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Approximate Data Aggregation in Sensor Equipped IoT Networks

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Abstract: As Internet-of-Things (IoT) networks provide efficient ways to transfer data, they are used widely in data sensing applications. These applications can further include wireless sensor networks. One of the critical problems in sensor-equipped IoT networks is to design energy efficient data aggregation algorithms that address the issues of maximum value and distinct set query. In this paper, we propose an algorithm based on uniform sampling and Bernoulli sampling to address these issues. We have provided logical proofs to show that the proposed algorithms return accurate results with a given probability. Simulation results show that these algorithms have high performance compared with a simple distributed algorithm in terms of energy consumption.

Key words: data aggregation; sampling; Internet-of-Things (IoT) networks

1 Introduction

As the urban population snowballs, the smart city has become inevitable to solve many day-to-day problems. These problems include power supply, disaster prediction, and traffic maintenance[1–4]. Some of the smart city applications that are already being used are parking services, intelligent light systems, and water conservation. For the better utilization of natural resources, we should incorporate these applications even in rural areas. The fundamental working principle of a smart city application is that there are various sensors deployed all over the city that are used for collecting data. This data helps us understand information at a city level and hence the data should be well-spread.

Similar to the smart city, we are also focusing on smart home applications[5–10]. These applications are based on the fact that in today’s world all the home electronics are connected to the internet. A network with such connected devices is called Internet-of-Things (IoT). Recent devices like Alexa and Google home are built on such a network. These devices interact with all the other devices connected to the same network. Since not all devices are the same, we need a way to collect sensory data from different sensors. The primary objective of any IoT network is to reduce cost and provide faster access to data. One of the distinct challenges in these applications is the deployment of a considerable number of sensing devices.

It is clear that sensors are the building blocks of any IoT network. However, a network with sensors has some drawbacks such as the issue of dynamic traffic, adding new service, adaptive to channel condition, and ever-changing user requirements. Having self-configurable sensors helps address some of these issues. Additionally, many algorithms have been proposed to solve the issues of routing, topology control, and time synchronization[11–24]. Using sensors in IoT networks reduces the communication cost but increases the processing cost. We deploy sensors in our network because they collect data over a long period and could be placed over a long period of time and could be
placed over a vast network. Hence, the data collected from these sensors is huge and requires high processing power to aggregate and analyze the data. Hence, if the data aggregation problem is addressed at the sensor level, we do not have to deal with extensive data. However, adding data aggregation functionality to the sensor might consume a lot of sensor’s energy. This further raises the energy consumption issue as the aggregation costs much energy and the sensors are not equipped with huge amounts of power supply. According to Ref. [25], cost of transmitting one bit of data is equivalent to the energy cost of executing 1000 instructions. Therefore, reducing data transmission is one of the major ways to decrease the energy cost in IoT. Hence, it is critical to design energy efficient data aggregation methods for sensor equipped IoT networks.

In this paper, we study two kinds of aggregation queries: maximum query and distinct set query. The maximum query is to calculate the maximum of all the sensory data. The distinct set query is to calculate the unique values in the sensory data. Both the queries are critical for an IoT. These two queries can be widely used in practice. For example, in the field of environmental monitoring, the maximum value query can be used to acquire the most serious level of pollution. While the user may get all the pollution levels in the monitored area using the distinct-set query. Therefore, the energy efficient data aggregation model should accommodate both queries in its development.

In practice, exact query results are not always necessary. Approximate query results may also be acceptable for some applications [26, 27]. Therefore, in this paper, we propose two algorithms to process approximate maximum queries and distinct-set queries. These algorithms are based on uniform sampling and Bernoulli sampling, respectively. Proposed algorithms will return the exact query results with probability not less than $1 - \delta$, where $\delta$ is a real number and its value can be arbitrarily small. In summary, the main contributions of our paper can be summarized as follows:

1. Mathematical estimators for the two aggregation operations are provided.

2. The mathematical methods to determine the required sample size and sample probability for calculating approximate maximum value and approximate distinct-set are designed.

