Core-collapse supernova simulations in one and two dimensions: comparison of codes and approximations

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ABSTRACT

We present spherically symmetric (1D) and axisymmetric (2D) supernova simulations for a convection-dominated $9 M_{\odot}$ and a $20 M_{\odot}$ progenitor that develops violent activity by the standing-accretion-shock instability (SASI). We compare in detail the AENUS-ALCAR code, which uses fully multidimensional two-moment neutrino transport with an M1 closure, with a ray-by-ray-plus (RbR+) version of this code and with the PROMETHEUS-VERTEX code that employs RbR+ two-moment transport with a Boltzmann closure. Besides testing consequences of ignored non-radial neutrino-flux components in the RbR+ approximation, we also discuss the influence of various transport ingredients applied or not applied in recent literature, namely simplified neutrino-pair processes, neutrino-electron scattering, velocity-dependent and gravitational-redshift terms, and strangeness and many-body corrections for neutrino-nucleon scattering. ALCAR and VERTEX show excellent agreement in 1D and 2D despite a slightly but systematically smaller radius ($\sim 1$ km) and stronger convection of the proto-neutron star with ALCAR. As found previously, the RbR+ approximation is conducive to explosions, but much less severely in the convection-dominated $9 M_{\odot}$ case than in the marginally exploding $20 M_{\odot}$ model, where the onset time of explosion also exhibits big stochastic variations, and the RbR+ approximation has no distinctly stronger supportive effect than simplified pair processes or strangeness and many-body corrections. Neglecting neutrino-electron scattering has clearly unfavorable effects for explosions, while ignoring velocity and gravitational-redshift effects can both promote or delay the explosion. The ratio of advection timescale to neutrino-heating timescale in 1D simulations is a sensitive indicator of the influence of physics ingredients on explosions also in multidimensional simulations.

Key words:
Radice et al. (2017), the dependence on the progenitor (e.g. Sukhbold et al. (2016)) and its pre-collapse asymmetries (e.g. Müller et al. 2017), the quest for a universal explosion criterion (e.g. Ertl et al. 2016), the information contained in the emitted neutrinos (e.g. Tamborra et al. 2017), gravitational waves (e.g. Richers et al. 2017a), and remnant structure (e.g. Ono et al. 2013), or the role of rotation and magnetic fields (e.g. Obergaulinger & Aloy 2017).

All these research lines depend in some way or another on results obtained from time-dependent models of the supernova core, which are provided by multidimensional (neutrino-) hydrodynamics simulations. One of the major obstacles for detailed CCSN models is the neutrino transport. Ideally, the latter requires to solve along with the three-dimensional hydrodynamics equations a Boltzmann equation depending on six phase-space coordinates. However, since this is not possible with present supercomputer capabilities, at least not for reasonable resolution and number of time steps, various schemes and approximations have been proposed in the past, which differ considerably concerning their computational efficiency and accuracy.

The computationally most efficient methods of handling neutrino processes are light-bulb (e.g. Hanke et al. 2012; Couch & Ott 2015) and leakage (Ruffert et al. 1996; O’Connor & Ott 2010; Perego et al. 2016) schemes, which add almost no cost to the hydrodynamics equations since the neutrino source terms entering the latter are estimated based on the hydro-/thermodynamic information only. More involved schemes, such as the fast multigroup transport (Müller & Janka 2015) or the one designed by Scheck et al. (2008), solve the conservation equation for neutrino energy assuming a pre-defined profile for the ratio of the flux density to energy density. Another scheme with relatively high computational efficiency is the isotropic diffusion source approximation (IDSA; Liebendorfer et al. 2009; Takiwaki et al. 2014; Siwa et al. 2016) that decomposes the neutrino distribution into a trapped and a free-streaming component, which are separately evolved using a parabolic diffusion equation and an elliptic equation, respectively. One of the most often used schemes, also outside of CCSN theory, is flux-limited diffusion (FLD; Levermore & Pomraning 1981; Dessart et al. 2006; Bruenn et al. 2016; Dolence et al. 2015), which evolves the energy density, $E$, of neutrinos assuming the flux density, $F$, to be given locally by a generalized (“flux-limited”) diffusion law that limits the flux to remain causal, $F \leq cE$ (c being the speed of light).

More recently, the so-called M1 scheme (Minerbo 1978; Levermore 1984; Pons et al. 2000; Audit et al. 2002; Shibata et al. 2011) was implemented in a number of neutrino-transport codes (O’Connor 2015; Foucart et al. 2015; Sekiguchi et al. 2015; Just et al. 2015; Kurida et al. 2016; Skinner et al. 2016). Being closely related to FLD, M1 evolves additionally to the latter the flux densities. The resulting two-moment system — energy and flux density are the 0th- and 1st-order angular moments of the specific neutrino intensity — are augmented with a local closure relation expressing the 2nd-order moments (i.e. the neutrino pressure tensor) as a function of the evolved moments. The M1 scheme comes with the convenient property of being hyperbolic, which, together with the fact that fluid velocities are high in CCSNe, allows to advance the equations using explicit time stepping. This, in turn, makes it computationally feasible to solve the unconstrained, fully multidimensional transport equations.

Finally, in contrast to the aforementioned approximate schemes, more involved but also considerably more expensive schemes, so-called Boltzmann solvers, attempt to resolve the full angular dependence of the radiation field. This can be achieved, e.g. by employing tangent ray (Burrows et al. 2000; Rampp & Janka 2002; Buras et al. 2006), discrete ordinate (Mezzacappa & Bruenn 1993; Yamada et al. 1999; Liebendorfer et al. 2004; Nagakura et al. 2018), or Monte Carlo methods (Janka & Hillebrandt 1989; Richers et al. 2017b; Foucart 2018) to evolve the Boltzmann equation either directly or coupled to a two-moment system.

One measure to make expensive solvers more affordable in axisymmetric (2D; e.g. Buras et al. 2006; Müller et al. 2012a; Bruenn et al. 2016; Nakamura et al. 2015) and three-dimensional (3D; e.g. Takiwaki et al. 2014; Nelson et al. 2015a; Lentz et al. 2015; Müller et al. 2017; Wongwathanarat et al. 2017) simulations is to employ the ray-by-ray (+plus) approximation (RbR+), which neglects the evolution of the non-radial flux components and therefore allows to evolve basically one-dimensional (1D) transport problems on each angular bin along the radial directions in a quasi-decoupled manner. This leads to a major boost of computational performance, particularly for a scheme where the time integration requires the inversion of matrices spanned over the entire grid, because then the total number of operations is reduced by much more than a factor of three and the code can be efficiently parallelized. The ray-by-ray approach is naturally motivated by the rather spherical geometry of the SN core and resulting subdominance of non-radial neutrino fluxes, and by the notion that local anisotropies enhanced by RbR+ could average out and thus only leave a small imprint on the angle- and time-averaged dynamics (Buras et al. 2006). On the other hand, the suspicion was raised (Dolence et al. 2015; Sumiyoshi et al. 2015) that the RbR+ approximation could enhance the feedback between the fluid flow and neutrino radiation, which could induce explosions more readily. However, so far the RbR+ approximation was investigated self-consistently, i.e. by comparing time-dependent simulations with and without using RbR+, only using one code (Skinner et al. 2016) and more detailed exploration is warranted.

In addition to the large number of available numerical approximations of the neutrino transport, probably an even larger number of (micro-)physics ingredients exists in various degrees of sophistication. For instance, while some studies use a large set of state-of-the-art neutrino interactions (e.g. Nelson et al. 2015a; Lentz et al. 2015; Ko klake et al. 2018), many other studies rely on a smaller and more basic set of interactions (often adopted from Bruenn 1985) and neglect or simplify numerically cumbersome reactions that couple different neutrino energies, such as inelastic scattering of neutrinos off electrons and positrons as well as pair processes (e.g. electron-positron annihilation and nucleon-nucleon bremsstrahlung). Similarly, frame-dependent effects such as Doppler- and gravitational energy-shifts are sometimes neglected for numerical convenience by dropping all velocity-dependent terms and energy derivatives in the evolution equations. Other features, such as strangeness (Horowitz 2002) or many-body (Horowitz et al. 2017) corrections to the neutral-current cross sections for...
neutrino-nucleon scattering can be implemented rather easily and have recently found their way into a number of simulations (e.g. Melson et al. 2015a; Bollig et al. 2017; Buras et al. 2018; Kotake et al. 2018).

With the growing number of numerical models, also the demand grows to perform systematic comparisons between codes, methods and approximations. Method comparisons, such as the ones conducted by Liebendörfer et al. (2005), Lenz et al. (2012b), Müller et al. (2012a), Richers et al. (2017b), might not only help improving the employed algorithms on all sides, e.g. by unmasking previously unknown deficiencies. They may also prove useful to other groups in locating the reasons why their codes give different results. Moreover, they may provide instrumental help in developing and gauging code extensions or new codes. Unfortunately, going beyond spherically symmetric models when comparing time-dependent CCSN simulations is significantly more challenging because of the appearance of (numerical and physical) perturbations, fluid instabilities, turbulence, and stochasticity. The last mentioned issue, the importance of physical perturbations, fluid instabilities, turbulence, and challenging because of the appearance of (numerical and physical) perturbations, fluid instabilities, turbulence, and stochasticity. The last mentioned issue, the importance of physical perturbations, fluid instabilities, turbulence, and stochasticity. The last mentioned issue, the importance of physical perturbations, fluid instabilities, turbulence, and stochasticity.

In the following we outline the main features of the employed simulation tools, Alcar and VERTEX. For in-depth descriptions of these codes we refer the reader to Just et al. (2015) as well as Rampp & Janka (2002) and Buras et al. (2006), respectively.

