The Spin-Dependent Structure Functions of Nucleons and Nuclei.

F.C Khanna\textsuperscript{a} and A.Yu. Umnikov\textsuperscript{b}

\textsuperscript{a}University of Alberta, Edmonton, Alberta T6G 2J1, Canada and TRIUMF, 4004 Wesbrook Mall, Vancouver, BC, Canada, V6T 2A3.

\textsuperscript{b} INFN, Sezione di Perugia and Department of Physics, University of Perugia, via A. Pascoli, Perugia, I-06100, Italy.

Abstract

We discuss connection between spin-dependent SFs of nucleons and nuclei in the deep inelastic lepton scattering. The case of the deuteron is studied in detail in the Bethe-Salpeter formalism.

1 Brief introduction

In the present talk we discuss the spin-dependent structure functions (SF) of nuclei and their relation to those of nucleon. Our main focus will be the deuteron, which we study in detail in the covariant Bethe-Salpeter formalism. Why it so important and interesting to study the nuclear effects in the SFs?

First, nuclei are the only source of the experimental information about neutron SFs, including the spin-dependent ones. To obtain this information, it is important to understand how nucleons are bound in the nuclei and how this binding affects their SFs. An accurate method to extract the neutron SFs from the nuclear data must be an essential part of the consistent analysis of the nucleon SFs. Second, a physics governing the processes with the participation of the nuclei is extremely interesting by itself. For instance, a spin-1 nucleus, such as a deuteron, has extra spin-dependent SFs than nucleons, i.e. $b_{12}^D$. Another example, nuclei as a slightly relativistic and weakly bound systems allows for more progress than the hadrons, in studying the covariant bound state problem. In certain situations the covariant approach gives results noticeably different from the nonrelativistic ones. For the spin-dependent SFs such situation is a calculation of the $b_{12}^D$. And third, our interest in the study of the reactions with the deuteron is in part motivated by the future and ongoing experiments. In particular, very recently we started a study of the chiral-odd SF $h_{1}^D$.

\textsuperscript{1}Talk at the Circum-Pan-Pacific Workshop on High Energy Spin Physics’96, October 2-4, 1996, Kobe, Japan.
2 Spin-dependent SF of nucleon, \( g_1^N \).

For recent reviews about the nucleon spin-dependent SFs see refs. [1, 2].

2.1 Basic formulae

The differential cross section for the polarized electron-nucleon scattering has the form:

\[
\frac{d^2\sigma}{d\Omega dE'} = \frac{\alpha^2 E'}{2m q^2 E} L^{\mu\nu} W^{\mu\nu},
\]

where \( \alpha = e^2/(4\pi) \), \( q = (\nu, 0, 0, -\sqrt{\nu^2 + Q^2}) \) is the momentum transfer, \( Q^2 = -q^2 \), \( m \) is the nucleon mass, \( E(E') \) is the energy of the incoming (outgoing) electron, \( L^{\mu\nu} \) and \( W^{\mu\nu} \) are the leptonic and hadronic tensors. The most general expression of \( W^{\mu\nu} \) is

\[
W_N^{\mu\nu}(q, p) = (2) \left( -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right) F_1^N(x_N, Q^2) + \left( p_{\mu} - q_{\mu} \frac{pq}{q^2} \right) \left( p_{\nu} - q_{\nu} \frac{pq}{q^2} \right) \frac{F_2^N(x_N, Q^2)}{pq} + \left( \frac{im}{pq} \epsilon_{\mu\nu\alpha\beta} q^\alpha \right) \left\{ S^\beta \left( g_1^N(x_N, Q^2) + g_2^N(x_N, Q^2) \right) - p^\beta \frac{(S q)}{pq} g_2^N(x_N, Q^2) \right\},
\]

where \( x_N = Q^2/(2pq) \) (in the rest frame of the nucleon \( x_N = Q^2/(2m\nu) \)) and \( S \) is the nucleon spin.

