Full Diversity Blind Signal Designs for Unique Identification of Frequency Selective Channels

Jian-Kang Zhang  
Department of Electrical and Computer Engineering,  
McMaster University, Hamilton, Ontario, Canada.  
Email: jkzhang@mail.ece.mcmaster.ca

Chau Yuen  
Institute for Infocomm Research, Room 03-07  
21 Heng Mui Keng Terrace, Singapore 119613.  
Email: cyuen@i2r.a-star.edu.sg

Abstract—In this paper, we develop two kinds of novel closed-form decompositions on phase shift keying (PSK) constellations by exploiting linear congruence equation theory: the one for factorizing a \(pq\)-PSK constellation into a product of a \(p\)-PSK constellation and a \(q\)-PSK constellation, and the other for decomposing a specific complex number into a difference of a \(p\)-PSK constellation and a \(q\)-PSK constellation. With this, we propose a simple signal design technique to blindly and uniquely identify frequency selective channels with zero-padded block transmission under noise-free environments by only using the first two block received signal vectors. Furthermore, a closed-form solution to determine the transmitted signals and the channel coefficients is obtained. In the Gaussian noise and Rayleigh fading environment, we prove that the newly proposed signaling scheme enables non-coherent full diversity for the Generalized Likelihood Ratio Test (GLRT) receiver.

I. INTRODUCTION

In this paper, we consider wireless communication systems with a single transmitting antenna and a single receiving antenna which transmit data over a frequency-selective fading channel. The systems which we consider mitigate the inter-symbol interference generated by the channel by transmitting the data stream in consecutive equal-size blocks, which are subsequently processed at the receiver on a block-by-block basis, see, e.g., [1]–[3]. In order to remove interblock interference, some redundancy is added to each block before transmission. There are several ways to add redundancy (e.g., [1], [2]), but in this paper we will focus on block-by-block communication systems with zero-padding redundancy; e.g., [1]–[3].

When the receiver possesses perfect knowledge of the channel and employs maximum likelihood (ML) detection, it was shown [3], [4] that such a system not only enables full diversity but also provide the maximum coding gain. Unfortunately, perfect channel state information at the receiver, in practice, is not easily attainable. If the coherent time is sufficiently long, then, the transmitter can send training signals that allow the receiver to estimate the channel coefficients accurately. For mobile wireless communications, however, the fading coefficients may change so rapidly that the coherent time may be too short to allow reliable estimation of the coefficients, especially in a system with a large number of antennas. Therefore, the time spent on sending training signals cannot be ignored because of the necessity of sending more training signals for the accurate estimation of the channel [5], [6].

To avoid having to transmit training signals, considerable research efforts have been directed to develop techniques of “blind channel estimation” [7], [8] recently. These techniques identify and estimate the transmission channel using only the received (perhaps noisy) signals at the receiver. The essence of these algorithms is to exploit the structure of the channel and/or the property of transmitted signals. The subspace method is one such method exploiting the channel structure and the second order statistics of input signals [8], [9]. In digital communication applications, the input signals have finite alphabet property such as the constant modulus for PSK modulation or integers for QAM, which can be further exploited to estimate the channel with shorter coherent time than the subspace method by numerically solving some optimization problems [10]. Thus far, all currently available blind methods without either training or pilots for the frequency selection channel estimation incur scale ambiguity and as a consequence, cannot uniquely identify the channel coefficients. In addition, wireless communication applications demand the accurate estimate of the channel as well as of the signal. The resulting scale ambiguity from the channel estimation will result in significant error probability for the signal estimation even in a noise-free environment.

To resolve this issue, we propose a novel signaling and transmitting technique for the frequency selective channel with zero-padded block transmission, in which neither the transmitter or the receiver knows the channel state information. Our main contributions in this paper are as follows:

1) A novel signal design technique using a pair of co-prime \(p\)-PSK and \(q\)-PSK constellations for the first two block transmissions is proposed to blindly and uniquely identify frequency selective channels with zero-padded block transmission by only processing the first two block received signals. Furthermore, a closed-form solution to determine the transmitted signals and the channel coefficients is obtained by utilizing the linear congruence equation theory.