3. Distributed algorithms for approximate maximum value and approximate distinct-set are provided. The energy costs of these algorithms are analyzed.

4. Extensive simulation results are presented which show the proposed algorithms perform significantly better than a simple distributed algorithm in the aspect of energy consumption.

The rest of the paper is organized as follows. Section 2 defines the problem. Section 3 provides the mathematical proof for the $\delta$-approximate aggregation algorithms. Section 4 explains the proposed $\delta$-approximate aggregation algorithms. Section 5 shows the simulation results and the related works are discussed in Section 6. Section 7 concludes the paper.

2 Problem Definition

Suppose we have an IoT network with $n$ sensor nodes and $s_i$ is the sensory data of node $i$ at time $t$. $S_t = \{s_{t_1}, s_{t_2}, \ldots , s_{t_n}\}$ is the set of all the sensory data at time $t$ and $\text{Dis}(S_t) = \{s_{t_1}^{d_1}, s_{t_2}^{d_2}, \ldots \}$ contains the distinct values in $S_t$. For example, if $S_t = \{s_{t_1}, s_{t_2}, s_{t_3}, s_{t_4}, s_{t_5}\}$ and $s_{t_1} = 1, s_{t_2} = 1, s_{t_3} = 2, s_{t_4} = 3, s_{t_5} = 3$, then $\text{Dis}(S_t) = \{1, 2, 3\}$.

In this paper, we address maximum and distinct set queries by performing $\text{max}$ and $\text{distinct set}$ operations, respectively. The definition of these operations are as follows:

1. The exact maximum value denoted by $\text{Max}(S_t)$ satisfies $\text{Max}(S_t) = \max\{s_{t_i} \in S_t | 1 \leq i \leq n\}$.

2. The exact distinct-set of $S_t$ denoted by $\text{Dis}(S_t)$ satisfies $\forall s \in S_t, \exists x, y \in \text{Dis}(S_t), s = x$ and $\forall s^d_x, s^d_y \in \text{Dis}(S_t), x \neq y \Rightarrow s^d_x \neq s^d_y$.

Obviously, the following steps can be used to solve the max and distinct set aggregation problems.

1. Arrange all the nodes in the network in the form of an aggregation tree where the sink node broadcasts the aggregation operation.

2. All the nodes submit their sensory data to the sink node along the aggregation tree.

3. The intermediate nodes in the aggregation tree calculate the partial results during the data transmission.

Although this method results in accurate aggregation results, it will also lead to huge communication and computation cost. Hence, we propose a $\delta$-approximation to the results that can be achieved by the above said aggregation operations. Let $I_t$ and $\tilde{I}_t$ are the accurate and approximate aggregation results at the time “$t$”, respectively. The definition of the $\delta$-estimator is as follows:

**Definition 1** ($\delta$-estimator) For any $\delta$ (0 $\leq \delta$ $\leq 1$), $\tilde{I}_t$ is called the $\delta$-estimator of $I_t$ if $\Pr(\tilde{I}_t \neq I_t) \leq \delta$. 

According to Definition 1, the problem of computing \( \delta \)-approximate maximum value and \( \delta \)-approximate distinct-set is defined as follows.

**Input:** (1) A sensor equipped IoT network; (2) The sensory data set \( S_t \); and (3) Aggregation operator \( \text{Agg} \in \{ \text{Max}, \text{Dis} \} \) and \( \delta (0 \leq \delta \leq 1) \).

**Output:** \( \delta \)-approximate aggregation result of \( \text{Agg} \).

### 3 Preliminaries

In this paper, we use two sampling techniques to sample the raw data in the network, which are uniform sampling and Bernoulli sampling, respectively. The preliminaries of computing \( \delta \)-approximate maximum value and \( \delta \)-approximate distinct-set are presented in the following subsections.

#### 3.1 Uniform sampling-based approximate aggregation

Let \( u_1, u_2, \ldots, u_m \) to denote \( m \) simple random samplings with replacement from sensory data set \( S_t \), \( U(m) = \{ u_1, u_2, \ldots, u_m \} \) is a uniform sample of \( S_t \) with sample size \( m \), we have the following conclusions.

