Viable Algorithmic Options for Creating and Adapting Emergent Software Systems

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Abstract: Given the complexity of modern software systems, it is of great importance that such systems be able to autonomously modify themselves, i.e., self-adapt, with minimal human supervision. It is critical that this adaptation both results in reliable systems and scales reasonably in required memory and runtime to non-trivial systems. In this paper, we apply computational complexity analysis to evaluate algorithmic options for the reliable creation and adaptation of emergent software systems relative to several popular types of exact and approximate efficient solvability. We show that neither problem is solvable for all inputs when no restrictions are placed on software system structure. This intractability continues to hold relative to all examined types of efficient exact and approximate solvability when software systems are restricted to run (and hence can be verified against system requirements) in polynomial time. Moreover, both of our problems when so restricted remain intractable under a variety of additional restrictions on software system structure, both individually and in many combinations. That being said, we also give sets of additional restrictions that do yield tractability for both problems, as well as circumstantial evidence that emergent software system adaptation is computationally easier than emergent software system creation.

1 Introduction

Given the complexity of modern software systems and the increasing need to modify such systems to handle both unplanned changes in system requirements and varying
operating environments, such systems must be able to autonomously modify themselves, i.e., self-adapt, with minimal human supervision [1, 2, 3]. Over the last 25 years, a great deal of research has been done on self-adaptive software systems, and a number of such systems based on various types of adaptation controllers (e.g., MAPE-K feedback loop [4], model-predictive [5], general control-theoretic [6]) have been created (see also [1, 7, 8]). Certifying that adaptation does not cause the resulting systems to violate functional and/or non-functional system requirements, i.e., validation and verification (V&V), is a major research challenge [8, page 21]. Such V&V must be done quickly enough that the proposed adaptations are still valid (to handle rapidly-changing operating environments). This is further complicated by the requirement that enough computational effort be put into the adaptation space search process to ensure that the adapted system optimizes system performance as much as possible (to satisfy system user expectations).

A valuable complement to algorithm development work would be to establish what the general algorithmic options are for software system creation and adaptation algorithms that are guaranteed to satisfy specified functional and non-functional system requirements while optimizing adapted system performance. This can be done using the tools and techniques of computational complexity analysis [9, 10, 11]. The results of such analyses can be used not only to establish those situations in which known algorithms are the best possible but also to guide the development of new algorithms (by highlighting relative to which types of efficient solvability such algorithms can and cannot exist).

A good test case for the utility of such complexity analyses would be a type of self-adaptive systems called emergent software systems [12, 13]. Such systems initially self-assemble from a provided library of software components to satisfy basic system requirements, and then continuously self-adapt with the assistance of a learning algorithm that uses collected system environment events and performance metrics to optimize a reward function such as system runtime or memory usage. An emergent web server based on a library of 30 components has been constructed under this paradigm and seems to function well [13]. However, as noted on [12, page 14],

...[the] learning algorithm, which is based on an exhaustive exploration phase, is not designed to scale up to large systems with thousands of compositions, but rather serves as a proof of concept and useful baseline against which to compare more sophisticated algorithms. This is an active research area for which we continue to develop more efficient and scalable solutions ...

Ongoing work has focused on exploiting such strategies as making components encode single responsibilities and dividing large systems into distributed collections of smaller
(and hopefully more manageable) subsystems \[14, 15\]. It is precisely at this current
stage of new algorithm development that computational complexity analyses might
be most useful.

\section*{1.1 Previous Work}

Computational complexity analyses have been done previously for component-based
software system creation by component selection \[16, 17\] and component selection
with adaptation \[18\], with \[18, 17\] having the additional requirement that the number
of components in the resulting software system be minimized. Given the intractability
of all of these problems, subsequent work has focused on efficient approximation algo-
rithms for component selection. Though it has been shown that efficient algorithms
that produce software systems whose number of components is within a constant mul-
tiplicative factor of optimal are not possible in general \[19\], efficient approximation
algorithms are known for a number of special cases \[20, 21, 19\]. All of these analyses
assume that any component can be composed with any other component, in the
sense that function definitions not given in a component \(c\) can be obtained by compo-
sition of \(c\) with any other components that have the required function definitions, i.e.,
component composition is not regulated using component interfaces. Moreover, none
of the formalizations used include specification of the internal structure of software
systems, system requirements, and components that are detailed enough to allow in-
vestigation of restrictions on these aspects that could make component selection or
component selection with adaptation tractable.

Only one analysis to date has incorporated a component model which allows
investigation of the tractability effects of restrictions on system requirement, com-
ponent internal, and software system structure, namely that in in \[22\] (subsequently
reprinted as \[23\]; see also \[24\] for the full version with proofs of results). The focus in
this work was on exact polynomial-time solvability and fixed-parameter tractability
of component-based software system creation and adaptation, where system adaptation
is in response to changes in the functional system requirements. The authors showed
that both of these problems are intractable in general and remain so under a variety
of restrictions on system requirement, component, and software system structure.
This was done relative to radically restricted components (single \texttt{if-then-else} blocks)
and software systems (two-level reactive systems). While components in this model
cannot arbitrarily compose in the sense described above, component composition was
implicitly regulated by the the combination of two-level system structure and a pa-
rameter restricting the number of types of components in a system, and there was
no notion of a component interface, let alone interfaces providing state variables or
functions with input parameters and/or returned values.
1.2 Summary of Results

In this paper, we present the first computational complexity analyses of the problems of emergent software system creation and adaptation. These problems can be stated informally as follows (and are described in more detail in Section 2):

**Emergent Software Creation (ESCCreate):** Derive (possibly by using a system environment function \(Env\)) an emergent software system \(S\) relative to given libraries \(L_{int}\) and \(L_{comp}\) of software interfaces and components that satisfies a given set \(R\) of software requirements and has the best possible value for a specified reward function \(Rew\).

**Emergent Software Adaptation (ESAdapt):** Given an emergent software system \(S\) relative to given libraries \(L_{int}\) and \(L_{comp}\) of software interfaces and components that satisfies a given set of software system requirements \(R\), derive (possibly by using a system environment function \(Env\)) an emergent software system \(S'\) relative to \(L_{int}\) and \(l_{comp}\) that satisfies \(R\) and has the best possible value for a specified reward function \(Rew\).

We consider the following types of efficient solvability (described in more detail in Section 3.1):

1. Polynomial-time exact solvability, such that a polynomial-time algorithm produces the correct output for a given input either (a) all the time \[10\] or (b) when such an output is known to exist (promise solvability).

2. Polynomial-time approximate solvability, such that a polynomial-time algorithm produces the correct output for a given input either (a) in all but a small number of cases \[25\] or (b) with a high probability \[26\], or, in the case of a problem that requires an output that optimizes some cost measure, (c) produces an output for a given input whose cost is within some arbitrarily small fraction of optimal \[27\].

3. Effectively polynomial-time exact restricted solvability (e.g., fixed-parameter (fp-)tractability \[9\]), such that an algorithm produces the correct output for a given input in what is effectively polynomial time when certain aspects of that input are of restricted value, e.g., the number of components in the given library or any valid assembled software system is small.

Relative to these types of solvability and various conjectures that are widely believed to be true within the Computer Science community, e.g. \(P \neq NP\) \[28, 10\], we prove the following results (Section 3):
• Neither ESCreate nor ESAdapt is solvable by an algorithm (regardless of run-
time or memory usage) that gives the correct output for every input, i.e., both
of these problems are unsolvable in the same sense as Turing’s classic Halting
Problem [29] (Sections 3.2.1 and 3.3.1 respectively).

• Neither ESCreate nor ESAdapt is solvable by a polynomial-time algorithm in
the sense (1a) above and ESCreate is not solvable by a polynomial-time algo-

"rithm in sense (1b) above, even when software systems are restricted to run
in and hence can be verified against requirements in polynomial time (Sections
3.2.2 and 3.3.2 respectively).

• Neither ESCreate nor ESAdapt is solvable by a polynomial-time algorithm in
the senses (2a), (2b), or (2c) above, even when software systems are restricted
to run in and hence can be verified against requirements in polynomial time
(Sections 3.2.3 and 3.3.3 respectively).

• Neither ESCreate nor ESAdapt is fixed-parameter tractable in sense (3) above
relative to restrictions of the values of the following aspects of the input:

- The number of software component interfaces in \( L_{\text{int}} \).
- The number of software components in \( L_{\text{comp}} \).
- The maximum number of components implementing an interface.
- The maximum number of interfaces provided by a component.
- The maximum number of interfaces required by a component.
- The maximum number of components in a software system.

This fixed-parameter intractability holds both when software systems are re-
stricted to run in and hence can be verified against requirements in poly-
nomial time and relative to many combinations of the aspects listed above, often
when aspects are restricted to small constant values. That being said, there
are several combinations of aspect-restrictions that do yield fixed-parameter
tractability (Sections 3.2.4 and 3.3.4 respectively).

All of the results above hold for ESCreate (except those related to cost-
inapproximability) relative to any choices of \( \text{Env}() \) and \( \text{Rew}() \) and for ESAdapt
(and ESCreate for cost-inapproximability) relative to any choice of \( \text{Env}() \) and one of
two specified reward functions, namely \( \text{Rew}_{\#\text{comp}}(S) \) (the number of components in
a software system) and \( \text{Rew}_{\text{CodeB}}(S) \) (the total size of the interface and component
code is a software system). This, in combination with the unresolved fixed-parameter
status of certain aspect-combinations at this time, suggests that emergent software
adaptation may in general be easier to do than emergent software creation (Section 4.2).

The list above may, at first glance, be read as saying that emergent software system creation and adaptation are not possible under any circumstances. However, this bleak interpretation is very much contrary to our intent. We consider only a small subset of possible restrictions on emergent software systems (Table 1), and our analysis, though complete with respect to some of these restrictions (Tables 5-7), is still incomplete with respect to the whole subset, let alone the universe of possible restrictions. The successes in real-world emergent software systems to date show that tractability is indeed possible in some circumstances. The key issue now is to determine in detail those circumstances in which tractability does and does not hold. Our results should thus be seen not as final statements but rather interim guidelines on how to address this issue. We see the involvement of software engineers in this process as essential (Section 4.3). To this end, we have tried to make the reasoning used to derive our results (both in general (Section 3.1) and in our proofs) accessible to software engineers, to make plain to them the rather limited circumstances under which our results hold and thus enable them to break these results by suggesting additional restrictions relative to which real-world emergent software system creation and adaptation are provably tractable.

Before we close out this subsection, two issues with respect to the results listed above should be noted. First, though certain notations (e.g., our conception of software system requirements) and some of the general ideas underlying proof techniques developed in [22, 24] are re-used in this paper, none of the results for component-based software system creation derived in [22, 24] carry over. This is because the restrictions on overall system structure and the number of types of components in a system critical to the proofs of those results in [22, 24] have no analogues in problems ESCreate and ESAAdapt investigated here. Second, all results in this paper are derived relative to the classic Turing machine model of computation, and hence do not directly address issues of efficient solvability or unsolvability under other models of computation such as quantum computers. That being said, as will be discussed in Section 4.3, our results indirectly imply certain consequences for the efficient solvability of ESCreate and ESAAdapt under such alternative models of computation.

1.3 Organization of the Paper

Our paper is organized as follows. In Section 2 we summarize the emergent software system model given in [12, 13] and formalize the problems of emergent software system creation and adaptation. In Section 3 we first in Section 3.1 describe several popular conceptions of efficient solvability and then in Sections 3.2 and 3.3 assess the efficient solvability of emergent software system creation and adaptation, respectively, relative
to each of these conceptions. In order to focus in the main text on the implications of our results, proofs of several of these results are given in an appendix. Our results are summarized and discussed in Section 4. Finally, our conclusions and directions for future work are given in Section 5.

2 Formalizing Emergent Software Creation and Adaptation

In this section, we first review the basic entities in the model of emergent software given in [12, 30, 13] — namely, software system requirements, interfaces and components, component-based software systems, and emergent software systems. We then formalize two computational problems associated with emergent software system creation and adaptation.

The basic entities in our model are formalized as follows:

- **Software system requirements**: The requirements will be a set $R = \{r_1, r_2, \ldots, r_{|R|}\}$ of input-output pairs where each pair $r_j = (i_j, o_j)$ consists of an input $i_j$ defined by a particular sequence of truth-values $i_j = \langle v_{i_1}(x_1), v_{i_2}(x_2), \ldots, v_{i_j}(x_{|X|}) \rangle$, $v_{i_j}(x_k) \in \{True, False\}$, relative to each of the Boolean variables $x_k$ in set $X = \{x_1, x_2, \ldots, x_{|X|}\}$ and an output $o_j$ from set $O = \{o_1, o_2, \ldots, o_{|O|}\}$. As such, these are functional requirements describing wanted system input-output behaviors and correspond to the pre-specified abstract goal of the system [15, page 3]. An example set of software system requirements is given in part (a) of Figure 1.

- **Interfaces and Components**: We use the runtime component model underlying the Dana programming language as specified in [30, 13]. In particular, following [13, Page 335], let an interface be a set of function prototypes, each comprising a function name, return type and parameter types, and a set of transfer fields, typed pieces of state that persist across alternate implementations of the interface during runtime adaptation. A component has one or more provided interfaces and zero or more required interfaces; the component has implementations for all functions specified in the provided interfaces, and these implementations in turn call upon functions and transfer fields specified in the required interfaces. We will assume that all available interfaces and components are stored in libraries $L_{int}$ and $L_{comp}$, respectively. Example interface and component libraries relative to the software system requirements in part (a) of Figure 1 are given shown in parts (b) and (c) of Figures 1 and 2, respectively.
| req. | x₁ | x₂ | x₃ | x₄ | x₅ | output |
|------|----|----|----|----|----|--------|
| r₁   | T  | T  | T  | T  | T  | 2      |
| r₂   | T  | F  | F  | F  | T  | 1      |
| r₃   | F  | F  | F  | F  | F  | 2      |
| r₄   | F  | F  | F  | F  | F  | 2      |
| r₅   | T  | T  | T  | F  | T  | 3      |

(a)

interface intSystem {
    interface intProc1 {
        void systemMain(Input I)  void callProc1(Input I)
    }
}

interface intProc2 {
    interface intProc3 {
        void callProc2(Input I)  void callProc3(Input I)
    }
}

interface intProc {
    void proc(Input I)
}

(b)

Figure 1: An Example Emergent Software System (Adapted from [22, Figure 1]).
(a) Software requirements $R = \{r_1, r_2, r_3, r_4, r_5\}$ defined on Boolean variables $X = \{x_1, x_2, x_3, x_4, x_5\}$ and output-set $O = \{1, 2, 3\}$. (b) A sample component interface library $L_{int}$ consisting of five interfaces.

Note that the interfaces and components in Figures 1 and 2 (and indeed all subsequent interfaces and components specified in this paper) are described using a notation approximating that of the Dana programming language, e.g., [13, Figure 2]. This is done to ensure both that the proofs of results given here do not invoke programming language features more powerful than those available in Dana and that these results are hence applicable to emergent software systems as described in [12, 30, 13], which are written in Dana.

• **Component-based software systems:** We use the model of component-based software systems specified in [12, 30, 13]. In particular, given interface and component libraries $L_{int}$ and $L_{comp}$ and a base component $c \in L_{comp}$ implementing a *main* function, a **valid component-based software system** $S$ consists of a set of component-choices from $L_{comp}$ including $c$ that not only implements all required interfaces of $c$ but also recursively implements all re-
component System1
    provides intSystem
    requires intProc1, intProc2, intProc3 {
    void systemMain(Input I) {
        if $v_I(x_1)$ then callProc1(I)
        elsif $v_I(x_5)$ then callProc2(I)
        else callProc3(I)
    }
}

component Proc1
    provides intProc1
    requires intProc {
    void callProc1(Input I) {
        proc(I)
    }
}

component Proc3
    provides intProc3
    requires intProc {
    void callProc3(Input I) {
        proc(I)
    }
}

component ProcA
    provides intProc {
    void proc(Input I) {
        if $v_I(x_4)$ then output 2
        elsif not $v_I(x_3)$ then output 1
        elsif $v_I(x_5)$ then output 3
        else $a_1$
    }
}

component ProcC
    provides intProc {
    void proc(Input I) {
        if $v_I(x_4)$ then output 2
        else output 2
    }
}

component ProcB
    provides intProc {
    void proc(Input I) {
        if not $v_I(x_2)$ then output 1
        elsif not $v_I(x_4)$ then output 2
        else output 3
    }
}

component ProcD
    provides intProc {
    void proc(Input I) {
        output 2
    }
}

Figure 2: An Example Emergent Software System (Cont’d). (c) A sample software component library $L_{comp}$ consisting of ten components.
Figure 3: An Example Emergent Software System (Cont’d). (d) Component wiring trees corresponding to two valid component-based software systems for $R$ in part (a) based on $L_{int}$, $L_{comp}$, and base component $Base \in L_{comp}$ given in parts (b) and (c). Note that interfaces $intSystem$ and $intProc$ can each be implemented by multiple components (namely, System1 and System2 ($intSystem$) and ProcA, ProcB, ProcC, and ProcD ($intProc$)) and are hence key in allowing component choices yielding different software systems; this is acknowledged by putting dashed boxes around these interfaces in the component wiring trees diagrams.

Queried interfaces of those component-choices and their sub-component-choices, if any. The interface-connections between these components are called wirings. In order to allow data transfer fields to hold different values relative to different implementations of an interface by the same component, these implementations are done relative to copies of that component. Moreover, in cases where a component provides multiple interfaces, different implementations of that component relative to two of those interfaces are done relative to reduced copies of that component, both of which only contain code for and thus provide only those services in the component specified by their respective implementing interfaces.

Any valid component-based software system has an associated directed vertex- and arc-labeled tree in which the vertices are components, the arcs are the wirings, and the vertices and arcs are labeled with the names of the associated components and interfaces from $L_{comp}$ and $L_{int}$, respectively; let this tree be called the component wiring tree $T$ associated with $S$. The component wiring trees of two valid component-based software systems relative to $R$, $L_{int}$, $L_{comp}$, and base component $Base \in L_{comp}$ given in parts (a)–(c) of Figures 1 and 2 are given in Figure 3. Note that such a $T$ has a single root vertex (namely, base component $c$), exactly one directed path from this root to any vertex, and
is labeled such that a component vertex-label does not occur more than once on any directed path from the root to a leaf, i.e., there are no recursive dependencies [12, page 10]. This prevents the possibility of infinite-depth software systems resulting from the interface-component implementation sequence between two same-label components on such a path being repeated an infinite number of times.

