Relativistic collisions of highly-charged ions: electron capture via electron-positron pair production

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Abstract. We have developed a relativistic version of the distorted-wave theory (RCDWEIS) to simulate the process of electron capture via pair production. Results are compared with experimental cross section data for La$^{57^+}$ impact on Gold, Silver and Copper targets. An observed enhancement with increasing $Z_T$ is evident, with the results found to be in better agreement with experiment than previous similar simulations. The theory is extended to give total cross sections for the interaction of U$^{28^+}$ and a bare Argon ion.

The advent of relativistic highly-charged ion accelerators, such as the relativistic heavy-ion collider (RHIC) at Brookhaven or the ESR at GSI Darmstadt has excited a new wave of interest by both theorists and experimentalists into the process of electron-positron pair production. These accelerators are designed to probe nuclear and atomic Physics under extreme conditions of high-charge and energy: the quark-gluon plasma [1] and QED corrections [2], for example. Under these conditions the vacuum interactions become significant, and the creation of electron-positron pairs arises. Since the ions are extremely highly charged, the electron created can be captured into the orbit of the nucleus while the positron is strongly repelled.

Gould [3] and Anholt & Gould [4] were the first to document the case for electron-positron pair production with simultaneous capture of the electron into the shell of one of the colliding ions (CPP). Subsequent work, e.g. Becker [5], [6] modelled the process as the excitation of the target electron from a negative energy state by the action of the passing projectile, whereas e.g. Eichler [7] treated the process as the transfer of the electron from the negative energy state of the target to the bound state of the projectile; both these models lack symmetry. Deco & Rivarola [8] found a two-centre description of the continuum positron was necessary, so as to account for interference effects arising from the creation of the electron-positron pair: an undistorted wavefunction was employed for the bound electron and a semi-relativistic Furry wavefunction, distorted by a scalar factor, adopted for the positron.

The process of electron capture via pair production can be represented schematically as follows

$$p^{Z_P^+} + T^{Z_T^+} \rightarrow p^{(Z_P-1)^+} + e^+ + T^{Z_T^+}. \quad (1)$$

Through the use of crossing symmetries, where the electron-positron interaction is assumed to be much weaker than their respective interactions with the highly-charged ions, process (1) is shown to be mathematically equivalent to the time-reversed ionization process, i.e.

$$p^{(Z_P-1)^+} + T^{Z_T^+} \rightarrow e^- + p^{Z_P^+} + T^{Z_T^+}. \quad (2)$$
A semirelativistic wavefunction is used to approximate the bound state,

\[ \Phi_i = \Phi_{0i} + \Phi_{1i} \]  

where

\[ \Phi_{0i} = Z_T^{|3/2|} e^{-Z_T r - i E_{si} t} \omega_i, \]  

and

\[ \Phi_{1i} = Z_T^{|3/2|} \left( -\frac{i}{2} \alpha \cdot \nabla_{rr} e^{-Z_T r} \right) e^{-i E_{si} t} \omega_i, \]  

\( \alpha \) is a Dirac matrix. In order to approximate the effect of the projectile on the bound state a distortion term must be introduced; this is defined as

\[ \mathbf{L}'_i = \mathbf{L}'_{0i} + \mathbf{L}'_{1i} \]  

and can be given in terms of a scalar and a vector component

\[ \mathbf{L}'_{0i} = N(\nu_p) \, _1F_1(\nu + 1; i \gamma (\nu r' + \mathbf{v} \cdot \mathbf{r}')) \mathbf{I} \]  

and

\[ \mathbf{L}'_{1i} = N(\nu_p) \mathbf{S}^{-1} \left( -\frac{i}{2\gamma c} \alpha \cdot \nabla_{r} \, _1F_1(\nu; 1; -i \gamma (\nu r + \mathbf{v} \cdot \mathbf{r})) \right) \mathbf{S}. \]  

where \( N(\nu_p) = \exp(\pi \nu_p/2) \Gamma(1 - i \nu_p), \) \( r' \) is the position vector of the electron in the target frame with relative velocity \( \mathbf{v} \), and \( \mathbf{S} \) is the operator that transforms the wavefunction from the projectile frame to the target frame, and satisfies

\[ \mathbf{S} = \left( \frac{1}{2} + \frac{1}{2} \gamma \right)^{1/2} (\mathbf{I} - x \alpha \cdot \hat{\mathbf{v}}) \]  

with \( \mathbf{I} \) denoting the unit matrix, \( x = v \gamma c^{-1}(\gamma + 1)^{-1} \) and \( \gamma = (1 - v^2/c^2)^{-1/2} \). The continuum wavefunction and subsequent distortion factors are derived in an analogous manner [9]. Retaining terms of first order, the initial and final state relativistic continuum distorted wave eikonal initial state (RCDWEIS) wavefunctions are now defined as

\[ \chi_i = \mathbf{L}'_{0i} \Phi_{0i} + \mathbf{L}'_{0i} \Phi_{1i} + \mathbf{L}'_{1i} \Phi_{0i}, \]  

and

\[ \chi_f = \mathbf{L}'_{0f} \Phi_{0f} + \mathbf{L}'_{0f} \Phi_{1f} + \mathbf{L}'_{1f} \Phi_{0f}. \]  

