TOWARDS A SOLUTION OF THE COSMOLOGICAL DOMAIN WALLS PROBLEM

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We show that all kinds of biasing of cosmological phase transitions produce qualitatively new type of domain wall networks. The biased networks consist of compact, finite size, bag-like wall structures and exhibit a generic instability. The surface of biased networks disappears exponentially fast after a limited period of scaling. We argue that fluctuations of the background make the network unstable even in the case of the “symmetric on the average” initial distribution. We observe that the variation in parameters of the potential, like its height, can influence the lifetime of the wall network, contrary to the standard beliefs.

I. Introduction

Topological defects provide cosmologists with a set of intriguing mechanisms for structure formation which are quite different in nature to the standard inflationary paradigm. Defects form at spontaneous symmetry breaking phase transitions in the early universe, and their subsequent field ordering dynamics can perturb the matter and radiation content of the universe, leaving characteristic signals in both the present day matter distribution and the microwave background.

The nature of the defect is determined by the topology of the vacuum manifold following the phase transition. A $\mathbb{Z}_2$ manifold with two disconnected vacua leads to domain walls, an $S^1$ manifold to cosmic strings, an $S^2$ manifold to monopoles, and an $S^3$ manifold to cosmological texture. Much recent attention has focused on the gauged cosmic string, and global texture scenarios. However, cosmological domain walls have long been considered unworkable. Zel’dovich, Kobzarev and Okun noticed in 1975 that the energy density of a domain wall network will eventually come to dominate that of matter or radiation. More recent attempts to save domain wall scenarios, such as the so-called “late-time” phase transitions, are now known to be in conflict with observations of the microwave background.

The most complete study of the dynamics of domain wall networks was given by Press, Ryden and Spergel (PRS). These authors studied the time evolution, during the era of matter domination, of topological domain wall kinks in a scalar field with a potential energy possessing two minima, both degenerate in energy. They showed conclusively that such networks rapidly evolved into long domain walls stretching across the universe whose surface area, and, hence, energy density, persisted for a long time. This persistence, or scaling behavior, led both to the relatively rapid domination of the energy density of the universe by these walls and to large distortions in the cosmic microwave background. Both of these results are incompatible with observations.

It did appear, therefore, that domain walls could not have formed in the early universe. However, these results were based on very special assumptions about the initial conditions of the domain wall network. To be precise, these authors initialized their networks by allowing the computer to randomly choose, at any point on the lattice, either one vacuum state or the other with each vacuum weighted with equal probability. However, if, for some reason, the two vacua were to be given different, or biased, probabilities, then these conclusions could be dramatically altered. Furthermore, it has become clear in recent years that non-equilibrium phase transitions, which can occur in realistic models of the early universe, generically lead to a biased choice of vacuum state. There seems to be every reason then to restudy the case of the walls with the modification that the domain wall network be initialized using biased vacuum probabilities.

II. Biased Transitions

To illustrate the way the biased phase transitions can actually occur we shall discuss in some de-
tail the post-inflationary nonthermal phase transitions in a weakly coupled, out-of-equilibrium scalar field. The inflationary epoch which we presume to precede the radiation dominated (RD) epoch is here the usual large-scale inflation with the Hubble parameter of the order of $10^{14}$ GeV. Each scalar field which lives in the Universe undergoing very rapid, quasi-exponential expansion is subject to large quantum fluctuations at all wavelengths, the overall magnitude of which is set by $<\phi>^2 \approx H_i^2 \delta H_i$, where $\delta$ is the duration of the inflationary epoch and $H_i$ the Hubble parameter during inflation, in massless case, and $<\phi>^2 \approx \frac{H_i^2}{m^2}$ in the case of a field of a mass $m$. More precisely, the magnitude of fluctuations (power spectrum) grows like $\frac{1}{k^2}$, so becomes very large for large wavelength components of the field. The result is, that these large wavelength components correlate the observables measured for the field over distances as large as the blown-up inflationary horizon, while the small short-wavelength components decorrelate locally the measurements, amounting to small-scale inhomogeneities. To see implications this picture has for phase transitions, let us assume that the field has a double-well potential, the height of the barrier between vacua being $V_o$ and the mass of the field - $m$. Then, after inflation, in the RD epoch, when the causal FRW horizon grows larger than $\ell = \frac{1}{m}$, in each causal volume the local field starts rolling towards the nearest minimum of the potential. If there would be no fluctuations, the initial conditions for rolling would be the same in all horizon volumes, and the field would assume the same vacuum state everywhere. However, if the characteristic scale of fluctuations, $H_i$, is at least comparable to the distance between the minima of the potential, there is a significant probability that in randomly chosen volumes the fluctuations have caused different minima to become local vacuum states. Now, whether the probability of choosing one, say (+), vacuum over the other is 1/2 or not, it is decided by the position of the background (long-wavelengths) part of the field with respect to the symmetric point in the field space - the point equally distant from both minima. To decide where the background is located, one should realize that the background itself is also a stochastic field. In fact, let us divide the quantum field, including all the fluctuations born during inflation, into 2 parts with respect to the scale $\ell$: the short-wavelength part $F_\ell$ consisting of fourier components with $H_i^{-1} < \lambda < \ell$ and the long-wavelength part $B_\ell$ with $\ell < \lambda < L$ where $L$ is the blown-up inflationary horizon. With such a division it is clear that $F_\ell$ measures the difference or decorrelation of the field between two points at distance $\ell$ apart, or equivalently between “mean” (in the rms sense) values of the field in two boxes of size $\ell$ each. On the other hand, the field $B_\ell$ measures the common part of the field, which looks like a constant contribution if one moves over distances $\ell$ and smaller. Of course, also $B_\ell$ is in principle a quantum, or - at the level of quasi-classical description - a stochastic field, which can be characterized by means of some probability distribution, or through moments of the distribution, like the mean or standard deviation. In first approximation, we can regard the $B_\ell$ field as a constant classical background whose value is chosen according to a gaussian distribution with the width equal to the rms value of the underlying quantum field

