Precision tests of QED in strong fields: experiments on hydrogen- and helium-like uranium

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Abstract. In this contribution, we present an experimental study carried out at GSI Darmstadt devoted to investigation of quantum electrodynamical (QED) effects for the ground states in hydrogen- and helium-like uranium. In the experiment, X-ray spectra following radiative recombination of free electrons with bare and H-like uranium ions ($^{92+}$U, $^{91+}$U) were measured at the electron cooler of the ESR storage ring. Utilizing clean and favorable experimental conditions present at the electron cooler, we were able to obtain very accurate values for the ground-state binding energies from the observed X-ray transitions. When compared with theory, our results provide the most stringent test of bound-state QED for one- and two-electron systems in the strong-field regime.

1. Introduction

Accurate investigations of high-Z simple atomic systems have experienced substantial progress during recent years both from experimental as well as from theoretical sides. In high-Z few-electron ions, bound electrons are exposed to extreme electric fields of heavy nuclei and the influence of quantum electrodynamical and relativistic effects on atomic structure is very pronounced (in comparison with the low-Z domain) which provide favorable conditions for study of the bound-state QED in the strong field regime. Experimentally, accurate measurements of x-ray transitions from bound or continuum states into the ground state of the heavy ion-electron systems represent most direct approach for deducing characteristic QED phenomena in intense fields and a comparison of predicted with experimentally determined level energies provides a critical test of theory in this domain.

Accurate binding-energy (transition) measurements have been performed for a number of hydrogen-like ions \cite{1, 2, 3} as well as for helium-like \cite{4} and lithium-like systems \cite{5, 6, 7}. In particular, the experiments for Li-like ions have achieved highest precision which is by about an order of magnitude larger than the one achieved by theoretical calculations \cite{7}. The latter
are limited mostly by the nuclear size uncertainties as well as by yet unevaluated two-loop lamb shift contributions.

In the following, we present an experimental study conducted at the electron cooler of the ESR storage ring, dedicated to the investigation of QED effects in the heaviest one- and two-electron systems available for experimental studies, i.e., hydrogen- and helium-like uranium. In the experiment, for the first time a combination of deceleration technique and the 0 deg spectroscopy at the electron cooler was utilized; which enabled us to obtain a precise value for the ground state Lamb shift in hydrogen-like uranium and to improve accuracy 3 times compared to the most accurate value available up to now [3]. Here, we like to note, that in combination with very accurate results obtained for the 2s binding energy in lithium-like ions, the present result for the H-like uranium might provide the way to disentangle between nuclear, one- and multi-electron contributions to the 2s binding energy [8]. In addition, for the first time, the two-electron contribution to the ground-state binding energy in He-like uranium was obtained by measuring the ionization potential of the He-like system with respect to the H-like one: the experimental technique introduced at the Super-EBIT device which exploits radiative recombination (RR) transitions into the vacant K shell of H- and He-like high-Z ions [4].

2. Experiment
For the experiment, bare and H-like uranium ions were injected, in alternate order, at the initial energy of close to 360 MeV/u into the ring and cooled by an electron beam of 300 mA. After the initial accumulation and cooling, the ions were decelerated to the final beam energy of 43.6 MeV/u using a technique which is routinely available at the ESR. At the final stage of beam handling the electron cooler was switched on again at the energy which corresponds to the energy of the decelerated ions. The cooler current and voltage applied were about 100 mA and 23 kV respectively. The electron cooling guarantees a well defined constant beam velocity, generally of the order of $\Delta \beta / \beta \approx 10^{-5}$ as well as a reduction of the beam emittance. In addition, at low energies, all uncertainties associated with Doppler corrections are strongly reduced compared to high-energy beams. Also, for decelerated ions the bremsstrahlung intensity caused by the cooler electrons is strongly reduced (due to a relatively small cooler voltage of 27 kV and current of 100 mA). Consequently, very clean conditions for x-ray spectroscopy are present at the cooler section.