1. \( u_i \) and \( u_j \) are independent with each other for all \( 1 \leq i \neq j \leq m \).
2. \( \Pr(u_i = s_{ij}) = \frac{1}{n} \) for any \( 1 \leq i \leq m, 1 \leq j \leq n \).

Based on the above two conclusions, we have the following lemma.

**Lemma 1** For any given value \( x \in \text{Dis}(S_t) \), we have

\[
\Pr(x \notin U(m)) = \left(1 - \frac{n_x}{n}\right)^m,
\]

where \( n_x \) is the number of appearance of value \( x \) in \( S_t \).

**Proof** \( \Pr(x \notin U(m)) = \Pr(u_1 \neq x \land u_2 \neq x \land \ldots \land u_m \neq x) \). Since all the samples \( u_1, u_2, \ldots, u_m \) are independent with each other, we have

\[
\Pr(x \notin U(m)) = \prod_{i=1}^{m} \Pr(u_i \neq x) = (\Pr(u_1 \neq x))^m.
\]

Moreover, we have

\[
\Pr(u_1 \neq x) = 1 - \Pr(u_1 = x) = 1 - \frac{n_x}{n}.
\]

Then this lemma is proved. \( \blacksquare \)

To obtain \( \delta \)-approximate maximum value, the mathematical estimator needs to be defined firstly. Let \( \text{Max}(S_t)_u \) denote the uniform sampling-based estimator of \( \text{Max}(S_t) \). Then \( \text{Max}(S_t)_u \) is defined as

\[
\text{Max}(S_t)_u = \text{Max}(U(m)).
\]

Based on Lemma 1, we have the following theorem.

**Theorem 1** \( \text{Max}(S_t)_u \) is a \( \delta \)-estimator of \( \text{Max}(S_t) \) if

\[
m \geq \frac{\ln \delta}{\ln \left(1 - \frac{n_{\text{min}}}{n}\right)},
\]

where \( n_{\text{min}} \) is the number of appearance of the least appearing data.

**Proof** Based on the condition, we have

\[
m \ln \left(1 - \frac{n_{\text{min}}}{n}\right) \leq \ln \delta,
\]

\[
\left(1 - \frac{n_{\text{min}}}{n}\right)^m \leq \delta.
\]

According to Lemma 1, we have

\[
\Pr(\text{Max}(S_t) \notin U(m)) = \left(1 - \frac{n_{\text{Max}(S_t)}}{n}\right)^m
\]

where \( n_{\text{Max}(S_t)} \) is the number of appearance for the maximum value in \( S_t \). Since \( n_{\text{Max}(S_t)} \geq n_{\text{min}} \), we have

\[
\Pr(\text{Max}(S_t) \notin U(m)) \leq \left(1 - \frac{n_{\text{min}}}{n}\right)^m \leq \delta.
\]

Then this theorem is proved. \( \blacksquare \)

Let \( \text{Dis}(S_t)_u \) denote the uniform sampling-based estimator of exact result \( \text{Dis}(S_t) \). Then \( \text{Dis}(S_t)_u \) is defined as

\[
\text{Dis}(S_t)_u = \text{Dis}(U(m)).
\]

Based on Lemma 1, we also have the following theorem.

**Theorem 2** \( \text{Dis}(S_t)_u \) is a \( \delta \)-estimator of \( \text{Dis}(S_t) \) if

\[
m \geq \frac{\ln(1 - (1 - \delta)^{n_{\text{min}}/n})}{\ln \left(1 - \frac{n_{\text{min}}}{n}\right)}.
\]

**Proof** First, we have

\[
1 - \left(1 - \frac{n_{\text{min}}}{n}\right)^m \leq 1 - (1 - \delta)^{n_{\text{min}}/n},
\]

\[
1 - \left(1 - \frac{n_{\text{min}}}{n}\right)^m \leq 1 - \delta,
\]

\[
1 - \left(1 - \left(1 - \frac{n_{\text{min}}}{n}\right)^m\right) \leq \delta.
\]