The post form of the transition amplitude is chosen

\[ A(b) = -i \int_{-\infty}^{t^+} dt \langle \mathbf{W}_f \chi_f | \chi_i \rangle, \]  

with the full overlap defined as

\[ \langle \mathbf{W}_f \chi_f | \chi_i \rangle = \left\langle \left[ (-i c\alpha \cdot \nabla \phi_{0f}) \mathbf{L}'_{1f} + \mathbf{S} (-i c\alpha \cdot \nabla \mathbf{L}'_{0f}) \right] \Phi_{1f} \right. \]
\[ + \left( \mathbf{V}_T + \mathbf{S}^2 V'_T \right) \left( \mathbf{L}'_{0f} \phi_{1f} + \mathbf{L}'_{1f} \phi_{0f} \right) \]  
\[ \left. e^{it \nu c - \gamma r} \sqrt{\nu} \mathbf{S}_{r} \omega_{f}^{-1} \right| \mathbf{L}'_{0i} \phi_{0i} + \mathbf{L}'_{0i} \phi_{1i} + \mathbf{L}'_{1i} \phi_{0i}, \]  

where \( \mathbf{v}_e \) is the electron velocity. The total cross section is obtained from integration over the ejectile and projectile momentum (or velocity), and takes the form

\[ \sigma_{cpp} = \sum_{\text{spins}} \frac{1}{2\pi(\gamma v)^2} \int_0^c dv_+ \gamma_+ v_+^2 \int_0^\pi d\theta \sin \theta \int |d\eta| T(\eta)|^2, \]  

(14)
Figure 1. Scaled cross sections in microbarns, for pair-production with electron capture by fully stripped Lanthanum ions (La$^{57+}$) striking thin foils of Copper ($Z_T=29$), Silver ($Z_T=47$) and Gold ($Z_T=79$). Comparison with RCDWEIS theory for capture to the 1s-state. The graph on the left illustrates the results of Lee [11], with the right-hand graph showing the revised results.

with $T(\eta)$ defined as the T-matrix, cf [9], and where we sum over all the spin states of the electron and positron pair.

In computing the cross sections for CPP, Mullan [10] and Lee [11] had to adapt their codes for ionization; by noting that a positron in the final state with energy $\epsilon_+$ and momentum $p_+$ travelling forward in time is equivalent to an electron with energy $-\epsilon_+$ and momentum $-p_+$ in the initial state, the following changes were made

\[ v_e \rightarrow -v_+, \quad v'_e \rightarrow -v'_+, \quad \epsilon_f \rightarrow -\epsilon_+, \quad \epsilon'_f \rightarrow -\epsilon'_+, \quad Z_T \leftrightarrow Z_p. \] (15)

where $\epsilon'_+$ is defined as the energy of a negative energy positron referred to the target frame. With the Sommerfeld parameter defined as $\omega = \alpha Z E/pc$, equations (14) also imply that $\omega_T \rightarrow -\omega_+$ and $\omega'_p \rightarrow -\omega'_+$, where $\omega_+ = Z_T/v_+$ and $\omega'_+ = Z_p/v'_+$. The experiments of Belkacem et al [12], [13], and [14] were for fully stripped Lanthanum ions striking thin foils of copper, silver and gold with collision energies of $E = 0.405, 0.956$ and $1.300$ GeV/u, the results (Figure 1) show a clear enhancement with increasing $Z_T$. Unlike previous models, i.e. the Relativistic Distorted-Wave Born (RDWB) approximation [15] and the target-centred Born approximation (OBK) [7], in which a suppression of the scaled cross section as $Z_T$ increases is evident, the correct ordering of the total cross section with respect to the nuclear charge has been obtained in this RCDWEIS treatment, with the results showing the same observed enhancement with increasing $Z_T$ as in [12]-[14].

Whilst studying the analysis and the subsequent codes of Lee [11], it was noticed that although terms were defined correctly in the literature by Lee, this was not transferred accurately to the code. It was also discovered that the level of accuracy of Lee’s results was not sufficient, this was possibly due to the restrictive computer processing speeds which were available at the time.

Examination of the individual contributions of the T-matrix terms, to the doubly differential cross sections, indicated that the effects due to terms of the order of $(\alpha Z)^2$ are very important for a description of the CPP process, they are found to have a large effect in reducing the total cross sections. The revised results, for these total cross sections, obtained after making the relevant changes to the code and increasing the quadrature points for the integration subroutines, are outlined in Figure 1. Although the data for the total cross sections are still outside the
Figure 2. Scaled cross section, $\sigma_{\text{CPP}}/Z^2_T$ in microbarns, for the process of pair production with electron capture, with a $\text{U}^{28+}$ ion in collision with a $\text{Ar}^{18+}$ ion.

We present in Figure 2 cross sections for the process of CPP involving the interaction of a $\text{U}^{28+}$ ion with a bare Argon ion ($Z=18$). The effective charge of the $\text{U}^{28+}$ ion has been calculated using the Slater Rule and is found to be $Z=35.7$. Consequently, the $\text{U}^{28+}$ ion is treated as a fully stripped ion with a charge of $Z=35.7$. Other theoretical and experimental results are available for projectile electron loss and capture in collisions of $\text{U}^{28+}$ with various gas targets, [16] and [17] respectively, but as of yet no data exists for this case of CPP.

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