Superon-graviton model and supersymmetric structure of spacetime and matter

KAZUNARI SHIMA\textsuperscript{a}, MOTOMU TSUDA\textsuperscript{a} and MANABU SAWAGUCHI\textsuperscript{b}

\textsuperscript{a}Laboratory of Physics, Saitama Institute of Technology
Okabe-machi, Saitama 369-0293, Japan

\textsuperscript{b}High-Tech Research Center, Saitama Institute of Technology
Okabe-machi, Saitama 369-0293, Japan

Abstract

A new Einstein-Hilbert type (SGM) action describing gravitational interaction of Nambu-Goldstone (N-G) fermion of nonlinear supersymmetry (NL SUSY) is obtained by performing the Einstein gravity analogue geometrical arguments in high symmetric four dimensional (SGM) spacetime. All elementary particles except graviton are regarded as the composite eigenstates of SO(10) super-Poincaré algebra (SPA) composed of the fundamental N-G fermion “superons” of NL SUSY. Some phenomenological implications of the composite picture of SGM, the linearization of SGM and $N = 2$ Volkov-Akulov model are discussed.

\textsuperscript{*}Invited Talk at Advanced Study Institute, Praha-Spin-02, Praha, Czech Republic, July 14-27, 2002

\textsuperscript{a}e-mail: shima@sit.ac.jp
\textsuperscript{b}e-mail: tsuda@sit.ac.jp
\textsuperscript{c}e-mail: sawa@sit.ac.jp
1 Introduction

Despite the success of the standard model (SM) as a unified model for the strong and the electroweak interaction, there still remain many unsolved problems, e.g. it can not explain the particle quantum numbers (\(Q, I, Y, \text{color, i.e.} U(1) \times SU(2) \times SU(3) \text{ gauge structure}\)), the three-generations structure and contains more than 28 parameters (even disregarding the mass generation mechanism for neutrino). The gravitational interaction is not considered. SM and GUT equipped minimally with supersymmetry (SUSY) have improved the situations in some points, but they are still pathological on the proton decay problem and less predictive due to more than 100 parameters.

Although SUSY\(^1\) is an essential notion to unify spacetime and matter, unfortunately SO(8) SUGRA in four dimensional spacetime is too small to accommodate all observed particles as elementary fields. The straightforward extension to SO\((N \geq 9)\) SUGRA has a difficulty due to so called the no-go theorem on the coupling of gravitation and the massless elementary high spin\((> 2)\) gauge field.

However it is well known that in the monopole phase, (i.e. at the very short distances of spacetime), the degrees of freedom (dimensions) of spacetime are fused with the dimensions of (the linear representation of) the local symmetry, which allows to define a unified (composite) field strength of the monopole configuration through the symmetry breaking \(SU(2) \times SO(3,1) \rightarrow U(1) \times SO(3,1)\)[2]. These phenomena suggest that spacetime itself would reveal unfamiliar features at the short distance by the identification\((fusion)\) of the symmetries of spacetime with those of matter and that these ultimate spacetime would be specified by a certain unified (composite) field strength\((curvature)\), where the no-go theorem becomes irrelevant in a sense that the fundamental Lagrangian with \(N \geq 9\) SUSY may be written down. Also, we think that from the viewpoint of simplicity and beauty of nature it is interesting to attempt the accommodation of all observed particles in a single irreducible representation of a certain algebra\(\text{(group)}\), especially for spacetime having a certain boundary \((\text{boundary condition})\). The fundamental theory should be given by only the geometrical arguments of high symmetrical spacetime and its spontaneous breakdown, which is encoded in the geometrical argument of spacetime by itself. In this talk we would like to present a model along this scenario.

