Gravitational form factors of a spin one particle

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We define the form factors of the quark and gluon symmetric energy-momentum tensor (EMT). The static EMT is related to the spatial distributions of energy, spin, pressure and shear forces. They are obtained in the form of a multipole expansion. The relations between gravitational form factors and the generalised parton distributions are given.

I. INTRODUCTION

The gravitational form factors (GFFs) contains the information of the spatial distributions of energy, spin, pressure and shear forces inside the system \cite{1}. The GFFs are defined through the matrix elements of the symmetric energy-momentum tensor (EMT). More details can be found in the recent papers \cite{2, 3}. For spin one particles, the GFFs, or EMT FFs, have been discussed in the literature \cite{4–6}, but, to our best knowledge, EMT-nonconserving FFs are either not discussed \cite{4, 5} or incomplete \cite{6}. Thus we introduced a definition for individual quark and gluon EMT FFs for spin one particles in Sec. II. In Breit frame, we find that that matrix elements of EMT can be expressed in terms of the multipole expansion for energy density, pressure and shear forces distributions, see Sec. III. By considering the Mellin moments of the vector generalised parton distributions (GPDs), the sum rules between the GPDs and EMT FFs are found in Sec. IV.

The EMT of QCD can be obtained by varying the action $S_{\mathrm{grav}}$ of QCD coupled to a weak classical torsionless gravitational background field with respect to the metric $g_{\mu\nu}(x)$ of this curved background field according to \cite{2, 7}

\[ \hat{T}_{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S_{\mathrm{grav}}}{\delta g^{\mu\nu}(x)} \] (1)

where $g$ denotes the determinant of the metric (the signature of the metric we use is $+---$). This procedure yields a symmetric Belinfante-Rosenfeld EMT. The quark and gluon contributions to the total EMT operator are given by

\begin{align}
T_{\mu\nu}^{q} &= \frac{1}{4} \left[ \bar{\psi}_q \left( -i \bar{D}^\mu \gamma^\nu - i \bar{D}^\nu \gamma^\mu + i \bar{D}_\rho \gamma^\mu \gamma^\nu \right) \psi_q - g^{\mu\nu} \bar{\psi}_q \left( -\frac{i}{2} \bar{\gamma}^\alpha \gamma^\alpha + \frac{i}{2} \bar{\gamma}^\rho \gamma^\rho - m_q \right) \psi_q \right], \quad (2a) \\
T_{\mu\nu}^{g} &= F^{a,\mu\eta}_{\nu} F^{a,\nu}_{\eta} + \frac{1}{4} g^{\mu\nu} F^{a,\eta\kappa} F^{a,\kappa\eta} . \quad (2b)
\end{align}

Here $\bar{D}_\mu = \bar{\partial}_\mu + ig t^a A^a_\mu$ and $\bar{D}_\mu = \bar{\partial}_\mu - ig t^a A^a_\mu$ with arrows indicating which fields are differentiated, $F^{a,\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$ and the SU(3) color group generators satisfy the algebra $[t^a, t^b] = if^{abc} t^c$ and are normalized as $\text{tr}(t^a t^b) = \frac{1}{2} \delta^{ab}$. The total EMT is conserved

\[ \partial^\mu \hat{T}_{\mu\nu} = 0, \quad \hat{T}_{\mu\nu} = \sum_q \hat{T}_{\mu\nu}^q + \hat{T}_{\mu\nu}^g . \quad (3) \]

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Table I: The notations of EMT FFs in the literature (in \cite{1} these are generalised form factors, not exactly EMT FFs) and their values in free theory obtained by the Proca Lagrangian. In Ref. \cite{3} there is a sign mistake in the term corresponding to our $(e_\mu \Delta_\nu + e_\nu \Delta_\mu) \gamma^\nu \cdot P$ in $E^a(t)$'s coefficient. In Ref. \cite{4}, the authors missed one term which should be corresponding to $\bar{c}_a^2$ term here. The result of Ref. \cite{10}, which appeared during the preparation of this paper, coincides with our result.

| this work | $A_0$ | $A_1$ | $D_0$ | $D_1$ | $J$ | $E$ | $\bar{f}$ | $\bar{c}_0$ | $\bar{c}_1$ |
|-----------|-------|-------|-------|-------|-----|-----|-------|-------|-------|
| free theory | 1 | 0 | $1 + 4h$ | 0 | 1 | 1 | 0 | 0 | 0 |
| Holstein \cite{4} | $F_1$ | $4F_3$ | $-2F_2$ | $8F_6$ | $F_3$ | $-2F_4$ | $-$ | $-$ | $-$ |
| Abidin \cite{5} | $A$ | $-2E$ | $C$ | $-8F$ | $A + B$ | $D$ | $-$ | $-$ | $-$ |
| Taneja \cite{6} | $\mathcal{G}_1$ | $-2\mathcal{G}_2$ | $-\mathcal{G}_1$ | $-2\mathcal{G}_4$ | $\frac{1}{2}\mathcal{G}_5$ | $-\mathcal{G}_6$ | $\frac{1}{4}\mathcal{G}_7$ | $\frac{1}{4}\mathcal{G}_7 + \mathcal{G}_8$ | $-$ |
| Cosyn \cite{10} | $\mathcal{G}_1$ | $-2\mathcal{G}_2$ | $-\mathcal{G}_1$ | $-2\mathcal{G}_4$ | $\frac{1}{2}\mathcal{G}_5$ | $-\mathcal{G}_6$ | $\frac{1}{4}\mathcal{G}_7$ | $\frac{1}{4}\mathcal{G}_7 + \mathcal{G}_8$ | $-2\mathcal{G}_9$ |
| Cosyn \cite{9} generalised form factors | $A^2_{2,0}$ | $-2C^2_{2,0}$ | $-4F^2_2$ | $-8G^2_2$ | $\frac{1}{4}B^2_{2,0}$ | $D^2_{2,1}$ | $E^2_{2,0}$ | $-$ | $-$ |

