Collapse of Composite Tubes under Uniform External Hydrostatic Pressure

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Abstract

This paper describes an experimental and a theoretical investigation into the collapse of 22 circular cylindrical composite tubes under external hydrostatic pressure. The investigations were on the collapse of fibre reinforced plastic tube specimens made from a mixture of three carbon and two E-glass fibre layers. The theoretical investigations were carried out using an in-house finite element computer program called BCLAM, together with the commercial computer package, namely ANSYS. It must be emphasised here that BS 5500 does not appear to exclusively cater for the buckling of composite shells under external hydrostatic pressure, so the work presented here is novel and should be useful to industry.

The experimental investigations showed that the composite specimens behaved similarly to isotropic materials previously tested, in that the short vessels collapsed through axisymmetric deformation while the longer tubes collapsed through non-symmetric bifurcation buckling. Furthermore it was discovered that the models failed at changes of the composite lay-up due to the manufacturing process of these models. These changes seemed to be the weak points of the specimens.

Keywords: circular cylinder, buckling, axisymmetric yield, composite, external pressure, finite elements, BCLAM, ANSYS.

1. Introduction

The surface area of the Earth is about 196.9 million sq.mi (510 million km$^2$), and about 75% of it is covered by water, but despite this, only about 0.1% of the oceans’ bottoms have been explored [1]. Such research has found large quantities of precious metals and minerals, together with large quantities of methane hydrates. In the case of methane, Dickens et al [1] have found deep-sea frozen methane hydrates lying under the ocean bottoms at depths of 2 miles (3.22km) or more. According to Dickens et al, the quantity of this methane is about 10,000 billion tonnes and if it is divided amongst all of mankind, it will amount to about 1,670 tonnes per person on Earth, or in monetary terms about $1,250,000 per person on Earth. Thus, it is necessary to exploit the oceans’ bottoms, which are as deep as 7.16 miles (11.52 km).

Now a large submarine can only dive to a depth of about 400m (1312 ft), but the deepest part of the oceans is 29 times deeper than this. As the submarine dives deeper and deeper
into the oceans, the external hydrostatic pressure increases, so that the wall thickness has to be increased. Eventually, the wall thickness becomes so large that the vessel has no reserve buoyancy and will sink like a stone to the very bottom of the ocean [2]. Ross [2] has found that for a submarine of internal diameter 10m (32.81ft) and constructed in high-tensile (HY80) steel, the thickness of its hull will be about 2.3 m (7.58ft), if it is to be designed to dive to the bottom of the Mariana’s Trench! The only way to overcome this problem is to use a material with a higher strength: weight ratio than high-strength metals. Such materials are Glass Fibre Reinforced Plastic (GFRP) and Carbon Fibre Reinforced Plastic (CFRP). There are many other composites, which are suitable, but these are cheaper than many of the other composites and have been used successfully for other structures.

2. Background

2.1 Buckling
Under uniform external pressure a long thin-walled circular cylinder can fail by non-symmetric bifurcation buckling or shell instability, at a fraction of that to cause axisymmetric yield [2]. To increase the buckling resistance of such vessels, they are usually stiffened by ring stiffeners, spaced at suitable distances apart. If the ring stiffeners are not strong enough, the entire vessel can collapse bodily by a mode called general instability. Another mode of failure is called axisymmetric deformation, where the vessel keeps its circular form while imploding.

For long thin vessels, the shell instability failure mode usually occurs at a much smaller pressure than that required to cause axisymmetric yield. The extremely long thin circular cylindrical vessel will fail in a flattening or ovalling mode, but the shorter circular cylinder will buckle in a lobar mode. In this paper, we will concern ourselves only with shell instability and axisymmetric deformation.

2.1.1 Thinness Ratio and Plastic Knockdown Factor. Windenburg and Trilling [3] introduced a thinness ratio (λ) to consider the fall in theoretical elastic buckling pressures due to the fact that shorter vessels failed by inelastic instability. The effects of initial out-of-roundness of these cylinders can cause a further drop in their buckling pressures than that predicted by elastic theory for perfect vessels. This factor is needed because even slight deviations from the perfect geometry can reduce the buckling pressure of a pressure vessel considerably. With the thinness ratio, it is also required to determine the so-called plastic knockdown factor (PKD) from the experimental results, so that the Design Chart can be made.

2.2 Finite Element Method (FEM)
In the theoretical analyses of these vessels, we will consider shell instability using an in-house finite element computer program called BCLAM, together with the well-known computer package ANSYS.

