The Condensation of Dibaryons in Nuclear Matter and Its Possible Signatures in Heavy Ion Collisions

Amand Faessler\textsuperscript{1},
M. I. Krivoruchenko\textsuperscript{1,2} and B. V. Martemyanov\textsuperscript{1,2}

\textsuperscript{1}Institut für Theoretische Physik, Universität Tübingen, Auf der Morgenstelle 14
D-72076 Tübingen, Germany

\textsuperscript{2}Institute for Theoretical and Experimental Physics, B.Cheremushkinskaya 25
117259 Moscow, Russia

Abstract

We consider the thermodynamics of the matter made of equal number of neutrons and protons and of scalar dibaryons. They interact via the exchange of scalar and vector mesons. The interaction is taken into account in the mean field approximation. The condensation of dibaryons in this matter and the phase transition of matter to quark matter are considered. Possible signatures of dibaryons in Heavy Ion Collisions are speculated on.

\textbf{PACS}:14.20.Pt, 21.65.+f

\textbf{keywords}: nuclear matter, dibaryon, condensation
1 Introduction

The possible existence and properties of a narrow dibaryon $d'(T = 0, J^P = 0^-)$ are discussed widely in the last years both from experimental [1,2,3] and theoretical [4,5] point of view. Being bosons such particles could condense in nuclear matter under proper conditions. This phenomenon happens when the nucleon chemical potential grows up to half of the mass of the dibaryon. Dibaryons are weakly produced due to the reactions

\begin{align}
    n + p &\to d' + \pi^0 \\
    n + n &\to d' + \pi^- \\
    p + p &\to d' + \pi^+ \quad (1.1)
\end{align}

which put them in chemical equilibrium with protons and neutrons. At zero temperature the condensation of dibaryons in nuclear ($n = p$) and neutron matter was studied in Refs.[6,7,8]. For the case of $d'$ the condensation starts when the baryon number density $\rho$ grows up to approximately $3\rho_0$ (here $\rho_0$ is the normal nuclear density). Such conditions (zero temperatures and high densities) are hard to imagine in terrestrial experiments. In neutron stars the densities of the central core can be even larger but it is difficult to see any clear signal [7] of the dibaryon condensation in neutron stars.

High densities of nuclear matter can in principle occur in Heavy Ion Collisions (HIC). But in that case not only the densities but also the temperatures are high. So we should extend the consideration of the dibaryon condensation to finite temperatures. Secondly we should take into account the possible phase transition of nuclear matter into quark matter.

Below we will describe our model of nuclear matter with dibaryons in section 2, consider the thermodynamical quantities of nuclear matter in section 3 (condensation curve), describe the model for quark matter in section 4 (phase transition) and speculate on possible signatures of dibaryons in HIC in the conclusions.

2 The model of nuclear matter with dibaryons

We consider an extension of the Walecka model [8] by including dibaryon fields to the Lagrangian density

\begin{align}
    L = \bar{\Psi}(i\gamma^{\mu} - m_N - g_\sigma\sigma - g_\omega\omega)\Psi + \frac{i}{2}(\partial_\mu\sigma)^2 - \frac{1}{4}m_\sigma^2\sigma^2 - \frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}m_\omega^2\omega^2_\mu + (\partial_\mu - ih_\omega\omega_\mu)\varphi^* (\partial_\mu + ih_\omega\omega_\mu)\varphi - (m_D + h_\sigma^2)\varphi^*\varphi \quad (2.2)
\end{align}

Here, $\Psi$ is the nucleon field, $\omega_\mu$ and $\sigma$ are the $\omega$- and $\sigma$-meson fields, $F_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$ is the field strength tensor of the vector field; $\varphi$ is the dibaryon field, for which we assume
that it is a isoscalar-pseudoscalar field. This assumption includes the interesting case of the d'-dibaryon. The values $m_\omega$ and $m_\sigma$ are the $\omega$- and $\sigma$-meson masses and $g_\omega$, $g_\sigma$, $h_\omega$, $h_\sigma$ are the coupling constants of the $\omega$- and $\sigma$-mesons with nucleons ($g$) and dibaryons ($h$).

