Mascarenhas, Walter F.
The stability of barycentric interpolation at the Chebyshev points of the second kind.
(English) Zbl 1305.65090 Numer. Math. 128, No. 2, 265-300 (2014).

The paper provides a detailed discussion of the problem of polynomial interpolation at the Chebyshev points of the second kind using the two well-known barycentric interpolation formulas from the point of view of numerical stability and rounding error propagation. The main results are: The first barycentric formula has stability problems. The root cause of these problems is discussed; it turns out that they can be overcome at the expense of severely increased run times. Assuming a specific normalization of the weights such that all weights are exactly representable in finite precision IEEE floating point arithmetic, the second barycentric formula behaves much better.

Reviewer: Kai Diethelm (Braunschweig)

MSC:
65D05 Numerical interpolation
65G50 Roundoff error
41A05 Interpolation in approximation theory
41A10 Approximation by polynomials

Keywords:
barycentric interpolation; Chebyshev points; finite precision arithmetic; numerical stability; rounding error propagation; finite precision IEEE floating point arithmetic

Software:
MPFR; Boost C++ Libraries; Boost; mctoolbox

Full Text: DOI arXiv

References:
[1] Berrut, J.-P., Rational functions for guaranteed and experimentally well conditioned global interpolation, Comput. Math. Appl., 15, 1-16, (1988) · Zbl 0646.65006 · doi:10.1016/0898-1221(88)90067-3
[2] Berrut, J.-P., Trefethen, L.N.: Barycentric Lagrange interpolation, SIAM Review, 46, No. 3, 501-517 (2004) · Zbl 1061.65006
[3] Bos, L.; Marchi, S.; Hormann, K., On the Lebesgue constant of berrut’s rational interpolant at equidistant nodes, J. Comput. Appl. Math., 236, 504-510, (2011) · Zbl 1231.41003 · doi:10.1016/j.cam.2011.04.004
[4] The boost library website: http://www.boost.org/ · Zbl 0061.28409
[5] Demming, R., Duffy, D.: Introduction to the Boost C++ Libraries—Volume 2—advanced libraries. Datasim. ISBN 978-94-91028-02-1 (2012)
[6] Cormen, T.H., et al.: Introduction to algorithms (1st edn). MIT Press and McGraw-Hill, USA, (1990) · Zbl 1158.68538
[7] Floater, M., Hormann, K., Barycentric rational interpolation with no poles and high rates of approximation, Numer. Math., 107, 315-331, (2007) · Zbl 1221.65002 · doi:10.1007/s00211-007-0093-y
[8] Feller, W.: An introduction to probability theory and its applications, vol. 2, 3rd edn. Wiley, New York (1971) · Zbl 0219.60003
[9] Henrici, P.: Elements of numerical analysis. Wiley, New York (1964) · Zbl 0149.10901
[10] Higham, N.J., The accuracy of floating point summation, SIAM J. Scient. Comput., 14, 783-799, (1993) · Zbl 0788.65053 · doi:10.1137/0914050
[11] Higham, N.J., The numerical stability of barycentric Lagrange interpolation, IMA J. Numer. Anal., 24, 547-556, (2004) · Zbl 1067.65016 · doi:10.1093/imanum/24.4.547
[12] Higham, N.J.: Accuracy and stability of numerical algorithms, 2nd edn. SIAM, Philadelphia (2002) · Zbl 1011.65010 · doi:10.1137/1.978089878718027
[13] Kahan, W., Further remarks on reducing truncation errors, Comm. ACM, 8, 40, (1965) · doi:10.1145/363707.363723
[14] Rump, SM, Ultimately fast accurate summation, SIAM J. Scient. Comput., 31, 3466-3502, (2009) · Zbl 1202.65033 · doi:10.1137/080738490
[15] Priest, D.: Algorithms for arbitrary precision floating point arithmetic. In: Kornerup, P., Matula, D. (eds.) Proceedings of the 10th Symposium on Computer Arithmetic, Grenoble, France, pp. 132-145. IEEE Computer Society Press, Piscataway (1991)

[16] Priest, D.: On properties of floating point arithmetics: numerical stability and the cost of accurate computations, Ph.D. thesis, Mathematics Department, University of California at Berkeley, CA (1992) - Zbl 0788.65053

[17] Fousse, L, Hanrot, G, Lefèvre, V, Pélissier, P, Zimmermann, P, MPFR: a multiple-precision binary floating-point library with correct rounding, ACM Trans Math Softw, 33, 13, (2007) - Zbl 1365.65302 - doi:10.1145/1236463.1236468

[18] Salzer, H, Lagrangian interpolation at the Chebyshev points $x_{n,\nu} = \cos \frac{\nu \pi}{n}, \nu = 0(1)n$; some unnoted advantages, Comput. J., 15, 156-159, (1972) - Zbl 0242.65007 - doi:10.1093/comjnl/15.2.156

[19] Taylor, WJ, Method of Lagrangian curvilinear interpolation, J. Res. Nat. Bur. Stand., 35, 151-155, (1945) - Zbl 0061.28409 - doi:10.6028/jres.035.006

[20] Trefethen, L.N.: Approximation theory and approximation practice. SIAM, Philadelphia (2013) - Zbl 1264.41001

[21] Webb, M, Trefethen, LN; Gonnet, P, Stability of barycentric interpolation formulas for extrapolation, SIAM J. Scient. Comput., 34, 3009-3015, (2012) - Zbl 1261.65015 - doi:10.1137/110848797

[22] Werner, W, Polynomial interpolation: Lagrange versus Newton, Math. Comp., 43, 205-207, (1984) - Zbl 0566.65009 - doi:10.1090/S0025-5718-1984-0744331-0

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.