Learning Sentimental Influences from Users’ Behaviors

Shenghua Liu, Houdong Zheng, Huawei Shen, Xiangwen Liao, Xueqi Cheng

1CAS Key Laboratory of Network Data Science & Technology
Institute of Computing Technology, Chinese Academy of Sciences
2Mathematics and Computer Science School, Fuzhou University
liushenghua@ict.ac.cn, shenghuawei@ict.ac.cn, liaoxw@fzu.edu.cn, cxq@ict.ac.cn

ABSTRACT
Modeling interpersonal influence on different sentimental polarities is a fundamental problem in opinion formation and viral marketing. There has not been seen an effective solution for learning sentimental influences from users’ behaviors yet. Previous related works on information propagation directly define interpersonal influence between each pair of users as a parameter, which is independent from each others, even if the influences come from or affect the same user. And influences are learned from user’s propagation behaviors, namely temporal cascades, while sentiments are not associated with them. Thus we propose to model the interpersonal influence by latent influence and susceptibility matrices defined on individual users and sentiment polarities. Such low-dimensional and distributed representations naturally make the interpersonal influences related to the same user coupled with each other, and in turn, reduce the model complexity. Sentiments act on different rows of parameter matrices, depicting their effects in modeling cascades. With the iterative optimization algorithm of projected stochastic gradient descent over shuffled mini-batches and Adadelta update rule, negative cases are repeatedly sampled with the distribution of infection frequencies users, for reducing computation cost and optimization imbalance. Experiments are conducted on Microblog dataset. The results show that our model achieves better performance than the state-of-the-art and pair-wise models. Besides, analyzing the distribution of learned users’ sentimental influences and susceptibilities results some interesting discoveries.

Keywords
opinion propagation; influence; susceptibility; cascade

1. INTRODUCTION
Collective opinions concisely form by a repeated process that a user who sees and agrees with a sentimental content, forwards, shares, or ‘Likes’ to feed her virtual community, resulting in a temporal cascade of users’ behaviors. As such, users who have taken actions become infective to others in their communities, encouraging their communities to a certain extend to take the same action by interpersonal influences. Each pair of users has a specific influence, especially when there exits some kind of social relation [19, 2]. Therefore, both opinion formation [39, 5] and viral marketing [31, 27] see the importance of learning sentimental influences between users, with which one can better model the dynamics of cascades [19], and maximize influence [21, 14, 6].

Figure 1: Motivations underlying our model. Example of cascades to illustrate the overfitting problem suffered by pair-wise models.

Among the existing works, it seldom sees an effective one for estimating sentimental influence between pairs of users as far as we are concerned. Most of studies focus on the process how users repeatedly update their opinions, the consensus they can reach [10, 34, 5], and opinion influence maximization [14], assuming that the interpersonal influences as edge weights are given or equally assigned. In the related domain of information propagation, influences learning has been studied, without consideration of different propagation behaviors on sentiments, although some studies modeled them on the distribution of content topics [29, 11]. Moreover, Goyal et. at [19] “learned” interpersonal influences by counting the successful propagation pairs. And with Bernoulli or Jaccard Index model, they estimated the influences as propagation probabilities. However, there usually record the times of users getting infected, while such successful propagation pairs that user $v$ infects user $u$ are rarely observed, or hardly traced. It limits the application of such a method. And NetInf [19] used an exponential or power-law form of incubation time between a pair of infected users to estimate the interpersonal influence, with empirically assigned parameters. Afterward, more novel models were proposed to learn interpersonal influence by maximizing the likelihood of observed cascades, which were proved to be more effective [33, 15, 18]. However, they used a free scalar parameter directly defined on a pair of users to represent the interpersonal influence. On one hand, the parameters are independent, even if the influences are acted by
or apply to the same user. On the other, such a pair-wise parameter cannot be trained if there are no observations of propagations between the user pair, e.g. users $c$ and $d$ do not appear in the same cascade, infected in Cascade1 and Cascade2 respectively, as Figure 1 shows. In that case, even though $c$ and $d$ have a relationship, and $c$, $e$ and $d$ form a social triangle, a zero or some empirically small constant is assigned, implying that it is never or seldom to successfully propagate between them in the future. In another way, Aral and Walker proposed to model the interpersonal influence by engineered features and corresponding linear coefficients learned for individual users other than user pairs directly. But users’ properties may not be available or easy to extract in other applications.

To fill the blank of previous works, we thus propose to learn distributed representations of users’ influences and susceptibility on sentiments. With such representations, we model the interpersonal influences and their decaying with the elapsed time in the hazard function of survival model, and maximize the likelihood of the observed behaviors of users taking or not taking the actions in sentimental cascades. Hence, the interpersonal influences, between two pairs of users, and with the same acting or applied user, can be coupled due to the corresponding representation of the same user. For example, the interpersonal influence of users $c$ to $d$, couples with that of users $c$ to $e$ by the corresponding representation defined on user $c$ as in Figure 1. Besides, it requires much fewer parameters, i.e., $O(n)$ for $n$ users, instead of $O(n^2)$ parameters for user pairs, beneficial to reducing the model complexity, and in turn combating the overfitting problem of assigning an empirical propagation probability. Moreover, the number of infected users in a cascade is usually much less than that of uninfected ones as negative cases. Considering all the negative users prevents the model from being applied to a real large dataset and balancing the optimization. Thus a negative sampling is employed to consider the expectation of negative cases instead, emphasizing the frequently infected users in other cascades. Finally a mini-batch Stochastic Gradient Decent (SGD) algorithm with Projected Gradient (PG) is designed to learn the model, and Adadelta is used to adjust the learning rate adaptively. In such a scheme, negative sampling is repeated in each iteration with a small number of samples each time, to approximate the expectation.