2) In the Gaussian noise and Rayleigh fading environment, we prove that the newly proposed scheme enables full diversity for the GLRT receiver.
Here, we should point out that the similar hybrid signaling schemes [11]–[15] were used to eliminate the ambiguity of the blind orthogonal space-time block codes.

Notation: Column vectors and matrices are boldface lowercase and uppercase letters, respectively; the matrix transpose, the complex conjugate, the Hermitian are denoted by $(\cdot)^T, (\cdot)^*, (\cdot)^H$, respectively; $I_N$ denotes the $N \times N$ identity matrix; $\gcd(m, n)$ denotes the greatest common divisor of positive integers $m$ and $n$. Particularly when $\gcd(m, n) = 1$, we say that $m$ and $n$ are coprime integers; $\varphi(n)$ denotes the number of all positive integers that do not exceed $n$ and are prime to $n$. The $(i, j)$th element of matrix $A$ is denoted by $a_{ij}$.

II. CHANNEL MODEL AND BLIND SIGNAL DESIGN

If the channel is assumed to be of length at most $L$ (i.e., if $L$ is an upper bound on the delay spread), then, the block transmission system with zero-paddling operate as follows: First, $L - 1$ zeros are append to $x$ to form $x'$ which is of length $P = K + L - 1$. The elements of $x'$ are serially transmitted through the channel. The channel impulse response is denoted by $h = [h_0, h_1, \ldots, h_{L-1}]^T$. The length $P$ received signal vector $r$ can be written as

$$r = Hs + \xi,$$

where $r$ is a $P \times 1$ received signal vector, $s$ is a $K \times 1$ transmitted signal vector, $\xi$ denotes the $P \times 1$ vector of noise samples at the receiver, and $H$ denotes the $P \times K$ Toeplitz matrix [1]–[3]

$$H = \begin{pmatrix}
    h_0 & 0 & \ldots & 0 \\
    h_1 & h_0 & \ldots & 0 \\
    \vdots & \ddots & \ddots & \vdots \\
    h_{L-1} & \ldots & h_1 & h_0 \\
    0 & \ddots & \ddots & \ddots \\
    \vdots & \ddots & \ddots & \ddots \\
    0 & \ldots & 0 & h_{L-1}
\end{pmatrix}_{P \times K}.$$

For blind channel estimation, it is convenient to rewrite the channel model (1) as

$$r = T(s)h + \xi,$$

where we have used the following fact $Hs = T(s)h$ with $T(s)$ defined by

$$T(s) = \begin{pmatrix}
    s_1 & 0 & \ldots & 0 \\
    s_2 & s_1 & \ldots & 0 \\
    \vdots & \ddots & \ddots & \vdots \\
    s_K & \ldots & s_2 & s_1 \\
    0 & \ddots & \ddots & \ddots \\
    \vdots & \ddots & \ddots & \ddots \\
    0 & \ldots & 0 & s_K
\end{pmatrix}_{P \times L}.$$

Now we assume that during a $T$ block transmission period, the channel coefficients keep constant and after that, will randomly change. Our novel blind modulation scheme is described as follows: During the first block transmission, each symbol of a transmitted signal vector $s = x$ is chosen from $p$-PSK constellation $\mathcal{X}$; i.e., $r_1 = T(x)h + \xi_1$, $x \in \mathcal{X}^K$. During the second block transmission, each symbol of a transmitted signal vector $s = y$ is chosen from $q$-PSK constellation $\mathcal{Y}$; i.e., $r_2 = T(y)h + \xi_2$, $y \in \mathcal{Y}^K$, where $p$ and $q$ are coprime. Then, during the $i$th block transmission for $3 \leq i \leq T$, each symbol of a transmitted signal vector $s = z_i$ can be chosen from any constellation $\mathcal{Z}$; i.e., $r_i = T(z_i)h + \xi_i$, $z_i \in \mathcal{Z}^K$. Collecting all the $T$ block received signals, we have

