Hierarchical forecasting with a top-down alignment of independent level forecasts

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Abstract

Hierarchical forecasting with intermittent time series is a challenge in both research and empirical studies. Extensive research focuses on improving the accuracy of each hierarchy, especially the intermittent time series at bottom levels. Then hierarchical reconciliation could be used to improve the overall performance further. In this paper, we present a \textit{hierarchical-forecasting-with-alignment} approach that treats the bottom level forecasts as mutable to ensure higher forecasting accuracy on the upper levels of the hierarchy. We employ a pure deep learning forecasting approach N-BEATS for continuous time series at the top levels and a widely used tree-based algorithm LightGBM for the intermittent time series at the bottom level. The \textit{hierarchical-forecasting-with-alignment} approach is a simple yet effective variant of the bottom-up method, accounting for biases that are difficult to observe at the bottom level. It allows suboptimal forecasts at the lower level to retain a higher overall performance. The approach in this empirical study was developed by the first author during the M5 Forecasting Accuracy competition, ranking second place. The method is also business orientated and could benefit for business strategic planning.

\textit{Keywords:} M5 competition; Forecasting reconciliation; Hierarchical forecasting; Hierarchical alignment.

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1. Introduction

With the rise of business oriented time-series data collection across industries, improving forecasting performance becomes vital for business success. Among the vast forecasting techniques, conventional statistical forecasting methods are dominant in the forecasting domain (for a survey in this area see Hyndman and Athanasopoulos (2018), while for an encyclopedia review of forecasting theory and applications, see Petropoulos et al. (2021)). Machine learning algorithms are widely used in time series forecasting, notably long short term memory (LSTM) neural networks and their variations, deep neural architectures such as N-BEATS (Oreshkin et al., 2019), or the Temporal Fusion Transformer (Lim et al., 2019). The top-ranking method in the M4 competition (Smyl, 2020) suggested that hybrid models combining statistical approaches and machine learning methods could further improve the forecasting accuracy.

Although statistical approaches tend to be more interpretable, a cost-efficient forecast usually requires expert information in e.g., features engineering and model specification. Those approaches work well with individual time series, but it is hard to scale them up with sizeable related time series sets. On the contrary, industrial machine learning platforms with pipelines provided for global forecasting solutions do not necessarily achieve the best optimality without extensive tuning. Still, they are easy to scale up for massive amounts of time-series data. It is thus appealing for industrial forecasters to adopt machine learning approaches.

Intermittent time series, which contains many zeros in the data, is a typical case in demand forecasting. Such time series are difficult to forecast because it is toilsome to make realistic assumptions across all the time series horizons. Dating back to the 1970s, researchers observed that conventional time series methods, such as exponential smoothing, did not perform well for forecasting intermittent demand time series (Croston, 1972). A straightforward approach, known as the Croston’s method, was proposed by splitting the time series into the non-zero component and stochastic component. The two parts were then forecasted separately and the final forecast was factorized by the two individual forecasts in the end. See e.g., Croston (1972), Rao (1973), and Shenstone and Hyndman (2005) for the related study. Although such a method has been proven to be helpful, it often produces forecast bias (Levén and Segerstedt, 2004). Many variants were proposed to improve the accuracy (Shenstone and Hyndman, 2005; Kourentzes and Athanasopoulos, 2021), or reduce the bias (Syntetos and Boylan, 2005; Nikolopoulos et al., 2011) of Croston’s method.

Attracted by the nonparametric and assumption-free nature of neural networks, researchers shift the focus to neural networks (Caner Turkmen et al., 2020), and directly construct neural networks for time series with particular patterns, most notably as short intermittent time series (Kourentzes, 2013) and long lumpy time series (Gutierrez et al., 2008). One widely known approach is DeepAR - an architecture
based on autoregressive recurrent networks (Salinas et al., 2020).

