Effects of quantum statistical pressure and exchange correlation on the low frequency electromagnetic waves in degenerate Fermi-Dirac pair-ion plasma

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Abstract

The low frequency, long wavelength electromagnetic waves, viz, shear Alfvén wave in quantum electron-positron-ion magneto plasmas, have been examined using quantum magneto hydrodynamic model. In this model, we have considered electrons and positrons are to be magnetized as well as degenerate whereas ions are magnetized but classical. We have also included the effects of exchange correlation terms which appear entirely the dynamic equations of electrons and positrons. The whole treatment is done using multi-fluid model. Our object is to study the shear Alfvén waves propagating in above said system of plasma. For that we have derived the modified dispersion relation of the shear Alfvén waves. Results are relevant to the terrestrial laboratory astrophysics.

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I. INTRODUCTION

At about $10^{-6} < t < 10$ sec time after the big bang when the universe was evolving lepton epochs is believed to exist. In this particular epoch temperatures of $10^9 < T < 10^{13}$ K are speculated to causing the annihilation of the hadrons and antihadrons pairs which formed matter composed of the electrons, positrons, and photons in thermodynamic equilibrium\(^1\). Pair plasmas consist of electrons and positrons (EP) inherent unique properties due to their mass and charge symmetry and are known to exist abundantly when early stars formation was taking place\(^2\)\(^-\)\(^7\). Their omnipresence is also accepted in the interior of accretion disks surrounding black holes, magnetospheres of the neutron star and pulsar, environments like the bipolar outflows (jets), active galactic nuclei, polar regions of neutron stars, at the center of Milky Way galaxy etc.\(^8\)\(^-\)\(^10\) In order to understand the physical phenomena happening hundred of light years away, EP plasmas are created by interaction of ultraintense laser with the solid targets. Scientists are hopeful that these interactions will lead to creation of highly dense laboratory electron positron plasmas ($n \sim 10^{26} m^{-3}$)\(^11\)\(^,\)\(^12\). Therefore theoretical investigations involving pair plasma is also needed for the advancement of laboratory astrophysics.

Presence of ions, in EP plasmas has also been identified in both laboratory and astrophysics\(^13\), which breaks the symmetry of equal mass and number of pair particles eventually leading to the new and interesting avenue of research for the scientists\(^9\)\(^,\)\(^10\).

Whereas Small temperature differences and also some nonlinear phenomena which emerge naturally during the evolution of pair particles may usually cause this asymmetric behavior in the experiments. On the other hand small contamination of much heavier immobile ion, or small mass difference of the pair particles can also produce asymmetries\(^9\)\(^,\)\(^10\). In electron-positron-ion (EPI) plasmas, physical phenomena like waves and instabilities can occur at both fast (high frequency) and slow (low frequency) time scales. Research has been carried out to study the both relativistic and non-relativistic pair plasmas astrophysical nature and produced laboratory\(^9\)\(^,\)\(^13\)\(^-\)\(^15\). While in the environment of neutron stars pair plasmas are speculated to be highly degenerate and ultradense that is why a rigorous investigation for example in the frame work of quantum hydrodynamics of EPI degenerate plasma has been made over the past few years\(^9\).

While shear Alfvén wave features of magnetized plasmas are considered one of the im-
portant waves in plasmas due to its wide applications in both lab and astrophysical environments. For their highly speculated importance Alfvén waves’ propagation in electron positron plasmas with and without ions have been extensively studied. For example since these waves damp much slowly than the Langmuir or magneto-acoustic waves so are thought to cause emission of electromagnetic radiations from the magnetosphere of pulsars \[14, 16, 17\].

Cerenkov radiation interaction with the plasma particles are speculated to as the reason for the excitation shear Alfv waves however still there are discrepancies and unexplainable features which require further investigation\[18\]. on the other hand as despite extensive theoretical modelling, our knowledge of pair plasmas is still speculative, owing to the extreme difficulty in recreating neutral matter-antimatter plasmas in the laboratory.