Given a set $R$ of software system requirements, a valid component-based software system $S$ is a **working component-based software system relative to** $R$ if for each input-output pair $(i_j, o_j) \in R$, the output of $S$ given input $i_j$ is $o_j$. For example, the software system on the left in Figure 3 satisfies all requirements in $R$ given in Figure 1(a) and hence is a working component-based software system relative to $R$, but the software system on the right is not (because it produces different outputs (3, 1, 1, and 2, respectively) for requirements $r_1$, $r_3$, $r_4$, and $r_5$).

- **Emergent software system**: As defined in [12, 13], an **emergent (component-based) software system** is one that, given initial functional software systems requirements $R$, interface and component libraries $L_{\text{int}}$ and $L_{\text{comp}}$, and a base component $c \in L_{\text{comp}}$, self-assembles and self-adapts as necessary to optimize system performance as its running environment changes over time. For a software system $S$, a system’s running environment and performance are quantified in terms of events and metrics whose values are sampled at discrete times during system operation [12, pages 10–11]; aspects of system performance used by a learning algorithm to guide both self-assembly and self-adaptation are in turn summarized in a reward function. As currently implemented [12], the initial self-assembly phase creates a list of all working software systems relative to the given $L_{\text{int}}$, $L_{\text{comp}}$, and $R$, where each system is described by a unique ID string that lists all components in the system and their interconnections. During the subsequent self-adaptation phase, the learning algorithm searches over this list to find appropriate alternatives to the currently-running system that might help optimize system performance [12, Section 3.2.1].

In this paper, we will assume that a system’s running environment and performance are sampled using an environment function $Env()$ that maps a given system onto a collection of events and metric-values and that the reward function $Rew()$ maps the latest accumulated performance metric values for a system onto a single positive integer value. For simplicity, we shall further assume that $Env()$ and $Rew()$ are computable in time polynomial in the size of $S$, $Rew(S)$ is optimized by minimization, i.e., smaller values of $Rew(S)$ are preferred, and $Rew()$ is used to choose among but does not alter the set of possible working
software systems relative to a given $L_{\text{int}}$, $L_{\text{comp}}$, and $R$.

We can now formalize computational problems corresponding to emergent software adaptation as conceived in \cite{12,13}:

**Emergent Software Creation (ESCreate)**

*Input*: Software system requirements $R$, interface and component libraries $L_{\text{int}}$ and $L_{\text{comp}}$; a base component $c \in L_{\text{comp}}$, and reward and environment functions $Rew()$ and $Env()$.

*Output*: A working component-based software system $S$ based on $c$ relative to $L_{\text{int}}$, $L_{\text{comp}}$, and $R$ that has the smallest value of $Rew(S)$ over all working systems based on $c$ relative to $L_{\text{int}}$, $L_{\text{comp}}$, and $R$, if any working system exists, and special symbol $\perp$ otherwise.

**Emergent Software Adaptation (ESAdapt)**

*Input*: Software system requirements $R$, interface and component libraries $L_{\text{int}}$ and $L_{\text{comp}}$; a working component-based software system $S$ based on component $c \in L_{\text{comp}}$ relative to $R$, $L_{\text{int}}$, and $L_{\text{comp}}$, and reward and environment functions $Rew()$ and $Env()$.

*Output*: A working component-based software system $S'$ based on $c$ relative to $L_{\text{int}}$, $L_{\text{comp}}$, and $R$ that has the smallest possible value of $Rew(S')$ over all working systems based on $c$ relative to $L_{\text{int}}$, $L_{\text{comp}}$, and $R$.

Problem ESCreate corresponds to the initial creation of a working emergent software system, while ESAdapt corresponds to each subsequent modification of the system to optimize the reward function as the system’s running environment changes over time. These problems do not correspond directly to emergent system creation and adaptation as currently implemented, in that ESCreate only returns one rather than all working software systems relative to $L_{\text{int}}$, $L_{\text{comp}}$, $R$, ESCreate optimizes the performance of this returned system relative to $Rew()$, and ESAdapt only has access to $S$ and not the complete list of working software systems relative to $L_{\text{int}}$, $L_{\text{comp}}$, and $R$.

However, as will be discussed in Section 4.2, complexity results for ESCreate and ESAdapt can still be used to investigate both current and potential implementations of emergent software systems.

Two additional notes are in order about our definitions of ESCreate and ESAdapt. First, the given $L_{\text{int}}$, and $L_{\text{comp}}$ in an input of ESCreate may not allow a working

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1 An additional difference and possible source of confusion is that the term “adaptation” in \cite{12} refers to the process of changing the components and interconnections in the currently-running software system to those in the candidate selected by the learning algorithm. In this paper, we shall instead use “adaptation” to refer to the overall self-adaptation process described by ESAdapt, which may involve multiple episodes of adaptation in the sense of \cite{12} to achieve an optimal system under $Rew()$.\[1\]
software system relative to the given R but there is always a working software system for any input of ESAdapt — namely, S. Second, Rew() and Env() are part of the input for both ESCreate and ESAdapt and must be included in any instance of these problems, i.e., it cannot be the case that Env() and/or Rew() are empty. That being said, Env() (as well as S in the case of ESAdapt) need not necessarily be used by any algorithm solving these problems but are provided as part of the problem inputs to make results derived here relevant to emergent systems as described in [12, 13] (see Section 4 for further discussion on the latter point).

In this paper, we will consider the following versions of Rew():

- \( \text{Rew}_{\#\text{comp}}(S) = \) the number of components in S.
- \( \text{Rew}_{\text{CodeB}}(S) = \) the size of the codebase of S, i.e., the total number of lines of code in the interfaces and components comprising S.

Relative to a particular reward function \( \text{Rew}_X() \), we refer to our problems above as ESCreate under \( \text{Rew}_X() \) and ESAdapt under \( \text{Rew}_X() \), respectively. Note that results derived relative to \( \text{Rew}_{\#\text{comp}}() \) and \( \text{Rew}_{\text{CodeB}}() \) have broad applicability as the values of these functions correlate with the values of at least some of the reward functions studied to date in emergent software systems, e.g., system response time [12, page 11]. As none of our result proofs rely on Env(), we need not specify its form further.

Let us now illustrate problems ESCreate and ESAdapt relative to the example emergent software system described in Figures 1–3:

- **Valid Software Systems relative to \( L_{\text{int}} \) and \( L_{\text{comp}} \):** As each interface proc1, proc2, and proc3 in software systems based on component System1 can be implemented with any of the components ProcA, ProcB, ProcC, or ProcD, there are \( 4^3 = 64 \) valid software systems relative to the general structure on the left in part (d) of Figure 3. By analogous reasoning relative to component System2 and interface proc1, there are 4 valid software systems relative to the general structure on the right. Hence, there are in total 68 valid software systems relative to \( L_{\text{int}} \) and \( L_{\text{comp}} \).

- **Working Software Systems relative to R:** The reader can verify that (i) there are no working software systems relative to R that incorporate component System2 and (ii) all working software systems relative to R that incorporate component System1 must implement interface Proc1 with component ProcA and interface Proc3 with either of the components ProcC or ProcD, and that interface Proc2 (as its implementing component code is never executed) can be implemented by any of the components ProcA, ProcB, ProcC or ProcD. Thus, the
implementations of Proc1, Proc2, and Proc3 in System1 that yield working software systems are (ProcA, ProcA, ProcC), (ProcA, ProcA, ProcD), (ProcA, ProcB, ProcC), (ProcA, ProcB, ProcD), (ProcA, ProcC, ProcC), (ProcA, ProcC, ProcD), (ProcA, ProcD, ProcC), and (ProcA, ProcD, ProcD), giving in total 8 working software system relative to R.

- **Output of ESCreate and ESAdapt under different Rew():** As all working software systems relative to R have 8 components, ESCreate under Rew#comp() can return any of them; however, as the system with the smallest codebase implements both Proc2 and Proc3 with ProcC, it is this system that would be selected by ESCreate under Rew_CodeB(). By analogous reasoning, ESAdapt under Rew#comp can return any of the 8 working software systems given system S drawn from the same set; however, ESAdapt under Rew_CodeB() must (regardless of the choice of the given S) return the system that implements both Proc2 and Proc3 with ProcC.

This concludes our formalization of emergent software system creation and adaptation. A reasonable conjecture at this point is that ESAdapt will be easier to solve than ESCreate, given that the former is given a working system as part of its input. This will be assessed below.

### 3 Results

In this section, we will use computational complexity analysis to assess viable algorithmic options for efficient emergent software system creation and adaptation. This will be done relative to various desirable types of efficient solvability described in Section 3.1. The results of our analyses for emergent software creation and adaptation are given in Sections 3.2 and 3.3 respectively.

It turns out that both ESCreate and ESAdapt are unsolvable in the most general possible case — that is, neither ESCreate nor ESAdapt have algorithms that always return the correct output for an input incorporating any possible choices of Env() and Rew() (in the case of ESCreate) or any possible choice of Env() and either of Rew#comp() or Rew_CodeB() (in the case of ESAdapt) and in which there are no restrictions on the form, size, or running times of L_int and L_comp, their member interfaces and components, or any software systems created using L_int and L_comp (see Sections 3.2.1 and 3.3.1 respectively). Hence, the remainder of our analyses will be done relative to restricted versions of these problems in which candidate component-based software systems run and hence can be verified against given system requirements in polynomial time.
3.1 Types of Efficient Solvability

Consider the following desirable forms of solvability:

1. **Polynomial-time exact solvability**: An exact polynomial-time algorithm is a deterministic algorithm whose runtime is upper-bounded by $c_1|x|^c_2$, where $|x|$ is the size of the input $x$ and where $c_1$ and $c_2$ are constants, and is always guaranteed to produce the correct output for all inputs. A problem that has such an algorithm is said to be polynomial-time tractable. Polynomial-time tractability is desirable because runtimes increase slowly as input size increases, and hence allow the solution of larger inputs.

   It is possible that the computational difficulty of a problem may be inflated in general by inputs that have no solutions, and hence force any algorithm to exhaustively consider all possible candidate solutions. In such cases, it is useful to assess whether a problem is polynomial-time exact promise solvable — that is, whether that problem is exactly solvable in polynomial time on those inputs which are guaranteed to have solutions, where these guarantees are known as promises.

2. **Polynomial-time approximate solvability**: A polynomial-time approximation algorithm is an algorithm that runs in polynomial time in an approximately correct (but acceptable) manner for all inputs. There are a number of ways in which an algorithm can operate in an approximately correct manner. Three of the most popular ways are as follows:

   (a) **Frequently Correct (Deterministic)** [25]: Such an algorithm runs in polynomial time and gives correct solutions for all but a very small number of inputs. In particular, if the number of inputs for each input-size $n$ on which the algorithm gives the wrong or no answer (denoted by the function $err(n)$) is sufficiently small (e.g., $err(n) = c$ for some constant $c$), such algorithms may be acceptable.

   (b) **Frequently Correct (Probabilistic)** [26]: Such an algorithm (which is typically probabilistic) runs in polynomial time and gives correct solutions with high probability. In particular, if the probability of correctness is $\geq 2/3$ (and hence can be boosted by additional computations running in polynomial time to be correct with probability arbitrarily close to 1 [31, Section 5.2]), such algorithms may be acceptable.

   (c) **Approximately Optimal** [27]: Such an algorithm $A$ runs in polynomial time and gives a solution $A(x)$ for an input $x$ whose value $val(A(x))$ is guaranteed to be within a multiplicative factor $f(|x|)$ of the value $v_{OPT}(x)$.
of an optimal solution for \( x \), i.e., \(|v_{OPT}(x) - val(A(x))| \leq f(|x|) \times v_{OPT}(x)\) for any input \( x \) for some function \( f() \). A problem with such an algorithm is said to be polynomial-time \( f(|x|) \)-approximable. In particular, if \( f(|x|) \) is a constant very close to 0 (meaning that the algorithm is always guaranteed to give a solution that is either optimal or very close to optimal), such algorithms may be acceptable.

3. **Effectively polynomial-time exact restricted solvability**: Even if a problem is not solvable in any of the senses above, a restricted version of that problem may be exactly solvable in close-to-polynomial time. Let us characterize restrictions on problem inputs in terms of a set \( K = \{k_1, k_2, \ldots, k_{|K|}\} \) of aspects of the input. For example, possible restrictions on the inputs ESCreate could be the number of given software requirements, the number of components in \( L_{comp} \), and the maximum number of components in a working software system relative to \( L_{int}, L_{comp} \), and \( R \) (see also Table 1 in Section 3.2.4). Let each such aspect be called a parameter.

One of the most popular ways in which an algorithm can operate in close-to-polynomial time relative to restricted inputs is **fixed-parameter (fp-) tractability**. Such an algorithm runs in time that is non-polynomial purely in terms of the parameters in \( K \), i.e., in time \( f(K)|x|^c \) where \( f() \) is some function, \(|x|\) is the size of input \( x \), and \( c \) is a constant. A problem with such an algorithm for parameter-set \( K \) is said to be **fixed-parameter (fp-)tractable relative to \( K \)**. Fixed-parameter tractability generalize polynomial-time exact solvability by allowing the leading constant \( c_1 \) of the input size in the runtime upper-bound of an algorithm to be a function of \( K \). Though such algorithms run in non-polynomial time in general, for inputs in which all the parameters in \( K \) have very small constant values and \( f(K) \) thus collapses to a possibly large but nonetheless constant value, such algorithms (particularly if \( f() \) is suitably well-behaved, \( e.g., (1.2)^{k_1+k_2} \)) may be acceptable.

In the following two subsections, we shall evaluate the algorithmic options for ESCreate and ESAdapt, respectively, relative to each of these types of solvability. Our unsolvability proofs will use reductions between pairs of problems, where a reduction from a problem \( \Pi \) to a problem \( \Pi' \) is essentially an efficient algorithm \( A \) for solving \( \Pi \) which uses a hypothetical algorithm for solving \( \Pi' \). Reductions are useful by the following logic:

- If \( \Pi \) reduces to \( \Pi' \) and \( \Pi' \) is efficiently solvable by algorithm \( B \) then \( \Pi \) is efficiently solvable (courtesy of the algorithm \( A' \) that invokes \( A \) relative to \( B \)).
• If \( \Pi \) reduces to \( \Pi' \) and \( \Pi \) is not efficiently solvable then \( \Pi' \) is not efficiently solvable (as otherwise, by the logic above, \( \Pi \) would be efficiently solvable, which would be a contradiction).

We will use the following three types of reducibility:

**Definition 1** [17, Section 3.1.2] Given decision problems \( \Pi \) and \( \Pi' \), i.e., problems whose answers are either “Yes” or “No”, \( \Pi \) polynomial-time (Karp) reduces to \( \Pi' \) if there is a polynomial-time computable function \( f() \) such that for any instance \( x \) of \( \Pi \), the answer to \( \Pi \) for \( x \) is “Yes” if and only if the answer to \( \Pi' \) for \( f(x) \) is “Yes”.

**Definition 2** [17, Section 3.1.2] Given search problems \( \Pi \) and \( \Pi' \), i.e., problems whose answers are actual solutions rather than just “Yes” or “No”, \( \Pi \) polynomial-time (Levin) reduces to \( \Pi' \) if there is a pair of polynomial-time functions \( f() \) and \( g() \) such that for any instance \( x \) of \( \Pi \), the answer to \( \Pi \) for \( x \) is \( g(x, y) \) if and only if the answer to \( \Pi' \) for \( f(x) \) is \( y \).

**Definition 3** [32] Given parameterized decision problems \( \Pi \) and \( \Pi' \), \( \Pi \) parameterized reduces to \( \Pi' \) if there is a function \( f() \) which transforms instances \( \langle x, K \rangle \) of \( \Pi \) into instances \( \langle x', K' \rangle \) of \( \Pi' \) such that \( f() \) runs in \( f'(K) \cdot |x|^c \) time for some function \( f'(\cdot) \) and constant \( c \), \( k' = g_K(K) \) for each \( k' \in K \) for some function \( g_K(\cdot) \), and for any instance \( \langle x, K \rangle \) of \( \Pi \), the answer to \( \Pi \) for \( \langle x, K \rangle \) is “Yes” if and only if the answer to \( \Pi' \) for \( f(\langle x, K \rangle) \) is “Yes”.

Our reductions will be from versions of the following problems:

**Turing Machine Halting (TM Halting)** [29]

*Input:* A Turing Machine \( M \) and a binary string \( x \).

*Question:* Does \( M \) halt when given \( x \) as input?

**Dominating set** [10, Problem GT2]

*Input:* An undirected graph \( G = (V, E) \) and a positive integer \( k \).

*Question:* Does \( G \) contain a dominating set of size \( k \), i.e., is there a subset \( V' \subseteq V \), \( |V'| = k \), such that for all \( v \in V \), either \( v \in V' \) or there is at least one \( v' \in V' \) such that \((v, v') \in E\)?

**Optimal Dominating set (Dominating set\(^{OPT}\))**

*Input:* An undirected graph \( G = (V, E) \).

*Output:* A dominating set in \( G \) of minimum size.

For each vertex \( v \in V \) in a graph \( G \), let the complete neighbourhood \( N_C(v) \) of \( v \) be the set composed of \( v \) and the set of all vertices in \( G \) that are adjacent to \( v \) by

\[ N_C(v) = \{ v \} \cup \{ u \in V \mid (v, u) \in E \} \]
a single edge, i.e., $v \cup \{u \mid u \in V \text{ and } (u,v) \in E\}$. We assume below for each instance of DOMINATING SET an arbitrary ordering on the vertices of $V$ such that $V = \{v_1,v_2,\ldots,v_{|V|}\}$. Note that only the first of the three problems above is provably unsolvable (indeed, unsolvable in the sense that there can be no algorithm period that returns the correct output for every input [34, Section 9.2.4]). Versions of the others are only known to be unsolvable relative to the types of efficient solvability listed at the start of this subsection modulo the conjectures $P \neq NP$ and $FPT \neq W[2]$; however, this is not a problem in practice as both of these conjectures are widely believed within computer science to be true [9, 28].

As we shall often see in the following two sections, a single reduction may imply multiple results. For example, with respect to the third of the solvability options described above, additional and sometimes stronger fp-tractability and intractability results can often be derived using the following three lemmas.

**Lemma 1** [35, Lemma 2.1.30] If problem $\Pi$ is fp-tractable relative to parameter-set $K$ then $\Pi$ is fp-tractable for any parameter-set $K'$ such that $K \subset K'$.

**Lemma 2** [35, Lemma 2.1.31] If problem $\Pi$ is fp-intractable relative to parameter-set $K$ then $\Pi$ is fp-intractable for any parameter-set $K'$ such that $K' \subset K$.

