BIMETRIC GRAVITY AND DARK MATTER

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We review some recent proposals for relativistic models of dark matter in the context of bimetric gravity. The aim is to solve the problems of cold dark matter (CDM) at galactic scales, and to reproduce the phenomenology of the modified Newtonian dynamics (MOND), while still being in agreement with the standard cosmological model Λ-CDM at large scales. In this context a promising alternative is dipolar dark matter (DDM) in which two different species of dark matter particles are separately coupled to the two metrics of bigravity and are linked together by an internal vector field. The phenomenology of MOND then results from a mechanism of gravitational polarization. Probably the best formulation of the model is within the framework of recently developed massive bigravity theories. Then the gravitational sector of the model is safe by construction, but a ghostly degree of freedom in the decoupling limit is still present in the dark matter sector. Future work should analyze the cosmological solutions of the model and check the post-Newtonian parameters in the solar system.

1 Introduction and motivation

1.1 Problem of dark matter at galactic scales

The standard model of cosmology Λ-CDM is widely held to be an excellent description of reality at large cosmological scales. Impressive observational successes of this model include the fit of the anisotropies of the cosmic microwave background (CMB), the baryon acoustic oscillations, the formation of large scale structures and the accelerated expansion of the Universe. However some fundamental issues remain: (i) The measured value of the cosmological constant Λ looks unnatural from a quantum field perspective; (ii) The weakly interacting particles envisaged as candidates for the cold dark matter (CDM) are still undetected in the laboratory; (iii) The model Λ-CDM falls short in explaining the observed regularities in the properties of dark matter halos around galaxies.

Regarding point (iii) we have in mind the baryonic Tully-Fisher relation between the observed luminous mass and the asymptotic rotation velocity of spiral galaxies, the analogous Faber-Jackson relation for elliptical galaxies, and, very important, the tight correlation between the mass discrepancy (i.e. the presence of dark matter) and the involved acceleration scale. In the prevailing view, these issues should be resolved once we understand the complicated baryonic processes (e.g. supernova winds and outflows from a central supermassive black hole) that affect galaxy formation and evolution. Note that with such foreseeable explanation, conclusive tests of
the Λ-CDM model become difficult since any new failure can be attributed to some further unknown aspect of baryonic physics. More importantly, this explanation is challenged by the fact that galactic data are — mysteriously enough — in excellent agreement with the MOND (MOdified Newtonian Dynamics) empirical non relativistic formula. A relativistic MOND theory is required to address issues concerning cosmology and gravitational lensing. Many relativistic extensions for MOND have been proposed, including a tensor-vector-scalar (TeVeS) theory, a bimetric theory, non-canonical Einstein-Æther theories, a Galileon theory, a Khronon theory, a modified dark matter theory. Most theories have difficulties at reproducing the cosmological observations, notably the full spectrum of CMB anisotropies. An exception is the modified dark matter approach, which agrees with the model Λ-CDM at first order cosmological perturbations. This approach, also called dipolar dark matter (DDM), is motivated by the dielectric analogy of MOND — that MOND represents the gravitational analogue of the Gauss law of electrostatics in dielectric non-linear materials. A natural formulation of the model is based on a bimetric extension of general relativity (GR), where two species of dark matter particles are respectively coupled to the two metrics, and are linked by an internal vector field generated by the mass of these particles. In this model the phenomenology of MOND emerges naturally and elegantly from a mechanism of gravitational polarization.

1.2 Bimetric massive gravity theories

Bimetric theories have been extensively investigated in the quest of a consistent massive gravity theory. From a theoretical point of view, the existence of a graviton mass is a very important fundamental question. At the linear level, there is a unique mass term which ensures the absence of ghosts in the theory, namely the Fierz-Pauli action. Even though theoretically consistent, this linear theory suffers from the van Dam-Veltman-Zakharov (vDVZ) discontinuity, which reflects the fact that one does not recover GR in the limit of vanishing graviton mass. In other words, this discontinuity can be traced to the coupling of the additional longitudinal graviton to the matter field in the mass going to zero limit. Vainshtein very soon realized that the nonlinearities of the theory become actually stronger in the vanishing mass limit, pointing that these nonlinearities might cure the vDVZ discontinuity, which required the nonlinear completion of the Fierz-Pauli theory. Unfortunately, this task seemed to face the inevitable problem of reintroducing the ghost-like instability.

