Giant tunable nonreciprocity of light in Weyl semimetals

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The propagation of light in Weyl semimetal films is analyzed. The magnetic family of these materials is known by anomalous Hall effect, which being purely intrinsic and universal depends only on the separation of the Weyl nodes in momentum space, allowing one to create strong gyrotropic and nonreciprocity effects without external magnetic field. The existence of nonreciprocal waveguide electromagnetic modes in magnetic Weyl semimetal films in the Voigt configuration is predicted. These waves are shown to exist in the two frequency regions one of which lies below the plasma frequency where the negative refraction in Weyl semimetal can be observed. The magnitude of their nonreciprocity depends both on the internal Weyl semimetal properties, the separation of Weyl nodes, and external factor, the optical contrast between the media surrounding the film. According to our estimations the anomalous Hall effect parameters in the real magnetic Weyl semimetals are by three orders of magnitude larger than in typical magnetic dielectrics. By tuning the Fermi level in Weyl semimetals one can vary the operation frequencies of the waveguide modes from THz to Mid-IR ranges and enhance their nonreciprocity by shifting to THz range. Our findings pave the way to the design of compact, tunable and effective nonreciprocal optical elements.

I. INTRODUCTION

Weyl semimetals (WSs) being topologically nontrivial phase of matter have recently attracted a significant attention due to their massless bulk fermions and protected Fermi arc surface states with the corresponding topological transport phenomena [1–6]. WS band structure contains even number [7] of nondegenerate band-touching points (Weyl nodes) which are topologically stable and can be regarded as magnetic monopoles and antimonopoles in the momentum space with positive or negative chiral charges and corresponding non-zero Chern numbers acting as the source and drain for the Berry curvature field [8, 9]. The topological protection of massless fermions in WS against weak perturbations follows from the separation of the individual Weyl nodes with opposite topological charges in momentum space, as the chiral Weyl nodes can only be destroyed by chirality mixing, which requires two opposite chirality Weyl nodes to meet. Such a separation demands breaking of either time-reversal ($\mathcal{T}$) or inversion ($\mathcal{P}$) symmetry, or both [2]. In WSs with lack of $\mathcal{P}$ symmetry the Weyl nodes separation is roughly proportional to the strength of the spin-orbit coupling (SOC), which indicates the crucial role played by SOC in the formation of WSs [10]. By contrast, in $\mathcal{T}$ and $\mathcal{P}$ invariant bulk Dirac semimetals (BDSs), where according to Kramers theorem all bands are doubly-degenerate, the massless bulk fermions require additional crystal symmetries to be stable [11].

The realization of a BDS phase in Na$_3$Bi, Cd$_3$As$_2$ and ZrTe$_5$ compounds was predicted [12, 13] and confirmed experimentally [14–18]. WS phase natural realizations contain the family of $\mathcal{T}$ broken magnetic materials including pyrochlore iridates Y$_2$IrO$_7$, Eu$_2$IrO$_7$ [19, 20], ferromagnetic spinels HgCr$_2$Se$_4$ [21], and half-metallic ferromagnets Co$_3$S$_2$Sn$_2$, Co$_3$S$_2$Se$_2$ [22–24]. WS family of $\mathcal{P}$ broken nonmagnetic materials includes noncentrosymmetric compounds TaAs, TaP, NbAs and NbP [25–33] (the detailed WS classification can be found in reviews [34–36]). Moreover, in some compounds, e.g., WTe$_2$ [37, 38] and MoTe$_2$ [39, 40], the tilt of the Weyl cones exceeds the Fermi velocity giving rise to II–type WS with open Fermi surface and new type of Weyl fermions at the boundary between electron and hole pockets [37].

