The Benktander claim reserving method, combining chain ladder method and Bornhuetter-Ferguson method using optimal credibility

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Abstract. Based on the settlement period for insurance claim, insurance is divided into 2 types of business which are short-tail business (settlement period <1 year) and long-tail business (settlement period ≥1 year). In long-tail business, it is important for insurance company to have claim reserve in order to settle claims in the future. Claim reserve modelling is done using chain ladder method that is based on the trend of paid claims. Another method that is often used is Bornhuetter-Ferguson which is based on paid claims and also premium. In this paper, Benktander method that combines chain ladder and Bornhuetter-Ferguson using optimal credibility is introduced. Optimal credibility is obtained through minimum mean squared error and minimum variance. Benktander provides moderate reserve compared to chain ladder and Bornhuetter-Ferguson.

Keywords: Claim reserve, chain ladder, Benktander, Bornhuetter-Ferguson, optimal credibility

1. Introduction
Some claims are settled (paid) by the insurance companies not long after the claims are reported. However, there are some insurance companies that need longer time to settle customers’ claim. Based on the settlement period for insurance claim, there are two types of insurance business. The first type is short-tail business which takes less than one year to settle claims, such as motorbike insurance that covers being fired or stolen. Another type is long-tail business which takes more than one year of claim settlement such as motorbike insurance that includes accident [1]. The claim settlement of this type takes a longer time due to the validation process. Outstanding claims arise in long-tail business due to long interval between claims being reported and claims being settled. Outstanding claim is the amount of claims that have not been settled due to long interval between claims being reported and settling time of claims. In reserving, outstanding claims have significant roles since insurance company are expected to hold reserves in order to pay future claims.

Two commonly known methods for claim reserving in non-life insurance are chain ladder and Bornhuetter-Ferguson. While chain ladder is defined by age-to-age factors (average of two consecutive development periods’ cumulative paid claims), this paper is based on loss ratio (average ratio of incremental paid claims to exposure for each period). Bornhuetter-Ferguson is burning cost estimate of the total ultimate claims of an origin period based on experience-based. If both claim reserves are combined using credibility, the combined claim reserves is expected to be more reliable since both
information from past claims and premium are involved. This method is called Benktander method [2]. The main contribution of this paper is Theorem 3.1, which provides an optimal credibility weight for combining Bornhuetter-Ferguson and chain-ladder. The optimal credibility weights minimize the variance of the claims reserve and the Mean Squared Error (MSE).

The organization of the paper is as follows. Section 2 contains introduction and the definition of the mentioned claim reserving methods. Section 3 defines the Benktander method claims reserves that combines the previous two methods using optimal credibility. Formulas for the optimal credibility weights, which minimize MSE of the loss ratio reserves are explained [3]. The remaining specified parameters are chosen where they minimize the variance of the optimal claims reserve. This is desirable due to the lower variances. The minimum variance optimal credible claims reserve is parameter-free. Section 4 presents numerical examples. Finally, section 5 points out steps of claim reserving using Benktander method with optimal credibility.

2. The chain ladder and Bornhuetter-Ferguson claims reserves
Let be the number of origin periods, where historical data on paid claims is available. Let , be the paid claims from origin period reported in period . Given assumption that after development periods all claims incurred in an origin period are known, the amount is the total ultimate claims from origin period . At the end of the current calendar year only the amount is known. The required amount for the incurred but unpaid claims of period , called -th period claims reserve, is equal to for . The total required amount of incurred but unpaid claims over all periods.

Suppose that the measure of exposure is the premium from the origin period . Our analysis is based on development year loss ratios representing incremental amount of expected paid claims per unit of premium in each development period, which are defined by the following.

The sum of development year loss ratios from all accident years are used to estimate ELR (Expected Loss Ratio), that is, . If premium collected by the company in year with ELR, the expected amount of money covered by the company to settle claim based on premium is gained. This expected amount is called burning cost , defined as the following.

Loss ratio payout factor is defined by the following.

