A solitary superconductivity in layered ferromagnet-superconductor heterostructures

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Abstract. An electron-electron interaction in a ferromagnet (F) was neglected in the standard approach to the proximity effect theories for the layered FS structures (here S is used for a superconductor). Actually this interaction exists, but it is suppressed by strong exchange field and can reveal itself if an exchange fields of adjacent F layers compensate each other. In the clean FS structures limit the preceding consideration of this interaction could explain a surprisingly high critical temperature $T_c$ in the short-periodic Gd/La superlattice. Here we analyze this problem for dirty case based on the Usadel equations solutions changing parameters of asymmetrical FS systems (thicknesses of layers, boundary transparencies, etc). Taking into account an electron-electron interaction leads to an appearance of hidden superconductivity of F layers which can manifest itself in the proximity effect conditions. It is especially expressed if magnetizations in F layers have opposite signs. The solitary superconductivity is also predicted for asymmetrical dirty FSF trilayers.

1. Introduction

In artificial layered structures consisting of the superconductor (S) and ferromagnet (F) layers the interplay between the S and F parameter orders can lead to a number of striking phenomena [1–3]. For example, the nonmonotonic behaviors of the critical temperature and the Josephson current as a function of the ferromagnetic layer thickness appears (see reviews [1–3] and references therein). As limit cases the re-entrant and periodically re-entrant superconductivity was predicted in works [4, 5]. Later the re-entrant superconductivity experimentally was discovered in bilayers V/Fe [6] and Nb/Cu$_{1-x}$Ni$_x$ [7]. Note, an appearance of superconductivity as solitary peak in dependence of critical temperature $T_c$ versus the F layer thickness $d_f$ was recently theoretically proposed for clean asymmetrical FS system [8, 9]. Most recently, the appearance of peculiar solitary superconductivity caused by external magnetic field is predicted for the F$_1$F$_2$S system [10].

These FS layered heterostructures can be perspective due to possible superconducting spin switch applications. Thus the spin valve device based on the three layered FS systems switched by weak external magnetic field was proposed in works [11–13]. Note, the superconducting switch based on the four-layered F$_1$S$_1$F$_2$S$_2$ system can have up to seven different states [14].

An electron-electron interaction in a ferromagnet (F) was neglected in the standard approach to the proximity effect theories for the layered SF structures (see for example [1–3] and references therein). In other words a superconducting order parameter $\Delta_f$ and an electron-electron interaction constant $\lambda_f$ were taken as zero for a ferromagnet. Actually this interaction exists, but it is “hidden” because it is suppressed by strong exchange field $I$. This interaction can reveal itself...
if an exchange field could be “disabled”. So, in this imaginary case a ferromagnet transforms to normal metal and that “unhidden” electron-electron interaction can lead to superconducting correlations and, therefore, a superconductivity onset with critical temperature $T_{cf}$, estimated by the standard BCS expression.

Previously we have shown that a consideration of this interaction in the clean symmetrical F/S structures limit [15–17] can explain a surprisingly high critical temperature $T_c$ ($T_c \sim 5\,\text{K}$) and an independence of $T_c$ from the F layer thickness in the short-periodic Gd/La superlattice [18, 19] for case when magnetizations in adjacent F layers were antiparallel (AP). The solutions of Eilenberger equations were obtained in Cooper limit case for ideal FS boundaries transparencies. At these conditions the full compensation of pair-breaking effect of exchange field and sharing all interactions over sample are possible for thin layered symmetrical systems [15,16]. Moreover, for clean asymmetrical FS structures (FSF’ [16], FSFS’ [20], superlattice [17]) we have also predicted an appearance of solitary peak in the dependence $T_c(d_f)$ at fixed $d_f$. This peak was centered at $d_f = d_{f^r}$, its height and its width strongly depend from the FS structure parameters. By analogy with re-entrant superconductivity, we called this phenomenon as a solitary re-entrant superconductivity or more appropriately a solitary superconductivity. Last term will be frequently used below.

In this paper we consider a dirty limiting case and analyze solutions of boundary value problem for the Usadel function, changing different parameters of asymmetrical $F_1S$ trilayer (thicknesses of layers, boundary transparencies, and so on). Taking into account an electron-electron interaction can lead to an appearance of “hidden” superconductivity of F layers which can be manifest itself in the proximity effect conditions. It will be especially expressed if exchange fields in both F layers have opposite signs. We discuss influence of the electron-electron interaction on the critical temperature and predict the solitary superconductivity for dirty asymmetrical $F_1S$ trilayer too.

2. Theoretical background
For dirty FS system we will use Usadel-like equations [21]. The order parameter $\Delta_{s,f}$ and electron-electron interaction $\lambda_{s,f}$ were taken into account for both metals S and F, respectively.

