - 2 \mu_\Sigma \Sigma + \Sigma \Sigma^\dagger \right] , \]  
\[ \mathcal{L}_B = \text{Tr} \overline{B} (i \mathcal{D} - m_B) B - D_B \text{Tr} \overline{B} \gamma^\mu \gamma^5 \{ A_\mu, B \} - F_B \text{Tr} \overline{B} \gamma^\mu \gamma^5 [ A_\mu, B ] , \]  
\[ \mathcal{L}_D = \mathcal{D} (i \mathcal{D} - m_D) D + \mathcal{G}_D \mathcal{D} \gamma^5 \mathcal{A} D , \]  
\[ \mathcal{L}_P = \overline{P} (i \mathcal{D} - m_P) P + \mathcal{G}_P \overline{P} \gamma^5 \mathcal{A} P , \]  
\[ \mathcal{L}_{O_1} = \text{Tr} \overline{O}_1 (i \mathcal{D} - m_{O_1}) O_1 - D_{O_1} \text{Tr} \overline{O}_1 \gamma^\mu \gamma^5 \{ A_\mu, O_1 \} - F_{O_1} \text{Tr} \overline{O}_1 \gamma^\mu \gamma^5 [ A_\mu, O_1 ] , \]  
\[ \mathcal{L}_{O_2} = \text{Tr} \overline{O}_2 (i \mathcal{D} - m_{O_2}) O_2 - D_{O_2} \text{Tr} \overline{O}_2 \gamma^\mu \gamma^5 \{ A_\mu, O_2 \} - F_{O_2} \text{Tr} \overline{O}_2 \gamma^\mu \gamma^5 [ A_\mu, O_2 ] , \]  
\[ \mathcal{L}_{\Lambda_1} = \overline{\Lambda}_1 \gamma \Lambda_1 + m_{\Lambda_1} \overline{\Lambda}_1 \Lambda_1 , \]  
\[ \mathcal{L}_Q = (\mathcal{D}_\mu Q) (\mathcal{D}^\mu Q) - m_Q^2 Q \bar{Q} , \]  
\[ \mathcal{L}_S = \overline{S} (i \mathcal{D} - m_S) S + \mathcal{G}_S \overline{S} \gamma^5 \mathcal{A} S , \]  
\[ \mathcal{L}_T = \overline{T} (i \mathcal{D} - m_T) T + \mathcal{G}_T \overline{T} \gamma^5 \mathcal{A} T . \]  

In the above equations, \( m_B, m_P \) etc are hadrons masses in the chiral limit.

Keeping the flavor indices explicitly, we get

\[ \mathcal{L}_\Sigma = \frac{F^2_\pi}{4} \left[ (\partial_\mu \Sigma^i) (\partial^\mu \Sigma^j) - 2 \mu_\Sigma \Sigma^i \Sigma^j + \Sigma^i \Sigma^j \right] , \]  
\[ \mathcal{L}_B = \overline{B}_j (i \mathcal{D} - m_B) B^i_j + i \overline{B}_j \gamma^\mu V^j \mu, k \bar{B}^k_i + i \overline{B}_j \gamma^\mu B^k_i \bar{V}^j \mu, k , \]  
\[ + (D_B + F_B) \overline{B}_\alpha \gamma^\mu \gamma^5 \{ A^\alpha_{\mu, i} B^i_k \} + (D_B - F_B) \overline{B}_\alpha \gamma^\mu \gamma^5 \{ B^i_k A^\alpha_{\mu, i} \} , \]  
\[ \mathcal{L}_D = \overline{D}_{ijk} (i \mathcal{D} - m_D) D^{ijk} + i \overline{D}_{ijk} \gamma^\mu D^{ijk} \mu, a + i \overline{D}_{ijk} \gamma^\mu D^{ijk} \mu, a + i \overline{D}_{ijk} \gamma^\mu D^{ijk} \mu, a + i \overline{D}_{ijk} \gamma^\mu D^{ijk} \mu, a , \]  
\[ + \mathcal{G}_D \overline{D}_{ijk} \gamma^\mu A^a_{\mu, i} B^{\alpha, j} , \]  
\[ \mathcal{L}_P = \overline{P}^{ijk} (i \mathcal{D} - m_P) P^{ijk} + i \overline{P}^{ijk} \gamma^\mu P^{ijk} \mu, a \bar{V}^{\alpha, j} \mu, a + i \overline{P}^{ijk} \gamma^\mu P^{ijk} \mu, a \bar{V}^{\alpha, j} \mu, a , \]  
\[ + \mathcal{G}_P \overline{P}^{ijk} A^a_{\mu, a} \bar{B}^{\alpha, i} , \]  
\[ \mathcal{L}_{O_1} = \overline{O}_{ij} (i \mathcal{D} - m_{O_1}) O_{ij} + i \overline{O}_{ij} \gamma^\mu V^j \mu, k \bar{O}_{ij}^k \bar{V}^j \mu, k , \]  
\[ + (D_{O_1} + F_{O_1}) \overline{O}_{ij} \gamma^\mu \gamma^5 \{ A^a_{\mu, i} O_{kj}^b \} + (D_{O_1} - F_{O_1}) \overline{O}_{ij} \gamma^\mu \gamma^5 \{ O_{ij}^a A^a_{\mu, i} \} , \]  
\[ \mathcal{L}_{O_2} = \overline{O}_{ij} (i \mathcal{D} - m_{O_2}) O_{ij} + i \overline{O}_{ij} \gamma^\mu V^j \mu, k \bar{O}_{ij}^k \bar{V}^j \mu, k , \]  
\[ + (D_{O_2} + F_{O_2}) \overline{O}_{ij} \gamma^\mu \gamma^5 \{ A^a_{\mu, i} O_{ij}^b \} + (D_{O_2} - F_{O_2}) \overline{O}_{ij} \gamma^\mu \gamma^5 \{ O_{ij}^a A^a_{\mu, i} \} , \]  
\[ \mathcal{L}_{\Lambda_1} = \overline{\Lambda}_1 \gamma \Lambda_1 + m_{\Lambda_1} \overline{\Lambda}_1 \Lambda_1 , \]  
\[ \mathcal{L}_Q = (\mathcal{D}_\mu Q) (\mathcal{D}^\mu Q) + \partial_\mu Q \bar{V}^{\alpha, j} \mu, a + \partial_\mu Q \bar{V}^{\alpha, j} \mu, a + \partial_\mu Q \bar{V}^{\alpha, j} \mu, a , \]  
\[ + \mathcal{G}_S \overline{S} \gamma^\mu \gamma^5 \{ O_{ij}^a A^a_{\mu, i} \} , \]  
\[ \mathcal{L}_T = \overline{T} (i \mathcal{D} - m_T) T + \mathcal{G}_T \overline{T} \gamma^5 \mathcal{A} T . \]
D. Interaction between different matter fields