II. DEFINITION OF EMT FORM FACTORS

We use the covariant normalisation $\langle p', \sigma'| p, \sigma \rangle = 2p^0 (2\pi)^3 \delta^{(3)}(p' - p)\delta_{\sigma \sigma'}$ of one-particle states, and introduce the kinematic variables $P = \frac{1}{2}(p' + p)$, $\Delta = p' - p$, $t = \Delta^2$. The EMT form factors of a spin-1 particle in QCD we define as\footnote{We chose the naming of the form factors in line with the naming used in Refs. \cite{1,2} for spin-0 and spin-1/2 particles.}

\begin{equation}
\langle p', \sigma'| \hat{T}^a_{\mu\nu}(x)| p, \sigma \rangle = \left[ 2P_{\mu}P_{\nu}\left( -\epsilon^* \cdot \epsilon A^a_0(t) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m^2} A^a_1(t) \right) \\
+ 2\left[ P_{\mu}(\epsilon^*_\nu \cdot P + \epsilon_{\nu} \epsilon^* \cdot P) + P_{\nu}(\epsilon^*_\mu \cdot P + \epsilon_{\mu} \epsilon^* \cdot P) \right] J^a(t) \\
+ \frac{1}{2}(\Delta_{\mu} \Delta_{\nu} - g_{\mu\nu} \Delta^2) \left( \epsilon^* \cdot \epsilon D^a_0(t) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m^2} D^a_1(t) \right) \\
+ \frac{1}{2}(\epsilon_{\mu} \epsilon^*_\nu + \epsilon^*_\mu \epsilon_{\nu}) \Delta^2 - (\epsilon^*_\mu \Delta_{\nu} + \epsilon^*_\nu \Delta_{\mu}) \epsilon \cdot P \\
+(\epsilon_{\mu} \Delta_{\nu} + \epsilon_{\nu} \Delta_{\mu}) \epsilon^* \cdot P - 4g_{\mu\nu} \epsilon^* \cdot P \epsilon \cdot P \right] E^a(t) \\
+ \left( \epsilon_{\mu} \epsilon^*_\nu + \epsilon^*_\mu \epsilon_{\nu} - \frac{\epsilon^* \cdot \epsilon}{2} g_{\mu\nu} \right) m^2 f^a(t) \\
+ g_{\mu\nu} \left( \epsilon^* \cdot m^2 \bar{c}_0^a(t) + \epsilon^* \cdot P \epsilon \cdot P \bar{c}_1^a(t) \right) \right] e^{i(p' - p)x},
\end{equation}

where $a = g, u, d, \ldots$ and the polarization vectors $\epsilon_{\mu}^* = \epsilon_{\mu}(p', \sigma')$, $\epsilon_{\mu} = \epsilon_{\mu}(p, \sigma)$, $\sigma = \pm 1, 0$. There are 9 GFFs for each quark flavour or gluon for a spin one particles. The 6 quark and gluon from factors (FFs) $A^a_{0,1}$, $D^a_{0,1}$, $J^a$ and $E^a(t)$ are individually EMT-conserving, and the other 3 FFs, $\bar{f}^a$ and $\bar{c}_a^2(t)$, are not. As discussed in Ref. \cite{2}, all the individual quark and gluon FFs depend on the renormalisation scale which we do not indicate for brevity. Due to EMT conservation, Eq. (3), the constraint $\sum_a f^a(t) = 0$ and $\sum_a \bar{c}_{0,1}^a(t) = 0$ hold, and the total form factors $A_{0,1}(t)$, $D_{0,1}(t)$, $J(t)$, $E(t)$ are renormalisation scale invariant where we defined $A_0(t) = \sum_a A^a_0(t)$ and analogously for other
form factors. Some of the notations for EMT FFs in the literature are listed in the Table I. The generalised form factors in [9] are connected with the gravitational form factors as shown in Table I.

### A. EMT form factors in free field theory

In the free field theory, the massive spin one particles are described by the Proca Lagrangian,

\[ \mathcal{L} = -\frac{1}{4} U_{\mu\nu} U^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu, \]

where \( A_\mu \) is a massive vector field and the field tensor is

\[ U_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]

The EMT corresponding to the Proca Lagrangian is given by

\[ \hat{T}_{(\text{Proca})}^{\mu\nu} = -U_{\rho\mu} U^{\rho\nu} - g^{\mu\nu} \mathcal{L} + m^2 A_\mu A^\nu. \]

The action \( S_{\text{grav}} \) can be modified by adding a non-minimal term for interaction with the gravity:

\[ S_{\text{grav}} = \int d^4x \sqrt{-g} \left( -\frac{1}{4} U_{\mu\nu} U^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu + \frac{1}{2} h R A^2 \right) \]

Here, \( R \) is the Riemann scalar. With this term added, the EMT in the free field theory becomes:

\[ \hat{T}_{\mu\nu}^{\text{improve}} = \hat{T}_{\mu\nu}^{(\text{Proca})} + \theta_{\mu\nu}^{\text{improve}}, \]

with \( \theta_{\mu\nu}^{\text{improve}} = -h(\partial_\mu \partial_\nu - g_{\mu\nu} \partial^2) A^2. \)

The value of the parameter \( h \) depends on the physics problem one is considering. With this improved EMT, one can obtain the free theory values of the total FFs [4] as shown in Table I.

### III. THE STATIC EMT AND STRESS TENSOR

Before discussing the components of EMT in Eq. (4), let us review the spin and quadrupole operators. For particles with spin \( S \geq 1 \), the quadrupole operator is the \((2S + 1) \times (2S + 1)\) matrix:

\[ \hat{Q}^{ij} = \frac{1}{2} \left( \hat{S}^i \hat{S}^k + \hat{S}^k \hat{S}^i - \frac{2}{3} S(S+1) \delta^{ik} \right), \quad (i, j, k = 1, 2, 3), \]

which is expressed in terms of the spin operator \( \hat{S}^i \). The spin operator can be expressed in terms of the SU(2) Clebsch-Gordan coefficients (in the spherical basis):

\[ \hat{S}^\lambda_{\sigma\sigma'} = \sqrt{S(S+1)} C^S_{\sigma \lambda} \hat{S}^\lambda_{\sigma}, \quad (\lambda = 0, \pm 1, \sigma, \sigma' = 0, \cdots, \pm J). \]

