Direct CP Asymmetry of $B \to X_{d,s} \gamma$ in a model with Vector quarks

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Abstract

We investigate the effect of vector quarks on the inclusive decays $B \to X_{d,s} \gamma$. We show that the branching ratio of $B \to X_{d} \gamma$ can differ sizably from the SM and MSSM predictions, being enhanced to present experimental observability or suppressed such that present runs of the B factories would not observe it. Current measurements of the direct CP asymmetry ($A_{CP}$) for $B \to X_{s} \gamma$ are sensitive to the contribution from $B \to X_{d} \gamma$. For a sufficiently enhanced BR($B \to X_{d} \gamma$) we show that the dominant contribution to the combined asymmetry may be from $B \to X_{d} \gamma$. Thus any large value for $A_{CP}$ should not immediately be attributed to $B \to X_{s} \gamma$, which stresses the importance of good K/π separation.

Keywords: CP Asymmetry, Rare B decay, Vector quarks
1 Introduction

Theoretical studies of rare decays of $b$ quarks have attracted increasing attention since the start of the physics program at the $B$ factories at KEK and SLAC. Both $B$ factories are running to expectations and in excess of 30 fb$^{-1}$ of data has been accumulated by each experiment. The much anticipated measurement of sin$2\phi_1$ has established CP violation in the $B$ system \cite{1,2}.

Many rare decays will be observed for the first time over the next few years, and theoretical studies of such decays in the context of models with new physics will continue to intensify. In this paper we are concerned with the decays $b \to d\gamma$ and $b \to s\gamma$ in a model with vector (or singlet) quarks. In such a framework the CKM matrix is necessarily non–unitary, leading to flavour changing neutral currents at tree–level.

It has been known for a considerable time that the inclusive decay $B \to X_s\gamma$ is a sensitive probe of new physics \cite{3}. It has been measured at CLEO \cite{4,5}, ALEPH \cite{6} and BELLE \cite{7}. The current branching ratio (BR) is in agreement with the Standard Model (SM) expectation \cite{8} but leaves room for new physics contributions. Preliminary measurements of the CP asymmetry have been made in the inclusive channel $B \to X_s\gamma$ \cite{9} and the exclusive channel $B \to K^*\gamma$ \cite{10,11}. Results with higher precision are expected to come from the $B$ factories in the future, especially after a possible luminosity upgrade.

Transitions of the form $b \to d\gamma$ have so far remained unobserved and in the SM are suppressed relative to $b \to s\gamma$ by a factor $|V_{td}/V_{ts}|^2 \approx 1/20$. Experimental upper limits exist for the branching ratios (BRs) of the exclusive decay channels, $B \to \rho^0\gamma$ and $B \to \rho^+\gamma$. CLEO \cite{10} obtains $\leq 1.7 \times 10^{-5}$ and $\leq 1.3 \times 10^{-5}$ respectively, with corresponding measurements by BELLE \cite{11} of $\leq 1.06 \times 10^{-5}$ and $\leq 0.99 \times 10^{-5}$.

There is considerable motivation for measuring the BR and CP asymmetry ($A_{CP}$) of the inclusive channel $B \to X_d\gamma$:

(i) It provides a theoretically clean way of measuring $V_{td}$, as proposed in \cite{12,13}.

(ii) $A_{CP}$ in the SM is sizeable, and much larger than that for $b \to s\gamma$ \cite{13}.

(iii) $A_{CP}$ is sensitive to new physics at the weak scale \cite{14,18}.

(iv) $b \to d\gamma$ transitions sizably affect the measurements of $A_{CP}$ for $b \to s\gamma$ \cite{9}. Therefore knowledge of $A_{CP}$ for $b \to d\gamma$ is essential, in order to compare experimental data with the theoretical prediction in a given model \cite{18}.

(v) $A_{CP}$ for the combined signal of $B \to X_s\gamma$ and $B \to X_d\gamma$ is expected to be close to zero in the SM \cite{19,20}, due the real Wilson coefficients and the unitarity of the CKM matrix. Both of these conditions can be relaxed in models beyond the SM.