2 Superon-Graviton Model (SGM)

Among single irreducible representations of all SO\((N)\) extended super-Poincaré\(\text{(SP)}\) symmetries, the massless irreducible representations of SO\((10)\) SP algebra\(\text{(SPA)}\)
is the only one that accommodates minimally all observed particles including the graviton. By considering that (i) for the massless case the algebra of the supercharges of SO(10) SPA in the light-cone frame can be recasted as those of the creation and annihilation operators of fermions and (ii) 10 generators $Q^N(N = 1, 2, ..., 10)$ of SO(10) SPA are the fundamental representations of SO(10) internal symmetry and decomposed $10 = 5 + 5^*$ with respect to SU(5) following $SO(10) \supset SU(5)$ and span $2 \cdot 2^{10}$ dimensional massless irreducible representation of SO(10) SPA, we can regard 10 generators $\mathbf{10} = \mathbf{5} + \mathbf{5}^*$ as the fundamental massless objects; a superon-quintet and an antisuperon-quintet with spin $\frac{1}{2}$ and that all the helicity states are the massless (gravitational) eigenstates of spacetime and matter with SO(10) SP symmetric structure, which are composed of superon. We regard (broken) SO(10) SP symmetry is to spacetime and matter (nature) what O(4) symmetry is to the relativistic hydrogen atom. To survey the physical implications of superon-graviton model (SGM) for spacetime and matter we assign the following SM quantum numbers to superons and adopt the following symbols (and the conjugates for anti-supersons).

$$\mathbf{5} = \left[ Q_a (a = 1, 2, 3), Q_m (m = 4, 5) \right] = \left[ (3, 1: -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}), (1, 2: 1, 0) \right], \quad (1)$$

where we have specified $(SU(3), SU(2); electric charges)$. Superon-quintet satisfy the Gell-Mann–Nishijima relation; $Q_e = I_z + \frac{1}{2}(B - L)$. Accordingly all $2 \cdot 2^{10}$ helicity states (up to helicity 3) are specified uniquely with respect to $(SU(3), SU(2); electric charges)$. Here we assume boldly an ideal super Higgs-like mechanism, i.e. all unnecessary (for SM) higher helicity states become massive by absorbing the lower helicity states in $SU(3) \times SU(2) \times U(1)$ invariant way via $SO(10)$ SPA upon the Clifford vacuum $\rightarrow [SU(3) \times SU(2) \times U(1)] \rightarrow [SU(3) \times U(1)]$. We have carried out the recombinations of the helicity states and found surprisingly that all the massless states necessary for the SM with three generations of quarks and leptons appear in the surviving massless states specified by the superon contents. For three generations of leptons $[(\nu_e, e), (\nu_\mu, \mu), (\nu_\tau, \tau)]$, we take

$$\left[ (Q_m \varepsilon_{ln} Q_l^c Q_m^*), (Q_m \varepsilon_{ln} Q_l^c Q_n^* Q_a Q_a^*), (Q_a Q_a^* Q_b Q_b^* Q_m^*) \right] \quad (2)$$

and for three generations of quarks $[(u, d), (c, s), (t, b)]$, we have uniquely

$$\left[ (\varepsilon_{abc} Q_b^c Q_c^* Q_m^*), (\varepsilon_{abc} Q_b^c Q_c^* Q_l \varepsilon_{mn} Q_m^* Q_n^*), (\varepsilon_{abc} Q_a^* Q_b^* Q_c^* Q_d Q_m^*) \right] \quad (3)$$

and their conjugates respectively. For $SU(2) \times U(1)$ gauge bosons $[W^+, Z, \gamma, W^-]$, $SU(3)$ color-octet gluons $[G^a(a = 1, 2, ..., 8)]$, $[SU(2)$ Higgs Boson], $[(X, Y)]$ lepto-quark bosons in GUTs, and a color- and SU(2)-singlet neutral gauge boson from $\mathbf{2} \times \mathbf{2}^*$ (called S boson) we have $[Q_4 Q_4^*, \frac{1}{\sqrt{2}}(Q_4 Q_4^* \pm Q_5 Q_5^*), Q_5 Q_5^*, [Q_1 Q_1^*, Q_2 Q_2^*, ...]$, $[\varepsilon_{abc} Q_a Q_b Q_c Q_m], [Q_a Q_m^*]$, and $Q_a Q_a^*$, (and conjugates) respectively.
Among predicted new particles one lepton-type electroweak-doublet ($\nu, \Gamma^-$) with spin $3/2$ with the mass of the electroweak scale ($\leq$ Tev), one neutral gauge boson $S$ and one doubly charged lepton $E^{2-}$ are color singlets and can be observed directly. Further studies are needed to estimate the masses of $S$ and $E^{2-}$.

