We present a panoramic view on various attempts to "solve" the problems of quantum measurement and macro-objectivation, i.e. of the transition from a probabilistic quantum mechanic microscopic world to a deterministic classical macroscopic world.

Keywords: macro-objectivation, quantum coherence, foundations of quantum mechanics

I. INTRODUCTION

Quantum Mechanics (QM) represents nowadays one of the pillars of modern physics: so far a huge amount of theoretical predictions deriving from this theory has been confirmed by very accurate experimental data, while the theory is at the basis of a large spectrum of researches ranging from solid state physics to cosmology, from biophysics to particle physics. Furthermore, in the last years the possibility of manipulating single quantum states has fostered the development of promising quantum technologies as quantum information (calculus, communication, etc.), quantum metrology, quantum imaging, ...

Nevertheless, even after a pluri-decennial debate many problems related to the foundations of this theory persist, like non-local effects of entangled states, wave function reduction and the concept of measurement in Quantum Mechanics, the transition from a microscopic probabilistic world to a macroscopic deterministic world described by classical mechanics (macro-objectivation) and so on. Problems that, beyond their fundamental interest in basic science, now also concern the impact of these developing technologies.

It is also worth to mention that this debate expanded beyond the physicists community, involving since its beginning philosophers with epistemological interests, as Popper or Feyerabend, and the development of a quantum logic.
In this paper, we wish, without any pretension of being exhaustive due to the huge published material on the subject, give a summary introduction to the main attempts for solving the measurement problem (and the related one of macro-objectivation) in QM. In our purpose this should represent a summary addressed not only to physicists, but to any scientist interested in this problematic. Since we intend to present the main attempts to solve the problem, we will group different similar proposals without to much attention to nuances distinguishing among them (even if for the authors of different proposals these ”nuances” may have a large relevance). A large bibliography is provided that will allow the interested reader to deepen specific interpretations or models and appreciate these differences. Furthermore, being addressed to a general audience, we will avoid too technical distinctions and details, preferring some simplification for improving the readability to an absolute precision in defining the concepts (of course when this does not lead to errors or misunderstandings).

II. THE MACRO-OBJECTIVATION AND MEASUREMENT PROBLEM IN QUANTUM MECHANICS

A. The Von Neumann chain

One of the most characteristic properties of QM is the superposition principle, i.e. the fact that a linear superposition of states (vectors of a Hilbert space) describing the system is still a valid state of the system. This assumption is at the basis of interference properties and probabilistic structure of the theory and it is a pervasive QM characteristic.

When extended to many particle systems this assumption leads to situations where the multi particle states cannot be factorized in single particle ones, a property called entanglement. This property leads to several counter-intuitive, paradoxical aspects of QM, as non-locality, EPR effect and so on and was define by Schrödinger the ”the characteristic trait of quantum mechanics”.

The problem of macro-objectivation derives by the fact that evolution equation in Standard Quantum Mechanics (SQM) is linear and thus requires that a macroscopic system interacting with a state in a superposition becomes entangled with it.

For example let us consider a macroscopic measurement apparatus described by the state $|\chi_0\rangle$ (i.e. a wave function as complicated as necessary), which interacts with the (microscopic) states $|\phi_1\rangle$ and $|\phi_2\rangle$.

The interaction, representing the measurement and lasting a time interval $\Delta t$, can be described by a linear evolution operator $U(\Delta t)$. The results of the measurement are then
\[ |\chi_0\rangle|\phi_1\rangle \rightarrow U(\Delta t)|\chi_0\rangle|\phi_1\rangle = |\chi_1\rangle|\phi_1\rangle \tag{1} \]

and

\[ |\chi_0\rangle|\phi_2\rangle \rightarrow |\chi_2\rangle|\phi_2\rangle \tag{2} \]

where the states \(|\chi_{1(2)}\rangle\) of the measuring apparatus represent the situations where a pointer denotes to have measured the state \(|\phi_{1(2)}\rangle\).

If \(|\chi_0\rangle\) interacts with the superposition state

\[ a|\phi_1\rangle + b|\phi_2\rangle \tag{3} \]

because of linearity of the evolution equation, one has

\[ |\chi_0\rangle[a|\phi_1\rangle + b|\phi_2\rangle] \rightarrow [a|\chi_1\rangle|\phi_1\rangle + b|\chi_2\rangle|\phi_2\rangle] \tag{4} \]

which is an entangled state involving the macroscopic apparatus as well.

Of course at a macroscopic level we do not perceive anything which can be thought as a superposition of two macroscopic situations, for example if the measuring apparatus has a pointer that is up or down according to if it has measured a property 1 or 2, we always observe the pointer in one well defined position and never in a undefined superposition of pointer up and down at the same time.

A very illuminating example of this problem was proposed by Schrödinger. Let us consider a box with a cat inside. In the box there is also a measurement apparatus that measures a property of a quantum system, which is in a superposition state for the measured observable. According to the value of the measurement the apparatus open or not a poison bottle. Thus, in this case the von Neumann chain includes the quantum system, the measuring apparatus and the poison bottle. But at the end also the cat is involved: if the poison has been diffused the cat dies, otherwise it survives. The result of this analysis is therefore that we have a superposition of cat alive and dead, which looks rather a paradoxical situation. From this example in the literature a superposition of two macroscopic states is usually dubbed a "Schrödinger cat".

