Robust states of ultracold bosons in tilted optical lattices

Moritz Hiller1,2, Hannah Venzl1, Tobias Zech1, Bartłomiej Oleś3, Florian Mintert1,4 and Andreas Buchleitner1

1 Physikalisches Institut, Albert-Ludwigs-Universität Freiburg, Hermann-Herder-Str. 3, 79104 Freiburg, Germany
2 Institute for Theoretical Physics, Vienna University of Technology, Wiedner Hauptstraße 8-10/136, A-1040 Vienna, Austria
3 Marian Smoluchowski Institute of Physics and Mark Kac Complex Systems Research Center, Jagiellonian University, Reymonta 4, 30-059 Kraków, Poland
4 FRIAS, Freiburg Institute for Advances Studies, Albert-Ludwigs-Universität Freiburg, Albertstr. 19, 79104 Freiburg, Germany

E-mail: moritz.hiller@physik.uni-freiburg.de

Received 20 December 2010, in final form 27 February 2012
Published 19 April 2012
Online at stacks.iop.org/JPhysB/45/095301

Abstract
We identify regular structures in the globally chaotic spectra of an interacting bosonic quantum gas in tilted periodic potentials. The associated eigenstates exhibit strong localization properties on the lattice and are dynamically robust against external perturbations.

1. Introduction
Ultracold atoms in optical lattices provide a versatile testing ground for the physics of interacting many-body quantum systems, ranging from the characteristic properties of many-particle ground states [1], over atomic transport [2, 3], to the emergence of collective properties, and thermodynamic behaviour [4, 5]. The complexity of the many-body dynamics of such systems increases rapidly with the number of particles and lattice sites and has its characteristic spectral counterpart in a highly irregular parametric evolution of the energy levels [6–9]. This implies a high sensitivity of the time evolution with respect to changes in the initial conditions and/or perturbations of the generating Hamiltonian, and renders control of generic many-particle dynamics an extremely challenging task. However, nonlinear coupling can also give rise to the emergence of stable collective modes which opens new perspectives for robust control [10, 11]. In our present contribution, we will identify such modes for the tilted Bose–Hubbard Hamiltonian (BHH), and demonstrate their pronounced localization properties as well as their extraordinary robustness against perturbations. In contrast to previous experimental and theoretical studies on bound-atom states (see, e.g., [12–14]), the stability of these solutions is not a mere consequence of energy separation (by a spectral gap) due to the presence of interatomic interactions. Instead, we find bound states that exist within the bulk of the spectrum which can be considered chaotic in the sense of random matrix theory [15].

2. Model
The simplest quantum-mechanical many-body description of ultracold bosons in a lattice is the BHH [16] that incorporates both the tunnelling of individual particles between neighbouring sites and their pairwise on-site interaction. When the one-dimensional lattice is subject to an additional static tilt (due to, e.g., gravitation), the Hamiltonian takes the form

\[ \hat{H} = -J \sum_{j=1}^{M-1} (\hat{a}_{j+1}^\dagger \hat{a}_j + \text{h.c.}) + \frac{U}{2} \sum_{j=1}^{M} \hat{n}_j (\hat{n}_j - 1) + \frac{F}{M} \sum_{j=1}^{M} \hat{l} \hat{n}_j, \]

where \( \hat{a}_j \) (\( \hat{a}_j^\dagger \)) annihilates (creates) a particle in the Wannier state localized at the \( j \)th site, \( \hat{n}_j = \hat{a}_j^\dagger \hat{a}_j \) is the associated number operator and \( M \) specifies the length of the lattice. Here we consider a tilt around the centre of the lattice, and the on-site term \( \hat{l} \) hence takes the values \( \hat{l} = -M/2 + l \) for even and \( \hat{l} = -(M + 1)/2 + l \) for odd \( M \). The BHH has two
The energy $E = \langle \hat{H} \rangle$ and the total number of particles $N = \langle \sum \hat{n}_i \rangle$. The parameters $J$, $U$, and $F$ describe the tunnelling strength, the on-site interaction and the static tilting field, respectively.

The model is based on a single (lowest) band approximation for the optical lattice [16]. This assumption is valid provided the kinetic energy, the interaction strength and the local chemical potential (resulting from the tilt) are sufficiently small not to excite higher Bloch bands. Therefore, the lattice needs to be sufficiently deep [1, 16], to induce large band gaps, and the interaction energy must be smaller than the single-particle ground-state energy, in order not to modify the single-particle wavefunction considerably.