The exclusive decays (e.g. $B \to \rho^0, \omega^0\gamma$ and $B \to \rho^+\gamma$) are expected to be observed for the first time at the $B$ factories. A measurement of the inclusive decay, although challenging, may be feasible due to the improved $K/\pi$ separation which is necessary to reduce the large background from $b \to s\gamma$ decays. The much larger $A_{CP}$ of $b \to d\gamma$ with respect to $b \to s\gamma$ (in the SM) is expected to compensate for its smaller branching ratio in terms of experimental observability.
The contribution of vector (i.e. \(SU(2)\)–singlet) quarks \([21, 22]\) to B physics processes has received increasing attention over the last few years \([23–30]\). Such particles appear in Grand Unified Theories (GUTs) \([23]\) and mix with the ordinary quarks, thus rendering the CKM matrix non–unitary. We will be working in the context of a model with one \(U\)– and one \(D\)–type vector quark. The presence of such particles affects the decays \(B \rightarrow X_s \gamma\) and \(B \rightarrow X_d \gamma\). Due to less stringent experimental constraints on the VQ parameters, the latter decay in particular may have a BR and \(A_{CP}\) very different from that expected in the SM and the Minimal Supersymmetric Standard Model (MSSM).

Our work is organised as follows: In section 2 we introduce our formalism and the vector quark model (VQ–model). In section 3 we outline our approach to calculate the VQ–contributions to the BRs and \(A_{CP}\) of \(b \rightarrow d \gamma\) and \(b \rightarrow s \gamma\). Section 4 presents the numerical results and section 5 contains our conclusions.

2 Vector quarks and the decays \(b \rightarrow s,d \gamma\)

There is much theoretical and experimental motivation to study the ratio

\[
R = \frac{BR(B \rightarrow X_d \gamma)}{BR(B \rightarrow X_s \gamma)}
\]

(1)

because it provides a clean handle on the ratio \(|V_{td}/V_{ts}|^2\) \([13]\). In the context of the SM, \(R\) is expected to be in the range \(0.017 < R < 0.074\), corresponding to \(BR(B \rightarrow X_d \gamma)\) of order \(10^{-5}\). \(R\) stays confined to this range in many popular models beyond the SM. This is because new particles such as charginos and charged Higgs bosons in the MSSM contribute to \(b \rightarrow s(d) \gamma\) with the same CKM factors. Therefore \(C_7\) is universal to both decays and cancels out in the ratio \(R\). In a model with vector quarks this is not the case, and we shall see that \(R\) can be suppressed or enhanced with respect to the SM\(^1\).

The dominant source of CP–Asymmetry in the decay modes \(B \rightarrow X_{s/d} \gamma\) is direct CP–violation. Mixing induced CP–violation is strongly suppressed by a factor of \(m_{s/d}/m_b\). The interference between mixing and decay necessary for this kind of CP–violation can only occur between identical final states. The photons from \(b \rightarrow s/d \gamma\) are predominantly left–handed, while those from \(\bar{b} \rightarrow \bar{s}/\bar{d} \gamma\) are predominantly right–handed, so interference is possible only between strongly suppressed contributions. (Since the weak interaction is left–handed, the spin flip in the quark line in the penguin loop must occur on an external leg, and this is proportional to the mass of the external quark involved.)

Mixing induced CP–violation becomes important for \(B \rightarrow X_{s/d} \gamma\) only if new, right–handed weak interactions are possible, because then the spin flip can occur inside the loop \([31]\). This is not relevant for the model we consider because our Vector Quarks are \(SU(2)\)–singlets and do not introduce right–handed couplings to the weak bosons.

The direct CP–Asymmetry is given by

\[
A_{CP}^{d/s} = \frac{\Gamma(B \rightarrow X_{d(s)} \gamma) - \Gamma(B \rightarrow X_{d(s)} \pi(\gamma))}{\Gamma(B \rightarrow X_{d(s)} \gamma) + \Gamma(B \rightarrow X_{d(s)} \pi(\gamma))} = \frac{\Delta \Gamma_{d/s}}{\Gamma_{d/s}^{tot}}
\]

(2)

\(^1\) SUSY models with non–flavour diagonal SUSY–breaking terms can also suppress or enhance \(R\) \([22, 33]\).
In the SM $A_{CP}^{d\gamma}$ is expected to lie in the range $-7\% \leq A_{CP}^{d\gamma} \leq -35\%$ [13], where the uncertainty arises from varying the Wolfenstein parameters $\rho$ and $\eta$ in their allowed ranges. Also included is the dependence of $A_{CP}^{d\gamma}$ on the scale $\mu_b$ which arises from varying $m_u/2 \leq \mu_b \leq 2m_b$. For definiteness we fix $\mu_b = 4.8$ GeV, and find $-5\% \leq A_{CP}^{d\gamma} \leq -28\%$. Therefore $A_{CP}^{d\gamma}$ is much larger than $A_{CP}^{\gamma}(\lesssim 0.6\%)$.

