# Predicting the bearing capacity of pile installed into cohesive soil concerning the spatial variability of SPT data (A case study)

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**ABSTRACT**

Nowadays, in situ tests have played a viable role in geotechnical engineering and construction technology. Besides lab tests conducted on undisturbed soil samples, many different kinds of in-situ tests were used and proved to be more efficient in foundation design such as pressuremeter PMT, cone penetration test CPT, standard SPT, etc. Among them, a standard penetration test (SPT for short) is easy to carry out at the site. For decades, it has proved reliable to sandy soil, but many viewpoints and opinions argued that the test was not appropriately applicable to cohesive soil because of scattered and dispersed data of SPT blow counts through different layers. This paper firstly studies how reliable the SPT data can predict the physical and mechanical properties; secondly, the soil strength is determined in terms of corrected N-SPT values, and finally the bearing capacity of a pile penetrating cohesion soil. By analyzing data from 40 boreholes located in 18 projects in Ho Chi Minh City, South VietNam, coefficients of determination between SPT numbers and physical and mechanical properties of different soil kinds are not the same: $R^2 = 0.623$ for sand, $=0.363$ for sandy clay and $=0.189$ for clay. The spatial variability of soil properties is taken into account by calculating the scale of fluctuation $\theta = 4.65m$ beside the statistically-based data in horizontal directions. Finally, the results from two theoretical approaches of predicting pile bearing capacity were compared to those of finite element program Plaxis 3D and static load test at site. Correlation between the capacity computed by using corrected N-values instead of soil strength and results of static load test has proved to be well suitable in evaluating the bearing capacity of driven and jack-in piles, particularly installing in the cohesive soil using the SPT blows.

**1. Introduction**

The bearing capacity of a pile installed in a soil foundation can be predicted by several different methods. Some methods used directly soil properties, i.e., shear strength parameters (cohesion and internal friction angle) obtained by a lab test on undisturbed soil samples; other indirect methods used data from in situ tests to characterize the soil strength as converted properties, then apply them to analytical formulas to estimate bearing capacity of the pile. SPT is an in-situ test that in decades hitherto was proposed to be reliable to sandy soil.

For sandy soil, there is a real significant correlation between internal friction angle and N
value (Hatanaka & Uchida, 1996). SPT can be used for predicting the bearing capacity of shallow footings in which, bearing capacity factor for the depth of footings Nq, for friction Nγ were calculated from regression equations founded by different authors. In the research, the energy ratio is 60 percent, corrected SPT value N60 can be used for estimating the allowable contact pressure concerning a 30% probability of exceeding a settlement of 25 mm.

For clayey and cohesive soil, it is rather complicated to ensure the practicability of SPT for predicting soil properties. Due to scattered data in the correlation between SPT values and the increase of pore water pressure as the rod plunged into clay layers. Moreover, low permeability of soil may lead to appear a temporary resistance for driving, higher number of blows will be obtained. It might have generally been proposed that SPT is rather unreliable to this kind of fine-grained soil. Nevertheless, SPT has still been a main in-situ test in soil investigation reports, even for large scale projects. Mahmoud (2013) studied the reliability of using SPT in predicting geotechnical properties of silty clay and sandy soil. The results indicated not only physical properties but shear strength parameters such as cohesion and internal friction angle had a significant correlation to corrected N SPT number, even silty clay.

Variability of soil can be described clearly by the coefficient of variation (COV). This coefficient varied to a very wide range, depending on soil type. Samples from large data set at the different locations had the COV (ratio of standard deviation divided by mean) from more than 40% to 150% for some conventional properties (Phoon & Kulhawy, 1999).

Table 1
Wide range of coefficient of variation for clay soil

| Clay Layer | Depths (m) parallel to surface | Depths (m) Horizontal plans | Statistics  | N-SPT |
|------------|-------------------------------|----------------------------|-------------|-------|
| Layer 1    | 2 - 4                         | 1 - 3                      | Mean        | 2.6   |
|            |                               |                            | St. Dev     | 3.9   |
|            |                               |                            | COV (%)     | 150   |
| Layer 2    | 9 - 11                        | 8 - 10                     | Mean        | 8.2   |
|            |                               |                            | St. Dev     | 10.4  |
|            |                               |                            | COV (%)     | 127   |
| Layer 3    | 28 - 42                       | 28 - 42                    | Mean        | 8.9   |
|            |                               |                            | St. Dev     | 4.1   |
|            |                               |                            | COV (%)     | 47    |

Source: The researcher’s data analysis

On the other hand, the soil has inherent uncertainties due to spatial variability and its response to load and effects. The response of sand is quite different as compared to that of clay. Kulhawy et al. (1983) pointed out 4 categories of uncertainties, relating to soil type; they are saturation status, devices, and techniques of testing, and other uncontrollable factors (bores, diameter, efficient energy delivered to hammer, robs and type of drilling equipment…etc.). There are more than 27 sources of uncertainties relating to soil characteristics (5 factors), water table (2 factors), equipment (7 or more factors), and more than 10 factors of the working condition at sites
(Zekkos, Bray, & Der Kiureghian, 2004). Therefore, regarding SPT, it should take into account uncertainties in the performance of the test. There were many research works for this content (Kulhawy & Mayne, 1990). Both ASTM D-1586, ASTM D6066-96 prescribed recommendations for this feature as well.