The interaction part of the chiral Lagrangians between different matter fields reads

\[ \mathcal{L}_{PAB} = C_{PAB}(\overline{P} \Gamma_P AB + \overline{B} \Gamma_P AP), \]  

\[ \mathcal{L}_{O_{1,AD}} = C_{O_{1,AD}} \bar{O}_1 A^\mu D_\mu + h.c., \]  

\[ \mathcal{L}_{O_{2,AD}} = C_{O_{2,AD}} \overline{\bar{O}_2} \gamma_5 A^\mu D_\mu + h.c., \]  

\[ \mathcal{L}_{O_{1,AB}} = C_{O_{1,AB}} \text{Tr}(\overline{O}_1 \gamma_5 \gamma^\mu \{A_\mu, B\} + \overline{B} \gamma_5 \gamma^\mu \{A_\mu, O_1\}) + \mathcal{H}_{O_{1,AB}} \text{Tr}(\overline{O}_1 \gamma_5 \gamma^\mu [A_\mu, B] + \overline{B} \gamma_5 \gamma^\mu [A_\mu, O_1]), \]  

\[ \mathcal{L}_{O_{1,AP}} = C_{O_{1,AP}} (\overline{O}_1 \Gamma_P AP + \overline{B} \Gamma_P AO_1), \]  

\[ \mathcal{L}_{O_{2,AB}} = C_{O_{2,AB}} \text{Tr}(\overline{O}_2 \gamma^\mu \{A_\mu, B\} + \overline{B} \gamma^\mu \{A_\mu, O_2\}) + \mathcal{H}_{O_{2,AB}} \text{Tr}(\overline{O}_2 \gamma^\mu [A_\mu, B] + \overline{B} \gamma^\mu [A_\mu, O_2]), \]  

\[ \mathcal{L}_{O_{2,AP}} = C_{O_{2,AP}} (\overline{O}_2 \Gamma_P \gamma_5 \gamma^\mu \gamma_5 \gamma^\xi [A_\mu, O_2], O_2), \]  

\[ \mathcal{L}_{O_{2,AO}} = C_{O_{2,AO}} \text{Tr}(\overline{O}_2 \gamma^\mu \{A_\mu, O_1\} + \overline{O}_1 \gamma^\mu \{A_\mu, O_2\}) + \mathcal{H}_{O_{2,AO}} \text{Tr}(\overline{O}_2 \gamma^\mu [A_\mu, O_1] + \overline{O}_1 \gamma^\mu [A_\mu, O_2]), \]  

\[ \mathcal{L}_{A_{1,AB}} = C_{A_{1,AB}} \text{Tr}(\overline{A}_1 \Gamma_A AB + \overline{B} \Gamma_A AA_1), \]  

\[ \mathcal{L}_{A_{1,AO}} = C_{A_{1,AO}} \text{Tr}(\overline{A}_1 \Gamma_A AO_1 + \overline{O}_1 \Gamma_A AA_1), \]  

\[ \mathcal{L}_{A_{1,AO}} = C_{A_{1,AO}} \text{Tr}(\overline{A}_1 \Gamma_A AO_1 + \overline{O}_1 \Gamma_A AA_1), \]  

\[ \mathcal{L}_{SQB} = C_{SQB} \overline{S} T_S \bar{Q} B + h.c., \]  

\[ \mathcal{L}_{SPQ} = C_{SPQ} \overline{S} T_S P Q \bar{Q} + h.c., \]  

\[ \mathcal{L}_{SQQ_1} = C_{SQQ_1} \overline{S} T_S \bar{Q} O_1 + h.c., \]  

\[ \mathcal{L}_{SQQ_2} = C_{SQQ_2} \overline{S} T_S \bar{Q} O_2 + h.c., \]  

\[ \mathcal{L}_{TQB} = C_{TQB} \overline{T} \Gamma_T B \bar{Q} + h.c., \]  

\[ \mathcal{L}_{TQO_1} = C_{TQO_1} \overline{T} \Gamma_T O_1 \bar{Q} + h.c., \]  

\[ \mathcal{L}_{TQO_2} = C_{TQO_2} \overline{T} \Gamma_T \gamma_5 O_2 \bar{Q} + h.c., \]  

\[ \mathcal{L}_{TQA_1} = C_{TQA_1} \overline{T} \Gamma_T Q A_1 + h.c., \]  

\[ \mathcal{L}_{TAS} = C_{TAS} (\overline{T} \Gamma_T AS + \overline{S} T_S AT), \]  

where \( D^\mu \) is the Rarita-Schwinger spinor for the \( \Delta \) decuplet, the subscripts \( P, S, T \) are the parities of pentaquarks (anti-decuplet, anti-sextet, triplet, respectively), and the subscript \( SP \) is the product of \( S \) and \( P \). \( \Gamma_+ = \gamma_5 \), and \( \Gamma_- = 1 \).
With explicit flavor indices we have