The critical temperature $T_c$ at the second order transition for $F_1S$ trilayers is obtained from the set of the self-consistent equations [22] for the superconducting gaps $\Delta_{s,f}(r)$ in S and F layers, respectively

$$\Delta_s(r) \ln t = 2\pi T_c \Re \sum_{\omega>0} \left( F_s(r, \omega) - \frac{\Delta_s(r)}{\omega} \right), \quad (1)$$

$$\Delta_{fi}(r) (\ln t + \ln \frac{T_{cs}}{T_{f_i}}) = 2\pi T_c \Re \sum_{\omega>0} \left( F_{fi}(r, \omega) - \frac{\Delta_{fi}(r)}{\omega} \right), \quad i = (1, 2), \quad (2)$$

where $t = T_c/T_{cs}$ is the reduced critical temperature ($T_{cs}$ is the superconducting critical temperature for the bulk S material, $T_{f_i}$ is “virtual” critical temperature $T_{f_i}$ for normal metal corresponding F$_i$ material in which an exchange field is assumed be lacking, $I_i = 0$, $\omega$ is the Matsubara frequency.

The pair amplitude $F_{s,(i)}$ satisfies the Usadel-like equations [21,23,24] for S layer

$$\left[ \omega - \frac{D_s}{2} \frac{d^2}{dx^2} \right] F_s(x, \omega) = \Delta_s(x), \quad (3)$$

and for F layers

$$\left[ \omega - iI_i - \frac{D_{fi}(I_i)}{2} \frac{d^2}{dx^2} \right] F_{fi}(x, \omega) = \Delta_{fi}(x), \quad D_{fi}(I_i) = \frac{D_{fi}}{1 - i2I_i^2 f_i}, \quad (4)$$
where $D_{s,f_i}$ is the diffusion constant in corresponding layer, and $\tau_{f_i}$ is the elastic scattering time of non-magnetic impurities in $F_i$ layer. Note, that in the presence of exchange interaction the diffusion constant $D_{f_i}(I)$ in (4) is complex $[2,25]$ (this form of complex diffusion constant is uniquely determined in the microscopic approach given in $[10]$, the consistency of approach is confirmed from the fact that the complex diffusion constant enters both in the differential equations (4) and in the boundary conditions in the same manner $[10]$). For completeness sake we should mention variant of complex diffusion constant obtained in work $[26]$ and corresponding equations (4) and in the boundary conditions in the same manner $[10]$). For completeness sake

For brevity here we do also not describe the boundary conditions of Kupriyanov-Lukichev type $[28]$ for pair amplitude $F$, microscopically derived in the works $[10,25]$ and a process of solution of this boundary value problem. Moreover we use the same approximations as in our recent works $[10,29]$.

3. Results and discussion

In this section we present and discuss the numerical results for the $F_1SF_2$ systems. We only consider the case of antiparallel orientation (AP state) of the magnetizations in plane of $F$ layers. Similarly to $[16]$, we also assume highly transparent $SF$ interfaces ($\sigma_s = \sigma_f = 100$). All lengths related to the $S$ and $F_{1,2}$ layers are normalized on the coherence lengths $\xi_s = \sqrt{D_s/2\pi T_{cs}}$ and $\xi_{f1,2} = \sqrt{D_{f1,2}/T_{cf}}$, respectively.

We first discuss the symmetrical $F_1SF_2$ system when the thicknesses of $F$ layers are equal ($d_{f1} = d_{f2}$). In Fig. 1 the $T_c(d_f)$ dependence is shown at various ratio $T_{cs}/T_{cf}$. It is clearly seen that the critical temperature $T_c$ of the $FSF$ system is practically equal to the critical temperature $T_{cs}$ of isolated $S$ layer (see solid and dashed lines) at small thicknesses of $F$ layers $d_f \leq 0.3\xi_f$, when $T_{cf} \geq T_{cs}$. For comparison, the dependence $T_c(d_f)$ has a rapid initial decline in the absence of electron-electron interaction ($T_{cs}/T_{cf} = \infty$). Both dependencies are in agreement with the results $[8]$ for clean $FSF$ trilayers. The reason that $T_c \simeq T_{cs}$ (for small thicknesses) is easy to understand: the effective exchange field is compensated by the

![Figure 1](image-url)  

*Figure 1.* (Color online) The influence of the interelectronic interaction on the critical temperature $T_c$ of the $F_1SF_2$ system. The critical temperature $T_c$ vs thicknesses $d_f$ of $F$ layers at few values of parameter $T_{cs}/T_{cf}$. Other parameters of the system are $d_s/\xi_s = 1.25$, $l_s/\xi_s = 0.7$, $\sigma_s = \sigma_f = 100$, $l_{f1}/\xi_{f1} = l_{f2}/\xi_{f2} = 0.3$, $I_1/\pi T_{cs} = I_2/\pi T_{cs} = 6$. 