In accordance to the ideology of the quark-parton model, the SFs \( F_{1,2} \) and \( g_1 \) are proportional to the appropriate quark distributions on a part of the total longitudinal momentum of the nucleon, \( x \). For instance, if we denote the net spin carried by quarks as \( \Delta q(x, Q^2) \) \( (q = u, d, s) \) and introduce the following combinations:

\[
\Delta q_3(x, Q^2) \equiv \Delta u(x, Q^2) - \Delta d(x, Q^2),
\]
\[
\Delta q_8(x, Q^2) \equiv \Delta u(x, Q^2) + \Delta d(x, Q^2) - 2\Delta s(x, Q^2),
\]
\[
\Delta \Sigma(x, Q^2) \equiv \Delta u(x, Q^2) + \Delta d(x, Q^2) + \Delta s(x, Q^2),
\]

then the SFs of the proton(neutron) can be written as:

\[
g_{1,p,n}^N(x, Q^2) = \pm \frac{1}{12} \Delta q_3(x, Q^2) + \frac{1}{36} \Delta q_8(x, Q^2) + \frac{1}{36} \Delta \Sigma(x, Q^2). \tag{6}
\]

Important objects of the study of quark structure of the hadrons are the so-called sum rules for the SFs. The sum rules relate the moments of the SFs to
the fundamental (or sometimes not very fundamental) constants of the theory. Integrating eq. (6), the first moments of the proton (neutron) structure functions can be written in self-explaining notation:

\[ S^{p(n)} \equiv \int_0^1 dx g_1^{p(n)}(x, Q^2) = \]

\[ \frac{1}{12} \left( 1 - \frac{\alpha_s}{\pi} + \cdots \right) \left( \pm \Delta q_3 + \frac{1}{3} \Delta q_8 \right) + \frac{1}{9} \left( 1 - \frac{\alpha_s}{3\pi} + \cdots \right) \Delta \Sigma, \]

where the perturbative QCD corrections, to order \(O(\alpha_s)\), are also presented.

From the current algebra for asymptotic integrals we have \((Q^2 \to \infty)\):

\[ \Delta q_3 = 1.257 \pm 0.003, \quad \Delta q_8 = 0.59 \pm 0.02 \] (7)

The first constant is from the weak decay of the neutron and the second constant is from the decay of the hyperon. The “?” mark is due to the residual questions about SU(3).

The Bjorken sum rule is the most fundamental relation:

\[ S^p - S^n = \frac{1}{6} \left( 1 - \frac{\alpha_s}{\pi} + \cdots \right) \Delta q_3, \]

which numerically gives 0.187 ± 0.003 at \(Q^2 = 10 \text{ GeV}^2\) and 0.171 ± 0.008 at \(Q^2 = 3 \text{ GeV}^2\).

The Ellis-Jaffe sum rule is not so fundamental. Assuming that \(\Delta s = 0\) and, therefore, \(\Delta \Sigma = \Delta q_8 \approx 0.6\), we get at \(Q^2 = 10 \text{ GeV}^2 (3 \text{ GeV}^2)\):

\[ S_{EJ}^p = 0.171 \pm 0.004 \quad (0.161 \pm 0.004), \]

\[ S_{EJ}^n = -0.014 \pm 0.004 \quad (-0.010 \pm 0.004). \]

The spin-dependent SFs, \(g_1\), allow also to study spin content of the hadrons. Indeed, using eqs. (8) and experimental values of \(S^{p,n}\) (a fraction of) the nucleon spin carried by quarks, \(\Delta \Sigma\), can be determined. The total angular momentum (spin) of the nucleon consists not only of \(\Delta \Sigma\), but also:

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_z^q + L_z^G, \]

where \(\Delta G\) is the gluon spin contribution, \(L_z^{q(G)}\) is the quark (gluon) orbital angular momentum contribution. In naive quark model \(\Delta \Sigma = 1\) and others are zeros. In the relativistic quark model \(\Delta \Sigma = 0.75\) and \(L_z^q = 0.125\) and others are zeros. From the current algebra \(\Delta \Sigma \approx 0.6 \pm 0.1\), others are unknown.
2.2 Experiments for $g_1^N$

Both the SFs and the sum rules are the subject of intensive experimental studies in recent years. Table I presents measurements by various experimental groups.