Both codes employ a Godunov-type finite-volume scheme in spherical polar coordinates to solve the equations of Newtonian hydrodynamics with an effective general relativistic gravitational potential (Marek et al. 2006). The equations read (with vector indices running over radial, r, polar, θ, and azimuthal, φ, coordinate, and η0 = 0):

\[ \partial_t \rho + \nabla_j (\rho v^j) = 0, \]
\[ \partial_t (\rho v^j) + \nabla_j (\rho v^j v^l + P_{\alpha l}) = -\rho v^j \Phi + Q^M_{\alpha l}, \]
\[ \partial_t \epsilon + \nabla_j (\epsilon v^j + v^l P_{\alpha l}) = -\rho v^j \Phi + Q_E + v_j Q^M_{\alpha l}, \]

where \( \rho, v^l, \epsilon, \Phi, \) and \( \Phi \) are the baryon density, fluid velocity, electron fraction, total (kinetic plus internal) gas energy density, gas pressure, and gravitational potential, respectively, and the neutrino-related source terms are given by (with neutrino-energy coordinate \( \epsilon \) and baryonic mass constant \( m_B \))

\[ Q_N = -\alpha m_B \int_0^\infty (S^{(0)}_{\nu^0} - S^{(0)}_{\bar{\nu}^0}) \frac{d\epsilon}{\epsilon}, \]
\[ Q^\ell_M = -\frac{\alpha}{c^2} \sum_{\nu} \int_0^\infty S^{(1,\ell)}_\nu d\epsilon, \]
\[ Q_E = -\alpha \sum_{\nu} \int_0^\infty S^{(0)}_\nu d\epsilon, \]

in terms of the lapse function \( \alpha \) and the 0th- and 1st-order angular moments of the collision integral measured in the comoving (i.e. fluid rest) frame, \( S^{(0)}_{\nu^0} \) and \( S^{(1,\ell)}_\nu \), respectively, where the index \( \nu \) runs over all six neutrino species.

Both \( \Phi \) and \( \alpha \) are computed from radial profiles of angle averaged fluid- and neutrino-related quantities using the prescriptions of Marek et al. (2006) (for the case “A” potential) and Rampp & Janka (2002), respectively. We do not include spherical harmonics terms higher than the multipole in the gravitational potential, i.e. gravity is considered to be spherically symmetric.

The hydrodynamics module coupled to Alcar, Aenus (Obergaulinger 2008), offers several choices for spatial reconstruction methods, Riemann solvers, and time integration schemes. In this study, we employ spatial reconstruction as adapted from the piecewise-parabolic method (PPM; Colella & Woodward 1984) in the version by Mignone (2014) that retains high-order accuracy near the coordinate center and polar axis the HLLC Riemann solver everywhere except

1 More specifically, we use the cell-centered version “PPM5” of Mignone (2014) to reconstruct the primitive variables and employ flattening near strong shocks as in Colella & Woodward (1984) but no steepening near contact discontinuities. We do not perform characteristic tracing as in the original PPM method.
except near coordinate-aligned shocks where we switch to the HLLE solver (e.g. [Toro 1997]), and a dimensionally unsplit 2nd-order Runge-Kutta time-integration scheme.

The neutrino transport in both codes is based on the multi-group evolution of the specific (i.e. per unit of neutrino energy) energy density, $E$, and specific flux density, $F^\alpha$, both measured in the comoving frame of the fluid, for the three species electron-neutrinos $\nu_e$, electron-antineutrinos $\bar{\nu}_e$, as well as $\nu_x$, representative of the four remaining heavy-lepton neutrinos. The latter is governed by (suppressing indices $i$):

$$\partial_t E + \nabla_j \left( \alpha F^j + v^i E \right) + P^{ij} \nabla_i v_j + F^i \nabla_i \alpha$$

$$- \partial_t \left[ \frac{\varepsilon}{2} (P^{ij} \nabla_i \nu_j + F^i \nabla_i \alpha) \right] = \alpha S^{(0)}, \quad (3a)$$

$$\partial_t F^i + \nabla_j \left( \alpha E^2 P^{ij} + v^j F^i \right) + F^i \nabla_j v^j + c^2 E \nabla^i \alpha$$

$$- \partial_t \left[ \frac{\varepsilon}{2} (Q^{ijk} \nabla_j \nu_k + c^2 P^{ij} \nabla_j \alpha) \right] = \alpha S^{(1)}, \quad (3b)$$

where $P^{ij}$, $Q^{ijk}$ are the 2nd- and 3rd-order angular moment tensors, respectively. Compared to the purely Newtonian counterparts, Eqs. (3) contain corrections for general relativistic results.

In ALCAR, the higher-order moments $P^{ij}$ and $Q^{ijk}$ needed to close the set of moment equations are expressed as functions of the moments $E, F^\alpha$ by

$$\frac{P^{ij}}{E} = \frac{1 - \chi}{2} \delta^{ij} + 3 \chi - 1 \frac{n_\nu n_e}{n_\nu}, \quad (4a)$$

$$\frac{Q^{ijk}}{E} = \frac{f}{2} \left( n_\nu n_\nu \delta^{ij} + n_\nu n_\nu \delta^{ik} + n_\nu n_\nu \delta^{jk} + 5q - 3f \right) \frac{n_e n_e n_\nu}{n_\nu n_\nu n_\nu}, \quad (4b)$$

where $f \equiv |F|/|E|$, $n_\nu \equiv F^\nu/|F|$, and the quantities $\chi$ and $q$ are determined by the chosen one-dimensional closure. For the latter, we use the Mino-ber close-up and the explicit expressions for $\chi$ and $q$, respectively. The resulting hyperbolic system of equations is solved in close analogy to the hydrodynamics equations, namely on the same grid and with the same reconstruction scheme, and an HLL Riemann solver. The time integration of the potentially stiff interaction source terms, however, is done in a mixed explicit-implicit manner in order to ensure numerical stability (see Appendix [A] for details).

VERTEX solves the same set of equations, Eqs. (1) and (2), for hydrodynamics and neutrino transport, respectively, except for the following differences: VERTEX obtains the higher-order moments using a variable-Eddington-factor approach, i.e. by solving a (slightly simplified) Boltzmann equation in addition to the moment equations. This approach is comparable in accuracy to solving the Boltzmann equation directly for a spherically symmetric stellar background and is then constrained to be used in the RbR+ mode. The latter assumes that the radiation field is axysymmetric around each radial ray, leading to vanishing non-radial components of the flux density vector, $F^\alpha$, while lateral advection of neutrinos by the fluid as well as lateral neutrino-pressure forces are still taken into account (see Buras et al. [2006] for details). VERTEX solves Eqs. (3) using finite-difference methods and fully implicit time integration.

For the hydrodynamics part the well-known PROMETHEUS code (Pryzex et al. [1989]) is applied, which employs the original PPM method (Colella & Woodward [1984]) to integrate the hydrodynamics equations with 2nd-order Strang-type dimensional splitting.

### 2.2 Model setup and neutrino interaction channels

We consider two stellar progenitor models in this study, one with rather high zero-age main sequence (ZAMS) mass of $20 M_\odot$, model s20 (Woosley & Heger [2007]), and one with lower ZAMS mass of $9 M_\odot$ (model 9.0A of Woosley & Heger [2015]), which we denote here as model s9. The simulations are initialized using the density, temperature, and electron fraction from the progenitor data. The 2D models are set up by mapping from 1D simulations at around $15 - 20$ ms after bounce and adding random perturbations $\delta \rho \in [-10^{-3}, 10^{-3}] \rho$ to the density, $\rho$, in each grid cell. In doing so, we ignore recent findings (e.g. [Couch et al. 2015]) that rather strong perturbations with low angular order can be present before collapse, which may help initiating the shock runaway.

We employ the equation of state (EOS) “SFHo” of Steiner et al. [2013]. Since this EOS is constructed assuming nuclear statistical equilibrium (NSE), ideally we should replace it by a composition-dependent EOS linked to a nuclear reaction network once the temperature drops below $\sim 5$ GK. However, for the present study we avoid this additional level of complexity and employ the SFHo-table everywhere. Any neutrino interactions with light nuclear clusters (i.e. elements with mass numbers $A = 2 - 4$ except $\alpha$ particles) appearing in the SFHo EOS are ignored. The original SFHo table only allows electron fractions up to 0.6. This turned out to be insufficient for a subset of models in which transient, proton-rich bubbles arise in the gain region. To this end, we extended the original SFHo table in the subnuclear domain towards more proton-rich conditions using a 23-species NSE solver.

For the ALCAR simulations, the radial grid remains fixed during the simulations. The standard resolution has $N_r = 640$ radial zones with a width of $\Delta r = 300$ m below $r = 30$ km and growing by a constant factor per cell up to $\Delta r = 2$ km at $r = 300$ km and finally increasing by $\approx 2$% per cell up to the outer grid boundary at $r = 10^9$ cm. In the
Vertex simulations the radial grid at the time of bounce consists of 400 zones, the distribution of which results from a quasi-Lagrangian treatment of the collapse, while during the post-bounce evolution the radial grid remains fixed (i.e. Eulerian) but is manually refined multiple times around the neutrinosphere by using a density-based criterion. Typically, a final number of radial zones of \( N_r \sim 570 - 630 \) is reached at the end of the simulations. The angular grids are always uniform in \( \theta \), with the standard number of zones being \( N_\theta = 240 \) and 160 for Alcar and Vertex, respectively. The neutrino energy space is discretized using 15 energy groups distributed nearly logarithmically between 0 and 400 MeV (Alcar) or 380 MeV (Vertex).

With a spherical polar coordinate mesh the Courant time step would become prohibitively small near the coordinate center. We mitigate this problem by using a spherically symmetric core up to a certain radius. In Alcar, this radius decreases linearly in time from 10 km at 20 ms post bounce down to 7 km at about 0.5 s post bounce, whereafter it remains constant. In Vertex, the radius of the 1D core is 1.6 km at all times.

Our reference models include the following set of neutrino reactions (and corresponding reverse reactions):

- Electrons/positron captures by protons/neutrons following [Bruenn 1985] including the corrections by [Horowitz 2002] for weak magnetism and nucleon recoil.
- Isoenergetic scattering of all neutrino types on neutrons and protons following [Bruenn 1985] and likewise augmented with corrections for weak magnetism and nucleon recoil [Horowitz 2002].
- Electron captures by heavy nuclei as in [Bruenn 1985].
- Coherent scattering of all neutrino types off heavy nuclei including corrections due to the nuclear form factor and ion-ion correlations (cf. [Bruenn & Mezzacappa 1997; Horowitz 1997]).
- Inelastic scattering of all neutrino types off electrons and positrons [Yueh & Buchler 1977; Bruenn 1985; Cer- nohorsky 1994].
- Pair processes, namely pair production and annihilation of electron-type (i.e. \( \nu_e \) and \( \bar{\nu}_e \)) as well as heavy-lepton neutrinos via nucleon-nucleon bremsstrahlung and electron-positron annihilation and pair production as in [Hannestad & Raffelt 1998] and [Pons et al. 1998], respectively.

Motivated by a number of recent studies [Melson et al. 2015a; Bollig et al. 2017; Radice et al. 2017; O’Connor et al. 2017] we additionally include the following corrections in some models:

- Strangeness correction to the axial-vector coupling in neutral-current nucleon-nucleon interactions with \( g_A^s = -0.1 \) [Horowitz 2002, Horowitz et al. 2017].
- Axial response corrections to the neutrino-nucleon scattering cross section due to many-body effects as in [Horowitz et al. 2017] using the fit formula for \( S_A \).

