R parity violating contribution to $e^+ e^- (\mu^+ \mu^-) \rightarrow t \bar{c}$

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Abstract

In this article we consider the contribution of $R_p$ violating couplings to the process $e^+ e^- (\mu^+ \mu^-) \rightarrow t \bar{c}$ at high energy lepton collider. We show that the present upper bound on the relevant $R_p$ violating couplings obtained from low energy measurements would produce a few hundred to a thousand top-charm events at the next linear $e^+ e^- (\mu^+ \mu^-)$ collider. Hence, it should be possible to observe the rare process at future lepton collider.
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In the Standard Model (SM) flavor changing neutral current (FCNC) processes are suppressed since they are forbidden at the tree level because of the well known GIM mechanism [1]. The two higgs doublet extension of the SM usually incorporates a discrete symmetry [2] to guarantee tree level natural flavor conservation. In the minimal supersymmetric standard model (MSSM) [3] there are two higgs doublets, however supersymmetry forces one of them to couple to $U^c_L$ and the other to $D^c_L$ which forbids tree level FCNC processes. Some FCNC processes like $b \to s \gamma$ [4] get small but measurable contributions from heavy fermions in the loop both in the SM and the MSSM. A similar enhancement however is not expected in the case of top-charm transition. If MSSM is extended by the addition of R-parity ($R_p$) violating interactions [5-7] which violate either B or L but not both then flavor changing neutral current processes can occur at the tree level. In fact, $R_p$ violating couplings because of its complex flavor structure opens up the possibility of many FCNC processes at the tree level. The effect of $R_p$ violating couplings on some of these FCNC processes have already been considered in the literature [8,9,10,11]. The FCNC process involved in top-charm production is of special interest because top quark is the heaviest of all fermions and therefore flavor symmetry violation is expected to be maximum for this process. Recently, the authors of Ref.[10] have studied the effects of $R_p$ violating couplings on top-charm production at hadron collider. The relatively clean environment of a lepton collider compared to a hadron collider will make the detection of the signal for the rare process easier. In this article we shall consider the contribution of the L and $R_p$ violating interaction $L_I = -\lambda'_{ijk}(\bar{d}^k_R)(\bar{e}^j_L)e^i_L + h.c.$ to the process $e^+e^- (\mu^+\mu^-) \to t\bar{c}$ via the exchange of down squarks in the u channel. In the SM the process $e^+e^- (\mu^+\mu^-) \to t\bar{c}$ proceeds at one loop level via $e^+e^- (\mu^+\mu^-) \to \gamma^*, Z^* \to t\bar{c}$.
The effective (off shell) $\gamma t\bar{c}$ and $Zt\bar{c}$ vertices can be evaluated to one loop and used in the calculation. The production rate for this rare process has been calculated in the framework of SM and is predicted to be extremely small [12]. One reason being in SM $t\bar{c}$ production, unlike $b\bar{s}$ production, does not get a large contribution from heavy fermion in the loop. The MSSM contribution to the branching ratio for $e^+e^- \to t\bar{c}$ has also been calculated [13] and has been shown to be small compared to that from the SM. Hence any experimental detection of this process beyond the SM prediction will therefore point to the existence of new physics other than MSSM.

The above $R_\rho$ violating interaction has been written in the quark mass basis. It should however be noted that the squark and quark mass matrices are not diagonal in the same basis [14]. Let $\tilde{D}$ be the 6×6 mass matrix that diagonalizes the down squark mass matrix $M^2_d$ and $G_{ik}$ be the 6×3 matrix that relates the weak gauge eigenstates $\tilde{d}^k_R$ to the mass eigenstates $\tilde{d}^m$ ($\tilde{d}_R^k = \tilde{d}_R^m G_{mk}$, $m=1-6$ and $k=1-3$). The transition matrix for the process $e^+e^- \to t\bar{c}$ at the tree level is given by

$$M = \sum_{k,l,n} \frac{\lambda'_{12k} \lambda'_{13l}}{2(u - m^2_{\tilde{d}_n})} G_{nl} G_{nk}^* \bar{u}_{tL}(p_1') \gamma_\mu v_{eL}(p_2') \bar{v}_{eL}(p_2) \gamma^\mu u_{eL}(p_1).$$

(1)

$u_{eL}(p_1)$ and $\bar{v}_{eL}(p_2)$ are the incoming spinors for $e^-$ and $e^+$. $\bar{u}_{tL}(p_1')$ and $v_{eL}(p'_2)$ are the outgoing spinors for t and $\bar{c}$. $m_{\tilde{d}_n}$ is a generic mass of the down squarks and $u = (p_1 - p_2)^2 = (p'_1 - p_2)^2$.

