Network calculus for parallel processing

G. Kesidis, B. Urgaonkar, Y. Shan, S. Kamarava
CSE and EE Depts
The Pennsylvania State University
{gik2, buu1, yxs182, szk234}@psu.edu

J. Liebeherr
ECE Dept
University of Toronto
jorg@comm.utoronto.ca

Abstract

In this note, we present preliminary results on the use of “network calculus” for parallel processing systems, specifically MapReduce. We also numerically evaluate the “generalized” (strong) stochastic burstiness bound based on publicly posted data describing an actual MapReduce workload of a Facebook datacenter.

I. INTRODUCTION

Multi-stage parallel data processing systems are ubiquitous. In a single parallel processing stage, a job is partitioned into tasks (i.e., the job is “forked” or the tasks are demultiplexed); the tasks are then worked upon in parallel. Within parallel processing systems, there are often processing “barriers” (points of synchronization or “joins”) wherein all component tasks of a job need to be completed before the next stage of processing of the job can commence. The terminus of the entire parallel processing system is typically a barrier. Thus, the latency of a stage (between barriers or between the exogenous job arrival point to the first barrier) is the greatest latency among the processing paths through it. Google’s MapReduce [8] (especially its open-source implementation Apache Hadoop [18]) is arguably the most popular such framework. However, numerous other systems exhibit similar programming patterns and our work is relevant to them as well [19], [16], [11].

In MapReduce, jobs arrive and are partitioned into tasks. Each task is then assigned to a mapper for initial processing. The results of mappers are transmitted (shuffled) to reducers. Reducers combine the mapper results they have received and perform additional processing. The workloads associated with reducers may be unrelated to those of their tributary mappers. A barrier exists before each reducer (after its mapper-shuffler stage) and after all the reducers (after the reducer stage).

To achieve good interleaving of the principal resources consumed by the mapper (CPU/memory) and the shuffler (network bandwidth), these stages are made to work in a pipelined manner wherein the shuffler transmits partial results created by the mapper (as they are generated) rather than waiting for a mapper to entirely finish its task. Of course, the shuffler must follow the mapper at all times in the sense of being able to send only what the mapper has generated so far [14]. On the other hand, the barrier between the shuffler stage and the reducer stage is a strict one - a reducer may not begin any processing until all of its shuffler’s work is done.

In the following, we consider a single parallel processing stage. Our approach can be extended to create a model with separate queues for the mapper, the shuffler, and the reducer stages (or even more stages), but we restrict our attention to the interaction between the shuffler and the reducer stages. Specifically, each processor/server (and associated job queue) in our model represents a mapper stage.

There is substantial prior work on fork-join queueing systems particularly involving underlying Markov chains, e.g., [15]. Of course, there is also an enormous literature on parallel processing systems in general. Typically, parallel processing systems employ robust load balancing to minimize synchronization delays at the barriers. To this end, load balancing could proactively estimate throughputs along the parallel processing paths and proportionately size the workloads from tasks.
fed to them. Moreover, “straggler” (deemed excessively delayed) tasks at barriers can be (reactively) restarted or the entire job can be interrupted and restarted or additional can be allocated (e.g., more parallelism). See [13], [?] for recent discussions on the online management of a MapReduce parallel processing system. Some recent work on MapReduce systems [2], [14] focuses on the the pipelining between the mapper and shuffler, the latter formulating a proactive scheduling problem that jointly considers individual job workloads of both mapper and shuffler (assuming the shuffler load is known a priori).

In this note, we focus on the Mapper stage, where any initial job scheduling would be in play to achieve a “bounded burstiness” of the aggregate workload. We prove to claims based using “network calculus” for parallel processing systems. We then numerically evaluate the “generalized” (strong) stochastic burstiness bound (gSBB) of a publicly disseminated workload trace of a Facebook datacenter.

