Λ_b → Λℓ⁺ℓ⁻ decay in universal extra dimensions

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Abstract

We study the exclusive Λ_b → Λℓ⁺ℓ⁻ decay in the Appelquist, Chang, Dobrescu model within one universal extra dimension. We investigate the sensitivity of the branching ratio, lepton polarization and forward–backward asymmetry \( A_{FB} \) to the compactification parameter \( 1/R \). We obtain that the branching ratio changed about 50% compared to the SM value, when \( 1/R = 200 \text{ GeV} \) and zero position of the forward–backward asymmetry is shifted to the left compared to the SM result. Therefore measurement of the branching ratio of Λ_b → Λℓ⁺ℓ⁻ decay and determination of zero position of \( A_{FB} \) are very useful in looking for new physics in the framework of the UED models.

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1 Introduction

Flavor–changing neutral current (FCNC) \( b \to s(d)\ell^+\ell^- \) transitions are forbidden in the standard model (SM) at tree level that occur at loop level, and therefore provide consistency check of the SM at quantum level. These decays induced by the FCNC are also very sensitive to the new physics beyond the SM. New physics embedded into rare decays through the Wilson coefficients which can take values different from their SM counterpart or through the new operator structures in an effective Hamiltonian (see [1] and references therein).

Among the hadronic, leptonic and semileptonic decays, the last decay channels are very significant, since they are theoretically, more or less, clean, and they have relatively larger branching ratio. From the theoretical side there are many works in which the semileptonic decay channels due to \( b \to s(d)\ell^+\ell^- \) transitions are investigated. These decays contain many observables like forward–backward asymmetry \( A_{FB} \), lepton polarization asymmetries, etc., which are very useful and serve as a testing ground for the SM and looking for new physics beyond the SM [1]. From experimental side, BELLE [2, 3] and BaBar [4, 5] collaborations provide recent measurements of the branching ratios of the semileptonic decays due to the \( b \to s\ell^+\ell^- \) transitions, which can be summarized as:

\[
B(B \to K^*\ell^+\ell^-) = \begin{cases} (16.5^{+2.2}_{-2.2} \pm 0.9 \pm 0.4) \times 10^{-7} & [2] , \\ (7.8^{+1.9}_{-1.7} \pm 1.2) \times 10^{-7} & [4] , \\ \end{cases}
\]

\[
B(B \to K\ell^+\ell^-) = \begin{cases} (5.5^{+0.75}_{-0.70} \pm 0.27 \pm 0.02) \times 10^{-7} & [2] , \\ (3.4 \pm 0.7 \pm 0.3) \times 10^{-7} & [4] , \\ \end{cases}
\]

\[
B(B \to X_s\ell^+\ell^-) = \begin{cases} (4.11 \pm 0.83^{+0.85}_{-0.81}) \times 10^{-6} & [3] , \\ (5.6 \pm 1.5 \pm 0.6 \pm 1.1) \times 10^{-6} & [5] . \\ \end{cases}
\]

Another exclusive decay which is described at inclusive level by the \( b \to s\ell^+\ell^- \) transition is the baryonic \( \Lambda_b \to \Lambda\ell^+\ell^- \) decay. Unlike mesonic decays, the baryonic decays could maintain the helicity structure of the effective Hamiltonian for the \( b \to s \) transition [6]. Radiative and semileptonic decays of \( \Lambda_b \) such as \( \Lambda_b \to \Lambda\gamma, \Lambda_b \to \Lambda_e\ell\nu_{\ell}, \Lambda_b \to \Lambda\ell^+\ell^- \) (\( \ell = e, \mu, \tau \)) and \( \Lambda_b \to \Lambda\nu\bar{\nu} \) have been extensively studied in the literature [7]–[16]. More about heavy baryons, including the experimental prospects, can be found in [7, 18].

Among the various models of physics beyond the SM, extra dimensions attract special interest, because they include gravity in addition other interactions, giving hints on the hierarchy problem and a connection with string theory. The model of Appelquist, Cheng and Dobrescu (ACD) [19] with one universal extra dimension (UED), where all the SM particles can propagate in the extra dimension, are very attractive. Compactification of the extra dimension leads to Kaluza–Klein (KK) model in the four–dimension. In this model the only additional free parameter with respect to the SM is \( 1/R \), i.e., inverse of the compactification radius.

The restrictions imposed on UED are examined in the current accelerators, for example, Tevatron experiments put the bound about \( 1/R \geq 300 \text{ GeV} \). Analysis of the anomalous
magnetic moment [20], and $Z \to b\bar{b}$ vertex [21] also lead to the bound $1/R \geq 300 \text{ GeV}$.

Possible manifestation of UED models in the $K_L - K_S$ mass difference, parameter $\varepsilon_K$, $B-\bar{B}_0$ mixing, $\Delta M_{d,s}$ mass difference, and rare decays $K^+ \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \mu^+\mu^-$, $B \to X_{s,d}\bar{\nu}\nu$, $B_{s,d} \to \mu^+\mu^-$, $B \to X_s\gamma$, $B \to X_s$ gluon, $B \to X_s\mu^+\mu^-$ and $\varepsilon'/\varepsilon$ are comprehensively investigated in [22] and [23]. Exclusive $B \to K^+\ell^+\ell^-$, $B \to K^*\bar{\nu}\nu$ and $B \to K^\ast \gamma$ decays are studied in the framework of the UED scenario in [24].