The nontrivial bulk band topology of WSs manifests in a number of novel exotic physical effects such as the protected against weak perturbations Fermi arc surface states [41–45] that connect the projections of the Weyl nodes in the surface Brillouin zone, the chiral anomaly [7, 46–49] (nonconservation of the chiral charge transferred between Weyl nodes of opposite chirality) and related negative longitudinal magnetoresistance [6, 50, 51] quadratic in magnetic field which appears if parallel electric and magnetic fields are applied. Also WSs possess two basic phenomena related to the chiral anomaly: the chiral magnetic effect (CME) [6, 18, 52–55] and the anomalous Hall effect (AHE) [6, 56–58], which are closely related to the topological magnetoelectric effect in $\mathcal{T}$ invariant topological insulators [9]. The CME, manifested in $\mathcal{P}$ broken WS as the electrical currents induced along the magnetic field, hypothetically could be caused by only a magnetic external field and not be associated with the chiral anomaly [59]. However, in an equilibrium state, when all contributions from filled electronic states are taken into account, the static magnetic-field-driven current must vanish [60]. Thus the nonvanishing CME implies the nonzero chiral chemical potential (the difference between local chemical potentials in Weyl nodes), which can be realized only in the nonequilibrium
state dynamically generated by DC parallel electric and magnetic fields and associated with the chiral anomaly [36, 61]. While the dynamic CME with the violation of the chiral current conservation is the consequence of the chiral anomaly, the AHE in any $\mathcal{T}$ broken system being the Hall effect in the absence of a magnetic field, strictly speaking, may be not a part of the chiral anomaly in WS. We underline that due to the nonzero Chern numbers of the Weyl nodes magnetic WSs are distinguished from ordinary ferromagnets by a lack of spin-dependent charge carriers scattering (extrinsic factor) and Fermi-surface contributions to the AHE. Instead, the AHE in WSs is purely intrinsic and determined only by the distance between the Weyl nodes in momentum space [56]. However, this is true only for the I-type WSs which has a point-like Fermi surface, while the AHE in the II-type WSs with tilted conical spectrum around the Weyl node is not universal and can change sign as a function of the parameters quantifying the tilt [58]. Notice that the cubic lattice symmetry of the typical magnetic WS crystals, such as pyrochlore iridates [19, 20], enforces vanishing of the AHE due to the absence of a preferred axis. Nevertheless, the AHE can be recovered by applying an uniaxial strain that lowers the symmetry [57].

The effects caused by WSs nontrivial topology manifest in the optical [53, 60, 62–69] and electron density responses [70–83]. In particular, the AHE and CME giving rise to gyrotropic terms in dielectric function [84] which lead to the Faraday and Kerr magneto-optical effects in $\mathcal{T}$ broken WS [63, 64] and to the natural optical activity in $\mathcal{P}$ broken WS [53, 65, 85]. Moreover, the AHE, CME and corresponding photocurrents in WS can be generated by illuminating with circularly polarized light [66, 67, 86]. Nontrivial topology of $\mathcal{T}$ broken WSs also results in the chiral Fermi arc plasmons with hyperbolic iso-frequency contours [80, 81], in the chiral electromagnetic (EM) waves propagating at the vicinity of the magnetic domain wall in WS [87], in the transverse EM waves in a static magnetic field (helicons) [88], and in the unusual EM modes with a linear dispersion in a neutral WS [77, 78]. Besides, the AHE also makes the surface plasmon polaritons (SPPs) in WS chiral without applying an external magnetic filed (compare with [89]). Particularly, in Ref. [79] the behavior of SPP on the surface of WS is calculated at different orientations of the AHE vector ($\mathbf{b}$) and the direction of SPP propagation ($\mathbf{q}$). The existence of the nonreciprocal SPP in WS, which dispersion depends on the sign of the wave vector, is predicted in the Voigt configuration, when both $\mathbf{b}$ and $\mathbf{q}$ are in the plane of WS film, but perpendicular to each other.

The nonreciprocal unidirectional EM waves are widely known in magneto-optics, and dielectric waveguides with ferrite cores or substrates, as well as films of magnetic dielectrics (see reviews [90], [91], [92]) are usually used for their transmission. The theoretical description of nonreciprocal SPP was given in [93], [94], and the generalization for nonreciprocal waveguide (WG) modes in a film in the Voigt configuration was made in [95]. Nonreciprocal optical elements are used in optical radiation control systems to create unidirectional optical circuits [96], for the directed excitations in a ring laser [97], in a laser gyroscope to eliminate the capture of the frequencies of counterpropagating modes [98], as well as in fiber optic gyroscopes for the initial phase shift between the counter waves [99].