In the experiment, these conditions can be met since all the parameters $J$, $U$, and $F$ in the BHH are readily controlled [4]: while $J$ and $F$ are solely determined by the lattice geometry, the inter-atomic interaction $U$ can additionally be adjusted using Feshbach resonances [17, 18].

In the limit of large particle numbers $N \gg 1$, the quantum dynamics may be described by the mean-field counterpart of the BHH, the discrete Gross–Pitaevskii equation (GPE) (see, e.g., [19, 20]). At fixed lattice length $M$, the mean-field limit is approached by increasing the particle number and, at the same time, keeping the scaled interaction $UN$ at a constant value. As a mean-field approach, the discrete GPE does not explicitly contain the particle number $N$ and, hence, hardly covers effects related to the granularity of matter [5]. In the following, we are concerned with a rather small number of interacting bosons where such effects will be essential, and thus resort to the full many-body Hamiltonian (1).

5 We note that for small particle numbers, deviations of the GPE description from the many-body picture have been predicted, e.g., for a two-site lattice in [21, 22], where a comparison to the Bose–Hubbard dynamics was performed. In [23, 24], both the BHH and the continuous GPE were further compared to the multiconfigurational time-dependent Hartree for bosons method, a more advanced propagation scheme for the many-body Schrödinger equation. The applicability of the GPE can though be extended, when combined with phase-space methods, i.e. when one considers not a single mean-field trajectory but propagates an ensemble of GPEs that reflects the initial quantum-mechanical state (see, e.g., [25] and references therein).

3. Parametric level evolution

Tunnelling, on the other hand, and interaction and tilt, on the other, define two (incompatible) symmetries of the system Hamiltonian (1). Hence, if either one of these terms dominates, the many-particle eigenstates exhibit the relatively simple structure of Bloch waves (for $J \gg UN, F$) or Wannier states (for $UN \gg J, F$ or $F \gg J, UN$). In the generic case, however, when all three terms have comparable weight, good quantum numbers with an unambiguous labelling of the system eigenstates cannot be defined, since the energy levels exhibit a complicated parametric evolution with $U, J$ or $F$, and avoided crossings of variable size abound [8, 9, 26, 27]. This is the spectral manifestation of quantum chaos, and is nicely illustrated by the overwhelming number of energy levels in figure 1.

However, the figure also shows energy levels with constant slope over a wide interval of the static tilt $F$ (at fixed $U$ and $J$), which thus represent many-particle eigenstates with characteristic properties which are invariant under changes of $F$: by virtue of the Hellmann–Feynman theorem [28],

$$\frac{\partial E}{\partial F} = \left\langle \frac{\partial H}{\partial F} \right\rangle = \langle \sum \hat{n}_i \rangle,$$

(2)

where $\langle \cdot \rangle$ represents the expectation value with respect to a single such state, the slope of these energy levels defines a constant centre of mass of the many-particle wavefunction, and, hence, suggests invariant localization properties of the particles on the lattice, under changes of $F$. This is corroborated by the average maximal particle number (AMP) of these states on a single lattice site,

$$\text{AMP} = \frac{\max \langle \hat{n}_i \rangle}{N},$$

(3)

in figure 2, in comparison to the AMP of the chaotic, i.e. irregular states of the spectrum. The average $\langle \cdot \rangle$ in (3) runs over the respective sample of states, at fixed value of $F$. Clearly, in the irregular eigenstates, the particles are largely spread out over the lattice, with AMP $\lesssim 0.25$, while the solutions with constant slope in figure 1 are more strongly localized on the lattice. Whereas this localization is to some extent
present already for vanishing tilt, it becomes strongest once the corresponding energy levels cross the bulk, i.e. for tilt strengths $0.1 \lesssim F/J \lesssim 0.4$, where more than 60% of all $(N = 3)$ particles are localized on a single site. Because of their pronounced localization properties, that we will show to remain unchanged under variation of the tilt, we refer to these as solitonic states in the following [29]. In contrast to previously identified localized states [12–14], the presently discussed solitonic states are not merely a consequence of energetic isolation with respect to the remaining part of the spectrum. Finally, we note that the observed localization is consistent with the fact that there exist as many solitonic states as sites of the lattice—$M = 11$ in the case investigated in figures 1 and 2.

