Spontaneous symmetry breaking in lasers with periodically modulated gain and refractive index

D Dolinina¹, K Staliunas²,³,⁴ and A Yulin¹

¹ ITMO University, Department of Physics and Engineering, Saint-Petersburg 197101, Russia
² Vilnius University, Faculty of Physics, Laser Research Center, Saulėtekio Ave. 10, Vilnius, Lithuania
³ ICREA, Passeig Lluís Companys 23, 08010, Barcelona, Spain
⁴ UPC, Dep. de Física, Rambla Sant Nebridi 22, 08222, Terrassa (Barcelona) Spain

E-mail: d.dolinina@metalab.ifmo.ru

Abstract. The dynamics of light in active optical systems with periodic complex potential is considered using coupled modes approach where the field is approximated by two counter propagating waves. It is demonstrated that shifting the position of the imaginary part of the potential (effective gain) with respect to the real part of the potential (variation of the refractive index) one can control the effective gain/losses seen by the upper and the power modes. This effect can be used to control the radiation from the laser. The effect of the Kerr nonlinearity is also considered and it is shown that this can result in spontaneous symmetry breaking leading to the formation of the hybrid nonlinear states.

1. Introduction

Laser structures with periodically modulated refractive index, also referred as “distributed feedback” lasers (or “DFB” lasers), gain great attention over the past few decades [1–3]. The index grating induces rescattering of counterpropagating waves on index lattice and provides optical feedback for the laser. In such systems the working mode can be represented as two resonantly coupled counter propagating waves. At the end of the Brillouin zone the group velocity becomes equal to zero and thus this mode has the lowest radiative losses. This facilitate the selection of the modes and helps to achieve single-mode generation regime. Besides the index grating the periodical gain modulation also can couple counterpropagating waves and provide feedback [4, 5]. The combined index and gain coupling also was studied in details [6, 7] but very few works consider the effect of finite shift of the periodic modulations of the gain and the refractive index [8, 9] and, to the best of our knowledge, nonlinear effects have not been taken in account in such two-gratings laser systems. In the present paper we consider the bifurcations of the stationary states forming in the system in the absence and in the presence of the Kerr nonlinearity.

In this work we consider nonlinear active waveguide with periodically changed refractive index and periodically modulated effective gain. We would like to mention that to obtain periodically modulated gain one can use spatially uniform pump and periodic linear losses. The focus point of this paper is the symmetry breaking and the formation of the hybrid stationary states with dominating direction of the energy flow. It has been shown earlier that the similar can occur in nonlinear driven-dissipative systems where the coupling results not only in the dispersion
characteristics splitting but also affects the dissipation rate of the modes so that the modes have very different Q factors [10–12]. Such states are characterized by nonzero energy flux at one of the directions of the waveguide. Here we consider the aforementioned phenomena in active optical system and consider how the relative position of the real and imaginary parts of the periodic potential affect the dynamics and the symmetry breaking bifurcation.

2. Considered system and its mathematical model

The dynamics of the system schematically shown in Fig.1(a) is described by two counter-propagating waves approach and can be express mathematically by following system of equations:

\[
\begin{align*}
(\partial_t + \partial_x)U_+ &= (i\alpha - \gamma)(|U_+|^2 + 2|U_-|^2)U_+ + (i\sigma + \Gamma)U_- + PU_+, \\
(\partial_t - \partial_x)U_- &= (i\alpha - \gamma)(|U_-|^2 + 2|U_+|^2)U_- + (i\sigma^* + \Gamma)U_+ + PU_-,
\end{align*}
\]

where \(U_+\) and \(U_-\) are the slow varying complex amplitudes of the two counter-propagating waves, \(\gamma\) is the nonlinear losses, \(\alpha = \pm 1\) is the Kerr-nonlinearity coefficient, \(\sigma\) is the coupling coefficient of the counter-propagating waves caused by index grating, \(\Gamma\) is the gain coupling caused by gain grating and \(P\) is gain in the system. It is worth noting here, that \(\sigma\) can be complex \(\sigma = |\sigma|e^{i\theta}\), where \(\theta\) is phase difference between the gratings defined as \(\theta = \kappa\Delta x\), \(\kappa\) is the lattice constant and \(\Delta x\) is the shift of the real part of the periodic potential in respect to imaginary one.