A set of cascades with different sentiments are collected from Microblog, covering a group of users who interact at a frequent level. Comparing with the state-of-the-art models, including above Bernoulli and Jaccard Index estimation methods, and pair-wise models, our model achieves better performances on the tasks of predicting cascade dynamics, “who will be retweeted”, and cascade size with users’ representations. Besides, it can be seen that learning influences separately on different sentimental polarities, mostly benefits the performances on both tasks, even if more parameters are brought in. As last, users’ representations on different sentiments are analyzed as well. And we find that users may have different influences on different sentiments, and are susceptible to different polarities. The “original influentials” are creative to post original attractive messages, while the “secondary influentials” gain their influence credits by hunting and advertising for interesting messages that already exists in the system.

The rest of the paper is organized as follows. Section 2 studies the existing and related works, and the motivation and our model are described in section 3, which the parameter learning algorithm is given. At last, experiments and result analysis are reported in section 4 and section 5 concludes the whole work.

2. RELATED WORK

Sentiment propagation and opinion formation have attracted many research works. [10, 22] experimentally showed that users’ sentiments were influenced by that of others surrounding them on LiveJournal dataset and Facebook dataset separately. [4] used Granger causality analysis to show that sentiment change of audiences were related to the landscape of popular users in Twitter. As for modeling opinion dynamics, successful models were proposed, including Sznajd model [35], Deffuant model [9], and Hegselmann and Krause model [21], which produced agreeing results. Moreover, [22] extended Sznajd model to complex networks. Deffuant et al. modeled the process of opinion dynamics that randomly select two users, and change their opinions to reduce the difference. [22] modeled to change users’ opinions according to the arithmetic average of that of their neighbors, and Fortunato et al. extended the model with multi-dimensional opinion vector, instead of a scalar opinion value. Besides, Suchecki et al. studied Voter model in scale-free network, small-world network, and random network. A recent work [5] by Bindel et al. discovered that traditional models including DeGroot model finally converged to a state of consensus under a set of general conditions, while it is rare in real opinion dynamics. Hence it proposed to model with users’ intrinsic beliefs in a game theory, which counterbalanced the opinions at Nash equilibrium. In addition, Gionis et al. studied the overall positive opinion maximization problem, adopting the game model of opinion dynamics. [1] modeled that a user’s opinion was generated from her latent opinion distribution, based on self-excited Hawkes process influenced by her neighbors.

The body of above works is mostly on opinion dynamics and maximization, assuming the sentimental influences between connected users were equal. It does not confirm to our observations in real life, in which a minority of influential users infect an exceptional number of their peers [23], and there are a mass of easily influenced users [39]. Thus, as a fundamental problem, sentimental influences were ever estimated by counting under Bernoulli assumption, or a threshold rule [39]. As far as we know, an effective method of learning sentimental influences from users’ behaviors remains unexplored.

Nevertheless, there were quite a few successful works on estimating interpersonal influences in the related domain of information propagation. Some of them made efforts to extract features that are related to propagation probability and learned from the observed information cascades. Crane et al. [8] measured the response function of information propagation dynamics in social systems with endogenous and exogenous factors. Artzi et al. [8] predicted whether a user would respond to or retweet a message, i.e. get influenced, by classifying with demographic and content features. In a way other than feature extractions, Tang et al. [38] proposed topic factor graph (TFG) to model the generative process of the topic-level social influence on large networks, by finding a topic distribution for each user. And [29] proposed a
probabilistic factor graph to model the direct and indirect influences between adjacent and non-adjacent users of heterogeneous network. Saito et al. [33] learned the propagation probability between neighbors of a directed network under independent cascade model, using the orders of users getting influenced as training data. And Goyal et. al [19] proposed to estimate the interpersonal influences in a counting manner, with assumptions of Bernoulli model and Jaccard Index separately. They estimated the influences as propagation probabilities. NetInf [10] adopted both exponential and a power-law incubation time models with fixed parameters as pair-wise probability to infer the underlying network. Besides, there are also a series of works learning a propagation probability between any pair of users with survival model and its variants to infer underlying networks with the transmission rates. NetRate [15] used survival theory to model transmission rate between every pair of users, which was viewed as an edge weight for the pair. And [17] then modeled the hazard rate in survival model with additive and multiplicative risks separately to improve the performance of cascade size prediction. Afterward, InfoPath [15] was proposed to learn time-varying transmission rates for user pairs as the edge weights of the hidden dynamic network. Taken together, these methods work in a pair-wise manner, i.e., they learned the propagation probability between pairs of users, fundamentally different from the proposed method in this paper which focuses on inferring user-specific influence and susceptibility from historical cascades. The features in influence and susceptibility representations were analyzed by [29], showing that propagation probability were determined by the two feature vectors, and learned the correlations between users’ attributes to identify influential or susceptible users. [45] then proposed a sequence model to learn a user’s latent representation of influence and susceptibility, based on the orders of users’ getting infected. In this work, we propose to learn the distributed representations of a user on sentiments, and continuous time model is employed to consider the infected times of users and the effect of the elapsed time on interpersonal influences, rather than their orders only.

3. LEARNING SENTIMENTAL INFLUENCE

A cascade is the snapshot of a propagation process, recording the times that users take actions on the same target, such as a piece of information, or product. Users taking actions become infected, and may influence others since such actions are publicly visible or pushed to the related users on purpose, by the online service. Thus we define a cascade $C$ for actions on a target as a temporal sequence

$$C = \{(v_1, t_1), (v_2, t_2), \ldots, (v_N, t_N) \mid t_1 \leq t_2 \leq \ldots \leq t_N\},$$

where $v_i$ is the user who take the action at time $t_i$, and $N$ is the total number of infected users, i.e. cascade size. Since social networks are not always available or existing in many applications, such as blogs, Yelp, Youtube, and online shopping, to make our model generally applicable, network structures are ignored. That is to say, the influence between any pair of users is modeled, and a very small value of influences can capture the underlying disconnections of the user network, and vice versa. Moreover, in the following we can see that our model can honor social network as well in the objective. In addition, a special time $t_E > t_N$ is defined as the biggest time window which we observe cascade $C$ in, namely the time when we take the snapshot for cascade $C$.

