Phase transition into superconducting mixed state and de Haas - van Alphen effect

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Abstract

The Landau expansion for the free energy of the superconducting mixed state near the upper critical field in powers of the square modulus of the order parameter averaged over Abrikosov lattice is derived. The analytical calculations has been carried out in frame of Gor’kov formalism for 3-dimensional isotropic BCS model beyond the limits of quasiclassical approximation, another words with Landau quantization of the quasiparticle energy levels taken into consideration. The derivation is performed at low temperature and high enough but finite crystal’s purity. The effect of Pauli paramagnetic terms is taken into account. The quantum oscillations of the critical temperature, the order parameter’s amplitude and the magnetization (de Haas-van Alphen effect) in the mixed state are found. The limitation of validity of a mean field approach due to critical fluctuations (Ginzburg criterion) for the phase transition under consideration is established.

1 Introduction

The growing body of measurements (see the most recent publications \cite{1, 2, 3, 4}) demonstrates the de Haas - van Alphen effect in the mixed state of type-II superconductors. The theoretical description of this phenomenon is rather cumbersome and as a result we have a vast amount of publications devoted to the subject \cite{5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15}. The origination of theoretical troubles is a nondiagonality of the order parameter matrix in Landau representation preventing a correct derivation of the quasiparticle energy spectrum in the mixed state. The later can be calculated properly just near $H_{c2}$ \cite{13}. The intention to avoid of this problem leads to idea to develope a theory far below $H_{c2}$ where a vortex core radius is much smaller than the distance between vortices and one can assume the space constancy of the order parameter modulus. This region on the first sight intersects with field interval of the observations of the dHvA oscillations in the mixed state extending down to the fields of the order of 1/2 or even 1/5 of the upper critical field (see \cite{3}). However in the superconductors with very large Ginzburg-Landau parameter $\kappa$, which is of the order of 20-30 in typical for observation of dHvA in the mixed state materials, the field interval of observation should still be considered as relating to vicinity of the upper critical field where the distance between the vortices almost coincides with the superconducting coherence length or vortex core radius. The former starts to exceed the later only at the fields of the order of $H_{c2}/\kappa$. So, in fact we need in a theoretical description of dHvA effect in a vicinity of the upper critical field $(H_{c2} - H)/H_{c2} < 1$ where one can assume the field inside of superconductor as coinciding with homogeneous external field and use the Abrikosov ansatz for the periodic in space order parameter distribution.

Here we present the derivation of the Landau expansion of the free energy density in vicinity of the upper critical field in powers of the square modulus of the order parameter averaged over Abrikosov lattice,

$$F_s - F_n = \alpha \Delta^2 + \frac{\beta}{2} \Delta^4,$$

which is valid until $(H_{c2} - H)/H_{c2} < 1$ in spite of relative value of the order parameter amplitude and the distance between Landau levels (for more exact condition of validity \cite{1} see \cite{57}). The treatment of the problem
is carried out in frame of the Gor'kov formalism with Landau quantization taken into consideration. It has been done earlier for 2-dimensional electron gas either numerically or analytically but out of dHvA observation region when the temperature $T$ exceeds the distance between the Landau levels $\omega_c = H/m^*$. The reason for the treatment of the problem in the unsuitable region is divergencies of $\alpha$ and $\beta$ coefficients in the formula at temperature $T \to 0$ in absolutely clean limit. Unlike to these papers we shall treat the problem analytically for 3-dimensional isotropic BCS model and escape of the divergencies by the introducing a width of Landau level $\Gamma$ originating of impurity scattering or a specimen’s inhomogeneity. In addition to where $\alpha$ and $\beta$ coefficients has been determined neglecting the oscillating terms here we take them into account. The expressions for the functions describing the oscillating behaviour of the critical temperature, the order parameter’s amplitude and the magnetization in the mixed state are found. As in a normal metal the tiny oscillating corrections to the free energy are transformed into the big summands in the magnetic moment as the result of differentiation of very fast oscillating functions of the ratio of Fermi energy to the cyclotron frequency. It is shown that at the distance of the order of $\varepsilon_F/T_c$ oscillations from the upper critical field there is noticeable decrease (of the order of unity) of the amplitude of dHvA oscillations of the magnetization in the mixed state.

As was pointed out in the paper the Landau level splitting due to Pauli paramagnetic interaction of electron spin with external magnetic field can cause the change of character of the phase transition to the mixed state. Namely, in clean enough type-II superconductor the second order transition transforms into the first order transition at temperatures below some tricritical point depending of impurity concentration and determined by equation $\beta(H, T) = 0$. Here we confirm that there is a principal opportunity of such a kind phase diagramme at low enough temperatures $T < \Gamma$ when the Zeeman splitting of Landau levels exceeds a width of them: $\mu_e H > \Gamma$. However in view of very complicated analytical structure of the expression for $\beta(H, T)$ it is difficult to determine definitely its sign in the region mentioned above. One may assert only that at $\mu_e H < \Gamma$ the second order type transition takes place up to zero temperature.

