Stabilization study of a non-Linear self-regression model using Linear Approximation Technique

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Abstract. In this paper, stability in a polynomial model called a polynomial self-regression model with trigonometric boundaries was studied. A linear approximation technique was used, and the Box-Genghis method was used in data analysis, and time series modeling of numbers of people with media. Data were obtained from AL-Yarmouk Hospital in Baghdad AL-Karkh for the period (2011-2016) shows the numbers of patients with the disease through the technique of Linear approximation. Then we found the single point and the stability of the final cycle, and we tried to apply the results that we obtained on the correct data for audit.

Keywords. stability, dynamic, single, limit cycle

1. Introduction

The subject of time series is one of the important topics through which most sciences can be entered in the way of medical, natural, biological sciences, human development sociology and many other sciences. Time series analysis methods are among the most used statistical methods, where any phenomenon can be analyzed for periods. Time is equal and not equal. The high interest in the subject of time series is due to the urgent need for a reliable and highly effective forecasting system So that it can be relied upon to explain many phenomena in different Journals [4]. Our research focuses on studying the stability of a proposed model Polynomial called the polynomial self-regression model polynomial autoregressive model with trigonometric terms (PAR). This model usually has cyclic nonlinear behavior and limit cycles. And we will try to find the traits of the dross or the stability of the singular point Using the method of linear approximation for the proposed nonlinear model and some other statistical characteristics.

2. Basic concepts and principles

2.1 Time Series

time series is a collection of data a recorded for particular phenomenon during a specific period that was economical, social, or statistical, and arranged sequentially according to time. Usually, the periods between the observations anal the other are equal.

A mathematical time series defines a sequence of random variables, defined within the probability space. Multivariate and index t, which belongs to the T group and denotes the time series usually \( \{x(t); -\infty < t < \infty, t \in T\} \) or abbreviated \( \{x(t)\} \).
If it takes continuous values then the time series It is called "Continuous-time" if it takes intermittent value \( t = \alpha, \mp 1, \mp 2, \ldots \) then the series is called "Time-Discrete " symbolizes the god with the symbol \( \{x_t; t = \alpha, \mp 1, \mp 2, \ldots \} \).

2.2 The polynomial model
To be \{y(t)\} Time series, \{z_t\} White annoyances . Assuming that \( f(0) \) Into the equation:
\[
y(t)=f(y(t-1),y(t-p),z(t-1),\ldots,z(t-p))+z_t
\]
A \( p \)-degree polynomial We will get a polynomial self-regression model and have the following formula:
\[
y(t) = p(t-1),\ldots,y(t-p),z(t-1),\ldots,(t-q)) +z(t)
\]
If that \( p(0) \) is degree polynomial and \{z(t)\} White annoyances from a degree q.[7]

2.3 Exponential Autoregressive model
In [9], the Japanese researcher Ozaki defined the exponential regression model from the rank \( P \) as follows:
\[
x_t = \sum_{i=1}^{p} (\alpha_i + \pi e^{-t^2})x_{t-i} + Z_t
\]
Where \{\[Z_t\}\} is an independent and uniform distribution of white inconvenience, and \( i = 1,2,\ldots,p \) and \( \alpha_i, \pi \) Constant quantities represent model coefficients.

2.4 polynomial autoregressive model with trigonometric
This proposed model is a polynomial Self-regression model with the terms of trigonometric functions with the following form:
\[
x_t = \sum_{i=1}^{p} \alpha_i \cos\left(\frac{\pi}{2} x_{t-k}\right)^t + Z_t ; k = 1,2,3,\ldots
\]
Where, \{\[x_t\]\} is a time series, \( \alpha_i \) is constant, \( Z_t \) white inconveniences, and \( t = \alpha, \mp 1, \mp 2, \ldots \)

