THE CONTRIBUTION OF SPIRAL ARMS TO THE THICK DISK ALONG THE HUBBLE SEQUENCE

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ABSTRACT

The first mechanism invoked to explain the existence of the thick disk in the Milky Way Galaxy was the spiral arms. Up-to-date work summons several other possibilities that together seem to better explain this component of our Galaxy. All these processes must affect distinct types of galaxies differently, but the contribution of each one has not been straightforward to quantify. In this work, we present the first comprehensive study of the effect of the spiral arms on the formation of thick disks, looking at early- to late-type disk galaxies in an attempt to characterize and quantify this specific mechanism in galactic potentials. To this purpose, we perform test particle numerical simulations in a three-dimensional spiral galactic potential (for early- to late-types spiral galaxies). By varying the parameters of the spiral arms we found that the vertical heating of the stellar disk becomes very important in some cases and strongly depends on the galactic morphology, pitch angle, arm mass, and the arm pattern speed. The later the galaxy type, the larger is the effect on the disk heating. This study shows that the physical mechanism causing the vertical heating is different from simple resonant excitation. The spiral pattern induces chaotic behavior not linked necessarily to resonances but to direct scattering of disk stars, which leads to an increase of the velocity dispersion. We applied this study to the specific example of the Milky Way Galaxy, for which we have also added an experiment that includes the Galactic bar. From this study we deduce that the effect of spiral arms of a Milky-Way-like potential on the dynamical vertical heating of the disk is negligible, unlike later galactic potentials for disks.

Key words: galaxies: evolution – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure

1. INTRODUCTION

Simulations of galaxy formation are coming to the point where detailed processes of galaxies can be explored in greater detail than ever before, such that random and rotational velocities can be better studied and understood (Scannapieco et al. 2011). Details on disk potentials can be probed and compared with observations, and we are now able to shed some light on the evolution of galaxies beginning with small-scale stellar motions.

Dynamical heating of the Milky Way disk has now been known for over 60 yr, mainly through observations in the solar vicinity. From those observations we learned that stellar random motions correlate nicely with their ages; this is known as the age-σ relation (Wielen 1977; Binney et al. 2000). In particular, in the case of the Milky Way disk, it is known that the radial velocity dispersion is twice as much as the vertical dispersion and that the radial scale length of the thick disk is much shorter than that of the thin disk (Bensby et al. 2011).

Recent studies also show that many, if not all, edge-on spiral galaxies appear to host dual-disk systems (van der Kruit & Searle 1981; Gerssen & Shapiro Griffin 2012): a younger, dynamically colder and thinner component, the thin disk, and at least one older, (mainly) stellar, dynamically hotter and thicker component, the thick disk (Yoachim & Dalcanton 2006, 2008; Comerón et al. 2011).

It is still not straightforward to elucidate the mechanism responsible for the vertical heating of the disk from observations, especially because it might more likely be instead a combination of several possibilities. Among the mechanisms proposed, there are some that are external to the disk, such as hits by satellite galaxies or minor mergers (Quinn et al. 1993; Huang & Carlberg 1997; Velazquez & White 1999; Font et al. 2001; Benson et al. 2004; Villalobos & Helmi 2008; Di Matteo et al. 2011) or scattering by dark halo objects or globular clusters (Hänninen & Flynn 2002; Vande Putte et al. 2009). There are also some of internal origin, such as dynamical heating by direct encounters with giant molecular clouds (Spitzer & Schwarzschild 1951; Villumsen 1983; Lacey 1984; Lacey & Ostriker 1985; Carlberg 1987; Inoue & Saitoh 2014); heating by encounters with the potential produced by long-lasting spiral arms (Faure et al. 2014) or irregular and transient spiral structure (Barbanis & Wolftr 1967; Jenkins & Binney 1990; Fuchs 2001; Minchev & Quillen 2006); perturbations from stellar bars (Saha et al. 2010); dissolution of young stellar clusters (Kroupa 2002); or during an intense star formation phase in a period of intense accretion very early in the history of the Galaxy (Snath et al. 2014).

Because of the nature of these theories, the effects are dependent on galactic morphology, particularly the intrinsic mechanisms such as bars and spiral arms. Therefore, to deeply understand secular evolution of disk galaxies, it is critical to study dynamical heating in a good sample of different disk galaxy types. Finally, from the point of view of observations, the radial heating agent seems to vary exactly as expected if the agent were the spiral arms, which provides a good chance that the spiral structure has at least an important role as a heating mechanism in the plane of galactic disks (Gerssen & Shapiro Griffin 2012). The importance and influence of spiral arms, and even their very same nature, is still under debate. There is no straightforward observational proof yet of their effect on stars; however, they are nowadays considered to play a key role on large-scale galactic dynamics (Sellwood 2013), for a review, (Antoja et al. 2009; Lépine et al. 2011; Quillen et al. 2011;
Minchev et al. 2012; Roskar et al. 2012). One plausible observational example is the stellar features seen in the velocity space, known as “moving groups” (Proctor 1869; Wilson et al. 1923; Roman 1949; Eggen 1959, 1977, 1990, 1996a, 1996b; Soderblom & Mayor 1993; Majewski 1994; Majewski et al. 1996); these structures might become the first clear, although indirect, evidence of the effect of the spiral arms (Chereul et al. 1998, 1999; Dehnen 1998; Famaey et al. 2005, 2008; Pompéia et al. 2011; Antoja et al. 2012). Of course, the spiral arms are not the only, nor the preferred, mechanism to explain moving groups, the galactic bar being the other possibility.

In this work we focus on the very first proposal to explain the vertical heating (Wielen 1977), disregarded at some point in the history because of their negligible effect on the Milky Way disk: the dynamical heating by effect of the spiral arms. We attempt to isolate and quantify the contribution of the spiral arms to the disk heating of galaxies. We performed numerical simulations of test particles in a three-dimensional galactic potential that models spiral arms (Pichardo et al. 2003), adjusted to simulate spiral galaxies, from early to late types (Pérez-Villegas et al. 2012, 2013). We produced a setup with relaxed initial conditions for a stellar disk. Finally, we calculated the effect on the vertical heating of the stellar disk produced by the nonaxisymmetric large-scale structures. We have included a preliminary study on a self-gravitating potential, known as the PERLAS model (Pichardo et al. 2003, 2004), for the Milky Way Galaxy that includes the spiral arms and the galactic bar.

