Induced chiral supergravities in 2D

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We analyze actions for 2D supergravities induced by chiral conformal supermatter. The latter may be thought as described at the classical level by superspace actions invariant under super-reparametrization, super-Weyl and super-Lorentz transformations. Upon quantization various anomalies appear which characterize the non-trivial induced actions for the supergravitational sector. We derive these induced actions using a chiral boson to represent the chiral inducing matter. We show that they can be defined in a super-reparametrization invariant way, but with super-Weyl and super-Lorentz anomalies. We consider the case of (1,0) and (1,1) supergravities by working in their respective superspace formulations and investigate their quantization in the conformal gauge. The actions we consider arise naturally in off-critical heterotic and spinning strings. In the conformal gauge, they correspond to chiral extensions of the super-Liouville theory.

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1. Introduction

In the Polyakov approach to string theory [1], one has to consider the quantization of two dimensional matter coupled to gravity. The gravitational sector can be described by the vielbein field $e_\mu^a$ and has no propagating degrees of freedom in the critical dimensions.

In fact, it can be gauge fixed using reparametrizations, Weyl rescaling and Lorentz rotations to a given background value, up to some global configurations described by the moduli. Thus, there is no local dynamics for two dimensional gravity in the target spacetime critical dimensions, since in such dimensions all of the local symmetries used to gauge away the vielbein are free of anomalies. In the bosonic string, one can generalize the concept of dimension by the pair of numbers given by the values of the central charges, $c$ and $\bar{c}$, of the two copies of the Virasoro algebra generated by the matter stress tensor.

The critical dimensions correspond in this language to $c = \bar{c} = 26$, most easily realized using 26 scalar fields which are interpreted as the coordinates of the string moving in a 26 dimensional flat spacetime. Evidently, a similar point of view can be taken also for the various supersymmetric extensions of the bosonic string theory, where one replaces the two Virasoro algebras by the appropriate constraint algebra defining the given string theory, e.g. super-Virasoro⊗Virasoro for the heterotic string or super-Virasoro⊗super-Virasoro for the spinning string. By a string “off the critical dimensions”, one then refers to string models in which the matter central charges do not saturate the critical bound. For example, in the bosonic case for $c = \bar{c} \neq 26$ an anomaly appears in the Weyl symmetry [1], and the conformal factor of the two dimensional vielbein becomes dynamical. This can be seen computing the induced gravitational action, obtained by integrating out the matter fields in the path integral. The resulting action $I \sim cR \Box^{-1} R$ is not Weyl invariant and the reparametrization ghosts, which contribute $I_{gh} \sim -26R \Box^{-1} R$, are not sufficient to cancel the Weyl anomaly. Therefore, one is led to study the quantization of the action $cR \Box^{-1} R$, which reduces to the Liouville model in the conformal gauge, to be able to complete the string quantization off the critical dimensions. Much progress has been achieved in such a task for $c = \bar{c} \leq 1$. It begun with the works of Polyakov [2] and Knizhnik, Polyakov and Zamolodchikov [3], who used a light-cone gauge for the metric, and followed by the works of David [4] and Distler and Kawai [5], who instead used the more natural conformal gauge. In these works, exact results in the form of gauge invariant critical exponents were obtained. Eventually, dynamical triangulations and matrix models in the double scaling limit were successfully used to improve on these results [6], at least for their relevance.
to string theory, allowing to sum up the perturbative expansion in the topology of the worldsheet. Efforts are now being made to understand how to overcome the $c = 1$ barrier, see for example [7]. However, there is a more general way to interpret strings off the critical dimensions. Namely, one can imagine the situation in which the matter central charges $c$ and $\bar{c}$ are different from each other and both different from 26. This happens only if the matter fields living on the worldsheet are chiral. It is obviously an interesting and phenomenologically promising situation, since chiral structures on the worldsheet will typically induce chiral properties in spacetime. Examples are easily constructed using two dimensional Majorana-Weyl fermions, which have $c = \frac{1}{2}$ and $\bar{c} = 0$ (or vice-versa for the opposite chirality). New kind of anomalies arise in such a situation, namely the gravitational anomalies [8]. However, it is known that one can shift these anomalies into the Weyl and Lorentz sectors by using local counterterms [9]. It is in this latter form that the full effective action for the case of a Weyl fermion was first found by Leutweyler [10]. It can be parametrized as follows

$$e^{-I[e_{\mu}^a]} = \int (\mathcal{D}X_e) e^{-S[X,e_{\mu}^a]}$$

$$I[e_{\mu}^a] = \frac{1}{24\pi} \int d^2 xe \left( cR_1 \frac{1}{\Box} R_1 + \bar{c}R_2 \frac{1}{\Box} R_2 + 2a\omega_+ \omega_+ \right),$$

(1.1)

where $X$ represents a generic chiral conformal system with central charges $c$ and $\bar{c}$ used to induce the gravitational action. The coupling $a$ multiplies a local term and is not fixed by the requirement of diffeomorphism invariance, $R_1$ and $R_2$ are Lorentz non-invariant chiral pieces of the curvature scalar constructed out of the vielbein field and $\omega_a$ are the components of the spin connection (the flat index $a$ takes the values $(+, -)$, see appendix A for notation). Quantization of this induced action for chiral gravity (or, equivalently, chirally induced action for gravity!) has been recently investigated by Oz, Pawelczyk and Yankielowicz [11] in a light-cone gauge for the metric, and by Myers and Periwal [12] in the conformal gauge†.

In the present paper, we consider the case of off-critical heterotic and spinning string by first deriving the chirally induced action for the corresponding supergravities. Since the constraint algebra is made out of a left $N = 1$ super-Virasoro plus a right Virasoro algebra

† A mismatch between the results of refs. [11] and [12] is properly understood in [13], where it is checked that the stress tensor for the Lorentz field used in [11] is the one arising from (1.1), but with a fixed value of $a$. This value can in fact be left arbitrary.
for the heterotic string, and of a $N = 1$ super-Virasoro algebra in each chiral sector for the spinning string, the supergravities to consider are $(1, 0)$ and $(1, 1)$ respectively. We identify the chirally induced actions with a simple trick. We use chiral bosons to represent a general chiral system. We evaluate the gaussian path integral by square completion and from the knowledge of the action induced by a free scalar field we obtain the required result. The chiral boson we use is the one recently introduced in ref. [14], and consists of a scalar field with peculiar couplings to the background geometry. After having obtained the induced actions, we proceed as in David, Distler and Kawai [4][5] to analyze their quantization in the conformal gauge, where they reduce to local actions for the Weyl and Lorentz modes of the vielbein. We do not identify any new exact critical exponents, partly because we only give a local description of the model and do not address topological issues.

The structure of the paper is as follows. In sec. 2 we review the case of chiral gravity, explaining our strategy in a simpler context and in a way that generalizes to the supersymmetric case. In sec. 3 we discuss the off-critical heterotic string, presenting the chiral induced action for $(1, 0)$ supergravity and quantizing it in the conformal gauge using free fields. We check the independence on the gauge fixing choice by verifying that the Lorentz and Weyl anomalies cancel. This is a necessary requirement for the BRST invariance of the quantum theory. In sec. 4 we repeat our analysis for the $(1,1)$ case, i.e. for the spinning string. From one point of view, this analysis is simpler than that for the heterotic string, since the $(1,1)$ superspace is intrinsically non-chiral and much of the derivations are in close parallel with the bosonic case. Eventually, in sec. 5 we present our conclusions and an outlook. We explain our conventions and notations in the appendices, where we review the various superspaces and list few useful formulae employed in the main text.

2. Review of induced chiral gravity

The easiest way to obtain the gravitational action induced by chiral matter in 2d, eq. (1.1), is to use a chiral boson with central charges $c$ and $\bar{c}$ to represent the inducing matter system. The chiral boson we have in mind was introduced by one of us in ref. [14] and is described by the action

$$S[X, e_\mu^a] = \frac{1}{2\pi} \int d^2xe(\nabla_+ X \nabla_- X + \beta_1 R_1 X + \beta_2 R_2 X),$$

(2.1)
where \( R_1 \) and \( R_2 \) are chiral halves of the curvature scalar \( R = R_1 + R_2 \). The precise definition of \( R_1 \) and \( R_2 \) is to be found in appendix A, to which we refer also for further details about our notation. The coupling to the chiral curvature scalars induce improvement terms in the stress tensor, which is traceless and conserved when evaluated in flat space, and generates two copies of a Virasoro algebra with central charges \( c = 1 + 3\beta_1^2 \) and \( \bar{c} = 1 + 3\beta_2^2 \). This is seen as follows. From (2.1) we compute the stress tensor, defined as

\[
T_{ab} = \frac{2\pi}{e} \frac{\delta S}{\delta e_{\mu}^a} e_{\mu b}.
\] (2.2)

We evaluate it on-shell and in flat space, obtaining the following non-vanishing components

\[
T_{++} = -\frac{1}{2} \partial_+ X \partial_+ X + \frac{\beta_1}{2} \partial_+^2 X,
\]

\[
T_{--} = -\frac{1}{2} \partial_- X \partial_- X + \frac{\beta_2}{2} \partial_-^2 X.
\] (2.3)

Using now the propagator derived from the flat space limit of (2.1),

\[
\langle X(x)X(y) \rangle = -\log(\mu^2|x - y|^2)
\] (2.4)

with \( \mu \) an infrared cut-off, it is immediate to obtain

\[
c = 1 + 3\beta_1^2, \quad \bar{c} = 1 + 3\beta_2^2
\] (2.5)

for the central charges \( c \) and \( \bar{c} \) of the Virasoro algebras generated by the operator product expansions of \( T_{++} \) with itself and \( T_{--} \) with itself, respectively. This confirms that the system described by the action (2.1) has chiral properties.