$$\text{rms}(B_\ell) = \frac{H_i}{2\pi} \sqrt{\log\left(\frac{L}{\ell}\right)}$$

Departures from this zeroth-order approximation will be considered in the next section. It is clear now, that the probability that the classical position of the background would correspond to the equal probabilities of populating both vacua is close to zero, and that the choice of the vacuum in a postinflationary transition is in general biased, one of the vacua becomes more likely to be choosen than the other. Of course, in the region interpolating between different vacua a domain wall shall form with a large probability. The physics of so formed biased networks is summarized in the following section.

III. Evolution of the Network

The detailed procedure which has been used to perform numerical simulations, and a simple an-
alytical model which can be used to understand main qualitative features of the result have been described in reference. The focus here is to summarize the evolution of the surface energy density of the network of domain walls in the radiation dominated epoch (the results in the matter dominated epoch are qualitatively identical).

Two Dimensional Simulations. In this case we have studied the evolution of $A/V$ as a function of elapsed conformal time, $\eta$, for a number of initial bias probabilities between $p = 0.5$ and 0.4. Each simulation ran on a $1024 \times 1024$ lattice until either no more domain walls were found in the box, or $\eta$ exceeded 512 ($= L/2$).

For the $p = 0.5$ case we recover the scaling properties reported in PRS. Fitting the scaling portion (10 $< \eta < 100$) of the curve to the power law

$$A/V \propto \eta^\nu,$$

we find $\nu = -0.88 \pm 0.04$.

Moving away from the $p = 0.5$ case, one sees a dramatic departure from self-similar scaling. In each case there is an exponential cut-off in the ratio $A/V$ at some characteristic time. For the cases of $p$ close to 1/2, that is for $p = 0.457$ and 0.45, we find that the curves are well fitted by a function of the form

$$A/V \propto \eta^\nu e^{\eta/\bar{\eta}}.$$  

However, for the cases of $p$ near $1 - p_c = 0.407$, that is, $p = 0.425$ and 0.4, a simple exponential suffices:

$$A/V \propto e^{-\eta/\bar{\eta}}.$$  

Values for $\nu$ and $\bar{\eta}$, averaged over 5 runs for each value of $p$, are given in Table I.

To conclude, in the two-dimensional simulations we see persistent scaling behavior precisely at $p = 0.5$. For $p$ below 0.5 but above the critical threshold $1 - p_c = 0.407$, we see scaling for a finite time which is then exponentially cut-off at some conformal time $\bar{\eta}$. The value of $\bar{\eta}$, which becomes very large as $p \rightarrow 0.5$, decreases rapidly as $p$ approaches the critical threshold. Near $p = 0.407$, and below it, no scaling behavior is seen and the behavior is well described by a simple exponential for all conformal time.

| $p$   | $\nu$ | $\bar{\eta}$ |
|-------|-------|--------------|
| 0.5   | -0.88 | -             |
| 0.475 | -0.6  | 22.4         |
| 0.45  | -0.68 | 7.8          |
| 0.425 | -     | 3.2          |
| 0.4   | -     | 2.2          |

TABLE I: Fits to the plots of $A/V$ against $\eta$ for different initial bias probabilities, $p$, in two dimensions, using the functional forms (2) and (3) given in the text.

Three Dimensional Simulations. The three dimensional simulations are run on a $128^3$ grid. Again, the self-similar evolution seen in PRS for the $p = 0.5$ case is reproduced well in the time range $2u_0 < \eta < L/2$. Measuring the logarithmic slope of $A/V$ versus $\eta$ between these times we find $\bar{\nu} = -0.89 \pm 0.06$.