In spin one case, it is equivalent to

\[ \hat{S}^i_{\sigma\sigma'} = i \epsilon^{ijk} \hat{\epsilon}_\sigma^s \hat{\epsilon}_{\sigma'}^k, \quad (i, j, k = 1, 2, 3), \]

where \( \epsilon^\mu(0, \sigma) = (0, \pm 1, \sqrt{2}) \) is the rest frame spin-1 polarization vectors,

\[ \hat{\epsilon}_\pm = \mp \frac{1}{\sqrt{2}} (1, \pm i, 0), \quad \hat{\epsilon}_0 = (0, 0, 1), \]

\[ \hat{\epsilon}_0 = (0, 0, 1), \]

\[ \hat{\epsilon}_0 = (0, 0, 1), \]
Applying the boost operator $L(p)$ from the rest frame $k^\mu = (m,0,0,0)$ to any frame $p^\mu(= L^\mu_\nu k^\nu)$, one has

$$e^\mu(p,\sigma) = \left( \vec{p} \cdot \hat{\sigma} + \frac{\vec{p} \cdot \hat{\sigma}}{m(m+E)} \right),$$

where $\sigma = \{+, -, 0\}$, $m$ and $E = \sqrt{|p|^2 + m^2}$ are the rest mass and energy of the state.

In the Breit frame, the initial(final) momentum $p^\mu(p'^\mu)$ has the relation $P^\mu = (p^\mu + p'^\mu)/2 = (E, 0, 0, 0)$ and $\Delta^\mu = p'^\mu - p^\mu = (0, \vec{\Delta})$. So $\vec{p} = -\vec{p}' = -\vec{\Delta}/2$ and $p^0 = p'^0 = E = \sqrt{m^2 - t/4}$ with $t = \Delta^2$. In this frame, with the polarization vectors Eq. (15), Eq. (14) can be expressed as

$$\langle p', \sigma' | T_{\alpha}^{\mu(0)}(0) | p, \sigma \rangle = 2m^2 \mathcal{E}_0^\alpha(t) \delta_{\sigma', \sigma} + \hat{Q}^{k l} \Delta^k \Delta^l \mathcal{E}_2^\alpha(t),$$

(16a)

$$\langle p', \sigma' | T_{\alpha}^{\mu(0)}(0) | p, \sigma \rangle = i e^{k l m} \hat{S}^{k l}_\sigma \Delta^m J^{\alpha}(t),$$

(16b)

$$\langle p', \sigma' | T_{\alpha}^{\mu(0)}(0) | p, \sigma \rangle = \frac{1}{2} (\Delta^i \Delta^j - \delta^{ij} \vec{\Delta}^2) D_0^\alpha(t) \delta_{\sigma', \sigma} + \left( \Delta^i \Delta^k \hat{Q}^{k l} + \Delta^i \Delta^l \hat{Q}^{k l} - \Delta^2 \hat{Q}^{i j} - \delta^{i j} \Delta^k \Delta^l \hat{Q}^{k l} \right) D_2^\alpha(t)
\quad + \frac{1}{2m^2} (\Delta^i \Delta^j - \delta^{ij} \vec{\Delta}^2) \Delta^k \Delta^l \hat{Q}^{k l} D_3^\alpha(t)
\quad + \frac{1}{12} \left( t + \frac{1}{6} \Delta^i \Delta^j \delta_{\sigma', \sigma} - \frac{1}{6} \Delta^i \Delta^j \delta'_{\sigma', \sigma} - \frac{m^2}{2(m + E)} \right) \left( \Delta^i \Delta^k \hat{Q}^{k l} + \Delta^i \Delta^l \hat{Q}^{k l} \right)
\quad + \frac{1}{4} \left( \delta^{ij} - \frac{1}{2} \Delta^i \Delta^j \right) \Delta^k \Delta^l \hat{Q}^{k l} \right) \bar{f}^\alpha(t)
\quad + \delta^{i j} \left[ \frac{m^2}{6} + \frac{t}{12} + \frac{1}{2m^2} \left( \Delta^i \Delta^j - \delta^{ij} \vec{\Delta}^2 \right) \right] \bar{c}^\alpha_0(t)
\quad + \frac{1}{4} \left( 1 - \frac{t}{4m^2} \right) \left( \frac{t}{3} \delta_{\sigma', \sigma} + \Delta^k \Delta^l \hat{Q}^{k l} \right) \bar{c}^\alpha_1(t) \right),$$

(16c)

where $\hat{Q}^{k l} = \langle S, \sigma' | \hat{Q}^{k l} | S, \sigma \rangle$ are the matrix elements of the quadrupole operator and

$$\mathcal{E}_0^\alpha(t) = A_0^\alpha(t) + \frac{1}{4} \bar{f}^\alpha(t) - \bar{c}^\alpha_0(t)$$

$$\mathcal{E}_2^\alpha(t) = \frac{t^2}{12m^2} \left[ -5 A_0^\alpha(t) + 3 D_0^\alpha(t) + 4 J^\alpha(t) - 2 E^\alpha(t) + A_4^\alpha(t) + \frac{1}{2} \bar{f}^\alpha(t) + \bar{c}^\alpha_0(t) \right]$$

$$\frac{t^2}{24m^4} \left[ - A_5^\alpha(t) + D_0^\alpha(t) + 2 J^\alpha(t) - 2 E^\alpha(t) + A_4^\alpha(t) + \frac{1}{2} D_1^\alpha(t) + \bar{c}^\alpha_1(t) \right] + \frac{t^3}{192m^6} \left[ A_5^\alpha(t) + D_1^\alpha(t) \right]$$

$$- \frac{t^2}{4m^2} \left[ - A_0^\alpha(t) + D_0^\alpha(t) + 2 J^\alpha(t) - 2 E^\alpha(t) + A_4^\alpha(t) + \frac{1}{2} D_1^\alpha(t) + \frac{1}{4} \bar{c}^\alpha_1(t) \right]$$

$$+ \frac{t^2}{32m^4} \left[ A_4^\alpha(t) + D_1^\alpha(t) \right],$$

(17a)