Let’s now consider the gravitational action induced by chiral systems. In order to derive it, we write down the path integral for the chiral boson

\[
e^{-I[\epsilon_{\mu}^a]} = \int (D X)_e e^{-S[X, \epsilon_{\mu}^a]} \]

(2.6)

where the translational invariant measure \( (D X)_e \) is implicitly defined using the following ultralocal and reparametrization invariant metric on field space (see e.g. refs. [15], [5] and references therein)

\[
||\delta X||^2_e = \int d^2 x e(\delta X)^2, \quad \int (D\delta X)_e e^{-||\delta X||^2_e} = 1.
\] (2.7)
We compute this path integral by completing squares and shifting the field $X \to X + \frac{1}{2} \Box^{-1}(\beta_1 R_1 + \beta_2 R_2)$, so that it reduces to the path integral of a free scalar without background charges. This way we obtain the following induced action

$$I[e^a_\mu] = \frac{1}{24\pi} \int d^2 x e \left( R \frac{1}{\Box} R + 3(\beta_1 R_1 + \beta_2 R_2) \frac{1}{\Box} (\beta_1 R_1 + \beta_2 R_2) \right),$$

where the first term on the right hand side is due to the well-known contribution of a free boson (which has $c = \bar{c} = 1$) and the second term is due to square completion. Cross terms between $R_1$ and $R_2$ are local (up to boundary terms discarded in our local analysis)

$$\int d^2 x R_1 \frac{1}{\Box} R_2 = \int d^2 x \omega_+ \omega_-$$

and eq. (2.8) can be written as follows

$$I[e^a_\mu] = I[e^a_\mu; c, \bar{c}, a] \equiv \frac{1}{24\pi} \int d^2 x e \left( c R_1 \frac{1}{\Box} R_1 + \bar{c} R_2 \frac{1}{\Box} R_2 + 2a \omega_+ \omega_- \right),$$

with $c$ and $\bar{c}$ given by eq. (2.5). The parameter $a$ is ambiguous and whereas the above computation gives $a = 1+3\beta_1 \beta_2$, its value is generically related to the specific regularization procedure adopted for computing the induced action. It multiplies a local term, and it can be changed at will by adding a local counterterm of the same form to the effective action. It is not fixed by requiring general coordinate invariance and 2d rigid Lorentz invariance. Note that for $c = \bar{c}$, one can recover the local Lorentz invariance by choosing $a = c$, thus obtaining the non-chiral action for gravity

$$I_0[e^a_\mu; c] = \frac{c}{24\pi} \int d^2 x e \frac{1}{\Box} R.$$ 

We consider now the action in (2.10) as the gravitational action to be quantized in order to investigate properties of off-critical chiral strings. We again point out that such an action is manifestly reparametrization invariant because it is built from manifestly invariant objects (scalars), but contains Lorentz and Weyl anomalies. It gives dynamics to the Lorentz and Weyl modes of the vielbein. To see this explicitly, we choose the conformal gauge for the diffeomorphism group

$$e_\mu^{+\pm} = \exp(\sigma - i\lambda)\hat{e}_\mu^{+\pm}, \quad e_\mu^{-} = \exp(\sigma + i\lambda)\hat{e}_\mu^{-},$$

where $\sigma$ is a real scalar field and $\lambda$ is a real vector field. The field $\hat{e}_\mu^{+\pm}$ is the vielbein field in the conformal gauge, and $\hat{e}_\mu^-$ is the vielbein field in the ordinary gauge. The parameter $\sigma$ is the conformal factor, and $\lambda$ is the conformal vector field. We have

$$e_\mu^{+\pm} = \exp(\sigma - i\lambda(\mu^+ \pm)) \hat{e}_\mu^{+\pm}, \quad e_\mu^{-} = \exp(\sigma + i\lambda(\mu^-)) \hat{e}_\mu^{-},$$

where $\mu^+$ and $\mu^-$ are the ordinary coordinates of the vielbein field.

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where $\mu^+$ and $\mu^-$ are the ordinary coordinates of the vielbein field.
where $\hat{e}_\mu^a$ is a given background vielbein, and obtain from (2.10) the following Lorentz-
Weyl action

$$S[\lambda, \sigma, \hat{e}_\mu^a] \equiv I[e_\mu^a] - I[\hat{e}_\mu^a]$$

$$= \frac{1}{24\pi} \int d^2x \hat{e} \left[ \sigma \square \sigma (c + \bar{c} + 2a) + \lambda \hat{\square} \lambda (2a - c - \bar{c}) + \lambda \hat{\square} 2i(\bar{c} - c) - 2\hat{R}_1\sigma (c + a) - 2\hat{R}_2\sigma (\bar{c} + a) - 2i\hat{R}_1\lambda (a - c) - 2i\hat{R}_2\lambda (\bar{c} - a) \right].$$

(2.13)

This is a generalization of the Liouville action (with the cosmological constant set to zero)
induced by off-critical strings. Note that we have written our equations with an euclidean
signature for the worldsheet, where the Lorentz group is the compact group $U(1)$ and the
$\lambda$ field is compactified. In a minkowskian signature one should Wick rotate to $\varphi = i\lambda$
and forget about the compactness. These comments should be kept in mind, for example,
when discussing unitarity of the theory or its global properties.

We are now going to analyze the quantization of (2.13). For simplicity we proceed
in a stepwise fashion, first considering the quantization of the Lorentz field $\lambda$ and then
the quantization of the Weyl field $\sigma$. We will use methods similar to those employed by
David, Distler and Kawai, i.e. we will disregard the cosmological constant, which we have
already set to zero from the beginning, and employ free field quantization. Therefore, we
start anew by making explicit the dependence of the induced action on the Lorentz field
only. We set $e_\mu^\pm = \exp(-i\lambda)\hat{e}_\mu^\pm$ and $e_\mu^- = \exp(i\lambda)\hat{e}_\mu^-$ as a partial gauge choice, and
obtain from (2.10) the following action for the Lorentz field (we just have to set $\sigma = 0$ in
(2.13) and replace hats with tildes)

$$S_{Lor}[\lambda, \tilde{e}_\mu^a] \equiv I[e_\mu^a] - I[\tilde{e}_\mu^a]$$

$$= \frac{1}{24\pi} \int d^2x \tilde{e} \left[ \lambda \tilde{\square} \lambda (2a - c - \bar{c}) - 2i\hat{R}_1\lambda (a - c) - 2i\hat{R}_2\lambda (\bar{c} - a) \right].$$

(2.14)

Of course, the background $\tilde{e}_\mu^a$ now contains the Weyl field $\sigma$: $\tilde{e}_\mu^a = \exp(\sigma)\hat{e}_\mu^a$. From
this expression it looks that $\lambda$ behaves like a chiral boson with background charges, as in
(2.1). To be sure we have to check that no surprises arise from the correct path integral
measure that must be used in quantizing $\lambda$. Such a measure is the one induced by the
reparametrization invariant metric on the space of worldsheet vielbeins and it is determined
by

$$||\delta \lambda||_e^2 = \int d^2x e(\delta \lambda)^2 \rightarrow (D\lambda)_e.$$

(2.15)
Of course, we can substitute $\tilde{e}$ for $e$, since the determinant of the vielbein is Lorentz invariant, implying that $(\mathcal{D}\lambda)_e = (\mathcal{D}\lambda)_{\tilde{e}}$. We see that the functional measure for the Lorentz field coincides with the usual translational invariant measure used to quantize free scalars, as in (2.6), and we conclude that $\lambda$ can really be treated as a chiral boson. It is then immediate to write down the components of the stress tensor and their central charges (just by using the previously derived formulas for the chiral boson)

\[
T_{++} = \frac{1}{2} (\partial_+ \lambda)^2 + \frac{\beta_1}{2} \partial^2 \lambda, \quad \beta_1 = i \frac{c-a}{6} \left( \frac{12}{c + \bar{c} - 2a} \right)^{\frac{1}{2}}, \quad c_{\text{Lor}} = 1 - \frac{(c-a)^2}{c + \bar{c} - 2a},
\]

\[
T_{--} = \frac{1}{2} (\partial_- \lambda)^2 + \frac{\beta_2}{2} \partial^2 \lambda, \quad \beta_2 = i \frac{a - \bar{c}}{6} \left( \frac{12}{c + \bar{c} - 2a} \right)^{\frac{1}{2}}, \quad \bar{c}_{\text{Lor}} = 1 - \frac{(\bar{c}-a)^2}{c + \bar{c} - 2a},
\]

where we have rescaled the Lorentz field $\lambda \to \lambda \left( \frac{12}{c + \bar{c} - 2a} \right)^{\frac{1}{2}}$ to get a standard normalization for the propagator

\[
\langle \lambda(x)\lambda(y) \rangle = - \log(\mu^2 |x - y|^2).
\]