In this paper, we study the propagation of light in magnetic WS films in the Voigt configuration without an external magnetic field. The role of a magnetic field plays the AHE in WS. We predict not only the nonreciprocity of the SPP on both sides of a WS film but also the existence of the nonreciprocal WG EM modes inside the film. The dispersions of the WG modes were obtained within the two-band model accounting for the gapless nature of the Dirac spectrum. We also underline the key role played by the optical contrast between the media surrounding the film in the nonreciprocity magnitude of the predicted WG modes. Besides, the possibilities of varying the nonreciprocity magnitude and operation frequencies of these modes by tuning the Fermi level in WS are discussed. Finally, we compare the AHE parameters in some real WS compounds. Unlike conventional nonreciprocal elements based on the sources of a constant magnetic field, it is possible to achieve nonreciprocity effects in the WS due to the AHE that does not require external sources of magnetic field. Thus unidirectional optical circuits can become more compact due to the lack of need for a permanent magnet.

II. WELM SEMIMETALS OPTICAL RESPONSE

Generally, the nonreciprocity effects in the Voigt configuration, as well as the magneto-optical effects, arise in a $\mathcal{T}$ broken media, and the violation of $\mathcal{P}$ symmetry leads to the natural optical activity effects like in chiral media. In the case of BDS the breaking of $\mathcal{T}$ or $\mathcal{P}$ symmetry splits each doubly degenerated Dirac point into a pair of Weyl nodes of opposite chirality, which are separated in the momentum space by vector $\mathbf{b}$ or in energy space by $\hbar b_0$ (the chiral chemical potential). The first case corresponds to magnetic materials with strong spin-orbit interaction, and the second case can be realized in materials with noncentrosymmetric crystal structure. In the first case there can be the AHE with the currents across the electric field, and in the second case the CME may occur with the currents induced along the magnetic field. The manifestation of WS topological nature in the optical response can be described by the additional axion term in the EM action [59, 61, 100]

$$S_\theta = -\frac{e^2}{(4\pi^2hc)} \int dt \, d^3r \, \theta(\mathbf{r}, t) \, \mathbf{E} \cdot \mathbf{B},$$

where $\theta(\mathbf{r}, t) = 2(\mathbf{b} \cdot \mathbf{r} - b_0 t)$ is the axion angle. Varying this axion action with respect to the EM vector potential
where the first term corresponds to AME and the second one to the CME currents. These currents result in additional terms of the displacement vector \( \mathbf{D} \) in the Maxwell equations with:

\[
\mathbf{D} = \varepsilon \mathbf{E} + \frac{ie^2}{\pi \hbar \omega} 2 \mathbf{b} \times \mathbf{E} - \frac{ie^2}{\pi \hbar \omega} 2 \mathbf{b} \mathbf{B} 
\]

Thus to account for WS topological properties in the optical response one may use the standard form of Maxwell equations with \( \mathbf{D} = \tilde{\varepsilon}_2 \mathbf{E} \) taking WS dielectric tensor in the form:

\[
\tilde{\varepsilon}_2 = \begin{pmatrix} \varepsilon_2 & 0 & 0 \\ 0 & \varepsilon_2 & 0 \\ 0 & -i\varepsilon_2b & \varepsilon_2 \end{pmatrix} 
\]

where \( \varepsilon_2 \) is a BDS dielectric function and \( \varepsilon_{2b} \) is a nondiagonal component caused by the AHE and CME. Since the nonreciprocal properties are always associated with the Hall response, we will consider the magnetic WS in an equilibrium state without any external fields, with Weyl nodes separated only in momentum space (i.e., \( b_0 = 0 \)). For this case, as follows from Eq. (3), the nondiagonal component of the tensor (4) can be written as:

\[
i\varepsilon_{2b} = i2\varepsilon_2 b^2/(\pi \hbar \omega) = i\varepsilon_2 \Omega_b/\Omega, \quad (5)
\]

where \( \Omega_b = 2br_s/(k_F \pi \varepsilon_\infty) \), \( \Omega = \hbar \omega/E_F \), \( E_F \) is the Fermi level, \( k_F = E_F/\hbar v_F \) is the Fermi momentum, \( v_F \) is the Fermi velocity, \( \varepsilon_\infty \) is the effective dielectric constant taking into account all interband electronic transitions. In Ref. [79] the standard one-band Drude model was used for \( \varepsilon_2 \) accounting only for the intraband electronic transitions:

\[
\varepsilon_D = \varepsilon_\infty (1 - \Omega^2_p/\Omega^2) \quad (6)
\]