$$\mathcal{E}_2^\alpha(t) = - A_5^\alpha(t) + 2 J^\alpha(t) - E^\alpha(t) + \frac{1}{2} A_4^\alpha(t) + \frac{1}{2} \bar{f}^\alpha(t) + \frac{1}{2} \bar{c}^\alpha_0(t) + \frac{1}{4} \bar{c}^\alpha_1(t)$$

$$- \frac{t^2}{4m^2} \left[ - A_0^\alpha(t) + D_0^\alpha(t) + 2 J^\alpha(t) - 2 E^\alpha(t) + A_4^\alpha(t) + \frac{1}{2} D_1^\alpha(t) + \frac{1}{4} \bar{c}^\alpha_1(t) \right]$$

$$+ \frac{t^2}{32m^4} \left[ A_4^\alpha(t) + D_1^\alpha(t) \right],$$

(17b)

$$J^{\alpha}(t) = J^{\alpha}(t) + \frac{1}{2} \bar{f}^\alpha(t) - \frac{t}{4m^2} \left( J^{\alpha}(t) - E^\alpha(t) \right).$$

(17c)

$$D_0^\alpha(t) = \frac{4}{3} \bar{f}^\alpha(t) + \frac{t}{12m^2} \left[ 2 D_0^\alpha(t) - 2 E^\alpha(t) + D_1^\alpha(t) \right] - \frac{t^2}{48m^4} D_1^\alpha(t),$$

(17d)

$$D_2^\alpha(t) = - E^\alpha(t),$$

(17e)

$$D_3^\alpha(t) = \frac{1}{4} \left[ 2 D_0^\alpha(t) - 2 E^\alpha(t) + D_1^\alpha(t) \right] - \frac{t}{16m^2} D_1^\alpha(t).$$

(17f)

The details for obtaining Eq. (16) and (17) are shown in Appendix A.

Due to the constraint $\sum_\alpha \bar{f}^\alpha(t) = 0$ and $\sum_\alpha \bar{c}^\alpha_0(t) = 0$, the total quark + gluon EMT drop $\bar{f}^\alpha(t)$ and $\bar{c}^\alpha_0(t)$ terms, so do $\mathcal{E}_{0,2}, J$ and $D_{0,2,3}(t)$. The free theory values of the total EMT FFs are listed in Table III. The D-term is defined as

$$D \equiv D_0(0) = \frac{1}{3} - 4 h.$$  

(18)
Table II: The free theory values of the total EMT FFs.

| EMT FFs | $\mathcal{E}_0(t)$ | $\mathcal{E}_2(t)$ | $\mathcal{J}(t)$ | $D_0(t)$ | $D_2(t)$ | $D_3(t)$ |
|---------|-----------------|-----------------|----------------|-----------|-----------|-----------|
| free theory | 1 | 0 | 1 | $\frac{1}{4} - 4\hbar$ | -1 | 0 |

Following Ref. [1], the static EMT $T_{\mu\nu}(\vec{r}', \sigma', \sigma)$ of the spin-1 particle is defined by Fourier transforming the EMT in Eq. (16a,16b,16c) with respect to $\Delta$ as

$$T_{\mu\nu}^a(\vec{r}', \sigma', \sigma) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}'} \langle p', \sigma' | \hat{T}_{\mu\nu}^a(0) | p, \sigma \rangle,$$

where $r = |\vec{r}|$.

### A. $T^{00}$: Energy density

Due to the presence of the EMT-nonconserving terms $\bar{f}^a$ and $\bar{c}^a_0$, the energy density $T^{00}(\vec{r}, \sigma', \sigma)$ can only be defined for the total system. The multipole expansion of the energy density is defined as [13]

$$T^{00}(\vec{r}, \sigma', \sigma) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \langle p', \sigma' | \hat{T}_{00}^{00}(0) | p, \sigma \rangle,$$

where $\varepsilon_0(r) = m^2 \hat{E}_0(r)$, $\varepsilon_2(r) = -r \frac{d}{dr} \frac{d}{dr} \hat{E}_2(r)$,

$$\varepsilon_0(r) = \varepsilon_2(r) \hat{Q}_{ij} Y_{ij}^{00},$$

with:

$$\hat{E}_{0,2}(r) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \varepsilon_{0,2}(t)$$

(the definition of Eq. (23) is used for other FFs in the following), and the irreducible (symmetric and traceless) tensor of $n$-th rank are [13]:

$$Y_{n+1}^{i_1\ldots i_n} = \frac{(-1)^n}{(2n-1)!!} r^{n+1} \partial^{i_1} \ldots \partial^{i_n} \frac{1}{r}, \quad \text{i.e.} \quad Y_0 = 1, \quad Y_1^i = \frac{r^i}{r}, \quad Y_2^{ik} = \frac{r^i r^k}{r^3} - \frac{1}{3} \delta^{ik}, \quad \text{etc.}$$

Notes there are obvious relations $\delta^{i_1\ldots i_n} Y_{n+1}^{i_1\ldots i_n} = 0$ and $\int d\Omega Y_{2}^{ik} = 0$.

In Ref. [13], more general tensor quantities are introduced for a particle of arbitrary spin:

$$M_{n+k_1\ldots k_n}^{k_1\ldots k_n} = \int d^3 r \ r^n \ Y_{n+k_1\ldots k_n}^{k_1\ldots k_n} \ T^{00}(\vec{r}),$$

which correspond to $2^n$-multipoles of the energy distribution. Here $T^{00}(\vec{r}) = T^{00}(\vec{r}, \sigma, \sigma)$. Note, that only even $n$ are allowed by the $P$-parity conservation. Obviously,

$$M_0 = m A_0(0) = m,$$

which gives the normalisation

$$A_0(0) = \sum_a A_{0}^{a}(t) = 1,$$

The function $\varepsilon_2(r)$ gives the quadrupole distribution of the energy inside the particles and describe the deviation of the hadron's shape from the spherical one for the hadrons with spin larger than 1/2. Obviously it satisfies the condition $\int d^3 r \varepsilon_2(r) = 0$. For free spin-1 particle one obtains (see Table II) that the quadrupole energy distribution is zero. Intuitively clear result.
B. \( T^{0j} \): Spin distribution

