Atomic spin relaxation and spatial decoherence near metallic and superconducting surfaces

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Abstract. We derive atomic spin relaxation rates near metallic and superconducting surfaces. Our results are based on a quantum-theoretical treatment of electromagnetic radiation near absorbing bodies. We show that there exists an atom-surface distance for which the expected relaxation rate becomes maximal and we discuss its dependence on the skin depth of the substrate material. In view of this effect we examine the possible use of superconducting materials. Furthermore, we discuss the influence of absorbing materials on spatial coherences in optical lattices.

1. Introduction
Spontaneous emission rates associated with magnetic moment transitions at low frequencies in MHz range are known to be too small to be observed experimentally because the free-space mode density is far too small to have any measurable effect on a magnetic dipole [1]. However, if a magnetic dipole is located near an absorbing material, the density of available modes increases dramatically. Experiments with trapped neutral atoms near thick metallic or dielectric surfaces show trapping lifetimes on the order of merely seconds [2, 3, 4]. In order to understand this increase in the decay rate, one has to recall that the substrates are made of absorbing materials with a finite conductivity $\sigma(\omega)$ in case of metals or, in general, with a non-vanishing imaginary part of the dielectric permittivity. By the fluctuation-dissipation theorem, there is electromagnetic field noise associated with this absorption. These field fluctuations then can induce magnetic dipole transitions as predicted in [5, 6, 7, 8, 9].

In this article, we first briefly review the theoretical foundations of the derivation of such relaxation rates using the formalism of QED in dielectrics. We then discuss some possibilities of reducing spin relaxation by making metallic surfaces thin and explore the possible use of superconductors. Finally, we present some results on spatial decoherence above planar surfaces that are relevant for proposals in connection with quantum information processing using neutral atoms in optical lattices.

2. Field quantization and spin relaxation rates
Neutral atoms trapped in specially designed magnetic-field configurations near metallic bodies suffer from spin flips due to magnetic-field fluctuations. Finite conductivity (and therefore resistivity) in the substrate material causes noise to drive spin flips from trapped to anti-trapped atomic Zeeman sublevels. In order to quantify the fluctuation strength, a quantum theory of light
in the presence of absorbing materials has been developed [10, 11, 12, 13]. In this quantization scheme, the relevant field operator are related via a source-quantity representation to a bosonic vector field \( \hat{\mathbf{f}}(\mathbf{r}, \omega) \) which serves as the dynamical variables of the theory,

\[
\hat{\mathbf{E}}(\mathbf{r}, \omega) = i \frac{\omega^2}{c^2} \sqrt{\frac{\hbar}{\pi \varepsilon_0}} \int d^3 s \sqrt{\varepsilon_I(s, \omega)} \mathbf{G}(\mathbf{r}, s, \omega) \cdot \hat{\mathbf{f}}(s, \omega),
\]

\[
\hat{\mathbf{E}}(\mathbf{r}) = \int_0^\infty d\omega \hat{\mathbf{E}}(\mathbf{r}, \omega) + \text{h.c.}
\]

Here, \( \mathbf{G}(\mathbf{r}, s, \omega) \) denotes the Green function of the classical scattering problem associated with the geometric alignment of bodies described by a complex permittivity \( \varepsilon(\mathbf{r}, \omega) = \varepsilon_R(\mathbf{r}, \omega) + i\varepsilon_I(\mathbf{r}, \omega) \). Furthermore, the bilinear Hamiltonian

\[
\hat{H}_F = \int d^3 r \int_0^\infty d\omega \hbar \omega \hat{\mathbf{F}}(\mathbf{r}, \omega) \cdot \hat{\mathbf{f}}(\mathbf{r}, \omega),
\]

together with the commutation rules \([\hat{f}_i(\mathbf{r}, \omega), \hat{f}_j^\dagger(\mathbf{r}', \omega')] = \delta_{ij} \delta(\mathbf{r} - \mathbf{r}')\delta(\omega - \omega')\), generates the time-dependent Maxwell equations. Moreover, and most importantly for our purposes, the fluctuation-dissipation theorem (here written for the magnetic field) takes its familiar form

\[
\langle \hat{B}_i(\mathbf{r}, \omega) \hat{B}_j^\dagger(\mathbf{r}', \omega') \rangle = \frac{\hbar \mu_0}{\pi} \text{Im} \left[ \nabla \times \nabla \times \mathbf{G}(\mathbf{r}, \mathbf{r}', \omega) \right]_{ij} \delta(\omega - \omega') \times \left( \bar{n}_{\text{th}} + 1 \right)
\]

with the thermal occupation number \( \bar{n}_{\text{th}} = (e^{\hbar \omega/k_B T} - 1)^{-1} \).