For the implementation of the rates in Alcar we followed as closely as possible that of Vertex, which is described in the Appendix of [Rampp & Janka 2002]. However, all neutrino rates (as well as all other physics modules) in Alcar have been coded from scratch, i.e. no routines have been copied from Vertex. The numerical treatment of the source terms in Alcar differs from that of Vertex in that it, first, allows for three instead of only one flux-vector component, and second, is both explicit and implicit in time depending on the type of interaction and dynamic conditions, whereas it is fully implicit in Vertex. We provide more details on the implementation of the source terms in Alcar in Appendix A.

We note that the set of neutrino interactions chosen here is not as advanced as the one usually employed in Vertex (e.g. [Buras et al. 2006]). However, we chose this rather basic set to facilitate the comparison between the two codes and to allow other groups to compare with our results on the basis of a widely available repository of input physics.

### 2.3 Investigated models

We list the investigated models and their properties in Table 4. The axisymmetric reference models run with Alcar are s20-ref and s9-ref for the s20 and s9 progenitors, respectively, and evolve the fully multidimensional (i.e. not RBR-constrained) transport equations, Eqs. (3), using the aforementioned neutrino interactions (except strangeness and many-body corrections). Models with “str” and “mb” in their names additionally include the strangeness and many-body corrections, respectively. Models with “rbr” in their names adopt the RBR+ approximation in Alcar exactly as described in [Buras et al. 2006], namely by setting the lateral flux densities \( F_R = F_S = 0 \) (though \( F_N = 0 \) is fulfilled anyway in our non-rotating models) and setting the lateral source term for the evolution of \( \rho \nu^0 \), \( Q_{\nu}^{\theta} \), to

\[
Q_{\nu}^{\theta, \text{RBR}} = -\frac{\alpha}{3} \sum_{\nu} \int \frac{1}{r \sin \theta} \frac{\partial E_{\nu}}{\partial \theta} d\epsilon
\]

for trapping densities, \( \rho > 10^{-2} \text{g cm}^{-3} \), and to zero for lower densities. In this way, lateral advection and compression of energy and radial flux remain included in the transport equations as well as lateral neutrino-pressure forces in the gas-momentum equations.

The remaining modeling variations are: Turning off inelastic neutrino-electron scattering (“noenes”), ignoring velocity-dependent and gravitational redshift terms (“novel”) by setting \( v = 0 \) and \( \partial_\epsilon = 0 \) in Eqs. [4], and using the simplified description of (\( e^\pm \)-annihilation and bremsstrahlung) pair processes (“pp”) as suggested in [O’Connor 2015]. The pair-process simplification consists of, first, ignoring all pair processes for electron-type neutrinos entirely, and second, assuming that respective annihilation partners for pair-annihilation of \( \nu_e \) are in isotropic, local thermodynamic equilibrium (LTE) when computing the pair-process interaction kernel. The second assumption reduces the source terms for \( \nu_e \) pair processes to be formally equivalent to source terms for emission/absorption processes (see [O’Connor 2015] for more details). All three aforementioned simplifications are particularly appealing for energy-dependent transport schemes, because each of them entails dropping numerically complicated energy-bin coupling terms. We include these simplifications here, because

4 We note that this also includes the diffusive component of the HLL fluxes through the cell interfaces, which can be non-zero even if the cell-centered fluxes vanish.
they have rarely been tested so far in multidimensional simulations that differ only in the pattern (but not amplitude) of any of the three aforementioned simplifications.

The two Alcar models using RbR+ with and without neutrino-electron scattering (s20-rbr and s20-rbr-nones, respectively) are compared with the two Vertex models s20VX and s20VX-nones that contain exactly the same corresponding input physics.

All two-dimensional models are mapped from the same 1D reference model corresponding to the code and progenitor (e.g. s20-pp-str-mb-novel is mapped from s20-ref-1D) – hence the collapse and bounce are always simulated without any of the three aforementioned simplifications.

In order to obtain a rough idea about the influence of stochasticity, we perform for some models additional simulations that differ only in the pattern (but not amplitude) of initial random density perturbations. These simulations are
labeled by numbers at the end of the model names. When discussing these models below we will suppress these numbers if the point of concern holds for all simulations of the given model independently of the initial perturbation pattern.

All simulations are stopped either once shock expansion sets in or after 1s (0.9 s) of post-bounce evolution in ALCAR (VERTEX), except for model s20-pp that was continued beyond 1s because of optimistic heating conditions at post-bounce time $t_{pb} = 1$ s.

3 RESULTS: 1D MODELS

We begin by comparing spherically symmetric models. Although rather unspectacular, 1D models offer the most straightforward and computationally economic way to identify the basic impact of modeling variations, and they may reveal features that could remain obscured by stochasticity in multidimensional models. Since for non-exploding 1D models the progenitor dependence is less relevant, we only discuss the s20 models in this section but not the s9 models.

3.1 Comparison with Vertex

The phases of collapse and early post-bounce evolution are simulated for all 1D and 2D models with the same input physics, thus we only need to compare these phases for models s20-ref-1D and s20VX-1D. The main features of the collapse are displayed in Fig. 1, which shows the electron-fraction, $Y_e$, lepton fraction, $Y_l$, and entropy per baryon, $s$, in the stellar center as functions of the time-evolving density, $\rho$, at the same location, as well as profiles of $Y_e$ and $s$ along the enclosed mass coordinate $m_{enc}(r) = 4\pi \int_0^r \rho r^2 d\tilde{r}$. The core deletonization and heating, aided by neutrino-electron scattering, proceeds up to the trapping density of $\rho_c \sim 2 \times 10^{12}$ g cm$^{-3}$, above which $s$ and $Y_l$ remain essentially constant. The central values as well as the profiles agree well for both codes. The bounce commences $\approx 296$ ms (298 ms) after initialzation for the ALCAR (VERTEX) model at a mass coordinate of $m_{enc} \approx 0.57 M_\odot$ (0.56 $M_\odot$). In Fig. 2 we show profiles of various quantities as functions of the enclosed mass (for times at and short after bounce) and radius (for later times). As can be seen in Fig. 2, the profiles of hydrodynamic quantities agree almost perfectly between ALCAR and VERTEX within the first tens of milliseconds after bounce. At this early stage the only noteworthy differences between both codes are observed in the neutrino-related quantities, e.g. in the luminosity,

$$L \equiv \rho^2 \int_{\Omega_e} F_e d\Omega$$

and energy-averaged Eddington factor,

$$\langle \chi \rangle \equiv \frac{\int_{\Omega_e} \chi E^d d\Omega}{\int_{\Omega_e} E^d d\Omega}.$$  

6 By definition, the bounce happens at the time when the shock entropy reaches $3 k_B$/baryon.

a few milliseconds after bounce right when the burst of electron neutrinos is released by the outward traveling shock wave. The aforementioned differences are related to what is seen in the small inset of the top left panel in Fig. 1, namely that the $\nu_e$ burst in ALCAR has $\approx 25\%$ higher peak luminosity ($5.32 \times 10^{53}$ erg s$^{-1}$ vs. $4.24 \times 10^{53}$ erg s$^{-1}$) and contains $\approx 10\%$ more energy ($2.79 \times 10^{53}$ erg vs. $2.55 \times 10^{53}$ erg as integrated between $t_{pb} = 0$ ms and 10 ms) than the burst in VERTEX. This difference goes hand in hand with a transiently lower deletonization in the VERTEX run (see Fig. 2, left column, third panel at $t_{pb} = 3$ ms). The reason remains unknown; it might be related to the approximate two-moment closure for the transport employed by ALCAR. The impact of the more energetic burst on the subsequent evolution is difficult to quantify, but it must be relatively small given that the agreement between both codes remains very good at later times. We also point out that VERTEX exhibits a somewhat stronger diffusive broadening of the outward propagating $\nu_e$ burst compared to ALCAR. This may be connected to a finer radial grid, smaller time step, and higher-order spatial reconstruction scheme used for integrating the transport equations in ALCAR. This difference in the translatory behavior of the $\nu_e$ burst at large distances has no relevance for the model evolution.

Before comparing the post-bounce dynamics below the shock we take a look at the mass accretion rates measured at $r = 500$ km in Fig. 3. The basically perfect match between ALCAR and VERTEX provides confidence that the numerical treatment of gravity and the tabulated EOS is handled consistently in both codes and, equally important, ensures that any differences of dynamics seen below the shock are not the result of different conditions above the shock.

In Fig. 4 we show important global quantities as functions of time for the four ALCAR and VERTEX models with and without $\nu - e^\pm$ scattering. The trajectory of the shock $r_s$ given in the top right panel of Fig. 4 reveals that prompt shock expansion occurs for about 70-80 ms, whereafter the shock stalls and recedes. Not surprisingly, in the present 1D models the shock retraction takes place smoothly and monotonically, except for a bump at $t_{pb} \sim 0.22 – 0.24 s$ that is related to the drop in the mass accretion rate (cf. Fig. 3) due to the infalling Si/Si-O interface.

The neutrino luminosities, $L$, mean energy

$$\langle \epsilon \rangle \equiv \frac{\int_{\Omega_e} E^d d\Omega}{\int_{\Omega_e} E^d d\Omega},$$

both plotted in Fig. 4, at $r = 500$ km in the lab-frame as seen by an observer at infinity), the root-mean-squared (rms) en-

7 Since the shock is not a perfect discontinuity but is distributed over several radial zones, we compute the shock radius as the arithmetic average of the radii bracketing the extended, numerical shock.

8 We note that the mean energies computed as in Eq. (8) are at $r = 500$ km essentially identical to the mean energies of the neutrino flux (obtained through replacing in Eq. (8) $E$ by $F^*$ and $E/\epsilon$ by $F^*/\epsilon$) because $F^*/(cE)$ is very close to unity at this large radius.
we observe a slightly smaller radius, \( r_{\text{NS}} \), and higher temperature, \( T_{\text{NS}} \), of the PNS surface (defined by the location where \( \rho = 10^{12} \text{ g/cm}^3 \); cf. Fig. 4), higher mean energies, \( \langle e \rangle \), for all neutrino species, a less extended shock radius, \( r_s \), and gain radius, \( r_g \), as well as a higher mass-specific neutrino heating rate, \( Q_{\text{heat}}/M_g \), for the ALCAR models compared to the VERTEX models. Remarkably, however, the ratio of the characteristic timescales, \( \tau_{\text{adv}}/\tau_{\text{heat}} \) (shown in the bottom left panel in Fig. 4), which is a more meaningful measure of the proximity to explosion than the aforementioned quantities (e.g. Janka 2001), does not exhibit a clear trend in either direction. This means that the impact of a more compact configuration in ALCAR, which in the first place is detrimental to an explosive runaway because the gain layer and shock sit deeper in the gravitational well, is approximately compensated by higher luminosities and therefore more powerful neutrino heating.

