Abstract—Self consistent solution to electromagnetic (EM)-
circuit systems is of significant interest for a number of applica-
tions. This has resulted in exhaustive research on means to couple
them. In time domain, this typically involves a tight integration
(or coupling) with field and non-linear circuit solvers. This is
in stark contrast to coupled analysis of linear/weakly non-linear
circuits and EM systems in frequency domain. Here, one typically
extracts equivalent port parameters that are then fed into the
circuit solver. Such an approach has several advantages; (a) the
number of ports is typically smaller than the number of degrees
of freedom, resulting in cost savings; (b) is circuit agnostic; (c)
can be integrated with a variety of device models. Port extraction
is tantamount to obtaining impulse response of the linear EM
system. In time domain, the deconvolution required to effect
this is unstable. Recently, a novel approach was developed for
time domain integral equations to overcome this bottleneck. We
extend this approach to time domain finite element method, and
demonstrate its utility via a number of examples; significantly, we
demonstrate that self consistent solutions obtained using either
a fully coupled or port extraction is identical to the desired
precision for non-linear circuit systems. This is shown within a
nodal network. We also demonstrate integration of port extracted
data directly with drift diffusion equation to model device physics.

I. INTRODUCTION

The combined simulation of full-wave electromagnetics
solvers with circuit subsystems are of considerable interest
in a number of applications, including analysis of shielded or
packaged systems, active antenna design, small signal analysis
of active devices and design of high speed interconnects
[1–3]. It is driven by advances in design techniques that
permit fabrication of complex RF devices with active elements,
making it important to characterize radiative coupling effects
early in the design process. To that end, extensive work has
been done in developing coupled Maxwell-circuit solvers in
time and frequency domain, with the current state of the art
utilizing finite element [4] or integral equation methods [5, 6].
Here, we restrict our discussion to transient analysis as they
better resolve strongly nonlinear systems.

The predominant approach to transient analysis of EM-
circuit system is to solve the system self consistently [7].
This involves solving both the linear and non-linear system
at every time step. Obviously, the tight integration implies
that the solution is not circuit agnostic. Alternatives that
have been explored is to use frequency domain methods to
construct a transient “impulse response” that take the form
either as RC extraction [8]) or S parameter methods [9].
The extracted response is readily incorporated into a circuit
simulator. While this approach is somewhat effective, the
advantages and limitation are apparent; (a) the approach is
independent of the circuit system; (b) given the bandwidth of
excitation, the harmonics generated due to non-linearity and
need to capture early time behavior, the number of frequency
samples necessary can be very high; and (c) often only a subset
of these frequencies are used. In weakly non-linear systems
or when the coupling is not strong the errors accrued may
be tolerable. When analysis of the circuit system is possible
using frequency domain techniques (harmonic balance) under
the assumption of weakly non-linear systems, one often takes
recourse to using a Schur complement approach to couple EM
to circuit systems [10, 11]. In addition to being circuit agnostic,
this is computationally more efficient as there are fewer ports
than number of spatial degrees of freedom.

It follows, that developing such a methodology for transient
analysis will have the same benefits, in addition to potential
integration with multiphysics codes that model device physics.
Extracting port parameters of the EM system is analogous to
coupling its numerical impulse response. Doing this in time
domain is challenging because of known instabilities associ-
ated with deconvolution [12]. A recently proposed technique
[11] for solving coupled circuits with time domain integral
equation (TDIE) solvers overcame this fundamental bottle-
neck. Extending this technique to finite element based solvers
involves several changes in the extraction process. First, the
extracted signal manifests itself as a transient admittance in
the circuit system as opposed to an impedance in [11]. As
a result, the feed model used is changed to a current probe
as opposed to a delta-gap feed, leading to differences both
in coupling and in the extraction process. Finally, using a
finite element scheme allows for integration with different set
differential equations used to model the device subsystem; in
this paper, we demonstrate this capability via coupling with
a non-linear Drift-Diffusion equations to model a Schottky
diode. The specific details involved will be covered in depth
in Section III.

