Pang, C. H. Jeffrey
Set intersection problems: supporting hyperplanes and quadratic programming. (English)
Zbl 1311.90144
Math. Program. 149, No. 1-2 (A), 329-359 (2015).

Summary: For finitely many closed convex sets \( K_1, \ldots, K_r \) in a Hilbert space \( X \), the set intersection problem (SIP) is stated as:
\[
\text{Find } x \in K := \bigcap_{i=1}^{r} K_i, \text{ where } K \neq \emptyset.
\] (SIP)

A popular method of solving SIP is the method of alternating projections, where one iteratively projects a point through the sets \( K_i \) to find a point in \( K \). Another problem related to the SIP is the best approximation problem (BAP): Find the closest point to \( x_0 \) in \( K \), that is
\[
\min_{x \in X} \|x - x_0\| \text{ s.t. } x \in K := \bigcap_{i=1}^{r} K_i
\] (BAP)

for closed convex sets \( K_i, i = 1, \ldots, r \). – The aim of this paper is make use of the supporting hyperplanes generated in the projection process to accelerate the convergence to a point in \( K \). The author proves theoretical properties of the alternating projection method supplemented with the insight on supporting hyperplanes.

First, he proposes an algorithm for the BAP, proves norm convergence, and the finite convergence of the algorithm when \( K_i \subset \mathbb{R}^n \) have a local conic structure and satisfy a normal condition. Also, the author shows that the normal condition cannot be dropped. It is discussed the behavior of the algorithm in the case when the intersection of the closed convex sets is empty.

Second, he proposes modifications of the alternating projection algorithm for the SIP, and proves their convergence, and, the superlinear convergence of a modified alternating projection algorithm in \( \mathbb{R}^2 \).

Finally, the author proves the superlinear convergence of an algorithm for the SIP in \( \mathbb{R}^n \) under theoretically conditions.

It remains to be seen whether the theoretical properties in this paper translate to effective algorithms in practice.

MSC:
- 90C30 Nonlinear programming
- 90C59 Approximation methods and heuristics in mathematical programming
- 47J25 Iterative procedures involving nonlinear operators
- 47A46 Chains (nests) of projections or of invariant subspaces, integrals along chains, etc.
- 47A50 Equations and inequalities involving linear operators, with vector unknowns
- 52A20 Convex sets in \( n \) dimensions (including convex hypersurfaces)
- 41A50 Best approximation, Chebyshev systems
- 41A65 Abstract approximation theory (approximation in normed linear spaces and other abstract spaces)
- 46C05 Hilbert and pre-Hilbert spaces; geometry and topology (including spaces with semidefinite inner product)
- 49J53 Set-valued and variational analysis
- 65K10 Numerical optimization and variational techniques

Keywords:
Dykstra’s algorithm; best approximation problem; alternating projections; quadratic programming; supporting hyperplanes; superlinear convergence
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