Match Characteristics of Design Parameters of Longitudinal Flying Qualities and Pitch Agility

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Abstract. Modern high-performance fighter design should meet the requirements of both flying qualities and flight agility. In this paper, the influence of flying qualities design parameters on flight agility was analyzed and the match laws of Level 1 flying qualities and satisfied flight agility was summarized. The longitudinal match law indicates that Control Anticipation Parameter (CAP) is the main effect factor of initial pitch acceleration and Equivalent Frequency ($\omega_{sp}$) and Damping Ratio ($\xi_{sp}$) affect the stability of the aircraft after loading. With the descending of Control Anticipation Parameter (CAP), the range formed by $\omega_{sp}$ and $\xi_{sp}$ shrinks and finally comes to a point.

Introduction

Flying qualities and agility are two main evaluating assessments of modern high-performance fighter maneuvering characteristics. Flying qualities concentrate on the operating difficulty during different stages in the flight envelope while flight agility concentrates on the quality of different flight missions. Flight agility consists of maneuverability and controllability. Aircraft maneuverability can be studied by general particle flight mechanics method but controllability is closely related to the flying qualities. So the measurements of flight agility have a certain inherent relationship with flying qualities [1,2]. But the requirements of flying qualities should take both the control of flight path and flight attitude into account while the flight agility focused more on the control of flight attitude. In conclusion, the quantitative evaluation of flying qualities and agility differ although the two assessments both evaluate the maneuver ability of the aircraft. For example, Cooper-Harper rating scale only assesses the quality of the flight mission but not the agility directly. Modern high-performance fighter requires Level 1 flying qualities and a satisfied agility so that the aircraft could convert maneuvering states fast while completing the flight missions. So the study of the match laws of Level 1 flying qualities and satisfied flight agility is necessary. Most modern fighter aircrafts are equipped with advanced flight control system, the aircraft and the control system couple into a high-order closed-loop system. The research on the match characteristics of design parameters of aircraft flying qualities and agility become a new problem in the design of modern high-order fighter aircraft.

The research on the relationship between flying qualities and agility are now mainly based on the linear model with simple flight control system [3,4]. Hodgkinson studied the relationship between flying qualities and transient agility when simulating the air combat [5]. Riley took flying qualities as an assessment of aircraft lateral agility [6]. Fang and Shen summarized the relationship between flying qualities and pitch agility based on the open-loop linear model and obtained an approximate formula for calculating pitch agility [7]. After that, based on the six degree-of-freedom nonlinear model with yaw damper and pitch damper, Fang summarized the relationship between flying qualities and lateral agility.

Compared to classic PID system, MRDI (Model Reference Dynamic Inversion) flight control system MRDI directly controls the flying qualities of the aircraft [8,9,10]. In this paper, pitch agility was evaluated quantitatively based on the six degree-of-freedom aircraft model with MRDI control system and summarized the match law of design parameters.
Assessment Criteria

Assessment Criteria of Longitudinal Flying Qualities

Flying qualities concentrate on the controllability and stability of the aircraft, including longitudinal and lateral. Similarly, according to the form of motion, flight agility metrics were classified into axial agility, pitch agility and lateral agility\(^\text{[11]}\). Meanwhile, according to the time scale, flight agility can be divided into transient agility, functional agility and agility potential. The axial agility assesses the acceleration/deceleration performance and is related to the performance of engine. In the high-order closed-loop aircraft control system, heading control is mainly designed to eliminate the side sliding and is coupled with lateral control. So the axial agility and heading agility will not be discussed in this paper.

According to MIL-STD-1797A\(^\text{[12]}\), the equivalent pitch rate transfer-function for pilot control-deflection expresses as:

\[
\frac{q_{\text{RM}}}{cmd_{\text{pitch}}} = \frac{K_q}{\omega_{sp}^2} \frac{s + \frac{1}{T_{\theta 2}}}{s^2 + 2\xi_{sp}\omega_{sp}s + \omega_{sp}^2},
\]

where \(\omega_{sp}\) is the equivalent frequency of the short cycle and \(\xi_{sp}\) is the damping ratio. Equivalent-system parameter \(1/T_{\theta 2}\) is the consonance between attitude and path, which has an approximation with Control Anticipation Parameter (CAP):

\[
CAP = \frac{\omega_{sp}^2}{(V_T/g)(1/T_{\theta 2})},
\]

where \((V_T/g)(1/T_{\theta 2})\) can be approximated by \(n/a\).

According to MIL-STD-1797A\(^\text{[12]}\), for Class IV aircraft in Category A flight phases, Level 1 flying qualities should meet requirements that \(\omega_{sp} \geq 1 \text{rad/s}, 0.35 \leq \xi_{sp} \leq 1.3\) and \(0.28(\text{g}^{-1}\cdot\text{s}^{-2}) \leq 3.6(\text{g}^{-1}\cdot\text{s}^{-2})\).

