Upsilon states in magnetized nuclear matter

Amal Jahan C.S., Shivam Kesarwani, Sushruth Reddy P., Nikhil Dhale, and Amruta Mishra

Department of Physics, Indian Institute of Technology, Delhi, Hauz Khas, New Delhi – 110 016, India

Abstract

The mass modifications of the bottomonium states ($\Upsilon(NS), N = 1, 2, 3, 4$ and $\Upsilon(1D)$) in magnetized nuclear matter are studied using chiral effective model. The in-medium masses are calculated from the medium modification of the scalar dilaton field in the chiral effective model, which simulates the gluon condensates of QCD. The strengths of the wave functions (assumed to be harmonic oscillator wave functions) denoted by the parameter, $\beta$, of the $\Upsilon(NS), N = 1, 2, 3, 4$, are fitted from their observed decay widths to $e^+e^-$. The decay width for the channel, $\Upsilon(1D) \rightarrow e^+e^-$ yet been observed experimentally, has been predicted in the present work, by using the value of the parameter, $\beta$ for $\Upsilon(1D)$, interpolated from the $\beta$ versus mass relation, for the upsilon states. The effects of the isospin asymmetry of the nuclear medium on the masses of the upsilon states are investigated and are observed to be large for high densities. This should have observable consequences at the asymmetric heavy ion collisions at the Compressed baryonic matter (CBM) experiments at FAIR, GSI as well as at SPS, CERN. The study of the bottomonium states at CBM will however require access to higher energies than the energy regime planned at present. The effects of magnetic field on the masses of bottomonium states in nuclear matter are studied in the present work. These masses are investigated including the anomalous magnetic moments (AMM) of the nucleons, and compared to the results when the AMMs of nucleons are not taken into account. The effects of magnetic field as well as isospin asymmetry on the upsilon masses are observed to be large at high densities.
I. INTRODUCTION

The study of the in-medium properties of the hadrons has been a topic of intense research in the recent past, due to its relevance in ultra-relativistic heavy ion collision experiments. In these experiments, matter at high density and/or temperature is created and the experimental observables are affected by modifications of the hadrons in the strongly interacting medium. In ultra-relativistic heavy ion collision experiments, e.g., at RHIC at BNL and at LHC at CERN, large magnetic fields are also believed to be created [1]. This has initiated a lot of work to study the effects of strong magnetic fields on the properties of hadrons in the medium. The study of heavy flavoured hadrons [2] has also attracted a lot of attention in the recent years, as these are being (planned to be) investigated extensively in the heavy ion collision experiments. Within the QCD sum rule approach, the masses of the heavy quarkonium states (charmonium and bottomonium states) are modified in the hadronic medium, due to the medium modifications of the gluon condensates in QCD [3–6]. On the other hand, the medium modifications of the masses of the light vector mesons [7, 8], as well as of the open charm (bottom) mesons [9, 11] arise due to the modifications of the light quark condensates. In the literature, the open heavy flavour mesons have also been studied using the coupled channel approach [12–16], the quark meson coupling model [17], due to pion exchange with nucleons [18], the heavy meson effective theory [19], studying the heavy flavour meson as an impurity in nuclear matter [20], as well as, using a chiral effective model [21–27]. There have also been predictions for the heavy meson-nucleus bound states [28, 29]. The formation of such bound states could be possible, due to the attractive interaction of the heavy mesons in the nuclear medium [5, 22, 23, 29]. The huge magnetic fields created in ultra-relativistic heavy ion collision experiments has also initiated the study of the heavy flavour mesons in presence of strong magnetic fields [30–35].

