Numerical investigation on thermoelectric generators in an exhaust automotive line with aluminium foam

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Abstract. In the present paper a two-dimensional convective heat transfer problem in a partially filled channel with metal foam is numerically solved in steady state regime. An external thermoelectric generators (TEG) component is placed on the top surface of the channel. The numerical analyses are accomplished assuming the local thermal equilibrium (LTE) model to simulate the presence of the aluminum foam. The working fluid is exhaust gas with properties equal to the air for fixed temperature of the upper surface of the thermo-electric generator (TEG). The thermophysical properties are assumed temperature independent and the TEG component is considered as a solid with an internal energy generation. The Ansys-Fluent code is applied in order to resolve the governing equations for gas, porous media and TEG. Several mass flow rates of exhaust gas on the inlet section of the channel are considered. Different thicknesses of aluminum foam are assumed into the duct. The foam is characterized by different porosity equal to 0.90, 0.95, 0.97. Moreover, the number of pores per inch also changes and assumes the following values of 5, 20, 40. Results are showed in terms of temperature distributions, pressure drop, thermoelectric efficiency for different exhaust gas flow rates and metal foam characteristics and thicknesses. The results highlight that the use of metal foams significantly increases the heat transfer between the surface of exhaust gas tube and hot gas. Consequently, the effectiveness improves, and it increases between three-ten times with respect to the one for tube without metal foams. It is shown that the pore density does not affect the effectiveness.

1. Introduction
The traditional engines are characterized by a not very high efficiency as it assumes a value of about 25%. A quantity equal to 30% is dissipated in the cooling system, 5% for the load losses and 40% is lost as exhaust gas into the environment. The capture of the gas heat represents a way for increasing the efficiency and reducing in global warming. In recent years, a great attention is devoted to the development of thermoelectric generators (TEG). These components, through the physical phenomenon called "Seebeck Effect", can generate electric current depending on the temperature value.

An analytical study on the thermoelectric generation physical process was carried out by Stevens et al. [1]. The authors found a set of basic equations in order to understand the thermal and fluid dynamic behaviour of the TEG. The results demonstrated that a minor efficiency was obtain in corresponding to a number of conductors major than the ideal configuration under analysis. Meng et al. [2] accomplished a study on performance and design optimization of a TEG for automobile exhaust waste heat recovery. The results showed that a more uniform temperature profile was obtained for the counter flow respect to the parallel flow counterpart. This result is essential because the thermoelectric
unit output power decreases when the temperature is not uniform along the streamwise direction. Moreover, the additional lateral heat conduction effect also reduces the output power. The increment of the thermoelectric unit number, not by changing thermoelectric unit spacing, can enhance the output power of the system. A numerical and analytical study on Bismuth-telluride based thermoelectric devices was carried out by the authors Erturun et al. [3] and Zhang et al. [4]. A three-dimensional model was created for the analysis of the TEG performance. The TEG behaviour was influenced by the variation of the geometric dimensions of the module elements. In fact, power, efficiency and voltages increased with increasing of width and with decreasing of length. The geometry of the heat exchanger in which the exhaust gas passes is one of the main points of the design and development of a TEG. Bai et al. [5] carried out a numerical investigation on an octagonal thermoelectric generator. The TEG, constituted by 9 thermocouples on each side (54 mm in length), was studied in a 4-cylinder petrol engine. The application of metal foam in the heat exchangers of the TEG induced a greater heat transfer and a major noise absorption, as well as a benefit in terms of electrical power was realized. Six different heat exchanger configurations were analyzed by Bai et al. [6] both in experimental and numerical way. The structures under investigation were a hollow cavity, with inclined plates, parallel plates, separate plates with cavities, serial plates and tube structure. The results demonstrated that only the inclined plate and empty cavity structure, under the maximum power output condition, had a pressure drop less than 80 kPa and the largest pressure drop surpassed 190 kPa. In this case, a system with a differential pressure switch is necessary to bypass part of the exhaust gas. An analysis about a thermoelectric generator system for passenger vehicles under several drive cycles was carried out by Kim et al. [7]. A 1-D TEG system with 40 TEG modules was constructed for the analyses. The results demonstrated that the TEG system contributed to the enhancement of the output power in a range from 1.54 to 1.68%, depending on the drive cycles. The first work that employed metal foams to improve the system performances of a TEG was provided by Nithyanandam and Mahajan [8]. The investigation was performed numerically on a two-dimensional channel in the steady state. In the simulations the metal foam totally filled the channel. For assigned pore density, the maximum net electric power obtained with metal foam was higher than the one generated from exhaust gas in the case without metal foam.

