Chameleon dark matter stars

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We consider static, spherically symmetric equilibrium configurations consisting of interacting dark matter/dark energy and embedded in an external, homogeneous chameleon scalar field. With the coupling function, form of which is taken to meet cosmological observations, we estimate the effect of such a nonminimal coupling on the properties of dark matter compact configurations. We show that the masses and sizes of the resulting chameleon dark matter stars are smaller than those of systems with no field present.

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I. INTRODUCTION

During the past one and a half decades, a number of conclusive evidences have appeared for that the present Universe is expanding with some acceleration [1, 2]. Such an acceleration could not be explained only by ordinary matter from which visible stars and components of galaxies in the Universe are made of. Invisible form of matter called dark energy (DE), which works as a repulsive force against attracting gravity and drives the accelerated expansion, has been invoked. The observational data indicate that more than 70% of the energy density in the present Universe could be assigned to DE.

Another important notion used to explain the evolution of the present Universe is the gravitationally attractive invisible substance called dark matter (DM). Its contribution to the total energy density is estimated to be of order 23%. DM is clustered on scales of the order of galaxies and clusters of galaxies, and its existence is so far evident only via its gravitational interaction.

Despite the fact that the true origin of neither DM nor DE is currently known, various ways have been suggested to model them. The simplest approach is the so-called ΛCDM model, where DE is described by Einstein’s Λ-term, and DM is supposed to be a pressureless fluid (cold dark matter). However, the well-known cosmological constant problem related to this model suggests one to look for other ways to describe the accelerated expansion of the Universe. Perhaps one of the most promising approaches to address the origin of DE are theories that include various fundamental fields [1, 2]. Other possible ways discussed in the literature are modified (non-Einstein) gravity theories [3, 4] and models with extra space dimensions [5, 6].

Along these lines, cosmological models with DM and DE interacting with each other not only gravitationally but also by a direct coupling [7–13] (for a review, see, e.g., [2, 14]) have a number of interesting features, and allow one to solve some known problems, for example, the so-called coincidence problem.

On the other hand, the question arises: if the interacting DM and DE do exist, how such interaction will affect the properties of small-scale objects consisting of such substance? In the bulk of the literature, various localized compact objects consisting of dark energy [15–24] and of dark matter [25–29] have been studied. In these models, DE and DM interact either gravitationally or by direct coupling. The direct coupling has been considered in Ref. [27], where a static compact configuration consisting of DM coupled to a scalar field responsible for DE was studied. In that paper, the assumption was that the mass of dark matter particles depends on the scalar field, that leads to significant changes in properties of the resulting configurations, such as their masses and sizes.

In the present paper, we deal with another possibility when the mass of a scalar field which is responsible for the acceleration depends on the surrounding matter content. As the matter content, both ordinary (baryon) matter and dark matter can serve. The use of such scalar field, dubbed a “chameleon” scalar field, forms the grounds of the alternative model of dynamical dark energy which is proposed in Refs. [30–32]. With the assumption that the

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chameleon scalar field interacts directly with dark matter, one can construct a cosmological model of the present Universe which is consistent with all current observations \[10\].

We use the model of Ref. \[10\] to describe compact objects consisting of dark matter which interacts directly with dark energy. We will identify the effect of this kind of nonminimal coupling on masses and sizes of resulting compact objects.

The paper is organized as follows. In Sec. III the general set of equations is derived for equilibrium configurations consisting of interacting DM and DE. Choosing an equation of state of dark matter in the form of an ideal completely degenerate Fermi gas, in Sec. II B we write down equations for this particular case, and solve them numerically in Sec. III. In conclusion, we summarize the results obtained.

## II. EQUATIONS FOR EQUILIBRIUM CONFIGURATIONS

We study the effect of the presence of direct interaction between a cosmological chameleon scalar field and dark matter on compact configurations consisting of such dark matter. To evaluate the effect, we specify the interaction by using the model of cosmological evolution of the Universe of Ref. \[10\]. Our basic set up is that the dark matter, being embedded in an external, homogeneous chameleon scalar field, feels its presence not only gravitationally but also through the nonminimal coupling. Obviously, the characteristics of such mixed configurations will then be determined by the properties both of the dark matter and of the chameleon scalar field.

