Dynamical evolution of heavy quarkonia in a deconfined medium

Nicolas Borghi, Clément Gombeaud
Fakultät für Physik, Universität Bielefeld, Postfach 100131, D-33501 Bielefeld, Germany
(Dated: January 14, 2013)

We investigate some consequences of the possibility that heavy quarkonia in a quark-gluon plasma possess different (quasi-)bound states, between which transitions are possible. In particular, we show that the time-evolution eigenstates in the medium are mixtures of the vacuum eigenstates. This leads to abundance ratios of quarkonia that differ from those predicted in statistical models or in the sequential-melting picture.

PACS numbers: 25.75.Nq, 12.38.Mh, 14.40.Pq

I. INTRODUCTION

Heavy quarkonia provide a convenient testing ground for analytical approaches to hadron properties in QCD, both in the vacuum and in the presence of a (deconfined) medium. In the latter case, the original idea of a direct link between charmonium suppression and deconfinement has been refined into more involved predictions for the behaviors of the various states with rising temperature (for recent reviews see Refs. [3–5]).

Many predictions are formulated in a static picture, most noticeably in terms of threshold temperatures, above which a given state is entirely “melted”, while it remains intact below. More dynamical approaches to the dissociation and formation or recombination of quarkonia in a medium have been considered in various classical kinetic frameworks: à la Boltzmann, in Langevin or Fokker–Planck descriptions, or through rate equations.

A common feature of these studies is their focus on charmonia. This is quite natural, since this corresponds to most of the existing experimental results. Now, the general expectation is that \( cc \) pairs can only exist in a quark-gluon plasma (QGP) as either a \( J/\psi \) or an unbound system. Accordingly, the above mentioned studies consider these two possibilities only. On the other hand, it is thought that several bottomonia are still bound in a QGP just above the deconfinement temperature. This opens a richer spectrum of possible behaviors for \( bb \) pairs, especially if transitions can be induced between different bound states. In this Letter, we wish to take this latter possibility seriously, and to investigate some of the consequences of this ansatz, restricting ourselves to the external degrees of freedom of quarkonia, will be presented in a longer, more technical publication. Eventually, we discuss both our model with its underlying assumptions and our results in Section IV.

II. MODEL AND METHOD

In this Section, we describe our model for the heavy quarkonium states and the medium in which they are immersed, as well as for the coupling between both. Then we briefly introduce the equations that govern the dynamics of the \( QQ \)-state populations.

A. Quarkonium in a quark-gluon plasma as a dissipative system

Our purpose in the present study is to investigate the evolution of heavy quarkonium states under the influence of the thermal degrees of freedom of the plasma, and especially gluons. Thus, we do not consider (light) quarks, which would interact with the heavy quark or antiquark through non-thermal gluons: our medium is a gluon plasma.

We model this plasma as a static bath of harmonic excitations, whose frequencies span a continuum \( \{ \omega \_\lambda \} \). The bath is treated as a thermal reservoir, whose thermodynamic properties are not affected by the transitions between the various states of the embedded quarkonia.

The modeling of heavy quarkonia is far more delicate than that of the plasma. In this Letter, we wish to identify generic behaviors that follow from allowing quark-antiquark pairs to be in different (quasi-)bound states in a deconfined medium, between which medium-induced transitions are permitted. In that view, we deliberately adopt an admittedly simplistic quarkonium model, which relies on a minimal number of parameters, deferring more realistic modeling to further studies. In particular, we make a number of assumptions, which we shall further discuss in Section IV.

Our first model assumption is the existence of a non-relativistic in-medium quark-antiquark potential that admits bound states, thereby neglecting any imaginary part in the potential. To make the model sim-
ple, we consider an attractive Coulomb potential \( V(r) = -\alpha/r \), with its well-known spectroscopy\(^1\). As we want to investigate the possible influence of transitions between levels, we depart from the exact Coulombic spectroscopy for the states that are bound in the vacuum: \( J/\psi, \chi_c \) and \( \psi' \) on the one hand, \( \Upsilon, \chi_b, \Upsilon', \chi_b' \) and \( \Upsilon'' \) on the other one\(^2\). Those states are modeled with Coulomb wave functions, yet with the same energy with respect to the ground 1S-state as measured in the vacuum. This is clearly an approximation, which allows us to lift the degeneracy between e.g. 2S and 1P states, and thereby permit single-gluon transitions between them.