We study the mass modifications of the upsilon states ($\Upsilon(NS), N = 1, 2, 3, 4$ and $\Upsilon(1D)$) arising due to medium change of a scalar dilaton field, within a chiral effective model [36–38], which is incorporated in the model to simulate the gluon condensates of QCD. The chiral effective model has been used to study finite nuclei [37], hot hyperonic matter [38], in-medium properties of the light vector ($\omega, \rho$ and $\phi$) mesons [39], and the kaons and antikaons [40, 43] as well as to investigate the bulk matter in the interior of (proto) neutron stars [44]. The chiral SU(3) model has also been generalized to SU(4) as well as to SU(5) to derive the interactions of
the charm as well as bottom mesons with the light hadronic sector. Using these interactions, the open (strange) charm mesons [21–25], open (strange) bottom mesons [26, 27], the charmonium states [22, 23], the upsilon states [45] have been studied. Within the chiral effective model, the mass modifications of the open charm (bottom) mesons arise due to their interactions with the baryons and scalar mesons, whereas the mass modifications of the charmonium and bottomonium states arise due to interaction with the scalar dilaton field, which simulates the gluon condensates of QCD. Using the medium modifications of the charmonium states as well as the $D$ and $\bar{D}$ mesons as calculated within the chiral effective model, the partial decay widths of the charmonium states to $D\bar{D}$ in the hadronic medium [23] have been studied using a light quark creation model [46], namely the $^3P_0$ model [47]. The in-medium decay widths of the charmonium (bottomonium) to $D\bar{D}$ ($B\bar{B}$) have also been investigated using a field theoretic model for composite hadrons [48, 49]. Recently, the effects of magnetic fields on the masses of the open charm $D$ and $\bar{D}$ mesons [33], the open bottom mesons [34], as well as, the charmonium states [35] in asymmetric nuclear matter, have also been studied using the chiral effective model.

The outline of the paper is as follows: In section II, we describe briefly the chiral effective model used to investigate the medium modifications of the upsilon states in magnetized nuclear matter. These modifications arise due to medium change of the dilaton field, which mimics the gluon condensates of QCD. The effects of the anomalous magnetic moments of the nucleons on the bottomonium masses are also investigated for different values of the magnetic field. In section III, we discuss the results obtained for the in-medium masses of the bottomonium states in strong magnetic fields. In section IV, we summarize the findings of the present study.

II. IN-MEDIUM MASSES OF THE BOTTOMONIUM STATES

The medium modifications of the bottomonium masses in strongly magnetized asymmetric nuclear matter are studied using a chiral effective model [37]. The model is based on the nonlinear realization of chiral symmetry [50–52] and broken scale invariance [37–39]. The effective hadronic chiral Lagrangian density contains the following terms

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{BM} + \mathcal{L}_{\text{vec}} + \mathcal{L}_0 + \mathcal{L}_{\text{scalebreak}} + \mathcal{L}_{SB} + \mathcal{L}_{\text{mag}}. \quad (1)$$
In the above Lagrangian density, the first term $L_{\text{kin}}$ corresponds to the kinetic energy terms of the baryons and the mesons. $L_{BM}$ is the baryon-meson interaction term, $L_{vec}$ corresponds to the interactions of the vector mesons, $L_0$ contains the meson-meson interaction terms, $L_{\text{scalebreak}}$ is a scale invariance breaking logarithmic potential given in terms of a scalar dilaton field \cite{53}, $L_{SB}$ is the explicit chiral symmetry breaking term, and $L_{\text{mag}}$ is the contribution from the magnetic field, given as \cite{33,35,54,57}

$$L_{\text{mag}} = -\bar{\psi}_i q_i \gamma_\mu A^\mu \psi_i - \frac{1}{4} \kappa_i \mu_N \bar{\psi}_i \sigma^{\mu\nu} F_{\mu\nu} \psi_i - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}, \tag{2}$$

where, $\psi_i$ is the field operator for the $i$-th baryon ($i = p, n$, for nuclear matter, as considered in the present work), and the parameter $\kappa_i$ in the second term in equation (2) is related to the anomalous magnetic moment of the $i$-th baryon \cite{54,60}. The values of $\kappa_p$ and $\kappa_n$ are given as $3.5856$ and $-3.8263$ respectively, which are the values of the gyromagnetic ratio corresponding to the anomalous magnetic moments of the proton and neutron respectively.

