A Technical appendix

A.1 Properties of the subspace shrinkage Prior

To see how \( \omega \) shrinks the VAR towards the factor model, it is convenient to exploit the fact that if the rank of \( X \) is \( K \), the matrix \( X \) and the matrix \( F_K \) (i.e., the first \( K \) PCs of \( X \)) span the same column space \( C \). Moreover, notice that \( F_K = (F_q, F_{(q+1):K}) \) with \( F_{(q+1):K} \) storing the final \( K - q \) principal components of \( X \). Using these definitions and the result that \( \text{C}(X) = \text{C}(F_K) \), the corresponding projection matrices coincide:

\[
X \left( X'X + \frac{\omega}{1-\omega}X'(IT - \Phi_0)X \right)^{-1} X' = F_K \left( F_K'F_K + \frac{\omega}{1-\omega}F_K'(IT - \Phi_0)F_K \right)^{-1} F_K'.
\] (10)

Notice that conditional on a standard normalization, we have that \( F_K'F_K = I_K \) and \( F_q'F_{(q+1):K} = 0 \). This allows us to rewrite Eq. (11) as:

\[
F_K \left( I_K + \frac{\omega}{1-\omega} \begin{bmatrix} I_q & 0 \\ 0 & 0 \end{bmatrix} \right)^{-1} F_K' = (F_q, F_{q+1};K) \begin{bmatrix} I_q & 0 \\ 0 & (1-\omega)I_{K-q} \end{bmatrix} \begin{bmatrix} F_q' \\ F_{q+1};K \end{bmatrix} \]  \hspace{1cm} (12)

\[
= \Phi_0 + (1-\omega) \Phi_1, \hspace{1cm} (13)
\]

with \( \Phi_1 = F_{q+1};K (F_q'F_{q+1};K)^{-1} F_q' \). It is straightforward to show that

\[
\Phi = \Phi_0 + \Phi_1,
\]

where \( \Phi = X(X'X)^{-1}X' \) is the projection matrix of \( X \). Thus, we can substitute \( \Phi_1 = \Phi - \Phi_0 \) in Eq. (14) and multiply from the right with \( Y \) to arrive at:

\[
\mathbb{E}(XA|Y, \omega) = \omega \Phi_0 Y + (1-\omega) \Phi Y, \hspace{1cm} (15)
\]

which shows that the posterior mean of the regression function is a convex combination of the VAR fit, \( \Phi Y \), and the fit of the PC regression, \( \Phi_0 Y \).
A.2 Our implementation of the Minnesota prior

In our empirical work we set the dummies as follows (see Banbura, Giannone, and Reichlin, 2010):

\[
Y = \begin{pmatrix}
\text{diag}(\hat{a}_1, \ldots, \hat{a}_M) / \hat{\vartheta} \\
0_{M(p-1) \times M} \\
\text{diag}(\hat{\sigma}_1, \ldots, \hat{\sigma}_M) \\
0_{1 \times M}
\end{pmatrix}, \quad 
X = \begin{pmatrix}
J_p \otimes \text{diag}(\hat{\sigma}_1, \ldots, \hat{\sigma}_M) / \hat{\vartheta} & 0_{K \times 1} \\
0_{M \times K} & 0_{M \times 1} \\
0_{1 \times K} & \kappa
\end{pmatrix},
\]

with \( \hat{\sigma}_j (j = 1, \ldots, M) \) denoting the OLS residual standard deviation of an AR(\( p \)) model for \( y_{jt} \), the \( j^{th} \) variable in \( Y_t \), \( \hat{a}_j \) is the \( j^{th} \) diagonal element of \( A \), and \( J_p = \text{diag}(1, \ldots, p) \). Notice that this set of dummies includes the prior for the intercept which depends on the hyperparameter \( \kappa \). \( \kappa \) is set to a very small number (in our empirical work it equals 0.001), leading to a weakly informative prior for the intercepts. Notice that the prior precision of the Minnesota prior is given by \( X'X \) and the prior mean is equal to the OLS estimate based on the dummy observations: 

