Reliability-based design for earth-fill dams against heavy rains

Shin-ichi Nishimura i), Takayuki Shuku ii), Toshifumi Shibata iii) and Kazunori Fujisawa iv)

i) Professor, Okayama University, 3-1-1, Tsushima-naka, Kita-ku, Okayama, 700-8530, Japan.
ii) Assistant Professor, Okayama University, 3-1-1, Tsushima-naka, Kita-ku, Okayama, 700-8530, Japan.
iii) Lecturer, Okayama University, 3-1-1, Tsushima-naka, Kita-ku, Okayama, 700-8530, Japan.
iv) Associate Professor, Kyoto University, Kitashirakawa Oiwake-cho, Sakyo-ku, Kyoto, 606-8502, Japan.

ABSTRACT

To mitigate the disasters of decrepit earth-fill dams, improvement work is conducted. Since there is a recent demand for low-cost improvements, the development of a design method for optimum improvement work at a low cost is the final objective of this research. The reliability-based design approach is discussed as such a design method. Heavy rains are the most serious events for aged fill-dams, and sometimes overflows leading to breaching occur. From the estimated probability of overflow, the probability of submergence in the downstream is estimated for the reliability-based design. The remarkable point in this paper is that the probability of occurrences of heavy rains and the uncertainty of the soil properties are simultaneously considered for the design. Considering the disaster loss, the risk of submergence could be assessed.

Keywords: reliability-based design, earth-fill dam, overflow, flood simulation, statistical rainfall model

1 INTRODUCTION

Many earth-fill dams have been built for farm ponds in Japan. Some of the dams are getting old and decrepit, and have weakened. Every year, a number of them are damaged by heavy rains, and in a few worst cases, the dams are completely destroyed. To mitigate such disasters, improvement work is conducted on the most decrepit earth-fill dams. Since there is a recent demand for low-cost improvements, the development of a design method for optimum improvement work at a low cost is the final objective of this research. The reliability-based design approach is discussed as such a design method.

Heavy rains are the most serious events for aged fill-dams, and sometimes overflows occur. From the estimated probability of overflow, the probability of submerge in the downstream ought to be estimated for the reliability-based design. The remarkable point in this paper is that the probability of heavy rains and the uncertainty of the soil properties are simultaneously considered for the task. Considering the disaster loss, the risk of submergence can be obtained. Considering the improvement cost, including improvement of spillway, the expected total cost is evaluated. By comparing the expected total costs between the original and the improved states of the earth-fill dam, the effect of improvement work is assessed.

2 QUASI-RAINFALL MODEL

In this paper, the rainfall events continuing for 72 hours are simulated based on the annual maximum rainfall intensities obtained from the rainfall data records in Okayama City, Japan for a span of 45 years. A dam breaching almost happens within 24 hours on an empirical basis. To cover all cases, the longer consecutive rainfalls for 72 hours are used.

The cumulative distribution of the annual maximum rainfalls, and the sequence of the 72hours-rainfall, namely, rainfall patterns are statistically modeled based on the data record. The quasi-rainfalls are generated as the random numbers following the statistical model[1].

3 EVALUATION METHOD FOR THE PROBABILITY OF OVERFLOW

At first, the quantity of inflow, discharge and storage are calculated. The inflow equation is defined as follows [2]:

\[ Q_{in} = f_p r A / 3.6 \]  \hspace{1cm} (1)

where \( Q_{in} \) is inflow to the reservoir (m³/s), \( f_p \) is the peak runoff coefficient, \( r \) is the quasi-rainfall intensity (mm/h) generated following the statistical model described in chapter 2 and \( A \) is the area of the basin (km²). Uniform random numbers are used for \( f_p \) in the 0.7 to 0.8 range. The discharge equation for a
rectangular weir as used in this study is;

\[ Q_{\text{out}} = CB_s h^{\frac{1}{2}} \]  \hspace{1cm} (2)

where \( Q_{\text{out}} \) = discharge (m³/s), \( C \) = discharge coefficient, \( B_s = \) width of spillway, and \( h = \) static or piezometric head on a weir referred to the weir crest (m). The storage of water in the water reservoir \( V_r \) on the full water level is estimated as:

\[ V_r = A_w h \]  \hspace{1cm} (3)

where \( A_w \) = area of water reservoir (km²). The increasing rate of the storage \( V_r \) with the runoff is:

\[ dV_r / dt = Q_{\text{in}} - Q_{\text{out}} \]  \hspace{1cm} (4)

