Variation of NMEs of $0\nu\beta\beta$ for $^{48}\text{Ca}$ with different components of NN interaction

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Abstract. We examine the sensitivity of nuclear matrix elements (NMEs) for light-neutrino exchange mechanism of neutrinoless double beta decay ($0\nu\beta\beta$) for $^{48}\text{Ca}$ to the various components of two-nucleon interaction, GXPF1A, in $fp$ model space. It is found that the contribution in NMEs coming from the central component is close to contribution from total interaction. The spin-orbit and tensor components are found canceling the contribution of each other.

1 Introduction

Neutrinoless double beta decay ($0\nu\beta\beta$) is a rare weak process, known mainly for determining the nature of neutrino and neutrino mass [1]. The decay rate of this process depends on nuclear matrix elements (NMEs), which are calculated using theoretical nuclear many-body models. In the literature, the nuclear shell model (NSM) has been widely used to calculate NMEs [1].

It is well known that the total two-nucleon interaction is the sum of central (C), spin-orbit (SO), and tensor (T) interaction. The earlier calculations of NMEs in NSM have been done using total two-nucleon interaction [2–5]. In the present work, we examine the variation of NMEs for light neutrino exchange mechanism of $0\nu\beta\beta$ of $^{48}\text{Ca}$ with different components of two-nucleon interaction-GXPF1A [6] of $fp$-model space using closure approximation. The employed interaction is decomposed into its central, spin-orbit, and tensor components using spin-tensor decomposition (STD) method [7–10].

This article is organized as follows. In section 2, the theoretical formalism for NMEs for light neutrino exchange mechanism of $0\nu\beta\beta$ is given. The STD is discussed in section 3. Results and discussion are presented in section 4. The summary of this work is given in section 5.

2 Nuclear Matrix Elements of $0\nu\beta\beta$

The NMEs for light neutrino exchange mechanism of $0\nu\beta\beta$ can be presented as a sum of Gamow-Teller ($M_{GT}$), Fermi ($M_{F}$), and tensor ($M_{T}$) matrix elements [4]:

$$M_{\nu} = M_{GT} - \left(\frac{g_V}{g_A}\right)^2 M_{F} + M_{T}$$

where, $g_V = 1$ and $g_A = 1.27$. $M_{\nu}$ = $(f|\tau_{-1}\tau_{-2}O_{12}^T|i)$ with $\alpha = GT, F, T$ can be written in terms of two-body transition density (TBTD) and two-body matrix elements $(k_1, k_2, j_{\tau_{-1}}\tau_{-2}O_{12}^T|k_1, k_2, J)$ [4];

$$M_{\nu} = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} TBTD(f, i, k, J) (k_1', k_2', j_{\tau_{-1}}\tau_{-2}O_{12}^T|k_1, k_2, J)$$

where, $\tau_{-}$ is isospin lowering operator, $O_{12}^T$ is transition operator of $0\nu\beta\beta$ defined with spin ($\sigma$) and neutrino potential operator $(H_{\nu}(r, E_{\nu}))$ [2]; $O_{GT} = (\sigma_1 \sigma_2)H_{GT}(r, E_{\nu})$, $O_{F} = H_{F}(r, E_{\nu})$, and $O_{T} = (3(\sigma_1 \cdot \sigma_2)H_{T}(r, E_{\nu})$. Eq. (2), $k$ stands for the set of quantum numbers ($n, l, j$), $i$ and $|f\rangle$ refer to the ground state (0$^+$) of $^{48}\text{Ca}$ and $^{48}\text{Ti}$, respectively. Neutrino potential is defined as [3];

