Planetary Migration

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1 Introduction

Studies of planet migration derived from disc planet interactions began before the discovery of exoplanets (see Papaloizou et al., 2007; Baruteau et al., 2014, for reviews). The potential importance of migration for determining orbital architectures being realised, the field received greater attention soon after the initial discoveries of exoplanets. Early studies based on very simple disc models indicated very fast migration times for low mass planets that raised questions about its relevance. However, more recent studies, made possible with improving resources, that considered improved physics and disc models revealed processes that could halt or reverse this migration. That in turn led to a focus on special regions in the disc where migration could be halted. In this way the migration of low mass planets could be reconciled with formation theories. In the case of giant planets which have a nonlinear interaction with the disc, the migration should be slower and coupled to the evolution of the disc which is where attention needs to be focused.

This review is primarily concerned with processes where migration is connected with the presence of the protoplanetary disk as described above. However, it should be noted that migration may be induced by disc-free gravitational interactions amongst planets or with binary companions. This will be discussed relatively briefly.

2 State of the art opportunities and challenges

The classification of the main types of migration applicable to single planets: type I, type II and type III which apply for different planet masses and disc physics has been established. In addition systems of planets have been considered in order to investigate the origin and sustainability of commensurabilities. However, the disc models normally considered are primitive in comparison to those that are becoming available which involve magnetic fields, detailed vertical structure and dust. Access to steadily improving computer resources and numerical tools should enable studies of disc planet interactions in their context. We may then obtain a more complete picture of how and where migration stalls for low mass planets and the speed of migration of giant planets. After brief descriptions of the main types of migration in Sections 3-3.5 we give an outline of some of the issues that may be addressed with improved resources in future in Sections 4-4.4.

3 Disc planet interaction and the main types of migration

A planet of mass, $M_p$, orbiting within a protoplanetary disc interacts gravitationally with nearby disk gas (Baruteau et al., 2014). We shall assume the planet, moving in a circular Keplerian orbit with angular velocity, $\Omega_p$, at radius, $r_p$, is moving hypersonically with Mach number, $M$, in the range $10^{-50}$. The disk aspect ratio, or ratio of vertical semi-thickness to radius is $\sim 1/M$. Consider nearby material that is streaming past the planet supersonically on circular orbits with the minimum distance from the planet exceeding $2r_p/(3M)$. Assuming that pressure effects can be neglected we can view this material as undergoing a scattering as it passes by (see eg. Lin & Papaloizou, 1979, 1993; Papaloizou & Terquem, 2006).
Figure 1:  Left panel: A five Earth mass planet orbits in the type I migration regime while embedded in a laminar disk with a surface density profile similar to that of the minimum mass solar nebula (MMSN). The form of the surface density is indicated (light high dark low). Note the outer wake is stronger leading to a negative torque on the planet. Middle panel: A schematic illustration of the part of the horseshoe region in the neighbourhood of a low mass planet. The direction of flow along streamlines is indicated. Note that the streamlines that leave the plot at the bottom wrap around to become the streamlines entering the plot from the top. Right panel: A five Jupiter mass orbiting in a prominent gap in a disk with MHD turbulence is illustrated in the right panel. Note that the direction of orbital rotation is anticlockwise.

3.1 Scattering Calculation

Consider a ring of disk material orbiting interior to the planet at radius $r_p - b$, where $b$ is small. As a fluid element of the ring undergoes its closest approach, its trajectory is deflected through an angle $\delta$ given by

$$\tan(\delta/2) = \frac{GM_p}{bv^2}$$

where the relative speed $v = 3\Omega_p b/2$. For small $\delta$ the specific angular momentum transferred to the planet is

$$\Delta j = r_p v(1 - \cos \delta) \sim r_p v \delta^2/2 = 2r_p v \left(\frac{GM_p}{bv^2}\right)^2$$

Considering the interior disk to consist of an ensemble of rings the rate of angular momentum transfer to the planet is

$$\frac{dJ}{dt} = \int_0^{\infty} \Sigma (3\Omega_p b/2) \Delta j \, db = \frac{8}{27}\Sigma r_p^4 b^2 \Omega_p^2 (r_p/b_0)^3$$

where $\Sigma$ is the disk surface density, $b_0$, is the smallest allowed $b$ which may correspond to an inner disk edge and $q = M_p/M_\star$. A corresponding calculation and expression applies to material orbiting exterior to the planet with the transfer of angular momentum being from the planet to the disc which rotates more slowly.