Since the reported possible creation of dense electron-positron plasma, where the charged particles behave as a Fermi gas, and quantum mechanical effects might play a vital role in the dynamics of charge carriers. It is important to mention that the criterion of quantum interference of particles is satisfied by the lighter plasma particles (electrons and positrons) more easily alos the inclusion of the exchange-correlation potential with the quantum effects through the Bohmian force and the quantum statistical pressure may reflect the comprehensive study of a quantum plasma system \[19–26\]. The electron exchange and correlation effects in dense plasmas (by the electron half-spin particles) play a central role in the plasma dielectric response function \[27\]. Hence, it is highly expected that contribution of the electron-exchange potential along with the Bohm potential and the Fermi degenerate pressure would reshape the dispersion properties of Alfvén waves and the interaction potentials of the medium in quantum plasmas. Needless to mention electron exchange-correlation effects being inadequate have been paid lass attention whereas for dense plasmas systems with low temperature they can have dominant significance.

The influence of quantum statistical degeneracy pressure and exchange correlation effects on Shear Alfvén waves in degenerate Fermi-Dirac electron-positron ion plasma has not been investigated yet. Moreover since pure Alfvén waves propagate parallel to the magnetic field, are not effected in quantized plasmas, whereas low frequency shear Alfvén waves making a small angle can be influenced by quantum effects so quite different results than the classical Maxwellian plasmas can be expected.

Therefore we aim to model for the dispersion of shear Alfvén waves (SAWs) in non-
relativistic dense pair-ion plasmas with exchange correlation effects which are attributes of electrons and positrons only while doing so the processes leading to the pair creation and recombination have been ignored.

The manuscript is organized as follows: basic equations and the dispersion relations for the shear Alfvén waves propagating in electron-positron ions are presented in section II. Quantitative analysis and conclusions are given in Sec. III and Sec IV.

II. BASIC FORMULATION AND INSTABILITY ANALYSIS

To study the dispersion properties of shear Alfvén wave (SAW) making a small angle with static external magnetic field $\mathbf{B}_0 = B_0 \hat{z}$ and propagating in a degenerate Fermi-Dirac pair-ion plasma, we write quantum Euler equations for the $j$ species of electron positron and ion in quantum Fermi-Dirac plasmas [24]

$$\frac{\partial \mathbf{v}_j}{\partial t} = \frac{q_j}{m_j} \mathbf{E} + \mathbf{v}_j \times \omega_{cj} \hat{z} - \frac{\nabla P_{Fj}}{m_j n_{0j}} + \frac{\hbar^2}{4m_j^2 n_{0j}} \nabla (\nabla^2 n_{j1}) - V_{j, xc} \nabla n_{j1} \tag{1}$$

In above equation last term represents electron and positron exchange-correlation potential which is a complex function of Fermi particles density and is given as $V_{j, xc} = 0.985 e^2/\epsilon n_{j1}^{1/3} \ln \left(1 + 18.37 a_B n_{j1}^{1/3}\right)$ [28] is considered the attribute of the spin effects in dense systems. For the readers it is useful to find that for the degenerate plasma, these affects have been calculated comprehensively in “Statistical Physics” book by Landau and Lifshitz [29] while exchange correlations for proton interaction have been presented by Tsintsadze et al., [30]. Since this depends upon the number density, so we cannot ignore it in dense plasma environments. In Eq. (1) $a_B = e\hbar^2/m_je^2$ is the well-known Bohr atomic radius.

Equation (1) is general and conveniently written however later we will treat ions as classical particle. In equation (1) $\hbar = h/2\pi$ and $\omega_{cj} = q_j B_0/m_j c$ the cyclotron frequency, $q_j$ the charge, $m_j$ mass and $c$ is the velocity of light in a vacuum of the $j$th species. Here, $j = i$ (ion), $j = e$ (electron), $j = p$ (positron), $q_e = -e$, $q_p = +e$ and $q_i = Z_i e$, with $e$ being the magnitude of electronic charge and $Z_i$ is the number of charges on ions. In Eq.(1), $P_{Fj} = \frac{m_j v_{Fj}^2 n_{j1}^3}{3 n_{0j}}$ is pressure law for 3-dimensional Fermi gas [28], where $v_{Fj}^2 = \frac{6 k_B T_{Fj}}{m_j}$ is the Fermi speed; $k_B$ is the Boltzmann constant, $T_{Fj} = \frac{\hbar^2 (3\pi^2 n_{0j})^{2/3}}{2m_j}$ is Fermi temperature, $n_j = n_{0j} + n_{1j}$ the total number density with equilibrium number density $n_{0j}$ and perturbed number density $n_{1j}$.
of $j$th particles in the field of SAW. The ion component can be considered classical or quantum depending upon the relevant parameters. However, in most of the situations, ions are considered as cold fluid while describing the ion wave. In these dense quantum and semiclassical plasmas, the screened interaction potential cannot be characterized by the standard Debye-Huckel model according to the multiparticle correlations and the quantum-mechanical effects such as the Bohm potential, quantum pressure, and electron exchange terms since the average kinetic energy of the plasma particle in quantum plasmas is of the order of the Fermi energy \cite{28}. Thermal temperature of ions is small as compared to the electrons and positrons and therefore ignored.

