Anomalous dimensions for the interpolating currents of baryons

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The anomalous dimensions for the interpolating currents of baryons are indispensable inputs in a serious analysis of baryon QCD sum rules. However, the results in the literature are vague. In view of this, in this work, we investigate the one-loop anomalous dimensions for some interpolating currents such as those of $\Lambda_Q$ and proton. This work has more significance in pedagogy.

I. INTRODUCTION

Recently, LHCb updated a measurement of the lifetime of $\Omega_c$ with $\tau(\Omega_c^0) = 268 \pm 24 \pm 10 \pm 2$ fs\textsuperscript{1}, which is nearly four times larger than the world-average $\tau(\Omega_c^0) = 69 \pm 12$ fs\textsuperscript{2} in PDG2018. Theoretically, the lifetimes of weak-decay baryons can be explained by the framework of heavy quark expansion (HQE) (see\textsuperscript{3} for a review). Among others, the calculation of hadron matrix elements is an important part to decipher this puzzle. In this direction, some efforts has been made in our recent work\textsuperscript{4}.

We intend to perform a careful analysis on these hadronic matrix elements with QCD sum rules (QCDSR). QCDSR is a QCD-based approach to investigate the properties of hadrons. It connects the hadron phenomenology and QCD vacuum structure via a few universal parameters like quark condensates and gluon condensates.

It is usually thought that about 30\% uncertainty will be introduced in QCDSR. However, as can be seen in our recent works\textsuperscript{5, 6}, once a stable Borel region is found, the prediction of physical quantity can reach high precision.

In QCDSR, the Wilson coefficients of OPE should be evolved to the energy scale $\mu_0$ of low-energy limit\textsuperscript{7}. For example, for the two-point correlation function of baryon,

\begin{equation}
T\{J(x)\bar{J}(0)\} = \sum_n C_n(x^2)O_n(0),
\end{equation}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$n$ & $C_n(x^2)$ & $O_n(0)$ \\
\hline
1 & 1 & 1 \\
2 & 2 & 2 \\
3 & 3 & 3 \\
\hline
\end{tabular}
\caption{Table of Wilson coefficients}
\end{table}

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FIG. 1: The one-loop diagrams for the calculation of anomalous dimension. Here $F$, $G$ and $H$ respectively stand for the quark fields of $u$, $d$ and $Q$ for $\Lambda_Q$.

The coefficient functions $C_n(p^2)$ in the momentum space should contain the factor

$$\left[ \frac{\alpha_s(p^2)}{\alpha_s(\mu_0^2)} \right]^{2\gamma_J - \gamma_n},$$

where $\gamma_J$ and $\gamma_n$ are respectively the anomalous dimensions of the current $J$ and the operator $O_n$.

It can be seen that, the results of the anomalous dimensions are indispensable inputs in a serious analysis of baryon QCD sum rules, especially for the case of bottom baryons. However, the results in the literature are vague. In view of this, in this work, we calculate the anomalous dimensions for some interpolating currents such as those of $\Lambda_Q$ and proton.

There are 3 relevant diagrams, as can be seen in Fig. 1. Following the convention of Peskin in [8], the sum of the divergent parts of Fig. 1a-1c will be of the form

$$-\frac{4}{3} b \frac{g^2}{(4\pi)^2} \frac{\Gamma(2-d/2)}{(M^2)^{2-d/2}} \times J,$$

with $b$ some constant. Then

$$\gamma(g^2) = \frac{4}{3} (3 + 2b) \frac{g^2}{(4\pi)^2} + O(g^4),$$

$$\gamma_J = \frac{2}{3} (3 + 2b) / \beta_0,$$

where $\beta_0 = 11 - (2/3)n_f$ with $n_f$ the number of fermion flavors. In Eq. (3), the factor of $(4/3) \cdot 3$ arises from the wavefunction renormalization of three fermion fields.

The rest of this paper is arranged as follows. In Sec. II and III, we will calculate the anomalous dimensions for the interpolating currents of $\Lambda_Q$ and proton, respectively. We conclude our paper in the last section.