The principal contributions of this paper are (a) the develop-
ment of a method for extracting transient port parameters
in the EM-circuit interface for finite element systems, (b)
the demonstration that solutions obtained through this method
are identical to those obtained using a fully coupled solution
to solve precision, and (c) integration with device specific
differential equations. Furthermore, we also briefly demon-
strate the implementation of a Perfectly Matched Layer (PML)
system for mixed finite element electromagnetic solvers. Via
numerous examples, we will demonstrate the application of
these for analysis of linear and nonlinear circuits coupled
to EM systems. Where possible, we will show comparison
against data that exists in the literature (either measured or modeled). We note that while our results are obtained using a implicit mixed FEM system, the prescribed procedure is applicable to the traditional wave equation solvers. Our rationale for using mixed FEM as opposed to the usual wave equation is that the latter has a time growing null-space of the form $\nabla \psi(r)$, whereas the former has a null space of the form $\nabla \psi(r)$. In mixed FEM, the magnitude of the null-space excited depends on the threshold used for the iterative solver. That said, it should noted that a gauging constraint as described in [13–16] eliminates this null space. Finally, the nonlinearities are assumed to be lumped or pointwise. While the proposed method can potentially be used to isolate small nonlinearities, defining and solving for two auxiliary variables in addition to the regular field unknowns to achieve the same effect. The PML implementation used in this paper directly evaluates the convolution integrals. To do so, we define a stretched coordinate system via the following transform

$$
\Lambda(\omega) = \begin{pmatrix}
\frac{s_1 s_3}{s_2} & 0 & 0 \\
0 & \frac{s_2 s_3}{s_1} & 0 \\
0 & 0 & \frac{s_1 s_2}{s_3}
\end{pmatrix}
$$

with $s_i = 1 + \frac{\alpha_i}{\omega \epsilon_0}$ to match the absorbing layers to free space. Here, $\alpha_i$ are the components of an anisotropic conductivity that governs the field loss. Stretching coordinates in this manner alters Maxwell’s equations as follows in frequency domain:

$$
\nabla \times \mu^{-1} \Lambda(\omega)^{-1} \cdot \mathbf{B}(r, \omega) = J(r, \omega) + \epsilon(\mathbf{r}) \Lambda(\omega) \cdot j \omega \mathbf{E}(r, \omega)
$$

Obtaining a time marching scheme involves inverse Fourier transforming (2) to obtain

$$
\nabla \times \mu^{-1} L_2(t) \ast \mathbf{B}(r, t) = \mathbf{J}(r, t) + \epsilon_0 L_1(t) \ast \mathbf{E}(r, t)
$$

where $L_1(t) = \mathscr{F}^{-1} (j \omega \Lambda(\omega))$ and $L_2 = \mathscr{F}^{-1} (\Lambda(\omega)^{-1})$. We discretize these equations by testing (3a) with a $W^2_0(\mathbf{r})$ basis function and (3b) with $W^1_0(\mathbf{r})$. Furthermore, the convolution terms are evaluated as done in [20].

The behavior of the attached devices at each port can be described generally by operators $\mathcal{D}$, $\mathcal{F}$ and couples to the EM system through $C_{\text{CKT}}$, forming

$$
\mathcal{D} \circ \left[ \mathcal{J}_{\text{CKT}}^{\text{CT}}(r, t), \mathbf{e}(t) \right] + C_{\text{CKT}} \circ \left[ \mathbf{e}(t) \right] = \mathcal{F} \circ \left[ \mathbf{e}(t) \right].
$$

Where $\mathcal{D}$ and $\mathcal{F}$ are general nonlinear operators and $C_{\text{CKT}}$ is a coupling operator that relates quantities in the EM system to those in the attached device. For the results presented in this work, we restrict $\mathcal{D}$ to either be an circuit network implemented through Modified Nodal Analysis [21]; used entirely by itself or in conjunction system governed by a set of Drift-Diffusion equations to model diodes [22, 23]. In the case of the former, we temporally represent the voltage and circuit unknowns using a $p$th order backward Lagrange interpolation function $L_p(t - t_i)$. We note that this choice of representation is what is commonly used in contemporary implementations of MNA, but are in no way the only feasible choice. Upon using our chosen representation, we obtain the following system