Therefore, measurements in quantum mechanics would seem to require some process breaking the entanglement: among the possible outcomes only one will be realized and observed in the measurement process, i.e. a non-unitary evolution is needed. Only one state in the superposition survives the measurement process, i.e. in the previous
example the measuring apparatus will be found or in the situation described by $|\chi_1\rangle$, with probability $|a|^2$, or in the one described by the state $|\chi_2\rangle$, with probability $|b|^2$ and the measured state (if the measurement is non-destructive) will be, correspondingly, in the state $|\phi_1\rangle$ or $|\phi_2\rangle$, respectively, after the measurement process. This is called the wave function collapse. However, this request must be justified more precisely. We have to understand at which point of the measurement process the collapse occurs and how this collapse happens.

A first answer is to split the world into a macroscopic one following classical mechanics and a microscopic one following QM, at the moment of the measurement by a classical apparatus the wave function collapses in one of the possible states. This is substantially the one adopted by the Copenhagen school even if a clear definition of what is Copenhagen interpretation is difficult to give, since different physicists (as Bohr or Heisenberg) exposed different points of view. For example according to Bohr classical concepts are somehow autonomous from QM (in a way reminiscent of Kant’s transcendental arguments)\textsuperscript{11}. However this solution, even if perfectly useful for practical calculation of quantum processes, is weak from a conceptual point of view since it does not permit to identify the border between quantum and classical worlds. How many particles should a body have for being macroscopic? What about ”macroscopic” systems as superconductors exhibiting quantum properties? For many reasons this answer looks to be unsatisfactory.

Various different ideas have been considered for explaining/understanding decoherence at macroscopic level, without reaching for any of them a general consensus in the physicists community. Among them (without any purpose to be exhaustive) we can distinguish schemes where QM is ”interpreted”, without modifying the formalism, and schemes where the formalism of QM is modified or considered as incomplete. In the first group one can mention: the many worlds models\textsuperscript{30}, modal interpretations\textsuperscript{32–34}, decoherence and quantum histories schemes\textsuperscript{6,35–37,39}, transactional interpretation\textsuperscript{40}, ‘informational’ interpretation\textsuperscript{41,42} and many others (see for example\textsuperscript{29,52–56,58,84} and Ref.s therein).

In the second group: dynamical reduction models (where a non-linear modification of Schrödinger equation is introduced)\textsuperscript{45,47,50}, reduction by consciousness (wave function collapse happens at observer level)\textsuperscript{51} or hidden variable theories\textsuperscript{20,68}. In this last case, macroobjectivation problem simply does not exist since in these models the specification of the state by using state vectors is insufficient, there are further parameters (the hidden variables) that we ignore for characterizing the physical situation. The physical system is always in a well specified state (corresponding to one of the quantum mechanical states present in the superposition) univocally determined by the value of the hidden variables. However, it must be noticed that for contextual theories one can attribute an objective state only to those variables which are non-contextual.
In the following we will present some of the most interesting attempts to solve macro-objectivation problem according to our opinion.

III. INCOMPLETENESS OF QUANTUM MECHANICS

A first class of schemes consider QM as incomplete: thus one has to modify it by introducing some further element.

A. Hidden variable models

According to some authors, a pure state does not describe individual systems, but an ensemble of similarly prepared systems: thus the formalism of QM is applicable only to groups of similar events and not to isolated events. As, in this case, QM predicts nothing which is relevant to a single system: the probabilities $|a|^2$ and $|b|^2$ of finding the apparatus in the state $|\chi_1\rangle$ or in $|\chi_2\rangle$ (namely with the pointer indicating 1 or 2) merely represent the frequency distribution of the possible measurement for an ensemble with a given state preparation. This would strongly limit the predictivity of QM; individual events are often met in physical investigation.

Of course, the suggestion that quantum states should refer to ensembles of similarly prepared systems and that QM cannot describe individual systems opens the door for hidden variables theories. In fact, in this case is reasonable to assume that quantum mechanics is not a complete theory, but it is a stochastic approximation of some deterministic theory.

This is just the hidden variable program, mentioned in the previous section. In this case no real entanglement exists: hidden variables completely specify the state of the system. We do not have really superpositions and thus the macro-objectivation problem simply does not exist. For what concerns contextual HVT (a theory is said to be non contextual when the value of a quantity is determined regardless of which other quantities are simultaneously measured along with it), like de Broglie Bohm one, this solution concerns only non-contextual observables, but only non-contextual observables can really be considered objective.

However, Bell has demonstrated that a local HVT cannot reproduce all the results of QM and following experiments have confirmed (even if some remaining experimental loophole leaves some space for specific models) the results of QM. Nevertheless, these tests do not concern non-local (as de Broglie-Bohm or Nelson models) or Planck scale HVT (that represent also a very interesting attempt to reconsider the problem of quantum gravity).

Since the discussion about HVT and their tests has been the subject of a recent review paper of the authors, we
address the interested reader to this (for some more recent study on HVT see also\textsuperscript{21}). Nevertheless, it is worth noticing that these theories, at least in principle, can be tested experimentally and thus a future discrimination between them and SQM is envisageable\textsuperscript{89}.

B. The GRW model

Another possible solution of the macro–objectivation problem is to suppose a non-linear modification of evolution equation leading to the collapse of the wave function\textsuperscript{44–46}.