We assume the geometry of the problem that the SAW is propagating with low frequency $\omega$ on ion dynamics and obliquely to external magnetic field $B_0 = B_0 \hat{z}$ and the propagation vector $k$ of the wave makes a small angle $\theta$ with $z$ - direction and lies in $xz$ - plane i.e., $(k_x, 0, k_z)$ where $k_x = k \sin \theta$ and $k_z = k \cos \theta$. The velocity components of $j$th species in the field of SAW can be obtained from Eq.(1),

\[
v_{jx} = \frac{iq_j \left(\omega^2 F_j E_x + i\omega \omega_{cj} F_j E_y + V_{F_{Bxcj}}^2 k_z k_z E_z\right)}{m_j \omega \left(\omega^2 - V_{F_{Bxcj}}^2 k^2 - F_j \omega_{cj}^2\right)},
\]

\[
v_{jy} = \frac{iq_j \left[-i\omega \omega_{cj} F_j E_x + \left(\omega^2 - V_{F_{Bxcj}}^2 k^2\right) E_y - i\omega \omega_{cj} F_j V_{F_{Bxcj}}^2 k_z E_z\right]}{m_j \omega \left(\omega^2 - V_{F_{Bxcj}}^2 k^2 - F_j \omega_{cj}^2\right)},
\]

and

\[
v_{jz} = \frac{iq_j}{m_j \omega F_j} \left[\frac{V_{F_{Bxcj}}^2 k_x k_z F_j}{\omega^2 - V_{F_{Bxcj}}^2 k^2 - F_j \omega_{cj}^2} E_x + \frac{i\omega \omega_{cj} F_j}{\omega^2 - V_{F_{Bxcj}}^2 k^2 - F_j \omega_{cj}^2} E_y + \left(1 + \frac{V_{F_{Bxcj}}^2 k_z^2}{\omega^2 - V_{F_{Bxcj}}^2 k^2 - F_j \omega_{cj}^2}\right) E_z\right],
\]

where

\[V_{F_{Bxcj}}^2 = v_{Fj}^2 + v_{Bj}^2 + v_{xcj}^2,
\]

\[v_{Bj}^2 = \frac{\hbar^2 k^2}{4m_j^2},
\]

\[v_{xcj}^2 = 0.985n_{0j}^{1/3} \left(\frac{e^2}{\epsilon}\right) \left[1 + \frac{0.034 \times 18.37}{1 + 18.37 a_{Bj} n_{0j}^{1/3}}\right],
\]

and

\[F_j = 1 - \frac{V_{F_{Bxcj}}^2 k_z^2}{\omega^2}.
\]

The electric field $E$ and magnetic field $B$ of SAW in pair ion plasma are related by the following curl equations,

\[
\nabla \times B = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial E}{\partial t},
\]
and
\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \]  
(10)

where
\[ \mathbf{J} = \sum_j q_j n_0 j \mathbf{v}_j \]  
(11)
is the current density of the plasma particles due to the propagation of electromagnetic shear Alfvén wave.