For both programs, the possibility of using different material models, including orthotropic material properties is allowed for.

2.3 The composite tubes
The models used for the tests were manufactured from fibre reinforced plastic tubes, see Figure 1; they consisted of a mixture of carbon and glass fibre layers. The manufacturer was
Carbon Fibre Tubes Ltd, Hayling Island, UK. The lay-up was $0^\circ/90^\circ/0^\circ/90^\circ/0^\circ$; the carbon fibres were laid lengthwise ($0^\circ$) and the E-glass fibres circumferentially ($90^\circ$). The manufacturer describes the manufacturing process to be rather similar to that of making a Swiss roll. That is the layers were of cloth form and were wrapped around each other, with impregnated resin, in a circumferential manner.

![Image of carbon fibre tubes]

Figure 1: The models and their end bungs

For sealing the two ends of each specimen under external water pressure, two end bungs with ‘O’ rings had to be manufactured. These end bungs were manufactured out of mild steel and are shown in Figure 1.

The actual number of layers were 3 Carbon layers and 2 E-Glass layers and the layer thicknesses were calculated from the overall measured wall thickness of 1.8 mm, where

\[
\begin{align*}
E_1 &= \text{Young’s modulus of the layer in direction of the fibre} \\
E_2 &= \text{Young’s modulus of the layer perpendicular to the fibre direction} \\
G_{12} &= \text{in-plane shear modulus}
\end{align*}
\]

2.3.1 Material properties of the single layers. The properties of the single layers are shown in Table 1.

\[
\begin{align*}
\sigma_{yp} &= \text{yield stress in the direction of the fibre} \\
\nu_{12} &= \text{major Poisson’s ratio}
\end{align*}
\]

2.4 Experimental Analysis and the Design Charts

After calculating a series of theoretical buckling pressures and thinness ratios, with the aid of an in-house computer program, namely MisesNp, for the models made from the present composite, the respective buckling pressures of the models were determined experimentally. MisesNp adopts the von Mises analytical solution for simply-supported ends.
Table 1. Material Properties of the Composite (MPa)

|       | $E_1$  | $E_2$  | $G_{12}$ | $\sigma_{yp}$ | $\nu_{12}$ | No. of Layers | Thickness (mm) |
|-------|--------|--------|----------|---------------|------------|---------------|----------------|
| Carbon| 135000 | 10000  | 5000     | 1500          | 0.3        | 3             | 0.3857         |
| E-glass| 40000  | 8000   | 4000     | 1000          | 0.25       | 2             | 0.3214         |

This testing was done in a high-pressure tank, where each model was tested to failure and the experimental buckling pressure was noted. With the experimentally obtained values for the buckling pressure ($P_{exp}$) of the series it was possible to calculate the PKD:

$$PKD = \frac{P_{cr}}{P_{exp}},$$

where $P_{cr}$ = theoretical buckling pressure.

With the values of PKD and the previously calculated thinness ratios ($\lambda$) it was possible to generate a Design Chart, where $1/\lambda$ was plotted against PKD. This Design Chart could now be used to calculate the predicted (experimental) buckling pressure $P_{pred}$ for a pressure vessel out of the same material for untested vessels.

During the design process the factors PKD and $\lambda$ have to be calculated. Once the design chart has been produced, it is possible to obtain the PKD from this chart, which then can be used to calculate the predicted buckling pressure, namely $P_{pred}$, where:

$$P_{pred} = \frac{P_{cr}}{PKD}$$

The new value gives an approximation of the real buckling pressure that can now be used to predict the possible diving depth of the submersible.

3. Experimental Investigation

3.1 Equipment

The experiments were carried out in the laboratories of the University of Portsmouth. The following equipment was used:

- High-pressure test tank (maximum pressure 210 bar).
- Pressure pump, with pressure gauge (maximum pressure 410 bar).

Before the manufactured specimens were tested, they were checked for any defects due to the manufacturing process or any matters that could affect their performance during the testing. It was observed that all the test specimens had minor defects at the ends, when they were cut during manufacture. These defects had the appearance of small delaminations and cracks on the surface. The affected zones could be roughly sized between 0.5cm and 1cm lengths from the ends, for all specimens. Due to the supporting influence of the end bungs, that had shoulders of about 1cm penetrating into the specimens’ ends, it was decided that these defects had almost no influence on the results. Some of the specimens, especially the shorter ones, had bigger defects. These could spread almost over the whole length of the
specimens. The defects could have had a considerable influence on the strength of the models, particularly those with the larger defects and because of this; it was decided to carefully observe the behaviour of the specimens with the larger defects during the tests.

