Parameter Variation on Nonlinear Energy Sink attached to Multiple Degree of Freedom System

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The importance of lightweight structures has increased in the last years motivated by the goals of improving efficiency of vehicles and conserving resources. In most cases the loss of weight by using fewer or lighter materials is accompanied by a reduction of the structures stiffness having a strong effect on occurring vibrations. Even if the damping ratio of the structure remains unchanged, by reducing weight and stiffness the vibration amplitudes might increase if the vibration excitation stays the same. Damping elements might be introduced to the structure mitigating critical resonance gains of the structure to reduce these vibrations. In this work the dynamics of a linear multiple degrees of freedom structure with a nonlinear damping element are investigated. The damping element is chosen to have a nonlinear characteristic to allow for a situation adaptive design. Numerical parameter studies are carried out to optimize the resulting damping of the entire structure. The nonlinear equation of motion is solved by applying the Harmonic Balance Method. The evaluation of the performance of the nonlinear damping element is done by consideration of the nonlinear frequency response functions of the structure.

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1 Introduction

Lightweight structures are endangered by vibrations which might cause malfunction or damage. At the same time the expectations regarding comfort and noise pollution gain in importance. The most efficient way to mitigate vibrations of a mechanical structure is to deliberately introduce damping to it. Most damping effects have a nonlinear characteristic so that frequency response functions depend on the excitation amplitude. The aim of this work is to demonstrate how to make use of the amplitude dependence for an optimal design of a nonlinear damping element which is attached to a linear structure with weak damping. A second design parameter that will be investigated is the location of the damping element. Nonlinear frequency response functions will be computed and compared for different setups and a design optimization process will be shown.

2 Numerical Model

The structure which is investigated is a clamped-free beam of steel with a length of 1 m and a height of 5 cm which is excited by harmonic forcing with an amplitude of up to 1000 N at its free end. The Ansys-2D-model of the beam consists of 20 plane-183 elements and is shown in Fig. 1. The resulting numerical model contains 200 degrees of freedom. The modal damping of the beam is set to 0.01 for each mode.

The nonlinear damping element can be considered as a Coulomb friction element as shown in Fig. 2. The aim of the optimization is a setup of the beam and the friction element where vibration amplitudes of the beam’s free end are reduced by optimal utilization of the amplitude dependence of the friction element. The varied parameters for this optimization are the absolute value of the tangential force \( \mu F_N \) and the damping element location. 20 nodes at the bottom side of the FE structure are considered as possible locations for the friction element. The locations are numbered from the fixed to the free end as implied in Fig. 3.

In [1] it has been shown that in case of equations of motion with a scalable nonlinear function \( \kappa f_{\omega l}(u) = f_{\omega l}(\kappa u) \) the whole equations of motion are scalable. As the only nonlinearity within the numeric model is Coulomb friction, the Equations of motion

\[
\begin{align*}
M \ddot{z} + C \dot{z} + K z + \tilde{c}_k \mu F_N \text{sgn}(\dot{z}_k) &= \tilde{e}_l \hat{F} \cos(\Omega t) \quad (1)
\end{align*}
\]

are scalable by the absolute value of the tangential force \( \mu F_N \). In Eq. 1 \( M, C, K \) are the structural matrices of the beam, the friction element is attached to the \( k^{\text{th}} \) degree of freedom and the degree of freedom the excitation acts on is described in line \( l \) as \( \tilde{c}_k, \tilde{e}_l \) are the \( k^{\text{th}} \) and \( l^{\text{th}} \) unit vectors. With the normalized excitation amplitude and the normalized system response

\[
\begin{align*}
\hat{F} &= \frac{F}{\mu F_N} ; \quad \ddot{z} = \frac{1}{\mu F_N} z \\
\end{align*}
\]

Eq. 1 can be written as

\[
\begin{align*}
M \ddot{z} + C \dot{z} + K z + \tilde{c}_l \text{sgn}(\dot{z}_k) &= \tilde{e}_l \hat{F} \cos(\Omega t) \quad (2)
\end{align*}
\]

\[
\begin{align*}
M \ddot{z} + C \dot{z} + K z + \tilde{c}_l \text{sgn}(\dot{z}_k) &= \tilde{e}_l \hat{F} \cos(\Omega t) \quad (3)
\end{align*}
\]
The normalized tangential force of the friction element is finally approximated by

\[ \tilde{F}_T = \text{sgn}(\hat{z}_k) \approx \frac{2}{\pi} \arctan(10^6 \cdot \hat{z}_k) \]  

in order to provide differentiability of the tangential force. The factor $10^6$ in Eq. 4 was chosen to provide a good approximation to the sign-function. Fig. 4 shows a comparison of the sign-function and the approximation in Eq. 4 which are in good agreement.

### 3 Numerical Methods

The numerical model consists of 200 equations while the excitation force only acts on one degree of freedom and another degree of freedom is affected by the nonlinear friction element’s tangential force. The numerical problem is solved using the Harmonic Balance Method (HBM), assuming a periodic system response described by a Fourier-series including 9 harmonics in this work. The unknown Fourier-coefficients of the solution are determined using Newton’s method minimizing the Fourier-coefficients of the residual. The harmonics of the dynamic tangential force is evaluated using the Alternating Frequency-Time-domain (AFT) method [2]. As the nonlinearity of the system only affects one degree of freedom, the AFT-method only needs to be applied to this degree of freedom. The dynamics of the other degrees of freedom can directly be determined in the frequency domain. A predictor-corrector-continuation scheme was applied to the HBM-problem in order to compute Nonlinear Frequency Response Functions (NFRF).