After substitution of Eqs. (2-4) into Eq. (11), the current density becomes,
\[ \mathbf{J} = \sigma \mathbf{E} \]  
(12)

Where \( \sigma \) is the linear conductivity tensor given by
\[ \sigma = \sum_j \frac{i q_j^2 n_0 j}{m_j \omega} k_j \]  
(13)

where
\[ k_j = \begin{pmatrix} \omega^2 F_j & \iota \omega \omega_{\perp j} F_j & \frac{V_{FB \perp j}^2 k_x k_z}{\omega^2 - V_{FB \perp j}^2 k^2 - \omega_{\perp j}^2 F_j} \\ -\iota \omega \omega_{\perp j} F_j & \omega^2 - V_{FB \perp j}^2 k^2 - \omega_{\perp j}^2 F_j & -\iota \frac{\omega_{\perp j} V_{FB \perp j}^2 k_x k_z}{\omega^2 - V_{FB \perp j}^2 k^2 - \omega_{\perp j}^2 F_j} \\ \frac{V_{FB \perp j}^2 k_x k_z}{\omega^2 - V_{FB \perp j}^2 k^2 - \omega_{\perp j}^2 F_j} & \iota \mathbf{\omega}_{\perp j} V_{FB \perp j}^2 k_x k_z \frac{1}{F_j} (1 + \frac{V_{FB \perp j}^2 k_x k_z}{\omega^2 - V_{FB \perp j}^2 k^2 - \omega_{\perp j}^2 F_j}) & \end{pmatrix} \]  
(14)

Combining these curl equations, we may write
\[ \mathbf{D} \cdot \mathbf{E} = 0. \]  
(15)

\( \mathbf{D} \) gives the linear plasma dispersion relation due to electromagnetic shear Alfvén wave \((\omega, k)\) and is defined by
\[ \text{Det}[\mathbf{D}] = k^2 I - \mathbf{k k} - \frac{\omega^2}{c^2} \mathbf{\xi} = 0 \]  
(16)

where \( I \) is the unit dyadic and \( \mathbf{\xi} = I - \sum_j \left( \frac{\mathbf{\omega}_j}{\omega^2} \right) k_j \). Here, \( \omega_{pj} = \left( \frac{4 \pi n_0 j q_j^2}{m_j} \right)^{1/2} \) is the plasma frequency of \( j \)th species. The matrix form of Eq. (16) is
\[ \text{Det}[\mathbf{D}] = \text{Det} \begin{pmatrix} k_z^2 - \frac{\omega^2}{c^2} \varepsilon_{xx} & -\frac{\omega^2}{c^2} \varepsilon_{xy} & -k_z k_x - \frac{\omega^2}{c^2} \varepsilon_{xz} \\ -\frac{\omega^2}{c^2} \varepsilon_{yx} & k^2 - \frac{\omega^2}{c^2} \varepsilon_{yy} & -\frac{\omega^2}{c^2} \varepsilon_{yz} \\ -k_z k_x - \frac{\omega^2}{c^2} \varepsilon_{xz} & -\frac{\omega^2}{c^2} \varepsilon_{yz} & k_x^2 - \frac{\omega^2}{c^2} \varepsilon_{zz} \end{pmatrix} = 0. \]  
(17)
Here, we treat the electrons, positrons quantized and magnetized while the ions are non-quantum but magnetized. The components of the medium response function are,

$$
\epsilon_{xx} = 1 - \frac{\omega_{pe}^2 F_e}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{ce}^2 F_e)} - \frac{\omega_{pp}^2 F_p}{(\omega^2 - V_{FBxcp}^2 k_z^2 - \omega_{cp}^2 F_p)} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},
$$

(18)

$$
\epsilon_{yy} = 1 - \frac{\omega_{pe}^2 (1 - V_{FBxce}^2 k_z^2 / \omega^2)}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{ce}^2 F_e)} - \frac{\omega_{pp}^2 (1 - V_{FBxce}^2 k_z^2 / \omega^2)}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{cp}^2 F_p)} - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},
$$

(19)

$$
\epsilon_{zz} = 1 - \frac{\omega_{pe}^2}{\omega^2 F_e} \left( 1 + \frac{V_{FBxce}^2 k_z^2}{\omega^2 (\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{ce}^2 F_e)} \right) - \frac{\omega_{pp}^2}{\omega^2 F_p} \left( 1 + \frac{V_{FBxce}^2 k_z^2}{\omega^2 (\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{cp}^2 F_p)} \right) - \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2},
$$

(20)

$$
\epsilon_{xy} = i \frac{\omega_{pe}^2 F_e}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{ce}^2 F_e)} \frac{\omega_{ce}}{\omega} - i \frac{\omega_{pp}^2 F_p}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{cp}^2 F_p)} \frac{\omega_{cp}}{\omega} - i \frac{\omega_{pi}^2}{\omega^2 - \omega_{ci}^2} \frac{\omega_{ci}}{\omega},
$$