The investigation of the fluctuation effects is also undertaken in the present work. Unlike to the zero field case where the fluctuation region in vicinity of the critical temperature is of the order of $(T_c/\varepsilon_F)^3$ that is negligibly small, the critical region at low temperatures in vicinity of the upper critical field is of the order of $(T_c/\varepsilon_F)^2$. The period of critical temperature oscillations has the same order of magnitude. This means that the field interval for the reentrant normal metal-superconductor phase transitions originating of critical temperature oscillations is located in the critical region. Hence the reentrant behavior has the poor chances for observation. It is shown however that the quantum oscillations of $T_c(H)$ can be accessible for observation in lowest temperature region $T < \Gamma \ll \omega_c$.

The paper is organized as follows. The formula is derived in the next Section. The oscillating with magnetic field behavior of the $\alpha$ (or another words the oscillating behavior of the critical temperature) and $\beta$ coefficients are described in the following two Sections. The magnetizations oscillations in the superconducting mixed state are calculated in Section 5. The critical fluctuations are discussed in the Section 6.

2 Free energy

The difference of the superconductor - normal metal free energy densities expanded in powers of the order parameter $\Delta(r)$ and averaged over a superconductor volume $V$ has the form

$$F_s - F_n = \frac{1}{V} \int dr (F_s(r) - F_n) = \frac{1}{gV} \int dr |\Delta(r)|^2 - \frac{1}{\sqrt{V}} \int dr_1 dr_2 K_2(r_1, r_2) \Delta^*(r_1) \Delta(r_2) + \frac{1}{2V} \int dr_1 dr_2 r_3 dr_4 K_4(r_1, r_2, r_3, r_4) \Delta^*(r_1) \Delta(r_2) \Delta^*(r_3) \Delta(r_4).$$

(2)

Here $g$ is the constant of the pairing interaction. Neglecting all the vortex corrections that is valid in ultra clean limit $\Gamma < \omega_c$ considered here we shall use for the functions $K_2$ and $K_4$ the following expressions

$$K_2(r_1, r_2) = \frac{1}{2} \sum_{\sigma = \pm 1} \sum_{\nu} T \sum_{\nu} G^\sigma(r_1, r_2, \tilde{\omega}_\nu) G^{-\sigma}(r_1, r_2, -\tilde{\omega}_\nu),$$

(3)

$^1$ Planck’s constant $\hbar$, electron’s charge $|e|$ and velocity of light $c$ are chosen to be unity throughout the paper.
written by means of the electron Green functions $G^\sigma (r_1, r_2, \tilde{\omega}_\nu)$ in normal state under an external magnetic field. The later are related with the Green functions depending only on the relative coordinate $\tilde{G}^\sigma (r_1 - r_2, \tilde{\omega}_\nu)$ as

$$G^\sigma (r_1, r_2, \tilde{\omega}_\nu) = \exp \left( i \int_{r_1}^{r_2} A(s) ds \right) \tilde{G}^\sigma (r_1 - r_2, \tilde{\omega}_\nu).$$  

Making use this formula one can rewright the quadratic in $\Delta (r)$ terms in the formula (2) as

$$\left( \frac{1}{g} - \int dR \exp \left( -\frac{1}{2} H \varrho^2 \right) K_2 (R) \right) \frac{1}{V} \int d|\Delta (r)|^2,$$

where $\varrho^2 = R^2 - Z^2$ and

$$K_2 (R) = \frac{1}{2} \sum_{\sigma = \pm 1} T \sum_{\nu} \tilde{G}^\sigma (R, \tilde{\omega}_\nu) \tilde{G}^{-\sigma} (R, -\tilde{\omega}_\nu).$$  

As it was discussed in Introduction for the strong type-II superconducting materials the Abrikosov solution for the order parameter

$$\Delta (r) = \Delta f (r),$$  

$$f (r) = \sqrt{2} \sum_{\nu = \text{integer}} \exp \left( \frac{2 \pi \nu}{\alpha} y - \left( \frac{x}{\lambda} + \frac{\pi \nu}{\alpha} \right)^2 \right).$$

is valid in broad enough vicinity of the upper critical field. Here for simplicity we have chosen the Abrikosov square lattice solution in the Landau gauge $A (r) = (0, Hx, 0)$. The elementary cell edge length $a$ is such that $a^2 = \pi \lambda^2$, where $\lambda = H^{-1/2}$ is magnetic length. We assume that magnetic field is uniform and coincides with an external field. This is certainly true in vicinity of $H_{c2}$ for the superconductors with large Ginzburg-Landau parameter.

Using formulas (8), (8), (8) one can rewright (8) in the shape (9)

$$F_s - F_n = \alpha \Delta^2 + \frac{\beta}{2} \Delta^4,$$

where

$$\alpha = \frac{1}{g} - \int dR \exp \left( -\frac{1}{2} H \varrho^2 \right) K_2 (R)$$

and

$$\beta = \frac{1}{V} \int dr_1 dr_2 dr_3 dr_4 K_4 (r_1, r_2, r_3, r_4) f^*(r_1) f (r_2) f^*(r_3) f (r_4).$$

The amplitude of the order parameter $\Delta$ has to be found from the free energy minimum condition.

The $\alpha$ and $\beta$ coefficients are the oscillating functions of the magnetic field which will be obtained in the following two Sections.

