Novel methods for Solving Economic Dispatch of Security-Constrained Unit Commitment Based on Linear Programming

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Abstract. There are two stages in solving security-constrained unit commitment problems (SCUC) within Lagrangian framework: one is to obtain feasible units’ states (UC), the other is power economic dispatch (ED) for each unit. The accurate solution of ED is more important for enhancing the efficiency of the solution to SCUC for the fixed feasible units’ statues. Two novel methods named after Convex Combinatorial Coefficient Method and Power Increment Method respectively based on linear programming problem for solving ED are proposed by the piecewise linear approximation to the nonlinear convex fuel cost functions. Numerical testing results show that the methods are effective and efficient.

1. Nomenclatures
For the convenience of presentation, some notations are defined as follows.
- \( T \) commitment horizon in hours;
- \( I \) number of units;
- \( K \) number of buses with loads;
- \( L \) number of transmission lines;
- \( P(t) \) power generation by unit \( i \) at time \( t \);
- \( u_i(t) \) binary variable: \( u_i(t) = 1 \) if unit \( i \) is turned on or kept on at time \( t+1 \), else \( u_i(t) = -1 \);
- \( x_i(t) \) the number of hours that unit \( i \) has been up \( (x_i(t) \geq 1) \) or down \( (x_i(t) \leq -1) \);
- \( r_i \) the minimum number of hours that the unit \( i \) must be up;
- \( s_i \) the minimum number of hours that the unit \( i \) must be down;
- \( C(P(t)) \) fuel cost for producing power \( P(t) \) of the unit \( i \); \( C(P(t)) = 0 \) if \( P(t) \);
- \( S_i(x_i(t-1), u_i(t-1)) \) startup/shutdown cost of the unit \( i \);
- \( D_k(t) \) load at bus \( k \) at time \( t \);
- \( D(t) \) system demand or load at time \( t \);
- \( r_i(t) \) the spinning reserve requirement at time \( t \);
- \( r_i(t) \) the spinning reserve contribution at time \( t \) of unit \( i \);
2. Introduction

Unit commitment (UC) is still one of the most significant for independent system operators (ISOs) to clear the electric power market and for generation companies (GENCOMs) to analyze generating costs and determine the bidding strategies [1-6]. The security-constrained unit commitment (SCUC) by incorporating security-related transmission constraints into the UC becomes crucial due to more and more transactions being driven to be taken in open-access electric power market.

Since the unit commitment problem belongs to NP-hard mixed integer programming problems [7], it is very difficult to obtain the optimal feasible solution within acceptable shorter time. Many optimization methods have been applied to solve UC or SCUC problem such as Lagrangian relaxation (LR) and mixed integer linear programming [6-13]. The most obvious advantage of LR is that the system constraints can always be relaxed by introducing Lagrange multipliers associated with the security, the demand and the spinning reserve constraints such that the different units can be decoupled in the dual problem and can be easily solved. However, the most disadvantage of LR is that the primal feasible solution is very difficult to construct based on the dual solution if some complex constraints such as the security constraint are added [3-4, 9, 12].

Some uncertainty optimization methods such as Genetic Algorithm [14-19], Evolutionary Programming [20], Tabu Search [21], artificial intelligence [22-23] Particle Swarm Optimization [24], etc., have also been used to solve UC or SCUC or the economic dispatch problem.

Fuel cost of thermal units has long been considered to be convex quadratic or convex piecewise linear functions [8-12]. Recently, non-convex fuel cost function with valve-point has been put into use due to its more precise formulation for the generating cost of a thermal unit [16, 20, 22]. The different methods for solving UC or SCUC problem with different forms of fuel cost of thermal units should be researched.

The processes of obtaining a feasible solution to UC or SCUC problem can be partitioned into two stages within the LR framework: one is to obtain feasible unit commitment states (UC); the other is to dispatch power generation economically (ED). For fixed feasible unit states, the efficiency of solving economic dispatch with different forms of fuel cost of thermal units is very difficult but very important for enhancing the precision of solution and for analyzing generating costs and determining the bidding strategies in power systems.

Two novel methods based on linear programming for simplifying the formulation of the economic dispatch with convex or convex piecewise linear fuel cost functions are proposed in this paper, which are named after the convex combinatorial coefficient method and power increment method. An implementation of power increment method is given. Testing example shows that the methods proposed in this paper are very efficient and effective.

The rest of this paper is organized as follows. Section II gives the statement of economic dispatch for SCUC problem. The two methods for approximate linearization to economic dispatch for SCUC problem are proposed and verified is in section III. Section IV demonstrated the testing example. Section V concludes the whole paper.