(21)

$$
\epsilon_{xz} = - \left( \frac{\omega_{pe}^2 V_{FBxce}^2 k_z^2 k_z}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{ce}^2 F_e)} \right) - \left( \frac{\omega_{pp}^2 V_{FBxce}^2 k_z^2}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{cp}^2 F_p)} \right),
$$

(22)

$$
\epsilon_{yz} = -i \left( \frac{\omega_{ce} \omega_{pe}^2 V_{FBxce}^2 k_z^2}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{ce}^2 F_e)} \right) + i \left( \frac{\omega_{cp} \omega_{pp}^2 V_{FBxce}^2 k_z^2}{(\omega^2 - V_{FBxce}^2 k_z^2 - \omega_{cp}^2 F_p)} \right),
$$

(23)

$$
\epsilon_{zy} = -\epsilon_{xy}, \quad \epsilon_{zx} = \epsilon_{xz}, \quad \epsilon_{yz} = -\epsilon_{yz}.
$$

(24)

For the oblique SAW case the propagation vector and the electric field are parallel to each other and so contribution of y component of electric field can ignored to zero and so Eq. (17) can be reduced to

$$
Det[D] = Det \begin{pmatrix}
  k_z^2 - \frac{\omega^2}{c^2} \epsilon_{xx} & -k_z k_x - \frac{\omega^2}{c^2} \epsilon_{xz} \\
  -k_z k_x - \frac{\omega^2}{c^2} \epsilon_{xz} & k_x^2 - \frac{\omega^2}{c^2} \epsilon_{zz}
\end{pmatrix} = 0.
$$

(25)

Or, the above equation can be written as follows,

$$
\omega^2 (\epsilon_{xx} \epsilon_{zz} - \epsilon_{xz}^2) - c^2 k_z^2 \epsilon_{zz} - c^2 k_x^2 \epsilon_{xx} - 2c^2 k_z k_x \epsilon_{xz} = 0
$$

(26)

Firstly, the mass of electrons and positrons are ignored for being much lighter than the behaviour ions. Then, for frequency range $\omega^2 \ll \omega_{ci}^2 \ll \omega_{ce}^2 = \omega_{cp}^2$ and $\omega^2 \ll V_{FBxce}^2 k_z^2$, $\omega^2 \ll V_{FBxce}^2 k_z^2$ the components of medium response function gain following simplified form,

$$
\epsilon_{xz} = -\frac{\omega_{pe}^2}{\omega_{ce}^2} \frac{k_x}{k_z} - \frac{\omega_{pp}^2}{\omega_{cp}^2} \frac{k_x}{k_z}
$$

(27)
\[ \epsilon_{xx} = 1 + \frac{\omega_{pe}^2}{\omega_{ce}^2} + \frac{\omega_{pp}^2}{\omega_{cp}^2} + \frac{\lambda_{pi}^2}{\omega_{ci}^2}, \]  
(28) 
\[ \epsilon_{zz} = 1 + \frac{\omega_{pe}^2}{V_F^2 B_{exc} k_z^2} + \frac{\omega_{pp}^2}{\omega_{ce} k_z^2} + \frac{\omega_{pp}^2 k_x^2}{V_F^2 B_{exc} k_z^2} + \frac{\omega_{pp}^2 k_x^2}{\omega_{cp} k_z^2} - \frac{\lambda_{pi}^2}{\omega^2}, \]  
(29) 

Using Eqs. (27-29) into Eq. (26) we get

\[ A\omega^4 + B\omega^2 + C = 0 \]  
(30)

where

\[ A = \left\{ \left( 1 + \frac{c^2}{v_A^2} \right) \left( 1 + \frac{1}{k_z^2 \lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_{Aep}^2 k_z^2} \right) - \frac{c^4 k_z^2}{v_{Aep}^2 k_z^2} \right\} \]  
(31)

\[ B = -\left\{ \left( 1 + \frac{c^2}{v_A^2} \right) \omega_{pi}^2 + c^2 k_z^2 \left( 1 + \frac{1}{k_z^2 \lambda_{DFc}^2} \right) + c^2 k_x^2 \left( 1 + \frac{c^2}{v_A^2} \right) \right\} \]  
(32)

and

\[ C = c^2 k_z^2 \omega_{pi}^2 \]  
(33)