If $b \to d\gamma$ and $b \to s\gamma$ cannot be properly separated, then only $A_{CP}$ of a combined sample can be measured.\footnote{This is also true (though much less important numerically) for the total rate, where the $b \to d\gamma$ contribution should be subtracted from the “$b \to s\gamma$” sample as done e.g. in [5].} Neglecting the masses $m_s$ and $m_d$ and subleading CKM–factors $V_{us(d)}^* V_{ub}$ against $V_{ts(d)} V_{tb}$ (i.e. considering only the leading terms), it has been shown [18] that $A_{CP}^{\gamma}$ and $A_{CP}^{d\gamma}$ cancel each other\footnote{An improved analysis with non-vanishing quark masses showed that this cancellation still holds to a very high degree for both the inclusive and exclusive decays [34].}

In the presence of new physics such a cancellation does not occur, as was shown in [18] in the context of the effective SUSY model. As stressed in [18], a reliable prediction of $A_{CP}^{d\gamma}$ in a given model is necessary since it contributes to the measurement of $A_{CP}^{\gamma}$. The CLEO result [9] is sensitive to a weighted sum of CP asymmetries, given by:

$$A_{CP}^{\exp} = 0.965A_{CP}^{s\gamma} + 0.02A_{CP}^{d\gamma}$$

The latest measurement stands at $-27\% < A_{CP}^{\exp} < 10\%$ (90\% C.L.) [4]. The small coefficient of $A_{CP}^{d\gamma}$ is caused by the smaller $\text{BR}(B \to X_d\gamma)$ (assumed to be 1/20 that of $\text{BR}(B \to X_s\gamma)$) and inferior detection efficiencies.

If the detection efficiencies for both decays were identical, this measured quantity would coincide with the weighted sum of the asymmetries

$$A_{CP}^{s\gamma+d\gamma} = \frac{\text{BR}^{s\gamma} A_{CP}^{s\gamma} + \text{BR}^{d\gamma} A_{CP}^{d\gamma}}{\text{BR}^{s\gamma} + \text{BR}^{d\gamma}}.$$  

The two terms in eqs. (3,4) can be of equal or of opposite sign, i.e. they can contribute constructively or destructively to the combined asymmetry. Since $\text{BR}(B \to X_d\gamma)$ may be enhanced in the VQ–model, the relative strength of its contribution to $A_{CP}^{s\gamma+d\gamma}$ may be increased. The non–negligible contribution of $b \to d\gamma$ to this combined asymmetry should be verifiable at proposed future high luminosity runs of $B$ factories. For integrated luminosities of 200 $\text{fb}^{-1}$ ($2500 \text{ fb}^{-1}$), [34] anticipates a precision of 3\% (1\%) in the measurement of $A_{CP}^{\exp}$.

We shall be working in a model with an extra generation of vector quarks, $U$ and $D$. Both $U$ and $D$ are singlets under $SU(2)$ and in the interaction basis (denoted by $U'$ and $D'$) do not couple to the $W$. Their mass is generated by terms of the form:

$$-f_d^{i4} \bar{\psi}_L D'_R \phi - f_u^{i4} \bar{\psi}_L U'_R \phi + \text{h.c.},$$

where $f_d^{i4} (i = 1 \to 3)$ are Yukawa couplings and $\bar{\psi}_L = (\bar{u}_i, \bar{d}_i)_L$. Thus $U'$ ($D'$) will mix with the up (down) type quarks, resulting in the (undiscovered) mass eigenstates $U$ and $D$. The known quarks hence contain small amounts of the $U'$ and $D'$ weak eigenstates. A feature of the vector
quark model is the extended \(4 \times 4\) CKM matrix which is now not unitary. This can be clearly seen from the charged current interaction in the interaction basis:

\[
\mathcal{L} = \frac{g}{\sqrt{2}} (W^\mu_\mu J^{\mu+}_\mu + W^\mu_\mu J^{\mu-}_\mu)
\]

where

\[
J^{\mu-}_\mu = \overline{u}_L a_W \gamma^\mu d'_L = \overline{u}_L V_{\text{CKM}} \gamma^\mu d_L \quad \text{and} \quad u_L = \begin{pmatrix} u_L \\ c_L \\ t_L \\ U_L \end{pmatrix}, \quad d_L = \begin{pmatrix} d_L \\ s_L \\ b_L \\ D_L \end{pmatrix}
\]

