Quantum radiation from a partially reflecting moving mirror

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Abstract

We consider the quantum radiation from a partially reflecting moving mirror for the massless scalar field in 1+1 Minkowski space. Partial reflectivity is achieved by localizing a δ-type potential at the mirror’s position. The radiated flux is exactly obtained for arbitrary motions as an integral functional of the mirror’s past trajectory. Partial reflectivity corrections to the perfect mirror result are discussed.

1 Introduction

One of the most interesting prediction of the quantum field theory is the radiation produced by moving mirrors. A mirror can be practically seen
as a barrier potential in the vicinity of a certain spatial boundary. Moving mirrors effects can be placed thus in the larger context of that produced in exterior classical fields. Essentially, the phenomenon expresses the fact that in presence of mirror-like boundaries in (non-uniform) motion real quanta will be excited out from vacuum fluctuations.

For perfect mirrors, situation enjoys a certain technical definiteness. As infinite barriers are involved, the problem basically relies in assuring the quantum field to vanish on the corresponding boundaries. There are a lot of mathematical methods to deal with such problems, and a wealth of literature following this line can be found. (Perfect reflectivity seems also to have been the preferred choice for discussing a series of most profound aspects of the phenomenon: the connection with the Hawking effect [2]-[8] and non-trivial situations implied by the possibility of generating negative energy fluxes [9]-[14]). As concerns partial reflectivity, results are comparatively less numerous. Two main directions of research have to be mentioned here. One is that followed by Jaekel and Reynaud, based on the method developed in Refs. [15, 16]. Reflectivity properties of the mirror are described within a scattering approach, and the radiation is derived from the analysis of an $S$ matrix depending on the mirror’s trajectory. The method has a large generality, but it is associated with a major drawback: by its very nature it is ill suited for an accurate evaluation of the radiated flux density, as a local quantity, at arbitrary times. In addition, the analysis is carried out in frequency space, obscuring somewhat the trajectory dependence in coordinate space. Another direction is that of Barton et al. [17]-[19] (see also Ref. [20]). They proceed in the more conventional way of constructing the field operator via its modes
expansion, tacking into account explicitly dielectric properties of the mirror. As a common feature, however, their results are systematically obtain within the framework of perturbation theory, in first order in the mirror’s velocity.

To our best knowledge, the only calculation allowing an exact result for the radiated flux is that of Frolov and Singh [21] for spherical mirrors (in $D$-dimensional flat space) expanding with a constant acceleration. Semi-transparency is realized by means of a $\delta$-like potential. Our intention here to provide a two dimensional semitransparent mirror model based on a similar idea, allowing an exact treatment for arbitrary motions.

We shall focus on the massless scalar field. The essential piece in our investigation is the construction of the (in) modes defining the field operator in Heisenberg image. We obtain them by introducing a set of space time dependent reflection and transmission coefficients into those of the Fulling and Davies model for perfect reflectors [22]. Matching conditions on the mirror’s worldline force the coefficients to respect a first order differential equation, which determine them as integral functionals of the mirror’s past trajectory. It turns out that calculations can be exactly carried out for the Wightman function and the renormalized energy-momentum tensor (we restrict here to the canonical expression). Qualitatively, one finds that the radiated flux results as history dependent quantity, with past dependence extending in principle to minus infinity. Contributions of past motions appear exponentially damped with the proper time interval up to the emission moment, with the barrier strength as the proportionality factor. The perfect mirror result is recovered in the limit of infinite barrier strengths.

The paper is organized as follows. In Sec. 2 we construct the quantum
Let \((t, x)\) denote the coordinates in 1+1 Minkowski space time and \(x(t)\) denote the trajectory of the mirror. We write the barrier potential as

\[
V(t, x) = \frac{a}{\gamma(t)} \delta(x - x(t)), \quad \gamma = \left(1 - \left(\frac{dx}{dt}\right)^2\right)^{-\frac{1}{2}},
\]

where \(\delta\) is the Dirac distribution and \(a\) a positive constant corresponding to the barrier strength. The \(\gamma^{-1}\) contraction factor assures relativistic covariance of Eq. (2) below. We proceed to construct the in modes for the field \(\varphi(t, x)\) obeying

\[
(\Box + V(t, x))\varphi(t, x) = 0.
\]