It is interesting to reduce the pressure drop inside the system with metal foam and using the partially filled channel can be the right way to realize this issue. Research activity on this configuration is extremely poor and there is a lack of knowledge.

In this work, a numerical investigation is carried out on a convective heat transfer problem in a partially filled channel with aluminum foam. An external thermoelectric generator (TEG) is posed on the upper surface of the duct. The Darcy-Brinkman-Forchheimer assumption and LTE model between fluid and solid phases of the metal foam are considered. The exhaust gas is characterized by the same properties of the air for a fixed temperature of the TEG top surface. The thermoelectric generator is considered as a solid with an internal energy generation. Several inlet mass flow rates of exhaust gas, equal to 20 g/s, 60 g/s, 100 g/s, are assumed. Different thicknesses of metal foam are considered. The foams are characterized by a porosity equal to 0.90, 0.95, 0.97 and by number of pores per inch (PPI) of 5, 20, 40. The results are presented as temperature distributions, pressure drop and thermoelectric efficiency for different exhaust gas mass flow rates and metal foam thicknesses and parameters.

2. Governing equations and physical domain

The physical model is a thermoelectric generator applied to vehicle waste heat recovery with aluminum foam characterized by a steady turbulent regime in forced convection. The 2D illustration of the system with TEG under investigation is showed in the Figure 1. The length, $L$, and the height $2H$ of the channel are equal to 272 mm and 60 mm, respectively. The channel is constituted by several metal foam layers with a thickness $S_{mf}$ which value is decided respect to the half-height $H$ of the channel. The ratio $S_{mf}/H$ assumes the following values: 1/8, 3/8, 5/8, 7/8. The TEG length $L_{teg}$ is equal to 65 mm while its thickness $S_{teg}$ is 8.5 mm.

The Darcy–Forchheimer-Brinkman model and LTE hypothesis are considered to describe the behavior of the aluminum foam. The metal foam is assumed isotropic, homogeneous with thermal and physical properties of the fluid and solid phases are constant. The buoyancy force and the viscous dissipation
are neglected; in addition, the thermal contact resistances between the tube surface and foam is overlooked. Under the hypotheses described above and with reference to the system in Figure 1, the governing equations are:

- **Conservation of mass**
  \[ \nabla \cdot \mathbf{V} = 0 \]  

- **Conservation of momentum**
  \[ \frac{\rho}{\phi^2} \nabla \cdot \mathbf{V} = -\nabla p + \nabla \cdot \mathbf{t} - \frac{K}{K} \nabla - \frac{C_{p}}{K} |\mathbf{V}| \nabla \]  

with \( \mathbf{t} \) the viscous stress tensor, considering the turbulent Reynolds stress:

\[ \mathbf{t} = \tau_{i,j} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho \mu \partial^2 u_i \partial x_j \]  

The last two terms in eq. (2) are the Darcy and the Forchhaimer terms. In the zone in which the aluminum foam is not present, the last terms of momentum equation are equal to zero and \( \phi \) is equal to 1 because there is only fluid phase.

- **Conservation of energy**
  \[ \rho \mathbf{V} \cdot \nabla T = \left[ \phi \left( k_f + k_s \right) + (1 - \phi) k_t \right] \nabla^2 T \]  

in which \( T_f = T_s = T \) for the LTE assumption, and \( k_f, k_s \), and \( k_t \) are the thermal conductivity of fluid, solid and turbulent, respectively.

For the present work, \( k-\varepsilon \) model proposed by Launder and Spalding [9] is assumed. Turbulent dynamic viscosity is evaluated from the knowledge of kinetic energy of turbulence, \( k \), and turbulent kinetic energy dissipation rate, \( \varepsilon \), given from the following equations:

\[ \mu_t = \rho c_\mu f_\mu \left( \frac{k^2}{\varepsilon} \right); \quad k_t = \frac{c_p \mu_t}{Pr_t} \]  

![Figure 1. Physical Domain](image-url)
- Turbulence kinetic energy (k-equation):

\[
\frac{\partial}{\partial x}(\rho uk) + \frac{\partial}{\partial y}(\rho vk) = \frac{\partial}{\partial x}\left[\mu + \frac{\mu_t}{\sigma_t} \frac{\partial k}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu + \frac{\mu_t}{\sigma_t} \frac{\partial k}{\partial y}\right] + \frac{G_k}{k} + G_b - \rho \varepsilon - D
\]  

(6)

