Diffeomorphism invariant eigenvalue problem for metric perturbations in a bounded region

Valeri N. Marachevsky and Dmitri V. Vassilevich *

Department of Theoretical Physics, St.Petersburg University,
198904 St.Petersburg, Russia

Abstract

We suggest a method of construction of general diffeomorphism invariant boundary conditions for metric fluctuations. The case of $d + 1$ dimensional Euclidean disk is studied in detail. The eigenvalue problem for the Laplace operator on metric perturbations is reduced to that on $d$-dimensional vector, tensor and scalar fields. Explicit form of the eigenfunctions of the Laplace operator is derived. We also study restrictions on boundary conditions which are imposed by hermiticity of the Laplace operator.
1 Introduction

One of the main problems in quantum cosmology is to find a suitable set of boundary conditions for graviton perturbations (see monograph [1] and review [2]). The contribution of the so-called physical degrees of freedom to the one-loop prefactor of the wave function of the Universe was first evaluated by Schleich [3]. However, it is generally accepted now that the contribution of other (“non-physical”) components of the metric perturbations does not cancel that of the ghost fields [4, 5, 6, 7]. This means that to define the one-loop prefactor one needs boundary conditions for all components of the metric perturbations and ghosts. One possible choice is the Luckock–Moss–Poletti [8] mixed boundary conditions. These boundary conditions are local and they ensure hermiticity of the Laplace operator. However, these boundary conditions are not completely gauge invariant. Recently it was understood [3, 4] that for non totally geodesic boundary locality contradicts gauge invariance. Hence the non-local gauge invariant boundary conditions suggested by Barvinsky [10] look naturally. However, hermiticity of the Laplace operator for these boundary conditions was not proved. The same is true for another set [6] of non-local boundary conditions. Local gauge invariant boundary conditions were suggested [4] for 2d gravity with dynamical torsion, but it is not clear whether this result can be extended to higher dimensions.

The purpose of the present work is to define most general diffeomorphism invariant boundary conditions for the metric perturbations, to find eigenfunctions of the Laplace operator on a disk and to study restrictions imposed by hermiticity of the Laplacian. We propose to impose boundary conditions independently on gauge-fixed fields and gauge transformations. This gives a large family of gauge invariant boundary conditions (section 2). Next we reduce systematically the gauge-invariant eigenvalue problem for metric perturbations to lower spin problems. For covariant gauge conditions on a disk we find the eigenfunctions explicitly (sec. 3). As a last step (sec. 4), we consider the hermiticity condition for the Laplace operator on transverse traceless metric perturbations. All boundary conditions leading to hermitian Laplace operator are specified. A part of this boundary conditions has very unusual form. In Sec. 5 we discuss the results.
2 General construction

The Einstein general relativity being formulated as a theory of dynamical metric field exhibits the diffeomorphism invariance. The infinitesimal diffeomorphism transformations with the parameter $\xi_\mu$ act on metric fluctuations $h_{\mu\nu}$ as follows:

$$h_{\mu\nu} \rightarrow h_{\mu\nu} + (L\xi)_{\mu\nu}, \quad (L\xi)_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$$ (1)

where $\nabla$ is covariant derivative with respect to background metric $g_{\mu\nu}$. Throughout this paper we suppose that $g_{\mu\nu}$ is flat.

Consider boundary conditions for $h$:

$$B h|_{\partial M} = 0$$ (2)

with some boundary operators $B$. The boundary conditions (2) are diffeomorphism invariant if there is a boundary operator $B_\xi$ such that

$$BL\xi|_{\partial M} = 0$$ (3)

provided $\xi$ satisfies

$$B_\xi \xi|_{\partial M} = 0$$ (4)

This means that diffeomorphism transformations map the functional space defined by eq. (2) onto itself. For example, for the Barvinsky boundary conditions [10] the boundary operator is $B_\xi = 1$, which gives Dirichlet boundary conditions for $\xi$ and, consequently, for ghosts.

