Neutralino and chargino masses and related sum rules beyond MSSM

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A B S T R A C T
We study the implications of dimension five operators involving Higgs chiral superfields for the masses of neutralinos and charginos in the minimal supersymmetric standard model (MSSM). These operators can arise from additional interactions beyond those of MSSM involving new degrees of freedom at or above the TeV scale. In addition to the masses of the neutralinos and charginos, we study the sum rules involving the masses and squared masses of these particles for different gaugino mass patterns in presence of the dimension five operators. We derive a relation for the higgsino mixing mass parameter and tan β in the presence of the dimension five operators.

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1. Introduction

Supersymmetry (SUSY) is a leading candidate [1] for physics beyond the Standard Model (SM). In supersymmetric theories the Higgs sector, so essential for the internal consistency of the SM, is technically natural [2]. Supersymmetry is, however, not an exact symmetry in nature, and the manner in which SUSY is broken is not known. The necessary SUSY breaking can be introduced through soft supersymmetry breaking terms that do not disturb the stability of the hierarchy between the weak scale and the large (grand unified or Planck) scale. The simplest implementation of the idea of low energy broken supersymmetry is the minimal supersymmetric standard model (MSSM) obtained by introducing the supersymmetric partners of the SM states, and introducing an additional Higgs doublet with opposite hypercharge to that of SM Higgs doublet, in order to cancel the gauge anomalies and generate masses for all the fermions of the Standard Model, with soft supersymmetry breaking terms generated by a suitable supersymmetry breaking mechanism [3]. In order for broken supersymmetry to be effective in protecting the weak scale against large radiative corrections, the supersymmetric partners of the SM particles should have masses of the order of O (TeV).

Because of underlying gauge invariance and supersymmetry, the Higgs sector of the MSSM is highly constrained. The LEP lower bound [4] on the Standard Model Higgs boson is \( m_{h_{SM}} \gtrsim 114 \text{ GeV} \). Although the tree-level upper bound \( m_{h} \leq M_Z \) on the lightest Higgs boson of MSSM is violated by the LEP bound, there are large radiative corrections to the tree-level mass coming from the top–stop loops [5–7]. If these radiative corrections have to be significant, then one of the stop mass eigenstates has to be heavy. On the other hand, for these radiative corrections to account for the current lower limit on the lightest Higgs boson mass, or for the possible Higgs mass \( m_h \sim 125 \text{ GeV} \), as hinted by the Large Hadron Collider (LHC) experiments, the top squarks must be so massive that it makes the MSSM to be finely tuned. Alternatively, there must be large left–right mixing between scalar top quarks. Such large mixing is difficult to obtain in specific models, and can arise only in special regions in the parameter space [8]. Currently the allowed range of the lightest Higgs mass from the LHC experiments, which is of interest for supersymmetric models, extends from the LEP limit to around 130 GeV [9]. All this suggests that there may be additional degrees of freedom in the theory beyond those of the MSSM [10]. The effect of possible new degrees of freedom, evaluated in terms of effective dimension five and six operators have been found to be significant for the Higgs boson mass, see e.g. [11]. As pointed out by Dine, Seiberg and Thomas [10], at dimension five only two operators are relevant for the Higgs boson sector. Several aspects of dimension five operators have been studied in recent years, including neutralino and chargino sector in the context of dark matter [12].

It is interesting to note that there are several candidates for such additional physics beyond the MSSM [13–17]. If this new physics lies at an energy scale which is above the masses of the MSSM degrees of freedom (we call it \( M \)), then it is convenient to study the effects...
of such additional degrees of freedom by using an effective Lagrangian approach from which the physics at scale \( M \) has been integrated out. The most general superpotential for the MSSM, which involves only the Higgs chiral superfields, up to dimension five can be written as [10]

\[
W_2 = \mu H_u H_d + \frac{\lambda}{M} (H_u H_d)^2,
\]

where \( \mu \) is the higgsino mixing parameter in the superpotential of MSSM, \( M \) is an energy scale which is much above the typical masses of the superparticles of MSSM, and \( \lambda \) is a dimensionless coupling. It has been shown that the dimension five operator in (1.1) raises the lightest Higgs boson mass of MSSM above the LEP limit without fine tuning, and, hence, without loss of naturalness [10].

Apart from the supersymmetry conserving dimension five operator in (1.1), there is another dimension five operator which involves supersymmetry breaking and can be represented by a dimensionless chiral spurion superfield \( [10] \). How ever, if in Section 3 we present our results for the spectrum of charginos and neutralinos, and the effect of dimension five operator on this spectrum.

Further, we discuss sum rules involving the masses and squared masses of neutralinos and charginos which can be used to study the effect of the dimension five operator. We conclude with a summary in Section 4.

2. Neutralino and chargino mass matrices

2.1. Higgsino sector

The superpotential (1.1) leads, up to dimension five, to the following interaction Lagrangian involving only the higgsino (\( \tilde{H}_u, \tilde{H}_d \)) and the Higgs (\( H_u, H_d \)) fields [10]:

\[
\mathcal{L} = \mu (\tilde{H}_u \tilde{H}_d) - \frac{\epsilon_1}{\mu^*} \left[ 2(H_u H_d)(\tilde{H}_u \tilde{H}_d) + 2(\tilde{H}_u H_d)(H_u \tilde{H}_d) + (H_u \tilde{H}_d)(\tilde{H}_u H_d) \right] + \text{H.c.},
\]

where \( \mu \) is the higgsino mixing parameter in the superpotential of MSSM, \( \mu \) is an energy scale which is much above the typical masses of the superparticles of MSSM, and \( \lambda \) is a dimensionless coupling. It has been shown that the dimension five operator in (1.1) raises the lightest Higgs boson mass of MSSM above the LEP limit without fine tuning, and, hence, without loss of naturalness [10].