3.2 Electron scattering

Figure 4 also shows the corresponding 1D results obtained when switching off neutrino-electron scattering at about 20 ms after bounce for both ALCAR and VERTEX. In the present 1D models the impact on many quantities is only on the percent level. Nevertheless, the less efficient energy deposition without neutrino-electron scattering (cf. \( \eta_{\text{heat}} \) in Fig. 3) is sufficient to reduce the shock radius, \( r_s \), by a few km and to lead to a more sizable reduction of the advection timescale, \( \tau_{\text{adv}} \), and of the timescale ratio, \( \tau_{\text{adv}}/\tau_{\text{heat}} \), by 10–20%. The most notable (but for the dynamics rather irrelevant) difference concerning the neutrino emission appears for the mean energies, \( \langle e \rangle \), of the heavy-lepton neutrinos, which are several MeV higher without the efficient down-scattering process of neutrino-electron scattering (Fig. 3 left column, second panel from top).

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![Figure 1](https://example.com/figure1.png)

**Figure 1.** Comparison between ALCAR and VERTEX models of properties characterizing the collapse. The left panel shows the central electron (\( Y_e \)) and lepton (\( Y_l \)) fraction and central entropy per baryon (\( s \)) as functions of central density. The right panel shows profiles of \( Y_e \) and \( s \) with respect to the enclosed mass for times at which the central density reaches the indicated values.
3.3 Pair processes

Figure 5 summarizes the evolution of the same quantities as in Fig. 4 but for the remaining one-dimensional ALCAR models using the s20 progenitor.

We start by considering model s20-pp-1D that incorporates the simplified pair processes treatment, i.e. for electron-type neutrinos all pair processes are neglected and for pair-annihilation of $\nu_x$ neutrinos the corresponding annihilation partners are assumed to be in isotropic LTE. The top left plot in Fig. 5 shows that the luminosities of heavy-lepton neutrinos are reduced compared to the reference case by $\sim 10\%$ during the first $\sim 150$ ms of post-bounce evolution. The main reason for this reduction is most likely the approximate assumption that $\nu_x$ pair-annihilation targets are isotropically distributed in momentum space, whereas they actually become more and more forward peaked with increasing radius. This boosts the $\nu_x$ pair-annihilation rates while leaving the rates of the inverse (i.e. $\nu_x$ pair-production) reactions unchanged. We note that an impact of similar size has been found also by O'Connor (2015) for 1D models with electron-positron annihilation but without bremsstrahlung. Keeping in mind that four times the individual $\nu_x$ luminosity enters the total energy loss rate of the PNS, this reduction of neutron-star cooling during $t_{pb} \lesssim 150$ ms probably explains the observed increase of the neutron-star radius by $\sim 1 - 2$ km and of the shock radius by $\sim 3 - 10$ km compared to the reference model.

The luminosities and spectral properties of emitted electron-type neutrinos remain fairly unaffected by the pair-
process simplification. This helps understanding the similarly weak sensitivity of the specific heating rate, which can approximately be written as

\[
\frac{Q_{\text{heat}}}{M_{\text{g}}} \propto \frac{L_{\epsilon,\text{rms}}^2}{r_{\text{g}}^2}
\]

(13)

(where \(L_{\epsilon,\text{rms}} \equiv L_{\nu_e,\epsilon_{\text{rms},\nu_e}} + L_{\nu_\mu,\epsilon_{\text{rms},\nu_\mu}}\)), and the heating timescale, which roughly scales like

\[
\tau_{\text{heat}} \propto \frac{E_{\text{tot},\nu}}{M_{\text{g}}} \frac{r_{\text{g}}^2}{L_{\epsilon,\text{rms}}^2} \propto \frac{G M_{\text{NS}} r_{\text{g}}}{L_{\epsilon,\text{rms}}^2}
\]

(14)

(where we assumed the specific total energy of the gain region to be approximately proportional to the gravitational energy at the gain radius). The heating rate, heating efficiency and the advection timescale, on the other hand, show stronger deviations from the reference model, mainly because they are directly proportional to the mass in the gain layer, which itself is enhanced by \(\sim 10 - 20\%\) compared to the reference model. As a result, \(\tau_{\text{adv}}/\tau_{\text{heat}}\), which is most relevant for the explosion behavior, is enhanced by a comparable amount during almost the entire evolution.

We note that this result is not in tension with previous studies (and with the results for models s20-str-1D and s20-mb-1D discussed below) that report more favorable runaway conditions for cases in which the PNS contraction proceeds faster (e.g. [Marek et al. 2009b], O’Connor & Couch 2018 [Melson et al. 2015b], Bolli et al. 2017). This is because in those studies the faster PNS contraction is the result of changing physics ingredients different from the ones varied here, e.g., using a softer nuclear equation of state, general relativity instead of Newtonian gravity, or reduced neutrino-scattering opacity for all neutrinos. In those cases the accelerated PNS contraction came along with more favorable neutrino emission properties (i.e. higher values of \(L_{\epsilon,\text{rms}}\)) that overcompensated for the stronger gravitational binding. In the present case, on the other hand, the deceleration of PNS contraction due to artificially amplified annihilation of \(x\) pairs is not accompanied by a large enough reduction of \(L_{\epsilon,\text{rms}}\) to cause more pessimistic runaway conditions.

3.4 Strangeness and many-body corrections

The two 1D models, s20-str-1D and s20-mb-1D, both incorporate corrections that effectively reduce the opacities for neutrino-nucleon scattering, in particular in the crucial neutrino-decoupling region, by a few percent. As a result, we observe an enhancement of the neutrino luminosities for all three species by a similar amount, which is well in agreement with previous studies that incorporated these corrections (e.g. [Melson et al. 2015b], O’Connor et al. 2017). In the present one-dimensional case, this leads to smaller shock- and PNS-radii throughout, but slightly higher mean- and rms-energies of neutrinos and therefore, in combination with the increased luminosities, to slightly stronger neutrino heating, shorter \(\tau_{\text{heat}}\) and higher \(\tau_{\text{adv}}/\tau_{\text{heat}}\), thus slightly more favorable conditions for shock revival.

3.5 Velocity-dependent and gravitational redshift terms

For model s20-novel-1D, in which frame-dependent effects such as advection and Doppler-shift as well as gravitational redshift are ignored after \(t_{\text{pb}} \approx 20\,\text{ms}\), one immediately recognizes the significantly smaller heating rate as well as specific heating rate, reduced heating efficiency, longer heating timescale, and reduced timescale ratio compared to the reference case. At first glance, this seems counterintuitive in view of the fact that according to Fig. 5 the (lab-frame) luminosities agree well and the (lab-frame) mean energies are even higher compared to the reference model. However, both quantities are not sufficiently representative of the heating conditions in the gain layer, as is revealed by Fig. 6 that depicts radial profiles of the luminosities and rms-energies measured in the comoving frame \(L_{\text{com}}\), as the frame correction is in relative terms smaller for the rms-energies, \(\epsilon_{\text{rms}}\) (cp. Figs. 3 and 6) that exhibit hardly any reduction in model s20-novel-1D compared to the reference model. The most likely explanation is that the frame correction is in relative terms smaller for the rms-energies (transforming as \(L_{\text{lab}}^2 \approx L_{\text{com}}^2 (1 + v/c)\) for forward peaked radiation) than for the luminosities (transforming as \(L_{\text{lab}}^2 \approx L_{\text{com}}^2 (1 + 2v/c)\) for forward peaked radiation), such that gravitational redshift may approximately compensate for the effect of Doppler blueshift for the rms-energies.

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10 For model s20-novel-1D the distinction between the comoving frame and lab frame is meaningless and the evolved neutrino moments are used for plots of both comoving- and lab-frame quantities.
Figure 4. Comparison of global properties as functions of time for the spherically symmetric ALICAR and VERTEX models of the s20 progenitor whose names are displayed in the panel showing $\eta_{\text{heat}}$. Left panels show from top to bottom the luminosities, $L$, and mean energies, $\langle \epsilon \rangle$, both measured in the lab-frame by an observer at infinity, the rms-energies, $\epsilon_{\text{rms}}$, measured at the gain radius in the local comoving frame, the total and specific neutrino heating rate in the gain region, $Q_{\text{heat}}$ and $Q_{\text{heat}}/M_g$, respectively, and the ratio of advection timescale to neutrino heating timescale, $\tau_{\text{adv}}/\tau_{\text{heat}}$. Right panels show from top to bottom the shock (gain) radius, $r_s$, the radius, $r_{\text{NS}}$, and temperature, $T_{\text{NS}}$, of the PNS surface where $\rho = 10^{11}$ g cm$^{-3}$, mass of the gain region, $M_g$, heating efficiency, $\eta_{\text{heat}}$, (computed using Eq. (11) with the lab-frame luminosities given in this plot), and the heating (advection) timescales, $\tau_{\text{heat}}$ ($\tau_{\text{adv}}$). Insets show enlarged regions of the same plots. Labels including numbers denote the margin in units of the current y-axis by which the curves for a given neutrino species are shifted, e.g. curves labeled $\bar{\nu}_e - 0.1$. All radii and $T_{\text{NS}}$ show angle-averaged data. The curves are smoothed using running averages of 10 ms.
4 RESULTS: 2D MODELS

After having obtained an idea of the level of agreement between the two codes ALCAR and VERTEX and the impact of our modeling variations for the spherically symmetric case, we now investigate how these dependencies translate to the 2D axisymmetric case. Additionally, we will examine the impact of using the RbR+ approximation for the ALCAR code, comment on the level of stochasticity and numerical convergence, and contrast our results for the s20 models with others found in the literature.
Before comparing the models in detail, we first summarize some features common to most models. Figure 7 depicts snapshots of the isotropic-equivalent lab-frame luminosity, $4\pi r^2 \int L_{\text{lab}}(\nu, r) d\nu$, and the specific entropy taken at three post-bounce times for two ALCAr models and one VERTEX model, and Figs. 8 and 9 provide a summary of global properties for most investigated models. By switching to 2D, we allow the fluid to develop fluid instabilities such as PNS convection, SASI, and post-shock convection driven by neutrino heating.