3. The theoretical Side
3.1 Nonlinear Dynamical System
In dynamic systems covered by non-Linear differential equations, they have often used Approximation methods for converting it into a system of linear differential equations, and there are two methods of approximation that depend on the structure. The physical system of the kinematic system and these two methods are approximation by wise cutting and approximation to the local sin. The stability of the kinematic systems depends mainly on the behavior of the solution, whether it approaches one point or a closed curve when the solution is cyclical or not. Non-Linear systems have several characteristics, including:

1. The nature of the Jump, since \( c\dot{x}(t)\) are the decaying force and \( \propto x(t) + Bx^3(t) \) is the reference force of the oscillating force.
2. Reliability between the amplitude of the oscillator and the frequency, which Can be seen in models of random vibrations.
3. Having the final cycle and this behavior of the solution is clearly shown in the vender POL formula that formulated:
\[
\ddot{x}(t) - B (1 - x^2(t))\dot{x}(t) + \propto x(t) = 0
\]
Where \( \dot{x}(t) = \frac{dx(t)}{dt} \)
And one of the most important techniques used to bring non-linear systems closer to Linear local systems near a single point. The stationary solution to the system is to approximate the local linear technique.
and to demonstrate this technique. We have a non-Linear differential equation, and it is converted into a linear differential equation with the following steps:
1. Reducing the rank of the equation.
2. Writing it in the 'space form.
3. Finding the fixed solution of the system, as follows:

We do not have the following equation: (Vander pol equation):

\[ x'' - (1 - x^2)x' + x = 0 \]  
(4)

And put \( x' = y \) the equation (2-2) becomes as:

\[ y' = (1 - x^2)y - x \]  
(5)

By rewriting it as the state space, that \((x, y)^T\) is the state vector, and that the only fixed point in the system (2-3) is the origin point, and the fixed solution at this point is zero \((\dot{x} = 0, \dot{y} = 0)\).

Using the Taylor (2-3) system expansion around its original point, we obtain:

\[
\begin{align*}
\dot{x} &= o \ast x + 1 \ast y \\
\dot{y} &= \left(-1 - 2xy\right)x + \left(1 - x^2\right)y 
\end{align*}
\]  
(6)

The expansion of Taylor is rounded in equation (2-4) to the second term only, given that the functions \(X(x, y)\) and \(Y(x, y)\) are functions with two variables \(x, y\) and \(x(0, 0) = o, y(0, 0) = o\) at the original point:

\[
\begin{bmatrix}
0 & 1 \\
-1 - 2xy & 1 - x^2
\end{bmatrix}
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
= \begin{bmatrix}
x \\
y
\end{bmatrix}
\]  
(7)

And this system has an original point becomes as follows:

\[
\begin{bmatrix}
0 & 1 \\
-1 & 1
\end{bmatrix}
\begin{bmatrix}
x \\
y
\end{bmatrix}
= \begin{bmatrix}
x \\
y
\end{bmatrix}
\]  
(8)

That is:

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= -x + y
\end{align*}
\]

That is, the equation for Van DerpoL (2-3) and near to that fixed point becomes a linear equation as follows:

\[ x' - \dot{x} + x = 0 \]  
(9)

This technique can be applied single non – zero points as well since non-Linear kinematic systems include single zero and non – zero points. The stability of these systems is determined by the behavior of the solution path when approaching these isolated \(t \rightarrow \infty\), points, or moving away from them when \(t\) and here represents time [12],[8].

### 3.2 Stationary and Stability

In many engineering and physical matters, we come across processes that can be described as statistically, stable, this means that if we get views of a process of this type and it is done.

Divide them into groups of periods. The different sections of these observations appear similar, and with more accurate words. The statistical characteristics are constant and do not change with time. Random operations that act in this way are called phased operations [1]. It means that there is no growth or erosion of time series data, in other words. The data are spread around a fixed medium and have a
constant variance. That means \( x_t, x_2, x_3, ..., x_t \) it must have the same probability density function, that is, \( f(x_1, x_2, x_3, ..., x_t) = f(x_{t+k}, x_{2+k}, x_{3+k}, ..., x_{t+k}) \) since \( k \) it represents a real exception. And that the Joint probability distribution does not change with the change in the period or when the displacement is carried out in a fixed number. It has been shown by the word priestly that the varied operations are usually created through a stable system that reaches the steady-state after an appropriate period. Specific output, and therefore the system is mathematically stable. if the roots of the system equation in the form of the defaults factor lie all outside the unit circle, and that the characteristics of the characteristics' equation are all located within the unit circle [1].