$$
\mathcal{Y} \mathcal{V}^{\text{CKT}}(t) = \mathcal{F}^{\text{CT}}(t) + f_{\text{nl}}^{\text{CKT}}(\mathcal{V}^{\text{CKT}}, t).
$$

which is subsequently delta tested to obtain a time marching scheme. Here $\mathcal{V}^{\text{CKT}}$ is a vector containing both the nodal voltages and branch currents in the circuit, $\mathcal{F}^{\text{CT}}$ and $f_{\text{nl}}^{\text{CKT}}$ refer to the excitations due to linear and nonlinear components. The linearized form in (5) can be solved at each timestep using a multi-dimensional Newton-Raphson scheme similar to [4]. Similarly, when employing a drift diffusion operator to model diodes in the system, the currents due to electrons and holes $\mathbf{J}_n(r, t)$ and $\mathbf{J}_p(r, t)$ running through the device are related to carrier densities $n(r, t)$, $p(r, t)$ and potential $\phi(r, t)$ through

$$
\mathbf{J}_n(r, t) = qD_n \nabla n(r, t) + q\mu_n(\mathbf{E}(r, t))n(r, t)\nabla \phi(r, t)
$$

and

$$
\mathbf{J}_p(r, t) = qD_p \nabla p(r, t) - q\mu_p(\mathbf{E}(r, t))p(r, t)\nabla \phi(r, t)
$$

The rest of the paper will be structured as follows: Section II will detail the implementation of the mixed finite element system for the EM system and the MNA solver for the attached circuits; Section III will describe the technique involved in extracting a set of transient port parameters from the EM system and using it to solve the coupled problem; Finally, Section IV will contain a set of numerical examples to both validate the method and demonstrate its efficacy.

II. FORMULATION

A. Problem Statement

Consider an object $\Omega_{\text{EM}} \subset \mathbb{R}^3$ bounded by a surface $\partial \Omega_{\text{EM}}$ that describes the geometry of an electromagnetic object containing $N_p$ ports, each associated with a lumped circuit subsystem. The currents flowing across these ports are collectively represented as $\mathcal{J}_{\text{CKT}}(r, t)$ with $r \in \Omega_{\text{EM}}$. We assume that any voltage sources in the circuit system are bandlimited to some frequency range $[f_{\text{min}}, f_{\text{max}}]$ with $f_{\text{min}} > 0$. Furthermore, we assume that the amplitude of these sources are zero when $t \leq 0$.

B. Modelling Framework

We construct a Maxwell solver following a mixed finite element scheme using Whitney edge and face basis functions $\mathbf{E}(r, t) = \sum_{i=1}^{N_e} e_i(t) \mathbf{W}_e^1(r)$ and $\mathbf{B}(r, t) = \sum_{j=1}^{N_f} b_j(t) \mathbf{W}_f^2(r)$, where $N_e$ and $N_f$ are the number of edges and faces respectively of the tetrahedral mesh to discretize the domain; see [17] and references therein. The EM unknowns are represented in time as $\mathbf{e}(t) = \sum_{j=1}^{N_f} e_j N(t - t_j)$ and $\mathbf{b}(t) = \sum_{j=1}^{N_f} b_j N(t - t_j)$, and tested by $\mathbf{W}(t - t_i)$. Both of these functions are defined in [18]. A Newmark-$\beta$ time stepping stencil with $\gamma = 0.5$ and $\beta = 0.25$ is used to solve for $\mathbf{e}(t)$ and $\mathbf{b}(t)$ and an appropriately configured PML to truncate the computational domain. Contemporary implementations of PML systems follow the general framework first outlined by Berenger [19] with more recent additions, including the use of stretched coordinates [20]. The implementation of these systems is done by either directly evaluating the convolutions resulting from the use of a stretched coordinate system or defining and solving for two auxiliary variables in addition to the regular field unknowns to achieve the same effect. The
where $\mu_n$ and $\mu_p$ are field dependent mobility rates for the electrons and holes respectively; and $D_n$ and $D_p$ are corresponding diffusion coefficients. The currents and carrier densities are further related through a set of continuity equations:

\begin{align}
\frac{\partial n(r, t)}{\partial t} &= \nabla \cdot \mathbf{J}_n(r, t) / q - R + G \\
\frac{\partial p(r, t)}{\partial t} &= -\nabla \cdot \mathbf{J}_p(r, t) / q - R + G
\end{align}