We divide first the Minkowski plane into the left \((L)\) and right \((R)\) regions, corresponding to points \((t, x < x(t))\) and \((t, x > x(t))\), respectively. Let us write the in modes for the perfect mirror [2] as

\[
U_\omega^L(u, v) = e^{-i\omega u} - e^{-i\omega f(v)}, \quad U_\omega^R(u, v) = 0,
\]
\[ V^R_\omega(u, v) = e^{-i\omega v} - e^{-i\omega g(u)}, \quad V^L_\omega(u, v) = 0, \quad \omega > 0, \quad (4) \]

where \( L, R \) superscripts refer to the definition domains and null coordinates were used \((u = t-x, v = t+x)\). Functions \( f, g \) are chosen such that identities

\[ f(v) = u, \quad (5) \]
\[ g(u) = v, \quad (6) \]

are satisfied when \((u, v)\) moves along the trajectory.

Consider, for example, the \( U^L_\omega \) solution. \( e^{-i\omega f(v)} \) represents the totally reflected component of the incident right-moving wave \( e^{-i\omega u} \). Partial reflectivity requests introduction of a reflection factor, along with a transmitted component in \( R \). We modify thus expressions (3) as

\[ U^L_\omega(u, v) = e^{-i\omega u} - R^L_\omega(v) e^{-i\omega f(v)}, \quad (7) \]
\[ U^R_\omega(u, v) = T^R_\omega(u) e^{-i\omega u}. \quad (8) \]

For \( V^L,R_\omega \) we similarly set

\[ V^R_\omega(u, v) = e^{-i\omega v} - R^R_\omega(u) e^{-i\omega g(u)}, \quad (9) \]
\[ V^L_\omega(u, v) = T^L_\omega(v) e^{-i\omega v}. \quad (10) \]

The physical significance of \( R_{\omega}^{R,L}, T_{\omega}^{R,L} \) coefficients is clear. For points inside the \( L, R \) regions, the \( u, v \) dependence in Eqs. (9)-(11) automatically assures

\[ \Box U^L,R_\omega = \Box V^L,R_\omega = 0. \quad (11) \]

At the \( L-R \) boundary, for each pair of solutions \((\varphi^L_\omega, \varphi^R_\omega)_{\varphi = U,V}\) we impose the continuity condition

\[ \varphi^L_\omega = \varphi^R_\omega. \quad (12) \]
and the matching of the first order derivatives, in conformity with Eqs. (1), (2). This is most conveniently obtained by considering the relativistic generalization of the stationary case \( x(t) = \text{const} \). One finds

\[
\epsilon_{\mu\nu} U^\mu \partial_\nu \varphi_R^\omega - \epsilon_{\mu\nu} U^\mu \partial_\nu \varphi_L^\omega + a \varphi_R^{R/L} = 0, \tag{13}
\]

where \( \epsilon_{\mu\nu} \) is the totally antisymmetric tensor in two dimensions (\( \epsilon_{tx} = 1 \)) and \( U^\mu \) is the mirror’s two velocity \( (U^\mu U_\mu = 1) \). Equation (12) implies

\[
\mathcal{R}_\omega^L + \mathcal{T}_\omega^R = \mathcal{R}_\omega^R + \mathcal{T}_\omega^L = 1, \tag{14}
\]

and Eq. (13)

\[
\frac{d\mathcal{T}_\omega^R}{du} - \frac{d\mathcal{R}_\omega^L}{dv} + i\omega \left( 1 + \frac{df}{dv} \right) \mathcal{R}_\omega^L + a\gamma \mathcal{T}_\omega^R = 0, \tag{15}
\]

\[
\frac{d\mathcal{T}_\omega^L}{dv} - \frac{d\mathcal{R}_\omega^R}{du} + i\omega \left( 1 + \frac{dg}{du} \right) \mathcal{R}_\omega^R + a\gamma \mathcal{T}_\omega^L = 0, \tag{16}
\]

where all quantities refer to a given point on the trajectory.