$$z = Sh + \eta,$$

where $\eta = (\xi_1^T, \xi_2^T, \ldots, \xi_T^T)^T$ and

$$S = (T(x), T(y), T(z_1), \ldots, T(z_T))^T$$

for $x \in \mathcal{X}^K, y \in \mathcal{Y}^K, z_i \in \mathcal{Z}^K$. Throughout this paper, we make the following assumptions:

1) The channel coefficients $h_\ell$ for $\ell = 0, 1, \ldots, L - 1$ are samples of independent circularly symmetric zero-mean complex white Gaussian random variables with unit variances and remain constant for the first $PT$ ($T \geq 2$) time slots, after which they change to new independent values that are fixed for next $PT$ time slots, and so on.

2) The elements $\eta$ is circularly symmetric zero-mean complex Gaussian samples with covariance matrix $\sigma^2 I_P$.

3) During $PT$ observable time slots, $T$ consecutive blocks $z_i$ are transmitted with each entry $z_{ik}$ for $i = 3, 4, \ldots, T$ and $k = 1, 2, \ldots, P$ being independently and equally likely chosen from the constellation $\mathcal{Z}$, while components $x_k$ and $y_k$ for $k = 1, 2, \ldots, P$ in the previous two blocks are independently and equally likely chosen from the respective $p$-PSK and $q$-PSK constellations $\mathcal{X}$ and $\mathcal{Y}$, where $p$ and $q$ are coprime.

4) Channel state information is not available at either the transmitter or the receiver.

Our goal in this paper is to prove that our designed signaling scheme [3]

1) enables the unique identification of the channel and the transmitted signals for any given nonzero received signal vector $r$ in a noise-free case and

2) provides full diversity for the GLRT receiver in the Gaussian noise and Rayleigh fading environment.

III. BLIND UNIQUE IDENTIFICATION AND FULL DIVERSITY

In this section, we first develop some decomposition properties on a pair of PSK constellations and then, prove that our blind modulation scheme proposed in the previous section enables the unique identification of the channel coefficients and the transmitted signals in a noise-free case as well as full diversity for the GLRT receiver in a noise environment.
A. Decompositions of PSK Constellations

**Proposition 1**: Let two positive integers \(p\) and \(q\) be coprime. Then, for any integer \(k\) with \(0 \leq k < pq\), there exists a pair of \(x\) and \(y\) such that
\[
xy = \exp\left(\frac{2\pi k}{pq}\right).
\]  
Furthermore, \(s_p\) and \(s_q\) can be uniquely determined by
\[
x = \exp\left(\frac{2\pi k q^p(p^{-1})}{p}\right) \quad (5a)
\]
\[
y = \exp\left(\frac{2\pi k p^q(q^{-1})}{q}\right) \quad (5b)
\]

Proposition 1 tells us that any \(pq\)-PSK symbol can be uniquely factored into the product of a pair of the coprime \(p\)-PSK symbol and \(q\)-PSK symbol. This factorization was first discovered by Zhou, Zhang and Wong [11], [13]. Now, Proposition 1 significantly simplifies the original representation. The following property gives a necessary and sufficient condition for a complex number to be able to be decomposed into a difference of a pair of the coprime \(p\)-PSK symbol and \(q\)-PSK symbol.