Recent joint effort to construct a consistent ghost-free non-linear theory for massive gravity gave fruitful results, which have initiated a revival of interests. The theory can be further generalized to bigravity, or multigravity, by the inclusion of the corresponding additional kinetic terms. Another fundamental question in the context of massive gravity is the consistent coupling to the matter fields. If one couples the matter fields in a naive way additively to both metrics simultaneously, this immediately reintroduces the Boulware-Deser (BD) ghost. Moreover, quantum corrections at one loop dictate that the only consistent way of coupling the matter sector to the two metrics is either through a minimal coupling to just one metric (which preserves the technical naturalness of the theory), or through a composite effective metric built out of both metrics in a very specific way. An important consequence of the coupling through the effective metric is a possible way out of the no-go result for the flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric together with the propagation of the five physical degrees of freedom of the graviton sector without introducing ghost and gradient instabilities.

Massive gravity is replete of phenomenology. Especially, its potential application in cosmology received much attention. Even if the decoupling limit of the theory admits self-accelerating solutions, the full theory with Minkowski reference metric suffers from the no-go result for flat FLRW solution. The cosmology of the bigravity theory has more freedom and features due to the dynamics of the reference metric. Assuming that the matter fields couple
minimally to one metric and further assuming that the mass of the graviton is of the same order as the Hubble parameter today, the theory admits several interesting branches of solutions.

1.3 Content and summary

In Sec. 2 of this paper we shall review the initial bimetric model and the broad range of phenomenology it is able to predict. On the bad side of this model, we shall also discuss, in Sec. 2.3, with the help notably of the minisuperspace of the model, the likely presence of ghost instabilities. This will motivate the redefinition, in Sec. 3, of the gravitational sector of the model in a way to make it consistent with the beautiful framework of massive bigravity theories. Gladly, this move will substantially simplify the model. On the other hand, the dark matter sector will essentially remain the same as in . The mechanism of gravitational polarization, and recovery of the MOND equation, checked in Sec. 3.2, thus appear as a natural consequence of massive bigravity theory for this type of matter. Concerning ghosts, the new model is safe in the gravitational sector (by construction), but a ghostly degree of freedom in the decoupling limit is reintroduced in the dark matter sector . A crucial question to address in future work is whether the polarization mechanism can be realized in absence of ghosts.

2 Bimetric theory with two dark matter species

2.1 Relativistic action

A relativistic model involving, in addition to the ordinary matter simply described by baryons, two species of dark matter particles, was proposed in . A vector field \( A_\mu \) is sourced by the mass currents of dark matter and dubbed graviphoton. This vector field is crucial in order to ensure the stability of the dipolar medium. The gravitational sector is composed of two dynamical Lorentzian metrics \( g_{\mu\nu} \) and \( f_{\mu\nu} \). The baryons (representing in fact the full standard model of particle physics) are coupled in the usual way to the metric \( g_{\mu\nu} \). The two species of dark matter particles are respectively coupled to the two metrics \( g_{\mu\nu} \) and \( f_{\mu\nu} \). The gravitational-plus-matter action of the model reads (in geometrical units \( G = c = 1 \))

\[
S = \int d^4x \left\{ \sqrt{-g} \left( \frac{R_g - 2\lambda_g}{32\pi} - \rho_{\text{bar}} - \rho_g \right) + \sqrt{-f} \left( \frac{R_f - 2\lambda_f}{32\pi} - \rho_f \right) + \sqrt{-G_{\text{eff}}} \left[ \frac{R_{\text{eff}} - 2\lambda_{\text{eff}}}{16\pi\varepsilon} + (J_\mu^g - J_\mu^f)A_\mu + \frac{a_0^2}{8\pi} W(X) \right] \right\}.
\]

