Neutrino mass generation in the SO(4) model

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Abstract

Generation of neutrino mass in SO(4) model is proposed here. The algebraic structure of SO(4) is same as to that of SU(2)\textsubscript{L} × SU(2)\textsubscript{R}. It is shown that the spontaneous symmetry breaking results three massive as well as three massless gauge bosons. The standard model theory according to which there exist three massive gauge bosons and a massless one is emerged from this model. In the framework of SU(2)\textsubscript{L} × SU(2)\textsubscript{R} a small Dirac neutrino mass is derived. It is also shown that such mass term may vanish with a special choice. The Majorana mass term is not considered here and thus in this model the neutrino mass does not follow seesaw structure.

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1 Introduction:

In the framework of standard model the neutrino is considered as the massless particle. With this motivation it is also been assumed that no right handed neutrino can enter in the theory. The situation became changed when Pontecorvo proposed the neutrino oscillations leading to the nonzero neutrino mass \[1\]. After that the solar neutrino problem and atmospheric neutrino anomaly led to the concept of the existence of small non-zero neutrino mass. The neutrino mass physics took a revolutionary shape when at the Neutrino’98 (June, 1998) conference in Takayama, Japan, the Super-Kamiokande collaboration announced the discovery of oscillations of the cosmic ray neutrinos, which would clearly indicate the existence of neutrino mass. To explain the mass generation mechanism of the neutrino the standard model theory is needed to be extended. There are a number of such extension models incorporating the neutrino mass. There exist two classes of models where introduction of lepton number violating interactions leads to radiative generation of small neutrino mass. The first one is the Zee model \[2\] where a charged $SU(2)_L$ singlet field $\eta^+$ is added to the standard model along with a second Higgs doublet. In the second model, called Babu model \[3\], one charged field $\eta^+$ along with a doubly charged field $h^{++}$ are added to the standard model, but no second Higgs doublet is introduced. There is an another model \[4\] where the neutrino mass is generated without including the right handed neutrino in the theory. In this model an additional $SU(2)_L$ triplet Higgs field $(\Delta^{++}, \Delta^+, \Delta^0)$ is added and it breaks the lepton number by two units and lead to the Majorana mass \[5\] of the neutrinos. In fact, the question is not only how to generate the neutrino mass in the extension of standard model, but also how to fix the smallness of neutrino mass compared to the mass of the charged fermions. Another big question arises that the standard model without right handed neutrino has got the desirable property that anomaly cancellations imply charge quantization \[6\] if there is only one family of fermions. This property is lost in presence of right handed neutrinos, as well as of more than one fermion generation. It has been pointed out \[7\] that if the neutrino is considered as Majorana particle then anomaly cancellation would imply charge quantization regardless of the number of generations.

An elegant way to generate the neutrino mass is to include the right handed neutrinos in the model which leads to the left right symmetric model \[8, 9, 10\]. In the standard model $B - L$ is a global symmetry and cannot be gauged. But when the right handed neutrino is included in the model the $B - L$ becomes gaugable and the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is the gauge group of the left right symmetric model \[11, 12\]. The seesaw structure of neutrino...
mass emerges in this left-right symmetric model. In this method the neutrino mass is generated by the spontaneous breaking of global $B-L$ symmetry. It has also been proposed that the generation of Majorana neutrino mass may be a consequence of the creation of massless Goldstone boson, called majoron. The neutrinoless double beta decay process is supposed to emit the majoron, although such process is subjected to be verified. several experiments have searched for neutrinoless double beta decay, but clear evidence is yet to be found. This majoron can couple to the charged fermions. The upper bound of the strength of Majoron-electron coupling is calculated in the astrophysical consideration (according to standard model of sun) as $g_{eJ} \leq 10^{-10}$.

In the present article we have proposed a new mechanism to generate the neutrino mass, which would be simply the Dirac mass term. We have shown here that in the framework of $SO(4)$ model the Dirac mass term of neutrino might be very small without any introduction of Majorana mass term. Hierarchy of the masses in the fermionic sector could be explained well in this $SO(4)$ model. The $SO(4)$ model incorporates both left as well as right hand fields. In Sec-2 we have designed the model to rescue the standard model after the spontaneous symmetry breaking. In Sec-3 we have discussed the mass generation in the fermionic sector and shown how to generate neutrino mass using this left-right symmetric model.