Assessment Criteria of Pitch Agility

In this paper, the transient pitch agility of high-order aircraft is mainly discussed, typically regarded as the capability to generate quick angular motions in a timescale of 1-3 seconds. The flight agility is assessed with the following metrics:

Load Time \(t_{\text{max}}\): Time to pitch to maximum load factor.

Unload Time \(t_{\text{unload}}\): Time to pitch from maximum to zero load factor.

Match Law of Longitudinal Design Parameters

Match Law of Longitudinal Design Parameter CAP and \(\xi_{sp}\)

Load time and unload time are two very common metrics used to analyze the pitch agility of an aircraft. At each flight condition investigated the aircraft is trimmed to straight and level flight. Step inputs of maximum aft deflection are applied to the longitudinal stick and held until the load factor reaches the maximum. Forward stick is then applied to pitch down to zero load factor\(^\text{[13]}\). Based on the simulation time history, the load time \(t_{\text{max}}\) and unload time \(t_{\text{unload}}\) can be calculated.

A conclusion can be drawn from the discussion above that the pitch agility of an aircraft is closed related to parameters \(\omega_{sp}, \xi_{sp}\) and CAP. In the high-order aircraft with MRDI flight control system, the flying qualities were decided by the reference model. So the numerical simulation was performed using this six-freedom aircraft model to study the match law of flying qualities and agility. The initial flight condition was a straight and level flight at 6000m and 0.6Ma. Firstly \(\omega_{sp}\) was set to 5.5rad/s to study the effects of CAP and \(\xi_{sp}\) on the pitch agility. Table 1 shows twelve typical assessment results of different combinations.
Table 1. Assessment with Different Combination of $\xi_{sp}$ and CAP.

| $\xi_{sp}$ | CAP (g$^{-1}$s$^{-2}$) | Flying Qualities | Load Time (s) | Unload Time (s) |
|-----------|----------------|----------------|--------------|----------------|
| 0.5       | 1.5 Level 1     | 0.45           | 0.70         |
|           | 4.0 Level 2     | 0.35           | 0.46         |
| 0.7       | 1.0 Level 1     | 0.51           | 1.04         |
|           | 5.0 Level 2     | 0.34           | 0.44         |
| 1.0       | 2.0 Level 1     | 0.46           | 0.57         |
|           | 7.0 Level 2     | 0.33           | 0.41         |
| 1.5       | 3.0 Level 1     | 0.80           | 0.46         |
|           | 4.0 Level 2     | 0.67           | 0.44         |
| 2.0       | 1.5 Level 1     | 1.22           | 1.07         |
|           | 5.0 Level 2     | 0.81           | 0.42         |
| 2.5       | 1.0 Level 1     | 1.29           | 1.48         |
|           | 7.0 Level 2     | 0.86           | 0.42         |

As shown in Table 1, pitch agility differs if aircraft has different configurations of flying qualities. And the load time and the unload time of an aircraft with Level 1 flying qualities may be greater than that of an aircraft with Level 2 flying qualities.

![Figure 1. Longitudinal Overload Response Characteristic Of Different Flight Qualities.](image)

Figure 1 shows the responses of the aircraft with three typical combinations of design parameters. Configuration with combination $\xi_{sp}=0.7$, CAP=2(g$^{-1}$s$^{-2}$) is in Level 1 flying qualities and combination $\xi_{sp}=2$, CAP=1(g$^{-1}$s$^{-2}$) is in Level 2 flying qualities. Compare the two curves, the response speed of the first configuration with a greater CAP is faster. It can be seen that the load reaches and captures the maximum load factor at 1.0s with little damping. At 3.0s the aircraft start unloading and reaches 0 at 3.58s. The second configuration with greater $\xi_{sp}$ and smaller CAP has an increase on the load time because the response speed decreases with the change of parameters. The result shows that, of this configuration, the load time $t_{max}=1.56$s and the unload time $t_{unload}=1.34$s.

It can be also concluded that the aircraft cannot finish the maneuver with the configuration $\xi_{sp} < 0.3$ for the reason that a small damping ratio causes the unstability of flying with the maximum load. And the configuration $\xi_{sp} < 0.5$ results in poor agility although it fits the Level 1 requirements.

40 combinations of CAP and $\xi_{sp}$ was simulated in this paper and an envelope was formed as Figure 2. The solid lines are the envelopes of $0<t_{max} \leq 0.5$s and $0.5s < t_{max} \leq 1.0$s and the dotted lines are CAP and $\xi_{sp}$ criteria boundaries of flying qualities for Category A flight phases in MIL-STD-1797A[12].
Figure 2. Load Time and Flying Qualities Criteria.

From Figure 2 it can be seen that the pitch agility are related to longitudinal flying qualities but not exactly the same. The envelope of load time between 0.5s and 1.0s appears to be a banding shape. The influence of $\xi_{sp}$ increasing is larger than that of CAP increasing. When $\xi_{sp} < 1$, the aircraft has a good pitch agility even if it is in Level 2 flying qualities with a large CAP. The reason is that the CAP parameter is originally conceived as the ratio of the initial pitch acceleration to the final normal acceleration while $\xi_{sp}$ is conceived as the damping ratio. A large $\xi_{sp}$ will cause a greater damping and increase the time for the aircraft to reach the maximum load factor. So the $\xi_{sp}$ increasing will lead to a reduce of pitch agility.