\( A = (X'X)^{-1} X'Y \).
### Additional simulation results

In Sub-section 3 we showed that our approach is able to recover the true number of factors as long as $M$ is not too large. We stated that one of the reasons why we under-estimate the true number of factors in large dimensional models is due to the fact that the VAR is capable of mimicking factor dynamics in finite samples and the likelihood is not strongly informative about whether a factor model or a VAR should be used to model $y_t$. If the number of observations, however, increases, our approach should be able to infer the true number of factors. We provide simulation evidence to support this claim in Table 5. This table is the same as Table 1 with two main differences. First, we drop the models which have an informative prior on $\omega$ for brevity reasons. Second, and more importantly, we increase the length of the time series and set $T = 2500$.

**Table 5:** Simulation results for differing values of $q$ and $M$ and $T = 2500$. Averages across 100 replications from the factor model DGP

| $q$ = | 1 | 3 | 6 | 8 | 1 | 3 | 6 | 8 |
|------|---|---|---|---|---|---|---|---|
| $M = 30$ | 1.00 | 3.00 | 6.32 | 8.00 | 1.00 | 3.04 | 6.16 | 8.00 |
| $M = 60$ | 1.00 | 3.00 | 6.00 | 8.00 | 1.00 | 3.00 | 6.00 | 8.00 |
| $M = 120$ | 1.00 | 3.00 | 6.00 | 8.00 | 1.00 | 3.00 | 6.00 | 8.00 |

| Posterior mean of $\omega$ |
|---------------------------|
| $M = 30$ | 0.99 | 0.99 | 0.99 | 0.82 | 0.99 | 0.99 | 0.99 | 0.81 |
| $M = 60$ | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 |
| $M = 120$ | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 | 0.99 |

**Notes:** subVAR denotes the VAR coupled with the subspace shrinkage prior, Minn is the combination between subspace and Minnesota shrinkage while flat is the subspace shrinkage prior without additional shrinkage. Both models feature a flat prior on $\omega$. Each number is based on computing the mean of posterior medians across 100 replications from the respective DGPs. For $q$, we use the posterior median as our point estimate while for $\omega$ we use the posterior mean.

The table suggests that if the number of observations becomes large, our model detects the true number of factors very accurately. For all model sizes, the estimated number of factors is never far away from the true number of factors. This finding suggests that the accuracy of our approach, as expected, strongly depends on the relationship between the number of coefficients and the length of the time series. Since the dimension of the state space grows quadratically in $M$, substantial information in the likelihood is required to infer the true number of factors for $M = 120$. 

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### Table 6: Description of the Dataset