Let \( \tau \) denote the proper time of the mirror, and let us regard \( \mathcal{R}, \mathcal{T} \) coefficients as functions of \( \tau \) via \( u = u(\tau), v = v(\tau) \). Eliminating \( \mathcal{T}_\omega^{R,L} \) in favour of \( \mathcal{R}_\omega^{L,R} \) using Eq. (14), one finds that Eqs. (15) and (16) combined lead to the following differential equation for \( \mathcal{R}_\omega^{L,R} \)

\[
\frac{d}{d\tau} \mathcal{R}_\omega^{L,R}(\tau) + \left( -i\omega D_\beta^\pm(\tau) + \frac{a}{2} \right) \mathcal{R}_\omega^{L,R}(\tau) = \frac{a}{2}, \tag{17}
\]

where we introduced the Doppler factors \( (\beta = dx/dt) \)

\[
D_\beta^\pm(\tau) = \sqrt{\frac{1 \pm \beta(\tau)}{1 \mp \beta(\tau)}}. \tag{18}
\]
Plus and minus signs in $D_\beta^\pm$ correspond to $R, L$, respectively. For example, for uniform trajectories $\beta = \text{const}$ the solution reads

$$R_{L,R}^{L,R} = \frac{a/2}{-i\omega D_\beta^\pm + a/2},$$  \hspace{1cm} (19)

having a transparent interpretation in terms of Doppler shifts as observed in the mirror’s proper frame.

The general solution can be written

$$R_{L,R}^{L,R}(\tau) = R_{L,R}^{L,R}(\tau_0) \exp \left(i\omega \delta^\pm(\tau, \tau_0) - a(\tau - \tau_0)\right)$$

$$+ \frac{a}{2} \int_{\tau_0}^{\tau} d\tau' \exp \left(i\omega \delta^\pm(\tau, \tau') - a/2(\tau - \tau')\right),$$  \hspace{1cm} (20)

with $\tau_0$ fixed and

$$\delta^\pm(\tau_2, \tau_1) = \int_{\tau_1}^{\tau_2} d\tau' D_\beta^\pm(\tau').$$  \hspace{1cm} (21)

With definitions (18) one actually finds for $\delta^\pm$

$$\delta^+(\tau_2, \tau_1) = v(\tau_2) - v(\tau_1),$$  \hspace{1cm} (22)

$$\delta^-(\tau_2, \tau_1) = u(\tau_2) - u(\tau_1).$$  \hspace{1cm} (23)

We let now $\tau_0 \to -\infty$. The first term in Eq. (20) vanishes$^1$, hence

$$R_{L,R}^{L,R}(\tau) = \frac{a}{2} \int_{-\infty}^{\tau} d\tau' \exp \left(i\omega \delta^\pm(\tau, \tau') - a/2(\tau - \tau')\right).$$  \hspace{1cm} (24)

We take Eq. (24) as the definition for the reflection coefficients. Transmission factors follow from unitarity relation Eq.(14). This completes the derivation of the in modes.

$^1$|$R_{L,R}^{L,R}(-\infty)$| < 1 by the assumption of uniform velocity in the infinite past, see next.
An observation is appropriate here: by the real part of the exponential factors, the coefficients are determined mainly by the motion in an interval

\[ 0 < \tau - \tau' \lesssim a^{-1}. \]  

(25)

As an immediate consequence, consider a trajectory with constant velocity after some fixed proper time \( \tau_c \). Then for sufficiently late times

\[ \tau - \tau_c \gg a^{-1} \]  

(26)

non-uniformities in velocity before \( \tau_c \) can be ignored and one can set

\[ \delta^{\pm}(\tau, \tau') = D^{\pm}_{\beta}(\tau_c)(\tau - \tau'), \]  

(27)

which is equivalent to the stationary result (19).

The Heisenberg field is constructed along the usual lines as

\[
\varphi(z) = \int_{0+}^{\infty} \frac{d\omega}{(2\pi)2\omega} \left( a_{\omega}^{U} U_{\omega}^{R}(z) + a_{\omega}^{V} V_{\omega}^{R}(z) + H.c. \right), \]

(28)

for points \( z \equiv (u, v) \) in the right region, and similarly with \( R \rightarrow L \) for \( z \) in the left region. The commutations for the creation-annihilation in operators are

\[
[a_{\omega}^{U}, a_{\omega'}^{U+}] = [a_{\omega}^{V}, a_{\omega'}^{V+}] = 2\pi(2\omega)\delta(\omega - \omega'), \]

(29)

and zero in rest. To assure that canonical commutations are satisfied, we shall suppose there exists a finite time \( t_0 \) (which can be taken arbitrarily far in the past) such that the trajectory is uniform before \( t_0 \). Then, in the reference frame where the mirror is at rest at infinite past, the modes are given up to a certain moment by the \( \beta = 0 \) coefficients (19). Orthonormality
and completeness can be easily checked in this case. It follows that canonical commutations are satisfied on some spacelike three surface, and thus, by Eq. (2) they are respected in whole Minkowski space [24].