The overflow head \( h \) is determined from Equations (2), (3) and (4), and the maximum \( h \) within the 72 hours, is defined as the peak overflow head on the spillway. When \( h_p \) becomes greater than the design overflow head \( h_d \), the overflow occurs. Then, the probability of overflow is defined by Equation (5) as the times of \( h_p < h_d \) in the iterations of the Monte Carlo simulation3;

\[ P_f = Pr \{ h_p < h_d \} \]  \hspace{1cm} (5)

The results for four dams are described in Table 1.

## 4 FLOOD SIMULATION

### 4.1 Determination of hydrograph

Once the event of the overflows happens, the flood simulation must be done as a next step to estimate the damage in the downstream area. The discharge hydrographs are applied for the flood simulation as input waves. When the overflow occurs, the embankment is supposed to be failed as in Figure 1, which shows a vertical section of an embankment. This section of the flood way is to be assumed to keep a trapezoid shape. The overflow hydrograph from broken section is determined by the runoff discharge \( Q_{\text{dis}} \) is formulated as follows;

\[ Q_{\text{dis}} = \sqrt{A_r (\partial A_r / \partial h)} \]  \hspace{1cm} (6)

\[ U = Q_{\text{dis}} / A_r \]  \hspace{1cm} (7)

\[ dH / dt = -E (1 - \eta_r ) \]  \hspace{1cm} (8)

\[ E = \alpha (\tau - \tau_c ) \]  \hspace{1cm} (9)

\[ C_f = 2 / \left[ \log \{ R \exp \left( A_r / 1 \right) \} \right] \]  \hspace{1cm} (10)

\[ C_f = 2 / \left[ \log \{ R \exp \left( A_r / 1 \right) \} \right] \]  \hspace{1cm} (11)

where \( A_r \) is the cross-sectional area of broken section (m²), \( U \) is the flow velocity (m/s), \( H \) is the height of the bottom of failed section, \( h \) is the overflow head on the bottom of the failed section (m), \( E \) is the erosion rate of embankment material (m/s), \( n_p \) is the porosity of the embankment, \( \alpha \) is the erosion rate coefficient (m/s/Pa), \( r \) is the shear stress (Pa), \( \tau_c \) is the critical shear stress (Pa), \( \rho \) is the water density (kg/m³), \( \kappa \) is Karman constant, \( R \) is the hydraulic radius (m), \( A_r \) is the constants (=8.5) and \( \kappa_r \) is roughness height (m). To consider uncertain material values, the variabilities of the several soil parameters of the earth-fills are dealt with as probabilistic parameters. The normal random numbers are substituted for overflow length \( B \), \( \kappa_r \) and \( \alpha \). The angular degree of cross-section of the failed section \( \gamma \) is also provided as a probabilistic variable4. The average and the standard deviation of each parameter are described in Table 2. Although, essentially, parameter \( \tau_c \) is a probabilistic parameter, it is dealt with as a deterministic parameter (=0.01) for simplicity. Since the assumption means that the strength of the earth-fill against the erosion is not considered, it leads the results of the flood simulation to the safe side. Since the variability of \( n_p \) is small generally, it is assumed to be constant of 0.30 in this study. At the inflow cells of flood simulation, the flow velocity \( U \) is given for the downstream direction, and the overflow head is given as the water depth.