3.1.1 Model Failure. All models were observed before and after failure and pictures were taken for documentation. The most important information has already been given, but in addition to this, three other issues were found, as follows:

(1) Three major failure modes could be indicated during the testing. These failure modes could be related to the length of the models. The three failure modes were:
1. Elastic buckling.
2. Inelastic buckling (see Figure 2).
3. Axisymmetric ‘yield’ failure (see Figure 3).

The longest models failed due to elastic buckling. In this failure mode the test models buckled elastically, but with a subsequent reduction of pressure the model returned to its original shape without any visible signs of damage.

The medium sized models failed due to inelastic buckling. This meant that the deformation just prior to buckling was large, so that the material had ‘yielded’ and buckled inelastically. These models did not return to their original shapes after the pressure was decreased to zero.

The shortest models failed due to axisymmetric ‘yield’ failure. In this case, the models deformed on the whole circumference; no lobes were formed in this mode, and the vessels collapsed axisymmetrically. This failure mode can be seen in Figure 3.

(2) It was found that the defects that were seen during the pre-test observation had almost no influence on the collapse pressures of the models. All the cracks, fractures or delaminations that could be seen on the models after collapse, were found at different locations to the deformations that were found just after manufacture. A ‘micograph picture’ of a section, of part of an undamaged tube is shown in Figure 4.

Figure 2: Inelastic buckling of a model.
After a careful investigation of the failure modes of the models, some of the vessels were subjected to microscopic analysis, via a microscope (See Figure 4), where the layer composition was photographed with magnified pictures of the polished surface perpendicular to the rotational axis of each tube. From these pictures, it could be seen that the lay-up of the different layers varied considerably over the whole circumference.

After a check of the position of these lay-up modifications and the position of post-test fractures, it showed that both were in correlation. This could be seen on all specimens that fractured along the longitudinal axis.

4. **Theoretical Investigations**

As already stated in the previous section, the theoretical investigation was first done using BCLAM an in-house DOS-based computer program. In this case, a truncated conical shell element was used with two ring nodes at its ends. Each ring node had four degrees of freedom, three translational and one rotational. The assumed displacement form in the azimuthal direction was sinusoidal and this enabled explicit integration in the azimuthal direction, thereby considerably reducing computational time. In contrast to this the ANSYS solution [4], used the Shell99 finite element, which had 8 nodes, with 6 degree of freedoms per node. Three of the degrees of freedom were translational and the other three were rotational degrees of freedom.

The boundary conditions for both analyses were assumed to be simply-supported at the ‘ends’.
The results of the BCLAM & Shell99 analyses with the theoretically calculated properties are listed in Table 2, where the figures in parentheses represent the number of circumferential waves or lobes (n) that the vessels buckle into.

Note:
The thinness ratios had been calculated using the equivalent Young’s modulus in y-direction and the other parameters listed in Table 2, except for the wall thickness, which was set to 1.8mm for the ME specimens.

5. Theoretical and Experimental Results

From Table 2, it can be seen that the ANSYS results are more stable than those yielded by BCLAM, especially for the very long and the very short vessels. It appears that for the vessels of extreme length, BCLAM was numerically unstable and because of this, no design chart will be produced for BCLAM; only a design chart for ANSYS will be produced, as shown by Figure 5.

For example, with reference to numerical instability, in the case of the longer vessels, 10 conical shell elements were initially used for the BCLAM analysis and when this was increased to 12 conical shell elements, the calculated buckling pressure appeared to converge. However, when the number of elements was increased to 20, the value of the calculated buckling pressure appeared to wildly diverge; similar experiences were observed for the very short vessels. Thus, it was believed that BCLAM was unstable, probably due to a numerically unstable feature commonly called ‘locking’ [5].

With the experimentally obtained buckling pressures ($P_{exp}$) and the theoretically calculated buckling pressures ($P_{cr}$) of the previous sections, the plastic knockdown factors (PKD) for both approaches were calculated by the formula:

$$PKD = \frac{P_{cr}}{P_{pred}}$$

6. Conclusions

- The experimental and theoretical investigations were performed successfully on the described fibre reinforced plastic tubes. The main findings are listed as follows:

- Design Charts were created successfully for use with ANSYS, but not with BCLAM, because of the numerical instability of the latter. It should be emphasised here, that it appears that BS 5500 does not exclusively cater for the buckling of thin-walled shells made from composite materials and this makes the present study a novel one.