3. Formulation of Economic Dispatch Problem

After a feasible SCUC solution is obtained by solving the corresponding dual problem using Lagrangian Relaxation method, the power generation level of each unit within the commitment
horizon $T$ must be given, which is called economic dispatch (ED). ED of the SCUC problem is to minimize the total power generating cost of the following continuous programming problem for fixed feasible states of units $u_i(t), t = 1, 2, \cdots T; i = 1, 2, \cdots, I$:

$$\min \{ \sum_{t=1}^{T} \sum_{i \in I_u^t} \hat{C}_i(P_i(t)) \}$$

(1)

where $I_u^t$ is the set of all units with $u_i(t) = 1$ at period time $t$, and $\hat{C}_i(P_i(t))$ is in generically denoted as the linear approximation to the quadratic function

$$C_i(P_i(t)) = a_i [P_i(t)]^2 + b_i P_i(t) + c_i, P_i(t) \in [\underline{P}_i, \bar{P}_i]$$

(2)

subject to

(A) System level constraints:

(a) system demand

$$\sum_{i \in I_u^t} P_i(t) = D(t) = \sum_{k=1}^{K} D_k(t)$$

(3)

(b) spinning reserve

$$\sum_{i \in I_u^t} r_i(t) \geq P_i(t);$$

(4)

where

$$r_i(t) = \min \{ \bar{r}_i, \bar{P}_i - P_i(t) \}$$

(5)

(c) DC transmission constraints

$$-\bar{F}_l \leq \bar{F}_l(t) = \sum_{i \in I_l^t} \Gamma_{i,l} P_i(t) - \sum_{k=1}^{K} \Gamma_{l,k} D_k(t) \leq \bar{F}_l,$$

(6)

$l = 1, 2, \cdots, L$

(B) Generation level Constraints

$$\underline{P}_i \leq P_i(t) \leq \bar{P}_i$$

(7)

It is obviously that the solution of ED problem (1) is separately done at each schedule time $t$. It should be noted that for the unit $i$ with ramp rate constraint with ramp rate $\Delta_i$

$$|P_i(t) - P_i(t-1)| \leq \Delta_i$$

(8)

and the corresponding maximal power generation $\bar{P}_i$ and minimal generation $\underline{P}_i$ can be replaced by $\bar{P}_u$ and $\underline{P}_u$, respectively, i.e., $P_i(t)$ satisfies

$$\underline{P}_u = \max \{ P_i \cdot P_i(t-1) - \Delta_i \} \leq P_i(t) \leq \min \{ P_i(t-1) + \Delta_i \} = \bar{P}_u.$$ 

(9)

Thus, by the above preparations, the solution of (1) with constraints (3)-(7 or 9) can be done by solving $T$ quadratic programming problems:

$$(\text{LP0}) \quad J_0^* = \min_{\hat{P}_i(t)} \left\{ \sum_{i \in I_u^t} \hat{C}_i(P_i(t)) \right\}$$

(10)

Subject to (3)-(7 or 9).

For the simplicity of statement, (9) is still replaced by (7) unless otherwise specified.

4. Linearization of Economic Dispatch Problem

The model (10) subject to (3)-(7) is a programming problem with linear and nonlinear constraints, which is rather difficult to be solved. An approximate linear programming problem to model (10) will be constructed by exerting the convexity of fuel cost functions and the concavity of individual unit
spinning reserve functions (5) (Fig.1). The detailed process is as follows.

Firstly, \( M \) points of division on the fuel cost curve of the unit \( i \) are interpolated, they are

\[
(P_{i,1}, C_{i,1}), (P_{i,2}, C_{i,2}), \ldots, (P_{i,M}, C_{i,M})
\]  

(11)

Without loss of generality, we let \( \bar{P}_i - \bar{R}_i \) be one of points of division in the interval \([P_i, \bar{P}_i] \): \( P_{i,1}, \ldots, P_{i,M} \), denoted by \( P_{i,m} \) (otherwise, adding it to the set of points of division).

Assume that the points of division on the spinning reserve curve and on are

\[
(P_{i,1}, r_{i,1}), (P_{i,2}, r_{i,2}), \ldots, (P_{i,M}, r_{i,M})
\]  

(12)

and it is clear that the following relation holds

\[
\begin{cases}
  r_{i,n} = \bar{R}_i, & \text{if } 1 \leq n \leq m_i \\
  r_{i,n} = \bar{P}_i - P_{i,m_i} & \text{if } m_i \leq n \leq M
\end{cases}
\]  

(13)

For the sake of convenience, we allow all the numbers of internal points of division of power interval \([P_i, \bar{P}_i] \), i.e., \( M \). However, the results in this paper hold for different number of internal points of division.

Since \( r_i(t) \) is a function in single variable \( P_i(t) \), the feasibility of a solution to (10) subject to (3)-(7) is completely determined by \( P_i(t) \). Hence, \( r_i(t) \) is not considered as a decision variable.

\[\text{Fig.1 Spinning Reserve Contribution } r_i(t) \text{ of Unit } i\]

\[\text{Fig. 2 Fuel Cost Curve } C_i(P_i(t)) \text{ of Unit } i\]

The main idea of the first method proposed in this paper for solving economic dispatch-convex coefficient method, is to formulate a linear programming problem by exerting the convexity of fuel cost functions and concavity of individual spinning reserve functions to solve the problem approximately. In order to attain the expected goal, a lemma is given firstly.

