Reply to Comment on ‘Encoding many channels on the same frequency through radio vorticity: first experimental test’

F Tamburini$^{1,2,8}$, B Thidé$^{3,4}$, E Mari$^5$, A Sponselli$^1$, A Bianchini$^1$ and F Romanato$^{6,7}$

$^1$Department of Physics and Astronomy, University of Padua, Vicolo dell’Osservatorio 3, IT-35122 Padua, Italy
$^2$CIVEN, Via delle Industrie 5, Torre Hammon IT-30175, Venezia-Marghera, Italy
$^3$IRF, Swedish Institute of Space Physics, Box 537, Ångström Laboratory, SE-75121 Uppsala, Sweden
$^4$Scuola Galileiana di Studi Superiori, University of Padua, via VIII Febbraio 1848, IT-35122 Padua, Italy
$^5$CISAS, University of Padua, via Venezia 15, IT-35131 Padua, Italy
$^6$Department of Physics and Astronomy, University of Padua, via Marzolo 8, IT-35100 Padua, Italy
$^7$LaNN, Laboratory for Nanofabrication of Nanodevices, Venetonanotech, via Stati Uniti 4, IT-35100 Padua, Italy

E-mail: fabrizio.tamburini@gmail.com

New Journal of Physics 14 (2012) 118002 (8pp)
Received 13 April 2012
Published 7 November 2012
Online at http://www.njp.org/
doi:10.1088/1367-2630/14/11/118002

Abstract. Our recent paper (Tamburini et al 2012 New J. Phys. 14 033001), which presented results from outdoor experiments that demonstrate that it is physically feasible to simultaneously transmit different states of the newly recognized electromagnetic (EM) quantity orbital angular momentum (OAM) at radio frequencies into the far zone and to identify these states there, has led to a comment (Tamagnone et al 2012 New J. Phys. 14 118001).

$^8$Author to whom any correspondence should be addressed.

Content from this work may be used under the terms of the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
These authors discuss whether our investigations can be regarded as a particular implementation of the multiple-input–multiple-output (MIMO) technique. Clearly, our experimental confirmation of a theoretical prediction, first made almost a century ago (Abraham 1914 Phys. Z. XV 914–8), that the total EM angular momentum (a pseudovector of dimension length × mass × velocity) can propagate over huge distances, is essentially different from—and conceptually incompatible with—the fact that there exist engineering techniques that can enhance the spectral capacity of EM linear momentum (an ordinary vector of dimension mass × velocity). Our OAM experiments (Tamburini et al 2012 New J. Phys. 14 033001; Tamburini et al 2011 Appl. Phys. Lett. 99 204102–3) confirm the availability of a new physical layer for real-world radio communications based on EM rotational degrees of freedom. The next step is to develop new protocols and techniques for high spectral density on this new physical layer. This includes MIMO-like and other, more efficient, techniques.

Contents

1. Introduction 2
2. Field behaviour at far-zone distances 4
   2.1. Explicit derivation of the $r^{-2}$ angular momentum density fall-off at large distances from an arbitrary radiation source 5
3. Conclusions 7
Acknowledgment 8
References 8

1. Introduction

Our outdoor experiments in Padua and Venice, Italy, in June 2011, aimed at investigating certain fundamental physical properties of the electromagnetic (EM) field at radio frequencies and their behaviour in a realistic real-world setting [1], demonstrated that the newly recognized physical observable EM orbital angular momentum (OAM) [5–10]

(i) can propagate and be identified over long distances also in radio, and
(ii) provides a new physical layer that has the capability to simultaneously transfer independent information wirelessly before applying specific communication protocols or spectral density enhancement techniques.

This is due to the fact that angular momentum is a conserved physical observable that has different physical properties (a pseudovector of dimension length × mass × velocity) and can therefore be used for communication purposes other than the conserved quantity linear momentum (an ordinary vector of dimension mass × velocity) that is used today [11–23].