The 0k-components of the EMT are related to the spatial distribution of the spin. The angular momentum operator in QCD is defined according to the generators of Lorentz transformation\[14]\,

\[ J^i = \frac{1}{2} \epsilon^{ijk} \int d^3x M^{0jk}, \]

where \( M^{0ij} \) is the angular momentum density, expressible in terms of the energy-momentum tensor \( T^{\mu\nu} \) through

\[ M^{\alpha\nu} = T^{\alpha\nu} - T^{\alpha\nu} x^\nu. \]

From Eq. (16b), one gets

\[ T^{0j}(0) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{-i\Delta \cdot r}(p',\sigma' | T_a^{0j}(0)| p,\sigma). \]

According to Eq. (28), define the individual contributions of quarks and gluons to the particle spin \( J = 1 \) as \[2, 14, 15\],

\[ J^i_a(\vec{r},\sigma',\sigma) = \epsilon^{ijk} x_i J^{0j}(\vec{r},\sigma',\sigma). \]

Inserting the expression (30) (with Eq. (16b)) into Eq. (31) yields:

\[ J^i_a(\vec{r},\sigma',\sigma) = S_{\sigma'\sigma} \int d^3\Delta \left[ \left( \bar{J}^a(t) + \frac{2}{3} t \frac{d\bar{J}^a(t)}{dt} \right) \delta^j_{ij} + \left( \Delta^i \Delta^j - \frac{1}{3} \Delta^2 \delta^j_{ij} \right) \frac{d\bar{J}^a(t)}{dt} \right], \]

with \( \bar{J}^a(t) = \frac{\alpha}{\pi} J^a(t) \). Above equation has the form very similar to that for spin-1/2 particle \[2, 15\]. Note that the form factor \( J^a(t) \) contains the EMT non-conserving form factor \( \bar{f}^a(t) \). In the case of spin-0 and spin-1/2 the non-conserving form factors do not enter the spatial spin distribution.

Summing over quarks and gluons and integrating over the space yields

\[ \sum_a \int d^3r J^i_a(\vec{r},\sigma',\sigma) = \bar{S}_{\sigma'\sigma} J(0) = \bar{S}_{\sigma'\sigma}. \]

where the individual contributions \( J^a(0) \) add up to \( \sum_a J^a(0) = J(0) \) which satisfies the normalisation condition \( J(0) = J = 1 \). Obviously the free theory value \( J^{\text{free theory}}(t) = 1 \) in Table 1 satisfies this relation.

C. \( T^{ij} \) Stress tensor

In the sprit of Ref. [13], the stress tensor defined by the \( ij \)-components of EMT in Eq. (16b), can be written generically to the quadrupole order as:

\[ T^{ij}(\vec{r},\sigma',\sigma) = \int \frac{d^3\Delta}{2E(2\pi)^3} e^{i\Delta \cdot r}(p',\sigma' | T_a^{ij}(0)| p,\sigma) \]

\[ = p_0(r) \delta^{ij} \delta_{\sigma'\sigma} + s_0(r) Y_{ij}^{2j} \delta_{\sigma'\sigma} + p_2(r) \bar{Q}^{ij} + 2 s_2(r) \left[ \bar{Q}_q^q Y_{ij}^{2j} + \bar{Q}_q^p Y_{ip}^{2j} - \delta^{ij} \bar{Q}_q^q Y_{qq}^{2j} \right] \]

\[ - \frac{1}{m_2^2} \bar{Q}_q^q \partial_q \partial_q \left[ p_3(r) \delta^{ij} + s_3(r) Y_{ij}^{2j} \right], \]

where the (quadrupole) pressure and shear forces functions

\[ p_0(r) = \frac{1}{3} \partial^2 \bar{D}_0(r), \quad s_0(r) = -\frac{1}{2} \frac{d}{dr} \frac{1}{r} \partial^2 \bar{D}_0(r), \]

\[ p_2(r) = \frac{1}{3} \partial^2 \bar{D}_2(r), \quad s_2(r) = -\frac{1}{2} \frac{d}{dr} \frac{1}{r} \partial^2 \bar{D}_2(r), \]

\[ p_3(r) = \frac{1}{3} \partial^2 \bar{D}_3(r), \quad s_3(r) = -\frac{1}{2} \frac{d}{dr} \frac{1}{r} \partial^2 \bar{D}_3(r). \]
where \( \partial^2 = \frac{1}{\rho} \frac{d}{d\rho} r^2 \frac{d}{dr} \) is the radial part of 3D Laplace operator.

Comparing with Ref. [13], we get two additional terms of quadrupole order \( n = 2 \), which are \( p_2(r) \) and \( s_2(r) \) terms. The EMT conservation, \( \partial_{\mu} T^{\mu\nu}(x) = 0 \), implies the equilibrium equation for the static stress tensor

\[
\partial_{\mu} T^{\mu\nu}(x) = 0. \tag{36}
\]

For each of the first two quadrupole orders, it is easy to check that Eq. (35a),(35b),(35c) satisfy the differential equations

\[
\frac{2}{3} s'_n(r) + \frac{2}{r} s_n(r) + p'_n(r) = 0, \quad \text{with} \quad n = 0, 2, 3, \tag{37}
\]

which guarantee the general stability condition of Eq. (36).

Another three obvious relations,

\[
\int d^3 r p_n(r) = \frac{1}{3} \int d^3 r \partial^2 D_n(r) = 0, \quad \text{with} \quad n = 0, 2, 3, \tag{38}
\]

which shows how the internal forces balance inside a composed particle. It is a consequence of the EMT conservation, known as the von Laue condition [2, 3]. We also note that the multipole pressure and shear forces distributions \( (p_n(r), s_n(r)) \) satisfy the same stability equation (37) as the distributions in spherically symmetric case of spin-0 and spin-1/2 particles. Therefore all stability relations discussed in [2, 3] are valid also for non-spherically case of particles with higher spins.