For simplifying \( A, B \) and \( C \), here we have used \( v_A^2 = \frac{c^2 \omega_{pe}^2}{\omega_{pi}^2}, v_A^2 = \frac{c^2 \omega_{pp}^2}{\omega_{pi}^2}, v_A^2 = \frac{c^2 \omega_{e}^2}{\omega_{pp}^2} \),

\( \frac{1}{v_A^2} = \left( \frac{1}{v_{Ac}} + \frac{1}{v_{Ap}} + \frac{1}{v_{Aep}} \right), \frac{1}{v_{Aep}} = \left( \frac{1}{v_{Ac}} + \frac{1}{v_{Ap}} \right), \lambda_{DFc}^2 = \frac{V_F^2 B_{exc}}{\omega_{pe}^2}, \lambda_{DFc}^2 = \frac{V_F^2 B_{exc}}{\omega_{pp}^2}, \) and

\( 1/\lambda_{DFc}^2 = (1/\lambda_{DFc}^2 + 1/\lambda_{DFp}^2) \). For further simplification, using \( \lambda_{DFc}^2 k_z^2 \ll 1, v_A^2 \ll c^2 \) in above expressions of \( A, B \) and \( C \) we get

\[ A = \left\{ \frac{c^2}{v_A^2} \frac{1}{k_z^2 \lambda_{DFc}^2} + \frac{c^2}{v_A^2 v_{Aep}^2} \frac{k_x^2}{k_z^2} \right\} \]  
(34)

\[ B = -c^2 \left\{ \frac{\omega_{pi}^2}{v_A^2} + \frac{1}{\lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_A^2} \right\} \]  
(35)

\[ C = c^2 k_z^2 \omega_{pi}^2 \]  
(36)

Eq. (30) is quadratic in \( \omega^2 \) then

\[ \omega^2 = \frac{\left( \frac{\omega_{pi}^2}{v_A^2} + \frac{1}{\lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_A^2} \right) \pm \sqrt{\left( \frac{1}{\lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_A^2} - \frac{\omega_{pi}^2}{v_A^2} \right)^2 - 4 \omega_{pi}^2 c^2 k_x^2}} {2 \left( \frac{1}{v_A^2} \frac{1}{k_z^2 \lambda_{DFc}^2} + \frac{c^2}{v_A^2 v_{Aep}^2} \frac{k_x^2}{k_z^2} \right)} \]  
(37)

For real frequency the term in square root should be positive this implies that

\( \left( \frac{1}{\lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_A^2} - \frac{\omega_{pi}^2}{v_A^2} \right) \gg -4 \omega_{pi}^2 \frac{c^2 k_x^2}{v_A^2} \) then

\[ \omega^2 = \frac{\left( \frac{\omega_{pi}^2}{v_A^2} + \frac{1}{\lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_A^2} \right) \pm \left( \frac{1}{\lambda_{DFc}^2} + \frac{c^2 k_x^2}{v_A^2} - \frac{\omega_{pi}^2}{v_A^2} \right)} {2 \left( \frac{1}{v_A^2} \frac{1}{k_z^2 \lambda_{DFc}^2} + \frac{c^2}{v_A^2 v_{Aep}^2} \frac{k_x^2}{k_z^2} \right)} \]  
(38)

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for the propagation of SAW using upper sign term and after some simplification we get

$$\omega^2 = \frac{k^2v_A^2(1 + k^2\rho_{Fi}^2)}{1 + v_A^2/v_{Aep}^2k^2\rho_{Fi}^2}$$ \hspace{1cm} (39)

where \(\rho_{Fi}^2 = (c^2/v_{Ai}^2)\lambda_D^2Fc\). This is the dispersion relation of SAW in electron positron-ion plasmas modified by exchange-correlation potentials due to electrons and positrons which is modified from the one in ordinary classical plasmas [31].