In equations (1) and (2) of their comment, Tamagnone et al [2] model a multiple-input–multiple-output (MIMO) setup, referring to the number of independent linear momentum modes. In other words, a standard multiport/multimode approach using $N_{\text{ant}}$ antennas (linear momentum modes). This MIMO technique gives at most $\log_2 N_{\text{ant}}$ bits per photon. If, instead,
one uses the same number $N_{\text{ant}}$ of antennas to transfer information by utilizing angular momentum [24], i.e. a multistate approach, one will be able to produce beams with up to $N_{\text{ant}}$ bits per photon. Already this fact clearly illustrates that there is a fundamental, and essential, difference between the linear momentum MIMO approach, based on the utilization of translational EM degrees of freedom, and the new angular momentum (OAM) approach, based on the utilization of the rotational EM degrees of freedom.

In our experiments [1], the linearly polarized $l = 0$ OAM mode was transmitted by a Yagi antenna, whereas the likewise linearly polarized $l = 1$ OAM eigenstate mode was transmitted by a single, specially prepared, parabolic antenna. The fact that it was shown a few years ago [24] that it is also possible to use arrays of conventional linear momentum producing antennas to generate approximately pure $l \neq 0$ OAM eigenstates has nothing to do with the MIMO technique. MIMO is a transmission–reception enhancement technique based entirely on the manipulation of linear momentum and polarization and is not a new fundamental physical principle or a new physical layer [25]. Polarization and linear momentum have no direct relationship with OAM. And interferometry, as used in our Padua and Venice experiments [1], is a standard measurement method in experimental physics that predates the MIMO technique by many decades.

Hence, the alternative transmission scheme with two Yagi antennas suggested to mimic our physics experiment [1] (illustrated in figure 1 of the comment [2]), which amounts to transmitting and receiving two linear momentum states, is irrelevant for at least the following two reasons.

1. According to the Nyquist theorem, a linear superposition of only two linear momentum states cannot produce a pure $l \neq 0$ angular momentum eigenstate of the kind produced by a twisted parabolic antenna [24].
2. It is impossible to use only two Yagi antennas to produce the far-field superposition of an $l = 1$ vortex beam, as transmitted by a twisted parabolic antenna, and an $l = 0$ beam, as transmitted by a Yagi antenna, as in our experiments.

Hence, with the EM linear momentum experiment suggested in the comment [2] it would not be possible to deduce any information about the physical properties of EM angular momentum in the radio frequency domain, which was the whole purpose of our experiments, performed both in a controlled laboratory environment [4] and in a realistic outdoor environment [1]. The discussions in the comment based on the suggested alternative experiment are therefore not relevant in the context of our Padua and Venice experiments reported in [1] or our Uppsala experiments reported in [4], where we transmitted radio beams that indeed carried OAM.

The configuration proposed in the comment basically comprises taking the sum and the difference of two conventional linear momentum modes that are detected by a simple interferometer scheme. The authors claim that

‘In this way, we create the differential mode that will be shown to be similar to the ‘vortex’ mode of [1] as far as detection is concerned, while using standard antennas’.

Using this scheme, the authors not only describe a method of multiple linear momentum mode signal detection, but also discuss the power decay dependence. This discussion brought them to the conclusion that

‘Whichever is the method used to transmit the signals, . . . it is clear that the odd mode signal transmitted by the vortex antenna will also suffer from the additional $r^{-1}$ factor (or $r^{-2}$ in terms of signal power), for a fixed Rx antenna spacing $d$’.
By assuming erroneously only linear momentum propagation and implementing a mere lobe combination of two antennas transmitting linear momentum, Tamagnone et al [2] claim to find an additional suppression factor in the field that carries angular momentum. This is the reason why the intensity of their ‘differential mode’, which is thought to fall off as the fourth power of the distance $r$, has a different behaviour than the vortex field used in our experiments.