More to see the potential of SGM and to survey the evidence of the compositeness of matter in the low energy we interpret (reproduce) the Feynman diagrams of SM(GUT) in terms of the superon pictures, i.e. a single line of a propagating particle is replaced by multiple lines representing superons in the particle under two assumptions at the vertex: (i) the analogue of the OZI-rule of the quark model and (ii) the superon number conservation. Many remarkable new insights are obtained qualitatively, e.g. in SM; naturalness of the mixing of $K^0 - \bar{K}^0$, $D^0 - \bar{D}^0$ and $B^0 - \bar{B}^0$, no CKM-like mixings among the lepton generations, $\nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$ transitions beyond SM, strong CP-violation, small Yukawa couplings and no $\mu \rightarrow e + \gamma$ despite the compositeness, etc. and in (SUSY)GUT; proton is stable without R-parity by hand (i.e., absence of the dangerous diagrams), etc. SGM may be the most economic model. The arguments are group theoretical so far.

### 3 Fundamental Theory of Superon-Graviton Model (SGM)

By noting the supercharges $Q$ of Volkov-Akulov(V-A) model of the NL SUSY given by the supercurrents $J^\mu(x) = \frac{1}{2} \sigma^\mu \psi(x) - \kappa \{ \text{the higher order terms of } \kappa, \psi(x) \}$ satisfy the SP algebra, we find that the fundamental theory of SGM for spacetime and matter at (above) the Planck scale is SO(10) NL SUSY in the curved spacetime. This is the field-current identity and justifies our bold assumption that the generator (supercharge) $Q^N (N = 1, 2, \ldots 10)$ of SO(10) SPA in the light-cone frame represents the fundamental massless particle, superon with spin $\frac{1}{2}$. We have written down the SGM action by performing the similar arguments to Einstein general relativity theory (EGRT) in high symmetric four dimensional (curved) SGM spacetime, where NL SUSY N-G fermion degrees of freedom $\psi(x)$ (i.e. the coset space coordinates of superGL(4R)/GL(4R) representing N-G fermions) are embedded at every curved spacetime point:

$$ L_{SGM} = -\frac{c^3}{16\pi G} |w| (\Omega + \Lambda), $$(4)

where $|w| = \det w^a_\mu = \det(e^a_\mu + t^a_\mu)$, $t^a_\mu = (\kappa/2i) \sum_{j=1}^{10} (\bar{\psi}^j \gamma^a \partial_\mu \psi^j - \partial_\mu \bar{\psi}^j \gamma^a \psi^j)$, and $\kappa^{-1} = \frac{c^4 \Lambda}{16\pi G}$ is a fundamental volume of four dimensional spacetime and $\Lambda$ is a small cosmological constant related to the superon-vacuum coupling constant. $\Omega$ is a new scalar curvature analogous to the Ricci scalar curvature $R$ of EGRT, whose explicit
The commutators of two new NL SUSY transformations (5) on $\psi$ expression is obtained by just replacing $e^a_\mu(x)$ by $w^a_\mu(x)$ in Ricci scalar $R$. These results can be understood intuitively by observing that $w^a_\mu(x) = e^a_\mu(x) + t^a_\mu(x)$ defined by $\omega^a = w^a_\mu dx^\mu$, where $\omega^a$ is the NL SUSY invariant differential forms of V-A[3], is invertible and $s^{\mu\nu}(x) \equiv w^a_\mu(x)w^{a\nu}(x)$ are a unified vierbein and a unified metric tensor in SGM spacetime[4][5].

