String Representation of Field Correlators in the SU(3)-Gluodynamics

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Abstract

The string representation of the Abelian projected SU(3)-gluodynamics partition function is derived by using the path-integral duality transformation. On this basis, we also derive analogous representations for the generating functionals of correlators of gluonic field strength tensors and monopole currents, which are finally applied to the evaluation of the corresponding bilocal correlators. The large distance asymptotic behaviours of the latter turn out to be in a good agreement with existing lattice data and the Stochastic Model of the QCD vacuum.

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1 Introduction

Despite a lot of efforts towards the construction of string representations of various gauge theories possessing a confining phase (for recent work see e.g. [1, 2] and Refs. therein), up to now the main progress in the solution of this problem has been achieved only for the case of Abelian theories. Those included compact QED in 3D [3] and 4D [4, 5], and the Abelian Higgs Model (AHM) in the normal [6, 7] and dual phases, extended by external electrically charged particles [3].

As far as gluodynamics is concerned, the derivation and investigation of the local string effective action in this theory has been performed in Refs. [8, 1] by making use of the Stochastic Model of the QCD vacuum [9, 10]. However, in this approach, the main aspect of the problem of string representation has not been addressed, namely the task of derivation of the complete string partition function as an integral over string world-sheets. Indeed, the Stochastic Vacuum Model (SVM) enables one to derive the string effective action only for the world-sheet of the minimal area. The reason for that is that within SVM one looses the meaning of the path-integral average over the gluodynamics vacuum, substituting this average by the phenomenological one. As a consequence, it looks difficult to extract singularities corresponding to QCD strings out of the resulting vacuum correlation functions.

In the present Letter, we shall adopt another strategy for the construction of the string representation of SU(3)-gluodynamics. It is based on the so-called Abelian projection method [11, 12]. The essence of this method is a gauge fixing procedure, which reduces the original gauge group SU(N_c) to the maximal Abelian subgroup \([U(1)]^{N_c-1}\). After this partial gauge fixing, the diagonal elements of the matrix-valued vector potential transform under the remaining maximal Abelian subgroup as Abelian gauge fields, whereas the off-diagonal elements transform as charged matter fields. Such a gauge fixing diagonalizes a certain (composite) operator, which transforms by the adjoint representation of SU(N_c). The singularities, which might occur if some of the eigenvalues of this operator coincide, then play the role of magnetic monopoles w.r.t. the “photon” fields of diagonal elements of the vector potential.

In particular, for the SU(2) case, there exists only one type of monopoles, and thus the Abelian projected SU(2)-gluodynamics is simply the dual AHM with monopoles, whereas in the SU(3) case there emerge three types of monopoles, and the Abelian projection leads to a more sophisticated dual model. The resulting effective Lagrangian of this model constructed in Ref. [12] will play a central role in our further investigations. This Lagrangian is also of the AHM type and therefore possesses singularities similar to the Abrikosov-Nielsen-Olesen strings [13] in dual AHM. These singularities could be carefully accounted for on the path-integral level by making use of the corresponding duality transformation [14]. (Similar calculations have been recently also performed in Ref. [15].)

In this Letter, our main goal will be the construction of the string representation for the correlators of gluonic field strength tensors and monopole currents. These correlators play a central role in SVM and are extremely important for the calculation of various QCD processes, which include nonperturbative effects. As it was mentioned above, up to now these correlators were considered in QCD only on the phenomenological level. Their analytical evaluation leading to the corresponding string representations occurred to be possible for the case of the dual AHM with external electrically charged particles and has been performed in Ref. [2]. In the present Letter, we shall apply the Abelian projection method and the path-integral duality transformation to the construction of string representations for these correlators directly in the dual model of SU(3)-gluodynamics. Our calculations will thus partly parallel, but also generalize those of Ref. [2].
The outline of the Letter is as follows. In Section 2, we shall construct the string representation for the partition function of the Abelian projected $SU(3)$-gluodynamics. In Section 3, we shall construct analogous representations for the generating functionals of the gluonic field and monopole current correlators and then by making use of them obtain string representations for the bilocal correlators. Studying the large distance asymptotic behaviours of the latter we find them to be quite similar to those, measured in the lattice experiments in Ref. [16]. In particular, we find that the correlation length of the vacuum in SVM corresponds to the inverse mass of two diagonal dual gluons, which arise when the Abelian projection is performed.