- Turbulence dissipation (ε-equation):

\[
\frac{\partial}{\partial x}(\rho \varepsilon) + \frac{\partial}{\partial y}(\rho \varepsilon) = \frac{\partial}{\partial x}\left[\mu + \frac{\mu_t}{\sigma_t} \frac{\partial \varepsilon}{\partial x}\right] + \frac{\partial}{\partial y}\left[\mu + \frac{\mu_t}{\sigma_t} \frac{\partial \varepsilon}{\partial y}\right] + C_{\varepsilon 1} \frac{\varepsilon}{k} G_k - C_{\varepsilon 2} f_2 \frac{\varepsilon^2}{k} + E
\]

(7)

In equations (6) and (7), the first two terms represent transport of kinetic energy of turbulence and dissipation rate of kinetic energy by convection, respectively. Third and fourth terms represent transport of these quantities by diffusion. \(G_k\) is the rate of generation of turbulent kinetic energy due to mean velocity gradients. \(\rho \varepsilon\) is the destruction rate of turbulent kinetic energy. In addition, there are some extra terms denoted by \(D\) in \(k\)-equation and \(E\) in \(\varepsilon\)-equation to explain the behavior near the wall.

The evaluation of the parameters \(K\) and \(C_F\) was accomplished with the correlations presented in [10,11].

The present work studies the turbulent flow and the corresponding convective heat transfers of exhaust gas characterized by the same properties of the air in corresponding of the inlet temperature \(T_i\) equal to 723 K. The properties of the gas are reported in the Table 1.

The TEG component is made up of Bismuth-telluride Bi\(_2\)Te\(_3\) with these characteristics (Table 2):

| Table 1. Parameters of the exhaust gas |
| --- | --- | --- | --- |
| \(\rho\) (kg/m\(^3\)) | \(c_p\) (J/kgK) | \(k\) (W/mK) | \(\mu\) (kg/ms) |
| 0.43 | 1105 | 0.059 | 3.71 e-5 |

| Table 2. Parameters of Bi\(_2\)Te\(_3\) |
| --- | --- | --- | --- |
| \(\rho\) (kg/m\(^3\)) | \(c_p\) (J/kgK) | \(k\) (W/mK) | \(\mu\) (kg/ms) |
| 7740 | 165 | 3.00 | ----- |

3. Numerical model

The finite volume method is applied for obtaining the solutions for the governing equations. The code Ansys-Fluent 15.0 is used in order to accomplish the numerical simulations. The Semi Implicit Method for Pressure-Linked Equations (SIMPLE) algorithm is applied to solve the pressure-velocity coupling; the least square cell is carried out to estimate the gradient evaluation for the spatial discretization. The Pressure Stagerring Option (PRESTO) is used to calculate the staggered pressure on the faces of the control volume; for energy and momentum equations, the second order upwind scheme spatial discretization is applied. The standard \(k-\varepsilon\) model is accomplished in order to simulate the viscous behaviour. Convergence criteria are imposed equal to \(10^{-5}\) for the continuity, the velocity components and for \(k\) and \(\varepsilon\) while for the energy equal to \(10^{-8}\).

As numerical domain, it is adopted half of the entire physical model for the assumption of the symmetry, as showed in the Figure 2.

The height of the channel \(H\) is equal to 30 mm. There are four metal foam thickness values applied in the simulations such that the ratio \(S_{mf}/H\) is equal to 1/8, 3/8, 5/8, 7/8. Each thickness of aluminum foam is equal to 3.75 mm. The mesh is constituted by rectangular cells into the computational domain. Three different types of grids were studied for finding an independent solution from the mesh. They are made up of a number of nodes equal to 91012, 126006, 358362. For an inlet gas velocity of 10 m/s and for clean case (without metal foam), the estimation of the average Nusselt number \(Nu\) evaluated...
Table 3. Parameters of the aluminum metal foams

| PPI | ϕ  | K(m²) | C_r  |
|-----|----|-------|------|
| 5   | 0.90 | 1.74 e-7 | 0.078 |
| 5   | 0.95 | 2.17 e-7 | 0.099 |
| 5   | 0.97 | 2.52 e-7 | 0.096 |
| 20  | 0.90 | 0.90 e-7 | 0.088 |
| 20  | 0.95 | 1.19 e-7 | 0.010 |
| 20  | 0.97 | 1.42 e-7 | 0.087 |
| 40  | 0.90 | 0.53 e-7 | 0.084 |
| 40  | 0.95 | 0.56 e-7 | 0.098 |
| 40  | 0.97 | 0.52 e-7 | 0.094 |

Figure 2. Numerical Domain

on the TEG bottom surface, demonstrates that the grid with 126006 nodes, adopted for the simulations, had an error minor than 1% respect to the mesh with 358362 nodes, as showed in Table 4. The meshes for the system with aluminum foam layers have been created with the same criteria of the construction of the grid for the clean case.