- The tested models failed at locations were the lay-up of the single layers changed. These changes seem to reduce the strength of the composite in this location.
### Table 2. Experimental Results & the Theoretical Results.

| Cylinder No. | Overall Length $L_0$ (mm) | Mean Diameter (mm) | Layer Thickness (mm) | Out of Circularity (mm) | $1/\lambda$ | $P_{ex}$ (MPa) | $P_{cr}$ BCLAM (MPa) | $P_{cr}$ ANSYS (MPa) |
|--------------|--------------------------|--------------------|----------------------|------------------------|------------|----------------|----------------------|----------------------|
| ME1A         | 242.755                  | 52.54              | 1.8                  | 0.4398                 | .251       | 1.93           | 4.025(2)             | 5.872(6)             |
| ME2          | 220.295                  | 52.51              | 1.78                 | 0.3985                 | .265       | 2.07           | 4.19(3)              | 5.479(2)             |
| ME3          | 200.27                   | 52.53              | 1.79                 | 0.3956                 | .279       | 2.28           | 4.380(3)             | 6.666(2)             |
| ME4          | 180.49                   | 52.53              | 1.79                 | 0.3732                 | .296       | 2.38           | 4.546(3)             | 6.810(4)             |
| ME5          | 159.795                  | 52.55              | 1.79                 | 0.3582                 | .317       | 2.55           | 4.808(3)             | 6.955(4)             |
| ME6          | 139.625                  | 52.54              | 1.8                  | 0.3799                 | .343       | 2.83           | 5.282(3)             | 5.606(3)             |
| ME7A         | 120.325                  | 52.54              | 1.8                  | 0.3378                 | .375       | 3.59           | 5.909(3)             | 6.450(4)             |
| ME7B         | 119.965                  | 52.52              | 1.76                 | 0.3484                 | .375       | 3.38           | 5.602(3)             | 6.700(3)             |
| ME7C         | 120.015                  | 52.55              | 1.78                 | 0.3568                 | .375       | 3.65           | 5.759(3)             | 6.700(3)             |
| ME8A         | 100.275                  | 52.55              | 1.78                 | 0.3807                 | .419       | 4.34           | 6.861(3)             | 8.345(4)             |
| ME8B         | 100.265                  | 52.55              | 1.79                 | 0.3546                 | .419       | 4.34           | 6.950(3)             | 8.345(4)             |
| ME9A         | 80.29                    | 52.54              | 1.77                 | 0.3121                 | .483       | 5.52           | 5.039(3)             | 10.278(4)            |
| ME9B         | 80.19                    | 52.58              | 1.81                 | 0.3476                 | .484       | 5.65           | 9.259(4)             | 10.278(4)            |
| ME10A        | 60.295                   | 52.56              | 1.78                 | 0.4182                 | .591       | 8.27           | 16.181(5)            | 16.418(5)            |
| ME10B        | 60.44                    | 52.56              | 1.79                 | 0.4278                 | .59        | 8.27           | 12.695(4)            | 16.418(5)            |
| ME11A        | 50.305                   | 52.57              | 1.8                  | 0.378                  | .682       | 10.48          | 11.210(4)            | 25.757(6)            |
| ME11B        | 50.485                   | 52.54              | 1.78                 | 0.3927                 | .68        | 11.45          | 13.858(4)            | 25.757(6)            |
| ME12A        | 40.185                   | 52.53              | 1.76                 | 0.3889                 | .835       | 13.93          | 18.417(5)            | 62.830(7)            |
| ME12B        | 40.39                    | 52.51              | 1.76                 | 0.4167                 | .831       | 16.0           | 18.298(5)            | 62.830(7)            |
| ME13A        | 30.405                   | 52.49              | 1.72                 | 0.4942                 | 1.164      | 22.06          | 26.879(6)            | 732.160(14)          |
| ME13B        | 30.22                    | 52.54              | 1.77                 | 0.3847                 | 1.174      | 19.70          | 29.492(6)            | 732.160(14)          |
| ME14A        | 34.915                   | 52.55              | 1.8                  | 0.4882                 | .972       | 18.34          | 24.163(5)            | 157.160(9)           |
| ME14B        | 35.05                    | 52.54              | 1.78                 | 0.4296                 | .967       | 18.62          | 23.267(5)            | 157.060(9)           |
The Design Chart for the orthotropic composite material has, in general a slightly different shape to design charts obtained for isotropic materials.

The Design Chart created in this paper should prove useful to designers, providing the structures have the same characteristics as those investigated herein and a bigger safety factor is used.

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