Another mistake is their assumption ‘And hence (keeping $d$ constant)’, which invalidates the results in equations (8) and (9) of the comment. In fact, in all our experiments the field and its phase were measured at varying baseline distances $d$ between the two interferometer antennas and successful communication tests were made for interferometer baseline distances, varying from 1.0 to 4.5 m. This corresponds to receiver antenna Fraunhofer distances of 30 and 311 m, respectively; the Fraunhofer distance for the transmitting parabolic antenna was 10 m, whereas the OAM signal was received and identified at a distance of 442 m. This clearly invalidates the claims regarding the ‘far-enough-field’ made on page 5 of the comment.

We point out that by using full angular momentum modes instead of only linear momentum and spin angular momentum (polarization) modes, as in all presently existing radio communication methods, including MIMO, one opens a new physical layer on which one can apply multiport and other performance-enhancing techniques and develop new communication protocols in a similar fashion to what is done today for the linear momentum physical layer. For this reason, their claim

‘We will show that this experiment is similar in essence to the one of [1], further demonstrating that the use of the OAM vortex mode is unnecessary here, and also drawing important conclusions with regard to the applicability of the concept’

is another inconsistency in the authors’ argument.

2. Field behaviour at far-zone distances

That the intensity of the EM angular momentum density, independently of the mode number $l$, falls off as $r^{-2}$ far away from dipole radiation sources, i.e. exactly the same as for the density of the EM linear momentum density (usually referred to as the EM power density in the engineering literature), was shown theoretically nearly a century ago by Max Abraham [3]. On page 918 in this paper, Abraham writes (in English translation; the equation numbers refer to those of Abraham’s original paper in German [3]):

‘The electric vector $E$ has a radial component, which indeed falls off as $r^{-2}$ with increasing $r$, while the components of $E$ that are orthogonal to the radius vector fall off as $r^{-1}$. Hence, with increasing distance from the light source the light waves become transverse, and the linear momentum becomes parallel to the radius vector; it could, according to (4), therefore seem that the angular momentum in the wave zone would be equal to zero. However, one observes that this is not the case, if one determines the order of the quantities in question; while the longitudinal component of $E$ is of the order $r^{-2}$, the scalar product of $r$ and $E$ is still of order $r^{-1}$, just as $H$. From (4c) it therefore follows that the density of the angular momentum, as well as the densities of the linear momentum and energy are of the order $r^{-2}$.’

In his 1941 textbook, Stratton [26] calculates temporal Fourier transform expressions for $E(t, \mathbf{x})$ and $B(t, \mathbf{x})$, generated by arbitrary distributions of charge and current densities at
rest relative to the observer. In their textbook published in 1962, Panofsky and Phillips [11] present a variant form of these expressions, and also give them in ordinary space–time coordinates. In Jefimenko’s 1966 textbook [27], the Panofsky and Phillips expressions for the retarded $E$ and $B$ fields are given in chapter 15. These expressions are referred to as the Stratton–Panofsky–Phillips–Jefimenko equations [28].

Following this approach, one can show that the angular momentum density generated by any arbitrary distribution of charges and currents and not only by dipoles, falls off in precisely the same way as the Poynting vector (linear momentum density), namely as $r^{-2}$, whereas the integration surface grows as $r^2$, exactly compensating for this when integrated so that the total angular momentum carried by the EM field goes to a constant at infinity [18, 23, 28]. Precisely as the conserved physical quantity linear momentum, the likewise conserved physical quantity angular momentum can therefore transfer information wirelessly over large distances, even on cosmological scales [29].

2.1. Explicit derivation of the $r^{-2}$ angular momentum density fall-off at large distances from an arbitrary radiation source