In the present study of the in-medium upsilon masses in magnetized nuclear matter using the chiral SU(3) model, we use the mean field approximation, where all the meson fields are treated as classical fields. The coupled equations of motion for the non-strange scalar field $\sigma$, strange scalar field $\zeta$, scalar-isovector field $\delta$ and dilaton field $\chi$, are derived from the Lagrangian density and are given as

\[
\begin{align*}
    k_0 \chi^2 \sigma - 4k_1 \left(\sigma^2 + \zeta^2 + \delta^2\right) \sigma - 2k_2 \left(\sigma^2 + 3\sigma\delta^2\right) - 2k_3 \chi \sigma \zeta & - \frac{d}{3} \chi^4 \left(\frac{2\sigma}{\sigma^2 - \delta^2}\right) + \left(\frac{\chi}{\chi_0}\right)^2 m_\pi^2 f_\pi - \sum g_{\sigma i} \rho_i^\sigma = 0 \tag{3} \\
    k_0 \chi^2 \zeta - 4k_1 \left(\sigma^2 + \zeta^2 + \delta^2\right) \zeta - 4k_2 \zeta^3 - k_3 \chi \left(\sigma^2 - \delta^2\right) & - \frac{d}{3} \chi^4 \left(\frac{\chi}{\chi_0}\right)^2 \left[\sqrt{2}m_\pi^2 f_\pi - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi\right] - \sum g_{\zeta i} \rho_i^\zeta = 0 \tag{4} \\
    k_0 \chi^2 \delta - 4k_1 \left(\sigma^2 + \zeta^2 + \delta^2\right) \delta - 2k_2 \left(\delta^3 + 3\sigma^2\delta\right) + 2k_3 \chi \delta \zeta & + \frac{2}{3} d \chi^4 \left(\frac{\delta}{\sigma^2 - \delta^2}\right) - \sum g_{\delta i} \rho_i^\delta = 0 \tag{5} \\
    k_0 \chi \left(\sigma^2 + \zeta^2 + \delta^2\right) - k_3 \left(\sigma^2 - \delta^2\right) \zeta + \chi^3 \left[1 + \ln \left(\frac{\chi^4}{\chi_0^4}\right)\right] + 4k_4 \chi^3 & - \frac{4}{3} d \chi^3 \ln \left(\left(\frac{\sigma^2 - \delta^2}{\sigma_0^2}\right) \left(\frac{\chi}{\chi_0}\right)^3\right) + 2\chi \left(\frac{m_\pi^2 f_\pi}{\chi_0^2}\right) m_\pi^2 f_\pi \sigma + \left(\sqrt{2} m_\pi^2 f_k - \frac{1}{\sqrt{2}} m_\pi^2 f_\pi\right) \zeta = 0 \tag{6}
\end{align*}
\]
In the above, \( \rho_i (i = p, n) \) are the scalar densities for the nucleons. In the presence of magnetic field, the proton has contributions from the Landau energy levels. The number density and the scalar density of the proton are given as

\[
\rho_p = \frac{eB}{4\pi^2} \left[ \sum_{\nu=0}^{\nu_{\text{max}}} k_{f,\nu,1}^{(p)} + \sum_{\nu=1}^{\nu_{\text{max}}} k_{f,\nu,-1}^{(p)} \right] \tag{7}
\]

and

\[
\rho_p^s = \frac{eB \mu N B}{2\pi^2} \left[ \sum_{\nu=0}^{\nu_{\text{max}}} \frac{\sqrt{m_p^* + 2eB\nu + \Delta_p}}{\sqrt{m_p^* + 2eB\nu}} \ln \left| \frac{k_{f,\nu,1}^{(p)} + E_f^{(p)}}{\sqrt{m_p^* + 2eB\nu + \Delta_p}} \right| + \sum_{\nu=1}^{\nu_{\text{max}}} \frac{\sqrt{m_p^* + 2eB\nu - \Delta_p}}{\sqrt{m_p^* + 2eB\nu}} \ln \left| \frac{k_{f,\nu,-1}^{(p)} + E_f^{(p)}}{\sqrt{m_p^* + 2eB\nu - \Delta_p}} \right| \right] \tag{8}
\]

where, \( k_{f,\nu,\pm 1}^{(p)} \) are the Fermi momenta of protons for the Landau level, \( \nu \) for the spin index, \( s = \pm 1 \), i.e. for spin up and spin down projections for the proton. These Fermi momenta are related to the Fermi energy of the proton as

\[
k_{f,\nu,s}^{(p)} = \sqrt{E_f^{(p)} - \left( \sqrt{m_p^* + 2eB\nu + s\Delta_p} \right)^2}. \tag{9}
\]