Proto-neutron star convection is the consequence of unstable gradients of the lepton number and specific entropy developing below the PNS surface as a result of neutrino emission. Leptons trapped in the dense core of the PNS are shuffled into the overlying layers, where they provide additional pressure support and thereby slow down the PNS contraction. Correspondingly, we observe larger PNS radii, $r_{\text{NS}}$, and lower temperatures, $T_{\text{NS}}$, as well as reduced energies of emitted neutrinos, $\langle \epsilon \rangle$ and $\epsilon_{\text{rms}}$, in 2D compared to 1D at equal times (Buras et al. 2006, Dessart et al. 2006, Müller et al. 2012a, Brüenn et al. 2016, Radice et al. 2017).

While PNS convection already starts at about 20–40 ms after bounce, the gain layer remains fairly spherically symmetric for a much longer time, namely until about $t_{\text{pb}} \sim 0.15$ s ($\sim 0.1$ s) for the s9 (s20) progenitor models, as can be recognized in Figs. 8 and 9 for the s20 (s9) models by the correspondingly late rise of the lateral (i.e. carried by non-radial motions) kinetic energy integrated over the gain layer, $E_{\text{kin,g}}$. This means that until these times the conditions for efficient growth are met neither for SASI nor for post-shock convection. Since the hydrodynamic instabilities are triggered by random perturbations, the exact time of significant departure from spherical symmetry slightly varies between the models. It may be worth pointing out that in tests with ALCAr using the s20 model we experienced quite some sensitivity of this transition time to the numerical treatment: Using a low (i.e. linear) order for spatial reconstruction of the hydro variables or employing the more diffusive HLL solver led to a significantly earlier onset of non-radial flow activity.
4.1.1 s20 progenitor models

Since SASI growth is favored by short advection timescales (Blondin & Mezzacappa 2006; Foglizzo et al. 2007; Scheck et al. 2008), $\tau_{\text{adv}}$, efficient growth sets in once the post-shock configuration has become sufficiently compact. Around $150 - 200 \text{ ms}$ after bounce the exponential growth of SASI modes with periods comparable to $\tau_{\text{adv}}$ can be seen in the dipole component of the shock surface (cf. bottom left panel of Figs. 12), which is obtained from a decomposition of the latter into Legendre polynomials (see, e.g., Summa et al. 2016 for the exact definition of the multipole coefficients). The snapshots in the top row of Fig. 7 are taken at $t_{\text{pb}} \sim 180 \text{ ms}$ right around the time when the transition from the well-ordered, linear phase to the turbulent, non-linear phase of the SASI takes place. After this transition the evolution of the post-shock layer is characterized by parasitic instabilities (Rayleigh-Taylor and Kelvin-Helmholtz) whose turbulent mass motions tap energy from the SASI. Post-shock convection tends to be suppressed for non-exploding s20 runs because of the retreating shock radius and correspondingly short advection timescales, which act against the development of a sufficiently large negative entropy gradient and correspondingly short growth timescales for convection. This statement is backed by the circumstance that the $\chi_{\text{conv}}$-parameter characterizing the growth conditions of post-shock convection (see, e.g., Foglizzo et al. 2006; Summa et al. 2016 for the definition of this quantity and details regarding its interpretation and computation) remains below the critical value of $\approx 3$ (bottom right panel of Figs. 8 and 10). Nevertheless, since the critical condition for convection, $\chi_{\text{conv}} > 3$, holds strictly only for the linear regime, the existence of secondary convective instability associated with highly non-linear SASI activity is not excluded.

4.1.2 s9 progenitor models

For the models using the s9 progenitor, which are characterized by considerably smaller mass accretion rates with respect to the s20 models (see Fig. 3) and therefore a less compact shock surface, the situation is reversed in that the dominant fluid instability is not SASI but post-shock convection. Correspondingly, we observe a transition of the $\chi_{\text{conv}}$-parameter above the critical value of $\approx 3$ at about $t_{\text{pb}} \sim 0.1 \text{ s}$ (cf. Fig. 11), whereafter $\chi_{\text{conv}}$ remains $\gtrsim 3$ during the entire simulation. This strong progenitor dependence of instability regimes, with massive (low-mass) progenitors favoring SASI (convection) has been recognized before, e.g. in Müller et al. (2012b); Fernández et al. (2014).

4.2 Comparison with Vertex

We start by comparing the two ALCAR models that incorporate the RbR$^+$ approximation with (s20-rbr) and without (s20-rbr-nones) neutrino-electron scattering to the corresponding VERTEX runs (s20VX and s20VX-nones, respectively).

First of all, as visible in Fig. 8 and Table 1 the models without neutrino-electron scattering do not explode (at least not until the end of each simulation), while the ones including neutrino-electron scattering do explode, but rather late (i.e. several hundred milliseconds after the infall of the Si/Si-O interface at $\sim 230 \text{ ms}$). The agreement of both codes in clearly showing the impact of a relatively small $O(5\%)$ variation of neutrino-interaction rates (in the present case due to neutrino-electron scattering) is thus already encouraging. Moreover, for both codes the exploding models are characterized by a significant scatter in explosion times (cf. Table 1). The three ALCAR models with the same input physics but different random initial perturbation patterns cover a large range of explosion times of $t_{\text{exp}} = 0.48 - 0.92 \text{ s}$, while the two corresponding VERTEX runs explode also within this time interval, namely at $t_{\text{exp}} = 0.53 \text{ s}$ and 0.7 s. The overview of important global properties as functions of time in Fig. 8 reveals that the differences between both codes concerning the neutrino luminosities and energies, as well as the PNS surface temperature and radius remain on the few-percent level, as found in 1D. This suggests that PNS convection is operating in both codes consistently regarding its impact on PNS contraction and neutrino emission. Nevertheless, a closer look unfolds that the $\nu_\tau$ and $\nu_e$ luminosities now show a small enhancement in VERTEX relative to ALCAR for $t_{\text{pb}} \sim 0.15 \text{ s}$ that was not seen in 1D (see Fig. 3) and may therefore be connected to some discrepancy in the PNS convection. Indeed, an inspection of the radial profiles of the lateral kinetic energy densities and the neutrino luminosities in Fig. 9 reveals that the PNS convection in ALCAR proceeds somewhat differently with higher kinetic energies. Unfortunately, despite a dedicated analysis we were unable to track down the exact reason for this discrepancy and its detailed consequences. We can, however, already clearly assess that this enhancement in convective energy is not an artifact of the RbR$^+$ approximation, because model s20-ref1, which does not employ RbR$^+$, shows a similar behavior (cf. Fig. 3 and a more detailed discussion of the RbR$^+$ approximation in Sec. 4.5). It may be that the stronger PNS convection in ALCAR is related to a subtle difference in the PNS structure, which is systematically more compact ($\sim 1 \text{ km}$ difference in $r_{\text{NS}}$) than in VERTEX models in 1D (Fig. 3) as well as 2D. This might allow PNS convection to be enhanced in ALCAR by tapping the higher gravitational binding energy of the more compact PNS.

We do not deem the disagreement in the PNS convection zone to be overly significant concerning the explosion dynamics, because the shock radius, the neutrino heating rates, characteristic timescales, as well as most remaining properties are in good agreement apart from stochastic fluctuations. Assessing in detail the heating conditions in the gain layer and the shock evolution, and ultimately finding the reason why a simulation at a certain time exhibits shock expansion or not, is difficult given the complicated temporal behavior of heating-related diagnostic quantities such as $Q_{\text{heat}}$, $\theta_{\text{heat}}$, $E_{\text{kin},g}$, and $\tau_{\text{adv}}/\tau_{\text{heat}}$. At later times, $t_{\text{pb}} \gtrsim 0.3 \text{ s}$, we observe, consistently for both codes, that the exploding models in comparison to the non-exploding models exhibit temporal variations on longer timescales and with larger amplitudes. For instance, $\tau_{\text{adv}}/\tau_{\text{heat}}$, occasionally travels up and down between $\sim 0.3 - 0.6$ on timescales of hundreds of milliseconds for the exploding models, while for the non-exploding models it remains rather close to $\sim 0.3$ with relatively short and low-amplitude oscillations. The larger amplitudes and longer timescales of temporal variations suggest
Figure 7. 2D maps of the isotropic-equivalent luminosities, $4\pi \int F_r \, d\epsilon$, measured in the local lab-frame (left side of each panel) and specific entropy, $s$ (right side of each panel), for the Alcar reference model using the s20 progenitor (s20-ref; left column), the RbR+ counterpart of the latter (s20-rbr; middle column), and the corresponding Vertex model (s20VX1; right column) at an early time during the transition from the linear to the non-linear phase of the SASI (top row), at an intermediate time right after the infall of the Si/Si-O interface (middle row), and at a late time showing the failed or successful onset of shock runaway (bottom row).
Figure 8. Same as Fig. 4 but for the axisymmetric s20 models whose names are displayed in the panel for $T_{NS}$, and additionally providing the lateral kinetic energies in the gain region, $E_{\text{lat}}$, as well as the convection parameter, $\chi_{\text{conv}}$, in the bottom row. The compilation of models compares the RbR+ cases with Alcar and Vertex to the reference 2D model s20-ref1. The curves are smoothed using running averages of 10 ms.
that the exploding models hover in a state very close to criticality before they ultimately explode. Retaining in such a state means that small perturbations may have a large dynamic impact. A natural suggestion from this is that the temporal pattern observed for the heating-related properties is shaped more by stochasticity than systematics. This conjecture is supported when comparing models that were initialized with a different pattern of density perturbations (cp. thick and thin lines of same color in Fig. [5]): $\frac{\tau_{\text{heat}}}{\tau_{\text{shock}}}$, for instance, varies over a comparable range of values but with a considerably different temporal behavior for different initial perturbations. We infer that the time of shock runaway must be similarly affected by stochasticity, which has often been neglected in a number of previous studies assuming that it would only be relevant during collapse, turn out to be a crucial ingredient for initiating shock runaway for the s20 model (in qualitative agreement with Burrows et al. 2000), and second, the use of an explicit time integrator in Alcar as opposed to an implicit one in Vertex that tends to broaden sharp features. At this point it is thus not clear up to which level the observed small-scale features in Alcar are physical or not; however, the dynamical consequences seem to be marginal given the otherwise good agreement.