(6c)

(6d)

where $R$ and $G$ respectively denote the electron-hole recombination and the collision ionization rates. Finally, the carrier densities are related to the potential through Poisson’s equation

\begin{equation}
\nabla \cdot (\varepsilon \nabla \phi(r, t)) = -q (p(r, t) - n(r, t) + N_i(r, t))
\end{equation}

(6e)

where $N_i(r, t)$ refers to the doping concentration. Solutions to the drift diffusion system can be obtained by discretizing (6) using an appropriate finite element or finite difference method; see [22–24] and the references therein for a detailed analysis. The results presented in this paper only involve 1D drift-diffusion systems and as a result we discretize (6) using a corresponding 1D finite element system.

We describe the interaction between EM and device subsystems in two parts. First, we consider a device system modelled using MNA. In this instance, the quantities involved in the device system are voltages and currents, which need to be related to fields and current densities in the EM system. Specifically, if the $k$th FEM edge (denoted by $I_k$) is attached to the $j$th circuit subsystem, the current impressed on the EM system is given by

\begin{equation}
\langle W(t - t_i), J_{kj}^{\text{CKT}}(t) \rangle = \langle W(t - t_i), I_{j}^{\text{CP}}(t) \int_{|I_k|} \mathbf{I}_k \cdot \mathbf{W}_k^1 dr \rangle \\
= \langle W(t - t_i), I_{j}^{\text{CP}}(t) C_{kj} \rangle
\end{equation}

(7)

with $C_{kj}$ denoting a coupling coefficient that relates quantities in the device subsystem to the EM solver. Furthermore, $I_{j}^{\text{CP}}(t)$ refers to the magnitude of the current impressed by the circuit subsystem over the coupling edge $I_k$. Similarly, the voltage across the coupling branch can be related to the electric field across the $k$th FEM edge

\begin{equation}
\langle \delta(t - t_i), V_{jk}^{\text{CKT}}(t) \rangle = \langle \delta(t - t_i), e_{jk}(t) \int_{|I_k|} \mathbf{I}_k \cdot \mathbf{W}_k^2(r) dr \rangle \\
= \langle \delta(t - t_i), e_{jk}(t) C_{jk} \rangle.
\end{equation}

(8)

$C_{jk}$ here likewise denotes a coupling coefficient that relates quantities in EM solver to the device. We observe from (8) and (7) that our choice of testing/representation functions leads the two coupling coefficients to be identical. For a drift diffusion setup, the electric field at the location of the port is related to the electron and hole mobilities $\mu_n(E(r, t))$ and $\mu_p(E(r, t))$ respectively. Since the devices are assumed to be lumped, $E(r, t)$ at the location of the port can be used directly to compute the carrier mobilities, since the field is assumed to be spatially constant within the device. Likewise, we can use $\mathbf{J}_p(r, t)$ and $\mathbf{J}_n(r, t)$ to construct the net current passing through the diode, which can then be reintroduced to the EM system following (7).

### III. Extraction of the Numerical Impulse Response

The computational bottlenecks involved with solving a coupled system as described in the previous Section are twofold: (1) Resolving nonlinear elements in the circuit system involves performing a solve of the combined matrix equation and (2) changing any of the attached circuit subsystems would require the coupled problem to be solved again, despite the EM system remaining unaltered. A potential way to exploit the linearity of the EM system is to extract its impulse response at each EM-circuit interface and use it through (8) in the circuit solve. Unfortunately, it is well known that deconvolution required to implement this is unstable [12].

The key insight in the method proposed herein is as follows. The current deposited on a given port edge is represented in time through a linear combination of $N_j$ basis functions. As a result, given the EM response due to a single temporal basis function, we can exploit the linearity of Maxwell’s equations and reconstruct the field anywhere in the system. Since this sequence of operations only involves reconstructing the current at a given port in terms of basis functions by which it is represented in the coupled solve, the respective fields computed by both methods should be numerically indistinguishable.