Given the fact that the solitonic states are localized around one lattice site, we further check whether they also exhibit localization in Fock space. A measure for the latter is the inverse participation ratio (IPR), defined as

$$\text{IPR}(|\psi\rangle) = \sum_{j=1}^{N} |c_j|^4,$$

where the $c_j$ are the expansion coefficients of the state $|\psi\rangle$ in a given basis, and $N$ is the Hilbert-space dimension. The IPR represents the inverse number of basis states that are occupied by the state $|\psi\rangle$, and varies from unity—when $|\psi\rangle$ coincides with one basis state—to $1/N$, for $|\psi\rangle$ an equally weighted superposition of all basis states. In fact, the states highlighted in red in figure 1 are precisely those eleven states with the largest IPR, i.e. those which exhibit the strongest localization properties in Fock space. To quantify this statement, we calculate the IPR averaged over the eleven solitonic states, as well as for all non-solitonic states$^6$. For the latter, figure 2 shows a moderate and monotonic increase of the IPR with growing $F$, which can be attributed to the onset of Stark localization, as expected for a very large tilt (see, e.g., [30]). In contrast, the IPR of the solitonic states is about one order of magnitude larger than the IPR of the irregular states, for $0.15 \lesssim F/J \lesssim 0.35$. For stronger tilt, some of the solitonic states disappear which we discuss in section 5 further down.

A few words regarding the dependence of our observations on the system parameters are in order. As we will explain in detail in section 5, the presence of solitonic states is not linked to a specific set of parameter values, but we expect them quite generally to appear in a regime where the Bose–Hubbard spectrum can be considered primarily chaotic. This expectation is corroborated by figure 3, which shows the parametric evolution of two Bose–Hubbard lattices with (a) $N = 6$ particles in $M = 7$ lattice sites and (b) $N = 10$ particles in $M = 5$ lattice sites, and interaction values $U$ chosen in the chaotic regime [27]. In agreement with figure 1, we observe as many solitonic states (marked in red) as sites of the lattice—$M = 7$ and $M = 5$ in figures 3(a) and (b), respectively. In

$^6$ The fluctuations of AMP and averaged IPR for the solitonic states, as visible in figure 2, can be attributed to avoided crossings that locally affect the levels and are not washed out due to the rather small number of solitonic levels that enter the average. For the non-solitonic states, AMP and averaged IPR show less fluctuations, since the average is taken over significantly more states.

---

**Figure 2.** The average maximal population (AMP, red), equation (3), and the averaged inverse participation ratio (IPR, black), equation (4), of the eigenstates of the BHH, equation (1). The solid lines correspond to the average of the $M = 11$ eigenstates with the largest individual IPR, and the dashed lines to the average of all the other eigenstates. Within the range $0.1 \lesssim F/J \lesssim 0.4$, the solitonic states are strongly localized, both on the lattice and in the Fock space. The parameters are chosen as in figure 1, and the dimension of the Hilbert space is $N = 286$.

**Figure 3.** Two spectra of the BHH (equation (1)) as a function of the static tilt $F/J$ (cf figure 1). The system parameters are (a) interaction strength $U = 0.5$, tunnelling coupling $J = 1$, $N = 6$ particles, $M = 7$ lattice sites, with Hilbert-space dimension $N = 1716$ and (b) $U = 0.3$, $J = 1$, $N = 10$, $M = 5$ and $N = 1001$. In both panels, the $M$ eigenstates with the largest IPR (see equation (4)) in the Fock number basis are plotted in red. The insets show the spectrum on a larger scale.
the following, we fix $U = J = 1$, $M = 11$ sites and $N = 3$ bosons, as a typical case of well-developed quantum chaos in coexistence with solitonic states.

4. Dynamical stability

The invariance properties of the solitonic states spelled out by the constant slope of their energy levels have an additional expression in the comparatively small avoided crossings with the irregular states of the spectrum, as evident from figure 1(b). This implies small coupling matrix elements between solitonic and irregular eigenstates, and therefore suggests an enhanced stability of the solitonic states under variations of $F$.

To test this conjecture, we investigate the multi-particle dynamics under a linear ramp of the tilt, $F(t) = F_i + R t$, from the initial value $F(0) = F_i$ to the final value $F(\Delta t) = F_f$. That is, the tilt is varied with the slew rate

$$ R \equiv \frac{F_f - F_i}{\Delta t}, \quad (5) $$

for both solitonic and irregular eigenstates $|\Psi\rangle$ of the BHH chosen from the same energy range in the bulk of the spectrum, as shown in figure 4(a). The stability of the initially prepared states is characterized in terms of the time-dependent IPR (see equation (4)), for a given rate $R$.