This paper is organized as follows. The galactic models, initial conditions, and methodology are described in Section 2. The role of each one of the parameters of the model is studied with detail in Section 3, where we present calculations of dispersion velocity, the velocity ellipsoid, time evolution for spiral galaxies from early to late types, and an application to the Milky Way Galaxy. Finally, in Section 4, we present our conclusions.

2. METHODOLOGY AND NUMERICAL IMPLEMENTATION

The effect of the spiral arms over the stellar disk has been studied profusely in either N-body simulations (Sellwood & Binney 2002; Roskar et al. 2012, 2013; Kawata et al. 2014; Roca-Fàbrega et al. 2013) or with spiral patterns treated as perturbations to the axisymmetric background and modeling them as a density wave (De Simone et al. 2004; Minchev & Quillen 2006; Faure et al. 2014).

In N-body simulations, although self-consistent, it is not plausible to adjust a given specific galaxy or to isolate the effect, for example, of the arms or establish in detail the role that each one of the parameters that characterize the spiral pattern play over the vertical heating of the disk. On the other hand, when treated as steady spirals exerted as a perturbation to the axisymmetric potential, according to the hypothesis of Lin & Shu (1964, 1969), the heating is minimum and linked only to the resonant regions of the spirals (Lynden-Bell & Kalnajs 1972; De Simone et al. 2004), being more efficient in the radial direction (Sellwood & Carlberg 1984; Minchev & Quillen 2006).

Here we use spiral arms that, although steady, are very different in nature to those typically and widely employed in literature. The gravitational potential due to the spiral pattern is not a simple perturbation but is rather based on a mass density distribution. With this model of spiral arms our studies on the vertical heating of the disk contrast in general with the density wave approach (except, of course, for really small spiral arm masses and pitch angles).

For the orbital study we then employed three-dimensional galactic potentials to model normal spiral galaxies (Sa, Sb, and Sc). The motion equations are solved in the noninertial reference system of the spiral arms and in Cartesian coordinates ($x'$, $y'$, $z'$). The orbits are integrated for 5 Gyr with a Bulirsh–Stoer algorithm (Press et al. 1992), with a conservation of the Jacobi constant approximately up to $10^{–12}$. The disk heating is computed through the measure of the velocity dispersion at different times.

2.1. Models for Normal Spiral Galaxies

The models include an axisymmetric component (bulge, massive halo, and disk) as the background potential, formed by a Miyamoto & Nagai (1975) disk and bulge, and a massive halo (Allen & Santillán 1991). The parameters used to model normal spiral galaxies (Sa, Sb, and Sc) are presented in Table 1 (compiled by Pérez-Villegas et al. 2013).

Superposed to the axisymmetric components, for the spiral arm potential we employed a bisymmetric self-gravitating three-dimensional potential, based on a density distribution, called the PERLAS model (Pichardo et al. 2003). This potential consists of individual inhomogeneous oblate spheroids superposed along a logarithmic spiral locus (Roberts et al. 1979). Each spheroid has a similar mass distribution; that is, surfaces of equal density are concentric spheroids of constant semi-axis ratio. The model considers a linear fall in density within each spheroid. The minor and major semi-axes of each oblate spheroid are 0.5 and 1.0 kpc, respectively (this gives a width of the spiral arms of 2 kpc and height of 0.5 kpc from the disk plane), and the separation among the spheroid centers along the spiral locus is 0.5 kpc. The superposition of the spheroids begins and ends in the ILR and CR, respectively. The density falls exponentially along the spiral arm, where the radial scale length of the galactic disk is used depending on morphological type; see Table 1). The mass assigned to build the spiral pattern is subtracted from the disk mass to keep the given model invariable in mass. PERLAS is a more realistic potential because it is based on a density distribution and considers the force exerted by the whole spiral structure, obtaining a more detailed shape for the gravitational potential, unlike a two-dimensional local arm such as the tight-winding approximation (TWA) represented for a simple cosine function.

The nature of spiral arms is a matter of discussion nowadays, particularly their long-lasting or transient nature. We have performed experiments with constant, transient, gradual, and sudden presence of the spiral arms. Although the growth rate is an unknown parameter in galaxies, we have considered different cases to test. On one hand we produce a set of experiments where the total mass of the spiral arms is introduced at once ($t = 0$ Gyr). The second set of experiments inserts the spiral arm mass linearly in a time lapse of 1 Gyr. And a third set of experiments, for which the spiral arms are simulated as transient, they vanish and grow with a given periodicity.
2.2. Initial Conditions and Equilibrium of a Stellar Disk

The initial condition setup follows the Miyamoto–Nagai density profile we are imposing. This is to avoid transient effects induced by differences between the initial particle distribution and the imposed disk potential. In this manner, the initial condition for the stellar disk is given by

\[
\rho_{MN} = \frac{M_d}{4\pi} \left[ a_2 R + \sqrt{(a_2 + \sqrt{a_2^2 + b_2^2})^2 (z^2 + b_2^2)} \right]^2 \frac{(a_2 + \sqrt{a_2^2 + b_2^2})^2}{R^2 + (a_2 + \sqrt{a_2^2 + b_2^2})^2 (z^2 + b_2^2)}^{3/2},
\]

where \( M_d \) is the mass of the galactic disk and \( a_2 \) and \( b_2 \) are the radial and vertical scale lengths, respectively. These three parameters span a range of values in our simulations in order to capture different galactic morphologies and kinds of spiral arms.

To distribute the particles according to the Miyamoto–Nagai density law, we solved Equation (1) with a root finder method. This is done by expressing the density \( \rho(R, z) \) in terms of the ratios \( \rho(R, 0)/\rho(0, 0) \) and \( \rho(R, z)/\rho(R, 0) \), which provides us with an equation for \( R \) and \( z \) in terms of the density. The value of these ratios ranges from 0 to 1; therefore we can explore all the possible values of the density with a random function and solve it for \( R \) and \( z \).

To assign velocities to the particles, we follow the strategy proposed by Hernquist (1993), where velocities are distributed by an approximation using moments of the collisionless Boltzmann equation plus the epicycle approach.