The field operators can be expanded into $c$-number- and operator parts:

$$\begin{align*}
\sigma &= \sigma_c + \hat{\sigma}, \\
\omega_\mu &= g_\mu\omega_c + \hat{\omega}_\mu, \\
\varphi &= \varphi_c + \hat{\varphi}, \\
\varphi^* &= \varphi_c^* + \hat{\varphi}^*.
\end{align*}$$

The $c$-number parts of the fields $A = \sigma$, $\omega_\mu$, $\varphi$, and $\varphi^*$ are defined as expectation values $A_c = \langle A \rangle$ over the ground state of the system. The average values of the operator parts are zero by definition: $\langle \hat{A} \rangle = 0$.

The $\sigma$-meson mean field determines the effective nucleon and dibaryon masses in the medium

$$\begin{align*}
m_N^* &= m_N + g_\sigma \sigma_c, \\
m_D^* &= m_D + h_\sigma \sigma_c.
\end{align*}$$

The nucleon vector and scalar densities are defined by expectation values

$$\begin{align*}
\rho_{NV} &= \langle \bar{\Psi}\gamma_0\Psi \rangle, \\
\rho_{NS} &= \langle \bar{\Psi}\Psi \rangle.
\end{align*}$$

The vector and scalar density of the dibaryons are defined by

$$\begin{align*}
\rho_{DV} &= \langle \varphi^\ast i\partial_0\varphi - 2h_\omega\omega_0\varphi^\ast\varphi \rangle, \\
\rho_{DS} &= \langle \varphi^\ast\varphi \rangle.
\end{align*}$$

Neglecting the operator parts of the meson fields, we get the following expressions for the meson mean fields

$$\begin{align*}
\omega_c &= \frac{g_\omega \rho_{NV} + h_\omega \rho_{DV}}{m_\omega^2}, \\
\sigma_c &= -\frac{g_\sigma \rho_{NS} + h_\sigma 2m_D^* \rho_{DS}}{m_\sigma^2}.
\end{align*}$$

The nucleon vector and scalar densities are given by

$$\begin{align*}
\rho_{NV} &= \gamma \int \frac{dk}{(2\pi)^3} \left((\exp(\sqrt{m_N^2 + k^2 - \nu_N}) + 1)^{-1} - (\exp(\sqrt{m_N^2 + k^2 + \nu_N}) + 1)^{-1}\right), \\
\rho_{NS} &= \gamma \int \frac{dk}{(2\pi)^3} \sqrt{m_N^2 + k^2} \left((\exp(\sqrt{m_N^2 + k^2 - \nu_N}) + 1)^{-1} + (\exp(\sqrt{m_N^2 + k^2 + \nu_N}) + 1)^{-1}\right).
\end{align*}$$
The statistical factor $\gamma = 4 (2)$ for nuclear (neutron) matter. Here $\nu_N$ is the nucleon chemical potential (excluding electrostatic energy of nucleon in the vector field) $\nu_N = \mu_N - g_\omega \omega^c$.

The dibaryon vector and scalar densities are given by

$$\rho_{DV} = \gamma_D \int \frac{dk}{(2\pi)^3} ((\exp(\sqrt{m_D^2+k^2-\nu_D}) - 1)^{-1} - (\exp(\sqrt{m_D^2+k^2+\nu_D}) - 1)^{-1}) +$$

$$+ \rho^0_{DV} = \rho_{DV} + \rho^0_{DV},$$

$$\rho_{DS} = \gamma_D \int \frac{dk}{(2\pi)^3} \frac{1}{2\sqrt{m_D^2+k^2}} ((\exp(\sqrt{m_D^2+k^2-\nu_D}) - 1)^{-1} + (\exp(\sqrt{m_D^2+k^2+\nu_D}) - 1)^{-1}) +$$

$$+ \frac{1}{2m_D} \rho^0_{DV} = \rho_{DS} + \rho^0_{DS}. \quad \text{(2.14)}$$

Here $\gamma_D$ is the dibaryon statistical factor (1 for $d'$), $\nu_D$ is the dibaryon chemical potential (excluding electrostatic energy of dibaryon in the vector field) $\nu_D = \mu_D - h_\omega \omega^c$, $\nu_D \leq m_D^*$, $\rho^0_{DV}$ is the density of dibaryons in the condensate that is formed when $\nu_D = m_D^*$. The chemical equilibrium between nucleons and dibaryons means $\nu_D = 2\nu_N$ (we consider the case when electrostatic energy is not changed in the process of transition of two nucleons to dibaryon ($h_\omega = 2g_\omega$)).