2 String Representation for the Partition Function of the Dual Infrared $SU(3)$-Gluodynamics

In the absence of quarks, the partition function of the infrared effective model of confinement reads [12]

$$Z = \int D\bar{B}_\mu D\chi \delta \left( \sum_{a=1}^{3} \theta_a \right) \exp \left\{ -\int d^4x \left[ \frac{1}{4} \bar{F}_{\mu\nu}^2 + \sum_{a=1}^{3} \left[ \frac{1}{2} \left( \partial_\mu - ig\bar{e}_a \bar{B}_\mu \right) \chi_a \right]^2 + \lambda \left( |\chi_a|^2 - \eta^2 \right)^2 \right] \right\}, \quad (1)$$

where $\bar{F}_{\mu\nu} = \partial_\mu \bar{B}_\nu - \partial_\nu \bar{B}_\mu$ denotes the field strength tensor of the Abelian vector potential $\bar{B}_\mu = (\bar{B}_3^3, \bar{B}_8^8)$. These two (magnetic) fields, which are dual to the usual gluonic fields $A_\mu^3$ and $A_\mu^8$, acquire a mass $m = \sqrt{3} \eta g$ due to the Higgs mechanism.

Next, in Eq. (1), $\chi_a = |\chi_a| e^{i \theta_a}$, $a = 1, 2, 3$, are three complex scalar fields of monopoles possessing magnetic charges $g\bar{e}_a$, respectively. Here,

$$\bar{e}_1 = (1, 0), \quad \bar{e}_2 = \left( -\frac{1}{2}, -\frac{\sqrt{3}}{2} \right), \quad \bar{e}_3 = \left( -\frac{1}{2}, \frac{\sqrt{3}}{2} \right)$$

stand for the so-called root vectors, which define the lattice at which monopole charges $\bar{m}$ are distributed. Namely, $\bar{m} = g \sum_{a=1}^{3} \zeta_a \bar{e}_a$, where $\zeta_a$’s are some integers. Notice, that the partition function (1) has been derived under the assumption that the dominant contribution to it is brought about by the monopoles with the smallest magnetic charge, $\zeta_a = \pm 1$ (see e.g. discussion in Ref. [17]). The meaning of the root vectors can be better understood if we mention that after singling out of the maximal Abelian subgroup $U(1) \times U(1)$ by the redefinition of the $SU(3)$-generators $T_i \equiv \frac{1}{2} \lambda_i$, $i = 1, \ldots, 8$, in the following way

$$\bar{H} \equiv (H_1, H_2) = (T_3, T_8), \quad E_{+1} = \frac{1}{\sqrt{2}} (T_1 \pm iT_2), \quad E_{+2} = \frac{1}{\sqrt{2}} (T_4 \mp iT_5), \quad E_{+3} = \frac{1}{\sqrt{2}} (T_6 \pm iT_7),$$

they obey the following commutation relations

$$[\bar{H}, E_a] = \bar{e}_a E_a, \quad [\bar{H}, E_{-a}] = -\bar{e}_a E_{-a}.$$\[Inclusion of quarks is straightforward and can be done along the lines of Ref. [2]. However, similarly to AHM, those are unimportant for the problem of string representation of field correlators under study.\]
Thus, the root vectors can be interpreted as structural constants in the so-obtained algebra. Expanding the vector potential $A_\mu \equiv A_\mu^j T_j$ over the new set of generators, we see that these vectors define the $U(1) \times U(1)$ charges of off-diagonal gluons.

As it has been demonstrated in Ref. [12], due to the fact that the trajectories of the monopoles of all three kinds are not independent, there should exist a constraint $\sum_{a=1}^3 \theta_a = 0$ relating the monopole fields to each other. We have imposed this constraint by the introduction of a corresponding $\delta$-function into the R.H.S. of Eq. (1).