\[ \mathcal{L}_{PAB} = C_{PAB} P^{ijk} \Gamma_P A_i^a B_j^b \epsilon_{abk} + h.c., \]  
\[ \mathcal{L}_{O_{1, AD}} = C_{O_{1, AD}} O_{1a}^i A_j^b D^{ijk} \epsilon_{abk} + h.c., \]  
\[ \mathcal{L}_{O_{2, AD}} = C_{O_{2, AD}} O_{2a}^i i \gamma_5 A_j^b D^{ijk} \epsilon_{abk} + h.c., \]  
\[ \mathcal{L}_{O_{1, AB}} = (O_{1, AB} + \mathcal{H}_{O_{1, AB}}) O_{1a}^i \gamma_5 \gamma^\mu A_{\mu, b} B_i^b + \mathcal{H}_{O_{1, AB}} \]  
\[ \mathcal{O}_{1, AB} = (O_{1, AB} - \mathcal{H}_{O_{1, AB}}) O_{1a}^i \gamma_5 \gamma^\mu B_{\mu, b} A_i^b + h.c., \]  
\[ \mathcal{L}_{O_{1, AP}} = C_{O_{1, AP}} O_{1a}^i P^{ijk} P_{ij} \epsilon_{abk} + h.c., \]  
\[ \mathcal{L}_{O_{2, AB}} = (O_{2, AB} + \mathcal{H}_{O_{2, AB}}) O_{2a}^i \gamma_5 \gamma^\mu A_{\mu, b} B_i^b + \mathcal{H}_{O_{2, AB}} \]  
\[ \mathcal{L}_{O_{2, AP}} = C_{O_{2, AP}} O_{2a}^i P^{ijk} P_{ij} \epsilon_{abk} + h.c., \]  
\[ \mathcal{L}_{O_{2, AO_1}} = (O_{2, AO_1} + \mathcal{H}_{O_{2, AO_1}}) O_{2a}^i \gamma_5 \gamma^\mu A_{\mu, b} O_i^b + \mathcal{H}_{O_{2, AO_1}} \]  
\[ \mathcal{L}_{O_{2, AO_2}} = (O_{2, AO_2} - \mathcal{H}_{O_{2, AO_2}}) O_{2a}^i \gamma_5 \gamma^\mu O_{1b} A_{\mu, i}^b + h.c., \]  
\[ \mathcal{L}_{P} = C_{P} \bar{P} (\xi m \xi + \xi^4 m \xi^4) P, \]  
\[ L_{O_1} = \alpha_{O_1} \text{Tr} (d_1 O_1 \{ \xi m \xi + \xi^4 m \xi^4, O_1 \} + f_1 O_1 \xi m \xi + \xi^4 m \xi^4, O_1) \] 
\[ + \beta_{O_1} \text{Tr} (O_1 O_1) \tr (m \Sigma + \Sigma^4 m), \]  
\[ L_{O_2} = \alpha_{O_2} \text{Tr} (d_2 O_2 \{ \xi m \xi + \xi^4 m \xi^4, O_2 \} + f_2 O_2 \xi m \xi + \xi^4 m \xi^4, O_2) \] 
\[ + \beta_{O_2} \text{Tr} (O_2 O_2) \tr (m \Sigma + \Sigma^4 m), \]  
\[ L_S = \alpha_S \bar{S} (\xi m \xi + \xi^4 m \xi^4) S. \]  

III. MASS AND MAGNETIC MOMENT RELATIONS

A. Mass relations

We include the nonzero current quark mass correction, which induces mass splitting in the multiplet. These symmetry breaking terms for various pentaquark multiplets are

\[ L_P = \alpha_P \bar{P} (\xi m \xi + \xi^4 m \xi^4) P, \]  
\[ L_{O_1} = \alpha_{O_1} \text{Tr} (d_1 O_1 \{ \xi m \xi + \xi^4 m \xi^4, O_1 \} + f_1 O_1 \xi m \xi + \xi^4 m \xi^4, O_1) \] 
\[ + \beta_{O_1} \text{Tr} (O_1 O_1) \tr (m \Sigma + \Sigma^4 m), \]  
\[ L_{O_2} = \alpha_{O_2} \text{Tr} (d_2 O_2 \{ \xi m \xi + \xi^4 m \xi^4, O_2 \} + f_2 O_2 \xi m \xi + \xi^4 m \xi^4, O_2) \] 
\[ + \beta_{O_2} \text{Tr} (O_2 O_2) \tr (m \Sigma + \Sigma^4 m), \]  
\[ L_S = \alpha_S \bar{S} (\xi m \xi + \xi^4 m \xi^4) S. \]
Hence we have the mass relation
\[ O \]
which was first derived in Ref. [52]. The pentaquark octet come from heavy anti-quark only.

\[ P \]
Mixed pentaquark anti-decuplet

From the above mass splittings, we can derive the following mass relations.

\[ \Delta m_{O_{10}} = 2 \alpha_p m_s, \] (25a)
\[ \Delta m_{\Sigma_{10}} = \frac{2}{3} \alpha_p (2 \hat{m} + m_s), \] (25b)
\[ \Delta m_{\Xi_{10}} = \frac{2}{3} \alpha_p (2 \hat{m} + m_s), \] (25c)
\[ \Delta m_{\Omega_{10}} = 2 \alpha_p \hat{m}. \] (25d)

From the above mass splittings, we can derive the following mass relations.

\[ m_{N_{10}} - m_{\Sigma_{10}} = m_\Theta - m_{N_{10}}, \] (26a)
\[ m_{\Sigma_{10}} - m_{\Xi_{10}} = m_{N_{10}} - m_{\Sigma_{10}}. \] (26b)

These relations have already been derived using the chiral soliton model [19] and chiral Lagrangian approach [53, 54]. The equal splitting for anti-decuplet pentaquark was also discussed in Ref. [32].

