Controller Design for Compound Pendulum with PID and MRAC Switch Control

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Abstract. Compound pendulum swings with the reference trajectory accurately so that we could get the motion states of it easily. To ensure the compound pendulum tracking performance, we propose a switch controller of PID and model reference adaptive controller (PID+MRAC). According to its equations of motion, we can establish a suitable model and obtain correspond adaptive reference rules. We determine a suitable threshold, then compare the error and threshold to choose PID control or MRAC control. The same output results could be obtained from reference model and plant model by adjusting the gain. Similarly, we compare the preference of PID controller, MRAC and PID+MRAC switch controller to evaluate which one is better. The results show that the switch control is the best one and the MRAC control is better than the PID control.

1. Introduction
As a simple physical model, compound pendulum system has compound nonlinear characteristics. Compound pendulum can be seen as an approximation of many complex systems, such as the crane operation process, aircrafts carry items with the rope and so on [1]. All of them can be simplified as the pendulum system for analysis their motion states [2], and we can obtain the control law for specific problems and control well. Naturally, we must consider that the motion of the compound pendulum has the damping and driving force [3]. Otherwise, we will get the wrong model and wrong approximate results.

In many industrial fields, proportional-differential-integral (PID) control is widely used for its simple structure and reliable operation. Desired performance would be got by adjusting the PID gain. With the development of control theory [4], many new control methods have been proposed, including model reference adaptive control (MRAC) control [5], [6]. Compared to PID control, MRAC has better performance on overcoming disturbances (including disturbances to system parameters and disturbances to control system performance [7]). Although MRAC can overcome disturbances, static errors often exist in practical applications which results poor control effect.

Therefore, a PID+MRAC switch controller is used to solve the above problem. For the compound pendulum, the results show that PID+MRAC has better control performance than the others. The MRAC control is better than PID control.

2. Mathematical model of compound pendulum system
For the circular compound pendulum shown in Figure 1, we can assume it meets the following conditions:
The circular compound pendulum is subject to a certain amount of damping torque. The driving torque acts on the circular complex compound pendulum. Compound pendulum movements follow the law of rigid body rotation strictly. The equation of the compound pendulum system movements can be expressed as (1):

\[ I \ddot{\theta} + c \dot{\theta} + mdg \sin \theta = F(0.5L + d) \]  

(1)

Where \( m \) is the compound pendulum quality; \( I \) is the moment of inertia of the axis \( o \); \( c \dot{\theta} \) indicates the damping torque; \( F(0.5L + d) \) is the driving torque. For a specific compound pendulum system, the parameters are set as (2).

\[ m = 0.43 \text{kg}, L = 0.495 \text{m}, d = 0.023 \text{m}, c = 0.00035 \text{N} \cdot \text{m} \cdot \text{s}, I_0 = 9.008 \times 10^{-3} \text{kg} \cdot \text{m}^2 \]  

(2)

Then (1) can be rewritten as (3):

\[ \dot{\theta} + 0.04 \dot{\theta} + 10.78 \sin \theta = 30.03F \]  

(3)

Figure 1. Circular compound pendulum model

3. Controller design

3.1. PID controller

PID controller design rules can be described as: the input of PID controller is error signals \( e(t) \) and the output of PID controller is \( u(t) \). Therefore, we can get (4) as follows [7]:

\[ u(t) = K_p e(t) + \frac{1}{T_i} \int_0^t e(t) dt + T_d \frac{de(t)}{dt} \]  

(4)

Where \( K_p, T_i, T_d \) are the scale factor, integral time constant and differential time constant, respectively. Using difference equations to discretize (4), we can get (5), where \( k_p, k_i, k_d \) are scale factor, integral coefficient, differential coefficient.

\[ u(k) = k_p e(k) + k_i \sum_{i=1}^{k} e(i) + k_d de(k) \]  

(5)

3.2. MRAC controller

In order to simplify the pendulum system and allow it to be handled under a certain degree of accuracy, a relative simple method is used to deal with this problem. However, some assumptions listed below should be taken into account [8].

- The reference model is a linear time-invariant system and is fully controllable and fully observable.
- The dimensions of the reference model are the same as the adjustable system model.
- In the parameter adjustment adaptive system, all parameters of the adjustable system can be adjusted.
- During the adaptive adjustment process, the parameters of the adjustable system are only affected by the adaptive mechanism.
The initial deviation between the parameters of the reference model and the parameters of the adjustable system is unknown.

Generalized error vectors and output error vectors are measurable.

In the model reference adaptive control system, the desired dynamic characteristics are represented by the ideal model, so that the characteristics of the controlled system are consistent with the ideal model [9]. It is assumed that there are several parameters that can be adjusted in the adjustable system, for example, the parameters in the gain or feedback compensation network. When the external conditions change or disturbances occur, the characteristics of the controlled object also make corresponding changes. By detecting the error between the actual system and the ideal model, an adaptive mechanism adjusts the parameters of the adjustable system to compensate for the influence of the external environment or other disturbances on the system, and gradually makes the performance index reach to a minimum value.

For the complex pendulum system, we adopt an adaptive control law based on the local parameter optimization method (MIT control law). The system structure of model reference adaptive control with adjustable gain is shown in Figure 2, where the $K_c$ is the gain which can be changed during the adjustment process; $u$ is the input of the system; $y_m$ is the output of the reference model; $y_p$ is the output of the plant model; $e$ is the error which is the difference between $y_m$ and $y_p$.