Let us first consider the IPR in the instantaneous basis, i.e. in the Hamiltonian’s time-dependent eigenbasis that follows the evolution of the static field $F(t)$. It is shown in figure 4(b) and reveals the broadening of the initial state due to transitions to other modes. We observe a strikingly different behaviour for both initial conditions: whereas the IPR of the irregular states decays rapidly, the solitonic states’ IPR decreases significantly slower. We also find that the decay rate observed in figure 4(b) is essentially independent of $R$, after an initial transient. A complementary view on the stability can be obtained from the IPR in the initial (fixed) basis defined by $F_i$, see figure 5(a). For very large rates, $R \gg 1$, the change in the tilt $F$ occurs on a time scale much faster than the internal (tunnelling) dynamics on the lattice. Hence, in this diabatic regime, the system wavefunction cannot adapt to the changing potential. Consequently, the IPR remains largely unaffected and a difference between solitonic and irregular states is hardly visible. In contrast, as we reduce the ramp, we observe more and more pronounced drops of the IPR for both types of initial states. However, the drop in the IPR of the irregular states is about one order of magnitude larger than that of the solitonic ones, which spells out the stability of the latter.

We finally remark that the IPR’s sensitive dependence on the slew rate $R$, present in the instantaneous basis shown in figure 4(b), can be understood from linear response theory. To this end, we note that, for sufficiently short times, the inverse participation ratio $\text{IPR}(|\Psi(t)\rangle) = \sum_j |\langle \Psi_j | \hat{U}(t) | \Psi_0 \rangle|^4$ is essentially given by the square of the survival probability $P(t) = |\langle \Psi_0 | \hat{U}(t) | \Psi_0 \rangle|^2$ since $|\langle \Psi_j | \hat{U}(t) | \Psi_0 \rangle|^2 \approx 0$ for $j \neq 0$, where $\hat{U}(t)$ is the time-evolution operator. For linearly driven chaotic systems, quantum linear response theory predicts a super-Gaussian decay $P(t) \sim \exp[-R^2 t^4]$ [31], which suggests a scale invariance of $P(t)$ and hence of the IPR with respect to the scaled time $\hat{t} = t \sqrt{R}$. This is confirmed in figure 5(b) where for solitonic and irregular initial states, the same dependence is observed. We stress, however, that between the two groups of curves there is an offset of about one order of magnitude, that is, the absolute decay is much weaker for the solitonic states.

5. Generating mechanism

In the previous section, we numerically confirmed the dynamical stability of the solitonic states, as suggested by their parametric level evolution. We now turn to the discussion of the underlying mechanism [32]. The existence of eigenstates of a many-body system with all particles localized close to each other, despite the presence of repulsive inter-particle

7 Note that the latter exhibit oscillations of $1 - \text{IPR}$ around $\hat{t} = 1$, which are likely to result from the regularity of the integrable part of the spectrum.
interactions $U$, was experimentally first demonstrated for pairs of atoms [12]. Theoretical investigations of bound atom states as well as of bound quasi-particle states in nonlinear lattice systems (see, e.g., [33–39]) also revealed the existence of bound states including more than two (quasi-) particles [13, 14, 40–42]. Recently, also Bloch oscillations for initially localized states of interacting bosons were investigated [43]. Such repulsively bound many-particle states are formed as a consequence of the energy mismatch between the on-site interaction energy and the maximal kinetic energy that can be realized in the lowest energy band in the lattice. That is, they are energetically isolated from the remainder of the spectrum.

In contrast to that, for our presently solitonic states, which run through the bulk of the spectrum, also energetically allowed transitions (to the irregular states) are blocked, and, in addition, lead to an enhanced stability under perturbations. This can be understood in terms of the transitions between the energy eigenstates that are induced by the tilt, as discussed in the following. The time-independent part of the corresponding transition amplitude from some initial state $|\Psi_i\rangle$ to a final state $|\Psi_f\rangle$ is given by the corresponding matrix element of the centre-of-mass operator $\langle \Psi_f | \sum_i \hat{I}\hat{S}_i | \Psi_i \rangle$. As shown by our analysis in figure 2, in the case of an irregular state, the atoms are distributed over the entire lattice, while the solitonic states are distinguished by the fact that most of the atoms occupy the same lattice site. As a result, the solitonic states are approximate eigenstates of the centre-of-mass operator and hence the matrix element $\langle \Psi_f | \sum_i \hat{I}\hat{S}_i | \Psi_i \rangle$ with $|\Psi_f\rangle$ an irregular and $|\Psi_i\rangle$ a solitonic state becomes very small, since both are system eigenstates and thus mutually orthogonal. More intuitively, a transition from a solitonic to an irregular state requires the redistribution of essentially all atoms over the entire lattice. The tunnelling of an atom over more than a single lattice site, as well as the simultaneous tunnelling of more than one atom, is, however, negligible for typical lattice depths, since it corresponds to higher order processes in the BHH. Whereas such processes can occur as resonance phenomena for large interaction strengths $U$ [43], they are negligible in the presently discussed regime of intermediate interactions. Furthermore, in a tilted, infinitely long lattice (but with finite particle numbers), all states are localized [43] and in that case, even analytic estimates of the localization volume of the states (and thus of the above transition matrix elements) can be obtained via composite-particle eigenvectors (see [43]).

