Probabilistic forecasting of sudden power supply interruptions of electromechanical and heat-power equipment of regionally isolated electrotechnical complex

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Abstract. There is considered probabilistic forecasting of the most important characteristics of sudden power supply interruptions — damage and downtime of electromechanical and heat-power equipment — for process plants typical for installations of a regionally isolated electrotechnical complex. These specifications are considered as random functions of the power supply interruptions duration. A feature of the specifications considered is the presence of finite jumps when the power supply exceeds the critical values. Generalized theoretical equations are obtained for continuous functions of mathematical expectation and variance of process downtime and damage when there is any number of finite jumps of certain implementations and there is consideration of the discontinuity points as random variables with different distribution laws. The obtained specifications are used to optimize the cost of creating power supply systems considering their reliability.

1. Introduction
Reliability is one of the most important properties characterizing the quality of power supply of industrial facilities [1]. Failure-free performance is paramount of all the components of this complex feature. Among the types of electrical power supply system (EPSS) failures the most severe consequences for the technological process can be due to sudden power supply interruptions (SPSI). SPSI of some technological consumers can lead to large economic losses (damages), as well as create a danger to the personnel life and health, which requires the creation of specialized methods for modeling EPSS [2], designing protection systems and developing other approaches [3, 4]. SPSI are particularly dangerous for process plants that have a continuous production process and are used in in-line flow charts typical for networks with distributed generation [5, 6, 7]. Further, there is considered the class of networks with distributed generation corresponding to the features of the regionally isolated electrotechnical complex (RIEC) introduced under study in [2].

Life safety is not amenable to economic optimization and should be solved by rationing reliability.

The minimization of losses from SPSI is possible with the EPSS rational design, which involves probabilistic forecasting of SPSI economic consequences. Such forecasting makes an expedient decision in the process of choosing options for designing EPSS RIEC [8], including the component of damage into the generally accepted [9] annual reduced cost formula RC, which in its simplest form can be represented as follows:

\[ RC = E_N I + PC + D, \] rubles/year,

where \( E_N \) is a normative coefficient of investment efficiency;
I is one-time investment according to the considered option;
PC is annual production costs during normal operation;
D is the expected average annual damage from the SPSI.

The term damage means the expenditure of material and monetary resources, irrecoverable and not leading to improvement in qualitative and quantitative indicators of the object functioning, in excess of the minimum necessary to fulfill its tasks [10].

Reducing the damage from SPSI for one or another EPSS option is possible by increasing capital investments (using more reliable and therefore more expensive equipment, introducing redundancy, using means of protection and automation, etc.). The result is an increase in reduced costs.
2. The problem statement

For a number of productions SPSI does not lead to serious consequences for technological processes, causing only downtime of equipment and personnel for a period equal to the duration of the power supply interruption (PSI). To consider the damage component at the EPSS design stage does not make sense for them because of its smallness compared to other components. However, there are objects where such neglect is hardly acceptable.

These are technological installations with a continuous process of production and management: petrochemicals, oil refining, metallurgy, chemical production, etc. For such process plants technological damage exceeds losses that may occur in the EPSS, as a result the latter are usually neglected. As an example, it is enough to point to the EPSS drilling rigs.

Methods for determining damage from SPSI have been developed in detail [9] for enterprises which damage depend linearly on the PSI duration and can be determined by the amount of under-supplied electricity. However, such an approach is illegal for process plants with a continuous technological process (oil refining, petrochemical, chemical, and metallurgical, etc.).

The method for determining the damage from SPSI for enterprises with a continuous process is well developed [8]. However, its disadvantage is that the damage is considered as a certain deterministic function on the random duration of the $Te$ PSI. This approach leads to significant errors in calculations in the range $Te$, which corresponds to an abrupt change in damage.