The frequency of the linear guided modes in the system is given by the following dispersion characteristics:

\[
\omega_{1,2} = -iP \pm \sqrt{k^2 + (\sigma - i\Gamma)(\sigma^* - i\Gamma)}.
\]

The real part of \(\omega\) is responsible for oscillatory dynamics of the modes and the imaginary part defines the rate of exponential decay or growth of the modes. From the Fig.1(b-e) it is clearly seen that one of the modes has greater gain comparing to the second one at \(k\) near zero. It is important to note here, that if gain grating has zero gain in its minima then one of the eigenmodes has zero effective gain, as it can be seen in Fig.1(c,e).

As it is clearly seen from Fig.1(b,c) the upper mode has higher gain, but situation changes if gratings are shifted on \(\theta = \pi\). In this case the upper mode has lower gain comparing to the upper one, see Fig.1(d,e). A physical explanation of such difference is quite simple: if maxima of standing wave coincide with maxima of gain grating it experiences significantly higher gain comparing to the standing wave in gain grating minima. Therefore, it is possible to provide more gain for preferable mode only by changing the relative phase of the gratings.

Figure 1. (a) Schematic view of a waveguide with periodical index and gain gratings. The gratings have the same lattice constant, but can have nonzero relative phase \(\theta\) defined in the text. (b) Real part of dispersion characteristics with \(\theta = 0\). (c) Imaginary part of dispersion characteristics with \(\theta = 0\). (d) Real part of dispersion characteristics with \(\theta = \pi\). (e) Imaginary part of dispersion characteristics with \(\theta = \pi\).
We start analyzing our full system with the simplest case of real $\sigma$. When $\sigma$ is real it is convenient to introduce new basis of symmetric $U_s = \frac{U_+ + U_-}{\sqrt{2}}$ and antisymmetric $U_a = \frac{U_+ - U_-}{\sqrt{2}}$ modes. In terms of variables $U_s$ and $U_a$ the equations have the following form:

\begin{align}
(\partial_t + (\gamma - i\alpha)K - i\sigma)U_s + (\partial_x + (i\alpha - \gamma)M)U_a &= (P + \Gamma)U_s \\
(\partial_t + (\gamma - i\alpha)K + i\sigma)U_a + (\partial_x + (i\alpha - \gamma)M)U_s &= (P - \Gamma)U_a,
\end{align}

where $K = \frac{3}{2}(|U_s|^2 + |U_a|^2)$, $M = \text{Re}(U_s^*U_a^*)$. If the field does not depend on coordinate, the stationary solution can be found in the form $\vec{U}_s = (U_s \neq 0, U_a = 0)^T$ and $\vec{U}_a = (U_s = 0, U_a \neq 0)^T$. It can be clearly seen that the second mode has lower gain $(P - \Gamma)$ comparing to the first mode and if $P = \Gamma$ it has no gain. Such low-gain mode in linear case corresponds to the lowest dispersion branch with $\theta = 0$ and the highest one with $\theta = \pi$. In nonlinear case one more type of solutions can exist with both modes not equal to zero, such solutions will be referred as hybrid states.

3. Stationary spatially uniform states

We numerically found the full set of the stationary solutions, the bifurcation diagram showing intensity ($W = |U_b|^2 + |U_d|^2 = |U_+|^2 + |U_-|^2$) dependence on gain $P$ for nonlinear symmetric, antisymmetric and hybrid states is shown in upper panel of Fig. 2(a) by blue, black and green lines respectively. Let us note here that symmetric state exist even for negative gain because it has full gain $P + \Gamma$ while the antisymmetric state has a threshold because its effective gain is $P - \Gamma$. As it can be seen from Fig. 2(a) at some point the hybrid state branches off from the symmetric state. This new mode can be interpreted as parametric excitation of the antisymmetric component. It is important to note here that because this new state appears as a result of spontaneous symmetry breaking it has non-equal amplitudes of counter-propagating waves $|U_+| \neq |U_-|$, as it can be seen in lower panel of Fig. 2(a), and such states have nonzero energy flow. Since the energy flux can be directed either to the right or to the left, the bifurcation branch of the hybrid states is double degenerated.