One might now wonder whether, similarly to the solitonic states, there are system eigenstates where (to a good approximation) all atoms but one are located on a single site, and the remaining particle is localized on some other site. Indeed, we find such states, that show comparable features as the solitonic states described so far, and they will be referred to as solitonic states of second order. For the case of three atoms, such states have also been analysed in [14]. For a transition from a solitonic state of first order to a solitonic state of second order, only tunnelling of a single atom over a single site is necessary, and hence one expects significantly larger coupling as compared to irregular states. Thus, in the simplest approximation one can treat two such states as a two-level system of (idealized) Fock states $|\ldots, 0, N, 0, \ldots\rangle$ and $|\ldots, 1, N-1, 0, \ldots\rangle$. Assuming vanishing coupling $J = 0$, their energy would coincide only at $F = U(N-1)$ which, for $U_2 = 1$, is far outside the regime in which solitonic states exist (see figures 1 and 2). Once we take into account the tunnelling, an avoided crossing of width $2J\sqrt{N}$ emerges between the two states and thus affects the stability of the solitonic states for tilts beyond the threshold value $F_t \approx U(N-1) - J\sqrt{N}$. For the parameter values used in figures 1 and 2, this corresponds to $F_t \approx 0.27$, and this is indeed where the solitonic states start to dissolve.

If, for larger $N$, more particles are bound in a solitonic state, more have to undergo tunnelling processes in order to transform into an irregular state. From this point of view, the stability of the solitonic states is expected to be enhanced with increasing particle number at constant interaction strength.
related aspects. Due to the comparably small particle number scope of this paper, we would like to shortly address some on the chaotic sea. The spectrum of the BHH (equation (1)) for fixed effective interaction $UN = 3$ and three different particle numbers $N$, plotted as a function of the static tilt $F/J$ (cf figure 1). For better comparability, the energies are additionally scaled by $N$. The system parameters are tunnelling coupling $J = 1$, $M = 3$ lattice sites, and particle numbers (Hilbert-space dimension) (a) $N = 10$ ($N = 66$), (b) $N = 20$ ($N = 231$) and (c) $N = 50$ ($N = 1326$). In all panels, only the six eigenstates with the largest IPR (see equation (4)) in the Fock number basis are plotted in red to keep the graphical presentation clear. We note, though, that the number of solitonic states increases with $N$. That level which crosses the bulk with negative slope and, in all three panels, is marked in red up to around $F/J \approx 0.6$ corresponds to a solitonic state of first order.

We first remark that in all cases, solitonic states are indeed observed: we highlighted the six levels with the highest IPR and note that in all three panels, the red level that crosses the bulk with negative slope and that is marked in red up to around $F/J \approx 0.6$ corresponds to one of the three solitonic states of first order. Second, qualitatively speaking, larger particle numbers result in a smaller effective Planck constant: that is, for increasing $N$ and $UN = \text{const}$, more quantum states share the available, $N$-independent phase space. Thus, also more states can exist within the island, i.e. an increased number of solitonic states of higher order are expected. This, as well, is confirmed by figure 6: for $N = 10$ particles, we find essentially one solitonic state with negative slope, corresponding to localization on the first lattice site, with negative single-particle on-site energy $F/ = -F$, see below equation (1). In contrast, for $N = 20$, two such levels can be clearly distinguished, while for $N = 50$ already three such states are among the six levels with the highest IPR. Finally, the detailed inspection of the spectra shown in figure 6 reveals that the critical tilt strength $F_t$, at which the solitonic state of highest order disappears, displays a weak dependence on the particle number $N$. Namely, for $N = 10$ ($N = 50$), we determine $F_t$ to be approximately $F/J \approx 0.65$ ($F/J \approx 0.75$). This indicates that for increasing $N$, the criterion for $F_t$ should be based on the following semiclassical consideration instead of the simplified picture given above: as the tilt $F$ is increased, the stable island (associated with localization on the corresponding lattice site) shrinks and, thus, fewer and fewer solitonic states can reside on it. Finally, at a certain value of the tilt $F$, given by the $(N$-independent) mean-field equations, the island vanishes and this determines the critical value $F_t$ at which the solitonic state of highest order disappears.

