The CMB modulation from inflation

David H. Lyth
Consortium for Fundamental Physics,
Cosmology and Astroparticle Group, Department of Physics,
Lancaster University, Lancaster LA1 4YB, UK

May 10, 2014

Abstract

Erickcek, Kamionkowski and Carroll proposed in 2008 that the dipole modulation of the CMB could be due to a very large scale perturbation of the field \( \phi \) causing the primordial curvature perturbation. We repeat their calculation using weaker assumptions and the current data. If \( \phi \) is the inflaton of any single-field inflation with the attractor behaviour, the asymmetry is almost certainly too small. If instead \( \phi \) is any curvaton-type field (ie. one with the canonical kinetic term and a negligible effect during inflation) the asymmetry can agree with observation if \( |f_{\text{NL}}| \) in the equilateral configuration is \( \simeq 10 \) for \( k^{-1} = 1 \text{ Gpc} \) and \( \lesssim 3 \) for \( k^{-1} = 1 \text{ Mpc} \). An \( f_{\text{NL}} \) with these properties can apparently be obtained from the curvaton with an axionic potential. Within any specific curvaton-type model, the function \( f_{\text{NL}}(k_1, k_2, k_3) \) required to generate the asymmetry would be determined, and could perhaps already be confirmed or ruled out using existing Planck or WMAP data.

1 Introduction

In 1978 Grishchuk and Zel’dovich investigated the effect of a very large-scale enhancement of the spectrum of the primordial curvature perturbation, upon the CMB anisotropy [1]. Within the observable universe the enhancement is expected to give an approximately linear function of position, but the linear component has no effect upon the observed CMB anisotropy. The leading observable effect is expected to come from the component that is a quadratic function of position; it gives an enhancement to the CMB quadrupole, called the Grishchuk-Zel’dovich effect, which is not observed leading to an upper bound on the magnitude of the quadratic component.

According to present thinking the primordial density perturbation comes from the perturbation of some field \( \phi \), which is generated from the vacuum
fluctuation during inflation. We are therefore talking about a very large scale contribution to the spectrum of $\phi$. This will generate a contribution to the spectrum of the curvature perturbation, giving the GZ effect, but it will also generate a very large scale contribution $\delta\phi_L(x)$ to $\phi$. As was pointed out by Erickcek, Kamionkowski and Carroll (EKC) in 2008 [2], the linear component of $\delta\phi_L$ will have a potentially observable effect; it will make $\zeta$ statistically anisotropic within the observable universe, generating a dipole modulation of the CMB anisotropy for which there is now some evidence [3, 4]. I call this the EKC effect.

EKC looked at two possibilities for $\phi$; that it is the inflaton of slow-roll inflation or that it is the curvaton [7] with a quadratic potential. In this paper we allow $\phi$ to be either the inflaton of any single-field model of inflation with the attractor behaviour, or any curvaton-type field. The latter could be the curvaton with a generic potential or more generally any field with the canonical kinetic term which has a negligible effect during inflation.

We begin in Section 2 by recalling the dipole modulation and its presumed origin. In Section 3 we recall the concept of a quasi-local contribution to $f_{NL}$, that is generated when the potential of $\phi$ is not quadratic (self-interaction of $\phi$). In Section 4 we calculate $A(k)$ with $\phi$ the inflaton and with $\phi$ a curvaton-type field. The normalization of $A(k)$ is proportional to the gradient of $\delta\phi_L$, which at this stage is not constrained.

In Section 5 we obtain an upper bound on the gradient of $\delta\phi_L$, assuming that the observable universe occupies a typical location within a region that encloses all significant wavelengths of $\delta\phi_L$. The bound is obtained by requiring that (i) the GZ effect on the quadrupole is not observed and (ii) the expectation value of $\zeta^2$ (for a random location of the observable universe) is not too much bigger than 1. In the Conclusion we summarise our result and consider alternative proposals for generating the dipole modulation. In an Appendix we describe the treatment of the GZ effect by Erickeck et. al. [2], which is different from ours.

2 CMB asymmetry from statistical inhomogeneity of $\zeta$

The CMB anisotropy has been analysed to search for a dipole modulation of a statistically isotropic quantity $\Delta T_{iso}$:

$$\Delta T(\hat{n}) = (1 + A\hat{p} \cdot \hat{n}) \Delta T_{iso}(\hat{n}),$$

where the unit vector $\hat{n}$ is the direction in the sky, the unit vector $\hat{p}$ is fixed. ‘Statistically isotropic’ here means that the correlators of $\Delta T_{iso}$ within a disk on the sky are independent of the location of that disc.

1The perturbations of two or more fields might be involved but we do not consider that possibility.
Using Eq. (1) for $\ell < \ell_{\text{max}} = 64$, and $\Delta T_{\text{iso}}$ for higher $\ell$, Ref. [3] uses WMAP data to find $|A| = 0.07 \pm 0.02$.\(^2\) Smoothing on a 5° scale (corresponding to $\ell_{\text{max}} \sim 12$) Ref. [4] uses Planck data to find the same result for $A$. Using a different method, and without making an a posteriori choice for $\ell_{\text{max}}$, Ref. [5] argues that such results are not statistically significant, but in this paper we take them to represent a real effect.

We assume that the dipole modulation of $\Delta T$ comes from statistical inhomogeneity of $\zeta$ within the observable universe. Since $\Delta T$ depends mostly on conditions at the last scattering surface at distance $x_{\text{ls}} = 14$ Gpc we need

$$\zeta_k(x) = (1 + A(k) \hat{p} \cdot x/x_{\text{ls}} + \cdots) \zeta_k(0),$$

(2)

corresponding to

$$P^{1/2}_\zeta(k, x) = (1 + A(k) \hat{p} \cdot x/x_{\text{ls}} + \cdots) P^{1/2}_\zeta(k, 0).$$

(3)

The dots in Eqs. (2) and (3) indicate contributions of higher order in $x$, that must be smaller than the linear term for $x < x_{\text{ls}}$. An equivalent definition of of $A(k)$ is

$$A(k) = \frac{\| \nabla P^{1/2}_\zeta(k, 0) \|}{P^{1/2}_\zeta(k, 0)}.$$  

(4)

CMB multipoles of order $\ell$ probe $k \sim \ell/x_{\text{ls}}$ and for $k^{-1}$ in the range $x_{\text{ls}}/60$ to $x_{\text{ls}}$, and we need $|A(k)| = 0.07 \pm 0.02$. On the much smaller scale $k^{-1} \sim 1$ Mpc the distribution of distant quasars requires $|A(k)| < 0.015$ (99% confidence level) [6]. Therefore, if the dipole modulation of $\Delta T$ is generated by the dipole modulation of $\zeta$, we should write instead of Eq. (1)

$$\Delta T(\hat{n}) = (1 + A(k) \hat{p} \cdot \hat{n}) \Delta T_{\text{iso}}(\hat{n}),$$

(5)

the expression applying for multipoles $\ell \simeq x_{\text{ls}} k$ or equivalently angular scales $\Delta \theta \simeq (x_{\text{ls}} k)^{-1}$.