**Lemma 1**: If \( P^*_i(t) (i \in I^*_i) \) is a feasible solution to the problem (LP0), then there must exists a group of real numbers \( \alpha^i_{1,1}, \ldots, \alpha^i_{1,M} \) such that

\[
P^*_i(t) = \sum_{m=1}^{M} \alpha^i_{1,m} P_{i,m}
\]  

(14)

\[
\sum_{m=1}^{M} \alpha^i_{1,m} = 1, i \in I^*_i; 0 \leq \alpha^i_{1,m} \leq 1
\]  

(15)
\[
\sum_{i \in I^+_t} \sum_{m=1}^M \alpha^i_{t,m} P_{t,m} = D(t) \quad (16)
\]
\[
\sum_{i \in I^+_t} \sum_{m=1}^M \alpha^i_{t,m} r_{t,m} \geq P_i(t) \quad (17)
\]
\[
\sum_{i \in I^+_t} \Gamma_{i,j} \sum_{m=1}^M \alpha^i_{t,m} P_{t,m} \geq \sum_{k=1}^K \Gamma_{i,k} D_k(t) - \bar{F}_i \quad (18)
\]
\[
\sum_{i \in I^+_t} \Gamma_{i,j} \sum_{m=1}^M \alpha^i_{t,m} P_{t,m} \leq \bar{F}_i + \sum_{k=1}^K \Gamma_{i,k} D_k(t) \quad (19)
\]

On the contrary, if there exists a group of real numbers such that (15)-(19) are satisfied, then \( \{P_i(t)\}_{i \in I^+_t} \) determined by (14) \((i = 1,2,\cdots,I)\) must be a feasible solution to the problem (LP0).

**Proof.** Firstly, if \( P_i(t) \) \((i = 1,2,\cdots,I)\) is a feasible solution to the problem (LP0), then \( P_i(t) \) must be between some two points of division, say, \( P_{i,m[i-1]} \leq P_i(t) \leq P_{i,m[i]} \). Therefore, there exists a real number \( \lambda^i_j : 0 \leq \lambda^i_j \leq 1 \) such that
\[
P_i(t) = \lambda^i_j P_{i,m[i-1]} + (1 - \lambda^i_j) P_{i,m[i]} \quad (20)
\]
\[
r_i(t) = \lambda^i_j r_{i,m[i-1]} + (1 - \lambda^i_j) r_{i,m[i]} \quad (21)
\]

The reason for (20) to hold is that the point \( \left( P_{i,m[i]}, r_{i,m[i]} \right) \) is always a division point. Hence, the real numbers
\[
\alpha^i_{t,m} = \begin{cases} 
0, & \text{if } m \neq m[i], m[i] - 1 \\
\lambda^i_j, & \text{if } m = m[i] - 1 \\
1 - \lambda^i_j, & \text{if } m = m[i] 
\end{cases} \quad (22)
\]

\((i \in I^+_t, m = 1,2,\cdots,M)\) are the desired numbers such that the constraints (15)-(19) are all satisfied.

On the contrary, assume that there are a group numbers \( \alpha^i_{t,m} \) \((i \in I^+_t, m = 1,2,\cdots,M)\) such that (15)-(19) hold. Then, \( \alpha^i_{t,m} \) \((i \in I^+_t, m = 1,2,\cdots,M)\) are combinatorial coefficients, and power values \( P_i(t) \) \((i \in I^+_t)\) defined by (15) satisfies (3), (6) and (7). Since the individual unit spinning reserve contribution \( r_i(t) \) defined by (5) is concave function in \( P_i(t) \) on the closed interval \([\bar{P}_i, \bar{P}_i]\), we have
\[
r_i(t) \geq \sum_{m=1}^M \alpha^i_{t,m} r_{i,m} \quad (23)
\]

Hence
\[
\sum_{i \in I^+_t} r_i(t) \geq \sum_{i \in I^+_t} \sum_{m=1}^M \alpha^i_{t,m} r_{i,m} \geq P_i(t) \quad (24)
\]
i.e., the system spinning reserve constraint (4) holds. Thus, the lemma is proved. Q.E.D

**Notes:** Lemma 1 manifests that one solution to economic dispatch problem (LP0) can be defined by a group of proper combinatorial coefficients, but is not unique. Such non-uniqueness cause a difficulty in calculating the fuel cost, since only two real numbers are not zeros while all others must be zeros when the combinatorial coefficient method is used to calculate the fuel cost (14).

The difficulty is solved by the following theorem.
Theorem 1 (Convex Combinatorial Coefficient Method): Considering the following linear programming problem

\[ \text{(LP1)} \quad J^*_1 = \sum_{i=1}^{M} \sum_{m=1}^{I^*_{i,m}} \alpha_{i,m}^* P_{i,m} \quad (25) \]

subject to the constraints (15)-(19). \( J^*_1 \) is the optimal value of (LP1), \( \alpha_{i,m}^* \) \((i \in I^*, m = 1,2,\cdots,M)\) denotes a optimal solution to (LP1). Then, the power value \( \{ \hat{P}_i^*(t) \}_{i \in I^*_t} \) defined by (26)

\[ \hat{P}_i^*(t) = \sum_{m=1}^{M} \alpha_{i,m}^* P_{i,m} \quad (26) \]

is an optimal solution to the problem (LP0).

Proof. Firstly, we will show that the two numbers \( J^*_0 \) and \( J^*_1 \) equals.