In doing so we have assumed $c + \bar{c} - 2a > 0$. The same holds also for $c + \bar{c} - 2a < 0$, but in this case we have to remember that with the field redefinition we have automatically performed a Wick rotation on $\lambda$. The Lorentz field has the effect of leveling up the chiral mismatch between the matter central charges $c$ and $\bar{c}$, namely

\[
c + c_{\text{Lor}} = \bar{c} + \bar{c}_{\text{Lor}} = 1 + \frac{c\bar{c} - a^2}{c + \bar{c} - 2a}.
\]

Note that $a$ is left arbitrary. We can derive once more these results by path integrating the Lorentz field with the measure in (2.15). Completing the squares and using the translational invariance of the measure we obtain

\[
e^{-I_{\text{Lor}}[\tilde{e}_\mu^a]} = \int (\mathcal{D}\lambda)_{\tilde{e}} e^{-S_{\text{Lor}}[\lambda, \tilde{e}_\mu^a]} \]

\[
I_{\text{Lor}}[\tilde{e}_\mu^a] = I[\tilde{e}_\mu^a; c_{\text{Lor}}, \bar{c}_{\text{Lor}}, a_{\text{Lor}}]
\]

where the functional on the right hand side of the second equation was defined in (2.10), $c_{\text{Lor}}$ and $\bar{c}_{\text{Lor}}$ are as in (2.16), and

\[
a_{\text{Lor}} = 1 + \frac{(c-a)(\bar{c}-a)}{c + \bar{c} - 2a}.
\]

In principle the value of $a_{\text{Lor}}$ could be changed by adding a local counterterm to $I_{\text{Lor}}$, but this is not necessary since we can check that $a + a_{\text{Lor}} = c + c_{\text{Lor}}$. It is the correct value which secures background Lorentz invariance

\[
I[\tilde{e}_\mu^a] + I_{\text{Lor}}[\tilde{e}_\mu^a] = I_0[\tilde{e}_\mu^a, c + c_{\text{Lor}}].
\]
The functional on the right hand side of this equation was defined in (2.11). Since \( a \) is still a free parameter, one could fix it by requiring \( c + c_{\text{Lor}} = 26 \), implying that the Weyl invariance is recovered once the reparametrization ghosts are introduced. We will avoid such a fine tuning, so that we are still left to quantize the Weyl mode \( \sigma \). This can be done by repeating the analysis of David, Distler and Kawai, and goes as follows. We complete the specification of the conformal gauge by setting \( \bar{e}_\mu^a = \exp(\sigma) e_\mu^a \) and take into account the well-known contribution of the reparametrization ghosts to the induced gravitational action

\[
I_{gh}[\bar{e}_\mu^a] = I_0[\bar{e}_\mu^a; -26].
\] (2.22)

The reparametrization ghosts are non-chiral and this fact has allowed us to postpone their inclusion up to now. Introducing them at an earlier stage would not have affected the previous analysis. The combined matter, Lorentz and ghost fields induce the non-chiral action \( I_0[\bar{e}_\mu^a, c + c_{\text{Lor}} - 26] \) which in turn produces the following Liouville action for the Weyl field

\[
S_{\text{Liou}}[\sigma, \dot{\sigma}] = I_0[\bar{e}_\mu^a; c + c_{\text{Lor}} - 26] - I_0[\dot{e}_\mu^a; c + c_{\text{Lor}} - 26] = (c + c_{\text{Lor}} - 26) \frac{1}{6\pi} \int d^2 x \dot{\sigma} \left[ \sigma \square \sigma - \dot{R} \sigma \right].
\] (2.23)

This Liouville action should be quantized with the measure induced by the distance function

\[
||\delta \sigma||_e^2 = \int d^2 x e(\delta \sigma)^2 = \int d^2 x \dot{e} e^\sigma (\delta \sigma)^2 \rightarrow (D \sigma)_\dot{e}
\] (2.24)

which is derived from the reparametrization invariant metric on the space of worldsheet vielbeins. The problem with this measure is that it is not invariant under shifts of the field \( \sigma \) and it is not known how to use it to quantize the theory. The best one can do is to quantize the Weyl field using the shift invariant measure obtained from

\[
||\delta \sigma||_{\dot{e}}^2 = \int d^2 x \dot{e}(\delta \sigma)^2 \rightarrow (D \sigma)_{\dot{e}}
\] (2.25)

and including a jacobian \( J \) which relates (2.24) to (2.25) [5]

\[
(D \sigma)_e = J(D \sigma)_{\dot{e}}.
\] (2.26)

This jacobian is formally given by

\[
J = \det e^\sigma
\] (2.27)
and was computed in [16] using an heat kernel regularization. The heat kernel used was the one corresponding to the scalar laplacian, which is the second functional derivative of the Liouville action (2.23) (i.e. it corresponds to the kinetic operator for the Weyl field). This means, in practice, that to compute the jacobian $J$ one has to compute the gravitational action induced by a single scalar field. In fact, the two computations are similar and the only difference is a normalization factor easily taken into account. The result is thus given by

$$J = \exp\left(- (I_0[\hat{\epsilon}_{\mu}^a, 1] - I_0[\hat{\epsilon}_{\mu}^a, 1])\right). \tag{2.28}$$

The subtraction of the constant $I_0[\hat{\epsilon}_{\mu}^a, 1]$ insures that $J = 1$ for $\sigma = 0$. Including the contribution of this jacobian in (2.23) brings us to consider the following path integral for the quantization of the Weyl mode

$$e^{-I_{Lio}[\hat{\epsilon}_{\mu}^a]} = \int (D\sigma) \hat{\epsilon} e^{-S_{Lio}[\sigma, \hat{\epsilon}_{\mu}^a]} \tag{2.29}$$

which gives

$$I_{Lio}[\hat{\epsilon}_{\mu}^a] = I_0[\hat{\epsilon}_{\mu}^a, c_{Lio}]$$

$$c_{Lio} = 26 - c - c_{Lor}. \tag{2.30}$$

It implies that with the inclusion of the Weyl field the total central charge vanish in each chiral sector

$$c_{tot} = c + c_{Lor} - 26 + c_{Lio} = 0$$

$$\bar{c}_{tot} = \bar{c} + \bar{c}_{Lor} - 26 + \bar{c}_{Lio} = 0, \tag{2.31}$$

or, equivalently,

$$I[\hat{\epsilon}_{\mu}^a] + I_{Lor}[\hat{\epsilon}_{\mu}^a] + I_{gh}[\hat{\epsilon}_{\mu}^a] + I_{Lio}[\hat{\epsilon}_{\mu}^a] = 0. \tag{2.32}$$

It is tempting to identify $c' \equiv c + c_{Lor}$ as the central charge $c$ appearing in the formulas for non-chiral gravity [3][4][5], carrying over all of the corresponding expressions for critical indices and observing that the barrier $c = 1$ is replaced by $c' = 1$. It implies that in term of our $c$, the barrier is $a$-dependent and can be avoided even for $c > 1$ by properly choosing $a$. However, this conclusion seems too naïve to us, and while it is partly confirmed in [12], we believe that more analysis is needed to understand: i) the topological issues connected to superselection rules for the Lorentz field found by Myers and Periwal, especially from the point of view of path integrals, ii) unitarity of the theory, since the imaginary coupling and the possibility of getting $c_{Lor}$ and $c'$ negative by choosing $a$ looks suspicious. We stress once
more that the quantization of the gravitational sector in the conformal gauge has resulted in a Lorentz-Weyl theory with central charges $c_{\text{grav}} \equiv c_{\text{Lor}} + c_{\text{Liou}}$ and $\bar{c}_{\text{grav}} \equiv \bar{c}_{\text{Lor}} + \bar{c}_{\text{Liou}}$ obeying $c_{\text{grav}} = 26 - c$ and $\bar{c}_{\text{grav}} = 26 - \bar{c}$. We have shown this by quantizing the Lorentz field first and the Weyl field afterwards. The vanishing of the total central charges is needed for the consistent quantization of the gravitational sector, since background Lorentz and Weyl symmetries correspond to a change of the gauge slice required in fixing the reparametrization invariance. Of course, the final result on the vanishing of the total central charges is independent of any stepwise analysis [12]. In the sequel we will develop a similar analysis for the (1, 0) and (1, 1) supersymmetric cases and will not dwell further on the interesting but difficult topological issues mentioned above, nor on the question of unitarity.