**Proposition 2**: Let \(w \neq 0\) be a given non-zero complex number and \(w = |w|e^{j\theta}\). Then, there exists a pair of \(x \in X\) and \(y \in Y\) satisfying equation
\[
x - y = w
\]  
if and only if there exist three integers \(m, n\) and \(k\) such that
\[
\theta = \pi(pm + qm) + k\pi + \frac{\pi}{2} \quad (7a)
\]
\[
|w| = (-1)^{k+1} \sin\left(\frac{\pi(pm - qm)}{pq}\right). \quad (7b)
\]

Furthermore, under the condition \(7\), if \(p\) and \(q\) are coprime, then, equation \(6\) has the unique solution that can be explicitly determined as follows:

1) If \(w = 0\), then, \(x = y = 1\).
2) If \(w \neq 0\), then, \(x\) and \(y\) are given by
\[
x = \exp\left(\frac{2\pi k q^p(p^{-1})}{p}\right) \quad (8a)
\]
\[
y = \exp\left(\frac{2\pi k p^q(q^{-1})}{q}\right) \quad (8b)
\]
with integer \(\ell = pq\left(\frac{\theta}{\pi} - \frac{1}{2}\right)\).

The proofs of Propositions 1 and 2 are omitted because of space limitation.

B. Blind unique identification of the channel

Our main purpose in this subsection is to prove that the blind signaling scheme \(3\) is capable of uniquely identifying the channel coefficients and the transmitted symbols. To do this, let \(u\) and \(v\) be two consecutive block received signal vectors in the first two block transmission from the channel model \(2\) in a noise-free environment; i.e.,
\[
u = T(x)h \quad x \in X^K, \quad (9)
\]
\[
v = T(y)h \quad y \in Y^K. \quad (10)
\]

Now, we formally state the first result.

**Theorem 1**: (Unique Identification) Let \(u = (u_1, u_2, \ldots, u_p)^T\) and \(v = (v_1, v_2, \ldots, v_p)^T\) be the first two consecutive block nonzero received signal vectors given by \(9\) and \(10\), respectively, and \(p\) and \(q\) be co-prime. If \(r\) denotes the maximum integers such that \(u_1v_1 = u_2v_2 = \cdots = u_r v_r = 0\), then, \(h_0 = h_1 = \cdots = h_{r-1} = 0\). In addition, the other remaining channel coefficients \(h_r, h_{r+1}, \ldots, h_{L-1}\) and all the transmitted symbols in \(x\) and \(y\) can be uniquely determined as follows.

1) Let \(w_1\) be defined by \(w_1 = \frac{u_{r+1}}{v_{r+1}} = |w_1|e^{j\theta_1}\) and \(\ell_1 = \frac{\exp\theta_1}{2\pi}\). Then, we have
\[
x_1 = \exp\left(\frac{2\pi \ell_1 q^p(p^{-1})}{p}\right) \quad (11a)
\]
\[
y_1 = \exp\left(-\frac{2\pi \ell_1 p^q(q^{-1})}{q}\right) \quad (11b)
\]
\[
h_r = x_1^ru_{r+1}. \quad (11c)
\]

2) For \(1 < m \leq L - r\), let \(w_m\) be defined by
\[
w_m = x_m^T\left(\tau_{r+m} - \sum_{i=1}^{m-2} h_{r+i}x_{m-i}\right)\]
\[
v_m = y_m^T\left(v_{r+m} - \sum_{i=1}^{m-2} h_{r+i}x_{m-i}\right). \quad (12)
\]

a) If \(w_m = 0\), then, we have
\[
x_m = x_1 \quad (13a)
\]
\[
y_m = y_1 \quad (13b)
\]
\[
h_{r+m-1} = x_1^T(u_{r+m} - \sum_{i=1}^{m-2} h_{r+i}x_{m-i}). \quad (13c)
\]

b) If \(w_m \neq 0\), let \(w_m = |w_m|e^{j\theta_m}\) and \(\ell_m = \frac{\exp\theta_m}{2\pi}\). Then, we have
\[
x_m = x_1 \exp\left(\frac{2\pi \ell_m q^p(p^{-1})}{p}\right) \quad (14a)
\]
\[
y_m = y_1 \exp\left(\frac{2\pi \ell_m p^q(q^{-1})}{q}\right) \quad (14b)
\]
\[
h_{r+m-1} = x_1^T(u_{r+m} - \sum_{i=1}^{m-2} h_{r+i}x_{m-i}). \quad (14c)
\]