Thus we proceeded as follows: once the density profile has been established, it is necessary to obtain the rotational velocity. This can be derived from \( \Phi \), the gravitational potential of the model

\[
\Omega_c(R) = \left( \frac{1}{R} \frac{\partial \Phi}{\partial R} \right)^{1/2}
\]

and \( v_c = R \Omega_c(R) \); therefore the circular velocity is given by

\[
v_c(R) = \left( \frac{R}{R} \frac{\partial \Phi}{\partial R} \right)^{1/2}.
\]

Once \( \Omega_c \) is known at any radius, we obtain

\[
\kappa = \sqrt{4\Omega_c^2 + R \frac{\partial^2}{\partial R^2}}
\]

known as the epicyclic frequency, necessary to calculate the velocity dispersion at \( R \) and to correct for the asymmetric drift.

To achieve the requirement for the stellar disk to be in equilibrium, it is necessary to introduce a given dispersion in the velocity as a function of \( R \). The velocity dispersions in the three polar coordinates are

\[
\sigma_R = 3.358 \frac{\Sigma(R) Q}{\kappa}
\]

\[
\sigma_\phi = \frac{1}{2} \frac{\sigma_R \kappa}{\Omega_c}
\]

\[
\sigma_z = \sqrt{\pi G \Sigma b_2},
\]

where \( \kappa \) is the epicyclic frequency, \( \Sigma(R) \) is the surface density, \( b_2 \) is the vertical scale length of the disk, and \( Q \) is the known Toomre parameter. According to Toomre (1964), local stability requires \( Q > 1 \); we chose \( Q = 1.1 \) and found this value to be sufficient for the three galaxy types. In this way the velocity dispersion depends on the mass of the components that form each galaxy.

The asymmetric drift correction, defined as

\[
\left( v_\phi \right)^2 = v_c^2 - \sigma_\phi^2 - \sigma_R^2 \left( 1 - 2 \frac{R}{\Omega_c} \frac{\partial \Sigma}{\partial R} \right)
\]

is a correction that has to be implemented in the setup for the initial conditions, given the fact that stellar orbits are not in general circular; instead, orbits follow epicycles around a guiding point at the position of the circular orbit, and these epicycles are characterized by the epicyclic frequency \( \kappa \).
Finally, the particles are distributed in the velocity space as

\[
\begin{align*}
\mathbf{v}_\phi &= \langle v_\phi \rangle \pm x\sigma_\phi \\
\mathbf{v}_R &= \langle v_R \rangle \pm x\sigma_R \\
\mathbf{v}_z &= \langle v_z \rangle \pm x\sigma_z
\end{align*}
\]

where \( x \) is a random number between 0 and 1, \( \langle v_\phi \rangle \) is given by Equation (8), and the average radial and vertical velocities are taken as \( \langle v_R \rangle = \langle v_z \rangle = 0 \).

2.3. Dispersion Analysis

The disk heating is often referred to as the increase in the velocity dispersion over the lifetime of a star. Any disk thickening is then related to an increase in the vertical velocity dispersion of the disk stars. In this paper we analyze the spiral arm effects on the stellar disk, based on the study of the vertical velocity dispersion and its dependence with the parameters that characterize the spiral pattern.

The vertical velocity dispersion \( \sigma_z \) is then calculated in the simulations by dividing the plane \( z = 0 \) into 1 kpc bins and computing, as usual, the squared root of the average squared vertical velocity for all the particles that fall into a given bin. This provides us with the vertical velocity dispersion as a function of \( R \). In order to establish the contribution of the spiral arms to the disk thickness we also compute \( \sigma_z \) as a function of time by measuring the velocity dispersion at a fixed radius \( R \) for every time code unit (in this case, every 100 Myr across 5 Gyr in the simulation).

2.4. Control Simulations: Testing the Initial Conditions Equilibrium

As described in Section 2.2, the first goal is to build an initial stellar disk in equilibrium to be sure that any change seen in radial and vertical velocity dispersion is strictly due to the interaction of the spiral arms with the stellar disk and not originated by a spurious nonrelaxed initial conditions setup.

For this test a control simulation was produced with only the axisymmetric components for the potential model. Stars are run by 5 Gyr for all galaxy types. Figure 1 shows \( \sigma_z \) as a function of \( R \) at different stages in the temporal evolution for an Sa, Sb, and Sc galaxy.

From the figure it is clear that the vertical velocity dispersion does not evolve or deviate from the initial dispersion, as expected for a disk in equilibrium with the axisymmetric potential.

3. RESULTS

We present in this section a set of controlled experiments to study the dynamical heating of disks on spiral galaxies, considering the spiral arms as the driver. The general purpose is to shed some light on the relative importance of these large-scale structures to other sources of dynamical heating in different morphological types. The experiments include studies of different structural and dynamical parameters of the spiral arms, such as different pitch angles, total spiral arm masses, angular speeds, transient, and one final case modeling the Milky Way Galaxy (with preliminary results that will be better developed in a future work).

3.1. Dependence of the Disk Heating Induced by Spiral Arms with the Galactic Morphology

Spiral galaxies present a wide variety of morphological types from massive bulge-dominated galaxies to practically bulgeless disks, spanning a wide range of values for the parameters that characterize different galaxy types.

Figure 2 shows the velocity dispersion, \( \sigma_z \), as a function of the galactocentric radius, \( R \), at different times in the simulation for our galactic models: Sa, Sb, and Sc (introduced in Section 2). For these three simulations the mass of the spiral arms, \( M_{\text{arms}} \), is 5% of the total disk mass with a pitch angle of 40° for the Sa galaxy, 45° for the Sb galaxy, and 40° for the Sc galaxy. For these experiments we have employed the largest spiral arm masses and pitch angles for plausible (nonfully chaotic) galactic models to identify clearly the spiral arm effects, if any. The three plots show a distinct increase in the vertical velocity dispersion caused by the spiral arms. Additionally, from Figure 2 it is clear that the change in \( \sigma_z \) with respect to the initial dispersion is smaller for the Sa galaxy and grows with the morphological type, being much larger for the Sc galactic model.