where \( \Omega_p^2 = 2r_sg^2/(3\pi \varepsilon_\infty) \) denotes the bulk plasma frequency constant normalized to the Fermi level, \( r_s = e^2/\hbar v_F \) is the effective fine structure constant, and \( g \) is the degeneracy factor (the number of nondegenerated Weyl nodes). To describe the dielectric response in BDS more accurately one should use the two-band model taking into account the interband electronic transitions in the Dirac cone. As we have shown in Ref. [101] according to this model a BDS dielectric function in the random phase approximation at zero temperature has the form:

\[
\varepsilon_2 = \varepsilon_b - \frac{2r_sg}{3\pi \Omega^2} + \frac{r_sg}{6\pi} \left[ \ln \left( \frac{4\Lambda^2}{\Omega^2 - 4} \right) + i\pi\theta(\Omega - 2) \right], \quad (7)
\]

where \( \Lambda = E_c/E_F \) (\( E_c \) is the cutoff energy beyond which the Dirac spectrum is no longer linear), \( \varepsilon_b \) is the effective background dielectric constant accounting the contributions from all bands below the Dirac cone. In Ref. [101] we obtained \( \varepsilon_b = 6.2 \) for \( g = 24 \) and \( \varepsilon_\infty = 13 \) (Eu$_2$IrO$_7$ [20]). The difference between Eq. (6) and Eq. (7) (see Fig. 1(a)) is manifested at frequencies above the Fermi level when the dielectric behavior (\( \varepsilon > 1 \)) occurs and WG modes can exist.

### III. NONRECIPROCAL WAVES

Let us consider the propagation of EM waves in the WS film with Weyl pairs, where in each pair nodes are separated in momentum space by the wave vector \( \mathbf{b} \). In the Voigt configuration, where nonreciprocal solutions can be found, the waves propagate in the plane of the film but perpendicular to the magnetic field (see Fig. 2). In our case the AHE plays the role of “internal magnetic field” with the direction determined by the vector \( \mathbf{b} \). The WS film is asymmetrically bounded by two semi-infinite dielectric media with dielectric functions \( \varepsilon_1 \) and \( \varepsilon_3 \). As

\[
\varepsilon_{12} = \begin{pmatrix} \varepsilon_1 & 0 & 0 \\ 0 & \varepsilon_2 & i\varepsilon_{2b} \\ 0 & i\varepsilon_{2b} & \varepsilon_2 \end{pmatrix}
\]
it will be shown below, for the nonreciprocal WG modes it is important that \( \varepsilon_1 \neq \varepsilon_3 \). The wave equation \( \nabla \times (\nabla \times \mathbf{E}) - k_0^2 \varepsilon \mathbf{E} = 0 \) with the vacuum wave vector \( k_0 = \omega/c \) for the considered system in the Voigt configuration has the form

\[
\begin{pmatrix}
q^2 + k_V^2 - k_0^2 \varepsilon_2 & 0 & 0 \\
0 & k_V^2 - k_0^2 \varepsilon_2 & qk_V - k_0^2 \varepsilon_{2b} \\
0 & qk_V + k_0^2 \varepsilon_{2b} & q^2 - k_0^2 \varepsilon_2
\end{pmatrix}
\begin{pmatrix}
E_x \\
E_y \\
E_z
\end{pmatrix}
= 0,
\]

where \( q \parallel y \) is the wave vector of EM waves, \( k_V = \sqrt{k_V^2 \varepsilon - q^2} \), and \( \varepsilon_V = \varepsilon_2 - \varepsilon_{2b}/\varepsilon_2 \) are the Voigt wave vector and dielectric function, respectively, which determine a light behavior inside the film in the considered configuration. This function has the resonance at the plasma frequency (\( \varepsilon_2 = 0 \)) which leads to the splitting of the WG modes region (\( \varepsilon > 1 \)) into two parts one of which lies below the plasma frequency (Fig. 1(b)). Such an anomalous penetration of light into a metal at frequencies below the plasma one is well known in the case of external magnetic field and is connected with the modification of the plasma frequency by the cyclotron resonance [102]. Interestingly that this phenomenon is accompanied by the effect of negative refraction, which can be observed not only in metamaterials but also in any gyrotropic (magnetic or chiral) system [103–106]. In WSs the similar effects of negative refraction, which can be observed not only in metamaterials but also in any gyrotropic (magnetic or chiral) system [103–106]. In WSs the similar phenomena can be observed, which has been recently predicted in Ref. [107, 108]. Notice that the large WG region above the plasma frequency is the manifestation of the interband transitions in the gapless WS Dirac spectrum, while the anomalous WG region below it, is typical for any magnetic metal, and it will remain the same if we use one-band model (6) instead of two-band one (7).