As a matter of fact, for any feasible solution to (LP1), say, \( \alpha_{i,m}^* \) \((i \in I^*_t, m = 1,2,\cdots,M)\),

\[ P_i(t) = \sum_{m=1}^{M} \alpha_{i,m}^* P_{i,m} \quad \text{defined by (26)} \]

is a feasible solution to (LP0) by Lemma 1. By the convexity of fuel cost functions of units, we have

\[ \hat{C}_i(P_i(t)) \leq \sum_{m=1}^{M} \alpha_{i,m}^* P_{i,m} \quad (27) \]

and

\[ J^*_0 \leq \sum_{i \in I^*_t} \hat{C}_i(P_i(t)) \leq \sum_{i \in I^*_t} \sum_{m=1}^{M} \alpha_{i,m}^* P_{i,m} \quad (28) \]

Therefore,

\[ J^*_0 \leq J^*_1 \quad (29) \]

On the contrary, for an optimal solution \( \{ P_i(t) \}_{i \in I^*_t} \) to (LP0), by Lemma 1, there must exist a group of combinatorial coefficients \( \alpha_{i,m}^* \) \((i \in I^*_t, m = 1,2,\cdots,M)\), which is a feasible solution to (LP1) such that

\[ P_i(t) = \sum_{m=1}^{M} \alpha_{i,m}^* P_{i,m} \quad \text{is defined by (22)}, \quad \text{we have} \]

\[ \hat{C}_i(P_i(t)) = \sum_{m=1}^{M} \alpha_{i,m}^* C_{i,m} \quad (30) \]

Thus,

\[ J^*_0 = \sum_{i \in I^*_t} \hat{C}_i(P_i(t)) = \sum_{i \in I^*_t} \sum_{m=1}^{M} \alpha_{i,m}^* P_{i,m} \geq J^*_1 \quad (31) \]

By (29) and (31), we have shown

\[ J^*_0 = J^*_1 \quad (32) \]

Secondly, we will show that if \( \alpha_{i,m}^{**} \) \((i \in I^*, m = 1,2,\cdots,M)\) is the optimal solution to (LP1), then \( \{ P_i^*(t) \}_{i \in I^*_t} \) defined by (26) is an optimal solution to (LP0).

In fact, \( \{ P_i^*(t) \}_{i \in I^*_t} \) defined by (26) is a feasible solution to (LP0) due to \( \{ \alpha_{i,m}^{**} \}_{i \in I^*_t, m = 1,2,\cdots,M} \)

is a feasible solution to (LP1) by Lemma1. Thus, combining the convexity of \( \hat{C}_i(P_i(t)) \) \((i \in I^*_t)\), we have
\[ J_0' \leq \sum_{i \in I^u} \hat{C}_i(P_i(t)) \leq \sum_{i \in I^u} \sum_{m=1}^{M} \alpha_{i,m}^u P_{i,m} = J_1' \]  

(33)

Since \( \{P_i(t')\}_{i \in I^u} \) is a feasible solution to (LP0), and the proved result (32), we know that \( \{P_i(t')\}_{i \in I^u} \) is the optimal solution to (LP0).

**Theorem 1** shows that we can solve the approximation economic dispatch by solving a linear programming problem (LP1) with constraints (15)-(19).

The linear problem has \( M \cdot |I^u| \) decision variables, and \( |I^u| + 2M |I^u| + 2L + 2 \) linear constraints (2M |I^u| of which, are boundary constraints), where \( |I^u| \) is the size of the set \( I^u \). The problem (LP1) can be changed into another linear programming problem by exerting Theorem 1 and the convexity of fuel cost functions of units.

Let \( \rho_{i,n} \) denote the slope of the secant connected \( (P_{i,n}, C_{i,n}) \) and \( (P_{i,n+1}, C_{i,n+1}) \) on the fuel cost curve, i.e.,

\[
\rho_{i,n} = \frac{C_{i,n+1} - C_{i,n}}{P_{i,n+1} - P_{i,n}}, \quad n = 1, 2, \ldots, M - 1
\]

(34)

Any feasible solution \( \{P_i(t')\}_{i \in I^u} \) to economic dispatch problem (LP0) must be between two points of division. If for some integer \( m \in \{1, 2, \ldots, M\} \) (35) holds,

\[ P_{i,m} \leq P_i(t') \leq P_{i,m+1} \quad (35) \]

then a group of variables \( \Delta P_{i,n} \), which is called power increment, satisfies (36).

\[
\begin{align*}
\Delta P_{i,n} &= P_{i,n+1} - P_{i,n}, \quad if \quad 1 \leq n < m \\
\Delta P_{i,n} &= P_i(t') - P_{i,n}, \quad if \quad n = m \\
\Delta P_{i,n} &= 0, \quad if \quad m < n \leq M - 1
\end{align*}
\]

(36)

Adding all power increments, we have

\[ P_i(t') = P_{i,1} + \sum_{n=1}^{M-1} \Delta P_{i,n} = P_{i,1} + \sum_{n=1}^{M-1} \Delta P_{i,n} \quad (37) \]

Since the power value \( P_i(t') \) and power increments can be expressed each other due to (36) and (37), the second method (called the power increment method) changing economic dispatch (LP0) into a linear programming proposed in this paper directly considers power increments as the decision variables. Thus, the following theorem is obtained.