To conclude this section, we summarize the strategy just presented, since we will follow it closely in the announced supersymmetric extensions. First, we construct a chiral boson to represent a conformal system with arbitrary central charges $c$ and $\bar{c}$. Then, we use it to induce the gravitational action containing the expected Weyl and Lorentz anomalies. This is the action which should be quantized in order to describe off-critical string theories. Then, we proceed to discuss some aspects of this quantization using the free field approach pioneered by David, Distler and Kawai. We check that at the quantum level the Lorentz mode behaves as a chiral boson. This allows us to directly apply formulae already in our hands. A similar analysis then applies to the Weyl mode, which is non-chiral in the (0, 0) and (1, 1) cases. In the (1, 0) case, an unavoidable chiral structure for the Weyl mode is present, as dictated by the chiral structure of the superspace itself. Looking at the total central charges we check that the background Lorentz and Weyl symmetries hold after quantization, a necessary requirement for gauge independence.

3. Induced (1, 0) chiral supergravity

To identify the induced action for the heterotic string off-critical dimensions, we are going to parallel the previous analysis in the (1, 0) superspace, reviewed for convenience in appendix B. To start with, we describe a chiral boson in superspace. Of course, the usual free scalar superfield $X$ with action

$$S_f[X, E_M^A] = \frac{1}{2\pi} \int d^3Z e_+ \nabla_+ X \nabla_- X \tag{3.1}$$
is already chiral because of the chiral nature of $(1,0)$ superspace. It contains a scalar field plus a left moving Majorana-Weyl fermion. The corresponding stress tensor generates a left superconformal algebra with $c = \frac{3}{2}$ and a right conformal algebra with $\bar{c} = 1$. To achieve arbitrary central charges we consider the coupling of the field $X$ to the background curvatures $R_1^+$ and $R_2^+$ described in appendix B, so that the stress tensor will acquire improvement terms

$$S[X, E_M^A] = \frac{1}{2\pi} \int d^3 Z e_+(\nabla_+ X \nabla_- X + \beta_1 R_1^+ X + \beta_2 R_2^+ X).$$

(3.2)

The super-stress tensor $T_{-A}^B$ is defined by

$$\delta S = -\frac{1}{\pi} \int d^3 Z e_+ T_{-A}^B H_B^A (-1)^A$$

(3.3)

with $H_A^B = \delta E_A^M E_M^B$ running over the set of independent variations ($H_+^A, H_-^-, H_-^+$) (the variations $H_A^B$ are not all independent because the constraints defining the superspace must be satisfied). Employing the list of dependent variations reported in appendix B, we obtain the following non-vanishing components of the stress tensor evaluated on-shell and in flat superspace

$$T \equiv T_{-+}^- = -\frac{1}{2} D_+ X \partial_+ X - \frac{1}{2} \beta_1 \partial_+ D_+ X,$$

$$\bar{T} \equiv T_{-}^{-+} = -\frac{1}{2} \partial_\pm X \partial_\pm X - \frac{1}{2} \beta_2 \partial_\pm^2 X.$$  

(3.4)

These components of the stress tensor generate through the operator product expansion a superconformal algebra with $c = \frac{3}{2} + 3\beta_1^2$ and a conformal algebra with $\bar{c} = 1 + 3\beta_2^2$. To verify this, one just need to use the propagator which follows from the flat space limit of (3.1)

$$\langle X(Z_1) X(Z_2) \rangle = -\log (\mu^2 z_{12}^+(z_1^- - z_2^-)),$$

(3.5)

where $z_1^+ = z_1^+ - z_2^+ - \theta_1^+ \theta_2^+$, and perform the necessary Wick contractions.

We now compute the $(1,0)$ supergravitational induced action. We first need to recall that for $c = \bar{c}$ the super-Weyl anomalous action was obtained in [17] and reads

$$I_0[E_M^A; c] = \frac{c}{24\pi} \int d^3 Z e_+ R^+ \frac{1}{\Box + \frac{1}{4} R^+ \nabla_+ R^+} \nabla_+ R^+$$

(3.6)

where $\Box = \frac{1}{2} (\nabla_+ \nabla_- + \nabla_- \nabla_+)$. It can be cast in a more convenient form which is reminiscent of the non-chiral bosonic induced action

$$I_0[E_M^A; c] = \frac{c}{24\pi} \int d^3 Z e_+ R^+ \frac{1}{\nabla_+ \nabla_-} R^+.$$  

(3.7)
Using this piece of information and the chiral boson constructed above, we are able to obtain the general \((1,0)\) induced action with super-Weyl and super-Lorentz anomalies. It is given by

\[
I_{[E_M^A]} = I_{[E_M^A; c, \bar{c}, a]} \equiv \frac{1}{24\pi} \int d^3 Z e_+ \left( c R_1^+ \frac{1}{\nabla_+ \nabla_-} R_1^+ + c R_2^+ \frac{1}{\nabla_+ \nabla_-} R_2^+ + 2a \Omega_+ \Omega_- \right)
\]

where \(c\) and \(\bar{c}\) are the central charges characterizing the two chiral symmetry algebras of the inducing matter, and \(a\) is a coupling left unfixed by the requirement of reparametrization invariance (it can be changed at will by adding a local counterterm of the form \(\Omega_+ \Omega_-\) to the induced action). This result can be obtained by the following considerations. A system with \(c = \bar{c} = \frac{3}{2}\) can be constructed by setting \(\beta_1 = 0\) and \(\beta_2 = \frac{1}{\sqrt{6}}\) in (3.2). It induces (3.7) with \(c = \frac{3}{2}\) once we use the path integral measure derived from the super-reparametrization invariant norm

\[
||\delta X||_E^2 = \int d^3 Z e_+ (\delta X)^2 \rightarrow (DX)_E
\]

joined with the requirement of fixing local ambiguities in the induced action by imposing Lorentz invariance. This is so because the path integral measure is invariant under super-reparametrizations, leaving only the possibility of Lorentz and Weyl anomalies. The Lorentz anomalies are eliminated by the requirement of preserving the corresponding Ward identities, which is possible for \(c = \bar{c}\), leaving only Weyl anomalies, correctly contained in (3.6). On the other hand, if we evaluate by square completion the path integral so constructed, we obtain by consistency the form of the gravitational action induced by the free scalar superfield

\[
e^{-I_f[E_M^A]} = \int (DX)_E e^{-S_f[X,E_M^A]}
\]

\[
I_f[E_M^A] = I[E_M^A; 3/2, 1, a_f].
\]

In this specific computation \(a_f = \frac{3}{2}\), but, as already mentioned, \(a_f\) multiplies a local term and its value can be changed at will by adding a local counterterm of the same form. There is no natural choice for it, as can be seen by rederiving \(I_f[E_M^A]\) using the \(c = \bar{c} = 1\) system described by (3.1) with \(\beta_1 = \frac{i}{\sqrt{6}}, \beta_2 = 0\). Proceeding once more as described above gives \(a_f = 1\) and shows that the value of \(a_f\) is not uniquely determined by the symmetries of the model. Armed with the knowledge of \(I_f[E_M^A]\), it is immediate to prove eq. (3.8) in full generality. This equation is the \((1,0)\) supersymmetric generalization of the Leutweyler action (1.1). In the superconformal gauge defined by

\[
E_{+}^M = \exp \left( \frac{i}{2} L - \frac{1}{2} W \right) \hat{E}_{+}^M, \quad E_{-}^M = \exp(-iL - W) \hat{E}_{-}^M
\]

(3.11)

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it produces the following extension of the super-Liouville action

\[ S[L, W, \hat{E}_M^A] \equiv I[E_M^A] - I[\hat{E}_M^A] \]

\[ = \frac{1}{24\pi} \int d^3Z \hat{e}_+ \left[ W \hat{\nabla}_+ \hat{\nabla}_-=W(c+\bar{c}+2a) + L \hat{\nabla}_+ \hat{\nabla}_-=L(2a-c-\bar{c}) \right. \]

\[ + L \hat{\nabla}_+ \hat{\nabla}_-=W 2i(\bar{c}-c) + 2\hat{R}_1^+ W(c+a) + 2\hat{R}_2^+ W(\bar{c}+a) \]

\[ + 2i\hat{R}_1^+ L(a-c) + 2i\hat{R}_2^+ L(\bar{c}-a) \]. \quad (3.12)

We now discuss the quantization of this (1, 0) chiral induced supergravity. First of all we have to consider the inclusion of the super-reparametrization ghosts. It is well-known that such a ghost system has \( c_{gh} = -15 \) and \( \bar{c}_{gh} = -26 \), so that we need to add to the matter induced action the following ghost contribution

\[ I_{gh}[E_M^A] = I[E_M^A; -15, -26, a_{gh}]. \quad (3.13) \]

A second thing we have to take into account is the correct path integral measure required for quantization. To path integrate over the Lorentz and Weyl fields we have to use the measures obtained from the super-reparametrization invariant norm on the space of worldsheet supervielbeins. The latter induces norms for the Lorentz and Weyl fields which are similar to the one for a scalar superfield

\[ ||\delta L||^2_E = \int d^3Z \hat{e}_+(\delta L)^2 = \int d^3Z \hat{e}_+ \exp \left( \frac{i}{2} L + \frac{3}{2} W \right) (\delta L)^2 \rightarrow (DL)_E \]

\[ ||\delta W||^2_E = \int d^3Z \hat{e}_+(\delta W)^2 = \int d^3Z \hat{e}_+ \exp \left( \frac{i}{2} L + \frac{3}{2} W \right) (\delta W)^2 \rightarrow (DW)_E. \quad (3.14) \]