With $p(q)$ denoting the set of FEM edges associated with the port $q$, we define an excitation vector $e_q(t)$ defined as follows

\begin{equation}
e_{q,k}(t) = \begin{cases} N(t - t_\delta) & k \in p(q) \\ 0 & \text{otherwise} \end{cases}
\end{equation}

(9)

where $\delta$ is the timestep at which the excitation is applied. $e_q$ is then used to define the forcing function $J_{j}^{\text{CKT}}(t)$ through (7) with $J_{j}^{\text{CKT}}(t) = C_{kj} e_q(t)$. This function is then applied to the RHS of (3) to obtain a solution vector $\mathbf{x}^d$. In order to solve the device equations, however, we only require the coefficients associated with each port, allowing us to construct a matrix $G_{kq} = x^k p(q)$ of dimensions $N_p \times N_p \times N_i$. Each column of $G$ represents a discrete impulse response for a pulse centered at the edge $p(q)$ measured at the $k$th edge. As a result, constructing the electric field at port $k$, in response to an arbitrary set of currents can be done by simply summing the convolutions of $G_{kq}$ with $I_q$ for each attached circuit port. The reconstructed fields can then be related using the appropriate coupling equations to quantities the device subsystems. For instance, if the attached port is governed through MNA, the voltage across the $j$th port $V_{jk}^{\text{CKT}}(t)$ in (8) can now be written in terms of $I_{j}^{\text{CP}}(t)$

\begin{equation}
V_{jk}^{\text{CKT}}(t) = \langle \delta(t - t_i), C_{jk} \sum_{q=1}^{N_p} G_{kq}(t) \ast I_{q}^{\text{CP}}(t) \rangle.
\end{equation}

(10)

yielding a standalone matrix equation for the device system.
IV. RESULTS

The numerical experiments presented in this section will be organized as follows: Sections IV-B and IV-C will compare results obtained using the port extraction technique described in Section III against existing results in the literature for both linear and nonlinear circuit systems. Section IV-E will highlight three key facts about the proposed method; first, we demonstrate that the solutions obtained through port extraction are numerically identical to their fully coupled counterparts; second, we show that the extraction procedure is circuit agnostic; and finally, we compare the complexity of the port extracted solve to a traditional fully coupled setup.

For the results presented in the remainder of this section, $N_t$ denotes the number of timesteps that the simulation is run over and $N_{EM}, N_{CKT}$ denote the numbers of EM and circuit unknowns respectively in the system. Unless specified otherwise, voltage sources are defined using $v(t) = \cos(2\pi f_0 t) e^{-t/\tau^2}$ where $\sigma = 3 \times (2\pi f_{bw})^{-1}$, with $f_{max} = f_0 + f_{bw}$. The timestep size $\Delta t = (30 f_{max})^{-1}$. Finally, GMRES was used to solve the system iteratively to a tolerance of $10^{-12}$.

A. Input Impedance of a Monopole Antenna

In this first example, we validate our technique by analysing a cylindrical monopole suspended above an infinite ground plane. Specifically, the monopole has a length of 5 cm, radius of 1.52 mm and is suspended 1.6 mm above a conducting square of side length 10 cm, as shown in Fig. 1a. To mimic an infinite ground plane, the truncating walls of the simulation domain are in direct contact with the ends of the square. The ground plane is coupled to the cylinder by a single, vertically oriented edge, across which is connected a driving circuit given by a time varying voltage source connected in series to a 100 $\Omega$ resistor. The voltage fed to the resistor is assumed to be a modulated Gaussian with center frequency $f_0 = 2.5$ GHz and bandwidth $f_{bw} = 2$ GHz. The timestep size $\Delta t$ was set to be $(30 f_{max})^{-1}$. The mesh used to discretize the domain had an average edge length of $(20 f_{bw})^{-1}$, resulting in $N_{EM} = 512,436$ and the simulation was run for $N_t = 2001$. The setup is geometrically identical to an example in [7] and looking at Fig 1b, we see good agreement between the admittance curves generated through port extraction and a coupled time domain solver for the same simplified probe model. The solve time per timestep performing the extraction as detailed in Section III was approximately 8 seconds per timestep, with the subsequent circuit solve completing its entire run of 2001 timesteps in under 20 ms.