**Lemma 3** [35, Lemma 2.1.35]. If problem is NP-hard when all parameters in parameter-set $K$ have constant values then $\Pi$ cannot be fp-tractable relative to any subset of $K$ unless $P = NP$.

There are a variety of techniques for creating a reduction from a problem $\Pi$ to a problem $\Pi'$ ([10, Section 3.2]; see also [33, Chapters 3 and 6]). One of these techniques is component design, in which an instance of $\Pi'$ constructed by a reduction is structured as mechanisms that generate candidate solutions for the given instance of $\Pi$ and check these candidates to see if any are actual solutions. We have already seen in the example software systems given in Figure 3 how interfaces with different implementing components (in that case, interfaces intSystem and intProc) can be used to generate choices when constructing a component-based software system. In subsequent subsections, we will use this and other features of interfaces and components under the Dana runtime model as described in Section 2 to structure mechanisms that generate candidate solutions (i.e., valid component-based software systems corresponding to vertex-sets of size $k$ in a given graph $G$) and check these candidates to see if they are actual solutions (e.g., working component-based software systems relative $R$ corresponding to dominating sets of size $k$ in $G$) in many of the reductions underlying our results for problems ESCreate and ESAadapt.
3.2 Results for Emergent Software Creation

Many of the results derived in this section for ESCreate will actually be derived relative to the following problem:

**Component-based Software Creation (CSCreate)**

*Input*: Software system requirements $R$, interface and component libraries $L_{int}$ and $L_{comp}$, and a base component $c \in L_{comp}$.

*Question*: Is there a working component-based software system $S$ based on $c$ relative to $L_{int}$, $L_{comp}$, and $R$?

Note that each input for ESCreate has a corresponding input to CSCreate (namely, the input to ESCreate without $Rew()$ and $Env()$). Moreover, any algorithm $A$ that solves ESCreate under some $Rew()$ can be also used to solve CSCreate (namely, if $A$ run on the given input $x$ for CSCreate produces a working system, output “Yes”, otherwise output “No”). This yields the following useful observation.

**Observation 1** For any choice of $Rew()$ and $Env()$, if there is an algorithm $A$ of solvability type $T$ for ESCreate under $Rew()$ than there is an algorithm $A'$ of solvability type $T$ for CSCreate.

3.2.1 Unsolvability of Unrestricted Emergent Software Creation

We start off by considering if problem ESCreate is solvable in the most general possible case — that is, if ESCreate has an algorithm that always returns the correct output for an input incorporating any possible choices of $Env()$ and $Rew()$ and in which there are no restrictions on the form, size, or running times of $L_{int}$ and $L_{comp}$, their member interfaces and components, or any software systems created using $L_{int}$ and $L_{comp}$. It turns out that such an algorithm cannot exist.

**Result A.1** For any choice of $Rew()$ and $Env()$, ESCreate is unsolvable.

**Proof**: Consider the following polynomial-time Karp reduction from TM HALTING to CSCreate: given an instance $I = \langle M, x \rangle$ of TM HALTING, construct an instance $I' = \langle R, L_{int}, L_{comp}, c \rangle$ of CSCreate in which $X = \{x_1\}$ and $O = \{1\}$, there is a single input-output pair $r$ in $R$ such that for $r = (True, 1)$, $L_{int}$ consists of the single interface

```java
interface base {
    void main(Input I)
}
```

and $L_{comp}$ consists of the single component.
component Base provides base {
    void main(Input I) {
        <CODEM(x)>
        output 1
    }
}

where <CODEM(x)> is the Dana code simulating the computation of $M$ on input $x$. As Dana contains both loops and conditional statements, it can readily simulate $M$ on input $x$ using code that is of size polynomial in the sizes of the given descriptions of $M$ and $x$. Finally, let $c$ be component Base in $L_{comp}$. Note that the instance of CSCreate described above can be constructed in time polynomial in the size of the given instance of TM Halting. To conclude the proof, observe that the only possible component-based system for the constructed instance of CSCreate based on $c$ is that consisting of Base itself, and that this system satisfies the sole input-output constraint in $R$ if and only if $M$ halts on input $x$ for the given instance of TM Halting. It is known that TM Halting cannot have an algorithm that is correct for all possible $\langle M, x \rangle$ instances [34, Section 9.2.4], and hence is unsolvable. Hence, the reduction above implies in turn that CSCreate cannot have an algorithm either. The unsolvability result for ESCreate then follows by contradiction from Observation 1.

This result is especially disconcerting as it holds relative to not just some choices but every possible choice of $Env()$ and $Rew()$ (this is because the proof of this result ignores these functions entirely). However, it is ultimately not surprising, given the computational power inherent in the Dana programming language and the folklore result that a number of problems in software engineering, e.g., checking if a software system satisfies a set of given requirements, are known to be unsolvable as a consequence of Rice’s Theorem [34, Section 9.3.3].

That being said, restricted versions of ESCreate may yet have correct and even efficient algorithms. One reasonable such restriction is that any candidate component-based software system $S$ created from a given $L_{int}$ and $L_{comp}$ runs in time polynomial in the input size $|I|$ and hence can be checked against the system requirements in $R$ in time polynomial in the size of $R$ (as $|I| < |R|$), i.e., created software systems not only operate but can also be verified quickly. Indeed, such a restriction is implicit in the requirement that emergent software systems be autonomously verifiable at runtime [12, page 5]. In the remainder of our analyses in this paper, we will assume ESCreate and CSCreate to be so restricted, and will denote these restricted versions as ESCreate_{poly} and CSCreate_{poly}, respectively.
3.2.2 Polynomial-time Exact Solvability of Restricted Emergent Software Creation

We now consider if ESCreate\textsubscript{poly} is efficiently solvable in the first of the senses listed at the start of Section 3.1 — namely, polynomial-time exact solvability and polynomial-time exact promise solvability. One might initially think that, given the somewhat radical nature of the restriction on ESCreate proposed at the end of the previous subsection, ESCreate so restricted is now efficiently solvable in both of these senses. However, this turns out not to the case.

These intractability results are shown using the following reduction. This reduction creates valid component-based systems with component wiring trees of the form shown in Figure 4 in which the multiply implemented interfaces cond\textsubscript{1}, cond\textsubscript{2}, ..., cond\textsubscript{k} are used to create valid software systems corresponding to all possible vertex-sets of size \(k\) in the graph \(G\) in the given instance of DOMINATING SET. As each input-output pair in the constructed \(R\) corresponds to a vertex-neighbourhood in \(G\), the code in component \textit{Base} ensures that working software systems correspond to dominating sets of size \(k\) in \(G\).

\textbf{Lemma 4} DOMINATING SET polynomial-time \textit{Karp} reduces to CSCreate\textsubscript{poly}.

\textbf{Proof:} Given an instance \(I = \langle G = (V, E), k \rangle\) of DOMINATING SET, construct the following instance \(I' = \langle R, L_{\text{int}}, L_{\text{comp}}, c \rangle\) of CSCreate\textsubscript{poly}: Let \(X = \{x_1, x_2, \ldots, x_{|V|}\}\), i.e., there is a unique Boolean variable corresponding to each vertex in \(V\), and \(O = \{0, 1\}\). There are \(|V|\) input-output pairs in \(R\) such that for \(r_j = (i_j, o_j), 1 \leq j \leq |V|, v_{i_j}(x_k) = \text{True}\) if \(v_k \in N_C(v_j)\) and is \text{False} otherwise and \(o_j = 1\). Let \(L_{\text{int}}\) consist of \(k + 1\) interfaces broken into two groups:

1. A single interface of the form
interface base {
    void main(Input I)
}

2. A set of $k$ interfaces of the form

interface condJ {
    Boolean inSetJ(Input I)
}

for $1 \leq J \leq k$.

Let $L_{comp}$ consist of $k|V| + 1$ components broken into two groups:

1. A single component of the form

   component Base provides base
       requires cond1, cond2, ..., condk {
           void main(Input I) {
               if inSet1(I) then output 1
               elsif inSet2(I) then output 1
               ...
               elsif inSetk(I) then output 1
               else output 0
           }
       }
   }

2. A set of $k|V|$ components of the form

   component InSetJK provides condJ {
       Boolean inSetJ(Input I) {
           return v_I(x_K)
       }
   }

   for $1 \leq J \leq k$ and $1 \leq K \leq |V|$.
Note that in $L_{comp}$, there are $|V|$ implementations of each cond-interface. Finally, let $c$ be component $\text{Base}$ in $L_{comp}$. Note that the instance of $\text{CS}\text{Create}_{poly}$ described above can be constructed in time polynomial in the size of the given instance of DOMINATING SET; moreover, as there is only a $(k+1)$-clause if-then statement block and no loops in the component code and $k \leq |V| < |I|$, any candidate component-based software system created relative to $L_{int}$, $L_{comp}$, and $c$ runs in time linear in the size of input $I'$.

Let us now verify the correctness of this reduction:

- Suppose that there is a dominating set $D$ of size at most $k$ in the given instance of DOMINATING SET. We can then construct a component-based software system consisting of $c$ and the $|D|$ InSet-components corresponding to the vertices in $D$; the choice of which interface to implement for each vertex is immaterial, and if there are less than $k$ vertices in $D$, the final $k - |D|$ required cond-interfaces can be implemented relative to InSet-components corresponding to arbitrary vertices in $D$. Observe that for each $(i_j, o_j) \in R$, this software system produces output $o_j$ given input $i_j$.

- Conversely, suppose that the constructed instance of $\text{CS}\text{Create}_{poly}$ has a working component-based software system based on $c$ relative to $L_{int}$, $L_{comp}$, and $R$. In order to correctly accommodate all input-output pairs in $R$, the $k$ if-then statements in $c$ must implement InSet-components whose corresponding vertices form a dominating set in $G$ of size at most $k$. Hence, the existence of a working component-based software system for the constructed instance of $\text{CS}\text{Create}_{poly}$ implies the existence of a dominating set of size at most $k$ for the given instance of DOMINATING SET.

This completes the proof.

**Result A.2** For any choice of $\text{Rew}()$ and $\text{Env}()$, if $\text{ES}\text{Create}_{poly}$ is polynomial-time exact solvable then $P = NP$.

**Proof:** Given the $NP$-hardness of DOMINATING SET, the reduction in Lemma 4 implies that $\text{CS}\text{Create}_{poly}$ is $NP$-hard, and hence not solvable in polynomial time unless $P = NP$. The polynomial-time intractability result for $\text{ES}\text{Create}_{poly}$ then follows by contradiction from Observation 1.

**Result A.3** For any choice of $\text{Rew}()$ and $\text{Env}()$, if $\text{ES}\text{Create}_{poly}$ is polynomial-time exact promise solvable then $P = NP$.
Proof: Suppose that for some choice of \( \text{Rew}(\cdot) \) and \( \text{Env}(\cdot) \), \( \text{ESC}_\text{Create}_\text{poly} \) is polynomial-time promise solvable by an algorithm \( A \). Consider the following algorithm for \( \text{Dominating set} \):

1. Given an instance \( I = \langle G = (V, E), k \rangle \) of \( \text{Dominating set} \), construct an instance \( I' = \langle R, L_{\text{int}}, L_{\text{comp}}, \text{Rew}(\cdot), \text{Env}(\cdot), c \rangle \) of \( \text{ESC}_\text{Create}_\text{poly} \) using the reduction from \( \text{Dominating set} \) to \( \text{CSC}_\text{Create}_\text{poly} \) described in Lemma 4 to create \( R, L_{\text{int}}, L_{\text{comp}} \), and \( c \).

2. Run \( A \) on \( I' \) to produce output \( O' \) for \( \text{ESC}_\text{Create}_\text{poly} \).

3. As specified in the converse part of the proof of correctness of the reduction in Lemma 4, use the invoked if-then components in \( O' \) to derive a candidate solution \( O \) for the given instance of \( \text{Dominating set} \).

4. If \( O \) is a correct solution for \( I \), output “Yes”; otherwise, output “No” (as by the definition of promise solvability, if the answer was “Yes” then \( A \) would have had to output \( O' \) such that \( O \) was a correct solution to the given instance of \( \text{Dominating set} \)).

As all steps in this algorithm run in polynomial time, the above is a polynomial-time algorithm for \( \text{Dominating set} \). However, given the \( NP \)-hardness of \( \text{Dominating set} \), this would imply that \( P = NP \), completing the proof.

3.2.3 Polynomial-time Approximate Solvability of Restricted Emergent Software Creation

We now consider if \( \text{ESC}_\text{Create}_\text{poly} \) is efficiently approximately solvable in either of the three senses (frequently correct (deterministic), frequently correct (probabilistic), or approximately optimal) listed at the start of Section 3.1. As can be seen below, the polynomial-time exact intractability of \( \text{ESC}_\text{Create}_\text{poly} \) proved in the previous section rules out all three of these types of efficient approximability.

We start by considering the two types of frequently correct approximability.

Result A.4 For any choice of \( \text{Rew}(\cdot) \) and \( \text{Env}(\cdot) \), if \( \text{ESC}_\text{Create}_\text{poly} \) is solvable by a polynomial-time algorithm with a polynomial error frequency (i.e., \( \text{err}(n) \) is upper bounded by a polynomial of \( n \)) then \( P = NP \).

\( ^3 \)It may initially seem puzzling why, in light of Observation 1, we here directly evaluate the polynomial-time promise solvability of \( \text{ESC}_\text{Create}_\text{poly} \). This is necessary because the promise solvability of any decision problem such as \( \text{CSC}_\text{Create}_\text{poly} \) is established by the trivial constant-time algorithm which always answers “Yes” (and hence is always correct if a solution exists).
Proof: That the existence of such an algorithm for CSCreate<sub>poly</sub> implies \( P = NP \) follows from the \( NP \)-hardness of CSCreate<sub>poly</sub> (which is established in the proof of Result A.1) and Corollary 2.2. in [25]. The polynomial-time inapproximability result for ESCreate<sub>poly</sub> then follows by contradiction from Observation 1.

**Result A.5** For any choice of \( \text{Rew}(\cdot) \) and \( \text{Env}(\cdot) \), if \( P = BPP \) and ESCreate<sub>poly</sub> is polynomial-time solvable by a probabilistic algorithm which operates correctly with probability \( \geq 2/3 \) then \( P = NP \).

Proof: It is widely believed that \( P = BPP \) [31, Section 5.2] where \( BPP \) is considered the most inclusive class of decision problems that can be efficiently solved using probabilistic methods (in particular, methods whose probability of correctness is \( \geq 2/3 \) and can thus be efficiently boosted to be arbitrarily close to one). Hence, if CSCreate<sub>poly</sub> has a probabilistic polynomial-time algorithm which operates correctly with probability \( \geq 2/3 \) then CSCreate<sub>poly</sub> is by definition in \( BPP \). However, if \( BPP = P \) and we know that CSCreate<sub>poly</sub> is \( NP \)-hard by the proof of Result A.2, this would then imply by the definition of \( NP \)-hardness that \( P = NP \). The polynomial-time inapproximability result for ESCreate<sub>poly</sub> then follows by contradiction from Observation 1.

To assess cost-approximability, we need the following problem.

**Optimal Component-based Software Creation (CSCreate<sup>OPT</sup>)**

*Input*: Software system requirements \( R \), interface and component libraries \( L_{int} \) and \( L_{comp} \), a base component \( c \in L_{comp} \), and a reward function \( \text{Rew}(\cdot) \).

*Output*: A working component-based software system \( S \) based on \( c \) relative to \( L_{int} \), \( L_{comp} \), and \( R \) that has the smallest value of \( \text{Rew}(S) \) over all working systems based on \( c \) relative to \( L_{int} \), \( L_{comp} \), and \( R \), if such a system exists, and special symbol \( \perp \) otherwise.

Let CSCreate<sub>poly</sub><sup>OPT</sup> be the version of CSCreate<sup>OPT</sup> such that any component-based system \( S \) runs in time polynomial in the input size \( |I| \). Note that each input for ESCreate<sub>poly</sub> has a corresponding input to CSCreate<sub>poly</sub><sup>OPT</sup> (namely, the input to ESCreate<sub>poly</sub> without \( \text{Env}(\cdot) \)). Moreover, any algorithm \( A \) that solves ESCreate<sub>poly</sub> under some \( \text{Rew}(\cdot) \) can also be used to solve CSCreate<sub>poly</sub><sup>OPT</sup> (namely, return whatever \( A \) run on the given input \( x \) for CSCreate<sub>poly</sub><sup>OPT</sup> produces). This yields the following useful observation.

**Observation 2** For any choice of \( \text{Rew}(\cdot) \) and \( \text{Env}(\cdot) \), if there is an algorithm \( A \) of solvability type \( T \) for ESCreate<sub>poly</sub> then there is an algorithm \( A' \) of solvability type \( T \) for CSCreate<sub>poly</sub><sup>OPT</sup> under \( \text{Rew}(\cdot) \).
Figure 5: General structure of valid software systems created by the reduction in the proof of Lemma 5. Note that indices $J$ and $x$ in $\text{InSetJx}$ are such that $1 \leq J, x \leq |V|$. Following the convention in Figure 3, interfaces with multiple implementing components are enclosed in dashed boxes.

We first give a reduction that will be used to establish the cost-inapproximability of ESCreate under $\text{Rew}_\#\text{comp}()$. This reduction builds on that in Lemma 4 by further exploiting the ability of interfaces to be implemented by multiple components to allow a set of $\text{BaseJ}$ components that effectively encode all possible candidate dominating sets of size 1 to $|V|$ in $G$ (see Figure 5).

**Lemma 5** Dominating set $\text{set}^{OPT}$ polynomial-time Levin reduces to $\text{CSCreate}^{OPT}$ under $\text{Rew}_\#\text{comp}()$ such that there is a dominating set of size $k$ for the given instance of Dominating set $\text{set}^{OPT}$ if and only if there is a working component-based software system $S$ with reward value $\text{Rew}_\#\text{comp}(S) = k + 2$ for the constructed instance of $\text{CSCreate}^{OPT}$.

**Proof:** Given an instance $I = \langle G = (V, E) \rangle$ of Dominating set $\text{set}^{OPT}$, construct the following instance $I' = \langle R, L_{\text{int}}, L_{\text{comp}}, c \rangle$ of $\text{CSCreate}^{OPT}$: Let $R$ be as in the proof of Lemma 4. Let $L_{\text{int}}$ consist of $|V| + 2$ interfaces broken into three groups:

1. A single interface of the form

   ```
   interface topBase {
       void main(Input I)
   }
   ```

2. A single interface of the form

   ```
   ```
interface base {
    void mainBase(Input I)
}

3. A set of $|V|$ interfaces of the form

interface condJ {
    Boolean inSetJ(Input I)
}

for $1 \leq J \leq |V|$.