### 3 Critical temperature oscillations

The critical temperature as a function of magnetic field at $T \ll T_c$ have been found in the paper by L.Gruenberg and L.Guenther [13]. It is determined by the equation

$$\alpha (H, T) = 0,$$

where

$$\alpha (H, T) = \frac{N_0}{2} \left\{ \frac{H - H_{c2o}}{H_{c2o}} + \frac{T^2}{T_c^2} S_0 - 2 \pi^{3/2} \left( \frac{\omega_c}{\mu} \right)^{1/2} S_1 - 2^{3/2} \pi^{1/2} \omega_c \frac{\mu}{S_2} \right\},$$

with

$$S_0 = \int d^2 r \frac{1}{2} |\Delta (r)|^2,$$

$$S_1 = \int d^2 r \frac{1}{2} |\Delta (r)|^2 \frac{1}{2} \int d^2 r \frac{1}{2} |\Delta (r)|^2,$$

$$S_2 = \int d^2 r \frac{1}{2} |\Delta (r)|^2 \frac{1}{2} \int d^2 r \frac{1}{2} |\Delta (r)|^2 \frac{1}{2} \int d^2 r \frac{1}{2} |\Delta (r)|^2,$$

$$\omega_c = \pi \lambda^2.$$
\[ S_1 = \frac{2\pi T}{\omega_c} \Re \sum_{n=1}^{\infty} \sum_{\nu=0}^{\infty} \exp \left( -\frac{4\pi n(\tilde{\omega}_\nu + i\mu_e H)}{\omega_c} \right), \]  
\[ S_2 = \frac{2\pi T}{\omega_c} \Re \sum_{n=1}^{\infty} \sum_{m>n}^{\infty} \sum_{\nu=0}^{\infty} (-1)^{m+n} \exp \left( -\frac{2\pi (m+n)(\tilde{\omega}_\nu + i\mu_e H)}{\omega_c} \right) \frac{\cos \left( \frac{2\pi |n-m|\mu_e}{\omega_c} - \frac{\pi}{2} \right)}{|n-m|^{1/2}}, \]  
\[ H_{c2o} = \frac{\pi^2 e^2 T^2}{2\gamma v_F}. \]

Here, \( \mu \) is chemical potential, \( v_F \) is Fermi velocity, \( \Gamma \) is Landau level width, \( \mu_e \) is the electron’s magnetic moment, \( N_0 = m * k_F/2\pi^2 \) is the normal metal electron density of states per one spin projection. \( S_0 \) is slow (logarithm) function of magnetic field with numerical value of the order of unity.

The substitution of the first two terms of (13) in the equation (13) reproduces of the quasiclassical result found by L.Gor’kov [17]

\[ H = H_{c2o}(1 - \frac{T^2}{T^2_0} S_0) \]  

for the upper critical field at temperature tending to zero. The third and the fourth terms in the formula (13) give correspondingly nonoscillating and oscillating corrections to Gor’kov’s expression owing to quantization of the quasiparticle levels. For the further considerations it will be convenient to represent the functions \( S_1 \) and \( S_2 \) in more simple analytical form.

The summation over \( \nu \) in (14) yields

\[ S_1 = \frac{\pi T}{\omega_c} \Re \sum_{n=1}^{\infty} \frac{\exp \left( -\frac{4\pi na}{\omega_c} \right)}{\sinh \frac{4\pi n\omega_c}{\omega_c}}, \]  

where

\[ a = \Gamma + i\eta \omega_c, \]  

and

\[ \eta \omega_c = \mu_e H - \frac{q}{2} \omega_c, \]

is the Zeeman energy deviation from a half integer number \( q/2 \) of distances \( \omega_c \) between Landau levels. For noninteracting electron gas in a perfect crystal \( a = 0 \) and

\[ S_1 \approx \frac{1}{4\pi} \ln \frac{\omega_c}{4\pi^2 T}, \]

diverges at \( T \to 0 \). This result formally means an existance of superconductivity of noninteracting electron gas in a perfect crystal at \( T = 0 \) in arbitrary large field [16].

In the opposite limit \( \pi T \ll |a| \) when

\[ S_1 = \Re \sum_{n=1}^{\infty} \left( \frac{1}{4\pi n} - \frac{2\pi^3 n}{3} \left( \frac{T}{\omega_c} \right)^2 \right) \exp \left( -\frac{4\pi na}{\omega_c} \right), \]

At \( |a| \ll \omega_c/4\pi \) it is

\[ S_1 = \frac{1}{4\pi} \ln \frac{\omega_c}{4\pi |a|} - \frac{\pi}{24} \left( \frac{T}{|a|} \right)^2. \]

For \( 4\pi \Gamma \sim \omega_c \) one can estimate sum (13) by its first term

\[ S_1 = \frac{\pi T}{\omega_c} \sinh \frac{4\pi \omega_c}{\omega_c} \exp \left( -\frac{4\pi \Gamma}{\omega_c} \right) \cos 4\pi \eta. \]