### D. EMT-nonconserving terms

The EMT-nonconserving terms in Eq. (41) violate the EMT conservation \( \partial_{\mu} T^{\mu\nu}(x) = 0 \) as

\[
\langle p', \sigma'| \partial_{\mu} T^{\mu\nu}_{\mu
u}(x)|p, \sigma \rangle = i \Delta^x \langle p', \sigma'| T^{\mu\nu}_{\mu
u}(x)|p, \sigma \rangle = i e^{i \Delta x} \left[ \left( e \cdot D\epsilon^0_s + e^s \cdot \Delta \epsilon^0 - \frac{e^s}{2} \Delta \right) m^2 f^0(t) + \Delta \epsilon^0 \right], \tag{39}
\]

In Breit frame, the 0-component of Eq. (39) is

\[
\langle p', \sigma'| \partial_{\mu} T^{\mu0}_{\mu0}(x)|p, \sigma \rangle = i e^{i \Delta x} \left( e \cdot D\epsilon^0_s + e^s \cdot \Delta \epsilon^0 \right) m^2 f^0(t) = 0, \tag{40}
\]

and the \( j \)-component of Eq. (39) is

\[
\langle p', \sigma'| \partial_{\mu} T^{\mu j}_{\mu j}(x)|p, \sigma \rangle = i e^{i \Delta x} \left[ \left( e \cdot D\epsilon^0_s + e^s \cdot \Delta \epsilon^0 - \frac{e^s}{2} \Delta \right) m^2 f^0(t) + \Delta \epsilon^0 \right] \]

\[
= i e^{i \Delta x} \left[ D^j \delta_{\rho \sigma} m^2 \left[ \frac{f^0(t)}{6} - \bar{c}_0^0(t) + \frac{t}{12m^2} \left[ - f^0(t) + 2 \bar{c}_0^0(t) + \bar{c}_0^0(t) \right] - \frac{t^2}{48m^2} \bar{c}_0^0(t) \right] - 2mE \Delta^j \bar{Q}^j f^0(t) + \Delta^j \Delta^k \Delta^l \bar{Q}^j k l \right] \frac{1}{2} \bar{c}_0^0(t) + \frac{1}{4} \bar{c}_0^0(t) + \frac{t}{16(m + E)^2} f^0(t) - \frac{t}{16m^2} \bar{c}_0^0(t) \right], \tag{41}
\]

The stability equation for the quark part of the stress tensor has the form:

\[
\frac{\partial T^{ij}_{\mu
u}(x)}{\partial x^\mu} + f^i(x) = 0. \tag{42}
\]
This equation can be interpreted (see e.g discussion in [16]) as the equilibrium equation for quark internal stress and external force (per unit of the volume) \( f^i(x) \) acting on quark subsystem from the side of the gluons. From Eq. (41) one sees that the corresponding force depends on the polarisation of the spin-1 particle through the quadrupole spin operators \( \vec{Q}^{ij} \).

**IV. SUM RULES: GPD AND GFF**

By considering the Mellin moments of the vector generalised parton distributions (GPDs) [17], the sum rules between the GPDs and EMT FFs are found in Ref. [8, 9]. The sum rules in Ref. [5] contain only for conserving EMT FFs (6 out total 9 FFs). In recent paper [9], the polynomiality sum rules for all leading-twist quark and gluon generalised parton distributions of spin-1 targets are given. The generalised form factors in the polynomiality condition for GPDs of spin-1 particles in [9] are connected to the gravitational form factors as shown in Table II.

The quark and gluon vector GPDs are introduced in Ref. [17] for deuteron as:

\[
\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p', \sigma' | \bar{\psi}_q(z^- e^{i\frac{1}{2}z}) \gamma^+ \psi_q(z) | p, \sigma \rangle |_{z^+=0, \, \vec{z}_\perp = 0} = -\left( e^{*} \cdot \epsilon \right) H^q_1 \left( \frac{e^{*+} \left( \epsilon \cdot P \right) + e^{*+} \left( \epsilon \cdot P \right) H^q_2}{M^2} - 2 \left( \epsilon \cdot P \right) \left( e^{*+} \left( \epsilon \cdot P \right) H^q_3 \right) \right) \]

\[
\frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p', \sigma' | \gamma^+ \psi_q(z^- e^{i\frac{1}{2}z}) \gamma^+ \psi_q(z) | p, \sigma \rangle |_{z^+=0, \, \vec{z}_\perp = 0} = -\left( e^{*} \cdot \epsilon \right) H^q_1 \left( \frac{e^{*+} \left( \epsilon \cdot P \right) + e^{*+} \left( \epsilon \cdot P \right) H^q_2}{M^2} - 2 \left( \epsilon \cdot P \right) \left( e^{*+} \left( \epsilon \cdot P \right) H^q_3 \right) \right) \]

\[
\frac{1}{P^+} \int \frac{dz^-}{2\pi} e^{ixp^+z^-} \langle p', \sigma' | F^{b, \eta}(z^- e^{i\frac{1}{2}z}) F^{b, \eta}(z) | p, \sigma \rangle |_{z^+=0, \, \vec{z}_\perp = 0} = -\left( e^{*} \cdot \epsilon \right) H^q_1 \left( \frac{e^{*+} \left( \epsilon \cdot P \right) + e^{*+} \left( \epsilon \cdot P \right) H^q_2}{M^2} - 2 \left( \epsilon \cdot P \right) \left( e^{*+} \left( \epsilon \cdot P \right) H^q_3 \right) \right) \]

where \( q = u, d, s, \ldots \). Integrating over \( x \) of Eq. (43a), one gets the conventional form factor decomposition of the vector current for a spin-1 particle,

\[
\langle p', \sigma' | \bar{\psi}_q(0) \gamma^\mu \psi_q(0) | p, \sigma \rangle = -2 \left( e^{*} \cdot \epsilon G^q_1(t) + 2G^q_3(t) \left( \frac{e^{*+} \left( \epsilon \cdot P \right)}{m^2} \right) \right) P^\mu + 2G^q_2(t) \left( e^{*} \cdot \epsilon \cdot P + e^{*+} \epsilon \cdot P \right) .
\]

So, for the quark GPDs, one has [17]

\[
\int_{-1}^{1} dx H^q_i(x, \xi, t) = G^q_i(t) \quad (i = 1, 2, 3),
\]

\[
\int_{-1}^{1} dx H^q_i(x, \xi, t) = 0 \quad (i = 4, 5).
\]

The charge, magnetic, and quadrupole form factors can be expressed in terms of \( G_i = \sum q G^q_i \) as (\( \eta = -t/4m^2 \))

\[
G_C(t) = G_1(t) + \frac{2}{3} \eta G_Q(t),
\]

\[
G_M(t) = G_2(t),
\]

\[
G_Q(t) = G_1(t) - G_2(t) + (1 + \eta) G_3(t),
\]

normalised by the charge \( G_C(0) = 1 \), magnetic moment \( G_M(0) = \mu_{S=1}/(2m) \), and quadrupole moment \( G_Q(0) = Q_{S=1}/m^2 \).