For an embedded low mass planet the cut off distance $b_0$ should be the distance at which the flow becomes supersonic, thus $b_0 = 2r_p/(3M)$. For large mass planets that make a gap in the disc, $b_0$ should be the distance to the gap edge. The net torque on the planet then depends on the difference between contributions from the inner and outer disc.

3.2 The effect of coorbital material

Coorbital material no longer streams past the planet but undergoes horseshoe turns, reversing direction as it approaches the planet (e.g. as in going from A to A’ in Fig. 2 (middle panel). As such a turn moves disc material from exterior to interior to the planet, angular momentum is transferred to it. After moving around the horseshoe material that was at A’ undergoes a close turn from from C to C’. Consequently angular momentum is removed from the planet. It then moves round the horseshoe back to A. For a non zero net torque there must be a difference between the two types of closely approaching material. This has
to be brought about by rapid enough transport of angular momentum and possibly heat brought about by material outside the coorbital region. If this does not occur the torque saturates as the two types of approach cancel each other out. The unsaturated torque on low mass planets is found to depend on the gradients of the inverse specific vorticity, $\Sigma/\Omega_p$, and the entropy if the material is not barotropic or efficiently cooled. A well known example is the planet trap (Masset et al., 2006) where the surface density, and hence $\Sigma/\Omega_p$, decreases rapidly inwards. Then angular momentum transport associated with the horseshoe turn $A \rightarrow A'$ will exceed that from the turn $C \rightarrow C'$ in Fig. 1, resulting in net transport to the planet that can halt inward migration. Note that coorbital dynamics is also affected by entropy gradients (Paardekooper & Mellema 2006), magnetic fields (eg. Terquem 2003; Guilet et al., 2013) and heat transport between the planet and coorbital material (Masset 2017).

3.3 Type I migration

This affects planets with masses up to those characteristic of ice giants embedded in a protoplanetary disc. To estimate the migration rate we use equation (3) with, $b_0 = 2r_p/(3M)$. The difference between outer and inner torques introduces a factor $\sim 5/M$, which measures the departure from strict symmetry, with the torque from the outer disc being larger (see Fig1). Thus the total torque acting on the planet is estimated as

$$\frac{d\Omega}{dt} \sim -(8/27)\Sigma \nu r_p^2 q^2 \Omega_p^3 (r_p/b_0)^3 (5/M) \Rightarrow -5\Sigma \nu r_p^2 q^2 \Omega_p^2 M^2,$$

leading to the the inward migration rate

$$\tau_{\text{in}} \sim 10^5 y. \left(M_4 (4\pi \Sigma r_p^2)^{-1} (500)^{-1}\right) \left(2\pi \Omega_p^{-1} (12y.)^{-1}\right) \left(3 \times 10^{-5} q^{-1}\right) \left(400 M^{-2}\right).$$

The effect of coorbital material should be included but this is ineffective in locally isothermal discs in which $\Sigma/\Omega_p$ is constant such the MMSN. The estimate (3) then indicates a rapid inward migration time scale $\sim 10^5 y.$ for a 10$M_\oplus$ planet at 5 astronomical units (au). This is much shorter than protoplanetary disc lifetimes $\sim 10^6 y.$, leading some to arbitrarily drop planetary migration from consideration (eg. Hansen & Murray 2012). However, this neglects coorbital torques in more general disc models that could reverse or halt this migration (eg. see discussion of a planet trap in Section 3.2). Thus instead of migrating inwards protoplanets stall at preferred locations where mass growth may occur.

3.4 Type II migration

This applies to protoplanets massive enough to make a gap in the disc. Typically, their mass should exceed that of Saturn. (for further discussion see the review by Papaloizou & Terquem [2006]). The surface density profile adjusts so that angular momentum exchange with the planet, which repels disc material, is balanced by internal transport through the disc at gap edges. Denoting the angular momentum flow through the disc by, $F$, making use of equation (3) this balance requires that at an edge

$$F = (8/27)\Sigma \nu r_p^2 q^2 \Omega_p^2 (r_p/b_0)^3.$$

The cut off distance is the semi-width of the gap. We estimate a minimum value, $b_0 = (3/2)r_H = (3/2)(q/3)^{1/3}r_p$, where $r_H$ is the Hill radius. For any realisable gap with larger $b_0$ we have

$$F \leq 0.26 \Sigma \nu r_p^2 q \Omega_p^2.$$

For a disc in which angular momentum transport results from an effective kinematic viscosity, $\nu$, assumed to result from a form of turbulence, $F = 3\pi \nu r_p^2 \Omega_p$. From (7) we then get