Like in the case of external magnetic field, since carriers drifting parallel to the applied field do not experience a magnetic force, in the Voigt configuration the TE-polarized (s) EM waves will not feel the AHE. Thus we consider only the TM-polarized (p) waves with field components \( H_x, E_y, E_z \) and magnetic field in the form \( H_s(r, t) = H_x(z) e^{i(\omega t - qy)} \) where \( H_x(z) \) in the media with \( \varepsilon_1 (z > d), \varepsilon_2 (0 < z < d), \) and \( \varepsilon_3 (z < 0) \) (see Fig. 2) is expressed as \( H_{1x}(z) = H_1 e^{-ik_1z}, H_{2x}(z) = H_2 e^{ik_2z} + \tilde{H}_2 e^{-ikVz}, \) and \( H_{3x}(z) = H_3 e^{ik_3z} \), respectively. These fields correspond to the WG modes propagating inside the film with the Voigt wave vector \( k_V \) and exponentially decaying out of it. Employing the boundary conditions at the two interfaces \( z = d \) and \( z = 0 \) we obtain the dispersion relation for the TM waves in the Voigt configuration (compare with Ref. [95]):

\[
[k_1k_3 (\varepsilon_2^2 - \varepsilon_{2b}^2) + k_2^2 \varepsilon_1 \varepsilon_3 \pm \varepsilon \varepsilon_{2b} (k_1 \varepsilon_3 - k_3 \varepsilon_1)] \tan (k_Vd) + k_V \varepsilon_2 (k_1 \varepsilon_3 + k_3 \varepsilon_1) = 0,
\]

where \( k_{1,2,3} = \sqrt{q^2 - k_0^2 \varepsilon_{1,2,3}} \). Since this expression depends on the sign of the wave vector \( q \), the TM waves in the considered configuration will be nonreciprocal, which means that at certain frequencies they may propagate only forward (\( p_+ \)) but at another frequencies only backward (\( p_- \)). For BDS films without Weyl features the dispersion relations have a standard form [101]: for the TM waves

\[
[k_1k_3 (\varepsilon_2^2 - \varepsilon_{2b}^2) + k_2^2 \varepsilon_1 \varepsilon_3] \tan (k_2d) + k_2 \varepsilon_2 (k_1 \varepsilon_3 + k_3 \varepsilon_1) = 0
\]

and for the TE waves

\[
[k_1k_3 - k_2^2] \tan (k_2d) + k_2 (k_1 + k_3) = 0,
\]

where \( k_2 = \sqrt{k_0^2 \varepsilon_2 - q^2} \) and \( k_{l,3} = \sqrt{q^2 - k_0^2 \varepsilon_{l,3}} \). In WS film the TE waves obey the Eq. (11) but the TM modes defined by the Eq. (9), which in the absence of the AHE turns into Eq. (10) in the limit \( \varepsilon_{2b} \to 0 \) and by successive substitutions: \( k_2 \to ik_2, \) then \( k_V \to k_2 \). The dispersion of SPP in WS or BDS films can be obtained from the Eq. (9) or Eq. (10) by substitution \( k_V \to ikV \) and \( k_2 \to ik_2, \) respectively.

Using the Eqs. (4), (5), (7), (9)–(11) on Fig. 3 at the same model parameters as for Fig. 1 we compared the dispersions of light and SPP in BDS (\( \varepsilon_2 \)) film and WS (\( \varepsilon_2 \)) film in the Voigt configuration (b) with thicknesses \( d = 0.5 \mu m \) on the semi-infinite dielectric substrate with \( \varepsilon_3 = 4 \), while the medium above the films with \( \varepsilon_1 = 1 \). SPP_\( \varepsilon_1 \) and SPP_\( \varepsilon_3 \) are the forward and backward nonreciprocal SPP, respectively. SPE denotes the interband Landau damping region. The parameters of BDS and WS are the same as for Fig. 1.
the nonreciprocity effect of TM waves in the WS film is determined not only by the component \( \varepsilon_{2b} \), but also by the term \((k_1\varepsilon_3 - k_3\varepsilon_1)\). Therefore, the nonreciprocity effect grows with the optical contrast \(|\varepsilon_1 - \varepsilon_3|\) between the media above and below the WS film. This can be understood from the fact that nonreciprocal SPP excited on both sides of the film will compensate each other if the media from both sides are the same. In the measure of the optical contrast between these media, the nonreciprocity effect will appear in the collective SPP or WG TM modes propagating along the film. In order to demonstrate these phenomena we considered the case of ordinary contrast, when the WS film lies on a dielectric substrate, and the case of high contrast, when the substrate is metallic.

