Research Article

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Large amplitude vibration of doubly curved FG-GRC laminated panels in thermal environments

Abstract: A study on the large amplitude vibration of doubly curved graphene-reinforced composite (GRC) laminated panels is presented in this paper. A doubly curved panel is made of piece-wise GRC layers with functionally graded (FG) arrangement along the thickness direction of the panel. A GRC layer consists of polymer matrix reinforced by aligned graphene sheets. The material properties of the GRC layers are temperature dependent and can be estimated by the extended Halpin-Tsai micromechanical model. The modelling of the large amplitude vibration of the panels is based on the Reddy’s higher order shear deformation theory and the effects of the von Kármán geometric nonlinearity, the panel-foundation interaction and the temperature variation are included in the derivation of the motion equations of the panels. The solutions for the large amplitude vibration of the doubly curved FG-GRC laminated panels are obtained by applying a two-step perturbation approach. A parametric study is carried out to determine the influences of foundation stiffness, temperature variation, FG distribution pattern, in-plane boundary condition and panel curvature ratio on the natural frequencies and the nonlinear to linear frequency ratios of the doubly curved FG-GRC laminated panels.

Keywords: Nanocomposites; Functionally graded materials; Temperature-dependent properties

1 Introduction

Doubly curved panels have many important engineering applications. During their service life, these panels may be subjected to different combinations of loading and environmental conditions which can cause the panels to experience large amplitude vibration with the panel deflection being in the order of the panel thickness. Therefore, it is necessary to fully understand the nonlinear vibration behaviors of doubly curved panels in thermal environments in engineering design and practice. Many studies have been carried out on the nonlinear vibration behavior of isotropic and composite laminated curved panels [1–8]. It is observed that an acceptable agreement for flat plates can be achieved by different researchers. However, the results for a curved panel exist large discrepancies which may be due to the hardening behavior (i.e. increase in vibration amplitude leads to the increase of the nonlinear frequency) or the softening behavior of the panel (i.e. increase in vibration amplitude leads to the decrease of the nonlinear frequency) [1–8].

Advanced composite materials possess unique features that many of the conventional materials do not have. We have witnessed the increased use of advanced composite materials, including the functionally graded material (FGM) [9], as key structural members in various engineering applications. Shen [10] first proposed to use FGM concept to nanocomposite structures in order to fully utilize the effect of nano filler reinforcement in composite structures. Shen and his co-authors and other research teams further studied the linear and nonlinear vibration characteristics of FGM curved panels [11–17] and FG carbon nanotube reinforced composite (CNTRC) curved panels [18–22] subject to temperature changes and/or resting on elastic foundations. Since the discovery of graphene by Geim and Novoselov in 2004 [23], extensive studies on graphene have been conducted by many researchers and the extraordinary material properties of graphene have been widely reported [24–28]. Due to these remarkable properties, graphene has become one of the ideal reinforcement agents in creating advanced polymer composites [29]. For graphene-based nanocomposites, one kind
is graphene platelet reinforced composite (GPLRC) where both the polymer matrix and graphene platelets (GPLs) are assumed to be isotropic and independent of temperature. In essence, GPL reinforced composites belong to particle reinforced composites. The GPLRC model is relatively simple and was adopted by many researchers, for example, the vibration analysis for doubly-curved panels reinforced by GPLs was reported by Wang et al. [30, 31] and Fazelzadeh et al. [32]. It is noted that graphene sheets have anisotropic and temperature dependent material properties [24–28] and it is possible to align graphene sheets in the polymer matrix that can result in better reinforcement effect for the graphene-based composites [33–35]. Shen et al. [36] first proposed a functionally graded graphene reinforced composite (GRC) model where aligned graphene reinforcements are anisotropic and the material properties of both the polymer matrix and graphene sheets are assumed to be temperature dependent. As reported by Lei et al. [37] the GRC model is more accurate than GPLRC model and was adopted by many researchers [38–43].

This paper will investigate the nonlinear free vibration behavior of doubly curved GRC laminated panels resting on elastic foundations in thermal environments. The panels with the piece-wise functionally graded GRC laminar layer pattern are considered in the study. The novelty of this study lies in the account of both the functionally graded material configurations and the temperature dependent properties in the nonlinear vibration analyses of FG-GRC laminated doubly curved panels. The extended Halpin-Tsai micromechanical model is applied to estimate the material properties of the GRC layers. The Reddy’s third order shear deformation shell theory is employed to derive the motion equations for the GRC laminated panels. Note that the motion equations of the panels also include the effects of the von Kármán geometric nonlinearity, the foundation support and the temperature variation. The boundary conditions of the panels are assumed to be simply supported. A two-step perturbation approach is employed to determine the nonlinear frequencies of doubly curved GRC laminated panels. The large amplitude vibration behavior of doubly curved FG-GRC laminated panels subject to the influence of foundation support and temperature variation is discussed in detail.
One of the key issues in structural analysis of graphene reinforced composites is the thermomechanical property evaluation of the composite. The Halpin-Tsai micromechanical model [44] is employed to estimate the effective material properties of the GRC layers in this study as we assume that the graphene sheets are aligned in the polymer matrix to form aligned 2D reinforcement agents. Due to incomplete stress transfer between graphene sheets and polymer matrix resulting from surface effect, strain gradients effect and intermolecular effect, the Halpin-Tsai model needs to be modified to account for these effects [45].

In the present study, the graphene reinforcement is either zigzag (refer to as 0-ply) or armchair (refer to as 90-ply). Based on the extended Halpin-Tsai model, the effective Young’s moduli and the shear modulus of the GRC layer can be expressed as [36]

\[ E_{11} = \eta_1 \left( \frac{1 + 2\left(a_G/h_G\right)\gamma_{G11}^G V_G}{1 - \gamma_{G11}^G V_G} \right) E^m \] (1a)

\[ E_{22} = \eta_2 \left( \frac{1 + 2\left(b_G/h_G\right)\gamma_{G22}^G V_G}{1 - \gamma_{G22}^G V_G} \right) E^m \] (1b)

\[ G_{12} = \eta_3 \left( \frac{1}{1 - \gamma_{G11}^G V_G} \right) G^m \] (1c)

where \( a_G, b_G \) and \( h_G \) are the length, the width and the effective thickness of the graphene sheet, and

\[ \gamma_{G11}^G = \frac{E_{11}^G/E^m - 1}{E_{11}^G/E^m + 2a_G/h_G} \] (2a)

\[ \gamma_{G22}^G = \frac{E_{22}^G/E^m - 1}{E_{22}^G/E^m + 2b_G/h_G} \] (2b)

\[ \gamma_{G12}^G = \frac{G_{12}^G/G^m - 1}{G_{12}^G/G^m} \] (2c)

where \( E^m \) and \( G^m \) are the elasticity modulus and shear modulus of the polymer matrix. Besides, \( E_{11}^G, E_{22}^G \) and \( G_{12}^G \) indicate the elasticity moduli and shear modulus of graphene sheet. We can see that the only difference between Eq. (1) and the conventional Halpin-Tsai model is the presence of efficiency parameters \( \eta_i (i=1,2,3) \). These parameters are obtained by matching the data which are evaluated by MD simulations [46] and the Halpin-Tsai model. In Eq. (1), \( V_G \) and \( V_m \) are the volume fractions of graphene and matrix, which should satisfy the partition of unity condition \( V_G + V_m = 1 \).

Poisson’s ratio \( \nu_{12} \) and the mass density \( \rho \) of the GRC layer may easily be expressed according to the conventional rule of mixtures

\[ \begin{bmatrix} \nu_{12} \\ \rho \end{bmatrix} = \begin{bmatrix} \nu_{12}^G & \nu_m \\ \rho^G & \rho_m \end{bmatrix} \begin{bmatrix} V_G \\ V_m \end{bmatrix} \] (3)

where \( \nu_{12}^G, \rho^G \) and \( \nu_m, \rho_m \) are the Poisson’s ratios and mass densities of the graphene and matrix, respectively. They are assumed to be weakly dependent on temperature variation.

Note that the material properties of the GRC layers estimated in Eq. (1) are temperature dependent as the material properties of the graphene sheets and the polymer matrix are both temperature dependent. According to Schapery model [47], the thermal expansion coefficients of the GRC layers can be expressed by

\[ a_{11} = \frac{V_G E_{11}^G a_{11}^G + V_m E_m a_m}{V_G E_{11}^G + V_m E_m} \] (4a)

\[ a_{22} = (1 + \nu_{12}^G) V_G a_{22}^G + (1 + \nu_m) V_m a_m - \nu_{12} a_{11} \] (4b)

in which \( a_{11} \) and \( a_{22} \) are the longitudinal and transverse thermal expansion coefficients of the GRC layers, \( a_{11}^G, a_{22}^G \) and \( a_m \) are thermal expansion coefficients, respectively, of the graphene and matrix.