The self-consistency equations have the form

$$m_N^* = m_N - \frac{g_\sigma}{m_\sigma} (g_\sigma \rho_{NS} + h_\sigma (2m_D^* \rho_{DS}^0 + \rho_{DV}^0)), \quad \text{(2.16)}$$

$$m_D^* = m_D - \frac{h_\sigma}{m_\sigma} (g_\sigma \rho_{NS} + h_\sigma (2m_D^* \rho_{DS}^0 + \rho_{DV}^0)). \quad \text{(2.17)}$$

The total baryon number density equals $\rho_{TV} = \rho_{NV} + 2 \rho_{DV}$.

We use here the following procedure of solving the self-consistency equations. We start with fixing the effective nucleon mass $m_N^*$. The value of effective dibaryon mass then simply follows from eqs. \(2.16, 2.17\)

$$m_D^* = m_D - \frac{h_\sigma}{g_\sigma} (m_N - m_N^*). \quad \text{(2.18)}$$

Further we assume that the condensation takes place and find the nucleon chemical potential: $\nu_N = m_D^*/2$. Then we find the condensate dibaryon density $\rho^0_{DV}$ from eq.\( (2.16)\). If it is positive the self-consistency equation is solved for a given point. If it is negative we take $\rho^0_{DV} = 0$ and scan $\nu_N = \nu_D/2$ from 0 to $m_D^*/2$ to make the eq.\( (2.16)\) true. After this procedure all the parameters of the matter are defined and we are ready to calculate various thermodynamical quantities.
3 Thermodynamical quantities of nuclear matter. Condensation curve

The energy density $\varepsilon = \langle T_{00} \rangle$ given by average value of the $T_{00}$ component has the form

$$\varepsilon = \gamma \frac{dk}{(2\pi)^3} \left( \sqrt{m_N^2 + k^2 + g_\omega \omega_c} \exp \left( \sqrt{m_N^2 + k^2 - \nu_N} \right) + 1 \right)^{-1} + \gamma_d \frac{dk}{(2\pi)^3} \left( \sqrt{m_N^2 + k^2 - g_\omega \omega_c} \exp \left( \sqrt{m_N^2 + k^2 + \nu_N} \right) + 1 \right)^{-1}$$

$$+ \left( \sqrt{m_N^2 + k^2 - h_\omega \omega_c} \exp \left( \sqrt{m_N^2 + k^2 - \nu_D} \right) - 1 \right)^{-1} + \left( m_\sigma^2 + h_\omega \omega_c \right) \rho_{DV}^0 + \frac{1}{2} m_\sigma^2 \sigma_c^2 - \frac{1}{2} m_\omega^2 \omega_c^2. \tag{3.19}$$

The last two terms are here the contributions of the classical $\omega$- and $\sigma$-meson fields to the energy density.

The hydrostatic pressure $p = -\frac{1}{3} \langle T_{ii} \rangle$ has the form

$$p = \frac{1}{3} \gamma \frac{dk}{(2\pi)^3} \left( \frac{k^2}{\sqrt{m_N^2 + k^2}} \exp \left( \sqrt{m_N^2 + k^2 - \nu_N} \right) + 1 \right)^{-1}$$

$$+ \frac{k^2}{\sqrt{m_N^2 + k^2}} \exp \left( \sqrt{m_N^2 + k^2 + \nu_N} \right) + 1 \right)^{-1}$$

$$+ \frac{1}{3} \gamma_d \frac{dk}{(2\pi)^3} \left( \frac{k^2}{\sqrt{m_N^2 + k^2}} \exp \left( \sqrt{m_N^2 + k^2 - \nu_D} \right) - 1 \right)^{-1}$$

$$+ \frac{k^2}{\sqrt{m_N^2 + k^2}} \exp \left( \sqrt{m_N^2 + k^2 + \nu_D} \right) - 1 \right)^{-1}$$

$$- \frac{1}{2} m_\sigma^2 \sigma_c^2 + \frac{1}{2} m_\omega^2 \omega_c^2. \tag{3.20}$$

Because dibaryons in the condensate are at rest they do not contribute to the pressure.

