The Radiative Charmed Baryon Decay $\Xi_{c2}^{0*} \rightarrow \Xi_{c1}^{0} \gamma$

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Abstract

V-spin symmetry ($s \leftrightarrow d$ symmetry) forbids the radiative decay $\Xi_{c2}^{0*} \rightarrow \Xi_{c1}^{0} \gamma$ in the SU(3) limit. The quark mass term breaks V-spin symmetry and the leading nonanalytic contribution to the radiative decay amplitude is computable in heavy baryon chiral perturbation theory. The radiative decay branching ratio is determined by the coupling constant $g_2$ and at leading order in chiral perturbation theory is given by $Br(\Xi_{c2}^{0*} \rightarrow \Xi_{c1}^{0} \gamma) = 1.0 \times 10^{-3} g_2^2$. Measurement of this branching fraction will determine $|g_2|$.

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Recently CLEO reported the discovery of $\Xi_{c2}^{0*}$ ($J^\pi = \frac{3}{2}^+$, sextet) with a mass $m(\Xi_{c2}^{0*}) = 2643 \pm 2$ MeV [1]. The dominant decay mode of $\Xi_{c2}^{0*}$ is to $\Xi_{c1}$ ($J^\pi = \frac{1}{2}^+$, anti-triplet) and a pion. Radiative decay $\Xi_{c2}^{0*} \rightarrow \Xi_{c1}\gamma$ is forbidden in the SU(3) limit by V-spin symmetry (a symmetry of strong and electromagnetic interactions under the interchange of strange and down quarks), an SU(2) subgroup of flavor SU(3). We point out that the leading contribution to the radiative decay amplitude of $\Xi_{c2}^{0*}$ is nonanalytic in quark masses ($\sim \mathcal{O}(m_q^{1/2})$), finite and computable in heavy baryon chiral perturbation theory. This leading order calculation gives a radiative decay branching ratio $Br(\Xi_{c2}^{0*} \rightarrow \Xi_{c1}\gamma) = 1.0 \times 10^{-3} g_2^2$, where $g_2$ is the $6^{(*)}6^{(*)}\pi$ coupling in the heavy baryon chiral Lagrangian. Using the value of $g_2$ in the large $N$ limit of QCD (where $N$ is the number of colors) yields a branching ratio of $\sim 1\%$. Although it may be difficult to observe a branching fraction of $10^{-2}$ at CLEO [2], the E781 experiment at Fermilab may be able to reach the $10^{-3}$ level [3]. This would be sufficient to determine $|g_2|$ to an accuracy of $\sim 30\%$. Previous work on radiative charmed baryon decays found that the decay amplitude of $\Xi_{c2}^{0*} \rightarrow \Xi_{c1}\gamma$ is small [4]. However, measurement of this branching ratio will be one of the only ways to determine $|g_2|$. Since $g_2$ enters in many heavy baryon loop calculations, its determination is vital if we wish to go beyond tree level in the heavy baryon sector. Furthermore, it is important to test the large $N$ and quark model predictions for axial coupling constants such as $g_2$.

Physics of hadrons containing a single heavy quark simplifies significantly in the limit where the mass of the heavy quark becomes infinitely greater than the scale of strong interactions [5]. In the heavy quark limit, heavy hadrons can be classified according to the spins of the light degrees of freedom, $s_\ell$. The lowest lying charm baryons contain the $s_\ell = 0(J^\pi = \frac{1}{2}^+)$ states which transform as $\mathbf{3}$ under flavor SU(3) ($\Lambda_{c0}^0$, $\Xi_{c0}^0$ and $\Xi_{c1}^+$), and the $s_\ell = 1(J^\pi = \frac{1}{2}^+, \frac{3}{2}^+)$ states which transform as $\mathbf{6}$ under flavor SU(3) ($\Sigma_{c+}^{++(*)}, \Sigma_{c+}^{+ (*)}, \Sigma_{c0}^{0(*)}, \Xi_{c2}^{++(*)}, \Xi_{c2}^{0(*)}$ and $\Omega_{c0}^{0(*)}$) [6]. It is convenient to introduce two superfields $T_i(v)$ (which transforms as $\mathbf{3}$) and $S^{ij}(v)$ (which transforms as $\mathbf{6}$), where $v$ (the velocity of charmed baryons) is
conserved in the heavy quark limit. The two superfields are

\[ T_i(v) = \frac{1 + \psi}{2} B_i , \]

\[ S^{ij}_\mu(v) = \frac{1}{\sqrt{3}} (\gamma_\mu + v_\mu) \gamma_5 \left( \frac{1 + \psi}{2} B_{ij} + \frac{1 + \psi}{2} B_{ij}^* \right) . \]

(1)

Here

\[ B_1 = \Xi^0_{c1} , \quad B_2 = -\Xi^+_{c1} , \quad B_3 = \Lambda^0_c , \quad (2) \]

while

\[ B_{11} = \Sigma^{++}_c , \quad B_{12} = \frac{1}{\sqrt{2}} \Sigma^+_c , \quad B_{22} = \Sigma^0_c ; \]

\[ B_{13} = \frac{1}{\sqrt{2}} \Xi^+_{c2} , \quad B_{23} = \frac{1}{\sqrt{2}} \Xi^0_{c2} , \quad B_{33} = \Omega^0_c ; \]

(3)

with the corresponding \( B^{(s)}_{ij} \) fields for spin-\( \frac{3}{2} \) partner of 6 .