The doubly curved GRC laminated panel is subjected to a transverse dynamic load \( q(X, Y, t) \) in a thermal environment. Within the framework of the Reddy’s third order shear deformation shell theory [48] and considering the effects of the von Kármán geometric nonlinearity, the panel-foundation interaction and the temperature variation, we can derive the motion equations for the doubly curved panel as follows

\[ \hat{L}_{11}(\mathbf{W}) - \hat{L}_{12}(\mathbf{V}_x) - \hat{L}_{13}(\mathbf{V}_y) + \hat{L}_{14}(\mathbf{F}) - \hat{L}_{15}(\mathbf{N}^T) \] (5)

\[ - \hat{L}_{16}(\mathbf{M}^T) - \frac{1}{R_1} \hat{\mathbf{F}}_{yy} - \frac{1}{R_2} \hat{\mathbf{F}}_{xx} + (K_1 \mathbf{W} - K_2 \mathbf{V}^2) \] (6)

\[ = \hat{L}(\mathbf{W}, \mathbf{F}) + \hat{L}_{17} \left( \frac{\partial^2 \mathbf{W}}{\partial t^2} - \left( I_3 \frac{\partial^3 \mathbf{X}}{\partial \mathbf{X} \partial t^2} + I_5 \frac{\partial^3 \mathbf{Y}}{\partial \mathbf{Y} \partial t^2} \right) \right) + q \] (7)

\[ \hat{L}_{21}(\mathbf{F}) + \hat{L}_{22}(\mathbf{V}_x) + \hat{L}_{23}(\mathbf{V}_y) - \hat{L}_{24}(\mathbf{W}) - \hat{L}_{25}(\mathbf{N}^T) \] (8)

\[ + \frac{1}{R_1} \hat{\mathbf{W}}_{yy} + \frac{1}{R_2} \hat{\mathbf{W}}_{xx} = - \frac{1}{2} \hat{L}(\mathbf{W}, \mathbf{W}) \] (9)

\[ \hat{L}_{31}(\mathbf{W}) + \hat{L}_{32}(\mathbf{V}_x) - \hat{L}_{33}(\mathbf{V}_y) + \hat{L}_{34}(\mathbf{F}) - \hat{L}_{35}(\mathbf{N}^T) \] (10)

\[ - \hat{L}_{36}(\mathbf{S}^T) = \hat{I}_3 \frac{\partial^3 \mathbf{X}}{\partial \mathbf{X} \partial t^2} - \hat{I}_3 \frac{\partial^3 \mathbf{Y}}{\partial \mathbf{Y} \partial t^2} \] (11)

\[ \hat{L}_{41}(\mathbf{W}) - \hat{L}_{42}(\mathbf{V}_x) + \hat{L}_{43}(\mathbf{V}_y) + \hat{L}_{44}(\mathbf{F}) - \hat{L}_{45}(\mathbf{N}^T) \] (12)

\[ - \hat{L}_{46}(\mathbf{S}^T) = \hat{I}_5 \frac{\partial^3 \mathbf{W}}{\partial \mathbf{X} \partial \mathbf{Y} \partial t^2} - \hat{I}_3 \frac{\partial^3 \mathbf{Y}}{\partial \mathbf{Y}^2 \partial t^2} \] (13)

in which

\[ \hat{L}_{17} = - \hat{I}_1 - \left( \frac{\partial^2}{\partial \mathbf{X}^2} + \frac{\partial^2}{\partial \mathbf{Y}^2} \right) \] (14)
where a comma denotes partial differentiation with respect to the corresponding coordinates, and \( \bar{W} \) is the transverse displacement, \( \bar{\Psi}_x \) and \( \bar{\Psi}_y \) are the rotations of the normals to the middle surface with respect to the \( Y \)- and \( X \)-axes, \( F \) is the stress function defined by \( \bar{N}_x = \partial^2 F/\partial Y^2 \), \( \bar{N}_y = \partial^2 F/\partial X^2 \) and \( \bar{N}_{xy} = -\partial^2 F/\partial X \partial Y \). \( \bar{L}(\cdot) \) and \( \bar{L}(\cdot) \) are the linear and nonlinear operators as defined in Shen [49], and \( \bar{L}(\cdot) \) contains the geometric nonlinearity terms in the von Kármán sense, and can be given by

\[
\bar{L}(\cdot) = \frac{\partial^2}{\partial X^2} \frac{\partial^2}{\partial Y^2} - 2 \frac{\partial^2}{\partial X \partial Y} \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} \frac{\partial^2}{\partial X^2} \tag{10}
\]

The panel is assumed to be in a constant temperature field at an isothermal state. The terms associated with the superscript \( T \) in Eqs. (5)-(8) contain the effect of temperature variation, \( \bar{N}^T \) are the thermal forces, \( \bar{M}^T \) are the thermal moments and \( \bar{F}^T \) are the higher order thermal moments. The effect of the panel-foundation interaction is included in the terms associated with \( \bar{R}_1 \) and \( \bar{R}_2 \) in Eq. (5). The terms \( I_j, I_\tilde{y} \) and \( I_x \) as well as \( \bar{M}^T, \bar{N}^T \) and \( \bar{F}^T \) are given in detail in Appendix A. We can use Eqs. (5)-(8) to analyse the case for GRC laminated cylindrical panels by setting \( R_2 = R \) and \( R_1 = \infty \) and for GRC laminated square spherical panels by setting \( R_1 = R_2 = R \).

Besides the governing equations (5)-(8), it is necessary to deal with different boundary conditions to solve the boundary-value problem. In the current study, the four curved edges of the panel are assumed to be simply supported, and the associate boundary conditions are

\[
\begin{align*}
\bar{W} = \bar{\Psi}_y = \bar{M}_x = \bar{P}_x &= 0 \quad \text{(at } X = 0, a) \tag{11a} \\
\bar{W} = \bar{\Psi}_x = \bar{M}_y = \bar{P}_y &= 0 \quad \text{(at } Y = 0, b) \tag{11b}
\end{align*}
\]

where \( \bar{M}_x \) and \( \bar{M}_y \) are the bending moments and \( \bar{P}_x \) and \( \bar{P}_y \) are the higher order moments as defined in Reddy and Liu [48].

Two in-plane boundary conditions, i.e. movable and immovable, are considered. For movable in-plane boundary conditions, one has

\[
\begin{align*}
\bar{N}_x &= 0 \quad \text{(at } X = 0, a) \tag{11c} \\
\bar{N}_y &= 0 \quad \text{(at } Y = 0, b) \tag{11d}
\end{align*}
\]

and for immovable in-plane boundary conditions, one has

\[
\begin{align*}
\bar{U} &= 0 \quad \text{(at } X = 0, a) \tag{11e} \\
\bar{V} &= 0 \quad \text{(at } Y = 0, b) \tag{11f}
\end{align*}
\]

where \( \bar{U} \) and \( \bar{V} \) are the plate displacements in the \( X \) and \( Y \) directions.

The movable conditions of Eqs. (11c) and (11d) can be imposed in the average sense as

\[
\int_{0}^{b} \bar{N}_x dY = 0, \quad \int_{0}^{a} \bar{N}_y dX = 0 \tag{12}
\]

Also, the immovable conditions of Eqs. (11e) and (11f) are fulfilled in the average sense as

\[
\begin{align*}
\int_{0}^{b} \int_{0}^{a} \bar{U} dX dY = 0, & \quad \int_{0}^{b} \int_{0}^{a} \bar{V} dY dX = 0 \tag{13} \\
\int_{0}^{b} \int_{0}^{a} \left( A_{11}^* \frac{\partial^2 F}{\partial Y^2} + A_{12}^* \frac{\partial^2 F}{\partial X^2} \right) dX dY &= 0 \tag{14a} \\
+ \left( B_{11}^* - \frac{4}{3h^2} E_{11} \right) \frac{\partial \bar{W}}{\partial X} + \left( B_{12}^* - \frac{4}{3h^2} E_{12} \right) \frac{\partial \bar{W}}{\partial Y} \tag{14b} \\
- \frac{4}{3h^2} \left( E_{11}^* \frac{\partial^2 \bar{W}}{\partial X^2} + E_{12}^* \frac{\partial^2 \bar{W}}{\partial Y^2} \right) + \frac{\bar{W}}{R_1} - \frac{1}{2} \left( \frac{\partial \bar{W}}{\partial Y} \right)^2 \\
- \left( A_{11}^* \bar{N}_x + A_{12}^* \bar{N}_y \right) & \quad dX dY = 0
\end{align*}
\]

In the above equations, the reduced stiffness matrices \( [A_{ij}^*] \), \( [B_{ij}^*] \), \( [D_{ij}^*] \), \( [E_{ij}^*] \), \( [F_{ij}^*] \) and \( [H_{ij}^*] \) are defined in Appendix B.

It should be noticed that, in the current study, either governing equations (5)-(8) or boundary conditions (11a)-(11f) are different from that used in [50]. For nonlinear problems the superposition principle is no longer valid. Hence, each nonlinear boundary value problem with different governing equations or boundary conditions should be solved separately.