3.1 Motivations

Interpersonal influences are quite different especially when existing some social relationships. Most existing works intuitively model the interpersonal influences in a pair-wise manner with $n^2$ independent variables to learn, assuming that interpersonal influence between different pairs of users are independent from each other, even if the influences are related to a common user. Such an overfitting problem becomes severe, when there is not observed any propagation between a pair of connected users. Taking Figure 1 as an example, two cascades $\{(a, t_1^a), (c, t_1^c), (c, t_1^d), (f, t_1^f)\}$ and $\{(a, t_2^a), (b, t_2^b) (c, t_2^c), (f, t_2^f) (d, t_2^d)\}$ are observed. It is not seen that user $c$ is infected before user $d$ did, even though there is a social link from user $c$ to user $d$. In such a case, most existing models took the propagation probability or transmission rate between them as zero, or some empirically small value [20], implying that it would never or seldom see successful propagation between the two users in the future. Nevertheless, with the witness of propagation from users $c$ to $e$ in one cascade, and from users $e$ to $d$ in another, user $c$ probably influences user $d$ like the triangle pattern in friendship relations. Thus, with the distributed representations of influence and susceptibility defined for every user, interpersonal influences can be correlated by the shared representations of the same user. As shown in the example of Figure 1 the influences between user pairs $(c, d)$ and $(c, e)$ are coupled with a shared representation of user $c$’s influence. And the interpersonal influence from user $c$ to $d$ can be intuitively estimated by the learned representations of $c$’s influence and $d$’s susceptibility, other than a small empirical constant or zero.

At last, not all the users take actions in a cascade, so let the total number of users be $M$, and there are always a large number of users immune to a contagion, i.e., $M - N \gg N$, who are treated as negative cases and informative to reflect the interpersonal influences from the infected users to them. Without network constants, considering all the uninfected users takes much more computational costs, even unable to tackle. Moreover, the severe imbalanced positive (infected) cases and negative (uninfected) cases make the negative likelihood dominate the optimization of the whole objective, losing focus on positive cases as the following.

$$\max_c \sum_c \ln L = \sum_c \ln L^{c}_{pos} + \sum_c \sum_{M' - N^c} \ln L^{c}_{neg} \text{(dominate)}$$

where superscript $c$ is used to show that the values are related to cascade $C$. It is seen that the right term in summation easily dominates the objective, since $M$ is relatively very large. So we use sampled users as negative cases. Nevertheless, the infected frequency of a user indicates how easily she could get infected again. Thus observing a frequently infected user immune to a contagion, provides more information in the likelihood. And sampling negative cases from the distribution of users’ infected frequencies is then a better choice for learning influences.

3.2 Survival Analysis Model

We begin to briefly introduce the preliminary knowledge on Survival Analysis Model [26] [17]. We consider the hap-
pening time $T$ of a user taking the action as a continuous random variable, defined over $[0, \infty)$. Let $f(t)$ and $F(t)$ denote the probability density function (p.d.f) and the cumulative density function (c.d.f) separately. And the probability $Pr(T \leq t) = F(t)$. So, the probability of a user not taking the action until time $t$ is defined by the survivor function

$$S(t) = Pr(T \geq t) = 1 - F(t) = \int_{t}^{\infty} f(x)dx.$$ 

A hazard function $h(t)$ is defined as the instantaneously infecting rate in time interval $[t, t + \varepsilon)$, where $\varepsilon$ is an infinitesimal elapsed time, given a user survives until time $t$.

$$h(t) = \lim_{\varepsilon \to 0} \frac{Pr(t \leq T < t + \varepsilon | T > t)}{\varepsilon} = \frac{f(t)}{S(t)}$$

Noticing that $f(t) = -S'(t)$ and $S(0) = 1$, the survivor function can be expressed as

$$\ln S(x) = - \int_{0}^{t} h(x)dx.$$ 

### 3.3 Modeling sentimental cascades

With the analysis above, we model the interpersonal influence by two non-negative $K \times D$ matrices $I_j$ and $S_i$ defined on each user $v_i$, where $K$ is the number of sentiment classes, and $D$ is the dimension of users’ representations on each sentiment class. For a message with sentimental opinion, we define a one-hot vector $o$ with $K$ dimensions, representing its exclusive sentiment class. Thus, for a cascade with sentiment $o$, the transmission rate function $\phi(\cdot)$ from users $v_j$ to $v_i$, is defined as equation (1), which indicates the likelihood of successful propagation between them. Although the original concept of transmission rate is not necessarily between 0 and 1, we scale it for regularization.

$$\phi(I_j, S_i, o) = 1 - \exp(-o^T I_j S_i^T o)$$  \hspace{1cm} (1)

where matrices $I_j$ and $S_i$ are parameters to separately capture the influence of user $v_j$ and the susceptibility of user $v_i$. Let $H_{ji}$ denote the set of parameters $(I_j, S_i, o)$ for simplification. With transmission rate $\phi(H_{ji})$, we can define the hazard function from users $v_j$ to $v_i$ in Survival Analysis Model, at time $t$ as follows.

$$h(t|t_j; \phi(H_{ji})) = \phi(I_j, S_i, o) \frac{1}{t - t_j + 1}.$$  \hspace{1cm} (2)

where $t - t_j + 1$ depicts the hazard function monotonously decaying with the time elapsed from $t_j$, and adding 1 avoids unbounded hazard rate due to a zero or infinitesimal value of $t - t_j$. Noticing that equation (2) holds only when $t \geq t_j$, we define hazard rate $h(t|t_j; \phi(H_{ji})) = 0$, when $t < t_j$, namely, user $v_j$ has not been infected at time $t$. Moreover, we can consider social network by defining hazard function $h(t|t_j; \phi(H_{ii})) = 0$ as well, if user $v_i$ and user $v_j$ are not connected.