## 2 Mass generation of Gauge bosons:

It is well known that the $SO(4)$ and $SU(2) \times SU(2)$ have the same Lie Algebraic structures. Therefore, to consider the left-right symmetric $SU(2)_L \times SU(2)_R$ model we can consider simply the $SO(4)$ model. Such model can be thought of as a generalization of the standard model of the electro-weak theory. It is quite reasonable to construct the Lagrangian as,

$$
\mathcal{L} = \frac{1}{2} (\partial_\mu \sigma)^2 + \frac{1}{2} \sum_{i=1}^{3} (\partial_\mu \phi_i)^2 - \mu^2 [\sigma^2 + \sum_{i=1}^{3} \phi^2] - \frac{\lambda}{4} [\sigma^2 + \sum_{i=1}^{3} \phi^2]^2
$$

(1)

where,

$$
\sigma \rightarrow \sigma + \phi_i \Lambda_i \quad \phi_i \rightarrow \phi_i + \varepsilon_{ijk} \Lambda_j \phi_j - \sigma \Lambda_i
$$

(2)

If we consider a new function $\Phi$ s.t.

$$
\Phi = \begin{pmatrix}
\sigma + i\phi_3 \\
-\phi_2 + i\phi_1 \\
\phi_2 + i\phi_1 \\
\sigma - i\phi_3
\end{pmatrix}
$$

(3)

then the Lagrangian takes the form

$$
\mathcal{L} = \frac{1}{4} Tr[(\partial_\mu \Phi^\dagger)(\partial^\mu \Phi)] - \frac{\mu^2}{2} Tr[\Phi^\dagger \Phi] - \frac{\lambda}{8} (Tr[\Phi^\dagger \Phi])^2
$$

(4)
Our aim is to generate the mass of the gauge bosons. In the framework of standard model theory a mass is produced when a gauge symmetry is spontaneously broken. To make the Lagrangian, given by the equation (4), a gauge invariant we replace the covariant derivative $\partial_\mu$ by $D_\mu$ as follows:

$$D_\mu = \partial_\mu + g \sum_{\alpha=1}^{3} \frac{\tau_\alpha}{2} L_\mu^\alpha - g' \sum_{\alpha=1}^{3} \frac{\tau_\alpha}{2} R_\mu^\alpha$$  \hspace{1cm} (5)

Here $L_\mu^\alpha$ and $R_\mu^\alpha$ are the gauge fields associated with $SU(2)_L$ and $SU(2)_R$ respectively. The $\Phi$ defined by the equation (3) has the transformation properties as follows:

$$\Phi' = U_L \Phi U_R^\dagger$$  \hspace{1cm} (6)

where,

$$U_L \equiv e^{i\Lambda_L^{\frac{\gamma}{2}}} \hspace{1cm} U_R \equiv e^{i\Lambda_R^{\frac{\gamma}{2}}}$$

We would like to design the model in such a way that it generates the masses of all gauge bosons, all known fermions and in addition the neutrino masses. In this model the gauge symmetry is spontaneously broken when $\Phi$ takes up the vacuum expectation value, i.e.,

$$\langle \Phi \rangle = v \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$  \hspace{1cm} (7)

A small perturbation about $\langle \Phi \rangle$ generates the gauge boson mass. The Lagrangian for the mass term is obtained as

$$\mathcal{L}_{\text{mass}} = -\frac{v^2}{8c_W^2} \sum_{\alpha} (c_W L_\mu^\alpha - s_W R_\mu^\alpha)^2$$  \hspace{1cm} (8)

The above Lagrangian shows that we lead to a situation in which three massive fields $c_W L_\mu^\alpha - s_W R_\mu^\alpha$ along with three massless fields $s_W L_\mu^\alpha + c_W R_\mu^\alpha$ are present. From the equation (8) the Lagrangian for the mass terms can also be written as

$$\mathcal{L}_{\text{mass}} = \mathcal{L}_{\text{mass}}^1 + \mathcal{L}_{\text{mass}}^2$$  \hspace{1cm} (9)

where,

$$\mathcal{L}_{\text{mass}}^1 = -\frac{v^2}{8} [(L_\mu^1)^2 + (L_\mu^2)^2] - \frac{v^2}{8c_W^2} [c_W L_\mu^3 - s_W R_\mu^3]^2 + 0]s_W L_\mu^3 + c_W R_\mu^3]^2$$ \hspace{1cm} (9a)

$$\mathcal{L}_{\text{mass}}^2 = -\frac{v^2}{8} [(R_\mu^1)^2 + (R_\mu^2)^2]$$ \hspace{1cm} (9b)