Now we are in a position to present the results for the thermodynamical quantities of nuclear matter. For our purpose the most interesting thing is the condensation curve i.e. the curve on the temperature- baryon density plane where the condensation of dibaryons starts either at increasing the baryon density or at lowering the temperature. The calculations are made for the standard values of RMF model parameters[9]. The coupling constants of the dibaryon $d'(2060)$ to $\sigma-$ and $\omega-$ mesons were taken to be[8]

$$h_\omega = 2g_\omega \quad h_\sigma = 1.6g_\sigma$$

The results are shown on the Fig.1 (condensation curve). It means e.g. that at zero temperature ($T = 0$) one has to compress to $\rho_{TV} = 0.54 fm^{-3}$ ($\approx 2.7$ of saturation density) to obtain the transition to the dibaryon condensate if the dibaryon mass is $m_D = 2060 MeV$. Also we show on Fig.1 the so called critical curve. It is the curve that shows the boundary of applicability of our model. Precisely it is the place where the effective mass of the nucleon goes to zero. On the right of the critical curve our model
cannot be applied. We hope that the more elaborated model will move this curve to the right in the unphysical region where the nuclear matter transforms to the quark matter for example. And finally we show on Fig.1 the phase transition curve i.e. the place where the nuclear matter (with or without condensed dibaryons) transforms to quark matter. Now we are coming to the description of this curve.

4 The model of quark matter. Phase transition

The nuclear matter we are considering is isotopically symmetric ($\rho_p = \rho_n$) and has no net strangeness ($\rho_s = 0$). At the beginning of the phase transition to quark matter isospin and strangeness are conserved. Therefore we will compare the thermodynamical quantities of nuclear matter with those of quark matter consisting of equal number of $u$ and $d$ quarks and no net strange quarks. The reliable temperatures in our consideration (we are interested in nuclear matter with condensed dibaryons) will be below 50 MeV. For such temperatures the contribution of pions, kaons etc. to the thermodynamical quantities of nuclear matter and the contribution of strange quarks, gluons etc. to the thermodynamical quantities of quark matter can be neglected. Then the equation of state (EOS) of nuclear matter $p_N(\mu_N, T)$ is described by eq. (3.20) and the equation of state of quark matter $p_q(\mu_q, T)$ is the following one\[11\]

$$p_q = \left( \frac{1}{2\pi^2} \mu_q^4 + \mu_q^2 T^2 \right)(1 - \frac{2\alpha_c}{\pi}) + \frac{16}{3\pi} \mu_q^2 T - B. \tag{4.21}$$

Here $\alpha_c$ is the constant of gluonic interaction of quarks. The first term in eq.(4.21) is the pressure of massless $u$ and $d$ quarks, the second term is the plasmon contribution to the pressure and the third term is the vacuum pressure that is responsible for confinement (MIT bag model[11]). The constants $\alpha_c$ and $B$ were taken to be

$$\alpha_c = 0.6 \quad B = 96 \text{ MeV}/\text{fm}^3$$

that ensures the stability of massive neutron stars against the transition to strange stars[12].

The phase transition of nuclear matter to quark matter starts when the pressure of quark matter becomes equal to the pressure of nuclear matter at equal baryon chemical potentials and temperatures (Gibbs criterion, here we assume that the phase transition is of the first order)

$$p_N(\mu_N, T) = p_q(\mu_q, T) \quad \mu_N = 3\mu_q$$

The typical dependences of $p_N$ and $p_q$ on $\mu_N$ for fixed $T = 50$ MeV are shown on Fig.2. The point of crossing of lines is the point of phase transition at given temperature. The
full phase transition curve is shown on Fig.1. As it follows from Fig.1 at temperatures above $36\text{MeV}$ the dibaryon condensation does not occur: increasing the density for fixed temperatures above $36\text{MeV}$ we come to the quark matter rather than to dibaryon condensation. The region on the $(T, \rho_{TV})$ plot where dibaryon condensate is formed ($p, n, d'$ in Fig.1) lies therefore below $36\text{MeV}$ for temperatures and above $0.54\text{ fm}^3$ for densities ($2.7$ of normal nuclear density).

In conclusion let us speculate on the possible signatures of dibaryon condensate formation in HIC.