Similarly, for the pentaquark octet \( O_2 \)

\[ \Delta m_{N_{s_2}} = [2 \beta_{O_2} + \alpha_{O_2} (d + f)](2 \hat{m}) + [\beta_{O_2} + \alpha_{O_2} (d - f)](2 m_s), \] (27a)
\[ \Delta m_{\Sigma_{s_2}} = (\beta_{O_2} + \alpha_{O_2} d)(4 \hat{m}) + 2 \beta_{O_2} m_s, \] (27b)
\[ \Delta m_{\Xi_{s_2}} = [2 \beta_{O_2} + \alpha_{O_2} (d - f)](2 \hat{m}) + [\beta_{O_2} + \alpha_{O_2} (d + f)](2 m_s), \] (27c)
\[ \Delta m_{\Lambda_{s_2}} = (\beta_{O_2} + \frac{1}{3} \alpha_{O_2} d)(4 \hat{m}) + (\beta_{O_2} + \frac{4}{3} \alpha_{O_2} d)(2 m_s). \] (27d)

Hence we have the mass relation

\[ 2 M_{N_b} + 2 M_{\Xi_b} = 3 M_{\Lambda_b} + M_{\Sigma_b}, \] (28)

which was first derived in Ref. [52]. The pentaquark octet \( O_1 \) has similar expression. The mass relations for ideally mixed pentaquark anti-decuplet \( P \) and pentaquark octet \( O_1 \) have been discussed in Ref. [53].

For the heavy pentaquark anti-sextet \( S_c \) and \( S_b \) we get

\[ \Delta m_{\Xi_{s_Q}} = 2 \alpha_{s_Q} \hat{m}, \] (29a)
\[ \Delta m_{\Sigma_{s_Q}} = \alpha_{s_Q} (m + m_s), \] (29b)
\[ \Delta m_{\Theta_{s_Q}} = 2 \alpha_{s_Q} m_s. \] (29c)

\[ M_{\Xi_{s_Q}} - M_{\Sigma_{s_Q}} = M_{\Sigma_{s_Q}} - M_{\Theta_{s_Q}}. \] (30)

The heavy pentaquark mass splittings have been discussed in Refs [20, 21, 47, 49]. Especially in the diquark model it is very simple to derive this mass relation with the Hamiltonian \( H_s = M + n_s (m_s + \alpha) \).

### B. Heavy pentaquark magnetic moment relations

As in Refs. [52, 56, 57], we can derive the magnetic moment relations of heavy pentaquark anti-sextet [47] in Jaffe and Wilczek’s model. Interested readers are referred to Refs. [47, 52, 58, 59, 60] for details. Here we list the results only.

\[ \mu_{\Xi_{s_Q}}^0 + \mu_{\Xi_{s_Q}}^{-} = 2 \mu_{\Xi_{s_Q}}^{-}, \] (31a)
\[ \frac{3 \mu_{\Theta_{s_b}} - \mu_{\Sigma_{s_b}} - 2 \mu_{\Xi_{s_b}}}{2 \mu_{\Xi_{s_b}}} = \mu_{\Xi_{s_b}}^{-}, \mu_{\Xi_{s_b}}^{-}, \] (31b)
\[ \mu_{\Xi_{s_b}}^{+} + \mu_{\Xi_{s_b}}^{-} = 2 \mu_{\Xi_{s_b}}^{0}, \] (31c)
\[ \frac{3 \mu_{\Theta_{s_b}} - \mu_{\Sigma_{s_b}} - 2 \mu_{\Xi_{s_b}}}{2 \mu_{\Xi_{s_b}}} = \mu_{\Xi_{s_b}}^{+}, \mu_{\Xi_{s_b}}^{-}. \] (31d)

These relations hold for both \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+ \) anti-sextet in JW’s model.

The magnetic moments of \( J^P = \frac{1}{2}^- \) heavy pentaquark triplet in the diquark model are all identical because they come from heavy anti-quark only.
IV. POSSIBLE STRONG DECAYS AND COUPLING CONSTANTS

Besides the possible decays of pentaquarks into conventional hadrons, we also consider the strong interactions and possible transitions between pentaquark multiplets. If pentaquarks are bound by flux tubes and have the non-planar diamond structure as suggested in [34], then the possible transitions between pentaquarks might get enhanced because of the special stable structure although the decay phase space is smaller. Expanding the interaction terms in the previous section we obtain the coupling constants for different decay modes. We present the results up to one pseudoscalar meson field. Since some interaction terms have similar flavor structure, it’s enough to consider the following pieces: $L_{A_1AO_1}$, $L_{O_{1,AP}}$, $L_{O_{2,SOQ}}, L_{SQP}, L_{TQB}$, and $L_{TAS}$.

A. Possible strong decays of anti-decuplet pentaquark $P_{ij}$

$SU(3)$ symmetry forbids the anti-decuplet to decay into the $\Delta$ decuplet and pion octet or the $A_1\pi$. The chiral Lagrangian and couplings of the anti-decuplet with pseudoscalar meson octet $M$ and nucleon octet $B$ can be found in Refs. [52, 53].

The anti-decuplet pentaquarks $P_{ij}$ and the octet $O_1$ pentaquarks lie close to each other [20]. Especially some states are nearly degenerate and mix ideally. So the strong interaction between these two multiplets is very important. One example is the identification of $N(1440)$ and $N(1710)$ as nucleon-like pentaquarks in the diquark model [20]. Such a big mass splitting after the diagonalization of the mixing mass matrix will allow the pionic transition to occur kinematically. We collect the couplings of the pentaquark anti-decuplet with even-parity pentaquark octet and pseudoscalar octet in Table I.

We want to emphasize that the odd-parity pentaquark octet $O_2$ lies much lower than the anti-decuplet. Pionic decay modes $P \rightarrow O_2 \pi$ are allowed kinematically in many channels. The coupling constants can also be found from Table I.

Replacing the octet $O_1$ with corresponding $B^3_1$ in Table I one gets the coupling constants of pentaquark anti-decuplet with nucleon octet and pseudoscalar meson octet. We note that there is a sign difference in some terms compared with those in Ref. [52].

B. Possible strong decays of light pentaquark octet $O_{1,2}$

The couplings of light pentaquark octet $O_{1,2}$ with pseudoscalar meson octet $M$ and nucleon octet $B$ can be found in Ref. [52, 53].

The even-parity and odd-parity octet pentaquarks can also decay into the $\Delta$ decuplet and the pseudoscalar meson octet. Jaffe and Wilczek pointed out that the decay mode $\Xi^{-} \rightarrow \Xi^{\ast 0}\pi^{-}$ observed by NA49 Collaboration may indicate the possible existence of the even-parity octet [20]. Since these decay modes can be measured in the near future, we present the couplings of the octet pentaquarks $O_{1,2}$ with the decuplet baryon and the pion octet in Table II.