In the case of ordinary contrast we compared the dispersions of WG modes and SPP in BDS and WS films with thickness \( d = 0.5 \mu m \), as well as in the film of magnetic dielectric (MD) with thickness \( d = 80 \text{ nm} \) and the same direction of magnetization as in WS. All the films are placed on a semi-infinite dielectric substrate (\( \varepsilon_3 = 4 \)). The optical response of the MD we described by the same dielectric tensor (4) as for the WS but with frequency independent components (\( \varepsilon_\infty = 13, \varepsilon_{2b} = 4 \)). The medium above the films for all cases with \( \varepsilon_1 = 1 \). The parameters of BDS and WS are the same as for Fig. 1.

In the case of metallic substrate (we take silver with \( \varepsilon_3 = 3.7 - \Omega_p^2/\Omega^2 \), where \( \Omega_p = \hbar \omega_p/\epsilon_F \) and \( \hbar \omega_p = 9.2 eV \)) at the considered frequencies its dielectric constant is very large and negative (\( \varepsilon_3 \sim -10^3 \)) which leads to a high optical contrast, and hence to a strong nonreciprocity effect. In the BDS film on the metallic substrate, the WG TM and TE modes swap places by frequency, and also only the high (in-phase) SPP branch

![Figure 4](image-url)

**FIG. 4.** The dispersions of light and SPP in BDS (\( \varepsilon_2 \)) film with \( d = 1.25 \mu m \) (a), in WS (\( \varepsilon_2 \)) film with \( d = 1.25 \mu m \) and in MD film with \( d = 80 \text{ nm} \) in the Voigt configuration. (a)-(c) the case of the ordinary contrast when all the films on the semi-infinite dielectric substrate (\( \varepsilon_3 = 4 \)), (d)-(f) the case of the high contrast when all the films on the semi-infinite silver substrate (\( \varepsilon_3 = 3.7 - \Omega_p^2/\Omega^2 \)), where \( \Omega_p = \hbar \omega_p/\epsilon_F \) and \( \hbar \omega_p = 9.2 eV \)). The WG TE (s) modes do not feel the AHE and WG TM (p) modes in (b)-(c), (e)-(f) become nonreciprocal: (SPP\( > \), p\( > \)) and (SPP\( < \), p\( < \)) are the forward and backward nonreciprocal (SPP, WG TM modes), respectively. The MD dielectric tensor (\( \varepsilon_2 \)) is the same as for the WS but with frequency independent components (\( \varepsilon_\infty = 13, \varepsilon_{2b} = 4 \)).
exists (Fig. 4(d)). In the WS film on the metal there is really a large difference between the dispersion of the nonreciprocal waves propagating in the opposite directions (Fig. 4(e)). In particular, the TM mode in one direction remains WG (p>) and in the opposite direction it becomes evanescent (SPP<). Also in this case the in-phase SPP splits to the pair of the nonreciprocal SPP with very high difference between SPFF> and SPP<. The similar behavior demonstrates the MD on the metal, where the WG TM and TE modes also swap places by frequency, the strong nonreciprocity effect of the WG TM modes takes place, and the nonreciprocal SPP arise due to the metallic substrate (Fig. 4(f)).

IV. DISCUSSION

For both ordinary and high optical contrasts between the media above and below the films, in the WS film the nonreciprocal WG modes and SPP can exist, similar to the waves which can be observed in WG with ferrite rods or MD films. However, in contrast to a MD film, in the WS film the WG modes frequency depends nonlinearly on the wave vector, and also there are two WG regions, one of which lies below the plasma frequency where the negative refraction in WS can be observed [107]. But the main advantage of WS over MD films is the magnitude of the nonreciprocity effect. In the WS it depends not only on the surrounding media optical contrast, but also on the separation of the Weyl nodes in momentum space $2b = \Omega_b k_F \pi \varepsilon_\infty / r_s$. For all the figures we took model parameters $E_F = 150 \text{meV}$, $g = 24$, $\Omega_b = \Omega_p \approx 0.93$, i.e., $2b \approx 0.4 \text{Å}^{-1}$ and $\varepsilon_{2b}(\Omega = \Omega_p) = 12$. Such a characteristics can be observed in the real compounds listed in Table I.