In the present work we study the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in the UED model. The plan of the paper is as follows. In section 2 we briefly discuss the main ingredients of ACD model and study the rare $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in it. Section 3 is devoted to the numerical analysis and conclusions.

## 2 Theoretical background for the $\Lambda_b \to \Lambda\ell^+\ell^-$ decay in universal extra dimension model

Before presenting a detailed derivation of the matrix element of $\Lambda_b \to \Lambda\ell^+\ell^-$ decay, let us discuss the main ingredients of ACD model, which is the minimal extension of the SM in $4+\delta$ dimensions, and we consider the simplest case $\delta = 1$. The five–dimensional ACD model with a single UED uses orbifold compactification. The fifth dimension that is compactified in a circle of radius $R$, with points $y = 0$ and $y = \pi R$ that are fixed points of the orbifolds. Generalization to the SM is realized by the propagating fermions, gauge bosons and the Higgs fields in all five dimensions. The Lagrangian can be written as

$$\mathcal{L} = \int d^4x dy \left\{ \mathcal{L}_A + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_Y \right\},$$

where

$$\mathcal{L}_A = -\frac{1}{4} W_{MN}^a W_{MN}^a - \frac{1}{4} B_{MN} B_{MN},$$

$$\mathcal{L}_H = \left( \mathcal{D}_M \phi \right)^\dagger \mathcal{D}_M \phi - V(\phi),$$

$$\mathcal{L}_F = \tilde{Q} \left( i\Gamma_M \mathcal{D}_M \right) Q + \bar{u} \left( i\Gamma_M \mathcal{D}_M \right) u + \mathcal{D} \left( i\Gamma_M \mathcal{D}_M \right) \mathcal{D},$$

$$\mathcal{L}_Y = -\tilde{Q} \bar{\nu} \phi u - \tilde{Q} \bar{\nu} \phi D + \text{h.c.}.$$  

Here $M$ and $N$ running over $0,1,2,3,5$ are the five–dimensional Lorentz indices, $W_{MN}^a = \partial_M W_N^a - \partial_N W_M^a + \bar{g} \varepsilon^{abc} W_M^b W_N^c$ are the field strength tensor for the $SU(2)_L$ electroweak gauge group, $B_{MN} = \partial_M B_N - \partial_N B_M$ are that of the $U(1)$ group, and all fields depend both on $x$ and $y$. The covariant derivative is defined as $\mathcal{D}_M = \partial_M - ig W_M^a T^a - ig' B_M Y$, where $\bar{g}$ and $\bar{g}'$ are the five–dimensional gauge couplings for the $SU(2)_L$ and $U(1)$ groups. The five–dimensional $\Gamma_M$ matrices are defined as $\Gamma^\mu = \gamma^\mu$, $\mu = 0,1,2,3$ and $\Gamma^5 = i\gamma^5$.

In the case of a single extra dimension with coordinate $x_5 = y$ compactified on a circle of radius $R$, a field $F(x,y)$ would be periodic function of $y$, hence can be written as

$$F(x,y) = \sum_{n=-\infty}^{+\infty} F_n(x) e^{iny/R}.$$
The Fourier expansion of the fields are

\[ B_\mu(x, y) = \frac{1}{\sqrt{2\pi R}} B_\mu^{(0)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B_\mu^{(n)}(x) \cos \left( \frac{ny}{R} \right), \]

\[ B_5(x, y) = \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} B_5^{(n)} \sin \left( \frac{ny}{R} \right), \]

\[ Q(x, y) = \frac{1}{\sqrt{2\pi R}} Q_L^{(0)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ Q_L^{(n)} \cos \left( \frac{ny}{R} \right) + Q_R^{(n)} \sin \left( \frac{ny}{R} \right) \right], \]

\[ U(D)(x, y) = \frac{1}{\sqrt{2\pi R}} U_R^{(0)} + \frac{1}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \left[ U_R^{(n)} \cos \left( \frac{ny}{R} \right) + U_L^{(n)} \sin \left( \frac{ny}{R} \right) \right]. \]

Under parity transformation \( P_5 : y \rightarrow -y \) fields having a correspondent in the four-dimensional SM should be even, so that their zero-mode in the KK can be interpreted as the ordinary SM field. Fields having no correspondent in the SM should be odd. From this expansion we see that fifth component of the vector field is odd under \( P_5 \) transformation.

One important property of ACD model is the KK parity is conserved. The parity conservation leads to the result that there is no tree level contribution of KK modes in low energy processes (at the scale \( \mu \ll 1/R \)) and single KK excitation cannot be produced in ordinary particle interaction. Finally note that in the ACD model there are three additional physical scalar modes \( a_n^{(0)} \) and \( a_n^\pm \).