Let \( n_{s_{ij}}^d \) to denote the number of appearance for \( s_{ij}^d \), then we have

\[
1 - \left(\prod_{i=1}^{\lvert \text{Dis}(S_t)_u \rvert} \left(1 - \left(1 - \frac{n_{s_{ij}}^d}{n}\right)^m\right)\right) \leq \delta.
\]

since \( n_{\text{min}} \leq n_{s_{ij}}^d \). Moreover, according to Lemma 1, we have

\[
1 - \left(\prod_{i=1}^{\lvert \text{Dis}(S_t)_u \rvert} \left(1 - \Pr(s_{ij}^d \notin U(m))\right)\right) \leq \delta.
\]

\[
1 - \left(\prod_{i=1}^{\lvert \text{Dis}(S_t)_u \rvert} \Pr(s_{ij}^d \in U(m))\right) \leq \delta.
\]

\[
1 - \Pr(\text{Dis}(S_t)_u = \text{Dis}(S_t)) \leq \delta.
\]

\[
\Pr(\text{Dis}(S_t)_u \neq \text{Dis}(S_t)) \leq \delta.
\]
Then this theorem is proved. ■

3.2 Bernoulli sampling-based approximate aggregation

Let \( B(q) = \{b_1, b_2, \ldots, b_{|B(q)|}\} \) denote a Bernoulli sample of data set \( S_t \) with sample probability \( q \). Then we have the following lemma.

**Lemma 2** For any given value \( x \in \text{Dis}(S_t) \), we have

\[
\Pr(x \notin B(q)) = (1 - q)^{n_x},
\]

where \( n_x \) is the number of appearance of value \( x \) in \( S_t \).

**Proof** Without loss of generality, we assume \( s_{t1} = s_{t2} = \cdots = s_{t_n} = x \), then we have \( \Pr(x \notin B(q)) = \Pr(s_{t1} \notin B(q) \land s_{t2} \notin B(q) \land \cdots \land s_{t_n} \notin B(q)) \). Therefore, we have

\[
\Pr(x \notin B(q)) = (\Pr(s_{t1} \notin B(q)))^{n_x}.
\]

According to the definition of Bernoulli sampling, we have

\[
\Pr(s_{t1} \notin B(q)) = 1 - \Pr(s_{t1} \in B(q)) = 1 - q.
\]

Then this lemma is proved. ■

Let \( \text{Max}(S_t)_b \) denote the Bernoulli sampling-based estimator of exact value \( \text{Max}(S_t) \). \( \text{Max}(S_t)_b \) is defined as

\[
\text{Max}(S_t)_b = \text{Max}(B(q)).
\]

Based on Lemma 2, we have the following theorem.

**Theorem 3** \( \text{Max}(S_t)_b \) is a \( \delta \)-estimator of \( \text{Max}(S_t) \) if

\[
q \geq 1 - (\delta)^{1/n_{\min}}.
\]

**Proof** Based on the condition, we have

\[
(1 - q)^{n_{\min}} \leq \delta.
\]

According to Lemma 2, we have

\[
\Pr(\text{Max}(S_t)_b \notin B(q)) = (1 - q)^{n_{\text{Max}(S_t)}},
\]

where \( n_{\text{Max}(S_t)} \) is the number of appearance for the maximum value in \( S_t \). Since \( n_{\text{Max}(S_t)} \geq n_{\min} \), we have

\[
\Pr(\text{Max}(S_t)_b \notin B(q)) \leq (1 - q)^{n_{\min}} \leq \delta.
\]

Then this theorem is proved. ■

Let \( \text{Dis}(S_t)_b \) denote the Bernoulli sampling-based estimator of exact result \( \text{Dis}(S_t) \). Then \( \text{Dis}(S_t)_b \) is defined as

\[
\text{Dis}(S_t)_b = \text{Dis}(B(q)).
\]

Based on Lemma 1, we have the following theorem.