$$36 \nu/(r_p^2 \Omega_p) \leq q.$$

This is marginally satisfied for a Saturn mass planet for a typical value of $\nu$. For a local gap to be produced we also require the planets gravity to dominate pressure effects within the Hill sphere leading to the requirement that $r_H/r_p = (q/3)^{1/3} > (1/M)$ (see eg. Lin & Papaloizou [1993]). A five Jupiter mass orbiting in a prominent gap in a turbulent disk is illustrated. In Fig1 when there is a deep gap, even though there may be mass
flow across it, torque balance across the gap results in radial migration on the same time scale as the general local evolu-
tion of the disk as indicated by Lin & Papaloizou 1986 (for more recent discussion of this aspect see Robert et al. 2018 and Sardoni et al. 2020). This is typically $2 \times 10^5 y$ in standard models resembling the MMSN with $h = (M)^{-1} = 0.05$ at 5 au (see eg. Nelson et al. 2000). However, this time increases with the local orbital period.

3.5 Type III Migration

This very rapid migration can occur for planets that form partial gaps in sufficiently massive discs (Masset & Papaloizou 2003 Peplinski et al. 2008). It involves a feedback process operating through coorbital torques that greatly amplifies the effect of an already existing possibly weak torque, $-T$.

Suppose the planet migrates inwards at a rate $\dot{a}$ (a corresponding parallel discussion applies for outward migration). Matter flows through the horseshoe region via the separatrix at a rate $\dot{m} = 2\pi \Sigma r_p |\dot{a}|$. This produces a torque acting on the planet of magnitude $|\delta J| = 2\pi \Sigma |\dot{a}| r_p^2 \Omega_p x_s$, where $x_s$ is the half width of the horseshoe region. As the planet moves it drags coorbital material with it. Noting that the mass in this region is $4\pi \Sigma x_s r_p x_s$, where $\Sigma_s$ is the surface density in the gap. This is added to the planet mass when considering torque balance which gives

$$
\frac{1}{2} (M_p + 4\pi \Sigma x_s r_p x_s) r_p \Omega_p |\dot{a}| = |\delta J| - 2\pi \Sigma |\dot{a}| r_p^2 \Omega_p x_s - T. \quad \text{Hence} \quad \frac{1}{2} (M_p - \delta m) r_p \Omega_p |\dot{a}| = -T. \quad (9)
$$

where, $\delta m = 4\pi \Sigma x_s r_p x_s (\Sigma - \Sigma_s)$, is the coorbital mass deficit. When the coorbital mass deficit approaches the planet mass migration can increase rapidly, the direction determined by the seeding torque $-T$. In practice the migration rate can increase until the time to migrate through the horseshoe region, $2x_s/|\dot{a}|$, becomes comparable to the horseshoe libration period $8\pi r_p/(3\Omega_p x_s)$. This leads to a characteristic fast migration time

$$
r_p/|\dot{a}| = (2/3)(r_p/x_s)^2 P_{orb} = (8/3^{7/3}) q^{-2/3} P_{orb}, \quad (10)
$$

where we adopt $x_s = 3r_H/2$ and $P_{orb}$ is the orbital period.

3.6 High eccentricity migration

After the protoplanetary disc has dispersed secular gravitational interactions amongst the planets in a multi-
planetary system may lead to a planet on an orbit with very high eccentricity which has close approaches to the central star leading to tidal interaction that circularises the orbit which conserves its angular momentum (eg. Wu 2011 Naoz et al. 2013 Papaloizou 2020). The semi-major axis may thus undergo a large decrease leading to an orbit with a period of a few days. High orbital inclinations relative to the initial orbital plane may also be generated. In principle the Kozai-Lidov mechanism could attain a similar result through the action of a binary companion in a highly inclined orbit. However, it has been argued that there are not enough companions of the required type for this to be viable except in a small number of cases (see Dawson & Johnson 2018 for a review).

