Electromagnetic Dark Energy and Gravitoelectrodynamics of Superconductors

Clovis Jacinto de Matos*

February 1, 2008

Abstract

It is shown that Beck and Mackey electromagnetic model of dark energy in superconductors can account for the non-classical inertial properties of superconductors, which have been conjectured by the author to explain the Cooper pair’s mass excess reported by Cabrera and Tate. A new Einstein-Planck regime for gravitation in condensed matter is proposed as a natural scale to host the gravitoelectrodynamic properties of superconductors.

1 Introduction

We start from the unsolved problem of the Cooper pairs excess of mass. A conjecture involving a gravitomagnetic London moment in superconductors to account for this anomaly is reviewed. Afterwards the relation of this new phenomena in superconductors with the spontaneous breaking of the principle of general covariance and its relation with the problem of the graviton mass and of dark energy is briefly discussed. The link with Beck’s electromagnetic model for dark energy in superconductors is then established. Assuming that different superconductive materials host different vacuum energy densities allows to account very accurately for the conjectured gravitomagnetic fields in superconductors. It appears that this new phenomena takes place at the Einstein-Planck scale defined from the five fundamental constants of nature: $c, \hbar, G, k, \Lambda$. Unexpectedly this seems to be the natural scale for an intermediate regime of quantum-gravity.

*ESA-HQ, European Space Agency, 8-10 rue Mario Nikis, 75015 Paris, France, e-mail: Clovis.de.Matos@esa.int
2 Cooper pairs mass excess

In 1989 Cabrera and Tate, through the measurement of the London moment, reported an anomalous Cooper pair mass excess in thin rotating Niobium superconductive rings.

\[
\frac{m^* - m}{m} = \frac{\Delta m}{m} = 9.2 \times 10^{-5} \tag{1}
\]

where \( m^* = 1.82203 \times 10^{-30} \text{ Kg} \) is the Cooper Pair mass experimentally measured, and \( m = 1.82186 \times 10^{-30} \text{ Kg} \) is the theoretical mass of Cooper pairs including relativistic corrections [1][2].

In an attempt to explain this anomalous excess of mass the author conjectured that the Cooper pairs mass do not increase but that instead an additional gravitomagnetic London-type moment must be taken into account in the quantization of the Cooper pairs canonical momentum [3][4].

\[
B_g = \frac{\Delta m}{m} 2\omega = 1.84 \times 10^{-4} \omega \tag{2}
\]

Where \( \omega \) is the superconductor’s angular velocity and \( B_g \) is the gravitomagnetic field both expressed in \( \text{Rad/s} \). The Gravitomagnetic London moment received some preliminary experimental confirmation in dedicated experiments conducted at the Austrian Research Centres (ARC) in Seiersdorf [8][9]. In the rest of the text we will refer to the coupling between \( B_g \) and \( \omega \) in rotating superconductors as

\[
\chi = \frac{B_g}{\omega} = 2 \frac{\Delta m}{m} = 1.84 \times 10^{-4} \tag{3}
\]

First steps to understand this anomalous coupling, in the framework of a spontaneous breaking of the Principle of General Covariance (PGC) in superconductors, lead to consider a massive spin one graviton to convey the gravitoelectromagnetic interaction, and a set of Einstein-Maxwell-Proca equations to describe the gravitoelectrodynamics of superconductors [5][6][7]. Solving these equations for gravitomagnetic fields we find the gravitomagnetic London moment expressed as a function of a density of energy \( \rho^* \) contained in the superconductive ring, and of the square of the graviton Compton wavelength \( \lambda_g \).

\[
\chi = \frac{8\pi G}{c^4} \rho^* \lambda_g^2 \tag{4}
\]

Novello and others [10][11][12] proposed a link between the Cosmological Constant (CC), \( \Lambda \), and a massive graviton,

\[
\frac{1}{\lambda_g^2} = \left( \frac{m_g c}{\hbar} \right)^2 = \frac{2}{3} \Lambda \tag{5}
\]
coming as a natural consequence of the equations of motion of a massive graviton propagating in a de-Sitter background. On the other side a non-vanishing cosmological constant can be interpreted in terms of a non-vanishing vacuum energy called dark energy.