3.3 Singular point

The single point of the form \( \varepsilon \) is defined;

\[
x_t = f(x_{t-1}, ..., x_{t-p})
\]

(10)

it is the one that the path of the model closes above, and the approach is either when \( t \to \infty \) or \( t \to -\infty \).

If the path approach is when \( t \to \infty \) it is called single stable point. And if the path approaches a point \( \varepsilon \) when \( t \to \infty \), then a single point is called unstable. And the necessary condition and what is sufficient for \( \varepsilon \) is to achieve the relationship: [10] [12], \( \varepsilon = f(\varepsilon) \).

3.4 Limit cycle

Let us have the following model: \( x_t = f(x_{t-1}, ..., x_{t-p}) \).

Limit cycle of the above model is defined as the isolated and closed path. \( x_t, x_{t+1}, x_{t+2}, ..., x_{t+q} = x_t \)

Since \( q \) represents a positive integer. The isolated path is defined as any path that begins very close to Limit cycle and approaches it either when \( t \to \infty \) or \( t \to -\infty \) if it is approaching it when \( t \to \infty \) it is called a stable Limit cycle, but if the approach is when \( t \to -\infty \) it is called an unstable limit cycle, but the closed path is if the initial values \( x_1, x_2, x_3, ..., x_p \) belong to the Limit cycle, then \( (x_{1+k}, ..., x_{p+k}) = (x_1, ..., x_p) \) for each positive integer \( K \) whereas \( P \) is the period to the Limit cycle, and it is the closed path points, and that \( q \) and \( k \) represent appositive integer [10].

3.5 Find the stability of non-linear models using Linear approximation

The Linear approximation method was proposed to find the stability of the non-Linear models based on the approximation technique in the previous paragraph that was applied to the non-linear exponential time series models by the researcher Ozaki. The method is summarized in two stages:

First stage: Finding the non-zero single Point of the non-Linear Model.

Second Stage: Stability test of this point using Linear approximation technique.

3.6 stability of the Polynomial self-regression model with the Limits of trigonometric functions

Let us have the following form:

\[
x_t = \sum_{i=1}^{\infty} a_i [\cos \left( \frac{\pi}{2} x_{t-k} \right)]^i + Z_t; k = 1, 2, ...
\]

(11)

As \( a_i \) constant and \( Z_t \) white inconveniences.

We assume that \( k = 1 \) (i.e. we will get Polynomials with trigonometric function and \( x_{t-1} \)) that is:

\[
x_t = \sum_{i=1}^{\infty} a_i [\cos \left( \frac{\pi}{2} x_{t-1} \right)]^i + Z_t
\]

(12)
3.7 Find the single point of the proposed model
To have the model defined in equation (2-10), we assume the effect of \( \{z_t\} \) equals zero and by using the definition of a single point, and by assuming that \( \mathbb{E}^n \rightarrow o \) for each \( n \geq A \) and \( j = 1,2,3,4 \) in equation (2-10) and using the decomposition of Taylor we get:

\[
\varepsilon^2 + \frac{1}{B} \varepsilon - \frac{A}{B} = 0
\]

(13)

Where as:
\[
A = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4
\]
\[
B = \frac{\pi^2}{8} (\alpha_1 + 2 \alpha_2 + 3 \alpha_3 + 4 \alpha_4)
\]

That is:
\[
\varepsilon = \frac{-1}{B} \pm \sqrt{\left(\frac{1}{B}\right)^2 + 4 \frac{A}{B}}
\]