Before continuing, we need to be precise about the meaning of Eqs. (2) and (3). For a cosmological perturbation $g(x)$, the correlators $\langle g(x) \rangle$, $\langle g(x) g(y) \rangle$ and so on are defined as averages over some ensemble, with the actual value of $g(x)$ corresponding to a typical realisation of the ensemble.\(^3\) Under the usual assuming that the perturbation originates as a vacuum fluctuation, $\langle \rangle$ is the quantum expectation value of the corresponding operator. If the correlators are invariant under rotations the perturbation is said to be statistically isotropic and if they are invariant under displacements it is said to be statistically homogeneous. In the latter case the expectation values can be taken as

\(^2\)On a given scale we can make $A$ positive by the choice of $\hat{p}$, but we want to allow for a possible change in sign of $A$ going from large to small scales, and to allow the simplest presentation of the calculation we will not demand that $A$ is positive on large scales.

\(^3\)We adopt the usual device of allowing $g$ to represent the actual value or else to run over all realisations according to the context. Correlators between different perturbations are defined in the same way.
spatial averages for a single realisation (in particular the one corresponding to the observed universe) so that for example $\langle g(x)g(x + y) \rangle$ is the average over $y$.

The correlators should be defined within a finite box \[9\]. (This regulates long-wavelength diverges in integrals that arise when correlators of products of perturbations are calculated. In the case of the inflationary cosmology the box should be within the inflated patch around us.) Within the box one uses a Fourier series that is approximated as a Fourier integral. The region of interest should fit comfortably into the box so that physically significant wavenumbers satisfy $kL \gg 1$ where $L$ is the box size. For a generic perturbation

$$g_k = \int g(x)e^{-i k \cdot x}d^3x.$$  \hspace{1cm} (6)

To define the spectrum, bispectrum etc. one assumes statistical homogeneity, and we will also assume statistical isotropy. The spectrum is defined by

$$\langle g_k g_{k'} \rangle = (2\pi)^3\delta^3(k + k')(2\pi^2/k^3)P_g(k),$$  \hspace{1cm} (7)

the bispectrum by

$$\langle g_{k_1} g_{k_2} g_{k_3} \rangle = (2\pi)^3\delta^3(k_1 + k_2 + k_3)B_g(k_1, k_2, k_3),$$  \hspace{1cm} (8)

and similarly for higher correlators.

In \[9\] it is proposed that the box size for cosmology should usually be the smallest one that comfortably contains the observable universe. Demanding say one percent accuracy the ‘minimal box’ size is presumably $L \sim 100x_{ls}$ corresponding to $\ln(L/x_{ls}) \sim 5$. The use of the minimal box avoids assumptions about inflation long before the observable universe leaves the horizon, and allows one to keep only the leading term when evaluating the correlators of products of perturbations.

The minimal box is appropriate for defining the spectrum etc. of perturbations that can be taken to be statistically homogeneous within it. But to handle $\zeta_k(x)$, one should use a small box centred on $x$ with size much smaller than $x_{ls}$. Within the box, $x$ can be regarded as constant and $\zeta_k(x)$ can be regarded as a statistically homogeneous perturbation with spectrum $P_c(k, x)$. Since the box size is $\ll x_{ls}$, $\zeta_k(x)$ is defined only for $1/k \ll x_{ls}$ which means that it can only be used to describe $\Delta T$ on small angular scales corresponding to multipoles $\gg 1$. That is as it should be, because the definition of $\Delta T_{iso}$ given after Eq. (1) makes sense only on these scales.

Going to the other extreme, one can assume that the inflated patch around us is big enough to allow the use of a big box containing all significant wavelengths of $\zeta_k(x)$ considered as a function of $x$ (equivalently, all significant wavelengths of the perturbation $\delta \phi_L(x)$) that generates $\zeta_k(x)$.

In this paper we first recall the generation of $\zeta$ without statistical inhomogeneity. Then we see how to generate the statistically inhomogeneous quantity $\zeta_k(x)$ using a small box. Finally, we consider a big box which allows us to place an upper bound on the gradient of $\delta \phi_L$ at a typical location, taken to be our own.
3 Generating $\zeta$ without statistical inhomogeneity

In this section we recall the standard description of $\zeta$ and its generation from the perturbation of some field $\phi$. We begin with the definition of $\zeta$, which makes no reference to its stochastic properties (and therefore invokes no box). The curvature perturbation $\zeta$ is taken to be smooth on some comoving scale $x_{\text{smooth}}$ that is shorter than any of interest (i.e. it is taken to only have modes with $k \lesssim x_{\text{smooth}}^{-1}$). Also, $\zeta$ is defined only while the smoothing scale is outside the horizon ($x_{\text{smooth}} \gg 1/aH$). Using the comoving threads of spacetime and the slices of constant energy density, $\zeta$ is defined by

$$\zeta(x, t) \equiv \delta(\ln a(x, t)) \equiv \ln[a(x, t)] - \ln[a(t)],$$

(9)

where $a(x, t)$ is the scale factor such that a comoving volume element has volume $\propto a^3$. Here $a(t)$ is the scale factor in the background universe that is invoked to define perturbations.

In the early universe $\zeta$ may be time-dependent, but if we take the smoothing scale to be the shortest cosmological scale it has reached some time-independent value $\zeta(x)$ at least by the time that the smoothing scale is approaching horizon entry. (By ‘cosmological scales’ we mean those that are probed by the CMB anisotropy and high redshift galaxy surveys, corresponding $e^{-15}x_{ls} < k^{-1} < x_{ls}$. If $\phi$ is the inflaton of single-field inflation, $\zeta$ already has the final value soon after $x_{\text{smooth}}$ leaves the horizon; in the opposite case that $\phi$ is a curvaton-type field the final value is reached only at some epoch after inflation.

The curvature perturbation $\zeta(x, t)$ generated by $\phi$ is given by the non-linear $\delta N$ formula [13, 14]

$$\zeta(x) \equiv \delta(\ln a(x, t)) = \delta(\ln a(x, t)/a(t_1)) \equiv \delta N(\phi(x, t_1)).$$

(10)

The function $N(\phi(x, t_1))$ is the number of $e$-folds of expansion at position $x$ between a time $t_1$ during inflation after the smoothing scale has left the horizon, with $\phi$ has the assigned value, and a time $t$ at which the energy density has a fixed value. The field $\phi(x, t_1)$ is defined on a ‘flat’ slice of spacetime (one on which the scale factor has a fixed value), and $\zeta$ is independent of the choice of $t_1$.