| ID FRED Code | Description | Transformation Codes | S M L XL |
|--------------|-------------|----------------------|---------|
| GDPC1        | Real Gross Domestic Product | 5 X X X X |         |
| PCEC96       | Real Personal Consumption Expenditures | 5 X X X X |         |
| PCDN         | Real Personal Consumption Expenditures: Nonfarm Dollars | 5 X X X |         |
| GDPCC1       | Real Gross Private Domestic Investment | 5 X X X X |         |
| FFx          | Real private fixed investment | 5 X X X |         |
| Y01B1Q1Q027SBEA | Real Gross Private Domestic Investment: Fixed Investment: Nonresidential Equipment | 5 X X X |         |
| FFx          | Real private fixed investment: Nonresidential | 5 X X X |         |
| FFx          | Real private fixed investment: Residential | 5 X X X |         |
| ANR4BEQ1Q16SBEA | Shares of gdp's domestic product: Gross private domestic investment: Change in private inventories | 1 X X X |         |
| Y01B1Q1Q027SBEA | Real Government Consumption Expenditures and Gross Investment: Federal | 5 X X X |         |
| A281Q1Q027SBEA | Real Government Consumption Expenditures and Gross Investment: Federal | 5 X X X |         |
| PGRECPFx     | Real Federal Government Current Receipts | 5 X X X |         |
| SLEB         | Real government state and local consumption expenditures | 5 X X X |         |
| EPXGSCC1     | Real Exports of Goods and Services | 5 X X X |         |
| USXGSCC1     | Real Imports of Goods and Services | 5 X X X |         |
| DPIPC06      | Real Disposable Personal Income | 5 X X X |         |
| OCTNF3       | Nonfarm Business Sector: Real Output | 5 X X X |         |
| OCTNF3       | Business Sector: Real Output | 5 X X X |         |
| INDEPDRO     | B’Totital Industrial Production Index (Index 2012=100) | 5 X X X |         |
| INDEPLAL      | B’Final industrial Production: Final Products (Market Group) (Index 2012=100) | 5 X X X |         |
| IPCOUSDG      | CPI:Consumer Goods Industrial Production: Consumer Goods (Index 2012=100) | 5 X X X |         |
| IPMSAT       | Metals (Index 2012=100) | 5 X X X |         |
| IPDMAT       | Durable Materials (Index 2012=100) | 5 X X X |         |
| IPDMAT       | Non durable Materials (Index 2012=100) | 5 X X X |         |
| IPDMAT       | Durable Consumer Goods (Index 2012=100) | 5 X X X |         |
| IPDHH110SQ    | Durable Goods: Automotive products (Index 2012=100) | 5 X X X |         |
| IPDHH110SED  | Non durable Consumer Goods (Index 2012=100) | 5 X X X |         |
| IPBUSQ        | Business Equipment (Index 2012=100) | 5 X X X |         |
| IPHH122SQ     | Consumer energy products (Index 2012=100) | 5 X X X |         |
| NUMiëNS       | Capacity Utilization: Manufacturing (SIC) (Percent of Capacity) | 5 X X X |         |
| IPH11012SB    | Industrial Production: Manufacturing (SIC) (Index 2012=100) | 5 X X X |         |
| IPH11022SB    | Industrial Production: Residential Utilities (Index 2012=100) | 5 X X X |         |
| IPFUELS       | Industrial Production: Fuel (Index 2012=100) | 5 X X X |         |
| PAYEM        | Emp:Nonfarm All Employees: Total nonfarm (Thousands of Persons) | 5 X X X |         |
| UFSPY        | All Employees: Total Private Industries (Thousands of Persons) | 5 X X X |         |
| MANEMP       | All Employees: Manufacturing (Thousands of Persons) | 5 X X X |         |
| USVRY        | All Employees: Service-Providing Industries (Thousands of Persons) | 5 X X X |         |
| USGQGD       | All Employees: Goods-Producing Industries (Thousands of Persons) | 5 X X X |         |
| DMANEMP      | All Employees: Durable goods (Thousands of Persons) | 5 X X X |         |
| DSMANEMP     | All Employees: Non durable goods (Thousands of Persons) | 5 X X X |         |
| USCOSS       | All Employees: Construction (Thousands of Persons) | 5 X X X |         |