**Theorem 2 (Power Increment Method)**: Consider the following linear program

\[ (LP2) \quad J_2^* = \min \sum_{i \in I^u} \sum_{n=1}^{M} \rho_{i,n} \Delta P_{i,n} \quad (38) \]

subject to

\[ 0 \leq \Delta P_{i,n} \leq P_{i,n+1} - P_{i,n} \quad (39) \]

\[ \sum_{i \in I^u} \Delta P_{i,n} = D(t) - \sum_{i \in I^u} P_i \quad (40) \]

\[ \sum_{i \in I^u} \sum_{n=1}^{M-1} \Delta P_{i,n} \leq \sum_{i \in I^u} \bar{P}_i - P_i(t) \quad (41) \]
Then there is a relation between problem (LP0) and (LP2):

If \( \{P_i(t)\}_{i \in I_i'} \) is an optimal solution to (LP0), then power increments \( \{\Delta P_{i,n}\}_{i \in I_i', n=1,2,\ldots,M-1} \) defined by (36) using \( \{P_i(t)\}_{i \in I_i'} \) constructs an optimal solution to (LP2);

On the contrary, if \( \{\Delta P_{i,n}\}_{i \in I_i', n=1,2,\ldots,M-1} \) is an optimal solution to (LP2), then \( \{P_i(t)\}_{i \in I_i'} \) defined by (37) is an optimal solution to (LP0).

**Proof.** The theorem will be shown by three parts.

**Part 1:** we will prove the fact: If \( \{P_i(t)\}_{i \in I_i'} \) is an optimal solution to (LP0), then the vector of power increments \( \{\Delta P_{i,n}\}_{i \in I_i', n=1,2,\ldots,M-1} \) defined by (36) will be used to construct one feasible solution to (LP2).

In fact, if \( \{\Delta P_{i,n}\}_{i \in I_i', n=1,2,\ldots,M-1} \) is an optimal solution to (LP0), then by comparing, (35), (36) and (39), \( \{P_i(t)\}_{i \in I_i'} \) defined by (36) satisfies (39), while (37) can be obtained from (36). The constraints (40), (42) and (43) are the results by substituting (37) into (3) and (6). We will show that (41) is also satisfied by such power increments \( \{\Delta P_{i,n}\}_{i \in I_i', n=1,2,\ldots,M-1} \). In fact, according to the (5) and the definition of \( m_i \), we have

\[
r_i(t) = \begin{cases} 
\bar{r}_i, & \text{if } P_i(t) \leq P_{i,m_i} \\
\bar{P} - P_i(t), & \text{if } P_i(t) > P_{i,m_i}
\end{cases}
\]

(44)

\[
\bar{P}_i = \bar{r}_i + P_{i,m_i}
\]

(45)

By (36), we have

\[
\sum_{n=m_i}^{M-1} \Delta P_{i,n} = \begin{cases} 
0, & \text{if } P_i(t) \leq P_{i,m_i} \\
P_i(t) - P_{i,m_i}, & \text{if } P_i(t) > P_{i,m_i}
\end{cases}
\]

(46)

Substituting (45) and (46) into (44), we have

\[
r_i(t) = \bar{r}_i - \sum_{n=m_i}^{M-1} \Delta P_{i,n}
\]

(47)

Therefore, (41) can be obtained from (4) and (47).

We also have the result (48)
\[ J_0^* = \sum_{i \in I'_2} \hat{C}_i(P(t)) \]
\[ = \sum_{i \in I'_2} C_i(P) + \sum_{i \in I'_2} \sum_{n=1}^{M-1} \rho_{i,n} \Delta P_{i,n} \]
\[ = \sum_{i \in I'_2} C_i(P) + J_2 \]
\[ \geq \sum_{i \in I'_2} C_i(P) + J_2^* \] (48)

**Part 2:** We will show that the fact: if the vector of power increments \( \{ \Delta P_{i,n} \}_{i \in I'_2, n=1,2,\ldots,M-1} \) is an optimal solution to (LP2), then the vector of corresponding power values \( \{ P_i(t) \}_{i \in I'_2} \) defined by (37) is a feasible solution to (LP0).

Firstly, the following phenomenon does not exist: there exist some unit \( i_0 \in I'_2 \) and two integers \( n_1, n_2 \in \{ 1,2,\ldots,M-1 \} \) such that \( n_1 < n_2 \), but
\[ 0 \leq \Delta P_{i_0,n_1} < P_{i_0,n_1} - P_{i_0,n_2} \quad \text{and} \quad \Delta P_{i_0,n_2} > 0 \] (49)

If (9) holds, we let
\[ \Delta P_{i,n} = \Delta P_{i,n_1} + \delta_{i,n_1}, \Delta P_{i,n_2} = \Delta P_{i,n_1} - \delta_{i,n_1} \] (50)

Where
\[ 0 < \delta_{i,n_1} \leq \min \{ P_{i_0,n_1+1}, P_{i_0,n_1} + \Delta P_{i_0,n_1}, \Delta P_{i_0,n_2} \} \] (51)

and \( n = 1,2,\ldots,M-1, n \neq n_1, n \neq n_2 \), let
\[ \Delta P_{i,n} = \Delta P_{i,n}, \quad n = 1,2,\ldots,M-1, n \neq n_1, n \neq n_2 \] (52)

It is obviously that \( \{ \Delta P_{i,n} \}_{i \in I'_2, n=1,2,\ldots,M-1} \) is still a feasible solution to (LP2) defined by (49)-(52).