5 Conclusion

The accessible temperatures and densities of nuclear matter in HIC are not very well known. Fig.3 shows our calculations of the temperature of compressed nuclear matter formed in HIC as a function of the density of compressed matter (different energies per nucleon of colliding ions). We have used the EOS of nuclear matter with dibaryons and a one-dimensional shock wave model[13] (Rankin-Hugoniot-Taub equation). The compression curve lies above the region of condensation (in ”$p,n$” or ”$Q$”) shown on Fig.1. On the first sight we should conclude that the dibaryon condensate never forms in HIC. But the compressed matter being formed evolves further and its evolution is governed by two effects: by the cooling due to emission of particles (mainly pions) and by the decompression (that also results in the cooling). If the radiation cooling prevails over the cooling due to the decompression, the matter could evolve into the region of the formation of the dibaryon condensate ($p, n, d'$ in Fig.1). If so, the significant part of nucleons will transform to dibaryons (up to 70%). The dibaryons could survive in the process of decompression ($\Gamma_{d'} \approx 0.5\text{MeV}$) and then decay as free particles in one of the following ways

$$d' \rightarrow nn\pi^+$$
$$d' \rightarrow np\pi^0$$
$$d' \rightarrow pp\pi^-.$$  \hspace{1cm} (5.22)

The pions in these decays have the spectrum with the characteristic kinetic energy about $40-50\text{MeV}$, if the dibaryon has a mass about $2060\text{MeV}$. Then these pions could form an excess in the temperature spectrum of all pions produced in HIC. In order to these pions could be observed they should not be thermalised in the nuclear matter like the other pions (direct ones, pions from different other resonances not considered in this paper). The principal possibility of this is due to the relatively long lifetime of $d'$-dibaryon - it could decay after the decay of nuclear matter.
So, we conclude, there is a region of temperatures and densities where the dibaryon condensate forms in nuclear matter. If formed in HIC, dibaryons could give a signature in form of $40 - 50MeV$ kinetic energy pions -the products of their decays.

The authors acknowledge the discussions with Drs. D.Kosov and L.Sehn. Two of us (M.I.K and B.V.M.) are grateful to the Institute for Theoretical Physics of University of Tuebingen for hospitality and financial support. This work was supported by the Deutsche Forschungsgemeinschaft under contract No FA67/20-1.

References

[1] R.Bilger et al. Phys.Lett. B269 (1991) 247.
[2] R.Bilger, H.A.Clement and M.G.Schepkin, Phys.Rev.Lett. 71 (1993) 42.
[3] H.Clement, M.Schepkin, G.J.Wagner and O.Zaboronsky, Phys.Lett. B337 (1994) 43.
[4] L.Ya.Glozman, A.Buchmann and A.Faessler, J.Phys. G20 (1994) L49.
[5] G.Wagner, L.Ya.Glozman, A.J.Buchmann and Amand Faessler, Nucl.Phys. A594 (1995) 263.
[6] A.M. Baldin et.al., Dokl.Acad.Sc. USSR 279 (1984) 602; St. Mrowczynski, Phys.Lett.B152 (1985) 299; A.V.Chizov et.al., Nucl.Phys. A449 (1986) 660.
[7] M.I.Krivoruchenko, JETP Letters 46 (1987) 3; R.Tamagaki, Progr.Theor.Phys. 85 (1991) 321; A.Olinto, P.Haensel and J.Frieman, Preprint FERMILAB-PUB-91-176-A, 1991.
[8] Amand Faessler, A.Buchmann, M.I.Krivoruchenko and B.V.Martemyanov, Phys.Lett. B391 (1997) 255.
[9] J.D.Walecka, Ann. Phys. (N.Y.) 83 (1974) 491.
[10] B.A.Freedman and L.D.McLerran, Phys.Rev. D16 (1978) 1130, 1147, 1169.
[11] A.Chodos, R.L.Jaffe, K.Johnson, C.B.Thorn and V.F.Weisskopf, Phys.Rev. D9 (1974) 3471.
[12] M.I.Krivoruchenko and B.V.Martemyanov, Astrophys. J. 378 (1991) 628; Nucl.Phys. B24 (1991) 134c.
Figure captions

**Fig.1** The condensation, phase transition and critical curves on the temperature($T$) - baryon number density($\rho_{TV}$) plot. The regions of nuclear matter($n, p$), nuclear matter with condensed dibaryons($n, p, d'$) and quark matter($Q$) are marked. The critical curve shows the boundary of applicability of our model.

**Fig.2** The example of nuclear and quark matter equations of state. The pressures of nuclear matter($p_N$) and quark matter($p_q$) are shown as the functions of baryon chemical potential($\mu_N$) at the temperature $T = 50$ MeV. The crossing point is the point of phase transition from nuclear matter to quark matter.

**Fig.3** The temperature of the shock wave($T$) as a function of baryon density ($\rho$) in ion collisions for mean field model of nuclear matter with dibaryons.