Finite element simulation of flexible ropes

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Finite element simulation of flexible ropes

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Abstract. The paper deals with examples of solving the equilibrium problem of flexible ropes having significant displacements, which shape is determined by the connections and load. The axis position of the flexible rope is described by the Cartesian coordinate vector. The stress state of the rope is characterized by the force in the rope. At present, the methods of calculating the cable systems are based on a differential formulation. Variational formulations of such problems and the schemes of the finite element method constructed on their basis are proposed. Examples of solution of model problems are given.

1. Introduction
The theory of a flexible ropes is considered, in which the stress-strain state is described by the coordinates of the rope axis and the forces in the rope [1, 2]. At present, numerical methods [3] and engineering approaches [4] based on the differential formulation of the problem are used to solve various problems of calculating cable systems. At the same time, in order to use modern universal numerical methods, for example, the finite element method, a variational statement of the problem is required.

To form the variational statement of the problem, one can apply a formal mathematical procedure, which is used [5] to obtain known variational principles of problems in the theory of elasticity. On the basis of this approach, in 1991 a mixed variational formulation of the flexible ropes equilibrium problem was developed [6].

The complexity of constructing the scheme of the finite element method is related to the geometric nonlinearity and the mixed nature of the problem. To solve a mixed problem, it is proposed to use the approach used in "hybrid finite element methods" [7, 8] or in "mixed finite element method with the elimination of some unknowns at the element level" [9, 10]. With this in mind, schemes of the finite element method were constructed [11, 12], in which the forces are calculated at the element level, and when solving the global system of algebraic equations, the coordinates of the axes of the ropes are calculated. And because of the geometric nonlinearity, an iterative procedure is used. Examples of solving test problems by the proposed finite element method are given below.

2. Differential Formulation
The axis position of the flexible ropes is described as vectors of its Cartesian coordinates \( \mathbf{x} = \{x_1, x_2, x_3\}^T \). The stress state in the ropes is characterized by the force \( T (T \geq 0) \). Coordinates along the ropes axis are denoted by \( s \). We assume one end of the ropes is fixed and the other is attached where the load is applied.

Here is the differential formula for the flexible inextensible ropes equilibrium problem [1, 2] in the form of:
static equation:
\[ \frac{d}{ds} \left( T \frac{dx}{ds} \right) + p = 0, \]  
(1)

geometric equation:
\[ \left( \frac{dx}{ds} \right)^T \frac{dx}{ds} = 1, \]  
(2)

static boundary condition:
\[ T \frac{dx}{ds} \bigg|_{s=l} = F_i, \]  
(3)

geometric boundary condition:
\[ x|_{s=0} = x_0. \]  
(4)

3. Variational Formulation

For this task we write the variational formula in the form of a stationary condition of the functional [6, 11, 12]:
\[ B_i(x, T) = \frac{1}{2} \int_0^1 \left( \left( \frac{dx}{ds} \right)^T \frac{dx}{ds} - 1 \right) ds - \int_0^1 x^T p \, ds - x^T F_i \bigg|_{s=l} = 0, \]  
(5)

depending on the force \( T \) and the vector of the ropes coordinate axis \( x \), a predetermined initial value of the boundary point is given (4).

The function (5) is combined: the unknowns are force \( T \) and the coordinates of the ropes axis \( x \). To solve this problem we use the mixed technique in [7-10], where the computational area is divided into sub-areas, and it is expected that a portion of the equations mixed method is calculated in each of these sub-areas separately.

We split the ropes into \( n \) sub-areas - elements. We assume that the variational equation for each element is:
\[ \int_0^1 \delta T \left( \left( \frac{dx}{ds} \right)^T \frac{dx}{ds} - 1 \right) ds = 0, \quad r = 1, 2, ..., n \]  
(6)

In the case the solution of the variational problem (5), the integral identity should be considered as:
\[ \int_0^1 \left( \frac{\delta dx}{ds} \right)^T T \frac{dx}{ds} ds - \int_0^1 \delta x^T p \, ds - \delta x^T F_i \bigg|_{s=l} = 0, \]  
(7)

4. The Finite Element Method

Here is the scheme of the finite element problem the method is based on the variational formula (6), (7) [11]. The coordinates of \( x \) approximation and \( T \) forces take the finite element in the form of:
\[ x = N^r \cdot X^r, \quad T = \left[ 1 \right] \cdot t^r \quad \in \ell^r, \]  
(8)

where \( N^r \) – the matrix of shape functions; \( X^r \) – Coordinate vector \( r \) element nodes; \( t^r \) – The value of the forces in \( r \) element (each portion of force \( T \) is made a constant function).