One can construct general diffeomorphism invariant boundary conditions in the following way. Consider a linear background gauge condition $\chi$. Denote the metric fluctuations satisfying the equation $\chi(h) = 0$ by $h^\perp$. For any $h$ there is a unique decomposition

$$h = h^\perp + L\nu$$ (5)

with complementary non-local projectors $P^\perp$ and $P^L$ such that $h^\perp = P^\perp h$, $L\nu = P^L h$. The following boundary operator

$$B h = B^\perp P^\perp h + B_\xi L^{-1} P^L h$$ (6)
defines diffeomorphism invariant boundary conditions for arbitrary $B^\perp$ and $B_\xi$. Note that $L$ is invertible on $P^L h$. In other words, the boundary conditions (5) are imposed independently on gauge-fixed components $h^\perp$ and pure gauge degrees of freedom.

Up to this point, gauge condition $\chi$ was arbitrary. We shall use the relativistic gauge condition

$$\chi(h)_\mu = \nabla^\nu h_{\mu\nu} - \alpha \nabla_\mu h^\nu$$  \hspace{1cm} (7)

with a real parameter $\alpha$. This choice is most convenient because the Laplace operator $\Delta = \nabla^\mu \nabla_\mu$ commutes with the projectors $P^\perp$ and $P^L$, and

$$\Delta : \{h^\perp\} \rightarrow \{h^\perp\}, \quad \Delta Lv = L\Delta v$$  \hspace{1cm} (8)

This means that one can consider separately the eigenvalue problem for the Laplace operator on the spaces $h^\perp$ and $Lv$. The Laplace operator in the latter space is reduced to the vector Laplace operator, which was studied in literature in detail (see e.g [1, 8]). Furthermore, $h^\perp$ can be decomposed in transverse traceless fluctuations $h^{TT}$ and a part depending on a scalar field:

$$h^\perp_{\mu\nu} = h^{TT}_{\mu\nu} + h^\perp(\sigma), \quad h^\perp(\sigma) = \left(g_{\mu\nu} \frac{(1 - \alpha)\Delta}{\alpha(d + 1) - 1} + \nabla_\mu \nabla_\nu\right) \sigma$$  \hspace{1cm} (9)

$h^{TT}_{\mu\nu} = 0, \nabla^\mu h^{TT}_{\mu\nu} = 0$. The decomposition (9) commutes with the Laplace operator:

$$\Delta : \{h^{TT}\} \rightarrow \{h^{TT}\}, \quad \Delta h^\perp(\sigma) = h^\perp(\Delta \sigma)$$  \hspace{1cm} (10)

Thus the eigenvalue problem for the Laplace operator on $h^\perp$ is reduced to the eigenvalue problem on $h^{TT}$ and relatively simple scalar eigenvalue problem.

In the following section we shall study the eigenvalue equation for the Laplace operator on $h^{TT}$.

### 3 Transverse traceless metric perturbations on a disk

Consider $d + 1$ dimensional unit disk $M$ with the metric

$$ds^2 = dr^2 + r^2 d\Omega^2, \quad 0 \leq r \leq 1,$$  \hspace{1cm} (11)
where \(d\Omega^2\) is the metric on unit sphere \(S^d\). We shall need explicit expressions for the Laplace operator acting on scalar field \(\phi\), vector \(A_\mu\) and rank two symmetric tensor \(h_{\mu\nu}\):

\[
\Delta \phi = (\partial_\nu^2 + \frac{d}{r} \partial_\nu + (d) \Delta) \phi \tag{12}
\]

\[
(\Delta A)_0 = (\partial_\nu^2 + \frac{d}{r} \partial_\nu + (d) \Delta - \frac{d}{r^2}) A_0 - \frac{2}{r} (d) \nabla^i A_i,
\]

\[
(\Delta A)_i = (\partial_\nu^2 + \frac{d-2}{r} \partial_\nu + (d) \Delta - \frac{d-1}{r^2}) A_i + \frac{2}{r} \partial_i A_0 \tag{13}
\]