With $p \neq 0.5$, we see qualitative features in the three dimensional runs similar to those of the two dimensional cases. The one major difference is that the turn-over into an exponential decay is seen to occur much earlier, so much so in fact that all the cases considered here ($p = 0.47, 0.48, 0.49$) are well fitted by a simple exponential curve with no initial pseudo-scaling regime. Using a fitting function of the form of equation (4), the values of the $\bar{\eta}$ for each $p$ are given in Table II.

| $p$ | $\bar{\eta}$ |
|-----|---------------|
| 0.49| 5.4           |
| 0.48| 2.8           |
| 0.47| 1.8           |

TABLE II: Fits to the plots of $A/V$ against $\eta$ for different initial bias probabilities, $p$, in three dimensions using the functional form (4) given in the text.
To conclude, in the three dimensional case we see persistent scaling behavior precisely at $p = 0.5$. For any value of $p$ below 0.5, little or no scaling behavior is seen and the behavior is well described by a simple exponential for all conformal time. 

*Background Fluctuation* 

The new result which we want to discuss here is the influence of the fluctuations of the background on the behavior of the network. In the simulations described so far the background value of the stochastic field was assumed to be a constant randomly located in the field space (the freedom of choosing this background value is reflected in the freedom of choosing the bias parameter $p$). However, we know from the previous section that in fact the background is not strictly homogeneous, as there must be fluctuations around the zeroth-order approximation constant background value. In the simulations whose results are depicted in Figure 1 we are simulating the fluctuations of the background through fluctuations of the probability $p$. At each lattice site we draw $p$ according to the gaussian distribution with some central value $p = p_0$ and the width equal to $\sigma = 0.25p_0$. As one can see from the plot, this has a dramatic impact on the behavior of the equilibrium network with previously constant value $p = p_0 = 0.5$. Now, with fluctuating $p$, the network becomes unstable, and its surface again decays quasi-exponentially. 

Intuitively one can understand this result noticing, that the average value of the probability over a randomly chosen small subvolume of the lattice is typically different from $p_0 = 0.5$, hence each small part of the network behaves like a biased network, and as these subnetworks are unstable, averaging over many subnetworks cannot produce the equilibrium behavior, despite the fact that the sub-probabilities average to $p_0 = 0.5$.

The other case shown in the picture (dotted curves, $p = 0.3$) illustrates the general tendency of biased networks to persist for a longer period of time (slower decay rate) when the probability (in fact - the background field configuration) fluctuates between different lattice sites. The fluctuating $p = 0.5$ case shows that in reality all the networks formed during actual cosmic transitions will be biased, hence after a finite scaling period, typically rather short, will disappear dissipating their surface energy in the form of scalar (maybe also gravitational) waves.

**Wall Thickness** 

The next observation which can have some significance for the problem of the possible impact of biased networks on structure formation is that the values of $\nu$ and $\bar{\eta}$ were found to be weakly dependent on the value of $w_0$, the domain wall thickness. Changing the wall thickness affects the network evolution through two closely balanced effects. Increasing the height of the potential barrier between the two vacua, $V_0 = \frac{1}{2} \frac{\pi^2}{w_0}$, makes transition from one vacuum to the other energetically less favorable. But, this is compensated somewhat, not necessarily exactly, by the increased domain wall surface tension, $\sigma = \frac{1}{2} \frac{\pi^2}{w_0}$. However, for a quartic potential, the value of $V_0$ clearly does not scale out of the equations of motion, so this balance need not be exact. (In the domain wall scenario, the scalar field is off the vacuum manifold at many points throughout space, making the evolution more dependent on the relative values of $V_0$ and $\phi_0$ than, say, a texture scenario, where the non-linear sigma model works well, and evolution is effectively independent of the potential barrier between vacua.) This is illustrated in the Figure 2.
IV. Conclusions

From the presented results it becomes clear that biased transitions solve the domain walls problem very efficiently and naturally due to the generic instability of the network with respect to the departure from \( p = 0.5 \) and with respect to fluctuations of the background.

The interesting and important for structure formation problem which remains to be clarified is whether there is a possibility of having unstable networks with scaling periods long enough to show up on the \( \frac{\Delta \rho}{\rho} \) curve. This would be an interesting possibility in the context of hot dark matter scenarios. The other point which has to be considered more detaily is interaction of biased networks with matter.

Recently we became aware of the work of S. Larsson, S. Sarkar and P. White, which confirms and extends our investigations of biased networks.

Acknowledgments

This work has been supported in part by Polish Committee for Scientific Research – KBN Grant, and by the EEC under the “Flavourdynamics” Network.

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