In sandy soil, N is largely affected by overburden pressure. The more value of this pressure is, the correction factors will be higher, approximately proportioning to square of relative density $D_r$ (Meyerhoff, 1976) as expressed in the following formula:

$$ N = D_r^2 (a + b \cdot p') $$

(1)

where $a$ and $b$ are factors of material dependence and $p'$ is the mean effective stress (Kudmetha & Dey, 2012). Then the correction factor due to overburden pressure, namely $C_N$, is

$$ C_N = \frac{N_{\text{correct}}}{N} = \frac{D_r^2 (a+b \cdot 100)}{D_r^2 (a+b \cdot p')} = \frac{a+b \cdot 100}{a+b \cdot p'} $$

(2)

The correction factor $C_N$ by Skempton (1986) defined in Eq. (2) is

$$ C_N = \frac{n}{1+0.01 \sigma_{vo}} $$

(3)

where $\sigma_{vo}$ is overburden pressure in kPa, $n = 2$ for loose sand and $n = 3$ for dense sand.

For decades, numerous research works tried to determine the reliability of SPT data in predicting bearing capacity of pile installed into cohesive soil. For analysis of reliability, it is necessary to quantify uncertainties by statistical parameters (i.e., standard deviation, mean, and the law of distribution). Some authors proposed a scheme of reliability-based design in geotechnical engineering (Honjo, 2011).

For examining correlations between properties in soil foundation, the scale of fluctuation should be determined. That is a distance in the soil foundation within which there is an autocorrelation for a specific property (Vanmarcke, 1977). According to Duong (2017), Vanmarcke’s theories can be used to quantify the spatial variability of soil properties in estimating the allowable bearing capacity of the bored pile. The result is a regression formula as below:

$$ Q_a = 4346.3 - 5202.1 \rho_{12} + 34.9 \theta + 353.3 \text{COV}_c + 357.7 \text{COV}_\phi - 565.1 \text{COV}_E - 2.9 \beta + 0.14 P_u $$

(4)

where $\rho_{12}$ is the coefficient of correlation between the two zones 1 and 2 along the pile shaft and $\theta$ is scale of fluctuation in vertical direction. COV(c) COV($\phi$) is coefficient of variation of cohesion and internal friction angle, respectively; $\beta$ is reliability index and $P_u$ is the ultimate bearing capacity of single pile.

In this paper, SPT data are corrected to use indirectly in predicting the bearing capacity of a driven pile. By using regression analysis over a large amount of data, correlation between SPT and soil properties such as water content, depth of testing, modulus of elasticity, Atterberg limit, etc. of samples taking from 40 boreholes of 18 projects in Ho Chi Minh City were found, soil strength parameters converted from corrected SPT numbers together with physical properties of soil was also determined. At least three approaches of computing bearing capacity of pile installed into foundation are used in which layer of cohesive soil was dominant. Results obtained were compared to that of a static load test.
2. Method

2.1. Literature reviews on N-SPT data

Undisturbed soil samples were brought to the laboratory to identify physical and mechanical properties. Conventional tests, e.g. Atterberg Limits test, direct shear test, unconfined compression test, etc. were sufficient for supplying materials for computing bearing capacity of footings and single pile; N data were in-situ test and they were raw data recorded at a site without any correction. Depths of in situ testing were noted together with N-blow counts.

Bazaraa’s formula can be chosen to correct the raw N numbers (Civilblog, 2013). The first correction is due to the overburden pressure and the second is only for silty and fine-grained soil. There was no other correction for borehole diameter or for energy delivered to the rod penetrating soil layers. Therefore, the first uncertainty was the energy percentage delivered to the penetrator. Besides, there was no evidence that the site test had the same efficiency at every size of boreholes; and how about the rod length effects, etc… In general, the abovementioned issues could be taken into account by the formula as below:

\[ N_{60} = N_{\text{field}} \cdot C_E \cdot C_B \cdot C_S \cdot C_R \]  

(5)

where \( N_{\text{field}} \) is the blow counts recorded at the site; \( C_E \) is the factor of energy correction, \( C_B \) _factor of boreholes, \( C_S \) _factor of soil sample size, and \( C_R \) is the factor of rod length. Many different authors had pointed out the different levels of energy ratio ER (that was defined to be a ratio of measured energy divided by theoretical energy. Aoki and Velloso (1975) accepted ER = 70%; Shioi and Fukui (1982) suggested ER=55%, Meyerhoff (1976), ER=55%. A number ER= 55% was proposed to be appropriate to use in pile bearing capacity for projects in Viet Nam (Hoang & Tam, 2016).