$$\rho_{\text{vac}} = \frac{c^4}{8\pi G}\Lambda \quad (6)$$

The small experimental value of the CC $\Lambda = 1.29 \times 10^{-52}[1/m^2]$ [13] and its origin remain a deep mystery. This is often call the CC problem, since at Planck scale the vacuum energy density should be of the order of $10^{120}$, in complete contradiction with measured value. Putting Equ.(5) and Equ.(6) into Equ.(4) and rearranging we get:

$$\chi = \frac{3}{2} \frac{\rho^*}{\rho_{\text{vac}}} \quad (7)$$

From Equ.(7) we formulate the hypothesis that $\rho^*$ corresponds to the density of dark energy contained in a given superconductor. It should be stressed that this hypothesis deviates significantly from the initial attempt in [6] to understand the gravitomagnetic London moment, in which $\rho^*$ was considered as being the Cooper pairs mass density [14].

### 3 Electromagnetic Dark Energy in Superconductors

To solve the CC problem Beck and Mackey proposed a model of dark energy based on electromagnetic vacuum fluctuations creating a small amount of vacuum energy density exactly equal to the cosmological vacuum energy density. They assume that in a superconductor virtual photons, with energy $\epsilon = \frac{1}{2}h\nu$, can exist in two different phases: A *gravitationally active* phase where they contribute to the dark energy density, and a *gravitationally inactive* phase where they do not contribute to the dark energy density [15][16][17][18]. The transition between the two phases is defined by a cutoff frequency, $\nu_c$. They constructed a Ginzburg-Landau type theory for the number density of gravitationally active photons in superconductors. In this way they obtain a finite dark energy density in superconducting materials dependent on a cutoff frequency $\nu_c$:

$$\rho^* = \frac{1}{2} \frac{\pi h}{c^3} \nu_c^4 \quad (8)$$
Equaling Equ. (8), where $\rho^*$ is the dark energy density in superconductors, to Equ. (6), where $\rho_{\text{vac}}$ is the density of dark energy in the Universe, the cosmological cutoff frequency in superconductors is estimated to be of the order of $\nu_c \simeq 2.01\,THz$. An experimental effort is currently undergoing at UCL and Cambridge to measure this cutoff frequency through the measurement of the spectral density of the noise current in resistively shunted Josephson junctions [19]. The formal attribution of a temperature $T$ to the virtual photons underlying dark energy is done by comparing their energy to the one of ordinary photons in a bath at temperature $T$.

$$\frac{1}{2}h\nu = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1}$$

This condition is equivalent to

$$h\nu = \ln 3kT$$

Using Equ. (10), the critical temperature $T_c$ in the Beck-Ginzburg-Landau model corresponds to a critical frequency

$$\nu_c = \ln 3\frac{kT_c}{\hbar}$$

Putting the cosmological cutoff frequency estimated above, $\nu_c = 2.01\,THz$ into Equ. (11) we find the associated critical temperature, $T_c = 87.49\,K$, which is characteristic of $High-T_c$ superconductors.

In the present work we consider that the critical temperature, $T_c$, characterizing a given superconductive material, will define its respective cutoff frequency, $\nu_c$, through Equ. (11), which in turn will determine the density of dark energy in the superconductor through Equ. (8). $T_c$ being different for different superconducting materials, $\nu_c(T_c)$ and $\rho^*(T_c)$, will accordingly adopt different values in different superconducting materials.