B. Input impedance of a strip above a Finite Ground Plane

We consider a conducting strip suspended over a finite ground plane, as specified in Fig. 2a. The coupling between the EM system and the driving circuit is achieved across a vertical 1 cm edge going from the conducting plane to the strip. The circuit is assumed to be a Thévenin source characterized by $f_0 = 1$ GHz and $f_{bw} = 999$ MHz connected in series to a 100 $\Omega$ resistor. The simulation domain is discretized using a tetrahedral mesh with approximate average edge length set to $(20 f_{max})^{-1}$, yielding $N_{EM} = 2,000,936$. The system was run for $N_t = 4000$ timesteps (with each timestep taking approximately 13 seconds to converge) and the port parameters were extracted through Fourier transforms of the time-series data. As is evident from Fig. 2b, the radiated power curve shows very good agreement to measured data and FD-TD.

C. Microstrip Amplifier

Next, we validate the proposed technique for nonlinear circuit systems by comparing the reflection coefficient and gain for a microstrip amplifier. The geometry and driving circuits are exactly as in [4] and we obtain the $S$ parameters through small signal analysis, with $f_0 = 5.5$ GHz, $f_{bw} = 3.5$ GHz and $f_{max} = f_0 + f_{bw}$. The tetrahedral mesh used to discretize the domain has an average edge length of $(15 f_{max})^{-1}$ with $N_{EM} = 5,134,732$. The data used to compute the scattering parameters was obtained by running this setup for $N_t = 6000$ timesteps. We note from Fig. 3b that the measured $S$ parameters show good agreement to results from [4]. Extracting this
D. Microstrip Rectifier modelled through Drift-Diffusion

Until now we have demonstrated the use of port extraction on linear and nonlinear systems connected to nodal circuit networks. Next, we aim to show that the proposed method works with systems where the devices are governed nonlinear differential equations. The system under analysis is a microstrip rectifier circuit as shown in Fig. 4a. The thickness of the board was 1 mm and the relative permittivity of the substrate 2.65. A HSMS-282B diode is placed across port $P$ (with physical parameters for (6) as in [25]) with a 10 pF filter capacitor attached to a variable load across port $C$. The input source was assumed to be a modulated Gaussian

with $f_0 = 2.45$ GHz and $f_{bw} = 0.25$ GHz. We performed two experiments on a microstrip rectifier circuit: (1) First, the Schottky diode in the layout was modeled using an equivalent circuit network, mimicking a similar setup simulated on ADS. (2) Next, using the same extracted port response as in the first experiment, we modeled the diode using a set of Drift-Diffusion [25] equations in (6). In each case, the conversion efficiency of the rectifier

\[
\eta = \frac{P_{DC}}{P_{source}} \cdot 100\% \tag{11}
\]

where $P_{DC}$ denotes the power measured at the output end $P_{source}$ the corresponding quantity at the source was compared against data from [25]. As is evident in Fig. 4b, in the first experiment, results obtained through the proposed method agree well with corresponding results obtained through ADS EM Co-simulation. In the second experiment, our results better match measured data of the rectifier circuit than the corresponding efficiency curve predicted by ADS. We emphasize the fact that the results from the equivalent circuit do not agree with experimental measurements, due to the network not being representative of the actual diode for the parameters chosen, thereby illustrating a situation where the ability to couple
(a) Schematic of the microstrip rectifier. All dimensions are in mm.

(b) Computed conversion efficiency for a diode rectifier with a simulation done through a physical model compared to ADS [25].

Fig. 4: Geometry layout and comparison of conversion efficiency for a microstrip rectifier circuit.

E. Strip above a Finite Ground Plane driven by different circuits

In keeping with objectives stated earlier, we first extracted the port parameters following the procedure in Section III for the example used in Section IV-B with $\Delta t = 16$ ps. This extracted response was then attached to a Chebyshev filter and a Diode Mixer circuit respectively, and the obtained port voltages in time were compared to equivalent results obtained from a direct solution of the coupled system.

1) Chebyshev filter: First, we use the extracted transient port parameters on a Chebyshev filter as shown in Fig. 5a. $V_s(t)$ was characterized by $f_0 = 1.5$ GHz and $f_{bw} = 0.5$ GHz.

(b) Plot of the port voltage for the Chebyshev filter from Fig. 5a obtained through the fully coupled and extracted responses.