Table 2

| Formula by                        | Skin friction                                      | Point bearing                                      |
|-----------------------------------|----------------------------------------------------|----------------------------------------------------|
| Aoki and Velloso (1975)           | \( Q_s = \frac{ak}{3.5} N_i \) \( a=14, k=1 \) (sand); | \( Q_p = \frac{k}{1.75} N_b \) \( a=60, k=0.2 \) (clay) |
| Shioi and Fukui (1982)            | Sand: \( Q_s = 2N_i \) Clay: \( Q_s = 10N_i \)   | Sand: \( Q_p = (1+0.04 \frac{D_p}{B})N_b \) Clay: \( Q_p = (1+0.06 \frac{D_p}{B})N_b \) |
| Meyerhoff (1976)                  | \( n_s = 1 \) \( Q_s = n_s N_i \) Non-displacement pile; \( n_s = 2 \) Displacement pile | \( Q_p = 0.4N_b C_1 C_2 \) \( C_1, C_2 \) factors dependent to ratio D/B (i.e., diameter to pile length) |
| Bazaraa and Kurkur (1986)         | \( Q_s = n_s N_i \) \( n_s=2\sim 4 \)             | \( Q_b = n_b N_b \) \( n_b=0.06\sim 0.2 \) \( N_b \) average of N taken 1B above and 3.75B below pile tip |

Source: Hoang and Tam (2016)
2.2. Correction factors

Bazaraa (1967) proposed the following corrections to obtain the actual count \( N \), based on the overburden pressure (\( N \) is modified to be \( N' \)) (Civilblog, 2013):

For \( p_o \leq 75 \text{ kPa} \)

\[
N' = \frac{4N}{1 + 0.04p_o}
\]  
(5a)

For \( p_o > 75 \text{ kPa} \)

\[
N' = \frac{4N}{3.25 + 0.01p_o}
\]  
(5b)

where \( N' \) is corrected \( N \) value; \( N \) is the observed \( N \)-value; \( p_o \) is overburden pressure, (kPa) = \( \gamma D \); \( D \) is depth of penetration (m); \( \gamma \) is unit weight of soil at the time of testing.

If the stratum (during testing) consists of fine sand & silt below water table, the corrected \( N \)-value (or \( N' \)) has to be further corrected to get the final corrected value \( N'' \) as below:

\[
N'' = 15 + \frac{1}{2} (N' - 15)
\]  
(6)

As such, there are two steps of correction against \( N \) numbers for every usage for all geotechnical computations.

2.3. Spatial variability – Scale of fluctuation

Pile bearing capacity includes shaft friction along the length \( u_{D1} \) and point bearing within \( u_{D2} \) as described in Figure 1.

![Figure 1](image)

**Figure 1.** Diagram for computation scale of fluctuation

Corrected SPT numbers were assumed to display as in Figure 1b. Correlation between the two zone \( u_{D1} \) and \( u_{D2} \) was characterized by a correlation factor as follows:
\[
\rho_{\mu_1, \mu_2} = \frac{D_2^2 \Gamma^2(D_{12}) - D_1^2 \Gamma^2(D_1) - D_2^2 \Gamma^2(D_2)}{2D_1D_2 \Gamma(D_1) \Gamma(D_2)}
\]

(7)

where \(\Gamma^2(\Delta z)\) denoted variance reduction factor for spatial average for interval \(\Delta z\) of pile shaft, determined by Vanmarcke (1983):

\[
\Gamma^2(\Delta z) = \frac{1}{2} \left( \frac{\theta}{\Delta z} \right)^2 \left[ \frac{2\Delta z}{\theta} - 1 + \exp\left( -\frac{2\Delta z}{\theta} \right) \right]
\]

(8)

- The scale of fluctuation \(\theta\) being defined as the distance within which some specific soil property have significant correlation from point to point (Vanmarcke, 1977, 1983) and variance reduction factor \(\Gamma^2(\Delta z)\) will be applied simultaneously to calculate:
  - Skin friction using the average value of corrected SPT blows within the scale of fluctuation;
  - Point bearing using the average value of corrected SPT number from 1D below pile tip and 4D above the level of pile tip;
  - Reliability using standard deviation, which is the square root of variance multiplied by \(\Gamma^2(\Delta z)\);
  - Regression Formula between bearing capacity and several predictors (i.e., independent variables and parameters), somewhat like the abovementioned formula (4) studied by Duong (2017).

### 2.4. Pile bearing capacity considering the spatial variability

As abovementioned remarks, steps for estimating bearing capacity for pile will consider two main issues: correction and spatial variability (both vertical and horizontal direction). Suitable approach will be suggested as below:

- N-SPT blow counts will be corrected first, two kinds of correction are obligatory: due to depth (sand) and due to fine grained soil and silty sand (clayey soil);
- Determine scale of fluctuation of SPT numbers \(N\). According to Vanmarcke (1977), the scale of fluctuation \(\theta\) may approximately equals to 0.8(\(\bar{d}\)) where

\[
\bar{d} = \frac{1}{n} \sum_{i=1}^{n} d_i
\]

(9)

where \(d_i\) as shown in Figure 1 is intersections of fluctuating property and its trend function (layers \(d_1, d_2 \ldots d_i\) is less than the scale of fluctuation \(\theta\)). Computation was conducted on sublayers which are smaller than \(\theta\).

- Determine characteristic length \(L\) (Cherubini & Vessia, 2010) \(L=D+B\) in which \(D\) is the embedment depth and \(B\) is foundation width (i.e., pile diameter);
- Compute \(\Gamma^2(L)\), denoted variance reduction factor, using Vanmarcke’s formula (8);
- Within the scale of fluctuation, the average value of corrected N SPT number was used to compute friction component of pile bearing capacity;
- Compute friction component of bearing capacity for individual segments (incremental
length of the pile) and point bearing component of bearing capacity of the pile;

- In the numerical model, layers will be divided into sub-layers that based upon the scale of fluctuation within which, soil strength is indirectly determined by corrected N-values;

- If reliability index of bearing capacity is required, compute variance reduction factor (i.e., variance multiplied by variance reduction factor), standard deviation, and the average value (reliability index was defined as the ratio between average value divided by the standard deviation for a specified limit function).