Obviously, a single gluon cannot induce a transition between color-neutral \( Q\bar{Q} \) states. We thus further assume that our spectrum of color-singlet states is accompanied by a parallel spectrum of color-octet states, which for the sake of simplicity we shall denote similarly to their singlet counterparts.

Eventually, we have to model the unbound \( Q\bar{Q} \) pairs, which should normally constitute a continuum. Consistency with our model for in-medium bound states through a real potential implies that the states in that continuum have higher energies than the bound states, i.e. the latter are \textit{always} energetically favored, which is far from granted. This is even more a problem with the Coulomb potential, which admits arbitrarily large bound states with high principal quantum number — which in the static Debye-screening picture would appear as unbound. To get rid of those states, we fix the \textit{dissociation} threshold by considering bound states of the Coulomb potential as describing unbound pairs: 2S and 1P states (resp. 3S and 2P states) and the higher excited states for \( c\bar{c} \) (resp. \( b\bar{b} \)) pairs. A minimal approach to mimic the unbound character of such states consists in forbidding transitions from them back to the bound ones. In our computations, we have fully discarded the scattering solutions of the Coulomb potential, and considered two or three levels of “dissociated” states, with at least two states per level, to estimate the error on our result.

To leave room for the possible “recombination” of heavy quark and antiquark into a quarkonium state\(^2\), we also slightly modified the model by allowing transitions from the lowest dissociated states to bound ones. The plots we present in Section III are for results from this variant of our model. Further plots will be shown elsewhere\(^3\).

Eventually, we need to specify the coupling between a \( Q\bar{Q} \) pair and the plasma. We assume \textit{dipolar coupling}, that is, the gluons only interact through their chromoelectric field. Incidentally, we need also assume that the Bohr frequencies between \( Q\bar{Q} \) states are included in the continuum of bath frequencies, so that transitions between states can be induced.

### B. Evolution of the \( Q\bar{Q} \) states in the plasma

Now that we have specified the ingredients of our model, we can turn to the time evolution. Further details will be given in a longer publication\(^3\), here we shall merely outline the calculation.

We expect that the state of the quark-antiquark system at a given time should be a statistical superposition of (vacuum) eigenstates. Then a natural approach is to use the master-equation formalism\(^2\), Chapter 4]. Within the decorrelation approximation — i.e., technically, assuming that the density matrix of the whole system factorizes into the product of the density matrix \( \rho_{Q\bar{Q}} \) of the \( Q\bar{Q} \) pair and that of the plasma at every time —, which amounts to considering an expansion up to second order in the coupling potential between the quark-antiquark pair and the plasma, the \textit{populations} (diagonal elements) of \( \rho_{Q\bar{Q}} \) are governed by the coupled Einstein equations

\[
\frac{d\rho_{Q\bar{Q}}}{dt}(t) = -\sum_{k\neq i} \Gamma_{i\rightarrow k} \rho_{ii}(t) + \sum_{k\neq i} \Gamma_{k\rightarrow i} \rho_{kk}(t),
\]

where the transition rates \( \Gamma_{i\rightarrow k} \) between \( Q\bar{Q} \) levels follow from Fermi’s golden rule (except for those we set to zero, to mimic the continuum, as explained above). These rates involve a sum over the states of the QGP, weighted by their respective probabilities. This introduces a dependence of all \( \Gamma_{i\rightarrow k} \) on the plasma temperature \( T \).

In the following Section, we present solutions to these evolution equations for the \( c\bar{c} \) and the richer \( b\bar{b} \) systems.