Table I.

| Experiment | Year   | Target | $\sim Q_2^2$ GeV$^2$ | $S_{Target}$ |
|------------|--------|--------|-----------------------|--------------|
| E80/E130   | 1976/83| p      | 5                     | 0.17 $\pm$ 0.05 |
| EMC        | 1987   | p      | 11                    | 0.123 $\pm$ 0.013 $\pm$ 0.019 |
| SMC        | 1993   | d      | 5                     | 0.023 $\pm$ 0.020 $\pm$ 0.015 |
| SMC        | 1994   | p      | 10                    | 0.136 $\pm$ 0.011 $\pm$ 0.011 |
| SMC        | 1995   | d      | 10                    | 0.034 $\pm$ 0.009 $\pm$ 0.006 |
| E142       | 1993   | n ($^3$He) | 2                   | -0.022 $\pm$ 0.011 |
| E143       | 1994   | p      | 3                     | 0.127 $\pm$ 0.004 $\pm$ 0.010 |
| E143       | 1995   | d      | 3                     | 0.042 $\pm$ 0.003 $\pm$ 0.004 |
| HERMES     | 1996   | n ($^3$He) | 3                   | -0.032 $\pm$ 0.013 $\pm$ 0.017 |

From the SMC and E143 data the Bjorken sum rule is:

$$(S^p - S^n)_{SMC} \approx 0.199 \pm 0.038 \quad \text{at} \quad Q_2^2 = 10 \quad \text{GeV}^2, \quad (14)$$

$$(S^p - S^n)_{E143} \approx 0.163 \pm 0.010 \pm 0.016 \quad \text{at} \quad Q_2^2 = 3 \quad \text{GeV}^2, \quad (15)$$

i.e. the sum rule is confirmed with 10 % accuracy. From Table I it is clear that the Ellis-Jaffe sum rules are broken.

As to the spin content, (13), only one piece, $\Delta \Sigma$, can be extracted from the data for the integrals of SFs. The world data from Table I gives:

$$\Delta \Sigma \approx 0.3 \pm 0.1, \quad (16)$$

which is larger than the first result of EMC, $\Delta \Sigma = 0.12 \pm 0.094 \pm 0.138 \approx 0$, but still lower than quark model estimates.

In addition to the perturbative corrections in eq. (7), various other corrections, such as the kinematic mass corrections, $\sim m^2/Q^2$ and higher twist corrections, $\sim 1/Q^2$, are discussed.

3 Nucleons and nuclei

Note that actual data for the neutron is not presented in Table I, only the data for lightest nuclei. A simple formula is used to obtain $g_1^n$ from the combined
proton and deuteron data:
\[ g_D^1 = \left( 1 - \frac{3}{2} w_D \right) (g_p^1 + g_n^1), \tag{17} \]
where \( w_D \) is the probability of the \( D \)-wave state in the deuteron. Depending on the model, \( w_D = 0.04 - 0.06 \). Similarly, the neutron SF is obtained from the \( ^3\!He \) data:
\[ g_{^3\!He}^1 = \left( P_S + \frac{1}{3} P_{S'} - P_D \right) g_n^1 + \left( \frac{2}{3} P_{S'} - \frac{2}{3} P_D \right) g_p^1, \tag{18} \]
where \( P_S, P_{S'} \), and \( P_D \) are weights of the \( S, S' \) and \( D \) waves in \( ^3\!He \), respectively.

Typical (model-dependent) values of these weights are \( P_S \approx 0.897, P_{S'} = 0.017 \) and \( P_D = 0.086 \).

It is important to realize that the real connection between the nucleon and nuclear SFs is more complex than given by formulae like eq. (17) and (18). Studies of the last decade show importance of the proper separation of the binding, Fermi motion and the off-mass-shell effects in the procedure of extracting the neutron SFs from the nuclear data (see refs. [3, 4, 5] and references therein). However, effects of the Fermi motion are sometimes estimated by the experimental groups, other effects are always neglected. Such a way of action can be phenomenologically more or less safe at the present level of accuracy of the experiments, but not in general.