The number density and the scalar density of neutrons are given as

\[
\rho_n = \frac{1}{4\pi^2} \sum_{s=\pm 1} \left\{ \frac{2}{3} k_{f,s}^{(n)} + s\Delta_n \left[ (m_n^* + s\Delta_n)k_{f,s}^{(n)} + E_f^{(n)} 2\arcsin \left( \frac{m_n^* + s\Delta_n}{E_f^{(n)}} \right) - \frac{\pi}{2} \right] \right\} \tag{10}
\]

and

\[
\rho_n^s = \frac{m_n^*}{4\pi^2} \sum_{s=\pm 1} \left[ k_{f,s}^{(n)} E_f^{(n)} - (m_n^* + s\Delta_n)^2 \ln \left| \frac{k_{f,s}^{(n)} + E_f^{(n)}}{m_n^* + s\Delta_n} \right| \right] \tag{11}
\]

In the above, the Fermi momentum, \( k_{f,s}^{(n)} \) for the neutron with spin projection, \( s (s = \pm 1 \) for the up (down) spin projection), is related to the Fermi energy for the neutron, \( E_f^{(n)} \) as

\[
k_{f,s}^{(n)} = \sqrt{E_f^{(n)} - (m_n^* + s\Delta_n)^2}, \tag{12}
\]

where \( \Delta_i = -\frac{1}{2}\kappa_i \mu N B \), where, \( \kappa_i \), is as defined in the electromagnetic tensor term in the Lagrangian density given by \( (2) \). For given value of the baryon density, \( \rho_B \), and the isospin asymmetry parameter, \( \eta = (\rho_n - \rho_p)/(2\rho_B) \), the values of the scalar fields are solved from the equations \( (3), (4), (5) \) and \( (6) \).
The medium modifications of the masses of the bottomonium states in the nuclear medium arise due to the modification of the gluon condensates, which is calculated within the chiral SU(3) model, from the medium change of the expectation value of the dilaton field. Equating the trace of the energy momentum tensor of QCD in the massless quarks limit \[61\], using the one loop beta function, with \( N_c = 3 \) and \( N_f = 3 \), to that of the chiral effective model as used in the present work, leads to the relation of the dilaton field to the scalar gluon condensate as given by \[35\]

\[
\left\langle \frac{\alpha_s}{\pi} G_{\mu\nu}^a G^{\mu\nu a} \right\rangle = \frac{8}{9} (1 - d) \chi^4
\]  

(13)

The mass shift of the bottomonium state arises due to the medium modification of the scalar gluon condensate, and hence due to the change in the value of the dilaton field, and is given as \[22, 23\]

\[
\Delta m_\Upsilon = \frac{4}{81} (1 - d) \int dk^2 \left( \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 \right) \frac{k}{k^2/m_b + \epsilon} \left( \chi^4 - \chi_0^4 \right),
\]  

(14)

where

\[
\left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 = \frac{1}{4\pi} \int \left| \frac{\partial \psi(\vec{k})}{\partial \vec{k}} \right|^2 d\Omega,
\]  

(15)

\( m_b \) is the mass of the bottom quark, taken as 5.36 GeV, \( m_\Upsilon \) is the vacuum mass of the bottomonium state and \( \epsilon = 2m_b - m_\Upsilon \). The values of the dilaton field in the nuclear medium and in the vacuum are \( \chi \) and \( \chi_0 \) respectively. \( \psi(\vec{k}) \) is the wave function of the bottomonium state in the momentum space, normalized as \( \int \frac{d^3k}{(2\pi)^3} |\psi(\vec{k})|^2 = 1 \) \[62\]. The wave functions of the bottomonium states can be obtained using Fourier transformations of the wave functions in the co-ordinate space, which are assumed to be harmonic oscillator wave functions \[46\] and are given as

\[
\psi_{Nlm}(\vec{r}) = N_{NI} \times (\beta^2 r^2)^{\frac{3+1}{2}} \exp \left( -\frac{1}{2} \beta^2 r^2 \right) L_{N-1}^{l+\frac{1}{2}} \left( \beta^2 r^2 \right) Y_{lm}(\theta, \phi) \equiv R_{NI}(r)Y_{lm}(\theta, \phi)
\]  