### A. Lagrangian and general set of equations

General form of the Lagrangian of the system under consideration can be written as follows:

\[
L = -\frac{c^4}{16\pi G} R + \frac{\Delta}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) + f(\varphi) L_{DM} .
\]

Here \(\varphi\) is the real scalar field with the potential \(V(\varphi)\), \(\Delta = \pm 1\) corresponds to the usual or phantom scalar field, respectively, \(L_{DM}\) is the Lagrangian of dark matter. The case \(f = 1\) corresponds to absence of a direct coupling between dark matter and scalar field. However, even in this case the two sources are still coupled via gravity. Cosmological equations of Ref. \[10\] can be obtained from the Lagrangian \[1\] by taking \(\Delta = +1\) and \(L_{DM} = -\varepsilon\), where \(\varepsilon\) denotes the energy density of dark matter.

The Lagrangian \[1\] contains two functions of the scalar field: the potential energy \(V(\varphi)\) and the nonminimal coupling function \(f(\varphi)\). Their form is specified proceeding from some general field-theoretical considerations in such a way as to be compatible with current astronomical and cosmological observations, and at the same time they should not contradict laboratory experiments \[10, 30–32\]. Following Ref. \[10\], we take (in units \(c = \hbar = 1\))

\[
f(\varphi) = \exp[\beta(\varphi - \varphi_0)/M_{Pl}], \quad V(\varphi) = M_\varphi^4 (M_{Pl}/\varphi)^\alpha ,
\]

where \(M_{Pl}\) is the Planck mass, \(\alpha, \beta, \text{and } \varphi_0\) are positive quantities, with \(\beta \sim \mathcal{O}(1)\), and the mass scale is tuned to \(M_\varphi \sim 10^{-3}\) eV in order for acceleration to occur at the present epoch. In this model, \(\varphi_0\) corresponds to the current value of the cosmological scalar field. Note that the potential \(V(\varphi)\) is an example of a tracker potential in quintessence models \[32\]. Moreover, the above functions \[2\] are similar to those used within the framework of chameleon cosmology \[30, 32\] where the direct coupling between a cosmological scalar field and ordinary (baryon) matter takes place.

In general, the presence of interaction between DM and DE may have potentially observable cosmological implications \[6, 12\]. On the other hand, at astrophysical scales, due to gravitational instabilities, interacting DM and DE may form compact dense objects composed primarily of dark matter. Clearly, physical properties of such mixed configurations will depend on the form both of the coupling function and of the potential energy of the scalar field. Using the particular form of interaction described by the Lagrangian \[1\] with the scalar functions determined by \[2\], we clarify the question of how the presence of the nonminimal coupling influences masses and sizes of the resulting compact gravitating configuration.

For this purpose, we start from a configuration consisting only of fermionic dark matter, with no scalar field present. Such objects can support themselves against gravitational contraction by the degeneracy pressure of fermions obeying Pauli principle, in the same way as in the case of neutron stars and white dwarfs. After introducing the nonminimal coupling to this system, we will track changes in its properties as they depend on the parameters appearing in the functions \[2\].

We will use Einstein’s gravitational equations that allows to take into account relativistic effects. As a matter source in these equations, we take the energy-momentum tensor which is obtained by varying the matter part of the
Lagrangian (1) with respect to the metric:

\[ T^k_i = f \left[ (\varepsilon + p)u_i u^k - \delta^k_i p \right] + \Delta \partial_i \varphi \partial^k \varphi - \delta^k_i \left[ \frac{\Lambda}{2} \partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right], \]  

where \( \varepsilon \) and \( p \) are the energy density and the pressure of a dark matter fluid, respectively, and \( u^i \) is the four-velocity.

To describe a compact spherically symmetric configuration, we take static metric of the form

\[ ds^2 = e^{\nu} d(x^0)^2 - e^{\lambda} dr^2 - r^2 d\Omega^2, \]

where \( \nu \) and \( \lambda \) are functions of the radial coordinate \( r \), the time coordinate \( x^0 = ct \), and \( d\Omega^2 \) is the metric on the unit two-sphere. Using this metric and the energy-momentum tensor (3), we obtain three equations (hereafter we use \( c = \hbar = 1 \)): First and second are the \( (0,0) \) and \( (1,1) \) components of the Einstein equations,

\[ G^0_0 = -e^{-\lambda} \left( \frac{1}{r^2} - \frac{\nu'}{r} \right) + \frac{1}{r^2} = 8\pi G \left( f \varepsilon + \frac{\Lambda}{2} e^{-\lambda} \varphi^2 + V \right), \]
\[ G^1_1 = -e^{-\lambda} \left( \frac{1}{r^2} + \frac{\nu'}{r} \right) + \frac{1}{r^2} = 8\pi G \left( -fp - \frac{\Lambda}{2} e^{-\lambda} \varphi^2 + V \right), \]

and the third one follows from the law of conservation of energy and momentum, \( T^k_i = 0 \). Taking the \( i = 1 \) component of this equation gives

\[ \frac{dp}{dr} = -(\varepsilon + p) \left( \frac{1}{2} \nu' + \frac{1}{f} \frac{df}{d\varphi} \varepsilon' \right). \]