The zero-mode is either right-handed or left-handed. The nonzero-modes come in chiral pair. This chirality is a consequence of the orbifold boundary conditions.

Lagrangian of the ACD model can be obtained by integrating over \( x_5 = y \)

\[ \mathcal{L}_A(x) = \int_0^{2\pi R} \mathcal{L}_5(x, y) dy. \]

Note that the zero-mode remains massless unless we apply the Higgs mechanism. All fields in the four-dimensional Lagrangian receive the KK mass \( n/R \) on account of the derivative operator \( \partial_5 \) acting on them. The relevant Feynman rules are derived in [22] and for more details about the ACD model we refer the interested reader to [22] and [23].

After this preliminary introduction, let us come back and discuss the main problem. In this section we present the matrix element of \( \Lambda_b \rightarrow \Lambda \ell^+\ell^- \) decay, as well as expressions of the branching ratio, forward–backward asymmetry and lepton polarizations.

At quark level, \( \Lambda_b \rightarrow \Lambda \ell^+\ell^- \) decay is described by \( b \rightarrow s\ell^+\ell^- \) transition. Effective Hamiltonian governing this transition in the SM with \( \Delta B = -1, \Delta S = 1 \) is described in terms of a set of local operators

\[ \mathcal{H} = \frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) \mathcal{O}_i(\mu), \]

where \( G_F \) is the Fermi constant, \( V_{ij} \) are the elements of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Explicit forms of the operators, which are written in terms of quark and gluon fields can be found in [24].
The Wilson coefficients in (1) have been computed at NNLO in the SM in [25]. At NLO the coefficients are calculated for the ACD model including the effects of KK modes, in [22] and [23], which we have used in our calculations. It should be noted here that, there does not appear any new operator in the ACD model, and therefore, new effects are implemented by modifying the Wilson coefficients existing in the SM, if we neglect the contributions of the scalar fields, which are indeed very small.

At $\mu = O(m_W)$ level, only $C_2^{(0)}, C_7^{(0)(m_W)}, C_8^{(0)(m_W)}, C_9^{(0)(m_W)}$ and $C_{10}^{(0)(m_W)}$ are different from zero, and the remaining coefficients are all zero.

In the following we do not consider the contribution to $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay from the lepton pair being created from $\bar{c}c$ resonance due to the $O_2$ operators. It can be removed by applying appropriate cuts to invariant dilepton mass around mass of the resonance.

In the SM, at quark level, $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is described with the help of the operators $C_7, C_9$ and $C_{10}$ as follows:

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{4\sqrt{2}} V_{tb} V_{ts}^* \left\{ C_7 \bar{s} i \sigma_{\mu\nu} (1 + \gamma_5) q^\nu \bar{\ell} \gamma^\mu \ell + C_9 \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \ell \right\} + C_{10} \bar{s} \gamma_\mu (1 - \gamma_5) b \bar{\ell} \gamma^\mu \gamma_5 \ell \right\}.$$  

(2)

The renormalization scheme independent coefficient $C_7^{(0)\text{eff}}$ [26] is given by

$$C_7^{(0)\text{eff}}(\mu_b) = \eta^{16}_{23} C_7^{(0)}(\mu_W) + \frac{8}{3} \left( \eta^{14}_{23} - \eta^{16}_{23} \right) C_8^{(0)}(\mu_W) + C_2^{(0)}(\mu_W) \sum_{i=1}^{8} h_i \eta^{a_i},$$  

(3)

where

$$\eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)},$$

and

$$C_2^{(0)}(\mu_W) = 1, \quad C_7^{(0)}(\mu_W) = -\frac{1}{2} D'(x_t, 1/R), \quad C_8^{(0)}(\mu_W) = -\frac{1}{2} E'(x_t, 1/R),$$

with the superscript (0) referring to leading log approximation, and coefficients $a_i$ and $h_i$ are given as

$$a_1 = \frac{14}{23}, \quad a_2 = \frac{16}{23}, \quad a_3 = \frac{6}{23}, \quad a_4 = -\frac{12}{23}, \quad a_5 = 0.4086, \quad a_6 = -0.4230,$$

$$a_7 = -0.8994, \quad a_8 = 0.1456$$

$$h_1 = 2.2996, \quad h_2 = -1.0880, \quad h_3 = -\frac{3}{7}, \quad h_4 = -\frac{1}{14}, \quad h_5 = -0.6494, \quad h_6 = -0.0380,$$