We will assume here that the \( R \)-parity, \( R_P = (-1)^{3B+2L+2S} \), is conserved, leading to a stable lightest supersymmetric particle (LSP). In the models that we will consider, it is the lightest neutralino, and thus the other \( R \)-odd particles will finally decay to it. Here we will study the effects of the dimension five operator (1.1) on the spectrum of neutralinos and charginos. In this work we will concentrate on different supersymmetry breaking mechanisms, which lead to different mass patterns for the gaugino mass parameters, and the implications of the dimension five operator (1.1) for these mass patterns. In particular, we will demonstrate that using sum rules specific for the neutralino and chargino sector, one could distinguish between different breaking patterns in presence of the dimension 5 operator. We will also derive a formula for the \( \mu \)-parameter as a function of \( \tan \beta \), and we will also consider determining the amount of the dimension five contribution using the sum rules.

In Section 2 we write down the mass matrices for the neutralinos and charginos in the presence of the dimension five operator (1.1). We review the experimental constraints on the parameters of the neutralino and chargino mass matrices, and discuss relevant aspects of different patterns for the soft supersymmetry breaking gaugino mass parameters that arise in models of low energy supersymmetry. In Section 3 we present our results for the spectrum of charginos and neutralinos, and the effect of dimension five operator on this spectrum. Further, we discuss sum rules involving the masses and squared masses of neutralinos and charginos which can be used to study the effect of the dimension five operator. We conclude with a summary in Section 4.
Bounds on $\epsilon_1$ have been discussed in [12]. The dimension five operator (1.1) causes a shift in the mass of the lightest Higgs boson, and if one assumes shift in $m_{\tilde{g}}$ to be at most 20%–30%, then $\epsilon_1$ is constrained to values smaller than 0.05 [12]. Larger shifts in the Higgs mass could in principle disrupt the vacuum stability by creating a new global minimum for the potential. This issue was examined in [18], and a criterion was found to exclude transitions to such a vacuum. Furthermore, $\epsilon_1$ is restricted by the scale of new physics appearing beyond the MSSM. If the scale of new physics is taken to be $M/\lambda > 5$ TeV, then using (2.2) one arrives at a limit $|\epsilon_1| \lesssim 0.04$ for $\mu = 200$ GeV, whereas $M/\lambda > 2$ TeV allows for $|\epsilon_1| \lesssim 0.1$. This limit is further increased at larger values of $\mu$. However, for large $\mu$ the lightest neutralino and chargino are mostly gauginos and the contribution from $\epsilon_1$ to their masses is much less significant. In the following we limit our discussion to $|\epsilon_1| \lesssim 0.1$.

We note that the dimension five operator (1.1) contributes to the lower right $2 \times 2$ submatrix of the neutralino mass matrix (2.3). Furthermore, this operator also contributes to the $(2, 2)$ element of the chargino mass matrix. We have included the most significant MSSM one-loop radiative corrections to the neutralino and chargino mass matrices in our analysis. Although these loop corrections are small (of the order of few GeV), these corrections in the $(3, 3)$ and $(4, 4)$ elements of the neutralino mass matrix can be important, since these elements vanish in the absence of dimension five contribution.

2.2. Experimental constraints

Collider experiments have searched for the supersymmetric partners of the Standard Model particles. No supersymmetric partners of the SM particles have been found in these experiments. At present only lower limits on their masses have been obtained. In particular, the search for the lightest chargino state at LEP has yielded lower limits on its mass [19]. The limit depends on the spectrum of the model [20]. Assuming that $m_0$ is large, from the chargino pair production one obtains the lower bound

$$M_{\tilde{\chi}_1^\pm} \gtrsim 103 \text{ GeV.} \quad (2.5)$$

For small $m_0$, the bound is lowered, so that for $m_0 < 200$ GeV, but $m_0 > m_{\tilde{\chi}_1^\pm}$, the limit becomes [20]

$$M_{\tilde{\chi}_1^\pm} \gtrsim 85 \text{ GeV.} \quad (2.6)$$

For the parameters of the chargino mass matrix (2.5) implies an approximative lower limit [21,22]

$$M_{\tilde{\chi}_1^\pm}, \mu \gtrsim 100 \text{ GeV}. \quad (2.7)$$

The limits (2.7) on the parameters $M_2$ and $\mu$ are found from scanning over the MSSM parameter space and are thus model independent.

Another important constraint for parameters in the SUSY models comes from the mass of the lightest Higgs boson. The current lower limit on the mass of the lightest Higgs boson from LEP is 144.4 GeV. Including theoretical uncertainties from NNLO and higher corrections [23] will decrease the limit by around 3 GeV, and in our calculations we will use the lower limit of 111 GeV. The LHC experiments have found indications for a particle with $m \sim 125$ GeV. Since this needs to be confirmed, we do not impose this mass constraint, but we will discuss the case of such a Higgs boson.