The total field angular momentum about a point $x_0$ carried by an EM beam propagating in free space is defined as the integral over the volume, outside the source region, where the fields are non-zero (the emitted ‘pulse’) [11–23],

$$J^\text{field}(t, x_0) = \int_V d^3x \ h^\text{field}(t, x, x_0),$$  \hspace{1cm} (1)

where

$$h^\text{field}(t, x, x_0) = (x - x_0) \times g^\text{field}(t, x)$$  \hspace{1cm} (2)

is the EM angular momentum density and, according to the Planck relation,

$$g^\text{field} = \varepsilon_0 (E \times B)$$  \hspace{1cm} (3)

is the EM field linear momentum density (SI units). For a radio beam such as that used in our Padua and Venice experiments [1], the total EM field angular momentum $J^\text{field}(t, x_0)$ about the moment point $x_0$ is to a good approximation given by the sum of the EM field spin angular momentum $\Sigma^\text{field}(t)$, which is an intrinsic quantity (does not depend on the choice of moment point $x_0$), and the EM field OAM $L^\text{field}(t, x_0)$, which is an extrinsic quantity (depends on the choice of moment point $x_0$) [30]

$$J^\text{field}(t, x_0) = L^\text{field}(t, x_0) + \Sigma^\text{field}(t).$$  \hspace{1cm} (4)

The EM field OAM about the origin $x_0 = 0$ is given by the following exact, manifestly gauge-invariant formula [[23], chapter 4]:

$$L^\text{field}(t, 0) = \int_V d^3x \ x \times \rho(t, x) A^\text{total}(t, x)$$  \hspace{1cm} (5)

$$+ \varepsilon_0 \int_V d^3x \ x \times [(\nabla A^\text{total}(t, x)) \cdot E(t, x)]$$  \hspace{1cm} (6)

$$- \varepsilon_0 \oint_S d^2x \ \hat{n} \cdot E(t, x)[x \times A^\text{total}(t, x)],$$  \hspace{1cm} (7)
where $A^{\text{rot}}$ is the rotational part (in the Helmholtz decomposition sense) of the EM vector potential $A$. In free space, and for a single temporal Fourier component with angular frequency $\omega$, formula (5) can be written in terms of the angular momentum operator $\hat{L} = -i\hbar \vec{x} \times \nabla$, as the cycle average [\cite{23,30}, chapter 4]

$$\langle \vec{L}^{\text{field}}(\mathbf{o}) \rangle = \frac{\varepsilon_0}{2\hbar \omega} \sum_{i=1}^{3} \int_{V} d^3 x (E_i)^* \hat{L} E_i. \quad (8)$$

Let us now consider a source consisting of an arbitrary charge density $\rho$ and an arbitrary current density and $j$. Let us also introduce the integrals of them over the source volume $V'$

$$q(t') = \int_{V'} d^3 x' \rho(t', x'), \quad (9a)$$
$$I(t') = \int_{V'} d^3 x' j(t', x'), \quad (9b)$$

where $t'$ is the retarded time and $x'$ is a source point in $V'$ with its barycentre at the origin $x_0 = 0$. Assuming that the observer is located at the field point $x$ at a distance $|x - x'|$ that is much larger than the extent of the source volume, i.e. $\inf |x - x'| \gg \sup |x'|$, and that the paraxial approximation can be applied, i.e. that the direction of the wave vector $\vec{k}$ seen by the distant observer can be approximated by that of the unit vector $\hat{n}$ from the barycentre of the source volume so that

$$\vec{k} \equiv \hat{k} \equiv k \frac{x - x'}{|x - x'|} \approx k \frac{x}{|x|} \equiv k \hat{n} = k \hat{r}, \quad (10)$$

one can show [\cite{23,28}] that the cycle averaged EM angular momentum density radiated from this arbitrary source is

$$\langle \vec{h}^{\text{field}}(\mathbf{o}) \rangle = \frac{1}{32\pi^2 \varepsilon_0 c^3} \left( \frac{\hat{n} \times \text{Re} \{(cq + I_n)\hat{r}^*\}}{cr^2} + \frac{\hat{n} \times \text{Re} \{(cq + I_n)\hat{r}^*\}}{r^3} + \cdots \right). \quad (11)$$

From this expression, valid for EM fields generated by arbitrary charge and current density sources, we see that the OAM to leading order falls off as $r^{-2}$ with distance $r = |x|$. When this OAM density is integrated over the (finite) volume $V$ occupied by the fields (i.e. over the ‘pulse’), the integration surface element grows as $r^2$, exactly compensating for this fall-off so that the total radiated angular momentum goes to a constant at very large distances. It is well known that the linear momentum density (Poynting vector) has precisely this far-zone behaviour and that this property is the fundamental physical basis for today’s linear momentum radio communications. Since angular momentum has the same far-zone behaviour, angular momentum can also be used for wireless information transfer over huge distances.