The SGM action (4) is invariant at least under global SO(10), ordinary GL(4R), the following new NL SUSY transformation;

$$\delta \psi^i(x) = \zeta^i + i\kappa(\bar{\zeta}^j\gamma^\rho\psi^j(x))\partial_\rho \psi^i(x), \quad \delta e^a_\mu(x) = i\kappa(\bar{\zeta}^j\gamma^\rho\psi^j(x))\partial_\rho e^a_\mu(x),$$

(5)

where $\zeta^i, (i = 1,..10)$ is a constant spinor and $\partial_\rho e^a_\mu(x) = \partial_\rho e^a_\mu - \partial_\mu e^a_\rho$, the following GL(4R) transformations due to (5);

$$\delta \zeta w^a_\mu = \xi^\nu \partial_\nu w^a_\mu + \partial_\nu \zeta^\nu w^a_\mu, \quad \delta s_{\mu\nu} = \xi^\kappa \partial_\kappa s_{\mu\nu} + \partial_\mu \xi^\kappa s_{\kappa\nu} + \partial_\nu \xi^\kappa s_{\mu\kappa},$$

(6)

where $\xi^\rho = i\kappa(\bar{\zeta}^j\gamma^\rho\psi^j(x))$, and the following local Lorentz transformation on $w^a_\mu$;

$$\delta_L w^a_\mu = \epsilon^b_a w^b_\mu$$

(7)

with the local parameter $\epsilon_{ab} = (1/2)\epsilon_{[ab]}(x)$ or accordingly on $\psi$ and $e^a_\mu$

$$\delta_L \psi(x) = -\frac{i}{2} \epsilon_{ab} \sigma^{ab}\psi, \quad \delta_L e^a_\mu(x) = \epsilon^b_a e^b_\mu + \frac{k}{4} \epsilon^{abcd}\gamma_5 \gamma_d \psi(\partial_\mu \epsilon_{bc}).$$

(8)

The commutators of two new NL SUSY transformations (5) on $\psi(x)$ and $e^a_\mu(x)$ are GL(4R), i.e. new NL SUSY is the square-root of GL(4R), e.g.

$$[\delta_{\zeta_1}, \delta_{\zeta_2}] \psi = \Xi^a \partial_\mu \psi, \quad [\delta_{\zeta_1}, \delta_{\zeta_2}] e^a_\mu = \Xi^\rho \partial_\rho e^a_\mu + e^a_\mu \partial_\rho \Xi^\rho,$$

(9)

where $\Xi^a = 2i\kappa(\bar{\zeta_2}^\mu \zeta_1^a - \xi_1^a \xi_2^\mu e^a_\mu(\partial_\rho e^a_\rho))$. They show the closure of the algebra.

SGM action (4) is invariant at least under[7] [global NL SUSY] $\otimes$ [local GL(4,R)] $\otimes$[local Lorentz] $\otimes$[global SO(N)], which is isomorphic to SO(10)SP corresponding to the linear representation of SGM. Now we have written down $N = 10$ SUSY theory including graviton, which has circumvented the no-go theorem so far.

4 Toward Low Energy Theory of SGM

The linearization of such a high nonlinear theory is inevitable to obtain a renormalizable field theory which is equivalent.

As a flat space limit of SGM, we have shown that $N = 2$ V-A model is equivalent to the spontaneously broken $N = 2$ linear SUSY(L SUSY) vector $J^P = 1^-$ gauge supermultiplet model with spontaneously broken SU(2) structure[8]. The linearization of $N = 1$ V-A model has been carried out and shows that it is equivalent.
to \( N = 1 \) scalar supermultiplet or \( N = 1 \) axial vector gauge supermultiplet of L SUSY. We conjecture that any global L SUSY (unified) model is equivalent to a NL SUSY model. These results are favorable to the SG M scenario based upon the composite (eigenstates) nature of all elementary particles except graviton.