If we make the simlifying assumption that the down squark mass eigenstates are degenerate then the expression for M reduces to

$$M = \sum_k \frac{\lambda'_{12k} \lambda'_{13k}}{2(u - m^2_{\tilde{d}_n})} \bar{u}_{tL}(p_1') \gamma_\mu v_{eL}(p_2') \bar{v}_{eL}(p_2) \gamma^\mu u_{eL}(p_1).$$

(2)

This assumption is however is not very realistic since one of the down squark mass eigenstates usually tends to be much lighter than the others because
of the large radiative corrections to the down squark mass matrix induced by the top quark. However the upper bounds on $R_p$ violating couplings has been derived in Ref. 8 assuming that the down squarks are degenerate and we shall therefore stick to it in the present work. The determination of the bounds on $R_p$ violating couplings from low energy experiments, in the case where the down squarks are not degenerate and there is considerable mixing between them, is rather complicated.

We shall assume that the operating CM energy $\sqrt{s}$ of the next linear $e^+e^-$ collider is large enough ($\sqrt{s}=500$ GeV or 1000 GeV) so that we can ignore the mass of incoming $e^+, e^-$ and outgoing $\bar{c}$. The square of the invariant transition matrix element for Left-handed (LH) $e^-$ incident on unpolarized $e^+$ is given by

$$|M|^2 = 2 N_c \frac{(\lambda'_{12k} \lambda'_{13k})(\lambda'_{12l} \lambda'_{13l})}{(u - m_d^2)^2} (p_1.p'_2)(p'_1.p_2) \quad (3)$$

In the above the repeated indices $k$ and $l$ are assumed to be summed over. For $\sqrt{s} \gg m_c, m_e$ we can make use of the approximate relation $(p_1.p'_2)(p'_1.p_2) \approx \frac{u(u - m_c^2)}{4}$. Note that the numerator does not contain any term linear in the incoming or outgoing fermion masses. Such terms can only arise from the interference between two currents of opposite chirality which is absent in our case. The differential scattering cross-section for LH $e^-$ incident on unpolarized $e^+$ turns out to be

$$\frac{d\sigma}{dt} = \frac{N_c}{32\pi s^2} |\lambda'_{12k} \lambda'_{13k}|^2 \frac{u(u - m_t^2)}{(u - m_d^2)^2} \quad (4)$$

where $t = -\frac{1}{2}(s - m_t^2)(1 - \cos \theta)$ and $u = -\frac{1}{2}(s - m_t^2)(1 + \cos \theta)$. $\theta$ is the angle between the incoming $e^-$ and outgoing $t$ quark in the CM frame. The repeated index $k$ is assumed to be summed over. Integrating over all angles
the total cross-section for $e^+e^- \rightarrow t\bar{c}$ is given by

$$
\sigma(s) = \frac{N_c}{32\pi s^2} |\lambda'_{12k}\lambda_{13k}^*|^2 [(s - 2m_t^2) + \frac{m_d^2}{s + m_d^2 - m_t^2} - (2m_d^2 - m_t^2) \ln \frac{s + m_d^2 - m_t^2}{m_d^2}].
$$

(5)

The most stringent bound on $|\lambda'_{12k}\lambda_{13k}^*|^2$ follows from low energy measurements of $K^+ \rightarrow \pi^+\nu\bar{\nu}$ [15], $b \rightarrow s\nu\bar{\nu}$ [15] and $\nu_e$ mass [16]. The bound obtained in this way depends on the value of $m_d$. For $m_d \approx 200$ GeV the upper bound [8] is given by $|\lambda'_{12k}\lambda_{13k}^*|^2 \leq 3.4 \times 10^{-4}$. The bound ($|\lambda'_{12k}\lambda_{13k}^*|^2 \leq 0.0512$) that follows from flavor conserving processes like $A_{FB}$ and atomic parity violation is much weaker [5,17]. For $m_d \approx 200$ GeV, $m_t \approx 175$ GeV and $N_c \approx 3$ the cross-section $\sigma(e^+e^- \rightarrow t\bar{c})$ turns out to be 8.77 (3.32) fb at $\sqrt{s} = 500(1000)$ GeV. With an integrated luminosity of 50 (300) fb$^{-1}$ [18] per year at $\sqrt{s} = 500(1000)$ GeV and a detection efficiency of 70% about 310 (700) $t\bar{c}$ events are expected at an $e^+e^-$ collider. The effect of QCD corrections to leading order will increase the cross-section by a factor of $(1 + c\frac{\alpha(s)}{\pi})$ where $c$ is a number of order unity. However since $\frac{\alpha(s)}{\pi}$ is rather small for $\sqrt{s} \approx 500(1000)$ GeV the effect of QCD corrections will be almost inappreciable. At LEP2 where unpolarized $e^+$ and $e^-$ beams will collide at $\sqrt{s} = 200$ GeV the cross-section for $e^+e^- \rightarrow t\bar{c}$ is .9 fb which is only one tenth of its value at $\sqrt{s} = 500$ GeV. In fig. 1 we have plotted the cross-section for the process $e^+e^- \rightarrow t\bar{c}$ as a function of $m_d$ for $\sqrt{s} = 500$ GeV and 1000 GeV, keeping $|\lambda'_{12k}\lambda_{13k}^*|^2$ fixed at $3.4 \times 10^{-4}$. We find that even for $m_d \approx 600$ GeV the cross-section is large enough to produce about 35 $t\bar{c}$ events at $\sqrt{s} = 500$ GeV.