Loss ratio reserve factor is defined by the following.
In general, the ultimate cumulative claim by the chain ladder method is predicted from multiplication of ultimate claims in the latest accident year with age-to-age factors. Age-to-age factors will be credible in predicting claims reserves if claims settled in previous years reflect future claims. If amount of claims in previous years are unable to predict the future claims (e.g. unstable claims between development years), modification of a more credible chain ladder method is required. In this research chain ladder method will be modified with loss ratio. In other words, the predicted claim reserves will be calculated based on the cumulative claim paid to date and loss ratio. Cumulative claim paid to date from accident year \(i\) \((C_{i,n-i})\) is proportion of paid ultimate cumulative claim. The proportion is defined by loss ratio payout factor. Therefore, claims reserves based on chain ladder with loss ratio is defined by the following.

\[
\hat{R}_i^{CL} = \frac{q_i}{p_i} C_{i,n-i}
\]

(5)

Note that in the Bornhuetter-Ferguson method, the claim reserve is defined as the proportion of the ultimate cumulative claim that has not been paid. In general, the proportion used relates to age-to-age factors. Similar to those described in the preceding sections, if the claims in previous years are unable to reflect the substantial claims to come, modifications of the more credible Bornhuetter-Ferguson method are required. The proportion is defined by the loss ratio (precisely the loss ratio reserve factor). In other words, the claim reserve is unpaid burning cost. Therefore, the claim reserves based on Bornhuetter-Ferguson method with loss ratio is as follows.

\[
\hat{R}_i^{BF} = q_i \times V_i \times ELR
\]

(6)

This estimate from chain ladder method and Bornhuetter-Ferguson will be used to estimate claim reserves based on Benktander method which combines those two method using optimal credibility.

3. Benktander method

The Benktander method combines the chain ladder method and the Bornhuetter-Ferguson method by giving weight to each method. The Benktander method uses the credibility factor in combining the chain ladder and Bornhuetter-Ferguson methods. Let the notation \(R_{GB}\) is claims reserves under the Benktander method, \(R_{CL}\) is the claims reserves based on chain ladder method and \(R_{BF}\) is claims reserves based on the Bornhuetter-Ferguson method. Let \(c\) be the credibility factor. Since the Benktander method incorporates the chain ladder method and the Bornhuetter-Ferguson method using the credibility factor, the claim reserves under the Benktander method is defined as follows.

\[
R^{GB} = cR^{CL} + (1 - c)R^{BF}
\]

(7)

**Theorem 3.1.** Assume that burning cost \(U_i^{BC}\) is independent with \(C_{i,n-i}\) and \(R_i\), also \(E[U_i^{BC}] = E[C_{i,n}] = E[C_{i,n-i}]\). Credibility that minimizes mean squared error \(mse(R^{GB}) = E(R^{GB} - R)^2\) is the following

\[
c_i^* = \frac{p_i Cov[C_{i,n-i}, R_i]}{q_i Var[C_{i,n-i}]} + \frac{p_i q_i Var[U_i^{BC}]}{p_i Var[U_i^{BC}]}
\]

(8)

where \(i = 1, \ldots, n\).

From the formulation of \(c_i^*\), note that:

- Since \(\frac{p_i}{q_i}\) increases as losses emerge, then \(c_i^*\) increases as losses emerge.

\[
\frac{p_i}{q_i}
\]


The $\text{Cov}[C_{i,n-1}, R_i]$ term measures the covariance of the losses paid to date, and the unpaid losses for accident year $i$. The larger the covariance, the larger $c_i^*$. This is reasonable given that large covariance implies that $c_i^*$ is predictive of $R_i$.

- If the variance of losses paid to date, $\text{Var}[C_{i,n-1}]$ is high, then the fractional term is small, resulting in a low credibility. This is logical, if $c_i^*$ is volatile, chain ladder method is not reliable.

- Finally, if the $\text{Var}[U_i^{BC}]$ term is large, then $c_i^* \sim \frac{p_i \cdot p_i}{p_i + t_i^*} \sim 1$. This is reasonable, since $U_i^{BC}$ is the complement. If the variability of the complement is large, the chain ladder is more reliable.

**Theorem 3.2.** With assumption that $\frac{C_{i,n-1}}{C_{i,n}} | C_{i,n}$ has beta distribution with parameters $p_i$ and $q_i$, credibility factor that minimizes mean squared error $\text{mse}(R^{GB}) = E(R^{GB} - R)^2$ is as the following

$$t_i = \frac{E[\alpha_i^2(C_{i,n})]}{\text{Var}[U_i^{BC}] + \text{Var}[C_{i,n}] - E[\alpha_i^2(C_{i,n})]}$$

where $i = 1, \ldots, n$.

In Theorem 3.2, the optimal credibility obtained is not as complex as the credibility obtained in Theorem 3.1 which involves covariance and variance of cumulative claim paid to date, as well as claim reserves. But in Theorem 3.2, $t_i$ still requires $E[\alpha_i^2(C_{i,n})], \text{Var}[U_i^{BC}]$ and $\text{Var}[C_{i,n}]$. Hereinafter, Theorem 3.3 will discuss another form of $t_i$ which minimizes the claim reserves variance of the Benktander method $\text{Var}[R_i^{GB}]$.