The substitution (24) into equations (13), (12) provides us by the averaged over oscillations behavior of upper critical field at low temperatures

\[ \bar{H}_{c2}(T) = H_{c2o} \left\{ 1 + \frac{1}{2} \sqrt{\frac{\pi \omega_c^2}{\mu}} \left( 1 - \frac{8\pi^4}{3} \left( \frac{T}{\omega_c} \right)^2 \right) \exp \left( -\frac{4\pi \Gamma}{\omega_c^2} \right) \cos 4\pi \eta \right\}, \]
Here \( \omega_{c2} = H_{c2}/m^* \) is the cyclotron frequency at the upper critical field. Let us denote the number of Landau levels at \( H = H_{c2} \) disposed below the Fermi level \( \varepsilon_F \) as

\[
n_{c2} = \frac{\varepsilon_F}{\omega_c} = \frac{1}{2\pi^2} \left( \frac{\varepsilon_F}{T_c} \right)^2. \tag{26}
\]

The number \( n_{c2} \) in typical materials with small Fermi energy and relatively high \( T_c \) where dHvA effect in the mixed state has been observed is of the order of one hundred. So, the upper critical field at zero temperature is noticably larger (\( \sim n_{c2}^{-1/2} \)) than its quasiclassical value (16). The decreasing \( H_{c2} \) with temperature follows \( T^2 \) law. This parabolic dependence in the region

\[
T \ll \frac{\omega_c}{2\pi^2} = \frac{T_c^2}{\varepsilon_F}
\]

is much faster than Gorkov’s parabola (17).

For \( S_2 \) the summation over \( \nu \) in (14) produces

\[
S_2 = \frac{\pi T}{\omega_c} \Re \sum_{l=1}^{\infty} \left( -\frac{1}{\sqrt{l}} \right) \exp \left( -\frac{2\pi l (\Gamma + i\mu_e H)}{\omega_c} \right) \cos \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \sum_{n=1}^{\infty} \exp \left( -\frac{4\pi n (\Gamma + i\mu_e H)}{\omega_c} \right) \frac{\sinh \pi \omega_c T}{\omega_c}.
\tag{27}
\]

Changing the summation variables to \( n \) and \( m = n - l \) we get

\[
S_2 = \frac{\pi T}{\omega_c} \Re \sum_{l=1}^{\infty} \left( -\frac{1}{\sqrt{l}} \right) \exp \left( -\frac{2\pi l (\Gamma + i\mu_e H)}{\omega_c} \right) \cos \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \sum_{n=1}^{\infty} \exp \left( -\frac{4\pi n (\Gamma + i\mu_e H)}{\omega_c} \right) \frac{\sinh \pi \omega_c T}{\omega_c}.
\tag{28}
\]

Similar to \( S_1 \) for noninteracting electron gas in a perfect crystal \((\mu_e H = \omega_c, \Gamma = 0)\) the expression for \( S_2 \) is diverged at \( T \to 0 \) (see also (13)).

In more realistic case \( 4\pi \Gamma \sim \omega_c \) one can keep only the first term in the sum over \( n \):

\[
S_2 = \frac{\pi T}{\omega_c} \sum_{l=1}^{\infty} \left( -\frac{1}{\sqrt{l}} \right) \exp \left( -\frac{2\pi l (\Gamma + i\mu_e H)}{\omega_c} \right) \cos \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \cos \left( 2\pi \frac{l}{\omega_c} H \frac{l + 2}{\omega_c} \right).
\tag{29}
\]

Substitution of (24) and (29) into (12) and the resolution of the equation \( \alpha(H, T) = 0 \) in respect of \( T \) gives the oscillating behavior of critical temperature. The simple estimation shows that the amplitude of oscillations at \( T \sim \omega_{c2} \) is of the order of

\[
\frac{\delta T_{osc}}{T_c} \sim \left( \frac{T_c}{\varepsilon_F} \right)^2.
\tag{30}
\]

At lower temperatures the amplitude of oscillations is somthat larger. The period of one oscillation \( \delta H \) is easy to find from

\[
1 = \delta n = \delta \frac{\varepsilon_F}{\omega_c} = -\frac{\varepsilon_F}{\omega_c} \frac{\delta H}{H}
\]

So,

\[
\frac{\delta H}{H_{c2}} \approx 2\pi^2 \left( \frac{T_c}{\varepsilon_F} \right)^2.
\tag{31}
\]

As we shall see later this value coincides with the critical region in vicinity of upper critical field at low temperatures. Hence the critical temperature oscillating behavior as a function of magnetic field exists only in frames of mean field approximation and has poor chances to be observable. On the other hand outside the narrow region of strong critical fluctuations in the mixed superconducting state the function

\[
\alpha(H, T) = \tilde{\alpha}(H, T) + \alpha_{osc}(H, T)
\tag{32}
\]

consists of smooth function

\[
\tilde{\alpha}(H, T) = \frac{N_0 H - \tilde{H}_{c2}(T)}{H_{c20}}
\tag{33}
\]
This expression can be positive or negative depending on the relation between the values of $\Gamma$ and $\omega_c$ (see (25)) and fast oscillating function of magnetic field where

$$f_n = \frac{\mu}{\omega_c} \sum_{l=1}^{\infty} \frac{(-1)^l \exp \left( -\frac{2\pi l (l+2)}{\omega_c} \right)}{\sqrt{l} \sinh \frac{2\pi l (l+2) T}{\omega_c}} \cos \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \cos \left( \frac{2\pi \mu_e H (l+2)}{\omega_c} \right)$$

(34)

giving the main contribution to the oscillating part of the magnetization.

