Calculation and analysis of gyroplane trimming characteristics

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Abstract. In order to analyse the trimming characteristics of gyroplane, a complete nonlinear dynamic equation is established, which is a seven-degree of freedom model. By using constraints, the quantity of unknown variables is equal to the quantity of equations. In the process of solving the equation, Levenberg-Marquardt algorithm is applied. After comparing the results of mathematical model to the flight test, it can be concluded that the seven degree of freedom model is valid.

Nomenclature

\[ C_T = \text{thrust coefficient} \]
\[ C_H = \text{drag coefficient} \]
\[ C_Y = \text{lateral force coefficient} \]
\[ C_Q = \text{torque coefficient} \]
\[ C_{L,HT} = \text{lift coefficient of horizontal tail} \]
\[ C_{D,HT} = \text{drag coefficient of horizontal tail} \]
\[ C_{L,VT} = \text{lift coefficient of vertical tail} \]
\[ C_{D,VT} = \text{drag coefficient of vertical tail} \]
\[ e = \text{offset value} \]
\[ M_S = \text{mass moment} \]
\[ Q_{PR} = \text{propeller torque} \]
\[ S_{HT} = \text{effective area of horizontal tail} \]
\[ S_{VT} = \text{effective area of vertical tail} \]
\[ T_{PR} = \text{propeller thrust} \]
\[ Q_{PR} = \text{propeller torque} \]
\[ (X_{PR}, Y_{PR}, Z_{PR}) = \text{coordinates of propeller hub centre} \]
\[ \delta_t = \text{throttle position} \]
\[ \eta_x = \text{lateral directional control of main rotor} \]
\[ \eta_r = \text{longitudinal control of the main rotor} \]
\[ \delta_r = \text{rudder deviation} \]
\[ \theta = \text{angle of pitch} \]
\[ \Omega = \text{rotation speed of main rotor} \]

1. Introduction

The gyroplane, or autogiro, is a class of rotorcraft, which is different from the helicopter. In fact, it was the first powered, heavier-than-air aircraft to fly successfully other than a conventional airplane. [1] Besides, it paved way for the development of helicopter both in engineering field like introducing cyclic pitch control and blades attached to the rotor hub by means of a hinge and in theory field. Unfortunately, from WWII broken out, helicopter attracts more and more attentions, while gyroplane gradually lost its attractions from the public and researchers.
Recent years, gyroplane meets new development opportunity since people realize its potential value. Compared to the helicopter and the fixed-wing aircraft, it owns many advantages, for example, excellent take-off and landing performance, simple structure, low cost, good safety, small noise etc.

From a scientific perspective, compering to substantial researches of other types of aircrafts, there have been a few studies of gyroplane. Although there are some engineering researches before WWII when rotor research has not focus on helicopter, they just own historical significances nowadays. Until 1991, UK CAA organized a program of research on gyroplane because of numerous fatal accidents. Funded by UK CAA, a team from Glasgow University tried to find the factors influencing gyroplane stability, control and handling quality. [2-4] However, there is no specific description about the mathematical model on their papers. Considering the few researches on this issue, there is a need to do more research to help the gyroplane technique develop. In aerodynamic research, it is a fundamental but vital step to establish a suitable equilibrium equation to analyze the trimming characteristics of gyroplane. This paper illustrates a seven-degree of freedom mathematical model to simulate gyroplane dynamic problem. Through using this model to analyze a kind of gyroplane named VPM M16, comparing the results from the model and the flight test, it can be concluded that this model is reliable and this mathematical model can be used to typical gyroplane.

![Gyroplane Diagram](image1)

**Figure 1.** Autogiro rotor (a) provides lift, with forward propulsion being provided by a conventional propeller, compared to the helicopter (b) where the rotor provides both lift and propulsion. [1]

2. Mathematical model

The most significant difference between gyroplane and helicopter is whether the main rotor connects to the engine. For gyroplane, its main rotor rotates only because of the inflow. So, for gyroplane, when it is in forward flight, the main rotor tilts backward, while for helicopter, its main rotor tilts forward. Gyroplane main rotor provides lift, with forward propulsion being provided by a conventional propeller, compared to the helicopter where the main rotor provides both lift and propulsion. The difference is shown on figure 1.

The gyroplane can be divided into five subsystems, main rotor, fuselage, horizontal tail, vertical tail and propeller. Through calculating the force and torque of every subsystem, summing them up together, we can get the final forces and torques.

2.1. Main rotor

Main rotor is the most complex part of the gyroplane. These assumptions are made when deriving the rotor aerodynamic force: the blade is rigid without considering the shimmy effect. In this way, the rotor aerodynamic model is composed of rotor force and torque model, rotor waving model and rotor induced velocity model. The blade twist angle is linearly distributed, and the incoming flow is evenly distributed.