Another challenge in the supply chain or online sales industry is simultaneously forecasting an extensive collection of time series. Such time series collection usually involves a hierarchical structure from different cross-sectional levels. For example, the dataset in the M5 competition (Makridakis et al., 2021) consists of 3049 products, which are sold across ten stores located in three states in the U.S. The products are further classified into three product categories and seven product departments. Notably, information from different cross-sectional levels may be crucial for improving overall forecasting performance. Although the practitioners find incorporating the cross-sectional information improves the overall forecasting performance, a challenge exists that the forecasts on each hierarchy should be coherent with the aggregation structure of the collection of time series. A hierarchical forecasting method could utilize the structural information to improve the overall forecasting performance and retain the forecast coherence. See Hyndman and Athanasopoulos (2018) for an introduction to hierarchical forecasting.

A common phenomenon of hierarchical time series sets in the sales industry is that the lowest level of the hierarchy exhibits a strong intermittent pattern, and the upper hierarchy levels contain forecastable components such as the trend or seasonality aggregated by the bottom level. Early research shows that spatial aggregation for intermittent time series improves the overall forecast accuracy (Zufferey et al., 2016). Similar results are also found with temporal aggregated intermittent time series in e.g., Nikolopoulos et al. (2011) and Kourentzes and Athanasopoulos (2021).

To this end, we consider a typical business planning scenario. Under certain circumstances, managers at the headquarter or the investors may not be interested in the forecasts of a particular product in a specific store, but they are more concerned with the state level’s forecasts for all products, which affect the overall revenue. By further incorporating the product information as explanatory variables into the bottom level models, we argue that decision-makers or business planners may benefit by further forecasting performance improvement, especially at the top levels, from hierarchical forecasting with intermittent time series, than by merely using individual forecasting models for the bottom level intermittent data.

In this paper, we describe and analyze an approach proposed by the first author, who utilizes standard machine-learning toolchains with a careful forecasting alignment scheme achieving the second rank of the M5 Forecasting Accuracy competition. Unlike the forecasting reconciliation approach that designs optimal proportions for the aggregations to guarantee the coherence across multi-hierarchy forecasts, the proposed hierarchical-forecasting-with-alignment approach adjusts the sum of the forecasts of the bottom level based on those aggregated at the top levels by selecting a tuning parameter at the bottom level model so that the sum of the forecasts produced across multiple forecasting horizons are aligned. Our analysis finds it particularly useful when there is difficulty in finding an optimal reconciliation as suggested in e.g., Wickramasuriya et al. (2019). A vital feature of this approach is that it improves the
overall forecast accuracy by allowing some suboptimal forecasts at the lower level but retaining high forecasting accuracy at the top levels.

We organize the remainder of the paper as follows. Section 2 presents the preliminary for models used in our paper. The methodology is described in Section 3. Section 4 describes the implementation details for the M5 competition. An ex-post analysis is provided in Section 5 and Section 6 concludes the paper.

2. Preliminary

This section briefly describes two machine learning algorithms, i.e., N-BEATS (Oreshkin et al., 2019) and LightGBM (Ke et al., 2017), that are used for the proposed hierarchical-forecasting-with-alignment framework. Both methods are robust and widely used in the machine learning community.

The N-BEATS (Oreshkin et al., 2019) is a pure deep learning approach with a deep neural architecture based on backward and forward residual links. It utilizes a very deep stack of fully-connected layers that consist of 30 stacks of depth 5 with a total depth of 150 layers. The N-BEATS constructs a doubly residual stacking architecture with two residual branches. One residual branch is running over the backcast prediction of each layer, and the other one is running over the forecast branch of each layer. This architecture improves the interoperability of deep learning models by especially allowing for trend and seasonality decomposition. A desired feature for practitioners is that the N-BEATS model does not require massive data sources, nor complex data transformation or feature engineering, making the model extremely easy to use and deploy in the industry.

The performance of the N-BEATS model relies on the ensembling step, a forecasting combination technique that combines the forecasts from different models. The final forecasts provided by the N-BEATS are based on multi-scale aggregation with the median of bagging selected models. The model pool for ensembling consists of N-BEATS models fitted on one or several of the sMAPE, MASE and MAPE metrics and a selection of different window lengths e.g., $2 \times h$, $3 \times h$, ..., $7 \times h$, respectively, where $h$ is the forecast horizon.

