Hydrodynamics for relativistic heavy ion collisions

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Simulations of the viscous hydrodynamic model for relativistic heavy-ion collisions at RHIC and LHC energies are presented. Results for spectra, femtoscopy radii, and transverse momentum fluctuations are favorably compared to the experimental data. Effects of the local charge conservation on correlation observables are also studied.

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1. Introduction

The expansion of the fireball in heavy-ion collisions generates collective, azimuthally asymmetric transverse flow which can be described quantitatively in hydrodynamic calculations [1, 2, 3, 4, 5, 6]. Due to large velocity gradients in the system, an important role in the hydrodynamic model is played by shear and bulk viscosities [8, 9, 10]. The asymmetry of the final flow is largely determined by fluctuations of the initial geometry of the fireball [11, 12]. We apply a 3 + 1-dimensional viscous hydrodynamic model [13] to Pb-Pb collisions at 2.76 TeV using averaged initial conditions, and to Au-Au collisions at 200 GeV using fluctuating initial conditions. Also, to estimate the collective flow in p-Pb and d-Pb interactions at the LHC, event-by-event simulations are used [14]. The angular dependence in the two-dimensional dihadron correlation functions for particles with soft momenta are determined by the collective flow [15]. The short-range charge dependent structures in the same side ridge can be explained largely as an effect of the local charge conservation at hadronization [16].

Initial conditions must be provided for the hydrodynamic evolution. No first-principle calculation of the formation and thermalization of the dense matter in the early stage exists yet. In our simulations we use the optical
Glauber model for the averaged initial conditions, and its Monte Carlo implementation [17] for the fluctuating initial conditions. The parameters of the model are adjusted to the RHIC data [5], with the default values of the shear and bulk viscosity coefficients $\eta/s = 0.08$ and $\zeta/s = 0.04$ (bulk viscosity is present only in the hadronic phase). The emission of particles and resonance decays at freeze-out are performed via the event generator THERMINATOR [18].

2. Averaged initial conditions

The use of the averaged conditions in $3 + 1$-dimensional viscous hydrodynamics leads to a satisfactory description of pseudorapidity distributions, transverse momentum spectra, elliptic flow, and the femtoscopic radii in Au-Au interaction at the top RHIC energies [13]. Hydrodynamic expansion with a hard equation of state with cross-over generates a strong transverse flow. It is interesting to note that the shear viscosity leads to a reduction of the odd component of the directed flow.

![Figure 1](image.png)

Fig. 1. (left) Preliminary data on $\pi^+$, $K^+$, $p$ transverse momentum spectra [19] and (right) the femtoscopic radii measured by the ALICE Collaboration [20], compared to viscous hydrodynamic results (from [21]).
In Pb-Pb collisions at $\sqrt{s} = 2.76$ TeV the transverse collective flow is even stronger. One observes a significant shift of the spectra of protons towards higher $p_T$, consistent with the preliminary ALICE data (Fig. 1). On the other hand, the calculated spectra of pions and kaons are too flat. A slightly better description is obtained with a larger value of the bulk viscosity, $\zeta/s = 0.08$. We find that an initial energy density compatible with the observed charged particle pseudorapidity distribution leads to approximate plateau in the rapidity distributions of identified particles. The model reproduces very well the femtoscopic radii, which indicates that the basic features of the space-time evolution of the system are realistic.

3. Fluctuating initial conditions

Event by event fluctuations in the initial conditions induce additional deformations of the transverse profile of the fireball. The flow coefficients of the elliptic ($v_2$) and triangular ($v_3$) flow are determined by the initial eccentricity and triangularity of the fireball [11, 12, 6]. Relatively, the most violent fluctuations occur in peripheral collisions or in small systems. For p-Pb and d-Pb collisions at the LHC energies the fireball is large enough to allow for a noticeable stage of collective expansion. The initial deformation of the density profile is large, especially in d-Pb interactions (Fig. 2). The generated elliptic and triangular flows are comparable to values observed in peripheral Pb-Pb collisions and could be measured in future experiments [14]. If such a scenario is realized, the spectra measured in p-Pb and d-Pb collisions should be strongly modified by final state (hydrodynamic) interactions, which means that such observations could not be used as reference data when looking for medium modifications expected in Pb-Pb collisions.

In Au-Au collisions at 200 GeV our calculations give elliptic and triangular flows similar as in other calculations [6, 25, 26, 27], showing a reduction of the flow asymmetry with an increasing shear viscosity. An additional effect of the hydrodynamic response to fluctuations is seen in the fluctuations of the average momentum in an event [23, 24]. Both the shape and the size of the initial fireball fluctuates event by event. While the role of the shape fluctuations in generating azimuthally asymmetric flow is widely discussed, the size fluctuations of the fireball are less studied. Our simulations show that an anti-correlation is seen between the r.m.s radius of the fireball and the average transverse momentum of the emitted particles. The experimental measure of the transverse momentum fluctuations eliminates statistical fluctuation due the finite multiplicity in the events. The remaining scaled transverse momentum fluctuations are proportional to the scaled fluctuations of the fireball size $\Delta p_T/\langle p_T \rangle \simeq 0.3 \Delta r/\langle r \rangle$. The size fluctuations at each centrality are predicted in the Glauber model. The calculated $p_T$
Fig. 2. (left) Initial entropy density in the transverse plane in a d-Pb collision. (right) Elliptic flow coefficient predicted in p-Pb collisions at 4.4 TeV and d-Pb collisions at 3.11 TeV (from [13]).

Fig. 3. The experimental data for $\langle \Delta p_{Ti} \Delta p_{Tj} \rangle^{1/2} / \langle p_{T} \rangle$ from the PHENIX Collaboration [22] (squares) compared to simulations with event-by-event viscous hydrodynamics (dots) and to the approximate result from Ref. [23] for perfect hydrodynamics with smooth conditions (crosses) (from [24]).

fluctuations are similar as in the data, although some overestimate of the fluctuations can be seen (Fig. 3). We have verified that the hydrodynamic response is not noticeably modified by a change in the freeze-out temperature, the bulk and shear viscosities, or the core-corona effects. The scaled transverse momentum fluctuations grow with increasing upper momentum cut-off used in the analysis. The same behavior is seen in our calculation, where it originates from the hydrodynamic response to the size fluctuations.

4. Non-flow charge correlations

The assumption that charge formation happens late in the evolution (at hadronization) explains the observed charge balance functions in relative
pseudorapidity \[28, 29, 30, 31\]. A similar effect is expected and observed in the relative azimuthal angle of the emitted particles \[32, 31\]. By generalizing these arguments to the two-dimensional dihadron correlation function in \(\Delta \phi - \Delta \eta\), we expect the appearance of a two-dimensional same-side peak for the unlike-sign pairs.

![Dihadron correlation functions for unlike- and like-sign pairs without (upper row) or with (lower row) charge balancing mechanism implemented at the end of the hydrodynamic evolution (from [16]).](image)

After adding such a charge balancing effect to the particle emission at freeze-out [16], qualitatively the same correlations are formed as observed experimentally [33] (Fig. 4). This non-flow effect is added to the dominant structure from the collective flow in the correlation functions. For the like-sign pairs the same-side ridge is nearly flat, showing essentially no additional structure. The charge-balancing also leads to non-flow correlations, increasing the observed flow coefficients \(v_2\) and \(v_3\).

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