(14)

3.8 Single point of the proposed grade of rank \( k \) (\( k=2,3,... \))
To find the single point of the proposed model (2–9) of rank \( k \), we neglect the effect of \( Z_t \) on the form (2–9), so we obtain:

\[
x_t = \sum_{j=1}^{\infty} \alpha_j \left[ \cos \left( \frac{\pi}{2} x_{t-k} \right) \right]^j; k = 2,3, ...
\]

Using the definition of a single point, we obtain:

\[
\varepsilon = \sum_{j=1}^{\infty} \alpha_j \left[ \cos \left( \frac{\pi}{2} \varepsilon \right) \right]^j
\]

The individual points are not flogged to any rank, and we follow a formula similar to the way the single point is found when \( k=1 \).

3.9 Non–Zero single point stability of the polynomial model (proposed)
We will attempt to test the stability of the non – zero single point using the local linear approximation method near the single point as follows:
Let us have the model defined in (2–10) to assume the effect of \( Z_t \) non – existent and using the Taylor decoder and compensation for \( x_t = \varepsilon + \varepsilon_t \) where \( \varepsilon \) are a very small amount and that \( \forall n \geq 2; \varepsilon_t^{n-1} \rightarrow o, j = 1,2,3,4 \) we get:

\[
\varepsilon + \varepsilon_t = A - B(\varepsilon^2 + 2 \in \varepsilon_{t-1})
\]

(15)

Where as:
\[
A = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4
\]
\[
B = \frac{\pi^2}{8} (\alpha_1 + 2 \alpha_2 + 3 \alpha_3 + 4 \alpha_4)
\]

Since then:
\[
\varepsilon = \frac{-1}{B} + \sqrt{\left(\frac{1}{B}\right)^2 + 4 \frac{A}{B}}
\]

With compensation in (15), we obtain:
\[ \varepsilon + \varepsilon_t = A - B(\varepsilon^2 + 2 \in \varepsilon_{t-1}) \]  

(16)

Accordingly, the model is stable if it is:

\[ |1 - \sqrt{1 + 4AB}| < 1 \]  

(17)

### 3.10 Limit cycle stability

We will find the stability of the limit cycle based on the following two views:

Theorem (1): the limit cycle in turn \( a, x_{t+1}, x_{t+2}, \ldots, x_{t+q} \), of the exponential regression from the first order is stable in orbit if the following condition is met:

\[ \left| \prod_{j=1}^{q} [\sin_j(1 - 2x_t^2 - j)e^{-x_t^2 - j}] \right| < 1 \]

The proof: Note [9]

Theorem (2): The limit cycle in turn \( q \) of the form \((1 - 13)\) is stable if it is:

\[ \left| \frac{\varepsilon_t + q}{\varepsilon_t} \right| < 1 \]

Proof: Note [10]

#### 3.10.1 Issue (1)

The limit cycle of the cycle \( q \) (if any) of the proposed model when \( p = 1 \) is stable if the condition is met:

\[ \left| \prod_{j=1}^{q} \left[ \cos \left( \frac{\pi}{2} x_{t-1} \right) \right] \right| < 1 \]

The proof:

We assume that the proposed model has a limit cycle in the period \( q \) and \( q > 1 \) as:

\[ x_t, x_{t+1}, x_{t+2}, \ldots, x_{t+q} = x_t \]

It is a closed and isolated path.