Combined with the rotor blade element theory and the momentum theory, the aerodynamic model of the rotor is established by the formula derived from the blade radius and the azimuth integral along the blade. The integral format rotor aerodynamic model is based on the rotor lift line theory. The aerodynamic load of the section is calculated according to the flow velocity and elevation of the air
foil section. Then the aerodynamic model of the rotor aerodynamic force in the wind shaft system is derived. The expressions of the rotor tension, rear force, lateral force and rotor torque are as (1):

\[
\begin{align*}
T &= C_T \rho \pi R^2 (\Omega)^2 \\
H &= C_H \rho \pi R^2 (\Omega)^2 \\
Y &= C_Y \rho \pi R^2 (\Omega)^2 \\
M &= \frac{k}{2} M_s\Omega^2 \epsilon_1 \beta \epsilon_1 \\
L &= \frac{k}{2} M_s\Omega^2 \epsilon_1 \beta \epsilon_1 \\
Q &= \frac{k}{2} M_s\Omega^2 \epsilon_1 \beta \epsilon_1
\end{align*}
\]

These coefficients are derived from Johnson’s book. [5]

2.2. Fuselage
The aerodynamic fuselage model is built based on wind tunnel test data. [6] The forces and the torques are as (2) (3):

\[
\begin{align*}
[F_x] &= [C_x] \frac{1}{2} \rho (u_F^2 + v_F^2 + w_F^2) \pi R^2 \\
[F_y] &= [C_y] \frac{1}{2} \rho (u_F^2 + v_F^2 + w_F^2) \pi R^2 \\
[F_z] &= [C_z] \frac{1}{2} \rho (u_F^2 + v_F^2 + w_F^2) \pi R^2 \\
[M_x] &= [C_L] \frac{1}{2} \rho (u_F^2 + v_F^2 + w_F^2) \pi R^2 \\
[M_y] &= [C_M] \frac{1}{2} \rho (u_F^2 + v_F^2 + w_F^2) \pi R^2 \\
[M_z] &= [C_N] \frac{1}{2} \rho (u_F^2 + v_F^2 + w_F^2) \pi R^2
\end{align*}
\]

All of these aerodynamic coefficients $C_x$, $C_y$, $C_z$, $C_L$, $C_M$, $C_N$ are processed through the conversion and the non-dimension of the rotor radius, which are related to the angle of attack and the sideslip angle.

2.3. Horizontal tail
The aerodynamic horizontal tail model is built based on the combination of the wind tunnel test data and theory. In the wind axis, the lift and drag on the horizontal tail are as formula (4) (5). Then the conservation is conducted from the wind axis to the body axis. Lift and drag coefficient of the horizontal tail are related to the angle of attack and sideslip angle.

\[
\begin{align*}
L_{HT} &= \frac{1}{2} \rho (u_{HT}^2 + v_{HT}^2 + w_{HT}^2) S_{HT} C_{L-HT} \\
D_{HT} &= \frac{1}{2} \rho (u_{HT}^2 + v_{HT}^2 + w_{HT}^2) S_{HT} C_{D-HT}
\end{align*}
\]

2.4. Vertical tail
The aerodynamic vertical tail model is built similarly to the horizontal tail. The lift and drag coefficient are as formula (6) (7).

\[
\begin{align*}
L_{VT} &= \frac{1}{2} \rho (u_{VT}^2 + v_{VT}^2 + w_{VT}^2) S_{VT} C_{L-VT} \\
D_{VT} &= \frac{1}{2} \rho (u_{VT}^2 + v_{VT}^2 + w_{VT}^2) S_{VT} C_{D-VT}
\end{align*}
\]

Lift and drag coefficients of the vertical tail are related to the angle of attack and sideslip angle.

2.5. Propeller
The propeller model is built based on the parameters i.e. thrust, output power etc. provided by the engine manufacturer. In the body coordinate system, the aerodynamic forces and torques of the propeller are as (8) (9):

\[
\begin{align*}
\begin{bmatrix}
F_{xPR} \\
F_{yPR} \\
F_{zPR}
\end{bmatrix} &=
\begin{bmatrix}
T_{PR} \\
0 \\
0
\end{bmatrix} \\
\begin{bmatrix}
M_{xPR} \\
M_{yPR} \\
M_{zPR}
\end{bmatrix} &=
\begin{bmatrix}
Q_{PR} \\
0 \\
0
\end{bmatrix} +
\begin{bmatrix}
0 & -z_{PR} & y_{PR} \\
z_{PR} & 0 & -x_{PR} \\
y_{PR} & x_{PR} & 0
\end{bmatrix}
\begin{bmatrix}
F_{xPR} \\
F_{yPR} \\
F_{zPR}
\end{bmatrix}
\end{align*}
\]

(8)

(9)

2.6. Resultant forces and resultant torques

By adding up all of the forces and torques of these subsystems, the total force as (10) and moment as (11) of the gyroplane can be gotten.

\[
\begin{align*}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} &=
\begin{bmatrix}
F_c \\
F_y \\
F_z
\end{bmatrix} +
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} +
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} +
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} +
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} \\
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} &=
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} +
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} +
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} +
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix}
\end{align*}
\]