3 The radiation

We are interested in the in-vacuum renormalized expectation values \( \langle T_{\mu\nu}\rangle_{\text{ren}} \) of the canonical energy-momentum operator

\[
T_{\mu\nu} = \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} g^{\alpha\beta} \partial_\alpha \varphi \partial_\beta \varphi,
\]

at points off the trajectory. We need the in-vacuum Wightman function

\[
D^+(z, z') = \langle 0, \text{in} | \phi(z) \phi(z') | 0, \text{in} \rangle,
\]

with the in vacuum defined by

\[
a_U^V |0, \text{in}\rangle = a_V^U |0, \text{in}\rangle = 0, \quad \omega > 0.
\]

Let \( D_0^+(z, z') \) denote the free field Wightman function, and let’s introduce the renormalized value

\[
D^+_{\text{ren}}(z, z') = D^+(z, z') - D_0^+(z, z').
\]

Then

\[
\langle T_{uu}(u, v) \rangle_{\text{ren}} = \lim_{u' \to u} \lim_{v' \to v} \partial_u \partial_{u'} D^+_{\text{ren}}(u, v, u', v'),
\]

\[
\langle T_{vv}(u, v) \rangle_{\text{ren}} = \lim_{u' \to u} \lim_{v' \to v} \partial_u \partial_{v'} D^+_{\text{ren}}(u, v, u', v').
\]
Space time coordinates symmetrization proves unnecessary here, as Eqs. (34), (35) will lead to real results. The mixed $uv$, $vu$ components vanish identically.

Let $D_R^+$ stand for $D^+(z, z')$ when both $z, z'$ belong to $R$, and analogously for $D_L^+$. It is convenient to introduce $\tau_u, \tau_v$ as the inverse functions of $u, v$

$$\tau_u(u(\tau)) = \tau, \quad \tau_v(v(\tau)) = \tau.$$  \hspace{1cm} (36)

Simple calculations yield (note that $\omega$ dependence in $R^{L,R}_\omega$ makes the frequency integrations similar to that in the free field case)

$$D_R^+ = D_0^+ + D_{R1}^+ + D_{R2}^+,$$  \hspace{1cm} (37)

where ($\epsilon \to 0_+$)

$$D_{R1}^+ = \frac{a}{8\pi} \int_{-\infty}^{\tau_u} d\tau_1 \ln \left( (u(\tau_1) - u' - i\epsilon)(v(\tau_1) - v' - i\epsilon) \right)$$

$$\times \exp \frac{a}{2}(\tau_1 - \tau_u)$$

$$+ \frac{a}{8\pi} \int_{-\infty}^{\tau_u'} d\tau_1 \ln \left( (u - u(\tau_1) - i\epsilon)(v - v(\tau_1) - i\epsilon) \right)$$

$$\times \exp \frac{a}{2}(\tau_1 - \tau_u'),$$  \hspace{1cm} (38)

and

$$D_{R2}^+ = -\frac{a^2}{16\pi} \int_{-\infty}^{\tau_u} d\tau_1 \int_{-\infty}^{\tau_u'} d\tau_2 \ln \left( (u(\tau_1) - u(\tau_2) - i\epsilon) \right)$$

$$\times (v(\tau_1) - v(\tau_2) - i\epsilon) \exp \frac{a}{2}(\tau_1 + \tau_2 - \tau_u - \tau_u').$$  \hspace{1cm} (39)
The explicit form of \(D_0^+\) is of no relevance here. \(\tau_u\) is a shorthand for \(\tau_u(u)\), and similarly for \(\tau_{u'}\). \(D_L^+\) can be obtained from \(D_R^+\) with the substitutions

\[u \leftrightarrow v, \quad u' \leftrightarrow v', \quad \tau_u \leftrightarrow \tau_v.\]  

(40)

We shall take advantage in the following of the \(R - L\) formal symmetry above to refer to \(R\) quantities only.