Let $L_{comp}$ consist of $|V|^2 + |V| + 1$ components broken into three groups:

1. A single component of the form

    component TopBase provides topBase requires base {
        void main(Input I) {
            mainBase(I)
        }
    }

2. A set of $|V|$ components of the form

    component BaseJ provides base
        requires cond1, cond2, ..., condJ {
            void mainBase(Input I) {
                if inSet1(I) then output 1
                elsif inSet2(I) then output 1
                ...
                elsif inSetJ(I) then output 1
                else output 0
            }
        }

    for $1 \leq J \leq |V|$.

3. A set of $|V|^2$ components of the form
component InSetJK provides condJ {
  Boolean inSetJ(Input I) {
    return v_I(x_K)
  }
}
for 1 \leq J \leq |V| and 1 \leq K \leq |V|.

Note that in \( L_{comp} \), there are \(|V|\) implementations of the base-interface and \(|V|\) implementations of each cond-interface. Finally, let \( c \) be component TopBase in \( L_{comp} \). Note that the instance of CSCreate\textsuperscript{OPT}_{poly} described above can be constructed in time polynomial in the size of the given instance of DOMINATING SET\textsuperscript{OPT},; moreover, as there is only an at most \((|V| + 1)\)-clause if-then statement block and no loops in the component code and \(|V| < |I|\), any candidate component-based software system created relative to \( L_{int}, L_{comp}, \) and \( c \) runs in time linear in the size of input \( I' \).

Let us now verify the correctness of this reduction:

- Suppose that there is a dominating set \( D \) of size \( k \) in the given instance of DOMINATING SET. We can then construct a component-based software system consisting of \( c \), component \( \text{base}_k \), and the \( k \) InSet-components corresponding to the vertices in \( D \); the choice of which interface to implement for each vertex is immaterial. Observe that for each \((i_j, o_j) \in R\), this software system produces output \( o_j \) given input \( i_j \); moreover, \( \text{Rew}_{\#\text{comp}}(S) = k + 2 \).

- Conversely, suppose that the constructed instance of CSCreate\textsuperscript{OPT}_{poly} has a working component-based software system based on \( c \) relative to \( L_{int}, L_{comp}, \) and \( R \) such that \( \text{Rew}_{\#\text{comp}}(S) = val \)\(^4\). As \( c \) is component TopBase which requires a Base component and this Base component requires some number of InSet components, this system is comprised of components TopBase, Base(\( val - 2 \)), and \( val - 2 \) InSet components. In order to correctly accommodate all input-output pairs in \( R \), the \( (val - 2) \) if-then statements in Base(\( val - 2 \)) must implement Inset-components whose corresponding vertices form a dominating set in \( G \) of size at most \( val - 2 \). Hence, the existence of a working component-based software system \( S \) such that \( \text{Rew}_{\#\text{comp}}(S) = val \) for the constructed instance of CSCreate\textsuperscript{OPT}_{poly} implies the existence of a dominating set of size \( val - 2 \) for the given instance of DOMINATING SET.

\(^4\)Note that the existence of at least one such a working system is guaranteed for all instances of CSCreate\textsuperscript{OPT}_{poly} constructed as described above (namely, the system consisting of components TopBase and Base(\(|V|\)) and the \(|V|\) components InSetJJ for \( 1 \leq J \leq |V| \), which corresponds to the dominating set consisting of all vertices in \( V \)). This is necessary for our reduction, as each instance of DOMINATING SET\textsuperscript{OPT} has at least one dominating set (namely, \( V \)), and cannot correspond to a constructed instance of CSCreate\textsuperscript{OPT}_{poly} whose solution is \( \perp \).
To complete the proof, note that the required functions $f()$ and $g()$ in the definition of a Levin reduction correspond respectively to the algorithm given at the beginning of this proof for constructing an instance of $\text{CSCreate}^\text{OPT}_{\text{poly}}$ under $\text{Rew}^\text{#comp}()$ from the given instance of $\text{DOMINATING SET}^\text{OPT}$ and the algorithm implicit in the converse clause of the proof of reduction correctness above for constructing a dominating set from a valid component-based software system for the constructed instance of $\text{CSCreate}^\text{OPT}_{\text{poly}}$.

The reduction above can also be used to establish the cost-inapproximability of $\text{ESCreate}$ under $\text{Rew}_{\text{CodeB}}()$.

Lemma 6 DOMINATING SET$^\text{OPT}$ polynomial-time Levin reduces to $\text{CSCreate}^\text{OPT}_{\text{poly}}$ under $\text{Rew}_{\text{CodeB}}()$ such that there is a dominating set of size $k$ for the given instance of DOMINATING SET$^\text{OPT}$ if and only if there is a working component-based software system $S$ with reward value $\text{Rew}_{\text{CodeB}}(S) = 9k + 16$ for the constructed instance of $\text{CSCreate}^\text{OPT}_{\text{poly}}$.

Proof: In the proof of Lemma 5, observe that for a dominating set of size $k$ in the given instance of DOMINATING SET$^\text{OPT}$, a software system $S$ for the constructed instance of $\text{CSCreate}^\text{OPT}_{\text{poly}}$ consists of components $\text{TopBase}$ and $\text{Base}_k$, $k$ $\text{InSet}$ components, interfaces $\text{topBase}$ and $\text{base}$, and $k$ $\text{cond}$ interfaces. The total number of lines of code in this system and hence the value of $\text{Rew}_{\text{CodeB}}(S)$ is therefore $5 + (k + 5) + 5k + 3 + 3 + 3k = 9k + 16$. The result then follows by a slight modification to the proof of correctness of the reduction described in Lemma 5.

Result A.6 For any choice of $\text{Env}()$, if $\text{ESCreate}^\text{OPT}_{\text{poly}}$ under $\text{Rew}^\text{#comp}()$ is polynomial-time $c$-approximable for any constant $c > 0$ then $P = NP$.

Proof: Observe that in the proof of the reduction in Lemma 5, the size $k$ of a dominating set in $G$ in the given instance of DOMINATING SET$^\text{OPT}$ is always a linear function of the value of $\text{Rew}^\text{#comp}(S)$ in the constructed instance of $\text{CSCreate}^\text{OPT}_{\text{poly}}$, i.e., $k = \text{Rew}^\text{#comp}(S) - 2$. This means that a polynomial-time $c$-approximation algorithm for $\text{CSCreate}^\text{OPT}_{\text{poly}}$ under $\text{Rew}^\text{#comp}()$ for any constant $c$, when combined with the reduction from DOMINATING SET$^\text{OPT}$ to $\text{CSCreate}^\text{OPT}_{\text{poly}}$ described in the proof of Lemma 5, implies the existence of a polynomial-time $3c$-approximation algorithm for DOMINATING SET$^\text{OPT}$ (as $c \times \text{Rew}^\text{#comp}(S) = c \times (k + 2) \leq c \times (k + 2k) \leq 3c \times k$ for $k \geq 1$).

However, if DOMINATING SET$^\text{OPT}$ has a polynomial-time $c$-approximation algorithm for any constant $c > 0$ then $P = NP$ [36], which means that $\text{CSCreate}^\text{OPT}_{\text{poly}}$ under $\text{Rew}^\text{#comp}()$ cannot have a polynomial-time $c$-approximation algorithm for any $c > 0$ unless $P = NP$. The polynomial-time inapproximability result for $\text{ESCreate}^\text{OPT}_{\text{poly}}$ under $\text{Rew}^\text{#comp}()$ then follows by contradiction from Observation 2.
Table 1: Parameters for emergent software creation problems.

| Parameter | Description                        |
|-----------|-----------------------------------|
| $|L_{int}|$ | # available interfaces            |
| $|L_{comp}|$ | # available components            |
| $I_{ci}$  | Maximum # components implementing an interface |
| $C_{pi}$  | Maximum # provided interfaces per component |
| $C_{ri}$  | Maximum # required interfaces per component |
| $S_{comp}$| Maximum # components in a valid system |
| $S_{depth}$ | Maximum depth of component wiring tree |

**Result A.7** For any choice of $Env()$, if $ESCreat_{poly}e^{OPT}$ under $Rew_{CodeB}()$ is polynomial-time $c$-approximable for any constant $c > 0$ then $P = NP$.

**Proof:** Observe that in the proof of the reduction in Lemma 6, the size $k$ of a dominating set in $G$ in the given instance of $DOMINATING\ set^{OPT}$ is always a linear function of the value of $Rew_{CodeB}(S)$ in the constructed instance of $CSCreat_{poly}e^{OPT}$, i.e., $k = \frac{Rew_{CodeB}(S) - 16}{9}$. This means that a polynomial-time $c$-approximation algorithm for $CSCreat_{poly}e^{OPT}$ under $Rew_{CodeB}()$ for any constant $c$, when combined with the reduction from $DOMINATING\ set^{OPT}$ to $CSCreat_{poly}e^{OPT}$ described in the proof of Lemma 6, implies the existence of a polynomial-time $25c$-approximation algorithm for $DOMINATING\ set^{OPT}$ $(as\ c \timesRew_{CodeB}(S) = c \times (9k + 16) \leq c \times (9k + 16k) \leq 25c \times k$ for $k \geq 1$). However, if $DOMINATING\ set^{OPT}$ has a polynomial-time $c$-approximation algorithm for any constant $c > 0$ then $P = NP$ [36], which means that $CSCreat_{poly}e^{OPT}$ under $Rew_{CodeB}()$ cannot have a polynomial-time $c$-approximation algorithm for any $c > 0$ unless $P = NP$. The polynomial-time inapproximability result for $ESCreat_{poly}e^{OPT}$ under $Rew_{CodeB}()$ then follows by contradiction from Observation 2.

### 3.2.4 Fixed-parameter Tractability of Restricted Emergent Software Creation

Given the plethora of intractability results in the previous three subsections, we now consider to what extent and relative to which parameters $ESCreat_{poly}e^{OPT}$ is and is not fp-tractable. In our analyses below, we will focus on parameter-sets $K$ drawn from the parameters listed in Table 1. These parameters can be divided into four main groups:

1. Parameters characterizing interface and component libraries ($|L_{int}|, |L_{comp}|$);
2. Parameters charactering interfaces (I_{ci});

3. Parameters charactering components (C_{pi}, C_{ri}); and

4. Parameters characterizing component-based software systems (S_{comp}, S_{depth}).

We first consider those parameter-sets which yield fp-intractability.

**Result A.8** For any choice of Rew() and Env(), if \langle |L_{int}|, C_{pi}, C_{ri}, S_{comp}, S_{depth} \rangle - ESCreate_{poly} is fp-tractable then \(FPT = W[2] \).

**Proof:** Given the \(W[2]\)-hardness of \(\langle k \rangle\)-DOMINATING SET, the reduction in Lemma 4 implies that CSCreate_{poly} is \(W[2]\)-hard when \(C_{pi} = 1, S_{depth} = 2, C_{ri} = k\), and \(|L_{int}| = S_{comp} = k + 1\) and hence not fp-tractable relative to these parameters unless \(FPT = W[2]\). The fp-intractability result for ESCreate_{poly} then follows by contradiction from Observation 1.

The reductions underlying the following three results exploit the tricks previously used to such good effect in Lemmas 4 and 5 as well as other features of our software component model. The reduction underlying Result A.9 reduces the number of InSet components by invoking a larger encoding of candidate dominating sets and more complex but still polynomial-time checking computations in the Base component. The reduction underlying Result A.10 reduces the number of interfaces required by any component to a constant by splitting the creation of the candidate dominating sets in component Base in the reduction in the proof of Result A.9 over multiple components. Finally, the reduction underlying Result A.11 reduces the number of components in \(L_{comp}\) to 3 by exploiting the ability of components providing multiple interfaces to provide only the code required by an interface in that interface’s copy of the component. Readers interested in details can consult the full proofs of these results in the appendix.

**Result A.9** For any choice of Rew() and Env(), if \langle I_{ci}, C_{pi}, S_{depth} \rangle - ESCreate_{poly} is fp-tractable then \(P = NP\).

**Result A.10** For any choice of Rew() and Env(), if \langle I_{ci}, C_{pi}, C_{ri} \rangle - ESCreate_{poly} is fp-tractable then \(P = NP\).

**Result A.11** For any choice of Rew() and Env(), if \langle |L_{comp}|, I_{ci}, S_{depth} \rangle - ESCreate_{poly} is fp-tractable then \(P = NP\).

We now consider those parameter-sets that yield fp-tractability. All of these results are based on the same brute-force solution enumeration algorithm relative to different worst-case runtime analyses.
Result A.12 For any choice of $\text{Rew}()$ and $\text{Env}()$, $\langle I_{ci}, C_{ri}, S_{\text{depth}} \rangle$-$\text{ESCreate}_{\text{poly}}$ is fp-tractable.

Proof: The largest possible component-based software system relative to a given $L_{\text{int}}$ and $L_{\text{comp}}$ has a component wiring tree rooted at base component $c$ with branching factor $C_{ri}$ and depth $S_{\text{depth}}$. This tree has $(C_{ri})^{S_{\text{depth}} - 2}$ non-root vertices, each corresponding to an interface required by a component. As each of these interfaces can be implemented by at most $I_{ci}$ components, there are at most $(I_{ci} + 1)(C_{ri})^{S_{\text{depth}}}$ possible component-based software systems of depth at most $S_{\text{depth}}$ based on $c$ (the “+ 1” term at the lowest level denotes labeling a vertex $v$ with a special symbol that triggers deletion all descendent-vertices of $v$).

Consider the algorithm that exhaustively generates all such systems and for each system $S$, (i) determines if $S$ is a working system relative to $R$ and, if so, (2) computes reward value $\text{Rew}(S)$. The output of this algorithm is the working system with the lowest or highest reward value, depending on the intent of $\text{Rew}()$. Given the above and our assumption that a candidate component-based software system $S$ can be checked against software system requirements $R$ in time polynomial in the sizes of $S$ and $R$, this algorithm runs in fp-time relative to $I_{ci}$, $C_{ri}$, and $S_{\text{depth}}$, completing the proof of this result.

Result A.13 For any choice of $\text{Rew}()$ and $\text{Env}()$, $\langle I_{ci}, S_{\text{comp}} \rangle$-$\text{ESCreate}_{\text{poly}}$ is fp-tractable.

Proof: As no path from the root to a leaf in the wiring component trees for our software systems can contain duplicate component vertex-labels, the length of the longest path in such a tree from base component $c$ is bounded by $S_{\text{comp}}$; this means that $S_{\text{depth}} \leq S_{\text{comp}}$. Moreover, as implementing each required interface adds a component to the software system, $C_{ri} \leq S_{\text{comp}} - 1 < S_{\text{comp}}$. Given these two observations, this result then follows from the algorithm in the proof of Result A.12.

Result A.14 For any choice of $\text{Rew}()$ and $\text{Env}()$, $\langle |L_{\text{int}}|, |L_{\text{comp}}| \rangle$-$\text{ESCreate}_{\text{poly}}$ is fp-tractable.

Proof: As no path from the root to a leaf in the wiring component trees for our software systems can contain duplicate component vertex-labels, the length of the longest path in such a tree from base component $c$ is bounded by $|L_{\text{comp}}|$; this means that $S_{\text{depth}} \leq |L_{\text{comp}}|$. Moreover, as a component cannot require the same interface twice, $C_{ri} \leq |L_{\text{int}}|$. Given these two observations, this result then follows from the algorithm in the proof of Result A.12.

Note that for each of the parameter-sets in Results A.12–A.14, $\text{ESCreate}_{\text{poly}}$ is fp-intractable relative to each non-empty subset of these parameter-sets. Hence, these
fp-tractability results are all minimal, in the sense that no subsets of the parameters in their associated parameter-sets yield fp-tractability.

### 3.3 Results for Emergent Software Adaptation

Many of the results derived in this section for ESAdapt will actually be derived relative to the following problem:

**Component-Based Software Adaptation (CSAdapt)**

*Input:* Software system requirements $R$, interface and component libraries $L_{int}$ and $L_{comp}$, a working component-based software system $S$ based on component $c \in L_{comp}$ relative to $R$, $L_{int}$, and $L_{comp}$, reward function $Rew()$, and an integer $k$.

*Question:* Is there a working component-based software system $S'$ based on $c$ relative to $L_{int}$, $L_{comp}$, and $R$ such that $Rew(S') \leq k$?

Note that each input for ESAdapt has a corresponding input to CSAdapt (namely, the input to ESAdapt without $Env()$). Moreover, any algorithm $A$ that solves ESAdapt under some $Rew()$ can be also used to solve CSAdapt (namely, if $A$ run on the given input $x$ for CSAdapt produces a working system $S'$ such that $Rew(S') \leq k$, output “Yes”, otherwise output “No”). This yields the following useful observation.

**Observation 3** For any choice of $Rew()$ and $Env()$, if there is an algorithm $A$ of solvability type $T$ for ESAdapt under $Rew()$ than there is an algorithm $A'$ of solvability type $T$ for CSAdapt under $Rew()$.

It is very important to note that CSAdapt (unlike CSCreate in Section 3.2) explicitly invokes $Rew()$; hence, all of our results proved by invoking Observation 3 (i.e., all results proved in this section) are relative to specific $Rew()$, namely, either $Rew_{\#comp}()$ or $Rew_{CodeB}()$. This has some interesting consequences, which will be discussed further in Section 4.

#### 3.3.1 Unsolvability of Unrestricted Emergent Software Adaptation

We start off by considering if problem ESAdapt is solvable in the most general possible case — that is, if ESAdapt has an algorithm that always returns the correct output for an input incorporating any possible choice of $Env()$ and in which there are no restrictions on the form, size, or running times of $L_{int}$ and $L_{comp}$, their member interfaces and components, or any software systems created using $L_{int}$ and $L_{comp}$. Analogous to ESCreate in Section 3.2.1 this once again turns out not to be the case, though we only show unsolvability at this time for ESAdapt under either $Rew_{\#comp}()$ or $Rew_{CodeB}()$.  

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Figure 6: General structure of valid software systems created by the reduction in the proof of Result B.1. The “twin” component-based system with an extra component and extra code that is given as $S$ in the reduction is on the left, and is forbidden as output by appropriate values of $k$ under $\text{Rew}_{\#\text{comp}}()$ and $\text{Rew}_{\text{CodeB}}()$ in favor of the component-based system on the right. Following the convention in Figure 3, interfaces with multiple implementing components are enclosed in dashed boxes.

Let us first consider ESAdapt under $\text{Rew}_{\#\text{comp}}()$. In the following, we modify $L_{\text{int}}$ and $L_{\text{comp}}$ in the reduction in the proof of Result A.1 such that any working component-based software system has a new topmost component $\text{topBase}$ and there are thus now two possible working component-based software systems, one of which is the given “twin” component-based system $S$ with extra component ($\text{Base1}a$) that can be disallowed as a possible solution by appropriately setting the value of $k$ relative to reward function $\text{Rew}_{\#\text{comp}}()$ (see Figure 6).

