Elliptic flow in $pp$-collisions at the LHC

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Abstract

We consider collective effects in $pp$-collisions at the LHC energies related to presence of the large orbital angular momentum in the initial state and role of this orbital momentum in the elliptic flow behavior.
Introduction

There were several collective effects in nuclear collisions observed at RHIC and various experimental observables relevant to their studies have been measured. In particular, there was observed ridge structure in the near-side two-particle correlation function in the secondary particles production. Similar structure in the two-particle correlation function was observed by the CMS Collaboration [1]. This is not an expected result. Commonly, $pp$–collisions were treated as a kind of “elementary” ones or the reference process for detecting deconfined phase formation in $AA$-collisions. It becomes evident that such a view should be corrected.

The experimental observations of RHIC and LHC on the ridge in the two-particle correlation functions demonstrate that the hadronic matter is strongly correlated and reveals high degrees of coherence when it is well beyond the critical values of density and temperature. The other anisotropic flow measurements at RHIC also demonstrated such a collective behavior.

In this note we discuss the elliptic flow in proton collisions on the base of approach [2] which was applied for discussion of the directed flow and the ridge in proton collisions [3]. This particular approach is based on the non-perturbative hadron dynamics. It should be noted that the values of transverse momenta in the relevant experimental data are not too large. Despite that there are considerations of the experimental results rendering to the perturbative QCD and parton radiation in the medium.

1 Elliptic flow in peripheral proton collisions

We consider in this section peripheral hadronic collisions. These collisions suppose that the impact parameter is different from zero. The mechanism of selecting such events in proton collisions will be considered further for the LHC energies. Here we adopt that impact parameter is non-zero and the reaction plane can therefore be determined by the cumulant method.

There are several experimental probes of collective dynamics. A most widely discussed one is the elliptic flow $v_2(p_\perp) \equiv \langle \cos(2\phi) \rangle_{p_\perp} = \frac{\langle p_x^2 - p_y^2 \rangle}{p_\perp^2}$, (1)

which is the second Fourier moment of the azimuthal momentum distribution of the particles with a fixed value of $p_\perp$. The azimuthal angle $\phi$ is an angle of the detected particle with respect to the reaction plane, which is spanned by the collision axis $z$ and the impact parameter vector $b$. The impact parameter vector $b$ is directed along the $x$ axis. Averaging is taken over large number of the events.
Elliptic flow can be expressed in covariant form in terms of the impact parameter and transverse momentum correlations as follows

\[
v_2(p_\perp) = \left( \frac{\hat{b} \cdot p_\perp}{p^2_\perp} \right)^2 - \left( \frac{\hat{b} \times p_\perp}{p^2_\perp} \right)^2,
\]

where \( \hat{b} \equiv b/b \). To get some hints on the possible behavior of the elliptic flow in proton collisions, it is useful to recollect what is known on this observable from nuclear collisions experiments. Integrated elliptic flow \( v_2 \) at high energies is positive and increases with \( \sqrt{s_{NN}} \). The differential elliptic flow \( v_2(p_\perp) \) increases with \( p_\perp \) at small values of transverse momenta, then it becomes flatten in the region of the intermediate transverse momenta and decreases at large \( p_\perp \).

It is also useful to apply a simple geometrical ideas which imply existence of the elliptic flow in hadronic reactions. Geometrical notions for description of multiparticle production in hadronic reactions were used by many authors, e.g by Chou and Yang in [4]. In the peripheral hadronic collisions the overlap region has different sizes along the \( x \) and \( y \) directions. According to the uncertainty principle we can estimate the value of \( p_x \) as \( 1/\Delta x \) and correspondingly \( p_y \sim 1/\Delta y \) where \( \Delta x \) and \( \Delta y \) characterize the size of the region where the particle originate from. Taking \( \Delta x \sim R_x \) and \( \Delta y \sim R_y \), where \( R_x \) and \( R_y \) characterize the sizes of the almond-like overlap region in transverse plane, we can easily obtain proportionality of \( v_2 \) in collisions with fixed initial impact parameter to the eccentricity of the overlap region, i.e.

\[
v_2 \sim \frac{R_y^2 - R_x^2}{R^2_x + R^2_y}.
\]

The presence of correlations of impact parameter vector \( b \) and \( p_\perp \) in hadron interactions follows also from the relation between impact parameters in the multiparticle production:

\[
b = \sum_i x_i b_i.
\]

Here \( x_i \) stand for Feynman \( x_F \) of \( i \)-th particle, the impact parameters \( b_i \) are conjugated to the transverse momenta \( p_i \). The above considerations are based on the uncertainty principle and angular momentum conservation, but they do not preclude the existence of the dynamical description, which will be discussed in the next section.