The N-BEATS model has already been proved to be robust and implemented on industry-level platforms. We use the GluonTS’ implementation (Alexandrov et al., 2020) in this analysis. The notable difference between GluonTS’ implementation and the original N-BEATS is splitting the training and forecasting series. The N-BEATS picks a random forecast point from the historical range of length $L_h$ for each selected time series, immediately preceding the last point in the train part of the time series. This method requires further tuning of the hyperparameter $L_h$. However, GluonTS does not use the $L_h$ parameter and cuts time windows by randomly selecting a time series and a starting point on that time series, which reduces the computation burden.
The LightGBM (Ke et al., 2017) is also a widely used machine learning algorithm based on gradient boosting decision trees. The decision tree algorithm was developed for machine learning tasks such as regression and classification. It partitions the input variables into tree structures, and the final prediction is decided based on the tree of input variables. The decision tree is a weak learner due to its simple decision strategy. Gradient boosting algorithms are then used to ensemble the predictions in a step-wise fashion to improve the prediction performance. The LightGBM further enhances the efficiency and scalability of gradient boosting libraries, such as XGBoost (Chen and Guestrin, 2016), for large datasets with high-dimensional features.

3. Methodology

3.1. Improved N-BEATS ensembles for upper levels

Hierarchical forecasting aims to build suitable forecasting models at different levels and prevent overfitting at all levels using the hierarchical structure. We first train the N-BEATS model using the top five levels data. Although the data from the top five levels are used to train the N-BEATS model, only the forecasts at the top level are employed for the hierarchical alignment described in Section 3.3. The other four levels were served as a cross check.

During the M5 competition, the first author experimented with the N-BEATS ensembles for the top-level time series during the validation time frame with different epochs. It is observable that the N-BEATS model started to overfit after approximately 12 epochs. For that reason, N-BEATS ensembles for the top-level forecast with 10 epochs were chosen for the final evaluation models. A comparison with other single forecasting models on the top level was not made due to the length of the competition time and the limited computing resources available on the Kaggle platform. But according to the reported score by the competitors 1 on the Kaggle platform, single models like the LightGBM at the top levels do not achieve better accuracy.

The stochastic gradient descent (SGD) algorithm used in the N-BEATS model has weak learning stability. The learning accuracy is very sensitive to the learning rate, and the convergence rate is slow. The lookahead optimizer (Zhang et al., 2019) is adopted to the N-BEATS model trainer further to improve the training accuracy and learning stability. Unlike the default stochastic searching scheme used in the SGD, the lookahead algorithm chooses a parameter search direction by looking ahead at the sequence of weights generated by the standard optimizer provided by the GluonTS package.

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1 Available online at https://www.kaggle.com/c/m5-forecasting-accuracy/discussion/134712
3.2. Bias-adjusted LightGBM model for the bottom level

At the bottom level, we use a bias-adjusted LightGBM to model the intermittent time series. The root mean squared error (RMSE) loss is used in the LightGBM as follows,

\[
RMSE = \sqrt{\frac{1}{h} \sum_{t=n+1}^{n+h} (Y_t - \hat{Y}_t)^2},
\]

where \(Y_t\) is the true value, \(\hat{Y}_t\) is the forecast, \(h\) is the forecasting horizon, and the residual is defined as \(e_t = Y_t - \hat{Y}_t\). We further use a customized gradient for the RMSE loss as follows

\[
\text{gradient} = \begin{cases} 
-2e_t & e_t < 0, \\
-2\lambda e_t & e_t \geq 0 
\end{cases}
\]

(1)

where \(\lambda > 0\) is a tuning parameter named the loss multiplier to allow for an asymmetric loss. When \(e_t \geq 0\), (i.e., the forecast is lower than the true value), the loss multiplier magnifies (\(\lambda > 1\)) or minifies (\(0 < \lambda < 1\)) the gradient of the loss function during the learning process. When \(\lambda = 1\), the customized gradient reduces to the conventional symmetric gradient. To this end, the loss multiplier controls the bias during the training process to achieve a bias-variance trade-off effect for the bottom level model. The customized gradient with the tuning parameter is particularly useful for the final hierarchical alignment step described in Section 3.3. The corresponding Hessian is

\[
\text{Hessian} = \begin{cases} 
2 & e_t < 0, \\
2\lambda & e_t \geq 0 
\end{cases}
\]