II. THE ANOMALOUS DIMENSION FOR $\Lambda_Q$

The interpolating current for $\Lambda_Q$ is usually taken as:

$$J_{\Lambda_Q} = \epsilon_{abc}(u^T_\alpha C\gamma_5 d_b)Q_c,$$
where $Q$ denotes a bottom or charm quark, $a, b, c$ are the color indices and $C$ is the charge conjugate matrix. In Eq. (1), the system of $u$ and $d$ quarks has spin and parity of $0^+$, while the whole baryon has $J^P = 1/2^+$. For Fig. 1a, it can be shown that
\[
\text{Fig. 1a} = \left( -\frac{8}{3} \right) \times ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times J_{\Lambda Q}.
\] (5)

For Figs. 1b, a new structure emerges because of
\[
(\bar{\psi}_1 \gamma^\mu \gamma^\nu \psi_2) \gamma_\mu \gamma_\nu \psi_3 = 4 \left( \bar{\psi}_1 \psi_2 \right) \psi_3 - \left( \bar{\psi}_1 \sigma^{\mu\nu} \psi_2 \right) \sigma_{\mu\nu} \psi_3.
\] (6)

It turns out that
\[
\text{Fig. 1b} = \frac{-1}{6} \times ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times \epsilon_{abc} [4 (u_a^T C \gamma^5 d_b) Q_c - (u_a^T C \gamma^5 \sigma^{\mu\nu} d_b) \sigma_{\mu\nu} Q_c].
\] (7)

Similarly,
\[
\text{Fig. 1c} = \frac{-1}{6} \times ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times \epsilon_{abc} [4 (d_b^T C \gamma^5 u_a) Q_c - (d_b^T C \gamma^5 \sigma^{\mu\nu} u_a) \sigma_{\mu\nu} Q_c].
\] (8)

Therefore
\[
\text{Fig. 1b + Fig. 1c} = 2 \times \left( -\frac{2}{3} \right) \times ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times \epsilon_{abc} (u_a^T C \gamma^5 d_b) Q_c.
\] (9)

In the above, we have used
\[
\epsilon_{abc}(u_a^T C \gamma^5 d_b) Q_c = \epsilon_{abc}(d_b^T C \gamma^5 u_a) Q_c,
\] (10)
\[
\epsilon_{abc}(d_b^T C \gamma^5 \sigma^{\mu\nu} u_a) \sigma_{\mu\nu} Q_c = -\epsilon_{abc}(u_a^T C \gamma^5 \sigma^{\mu\nu} d_b) \sigma_{\mu\nu} Q_c.
\] (11)

Finally, the counterterm for the current is obtained as
\[
-\delta J_{\Lambda Q} = \text{Fig. 1a} + \text{Fig. 1b} + \text{Fig. 1c} = -4 \times ig^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times J_{\Lambda Q}
\]
\[
= 4 \times \frac{g^2}{(4\pi)^2} \left( \frac{\Gamma(2 - d/2)}{(M^2)^{2-d/2}} \right) \times J_{\Lambda Q}.
\] (12)

where $M$ is the reference scale of renormalization. As can be seen in Eq. (2), $b = -3$. Therefore, the anomalous dimension for the current of $\Lambda Q$ is $-2/\beta_0$. However, it seems that our result is different from that in [9].

III. THE ANOMALOUS DIMENSION FOR PROTON

The interpolating current for proton is usually taken as
\[
J_p = \epsilon_{abc}(u_a^T C \gamma^\mu u_b) \gamma_5 \gamma_\mu d_c.
\] (13)
where the system of two \( u \) quarks has spin and parity of \( 1^+ \), while the whole baryon has \( J^P = 1/2^+ \).