The dependence of the effect of the spiral arms with the morphology is such that for an Sc galaxy the effect is evident in the spatial distribution of the stellar disk particles. Figure 3 shows the \( x-z \) projection of the stellar distribution plotted at \( t = 0, t = 2.5 \) Gyr, and \( t = 5 \) Gyr. Additionally, a thickening of the disk is discernible during the orbital evolution when compared with the initial distribution.

The thickness can be quantified by computing the root mean square of the coordinate \( z \), i.e., \( \epsilon_{\text{rms}} = \sqrt{\langle z^2 \rangle} \). Figure 4 shows the thickness as a function of \( R \) for the stellar disks in Figure 3.

From this first set of experiments, we conclude that the sharpest effect on the velocity dispersion is present on the latest morphological types. It is worth mentioning here that, although we are separating the models in Sa, Sb, and Sc galaxies, with strong gaps in between the different models in the initial scale height, the results on the disk heating driven by spiral arms presented here are more general; that is, the observed heating results are significant in thinner disks, which, as a consequence, has implications on galaxy types; in this case, particularly on later types.

For the earliest type (Sa), the isolated effect of spiral arms corresponds to a maximum increment of 7% of the initial velocity dispersion, this considering the most massive and the largest pitch angles possible to produce plausible galactic models. Likewise, for the intermediate galaxy type (Sb), the isolated effect of spiral arms corresponds to a maximum increment of 20% of the initial velocity dispersion, again considering the most massive and the largest pitch angles possible to produce plausible galactic models. Finally, for the latest type (Sc), the isolated effect of spiral arms corresponds to a maximum of 62% of the initial velocity dispersion for this example, where we have not used the maximum plausible parameters for the spiral arms. When the maximum pitch angle and mass is employed, the percentage goes up to almost 90% of the initial velocity dispersion.

Because a visible effect on the vertical dispersion when spiral arms are included is considerably larger for the latest galaxy types, compared with early and intermediate types, in the next sections we concentrate on a more detailed study of the disk thickening, focusing only on the late-type galaxies.
3.1.1. Pitch Angle Effect

The pitch angle is one of the most influential structural parameters that characterize spiral patterns. In this section a range of values is explored in order to quantify the dependence of the increment on the vertical velocity dispersion with pitch angle in the most affected galactic models that are the latest types.

Figure 5 shows three plots of $\sigma_z$ versus $R$, where the mass of the spiral arms is set to a constant for each plot and the pitch angle varies according to Table 1. Each plot has the initial dispersion curve $\sigma_z(R, t = 0)$ and the dispersion after a 5 Gyr evolution $\sigma_z(R, t = 5 \text{ Gyr})$ for each pitch angle value.

From Figure 5, it is clear that regardless of the mass of the spiral arms, the vertical velocity dispersion increases notably with the pitch angle. The less massive the spiral arms, the smaller is their effect in general, as expected. Indeed, for spiral arm masses smaller than $\sim 1\%$ of the disk, the contribution of spiral arms to the dynamical heating becomes negligible.

3.1.2. Spiral Arm Mass Effect

As is shown in the previous section, the effect of the pitch angle can be significant to the disk thickening. It is also intuitively clear that this scales with the mass of the spiral arms. To address this point, we produced several experiments, condensed in Figure 6, that show the disk-thickening

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**Figure 1.** Test for the initial conditions equilibrium: velocity dispersion of the stellar disk in the axisymmetric potential only, as a function of $R$, along 5 Gyr time evolution for each galaxy type.

**Figure 2.** As in Figure 1, but including now the spiral arms contribution to the potential. The vertical velocity dispersion is plotted as a function of $R$ for 5 Gyr time evolution.
dependence on the spiral arm mass and the pitch angle together. For this purpose, first we identified the radius $R_{\text{max}}$ at which the maximum difference between the final and initial vertical velocity dispersion occurs. We find that this occurs at approximately $R_{\text{max}} = 1.5 \text{kpc}$ for this model. We then keep $R$ constant at that value and plot $\Delta \sigma_z$ there as a function of the pitch angle. This will show us the tendency seen in the previous section. Now repeating this for the three spiral arm masses employed will show how $\Delta \sigma_z$ scales with this second parameter.

Figure 6 shows $\Delta \sigma_z$ as a function of the pitch angle for three different masses of the spiral arms. Here it is clear that both parameters, the pitch angle and the mass of the spiral arms, affect considerably the disk-thickening effect driven by the spiral structure.

We produce with these results an empirical functional relation between $\Delta \sigma_z$ and the pitch angle. The fit of data plotted in Figure 6 is made by noting that $\Delta \sigma_z$ increases slowly at small angles, then the slope of the curve grows with the angle and flattens for the largest values of the pitch angle. This behavior could be interpreted as a saturation effect of $\Delta \sigma_z$ after a certain time, this time being shorter for higher angles and masses. The saturation effect is also seen in a $\sigma$–time relation (Carlberg et al. 1985; Seabroke & Gilmore 2007; Soubiran et al. 2008), where the dispersion remains constant after $\sim 5 \text{ Gyr}$. Based on this observation, a nice fit to the results would be a Boltzmann sigmoidal function, which is characterized by displaying a progression from small beginnings that accelerates and approaches a climax over the independent variable. The Boltzmann sigmoidal function for this particular case, is defined by

$$
\Delta \sigma_z = A_2 + \frac{(A_1 - A_2)}{1 + \exp\left((x - x_0)/d\right)}
$$

where

$$x = \text{Pitch Angle} (^\circ)$$

$$x_0 = \text{center} (^\circ)$$

$$d = \text{width} (^\circ)$$

$$A_1 = \text{initial } \Delta \sigma_z \text{ value (km s}^{-1}\text{)}$$

$$A_2 = \text{final } \Delta \sigma_z \text{ value (km s}^{-1}\text{)}$$

The center $x_0$ is the pitch angle at which $\Delta \sigma_z$ is halfway between $A_1$ and $A_2$. The width $d$ is related to the steepness of the curve, with a larger value denoting a shallow curve.

Figure 7 shows the fit of the sigmoidal function (Equation (10)) to the data. We see that this function reproduces well the behavior of $\Delta \sigma_z$ with the pitch angle for the three masses of spiral arms used in our simulations.