(16)

where \( \beta = \sqrt{M\omega/\hbar} \) characterizes the strength of the harmonic potential, \( M = m_b/2 \) is the reduced mass of bottom quark - bottom antiquark system, \( L_{N-1}^l (\beta^2 r^2) \) is the associated Laguerre Polynomial. The wave functions in the co-ordinate space are normalized as

\[
\int d^3r |\psi_{Nlm}(\vec{r})|^2 = 1,
\]  

(17)

with \( \int_0^\infty |R_{NI}(r)|^2 r^2 dr = 1 \) determining the normalization constants \( N_{NI} \), and, \( Y_{lm}(\theta, \phi) \) are the spherical harmonics satisfying the orthonormality condition \( \int Y_{lm}(\theta, \phi) Y_{lm'}(\theta, \phi) d\Omega = \delta_{ll'} \delta_{mm'} \).
The strengths of the harmonic oscillator potential, $\beta$, for the bottomonium states $\Upsilon(NS), N = 1, 2, 3, 4$ are calculated from their observed leptonic decay widths, $\Upsilon(NS) \rightarrow e^+e^-$. The expression for the decay width, $\Gamma(\Upsilon(NS) \rightarrow e^+e^-)$ is given as \[45, 49, 63\]
\[
\Gamma(\Upsilon(NS) \rightarrow e^+e^-) = \frac{4\alpha^2}{9m_{\Upsilon(NS)}^2}|R_{NS}(r = 0)|^2,
\]
where $\alpha = 1/137$, $m_{\Upsilon(NS)}$ is the vacuum mass of the bottomonium state $\Upsilon(NS)$, and $R_{NS}(r)$ is the radial part of the wave function for this state. The values of $\beta$ calculated are 1309.2, 915.4, 779.75 and 638.6 MeV fitted from the observed leptonic decay widths of 1.34, 0.612, 0.443 and 0.272 keV for the $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, respectively. The vacuum masses of the states, $\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, are 9460.3, 10023.26, 10355.2 and 10579.4 MeV respectively. The $\beta$ values as stated above for these states are observed to be smaller for larger masses of the upsilon state. We use the value of the harmonic oscillator potential strength parameter, $\beta$ for $\Upsilon(1D)$ (vacuum mass of 10163.7 MeV) as 858 MeV, which is the value obtained from interpolation from the plot of mass versus $\beta$ for the upsilon states. The leptonic decay width for $\Upsilon(1D)$ can be calculated by using the formula \[63\]
\[
\Gamma(\Upsilon(ND) \rightarrow e^+e^-) = \frac{25\alpha^2}{18m_{\Upsilon(ND)}^2m_b^2}|R_{ND}''(r = 0)|^2.
\]
With the value of $\beta$ as 858 MeV for $\Upsilon(1D)$, the decay width $\Gamma(\Upsilon(1D) \rightarrow e^+e^-)$ is found to be 0.00715 keV. This value is very small, as compared to the values for the leptonic decay widths of the $\Upsilon(NS)$ states. The value for $\Gamma(\Upsilon(1D) \rightarrow e^+e^-)$ obtained in the present study may be compared with the value of 0.02 keV calculated in Ref. \[63\] using a potential model for study of the heavy quarkonium states.

In the next section we shall present the results obtained for the in-medium upsilon masses in asymmetric nuclear matter in presence of strong magnetic fields.

**III. RESULTS AND DISCUSSIONS**

In this section, we first investigate the effects of magnetic field, density and isospin asymmetry of the magnetized nuclear medium on the dilaton field $\chi$, which mimics the gluon condensates of QCD, within the chiral SU(3) model. The in-medium masses of upsilon states
(ϒ(NS), N = 1, 2, 3, 4, ϒ(1D)), are then calculated from the value of χ in the nuclear medium using equation (14).