### 3 Solution procedure

The nonlinear vibrations of flat or cylindrical or doubly curved panels are different nonlinear problems. A two-step perturbation approach was developed by Shen [49]
and was successfully to solve different kinds of nonlinear problems of beams, plates and shells by many research teams [51–61]. To use this two-step perturbation approach to solve the large amplitude vibration problem of doubly curved FG-GRC laminated panels, the motion equations (5) to (8) can be expressed in dimensionless forms as

\[
L_{11}(W) - L_{12}(Ψ_x) - L_{13}(Ψ_y) + γ_{14}L_{14}(F) - L_{16}(Μ^2) = \eta^{-1}γ_{14}F_{xy} - η^{-1}γ_{0}W_{x}F_{yy} + (K_{1}W - K_{2}Ψ_{x}^{2}W) = γ_{14}\beta^2(L(W, F) + L_{17}\left(\frac{\partial^2 W}{\partial t^2}\right) + \left(\frac{γ_{81}3\Psi_{x}}{\partial x\partial t^2} + \frac{γ_{82}\beta}{\partial y\partial t^2}\right) + λ_{q}
\]

\[
L_{21}(F) + γ_{24}L_{22}(Ψ_x) + γ_{24}L_{23}(Ψ_y) - γ_{24}L_{24}(W) + \eta^{-1}γ_{24}W_{,xx} + η^{-1}γ_{24}Ψ_{x}^{2}W_{,yy} = -\frac{1}{2}\gamma_{24}Ψ_{x}^{2}L(W, W)
\]

\[
L_{31}(W) + L_{32}(Ψ_x) - L_{33}(Ψ_y) + γ_{14}L_{34}(F) - L_{36}( fillColor=transparent
\]

\[
L_{41}(W) - L_{42}(Ψ_x) + L_{43}(Ψ_y) + γ_{14}L_{44}(F) - L_{46}( fillColor=transparent
\]

\[
with \quad L_{17}() = γ_{170} + \left(γ_{171}\frac{\partial^2}{\partial x^2} + γ_{172}\frac{\partial^2}{\partial y^2}\right)
\]

and the other dimensionless linear operators \(L_{ij}()\) are given in Shen [49]. In Eqs. (15)-(19), the non-dimensional parameters are defined by

\[
x = \frac{πX}{a}, \quad y = \frac{πY}{b}, \quad β = \frac{a}{b},
\]

\[
η = \frac{π^2R_{2}^{2}}{α^2}[D_{11}D_{22}^*A_{11}^*A_{22}^*]^{1/4}, \quad γ_{0} = \frac{R_{2}}{K_{1}}
\]

\[
W = \frac{W}{[D_{11}^*D_{22}^*A_{11}^*A_{22}^*]^{1/4}}, \quad (Ψ_{x}, Ψ_{y}) = \frac{a}{π}[D_{11}^*D_{22}^*A_{11}^*A_{22}^*]^{1/4}, \quad F = \frac{F}{[D_{11}^*D_{22}^*]^{1/2}},
\]

\[
(M_{x}, P_{x}) = \frac{a^2}{π^2}[D_{11}D_{12}^*D_{22}^*A_{11}^*A_{22}^*]^{1/4} \left(\frac{1}{\bar{M}_{x}}, \frac{4}{3h^2}P_{x}\right),
\]

\[
t = \frac{πt}{a} \sqrt{E_{0}/ρ_{0}}, \quad ω_L = Ωt = \frac{a}{π} \sqrt{ρ_{0}/E_{0}}, \quad γ_{14} = \left[D_{22}^*/D_{11}\right]^{1/2},
\]

\[
\gamma_{24} = \left[A_{11}^*/A_{22}^*\right]^{1/2}, \quad γ_{5} = \left[A_{12}^*/A_{22}^*\right]
\]

\[
(γ_{71}, γ_{72}) = (A_{11}^*, A_{12}^*) R_{2} \left[A_{11}^*A_{22}^*/D_{11}D_{22}\right]^{1/4},
\]

\[
(γ_{74}, γ_{75}, γ_{77}, γ_{78}) = \frac{a^2}{π^2hD_{11}}(D_{x}^*, D_{y}^*, \frac{4}{3h^2}F_{x}^*, \frac{4}{3h^2}F_{y}^*),
\]

\[
γ_{170} = -\frac{I_{1}E_{0}a^2}{π^2ρ_{0}D_{11}^*},
\]

\[
(γ_{79}, γ_{72}, γ_{78}, γ_{72}, γ_{81}, γ_{82}, γ_{83}, γ_{84}, γ_{71}, γ_{72}) = (-\bar{I}_{3}, -\bar{I}_{3}, -\bar{I}_{3}, I_{1}, I_{2}, I_{3}, I_{3}, I_{1}) \frac{E_{0}}{ρ_{0}D_{11}^*},
\]

\[
(K_{1}, k_{1}) = \mathcal{K}_{1} \left(\frac{a^4}{π^4D_{11}^*}, \frac{b^4}{E_{0}h^2}\right),
\]

\[
(K_{2}, k_{2}) = \mathcal{K}_{2} \left(\frac{a^2}{π^2D_{11}^*}, \frac{b^2}{E_{0}h^2}\right),
\]

\[
λ_{q} = \frac{q^2a^4}{π^4D_{11}^*[D_{11}^*D_{22}^*A_{11}^*A_{22}^*]^{1/4}}
\]

in which \(E_{0}\) and \(ρ_{0}\) are the material properties of the polymer matrix \(E_{m}\) and \(ρ_{m}\) at room temperature \((T_{0}=300\, K)\), and the terms \(A_{11}^*, D_{x}^*, F_{x}^*\), etc. are defined by

\[
\begin{bmatrix}
A_{x}^* \\
A_{y}^* \\
D_{x}^* \\
D_{y}^* \\
F_{x}^* \\
F_{y}^*
\end{bmatrix}
\]

\[
\Delta T
\]

\[
= -\sum_{k=1}^{N} \int_{h_{k-1}}^{h_{k}} \left[\frac{A_{x}}{A_{y}} (1, Z, Z') ΔTdz\right]
\]

Based on Eq. (20), the boundary conditions of Eqs. (11a) and (11b) can be written in non-dimensional forms as

\[
W = Ψ_{y} = M_{x} = P_{x} = 0 \quad (at x = 0, π)
\]

\[
W = Ψ_{x} = M_{y} = P_{y} = 0 \quad (at y = 0, π)
\]

and the movable in-plane boundary conditions of Eqs. (11c) and (11d) become

\[
\int_{0}^{π} \frac{∂^2 F}{∂y^2} dy = 0 \quad (at x = 0, π)
\]

\[
\int_{0}^{π} \frac{∂^2 F}{∂x^2} dx = 0 \quad (at y = 0, π)
\]

and the immovable in-plane boundary conditions of Eqs. (11e) and (11f) become

\[
\int_{0}^{π} \int_{0}^{π} \left(\frac{2}{724}a^2\frac{∂^2 F}{∂y^2} - 75\frac{∂^2 F}{∂x^2}\right)
\]

\[
+ γ_{24} \left(\frac{7511}{261} \frac{∂Ψ_{x}}{∂x} + γ_{223}β \frac{∂Ψ_{y}}{∂y}\right)
\]

\[
- γ_{24} \left(\frac{7611}{244} \frac{∂^2 W}{∂x^2} + \frac{244a^2}{3h^2} \frac{∂^2 W}{∂y^2}\right) + η^{-1}γ_{70}γ_{74}W
\]
We introduce
\[ \frac{1}{2} \frac{\partial^2 W}{\partial x^2} + \eta^{-1} (\gamma_{24} \gamma T_1 - \gamma_{35} \gamma T_2) \Delta T \] dx \, dy = 0 \]
(at \( x = 0, \pi \))
\[ \int_0^\pi \int_0^\pi \left[ \frac{\partial^3 F}{\partial x^2} - \gamma_2 \frac{\partial^2 F}{\partial y^2} \right] \, dx \, dy \] (22f)
\[ + \gamma_{24} \left( \gamma_{242} \frac{\partial^2 W}{\partial x^2} + \gamma_{242} \frac{\partial^2 W}{\partial y^2} \right) - \gamma_{24} \left( \gamma_{242} \frac{\partial^2 W}{\partial x^2} + \gamma_{242} \frac{\partial^2 W}{\partial y^2} \right) + \eta^{-1} \gamma_{24} W \]
\[ - \frac{1}{2} \gamma_{24} \frac{\partial W}{\partial y} \left( \frac{\partial W}{\partial y} \right) \right] \, dy \, dx = 0 \]
(at \( y = 0, \pi \))

where \( \gamma_{ij} \) are defined in Shen [49].