4 The Planck-Einstein Regime of Gravitation

Substituting Equ. (6), Equ. (8) and Equ. (11) into Equ. (7) we end with:

$$\chi = \frac{3\ln^4 3}{4\pi} \frac{k^4 G}{c^5 \hbar^3 \Lambda} T_c^4$$

Defining the Planck-Einstein temperature, $T_{PE}$

$$T_{PE} = \frac{1}{k} \left( \frac{c^5 \hbar^3 \Lambda}{G} \right)^{1/4} = 60.71\,K$$

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Equ.(12) can be written in the following form:

$$\chi = \frac{3 \ln^4 3}{4\pi} \left( \frac{T_c}{T_{PE}} \right)^4$$

(14)

As indicated above, in the present work, we release the constraint, with respect to Beck and Mackey initial model, that all superconductive materials host the same cosmological vacuum energy density. We are thus assuming that the cutoff frequency is directly proportional to the fourth power of the critical temperature of a given superconductor, and is different for different superconductive materials. A direct consequence of this assumption on Beck and Mackey initial model is that the critical temperature defining the gravitational activity of virtual photons could also be equal to the usual critical temperature characterizing the superconductive state. Thus substituting the critical transition temperature of Niobium, $T_c = 9.25K$, into Equ.(14) we find a coupling between the gravitomagnetic field and the angular velocity of a rotating superconductive Niobium ring:

$$\chi = 1.87 \times 10^{-4}$$

(15)

which is showing to be extremely close to the above conjectured coupling based on Cabrera and Tate’s measurements of the Cooper pairs mass in Niobium.

$$\chi = 2 \frac{\Delta m}{m} = 1.84 \times 10^{-4}$$

(16)

Let us now examine this Coupling for different superconductors starting with Aluminium and ending with High-$T_c$ superconductors like YBCO.

| Superconductive material | $T_c[K]$ | $\chi$ |
|--------------------------|----------|--------|
| $Al$                     | 1.18     | $4.96 \times 10^{-8}$ |
| $In$                     | 3.41     | $3.46 \times 10^{-6}$ |
| $Sn$                     | 3.72     | $4.90 \times 10^{-6}$ |
| $Pb$                     | 7.2      | $6.88 \times 10^{-5}$ |
| $Nb$                     | 9.25     | $1.87 \times 10^{-4}$ |
| $High - T_c$             | 79.06    | 1      |
| $BSCCO$                  | 87.5     | 1.5    |
| $YBCO$                   | 94       | 2      |

Table 1: Coupling between the gravitomagnetic field and the angular velocity, $\chi$, of different superconductive materials.
We note that for YBCO, with $T_c = 94K$, the gravitomagnetic London moment transforms exactly into the classical gravitational Larmor theorem [20]:

$$B_g = 2\omega$$

At this point one remark is in order: Our theoretical derivation presented in this paper strictly speaking holds only for conventional low-$T_c$ superconductors, because we are using simple Ginzburg-Landau models and BCS type arguments for both the superconductor and the dark energy model [18].

Before concluding it is interesting to note that the Planck-Einstein scale (involving the fundamental constants: $\Lambda, \hbar, c, k, G$) corresponds to the geometric mean between cosmological physics (involving the fundamental constants: $\Lambda, \hbar, c, k$) and high energy particle physics (involving the fundamental constants: $\hbar, c, k, G$), just in the scale domain relevant for condensed matter physics at low temperatures. It is unexpected to have a possible quantum regime of gravity at this scale [21][22][23].