To solve this problem, it is advisable to use the methods of the random functions theory [10] and probability theory [11]. In this case, a mathematical model should be constructed and generalized theoretical equations should be obtained for continuous expectation functions and variance of process downtime and damage from SPSI in the presence of any number of finite discontinuities of certain implementations and considering the discontinuity points as random variables with different distribution laws.

3. Theory

Usually technological reserves are used in major industries, and they keep the process parameters within acceptable limits at PSI even if there is no power supply. This is also due to rather large inertia of thermal and some other chemical processes. Therefore, if PSI has short duration, productivity decreases at such productions (sometimes to zero), but it is restored gradually after there is power supply (usually not immediately). Thus, the downtime $Td$ of the production process exceeds the PSI duration. It is not possible to assess the resulting damage in terms of the amount of unreleased electricity.

The possibilities of technological reserve are not unlimited. If the PSI is long-lasting, the process personnel have to take measures for the safe shutdown of the production. At the same time, the technological parameters go beyond the established limits, the technological process is cranked down, and it will take considerable time to restore the technological mode after power restoration.

This time is necessary to bring the parameters regulating the mode (pressure, temperature, reagent stock, etc.) to the minimum necessary values. Moreover, in some cases it takes time to restore the certain units and to set up the control system. The duration of this period depends on a number of objective and subjective factors. The objective factor is the degree of cooling of the process equipment, products and reagents, the ambient temperature, the rate of growth of various technological process parameters, etc [2]. The subjective factor is operating experience, correctness, timeliness and efficiency of implementation of measures to manage the technological mode.

The PSI duration, caused a sharp increase in the downtime of the process, will be called the critical $Te_{CR}$. For various technological installations, it ranges from a few seconds to several ten minutes.

As a result, the downtime of the technological process $Td$, which, when the PSI duration is less critical ($Te \leq Te_{CR}$) slightly exceeded this duration, is calculated in minutes, when the PSI duration exceeded the allowable value ($Te > Te_{CR}$) it sometimes reaches tens of hours. A further increment of PSI duration beyond the value $Te_{CR}$ increases the $Td$ downtime relatively little – only by this increment (Figure 1).

The damage from SPSI is usually directly proportional to the duration of the process downtime. Therefore, there is a problem to determine the components of the process downtime depending on the
SPSI duration. These components are not difficult to determine when the PSI has already occurred and its consequences are known. However, at the design stage of the EPSS enterprises, these components cannot be reliably known due to their random nature.

In fact, the dependence of the downtime (as well as damage) on the duration of the power supply interruption is a random function of $t_D(t_i)$. Consideration of the quantitative characteristics of the effects of power supply interruption as random functions is the most common approach to the task and does not emphasize the nature of the interaction between random variables, but also follows it in the development process.

The main characteristics of a random function are [10] the expectation function (ME), the variance, and the autocorrelation function. The determination of these characteristics is based on the compilation of data for certain implementations. The task of obtaining the largest possible number of implementations in order to calculate more accurately the required characteristics - first of all, the ME of a downtime random function can be solved by expert estimation method in combination with data on the actual PSI consequences. Considering that the amount of initial data is limited, it is not the characteristics of the random function that can be determined, but their estimates.

Thus, the ME function represents the ME system of estimating a random function obtained for the $k$-th section ($k$-th time value $t_i$) using the well-known equation [10], which can be written in the form of:

$$\hat{m}_{t_{jk}}(t_{EK}) = \frac{\sum_{i=1}^{n} t_{jk}(t_{EK})}{n}$$

where $n$ is the number of implementations of the random function;

$\hat{m}_{t_{jk}}(t_{EK})$ is ME rating of the technological process downtime, corresponding to the duration of the power supply interruption equal to $t_{EK}$;

$t_{Djk}(t_{EK})$ is the value of $i$-th implementation of the random function corresponding to the time $t_{EK}$.