Equations (15) to (18) can be separated into two sets of differential equations which are then solved in sequence. The first set of differential equations is for the nonlinear thermal bending problem and can be solved using the same method as reported in [62], and the second set of differential equations are used to obtain the homogeneous vibration solution on the initial deflected panel. A two-step perturbation technique is applied to determine this homogeneous solution. We assume that the perturbation equations for the displacements and the forces with a small perturbation parameter \( \epsilon \) which has no physical meaning in the first step are given by

\[ W(x, y, t) = \sum_{j=1}^{m} e^j W_j(x, y, t), \] (23)
\[ \Psi_x(x, y, t, \epsilon) = \sum_{j=1}^{m} e^j \Psi_{x_j}(x, y, t), \]
\[ \Psi_y(x, y, t, \epsilon) = \sum_{j=1}^{m} e^j \Psi_{y_j}(x, y, t), \]
\[ F(x, y, t, \epsilon) = \sum_{j=1}^{m} e^j f_j(x, y, t), \]
\[ \lambda_0(x, y, t, \epsilon) = \sum_{j=1}^{m} e^j \lambda_j(x, y, t), \]

We introduce \( \tau = \epsilon t \) to improve the perturbation solution process for solving the large amplitude vibration problem. In order to satisfy the simply supported boundary conditions in the space domain, the first order solution of the panel is assumed to have the form

\[ w_1(x, y, t) = A_{11}^{(1)}(t) \sin mx \sin ny \] (24)

where \((m, n)\) is the number of waves of vibration mode in the \(X\) and \(Y\) directions. The initial conditions are assumed to be

\[ W|_{t=0} = \frac{\partial W}{\partial t}|_{t=0} = 0, \quad \Psi_x|_{t=0} = \frac{\partial \Psi_x}{\partial t}|_{t=0} = 0, \] (25)

\[ \Psi_y|_{t=0} = \frac{\partial \Psi_y}{\partial t}|_{t=0} = 0 \]

Taking into consideration of Eq. (23), a set of perturbation equations are obtained by collecting the terms of the same order of \( \epsilon \) in Eqs. (15)-(18). Eq. (24) is then applied as the first step solution to the perturbation equations and following a step by step approach, we can obtain the 4th order asymptotic solutions as

\[ W(x, y, t) = \epsilon A_{11}^{(1)}(t) \sin mx \sin ny + (\epsilon A_{11}^{(1)}(t))^3 [a_{331} \sin 3mx \sin ny + a_{313} \sin mx \sin 3ny + O(\epsilon^4)] \]

\[ \Psi_x(x, y, t) = [\epsilon A_{11}^{(1)}(t)c_{111} + \left( \epsilon \frac{\partial^2 A_{11}^{(1)}(t)}{\partial t^2} \right) c_{311}] \]

\[ \cdot \cos mx \sin ny + (\epsilon A_{11}^{(1)}(t))^3 [c_{331} \cos 3mx \sin ny + c_{313} \cos mx \sin 3ny] + O(\epsilon^4) \]

\[ \Psi_y(x, y, t) = [\epsilon A_{11}^{(1)}(t)d_{11} + \left( \epsilon \frac{\partial^2 A_{11}^{(1)}(t)}{\partial t^2} \right) d_{311}] \]

\[ \cdot \sin mx \cos ny + (\epsilon A_{11}^{(1)}(t))^3 [d_{331} \cos 3mx \cos ny + d_{313} \cos mx \cos 3ny] + O(\epsilon^4) \]

\[ F(x, y, t) = -B_{00}^{(0)} y^2 / 2 - b_{00}^{(0)} x^2 / 2 + (\epsilon A_{11}^{(1)}(t)) \left[ -B_{00}^{(1)} y^2 / 2 - b_{00}^{(1)} x^2 / 2 + b_{111} \sin mx \sin ny \right] \]

\[ + \left( \epsilon \frac{\partial^2 A_{11}^{(1)}(t)}{\partial t^2} \right) b_{311} \sin mx \sin ny \]

\[ + (\epsilon A_{11}^{(1)}(t))^2 \left[ -B_{00}^{(2)} y^2 / 2 - b_{00}^{(2)} x^2 / 2 + b_{202} \cos 2ny \right] \]

\[ + b_{220} \cos 2mx \]

\[ + (\epsilon A_{11}^{(1)}(t))^3 [b_{331} \sin 3mx \sin ny + b_{313} \sin mx \sin 3ny] + O(\epsilon^4) \]

\[ \lambda_0(x, y, t) = \epsilon \left[ g_{31} A_{11}^{(1)}(t) + g_{30} \left( \frac{\partial^2 A_{11}^{(1)}(t)}{\partial t^2} \right) \right] \]

\[ \cdot \sin mx \sin ny \]

\[ + (\epsilon A_{11}^{(1)}(t))^2 \left[ g_{220} \cos 2mx + g_{202} \cos 2ny \right] \]

\[ + (\epsilon A_{11}^{(1)}(t))^3 [g_{331} \sin mx \sin ny] + \ldots \]

It is worth noting that the perturbation series is a divergent series. Which order solution is closer to the real solution needs to be determined by experimental verification or by comparing with the theoretical exact solution. Contrary to Zhang’s conclusion [63], there is no such thing as a
higher order perturbation solution being more correct than a lower order solution.

In Eqs. (26)-(30), \( r \) is replaced back by \( t \). It is noted that the small perturbation parameter \( \varepsilon \) is replaced by \((\varepsilon A_{11}^{(1)})\) in the second step. For free vibration analysis, applying Galerkin procedure to Eq. (30), we have

\[
\begin{align*}
\frac{d^2 (\varepsilon A_{11}^{(1)})}{dt^2} + g_{31} (\varepsilon A_{11}^{(1)}) + g_{32} (\varepsilon A_{11}^{(1)})^2 \\
+ g_{33} (\varepsilon A_{11}^{(1)})^3 = 0
\end{align*}
\]

in which the terms \( g_{ij} \) are given in Appendix C. Eq. (31) can be solved to obtain the nonlinear frequency of the panel as follows

\[
\omega_{NL} = \omega_{1} \left[ 1 + \frac{9g_{31}g_{13} - 10g_{32}^2}{12g_{31}} A \right]^{1/2}
\]

where \( \omega_{1} = \left[ g_{31}/g_{30} \right]^{1/2} \) is the dimensionless linear frequency, and \( A = W_{\text{max}} = W_{\text{max}}/|D_{11} D_{22} A_{11} A_{22}|^{1/4} \) is the dimensionless maximum amplitude of the panel. It is worth noting that Eqs. (31) and (32) are similar in form to those of the cylindrical panels [50], but have different contents, as shown in Appendix C.

### Table 2: Temperature-dependent efficiency parameters of graphene/PMMA nanocomposites [36]

| \( T \) (K) | \( V_{G} \) | \( \eta_1 \) | \( \eta_2 \) | \( \eta_3 \) |
|-----------|-----------|-----------|-----------|-----------|
| 300       | 0.03      | 2.929     | 2.855     | 11.842    |
| 300       | 0.07      | 3.013     | 2.966     | 15.944    |
| 300       | 0.09      | 2.647     | 2.609     | 32.816    |
| 300       | 0.11      | 2.311     | 2.260     | 33.125    |
| 400       | 0.03      | 2.977     | 2.896     | 13.928    |
| 400       | 0.05      | 3.128     | 3.023     | 15.229    |
| 400       | 0.07      | 3.060     | 3.027     | 22.588    |
| 400       | 0.09      | 2.701     | 2.603     | 28.869    |
| 400       | 0.11      | 2.405     | 2.337     | 29.527    |
| 500       | 0.03      | 3.388     | 3.382     | 16.712    |
| 500       | 0.05      | 3.544     | 3.414     | 16.018    |
| 500       | 0.07      | 3.462     | 3.339     | 23.428    |
| 500       | 0.09      | 3.058     | 2.936     | 29.754    |
| 500       | 0.11      | 2.736     | 2.665     | 30.773    |

### 4 Numerical results and discussion

In this section, numerical results for the nonlinear vibration of doubly curved GRC laminated panels in thermal environments are obtained. It is noted that the material properties of graphene sheets are anisotropic [24–26] and temperature dependent [27]. We select zigzag (refer to as 0-ply) graphene sheets with effective thickness \( h_{G} = 0.188 \) nm and \( \rho_{G} = 4118 \) kg/m\(^3\) as reinforcement agents. Lin et al. [46] performed a molecular dynamics simulation to evaluate the material properties of graphene sheets at different temperatures. These material properties at three different temperature levels are provided in Table 1. It must be pointed out that the Young’s modulus of single-layer graphene sheet is not a constant which depends on the value of its effective thickness [64]. For example, it has been reported that Young’s modulus of single-layer graphene sheet is estimated to be about 1 TPa. This is due to the fact that the effective thickness of graphene sheet is taken to be 0.34 nm [65, 66], otherwise the Young’s modulus may reach 2.47 TPa which is significantly larger than that reported in [65, 66], when the effective thickness of graphene sheet is only 0.129 nm [27]. In the MD work of Lin et al. [46], the calculated effective thickness of the graphene sheet is 0.188 nm according to the research findings of Shen et al. [27]. Therefore, the predicted Young’s moduli of the graphene sheet (see in Table 1) reach around 1.8 TPa. The graphene efficiency parameters \( \eta_1, \eta_2 \) and \( \eta_3 \) used in the extended Halpin-Tsai micromechanical model are given in Table 2 which are obtained by comparing the GRC moduli from the MD simulations and from the Halpin–Tsai model, as previously reported in Shen et al. [36]. We assume that \( G_{13} = G_{23} = 0.5G_{12} \). We select Poly (methyl methacrylate), referred to as PMMA, for the matrix. The material properties of PMMA are assumed to be \( \rho^{m} = 1150 \) kg/m\(^3\), \( v^{m} = 0.34 \), \( a^{m} = 45(1+0.0005\Delta T) \times 10^{-6} \) K and \( E^{m} = (3.52-0.0034\Delta T) \) GPa, in which \( T = T_{0} + \alpha \Delta T \) and \( T_{0} = 300 \) K (room temperature). Hence, we have \( a^{m} = 45.0 \times 10^{-6} \) K and \( E^{m} = 2.5 \) GPa when \( T = 300 \) K.