In what follows, we shall for simplicity consider the model (1) in the so-called London limit, i.e. at $\lambda \to \infty$. In this limit, which corresponds to infinitely heavy monopole fields, the radial parts of the latter can be integrated out, and we are left with the following partition function

$$Z = \int \mathcal{D} \tilde{B}_\mu \mathcal{D} \theta^\text{sing}_a \mathcal{D} \theta^\text{reg}_a Dk \delta \left( \sum_{a=1}^3 \theta^\text{sing}_a \right) \exp \left\{ \int d^4x \left[ -\frac{1}{4} \tilde{F}^2_{\mu\nu} - \frac{\eta^2}{2} \sum_{a=1}^3 \left( \delta_\mu \theta_a - g \varepsilon_a \tilde{B}_\mu \right)^2 + ik \sum_{a=1}^3 \theta^\text{reg}_a \right] \right\}. \tag{2}$$

Similarly to AHM [3, 4, 5], in the model (1), there exist string-like singularities (closed vortices) of the Abrikosov-Nielsen-Olesen type. That is why, in Eq. (2) we have decomposed the total phases of the monopole fields into a singular and regular part, $\theta_a = \theta^\text{sing}_a + \theta^\text{reg}_a$, and imposed the constraint of vanishing of the sum of regular parts by introducing the integration over the Lagrange multiplier $k(x)$. Analogously to the dual AHM, in the model (2), $\theta^\text{sing}_a$'s describe a given electric string configuration and are related to the world-sheets $\Sigma_a$'s of strings of three types via the equations

$$\varepsilon_{\mu\nu\lambda\rho} \partial_\lambda \partial_\rho \theta^\text{sing}_a(x) = 2\pi \Sigma^a_{\mu\nu}(x) \equiv \int d\sigma \exp \left( x_a(\xi) \right) \delta(x - x_a(\xi)). \tag{3}$$

Here, $x_a \equiv x^a_\mu(\xi)$ is a four-vector, which parametrizes the world-sheet $\Sigma_a$, and $\xi = (\xi^1, \xi^2)$ stands for the two-dimensional coordinate.

The path-integral duality transformation of the partition function (2) is parallel to that of Ref. [3]. The only nontriviality brought about by the additional integration over the Lagrange multiplier occurs to be apparent due to the explicit form of the root vectors. Indeed, let us first cast Eq. (2) into the following form

$$Z = \int \mathcal{D} \tilde{B}_\mu Dk e^{-\frac{1}{4} \int d^4x \tilde{F}^2_{\mu\nu}},$$

$$\cdot \int \mathcal{D} \theta^\text{sing}_a \mathcal{D} \theta^\text{reg}_a \mathcal{D} C^a_\mu \exp \left\{ \int d^4x \left[ -\frac{1}{2\eta^2} \left( C^a_\mu \right)^2 + iC^a_\mu \left( \partial_\mu \theta_a - g \varepsilon_a \tilde{B}_\mu \right) + ik \sum_{a=1}^3 \theta^\text{reg}_a \right] \right\} \tag{4}$$

and carry out the integration over the $\theta^\text{reg}_a$'s. In this way, one needs to solve the equation $\partial_\mu C^a_\mu = k$, which should hold for an arbitrary index $a$. The solution to this equation reads

$$C^a_\mu (x) = \frac{1}{2} \varepsilon_{\mu\nu\lambda\rho} \partial_\lambda h^a_{\lambda\rho} (x) - \frac{1}{4\pi^2} \frac{\partial}{\partial x_\mu} \int dy \frac{k(y)}{(x - y)^2},$$

where $h^a_{\lambda\rho}$ stands for the Kalb-Ramond field of the $a$-th type. Next, making use of the constraint $\sum_{a=1}^3 \theta^\text{sing}_a = 0$, replacing then the integrals over $\theta^\text{sing}_a$'s by the integrals over $x^a_\mu(\xi)$'s by virtue of Eq.
(3) and omitting for simplicity the Jacobians emerging during such changes of the integration variables, we arrive at the following representation for the partition function

\[ Z = \int D\vec{B}_\mu e^{-\frac{1}{4}\int d^4xF_{\mu\nu}^2}. \]

\[ \cdot \int Dk \exp \left\{ \frac{1}{4\pi^2} \int d^4xdy \left[ -\frac{3}{2\eta^2} \frac{k(x)k(y)}{(x-y)^2} + ig \left( \frac{\partial}{\partial x_\mu}(x-y)^2 \right) \sum_{a=1}^{3} \xi_a \vec{B}_\mu(x) \right] \right\}. \]

\[ = \int D\bar{x}_\mu(\xi) \delta \left( \sum_{a=1}^{3} \Sigma_{\mu\nu}^a \right) \int Dh^{a}_{\mu\nu} \exp \left\{ \int d^4x \left[ -\frac{1}{12\eta^2} \left( H^{a}_{\mu\nu,\lambda} \right)^2 + i\pi h^{a}_{\mu\nu,\lambda} \Sigma_{\mu\nu}^a - \frac{ig}{2} \varepsilon_{\mu\nu,\lambda\rho} \xi_a \vec{B}_\mu \partial_\nu h^{a}_{\mu} \right] \right\}, \]

