A METHOD FOR SEISMIC DESIGN OF
RC FRAME BUILDINGS USING FUNDAMENTAL MODE
AND PLASTIC ROTATION CAPACITY

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ABSTRACT

A seismic design method is proposed for RC frame buildings, with focus on two of the seven virtues of earthquake resistant buildings, namely deformation capacity and desirable collapse mechanism. Fundamental lateral translation mode of the building and plastic rotation capacity of beams are included as input to estimate lateral force demand. Guidelines are provided to proportion beam and column cross-sections through: (a) closed-form expressions of flexural rigidities to maximize participation of the fundamental mode, and (b) relative achievable plastic rotation capacity using current design and detailing practice. This method is seen to surpass two prominent displacement-based design methods reported in literature. Results of nonlinear static pushover and nonlinear time history analyses of buildings of three different heights designed by this and the said two methods are used to make a case for the proposed method; the proposed method is able to control plastic rotation demand in beams and provide at least 20% more lateral deformation capacity than the said methods.

INTRODUCTION

The seven virtues of earthquake resistant buildings (ERBs) are (Figure 1): (1) regular structural configuration, (2) at least a minimum lateral stiffness, (3) sufficient lateral strength, (4) good overall lateral ductility, (5) large overall lateral deformability, (6) desirable collapse mechanism, and (7) large energy dissipation capacity. In the traditional force-based earthquake resistant design of RC buildings, most design codes have provisions to meet directly the first three virtues and the fourth through prescriptive ductile detailing. But, the last three virtues are not in direct focus in current force based design practice. Because earthquake ground shaking imposes lateral displacement demand on structures and inputs energy to them at their base, the last three virtues are essential. Eventually, design codes should guide designers to meet these three virtues also. Studies should be undertaken and design methods suggested towards achieving this intent. Literature indicates that many studies have attempted this [1-10]. Two prominent studies, whose variants have been adopted in various other studies, are: (1) Direct Displacement Based Design (DDBD) [11-13], and (2) Performance-based Plastic Design (PBPD) [14]. Of these two methods, the latter has attempted to bring in the last three virtues, though in a simple way. The important merits and limitations of these two methods are summarized in Table 1.

Further, it is customary in the seismic coefficient method of the traditional force based design to consider the fundamental lateral translational mode to be the dominant mode. If this assumption can be realised through appropriate proportioning of lateral stiffness and associated lateral strength of buildings along their height, the method can be used readily by practising structural engineers. Thus, improved performance of frame buildings can be achieved, if buildings have: (a) rotation demands at plastic hinges less than those which can be provided practically, and (b) overall lateral deformation capacity more than that imposed by earthquake shaking. The former is contingent on the latter.

Figure 1: Seven virtues of earthquake resistant buildings.

PAST STUDIES AND GAPS

Behaviour of reinforced concrete (RC) moment frame (MF) buildings in past earthquakes during severe earthquake shaking are known to have failed for want of deformation capacity [15-18]. Factors that reduce lateral deformation capacity include: (1) un-accounted contribution of higher modes of oscillation, (2) unsuitable slenderness ratio l/d of members, (3) no preclusion of shear failure through capacity design, (4) formation of local mechanisms involving large plastic deformation demand in beams and columns, and (5) insufficient plastic rotation capacity θpc in beams [14,19-23].

Attempts were made to improve design methods to address these factors. Members were designed to sustain combined effects of fundamental and higher modes of oscillation [13,24]. But, till date, no method explicitly proportions the

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member sizes and strengths to make the fundamental mode of oscillation become the dominant mode, with at least 80% mass (say) participating in just the fundamental mode alone. While codes specify limits on maximum slenderness $l/d$ of beams, the values specified (20-26) makes beams too flexible to sustain good inelastic action [25]. Limiting lateral displacement in each storey under service loads and designing members by capacity design are practiced routinely now. But, low column-to-beam flexural strength ratios (~1.4) are recommended in design codes [26,27], even though higher values (2.2-2.8) are recommended in literature [28,29].

Further, in the recent past, plastic rotation capacity $\theta_{pb}$ was recommended as a design input in a design method [14,30], but adequate provisions to limit the plastic rotation demand $\theta_{pb}$ was not integrated into the design method. Consequently, results of response history analyses show that the actual $\theta_{pb}$ demands are much higher than $\theta_{pb}$ [14]. $\theta_{pb}$ can be limited to $\theta_{bc}$, if more beams are made to participate in the collapse mechanism. More beams participate in the response, if relative stiffness and strength of beams and columns are proportioned appropriately, thereby reducing the possibility of concentration of plastic actions in limited beams and columns. Furthermore, if the fundamental mode shape can be related to $\theta_{pb}$ and if $\theta_{pb}$ is ensured to be less than a fraction of $\theta_{bc}$ during design stage, most beams can utilise fully the $\theta_{bc}$; in turn, this will help maximise the lateral deformation capacity of buildings. Hence, quantifying the available $\theta_{bc}$ is the first step. Typically, RC beams designed and detailed by current seismic design codes have $\theta_{bc}$ in the range 0.015–0.030 rad [31-33]. These values are small owing to many factors, like: (1) large flexural rigidity of beams owing to heavy gravity loads, and (2) stiffness degradation and strength deterioration of RC beams under reverse cyclic response during strong earthquake shaking [34]. Until such time, $\theta_{bc}$ is increased through new design and/or detailing strategies, it is prudent to have design guidelines such that $\theta_{pb}$ is restricted to within $\theta_{bc}$. Thus, design methods should formally recognise the limited $\theta_{bc}$ made available when members are designed and detailed by the current methods. Design methods are available, which use $\theta_{bc}$ of beams as design input to estimate the lateral force demand of the building [10,14,30]. But, specific design guidelines are not available to ensure that $\theta_{pb}$ does not exceed $\theta_{bc}$ of beams.

A single method is not available yet, which: (1) proportions stiffness and strength of members, to maximize the contribution of the fundamental mode, and (2) uses the limited $\theta_{bc}$ available in beams as design input and ensures that $\theta_{pb}$ does not exceed it during strong earthquake shaking. To address these challenges and the overcome limitations in a holistic way, three actions are required in design, namely: (1) proportion member sizes considering a single mode, the fundamental lateral mode, reducing effects of higher modes; (2) design all beams along the height of the building so that they have near uniform $\theta_{bc}$, and (3) include $\theta_{bc}$ as a design input so as to ensure that $\theta_{pb}$ does not exceed $\theta_{bc}$ of beams.

| S.No. | Parameter | DDBD | PBPD |
|-------|-----------|------|------|
| 1     | Basis of Design Method | Target Deformability | Energy Dissipation |
|       | (a) Strength | Focus is on 5th Virtue of ERBs, i.e., overall lateral deformability. | Focus is on 7th Virtue of ERBs, namely energy dissipation. |
|       | (b) Shortcoming | Method does not directly address the 7th Virtue of ERBs. Further, the Method involves significant iteration especially when beams have large slenderness ratio. | Method employs a number of assumptions. |
| 2     | Design Lateral Force | Inter-Storey Drift | Plastic Rotation Capacity at Beam Ends |
|       | (a) Strength | Parameter familiarly used in traditional design. | Plastic rotation at beam ends is related to design lateral force. Focus is indirectly on 5th Virtue of ERBs, i.e., overall lateral deformability. |
|       | (b) Shortcoming | Many other factors also affect inter-storey drift, e.g., cracking, and not just geometric dimensions of members. | Parameter not easy to control at beam ends along the entire height of the building. |
| 3     | Distribution of Design Lateral Load along Height of Building | An empirical distribution of Inter-storey drift determines the said distribution | An empirical distribution |
|       | (a) Strength | Focus is directly on 6th Virtue of ERBs, i.e., collapse mechanism. | Focus is directly on 6th Virtue of ERBs, i.e., collapse mechanism. |
|       | (b) Shortcoming | Assumed distribution may not reflect the distribution of inter-storey drift demand. | The mechanism considered is an ideal one, and may be difficult to realise. |
| 4     | Mitigating Effects of Higher Modes | Yes | Yes |
|       | (a) Strength | Empirical reduction factor is used to account for increased demand. | Effects considered implicitly by an empirical distribution of lateral force demand along height. |
|       | (b) Shortcoming | No attempt is made to enhance the contribution of first mode. | No attempt is made to enhance the contribution of first mode. |
But, even when a single mode, namely the fundamental lateral mode, is made to dominate, buildings can deform in shear, linear or flexure type lateral profiles. If they deform in linear mode, \( \theta_{ld} \) is uniform along its height, thereby improving its lateral deformation capacity. But, to make buildings have large modal mass in this mode, buildings should deform in shear mode; in such a case, \( \theta_{ld} \) is unduly large in few storeys near the base of buildings. This paper presents an analytical method that balances these competing requirements by identifying a fundamental mode shape \( \{ \phi \}_1 \) of the building, which has large modal mass participation and which gives near-uniform \( \theta_{ld} \) in beams along the height. Also, the method uses the limited \( \theta_{ls} \) available in beams as design input.