With respect to the experimental viability of the investigated setup, it is fair to say that not only the lattice geometry is under exquisite control by the experimentalists, but also interaction strength and particle detection: in typical optical lattice-based experiments, the ratio $U/J$ can be

---

8 We note that the parametric level dynamics presented in figures 1 and 3 obey this requirement, i.e. $UN = 3$ in the three cases shown. The mean-field dynamics is not the same, however, among lattices of different size $M$: given the fact that the lattice lengths $M$ were on purpose chosen differently, such as to demonstrate the general existence of solitonic states for various configurations, the respective phase spaces have a different dimension of $2M$ [19] and are thus not identical.

9 For a two-site Bose-Hubbard system, a similar strategy was successfully applied in [48] (see also references therein), and a tilted chaotic three-mode system has been studied in the mean-field limit in [49, 50].
rapidly switched off within about one hundred μs and was used to accommodate for gravitation. Presumably, this levitation can as well be tuned to a nonzero value so as to adjust the strength of the gravitational forces, which would amount to controlling the lattice tilt F. Finally, we note that the detection efficiency has reached the single-atom level [52, 53], which permits a very precise counting of occupation numbers. Thus, the experimental detection of solitonic states should be within reach.

6. Conclusion

In summary, we have studied the dynamical evolution of ultracold bosons confined on a one-dimensional optical lattice, which is subject to a tilt. Our present work focused on the parameter regime in which kinetic, inter-particle interaction and on-site energy are balanced. In this regime, where the corresponding Bose–Hubbard Hamiltonian with an additional tilt term can be characterized as being primarily chaotic in a spectral sense [26, 27, 54], we have identified regular structures in the parametric level evolution. Associated with these structures are the solitonic states, which are distinguished by a strong localization on the lattice as well as in the Fock space, which barely changes with the tilt strength. Unlike other studies on energetically isolated bound states [12–14], this property results from the weak coupling of the solitonic states to the bulk of energy levels [32].

Based on the inverse participation ratio, evaluated in the fixed as well as in the adiabatic basis of the Bose–Hubbard Hamiltonian, we compared the dynamical stability of these solitonic states to an ensemble of neighbouring states in the bulk of the spectrum, as the tilt was ramped with different slew rates. We observed a drastically increased robustness of the solitonic states, spelled out by a significantly larger inverse participation ratio at the end of the dynamical evolution.

The present work belongs to that branch of ultracold atom physics which is concerned with comparatively small particle numbers. This direction has recently seen substantial experimental advances, not at last in the resolution of imaging techniques [52, 53] that would allow us to directly identify the solitonic states. Their remarkable dynamical robustness makes them excellent candidates, e.g., for the preparation of stable quantum few-body states, in a parameter regime where, due to the generic presence of spectral chaos, a substantial portion of the eigenstates sensitively depends on system parameters such as the tilt. On the other hand, provided that coherent superpositions of solitonic states can be prepared, reasonably stable tunnel dynamics of more than one boson [43] would be realizable for relatively small interaction values. Finally, in light of the mean-field limit discussed in the previous section, the crossover in stability of these states from the few- to the many-body regime might be explored.

Acknowledgments

We acknowledge financial support of the Deutsche Forschungsgemeinschaft DFG via the Research Unit 760, and through a personal grant (FM). Support by the EU COST Action MP1006 ‘Fundamental Problems in Quantum Physics’ is acknowledged. BO gratefully acknowledges financial support of the European Science Foundation within the QUDEDIS program and of the Polish Government (scientific funds 2008–2011).