where \( H^{a}_{\mu\nu,\lambda} = \partial_\mu h^{a}_{\nu,\lambda} + \partial_\lambda h^{a}_{\mu,\nu} + \partial_\nu h^{a}_{\lambda,\mu} \) stands for the field strength tensor of the Kalb-Ramond field \( h^{a}_{\mu\nu} \). Clearly, due to the explicit form of the root vectors, the sum \( \sum_{a=1}^{3} \xi_a \vec{B}_\mu \) vanishes, and the integration over the Lagrange multiplier thus yields an inessential determinant factor. Notice also, that due to Eq. (3), the constraint \( \sum_{a=1}^{3} \theta^{a}_{\text{sing}} = 0 \) resulted into a constraint for the world-sheets of strings of three types \( \sum_{a=1}^{3} \Sigma_{\mu\nu}^a = 0 \). This means that actually only the world-sheets of two types are independent of each other, whereas the third one is unambiguously fixed by the demand that the above constraint holds.

Integrations over the dual gauge field \( \vec{B}_\mu \) as well as over the Kalb-Ramond fields are now straightforward. Referring the reader for the details to Ref. [2] and taking into account that due to the closeness of the world-sheets in our case all the boundary terms vanish, we finally arrive at the desired string representation

\[ Z = \int D\bar{x}_\mu(\xi) \delta \left( \sum_{a=1}^{3} \Sigma_{\mu\nu}^a \right) \exp \left[ -\frac{gn^3}{4} \left\{ \frac{3}{2} \int_{\Sigma_1} d\sigma_{\mu\nu}(x_{\alpha}(\xi)) \int_{\Sigma_2} d\sigma_{\mu\nu}(x_{\alpha}(\xi')) \frac{K_1(m|x_{\alpha}(\xi) - x_{\alpha}(\xi')|)}{|x_{\alpha}(\xi) - x_{\alpha}(\xi')|} \right] \right], \]

(5)

where \( K_1 \) stands for the modified Bessel function. Finally, it is possible to integrate over one of the three world-sheets, for concreteness \( x_{\mu}^3(\xi) \). This yields the expression for the partition function in terms of the integral over two independent string world-sheets

\[ Z = \int D\bar{x}_\mu(\xi) D\bar{x}_\mu^2(\xi) \exp \left\{ -\frac{gn^3}{2} \left\{ \frac{3}{2} \int_{\Sigma_1} d\sigma_{\mu\nu}(x_{1}(\xi)) \int_{\Sigma_2} d\sigma_{\mu\nu}(x_{1}(\xi')) \frac{K_1(m|x_{1}(\xi) - x_{1}(\xi')|)}{|x_{1}(\xi) - x_{1}(\xi')|} \right\} \right. \]

\[ + \int_{\Sigma_1} d\sigma_{\mu\nu}(x_{1}(\xi)) \int_{\Sigma_2} d\sigma_{\mu\nu}(x_{2}(\xi')) \frac{K_1(m|x_{1}(\xi) - x_{2}(\xi')|)}{|x_{1}(\xi) - x_{2}(\xi')|} \]

\[ + \left. \int_{\Sigma_2} d\sigma_{\mu\nu}(x_{2}(\xi)) \int_{\Sigma_2} d\sigma_{\mu\nu}(x_{2}(\xi')) \frac{K_1(m|x_{2}(\xi) - x_{2}(\xi')|)}{|x_{2}(\xi) - x_{2}(\xi')|} \right\}. \]

(6)
According to Eq. (6), in the language of the effective string theory, the partition function (2) has the form of two independent string world-sheets, which (self-)interact by the exchanges of massive dual gauge bosons.

Notice, that the string tension of the Nambu-Goto term and the inverse bare coupling constant of the rigidity term, corresponding to each of the three terms in the string effective action standing in the exponent on the R.H.S. of Eq. (6) are similar to those of Ref. and read

\[ \sigma = \pi \eta^2 \ln \frac{2M}{\gamma m}, \]

and

\[ \frac{1}{\alpha_0} = -\frac{\pi}{8g^2}, \]

respectively. Here, \( \gamma = 1.781... \) is the Euler’s constant, and \( M = 2\sqrt{2}\lambda\eta \) is the monopole mass following from Eq. (1), which serves as an UV momentum cutoff. Obviously, due to their non-analyticity in \( g \), both of these quantities are manifestly nonperturbative. The second important observation is that \( \alpha_0 < 0 \), which signifies on the stability of the string world-sheets [18, 5].