Now we show explicitly by the heuristic and practical arguments that for \( N = 2 \) SUSY a SUSY invariant relation between component fields of a vector supermultiplet of L SUSY and N-G fermions of the V-A model of NL SUSY is written by using only three arbitrary dimensionless parameters, which can be recast as the vacuum expectation values of auxiliary fields in the vector supermultiplet of L SUSY. We denote in this paper the component fields of an \( N = 2 \) U(1) gauge supermultiplet as follows; namely, \( A \) and \( B \) for two physical scalar fields, \( A_a \) for a U(1) gauge field and \( \lambda' \) \((i = 1,2)\) for two Majorana spinors in addition to \( F, G \) and \( D \) for three auxiliary scalar fields at least for a free vector supermultiplet required from the mismatch of the degrees of freedom between bosonic and fermionic physical fields. The component fields indeed belong to representations of a rigid SU(2) \[13\]; namely, \( \lambda \) and \((F, G, D)\) belong to representations 2 and 3 of SU(2) respectively while other fields are singlets. In Ref.[8] we have linearized \( N = 2 \) NL SUSY in the manifestly SU(2) covariant form. The L SUSY transformations of these component fields generated by constant (Majorana) spinor parameters \( \zeta^i \) are

\[
\begin{align*}
\delta A &= \bar{\zeta}^1 \lambda^1 + \bar{\zeta}^2 \lambda^2, \\
\delta B &= \bar{i} \zeta^1 \gamma_5 \lambda^1 + \bar{i} \zeta^2 \gamma_5 \lambda^2, \\
\delta A_a &= -\bar{i} \zeta^1 \gamma_a \lambda^2 + \bar{i} \zeta^2 \gamma_a \lambda^1, \\
\delta \lambda^1 &= \{(F + i \gamma_5 G) - \bar{i} \gamma \bar{\theta}(A + i \gamma_5 B)\} \zeta^1 - i F_{ab} \sigma^{ab} \zeta^2 + i \gamma_5 \zeta^2 D, \\
\delta \lambda^2 &= \{(F - i \gamma_5 G) - \bar{i} \gamma \bar{\theta}(A + i \gamma_5 B)\} \zeta^2 + i F_{ab} \sigma^{ab} \zeta^1 + i \gamma_5 \zeta^1 D, \\
\delta \bar{F} &= -\bar{i} \zeta^1 \bar{\gamma} \bar{\theta} \lambda^1 - \bar{i} \zeta^2 \bar{\gamma} \bar{\theta} \lambda^2, \\
\delta \bar{G} &= \bar{\zeta}^1 \gamma_5 \bar{\theta} \lambda^1 - \bar{\zeta}^2 \gamma_5 \bar{\theta} \lambda^2, \\
\delta D &= \bar{\zeta}^1 \gamma_5 \bar{\theta} \lambda^2 + \bar{\zeta}^2 \gamma_5 \bar{\theta} \lambda^1,
\end{align*}
\]

which satisfy a closed off-shell algebra.

On the other hand, in the \( N = 2 \) V-A model we have a NL SUSY transformation laws of (Majorana) N-G fermions \( \psi^i \) generated by \( \zeta^i \),

\[
\delta \psi^i = \frac{1}{\kappa} \bar{\zeta}^i - i \kappa (\bar{\zeta}^j \gamma^a \psi^j) \partial_a \psi^i, \tag{10}
\]

where and hereafter \( \kappa \) is a constant whose dimension is \((\text{mass})^{-2}\). Eq.(10) also satisfies the off-shell commutator algebra without a U(1) gauge transformation.

From above L and NL SUSY transformations a SUSY invariant relation between the component fields of the \( N = 2 \) vector supermultiplet and the N-G fermion fields \( \psi^i \) is obtained at the leading orders of \( \kappa \) as follows: Indeed, adopting an ansatz

\[
\begin{align*}
\lambda^1 &= (\xi + \bar{i} \theta \gamma_5) \psi^1 + (\eta + \bar{i} \varphi \gamma_5) \psi^2 + \ldots, \\
\lambda^2 &= (\xi' + \bar{i} \theta' \gamma_5) \psi^1 + (\eta' + \bar{i} \varphi' \gamma_5) \psi^2 + \ldots. \tag{11}
\end{align*}
\]