We consider a ground-state atom in a hyperfine magnetic state \(|i\rangle\) that is trapped at a position \( \mathbf{r}_A \) near the surface. The Hamiltonian describing its transition to a state \(|f\rangle\) is given in the rotating-wave approximation by [7]

\[
\hat{H} = \hat{H}_F + \hbar \omega_A \hat{\sigma}_z - \mu_B g_S \left( \langle f | \hat{S}_q(\mathbf{r}_A) | i \rangle \hat{\sigma} \hat{B}_q(\mathbf{r}_A) + \text{h.c.} \right),
\]

where \( \hat{S} \) is the electronic spin operator (the nuclear magnetic moment can be neglected), \( \hat{\sigma} = |i\rangle \langle f| \) denotes the atomic lowering operator, and \( \hat{\sigma}_z = \frac{1}{2}(|i\rangle \langle i| - |f\rangle \langle f|) \). After solving the Heisenberg equations of motions, we obtain for the spin flip rate in Markov approximation [7]

\[
\Gamma = \frac{2\mu_0 \mu_B g_S^2}{\hbar} \langle f | \hat{S}_p | i \rangle \langle i | \hat{S}_q | f \rangle \text{Im} \left[ \nabla \times \nabla \times \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, \omega_A) \right]_{pq} \times \left( \bar{n}_{\text{th}} + 1 \right),
\]

in which we observe the appearance of the fluctuation-dissipation theorem (4). The relaxation rate (6) is seen to depend, via the dyadic Green function \( \mathbf{G}(\mathbf{r}_A, \mathbf{r}_A, \omega_A) \), on the geometry and the electromagnetic properties of the substrate material. For metals at low frequencies, the Drude relation \( \varepsilon(\omega) = 2\text{i} \left( \frac{\omega}{\omega_0} \right)^2 \) between the dielectric permittivity and the skin depth \( \delta \) is valid.

We have considered two experimentally relevant situations in which \(^{87}\text{Rb} \) atoms in their \(^5S_{1/2}, F = 2, m_F = 2 \) ground state were magnetically trapped near a wire [2] or a thin metallic film [4]. The necessary Green functions for the multilayered cylindrical and planar geometries have been taken from [14] and [15], respectively. The wire used in [2] was a solid 185\,\mu\text{m} Cu
Figure 1. Lifetime in seconds as a function of atom-surface distance. Left figure: Al coated Cu wire with 240 \( \mu \)m radius as used in [2]. Right figure: 2\( \mu \)m-thick planar Cu layer on silicon substrate as used in [4]. Crosses denote experimental values. The calculated lifetimes in solid lines (——) are taken from [7] and [8], respectively. Other parameters in the text.

cylinder with a 55\( \mu \)m Al coating. The temperature of the wire was 380K and the bias magnetic field was adjusted such that the spin flip frequency was \( f = 560 \)kHz. At this frequency the skin depth \( \delta \) of copper is roughly 85\( \mu \)m whereas the skin depth of aluminium is 110\( \mu \)m. The dependence of the lifetime \( \tau = 1/\Gamma \) on the atom-surface distance is shown on the left in figure 1 [7]. The experiment reported in [4] used a Cu layer of 2\( \mu \)m thickness above a thick silicon substrate. The chosen transition frequency of 400kHz corresponds to a skin depth of \( \delta = 103\mu \)m, the substrate was held at a temperature of 400K. The calculated lifetime is shown on the right in figure 1 [8] where losses due to background gas collisions have been included. In both cases the calculations agree very well with the experimental data.

The lifetime depends on three independent length scales: the atom-surface distance \( d \), the skin depth of the chosen substrate material \( \delta \) and the radius of the wire or the thickness of the planar layer \( h \), respectively. There are several asymptotic regimes in which analytical formulas for the behaviour of the lifetime can be given. For example, if the skin depth is the shortest length scale, i.e. \( \delta \ll d, h \), the lifetime scales as \( d^4 \) meaning that for long lifetimes the atom should be far away from the substrate.

An interesting situation occurs when we look at the dependence on the skin depth \( \delta \) of the chosen substrate material. Since the effective volume of the radiating material and thus the number of fluctuating dipoles grows with increasing skin depth, whereas the strength of the fluctuations decreases, we expect a minimum in the observed lifetime of a trapped atom. This behaviour is depicted in figure 2. For \( \delta \ll h, d \), the lifetime scales as \( \tau \propto \delta^{-1} \), whereas for \( \delta \gg d \), it increases with \( \tau \propto \delta^2 \). One observes that there is a pronounced minimum in the lifetime as a function of skin depth. We find this minimum at \( \delta_{\min} \simeq d \) for thick films, whereas for thin films it is at \( \delta_{\min} \simeq \sqrt{hd} \).