2 Peripheral inelastic proton-proton interactions at the LHC energies

The particle production mechanism proposed in the model \([2, 5]\) takes into account the geometry of the overlap region and dynamical properties of the transient state in hadron interaction. This picture assumes deconfinement at the initial stage of interaction. The transient state here appears as a rotating medium of massive quarks and pions which hadronize and form multiparticle final state at the freeze-out stage. Essential point for this rotation is the presence of a non-zero impact parameter in the collision.

In this section we consider unitarity saturation as a dynamical mechanism leading to the peripheral nature of inelastic collisions at the LHC energies. We also discuss the limiting energy dependence of the anisotropic flows resulting from unitarity saturation in central collisions (\(b = 0\)).

In the unitarization approach (\(U\)-matrix) the elastic scattering matrix in the impact parameter representation has the form:

\[
S(s, b) = \frac{1 + iU(s, b)}{1 - iU(s, b)},
\]

where \(S(s, b) = 1 + 2if(s, b)\) and \(U(s, b)\) is the generalized reaction matrix, which is considered to be an input dynamical quantity similar to the eikonal function. Unitarity equation rewritten at high energies for the elastic amplitude \(f(s, b)\) has the form

\[
\text{Im} f(s, b) = h_{el}(s, b) + h_{inel}(s, b)
\]

where the inelastic overlap function

\[
h_{inel}(s, b) = \frac{1}{4\pi} \frac{d\sigma_{inel}}{db^2}
\]

is the sum of all inelastic channel contributions. Inelastic overlap function is related to \(U(s, b)\) according to Eqs. \((5)\) and \((6)\) as follows

\[
h_{inel}(s, b) = \frac{\text{Im} U(s, b)}{|1 - iU(s, b)|^2},
\]

this function is the probability distribution over impact parameter of the inelastic interactions:

\[
\sigma_{inel}(s) = 8\pi \int_0^\infty db \, b \, \frac{\text{Im} U(s, b)}{|1 - iU(s, b)|^2}.
\]

It should be noted that

\[
\text{Im} U(s, b) = \sum_{n \geq 3} \tilde{U}_n(s, b),
\]
where $\bar{U}_n(s, b)$ is a Fourier–Bessel transform of the function $\bar{U}_n(s, t)$, the form of this function can be found in [5].

Since the form of the impact parameter dependence of the function function $h_{inel}(s, b)$ evolves from a central to the peripheral one at the LHC energies [6] and this function tends to zero at $b = 0$ and $s \rightarrow \infty$, the mean multiplicity

$$\langle n \rangle(s) = \frac{\int_{0}^{\infty} bdb \langle n \rangle(s, b) h_{inel}(s, b)}{\int_{0}^{\infty} bdbh_{inel}(s, b)}$$

obtains a main input from the collisions with non-zero impact parameters. One can suggest therefore that the events with average and higher multiplicity at the LHC energy $\sqrt{s} = 7$ TeV correspond to the peripheral hadron collisions [6]. Thus, at the LHC energy $\sqrt{s} = 7$ TeV there is a dynamical selection of the peripheral region in impact parameter space responsible for the inelastic processes. In the nuclear reactions similar selection of the peripheral collisions is provided by the relevant experimental adjustments.

Of course, the standard inclusive cross-section for unpolarized particles being integrated over impact parameter $b$, does not depend on the azimuthal angle of the detected particle transverse momentum. It can be written with account for $s$–channel unitarity in the following form

$$\frac{d\sigma}{d\omega} = 8\pi \int_{0}^{\infty} bdb \frac{I(s, b, \omega)}{|1 - iU(s, b)|^2}.$$  \hspace{1cm} (10)

When the impact parameter vector $b$ and transverse momentum $p_\perp$ of the detected particle are fixed the function $I$ does depend on the azimuthal angle $\phi$ between vectors $b$ and $p_\perp$. It should be noted that the impact parameter $b$ is the variable conjugated to the transferred momentum $q = p_a' - p_a$ between two incident channels which describe production processes of the same final multiparticle state. The dependence on the azimuthal angle $\phi$ can be written in explicit form through the Fourier series expansion

$$I(s, b, y, p_\perp) = \frac{1}{2\pi} I_0(s, b, y, p_\perp)[1 + \sum_{n=1}^{\infty} 2\bar{v}_n(s, b, y, p_\perp) \cos n\phi].$$  \hspace{1cm} (11)