Similarly, since the odd-parity octet $O_2$ is lower than the even-parity octet $O_1$, pionic decay modes $O_1 \rightarrow O_2 \pi$ are allowed kinematically in some channels. The couplings are collected in Table I. One can also get the coupling constants of pentaquark octet with nucleon octet and pseudoscalar meson octet from Table II with special $b$ and proper replacement [52, 53].

It’s straightforward to derive the coupling of pentaquark singlet $\Lambda_1$ with pentaquark octet $O_{1j}$ and pseudoscalar meson octet $\pi_j^1$:

$$L_{\Lambda_1 AO_1} = -\frac{1}{F_\pi}C_{\Lambda_1 AO_1}\sum_{A_1}(\phi_{\pi_0}^{\ast}S_{8,1}^{\ast} + \phi_{\pi_0}^{\ast}S_{8,1}^{-\ast} + \phi_{\pi_0}^{\ast}S_{8,1}^{+})$$

$$+ \phi_{K_0}^{\ast}S_{8,1}^{\ast} + \phi_{K_0}^{\ast}S_{8,1}^{-\ast} + \phi_{K_0}^{\ast}S_{8,1}^{+} + h.c. \ .$$

(32)

C. Possible strong decays of heavy pentaquarks

The interaction of heavy pentaquarks with the heavy vector meson and nucleon octet has the same flavor structure as in the case of heavy pseudoscalar mesons. It is interesting to note that the heavy pentaquark observed by H1 Collaboration sits right on the threshold of $\Delta$ and $D$ meson. One may wonder whether this resonance is affected largely by the threshold behavior. However, in the $SU(3)$ symmetry limit the heavy pentaquark anti-sextet can not decay into an decuplet and a heavy pseudoscalar meson. In other words, this state can not be explained as a coupled
channel effect between $D^*-p$ and $D\Delta$ through t-channel pion exchange. The anti-sixtlet will not decay into $\Lambda_1$ plus a heavy meson. Similarly, the heavy triplet will not decay into the $\Delta$ decuplet plus a heavy meson or the anti-decuptet plus a heavy meson.

The interaction between heavy pentaquarks, nucleon octet $B$ and pseudoscalar meson octet $M$ are discussed in [48, 49]. In Jaffe and Wilczek’s diquark model, the odd-parity heavy pentaquark triplet is much lower than the even-parity heavy sextet. Pionic decays $S \to T \pi$ may happen in many channels. It will be very interesting to explore this kind of decay process experimentally. Now the heavy quark acts as a spectator. We collect the relevant coupling constants in Table IV.

Similarly, the heavy triplet will not decay into the $\Delta$ decuplet plus a heavy meson or the anti-decuptet plus a heavy meson.

For completeness, we also consider the interaction within the same pentaquark multiplet arising from these terms: $G_P\bar{P}\gamma_5AP$, $(D_O + F_O)\text{Tr}((\bar{O}\gamma_\mu)A_\mu O) + (D_O - F_O)\text{Tr}((\bar{O}\gamma_\mu)A_\mu O)$, $G_S\bar{S}\gamma_5AS$ and $G_T\bar{T}\gamma_5AT$. The coupling constants for pentaquark octet $O_1$ or $O_2$ can be found in Table V through simple replacement. We collect other couplings in Table VIII-X. All these processes might be forbidden by kinematics.

\subsection*{D. Interaction of pentaquarks within the same multiplet}

For completeness, we also consider the interaction within the same pentaquark multiplet arising from these terms: $G_P\bar{P}\gamma_5AP$, $(D_O + F_O)\text{Tr}((\bar{O}\gamma_\mu)A_\mu O) + (D_O - F_O)\text{Tr}((\bar{O}\gamma_\mu)A_\mu O)$, $G_S\bar{S}\gamma_5AS$ and $G_T\bar{T}\gamma_5AT$. The coupling constants for pentaquark octet $O_1$ or $O_2$ can be found in Table V through simple replacement. We collect other couplings in Table VIII-X.

\section*{E. Decay widths}

There are four types of interaction terms corresponding to four kinds of Lorentz structures depending on the parities of the pentaquarks.

\begin{align}
& a_1\bar{F}_1\gamma_5\gamma^\mu\partial_\mu M F_2 \\
& a_2\bar{F}_1\gamma^\mu\partial_\mu M F_2 \\
& a_3\bar{F}_1\gamma_5\bar{Q} F_2 \\
& a_4\bar{F}_1\bar{Q} F_2,
\end{align}

where $F_1$, $F_2$ denotes initial and final fermions respectively, $M$ and $Q$ are the pseudoscalar mesons. The corresponding decay widths are

\begin{align}
\Gamma_1 &= \frac{|p^*|^2}{8\pi m_1^2} a_1^2 (m_1 + m_2)^2 [(m_1 - m_2)^2 - m_M^2] \\
\Gamma_2 &= \frac{|p^*|^2}{8\pi m_1^2} a_2^2 (m_1 - m_2)^2 [(m_1 + m_2)^2 - m_M^2] \\
\Gamma_3 &= \frac{|p^*|^2}{8\pi m_1^2} a_3^2 [(m_1 - m_2)^2 - m_Q^2] \\
\Gamma_4 &= \frac{|p^*|^2}{8\pi m_1^2} a_4^2 [(m_1 + m_2)^2 - m_Q^2],
\end{align}

where $p^*$ is the meson momentum in the parent particle $F_1$ rest frame.

\begin{equation}
|p^*|^2 = \frac{1}{4m_1^2} [m_1^2 - (m_2 + m_M)^2][m_1^2 - (m_2 - m_M)^2].
\end{equation}

For example, $\Theta^+$ width reads

\begin{align}
\Gamma_{\Theta^+} &= 2\Gamma_{\Theta^+ \to K^+n} = 2\Gamma_{\Theta^+ \to K^0p} \\
&= \frac{C^2_{\Theta A}\bar{P}_p}{4\pi F_2^2 m_\Theta} (m_\Theta + m_N)^2 [(m_\Theta - m_N)^2 - m_K^2].
\end{align}