\[
(\Delta h)_{00} = (\partial_\nu^2 + \frac{d}{r} \partial_\nu + (d) \Delta - \frac{2d}{r^2}) h_{00} - \frac{4}{r} (d) \nabla_i h^i_0 + \frac{2}{r^2} h^i_0
\]

\[
(\Delta h)_{0i} = (\partial_\nu^2 + \frac{d-2}{r} \partial_\nu + (d) \Delta - \frac{2d+1}{r^2}) h_{0i} + \frac{2}{r} \partial_i h_{00} - \frac{2}{r} (d) \nabla^k h_{ik}
\]

\[
(\Delta h)_{ik} = (\partial_\nu^2 + \frac{d-4}{r} \partial_\nu + (d) \Delta - \frac{2d-4}{r^2}) h_{ik} + \frac{2}{r} (d) \nabla_i h_{0k} + (d) \nabla_k h_{0i} + \frac{2}{r^2} g_{ik} h_{00} \tag{14}
\]

where \((d)\nabla\) and \((d)\Delta\) are the \(d\)-dimensional covariant derivative and the Laplace operator with respect to \(d\)-dimensional metric \(g_{ik}\), respectively.

Let us expand \(h_{00}\), \(h_{0i}\) and \(h_{ik}\) in sums of irreducible harmonics on \(S^d\):

\[
h_{00} = s_1
\]

\[
h_{0i} = \partial_i s_2 + u_{1,i}
\]

\[
h_{ik} = (d) \nabla_i u_{2,k} + (d) \nabla_k u_{2,i} + (d) \nabla_i (d) \nabla_k - \frac{1}{d} g_{ik} (d) \Delta) s_3
\]

\[
- \frac{1}{d} g_{ik} s_1 + h^i_{ik}
\]

Here \(s_A\) are scalar fields, \(u_{A,i}\) are transversal vectors, \((d)\nabla^i u_{A,i} = 0\), and \(h^i_{ik}\) is transversal traceless tensor, \(h^i_{ij} g^{ij} = 0\), \((d)\nabla^i h^j_{ik} = 0\). In the expansions \([15]\) we used the fact that the \(d + 1\) dimensional trace of \(h_{\mu\nu}\) is zero. Up to some redefinitions the representation \([13]\) is the same as was used by other authors \([1]\). Here however we shall apply it to transversal fields only. The \(d + 1\) dimensional transversality condition, \(\nabla^\mu h_{\mu\nu} = 0\), reads:

\[
\nu = 0 : \quad 0 = (d) \nabla^i h_{0i} + (\partial_0 + \frac{d}{r}) h_{00} - \frac{1}{r} h^i_i
\]
Due to the equations (16) the fields $s_A$ and $u_A$ can be expressed via single independent function of each kind. General solution of (16) has the following form:

$$h_{T T}^{\mu \nu} = h_{\mu \nu}^{tt} + h_{\mu \nu}(u) + h_{\mu \nu}(s)$$

(17)

$$h_{0i}(u) = -(\frac{d}{r^2} + \frac{d - 1}{r})ru_i$$

$$h_{ik}(u) = r(\partial_0 + \frac{d - 1}{r})(\frac{d}{r} \nabla_i u_k + \frac{d}{r} \nabla_k u_i)$$

(18)

$$h_{00}(s) = -(d - 1)(\frac{d}{r^2} + \frac{d}{r})r^2s;$$

$$h_{0i}(s) = -(d - 1)\partial_i(\partial_0 + \frac{d - 1}{r})(\frac{d}{r} \Delta + \frac{d}{r^2})r^2s;$$

$$h_{ik}(s) = (\frac{d}{r} \nabla_i \nabla_k - \frac{1}{d} g_{ik})(\frac{d}{r} \Delta)(d\partial_0^2 + \frac{2d^2 - 5d}{r^2} \partial_0 + \frac{d(d - 3)}{r^2})$$

$$+ (\frac{d}{r} \Delta)r^2s - \frac{d - 1}{d} g_{ik}(\frac{d}{r} \Delta)(\frac{d}{r} \Delta + \frac{d}{r^2})r^2s.$$  

(19)

The components $h_{00}^{tt}$, $h_{0i}^{tt}$, $h_{00}(u)$ vanish identically.