However, since \( \rho_{i,n_1} < \rho_{i,n_2} \), we have
\[ J_2^* = \sum_{i \in I'_2} \sum_{n=1}^{M} \rho_{i,n} \Delta P_{i,n} \]
\[ = \left( \sum_{i \in I'_2} \sum_{n=1}^{M} \rho_{i,n} \Delta P_{i,n} \right) + \left( \rho_{i_0,n_1} \Delta P_{i_0,n_1} + \rho_{i_0,n_2} \Delta P_{i_0,n_2} \right) \]
\[ = \sum_{i \in I'_2} \rho_{i,n} \Delta P_{i,n} + \left( \rho_{i_0,n_1} - \rho_{i_0,n_2} \right) \delta_{i_0,n_1} < J_2^* \] (53)

This contradicts that \( \{ \Delta P_{i,n} \}_{i \in I'_2, n=1,2,\ldots,M-1} \) is an optimal solution to (LP2). Thus, (49) does not hold, i.e., \( \{ \Delta P_{i,n} \}_{i \in I'_2, n=1,2,\ldots,M-1} \) satisfies (36).

Therefore, the optimal solution \( \{ \Delta P_{i,n} \}_{i \in I'_2, n=1,2,\ldots,M-1} \) to (LP2) satisfies (36), the corresponding
\[ \{ P_i(t) \}_{i \in \mathcal{I}} \text{ defined by (37) satisfies (3) and (6). According to (44), (45) and (36) we have} \]
\[ r_i(t) = \bar{r}_i - \sum_{n=m}^{M-1} \Delta P_{i,n} \]  
(54)

Comparing (41) with (53), (4) holds.

Hence, \( \{ P_i(t) \}_{i \in \mathcal{I}} \) defined by (37) is a feasible solution to (LP0) and,

\[ J_0^* \leq J_0 = \sum_{i \in \mathcal{I}} \sum_{n=1}^{M-1} \tilde{C}_i(P_i(t)) \]
\[ = \sum_{i \in \mathcal{I}} C_i(P_i) + \sum_{i \in \mathcal{I}} \sum_{n=1}^{M-1} \Delta P_{i,n} \]  
(55)

\[ \leq \sum_{i \in \mathcal{I}} C_i(P_i) + J_2^* \]

**Part 3:** By (48) and (55), we have

\[ J_2^* = J_2, J_0^* = J_0 \]  
(56)

Combining part 1, part 2 and (55), Theorem 2 is proved.

5. Implementation of Power Incremental Method

In order to implement the two methods proposed in this paper, the set \( \mathcal{I}_i^n \) of units in the SCUC problem will be classified into three categories at time \( t \): \( E_{1t} \) is the set of units in normal generating state without ramp rate constraints; \( E_{2t} \) the set of units at the first/last generating hour with minimum generation; \( E_{3t} \) the set of units with fixed generating levels at time \( t \). The generation levels of the units in \( E_{2t} \) are constrained to their minimums and those associated with \( E_{3t} \) are required to be fixed or very difficult to be adjusted. In order to obtain better economic dispatch of power generation of units, \( E_{3t} \) can be further divided into four categories as follows

\[ E_{3t} = \{ i \mid i \in E_{3t}, i \in E_{3t} \land i \in E_{2t} \} \]
\[ E_{1t} = \{ i \mid i \in E_{3t}, \bar{P}_m \leq \bar{P}_i \leq \bar{P}_i - \bar{r} \} \]
\[ E_{2t} = \{ i \mid i \in E_{3t}, \bar{P}_i - \bar{r} < P_a \} \]
\[ E_{3t} = \{ i \mid i \in E_{3t}, P_a < \bar{P}_i - \bar{r} < P_a \} \]  
(57)

According to Theorem 2, we have the following theorem 3:

**Theorem 3 (Implementation of Power Increment Method):** The economic dispatch problem (LP2) can be solved approximately by dealing with the following simplified form, i.e., (LP3), of (LP2)

\[ \text{(LP3): } J_3^* = \min_{\Delta P^{k_{i,m}}, \Delta P^{k_{i,j}}} \left[ \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{m=1}^{M_i} \omega_i^{m} \Delta P_{i,m} + \sum_{k=1}^{3} \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{j=1}^{M_i} \rho_{i,j}^{k} \Delta P_{i,j}^{k} \right] \]  
(58)

subject to

\[ \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{m=1}^{M_i} \Delta P_{i,m} + \sum_{k=1}^{3} \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{j=1}^{M_i} \Delta P_{i,j}^{k} = S_{d}(t) \]  
(59)

\[ \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{m=1}^{M_i} \Delta P_{i,m} + \sum_{k=1}^{3} \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{j=1}^{M_i} \Delta P_{i,j} + \sum_{i \in \mathcal{I}_{E_{1}}} \sum_{j=1}^{M_i} \Delta P_{i,j} \leq S_{r}(t) \]  
(60)
\[ \sum_{i \in E_{m}} \sum_{m=1}^{M} \Gamma_{i} \Delta P_{i,m} + \sum_{k=1}^{3} \sum_{i \in E_{m}} \sum_{j=1}^{M-1} \Gamma_{i} \Delta P_{i,j} \geq S_{D}^{i} \]  \hspace{1cm} (61)  \\
\[ \sum_{i \in E_{m}} \sum_{m=1}^{M} \Gamma_{i} \Delta P_{i,m} + \sum_{k=1}^{3} \sum_{i \in E_{m}} \sum_{j=1}^{M-1} \Gamma_{i} \Delta P_{i,j} \leq S_{i}^{2} \]  \hspace{1cm} (62)  \\
\[ 0 \leq \Delta P_{i,m} \leq P_{i,m+1} - P_{i,m}, m = 1, 2, \ldots, M - 1, i \in E_{m} \]  \hspace{1cm} (63)