### 4.3 Electron scattering, pair processes, strangeness and many-body corrections

As already pointed out in the previous section the relatively small effects associated with neutrino-electron scattering, which has often been neglected in a number of previous studies assuming that it would only be relevant during collapse, turn out to be a crucial ingredient for initiating shock runaway for the s20 model (in qualitative agreement with Burrows et al. 2018). For the s9 progenitor models (see Table 4 and Fig. 11 for an overview of global properties as functions of time) we see the same strong sensitivity, i.e. models exploding around $t_{\text{pb}} \sim 0.3 - 0.5 \text{s}$ (s9-ref, s9-rbr) are turned into non-exploding models (s9-nones, s9-rbr-nones) when ig-
Figure 10. Same as Fig. 4 but for the axisymmetric s20 models whose names are displayed in the panel for TNS, and additionally providing the lateral kinetic energies in the gain region, E^{lat,kg}_\text{gain}, as well as the convection parameter, \( \chi_{\text{conv}} \), in the bottom row. The compilation of models compares different variations of the physics input of the ALCAR simulations with the reference model s20-ref1. The curves are smoothed using running averages of 10 ms.
Figure 11. Same as Fig. 4 but for the axisymmetric s9 models whose names are displayed in the panel for $\eta_{\text{heat}}$, and additionally providing the lateral kinetic energies in the gain region, $E_{\text{lat}}$, as well as the convection parameter, $\chi_{\text{conv}}$, in the bottom row. The suite of models compares variations of the input physics to the reference cases of the $9 M_\odot$ ALCAr simulations, s9-ref1 and s9-ref2. The curves are smoothed using running averages of 10 ms.
noring neutrino-electron scattering. These results demonstrate once more how sensitive the onset of shock runaway can be to variations in the neutrino interaction rates.

In view of the substantial impact of neutrino-electron scattering it seems not too astonishing that we see a similarly strong impact of other variations of the neutrino rates considered in this study, both for the s20 (see Fig. 10) and s9 (see Fig. 11) models: The simplified treatment of pair processes (used in all models labeled by “pp” in their names), as well as the scattering-opacity reducing strangeness and many-body corrections (including “str” and “mb” in their names, respectively) are all conducive to explosions, i.e. each modification comes along with a visible boost of the timescale ratio, \( \frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \), as well as (in the case of a successful explosion) a shift towards earlier times of shock expansion. This shift is naturally larger in the s20 models, because the reference s20 model (s20-ref) lacks an explosion while the reference s9 model (s9-ref) already explodes around \( t_{\text{pb}} \sim 0.4 \text{s} \). Combining all three of these variations turns a robustly (in the sense of being independent of stochasticity) non-exploeding model (s20-ref) into a robustly exploding model (s20-pp-str-mb), with explosion times between 0.38-0.50 s, cf. Table 1.

Crudely judging from the explosion times (cf. Table 1), the impact of the pair-process simplification is roughly comparable to that of the combination of the two physically motivated opacity corrections (“str” and “mb”; cp. models s9-ref, s9-str-mb, and s9-pp) and slightly stronger than that of neutrino-electron scattering (cp. models s20-rrb-pp, s20-rbr, and s20-rbr-pp-ones).

Finally, we point out an interesting feature: The impact of each microphysics variation on the explosion behavior is qualitatively correctly predicted in 1D by the timescale ratio, \( \frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \), in the sense that all modifications, which lead to larger values of \( \frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \) in 1D, trigger an earlier onset of explosion, or an explosion at all, in 2D (see for the s20 models Figs. 5 and Sec. 3, as well as for the s9 models the dotted lines in the panel for \( \frac{\tau_{\text{adv}}}{\tau_{\text{heat}}} \) in Fig. 11 showing the behavior of the corresponding 1D models). In contrast, the shock trajectory of 1D models is not a good indicator of more/less favorable runaway conditions: Both strangeness and many-body corrections reduce the shock radius in 1D, while the pair-process simplification increases the latter (cf. Fig. 7), but all three of these physics variations help initiating a shock runaway in 2D.

4.4 Velocity-dependent and gravitational redshift terms

Concerning the prospects of shock runaway, the purely negative impact of neglecting velocity-dependent as well as gravitational redshift terms in the transport seen in 1D seems to be partially alleviated in 2D, albeit in a model-dependent fashion: An early exploding s20 model (s20-pp-str-mb) explodes even earlier with these modifications (s20-pp-str-mb-novel), a late exploding s20 model (s20-rbr) then lacks an explosion (s20-rbr-novel), and the reference s9 model (s9-ref) seems almost unaffected (s9-novel).

These results are most likely connected to the reduced convective activity in the PNS, which can be inferred from the smaller values of absolute velocities, \(|v|\), in the PNS convection zone displayed for a representative time in Fig. 6. The consequence is a PNS that contracts faster, is hotter, and therefore emits neutrinos with higher rms-energies (see \( \tau_{\text{NS}}, T_{\text{NS}}, \) and \( \epsilon_{\text{rms}} \) in Figs. 8 and 11). For a less massive PNS these explosion facilitating consequences seem to be able to (over-)compensate for the missing Doppler blueshift in the infalling material, because the infall velocities are lower compared to a more massive PNS (see profiles of the radial velocity, \( v_r \), in Fig. 4). This dependence on the PNS mass at the time of close proximity to criticality might explain why the runaway conditions become more (less) optimistic for the originally early (late) exploding model when switching off the considered terms. In the models with the less massive s9 progenitor, all velocities, including those in the PNS convection zone, are lower to begin with, which might explain the almost vanishing net impact on the shock trajectory in this case.

The actual reason for the less efficient PNS convection when switching off the considered terms is not determined easily. We suspect the following: When neglecting velocity-dependent terms in the moment equations, neutrinos (i.e. their energy and lepton number) are not advected with the flow as they should be deep inside the PNS. Instead, neutrinos carried in a bubble that is about to rise in response to an unstable stratification effectively leak out of the bubble and are left behind. Since near the bottom of the PNS convection zone the neutrino lepton number can be a sizable fraction of \( Y_e \), these neutrino losses effectively reduce the lepton number of the bubble and therefore the tendency of the latter to rise further.

4.5 Ray-by-ray-plus approximation

In this section we compare the impact of using the RbR+ approximation on the PNS convection and neutrino emission as well as on the explosion dynamics. 4.5.1 PNS convection and neutrino emission

We start by addressing the question if using the RbR+ approximation might have a significant impact on the PNS convection and, in turn, on the neutrino emission from the depletonizing PNS. Figure 9 compares the angle-averaged, radial profiles of (comoving-frame) luminosities, \( 4\pi r^2 \int F_\nu \, dr \) for all evolved neutrinos species, \( \nu \in \{\nu_e, \bar{\nu}_e, \nu_x\} \), as well as the radial profiles of the convective lateral kinetic energies, \( \rho v^2 / 2 \), at two representative times for models s20-ref and s20-rbr. The convectively unstable region is characterized by large values of \( \rho v^2 / 2 \). The neutrino luminosities – all the way from inside the PNS up to the saturation (i.e. gain) radius – as well as the convective energies are very similar for both models (apart from small temporal fluctuations) even at late post-bounce times. Moreover, as can be seen in Figs. 9 and 10 showing functions of time for the s20 models and Fig. 11 for the s9 models, also the mean and rms-energies of emitted neutrinos are barely affected. These results suggest that RbR+ has a negligible impact on PNS convection and on the angle-averaged neutrino emission, at least much less than, e.g., neglecting velocity-dependent terms in the transport (cf. Sec. 4.4). This result is not too astonishing considering that neutrinos deep inside the PNS are strongly coupled to the medium such that, first, the lateral advection fluxes included in the RbR+ approximation strongly

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4.5.2 Impact on explosion behavior of s20 models

While the reference model, s20-ref, is not showing any indication of shock runaway during ~ 1 s of post-bounce evolution, the corresponding RbR+ model, s20-rbr (which was compared against the corresponding Vertex model in Sec. 4.2), does explode, though rather late and with a substantial scatter in the time of shock runaway. Likewise, switching to the simplified pair-process treatment, which itself promotes explodability, yields late explosions with (s20-pp) and early explosions with (s20-rbr-pp) the RbR+ treatment. In our set of s20 models, the net effect of RbR+ regarding the explodability is comparable to using the pair-process simplification together with either the strangeness (s20-pp-str) or the many-body correction (s20-pp-mb) but smaller than using all three of those together (s20-pp-str-mb).

In order to understand what pushes the RbR+ models closer to criticality, we consider Fig. 12 which shows for several models the excess/shortage of neutrino fluxes, rms-energies, temperatures, and mass fluxes close to the north pole (i.e. averaged over $\theta < \pi/10$) with respect to their averages over the full sphere, as well as the normalized coefficients corresponding to dipole and quadrupole deformation modes obtained from a Legendre expansion of the shock surface. All exploding and non-exploding s20 models with RbR+ (including the ones not plotted in Fig. 12) feature a sustained polar enhancement of temperatures, mass-fluxes (albeit with large temporal fluctuations) and electron-type

Figure 12. Impact of RbR+. Each panel in the first three rows shows for a given quantity at a given radius the value of this quantity averaged over $\theta < \pi/10$, normalized to the average of this quantity over the full sphere. The first row shows this pole-to-sphere ratio for the energy-integrated radial fluxes (left panel) and rms-energies (right panel) summed over both species $\nu_e$ and $\bar{\nu}_e$ and measured at the gain radius in the comoving frame. The second row shows the same quantities but just for the $\nu_x$ neutrinos. The third row shows the pole-to-sphere ratio for the temperature (left panel) and radial mass-flux density (right panel) at the PNS surface (defined by the radius at which $\rho = 10^{11}$ g cm$^{-3}$). The bottom row shows the Legendre coefficients of the dipole (left panel) and quadrupole (right panel) deformation moments of the shock surface normalized to the monopole moments. All curves are smoothed using running averages of 10 ms.

dominate the lateral diffusion fluxes, and second, lateral neutrino-momentum transfer to the fluid is well described by using pressure derivatives associated with assumed isotropic neutrino distributions, as applied in the RbR+ treatment (Buras et al. 2006).
neutrino fluxes that is visibly smaller without RbR+, and these features are accompanied by higher shock-oscillation amplitudes and therefore more optimistic runaway conditions.

These results suggest that neutrino heating can have a supportive effect to the otherwise purely fluid-dynamical advective-acoustic/advective cycle of the SASI (Blondin et al. 2003; Blondin & Mezzacappa 2006): Sloshing motions triggered by the SASI, in which gain material is dragged from one hemisphere to the other in an oscillatory fashion, are accompanied by accretion hot spots at the poles of the PNS, and these hot spots, in turn, provide additional energy to the ascending material by means of enhanced neutrino heating. This positive feedback is apparently more efficient with RbR+ as neutrino fluxes stemming from these hot spots remain more collimated when reaching the gain layer, causing more energy to be pumped into the fastest, axis-near material that carries most of the kinetic energy.