Fig. 5: Circuit description and comparison of port voltages between the port extraction and fully coupled methods for a linear circuit system.

The timestep size in the circuit system was set to the same size used in the extraction of the EM response. The comparison of the port voltages as a function of time are shown in Fig. 5b. The $L^2$ error between the two solutions was $3.1 \times 10^{-12}$.

2) Diode Mixer: Next, we use extracted port parameter with a nonlinear Diode Mixer as shown in Fig. 6a. The diode between nodes 3 and 7 has a saturation current $I_s = 2$ nA, emission coefficient $\eta = 2.0$ and $k_B T/q = 25.6$ mV. The RF and LO sources were assumed to be sine waves of magnitude 0.4 V with frequencies 900 MHz and 800 MHz respectively. The current across the diode was modelled using the Shockley equation and the bias voltage was set to 0.7 V to activate the diode. The relative $L^2$ error between the two curves in Fig. 6b was $4.7 \times 10^{-12}$.

3) Computational Complexity: Finally, we discuss the asymptotic cost complexity of the proposed method. Let $N_{EM}$ and $N_{CKT}$ denote the number of degrees of freedom of the EM and circuit systems, respectively; $N_t$ the number of timesteps; $N_{NL}$ the number of nonlinear iterations per time step; $N_p$ the number of circuit ports and $N_{GMRES}$ the number of matrix multiplications required to solve the linearized system denoted in the superscript. The cost for solving the fully coupled system is

$$C_{\text{coupled}} = O\left(N_t N_{NL}^{\text{coupled}} N_{GMRES}^{\text{coupled}} (N_{EM} + N_{CKT})\right).$$

(12)
On the other hand, the cost of port extraction is

\[
C_{\text{PE}} = C_{\text{PE},1} + C_{\text{PE},2}
= O(N_t N_p N_{\text{PE,GMRES}} N_{\text{EM}}) + O(N_{\text{NL}} N_{\text{NL,GMRES}} N_{\text{CKT}})
\]

(13)

Before, we proceed, we note the following. Typically, \(N_{\text{EM}} \gg N_{\text{CKT}}\). As a result, we ignore the cost of computing the Jacobian in (12). In (13), the first portion refers to the cost of exciting each port and obtaining the corresponding response at other ports. It is a one-time cost and not incurred as one marches through (indeed, it is the characteristic of the EM systems and circuit agnostic). The second term in (13), is the cost of non-linear solve at each port. Typically, the number of non-linear solves, \(N_{\text{NL,coupled}} N_{\text{GMRES}}\) in (12), is significantly larger than \(N_{\text{NL}} N_{\text{PE,GMRES}}\) as it involves a fully coupled solve involving all the degrees of freedom in the system. In order to meaningfully compare computational costs, it is important to incorporate the contributions of both \(C_{\text{PE,1}}\) and \(C_{\text{PE,2}}\) against \(C_{\text{coupled}}\). To do this, we considered the solve time per nonlinear iteration within a single timestep of each solve for the example in Section IV-E2, i.e the finite ground plane monopole antenna driven by a diode mixer circuit. This includes the cost for evaluating a single timestep in the extraction process as well as the cost of computing \(N_{\text{NL}}\) nonlinear iterations, where \(N_{\text{NL}}\) is the average number of nonlinear iterations required to achieve convergence per timestep. In this example, \(N_{\text{NL}} = 6\), per time step. Extracting the impulse response took 13s per \(N_t\), i.e., \(C_{\text{PE,1}} = 13 N_t\). What is compared in Fig. 7 are \(C_{\text{PE,2}}/N_t\) and \(C_{\text{coupled}}/N_t\). As is evident, \(C_{\text{PE,2}} \ll C_{\text{coupled}}\).

V. CONCLUSION

In this paper, we have demonstrated a technique to extract transient port parameters from a coupled EM-circuit solver. We have shown that the technique is stable, circuit agnostic, computationally efficient and produces solutions that are numerically identical to those obtained through a traditional fully coupled solve. The extension of this method to more sophisticated domain-decomposition solvers, application to MIMICs and resolution of continuous nonlinear material distributions in the EM system will be explored in subsequent papers.

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