### 4.2 Flood simulation method

As basic governing equations, two-dimensional shallow water equations are employed, in which the flow velocity is assumed to be equally distributed along the vertical axis.

\[ \frac{\partial U}{\partial t} + \frac{\partial F}{\partial x} + \frac{\partial G}{\partial y} = S \]  \hspace{1cm} (12)

\[ U = \begin{pmatrix} h \\ u h \\ v h \end{pmatrix}, \quad F = \begin{pmatrix} u h \\ u^2 h + g h^2 / 2 \\ v u h \end{pmatrix}, \quad G = \begin{pmatrix} u h \\ v u h \\ v^2 h + g h^2 / 2 \end{pmatrix} \]  \hspace{1cm} (13)
in which \( n \) is Manning’s ratio. The equations are solved by the finite volume method (FVM), employing two dimensional rectangular cells. The FVM is the numerical method based on the integral type equation, and the analytical area is divided into finite number of cells. A group of cells objective for analysis is defined as a “control volume”. An example of the control volume is exhibited in Figure 2. The cell objective for the calculation is defined as \( \Omega \), and surrounding cell are defined as \( R \) in the figure. Following this definition, the governing equation is derived.

\[
\frac{dU_i}{dt} = -\frac{1}{\Omega_i} \sum_{j=1}^{4} \mathbf{E} \cdot \mathbf{n} \Delta \Gamma_{ij} + \mathbf{S}_i
\]  

(17)

in which \( \mathbf{E} = \mathbf{F} + \mathbf{G} \), \( \mathbf{n} \) is unit normal outward vector, \( \Omega \) is area of the cell, and \( \Delta \Gamma \) is the length of the boundary. The numerical flux \( \mathbf{E} \cdot \mathbf{n} \) is calculated by the HLL Riemann solver. Boundary conditions and input cells are shown in Figure 3. In input cells, \( h \) and \( v \) are obtained from hydrograph described in Chapter 4.1.

\[
S = S_o + S_f = \left\{ \begin{array}{l}
0 \\
ghS_{x0} \\
ghS_{y0}
\end{array} \right. + \left\{ \begin{array}{l}
0 \\
-ghS_{x1} \\
-ghS_{y1}
\end{array} \right. 
\]  

(14)

in which \( t \) is the time, \( x \) and \( y \) are the horizontal orthogonal axes, \( h \) is the water depth, \( u \) and \( v \) are flow velocities for \( x \) and \( y \) directions, \( g \) is the acceleration of gravity, and \( S_{x0} \) and \( S_{y0} \) are the inclinations along the \( x \) and \( y \) axes of the river bottom, \( S_{x1} \) and \( S_{y1} \) are the inclinations of friction. Inclination \( S_{x0} \) is obtained from the height of the bottom, \( z_b \), and the positive direction of inclination is defined to be downstream, namely,

\[
S_{x0} = -\frac{\partial z_b}{\partial x}, \quad S_{y0} = -\frac{\partial z_b}{\partial y}
\]

(15)

The inclinations of the friction are defined from Manning’s formula,

\[
S_{x1} = \frac{n^2u\sqrt{u^2+v^2}}{h^{4/3}}, \quad S_{y1} = \frac{n^2v\sqrt{u^2+v^2}}{h^{4/3}}
\]  

(16)

5 RELIABILITY-BASED DESIGN

The probability of submerge in a cell \( i \) per year is defined as:

\[
P_i = \frac{N_E}{j=1} (P_{f,j,p}) \times P_{O,i}(i)
\]

(18)

in which the subscript \( j \) corresponds to the event, which is the combination of the failure dams, \( N_E \) is number of events, \( P_f \) is probability of overflow, and \( P_s \) is the probability of submerge under the condition that the overflow happens.

The expected total cost within the lifetime period = \( t \) is given by the following equations.

\[
C_T = C_0 + \sum_{i=1}^{M} C_{f,i} \times E[n_i]
\]

(19)

\[
e[n_i] = \sum_{k=1}^{b_i} P_{o,i}(1 - P_{o,i})^{k-1} \{1 + (t_i \times P_{f,i})(k)_{i} \} (\text{Original})
\]

(20)

in which \( M \) is number of cells, \( C_T \) is the expected total cost \( n \) is the frequency of overflows within a lifetime span of \( t \) (years), \( P_o \) and \( P_f \) are the probabilities of overflow a year corresponding to the original and the improved states of the embankment, respectively, \( C_0 \) is the cost of the improvement, and \( C_f \) is the damage loss due to flooding, \( P_o \) and \( P_f \) coincide with \( P_i \) derived from Equation (18), corresponding to the two states.