with \( \xi, \eta, \theta, \varphi, \xi', \eta', \theta' \) and \( \varphi' \) being eight arbitrary real parameters which are the most general one for the dimensionless case, we substitute (11) and (10) into the
The results are

\[ A = \frac{1}{2} \kappa \xi (\bar{\psi}^1 \psi^1 + \bar{\psi}^2 \psi^2) + \frac{i}{2} \kappa \theta (\bar{\psi}^1 \gamma_5 \psi^1 - \bar{\psi}^2 \gamma_5 \psi^2) + i \kappa \varphi \bar{\psi}^1 \gamma_5 \psi^2 + \ldots , \]

\[ B = \frac{i}{2} \kappa \xi (\bar{\psi}^1 \gamma_5 \psi^1 + \bar{\psi}^2 \gamma_5 \psi^2) - \frac{1}{2} \kappa \theta (\bar{\psi}^1 \psi^1 - \bar{\psi}^2 \psi^2) - \kappa \varphi \bar{\psi}^1 \psi^2 + \ldots , \]

\[ A_a = -i \kappa \xi \bar{\psi}^1 \gamma_a \psi^2 + \kappa \theta \bar{\psi}^1 \gamma_5 \gamma_a \psi^2 - \frac{1}{2} \kappa \varphi (\bar{\psi}^1 \gamma_5 \gamma_a \psi^1 - \bar{\psi}^2 \gamma_5 \gamma_a \psi^2) + \ldots , \]

\[ \lambda^1 = (\xi + i \theta \gamma_5) \psi^1 + i \varphi \gamma_5 \psi^2 + \ldots , \]

\[ \lambda^2 = (\xi - i \theta \gamma_5) \psi^2 + i \varphi \gamma_5 \psi^1 + \ldots , \]

\[ F = \xi \left\{ \frac{1}{\kappa} - i \kappa (\bar{\psi}^1 \theta \psi^1 + \bar{\psi}^2 \theta \psi^2) \right\} - \kappa \theta (\bar{\psi}^1 \gamma_5 \theta \psi^1 - \bar{\psi}^2 \gamma_5 \theta \psi^2) \]

\[ - \kappa \varphi \partial_a (\bar{\psi}^1 \gamma_5 \gamma^a \psi^2) + \ldots , \]

\[ G = \theta \left\{ \frac{1}{\kappa} - i \kappa (\bar{\psi}^1 \theta \psi^1 + \bar{\psi}^2 \theta \psi^2) \right\} + \kappa \xi (\bar{\psi}^1 \gamma_5 \theta \psi^1 - \bar{\psi}^2 \gamma_5 \theta \psi^2) \]

\[ - i \kappa \varphi \partial_a (\bar{\psi}^1 \gamma_5 \gamma^a \psi^2) + \ldots , \]

\[ D = \varphi \left\{ \frac{1}{\kappa} - i \kappa (\bar{\psi}^1 \theta \psi^1 + \bar{\psi}^2 \theta \psi^2) \right\} + \kappa \xi \partial_a (\bar{\psi}^1 \gamma_5 \gamma^a \psi^2) \]

\[ + i \kappa \theta \partial_a (\bar{\psi}^1 \gamma_5 \gamma^a \psi^2) + \ldots , \]

in which the three arbitrary real parameters \( \xi \), \( \theta \) and \( \varphi \) are involved. The first term \(-i \kappa \xi \bar{\psi}^1 \gamma_a \psi^2 \) in Eq.(13) shows the vector nature of the U(1) gauge field as shown in [8]. Also Eqs.(18) to (20) for the auxiliary fields \( F \), \( G \) and \( D \) have the form which is proportional to a determinant \( |w| = \text{det}(w^a_b) \) in the \( N = 2 \) V-A model (with \( w^a_b \) being defined by \( w^a_b = \delta^a_b + t^a_b \) and \( t^a_b = -i \kappa^2 \bar{\psi}^i \gamma^a \partial_b \psi^i \) plus total derivative terms at least at the leading orders of \( \kappa \): namely, \( F = (\xi/\kappa) \times [ \text{leading terms of } |w| ] + [ \text{tot. der. } ] + \ldots , \) etc. In addition, the first terms in Eqs.(18) to (20) or the SUSY transformations of Eqs.(16) and (17) show that \( \xi/\kappa \), \( \theta/\kappa \) and \( \varphi/\kappa \) correspond to the vacuum expectation values of the auxiliary fields \( F \), \( G \) and \( D \).