- Reliability of SPT numbers will be assessed by comparing indirectly predicted value of pile bearing capacity using either Meyerhoff’s formulas and finite element modeling software and that of static load test;

- Compute friction component of bearing capacity for individual segments (incremental length of pile) and point bearing component of bearing capacity of pile;

- In numerical model, layers will be divided into sub-layers that based upon the scale of fluctuation within which, soil strength is indirectly determined by corrected N-values;

- If reliability index of bearing capacity is required, compute variance reduction factor (i.e., variance multiplied by variance reduction factor), standard deviation and average value (reliability index was defined as ratio between average value divided by standard deviation for a specified limit function);

- Applicability of SPT numbers will be assessed by comparing indirectly predicted value of pile bearing capacity using either Meyerhoff’s formulas and finite element modeling software and that of static load test.

**Table 3**

Data collection for regression analysis

| Project No | Depth of samples Z (m) | Water content ω | Dried γd (kN/m³) | Initial void ratio e | Plastic index Ip | Overburden pressure p, kPa | Elastic modulus E (kPa) | Raw SPT N | Corrected N' | Corrected N'' |
|------------|------------------------|------------------|-------------------|---------------------|-----------------|-------------------------|------------------------|------------|--------------|--------------|
| 2          | 2                      | 0.38             | 13.02             | 1.07                | 0.24            | 35.3                    | 22.61                  | 2          | 4            | 10           |
| 2          | 4                      | 0.39             | 13.03             | 1.06                | 0.25            | 70.8                    | 21.49                  | 2          | 3            | 9            |
| 2          | 6                      | 0.29             | 14.55             | 0.84                | 0.17            | 88.4                    | 24.69                  | 5          | 5            | 10           |
| 2          | 6                      | 0.33             | 14.19             | 0.90                | 0.19            | 78.6                    | 23.26                  | 5          | 5            | 10           |
| …          |                        |                  |                   |                     |                 |                         |                        |            |              |              |
| 18         | 48                     | 0.20             | 16.68             | 0.63                | 0.25            | 520                     | 45.92                  | 33         | 16           | 16           |
| 18         | 50                     | 0.19             | 16.61             | 0.61                | 0.18            | 539                     | 48.68                  | 34         | 16           | 16           |
| 18         | 52                     | 0.18             | 16.60             | 0.62                | 0.19            | 559                     | 42.70                  | 37         | 17           | 16           |
| 18         | 54                     | 0.18             | 16.74             | 0.60                | 0.17            | 578                     | 45.15                  | 31         | 14           | 15           |

Source: Truong (2017)
3. Results

3.1. Data collection and soil strength in terms of corrected N blows

To assess the reliability of SPT numbers in predicting the physical properties and strength of cohesive soil, the procedure is:

- Classify data into 4 groups: medium sand, silty sand, clayey sand, and sandy clay.
- Soil data of 40 boreholes taken from 18 projects were tabulated as in Table 3, in which soil was again classified into three groups: non-cohesive soil (sand), fine soil (clay), and cohesive soil (both clayey sand and sandy clay) for clearly physical properties.
- Because SPT data must be corrected by transforming N into N’ for sand and N’ into N” for clay, depth of sampling was taken into account in all regression equations.

Table 4
Collected Data of 18 projects

| Type                        | Project Number                  | Number of samples | State of soil               |
|-----------------------------|---------------------------------|-------------------|-----------------------------|
| Sand                        | 2, 6, 10, 17, 18 (BH1)          | 20                | Medium density              |
| Clayey sand, sandy clay     | 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 13, 16, 17, 18 (BH1) | 185               | Mainly plastic              |
| Clay                        | 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12, 16, 17, 18 (BH2) | 233               | Semi solid to stiff, low plasticity |

Source: Truong (2017)

In order to find out the correlation between SPT numbers and soil physical properties such as unit weight $\gamma$, moisture $\omega$, void ratio $\varepsilon$, modulus of elasticity $E$, plasticity index $I_p$ and shear strength ($c_u$, $\phi$), data were tabulated as described in Table 3 for each soil group.

![Figure 2. Data analysis tool in Excel (Truong, 2017)](image)

With level of confidence is 95%, corrected N is chosen to be dependent variable $Y$ and independent variables $X$s.
Figure 3. Regression analysis window for correlation between SPT and other properties

- N’ will be correction value for sand and N” for both clayey sand and sandy clay (cohesive soil in general). For predicting indirectly the bearing capacity of the pile, the bearing component at pile tip, and the friction along the shaft are converted in terms of N’ or N”. With soil strength parameters (i.e., cohesion and internal friction angle) and physical properties related to SPT number, a regression equation will be obtained and used in evaluating the bearing capacity of soil foundation or driven pile.