Here the matrix \(a_W\) is \(\text{Diag}(1,1,1,0)\), reflecting the fact that the charged current couplings to the known quarks are flavour diagonal, while the vertex \(W-U'\) is absent. In the mass basis the vertices \(W-U-d\) (\(d\) denoting any down–type quark) are generated and thus an extra diagram with an internal \(U\) quark contributes to the decay \(b \to s\gamma, d\gamma\). The \(4 \times 4\) CKM matrix is then simply given by

\[
V_{\text{CKM}} = U^\dagger_L a_W U^d_L
\]

where \(U^\dagger_L\) and \(U^d_L\) rotate the mass eigenstates to the interaction eigenstates. Clearly \(V_{\text{CKM}}\) is not unitary due to the presence of \(a_W\). A consequence of this is the generation of FCNC vertices, where the effective coupling for the vertex \(Z-d_i-d_j\) is \((V^\dagger V)_{ij}\). This gives rise to a further class of diagrams that contributes to \(b \to s\gamma, d\gamma\) (c.f. section 3).

These FCNC vertices are strongly constrained by current experiments. The vertices \(Z-b\)–\(s\) and \(Z-b\)–\(d\) are e.g. constrained by the non–observance of \(B \to X_d l^+ l^-\) and \(B \to X_d l^+ l^-\), respectively \([25]\). (Note that the first evidence for \(B \to K\mu^+\mu^-\) has recently been reported by the BELLE collaboration \([26]\).)

We now briefly summarise the previous works which considered the effect of vector quarks on \(B \to X_d(s)\gamma\). (Work on these decays in other models has already been summarised in \([18]\).)

\([24,27]\) studied a model with a \(D\) type vector quark which induces \(Z\) and \(h\) FCNC contributions to \(B \to X_d(s)\gamma\). It was shown that these contributions have negligible impact on \(B \to X_s\gamma\), but can enhance or suppress \(R\). Implications for \(A_{CP}^{s\gamma}\) were found to be small.

The effect of the \(U-W\) contribution on \(A_{CP}^{s\gamma}\) was considered in \([24]\). In a model with both \(U\) and \(D\) vector quarks (where the \(D\)–contribution could actually be neglected) \([28]\), the \(U-W\) mediated contribution to \(\text{BR}(B \to X_s\gamma)\) was shown to permit BRs anywhere within the experimental limits. More accurate measurements of \(\text{BR}(B \to X_s\gamma)\) at the B factories will further restrict the available parameter space for VQ–models.

To our knowledge the combined effect of \(U\) and \(D\) vector quarks on \(\text{BR}(B \to X_d\gamma)\) has not yet been considered. In addition, no analysis of \(A_{CP}^{d\gamma}\) in any model with vector quarks has been carried out. Given the importance of the measurement of the combined asymmetry \(A_{CP}^{s\gamma+d\gamma}\), we will also study the correlation of the individual asymmetries \(A_{CP}^{s\gamma}\) and \(A_{CP}^{d\gamma}\) as a function of the VQ parameters.

\footnote{It is obvious where \(u_L\) denotes the 4–vector of up–type quarks and where it denotes the up–quark only}
3 Direct CP Asymmetry in $B \to X_{d,s} \gamma$

In this section we explore the effect of $U$ and $D$–type vector quarks on both $\text{BR}(B \to X_{d,s} \gamma)$ and the direct CP asymmetries, $\mathcal{A}_{CP}^{d,s \gamma}$. Diagrams contributing to $b \to s,d \gamma$ in the VQ–model are shown in fig. 1.