References

[1] Dalfovo F, Giorgini S, Pitaevskii L P and Stringari S 1999 Theory of Bose–Einstein condensation in trapped gases Rev. Mod. Phys. 71 463
[2] Fertig C D, O’Hara K M, Huckans J H, Rolston S L, Phillips W D and Porto J V 2005 Strongly inhibited transport of a degenerate 1D Bose gas in a lattice Phys. Rev. Lett. 94 120403
[3] Ponsonare A V, Madronero J, Kolovsky A R and Buchleitner A 2006 Atomic current across an optical lattice Phys. Rev. Lett. 96 050404
[4] Morsch O and Oberthaler M 2006 Dynamics of Bose–Einstein condensates in optical lattices Rev. Mod. Phys. 78 179
[5] Bloch I, Dalibard J and Zwerger W 2008 Many-body physics with ultracold gases Rev. Mod. Phys. 80 885
[6] de Filippo S, Fusco Girard M and Salerno M 1989 Avoided crossing and nearest-neighbour level spacings for the quantum DST equation Nonlinearity 2 477
[7] Chelles A 1996 Nearest-neighbour level spacings for the non-periodic discrete Schrödinger equation J. Phys. A: Math. Gen. 29 4515
[8] Kolovsky A R and Buchleitner A 2004 Quantum chaos in the Bose–Hubbard model Europhys. Lett. 68 632
[9] Hiller M, Kottos T and Geisel T 2009 Wave-packet dynamics in energy space of a chaotic trimeric Bose–Hubbard system Phys. Rev. A 79 023621
[10] Chirikov B V 1979 A universal instability of many-dimensional oscillator systems Phys. Rep. 52 263
[11] Buchleitner A, Delande D and Zakrzewski J 2002 Non-dispersive wave packets in periodically driven quantum systems Phys. Rep. 368 409
[12] Winkler K, Thalhammer G, Lang F, Grimm R, Denschlag J H, Daley A J, Kantan A, Büchler H P and Zoller P 2006 Repulsively bound atom pairs in an optical lattice Nature 441 853
[13] Dorignac J, Eilbeck J C, Salerno M and Scott A C 2004 Quantum signatures of Breather–Breather interactions Phys. Rev. Lett. 93 025504
[14] Valiente M, Petersyan D and Saenz A 2010 Three-body bound states in a lattice Phys. Rev. A 81 011601(R)
[15] Hauke P 2000 Quantum Signatures of Chaos 2nd edn (Berlin: Springer)
[16] Jaksh D, Bruder C, Cirac I J, Gardiner C W and Zoller P 1998 Cold bosonic atoms in optical lattices Phys. Rev. Lett. 81 3108
[17] Inouye S, Andrews M R, Stenger J, Müllner H-J, Stamper-Kurn D M and Ketterle W 1998 Observation of Feshbach resonances in a Bose–Einstein condensate Nature 392 151
[18] Chin C, Grimm R, Julienne P and Tiesinga E 2010 Feshbach resonances in ultracold gases Rev. Mod. Phys. 82 1225
[19] Eilbeck J C, Lomdahl P S and Scott A C 1985 The discrete self-trapping equation Physica D 16 318
[20] Smerzi A, Fantoni S, Giovanazzi S and Shenoy S R 1997 Quantum coherent atomic tunneling between two trapped Bose–Einstein condensates Phys. Rev. Lett. 79 4950
[21] Vardi A and Anglin J R 2000 Bose–Einstein condensates beyond mean field theory: quantum backreaction as decoherence Phys. Rev. Lett. 86 568
[22] Anglin J R and Vardi A 2001 Dynamics of a two-mode Bose–Einstein condensate beyond mean-field theory Phys. Rev. A 64 013605
[23] Sakmann K, Streltsov A I, Alon O E and Cederbaum L S 2007 Exact quantum dynamics of a bosonic Josephson junction Phys. Rev. Lett. 103 220601
[24] Sakmann K, Streltsov A I, Alon O E and Cederbaum L S 2010 Quantum dynamics of attractive versus repulsive bosonic Josephson junctions: Bose–Hubbard and full-Hamiltonian results Phys. Rev. A 82 013620
[25] Chuchem M, Smith-Mannschott K, Hiller M, Kottos T, Vardi A and Cohen D 2010 Quantum dynamics in the bosonic Josephson junction Phys. Rev. A 82 053617
[26] Kolovsky A R and Buchleitner A 2003 Floquet–Bloch operator for the Bose–Hubbard model with static field Phys. Rev. E 68 056213
[27] Venzl H 2011 Ultracold bosons in tilted optical lattices—impact of spectral statistics on simulability, stability, and dynamics PhD Thesis Albert-Ludwigs Universität Freiburg, Germany (http://www.freidok.uni-freiburg.de/volltexte/8126/)
[28] Feynman R P 1959 Forces in molecules Phys. Rev. 56 340
[29] Zakrzewski J, Buchleitner A and Delande D 1997 Non-dispersive wave packets as solitonic solutions of level dynamics Z. Phys. B 103 115