The deuteron is the most appropriate target to study the neutron SFs, since it has a well-known structure and well-studied wave function or relativistic amplitude. Besides all other effects such as meson exchanges, binding of the nucleons, off-mass-shell corrections, shadowing, etc, are minimal. Even in the case of \( ^3\!He \) the situation is known to be different [6, 7]. Indeed, eq. (18) or even more sophisticated convolution formula violate the fundamental Bjorken sum rule for the \( ^3\!He-^3\!H \) pair at the 3-5% level, which is a serious indication of other degrees of freedom involved in the process. Once again, this fact is completely ignored by the experimental groups reporting the results for the neutron SFs from experiments with \( ^3\!He \). In what follows we consider the nuclear effects in the spin-dependent SFs of the deuteron. Results of our studies make us certain that an accurate extraction of the neutron spin structure function, \( g_n^1 \), is possible.

Considering nuclei as a complex system of interacting nucleons and mesons, we calculate the nuclear SFs in terms of the structure functions of its constituents, nucleons and mesons, and in the Bethe-Salpeter formalism for the deuteron amplitude. For the spin-independent SFs, \( F_2^D \), the mesonic contributions to the SF
is important (although quite small) for the consistency of the approach, since the mesons carry a part of the total momentum of the nuclei (see [8] and references therein). However, for the spin-dependent SFs explicit contribution of mesons is not important. Rather, their presence manifests via binding of nucleons in nuclei. This is why we consider only nucleon contributions to the spin-dependent SFs.

We start with the general form of the hadron tensor of the deuteron with the total angular momentum projection, $M$, keeping only leading twist SFs:

$$W^D_{\mu\nu}(q, P_D, M) = (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}) F^D_1(x_D, Q^2, M) + \left( P_{D\mu} - q_{\mu} \frac{P_Dq}{q^2} \right) \left( P_{D\nu} - q_{\nu} \frac{P_Dq}{q^2} \right) \frac{F^D_2(x_D, Q^2, M)}{P_Dq} + \frac{i M_D}{P_Dq} \epsilon_{\mu\nu\alpha\beta} q^\alpha S_D^\beta(M) g^D_1(x_D, Q^2),$$

where $x_D = Q^2/(2P_Dq)$ (in the rest frame of the deuteron $x_D = Q^2/(2M_D\nu)$), $S_D(M)$ is the deuteron spin and $F^D_{1,2}$ and $g^D_1$ are the deuteron SFs. Averaged over $M$ this expression leads to the well-known form of the spin-independent hadron tensor which is valid for hadron with any spin:

$$W^D_{\mu\nu}(q, P_D) = \frac{1}{3} \sum_M W^D_{\mu\nu}(q, P_D, M)$$

$$= (-g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2}) F^D_1(x_D, Q^2) + \left( P_{D\mu} - q_{\mu} \frac{P_Dq}{q^2} \right) \left( P_{D\nu} - q_{\nu} \frac{P_Dq}{q^2} \right) \frac{F^D_2(x_D, Q^2)}{P_Dq},$$

where $F^D_{1,2}(x_D, Q^2)$ are the result of averaging of the SF $F^D_{1,2}(x_D, Q^2, M)$.

To separate $g^D_1$ we can use one of the following projectors [9]:

$$R^{(1)}_{\mu\nu} = i \epsilon_{\mu\nu\alpha\beta} q^\alpha S_D^\beta(M), \quad R^{(2)}_{\mu\nu} = \frac{i (S_D(M)q)}{P_Dq} \epsilon_{\mu\nu\alpha\beta} q^\alpha P_D^\beta.$$

In the limit $Q^2/\nu^2 \to 0$:

$$g^D_1 = \frac{R^{(1)\mu\nu} W^D_{\mu\nu}}{2\nu} = \frac{R^{(2)\mu\nu} W^D_{\mu\nu}}{2\nu}. \quad (22)$$

The nucleon contribution to the deuteron SFs is presented by the triangle graph, written in terms of the Bethe-Salpeter amplitude of the deuteron [8]:

$$W(p_1, q)$$

$$P_D \quad P_1 \quad P_2 \quad P_D$$
where $\hat{W}$ is the appropriate operator, describing the scattering on the constituent nucleon. Neglecting small correction due to the “nucleon deformation” \cite{10} it can be written down as:

$$
\hat{W}_{\mu\nu}(q, p) = \hat{W}_{\{\mu\nu\}}(q, p) + \hat{W}_{[\mu\nu]}(q, p),
$$

where $\{\ldots\}$ and $[\ldots]$ denote symmetrization and antisymmetrization of indices, respectively, $W^{N}_{\{\mu\nu\}}$ is the spin-independent part of the hadron tensor of the nucleon and $g^N_{1}(q, p) = g^N_{1}(x, Q^2)$ is the spin-dependent nucleon SF.