The validation analysis is accomplished comparing the present model with the results given by Nithyanandam and Mahajan [8] for heat sinks with metal foam. This comparison in terms of the pressure drop and average wall temperature for aluminum foam with pore density PPI=20 is presented in Table 5. A good agreement between the present data and the ones given in [8] is noted.

Table 4. Grid Independence

| Nodes Number | Nu  | % error |
|--------------|-----|---------|
| 91012        | 309.69 | 1.3%    |
| 126006       | 310.57 | 0.99%   |
| 358362       | 313.67 | ----    |

Table 5. Comparison between the present model and the data given in [8]

| V[m/s] | Δp[Pa] [8] | Δp[Pa] Present work | T_w[K] [8] | T_w[K] Present work |
|--------|------------|---------------------|-----------|---------------------|
| 1.0    | 31.30      | 34.94               | 308.7     | 308.1               |
| 1.2    | 45.00      | 47.12               | 307.4     | 307.6               |
| 1.4    | 59.65      | 61.76               | 306.5     | 307.1               |
| 1.6    | 77.82      | 77.82               | 305.9     | 306.7               |
| 1.8    | 93.88      | 94.58               | 305.4     | 306.3               |
In this study, three different exhaust gas mass flow rates $\dot{m}$ are used, equal to 20 g/s, 60 g/s and 100 g/s. The boundary conditions applied for the study of the problem are the following: a fixed mass flow rate with a temperature $T_{in}$ equal to 723 K is considered at the inlet section; a fixed temperature $T_{cold}$ equal to 313 K is imposed on the top surface of the TEG; the overflow condition is considered to the exit; the other boundary lines are assumed adiabatic. In Table 6 are summarized the boundary conditions for temperature and velocity variables.

| LINES                          | VELOCITY CONDITIONS | TEMPERATURE CONDITIONS |
|--------------------------------|---------------------|------------------------|
| Inflow (x=0, 0≤y≤H)           | $\dot{m}$           | $T_{in}$               |
| Outflow (x=L, 0≤y≤H)          | $\frac{\partial u}{\partial x} = 0$ ; $v = 0$ | $\frac{\partial T}{\partial x} = 0$ |
| Symmetry axis (0≤x≤L, y=0)    | $\frac{\partial u}{\partial y} = 0$ ; $v = 0$ | $\frac{\partial T}{\partial x} = 0$ |
| Top surface of the TEG (y=H+S_{teg}) | -                  | $T_{cold}$             |
| Other wall surfaces           | No-Slip             | Adiabatic              |

Inside the TEG, a source term is set equal to the electric power per unit of volume:

$$P_e = \eta \dot{Q}$$

where $\eta$ is the efficiency of the system and it is equal to:

$$\eta = \frac{T_{hot} - T_{cold}}{T_{hot}} \frac{\sqrt{1+ZT} - 1}{\sqrt{1+ZT} + T_{cold} / T_{hot}}$$

in which $T_{hot}$ is the temperature on the bottom surface of TEG and the thermoelectric figure of merit $ZT$ is assumed equal to 1.5.

The input data are: the inlet temperature and mass flow rate of the exhaust gas, $T_{in}$ and $\dot{m}$, respectively; the top surface temperature of the TEG, $T_{cold}$; the height of the TEG, $S_{teg}$, the metal foam thickness, $S_{mf}$, the porosity, $\phi$, and the pore density, PPI; the permeability, $K$, the inertia coefficient, $C_F$, the thermophysical proprieties of metal foam and exhaust gas. The output are given in terms of external temperature of the tube, i.e. the lower temperature of the TEG, $T_{hot}$, the pressure losses, $\Delta p$, and the effectiveness, $\eta$.

### 4. Results and Discussions

The thermal and fluid dynamic analyses are accomplished for a fixed exhaust gas flow rate, equal to 20 g/s, 60 g/s and 100 g/s and for temperature of 723 K. The fluid regime is turbulent, and the LTE model is assumed for executing the estimation of the energy equation. In Table 7 the values of the main parameters in the simulations to carried out the presented results.

| Mass flow rate $m$ [g/s] | Porosity, $\phi$ | Pore density, PPI | Metal foam thickness referred to channel height S/H |
|--------------------------|------------------|-------------------|-----------------------------------------------|
| 20                       | 0.90             | 5                 | 1/8                                           |
| 60                       | 0.95             | 10                | 3/8                                           |
| 100                      | 0.97             | 20                | 5/8                                           |
|                           |                  |                   | 40                                            |