**Result B.1** For any choice of $\text{Env}()$, ESAdapt under $\text{Rew}_{\#\text{comp}}()$ is unsolvable.

**Proof:** Consider the following polynomial-time Karp reduction from TM HALTING to CSAdapt: given an instance $I = \langle M, x \rangle$ of TM HALTING, construct an instance $I' = \langle R, L_{\text{int}}, L_{\text{comp}}, S, c, \text{Rew}() = \text{Rew}_{\#\text{comp}}(), k \rangle$ of CSAdapt in which $X = \{x_1\}$ and $O = \{1\}$, there is a single input-output pair $r$ in $R$ such that for $r = (\text{True}, 1)$, $L_{\text{int}}$ consists of the three interfaces

```java
interface topBase {
    void main(Input I)
}

interface base1 {
    void main1(Input I)
}
```

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interface base2 {
    void base1a(Input I)
}

and $L_{comp}$ consists of the four components

component TopBase provides topBase requires base1 {
    void main(Input I) {
        main1(I)
    }
}

component Base1 provides base1 requires base2 {
    void main1(Input I) {
        base1a(I)
    }
}

component Base1a provides base2 {
    void base1a(Input I) {
        output 1
    }
}

component Base2 provides base1 {
    void main1(Input I) {
        <CODEM(x)> 
        output 1
    }
}

where $<CODEM(x)>$ is the Dana code simulating the computation of $M$ on input $x$. As noted previously in the proof of Result A.1, Dana contains both loops and conditional statements and can readily simulate $M$ on input $x$ using code that is of size polynomial in the sizes of the given descriptions of $M$ and $x$. Finally, let $c$ be component TopBase in $L_{comp}$, $S$ be the software system based on components TopBase, Base1 and Base1a, and $k = 2$. Note that the instance of CSAdapt described above can be constructed in time polynomial in the size of the given instance of TM HALTING. To conclude the proof, observe that the only possible component-based system for the constructed instance of CSAdapt which has only the two components required by the value of $k$
is that consisting of TopBase and Base2, and that this system satisfies the sole input-output constraint in $R$ if and only if $M$ halts on input $x$ for the given instance of TM HALTING. It is known that TM HALTING cannot have an algorithm that is correct for all given $<M, x>$ instances ([29]; see also [34, Section 9.2.4]), and hence is unsolvable. The reduction above implies in turn that CSAdapt under $Rew_{\#comp}()$ cannot have an algorithm either. The unsolvability result for ESAdapt under $Rew_{\#comp}()$ then follows by contradiction from Observation 3.

We can in turn use a version of the reduction above to prove unsolvability of ESAdapt under $Rew_{CodeB}()$ by “padding” the code of component Base1a to ensure that the given component-based system $S$ cannot be a solution relative to an appropriate value of $k$.

**Result B.2** For any choice of $Env()$, ESAdapt under $Rew_{CodeB}()$ is unsolvable.

**Proof:** Consider a modification of the reduction in the proof of Result B.1 in which component Base1a is instead

```plaintext
component Base1a provides base2 {
  void base1a(Input I) {
    <BLOCK>
    output 1
  }
}
```

where $<BLOCK>$ consists of $|<CODE(M(x))>|$ copies of the statement $x = 1$ and $k = |<CODE(M(x))>| + 16$. To conclude the proof, observe that the only possible component-based system for the constructed instance of CSAdapt which has the code-base length required by the value of $k$ is that consisting of TopBase and Base2, and that this system satisfies the sole input-output constraint in $R$ if and only if $M$ halts on input $x$ for the given instance of TM HALTING. It is known that TM HALTING cannot have an algorithm that is correct for all given $<M, x>$ instances [34, Section 9.2.4], and hence is unsolvable. The reduction above implies in turn that CSAdapt under $Rew_{CodeB}()$ cannot have an algorithm either. The unsolvability result for ESAdapt under $Rew_{CodeB}()$ then follows by contradiction from Observation 3.

Unlike ESCreate in Section 3.3.1, the two results above only hold relative to some choices of $Env$ and $Rew()$ — namely, every possible choice of $Env()$ paired with either of $Rew_{\#comp}()$ or $Rew_{CodeB}()$ (this is because the proofs of these results only ignore $Env()$ and not both $Env()$ and $Rew()$). That being said, restricted versions of ESAdapt under $Rew_{\#comp}()$ and $Rew_{CodeB}()$ may yet have correct and even efficient algorithms. Thus, in the remainder of this paper, we shall assume that ESAdapt and
CSAdapt are (as were ESCreate and CSCreate in Section 3.2.1) restricted such that any component-based software system $S$ and $S'$ relative to given $L_{int}$ and $L_{comp}$ run in time polynomial in input size $|I|$ and hence can be checked against the system requirements in $R$ in time polynomial in the sizes of $S$ and $R$, i.e., created software systems not only operate but can also be verified quickly. We will denote these restricted versions as ESAdapt$_{poly}$ and CSAdapt$_{poly}$, respectively.

### 3.3.2 Polynomial-time Exact Solvability of Restricted Emergent Software Adaptation

We now consider if ESAdapt$_{poly}$ is efficiently solvable in the first of the senses listed at the start of Section 3.1 — namely, polynomial-time exact solvability and polynomial-time exact promise solvability. As was the case with ESCreate$_{poly}$ in Section 3.2.2, it turns out that ESAdapt$_{poly}$ is also polynomial-time exact intractable, with the difference that this intractability is proven relative to specific rather than any reward functions (namely, $Rew_{\#comp}()$ and $Rew_{CodeB}()$).

**Lemma 7** Dominating set polynomial-time Karp reduces to CSAdapt$_{poly}$ under $Rew_{\#comp}()$.

**Proof:** Given an instance $I = \langle G = (V, E), k \rangle$ of DOMINATING SET, construct the following instance $I' = \langle R, L_{int}, L_{comp}, S, c, Rew() = Rew_{\#comp}(), k' \rangle$ of CSAdapt$_{poly}$: Without loss of generality, assume $k < |V|$. Let $R$, $L_{int}$, $L_{comp}$, and $c$ be as in the proof of Lemma 5 and $S$ be the software system based on TopBase, Base($|V|$), and the components InSet$_{JJ}$ for $1 \leq J \leq |V|$. Finally, let $k' = k + 2$. Note that this instance CSAdapt$_{poly}$ can be constructed in time polynomial in the size of the given instance of DOMINATING SET; moreover, as there are only a $(k + 1) \leq (|V| + 1) < (|I| + 1)$-clause if-then statement block and two single-level loops in the component code that each execute at most $|V| < |I|$ times, any component-based software system created relative to $L_{int}$, $L_{comp}$, and $Ec$ runs in time linear in the size of input $I'$.

Let us now verify the correctness of this reduction:

- Suppose that there is a dominating set $D$ of size $k$ in the given instance of DOMINATING SET. We can then construct a component-based software system $S'$ consisting of $c$ and component Base$k$, in which the $k$ cond-interfaces in Base$k$ are implemented by InSet components corresponding to the vertices in $D$. Observe that for each $(i_j, o_j) \in R$, $S'$ produces output $o_j$ given input $i_j$; moreover, $Rew_{\#comp}(S') = k + 2 \leq k'$.
- Conversely, suppose that the constructed instance of CSAdapt$_{poly}$ has a working component-based software system $S'$ based on $c$ relative to $L_{int}$, $L_{comp}$,
and $R$ such that $Rew_{\#comp}(S') \leq k' = k + 2$. Such a system $S'$ cannot include component $Base(|V|)$ as in $S$; because in that case, $Rew_{\#comp}(S')$ would have the value $|V| + 2 \not\leq k'$. Therefore, $S'$ must include one of the component $BaseJ$ for $1 \leq J \leq k$; let us call this component $BaseM$. As in the proof of Lemma 5 in order to correctly accommodate all other input-output pairs in $R$, the $M$ if-then statements in $BaseM$ must implement $InSet$-components whose corresponding vertices form a dominating set in $G$ of size at most $k$ in $G$. Hence, the existence of a working component-based software system for the constructed instance of $ CSCreate_{poly}$ implies the existence of a dominating set of size at most $k$ for the given instance of $ Dominating set.$

This completes the proof.

**Lemma 8** Dominating set polynomial-time Karp reduces to $CSAdapt_{poly}$ under $Rew_{CodeB}()$.

**Proof:** In the proof of Lemma 7, observe that for a dominating set of size $k$ in the given instance of $ Dominating set^{OPT}$, a software system $S'$ for the constructed instance of $CSAdapt_{poly}$ consists of components $TopBase$ and $Basek$, $k$ $InSet$ components, interfaces $topBase$ and $base$, and $k$ $cond$ interfaces. The total number of lines of code in this system and hence the value of $Rew_{CodeB}(S')$ is therefore $5 + (k + 5) + 5k + 3 + 3 + 3k = 9k + 16$. The result then follows by a slight modification to the proof of correctness of the reduction described in Lemma 7.

**Result B.3** For any choice of $Env()$, if $ESAdapt_{poly}$ under $Rew_{\#comp}()$ is polynomial-time exact solvable then $P = NP$.

**Proof:** Given the $NP$-hardness of $ Dominating set$, the reduction in Lemma 7 implies that $CSAdapt_{poly}$ under $Rew_{\#comp}()$ is $NP$-hard, and hence not solvable in polynomial time unless $P = NP$. The polynomial-time intractability result for $ESAdapt_{poly}$ under $Rew_{\#comp}()$ then follows by contradiction from Observation 3.

**Result B.4** For any choice of $Env()$, if $ESAdapt_{poly}$ under $Rew_{CodeB}()$ is polynomial-time exact solvable then $P = NP$.

**Proof:** Given the $NP$-hardness of $ Dominating set$, the reduction in Lemma 8 implies that $CSAdapt_{poly}$ under $Rew_{CodeB}()$ is $NP$-hard, and hence not solvable in polynomial time unless $P = NP$. The polynomial-time intractability result for $ESAdapt_{poly}$ under $Rew_{CodeB}()$ then follows by contradiction from Observation 3.

Note that, unlike for $ESCcreate_{poly}$, evaluating the polynomial-time exact promise solvability of $ESAdapt_{poly}$ is not possible. This is because (as noted previously in
Section 2) any version of ESAdapt\textsubscript{poly} promising a working software system for the given input will always have at least one such system — namely, \(S\).

### 3.3.3 Polynomial-time Approximate Solvability of Restricted Emergent Software Adaptation

We now consider if ESAdapt\textsubscript{poly} is efficiently approximately solvable in either of the three senses (frequently correct (deterministic), frequently correct (probabilistic), or approximately optimal) listed at the start of Section 3.1. Once again, as with ESAdapt\textsubscript{poly}, this turns out not to be the case. The proofs of Results B.5–B.8 below are analogous to the proofs for Results A.2 and A.3 in Section 3.2, only this time relative to the proofs of Results B.3 and B.4 and Observation 3.

**Result B.5** For any choice of \(Env()\), if ESAdapt\textsubscript{poly} under \(Rew_{\#\text{comp}}()\) is solvable by a polynomial-time algorithm with a polynomial error frequency (i.e., \(err(n)\) is upper bounded by a polynomial of \(n\)) then \(P = NP\).

**Result B.6** For any choice of \(Env()\), if ESAdapt\textsubscript{poly} under \(Rew_{\text{CodeB}}()\) is solvable by a polynomial-time algorithm with a polynomial error frequency (i.e., \(err(n)\) is upper bounded by a polynomial of \(n\)) then \(P = NP\).

**Result B.7** For any choice of \(Env()\), if \(P = BPP\) and ESAdapt\textsubscript{poly} under \(Rew_{\#\text{comp}}()\) is polynomial-time solvable by a probabilistic algorithm which operates correctly with probability \(\geq 2/3\) then \(P = NP\).

**Result B.8** For any choice of \(Env()\), if \(P = BPP\) and ESAdapt\textsubscript{poly} under \(Rew_{\text{CodeB}}()\) is polynomial-time solvable by a probabilistic algorithm which operates correctly with probability \(\geq 2/3\) then \(P = NP\).

To assess cost-approximability, we need the following problem.

**Optimal Component-based Software Adaptation** (CSAdapt\textsubscript{OPT})

**Input:** Software system requirements \(R\), interface and component libraries \(L_{\text{int}}\) and \(L_{\text{comp}}\), a working component-based software system \(S\) based on component \(c \in L_{\text{comp}}\) relative to \(R\), \(L_{\text{int}}\), and \(L_{\text{comp}}\), and a reward function \(Rew()\).

**Output:** A working component-based software system \(S'\) based on \(c\) relative to \(L_{\text{int}}\), \(L_{\text{comp}}\), and \(R\) that has the smallest possible value of \(Rew(S)\) over all working systems based on \(c\) relative to \(L_{\text{int}}\), \(L_{\text{comp}}\), and \(R\).

Let CSAdapt\textsubscript{OPT} be the version of CSAdapt\textsubscript{OPT} such that given and candidate component-based systems \(S\) and \(S'\) run in time polynomial in the input size \(|I|\). Note that each input for ESAdapt\textsubscript{poly} has a corresponding input to CSAdapt\textsubscript{OPT}.
(namely, the input to ESAdapt\textsubscript{poly} without \textit{Env}()). Moreover, any algorithm \textit{A} that solves ESAdapt\textsubscript{poly} under some \textit{Rew}() can also be used to solve CSAdapt\textsubscript{OPT} (namely, return whatever \textit{A} run on the given input \textit{x} for CSAdapt\textsubscript{OPT} produces). This yields the following useful observation.

**Observation 4** For any choice of \textit{Rew}() and \textit{Env}(), if there is an algorithm \textit{A} of solvability type \textit{T} for ESAdapt\textsubscript{poly} under \textit{Rew}() than there is an algorithm \textit{A}' of solvability type \textit{T} for CSAdapt\textsubscript{OPT} under \textit{Rew}().

Observe that the reductions in the proof of Lemmas 7 and 8 based on that in the proof of Lemma 5 only add a working software component-based software system \textit{S} and do not change any other details of the constructed \textit{R}, \textit{L}_{\text{int}}, \textit{L}_{\text{comp}}, and \textit{c}. Hence, the following hold by slight modifications to the proofs of Lemmas 5 and 6 and Results A.6 and A.7

**Lemma 9** Dominating set\textsuperscript{OPT} polynomial-time Levin reduces to CSAdapt\textsubscript{OPT} under \textit{Rew}_{\#\text{comp}}() such that there is a dominating set of size \textit{k} for the given instance of Dominating set\textsuperscript{OPT} if and only if there is a working component-based software system \textit{S} with reward value \textit{Rew}_{\#\text{comp}}(\textit{S}) = \textit{k} + 2 for the constructed instance of CSAdapt\textsubscript{OPT}.

**Lemma 10** Dominating set\textsuperscript{OPT} polynomial-time Levin reduces to CSAdapt\textsubscript{OPT} under \textit{Rew}_{\text{CodeB}}() such that there is a dominating set of size \textit{k} for the given instance of Dominating set\textsuperscript{OPT} if and only if there is a working component-based software system \textit{S} with reward value \textit{Rew}_{\text{CodeB}}(\textit{S}) = 9\textit{k} + 16 for the constructed instance of CSAdapt\textsubscript{OPT}.

**Result B.9** For any choice of \textit{Env}(), if ESAdapt\textsubscript{OPT} under \textit{Rew}_{\#\text{comp}}() is polynomial-time \textit{c}-approximable for any constant \textit{c} > 0 then \textit{P} = \textit{NP}.

**Result B.10** For any choice of \textit{Env}(), if ESAdapt\textsubscript{OPT} under \textit{Rew}_{\text{CodeB}}() is polynomial-time \textit{c}-approximable for any constant \textit{c} > 0 then \textit{P} = \textit{NP}.

### 3.3.4 Fixed-parameter tractability of Restricted Emergent Software Adaptation

Given the plethora of intractability results in the previous three subsections, we now consider to what extent and relative to which parameters ESAdapt\textsubscript{poly} is and is not fp-tractable. Our results in this section are derived relative to the parameters listed
in Table 1 for ESCreate_{poly} in Section 3.2.4 and use versions of the proofs of Results A.8, A.9, A.10, and A.11 as modified by the “twinning” and “padding” tricks used in the proofs of Results B.1 and B.2. The former trick works relative to the results listed above derived by reductions from DOMINATING SET, as any instance of DOMINATING SET always has a trivial dominating set consisting of the set V of all vertices in the given graph G which can be used to structure the “twin” given component-based software system S. We show how this is done relative to the proof of Result A.8 for ESAdapt_{poly} under Rew_{#comp}() and Rew_{CodeB}() and leave the details of the other proofs to the reader.

Result B.11 For any choice of Env(), if ⟨|L\_int|, C_{pi}, C_{ri}, S_{comp}, S_{depth}⟩-ESAdapt_{poly} under Rew_{#comp}() is fp-tractable then FPT = W[2].

Proof: Consider the following polynomial-time Karp reduction from DOMINATING SET to CSAdapt_{poly} under Rew_{#comp}(): given an instance I = ⟨G = (V, E), k⟩ of DOMINATING SET, construct an instance I' = ⟨R, L\_int, L_{comp}, S, c, Rew() = Rew_{#comp}(), k'⟩ of CSAdapt_{poly} in which X = \{x_1, x_2, \ldots, x_{|V|}\}, i.e., there is a unique Boolean variable corresponding to each vertex in V, and O = \{0, 1\}. There are |V| input-output pairs in R such that for r_i = (i_j, o_j), 1 \leq j \leq |V|, v_i(x_k) = True if v_k \in N_C(v_j) and is False otherwise and o_j = 1. Let L\_int consist of k + 3 interfaces broken into four groups:

1. A single interface of the form

```
interface topBase {
    void main(Input I)
}
```

2. A single interface of the form

```
interface base1 {
    void main1(Input I)
}
```

3. A single interface of the form

```
interface base2 {
    void base1a(Input I)
}
```
4. A set of $k$ interfaces of the form

```java
interface condJ {
    Boolean inSetJ(Input I)
}
```

for $1 \leq J \leq k$.

