In this paper a new theory of Dark Matter is proposed. Experimental analysis of several Galaxies show how the non-gravitational contribution to galactic Velocity Rotation Curves can be interpreted as that due to the Cosmological Constant $\Lambda$. The experimentally determined values for $\Lambda$ are found to be consistent with those expected from Cosmological Constraints. The Cosmological Constant is interpreted as leading to a constant energy density which in turn can be used to partly address the energy deficit problem (Dark Energy) of the Universe. The work presented here leads to the conclusion that the Cosmological Constant is negative and that the universe is de-accelerating. This is in clear contradiction to the Type Ia Supernovae results which support an accelerating universe.
1 Dark Matter

The problem of missing or Dark Matter, namely that there is insufficient material in the form of stars to hold galaxies and clusters together, has been known since the pioneering work of Bessel, Zwicky and most recently Rubin [1].

The existence of non-luminous Dark Matter was first inferred in 1984 by Fredrich Bessel from gravitational effects on positional measurements of Sirius and Procyon. In 1933, Zwicky concluded that the velocity dispersion in Rich Clusters of galaxies required 10 to 100 times more mass to keep them bound than could be accounted for by luminous galaxies themselves.

Finally, Trimble [2] noted that the majority of galactic rotation curves, at large radii, remain flat or even rise well outside the radius of the luminous astronomical object.

The missing Dark Matter has been traditionally explained in terms of Dark Matter Halo’s [3], although none of the Dark Matter Halo models have been very successful in explaining the experimental data [4, 5].

This paper will describe the missing matter (Dark Energy) in terms of a Cosmological Constant which leads to a constant energy density.

The experimental determination of galactic velocity rotation curves (VRC) has been one of the most important approaches used to estimate the "local" mass (energy) density of the Universe. Several sets of data from VRC’s will be analysed and the contribution due to the Cosmogical Constant determined.

2 Constraints on the value of the Cosmological Constant

It is interesting to estimate the allowed range of values for the Cosmological Constant within the constraints of General Relativity and observational astronomy, (for a comprehensive review, see Bahcall et.al. [6]).

Starting from a General Relativity point of view, the Friedman energy equation is given by:

\[
1 = \frac{8\pi G_N \rho_{\text{matter}}}{3 H^2} - \frac{kc^2}{R^2 H^2} + \frac{c^2\Lambda}{3H^2},
\]

where the Hubble Constant is denoted by \( H \), the curvature term by \( k \) and \( G_N \) denotes...
the Newton gravitational constant. Eq.(1) can be rewritten as
\[ 1 = \Omega_m + \Omega_k + \Omega_\Lambda \] (2)
Here the relative contributions to the energy density of the universe are given by, the mass, curvature and Cosmological Constant.
If we assume that the curvature contribution is small:
\[ 1 = \Omega_{\text{Matter}} + \Omega_\Lambda \] (3)

In order to satisfy equation (3), it was surprising to discover that only a narrow range of values for the observed Cosmological Parameters were allowed. A ”reasonable” set of parameters consistent with observation are:
\[ H_0 = 100Kms^{-1}Mpc^{-1}, \rho_{\text{matter}} = 5 \times 10^{-30}gcm^{-3}, \frac{\Omega_\Lambda}{\Omega_{\text{matter}}} = 4.3 \] (4)

and \( \Omega_{\text{Matter}} + \Omega_\Lambda = 1.4 \) (here we assume a small value for the curvature \( \sim 0.4 \).
(For an authoritative review of the matter/energy sources of the universe, see Turner [7]).

It was found that observational constraints placed upon the range of values for the cosmological parameters lead to a surprisingly narrow range of possible values for the Cosmological Constant, the range being given by:
\[ 10^{-56} < |\Lambda| < 5 \times 10^{-55}cm^{-2}. \] (5)

3 Experimental Results
It was shown [10], within the Weak Field Approximation, that the Cosmological Constant at large radii could be determined from galactic velocity rotation curves. This contribution is given by:
\[ v_\Lambda^2(r) = v_{\text{obs}}^2(r) - v_{\text{mass}}^2(r), \text{ leading to,} \]
\[ v_\Lambda^2/r = \frac{c^2\Lambda r}{3}, \text{ at large } r \] (6)
The results obtained by this analysis are shown in Table 1.

The experimental values obtained for the Cosmological Constant fall within the range determined from General Relativity and observational constraints. While the initial results are promising, a thorough and systematic analysis of galactic rotation curves needs to be undertaken in order to confirm the trend.