Here \( R_g, R_f \) and \( R_{\text{eff}} \) are the Ricci scalars of the \( g \) and \( f \) metrics, and of an effective composite metric \( G_{\mu\nu}^{\text{eff}} \) defined non-perturbatively from \( g_{\mu\nu} \) and \( f_{\mu\nu} \) by

\[
G_{\mu\nu}^{\text{eff}} = g_{\mu\rho}X^\rho_\nu = f_{\mu\rho}Y^\rho_\nu,
\]

where the square root matrix is defined by \( X = \sqrt{g^{-1}f} \), together with its inverse \( Y = \sqrt{f^{-1}g} \). Notice that \( G_{\mu\nu}^{\text{eff}} \) can be shown to be automatically symmetric. The dimensionless coupling constant \( \varepsilon \) measures the strength of the interaction between the two sectors \( g \) and \( f \). We inserted three different cosmological constants \( \lambda_g, \lambda_f \) and \( \lambda_{\text{eff}} \) in the \( g, f \) and interaction sectors. They will be ultimately related to the observed cosmological constant \( \Lambda \).

Baryons and dark matter particles are described by pressureless fluids with conserved scalar densities \( \rho_{\text{bar}}, \rho_g \) and \( \rho_f \). In addition \( J_\mu^g \) and \( J_\mu^f \) stand for the mass currents of the two types

\^ Actually the effective composite metric was defined in \(^{20}\) by the different condition

\[
G_{\mu\nu}^{\text{eff}} = g_{\rho\sigma}g_{\mu\nu}f_{\rho\sigma} = G_{\rho\sigma}g_{\mu\nu}f_{\rho\sigma}.
\]

It has been shown \(^{44}\) that this condition is equivalent to the requirement (2).
of dark matter particles, defined by

$$\sqrt{-G_{\text{eff}}} J^\mu_g = \sqrt{-g} J^\mu_g, \quad \sqrt{-G_{\text{eff}}} J^\mu_f = \sqrt{-f} J^\mu_f,$$

where $J^\mu_g = \rho_g u^\mu_g$ and $J^\mu_f = \rho_f u^\mu_f$ are the conserved dark matter currents associated with the respective metrics $g$ and $f$, thus obeying $\nabla_\mu J^\mu_g = 0$ and $\nabla_\mu J^\mu_f = 0$.

The vector field $A_\mu$ obeys a non-canonical kinetic term $W(X)$ where

$$W(X) = -\frac{G^{\mu\rho}_{\text{eff}} \sigma^\alpha_{\mu\rho} \mathcal{F}_{\mu\nu} \mathcal{F}_{\rho\sigma}}{2a_0^2},$$

with $\mathcal{F}_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Note that the vector field strength in (4) has been rescaled by the MOND acceleration $a_0 \approx 1.2 \times 10^{-10}$ m/s$^2$. The function $W(X)$ is determined phenomenologically, but in principle it should be interpreted within some more fundamental theory. In the limit $X \ll 1$, which corresponds to the MOND weak-acceleration regime below $a_0$, we impose

$$W(X) = X - \frac{2}{3} X^{3/2} + \mathcal{O}(X^2).$$

On the other hand we also impose that when $X \gg 1$, corresponding to the strong-acceleration regime much above $a_0$,

$$W(X) = A + \frac{B}{X^b} + \mathcal{O}\left(\frac{1}{X^b}\right),$$

where $A$ and $B$ are constants and $b > 0$.

2.2 Phenomenological predictions

We now review the rich phenomenology of this model, which is quite successful at different scales and in different regimes. We refer to [20] for the details.