4 Oscillations of $\beta(H,T)$

Instead of formula (11) it is more suitable to use an expression for $\beta$ in the so called magnetic sublattices representation (see [13])

$$\beta(H,T) = \frac{1}{2} \sum_{\sigma=\pm 1} T \sum_{n,n',m,m'} \int \frac{dk_z}{2\pi} G^{-\sigma}(\xi_{nk_z}, -\tilde{\omega}_\nu) G^\sigma(\xi_{nk_z}, \tilde{\omega}_\nu) G^{-\sigma}(\xi_{nk'_z}, -\tilde{\omega}_\nu) G^\sigma(\xi_{nk'_z}, \tilde{\omega}_\nu) F_{nm'm'}^{nn'}$$

(35)

where

$$G^{\sigma}(\xi_{nk_z}, \tilde{\omega}_\nu) = \frac{1}{i\tilde{\omega}_\nu - \xi_{nk_z} + \sigma \mu_e H}$$

(36)

$$\xi_{nk_z} = \omega_c(n + \frac{1}{2}) + \frac{k^2}{2m^*} - \mu,$$

(37)

$$F_{nm'm'}^{nn'} = \int \frac{d\vec{q}}{(2\pi)^2} f_{nm}(\vec{q}) f_{n'm'}^*(\vec{q}) f_{nm}(\vec{q}) f_{n'm'}(\vec{q})$$

(38)

and $f_{nm}(\vec{q})$ are the matrix elements of functions (9).

In view of extremely complicated structure of general formula (35) we are limited ourselves by the analysis of diagonal terms $n = n' = m = m'$ giving the main contribution to the expression (15) at $\Gamma < \omega_c$. For this case as was shown in the paper [13] $F_{nn'}^{nm} \approx 1/(2\pi)^2 n$ and

$$\beta(H,T) = \frac{1}{2(2\pi)^2} \sum_{\sigma=\pm 1} T \sum_{\nu=0}^{\infty} \int \frac{dk_z}{2\pi n} \left( G^{\sigma}(\xi_{nk_z}, \tilde{\omega}_\nu) G^{-\sigma}(\xi_{nk_z}, -\tilde{\omega}_\nu) \right)^2.$$ 

(39)

Let us represent

$$\beta(H,T) = \tilde{\beta}(H,T) + \beta^{osc}(H,T)$$

(40)

as the sum of smooth nonoscillating function and fast oscillating function of magnetic field. For the calculation of nonoscillating part of $\beta$ following the papers [13, 14] one can substitute the summation over $n$ by the integration according to

$$\frac{1}{2\pi} \sum_{n=0}^{\infty} \int \frac{dk_z}{2\pi} = \int_0^{\infty} \frac{dn}{2\pi \lambda^2} \int \frac{dk_z}{2\pi} = N_0 \int d\xi \int_0^{\pi/2} \sin \theta d\theta$$

Making use $n \approx \mu \sin^2 \theta / \omega_c$ and performing the integration over $\theta$ we get

$$\tilde{\beta}(H,T) = \frac{\omega_c}{8\pi \mu} \sum_{\sigma=\pm 1} T \sum_{\nu} N_0 \int d\xi \left( G^{\sigma}(\xi, \tilde{\omega}) G^{-\sigma}(\xi, -\tilde{\omega}) \right)^2,$$

(41)

The integration over $\xi$ yields

$$\tilde{\beta}(H,T) = N_0 \frac{\omega_c \ln \frac{\mu}{\omega_c}}{4\mu} \sum_{\nu=0}^{\infty} \frac{\tilde{\omega}^3 - 3\tilde{\omega}(\mu_e H)^2}{\tilde{\omega}^2 + (\mu_e H)^2}.$$ 

(42)

This expression can be positive or negative depending on the relation between the values of $\Gamma$ and $\mu_e H$. At $T = 0$ $K$ when it is possible to change the summation to the integration we obtain

$$\tilde{\beta}(H,0) = N_0 \frac{\omega_c \ln \frac{\mu}{\omega_c}}{16\pi \mu} \frac{\Gamma^2 - (\mu_e H)^2}{(\Gamma^2 + (\mu_e H)^2)^2},$$

(43)
This result demonstrates the positiveness of $\bar{\beta}$ up to zero temperature at $T > \mu_e H$. In the opposite case $\bar{\beta}$ at $T = 0$ $K$ is negative. Hence below some temperature $\sim \mu_e H$ the second order type transition to the superconducting state starts to be of the first order. Because we have thrown out the nondiagonal terms it is difficult to say definitely is it the case or not and if it is so what is the temperature of tricritical point. Leaving this problem for future experimental and theoretical investigations let us assume positive value of

$$\bar{\beta}(H, 0) \approx N_0 \frac{\omega_c \ln \frac{\mu}{\omega}}{16\pi \mu \Gamma^2},$$

(44)

which is valid at least at the fulfillment of the conditions $T, \mu_e H < \Gamma < \omega_c$.