Using the explicit expressions (12), (13) and (14) one can demonstrate by straightforward computations that

$$\Delta h_{\mu \nu}(u_i) = h_{\mu \nu}(\Delta u_i), \quad \Delta h_{\mu \nu}(s) = h_{\mu \nu}(\Delta s).$$

(20)

We see, that the eigenfunctions of the Laplace operator $\Delta$ on the space of transverse traceless tensors can be defined through the eigenfunctions of the same operator on the so called physical components $s$, $u$ and $h^{tt}$ of scalar, vector and tensor fields. The spectrum of the Laplace operator on the latter components is well known (see e.g. monograph [1]). Let us represent $h^{tt}$, $u$ and $s$ as Fourier series of hyperspherical harmonics on $S^d$:

$$h_{ik}^{tt}(x^{\mu}) = \sum_{(l)} H_{(l)}(r) Y_{ik}^{tt(l)}(x^i),$$

$$u_k(x^{\mu}) = \sum_{(l)} U_{(l)}(r) Y_k^{u(l)}(x^i),$$

$$s(x^{\mu}) = \sum_{(l)} S_{(l)}(r) Y_s^{s(l)}(x^i).$$

(21)
The spectrum of the $d$-dimensional Laplace operator on these harmonics is well known [11]:

\[
^{(d)} \Delta Y^{s(l)}(x_i) = -\frac{1}{r^2} l(l + d - 1) Y^{s(l)}(x_i), \quad l \geq 0
\]

\[
^{(d)} \Delta Y^{u(l)}_{k}(x_i) = -\frac{1}{r^2} [l(l + d - 1) - 1] Y^{u(l)}_{k}(x_i), \quad l \geq 1
\]

\[
^{(d)} \Delta Y^{tt(l)}_{jk}(x_i) = -\frac{1}{r^2} [l(l + d - 1) - 2] Y^{tt(l)}_{jk}(x_i) \quad l \geq 2
\]

(22)

The corresponding degeneracies are

\[
D_{s}^{l} = \frac{(2l + d - 1)(l + d - 2)!}{l!(d - 1)!}
\]

\[
D_{u}^{l} = \frac{l(l + d - 1)(2l + d - 1)(l + n - 3)!}{(d - 2)!(l + 1)!}
\]

\[
D_{tt}^{l} = \frac{(d + 1)(d - 2)(l + d)(2l + d - 1)(l + d - 3)!}{2(d - 1)!(l + 1)!}
\]

(23)

Note, that the $d + 1$-dimensional Laplace operators (13) and (14) map the spaces of $d$-dimensional transverse vectors and transverse traceless tensors onto themselves. The eigenvalue equations for the $d + 1$ dimensional Laplace operator $\Delta$ are reduced to ordinary differential equations for the functions $H_{l}(t), U_{l}(t)$ and $S_{l}(t)$. One can easily find the eigenfunctions:

\[
 s_{l,\lambda} \propto r^{(1-d)/2} J_{(d-1)/2+l}(r\lambda)Y^{s(l)}(x^{i}),
\]

\[
 u_{l,\lambda k} \propto r^{(3-d)/2} J_{(d-1)/2+l}(r\lambda)Y^{u(l)}(x^{i}),
\]

\[
 h_{tt}^{l,\lambda k} \propto r^{(5-d)/2} J_{(d-1)/2+l}(r\lambda)Y^{tt(l)}(x^{i}),\quad l = 2, 3, 4, ...
\]

(24)

The values $l = 0, 1$ are excluded because the corresponding harmonics generate zero modes of the mappings $s \rightarrow h^{TT}$ and $u \rightarrow h^{TT}$. The eigenvalues $-\lambda^2$ of the Laplace operator $\Delta$ are defined by boundary conditions. Degeneracies of each eigenvalue are given by (23).