- Cosmology. We study the cosmology of the model by expanding the two metrics around two homogeneous and isotropic FLRW background metrics,

$$d\bar{s}_g^2 = a_g^2 [-d\eta^2 + \gamma_{ij} dx^i dx^j],$$

$$d\bar{s}_f^2 = a_f^2 [-d\eta^2 + \gamma_{ij} dx^i dx^j],$$

where $\eta$ is the conformal time and $a_g$ and $a_f$ are the two different scale factors. We show that the consistency of the equations in the background, where the two metrics superpose, is ensured provided that the background densities obey,

$$\bar{\rho}_b = \frac{(\alpha - 1)(\varepsilon - 1)}{\alpha + \varepsilon} \bar{\rho},$$

where $\alpha = a_g/a_f$ is constant, and $\bar{\rho}$ is the common density of the two dark matter fluids in the background. The cosmological constants should also be related to the observed cosmological constant $\Lambda$, through the relations,

$$\lambda_g = \Lambda, \quad \lambda_f = \alpha^2 \Lambda, \quad \lambda_{\text{eff}} = \alpha \Lambda.$$

Then, at first order cosmological perturbations around the FLRW metrics, we define the $g$-sector as being the observable one because it is where the baryons live and light propagates, and all observations take place. We thus study the perturbation equations in this sector. We define some effective dark matter quantities that modify the $g$-type dark matter particles taking into account their interaction with the other sector. Using these new variables we find that the perturbation equations in the $g$ sector take exactly the same form as those of the $\Lambda$-CDM model. Thus the model turns out to be indistinguishable from $\Lambda$-CDM up to first order perturbation, and is then fully consistent with the observed fluctuations of the CMB.
MOND. At galactic scales, in a regime of small accelerations $a \ll a_0$, the potential function $W$ takes the form (5). Based on a particular solution for the equation of the vector field $A_\mu$, the two dark matter fluids can be described as a polarizable dipolar medium. The MOND equation is then recovered as a result of a mechanism of gravitational polarization which appears as a natural consequence of the model. Moreover the dipolar dark matter medium undergoes stable plasma-like oscillations. We shall give some more details of this polarization mechanism in Sec. 3.2 at the occasion of the next model based on massive bigravity. However note that the next model has essentially the same predictions with regards to MOND phenomenology as the model. There is though a difference, in that in the model we have to assume that the coupling constant $\varepsilon$ in the action (1) tends to zero, while no particular requirement will be necessary in the next model.

Solar System. As we modify GR we have to check the post-Newtonian limit in the Solar System, for which the potential function $W$ in the action takes the form (6). We have shown that when we expand the model at first post-Newtonian order, the parametrized post-Newtonian (PPN) parameters are exactly the same as in GR. A crucial point for this test is a non-linear effect present in our definition of the effective metric (2) and which permits to recover the correct value for the parameter measuring the amount of non-linearity, $\beta_{\text{PPN}} = 1$. Thus the model passes the Solar System tests and is viable.

2.3 Minisuperspace and ghosts

Unfortunately, the previous model proposed in suffers from an harmful ghost instability. The source of its origin is multifaceted. First of all, the presence of the square root of the determinant of $G_{\text{eff}}$ in the action (1) corresponds to ghostly potential interactions,

$$\sqrt{-G_{\text{eff}}} = \sqrt{-g\sqrt{-f}}.$$  \hfill (10)

Already at the linear order around a flat background, posing $g_{\mu\nu} = (\eta_{\mu\nu} + h_{\mu\nu})^2$ and $f_{\mu\nu} = (\eta_{\mu\nu} + \ell_{\mu\nu})^2$, these potential interactions do not preserve the Fierz-Pauli tuning, with $\cdots$ denoting the trace as usual,

$$\sqrt{-G_{\text{eff}}} = 1 + \frac{1}{2} [h + \ell] + \frac{1}{8} \left( [h + \ell]^2 - 2 [h^2 + \ell^2] \right)$$

$$+ \frac{1}{48} \left( [h + \ell]^3 - 6 [h + \ell] [h^2 + \ell^2] + 8 [h^3 + \ell^3] \right) + \cdots,$$  \hfill (11)

yielding a linear ghost instability already present at the scale

$$m^2 M_{Pl}^2 \sqrt{-G_{\text{eff}}} \sim m^2 M_{Pl}^2 (\Box \pi)^2$$

$$= \frac{m^2}{\Lambda_3^3},$$  \hfill (12)