The same implementations determine [4] estimates of the variance of a random function

$$\hat{D}_{t_{jk}}(t_{EK}) = \frac{\sum_{i=1}^{n} [t_{jk}(t_{EK}) - \hat{m}_{t_{jk}}(t_{EK})]^2}{n - 1}$$

and estimates of correlation moments:

$$R(\hat{t}_{k}, \hat{t}_{j}) = \frac{\sum_{i=1}^{n} [t_{jk}(t_{EK}) - \hat{m}_{t_{jk}}(t_{EK})] \cdot [t_{jk}(t_{Ek}) - \hat{m}_{t_{jk}}(t_{Ek})]}{n - 1}$$

where $\hat{m}_{t_{jk}}(t_{Ek})$ is the ME estimate for the technological process downtime corresponding to the duration $t_{Ek}$;

$t_{Djk}(t_{Ek})$ is the value of the $i$-th implementation of the random function corresponding to the time $t_{Ek}$.

Such a method to determine the characteristics of a random function is based on replacing it with a system of a sufficiently large number of random variables. Each random variable entering the system is a "cross section" of a family of implementations for a fixed value of the argument — the power outage time. The order of obtaining characteristics is illustrated in Figures 1 and 2.

The implementation of a random function (there are 5 in the example) is shown in Figure 1, and Figure 2 graphically (stepped line 1 and broken line 2) reproduces the estimate of the ME of a random function, which is based on the graphs in Figure 1. There are points of final discontinuity for graphs of implementations. The final discontinuity points are observed when the PSI time values are equal to the critical $T_{ECR}$. Due to the fact that value $T_{ECR}$ for each implementation is random, it does not match on different graphs (the discontinuity points in Figure 1 are observed only for two realizations - 1 and 3).

Analysis of Figure 2 shows that with an increase in the number of implementations, the number of discontinuity points of the generalized dependence will increase. Due to the fact that the PSI critical time is a continuous random variable, the number of discontinuities will tend to infinity, the dimensions of the graph "steps" will be zero, which in the limit will ensure the continuity of the generalizing dependence $m_{to}(t_{i})$ even in the area of discontinuities of certain implementations. Such a continuous "smoothing" dependence can be developed by applying well-known methods of leveling empirical functions, for example, using the "least squares method." However, the application of this or
any other method will be the most rational if the theoretical form of the smoothing function is known and it is to determine only the value of the parameters that fit best the experimental data.

We define the theoretical form of the function $m_D(t_E)$ in the segment where finite discontinuities of certain implementations are observed.

One denotes:
- $m_{t_E1}(t_{Ej})$ is the ME function of the random function in point $t_{Ej}$ provided that the discontinuity point of all realizations is to the right of the section $t_{Ej}$;
- $m_{t_E2}(t_{Ej})$ is the ME function of the random function at the point $t_{Ej}$ provided that the discontinuity point of all realizations is to the left of the section $t_{Ej}$;
- $T_{E_C}$ is the random coordinate of the discontinuity point on the PSI duration axis;
- $F(t_{E_C})$ is integral function of the probability distribution of the random coordinate of the point of discontinuity.

Using the equation for estimating the ME of a random function for an arbitrarily taken within the interval of discontinuities of realizations of the section $t_{Ej}$ and applying the Chebyshev theorem [10], there is obtained:

$$m_{t_E}(t_E) = m_{t_E1}(t_E)[1 - F(t_{E_C})] + m_{t_E2}(t_E)F(t_{E_C})$$

$$= m_{t_E1}(t_E) + [m_{t_E2}(t_E) - m_{t_E1}(t_E)]F(t_{E_C}) =$$

$$= m_{t_E1}(t_E) + m_{t_E2}(t_E)F(t_{E_C}),$$

where $m_{t_E2}(t_E)$ is the ME of the difference function $t_{n2}(t_E)$ and $t_{n1}(t_E)$.

