Tackling gravity wave confusion noise with template optimizers

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Abstract. The Mock LISA Data Challenge 4.0 simulated the joint two-year recording of gravitational wave signals from mergers of spinning black holes, extreme mass ratio inspirals, Galactic white dwarf binaries, bursts from cosmic strings, and a stochastic background—all over LISA instrument noise. We analysed this data using a global multi-start box and bound optimization scheme, incorporating multi-dimensional Nelder Mead simplex 2 optimization. Our scheme identified 2658 binaries. Of these, 2246 were found to systematically decompose the power in a strong spinning black hole merger into a “white dwarf binary transform”. The remaining 416 binaries were identified with a false alarm rate of ∼23%.

1. Introduction
The Laser Interferometer Space Antenna (LISA) was a proposed joint venture of ESA and NASA to observe gravitational waves in the low frequency range from $10^{-4} - 10^{-1}$ Hz [1]. The proposed configuration was a constellation of 3 spacecraft in heliocentric orbit. The constellation forms an interferometer from a nearly equilateral triangle with 5 million km arms. The likely sources of gravitational radiation in this range are supermassive black hole mergers at cosmological distances, extreme mass ratio inspirals of compact objects into supermassive black holes, and the Galactic population of close white dwarf binaries. A stochastic background of gravitational waves and bursts from cosmic strings are less likely, but nevertheless possible sources. All of these sources will be recorded by LISA simultaneously, and this forms the main challenge of LISA data analysis—how to disentangle all of these sources.

The Mock LISA Data Challenges were created to encourage the data analysis community to develop solutions to the problem of identifying all of these sources [2]. There have been 5 challenges associated with the MLDC. Challenge 1 introduced several source types, each with its own data stream [2]. The Challenge 2 included a data stream containing a Galactic population of $\sim 26 \times 10^6$ white dwarf binaries, although all the systems were monochromatic (i.e. $\dot{f} = 0$) [3]. The most successful search algorithm to date was able to extract $\sim 20,000$ binaries, many of which had cross-correlations greater than 0.9 [3]. Unfortunately, this code [4, 5] has been lost [6], and is no longer in use. Following a reprise of Challenge 1 that saw the introduction of new algorithms [7], Challenge 3 incorporated chirping and mass-transferring systems. The three submissions to Challenge 3 had varying degrees of success, recovering 494 [8], 1940 [9], and 14,838 [10], but with somewhat low cross-correlations with the true population [11]. In Challenge 4, a single two year data stream was simulated containing Galactic binaries, supermassive
black hole mergers, extreme mass ratio inspirals, bursts from cosmic strings, and a stochastic background. We have developed an analysis algorithm to identify Galactic binaries from within the LISA data stream [11]. We tested our algorithm against Challenge 3 of the MLDC, which had only the Galactic population of white dwarf binaries and recovered 7131 binaries with cross-correlations greater than 0.9. When applied to the Challenge 4 data, the additional signals from the supermassive black hole mergers have proven to be problematic and significantly reduced the efficacy of our algorithm. These signals must be removed for the algorithm to be fully successful.

2. The Algorithm

The gravitational wave signal from a close white dwarf binary can be described by 8 parameters. The biggest challenge with identifying individual close white dwarf binaries from within the combined Galactic signal of 30 million sources is that a global fit requires searching through an $8N$-dimensional space for the optimal fit with the data, but $N$ is an unknown number to be determined by the analysis. Previous solutions to this problem have included fixing $N$, or through single signal subtraction schemes where individual binaries are identified and subtracted from the signal [10, 12, 13]. We use an adaptive multi-dimensional/multi-signal layered reduction approach. We work in the frequency domain using the rigid adiabatic approximation for our template models and process only isolated data snippets covering the frequency spread of a single signal. We adaptively add signals to the single-signal solution parameter space if the signals are found to be overlapping.

A match between template and signal is determined by minimizing the value of $\chi^2$. The minimization technique that we use is the Nelder-Mead simplex 2 (NM2) algorithm. The Nelder-Mead optimization technique was proposed in 1965 by John Nelder and Roger Mead [14] in order to minimize an objective function in a many-dimensional space without the use of derivatives. As first proposed, the algorithm scaled with the dimension $n$ of the space as $O(n^2)$. Recent advances in computing have allowed the implementation of the simplex 2 algorithm, an improvement over the initial one, with an $O(n)$ scaling.

This improvement has allowed us to more efficiently search over larger parameter spaces without relying on more costly MCMC techniques. We use a multi-start global optimization scheme in which we invoke the NM2 at multiple starting points in parameter space and record the many local minima that are obtained. The global minimum is then found by comparing the local minima. We guide and tune the multi-start by using a box-and-bound method in which we restrict the search over parameter space by resampling from an initial random distribution of sampling points. The $\chi^2$ surface is sampled with a coarse grid for an initial distribution and the probability of resampling points is weighted by the $\chi^2$ values from the initial sample. In this way we focus our multi-start distribution on regions of parameter space where the solution is most likely to be found. This can be seen in Figure 1.