In particular we detail here a little the model that has been proposed by Ghirardi-Rimini-Weber\textsuperscript{45}.

The idea consists in considering an extension of quantum mechanics where the wave function suddenly randomly collapses according to

\[ \Psi(x_1, ..., x_N)j(x - x_i)/R \] (5)

where

\[ j(x - x_i) = A \exp\left[-(x - x_i)^2/(2a)^2\right] \] (6)

and

\[ |R(x)|^2 = \int dx_1...dx_N |\Psi(x_1, ..., x_N)j(x - x_i)|^2 \] (7)

is a normalisation factor. \( x_i \) is the specific coordinate of the \( i \)th particle of the system (the one whose coordinate is "fixed" by the collapse).

The probability of the collapse is given, for each particle, by \( 1/\tau \), where \( \tau \) can be fixed to be \( \approx 10^{15}s \approx 10^8 \) years.

For the constant \( a \), which appears in Eq. (6), GRW suggested \( a \approx 10^{-7} \) m.

Finally, the collapse centre \( x \) is randomly chosen with probability distribution \( |R(x)|^2 \).

The effect of the collapse described by Eq. (5) is that one position co-ordinate is fixed. If the state is a superposition of different macroscopic states the system will thus collapse on one of them (the one where particle \( i \) has a position \( x_i \)) corresponding to different position co-ordinate for the particles belonging to the system, for example to different co-ordinates of the pointer of a measuring apparatus.

The process happens with a very small probability for a single particle (and thus does not modify QM predictions for few particles systems), but for a macroscopic system, where the number of particles is \( N \approx 10^{24} \), the probability
\( N/\tau \) is extremely high and the collapse is almost immediate.

C. Extensions of GRW model

The original GRW model does not express the stochastic part of evolution in a compact mathematical form. This has been obtained in many other models where one introduces dynamical equations that modify the Schrödinger one including a description of the wave function collapse.

For example one can consider the dynamical equation of ref. 47, which leads to the non-unitary evolution of the state (for the sake of simplicity the unitary part is neglected):

\[
|\Psi(t)\rangle = \exp[-1/(4\lambda t)(B(t) - 2\lambda tA)^2] |\Psi(0)\rangle \tag{8}
\]

\( B(t) = \int_0^t dt' w(t'), \) \( w(t) \) being a white noise function and \( A \) is an opportune operator, whose eigenstates are the ones to which the collapse occurs and \( \lambda \) is a parameter (\( \approx 10^{-16} s \)).

Let us now suppose that the initial state \( |\Psi(0)\rangle \) can be written as a superposition of two eigenstates \( |a_i\rangle \) of \( A \) (corresponding to the eigenvalues \( a_i \) respectively) as

\[
|\Psi(0)\rangle = a|a_1\rangle + b|a_2\rangle \tag{9}
\]

then the evolution of Eq. (8) would lead to

\[
|\Psi(t)\rangle = a \exp[-1/(4\lambda t)(B(t) - 2\lambda t a_1)^2] |a_1\rangle + b \exp[-1/(4\lambda t)(B(t) - 2\lambda t a_2)^2] |a_2\rangle \tag{10}
\]

showing how a state evolves under a particular noise \( B(t) \). The evolution is not unitary, so statevectors evolving under different \( B(t) \) have different norms. States with larger norm are more likely and the probability density for \( B(t) \) to be the actual noise is:

\[
P(B) = \langle \Psi(t)|\Psi(t)\rangle = |a|^2 \exp[-1/(2\lambda t)(B(t) - 2\lambda t a_1)^2] + |b|^2 \exp[-1/(2\lambda t)(B(t) - 2\lambda t a_2)^2] \tag{11}
\]

The most probable \( B(t) \) are thus \( B(t) = 2\lambda a_1 t \) and \( B(t) = 2\lambda a_2 t \). If the first is the actual one the state collapses to \( |a_1\rangle \) with probability \( |a|^2 \) as the second component is exponentially suppressed (and in practice disappears as \( t \to \infty \)) as

\[
|\Psi(t)\rangle = a|a_1\rangle + b \exp[-\lambda t(a_1 - a_2)^2] |a_2\rangle \tag{12}
\]
On the other hand, if $B(t) = 2\lambda a^2 t$ the collapse is to $|a_2\rangle$ with probability $|b|^2$.

Thus, the collapse happens into a state or into the other according to how the noise has fluctuated: it remains however unexplained why the noise fluctuated that or the other way. This could be addressed identifying the noise with some physical process (for example gravitational fluctuations) and is demanded to a future theory.

This example represents a very simple model for a stochastic equation. A variety of more sophisticated models have been proposed\cite{GRW}. Even if we will not discuss advantages and defects of all of them, nevertheless we would like to point out some general properties of this kind of models.

One first interesting point is that if we choose the operator $A$ (or better a class of operators generalising Eq. 8) to be of the form

$$A(x, t) = \frac{1}{(\pi a^2)} \int dy N(y, t) \exp\left[-\frac{(x - y)^2}{2a^2}\right]$$

where $x$ is the position and $N$ the particles number operator, we recover a model very similar to the original GRW.

Another interesting characteristic of all the collapse models is that a narrowed wave function increases its energy because of uncertainty principle: this leads to a violation of energy conservation. This can be checked looking to the evolution of the average value of the Hamiltonian.