It is necessary to compare three values:

- Parameters of soil properties (physical parameters and soil strength) in terms of SPT numbers (this study);
- Shear strength of the conventionally obtained lab tests.
- Shear strength of previous studies about correlations between soil properties (physical, mechanical properties and compressibility).

Then the most appropriate values to use in foundation engineering are selected.

3.2. Correlation equations

Some results were described in Tables 4, 5 and equations of multivariable regression are shown in Figure 4:

Figure 4. Regression statistics and variance analysis ANOVA (Truong, 2017)
Results of multivariable linear regression for different kinds of soil (Sand, Clay, Sandy Clay and Clayed Sand) are tabulated in Table 5.

**Table 5**

Significance of the Analysis of Variance (ANOVA) for sand, sandy clay/clayey sand, clay

| Type                      | Regression Statistics | Analysis of variance (ANOVA) |
|---------------------------|-----------------------|-----------------------------|
|                           | $R^2$                 | Adjusted $R^2$              | $F$  | Significance F |
| Sand (20 samples)         | 0.623                 | **0.456**                   | 6.194 | 0.003          |
| Clayey sand, Sandy clay (185 samples) | 0.363                 | **0.341**                   | 16.905 | 2.07E-15      |
| Clay (233 samples)        | 0.189                 | **0.171**                   | 10.571 | 3.86E-9       |

Source: Regression equation for sandy soil

$$N' = 224.9 - 0.13Z + 0.022 \omega - 7.89 \gamma_d - 139.67\epsilon + 0.02E$$  \hspace{1cm} (10)

where $Z$ is the depth of sampling, other symbols are given in Table 3.

In multivariable linear regression analysis, $R^2$ adjusted is used instead of ($R$ Squared). $R^2$ adjusted = **0.456** (or $R$ = 67.5%) indicated that the predictors or a few independent variables as prescribed only explained nearly 46% dependent variables $N'$. It meant that more than 54% was due to other uncertainties in measurement, errors in lab tests etc. These properties of sand were rather reliable in predicting $N'$.

As for silty sand, results from observed 70 samples showed no correlation found.

**Regression formula for clayey sand and sandy clay**

$$N' = 9.955 + 0.036Z - 0.026\omega + 0.813\gamma_d + 0.098(I_p) + 0.009E$$  \hspace{1cm} (11)

$R^2$ (R Squared adjusted) = 0.341 (slightly smaller as that of Sand with R Squared adjusted = 0.456) indicated that the predictors or a few independent variables as prescribed only explained nearly 34% dependent variables $N'$. It meant that more than 66% was due to other uncertainties in measurement, errors in lab tests, etc. These properties were relatively reliable in predicting $N'$.

**Regression formula for clay**

$$N'' = 78.629 - 0.044Z - 82.65\omega - 2.81\gamma_d + 7.31(I_p) + 0.048E$$  \hspace{1cm} (12)

$R^2$ (R Squared adjusted) = 0.171 indicated that the predictors or a few independent variables as prescribed only explained nearly 17% dependent variables $N'$. It meant that more than 83% was due to other uncertainties in measurement, errors in lab tests, etc.

These properties were weakly reliable in predicting $N''$. Based on soil data and corrected numbers of SPT data, by conducting multivariable regression analysis, some results are:

- Sand: relatively usable
- Sandy Clay/ Clayey Sand: tentatively usable
• Clay: tentatively usable with remarkable caution.

• Based on $R^2$ adjusted ($R^2$ adjusted = 0.456 for sand and = 0.341 for clayey sand and sandy clay; for clay, $R^2$ adjusted = 0.171) SPT is reliable for sand and clayey sand, and *weakly reliable for clay* in a multi variable regression model.

• Although the R squared was relatively small, but strongly related to each other, expressed in very small value Significant F in the most right column of Table 6

**Table 6**
Regression statistics and variance analysis ANOVA (Truong, 2017)

| Source: Truong (2017) |
|------------------------|

- For sand moisture, dried density, initial void ratio and overburden pressure (depth of samplings) affected most to corrected SPT numbers instead of recording data without correction.

- For clay, depth of sampling, plasticity index did not affect SPT both $N'$ and $N''$. Cohesion and modulus of deformation E had a slight effect on $N''$. This might be unclear.

- For cohesive soil, depth of sampling affected most significant the corrected SPT numbers. Hence, a correction was obviously necessary.

- Corrected SPT numbers were applicable for both sand ($N'$) and clay ($N''$), with different coefficients of determination.

### 3.3. Single-variable regression analysis for shear strength parameters of cohesive soil

At 95% confidence level: there was a weak correlation between corrected SPT number $N''$ and many different soil parameters altogether but it was a significant correlation between $N''$ (or $N'$) and E, c, φ. Regression formulas for the main parameters of soil strength and compressibility for different soils are in Table 7.