Taking Equation (10) as the functional form for the $\Delta \sigma_z$–pitch angle relation, in Table 2, we summarize the parameters that describe the fits plotted in Figure 7 for the three spiral arm masses.
3.1.3. Gradually Increasing the Mass of the Spiral Arms

As explained before, the spiral structure in our galactic potential models is imposed and fully introduced since the starting point of the simulation. However, one might wonder if spurious effects (such as a drastic dispersion increase in the early stages of the simulation) are introduced with this method; also, in real galaxies the birth and death of spiral arms is most probably not a sudden process.

In order to explore these two scenarios, we prepared a set of representative simulations modifying the model in such a way that the arms are allowed to have a growing period until the total assigned mass is reached. For this set of simulations we started with an Sc galaxy with different combinations of values for the spiral arm mass and the pitch angle. We chose two radial positions and measured the evolution of \( s_z \) with time as shown in Figure 8. In this figure the temporal evolution of \( s_z \) for the sudden full-mass arms is compared with a model with linear (1 Gyr period) growing mass. It is clear that there is a shift in the dispersion achieved at the end of the simulation, but, most important, for both kinds of imposed spiral arms the velocity dispersion increases at roughly the same rate.

With this simple experiment we are not pretending to capture the great variety of processes that lead to the formation of a real spiral pattern, nor their time lapse of full action, but this is still likely a better approximation than a sudden action of spiral arms. With this exercise we notice, however, that, for any radial position, the vertical dispersion is only shifted when using spiral arms that have a growing period. This does not suppose important differences with our previous simulations or results derived from them.

3.1.4. Varying the Angular Velocity of the Spiral Pattern

The vertical heating of the stellar disk produced by the spiral arms depends on the parameters of the nonaxisymmetric structure, particularly for the late galaxy types. By varying the pitch angle and the mass of the arms we have found a correlation between these parameters and the thickness of the stellar disk.

In this section, we now study the pattern speed effect on the disk vertical heating. For this purpose, we ran a set of simulations with different values of the spiral pattern for the latest types of galaxies, Sb and Sc, that are more clearly affected for the structural parameters of the arms. In Figure 9 we show first the evolution of the vertical velocity dispersion \( \sigma_z \) over time up to 5 Gyr for different values of the pattern speed, \( \Omega \), for an Sc galaxy. There seems to be a clear relation between the angular velocity and the dynamical heating. The velocity dispersion increases for slower rotating patterns.

Figures 10 and 11 summarize the results of the set of simulations when varying the pattern speed for Sb and Sc galaxies. The final stage of \( \sigma_z \) is shown across the entire disk for each one of the pattern speeds used. As expected, the final vertical velocity dispersion and disk thickness are larger for slower rotating arms, independent of the galaxy type. Smaller values of \( \Omega \) allow the spiral arms to heat the stellar disk more.
efficiently, likely due to the minor relative angular velocity between the arms and the stars, this allows the stars to interact with the potential of the arms for longer periods of time.

Other studies link the heating of the stellar disk and the role of the pattern speed only to the resonant regions of the spirals (Lynden-Bell & Kalnajs 1972; De Simone et al. 2004); in contrast, in this study we find that the heating, radial and vertical, occurs along the entire length of the spiral arms.

On the other hand, N-body simulations have been shown to develop transient spiral structure that spans a range of pattern speeds (Sellwood & Binney 2002; Roskar et al. 2012), but as the arms are transient, and it is not possible to isolate its effect in N-body simulations, it is difficult to establish a dependence of the heating on the pattern speed.

The experiments and the model shown here allow us to establish a relation between the vertical heating of the stellar disk and the pattern speed.

3.1.5. The $\sigma$–Time Relation

It is already known that the age and velocity dispersion of stars are correlated. This has been established from observations in the solar neighborhood, as well as from numerical simulations (Holmberg et al. 2009; Roskar et al. 2013; Martig et al. 2014). The $\sigma$–$t$ relation shows a smooth, general increase of the velocity dispersion with time and is best parameterized by a power law with exponents ranging between 0.2 and 0.5 (Gerssen & Shapiro Griffin 2012).

We explore the $\sigma$–$t$ relation in our simulations to find out if the velocity dispersion in the stellar disk due to the spiral arms

| $M_{\text{arms}}$ (%$M_{\text{disk}}$) | $x_0$ (°) | $d$ (°) | $A_1$ (km s$^{-1}$) | $A_2$ (km s$^{-1}$) |
|---|---|---|---|---|
| 1% | 64.73 | 24.05 | -0.33 | 5.32 |
| 3% | 54.73 | 14.95 | -0.057 | 24.84 |
| 5% | 35.68 | 6.82 | 1.69 | 28.44 |

Note. Parameters that combined with Equation (10), describe the $\Delta\sigma$–pitch angle relation. Center $x_0$, width $d$, initial $\Delta\sigma$ value $A_1$, and final $\Delta\sigma$ value $A_2$. For the three spiral arm masses (in percentage of the disk mass $M_{\text{disk}}$).

Figure 8. $\sigma$–$t$ relation that shows, regardless of the radial position, a continuous heating for both kinds of imposed spiral arms: those that grow with time (blue lines) and those totally formed since the beginning (black lines).
fits with a power law $t^\alpha$ and, more important, to establish a range of values for $\alpha$.

To measure the time evolution of $\sigma_z$ in our simulations, first we locate the radius $R_{\text{max}}$ at which the maximum increase in the vertical velocity dispersion occurs. This radius is $R_{\text{max}} = 1.5\, \text{kpc}$ and is the same for all the simulations with the Sc galaxy, independent of the pitch angle or the mass of the spiral arms.

Because a log–log plot is useful to recognize a possible power law relationship, Figure 12 shows the log$(t)$–log$(\sigma_z)$ dependency. Most of the points fall on a straight line; this reveals behavior of the form $\sigma_z \propto t^\alpha$.

Figure 13 shows the evolution of $\sigma_z$ with time at $R = R_{\text{max}}$, $M_{\text{arms}}/M_{\text{disk}} = 0.05$ and at different pitch angles: 20°, 30°, 40°, and 50°. The black line in each plot is the best fit of the data with a power law of the form $\sigma_z \propto t^\alpha$. We made the same analysis for a spiral arm mass of $M_{\text{arms}}/M_{\text{disk}} = 0.03$; an interesting outcome is that the value of $\alpha$ is independent of the spiral arm mass. Different masses for the spiral arms will just change the proportionality constant in the relation $\sigma_z \propto t^\alpha$. Consequently, $\alpha$ depends only on the pitch angle, and for the angles used in our simulation, $\alpha$ varies within the range 0.27–0.56 for Sc galaxies.