However, the path integral measures \((DL)_E\) and \((DW)_E\) are not translational invariant since the superdeterminant \( e_+ \) depends on the Lorentz and Weyl fields themselves. It is not clear how to use these measures to compute directly the path integral. Instead, one can use the translational invariant measures obtained from

\[ ||\delta L||^2_E = \int d^3Z \hat{e}_+ (\delta L)^2 \rightarrow (DL)_E \]

\[ ||\delta W||^2_E = \int d^3Z \hat{e}_+ (\delta W)^2 \rightarrow (DW)_E. \quad (3.15) \]

These translational invariant measures are related to the super-reparametrization invariant ones by a jacobian factor

\[ (DL)_E = J(DL)_E, \quad (DW)_E = J(DW)_E \quad (3.16) \]
formally given by \( J = \text{sdet} \exp\left(\frac{i}{2}L + \frac{3}{2}W\right)\). We compute \( J \) in a way similar to the bosonic case treated in refs. [16] and briefly reviewed in the previous section. Accordingly, we should calculate the jacobian \( J \) using the heat-kernel regularization corresponding to the “laplacian” \( \nabla^+ \nabla^- \) which is the kinetic operator for a scalar superfield (with the necessary modification to obtain a truly elliptic differential operator). It is clear that such a computation is identical to the one that must be done to derive the gravitational action induced by a free scalar superfield, where all of the anomalous dependence on the Lorentz and Weyl modes of the supervielbein is contained in the path integral measure (3.9). In fact, one could derive these anomalies by computing the heat-kernel regulated jacobian corresponding to infinitesimal symmetry transformations and obtain the induced action by integrating the corresponding anomalous Ward identities. The only difference between such an induced action and our jacobian \( J \) is the overall normalization. Recalling eq. (3.10), we can immediately write down the result

\[
J = \exp\left(-\left(I[E_M^A; 3/2, 1, a_J] - I[\hat{E}_M^A; 3/2, 1, a_J]\right)\right) \tag{3.17}
\]

where the last term in the exponent is a constant included to normalize \( J \) to unity for \( L = W = 0 \). As could have been expected, the expression in the exponent has the structure of the Lorentz-Weyl action in (3.12). Note that there is no canonical value for the parameter \( a_J \). To summarize, we collect in one expression all terms containing a Lorentz and Weyl dependence

\[
I'[E_M^A] \equiv I[E_M^A] + I_{gh}[E_M^A] - 2\log J
= I[E_M^A; c, \bar{c}, a] + I[E_M^A; -15, -26, a_{gh}]
+ 2(I[E_M^A; 3/2, 1, a_J] - I[\hat{E}_M^A; 3/2, 1, a_J]) \tag{3.18}
\]

where on the right hand side of the top line the first term is due to the inducing matter, the second to the ghosts for the conformal gauge and the third one to the jacobians in (3.16). This action can be rewritten in a more compact way as

\[
I'[E_M^A] = I[E_M^A; c', \bar{c}', a'] - 2I[\hat{E}_M^A; 3/2, 1, a_J] \tag{3.19}
\]

where

\[
c' = c - 15 + 3 \\
\bar{c}' = \bar{c} - 26 + 2 \\
a' = a + a_{gh} + 2a_J. \tag{3.20}
\]
It should be quantized with the translational invariant measures (3.15). We will consider first the quantization of the Lorentz field, as already explained in the bosonic case. To extract from (3.19) the part depending on the Lorentz mode we set

\[ E_+^M = \exp \left( \frac{i}{2} L \right) \tilde{E}_+^M, \quad E_-^M = \exp(-iL)\tilde{E}_-^M, \quad (3.21) \]

where the vielbein \( \tilde{E}_A^M \) now contains the Weyl field \( W \), and obtain from the relevant piece of (3.19) the following Lorentz action

\[ S_{Lor}[L, \tilde{E}_M^A] \equiv I[E_M^A; c', \bar{c}', a'] - I[\tilde{E}_M^A; c', \bar{c}', a'] \]

\[ = \frac{1}{24\pi} \int d^3\tilde{e} \left[ L \tilde{\nabla}_+ \tilde{\nabla}_- L(2a' - c' - \bar{c}') + 2i\tilde{R}_1^+ L(a' - \bar{c}') + 2i\tilde{R}_2^+ L(\bar{c}' - a') \right]. \quad (3.22) \]

Clearly, the Lorentz field behaves as a \((1,0)\) chiral boson, and since we have already taken care of the nontrivial part of the path integral measure, we can immediately compute

\[ e^{-I_{Lor}[\tilde{E}_M^A]} = \int (DL)_{\tilde{E}} e^{-S_{Lor}[L, \tilde{E}_M^A]} \]

\[ I_{Lor}[\tilde{E}_M^A] = I[\tilde{E}_M^A; c_{Lor}, \bar{c}_{Lor}, a_{Lor}] \]

with the values of the central charges given by

\[ c_{Lor} = \frac{3}{2} - \frac{(c' - a')^2}{c' + \bar{c}' - 2a'} \]

\[ \bar{c}_{Lor} = 1 - \frac{(\bar{c}' - a')^2}{c' + \bar{c}' - 2a'}. \quad (3.24) \]

Here above we have used for simplicity the measure \((DL)_{\tilde{E}}\) defined in the obvious way. Clearly the relation between this measure and the ones previously defined is as follows

\[ (DL)_{E} = J_1(DL)_{\tilde{E}}, \quad J_1 = \exp(-I[E_M^A; 3/2, 1, aJ] - I[\tilde{E}_M^A; 3/2, 1, aJ]) \]

\[ (DL)_{\tilde{E}} = J_2(DL)_{\tilde{E}}, \quad J_2 = \exp(-I[\tilde{E}_M^A; 3/2, 1, aJ] - I[\tilde{E}_M^A; 3/2, 1, aJ]). \quad (3.25) \]

Of course, \( J = J_1 J_2 \). Note that the values of the Lorentz central charges depend on the arbitrary parameter \( a' \), in a way similar to the bosonic case. Note also that there is no particular value of \( a_{Lor} \) which looks natural, so far. However, we will soon discover that the Ward identities for background Lorentz and Weyl invariance will fix a unique value for such a constant. Now, we are left to quantize the Weyl field. We complete the specification of the conformal gauge by setting

\[ \tilde{E}_+^M = \exp \left( -\frac{1}{2} W \right) \tilde{E}_+^M, \quad \tilde{E}_-^M = \exp(-W)\tilde{E}_-^M \quad (3.26) \]
and collect all the terms left over after the Lorentz integration

\[ I''[\tilde{E}_M^A] \equiv I[\tilde{E}_M^A] + I_{gh}[\tilde{E}_M^A] + I_{\text{Lor}}[\tilde{E}_M^A] - \log J_2. \quad (3.27) \]

There is only one jacobian here since the one for the Lorentz field was used to construct the measure \((DL)\tilde{E}\) in (3.23) and is effectively incorporated in \(I_{\text{Lor}}[\tilde{E}_M^A]\). We rewrite the action (3.27) in a compact form

\[ I''[\tilde{E}_M^A] = I[\tilde{E}_M^A; c'', \bar{c}'', a''] - I[\tilde{E}_M^A; 3/2, 1, a_J] \quad (3.28) \]

where

\[ c'' = c - 15 + c_{\text{Lor}} + \frac{3}{2} \]
\[ \bar{c}'' = \bar{c} - 26 + \bar{c}_{\text{Lor}} + 1 \]
\[ a'' = a + a_{gh} + a_{\text{Lor}} + a_J, \quad (3.29) \]

and use it to obtain the following super-Liouville action

\[
S_{\text{Liou}}[W, \hat{E}] \equiv I[\tilde{E}_M^A; c'', \bar{c}'', a''] - I[\tilde{E}_M^A; c'', \bar{c}'', a''] \\
= \frac{1}{24\pi} \int d^3 \hat{e}_+ \left[ W \hat{\nabla}_+ \hat{\nabla}_- W (c'' + \bar{c}'' + 2a'') + 2\hat{R}_1^+ W (c'' + a'') + 2\hat{R}_2^+ W (\bar{c}'' + a'') \right].
\]

(3.30)

It is immediate to quantize it by evaluating the corresponding path integral

\[
e^{-I_{\text{Liou}}[\tilde{E}_M^A]} = \int (DL)\tilde{E} \ e^{-S_{\text{Liou}}[W, \tilde{E}_M^A]} \quad (3.31)
\]

\[ I_{\text{Liou}}[\tilde{E}_M^A] = I[\tilde{E}_M^A; c_{\text{Liou}}, \bar{c}_{\text{Liou}}, a_{\text{Liou}}] \]

where the values of the central charges are given by

\[
c_{\text{Liou}} = \frac{3}{2} - \frac{(c'' + a'')^2}{c'' + \bar{c}'' + 2a''} \quad (3.32)
\]
\[
\bar{c}_{\text{Liou}} = 1 - \frac{(\bar{c}'' + a'')^2}{c'' + \bar{c}'' + 2a''}.
\]