II. SINGLE-STAGE, FORK-JOIN SYSTEM

Consider single-stage fork-join (parallel processing) system, modeled as a bank of \( K \) parallel queues, with queue-\( k \) provisioned with service/processing capacity \( s_k \). Let \( A \) be the cumulative input process of work that is divided among queues so that the \( k \)th queue has arrivals \( a_k \) and departures \( d_k \) in such a way that \( \forall t \geq 0, \)

\[
A(t) = \sum_k a_k(t).
\]

Define the virtual delay processes for hypothetical departures from queue \( k \) at time \( t \geq 0 \) as

\[
\delta_k(t) = t - a_k^{-1}(d_k(t)),
\]

where we define inverses \( a_k^{-1} \) of non-decreasing functions \( a_k \) as continuous from the left so that

\[
a_k(a_k^{-1}(v)) = a_k^{-1}(a_k(v)) = v.
\]

In following definition of the cumulative departures, \( D \), the output is determined by the most lagging (straggling) queue/processor: \( \forall t \geq 0, \)

\[
D(t) = A(t - \max_k \delta_k(t)) = A\left(\min_k a_k^{-1}(d_k(t))\right).
\]

Note that in the case of continuous, fluid arrivals (e.g., piecewise linear \( A \)), this definition of departures \( D \) corresponds to periods of continual, possibly perpetual, barriers (synchronization times). In the case of discrete arrivals (piecewise constant \( A \) with jump discontinuities at arrival instances), then the barriers are discrete.

Define the convolution \((\ast)\) (and deconvolution, \(\ominus\)) identity as

\[
u_\infty(t) = \begin{cases} 
0 & \text{if } t \leq 0 \\
\infty & \text{if } t > 0
\end{cases}
\]

and

\[
d_{\max,k} = \min\{z \geq 0 : \forall x \geq 0, s_{\min,k}(x) \geq (b_{\in,k} \ast \Delta z u_\infty)(x) = b_{\in,k}(t - z)\}
\]

where the delay operator \((\Delta_d g)(t) \equiv g(t - d)\) and \(d_{\max,k}\) is the largest horizontal difference between \(b_{\in,k}\) and \(s_{\min,k}\) [6].
A queue $q$ with service $s$ has a (non-decreasing) minimum service curve $s_{\text{min}}$ \(i.e., s \gg s_{\text{min}}\) if for all (cumulative) arrivals $a$ and all time $t$, its cumulative departures

\[ d(t) \geq (s_{\text{min}} \ast a)(t) := \min_{v \leq t} a(v) + s_{\text{min}}(t - v). \]

**Claim 1.** If $s_{\text{min},k}$ is the minimum service curve of the $k$th queue whose arrivals $a_k \ll b_{\text{in},k}$ (conform to burstiness curve $b_{\text{in},k}$), then for all $t \geq 0$,

\[ D(t) \geq A(t - \max_k d_{\text{max},k}). \]

**Remark:** This claim simply states that the maximum delay of the whole system (from $A$ to $D$) is the maximum delay among the queues. Equivalently, the minimum service curve from $A$ to $D$ is $\Delta_d u_\infty$, where $d := \max_k d_{\text{max},k}$.

**Proof:** By hypothesis, \(\forall t \geq v \geq 0\) and \(\forall k\),

\[ s_{\text{min},k}(t - v) \geq b_{\text{in},k}(t - v - d_{\text{max},k}) \geq a_k(t - d_{\text{max},k}) - a_k(v) \]

Thus, \(\forall t \geq v \geq 0\) and \(\forall k\),

\[ a_k(v) + s_{\text{min},k}(t - v) \geq a_k(t - d_{\text{max},k}) \]

\[ \Rightarrow (a_k \ast s_{\text{min},k})(t) \geq a_k(t - d_{\text{max},k}) \]

\[ \Rightarrow a_k^{-1}((a_k \ast s_{\text{min},k})(t)) \geq t - d_{\text{max},k} \]

where we have used the fact that, \(\forall k\), $a_k$ are nondecreasing. Thus,

\[ D(t) = A \left( \min_k a_k^{-1}(d_k(t)) \right) \geq A \left( \min_k a_k^{-1}((a_k \ast s_{\text{min},k})(t)) \right) \geq A \left( \min_k t - d_{\text{max},k} \right) = (A \ast \Delta_d u_\infty)(t), \]

where we have used the fact that $A$ is nondecreasing.