The dilaton field χ as modified in the asymmetric nuclear medium in the presence of strong magnetic fields, has been discussed in detail in Ref. [35]. The variations of the dilaton field χ with magnetic field, baryon density, and isospin asymmetry, within the chiral SU(3) model, are obtained by solving the coupled equations of motion of the scalar fields, σ, ζ, δ and χ. The number density and scalar density of the proton have contributions from the Landau energy levels in the presence of the magnetic field. The value of χ is observed to decrease with increase in density. When the anomalous magnetic moments (AMM) of the nucleons are taken into consideration, at a given density, with increase in magnetic field, χ is observed to attain a higher value and thus the shift from the vacuum value decreases. The mass shifts from vacuum value, on the other hand, are observed to increase with increasing magnetic field, when the anomalous magnetic moments of the nucleons are not taken into account. As the isospin asymmetry of the medium increases, the scalar field χ is also observed to increase. For the case of η=0.5, the medium comprises of only neutrons, and hence the only effect of magnetic field is due to the anomalous magnetic moment of the neutrons. Hence, in the case when the AMM effects are not taken into consideration, the value of the dilaton field remains independent of the magnetic field. This leads to the mass shift of the upsilon states to be independent of the magnetic field for η=0.5, when AMM effects are not taken into account.

The masses of the bottomonium states, ϒ(1S), ϒ(2S), ϒ(3S), ϒ(4S), and ϒ(1D), in magnetized nuclear matter, as calculated from the medium change of the dilaton field, are plotted as functions of the baryon density (in units of nuclear matter saturation density) in figures 1, 2, 3, 4, and, 5 respectively. These are plotted for values of eB as 4m^2_π, 8m^2_π, 10m^2_π, and, 12m^2_π. The masses of the bottomonium states are illustrated for the values of the isospin asymmetry parameter as η=0, 0.3 and 0.5, with (without) accounting for the anomalous magnetic moments (AMM) of the nucleons.

The mass modifications of bottomonium states are found to be larger in symmetric nuclear matter than in asymmetric nuclear matter. As value of η increases the mass drop decreases. For a particular magnetic field, at a fixed density the effect of anomalous magnetic moments results in smaller mass modifications for the bottomonium states in comparison to the case.
FIG. 1: (Color online) The mass shift of Υ(1S) plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, \( \eta \), including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 2: (Color online) The mass shift of $\Upsilon(2S)$ plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $\eta$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 3: (Color online) The mass shift of Υ(3S) plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $\eta$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 4: (Color online) The mass shift of Υ(4S) plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $η$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
FIG. 5: (Color online) The mass shift of $\Upsilon(1D)$ plotted as a function of the baryon density in units of nuclear matter saturation density, for different values of the magnetic field and isospin asymmetry parameter, $\eta$, including the effects of the anomalous magnetic moments of the nucleons. The results are compared to the case when the effects of anomalous magnetic moments are not taken into consideration (shown as dotted lines).
when we do not incorporate anomalous magnetic moment effects.

For a fixed value of $\eta$, when we account for the effects of anomalous magnetic moments of the nucleons, as magnetic field increases, the mass modifications of bottomonium states mostly remain constant with negligible variation at lower densities. In the same case at higher densities, the mass drop decreases as magnetic field becomes larger. On the other hand, when AMM effects are not taken into account, in symmetric nuclear matter, the mass drop increases at higher densities and at larger magnetic fields. For asymmetric nuclear matter, without AMM, the mass modifications of bottomonium states at a fixed density remains mostly independent of magnetic field. Particularly for the case of $\eta = 0.5$ when the medium is devoid of protons, magnetic field has no effect on mass modifications, since neutrons respond to the magnetic field only due to their anomalous magnetic moment.

At a given magnetic field, as baryon density increases, there is a general drop in the value of $\chi$ and hence in general it is found that the mass shifts of all bottomonium states steadily increase as functions of density at a fixed value of magnetic field. As density increases the effect of magnetic field is more prominent. It has to be noted that, for a given density, magnetic field, and, isospin symmetry, the ratio of the magnitudes of the mass shifts for the bottomonium states turns out to be the ratio of the magnitudes of the integrals (given in equation (14), calculated from their respective wave functions in momentum space. The mass drop of $\Upsilon(1S)$ turns out to be extremely small whereas $\Upsilon(1D), \Upsilon(2S), \Upsilon(3S)$ are observed to have larger mass drops, the largest being for $\Upsilon(4S)$.