| USERS        | All Employees: Education & Health Services (Thousands of Persons) | 5 X X X |         |
| USFIRE       | All Employees: Financial Activities (Thousands of Persons) | 5 X X X |         |
| USMFO        | All Employees: Information Services (Thousands of Persons) | 5 X X X |         |
| USBFS        | All Employees: Professional & Business Services (Thousands of Persons) | 5 X X X |         |
| USLAH        | All Employees: Leisure & Hospitality (Thousands of Persons) | 5 X X X |         |
| USMOR        | All Employees: Other Services (Thousands of Persons) | 5 X X X |         |
| USMINE       | All Employees: Mining and Logging (Thousands of Persons) | 5 X X X |         |
| USPSU        | All Employees: Trade, Transportation & Utilities (Thousands of Persons) | 5 X X X |         |
| USOOG        | All Employees: Government (Thousands of Persons) | 5 X X X |         |
| USHRADE      | All Employees: Retail Trade (Thousands of Persons) | 5 X X X |         |
| USWTRADE     | All Employees: Wholesale Trade (Thousands of Persons) | 5 X X X |         |
| CES0893000001 | Government: Federal (Thousands of Persons) | 5 X X X |         |
| CES0893000001 | Government: State Government (Thousands of Persons) | 5 X X X |         |
| CES0893000001 | Government: Local Government (Thousands of Persons) | 5 X X X |         |
| CE1604       | Civilian Employment (Thousands of Persons) | 5 X X X |         |
| CIVPAR1      | Civilian Labor Force Participation Rate (Percent) | 5 X X X |         |
| UNRATE       | Civilian Unemployment Rate (Percent) | 5 X X X |         |
| UNRATESTX    | Unemployment Rate less than 27 weeks (Percent) | 5 X X X |         |
| UNRATESTX    | Unemployment Rate for more than 27 weeks (Percent) | 5 X X X |         |
| LNS14000012   | Unemployment Rate - 16 to 19 years (Percent) | 2 X X X |         |
| LNS14000025   | Unemployment Rate - 20 years and over, Men (Percent) | 2 X X X |         |
| LNS14000026   | Unemployment Rate - 20 years and over, Women (Percent) | 2 X X X |         |
| UEMPTLY       | Number of Civilian Unemployed - Less Than 5 Weeks (Thousands of Persons) | 5 X X X |         |
| UEMPSTLY514   | Number of Civilian Unemployed for 5 to 14 Weeks (Thousands of Persons) | 5 X X X |         |
| UEMPSTLY526   | Number of Civilian Unemployed for 15 to 26 Weeks (Thousands of Persons) | 5 X X X |         |
| UEMPSTLY704   | Number of Civilian Unemployed for 27 Weeks and Over (Thousands of Persons) | 5 X X X |         |
| AWHRMAN      | Average Weekly Hours of Production and Nonsmokers: Manufacturing (Hours) | 1 X X X |         |
| AWHRMAN      | Average Weekly Overtime Hours of Production and Nonsmokers: Manufacturing (Hours) | 2 X X X |         |
| HWLx         | Help-Wanted Index | 1 X X X |         |
| CES0660000007 | Average Weekly Hours of Production and Nonsmokers: Goods-Producing | 2 X X X |         |
| CLMASS       | Initial Claims | 2 X X X |         |
| HOUST        | Housing Starts: Total: New Privately Owned Housing Units Started | 5 X X X |         |
| HOUSTDF      | Privately Owned Housing Starts: 5-Unit Structures or More | 5 X X X |         |
| HOUSTJW      | Housing Starts in Midwest Census Region (Thousands of Units) | 5 X X X |         |
| HOUSTNE      | Housing Starts in Northeast Census Region (Thousands of Units) | 5 X X X |         |
| HOUSTS       | Housing Starts in South Census Region (Thousands of Units) | 5 X X X |         |
| HOUSTW       | Housing Starts in West Census Region (Thousands of Units) | 5 X X X |         |
| RSFAM        | Real Retail and Food Services Sales (Millions of Channel 2012 Dollars) | 5 X X X |         |
| AMDSINOx     | Real Manufacturers New Orders: Durable Goods (Millions of 2012 Dollars) | 5 X X X |         |
| AMDSDUOx     | Real Value of Manufactures Unfilled Orders for Durable Goods Industries | 5 X X X |         |