We now consider a stationary stochastic model of this single-stage system, see Figure 1. To simplify matters, we assume the workload process $A$ has strong ("generalized") stochastically bounded burstiness (gSBB) [12], and leave to future work generalizations to non-stationary settings assuming only (weak) stochastically bounded burstiness [17] or bounded log moment-generating function [1].

**Claim 2.** In the stationary regime at time $t \geq 0$, if

A1 service to queue $k$, $s_k \gg s_{\text{min},k}$ where

\[ \forall v \geq 0, \quad s_{\text{min},k}(v) := v \mu_k; \]

1If we assume that $d(t) \geq (s_{\text{min}} \ast a)(t)$ only for times $t$ when $d(t)$ is strictly increasing, then one can show that $d(t) \geq (s_{\text{min}} \ast a)(t)$ for all time $t$ by simple (pathwise) coupling argument comparing the queue $q = a - d$ to that of one with the same arrivals and service rate exactly $s_{\text{min}}, q_0(t) = \max_{v \leq t} a(t) - a(v) - s_{\text{min}}(t - v)$ for all $t \geq 0$ (or some time-origin other than 0). Note that the condition that $d(t)$ is strictly increasing is used rather than the queue backlog is positive ($q(t) > 0$) because if the arrivals are fluid, then the queue may be empty ($q(t) = 0$) while $d(t)$ is strictly increasing, \(i.e.,\) while departures are occurring at time $t$. 


π is the degree of parallelism

∀k, ∃ small ε_k > 0 such that ∀v ≤ t
\[ |a_k(t) - a_k(v) - \frac{\mu_k}{M}(A(t) - A(v))| \leq \varepsilon_k \text{ a.s.,} \]

\( M := \sum_k \mu_k; \)

the total arrivals have strong stochastically bounded burstiness [12],
\[ P(\max_{v \leq t} A(t) - A(v) - M(t - v) \geq x) \leq \Phi(x), \]

where \( \Phi \) decreases in \( x > 0; \)

then ∀x > 2M \( \min_k \varepsilon_k / \mu_k, \)
\[ P(A(t) - D(t) \geq x) \leq \Phi(x - 2M \min_k \varepsilon_k / \mu_k). \]

Remark: By A2, the mapper divides arriving work roughly proportional to minimum allocated service resources \( \mu_k \) to queue \( k \), i.e., strong load balancing.

Proof:
\[
\begin{align*}
P(A(t) - D(t) \geq x) &= P(A(t) - A(\min_k a_k^{-1}(d_k(t))) \geq x) \\
&= P(\min_k a_k^{-1}(d_k(t)) \leq A^{-1}(A(t) - x) =: t - z) \\
&= P(\forall k, d_k(t) \leq a_k(t - z)) \\
&= P(\forall k, a_k(t) - d_k(t) \geq a_k(t) - a_k(t - z) =: x_k), \\
&\leq P(\forall k, \max_{v \leq t} a_k(t) - a_k(v) - (t - v)\mu_k \geq x_k)
\end{align*}
\]

where we have used the fact that \( A \) and the \( a_k \) are nondecreasing (cumulative arrivals) and the inequality is by A1. Also, we have defined non-negative random variables \( z \) and \( x_k \) such that...
\[ \sum_k x_k = x = A(t) - A(t - z). \] So by using A2 then A3, we get

\[
P(A(t) - D(t) \geq x) \\
\leq P(\forall k, \max_{v \leq t} \frac{\mu_k}{M}(A(t) - A(v)) + \varepsilon_k - (t - v)\mu_k \\
* \geq \frac{\mu_k}{M}x - \varepsilon_k) \\
= P(\forall k, \max_{v \leq t} (A(t) - A(v)) - (t - v)M \\
\geq x - 2\frac{M}{\mu_k} \varepsilon_k) \\
= P(\max_{v \leq t} (A(t) - A(v)) - (t - v)M \\
\geq x - 2M \min_k \varepsilon_k / \mu_k) \\
\leq \Phi(x - 2M \min_k \varepsilon_k / \mu_k).
\]