**Theorem 3.3.** With assumption that $\text{Var}[C_{i,n}] = \text{Var}[U_i^{BC}]$, credibility factor that minimizes mean squared error $\text{mse}(R^{GB}) = E(R^{GB} - R)^2$ and variance $\text{Var}[R_i^{GB}]$ is as following where $i = 1, \ldots, n$.

$$c_i^* = \frac{p_i}{p_i + t_i^*}$$

$$t_i^* = \sqrt{p_i}$$

(10)

Through this theorem, the optimal credibility that will be assigned to chain ladder and Bornhuetter-Ferguson method has been obtained. Therefore, claim reserves based on Benktander method, $R_i^{GB}$, is as follows.

$$R_i^{GB} = \frac{p_i}{p_i + \sqrt{p_i} \cdot R_i^{CL} + \left(1 - \frac{p_i}{p_i + \sqrt{p_i}}\right) R_i^{BF}}$$

(11)

4. **Numerical example**

Optimal credibility in Benktander method is applied to a long-tail business (private passenger auto liability/medical) claim reserves in United State. The available data is claim settled from 1999 to 2008. The claim is settled in US Dollar (thousand). Total premium collected by those companies in every accident year is also known [4]. Table 1 and table 2 represent run-off triangle of the data in incremental and cumulative, respectively.

Claim reserves based on Benktander method with optimal credibility is predicted in following steps.
Table 1. Run-off Triangle in incremental (thousand USD).

| Accident year | Development year |
|---------------|------------------|
| 0  | 20,506 | 36,268 | 43,467 | 47,391 | 49,454 | 50,411 | 50,880 | 51,146 | 51,266 | 51,366 |
| 1  | 22,180 | 39,076 | 46,521 | 50,711 | 52,888 | 53,901 | 54,443 | 54,710 | 54,871 |
| 2  | 23,047 | 40,194 | 47,894 | 52,215 | 54,512 | 55,553 | 56,057 | 56,331 |
| 3  | 24,131 | 41,878 | 49,966 | 54,470 | 56,889 | 57,955 | 58,479 |
| 4  | 24,107 | 41,413 | 49,126 | 53,626 | 56,002 | 57,147 |
| 5  | 24,368 | 41,512 | 49,207 | 53,796 | 56,143 |
| 6  | 25,051 | 42,608 | 50,571 | 55,112 |
| 7  | 25,583 | 43,589 | 51,659 |
| 8  | 27,198 | 46,283 |
| 9  | 26,977 |

Table 2. Run-off Triangle in cumulative (thousand USD).

| Accident year | Development year |
|---------------|------------------|
| 0  | 20,506 | 56,774 | 100,241 | 147,632 | 197,086 | 247,497 | 298,377 | 349,523 | 400,789 | 452,155 |
| 1  | 22,180 | 61,256 | 107,777 | 158,488 | 211,376 | 265,277 | 319,720 | 374,430 | 429,301 |
| 2  | 23,047 | 63,241 | 111,135 | 163,350 | 217,862 | 273,415 | 329,472 | 385,803 |
| 3  | 24,131 | 66,009 | 115,975 | 170,445 | 227,334 | 285,289 | 343,768 |
| 4  | 24,107 | 65,520 | 114,646 | 168,272 | 224,274 | 281,421 |
| 5  | 24,368 | 65,880 | 115,087 | 168,883 | 225,026 |
| 6  | 25,051 | 67,659 | 118,230 | 173,342 |
| 7  | 25,583 | 69,172 | 120,831 |
| 8  | 27,198 | 73,481 |
| 9  | 26,977 |

4.1. Loss ratio and ELR
Applying equations mentioned in preceding section, the calculated ELR is 6,459 and loss ratio payout factor & reserve factor are following (table 3). Table 3 represent loss ratio payout factor ($p_l$) and loss ratio reserve factor ($q_l$) for each year. As time goes by $p_l$ will decrease and $q_l$ increase.

4.2. Claim reserves using chain ladder and Bornhuetter-Ferguson with loss ratio
Since both loss ratios have been obtained, claim reserves on both methods can be calculated. Based on table 4 and table 5 we see that claim reserved for Bornhuetter-Ferguson method is lower than chain ladder method for each accident year.