The explicit expressions of the deuteron SFs in terms of the Bethe-Salpeter amplitude, $\Psi_{M}(p_0, p)$, are given by:

$$
F^D_2(x_N, Q^2, M) = i \sum_{\mu, \nu} \frac{d^4p}{(2\pi)^4} \frac{F^N_2 \left( \frac{x_Nm}{p_{10} + p_{13}}, Q^2 \right)}{2M_D} \text{Tr} \left\{ \bar{\Psi}_M(p_0, p)(\gamma_0 + \gamma_3)\gamma_5\Psi_M(p_0, p)(\hat{p}_2 - m) \right\},
$$

$$
g^D_1(x_N, Q^2) = i \sum_{\mu, \nu} \frac{d^4p}{(2\pi)^4} \frac{g^N_1 \left( \frac{x_Nm}{p_{10} + p_{13}}, Q^2 \right)}{2M_D} \text{Tr} \left\{ \bar{\Psi}_M(p_0, p)(\gamma_0 + \gamma_3)\gamma_5\Psi_M(p_0, p)(\hat{p}_2 - m) \right\} |_{M=1},
$$

where $p_{10}$ and $p_{13}$ are the time and 3-rd components of the struck nucleon momentum. Averaging over the projection $M$ has not been done in eq. (26), since we use the present form later to calculate the SF $b^D_{1,2}$. Then two independent “SFs”, with $M = \pm 1$ and $M = 0$ are obtained:

$$
F^D_2(x_N, Q^2) = \frac{1}{3} \sum_{M=0, \pm 1} F^D_2(x_N, Q^2, M),
$$

$$
F^D_2(x_N, Q^2, M = +1) = F^D_2(x_N, Q^2, M = -1).
$$

A method to calculate numerically expressions like (26) and (27) is presented in ref. \cite{9}. The important details of the calculations are:

1. A realistic model for the Bethe-Salpeter amplitudes is essential for a realistic estimate of the nuclear effects. We use a recent numerical solution \cite{8} of the ladder Bethe-Salpeter equation with a realistic exchange kernel.
2. The Bethe-Salpeter amplitudes and, therefore, eqs. (26)-(27) have a non-trivial singular structure. These singularities must be carefully taken into account.

3. The BS amplitudes are numerically calculated with the help of the Wick rotation. Therefore, the numerical procedure for inverse Wick rotation must be applied.

Calculations with the realistic BS amplitudes result in the behavior of the $g_D^1$ very similar to other calculations [11, 12]. The result is presented in Fig. 1 in the form of the ratio, $g_D^1/g_N^1$. Dotted curve presents non-relativistic calculations with the Bonn wave function, solid curve the BS result. The dashed curve is the illustrative result for the non-relativistic calculations utilizing the BS axial-vector density. Last curve, dot-dashed, in Fig. 1 corresponds to the naive formula (17). Despite seemingly drastic difference in the ratio $g_D^1/g_N^1$ given by (17) and realistic calculations, the typical experimental errors today are larger. (Large fluctuations of the ratio at $x < 0.7$ are not too important. They correspond to zeros of the nucleon SF which are slightly shifted by the convolution formula.) Indeed, in Fig. 2 we present representative example of data (SMC-1994), together with two fits of these data (dashed lines). The solid lines present results of exact extraction of the nucleon SF from the present deuteron data. We see that curves for deuteron and nucleon both do not contradict the experiment.

However, in certain kinematical conditions effects can be bigger. For instance, lately much interest is devoted to the discussion of the Gerasimov-Drell-Hearn
sum rule for the proton and the neutron SFs at small $Q^2$ in general and at $Q^2 = 0$ in particular (see reviews [1, 2] and references therein). Very important contribution to the study of the neutron SFs is expected from the Jefferson Lab groups [13], where experiments with the deuterons and $^3$He are planned in the intervals $Q^2 \sim 0.15 - 2$ GeV$^2$. Analysis of the deuteron SFs in this interval of $Q^2$, the nucleon “resonances” region, shows that effect of the binding and Fermi motion is much larger here than in the deep inelastic regime. An example of the calculation of the deuteron structure function, $g_D(x, Q^2)$, is presented in Fig. 3a (dashed line) at $Q^2 = 1.0$ GeV$^2$. It is compared with the nucleon SF, $g_N(x, Q^2)$, input into the calculation. In the areas of resonance structures in $g_N(x, Q^2)$, the deuteron SF differs up to 50%! In Fig. 3b we present a comparison of the neutron SF, $g_n^D$ (solid line, input into calculations in Fig. 3a), with the “neutron” SF “extracted” by means of the naive formula (17) (dashed line). We see that these two functions have nothing in common.