![Figure 2. System structure of model reference adaptive control with adjustable gain](image)

For compound pendulum system described above, we can use the (6) to show.

$$u = k_1u_c - k_2\dot{\theta} - k_3\theta$$

(6)

Where $k_1$ is the adaptive control gain; $k_2, k_3$ is the feedback gain; $u_c$ is the given input; $\theta$ is the controlled output. Then we take the (6) into the (3), we can get the (7):

$$\dot{\theta} + 0.04\dot{\theta} + 10.78\sin \theta = k_1u_c - k_2\dot{\theta} - k_3\theta$$

(7)

The adaptive control law of each control parameter is shown as (8):

$$k_i = -\gamma \cdot e \cdot \frac{\partial e}{\partial k_i} \quad i = 1, 2, 3$$

(8)

Where $\gamma$ is the design parameter; $e$ is the difference between the output of the controlled object and the output of reference model; $\frac{\partial e}{\partial k_i}$ is the sensitivity. Our object is to predict the corresponding control parameters by the error of plant model and reference model. The reference model of compound pendulum system is written as (9):

$$\dot{\theta} + 20\dot{\theta} + 100\sin \theta = 100u$$

(9)

Combined with the above formulas, the sensitivity of each parameter is (10):

$$\frac{\partial \dot{e}}{\partial k_1} + 20(\frac{\partial e}{\partial k_1})' + 100(\frac{\partial e}{\partial k_1}) = u_c$$

(10)
3.3. PID+ MRAC switch controller

The switch controller structure of PID+ MRAC is shown in Figure 3, where the u, ym, yp, e are the same as described above. The difference from the MRAC control structure and PID+ MRAC control structure is the switch of the system feedback, which select the corresponding control method to control [10]. The switch will use the magnitude of the error to choose control this system via PID or via MRAC. Therefore, we can get better dynamic performance and steady state performance.

\[
u(t) = \begin{cases} 
  k_1u_c - k_2 \dot{\theta}(t) - k_3\theta(t) & e \geq \alpha \\
  k_p[e(t) + k_i \int_0^t e(t)dt + k_d \frac{de(t)}{dt}] & \text{else}
\end{cases}
\]

(11)

Similarly, we can get the control law (11), where \( \alpha \) is the threshold which set to different values for different systems; the remaining parameters have the same meaning as the above parameters.

4. Simulation analysis

In this section, we use the above three controllers to simulate the compound pendulum system for simulation. We set the angle as 60° and angle conversion is necessary for the final display. The threshold \( \alpha \) is set as 1.2 to meet the control request. And the input \( u \) is step signal is the sample time 1.

In order to check the anti-interference performance of the system, a sine signal with a duration of 1s was added to interfere at \( t=7 \) s. The results are shown in figure 4.

Figure 3. System structure of PID+ MRAC switch control

Figure 4. The corresponding step response of three methods
From figure 4, we can get the messages as follow:
- PID + MRAC control is the best and MRAC control is better than PID control
- The threshold $\alpha$ makes the PID + MRAC curve coincides the MRAC curve before 1s, then the PID + MRAC curve has a smaller overshoot than the MRAC curve and faster response.
- The adjustment of the parameters has not yet reached an optimal level, which results a very long adjustment time when the disturbances produce.

5. Conclusion
This paper has proposed a PID+ MRAC controller to control the compound pendulum. Then, three control method methods are used to compare the effect of the control. Compared PID control and MRAC control, the results show that the PID+ MRAC has the best performance of the control. But it should be note that the adjustment of parameters is not good. This study would give readers more ideas to design excellent controllers for different systems.

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References
[1] Li Q R, Tao W H, Sun N, et al. Stabilization Control of Double Inverted Pendulum System[C] International Conference on Innovative Computing Information and Control. IEEE Computer Society, 2008:417.
[2] Rsetam K, Cao Z, Man Z, et al. Optimal second order integral sliding mode control for a flexible joint robot manipulator[C]/ IECON 2017 -, Conference of the IEEE Industrial Electronics Society. IEEE, 2017:3069-3074.
[3] Sun H, Zhao Y, Liu G, et al. Simulation of the nonlinear behavior of compound pendulum's oscillation based on MATLAB[J]. Journal of Shangqiu Normal University, 2012.
[4] Zuo X, Liu J W, Wang X, et al. Adaptive PID and Model Reference Adaptive Control Switch Controller for Nonlinear Hydraulic Actuator[J]. Mathematical Problems in Engineering, 2017, 2017(4):1-15.
[5] Zhou X, Yang C, Cai T. A Model Reference Adaptive Control/PID Compound Scheme on Disturbance Rejection for an Aerial Inertially Stabilized Platform[J]. 2016, 2016(5):1-11.
[6] Peng Y Q, Luo J, Zhuang J F, et al. Model reference fuzzy adaptive PID control and its applications in typical industrial processes[C]/ IEEE International Conference on Automation and Logistics. IEEE, 2008:896-901.
[7] Zhen-Shun W U, Yao J J, Yue D H. A self-tuning fuzzy PID controller and its application[J]. Journal of Harbin Institute of Technology, 2004, 36(11):1578-1580.
[8] Zinober A. Adaptive Control: The Model Reference Approach[J]. Electronics & Power, 1980, 26(3):263.
[9] Zhang D, Wei B. Design of a joint control system for serial mechanical arms based on PID and MRAC control[C]// Intelligent Robot Systems. IEEE, 2016:91-96.
[10] Lei Y, Ze-Tao L I. Adaptive fuzzy PID control of temperature system of greenhouse[J]. Control Engineering of China, 2014.