**Figure 2.** Bifurcation diagrams for spatially uniform states. Upper panels show dependence of the full field intensity of states on gain $W = |U_b|^2 + |U_d|^2 = |U_+|^2 + |U_-|^2$; dashed lines indicate dynamically unstable states. Lower panels demonstrate intensities of counterpropagating waves; by solid and dash-dotted different branches from corresponding upper panel are shown. (a) Upper panel shows all kinds of the uniform states with $\theta = 0$. The lower panel demonstrates field structure in terms of counterpropagating waves for symmetric and hybrid states (solid and dash-dotted lines, respectively). (b) Upper panel shows destruction of symmetry breaking bifurcation from panel (a) with $\theta = 0.05\pi$. The lower panel shows $|U_\pm|^2$ for branches from the upper panel. (c) The same as in (b), but $\theta = \pi/2$. (d) Restoring of symmetry breaking bifurcation at $\theta = \pi$. Other parameters are: $\alpha = -1$, $\gamma = 1$, $\Gamma = 0.1$, $|\sigma| = 0.3$. 

We start analyzing our full system with the simplest case of real $\sigma$. When $\sigma$ is real it is convenient to introduce new basis of symmetric $U_s = \frac{U_+ + U_-}{\sqrt{2}}$ and antisymmetric $U_a = \frac{U_+ - U_-}{\sqrt{2}}$ modes. In terms of variables $U_s$ and $U_a$ the equations have the following form:

\begin{align}
(\partial_t + (\gamma - i\alpha)K - i\sigma)U_s + (\partial_x + (i\alpha - \gamma)M)U_a &= (P + \Gamma)U_s \\
(\partial_t + (\gamma - i\alpha)K + i\sigma)U_a + (\partial_x + (i\alpha - \gamma)M)U_s &= (P - \Gamma)U_a,
\end{align}

where $K = \frac{3}{2}(|U_s|^2 + |U_a|^2)$, $M = \text{Re}(U_s^*U_a^*)$. If the field does not depend on coordinate, the stationary solution can be found in the form $\vec{U}_s = (U_s \neq 0, U_a = 0)^T$ and $\vec{U}_a = (U_s = 0, U_a \neq 0)^T$. It can be clearly seen that the second mode has lower gain $(P - \Gamma)$ comparing to the first mode and if $P = \Gamma$ it has no gain. Such low-gain mode in linear case corresponds to the lowest dispersion branch with $\theta = 0$ and the highest one with $\theta = \pi$. In nonlinear case one more type of solutions can exist with both modes not equal to zero, such solutions will be referred as hybrid states.

3. Stationary spatially uniform states

We numerically found the full set of the stationary solutions, the bifurcation diagram showing intensity ($W = |U_b|^2 + |U_d|^2 = |U_+|^2 + |U_-|^2$) dependence on gain $P$ for nonlinear symmetric, antisymmetric and hybrid states is shown in upper panel of Fig. 2(a) by blue, black and green lines respectively. Let us note here that symmetric state exist even for negative gain because it has full gain $P + \Gamma$ while the antisymmetric state has a threshold because its effective gain is $P - \Gamma$. As it can be seen from Fig. 2(a) at some point the hybrid state branches off from the symmetric state. This new mode can be interpreted as parametric excitation of the antisymmetric component. It is important to note here that because this new state appears as a result of spontaneous symmetry breaking it has non-equal amplitudes of counter-propagating waves $|U_+| \neq |U_-|$, as it can be seen in lower panel of Fig. 2(a), and such states have nonzero energy flow. Since the energy flux can be directed either to the right or to the left, the bifurcation branch of the hybrid states is double degenerated.
The hybrid states appear as a result of the pitchfork bifurcation and the points where such bifurcation takes place is easy to define. The symmetric and antisymmetric states become parametrically unstable at following values of gain:

\[ P_{SSB}^\pm = \pm \frac{(2\Gamma^2\gamma + \alpha\Gamma\sigma + 3\gamma\sigma^2)}{\Gamma\gamma - \alpha\Gamma}, \]  

where \( P_{SSB}^+ \) and \( P_{SSB}^- \) are bifurcation points for symmetric and antisymmetric states respectively. From this expression one can verify that regardless of zero or \( \pi \) phase difference between the gratings and of Kerr nonlinearity sign spontaneous breaking bifurcation can take place both on symmetric or antisymmetric states.

The system dynamics becomes richer when \( \sigma \) is complex. In terms of the counterpropagating waves the corresponding linear eigenmodes have the form \( \vec{U}_{1,2} = (\pm \sqrt{\sigma - i\Gamma}, \sqrt{\sigma^* - i\Gamma})^T \), which shows that the symmetry gets broken. Because of such symmetry breaking caused by the phase shift between the gratings the double degenerated bifurcation branches of stationary states split in two, as it can be seen in Fig.2(b). The symmetric state with nonzero phase shift \( \theta \) becomes slightly asymmetric and spontaneous symmetry breaking bifurcation transforms into a fold bifurcation. This splitting becomes more significant as \( \theta \) grows to \( \pi/2 \), see Fig.2(c). The bifurcation branches become degenerated again, as phase shift approaches \( \theta = \pi \) and eigenmodes become symmetric what results in restoring of symmetry breaking bifurcation, as it can be seen in Fig2(d). It is important to note, that here we considered appearance of hybrid modes from the upper dispersion branch with higher gain at \( \theta = 0 \), but as \( \theta \to \pi \) the upper dispersion branch becomes low-gain mode, so the threshold appears.

As it can be seen from Fig.2 both types of hybrid states with spontaneously and forced broken symmetry can be dynamically stable, what allow us to develop our research with search and studying different localized structures such as switching waves and solitons.

4. Conclusion

The rich dynamics of the electromagnetic field in the systems with complex periodic potentials and Kerr nonlinearity can be used to achieve more control over the stationary lasing states. For instance the effects discussed in the paper can potentially be used to affect the nonlinear patterns forming in the lasers, switching their radiation frequency and the direction of the emission.

5. Acknowledgments

The work was supported by Russian Fund for Basic Research (Grant “Aspiranty” No. 20-32-90227).

[1] Kogelnik H and Shank C 1972 *Journal of applied physics* **43** 2327–2335
[2] Kazarinov R and Henry C 1985 *IEEE Journal of Quantum Electronics* **21** 144–150
[3] Poudavoud N, Mayer A, Buchm"uller M, Brinkmann K, Haeger T, Hu T, Heiderhoff R, Shutsko I, G"orn P, Chen Y et al. 2018 *Advanced Materials Technologies* **3** 1700253
[4] Luo Y, Nakano Y, Tada K, Inoue T, Hosomatsu H and Iwaoka H 1990 *Applied physics letters* **56** 1620–1622
[5] Luo Y, Nakano Y, Tada K, Inoue T, Hosomatsu H and Iwaoka H 1991 *IEEE journal of quantum electronics* **27** 1724–1731
[6] David K, Morthier G, Vankwikelberge P and Baets R 1990 *Electronics Letters* **26** 238–239
[7] Fessant T 1998 *JOSA B* **15** 2689–2699
[8] Cardimona D, Sharma M, Kovavis V and Gavrielides A 1995 *IEEE journal of quantum electronics* **31** 60–66
[9] Kwon K Y 1996 *IEEE journal of quantum electronics* **32** 1937–1949
[10] Krasikov S, Bogdanov A and Iorsh I 2018 *Physical Review B* **97** 224309
[11] Dolinina D and Yulin A 2020 *Optics Letters* **45** 3781–3784
[12] Dolinina D and Yulin A 2021 *arXiv preprint arXiv:2102.07526*