|                         | Einstein scale | Planck-Einstein Scale | Planck scale |
|-------------------------|----------------|-----------------------|--------------|
| $\Lambda, \hbar, c, k$  | $\Lambda, \hbar, c, k G$ | $c, \hbar, k, G$       |
| Temperature [K]         | $T_E = \frac{1}{k} \sqrt{\frac{\hbar^2}{c^3}} \Lambda$ | $T_{PE} = \sqrt{T_E T_P}$ | $T_P = \frac{1}{k} \sqrt{\frac{\hbar c^5}{G}}$ |
| 2.95 $\times 10^{-39}$ | 60.71          | 1.42 $\times 10^{34}$ |
| Time [s]                | $t_E = \sqrt{\frac{1}{c^2}} \Lambda$ | $t_{PE} = \sqrt{t_E t_P}$ | $t_P = \frac{\hbar c^5}{G}$ |
| 2.58 $\times 10^{43}$  | 1.26 $\times 10^{-13}$ | 5.38 $\times 10^{-44}$ |
| Length [m]              | $l_E = \frac{1}{\Lambda}$ | $l_{PE} = \sqrt{l_E l_P}$ | $l_P = \frac{\hbar c^5}{G^2}$ |
| 8.8 $\times 10^{25}$   | 3.77 $\times 10^{-9}$ | 1.61 $\times 10^{-39}$ |
| Mass [Kg]               | $M_E = \sqrt{\frac{k^2 \Lambda}{c^4}}$ | $M_{PE} = \sqrt{M_E M_P}$ | $T_P = \sqrt{\frac{\hbar c^5}{G}}$ |
| 5.53 $\times 10^{-95}$ | 9.32 $\times 10^{-39}$ | 2.17 $\times 10^{-8}$ |
| Energy [J]              | $E_E = \sqrt{\frac{c^2 \hbar^2}{\Lambda}}$ | $E_{PE} = \sqrt{E_E E_P}$ | $E_P = \sqrt{\frac{\hbar c^5}{G^2}}$ |
| 4.07 $\times 10^{-78}$ | 8.38 $\times 10^{-22}$ | 1.96 $\times 10^9$ |
| Energy density [J/m$^3$] | $\rho_E = \sqrt{\frac{c^2 \hbar^2 \Lambda^4}{G}}$ | $\rho_{PE} = \sqrt{\rho_E \rho_P}$ | $\rho_P = \sqrt{\frac{14 \hbar^2}{G^4 \Lambda^4}}$ |
| 5.26 $\times 10^{-130}$ | 3.73 $\times 10^{-9}$ | 4.6 $\times 10^{113}$ |

Table 2: Cosmological, versus Planck-Einstein, versus Planck scales.

Explicitly one has the following formulas at the Planck-Einstein scale:

$$E_{PE} = kT_{PE} = \left(\frac{c^7 \hbar^3 \Lambda}{G}\right)^{1/4} = 5.25 [meV]$$

(18)

$$m_{PE} = \frac{E_{PE}}{c^2} = \left(\frac{\hbar^3 \Lambda}{cG}\right)^{1/4} = 9.32 \times 10^{-39} [Kg]$$

(19)
\[ l_{PE} = \frac{\hbar}{M_{PE}} = \left( \frac{\hbar G}{c^3 \Lambda} \right)^{1/4} = 0.037[mm] \] (20)

\[ t_{PE} = \frac{l_{PE}}{c} = \left( \frac{\hbar G}{c^3 \Lambda} \right)^{1/4} = 1.26 \times 10^{-13}[s] \] (21)

\[ \rho_{PE} = \frac{E_{PE}}{l_{PE}^3} = \frac{c^4 \Lambda}{G} = 104[eV/mm^3] \] (22)

One readily notices that the numerical values of Planck-Einstein quantities correspond to typical time, length or energy scales in superconductor physics, as well as to typical energy scales for dark energy.

5 Conclusions

In conclusion, Table 1 shows that the effective laws of inertia in superconducting cavities deviate from the laws of classical mechanics, recovering however the classical regime in the limit of YBCO cavities. Above, \( T_c = 94K \), it is not clear if the classical gravitational Larmor theorem is affected as indicated in our model. To investigate this interpretation, from an experimental point of view, it is recommended to probe classical Coriolis forces on test masses moving inside rotating superconductive cavities.

The non-classical laws of inertia in superconductive cavities arise at the Einstein-Planck regime of gravitation, which appears to correspond to scales relevant in the domain of condensed matter physics at low temperatures. It is worth noting that the non-classical rotational inertia exhibited by supersolids could be another experimental evidence that quantum materials contain a dark energy density inferior to its cosmological value [24].

Important questions, related to the proposed approach to dark energy in superconductors, are open and deserves further investigation: What couples (decouples) electromagnetic vacuum energy to (from) gravity? Is Planck-Einstein physics only of interest to superconductor’s physics, or is it also relevant in other domains of condensed matter physics?

6 Acknowledgements

The author would like to thank Prof. Christian Beck, for useful correspondence on the subject of the present paper, and for developing, together with Prof. M. C. Mackey, the concept of electromagnetic dark energy in superconductors.
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