Let us watch carefully the terms $\mathcal{L}_{\text{mass}}^1$ and $\mathcal{L}_{\text{mass}}^2$. The terms $\mathcal{L}_{\text{mass}}^1$ looks like the SM Lagrangian for mass terms of gauge bosons. Only difference is that there exists $R_\mu^3$ field instead
of the field $B_\mu$. We know the generator $T_3^R$ is associated with $R_3^3$ whereas $B_\mu$ corresponds to $Y$. In the $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ framework Gellmann-Nishigima type of relation

$$Y = 2T_3^R + (B - L)$$

is well known and in this model the Majorona neutrino mass is generated by the spontaneous breaking of $B - L$ symmetry. But we don’t incorporate the Majorana mass term as $B - L$ maintains a perfect symmetry in the particle world so far the occurrence of double beta decay process is established. Therefore in our model we simply identify

$$Y = 2T_3^R$$

and then the equation (9a) becomes

$$L_{mass} = -\frac{v^2}{8}[(W_1^1)^2 + (W_2^2)^2] - \frac{v^2}{8c_w^2}[c_w W_3^3 - s_w B_\mu]^2 + 0[2 W_3^3 + c_w B_\mu]^2$$

with taking $L_1^1 = W_1^1$, $L_2^2 = W_2^2$, $L_3^3 = W_3^3$ and $R_3^3 = B_\mu$. Thus the standard model is rescued by the equation (12), but in addition to that we get the mass term of $R_1^1$ and $R_2^2$ in equation (9b) that is beyond the standard model scenario. Even no such right hand gauge boson fields have been detected experimentally, although their existence cannot be ruled out. At least we can say that all the known particles, predicted in the standard model theory, are incorporated in the $SO(4)$ model.

3 Mass generation of fermionic sector:

Let us now consider the fermionic sector. It is quite clear that the fermion contents in this model are given by

$$\left( \begin{array}{l} \nu \\ l \end{array} \right)_L \equiv \left( \begin{array}{l} u \\ d \end{array} \right)_L \equiv (2, 1) \quad \left( \begin{array}{l} \nu \\ l \end{array} \right)_R \equiv \left( \begin{array}{l} u \\ d \end{array} \right)_R \equiv (1, 2)$$

where, $\nu, l, u$ and $d$ stand for neutrino, lepton, up and down quark respectively of all three generations.

In the $SO(4)$ model the Yukawa term of the Lagrangian is taken as

$$L^Y = \sum_f g_f \left( \begin{array}{c} u^f \\ d^f \end{array} \right)_L \left( \begin{array}{c} v \\ 0 \\ 0 \end{array} \right) \left(1 - x_f \tau_2\right) \left( \begin{array}{c} u^f \\ d^f \end{array} \right)_R + h.c.]$$

where, $\left( \begin{array}{c} u^f \\ d^f \end{array} \right)$ represents a general fermionic doublet and $x_f$ represents a real number lying between -1 to 1. From the above Lagrangian we obtain the fermionic mass term as

$$L^Y_{mass} = \sum_f \frac{g_f v}{\sqrt{2}} [(1 - x_f)\overline{u^f} u^f + (1 + x_f)\overline{d^f} d^f]$$

(14)
Clearly the difference of the masses of up and down quark (of all generations) arise due to the factor $|x_f| = \frac{m_u}{m_d}$. Let us now study the leptonic sector. The mass term generated in the above manner becomes

$$L_{\text{mass}}^l = g_l v \sqrt{2} [(1 - x_l) \overline{\nu} \nu + (1 + x_f) \overline{U}]$$

(Note that in the equation (15) $\nu$ denotes the neutrino of generation $l$ lepton. For the convenience of notation the generation index $l$ is simply dropped from the suffix of $\nu$.]

Thus the mass of the neutrino becomes

$$m_{\nu_l} = \frac{g_l v}{\sqrt{2}} (1 - x_l)$$

that is to be identified as the Dirac mass of the neutrino. We see that in the SO(4) model Dirac mass of neutrino can be produced without any difficulty. Also in the leptonic sector there is a scope to recover the standard model theory by assuming $x_l = 1$ leading zero neutrino mass. Therefore, it is quite clear that $x_f$ plays a key role to fix the mass of quark and lepton sector for a given generation. Although we cannot establish any rule to fix the nature of $x_f$ for different quarks and leptons, but at least we can say that for $0 < x_l \ll 1$ a small neutrino mass is obtained.

## 4 Discussion

The model that we have discussed here is not only simple, but it also rescues the standard model of the electroweak theory with some special choices. If we look on the well known $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ we see that the Majorana mass term of the neutrino is generated by the spontaneous breaking of global $B-L$ symmetry and the unbroken symmetry is $U(1)_Q$. Such symmetry breaking occurs in two steps. But in our model the symmetry is broken once from $SU(2)_L \times SU(2)_R$ to $\times SU(2)_R$, not to any $U(1)$ symmetry. That is the unbroken symmetry in our model is $\times SU(2)_R$. The generator $Y$ is one of three generators of a $SU(2)$ group and ultimately we get $Q$ through the Gellmann-Nishigima relation as in the usual standard model case. However, we cannot detect the fields associated to the other two generators. Another simplicity of $SO(4)$ model is the absence of Majorana type of mass. In this model the Dirac mass term may be very small and thus the smallness of the neutrino mass can be explained without introducing any Majorana mass term. The seesaw structure of the neutrino mass is not emerged in this model. Therefore, the model can be considered as an alternative of the seesaw mechanism.
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