Frozen flow or not? Investigating the predictability of the atmosphere.

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Abstract. The spatial description of aberrations produced by atmospheric turbulence for astronomical applications is agreed to be well modeled by the Kolmogorov theory of energy cascades. Instances of these aberrations can be easily generated using closed-form analytic models based on power-spectra following the von Kármán model. However, in astronomical applications it is also the temporal evolution of these aberrations which is of interest. The description of this aspect is less well established, and is generally assumed to follow the Taylor Hypothesis over large time-scales as an approximation. This can be described as follows: if the aberrations are considered part of a flow that, for example, moves over a telescope aperture, then from the Lagrangian perspective the aberrations are static and as described by an instance of the spatial description described earlier. However, from the Euclidean perspective, the aberrations seem to just translate. Hence the alternative description of the Taylor Hypothesis as “frozen flow”. This work summarizes the relevant findings of the applicability of frozen flow for the astronomical context, and demonstrates the potential benefits as well as shortcomings of using the frozen flow model for temporal evolution of these aberrations.

1. Introduction
The temporal evolution of turbulence is a difficult topic, since the mechanisms are complex and can be a combination of stochastic and predictable phenomena. In the context of aberrations produced by the atmosphere and as viewed by astronomical telescopes, these can be produced by turbulence due to mixing of variable temperature air parcels. The description of these aberrations in space and time is then important for understanding the effects on the quality of light received by the telescope: the total intensity (scintillation) and distribution of light in the PSF (seeing) are two examples of aberration-caused phenomena.

The spatial description of aberrations is still not well understood, but generally considered to be adequately modeled in the free atmosphere by theories based on the Kolmogorov principle of energy cascades that describe energy transfer on various spatial scales. By modifying the models of Tatarski and other workers, the representation of spatial correlations in the aberrations can be described by isotropic power-spectra using a small number of parameters: typically $r_0$ and $L_0$ are considered sufficient with a power law form $\sim \nu^{-11/3}$ between scales of $\sim 1 - 100$ m to $\sim 1$ mm. The assumptions generally made are that the three-dimensional mixing can be represented in terms of optical path length change from a mean by a two-dimensional “phase screen”. Such a phase screen can exist at any altitude, and there may be several. Collectively, these are called “layers” and each layer has its own independent parameter values.
However, the temporal description of how aberrations evolve is more complex. The description of layers and the consideration of how turbulence evolves, immediately suggests the concept of layers translating horizontally above a fixed point on the surface. This point can be within a telescope aperture, for example. Then the cumulative aberrations received by the telescope will alter based on the translation of the layers and the changes in mixing associated with each layer. These are called flow and boiling, respectively. A lack of wind, for example at the surface, can still have boiling associated with that layer. Correspondingly, for very high wind speeds, the flow\(^1\) that translates dominates: from the Eulerian perspective, the layer with high wind speed associated with it is effectively frozen and simply moves over the aperture, while from the Lagrangian perspective the turbulent mixing is static. This latter description is called the Taylor Hypothesis \(^1\) and allows for the optical path length at any one point, \(x\), in the telescope aperture to be defined per layer \(j\) by \(\text{OPD}_j(\mathbf{x},t+\delta t) = \text{OPD}_j(\mathbf{x}+\mathbf{v}_j\delta t, t)\). The AMS glossary\(^2\) states the hypothesis is,

An assumption that advection contributed by turbulent circulations themselves is small and that therefore the advection of a field of turbulence past a fixed point can be taken to be entirely due to the mean flow.

Since this neglects boiling, and presumes an isotropic and homogeneous turbulence process that leads to equivalent mixing and therefore refractive index change, it is also insufficient. Using published data, here a review is carried out to examines the quality of the frozen flow assumption from the perspective of astronomical adaptive optics.

2. Laboratory-based results on mixing processes
In a series of balloon-borne experiments, Coulman \textit{et al.}\(^3\) investigated the mixing processes that are associated with small-scale temperature fluctuations, as these lead to variations in atmospheric refractive index. The general conclusion was that there was evidence of strong mixing at the edges of well developed turbulent regions. At these edges, which were relatively confined in altitude, the mixing was undeveloped and so aberrations would be imparted on light traveling through them. Within the regions, the mixing was often fully developed and homogeneous, and the potential temperature gradient approximately equal to zero. This can be translated to a stable situation which has little or no refractive index variation. Thus the concept of layers is preserved at the edges of these regions.