| BUSINVx      | Total Business Inventories (Millions of Dollars) | 5 X X X |         |
| ISSRATx      | Total Business: Inventories to Sales Ratio | 2 X X X |         |
| PCECTPE      | Personal Consumption Expenditure: Chain-type Price Index | 6 X X X |         |
| PCEFLPE      | Personal Consumption Expenditure: Chain-type Price Index | 6 X X X |         |
| G2DFCTPI     | Gross Domestic Product: Chain-type Price Index | 6 X X X |         |
| G3DFCTPI     | Gross Private Domestic Investment: Chain-type Price Index | 6 X X X |         |
| IFDSB         | Business Sector: Implicit Price Deflator (Index 2012=100) | 5 X X X |         |
| ID | FEER Code | Description | Transformation Codes | S | M | L | XL |
|----|-----------|-------------|----------------------|---|---|---|----|
| 91 | DGDSRG3Q086SBEA | Personal consumption expenditures: Goods | 6 | X | X |
| 92 | DGDSRG3Q086SBEA | Personal consumption expenditures: Services | 6 | X | X |
| 93 | DGDSRG3Q086SBEA | Personal consumption expenditures: Non-durables goods | 6 | X | X |
| 95 | DGDSRG3Q086SBEA | Personal consumption expenditures: Services: Household consumption expenditures | 6 | X | X |
| 96 | DMTRGRGQ086SBEA | Personal consumption expenditures: Durable goods: Motor vehicles and parts | 6 | X | X |
| 97 | FMTRGQ086SBEA | Personal consumption expenditures: Durables: Furnishings and durable household equipment | 6 | X | X |
| 98 | DREGQ086SBEA | Personal consumption expenditures: Durables: Recreational goods and vehicles | 6 | X | X |
| 99 | DGDSRG3Q086SBEA | Personal consumption expenditures: Goods: Other durables goods | 6 | X | X |
| 100 | DFXARGQ086SBEA | Personal consumption expenditures: Non-durables goods: Food and beverages purchased for off-premises consumption | 6 | X | X |
| 101 | DCLORQ086SBEA | Personal consumption expenditures: Non-durables goods: Clothing and footwear | 6 | X | X |
| 102 | DQGRQ086SBEA | Personal consumption expenditures: Non-durables goods: Gasoline and other energy goods | 6 | X | X |
| 103 | DGDSRG3Q086SBEA | Personal consumption expenditures: Other non-durables goods | 6 | X | X |
| 104 | DHTRGRGQ086SBEA | Personal consumption expenditures: Services: Housing and utilities | 6 | X | X |
| 105 | DHTRGRGQ086SBEA | Personal consumption expenditures: Services: Health care | 6 | X | X |
| 106 | DTRSGQ086SBEA | Personal consumption expenditures: Transportation services | 6 | X | X |
| 107 | DTRGQ086SBEA | Personal consumption expenditures: Recreation services | 6 | X | X |
| 108 | DPXARGQ086SBEA | Personal consumption expenditures: Services: Food services and accommodations | 6 | X | X |
| 109 | DFSRGQ086SBEA | Personal consumption expenditures: Financial services and insurance | 6 | X | X |
| 110 | DTSRGQ086SBEA | Personal consumption expenditures: Other services | 6 | X | X |
| 111 | CPFURS | Consumer Price Index for All Urban Consumers: All Items | 6 | X | X | X |
| 112 | CPFURE | Consumer Price Index for All Urban Consumers: All Items Less Food & Energy | 6 | X | X | X |
| 113 | WPSFD49027 | Producer Price Index by Commodity for Finished Goods | 6 | X | X |
| 114 | FICPOC | Producer Price Index for All Commodities | 6 | X | X |
| 115 | WPSFD40052 | Producer Price Index by Commodity for Finished Consumer Goods | 6 | X | X |
| 116 | FPFDCC | Producer Price Index by Commodity Industrial Commodities | 6 | X | X |
| 117 | WPSFDC | Producer Price Index by Commodity Intermediate Materials: Supplies & Components | 6 | X | X |
| 118 | WPSFUG | Producer Price Index by Commodity for Fuels and Related Products and Power | 5 | X | X |
| 119 | 0LPBICEx | Real Crude Oil Price: West Texas Intermediate (WTI) - Cushing, Oklahoma | 5 | X | X |
| 120 | WPSFD6 | Producer Price Index: Crude Materials for Further Processing | 6 | X | X |