And then the survivor function $S(t|t_j; \phi(H_{ji}))$ of user $v_i$ surviving later than time $t$ and under the influence of user $v_j$, satisfies

$$\ln S(t|t_j; \phi(H_{ji})) = - \int_{0}^{t} h(x|t_j; \phi(H_{ji}))dx$$

$$= \phi(I_j, S_i, o) \cdot \ln(t - t_j + 1)$$  \hspace{1cm} (3)

Finally, the probability density function of user $v_i$ happening (getting infected) at time $t$, given user $v_j$ happening (infected) at time $t_j$ is calculated as follows.

$$f(t|t_j; \phi(H_{ji})) = h(t|t_j; \phi(H_{ji})) \cdot S(t|t_j; \phi(H_{ji})).$$

With the assumption that a user is only infected by one of the previously infected ones [17], the likelihood of user $v_i, i > 1$, being infected at time $t_j$ in a cascade is

$$f(t_i|t_j; \phi(H)) = \prod_{j:t_j < t_i} h(t_i|t_j; \phi(H_{ji})) \cdot \prod_{k:t_k < t_j} S(t_i|t_k; \phi(H_{ki})).$$  \hspace{1cm} (4)

So the joint likelihood of observing the whole cascade, given the user $v_1$ firstly taking the action at time $t_1$ is

$$f(t \setminus t_1|t_1; \phi(H)) = \prod_{i:1 < i < t_1} h(t_i|t_j; \phi(H_{ji})) \cdot \prod_{k:t_k < t_1} S(t_i|t_k; \phi(H_{ki})).$$

Considering the negative cases that users are not infected at the end, the probability of user $v_i$ surviving later than time $t_E$ is

$$S(t_E|t; \phi(H)) = \prod_{i:1 \leq t_i < t_E} S(t_E|t_i; \phi(H_{ii})).$$

And the log-likelihood of a cascade is as follows, considering negative cases.

$$\ln L(I, S; o) = \sum_{i:1 < i < t_1} \ln \left( \sum_{j:t_j < t_i} \phi(I_j, S_i, o) \frac{1}{t_i - t_j + 1} \right) - \sum_{i:1 < i < t_1} \sum_{k:t_k < t_i} \phi(I_k, S_i, o) \cdot \ln(t_i - t_k + 1) - \sum_{i:t_i \geq t_E} N \sum_{j=1}^{N} \phi(I_j, S_i, o) \cdot \ln(t_E - t_j + 1)$$

There are a large number of negative users comparing to the number of infected ones in a cascade. Maximizing the likelihood of all the negative cases limits the scalability of
our model, and the imbalance between positive and negative cases may mislead the optimization direction. Thus we sample $L$ users as negative cases according to the distribution $P(u) \propto R_u^{-\alpha}$, where $R_u$ is the frequency of user $u$ infected in cascades. It is worth noticing that sampling negative cases are repeated in every optimization iteration to honor the expectation. To give a direct understanding of the likelihood, the dependencies are concisely represented in Figure 2.

Finally, the optimization problem of learning users’ sentimental influences and susceptibilities

$$\min_{\mathbf{I}, \mathbf{S}} \quad -\sum_c \ln \mathcal{L}^c(\mathbf{I}, \mathbf{S}; \mathbf{o}^c) \quad (5a)$$

subject to $I_{ki} \geq 0, S_{ki} \geq 0, \forall k, i.$

where superscript $c$ is used to indicate that the value or function are related to cascade $C$.

### 3.4 Optimization

Optimization algorithm is the key to learn the distributed representations of users’ influences. First of all, the gradients of transmission rate function $\mathbf{I}$ on $\mathbf{I}_v$ and $\mathbf{S}_v$ are $K \times D$ matrices.

$$\frac{\partial \phi(\mathbf{I}_v, \mathbf{S}_v, o)}{\partial \mathbf{I}_v} = (1 - \phi(\mathbf{I}_v, \mathbf{S}_v, o)) \mathbf{o}^T \mathbf{S}_v$$

$$\frac{\partial \phi(\mathbf{I}_v, \mathbf{S}_v, o)}{\partial \mathbf{S}_v} = (1 - \phi(\mathbf{I}_v, \mathbf{S}_v, o)) \mathbf{o}^T \mathbf{I}_v$$

where only the $k$-th row in both matrices can have non-zero gradients, because a cascade belongs to the $k$-th sentiment class, i.e., $o_k = 1$. Furthermore, if user $v$ gets infected in a cascade, $t_1 \leq t_v \leq t_N$, the gradients of the log-likelihood on matrix $\mathbf{I}_v$ may have non-zero gradients. And the gradients of the log-likelihood on matrix $\mathbf{S}_v$ may be non-zero, if user $v$ is infected and $t_1 < t_v \leq t_N$, or she is a negative user. Otherwise, the gradients are always zeros. As the negative cases for a cascade is repeatedly sampled in every iteration, we define $[V_v]_r$ as the set of negative users at the $r$-th iteration of algorithm for cascade $C$.

$$[V_v]_r = \{v_i \sim P(u)\}_L,$$

where $L$ is the set size.

Therefore, the gradients of the objective function $\mathbf{I}^c$ on matrices $\mathbf{I}_v$ and $\mathbf{S}_v$ are as follows.

$$g_{I_v} = -\sum_c 1(t_c^v \leq t_N) \frac{\partial \mathcal{L}^c(\mathbf{I}, \mathbf{S}; \mathbf{o}^c)}{\partial \mathbf{I}_v}$$

$$g_{S_v} = -\sum_c 1(t_1 < t_c^v \leq t_N) \frac{\partial \mathcal{L}^c(\mathbf{I}, \mathbf{S}; \mathbf{o}^c)}{\partial \mathbf{S}_v}$$

$$+ \sum_{j=1}^{N_v} \sum_c (1 - \phi(\mathbf{I}_j, \mathbf{S}_j, \mathbf{o}^c)) \cdot (\mathbf{I}_j, \mathbf{S}_j, \mathbf{o}^c) \cdot \mathbf{I}_j$$

where $1(\cdot)$ is an indicator function, putting $1$ if the argument is true, and $0$ otherwise. $g_{I_v}$ and $g_{S_v}$ are $K \times D$ matrices, containing partial derivatives of objective function $\mathbf{I}^c$ on each elements of matrices $\mathbf{I}_v$ and $\mathbf{S}_v$ separately.