Comparison studies are carried to verify the correctness of the present solution method. The fundamental fre-
Table 3: Comparison of linear frequency $\hat{\Omega} = \Omega (a^2/h) \sqrt{\rho_0/E_0}$ for double curved CNT/PmPV panels ($a/b = 1, a/h = 20, a/R_1 = b/R_2 = 0.5, T = 300 K$)

| $V_\varphi$ | Source | UD | FG-Λ | FG-V | FG-X | FG-O |
|------------|--------|----|------|------|------|------|
| 11         | Pouresmaeili et al. [67] Present | 20.2381 | 18.2514 | 18.5425 | 22.4320 | 17.1397 |
| 14         | Pouresmaeili et al. [67] Present | 20.2587 | 18.8587 | 18.7062 | 24.0664 | 16.9807 |
| 17         | Pouresmaeili et al. [67] Present | 21.7298 | 20.0403 | 19.9679 | 26.1019 | 18.1251 |

The dimensionless frequency is defined by $\hat{\Omega} = \Omega (a^2/h) \sqrt{\rho_0/E_0}$, with $\rho_0$ and $E_0$ being the reference values of PmPV at room temperature $T = 300 K$. In Table 3, the extended Voigt model (rule of mixture) is adopted and the CNT efficiency parameters are taken to be $\eta_1 = 0.149, \eta_2 = \eta_3 = 0.934$ for the case of $a_1 = 0.11$, and $\eta_1 = 0.150, \eta_2 = \eta_3 = 0.941$ for the case of $V_{CN}^* = 0.14$, and $\eta_1 = 0.149, \eta_2 = \eta_3 = 1.381$ for the case of $V_{CN}^* = 0.17$. It can be seen that for the UD, FG-Λ, FG-V and FG-X cases the results of Pouresmaeili and Fazelzadeh [67] are lower than the present solutions whereas for the FG-O case the results of Pouresmaeili and Fazelzadeh [67] are higher than the present solutions. As a second example, the dimensionless fundamental frequencies for Al/ZrO$_2$ doubly curved panels, which have ceramic-rich outer surface and metal-rich inner surface, are calculated and compared in Table 4 with the analytical hybrid Laplace–Fourier transformation results of Kiani et al. [11] and the isoparametric finite element approach results of Kar and Panda [13]. The conventional Voigt model was adopted by Kiani et al. [11] and Kar and Panda [13]. The value of $N$ in Table 4 is the index of volume fraction. The material properties of the panels do not include the effect of temperature with the properties of Aluminum being $E_m = 70$ GPa, $\nu_m = 0.3$ and $\rho_m = 2702$ kg/m$^3$ and Zirconia being $E_c = 151$ GPa, $\nu_c = 0.3$, and $\rho_c = 3000$ kg/m$^3$. The third comparison study is presented in Figure 2 on the nonlinear-to-linear frequency ratios $\omega_0/\omega_1$ for a $(0/90/0)$ laminated square spherical panel from the present method and from the FEM results of Singh and Panda [4] based on a higher order shear deformation theory. The panel is of movable in-plane boundary condition. The geometric parameters and the material properties used in the computation study are: $a/b = 1, a/h = 100, R_1/a = R_2/b = 5$ $(m, n) = (1, 1)$ and $G_{12} = 0.6 E_{22}, v_{12} = 0.25$. Only the vibration mode of $(m, n) = (1, 1)$ is considered. The three comparison studies have shown that good agreement is achieved between the results from the present solution method and from existing research work in the open literature.

Tables 5-7 and Figures 3-7 present the numerical results for the large amplitude vibration of doubly curved GRC laminated panels with $h = 2$ mm, $a/b = 1.0, a/h = 10$ and 20. Note that GRCs may contain the volume fraction of graphene reinforcement by up to 21% [68] and in this study the maximum graphene volume fraction is 11%. Four FG and one UD GRC laminated doubly curved panels are considered. The UD GRC panel consists of 10 GRC layers with the graphene volume fraction for each layer being identical, i.e. $V_G = 0.07$. The four FG-GRC panels consist of 10 GRC layers with piece-wise graphene volume fractions, i.e. the
Large amplitude vibration of doubly curved FG-GRC laminated panels in thermal environments

Figure 3: Frequency-amplitude curves for doubly curved (0/90/0/90/0) GRG panels with different graphene distribution patterns

Figure 4: Influence of temperature variation on the frequency-amplitude curves of doubly curved (0/90/0/90/0) GRG laminated panels

Figure 5: Influence of panel curvature ratio \( \alpha/R_1 \) on the frequency-amplitude curves of doubly curved (0/90/0/90/0) GRG laminated panels

Figure 6: Influence of foundation stiffness on the frequency-amplitude curves of doubly curved (0/90/0/90/0) GRG laminated panels resting on elastic foundations

Figure 7: Influence of in-plane boundary conditions on the frequency-amplitude curves of doubly curved (0/90/0/90/0) GRG laminated panels

FG-A type with \([0.03]/(0.05)/[0.07]/(0.09)/[0.11]/[0.2] \), the FG-V with \([0.11]/[0.09]/[0.07]/[0.05]/[0.03]/[0.2] \), the FG-O type with \([0.03]/[0.05]/[0.07]/[0.09]/[0.11]/[0.3] \) and the FG-X type with \([0.11]/[0.09]/[0.07]/[0.05]/[0.03]/[0.5] \). Note that the total graphene volume fraction for all considered GRG laminated panels are the same. The non-dimensional frequency parameter \( \tilde{\Omega} = \Omega (b^2/h) \sqrt{\rho_0/E_0} \) is used in this study, where \( \rho_0 \) and \( E_0 \) are the reference values of \( \rho \) and \( E \) at \( T = 300 \) K. The panels are simply supported with immovable in-plane condition, unless stated otherwise.

Table 5 presents the natural frequencies of \((0)_{10}, (0/90/0/90/0)_5 \) and \((0/90)_{37} \) doubly curved GRG laminated panels having \( b/h = 20 \) and \( \alpha/R_1 = b/R_2 = 0.4 \) with environmental temperature \( T = 300 \) and 400 K. It is observed that increasing the environmental temperature will result in the decrease in the natural frequencies of the panels.
Table 4: Comparisons of fundamental frequencies \( \tilde{\omega} = \Omega (b^2 / h) \sqrt{\rho c / E_c} \) for double curved Al/ZrO\(_2\) panels \((a/b = 1, b/h = 10, R_1 = 2R_2)\)

| \(R_2/b\) | \(\tilde{\omega}\) | \(N\) |
|----------|-----------------|------|
| 3        | Kiani et al. [11] | 6.2330 5.6206 5.3523 5.1435 4.9642 4.8300 |
|          | Kar and Panda [13] | 6.2277 5.6191 5.3501 5.1377 4.9540 4.8214 |
|          | Present\(^a\) | 6.2806 5.6761 5.4091 5.1976 5.0096 4.8711 |
|          | Present\(^b\) | 6.2806 5.5405 5.2983 5.1156 4.9376 4.8078 |
| 5        | Kiani et al. [11] | 5.9412 5.3492 5.0960 4.9077 4.7515 4.6244 |
|          | Kar and Panda [13] | 5.9394 5.3505 5.0963 4.9039 4.7428 4.6174 |
|          | Present\(^a\) | 5.9585 5.3732 5.1211 4.9302 4.7668 4.6380 |
|          | Present\(^b\) | 5.9585 5.2455 5.0205 4.8577 4.6999 4.5768 |
| 10       | Kiani et al. [11] | 5.8128 5.2310 4.9851 4.8062 4.6599 4.5354 |
|          | Kar and Panda [13] | 5.8127 5.2333 4.986 4.8028 4.6515 4.5288 |
|          | Present\(^a\) | 5.8173 5.2403 4.9946 4.8129 4.6607 4.5360 |
|          | Present\(^b\) | 5.8173 5.1157 4.8985 4.7448 4.5961 4.4759 |
| 20       | Kiani et al. [11] | 5.7802 5.2018 4.9580 4.7816 4.6376 4.5135 |
|          | Kar and Panda [13] | 5.7805 5.2041 4.9589 4.7782 4.6291 4.5068 |
|          | Present\(^a\) | 5.7815 5.2065 4.9625 4.7831 4.6337 4.5102 |
|          | Present\(^b\) | 5.7815 5.0826 4.8675 4.7163 4.5700 4.4505 |
| 100      | Kiani et al. [11] | 5.7698 5.1932 4.9504 4.7749 4.6315 4.5071 |
|          | Kar and Panda [13] | 5.7701 5.1954 4.951 4.7712 4.6227 4.5003 |
|          | Present\(^a\) | 5.7699 5.1956 4.9522 4.7736 4.6251 4.5019 |
|          | Present\(^b\) | 5.7699 5.0718 4.8575 4.7073 4.5617 4.4424 |

\(^a\) Voigt model; \(^b\) Mori-Tanaka model.

due to the stiffness of the panels being reduced at a higher temperature. Results in Table 5 also reveal that the panel with the FG-X reinforcement pattern has the largest, while the panel with the FG-O pattern has the smallest natural frequencies in the five considered cases. Like in the case of cylindrical panels [50], the doubly curved GRC panels with \((0/90/0/90/0)_{S}\) and \((0/90)_{5T}\) lamination arrangements have the same fundamental frequencies which are slight higher than the ones for the panels with \((0)_{10}\) lamination arrangement at \(T = 300\) K.