### III. Numerical Example

There is on-going work characterizing workload in MapReduce parallel-processing systems for purposes of performance evaluation [7]. Figure 3 of [4] depicts a week-long trace of the total number of arriving jobs to a MapReduce system operated by Facebook. Clearly, the job rate exhibits “time-of-day” periodicity in its mean and variance. It can be simply modeled as a bounded AR(1) (bounded, two-parameter autoregressive) process with (deterministically) time-varying parameters. A day-long trace of the data of individual jobs from which Figure 3 of [4] was partially derived is publicly available at [9]. From this dataset, we depict the aggregate job arrival rate, by ten-minute intervals (i.e., 144 time samples), in Figure 2. Here, we see that the data is roughly stationary.

![Fig. 2: The aggregate job arrival rate from [9]](image-url)

Moreover, Table 1 of [4] identifies ten different Facebook job types (i.e., ten rows)\(^2\). In column 1, the number of observed jobs \(n_j\) of type \(j\) is given. Also, the mean number of “task-seconds” per type-\(j\) job for the mapper stage, \(w_j\), is given in the “Map time” column\(^3\). With this information,

\(^2\)Identified through clustering based on the features (columns) identified in this table. Clustering is also used in [3].
\(^3\)We divided “Map time” by 600s consistent with the ten-minute sampling of the aggregate number of jobs in Figure 2.
we can develop an aggregate workload model to the mapper stage, $A$, assuming that at each point in time (of Figure 3 of [4] or our Figure 2), the types of jobs arriving are distributed as in column 1 of Table 1 of [4]. Timing information associated with workloads, including the total execution duration of the individual jobs, are not given in the datasets available at [9]. Execution times are provided for individual jobs of CMU’s OpenCloud Hadoop cluster [5] (so the previous assumption would not be necessary were we to model this dataset).

Figure 3 depicts a typical generated trace of the cumulative workload $A$ to the mapper. Figure 4 depicts the queue process $Q$ corresponding to this cumulative workload trace $A$ and service rate $M = 600$ (which is the slope of the line in Figure 3).

![Cumulative arrivals and service curve](image)

**Fig. 3:** Cumulative workload generated using the job arrival data from [9] and the job workload-distribution data of Table 1 of [4]

Figure 5 depicts the gSBB bound $\Phi$ at service rate $M = 600$ (recall assumption A3 of Claim 2) using the day-long raw trace given in Figure 2 and based on multiple samplings of the average “Map time” data of Table 1 of [4] as described above. The vertical lines represent confidence bars based on 30 independent trials (generated sample paths of the cumulative workload, $A$, as Figure 3).

**IV. DISCUSSION**

Typically, the amount of allocated parallelism of a job at a stage is based on the size of the job’s input data-set to that stage, as that information is readily available operationally online. The execution time for the component tasks will, of course, greatly depend on other factors such as algorithmic/computational complexity. This is evident in Facebook rows 4 and 5 of Table 1 of [4], where two jobs have about the same mean input data size ($\approx$ 400KB in Input column) but significantly different mean Map times (one is roughly double the other). This said, it’s likely that the *same algorithm* will be applied for all tasks of a given job so that effective load balancing from job to task typically may be achieved, *i.e.*, when $\forall k, l, \mu_k = \mu_l$ in Claim 2. Note that Claim 2 allows for processors of different capacities $\mu$, as considered in [10].
Fig. 4: The queue backlog corresponding to arrivals of Figure 3 and service rate $M = 600$

Fig. 5: The gSBB bound $\Phi$ of the aggregate mapper workload of the Facebook trace described in [4] at service rate $M = 600$

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