Plugging the values of the various central charges in these formulae, one can verify that we can achieve

\[ c + c_{\text{Lor}} - 15 + c_{\text{Liou}} = \bar{c} + \bar{c}_{\text{Lor}} - 26 + \bar{c}_{\text{Liou}} = 0 \quad (3.33) \]

by suitably choosing \(a_{\text{Lor}}\) in \(a''\). Let’s see this in a more detailed way. One can first check that \(\bar{c}'' = c''\). Then, fixing \(a_{\text{Lor}}\) in \(a''\) to obtain \(a'' = c''\) gives

\[
c_{\text{Liou}} = \frac{3}{2} - c'' \quad (3.34)
\]
\[
\bar{c}_{\text{Liou}} = 1 - c'',
\]

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and, consequently, eq. (3.33). Thus, there is a unique value for \( a_{\text{Lor}} \) consistent with the Lorentz and Weyl background invariance, namely \( a_{\text{Lor}} = c'' - (a + a_{gh} + a_{J}) \). Note that with this specific value of \( a_{\text{Lor}} \) the Liouville action (3.30) takes the following simple form

\[
S_{\text{Liou}} = \frac{c''}{6\pi} \int d^3Z \hat{e}_+ [W \hat{\nabla}_+ \hat{\nabla}_- W + \hat{R}^+ W].
\] (3.35)

A final thing to take care of is to fix \( a_{\text{Liou}} = -(a + a_{gh} + a_{\text{Lor}}) \) in (3.31) to recover the full background Lorentz and Weyl invariance, so that we obtain

\[
I[\hat{E}_M{}^A] + I_{\text{Lor}}[\hat{E}_M{}^A] + I_{gh}[\hat{E}_M{}^A] + I_{\text{Liou}}[\hat{E}_M{}^A] = 0.
\] (3.36)

We have thus seen that for the (1, 0) induced supergravity the quantization of the Lorentz-Weyl theory gives central charges \( c_{\text{grav}} = 15 - c \) and \( \bar{c}_{\text{grav}} = 26 - \bar{c} \), together with a unique value for the couplings \( a_{\text{Lor}} \) and \( a_{\text{Liou}} \). These results are necessary to show the consistency of the quantization of the supergravitational sector, since the background Lorentz and Weyl invariance corresponds to a change of the gauge slice. The analysis has been very similar to the one carried out in the bosonic case, even though the chiral nature of (1, 0) superspace has made things slightly more complicated.

4. Induced (1, 1) chiral supergravity

The (1, 1) superspace is the natural arena for the description of the spinning string as long as one does not need fermionic vertex operators [18]. Its non-chiral structure makes it easier to discuss the Lorentz and Weyl anomalous action for supergravity induced by chiral supermatter as well as its quantization, and we can follow closely the case of non-critical bosonic strings.

We start with a bosonic superfield \( X \) coupled to the chiral background scalars \( R_1 \) and \( R_2 \) (see appendix C)

\[
S[X, E_M{}^A] = \frac{1}{2\pi} \int d^4Z E(\nabla_+ X \nabla_- X + \beta_1 R_1 X + \beta_2 R_2 X).
\] (4.1)

It describes a chiral boson in superspace as can be checked by computing its stress tensor \( T_A{}^B \) defined by

\[
\delta S = -\frac{1}{\pi} \int d^4Z E T_A{}^B H_B{}^A(-1)^A,
\] (4.2)
with $H_A^B$ running over the set of the six independent variations ($H_{\pm}^\pm, H_{\pm}^\mp, H_{+}^+, H_-^-$).

Using the list of the dependent variations given in appendix C, one gets the following non-vanishing components of the stress tensor evaluated on-shell and in flat superspace

$$
T \equiv T_{++}^- = -\frac{1}{2}D_+X\partial_+X - \frac{1}{2}\beta_1\partial_+D_+X, \quad c = \frac{3}{2} + 3\beta_1^2,
$$
$$
\bar{T} \equiv T_{--}^+ = \frac{1}{2}D_-X\partial_-X + \frac{1}{2}\beta_2\partial_-D_-X, \quad \bar{c} = \frac{3}{2} + 3\beta_2^2.
$$

We have reported also the central charges of the corresponding super-Virasoro algebras. Their values can be checked using the propagator

$$
\langle X(Z_1)X(Z_2) \rangle = -\log(\mu^2 z_{12}^+ z_{12}^-),
$$

where $z_{12}^\pm = z_1^\pm - z_2^\pm$ and $\theta_1^\pm = z_1^- - z_2^+ - \theta_1^- \theta_2^+$, and computing the relevant operator product expansions. This is enough to make sure that eq. (4.1) describes a chiral system. We now use this chiral boson to represent superconformal systems with arbitrary central charges $c$ and $\bar{c}$. After recalling the form of the effective action due to a $c = \bar{c}$ system [19]

$$
I_0[E^A_M; c] = \frac{c}{24\pi} \int d^4 Z \frac{1}{\nabla_+ \nabla_-} R,
$$

it is a simple task to obtain the Lorentz and Weyl anomalous effective action induced by chiral superconformal systems. It reads

$$
I[E^A_M] = I[E^A_M; c, \bar{c}, a] \equiv \frac{1}{24\pi} \int d^4 Z E \left( c R_1 \frac{1}{\nabla_+ \nabla_-} R_1 + \bar{c} R_2 \frac{1}{\nabla_+ \nabla_-} R_2 + 2a \Omega_+ \Omega_- \right).
$$

In the superconformal gauge

$$
E_{\pm}^M = \exp \left( \pm \frac{i}{2} L - \frac{1}{2} W \right) \hat{E}_{\pm}^M,
$$

it generates the following action for the Lorentz and Weyl modes $L$ and $W$ of the super-vielbein

$$
S[L, W] = I[E^A_M] - I[\hat{E}^A_M]
$$
$$
= \frac{1}{24\pi} \int d^4 Z \hat{E} \left[ W \hat{\nabla}_+ \hat{\nabla}_- W (c + \bar{c} + 2a) + L \hat{\nabla}_+ \hat{\nabla}_- L (2a - c - \bar{c}) \\
+ L \hat{\nabla}_+ \hat{\nabla}_- W 2i(\bar{c} - c) + 2\hat{R}_1 W (c + a) + 2\hat{R}_2 W (\bar{c} + a) \\
+ 2i\hat{R}_1 L (a - c) + 2i\hat{R}_2 L (\bar{c} - a) \right].
$$
Again we discuss the quantization of this (1, 1) supersymmetric Lorentz-Weyl action by taking care of the Lorentz field first. To do this in a simple way, we re-insert the Weyl field $W$ back into the background, i.e. we set $E_{\pm}^M = \exp(\pm i \frac{1}{2} L) \tilde{E}_{\pm}^M$ in (4.6), and obtain

$$S_{Lor}[L, \tilde{E}_M^A] \equiv I[E_M^A] - I[\tilde{E}_M^A]$$

$$= \frac{1}{24\pi} \int d^4 Z \tilde{E} \left[ L \tilde{\nabla}_+ \tilde{\nabla}_- L (2a - c - \bar{c}) + 2i \tilde{R}_1 L (a - c) + 2i \tilde{R}_2 L (\bar{c} - a) \right].$$

(4.9)

Path integrating over $L$ gives

$$e^{-I_{Lor}[\tilde{E}_M^A]} = \int (DL)_{\tilde{E}} e^{-S_{Lor}[L, \tilde{E}_M^A]}$$

$$I_{Lor}[\tilde{E}_M^A] = I[\tilde{E}_M^A; c_{Lor}, \bar{c}_{Lor}, a_{Lor}]$$

(4.10)

where

$$c_{Lor} = \frac{3}{2} - \frac{(c - a)^2}{c + \bar{c} - 2a}, \quad \bar{c}_{Lor} = \frac{3}{2} - \frac{(\bar{c} - a)^2}{c + \bar{c} - 2a}, \quad a_{Lor} = \frac{3}{2} + \frac{(c - a)(\bar{c} - a)}{c + \bar{c} - 2a}.$$ (4.11)

Note that we have used the measure derived from

$$||\delta L||_E^2 = \int d^4 Z E(\delta L)^2 \to (DL)_E$$

(4.12)

which satisfies $(DL)_E = (DL)_{\tilde{E}}$ because the superdeterminant of the vielbein in the (1, 1) superspace is Lorentz invariant. As a consequence of such a quantization we obtain

$$c + c_{Lor} = \bar{c} + \bar{c}_{Lor} = a + a_{Lor} = \frac{3}{2} + \frac{c\bar{c} - a^2}{c + \bar{c} - 2a}.$$ (4.13)

where the coupling $a$ is left arbitrary. Now, it remains to quantize the Weyl mode and this can be done by repeating the analysis of [20], which implies that the total central charges vanish

$$c_{tot} = c + c_{Lor} - 15 + c_{Liou} = 0,$$

$$\bar{c}_{tot} = \bar{c} + \bar{c}_{Lor} - 15 + \bar{c}_{Liou} = 0.$$ (4.14)