To find the oscillating part of $\beta(H, T)$ at the same conditions let us apply to the expression (37) the Poisson summation formula

$$\beta^{osc}(H, T) = N_0 \frac{\omega_c^{3/2}}{2^{5/2} \pi \mu^{3/2}} \Re \sum_{\sigma = \pm 1} T \sum_{T} \int_{-\infty}^{\infty} \frac{d\tau}{\omega_c} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi n(\varepsilon, k_z)} \sum_{l=1}^{\infty} e^{2\pi i n} (G^\sigma (\varepsilon - \mu, \bar{\omega}_\nu) G^{-\sigma} (\varepsilon - \mu, -\bar{\omega}_\nu))^2,$$

(45)

here $\varepsilon = \xi_n k_z + \mu$. Subsequent calculations follows by well known Lifshits, Kosevich derivation [18] of normal metal de Haas - van Alphen oscillations. Performing the saddle point integration over $k_z$ we get

$$\beta^{osc}(H, T) = N_0 \frac{\omega_c^{3/2}}{2^{5/2} \pi \mu^{3/2}} \Re \sum_{\sigma = \pm 1} T \sum_{T} \int_{-\infty}^{\infty} \frac{d\tau}{\omega_c} \exp \left(2\pi i \frac{T}{\omega_c} \xi - \frac{i\pi}{4}\right) \times$$

$$\int_{-\infty}^{\infty} d\xi \left(\bar{\omega}_\nu - \xi + \sigma \mu_e H^2 (-i\bar{\omega}_\nu - \xi - \sigma \mu_e H)^2 \right).$$

(46)

The integration over $\xi$ yields

$$\beta^{osc}(H, T) = N_0 \frac{\omega_c^{3/2}}{2^{5/2} \pi \mu^{3/2}} \Re \sum_{l=1}^{\infty} \frac{(-1)^l}{\sqrt{l}} \cos \left(2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4}\right) \Re T \sum_{\nu=0}^{\infty} \exp \left(-2\pi l \frac{\bar{\omega}_\nu + i\mu_e H}{\omega_c}\right) \times$$

$$\left\{ \frac{1}{(\bar{\omega}_\nu + i\mu_e H)^2} + \frac{2\pi l}{\omega_c (\bar{\omega}_\nu + i\mu_e H)^2}\right\}.$$

(47)

At the fulfillment of the conditions $T, \mu_e H < \Gamma < \omega_c$ one can rewrite this expression in more simple form:

$$\beta^{osc}(H, 0) = N_0 \frac{\omega_c^{3/2}}{2\pi (2\mu)^{3/2}} \Re \sum_{l=1}^{\infty} \frac{(-1)^l}{\sqrt{l}} \cos \left(2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4}\right) \exp \left(-2\pi l \frac{\Gamma}{\omega_c}\right) I \left(2\pi l \frac{\Gamma}{\omega_c}\right),$$

(48)

where

$$I(x) = \int_{0}^{\infty} dy \left\{ \frac{1}{(y+1)^2} + \frac{x}{(y+1)^2}\right\} \exp (-xy).$$

Thus, as it is expected, the oscillating part of $\beta$ has been published in [18]. Although the results of that paper are in general qualitatively correct there there are the errors in the values of $\beta$ and $\beta^{osc}$ (compare formula (3) in [13] and (14), (48) in the present article).

5 Magnetizations oscillations

Minimization of the expression (1) over $\Delta$ yields

$$F_s = F_n - \frac{\alpha^2}{2\beta},$$

(49)
Now we can calculate the magnetization

\[ M_s = -\frac{\partial F_s}{\partial H}. \]  

(50)

Keeping at the differentiation only the fast oscillating terms we get

\[ M_s^\text{osc} \simeq M_n^\text{osc} + \frac{\bar{\alpha} \partial \alpha^\text{osc}}{\beta \partial H} - \frac{\bar{\alpha}^2 \partial \beta^\text{osc}}{2\beta^2 \partial H}. \]  

(51)

Here \( M_n^\text{osc} \) is the oscillating part of normal metal magnetization \[13\] including Dingle factor \( \exp(-2\pi i \Gamma/\omega_c) \) due to scattering by imperfections

\[ M_n^\text{osc} = \sum_{l=1}^{\infty} M_l. \]  

(52)

Here

\[
M_l = N_0 \frac{\omega^2}{2^{1/2}\pi H} \left( \frac{\mu}{\omega_c} \right)^{1/2} \frac{(-1)^{l+1} \lambda_l}{l^{3/2} \sinh \lambda_l} \sin \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \cos \left( 2\pi l \frac{\mu_c H}{\omega_c} \right) \exp \left( -2\pi l \frac{\Gamma}{\omega_c} \right),
\]

(53)

\[
\lambda_l = \frac{2\pi^2 l \Gamma}{\omega_c}.
\]

(54)