In the above equations, the prime denotes differentiation with respect to \( r \). Note that Eq. (7) differs from usual equation of hydrostatic equilibrium by the presence of an extra term in the right-hand side which is associated with the coupling function \( f \). The appearance of this term is related to the choice \( L_{DM} = -\varepsilon \). Another possible choice, \( L_m = p \), used in Refs. 23-24, 34, 35 implies that the term containing \( f \) in the right-hand side of Eq. (7) is absent (on this subject see the discussion in [35]).

Equations (5)-(7) must be supplemented by an equation for the scalar field which follows from the Lagrangian (1):

\[ \frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^i} \left[ \sqrt{-g} g^{ij} \partial \varphi \partial x^j \right] = -\Delta \left( \frac{dV}{d\varphi} + \frac{\varepsilon}{f} \frac{df}{d\varphi} \right). \]

Thus, we have five unknown functions \( \nu, \lambda, \varphi, \varepsilon, \) and \( p \). Keeping in mind that \( \varepsilon \) and \( p \) are related by an equation of state, there remains only four unknown functions. To determine these functions, we have four equations: the two Einstein equations (5) and (6), the scalar-field equation (8), and the equation of hydrostatic equilibrium (7) (the modified Tolman-Oppenheimer-Volkoff equation).

B. The case of fermionic dark matter

In order to perform calculations, we should choose some equation of state for dark matter. To elucidate how the interaction between DM and DE affects the characteristics of compact configurations, we will restrict ourselves to some simplest form of equation of state. Namely, we assume that dark matter is an ideal completely degenerate Fermi gas at zero temperature. Its equation of state is obtained using usual expressions for the energy density and pressure [28, 36]:

\[ \varepsilon = \frac{1}{\pi^2} \int_0^{k_F} k^2 \sqrt{m_f^2 + k^2} dk = \frac{m_f^4}{8\pi^2} \left[ z \sqrt{1 + z^2 (1 + 2z^2)} - \sinh^{-1}(z) \right] \equiv m_f^4 \tilde{\varepsilon}, \]
\[ p = \frac{1}{3\pi^2} \int_0^{k_F} \frac{k^4}{\sqrt{m_f^2 + k^2}} dk = \frac{m_f^4}{24\pi^2} \left[ z \sqrt{1 + z^2 (2z^2 - 3)} + 3 \sinh^{-1}(z) \right] \equiv m_f^4 \tilde{p}. \]

Here \( m_f \) is the fermion mass, \( k_F \) is the Fermi momentum, \( z = k_F/m_f \) is the relativity parameter, \( \tilde{\varepsilon} \) and \( \tilde{p} \) are the dimensionless energy density and pressure expressed in units of \( m_f^4 \).

In two limiting cases, this equation of state can be represented in simple power-law forms: (i) in the nonrelativistic case, \( z \ll 1 \), we get the polytropic law, \( \tilde{p} \propto \tilde{\varepsilon}^{5/3} \); (ii) in the ultrarelativistic case, \( z \gg 1 \), we have \( \tilde{p} = \tilde{\varepsilon}/3 \).
We introduce dimensionless variables for the current radius \( r \) and mass \( M(r) \) of the configuration under consideration [28]:

\[
\frac{\tilde{M}}{M_L} = r \quad \text{with} \quad M_L = \frac{M_0}{m_f^3}, \quad R_L = \frac{R_0}{m_f^2}.
\] (11)