The function $I_0(s, b, \xi)$ satisfies to the following sum rule

$$\int I_0(s, b, y, p_\perp) dp_\perp dy = \langle n \rangle(s, b) \text{Im} U(s, b),$$ \hspace{1cm} (12)

where $\bar{n}(s, b)$ is the mean multiplicity depending on impact parameter. Thus, the bare anisotropic flow $\bar{v}_n(s, b, y, p_\perp)$ is related to the measured flow $v_n$ as follows

$$v_n(s, b, y, p_\perp) = w(s, b)\bar{v}_n(s, b, y, p_\perp).$$
where the function $w(s, b)$ is

$$w(s, b) \equiv |1 - iU(s, b)|^{-2}.$$  

In the above formulas the variable $y$ denotes rapidity, i.e. $y = \sinh^{-1}(p/m)$, where $p$ is a longitudinal momentum. Thus, we can see that unitarity corrections are mostly important at small impact parameters, i.e. they modify anisotropic flows at small centralities, while peripheral collisions are almost not affected by unitarity. The following limiting behavior of $v_n$ at $b = 0$ can easily be obtained:

$$v_n(s, b = 0, y, p_\perp) \to 0$$

at $s \to \infty$ since $U(s, b = 0) \to \infty$ in this limit.

In conclusion, one should note that inelastic events at the LHC energies come mainly from the non-zero values of the impact parameter. It is the result of approaching to unitarity saturation.

### 3 Coherent rotation of transient matter and elliptic flow

Before turning on discussion of anisotropic flows, it is useful to note that the energy dependence of the average transverse momentum was considered in [5].

The mechanism is the following: the nonzero orbital angular momentum leads to coherent rotation of a transient matter located in the overlap region as a whole in the $xz$-plane. This rotation is similar to the rotation of a liquid where strong correlations between particles momenta exist.

In this mechanism of hadron production, the valence constituent quarks excite a part of the cloud of virtual massive quarks and those quarks subsequently hadronize and form the multiparticle final state.

Here we use this mechanism for evaluation of the elliptic flow in $pp$-interactions. It was suggested in [5] that the rotation of transient matter will affect the average transverse momentum of secondary hadrons produced in proton-proton collisions. A coherent rotation of transient liquid-like state results in a contribution to the transverse momentum in the form:

$$\Delta p_T = \kappa L(s, b),$$  

(13)

where $L(s, b)$ is the orbital angular momentum and $\kappa$ is a constant which has a dimension of inverse length. Note that the power-like dependence of the average transverse momentum at high energies has been obtained in good agreement with experimental data [5].
Going further, one should note that, in fact, the rotation gives a contribution to the $x$-component of the transverse momentum and does not contribute to the $y$-component of the transverse momentum, i.e.

$$\Delta p_x = \kappa L(s, b) \quad (14)$$

while

$$\Delta p_y = 0. \quad (15)$$

Thus, assuming that $p_x = p_0 + \Delta p_x$ ($\Delta p_x \ll p_0$) and $p_y = p_0$, the effect of the rotating transient matter contribution into the elliptic flow can be calculated by analogy with the calculation of the average transverse momentum performed in [5]. The resulting integrated elliptic flow increases with energy

$$v_2 \propto s^{\delta_C},$$

where $\delta_C = 0.207$. It should be noted here that transient matter consists of virtual constituent quarks strongly interacting by pion (Goldstone bosons) exchanges.

The incoming constituent quark has a finite geometrical size determined by the radius $r_Q$ and interaction radius $R_Q$ ($R_Q > r_Q$). Former one is determined by the chiral symmetry breaking mechanism and the latter one — by the confinement radius. Meanwhile, it is natural to suppose on the base of the uncertainty principle that size of the region where the virtual massive quark $Q$ is knocked out from the cloud is determined by its transverse momentum, i.e. $\bar{R} \simeq 1/p_{\perp}$. However, it is evident that $\bar{R}$ cannot be larger than the interaction radius of the valence constituent quark $R_Q$. It is also clear that $\bar{R}$ should not be less than the geometrical size of the valence constituent quark $r_Q$ for this mechanism be a working one. When $\bar{R}$ becomes less than $r_Q$, this constituent quark mechanism does not work anymore and one should expect vanishing collective effects in the relevant region of the transverse momentum.