$$h_7 = -0.0185, \quad h_8 = -0.0057$$

The functions $D'$ and $E'$, which describe electromagnetic and chromomagnetic penguins, respectively, are calculated in [22] and [23], and lead to the following results (see also [24]).
\[ D'_0(x_t) = -\frac{(8x_t^3 + 5x_t^2 - 7x_t)}{12(1 - x_t)^3} + \frac{x_t^2(2 - 3x_t)}{2(1 - x_t)^4} \ln x_t, \]  
\[ E'_0(x_t) = -\frac{x_t(x_t^2 - 5x_t - 2)}{4(1 - x_t)^3} + \frac{3x_t^2}{2(1 - x_t)^4} \ln x_t, \]  
\[ D'_n(x_t, x_n) = \frac{x_t[-37 + 44x_t + 17x_t^2 + 6x_n^2(10 - 9x_t + 3x_t^2) - 3x_n(21 - 54x_t + 17x_t^2)]}{36(x_t - 1)^3} \]  
\[ + \frac{x_n(2 - 7x_n + 3x_n^2)}{6} \ln \frac{x_n}{1 + x_n} \]  
\[ + \frac{(2 - x_n - 3x_t)[x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n + (10 - x_t)x_nx_t]}{6(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n}, \]  
\[ E'_n(x_t, x_n) = \frac{x_t[-17 - 8x_t + x_t^2 - 3x_n(21 - 6x_t + x_t^2) - 6x_n^2(10 - 9x_t + 3x_t^2)]}{12(x_t - 1)^3} \]  
\[ - \frac{1}{2}x_n(1 + x_n)(-1 + 3x_n) \ln \frac{x_n}{1 + x_n} \]  
\[ + \frac{(1 + x_n)[x_t + 3x_t^2 + x_n^2(3 + x_t) - x_n + (10 - x_t)x_nx_t]}{2(x_t - 1)^4} \ln \frac{x_n + x_t}{1 + x_n}, \]

where the functions with and without \( x_n \) correspond to the KK and SM excitation contributions, respectively. Using the prescription presented in [22] and [23], and summing over \( n \), we get the following expressions:

\[ \sum_{n=1}^{\infty} D'_n(x_t, x_n) = \frac{x_t[37 - x_t(44 + 17x_t)]}{72(x_t - 1)^3} \]  
\[ + \frac{\pi M_W R}{12} \left[ \int_0^1 \frac{dy}{y} (2y^{1/2} + 7y^{3/2} + 3y^{5/2}) \coth(\pi M_W R \sqrt{y}) \right] \]  
\[ - \frac{x_t(2 - 3x_t)}{(x_t - 1)^4} J(R, -1/2) \]  
\[ - \frac{1}{(x_t - 1)^4} \left\{ x_t(1 + 3x_t) + (2 - 3x_t)[1 - (10 - x_t)x_t] \right\} J(R, 1/2) \]  
\[ - \frac{1}{(x_t - 1)^4} \left\{ (2 - 3x_t)(3 + x_t) + 1 - (10 - x_t)x_t \right\} J(R, 3/2) \]  
\[ - \frac{(3 + x_t)(1 + 3x_t)}{(x_t - 1)^4} J(R, 5/2) \]  
\[ \sum_{n=1}^{\infty} E'_n(x_t, x_n) = \frac{x_t[17 + (8 - x_t)x_t]}{24(x_t - 1)^3} \]  
\[ + \frac{\pi M_W R}{4} \left[ \int_0^1 \frac{dy}{y} (y^{1/2} + 2y^{3/2} - 3y^{5/2}) \coth(\pi M_W R \sqrt{y}) \right] \]  
\[ - \frac{x_t(1 + 3x_t)}{(x_t - 1)^4} J(R, -1/2) \]
\[ + \frac{1}{(x_t - 1)^4} [x_t (1 + 3x_t) - 1 + (10 - x_t)x_t] J(R, 1/2) \]
\[ - \frac{1}{(x_t - 1)^4} [(3 + x_t) - 1 + (10 - x_t)x_t] J(R, 3/2) \]
\[ + \frac{(3 + x_t)}{(x_t - 1)^4} J(R, 5/2) \], \quad (9)\]

where

\[ J(R, \alpha) = \int_0^1 dy y^\alpha \left[ \coth(\pi M_W R \sqrt{y}) - x_t^{1+\alpha} \coth(\pi m_t R \sqrt{y}) \right] . \quad (10)\]

The Wilson coefficient \( C_9 \) in the ACD model and in the NDR scheme is

\[ C_9(\mu) = P_0^{\text{NDR}} + \frac{Y(x_t, 1/R)}{\sin^2 \theta_W} - 4Z(x_t, 1/R) + P_E E(x_t, 1/R) , \quad (11)\]

where \( P_0^{\text{NDR}} = 2.60 \pm 0.25 \) [27] and the last term is numerically negligible. The functions \( Y(x_t, 1/R) \) and \( Z(x_t, 1/R) \) are defined as:

\[ Y(x_t, 1/R) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n) , \]
\[ Z(x_t, 1/R) = Z_0(x_t) + \sum_{n=1}^{\infty} C_n(x_t, x_n) , \quad (12)\]

with

\[ Y_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t - 4}{x_t - 1} + \frac{3x_t}{(x_t - 1)^2} \ln x_t \right] , \]
\[ Z_0(x_t) = \frac{18x_t^4 - 163x_t^3 + 259x_t^2 - 108x_t}{144(x_t - 1)^3} + \left[ \frac{32x_t^4 - 38x_t^3 - 15x_t^2 + 18x_t}{72(x_t - 1)^4} - \frac{1}{9} \right] \ln x_t \]
\[ C_n(x_t, x_n) = \frac{x_t}{8(x_t - 1)^2} \left[ x_t^2 - 8x_t + 7 + (3 + 3x_t + 7x_n - x_t x_n) \ln \frac{x_t + x_n}{1 + x_n} \right] , \quad (13)\]

and

\[ \sum_{n=1}^{\infty} C_n(x_t, x_n) = \frac{x_t(7 - x_t)}{16(x_t - 1)} - \frac{\pi M_W R x_t}{16(x_t - 1)^2} \left[ 3(1 + x_t) J(R, -1/2) + (x_t - 7) J(R, 1/2) \right] . \quad (14)\]