Let $L_{comp}$ consist of $k|V| + 4$ components broken into five groups:

1. A single component of the form

```java
component TopBase provides topBase requires base1 {
    void main(Input I) {
        main1(I)
    }
}
```

2. A single component of the form

```java
component Base1 provides base1 requires base2 {
    void main1(Input I) {
        base1a(I)
    }
}
```

3. A single component of the form

```java
component Base1a provides base2 {
    void base1a(Input I)
    requires cond1, cond2, ..., condk {
        if inSet1(I) then output 1
        elsif inSet2(I) then output 1
        ...
        elsif inSetk(I) then output 1
        else output 1
    }
}
```
4. A single component of the form

```plaintext
component Base2 provides base1
    requires cond1, cond2, ..., condk {
    void main1(Input I) {
        if inSet1(I) then output 1
        elsif inSet2(I) then output 1
        ...
        elsif inSetk(I) then output 1
        else output 0
    }
}
```

5. A set of \(k|V|\) components of the form

```plaintext
component InSetJK provides condJ {
    Boolean inSetJ(Input I) {
        return v_I(x_K)
    }
}
```

for \(1 \leq J \leq k\) and \(1 \leq K \leq |V|\).

Note that in \(L_{\text{comp}}\), there are \(|V|\) implementations of each cond-interface. Finally, let \(c\) be component TopBase in \(L_{\text{comp}}\), \(S\) be the component-based software system composed of TopBase, Base1, Base1A, and InSetJJ for \(1 \leq J \leq k\), and \(k' = k + 2\).

Note that the instance of CSAdapt\text{poly} described above can be constructed in time polynomial in the size of the given instance of DOMINATING SET; moreover, as there is only a \((k+1)\)-clause if-then statement block and no loops in the component code and \(k \leq |V| < |I|\), any candidate component-based software system created relative to \(L_{\text{int}}, L_{\text{comp}}\), and \(c\) runs in time linear in the size of input \(I'\).

Let us now verify the correctness of this reduction:

- Suppose that there is a dominating set \(D\) of size at most \(k\) in the given instance of DOMINATING SET. We can then construct a component-based software system \(S'\) consisting of \(c, Base2\), and the \(|D|\) InSet-components corresponding to the vertices in \(D\); the choice of which interface to implement for each vertex is immaterial, and if there are less than \(k\) vertices in \(D\), the final \(k - |D|\) required cond-interfaces can be implemented relative to InSet-components corresponding to arbitrary vertices in \(D\). Observe that for each \((i_j, o_j) \in R\), this software system produces output \(o_j\) given input \(i_j\) and \(Rew_{\#comp}(S') = k + 2\).
Conversely, suppose that the constructed instance of CSAdapt\textsubscript{poly} has a working component-based software system $S'$ based on $c$ relative to $L_{\text{int}}, L_{\text{comp}}$, and $R$ such that $\text{Rew}_{\#\text{comp}}(S') \leq k' = k + 2$. This system cannot incorporate components \texttt{Base1} and \texttt{Base1a}, as this would result in $\text{Rew}_{\#\text{comp}}(S') = k + 4 \not\leq k' = k + 2$. Hence, $S'$ must include component \texttt{Base2}. In order to correctly accommodate all input-output pairs in $R$, the $\leq k$ if-then statements in \texttt{Base2} must implement InSet-components whose corresponding vertices form a dominating set in $G$ of size at most $k$. Hence, the existence of a working component-based software system $S'$ for the constructed instance of CSAdapt\textsubscript{poly} under $\text{Rew}_{\#\text{comp}}()$ such that $\text{Rew}_{\#\text{comp}}(S') \leq k'$ implies the existence of a dominating set of size at most $k$ for the given instance of DOMINATING SET.

This completes the proof of correctness of the reduction. Given the $W[2]$-hardness of $\langle k \rangle$-DOMINATING SET, this reduction implies that CSAdapt\textsubscript{poly} under $\text{Rew}_{\#\text{comp}}()$ is $W[2]$-hard when $C_{\pi} = 1$, $S_{\text{depth}} = 4$, $C_{ri} = k$, and $|L_{\text{int}}| = S_{\text{comp}} = k + 3$ and hence not fp-tractable relative to these parameters unless $FPT = W[2]$. The fp-intractability result for ESAdapt\textsubscript{poly} under $\text{Rew}_{\#\text{comp}}()$ then follows by contradiction from Observation 3.

**Result B.12** For any choice of $\text{Env}()$, if $\langle \mid L_{\text{int}} \mid, C_{\pi}, C_{ri}, S_{\text{comp}}, S_{\text{depth}} \rangle$-ESAdapt\textsubscript{poly} under $\text{Rew}_{\text{CodeB}}()$ is fp-tractable then $FPT = W[2]$.

**Proof:** Observe that in the reduction in the proof of Result B.11, if $k' = 6k + 15$, the only possible working component-based software system is that including \texttt{TopBase} and \texttt{Base2}. The result then holds by a modified version of the proof of Result B.11.

**Result B.13** For any choice of $\text{Env}()$, if $\langle I_{ci}, C_{\pi}, S_{\text{depth}} \rangle$-ESAdapt\textsubscript{poly} under $\text{Rew}_{\#\text{comp}}()$ is fp-tractable then $P = NP$.

**Result B.14** For any choice of $\text{Env}()$, if $\langle I_{ci}, C_{\pi}, S_{\text{depth}} \rangle$-ESAdapt\textsubscript{poly} under $\text{Rew}_{\text{CodeB}}()$ is fp-tractable then $P = NP$.

**Result B.15** For any choice of $\text{Env}()$, if $\langle I_{ci}, C_{\pi}, C_{ri} \rangle$-ESAdapt\textsubscript{poly} under $\text{Rew}_{\#\text{comp}}()$ is fp-tractable then $P = NP$.

**Result B.16** For any choice of $\text{Env}()$, if $\langle I_{ci}, C_{\pi}, C_{ri} \rangle$-ESAdapt\textsubscript{poly} under $\text{Rew}_{\text{CodeB}}()$ is fp-tractable then $P = NP$.

**Result B.17** For any choice of $\text{Env}()$, if $\langle \mid L_{\text{comp}} \mid, I_{ci}, S_{\text{depth}} \rangle$-ESAdapt\textsubscript{poly} under $\text{Rew}_{\#\text{comp}}()$ is fp-tractable then $P = NP$. 

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Result B.18 For any choice of Env(), if \(|L_{\text{comp}}, I_{ci}, S_{\text{depth}}|\)-ESAdapt\(_{\text{poly}} \) under \(\text{Rew}_{\text{CodeB}}()\) is fp-tractable then \(P = NP\).

The next three results follow from the algorithms in Results A.12–A.14, respectively, in which all candidate working systems are generated and hence can be evaluated under any Rew() of interest that is computable in time polynomial in the size of given system \(S\).

Result B.19 For any choice of Rew() and Env(), \(|I_{ci}, C_{ri}, S_{\text{depth}}|\)-ESAdapt\(_{\text{poly}}\) is fp-tractable.

Result B.20 For any choice of Rew() and Env(), \(|I_{ci}, S_{\text{comp}}|\)-ESAdapt\(_{\text{poly}}\) is fp-tractable.

Result B.21 For any choice of Rew() and Env(), \(|L_{\text{int}}, |L_{\text{comp}}|\)-ESAdapt\(_{\text{poly}}\) is fp-tractable.

Given the fp-intractability results we have at this time, none of these fp-tractability results are known to be minimal in the sense described at the end of Section 3.2.4.

4 Discussion

In this section, we will first summarize our results for the problems ESCreate and ESAdapt (Section 4.1). We then discuss the implications of these results for real-world implementations of emergent software systems (Section 4.2). Several points raised in this discussion motivate the inclusion of new results; so as not to disturb the flow of the discussion, proofs when required are given in the appendix. Finally, we conclude with some general thoughts and caveats on how best to interpret and use the results of computational complexity analyses such as ours within software engineering (Section 4.3).

4.1 Summary of Results

Our results establish that neither of our investigated problems ESCreate or ESAdapt is solvable exactly by any algorithm when no restrictions are placed on the structure or operation of the derived software systems (Results A.1, B.1, and B.2). This intractability still holds for both problems relative to polynomial-time exact and exact promise solvability (Results A.2, A.3, B.3, and B.4) and all three of the considered forms of polynomial-time approximability (Results A.4–A.7 and B.5–B.10) even when software systems are restricted to run in time polynomial in the sizes of their inputs.
Table 2: A Detailed Summary of Our Parameterized Complexity Results for ESCreate\textsubscript{poly} and ESAdapt\textsubscript{poly} under Rew\textsubscript{#comp}() and Rew\textsubscript{CodeB}(). a) Summary for ESCreate\textsubscript{poly}. Each column in this table is a result which holds relative to the parameter-set consisting of all parameters with a @-symbol in that column. If the result holds when a particular parameter has a constant value $c$, that is indicated by $c$ replacing @ for that parameter in that result’s column. Fp-intractability results are given first and fp-tractability results (shown in bold) are given last.

|   | A.8 | A.9 | A.10 | A.11 | A.12 | A.13 | A.14 |
|---|-----|-----|------|------|------|------|------|
| $L_{int}$ | @   | –   | –    | –    | –    | –    | @    |
| $L_{comp}$ | –   | –   | 3    | –    | –    | –    | @    |
| I\textsubscript{ci} | –   | 2   | 2    | 2    | @    | @    | –    |
| C\textsubscript{pi} | 1   | 1   | 1    | –    | –    | –    | –    |
| C\textsubscript{ri} | @   | –   | 2    | –    | @    | –    | –    |
| S\textsubscript{comp} | @   | –   | –    | –    | –    | @    | –    |
| S\textsubscript{depth} | 2   | 3   | 2    | –    | @    | –    | –    |

Moreover, both problems when so restricted remain fixed-parameter intractable relative to all parameters listed in Table 1 (Results A.8–A.11 and B.11–B.18), both individually and in many combinations and even when many parameters have values that are small constants. That being said, there are several combinations of parameters that yield fp-tractability for our problems (see Tables 2–4).

Our intractability results have a surprisingly broad applicability. This is because all results (excluding polynomial-time exact promise unsolvability and cost-inapproximability) are derived relative to two underlying problems — namely, CSCreate (which asks for any working software system $S$ relative to the given interface and component libraries $L_{int}$ and $L_{comp}$ and software system requirements $R$) and CSAdapt (which asks for a working software system $S'$ relative to given $L_{int}$, $L_{comp}$, $R$, and $S$ such that $Rew(S') \leq k$ for some given $k$). The use of CSCreate and CSAdapt has two consequences. First, as CSCreate does not invoke $Env()$ or $Rew()$, intractability results derived relative to CSCreate hold for ESCreate relative to each possible choice of $Env()$ and $Rew()$ and hence ESCreate in general. By analogous reasoning, as CSAdapt does not invoke $Env()$ but does explicitly invoke $Rew()$ relative to given bound $k$, intractability results derived relative to CSAdapt hold for ESAdapt relative to only some possible choices of $Env()$ and $Rew()$ — namely, any possible $Env()$ and one of either $Rew\textsubscript{#comp}()$ or $Rew\textsubscript{CodeB}()$. However, as these cases must be solved by any algorithm that solves ESAdapt with arbitrary input $Env()$ and $Rew()$, these intractability results also hold for ESAdapt in general. Second, as
Table 3: A Detailed Summary of Our Parameterized Complexity Results for ESCreate\textsubscript{poly} and ESAdapt\textsubscript{poly} under Rew\textsubscript{\#comp}() and Rew\textsubscript{CodeB}() (Cont’d). b) Summary for ESAdapt\textsubscript{poly} under Rew\textsubscript{\#comp}().

|    | B.11 | B.13 | B.15 | B.17 | B.19 | B.20 | B.2 |
|----|------|------|------|------|------|------|----|
| | L\textsubscript{int} | @     | –     | –     | –     | –     | @   |
| | L\textsubscript{comp} | –     | –     | 5     | –     | –     | @   |
| | I\textsubscript{ci} | –     | 2     | 2     | @     | @     | –   |
| | C\textsubscript{pi} | 1     | 1     | 1     | –     | –     | –   |
| | C\textsubscript{ri} | @     | –     | 2     | –     | @     | –   |
| | S\textsubscript{comp} | @     | –     | –     | –     | –     | @   |
| | S\textsubscript{depth} | 3     | 4     | –     | 3     | @     | –   |

Table 4: A Detailed Summary of Our Parameterized Complexity Results for ESCreate\textsubscript{poly} and ESAdapt\textsubscript{poly} under Rew\textsubscript{\#comp}() and Rew\textsubscript{CodeB}() (Cont’d). c) Summary for ESAdapt\textsubscript{poly} under Rew\textsubscript{CodeB}().

|    | B.12 | B.14 | B.16 | B.18 | B.19 | B.20 | B.2 |
|----|------|------|------|------|------|------|----|
| | L\textsubscript{int} | @     | –     | –     | –     | –     | –     | @ |
| | L\textsubscript{comp} | –     | –     | 5     | –     | –     | @   |
| | I\textsubscript{ci} | –     | 2     | 2     | @     | @     | –   |
| | C\textsubscript{pi} | 1     | 1     | 1     | –     | –     | –   |
| | C\textsubscript{ri} | @     | –     | 2     | –     | @     | –   |
| | S\textsubscript{comp} | @     | –     | –     | –     | –     | @   |
| | S\textsubscript{depth} | 3     | 4     | –     | 3     | @     | –   |

neither CSCreate nor CSAdapt optimizes derived software systems relative to Rew(). the observed forms of intractability for ESCreate and ESAdapt cannot be attributed to optimization of derived software systems relative to Rew().

4.2 Implications for Real-world Emergent Software Systems

The foregoing is all well and good for our problems ESAdapt and ESCreate. However, how applicable are our results to real-world emergent software system creation and adaptation? Given the use of CSCreate (which ignores Rew()) to derive results for ESCreate, all of our results, both for tractability and intractability, apply directly to real-world emergent system creation. The situation is the same for ESAdapt relative to real-world emergent system adaptation if one is only given the information in the
stated input to ESAdapt. However, this is not the case in the real-world emergent software system described in [13, 12], where the adaptation process is also given the complete list of working software systems created relative to $L_{int}$, $L_{comp}$, and $R$ (see Section 2). If this list is small enough (i.e., of size polynomial in the sizes of $L_{int}$, $L_{comp}$, and $R$), then a linear scan of this list can determine a candidate with improved (indeed, optimal) performance relative to $Rew()$ and adaptation can be done in time polynomial the sizes of $L_{int}$, $L_{comp}$, $R$, and $S$. The issue of whether an intractable problem can have some fixed part of its input (e.g., $L_{int}$, $L_{comp}$, $R$, and $c$) preprocessed (probably in non-polynomial time) to create information of polynomial size that can be used to solve subsequent instances of the problem with a varying part (e.g., $S$, $Rew()$, and $k$) in polynomial time is addressed by the complexity framework given in [37]. At present, relative to an admittedly artificial and problem-specific reward function, we have the following result.

**Result B.22 :** If ESAdapt$_{poly}$ can have $L_{int}$, $L_{comp}$, $R$, and $c$ preprocessed to create polynomial-size information that can be used to solve ESAdapt$_{poly}$ instances of arbitrary $S$, $Rew()$, and $k$ in polynomial time then the Polynomial Hierarchy $PH$ collapse, i.e., $PH = \Sigma^p_2$.

There is good reason to believe that this result holds, as it is widely believed within computer science that the Polynomial Hierarchy does not collapse, let alone to a level as low in the Hierarchy as $\Sigma^p_2$. Note that this result applies to any polynomial-length information obtained by preprocessing, and not just as in [12, 13] a list of working software systems relative to $L_{int}$, $L_{comp}$, $c$, and $R$. Though Result B.22 does not invalidate the adaptation strategy employed in [12, 13], it does suggest that it is not universally applicable to all instances of emergent system adaptation, and that further complexity-theoretic work is necessary to determine those situations in which it does and does not work.

Let us now consider some additional implications of our results:

- All of our polynomial-time and fixed-parameter results for runtime-restricted ESCreate and ESAdapt hold when valid software systems are restricted to run (and hence can be verified against system requirements) in time polynomial (and indeed, as noted in our proofs, linear) in the size of the given input. This suggests that the basic computational difficulty of these problems is tied not so much to V&V as the acts of creating working software systems from interface and component libraries relative to a set of functional system requirements (in the case of ESCreate) and attempting to merely improve (and not necessarily optimize) the performance of an existing working system (in the case of ESAdapt).
• Our fixed-parameter intractability and tractability results for both of our problems relative to the parameter-set \( \{I_{ci}, C_{pi}, C_{ri}, S_{depth}, S_{comp}\} \) are complete, in the sense that the fp-status of our problems relative to each of the subsets of this parameter set is known (see Tables 5–7). Moreover, our fp-tractability results are not only all based on different runtime analyses of the same brute-force candidate software system algorithm but are all minimal, in the sense that no subset of the parameters invoked in any of these fp-tractability results yields fp-tractability. This suggests that, short of additional restrictions, the proof-of-concept brute-force enumeration algorithm proposed in [12, 13] for creating valid software systems may in fact be the best possible.

• As all of our intractability results hold when component interfaces support specialized rather than universal composability, components are small, and components comply with the single-responsibility design pattern, claims that these attributes help tame the state space explosion when creating valid software systems from interface and component libraries ([12, pages 4 and 5] and [15, page 2], respectively) should be seen as incomplete, in the sense that additional restrictions may be required to account for observed efficient operation. The same holds relative to claims of the proposed linear bandit learning algorithm working with high probability [13, page 340] given our polynomial-time probabilistic inapproximability results for ESAdapt.

Though the situation may change in future as additional results are derived, the above does suggest that emergent software system adaptation may indeed be computationally easier than emergent software system creation, and that efficient adaptation (after an initial computationally costly but avoidable system creation) may be possible in emergent software systems.

The above (in particular, the second and third bullet-points) very much begs the question of under what additional restrictions such efficient adaptation or creation might be possible. Answering this question will aid both designing the best possible emergent software systems and explaining when and why existing systems do perform well (e.g., the claims listed in point (3) above). Known sets of aspect restrictions that yield fp-tractability like those reported in this paper are a start, but given the observed prevalence of fp-intractability, more aspects need to be analyzed. A good source for these would be restrictions that “break” our intractability reductions, e.g., restricting the number of instances of required interfaces in a component that can be implemented by multiple components. One should also not underestimate the value of additional broad restrictions like our requirement that derived software systems run in polynomial time. That such restrictions are critical is demonstrated by the three results below, which follow from the reductions in the proofs of Results A.1,
B.1, and B.2 (in which the values of $|L_{\text{int}}|$, $|L_{\text{comp}}|$, $I_{\text{ci}}$, $C_{\text{pi}}$, $C_{\text{ri}}$, $S_{\text{comp}}$, and $S_{\text{depth}}$ are all very small constants) and the definition of fp-tractability.