**Table 7**
Regression formulas for converting $N$-values to soil properties (units in SI, i.e., kPa and degree)

| Soil type | Confidence level |
|-----------|------------------|
| **Sand**  | **85%**          | **95%**          |
| E         | E=111.5+1.826N'  | E=118.8+2.431N' |
| φ         | φ =32.6°-0.01N'  | φ =32.93°-.02°N'|
| E         | E=57.67+2.38N'' | E=61.63+2.37N'' |
| **Clayey sand/ Sandy** | | |
| C         | C=0.134+0.001N'' | C=0.141+0.002N''|
| φ         | φ =19.81+0.52N'' | φ =20.33+0.56N''|
| Soil type | Confidence level | 85% | 95% |
|-----------|------------------|-----|-----|
| Clay      | E                | E=52.6+1.7N'' | E=56.7+1.939 N'' |
| Clay      | C                | C=0.367+0.02N'' | C=0.407+0.022N'' |
| Clay      | φ                | φ=15.89+0.36N'' | φ=16.5+0.39N'' |
| Clay      | E                | E=52.6+1.7N'' | E=56.7+1.939N'' |

Source: Truong (2017)

Results obtained from the abovementioned regression analysis can be compared to those of previous works conducted by Mahmoud (2013) in which shear strength of silty clay with sand soil can be calculated as following equations:

\[
φ \text{ (in degree)} = 0.209N'' + 19.68 \\
C \text{ (in kG force/cm}^2\text{)} = 0.014 N'' - 0.18
\]

where N'' is the corrected SPT numbers, E in kPa.

4. Prediction of the bearing capacity of pile as per TCVN 10304: 2014 (Ministry of Sciences and Technology, Vietnamese Government, 2014) concerning the spatial variability

Mean values of corrected SPT N-values will be estimated in sub-layers which should be smaller than scale of fluctuation \( \theta \) (Figure 5). Soil properties within scale of fluctuation are chosen by using converted values from corrected SPT numbers.

**4.1. Soil profile**

Soil profile are shown in Figure 5. A designed pile will be installed through 5 soil layers. Clayey soil is dominant. Average depth is from formula (9), or

\[
\bar{d} = \frac{5.7 + 1.9 + 7.5 + 6 + 8}{5} = 5.82 m
\]

Scale of fluctuation was taken \( \theta = 0.8 \bar{d} =4.65 \) m. Pile bearing capacity can be calculated appropriately by using soil properties within a distance \( d_i < 0 \).
In general, ultimate bearing capacity is calculated by formula as below:

\[ Q_s = q_bA_b + \mu \sum (f_{c,i}l_{c,i} + f_{s,i}l_{s,i}) \]  

(13)

where, \( q_b \) is point bearing resistance in kPa, \( A_b \) and \( \mu \) are cross section of pile tip and perimeter of pile section, respectively. \( f_{c,i} \) and \( l_{c,i} \) are skin friction and length of \( i \)th pile segment penetrating in clay, respectively; and the second term in parentheses of the formula (13) is for sand \( i \)th layers. All the resistance of soil at pile tip \( q_b \) or shaft friction \( f_{c,i} \) along the pile mantle are calculated by the modified N-SPT numbers, especially within the scale of fluctuation \( \theta \) (i.e., black vertical line).

**4.2. Bearing capacity suggested by the Meyerhoff’s formula**

This formula has still been used popularly in Vietnam as an alternative for comparison purpose (TCVN 10304: 2014, Ministry of Sciences and Technology, Vietnamese Government, 2014). Soil strength will be used to compute shaft pile friction and point bearing.

For pile installing into sandy soil,

\[ f_{s,i} = K_s \sigma_v \tan \delta \]  

(14)

\[ q_b = \sigma_v N_q + c.N_c \]  

(15)

where \( N_q \) \( N_c \) are bearing capacity factor, dependent on friction angle of soil; \( K_s \) is coefficient of lateral pressure, \( K_s = (1-1.2)(1-\sin \phi') \) for driven pile; \( \tan(\delta) \) is coefficient of external friction between soil and pile shaft.

Appendix G of TCVN 10304:2014 (Ministry of Sciences and Technology, Vietnamese Government, 2014) also recommended a Meyerhoff’s formula of skin friction and point bearing resistance using SPT N-values, as below:

\[ f_{s,i} = \frac{10.N_{s,i}}{3} \]  

(16)
For pile installing into cohesive soil

\[ f_{c,i} = \alpha_p \cdot f_{L} \cdot c_{u,i} \] (18)

\[ q_b = 9 \cdot c_u \] (19)

where \( \alpha_p \) is factor applied for driven pile, depended on the ratio of undrained strength to average effective stress (in spreadsheet denoted \( \alpha_p \)); \( f_L \) is a factor relating to the slenderness length \( h/D \) (\( D= \) diameter) of pile.

These abovementioned formulas were applied to driven or jacked-in pile. For considering spatial variability, each soil layer will be chosen as \( d_i \) as in Figure 2. In case the scale of fluctuation is taken into account, friction and point bearing will be computed within that scale of fluctuation \( \theta \).