![Figure 2.](image-url)

The same effects appear in $^3$He [15]. The presented example, shows that in every particular situation one has to consider the nuclear effects and take into account corresponding corrections to the SFs.

Mathematically the problem of extraction of the neutron SF from the deuteron data is formulated as a problem to solve the inhomogeneous integral equation (27) for the neutron SF with a model kernel and experimentally measured left hand side $g_D^P$. Recently we proposed a method to extract the neutron SF from the

---

2Depending on the model, some additive corrections could be taken into account.
deuteron data within any model, giving deuteron SF in the form of a "convolution integral plus/minus additive corrections" [4]. The principal advantages of the method, compared with the smearing factor method, are the following. (i) Only analyticity of the SF need be assumed, (ii) the method allows us to elaborate on the spin-dependent SF, where the traditional smearing factor method does not work.

![Figure 3.](image)

4 Other spin-dependent structure functions

4.1 SFs for spin-1 hadron, $b_{1,2}^D$

The SF $b_1^D$ is defined by (see ref. [10] and references therein):

$$b_2(x_N, Q^2) = F_2^D(x, Q^2, M = +1) - F_2^D(x, Q^2, M = 0), \quad \text{(30)}$$

Note, the SF $F_2^D(x, Q^2, M)$ is independent of the lepton polarization, therefore, both SFs, $F_2^D$ and $b_2^D$, can be measured in experiments with an unpolarized lepton beam and polarized deuteron target. In view of eq. (29), only one of the SFs $F_2^D(x, Q^2, M)$ is needed, in addition to the spin-independent $F_2^D(x, Q^2)$, in order to obtain $b_2(x, Q^2)$. The other SF, $b_1^D$, is related to the deuteron SF $F_1^D$, the same way as $b_2^D$ is related to $F_2^D$, via eqs. (28), and $b_2^D = 2xb_1^D$. 

10
Sum rules for the deuteron SFs $b_1^D$ and $b_2^D$ are a result of the fact that the vector charge and energy of the system are independent of the spin orientation:

$$\int_0^1 dx_D b_1^D(x_D) = 0, \quad \int_0^1 dx_D b_2^D(x_D) = 0. \quad (31)$$

These sum rules were suggested by Efremov and Teryaev [17].

The SFs $b_1^D$ and $b_2^D$ are calculated within two approaches as well. The results are shown in Fig. 4 a) and b). The behavior of the functions in Fig. 4 a) suggests the validity of the first of sum rules (31). At the same time, the nonrelativistic calculation for $b_2^D$ in Fig. 4 b) (dotted line) obviously does not satisfy the second sum rule. The main difference of the relativistic and nonrelativistic calculations is at small $x$, where these approaches give different signs for the SFs. To check a model dependence of the nonrelativistic calculations, we also performed calculations with the “softer” deuteron wave function (with cut-off of the realistic wave function at $|p| = 0.7$ GeV). Corresponding SFs are shown in Fig. 4 a) and b) (dashed line). It also does not affect the principle conclusion that the nonrelativistic approach violates the sum rules.
4.2 Chiral-odd SF $h_1^D$