Claims that $l \neq 0$ OAM modes fall off faster with distance $r$ from the source than the $l = 0$ modes are therefore unfounded and contradict the established electrodynamics literature. It is well known that beams carrying different OAM states, e.g. Laguerre–Gaussian beams, have a beam widening that depends on the square root of the OAM mode $l$. We also point out that in our experiments we used a twisted (OAM) beam with $l = 1$. 

\text{New Journal of Physics 14} (2012) 118002 (http://www.njp.org/)
3. Conclusions

We have pointed out inconsistencies in the claims made in the comment [2] on our paper [1]. The fact that EM angular momentum is a unique, fundamental physical observable carried by the EM field and is represented by a pseudovector with dimension length × mass × velocity, while MIMO is a multiport engineering technique for enhancing the information-carrying capability of linear momentum, which is another unique, fundamental physical observable represented by an ordinary vector that has the dimension mass × velocity, shows that OAM radio has nothing to do with MIMO radio.

In many cases, approximate OAM eigenmodes and superposition of them can be generated by an array of conventional linear-mode antennas [24]. However, the fact that the same array can also be configured to generate MIMO radio does not of course mean that the two methods are identical. This apparent similarity might mislead workers who are not familiar with the whole set of conserved quantities of the EM field and their physical meaning.

One can easily show that the properties of the far-zone field generated by two transmitting antennas, one (a Yagi) generating $l = 1$ OAM, and one (a modified parabolic antenna) generating $l = 0$ (no OAM) cannot be emulated by the multiport setup proposed in the comment and that therefore the claims in the comment have no bearing on our results.

The authors of the comment show that at large distances from the radiation source of this two-port field falls off as $r^{-4}$ whereas it is well known from the standard literature that the angular momentum density, for any mode $l$, falls off as $r^{-2}$, i.e. at precisely the same rate as the linear momentum density (the Poynting vector) does.

Moreover, the MIMO technique often utilizes wave polarization to double the number of channels all the way to the far zone. Since wave polarization and the spin part of the angular momentum are two aspects of the very same physical quantity, it would not, if the authors of the comment were correct, be possible to use polarization in radio communications since then the intensity would fall off as $r^{-4}$ and therefore not be usable. This is yet another inconsistency in the argument of Tamagnone et al.

It is clear that the MIMO technique approach to enhance the properties of linear momentum beams has nothing to do with the OAM beam transmission used in our experiments.

The authors of the comment seem to believe that all the authors did was to perform one experiment on 24 June 2011 where we simply measured the relative phase between two points and two points only in the phase front of the transmitted beam. They also seem to believe that the authors based all claims in the paper on this single measurement alone and that the purpose of our experiments was only that of transmitting two channels on the same frequency. From the above it should be clear that this is not the case and we therefore reiterate emphatically that prior to the Venice experiments, careful systematic experiments were performed in Padua, with the same equipment used in the Venice experiments. In the systematic Padua experiments, the OAM/vorticity was confirmed by repeated measurements of phase and amplitude differences in a large number of points in the phase front surrounding the linear momentum intensity minimum in the same manner as in the controlled laboratory experiments in the Uppsala Ångström Laboratory anechoic chamber [4].

As already pointed out, our proposal to use the full angular momentum modes instead of only linear momentum and spin angular momentum (polarization) opens a new physical layer on which one can apply enhancing techniques and develop new communication protocols allowing very high spectral densities. Moreover, classical and quantum field theory show that linear and angular momentum can be carried by massless bosons, even if in classical mechanics these
quantities are proportional to a mass term. Work along these lines is in progress and will be reported elsewhere.

Acknowledgment

We thank Carlo G Someda for very helpful discussions during the preparation of this reply.