(10)

(11)

Considering the main rotor is an independent freedom in this case, there is a group of seven-degree of freedom equilibrium equations as (12) (13) (14).

\[
\begin{align*}
\begin{bmatrix}
F_x \\
F_y \\
F_z
\end{bmatrix} +
\begin{bmatrix}
1 & 0 & 0 \\
0 & \cos\phi & \sin\phi \\
0 & -\sin\phi & \cos\phi
\end{bmatrix}
\begin{bmatrix}
\cos\theta & 0 & -\sin\theta \\
0 & 1 & 0 \\
\sin\theta & 0 & \cos\theta
\end{bmatrix}
\begin{bmatrix}
m_{e,g}
\end{bmatrix} &=
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \\
\begin{bmatrix}
M_x \\
M_y \\
M_z
\end{bmatrix} &=
\begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \\
Q_x &=
0
\end{align*}
\]

(12)

(13)

(14)

3. Calculation of trim equations

Since there has been the seven-degree of freedom trim model, the next step is to solve this problem. This trim calculation is essentially to solve nonlinear equation. The nonlinear flight dynamic model is

\[
\dot{x} = f(x,u)
\]

(15)

where \(x\) is the state vector, \(u\) is the control vector as (16) (17)

\[
x = (u, v, w, \theta, \phi, \varphi, \Omega)
\]

(16)

\[
u = (\delta_\alpha, \eta_1, \eta_2, \delta_\alpha, \theta, \phi, \Omega)
\]

(17)

Using the constraints from Padfield [7], the ground velocity, the flight path angle, the velocity of flight deflecting path angle and sideslip angle are defined. Then the problem becomes solving seven-variable nonlinear equations as (18).

\[
g_i(y_1, y_2, y_3, y_4, y_5, y_6, y_7) = 0, i = 1...7
\]

(18)

where \(y\) contains variables as (19)

\[
y = (\delta_\alpha, \eta_1, \eta_2, \delta_\alpha, \theta, \phi, \Omega)
\]

(19)
Considering the advantages of LMA (Levenberg-Marquardt algorithm) [8], the problem can be solved by it.

4. Mathematical model validation

In this paper, the example gyroplane is VPM M16, which is a normal configuration gyroplane. There are parameters of VPM M16 in table 1.

Inputting these data into the model on Matlab and calculating the state variables and control variables in different ground speed situations. By comparing the variables from the model to the flight test [7], it can be concluded that the model is valid although there are some deviations. The comparison is shown on figure 2-6.

Rotor speed is consistent with the flight test counterpart both in tendency and number. The deficiencies of rotor speed between the model and the test may be due to the blade twist, which may manifest itself aerelasticity in flight and is not incorporated in the rigid-blade model. Longitudinal and latitudinal tilt of main rotor compares favourably with the flight measurement especially at the high speed. Although there is a uniform error across the speed, this is less than five degrees. The pitch attitude shows the same trend which is downward as the speed increases. It is easy to understand this trend because as the speed increases, the inflow increases, to maintain the same lift, there is fewer need of angle of attack of main rotor. As for the roll attitude, the prediction of this model is not precious, but the results both in model and flight test are less than 2 degree and this number is almost not affected by velocity.

| Item                        | Value |
|-----------------------------|-------|
| Mass (kg)                   | 450   |
| Ixx (kg.m²)                 | 195   |
| Iyy (kg.m²)                 | 637   |
| Izz (kg.m²)                 | 442.5 |
| Ixz (kg.m²)                 | -46   |
| Airfoil                     | NACA 8H12 |
| Main Rotor Blade Number     | 2     |
| Radius (m)                  | 4.267 |
| Flapping Inertial (kg.m²)   | 51.6  |
| Chord (m)                   | 0.22  |
| Radius (m)                  | 0.86  |
| Chord (m)                   | 0.1   |
| Fuselage Side Area (m²)     | 1.4   |
| Plan Fuselage Area (m²)     | 1.6   |
| Fuselage Frontal Area (m²)  | 0.59  |

Table 1. Parameters of VPM M16[9].

Figure 2. Rotor Speed Comparison.
5. Conclusion
By establishing five subsystems model separately and summing up the forces and torques together, there is a seven-degree-freedom model to simulate the trimming characteristics. Although there are some deviations between the model results and the flight test data, the differences are acceptable and explainable. Consequently, the seven-degree-freedom model is valid for simulating a typical gyroplane. The values about trimming showed a logical trend as the velocity changes. Consequently, the mathematical model is suitable to analyse a typical gyroplane aerodynamic problem.

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**Figure 3.** Longitudinal Tilt Angle of Main Rotor Comparison.

**Figure 4.** Lateral Tilt Angle of Main Rotor Comparison.

**Figure 5.** Pitch Attitude Comparison.

**Figure 6.** Roll Attitude Comparison.
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