If the mass of $\Lambda_1$ is around 1405 MeV [52], it decays into $\pi\Sigma$ only. Its width is

\begin{align}
\Gamma_{\Lambda_1} &= 3\Gamma_{\Lambda_1 \to \pi^+\Sigma^-} \\
&= \frac{3C^2_{\Lambda A}\bar{P}_\pi}{8\pi F_2^2 m_{\Lambda_1}} (m_{\Lambda_1} - m_\Sigma)^2 [(m_{\Lambda_1} + m_\Sigma)^2 - m_\pi^2].
\end{align}
Even with $|C_{A,AB}| = 10|C_{P,AB}|$, hence $\Gamma_{A_1}/\Gamma_{A_0} \approx 43$, $A_1$ is still not a broad resonance assuming the current experimental upper bound of $\Theta^+$ width.

The ratio $BR(\Xi^{−−}_{10} \to \Sigma^− K^−)/BR(\Xi^{−−}_{10} \to \Xi^− π^−)$ are different for positive and negative parity pentaquark, which is independent of models. This property has been proposed to determine the parity of pentaquark anti-decuplet in Ref. [52]. If we assume $\Gamma(\Xi^{−−}_{10} \to \Xi^− π^−)$ is significantly smaller than $\Gamma(\Xi^{−−}_{10} \to \Sigma^− K^−)$ and $\Gamma(\Xi^{−−}_{10} \to \Xi^− π^−)$ because of phase space suppression, then the ratio $\Gamma_{\Xi^{−−}_{10}}/\Gamma_{\Theta^+}$ is about 4.1 for positive parity and 2.0 for negative parity with $m_{\Xi^{−−}_{10}} = 1862$ MeV. If their widths can be measured accurately, the parity of the anti-decuplet can be determined [52].

V. SUMMARY AND DISCUSSIONS

We have constructed the chiral Lagrangian involving six $SU(3)_f$ pentaquark multiplets. In the framework of Jaffe and Wilczek's diquark model, these pentaquark multiplets include one even-parity anti-decuplet, one even-parity octet, one odd-parity octet, one odd-parity singlet, one even-parity heavy anti-sextet, and one heavy triplet. However our discussion relies on the $SU(3)$ symmetry only. Therefore the results are general and not limited to this particular model.

After taking into account of the symmetry breaking correction from the non-zero quark mass, we have derived the Gell-Mann–Okubo mass relations for different pentaquark multiplets. Similarly, we have also derived the Coleman-Glashow relations for heavy pentaquark magnetic moments. We have discussed the couplings of pentaquarks with other pentaquarks and pseudoscalar mesons. We have also investigated the possible decays of pentaquarks into the $\Delta$ decuplet and pseudoscalar mesons.

If symmetry and kinematics allow, the most efficient decay mechanism of pentaquarks is for the four quarks and one anti-quark to regroup with each other into a three-quark baryon and a meson. This is in contrast to the $^3P_0$ decay models for the ordinary hadrons. This regrouping is coined as the "fall-apart" mechanism, which leads to selection rules in the octet pentaquark decays. This "fall-apart" decay mechanism can be taken care of in the chiral Lagrangian formalism through keeping the flavor indices explicitly [52, 53]. The couplings of two octet baryons with a pseudoscalar mesons with the general $F/D$ flavor structure is presented in Table III. It is pointed out that the "fall-apart" mechanism requires $b = \frac{1}{2}$ for the even-parity pentaquark octet decays into nucleon octet and pseudoscalar meson [52]. In contrast, this mechanism requires $b = -1$ for the odd-parity pentaquark octet decays into nucleon octet and pseudoscalar meson [52].

We collect all the possible decay modes of $\Theta^+$, $\Xi^{−−}_{10}$ and $\Theta^0_c$ in Table XI with corresponding coupling constants in JW's model. We find that $\Xi^{−−}_{10}$ can also decay into $\Xi^{−−}_{8,2}$ via the emission of a $\pi^−$. The heavy pentaquark $\Theta^0_c$ has four decay channels, $D^− p$, $D^0 n$, $D^− p$ and $D^0 n$. The decay modes and couplings of the other exotic anti-decuplet members and anti-sextet are also included in the table. Using the the mass of $\Theta^0_c$ from H1 experiment as a constraint, we have updated our old mass estimate of heavy pentaquarks in [47] and use the new values to analyze the possible decay modes in Table XI. Hopefully our present study may help the future experimental discovery of those missing pentaquarks.

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| Θ\(^+\) | N\(_{10}^{+}\) | N\(_{10}^{0}\) | Σ\(_{10}^{+}\) |
|---|---|---|---|
| K\(^+\)Σ\(_{8,1}\) | 1 | π\(^+\)Σ\(_{8,1}\) | 1 |
| K\(^0\)Σ\(_{8,1}\) | -1 | π\(^0\)Σ\(_{8,1}\) | -1 |
| K\(^-\)Σ\(_{8,1}\) | 1 | π\(^-\)Σ\(_{8,1}\) | 1 |
| K\(^0\)Ω\(_{8,1}\) | -1 | π\(^0\)Ω\(_{8,1}\) | -1 |
| K\(^-\)Ω\(_{8,1}\) | 1 | π\(^-\)Ω\(_{8,1}\) | 1 |
| K\(^0\)Ω\(^\pm\) | -1 | π\(^0\)Ω\(^\pm\) | -1 |
| K\(^-\)Ω\(^\mp\) | 1 | π\(^-\)Ω\(^\mp\) | 1 |

**Table I:** Couplings of the pentaquark anti-decuplet \(P_{ijk}\) with the pentaquark octet \(O_{1j}^i\) and pseudoscalar meson octet \(π_j^i\). The universal coupling constant \(-\frac{1}{π}C_{O_{1A}}\) is omitted.