Without loss of generality, in this section, we assume that the current is built from quark fields of three distinct flavors \( F, G, H \):

\[
J_p = \epsilon_{abc}(F_a^T C\gamma^\mu G_b)\gamma_5\gamma_\mu H_c.
\] (14)

For Fig. 1a, it can be shown that

\[
\text{Fig. 1a} = \left(-\frac{2}{3}\right) \times i g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times J_p.
\] (15)

For Fig. 1b, a new structure emerges because of

\[
(\bar{\psi}_1 \gamma^\mu \gamma^\nu \psi_2)\gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho \psi_3
= (\bar{\psi}_1 \gamma^\mu \gamma^\rho \psi_2)\gamma_5 (2g_{\mu\nu} \gamma_\mu - 2g_{\mu\rho} \gamma_\nu + \gamma_\rho \gamma_\mu \gamma_\nu) \gamma_\rho \psi_3
= 10 (\bar{\psi}_1 \gamma^\mu \psi_2)\gamma_5 \gamma_\mu \gamma_\psi_3 - 6 (\bar{\psi}_1 \gamma^5 \gamma_5 \psi_2)\gamma_\mu \psi_3.
\] (16)

It turns out that

\[
\text{Fig. 1b} = \left(-\frac{1}{6}\right) \times i g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times \epsilon_{abc}[10 (F_a^T C\gamma^\mu G_b)\gamma_5\gamma_\mu H_c - 6 (F_a^T C\gamma^\mu \gamma_5 G_b)\gamma_\mu H_c].
\] (17)

Similarly,

\[
\text{Fig. 1c} = \left(-\frac{1}{6}\right) \times i g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times \epsilon_{abc}[10 (G_a^T C\gamma^\mu F_b)\gamma_5\gamma_\mu H_c - 6 (G_a^T C\gamma^\mu \gamma_5 F_b)\gamma_\mu H_c].
\] (18)

Therefore

\[
\text{Fig. 1b + Fig. 1c} = 2 \times \left(-\frac{5}{3}\right) \times i g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times \epsilon_{abc}(F_a^T C\gamma^\mu G_b)\gamma_5\gamma_\mu H_c.
\] (19)

In the above, we have used

\[
\epsilon_{abc}(G_a^T C\gamma^\mu F_b)\gamma_5\gamma_\mu H_c = \epsilon_{abc}(F_a^T C\gamma^\mu G_b)\gamma_5\gamma_\mu H_c,
\] (20)

\[
\epsilon_{abc}(G_a^T C\gamma^\mu \gamma_5 F_b)\gamma_\mu H_c = -\epsilon_{abc}(F_a^T C\gamma^\mu \gamma_5 G_b)\gamma_\mu H_c.
\] (21)

Finally, the counterterm for the current is obtained as

\[
-\delta J_p = \text{Fig. 1a + Fig. 1b + Fig. 1c} = -4 \times i g^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^4} \times J_p
= 4 \times \frac{g^2}{(4\pi)^2} \frac{\Gamma(2 - d/2)}{(M^2)^2 - d/2} \times J_p.
\] (22)

It happens that \( \delta J_p = \delta J_{\Lambda_Q} \) and thereby \( \gamma_{J_p} = \gamma_{J_{\Lambda_Q}} = -2/\beta_0 \). Our result is consistent with that of Peskin in [8].
IV. CONCLUSIONS

The anomalous dimensions for the interpolating currents of baryons are indispensable inputs in a serious analysis of baryon QCD sum rules. However, the results in the literature are vague. In view of this, in this work, we have investigated the anomalous dimensions for some interpolating currents such as those of $\Lambda_Q$ and proton.

In this work, we do not consider the interpolating current of $\Delta$, which has the quark content of $uuu$ and spin-3/2. We feel that the interpolating current of $\Delta$ written in terms of Dirac gamma matrices

$$(J_\Delta)^\mu = \epsilon_{abc}(u^T_a C\gamma^\mu u_b)u_c,$$  \hspace{1cm} (23)

is cumbersome so that it is not easy to see clearly the completely symmetrical relation among these three $u$ quarks. It seems that the prescription in [8] is still a better choice.

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