It is also worth noting, that according to Eq. (6), the energy density corresponding to the obtained effective nonlocal string Lagrangian increases not only with the distance between two points lying on the same world-sheet, but also with the distance between two different world-sheets. This means that also the ensemble of strings as a whole displays confining properties.

### 3 String Representation of Field and Current Correlators

This Section is the main part of the present Letter. Here, we shall derive string representations for field correlators of gluonic field strength tensors and monopole currents. Let us start with the string representation for the generating functional of the correlators of gluonic field strength tensors. In the London limit, this object reads

\[
Z \left[ \vec{S}_{\alpha \beta} \right] = \int D\vec{B}_\mu D^{\text{sing}} \theta_a D^{\text{reg}} \theta_a Dk \delta \left( \sum_{a=1}^{3} \theta_a^{\text{sing}} \right) \cdot \exp \left\{ -\int d^4 x \left[ -\frac{1}{4} \vec{F}_{\mu \nu}^2 - \frac{\eta^2}{2} \sum_{a=1}^{3} \left( \partial_\mu \theta_a - g \varepsilon_a \vec{B}_\mu \right)^2 + ik \sum_{a=1}^{3} \theta_a^{\text{reg}} + i\vec{S}_{\mu \nu} \vec{F}_{\mu \nu} \right] \right\},
\]

where \( \vec{S}_{\mu \nu} \) stands for the source of the field strength tensor \( \vec{F}_{\mu \nu} \equiv \frac{1}{2} \varepsilon_{\mu \nu \lambda \rho} \vec{F}_{\lambda \rho} \), which is nothing else but the field strength of the usual gluonic field \( \vec{A}_\mu = (A_3^\mu, A_8^\mu) \). Performing with Eq. (7) the same path-integral duality transformation as the one applied in the previous Section, we arrive at the following string representation for this generating functional

\[
Z \left[ \vec{S}_{\alpha \beta} \right] = \exp \left( -\int d^4 x \vec{S}_{\mu \nu}^2 \right) \int D^{\text{sing}} \xi D^{\text{reg}} \xi Dk \delta \left( \sum_{a=1}^{3} \Sigma_a \right) \cdot \exp \left[ -\int d^4 x d^4 y \left( \pi \Sigma_{\lambda \nu}^{\text{sing}}(x) - ig \varepsilon_a \vec{S}_{\lambda \nu}(x) \right) D^{\text{sing}} \mu \nu (x-y) \left( \pi \Sigma_{\mu \rho}^{\text{reg}}(y) - ig \varepsilon_b \vec{S}_{\mu \rho}(y) \right) \right],
\]

where \( D^{\text{sing}}_{\lambda \nu, \mu \rho} (x) \) denotes the propagator of the Kalb-Ramond field \( h^{\text{a}}_{\mu \nu} \), defined as (cf. Ref. [2])
\[ D^{ab}_{\lambda\nu,\mu\rho}(x) \equiv \delta^{ab} \left[ D^{(1)}_{\lambda\nu,\mu\rho}(x) + D^{(2)}_{\lambda\nu,\mu\rho}(x) \right], \]

where

\[ D^{(1)}_{\lambda\nu,\mu\rho}(x) = \frac{g\eta^3}{8\pi^2} \sqrt{\frac{2}{3}} \frac{K_1}{|x|} \left( \delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\lambda\nu}\delta_{\mu\rho} \right), \]

\[ D^{(2)}_{\lambda\nu,\mu\rho}(x) = \frac{\eta}{4\pi^2 g x^2} \sqrt{\frac{2}{3}} \left\{ \left[ \frac{K_1}{|x|} + \frac{m}{2} (K_0 + K_2) \right] \left( \delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\lambda\nu}\delta_{\mu\rho} \right) + \frac{1}{2|x|} \left[ 3 \left( \frac{m^2}{4} + \frac{x^2}{12} \right) K_1 + \frac{3m}{2|x|} (K_0 + K_2) + \frac{m^2}{4} K_3 \right] \right. \]