In order to find gauge invariant boundary conditions for the metric fluctuations on a disk one should choose some (arbitrary) boundary conditions for the fields $v, \sigma, s, u$ and $h^{tt}$ entering the decompositions (3), (4) and (17). The equations (8), (10), (20), (24) define the spectrum of the Laplace operator.
4 Selfadjointness analysis

Let us find the boundary conditions for $h^{TT}$ which lead to selfadjoint Laplacian. We shall use ordinary inner product without surface terms in the space of rank two symmetric tensor fields:

$$<h', h> = \int d^{d+1}x \sqrt{g} g^{\mu \nu} g^{\eta \kappa} h'_\mu h_{\nu \kappa}$$

(25)

The selfadjointness means that $<h', \Delta h> = <\Delta h', h>$ provided $h'$ and $h$ satisfy some boundary conditions. This is equivalent to vanishing of the following surface integral

$$\int_{\partial M} g^{\mu \nu} g^{\eta \kappa} (h'_\mu \partial_0 h_{\nu \kappa} - h_{\mu \eta} \partial_0 h'_{\nu \kappa}) = 0$$

(26)

Due to orthogonality of the tensor harmonics on $S^d$ the equation (26) should be satisfied by all fields $s$, $u$ and $h^{tt}$ and all values of $l$ in the decompositions (19) and (21) independently. Let us suppose that the boundary conditions are $SO(d)$ invariant. Thus the equation (26) generates some restrictions on boundary conditions for the coefficient functions $H(l)$, $U(l)$ and $S(l)$ for any $l$.

Consider first the $h^{tt}$ fluctuations. The integral (26) vanishes if $H(l)$ satisfies one of the following boundary conditions:

$$H(l)|_{\partial M} = 0 \quad \text{or} \quad (\partial_0 + C^{tt}_l) H(l)|_{\partial M} = 0$$

(27)

with arbitrary constants $C^{tt}_l$. The boundary conditions (27) are very general. This means that the boundary conditions for the so called physical degrees of freedom are not restricted by the invariance or selfadjointness requirements and are defined by physics only.

Consider now the $h(u)$ fluctuations. One can check the equation (26) on the eigenfunctions (21), (24). Namely, we can suppose that $u$ and $u'$ correspond to some fixed eigenvalues of $\Delta$ and $r^2 (d) \Delta$. Denote the corresponding quantum numbers as $\lambda$, $l$, $\lambda'$ and $l'$. Due to orthogonality of spherical harmonics the integral in (26) vanishes identically for $l' \neq l$. Hence we put $l' = l$. Consider first the Neumann boundary conditions for $U(l)$:

$$(\partial_0 + C^u_l) U(l)|_{\partial M} = 0$$

(28)

\footnote{Strictly speaking, this equation means that the Laplacian is symmetric. For selfadjointness, one should also require that an operator and its adjoint have coinciding domains of definition.}
To preserve the $SO(d)$ invariance of the boundary conditions $C^u_l$ should be constant. By substituting (28) in (26) we obtain the following condition

$$(\lambda^2 - \lambda'^2)(C^u_l - d + 1)(l(l + d - 1) - d) \int_{\partial M} d^d x \sqrt{(d)} g^{ij} u^i u'^j = 0$$  \hspace{1cm} (29)$$

Since $\lambda$ and $\lambda'$ have, in general, different values, the condition (29) implies that $C^u_l = d - 1$. One can also check that (26) holds if $u_i$ satisfies Dirichlet boundary conditions. Thus to give selfadjoint Laplace operator, $U(\ell)$ should satisfy one of the two following boundary conditions:

$$U(\ell)|_{\partial M} = 0 \text{ or } (\partial_{\nu} + d - 1)U(\ell)|_{\partial M} = 0$$ \hspace{1cm} (30)$$