Renormalization amounts to ignore \(D_0^+\) in Eq. (37). The double \(v\) derivative (34) is trivial

\[\langle T^R_{vv}(u, v)\rangle_{\text{ren}} = 0.\]  

(41)

The \(uu\) component (35) requires some effort. Calculations are outlined in Appendix A. One finds the \(v\) independent quantity

\[\langle T^R_{uu}(u, v)\rangle_{\text{ren}} = -\frac{1}{4\pi} \frac{1 - \beta^2(\tau_u)}{(1 - \beta(\tau_u))^2} \times T^R(\tau_u),\]  

(42)

where

\[T^R = T^R_I + T^R_{II},\]  

(43)

with

\[T^R_I(\tau_u) = \frac{a^2}{4} \int_{-\infty}^{\tau_u} \int_{-\infty}^{\tau_u} d\tau_1 d\tau_2 \left[ \partial_{\tau_1} \partial_{\tau_2} \ln \frac{v(\tau_1) - v(\tau_2)}{u(\tau_1) - u(\tau_2)} \right] \times \exp \frac{a}{2}(\tau_1 + \tau_2 - 2\tau_u),\]  

(44)

and

\[T^R_{II}(\tau_u) = -\frac{a}{2} \int_{-\infty}^{\tau_u} \int_{-\infty}^{\tau_u} d\tau_1 d\tau_2 \left[ \partial_{\tau_1} \partial_{\tau_2} \frac{\dot{u}(\tau_1) - \dot{u}(\tau_2)}{u(\tau_1) - u(\tau_2)} \right] \times \exp \frac{a}{2}(\tau_1 + \tau_2 - 2\tau_u).\]  

(45)
The overdot represents differentiation with respect to proper time. Coincidence limits in the integrands are discussed in Appendix B.

Equations (42)-(45) represent our main result. They explicitly show that for finite barrier strengths the radiated energy-momentum density at a given point

$$\langle T^R_{tt}(u, v) \rangle_{\text{ren}} = -\langle T^R_{tx}(u, v) \rangle_{\text{ren}} \equiv \langle T^R_{uu}(\tau_u) \rangle_{\text{ren}}$$

(46)

depends on the entire mirror’s past history. Past dependence extends up to the retarded (emission) time $\tau_u$, marking the intersection of the trajectory with the past light cone at $(u, v)$. As for $R, T$ coefficients, exponential factors restrict the influence of past motions to an effective interval $\sim a^{-1}$ before $\tau_u$.

Perfect reflectivity naturally results by letting $a \to \infty$. To evaluate the limits, it is convenient to use

$$\lim_{a \to \infty} a \int_{-\infty}^{\tau} d\tau' f(\tau') \exp a(\tau' - \tau) = \lim_{\tau' \to \tau} \int_{\tau' \to \tau} f(\tau'),$$

(47)

which makes apparent disappearance of history dependence. For Eqs. (44), (45) situation is as follows. Let $\alpha$ denote the mirror’s proper acceleration. Then coincidence limit (44) in $T^R_I$ can be rewritten (we assume the acceleration is continuously differentiable at $\tau_u$, see Appendix B)

$$\lim_{a \to \infty} T^R_I(\tau_u) = \frac{1}{3} \dot{\alpha}(\tau_u).$$

(48)

For $T^R_{II}$ component, suffices to note that the double integral is multiplied by only one power of $a$, so that it brings no contribution. Combining Eqs. (48), (42), one recovers the well known perfect mirror result [22, 23].

\footnote{A similar conclusion was reached, e.g., in Refs. [17, 13].}
Suppose now that $\alpha$ is finite, but sufficiently large that no significative changes in velocity occur in the effective interval before $\tau_u$. In other words, we are restricting to slow motions on a proper time scale $\sim a^{-1}$. Then the brackets can be Taylor expanded around $\tau_1 = \tau_2 = \tau_u$, followed by a term by term integration. The result is the series

$$T^R = T^R_0 + \sum_{n=1}^{\infty} \left( \frac{2}{a} \right)^n T^R_n,$$

(49)

with $T^R_n$ coefficients completely determined as polynomials in $\alpha$ and its first $n + 1$ derivatives at $\tau_u$. $T^R_0$ equals precisely the perfect mirror contribution (48). Higher terms naturally describe semitransparency corrections. We give below the first two coefficients (for arbitrary orders, evaluation of the coincidence limits in the Taylor expansion proves quite laborious)

$$T^R_1 = \frac{1}{6} \ddot{\alpha} - \frac{1}{6} \dddot{\alpha},$$

(50)

$$T^R_2 = \frac{37}{420} \alpha^2 \dot{\alpha} - \frac{1}{6} \dot{\alpha}^2 - \frac{1}{6} \ddot{\alpha} \alpha + \frac{11}{120} \dddot{\alpha},$$

(51)

It is interesting to note that both corrections can be accommodated into the perfect mirror result by replacing $\alpha$ in (48) with an “effective acceleration” containing supplemental $\dot{\alpha}, \ddot{\alpha}$ dependent terms.