\[ \cdot \left( \delta_{\lambda\mu}x_{\mu}x_{\nu} + \delta_{\mu\nu}x_{\lambda}x_{\rho} - \delta_{\mu\lambda}x_{\nu}x_{\rho} - \delta_{\nu\rho}x_{\mu}x_{\lambda} \right), \]

with \( K_n \equiv K_n(|x|) \) standing for the modified Bessel functions. In the last exponent on the R.H.S. of Eq. (8), the term

\[ -\pi^2 \int_{\Sigma_a} d\sigma_{\lambda\nu} (x_a(\xi)) \int_{\Sigma_a} d\sigma_{\mu\rho} (x_a(\xi')) D^{(2)}_{\lambda\nu,\mu\rho} (x_a(\xi) - x_a(\xi')) \]

can be rewritten as a boundary one and thus vanishes due to the closeness of the string world-sheets.

The string representation for the bilocal correlator of the field strength tensors can now immediately be read off from Eq. (8) by making use of the equality \( \varepsilon^i_a \varepsilon^j_a = \frac{2}{3} \delta^{ij} \), where \( i, j = 1, 2 \) are the \( U(1) \times U(1) \) indices referring to the generators \( T_3, T_8 \). We obtain

\[ \langle \tilde{F}_{\lambda\nu}^i(x) \tilde{F}_{\mu\rho}^j(y) \rangle = \frac{1}{Z[0]} \frac{\delta^2 \mathcal{Z} \left[ \tilde{S}_{\alpha\beta} \right]}{\delta S^i_{\lambda\nu}(x) \delta S^j_{\mu\rho}(y)} \bigg|_{\tilde{S}_{\alpha\beta} = 0} = \delta^{ij} \left( \delta_{\lambda\mu}\delta_{\nu\rho} - \delta_{\lambda\nu}\delta_{\mu\rho} \right) \delta(x-y) + g^2 D^{aa}_{\lambda\nu,\mu\rho}(x-y) - 4\pi^2 g^2 \varepsilon^i_a \varepsilon^j_b \int_{\Sigma_c} d\sigma_{\alpha\beta} (x_c(\xi)) \int_{\Sigma_d} d\sigma_{\gamma\zeta} (x_d(\xi')) D^{ac}_{\alpha\beta,\lambda\nu} (x_c(\xi) - x) D^{bd}_{\gamma\zeta,\mu\rho} (x_d(\xi') - y) \bigg|_{x_a(\xi)} \right), \]

where

\[ \langle \ldots \rangle_{x_a(\xi)} \equiv \frac{\int d x^a_c(\xi) \delta \left( \sum_{a=1}^3 \Sigma^a_{\mu\nu} \right) \ldots \exp \left[ -\frac{g\eta^3}{4} \sqrt{\frac{3}{2}} \int_{\Sigma_a} d\sigma_{\mu\nu} (x_a(\xi)) \int_{\Sigma_a} d\sigma_{\mu\nu} (x_a(\xi')) K_1(|x_a(\xi) - x_a(\xi')|) \right] \bigg|_{x_a(\xi)} \int d x^a_c(\xi) \delta \left( \sum_{a=1}^3 \Sigma^a_{\mu\nu} \right) \exp \left[ -\frac{g\eta^3}{4} \sqrt{\frac{3}{2}} \int_{\Sigma_a} d\sigma_{\mu\nu} (x_a(\xi)) \int_{\Sigma_a} d\sigma_{\mu\nu} (x_a(\xi')) K_1(|x_a(\xi) - x_a(\xi')|) \right] \bigg|_{x_a(\xi)} \right) \]

defines the average over the string world-sheets, and the term with the \( \delta \)-function on the R.H.S. of Eq. (9) corresponds to the free contribution to the correlator.
At this point, let us recall that the theory (1) is a large distance effective theory of confinement. This means that only the large-distance asymptotic behaviours of the correlator (9) should be compatible with those of SU(3)-gluodynamics. At such distances, i.e. at \(|x| \gg \frac{1}{m}\), the propagator of the Kalb-Ramond field \(D^{ab}_{\lambda \nu \rho}(x)\) has the order of magnitude \(g^2 \eta^4\), and therefore the last term on the R.H.S. of Eq. (9) can be disregarded w.r.t. the second one, provided that the following inequality holds

\[
g \eta^2 \cdot \max_a |\Sigma_a| \ll 1,\tag{11}
\]

where \(|\Sigma_a|\) stands for the area of the world-sheet \(\Sigma_a\). From now on, we shall consider the case of small enough \(\eta\) and/or \(g\), for which inequality (11) takes place, and thus only the second term on the R.H.S. of Eq. (9) is sufficient.