A high energy $\mu^+\mu^-$ collider would also provide a relatively clean environment to look for flavor changing top-charm events. For $m_d \approx 200$ GeV the
upper bound on $|\lambda'_{22k}\lambda'_{23k}|^2$ is given by $7.5 \times 10^{-4}$. Hence a $\mu^+\mu^-$ linear collider operating at $\sqrt{s} = 500(1000)$ GeV with an integrated luminosity of 50 (300) fb$^{-1}$ [19] and a 70% detection efficiency is expected to produce about 665 (1480) $t\bar{c}$ events. Note that although the process $e^+e^- (\mu^+\mu^-) \to t\bar{c}$ has been analysed here from the standpoint of $R_p$ violating interactions the same could also arise from flavor violating leptoquark interactions. Top charm production at a high energy $e^+e^-$ or $\mu^+\mu^-$ collider has several phenomenological vantage points. First the experimental signature of the final state is extremely clean, a fat jet recoiling against a relatively thin jet. Second the relatively clean environment of a lepton collider causes the background to be small and enhances the detection efficiency of the signal. The upshot of this discussion is that the present low energy bounds on $R_p$ violating couplings would produce a sufficient number of top charm events at a high energy lepton collider that could be easily detected.

A null result on top-charm production would be somewhat discouraging since we would be losing good opportunity to learn about flavour changing neutral current events in the up-type quark sector. However, it could be used to derive an upper bound on $|\lambda'_{12k}\lambda'_{13k}|^2$. A precise estimation of the bound depends crucially on the possible background events. Consider the decay of the $t\bar{c}$ system: $t + \bar{c} \to W + b + \bar{c}$. Now the W boson can decay in the leptonic mode through $W \to l + \nu$ or in the hadronic channel as $W \to q + \bar{q}'$. For the charged lepton mode, the signal contains a high energy charged lepton, a large missing energy from the $\nu$, a b-jet and a c-jet. The b-jet can be tagged in the final state but the c-jet can be mimicked in the light quarks. On the other hand, for the hadronic channel, the signal contains 4-jet events. The same signal can also be produced through $e^+e^- \to W^+W^-$ followed by the decay of one $W$ in the leptonic mode and the other in the hadronic mode.
Alternatively, both the W's can decay in the hadronic mode. The \(W^+W^-\) production cross-section [20] is 7.38(2.83) pb at \(\sqrt{s} = 500(1000)\) GeV and the branching ratio of \(W^- \rightarrow b\bar{c}\) is \(\sim .0007\). Hence, the background events arising out of \(W^+W^-\) production will be highly suppressed compared to the signal. On the contrary, the other possible source of background arising from \(e^+e^- \rightarrow b\bar{b}\) followed by the decay \(b \rightarrow c\ell\nu\) has a quite large cross-section \(\sim 0.1\) pb (.01 pb) at \(\sqrt{s} = 500(1000)\) GeV. With a luminosity of 50(300) \(fb^{-1}\) at \(\sqrt{s} = 500(1000)\) GeV the signal to background ratio is \(\frac{S}{\sqrt{S+B}} \approx 1.6(4.5)\). The energy of the final state charged lepton produced from the decay of b-quarks is much less \((m_b - m_c)\) than the lepton produced from the decay of W boson \((\sim m_W)\). By applying suitable energy cuts on the final state charged lepton it is possible to isolate the signal from the background.

In conclusion in this work we have shown that the present low energy bound on \(R_p\) violating couplings would produce a few hundred to a thousand top-charm events at the next linear \(e^+e^- (\mu^+\mu^-)\) collider. Given the clear environment of lepton collider it should be possible to observe this rare FCNC process.
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Figure Captions

Fig 1. Cross-section for $e^+e^- \rightarrow t\bar{c}$ plotted against $m_{\bar{d}}$ at $\sqrt{s} = 500$ (1000) GeV.
$\sqrt{s} = 500$ GeV

$\sqrt{s} = 1000$ GeV

Fig. 1