Table 8
Illustrated spreadsheet for computing abutment B pile bearing capacity using Meyerhoff’s formula and corrected \( N' \), borehole BH-1

| No. | Soil layers      | Thickness | \( \gamma \) | \( G'_{vo} \) | \( \phi \) | \( c \) | N-SPT | \( a_p \) | \( C_u,i \) | \( k_2 \) | \( f_i \) | Axq | Qi | WITH CORRECTED N VALUES | 8.4 | M | (D=0.35) |
|-----|------------------|-----------|-------------|--------------|---------|-------|-------|-------|---------|-------|-------|-----|-----|----------------------|-----|---|-------------|
| 1   | Firmly plastic clay | 2 | 19.8 | 19.8 | 16.8 | 2.4 | 7.75 | 1 | 8.39 | 2 | 8.4 | 2.8 | 23.5 | 8.4 | 0.35 | M |
| 2   | Firmly plastic clay | 2 | 19.8 | 59.4 | 16.8 | 2.4 | 13.9 | 1 | 20.31 | 2 | 20.3 | 2.8 | 56.9 | 8.4 | 0.35 | M |
| 3   | Firmly plastic clay | 1.7 | 19.8 | 96.0 | 16.8 | 2.4 | 13.4 | 1 | 31.33 | 2 | 31.3 | 2.4 | 74.6 | 8.4 | 0.35 | M |
| 4   | Granular soil, dense | 1.9 | 2.02 | 13.21 | 31.3 | 0 | 13.6 | 0.63 | 80.29 | 2 | 50.5 | 2.7 | 134.2 | 8.4 | 0.35 | M |
| 5   | Plastic clayey sand | 0.8 | 1.05 | 15.54 | 25.2 | 1.1 | 12.9 | 0.68 | 74.24 | 2 | 50.2 | 1.1 | 56.2 | 8.4 | 0.35 | M |

Friction component (kN) \textbf{345.3}

| Friction component (kN) | 345.3 |
|-------------------------|-------|

Point bearing component

| \( N_p \) | \( k_1 \) | \( q_b \) | \( A_b \) | \( Q_b \) |
|-----------|---------|-------|-------|-------|
| kPa       | m²      | kN   |
| 13.2      | 400     | 5260 | 0.12  | 644.4 |

Total bearing capacity (kN) \textbf{989.7}

Source: Truong (2017)

If the uncorrected \( N \) values are used, bearing capacity equals approximately to 85.86 tons with:

- Friction component: 282.6 kN (-18.1% as compared to that of using corrected \( N \)).
Point bearing component: 572.8 kN (-11.1% as compared to that of using corrected N).

4.3. Bearing capacity suggested by Architectural Institute of Japan (item G.3.2 of the TCVN 10304: 2014) (Ministry of Sciences and Technology, Vietnamese Government, 2014)

Appendix G of National Standard (TCVN 10304: 2014, Ministry of Sciences and Technology, Vietnamese Government, 2014) described steps to apply the main contents of Recommendations for Design of Building Foundation (Architectural Institute of Japan issued in 1988, hereinafter denoted AIJ for short) in predicting the bearing capacity of pile, both driven and bored piles. SPT data were used to indirectly compute friction $f_s,i$ and point bearing resistance $q_b$ by formulas (15) and (17) where $N_{s,i}$ is average number of SPT in the $i^{th}$ soil layer; $N_p$ is average value of SPT blow counts taken within a zone 1D below the pile tip level and 4D above level of the pile tip. An Excel spreadsheet for computing bearing capacity was displayed as in Table 9 below:

**Table 9**

Illustrated spreadsheet for computing abutment B pile bearing capacity using recommendation of AIJ and corrected $N'$, borehole BH-1

| Soil layers            | thickness | $\gamma$ | $\sigma_{vo}$ | SPT | $a_p$ | $f_l$ | $C_{u,i}$ | $f_i$ | $A_x$ | $Q_s$ |
|------------------------|-----------|----------|---------------|-----|-------|------|-----------|------|------|-------|
| Firmly plastic clay    | 2         | 19.8     | 19.8          | 7.8 | 0.5   | 48.44| 24.2      | 2.8  | 67.8 |
| Firmly plastic clay    | 2         | 19.8     | 59.4          | 14.0| 0.5   | 87.29| 43.6      | 2.8  | 122.2|
| Firmly plastic clay    | 1.7       | 19.8     | 96.03         | 13.4| 0.5   | 83.99| 42.0      | 2.38 | 100  |
| Granular soil, dense   | 1.9       | 20.2     | 132.1         | 13.6| 0.7   | 85.26| 04.5      | 2.66 | 12.1 |
| Plastic clayey sand    | 0.8       | 10.5     | 155.4         | 13.0| 0.8   | 81.21| 65.6      | 1.12 | 73.5 |

Friction component (kN) **375.6**

| Point bearing component | $N_p$ | $k_1$ | $q_b$ | $A_b$ | $Q_b$ |
|-------------------------|-------|------|------|-------|-------|
|                         | kPa   | m2   | kN   |       |       |
| Friction                | 13.2  | 400  | 5260 | 0.12  | 88.2  |

Total bearing capacity (kN) **463.8**

Source: Truong (2017)

For uncorrected $N$ blow counts, total bearing capacity equals approximately to 378 kN with:

- Friction component: 305 kN (-18.8% as compared to that of using corrected N).
- Point bearing component: 73 kN (-17.2% as compared to that of using corrected N).

4.4. Bearing capacity determined by numerical method

For comparison purposes, a Plaxis 3D model was used in Figure 6. Mohr-Coulomb (MC) soil behavior model was chosen because of its relevancy to the bearing capacity problem, partly because of limited data from soil reports (without results from triaxial compression tests). Data of
soil properties input into software were converted from corrected SPT N-values, as abovementioned regression equations of Table 7. Calibration for the model was disregarded in case accepting a linear proportion factor between measured bearing capacity and computed one.