For example, for the Ref.\cite{GRW} model, this gives

$$\langle H(t) \rangle = E(0) + \frac{3}{4\lambda t^2} \frac{\hbar^2}{2ma^2}$$

where $\lambda$ and $a$ are the parameters previously defined, $m$ is the particle mass and $n$ the number of particles.

For, let say, $10^{24}$ nucleons one has an energy increasing of 0.05 attoJoule/s, corresponding to a temperature increase of 0.001$^o$K since the beginning of the universe! Even if this is a very small amount, somehow a particle could suddenly gain a large quantity of energy and there is some hope of detecting this phenomenon. For example, an analysis\cite{GRW} has been done of data collected in some experiment\cite{(5)} (originally addressed to double $\beta$ decay search), where one has searched for X rays which can be attributed to ionisation of one electron, which had got energy by a collapse. The result of such an analysis is a limit on $\tau$, which indicates that electrons must collapse much less rapidly than protons (whose collapse value is fixed in the model by the request of having a correct wave function reduction for macroscopic bodies). This result has been related to the fact that the electron mass is a factor 1836 smaller than the proton one and has led to speculations about the fact that collapse could be related to quantum gravity\cite{GRW}, as a mass dependent coupling would seem to point out (for possible relations between gravity and wave function collapse, see also\cite{other}).

On the other hand, experimental tests of these models based on interference concerning mesoscopic objects are
much more difficult to be realised.

Finally, one can also notice that the collapse is a non-local process: an entangled state collapse could also instantly involve two very far components of the system, however no faster-than-light information can be transmitted using these non-local effects.

Nevertheless, the definition of ”instantly” would require a preferred frame where the collapse happens. It has been demonstrated that this would not lead to any experimental result in disagreement with special relativity predictions\textsuperscript{59}. However, it looks rather peculiar that the collapse of entangled states is such not to permit any faster than light transmission and at the same time it carries along non relativistically invariant phenomena. A relativistic collapse model would be rather desirable.

Some attempts in this sense have been done\textsuperscript{47,49}. For example, in Ref.\textsuperscript{62} the operator $A$ in Eq. (8) is substituted by a scalar field. The collapse then works as follows: a fermion in a certain superposition is entangled with the scalar field too. The ”noise” causes the collapse of the scalar field and this involves the fermion. Incidentally, one can notice that in this case which detection triggers the collapse and where and when the collapse takes place become frame-dependent, but of course these are not measurable properties. The real problem of the model is that the collapse originates an infinite increasing of energy (due to creation of scalar particles), which cannot be eliminated. Some progress has been obtained\textsuperscript{47,49} (e.g. by introducing tachyonic fields), but a satisfactory solution is still missing.

In conclusion, it is worth to emphasize that, as for HVT theories, also in this case a final answer on these models will come from experimental tests of them.

D. Reduction by consciousness

Somehow also the interpretation of a reduction by consciousness of Wigner\textsuperscript{51} can be considered as a scheme where QM is not complete, indeed consciousness acquires an extra-physical role and cannot be described by the theory.

In little more detail the Wigner’s argument: He analysed the von Neumann sequence of Eq. (4) supposing that at the end a friend of his looks at the experimental apparatus. In principle also Wigner’s friend is described by a very complex wave function, initially $|F_0\rangle$. After observation one should have the entangled state

$$|\chi_1\rangle|\phi_1\rangle|F_1\rangle + |\chi_2\rangle|\phi_2\rangle|F_2\rangle$$

(15)

where now even the Wigner’s friend is in a superposition. Then Wigner asks his friend about the result: he is sure of obtaining a well precise answer. What does it means this? Wigner cannot assume (if he does not want to assume an
extreme solipsistic attitude) that his question causes the collapse of his friend wave function, then the collapse must be happened at some point of the von Neumann sequence. But which is the more distinctive point of the sequence? According to Wigner’s answer this point is when a perception happens. Then the sequence is interrupted when his friend observes the apparatus: it is perception to cause the collapse.

Of course, such a point of view considers the mind out of physical world, than can be considered a weak point. Furthermore, what about the universe evolution? The universe is remained in an extremely complex entangled state up to when the first jelly-fish had a first foggy perception of it? Or it has been bound to wait for the transition between homo erectus and homo sapiens?

IV. COLLAPSE-FREE APPROACHES

In these approaches the main idea is that the mathematical formalism of QM is sufficient as it stands, no changes have to be added to it.

A. The many worlds models

According to this interpretation, due to Everett, one supposes that every quantum possibility realises even at a macroscopic level, but in different non-comunicating universes: thus no interference can be observed for macroscopic bodies.

Another similar hypothesis suggests that the splitting happens at the level of mind. One has different minds with different perceptions corresponding to different component of the state. Of course the minds of different observers must be correlated in order to observe the same result.

Deutsch has supported many world interpretation on the basis that it gives an explanation of advantages of quantum computation as a "parallel" calculation in different worlds; furthermore he claims, against current opinion, that Everett scheme has empirical differences with Standard QM (as "superposition of distinct states of consciousness"), a position that received various critics. Indeed, for of the other authors testing such a hypothesis is impossible.

Although it is surely charming and solves someway the macro-objectivation problem, however Occam’s razor, asserting that we have to refuse a theory that introduced unnecessary elements, could apply to it: indeed, in many worlds interpretation one introduces the hypothesis of a continuous generation of infinite splitting worlds without a real necessity of doing it (other possible explanations are available). Furthermore, the splitting should concern every
measurement process only, but not other processes where interference appears. However, the distinction between the two processes is not always evident.