\[ 0 \leq \Delta P_{i,j}^{k} \leq P_{i,j+1}^{k} - P_{i,j}^{k}, j = 1, \ldots, M_{i,3}^{k} - 1, i \in E_{3}, k = 1, 2, 3 \]  \hspace{1cm} (64)

If the optimal solution to (LP3) is \[ \{ \Delta P_{i,j}^{k} \} \}_{i \in E_{m}, m=1,2, \ldots, M-1}, \{ \Delta P_{i,j}^{k*} \} \}_{i \in E_{m}^{k*}, j=1, \ldots, M_{m}^{k*}, k=1,2,3} \]  \hspace{1cm} (65)

Then the power and individual spinning reserve of each unit \( i \in E_{1} \cup E_{3}^{1} \cup E_{3}^{2} \cup E_{3}^{3} \) is

\[ P_{i}^{r} (t) = P_{i} + \sum_{m=1}^{M-1} \Delta P_{i,m}^{*}, i \in E_{3}^{1} \]  \hspace{1cm} (66)

\[ P_{i}^{*} (t) = P_{i} + \sum_{m=1}^{M-1} \Delta P_{i,m}^{*}, i \in E_{3}^{2}, k=1,2,3 \]  \hspace{1cm} (67)

\[ \bar{r}_{i}^{*} (t) = \bar{r}_{i}, i \in E_{3}^{1} \]  \hspace{1cm} (68)

\[ \bar{r}_{i}^{*} (t) = \bar{P}_{i} - P_{i} - \sum_{j=1}^{M_{i,3}^{k*} - 1} \Delta P_{i,j}^{k*}, i \in E_{3}^{2}, k=1,2,3 \]  \hspace{1cm} (69)

\[ \bar{r}_{i}^{*} (t) = \bar{P}_{i} - P_{i} - \sum_{j=1}^{M_{i,3}^{k*} - 1} \Delta P_{i,j}^{k*}, i \in E_{3}^{3} \]  \hspace{1cm} (70)

where

\[ S_{D}^{i} = D(t) - \sum_{i \in E_{1}} P_{i} - \sum_{i \in E_{2}} P_{i} - \sum_{k=1}^{3} \sum_{i \in E_{m}^{k}} P_{i} - \sum_{k=1}^{3} \sum_{i \in E_{m}^{k*}} P_{i} \]  \\
\[ S_{i}^{j} = \sum_{i \in E_{1}} \bar{r}_{i} + \sum_{k=1}^{3} \sum_{i \in E_{m}^{k}} \bar{r}_{i} + \sum_{i \in E_{m}^{k*}} (\bar{P}_{i} - P_{i}) - P_{i} (t) \]  \\
\[ S_{j}^{l} = \bar{F}_{i} + \sum_{k=1}^{3} \sum_{i \in E_{m}^{k}} D_{i} (t) + \sum_{i \in E_{m}^{k*}} \Gamma_{i} P_{i} + \sum_{k=1}^{3} \sum_{i \in E_{m}^{k}} \Gamma_{i} P_{i} \]  \\
\[ S_{j}^{l} = \bar{F}_{i} + \sum_{k=1}^{3} \sum_{i \in E_{m}^{k}} D_{i} (t) + \sum_{i \in E_{m}^{k*}} \Gamma_{i} P_{i} + \sum_{k=1}^{3} \sum_{i \in E_{m}^{k}} \Gamma_{i} P_{i} \]  \\
\[ \rho_{i,m}^{j} = \frac{C_{i} (P_{i,m+1}) - C_{i} (P_{i,m})}{P_{i,m+1} - P_{i,m}}, m = 1, \ldots, M - 1, i \in E_{1} \]  \\
\[ \rho_{i,j}^{k} = \frac{C_{i} (P_{i,j+1}^{k}) - C_{i} (P_{i,j}^{k})}{P_{i,j+1}^{k} - P_{i,j}^{k}}, j = 1, \ldots, M_{i,3}^{k} - 1, k = 1,2,3 \]  

and \( m, n \) are the indexing number of point of division of each fuel cost curve, \( M \) is the corresponding total number of points of division; \( M_{i,3}^{k} (k = 1,2,3) \) is indexing number of point of division of each fuel cost curve of the \( k \)-th type of units \( E_{3}^{k} \) with ramping constraint; \( m_{r} (i \in E_{m}) \) and \( j_{r}^{*} (i \in E_{3}^{3}) \) are the indexing number of point of division \( \bar{P}_{i} - \bar{r}_{i} \), respectively.
6. Numerical Testing

Example: This example is originated from [5]. The system parameters are summarized in Table 1-4. The percentage of system load drawn by each load bus is given in Table 2 with $D_k(t) = D(t)\sigma_k$, where $k$ is the index of load bus and the system loads are listed in Table 3. The reserve requirements are defined as 10% of the system load at each hour. The basic unit parameters are shown in Table 4 with Units 1-5 having minimum generation constraints at the first/last up hour and ramping constraints. The ramp rates of units’ 1-unit 5 are 100MW, 120MW, 120MW, 190MW and 190MW, respectively. The power grid is illustrated in Fig. 3.

