BOSE-EINSTEIN CONDENSATION OF CHARGED PARTICLES
AND SELF-MAGNETIZATION

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We discuss the Bose-Einstein condensation of relativistic vector charged particles in a strong external magnetic field in very dense matter, as may be paired spin-up electrons. We show that for electrons such systems may maintain self-consistently magnetic field strengths in the interval $10^{10} - 10^{13}$ Gauss. This could be the origin of large magnetic fields in some white dwarfs, but may also impose bounds due to the arising of strong anisotropy in the pressures, which may produce a transverse collapse of the star.

1. Introduction

In the paper\textsuperscript{1} was shown that a relativistic gas under the influence of a magnetic field $B$ of order of the quantum electrodynamics limit of $m_e^2/e \sim 10^{13}$ Gauss and for densities $N \leq 10^{30}$ cm$^{-3}$ becomes unstable and collapses, since the pressure perpendicular to the field vanishes. Physically, the system, infinitely degenerate with regard to the orbit’s center quantum number, becomes otherwise one dimensional, with all the electrons falling on the Landau ground state $n = 0$. The magnetic “Bohr radius” being of order $\sqrt{\hbar c/eB} = 10^{-2}$Å. Its spectrum looks equivalent to that of a free unidimensional particle moving along the external field. under these conditions the system is unable to exert any transverse pressure.

Looking at the problem from another side, in a recent paper\textsuperscript{4} has been shown that an electron gas confined to the Landau ground state cannot be in $\beta$-decay equilibrium in a neutron star due to an incompatibility among the spin orientation of the particles involved. The suggestion is given in\textsuperscript{5} that a bosonization of the
electron gas may take place, which would mean a solution to the above-mentioned problem. We will discuss here the behavior of the bosonic gas whose constituent particles are relativistic bosons, which we suggest may represent the electron pairs, under such extreme conditions of confinement to the $n = 0$ Landau ground state. As is known in normal superconductors, scalar pairing condensates occur in absence of a magnetic field (Cooper pairs). The pairing leading to bosonization is mediated by some interaction as is the Coulomb exchange interaction among electrons, for instance, in the ferromagnetic pairing in atoms. In addition to bosonization, another effect could be associated with a small change in the new particle mass, that is, the final particle would have a mass smaller than twice that of the initial unpaired particles since the whole relativistic energy is reduced due to inertial effects. In view of this, we shall assume that this binding energy is not significant.

When a magnetic field is applied, the superconductivity (condensate) is destroyed at some critical magnetic field, (the Schafroth critical field). But if the magnetic field increases largely enough to have a significant fraction of its density in the Landau ground state, it has been suggested that the condensate reappears as a consequence of some interaction compatible with a spin-one vector pairing. This would lead to a superconductive-ferromagnetic behavior.

Such particles would carry twice the charge of the electron and an effective mass which in principle we take of order twice of that of the electron, although some corrections must be introduced however, due to effects coming from the large density and the magnetic field. Then the system may be treated by following the same formalism used in previous references. As a consequence of condensation the system would behave as ferromagnetic and under the action of an external field $H$, a magnetization $\mathcal{M}$ arises, leading us to define a microscopic magnetic field $B = H + 4\pi M$. The interesting point here is that, due to the positive character of $\mathcal{M}$ it may occur that $B \sim 4\pi M$, or $H \ll 4\pi M$ i.e., the microscopic magnetic field be produced by self-magnetization.

2. The condition of self-magnetization

We shall propose that the relativistic paired electron system behaves as a vector particle with energy eigenvalues $\epsilon_0(p_3) = \sqrt{p_3^2c^2 + M^2c^4 - 2eBHc}$, for the Landau ground state $n = 0$ (where we take $M = 2m_e$, $m_e$ being the electron mass), and $\epsilon_n(p_3) = \sqrt{p_3^2c^2 + M^2c^4 + 4eBHc(n + \frac{1}{2})}$ for the excited states $n = 1, 2, \ldots$. We observe that the magnetic field introduces an effective mass for vector bosons $M_0 = \sqrt{M^2 - 2eBHc}/c^3$ in the ground state such that as $B$ increases, $M_0$ decreases. This leads to an effective magnetic moment in the ground state $m = e\hbar/2M_0c$. The magnetic mass is $M_n = \sqrt{M^2 + 2eBHc(n + \frac{1}{2})}/c^3$ for the excited states, which increases with $B$ and $n$.

In Ref. we have shown that Bose-Einstein condensation, in the sense of a large population in the Landau ground state having its momentum along the magnetic field equal to zero or very small, occurs for scalar and vector particles in presence of
a strong magnetic field. We name \( n^+ \) and \( n^- \) the density of particles and antiparticles, respectively, in the ground state. We expect then most of the population of particles to be around the ground state, since for small temperatures \( n^+ \) is vanishing small and \( n^- \) is a bell-shaped curve with its maximum at \( p_3 = 0 \). We will define \( \mu' = \mu - M_0 c^2 \) and recall the procedure followed in 2. We call \( p_0 (\gg \sqrt{-2M_0 \mu'}) \) some characteristic momentum. Taking by symmetry the density of particles minus antiparticles (the latter will vanish as \( -\mu' \ll T \)) we have in a small neighborhood of \( p_3 = 0 \),

\[
N_0 = \frac{2eBT}{2\pi^2\hbar^2 c} \int_0^{p_0} \frac{dp_3}{\sqrt{p_3^2 c^4 + M_0^2 c^4 + 2\epsilon B c\hbar c - \mu}} - \int_0^{p_0} \frac{dp_3}{\sqrt{p_3^2 c^4 + M_0^2 c^4 + 2\epsilon B c\hbar c + \mu}}
\]

where \( N = N_0 + \delta N \) and \( \delta N \) is the density in the interval \([p_0, \infty] \). Actually as \( \mu' \to 0 \), \( N_0 \to N \) and \( \delta N \) is very small. We get then

\[
\mu' \simeq \frac{e^2 B^2 T^2 M_0}{2\pi^2 N^2 a^4 c^2}.
\]