For symmetric nuclear matter ($\eta = 0$), at $\rho_B = \rho_0(4\rho_0)$, including the effects of AMM, the mass shifts (in MeV) of $\Upsilon(1S)$ are obtained as $-0.716(-2.57)$, $-0.7163(-2.463)$, $-0.7145(-2.41)$ and $-0.722(-2.32)$ for magnetic fields $4m_\pi^2$, $8m_\pi^2$, $10m_\pi^2$, and $12m_\pi^2$ respectively. Under the same conditions, the mass shifts of $\Upsilon(1D)$ are found to be $-8.09(-29.254)$, $-8.09(-28.04)$, $-8.06(-27.43)$, $-8.149(-26.435)$ and those of $\gamma(2S)$ is obtained as $-6.82(-24.494)$, $-6.826(-23.475)$, $-6.81(-22.96)$ and $-6.88(-22.12)$. The mass shifts of $\Upsilon(3S)$ are $-24.28(-87.15)$, $-24.29(-83.53)$, $-24.23(-81.7)$ and $-24.47(-78.73)$ and of $\Upsilon(4S)$, these have values of $-98.66(-354.17)$, $-98.7(-339.44)$, $-98.45(-332)$ and $-99.46(-319.9)$ respectively for the corresponding magnetic fields. The effects of magnetic field on the in-medium upsilon masses are observed to be marginal for small densities, upto
around nuclear matter saturation density, whereas the effects are observed to be larger at higher densities. The effects of the anomalous magnetic moments (AMM) are seen to be appreciable at higher densities for larger values of the magnetic field. The mass shifts of $\Upsilon(1S)$, $\Upsilon(1D)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, at density $4\rho_0$, with AMM effects in symmetric nuclear matter, as stated above, may be compared with the values of $-2.707$, $-37.22$, $-25.8$, $-91.82$, and $-373.13$ for $eB = 4m^2_\pi$ and $-3$, $-34.21$, $-28.64$, $-101.92$ and $-414.17$ for for $eB = 12m^2_\pi$, when the effects from AMM are not taken into account.

For asymmetric nuclear matter ($\eta = 0.3$), at $\rho_B = \rho_0(4\rho_0)$, including the effects of AMM, the mass shifts (in MeV) of $\Upsilon(1S)$, $\Upsilon(1D)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, are observed to be $-0.688(-2.535)$, $-7.77(-28.85)$, $-6.56(-24.15)$, $-23.33(-85.95)$, and $-94.79(-349.26)$ for the $eB = 4m^2_\pi$ and, $-0.666(-2.206)$, $-7.52(-25.1)$, $-6.35(-21.02)$, $-22.59(-74.79)$, and, $-91.78(-303.92)$ for the $eB = 12m^2_\pi$.

IV. SUMMARY

The medium modifications of the masses of the bottomonium states ($\Upsilon(1S)$, $\Upsilon(2S)$, $\Upsilon(3S)$, $\Upsilon(4S)$, $\Upsilon(1D)$) in strongly magnetized hadronic matter are investigated using a chiral effective model. The variation in the masses of bottomonium states are due to the modification of dilaton field with change in baryon density, isospin asymmetry as well as magnetic field. The in-medium upsilon masses are studied accounting for the anomalous magnetic moments of the nucleons and are compared to the case when these effects are not taken into consideration. The modifications due to magnetic field are observed to be rather small at low baryonic densities. The effects of magnetic field and isospin asymmetry in causing the mass modifications are significant at high densities, with excited states of bottomonia showing larger mass drop than the ground state. Anomalous magnetic moment effects are observed to be more pronounced with increase in isospin asymmetry of the medium, and, are larger at higher densities. The density effects are found to be the dominant medium effects, as compared to the effects due to isospin asymmetry and the magnetic field.
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