In WS the separation of the Weyl nodes in momentum space can be so large $2b \approx 0.5 \text{Å}^{-1}$ (Co$_3$S$_2$Se$_2$) that the AHE dielectric tensor component $\varepsilon_{2b} \approx 2.3 / \hbar \omega [\text{eV}]$ even in the optical range ($\hbar \omega \sim 2 \text{eV}$) may be of the order of $\varepsilon_{2b} \sim 1$. While for the typical MD film (bismuth iron garnet) in the optical range $\varepsilon_{2b} = 0.003$ [110] is by three orders of magnitude less than in some WS films. However, while bismuth iron garnet retains the magnetization at room temperature, all the WS listed in Table I can be used only at $T < T_C \sim 150 \text{K}$. Notice that the highest frequency of the weakly damped WG modes in WS is determined by the value $2E_F$ above which there is interband Landau damping region (see imaginary part of Eq. (7))). Besides, when the Fermi level is high enough that the Fermi surfaces enclosing the two Weyl nodes with opposite topological charges merge, the magnitude of the AHE and corresponding WG modes nonreciprocity may dramatically change [56]. Thus tuning the Fermi level in WS one can vary the operation frequencies of the predicted nonreciprocal waves from THz to Mid-IR ranges obtaining very strong nonreciprocity with $\varepsilon_{2b}$ from $\sim 170$ to $\sim 10$, respectively. Notice that due to the scalability of Eqs. (9)–(11) $E_F = 150 \text{meV}$, $d = 0.5 \mu\text{m}$ and $E_F = 60 \text{meV}$, $d = 1.25 \mu\text{m}$ gives the same WG modes, so for each Fermi level one should just choose the appropriate film thickness.

| Compounds          | $E_F (\text{meV})$ | $g$  | $2b(\text{Å}^{-1})$ | $\varepsilon_{2b}$ | $\omega_0 (\text{THz})$ |
|--------------------|-------------------|-----|---------------------|-------------------|------------------|
| Y$_2$IrO$_7$ [19]  | 10                | 24  | 0.37                | 170               | 2.4              |
| Eu$_2$IrO$_7$ [20] |                   |     |                     |                   |                  |
| Co$_3$S$_2$Sn$_2$  | 60                | 12  | 0.4                 | 36                | 14.5             |
| Co$_3$S$_2$Se$_2$  | 110               | 12  | 0.5                 | 21                | 26.6             |

TABLE I. The list of magnetic WSs with different Fermi levels $E_F$, numbers of nondegenerated Weyl nodes $g$, separations of the Weyl nodes in momentum space $2b$, corresponding dielectric tensor AHE components $\varepsilon_{2b}$ taken at $\hbar \omega = E_F$, and operation frequencies $\omega_0$.

V. CONCLUSION

In summary, we predict the existence of nonreciprocal WG modes in magnetic WS films in the Voigt configuration without an external magnetic field. The role of a magnetic field plays the AHE in WS, which being purely intrinsic and universal depends only on the separation of the Weyl nodes in momentum space. The nonreciprocity value also depends on the optical contrast between the media surrounding a WS film, particularly, a metallic substrate leads to a significant increase of the nonreciprocity due to the high optical contrast with the medium above the film. We show that the nonreciprocal WG modes may exist in the two frequency regions one of which lies below the WS plasma frequency where the negative refraction in WS can be observed. We provide the AHE parameters of the real WS materials where a strong nonreciprocity can be observed even without a help of the surrounding media optical contrast. Such a high values of the AHE in WS may be useful not only for the nonreciprocity but also for the gyrotropic effects. Moreover, tuning the Fermi level in WS one can vary the operation frequencies of the WG modes from THz to Mid-IR ranges and enhance their nonreciprocity by shifting to THz range. Thus WSs allow to realize giant tunable gyrotropic and nonreciprocity effects for a propagating light which paves the way to the design of compact, tunable and effective nonreciprocal optical elements.

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