Interactions of heavy hadrons with soft pions and photons (of energy \( \ll \Lambda_\chi \sim 1 \) GeV) can be described by using both heavy quark symmetry and chiral symmetry. At lowest order in the heavy quark expansion and chiral expansion the heavy baryon chiral Lagrangian is given by [6]

\[ \mathcal{L} = \frac{f^2}{8} Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + \bar{T}^i i v \cdot \partial T_i - \bar{S}^{ij}_\mu i v \cdot \partial S^{ij}_\mu + \Delta \bar{T}^i T_i \]

\[ + i g_2 \epsilon_{\mu\nu\rho} \bar{S}^{ij}_\mu \rho \cdot \nu (A^{\lambda})_k^i S^{jk}_\lambda + g_3 (\epsilon_{ijk} \bar{T}^i (A^{\mu})^j_l S^{kl}_\mu + h. c. ) , \]

(4)

where \( \Delta \) is the mass difference between the 6 and 3 states, and \( D_\mu \) is the chiral covariant derivative. The vector and axial vector chiral fields \( V_\mu \) and \( A_\mu \) are formed
from the pseudo-Goldstone boson fields

\[ V_\mu = \frac{1}{2} (\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger) , \]

\[ A_\mu = \frac{i}{2} (\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger) , \]  

(5)

and \( \xi^2 = \Sigma = \exp \left( \frac{2iM}{f} \right) \), where \( M \) is the meson octet

\[ M = \begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
\pi^- \\
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta \\
K^- \\
\bar{K}^0 \\
-\sqrt{\frac{2}{3}} \eta
\end{pmatrix}, \]  

(6)

and \( f \approx 132 \text{ MeV} \) is the pion decay constant.

The axial coupling constants \( g_2, g_3 \) are unknown parameters in the effective theory and must be determined experimentally. It has been shown that in the large \( N \) limit they are given by \( g_3 = \sqrt{\frac{3}{2}} g_A, \ g_2 = -\frac{3}{2} g_A [7,8] \), where \( g_A \) is the nucleon axial coupling. (Experimentally \( g_A = 1.25 \).) One expects that the deviations from these \( N = \infty \) relations occur at the \( 1/N \) level, i.e., \( \sim 30\% \) [8]. (The nonrelativistic quark model gives \( g_3 = \sqrt{2}, \ g_2 = -2 \).) The total hadronic decay width of \( \Xi_{c2}^0 \rightarrow \Xi_{c1}^0 \pi^0, \Xi_{c1}^+ \pi^- \) is given at tree level by

\[ \Gamma_0 = \frac{g_3^2 |p_\pi|^3}{8\pi f^2} . \]  

(7)

The CLEO result \( \Gamma_0 < 5.5 \text{ MeV} [1] \) gives an upper bound \( |g_3| < 1.4 \). At present there is no experimental information on \( g_2 \). It is important to determine \( g_2 \), not only because \( g_2 \) enters in many heavy baryon loop computations, but also because this will indicate how well the large \( N \) and quark model predictions work in the heavy baryon sector.
We now turn to the radiative decays of the $J^P = \frac{3}{2}^+$ baryons in the $6$ of SU(3). The leading contribution to $6 \to \overline{3}\gamma$ is the magnetic dipole (M1) radiation with the electric dipole (E2) component suppressed by a factor of $1/m_c^2$ [9]. In chiral Lagrangian the M1 transition arises from a dimension 5 operator

$$\mathcal{L} = a\frac{\alpha}{\Lambda_x}T^iQ_i^j\gamma^\mu\gamma_5S^{kl\mu}F_{\mu\nu}\epsilon_{ijk},$$

where

$$Q = \frac{1}{2}(\xi Q\xi^\dagger + \xi^\dagger Q\xi),$$

$Q$ is the charge matrix for the light quarks

$$Q = \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix},$$

and $a$ is some unknown constant of order $O(1)$ by dimensional analysis. This term gives the leading contribution to the radiative decays $\Sigma_c^{0(*)} \to \Lambda_c^0\gamma$ and $\Xi_c^{+(*)} \to \Xi_c^{+}\gamma$. However, it doesn’t contribute to $\Xi_c^{0(*)} \to \Xi_c^{0}\gamma$ because of the V-spin symmetry. The V-spin symmetry is broken by quark masses ($m_d \neq m_s$), and the leading contribution to $\Xi_c^{0(*)} \to \Xi_c^{0}\gamma$ arises from the 1-loop graph (fig. 1) by keeping the masses of $K$ and $\pi$ in the loop. This gives rise to an amplitude of order $\mathcal{O}\left(m_q^{1/2}\right)$. An explicit calculation gives the partial width for $\Xi_c^{0}\gamma$:

$$\Gamma_{\gamma} = \frac{E^3}{6\pi} \left(\frac{eg_2g_3}{16\pi^2f^2}\right)^2 \left[\frac{1}{2}E\log\frac{m_K^2}{m_\pi^2} - \left(J(m_K^2, E) - J(m_\pi^2, E)\right)\right]^2,$$

where

$$J(m^2, E) = \int_0^1 dx\sqrt{m^2 - x^2E^2 - i\epsilon}\left(\pi - 2\tan^{-1}\frac{xE}{\sqrt{m^2 - x^2E^2 - i\epsilon}}\right),$$

and $E_\gamma$ is energy of the photon. (In the limit $\Delta \to 0$, $J(m^2) \to \pi m$.) From

\[\text{† The same calculation gives the width of } \Xi_c^{0}\gamma. \text{ (Pionic transitions from } \Xi_c^{0}\text{ to } \Xi_c^{1}\text{ are kinematically forbidden.)}\]
equations (11) and (12), we arrive at the branching ratio for \( \Xi_{c2}^{0*} \to \Xi_{c1}^0 \gamma \)

\[
Br(\Xi_{c2}^{0*} \to \Xi_{c1}^0 \gamma) = \frac{\Gamma_\gamma}{\Gamma_\pi} = \frac{\alpha g_2^2}{3\pi} \frac{E_\gamma^3}{|\vec{p}_\pi|^3} \left[ \frac{1}{2} E_\gamma \log \frac{m_K^2}{m_\pi^2} - (J(m_K^2, E_\gamma) - J(m_\pi^2, E_\gamma)) \right]^2
\]

(13)

Inserting the large \( N \) value for \( g_2 \) yields a radiative branching ratio of \( \sim 1/3 \% \). This is probably within the reach of E781, and may also be seen at CLEO with some luck.

The leading counter terms which contribute to \( \Xi_{c2}^{0*} \to \Xi_{c1}^0 \gamma \) have the form

\[
\mathcal{L} = b_1 \frac{e}{\Lambda^2} \bar{T}^i Q^i_j (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)^k \gamma^\mu \gamma_5 S^{km,\nu} F_{\mu\nu} \epsilon_{ijk}
\]

\[
+ b_2 \frac{e}{\Lambda^2} \bar{T}^i Q^j_m (\xi m_q \xi + \xi^\dagger m_q \xi^\dagger)^l \gamma^\mu \gamma_5 S^{kl,\nu} F_{\mu\nu} \epsilon_{ijk},
\]

(14)

where \( m_q \) is the light quark mass matrix

\[
\begin{pmatrix}
m_u \\
\vdots \\
m_d \\
\vdots \\
m_s
\end{pmatrix}
\]

(15)

and \( b_1, b_2 \) are unknown constants of order \( O(1) \). These contributions are suppressed by a factor of order \( O\left( m_s^{1/2} \right) \) relative to the leading nonanalytic piece.

There are also \( O\left( \frac{1}{m_c} \right) \) SU(3) breaking corrections to the above result. From naive dimensional analysis we expect that these higher order contributions are down by a factor of \( O\left( \frac{m_K}{m_s}, \frac{m_K}{\Lambda} \right) \) and therefore we expect our result to hold with an approximately 30% uncertainty.

The electric quadrupole (E2) contribution to \( \Xi_{c2}^{0*} \to \Xi_{c1}^0 \gamma \) violates both the heavy quark spin symmetry and flavor SU(3). (The E2 transition to decay \( \Sigma_c^* \to \))
Λ_cγ was considered in ref. [9]. Formally, the leading contribution to the E2 amplitude comes from the same graph (fig. 1) as the M1 amplitude but with the $6^* - 6$ mass difference explicitly retained. A straightforward calculation gives the ratio of the E2 amplitude to the M1 amplitude to be approximately 1%. (The mass splitting between the $6^*$ and 6 states is taken to be $\sim 64$MeV [10].) This is too small to be seen in the near future.

To conclude, we have studied the SU(3) breaking charm baryon decay $\Xi_{c2}^{0*} \to \Xi_{c1}^{0} \gamma$ at leading order in chiral perturbation theory and found a branching ratio of $1.0 \times 10^{-3} g_2^2$, where $g_2$ is the $6^{(*)}6^{(*)}\pi$ axial coupling constant in the heavy baryon chiral Lagrangian. We estimate that the theoretical uncertainty of our result is approximately 30%. Measurement of this branching fraction may prove to be the best way (and perhaps the only way) to determine $|g_2|$.

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Fig. 1: Feynman diagram contributing to $\Xi_{c2}^{0*} \rightarrow \Xi_{c1}^{0} \gamma$ at leading order in heavy baryon chiral perturbation theory.