The LHC experiments have obtained constraints on the squark and gluino masses. The ATLAS and CMS preliminary results indicate [24] that in the gravity mediated breaking the gluino mass limit is close to 1 TeV for a number of channels. Since this limit is model dependent, in the plots we will show the ranges for gaugino mass parameters satisfying Eq. (2.7) but keep in mind that the small gaugino mass parameters may violate the experimentally measured gluino mass.

2.3. Gaugino mass patterns

Having constrained the parameters $M_2$ and $\mu$, which enter the chargino as well as the neutralino mass matrix, we now turn to the theoretical models for the supersymmetry breaking gaugino mass parameters $M_1, M_2,$ and $M_3$. Theoretically, a simple set of patterns has emerged for these SUSY breaking parameters, which can be described as follows. Here we will briefly list the mass patterns. A more detailed discussion can be found e.g. in [25,26].

2.3.1. Gravity mediated breaking

The first pattern, which has been the object of extensive studies, is the one which arises in the gravity mediated supersymmetry breaking models, usually referred to as the mSUGRA pattern. In the gravity mediated minimal supersymmetric standard model, the soft gaugino masses $M_i$ and the gauge couplings $g_i$ satisfy the renormalization group equations (RGEs) ($|M_3| \equiv |M_{\tilde{g}}|$, the tree-level gluino mass)

$$16\pi^2 \frac{dM_i}{dt} = 2b_i M_i g_i^2, \quad b_i = \left(\frac{33}{5}, 1, -3\right), \quad (2.8)$$

$$16\pi^2 \frac{dg_i}{dt} = b_i g_i^3 \quad (2.9)$$

at the leading order, where $i = 1, 2, 3$ refer to the $U(1)_Y$, $SU(2)_L$ and the $SU(3)$ gauge groups, respectively. Furthermore, $g_1 = \frac{5}{3} g', g_2 = g$, and $g_3$ is the $SU(3)_C$ gauge coupling. With the boundary conditions ($\alpha_i = g_i^2/4\pi, i = 1, 2, 3$)

$$M_1 = M_2 = M_3 = m_{1/2}, \quad (2.10)$$

$$\alpha_1 = \alpha_2 = \alpha_3 = \alpha_\beta \quad (2.11)$$

at the GUT scale $M_G$, the RGEs (2.8) and (2.9) imply that the soft supersymmetry breaking gaugino masses scale like gauge couplings:
\[ \frac{M_1(M_Z)}{\alpha_1(M_Z)} = \frac{M_2(M_Z)}{\alpha_2(M_Z)} = \frac{M_3(M_Z)}{\alpha_3(M_Z)} = \frac{m_{1/2}}{\alpha_G}. \]  
(2.12)

After including radiative corrections, the ratios for gaugino masses are

\[ M_1 : M_2 : M_3 \simeq 1 : 9 : 6.2. \]  
(2.13)

This pattern is typical of any scheme obeying Eqs. (2.8) and (2.10). Note that the gluino mass used above is the running mass evaluated at the scale of the gluino mass, whereas the gaugino mass parameters \( M_1 \) and \( M_2 \) are running parameters evaluated at the weak scale \( M_Z \). Using the ratio (2.13) and the lower limit (2.7), we have the constraint

\[ M_1 \gtrsim 50 \text{ GeV}. \]  
(2.14)

in the gravity mediated supersymmetry breaking models.

We note that in the gravity mediated supersymmetry breaking models, the parameter \( \mu \) is not constrained. As such \( |\mu| \) can be smaller or larger than \( M_1, M_2 \). If \( |\mu| \gg M_1, M_2 \), then the lightest neutralino is mostly a gaugino, whereas in the opposite case \( |\mu| \ll M_1, M_2 \), it is dominantly a higgsino.

### 2.3.2. Anomaly mediated breaking

A second pattern of gaugino masses, which is distinct from the mSUGRA pattern, arises in anomaly mediated supersymmetry breaking models (AMSB). Since the soft supersymmetry breaking parameters are determined by the breaking of the scale invariance, they can be written in terms of the beta functions and anomalous dimensions in the form of relations which hold at all energies. In MSSM, the pure anomaly mediated contributions to the supersymmetry breaking gaugino masses can be written as [27]

\[ M_3 = \frac{\beta_g}{\beta_a} m_{3/2}. \]  
(2.15)

where \( m_{3/2} \) is the gravitino mass, \( \beta_g \)'s are the relevant \( \beta \) functions. We note that the gaugino masses are proportional to their corresponding gauge group \( \beta \) functions with the lightest supersymmetric particle being mainly a wino.

However, it turns out that the pure scalar mass-squared anomaly contribution for sleptons is negative [28]. As implied, we have the constraint

\[ |\mu| \gtrsim 280 \text{ GeV}. \]  
(2.17)

This to be contrasted with the corresponding result (2.14) for the gravity mediated supersymmetry breaking. We further note that in the anomaly mediated supersymmetry breaking mechanism, the higgsino parameter \( \mu \) cannot be smaller than \( M_1 \) due to the constraints following from electroweak symmetry breaking condition [29]. This implies that the dominant component of the lightest neutralino will be a gaugino. Thus, the effect of the dimension five operator on the lightest neutralino mass will be small, since it affects the higgsino component only.