The effective Hamiltonian for $b \to d \gamma$ is given by

$$H_{eff} = -\frac{4G_F}{\sqrt{2}} V_{td}^* V_{tb} \sum_{i=1}^{8} C_i(\mu_b) Q_i(\mu_b)$$

where $Q_i(\mu_b)$ are the current density operators for the $\Delta B = 1$ transitions and $C_i(\mu_b)$ are their Wilson coefficients. The relevant operators for $b \to d \gamma$ decay are given by

$$Q_2 = \bar{d}_L \gamma^\mu c_L \bar{c}_L \gamma^\mu b_L,$$

$$Q_7 = \frac{\kappa}{16\pi^2} m_b \bar{d}_L \sigma^{\mu\nu} b_R F_{\mu\nu},$$

$$Q_8 = \frac{g_s}{16\pi^2} m_b \bar{d}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a.$$ 

The analogous formulae for the $b \to s \gamma$ decay can obtained from Eqs. (9) and (10) by making the replacement $d \to s$.

We use formula (27) from [20]:

$$\mathcal{A}_{CP}^{d(s)\gamma} = \frac{\alpha_s(m_b)}{|C_7|^2} \left\{ \frac{40}{81} \Im[(1 + \Delta_{d(s)}) C_2 C_7^*] - \frac{4}{9} \Im[C_8 C_7^*] + \frac{8z}{3} [v(z) + b(z, \delta)] \right\} \left\{ \Im[(1 + \epsilon_{d(s)} + \Delta_{d(s)}) C_2 C_7^*] + \frac{8z}{27} b(z, \delta) \Im[(1 + \epsilon_{d(s)} + \Delta_{d(s)}) C_2 C_8^*] \right\}$$

where in our notation $\Delta_x = (z_{xb} - V_{ux}^* V_{Ub})/(V_{ux}^* V_{ub})$ for $(x = d, s)$.

The expressions for the Wilson coefficients at the $M_W$ scale may be divided into the charged current and neutral current mediated contributions, where only the former exist in the SM. Contrary to the SM case, the charged current mediated contribution contains a term stemming from a loop with the $U$ vector quark.

Thus at the $M_W$ scale we have:

$$C_7 = C_7^W + C_7^ZH$$
We will use the leading order expressions for $C_7$ (with the constant terms included in the Inami–Lim functions) which can be found in [29].

In the SM the CKM–factors are customarily included in the pre-factor of the effective Hamiltonian. This is possible because the top–quark loop is the dominant contribution to $C_7$ and the CKM–factor $V_{cb}^*V_{tb}$ in $C_2$ can be reexpressed in terms of $V_{ts}^*V_{tb}$ utilizing CKM unitarity.

In the vector quark model this is not possible anymore. For $C_7$, the constant terms in the Inami–Lim functions that are cancelled in the SM due to CKM unitarity, have to be taken into account. Also in replacing $V_{cb}^*V_{tb}$ in $C_2$, additional terms arising from CKM non–unitarity are generated. This results in different Wilson coefficients for $b \to s\gamma$ and $b \to d\gamma$.

For comparison with standard calculations, we keep the term $V_{td(s)}^*V_{tb}$ in the pre-factor of $H_{eff}$ and therefore have to divide all but the top–quark contributions to the Wilson coefficients by this factor.

We define $C_7^{s\gamma}$ and $C_7^{d\gamma}$ as the Wilson coefficient for $b \to s\gamma$ and $b \to d\gamma$ respectively. The magnitude of $|C_7^{s\gamma}|$ is constrained by measurements of BR($B \to X_s\gamma$) [28].

In contrast, $C_7^{d\gamma}$ is only very weakly constrained due to the non–observation of $b \to d\gamma$. Recent direct searches for the exclusive decay $B \to \rho\gamma$ give an improved upper bound on $R_{excl}$ for exclusive decays of $R_{excl} \lesssim 0.19$ (90% C.L.) [11]. One expects a weaker bound on the inclusive $R$, since estimates of $R_{excl}/R$ are smaller than 1 [27].

On the other hand the presence of $b \to d\gamma$ events in the samples of the inclusive measurements of BR($B \to X_s\gamma$) [3] also constrains $R$. This bound, however, is not relevant for our analysis, since in the vector quark model the prediction for $b \to s\gamma$ can easily be lowered to account for a higher admixture of $b \to d\gamma$ in the experimental sample.

4 Numerical Results

We vary the vector quark parameters in the ranges given in table [1]. The Wolfenstein parameters $\rho$ and $\eta$ quantify the current uncertainty in the CKM parameters related to the mixing of the first and the third generation of quarks. Our results are virtually independent of our choice of $m_{U/D}$ and $M_H$.