with fixed thickness of \( F \) trilayer. The solitary peaked superconductivity \( T_{cs}(d_f) \) at various value of the ratio \( T_{cs}/T_{cf} \) with fixed thickness of \( F_2 \) layer \( d_{f2} = 0.8\xi_{f2} \). Other parameters of the system are \( d_s/\xi_s = 1.25, l_s/\xi_s = 0.7, \sigma_s = \sigma_f = 100, l_{f1}/\xi_{f1} = l_{f2}/\xi_{f2} = 0.5, I_1/\pi T_{cs} = I_2/\pi T_{cs} = 2 \).

Antiparallel mutual orientation of the magnetizations \( (I_{eff} \simeq 0) \) and it leads to contact of two “same superconductors” with \( T_{cf} \approx T_{cs} \) due to “hidden” electron-electron pairing interaction in F layers. For greater thicknesses \( (d_f \gtrsim l_f) \) the difference with clean case appears owing to the increasing role of the elastic scattering time on non-magnetic impurities in F layers.

Then we consider the asymmetrical \( F_1SF_2 \) system.

For investigation of the solitary superconductivity we will examine the case of very weak exchange fields \( I = 2\pi T_{cs} \). So, in Fig. 2 the influence of the electron-electron interaction on the phase diagrams with the solitary superconductivity is presented for the case of soft ferromagnet. We remark that the \( F_1SF_2 \) system is asymmetrical: the thickness of the \( F_1 \) layer can be only changed but the \( F_2 \) layer has fixed thickness \( (d_{f2} = 0.8\xi_{f2}) \). We found parameters when solitary peak is pronounced. In this case we observe decreasing the maximum of the critical temperature with increasing the \( T_{cs}/T_{cf} \) parameter. We can find that the superconductivity completely disappears in this system when \( T_{cs} > 5.4T_{cf} \). Thus we conclude that the appearance of the solitary superconductivity in dirty FSF trilayers can be say about a manifestation of “hidden superconductivity” in the F layers, but it requires the fulfillment of a number of the sufficiently specific conditions. We list them here once again: the FSF system should be in the AP state (magnetizations \( M_1 \) and \( M_2 \) are antiparallel); the “virtual” critical temperature \( T_{cf} \) must be comparable with the critical temperature \( T_{cs} \) of the S layer; the thickness of the S layer should be of the order of the coherence length \( \xi_s \) for a more effective compensation of the exchange fields. At the same time, the \( F_1S \) and \( SF_2 \) interfaces should be highly transparent; if the the S layer is thin, it is desirable to use the soft ferromagnets (with a low Curie temperature) such as \( Cu_{1-x}Ni_x \) alloy [30].

Figure 2. (Color online) The phase diagrams with the solitary superconductivity for the \( F_1SF_2 \) trilayer. The solitary peaked superconductivity \( T_{cs}(d_f) \) at various value of the ratio \( T_{cs}/T_{cf} \) with fixed thickness of \( F_2 \) layer \( d_{f2} = 0.8\xi_{f2} \). Other parameters of the system are \( d_s/\xi_s = 1.25, l_s/\xi_s = 0.7, \sigma_s = \sigma_f = 100, l_{f1}/\xi_{f1} = l_{f2}/\xi_{f2} = 0.5, I_1/\pi T_{cs} = I_2/\pi T_{cs} = 2 \).

4. Conclusions

In this work we considered the asymmetrical \( F_1SF_2 \) trilayers. Our theoretical approach takes into account of the electron-electron interaction in both F layers. We showed that in dirty limit, the asymmetry and electron-electron pairing interaction can lead to an appearance of the solitary superconductivity. In particular, we found that the kind of dependence \( T_{cs}(d_f) \) at small F layers thicknesses may indicate about possible presence of electron-electron interaction in ferromagnets. Obtained results for dirty \( F_1SF_2 \) trilayer are in qualitative agreement with ones for clean limiting case [8]. But there are important differences. So, in clean systems we should...
take into account the transverse Fulde-Ferrell-Larkin-Ovchinnikov states, which can be neglected in dirty case. The solitary peak of superconductivity in clean case [8] has always a maximum at $d_f^1 = d_f^2$, but the dirty FSF system has, as rule, asymmetrical condition on position of this maximum. So, the unconventional experimental behavior of Gd/La system [18, 19] can be explained only in “clean” model for F metals with one-domain structure.

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