### III. RESULTS

The Gedankenexperiment we discussed in Section I amounts to picking out an initial condition at \( t = 0 \) for Eqs. (1), for instance \( \rho_{ii}(t=0) = 1 \) for the ground 1S state and 0 for the excited levels, and to solve the coupled system.

A first observation, independent from the initial condition, follows directly from the structure of the coupled evolution equations: the latter are not diagonal. As a consequence, the populations corresponding to the vacuum eigenstates of the potential do not constitute an eigenstate of the “evolution operator” for the vector of populations that can be read off Eqs. (1); rather, they are linear combinations of the latter. That is, the different quarkonium states do not evolve independently from each other, but they are coupled together by the medium. For instance, even if \( \Upsilon' \) is constantly either dissociated or decaying into \( \chi_b \), at the same time it is recreated, with different rates, through the excitation of \( \chi_b \) or — when

---

1. We take \( \alpha = 0.4 \) for \( c\bar{c} \) pairs and a smaller \( \alpha \approx 0.25 \) for \( b\bar{b} \) pairs, to account for the running of the coupling constant and the smaller size of the ground state.

2. For given \( S \) and \( L \) quantum numbers, we consider for simplicity a single \((2L+1)\)-fold degenerated state.

3. For given \( S \) and \( L \) quantum numbers, we consider for simplicity a single \((2L+1)\)-fold degenerated state.

we allow for that possibility — through recombination of an unbounded $b\bar{b}$ pair. Thus we cannot have the total disappearance of a state in the plasma at finite times, as implied by static approaches to quarkonium suppression.

Let $\Gamma$ denotes the smaller (in absolute value) of the eigenvalues of the evolution operator for the populations. The medium-induced mixing of states is such that after some transient behavior, which depends on the specific choice of initial condition, the populations $\rho_{ii}^Q$ of all bound states (and of the unbound states that are allowed to recombine) evolve with the same characteristic time scale $\Gamma^{-1}$. We illustrate this in Fig. 1, which shows the time evolution of the populations of $c\bar{c}$ (left) and $b\bar{b}$ (right) states at respectively $2T_c$ and $5T_c$. The characteristic evolution times for these systems at the chosen temperatures within our model are respectively 3.4 and 3.8 fm/c. Quite obviously, for a given system $\Gamma^{-1}$ decreases with rising temperature.

Past the transient regime, the populations of the various bound and recombining states are “equilibrated” with each other, in the sense that the population ratios remain stable. (There is however no strict equilibrium, since $Q\bar{Q}$ pairs are consistently lost to non-recombining states). These “stationary” population ratios depend on the plasma temperature, as shown for $b\bar{b}$ pairs in Fig. 2. The stationary ratios differ significantly from those for thermally equilibrated levels, which is easily understandable. The thermal ratios are those which make Eqs. (1) stationary with all transition rates fulfilling the detailed balance condition

$$\Gamma_{i\to k} e^{-E_i/T} = \Gamma_{k\to i} e^{-E_k/T} \quad \forall i, k.$$  \hspace{1cm} (2)

Here, the condition is obeyed only by bound ($\Upsilon, \chi_b, \Upsilon'$) and recombining ($\chi_b', \Upsilon''$) states, but not by the unbound ones, from which there are no transition. It is thus normal that the resulting stationary ratios diverge from the thermal ratios.

We have checked that when we do not suppress back transitions from the unbound states, but set them according to condition (2), then the stationary abundance ratios are the same as in thermal equilibrium.

Given the crudeness of our model, which we wish to further discuss in the next Section, we have not attempted at this stage to make predictions for observables in real nucleus-nucleus collisions, which would anyway necessitate some extra modeling of the plasma kinetics as well as accounting for the hadronic phase.

IV. SUMMARY AND DISCUSSION

We have examined the dynamics of the populations of heavy quarkonium states in a static QGP within the
quantum-mechanical master-equation formalism, assuming that some elements of the quarkonium spectroscopy survive in the plasma, in particular that different bound states exist, between which transitions can be induced by the medium. Under this assumption, we find that the bound states are mixed together by the medium, so that they all evolve with the same characteristic time scale. This differs from the usual picture of sequential melting, inasmuch as no bound state can totally disappear while others would survive.