Our aim below will be to compare the bilocal correlator (9) in the approximation (11) with the one of SVM, parametrized by two Lorentz structures as follows [9, 10]

\[
\langle \tilde{F}^i_{\lambda \nu}(x) \tilde{F}^j_{\mu \rho}(0) \rangle = \delta^{ij} \left\{ \left( \delta_{\lambda \mu} \delta_{\nu \rho} - \delta_{\lambda \rho} \delta_{\nu \mu} \right) D \left( x^2 \right) + \frac{1}{2} \left[ \partial_\lambda \left( x_\mu \delta_{\nu \rho} - x_\rho \delta_{\nu \mu} \right) + \partial_\nu \left( x_\rho \delta_{\lambda \mu} - x_\mu \delta_{\lambda \rho} \right) \right] D_1 \left( x^2 \right) \right\}, \tag{12}
\]

i.e. to find the coefficient functions \(D\) and \(D_1\). Let us point out once more, that Eq. (12) is nothing else, but the correlator of two usual gluonic field strength tensors, \(F^3_{\mu \nu}(A)\) and/or \(F^8_{\mu \nu}(A)\).

Direct comparison of Eqs. (9) and (12) then yields

\[
D \left( x^2 \right) = \frac{m^3}{4 \pi^2} \frac{K_1}{|x|} \rightarrow \frac{m^4}{4 \sqrt{2} \pi^2} \frac{e^{-m|x|}}{(m \ |x|)^{\frac{3}{2}}}, \quad |x| \gg \frac{1}{m}, \tag{13}
\]

and

\[
D_1 \left( x^2 \right) = \frac{m}{2 \pi^2 x^2} \left[ \frac{K_1}{|x|} + \frac{m}{2} \left( K_0 + K_2 \right) \right] \rightarrow \frac{m^4}{2 \sqrt{2} \pi^2} \frac{e^{-m|x|}}{(m \ |x|)^{\frac{3}{2}}}, \quad |x| \gg \frac{1}{m}. \tag{14}
\]

We now see that the obtained functions (13) and (14) coincide with those from Ref. [9]. Obviously, the bilocal correlator (12) is nonvanishing only for the gluonic field strength tensors of the same kind, i.e. for \(i = j = 1\) or \(i = j = 2\). It is straightforward to show, that this property remains to be valid if one accounts for the last term on the R.H.S. of Eq. (9).

Hence, for these diagonal correlators, the vacuum of the model (1) in the London limit exhibits a nontrivial correlation length, \(T_g = \frac{1}{m}\). Notice also, that the large distance asymptotic behaviours (13) and (14) of the correlation functions \(D\) and \(D_1\) correspond to those obtained for the SU(3)-gluodynamics in the lattice experiments [10]. Namely, both of them decrease exponentially at the distance \(T_g\), and secondly \(D_1 \ll D\) due to the preexponential power-like behaviours.

Finally, it is quite instructive to rederive the coefficient function (13) from the string representation for the correlator of two monopole currents, \(\tilde{j}_\mu = -g \eta^2 \tilde{\varepsilon}_a \left( \partial_\mu \theta_a - g \left( \tilde{\varepsilon}_a \tilde{B}_\mu \right) \right)\). This can be done by virtue of the equation [10]

\[
\langle j^i_\beta(x) j^j_\sigma(y) \rangle = \delta^{ij} \left( \frac{\partial^2}{\partial x_\mu \partial y_\mu} \delta_{\beta \sigma} - \frac{\partial^2}{\partial x_\beta \partial y_\sigma} \right) D \left( \left( x - y \right)^2 \right), \tag{15}
\]

where

\[g \eta^2 \cdot \max_a |\Sigma_a| \ll 1,\]
which can be obtained from the equations of motion. Besides that, it is also useful to derive the string representation for the generating functional of the monopole current correlators itself, which can then be applied to a derivation of the bilocal correlator. Such a generating functional reads

\[ \hat{Z} [\vec{J}_\mu] = \int \mathcal{D} \vec{B}_\mu \mathcal{D} \theta^{\text{sing}}_a \mathcal{D} \theta^{\text{reg}}_a \mathcal{D} k \delta \left( \frac{3}{\sum_a} \theta^{\text{sing}}_a \right) \cdot \exp \left\{ \int d^4 x \left[ -\frac{1}{4} \bar{F}_{\mu \nu}^2 - \frac{\eta^2}{2} \sum_a \left( \partial_\mu \theta_a - g \bar{\varepsilon}_a \vec{B}_\mu \right)^2 + ik \sum_a \theta^{\text{reg}}_a + \vec{j}_\mu \vec{J}_\mu \right] \right\} . \]