**PROPOSED METHOD OF DESIGN**

The Proposed Design (PD) method is meant for seismic design of low-rise RC MF buildings. Its salient facets are: (a) proportioning stiffness and strength of members to make buildings respond to earthquake shaking primarily in their fundamental lateral translational mode with large (>80%) modal mass, and with near-uniform distribution of plastic rotation demand \( \theta_{ld} \) at beam ends, and (b) using the limited plastic rotation capacity \( \theta_{ls} \) available in beams as a design input to estimate the Lateral Force Demand on buildings. The method assumes: (a) uniform distribution of mass \( m \) in storey \( i \) of an \( N \) -storey building, and (b) uniform heights of all storeys except the first (with \( \eta = L_1/L_0 \), being the ratio of centerline heights of 1st and \( i^{th} \) storeys). The PD method involves five steps (Figure 2).

**Step 1:** Choose Fundamental Mode Shape and Proportion Stiffness of Members

The sub-steps involved in proportioning of members of buildings are:

**Step 1a:** Select regular grid in plan and elevation of the building.

**Step 1b:** Select a Modal Mass \( M_1^* \) (over 80%) desired in fundamental lateral mode. With \( M_1^* \) as input, solve Eq.(1) numerically to obtain the non-dimensional mode shape parameter \( \alpha \) (for \( \alpha>0 \), \( \alpha=0 \) and \( \alpha<0 \), Eq.(2) gives shear-, linear- and flexure-type mode shapes, respectively). Eq.(1) is derived considering fundamental lateral mode shape as per Eq.(2). Figure 3 shows \( \alpha \) for \( \eta = 1 \).

\[
\{ \phi \}_1 = \begin{bmatrix}
\lambda_1 \\
\lambda_2 \\
\vdots \\
\lambda_N \\
\rho_1 \\
\vdots \\
\rho_{N-1} \\
\rho_N
\end{bmatrix}
= \begin{bmatrix}
\eta + \sum_{j=1}^{N-1} (1-\alpha)^j/L_{jN} \\
(1-\alpha)^{N-1}/L_i \\
\vdots \\
(\eta/L_{i1}) \\
(\eta/L_{i1}) \\
\vdots \\
\eta \\
\end{bmatrix}
\]

(2)

\[
M = \begin{bmatrix}
\frac{\eta}{n} \left[ \frac{(1-\alpha)^N}{N} \right] - \frac{1-(1-\alpha)^N}{N} \\
\frac{1-(1-\alpha)^N}{N} \\
\frac{1-(1-\alpha)^N}{N} \\
\frac{1-(1-\alpha)^N}{N} \\
\frac{1-(1-\alpha)^N}{N} \\
\end{bmatrix}
= M_1^*
\]

(1)
Figure 3: Mode shape coefficient \( \alpha \) for different desired \( M_i^* \) of buildings with uniform mass and storey height.

**Step 1c:** Obtain the fundamental lateral mode shape \{ \( \phi \) \} using Eq.(2) and \( \alpha \) from Step 1b. Typically, \{ \( \phi \) \} has 2N mode shape coefficients \( \rho \) and \( \lambda \) corresponding to lateral translational and rotational degrees of freedom at each floor \( i \) is taken as \((\rho - \rho_{i-1})/\rho_i\) to simplify derivation of Eqs.(4) to (8).

**Step 1d:** Identify flexible lower storeys (i.e., \( K/K_{i+1} < 1 \)) from Eq.(3) using \( \rho \) from Step 1c:

\[
K_i = \frac{\sum_{j=i}^{N} m_j \rho_j}{m_i \rho_i} \left( \frac{\rho_i - \rho_{i-1}}{\rho_i - \rho_{i-2}} \right)
\]

(3)

where \( m_i \) is the seismic mass lumped at floor \( i \) and \( K_i \) the lateral translational stiffness of storey \( i \). If more than 0.2N storeys of the \( N \)-storey building have flexible lower storeys (i.e., \( K/K_{i+1} < 1 \)), then select smaller \( M_i^* \). By sacrificing some \( M_i^* \), the number of storeys with flexible lower storeys reduces (Figure 4). Thus, choice of \( M_i^* \) also controls stiffness proportioning.

![Flexible Lower Storeys](image)

**Figure 4:** Influence of mode shape parameter \( \alpha \) on number of flexible lower storeys in a 12-storey building (with equal storey height and storey mass) and fundamental modal mass \( M_i^* \).

**Step 1e:** Choose sizes of columns and beams in the top storey based on gravity load considerations, and thereby their gross moments of inertia \( I_{ci} \) and \( I_{bi} \) respectively. When doing so, keep \( l/d \) ratio of members in the range 10:14; this reduces design iterations and leads to plastic rotation capacity \( \theta_{pl} \) in the practical range.

**Step 1f:** Estimate required gross moments of inertia \( I_{ci} \) and \( I_{bi} \) of columns and beams, respectively, in each lower storey \( i \), starting from the \((N-1)th\) storey and going downwards, using Eq.(4) and Eq.(5) as shown below:

\[
I_{ci} = \frac{I_{ci,N}}{\kappa_{ci}} \left[ \sum_{j=1}^{N} \rho_j m_j \right] \left( \begin{array}{c} \rho_i m_i \\ \rho_{i-1} m_{i-1} \end{array} \right) L_{ci} \left( \frac{I_{ci} K_{ci}}{I_{ci,N} K_{ci,N}} \right)^2 \left( 1 + \frac{2 \lambda_{N-1}}{\lambda_i} \right) \left( \frac{1 - \lambda_{N-1}}{\lambda_i} \right) \left( \frac{l_{ci} K_{ci}}{I_{ci} K_{ci,N}} \right) \frac{1}{\rho_i} \frac{1}{\rho_{i-1}} \frac{1}{\rho_{i-2}}
\]

(4)

\[
I_{bi} = \frac{I_{bi,N}}{\kappa_{bi}} \left[ \sum_{j=1}^{N} \rho_j m_j \right] \left( \begin{array}{c} \rho_i m_i \\ \rho_{i-1} m_{i-1} \end{array} \right) L_{ci} \left( \frac{I_{ci} K_{ci}}{I_{ci,N} K_{ci,N}} \right)^2 \left( 1 + \frac{2 \lambda_{N-1}}{\lambda_i} \right) \left( \frac{1 - \lambda_{N-1}}{\lambda_i} \right) \left( \frac{l_{ci} K_{ci}}{I_{ci} K_{ci,N}} \right) \frac{1}{\rho_i} \frac{1}{\rho_{i-1}} \frac{1}{\rho_{i-2}}
\]

(5)

where \( A_1 \), \( A_2 \) and \( A_3 \) are given by:

\[
A_1 = \begin{cases} 
1 + \frac{2 \lambda_{N-1}}{\lambda_i} & 1 < i \leq N \\
1 + \frac{2 \lambda_{N-1}}{\lambda_i} & i = 1
\end{cases}
\]

(6)

\[
A_2 = \left( \frac{\lambda_{N-1} + \lambda_i}{3 \lambda_N} \right) \left( \begin{array}{c} \lambda_{N-1} + \lambda_i \\ \lambda_{N-1} + \lambda_i \\ \lambda_{N-1} + \lambda_i \end{array} \right) \left( \begin{array}{c} 2 \\ 3 \\ 3 \end{array} \right) \left( \begin{array}{c} I_{ci} K_{ci} \\ I_{ci} K_{ci} \\ I_{ci} K_{ci} \end{array} \right)
\]

(7)

\[
A_3 = \begin{cases} 
1 & 1 < i \leq N \\
1 & i = 1 \\
\frac{\lambda_{N-1} I_{ci} K_{ci}}{3 \lambda_N} & i = 1
\end{cases}
\]

(8)

in which \( L_{ci} \) and \( L_b \) are centerline heights of columns in storey \( i \) and lengths of beams at floor \( i \), respectively; \( \kappa_{ci} \) and \( \kappa_{bi} \) are \( \kappa_{eff} \) ratios of these columns and beams, respectively; and \( m_i \) seismic mass lumped at floor \( i \). Eqs.(4) to (8) are derived using characteristic Eigen equation, in which the stiffness matrix of members (i.e., beams and columns) is written considering only flexural deformations (as opposed to considering both flexural and shearing deformations when writing the stiffness matrix) [35]; this consideration is acceptable for members with \( l/d \) in the range 10:14. For initial proportioning, \( \kappa_{ci} \) and \( \kappa_{bi} \) are taken as 0.50 and 0.35, respectively, as these values are seen to reduce design iterations required, if any. Member sizes are rounded off to nearest 50mm.
Step 1g: Since member sizes are rounded off in Step 1f, the dynamic characteristics of the building change slightly. Hence, update \( T_i \) and \( \{\phi\}_i \) of the building using modal analysis with new member sizes as determined in Step 1f. And, estimate \( K_i \) of each storey \( i \) using Eq.(9) [36].

\[
\begin{bmatrix}
K_N \\
K_{N-1} \\
\vdots \\
K_1
\end{bmatrix} = \begin{bmatrix}
\frac{\alpha^2 m_a \rho_N}{\rho_N - \rho_{N-1}} \\
\frac{\alpha^2 \sum_{j=N-1}^{N} m_j \rho_j}{\rho_{N-1} - \rho_{N-2}} \\
\vdots \\
\frac{\alpha^2 \sum_{j=1}^{1} m_j \rho_j}{\rho_1}
\end{bmatrix}
\]  

(9)

where \( \rho_1 \), the mode shape coefficients corresponding to lateral translation degree of freedom, is obtained from modal analysis and not from Eq.(2), and \( \alpha = 2 \pi T_i \). Then, estimate the initial lateral translational stiffness \( K_{\text{initial}} \) of the building as:

\[
K_{\text{initial}} = \frac{1}{B_1 + \frac{h_1^* - h_i}{h_{i+1} - h_i}} B_2 - B_1
\]  

(10)

where the effective height \( h_1^* \) of the building is given by

\[
h_1^* = \frac{\{\phi\}_1^T [M] [h_1]_{\text{initial}}}{\{\phi\}_1^T [M] [\phi]_{\text{initial}}}
\]  

(11)

in which \( h_{i+1} \) and \( h_i \) are heights from the base of the building to floors \( i \) and \( i+1 \) between which \( h_1^* \) is located (Figure 4), \( h_j \) is height \( j \) from base of the building to floor \( j \) and \( \{\phi\}_1 \) the fundamental lateral translational mode shape:

\[
B_1 = \sum_{q=1}^{p=1} \left( \frac{1}{K_q} \right) \sum_{j=q}^{N} \left[ \frac{m_j \rho_j}{\sum_{p=1}^{N} m_p \rho_p} \right]
\]  

(12)

Step 1h: Examine adequacy of sizes chosen of members by checking if the drift demand is within the allowable limits under the design seismic lateral force \( H_{\text{design}} \) given in the seismic design code. If the lateral drift is more, then increase \( L_N \) of columns and/or \( L_{IO} \) of beams in the top storey in Step 1e until the lateral drift limit is less than the allowable limit. \( H_{\text{design}} \) estimated in this step is NOT used in the strength design of the building.

Step 2: Estimate Lateral Force Demand

Assume that: (a) lateral force-displacement response is elastic-perfectly plastic (Figure 5), (b) all storeys sustain equal inter-storey drift, and (c) all columns remain elastic, and (d) all beams form plastic hinges. Estimate lateral force demand on the building as a function of \( \theta_{\text{pc}} \) of beams by the following procedure:

Step 2a: Estimate the Elastic Maximum Lateral Force \( H_e \) as:

\[
H_e = Z I \left( \frac{S_n}{g} \right) W
\]  

(14)

where \( Z \) is the Zone Factor, \( I \) the importance factor, \( (S_n/g) \), the spectral acceleration at \( T_1 \) (corresponding to Maximum Considered Earthquake (MCE)) and \( W \) the total seismic weight of the building [37-38].

Step 2b: Estimate the Elastic Lateral Displacement Demand \( \Delta_e \) as:

\[
\Delta_e = \frac{H_e}{K_{\text{initial}}}
\]  

(15)

Using Equal Displacement Rule [39], the inelastic lateral displacement demand \( \Delta_f \) of a flexible building is given as:

\[
\Delta_f = \Delta_e.
\]  

(16)

Step 2c: Estimate the Plastic Lateral Displacement Capacity \( \Delta_{\text{pc}} \) as:

\[
\Delta_{\text{pc}} = h_1^* \theta_{\text{pc}} \left( \frac{E}{\rho} \right) \bar{L}_b
\]  

(17)

![Figure 5: Expected damage in members, and lateral force-displacement response, of buildings.](image-url)
where for flexible building \((T_1 > 0.5s)\) \(h_1^*\) is as per Eq.(11) and \(L_0^*\) the distance between plastic hinges in beam, and \(L_0\) centerline length of beam bay. Choosing a safety factor \(\gamma_0\) of 2.0 for plastic rotation capacity \(\theta_{pbc}\) of beams in flexible buildings and of 1.5 in stiff buildings, the \(Design\ Lateral\ Plastic\ Displacement\ Capacity\ \Delta_{pdes}\) is:

\[
\Delta_{pdes} = \frac{\Delta_{pc}}{\gamma_0} = h_1^* \left(\frac{\theta_{pbc}^*}{\gamma_0} \frac{L_0^*}{L_0}\right)
\]

\((18)\) 

\(\gamma_0\) is calibrated using results of the time history analysis of 12 buildings (of 4-, 8- and 12-storeys) subjected to a suite of 50 ground motions.

**Step 2d:** Estimate the Yield Displacement Capacity \(\Delta_y\) of flexible buildings \((T_1 > 0.5s)\) as:

\[
\Delta_y = \Delta_{pdes} - \left[\theta_{pbc} \left(\frac{L_0^*}{L_0}\right)\right], \quad \gamma_0
\]

\(of\ stiff\ buildings\ \((T_1 < 0.5s)\)\) using work balance equation as:

\[
\Delta_y = -\Delta_{pdes} + \sqrt{(\Delta_{pdes})^2 - (\Delta_y)^2}
\]

\((19)\) 

**Step 2e:** Estimate the Overstrength Lateral Force Demand \(H_\Omega\) as:

\[
H_\Omega = K_{initial} \Delta_y
\]

Estimate the Design Lateral Force Demand \(H_D\) as

\[
H_D = \frac{H_\Omega}{\Omega}
\]

\((20)\)

\(\Omega\) (=1/0.9) is the overstrength factor (in which 0.9 is the resistance factor).

**Step 2f:** Ensure \(\Delta_y\) obtained is within the limits

\[
0.45\varepsilon_y \left(\frac{L_0}{d}\right) h_1^* \leq \Delta_y \leq 0.55\varepsilon_y \left(\frac{L_0}{d}\right) h_1^*
\]

\((21)\)

where \(\varepsilon = E/\varepsilon\) and \((L/d)\) are yield strain of flexural reinforcement in beams and average \((L/d)\) ratios of all beams in the building, respectively [13]. To minimize the number of iterations needed to match \(E_L\) assumed (Step 1) and \(E_L\) estimated (Step 4), ensure that \(\Delta_y\) is within the specified limit. If not, change \((L/d)\) ratio of beams and choose a new \(h_1^*\) in Step 1e.

**Step 3:** Proportioning Member Strengths

**Step 3a:** Perform linear elastic structural analysis, and obtain:

(a) flexural demands \(M_{b(D+pL+H_p)}\) in beams, and (b) flexural \(M_{c(D+pL+H_p)}\) and axial \(P_{D+pL+H_p}\) demands in columns when the building is subjected to the combined action of \(D\) (Dead Load), \(pL\) (a fraction of Live Load) considered to estimate the seismic mass) and \(H_D\) (Earthquake Load given by Eq.(22)).

**Step 3b:** Design members considering: (1) demands on columns and beams amplified by \(1/\beta_c\) and \(1/\beta_b\), respectively, to account for excessive demands arising out of whiplash effect in upper storeys and shear mode effect in lower storeys; (2) \(\beta_b\) taken as 0.5 in first storey and 0.7 in the others, and \(\beta_c\) as 1.0 up to two-thirds height of the building, and then linearly reducing to 0.5 at the roof in 12-storey buildings and to 0.6 in 8-storey buildings \((\beta_c\)

is calibrated using results of 360 nonlinear time history analyses); and (3) Capacity Design principle to ensure flexural yielding precedes shear failure in members. Thus, the design moment capacities \(M_{cd,req}\) and \(M_{bd,req}\) required in columns and beams, respectively, are taken as:

\[
M_{bd,req} = \frac{1}{\phi} \frac{M_{b(D+pL+H_p)}}{\beta_b} \bigg|_{P=0},\quad \text{and}
\]

\[
M_{cd,req} = \frac{1}{\phi} \frac{M_{c(D+pL+H_p)}}{\beta_c} \bigg|_{P=0},\quad \text{and}
\]

\((22)\) 

\(\phi\) is capacity reduction factor as defined in seismic design code [26].

**Step 4:** Updating Member Stiffness

**Step 4a:** Re-evaluate \(E_L\) of members as \(M_1/\phi\), where \(M_1\) - \(\phi\) curve is obtained using characteristic \(\sigma\)-\(\varepsilon\) curves of concrete and reinforcing steel [40]. Compare these values with \(E_L\) taken in Step 1 of beams and columns as \(0.5E_{I_{gross}}\) and \(0.5E_{I_{gross}}\), respectively.

**Step 4b:** If \(E_L\) is away by more than 10% of that considered in Step 1, repeat the analysis and building redesigned. Experiences from design of buildings indicate that one iteration is sufficient if: (a) yield displacement \(\Delta_y\) from Eq.(19) is within the limits given in Eq.(23), and (b) \(\Delta/\Delta_y\) is in the range 1.5–2.5.

**Step 5:** Detailing Members

**Step 5a:** Detail all members as per ductile detailing requirements given in design code.

![Figure 6: Elevations and plans of buildings considered in the study.](image)

**NUMERICAL STUDY**

**Details of Study Buildings**

Three RC buildings of 4-, 8- and 12-storeys are considered as study buildings whose details are available in literature (Figure 6) [14, 41]. Buildings are designed by the PD method and two other state-of-the-art design methods, namely Direct Displacement Based Design (DDBD) [8, 13] and Performance
Based Plastic Design (PBPD) methods [30]. The inputs and assumptions made in design are listed in Tables 2 to 4. Also, the following assumptions are made: (1) Base of first storey columns is fixed. (2) Gravity loads considered are: (a) uniform floor dead load of 8.38 kN/m² (=175psf), and (b) uniform floor live load of 2.39 kN/m² (=50 psf). (3) Partitions do not participate in lateral load transfer. (4) Buildings are located in a high seismic zone (Los Angeles, CA, USA), for which $S_{ms}$ and $S_{m1}$ are 1.5g and 0.9g, respectively [38]; the expected severe intensity of shaking corresponds to $S_{ms}$ of 1.5g and $S_{m1}$ of 0.9g (Soil Class: $S_D$). (5) Members are designed as per ACI 318–14 with: (a) concrete of cylinder compressive strength of 34.47 MPa (=5 ksi) in beams and 48.26 MPa (=7 ksi) in columns, and (b) flexural reinforcement steel with yield stress of 413.7 MPa (=60 ksi) in beams and of 517.12 (=75 ksi) in columns, and (c) shear reinforcement steel with yield stress of 413.7 MPa (=60 ksi) in both beams and columns. Cross-sectional details of members, ratios of effective to gross rigidities of members and reinforcement provided in members of buildings designed using PD and DDBD methods are available in Annex A; the same for buildings designed using PBPD method is available in literature [14].

**Table 2: Design inputs and assumptions made in PD method.**

| S.No. | Input and Assumption | 8 storey | 12 storey |
|-------|----------------------|----------|----------|
| **Step 1: Choosing Fundamental Mode & Proportioning Member Stiffness** | | | |
| 1.1   | $\alpha$ used to choose $[\phi]_1$ | 0.15 | 0.07 |
| 1.2   | Mass Participation for chosen $[\phi]_1$ | 87.8% | 84.0% |
| 1.3   | Initial $EI_{eff}/EI_{gross}$ of beams | 0.3 | 0.3 |
| 1.4   | $L_b/d$ ratio of beams in top storey | 12.2 | 12.2 |
| 1.5   | Average $L_b/d$ ratio of beams | 10.0 | 9.4 |
| 1.6   | Acceptable yield displacement (in m) (Eq. (10)) | 0.210-0.256 | 0.287-0.351 |
| **Step 2: Estimating Lateral Force Demand** | | | |
| 2.1   | Plastic Rotation Capacity (rads) of beams | 0.03 | 0.03 |
| 2.2   | Safety Factor $\gamma_\theta$ for rotation capacity | 2.0 | 2.0 |
| **Step 3: Proportioning Member Strength** | | | |
| 3.1   | Beams are designed using $\beta_\delta$ in the range of 0.5-1.0 to ensure that they yield and to control whiplash effect. All columns are designed using $\beta_\delta$ of 0.5 to ensure that they do not yield. | | |

**Table 3: Design inputs and assumptions made in DDBD method [13].**

| S.No. | Input and Assumption | 8 storey | 12 storey |
|-------|----------------------|----------|----------|
| **Estimating Lateral Force Demand** | | | |
| 2.1   | Critical Inter-storey drift | 2.5% | 2.0% |
| 2.2   | $L_b/d$ ratio of beams | 9 | 7 |
| 2.3   | $EI_{eff}/EI_{gross}$ of non-yielding columns | 0.5 | 0.5 |
| 2.4   | (1) Distribution along building height of inter-storey drift demand (to estimate lateral displacement demand $\Delta_d$ of the building) is as per literature [13]. (2) Design lateral force estimated using yield displacement and $\Delta_d$. (3) All beams yield. | | |
| **Proportioning Member Strength** | | | |
| 3.1   | (1) Flexural demand is estimated in beams using equilibrium-based analysis considering lateral loads alone. (2) Columns are designed for combined actions of flexural demand (estimated using lateral loads alone) and axial demand (estimated from gravity loads alone). (3) Capacity protection factor used to ensure columns do not yield [8]. | | |
Modeling Details

Typical 2D interior frames oriented along X-direction (Figure 6) are considered to assess seismic performance of the study buildings. Commercially available Perform 3D structural analysis software (version 5) [42] is used. Members are modeled using linear elements. The bases of first storey columns are considered to be fixed. Also, all nodes are restrained from moving in the out-of-plane direction. Ratios of effective to gross flexural rigidities of structural elements vary. These and the cross-section details of all members are listed in Annex A. Also, seismic mass is lumped at the beam-column joints. Lumped $M-\theta$ plastic hinges are used to model inelasticity (flexural yielding). $M-\theta$ responses of beams and $P-M-\theta$ responses of columns are estimated based on span, longitudinal reinforcement and confinement offered by transverse reinforcement. The force-deformation backbone curves of the lumped plastic hinges are idealized using a tri-linear relation with strength loss. Strength and stiffness degrading hysteretic loops of yielding actions are modelled using cyclic degradation energy factor $e$, it denotes the ratio of the area of degraded hysteretic loop to the area of elastic perfectly-plastic hysteretic loop. $e$ is linearly reduced: (a) from 100% to 60% between first yield point and ultimate strength point and (b) from 60% to 20% between ultimate strength point and residual strength point of idealized tri-linear relation, depending on where the unloading starts. This hysteresis model accounts for cyclic strength reduction [43]. The reduction in stiffness is as per literature [44]. Shear failure of members is precluded through capacity design and detailing. Beam-column joints are considered to be stiff and strong. Rayleigh damping of 5% between 0.9$T_1$ and 0.25$T_1$ (as recommended in the manual of Perform 3D) is used.

Methods of Analyses

The dynamic characteristics of the buildings are estimated using modal analysis of buildings. Performances of designed buildings are assessed by both Nonlinear Static (NSA) and Nonlinear Time History Analyses (NTHA). NSA is used to obtain the lateral force-displacement response of buildings and the lateral displacement capacity; when reporting the lateral force-displacement response, the lateral displacement at the effective height $h'$ of the building is used, to ensure consistency between the lateral force-displacement curve considered in the analysis and design stages. The performance point of a building is obtained using equivalent linearization procedure [45]. In NTHA, each building is subjected to a suite of 30 ground motions (Table 5) [46-48], which are selected to have significant randomness (i.e., coefficient of variation) in their characteristics [48] (Table 6). Further, 30 ground motions are selected to limit epistemic uncertainty related to selection of ground motions. Ground motions are scaled using spectral scaling method to ensure the buildings are subjected to the MCE level of earthquake shaking. $P-\Delta$ effects are considered in both NSA and NTHA.

During NSA and NTHA, stated ‘failure’ of buildings denotes at least one structural element reaching any one of the limit states: (1) exhausting plastic rotation capacity $\theta_{pc}$ of beams, and (2) reaching ultimate compressive strain $\varepsilon_{cu}$ of confined concrete in columns. Exhastuing $\theta_{pc}$ of beams may not lead to collapse of buildings, but only may lead to disruption in the gravity load path resulting in increased demand in few columns. In contrast, crushing failure of columns by reaching $\varepsilon_{cu}$ of confined concrete can lead to local failure, and even, global failure of buildings. Thus, the said limit states are considered to assess the guaranteed capacity of buildings. Further, average and maximum estimates of plastic rotation demand and inter-storey drift demand are obtained using NTHA results of 27 of the 30 ground motions; 3 outlier data points on the higher side are ignored. Ignoring outliers is acceptable as the general acceptance criteria used in design codes allow failure of certain percentile of samples (e.g., definitions of minimum specified loads and material strengths).

For interpreting results of NTHA, the uniformity in the variation of responses (such as plastic rotation demand and inter-storey drift demand) is quantified along the building height; data of N-2 responses is used to estimate their $CoV^2/N$ is the number of storeys in the building). Effectively, $CoV$ is estimated without considering responses of the first and top storeys of a building, because responses of these storeys are significantly influenced by either the fixity of columns at the base or discontinuity of members at the roof level [49].

| S.No. | Input and Assumption | Building |
|-------|----------------------|----------|
|       |                      | 8 storey | 12 storey |
| 1.1   | Yield lateral drift  | 0.5%     | 0.5%     |
| 1.2   | Ultimate lateral drift for severe shaking | 3% | 3% |
|       | Ultimate lateral drift for design shaking | 2% | 2% |
| 1.3   | Plastic Rotation Capacity (rads) of beams | 0.025 | 0.026 |
| 1.4   | (1) All beams yield, and sustain nearly same plastic rotation demand. | |
|       | (2) Inter-storey drift demand is uniform along the height. | |

Table 4: Design inputs and assumptions made in PBPD method [14].
Table 5: List of 30 GMs [46-48].

| No. | Event          | Station                  | Year | \(M_w\) | PGA (g) | Epicentral distance (km) |
|-----|----------------|--------------------------|------|---------|---------|--------------------------|
| 1   | Kern County    | Taft                     | 1952 | 7.36    | 0.159   | 38.9                     |
| 2   | San Fernando   | Palmdale Fire Station    | 1971 | 6.60    | 0.133   | 25.4                     |
| 3   |                | Lake Hughes              |      |         | 0.144   | 25.8                     |
| 4   | Tabas          | Dayhook                  | 1978 | 7.35    | 0.324   | 13.9                     |
| 5   |                | Plaster City             |      |         | 0.042   | 31.7                     |
| 6   | Imperial Valley| Niland Fire Station      | 1979 | 6.50    | 0.069   | 35.9                     |
| 7   |                | Delta                    |      |         | 0.351   | 43.6                     |
| 8   |                | Coachella Canal #4       |      |         | 0.115   | 49.3                     |
| 9   | Park Field     | Cholame 3W               |      |         | 0.078   | 30.4                     |
| 10  |                | Gold Hill 3E             | 1983 | 6.40    | 0.094   | 29.2                     |
| 11  |                | Fault Zone 3             |      |         | 0.139   | 36.4                     |
| 12  |                | Fault Zone 10            |      |         | 0.073   | 30.4                     |
| 13  | Superstition hills | Wildlife Lique. Array | 1987 | 6.30    | 0.207   | 24.7                     |
| 14  | Loma Prieta    | Hollister-South Pine     | 1989 | 6.90    | 0.273   | 47.9                     |
| 15  |                | Red Wood City            |      |         | 0.091   | 32.6                     |
| 16  | Salinas-John and Work |                       |      |         | 0.154   | 44.6                     |
| 17  | Cape Mendocino | Eureka-Myrtle and West   | 1992 | 7.10    | 0.116   | 23.6                     |
| 18  |                | Fortuna Boulevard        |      |         | 0.152   | 24.9                     |
| 19  | Landers        | Fire Station             | 1992 | 7.30    | 0.076   | 37.5                     |
| 20  |                | Palm Springs Airport     |      |         | 0.171   | 23.2                     |
| 21  |                | Desert Hot Spring        |      |         | 0.087   | 36.3                     |
| 22  | Northridge     | Lake Hughes #1           | 1994 | 6.70    | 0.230   | 47.6                     |
| 23  |                | Downey-Co Maint. Bldg.   |      |         | 0.133   | 41.9                     |
| 24  |                | LA 116th Street School   |      |         | 0.483   | 7.08                     |
| 25  | Kobe           | Nishi-Akashi             | 1995 | 6.90    | 0.251   | 22.5                     |
| 26  |                | Kakogawa                 |      |         | 0.214   | 24.8                     |
| 27  | Hector Mine    | Hector                   | 1999 | 7.13    | 0.265   | 11.6                     |
| 28  | Chi Chi        | TCU 047                  | 1999 | 7.62    | 0.298   | 35.0                     |
| 29  | Chamoli        | Gopeshwar                | 1999 | 6.8     | 0.359   | 8.7                      |

Table 6: Statistical variation of ground motion characteristics [48].

| Quantities                  | Epicentral Distance (km) | PGA (g) | Significant Duration (s) | Frequency corresponding to peak Fourier amplitude (Hz) |
|-----------------------------|--------------------------|---------|--------------------------|-----------------------------------------------------|
| Minimum                     | 7.08                     | 0.042   | 8.4                      | 0.21                                                |
| Maximum                     | 49.30                    | 0.483   | 50.33                    | 2.64                                                |
| Mean                        | 30.48                    | 0.188   | 19.72                    | 1.28                                                |
| CoV (%)                     | **36.9**                 | **58.8**| **51.8**                 | **52.9**                                            |

RESULTS

Overall Responses

Figure 7 shows lateral force-displacement curves of the study buildings obtained from NSA, and Table 7 lists the following results of buildings designed by the different methods: (a) lateral translational stiffness, (b) strength capacity, (c) lateral drift capacity and demand, and (d) total energy. The salient observations from the NSA of buildings designed by the three methods are:

(1) Buildings designed by PD method have the least \( K_{initial} \) (most flexible), and those by DDBD method the highest (most stiff). \( K_{initial} \) of buildings are different owing to differences in: (a) member sizes, and (b) ratio of effective to gross flexural rigidities of members.

(2) The 8-storey and 12-storey buildings designed by PD method and DDBD method have the highest lateral strength, respectively; lateral strength is lowest in buildings designed by PBPD method, because the method considers least design lateral force.
(3) Buildings designed by PD method have highest lateral drift capacity (at least 20% more), because both stiffness and strength are proportioned explicitly; buildings designed by PBPD method have lowest lateral drift capacity.

(4) Total energy stored in the buildings (estimated as area under the lateral force-displacement curve) (Table 7) is highest in buildings designed by PD and DDBD methods in 8- and 12-storey buildings, respectively; buildings designed by PBPD method have lowest total energy.

Performance of Buildings

Acceptability of the design of a building is examined by the number of ground motions that the building withstands without exceeding $\theta_{pc}$ of beams ($= 0.03$ rads). Table 8 lists the number of instances when $\theta_{pc}$ is exceeded in buildings when resisting MCE level earthquake shaking; it is estimated using the counted statistics method (as in [50]). Also, it provides results from NTHA along with the number of ground motions that cause yielding of columns. And, Figure 8 shows number ground motions that cause yielding of members designed by the three methods. The salient observations are:

(1) Buildings designed by PD and DDBD methods withstand about 90% of ground motions (i.e., at least 27 of 30 ground motions) without exceeding $\theta_{pc}$ of beams; those designed by PBPD method withstand only ~70% of ground motions, respectively.

(2) All 30 ground motions result in yielding of columns in 12-storey buildings designed by DDBD method, because design of columns is based on axial demand from gravity load analysis, and flexural demand from lateral load analysis. Also, the method uses a capacity protection factor to prevent yielding of columns. Notwithstanding this, the method underestimates demand on columns in exterior bays (where axial demand on columns changes significantly due to overturning action under earthquake shaking); consequently, exterior columns sustain significant yielding in 12-storey buildings. Thus, the design of columns is inadequate to prevent yielding of columns. This observation is consistent with results present in literature [51].

(3) Only 1 (of 30) ground motion causes yielding of columns in buildings designed by the PD method. And, no more than 4 (of 30) ground motions cause yielding of columns in buildings designed by PBPD method.

![Figure 7: Lateral force-displacement response of buildings designed by the three methods.](image)

**Table 7: Lateral drift capacity and demand of buildings designed by the three methods.**

| Buildings | Design Method | Drift Demand (%) | $K_{init}$ (kN/m) | Strength (kN) | Drift Capacity (%) | Total Energy (kNm) |
|-----------|---------------|------------------|-------------------|--------------|-------------------|-------------------|
| 4-storey  | DDBD          | 2.26             | 33,442            | 2,700        | 3.00              | 929               |
|           | PBPD          | 2.60             | 27,365            | 1,849        | 2.72              | 510               |
|           | PD            | 2.41             | 25,027            | 3,719        | 3.88              | 1,577             |
| 8-storey  | DDBD          | 1.88             | 11,228            | 1,793        | 2.86              | 969               |
|           | PBPD          | 2.51             | 9,300             | 777          | 2.38              | 386               |
|           | PD            | 2.32             | 7,491             | 1,975        | 3.54              | 1,225             |
| 12-storey | DDBD          | 1.50             | 13,484            | 2,428        | 2.76              | 1,900             |
|           | PBPD          | 2.23             | 8,163             | 853          | 2.33              | 626               |
|           | PD            | 2.24             | 5,837             | 2,068        | 3.32              | 1,813             |
Figure 8: Number of ground motion that causes yielding in each beam and column of buildings designed by the three methods.

Table 8: Performance of buildings designed by the three methods.

| Buildings | Design Method | Number of instances when \( \theta_{pb} \) is exceeded (%) | Number of Ground Motions |
|-----------|---------------|----------------------------------------------------------|--------------------------|
|           |               | Sustained safely than do not lead to yielding of Columns | that lead to yielding of Columns |
| 4-storey  | DDBD          | 5             | 28  | 24  | 6  |
|           | PBPD          | 30            | 21  | 26  | 4  |
|           | PD            | 3             | 29  | 29  | 1  |
| 8-storey  | DDBD          | 10            | 27  | 13  | 17 |
|           | PBPD          | 13            | 25  | 30  | -  |
|           | PD            | 10            | 27  | 29  | 1  |
| 12-storey | DDBD          | 3             | 29  | -   | 30 |
|           | PBPD          | 23            | 29  | 30  | -  |
|           | PD            | 3             | 29  | 30  | -  |

Plastic Rotation Demand

The PD and PBPD methods use available plastic rotation capacity \( \theta_{pb} \) of beams as input to estimate design lateral force of buildings (Tables 2 and 4). The PD method uses a safety factor to decide the design capacity \( \theta_{pbdes} = \theta_{pb} / \gamma \) from the available capacity \( \theta_{pb} \). In contrast, PBPD method uses the available capacity \( \theta_{pb} \) as the design capacity \( \theta_{pbdes} \). Table 9 shows these values along with the maximum \( \theta_{pbmax} \) and average \( \theta_{pbavg} \) of absolute plastic rotation demands in beams when resisting at least 27 (of the 30) ground motions without exceeding \( \theta_{pb} \). In addition, the PD and PBPD methods assume all beams to undergo nearly similar \( \theta_{pb} \) under severe ground shaking. To examine the validity of this assumption, the average \( \theta_{pbavg,storey} \) and maximum \( \theta_{pbmax,storey} \) plastic rotation demands are examined on beams in each storey (Figure 9); CoVs of these values are listed in Table 9. The salient observations on plastic rotation demands in buildings designed by the three methods are:

(1) DDBD method: The plastic rotation demands in beams in 12-storey buildings show the smallest \( \theta_{pbmax} \). Also, these
values are: (a) less than the available $\theta_{pbc}$ of 0.03 rads, and
(b) near uniform along the height, even though significant
yielding of columns is observed (Figure 9).

(2) PBPD method: The plastic rotation demand $\theta_{pb_d,max}$ in
beams exceeds $\theta_{pbc}$ in 5-9 ground motions, because this
method uses available $\theta_{pbc}$ itself as the design value. This
highlights the need to use a safety factor of plastic rotation
capacity in design to limit the plastic rotation demand in
beams. Also, beams in 8-storey building have the largest
$\theta_{pb_d,max}$. In most storeys of 8-storey and 12-storey
buildings, $\theta_{pb_d,max,storey}$ exceed the available $\theta_{pbc}$ of 0.03
rad. Also, $\theta_{pb_d,max}$ is concentrated in the first few storeys.
Thus, the plastic actions (and hence damage) are localised
in buildings designed by PBPD method. Thus, $\theta_{pbc}$ of
beams is exceeded and the assumptions that plastic
rotation demand is uniform along the height is violated.

(3) PD method: It uses a safety factor $\gamma$ on available plastic
rotation capacity $\theta_{pbc}$ to estimate lateral force demand on
buildings. Hence, the plastic rotation demand in beams is
less than the available $\theta_{pbc}$; this is not observed in any
other method. Further, plastic rotation demands in beams
are almost uniform along the height. Thus, as in buildings
designed by the DDBD method, damage in building
design using PD method is well distributed along the
building height.

![Figure 9: Variation in plastic rotation in beams along the height of 4-, 8- and 12-storey buildings designed by the three methods: (a) average of maximum rotation, and (b) absolute maximum rotation.](image)

| Buildings | Methods | Capacity $\theta_{pbc}$ | Design Capacity $\theta_{pbdes}$ | Maximum Demand $\theta_{pb_d,max}$ | Average Demand $\theta_{pb_d,avg}$ | Maximum Demand $\theta_{pb_d,max,storey}$ | Average Demand $\theta_{pb_d,avg,storey}$ |
|-----------|---------|------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
| 4-storey  | DDBD    | -                      | -                                | 2.94                             | 1.29                             | 5.0                              | 6.3                              |
|           | PBPD    | 2.40                   | 2.40                             | 3.70                             | 1.93                             | 1.7                              | 1.6                              |
|           | PD      | 3.00                   | 2.00                             | 1.91                             | 0.86                             | 4.6                              | 18.2                             |
| 8-storey  | DDBD    | -                      | -                                | 2.95                             | 1.07                             | 7.0                              | 12.0                             |
|           | PBPD    | 2.50                   | 2.50                             | 3.70                             | 1.59                             | 16.4                             | 20.4                             |
|           | PD      | 3.00                   | 1.50                             | 3.05                             | 1.02                             | 21.7                             | 14.4                             |
| 12-storey | DDBD    | -                      | -                                | 1.92                             | 0.76                             | 10.7                             | 11.9                             |
|           | PBPD    | 2.60                   | 2.60                             | 3.63                             | 1.38                             | 23.9                             | 34.4                             |
|           | PD      | 3.00                   | 1.50                             | 2.60                             | 1.01                             | 7.5                              | 15.8                             |

Table 9: Plastic rotations assumed in design of beams and those obtained from NTHA of buildings designed by the three methods.
Displacement Demand (Inter-storey Demand)

The DDBD, PBPD and PD methods use lateral displacement demand $\Delta s$ on building to estimate design lateral force. DDBD method uses critical inter-storey drift demand and distribution of inter-storey drift demand along the building height. PBPD method assumes appropriate values. The absolute maximum $\Delta d_{\text{max}}$ and average $\Delta d_{\text{avg}}$ demands estimated from NTHA with at least 27 (of 30) ground motions are listed in Table 10. In 4-storey designed by PD method, the displacement demand is within the values assumed in design. But, the displacement demand is 11% more than that assumed in the design of the same building using the PBPD method. To estimate lateral force demand on buildings: (a) PBPD and PD methods assume uniform distribution of inter-storey drift demand along the height, and (b) DDBD method assumes either uniform or gradually reducing inter-storey drift demand profile along the height, depending on number of storey. To assess the validity of the assumptions made, the variations of $\delta_{\text{max}}$ and $\delta_{\text{avg}}$ are studied along height (Figures 10) and of their CoVs (Table 10). The salient observations on the maximum inter-storey drift demands in buildings designed by the three methods are:

1. **DDBD method**: $\Delta s$ assumed in design and $\Delta d_{\text{max}}$ demand from analysis differ by less than 16%. Further, $\delta_{\text{avg}}$ is less than the critical inter-storey drift assumed in design to estimate lateral force demand obtained from NTHA, but $\delta_{\text{max}}$ is more, but almost uniform along the height (Table 10).

2. **PBPD method**: $\Delta s$ and $\Delta d_{\text{max}}$ differ by up to +11%. Also, inter-storey drift demand is concentrated in the first few storeys of 12-storey building. Thus, the inter-storey drift demand assumed in design does not match with that obtained from NTHA.

3. **PD method**: $\Delta s$ and $\Delta d_{\text{max}}$ differ by less than 16%. Also, the inter-storey drift demand ($\delta_{\text{max}}$ and $\delta_{\text{avg}}$) is nearly uniform along the height; this is reflected by the CoV values also (Table 10).

![Figure 10: Variation in inter-storey drift along the height of 4-, 8- and 12-storey buildings designed by the three methods: (a) average of maximum inter-storey drift, and (b) absolute maximum inter-storey drift.](image)

![Table 10: Lateral displacement demand estimated in design and those obtained from NTHA of buildings designed by the three methods.](table)

| Buildings | Methods | Lateral displacement (%) | CoV (%) of distribution of |
|-----------|---------|--------------------------|---------------------------|
|           | Design stage $\Delta s$ | Average $\Delta d_{\text{avg}}$ | Maximum $\Delta d_{\text{max}}$ | Average $\delta_{\text{avg}}$ | Maximum $\delta_{\text{max}}$ |
| 4-storey  | DDBD    | 2.50                     | 1.62                      | 2.70                      | 5.8                        | 3.3                        |
|           | PBPD    | 3.00                     | 2.04                      | 3.33                      | 3.4                        | 1.9                        |
|           | PD      | 3.03                     | 1.79                      | 2.35                      | 7.7                        | 5.7                        |
| 8-storey  | DDBD    | 2.02                     | 1.26                      | 1.87                      | 7.0                        | 6.8                        |
|           | PBPD    | 3.00                     | 1.60                      | 2.88                      | 10.2                       | 10.9                       |
|           | PD      | 2.32                     | 1.64                      | 2.50                      | 3.3                        | 6.8                        |
| 12-storey | DDBD    | 1.49                     | 1.06                      | 1.74                      | 4.9                        | 6.5                        |
|           | PBPD    | 3.00                     | 1.35                      | 2.52                      | 19.6                       | 21.6                       |
|           | PD      | 2.24                     | 1.50                      | 2.60                      | 5.9                        | 6.9                        |
Table 11: Bill of quantities.

| Buildings  | Methods | Volume of Concrete (m³) | Weight of Longitudinal steel (tonnes) |
|------------|---------|-------------------------|--------------------------------------|
| 4-storey   | DDBD    | 81.4                    | 9.4                                  |
|            | PBPD    | 78.7                    | 7.6                                  |
|            | PD      | 69.5                    | 18.4                                 |
| 8-storey   | DDBD    | 107.3                   | 12.8                                 |
|            | PBPD    | 75.8                    | 9.2                                  |
|            | PD      | 91.5                    | 18.9                                 |
| 12-storey  | DDBD    | 187.6                   | 22.8                                 |
|            | PBPD    | 116.3                   | 14.4                                 |
|            | PD      | 154.6                   | 29.0                                 |

Table 12: Overall rating of the buildings designed using the three methods.

| Performance Index               | DDBD | PBPD | PD  |
|---------------------------------|------|------|-----|
| Lateral Strength                | Largest | Least | Large |
| Lateral Deformability           | Large | Smallest | Largest |
| Damage Distribution             | Largest | Smallest | Largest |
| Damage at undesirable locations | Highest | Least | Least |
| Plastic Rotation Demand Demand  | Small | *Largest | Small |
| Lateral Displacement Demand     | Small | Least | Medium |
| Inter-storey Drift Demand       | Small | *Largest | Large |
| Material required               | Highest | Least | Highest |

* Concentrated in a few storeys

Summary

The results suggest that buildings designed by PD method demonstrate the best seismic performance and those designed by the PBPD method the worst. The overall ratings of the three methods are presented in Table 12 based on different considerations. The PD method provides an acceptable building using: (a) the properties of the first translational mode alone, and (b) the maximum plastic rotation capacity of beams, such that plastic rotation demand in beams are limited to levels within practically achievable values and are uniform along the height: the maximum plastic rotation demand (averaged over all beams in a storey) is 77%–84% of the design values in 8- and 12-storey buildings.

CONCLUSIONS

The salient conclusions of this study are:

1. A new method is proposed for seismic design of moment frame low-rise buildings. The design method considers:
   (a) A single mode, namely the fundamental lateral translational mode, and
   (b) Plastic rotation capacity of beams as a design input, with a safety factor of 2.0 on available plastic rotation capacity in them.

The resulting building possesses good seismic performance – desirable mechanism and large deformability. Further, results of the numerical study highlight the efficacy of the proposed design method in limiting: (a) the contribution of higher modes of oscillation in the seismic response of buildings and (b) the plastic rotation demand \( \theta_{pl} \) is successfully restricted to within to practically achievable \( \theta_{pl} \) and available in members, which is considered as design input.

2. Based on numerical study presented, the relative performances of buildings designed by the proposed and two other methods show that the Proposed Design method is: (a) better than the DDBD, and (b) significantly better than the PBPD method in controlling seismic behavior of buildings.

This method is not applicable to tall RC MF buildings, because it is difficult to make first mode dominate in such buildings.

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Cross-sectional details of members, ratios of effective rigidity to gross rigidity of beam and longitudinal reinforcement in members of building designed using Direct Displacement Based Design method and Proposed Design method are listed in Table A.1. The details of buildings designed using Performance Based Plastic Design method are available in literature [14].

### Table A.1: Details of building designed using DDBD and PD methods

| Storey | DDBD Method: 4-Storey | PD Method: 4-Storey |
|--------|------------------------|---------------------|
|        | B  | E·C | I·C | B  | Br·Bb | E·C | I·C | B  | Br·Bb | E·C | I·C |
| 4      | 400×850 | 850 | 850 | 0.18 | 0.49 | 0.49 | 0.8 | 1.6 | 400×650 | 700 | 850 | 0.24 | 1.12,0.54 | 4.3 | 4.3 |
| 3      | 400×850 | 850 | 850 | 0.32 | 0.88 | 0.88 | 0.8 | 1.6 | 400×850 | 700 | 850 | 0.33 | 1.47,0.98 | 4.3 | 4.3 |
| 2      | 400×850 | 850 | 850 | 0.42 | 1.15 | 1.15 | 0.8 | 1.6 | 400×850 | 700 | 850 | 0.44 | 2.18,1.68 | 4.3 | 4.3 |
| 1      | 400×850 | 850 | 850 | 0.47 | 1.29 | 1.29 | 0.8 | 1.6 | 400×650 | 700 | 850 | 0.36 | 2.31,1.25 | 4.3 | 4.3 |

| Storey | DDBD Method: 8-Storey | PD Method: 8-Storey |
|--------|------------------------|---------------------|
|        | B  | E·C | I·C | B  | Br·Bb | E·C | I·C | B  | Br·Bb | E·C | I·C |
| 8      | 450×600 | 750 | 750 | 0.11 | 0.30 | 0.30 | 1.7 | 0.8 | 300×450 | 550 | 650 | 0.24 | 1.66,0.81 | 3.4 | 3.0 |
| 7      | 450×600 | 750 | 750 | 0.21 | 0.59 | 0.59 | 1.7 | 0.8 | 450×550 | 600 | 700 | 0.21 | 1.11,0.69 | 3.5 | 3.1 |
| 6      | 450×600 | 750 | 750 | 0.30 | 0.87 | 0.87 | 1.7 | 0.8 | 450×600 | 650 | 750 | 0.24 | 1.16,0.87 | 3.0 | 2.7 |
| 5      | 450×600 | 750 | 750 | 0.38 | 1.10 | 1.10 | 1.7 | 0.8 | 450×650 | 650 | 750 | 0.26 | 1.19,0.94 | 3.0 | 2.7 |
| 4      | 450×600 | 750 | 750 | 0.45 | 1.29 | 1.29 | 1.7 | 0.8 | 450×650 | 650 | 750 | 0.30 | 1.42,1.17 | 3.0 | 2.7 |
| 3      | 450×600 | 750 | 750 | 0.50 | 1.45 | 1.45 | 1.7 | 0.8 | 450×650 | 650 | 750 | 0.35 | 1.68,1.42 | 3.0 | 2.7 |
| 2      | 450×600 | 750 | 750 | 0.54 | 1.56 | 1.56 | 1.7 | 0.8 | 450×650 | 650 | 750 | 0.40 | 2.04,1.79 | 3.0 | 2.7 |
| 1      | 450×600 | 750 | 750 | 0.56 | 1.62 | 1.62 | 1.7 | 0.8 | 450×650 | 650 | 750 | 0.29 | 1.73,1.09 | 3.0 | 2.7 |

| Storey | DDBD Method: 12-Storey | PD Method: 12-Storey |
|--------|------------------------|---------------------|
|        | B  | E·C | I·C | B  | Br·Bb | E·C | I·C | B  | Br·Bb | E·C | I·C |
| 12     | 450×750 | 800 | 800 | 0.12 | 0.31 | 0.31 | 1.0 | 1.8 | 300×450 | 550 | 650 | 0.25 | 1.76,0.85 | 2.1 | 3.0 |
| 11     | 450×750 | 800 | 800 | 0.18 | 0.48 | 0.48 | 1.0 | 1.8 | 350×600 | 650 | 750 | 0.21 | 1.20,0.68 | 1.9 | 2.3 |
| 10     | 450×750 | 800 | 800 | 0.23 | 0.63 | 0.63 | 1.0 | 1.8 | 450×600 | 650 | 800 | 0.22 | 1.11,0.79 | 1.9 | 2.0 |
| 9      | 450×750 | 800 | 800 | 0.28 | 0.78 | 0.78 | 1.0 | 1.8 | 450×650 | 700 | 800 | 0.24 | 1.15,0.88 | 2.1 | 2.0 |
| 8      | 450×750 | 800 | 800 | 0.33 | 0.92 | 0.92 | 1.0 | 1.8 | 450×750 | 700 | 800 | 0.25 | 1.15,0.91 | 2.1 | 2.0 |
| 7      | 450×750 | 800 | 800 | 0.37 | 1.04 | 1.04 | 1.0 | 1.8 | 450×650 | 750 | 800 | 0.29 | 1.37,1.13 | 2.3 | 2.0 |
| 6      | 450×750 | 800 | 800 | 0.41 | 1.14 | 1.14 | 1.0 | 1.8 | 450×650 | 750 | 800 | 0.32 | 1.54,1.30 | 2.3 | 2.0 |
| 5      | 450×750 | 800 | 800 | 0.44 | 1.23 | 1.23 | 1.0 | 1.8 | 450×650 | 750 | 800 | 0.35 | 1.69,1.46 | 2.7 | 2.4 |
| 4      | 450×750 | 800 | 800 | 0.47 | 1.31 | 1.31 | 1.0 | 1.8 | 450×650 | 750 | 800 | 0.38 | 1.83,1.60 | 2.7 | 2.4 |
| 3      | 450×750 | 800 | 800 | 0.49 | 1.37 | 1.37 | 1.0 | 1.8 | 450×650 | 750 | 800 | 0.40 | 1.95,1.71 | 2.7 | 2.4 |
| 2      | 450×750 | 800 | 800 | 0.12 | 0.31 | 0.31 | 1.0 | 1.8 | 300×450 | 550 | 650 | 0.25 | 1.76,0.85 | 2.1 | 3.0 |
| 1      | 450×750 | 800 | 800 | 0.18 | 0.48 | 0.48 | 1.0 | 1.8 | 350×600 | 650 | 750 | 0.21 | 1.20,0.68 | 1.9 | 2.3 |