We can continue to obtain higher order terms in the SUSY invariant relations: After some calculations we obtain the relation between \( \lambda^i \) and the N-G fermion fields \( \psi^i \) at \( O(\kappa^2) \) as

\[ \lambda^1 = (\xi + i \theta \gamma_5) \psi^1 + i \varphi \gamma_5 \psi^2 - \frac{i}{2} \kappa^2 \xi \{ (\bar{\psi}^1 \theta \psi^1) \psi^1 - (\bar{\psi}^1 \gamma_5 \theta \psi^1) \gamma_5 \psi^1 \]
\[
\psi_1^a \left( \gamma^a \psi_1 + \left( \gamma^5 \partial_a \psi_1 \right) \gamma^5 \gamma^a \psi_1 \right) + \ldots,
\]
and \( \lambda^2 \) is obtained by exchanging the indices 1 and 2 and by replacing \( \theta \) with \(-\theta\) in Eq. (21). We can also construct the SUSY invariant relation with respect to the bosonic fields of the linear supermultiplet at \( O(\kappa^3) \). In principle, we can further continue to obtain higher order terms in the SUSY invariant relation following this approach. However, it will be more useful to extend the superfield formalism Refs. [9] [11] [12] to \( N = 2 \). Remarkably, Eqs. (13) to (20) (and also (21), etc.) reduce to that of the \( N = 1 \) SUSY by imposing, e.g. \( \psi^2 = 0 \): Indeed, when \( \xi = 1 \) and \( \theta = \varphi = 0 \), they become that of the scalar supermultiplet obtained in Ref. [10]. When \( \varphi = 1 \) and \( \xi = \theta = 0 \), they reduce to that of the U(1) gauge supermultiplet obtained in Refs. [9] [12].

Now we consider an action which is invariant under L SUSY.

\[
S_{\text{lin}} = \int d^4x \left[ \frac{1}{2} \left( \partial_a A \right)^2 + \frac{1}{2} \left( \partial_a B \right)^2 - \frac{1}{4} F_{ab}^2 + \frac{i}{2} \bar{\lambda} \gamma^a \lambda^i + \frac{1}{2} \left( F^2 + G^2 + D^2 \right) - \frac{1}{\kappa} (\xi F + \theta G + \varphi D) \right],
\]

(22)

where \( \xi, \theta \) and \( \varphi \) are three arbitrary real parameters satisfying \( \xi^2 + \theta^2 + \varphi^2 = 1 \). The last three terms proportional to \( \kappa^{-1} \) is an analogue of the Fayet-Iliopoulos D term in the \( N = 1 \) theories [14]. The field equations for the auxiliary fields \( F, G \) or \( D \) are \( F = \xi/\kappa, G = \theta/\kappa \) or \( D = \varphi/\kappa \) indicating a spontaneous SUSY breaking. Substituting (13) to (20) into the linear action \( S_{\text{lin}} \) of (22), we can show immediately that \( S_{\text{lin}} \) coincides with the following V-A action \( S_{\text{VA}} \) up to and including \( O(\kappa^0) \); namely, \( S_{\text{VA}} = -\frac{1}{2\kappa^2} \int d^4x |w| = -\frac{1}{2\kappa^2} \int d^4x [1 + \tilde{t}^a a + \ldots] \), which is invariant under (14).