Although we have presented the time evolution of $\sigma_z$ at $R = R_{\text{max}}$, it is possible to make the measure at any value of $R$. Figure 14 shows the same analysis at a different radial position, and this time corresponds to half of the arm length, $R = 4.2\, \text{kpc}$. We notice that measuring the time evolution of $\sigma_z$ at different radius gives us the same power law behavior.
With a similar analysis for Sb galaxies, where $R_{\text{max}} = 2.5$ kpc, we again find that a power law provides a nice fit to the data. Figure 15 shows the $\sigma_z$ temporal evolution for pitch angles of $36^\circ$ and $45^\circ$, and the best fits are reached with values for $\alpha$ of 0.32 and 0.37, respectively. Smaller angles than those give us plots with more scattered points, where a power law fit is not straightforward to obtain; that is, we are not able to establish the value of $\alpha$ for angles smaller than $36^\circ$. For Sb galaxies we can only provide an upper bound for $\alpha$ of $\sim 0.37$.

Considering that the pitch angles we are employing here represent the maximum plausible values of pitch angle and spiral arm mass for each type of galaxy before chaos...
dominates, the values of $\alpha$ presented here would represent an upper bound for the contribution of the spiral arms to the vertical velocity dispersion of stars in each galaxy type.

3.1.6. Velocity Ellipsoid

For the dynamical evolution analysis of the stellar disk we applied a classic method derived from the distribution of the velocity dispersions. The axes of such a distribution define the known as the “velocity ellipsoid,” and this is characterized by the two axes ratios: $\sigma_z/\sigma_R$ and $\sigma_\phi/\sigma_R$.

The shape of the velocity ellipsoid seems to show a trend in $\sigma_z/\sigma_R$ with the morphological type (Gerssen & Shapiro Griffin 2012). Given the three types of galaxies we are using in this work and the parameters given in Table 1 that define them, we are able to measure the shape of the velocity ellipsoid and compare it with the morphological type. In Figure 16 we plot this ratio as measured at the final stage of our simulations. The value $\sigma_z/\sigma_R$ decreases with galaxy type, and the trend continues over time.

In Figure 17 we follow the temporal evolution for the shape of the velocity ellipsoid. We see that for Sa and Sb galaxies, $\sigma_z$ is always greater than $\sigma_R$, and the presence of spiral arms does not seem to alter this tendency. The initial fall in the curves indicates that although $\sigma_z$ and $\sigma_R$ both increase with time, $\sigma_R$ does so in a greater proportion than $\sigma_z$, especially for late-type galaxies. This heating in the $R$ direction is similar to that seen in Figure 2.

The shape of the velocity ellipsoid evolves toward lower values and settles down to be nearly constant and fluctuates around some equilibrium value, a behavior already noticed in other numerical work (Sellwood 2008).

Looking at the values of the plots in Figure 17, we see that the velocity dispersion ratio falls to half its initial value for Sa and Sb galaxies, with $\sigma_z$ always greater than $\sigma_R$, but the final value of $\sigma_z/\sigma_R$ falls lower than one for Sc galaxies. This indicates that the relative increase of $\sigma_R$ for Sc galaxies is greater than that seen on Sa and Sb galaxies. This means that the heating effect of the spiral arms in Sc galaxies, apart from being more notorious in the $z$ direction, is also greater in the $R$ direction compared to the other galaxy types.

In previous sections we have shown that the vertical heating of the stellar disk strongly depends on the pitch angle of the spiral pattern. Consequently, the disk thickness in our simulations grows with the value of the pitch angle, as seen in Figure 5 and Equation (10). As shown in Jenkins & Binney (1990), where the ratio $\sigma_z/\sigma_R$ depends on the spiral structure, here we are able to measure the shape of the velocity ellipsoid and its dependence on the pitch angle. Because the ratio of the velocity dispersion reaches a nearly constant value only after a certain period of time, under the influence of the spiral pattern (Figure 17), we computed the ratio $\sigma_z/\sigma_R$ at the final stage in our simulations (5 Gyr) for several pitch angles, for the Sc galaxy. In Figure 18 we show how the shape of the velocity ellipsoid would be as a function of the pitch angle. It decreases first as the pitch angle grows, but after a 40° angle, it fluctuates around some constant value.

For small pitch angles (< 20°), the radial and vertical dispersions are nearly the same, and this relation continues after 5 Gyr, despite the presence of the spiral pattern. This is because even when $\sigma_z$ and $\sigma_R$ increase with time, the growth rate is small in both directions compared with the initial dispersions. On the other hand, Figure 18 shows that for larger pitch angles, not only the increment in $\sigma_z$ is considerable, as is found in Section 3.1.1, but the heating in the $R$ direction is, remarkably, even greater than that in the $z$ direction. This is expected from a structure that rotates in the plane of the galaxy, but here we have found that it has a considerable effect in both the radial
and vertical directions and varies sensibly with the parameters of the spiral pattern.

### 3.1.7. Transient Spiral Arms

All results presented in the previous sections are obtained from simulations that assume long-lived spiral arms. However, it is a result from \( N \)-body simulations that the spiral arms might rather be transient features in general (Grand et al. 2012; Kawata et al. 2011; Sellwood 2011).

In this section we present some experiments with transient spiral arms to quantify the effect on the vertical structure of the stellar disk and to study possible differences with our previous results. The adjustability of the galactic models allows us to emulate transient and recurrent spiral arms by making them grow and disappear periodically. The lifetime of the spiral patterns is hard to determine in \( N \)-body simulations, and even in the cases where it can be determined with Fourier modes analysis, it is different for every simulation. We take two different periods for a simple experiment, one with 100 Myr and the other with 500 Myr (Grand et al. 2014).

The simulations are made with an Sc galactic model with a spiral arm pitch angle of 50°, \( M_{\text{arms}}/M_{\text{disk}} = 0.05 \), and two different lifetimes for the transient spiral arms. Figure 19 shows the vertical velocity dispersion \( \sigma_z \) as a function of \( R \) for the 5 Gyr evolution time with the transient spiral arms. Even with this kind of spiral pattern that is not always present in the disk, grows, and disappears periodically, we can see that \( \sigma_z \) keeps increasing with time for both lifetimes used, as it does in our previous simulations with longer-lasting spiral arms.