Also, the problem of how the basis problem (i.e. how to choose in which basis the splitting of worlds happens) is still under discussion.\textsuperscript{73}

For some recent works concerning many worlds interpretation see Ref.\textsuperscript{71}.

\section*{B. Decoherence}

A different, albeit connected, point of view is the one known as quantum decoherence.

The starting point of this interpretation is considering how one can perform a measurement showing a macroscopic superposition.

For example, let us consider a system composed of two subsystems dubbed $A$ and $S$.

We suppose that in $A$ one can perform measurements only on compatible variables corresponding to different eigenspaces $A_k$. Then it exists, as one could show, an operator $T$ that has different eigenvalues $t_k$ for each eigenspace $A_k$ (where $P^A_k$ will denote the projector on this eigenspace). Every function on $A$ can be written as a function $f(T)$ of $T$.

Let us now measure the mean value of the operator $O^S f(T)$, where $O^S$ acts on the subspace $S$ and $f(T)$ on $A$. In the following $I^S, I^A$ are the identity operators on the subspaces $S, A$.

Then

$$\langle \Psi | O^S f(T) | \Psi \rangle = \langle \Psi | O^S f(T) I^S \otimes I^A | \Psi \rangle = \langle \Psi | O^S f(T) \sum_{i,k} P^S_i P^A_k | \Psi \rangle = \sum_k f(t_k) \langle \Psi | O^S P^A_k | \Psi \rangle \quad (16)$$

Let us now consider a statistical mixture of the normalised states $\frac{P^A_k | \Psi \rangle}{||P^A_k | \Psi \rangle||}$ with weights $p_k = ||P^A_k | \Psi \rangle||^2$, then

$$\langle O^S f(T) \rangle = \sum_k p_k \langle \Psi | P^A_k O^S f(T) P^A_k | \Psi \rangle = \sum_k f(t_k) \langle \Psi | O^S (P^A_k)^2 | \Psi \rangle = \sum_k f(t_k) \langle \Psi | O^S P^A_k | \Psi \rangle \quad (17)$$

which cannot be distinguished by the pure state result of Eq. (16)

The result of this analysis is that if we are bound to measure only compatible observables for a subsystem, then the pure state $| \Psi \rangle$ cannot be distinguished by a statistical mixture.

Furthermore, this implies that we cannot neglect (or limit our measurements to compatible variables) any of the constituents of the von Neumann sequence, if we want to distinguish a pure state from a statistical mixture.

The idea of quantum decoherence\textsuperscript{35–39} is that the interaction with environment makes practically impossible to identify interference for macroscopic systems as a huge amount of subsystems are rapidly involved and therefore
considering all the constituents of the von Neumann sequence becomes practically impossible (somehow related to this scheme are the models based on master equations\textsuperscript{50}). Performing correlation measurements on a macroscopic system is "de facto" impossible and thus one cannot show "de facto" a macroscopic superposition. This can be restated by asserting that after interaction with environment a pure state is transformed in statistical mixture when environment degrees of freedom are traced out. e.g. the state of Eq.4 (where now the states $|\chi\rangle$ represent the environment), when considering orthogonality of environment states, will be described by:

$$\rho_{\text{red}} = Tr^{\text{Env}} \rho_{\text{system+Env}} = |a|^2 |\phi_1\rangle\langle \phi_1| + |b|^2 |\phi_2\rangle\langle \phi_2|$$

where

$$\rho_{\text{system+Env}} = |a|^2 |\phi_1\rangle\langle \phi_1| \otimes |\chi_1\rangle\langle \chi_1| + |b|^2 |\phi_2\rangle\langle \phi_2| \otimes |\chi_2\rangle\langle \chi_2|$$

However, Bell objected that in any case this leads only to a valid for all practical purposes theory, which however does not solve quantum measurement problem definitively. One could always suppose to be able to prepare a very smart experiment which would permit to show macroscopic superpositions.

The answer of decoherence scheme supporters is the attempt of showing that such an experiment cannot be even envisaged, for it would require either an infinite components apparatus or an infinite measurement time. Many models have been studied for supporting this statement, but no general prove of it has yet been found.

Another objection\textsuperscript{72} is that within QM the correspondence between statistical ensembles and statistical operators is infinitely many to one. Thus, even when accepting that the statistical operator to be used is the one of Eq.\textsuperscript{19} there is no reason to interpret it as describing the statistical ensemble Eq.\textsuperscript{18}.

For some recent development of decoherence schemes, as Quantum Darwinism (i.e. the redundant recording of information about the preferred states of a decohering system by its environment), see\textsuperscript{74}.

C. Quantum Histories

Decoherence models are related to the quantum histories formulation of Quantum Mechanics\textsuperscript{6,35}, which somehow try to give a more precise description of the process of measurement in this framework. The same objections, just quoted at the end of previous subsection, also pertain this approach.

This formulation starts by the hypothesis that measurements have to be treated as every other interaction.

The various properties will be specified by a projector operator $P_k$, e.g. if the particle has spin in the direction $z$, this property is specified by the projector on the eigenfunction corresponding to the spin in the direction $z$ (for the
sake of generality, in the following $P_k$ is going to be considered as a generalised projector, projecting in a certain set of eigenstates corresponding to same interval of the eigenvalues).