In the $s$–sector we use $|V_{Us}^*V_{Ub}| < 0.004$ which is 1/3 smaller than the bound derived from BR$^{s\gamma}$ in [28], we take this smaller value in the light of recently improved measurements of $b \to s\gamma$. The non–unitarity parameter $z_{sb}$ ($z_{db}$) is constrained from non–observation of $B \to X_s\ell^+\ell^-$ ($B \to X_d\ell^+\ell^-$) [25].

Our results depend crucially on the magnitude of $|V_{Ud}^*V_{Ub}|$. Bounds from CKM unitarity on this quantity are rather weak (of the order of the uncertainty in the CKM parameters in the $d$–sector) and $B \to X_d\gamma$ has not yet been observed, so the only restrictions on this parameter come from $B^0\bar{B}^0$–mixing [22] [24]. The exact bounds on $|V_{Ud}^*V_{Ub}|$ from $B^0\bar{B}^0$–mixing are not clear and depend on the tree level $Z$–mediated contributions as well as the rather large uncertainties in the SM prediction of $B^0\bar{B}^0$–mixing (bag parameter, etc.). We give results for different choices of $|V_{Ud}^*V_{Ub}|$ up to 0.01 ($\approx |V_{ts}^*V_{tb}|$). A more detailed study of $B^0\bar{B}^0$–mixing in the VQ model will be presented elsewhere [27].

In figure 2 we plot $A_{CP}^{s\gamma}$ against $A_{CP}^{d\gamma}$. It can be seen that while $A_{CP}^{s\gamma}$ does not substantially differ from its SM value, $A_{CP}^{d\gamma}$ can vary over a much larger range. We restrict ourselves to
Figure 2: CP asymmetry of $b \rightarrow d\gamma$ against CP asymmetry of $b \rightarrow s\gamma$. The VQ-parameters are varied in the ranges given in table 1; each point in the diagram corresponds to one random point in parameter space.

| figure# | $\rho$ | $\eta$ | $m_{U/D}$ | $M_H$ | $|V_{U,s}^* V_{U/b}|$ | $|V_{U,d}^* V_{U/b}|$ | $|z_{sb}|$ | Arg $z_{sb}$ | $|z_{db}|$ | Arg $z_{db}$ |
|---------|-------|-------|---------|------|-----------------|-----------------|--------|-------------|--------|-------------|
| 2 (min) | -0.1  | 0.2   | 250     | 100  | 0               | 0               | 0      | 0           | 0      | 0           |
| 2 (max) | 0.4   | 0.5   | 1000    | 200  | 0.004           | 0.008           | 8.1 \cdot 10^{-4} | 2\pi    | 0.001       | 2\pi    |
| 3–6     | 0.4   | 0.5   | 500     | 100  | 0.004           | variable        | 8.1 \cdot 10^{-4} | 0       | 0.001       | $\pi$   |

Table 1: Model parameter ranges
points in parameter space where $|A_{d\gamma}^{CP}| < 45\%$. Asymmetries greater than 50\% are attainable, but these correspond to cases with a virtually complete cancellation of the SM contribution by the new VQ contribution ($|C_{d\gamma}^{7}| \ll 0.3$). Such large asymmetries are untrustworthy, since our formula for the CP asymmetry (2) breaks down in these cases.

The correlation between $A_{d\gamma}^{CP}$ and $BR^{d\gamma}$ is studied in detail in figure 3, where it can be seen that $|A_{d\gamma}^{CP}| > 45\%$ occurs only for $BR^{d\gamma} < 10^{-6}$. Branching ratios of this magnitude would require $\gg 10^8 b\bar{b}$ pairs to be detected which is beyond the discovery potential of current $B$ factories. Note that figure 3 was obtained with fixed values for the CKM parameters $\rho$ and $\eta$ as given in table 1. Varying $\rho$ and $\eta$ shifts the contours in figure 3 vertically by the amount of uncertainty in the SM prediction of $A_{d\gamma}^{CP}$.