Fig. 3 The power grid with 31 buses, 16 units, 43 transmission lines and 11 load centers

A feasible unit commitment is listed in Table 5 obtained from modifying the corresponding infeasible unit states after 50 dual iterations using Standard Lagrangian Relaxation method. Then the economic dispatch of power of all generating units by solving economic dispatch problem (LP3) is obtained, the total generating costs is $1114010$, the dual lower bound is $1111701.8$, the duality gap is 0.21%. Smaller duality gap presented the better effectiveness and efficiency of the proposed methods in this paper.

| Line: from -> to | Capacity (MW) | Line: from -> to | Capacity (MW) |
|-----------------|---------------|-----------------|---------------|
| 1-2             | 1000          | 16-18           | 1200          |
| 1-12            | 1000          | 16-19           | 800           |
| 2-13            | 1000          | 17-21           | 1200          |
| 3-14            | 2000          | 18-25           | 2500          |
| 3-15            | 2000          | 19-26           | 250           |
| 4-6             | 1500          | 19-31           | 200           |
| 5-6             | 1500          | 20-24           | 1000          |
| 6-7             | 1200          | 20-28           | 1000          |
| 6-18            | 1200          | 20-30           | 1000          |
| 7-16            | 1200          | 21-26           | 900           |
| 7-17            | 1200          | 22-26           | 1250          |
| 8-22            | 1000          | 23-27           | 1250          |
| 9-23            | 1000          | 24-25           | 1000          |
| 10-14           | 1000          | 25-31           | 250           |
| 11-15           | 1000          | 26-27           | 1200          |
| 12-20           | 1000          | 26-29           | 800           |
| 13-18           | 1000          | 26-31           | 600           |
| 13-20           | 1000          | 28-30           | 1000          |
| 14-18           | 1780          | 30-31           | 700           |
| 15-18           | 1780          |                 |               |
Table 2 Percentage of system load drawn by each load bus

| Bus | Percentage (σ_k) | Bus | Percent (σ_k) |
|-----|------------------|-----|---------------|
| 1   | 0.024            | 7   | 0.265         |
| 2   | 0.024            | 8   | 0.062         |
| 3   | 0.361            | 9   | 0.024         |
| 4   | 0.036            | 10  | 0.048         |
| 5   | 0.012            | 11  | 0.12          |
| 6   | 0.024            |     |               |

Table 3 System load by hours

| Hour | Loads (MW) | Hour | Loads (MW) |
|------|------------|------|------------|
| 1    | 2502       | 13   | 7995       |
| 2    | 2441       | 14   | 7201       |
| 3    | 2197       | 15   | 6591       |
| 4    | 2075       | 16   | 6225       |
| 5    | 2502       | 17   | 6652       |
| 6    | 3418       | 18   | 7812       |
| 7    | 4809       | 19   | 8056       |
| 8    | 5859       | 20   | 7079       |
| 9    | 6957       | 21   | 5188       |
| 10   | 7690       | 22   | 4028       |
| 11   | 8056       | 23   | 3174       |
| 12   | 8300       | 24   | 2807       |

Table 4 Basic generator parameters

| Unit | \( \bar{r}_i (MW) \) | \( P_i (MW) \) | \( \bar{P}_i (MW) \) |
|------|------------------|---------------|---------------------|
| 1    | 100              | 300           | 1315                |
| 2    | 120              | 360           | 1578                |
| 3    | 120              | 360           | 1578                |
| 4    | 190              | 360           | 1578                |
| 5    | 190              | 100           | 1815                |
| 6    | 1500             | 300           | 1815                |
| 7    | 800              | 240           | 1052                |
| 8    | 500              | 150           | 657.5               |
| 9    | 500              | 100           | 605                 |
| 10   | 150              | 45            | 197.3               |
| 11   | 300              | 90            | 394.5               |
| 12   | 600              | 120           | 726                 |
| 13   | 750              | 150           | 907.5               |
| 14   | 175              | 52            | 229.6               |
| 15   | 200              | 60            | 263                 |
| 16   | 600              | 120           | 726                 |

Table 5 Feasible unit commitment states

| Units | Unit commitment states at each hour |
|-------|------------------------------------|
| Hour 1|                                    |
| 1     | 1 0 0 0 0 0 1 1 1 1 1 1 1 1 1       |
| 2     | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1|
| 3     | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1|
| 4     | 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1|
| Hour 24|                                   |
7. Conclusions
By using the convexity of the generating cost function and the concavity of spinning reserve contribution of each individual unit, and the linearity of systems constraints such as load balance, system spinning requirement and security constraints, the ED problem is then skillfully transformed into a linear program. Several theorems guarantee the proposed methods to be reasonable. Numerical testing result shows that the methods are effective and efficient.

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