For the subsequent terms in (51) making use \[13\], [14] and approximative expressions (44) and (48) for the smooth and oscillating parts of \( \beta \) we get

\[
\frac{\bar{\alpha} \partial \alpha^\text{osc}}{\beta \partial H} = -N_0 \frac{2^{7/2} \pi^3 / 2 \Gamma^2}{H} \frac{\mu}{\omega_c \ln \frac{\mu}{\omega_c}} \frac{\bar{H}_{c2}(T) - H}{H_{c2o}} \times \frac{\omega_c}{\mu} ln \frac{\mu}{\omega_c} \left( \frac{\bar{H}_{c2}(T) - H}{H_{c2o}} \right)^2 \times \sum_{l=1}^{\infty} (-1)^{l+1} \frac{l^{3/2}}{(l+2)^2} \sinh \lambda_{l+2} \sin \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \cos \left( 2\pi(l+2) \frac{\mu_c H}{\omega_c} \right) \exp \left( -2\pi(l+2) \frac{\Gamma}{\omega_c} \right).
\]

(55)

\[
\frac{-\bar{\alpha}^2 \partial \beta^\text{osc}}{2\beta^2 \partial H} = N_0 \frac{2^{5/2} \pi^2 \Gamma^2}{H} \frac{\mu^{3/2}}{\omega_c^3 / 2 \ln \frac{\mu}{\omega_c}} \left( \frac{\bar{H}_{c2}(T) - H}{H_{c2o}} \right)^2 \times \sum_{l=1}^{\infty} (-1)^{l+1} \frac{l^{1/2}}{2} \sin \left( 2\pi l \frac{\mu}{\omega_c} - \frac{\pi}{4} \right) \exp \left( -2\pi l \frac{\Gamma}{\omega_c} \right) I \left( 2\pi l \frac{\Gamma}{\omega_c} \right).
\]

(56)

The corrections to the dHvA amplitude in the mixed superconducting state have the same structure as the normal metal part: each term in the sums (55), (66) oscillates with the frequency \( l\mu/\omega_c \) corresponding to the extremal crosssection of the Fermi surface. From these expressions at \( 4\pi \Gamma \sim \omega_c \) it is easy to see that until

\[ \frac{\bar{H}_{c2}(T) - H}{H_{c2o}} < \sqrt[4]{\frac{\omega_c}{\mu} \ln \frac{\mu}{\omega_c}} \]  

(57)

the following inequality takes place

\[
|M_n^\text{osc}| > \left| \frac{\bar{\alpha} \partial \alpha^\text{osc}}{\beta \partial H} \right| > \left| -\frac{\bar{\alpha}^2 \partial \beta^\text{osc}}{2\beta^2 \partial H} \right|.
\]

(58)

So, in the limit of validity of the condition (57) one can neglect of the third term in the formula (51) and treat the (55) as the main mixed state correction to the oscillating part of magnetization in the normal state. Summing up (52) and (55) we obtain

\[ M_s^\text{osc} = M_n^\text{osc} + \frac{\bar{\alpha} \partial \alpha^\text{osc}}{\beta \partial H} = \sum_{l=1}^{\infty} M_l M_{sl}, \]

(59)

\[
M_{sl} = 1 - \frac{1}{\sqrt{\pi}} \left( \frac{4\pi \Gamma}{\omega_c} \right)^2 \frac{\mu^{1/2} \ln \frac{\mu}{\omega_c} \bar{H}_{c2}(T) - H}{H_{c2o}} \frac{l^2 \lambda_{l+2} \sinh \lambda_l}{(l+2) \lambda_l \sinh \lambda_{l+2}} \cos \left( 2\pi l \frac{\mu_c H}{\omega_c} \right) \exp \left( -4\pi \frac{\Gamma}{\omega_c} \right),
\]

(60)
This expression presents the main result of the paper. Looking on (60) one can conclude that at $\mu_e H \approx \omega_c/2$ as well as at $\mu_e H \ll \omega_c/2$ the amplitude of de Haas - van Alphen effect in the superconducting mixed state is less than in the normal state. For the intermediate values of Zeeman splitting like $\mu_e H \approx \omega_c/4$ the amplitude of the magnetization oscillations in the mixed state can be even larger than in the normal state.

The region of validity of the result (59) is determined by the inequality (57) which can be rewritten as (see (26))

$$\frac{\bar{H}_{c2}(T) - H}{H_{c2}} < \frac{\ln n_{c2}}{\sqrt{n_{c2}}}.$$  (61)

If we remember that period of oscillations (31) is of the order of

$$\frac{\delta H}{H_{c2}} \approx \frac{1}{n_{c2}}$$  (62)

that means the formula (59) describes the oscillating part of magnetization in the field interval corresponding to

$$\sqrt{n_{c2}} \ln n_{c2}.$$  (63)

novations below $H_{c2}$. This number in the typical for observation dHvA effect in the superconducting mixed state materials is of the order of several tens. Out this field interval the corrections to the normal metal amplitude of any order $\sim (H_{c2} - H)^n$ shall be the same order of magnitude and the result (59) is inapplicable to the mixed state dHvA effect description. It is worth to be noted here that the region (63) coincides with the region of the existence of gapless superconductivity found in the paper [13]. So, out the field interval (57) below upper critical field the de Haas van Alphen effect in the superconducting mixed state has to be negligibly small.