where $\Lambda_3 = (M_{Pl}m^2)^{1/3}$ and $\pi$ encodes the helicity-0 degree of freedom of the massive graviton with mass $m$. The ghost corresponds to a very light degree of freedom, and hence the theory cannot be used as an effective field theory. The other source of ghostly interactions in the model is originated in the presence of three kinetic terms. Studying the theory in the minisuperspace immediately reveals the ghostly interactions of the considered kinetic terms. The dangerous sub-Lagrangian due to kinetic interactions is given by

$$\mathcal{L}_{\text{kin}} = \frac{M_{Pl}^2}{2} \sqrt{-g} R_g + \frac{M_{\text{eff}}^2}{2} \sqrt{-G_{\text{eff}}} R_{\text{eff}}.$$  \hfill (13)

As a first diagnostic, we can assume the two metrics to be of the mini-superspace form

$$ds^2_g = g_{\mu\nu} dx^\mu dx^\nu = -n_g^2 dt^2 + a_g^2 dx^2,$$  \hfill (14a)
\[ ds^2_f = f_{\mu\nu}dx^\mu dx^\nu = -n_f^2 dt^2 + a_f^2 dx^2, \] (14b)

where \( n_g, n_f \) are the lapse functions and \( a_g, a_f \) are the scale factors of the respective metrics. The effective metric \( g_{\text{eff}} \) in the mini-superspace becomes

\[ ds^2_{g_{\text{eff}}} = G_{\mu\nu}dx^\mu dx^\nu = -n_{\text{eff}}^2 dt^2 + a_{\text{eff}}^2 dx^2, \] (15)

where the effective lapse and effective scale factor correspond to \( n_{\text{eff}} = \sqrt{n_g n_f} \) and \( a_{\text{eff}} = \sqrt{a_g a_f} \). We compute the conjugate momenta for the scale factors and get

\[ p_g = -6M_P^2 a_g^2 H_g - \frac{3}{2} M_P^2 a_f n_{\text{eff}} \frac{d}{dt} (H_g n_g + H_f n_f), \]
\[ p_f = -\frac{3}{2} M_P^2 a_f n_{\text{eff}} (H_g n_g + H_f n_f), \] (16)

with the conformal Hubble factors \( H_g = \frac{\dot{a}_g}{a_g n_g} \) and \( H_f = \frac{\dot{a}_f}{a_f n_f} \) respectively. After performing the Legendre transformation we obtain the Hamiltonian in the mini-superspace:

\[ H_{\text{kin}}^{\text{eff}} = -\frac{(p_g a_g - p_f a_f)}{12 M_P^2 a_g^3} n_g^2 - \frac{p_f^2 a_g n_{\text{eff}}}{3 M_P^2 a_g^3}. \] (17)

As one can see immediately, the Hamiltonian is highly non-linear in the lapses \( n_g \) and \( n_f \), signalling the presence of the BD ghost degree of freedom already in the mini-superspace. Summarising, the model proposed in 20 cannot represent a consistent theory for dipolar dark matter in bigravity due to the ghostly contribution in form of the cosmological constant for \( G_{\text{eff}} \), and the kinetic term \( \sqrt{-G_{\text{eff}} R_{\text{eff}}} \).

3 Model based on massive bigravity theory

3.1 Covariant theory

The previous model 20 is plagued by harmful ghosts, but it remains that the phenomenology, especially at galactic scales (i.e. MOND), calls for a more fundamental theory. We now look for a consistent coupling of the dark matter particles within the framework of massive bigravity theory and the restrictions made in 32,36,37 concerning matter fields. We thus propose a new model, whose dark matter sector is essentially the same as in the previous model 20, but whose gravitational sector is now based on ghost-free massive bigravity theory. As bigravity theory represents essentially a unique consistent deformation of GR, we think that the new model could represent an important step toward a more fundamental theory of dark matter à la MOND in galactic scales.