In order to obtain the two-electron contribution in He-like uranium, a relative measurement of radiative recombination (RR) into the K-shell of initially bare and H-like ions is needed, therefore, we changed periodically the two charge states during the experiment. Moreover, within the cooler section, the trajectory for both ion species must be the same. Since the beam energy at the ESR is determined by the cooler voltage, identical beam energies for both ion species are guaranteed. Also, the trajectories of the ion beams inside the cooler section are well controlled. Even, a slight misalignment between the beams of bare and H-like ions (e. g. 1 mm) does not affect the final accuracy of the experiment. Here, we profit from the 0° geometry of our x-ray setup which is rather insensitive to an uncertainty in the observation angle.

X-rays emitted via the radiative recombination inside the electron cooler were detected using three independent strips of a segmented germanium detector, each furnished with an individual readout. The detector was mounted 4.1 m downstream of the midpoint of the 2.5m long straight cooling section and could be moved vertically by means of a stepping motor (see Fig. 1). No x-ray spectra were recorded during the beam accumulation periods. Only after the completion of a whole cycle (including deceleration) the detector was placed in the measurement position and data accumulation was started. The measuring time per cycle was limited by the capture rate in the cooler to typically few minutes.

During the data accumulation the detector was placed close to the ion beam, resulting in observation angles of the ion-beam/electron-beam interaction zone of 0.35°, 0.53° and 0.71° for
strip number 1, strip number 2 and strip number 3, respectively, with an angular acceptance of ±0.17°. The resulting accuracy of the observation angle $\Delta \theta$ amounts to $\Delta \theta = 0.02°$. The shift of the observed photon energy between two neighboring strips due to the Doppler effect amounted to about 1 eV. The Doppler broadening was negligible due to the observation angle of close to 0°. In addition, the photons were recorded in coincidence with down-charged uranium ions, as produced by the capture of one electron in the cooler. The down-charged ions were registered in a gas-filled multiwire proportional counter (MWPC) which was installed in a pocket behind the first dipole magnet downstream of the electron cooler.

3. Results and comparison with theory

In Fig. 2 we present calibrated coincident x-ray spectra as observed in the experiment for capture into bare and H-like uranium ions at an energy of 43.6 MeV/u. The spectra are almost background free, since they were recorded in coincidence with down-charged ions. The most intense lines observed can be attributed to direct transition of electrons into the K-shell of the projectile ions (K-RR) and to characteristic $L \rightarrow K$ (Lyman $\alpha$) transitions. due to the observation angle of approximately 0°, the characteristic Ly$\alpha$ transitions and the K-RR line with energies of about 100 and 130 keV in the emitter frame, are blue shifted and appear at energies close to 130 and 170 keV respectively. The radiative recombination transitions into the ground state of bare and H-like projectiles can be exploited for a determination of the two-electron contribution to the ground state binding energy in He-like uranium; the difference in the centroid energies for such transitions equals to the difference in the ionization potential between the H- and He-like ions formed by the recombination process [4] which gives exactly the two-electron contribution to the ground state energy of He-like uranium. In order to achieve the desirable precision, an accurate determination of the x-ray energies is needed. Although the intrinsic linewidth of the Ge(i) detector for the energy range of relevance is about 700 eV, the small energy difference between two closely spaced lines can be determined with high accuracy [9]. In order to take advantage of this property, a projectile energy of 43.59 MeV/u was chosen. At this particular beam energy the Doppler shift close to 0 deg allowed us to place the 177.213 keV $\gamma$-ray line of $^{169}$Yb, used for calibration, just in between the K-RR lines for H- and He-like uranium (see Fig. 2). In order to gain control over possible electronic drifts the detector was regularly calibrated, in 2 to 4 h intervals. The data were divided into individual groups and were analyzed separately. The RR line centroid positions were determined from least-squares fits using a Gaussian peak shape with a shelf on the low energy side [9]. Since, the RR lines for the bare and the H-like ions were measured in individual runs, the RR centroid energies were always determined relative to the closely spaced 177.213 keV $\gamma$-ray line of $^{169}$Yb. Using the 177.213 keV calibration line as a reference, the relative energy separation for RR into the
bare and H-like ions was determined for each detector segment separately to $3059 \pm 22$ eV, $3029 \pm 17$ eV, and $3098 \pm 36$ eV, giving a weighted mean value of $3047.91 \pm 12.6$ eV. Finally, in order to obtain the difference between ionization potentials for H- and He-like uranium, the energy difference between the K-RR lines for capture into bare and into H-like uranium ions, extracted from the x-ray spectra (observed in the laboratory system) has to be transformed into the rest frame of the ions according to the Doppler formula: 