3) For \(L - r + 1 \leq m \leq K\), we have
\[
x_m = \frac{u_{m+r} - \sum_{i=1}^{L-r-1} h_{L-i}x_{m-L+i}}{h_r} \quad (15a)
\]
\[
y_m = \frac{u_{m+r} - \sum_{i=1}^{L-r-1} h_{L-i}y_{m-L+i}}{h_r} \quad (15b)
\]

**Proof**: Basically, the proof of Theorem 1 captures the following four steps.
Step 1. First, we consider the first received signals in each block. In this case, we have
\[ h_0 x_1 = u_1, \quad \text{(16a)} \]
\[ h_0 y_1 = v_1. \quad \text{(16b)} \]

Therefore, if either \( u_1 \) or \( v_1 \) is zero, then, \( h_0 \) is zero. Similarly, we can obtain \( h_1 = h_2 = \cdots = h_{r-1} = 0 \) if \( u_1 v_1 = u_2 v_2 = \cdots = u_r v_r = 0 \).

Step 2. We continue to proceed the \((r+1)\)-th received signals for each block. In this case, we have received
\[ h_r x_1 = u_{r+1}, \quad \text{(17a)} \]
\[ h_r y_1 = v_{r+1}. \quad \text{(17b)} \]

Since \( u_{r+1} v_{r+1} \neq 0 \), eliminating \( h_1 \) from (17) results in
\[ x_1 y_1^* = \frac{u_{r+1}}{v_{r+1}} w_1. \quad \text{(18)} \]

Now, by Proposition 1, \( x_1 \) and \( y_1 \) can be uniquely determined by (17a) and (17b), respectively, and thus, \( h_r \) is uniquely determined by (17c).

Step 3. Let us consider the \((r+2)\)-th received signals for each block:
\[ h_{r+1} x_1 + h_r x_2 = u_{r+2}, \quad \text{(19a)} \]
\[ h_{r+1} y_1 + h_r y_2 = v_{r+2}. \quad \text{(19b)} \]

Eliminating \( h_{r+1} \) from (19) yields
\[ x_1 x_2 - y_1 y_2 = \frac{u_{r+2} x_2^* - v_{r+2} y_2^*}{h_r} w_2. \quad \text{(20)} \]

Since \( x_i \in \mathcal{X} \) and \( y_i \in \mathcal{Y} \) for \( i = 1, 2 \), we have \( x_1 x_2 \in \mathcal{X} \) and \( y_1 y_2 \in \mathcal{Y} \) too. Now, by Proposition 2, \( x_2 \) and \( y_2 \) can be uniquely determined by (13a) or (14a) and (13b) or (14b), respectively, and thus, \( h_{r+1} \) is uniquely determined by (13c) or (14c) with \( m = 2 \).

In general, we proceed to consider determining the \((r+m)\)-th channel coefficient for \( 2 < m \leq L - r - 1 \). In this case, we have received
\[ h_{r+m-1} x_1 + h_{r+m-2} x_2 + \cdots + h_r x_m = u_{r+m}, \quad \text{(21a)} \]
\[ h_{r+m-1} y_1 + h_{r+m-2} y_2 + \cdots + h_r y_m = v_{r+m}. \quad \text{(21b)} \]

This is equivalent to
\[ h_{r+m-1} x_1 + h_r x_m = u_{r+m} - \sum_{i=1}^{m-2} h_{r+i} x_{m-i}. \quad \text{(22a)} \]
\[ h_{r+m-1} y_1 + h_r y_m = v_{r+m} - \sum_{i=1}^{m-2} h_{r+i} y_{m-i}. \quad \text{(22b)} \]