The ++ components of Eq. (2) are,

\[
T^{++}_q(x) = \frac{1}{2} \bar{\psi}_q \left( -i \gamma^+ \gamma^+ + i \vec{D}^+ \gamma^+ \right) \psi_q(x),
\]

\[
T^{++}_g(x) = F^{b, \eta} F^{b, \eta +}(x),
\]
or,

\[ T_{q}^{++}(0) = (P^+)^2 \int dx \int \frac{dz}{2\pi} e^{ixP^+z} \left[ \bar{\psi}_q(-\frac{1}{2}z)\gamma^+\psi_q(\frac{1}{2}z) \right]_{\gamma^+=0, z_\perp=0}, \]

\[ = \left[ 2(P^+)^2 \left( -\epsilon^* \cdot \epsilon A_0^q(t) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m^2} A_1^q(t) \right) + \frac{1}{2}(\Delta^*)^2 \left( \epsilon^* \cdot \epsilon D_0^q(t) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m^2} D_1^q(t) \right) \right. \]

\[ + \frac{1}{2}(\Delta^*)^2 \left( \epsilon^* \cdot \epsilon D_0^q(t) + \frac{\epsilon^* \cdot P \epsilon \cdot P}{m^2} D_1^q(t) \right) \]

\[ + 4P^+ \left( \epsilon^* \cdot \epsilon P + \epsilon^* \epsilon^* \cdot P \right) J^q(t) \]

\[ + \left[ \epsilon^* \epsilon^* \Delta^2 - 2\epsilon^* \epsilon^* \Delta^2 \epsilon \cdot P + 2\epsilon^* \epsilon^* \Delta^2 \epsilon \cdot P \right] E^q(t) \]

\[ + 2\epsilon \epsilon^* \epsilon^*^* m^2 \bar{f}^q(t), \] (48a)

\[ T_{g}^{++}(0) = P^+ \int dx \int \frac{dz}{2\pi} e^{ixP^+z} \left[ F_{b,\eta}(\frac{1}{2}z) F_{\eta,\bar{b}}(\frac{1}{2}z) \right]_{\gamma^+=0, z_\perp=0}. \] (48b)

where \( T_{g}^{++}(0) \) is similar with \( T_{q}^{++}(0) \). Compare Eq. (48) with (43), we get the polynomiality property of vector GPDs as

\[ \int_{-1}^{1} dx dH^q_1(x, \xi, t) = A_0^q(t) - \xi^2 D_0^q(t) + \frac{t}{6m^2} E^q(t) + \frac{1}{3} \bar{f}^q(t), \] (49a)

\[ \int_{-1}^{1} dx dH^q_2(x, \xi, t) = 2J^q(t), \] (49b)

\[ \int_{-1}^{1} dx dH^q_3(x, \xi, t) = -\frac{1}{2} [A_1^q(t) + \xi^2 D_1^q(t)], \] (49c)

\[ \int_{-1}^{1} dx dH^q_4(x, \xi, t) = -2\xi E^q(t), \] (49d)

\[ \int_{-1}^{1} dx dH^q_5(x, \xi, t) = \frac{t}{2m^2} E^q(t) + \bar{f}^q(t), \] (49e)

for the quark parts and

\[ \int_{-1}^{1} dx dH^q_1(x, \xi, t) = A_0^q(t) - \xi^2 D_0^q(t) + \frac{t}{6m^2} E^q(t) + \frac{1}{3} \bar{f}^q(t), \] (50a)

\[ \int_{-1}^{1} dx dH^q_2(x, \xi, t) = 2J^q(t), \] (50b)

\[ \int_{-1}^{1} dx dH^q_3(x, \xi, t) = -\frac{1}{2} [A_1^q(t) + \xi^2 D_1^q(t)], \] (50c)

\[ \int_{-1}^{1} dx dH^q_4(x, \xi, t) = -2\xi E^q(t), \] (50d)

\[ \int_{-1}^{1} dx dH^q_5(x, \xi, t) = \frac{t}{2m^2} E^q(t) + \bar{f}^q(t), \] (50e)

for the gluon part. Note that, in \( H_{1,5}^q (H_{1,5}^g) \) it contains the EMT-nonconserving GFFs \( \bar{f}^q (\bar{f}^g) \). Thus it is useful to rewrite them as

\[ \int_{-1}^{1} dx \left[ H^q_1(x, 0, t) - \frac{1}{3} H^q_5(x, 0, t) \right] = A_0^q(t), \] (51a)

\[ \int_{-1}^{1} dx \left[ H^g_1(x, 0, t) - \frac{1}{3} H^g_5(x, 0, t) \right] = A_0^g(t). \] (51b)
In the GPD approach, the normalisation for $A_0$ in Eq. (27) is the energy-momentum sum rule,

$$\int_{-1}^{1} dx \left[ \sum_q x H_1^q(x,0,0) + H_1^q(x,0,0) - \frac{1}{3} \left( \sum_q x H_2^q(x,0,0) + H_2^q(x,0,0) \right) \right] = 1,$$

and the normalisation for $J$ in Eq. (33) gives the sum rule,

$$\frac{1}{2} \int_{-1}^{1} dx \left[ \sum_q x H_2^q(x,0,0) + H_2^q(x,0,0) \right] = 1.$$

V. SUMMARY

In this paper, we formulate the EMT form factors for a spin-1 hadrons. The energy density, spin distribution and stress tensor are given. The pressure and shear forces functions are found in terms of multipole expansion as the spin-1 particle is not spherically symmetric. The sum rules between the GFFs and GPDs are derived.