3. Tuning on MLDC 3.1.0

We tuned our algorithm on the data set from MLDC Challenge 3 that contained only Galactic binaries. The MLDC 3.1.0 data set contained 30 million close white dwarf binaries and included systems with a drifting frequency. The change in frequency could be in either direction and was due to either gravitational wave inspiral ($f > 0$) or stable mass transfer ($f < 0$). With a run time of 15840 CPU-hours on the UTB cluster Futuro, the algorithm identified 7131. We determined the efficiency of the algorithm by using the cross-correlation tool from the MLDC assessment suite and identified any source that was found with a cross-correlation greater than 0.9 as a successful recovery. This allowed us to impose thresholds in amplitude and frequency in order to eliminate many false alarms. The thresholds are shown in Figure 2. With these thresholds, we determined a 98.1% efficiency of detection and had 6996 binaries with a cross-correlation greater than 0.9.
**Figure 1.** An example of the box-and-bound method in the two-dimensional slice of parameter space for sky location. The initial distribution of multi-start points is randomly distributed throughout the parameter space, but after a few resamplings, the distribution has collapsed about the true values shown by crosses.

**Figure 2.** Recovered binaries in the frequency-amplitude plane of parameter space from the tuning runs on MLDC 3.1.0. The found binaries are shown in light grey (green in the online version), while missed binaries are shown in dark grey (red in the online version). The frequency and amplitude thresholds are also shown.
4. Application to MLDC 4
The data set that was provided with the fourth round of the Mock LISA Data Challenge was known as the “whole enchilada.” This is because it was a single set containing all sources of the separate MLDC 3 challenges combined together. These sources include supermassive black hole mergers, extreme mass ratio inspirals, a cosmological stochastic background, cosmic string bursts, and the full Galactic population of interacting and detached white dwarf binaries. Although our code was intended solely to operate on a cleaned data set, we applied it to the whole enchilada without removing any of the other signals. With a runtime on Futuro of 16810 CPU-hours, our code identified 8192 binaries. After thresholding based on tuning with known binaries from the training set, 2658 signals remained. However, 2246 of these in the very low frequency regime had a strange feature of nearly matching characteristics. We assumed that these binaries were the result of a flaw in the analysis and discarded them as well. This left us with 412 recovered binaries that we submitted to the MLDC task group—a rather poor performance. After comparison with the key-file for challenge 4 and applying the cross-correlation assessment, we found a detection efficiency of 77.1%. The missed-found plot for these binaries is shown in Figure 3.

![Figure 3](image-url)

**Figure 3.** Recovered binaries in the frequency-amplitude plane of parameter space from the application of our code to the MLDC 4 data set. The true Galaxy is shown in dark grey (blue in the online version), the missed or false binaries are shown in grey (red in the online version), and the found binaries are shown in light grey (green in the online version). The 2246 discarded binaries are circled in the lower right-hand corner.
5. The White Dwarf Binary Transform

After the key to the data was released, post-processing showed that a very strong supermassive black hole merger was placed just at the frequency range where the unusual population of binaries was found. We therefore hypothesized that these matching binaries were actually a white dwarf binary wavelet transform of the supermassive black hole binary. In order to check this result, we superposed the frequency-domain of the time-delay interferometry signal of the supermassive black hole binary with the unusual, discarded white dwarf binaries. We found that 1319 of the white dwarf binaries corresponded to a match with the supermassive black hole binary. This is shown in Figure 4. We then compared the time domain response of the black hole merger and the superposition of these false alarms in Figure 5. We notice a good visual match of the white dwarf binary wavelet transform with the generic waveform for a black hole merger. However, the match is qualitative at best and lacks the necessary fidelity to give a quantitative match to the exact waveform.

6. Discussion

We have demonstrated that our code is CPU-efficient, yet powerful enough to untangle source confusion in high numbers at low false alarm rates. However, this success is predicated on a clean
The barycentric time response of the white dwarf binary transform (above) compared with the true supermassive black hole binary merger signal (below). There is a qualitative match, but not a quantitative one.

data stream, and so a subtraction of loud supermassive binary black hole mergers is required in order to fully realize the potential of our technique. Despite this, our technique has yielded 412 signals at an efficiency of 77.1% in the presence of very strong source confusion from other source classes. We further presented qualitative evidence that our data analysis can also identify and decompose a strong binary black hole merger at low frequencies by means of a white dwarf binary transform. We acknowledge the accidental character of the white dwarf binary transform and therefore do not claim conclusive quantitative proof. It is up to future work to concentrate on this property and fine-tune the approach to completely and quantitatively decompose a black hole merger signal beyond doubt. We highlight the prospect of such an approach, as it may be able to render the frequency and time domain response of a spinning black hole merger without ever applying any kind of numerical template. If successful, this work would show that it may be possible to recover strong waveforms even if the most sophisticated template calculated today may mismatch the reality of a gravitational waveform. [15]

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