**Result A.15** For any choice of $\text{Rew}()$ and $\text{Env}()$, $\langle|L_{\text{int}}|, |L_{\text{comp}}|, I_{\text{ci}}, C_{\text{pi}}, C_{\text{ri}}, S_{\text{comp}}, S_{\text{depth}}\rangle$-ESCreate is unsolvable.

**Result B.23** For any choice of $\text{Env}()$, $\langle|L_{\text{int}}|, |L_{\text{comp}}|, I_{\text{ci}}, C_{\text{pi}}, C_{\text{ri}}, S_{\text{comp}}, S_{\text{depth}}\rangle$-ESAdapt under $\text{Rew}_{\text{#comp}}()$ is unsolvable.

**Result B.24** For any choice of $\text{Env}()$, $\langle|L_{\text{int}}|, |L_{\text{comp}}|, I_{\text{ci}}, C_{\text{pi}}, C_{\text{ri}}, S_{\text{comp}}, S_{\text{depth}}\rangle$-ESAdapt under $\text{Rew}_{\text{CodeB}}()$ is unsolvable.

Another demonstration is the above-noted solvability of ESAdapt in polynomial time if polynomial-size preprocessed information is available. Additional possible restrictions of this type that might be useful could be on the forms of environment or reward functions or the internal code structure and/or maximum code-length of components. It might also be of use to consider alternative problem formulations. For example, requiring that ESAdapt only derive a new system whose $\text{Rew}()-$performance is a constant-valued improvement over that of the given system $S$ (cf., the absolute rather than $S$-relative performance bound currently implemented by $k$ in CSAdapt) will not lower the computational complexity of ESAdapt relative to $\text{Rew}_{\text{#comp}}()$ (as the twinning trick used to derive these intractability results means that working systems differ in at most one component and hence in $\text{Rew}_{\text{#comp}}()-$value by at most one) but it may well work relative to other reward functions.

### 4.3 Computational Complexity Analysis and Software Engineering

Quite aside from the utility of intractability results for ESCreate and ESAdapt in answering questions about the operation of existing emergent software systems and the design of new ones, the mere existence of such results relative to standard types of intractability such as Turing unsolvability and $NP$- and $W$-hardness has additional far-reaching consequences. This is because of structural complexity theory, the subdiscipline of theoretical computer science that investigates the (non)inclusion relationships between various complexity classes [38, 40, 41, 42]. As complexity classes are typically defined relative to particular types of algorithms (e.g., $P$, $EXP$, and $PSPACE$ are the classes of problems solvable in polynomial time, exponential time, and polynomial space, respectively), standard intractability results typically imply many other intractability results. For example, given the imminent availability of functional quantum computers and the accompanying hoopla [43], one might think that polynomial-time intractability (in particular, $NP$-hardness) results like
Table 5: Systematic Parameterized Complexity Analysis of ESCreate\textsubscript{poly} and ESAdapt\textsubscript{poly} Relative to the Parameter-set \{I\textsubscript{ci}, C\textsubscript{pi}, C\textsubscript{ri}, S\textsubscript{depth}, S\textsubscript{comp}\}. a) Analysis for ESCreate\textsubscript{poly}. In this table, a $\sqrt{X}$ symbol indicates that ES-create relative to the parameter-set composed of the union of the row and column parameter-sets indexing that entry is fp-(in)tractable. The original results are subscripted with the result-number followed by a star; all other results are subscripted by number of the result they are derived from relative to Lemmas 1 and 2.

|     | $S_{\text{comp}}$ | $S_{\text{depth}}$ | $S_{\text{comp}} \cdot S_{\text{depth}}$ |
|-----|-------------------|---------------------|------------------------------------------|
|     | I\textsubscript{ci} | X\textsubscript{A.9} | $\sqrt{A.13}$ | $\sqrt{A.13}$ |
|     | C\textsubscript{pi} | X\textsubscript{A.8} | X\textsubscript{A.9} | X\textsubscript{A.8} |
|     | C\textsubscript{ri} | X\textsubscript{A.8} | X\textsubscript{A.8} | X\textsubscript{A.8} |
|     | I\textsubscript{ci}, C\textsubscript{pi} | X\textsubscript{A.9} | $\sqrt{A.13}$ | $\sqrt{A.9}$ | $\sqrt{A.13}$ |
|     | I\textsubscript{ci}, C\textsubscript{ri} | X\textsubscript{A.10} | $\sqrt{A.13}$ | $\sqrt{A.12}$ | $\sqrt{A.12}$ |
|     | C\textsubscript{pi}, C\textsubscript{ri} | X\textsubscript{A.8} | X\textsubscript{A.8} | X\textsubscript{A.8} | X\textsubscript{A.8} |
|     | I\textsubscript{ci}, C\textsubscript{pi}, C\textsubscript{ri} | X\textsubscript{A.10} | $\sqrt{A.13}$ | $\sqrt{A.12}$ | $\sqrt{A.12}$ |

those given here are moot. However, it is widely believed in computer science that $NP \not\subseteq BQP$, where $BQP$ (Bounded-error Quantum Polynomial Time) is considered the largest class of problems solvable by usable quantum computer algorithms \[44\]. If any NP-hard problem is in $BQP$ then this conjecture is false \[44\], rendering such quantum solvability of NP-hard problems like ours very unlikely.

The above, in combination with the points raised and implications described earlier in Section 4 highlights why computational complexity analyses are useful in software engineering and moreover the context in which they can be used most productively — namely, as part of an ongoing dialogue between software engineers and theoretical computer scientists, in which questions raised by members of one group, e.g.,

- Why do our systems (not) work as well as they do?
- Does this restriction on our systems matter, and if so, how?
- How do we formulate relevant problems for analysis?
- What types of algorithms do (not) exist for our problems?

inspire both investigations by and new questions from the other.

Before we close out this subsection, two final caveats are in order. First, it is important to note that the brute-force search algorithms underlying all of our fp-tractability
Table 6: Systematic Parameterized Complexity Analysis of ESCreate\textsubscript{poly} and ESAdapt\textsubscript{poly} Relative to the Parameter-set \{\(I_{ci}, C_{pi}, C_{ri}, S_{depth}, S_{comp}\)\} (Cont’d). b) Analysis for ESAdapt\textsubscript{poly} under \(\textsf{Rew}_{\#\text{comp}}()\).

|                | \(S_{\text{comp}}\) | \(S_{\text{depth}}\) | \(S_{\text{comp}, S_{\text{depth}}}\) |
|----------------|----------------------|----------------------|-------------------------------------|
| \(\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}\n
results are not immediately usable in real-world emergent software systems because the running times of these algorithms are (to be blunt, ludicrously) exorbitant. This difficulty is also not as bad as it initially seems because it is well known within the parameterized complexity research community that once fp-tractability is proven relative to a parameter-set, surprisingly effective fp-algorithms can often be subsequently developed [45, 46]. This may involve either applying advanced fp-algorithm design techniques or adding additional parameters to minimal parameter-sets, and typically results in algorithm runtimes with greatly diminished non-polynomial terms and polynomial terms that are additive rather than multiplicative.

Second, and perhaps more importantly, the fixed-parameter (and indeed all of the theoretical) analyses in this paper are only intended to sketch out what types of efficient algorithms do and do not exist for our problems of interest and are not intended to be of immediate use. Given this, one may be tempted to conclude that our results and by proxy theoretical analyses in general are irrelevant. We believe that such a view is shortsighted at best and dangerous at worst. Not knowing the precise conditions under which existing emergent software system creation and adaptation algorithms work well may have serious consequences, e.g., drastically slowed software creation time and/or unreliable software operation, if these conditions are violated. These consequences would be particularly damaging in the case of the fully automatic operations underlying emergent software systems. Given that reliable software operation is crucial and efficient software creation and adaptation is at the very least desirable, the acquisition of such knowledge via a combination of rigorous empirical and theoretical analyses should be a priority. With respect to theoretical analyses, it is our hope that the techniques and results in this paper comprise a useful first step.
Table 7: Systematic Parameterized Complexity Analysis of ESCreate\textsubscript{poly} and ESAdapt\textsubscript{poly} Relative to the Parameter-set \(\{I_{ci}, C_{pi}, C_{ri}, S_{depth}, S_{comp}\}\) (Cont’d). c) Analysis for ESAdapt\textsubscript{poly} under \(Rew_{CodeB}\).

|               | \(S_{comp}\) | \(S_{depth}\) | \(S_{comp}, S_{depth}\) |
|---------------|--------------|---------------|--------------------------|
| \(-\)         | NPh          | \(X_{B.12}\) | \(X_{B.12}\) \(X_{B.12}\) |
| \(I_{ci}\)   | \(X_{B.14}\) | \(\sqrt{B.20}\) | \(X_{B.14}\) \(\sqrt{B.20}\) |
| \(C_{pi}\)   | \(X_{B.12}\) | \(X_{B.12}\) | \(X_{B.12}\) \(X_{B.12}\) |
| \(C_{ri}\)   | \(X_{B.12}\) | \(X_{B.12}\) | \(X_{B.12}\) \(X_{B.12}\) |
| \(I_{ci}, C_{pi}\) | \(X_{B.14}\) | \(\sqrt{B.20}\) | \(X_{B.14}\) \(\sqrt{B.20}\) |
| \(I_{ci}, C_{ri}\) | \(X_{B.16}\) | \(\sqrt{B.20}\) | \(\sqrt{B.19}\) \(\sqrt{B.19}\) |
| \(C_{pi}, C_{ri}\) | \(X_{B.12}\) | \(X_{B.12}\) | \(X_{B.12}\) \(X_{B.12}\) \(\sqrt{B.19}\) |
| \(I_{ci}, C_{pi}, C_{ri}\) | \(X_{B.16}\) | \(\sqrt{B.20}\) | \(\sqrt{B.19}\) \(\sqrt{B.19}\) |

5 Conclusions and Future Work

In this paper, we have applied computational complexity analysis to evaluate algorithmic options for emergent software system creation and adaptation relative to several popular types of exact and approximate efficient solvability. We have shown that neither problem is correctly and exactly solvable for all inputs when no restrictions are placed on the structure and operation of valid software systems. This intractability continues to hold relative to all examined types of efficient exact and approximate solvability when valid software systems are restricted to run in times polynomial in their input sizes. Moreover, both of our problems remain intractable under a variety of additional restrictions on valid software system structure, both individually and in many combinations. That being said, we give sets of additional restrictions that do yield tractability for both problems, as well as circumstantial evidence that emergent software system adaptation is computationally easier than emergent software system creation.

There are three promising directions for future research. The first of these is to extend our current analyses by (1) establishing the fp-status of our problems relative to all parameter-combinations from Table 1 and (2) considering additional parameters for and restrictions on emergent software systems. A productive source for the latter might be restrictions which “break” the reduction-tricks used in our intractability proofs. The second research direction is to consider additional computational problems associated with emergent software systems. Of particular interest here is the automated synthesis of both variants of existing components and new components from input-output examples \[47, 48\]. Analyses of these problems may also give insights into the more general problem of program synthesis \[49, 50, 51\].
The third research direction involves not so much emergent software systems but self-adaptive and component-based software systems in general. Invoking the terminology in the taxonomy of component models given in [52], emergent software systems have operation-based interfaces, provide horizontal and partial vertical component binding during composition, and explicitly distinguish between the required and provided interfaces of a component. These characteristics are not only exploited by our intractability result reductions but are also typical of many other component models [52 Table 2]. Hence, it would be of great interest to see if our analyses for emergent software systems could be extended to software system creation and adaptation problems arising relative to other proposed types of self-adaptive software systems [1 7 8] and within component-based software engineering in general [53 54 55].

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Let us first prove Result A.9. In the reduction below, instead of encoding a dominating set of size $k$ implicitly in the components implementing procedures $\text{InSet1}()$, $\text{InSet2}()$, ..., $\text{InSetk}()$ as in the reductions in the proofs of Lemmas 4 and 5, a candidate dominating set is encoded explicitly in binary-valued vector $vS$ in component $\text{Base}$ in the components used to implement the procedures $\text{vertexStatus1}()$, $\text{vertexStatus2}()$, ..., $\text{vertexStatus}(|V|)$. Moreover, checks to see if this candidate is an actual dominating set of size $k$ in $G$ are explicitly coded in component $\text{Base}$ (see Figure 7). This allows us to reduce the maximum number of interfaces...
Figure 7: General structure of valid software systems created by the reduction in the proof of Result A.9. Note that index $x$ in $\text{DomSetStatus}_x$ is such that $x \in \{0, 1\}$. Following the convention in Figure 3, interfaces with multiple implementing components are enclosed in dashed boxes.

provided by any component and the maximum number of components implemented by any interface to 1 and 2, respectively, while maintaining a maximum component depth of 3.

**Result A.9** For any choice of $\text{Rew}()$ and $\text{Env}()$, if $\langle I_{ci}, C_{pi}, S_{depth}\rangle$-$\text{ESC}_{\text{poly}}$ is fp-tractable then $P = NP$.

**Proof:** Consider the following polynomial-time Karp reduction from DOMINATING SET to $\text{CSC}_{\text{poly}}$: given an instance $I = \langle G = (V, E), k \rangle$ of DOMINATING SET, construct an instance $I' = \langle R, L_{\text{int}}, L_{\text{comp}}, c \rangle$ of $\text{CSC}_{\text{poly}}$ in which $R$ is the same as that given in the reduction in the proof of Lemma 4. Let $L_{\text{int}}$ consist of $|V| + 2$ interfaces broken into three groups:

1. A single interface of the form

   ```
   interface base {
       void main(Input I)
   }
   ```

2. A set of $|V|$ interfaces of the form

   ```
   interface vertStatJ {
       int vertexStatusJ()
   }
   ```

   for $1 \leq J \leq |V|$.
3. An interface of the form

```java
interface domSetStat {
    int domSetStatus()
}
```

Let $L_{comp}$ consist of $|V| + 3$ components broken into three groups:

1. A single component of the form

```java
component Base provides base
    requires vertStat1, vertStat2, ..., vertStat|V| {
        void main(Input I) {

        create integer array vS of length |V|
        vS[1] = vertexStatus1()
        vS[2] = vertexStatus2()
        ...
        vS[|V|] = vertexStatus|V|()

        numFound = 0
        for i = 1 to |V| do
            if vS[j] == 1 then
                numFound = numFound + 1
        if numFound == k then
            isCandidatekDomSet = True
        else
            isCandidatekDomSet = False

        if isCandidatekDomSet then
            numFound = 0
            for i = 1 to |V| do
                if v_I(x_i) and vS[i] == 1 then
                    numFound = numFound + 1
            if numFound > 0 then
                output 1
            else
                output 1
        }
    }
}
```
output 0
else
  output 0
}

2. A set of $|V|$ components of the form

component VertexStatusJ provides vertStatJ {
  requires domSetStat
  int vertexStatusJ() {
    return domSetStatus()
  }
}

for $1 \leq J \leq |V|$.

3. Two components of the form

component DomSetStatus0 provides domSetStat {
  int domSetStatus() {
    return 0
  }
}

component DomSetStatus1 provides domSetStat {
  int domSetStatus() {
    return 1
  }
}

Observe that in component Base, if entry $i$ of array vS has value 1, then vertex $i$ is in the candidate dominating set for $G$ encoded in vS. Finally, let $c$ be component Base in $L_{comp}$. Note that the instance of CSCreate poly described above can be constructed in time polynomial in the size of the given instance of DOMINATING SET; moreover, as there is only a $|V|$-length assignment statement block and two single-level loops in the component code that each execute at most $|V| < |I|$ times, any component-based software system created relative to $L_{int}$, $L_{comp}$, and $c$ runs in time linear in the size of input $I'$.

Let us now verify the correctness of this reduction:
Figure 8: General structure of valid software systems created by the reduction in the proof of Result A.10. Note that index \( x \) in \( \text{DomSetStatus}_x \) is such that \( x \in \{0, 1\} \). Following the convention in Figure 3, interfaces with multiple implementing components are enclosed in dashed boxes.

- Suppose there is a dominating set \( D \) of size at most \( k \) in the given instance of \textsc{Dominating set}; if \( |D| < k \), augment \( D \) with \( k - |D| \) arbitrary other vertices from \( G \) such that \( |D| = k \). Construct a component-based software system consisting of \( c \) and all \(|V|\) components \( \text{VertexStatus}_J \) for \( 1 \leq J \leq |V| \) in which the \( \text{domSetStat} \) interface in component \( \text{VertexStatus}_J \) is implemented by component \( \text{DomSetStatus}_1 \) if vertex \( J \) is in \( D \) and by component \( \text{DomSetStatus}_0 \) otherwise. Observe that for each \((i_j, o_j) \in R\), this software system produces output \( o_j \) given input \( i_j \).

- Conversely, suppose that the constructed instance of \( \text{CSCreate}_{\text{poly}} \) has a working component-based software system based on \( c \) relative to \( L_{\text{int}}, L_{\text{comp}}, \) and \( R \). In order to accommodate all input-output pairs in \( R \), the \(|V|\) \( \text{vertexStatus} \) components must implement their \( \text{domSetStat} \) interfaces such that the 1-values in array \( vS \) specify a dominating set of size \( k \) in \( G \). Hence, the existence of a working component-based software system for the constructed instance of \( \text{CSCreate}_{\text{poly}} \) implies the existence of a dominating set of size \( k \) for the given instance of \textsc{Dominating set}.

The reduction above is thus correct. Given the \( NP \)-hardness of \textsc{Dominating set}, this reduction implies that \( \text{CSCreate}_{\text{poly}} \) is \( NP \)-hard when \( I_{\text{ci}} = 2, C_{\text{pi}} = 1, \) and \( S_{\text{depth}} = 3 \) and hence by Lemma 3 not fp-tractable relative to these parameters unless \( P = NP \). The fp-intractability result for \( \text{ESCreate}_{\text{poly}} \) then follows by contradiction from Observation 1.

Let us now prove Result A.10. In the reduction below, we modify the reduction in the proof of Result A.9 to split the call to procedures \( \text{vertexStatus}_1() \), \( \text{vertexStatus}_2() \), \ldots, \( \text{vertexStatus}_{|V|}() \) in component \( \text{Base} \) over \(|V|\) base-
components (see Figure [5]). This still allows us to have the values of the maximum number of interfaces provided by any component and the maximum number of components implemented by any interface as 1 and 2, respectively, as in the previous reduction but trades off constant-valued component depth for a constant-valued maximum number of interfaces required by any component.

**Result A.10** For any choice of $Rew()$ and $Env()$, if $(I_{ci}, C_{pi}, C_{ri})$-ESC$i$_poly is fp-tractable then $P = NP$.