Here \( M_L \) and \( R_L \) are characteristic mass and radius of a fermionic configuration obtained by Landau (for details, see Ref. [28]). Also, introducing the dimensionless scalar field, \( \phi = \varphi / M_{Pl} \), and rewriting the metric function \( \lambda \) in terms of the Schwarzschild mass parameter \( M \) as \( e^{-\lambda} = 1 - 2GM(r)/r \), we represent Eqs. (3), (7), and (8) in the following dimensionless form:

\[
\frac{dp}{dr} = -(\varepsilon + p) \left\{ \frac{M - 4\pi r^3}{r(r - 2M)} \left[ -fp - \Delta/2(1 - 2M/r)\phi^2 + V \right] + \frac{1}{f} \frac{df}{d\phi} \phi' \right\},
\] (12)

\[
\frac{dM}{dr} = 4\pi r^2 \left[ f\varepsilon + \frac{\Delta}{2} \left( 1 - \frac{2M}{r} \right) \phi^2 + V \right],
\] (13)

\[
\phi'' + \frac{3}{2} \left[ \frac{1}{r} - \frac{1 - 8\pi r^2}{r - 2M} \right] \left[ -fp - \Delta/2(1 - 2M/r)\phi^2 + V \right] - 2\delta(M' - M/r) \right\} \phi' = \frac{\Delta}{1 - 2M/r} \left( \frac{dV}{d\phi} + \varepsilon \frac{df}{d\phi} \right).
\] (14)

For convenience, we drop hereafter the tilde. In obtaining Eq. (12) we have used Eq. (6). The functions \( f \) and \( V \) from \([2]\), which appear in these equations, take the following dimensionless form:

\[
f(\phi) = \exp[\beta(\phi - \phi_0)], \quad V(\phi) = \delta(1/\phi)^\alpha \quad \text{with} \quad \delta = (M_\varphi/m_f)^4.
\] (15)

Thus, to describe the static configuration under consideration we have obtained three equations \((12)-(14)\) with the scalar field functions \((15)\).

### III. NUMERICAL RESULTS

In this section, we solve the system of equations \((12)-(14)\) numerically for given \( \alpha, \beta, \) and \( \delta \) subject to the following boundary conditions in the vicinity of the center of the configuration \( r = 0 \),

\[
\varepsilon \simeq \varepsilon_c + \frac{\varepsilon_2}{2} r^2, \quad M \simeq M_3 r^3, \quad \phi \simeq \phi_c + \frac{\phi_2}{2} r^2,
\] (16)

where \( \phi_c \) and \( \varepsilon_c \) denote the central values of \( \phi \) and the energy density \( \varepsilon \) at \( r = 0 \). The coefficients \( \varepsilon_2, M_3, \) and \( \phi_2 \) can be found from Eqs. \((12)-(14)\). In solving these equations, we will use the dimensionless equation of state which is given parametrically by Eqs. \((6)\) and \((10)\) in terms of the variable \( z \).

We start the numerical procedure near the origin \( r \approx 0 \) and proceed to the point \( r = r_b \), where the pressure is zero. We refer to the obtained solutions as internal solutions. The mass contained inside the sphere of the radius \( r_b \) will be treated as the mass of the dark matter star.

As we suppose in Sec. \([11]\) our spherically symmetric configuration is taken to be embedded in an external, homogeneously distributed cosmological scalar field \( \varphi_0 \). To provide the smoothness of solutions along the radius, we require the internal solutions to sew solutions obtained for the region \( r > r_b \) characterized by a nonzero scalar field energy density. Thus, for \( r > r_b \) we proceed numerical solutions of equations \((13)-(14)\) retaining only the gravitational and scalar fields while the dark matter fluid is taken to be zero. At large distances, the scalar field reaches its asymptotic value \( \varphi_0 \) taken from \([10]\), which can be rewritten in our dimensionless variables as follows:

\[
\phi_0 \approx \frac{\alpha}{\sqrt{8\pi\beta}} \frac{\Omega_{DE}^{(0)}}{\Omega_{\text{DM}}^{(0)}}
\] (17)

Here \( \Omega_{DE}^{(0)} \) and \( \Omega_{\text{DM}}^{(0)} \) denote the dimensionless densities of dark energy and dark matter, respectively, measured in units of the current critical density. The factor \( 1/\sqrt{8\pi} \) appears here since we take \( M_{Pl} = 1/\sqrt{G} \) instead of the reduced Planck mass, \( M_{Pl} = 1/\sqrt{8\pi G} \), used in Ref. \([10]\). Also, as in \([10]\), we take the usual (nonphantom) scalar field, i.e., we set \( \Delta = +1 \).
FIG. 1: The mass-radius relation for configurations with equation of state given by (9)-(10) and functions $f, V$ from (15). The parameter $\beta = 0, 0.5, 1, 2, 2.3, 3, 5, 7$, from top to bottom. The curve labeled by $\beta = 0$ corresponds to configurations without a scalar field.