6 Critical fluctuations

The problem of critical behavior of type II superconductors near the upper critical field had been considered in many theoretical papers (see for instance [20] and references therein). We shall not be interested here by the solution of this problem as whole but just an estimation of a width of the fluctuational region at very low temperatures in very clean materials. To establish the region of importance of critical fluctuations under magnetic field near the zero temperature let us remind first how to estimate it in zero magnetic field near $T_c$. The most convinient for our purposes way is to compare the density of energy of critical fluctuations $F_{fl} \sim T/(\xi(T))^3$ with mean field energy density $F_{mf} \sim N_0(T - T_c)^2$ above critical temperature. Here

$$\xi(T) = \xi_0 \sqrt{\frac{T_c}{T - T_c}}$$

is the coherence length. The comparison of these values leads us to well known condition (Ginzburg criterion)

$$\frac{T - T_c}{T_c} < \left( \frac{T_c}{\xi_F} \right)^4$$  (64)

of importance of critical fluctuations. To get the similar condition near $H_{c2}$ at low temperatures one needs to take into account the one dimensional character of fluctuations along magnetic field direction such that the density of energy of critical fluctuations will be

$$F_{fl} \sim \frac{T}{2\pi \lambda^2 \xi(H)}.$$  (65)

Where $\lambda = H^{-1/2}$ is magnetic length coinciding at $H = H_{c2}(T = 0)$ with zero temperature coherence length $\xi_0 = v_F/2\pi T_c$. The field dependent coherence length above $H_{c2}$ is given by

$$\xi(H) \approx \lambda \sqrt{\frac{H_{c2}}{H - H_{c2}}}.$$  (66)
Here we have omitted the logarithmic corrections to $\xi(H)$ (see \[21\]).

The density of mean field free energy has the form

$$F_{mf} = \frac{\alpha^2}{2\beta} = 2\pi N_0 \frac{\varepsilon_F \Gamma^2}{\omega_c \ln \frac{\varepsilon_F}{\omega_c}} \left(\frac{H - H_{c2}}{H_{c2}}\right)^2$$

(67)

The comparison of equations (65) and (67) shows that the critical fluctuations contribution to the thermodynamical values surpass the corresponding mean field values at

$$\frac{H - H_{c2}}{H_{c2}} < \pi^2 \left(\frac{T_c}{\varepsilon_F}\right)^2 \left(\frac{T \omega_c \ln \varepsilon_F}{\omega_c^2}\right)^{2/3}$$

(68)

This inequality establishes the region of importance of critical fluctuations (Ginzburg criterion) above $H_{c2}$ at low temperatures. The applications of this criterion has its own limitations. The point is that it is valid for classical or thermal fluctuations with energies $\sim T$ larger than minimal frequency of the order parameter fluctuations

$$\frac{H - H_{c2}}{H_{c2}} < \frac{T}{T_c}.$$  

(69)

When temperature decreases, the region of importance of thermal fluctuations (68) shrinks slower ($\propto T^{2/3}$) than the region \[69\] of applicability of Ginzburg criterion. So, at lowest temperatures the formula \[68\] is inapplicable. Let us compare vicinities \[68\] and \[69\] say at

$$T \sim \Gamma \sim \frac{\omega_{c2}}{4\pi} \sim \frac{\pi T_c^2}{2\varepsilon_F^2}.$$  

(70)

For the region of importance of critical fluctuations \[68\] we get

$$\frac{H - H_{c2}}{H_{c2}} < \left(4\pi^4 \ln \frac{\varepsilon_F}{\omega_c}\right)^{2/3} \left(\frac{T_c}{\varepsilon_F}\right)^2$$

(71)

and the thermal fluctuations region \[69\] is limited by

$$\frac{H - H_{c2}}{H_{c2}} < \frac{\pi T_c}{2 \varepsilon_F^2}.$$  

(72)

We see that the region \[71\] is still parametrically more narrow than the region \[72\]. However due to the large numerical factor in \[71\] these regions can be proved of the same order of magnitude.

At lower temperatures we have to calculate the quantum fluctuation contribution. For the problem under discussion this contribution is exponentially small, so one can say that the region of the lowest temperatures is fluctuationless.

Turning back to the inequality \[68\] and taking into account the estimation \[71\] one can conclude that the region of importance of thermal fluctuations at least for temperatures above $T \sim \Gamma$ presented by \[70\] is given by inequality \[68\] that is broader than the period of critical temperature oscillations \[31\]:

$$\frac{\delta H}{H_{c2}} \approx 2\pi^2 \left(\frac{T_c}{\varepsilon_F}\right)^2$$

Thus in this temperature region the oscillations of critical temperature have poor chances for observations. On the other hand at temperatures

$$T < \Gamma < \frac{\omega_{c2}}{4\pi}$$

(73)

we have region free of fluctuations where one can hope to find the quantum oscillations of critical temperature.

7 Conclusion

The quantitative theory presented in the paper demonstrates the suppression of the de Haas van Alphen effect in the superconducting mixed state according the equations \[29\], \[30\]. The suppression is developed in the region below the upper critical field embracing up to hundred oscillations of magnetization \[33\] and limited by the inequality \[37\]. The quantum oscillations of the critical temperature are accessible for observation at very low temperatures \[23\].
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