Table 6 presents the effect of foundation stiffness on the natural frequencies of \((0/90/0/90/0)_{S}\) doubly curved GRC laminated panels with \(b/h = 10, R_1 = 2R_2 = 0.4\) and \(a/R_1 = 0.2\) and \(0.8\) under thermal environmental condition \(T = 300\) K. We consider two foundation models which are the Pasternak elastic foundation with \((k_1, k_2) = (1000, 100)\) and the Winkler elastic foundation with \((k_1, k_2) = (1000, 0)\). The panel without elastic foundation, i.e., \((k_1, k_2) = (0, 0)\), is also considered. Like in the case of cylindrical panels [50], the natural frequency is increased when the foundation stiffness is increased at room temperature.

The impact of the in-plane boundary conditions on the nonlinear vibration behavior of the \((0/90/0/90/0)_{S}\) doubly curved GRC laminated panels is investigated and the results are presented in Table 7. The panels have the geometric parameters of \(b/h = 10\) and \(a/R_1 = b/R_2 = 0.02\) and are subject to environmental temperature of \(T = 300\) and \(400\) K. The results in Table 7 show that when the panels are subject to room temperature, the fundamental frequencies for the panels with either the movable or the immovable in-plane boundary conditions are the same which is due to the fact that no initial in-plane thermal stresses are present in the panels in this case. However, when the panels are subject to temperature of \(T = 400\) K, in-plane compressive thermal stresses are introduced to the panels with immovable in-plane boundary condition. Like in the case of cylindrical panels [50], the nonlinear-to-linear frequency ratios for the panel with movable in-plane boundary condition are smaller than the one for the panel with immovable in-plane boundary condition.

Figure 3 depicts the frequency-amplitude curves for four FG and one UD doubly curved GRC laminated panels of \(b/h = 10, R_1 = 2R_2 = 0.02\) and \(a/R_1 = b/R_2 = 0.05\) at room temperature of \(T = 300\) K. The laminated arrangement of the panels is \((0/90/0/90/0)_{S}\). It is observed that amongst the five GRC panels, the FG-X panel has the highest fundamental frequency as it has the largest panel stiffness, while the FG-O GRC panel has the lowest fundamental frequency as it has
Table 5: Natural frequency $\tilde{\Omega} = \Omega (b^2 / h) \sqrt{\rho_0 / E_0}$ of double curved GRC laminated panels in thermal environments ($a/b = 1$, $b/h = 20$, $h = 2$ mm, $a/R_1 = b/R_2 = 0.4$)

| $T$ (K) | Lay-up   | (1,1)$^a$ | (1,2)     | (2,1)     | (2,2)     | (1,3)     |
|---------|----------|-----------|-----------|-----------|-----------|-----------|
| 300     | (0)$_{10}$ | 48.3014   | 82.6496   | 82.8618   | 118.5889  | 146.2361  |
|         | UD       | 45.0530   | 74.4394   | 74.7672   | 106.6529  | 131.3309  |
|         | FG-V     | 47.0308   | 76.4596   | 76.7165   | 109.7334  | 132.7072  |
|         | FG-Λ     | 49.2661   | 85.4987   | 85.7574   | 123.8480  | 149.7251  |
|         | FG-O     | 48.6463   | 71.4763   | 71.7414   | 101.4378  | 124.5773  |
| (0/90/0/90/0)$_S$ | UD   | 48.3017   | 82.7196   | 82.7923   | 118.5891  | 146.4664  |
|         | FG-V     | 45.0534   | 74.5451   | 74.6625   | 106.6530  | 131.6821  |
|         | FG-Λ     | 47.0316   | 76.5387   | 76.6388   | 109.7343  | 133.0248  |
|         | FG-X     | 49.2667   | 85.6444   | 85.6125   | 123.8484  | 150.1496  |
|         | FG-O     | 44.8650   | 71.1696   | 71.5993   | 101.4382  | 125.0769  |
| 400     | (0)$_{10}$ | 34.2138   | 57.5011   | 67.5649   | 94.5042   | 114.5018  |
|         | UD       | 36.1359   | 50.9012   | 61.1003   | 85.2160   | 101.3580  |
|         | FG-V     | 30.5796   | 51.8494   | 62.2123   | 86.4531   | 102.7173  |
|         | FG-Λ     | 36.2822   | 63.0136   | 71.6413   | 101.1963  | 120.8419  |
|         | FG-X     | 31.2439   | 45.8868   | 56.8932   | 78.1132   | 92.6827   |
| (0/90/0/90/0)$_S$ | UD   | 34.2140   | 61.7114   | 63.7426   | 94.5043   | 120.2617  |
|         | FG-V     | 36.1360   | 55.0395   | 57.4023   | 85.2169   | 107.2002  |
|         | FG-Λ     | 30.5811   | 51.6494   | 58.4583   | 86.4537   | 108.6451  |
|         | FG-X     | 36.2833   | 66.6612   | 67.4694   | 101.1972  | 126.1756  |
| (0/90)$_{5T}$ | UD   | 34.2139   | 62.7425   | 62.7280   | 94.5043   | 121.7073  |
|         | FG-V     | 36.1360   | 56.2466   | 56.2198   | 85.2167   | 108.9418  |
|         | FG-Λ     | 30.5810   | 57.2915   | 57.2412   | 86.4534   | 110.4587  |
|         | FG-X     | 36.2833   | 67.4694   | 67.4635   | 101.1972  | 127.3658  |

$^a$ vibration mode

the smallest panel stiffness. It is also observed that the FG-X panel has the lowest and the FG-O has the highest nonlinear to linear frequency ratios in the considered cases. We will focus our analysis on the doubly curved UD and FG-X GRC (0/90/0/90/0)$_S$ panels for the subsequent cases.

Figure 4 shows the effect of temperature variation on the large amplitude vibration behavior of doubly curved UD and FG-X laminated panels with $b/h = 10$ and $a/R_1 = b/R_2 = 0.05$. Note that the glass transition temperature of PMMA can be substantially increased when graphene sheets are added in PMMA [69] and in the current study, we will consider the environmental temperature variation from $T = 300$ to $500$ K. It can be seen that for the cases of $T = 300$ and $400$ K, the frequency-amplitude curve increases with increase in temperature for both UD and FG-X GRC panels, whereas for the case of $T = 500$ K, the frequency-amplitude curve of the FG-X GRC panel becomes higher than that of the UD GRC panel.

The influence of the curvature ratio $a/R_1$ on the frequency-amplitude curves of the doubly curved UD and FG-X panels of $b/h = 10$, $b/R_2 = 0.05$ and $a/R_1 = 0.1, 0.15$ and 0.2 at room temperature of $T = 300$ K is given in Figure 5. As previously reported in [1–3], the curves of nonlinear frequency as a function of amplitude of curved panels might be hardening or softening type. It is observed that the nonlinear frequency as a function of amplitude of the GRC laminated doubly curved panel is the softening type when $a/R_1 = 0.2$. For other cases, the nonlinear frequency...
Table 6: Effects of foundation stiffness on the natural frequency $\tilde{\Omega} = \Omega(b^2/h)\sqrt{\rho_0/E_0}$ of double curved (0/90/0/90/0)\textsubscript{SGRC} laminated panels. ($a/b = 1, b/h = 20, h = 2$ mm, $b/R_2 = 0.4, T = 300$ K)