The field is written

$$\phi(x, t) = \phi_0(t) + \delta\phi(x, t).$$

(11)

Expanding Eq. (10) gives [14]

$$\zeta(x) = N'(\phi_0(t_1))\delta\phi(x, t_1) + \frac{1}{2}N''(\phi_0(t_1))(\delta\phi(x, t_1))^2 + \cdots.$$  

(12)

To be more precise its gradient is $\lesssim x_{\text{smooth}}^{-1}$. We are not really invoking a Fourier transform here but use $k$ for ease of presentation. The same device will be used later without comment.
The spectrum $\mathcal{P}_\zeta$ and bispectrum $B_\zeta$ are defined by Eqs. (7) and (10). Instead of the latter one usually works with

$$f_{NL}(k_1, k_2, k_3) = \frac{5}{6} \frac{B_\zeta(k_1, k_2, k_3)}{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + 2 \text{ perms}},$$  \tag{13}$$

where $\mathcal{P}_\zeta(k) \equiv (2\pi^2/k^3)\mathcal{P}_\zeta(k)$.

Analyses of the data to obtain observational constraints on $\mathcal{P}_\zeta$ and $f_{NL}$ take $\zeta$ to be statistically homogeneous and isotropic within a box of at least minimal size. The constraints are obtained on the assumption that our location within the chosen box is typical, and they take $\Delta T$ to be statistically isotropic. For $\mathcal{P}_\zeta(k)$, observation [10] [11] gives $\mathcal{P}_\zeta^{1/2} \approx 5 \times 10^{-5}$ and

$$n(k) - \frac{1}{2} \equiv \frac{1}{\mathcal{P}_\zeta^{1/2}(k)} \frac{d\mathcal{P}_\zeta^{1/2}(k)}{d \ln k} = -0.040 \pm 0.007.$$  \tag{14}$$

If $f_{NL}$ is independent of $k_i$ then [12] $f_{NL} = 2.7 \pm 5.8$, but the bound is much weaker for generic $k_i$, roughly $|f_{NL}| \lesssim 100$. If $|f_{NL}| \gtrsim 1$ it will eventually be detected.

Except where stated we assume that $\phi$ has the canonical kinetic term when these scales are leaving the horizon during inflation. Then $\delta\phi_k$ is created from the vacuum fluctuation at the epoch of horizon exit $aH = k$. The spectrum is initially $\mathcal{P}_{\delta\phi}(k) = (H/2\pi)^2$ and $B_{\delta\phi}(k, k, k)$ is initially very small.

To keep $f_{NL}$ within the observational bound, the first term of Eq. (12) must dominate giving

$$\mathcal{P}_\zeta(k) = N''(\phi_0(t_1))(H(t_1)/2\pi)^2.$$  \tag{15}$$

Including the second term, $f_{NL}$ is given by [14] [15]

$$\frac{6}{5} f_{NL}(k_1, k_2, k_3) = \frac{N''(t_1)}{N''(t_1)} + \frac{(2\pi)^3 N''(t_1) B_{\delta\phi}(k_1, k_2, k_3, t_1)}{\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + 2 \text{ perms}}.$$  \tag{16}$$

For our purpose we can set $t_1 = t_k$ where $t_k$ is horizon exit for a scale $k$ which is taken to be the smoothing scale. Then

$$\mathcal{P}_\zeta(k) = N''(\phi_0(t_k))(H(t_k)/2\pi)^2,$$  \tag{17}$$

and

$$f_{NL}^{\text{qlocal}}(k) = \frac{5}{6} N''(\phi_0(t_k))/N''(\phi_0(t_k)),$$  \tag{18}$$

where $f_{NL}^{\text{qlocal}}(k) \equiv f_{NL}^{\text{qlocal}}(k, k, k)$ and the superscript qlocal (quasi-local) means that $\delta\phi$ is taken to be gaussian at horizon exit so that $B_{\delta\phi}(k, k, k, t_k) = 0$. (If $f_{NL}^{\text{qlocal}}$ is independent of $k$ it is called the local contribution.)

We suppose first that $\phi$ is the inflaton of slow-roll inflation [16]. In this case, $\zeta_k(t)$ achieves its final value promptly at $t = t_k$, which means that

$$\zeta_k = H(t_k)(\delta\zeta)_k = -H(t_k)\delta\phi_k(t_k)/\dot{\phi}_0(t_k),$$  \tag{19}$$
where $\delta t$ is the displacement of the slice of uniform energy density from the flat slice on which $\delta \phi$ is defined, and the second equality is valid to first order in $\delta \phi$. This gives

$$P_\zeta(k) = \left( \frac{H(t_k)}{\dot{\phi}_0(t_k)} \right)^2 P_{\delta \phi}(k, t_k)$$  \hfill (20)

$$P_{\delta \phi}(k, t_k) = \left( \frac{H(t_k)}{2\pi} \right)^2.$$  \hfill (21)

The slow-roll approximation corresponds to conditions on the scalar field potential, $\epsilon \ll 1$ and $|\eta| \ll 1$ where $\epsilon \equiv M_P^2 (V'/V)^2/2$ and $\eta \equiv M_P^2 V''/V$, and

$$\dot{\phi}_0(t_k) \approx -V'(\phi_0(t_k))/3H(t_k).$$  \hfill (22)

These imply

$$3M_P^2 H^2(t_k) \approx V(\phi_0(t_k)).$$  \hfill (23)

These equations make $P_\zeta(k)$ a function of $\phi_0(t_k)$. Using $k = a(t_k)H(t_k)$ and the good approximation $d \ln k \approx d \ln a = Hdt$ this gives

$$\frac{n(k) - 1}{2} = \frac{1}{P_{\zeta}^{1/2}(k)} dP_{\zeta}^{1/2}(k) \frac{\dot{\phi}}{H}$$  \hfill (24)

$$\approx \frac{\eta(t_k) - 3\epsilon(t_k)}{H(t_k)}.$$  \hfill (25)

Evaluating Eq. (18) gives

$$\frac{6}{5} f_{\text{local}}^{\text{local}}(k) \approx 2\epsilon(t_k) - \eta(t_k).$$  \hfill (26)

Barring a fine-tuned cancellation

$$|f_{\text{NL}}^{\text{local}}(k)| \approx 1 - n(k)$$  \hfill (27)

and in any case $|f_{\text{NL}}^{\text{local}}| \ll 1$. These expressions assume $B_{\delta \phi}(k, k, t_k) = 0$, which is not a good approximation with $\phi$ the inflaton because the first term of Eq. (16) is also very small. The full $f_{\text{NL}}$ \cite{17} still satisfies $|f_{\text{NL}}| \ll 1$ but we don’t need it.