We independently train separate models for the time series data at each store for the parallelization simplicity on the Kaggle computing platform. It is worth mentioning that no rolling or lagged demand features were used in the LightGBM training, in contrast to conventional statistical forecasting approaches. The motivation is to focus on the intermittent characteristics at the bottom level. We argue that the intermittent demand can be significantly influenced by factors like prices, time in a year, or special events rather than the historical demand information. This approach with non-time series features consistently achieves more stable forecast results. For the final forecasts, five different multipliers around the optimal loss multiplier were used to build five independent forecast models. The final forecasts at the bottom level are based on the mean ensemble of the five models. See Section 4 for the implementation details.

During the M5 competition, the first author also investigated the DeepAR (Salinas et al., 2020) approach for modeling the intermittent time series. Nonetheless, the experiments found it difficult to achieve better results than LightGBM models or obtain optimal reconciliation results compared with the
minimum trace (MinT) optimal reconciliation approach proposed in Wickramasuriya et al. (2019).

3.3. Aligning top level forecasts and aggregated bottom level forecasts

Given the independent forecasting results for the top level from stable N-BEATS forecasting models with the number of epochs = 10, the final step is to find the optimal bottom level forecasts produced by the LightGBM model. We tune the loss multiplier ($\lambda$) introduced for the LightGBM model in Section 3.2 so that the RMSE between N-BEATS forecasts and aggregated bottom level forecasts reaches a minimum. This step is called the hierarchical alignment, and the corresponding alignment metric is defined in Equation 2 as

$$\arg \min_{\lambda} \left\{ \frac{1}{h} \sum_{t=n+1}^{n+h} \left( \hat{Y}_{t}^{(\text{top})} - \text{Agg}(\hat{Y}_{t}^{(\text{bottom})}(\lambda)) \right)^2 \right\},$$

where $\text{Agg}(\cdot)$ is the aggregating method used at the bottom level, and we use the mean aggregation function throughout the M5 competition.

The hierarchical forecasting with alignment method is simple and applicable to any model appropriate to the bottom level. The loss multiplier $\lambda$ in Equation 2 is selected so that the sum of the forecasts produced across the entire forecasting horizon are aligned. It means that the objective function for the alignment depends on the forecasting horizon. During the M5 competition, a manual grid search method was used to find the optimal loss multiplier for the LightGBM at the bottom level, which can easily be automated in an industrial setting. This hierarchical alignment method only requires running the model for the top level once. The optimal loss multiplier could be obtained during the optimization stage of the bottom level model by examining the alignment metric in Equation 2. Note that in the evaluation time frame during the M5 competition, Equation 2 is used to determine optimal multiplier values because the actual future values are unknown. We further provide an ex-post analysis in Section 5 using the WRMSSE metric (Equation 4) when all data and results are disclosed after the competition.

It is worth mentioning that there are alternative ways for hierarchical alignment. In principle, this hierarchical alignment method should work for aligning all upper levels with the bottom level. The reason that we only align with the top level is twofold. First, the scoring metric in the M5 competition gives higher weights on upper levels because they contain fewer series than the bottom level. Second, the computational cost on the top level is much lower than aligning with several levels.

4. Implementation details

The hierarchical forecasting with alignment approach has been successfully applied to the M5 competition dataset, consisting of a hierarchical structure of daily sales data of total 42,840 series
Table 1: Overview of series per level and contribution to error metric for that level. Note that all hierarchical levels are equally weighted in the M5 competition.