4.3. Optimal credibility
The optimal credibility of Benktander method that minimizes mean squared error and variance is as follows (table 6). In table 7, we obtain optimal credibility for each year based on equation 10.
Table 3. Loss ratio payout factor and loss ratio reserve factor for each development year.

| Accident year | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|---------------|----|----|----|----|----|----|----|----|----|----|
| \( p_t \)     | 100% | 96% | 88% | 79% | 69% | 58% | 47% | 35% | 23% | 11% |
| \( q_t \)     | 0%  | 4%  | 12% | 21% | 31% | 42% | 53% | 65% | 77% | 89% |

Table 4. Claim reserves based on chain ladder method for each accident year.

| Accident year | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|---------------|----|----|----|----|----|----|----|----|----|----|
| Claim reserve | 0  | 19,854 | 52,847 | 92,174 | 127,611 | 162,320 | 196,579 | 222,561 | 241,459 | 207,882 |

Table 5. Claim reserves based on Bornhuetter-Ferguson method for each accident year.

| Accident year | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|---------------|----|----|----|----|----|----|----|----|----|----|
| Claim reserve | 0  | 6,715 | 20,137 | 39,755 | 65,104 | 98,505 | 133,988 | 175,737 | 198,773 | 218,931 |

Table 6. Optimal credibility for each accident year.

| Accident year | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|---------------|----|----|----|----|----|----|----|----|----|----|
| Optimal credibility | 0.5 | 0.49 | 0.48 | 0.47 | 0.45 | 0.43 | 0.41 | 0.37 | 0.33 | 0.25 |

Table 7. Claim reserves using Benktander method for each accident year.

| Accident year | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  |
|---------------|----|----|----|----|----|----|----|----|----|----|
| Claim reserve | 0  | 13,210 | 35,967 | 64,410 | 93,444 | 126,106 | 159,422 | 193,170 | 212,677 | 216,135 |

4.4. Claim reserves using Benktander method

Since optimal credibility and claim reserves based on chain ladder & Bornhuetter-Ferguson methods were obtained, claim reserves using Benktander method can be counted.

Note that the claim reserves obtained from the Benktander method are not as volatile as the claim reserves from chain ladder and Bornhuetter-Ferguson methods. In fact, the predicted claim reserves that are too excessive or too low is not good. If the predicted claim reserves is too large, it will incriminate the investment management part of a company because it needs to set aside large funds to pay for claims in the future. Conversely, if the predicted claim reserve is too low, prepared funds by the company may not be sufficient to pay claims in the future. Using the Benktander method that utilizes both information derived from chain ladder and Bornhuetter-Ferguson methods, the predicted claim reserves is not as volatile as the predicted claim reserves with only chain ladder or Bornhuetter-Ferguson methods alone. Thus, the total claim reserves that need to be prepared is the sum of the claim reserves of each accident year (13,210 + … + 216,135) which is $1,114,543,082,000.
5. Conclusion
The prediction using the optimal credibility in Benktander method is done with the following steps. First, given the run-off triangle in the form of incremental claim, a run-off triangle can be obtained in the form of a cumulative claim. Furthermore, we can generate loss ratio and Expected Loss Ratio (ELR) from each year of accident. Then, from the ratio of losses that have been generated, we can obtain loss ratio payout factor and loss ratio reserve factor. Applying both loss ratios, claim reserves of chain ladder and Bornhuetter-Ferguson can be calculated. Optimal credibility that minimizes the squared error mean and the variance of the Benktander method can be calculated. Finally, claim reserves using Benktander method with optimal credibility is calculated by assigning optimal credibility as weight to both claim reserves by chain ladder and Bornhuetter-Ferguson methods.

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References
[1] Olofsson M 2006 Stochastic Loss Reserving Testing the New Guidelines from the Australian Prudential Regulation Authority (APRA) on Swedish Portfolio Data Using a Bootstrap Simulation and the Distribution-Free Method by Thomas Mack Master Thesis (Sweden: Mathematical Statistics, Stockholm University) available at http://janroman.dhis.org/finance/Compendiums%20-Theses/Exjobb%20SU/report.pdf
[2] Hurlimann W 2009 Astin. Bull. 39 81-99
[3] Mack T 1994 Casualty Actuarial Society Forum 1 101-82
[4] Shapland M R, FCAS, FSA, MAAA and Xiao P 2016 CAS E-Forum available at http://www.casact.org/pubs/forum/16sforum/Shapland-Xiao-Industry-Data.xls