Instead of slow-roll inflation one can consider the most general single-field inflation paradigm, in which the unperturbed solution $\phi(t)$ is unique up to a time translation; in other words, $\dot{\phi}$ is a function of $\phi$ (attractor behaviour). Within this paradigm, one can still expect Eqs. (24) and (27) to apply. We have verified this explicitly for the case of k-inflation \cite{8}. The Lagrangian is an arbitrary function of $\phi$ and $\partial_\mu \phi \partial^\mu \phi$, and we will choose $\phi$ so that $\delta \phi$ has the canonical action for a free field.\footnote{In the notation of \cite{8}, $\delta \phi = v$.} The epoch $t_k$ at which $\delta \phi$ is generated from the vacuum fluctuation is given by $c_s k = aH$, and

$$P_{\delta \phi}(t_k, k) = c_s^{-3}(H(t_k)/2\pi)^3$$  \hfill (28)

$$P_\zeta(k) = \left( \frac{H}{2c_s M_P^2 |H|} \right)^2 \left( \frac{H}{2\pi} \right)^2.$$  \hfill (29)
From these and Eq. (19) we deduce for $N_\phi \equiv dN/d\phi$

$$N_\phi^2(t_k) = \frac{c_s^2(t_k)H^2(t_k)}{2M_p^2\dot{H}(t_k)} = \frac{H^2(t_k)}{\dot{\phi}^2(t_k)}. \tag{30}$$

The conditions $|\dot{H}/H^2| \ll 1, |\ddot{H}/HH| \ll 1$ and $|\dot{c}_s/Hc_s|$ are imposed and we then have

$$n(k) - 1 = -\frac{\dot{c}_s}{c_sH} + 4\frac{\dot{H}}{H^2} - \frac{\ddot{H}}{HH}, \tag{31}$$
$$\frac{6}{5}f_{\text{NL}}^{\text{local}}(k) = \frac{\dot{c}_s}{c_sH} + \frac{\dot{H}}{H^2} - \frac{1}{2}\frac{\ddot{H}}{HH}. \tag{32}$$

(The final relation has not been given before.)

In summary, Eq. (24) is satisfied for k-inflation, and so is Eq. (27) barring a cancellation, and in any case $|f_{\text{NL}}^{\text{local}}(k)| \ll 1$. The first equation holds whenever the epoch $t_k$ at which $\delta\phi$ is generated satisfies $k = f(t_k)a(t_k)H(t_k)$ with $|\dot{f}/f| \ll H$. The second equation holds if $n(k) - 1$ and $f_{\text{NL}}^{\text{local}}(k)$ are both linear combinations of small parameters with numerical coefficients roughly of order 1, and $|f_{\text{NL}}^{\text{local}}(k)| \ll 1$ then in any case holds. One can expect all of these features for any single-field inflation model with the attractor behaviour.

Finally, consider the case that $\phi$ is a curvaton-type field. In this case one expects barring cancellations $|f_{\text{NL}}^{\text{local}}(k)| \gtrsim 1$, which we will assume. (As we will see, $|f_{\text{NL}}^{\text{local}}(k)| \sim 10$ is actually required to generate the required asymmetry for $k^{-1} \sim \text{Gpc}$.) Then $f_{\text{NL}}^{\text{local}}$ is practically equal to the full quantity $f_{\text{NL}}$ and we will identify them. Also, the evolution of $\delta\phi_k$ after horizon exit is given by

$$H(t)\dot{\delta}\phi_k(t) = -V''(\phi_0(t))\delta\phi_k(t) - \frac{1}{2}V'''(\phi_0(t))[(\delta\phi(t))^2]_{k} - \cdots. \tag{33}$$

If $V$ is quadratic, the evolution is linear. Then $B_{\delta\phi}(k, k)$ remains zero if it is zero initially, which from Eq. (16) means that $f_{\text{NL}}(k)$ is a constant.

# 4 Generating $\zeta_k(x)$

Now we write

$$\phi(x, t) = \phi_0(x, t) + \delta\phi(x, t), \tag{34}$$
$$\phi_0(x, t) = \phi_0(t) + \delta\phi_L(x, t). \tag{35}$$

where $\delta\phi$ has $k > x_{ls}^{-1}$ and $\delta\phi_L$ has $k < x_{ls}^{-1}$. For the curvature perturbation we write

$$\zeta(x) = \zeta_{\delta\phi}(x) + \zeta_{GZ}(x) \tag{36}$$
$$\zeta_{\delta\phi}(x) \equiv N(\phi(x, t_k)) - N(\phi_0(x, t_k))$$
$$= N'(\phi_0(x, t_k))\delta\phi(x, t_k) + \cdots \tag{37}$$
$$\zeta_{GZ}(x) \equiv N(\phi_0(x, t_k)) - N(\phi_0(t_k))$$
$$= N'(\phi_0(t_k))\delta\phi_L(x, t_k) + \frac{1}{2}N''(\phi_0(t_k)) (\delta\phi_L(x, t_k))^2 + \cdots. \tag{38}$$
Each of the contributions $\zeta_{\phi}$ and $\zeta_{\text{GZ}}$ is independent of $t_k$ because they vary on different scales and $\zeta$ itself is independent of $t_k$. The first term of Eq. (37) dominates because $\zeta_{\phi}$ is almost gaussian, and we assume that the first term of Eq. (38) dominates which will be justified in the next section. Then $\delta \phi_L(x, t_k) \propto 1/N'(\phi_0(t_k))$, which from Eq. (17) is proportional to $H(t_k)$. Analogously with Eqs. (3) and (4) we write

$$\delta \phi_L(x, t_k) = B(k)(H(t_k)/2\pi)\hat{p} \cdot x/x_{ls} + \cdots$$

(39) 

$$H(t_k)/2\pi B(k)/x_{ls} \equiv |\nabla (\delta \phi_L(x, t_k))|_{x=0}.$$  (40)

(Remember that $t_k$ is a function of $k$ so that either can be used as an argument.)

In the next section, we derive an upper bound on $B$ on the assumption that the observable universe occupies a typical position, within a region big enough to contain all significant wavelengths of $\delta \phi_L$. If $\phi$ is a curvaton-type field the bound is

$$P_\zeta(k)B^2 \lesssim 4 \times 10^{-4}/|f_{\text{NL}}(k)|.$$  (41)

If instead $\phi$ is the inflaton, the right hand side is just $4 \times 10^{-4}$.