Previous results [10] reported for the value of the Cosmological Constant were 100 to 1000 times the "allowed value". This systematic error arose for two main reasons: the first by not taking the gradient of the curves at sufficiently large radii and the second by the lack of access to "real" experimental data leading to crude data analysis.

The results presented in this paper suffer from the second problem, i.e. all the gradients were obtained from the data in the published literature and not from raw experimental data i.e. M33 Corbelli & Salucci [4], NGC 3198 [8] and all others from [12].

However experience has taught us that a cursory look at rotation curves will determine which galaxies are candidates for explanation by a Cosmological Constant and which are not. Galaxies where the velocity rotation curve remains flat or rises at large radii, are immediate candidates. NGC 3198 is a good example, whereas others such as M33 [4] has clearly not relaxed to the Cosmological background, even at many times the galactic radii. A full explanation for M33 has to be sought in a different direction.

Finally, a simple calculation of the effective mass density due to the Cosmological Constant in NGC 3198, 

$$\rho_{\text{eff}} = -\frac{c^2 \Lambda}{4\pi G_N}$$  \hspace{1cm} (8)$$
leads to a value of $5.4 \times 10^{-29} \text{gcm}^{-3}$ which is comparable to the HI mass density [13] at the outer disk of NGC 3198 galaxy. This is further confirmation that the
Cosmological Constant effect can be seen at galactic scale lengths.

4 Accelerating or Decelerating Universe?

Recently there has been great interest in the Type Ia Supernovae results of Perlmutter et al \[11\] which suggest that the universe is accelerating.

In this section we will show that the Weak Field Approximation coupled with galactic velocity rotation curve data inevitably lead to a negative Cosmological Constant.

The equation for the VRC is given \[10\] by

\[
- \frac{v^2}{r} = - \frac{Gm}{r^2} + \frac{c^2 \Lambda}{3} r
\]  

(9)

We note that Eq.\( (9)\) is only strictly true for small and large radii, however it will serve to illustrate our arguments.

Using the Newtonian limit of Einstein field equations we derived equation (9). It is important to realize that the Cosmological Constant obeys the equation of state given by,

\[
P_\Lambda = -c^2 \rho_\Lambda,
\]  

(10)

where \(P_\Lambda\) is the pressure term due to \( \Lambda \). Taking the Newtonian limit in the absence of matter, \(T_{\mu\nu} = 0\), the differential equation for the static Newtonian potential becomes

\[
\nabla^2 \Phi = -c^2 \Lambda
\]  

(11)

leading to,

\[
\rho_{\text{eff}} = \rho_\Lambda + \frac{3P_\Lambda}{c^2} = -2\rho_\Lambda
\]  

(12)

If we arbitrary set \(\Phi = 0\) at the origin, then in spherical coordinates \([14]\) has the solution \(\Phi = -\frac{c^2 \Lambda}{6} r^2\) \([14]\). Thus, the Cosmological Constant leads to the following correction to the Newtonian potential

\[
\Phi = - \frac{Gm}{r} - \frac{c^2 \Lambda}{6} r^2
\]  

(13)

At small galactic radii the velocity versus radius contribution is well known and follows Newtonian physics. For large radii a negative Cosmological Constant gives a positive contribution to the VRC which is what is actually observed. On the other hand the effect of a positive Cosmological Constant would be to lower the rotation

\footnote{In Ref.\([14]\) eq.(15), there was a typographical sign error for one of the terms and also the negative pressure effect associated with \( \Lambda \) was not fully appreciated.}
curve below that due to matter alone.

The above simple argument, based on observational astronomy, allows only a negative Cosmological Constant as a possible explanation for the galactic velocity rotation curve data. This is in clear disagreement with the Type Ia supernovae results [11]. However, given the uncertainties in the determination of the deceleration parameter, $q_0$, derived from supernovae data [11] the approach outlined above has certain merits worth consideration.

In summary these are, the Cosmological Constant is determined from direct measurement unlike the Supernovae results, the experimentally determined value is the correct order of magnitude as that required from cosmological constraints, and finally a negative Cosmological Constant is consistent, and indeed a natural physical explanation, for the observed galactic velocity rotation curve data.

Finally, observations of global clusters of stars constrain the age of the universe and consequently place an observational limit on a negative Cosmological Constant [16] of

$$|\Lambda| \leq 2.2 \times 10^{-56} \, \text{cm}^{-2}. $$

(14)

Note, the Cosmological Constant derived from global cluster constraints is in agreement with the experimentally determined value derived from galactic velocity rotation curve data.