Once being applied to this object, path-integral duality transformation yields for it the following string representation

\[ \hat{Z} [\vec{J}_\mu] = \hat{Z}[0] \exp \left[ \frac{m^2}{2} \int d^4 x \vec{J}_\mu^2 (x) \right] \cdot \exp \left\{ g \varepsilon_{\lambda \nu \alpha \beta} \int d^4 x d^4 y \left[ -\frac{g}{8} \varepsilon_{\mu \rho \gamma \delta} \left( \frac{\partial^2}{\partial x_\alpha \partial y_\gamma} D^{aa}_{\lambda \nu, \mu \rho} (x) \right) \vec{J}_\beta (x) \vec{J}_\delta (y) + + \pi \varepsilon_a \Sigma^b_{\mu \rho} (y) \left( \frac{\partial}{\partial x_\alpha} D^{bb}_{\lambda \nu, \mu \rho} (x - y) \right) \vec{J}_\beta (x) \right] \right\} \right|_{a(x)} , \tag{16} \]

where \( \hat{Z}[0] \) is defined by Eq. (5). Then, the string representation for the bilocal correlator following from Eq. (16), reads

\[ \langle j^{ij}_\beta (x) j^{ij}_\alpha (y) \rangle = m^2 \delta^{ij} \delta_\beta \delta_\alpha (x - y) + g^2 \varepsilon_{\lambda \nu \alpha \beta} \varepsilon_{\mu \rho \gamma \sigma} \left\{ -\frac{1}{4} \delta^{ij} \frac{\partial^2}{\partial x_\alpha \partial y_\gamma} D^{aa}_{\lambda \nu, \mu \rho} (x - y) + + \pi \varepsilon_a \Sigma^b_{\mu \rho} \left( \frac{\partial}{\partial x_\alpha} D^{bb}_{\lambda \nu, \mu \rho} (x - y) \right) \right\} \right\} \right|_{a(x)} \tag{17} \]

By comparing of Eqs. (15) and (17), we recover, in the approximation (11), the coefficient function (13). This confirms the consistency of our calculations.

## 4 Summary

In the present Letter, we have generalized the results of Ref. [3] to the non-Abelian case of \( SU(3) \)-gluodynamics, by making use of the Abelian projection method [11, 12]. In particular, we have derived the string representation for the partition function of the effective infrared dual model of confinement, proposed in Ref. [12], in the London limit. It turned out to have the form of two independent string world-sheets, which interact with each other and also self-interact by virtue of the exchanges of the massive dual gauge bosons. The string tension of the Nambu-Goto term and the inverse bare coupling constant of the rigidity term following from the obtained string effective action are similar to those of Ref. [3]. Namely, both of these quantities are nonanalytic in the magnetic coupling constant, which means their nonperturbative nature, and, secondly, the
rigidity coupling constant turned out to be negative, which is sufficient for the stability of the string world-sheets \[8, 3\].

Finally, we have derived string representations for the generating functionals of correlators of gluonic field strength tensors and monopole currents. As it turns out, the vacuum of the model under study displays a nontrivial correlation length, which is seen in the correlators of diagonal (w.r.t. the $U(1) \times U(1)$ maximal Abelian subgroup) gluons of the same kind. This length is equal to the inverse mass of the dual gauge bosons. We have also established a correspondence to SVM and derived two coefficient functions, which parametrize the bilocal correlator in our model for the case of the above described nontrivial correlators. The infrared asymptotic behaviours of these functions turned out to be in agreement with the corresponding lattice data. Finally, by making use of the string representation for the bilocal correlator of the monopole currents and equations of motion, we have recovered the expression for one of the coefficient functions, thus confirming the consistency of the performed calculations.

In conclusion, the proposed approach gives a new status to SVM for the case of $SU(3)$-gluodynamics, by virtue of the methods of the Abelian projection and path-integral duality transformation. It also provides us with some new insights to the solution of the long standing problem of string representation of QCD.

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