Both $\zeta_{\phi}$ and $\zeta_{\text{GZ}}$ contribute to the CMB quadrupole, but the GZ effect that might have enhanced the quadrupole comes only from $\zeta_{\text{GZ}}$. We deal with it in the next section, but for now focus on $\zeta_{\phi}$. Evaluated in a box with size $\ll x_{ls}$ centred at position $x$ it gives

$$\zeta_k(x) = N'(\phi_0(x, t_k))\delta \phi_k(x, t_k) + \cdots$$  (42)

$$P_\zeta(k, x) = N'^2(\phi_0(x, t_k))(H(x, t_k)/2\pi)^2,$$  (43)

where $H(x, t) \equiv \dot{a}(x, t)/a(x, t)$ is the unperturbed quantity within the small box and we kept only the first term in evaluating Eq. (43). After insertion into Eq. (44), this gives $A(k)$. We will take the results of the previous section to apply to $\zeta_k(0)$ and $P_\zeta(k, 0)$.

We consider first the case that $\phi$ is the inflaton of single-field inflation. Then $P_\zeta(k, x)$ is a function of $\phi_0(x, t_k)$ and using Eq. (24) we have

$$\frac{\nabla P^{1/2}_\zeta(k, 0)}{P^{1/2}_\zeta(k, 0)} = \frac{1-n(k)}{2} \frac{H(t_k)}{\phi_0(t_k)} \nabla \phi(0, t_k).$$  (44)

Using Eqs. (44) and (40) this gives

$$A(k) = \frac{1-n(k)}{2} BP^{1/2}_\zeta(k).$$  (45)

Observational constraints on $n(k)$ easily allow $A(k)$ to have sufficient scale dependence [10], but the bound $P_\zeta(k)B^2 \lesssim 2 \times 10^{-4}$ makes $A(k)$ too small.

Now suppose instead that $\phi$ is a curvaton-type field. Since $\phi$ has a negligible effect during inflation, $H(x, t_k)$ is independent of $x$ and Eqs. (44) and (40) Eq. (44) give

$$A(k) = \frac{6}{5} f_{\text{NL}}(k) BP^{1/2}_\zeta(k).$$  (46)
Using Eq. (41),

$$|A(k)| \lesssim 0.018|f_{NL}(k)|^{1/2}. \quad (47)$$

To have $|A(k)| = 0.07 \pm 0.02$ on the Gpc scale we need $|f_{NL}(k)| \gtrsim 8$ on that scale. A tight observational bound on $f_{NL}(k)$ could be obtained using a shape for $f_{NL}(k_1, k_2, k_3)$ derived within a specific curvaton-type model (see [15][20] for the curvaton) but it would presumably be no tighter than the result $|f_{NL}| \sim 10$ that holds if $f_{NL}$ is a constant. We conclude that the linear GZ effect can account for the CMB asymmetry if $\zeta$ is generated by a curvaton-type field.

Before leaving this section we mention a perhaps simpler way of proceeding when $\phi$ is the curvaton. Instead of Eqs. (36)–(38) one can write

$$\zeta(x) = N(\phi(x, t_k) - N(\phi_0))$$

$$= N'(\phi_0(t_k)) (\delta \phi(x) + \delta \phi_L(x)) + \frac{1}{2} N''(\phi_0(t_k)) (\delta \phi(x) + \delta \phi_L(x))^2 + \cdots \quad (49)$$

$$\equiv (\zeta_S(x) + \zeta_L(x)) + \frac{3}{5} f_{NL}(k) (\zeta_S(x) + \zeta_L(x))^2 + \cdots \quad (50)$$

$$= \left(1 + \frac{6}{5} f_{NL}(k) \zeta_L(x)\right) \zeta_S(x) + \zeta_L(x) + \frac{3}{5} f_{NL}(k) \zeta_L^2(x) + \cdots, \quad (51)$$

where $\zeta_S \equiv N'(\phi_0(t_k)) \delta \phi(x)$ and $\zeta_L \equiv N'(\phi_0(t_k)) \delta \phi_L(x)$. The first term of Eq. (51) corresponds to $\zeta_{\delta \phi}$ of Eq. (36) and the other two terms correspond to $\zeta_{GZ}$ of Eq. (36). With $\phi$ the inflaton, Eq. (51) is still correct, but not very useful for calculating $P_\zeta(k, x)$ because $P_{\delta \phi}(k, t_k)$ within a small box depends on the position $x$.

5. **The view from a big box**

The calculation of the previous section we invoked only the observable universe corresponding to $x < x_{ls}$. The function $\delta \phi_L(x)$ was taken as a given quantity without discussing its origin.

In this section we assume that the nearly homogeneous patch containing the observable universe contains all significant wavelengths numbers of $\delta \phi_L(x)$. That is desirable because it allows $\delta \phi_L$ to be generated from the vacuum fluctuation like $\delta \phi$. With this assumption we will derive Eq. (11) if $\phi$ is a curvaton-type field, and the same bound without the $f_{NL}$ factor if $\phi$ is the inflaton.

Within the big box, $\zeta$ is statistically homogeneous. To proceed, we use Eq. (31), choosing $t_k = t_1$ where $t_1$ is the epoch of horizon exit for the scale $k_1^{-1} = 1$ Gpc.

We assume that the first term of Eq. (50) dominates. This makes $P_\zeta(k) = N^2 P_{\delta \phi}(k) \simeq (5 \times 10^{-5})^2$ for $k > x_{ls}^{-1}$, and $P_\zeta(k) = N^2 P_{\delta \phi_L}(k)$ for $k < x_{ls}$. Also, since $\zeta_k \simeq N' \delta \phi_k$ on cosmological scales, it is at least approximately gaussian on those scales, though its bispectrum etc. within the big box cannot
be calculated without further assumptions (i.e., we do not know how good is the approximation \( \zeta_k \approx N' \delta \phi_k \)).

If \( \phi \) is a curvaton-type field we expect \(| f_{\text{NL}}^{\text{local}}(k_1) | \simeq | f_{\text{NL}}(k_1) | \gtrsim 1 \). Then the condition that the first term of Eq. (50) dominates corresponds to \( f_{\text{NL}}^{2}(k_1) \langle \zeta^2 \rangle \lesssim 1 \). This is a bit stronger than the condition \( \langle \zeta^2 \rangle \lesssim 1 \) that is usually imposed when discussing the GZ effect. If instead \( \phi \) is the inflaton, \(| f_{\text{NL}}^{\text{local}}(k_1) | \ll 1 \) and it is weaker.

The condition \( \langle \zeta^2 \rangle \lesssim 1 \) is not strictly required because a nearly constant value of \( \zeta \) in the observable universe can be absorbed into the scale factor \( a(t) \). However, we do require \( \mathcal{P}_\zeta(k) \lesssim 1 \) so that the spatial curvature scalar within a region with size \( k^{-1} \) is \( \lesssim k \). Indeed, a violation of that condition would imply a strong spatial curvature which would invalidate the interpretation of \( x \) as a distance, for a typical region which we are supposed to occupy [21].