### 4 Dynamic analysis of the considered model and damping element design proposal

NFRFs have been computed for different normalized excitation amplitudes $\hat{F}$ covering the system states of pure locking of the friction element, pure sliding of the friction element, and the intermediate conditions. As the equations of motion have been normalized to the friction elements tangential force $\mu F_N$, the NFRFs only have to be computed once and the results can be scaled to different values of $\mu F_N$ afterwards. As 20 different damping element locations $k$ are considered, each resulting in a different set of equations of motion Eq. 3, the NFRFs have to be computed for each damping element location separately.

#### 4.1 Determination of the optimal damping element location

Fig. 7 shows 26 NFRFs for normalized excitation amplitudes $\hat{F}$ in the range $\hat{F} = [10^{-3}, 10^2]$ of the normalized system for the damping element attached to the node 12 as numbered in Fig. 3. It can be seen that the peaks of the NFRFs are located at different frequencies for high and low normalized excitation amplitudes. The aim of the optimization was defined to minimize the maximum dynamic displacement $x$ of the free end of the beam. If the normalized maximum dynamic displacement $\hat{x}$ of the free end of the beam is considered with respect to the corresponding normalized excitation amplitude, the derivative of the resulting curve given by differential quotients then describes the response magnification ratio of the maximum dynamic displacement to the excitation force. For a linear system, this ratio is always equal to 1. For the beam considered in this...
work with the friction element attached to the 20 different nodes in Fig. 3, the response magnification ratio resulting from the NFRFs for each of the damping element locations are shown in Fig. 7. The jittering at high excitation force amplitudes results from the NFRFs being represented by discrete points. It can be seen, that for all damping element locations for very low excitation amplitudes and neglecting the jittering also for very high excitation amplitudes the response magnification ratio is close to 1. Going from low to high excitation forces most of the response magnification ratios show a region with values below 1 followed by a narrow maximum which reaches values of up to 10 for several damping element locations. The result with the widest range of excitation force amplitudes with a response magnification ratio below 1 corresponds to location 12 of the damping element. This particular response magnification ratio is shown in Fig. 8. In Fig. 6 the corresponding maximum normalized displacement depending on the normalized excitation force amplitude is shown. It can be seen, that a wide range with a small response magnification ratio results in a wide range of excitation force amplitudes without significant response magnification. This vibration amplitude saturating response behaviour is the optimal exploitation of the friction element’s amplitude dependent damping characteristics.

### 4.2 Determination of optimal absolute tangential force

The range of excitation force amplitudes without significant response magnification ends where the response magnification ratio in Fig. 8 reaches a value higher than 1. In this example this is the case at \( \hat{F} = 0.4 \). where a vertical black line is drawn into Fig. 6. As the maximum expected physical excitation force was defined to be \( \hat{F} = 1000 \text{ N} \), the optimal absolute tangential force \( \mu F_N \) of the friction element can be derived from Eq. 2 as follows

\[
\mu F_N = \frac{\hat{F}}{\hat{F}} \times \frac{1000 \text{ N}}{0.4} = 2500 \text{ N}.
\]

Fig. 5: 26 NFRFs for normalized excitation amplitudes \( \hat{F} = [10^{-3}, 10^2] \) of the normalized system for the damping element attached to node 12.

Fig. 6: Maximum response of NFRFs for damping element attached to the beam at location 12 depending on excitation force amplitude, vertical black line at \( \hat{F} = 0.4 \).

Fig. 7: Response magnification ratio of the maximum dynamic displacement over the excitation force for 20 different damping element locations.

Fig. 8: Response magnification ratio of NFRFs for damping element attached to the beam at location 12.
As a consequence the normalized axes of Fig. 6 can directly be substituted by Eq. 2 which is in this case

\[ \vec{F} = \hat{F} \cdot 2500 \text{ N} \quad ; \quad \vec{z} = \hat{z} \cdot 2500 \text{ N} \]

Note that the unit of the normalized displacement vector \( \hat{z} \) is \( \frac{\text{m}}{\text{N}} \). As NFRFs have already been computed for the beam with the damping element attached at location 12 for the normalized system, the vertical axis of Fig. 5 can be substituted into physical displacements with Eq. 6, too.

## 5 Conclusion

A single Coulomb friction element has been attached to a linear multiple degree of freedom system. Making use of the scalability of the resulting equations of motion with respect to the friction element’s tangential force, the optimization of the friction element location was decoupled from the optimisation of the friction element’s tangential force. Nonlinear frequency response functions have been computed with a continuation implemented Harmonic Balance algorithm for various friction element locations and excitation amplitudes. At first the friction element location at the linear structure has been optimized for optimal exploitation of the amplitude dependence of the friction element’s damping performance. Knowing the maximum expected excitation of the linear structure, the optimal friction element’s tangential force can be determined from scaling relations. The precomputed results have been scaled according to the friction element’s tangential force which follow from the optimization.

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