Finally the Wilson coefficient \( C_{10} \), which is scale independent, is given by:

\[ C_{10} = -\frac{Y(x_t, 1/R)}{\sin^2 \theta_W} . \quad (15)\]

With these coefficients and the operators in (1) the inclusive \( b \to s \ell^+ \ell^- \) transitions have been studied in [22, 23].
The amplitude of the exclusive \( \Lambda_b \to \Lambda \ell^+ \ell^- \) decay is obtained by calculating the matrix element of the effective Hamiltonian for the \( b \to s \ell^+ \ell^- \) transition between initial and final baryon states \( \langle \Lambda | \mathcal{H}_{\text{eff}} | \Lambda_b \rangle \). It follows from Eq. (16) that the matrix elements

\[
\langle \Lambda | \bar{s} \gamma_\mu (1 - \gamma_5) b | \Lambda_b \rangle ,
\langle \Lambda | \bar{s} \sigma_{\mu \nu} (1 + \gamma_5) b | \Lambda_b \rangle ,
\]

are needed in order to calculate the \( \Lambda_b \to \Lambda \ell^+ \ell^- \) decay amplitude.

These matrix elements parametrized in terms of the form factors are as follows (see [14, 28])

\[
\langle \Lambda | \bar{s} \gamma_\mu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ f_1 \gamma_\mu + i f_2 \sigma_{\mu \nu} q^\nu + f_3 q_\mu \right] u_{\Lambda_b} ,
\]

\[
\langle \Lambda | \bar{s} \gamma_\mu \gamma_5 b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ g_1 \gamma_\mu \gamma_5 + i g_2 \sigma_{\mu \nu} \gamma_5 q^\nu + g_3 q_\mu \gamma_5 \right] u_{\Lambda_b} ,
\]

where \( q = p_{\Lambda_b} - p_{\Lambda} \).

The form factors of the magnetic dipole operators are defined as

\[
\langle \Lambda | \bar{s} \sigma_{\mu \nu} q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ f_1^T \gamma_\mu + i f_2^T \sigma_{\mu \nu} q^\nu + f_3^T q_\mu \right] u_{\Lambda_b} ,
\]

\[
\langle \Lambda | \bar{s} \sigma_{\mu \nu} \gamma_5 q^\nu b | \Lambda_b \rangle = \bar{u}_\Lambda \left[ g_1^T \gamma_\mu \gamma_5 + i g_2^T \sigma_{\mu \nu} \gamma_5 q^\nu + g_3^T q_\mu \gamma_5 \right] u_{\Lambda_b} .
\]

Using the identity

\[
\sigma_{\mu \nu} \gamma_5 = -\frac{i}{2} \epsilon_{\mu \nu \alpha \beta} \sigma^{\alpha \beta} ,
\]

the following relations between the form factors are obtained:

\[
f_1^T = -\frac{q^2}{m_{\Lambda_b} - m_{\Lambda}} f_3^T ,
\]

\[
g_1^T = \frac{q^2}{m_{\Lambda_b} + m_{\Lambda}} g_3^T .
\]

Using these definitions of the form factors, for the matrix element of the \( \Lambda_b \to \Lambda \ell^+ \ell^- \) we get

\[
\mathcal{M} = \frac{G_F}{4\sqrt{2} \pi} V_{tb} V_{ts}^* \frac{1}{2} \left\{ \bar{\ell} \gamma_\mu (1 - \gamma_5) \ell \bar{u}_\Lambda \left[ (A_1 - D_1) \gamma_\mu (1 + \gamma_5) + (B_1 - E_1) \gamma_\mu (1 - \gamma_5) 
\right.
\right.
\]

\[
+ i \sigma_{\mu \nu} q^\nu \left[ (A_2 - D_2)(1 + \gamma_5) + (B_2 - E_2)(1 - \gamma_5) \right]
\]

\[
+ q_\mu \left[ (A_3 - D_3)(1 + \gamma_5) + (B_3 - E_3)(1 - \gamma_5) \right] \right\} u_{\Lambda_b}
\]

\[
+ \bar{\ell} \gamma_\mu (1 + \gamma_5) \ell \bar{u}_\Lambda \left[ (A_1 + D_1) \gamma_\mu (1 + \gamma_5) + (B_1 + E_1) \gamma_\mu (1 - \gamma_5) 
\right.
\]

\[
+ i \sigma_{\mu \nu} q^\nu \left[ (A_2 + D_2)(1 + \gamma_5) + (B_2 + E_2)(1 - \gamma_5) \right]
\]