**III. RESULTS AND DISCUSSION**

Electron-positron ion plasma exist in very dense astrophysical environment with electron number density \(n_{0e} \sim 10^{27} cm^{-3}\) and in laboratory with \(n_{0e} \sim 10^{16} cm^{-3}\). Now we quantitatively analyse the results obtained in Sec. (2). In this study, the typical parameters for dense plasmas [2] that are relevant to astrophysical objects e.g., neutron stars and pulsar’s atmosphere have been used. In such environments, due to highly density and strong magnetic field of many orders higher than that of laboratory plasma, behaves exotically. The interaction between positrons and/or electrons in such plasmas is very weak due to Pauli blocking and they are more suited for quantum hydrodynamics. Therefore, we select the following typical electron number density \(n_{0e} \sim 1.5 \times 10^{22} cm^{-3}\), with very high external magnetic field \(B_0 \sim 10^{10} G\) and using physical constants in cgs system viz., \(c = 3 \times 10^{10} cm sec^{-1}\), \(m_e = 9.1 \times 10^{-28} g\), \(m_i = 1.67 \times 10^{-24} g\), and \(\hbar = 1.057 \times 10^{-27} erg sec\). Eq. (39) is plotted to investigate the changes in the dispersion characteristics of shear Alfvén waves for different variables that are: \((\omega/\omega_{ci} vs k\frac{\lambda_D}{\omega_{ci}})\), \((\omega/\omega_{ci} vs n_{oi}[cm^{-3}])\), \((\omega/\omega_{ci} vs B_0[G])\) and \((\omega/\omega_{ci} vs \theta[Degree])\).

Figure (1) shows the plot of Eq. (39) and increase in frequency \(\omega\) and phase speed of SAW with propagation vector \(k\) can be noticed. It represents that for any value of \(k\), the frequency \(\omega\) and phase speed of SAW increases due the inclusion of exchange - correlation Potential. The plot is a curve due to the oblique propagation of SAW with external magnetic field (see equation (39)). Thus oblique propagation of SAW effected by Fermi temperature and Exchange potential. For Alfvén wave propagating exactly along ambient magnetic field that is \(k_x = 0\) then we left with the dispersion relation \(\omega^2 = k^2v_A^2\), which cannot be effected by the contribution of Bohm, Fermi and Exchange potential.

Fig.(2) shows the increment in the frequency band with the increase of ion number density.
It also shows the growth of frequency with the consideration of Exchange-Potential. Fig.(3) explains that the Alfvén frequency decreases with strength of magnetic field as expected. But there are larger values of $\omega$ at every value of magnetic field with exchange potential than values of $\omega$ without exchange-potential. In Fig.(4) there is a large difference in the frequency of wave with and without exchange-potential on the increment of small value of $\theta$.

Concluding, dispersion relation of low frequency electromagnetic waves or shear Alfvén wave nonrelativistic Fermi-Dirac pair ion plasma have been derived and studied quantitatively. Overall the exchange correlation affects significantly modify the waves depression. These results are in particularly important for the terrestrial laboratory astrophysics as future ultra-intense lasers are believed to produce dense degenerate pair plasmas under favorable conditions.