The spin-dependent SFs $h_1$ of the nucleons and deuteron can not be measured in the inclusive deep inelastic scattering, but in the semi-inclusive process $^{18}$. In this sense these SFs are different from the SFs studied in the present paper. However, we present the results for these functions, since (i) they carry important information about the spin structure of the nucleons $^{18, 19}$ and the deuteron $^{21}$, (ii) the experiments are planned to measure them $^{20}$ and (iii) from the theoretical point of view structure function of the deuteron, $h_1^D$, is defined in a way very similar to the usual deep inelastic SFs $^{21}$:

$$h_1^D(x_N) = i \int \frac{d^4p}{(2\pi)^4} \frac{x_N m}{p_{10} + p_{13}} \frac{\text{Tr} \{ \bar{\Psi}_M(p_0, p) \gamma_5 \gamma_3 \gamma_0 \Psi_M(p_0, p)(\vec{p}_2 - m) \}}{2(p_{10} + p_{13})} \vert_{M=1}. \quad (32)$$

To calculate the realistic SF $h_1^P(x)$ we need the nucleon SFs $h_1^N(x)$. However, so far there is no existing experimental data for this function, and very little is known about the form of $h_1^N$ in theory. In the present paper we follow the ideas of ref. $^{18}$ to estimate $h_1^N$. Since the sea quarks do not contribute to $h_1^N$, its flavor content is simple:

$$h_1^N(x) = \delta u(x) + \delta d(x), \quad (33)$$

where $\delta u(x)$ and $\delta d(x)$ are the contributions of the u- and d-quarks, respectively $^{18, 19}$. Since the matrix elements of the operators $\propto \gamma_5 \gamma_3$ and $\propto \gamma_5 \gamma_3 \gamma_0$ coincide in the static limit, as a crude estimate we can expect that

$$\delta u(x) \sim \Delta u(x), \quad \delta d(x) \sim \Delta d(x), \quad (34)$$

where $\Delta u(x)$ and $\Delta d(x)$ are contributions of the u- and d-quarks to the spin of the nucleon, which is measured through the SF $g_1^N$. Correspondly, the simplest estimation for $h_1^N$

$$h_1^N(x) = \alpha \Delta u(x) + \beta \Delta d(x), \quad (35)$$

$$\alpha = \beta = 1 \quad (36)$$

should not be too unrealistic. In fact, the bag model calculation shows that difference between $\delta q$ and $\Delta q$ is typically only few percent $^{18}$. This analysis is
mostly a qualitative one, since it is limited by the case with one quark flavor and
does not pretend to describe a phenomenology.

To evaluate possible deviations from the simple choice of $h_1^N$, (33) with (36),
we suggest:

$$\alpha = \delta u / \Delta u, \quad \beta = \delta d / \Delta d,$$

(37)

where $\delta q$ and $\Delta q$ are the first moments of $\delta q(x)$ and $\Delta q(x)$, respectively ($q = u, d$). For $\delta u$ and $\delta d$ we can adopt the results from the QCD sum rules and the bag model calculations [19]. As to $\Delta u$ and $\Delta d$, we can use the experimental data analysis [1, 2] or theoretical results, e.g. the QCD sum rule results [19]. Thus, we estimate [21]:

$$\alpha = 1.5 \pm 0.5, \quad \beta = 0.5 \pm 0.5,$$

(38)

at the scale of $Q^2 = 1 \text{ GeV}^2$.

The realistic form of the distributions $\Delta u(x)$ and $\Delta d(x)$ can be taken from a fit to the experimental data for $g_1^N$. In our calculations we used parametrization from ref. [22]. At this point we have to realize that, in spite of expected relations (34), distributions $\delta q$ and $\Delta q$ are very different in their nature. Especially at $x \lesssim 0.1$, where $\Delta q$ probably contains a singular contribution of the polarized sea quarks,

Figure 5.
but δq does not. Therefore we expect eq. (35) to be a reasonable estimate in the region of the valence quarks dominance, say \( x \gtrsim 0.1 \). For completely consistent analysis, the parameters \( \alpha \) and \( \beta \), and the distributions \( \Delta u(x) \) and \( \Delta d(x) \) should be scaled to the same value of \( Q^2 \). However, for the sake of the unsophisticated estimates we do not go into such details.