We also point out the following aspect: one sees that corrections can generally both diminish or enhance the zeroth order flux. Consider, for example, a leftward accelerated trajectory on which the perfect mirror radiates a positive energy flux in $R$

$$\alpha < 0, \quad \dot{\alpha} < 0.$$

(52)

Then if the flux is increasing or not decreasing too fast as seen in the mirror proper frame (i.e. $\dddot{\alpha}$ is not too large), the leading correction Eq. (50)...
has always a diminishing effect. By contrast, for the absolute values of the negative energy flux radiated in $L$, the opposite is true.

We end with a comment on non-radiative motions. It is clear that non-radiativity results as a history dependent property. Consider a fixed trajectory up to proper time $\tau_0$. To find its non-radiative future extension, one should solve the integro-differential system

$$T^R(\tau) = T^L(\tau) = 0, \quad \tau > \tau_0,$$

(53)

where $\tau < \tau_0$ trajectory plays the role of Cauchy data. Unfortunately, a general treatment is analytically rather difficult, if not impossible, given non-linearities implied. We shall contend ourselves to point out the special case when both integrands in Eqs. (44), (45) vanish identically. Coincidence limit in $T^R_I$ readily gives a necessary condition as

$$\alpha(\tau) = \text{const.}$$

(54)

One can easily check that this is also sufficient. Hence no radiation occurs for constant acceleration trajectories, and this is what one should expect by conformal invariance of the theory. An observation is in order here. It may be argued that uniformly accelerated trajectories do not respect the constant velocity condition for $t \to -\infty$ as formulated in Sec. 2. From a practical point of view, however, this is not essential: the mirror can be considered stationary up to a certain moment, and only afterwards taken to accelerate. After a proper time large enough compared to $a^{-1}$ the motion will effectively count as non-radiative.

\[ \text{A similar situation happens for } D = 2 \text{ spherical mirrors in Ref. [21].} \]
4 An analogy with extended classical charges

We conclude by briefly discussing an analogy. History dependence of the emitted flux invites to a parallel with extended charges in classical electrodynamics. Consider an arbitrarily moving charge with characteristic length $r$. Retardation implies that the electromagnetic flux at a certain point is determined by the motion in some finite length interval $\Delta \tau \sim r$. One is led thus to the correspondence

$$r \sim a^{-1}. \quad (55)$$

It is further relevant to turn toward a dynamical aspect. It is a common remark that the quantum backreaction force experienced by perfect mirrors in two dimensions\cite{22} displays the same trajectory dependence as the radiation force\cite{25} acting on pointlike charges. This makes the well known $r \to 0$ pathologies, i.e. runaway trajectories and acausality, manifest too for perfect mirrors.

Our observation concerning similarity expressed by (55) is as follows. It was shown in Refs.\cite{26,27} that situation above changes when allowing for a transparency factor: more precisely, the unphysical behavior is absent when the transparency cut-off is sufficiently small compared to the mirror’s mass\footnote{The result is based on the $S$ matrix approach in Ref.\cite{13}. There are certain limitations in the proof, as approximations are made to linearize the dynamical equation. In our model, linearization shows there are no unphysicalities if $a/(4\pi m) < 1$, with $m$ the mirror’s mass.}.

Now, the cut-off in our model is some quantity of order $a$. These find a direct parallel in what happens for extended charges: the inverse of $r$ plays
an identical role, in the sense that unphysicalities disappear \cite{28} provided the ratio between the electromagnetic self energy \( \sim r^{-1} \) and the physical mass is small enough.