In figure 4 we plot the ratio of the branching ratios for $b \to d\gamma$ and $b \to s\gamma$ against the argument of $V_{Ud}^* V_{Ub}$, drawing curves for different values of $|V_{Ud}^* V_{Ub}|$. For real, positive $V_{Ud}^* V_{Ub}$, there is constructive interference with the SM contribution proportional to $V_{td}^* V_{tb} = A\lambda^3 (1 - \rho - i\eta) + O(\lambda^4)$. For real, negative $V_{Ud}^* V_{Ub}$ (corresponding to points around $\text{Arg}(V_{Ud}^* V_{Ub}) = \pi$) the interference is destructive. Note that the smallest values for $R$ are not obtained for the largest $|V_{Ud}^* V_{Ub}|$, because if $|V_{Ud}^* V_{Ub}| > |V_{td}^* V_{tb}|$, exact cancellation cannot occur.

In figure 5 we plot the combined CP asymmetry as defined in equation (4) against the argument of $V_{Ud}^* V_{Ub}$. We plot several curves for different values of $|V_{Ud}^* V_{Ub}|$ just like in figures 3 and 4. The extreme values for the combined asymmetry are obtained where $R$ is maximal and not — as could be naively expected — where the CP asymmetry of $b \to d\gamma$ becomes much larger than its SM value. This is because large CP asymmetries in $b \to d\gamma$ as seen in figure 3 always imply small values for $R$ which makes these points in parameter space unimportant for the combined asymmetry.

Note that in our analysis $BR^{s\gamma}$ and $A_{CP}^{b\gamma+s\gamma}$ are close to their SM values. The huge variations in $A_{CP}^{s\gamma+d\gamma}$ stem from the variation in $BR^{d\gamma}$. In wide ranges of our parameter space, $b \to d\gamma$ actually dominates the combined asymmetry! Any large signal observed in $A_{CP}^{s\gamma+d\gamma}$ (which is
Figure 4: $R$ against $\text{Arg} \, V_{Us}^* V_{Ub}$ for different values of $|V_{Us}^* V_{Ub}|$

Figure 5: Combined Asymmetry against $\text{Arg} \, V_{Us}^* V_{Ub}$ for different values of $|V_{Us}^* V_{Ub}|$

the experimentally relevant quantity) should not necessarily be attributed to $b \to s \gamma$ only. This motivates excellent $K-\pi$ separation which is crucial in distinguishing the two decay modes.

To obtain a rough estimate on the number of $b\bar{b}$ pairs $N_{b\bar{b}}$ required to establish CP violation to a given significance, following [19] we define the observability $\Omega_{d(s)\gamma} = (A_{CP}^{b\to d(s)\gamma})^2 \cdot \text{BR}^{d(s)\gamma}$, where $\Omega_{d(s)\gamma} \propto 1/N_{b\bar{b}}$. This observability is plotted in figure 4 against $\text{Arg} \, V_{Us}^* V_{Ub}$, where the horizontal lines indicate the SM observabilities. (We do not take into account different detection efficiencies for the two channels.) Despite the suppressed $\text{BR}^{d\gamma}$ in the SM, $A_{CP}^{b\to d\gamma}$ is much more observable than $A_{CP}^{b\to s\gamma}$. In most of the parameter space, $A_{CP}^{b\to d\gamma}$ has a higher observability than in the SM. The optimal observability occurs for the largest values of $R$ (compare fig. 4).
Figure 6: CP asymmetry and observability $\Omega$ against $\text{Arg } V_{Ud}^* V_{Ub}$

5 Conclusions

We have studied the effect of a single generation of vector quarks (VQ) on the short-distance component of the inclusive decays $B \to X_{d,s} \gamma$. We found that $A_{CP}^{b \to s \gamma}$ is relatively insensitive to VQ interactions if the experimental bounds on $\text{BR}^{s \gamma}$ are respected. For the decay mode $b \to d \gamma$ both the decay rate and the CP asymmetry can be significantly different from their SM values. Phenomenologically most salient is the fact that the decay rate can be increased up to current experimental observability or decreased beyond the sensitivity of the current $B$ factories. This variation of $\text{BR}^{d \gamma}$ has a profound impact on the experimentally relevant CP asymmetry of a combined sample of $b \to s,d \gamma$ which can actually be dominated by $A_{CP}^{b \to d \gamma}$. Observation of any $A_{CP}^{s,d \gamma}$ significantly different from zero is a clear sign of new physics, but the theoretical interpretation should await experimental separation of the individual channels.

Acknowledgements

The authors wish to thank A. Arhrib and Y. Okada for useful discussions and comments. S.R was supported by the Japan Society for the Promotion of Science (JSPS).
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