The framework of Stochastic Gradient Decent (SGD) over shuffled mini-batches is employed for efficient optimization. The mini-batch size is set $12$ cascades. In order to solve the non-negative constraints on parameters, Projected Gradient (PG) [28] is used to adjust the gradients. Let the parameter updates be $\Delta \mathbf{I}_v$ and $\Delta \mathbf{S}_v$ for each user $v$. And matrices $\mathbf{I}, \mathbf{S} \in \mathbb{R}^{K \times DM}$ are the concat of $\mathbf{I}_v$ and $\mathbf{S}_v$ for all users $v$. $M$ is the user count as defined previously. Thus the updates will be reduced by a rate $0 < \beta < 1$, namely, $\beta \Delta \mathbf{I}_v$ and $\beta \Delta \mathbf{S}_v$, if the following condition does not hold.

$$\mathcal{O}([E]_{r+1}) - \mathcal{O}([E]_r) \leq \sigma \cdot Tr(\nabla \mathcal{O}([E]_r)^T([E]_{r+1} - [E]_r))$$

(6)

where $[\cdot]_r$ means the parameter in the $r$-th iteration. With $\mathbf{E} = (\mathbf{I}, \mathbf{S}) \in \mathbb{R}^{K \times DM}$, $\mathcal{O}([E])$ is the simplified representation of objective function $\mathbf{E}$, $Tr(\cdot)$ is the trace of a matrix, and $\sigma$ is a constant between $0$ and $1$.

Moreover, since deciding learning rate is not trivial, so we choose Adadelta [11] to adaptively tune the learning rate. Let $\rho$ be decay rate and $\epsilon$ be a small constant. The accumulate gradients are

$$E[g_{I_v}]_r = \rho E[g_{I_v}]_{r-1} + (1 - \rho)[g_{I_v}]_r$$

$$E[g_{S_v}]_r = \rho E[g_{S_v}]_{r-1} + (1 - \rho)[g_{S_v}]_r$$

$$E[\Delta \mathbf{I}_v]^2]_r = \rho E[\Delta \mathbf{I}_v]^2]_{r-1} + (1 - \rho)[\Delta \mathbf{I}_v]^2]_r$$

$$E[\Delta \mathbf{S}_v]^2]_r = \rho E[\Delta \mathbf{S}_v]^2]_{r-1} + (1 - \rho)[\Delta \mathbf{S}_v]^2]_r$$

And with the definition of function $RMS[x] = \sqrt{E[x^2]_r + \epsilon}$, the update values are calculated as

$$[\Delta \mathbf{I}_v]_r = \frac{RMS[\Delta \mathbf{I}_v]_r}{RMS[g_{I_v}]_r} [g_{I_v}]_r$$

$$[\Delta \mathbf{S}_v]_r = \frac{RMS[\Delta \mathbf{S}_v]_r}{RMS[g_{S_v}]_r} [g_{S_v}]_r$$

Let the project function $\psi(x)$ be defined as projecting $x$ into non-negative space, namely, $\psi(x) = 0$ if $x < 0$; otherwise $\psi(x) = x$. Therefore, the algorithm of learning users’ sentimental influences is listed in Algorithm 1.

Algorithm 1 Algorithm of learning users’ sentimental influences.

Given $0 < \rho, \beta < 1$, constants $\sigma$ and $\epsilon$;

initialize parameters $\mathbf{I}_v$ and $\mathbf{S}_v$ for each user $v$;

Cascade set $C$.

Iteration index $\tau := 0$;

$E[g_{I_v}]_0, E[g_{S_v}]_0, E[\Delta \mathbf{I}_v]^2]_0, E[\Delta \mathbf{S}_v]^2]_0 = 0$;

repeat

Randomly shuffle $C$;

Split $C$ into groups by mini-batch size;

for each group do

Compute gradients $[g_{I_v}]_r, [g_{S_v}]_r$;

Accumulate gradients and updates:

$E[g_{I_v}]_r, E[g_{S_v}]_r, E[\Delta \mathbf{I}_v]^2]_r, E[\Delta \mathbf{S}_v]^2]_r$;

Parameter update values:

$[\Delta \mathbf{I}_v]_r, [\Delta \mathbf{S}_v]_r$;

Update $[\mathbf{I}_v]_{r+1} = \psi([\mathbf{I}_v]_r + [\Delta \mathbf{I}_v]_r)$;

Update $[\mathbf{S}_v]_{r+1} = \psi([\mathbf{S}_v]_r + [\Delta \mathbf{S}_v]_r)$;

while not Condition (6) do

decreasing update values:

$[\Delta \mathbf{I}_v]_r = \beta [\Delta \mathbf{I}_v]_r, [\Delta \mathbf{S}_v]_r = \beta [\Delta \mathbf{S}_v]_r$;

Update $[\mathbf{I}_v]_{r+1} = \psi([\mathbf{I}_v]_r + [\Delta \mathbf{I}_v]_r)$;

Update $[\mathbf{S}_v]_{r+1} = \psi([\mathbf{S}_v]_r + [\Delta \mathbf{S}_v]_r)$;

end while

$\tau := \tau + 1$

end for

until parameters converged, or maximum epoch.
4. EVALUATIONS

Microblog data is used to evaluate our model. To make the application more general, we assume that the retweeting relations and following relations are not available in the evaluations, only keeping the temporal sequence of users taking actions, i.e., retweet, and their infected times as the dataset. We then demonstrate the performance of our model at the well-known tasks, by comparing to the state-of-the-art models, and the learned sentimental influences are analyzed as well.