Let us have the model defined in equation (12) below:

\[ x_t = \sum_{j=1}^{\infty} \alpha_j \left[ \cos \left( \frac{\pi}{2} x_{t-k} \right) \right] + z_t \]

We take a special case when \( k = 1, j = 1 \) and generalize the idea in the same way.

\[ x_t = \alpha_1 \cos \left( \frac{\pi}{2} x_{t-1} \right) + Z_t \]

\[ x_t = \sum_{j=1}^{\infty} \alpha_j \left[ \cos \left( \frac{\pi}{2} x_{t-k} \right) \right] \]

We take a special case when \( k = 1, j = 1 \) and generalize the idea in the same way.

\[ x_t = \alpha_1 \cos \left( \frac{\pi}{2} x_{t-1} \right) + Z_t \]

\[ x_t = \alpha_1 \sum_{k=0}^{\infty} (-1)^k \frac{\pi x_{t-1}^{2k}}{2k!} + Z_t \]

(18)

Let \( x_t = x_s + \varepsilon_s \) and assume that \( |\varepsilon_{s-1}|^n \rightarrow 0 \) for each \( n \geq 2 \) and neglecting the effect \( \{z(t)\} \) we get study of the stability of a nonlinear self-regression model:
\[
x_s + \varepsilon_s = \alpha \left[ 1 - \frac{\pi^2}{2!} x_{s-1}^2 + \frac{\pi^4}{4!} x_{s-1}^4 - \frac{\pi^6}{6!} x_{s-1}^6 + \cdots \right] - \alpha \frac{2(\pi^2/2)^2}{2!} x_{s-1} \varepsilon_{s-1} + \cdots
\]

That is:

\[
\varepsilon_s = -\alpha_1 \frac{2(\pi^2/2)^2}{2!} x_{s-1} \varepsilon_{s-1} + o(x_{s-1} \varepsilon_{s-1})
\]

(19)

Leads to \( \varepsilon_s = \beta x_{s-1} \varepsilon_{s-1} \)

(20)

Assume that:

\[ \beta = -\alpha_2 \frac{2(\pi^2/2)^2}{2!} \]

\( T(x_{t-1}) = \beta x_{s-1} \)

\[ \Rightarrow \varepsilon_{t+q} = \prod_{i=1}^{q} T(x_{t+q-i}) \varepsilon_t \]

\[ \left| \frac{\varepsilon_{t+q}}{\varepsilon_t} \right| = \prod_{i=1}^{q} T(x_{t+q-i}) \]

For the sake of zero convergence this ratio must be less than one:

\[ \prod_{i=1}^{q} T(x_{t+q-i}) < 1 \]

Thus, the model has a stable orbital limit cycle if the following condition is met:

\[ \prod_{i=1}^{q} \beta(x_{t+q-i}) < 1 \]

(21)

And by this, the proof is made.

3.10.2 Issue (2)

The proposed model has a limit cycle if it is:

\[ \left| \frac{\varepsilon_{t+q}}{\varepsilon_t} \right| < 1 \]

The proof:

Let us have the model defined by the equation (18)

Neglecting the effect \{z_t\} and using the assumptions in the previous paragraphs, we obtained the following form:

\[ \varepsilon_t = (1 - \sqrt{1 + 4AB}) \varepsilon_{t-1} \]

Where:

\[ A = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 \]

\[ B = \frac{\pi^2}{8}(\alpha_1 + 2 \alpha_2 + 3 \alpha_3 + 4 \alpha_4) \]

We assume that:
We get:

\[ C = \left( 1 - \sqrt{1 + 4AB} \right) \]

That is:

\[ \frac{\varepsilon_{t+q}}{\varepsilon_t} = C^q \Rightarrow |\varepsilon_{t+q}| = |C^q| < 1 \]

And this condition is fulfilled if it is \(|C| < 1\). And then the proof ends.

Example 3.1

Let us have the following model:

\[ x_t = 0.398 \cos \left( \frac{\pi}{2} x_{t-1} \right) + z(t) \]

And refer to the stability condition of the proposed model, we get the single point \(\varepsilon = 0.29256\) and using the relationship (16) we get:

\[ \varepsilon_t = -0.07084 \varepsilon_{t-1} \]

And since \(|\lambda| < 1\) and therefore the model is stable.

The following figures show the stability of the model and by assuming different initial values.[15]

![Figure (1)](image)

**Figure (1)**. the serial series generated from the model

We notice from the above figures that the series generated by the model does not depend on the initial condition and that paths an approach to the limit cycle.