Next, using the procedure of finite element discretization [11], we obtain
\[ \left( X^r \right)^T H^r X^r - l^r = 0, \quad r = 1, 2, ..., n, \]  
(9)
where:

\[ H' = \int_{r} \left( \frac{dN'}{ds} \right)^{T} \left( \frac{dN'}{ds} \right) ds, \]

\[ \sum_{r=1}^{n} \left[ (\delta X')^{T} (H'X' - P') \right] = 0, \] (10)

\[ H' = t' H', \quad P' = \int_{r} (N')^{T} p \, ds + (N')^{T} F, \]

In (10)

Finally, for a system of finite elements from the assembly procedure for performing matrices and vectors in (10), we get:

\[ H, X - P = 0. \] (11)

The iterative procedure is follows for solving this problem. The initial value of the forces in the elements is input. We solve the system of algebraic equations (11), with unknown linear coordinates. To verify the correlated values (9) the following equation is used:

\[ t_{r}^{(k)} = t_{r}^{(k-1)} \cdot \left( (X')^{T} H' X' \right)^{-1/2}, \quad r = 1, 2, ..., n, \] (12)

where \( k \) is a number of iteration.

The calculated values \( t_{r}^{(k)} \) are used to recalculate the matrices and vectors (11). Iterations continue until the desired accuracy of the computation calculation is achieved.

5. Solution of the model tasks

5.1. Task 1

The simplest model task is to check the convergence of the method is the following: the ropes with fixed at ends and loaded by the vertical concentrated force in the middle of the span (Figure 1 – Figure 4). In order to calculate the force required we break the ropes into 2 finite elements. We use finite elements with linear approximation coordinates and the force on each portion is approximately constant. For forces up to 0.001 only 9 iterations are sufficient.

Figure 1. Task 1. A flexible rope with concentrated force. The computational model.

Figure 2. Task 1. A flexible rope with concentrated force. Form a flexible string, depending on the iteration step.
5.2. Task 2
In the second task, the ropes was loaded with vertically distributed loading. Figure 5 – Figure 9 shows the results of the calculations by varying the number of finite elements. For forces up to 0.001 only 11 iterations are required.

5.3. Task 3
A finite-element equilibrium modeling of the flexible extensible rope is performed on the basis of the variational formulation of the problem proposed in [12]. To compare the results, we considered the equilibrium problem of a steel cable, the solution of which in [3] was obtained by a numerical solution of the system of transcendental equations.
Figure 8. Task 2. A flexible rope with distributed load. The convergence of the coordinates.

Figure 9. Task 2. A flexible rope with distributed load. The convergence of forces.

The equilibrium problem of a steel cable under water is considered under different coordinates of the upper attachment point of the cable (see Figure 10 – Figure 14). The length of the cable is 300 m, the diameter is 10 cm. The cable has its own weight and Archimedean force. Stiffness in tension is accepted $1.099 \times 10^9$ N.

The following coordinates of the upper anchorage point of the cable were set: b) $X_B = Y_B = 200$ m; c) $X_B = Y_B = 210$ m; d) $X_B = Y_B = 214$ m; e) $X_B = Y_B = 215$ m.

Table 1. Comparison of the force in the cable, calculated by different methods.

|                  | The maximum effort $T$ in the cable at the coordinates $X_B = Y_B$, N |
|------------------|-------------------------------------------------|
|                  | 200 m   | 210 m   | 214 m   | 215 m   |
| The technique of [3] | 170 000 | 292 000 | 9 740 000 | 14 900 000 |
| The proposed FEM   | 188 135 | 325 603 | 9 744 881 | 14 921 017 |
| Relative error     | 9.64%   | 10.32%  | 0.05%    | 0.14%    |

Figure 10. Task 3. Scheme of steel cable.

6. Conclusion

The performed calculations show that the proposed finite element method allows the very first iterations to achieve correct ropes formation, and to calculate forces with an accuracy of 0.001 takes approximately 10 iterations. Moreover, increasing the number of finite elements is not affected by the number of iterations. The finite element method, taking into account the extensibility of the ropes, yielded practically identical results for the cable stretching specification by the coordinates of the point B ($X_B = Y_B = 214$ m and 215 m).

Thus, the proposed finite element method for the calculation of cable systems produced good results in solving known problems and can be used to calculate any cable systems.
Figure 11. Task 3. $X_B = Y_B = 200$ m.

Figure 12. Task 3. $X_B = Y_B = 210$ m.

Figure 13. Task 3. $X_B = Y_B = 214$ m.

Figure 14. Task 3. $X_B = Y_B = 215$ m.

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