4 Important questions/goals for future studies

4.1 Planet migration in discs with MHD winds and ordered magnetic fields

Hot Jupiters have masses $> 0.3M_J$ and orbital periods $< 10d$. They can be seen as a group in the top left of the left panel of Fig. 2. About one in ten giant planet systems contain a Hot Jupiter (see eg. Papaloizou 2020 and references therein). Disc migration has been proposed as a mechanism for moving them from beyond the ice line to their present close in orbits. For a review of the applicability of this mechanism as well as others involving gravitational interactions amongst planets or with binary companions leading to high eccentricity migration after the disc has dispersed see Dawson & Johnson 2018. Note that the left panel of Fig. 2 indicates a separation between giant planets with orbital periods $\sim 10d$ that may have formed beyond an ice line with later modest migration and the Hot Jupiters that underwent a more extreme form. Type II migration is the most likely mechanism applying to giant planets.
Figure 2: Left panel: The dependence of the planet mass, $M_p$, on orbital period, the data was taken from the NASA exoplanet archive: https://exoplanetarchive.ipac.caltech.edu/index.html (as of January 2019). Right panel: Magnetic energy contours in the coorbital region of a $\sim 7M_{\oplus}$ planet, with location indicated by a + sign, where there was an initial toroidal field. The gas flow around the horseshoe turns produces regions of field concentration. The black lines denote field lines and the white lines horseshoe separatrices (taken from Guilet et al. (2013)).

Baruteau et al. (2014) with Hot Jupiters potentially undergoing more extreme type III migration (see eg. Masset & Papaloizou 2003 [Papaloizou et al. 2007]).

As the time scale for type II migration at a few au is significantly less than the disc lifetime (eg. Nelson et al. 2000) a slowing down mechanism is needed. Advantage may be taken of the scaling of the migration rate with viscosity when there are regions of the disc where the viscosity is expected to be small. This requires consideration of recent models that include magnetic fields and non-ideal magnetohydrodynamics (MHD) (eg. Bai 2017; Rios et al. 2020). Three dimensional (3D) simulations show the disc inner region to be mainly laminar with large scale toroidal fields and wind-driven accretion. This is in contrast to earlier simulations that studied gap formation assuming ideal MHD (eg. the simulation in Nelson & Papaloizou 2003 illustrated in the right panel of Fig.1).

In the presence of MHD winds with negligible explicit viscosity, gap formation and type II migration are still expected but with the angular momentum loss rate from the outer disc required to supply the wind used in equation (7). No such balance applies to the inner disc if it is disconnected from the outer disc. However, magnetic linkage could affect this, enabling angular momentum transfer between the inner and outer disc independent of the planet. This may cause its migration to differ from that found for a viscous disc, potentially slowing it down. Simulations also need to be consider the role of the vertical shear instability (eg. Stoll & Kley 2015) and incorporate processes leading to photo-evaporation and disc dispersal in order to investigate final planetary configurations (eg. Alexander & Pascucci 2012).

Studies of type III migration also need to be done with recent disc models in 3D. Here there is significant material in the coorbital region, some of which may be associated with vortex production that can interrupt the migration (Lin & Papaloizou 2010) as well as accrete onto the planet. It has been assumed that the planet does not accrete significantly (eg. Peplinski et al. 2008), though significant accretion might inhibit the mechanism. It is possible that the planet fills its critical Hill sphere or Roche lobe and is unable to accept mass on a rapid type III migration time scale. We note that the thermal time scale of a planet filling its Roche lobe may be estimated as $t_{KH} \sim GM_p^3/(4\pi(2r_H/3)^3\sigma T_e^4)$, where $T_e$ is the effective temperature...
and $\sigma$ is Stefan’s constant. This may be written

$$t_{KH} \sim 81\pi M_p/(8P_{orb}^2\sigma T_e^4) \sim 1255(M_p/M_\odot)(P_{orb}/1y)^{-3}(T_e/(300K))^{-4}P_{orb},$$

which may plausibly exceed the characteristic time for type III migration given by (10) leading to thermally restricted accretion for a Saturn mass planet at $\sim 1au$. The issues highlighted here need to be investigated in more detail.

4.2 Tidal dissipation and high eccentricity migration

Orbital evolution produced as a result of tidal interaction with the central star may be important for giant planets brought close in by disc migration or high eccentricity migration occurring after the disc has dispersed. In the latter case, as the energy that has to be dissipated may greatly exceed the binding energy of the planet (Ivanov & Papaloizou, 2004), the interaction may be strongly nonlinear, potentially leading to disruption (Guillochon et al., 2011). In this context the issue of how much energy is dissipated in the planet and whether it can be efficiently radiated away is important for determining possible final orbital parameters of hot Jupiters. Further studies are needed.

4.3 Coorbital torques, saturation and halting type I migration

The role of coorbital torques in halting type I migration has been mentioned in Section 3.3. As the action of coorbital torques or horseshoe drag may determine disc locations where planets can grow it is important that they be investigated with the most accurate disc models that incorporate magnetic fields where relevant. These can prevent the torques from saturating when there is an effective viscosity (see Section 3.2) as shown for a disc with ideal MHD and a simple equation of state by Baruteau et al. (2011). However, what happens in the non ideal case with ordered fields and winds is unclear.