[30] Glück M, Kolovsky A R and Korsch H J 2002 Wannier–Stark resonances in optical and semiconductor superlattices Phys. Rep. 366 103
[31] Cohen D and Kottos T 2000 Quantum-mechanical nonperturbative response of driven chaotic mesoscopic systems Phys. Rev. Lett. 85 4839
[32] Venzl H, Zech T, Oles B, Hiller M, Mintert F and Buchleitner A 2010 Solitonic eigenstates of the chaotic Bose–Hubbard Hamiltonian Appl. Phys. B 98 647
[33] Ovchinnikov A A 1969 Localized long-lived vibrational states in molecular crystals Zh. Eksp. Teor. Fiz. 57 263 Ovchinnikov A A 1970 Localized long-lived vibrational states in molecular crystals Sov. Phys.—JETP 30 147
[34] Scott A C, Eilbeck J C and Gilhøj H 1994 Quantum lattice solitons Physica D 78 194
[35] Piil R and Mølmer K 2007 Tunneling couplings in discrete lattices, single-particle band structure, and eigenstates of interacting atom pairs Phys. Rev. A 76 023607
[36] Petrovyan D, Schmidt B, Anglin J R and Fleischhauer M 2007 Quantum liquid of repulsively bound pairs of particles in a lattice Phys. Rev. A 76 033606
[37] Wang L, Hao Y and Chen S 2008 Quantum dynamics of repulsively bound atom pairs in the Bose–Hubbard model Eur. Phys. J. D 48 229
[38] Weiss C and Breuer H-P 2009 Photon-assisted tunneling in optical lattices: ballistic transport of interacting boson pairs Phys. Rev. A 79 023608
[39] Jin L, Chen B and Song Z 2009 Coherent shift of localized bound pairs in the Bose–Hubbard model Phys. Rev. A 79 032108
[40] Miller P D, Scott A C, Carr J and Eitheck J C 1991 Binding energies for discrete nonlinear Schrödinger equations Phys. Scr. 44 509
[41] Wang W Z, Gammel J Tinka, Bishop A R and Salkola M I 1996 Quantum breathers in a nonlinear lattice Phys. Rev. Lett. 76 3598
[42] Schofield S A, Wyatt R E and Wolynes P G 1996 Computational study of many-dimensional quantum vibrational energy redistribution: I. Statistics of the survival probability J. Chem. Phys. 105 940
[43] Hoberman R, Krimmer D O, Haque M and Flach S 2010 Interaction-induced fractional Bloch and tunneling oscillations Phys. Rev. A 81 065601
[44] Franzosi R and Penna V 2003 Chaotic behavior, collective modes, and self-trapping in the dynamics of three coupled Bose–Einstein condensates Phys. Rev. E 67 046227
[45] Hiller M, Kottos T and Geisel T 2006 Complexity in parametric Bose–Hubbard Hamiltonians and structural analysis of eigenstates Phys. Rev. A 73 061604(R)
[46] Trimbach F, Withnau D and Korsch H J 2009 Beyond mean-field dynamics of small Bose–Hubbard systems based on the number-conserving phase space approach Phys. Rev. A 79 013608
[47] Ng G S, Hennig H, Fleischmann R, Kottos T and Geisel T 2009 Avalanches of Bose–Einstein condensates in leaking optical lattices New J. Phys. 11 073045
[48] Smith-Mannschott K, Chuchem M, Hiller M, Kottos T and Cohen D 2009 Occupation statistics of a Bose–Einstein condensate for a driven Landau–Zener crossing Phys. Rev. Lett. 102 230401
[49] Wang G-F, Ye D-F, Fu L-B, Chen X-Z, and Liu J 2006 Landau–Zener tunneling in a nonlinear three-level system Phys. Rev. A 74 033414
[50] Graefe E M, Korsch H J and Withnau D 2006 Mean-field dynamics of a Bose–Einstein condensate in a time-dependent triple-well trap: nonlinear eigenstates, Landau–Zener models, and stimulated Raman adiabatic passage Phys. Rev. A 73 013617
[51] Gustavsson M, Haller E, Mark M J, Danzl J G, Rojas-Kopeinig G and Nägerl H-C 2008 Control of interaction-induced dephasing of Bloch oscillations Phys. Rev. Lett. 100 080404
[52] Bakr W S, Gillen J I, Peng A, Folling S and Greiner M 2009 A quantum gas microscope for detecting single atoms in a Hubbard-regime optical lattice Nature 462 74
[53] Sherson J F, Weitenberg C, Endres M, Cheneau M, Bloch I and Kuhr S 2010 Single-atom-resolved fluorescence imaging of an atomic Mott insulator Nature 467 68
[54] Buchleitner A and Kolovsky A R 2003 Interaction-induced decoherence of atomic Bloch oscillations Phys. Rev. Lett. 91 253002