### 2.3.3. Mirage mediated supersymmetry breaking

A third simple gaugino mass pattern arises from the mirage (or mixed modulus) mediated supersymmetry breaking, which is a hybrid between anomaly mediated supersymmetry breaking and mSUGRA pattern. Mirage mediation is naturally realized in KKLT-type moduli stabilization [30] and its generalizations, a well-known example being KKLT moduli stabilization in type IIB string theory [31]. Phenomenology and cosmology of mirage mediation have been studied in [32–40]. Signatures of this scenario at LHC and the spectrum of neutralino mass in particular have been studied in [25,41].

The boundary conditions for the soft supersymmetry breaking gaugino mass terms can be written as [42]

\[ M_\alpha = M_0 \left[ 1 + \frac{\ln(\tilde{M}_P/m_{3/2})}{16\pi^2} b_\alpha g_\alpha^2 a \right], \]  
(2.18)

where \( M_0 \sim 1 \text{ TeV} \) is a mass parameter characterizing the moduli mediation, \( \tilde{M}_P \) is the reduced Planck mass, \( g_\alpha \) are the gauge couplings and \( b_\alpha \) the corresponding one-loop beta function coefficients, and \( \alpha = m_{3/2}/[M_0 \ln(\tilde{M}_P/m_{3/2})] = O(1) \) is a parameter representing the ratio of anomaly mediation to moduli mediation. In addition to \( M_0, \alpha \) and \( \tan \beta \), mirage mediation is parametrized by \( a_1 \) and \( c_1 \), for which we follow definitions of [42].

Throughout the Letter we have used the values \( c_1 = a_1 = 1 \). At low energies, the gaugino masses in mirage mediation can be written as

\[ \frac{M_\alpha(\mu)}{g_\alpha^2(\mu)} = \left( 1 + \frac{\ln(\tilde{M}_P/m_{3/2})}{16\pi^2} g_\alpha^2 c_{\text{GUT}} b_\alpha a \right) \frac{M_0}{g_{\text{GUT}}^2}. \]  
(2.19)

This leads to a unification of the soft gaugino masses at the mirage messenger scale [43].
Fig. 1. The lightest neutralino mass in mSUGRA at tree level, and with one-loop radiative corrections as a function of the gaugino mass parameter $M_1$. The blue solid line corresponds to the tree-level mass with $\epsilon_1 = 0$. The other curves are in order of increasing dash length: tree-level mass with $\epsilon_1 = 0.1$ (violet); one-loop mass with $\epsilon_1 = 0$ (ochre); and one-loop mass with $\epsilon_1 = 0.1$ (green). Here $\mu = 200 \text{ GeV}$ and $\tan \beta = 10$. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

\[
M_{\text{mir}} = M_{\text{GUT}} \left( \frac{m_{3/2}}{M_{\text{Pl}}} \right)^{\alpha/2},
\]

which is lower than GUT scale for positive values of $\alpha$. For $g^2_{\text{GUT}} \simeq 1/2$ the resulting low energy values yield the mirage mass pattern

\[
M_1 : M_2 : M_3 \simeq (1 + 0.66\alpha) : (2 + 0.2\alpha) : (6 - 1.8\alpha).
\]

Including the radiative corrections for the gaugino masses, we obtain

\[
M_1 : M_2 : M_3 \simeq 1 : 1.5 : 2.1 \quad \text{for } \alpha = 1,
\]

\[
M_1 : M_2 : M_3 \simeq 1 : 1.2 : 0.92 \quad \text{for } \alpha = 2,
\]

where we have used the value $M_0 = 1 \text{ TeV}$. Thus, for the mirage mediation, we find

\[
M_1 \gtrsim 67 \text{ GeV } \quad \text{for } \alpha = 1.
\]

\[
M_1 \gtrsim 83 \text{ GeV } \quad \text{for } \alpha = 2.
\]

Depending on the values of parameter, the lightest neutralino can be dominantly either a higgsino or a gaugino.

3. Numerical results and sum rules for neutralino and chargino masses

For large values of $\mu$, the lightest neutralino and chargino are almost pure gauginos. In this case, the corrections to the lightest neutralino and chargino masses from BMSSM operators are small, since they affect the higgsino sector. If, on the other hand, the $\mu$ parameter is small compared to the gaugino mass parameters, i.e. if the lightest neutralino and chargino are dominantly higgsinos, the BMSSM corrections to their masses can be significant. In the case when the lightest neutralino and chargino are dominantly gauginos, it may be possible to study the effects of dimension five operator by using the sum rules for the masses of all the neutralinos and charginos. We will demonstrate that sum rules involving the neutralino and chargino masses can be used to distinguish between the different SUSY breaking patterns in presence of dimension five operator.