The model is based on the bigravity-plus-matter action 44,45

\[ S = \int d^4x \left\{ \sqrt{-g} \left( \frac{M_g^2}{2} R_g - \rho_{\text{bar}} - \rho_g \right) + \sqrt{-f} \left( \frac{M_f^2}{2} R_f - \rho_f \right) \right. \]
\[ \left. + \sqrt{-g_{\text{eff}}} \left[ \frac{m^2}{4\pi} + \mathcal{A}_\mu \left( j^\mu_g - \frac{\alpha}{\beta} j^\mu_f \right) + \frac{a^2}{8\pi} \mathcal{W}(X) \right] \right\}, \] (18)

where \( M_g \) and \( M_f \) are two coupling constants, and \( m \) is the mass of the graviton. The ghost-free potential interactions between the two metrics \( g \) and \( f \) take the particular form of the square root of the determinant of the effective metric 32,48,

\[ g_{\mu\nu}^{\text{eff}} = \alpha^2 g_{\mu\nu} + 2\alpha\beta G_{\mu\nu}^{\text{eff}} + \beta^2 f_{\mu\nu}, \] (19)

where \( \alpha \) and \( \beta \) are arbitrary constants, which can always be restricted in the model (18) to satisfy \( \alpha + \beta = 1 \), and \( G_{\mu\nu}^{\text{eff}} \) denotes the effective metric of the previous model as defined by (2).
in terms of $X = \sqrt{g^{-1}f}$ and $Y = \sqrt{f^{-1}g}$. The square root of the determinant of this effective metric admits a closed-form expression,

$$\sqrt{-g_{\text{eff}}} = \sqrt{-g} \det(\alpha \mathbb{1} + \beta X) = \sqrt{-f} \det(\beta \mathbb{1} + \alpha Y),$$  \hspace{1cm} (20)

and can also be written with the help of the usual elementary symmetric polynomials $e_n(X)$ and $e_n(Y)$ of the square root matrices $X$ or $Y$ as

$$\sqrt{-g_{\text{eff}}} = \sqrt{-g} \sum_{n=0}^{4} \alpha^{4-n} \beta^n e_n(X) = \sqrt{-f} \sum_{n=0}^{4} \alpha^n \beta^{4-n} e_n(Y).$$  \hspace{1cm} (21)

We see that (20)–(21) corresponds to the right form of the acceptable potential interactions between the metrics $g$ and $f$. To recover the usual Newtonian limit for the motion of baryons with respect to the ordinary metric $g$ we must impose

$$M_g^2 + \frac{\alpha^2}{\beta^2} M_f^2 = \frac{1}{8\pi}.$$  \hspace{1cm} (22)

Finally the vector field $A_\mu$ is now coupled to the metric $g_{\text{eff}}^{\mu\nu}$ rather than to $G_{\text{eff}}^{\mu\nu}$, thus

$$X = -\frac{g^{\mu\rho}_{\text{eff}} g^{\nu\sigma}_{\text{eff}} F_{\mu\nu} F_{\rho\sigma}}{2a_0^2}.$$  \hspace{1cm} (23)

However the functional form of the non-canonical kinetic term $W$ in the MOND regime is the same as in the previous model, see (5), and we could also impose (6). Finally the mass currents to which is coupled the vector field in (18) are defined by

$$\sqrt{-g_{\text{eff}}} J^{\mu}_{g} = \sqrt{-g} J^{\mu}_{g}, \quad \sqrt{-g_{\text{eff}}} J^{\mu}_{f} = \sqrt{-f} J^{\mu}_{f},$$  \hspace{1cm} (24)

where, as before, $J^{\mu}_{g} = \rho_g u^{\mu}_{g}$ and $J^{\mu}_{f} = \rho_f u^{\mu}_{f}$. However, notice the presence of the factor $\alpha/\beta$ in the interaction term between $A_\mu$ and the current $j^{\mu}_{f}$ in (18). This factor can be interpreted as a ratio between the gravitational charge and the inertial mass of the $f$ particles when measured with respect to the $g$ metric.

### 3.2 Gravitational polarization & MOND

We now discuss the main point of this model, which is to allow a mechanism of gravitational polarization that permits to recover in a straightforward way the phenomenology of dark matter at galactic scales (MOND). In this respect the predictions of the new model are essentially the same as those of the previous model\(^\text{20}\).