Dropping the small short-scale contribution we have

\[
\langle \zeta^2 \rangle = \int_0^{x_{ls}^{-1}} \frac{dk}{k} \frac{d\zeta}{k} \mathcal{P}_\zeta(k).
\]  

We see that \( \mathcal{P}_\zeta(k) \lesssim 1 \) implies at least roughly \( \langle \zeta^2 \rangle \lesssim 1 \) unless the integral receives significant contributions from a very large range \( \Delta \ln k \gg 1 \). We will see that this would probably make \( A(k_1) \) too small. We therefore assume

\[
\int_0^{x_{ls}^{-1}} \frac{dk}{k} \frac{d\zeta}{k} \mathcal{P}_\zeta(k) \lesssim f_{\text{NL}}^{2}(k_1),
\]  

if \( \phi \) is a curvaton-like field with \( f_{\text{NL}}^{2}(k_1) \gtrsim 1 \). If instead \( \phi \) is the inflaton we set the right hand side equal to 1. In both cases the first term of Eq. (51) dominates for a typical value of \( \delta \phi_L(x) \), which means that the first term of Eq. (38) dominates as advertised.

We assume that our location within the big box is typical. Multiplying both sides of Eq. (40) by \( N' \) and squaring them gives

\[
\mathcal{P}_\zeta(k_1) B^2 \simeq \int_0^{x_{ls}^{-1}} \frac{dk}{k} (x_{ls} k)^2 \mathcal{P}_\zeta(k).
\]  

For the CMB multipoles, \( a_{\ell m}^2 \simeq \langle a_{\ell m}^2 \rangle = C_{\ell} \) with

\[
C_{\ell} = 4\pi \int_0^{\infty} T_{\ell}^{2}(k) \mathcal{P}_\zeta(k)dk/k,
\]  

where \( T_{\ell}(k) \) is \( \sim 1 \) for \( k^{-1} \sim x_{ls} \) and close to 1 for \( k^{-1} \gg x_{ls} \).

The cosmic variance of \( a_{\ell m}^2 \) is defined as the mean-square difference between \( a_{\ell m}^2 \) and \( C_{\ell} \) (i.e., \( \langle (a_{\ell m}^2 - C_{\ell})^2 \rangle \)). If \( \zeta_k \) were gaussian, \( a_{\ell m} \) would have a gaussian probability distribution and the cosmic variance would be \( 2C_{\ell}^2 \). Since \( \zeta_k \) is at least approximately gaussian we expect that to be at least approximately correct, but the precise cosmic variance cannot be calculated without further
information. The observed dipole modulation corresponds to a systematic bias of the observed \( a_{lm} \) away from \( C_\ell \), and Eq. (54) will ensure that the bias is allowed (for a typical observer) by the cosmic variance of \( C_\ell \). An investigation of how that comes about is beyond the scope of this paper.

Using the Sachs-Wolfe approximation, the GZ contribution to \( C_2 \) is

\[
C_2^{GZ} = \frac{4\pi}{25} \int_0^{x_{ls}^{-1}} dk \left( \frac{(k x_{ls})^2}{15} \right)^2 P_\zeta(k).
\]

(56)

We will require \( \sqrt{C_2^{GZ}} \) to be \( \lesssim 3 \) times the rms quadrupole found in [19], giving

\[
\int_0^{x_{ls}^{-1}} \frac{dk}{k} (k x_{ls})^4 P_\zeta(k) \lesssim (3.8 \times 10^{-4})^2.
\]

(57)

Using the Cauchy-Schwartz inequality, Eqs. (53), (54), and (57) imply if \( \phi \) is the curvaton

\[
B^2 P_\zeta(k_1) \lesssim 4 \times 10^{-4} |f_{NL}(k_1)|^{-1}.
\]

(58)

The result for \( \phi \) the inflaton is obtained by setting \( f_{NL}(k_1) = 1 \).

The bound (58) is saturated by choosing \( P_\zeta(k) \propto \delta(k-k_L) \) with \( (x_{ls} k_L)^2 \approx 4 \times 10^{-4} |f_{NL}(k_1)| \). Reducing \( x_{ls} k_L \) by a factor of 10 gives \( B^2 P_\zeta(k_1) \lesssim 10^{-6} \) which makes \( A(k_1) \) much too small. The same is true if we increase it by that factor, though this might be regarded as incompatible with \( x_{ls} k_L \ll 1 \).

Instead of a strong peaking one might assume a flat plateau: \( P_\zeta(k) \) constant in a range \( \ln k_1 - N_L < \ln k < \ln k_L \) with \( N_L \gg 1 \). Then the weakest bound is for \( (x_{ls} k_L)^2 \approx 8 \times 10^{-4} |f_{NL}(k_1)| \sqrt{N_L} \) which gives

\[
2N_L B^2 P_\zeta(k_1) |f_{NL}(k_1)| \lesssim 4 \times 10^{-4}.
\]

(59)

Since \( |f_{NL}(k_1)| \lesssim 100 \) we need \( N_L \lesssim 5 \). As before a value of \( x_{ls} k_L \) much below \( 10^{-2} \) is not allowed.

These examples suggest that we need \( P_\zeta(k) \) to grow sharply below some \( k = k_L \sim 10^{-2} x_{ls}^{-1} \), and to fall off when \( k^{-1} \) is not far below \( k_L \). It remains to be seen if this allows a plausible mechanism for generating the required large \( \delta \phi_L \) from the vacuum fluctuation.

We close this section by mentioning a different possibility for obtaining a spatial variation of the background field \( \phi_0(x) \). Instead of invoking an enhancement of \( P_{\delta \phi_L}(k) \), we can keep the usual fairly flat spectrum and suppose that we live at a special place within the large box. This possibility was considered in detail by Linde and Mukhanov [22] for the curvaton with a quadratic potential, who noticed that it could generate dipole modulation of the CMB. It is not clear how easily this scheme could keep the CMB quadrupole small enough, while generating the required asymmetry.

6 Conclusion

It was disappointing, though hardly a surprise, that the GZ effect was not seen when the CMB quadrupole was first observed. The dipole modulation...
of the CMB anisotropy may now be making up for that disappointment, by exhibiting the closely related EKC effect.

On the assumption that $\zeta$ is generated by some field $\phi$, we have found that the EKC effect almost certainly cannot generate the observed asymmetry if $\phi$ is the inflaton, but that it can do so if $\phi$ is a curvaton-type field. This is perhaps the first indication that the latter may be nature’s choice. The asymmetry can agree with observation if $|f_{\text{NL}}|$ in the equilateral configuration is $\approx 10$ for $k^{-1} = 1 \text{ Gpc}$ and $\lesssim 3$ for $k^{-1} = 1 \text{ Mpc}$. An $f_{\text{NL}}$ with these properties can apparently be obtained from the curvaton with an axionic potential [20]. Within any specific curvaton-type model, the function $f_{\text{NL}}(k_1, k_2, k_3)$ required to generate the asymmetry would be determined, and could perhaps already be confirmed or ruled out using existing Planck or WMAP data.