Since radiative corrections will be competing with the corrections coming from the dimension five operator, it is important to compare the magnitude of the $\epsilon_1$ corrections with one-loop radiative corrections. In Fig. 1 we have plotted the lightest neutralino mass in the mSUGRA pattern of gaugino masses with $\mu = 200 \text{ GeV}$, $\tan \beta = 10$. We have plotted the lightest neutralino mass at the tree level, with radiative corrections, with corrections coming only from $\epsilon_1$, and with both the radiative and $\epsilon_1$ corrections. The radiative corrections are calculated using small $\mu$ approximation [44,45]. Only the contributions from quark–squark loops are included and squark masses are taken to be 1 TeV. It is seen that radiative corrections and $\epsilon_1$ corrections are both generally a few GeV, but for large gaugino mass parameters they are of the opposite sign. At $M_1 = 1 \text{ TeV}$, the radiative and $\epsilon_1$ corrections are of similar magnitude (but opposite sign) for $\epsilon_1 = -0.04$. The kink in the BMSSM corrections shows that at the corresponding value of the parameter $M_1$, the lightest neutralino changes from an eigenstate containing a significant gaugino component to another mass eigenstate, which is almost a pure higgsino.

In Fig. 2 we show the lightest neutralino and chargino masses for several values of $\epsilon_1$, $\epsilon_1 = 0$, ±0.05, ±0.1. We have plotted these masses for the mSUGRA model. The dimension five operator causes a shift in the lightest Higgs mass which can bring it down below the current experimental limit [12]. We have excluded the parts of the graphs where $m_{\text{higgs}} < 111 \text{ GeV}$ in Figs. 2–6, when calculating the Higgs mass with SOFTSUSY [46] and shift caused by dim 5 operators is taken into account. Because for $\mu \ll M_1$, $M_2$ the higgsino sector strongly dominates the lightest neutralino and chargino masses, and thus the plot for mSUGRA is a representative for the mirage mediation models as well since the only difference in the masses in these models is due to the gaugino nonuniversality. It is seen that the effect of BMSSM operators in the case of mSUGRA pattern of gaugino masses is a few GeV, depending on the parameters. For $\tan \beta = 10$, Figs. 2(a) and (c), for positive $\epsilon_1 = 0.05$, there are experimentally allowed Higgs masses only for $M_1 > 450$. For $\epsilon_1 = 0.1$ Higgs is too light for all $M_1 < 1 \text{ TeV}$. Increasing $\tan \beta$ leads to a heavier Higgs, and the $M_1$ values shown in Figs. 2(b) and (d) are allowed. The effect of dimension five operator for small $M_1$ values is opposite for neutralino and chargino masses, while for large values of $M_1$, the neutralino mass is always smaller than what it is without the $\epsilon_1$ correction. For chargino mass the correction is positive for negative $\epsilon_1$ and it is negative for positive $\epsilon_1$. Thus, the effect of dimension five operator is enhanced for negative $\epsilon_1$ in the difference of chargino and neutralino.
Fig. 2. The lightest neutralino and chargino masses in mSUGRA for several values for the parameter $\epsilon_1 = \frac{\lambda}{M} \mu$. The blue solid line corresponds to $\epsilon_1 = 0$, and the thick dashed lines in order of increasing dash length represent $\epsilon_1 = 0.05$ (violet), $\epsilon_1 = 0.1$ (ochre). The thin dashed lines denote the lightest neutralino mass for $\epsilon_1 = -0.05$ (violet), $\epsilon_1 = -0.1$ (ochre), again in the order of increasing dash length. Here $\mu = 200$ GeV and one-loop radiative corrections are included. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Fig. 3. The difference between the lightest neutralino and the lightest chargino mass in mSUGRA plotted for several values of the parameter $\epsilon_1 = \frac{\lambda}{M} \mu$. The blue solid line corresponds to $\epsilon_1 = 0$, and the thick dashed lines in order of increasing dash length represent $\epsilon_1 = 0.05$ (violet), $\epsilon_1 = 0.1$ (ochre). The thin dashed lines correspond to $\epsilon_1 = -0.05$ (violet), $\epsilon_1 = -0.1$ (ochre), again in the order of increasing dash length. Here $\mu = 200$ GeV and one-loop radiative corrections are included. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

masses, as seen in Fig. 3. We have not shown the results for the AMSB case, since in the AMSB $\mu$ cannot be smaller than $M_1$ due to the electroweak symmetry breaking condition [29], and thus in this case the dimension five contribution is negligible to the lightest neutralino and chargino masses.

If the $\mu$ parameter is large compared to the soft gaugino masses, the two heaviest of the neutralinos are mostly higgsinos. The relative contribution of the dimension five operator to the mass for a heavy particle from the BMSSM operators is small. We conclude that if dimension 5 contribution to the masses of neutralinos and charginos is sizable, one cannot use purely the neutralino and chargino masses to determine the supersymmetry breaking mechanism. We, therefore, consider here two different sum rules involving neutralino and chargino masses and their squares. The dependence on gaugino masses enters these sum rules in a specific manner.