4.1 Data Description

Several strategies are taken to collect Microblog data from Sina Weibo\(^3\). We initially collected about 315.6 million records including posting, retweeting, and mentioning messages between Nov 1st, 2013 to Feb 28, 2014 from the timeline of 312,000 users sampled from Sina Weibo database. Since emoticons in cascade messages are usually used as the sentiment indicator, we filtered the messages with frequently used emoticons and active users, and crawled the full records of retweeting cascades of those remaining messages. Emoticons are split into positive sentiment set and negative sentiment set according to a dictionary of emoticons. And sentiments of messages are intuitively assigned according to emoticons in our experimental settings. Otherwise, one can use any reliable sentiment classifier, such as OpinionFinder\(^6\) to decide the sentiments. Meanwhile, retweeting relations are also extracted from the auto-generated contents, which help to preprocess data, and are used for ground truth in later evaluations. And we define the activeness \(A_v\) of a user \(v\) as the summation of the frequency of user \(v\) getting infected (retweeting others), \(A_v\), and that of user \(v\) influencing others (being retweeted), \(A_v\), in current cascades.

\[
A_v = A_v + A_v.
\]

Afterward, in a way of “onion peeling”, we repeated to delete for each cascade, the records of the users with activeness less than 5, and so did those of the users retweeting them. In each iteration, the cascades of sizes less than 8 are deleted as well, since very short cascades are considered as accidents.

| Time span | Total users | Total cascade size | Positive | Negative |
|-----------|-------------|-------------------|----------|----------|
| 10/31/13  | 6219        | 44021             | 325      | 412      |

| User activeness | Cascade size |
|-----------------|--------------|
| median          | 5            |
| mode            | 4            |
| median          | 37           |
| mode            | 10           |

With such a heuristic way, we finally get a set of cascades over a virtual community of active users from Oct 31, 2013 to Mar 3, 2014. As listed in Table 1(a), there are 6,219 users, and 44,021 cascade records totally. The number of cascade messages with positive emoticons is 325, and the number of those with negative ones is 412, keeping a balanced observations for learning sentimental influences. And Figure 3 illustrates the distributions of top frequently used emoticons in the messages of cascades, indicating their positive sentiments or negative sentiments. Furthermore, the median and mode values of the distribution of users’ activeness are 5 and 4 separately as in Table 1(b), indicating that users’ behaviors are not rarely observed in our dataset to guarantee a successful learning. And it also gives the median and mode of the distribution of cascade sizes as well, showing the sufficiency of involved users in a cascade. The cascades in the dataset are evenly split into 10 groups, and 10-fold cross testing are used for evaluations, alternatively with 9 of 10 groups as training, and the remaining one as testing.

4.2 Evaluation Models

In the experiments, we choose the following models for comparison.

- **CT Bernoulli** and **CT Jaccard** models\(^{19}\): They are continuous time models that the propagation probability \(P_{ab}\) from user \(a\) (infected) to \(b\) decays with the elapsed time. For a fair comparison, we use the same decaying function, i.e., \(P_{ab} = P_{ab0} / (t_b - t_a + 1)\), and the same assumption that a user is only infected by one of the infective users. CT Bernoulli model assumes that an initial propagation probability \(P_{ab0}\) follows Bernoulli distribution, i.e. the fraction of number of successful propagation over the total number of trials, from one user to another. And CT Jaccard model defines an initial propagation probability \(P_{ab0}\), in a form of Jaccard Index, which is the number of successful propagation divided by the total number of cascades with at least one infected between a pair of users. Since there only observes a temporal sequence of users getting infected in training dataset, we assume that successful propagation takes place from every earlier infected users to the current one.

- **NetRate**\(^{15}\): It directly define a scalar parameter as interpersonal influence between a pair of users, and learned them with Survival model. Since Jaccard Index was reported as a better estimator of propagation probability\(^{19}\), we use Jaccard Index to initialize the transmission rates at the beginning of the learning stage, to get a better fine tune.

- **CT LIS**: We ignore the differences of latent influence and susceptibility on sentiments of cascade messages, and define two \(D\)-dimensional vectors, \(I_v\) and \(S_v\), for

![Figure 3: The distribution of emoticon frequency.](image)
transmission rate function $\phi(\cdot)$ instead. Such parameters were ever defined by [35], which used a static way to model the orders of users’ behaviors. So we use “CT LIS” to indicate our upgrade version for continuous time model.

- **Sent LIS**: It is our model that learns sentimental influences considering all the negative cases. And we use “Sent LIS (neg sample)” to indicate ours with negative sampling.

### 4.3 Tasks and evaluation metrics

The following tasks are used to evaluate the effectiveness of our learned sentimental influences and the improvements comparing to the other models. And the metrics for each task are introduced as well.

**PCD**: predicting cascade dynamics. The happening times and infected users of cascade dynamics are both predictable by our model. However, in order to make the task simple and easy to evaluate, we design the task that aims at predicting whether a user will be infected at a given time $t$, knowing the previous truth, i.e. the users who have been infected, and their happening times before time $t$. Thus on one hand, the task can be treated as a set of binary classification problems, and we evaluate the results, with the infected users as the positive cases, and finally uninfected users until time $t_E$ as the negative ones. As for the positive cases, the likelihood of an infected user $v_i$ at a given time $t_i$, is given by $f(t_i; \phi(H_{u,v}))$. The likelihood of a negative user $v_i$ if she had been infected right after the positive ones, i.e., at time $t_i + \epsilon$, would be calculated as $f(t_i + \epsilon; \phi(H_{u,v}))$, where $\epsilon$ is a very small constant. Thus with the likelihood values for all the users, true positive (TP) rate and false positive (FP) rate can be calculated given any threshold. And then AUC (the area under the ROC curve) can be evaluated as $[12]$, where ROC is drawn with TP rate and FP rate as the coordinates.

On the other hand, given time $t$, and the observation of cascades before that time, we can calculate the infected likelihood for candidates $v$, by $f(t; \phi(H_{u,v}))$. Thus with ranking the candidates with their likelihood values, the top ones are the most probably infected, and a well-performed model can give a high rank to those users happened at the moment. In such a way, Mean Reciprocal Rank (MRR) [37] for rankings at all times of users getting infected in cascades is calculated as the metric.