In this paper, the improvement of the spillway is considered, and the improvement brings about a drastic increase in the discharge ability of the spillway. In
Equation (19), improvement cost $C_0$ is zero for the original state of the embankment, and it is assumed that when embankments are failed due to overflows, that they will be restored to the same level as the improved state of the embankment.

6 CASE STUDY

6.1 Outline of site

A group of earth-fill dams, A, B, C, and D, prescribed in Figure 4, is analyzed in this study. These dams were constructed in the valleys of a mountainous area. Profiles of the dams are described in Table 1. The broken sections of all ponds are same as in Figure 1.

Dams A and B and dams C and D are separated by a ridge, and are not affected by each other, while there are correlations for the probability of overflow between A and B or C and D. Since dam B is smaller than dam A, dam B is inevitably broken following the breaking of dam A. If the upstream dam, dam C, is broken, then dam D is inevitably broken. Dam D never breaks without also breaking dam C, according to the calculation results of the probability of overflow. Thus, dams C and D are denoted together as dam CD. The probabilities of overflow are expressed by Equation (21), considering the relationships among the dams.

\[
P_f(A \cap B) \neq 0, \quad P_f(\overline{A} \cap B) \neq 0, \quad P_f(A \cap \overline{B}) = 0 \\
P_f(C \cap D) \neq 0, \quad P_f(\overline{C} \cap D) = 0, \quad P_f(C \cap \overline{D}) = 0
\]  

(21)

The symbols A, B, C, and D are the sets employed to represent the events of breaching of each dams.

6.2 Result of flood simulation

The example of the discharge hydrographs from the dams are shown in Figure 5, which corresponds to the case in which dam CD breaks and the overflow depth is shown. As Manning’s ratio, the value, $n=0.026$ is assumed.

The expected maximum head of the flood discharge is described in Figures 6(a)-(c), which correspond to the cases, in which dams A and B break, CD breaks, and dams A, B, and CD break, respectively. In the figure, submerged depths smaller than 1.0 cm are disregarded.

In comparison among Figures 6 (a), (b) and (c), the

Figure 7. Spatial distributions of risk. (Unit: 1,000 JPY/m²)
heads of discharge of (a) and (b) are greater than (c), although the volume of discharged water is greatest among the three cases. The reason is that the flow velocity affects the results. This phenomenon means that damage in the downstream area is not necessarily correlated with the depth of submergence.

Figure 7 exhibits the spatial distributions of risk corresponding to the original and improved states of earth-fills. With improvement of the spillway, the risk is reduced dramatically, since the probability of overflow is predicted to be zero for Dam CD. The improved spillway has significantly larger value of discharge coefficient, \( C \), compared with that of the original one. The total risk and the expected costs are given in Table 3. The difference between the costs of the original and improved states means the effect of the improvement works. The value is evaluated as:

\[
335,055 - 314,170 = 20,885 \text{ (1,000 JPY)}
\]

7 CONCLUSIONS

(1) A generation method for quasi-rainfall, using random numbers, has been proposed in this research. The generated rainfall events were applied to calculate the inflow from the surrounding basin into the reservoirs.

(2) The probability of overflow caused by the heavy rains has been calculated for several earth-fill dams. The inflow and the discharge were estimated considering the effect of the reservoir storage. We assumed that overflows will occur when the maximum overflow head on the spillway bed becomes greater than the design overflow head. The probability of overflow was then determined by the Monte Carlo simulation. The probability of overflow of a dam in the studied site is calculated to be 3.47% in the maximum case.

(3) The variability of the several soil parameters of the earth-fill dams have been dealt with as probabilistic parameters to consider uncertain material values for producing the discharge hydrograph. Normal random numbers were assigned to the overflow length, the roughness height and the erosion rate coefficient. And the angular degree of the cross-section of the failed section was also provided as a probabilistic variable.

(4) The maximum head of submerge and the risk in the downstream area were estimated. Finally, the expected total costs were compared between current and improved states of the spillways, and the effect of the improvement work has been evaluated.

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