\[
+ q_\mu \left[ (A_3 + D_3)(1 + \gamma_5) + (B_3 + E_3)(1 - \gamma_5) \right] \right\} u_{\Lambda_b} ,
\]

\[
\]
where
\[ A_1 = \frac{1}{q^2} \left( f_1^T - g_1^T \right) (-2m_s C_7) + \frac{1}{q^2} \left( f_1^T + g_1^T \right) (-2m_b C_7) + (f_1 - g_1) C_{eff}^0, \]
\[ A_2 = A_1 (1 \to 2), \]
\[ A_3 = A_1 (1 \to 3), \]
\[ B_1 = A_1 \left( g_1 \to -g_1; g_1^T \to -g_1^T \right), \]
\[ B_2 = B_1 (1 \to 2), \]
\[ B_3 = B_1 (1 \to 3), \]
\[ D_1 = C_{10} (f_1 - g_1), \]
\[ D_2 = D_1 (1 \to 2), \]
\[ D_3 = D_1 (1 \to 3), \]
\[ E_1 = D_1 \left( g_1 \to -g_1 \right), \]
\[ E_2 = E_1 (1 \to 2), \]
\[ E_3 = E_1 (1 \to 3). \]

From these expressions it follows that \( \Lambda_b \to \Lambda \ell^+ \ell^- \) decay is described in terms of many form factors. It is shown in [29] that Heavy Quark Effective Theory reduces the number of independent form factors to two (\( F_1 \) and \( F_2 \)) irrelevant of the Dirac structure of the corresponding operators, i.e.,
\[ \langle \Lambda(p_{\Lambda}) | s \Gamma b | \Lambda(p_{\Lambda_b}) \rangle = \bar{u}_{\Lambda} \left[ F_1(q^2) + \gamma \Gamma u_{\Lambda_b} \right], \]
where \( \Gamma \) is an arbitrary Dirac structure and \( \nu^\mu = p_{\Lambda_b}^\mu/m_{\Lambda_b} \) is the four–velocity of \( \Lambda_b \).

Comparing the general form of the form factors given in Eqs. (22)–(20) with (23), one can easily obtain the following relations among them (see also [14, 15, 28])
\[ g_1 = f_1 = f_2^T = g_2^T = F_1 + \sqrt{\hat{r}_\Lambda} F_2, \]
\[ g_2 = f_2 = g_3 = f_3 = \frac{F_2}{m_{\Lambda_b}}, \]
\[ g_1^T = f_1^T = \frac{F_2}{m_{\Lambda_b}} q^2, \]
\[ g_3^T = \frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} + m_{\Lambda}), \]
\[ f_3^T = -\frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} - m_{\Lambda}), \]
where \( \hat{r}_\Lambda = m_{\Lambda_b}^2/m_{\Lambda_b}. \)

In what follows, we will be looking for the possible manifestation of UED theory in branching ratio, as well as in lepton polarizations. For this purpose, we present the decay rate for the \( \Lambda_b \to \Lambda \ell^+ \ell^- \) taking into account lepton polarizations.

In the rest frame of lepton \( \ell^- \) the unit vectors along the longitudinal, normal and transversal components of the \( \ell^- \) are chosen as:
\[ s_{L_\mu}^- = \left( 0, \vec{e}_{L^-} \right) = \left( 0, \frac{\vec{p}_-}{|\vec{p}_-|} \right), \]

\[ 8 \]
\[ s_N^\mu = (0, \vec{e}_N) = \left(0, \frac{\vec{p}_\Lambda \times \vec{p}_{-}}{|\vec{p}_\Lambda \times \vec{p}_{-}|}\right), \]
\[ s_T^\mu = (0, \vec{e}_T) = \left(0, \vec{e}_N^\perp \times \vec{e}_L^\perp\right), \]  
(26)

where \( \vec{p}_- \) and \( \vec{p}_\Lambda \) are the three–momenta of the leptons \( \ell^- \) and \( \Lambda \) baryon in the center of mass frame (CM) of \( \ell^- \ell^+ \) system, respectively.

The longitudinal component of the lepton polarization is boosted to the CM frame of the lepton pair by Lorentz transformation, yielding \( \left(s_L^\mu\right)_{CM} = \left(\frac{|\vec{p}_{-}|}{m_\ell}, \frac{E_\ell \vec{p}_{-}}{m_\ell |\vec{p}_{-}|}\right) \),

(27)

where \( E_\ell \) and \( m_\ell \) are the energy and mass of \( \ell^- \) in the CM frame. The remaining two unit vectors \( s_N^\mu, s_T^\mu \) are unchanged under Lorentz boost.