| Hierarchy level | Description                                                                 | Number of series |
|-----------------|------------------------------------------------------------------------------|------------------|
| 1               | All products, all stores, all states                                        | 1                |
| 2               | All products by states                                                      | 3                |
| 3               | All products by store                                                       | 10               |
| 4               | All products by category                                                     | 3                |
| 5               | All products by department                                                   | 7                |
| 6               | Unit sales of all products, aggregated for each State and category           | 9                |
| 7               | Unit sales of all products, aggregated for each State and department         | 21               |
| 8               | Unit sales of all products, aggregated for each State and category           | 30               |
| 9               | Unit sales of all products, aggregated for each store and department         | 70               |
| 10              | Unit sales of product x, aggregated for all stores/states                    | 3,049            |
| 11              | Unit sales of product x, aggregated for each State                           | 9,147            |
| 12              | Unit sales of product x, aggregated for each store                           | 30,490           |
| **Total**       |                                                                               | **42,840**       |

spanning 1,941 days. Table 1 depicts the hierarchical structure of the data and the number of series per aggregation level. One could observe from Table 1 that the upper levels contain much less series than the lower level.

The Root Mean Squared Scaled Error (RMSSE), which is a variant of the well-known Mean Absolute Scaled Error (MASE) (Hyndman and Koehler, 2006), is used for calculating the out-of-sample forecasting error for each series as described in Equation 3,

\[
RMSSE = \sqrt{\frac{1}{h} \sum_{t=h+1}^{n} (Y_t - \hat{Y}_t)^2}
\]

After estimating the RMSSE for all the 42,840 time series of the competition, the Weighted RMSSE (WRMSSE) is used by the organizer for the overall accuracy comparison defined in Equation 4

\[
WRMSSE = \sum_{i=1}^{42,840} \omega_i \times RMSSE_i
\]

where \(\omega_i\) is the weight of the series based on actual dollar sales of the product. It is worth mentioning that the actual future values \(Y_{n+1}, ..., Y_{n+h}\) were not available during the validation frame in the M5 competition.

Interestingly, one may notice that the RMSSE from hierarchical levels of view are equally weighted, which indicates that the overall accuracy in WRMSSE is primarily affected by the upper levels forecasts. Because all hierarchical levels are equally weighted with the M5 accuracy metric WRMSSE, it is easier to forecast the continuous time series at upper levels than to forecast massive intermittent time series at the bottom level. Based on the above concern, the hierarchical alignment scheme builds the forecasting
Table 2: Features for intermittent time series data at the bottom level used in the LightGBM.

| Feature         | Description                                                                                                          |
|-----------------|----------------------------------------------------------------------------------------------------------------------|
| item_id         | 3049 unique identifiers of items.                                                                                  |
| dept_id         | 16942 unique identifiers of department.                                                                             |
| cat_id          | 5652 unique identifiers of category, e.g., foods, household, hobbies.                                                |
| sell_price      | Price of item in store for given date.                                                                             |
| event_type      | 108 categorical events, e.g. sporting, cultural, religious.                                                         |
| event_name      | 157 event names for event_type, e.g. super bowl, valentine’s day, president’s day.                                  |
| event_name_2    | Name of event feature as given in competition data.                                                                   |
| event_type_2    | Type of event feature as given in competition data.                                                                   |
| snap_CA         | Binary indicator for SNAP information in CA.                                                                           |
| snap_TX         | Binary indicator for SNAP information in TX.                                                                           |
| snap_WI         | Binary indicator for SNAP information in WI.                                                                           |
| release         | Release week of item in store.                                                                                       |
| price_max       | Maximum price for item in store in the train data.                                                                   |
| price_min       | Minimum price for item in store in the train data.                                                                   |
| price_std       | Standard deviation of price for item in store in the train data.                                                    |
| price_mean      | Mean of price for item in store in train data.                                                                       |
| price_norm      | Normalized sell price. divided by the price_max.                                                                     |
| price_nunique   | Number of unique prices for item in store.                                                                         |
| item_nunique    | Number of unique items for a given price in store.                                                                   |
| price_diff_w    | Weekly price changes for items in store.                                                                             |
| price_diff_m    | Price changes of item in store compared to its monthly mean.                                                        |
| price_diff_y    | Price changes of item in store compared to its yearly mean.                                                         |
| tm_d            | Day of month.                                                                                                       |
| tm_w            | Week in year.                                                                                                       |
| tm_m            | Month in year.                                                                                                      |
| tm_y            | Year index in the train data.                                                                                       |
| tm_wm           | Week in month.                                                                                                      |
| tm_dw           | Day of week.                                                                                                        |
| tm_w_end        | Weekend indicator.                                                                                                   |

strategy focusing on the forecasting accuracy at the top level forecasts. It aligns them with the bottom level forecasts.