**Theorem 4** \( \text{Dis}(S_t)_b \) is a \( \delta \)-estimator of \( \text{Dis}(S_t) \) if

\[
q \geq 1 - (1 - (1 - \delta)^{n_{\min}/n})^{1/n_{\min}}.
\]

**Proof** According to the condition, we have

\[
(1 - q)^{n_{\min}} \leq 1 - (1 - \delta)^{n_{\min}/n},
\]

\[
(1 - (1 - q)^{n_{\min}})^{n/n_{\min}} \geq 1 - \delta.
\]

Moreover, according to Lemma 2, we have

\[
1 - \prod_{i=1}^{\text{Dis}(S_t)} (1 - \Pr(s^{d}_{ti} \notin B(q))) \leq \delta.
\]

Let \( n_{\min} \) denote the number of appearance for \( s^{d}_{ti} \), since \( n_{\min} \leq n_{\min} \), we have

\[
1 - \prod_{i=1}^{\text{Dis}(S_t)} (1 - (1 - q)^{n_{\min}^d}) \leq \delta.
\]

Then this theorem is proved. ■

4 \( \delta \)-Approximate Aggregation Algorithm

Theorems in Section 3 describe the calculation methods required for sampling size and probability. However, we need to address the following problems:

1. Broadcasting the sampling information by the sink node to the whole network.
2. Sampling the sensory data.
3. Transmission and aggregation of the partial aggregation results.

4.1 Uniform sampling-based aggregation algorithm

One of the naive methods to calculate sample size \( m \) can be described as follows:

1. The sink nodes generate and broadcast \( m \) random numbers \( \{1, 2, 3, \ldots, n\} \) into the network.
2. A sensor node identifies itself by the random number sent by the sink node, thereby receiving the sensory data.

This procedure needs huge energy cost due to the broadcasting information transmitted throughout network sensors. Hence, we need to develop a mechanism to reduce the energy cost for broadcasting. Therefore, to reduce the energy cost, we cluster the network into "k" clusters \( \{C_1, C_2, \ldots, C_k\} \) that are disjoint. By using the method proposed in Ref. [28], we organize the cluster heads in the network as a minimum hop-count spanning tree that has sink node as the root. We then perform uniform sampling algorithm proposed in Ref. [29]. We describe the algorithm as follows:
The sink node generates random numbers \( Y_i \) with the probability \( Pr(Y_i = 1) = \frac{|C_i|}{n} \) \((1 \leq i \leq m)\).

Let \( m_i \) be the sample size of \( C_i \). Then \( m_i \) is calculated by \( m_i = |\{Y_i | Y_i = 1\}|\).

The sink node sends the sample size \( \{m_i | 1 \leq l \leq k\} \) to each cluster head. Each cluster head samples the sensory data in its own cluster using the above naive sampling algorithm.

If the sensory data received by the \( l \)-th cluster head is \( U(m_i) \), it then calculates the partial aggregation result \( R(U(m_i)) \) based on the aggregation operation \( Agg \) by using the following method:

\[
R(U(m_i)) = \begin{cases} 
\text{Max}(U(m_i)), & \text{if } Agg = \text{Max}; \\
\text{Dis}(U(m_i)), & \text{else}.
\end{cases}
\]

Then \( R(U(m_i)) \) is transmitted along the spanning tree. To further reduce the transmission cost, the intermediate nodes also aggregate the received partial result while transmitting the sensory data. The whole process is explained in Algorithm 1.

According to the content in Section 3.1, we have

\[
m = \begin{cases} 
\left\lfloor \frac{\ln \delta}{\ln(1 - \frac{2n_{\text{max}}}{n})} \right\rfloor, & \text{if } Agg = \text{Max}; \\
\left\lfloor \frac{\ln(1 - \frac{n_{\text{min}}}{n})}{\ln(1 - \frac{2n_{\text{max}}}{n})} \right\rfloor, & \text{if } Agg = \text{Dis}.
\end{cases}
\]

Therefore, we have

\[
m = \begin{cases} 
O\left(\frac{\ln 1}{\delta}\right), & \text{if } Agg = \text{Max}; \\
O\left(\ln\left(\frac{1}{1 - (1 - \delta)^n_{\text{min}}/n}\right)\right), & \text{if } Agg = \text{Dis}.
\end{cases}
\]

In practice, \( |R_j| \) can be regarded as a constant.