From the trace of the neutralino mass matrix (2.3) one obtains the sum over the neutralino mass eigenvalues which we denote by $\sigma$. This can be written as

$$\sigma(\epsilon_1) \equiv \sum_{i=1}^{4} \eta_i m_{\tilde{\chi}_i^0} = M_1 + M_2 + 2 \frac{\epsilon_1}{\mu} \nu^2,$$

(3.1)
Fig. 4. The contribution arising from $\epsilon_1$ to the total sum of (3.1) in different supersymmetry breaking models. The solid blue line corresponds to AMSB; mSUGRA (violet), and mirage mediation with $\alpha = 1$ (ochre), and $\alpha = 2$ (green) models, respectively, are presented in the order of increasing dash length. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

Table 1

| $M_\tilde{g}$ (GeV) | 750   | 2000  |
|---------------------|-------|-------|
| mSUGRA              | $\tilde{\chi}_0^0$ | $\tilde{\chi}_1^0$ |
| AMSB                | $\tilde{\chi}_2^0$ | $\tilde{\chi}_2^0$ |
| mirage $\alpha = 1$ | $\tilde{\chi}_2^0$ | $\tilde{\chi}_2^0$ |
| mirage $\alpha = 2$ | $\tilde{\chi}_2^0$ | $\tilde{\chi}_2^0$ |

at leading order in $\epsilon_1$, where $\eta_i$ is the sign of the $i$th eigenvalue. This sum rule depends on the $\mu$ parameter through BMSSM operators, when $\epsilon_1$ is taken as an independent parameter. It should be noted that in most of the allowed parameter space the neutralino mass matrix has one negative eigenvalue (see Table 1 for the gluino masses we use in this work). This needs to be taken into account when evaluating the sum. An advantage of this sum rule is that in addition to the gaugino mass parameters and $\epsilon_1$, it depends only on the supersymmetric higgsino mixing parameter $\mu$.

Using relations (2.12), (2.15), (2.19), and (2.20) the gaugino mass parameters $M_1$ and $M_2$ can be expressed in terms of the gluino mass $M_\tilde{g}$ and coupling constant $\alpha_i$, both observable quantities. For mSUGRA, AMSB and mirage mediation the sum rule can then be written as, with $B = \ln(M_{\text{GUT}}/M_{\text{mir}})/(16\pi^2)$,

$$
\sigma_{\text{msUGRA}}(\epsilon_1) = \frac{M_\tilde{g}}{\alpha^3} \left( \alpha_1 + \alpha_2 \right) + 2 \frac{\epsilon_1}{\mu} v^2,
$$

$$
\sigma_{\text{AMSB}}(\epsilon_1) = \frac{M_\tilde{g}}{3} \left[ \frac{\alpha_2}{\alpha_3} + \frac{33}{5} \frac{1}{\alpha_3} \right] + 2 \frac{\epsilon_1}{\mu} v^2,
$$

$$
\sigma_{\text{mirage}}(\epsilon_1) = \frac{M_\tilde{g}}{\alpha_3} \left[ 1 - 3B \right]^{-1} \left[ \alpha_2(1 + B) + \alpha_1 \left( 1 + \frac{33}{5} B \right) \right] + 2 \frac{\epsilon_1}{\mu} v^2. \tag{3.2}
$$

In Fig. 4 we have plotted the magnitude of the dimension five contribution relative to the whole sum with two $\mu$ and $M_\tilde{g}$ values, $\mu = 200$, 500 GeV, and $M_\tilde{g} = 750$, 2000 GeV. The plotted quantities can be written in terms of observables as

$$
\frac{\sigma(\epsilon_1) - \sigma(0)}{\sigma(\epsilon_1)} = \frac{\sum_{i=1}^{4} \eta_i m_{\tilde{\chi}_i^0}}{\sum_{i=1}^{4} \eta_i m_{\tilde{\chi}_i^0}} M_\tilde{g}. \tag{3.3}
$$
where $\gamma_{SB}$ refers to the coefficient of $M_\tilde{g}$ in different gaugino mass patterns in Eq. (3.2). We have again taken account of the experimental limit for the Higgs mass by excluding the parts of the lines violating the limit of $m_h < 111$ GeV (when calculating $m_h$, we use $\tan \beta = 30$). AMSB is not allowed for the $\mu = 200$ GeV case due to the constraint $\mu > M_1$ in this model. In the sum $\sigma$ the dimension five contribution is inversely proportional to $\mu$, and the maximum percentage contribution is achieved with the lowest gluino mass. The contribution is largest for mSUGRA pattern, and smallest for mirage mediation with $\alpha = 2$. In our example with $M_\tilde{g} = 750$ GeV and $\mu = 200$ GeV, the contribution with $\epsilon_1 = -0.1$ varies between $-2.5\%$ and $-9\%$.

The Higgs mass is an important constraint for the breaking patterns that we have studied in this Letter. For the chosen values of $\tan \beta = 10, 30$, and gluino masses $M_\tilde{g} = 750$ GeV and 2 TeV, we have shown in Table 2 the values of $\epsilon_1$ for which $m_h = 125$ GeV. The smaller the $\epsilon_1$ parameter is, the heavier the Higgs is. For mSUGRA and mirage mediation with $\alpha = 2$ and for $\tan \beta = 30$, $m_\tilde{g} = 750$ GeV, the required $\epsilon_1$ would be smaller than $-0.1$.

From the trace of the squares of the neutralino and chargino mass matrices, one obtains a sum rule for the neutralino and chargino masses squared, which we denote by $\Sigma$:

$$
\Sigma(\epsilon_1) = 2 \sum_{i=1}^{2} m_{\chi_i}^2 - \sum_{i=1}^{4} m_{\tilde{\chi}_i^0}^2 = [M_{\tilde{\chi}_1^0}^2 - M_{\tilde{\chi}_1^+}^2] + 4M_{W}^2 - 2M_{Z}^2 + 4\epsilon_1 v^2 \sin 2\beta \beta.
$$

(3.4)

at leading order in $\epsilon_1$. This sum rule depends on $\tan \beta$ in addition to $M_1$, $M_2$ and $\epsilon_1$ but not on $\mu$. In this sense the sum rules (3.1) and (3.4) are complementary.