The effect of a large scale toroidal magnetic field has been considered in two dimensional (2D) model discs by Terquem (2003) for ideal MHD and Guilet et al. (2013) when there is enough magnetic diffusivity to enable disc gas to diffuse across field lines as horse shoe turns are executed (see the right panel of Fig. 2). In the former inviscid case, horseshoe turns are prevented through the operation of Alfvén resonances and the migration can reverse. In the latter case field concentrations can produce surface density changes that modify the torques potentially reversing migration (see Fig. 2). These simulations adopted a laminar viscosity with Reynolds’ number of $1.2 \times 10^5$ and Prandtl number of unity in most cases. This needs to be looked at in 3D models with non ideal MHD without applied viscosity. One might anticipate instabilities leading to vortex production for sufficiently large planet masses (eg. Li et al., 2009).

Another issue associated with horseshoe turns in 3D models is the role of vertical stratification (see McNally et al., 2020). They find that buoyancy oscillations lead to a speeding up inward migration. This needs to be considered with active MHD and thermal diffusion. Additional effects arise from heat transport connecting the planet and disc material (Masset, 2017). Asymmetries in the horseshoe region (note the displacement of the planet from the central stagnation point in the right panel of Fig. 2) and an offset between the circular motion of the planet and disc material lead to asymmetries in the heat transport and density structure that can produce migration stalling torques. The effects of accretion of mass and angular momentum and a magnetic connection should be studied in 3D.

4.4 Multiplanet systems, commensurabilities and conversion from inward to outward migration, convergent to divergent migration and the Grand Tack

Systems of planets often do not behave as if the components were independent. Convergent migration of planet pairs leads to mean motion resonances yet their incidence is modest, being about 30% of giant planet pairs with period ratios $< 3.5$ and 20% of all pairs with period ratio 3 are close to a commensurability (see eg. Papaloizou, 2020, for discussion). There are rare systems of three or more planets forming resonant chains. If convergent migration of components is common during or post formation some disruptive mechanism is required to disrupt them either before or after the disc dispersal, (eg. Izidoro et al., 2017). Up to now studies of disc planet interactions in these contexts have mostly adopted simple disc models and been in 2D.
The joint migration of Jupiter and Saturn was first studied by Masset & Snellgrove (2001). They found that the system switched from inward to outward migration after an initial rapid inward migration of Saturn caused the system to lock into a 3:2 commensurability. This scenario has been adopted in the Grand Tack scenario proposed to operate during the formation of the solar system (see eg. Pierens et al., 2014). The process which occurs when an inwardly migrating giant planet is caught up by a lighter one has been proposed as a mechanism that can push planets out to large distances (see Crida et al., 2009). These simulations involve type II migration as well as fast possibly type III migration of an outer lighter giant planet. Thus the issues highlighted in Section 4.1 apply. Consideration of realistic disc models with allowance for the mass growth fed by material in coorbital regions is needed to see if the outer planet always has the lowest mass as required in these scenarios.

4.4.1 Formation and subsequent evolution of commensurabilities

Giant planets with periods \( > 100d \) may not have migrated through large distances (see left panel of Fig.2). The absence of super Earths with bare cores composed of volatiles and the relative rarity of commensurabilities in two planet systems indicates the same applies to them (see eg. Papaloizou & Terquem, 2019; Papaloizou, 2020). But the existence of near commensurabilities supports a picture of at least restricted migration in some cases. The formation of low order commensurabilities through the convergent migration of two planets is well documented (see eg. Baruteau et al., 2014). However, initially convergent migration can be reversed through wake planet interactions when the density response produced by one planet reacts back on the other. This has been seen when giant planets interact with incoming super Earths (Podlewska-Gaca, et al., 2012) and for pairs of Neptune mass planets (Baruteau & Papaloizou, 2013). It results in a departure from commensurability and may help reduce the number of expected strict commensurabilities. These studies should be extended to incorporate the final stages of the evolution of the protoplanetary disk. The location and evolution of structural features constituting migration traps and interaction with planetesimals and dust will be important.

Summary

Studies of disc planet interactions began with simple models before the discovery of exoplanets about forty years ago. While studies of migration resulting from disc free processes such as gravitational interactions leading to high eccentricity migration followed by tidal interaction with the central star began shortly after the first discoveries of exoplanets. Advances in our understanding of protoplanetary discs coupled with improving numerical tools and computational resources offer the prospect of more realistic modelling of planet formation and early evolution. This will lead to improved comparisons with observations of structured protoplanetary discs (see eg. Francis & van der Marel, 2020) as well as improved predictions for the form of planetary systems.

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