$$E = E_{\text{lab}} \gamma (1 - \beta \cos \theta_{\text{lab}}).$$

Here, $E$ and $E_{\text{lab}}$ are the x-ray energies in the emitter and in the laboratory frame, respectively, $\theta_{\text{lab}}$ denotes the laboratory observation angle, and $\gamma$ is the relativistic Lorentz factor. According to the formula the energy in the emitter frame depends on the observation angle (in the laboratory system) and on the $\beta$ value of the projectile. The observation angles for strip number 1, strip number 2 and strip number 3 were $0.35^\circ$, $0.53^\circ$ and $0.71^\circ$, respectively, with an uncertainty of $\Delta \theta = 0.02^\circ$. This translates into a relative uncertainty $\Delta E/E = 10^{-6}$ and can therefore be neglected in the following. The value of $\beta$ was determined from the electron cooler voltage since the velocity of the cooling electrons defines the velocity of the stored ions. It is given by the relation 

$$(\gamma - 1)mc^2 = eU_e$$

where $e$ and $mc^2$ are the charge and the rest mass of the electron, respectively. $U_e$ is an effective electron acceleration voltage represented by the following formula 

$$U_e = U \times \frac{1}{1.0011 - 375I_c} [4].$$

The first term is the voltmeter reading corrected for a calibration of the cooler voltage and the second term represents the space charge correction. The latter was determined by a measurement of the Schottky revolution frequency of the circulating beam as function of the cooler current $I_c$. In our experiment $U = 23924$ V and $I_c = 100$ mA which gives an effective cooler voltage $U_e$ of 23913 V. From this a $\beta$ value of $\beta = 0.29565$ follows with an uncertainty of $5.8 \times 10^{-6} \Delta U_e$ where $\Delta U_e$ refers to the accuracy achieved in the calibration of the cooler voltage. Accordingly the Doppler correction factor is determined to 0.737 309 for the first strip, 0.737 317 for the second, and 0.737 327 for the third strip, with an uncertainty introduced by $\beta$ of less than $\pm 3.2 \times 10^{-5}$. The latter is the same for all the three different strips and corresponds to an uncertainty of $\pm 0.071$ eV on an absolute scale. Here we emphasize that because of the combination of the $0^\circ$ geometry and the deceleration technique the systematic uncertainties introduced by $\Delta \theta$ and $\Delta \beta$ do not affect the final accuracy achieved. In contrast,
Table 1. Comparison of our experimental result for He-like uranium with the calculations of Yerokhin et. al. [10]. All energies are given in eV.

|                | 1-photon exchange | 2-photon exchange non-QED | 2-photon exchange QED | 2eSE | 2eVP | ≥ 3-photon exchange | Total theory | Exp.    |
|----------------|--------------------|--------------------------|-----------------------|------|------|---------------------|--------------|---------|
|                |                    |                          |                       |      |      |                     |              |         |
|                | 2265.88(1)         | -12.09                   | -0.79                 | -9.78| 2.63 | 0.06(9)             | 2245.92(9)   | 2248 ± 9 |

the result is entirely limited by counting statistics. Therefore, from a Lorentz transformation into the emitter frame, we obtain a value of 2248 ± 9 eV for the two-electron contribution to the ground state ionization potential in He-like uranium.

In table 1, we compare our experimental result obtained for the two-electron contribution with the theoretical calculations of Yerokhin et al. [10]. Our result agrees very well with the theoretical prediction within the experimental uncertainty. This theory takes into account the electron-electron interaction complete to second order in α. Beyond the first-order one-photon-exchange contribution it also comprises the non-QED contribution of the two-photon exchange as well as the QED two-photon exchange part also called the box and the ladder diagram. Most important, the radiative two-electron QED contributions are considered to second order in α in a complete fashion. This two-electron Lamb shift comprises both the two-electron self energy (2eSE) and the two-electron vacuum polarization (2eVP) [10]. We have to add that the theoretical treatment of Yerokhin et al. [10] is in excellent agreement with a further theoretical approach by Persson et al. [11], based on relativistic many-body perturbation calculations, which also comprises all specific two-electron QED effects to second order in α. These calculations also show that the specific two-electron QED effects are almost completely unaffected by the uncertainties of the nuclear-charge radius, one of the most serious limitations for the QED tests in high-Z one-electron systems. As can be deduced from the experimental and theoretical results presented in the table the experimental data provide a meaningful test of the many-body non-QED part of the electron-electron interaction. Moreover, our accuracy is of the same size as the second-order two-electron self energy contribution.