$$H_{\nu}(r, E_{\nu}) = \frac{2R}{\pi} \int_{0}^{\infty} j_\lambda(q)q^2 dq$$

where, $E_{\nu}$, $E_i$, and $E_f$ are the energies of $^{48}\text{Sc}$, $^{48}\text{Ca}$ and $^{48}\text{Ti}$ respectively, $q$ is the momentum of the virtual Majorana neutrino and $r$ is the distance between the nucleons. $j_\lambda(q)$ is the spherical Bessel function with $p = 0$ and 2. In closure approximation one replaces $E_{\nu} - (E_i + E_f)/2 \rightarrow (E)$, where, $\langle E\rangle$ is the closure energy which takes care the effects of large number of excitation energy of states of intermediate nuclei (48Sc). Closure approximation removes the complications of calculating large number of excitation energy of states of $^{48}\text{Sc}$. As neutrino momentum ($q$) in the decay is high (~100-200 MeV), NMEs are not much sensitive with the excitation energy of $^{48}\text{Sc}$. So by replacing all excitation energy with a constant closure energy $\langle E\rangle$ gives NME with around 90% accuracy [3].

3 Spin Tensor Decomposition (STD)

Nucleons are intrinsic spin 1/2 fermions; therefore, the interaction between two-nucleon can be written as the linear sum of the scalar product of configuration space operator $Q$ and spin space operator $S$ of rank $k$ [7, 8];
\[ V = \sum_{k=0}^{3} V(k) = \sum_{k=0}^{3} Q^k S^k \]

where, rank \( k = 0, 1 \) and 2 represent central, spin-orbit and tensor force, respectively. Using the \( LS \)-coupled two-nucleon wave functions, the matrix element for each \( V^k \) can be calculated from matrix element for \( V^9 \):

\[
\langle (ab), LS : JM|V(k)|(cd), L'S' : JM \rangle = (2k + 1)(-1)^J \sum_{J'} (-1)^{J'}(2J' + 1) \left\{ \begin{array}{ccc} L & S & J' \\ L' & S' & J \end{array} \right\} \times \langle (ab), LS : JM|V|(cd), L'S' : JM \rangle \]

(5)

here, \( a \) includes quantum numbers \( n_a \) and \( l_a \).

4 Results and Discussion

![Figure 1](URL)

Figure 1. Contribution of various spin-parity (J\(^\pi\)) of decaying neutrons or final protons to the NMEs.

We have calculated TBTD in terms of two-nucleon transfer amplitudes (TNA) with 50 states of \( ^{46}\text{Ca} \) using method described in Ref. [4]. The TBMEs have been calculated with closure energy \( \langle E \rangle = 0.5 \text{ MeV} \) [3]. The effect of finite nucleon size (FNS), higher order currents (HOC) [2] have also been considered. The calculated NMEs are given in Table 1.

| Type            | C                  | C+SO              | C+SO+T             |
|-----------------|--------------------|-------------------|--------------------|
| \( M_F \)       | FNS+HOC            | -0.273            | 0.234              | -0.217              |
| \( M_{GT} \)    | FNS+HOC            | 0.933             | -0.828             | 0.790               |
| \( M_T \)       | FNS+HOC            | -0.081            | 0.070              | -0.076              |
| \( M^{\nu} \)   | FNS+HOC            | 1.021             | -0.904             | 0.848               |

It is found that the NMEs calculated with C interaction are near to NMEs calculated with total interaction. On the addition of SO part to C part, the sign of NMEs gets change but in absolute value they remain almost same. Similar effects are also seen when we add T part to C+SO part of two-nucleon interaction. Thus, we infer that SO and T parts negate the effect of each other.

Results of NMEs as a function of coupled spin-parity of decaying nucleons are shown in Fig. 1. It is found that the dominant contribution in NMEs comes from \( J^\pi = 0^+ \) and \( 2^+ \). But, their contribution are present with the opposite effect resulting in a small value of NMEs. Negligible contributions comes from other \( J^\pi \).

5 Summary

We have examined the sensitivity of NME for light-neutrino exchange mechanism of \( 0\nu\beta\beta \) for \( ^{48}\text{Ca} \) with various components we get using STD for GXPF1A interaction. It is found that the NMEs calculated with C part and total interaction are close to each other. SO and T parts negate the contribution of each other in the NMEs. Dominating contribution to NMEs comes from 0\(^+\) and 2\(^+\) spin-parity states of decaying nucleons.

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