We observe that \( \mu' \) is a decreasing function of \( T \) and vanishes for \( T = 0 \), where the "critical" condition \( \mu = M_0 c^2 \) is reached. As shown in 2 in that limit the Bose-Einstein distribution degenerate in a Dirac \( \delta \)-function, which means to have all the system in the ground state \( p_3 = 0 \). From (1) we may write the thermodynamic potential as

\[
\Omega = \frac{eBT}{2\pi^2\hbar^2 c} \sqrt{M_0^2 c^4 - \mu^2},
\]

while the magnetization is given approximately by

\[
\mathcal{M} = -\frac{\partial \Omega}{\partial B} = \frac{eN\hbar}{M_0 c},
\]

\[
B = 4\pi M = 4\pi \frac{eN\hbar}{M_0 c}.
\]

One can then state the condition for self-magnetization by writing the equation \( H = B - 4\pi M = 0 \). First, let us assume that \( N \sim 10^{32} \). Then \( \mathcal{M} \sim 10^{12} \) and \( B \sim 10^{13} \) G. Therefore, the condition for self-magnetization is satisfied. The system becomes a giant magnet whose stability is determined by the transverse pressure condition \( P_{\perp} \approx P_3 \), where \( P_{\perp} = -\Omega - B \mathcal{M} \). However, the estimate value of \( \Omega \) would depend on the fraction of electrons paired. Let us name \( N_u \), \( N_p = N - N_u \) the density of unpaired and paired electrons, respectively. If \( N_u \sim N_p \) then the dominating
pressure comes from the (unpaired) electron gas contribution, $\Omega \sim N M_0 \sim 10^{26}$, which is very close to $BM \sim 10^{25}$. Thus, one can affirm that in the region $N \sim 10^{32} - 10^{33}$, the condition $P_\perp \ll P_3$ holds and the bosonized ferromagnetic white dwarf star destabilizes and collapses. If $N_u \ll N_p$ so that the dominant pressure comes from the paired gas the collapse is also unavoidable since $P_\perp \sim 0$. As $\Omega$ is positive, its contribution to pressure is negative. The stability requires from a Fermion background.

There is still another point to be considered when $eB$ approaches to $M$. As $M_0$ decreases with increasing $B$, the magnetization $M$ increases with $B$, and would diverge for $M_0 \to 0$. For $eB\hbar/c^3$ close enough to $M$ one expects the main contribution to $B$ be produced by $M$. We get an equation similar to the one discussed in 1 for the $W$ condensate.

Let us write $2eB\hbar/M^2c^3 = x^2$ where $0 \leq x \leq 1$. For $x = 1$, we have the critical field $B_C = M^2c^3/2e\hbar \simeq 8.82 \times 10^{13}$ G. Then we can write $M_0 = M\sqrt{1 - B/B_C}$. We easily get

$$x^2 \sqrt{1 - x^2} = \frac{8\pi e^2 \hbar^2 N}{M^3 c^3} = A.$$  

By simple inspection we find that it has real solutions only for $A < 2\sqrt{3}/9 = A_1$. This means that $N \leq 10^{32}$. By solving the cubic equation (6) we find that for $A \ll 1$ these real solutions are $x_1 = \sqrt{A + A^2/2}$ and $x_2 = \sqrt{1 - A^2}$. The first solution means that $B$ increases with increasing $N$, up to the value $B_{\text{max}} = 2/3B_C$. In the second solution $B$ decreases for growing $N$, and its limit for $N \to 0$ being $B_C$. The last result has only meaning if interpreted as indicating that the expression for the magnetization (4) is incomplete. Actually, it must include the contribution from Landau states other than the ground state, which lead to a diamagnetic response to the field. The decreasing in population of the ground state is compensated by the increasing of the number of particles in Landau states with $n > 0$. Their contribution would compensate the increasing of the self-consistent field with increasing $N$ to keep $B < B_C$.

3. Conclusions

From all the previous reasoning we conclude that an electron system, as a white dwarf, can be hardly stable at fields of order $B_c$. In principle, such fields can be maintained self-consistently, but the possibility of a collapse is highly increased: the one-dimensional world created by the magnetic field is completely unstable. The previous results, if applied to the condensation of $\rho$ and $\omega$ mesons (for instance, in neutron stars), would be compatible with the self-magnetization condition for densities of order $10^{39}$ cm$^{-3}$ if the resulting magnetic fields are in the interval $10^{17} - 10^{19}$ G. Instabilities would lead to a collapse. This point deserves further research, as it provides additional arguments to those given in 7 against the claimed existence of the so-called magnetars, because those objects seem to be unstable under such $\rho$ and $B$ conditions.
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