In addition, we find that after some transient regime, the population ratios remain stationary, at values determined by the plasma temperature, yet different from the ratios for quarkonium states in thermal equilibrium. While the fact that vacuum eigenstates are no longer eigenstates in the QGP is model-independent, the actual predictions for the abundance ratios depend on the model parameters, and could be used to constrain the latter from experimental results.

Let us now discuss our model. Its key ingredient in our eyes is the ansatz of medium-induced transitions — with a large enough rate compared to the inverse of the plasma lifetime — between bound quarkonium states. This is the element which couples the evolutions of the various bound states together, irrespective of the details of their modeling or of the mechanism responsible for the transitions.

The latter is important inasmuch as it selects which states are coupled together. Here we wanted to consider, in analogy to quantum optics, transitions induced by the gauge bosons. This choice forced us to invoke some hazy “spectrum of color-octet states”, paralleling that of color-singlets — which is far from being granted, given the repulsive nature of the color-octet channel. Instead, we could have conjured non-perturbative effects, like some kind of soft color interactions [24], to instantly turn color octets into singlets in the medium. Such explanations seem to us to be as disputable as our choice in the present context. Alternatively, one could think of non-color-exchanging processes, as quasi-elastic collisions with off-shell quarks or gluons or scattering of photons, provided the latter happen with a sufficient rate.

If one accepts the possibility of medium-induced transitions between bound states, then the models for medium, quarkonia and their interaction that we have used are purposely the simplest ones one can think of, yet still realistic enough to illustrate some plausible phenomena. The orders of magnitude we obtain for the typical time scales for the evolution of quarkonium populations are reasonable, which justifies our choice a posteriori. The model allows us to compute transition rates between bound states (which is the reason why we have kept the Coulomb wave functions although with “wrong” energies), while these are not known in more realistic models, since such transitions have not been investigated before. For the dissociation widths of the 1S states, one could use the known results at leading [26] or next-to-leading order [27]: this would represent an improvement, yet only a partial one.

Note that the dissociation process that we consider, as everyone does, is a classical process, in which energy is transferred to the QQ pair. This is inherent to the description by a real potential similar to the vacuum one. It might turn out that a better description of the transition from bound QQ state to unbound quark and antiquark in a QGP should involve some tunneling through a barrier, as studied in hadronic matter in Ref. [28].

The master-equation approach we have used relies on a few assumptions, which we shall detail elsewhere [13]. In short, these amount to assuming — as is also done in Boltzmann, Langevin or Fokker–Planck formalisms — that the typical time scale of plasma correlations is small against the characteristic time scale of the QQ-plasma interaction, i.e. the formalism implicitly rests on a “weak-coupling” assumption. We are investigating alternate approaches that do not make use of this hypothesis [22], yet this seems only feasible at the cost of some alternative approximations.

Eventually, we have assumed dipolar coupling between a QQ pair and the plasma, discarding chromomagnetic or quadrupolar and higher order chromoelectric couplings. This is a large-wavelength approximation — which might be disputable for gluons that should resolve the structure of the bound states — that can be released when using a more realistic model of the quarkonia in the plasma.

V. ACKNOWLEDGMENTS

We gratefully thank J.-P. Blaizot, J.-Y. Ollitrault and H. Satz for enlightening discussions. C. G. acknowledges support from the Deutsche Forschungsgemeinschaft under grant GRK 881.

[1] N. Brambilla et al., Eur. Phys. J. C 71 (2011) 1534 [arXiv:1010.5827 [hep-ph]].

[2] T. Matsui and H. Satz, Phys. Lett. B 178 (1986) 416–422.

[3] R. Rapp, D. Blaschke and P. Crochet, Prog. Part. Nucl. Phys. 65 (2010) 209–266. [arXiv:0807.2470 [hep-ph]].