Considering a sequence of temporal instant $t_1, t_2, \ldots t_n$ and making some precise assertions about the properties of the physical system (which must be isolated) at the various instants (for example the particle has spin in the direction $z$ etc.), a quantum history is then a sequence of statements like:

$$P_{k_n} \exp[-i/\hbar H(t_n - t_{n-1})]P_{k_{n-1}} \cdot \exp[-i/\hbar H(t_{n-1} - t_{n-2})]...P_{k_1} \exp[-i/\hbar H(t_1 - t_0)]|\Psi(t_0)\rangle$$

where $|\Psi(t_0)\rangle$ specifies the initial state.

Projectors $P_{k_i}$ corresponding to the same set, denoted by the suffix $i$, are alternative and exhaustive, $P_{k_i} P_{k'_i} = \delta_{k_i, k'_i}$ and $\sum_{k_i} P_{k_i} = 1$. The probability associated at each history is then

$$P[t_n, k_n; \ldots t_1, k_1] = ||P_{k_n} \exp[-i/\hbar H(t_n - t_{n-1})]P_{k_{n-1}} \exp[-i/\hbar H(t_{n-1} - t_{n-2})]...P_{k_1} \exp[-i/\hbar H(t_1 - t_0)]|\Psi(t_0)\rangle||^2$$

Let us emphasise again that making an assertion about the system at a certain time $t_n$ does not require, in this formulation, having performed a measurement. The probabilities Eq. [21] refer to the objective fact that the system has the indicated properties ($k_1, \ldots, k_n$) at the different instants $t_1, \ldots, t_n$.

However, it is evident that we cannot include all the possible histories together, otherwise the laws of probability would not be respected. As a simple example let us consider the very simple history:

$$P_{k_1} \exp[-i/\hbar H(t_1 - t_0)]|\Psi(t_0)\rangle$$

If we sum on all the possible values of the index $k_1$ then $\sum_{k_1} P_{k_1} = 1$ and thus the sum over the probabilities concerning the different histories where the index $k_1$ is varied is 1:

$$\sum_{k_1} P[t_1, k_1] = \sum_{k_1} ||P_{k_1} \exp[-i/\hbar H(t_1 - t_0)]|\Psi(t_0)\rangle||^2 = 1$$

This simply means that among all the various possible alternatives, only one is realised.

But we could have chosen instead of the property $k_1$ another non compatible property. For example if $k_1$ is the $z$ spin component we could have chosen the $x$ spin component. In this case the sum of the two families of histories would have probability 2! It is evident that we cannot consider all the histories together, but we have to select a subsample of ”compatible” histories.
One can show that the necessary condition for two histories for being compatible is that the so-called decoherence functional

\begin{equation}
\langle \Psi(t_0)|exp[i/hH(t_1-t_0)]P_{j_1}exp[i/hH(t_2-t_1)]P_{j_2}exp[i/hH(t_3-t_2)]...exp[i/hH(t_n-t_{n-1})]P_{j_n}\rangle
\end{equation}

\begin{equation}
P_{k_n}exp[-i/hH(t_n-t_{n-1})]P_{k_{n-1}}exp[-i/hH(t_{n-1}-t_{n-2})]...P_{k_1}exp[-i/hH(t_1-t_0)]\Psi(t_0)\end{equation}

(24)

is zero as soon as at least one of the indexes \(j_i\) differs from the corresponding index \(k_i\). This is substantially the complementarity principle: in quantum mechanics we cannot make statements on the value of complementary observables (as position and momentum) at the same time.

If we consider a measurement apparatus, \(A\), then the history concerns properties of both the quantum system, \(S\), and the apparatus itself. If we are interested into the properties of the subsystem \(S\) only, the projectors appearing in the history concern the Hilbert space pertaining \(S\) only.

A set of histories \(P_{k_n}...P_{k_1}\) determines an evolution of the initial state \(|\Psi(t_0)\rangle\) of the form (where we sum over a subset of the possible values of \(k_1\), corresponding to the ones included in the specified history, which now we assume, more generally, to be specified by a set of possible values of the observable):

\begin{equation}
|\Psi(t_1)\rangle = \sum_{k_1} P_{k_1} exp[-i/hH(t_1-t_0)]|\Psi(t_0)\rangle = \sum_{k_1} \sum_{a\epsilon k_1} |\phi_a\rangle |\chi_a\rangle
\end{equation}

(25)

where we have written \(P_{k_1} = \sum_a |\phi_a\rangle \langle \phi_a|\) (we consider \(P_{k_1}\) as projecting in a certain set denoted by the index \(a\) and \(|\phi_a\rangle\) denotes the eigenfunctions basis of an observable in \(S\). \(|\chi_a\rangle = \langle \phi_a|exp[-i/hH(t_1-t_0)]\Psi(t_0)\rangle\) is a state of the Hilbert space \(A\) and is dubbed the \(A\) state relative to the \(S\) state \(|\phi_a\rangle\). Proceeding this way, at \(t = t_n\) we have:

\begin{equation}
|\Psi(t_n)\rangle = \sum_{k_1,...,k_n} \sum_{a_1\epsilon k_1} |\phi_{a_1}\rangle |\chi_{a_1}^{k_1,...,k_n}\rangle
\end{equation}

(26)

where inside brackets is the branch specified by indexes \(k_1,...,k_n\) (corresponding to the observable considered at times \(t_1,...,t_n\).