| $(k_1, k_2)$ | $a/R_1$ | (1,1)$^a$ | (1,2) | (2,1) | (2,2) | (1,3) |
|--------------|---------|-----------|-------|-------|-------|-------|
| (0, 0)       | 0.2     | UD        | 41.1721 | 76.5493 | 81.0143 | 115.8924 | 142.6105 |
|               |         | FG-V      | 37.8351 | 68.4411 | 72.8904 | 104.1301 | 128.0025 |
|               |         | FG-Λ      | 39.5992 | 69.7487 | 74.7134 | 106.5038 | 128.7642 |
|               |         | FG-X      | 42.4981 | 79.9146 | 84.0718 | 121.3349 | 146.4308 |
|               |         | FG-O      | 37.3048 | 64.7792 | 69.4611 | 98.3588 | 120.8690 |
| 0.8          | UD      | 64.4642   | 101.4007 | 86.7702 | 125.9762 | 160.0728 |
|               |         | FG-V      | 61.3089 | 93.2844 | 78.6432 | 113.8672 | 145.1027 |
|               |         | FG-Λ      | 63.4989 | 96.1479 | 80.9829 | 118.1785 | 147.4119 |
|               |         | FG-X      | 64.8215 | 102.8914 | 89.3618 | 130.7637 | 162.3642 |
|               |         | FG-O      | 61.5448 | 91.6238 | 75.9404 | 109.7616 | 139.7027 |
| (1000, 0)    | 0.2     | UD        | 50.3874 | 81.8504 | 86.0407 | 119.4462 | 145.5074 |
|               |         | FG-V      | 67.4690 | 74.3210 | 78.4375 | 108.0687 | 131.2189 |
|               |         | FG-Λ      | 49.1102 | 75.5272 | 80.1347 | 110.3581 | 131.9623 |
|               |         | FG-X      | 51.4760 | 85.0073 | 88.9267 | 124.7367 | 149.2571 |
|               |         | FG-O      | 47.2807 | 70.9640 | 75.4283 | 102.5191 | 124.2693 |
| 0.8          | UD      | 70.7063   | 105.4599 | 91.4809 | 129.2531 | 162.6590 |
|               |         | FG-V      | 67.8414 | 97.6801 | 83.8104 | 117.4797 | 147.9477 |
|               |         | FG-Λ      | 69.8272 | 100.4187 | 85.9253 | 121.6367 | 150.2136 |
|               |         | FG-X      | 71.0318 | 106.8950 | 93.9437 | 133.9262 | 164.9176 |
|               |         | FG-O      | 68.0557 | 96.0961 | 81.2803 | 113.5048 | 142.6548 |
| (1000, 100)  | 0.2     | UD        | 64.8411 | 104.1308 | 107.4558 | 144.4679 | 171.4938 |
|               |         | FG-V      | 62.7740 | 98.3161 | 101.4637 | 135.1959 | 159.5227 |
|               |         | FG-Λ      | 63.8533 | 99.2318 | 102.7823 | 137.0349 | 160.1359 |
|               |         | FG-X      | 65.6898 | 106.6346 | 109.7848 | 148.8911 | 174.7155 |
|               |         | FG-O      | 62.4587 | 95.8065 | 99.1586 | 130.8007 | 153.8491 |
| 0.8          | UD      | 81.6386   | 123.5538 | 111.8593 | 152.6759 | 186.2677 |
|               |         | FG-V      | 79.1696 | 116.9774 | 105.6720 | 142.8296 | 173.5434 |
|               |         | FG-Λ      | 80.8782 | 119.2755 | 107.3585 | 146.2942 | 175.4819 |
|               |         | FG-X      | 81.9200 | 124.7848 | 113.8866 | 156.6703 | 188.2702 |
|               |         | FG-O      | 79.3549 | 115.6610 | 103.6798 | 139.5779 | 169.0476 |

$^a$ vibration mode

as a function of amplitude of the GRC laminated doubly curved panels is the hardening type.

The impact of foundation stiffness on the frequency-amplitude curves of UD and FG-X doubly curved panels resting on elastic foundations is illustrated in Figure 6. The panel has $b/h = 10$ and $a/R_1 = b/R_2 = 0.05$ at $T = 300$ K. Two foundation models are considered. The foundation stiffnesses are the same as used in Table 6, i.e., $(k_1, k_2) = (1000, 100)$ is for the Pasternak elastic foundation, and $(k_1, k_2) = (1000, 0)$ is for the Winkler elastic foundation. As expected, increasing the foundation stiffness will result in the reduction of the frequency-amplitude curves of the panels.

Figure 7 presents the relationship between the in-plane boundary conditions and the frequency-amplitude curves of the doubly curved UD and FG-X panels with $b/h = 10$ and $a/R_1 = b/R_2 = 0.05$ at $T = 300$ K. The ‘movable’ and ‘immovable’ in-plane boundary conditions are considered. It is observed that the nonlinear-to-linear frequency ratios for panels with immovable in-plane boundary conditions are larger than the ones for the panels with movable in-plane boundary conditions.
Table 7: Effects of end condition on the nonlinear to linear frequency ratios $\omega_{NL}/\omega_{L}$ for double curved (0/90/0/90/0)$_{S}$ GRC laminated panels in thermal environments ($a/b = 1$, $b/h = 10$, $h = 2$ mm, $a/R_1 = b/R_2 = 0.02$)

| $T$ (K) | $\hat{\Omega}$ | $\Omega_{max}/h$ |
|---------|----------------|------------------|
|         | 0.2            | 0.4             | 0.6             | 0.8            | 1.0            |
|         | in-plane immovable boundary condition | | | | | |
| 300     | UD 28.0986     | 1.0232          | 1.0898          | 1.1926         | 1.3232         | 1.4742         |
|         | FG-V 25.3511   | 1.0258          | 1.0997          | 1.2129         | 1.3556         | 1.5195         |
|         | FG-Λ 25.4331   | 1.0249          | 1.0963          | 1.2059         | 1.3444         | 1.5039         |
|         | FG-X 29.5211   | 1.0191          | 1.0744          | 1.1607         | 1.2717         | 1.4016         |
|         | FG-O 23.5816   | 1.0292          | 1.1122          | 1.2382         | 1.3957         | 1.5752         |
| 400     | UD 21.9646     | 1.0333          | 1.1274          | 1.2688         | 1.4437         | 1.6414         |
|         | FG-V 19.6591   | 1.0364          | 1.1386          | 1.2910         | 1.4782         | 1.6887         |
|         | FG-Λ 25.4331   | 1.0388          | 1.1475          | 1.3087         | 1.5056         | 1.7262         |
|         | FG-X 23.8171   | 1.0260          | 1.1003          | 1.2141         | 1.3575         | 1.5222         |
|         | FG-O 17.5688   | 1.0455          | 1.1714          | 1.3555         | 1.5775         | 1.8237         |
|         | in-plane movable boundary condition | | | | | |
| 300     | UD 28.0986     | 1.0061          | 1.0242          | 1.0537         | 1.0936         | 1.1429         |
|         | FG-V 25.3511   | 1.0068          | 1.0269          | 1.0596         | 1.1037         | 1.1580         |
|         | FG-Λ 25.4331   | 1.0066          | 1.0262          | 1.0580         | 1.1011         | 1.1540         |
|         | FG-X 29.5211   | 1.0050          | 1.0199          | 1.0442         | 1.0773         | 1.1184         |
|         | FG-O 23.5816   | 1.0078          | 1.0308          | 1.0681         | 1.1183         | 1.1796         |
| 400     | UD 26.2383     | 1.0063          | 1.0249          | 1.0552         | 1.0963         | 1.1469         |
|         | FG-V 23.8420   | 1.0068          | 1.0271          | 1.0600         | 1.1044         | 1.1590         |
|         | FG-Λ 24.0202   | 1.0070          | 1.0278          | 1.0616         | 1.1071         | 1.1631         |
|         | FG-X 27.3880   | 1.0052          | 1.0208          | 1.0462         | 1.0807         | 1.1235         |
|         | FG-O 22.1643   | 1.0080          | 1.0314          | 1.0695         | 1.1205         | 1.1829         |

5 Concluding remarks

Applying a multi-scale approach, a large amplitude vibration analysis of doubly curved GRC laminated panels has been carried out. The panels are supported by an elastic foundation and in thermal environments. The piecewise functionally graded (FG) GRC layer arrangement is considered to achieve the UD, FG-X, FG-O, FG-V and FG-Λ laminated panels in this study. The extended Halpin-Tsai model is employed to evaluate the material properties of GRC layers which includes the thermal effect on both the graphene sheets and the polymer matrix. Results for large amplitude vibration of doubly curved UD and FG GRC laminated panels are obtained and discussed in detail. Like in the case of cylindrical panels, the rise of temperature will lead to the decrease of the natural frequencies and the increase of the nonlinear-to-linear frequency ratios for the doubly curved GRC laminated panels. On the other hand, the increase of foundation stiffness will result in the increase of the natural frequencies and the decrease of the nonlinear-to-linear frequency ratios of the panels. We observed that the FG-X panel has the highest fundamental frequency but the lowest nonlinear-to-linear frequency ratios and the FG-O panel has the lowest fundamental frequency but the highest nonlinear-to-linear frequency ratios in the considered cases. Unlike in the case of cylindrical panels where both UD and FG-GRC panels display a hardening nonlinearity, in some cases the frequency-amplitude curves of the doubly curved UD and FG-GRC laminated panels exhibit a softening nonlinear behavior at room temperature.

Acknowledgement: This study was supported by the National Natural Science Foundation of China under Grant 51779138 and the Australian Research Council under Grant DP140104156. The authors are very grateful for these financial supports.