Based on this picture we suspect that the rather significant impact of RbR+ observed in the present SASI-dominated models is fostered by the following circumstances: First, sloshing modes generate strong downflows always at the same location, namely near the poles, which facilitates the development of hot spots and creates locally enhanced neutrino fluxes that are always pointed towards the optimal direction for dynamical feedback. Second, sloshing modes cause downflows to be rather well synchronized with expansion flows, i.e. both take place on the same characteristic timescales (namely the advection timescale $\tau_{\text{adv}}$) and downflows are always succeeded by expansion flows.

### 4.5.3 Impact on explosion behavior of s9 models

We now consider the convection-dominated s9 models; see Figs. 11 and 13 for the corresponding time-dependent properties of selected models. Here, using RbR+ advances the onset time of shock runaway only by $\sim 0.02 - 0.13$ s in model s9-rbr compared to s9-ref (cp. Table 1), which is a rather small effect considering that neglecting neutrino-electron scattering prohibits the runaway entirely, both with (s9-rbr-nones) and without (s9-nones) using RbR+. The impact of RbR+ is thus much less extreme than in the s20 models, where switching off RbR+ delays the explosion by a much longer time than switching off neutrino-electron scattering (cp. models s20-rbr-pp, s20-pp, and s20-rbr-pp-nones, respectively).

The reduced impact of RbR+ for the s9 models becomes particularly obvious when comparing the two longest...
Comparing the time evolution of $\tau_{\text{adv}}/\tau_{\text{heat}}$ (Figs. [8] and [11]) between various models reveals, first, that the amplitudes of temporal fluctuations are lower in the convection-dominated s9 models than in the SASI-dominated s20 models, and second, that at least for the s20 models these amplitudes grow with the proximity of each model to the runaway threshold. This tendency is consistent with the observed time interval, $\Delta t_{\text{run}}$, within which the runaway times are dispersed for models differing only in the initial perturbation pattern (see Table [1]). The scatter seems to be greater for the s20 models than for the s9 models, and for the s20 models the scatter is larger for more marginally exploding models (e.g. s20-rbr, s20-pp-str) than for more robustly exploding models (e.g. s20-rbr-pp). These tendencies are in qualitative agreement with Cardall & Budiardja (2015), who examined stochasticity using a large number of simplified models and found larger dispersion of explosion times for SASI-dominated models than for convection-dominated models (see also Kazeroni et al. 2017).

The strong dependence of $\Delta t_{\text{exp}}$ on the progenitor and the input physics might be one reason why previous studies report quite diverse values of $\Delta t_{\text{exp}}$ (e.g. O’Connor & Couch 2018, Summa et al. 2016, Cardall & Budiardja 2015, Tak-waki et al. 2014). An additional reason might simply be that many studies, particularly the ones including computationally expensive neutrino transport such as ours, are forced to rely on rather poor statistics because they can only afford a small number of simulations.

Coming now back to the second question concerning numerical convergence: For the reference s20 model, s20-ref, and its counterpart including the RbR+ approximation, s20-rbr, we repeated the simulations with both increased and decreased resolutions in both radial and angular directions (see models ending with “hires”, “lores”, “hiθ” and “loθ” in Table [1], in some cases even multiple times with different initial perturbation patterns. We could not identify a systematic trend with varying the resolution, neither regarding the neutrino-emission properties nor the heating conditions in the gain layer nor the scatter in explosion times. In particular, for model s20-rbr the onset of explosion does not appear to be correlated with resolution in any direction: For all three angular resolutions with 80, 240, and 320 zones the scatter $\Delta t_{\text{exp}}$ in the times of shock runaway remains comparably high, namely, $\Delta t_{\text{exp}} = 0.33$ s, 0.44 s, and 0.43 s, respectively. Since model s20-rbr is quite marginal concerning its tendency to explode and would therefore probably be quite sensitive to numerical resolution, this suggests (although does not prove) that, at least, features the runaway is sensitive to are numerically converged.

### 4.7 Previous axisymmetric simulations of the s20 progenitor

Here we want to collect some results concerning the explosion behavior of axisymmetric simulations performed by various other groups using the s20 progenitor. A detailed comparison is, however, out of the scope of this paper.

O’Connor & Couch (2018) used M1 transport in cylindrical coordinates, a similar neutrino setup as applied here for model s20-pp but additionally neglecting neutrino-electron scattering, and employing the LS220 EOS (Latimer & Swesty 1991). In their models shock expansion sets in around 700 ms post bounce, while our model s20-pp explodes later and would probably not explode at all without neutrino-electron scattering. However, the LS220 EOS used in O’Connor & Couch (2018) is slightly softer than the SFHo EOS used here and might lead to earlier explosion times.

Formally the same setup and a similar M1 scheme as in O’Connor & Couch (2018), although in spherical coordinates, was used by Skinner et al. (2016). We were unable to
ascertain how Skinner et al. (2016) treated pair processes, e.g. if they ignored annihilation for electron-type neutrinos.

1. In 1D the agreement between ALCAR and VERTEX is found to be excellent concerning nearly all features. The only noteworthy differences are a more energetic neutrino burst, slightly (≤ 5%) higher luminosities, and a hotter and (by ≈ 1 km) more compact proto-neutron star (PNS) in ALCAR.

2. In the two examined 2D models of the SASI-dominated s20 progenitor with and without neutrino-electron-scattering, the agreement found between ALCAR in the RbR+ mode and VERTEX remains very good concerning the neutrino emission, the PNS contraction, the heating conditions, and the explosion times. The two last mentioned features are subject to substantial stochastic scatter, the degree of which is, consistently in both codes, stronger for the exploding models with neutrino-electron scattering than for the non-explosive models without. Similar to the 1D case, the PNS radius in the ALCAR models is again smaller throughout by ∼ 1 km compared to the VERTEX models. Although PNS convection has almost the same impact on the PNS radius and neutrino luminosities with ALCAR and VERTEX, the associated kinetic energies are higher by a factor of a few in ALCAR than in VERTEX. The origin of this discrepancy is not related to the RbR+ approximation and needs to be found in future work.

3. When comparing ALCAR models with and without the RbR+ approximation we could not find any significant impact of RbR+ on PNS convection and on the (angle-averaged) neutrino emission. Concerning the impact on the explosion behavior, the RbR+ approximation has a notable explosion-promoting impact for the SASI-dominated s20 models, while it shows only a small impact for the convection-dominated s9 models. We interpret this as a consequence of the tendency that linear sloshing modes can be more efficiently amplified by RbR-enhanced polar neutrino heating than more randomly rising convective bubbles. However, even in the SASI-dominated models the net impact of RbR+ remains reasonable and quantitatively comparable to typical modeling variations in the microphysics sector (see next item). Our results confirm previous suspicions (e.g. Sumiyoshi et al. 2015; Skinner et al. 2016) that RbR+ can facilitate explosions, but do so only partly because our convection-dominated models are only marginally affected and a corresponding comparison in the more realistic three-dimensional case has yet to be conducted. Additionally, one should keep in mind that any conclusions drawn from comparisons with the fully multidimensional M1 scheme bear the remaining uncertainty that is connected to the still incompletely known accuracy of the fully multidimensional M1 scheme.

4. Simplifying pair processes, including neutrino-electron scattering, as well as adopting the strangeness and many-body corrections all have, roughly in this order of relevance, a significant explosion-facilitating impact in our 2D models, while in 1D these modifications result in rather small changes of the shock radius. The timescale ratio, \( \tau_{\text{adv}} / \tau_{\text{heat}} \), in 1D predicts on a qualitative level remarkably well the impact of each modeling variation for the 2D models. In contrast, the shock radius can be shifted both to lower (“str” and “mb” models) or higher (“pp” models) values in 1D for microphysics variations that promote an explosion in 2D. This suggests that \( \tau_{\text{adv}} / \tau_{\text{heat}} \) is a more powerful diagnostic quantity than the shock trajec-

5 SUMMARY AND CONCLUSIONS

In this study we used 1D (spherically symmetric) and 2D (axisymmetric) models to compare the relatively new AENUS-ALCAR code Just et al. (2015), which incorporates the fully multidimensional M1 approximation for neutrino transport, against the well-established PROMETHEUS-VERTEX code Rampp & Janka (2002) Buras et al. (2006), which employs an accurate Boltzmann solver restricted to the ray-by-ray+ (RbR+) approximation that neglects non-radial neutrino flux components. We compared with VERTEX by mimicking the RbR+ approximation in ALCAR and we tested the RbR+ approximation by comparing to the fully multidimensional version of ALCAR. Moreover, we investigated the impact of other modeling variations in the neutrino transport that are frequently used by CCSN modelers, namely neglecting inelastic neutrino-electron scattering, simplifying pair-processes by assuming target neutrinos to be in isotropic equilibrium, applying strangeness and many-body corrections to neutrino-nucleon scattering, and ignoring velocity-dependent as well as gravitational redshift terms in the transport equations. In order to obtain information about the impact of stochasticity in our comparison study we repeated some of our simulations several times starting with different initial random perturbation patterns (but the same perturbation amplitudes). With respect to the questions raised in the introduction, we obtained the following results:
tory when estimating the impact of modeling variations using computationally less demanding 1D models.

5. Ignoring velocity-dependent terms and gravitational redshift in the transport equations reduces the neutrino fluxes noticeably and the rms-energies barely as measured in the comoving frame of infalling material in the gain region. In 2D these explosion-hampering features can, however, be (over-)compensated, because PNS contraction is accelerated owing to less vigorous PNS convection. The net effect on the explosion is case dependent and becomes more pessimistic for more massive PNSs, which imply higher velocities and stronger redshift: For an s20 model that originally explodes early when the PNS is still less massive, the explosion sets in earlier, while for an originally late exploding model the explosion lacks entirely. For the low-mass s9 models the aforementioned effects compensate and there is barely any visible impact on the shock trajectory.

6. The conditions in the gain layer (e.g. the lateral kinetic energy or timescale ratio) and the explosion times can be subject to substantial random variations in agreement with the findings by Cardall & Budiardja (2015) based on simplified models. The amplitudes of temporal fluctuations and the scatter in the explosion times increase with the proximity to criticality for a given model, and they turned out to be much greater for the SASI-dominated s20 models than for the convection-dominated s9 models.