4.1 Experimental Tests-Dark Matter Halo vs Cosmological Constant

It would be of some interest if it was possible to experimentally distinguish between the contribution of Dark Matter Halo’s and Dark Energy (Cosmological Constant) to galactic rotation curves.

We know that Dark Matter predicts a variation of mass at large radii given by [17],

$$M_{DM}(r) \propto r$$

(15)

while for Dark Energy due to a Cosmological Constant,

$$M_\Lambda(r) \propto r^3[\rho_\Lambda + (3P_\Lambda/c^2)].$$

(16)

With these different types of predictive variations it should be possible to design experimental tests to distinguish between the two phenomena.
5 Quark Hadron Phase transition

In this section which is of more speculative nature, working within the Extended Large Number Hypothesis, and using the experimentally determined Cosmological Constant, we will demonstrate how the energy density for the Quark - Hadron can be estimated.

However, it is useful to put into context the significance of the Cosmological Constant for many seemingly disparate branches of Physics. The figure 1 below shows the Cosmological Constant at the epicentre of Physics.

The diagram demonstrates a dichotomy whereby several branches of Physics need a non-zero Cosmological Constant in order to explain key physical phenomena, whilst in others a non-zero value presents a fundamental problem.

![Diagram showing Lambda at the epicentre of Physics](image)

Figure 1: $\Lambda$ at the epicentre of Physics

It is also noted here that while fundamental theories of Particle Physics such as the Standard Model, Quantum Field Theory and String Theory have many major predictive successes they all have problem with a high vacuum energy density. On the other hand while the Extended LNH is formulated from a naive theory it appears to correctly predict the correct vacuum energy density and other cosmological parameters. The Extended LNH relates the value of the Cosmological Constant to...
the effective mass given by:

\[ |\Lambda| = \frac{G_N^2 m_{eff}^6}{\hbar^4} = \frac{e^6 L_s^4}{\hbar^6} m_{eff}^6 \quad (17) \]

Matthews [18] pointed out that when using the Extended LNH to determine today’s cosmological parameters, the mass of the proton originally suggested by Dirac should be replaced by the energy density of the last phase transition of the Universe: Quark - Hadron.

Note that in equation (17) there are no free parameters, \( L_s \) is normalised to the gravitational constant and corresponds to the fundamental length of String Theory.

Using equation (17) and the Cosmological Constant determined from NGC 5033, the effective mass is given by

\[ m_{\text{Effective}} = 332 \text{MeV} \quad (18) \]

We will associate this value with the Quark - Hadron phase transition energy. (Other experimentally determined Cosmological Constant data give \( m_{QH} \) in the range 295 - 410 MeV). The experimentally determined value within the LNH predicts the correct order of magnitude for the phase transition.

The above result poses the question that it might be possible to gain insights on the quantum mechanical origin of the Universe, as Dirac [19, 20, 21] suggested, from direct observation of the present day Universe.

Finally, does the Cosmological Constant provide the key to the integration of the various Physics disciplines as Figure 1 suggests?

\section{Discussion}

Analysis of the galactic rotation curves show that the missing Galactic Dark Matter can be explained in terms of a Cosmological Constant.

This contribution can be considered a prime candidate for the "Dark Energy" which is smoothly distributed throughout space, and contributes approximately 70\% to the mass/energy of the Universe [7].

However, in order to support this thesis for the Cosmological Constant, thorough and systematic analysis of galactic velocity rotation data needs to take place.
It was shown how, within the Weak Field Approximation, that VRC data inevitably lead to a negative value for the Cosmological Constant in direct disagreement with the type Ia Supernovae data. Nevertheless, given the uncertainties in determining the deceleration parameter $q_0$ \[^{[3]}\], from the redshift-magnitude Hubble diagram using Type Ia supernovae as standard candles, we believe our approach is worth further consideration.

The experimental values determined for the Cosmological Constant are shown to lie within an acceptable range. These values, used within the Extended Large Number Hypothesis, predict values for the Quark-Hadron phase transition energy in the range 295-410 MeV.

It would be remarkable, if proved correct, that the Cosmological Constant could be directly determined from the analysis of galactic velocity rotation curves.

Equally remarkable, if proved correct, is the idea that astronomical observations can shed light on the last quantum mechanical phase transition of the Universe, namely the Quark-Hadron.

7 Acknowledgements

We would like to thank Paolo Salucci for invaluable discussions on Cosmological and Astronomical aspects related to this work and Alexander Love for useful comments on the manuscript. We also thank J. Hargreaves and D. Bailin for suggestions and useful discussions.

We also would like to thank Deja Whitehouse for proof reading this document.

George Kraniotis was supported for this work by PPARC.

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