Example 3.2

Let us have the following model:

\[ x_t = 2.787 \cos \left( \frac{\pi}{2} x_{t-1} \right) + z(t) \]

And from it we get \(\varepsilon = 1.26104\), and from the relationship (16) we get the form \(\varepsilon_t = -2.44025 \varepsilon_{t-1}\).
That is, $1 < |\lambda|$ which indicates that the stability condition has not been fulfilled, the form (2) is not stable.[15]

The following figures illustrate this:

![Graphs illustrating the serial series generated from the model(2)](image)

**Figure (2).** the serial series generated from the model(2)

4. The practical side

4.1 Introduction

During this time we will try to rely on the way (Box- Genghis) in the analysis and Modeling of the time series of numbers of People with media, where data was obtained from the Yarmouk Hospital in Baghdad/ AL-karkh for the period (2011- 2016) represented by the number of patients with the disease.

1. The first stage: diagnosing the model (Model-identification)
2. The second stage: estimating the parameters of the Model (Parameters Estimation)
3. Third stage: suitability test of the personalized Model (Diagnostic checking) [3].

4.2 Data analysis

Figure (1) represents the graph of the time series, where the axis of the Years is the time $t$, and the axis of the antibiotics represents the numbers of patients with this disease, which represents the studied time series $\{x_t\}$. As we note that the series fluctuates irregularly, which indicates that the series is not crossed in the middle and that the series follows a periodic, irregular system, that is, it repeats itself for a specific period.
The figures (2), (3) and (4) represent the fee for the self-correlation function, the partial self-correlation function, and a drawing for the incidence of this disease.
Where we note that the sequence is not related, and we find that more than 8% approximately of the Self-Correlation coefficients are outside the constraint $\pm \frac{1.92}{\sqrt{\text{df}}}$, but they are close to the natural distribution, as we note that most of the data are close to the main axis of the nature drawing.

4.3 Modeling
From the observation of the graph of the series, figure (1) we find that the series is not arranged in the Middle, and fluctuate periodically, almost irregularly.
In order to convert it into a time series ranging in the Middle, the differences were taken for it, that is:

$$Y_t = \Delta^d x_t \ldots$$

$d = 1, 2, \ldots$; $\Delta = (1 - B)$

$x_t$ represents the original series, $Y_t$ represents the transformed series. And B displacement posteriori trigger, which is known as the following:

$$B_r^r = x_{t-r} ; r = 1, 2, \ldots$$

And the shape $(s)$ represents the transformed sequence $(d = 1)$, and we find that the sequence is rounded around the medium.
Figure (7), the serial drawing of the converted data

Figure (8) is a self-correlation function for the converted data
As for the shapes (6), (7) and (8), the represents the drawing of the self and partial correlation function and the nature drawing, as we note that the serial data are not correlated and where we observe more than 12% of the self-correlation coefficients that are outside the constraint $\pm \frac{1.02}{\sqrt{n}}$ but they are close to the natural distribution.

Depending on the graphical description and correlation function diagram, we have noticed that the series possesses periodic, characteristics and irregular cycle swhich enables us to use non -Liner models to represent the series.

Since the model we have is a polynomial model, since:

$$y_t = \sum_{j=1}^{p} \alpha_j \left[ \cos \left( \frac{\pi}{2} y_{t-1} \right) \right]^{j} + Z_t$$

We will try to construct models form the first anal second ranks to demonstrate the stability of the model above using relationships obtained in the hypothetical side of the previous paragraphs.

let $p = 1$ be in the form above:
\[ y_t = \propto_1 \cos \left( \frac{\pi}{2} y_{t-1} \right) + Z_t \]

Using the Matlab program, we obtained the following form:

\[ y_t = 0.0811 \cos \left( \frac{\pi}{2} y_{t-1} \right) + Z_t \]

\[ \sigma^2 = 0.1476576 \]

\[ NBIC = -1.83249 \]