In Figure 20 we compare the final vertical velocity dispersion for both models of transient spirals against that obtained for nontransient spiral arms. The plots show only slight deviations between them and are not significative at all radii.

For a more detailed comparison we pick the radial zone at which the increase in vertical velocity dispersion is highest and plot the temporal evolution of \( \sigma_z \) for the transient and nontransient spiral arms and show it in Figure 21. Some slight differences are visible at the end of the simulation, and as this radial zone is where \( \Delta \sigma_z \) is maximum, the differences are even smaller for the rest of the disk.

Despite the different natures of the three types of spiral arms simulated, the induced stellar dynamical behavior is similar. This is because the transient spiral arms, although not always present in the disk, are recurrent and form very quickly once the previous spiral pattern disappears, as seen in the \( N \)-body simulations, leaving the stars under an almost constant influence.

### 3.2. Plausible Origin of the Vertical Stellar Heating

Based on the set of experiments presented here, we conclude that a spiral structure can excite, considerably, velocities in the \( z \) direction. Furthermore, we noticed that the physical mechanism causing the heating is different from simple resonant excitation. The spiral pattern induces chaotic behavior not linked necessarily to resonances, but rather to direct scattering of disk stars, which leads to an increase of the velocity dispersion.

In order to produce evidence to support this gravitational scattering interpretation, we performed the following analysis. For an Sc galaxy with spiral mass \( M_{\text{arms}} = 0.05 M_{\text{disk}} \) and a pitch angle of 50°, we took a sample of 458 particles that at the end of the simulation are part of the hot component (i.e., particles that experienced a significant increase in their velocity dispersion). By tracing back the initial positions, we reconstruct the orbit of each particle. To classify the orbits and differentiate them as regular or irregular, we implemented a known spectral method by Carpintero & Aguilar (1998). The method is able also to identify loop, box, and other resonant orbits. The results are presented in Figure 22.

This analysis shows that the orbital sample is completely dominated by irregular orbits, with no evidence of resonances from any kind. In this way we interpret the vertical heating as caused by gravitational scattering of stars by spiral arms, as already has been pointed out in other studies (Fujii et al. 2011).

### 3.3. Case of the Milky Way

Probably the hardest case to study, due to the plethora of information available, sometimes contradictory, sometimes model- or observation-dependent, is the Milky Way Galaxy. The existence of different types of disks or different types or vertical structures (the old and young thin disks and the thick disk) has been known, but not for long. What are the mechanisms that produce the heating of stellar orbits into the thick disk? What is the relative importance of each one? Is this present in all galaxies? Are the mechanisms that radially affect the disks effective in the vertical component?
A lot of the finest work on mechanisms to dynamically heat the disk has been done in the plane with steady two-dimensional potential models, studying fundamental physical phenomena such as the radial migration. Regarding the vertical structure, this is a subject that started several decades ago to be of interest in astrophysics. Only recently, thanks to the calculation power of the new generation of supercomputers, we have been able to produce more realistic and detailed simulations with test particles in steady potentials and with live models with improved resolution, all to study with unprecedented details the dynamics of the Galactic disk.

In this section, as a preliminary application to a specific galaxy, we have constructed a detailed density distribution, based on the model of the Milky Way Galaxy that includes spiral arms, with the purpose of exploring their isolated effect on the heating to the disk and its participation in the process of production of the thick disk. We compare our results with other recent work dedicated to Milky-Way-like potentials (Faure et al. 2014). Additionally, the experiments presented in this section for the Milky Way include one more experiment incorporating the calculations the Galactic bar. We are not pretending this is an extensive study of the Milky Way Galaxy, but rather a modest first approximation and preliminary results of only the vertical heating effects by spiral arms and bar on a detailed model of the Galaxy; in consequence, it is important to mention that we are not including all of the relevant references to this problem. In an ongoing work, we will provide a work fully dedicated to the Milky Way Galaxy in this context and the relevant references.

The parameters of the galactic models employed here allow us to model different morphological types and different spiral arm classes by changing the pitch angle, the mass, or even making them transient. In this section, we produce observationally motivated models of the Milky Way galaxy and study

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**Figure 20.** Comparison of $\sigma_z$ after 5 Gyr for transient spiral arms, 100 Myr lifetime, 500 Myr lifetime, and nontransient spiral arms.

**Figure 21.** Comparing the time evolution of $\sigma_z$ for transient spiral arms, 100 Myr lifetime, 500 Myr lifetime, and nontransient spiral arms.

**Figure 22.** Orbital classification on the disk particles that experienced a significant increase in their velocity dispersion. The histogram shows that the orbital sample is dominated by irregular orbits.

**Figure 23.** Effect of the Milky-Way-like spiral arms in the dynamical heating of the disk. (top) Initial and final vertical velocity dispersions after 5 Gyr evolution. (bottom) $\Delta \sigma_z$ (final–initial). The vertical heating is minimum over a small radial region.
the response of the stellar disk to the spiral arms. The parameters that describe the components that compose the axisymmetric background potential are constrained by recent estimates of the galactic rotational curve. We adopt a pitch angle of 15.5° for the spiral arms, a mass $M_{\text{arm}}/M_{\text{disk}} = 0.03$, and an initial time of 0.5 Gyr to increase the spiral arms up to their full mass. For the first simulation presented for the Milky Way, we do not include a bar in order to try to isolate the influence of the spiral pattern in the stellar disk (and to compare with other similar work).

Figure 23 (top panel) shows the initial and final vertical velocity dispersion after a 5 Gyr evolution for the Milky-Way-like model (halo, disk, bulge, and spiral arms). As seen for the case of early-type galaxies (i.e., small pitch angles and spiral arm masses), the change in velocity dispersion is small. Indeed, although the effect is not negligible, in the Milky-Way-like model experiment, with a pitch angle of 15°.5 and a mass $M_{\text{arm}}/M_{\text{disk}} = 0.03$, the spiral arms are not capable of heating the stellar disk. In Figure 23 (bottom panel) we plot the difference between $\sigma_z$ initial and final and show that the main heating is produced in the innermost region of the disk.