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[1] S. Wineberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity (Wiley and Sons, New York, 1972).
[2] G. Sarri, at el., nature communications, 6:6747, DOI: 10.1038/ncomms7747.
[3] F. Pacini, Nature London 219, 145 (1968).
[4] P Goldreich and W. H. Julian, Astrophys. J. 157, 869 (1969).
[5] M. J. Rees, Nature London 229, 312 (1971).
[6] C. M. Surko, M. Leventhal, and A. Passner, Phys. Rev. Lett. 62, 901 (1989).
[7] H. Boehmer, M. Adams, and N. Rynn, Phys. Plasmas 2, 4369 (1995).
[8] E. P. Liang, S. C. Wilks, and M. Tabak, Phys. Rev. Lett. 81, 4887 (1998).
[9] S.A. Khan, M. Ilyas, Z. Wazir and Z. Ehsan, Astrophys Space Sci DOI 10.1007/s10509-014-
1925-8.
[10] Z. Ehsan, N. L. Tsintsadze, H. A. Shah, R. M. G. M. Trines, and M. Imran Phys. Plasmas
23, 062125 (2016) and references therein.
[11] Glenzer, S.H., Landen, O.L., Neumayer, P., Lee, R.W., Widmann, K., Pollaine, S.W., Wallace,
R.J., Gregori, G., Höll, A., Bornath, T., Thiele, R., Schwarz, V., Kraeft, W.-D., Redmer, R.: 
Phys. Rev. Lett. 98, 065002 (2007)
[12] Ridgers, C.P., Brady, C.S., Duclous, R., Kirk, J.G., Bennett, K., Arber, T.D., Robinson,
A.P.L., Bell, A.R.: Phys. Rev. Lett. 108, 165006 (2012).
[13] R. Sabry, W. M. Moslem, P.K. Shukla, Eur. Phys. J. D 51, 233 (2009).
[14] A. D. Rogava, S. M. Mahajan, and V. I. Berezhiani Physics of Plasmas 3, 3545 (1996).
[15] G. A. Stewart and E. W. Laing, J. Plasma Phys. 47, 295 (1992); N. Iwamoto, Phys. Rev. E
47, 604 (1993).
[16] G. P. Zank and R. G. Greaves, Phys. Rev. E 51, 6079 (1995).
[17] A. B. Mikhailovskii, O. G. Onishchenko, and E. G. Tatarinov, Plasma Phys. Controlled Fusion
27, 527 (1985).
[18] A. D. Verga and C. F. Fontan, Plasma Phys. Controlled Fusion 27, 19 1985.
[19] N. L. Tsintsadze and L. N. Tsintsadze, EPL, 88, 35001 (2009); G. Manfredi, Fields Inst.
Commun. 46, 263 (2005). F. Haas, Europhys. Lett. 77, 45004 (2007)
[20] M. Shahmansouri, Phys. Plasmas 22, 092106 (2015). H. KhalilpourPhys. Plasmas 22, 122112
(2015).
[21] P. K. Shukla and B. Eliasson, Phys. Rev. Lett. 99, 096401 (2007).
[22] M. Ali, A. Hussain, and G. Murtaza, Phys. Plasmas 18, 092104 (2011).
[23] S. Noureen, G. Abbas, and H. Farooq Phys. Plasmas 24, 092103 (2017).
[24] M. J. Lee and Y. D. Jung Phys. Plasmas 24, 093301 (2017)
[25] D. B. Melrose, Quantum Plasmadynamics, Un-Magnetized Plasmas, Lecture Notes in Physics
Vol. 735 (School of Physics, University of Sydney, 2006).
[26] H. G. Craighead, Science 290, 1532 (2000); M. Shahid and G. Murtaza, Phys. Plasmas 20,
082124 (2013)
[27] N. Crouseilles, P. A. Hervieux, and G. Manfredi, Phys. Rev. B 78, 155412 (2008).
[28] A. Abdikian and Z. Ehsan PhysicsLetters A3 81 2939 (2017) and references therein; R. Maroof,
A. Mushtaq, and A. Qamar, Phys. Plasmas 23, 013704 (2016).

[29] L. D. Landau and E. M. Lifshitz, Statistical Physics, 2nd ed. Pergamon, Oxford, 1996.

[30] N. L. Tsintsadze, G. Murtaza and Z. Ehsan, Phys. Plasmas 13, 22103 (2006)

[31] A. Hasegawa, J. Geophys. Res. 81, 5083, doi:10.1029/JA081i028p05083 (1976).
Figure Captions

Fig. 1: Relationship of $[\omega/\omega_{ci} vs k_{\|}/\omega_{ci}]$ with Exchange-Potential (solid curve) and without Exchange-Potential (dashed curve) at $n_{0e} \sim 1.5 \times 10^{22} cm^{-3}$, $B_0 \sim 10^{10} G$, $\theta \sim 5[Degree]$.

Fig. 2: Relationship of $[\omega/\omega_{ci} vs n_{0i}[cm^{-3}]]$ with Exchange-Potential (solid curve) and without Exchange-Potential (dashed curve) at $\theta \sim 5[Degree]$, $B_0 \sim 10^{10} G$.

Fig. 3: Relationship of $[\omega/\omega_{ci} vs B_0[G]]$ with Exchange-Potential (solid curve) and without Exchange-Potential (dashed curve) at $n_{0e} \sim 1.5 \times 10^{22} cm^{-3}$, $\theta \sim 5[Degree]$.

Fig. 4: Relationship of $[\omega/\omega_{ci} vs \theta[Degree]]$ with Exchange-Potential (solid curve) and without Exchange-Potential (dashed curve) at $n_{0e} \sim 1.5 \times 10^{22} cm^{-3}$, $B_0 \sim 10^{10} G$. 