**Figure 1.** Graphs of certain realizations of random function of process downtime on interruption duration

**Figure 2.** Generalized graphs of realizations of random function of process downtime on interruption duration
If the dependence $t_0(t_E)$ has not one, but several points of discontinuity, each is a random variable with its own law of probability distribution $- F(t_{E1\text{CR}}), F(t_{E2\text{CR}}), F(t_{E3\text{CR}})$ and so on, the final equation for the function $m_{td}(t_E)$ in general form takes the form:

a) if there are two discontinuities of certain realizations

$$m_{tn}(t_E) = [1 - F(t_{E1\text{CR}})]m_{tn1}(t_E) + [F(t_{E1\text{CR}}) - F(t_{E2\text{CR}})]m_{tn2}(t_E) + F(t_{E2\text{CR}})m_{tn3}(t_E) =$$

$$= m_{tn1}(t_E) + [m_{tn2}(t_E) - m_{tn3}(t_E)] f(t_{E1\text{CR}}) + [m_{tn3}(t_E) - m_{tn2}(t_E)] f(t_{E2\text{CR}});$$

b) if there are $n$ discontinuities of certain realizations

$$m_{td}(t_E) = m_{td1}(t_E) + [m_{td2}(t_E) - m_{td1}(t_E)] f(t_{E1\text{CR}}) + \cdots + [m_{tdi+1}(t_E) - m_{tdi}(t_E)] f(t_{Ei\text{CR}}) + \cdots +$$

$$+[m_{tdn+1}(t_E) - m_{tdn}(t_E)] f(t_{E\text{CRn}}).$$

Where $F(t_{E1\text{CR}}), F(t_{E2\text{CR}}), F(t_{E3\text{CR}}), F(t_{E\text{CRn}})$ are the laws of critical time distribution, respectively, for the first, second, and, $i$ -th, $n$ -th of discontinuity points, expressed as an integral distribution function;

$m_{tn1}(t_E), m_{tn2}(t_E), m_{tn3}(t_E), m_{tnn}(t_E)$ is ME of the function under study with its approximation by a discontinuous function respectively, on the first, second, $i$ -th, and last segments of continuity.

Mostly, the critical time of power supply interruption is distributed according to the Gauss law with parameters: mathematical expectation $M[T_{E\text{CR}}]$ and standard deviation $\sigma[T_{E\text{CR}}]$.

In this case, the integral distribution function of the probabilities of the critical time is described on the basis of classical equations [11] and takes the following form:

$$F(t_{E\text{CR}}) = F\left(\frac{t_{E\text{CR}} - M[T_{E\text{CR}}]}{\sigma[T_{E\text{CR}}]}\right),$$

where there is normal probability distribution function

$$F = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-0.5t^2} dt.$$

Then for the case with one discontinuity of implementations, we obtain the general form of the function $m_{td}(t_E)$ for the normal law of the distribution of the critical time:

$$m_{tn}(t_E) = m_{tn1}(t_E) + [m_{tn2}(t_E) - m_{tn1}(t_E)] F\left(\frac{t_E - M[T_{E\text{CR}}]}{\sigma[T_{E\text{CR}}]}\right).$$

If the critical time of power supply interruption is distributed according to the law of uniform density in the range from the minimum $t_{E\text{CR_MIN}}$ and up to the maximum $t_{E\text{CR_MAX}}$ of possible values we obtain

$$m_{tn}(t_E) = m_{tn1}(t_E) + \frac{m_{tn2}(t_E) - m_{tn1}(t_E)}{t_{E\text{CR_MAX}} - t_{E\text{CR_MIN}}}(t_E - t_{E\text{CR_MIN}}).$$

If the critical time of power supply interruption is distributed according to the exponential law in the interval from the minimum possible value of the critical time $t_{E\text{CR_MIN}}$, we obtain

$$m_{tn}(t_E) = m_{tn1}(t_E) + [m_{tn2}(t_E) - m_{tn1}(t_E)] \left(1 - e^{-\frac{t_E - t_{E\text{CR_MIN}}}{M[T_{E\text{CR}}]}}\right).$$

Along with the ME, the random function of the downtime can be characterized by the variance. Knowing the variance, one can judge the degree of reliability of a point estimate of a random variable, and such an estimate in this case is the ME. In addition, the use of the variance values determines the ME interval estimates for different sections of a random function.