Conflict of Interests: The authors declare that there are no conflicts of interests with publication of this work.
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Appendix A

In Eqs. (5)-(8), the thermal forces, moments and higher order moments $\mathbf{N}^T$, $\mathbf{M}^T$ and $\mathbf{P}^T$ of the shell are evaluated by

$$
\begin{bmatrix}
N_x^T \\
N_y^T \\
N_{xy}^T
\end{bmatrix}
= \mathbf{A}_x
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix}
+ \frac{4}{3h^2}
\begin{bmatrix}
P_x^T \\
P_y^T \\
P_{xy}^T
\end{bmatrix}
$$

(A.1a)

and $\mathbf{S}^T$ are defined by

$$
\begin{bmatrix}
S_x^T \\
S_y^T \\
S_{xy}^T
\end{bmatrix}
= \mathbf{A}_y
\begin{bmatrix}
M_x \\
M_y \\
M_{xy}
\end{bmatrix}
$$

(A.1b)

in which $\Delta T = T - T_0$ is the change of temperature with respect to a predefined reference temperature $T_0$ at which the panel does not subject to any thermal strains, and

$$
\mathbf{A}_x =
\begin{bmatrix}
\alpha_{11} & \alpha_{12} & \alpha_{16} \\
\alpha_{12} & \alpha_{22} & \alpha_{26} \\
\alpha_{16} & \alpha_{26} & \alpha_{66}
\end{bmatrix}
$$

(A.2)

in which $\alpha_{ij}$ and are the thermal expansion coefficients for the $k$th ply, $\mathbf{Q}_{ij}$ are the transformed elastic constants for a GRC layer. Note that $Q_{ij} = Q_{ij}$ as zigzag or armchair graphene sheet is used in the GRC layer and

$$
\begin{align*}
Q_{11} &= \frac{E_{11}}{1 - \nu_{12} \nu_{21}}, & Q_{22} &= \frac{E_{22}}{1 - \nu_{12} \nu_{21}}, \\
Q_{12} &= \frac{\nu_{21} E_{11}}{1 - \nu_{12} \nu_{21}}, & Q_{16} &= 0, & Q_{26} &= 0, & Q_{44} &= G_{12}, & Q_{55} &= G_{13}
\end{align*}

(A.3)

where $E_{11}$, $E_{22}$, $G_{12}$, are the effective Young’s and shear moduli and $\nu_{12}$ and $\nu_{21}$ are the Poisson’s ratios of the GRC layer, respectively.

The inertias $I_i (i = 1, 2, 3, 4, 5, 7)$ are defined by

$$
(I_1, I_2, I_3, I_6, I_7)
$$

(A.4)

in which $\rho_k$ is the mass density of the $k$th ply, and

$$
\begin{align*}
\mathbf{F}^T &= \frac{1}{h_k} \int \rho_k(1, Z, Z^2, Z^4, Z^6)dZ \\
&= \sum_{k=1}^{N} \frac{h_k}{h_i} \int \rho_k(1, Z, Z^2, Z^4, Z^6)dZ
\end{align*}

$$
\begin{align*}
I_3 &= I_4 - \frac{T_3 T_2}{T_1}, & I_7 &= I_4 - \frac{T_7 T_2}{T_1}, & I_5 &= I_4 - \frac{T_5 T_2}{T_1}, \\
I_3' &= I_4 - \frac{T_3 T_2}{T_1}, & I_7' &= I_4 - \frac{T_7 T_2}{T_1}, & I_5' &= I_4 - \frac{T_5 T_2}{T_1},
\end{align*}

(A.5)

in which $c_4 = 4/(3h^2)$.

Appendix B

In Eqs. (14a) and (14b), $[A_{ij}]$, $[B_{ij}]$, $[D_{ij}]$, $[E_{ij}]$, $[F_{ij}]$ and $[H_{ij}]$ are the reduced stiffness matrices determined through relationships [70]

$$
\mathbf{A}^* = \mathbf{A}^{-1}, \quad \mathbf{B}^* = -\mathbf{A}^{-1} \mathbf{B}, \quad \mathbf{D}^* = \mathbf{D} - \mathbf{B} \mathbf{A}^{-1} \mathbf{B}, \quad \mathbf{E}^* = -\mathbf{A}^{-1} \mathbf{E}, \quad \mathbf{F}^* = \mathbf{F} - \mathbf{E} \mathbf{A}^{-1} \mathbf{E}, \quad \mathbf{H}^* = \mathbf{H} - \mathbf{E} \mathbf{A}^{-1} \mathbf{E}
$$

(B.1)

where $A_{ij}$, $B_{ij}$, $D_{ij}$, etc., are the panel stiffnesses defined by

$$
(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij})
$$

(B.2a)

$$
= \sum_{k=1}^{N} \frac{h_k}{h_i} \int (\mathbf{Q}_{ij})_k(1, Z, Z^2, Z^4, Z^6)dZ \\
= \sum_{k=1}^{N} \frac{h_k}{h_i} \int (\mathbf{Q}_{ij})_k(1, Z, Z^2, Z^4)dZ
$$

(B.2b)

in which $c_4 = 4/(3h^2)$.

Appendix C

In Eq. (31)

$$
g_{30} = -\left[ \gamma_{170} - (\gamma_{171} m^2 + \gamma_{172} n^2 b^2) \right]
$$

(C.1)
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\[
\gamma_{317} = \frac{g_{06}}{g_{00}} \gamma_{14724} + \frac{g_{05} \gamma_{317} g_{02} + \gamma_{78} n^2 \beta^2 g_{01}}{g_{00}} - \frac{\gamma_{81} m^2 g_{06} + \gamma_{78} n^2 \beta^2 g_{03}}{g_{00}} -\frac{\gamma_{14724} g_{05} g_{05} g_{05}}{g_{06}},
\]

\[g_{31} = G_{31} - D_{02} + 2 G_{32} \Phi(T), \quad g_{32} = D_{02} - D_{12} + 2 G_{33} \Phi(T), \quad g_{33} = G_{33} - D_{22},\]

in which

\[
G_{31} = g_{08} + \gamma_{14724} \frac{g_{05} g_{07}}{g_{06}} + [K_1 + K_2(m^2 + n^2 \beta^2)], \quad (C.2)
\]

\[
G_{32} = -\frac{2}{3 \pi^2 mn} \gamma_{14724} m^2 n^2 \beta^2 \left( \frac{\gamma_{78} + \gamma_{79}}{\gamma_{76}} + \frac{1}{4 m^2 \eta_{76}} \right) + 4 \frac{g_{05}}{g_{06}} (1 - \cos m \pi)/(1 - \cos n \pi)
\]

\[
G_{33} = \frac{1}{16} \gamma_{14724} \left( \frac{m^4}{\gamma_{77}} + \frac{n^4 \beta^4}{\gamma_{76}} \right),
\]

and other symbols are defined as in Shen [49]. Also

\[
\Phi(T) = \lambda + \Theta_2(\lambda)^2 + \Theta_3(\lambda)^3 + \cdots \quad (C.3)
\]

where (with \(m=n=1\))

\[
\lambda = \frac{16}{n^2 g_{08}} \left( \gamma_{73} m^2 + \gamma_{74} n^2 \beta^2 \right) \quad (C.4)
\]

\[
-\left( \gamma_{73} - \gamma_{76} \right) m^2 g_{102} + (\gamma_{74} - \gamma_{77}) n^2 \beta^2 g_{101} \right) \Delta T
\]

\[
\times \left( \frac{D_{11} D_{22} A_{11} A_{22}^*}{\Delta T} \right)^{1/6},
\]

\[
G_{08} = Q_{11} - D_{02},
\]

\[
\Theta_2 = \frac{1}{g_{08}} \left[ \frac{8}{3 \pi^2} \gamma_{14724} m^2 n^2 \beta^2 \left( \frac{\gamma_{78} + \gamma_{79}}{\gamma_{76}} + \frac{1}{4 m^2 \eta_{76}} \right)
\]

\[
+ 4 \frac{g_{05}}{g_{06}} \right] - D_{12},
\]

\[
\Theta_3 = 2 \Theta_2^2 - \frac{G_{33}}{G_{08}},
\]

and for the case of in-plane ‘movable’ condition

\[
B_{000} = b_{000} = B_{100} = b_{100} = B_{200} = b_{200} = 0, \quad (C.5)
\]

and for the case of in-plane ‘immovable’ condition

\[
B_{000} = \eta^{-1} \gamma_{71} \Delta T, \quad b_{000} = \eta^{-1} \gamma_{72} \Delta T, \quad (C.6)
\]

\[
B_{100} = \eta^{-1} \gamma_{24} \frac{\gamma_{70} + \gamma_{75}}{\gamma_{72} - \gamma_{75}}, \quad b_{100} = \eta^{-1} \gamma_{24} \frac{\gamma_{76}^2 + \gamma_{77} \gamma_{75}}{\gamma_{72} - \gamma_{75}},
\]

\[
B_{200} = -\frac{1}{8} \gamma_{24} \frac{\gamma_{75} m^2 + \gamma_{72} n^2 \beta^2}{\gamma_{72} - \gamma_{75}},
\]

\[
b_{200} = -\frac{1}{8} \gamma_{24} \frac{\gamma_{75} m^2 + \gamma_{72} n^2 \beta^2}{\gamma_{72} - \gamma_{75}},
\]