**Note added**

During finishing the present manuscript we became aware of recent Ref. [10] where the EMT form factors for spin-1 particles were also considered.

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**Appendix A: Breit frame formulae**

The matrix elements of the spin 1 quadrupole operator in Eq. (11),

$$\hat{Q}_{\sigma\sigma'}^{ik} = \frac{1}{2} \left( \hat{S}^i \hat{S}^k - \hat{S}^k \hat{S}^i - \frac{4}{3} \delta_{ik} \right)_{\sigma\sigma'}$$

$$= \frac{1}{3} \delta_{ij} \delta_{\sigma\sigma'} - \frac{1}{2} \left( \hat{\epsilon}_{\sigma'j} \hat{\epsilon}_{\sigma i} + \hat{\epsilon}_{\sigma'j} \hat{\epsilon}_{\sigma i} \right),$$

where $\sigma = \{ +, -, 0 \}$, and

$$\hat{S}^i \hat{S}^j - \hat{S}^j \hat{S}^i = i \epsilon^{ijk} \hat{S}^k.$$  \hspace{1cm} (A1)

In Breit frame, the initial(final) momentum $p^\mu(p'^\mu)$ has the relation $P^\mu = (p^\mu + p'^\mu)/2 = (E, 0, 0, 0)$ and $\Delta^\mu = p'^\mu - p^\mu = (0, \Delta)$. So $\vec{p} = -\vec{p}' = -\vec{\Delta}/2$ and $p^0 = p'^0 = E = \sqrt{m^2 - t}/4$ with $t = \Delta^2$. So initial and final polarization vectors are

$$\epsilon^\mu(p, \sigma) = \left( \frac{\Delta \cdot \hat{\epsilon}_\sigma}{2m}, \frac{\Delta \cdot \hat{\epsilon}_\sigma}{4m(m + E)} \right),$$

$$\epsilon^\mu(p', \sigma') = \left( \frac{\Delta \cdot \hat{\epsilon}_{\sigma'}}{2m}, \frac{\Delta \cdot \hat{\epsilon}_{\sigma'}}{4m(m + E)} \right).$$  \hspace{1cm} (A2)

$$\epsilon^\mu(p, \sigma) = \left( \frac{\Delta \cdot \hat{\epsilon}_\sigma}{2m}, \frac{\Delta \cdot \hat{\epsilon}_\sigma}{4m(m + E)} \right),$$

$$\epsilon^\mu(p', \sigma') = \left( \frac{\Delta \cdot \hat{\epsilon}_{\sigma'}}{2m}, \frac{\Delta \cdot \hat{\epsilon}_{\sigma'}}{4m(m + E)} \right).$$  \hspace{1cm} (A3)
So one can get the following useful relations (here $\hat{Q}_{kl} = \hat{Q}_{k\ell}^\dagger$ is the matrix element and $\epsilon'^\mu = \epsilon'^\mu(p', \sigma')$, $\epsilon'^\mu = \epsilon'^\mu(p, \sigma)$, and note $t = \Delta^2$),

$$
(\epsilon'^\sigma \cdot \Delta)(\epsilon'^\sigma \cdot \Delta) = \frac{t}{3} \epsilon'^\sigma - \hat{Q}_{kl} \Delta_k \Delta_l ,
$$
(A5a)

$$
\epsilon'^\sigma \cdot \epsilon^\sigma = \left( \frac{t}{6m^2} - 1 \right) \delta^\sigma_{\sigma'} + \frac{1}{2m^2} \hat{Q}_{kl} \Delta_k \Delta_l ,
$$
(A5b)

$$
\epsilon'^\sigma_{\sigma',0} \epsilon^\sigma_{\sigma',0} = \frac{t}{12m^2} \delta^\sigma_{\sigma'} + \frac{1}{4m^2} \hat{Q}_{kl} \Delta_k \Delta_l ,
$$
(A5c)

$$
\epsilon^\sigma \cdot \Delta = - \frac{E}{m} \epsilon^\sigma \cdot \Delta ,
$$
(A5d)

$$
\epsilon'^\sigma \cdot \Delta = - \frac{E}{m} \epsilon'^\sigma \cdot \Delta ,
$$
(A5e)

$$
(\epsilon'^\sigma \cdot \Delta)(\epsilon^\sigma \cdot \Delta) = - \frac{E^2}{m^2} \left( \frac{t}{3} \delta^\sigma_{\sigma'} + \hat{Q}_{kl} \Delta_k \Delta_l ^' \right) ,
$$
(A5f)

$$
\epsilon'^\sigma_{\sigma',0} \epsilon^\sigma \cdot \Delta = \frac{E}{2m^2} \left( \frac{t}{3} \delta^\sigma_{\sigma'} + \hat{Q}_{kl} \Delta_k \Delta_l ^' \right) ,
$$
(A5g)

$$
\epsilon^\sigma \cdot \Delta = - \epsilon'^\sigma_{\sigma',0} \epsilon^\sigma \cdot \Delta ,
$$
(A5h)

$$
\epsilon^\sigma \cdot \Delta - \epsilon^\sigma \cdot \Delta = \frac{iE}{m} \Delta_k \epsilon^{kji} \hat{Q}^j_{\sigma'\sigma} .
$$
(A5i)

$$
\epsilon^\sigma \cdot \Delta = \frac{iE}{m} \Delta_k \epsilon^{kji} \hat{Q}^j_{\sigma'\sigma} ,
$$
(A5j)

$$
\epsilon^\sigma_{\sigma',j} \epsilon^\sigma \cdot \Delta + \epsilon^\sigma_{\sigma',0} \epsilon^\sigma_{\sigma',j} = \frac{iE}{m} \Delta_k \epsilon^{kji} \hat{Q}^j_{\sigma'\sigma} ,
$$
(A5k)

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