Eliminating \( h_{r+m-1} \) from (22) yields
\[ x_1 x_m - y_1 y_m = \frac{x_1 \left( u_{r+m} - \sum_{i=1}^{m-2} h_{r+i} x_{m-i} \right)}{h_r} \]
\[ - \frac{y_1 \left( v_{r+m} - \sum_{i=1}^{m-2} h_{r+i} y_{m-i} \right)}{h_r} = w_m. \quad \text{(23)} \]

Now, by Proposition 2, \( x_m \) and \( y_m \) can be uniquely determined by (13a) or (14a) and (13b) or (14b), respectively, and thus, \( h_{r+m-1} \) is uniquely determined by (13c) or (14c).

Step 4. \( L - r - 1 < m \leq L \). In this case, since the channel coefficients have been determined by the previous three steps, we can determine the other remaining transmitted signals from the remaining received signals of each block. In this case, we have
\[ h_{L-1} x_{m-1} + h_{L-2} x_{m-2} + \cdots + h_r x_m = u_{r+m}, \quad \text{(24a)} \]
\[ h_{L-1} y_{m-1} + h_{L-2} y_{m-2} + \cdots + h_r y_m = v_{r+m}. \quad \text{(24b)} \]

From this we can obtain (15). This completes the proof of Theorem 1.

We would like to make the following observations on Theorem 1:

1) Theorem 1 not only tells us that the channel coefficients can be uniquely identified by only transmitting two block signals with each symbol selected from two co-prime PSK constellations, but also provides simple and closed-form solutions to both the channel coefficients and the transmitted symbols. The traditional blind method [9] based on the second order statistics requires a lot of data blocks. Even so, it still cannot uniquely identify the channel coefficients as well as the transmitted signals.

2) If we set \( p = 2^m \) and \( q = 2^m + 1 \) with \( m \) being a positive integer, then, it is clear that \( p \) and \( q \) are coprime. Therefore, Theorems 1 holds for such a pair of \( p \) and \( q \). In addition, if we want the original symbol sets \( \mathcal{X} \) and \( \mathcal{Y} \) to contain the same integer bits, we can delete the only one common element 1 from \( \mathcal{Y} \); i.e., \( \mathcal{Y} = \mathcal{Y} - \{1\} \). Thus, there are totally \( 2^m \) elements in the remaining set \( \mathcal{Y} \) and Theorems 1 still holds for such a pair of constellations \( \mathcal{X} \) and \( \mathcal{Y} \).

3) By Theorem 1, if we let \( m_0 \) denote the maximum positive integer such that \( u_{r+m} - \sum_{i=1}^{m-2} h_{r+i} x_{m-i} = 0 \) for \( m_0 < m \leq L - r \) but \( u_{r+m_0} - \sum_{i=1}^{m_0-2} h_{r+i} x_{m_0-i} = 0 \), then, the length of the channel is actually equal to \( m_0 - r + 1 \). Therefore, our blind modulation scheme enables the receiver to exactly determine the length of the channel by only utilizing the first two block received signals.

C. Full diversity

The GLRT requires neither the knowledge of the fading and noise statistics, nor the knowledge of their realizations [16]. The criterion can be simply stated as \( \hat{S} = \arg \max_x \{ z^H S \left( S^H S \right)^{-1} S^H z \} \). In fact, the GLRT projects the received signal \( z \) on the different subspaces spanned by \( S \) and then calculate the energies of all the projections and choose the projection that maximizes the energy. Now, in order to examine full diversity, for any pair of distinct codewords \( \mathbf{S} \) and \( \tilde{S} \), let \( \left( \frac{\mathbf{S}^H}{\tilde{S}^H} \right) \begin{bmatrix} \mathbf{S} \\ \tilde{S} \end{bmatrix} = \mathbf{A} \). Brehler and Varanasi [17] proved the following lemma.

Lemma 1: If matrices \( \mathbf{A} \) have full column rank for all pair of distinct codewords \( \mathbf{S} \) and \( \tilde{S} \), then, the code-book provides full diversity for the GLRT receiver.
Now, we are in position to formally state the second our main result.