The good agreement between ALCAR and VERTEX supports the reliability of both codes and lends credence to the rather new M1 method that is currently being used by several groups for simulating CCSNe. Concerning RbR+ in the 3D case, our findings suggest that the impact might be overall less dramatic than in 2D, because an artificial symmetry axis that fosters axis-parallel motions is absent.

The sensitive dependence of the onset of explosion on modeling variations seen in our 2D models once again (e.g. Buras et al. 2006; Müller et al. 2012a; Lentz et al. 2012b; Sumiyoshi et al. 2015; O’Connor & Couch 2018; Burrows et al. 2018; Richers et al. 2017b; Kotake et al. 2018) highlights the importance of using a sophisticated transport scheme in order to make quantitatively reliable predictions regarding the highly non-linear evolution of the SN core. Moreover, it suggests that a careful analysis is necessary when comparing models in the literature just based on their explosion times. On the one hand, large differences in explosion times might be caused just by some difference in a single neutrino rate and may therefore not necessarily reflect more serious code differences. On the other hand, a good agreement of explosion times may not necessarily imply that all differences in the physics input are small; it could just be that individual differences compensate each other and the net impact is small.

More detailed comparisons such as the present one are needed to understand differences in multidimensional CCSN results obtained worldwide. These comparisons should include more codes using different transport approximations (e.g. FLD, IDSA), as well as discretization schemes for the hydro (e.g. cartesian or cylindrical or spherical polar grid) and the transport (e.g. tangent-ray, discrete ordinate, Monte Carlo). Our results suggest that convection-dominated progenitors, which seem to come with an overall smaller level of stochasticity, might be more suited for such studies than SASI-dominated progenitors.

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the additional two variables are $Y$ tering processes the source terms entering the moment equa-
that is described below. bin coupling source terms wherever justified, in the manner
large. This is why we avoid an implicit integration of energy-
the computational cost would in this case soon become very
of the explicit integration of the non-local divergence terms)
that the time steps used in cobian possibly multiple times per integration step. Given
a non-linear system of equations of rank 3 $\times$ 4 were to be treated fully implicitly, at each spatial grid point
$N$ for the three species, four moments, and $\bar{\nu}$, neutrino-source terms, which may
$\delta U_{\nu,q}^n$ (with $\nu = e$, $\bar{e}$, $\nu$, and $q = 1, \ldots, N$, denoting the neutrino species and energy group, respectively) is the vector of hydrody-
namic and neutrino-transport variables at the old/new time step, $(\delta U)_{\nu,q}^{n+1}$ are the neutrino-source terms, which may
depend on both old and new variables, and $(\delta U)_{\nu,q}^n$ other represents all remaining terms, which depend only on the old
variables. Now if in Eq. (A1) all neutrino-interaction terms for the three species, four moments, and $N_e$ energy groups were to be treated fully implicitly, at each spatial grid point
a non-linear system of equations of rank $3 \times 4 \times N_e + 2$ (the additional two variables are $Y$, and $T$) would need to be
solved, which involves the inversion of a non-sparse Ja-
cobian possibly multiple times per integration step. Given
that the time steps used in ALCAR are rather short (because of the explicit integration of the non-local divergence terms) and the total number of integration steps is therefore large, the computational cost would in this case soon become very
large. This is why we avoid an implicit integration of energy-
bin coupling source terms wherever justified, in the manner
that is described below.

For all emission/absorption as well as iso-energetic scatter-
ing processes the source terms entering the moment equa-
tions, Eqs. (3), are given by:
\begin{align}
C_{\nu,\nu}^{(0)} & = \alpha_\nu \epsilon_\nu (E^eq - E), \quad \text{(A2a)} \\
C_{\nu,\bar{\nu}}^{(1),+} & = -c (\kappa_+^{\nu} + \kappa_+^{\bar{\nu}}) F^+, \quad \text{(A2b)}
\end{align}
where $E^eq$ is the equilibrium energy density corresponding to a Fermi-Dirac distribution, $\kappa_+^{\nu}$ is the sum of all absorp-
tion opacities corrected for stimulated absorption, and $\kappa_+$ is the sum of opacities for iso-energetic scattering. We follow [Rampp & Janka (2002)] for the computation of these opacities, except that we additionally account for weak magnetism and nucleon recoil [Horowitz (2002)] and, in selected models (cf. Sec. 2.3), for strangeness corrections and many-
body corrections in a way described in [Horowitz et al. (2017)]. For neutral-current reactions of $\nu_e$ neutrinos (repre-
senting the four heavy-lepton neutrinos) we use as effective
opacity the arithmetic average of the opacity of a heavy-
lepton neutrino and that of its antiparticle. In computing
the source terms, Eqs. (A2), the neutrino energy- and flux-
densities are treated implicitly and the opacities explicitly.
The equilibrium energy density, $E^eq$, is usually treated ex-
plitly, i.e. using old values of $Y_e$ and $T$. Occasionally, how-
ever, in regions with strong neutrino-matter coupling and
short fluid-dynamical timescales the difference between equi-
librium energies at the old and new time step becomes so
large that numerical oscillations would result for an explicit
treatment of $E^eq$. In these (rare) cases, which we detect us-
ing a criterion based on the density and the relative change of gas energy due to neutrino interactions, we perform an addi-
tional intermediate step to obtain improved values of $Y_e$ and $T$ (and therefore $E^eq$). In this step we solve Eqs. (A1)
for $Y_e$ and $T$ using implicit equilibrium energies and under
the simplifying assumption that the neutrino fluxes vanish.

Inelastic scattering of (all types of) neutrinos off elec-
trons and positrons is implemented as in [Yueh & Buchler (1977); Bruenn (1985); Rampp & Janka (2002)], i.e. using
a Legendre expansion in the scattering angle of neutrinos
up to 1st order. For the multidimensional generalization of
the formalism of [Yueh & Buchler (1977); Bruenn (1985), Rampp & Janka (2002)] (who assumed spherical symmetry and
therefore vanishing non-radial flux-vector components and
diagonal Eddington-tensor components) we use the expres-
sions given in [Cernohorsky (1994)]. We obtain the Leg-
edre coefficients of up to 1st order for each initial-state and
final-state neutrino energy and each neutrino species by
table interpolation. The scattering rates are computed explicit-
ly, i.e. using Legendre coefficients and neutrino moment-
s from the old time step. In order for the explicit in-
tegration to remain stable, the rates for inelastic neutrino-
lepton scattering are reduced at high densities (where these
rates are subdominant compared to charged-current reac-
tions for conditions considered in this paper) by a factor of
max\{1, $\rho/(5 \times 10^{12}$ g cm$^{-3}$)$/2$\} (following the suggestion by
O’Connor (2015)).

Finally, for pair processes we again follow [Rampp
& Janka (2002)] as closely as possible,$^{11}$ where electron-
positron annihilation is implemented based on [Pons et al.
(1998)] and nucleon-nucleon bremsstrahlung is implemented
based on [Hannestad & Raffelt (1998)]. We expand the in-
teraction kernel in $\cos \omega \equiv \mathbf{n} \cdot \bar{\mathbf{n}}$, where $\mathbf{n}$ and $\bar{\mathbf{n}}$ are the
propagation unit vectors of the considered neutrino and its

$^{11}$ We note the following misprints in the Appendix of [Rampp
& Janka (2002)], which are correctly accounted for in both simula-
tion codes, ALCAR and VERTEX: In Eq.(A.20), i.e. the multipole
expansion of the 1st-moment source term for pair-processes, the
factor $(2t + 1)$ must be replaced by 1, in Eq.(A.39) the quantity
erroneously denoted as production coefficient, $\phi^p_{10}$, is actually
the annihilation coefficient $\phi^a_{10}$, and in Eq.(A.41) the factor ($\hbar c$)$^3$ must be replaced by 1.
annihilation partner, respectively. The resulting expansion of the pair-process source terms for the moments $E$ and $F$ of a given neutrino species at energy $\epsilon$ reads (up to 2nd order in $\cos \omega$):

$$C_{\text{pair}}^{(0)} = \frac{8\pi^2 \epsilon^3}{(hc)^3} \int_0^\infty d\bar{\epsilon} \bar{\epsilon}^2 \left[ \phi_0^p (1 - M_0 - \overline{M}_0) + \phi_0^a M_0 \overline{M}_0 ight] + 3\phi_1^a M_{1,j} \overline{M}_1 + \frac{5}{2} \phi_2^a (3M_{2,jk} \overline{M}_{2,k} - M_0 \overline{M}_0),$$  

(A3a)

$$C_{\text{pair}}^{(1),i} = \frac{8\pi^2 \epsilon^2 \epsilon^3}{(hc)^3} \int_0^\infty d\bar{\epsilon} \bar{\epsilon}^2 \left[ -\phi_0^p M_i^1 - \phi_1^p \overline{M}_1^i ight] + \phi_0^a M_i^1 \overline{M}_0 + 3\phi_1^a M_{1,j}^i \overline{M}_{1,j} + \frac{5}{2} \phi_2^a (3M_3^{i,jk} \overline{M}_{2,jk} - M_i^1 \overline{M}_0),$$  

(A3b)

where

$$\{M_0, M_1^1, M_2^{ij}, M_3^{ijk}\} = \frac{1}{4\pi} \int d\Omega \mathcal{F}\{1, n^i, n^{ij}, n^{ijk}\}$$  

(A4)

are angular moments of the neutrino distribution function, $\mathcal{F}$, and overlined symbols denote the corresponding quantities of the annihilation partner. The coefficients $\phi_{0/1/2}$ are defined as in [Rampp & Janka 2002] and are taken into account up to 2nd (1st) order for $e^\pm$-annihilation (bremsstrahlung). They are computed using hydro variables at the old time step. In order to circumvent expensive matrix inversions, we also treat the neutrino moments in Eqs. (A3) explicitly in time for densities below $10^{13}$ g cm$^{-3}$. Above this density, we employ the simplified pair-process treatment (that is used for “pp” models at all densities; cf. Sec. 2.3), i.e. we ignore pair processes for electron-type neutrinos and assume that annihilation partners for $\nu_x$ neutrinos are in isotropic LTE. This allows to write the pair-process rates formally equivalent to those of emission/absorption processes, cf. Eqs. (A2), and to treat the rates implicitly in the neutrino moments without needing to perform matrix inversions. The simplification is justified by the circumstances that at high densities, $\rho > 10^{13}$ g cm$^{-3}$, charge-current reactions strongly dominate pair-processes for electron-type neutrinos, and $\nu_x$ neutrinos should be sufficiently close to isotropic LTE.

We have verified that the quenching of neutrino-electron scattering (as described above) as well as the simplification of pair processes at high densities has no significant impact on the results discussed in this paper.