[4] L. Kluberg and H. Satz in Relativistic Heavy-Ion Physics, Landolt-Börnstein New Series I/23A (Springer, 2009) [arXiv:0901.3831 [hep-ph]].

3 See Ref. [22], which appeared while this Letter was being finalized, for a study of the photoionization of the J/ψ by large magnetic fields in a QGP.
[5] R. Rapp and H. van Hees in *Quark-Gluon Plasma 4*, ed. by R. C. Hwa and X.-N. Wang (World Scientific, Singapore, 2010) pp. 111–206 [arXiv:0903.1096 [hep-ph]].

[6] D. Levin-Plotnik and B. Svetitsky, Phys. Rev. D 52 (1995) 4248–4250 [hep-ph/9503305].

[7] A. Polleri, T. Renk, R. Schneider and W. Weise, Phys. Rev. C 70 (2004) 044906 [nucl-th/0306025].

[8] L. Yan, P. Zhuang and N. Xu, Phys. Rev. Lett. 97 (2006) 232301 [nucl-th/0608010].

[9] B. K. Patra and V. J. Menon, Nucl. Phys. A 708 (2002) 353–364 [hep-ph/0112196].

[10] C. Young and E. Shuryak, Phys. Rev. C 79 (2009) 034907 [arXiv:0803.2866 [nucl-th]].

[11] L. Grandchamp, R. Rapp and G. E. Brown, Phys. Rev. Lett. 92 (2004) 212301 [hep-ph/0306077].

[12] X. Zhao and R. Rapp, preprint [arXiv:1102.2194 [hep-ph]].

[13] N. Borghini and C. Gombeaud, in preparation.

[14] S. Digal, P. Petreczky and H. Satz, Phys. Lett. B 514 (2001) 57–62 [arXiv:hep-ph/0105234].

[15] C. Y. Wong, Phys. Rev. C 72 (2005) 034906 [arXiv:hep-ph/0408020].

[16] F. Arleo, J. Cugnon and Y. Kalinovsky, Phys. Lett. B 614 (2005) 44–52 [arXiv:hep-ph/0410295].

[17] W. M. Alberico, A. Beraudo, A. De Pace and A. Molinari, Phys. Rev. D 75 (2007) 074009 [arXiv:hep-ph/0612062].

[18] D. Cabrera and R. Rapp, Phys. Rev. D 76 (2007) 114006 [arXiv:hep-ph/0611134].

[19] A. Mocsy and P. Petreczky, Phys. Rev. D 77 (2008) 014501 [arXiv:0705.2559 [hep-ph]].

[20] M. Laine, O. Philippsen, P. Romatschke and M. Tassler, JHEP 0703 (2007) 054 [arXiv:hep-ph/0611300].

[21] K. Nakamura et al. (Particle Data Group), J. Phys. G 37 (2010) 075021.

[22] R. L. Thews, M. Schroedter and J. Rafelski, Phys. Rev. C 63 (2001) 054905 [arXiv:hep-ph/0007323].

[23] C. Cohen-Tannoudji, J. Dupont-Roc and G. Grynberg, *Atom-Photon Interactions* (John Wiley & Sons, New York, 1998).

[24] A. Edin, G. Ingelman and J. Rathsman, Phys. Lett. B 366 (1996) 371–378 [hep-ph/9508386].

[25] K. Marasinghe and K. Tuchin, preprint arXiv:1103.1329 [hep-ph].

[26] M. E. Peskin, Nucl. Phys. B 156 (1979) 365–390.

[27] G. Bhanot and M. E. Peskin, *ibid.* 156 (1979) 391–416.

[28] Y. Park, K.-I. Kim, T. Song, S. H. Lee and C.-Y. Wong, Phys. Rev. C 76 (2007) 044907 [arXiv:0704.3770 [hep-ph]].

[29] D. Kharzeev, L. D. McLerran and H. Satz, Phys. Lett. B 356 (1995) 349–353 [hep-ph/9504338].

[30] N. Dutta, N. Borghini and C. Gombeaud, in preparation.