It remains to be seen if a plausible mechanism can be found for generating $\delta \phi_L$ from the vacuum fluctuation. On the other hand, it seems hard to come up with an alternative to the EKC effect. After ruling out various scenarios, [23] mention only five that might still be viable. These are (i) an inhomogeneous tilt $n(k, x) - 1$, (ii) a statistically inhomogeneous isocurvature perturbation [25], (iii) a statistically inhomogeneous tensor perturbation, (iv) asymmetry of the optical depth, (v) bubble collisions and (vi) non-trivial topology of the Universe. Of these, the first is identical with our version of the EKC effect (with possible generalisation to the case that $\zeta$ comes from a curvaton-type field with a non-canonical kinetic term, or from two or more field perturbations) and the second may be in conflict with Planck bounds on the isocurvature amplitude. The next two are only partially investigated in [23] and may also be in conflict with existing data while the last two have not been tried at all. There is also the proposal of [26], which replaces, during inflation, the usual Riemannian spacetime by what is called Randers spacetime. The $A(k)$ appears to be viable, with $A \propto 1/k^3$.

Since the first version of this paper appeared on arXiv.org there have been four more papers. That of [27] invokes a contraction of the universe, followed by an inflationary expansion with $\phi$ the inflaton. On scales leaving the horizon during the contraction, $\dot{H}/H^2$ is enhanced which sufficiently enhances $A(k)$. But the enhancement applies only to scales $k^{-1} > (\mathcal{H}_0)^{-1}$ (their notation) which for their best fit corresponds to $k^{-1} > 5.4 \text{ Gpc} \sim x_{\text{ls}}/3$. That is outside the required range $x_{\text{ls}}/60 < k^{-1} \ll x_{\text{ls}}$. (At $3\sigma$ though, they can have $1/\mathcal{H}_0 = x_{\text{ls}}/30$ which might work.) In [28] an implementation of the isocurvature scenario of [25] is proposed. In [29, 30] they consider the EKC effect with, among other things, the possibility of two or more curvaton-type fields.

---

6 They rule out the large-scale non-gaussianity proposal of [24] on the ground that it makes $A(k)$ scale-independent, but this scenario in fact generates only statistical anisotropy of $P_\zeta$ which cannot generate the CMB asymmetry. Figure 1 of that paper refers to $L = 2$ in their notation, not to $L = 1$ as stated in the caption. I thank Fabian Schmidt for clarification of this issue.

7 This dependence will be explained in a future version of [26]. (Personal communication from S. Wang.)

8 Personal communication from Yun-Song Piao.
Although all possibilities should be explored, it seems fair to say that there is at present no proposal which looks more plausible than the EKC effect.

Acknowledgements

I thank Andrew Liddle for valuable comments. The work is supported by the Lancaster-Manchester-Sheffield Consortium for Fundamental Physics under STFC grant ST/J000418/1.

A The EKC treatment of the GZ effect

In [2] the GZ effect is treated in a way that is different and less general than ours. For the case that \( \phi \) is the inflaton of slow-roll inflation, they consider only the first term of \( \zeta_{GZ} \) because they work to first order in \( \phi \). In that term they take \( \delta \phi_L \) to be sinusoidal, with the \( B = 0 \) so that the leading GZ effect is for the octupole. Requiring it to be less than the observed quantity they conclude that \( A(k) \) is too small.

For the case that \( \phi \) is the curvaton with a quadratic potential, they include also the second term of \( \zeta_{GZ} \). Inserting Eq. (39) this gives

\[
\zeta_{EKC-GZ}(x, t_k) = \frac{1}{2} N''(\phi_0(t_k)) B^2 (H(t_k)/2\pi)^2 x_{ls}^{-2} (x \cdot \hat{p})^2
\]

(60)

\[
= \frac{3}{5} |f_{NL}(k)| B^2 \mathcal{P}_\zeta(k) x_{ls}^{-2} (x \cdot \hat{p})^2.
\]

(61)

Using Eqs (2)–(4) of [2] with \( \Phi = -(3/5)\zeta \), this gives a contribution to the quadrupole given by

\[
|f_{NL}(k)| B^2 \mathcal{P}_\zeta(k) = 2.2 \times 10^{-4} \left( \frac{|a_{20}^{EKC}|}{1.8 \times 10^{-5}} \right),
\]

(62)

where the polar axis for \( a_{20} \) is along the \( \hat{p} \) direction. Barring a cancellation, \( |a_{20}^{EKC}| \) should be \( \lesssim \) the observed \( |a_{20}| \). That has yet to be extracted from the data, and EKC required instead that \( |a_{20}^{EKC}| \) be less than 3 times the rms value of \( a_{\ell m} \) found in [19] corresponding to \( |a_{20}^{EKC}| < 1.8 \times 10^{-5} \). With that assumption, Eq. (62) gives

\[
|f_{NL}(k)| B^2 \mathcal{P}_\zeta(k) < 2.2 \times 10^{-4}.
\]

(63)

This is essentially the same as our bound [58] but its status is very different because it ignores the first term of \( \zeta_{GZ} \). As we have seen, the total GZ contribution to the quadrupole can be much smaller than \( a_{20}^{EKC} \), indicating a cancellation between the first and second terms of \( \zeta_{GZ} \). In that regime, our bound [57] is a consequence of \( f_{NL}(k_1) \langle \zeta^2 \rangle \lesssim 1 \) which has nothing to do with the GZ effect.

I thank A. Erickcek for pointing this out to me.

The difference probably comes from our use of the Sachs-Wolfe approximation as opposed to their exact evaluation of the quadrupole.
References

[1] L. P. Grishchuk and I. B. Zel’dovich, Sov. Astron. 22 (1978) 125.

[2] A. L. Erickcek, M. Kamionkowski and S. M. Carroll, “A Hemispherical Power Asymmetry from Inflation,” Phys. Rev. D 78 (2008) 123520 [arXiv:0806.0377 [astro-ph]]. A. L. Erickcek, S. M. Carroll and M. Kamionkowski, “Superhorizon Perturbations and the Cosmic Microwave Background,” Phys. Rev. D 78 (2008) 083012 [arXiv:0808.1570 [astro-ph]].