The differential decay rate for \( \Lambda_b \rightarrow \Lambda \ell^+\ell^- \) decay along any spin direction \( \vec{\xi}^\pm \) along the \( \ell^\pm \) can be written as:

\[
\frac{d\Gamma(\vec{\xi}^\pm)}{d\hat{s}} = \frac{1}{2} \left(\frac{d\Gamma}{d\hat{s}}\right)_0 \left[1 + \left(\cal{P}_L^\pm \vec{e}_L^\mp + \cal{P}_N^\pm \vec{e}_N^\mp + \cal{P}_T^\pm \vec{e}_T^\mp\right) \cdot \vec{\xi}^\pm\right],
\]  
(28)

where \( (d\Gamma/d\hat{s})_0 \) corresponds to the unpolarized differential decay rate, \( \hat{s} = q^2/m_{\Lambda_b}^2 \) and \( \cal{P}_L, \cal{P}_N \) and \( \cal{P}_T \) represent the longitudinal, normal and transversal polarizations of \( \ell \), respectively, and has the following form:

\[
\left(\frac{d\Gamma}{d\hat{s}}\right)_0 = \frac{G^2 \alpha^2}{8192 \pi^3} |V_{tb} V_{ts}^*|^2 \lambda^{1/2}(1, r, \hat{s}) v \left[\cal{T}_0(\hat{s}) + \frac{1}{3} \cal{T}_2(\hat{s})\right],
\]  
(29)

where \( \lambda(1, r, \hat{s}) = 1 + r^2 + \hat{s}^2 - 2r - 2\hat{s} - 2r\hat{s} \) is the triangle function and \( v = \sqrt{1 - 4m_\ell^2/q^2} \) is the lepton velocity.

The polarizations \( \cal{P}_L, \cal{P}_N \) and \( \cal{P}_T \) are defined as:

\[
\cal{P}_i^{(\mp)}(\hat{s}) = \frac{d\Gamma(\vec{\xi}^\mp = \vec{e}_i^\mp)}{d\hat{s}} - \frac{d\Gamma(\vec{\xi}^\mp = -\vec{e}_i^\mp)}{d\hat{s}},
\]

where \( i = L, N, T \).

One of the efficient tools for establishing new physics effects is study of the forward–backward asymmetry \( A_{FB} \) which is defined as

\[
A_{FB} = \frac{\int_0^1 \frac{d\Gamma}{d\hat{s}dz} dz - \int_0^0 \frac{d\Gamma}{d\hat{s}dz} dz}{\int_0^1 \frac{d\Gamma}{d\hat{s}dz} dz + \int_0^0 \frac{d\Gamma}{d\hat{s}dz} dz},
\]

where \( z = \cos \theta \) dependence of the differential decay rate can be implemented by making the replacement

\[
\cal{T}_0(\hat{s}) + \frac{1}{3} \cal{T}_2(\hat{s}) \rightarrow \cal{T}_0(\hat{s}) + \cal{T}_1(\hat{s}) z + \cal{T}_2(\hat{s}) z^2,
\]
on the right-hand side of Eq. (28), and $\theta$ is the angle between $\Lambda_b$ and $\ell^-$ in the CM of leptons. Explicit expressions of $\mathcal{T}_0(\hat{s})$, $\mathcal{T}_1(\hat{s})$ and $\mathcal{T}_2(\hat{s})$ can be found in [14].

It is well known that in the $B \rightarrow K^* \ell^+ \ell^-$ decay the zero–position of $A_{FB}$ is practically independent of the form factors [28]. For this reason, determination of zero–position of $A_{FB}$, as well as it magnitude, is very promising in looking for new physics beyond the SM. Note also that the combined analysis of the lepton polarizations can give additional information about the existence of new physics since in the SM $P^+_L + P^-_L = 0$, $P^+_N + P^-_N = 0$ and $P^+_T - P^-_T \approx 0$ (in the $m_\ell \rightarrow 0$ limit). Therefore any nonzero value resulting from these combined polarizations can be considered as an confirmation of new physics.

3 Numerical analysis

In this section we present our numerical results for the polarization asymmetries $P_L$, $P_N$ and $P_T$ when one of the leptons is polarized. The values of the input parameters we use in our calculations are: $|V_{tb}V^*_{ts}| = 0.0385$, $m_\tau = 1.77\text{ GeV}$, $m_\mu = 0.106\text{ GeV}$. $m_b = 4.8\text{ GeV}$ [30], $m_t = 172.7\text{ GeV}$ [31] and $\tau_{B_0} = 1.527 \pm 0.008$.

From the expressions of asymmetries it follows that the form factors are the main and the most important input parameters necessary in the numerical calculations. The calculation of the form factors of $\Lambda_b \rightarrow \Lambda$ transition does not exist at present. But, we can use the results from QCD sum rules in corporation with HQET [29, 32]. We noted earlier that, HQET allows us to establish relations among the form factors and reduces the number of independent form factors into two. In [29, 32], the $q^2$ dependence of these form factors are given as follows

$$F(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2}.$$  

The values of the parameters $F(0)$, $a_F$ and $b_F$ are given in table 1.

|   | $F(0)$ | $a_F$   | $b_F$   |
|---|--------|---------|---------|
| $F_1$ | 0.462  | -0.0182 | -0.000176 |
| $F_2$ | -0.077 | -0.0685 | 0.00146  |

Table 1: Form factors for $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in a three parameter fit.

Note that the first analysis of the HQET structure of the $\Lambda_Q \rightarrow \Lambda_q$ transition is performed in [33] (see also [34]).