And when \( p = 2 \) that is:

\[ y_t = \propto_1 \cos \left( \frac{\pi}{2} y_{t-1} \right) + \propto_2 \left( \cos \left( \frac{\pi}{2} y_{t-1} \right) \right)^2 + Z_t \]

We obtained the following form:

\[ y_t = 0.0798 \cos \left( \frac{\pi}{2} y_{t-1} \right) + 0.0373 \left( \cos \left( \frac{\pi}{2} y_{t-1} \right) \right)^2 + Z_t \]

\[ \sigma^2 = 0.1722664 \]

\[ NBIC = -1.545764 \]

4.4 Stability

In this paragraph, we will attempt to apply the relationships (single Point and Limit cycle stability) obtained in the theoretical side of the models that were constructed to represent the studied sequence.

**EXAMPLE 4.4.1** Polynomial regression model of the first order

That is:

\[ y_1 = 0.0622 \cos \left( \frac{\pi}{2} y_{t-1} \right) + Z_t \]

Where as:

\[ \sigma^2 = 0.1688576 \]

\[ NBIC = -1.8428783 \]

\[ \propto = 0.0822 \]

i. The single Point

of the above model is by using the relationship (14) is:

\[ \varepsilon_1 = 0.06018272 \]

ii. Single point stability:

Using relationship (16), we get the following form:

\[ \varepsilon_{1t} = -0.0348714 \varepsilon_{t-1} \]

It is a first-order self – regression model and the characteristic equation for the model is:

\[ \nu + 0.0348714 = 0 \]

That is \( \lambda = -0.0348714 \) is the root of the equation, and it is clear that the above model is stable because \( |\lambda| < 1 \)

iii. Limit cycle stability:

The higher model has a stable orbital limit cycle if the condition is met:

\[ \left| \frac{\varepsilon_{t+q}}{\varepsilon_t} \right| < 1 \]

From the relationship (16) and the hypotheses in that paragraph we obtained:

\[ \varepsilon_t = A \varepsilon_{t-1} \]

From which we obtain:

\[ c = -0.0348625 \]

Therefore, the condition for the model to have a stable limit cycle is \( |C^q| < 1 \), and since \( |C| < 1 \) the amount \( |C^q| < 1 \) is fulfilled for all values \( q > 1 \)

So the model has a limit – cycles

**EXAMPLE 4.4.2** The polynomial regression model of the second-order that is:

\[ y_t = 0.0876 \cos \left( \frac{\pi}{2} y_{t-1} \right) + 0.0373 \left( \cos \left( \frac{\pi}{2} y_{t-1} \right) \right)^2 + Z_t \]

Whereas:
\[ \sigma_z^2 = 0.1512664 \]
\[ NBIC = -1.444782 \]
\[ \alpha_1 = 0.0876 \]
\[ \alpha_2 = 0.0474 \]
i. The single point of the above model is by using the relationship (14) is:
\[ \varepsilon = 0.12848034 \]
ii. The stability of a single point using the relationship (16), we get the following model:
\[ \varepsilon_t = -0.108305263 \varepsilon_{t-1} \]
It is a second-order subjective regression model, and the characteristic equation for the model is:
\[ \nu + 0.108305263 = 0 \]
That is \( \lambda = -0.108305263 \)
The root of the equation it is clear that the model is stable above because \(|\lambda| < 1\)
iii. Stability limit cycle the above form have Stability limit cycle if the condition is true:
\[ \left| \frac{\varepsilon_{t+q}}{\varepsilon_t} \right| < 1 \]
It is a relationship (16) and hypotheses we get .
\[ \varepsilon_t = A \varepsilon_{t-1} \]
From them we get.
\[ C = -0.108305263 \]
And so it is condition of stability limit cycle \( |C^q| < 1 \)
Since the \(|C| < 1\) the value \(|C^q| < 1\) all values will be fulfilled \( q > 1\) so the model has stability limit cycle

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