References

[1] Tamburini F, Mari E, Sponselli A, Thidé B, Bianchini A and Romanato F 2012 New J. Phys. 14 033001
[2] Tamagnone M, Crayre C and Perruisseau-Carrier J 2012 New J. Phys. 14 118001
[3] Abraham M 1914 Phys. Z. XV 914–8
[4] Tamburini F, Mari E, Thidé B, Barbieri C and Romanato F 2011 Appl. Phys. Lett. 99 204102–3
[5] Allen L, Barnett S M and Padgett M J 2003 Optical Angular Momentum (Bristol: Institute of Physics Publishing)
[6] Andrews D L 2008 Structured Light and Its Applications: An Introduction to Phase-Structured Beams and Nanoscale Optical Forces (New York: Academic)
[7] Molina-Terriza G, Torres J P and Torner L 2007 Nature Phys. 3 305–10
[8] Franke-Arnold S, Allen L and Padgett M 2008 Laser Photon. Rev. 2 299–313
[9] Torres J P and Torner L 2011 Twisted Photons: Applications of Light With Orbital Angular Momentum (Weinheim, DE: Wiley-VCH/Wiley)
[10] Padgett M and Bowman R 2011 Nature Phys. 5 343–8
[11] Panofsky W K H and Phillips M 1962 Classical Electricity and Magnetism 2nd edn (Reading, MA: Addison-Wesley)
[12] Podolsky B and Kunz K S 1969 Fundamentals of Electrodynamics (New York: Dekker)
[13] Eyges L 1972 The Classical Electromagnetic Field (New York: Dover)
[14] Landau L D and Lifshitz E M 1975 The Classical Theory of Fields (Oxford: Pergamon)
[15] Mendel L and Wolf E 1995 Optical Coherence and Quantum Optics (New York: Cambridge University Press)
[16] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1997 Photons and Atoms. Introduction to Quantum Electrodynamics (New York: Wiley)
[17] Low F E 1997 Classical Field Theory. Electromagnetism and Gravitation (New York: Wiley)
[18] Schwinger J, DeRaad L L Jr, Milton K A and Tsai W 1998 Classical Electrodynamics (Reading, MA: Perseus Books)
[19] Jackson J D 1999 Classical Electrodynamics 3rd edn (New York: Wiley)
[20] Griffiths D J 1999 Introduction to Electrodynamics 3rd edn (London: Prentice-Hall)
[21] Milton K A and Schwinger J 2006 Electromagnetic Radiation: Variational Methods, Waveguides and Accelerators (Berlin: Springer)
[22] Rohrlich F 2007 Classical Charged Particles 3rd edn (Singapore: World Scientific)
[23] Thidé B 2011 Electromagnetic Field Theory 2nd edn (Mineola, NY: Dover)
[24] Thidé B, Then H, Sjöholm J, Palmer K, Bergman J, Carozzi T D, Istomin Y N, Ibragimov N H and Khamitova R 2007 Phys. Rev. Lett. 99 087701
[25] Vucetic B and Yuan J 2003 Space–Time Coding (Chichester: Wiley)
[26] Stratton J A 1941 Electromagnetic Theory (New York: McGraw-Hill)
[27] Jefimenko O D 1966 Electricity and Magnetism. An Introduction to the Theory of Electric and Magnetic Fields (New York: Appleton-Century-Crofts)
[28] Thidé B, Lindberg J, Then H and Tamburini F 2010 arXiv:1001.0954 [physics.class-ph]
[29] Tamburini F, Thidé B, Molina-Terriza G and Anzolin G 2011 Nature Phys. 7 195–7
[30] van Enk S J and Nienhuis G 1992 Opt. Commun. 94 147–58

New Journal of Physics 14 (2012) 118002 (http://www.njp.org/)
Copyright of New Journal of Physics is the property of IOP Publishing and its content may not be copied or emailed to multiple sites or posted to a listserv without the copyright holder's express written permission. However, users may print, download, or email articles for individual use.