We note that the linearization of \( N = 2 \) SUSY in this paper can be discussed as a manifestly (rigid) SU(2) invariant form [9], which gives more concise expressions of the SUSY invariant relation (13) to (20) (and also (21), etc.). In these arguments, adopting the general ansatz (11) having the eight real dimensionless parameters with \( \kappa^0 \), we have explicitly shown that for \( N = 2 \) SUSY the SUSY invariant relation (13) to (20) (and also (21), etc.) is written by using only three arbitrary parameters, which can be recast as the vacuum expectation values of the auxiliary fields in the vector supermultiplet. These heuristic arguments are practical and show more general assumptions adopted for obtaining the SUSY invariant relation.

The analysis by using the NL SUSY superfield in curved spacetime [1] may be useful to carry out the computations to all orders and make the arguments transparent. From those arguments on the linearization of \( N = 1 \) and \( N = 2 \) SUSY, we speculate that any renormalizable (\( N \)-extended) global L SUSY (interacting) model is equivalent to the (\( N \)-extended) V-A model despite the difference of the number of the dynamical degrees of freedom. These results support the SGM scenario [3, 4] which
is a composite model of matter based on the global NL SUSY (generalized V-A[15]) model in curved spacetime. It is interesting that the appearance of vector (not axial) gauge field as a composite necessitates $N = 2$ NL SUSY, i.e. SU(2) structure (and its spontaneous breakdown to U(1)) in L SUSY. The compositeness of all elementary particles, i.e. composite picture of SGM may explain $SU(2) \times U(1)$ gauge structure in SM.

As for the linearization of SGM, we have recently obtained the SUSY invariant relations between $e^a_\mu(x)$, $\psi_\mu(x)$ and N-G field $\psi(x)$, e.g.

$$\psi_\mu(x) = \frac{\sqrt{\kappa}}{4} \gamma_\alpha \gamma^\rho \psi(x) \partial_{\nu} e^a_\mu(x), \quad (23)$$

which produce the closed algebra [16]. The details of SGM case will appear soon. The cosmology of SGM and SGM with extra dimensions, ...etc. are open.

The work of M. Sawaguchi is supported in part by the special research project of High-Tech Research Center of SIT.

References

[1] For a reference J. Wess and J. Bagger, *Supersymmetry and Supergravity* (Princeton Univ. Press, 1992).

[2] A. M. Polyakov, *JETP Lett.* 20, 194 (1974).

G. 't Hooft, *Nucl. Phys.* B79, 276 (1974).

[3] K. Shima, Z. Phys. C18, 25 (1983).

K. Shima, *European. Phys. J.* C7, 341 (1999).

[4] K. Shima, *Phys. Lett.* B501, 237 (2001).

[5] D.V. Volkov and V.P. Akulov, *Phys. Lett.* B46, 109 (1973).

[6] K. Shima, Proceeding of Institute of Mathematics of National Academy of Science of Ukraine (2002), 1-10, ed. by A. G. Nikitin, et.al.

[7] K. Shima and M. Tsuda, *Phys. Lett.* B507, 260 (2001).

[8] K. Shima, Y. Tanii and M. Tsuda, *Phys. Lett.* B in press, hep-th/0205178 (2002).

[9] E.A. Ivanov and A.A. Kapustnikov, *J. Phys.* A11, 2375 (1978).

E.A. Ivanov and A.A. Kapustnikov, *J. Phys.* G8, 167 (1982).
[10] M. Roček, *Phys. Rev. Lett.* **41**, 451 (1978).

[11] T. Uematsu and C.K. Zachos, *Nucl. Phys.* **B201**, 250 (1982).

[12] K. Shima, Y. Tani and M. Tsuda, *Phys. Lett.* **B525**, 183 (2002).

[13] P. Fayet, *Nucl. Phys.* **B113**, 135 (1976).

[14] P. Fayet and J. Iliopoulos, *Phys. Lett.* **B51**, 461 (1974).

[15] J. Wess, Festschrift fuer 60th Birthday of J. Lopszanski (1982).
    K. Shima and M. Tsuda, Class. Quantum Grav. in press (2002).

[16] K. Shima, M. Tsuda and M. Sawaguchi, in preparation.