The dimension 5 contribution in $\Sigma(\epsilon_1)$ decreases for increasing $\tan \beta$. The gaugino mass parameters $M_1$ and $M_2$ can again be expressed in terms of the gluino mass $M_\tilde{g}$ and coupling constants $\alpha_i$. For mSUGRA, AMSB and mirage mediation the sum rule can be written as

$$
\Sigma_{\text{mSUGRA}}(\epsilon_1) = \frac{M_\tilde{g}^2}{\alpha_3^2} \left( \alpha_2^2 - \alpha_1^2 \right) + 4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta.
$$

$$
\Sigma_{\text{AMSB}}(\epsilon_1) = \frac{M_\tilde{g}^2}{9} \left( \frac{3}{2} \right)^2 \left( \frac{\alpha_1^2}{\alpha_3^2} \right) + 4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta.
$$

$$
\Sigma_{\text{mirage}}(\epsilon_1) = \frac{M_\tilde{g}^2}{\alpha_3^2} \left[ 1 - 3B \right] \left( \alpha_2^2 (1 + B)^2 - \alpha_1^2 \left( 1 + \frac{33}{5} B \right) \right) + 4M_W^2 - 2M_Z^2 + 4\epsilon_1 v^2 \sin 2\beta.
$$

(3.5)

In Fig. 5 we have plotted the magnitude of the dimension five contribution relative to the whole sum with two $\tan \beta$ and $M_\tilde{g}$ values, $\tan \beta = 10, 30$ and $M_\tilde{g} = 750, 2000$ GeV. The plotted quantities can be written in terms of observables as

![Fig. 5. The contribution arising from $\epsilon_1$ to the total sum of (3.4) in different supersymmetry breaking models. The solid blue line corresponds to AMSB; mSUGRA (violet), and mirage mediation with $\alpha = 1$ (ochre), and $\alpha = 2$ (green) models, respectively, are presented in the order of increasing dash length. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)](image-url)
Fig. 6. The fraction of the contribution arising from $\epsilon_1$ to the total sum of (3.4) plotted as function of the ratio $M_2/M_1$ with $M_1 = 400$ GeV (long dashes, green), $M_1 = 200$ GeV (short dashes, purple), and $M_1 = 100$ GeV (solid line, ochre). On the horizontal axis $M_2/M_1 = 0.36$ corresponds to AMSB, $M_2/M_1 = 1.2$ to mirage mediation with $\alpha = 2$, $M_2/M_1 = 1.5$ to mirage mediation with $\alpha = 1$, and $M_2/M_1 = 1.9$ to mSUGRA. Here $\epsilon_1 = -0.1$ and $\tan \beta = 10$. Only the parts of the lines that agree with the experimental limit for the chargino mass (2.5) are shown. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

\[
\frac{\Sigma(\epsilon_1) - \Sigma(0)}{\Sigma(\epsilon_1)} = 2 \sum_{i=1}^{2} m_{\tilde{\chi}^0_i}^2 - \sum_{i=1}^{4} m_{\tilde{\chi}^0_i}^2 - \frac{\alpha_{SB}^2 M_\tilde{g}}{M_\tilde{g}},
\]

where $\alpha_{SB}$ is the supersymmetry breaking model dependent coefficient of $M_\tilde{g}^2$ in (3.5). As seen from Fig. 5 increasing $\tan \beta$ from 10 to 30 roughly halves the dimension five contribution. Larger $\tan \beta$ however allows larger positive values $\epsilon_1$ without violating the Higgs mass constraint. In contrast with $\sigma$, the maximum dimension five contribution of 10% is seen in the mirage mediation model with $\alpha = 2$, and in mSUGRA the contribution is the lowest of the four examined models. It is seen that for AMSB and mirage mediation with $\alpha = 2$ the contribution to $\sigma$ is opposite sign to the contribution to $\Sigma$, while for mSUGRA and mirage mediation with $\alpha = 1, \sigma$ and $\Sigma$ have the same sign.

By combining the sum rules Eq. (3.2) and (3.5) we obtain a relation for $\tan \beta$ and $\mu$ that is independent of $\epsilon_1$,

\[
\mu = \frac{2 \sum_{i=1}^{2} m_{\tilde{\chi}^0_i}^2 - \sum_{i=1}^{4} m_{\tilde{\chi}^0_i}^2 - \frac{\alpha_{SB}^2 M_\tilde{g}}{M_\tilde{g}} - 4 M_W^2 + 2 M_Z^2}{4 \tan^2 \beta}.
\]

This relation can be used for estimating the value of $\mu$ in BMSSM models if $\tan \beta$ is known. It should be noted that this formula does not exist without the BMSSM operator $\epsilon_1$. Thus a consistent value with other measurements may indicate the existence of the BMSSM operators. From precise measurements the value of $\epsilon_1$ can also be determined from Eq. (3.2) and (3.5) when $\mu$ or $\tan \beta$ are known.