Theorem 2: The blind modulation designed in Section III with $p$ and $q$ being coprime enables full diversity for the GLRT receiver.

Proof: By Lemma 1, we only need to prove that $(S, \tilde{S})$ has full column rank for any pair of distinct signal matrices $S$ and $\tilde{S}$. Otherwise, if there existed a pair of distinct codeword matrices $S$ and $\tilde{S}$ for which the matrix $(S, \tilde{S})$ does not have full column rank, then, the linear equations with respect to variables $h$ and $\tilde{h}$, \[
\begin{pmatrix}
S & \tilde{S} \\
\tilde{h} & -h
\end{pmatrix} \begin{pmatrix}
h \\
\tilde{h}
\end{pmatrix} = 0,
\]
would have a nonzero solution $h_0$ and $\tilde{h}_0$. Let $r_0 = Sh_0$. Then, we would also have $r_0 = Sh_0$. In other words, for a given nonzero received signal $r_0$, equation $r_0 = Sh$ has two distinct pair of solutions. By Theorem 1, we have that $T(x) = T(\tilde{x})$ and $T(y) = T(\tilde{y})$. Since $S \neq \tilde{S}$, there is a pair of distinct signal sub-matrices in $S$ and $\tilde{S}$, $T(z_i)$ and $T(\tilde{z}_i)$ for some $3 \leq i \leq L$. That being said, $z_i \neq \tilde{z}_i$. If we let $z_i = (z_{i1}, z_{i2}, \cdots, z_{iK})^T$ and $\tilde{z}_i = (\tilde{z}_{i1}, \tilde{z}_{i2}, \cdots, \tilde{z}_{iK})^T$, then, there exists a positive integer $k$ such that $z_{ik} \neq \tilde{z}_{ik}$ but $z_{i\ell} = \tilde{z}_{i\ell}$ for $\ell = 1, 2, \cdots, k - 1$. For notional simplicity, we use $B[M : N]$ to denote the sub-matrix of a matrix $B$ consisting of all the columns and the rows from $M$ to $N$. Then, we have the result (25), which is shown in the bottom of this page, where we have used the fact that $T(\tilde{z}_i)[k : K + k - 1] - T(z_i)[k : K + k - 1]$ is actually a $K \times K$ lower triangular matrix with the diagonal entries being all equal to $z_{ik} - \tilde{z}_{ik}$. Therefore, the sub-matrix of $(S, \tilde{S})$, \[
\begin{pmatrix}
\begin{bmatrix}
T(x_i)[1 : K] \\
T(\tilde{x}_i)[1 : K]
\end{bmatrix} & T(z_i)[k : K + k - 1] - T(\tilde{z}_i)[k : K + k - 1]
\end{pmatrix}
\]
is a $2K \times 2K$ invertible matrix and hence, $(S, \tilde{S})$ has full column rank, which contradicts with the previous assumption. This completes the proof of Theorem 2.

So far, we have shown that our blind modulation scheme not only enables the unique identification of the channel coefficients in the noise-free case but also full diversity in the noise environment. However, the traditional blind method [9] based on the second order statistics can provide neither the unique identification of the channel coefficients nor estimation of the transmitted signals with full diversity reliability. Similar to the Comment 2) on Theorem I, our Theorem 3 is also true for both a particular pair of $p = 2^m$ and $q = 2^m + 1$ and the derived pair of constellations.

IV. CONCLUSION

In this paper, we proposed a novel blind modulation tech- nique to uniquely identify frequency selective channels with zero-padded block transmission under noise-free environments by only processing the first two block received signal vectors. Furthermore, a closed-form solution to determine the transmitted signals and the channel coefficients was derived by using linear congruence equation theory. In the Gaussian noise and Rayleigh fading environment, we proved that our new scheme enables full diversity for the GLRT receiver.

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