![Figure 6. Plaxis model for determining bearing capacity of pile](image)

At-site determination of the ultimate bearing capacity of a pile has complied with item 7.3.2 of TCVN 10304: 2014 (Ministry of Sciences and Technology, Vietnamese Government, 2014) “Pile Foundation - Code for design building foundation and construction works” that admitted a settlement at failure as below:

\[ S = \xi S_{gh} \]  \hspace{1cm} (20)

where, \( S_{gh} \) is settlement at ultimate condition, taken as 40mm (item 7.3.2); \( \xi = 0.2 \). As such, ultimate bearing capacity will be the load at which the pile settlement equals to 8 mm:

![Figure 7. Ultimate bearing capacity of 35cm square pile from static load test and Plaxis model](image)

Ultimate bearing capacity by static load test is 676.8 kN, while this value is determined by yield point (big displacement at a constantly kept load) at \( P=473.7 \) kN (solid circle line in Figure 7).

Three piles with different configuration were considered: For borehole BH1 with 5 layers:
abutment B pile (square 35cm, L=8.4 m) and pile P58 (square 25cm pile, L=8.6 m). For borehole BH2, with 7 layers: pile P61 (square 30cm, L=12.2 m). Calculating spreadsheets are established as in Table 8, Table 9. Results are compared as shown in Figure 8:

![Figure 8](image_url)

**Figure 8.** Comparisons between ultimate bearing capacity of pile using Meyerhoff’s formula, AIJ, Plaxis and Static Load Test (Truong, 2017)

### 4.5. Discussion

- Correlation equations in Table 5 should be studied within the scale of fluctuation for higher $R^2$ (adj.) instead of the entire soil profile. But since the thickness of soil layers was smaller than the scale of fluctuation $\theta$, and $R^2$ (adj.) was very high, therefore, the calculation for bearing capacity would be implemented with sufficient and reasonable accuracy in practice. By applying the scale of fluctuation, the spatial variability is taken into account, so the bearing capacity for the pile is calculated with a more rigorous procedure.

- Regression equations for converting N-values into soil strength might have some errors. The most likely value may be computed by the square root of the sum of squares (or SRSS) law as follows:

$$C_{design} = \sqrt{(C_{labtest})^2 + (C_{convert})^2 + (C_{model})^2}$$  \hspace{1cm} (21)

in which, the first term belongs to soil properties obtained by conventional lab tests; the second term refers to measurement or correlation with corrected N-values, and the third term relates to formula of shaft friction and tip resistance (Cherubini & Vessia, 2010).

- Back analysis to calibrate the numerical model is necessarily conducted for obtaining the proper set of soil strength properties unless a linear correlation in the elastic domain is found.

- Load - displacement curve obtained by Plaxis indicated that soil foundation for the pile was still workable in the elastic domain. Meanwhile, results from the static load test showed a sharper trend in curvature, indicating a yielding point in the bearing capacity of the soil foundation.

- Bearing capacity computed by AIJ using directly corrected N-values proved to be close to that of the static load test (Figure 8). Furthermore, comparison on results of bearing capacity obtained by two approaches, (one from numerical finite element model _ Plaxis 3D, using converting data of soil properties from N-values_, and the other from static load test) pointed out
that there was a **linear** correlation between them as in Figure 9 as following:

![Figure 9. Results of bearing capacity obtained by Plaxis, A.I.J v/s by Static Load Test](image)

This may come to a suggestion that corrected N-values can be used tentatively in predicting the bearing capacity of pile installing into cohesive soil at a specific site, and AIJ formula using directly corrected N-values will be more predictable than other analytical approaches.

Single variable linear regression analysis at a level of confidence of 95% provides a set of converted parameters of soil strength-friction angle and cohesion_ which is possible to predict bearing capacity in a numerical model.

### 4. Conclusion

The bearing capacity of a pile installing into cohesion soil can be assessed by comparing the results obtained by theoretical formulas using converting corrected N-values, numerical model, and static load tests. Multivariable linear regression analysis showed a weak correlation between N-values and physical properties of the cohesive soil and depth of testing, but single variable linear regression model showed a significant correlation of corrected N values to cohesive soil strength (i.e., friction angle and cohesion) with a level of confidence 95%. The spatial variability is taken into account with the scale of fluctuation, \( \theta \), postulated by Vanmarcke (1977). The soil profile characterized by the numbers of SPT blows is divided into segments of which its length is smaller than \( \theta \). Two locally used approaches including the Meyerhoff’s formula, and recommendation suggested by the Architectural Institute of Japan, Appendix G, item G.3.2 (TCVN 10304: 2014, Ministry of Sciences and Technology, Vietnamese Government, 2014) were used in which the soil strength were indirectly converted from corrected N-values and assigned as input data into models; the results were compared to those of numerical model Plaxis 3D and static load test. Results indicated that the approach of AIJ using directly SPT data predicted a closer value of bearing capacity as compared to that of the static load test. Besides, the numerical model Plaxis 3D using indirect SPT data (i.e., model converted SPT data to soil strength and compressibility) pointed out a value of bearing capacity which was a highly linear correlation to the reliable result of the static load test. These results could help practitioners in estimating bearing capacity with SPT data with a satisfactory accuracy.

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