Table 2 shows the time series features used for the LightGBM model at the bottom level. The feature matrix includes all available categorical features like store_id and category_id. Furthermore, the time since the first sale of an item, the price, and derived price features (price_min, price_max, etc.) are also included. Events and SNAP data, and time features like day, week and month are also used in the model.

Table 3 and Table 4 document the parameter settings for the N-BEATS model and LightGBM model, respectively. The baseline features for the bottom level LightGBM model and hyperparameters were taken from a public Kaggle user notebook provided by Konstantin Yakovlev (https://www.kaggle.com/kyakovlev/m5-simple-fe). The feature matrix does not include any time rolling or lagged features and we find the non-time series features are of great importance for the intermittent time series forecasting with the LightGBM model. There are two hyperparameters, (i) the number of epochs for the
Table 3: Parameter settings for N-BEATS ensembles using GluonTS (Alexandrov et al., 2020)

| Parameter               | Description                                                                 | Value |
|-------------------------|-----------------------------------------------------------------------------|-------|
| num_stacks              | The number of stacks the network should contain.                           | 30    |
| widths                  | Widths of the fully connected layers with ReLu activation in the blocks.    | 512   |
| meta_prediction_length  | Forecast horizon $h$.                                                       | 28    |
| meta_bagging_size       | The number of models that share the parameter combination. Each of these models gets a random initialization. | 3     |
| meta_context_length     | The number of time units that condition the predictions.                   | $h \times \{3, 5, 7\}$ |
| meta_loss_function      | The loss function (metric) to use for training the models.                 | sMAPE |
| learning_rate           | Learning rate for each boosting round.                                      | 0.0006|
| epochs                  | The number of epochs used for the optimization algorithm.                  | \{10, 12\} |
| num_batches_per_epoch   | The number of batches in each epoch for the optimization algorithm.       | 1000  |
| batch_size              | The batch size used in the optimization.                                   | 16    |

Table 4: Parameter settings for the LightGBM model (Ke et al., 2017).

| Parameter       | Description                                                                 | Value |
|-----------------|-----------------------------------------------------------------------------|-------|
| objective       | The objective function to maximize with an optimization algorithm           | custom|
| metric          | Metric(s) to be evaluated on the evaluation set                             | rmse  |
| learning_rate   | Learning rate for each boosting round.                                      | 0.2   |
| lambda_l1       | $L_1$ norm penalty to prevent overfitting                                   | 0.5   |
| lambda_l2       | $L_2$ norm penalty to prevent overfitting                                   | 0.5   |
| bagging_freq    | Frequency for bagging at every $k$ iterations                               | 1     |
| bagging_fraction| The proportion of randomly selected data for the next $k$ iterations.       | 0.85  |
|(colsample_bytree)|The proportion for randomly selecting a subset of features on each tree.     | 0.85  |
| colsample_bynode | Proportion for randomly selecting a subset of features on each tree node.   | 0.85  |

N-BEATS model on the top level, and (ii) the optimal loss multiplier for the top-down alignment. The approach requires a minimal effort of parameter tuning. We use a simple grid search on the validation window to find the optimal hyperparameters.

Finally, in the evaluation phase of the M5 competition, the optimal multiplier value used on the evaluation time frame was 0.95 based on a manual grid search in the space of ($0, 2$]. To produce the final output, we further use the mean ensemble to obtain the results based on the five closest multipliers [0.90, 0.93, 0.95, 0.97, 0.99] in the grid around the optimal value of 0.95. This ensemble method reduces the high variance effect on the bottom level forecasts when we apply the LightGBM with the features used in Table 2. Although the decision-makers focus on the accuracy of top levels, the low variance forecasts on the bottom level also improve the overall performance. The extra ensemble step of the five models around the best $\lambda$ mitigates this high variance while preserving the low bias.
Table 5: Hierarchical view of forecasting errors for the aggregated forecasts with LightGBM. The comparison is based on different accuracy metrics during the evaluation phase of the competition. The top level N-BEATS models are evaluated with epochs of 10 and 12, respectively.