| Ξ\(_{8,1}\) | Ξ\(_{8,1}^0\) | Ξ\(_{8,1}^+\) | Ξ\(_{8,1}^-\) |
|---|---|---|---|
| π\(^-\)Ξ\(_{8,1}\) | -1 | π\(^-\)Ξ\(_{8,1}^+\) | -1 |
| π\(^-\)Ξ\(_{8,1}^0\) | -1 | π\(^-\)Ξ\(_{8,1}^0\) | -1 |
| π\(^-\)Ξ\(_{8,1}^-\) | -1 | π\(^-\)Ξ\(_{8,1}^-\) | -1 |
| π\(^-\)Ξ\(_{8,1}^\mp\) | -1 | π\(^-\)Ξ\(_{8,1}^\mp\) | -1 |
| K\(^0\)Ξ\(_{8,1}\) | -1 | K\(^0\)Ξ\(_{8,1}^+\) | -1 |
| K\(^0\)Ξ\(_{8,1}^0\) | -1 | K\(^0\)Ξ\(_{8,1}^0\) | -1 |
| K\(^0\)Ξ\(_{8,1}^-\) | -1 | K\(^0\)Ξ\(_{8,1}^-\) | -1 |
| K\(^0\)Ξ\(_{8,1}^\mp\) | -1 | K\(^0\)Ξ\(_{8,1}^\mp\) | -1 |

**Table II:** Couplings of the pentaquark octet \(O_{1j}^i\) with the baryon decuplet \(D_{ijk}\) and pseudoscalar meson octet \(π_j^i\). The universal coupling constant \(-\frac{1}{π}C_{O_{1A}}\) is omitted. Except the universal coupling constant, the couplings of \(O_2\) is the same.
TABLE V: Couplings of the heavy pentaquark anti-sextet $S_{ij}$ with the light pentaquark octet $O_{ij}^+$ and heavy flavor pseudoscalar meson triplet $Q_i$. The universal coupling constant $-\frac{1}{\sqrt{2}}C_{QO_{1j}}$ is omitted.
\( \Sigma_{0}^{\pm} (\Sigma_{8}^{\pm}) \quad \Sigma_{0}^{\mp} (\Sigma_{8}^{\mp}) \quad \Xi_{0}^{\pm} (\Xi_{8}^{\pm}) \)

| \( D^{0} (B^{+}) \bar{\Xi}_{0}^{-} \) | \( D^{0} (B^{+}) \Xi_{0}^{-} \) | \( D^{0} (B^{+}) \Xi_{8}^{-} \) | \( D^{0} (B^{+}) \Xi_{10}^{-} \) |
|-----------------|-----------------|-----------------|-----------------|
| \( \sqrt{\frac{1}{3}} \) | \( \sqrt{\frac{1}{3}} \) | \( -\sqrt{\frac{1}{3}} \) | \( -\sqrt{\frac{1}{3}} \) |
| \( D^{-} (B^{0}) \bar{\Xi}_{0}^{-} \) | \( D^{-} (B^{0}) \Xi_{0}^{-} \) | \( D^{-} (B^{0}) \Xi_{8}^{-} \) | \( D^{-} (B^{0}) \Xi_{10}^{-} \) |
| \( -\sqrt{\frac{1}{3}} \) | \( -\sqrt{\frac{1}{3}} \) | \( \sqrt{\frac{1}{3}} \) | \( \sqrt{\frac{1}{3}} \) |
| \( D_{-}^{*} (B_{0}^{+}) \Sigma_{0}^{+} \) | \( D_{-}^{*} (B_{0}^{+}) \Sigma_{8}^{+} \) | \( D_{-}^{*} (B_{0}^{+}) \Sigma_{10}^{+} \) | \( \sqrt{\frac{1}{3}} \) |
| \( D_{-}^{*} (B_{0}^{+}) \bar{\Sigma}_{0}^{+} \) | \( D_{-}^{*} (B_{0}^{+}) \bar{\Sigma}_{8}^{+} \) | \( D_{-}^{*} (B_{0}^{+}) \bar{\Sigma}_{10}^{+} \) | |\( \sqrt{\frac{1}{3}} \) |

**TABLE VI:** Couplings of the heavy pentaquark anti-sextet \( S_{ij} \) with the pentaquark anti-decplet \( P_{ijk} \) and heavy flavor pseudoscalar meson triplet \( \bar{Q} \). The universal coupling constant \( C_{SQP} \) is omitted.

\[ \begin{array}{c|c|c|c|c|c} \Sigma_{0}^{0} (\Sigma_{8}^{0}) & \Sigma_{0}^{0} (\Sigma_{8}^{0}) & \Xi_{0}^{0} (\Xi_{8}^{0}) \\ \hline D^{0} (B^{+}) \Sigma_{0}^{+} & 1 & D^{0} (B^{+}) \Sigma_{8}^{+} & 1 & D^{0} (B^{+}) \Sigma_{10}^{+} \\ D^{-} (B^{0}) \Sigma_{0}^{+} & -\sqrt{\frac{1}{3}} & D^{-} (B^{0}) \Sigma_{8}^{+} & -\sqrt{\frac{1}{3}} & D^{-} (B^{0}) \Sigma_{10}^{+} \\ D_{-}^{*} (B_{0}^{+}) \Sigma_{0}^{+} & \sqrt{\frac{1}{3}} & D_{-}^{*} (B_{0}^{+}) \Sigma_{8}^{+} & \sqrt{\frac{1}{3}} & D_{-}^{*} (B_{0}^{+}) \Sigma_{10}^{+} \\ \hline D^{0} (B^{+}) \Sigma_{0}^{0} & \sqrt{\frac{1}{3}} & D^{0} (B^{+}) \Sigma_{8}^{0} & \sqrt{\frac{1}{3}} & D^{0} (B^{+}) \Sigma_{10}^{0} \\ D^{-} (B^{0}) \Sigma_{0}^{0} & -\sqrt{\frac{1}{3}} & D^{-} (B^{0}) \Sigma_{8}^{0} & -\sqrt{\frac{1}{3}} & D^{-} (B^{0}) \Sigma_{10}^{0} \\ D_{-}^{*} (B_{0}^{+}) N_{0}^{0} & \sqrt{\frac{1}{3}} & D_{-}^{*} (B_{0}^{+}) N_{8}^{0} & \sqrt{\frac{1}{3}} & D_{-}^{*} (B_{0}^{+}) N_{10}^{0} \\ \hline \end{array} \]