The value of the quark interaction radius was obtained under analysis of the elastic scattering [7] and it has the following dependence on its mass

$$R_Q = \xi/m_Q \sim 1/m_{\pi} \quad (16)$$

where $\xi \simeq 2$ and therefore $R_Q \simeq 1 \text{ fm}$, while the geometrical radius of quark $r_Q$ is about $0.2 \text{ fm}$. It should be noted the region, which is responsible for the small-$p_{\perp}$ hadron production, has large transverse dimension and the incoming constituent quark excites the rotating cloud of quarks with different values and directions of their momenta in that case. Effect of rotation will therefore be smeared off over the volume $V_{\bar{R}}$ and then one should expect that $\langle \Delta p_x \rangle_{V_{\bar{R}}} \simeq 0$. Thus,

$$v_2^Q(p_{\perp}) \equiv \langle v_2 \rangle_{V_{\bar{R}}} \simeq 0 \quad (17)$$
at small $p_{\perp}$. When we proceed to the region of higher values of $p_{\perp}$, the radius $\bar{R}$ is decreasing and the effect of rotation becomes more and more prominent, incoming valence quark excites now the region where most of the quarks move coherently, in the same direction, with approximately the same velocity. The mean value $\langle \Delta p_x \rangle_{\bar{R}} > 0$ and
\[
v_2^Q(p_{\perp}) \equiv \langle v_2^Q \rangle_{\bar{R}} > 0
\]
and it increases with $p_{\perp}$. The increase of $v_2^Q$ with $p_{\perp}$ will disappear at $\bar{R} = r_Q$, i.e. at $p_{\perp} \geq 1/r_Q$, and saturation will take place. The value of transverse momentum where the flattening starts is about $1 GeV/c$ for $r_Q \simeq 0.2 fm$. At very large transverse momenta the constituent quark picture would not be valid and elliptic flow vanishes as it was already mentioned.

We discussed elliptic flow for the constituent quarks. Predictions for the elliptic flow for the particular hadrons depends on the supposed mechanism of hadronization. For the region of the intermediate values of $p_{\perp}$ the constituent quark coalescence mechanism \cite{8, 9} would be dominating one. In that case the values for the hadron elliptic flow can be obtained from the constituent quark one by the replacement $v_2 \rightarrow n_v v_2^Q$ and $p_{\perp} \rightarrow p_{\perp}^Q/n_v$, where $n_v$ is the number of constituent quarks in the produced hadron.

Typical qualitative dependence of elliptic flow in $pp$-collisions in this approach is presented in Fig.1

![Figure 1: Qualitative dependence of the elliptic flow $v_2$ on transverse momentum in pp-collisions.](image)

The centrality dependence of the elliptic flow is determined by dependence of the orbital angular momentum $L$ on the impact parameter, i.e. it should be decreasing towards high and low centralities. Decrease toward high centralities is evident since no overlap of hadrons should occur at high enough impact parameters. Decrease of $v_2$ toward lower centralities is specific prediction of the proposed mechanism based on rotation since central collisions with smaller impact parameters would lead to slower rotation or its complete absence in the head-on
collisions. Qualitative dependence of the elliptic flow on the impact parameter is similar to the curve depicted in Fig.1 where variable $b$ being used instead of transverse momentum.

**Conclusion**

We considered the elliptic flow in the proton-proton interactions at the LHC energies in the particular nonperturbative approach, where the origin of elliptic flow is associated with the effects of rotation of quark-pion transient matter. This mechanism of anisotropic flow might be a leading one in hadron collisions, since those have smaller geometrical extension and the probability of hydrodynamical generation of elliptic flow is lower compared to the collisions of nuclei.

At such high energies (LHC) the presence of reflective scattering mode serves as a trigger for the mainly peripheral nature of the inelastic interactions and the presence of large orbital angular momenta. Since correlations are maximal in the rotation plane, a narrow ridge should be observed in the two-particle correlation function $[3]$.

In conclusion, we would like to remark that performing studies of the multiparticle productions in $pp$-collisions at the LHC under the scope of the searches for the possible existence of the rotation effects would be of a significant interest. Such effects would be absent when the genuine quark-gluon plasma (gas of free quarks and gluons) being formed at the LHC energies because of all collective effects and the anisotropic flows, in particular, should disappear in this case.

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