**Proof:** Consider the following polynomial-time Karp reduction from DOMINATING SET to CSC$i$_poly: given an instance $I = (G = (V, E), k)$ of DOMINATING SET, construct an instance $I' = (R, L_{int}, L_{comp}, c)$ of CSC$i$_poly in which $R$ is the same as that given in the reduction in the proof of Lemma [4]. Let $L_{int}$ consist of $2|V| + 1$ interfaces broken into four groups:

1. A single interface of the form

   ```java
   interface base {
       void main(Input I)
   }
   ```

2. A set of $|V| - 1$ interfaces of the form

   ```java
   interface baseJ {
       void callBaseJ(Input I, int[] vS)
   }
   ```
   for $2 \leq J \leq |V|$.

3. A set of $|V|$ interfaces of the form

   ```java
   interface vertStatJ {
       int vertexStatusJ()
   }
   ```
   for $1 \leq J \leq |V|$.

4. An interface of the form

   ```java
   interface base {
       ```
Let $L_{comp}$ consist of $2|V| + 2$ components broken into five groups:

1. A single component of the form

   \begin{verbatim}
   component Base provides base
       requires vertStat1, base2 {
       void main(Input I) {
           create integer array vS of length |V|
           vS[1] = vertexStatus1()
           callBase2(I, vS)
       }
   }
   \end{verbatim}

2. A set of $|V| - 2$ components of the form

   \begin{verbatim}
   component BaseJ provides baseJ
       requires vertStatJ, base(J + 1) {
       void callBaseJ(Input I, int[] vS) {
           vS[J] = vertexStatusJ()
           callBase(J + 1)(I, vS)
       }
   }
   \end{verbatim}

   for $2 \leq J \leq |V| - 1$.

3. A single component of the form

   \begin{verbatim}
   component Base|V| provides base|V|
       requires vertStat|V| {
       void callBase|V|(Input I, int[] vS) {
   \end{verbatim}
vS[|V|] = vertexStatus(|V|)

numFound = 0
for j = 1 to |V| do
    if vS[i] == i then
        numFound = numFound + 1
    if numFound = k then
        isCandidatekDomSet = True
    else
        isCandidatekDomSet = False
if isCandidatekDomSet then
    numFound = 0
for i = 1 to |V| do
    if v_I(x_i) and vS[i] == 1 then
        numFound = numFound + 1
    if numFound > 0 then
        output 1
    else
        output 0
else
    output = 0
}

4. A set of |V| components of the form

    component VertexStatusJ provides vertStatJ {
        requires domSetStat
        int vertexStatusJ() {
            return domSetStatus()
        }
    }

for 1 ≤ J ≤ |V|.

5. Two components of the form
Finally, let \( c \) be component \( \text{Base} \) in \( L_{\text{comp}} \). Note that the instance of CSCreate_{poly} described above can be constructed in time polynomial in the size of the given instance of DOMINATING SET; moreover, as there are only single-level loops in the component code that each execute at most \(|V| < |I|\) times, any component-based software system created relative to \( L_{\text{int}}, L_{\text{comp}}, \) and \( c \) runs in time linear in the size of input \( I' \).

Observe that interfaces \( \text{base}, \text{base}_J \) for \( 1 \leq J \leq |V| \), and \( \text{domSetStat} \) and components \( \text{VertexStatus}_J \) for \( 1 \leq J \leq |V| \), \( \text{DomSetStatus}_0 \), and \( \text{DomSetStatus}_1 \) are the same as in the reduction in the proof of Result A.9. Moreover, interfaces \( \text{base} \) and \( \text{base}_J \), and components \( \text{Base} \) and \( \text{callBase}_J \), \( 2 \leq J \leq |V| \), effectively simulate interface \( \text{base} \) and component \( \text{Base} \) in the same reduction. Hence, the proof of correctness of the reduction in the proof of Result A.9 can, with slight modifications, also prove the correctness of the reduction described above. Given the \( NP \)-hardness of DOMINATING SET, this reduction implies that CSCreate_{poly} is \( NP \)-hard when \( I_{ci} = 2, C_{pi} = 1, \) and \( C_{ri} = 2, \) and hence by Lemma 3 not \( \text{fp-tractable} \) relative to these parameters unless \( P = NP \). The \( \text{fp-intractability} \) result for ESCreate_{poly} then follows by contradiction from Observation 1.

Let us now prove Result A.11. The reduction below modifies that given in the proof of Result A.9 such that there are still only two components \( \text{DomSetStatus}_0 \) and \( \text{DomSetStatus}_1 \) but now, each of these components contains customized versions of procedures \( \text{vertexStatus}_1(), \text{vertexStatus}_2(), \ldots, \text{vertexStatus}_{|V|}() \) instead of placing these procedures separately in components \( \text{VertexStatus}_1, \text{VertexStatus}_2, \ldots, \text{VertexStatus}_{|V|} \) (see Figure 9). By invoking the abilities of different interfaces to implement different copies of the same component and implementing components to provide to an interface only that code which is used by that interface, not only is the maximum component depth reduced by one but the size of the component library \( L_{\text{comp}} \) is reduced to a constant (namely, 3).
Figure 9: General structure of valid software systems created by the reduction in the proof of Result A.11. Note that index $x$ in $\text{DomSetStatus}_x$ is such that $x \in \{0, 1\}$. Following the convention in Figure 3, interfaces with multiple implementing components are enclosed in dashed boxes.

**Result A.11** For any choice of $\text{Rew}()$ and $\text{Env}()$, if $\langle |L_{\text{comp}}|, I_{ci}, S_{\text{depth}} \rangle$-ESCreate$_{\text{poly}}$ is fp-tractable then $P = NP$.

**Proof:** Consider the following polynomial-time Karp reduction from DOMINATING SET to CSCCreate$_{\text{poly}}$: given an instance $I = \langle G = (V, E), k \rangle$ of DOMINATING SET, construct an instance $I' = \langle R, L_{\text{int}}, L_{\text{comp}}, c \rangle$ of CSCCreate$_{\text{poly}}$ in which $R$ is the same as that given in the reduction in the proof of Lemma 4. Let $L_{\text{int}}$ consist of $|V| + 1$ interfaces broken into two groups:

1. A single interface of the form

   ```java
   interface base {
       void main(Input I)
   }
   ```

2. A set of $|V|$ interfaces of the form

   ```java
   interface vertStatJ {
       int vertexStatusJ()
   }
   ```

   for $1 \leq J \leq |V|$.

Let $L_{\text{comp}}$ consist of 3 components broken into three groups:

1. A single component of the form
component Base provides base
    requires vertStat1, vertStat2, ..., vertStat|V| { 
    void main(Input I) { 
        
        create integer array vS of length |V|
        vS[1] = vertexStatus1()
        vS[2] = vertexStatus2()
        ...
        vS[|V|] = vertexStatus|V|()
        
        numFound = 0
        for i = 1 to |V| do
            if vS[i] == i then 
                numFound = numFound + 1
        if num_found == k then
            isCandidatekDomSet = True
        else
            isCandidatekDomSet = False
        
        if isCandidatekDomSet then
            numFound = 0
            for i = 1 to |V| do
                if v_I(x_i) and vS[i] == 1 then
                    numFound = numFound + 1
            
            if numFound > 0 then
                output 1
            else
                output 0
        else
            output 0
    } 
}
2. Two components of the form

\[
\text{component DomSetStatus0 provides } \text{vertStat1, vertStat2, ... vertStat}|V| \{ \\
  \text{int vertexStatus1() \{ return 0 \}} \\
  \text{int vertexStatus2() \{ return 0 \}} \\
  \ldots \\
  \text{int vertexStatus}|V|() \{ return 0 \}
\}
\]

\[
\text{component DomSetStatus1 provides } \text{vertStat1, vertStat2, ... vertStat}|V| \{ \\
  \text{int vertexStatus1() \{ return 1 \}} \\
  \text{int vertexStatus2() \{ return 1 \}} \\
  \ldots \\
  \text{int vertexStatus}|V|() \{ return 1 \}
\}
\]

Observe that in component Base, if entry } i \text{ of array } vS \text{ has value 1 then vertex } i \text{ is in the candidate dominating set for } G \text{ encoded in } vS. \text{ Finally, let } c \text{ be component Base}
in $L_{comp}$. Note that the instance of CSCreate$_{poly}$ described above can be constructed in time polynomial in the size of the given instance of DOMINATING SET; moreover, as there are only a single $|V|$-length assignment block and two single-level loops in the component code that each execute at most $|V| < |I|$ times, any component-based software system created relative to $L_{int}$, $L_{comp}$, and $c$ runs in time linear in the size of input $I$.

Let us now verify the correctness of this reduction:

- Suppose there is a dominating set $D$ of size at most $k$ in the given instance of DOMINATING SET; if $|D| < k$, augment $D$ with $k - |D|$ arbitrary other vertices from $G$ such that $|D| = k$. Construct a component-based software system consisting of $c$ and $|V|$ copies of the domSetStatus components in which the vertStatJ interface in component Base is implemented by component DomSetStatus1 if vertex J is in $D$ and by component DomSetStatus0 otherwise. Observe that for each $(i_j, o_j) \in R$, this software system produces output $o_j$ given input $i_j$.

- Conversely, suppose that the constructed instance of CSCreate$_{poly}$ has a working component-based software system based on $c$ relative to $L_{int}$, $L_{comp}$, and $R$. In order to accommodate all input-output pairs in $R$, the $|V|$ vertStatJ interfaces in component Base must be implemented by domSetStatus components such that the produced values in array vS specify a dominating set of size $k$ in $G$. Hence, the existence of a working component-based software system for the constructed instance of CSCreate$_{poly}$ implies the existence of a dominating set of size $k$ for the given instance of DOMINATING SET.

The reduction above is thus correct. Given the $NP$-hardness of DOMINATING SET, this reduction implies that CSCreate$_{poly}$ is $NP$-hard when $|L_{comp}| = 3$, $I_{ci} = 2$, and $S_{depth} = 2$ and hence by Lemma 3 not fp-tractable relative to these parameters unless $P = NP$. The fp-intractability result for ESCreate$_{poly}$ then follows by contradiction from Observation 1.

Finally, let us prove Result B.22. This result addresses the feasibility of preprocessing a fixed portion of the input to an intractable problem to create polynomial-size information that can be used to solve future instances of that problem relative to a varying portion of the problem input in polynomial time. This will be evaluated using the computational complexity framework described in [37]. The input to a problem of interest will be considered as a pair $(x, y)$, where $x$ is the fixed part and $y$ is the varying part. As we will focus here on decision problems like CSCreate$_{poly}$ in the main text whose solution is either “Yes” or “No”, each such problem will be considered a language of pairs. An example of such a problem is
Figure 10: General structure of valid software systems created by the reduction in the proof of Result B.22. Note that index \( x \) in \( \text{TruthValue} \) is such that \( x \in \{ \text{True, False} \} \). Following the convention in Figure 3, interfaces with multiple implementing components are enclosed in dashed boxes.

\[
\text{C-SAT} = \{ \langle (f), (p) \rangle \mid p \text{ can be extended to a truth assignment satisfying } f \}
\]

where fixed part \( f \) is a formula in conjunctive normal form (a set \( C = \{ C_1, C_2, \ldots, C_n \} \) of clauses AND-ed together where each clause is a set of negated or unnegated Boolean variables, e.g., \((v_1 \text{OR} \neg v_2 \text{OR} v_4) \text{AND}(v_2 \text{OR} v_3)\)) over a set of Boolean variables \( V = \{ v_1, v_2, \ldots, v_m \} \) and varying part \( p \) is a partial truth assignment to the variables in \( V \). Consider the following reducibility between such problems.

**Definition 4** [37, Definition 2.8] A \( \| \sim \) reduction between two languages of pairs \( A \) and \( B \) is a triple \((f_1, f_2, g)\) where \( f_1 \) and \( f_2 \) are poly-size unary functions and \( g \) is a binary polynomial-time function such that for any pair of strings \( \langle x, y \rangle \) it holds that:

\[
\langle x, y \rangle \in A \text{ if and only if } \langle f_1(x, |y|), g(f_2(x, |y|), y) \rangle \in B
\]

It is known that C-SAT is hard under \( \| \sim \) reductions relative to a class \( \| \sim \) NP [37, Section 3.1] and hence not preprocessable in the sense above unless the Polynomial Hierarchy \( \text{PH} \) collapses, i.e., \( \text{PH} = \Sigma_3^p \) [37, Theorem 2.12]. We will now prove the result by a reduction based on that in the proof of Result A.9 (see Figure 10).

**Result B.22:** If \( \text{ESA}_{\text{poly}} \) can have \( L_{\text{int}}, L_{\text{comp}}, R, \) and \( c \) preprocessed to create polynomial-size information that can be used to solve \( \text{ESA}_{\text{poly}} \) instances of arbitrary \( S, \text{Rew}() \), and \( k \) in polynomial time then the Polynomial Hierarchy \( \text{PH} \) collapses, i.e., \( \text{PH} = \Sigma_2^p \).

\(^5\) Aficionados of [37] will protest that by Theorem 2.12, our result implies that the Polynomial Hierarchy only collapses to \( \Sigma_3^p \). However, when that theorem is combined with the Karp-Lipton Theorem [56] (the most typical version of which states that if \( \text{NP} \subseteq \text{P/Poly} \) then \( \text{PH} = \Sigma_2^p \)), we can strengthen Theorem 2.12 to give our stated result.
Proof: Consider the following \( \Vdash \) reduction from C-SAT to CSAdapt\textsubscript{poly}: given an instance \( I = \langle (f), (p) \rangle \) of C-SAT, construct an instance \( I' = \langle (R, L_{int}, L_{comp}, c), (S, Rew(), k) \rangle \) of CSAdapt\textsubscript{poly} in which \( R \) has a single input-output pair \( \{True, 1\} \). Let \( f' \) be the version of \( f \) in which there is a new Boolean variable \( x_{m+1} \) that is OR-ed into every clause and \( p' \) be the version of \( p \) such that \( x_{m+1} = False \). Let \( L_{int} \) consist of \( m + 3 \) interfaces broken into three groups:

1. A single interface of the form

   \[
   \text{interface Base} \{
   \quad \text{void main(Input I)}
   \}
   \]

2. A set of \( m + 1 \) interfaces of the form

   \[
   \text{interface varAssignJ} \{
   \quad \text{Boolean varAssignJ()}
   \}
   \]

   for \( 1 \leq J \leq m + 1 \).

3. An interface of the form

   \[
   \text{interface truthValue} \{
   \quad \text{Boolean truthValue()}
   \}
   \]

Let \( L_{comp} \) consist of \( m + 4 \) components broken into three groups:

1. A single component of the form

   \[
   \text{component Base provides base}
   \quad \text{requires varAssign1, varAssign2, ..., varAssignm} \{
   \quad \text{void main(Input I)} \{
   \quad \quad \text{create Boolean array vA of length m}
   \quad \quad vA[1] = varAssign1()}
   \}
   \]
vA[2] = varAssign2()
...
vA[m+1] = varAssign(m+1)()

if truth assignment vA satisfies f’ then
    output 1
else
    output 0

2. A set of \( m + 1 \) components of the form

\[
\text{component VarAssignJ provides varAssignJ \{ \\
    \text{requires truthValue} \\
    \text{Boolean varAssignJ() \{ \\
        \text{return truthValue()} \\
    \}}
\}}
\]

for \( 1 \leq J \leq m + 1 \).

3. Two components of the form

\[
\text{component TruthValueFalse provides truthValue \{ \\
    \text{Boolean truthValue() \{ \\
        \text{return False} \\
    \}}
\}}
\]

\[
\text{component TruthValueTrue provides truthValue \{ \\
    \text{Boolean truthValue() \{ \\
        \text{return True} \\
    \}}
\}}
\]

Observe that in component \text{Base}, if entry \( i \) of array \( vA \) has value \textsf{True} (\textsf{False}), then variable \( i \) in the candidate truth-assignment for \( f \) encoded in \( vA \) has value \textsf{True} (\textsf{False}). Let \( c \) be component \text{Base} in \( L_{comp} \) and \( S \) be the component-based software
system consisting of Base and VarAssign1, VarAssign2, ..., VarAssign(m+1) such that varAssign(m+1)() is implemented by TruthValueTrue and all other varAssignx(), 1 ≤ x ≤ m, have a random implementation relative to TruthValueFalse and TruthValueTrue. Observe that courtesy of the truth-setting of v_{m+1} encoded in S, each clause in f' is satisfied and S will output 1 on any input. Finally, let Rew(S') return the number of variable-assignments in the truth-assignment for f' encoded in component-based system S'' that differ from those in the partial truth-assignment p' and set k = 0. Note that the instance of CSAdapt_{poly} described above can be constructed in time polynomial in the size of the given instance of C-SAT and that this construction encodes the required functions f_1(), f_2(), and g() in the definition of ≤_{comp}-reducibility; moreover, any candidate software system created relative L_{int}, L_{comp}, and c runs in time linear in the size of input I'.

Let us now verify the correctness of this reduction:

• Suppose there is a satisfying assignment p'' that extends p and satisfies f. Construct a component-based software system S' consisting of c = Base and VarAssign1, VarAssign2, ..., VarAssign(m+1) such that varAssignx() for 1 ≤ x ≤ m, implements the TruthValue component corresponding to the value of v_x in p'' and varAssign(m+1)() implements TruthValueFalse. Observe that for the only requirement r ∈ R, this software system produces output 1 given input True; moreover, Rew(S') = 0 ≤ k = 0. Hence, the existence of a satisfying truth-assignment that extends p and satisfies f in the given instance of C-SAT implies the existence of a working component-based software system S' for the constructed instance of CSAdapt_{poly} such that Rew(S') ≤ k.

• Conversely, suppose that the constructed instance of CSAdapt_{poly} has a working component-based software system S' based on c relative to L_{int}, L_{comp}, and R such that Rew(S') ≤ k. As k = 0, by the definition of Rew(), this means that the truth-assignment to the variables in V encoded in S' cannot differ from any of the variable assignments in the partial truth-assignment p'. As v_{m+1} = False in p', this means that the truth-assignments to the remaining variables in V (i) are an extension of p and (ii) encode a truth assignment that satisfies f. Hence, the existence of a working component-based software system for the constructed instance of CSAdapt_{poly} such that Rew(S') ≤ k implies the existence of a satisfying truth-assignment that extends p and satisfies f in the given instance of C-SAT.

The reduction above is thus correct. Given the ||⇒NP-hardness of C-SAT, this reduction implies that CSAdapt_{poly} is ||⇒NP-hard as well and not preprocessable in the sense of [37] unless the Polynomial Hierarchy PH collapses, i.e., PH = Σ_2^p.
(see the discussion immediately prior to this proof). The same non-preprocessability result for ESAdaptpoly then follows by contradiction from Observation \[ \square \]