**WBR**: who will be retweeted. Microblog users get infected and take actions to retweet the message from one of their followees who posts or retweets it previously. Thus the task predicting “who will be retweeted” is a way to examine interpersonal influence under quantitative understanding. In the scene of multi-exposures, high interpersonal influence will have high probability to be forwarded. As such, given $(v_i, t_i)$, namely, user $v_i$ happened at time $t_i$, the infective user that $v_i$ retweets is

$$\arg\max_{j:t_j < t_i} f(t_j; \phi(H_{j,v_i}))$$

We therefore deal with the prediction task as a ranking problem of interpersonal influence. The user with higher rank is more probable to be retweeted. We evaluate the prediction performance by metrics of average Accuracy (Acc) of top-one prediction and MRR. The ground truth of retweets can be extracted from the content of Microblog messages. Larger values of Acc and MRR indicate better predictions.

**CSP**: Cascade size prediction. Cascade size prediction, as a key part of influence maximization and viral marketing, is one of the most important applications based on modeling cascade dynamics. In our settings of CSP task, we choose the first $P$ users and acting times of each cascade as the initialization, and predict the cascade size at time $t_N$, $t_N > t_P$, where $t_P$ is the acting time of the $P$-th user $v_P$. The simulation method is used to predict the cascade size by dynamics models. The prediction time span $t_N - t_P$ is evenly split and marked by time scales. Thus starting after time $t_P$, an infected user $v$ tries to influence an uninfected user $v$ at each scale $\tau_i > t_P$, with the probability

$$Pr(T \leq \tau_i | t_u; \phi(H_{u,v})) = \int_{\tau_{i-1}}^{\tau_i} f(t|t_u; \phi(H_{u,v}))dt \over S(\tau_{i-1}|t_u; \phi(H_{u,v}))$$

And if user $v$ is infected at time scale $\tau_i$ with such a sampling, she will be added as infected users at the following time scales. The simulations are repeated, and the average cascade size is reported as the prediction. Thus the prediction can be evaluated by mean absolute percentage error (MAPE), where a smaller value indicates a better prediction.

### 4.4 Evaluation results.

As the description of dataset, we split the whole datasets into 10 groups for cross testing. Thus each experiments are repeated 10 times, and the average metrics and the Standard Deviation (SD) are reported. And the dimension of users’ representations on a sentimental polarity is $D = 8$ in the following evaluations for computational efficiency.

![Figure 4: The ROC curves of evaluation models on PCD task.](image-url)
NetRate (Jaccard) is the best of the three, thanks to the significance test, p-value and 0.0265 in the metric of MRR, overwhelming other models in bold text. It is seen that our model achieves 0.0216 all the models, with the best and the second best MRRs and AUCs in bold text. It takes her advantages to “CT Bernoulli” in both metrics, and NetRate also achieves a better performance in both accuracies and MRRs in the estimation of propagation probability, as reported. In the measurement of binary classification, “Sent LIS” and “Sent LIS (neg sample)” both outperform the others in AUC, which are 0.8992 and 0.8983 separately, with the former achieving a slightly better result. Besides, the machine learning model NetRate can further tune the Jaccard Index to achieve better MRR and AUC values. The bold numbers are the best and second best MRRs and AUCs in bold text. It is seen that our model achieves 0.0216 and 0.0265 in the metric of MRR, overwhelming other models with significance test, p-value < 0.01. And our negative sampling model get the best, thanks to its effort in balancing positive and negative cases. By examining the results generated from “CT Bernoulli” and “CT Jaccard”, it shows a consistent result that Jaccard Index can beat the Bernoulli model in the estimation of transmission rates of users, showing our advantages in the remission of overfitting and model complexity reduction.

Table 2: Average MRRs and AUCs of PCD task for 10-fold cross testing.

|                | CT Bernoulli | CT Jaccard | NetRate (Jaccard) | CT LIS | Sent LIS | Sent LIS (neg sample) |
|----------------|--------------|------------|-------------------|--------|---------|----------------------|
| **Acc (±SD)**  |              |            |                   |        |         |                      |
| Average        | 0.0365 ± 0.0029 | 0.0364 ± 0.0036 | 0.0371 ± 0.0038 | 0.0196 ± 0.0039 | 0.0216 ± 0.0033 | 0.0265 ± 0.0044 |
| **MRR (±SD)**  |              |            |                   |        |         |                      |
| Average        | 0.8732 ± 0.0658 | 0.8621 ± 0.0802 | 0.8718 ± 0.0730 | 0.8793 ± 0.0207 | 0.8992 ± 0.0152 | 0.8983 ± 0.0156 |

Table 3: Average accuracies and MRRs of WBR task for 10-fold cross testing.

|                | CT Bernoulli | CT Jaccard | NetRate (Jaccard) | CT LIS | Sent LIS | Sent LIS (neg sample) |
|----------------|--------------|------------|-------------------|--------|---------|----------------------|
| **Acc (±SD)**  |              |            |                   |        |         |                      |
| Average        | 0.1221 ± 0.0365 | 0.3000 ± 0.0964 | 0.3050 ± 0.0961 | 0.4123 ± 0.0874 | 0.3840 ± 0.1255 | 0.3980 ± 0.1392 |
| **MRR (±SD)**  |              |            |                   |        |         |                      |
| Average        | 0.2592 ± 0.0703 | 0.4349 ± 0.1275 | 0.4354 ± 0.1273 | 0.4696 ± 0.0876 | 0.4282 ± 0.1269 | 0.4134 ± 0.1348 |

all the models, with the best and the second best MRRs and AUCs in bold text. It is seen that our model achieves 0.0216 and 0.0265 in the metric of MRR, overwhelming other models with significance test, p-value < 0.01. And our negative sampling model get the best, thanks to its effort in balancing positive and negative cases. By examining the results generated from “CT Bernoulli” and “CT Jaccard”, it shows a consistent result that Jaccard Index can beat the Bernoulli model in the estimation of transmission rates of users, showing our advantages in the remission of overfitting and model complexity reduction.