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**Appendix A**

We sketch here the steps leading to Eqs. (44), (45). Let

\[
 t_{1,2}(u, u') = \partial_u \partial_{u'} D_{\alpha \beta}^+(u, v, u', v').
\]  

(56)

It turns out that \( t_1, t_2 \) separately diverge for \( \epsilon \to 0, u' \to u \). An appropriate grouping of terms in their sum, however, makes divergences to cancel among themselves. We show below a way to do this. Observe that terms in \( t_2 \) can be organized in \( u, v \) dependent parts. We denote them as \( t^u_2, t^v_2 \), respectively. We further introduce a set of equivalent, but *formally* different expressions

\[
 t^u_{2\alpha} = t^u_{2\beta} = t^u_{2\gamma} = t^u_2, \quad t^v_{2\gamma} = t^v_2,
\]  

(57)

which we define as follows.
With a series of integrations by parts with respect to $\tau_1$, $t_2^u$ can be cast into the form

\[
t_{2u} = -\frac{a^2}{16\pi} \int_{-\infty}^{\tau_1} d\tau_1 \exp \frac{a}{2}(\tau_1 - \tau_u) \int_{-\infty}^{\tau_1} d\tau_1 \frac{du_1}{u(\tau_1) - u' - i\epsilon} \left\{ \int_{-\infty}^{\tau_u} d\tau_1 \frac{du_1}{u(\tau_1) - u(\tau_2) - i\epsilon} \right\}.
\]

(58)

Similar integrations with respect to $\tau_2$ lead to a second expression $t_{2\beta}^u$. An integration by parts with respect to $\tau_2$ in the double integral in Eq. (58) eliminates the single integration term. Let $t_{2\gamma}^u$ denote the resulting expression. Starting with $t_2^v$ and repeating the steps leading from $t_2^u$ to $t_{2\gamma}^u$, one obtains $t_{2\gamma}^v$.

We organize now the sum $t_1 + t_2$ as

\[
(t_{2\gamma}^v - t_{2\gamma}^u) + (t_{2\alpha}^u + t_{2\beta}^u + t_1).
\]

(59)

At this point one can set $\epsilon = 0$ and let $u' \to u$, in each parenthesis the divergences getting cancelled in a manifest way. The two parentheses correspond, in order, to $I$ and $II$ contributions in $\langle T_{uu}^{R}(u,v) \rangle_{\text{ren}}$.

**Appendix B**

We make here some observations concerning the coincidence limits in Eqs. (44), (45). Simple analysis shows that the quantities in the brackets are finite, provided the trajectory is at least of class $C^3$, respectively $C^4$. The coincidence limit in $T_{1}^{R}$ is of direct relevance for the perfect reflectivity limit. For $\tau_1, \tau_2 \to \tau$ it reads

\[
\frac{1}{6} \dddot{v}(\tau) - \frac{1}{6} \dddot{u}(\tau).
\]

(60)
An equivalent way of writing $T_{I,II}^R$ is to eliminate the double derivatives $\partial_{\tau_1} \partial_{\tau_2}$ by integrations by parts. Then coincidence limits involve only the first derivatives of $u, v$ in $T_{I}^R$, and the first two derivatives of $u$ in $T_{II}^R$. The same applies for the resulting boundary terms at $\tau_u$. The conclusion is that a $C^2$ trajectory suffices to assure the continuity of the flux. Note that this is not the case for perfect mirrors.

With the above choice, discontinuities in the acceleration manifest in $T_{II}^R$ in the boundary term at $\tau_u$. Consider a trajectory with acceleration continuous everywhere excepting the moment $\tau_0$, where

$$\ddot{u}(\tau_0+) - \ddot{u}(\tau_0-) = \Delta \dot{u}_0 \neq 0, \quad \tau_{0\pm} = \tau_0 + 0_\pm. \quad (61)$$

One finds this entails the flux discontinuity

$$\langle T_{uu}^R(u(\tau_0+), v) \rangle_{\text{ren}} - \langle T_{uu}^R(u(\tau_0-), v) \rangle_{\text{ren}} = \frac{a \cdot \Delta \dot{u}_0}{8\pi \dot{u}^3(\tau_0)}. \quad (62)$$

As $a$ approaches infinity, the quantity above is progressively canceled by the integration terms in $T_{II}^R$. In the limit, however, the discontinuity survives due to the $T_I^R$ contribution (when it assumes a distributional form cf. (60)).

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[28] See H. M. Franca, G. C. Marques and A. J. da Silva, Nuovo Cim. 48A, 65 (1978) and references therein. Proofs generally rely here too on the analysis of a linearized equation of motion.