The case of $h(s)$ can be analyzed along the same lines. We obtain the Laplace operator is selfadjoint only for Neumann boundary conditions:

$$(\partial_{\nu} + C^s_l)S(\ell)|_{\partial M} = 0$$

$$C^s_l = d - 1 + \frac{1}{d}(\Lambda_l \pm \sqrt{\Lambda^2_l - d\Lambda_l})$$ \hspace{1cm} (31)$$

where $\Lambda_l$ are eigenvalues of $-r^2 \, (d) \Delta$. These boundary conditions (31) are non-local and even non polynomial in tangential derivatives. However, due to the fact that the conditions (31) contain only first order normal derivatives, they define a well posed boundary problem for the Laplace operator. One can see this directly by using the decomposition (21) and analyzing ordinary differential equations for $S(\ell)$. This completes our analysis of boundary conditions for $h^{TT}$ leading to symmetric Laplace operator.

Strictly speaking, to use spectral representation for the Laplace operator in the path integral one should demonstrate selfadjointness. This might be done by using one of the criteria of monograph [12]. We postpone this tedious task to a future publication where we shall analyze path integral over metric perturbations. In any case, for boundary conditions other than listed here the Laplace operator is not symmetric and, hence, is not selfadjoint.

Boundary conditions for other components of the metric fluctuations can be analyzed along the same lines. Consider the gauge (7) with $\alpha \to (d+1)^{-1}$. Instead of $h^\perp(\sigma)$ it is convenient to introduce $h^\perp(\omega) = g_{\mu\nu}\omega(\omega)$. One can easily demonstrate that the surface integral (26) vanishes for Dirichlet boundary conditions as well as for Neumann boundary conditions with arbitrary
constant. Hence at least in this gauge selfajointness does not mean any restrictions on the trace part of metric fluctuations. In this section we will not consider the pure gauge part $L v$, because this fluctuations represent zero modes of the gravitational action.

5 Discussion and conclusions

In order to describe a physical theory the boundary conditions for metric fluctuations should satisfy some requirements. Let us discuss them in brief.

(i) Some part of boundary conditions is fixed by physics. Usually, these are the boundary conditions for the so called physical degrees of freedom, which correspond to $h^{tt}$ in our notations. Fortunately, these boundary conditions are not fixed by selfconsistency conditions considered in the present paper.

(ii) Diffeomorphism invariance. This is the basic invariance of gravitation. This property should be preserved to obtain meaningful quantum theory. If boundary conditions are not gauge invariant, than, for example, on-shell effective action becomes gauge-dependent \[13\]. One can satisfy the diffeomorphism invariance in a relatively simple way. One can choose the condition (6) with arbitrary conditions for the gauge-fixed part $h^\perp$ and gauge transformations. Furthermore, the eigenvalue problem for the Laplace operator on metric perturbations is reduced to eigenvalue problems for $h^{tt}$ and several $d$-dimensional vectors and scalars. On a disk, explicit form of the eigenfunctions was obtained.

(iii) The Laplace operator should be Hermitian with respect to inner product (25) without surface terms. Only if an operator $K$ is Hermitian with respect to an inner product $<\ ,\ >$, the path integral

$$\int D\phi \exp(-<\phi, K\phi>)$$

is defined by product of eigenvalues of $K$. We considered the Laplace operator and the inner product (25) because, first, they are the most simple choice, and second, because this construction is used in all actual computations, though sometimes implicitly. The boundary conditions (31) for $h^{TT}(s)$ are somewhat unusual. May be these boundary conditions do nevertheless admit a nice geometric interpretation. It could happen as well that a more careful
consideration of boundary terms in the action will modify drastically the results of the previous section.

Unfortunately, complicated form of the boundary conditions (31) prevents us from using powerful methods of evaluation of functional determinants designed recently [14, 15, 16, 17, 18].

Another way to determine proper boundary conditions may be to introduce arbitrary surface coupling and look for the renormalization group fixed points in the sense of recent works [19].

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