We will omit the description of such an analysis here since it is very similar to the one given in section 2 and is anyway reported in ref. [20].
5. Conclusions

We have derived supergravitational actions induced by superconformal chiral matter, namely the $(1,0)$ and $(1,1)$ chiral induced supergravities given by eqs. (3.8) and (4.6), respectively. These actions are the starting points to study off-critical heterotic and spinning strings. The chiral structure is particularly natural in the heterotic case since the heterotic string is chiral by its own nature (the constraint algebra which defines it, super-Virasoro⊗Virasoro, is chiral). A main tool in the derivation of these chiral gravitational actions was the chiral boson introduced in ref. [14]. It consists of a scalar (super)field with linear couplings to the chiral background curvature scalars, whose effect is to induce improvement terms in the stress tensor, so that arbitrary central charges $c$ and $\bar{c}$ can be obtained. The induced gravitational actions we have found have an interesting chiral structure. In the conformal gauge they give rise to generalizations of the Liouville action and give dynamics to both the Lorentz and Weyl modes of the vielbein. In a stepwise quantization, the Lorentz mode, which we quantize first, is seen to behave as a chiral boson with the effect of leveling up the chiral mismatch between the matter central charges. The Weyl mode is quantized afterwards, using the free field approach of David, Distler and Kawai. We have investigated only local properties of the model. Global properties are more subtle to analyze, but essential for a derivation of critical exponents along the line of reasoning of refs. [4][5]. An example of a global effect is the following one. It can be seen that the Lorentz field acquires superselection rules, derived in the bosonic case by Myers and Periwal using the $SL(2,C)$ invariance on the sphere as well as factorization for more complicated surfaces [12]. Such rules arise because winding sector gets excited when the Lorentz field is in the presence of a non-trivial topology (i.e. higher genus surfaces or punctures in the sphere). Similar superselection rules can also be derived in the supersymmetric cases using the proper supersymmetric generalization of the $SL(2,C)$ invariance group. Another property worth of analysis is unitarity. In fact, as noticed in [12], an interesting feature that arises from the quantization of chiral non-critical string is that the barrier $c \leq 1$ of non-chiral non-critical bosonic string can be avoided by properly choosing the coupling $a$ (see eq. (2.18)). However, the Lorentz field has imaginary coupling (at least in a euclidean signature for the worldsheet) and its central charges can become negative for certain values of $a$. This suggest that one should pay due attention to unitarity in a complete theory of chiral off-critical strings. Thus, it is clear that additional work is required to fully understand the quantization of these induced chiral (super)gravities. Certainly, they describe fascinating models which may teach us many useful lessons in string theory, chiral models and 2d gravity.
Appendix A. (0,0) superspace

We describe the local geometry of the two dimensional (0,0) superspace, that is to say of an usual Riemann surface, in such a way that immediately generalizes to the (1,0) and (1,1) superspaces. First of all, we denote the complex coordinates of the 2d flat space by \( x^\mu = (x^+, x^-) \) and their corresponding derivatives by \( \partial_\mu = (\partial_+, \partial_-) \). We also use the integration measure \( d^2x = dx^1 dx^2 \), where \( x^+ = x^1 + ix^2 \) and \( x^- = x^1 - ix^2 \).

A curved space is obtained by introducing Lorentz covariant derivatives, defined as
\[
\nabla_a = e_a^\mu \partial_\mu + \omega_a J, \quad a = (+, =)
\]
and constrained by
\[
[\nabla_+, \nabla_-] = -RJ,
\]
where \( a \) denotes Lorentz (flat) indices† and \( J \) is the Lorentz generator ( \([J, v_+] = v_+, [J, v_-] = -v_-, \text{ etc.}\) ). The Lorentz metric \( \eta_{ab} \) has \( \eta_{++} = \eta_{=+} = \frac{1}{2} \) as the only non-zero components. The constraint in (A.2) is solved for the spin connection \( \omega_a \) as a function of the vielbein \( e_a^\mu \)
\[
\omega_+ = -\frac{1}{e} \partial_\mu (ee_+^\mu), \quad \omega_- = \frac{1}{e} \partial_\mu (ee_-^\mu),
\]
where \( e \equiv \det e_a^\mu \) and \( e_a^\mu \) inverse of \( e_a^\mu \). The definition of the scalar curvature \( R \) then gives (\( e_a \equiv e_a^\mu \partial_\mu \))
\[
R = R_1 + R_2, \quad R_1 = -\nabla_+ \omega_- = -(e_+ - \omega_+) \omega_-, \quad R_2 = \nabla_- \omega_+ = (e_- + \omega_-) \omega_+.
\]
The scaling properties under Weyl (\( \sigma \)) and Lorentz (\( \lambda \)) transformations, given by
\[
e_\mu^+ \rightarrow \exp(\sigma - i\lambda)e_\mu^+, \quad e_\mu^- \rightarrow \exp(\sigma + i\lambda)e_\mu^-
\]
are easily derived from the above formulas, and read
\[
\omega_+ \rightarrow \exp(-\sigma + i\lambda)(\omega_+ - \nabla_+(\sigma + i\lambda)), \quad \omega_- \rightarrow \exp(-\sigma - i\lambda)(\omega_- + \nabla_-(\sigma - i\lambda)),
\]
\[
e \rightarrow \exp(2\sigma)e, \quad eR_1 \rightarrow e(R_1 - \Box(\sigma - i\lambda)), \quad eR_2 \rightarrow e(R_2 - \Box(\sigma + i\lambda)),
\]
\[
eR \rightarrow e(R - 2\Box\sigma),
\]
(A.6)

† The curved index \( \mu \) will always appear contracted in covariant formulas and thus no confusion should arise with the use of \((+, =)\) as flat indices. When choosing the conformal gauge, \( e_\mu^a = \delta_\mu^a \), flat and curved indices gets identified. This explains our notation.
where $\Box \equiv \nabla_\tau \nabla_\tau = \nabla_+ \nabla_- \nabla_\tau$ is the laplacian acting on scalars. Note that $R$ is Lorentz invariant and coincides, up to some normalization, with standard definitions which make use of the metric tensor $g_{\mu \nu} = e_{\mu}^{a} e_{\nu}^{b} \eta_{ab}$. With our normalization the Euler theorem for a surface of genus $g$ reads: $\int d^{2}x R = (2 - 2g) \pi$.

**Appendix B. (1, 0) superspace**

The rigid $(1, 0)$ superspace is described by the coordinates $Z^{M} = (\theta^{+}, x^{+}, x^{-})$, with $\theta^{+}$ fermionic and $x^{+}, x^{-}$ complex conjugate bosonic coordinates. The covariant derivatives of rigid superspace are denoted collectively by $D_{M} = (D_{+}, \partial_{++}, \partial_{--})$, with

$$D_{+} = \frac{\partial}{\partial \theta^{+}} + \theta^{+} \frac{\partial}{\partial x^{+}} \tag{B.1}$$

and the only non-trivial graded commutator is $\{D_{+}, D_{+}\} = 2 \partial_{+}$. The integration measure is denoted by $d^{3}Z = d^{2}xd\theta^{+}$.

The geometry of $(1, 0)$ supergravity is locally defined by introducing Lorentz covariant derivatives

$$\nabla_{A} = E_{A}^{M} D_{M} + \Omega_{A} J, \quad A = (+, +, -) \tag{B.2}$$

constrained by

$$\{\nabla_{+}, \nabla_{+}\} = 2 \nabla_{+}$$

$$[\nabla_{+}, \nabla_{-}] = R^{+} J. \tag{B.3}$$

Of the $9 + 3$ superfields contained in the supervielbein and spin connection, 7 are killed by the constraints, leaving 5 independent superfields, usually called prepotentials, describing $(1, 0)$ supergravity. We will not need the explicit solution of the constraints in terms of the prepotentials, which are described in [17]. It will be enough to note that the first constraint in (B.3) gives $E_{+}^{M}$ and $\Omega_{+}$ as functions of $E_{+}^{M}$ and $\Omega_{+}$, while the second one allows in particular to solve for the remaining components of the spin connections

$$\Omega_{+} = -\frac{1}{e_{+}} D_{M}(e_{+} E_{+}^{M}), \quad \Omega_{-} = (-1)^{M} \frac{2}{e_{+}} D_{M}(e_{+} E_{-}^{M}), \tag{B.4}$$

where $e_{+} = \text{sdet} E_{M}^{A}$, with $E_{M}^{A}$ inverse of $E_{A}^{M}$. Note that the superdeterminant $e_{+}$ transforms under Lorentz rotations as indicated by the Lorentz index. It is a bosonic
object. The second equation in (B.3) gives the definition of the curvature $R^+$, which can be naturally split in chiral pieces

$$R^+ = R_1^+ + R_2^+,$$

$$R_1^+ \equiv \nabla_+ \Omega_+ = (E_+ - \Omega_+) \Omega_+,$$

$$R_2^+ \equiv -\nabla_+ \Omega_+ = -\left( E_+ + \frac{1}{2} \Omega_+ \right) \Omega_+,$$  \hspace{1cm} (B.5)

where $E_A \equiv E_A^M D_M$. The constraints describing the local $(1, 0)$ superspace are manifestly super-reparametrization and local Lorentz covariant, i.e. covariant under

$$\nabla_A \to \hat{\nabla}_A = e^{K+ \Lambda} \nabla_A e^{-K-\Lambda},$$  \hspace{1cm} (B.6)

with $K \equiv K^M D_M$ and $\Lambda \equiv iLJ$. The superfields $K^M$ and $L$ describe super-reparametrizations and Lorentz rotations, respectively. In addition, Weyl transformations can be defined as [21]

$$E_+^M \to \hat{E}_+^M = \exp\left( -\frac{1}{2} W \right) E_+^M,$$

$$E_+^M \to \hat{E}_+^M = \exp(-W)E_+^M.$$

(B.7)