7/8
In the Figures 3(a), 3(b), the average temperature on the TEG bottom surface is showed as a function of the ratio $S_{mf}/H$ (reported in all graphs as $S/H$ for brevity) varying the flow rate value, and porosity. From the figure 3(a), it is possible to see as the temperature increases with increasing quantity of metal foam because its presence enhances the heat transfer. In addition, as expected, the average hot temperature $T_{hot}$ increases as the gas flow rate at the entrance augments. Moreover, for a fixed PPI and assigned exhaust gas flow rate, the temperature evaluated on the bottom surface of the TEG component decreases with increasing of porosity value, as possible to observe in the Figure 3(b).

![Figure 3](image1.png)

**Figure 3.** Average hot temperature as a function of $S_{mf}/H$ for different mass flow rates (a) and $\phi$ (b)

![Figure 4](image2.png)

**Figure 4.** $\Delta p$ for different $S_{mf}/H$ (a), PPI number (b), porosity (c)
In the Figure 4(a), 4(b), 4(c), the pressure drop $\Delta p$ is reported. One can observe from the Figure 4(a) that, in corresponding of 5 PPI and $\phi = 0.90$, the pressure drop, evaluated between the inlet and outlet section of the channel, increases with increasing of mass flow rate and with increasing of metal foam thickness because the metal foam produces a major friction to the fluid flow respect to the clean case. The diagram of Figure 4(b) shows $\Delta p$ for different number of PPI for $S_{mf}/H = 7/8$ and fixed porosity. It can be immediately observed that as the PPI number increases, the pressure drop and consequently the load losses also augment. Furthermore, from the Figure 4(c), one can notice that the increase of porosity causes a decrease in terms of pressure drop except at a flow rate of 100 g/s for which at porosity of 0.95 there is a greater mechanical loss. The effectiveness of the system $\eta$ is represented in the Figure 5(a), 5(b), 5(c), as a function of gas flow rate, PPI number, and $\phi$ respectively.

By the Figure 5(a), it is immediately evident that, in corresponding to a gas flow rate of 100 g/s and $S_{mf}/H = 1/8$, the effectiveness of the system assumes an increment equal to 166.7% up to a maximum of about 283.3% respect to the clean case. The results are in agreement with the values given in [8]. The Figure 5(b) demonstrates how the efficiency of the thermoelectric generator is not influence by the number of PPI. Instead, in the Figure 5(c), it is possible to observe that $\eta$ decreases with increasing of porosity value.

5. Conclusions

The results show that the average hot temperature evaluated on the bottom surface of the TEG component increases with increasing metal foam thickness and with decreasing the porosity. The results in terms of pressure drop demonstrate that this increases with the increment of the PPI number.
and with the aluminium foam thickness while the increment of porosity causes a decrease in the pressure drop. TEG effectiveness greatly benefits from the presence of metal foam, increasing its value by 283% compared to the case clean. Furthermore, the effectiveness improves with increasing exhaust gas flow rate; furthermore, the number of PPI does not influence the value of \( \eta \). In addition, the efficiency decreases with increasing of porosity value. The main conclusion is to observe that a mild thickness, 3/8 allows to have a sustainable pressure loss with an acceptable system efficiency.

6. Nomenclature

| Symbol | Description | Unit |
|--------|-------------|------|
| \( C_f \) | drag factor coefficient |  |
| \( d_f \) | fiber diameter | m |
| \( d_p \) | pore diameter | m |
| \( K \) | porous permeability | m\(^2\) |
| \( p \) | static pressure | Pa |
| \( Pe \) | electric power | W |
| \( PPI \) | number of pores per inch |  |
| \( \dot{Q} \) | thermal power | W |
| \( S \) | thickness | m |
| \( u \) | x-velocity | m/s |
| \( v \) | y-velocity | m/s |
| \( V \) | velocity vector | m/s |
| \( V \) | volume | m\(^3\) |
| \( ZT \) | thermoelectric figure of merit |  |

Greek symbols

| Symbol | Description | Unit |
|--------|-------------|------|
| \( \varepsilon \) | energy dissipation rate |  |
| \( \eta \) | thermoelectric efficiency |  |
| \( \tau \) | viscous stress tensor |  |
| \( \phi \) | porosity |  |

Subscripts

| Symbol | Description | Unit |
|--------|-------------|------|
| \( d_f \) | fiber diameter |  |
| cold | cold |  |
| hot | hot |  |
| in | inlet |  |
| mf | metal foam |  |
| \( t \) | turbulent |  |
| teg | thermoelectric generator |  |

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