The results of calculation of the nucleon and deuteron SFs, \( h_1^N \) (solid lines) and \( h_1^D \) (dashed lines), are shown in Fig. 5. The group of curves 1 represents case (36), which is a possible lower limit for \( h_1^{N,D} \) in accordance with our estimates (38). Curves 2 represent the case \( \alpha = 1.5, \ \beta = 0.5 \), which is close to the mid point results of the bag model and the QCD sum rules. The upper limit corresponding to the estimates (38) is presented by curves 3. For all cases the deuteron SF is suppressed comparing to the nucleon one, mainly because of the depolarization effect of the D-wave in the deuteron. This is quite similar to the case of the SFs \( g_1^N \) and \( g_1^D \). Note that our estimate of the nucleon SF \( h_1^N \), (38), gives systematically larger function than naive suggestion (36), the curves 1 in Fig. 5 which essentially corresponds to the estimate \( h_1^N \simeq (18/5) g_1^N \), neglecting possible negative contribution of the s-quark sea [1, 2]. The large size of the effect suggests that it can be detected in future experiments with the deuterons [20].

### 5 Brief conclusion

We have presented the results of our study of the spin-dependent structure functions of the deuteron. In particular, the leading twist \( g_1^D, b_2^D \) and \( h_1^D \) are considered. The issue of the extraction of the neutron structure functions from the deuteron data is addressed. The role of relativistic effects is studied and can be summarized as: (i) relativistic calculations give a slightly larger magnitude of the binding effects, (ii) the relativistic Fermi motion results in “harder” SF at high \( x \), and (iii) covariant approach is internally consistent, while the nonrelativistic approach is internally inconsistent and violates important sum rules.

### Acknowledgements

We wish to thank everyone who essentially contributed to studies included in this presentation, L.P. Kaptari, C. Ciofi degli Atti, Han-xin He and S. Scopetta. This work is supported in part by NSERC, Canada, and INFN, Italy.
References

[1] R.L. Jaffe, e-print archives: hep-ph/9603422 and hep-ph/9602236.
[2] Hai-Yang Cheng, e-print archive: hep-ph/9607254.
[3] L.P. Kaptari and A.Yu. Unnikov, Phys. Lett. B259 (1991) 155.
[4] A.Yu. Unnikov, F.C. Khanna and L.P.Kaptari, Z. Phys. A348 (1994) 211.
[5] W. Melnitchouk and A.W. Thomas, Phys. Lett. B377 (1996) 11.
[6] L.P. Kaptari and A.Yu. Unnikov, Phys. Lett. B240 (1990) 203.
[7] L. Frankfurt, V. Guzey, M. Strikman, Phys.Lett.B381:379-384,1996.
[8] A.Yu. Unnikov and F. Khanna, Phys. Rev. C49 (1994) 2311;
    A.Yu. Unnikov, L.P. Kaptari, K.Yu. Kazakov and F. Khanna, Phys. Lett. B334 (1994) 163.
[9] A.Yu. Unnikov, F. Khanna and L.P. Kaptari, e-print archives: hep-ph/9608549; to be published.
[10] W. Melnitchouk, A.W. Schreiber and A.W. Thomas, Phys. Rev. D49 (1994) 1183.
[11] L.P. Kaptari, K.Yu. Kazakov, A.Yu. Unnikov and B. Kämpfer, Phys. Lett. B321 (1994) 271.
[12] W. Melnitchouk, G. Piller and A.W. Thomas, Phys. Let. B346, 165 (1995).
[13] S.E. Kuhn et al., CEBAF Proposal No. 93-009; Z.E. Meziani et al., CEBAF Proposal No. 94-010.
[14] C. Ciofi degli Atti, L.P. Kaptari, S. Scopetta and A.Yu. Unnikov, Phys. Lett. B376 (1996) 309.
[15] C. Ciofi degli Atti, S. Scopetta, e-print archive: nucl-th/9606034; to be published.
[16] A. Yu. Unnikov, e-print archive: nucl-th/9605291.
[17] A.V. Efremov and O.V. Teryaev, Sov. J. Nucl. Phys. 36, 557 (1982).
[18] R.L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552; Nucl. Phys. B375 527.
[19] H.X. He and X. Ji, Phys. Rev. D 52 (1995) 2960;
    H.X. He and X. Ji, MIT-CTP-2551; e-print archive: hep-ph/9607408.
[20] The RSC proposal to RHIC, 1993; The HERMES proposal to HERA, 1993.
[21] A.Yu. Unnikov, Han-xin He and F. Khanna, e-print archives: hep-ph/9609353; to be published.
[22] A. Schäfer, Phys. Lett. B208, 175 (1988).