The Weyl transformation rules on the other components of the supervielbein and on the spin connection follow form the constraints. Altogether there are 5 local symmetries which are enough to locally gauge fix the full supergeometry to a given background value. The Weyl and Lorentz transformation on the relevant geometrical objects needed in the text are as follows

$$E_+^M \to \exp\left( \frac{i}{2} L - \frac{1}{2} W \right) E_+^M, \quad E_+^M \to \exp(-iL - W)E_+^M,$$

$$e_+ \to \exp\left( \frac{i}{2} L + \frac{3}{2} W \right) e_+,$$

$$\Omega_+ \to \exp\left( \frac{i}{2} L - \frac{1}{2} W \right) (\Omega_+ - E_+(iL + W)),$$

$$\Omega_+ \to \exp(-iL - W)(\Omega_+ - E_+(iL - W));$$

$$e_+ R_1^+ \to e_+ (R_1^+ - \nabla_+ \nabla_+ (iL - W)),$$

$$e_+ R_2^+ \to e_+ (R_2^+ + \nabla_+ \nabla_+ (iL + W)),$$

$$e_+ R_+ \to e_+ (R_+ + 2\nabla_+ \nabla_+ W).$$

(B.8)

To derive stress tensors form superspace actions, we need to vary the vielbein. Of course, not all variations are independent because of the constraints. Denoting the vielbein variations by $H_A^B = \delta E_A^M E_M^B$ and varying the torsion constraints which follow from (B.3),
one notices that the 5 variations \((H_+^+, H_=^+, H_=^+)\) can be taken as independent, while other variations are as follows

\[
\begin{align*}
H_+^+ &= \nabla_+ H_+^+ + \frac{1}{2}(\nabla_+ H_=^+ - \nabla_- H_+^+) \\
H_+^+ &= 2H_+^+ + \nabla_+ H_+^+ \\
H_+^+ &= \nabla_+ H_+^+ \\
H_=^+ &= \frac{1}{2}(\nabla_+ H_=^+ - \nabla_- H_=^+ ) \\
\delta \Omega_+ &= \nabla_+ H_=^+ - \nabla_- H_+^+ + H_+^A \Omega_A \\
\delta \Omega_- &= \nabla_+ H_=^+ - 2\nabla_- H_+^+ - \nabla_+ \nabla_- H_+^+ + H_=^A \Omega_A + H_+^+ R^+.
\end{align*}
\]

It is immediate to derive also

\[
\delta e_+ = -e_+(H_+^+ + H_=^+ + \nabla_+ H_+^+).
\]

**Appendix C. \((1,1)\) superspace**

The rigid \((1,1)\) superspace is described by the coordinates \(Z^M = (\theta^+, \theta^-, x^+, x^-)\) and susy covariant derivatives \(D_M = (D_+, D_-, \partial_+, \partial_-)\), with

\[
D_+ = \frac{\partial}{\partial \theta^+} + \theta^+ \frac{\partial}{\partial x^+}, \quad D_- = \frac{\partial}{\partial \theta^-} + \theta^- \frac{\partial}{\partial x^-}.
\]

The non-trivial graded commutators are \(\{D_+, D_+\} = 2\partial_+\) and \(\{D_-, D_-\} = 2\partial_-\) while the integration measure is given by \(d^4Z = d^2xd\theta^+d\theta^-\).

The geometry of \((1,1)\) supergravity is defined locally by the Lorentz covariant derivatives

\[
\nabla_A = E_A^M D_M + \Omega_A J, \quad A = (+, -, ++, =)
\]

constrained by

\[
\begin{align*}
\{\nabla_+, \nabla_+\} &= 2\nabla_+ \\
\{\nabla_-, \nabla_-\} &= 2\nabla_- \\
\{\nabla_+, \nabla_-\} &= RJ.
\end{align*}
\]

Of the 16+4 superfields contained in the supervielbein and spin connection, 14 are killed by the constraints, leaving 6 independent prepotentials describing \((1,1)\) supergravity. Again, we do not need the full solution in terms of the prepotential, which can be found in [22]. It is enough to note that the first two constraints in (C.3) gives \(E_A^M\) and \(\Omega_A\) with
lower bosonic indices as functions of those with lower fermionic indices, while the third constraint allows to solve for the remaining component of the spin connections as function of the vielbein components

$$\Omega_\pm = \mp \frac{2}{E} D_M(EE^*_M), \quad (C.4)$$

where \( E = \text{sdet} E^A_M \) and \( E^A_M \) inverse of \( E_A^M \). The scalar curvature \( R \) can be naturally split in chiral pieces

$$R = R_1 + R_2$$

$$R_1 \equiv \nabla_+ \Omega_- = \left( E_+ - \frac{1}{2} \Omega_+ \right) \Omega_- \quad (C.5)$$

$$R_2 \equiv \nabla_- \Omega_+ = \left( E_- + \frac{1}{2} \Omega_- \right) \Omega_+,$$

with \( E_A \equiv E^A_M D_M \). The above constraints describing the local (1,1) superspace are manifestly covariant under

$$\nabla_A \rightarrow \nabla'_A = e^{K+\Lambda} \nabla_A e^{-K-\Lambda}, \quad (C.6)$$

with \( K \equiv K^M D_M \) and \( \Lambda \equiv iLJ \) describing respectively super-reparametrizations and Lorentz rotations. In addition, Weyl transformations can be defined by [21]

$$E^*_\pm M \rightarrow \hat{E}^*_\pm M = e^{-\frac{i}{2}W} E^*_\pm M. \quad (C.7)$$

The rules on the other components of the supervielbein and on the spin connection follow form the constraints. Altogether there are 6 local symmetries which are enough to locally gauge fix the full supergeometry to a given background value. We list the following transformation properties of some geometrical objects under a Lorentz and Weyl transformation

$$E^*_\pm M \rightarrow \exp\left( \pm \frac{i}{2} L - \frac{1}{2} W \right) E^*_\pm M$$

$$E \rightarrow \exp(W) E$$

$$\Omega_\pm \rightarrow \exp\left( \pm \frac{i}{2} L - \frac{1}{2} W \right) (\Omega_\pm - E_\pm (iL \pm W)) \quad (C.8)$$

$$ER_1 \rightarrow E(R_1 - \nabla_+ \nabla_- (iL - W))$$

$$ER_2 \rightarrow E(R_2 + \nabla_+ \nabla_- (iL + W))$$

$$ER \rightarrow E(R + 2\nabla_+ \nabla_- W).$$

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To conclude, we report the dependent variations of the vielbein, spin connection and superdeterminant of the vielbein as function of the 6 independent vielbein variations \((H^+, H^-, H^{++}, H^{+-})\):

\[
\begin{align*}
H^{--} &= -\frac{1}{2} (\nabla_+ H^- + \nabla_- H^+) \\
H_{+-} &= \nabla_+ H^+ \\
H^{++} &= 2H^+ + \nabla_+ H^{++} \\
H_{+} &= -\frac{1}{2} (\nabla_+ H^- + \nabla_- \nabla_+ H^+ - H^+ R) \\
H_{++} &= \nabla_+ H^+ + \nabla_+ H^- - \frac{1}{2} (\nabla_+ H^+ + \nabla_- \nabla_+ H^- - H^- R) \\
H_{-} &= -\frac{1}{2} (\nabla_- H^{++} + \nabla_+ H^{--}) \\
H_{-} &= \nabla_- H^{--} \\
H_{-} &= 2H^- + \nabla_- H^- \\
H_{-} &= -\frac{1}{2} (\nabla_- H^{++} + \nabla_+ \nabla_- H^{--} + H^{--} R) \\
H_{-} &= \nabla_- H^- + \nabla_+ H^+ - \frac{1}{2} (\nabla_+ H^- + \nabla_- \nabla_+ H^+ + H^+ R) \\
\delta \Omega^+ &= 2\nabla_+ H^- - \nabla_+ H^+ - \nabla_- \nabla_+ H^- + H^- R + H^+ A \Omega_A \\
\delta \Omega^- &= -2\nabla_- H^+ + \nabla_+ H^{--} + \nabla_+ \nabla_- H^{++} + H^{--} R + H^+ A \Omega_A \\
\delta E &= -E (H^+ + H^- + \nabla_+ H^{++} + \nabla_- H^+).
\end{align*}
\]
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