According to Ref. [29], the communication cost and the energy cost of the above algorithm is \( O(\ln 1/\delta) \) if \( Agg = \text{Max} \), while the cost is \( O\left(\ln\left(\frac{1}{1 - (1 - \delta)^n_{\text{min}}/n}\right)\right) \) if \( Agg = \text{Dis} \). In practice, the value of \( n_{\text{min}} \) can be acquired by the background knowledge of the specific applications. For example, in the field of environmental monitoring, the user can get the value of \( n_{\text{min}} \) according to the historical data.

### 4.2 Bernoulli sampling-based aggregation algorithm

Unlike the uniform sampling-based aggregation algorithm, the sampling information of Bernoulli sampling-based aggregation algorithm utilizes only the sampling probability \( q \). Additionally, Bernoulli-based method provides a mechanism for each node in the network to do the sampling independently. Therefore, the following steps are used in the Bernoulli sampling-based aggregation algorithm to perform sampling and the network need not be divided into clusters.

1. Sink node broadcasts the sampling probability \( q \) in the network.

2. Each node generates a random number \( rand \) in the range of \([0, 1]\), submit its sensory data to the parent node if \( rand < q \).

When the intermediate nodes in the spanning tree receive the submitted sensory data, they will calculate the partial aggregation results using a similar method introduced in Section 4.1. These nodes then transmit the

### Algorithm 1 Uniform sampling-based aggregation algorithm

**Input:** \( \delta \), aggregation operator \( Agg \in \{\text{Max, DistinctSet}\} \)

**Output:** \( \delta \)-approximate aggregation results

```plaintext
1: if \( Agg = \text{Max} \) then
2: \( m = \left\lfloor \frac{\ln 1}{\ln(1 - \frac{2n_{\text{max}}}{n})} \right\rfloor \)
3: else
4: \( m = \left\lfloor \frac{\ln(1 - (1 - \delta)^n_{\text{min}}/n)}{\ln(1 - \frac{2n_{\text{max}}}{n})} \right\rfloor \)
5: end if
6: generate \( Y_i \) following \( Pr(Y_i = 1) = \frac{|C_i|}{n} \),
7: \( m_i = \{|Y_i | Y_i = 1|\} (1 \leq i \leq m, 1 \leq l \leq k) \), the sink sends \( m_i \) to each cluster head by multi-hop communication
8: for each cluster head of the clusters \( C_l \) \((1 \leq l \leq k)\) do
9: generate random numbers \( k_1, k_2, \ldots, k_m \), then broadcast inside the cluster
10: end for
11: for each cluster member of \( C_l \) \((1 \leq l \leq k)\) do
12: send sensory value to cluster head if its id belongs to \( \{k_1, k_2, \ldots, k_m\} \)
13: end for
14: for each cluster head of the clusters \( C_l \) \((1 \leq l \leq k)\) do
15: receive sample data \( U(m_i) \) and calculate partial result \( R(U(m_i)) \)
16: end for
17: for each node \( j \) in the spanning tree do
18: if \( j \) is the leaf node then
19: Send \( R_j \) to its parent node
20: else
21: Receive partial results \( R_{j1}, R_{j2}, \ldots, R_{jc} \) from its children
22: if \( Agg = \text{Max} \) then
23: \( R_j = \text{max}(R_{j1}, R_{j2}, \ldots, R_{jc}) \)
24: else
25: \( R_j = \bigcup_{i=1}^{j} R_{ji} \)
26: end if
27: if \( j \) is the sink node then
28: return \( R_j \)
29: else
30: Send \( R_j \) to its parent node
31: end if
32: end if
33: end for
```
partial results along the spanning tree. Similarly, during the process of transmitting partial aggregation results to the sink node along the spanning tree, the intermediate nodes in the spanning tree aggregate the received partial results. The process mentioned above is explained in detail in Algorithm 2.