WBR: With the extraction of retweeting relations from retweet content, the evaluation results of WBR task are reported in Table 3 based on the ground truth. The top-one accuracies (Acc) and MRRs of all cascades are averaged for the 10-fold cross tests as well, and the significance is tested. The bold numbers are the best and second best performances. Again we can see that with distributed representations of users, “CT Bernoulli”, “CT Jaccard” and NetRate limit their performance, comparing to the proposed models that learning distributed representations of users, showing our advantages in the remission of overfitting and model complexity reduction.

Table 4: Average accuracies and MRRs of WBR task for 10-fold cross testing.

|                | CT Bernoulli | CT Jaccard | NetRate (Jaccard) | CT LIS | Sent LIS | Sent LIS (neg sample) |
|----------------|--------------|------------|-------------------|--------|---------|----------------------|
| **Acc (±SD)**  |              |            |                   |        |         |                      |
| Average        | 0.0456 ± 0.0802 | 0.0755 ± 0.1255 | 0.0874 ± 0.1348 | 0.0730 ± 0.0152 | 0.0964 ± 0.0152 | 0.0961 ± 0.0152 |
| **MRR (±SD)**  |              |            |                   |        |         |                      |
| Average        | 0.1348 ± 0.0152 | 0.4354 ± 0.1273 | 0.4354 ± 0.1273 | 0.4696 ± 0.1269 | 0.4282 ± 0.1269 | 0.4134 ± 0.1348 |

Nevertheless, to show the differences of transmission rates learned by our model “Sent LIS (neg sample)” and pair-wise model NetRate, we separately estimate the transmission rates of ours on positive sentiment and negative sentiment, respectively, by latent sentimental influence and susceptibility matrices of Table 4. For each pair of users, there is a point with our transmission rate as X-coordinates, and that of NetRate as Y-coordinates. And we count the number of points falling in each lattice cell, as illustrated in Figure 5 (a) and (b), which cells are colored from cold color to warm color based on the point counts. Thus it is seen that a very warm and long line lying on the X-axis from 0.1 to 0.4 for both figures of positive sentiment and negative sentiment. It tells that a lot of overfitting transmission rates by NetRate assigning a zero or small constant, can be estimated by the distributed representations of users, which varies between different user pairs. Besides, the higher transmission rates from NetRate can also have a discriminative distribution in the transmission rates of ours, as those horizontally aligned warm cells show. And the same solution can also be concluded from Figure 5 (c) and (d). All above gives an evidence that our learned representations of users’ influences in cascade size prediction.

4.5 Analysis of users’ sentimental influences and susceptibilities

Besides the comparisons of evaluation models, we investigate our learned distributed representations of users on sentiments, matrices I, and S, for each user v. For each row in matrices I, and S, it is the representation of user v’s influence and susceptibility on the corresponding sentiment, denoted as “Positive I”, “Negative I”, “Positive S”, and
“Negative S”. And we use L1-norm of those row vectors to measure the degrees of influence and susceptibility on sentiments. Once more, we construct points of users with those L1-norm values as coordinates, and count the number of points falling into a predefined lattice cell. Thus the contour maps are draw accordingly in Figure 6. Figure 6 (a) and (b) are the contour maps of users’ influences v.s. susceptibility of each user with two matrices respectively. There are two peaks in both contour maps. It is interesting to see the peaks nearby “Positive I” and “Negative I” axes, which show that amount of influential users who are not susceptible to others as [2] claimed. We name them as original influentials in both positive sentiment and negative sentiment. On the other side, there are another part of influential users in the other two peaks located at the upper right of the contour maps, who are susceptible and active to retweet others’ messages, named secondary influentials in both sentiments. In another word, the secondary influentials may take a lot of efforts on retweeting attractive messages to gain their reputations and influences. And the original influentials focus on composing attractive and initial messages for the system. Thus the original influentials are the primitive power of the system to bring new resources, and the secondary influentials are good advertisers to let people get information.

Finally, we show a main peak in the contour maps of Figure 6 (c) and (d) in a 2-dimensional view, which give a distribution of users’ influences on positive sentiment and negative sentiment in (c), and that of users’ susceptibilities on both sentiments in (d). From Figure 6 (c), it is seen that users could have higher influences on positive sentiment, while lower ones on negative sentiment, and vice versa, although a certain amount of them have almost the same high influences on both sentiments. Figure 6 (d) gives the similar solution on susceptibilities, which some users are more sensitive to positive sentiments, and others are sensitive to negative ones. And it seems that more users have the same high susceptibilities on both sentiments than whom have the same high influences in the dataset.

We propose a model to learn the distributed representations of users’ influences on sentiments from their history behaviors. By explicitly characterizing the sentimental influence and susceptibility of each user with two matrices respectively, the model reduces the complexity of pair-wise models, and in turn remits the overfitting problem. We also design an effective algorithm to train the model based on maximizing logarithmic likelihood of information cascades. Adadelta method is used to estimate an efficient learning rate adaptively, and PG method guarantees the constants of non-negative parameters. Our model does not require the knowledge of social network structure, hence having wide applicability to the scenarios with or without explicit social networks. Explicit social network can be added as indicators in the likelihood of a user getting infected by the connected and infective ones. We evaluated the effectiveness of our model on Microblogging dataset from Sina Weibo, the largest social media in China. Experimental results demonstrate that our model consistently outperforms existing pair-wise methods at predicting cascade dynamics, “who will be retweeted”, and cascade size prediction. Moreover, with the analysis of users’ sentimental influences and susceptibilities, we find that there are two peaks in the contour maps, indicating original influentials and secondary influentials. The former only create initial and high-quality messages to influence others, while the latter attract others’ attentions by retweeting interesting messages. Besides, users may have different reactions on messages with different sentiments. In the future, we would like to apply the distributed representations of users to more imaginative applications.

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(a) Positive influence v.s. positive susceptibility

(b) Negative influence v.s. negative susceptibility

(c) Positive influence v.s. negative influence

(d) Positive susceptibilities v.s. negative susceptibility

Figure 6: Analysis of L1-norm of latent sentimental influences and susceptibilities.

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