In order to have an idea about the sensitivity of our results to the specific parametrization of the two form factors predicted by the QCD sum rules in corporation with the HQET, we also have used another parametrization of the form factors based on the pole model and compared the results of both models. The second set of form factors which have the dipole form predicted by the pole model are given as:

$$F_{1,2}(E_\Lambda) = N_{1,2} \left( \frac{\Lambda_{QCD}}{\Lambda_{QCD} + E_\Lambda} \right)^2,$$

Note that the first analysis of the HQET structure of the $\Lambda_Q \rightarrow \Lambda_q$ transition is performed in [33] (see also [34]).
where
\[
E_\Lambda = \frac{m_{\Lambda_b}^2 - m_\Lambda^2 - q^2}{2m_{\Lambda_b}},
\]
and \( \Lambda_{QCD} = 0.2, N_1 = 52.32 \) and \( N_2 \simeq -0.25N_1 \) [35].

In Figs. 1 and 2 we present the dependence of the branching ratio for the \( \Lambda_b \to \Lambda\mu^+\mu^- \)
and \( \Lambda_b \to \Lambda\tau^+\tau^- \) decays on the compactification parameter \( 1/R \), respectively. For a comparison, in these figures we also present the SM prediction for this decay using both set of form factors. From these figures we see that the branching ratios are larger compared to the SM result for both decay modes and practically they are insensitive to the form factors. The value of the branching ratio for \( 1/R = 200 \text{ GeV} \) is approximately 50% larger compared to that of the SM prediction. But this difference decreases as \( 1/R \) gets larger. Therefore measurement of the value of branching ratio of the \( \Lambda_b \to \Lambda\ell^+\ell^- \), \( \ell = \mu, \tau \) decays can serve as a good test for physics beyond the SM.

Analysis of the lepton polarizations leads to the following results:

- Maximum value of the difference between the SM and ADC models (for the minimum value of \( 1/R = 200 \text{ GeV} \) predictions, as far as longitudinal polarization is concerned, is about 10%.

- Practically, there is no difference between the predictions of the SM and ACD models for the \( \tau \) lepton with longitudinal polarization.

- These two models lead to the same result for the \( \mu \) lepton with transversal polarization case.

- Up to \( s = 0.6 \), the maximum difference in the predictions of these two models is about 12% for the \( \tau \) lepton with transversal polarization case.

- Normal polarization of lepton contributes very small and the difference between predictions of the two models for this polarization can never be measured in the experiments. For the \( \tau \) lepton case the difference \( (P_N)_{ACD} - (P_N)_{SM} \approx 0.5\% \) is quite a challenging to be measured.

From this discussion we conclude that measurement of polarizations of lepton is not useful for establishing the UED models.

We can now discuss the prediction of the ACD model for the forward–backward asymmetry. In Fig. 3 we show the dependence of of \( A_{FB} \) on \( \hat{s} \) at four fixed values of \( 1/R \) and SM, for the \( \Lambda_b \to \Lambda\mu^+\mu^- \) decay when the first set of form factors are considered. For the sake of completeness, we also present in this figure the forward–backward asymmetry prediction of SM. From this figure we see that the zero–position of \( A_{FB} \) is sensitive to the compactification parameter \( 1/R \), similar to the \( B \to K^*\ell^+\ell^- \) decay case and in all cases it is shifted to the left compared to that of the SM prediction. Therefore, experimental determination of zero–position \( A_{FB} \) can give invaluable information about new physics effects. It should be noted that the zero position of \( A_{FB} \) is practically insensitive to the choice of form factors and it coincides for both sets.
In conclusion, we have studied the rare $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay in the ACD model with a single universal extra dimension. We investigate the sensitivity of the branching ratio, lepton polarizations and lepton forward–backward asymmetry in the $\Lambda_b \rightarrow \Lambda \ell^+\ell^-$ decay to the compactification parameter $1/R$. We found that the branching ratio and zero–position of the forward–backward asymmetry are very sensitive to the presence of the compactification parameter $1/R$ and can be useful for establishing new physics effects due to the compactification of the fifth dimension.

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Figure captions

**Fig. 1** The dependence of the branching ratio for the \( \Lambda_b \to \Lambda \mu^+ \mu^- \) decay on the compactification parameter \( 1/R \), for the first set of form factors I and the second set of form factors II, respectively.

**Fig. 2** The same as Fig. 1, but for the \( \Lambda_b \to \Lambda \tau^+ \tau^- \) decay.

**Fig. 3** The dependence of the forward–backward asymmetry \( A_{FB} \) on \( \hat{s} \) at four fixed values of \( 1/R \) and in the SM, for the \( \Lambda_b \to \Lambda \mu^+ \mu^- \) decay.
Figure 1:

$10^6 \times B (\Lambda_0 \to \Lambda \mu^+ \mu^-)$

$1/R (GeV)$

Figure 2:

$10^7 \times B (\Lambda_0 \to \Lambda \tau^+ \tau^-)$

$1/R (GeV)$
Figure 3: