Dynamic behavior of the mechanical systems from the structure of a hybrid automobile

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Abstract. In introduction are presented solutions of planetary mechanisms that can be used in the construction of the hybrid automobiles where the thermal and electrical sources must be coupled. The systems have in their composition a planetary mechanism with two degrees of mobility at which are coupled a thermal engine, two revertible electrical machines, a gear transmission with four gears and a differential mechanism which transmits the motion at the driving wheels. For the study of the dynamical behavior, with numerical results, one designs such mechanisms, models the elements with solids in AutoCAD, and obtains the mechanical properties of the elements. Further on, we present and solve the equations of motion of a hybrid automotive for which one knows the dynamical parameters.

1. Introduction
In the last years all great constructors of automobiles promoted solutions of hybrid automotive as solution to reduce the noxae emitted by the thermal engines, especially in the urban environment. More and more countries, by financial facilitations or traffic restrictions, encourages the production or buying of hybrid or electrical automotive.

No matter the type of the hybrid automotive: mini-hybrid, semi-hybrid or totally hybrid, it has in its composition a source of thermal power and a source of electrical power.

In [1] are described three types of coupling systems of these two sources of power: mechanical systems, hydraulic systems and electrical systems. In the same paper are also presented the differential equations of motions for a planetary mechanism used by Toyota Corp. in the series production on the hybrid automobile Prius, launched in fabrication in 1997.

The idea of using the planetary mechanism in the coupling of power sources was extended in [2] also for other configurations of mechanisms. Another four such mechanisms are analyzed from the kinematic point of view, mechanisms which can be used for the coupling of the power sources. Also in [2] is analyzed the stability of the five studied mechanical systems. Authors established the conditions that must be fulfilled by certain mechanical, kinematical and dynamical parameters in order to assure a stable motion. Modifying the configuration of the mechanism, one also modifies its stability of the motion.

In the present paper we purpose to study, by comparison, the dynamical behavior of a hybrid automobile which contains a planetary mechanism with four elements, in two configurations, mechanism which sums the power of two electrical machines and a thermal engine, and a mechanical transmission that transmits the motion to the wheel of the automotive.
2. Solution of analyzed planetary mechanisms

The planetary mechanisms are mechanisms with two degrees of freedom and they can couple a thermal engine and two electrical machines. The solution of mechanical systems used on Prius automobile is that in Fig. 1.

![Figure 1. Mechanical system used at the Toyota Prius automobile.](image)

In this configuration of planetary mechanism with four elements, the thermal engine is coupled at the port-satellite lever,1, the electrical machine GE is coupled at the solar wheel 3, and the electrical machine ME at the crown 4 of the mechanism. The motion transmits to the main transmission of the automobile through a chain transmission and through a gear with two cylindrical gears \(z_5 - z_6\). The electrical machines ME and GE are used as motor or generator depending of the speed of the automobile and the electrical energy heaped in accumulator battery BAT. A controller controls the functioning modes, depending on the energy generated by the automobile.

In Fig. 2 we present another configuration of planetary mechanism with four elements.

![Figure 2. Planetary mechanism with four elements and double satellite.](image)

Contrary to the solution analyzed in [3], where the thermal engine MT acts the port-satellite lever, in this configuration it acts upon the solar wheel 4. The electrical machine GE acts the solar wheel 3, and the electrical machine ME acts the port-satellite lever 1.
3. Kinematic analysis of the studied mechanisms

For the mechanism in Fig. 1 we denote the number of teeth of the gears 2, 3, 4 by \( z_2, z_3, z_4 \), respectively. In the case of the mechanism in Fig. 2, we denote by \( z_2', z_3', z_4' \) the number of teeth of the double satellite 2, and by \( z_3, z_4 \) the number of teeth of the gears 3 and 4.

In both cases, the absolute angular velocities of the elements 1, ..., 4 are denoted by \( \omega_1, ..., \omega_4 \).

Applying the Willis relations for both mechanisms, we successively obtain the relation in Table 1.

| Number of teeth of the gears | Number of teeth of the gears |
|------------------------------|------------------------------|
| \( z_2 = 15 \), \( z_3 = 18 \), \( z_4 = 48 \) | \( z_2' = 45 \), \( z_3' = 30 \), \( z_3 = 20 \), \( z_4 = 35 \) |
| \( i_1 = 3.2 \); \( i_2 = -2.6 \) | \( i_1 = -1.1 \); \( i_2 = 2.6 \) |
| \( \omega_2 = -2.2\omega_1 + 3.2\omega_4 \), \( \omega_3 = 3.6\omega_3 - 2.6\omega_4 \) | \( \omega_2 = 2.1\omega_1 - 1.1\omega_4 \), \( \omega_3 = -1.6\omega_1 + 2.6\omega_4 \) |

In the selection of the number of teeth of the gears in the two cases, we took into account that the ratio \( i_z \), in modulus, has close values for both mechanisms in order to compare the results.

4. The equations of motion of the planetary mechanisms

Considering as known the configuration of the planetary mechanism, the number of teeth, the modulus and the thickness of the gears, one determines the masses \( m_1, m_2, m_3, m_4 \) of the elements and their moments of inertia \( J_1, J_2, J_3, J_4 \).

The expression of the mechanical power developed by the three motors reads

\[
P = M_1\omega_1 + M_3\omega_3 + M_4\omega_4
\]

where \( M_1, M_3, M_4 \) are the torques developed by the three motors, function of the configuration of the planetary mechanism (Fig. 1 or Fig. 2).

Keeping into account the relation (2), we obtain the expression

\[
P = [M_1 + M_3(1 - i_1)]\omega_1 + [M_3i_2 + M_4]\omega_4.
\]

The generalized forces \( Q_1, Q_4 \) are obtained from the relation

\[
\dot{\theta}_1 = [M_1 + M_3(1 - i_1)]\theta_1 + [M_3i_2 + M_4]\theta_4,
\]
from which it results

\[ Q_1 = M_1 + M_3(1 - i_2), \]  

\[ Q_4 = M_3 i_2 + M_4. \]  

To be out to obtain the Lagrange equations we start from the expression of the kinetic energy \( T \),

\[ T = \frac{1}{2} \left( J_1 \omega_1^2 + m_2 R_2^2 \omega_2^2 + J_4 \omega_4^2 \right) \]  

where \( R_2 \) is the radius at which there are the satellites 2.

The kinetic energy, keeping into account the relations (1) and (2), may be also written as

\[ T = \frac{1}{2} \left( A_1 \dot{\theta}_1^2 + A_2 \dot{\theta}_4^2 \right) + A_2 \dot{\theta}_1 \theta_4 \]

where

\[ A_1 = J_1 + J_2 \left(1 - i_2^2\right) + J_4 \left(1 - i_2^2\right) + m_2 R_2^2, \]

\[ A_{22} = J_2 i_2^2 + J_4 i_2^2 + J_4, \]

\[ A_{12} = J_2 \left(1 - i_2 \right) \dot{\theta}_1 + J_4 \left(1 - i_2 \right) \dot{\theta}_2. \]

In the Lagrange equations

\[ \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}_1} \right) - \frac{\partial T}{\partial \theta_1} = Q_1, \quad \frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\theta}_4} \right) - \frac{\partial T}{\partial \theta_4} = Q_4, \]

replacing the partial derivatives

\[ \frac{\partial T}{\partial \dot{\theta}_1} = A_1 \dot{\theta}_1 - A_{12} \dot{\theta}_4, \quad \frac{\partial T}{\partial \theta_1} = 0, \]

\[ \frac{\partial T}{\partial \dot{\theta}_4} = A_{12} \dot{\theta}_1 + A_{22} \dot{\theta}_4, \quad \frac{\partial T}{\partial \theta_4} = 0, \]

we obtain the system of differential equations

\[ A_1 \ddot{\theta}_1 + A_{12} \ddot{\theta}_4 = M_1 + M_3 \left(1 - i_2 \right), \quad A_{12} \ddot{\theta}_1 + A_{22} \ddot{\theta}_4 = M_3 i_2 + M_4, \]

from which it results the solution

\[ \theta_1 = \frac{\left[M_1 + M_3 \left(1 - i_2 \right)\right] A_{12} - \left[M_3 i_2 + M_4\right] A_{12}}{A_{11} A_{22} - A_{12}^2}, \quad \theta_4 = -\frac{\left[M_1 + M_3 \left(1 - i_2 \right)\right] A_{12} + \left[M_3 i_2 + M_4\right] A_{11}}{A_{11} A_{22} - A_{12}^2} \]

5. The equations of motion of the hybrid automobile

For the determination of the equations of motion of the hybrid automobile we proceed similarly to the determination of the equations of motion of the planetary mechanisms.

In this case the expression of the mechanical power is
\[ P = M_1 \omega_1 + M_2 \omega_2 + M_3 \omega_4 - P_R - P_a \]  

(18)

where the power consumed by the rolling resistance \( P_R \) has the expression

\[ P_R = f \cdot G_u \cdot v_a \]  

(19)

while the power consumed by the air resistance \( P_a \) has the expression

\[ P_a = \frac{1}{2} \rho c_x A v_a^2 \]  

(20)

We used the classical notations: \( v_a \) – the speed of the automobile, \( G_u \) – the weight of the automobile, \( f \) – the coefficient of the rolling resistance, \( \rho \) – the density of the air, \( c_x \) – the coefficient of air resistance, \( A \) – the transversal area of the automobile.

The sign ± in front of the expression \( M_1 \omega_3 \) was introduce because the electrical machine \( GE \) can be generator and then it consumes power (the sign –) or it can be motor and then it produces power (the sign +).

For the determination of the relation between the speed of the automobile and the angular velocity \( \omega_1 \) at the axle of the machine \( ME \) we start from the relation

\[ \omega_r = \frac{\omega}{i_s} \]  

(21)

where by \( \omega_1 \) we denoted the angular velocity of the wheel of automobile and by \( i_s \) the transmission ratio from the axle of the machine \( ME \) to the wheel of the automobile. Knowing the rolling radius \( r_r \) of the wheel of the automobile, expression (18) becomes

\[ P = M_1 \omega_1 + M_2 \omega_2 + M_3 \omega_4 - G_u f \frac{\omega}{i_s} r_r - \frac{1}{2} \rho c_x A \frac{\omega_1^3}{i_s^3} r_r^3. \]  

(22)

In the previous relation the angular velocity \( \omega_1 \), for the mechanism in Fig. 1 is \( \omega = \omega_1 \), while for the mechanism in Fig. 2 it is \( \omega = \omega_4 \).

Keeping into account the relation (2), relation (22) becomes

\[ P = [M_1 \pm M_2 (1 - i_2) \omega_2] + [\pm M_3 \omega_4] - G_u f \frac{\omega}{i_s} r_r - \frac{1}{2} \rho c_x A \frac{\omega^3}{i_s^3} r_r^3. \]  

(23)

From the expression of the elementary work it results the expressions of the generalized forces in the cases of the two mechanisms. For the mechanism in Fig. 1 the generalized forces are

\[ Q_i = M_1 \pm M_2 (1 - i_2), \]  

(24)

\[ Q_4 = \pm M_3 i_2 + M_4 - G_u f \frac{r_r}{i_s} - \frac{3}{2} \frac{\rho c_x A r_r^2}{i_s^2} \omega_4^2, \]  

(25)

while for the mechanism in Fig. 2 they are

\[ Q_i = M_1 \pm M_2 (1 - i_2) - G_u f \frac{r_r}{i_s} - \frac{3}{2} \frac{\rho c_x A r_r^2}{i_s^2} \omega_1^2, \]  

(26)

\[ Q_4 = \pm M_3 i_2 + M_4. \]  

(27)
The expression (8) of the kinetic energy completes with the kinetic energy of the automobile which, in a simplified model, is a mass \( m_0 \) in translational motion, 2 driving wheels with the inertial moments \( J_{RM} \) and 2 free wheels with the moment of inertia \( J_{RL} \). In the moments of inertia of the wheels we considered as included all the moments of inertia reduced to them (breaking system, the system of transmission of the rotational motion from the axle of \( ME \) to the wheels etc.). In this case the expression of the kinetic energy reads

\[
T = \frac{1}{2} \left( J_1 \omega_1^2 + m_2 R_2^2 \omega_2^2 + J_3 \omega_3^2 + J_4 \omega_4^2 + m_d \left( \frac{\omega}{i_s} \right)^2 \right) + 2 \left( J_{RM} + J_{RL} \right) \left( \frac{\omega}{i_s} \right)^2 \tag{28}
\]

Using, for the mechanism in Fig. 1, the notations

\[
A_{11} = J_1 + J_2 (1 - i_1)^2 + J_3 (1 - i_2)^2 + m_2 R_2^2, \quad A_{12} = J_2 i_2^2 + J_3 i_2^2 + J_4 + \frac{m_d r_2^2 + 2(J_{RM} + J_{RL})}{i_s^2},
\]
\[
A_{12} = J_2 (1 - i_1) \dot{i}_1 + J_3 (1 - i_2) \dot{i}_2,
\]
and, for the mechanism in Fig. 2, the notations

\[
A_{11} = J_1 + J_2 (1 - i_1)^2 + J_3 (1 - i_2)^2 + m_2 R_2^2 + \frac{m_d r_2^2 + 2(J_{RM} + J_{RL})}{i_s^2}, \quad A_{22} = J_3 i_2^2 + J_3 i_2^2 + J_4,
\]
\[
A_{12} = J_2 (1 - i_1) \dot{i}_1 + J_3 (1 - i_2) \dot{i}_2,
\]
we obtain the expression of the kinetic energy given by the relation (9).

From the Lagrange equations (13) it results, in a similar way, the system of differential equations (16) from which one obtains, for the mechanism in Fig. 1, the solution

\[
\ddot{\theta}_1 = \frac{M_1 \pm M_3 (1 - i_2)}{A_{11} A_{22} - A_{12}^2} [A_{12} \left[ \pm M_3 i_2 + M_4 - \frac{G_a f r}{i_s} - \frac{3pc_A r^3}{2s_i^3} \theta_i^2 \right] A_{12}] A_{12},
\]
\[
\ddot{\theta}_4 = \frac{M_1 \pm M_3 (1 - i_2)}{A_{11} A_{22} - A_{12}^2} \left[ M_1 \pm M_3 (1 - i_2) - \frac{G_a f r}{i_s} - \frac{3pc_A r^3}{2s_i^3} \theta_i^2 \right] A_{12} + \left[ \pm M_3 i_2 + M_4 \right] A_{11},
\]
while for the mechanism in Fig. 2, the solution

\[
\ddot{\theta}_1 = \frac{M_1 \pm M_3 (1 - i_2)}{A_{11} A_{22} - A_{12}^2} [A_{12} \left[ \pm M_3 i_2 + M_4 - \frac{G_a f r}{i_s} - \frac{3pc_A r^3}{2s_i^3} \theta_i^2 \right] A_{12}] A_{12},
\]
\[
\ddot{\theta}_4 = \frac{M_1 \pm M_3 (1 - i_2)}{A_{11} A_{22} - A_{12}^2} \left[ M_1 \pm M_3 (1 - i_2) - \frac{G_a f r}{i_s} - \frac{3pc_A r^3}{2s_i^3} \theta_i^2 \right] A_{12} + \left[ \pm M_3 i_2 + M_4 - \frac{G_a f r}{i_s} - \frac{3pc_A r^3}{2s_i^3} \theta_i^2 \right] A_{11},
\]

6. Integration of the equations of motion

For the integration of the equations of motion we need the mechanical properties of the elements. For this, we modeled with solids in AutoCAD the gears and the coupling elements. The gears were obtained with the AutoLisp functions described in [6]. All gears are manufactured from steel, have the same modulus \( m = 2.5 \) and the same thickness \( b = 20 \text{ mm} \). Both mechanism have three satellites.
With the aid of the AutoCAD command MASSPROP one obtains the geometric properties from which one retains the mass of the element and the inertial moment relative to the axis \( OZ \). In Table 2 are presented the obtain data.

### Table 2.

|                      | Data for the mechanism in Fig. 1 | Data for the mechanism in Fig. 2 |
|----------------------|----------------------------------|----------------------------------|
| \( m_1 \) | 0.2564 kg, \( J_1 = 1.372 \cdot 10^{-4} \text{kgm}^2 \) | \( m_1 = 0.4742 \text{kg}, J_1 = 1.333 \cdot 10^{-3} \text{kgm}^2 \) |
| \( m_2 \) | 0.5169 kg, \( J_2 = 9.7042 \cdot 10^{-4} \text{kgm}^2 \) | \( m_2 = 6.5688 \text{kg}, J_2 = 5.043 \cdot 10^{-2} \text{kgm}^2 \) |
| \( m_3 \) | 0.2007 kg, \( J_3 = 6.197 \cdot 10^{-5} \text{kgm}^2 \) | \( m_3 = 0.5468 \text{kg}, J_3 = 1.396 \cdot 10^{-4} \text{kgm}^2 \) |
| \( m_4 \) | 1.3086 kg, \( J_4 = 4.595 \cdot 10^{-3} \text{kgm}^2 \) | \( m_4 = 0.5468 \text{kg}, J_4 = 8.777 \cdot 10^{-4} \text{kgm}^2 \) |

Further on, we need the characteristics of the motors as functions of the angular velocity. The characteristic of the thermal engine is approximated with a function of second degree

\[
M_{ME} = \begin{cases} 
0 & \text{if } \omega < \omega_{10}, \\
M_{01} + a_1 \omega + b_1 \omega^2 & \text{if } \omega \geq \omega_{10}.
\end{cases}
\]  
(33)

For the characteristics of the electrical motors we preferred ([2]) a function of first degree. For the electrical machine \( ME \) the characteristic is

\[
M_{ME} = \begin{cases} 
M_{02} & \text{if } \omega < \omega_{20}, \\
M_{02} + a_2 \omega & \text{if } \omega > \omega_{20}.
\end{cases}
\]  
(34)

and for the electrical machine \( GE \) the characteristic is

\[
M_{GE} = \begin{cases} 
M_{03} & \text{if } \omega < \omega_{10}, \\
M_{03} + a_3 \omega & \text{if } \omega > \omega_{10}.
\end{cases}
\]  
(35)

In the previous relations \( M_{0i}, \omega_{0i}, a_i \), with \( i = 1, 2, 3 \), and \( b_i \) are constants which approximate the external characteristic given by the producer.

Considering an automobile with the mass \( m_a = 2000 \text{kg} \), with a coefficient of air resistance \( c_s = 0.34 \), a frontal area \( A = 2.22 \text{m}^2 \), the coefficient of rolling resistance \( f = 0.019 \) and an efficiency of the transmission \( \eta = 0.9 \), it is necessary a power of approximate 90 kW in order to reach a maximum speed of 180 km/h.

For the start-up and functioning in urban cycle one uses only the electric motor \( ME \). Considering that at 2000 rot/min ( \( \omega = 209.44 \text{rad/s} \) ) for the angular speed of \( ME \) the automobile reaches a speed of 60 km/h and the dynamic rolling radius has the value \( r_c = 0.305 \text{m} \), it results the value of the transmission ratio \( i_s \).

\[
i_s = \frac{\omega}{\omega_r} = \frac{\frac{\omega}{v_n}}{\frac{v_n}{r_c}} = \frac{209.44}{16.666} = 3.83 \]  
(36)

At maximum speed \( ME \) being directly coupled to the transmission will have the angular speed of 6000 rot/min.

Considering also that the thermal engine (a spark ignition one) has the angular speed of maximum power at 6000 rot/min, and for the reach of the maximum speed the energy is given by the thermal engine and by the two electrical machines feed from the accumulator battery, then from the relation
(2) it results that the angular speed of the electrical machine $GE$ is equal to those of the two motors, $\omega_3 = \omega_1 = \omega_4$ no matter the configuration of the coupling mechanism (Figs. 1 or 2).

In the expression of the mechanical power (3), since the angular speeds are equal, it results

$$M = M_{MT} + M_{GE} + M_{ME}$$  \hspace{1cm} (37)

If we choose the power of the thermal engine $MT$, $P_{MT} = 60$ kW at 6000 rot/min the driving torque is $M_{MT} = 95.49$ Nm. If in the expression (33) we consider three functioning points (95, 157.07), (110, 387.46) and (95.5, 628.31), then it results the expression of $MT$

$$M_{MT} = \begin{cases} 
0 \text{ if } \omega < 100 \text{ rad/s,} \\
68.588 + 0.210\omega - 0.00027\omega^2 \text{ if } \omega > 100 \text{ rad/s.}
\end{cases} \hspace{1cm} (38)$$

Similarly we determine the external characteristic of the two electrical machines.

For $ME$ we consider the functioning points (50, 209.44), (23.80, 628.32) and it results the characteristic

$$M_{ME} = \begin{cases} 
63.10 \text{ if } \omega < 209.44 \text{ rad/s,} \\
63.10 - 0.063\omega \text{ if } \omega > 209.44 \text{ rad/s.}
\end{cases} \hspace{1cm} (39)$$

For $GE$ we consider the functioning point (23.80, 628.32), (5, 1047.2) and it results

$$M_{GE} = \begin{cases} 
52 \text{ if } \omega < 628.32 \text{ rad/s,} \\
52 - 0.045\omega \text{ if } \omega > 628.32 \text{ rad/s.}
\end{cases} \hspace{1cm} (40)$$

We will have to calculate also the constants given by the relations (29) for the mechanism in Fig. 1 and relations (30) for the mechanism in Fig. 2. In Table 3 are presented the obtained data.

| Data for the mechanism in Fig. 1 | Data for the mechanism in Fig. 2 |
|----------------------------------|----------------------------------|
| $A_{11} = 0.006571$, $A_{22} = 12.889127$, $A_{12} = -0.007412$ | $A_{11} = 13.140103$, $A_{22} = 0.062842$, $A_{12} = -0.117074$ |

The integration of the equations of motion is performed with the aid of the fourth order Runge Kutta method. One transforms the system of two second order differential equations in a system of four first order differential equations.

7. Conclusions

In the paper are analyzed, from the kinematic and dynamic point of view, the hybrid automobiles which have in their composition a planetary mechanical system with four components. The planetary mechanism with four elements that couples a thermal engine and two electrical machines is a mechanism with two degrees of mobility.

Comparing to the classic solution adopted by Toyota Corp. on model Prius we present a second solution of planetary mechanism (with four elements and double satellite) which can be used in coupling of three power sources.

From the performed kinematic analysis, by comparison to the two mechanisms, it resulted that both solutions of mechanisms are compatible.

For the study of the motion of the planetary mechanism one uses a system of two second order differential equations obtained by the aid of Lagrange equations.

Considering the mechanical power consumed by an automotive to beat the rolling and air resistances, proceeding in a similar way to that of obtaining the equations of motion for the planetary
mechanism, it results a system of two second order differential equations which describe the motion of the hybrid automotive.

For the numerical integration of these differential equations one determined the mechanical properties of the elements, by modeling them with solids in AutoCAD, and the mechanical characteristics $M(\omega)$ of the three machines. The mechanical properties were determined in the case of the mechanism used by Toyota Corp. and in the case of the mechanism with double satellite.

For the determination of the powers of the thermal engine and electrical machines we considered an automobile with the mass of 2000 kg that can move with a maximum speed of 180 km/h. The solving of the equations of motion is performed by the fourth order Runge-Kutta method and by transformation of the system of two second order differential equations in a system of four first order differential equations.

In a future paper we will study the functioning modes of the hybrid automotive: the start-up in electric mode, light acceleration when the thermal engine has an angular speed greater than that of the electric motor, displacement with constant average speed, strong acceleration, displacement with constant high speed, recuperative breaking and reverse displacement.

Reference

[1] Popa D, Stănescu ND and Tudor I, 2013, The study systems used in coupling power source, Scientific Bulletin Automotive series 23 60–7

[2] Popa D and Stănescu ND, 2015, Some observations concerning the stability of the mechanical systems used in the construction of the summing power planetary mechanisms, RIAV XII

[3] Pandrea N and Popa D, 2007, The coupling systems of the power sources, used at hybrid automobile, The 2nd International Conference "Computational Mechanics and Virtual Engineering" COMEC 2007, Brasov

[4] Molina S, 1997, Mechanism design. The Practical Kinematics and Dynamics of Machinery. Ed. Pergamon, Great Britain,

[5] Norton RL, 2003, Design of Machinery: An Introduction to the Synthesis and Analysis of Mechanisms and Machines, McGraw-Hill.

[6] Collins JA, Busby HR and Staab GH, 2003, Mechanical Design of Machine Elements and Machines: A Failure Prevention Perspective, 2nd Edition, Wiley, New York

[7] Dresig H, Holzwei, 2010, Dynamics of Machinery: Theory and Applications, Springer New York

[8] Mabie HH and Reinholdt CF, 1987, Mechanisms and Dynamics of Machinery, 4th Edition, Wiley, New York

[9] Russell K, Shen Q and Sodhi RS, 2015, Kinematics and Dynamics of Mechanical Systems: Implementation in MATLAB and Sim Mechanics, CRC Press, Boca Raton