According to the results above, a conclusion can be drawn that with a same $\omega_{sp}$, the pitch agility can reach a better performance if $\text{CAP} > 1 (g^{-1} \cdot s^{-2})$ and $\xi_{sp}$ is around 1. The increase of $\xi_{sp}$ has a bad effect on the pitch agility. On the condition that the CAP increases to the Level 2 flying qualities, the aircraft can still has a good pitch agility if $0.5 < \xi_{sp} < 1$ in the configuration. But when $\xi_{sp} < 0.5$, the aircraft cannot finish the maneuver because of the reduce of the longitudinal stability.

The Match Value Set of Design Parameters

In this section, the initial flight condition was set to a straight and level flight at 6000m and 0.6Ma and research the match value set boundary of $\omega_{sp}$, $\xi_{sp}$ and CAP. Based on the conclusion above, the range of the parameter values were set to $3 \text{rad/s} \leq \omega_{sp} \leq 6 \text{rad/s}$ and $0.7 \leq \xi_{sp} \leq 5$ which are in the Level 1 flying qualities boundary. The CAP values was set to $0.7 \leq \text{CAP} \leq 7$ which include Level 1 and Level 2 flying qualities and guaranteed that the aircraft can finish the maneuvering. According to the simulation conducted with different combinations of the design parameters, the assessment results of pitch agility are shown in Figure 3 and Figure 4.

The two surfaces in Figure 3 show the change of load time with $\xi_{sp}$ and CAP in the condition $\omega_{sp} = 3 \text{rad/s}$ and $\omega_{sp} = 6 \text{rad/s}$. It can be seen that with the increase of $\xi_{sp}$ and decrease of CAP, the load time increases from 0.3s to 1.0s. And the change of load time is greater when $\omega_{sp} = 3 \text{rad/s}$. Compared with the configurations in which $\omega_{sp} = 6 \text{rad/s}$, the aircraft get smaller load time if $\text{CAP} > 4 (g^{-1} \cdot s^{-2})$ and $\xi_{sp} < 1$. 
In Figure 4, the body shows the criteria boundary in which the load time of the aircraft is less than 0.5s. The dotted lines are criteria boundaries of flying qualities for Category A flight phases in MIL-STD-1797A\cite{12}. The intersection is supposed to be the match value set of design parameters which meet both the Level 1 flying qualities standards and the pitch agility standards. It can be expressed as set:

$$\{\omega_{sp}, \zeta_{sp}, \text{CAP} | \tau_{max} < 0.5s, \text{CHR} \leq 3\}.$$ (3)

In Eq.3, $\tau_{max}$ is the load time and CHR is Cooper-Harper rating of the aircraft. Flying qualities are supposed to be Level 1 if CHR $\leq 3$.

As shown in Figure 4, it can be seen that the longitudinal flying qualities are related to the pitch agility but has a certain difference. With the increase of CAP, the range of combinations of design parameters ($\omega_{sp}, \zeta_{sp}$) which fit the Eq.3 externs. When CAP $> 3.6(g^{-1}s^{-2})$, the range continues to extern but the flying qualities come to Level 2 so the combinations do not fit the design requirement any more. When CAP $= 1.25(g^{-1}s^{-2})$, $\omega_{sp}=6$rad/s and $\zeta_{sp}=0.7$, the body shrinks to be a point. So on this aircraft, the combinations of parameters which meet Eq.3 are $1.0\text{rad/s} \leq \omega_{sp} \leq 6.0\text{rad/s}$, $0.7 \leq \zeta_{sp} \leq 1.3$, $1.25(g^{-1}s^{-2}) \leq \text{CAP} \leq 3.6(g^{-1}s^{-2})$.

Summary
(1) Flying qualities assess the ability to control both flight path and flight attitude while the flight agility focused more on the control of flight attitude. So the requirements of flying qualities are related to the flight agility but are not exactly the same. The combinations of parameters which meet the Level 1 flight quality may not meet the agility standards.

(2) Based on the maneuvering simulations, an envelope is summarized in which the combination of parameters meets both the longitudinal flying qualities standards and pitch agility standards. In the envelope, with the descending of the Control Anticipation Parameter(CAP), the range formed by $\omega_{sp}$ and $\zeta_{sp}$ shrinks and finally comes to a point which is combined with the smallest CAP, the smallest $\zeta_{sp}$ and the biggest CAP. The final result of range which meet the two standards is $1.0\text{rad/s} \leq \omega_{sp} \leq 6.0\text{rad/s}$, $0.7 \leq \zeta_{sp} \leq 1.3$, $1.25(g^{-1}s^{-2}) \leq \text{CAP} \leq 3.6(g^{-1}s^{-2})$.

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