3. Superconducting materials

A direct consequence of the behaviour as a function of \( \delta \) is that for any fixed atom-surface distance \( d \), there exist two possible choices of material for a chosen lifetime. For example, for an atom-surface distance of 50\( \mu \)m as in figure 2, there are two possible choices of 3\( \mu \)m and 100\( \mu \)m that lead to the same lifetime of 10s. The latter choice corresponds, at the frequency of 560kHz chosen in the figure, to the best metals at room temperature such as Cu (\( \delta = 85\mu \)m) or Al (\( \delta = 110\mu \)m). For the other choice, there is no normal metal that exhibits skin depths
as small as 3μm at that low frequencies. However, superconductors are possible candidates. In a material with superconducting gap Δ(T) at a finite temperature T, the fraction of thermally broken Cooper pairs is given by the Maxwell–Boltzmann distribution exp(−2Δ(T)/k_BT). Since typically Δ(0) ≃ k_BTc, at temperature moderately below Tc the normally-conducting fraction can be significant.

As an experimentally relevant example, we have a closer look at niobium (Nb) because it has a rather high critical temperature of Tc ≃ 9.3K [16] (or Tc ≃ 8.3K for 15nm thin films [17]). Measurements of the complex magnetic susceptibility have been reported in [18], from which one can infer that just above Tc, the skin depth at 560kHz is δN ≃ 15μm, whereas just below Tc it drops to δS ≃ 2μm. Hence, superconducting Nb films could be possible candidates for providing long spin flip lifetimes.

4. Spatial decoherence

In a similar fashion to relaxation rates, we can study spatial coherence properties of neutral atoms trapped near metallic surfaces. Consider the situation in which two counterpropagating laser beams form a standing-wave pattern parallel to the trapping surface. An atom located initially in one of the minima can coherently tunnel through the potential barrier formed by the laser beams and can create spatially nonlocal superposition states which could be used to encode quantum information into spatial degrees of freedom [19].

However, spin flips driven by uncorrelated noise at the two wells causes decoherence of the spatial superposition (note that in sufficiently strong trapping configurations also the |F = 2, mF = 1⟩ state is trapped so that a single spin flip does not necessarily causes an atom loss). Consider an atom being initially prepared in a coherent superposition of being located in two spatial positions 1 and 2 separated by a distance l at a distance d away from a planar substrate. Then, it can be shown that the off-diagonal matrix element (coherence) of the single-particle density matrix at some later time t can be written as [20]

\[
\rho_{12}(t) = e^{-\Gamma t} + (1 - e^{-\Gamma t}) \frac{2\mu_0(\mu_B g S)^2}{\hbar \Gamma} \langle i|\hat{S}_p|f⟩⟨f|\hat{S}_q|i⟩ \\
\times \text{Im}\{\nabla r_1 \times \nabla r_2 \times G(r_1, r_2, \omega_A)|p_0(\hat{n}_{th} + 1)\}
\]

where Γ is the spin flip rate (6). This coherence shows both an exponential decrease that is independent of the atom’s position and thus a purely local contribution, and a term that is
proportional to the spatial magnetic-field correlation tensor [21] which again is a consequence of the fluctuation-dissipation theorem (4).

The long-time behaviour of (7), i.e. when \( t \to \infty \), is shown in figure 3 as a function of the lateral separation \( l \) for different atom-surface distances \( d \). We have taken the spin-flip frequency to be 560kHz and the substrate temperature as 300K. One observes that the spatial coherence is maintained to a high degree (\( \rho_{12}(t) \gtrsim 0.9 \)) for separations up to the atom-surface distance, \( l \lesssim d \). Even for \( l \gtrsim 2d \) the spatial coherence does not drop below 0.5. These results are encouraging as they allow to robustly encode quantum information into spatial degrees of freedom over relatively large distances. Further investigations to experimentally relevant situations will be presented in [20].

5. Summary

We have presented a quantum-mechanical treatment of atomic spin relaxation rates and spatial decoherence of trapped neutral atoms near metallic and superconducting surfaces. The theoretical calculations agree very well with experimental data obtained in different groups and show that spin flips driven by dissipative processes in the substrate material are the main cause of atom loss. We have observed that for any fixed atom-surface distance there is a range of material (characterized by their skin depth) that leads to a minimum in the expected lifetime. This observation has consequences for future designs of microstructured atom chips. We have further investigated the possible use of superconducting surfaces. A particularly interesting candidate is niobium because of its high transition temperature of 9.3K for bulk Nb. From measurements we can infer that the skin depth of Nb just below the critical temperature reduces to about 1.5\( \mu \)m which, together with the reduction in thermal photon numbers with respect to room temperature, leads to an increase in trapping lifetime by three orders of magnitude.

Within the framework of electromagnetic-field quantization in absorbing materials we have also treated the problem of spatial decoherence which is relevant for future applications of quantum information storage and processing with neutral atoms. For a thick planar substrate, the coherence can be maintained for spatial separations that are comparable to the atom-surface distance. The dependence of the coherences on layer thickness and skin depth are subject of further investigations but we believe they will play an important role in the design of quantum information processing schemes with trapped neutral atoms.
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