[3] J. Hoftuft, H. K. Eriksen, A. J. Banday, K. M. Gorski, F. K. Hansen and P. B. Lilje, “Increasing evidence for hemispherical power asymmetry in the five-year WMAP data,” Astrophys. J. 699 (2009) 985 [arXiv:0903.1229 [astro-ph.CO]].

[4] P. A. R. Ade et al. [Planck Collaboration], “Planck 2013 results. XXIII. Isotropy and Statistics of the CMB,” arXiv:1303.5083 [astro-ph.CO].

[5] C. L. Bennett, R. S. Hill, G. Hinshaw, D. Larson, K. M. Smith, J. Dunkley, B. Gold and M. Halpern et al., “Seven-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Are There Cosmic Microwave Background Anomalies?,” Astrophys. J. Suppl. 192 (2011) 17 [arXiv:1001.4758 [astro-ph.CO]].

[6] C. M. Hirata, “Constraints on cosmic hemispherical power anomalies from quasars,” JCAP 0909 (2009) 011 [arXiv:0907.0703 [astro-ph.CO]].

[7] D. H. Lyth and D. Wands, “Generating the curvature perturbation without an inflaton,” Phys. Lett. B 524 (2002) 5 [hep-ph/0110002].

[8] J. Garriga and V. F. Mukhanov, “Perturbations in k-inflation,” Phys. Lett. B 458 (1999) 219 [hep-th/9904176].

[9] D. H. Lyth, “The curvature perturbation in a box,” JCAP 0712 (2007) 016 [arXiv:0707.0361 [astro-ph]].

[10] G. Hinshaw, D. Larson, E. Komatsu, D. N. Spergel, C. L. Bennett, J. Dunkley, M. R. Nolta and M. Halpern et al., “Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results,” arXiv:1212.5226 [astro-ph.CO].

[11] P. A. R. Ade et al. [Planck Collaboration], “Planck 2013 results. XVI. Cosmological parameters,” arXiv:1303.5076 [astro-ph.CO].

[12] P. A. R. Ade et al. [Planck Collaboration], “Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity,” arXiv:1303.5084 [astro-ph.CO].
[13] M. Sasaki and E. D. Stewart, “A General analytic formula for the spectral index of the density perturbations produced during inflation,” Prog. Theor. Phys. 95 (1996) 71 [astro-ph/9507001].

[14] D. H. Lyth and Y. Rodriguez, “The Inflationary prediction for primordial non-Gaussianity,” Phys. Rev. Lett. 95 (2005) 121302 [astro-ph/0504045].

[15] C. T. Byrnes, S. Nurmi, G. Tasinato and D. Wands, “Scale dependence of local $f_{NL}$,” JCAP 1002 (2010) 034 [arXiv:0911.2780 [astro-ph.CO]].

[16] A. D. Linde, “A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems,” Phys. Lett. B 108 (1982) 389; A. Albrecht and P. J. Steinhardt, “Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking,” Phys. Rev. Lett. 48 (1982) 1220.

[17] J. M. Maldacena, “Non-Gaussian features of primordial fluctuations in single field inflationary models,” JHEP 0305 (2003) 013 [astro-ph/0210603]. D. Seery and J. E. Lidsey, “Primordial non-Gaussianities from multiple-field inflation,” JCAP 0509 (2005) 011 [astro-ph/0506056]; D. Seery, K. A. Malik and D. H. Lyth, “Non-Gaussianity of inflationary field perturbations from the field equation,” JCAP 0803 (2008) 014 [arXiv:0802.0588 [astro-ph]].

[18] D. H. Lyth and I. Zaballa, “A Bound concerning primordial non-Gaussianity,” JCAP 0510 (2005) 005 [astro-ph/0507608]; I. Zaballa, Y. Rodriguez and D. H. Lyth, “Higher order contributions to the primordial non-Gaussianity,” JCAP 0606 (2006) 013 [astro-ph/0603534].

[19] G. Efstathiou, “A Maximum likelihood analysis of the low CMB multipoles from WMAP,” Mon. Not. Roy. Astron. Soc. 348 (2004) 885 [astro-ph/0310207].

[20] P. Chingangbam and Q. -G. Huang, “The Curvature Perturbation in the Axion-type Curvaton Model,” JCAP 0904 (2009) 031 [arXiv:0902.2619 [astro-ph.CO]]; Q. -G. Huang, “Negative spectral index of $f_{NL}$ in the axion-type curvaton model,” JCAP 1011 (2010) 026 [Erratum-ibid. 1102 (2011) E01] [arXiv:1008.2641 [astro-ph.CO]]; T. Kobayashi and T. Takahashi, “Runnings in the Curvaton,” JCAP 1206 (2012) 004 [arXiv:1203.3011 [astro-ph.CO]].

[21] M. Kopp, S. Hofmann and J. Weller, “Separate Universes Do Not Constrain Primordial Black Hole Formation,” Phys. Rev. D 83 (2011) 124025 [arXiv:1012.4369 [astro-ph.CO]].

[22] A. D. Linde and V. Mukhanov, “The curvaton web,” JCAP 0604 (2006) 009 [astro-ph/0511736].
[23] L. Dai, D. Jeong, M. Kamionkowski and J. Chluba, “The Pesky Power
Asymmetry,” arXiv:1303.6949 [astro-ph.CO].

[24] F. Schmidt and L. Hui, “Cosmic Microwave Background Power Asym-
metry from Non-Gaussian Modulation,” Phys. Rev. Lett. 110 (2013) 011301
[Publisher-note 110 (2013) 059902] [arXiv:1210.2965 [astro-ph.CO]].

[25] A. L. Erickcek, C. M. Hirata and M. Kamionkowski, “A Scale-Dependent
Power Asymmetry from Isocurvature Perturbations,” Phys. Rev. D 80
(2009) 083507 [arXiv:0907.0705 [astro-ph.CO]].

[26] Z. Chang and S. Wang, “Inflation and primordial power spectra at
anisotropic spacetime inspired by Planck’s constraints on isotropy of
CMB,” arXiv:1303.6058 [astro-ph.CO].

[27] Z. -G. Liu, Z. -K. Guo and Y. -S. Piao, “Obtaining the CMB anomalies
with a bounce from the contracting phase to inflation,” arXiv:1304.6527
[astro-ph.CO].

[28] J. McDonald, “Isocurvature and Curvaton Perturbations with Red Power
Spectrum and Large Hemispherical Asymmetry,” arXiv:1305.0525 [astro-
ph.CO].

[29] L. Wang and A. Mazumdar, “Small non-Gaussianity and dipole asymme-
try in the CMB,” arXiv:1304.6399 [astro-ph.CO].

[30] M. H. Namjoo, S. Baghram and H. Firouzjahi, “Hemispherical
Asymmetry and Local non-Gaussianity: a Consistency Condition,”
arXiv:1305.0813 [astro-ph.CO].