According to the analysis in Section 3.2, for the sample probability \( q \), we have
\[
q = \begin{cases} 
1 - (\delta)^{1/n_{\text{min}}}, & \text{if } \text{Agg} = \text{Max}; \\
1 - (1 - (1 - \delta)^{n_{\text{min}}/n})^{1/n_{\text{min}}}, & \text{if } \text{Agg} = \text{Dis}.
\end{cases}
\]

Similarly, the communication cost and the energy cost of the Bernoulli sampling-based \( \delta \)-approximate aggregation algorithm is \( O(n - n(1 - (1 - \delta)^{n_{\text{min}}/n})^{1/n_{\text{min}}}) \) if \( \text{Agg} = \text{Dis} \).

### Algorithm 2 Bernoulli sampling-based aggregation algorithm

**Input:** \( \delta \), aggregation operator \( \text{Agg} \in \{\text{Max}, \text{Dis}\} \)

**Output:** \( \delta \)-approximate aggregation results

1: if \( \text{Agg} = \text{Max} \) then
2: \[ q = 1 - (\delta)^{1/n_{\text{min}}} \]
3: else
4: \[ q = 1 - (1 - (1 - \delta)^{n_{\text{min}}/n})^{1/n_{\text{min}}} \]
5: end if
6: Sink node broadcasts \( q \) in the network
7: for each leaf node \( j \) in the spanning tree do
8: if rand < \( q \) then
9: Send its own sensory data to its parent node;
10: end if
11: end for
12: for each non-leaf node \( j \) in the spanning tree do
13: Receive partial results \( R_{j1}, R_{j2}, \ldots, R_{jC} \) from its children
14: if \( \text{Agg} = \text{Max} \) then
15: \[ R_j = \max(R_{j1}, R_{j2}, \ldots, R_{jC}) \]
16: else
17: \[ R_j = \bigcup_{i=1}^{C} R_{ji} \]
18: end if
19: if rand < \( q \) then
20: if \( \text{Agg} = \text{Max} \) then
21: \[ R_j = \max(R_j, \text{j.data}) \]
22: else
23: \[ R_j = R_j \cup \{\text{j.data}\} \]
24: end if
25: end if
26: if \( j \) is the sink node then
27: return \( R_j \)
28: else
29: Send \( R_j \) to its parent node
30: end if
31: end for
The third group of simulation is to compare the energy cost between the uniform sampling-based aggregation algorithm and the simple distributed algorithm. The steps of the simple distributed algorithm are as follows.

1) Collect all the raw sensory data.
2) Aggregate the partial results during the transmission.

We can see that the simple distributed algorithm can always return accurate aggregation results. For the uniform sampling-based aggregation algorithm, we set $\delta = 0.1$, $\frac{H}{n_{\min}} = 15$, and the network size changes from 500 to 1500. The results are listed in Fig. 3. We can see that for all these two algorithms, the energy cost increases with the increase of the network size. Moreover, the energy cost of the uniform sampling-based aggregation algorithm is much lower than that of the naive distributed algorithm since only a small number of nodes need to transmit their sensory data.

These results indicate that the uniform sampling-based aggregation algorithm performs much better in energy cost although it may return wrong aggregation results. Finally, with the increase in the network size, the energy cost of the simple distributed algorithm proliferates, while the energy cost for the uniform sampling-based aggregation algorithm almost remains the same. That is because the uniform sampling algorithm’s required sample size depends on the value of $\delta$ and $\frac{n_{\min}}{n}$ rather than the network size $n$ itself. This phenomenon also indicates that the uniform sampling algorithm is appropriate for large scale networks, which is verified by the results shown in Fig. 4.