Thus, if \(|\chi_{a_1}^{k_1,...,k_n}\rangle\) with different apices are orthogonal, we have decoherent histories, namely compatible each other. If the system is such that the \(S\) states \(|\phi_a\rangle\) are correlated to orthogonal states \(|\chi_{a_1}^{k_1,...,k_n}\rangle\), we have decoherence.

Up to here the histories formalism is just another way of formulating QM, without any solution of the macro-objectivation problem. However, the last point gives some clues how to proceed in order to give a solution of this problem: if the system becomes entangled with orthogonal states, decoherence appears. Nevertheless, the effective decoherence phenomenon is discussed in different ways by different authors.

Omnes and others use the histories together with the idea of environment decoherence\(^6\), described in the previous paragraph, and relate the macro-objectivation to the practical orthogonality of macroscopic states \(|\chi_{a_1}^{k_1,...,k_n}\rangle\). More
in detail they argue that the evolution of the composed system \( S \), quantum system, plus \( A \), apparatus, is such that starting a certain point it exists an observable such that its specifications establish a set of decoherent histories. The deterministic behaviour of the "classical" state \( A \) is due to the fact that among these histories, one has almost unit probability.

For Ref.\(^{37}\) quantum histories must be coupled to the many worlds interpretation: different incompatible histories realise in different worlds.

Finally, for Gell-Mann and Hartle\(^{39}\) one must limit histories to the whole universe: simplifying a bit, decoherence is due to the fact that it is clearly impossible to have evidences of superpositions of different states of the universe.

For some recent study in this framework see\(^{29}\).

### D. Informational interpretation

In the recent years emerged, largely motivated by quantum information studies\(^{3}\), a new approach based on the idea that quantum mechanics must be considered as a theory about "information", which is more fundamental that the concept of "substance".

As the former interpretations, but even more, also in this case there is not a univocal interpretation, but different authors inside this framework emphasize more or less some specific point.

For example in "bayesian interpretation" is suggested that "a quantum state is specifically and only a mathematical symbol for capturing a set of beliefs or gambling commitments"\(^{42}\), i.e. following bayesian subjectivist interpretation of (quantum) probabilities, one reaches a subjective interpretation of the quantum state.

For Ref.\(^{75}\) information must be considered as the fundamental basic entity, "the concept of a many-to-one state reduction is not a fundamental one but results from the practical impossibility to reconstruct the original state after the measurement". Thus, somehow this approach stops to attribute an ontology to the theory, it states that that QM gives rules about the information one can have, not on the objects themselves.

Similar ideas have been expressed in Ref.s\(^{76}\) as well.

Anyway, this approach relating to the idea that physics can be reduced to information contains several drawbacks and has been criticized by many authors\(^{1,77,78}\).
E. The modal interpretations

According to their proposers\textsuperscript{43,80} modal interpretations have the ambition to construe quantum mechanics as an objective, man-independent description of physical reality. This is achieved by stating that the relation between the formalism of quantum theory and physical reality is to be taken as probabilistic, i.e. the quantum formalism does not describe what actually is the case in the physical world, but rather provides us with a list of possibilities and their probabilities. As stated by Dieks\textsuperscript{80} "the state in Hilbert space is about possibilities, about what may be the case, about modalities".

Let us consider the bi-ortho-normal decomposition (whose existence is guaranteed by a theorem of Von Neumann) for a composed system $A - S$

$$|\Psi\rangle = \sum_{i,a} \sqrt{p_i} |\phi_i^S\rangle |\chi_i^A\rangle$$  \hspace{1cm} (27)

where the chosen states are such that

$$\langle \phi_i^S | \phi_j^S \rangle = \delta_{i,j}$$ \hspace{1cm} (28)$$

and

$$\langle \chi_a^A | \chi_b^A \rangle = \delta_{a,b}$$ \hspace{1cm} (29)

In modal interpretation the mathematical state represents situations with definite physical properties even if this state is a superposition of eigenstates of the corresponding observables, definite values are assigned to the observables associated to the projectors $|\phi_i^S\rangle \langle \phi_i^S|$. The physical situation corresponding to the mathematical description of Eq. 27 is that the partial system associated with Hilbert space $S$, taken by itself, possesses exactly one of the properties associated with the states $|\phi_i^S\rangle$. The distinct terms in Eq. 27 are treated as distinct branches, but one is always speaking of only one physical system at variance with many-world interpretation, where one has proliferation of systems. It is a fundamental property of this interpretation that a physical system always possesses exactly one of the possible values singled out by the form 27, even if they are not necessarily characterised by classical properties as position or momentum. For example for an isolated single particle system in the state $|\phi\rangle$, the applicable property would correspond, with probability 1, to the projector $|\phi\rangle \langle \phi|$. Therefore, the weight $p_i$ assumes the meaning of the fraction of the members of the set, which are in the factorised state $|\phi_i^S\rangle |\chi_i^A\rangle$. Wether we consider a quantum system $S$ and a measuring apparatus $A$, we have that modal inter-
pretation states that a fraction $p_i$ of apparatuses will be found in the well defined situation described by the state $|\chi_i^A\rangle$.