**TABLE VII:** Couplings of the heavy pentaquark triplet \( T^{i} \) with the pentaquark octet \( O_{i}^{j} \) and heavy flavor pseudoscalar meson triplet \( \bar{Q} \). The universal coupling constant \( C_{TQO} \) is omitted.

\[ \begin{array}{c|c|c|c|c|c} \Theta^{+} & N_{10}^{+} & N_{10}^{0} & N_{10}^{-} & \Sigma_{10}^{+} \\ \hline K^{+} N_{10}^{+} & \frac{1}{\sqrt{2}} & \pi^{+} N_{10}^{+} & -\frac{1}{\sqrt{2}} & \pi^{+} N_{10}^{0} \\ K^{0} N_{10}^{+} & \frac{1}{\sqrt{2}} & \pi^{0} N_{10}^{+} & -\frac{1}{\sqrt{2}} & \pi^{0} N_{10}^{0} \\ \eta_{0} \Theta^{+} & -\frac{1}{\sqrt{8}} & \eta_{0} N_{10}^{+} & -\frac{1}{\sqrt{8}} & \eta_{0} N_{10}^{0} \\ \hline K^{+} \Sigma_{10}^{+} & \frac{1}{\sqrt{2}} & K^{+} \Sigma_{10}^{0} & -\frac{1}{\sqrt{2}} & K^{+} \Sigma_{10}^{0} \\ K^{0} \Sigma_{10}^{+} & \frac{1}{\sqrt{2}} & K^{0} \Sigma_{10}^{0} & -\frac{1}{\sqrt{2}} & K^{0} \Sigma_{10}^{0} \\ \hline \end{array} \]

**TABLE VIII:** Couplings of the pentaquark anti-decplet \( P_{ijk} \) with pseudoscalar meson octet \( \pi_{i}^{j} \). The universal coupling constant \( -\frac{1}{\pi_{i}^{j}} G_{P} \) is omitted.
TABLE IX: Couplings of the heavy pentaquark anti-sextet $S_{ij}$ with pseudoscalar meson octet $\pi^j$. The universal coupling constant $-\frac{1}{\sqrt{2}}G_S$ is omitted.

|       | $\Xi_{0c}^-$ ($\Xi_{10b}$) | $\Xi_{0c}^0$ ($\Xi_{10b}^0$) | $\Xi_{0c}^0$ ($\Xi_{10b}^0$) |
|-------|---------------------------|---------------------------|---------------------------|
| $\pi^0\Xi_{0c}^-$ ($\Xi_{10b}^0$) | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\pi^-\Xi_{0c}^-$ ($\Xi_{10b}^0$) | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\eta\Xi_{0c}^-$ ($\Xi_{10b}^0$) | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ |
| $K^-\Sigma^-_{1c} (\Sigma_{10b}^-)$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |

|       | $\Sigma_{0c}^0$ ($\Sigma_{10b}^0$) | $\Sigma_{0c}^0$ ($\Sigma_{10b}^0$) | $\Theta_{0c}^0$ ($\Theta_{10b}^0$) |
|-------|---------------------|---------------------|---------------------|
| $\pi^0\Sigma_{0c}^0$ ($\Sigma_{10b}^0$) | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\pi^-\Sigma_{0c}^0$ ($\Sigma_{10b}^0$) | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $\eta\Sigma_{0c}^- (\Sigma_{10b}^0)$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ | $\frac{1}{\sqrt{6}}$ |
| $K^-\Xi_{0c}^- (\Xi_{10b}^-)$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |
| $K^0\Xi_{0c}^- (\Xi_{10b}^-)$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{\sqrt{2}}$ |

TABLE X: Couplings of the heavy pentaquark triplet $T^i$ with pseudoscalar meson octet $\pi^j$. The universal coupling constant $G_{TAT}$ is omitted.
### Table XI

| $\Theta^+$ | $\Xi_{10}^-$ | $\Xi_{5c}^-(\Xi_{5b}^-)$ | $\Theta_0^+(\Theta_0^\pm)$ |
|------------|-------------|-----------------|------------------|
| $K^+ n$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $Y$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $Y$ | $D^-(B^0)p$ | $-C_{SQB}$ | Y(Y) |
| $K^0 p$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $Y$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $Y$ | $\bar{D}^0(B^+\bar{n})$ | $C_{SQB}$ | Y(Y) |
| $K^+ N_{10}^p$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $D^-(B^0)0_{10}$ | $-C_{SQP}$ | N(N) |
| $K^0 N_{10}^p$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $D^0(B^+)0_{10}$ | $-\frac{1}{\sqrt{2}}C_{SQP}$ | N(N) |
| $\eta_0\Theta^+$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $D^-(B^+)\Theta^+$ | $C_{SQP}$ | N(N) |
| $K^+ n_{8,1}$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $D^-(B^+)n_{8,1}$ | $-C_{SQO1}$ | N(N) |
| $K^0 p_{8,1}$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $+$ | $D^0(B^+)p_{8,1}$ | $-C_{SQO1}$ | N(N) |
| $K^+ n_{8,2}$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $D^-(B^+)n_{8,2}$ | $-C_{SQO2}$ | N(N) |
| $K^0 p_{8,2}$ | $-\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $\frac{1}{\sqrt{2}}\Sigma_{PAB}$ | $N$ | $D^0(B^+)p_{8,2}$ | $-C_{SQO2}$ | N(N) |

Note: Y or N represents the decay mode which is kinematically allowed or forbidden in JW's model with the masses estimated in Ref. 21, 41, 43, 52. Y or N in the parentheses corresponds to the case of the heavy pseudoscalar meson being replaced by the heavy vector meson. Whenever the pentaquark lies very close to the threshold of the final state, we indicate this case with *.