| Hierarchy | Category | RMSSE Epoch 10 | RMSSE Epoch 12 | MEAN ERROR Epoch 10 | MEAN ERROR Epoch 12 | MAE Epoch 10 | MAE Epoch 12 | RMSE Epoch 10 | RMSE Epoch 12 |
|-----------|----------|----------------|----------------|---------------------|---------------------|--------------|--------------|----------------|----------------|
| 1         | All      | 6.2742 x 10^7  | 9.8031         | 56.3                | 305.5               | 1286.0       | 1379.9       | 1796.7         | 1991.2         |
| 2         | CA_1     | 6.0036         | 9.0086         | 17.2                | 239.9               | 74.4         | 78.4         | 101.1          | 108.9          |
| 3         | CA_2     | 1.055          | 2.8249 x 10^6  | 76.9                | 1.7                 | 225.1        | 217.8        | 358.4          | 330.8          |
| 4         | CA_3     | 0.179          | 0.1569         | 247.1               | 297.9               | 373.8        | 425.1        | 437.4          | 493.6          |
| 5         | CA_4     | 0.0614         | 0.0344         | 55.4                | 41.4                | 95.2         | 99.3         | 128.8          | 135.6          |
| 6         | TX_1     | 0.0606         | 0.0077         | 108.1               | 38.4                | 156.7        | 126.5        | 191.1          | 162.7          |
| 7         | TX_2     | 1.4627 x 10^-4 | 0.0018         | 1.3                 | 19.8                | 176.7        | 168.5        | 222.5          | 225.9          |
| 8         | TX_3     | 0.0019         | 0.0015         | 14.2                | 12.6                | 133.3        | 137.4        | 161.8          | 164.5          |
| 9         | Wl_1     | 0.0149         | 0.0022         | 84.5                | 32.1                | 162.4        | 148.0        | 195.6          | 179.7          |
| 10        | Wl_2     | 0.0097         | 0.0114         | 64.8                | 70.3                | 225.5        | 242.8        | 282.4          | 301.9          |
| 11        | Wl_3     | 0.0272         | 0.0203         | 101.3               | 87.6                | 186.9        | 143.2        | 219.1          | 179.8          |
| 12        | HOUSEHOLD| 0.0020         | 0.0022         | 44.8                | 169.5               | 881.7        | 943.8        | 1190.4         | 1237.2         |
| 13        | HOUSEHOLD| 0.0066         | 0.0010         | 35.5                | 44.7                | 292.1        | 338.5        | 446.8          | 489.0          |
| 14        | HOUSEHOLD| 0.0083         | 0.0043         | 99.1                | 88.2                | 162.9        | 151.1        | 191.4          | 174.7          |
| 15        | HOUSEHOLD| 0.0262         | 0.0223         | -117.1              | -109.4              | 230.5        | 229.9        | 320.6          | 333.2          |
| 16        | HOUSEHOLD| 0.0084         | 0.0233         | 240.4               | 401.5               | 740.1        | 807.6        | 900.7          | 957.0          |
| 17        | HOUSEHOLD| 0.0046         | 0.0008         | -31.4               | 13.1                | 113.3        | 109.4        | 174.9          | 160.3          |
| 18        | HOUSEHOLD| 1.2666         | 1.2309         | -35.2               | -35.5               | 39.2         | 38.4         | 51.7           | 52.1           |
| 19        | HOUSEHOLD| 0.0003         | 0.0064         | 18.2                | 86.9                | 224.3        | 264.3        | 333.8          | 366.3          |
| 20        | HOUSEHOLD| 0.0267         | 0.0175         | -56.4               | -45.6               | 103.5        | 97.0         | 153.1          | 150.3          |

During the evaluation frame of the M5 competition, the N-BEATS forecasts and the finally aggregated forecasts on the top five levels were visualized in Figure 1. Table 5 further depicts the forecasting error for the aggregated forecasts produced by LightGBM. The top level N-BEATS models are evaluated with epochs of 10 and 12, respectively. The mean error was explicitly reported to check whether the LightGBM has over or under-forecasted the actual values with the customized gradient in Equation 1. Specifically, if the forecast is greater than the actual values, the error will be positive, vice versa. We ran the model with the number of epochs from 1 to 12. We have noticed that it yields a better overall forecasting performance with 10 epochs, which was also used for the final submission of the M5 competition.