With this experiment we conclude that the vertical heating that spiral arms are able to produce in our Milky-Way-like Galaxy model is minimum, and it is located in a small radial region. This result represents a different one compared to that of Faure et al. (2014), who find some important effect on the vertical heating due to the spiral arms. To try to account for the origin of this discrepancy, further studies will be performed in a future work.

In a second experiment, we have also included a known mechanism that exerts strong secular radial and vertical evolution on galactic disks, the central bar. Although the present work is dedicated to the study of the effect of the spiral arms on stellar disk vertical heating, we have produced a careful experiment for the Milky Way Galaxy to explore to what extent the central bar can contribute to the vertical heating. In an ongoing paper we study in detail the vertical heating produced by the full galactic model of the Milky Way; here we present some preliminary results of this study.

We ran a simulation with the parameters representative of the Milky Way Galaxy, given in Table 3. Figure 24 shows the initial and final vertical velocity dispersion $\sigma_z$ across the stellar disk. Although the two nonaxisymmetric structures are present in the simulation, by comparing Figures 23 and 24, there is no doubt that the heating seen in the second one is produced by the galactic bar and that the only affected region seems to be the central part (similar results were obtained by Saha et al. (2010).

The effect of the galactic bar on the vertical velocity dispersion is considerable, with an increase in $\sigma_z$ up to 16 km s$^{-1}$, mostly in the inner 6 kpc of the disk.

### 4. CONCLUSIONS

With the use of detailed models for spiral galaxies, we produce an extensive study of disk vertical heating. The models include an axisymmetric component (bulge, massive halo, and disk) plus a density-based three-dimensional potential for the spiral arms, orbitally tested for self-consistency.

The main outcome of this study is that the spiral structure is capable of exciting moderate to high dispersion velocities in the z direction, inducing a relation between stellar age and $\sigma_z$ in all spirals except the earliest types. Although rather small, this mechanism works for the Milky Way Galaxy, assuming the parameters for the spiral arms known to this day. Consequently, by isolating the effect of the spiral arms on the vertical velocity dispersion of the stellar disk, we can conclude that spiral arms have the capability to contribute to the heating mechanism that gives rise to thick disks in spiral galaxies, from intermediate to late morphological types.

Therefore, the thickness of the stellar disk driven by the spiral arms goes from negligible to very important, depending on the characteristics of the spiral pattern, such as mass, pitch angle, and angular speed, along with the morphological galaxy type. By covering a whole set of values for these parameters in the test particle simulations, we conclude as follows.

1. The relative increase in vertical velocity dispersion is a function of the morphological type, being smaller for early-type galaxies (Sa type in the Hubble scheme) and larger from intermediate- to late-type galaxies. Although for the sake of clarity, we present our results by separating the models in approximately average Sa, Sb, and Sc galaxy types, we are aware that strong gaps are inbetween the different models in the initial scale height.

The results on the disk heating driven by spiral arms

| Parameter | Value | Reference |
|-----------|-------|-----------|
| $R_0$     | 8.5 kpc | 1         |
| $\theta_0$| 220 km s$^{-1}$ | 1         |
| $M_a$     | $1.41 \times 10^{10}$ $M_\odot$ | 1         |
| $M_d$     | $8.56 \times 10^{10}$ $M_\odot$ | 1         |
| $M_t$     | $80.02 \times 10^{10}$ $M_\odot$ | 1         |
| Disk scale length | 2.5 kpc | 2         |
| $b_1$     | 0.3873 kpc | 1         |
| $a_2$     | 5.3178 kpc | 1         |
| $b_2$     | 0.2500 kpc | 1         |
| $a_3$     | 12 kpc   | 1         |
| Locus     | Logarithmic | 3         |
| Arms number | 2, 4.5 |          |
| Pitch angle | 15:5 | 4         |
| $M_d/M_D$ | 3%   | ...       |
| Scale length | 2.5 kpc | disk-based |
| Patter speed ($\Omega_p$) | $-20$ km s$^{-1}$ kpc$^{-1}$ | 6         |
| Inner limit | 3.3 kpc | based on ILR |
| Outer limit | 12 kpc  | based on CR |

| Parameter | Value | Reference |
|-----------|-------|-----------|
| Major axis | 3.5 kpc | 2, 7       |
| Scale length | 1.7, 0.64, 0.44 kpc | 2 |
| Axial ratio | 0.64/1.7, 0.44/1.7 | ... |
| Mass | $1.41 \times 10^{10} M_\odot$ | ... |
| Pattern speed ($\Omega_p$) | 50 km s$^{-1}$ kpc$^{-1}$ | 6 |

References. (1) Allen & Santillán (1991), (2) Freudenreich (1998), (3) Seigar & James (1998), Seigar et al. (2006), (4) Drimmel (2000), (5) Grosbøl et al. (2002), Churchwell et al. (2009), Elmegreen & Elmegreen (2014), (6) Gerhard (2011), (7) Binney et al. (1997), Bissantz & Gerhard (2002).

* a Up to 100 kpc halo radius.
* b $b_1$, $a_2$, $b_2$, and $a_3$ are scale lengths.
Consequently, the spiral arms are key to determine the shape of the velocity ellipsoid.

6. Although in this work we cover the general properties of normal spiral galaxies, we took advantage of the adjustability of our model to represent the gravitational potential of the Milky Way Galaxy. Analyzing the change in velocity dispersion induced by the nonaxisymmetric structures of the Galaxy we found that the galactic spiral arms are not capable to induce an important thickness in the stellar disk and the increase in vertical velocity dispersion is small; therefore, from this study we conclude that the spiral arms play no role in producing a thick disk. If we add the galactic bar, on the other hand, the vertical velocity dispersion increases considerably, mostly within the region covered by the bar. This means that for the Milky Way, the bar is an important heating mechanism that should be considered in calculations, but mostly in the inner region of the disk.

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Figure 24. Effect of the Milky Way spiral arms + bar model on the vertical disk heating. (top) Initial and final vertical velocity dispersions after 5 Gyr evolution. (bottom) Δσz (final−initial). The vertical heating is appreciable, with an increment up to 16 km s−1 in the inner region of the disk.