A series of laboratory experiments by He \textit{et al.}\(^4\) on mixing shows that a modification of Taylor’s hypothesis to include a velocity RMS term can permit frozen flow-like behaviour to be used as an approximation for situations where \(v \ll \delta t\) and boiling is not a dominant term. (It can be large, however, in terms of \(\sigma_v\).) Then this is evidence, albeit limited, that the general description given in the introduction is not an unreasonable state for the atmosphere: layers of refractive index variation that are associated with sufficient wind velocity for the resulting aberrations to be described by a flow.

3. Telescope-based results
The main astronomical application to date regarding the assumption of the frozen flow hypothesis has been in adaptive optics. Accordingly, here the results from some key papers are summarized and brought together for an understanding of the current state-of-the-art in frozen flow applicability over small space- and time-scales. Key uses for frozen flow include,

- Predictive (forecasting) behaviour, in time and/or space,
- Sub-sampling wavefronts for increased resolution,

\(^1\) The Eulerian perspective is associated with a fixed point relative to the surface i.e. \(x(t) \equiv x(0)\), while the Lagrangian is associated with a point moving with the flow i.e. \(x(t) = x(0) + v\cdot t\).
• Characterization of ‘optical turbulence’,
• Improved accuracy of AO simulations.

Generally, there are two approaches to using the data from one or more AO wavefront sensors to analyze the existence of frozen flow in layer motion: using cross correlation of slope measurements between different sub-apertures (on one WFS or between multiple, identical WFS), or Fourier analysis of all slope measurements from a WFS. The three publications considered here are Schöck and Spillar\[5\], Poyneer \textit{et al.}\[6\], and Guesalaga \textit{et al.}\[7\]. A précis of the conclusions is shown in table 1.

| Telescope/site | Period  | Method | Temporal correlation |
|---------------|---------|--------|----------------------|
| Starfire, 1.5 & 3.5 m | 7 nights | XC     | 100 ms, XC→0.5 |
| Keck & Gemini-N | 125 nights | Fourier | 20–40% decay over 0.5 s |
| Gemini-S       | 3 campaigns | XC     | 100s of ms, XC→0.5 |

The results from Schöck and Spillar are summarized in figure 1 which demonstrates that unlike with frozen flow, the cross-correlation of slopes displays a linear decay. In figure 2 it is shown that there is a relationship between derived wind speed in a layer and the decay of cross-correlation from the expected frozen flow value with time. The work of Poyneer \textit{et al.} is consistent with these figures, and suggests frozen flow has time-limited application in the context of a 8 m aperture.
Figure 2. (From Guesalaga et al. [7]:) Demonstrating that cross-correlation decay is correlated with derived wind speed. The data from figure 1 was estimated to produce a cross (red in electronic figure), to show that the published work [5] is consistent with the analyses of Guesalaga et al.

From these works, Guesalaga et al. [7] state that for AO feedback loop latency (≈ 5 ms), the decay of the cross-correlation is ≤ 3%. Schöck & Spillar [5] find that for cross-correlation to ≥ 90%, time-scales of 25 ms are typical. Finally, Poyneer et al. state frozen flow is detectable for ≥ 94% of data analyzed and such flow contributed up to 70% of the ‘high-frequency’ power.

4. Summary
This short review of frozen flow has briefly investigated why the translation of layer-based phase screens in the atmosphere is a valid proposition for modelling aberrations in the context of the apertures of astronomical telescopes. The uses of frozen flow are varied, and do include the improved deconvolution of the point-spread-function [8] and predictive control [9]. The evidence suggests that frozen flow is partially valid, over small time scales (≤ 50 ms) over diverse nights at diverse observatories and sites. These then lead to the following questions as regards future research on this topic:

- The importance of accurate representation in adaptive optics simulations,
- Are decay rates always linear? Is this a sufficient temporal description at small time-scales?
- The variation over different spatial scales: are low- and high-spatial frequencies similar in behaviour?
- The lack of discovered cross-comparisons: no comparisons known by this author between techniques and/or instruments using the same and diverse datasets.

The Taylor Hypothesis of frozen flow is a convenient description for temporal evolution of phase screens that is valid over small time-scales and is partially valid over significant time-scales (≤ 100 ms). Given its ubiquitous application, establishing the limitations and where to stop using this model is important to understand its limitations.
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