Formally, at the points where the scalar field becomes equal to $\phi_0$ there is a nonzero gradient of the field. However, since for the values of the parameters $\alpha$, $\beta$, and $\delta$ being used here (see below) the field goes to $\phi_0$ only at $r \gg r_b$, the magnitude of the gradient part of the energy density $T_{00}'$ is always much smaller than the potential energy $V(\phi)$. This allows to consider the field in this region, to a good approximation, as homogeneous.

Using the above procedure, we define the mass of the configuration under consideration as the mass of all matter (dark matter plus dark energy) contained inside the sphere of the radius $r = r_b$. In Refs. [23, 24] configurations consisting of ordinary (baryon) matter nonminimally coupled to a cosmological chameleon scalar field were called “chameleon stars.” By analogy, let us call the configurations being considered here as chameleon dark matter stars, and denote their mass as $M_{CDMS} = M(r_b)$.

Another physically relevant parameter is given by the coordinate $\bar{r}$ associated with the proper radius of the star. This quantity is defined such as to be invariant with respect to spatial coordinate transformations preserving the spherical symmetry, and can be presented as follows:

$$\bar{r} = \int_0^r e^{\lambda/2} dr' = \int_0^r \left[1 - \frac{2M(r')}{r'}\right]^{-1/2} dr'.$$ (18)

Then, in dimensional variables, the proper radius is $R_{prop} = \bar{r} R_L$. Since $R_L$ is equal to half the Schwarzschild radius, then we require $\bar{r} > 2$ to avoid black hole configurations.

We turn now to a consideration of solutions obtained according to the above procedure. To perform numerical calculations, it is necessary to choose the values of the parameters $\alpha$, $\beta$, and $\delta$. Here we fix $\alpha = 0.2$, which has been used in the paper [10] in describing the evolution of the Universe. The value of the parameter $\delta$ appearing in (15) depends both on the mass scale parameter $M_\phi$ (in Ref. [10] it was taken as $M_\phi \sim 10^{-3}$ eV) and on the fermion mass $m_f$. Since at the moment it is not definitely known from which type of fermions gravitationally bound clumps of dark matter may consist of, various fermion particles are considered in the literature. This could be both superlight gravitinos with a mass of the order of $10^{-2}$ eV and superheavy WIMPs with a TeV mass scale [37]. Following [28], we assume for definiteness that $m_f$ lies in the range $1$ eV $\lesssim m_f \lesssim 10^2$ GeV. Then the value of the parameter $\delta$ is always considerably smaller than unity, and, as the numerical calculations indicate, the influence of the potential $V(\phi)$ from (15) on the solutions is negligibly small compared with other terms. I.e., the scalar field can be treated as having no potential energy on the scales associated with the fermion mass scale $m_f$.

Thus, we have only one free parameter $\beta \sim O(1)$. Since its explicit value does not follow from any first principles, we will vary its magnitude slightly, keeping track of changes in characteristics of compact configurations being
FIG. 2: Typical distributions of the dimensionless total energy density $T_0^0$ from (5) (left panel) and of the current masses $M_{\text{CDMS}} = M(\bar{r})$ (right panel) as functions of the relative invariant radius $\bar{r}/\bar{r}_b$ for different $\beta$. For both panels, the central dark matter energy density is taken as $\varepsilon_c = 10^{-2}$. In the right panel, the parameter $\beta$ runs the values 0, 1, 2, 3, 5, 7, from top to bottom.

considered here.

The results of numerical calculations of Eqs. (12)-(14) with the boundary conditions (16) are presented in Figs. 1 and 2. Figure 1 shows the mass-radius relation for different values of the parameter $\beta$. In plotting the curves, we have used the values of the dimensionless central density of dark matter $\varepsilon_c$ from (16) lying in the range $10^{-5} \leq \varepsilon_c \leq 10$.

In obtaining the solutions, we started from such central values $\phi_c$ at which the scalar field asymptotically, when $r \gg r_b$, went to the background value $\phi_0$ from (17). The numerical calculations indicate that $\phi_c$ must be always less than that of given by Eq. (17). Since $\phi_0$ depends on $\beta$, a situation may occur when for some $\beta$ and $\varepsilon_c$ it is necessary to set $\phi_c < 0$. It is therefore inevitable in this case that $\phi(r)$ would cross zero at some point, giving a singularity in the potential $V(\phi)$, that is physically unacceptable. That is why, in plotting the curves in Fig. 1, we restrict ourselves to only such values of $\varepsilon_c$ which provide the positivity of $\phi(r)$ everywhere along the radius. This implies that not all mass-radius curves have a maximum. There exists some critical value of $\beta \approx 2.3$ which still allows the curve to approach the maximum at some $\varepsilon_c$. At larger $\beta$, the maximum is absent.