5.2 Bernoulli sampling-based aggregation algorithm

The first group of simulations is about the relationship between $\delta$ and the sample probability. The results are presented in Fig. 5. The results show that the
sample probability increases with the decline of $\delta$. Moreover, the sample probabilities are much smaller than 1. For example, when $\delta = 0.01$, the sample probability is about 0.066 for deriving $\delta$-approximate maximum value. Therefore, our Bernoulli sampling-based algorithm also saves a great deal of energy. Similarly, the required sample size for the distinct-set aggregation is greater than that of the maximum value aggregation in the same condition.

The second group of simulations is about the relationship between $\delta$ and the energy cost. The results are shown in Fig. 6. Similarly, we can see that the energy cost increases with the decline of $\delta$ and the energy cost for the distinct-set aggregation is greater than that of the maximum value aggregation.

The third group of simulation is to compare the energy cost between the Bernoulli sampling-based aggregation algorithm and the simple distributed algorithm. For the Bernoulli sampling-based aggregation algorithm, we set $\delta = 0.1$ and $n_{\text{min}} = 67$. The network size varies from 500 to 1500. The results are listed in Fig. 7. Similarly, we can see for the same network size, the energy cost of the Bernoulli sampling-based aggregation algorithm is much lower than that of the simple distributed algorithm which indicates that Bernoulli sampling-based aggregation algorithm has high performance on energy consumption. Moreover, we can also see that the Bernoulli sampling-based aggregation algorithm has even better performance on large scale networks.

The fourth group of simulation is to compare the energy cost between the Bernoulli sampling-based aggregation algorithm and the uniform sampling-based aggregation algorithm. We set $\delta = 0.1$ and $\frac{n}{n_{\text{min}}} = 15$. 
In order to ensure the network connectivity when the network size is small, we set node’s communication to 60 m for this group of simulation. The results are shown in Fig. 4. We can see that for both the uniform sampling-based aggregation algorithm and the Bernoulli sampling-based aggregation algorithm, the energy cost increases with the increase of network size. Moreover, the Bernoulli sampling-based aggregation algorithm has lower energy cost when the network size is small, while the uniform sampling-based aggregation algorithm has lower energy cost when the network size is large. From the above results, we can see the Bernoulli sampling-based aggregation algorithm has the following advantages.

1. The Bernoulli sampling-based aggregation algorithm can be used in unclustered networks.
2. The Bernoulli sampling-based aggregation algorithm has lower energy cost in small scale networks.

6 Related Work

The sampling technique has been widely used, such as quantile calculation, data collection, and top-k query. For example, Ref. [32] is about an approximate algorithm to calculate the quantiles in wireless sensor networks. By using the sampling technique, Ref. [33] develops ASAP, which is an adaptive sampling-based method to do energy-efficient data collection in sensor networks. Reference [34] uses samples of past sensory data to define the problem of optimizing approximate top-k queries. However, all these techniques cannot be used in our problem because these operations differ a lot with the maximum query and distinct-set query.

The distinct-count query has been widely studied in many works, such as Refs. [35, 36]. Reference
Fig. 7 Energy cost comparison between the Bernoulli sampling-based aggregation algorithm and simple distributed algorithm.

[35] introduces an algorithm to calculate approximate distinct-count based on approximate frequency query results. Reference [37] is about range count queries in big IoT data. Reference [36] is about an algorithm to compute the approximate distinct-count. However, this algorithm is centralized and not appropriate for IoT networks. Moreover, all these works are for the distinct-count query, which is about the size of the distinct set rather than the content of the distinct set. Therefore, the above works still cannot be used in our problem.

7 Conclusion

In this paper, the approximate algorithms for the maximum value aggregation and distinct-set aggregation operations in sensor equipped IoT networks are proposed. These algorithms are based on the uniform sampling and Bernoulli sampling, respectively. We have proposed mathematical estimators for the two algorithms. Moreover, we have derived the values for the required sample size and the required sample probability for any given \( \delta \). Finally, an algorithm based on uniform sampling and an algorithm based on Bernoulli sampling are provided. Simulation results are shown for various \( \delta \) values and the network sizes. These simulation results indicate that the proposed algorithms have high performance in terms of the energy cost.

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