Besides the two-electron contribution to the ground state of He-like uranium, we derived the value for the 1s Lamb shift in H-like uranium by accurate determination of energies of intense characteristic $L \rightarrow K$($Ly_{\alpha}$) transitions observed in the spectra (see Fig. 2) as well as of the K-RR lines. Assuming that energies of the $L$-shell states are precisely known from the theory [12], the ground state Lamb shift can be deduced by comparison with the Dirac energy eigenvalue for the 1s-ground state of a point like nucleus. In order to obtain the $Ly_{\alpha}1$ centroid energy a similar technique as for the case of the two-electron contribution was applied. In the present case the $^{169}$Yb $\gamma$-line at 130.523 keV was used. For each detector strip, 2 independent values for the 1s binding energy were obtained, one from the $Ly_{\alpha}1$ transition and one from the K-RR line, respectively; 131814.1 ± 4.3 , 131821.9 ± 3.7, 131821.7 ± 6.2; 131812.96 ± 9.7, 131823.26 ± 7.6, 131848.5 ± 12.1. After the transformation into the emitter frame, the uncertainty resulting from $\Delta \beta = 2.9 \times 10^{-5}$ of 3.5 eV is introduced. The error resulting from an uncertainty in the observation angle introduces a negligible contribution of about 0.1 eV. Finally, taking a weighted average of the above values and comparing with the Dirac energy eigenvalue for the 1s-ground state of a point like nucleus we obtain a value of 460.2 ± 2.3 ± 3.5 eV for the ground state Lamb shift in H-like uranium. The uncertainty of 2.3 eV is of statistical nature whereas the one of 3.5 eV stems from the imprecision of the beam velocity determination. Besides, we estimate an uncertainty of 2 eV to account for possible systematic errors introduced by the line-shape analysis. Altogether, this results in 4.6 eV uncertainty for the 1s Lamb shift in H-like uranium. In table 2 we compare
Table 2. The Ground-state Lamb shift for H-like uranium. All values are given in eV.

|                |          |
|----------------|----------|
| Finite nuclear size | 198.81   |
| 1-st order QED    | 266.45   |
| 2-nd order QED    | -1.26(33) |
| Total theory [13, 14] | 464.26±0.5 |
| This work         | 460.2±4.6 |

Our experimental result for the 1s Lamb shift with the newest theoretical value [13, 14]. Several individual contributions to the total theoretical Lamb shift including the recently calculated 2-nd order QED corrections are listed separately as well. From the comparison we can state that our result provides a test of the first order (in $\alpha$) QED contributions at the 2% level and thus, represents the most precise test of the bound-state QED in high-Z one-electron systems. Besides, the present result is consistent with the values from former experiments [2, 3] and is almost 3 times more precise than the most accurate result reported up to now [3].

4. Conclusions
We have carried out an experimental study devoted to an accurate investigation of bound-state QED effects in the heaviest one- and two-electron ions. Exploiting the deceleration capability of the ESR storage in combination with 0 deg spectroscopy at the electron cooler allowed us to obtain accurate values for 1s Lamb shift in H-like uranium and for the first time, the two-electron contribution to the ground state binding energy in Helium-like uranium. Both results are in excellent agreement with the most recent theoretical predictions. For the case of He-like uranium, the present value represents most accurate determination of two-electron effects in the domain of high-Z He-like ions and the accuracy reaches already the size of the specific two-electron radiative QED corrections. For hydrogen-like uranium, our result is about 3 times more precise than the most accurate value available up to now, thus representing the most accurate test of the strong field QED for the most fundamental atomic system, in the absence of many-electron effects.

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