The gaugino mass pattern realized in Nature may well turn out to be a mixture of the patterns studied here. This possibility can be considered by a general study of the ratio of $M_1$ and $M_2$. In Fig. 6 we show the fraction of the contribution from the dimension five operator to the sum rule (3.4) for $\epsilon_1 = -0.1$ as a function of the ratio of the mass parameters $M_2$ and $M_1$. Although at $M_1 = 400$ GeV (and larger) the dimension five contribution remains at less than a few percent for all models, $M_1 = 100$ GeV can produce as high as a 20 percent dimension five contribution in mirage mediation with $\alpha = 2$ and a 10 percent contribution in mSUGRA. As expected, the contribution is highest near the point $M_2/M_1 = 1$, where the sum of the squares of the gaugino mass parameters cancels in the sum rule, thus making the sum completely independent of the gaugino masses. This point corresponds to mirage mediation with $\alpha = 2.17$. Consequently, mirage mediation models with $\alpha$ close to this value allow significant dimension five contributions, although the lower bound for the gluino mass restricts $M_1$ to 1 TeV range and above. The experimental limit for the chargino mass rules out $M_1$ lower than 280 GeV in AMSB, and the dimension five contribution remains at a few percent for all allowed values for the gaugino masses for this model.

The usefulness of the sum rules depends on the accuracy with which the masses can be measured. The experimental error in the measurement of the neutralino and the chargino masses has been discussed in e.g. [47] for the LHC and for a possible future linear collider. While the quoted accuracies are not precise enough for using the sum rules, we have calculated as an example the accuracy for 3.2 and 3.5 assuming 1% error in the measurement of the three heaviest neutralino masses and in both chargino masses, while neglecting the error in the lightest neutralino mass. Results are presented in Fig. 7. The accuracy of measuring $\Sigma$ is diminished by the negative contribution of the neutralinos in the sum as well as the squaring of the masses, although at low gluino masses the uncertainty is of the same order of magnitude as the maximum $\epsilon_1$ contribution in our range of $\epsilon_1 > -0.1$ in AMSB and mirage mediation models.

The accuracy of $\sigma$ is affected by the mass of the neutralino with negative contribution to the sum compared to the masses of the other three neutralinos. We note that the uncertainty in $\sigma$ differs significantly with respect to the $\mu$ parameter only in the case of mSUGRA, and is largely independent of the gluino mass for $\mu = 200$ GeV. Since the $\epsilon_1$ contribution is inversely proportional to $\mu$, the usefulness of $\sigma$ in the detection of any BMSSM effect is greater for lower values of $\mu$, for which the uncertainty is at 1% level for the whole gluino mass range (and in all models, excluding AMSB). As a comparison, the BMSSM contribution ranges from 1% to 4% for $\epsilon_1 = -0.05$, and from 2% to 9% for $\epsilon_1 = -0.1$, when $M_\tilde{g} = 750$ GeV and $\mu = 200$ GeV (Fig. 4).
Fig. 7. The quantities $\alpha$ and $\beta$ as a function of the gluino mass and their experimental uncertainties assuming 1% uncertainty in the measurement of three heaviest neutralino masses and of both chargino masses. The solid blue line corresponds to AMSB; mSUGRA (violet), and mirage mediation with $\alpha = 1$ (ochre), and $\alpha = 2$ (green) models, respectively, are presented in the order of increasing dash length. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this Letter.)

4. Summary

We have studied the contribution of the dimension five BMSSM operators involving chiral Higgs superfields to the neutralino and chargino masses. The contribution can be significant when the higgsino mixing parameter $\mu$ is small compared to the soft supersymmetry breaking gaugino mass parameters, as we have illustrated. If the $\mu$ parameter is large, its effect is negligible on the mass of the lightest neutralino, which is dominantly a gaugino. Thus, the sensitivity to the BMSSM operator studied here is very different in different supersymmetry breaking models, since in the mSUGRA and mirage mediation models the $\mu$ parameter can be small, while in the anomaly mediation models it is always larger than the gaugino mass parameters. The effect of the dimension five operators on the masses of the heavier neutralinos is relatively small as compared to the lightest neutralino mass, and thus more difficult to isolate.

We have examined whether the sum rule involving squares of the neutralino and chargino masses and the sum rule involving neutralino masses could be used for the detection of BMSSM by calculating the contribution of the dimension five parameter to the sums. We have shown that the two sum rules can be combined to derive a relation between $\mu$ and $\tan \beta$ which is valid in the presence of the studied dimension 5 BMSSM operator.

The accuracy of the neutralino and chargino mass measurements is a key issue in the usefulness of the sum rules. We have examined whether the sum rule $\Sigma$ involving squares of the neutralino and chargino masses and the sum rule $\sigma$ involving neutralino masses could be used for the detection of BMSSM by calculating the contribution of the dimension five parameter to the sum, and evaluating the accuracy to which the sum can be measured using the anticipated accuracies for neutralino and chargino measurements at a linear collider. For large $\mu$, the BMSSM effect contributes to the $\Sigma$-sum more significantly than to the lightest neutralino mass, but the cumulative error from the squares of the neutralino and chargino masses diminishes the accuracy of the total sum measurement. The uncertainty is at best of the same order of magnitude with the BMSSM contribution. The other sum rule $\sigma$ involving neutralino masses has the advantage of
having far less experimental uncertainty, and for our example accuracies, the measurement error would be smaller than the dimension five contribution to the sum rule.

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