Dispersion of the random function $D_{td}(t_E)$ when there is a single point of discontinuity of certain realizations is
D_{tD}(t_E) = D_{tD1}(t_i) + [D_{tD2}(t_E) - D_{tD1}(t_E)]F(t_{E\,CR}) + [m_{tD2}(t_E) - m_{tD1}(t_E)]^2F(t_{E\,CR})[1 - F(t_{E\,CR})].

In view of the fact that in the areas of continuity of certain realizations, the variance of the random function is approximately constant $D_{tD1}(t_E) \approx D_{tD2}(t_E) = D_{tD}$ the equation can be written differently:

$D_{tD}(t_E) = D_{tD} + F(t_{E\,CR})[m_{tD2}(t_E) - m_{tD1}(t_E)]^2[1 - F(t_{E\,CR})].$

Often, a more convenient analysis characteristic is used — the standard deviation of a random function, which is the square root of the variance of this function.

Analysis of the formula shows that the variance of the random function is minimal and equal to $D_{tD}$ in the continuity segments of certain realizations, since in this case one of the factors $F(t_{E\,CR})$ or $1 - F(t_{E\,CR})$ is zero. The maximum value of the dispersion can correspond to the ME of the critical time in the case of its symmetric distribution law ($t_{E\,CR}$) and equality of coefficients with the variable $t_E$ in functions $m_1(t_E)$ and $m_2(t_E)$, since the product $F(t_{E\,CR})[1 - F(t_{E\,CR})]$ has a maximum at $F(t_{E\,CR}) = 0.5$. The asymmetry of the critical time distribution law, the change in the ME of the difference between the functions $m_1(t_E)$ and $m_2(t_E)$ with a change in the argument $-m_{\Delta tn}(t_E) \neq$ const, the proximity of the intersection point of the functions $m_1(t_E)$ and $m_2(t_E)$ to the ME of the critical time will shift the maximum value of the dispersion in one direction or another from the latter.

4. Example of using

The construction of ME graphs and the standard deviation of the random function of the process downtime on the duration of the power supply interruption when there is one final discontinuity of the implementations is considered as an example (Figures 3 and 4), for one technological installation of the refinery. It is accepted in the example:

- for the first section $-m_{tn1}(t_E) = 4 t_E; D_{tn1}(t_E) = 0.15$ h$^2$;
- for the second segment $-m_{tn2}(t_E) = t_E + 10; D_{tn2}(t_E) = 0.15$ h$^2$;
- critical point with a coordinate distributed according to the Gauss law with the parameters $M[T_{E\,CR}] = 1$ h, $\sigma[T_{E\,CR}] = 0.1$ h (curve 2), $\sigma[T_{E\,CR}] = 0.2$ h (curve 3).

With these initial data ME random function of the downtime (curve 2, Figure 3) is described by the equation: $m_{tn}(t_E) = 4 t_E + (10 - 3 t_E)F(t_{E\,CR}^{'}-1)_{0.1}$

The variance of the random downtime function is described by the equation:

$D_{tD}(t_E) = 0.15 + (10 - 3t_E)^2 F(t_{E\,CR}^{'}-1)_{0.1}[1 - F(t_{E\,CR}^{'}-1)_{0.1}].$
5. Conclusion

1. The main characteristics of the SPSI effects - the damage and downtime of the process - for industries with a continuous process for plants of regionally isolated electrotechnical complex (RIEC), it is advisable to consider as a random function of the PSI duration.

2. When forecasting the use of discontinuous functions for estimating the SPSI effects can lead to significant systematic errors.

3. The study obtained generalized theoretical equations for continuous functions of expectation and variance of the process downtime and damage when there is any number of finite discontinuities of certain realizations and consideration of the discontinuity points as random variables with different distribution laws.
6. References

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