However, it must be noticed that in general the choice of the subsystems $A$ and $S$ is not univocal. If we have a quantum system in interaction with a measurement apparatus, it is rather obvious to consider the quantum system as $S$ and the apparatus as $A$. But if we consider, for example, a system of three distinguishable spin $1/2$ particles (for example three quarks of different flavour), we could consider particle 1 and 2 to form the system $A$ (for example a singlet) and particle 3 to form the system $S$. In this case we would have definite properties for the system particle 1+2 and for particle 3. On the other hand, we could have considered particle 1 and 3 to form the system $A$ (for example a singlet) and particle 2 to form the system $S$. In this case we would have definite properties for the system particle 1+3 and for particle 2. Therefore, we must be cautious in making statements about the objective properties belonging to a system, one cannot use traditional logic for making assumptions about properties of the subsystems: a new logic must be introduced (the so-called modal logic).

From the fact that the assignment of properties depends on the choice of $A$ and derives that the assignment of property is context dependent: the partial system associated with $S$ is assigned of a specific property in the context of its interaction with the environment represented by $A$.

Furthermore, a pure state is supposed to evolve according Schrödinger equation as a pure state, independently of the statements done about its properties at a certain time. But it remains unclear why it does not behave as a statistical mixture, if we reinterpret the bi-orthogonal decomposition, in the modal interpretation, as an inhomogeneous set. The modal interpretation, somehow, does not seem to offer a real objective existence to the properties of the system: the hypothetical subsystems seem to have only a conceptual and not a real status. Indeed, to take properties as existing in actuality would lead to some sort of hidden variable theory.

F. Relational Quantum Mechanics

In Relational Quantum Mechanics there are no observer independent states, nor observer independent values of physical quantities. The states are relative to the observer, which has not in principle the connotation of consciousness since any system can provide a frame of reference relative to which states and values are assigned. All observer are assumed to be equivalent. Similar to the recent developments described in subsection IV-D, Relational Quantum Mechanics describes information that one system can have about another.

As an example, let us imagine to have two observers A and B. A measures a dichotomic observable $O$ on a system
S, registering 1 and, thus, assigning the state |O, 1⟩.

B has the information that the measurement is taking place, and describes the state of A + S as \( \alpha |O, 1⟩ \otimes |A, 1⟩ + \beta |O, 0⟩ \otimes |A, 0⟩ \) where |A, 0⟩, |A, a⟩ are the pointer reading states. If B does not make a measurement it has to assign to the observable O of S 0 with probability \(|\beta|^2\) and 1 with probability \(|\alpha|^2\). Therefore, A and B assign different states to S.

A tentative answer to many questions that one can poses about this interpretation may be found in G.

G. Further interpretations

Beyond the most discussed models and interpretations presented in the previous sections, several others have appeared with a more limited success.

Among them one can mention:

- **Transational Interpretation**\(^{40}\): here a retarded offer wave (OW) \( \Psi \), formally corresponding to the usual quantum state vector, is emitted by a source. Then, depending on the experimental arrangement, components of the OW may be absorbed by one or more absorbers, each of them responding by sending an advanced (time-reversed) confirmation wave (CW) \( \Psi^* \), which travels back to the emitter. When there are N such CW responses, there are N incipient transactions in the form of OW/CW superpositions. The Echo amplitude \( \Psi \Psi^* \) at the locus of the emitter equals the Born probability. The exchange repeats until the exchange quantities (as energy) satisfy the quantum boundary conditions of the system.

- **Emergence of classical world from quantum one**: Laughlin in Ref.\(^{84}\) attributes the emergence of classical from the quantum world as deriving from the complexity (intended as a term denoting systems whose properties cannot be derived by the properties of their constituent) of measurement apparatuses. Nevertheless this "answer" may look somehow a tautology, answering to the question about why a classical apparatus does not show a quantum behaviour with "it is too complex for showing it". However, it is worth to mention here that recently Omnes proposed a similar idea, i.e. that a "non-unitary" evolution can be introduced when the "organization" of the macroscopic measuring apparatus is considered, even if a wave function collapse model was not yet proposed\(^{29}\), but demanded to future developments.

- **Belavkin’s scheme**: here the collapse of the wave function is the stochastic result of a deterministic unitary evolution when an interaction between a quantum and a classical system happens\(^{85}\), classicality being related to superselection rules stating that not all the observables are actually measurable (quantum superposition of certain
V. CONCLUSIONS

In this paper we have tried to present a panoramic view on various attempts to "solve" the problems of quantum measurement and macro-objectivation, i.e. of the transition from a probabilistic quantum mechanic microscopic world to a deterministic classical macroscopic world.

We have tried to be as unbiased as possible, presenting advantages and disadvantages of the various proposals and leaving to the reader to form (eventually using the large bibliography) an opinion on them. Of course a complete objectivity and, even more, exhaustivity in treating such arguments is impossible: we do apologize with authors that either are not satisfied of the description of their schemes or have not been quoted (properly).

Nevertheless, we hope this paper will represent a useful tool for who wants to approach this kind of studies.

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86. in the following we denote by QM quantum mechanics including all its extensions, by SQM quantum mechanics formalism without any extension or interpretation.

87. i.e. a theory where the choice of measurement settings in some place cannot affect in any way the result of a measurement performed at a space-like (i.e. not connected with a sub-luminal signal) distance.

88. non-locality that is anyway such not to allow superluminal signalling.
with the possible exception of dBB and Nelson models that are usually considered to be built to be equivalent to SQM (for some discussion on this point see\textsuperscript{19}).