For this purpose we are mainly interested to that particular combination of the two metrics $g$ and $f$ which is massless\(^\text{47}\). Working in the non-relativistic limit $c \to \infty$, we find that the massless combination reduces to an ordinary Poisson equation for the Newtonian potentials $U_g$ and $U_f$ associated with the two metrics, namely

$$\Delta \left( \frac{2M_g^2}{\alpha} U_g - \frac{2M_f^2}{\beta} U_f \right) = -\frac{1}{\alpha} (\rho^{*}_{\text{bar}} + \rho^{*}_g) + \frac{1}{\beta} \rho^{*}_f,$$  \hspace{1cm} (25)

where $\rho^{*}_{\text{bar}}$, $\rho^{*}_g$ and $\rho^{*}_f$ denote the ordinary Newtonian densities of the matter fluids. However, with massive (bi-)gravity the two sectors associated with the metrics $g$ and $f$ do not evolve independently but are linked together by a constraint equation which comes from the Bianchi identities. We find that this constraint reduces in the non-relativistic limit to

$$\nabla (\alpha U_g + \beta U_f) = 0.$$  \hspace{1cm} (26)
Combining (25)–(26) and using (22) we readily obtain the following Poisson equation for the Newtonian potential in the ordinary sector,

$$ \Delta U_g = -4\pi \left( \rho_{\text{bar}}^* + \rho_g^* - \frac{\alpha}{\beta} \rho_f^* \right). \quad (27) $$

Similarly we find that the equation governing the vector field, obtained by varying the action (18) with respect to $A_\mu$, reduces in the non-relativistic limit to a single Coulomb type equation,

$$ \nabla \cdot \left[ W_X \nabla \phi \right] = 4\pi \left( \rho_g^* - \frac{\alpha}{\beta} \rho_f^* \right), \quad (28) $$

where $W_X$ is the derivative of $W$ with respect to its argument $X$. Finally we have also at our disposal the equations of motion of the baryons and the dark matter particles. We look for explicit solutions of the equations of motion of dark matter in the form of plasma-like oscillations around some equilibrium configuration. Thanks to the constraint equation (26) we find that an equilibrium is possible, i.e. the dark matter particles are at rest, when the Coulomb force annihilates the gravitational force, namely

$$ \nabla U_g + \nabla \phi = 0. \quad (29) $$

At equilibrium we shall grasp a mechanism of gravitational polarization, i.e. we can define a polarization field which will be aligned with the gravitational field. Away from equilibrium we find that the polarization field undergoes stable plasma like oscillations (see for details). Finally, combining the three equations (27), (28) and (29) we easily deduce that the equation for the ordinary potential $U_g$ takes exactly the Bekenstein-Milgrom form,

$$ \nabla \cdot \left[ (1 - W_X) \nabla U_g \right] = -4\pi \rho_{\text{bar}}^*, \quad (30) $$

with MOND interpolating function $\mu = 1 - W_X$. Furthermore, thanks to the postulated form (5) of the function $W(X)$ we recover the correct deep MOND regime when $X \to 0$. We refer to for more details about this way of recovering the MOND phenomenology, which is of course reminiscent of the dielectric analogy of MOND.

Let us emphasize that gravitational polarization & MOND appear as natural consequences of this model, and are made possible by the constraint equation (26) linking together the two metrics of massive bigravity theory. Furthermore this requires that the gravitational force can be annihilated by some internal non-gravitational force, here chosen to be a vector field. This implies the existence of a coupling between the two species of dark matter particles living in the $g$ and $f$ sectors. Unfortunately, as has been shown in for the latter coupling is problematic as it yields a ghostly degree of freedom in the dark matter sector. Further work is needed to determine the exact mass of the ghost and see whether the required polarization mechanism and the ghost absence are compatible. On the phenomenology side, the cosmological implications of the model and the post-Newtonian parameters in the Solar System should be investigated in great details, as it has been done for the previous model.

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