The code is written in Python and reproducible Jupyter Notebooks running on the Kaggle platform are available at https://github.com/matthiasanderer/m5-accuracy-competition.

5. Ex-post analysis

The M5 competition used a two-phase testing strategy consisting of the validation and evaluation phases. In the evaluation phase, no actual future time series values were available to the public during the competition. After the competition, the optimal value of the bias multiplier \( \lambda \) could be determined by looking at the overall WRMSSE metric in Equation 4. We notice that the intervals that contain the optimal multiplier are different in the two phases.

We now check the alignment performance in terms of forecasting errors with different bias multipliers for the validation time frame with the disclosed actual future values. Figure 2 shows the forecasting errors (WRMSSE) at all hierarchical levels based on the bottom level of LightGBM forecasts. One can observe that (i) the lower levels, which also contain larger portions of data, produce higher forecasting
Figure 1: Visualizing the N-BEATS forecasts (blue) and the bottom-up aggregated forecasts (orange) on top five levels during the evaluation phase of the competition.
errors compared to them at upper levels; (ii) the overall WRMSSE on the validation time frame reaches a minimum of 0.5291 when $\lambda = 1.16$. The loss multiplier $\lambda$ was searched in the space of $(0, 2]$, and the WRMSSE is close to the result at the evaluation time frame with WRMSSE = 0.52816. This indicates that by alternating the tuning parameter $\lambda$ in the customized gradient function, one may lose some accuracy at the bottom level, but the upper levels’ accuracy has significantly improved. As a result, the overall accuracy is improved.

6. Conclusion

This paper proposes a hierarchical-forecasting-with-alignment approach that focuses on improving the point forecast accuracy for upper levels by aligning the high accuracy top level forecasts with the aggregated bottom level forecasts in a hierarchical time series setting. The proposed top-down alignment approach ensures low forecasting errors on the upper levels of the hierarchy and improves the overall forecasting performance for an equally weighted metric like WRMSSE. Our research sheds light on an orthogonal direction for forecasting reconciliation, as suggested in e.g., Wickramasuriya et al. (2019), allowing some suboptimal forecasts at the lower level while retaining the accuracy on upper levels.

The hierarchical forecasting with alignment approach is straightforward to implement in practice. Improving overall forecasting performance requires accurate forecasting on the top level of the hierarchy. We employ the state-of-the-art deep learning forecasting approach N-BEATS for continuous time series
at the top levels and a widely used tree-based algorithm LightGBM with non-time series features for
the bottom level intermittent time series. Both methods are easy to use and effortless to scale up with
massive time series. It is worth mentioning that the presented framework is general, and one could easily
replace N-BEATS and LightGBM with other appropriate forecasting algorithms.

One notable difference compared to other approaches in the M5 competition is that the approach
focuses on improving the forecasting accuracy on the continuous upper levels. We do not directly take the
forecasting accuracy from bottom level intermittent time-series as our central attention, and the bottom
level forecasts are treated as mutable to ensure the hierarchical alignment. However, special combination
techniques e.g., Kang et al. (2021) could improve the accuracy of intermittent time series forecasting.

Although we focus on the point forecast in this paper, the probabilistic forecast with the presented
scheme should be straightforward to implement with a corresponding probabilistic loss function. Another
direction for future research is finding the best combination of top-level and bottom-level models. At
the moment, the forecasting for the upper levels and the bottom level is done independently. To utilize
the information across hierarchical levels, we could consider a joint modeling scheme together with the
alignment approach, or alignment with multiple levels in the future study. Combining the top-down
alignment with other reconciliation methods is also possible but needs further investigation.

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