In Fig. 1 the curve labeled $\beta = 0$ corresponds to the case studied in [28] with no scalar field present. The points of the curve to the right of the maximum correspond to stable configurations: with increase of the mass of a star its radius decreases. The points of the curve to the left of the maximum refer to unstable configurations. The curves for configurations with the scalar field demonstrate the similar behavior for systems with $\beta \lesssim 2.3$ when the maximum is still present. At bigger values of $\beta$, the absence of maxima means that all configurations are stable.

By comparing the stable configurations with and without the scalar field, one can identify the changes brought about by the presence of the nonminimal coupling. Namely, with the increase of $\beta$ that corresponds to a stronger interaction between dark matter and dark energy, both the masses (at a fixed star radius) and the sizes (at a fixed star mass) of the systems with the field become smaller, as compared to the configurations without a scalar field. Moreover, the differences between masses and sizes in these two cases can be as high as the order of magnitude, at large values of $\beta$ (cf. Fig. 2).

These results agree with those we get when considering the radial distribution of the matter. The results of our calculations with some fixed central value of the dark matter energy density $\varepsilon_c$ are shown in Fig. 2. As one can see from the plots presented in the left panel, configurations with the scalar field are characterized by a lower concentration of the matter towards the center. Also, taking into account the fact that the systems with the scalar field are more compact as compared to the field-free configurations having the same $\varepsilon_c$, we eventually obtain configurations whose
masses are less than that of the system without a scalar field, as demonstrated in the right panel of Fig. 2.

Let us now derive an approximate formula for the dependence of the maximum mass of the stars on the parameter $\beta$. To do this, first we convert the dimensionless variables to the physical ones. We remind also that not all mass-radius curves have a maximum. So, for the configurations with $\beta \gtrsim 2.3$, we take the very left point of the mass-radius curve as corresponding to the maximum mass. Following \[28\] we take the characteristic fermion mass scale as $m_f = 1$ GeV, for which the characteristic Landau mass $M_L = 1.632 M_\odot$. Then the corresponding approximate expression for a maximum mass, $M_{\text{max}} = M_{\text{max}}(\beta)$, can be presented in the form

$$M_{\text{max}} \approx 0.627 M_\odot \left(\frac{1 \text{ GeV}}{m_f}\right)^2 e^{-0.1 \beta^{5/3}}. \quad (19)$$

This expression gives masses of the order of a stellar mass for the fermion mass $m_f \sim 1$ GeV. In the two extreme cases we have: (i) for superlight particles of mass $m_f \sim 1$ eV, we get very heavy and large configurations of mass $M_{\text{max}} \sim 10^{17} - 10^{18} M_\odot$ and the radius of the order of a typical galaxy cluster size, $R \sim 10^{24}$ cm; (ii) for superheavy fermions of mass $m_f \sim 100$ GeV, we get light and small objects of mass $M_{\text{max}} \sim 10^{-3} - 10^{-4} M_\odot$ and $R \sim 10^2$ cm.

To summarize the results, we have studied the model describing compact gravitating configurations consisting of interacting dark matter and dark energy. Dark energy is modeled by the chameleon scalar field $\phi$ which has the nonminimal coupling to dark matter described by the function $f(\phi)$ in the Lagrangian \[1\]. We specified the form of this function to provide agreement with current astronomical and cosmological observations \[1\]. The particular case of a compact configuration with dark matter represented as an ideal completely degenerate Fermi gas has been studied in detail. In order to elucidate the role of the scalar field, we made a comparison of dark matter configurations supported only by the Fermi gas with the chameleon dark matter stars. The result is that the parameter $\beta \sim \mathcal{O}(1)$ appearing in the coupling function $f(\phi)$ largely determines masses and sizes of the objects under consideration. In particular, as it is seen from Fig.\[1\] while $\beta$ increases, both the masses (at a fixed star radius) and the sizes (at a fixed star mass) of the chameleon dark matter stars tend to become smaller than those of configurations without a scalar field.

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