| 121 | FPFD3M | Producer Price Index: Commodities: Metals and metal products. Primary nonferrous metals | 6 | X | X |
| 122 | CPATPPSL | Consumer Price Index for All Urban Consumers: Apparel | 6 | X | X |
| 123 | CPTURS | Consumer Price Index for All Urban Consumers: Transportation | 6 | X | X |
| 124 | CPFDHDSL | Consumer Price Index for All Urban Consumers: Medical Care | 6 | X | X |
| 125 | CUSF00008AC | Consumer Price Index for All Urban Consumers: Commodities | 6 | X | X |
| 126 | CESF000008t | Real Average Hourly Earnings of Production and Nonproduction Employees: Construction | 5 | X | X |
| 127 | CESF000008t | Real Average Hourly Earnings of Production and Nonproduction Employees: Manufacturing | 5 | X | X |
| 128 | CPDI0NFB | Nonfarm Business Sector, Real Compensation Per Hour (Index 2012=100) | 5 | X | X |
| 129 | CESF00000008 | Average Hourly Earnings of Production and Nonproduction Employees | 6 | X | X | X |
| 130 | EODMFS | Effective Federal Funds Rate (Percent) | 2 | X | X | X |
| 131 | TBMIS | 3-Month Treasury Bill Secondary Market Rate (Percent) | 2 | X | X |
| 132 | TTSMS | 6-Month Treasury Bill Secondary Market Rate (Percent) | 2 | X | X |
| 133 | G1 | 1-Year Treasury Constant Maturity Rate (Percent) | 2 | X | X | X |
| 134 | GS10 | 10-Year Treasury Constant Maturity Rate (Percent) | 2 | X | X | X |
| 135 | AAA | Moody's Seasoned Aus Corporate Bond Yield (Percent) | 2 | X | X |
| 136 | BAA | Moody's Seasoned Aus Corporate Bond Yield (Percent) | 2 | X | X |
| 137 | BAA30YM | Moody's Seasoned Aus Corporate Bond Yield Relative to Yield on 10-Year Treasury | 1 | X |
| 138 | TBIS3Mx | 6-Month Treasury Bill Minus 3-Month Treasury Bill, secondary market (Percent) | 1 | X |
| 139 | GISTBMx | 1-Year Treasury Constant Maturity Minus 3-Month Treasury Bill, secondary market (Percent) | 1 | X |
| 140 | GISTBMx | 10-Year Treasury Constant Maturity Minus 5-Year Treasury Bill, secondary market (Percent) | 1 | X |
| 141 | G5 | 5-Year Treasury Constant Maturity Rate | 2 | X |
| 142 | TRBSFFM | 3-Month Treasury Constant Maturity Minus Federal Funds Rate | 1 | X |
| 143 | TFFFM | 5-Year Treasury Constant Maturity Minus Federal Funds Rate | X | X |
| 144 | AAFFM | Moody's Seasoned Aus Corporate Bond Minus Federal Funds Rate | 1 | X |
| 145 | MIRREAL | Real Mid Money Stock | 5 | X | X |
| 146 | MIRREAL | Real Mid Money Stock (Percent) | 5 | X |
| 147 | BUSL0Axx | Real Commercial and Industrial Loans, All Commercial Banks | 5 | X | X |
| 148 | AUSL0Axx | Real Consumer Loans at All Commercial Banks | 5 | X | X |
| 149 | NSREVSLx | Total Real Nontraditional Loans and purchased, Outstanding | 5 | X | X |
| 150 | REALLx | Real Estate Loans, All Commercial Banks | 5 | X | X |
| 151 | TOTLSx | Total Consumer Credit Outstanding | 5 | X | X |
| 152 | TOTRESNS | Total Reserve of Depository Institutions | 6 | X | X |
| 153 | NDORORNS | Reserve Of Depository Institutions, Nonreserve | 7 | X |
| 154 | DTCOLYVHBM | Consumer Motor Vehicle Loans Outstanding Owned by Finance Companies | 6 | X |
| 155 | DCTCHTBM | Total Consumer Loans and Leases Outstanding Owned and Securitized by Finance Companies | 6 | X |
| 156 | SVENT | Securities in Bank Credit at All Commercial Banks | 6 | X |
| 157 | TASHIOx | Real Total Assets of Household and Nonprofit Organizations | 5 | X |
| 158 | EKEUx | Switzerland / U.S. Foreign Exchange Rate | 5 | X | X |
| 159 | EXSFXa | Japa / U.S. Foreign Exchange Rate | 5 | X | X |
| 160 | EXSFXa | U.S. / U.K. Foreign Exchange Rate | 5 | X | X |
| 161 | EXACx | Canada / U.S. Foreign Exchange Rate | 5 | X | X |
| 162 | SP500x | S&P's Common Stock Price Index: Composite | 5 | X | X |
| 163 | EPOCBx | S&P's Common Stock Price Index: Industrials | 5 | X | X |
| 164 | SP400x | S&P's Composite Common Stock: Dividend Yield | 2 | X |

Notes: This table provides an overview of the dataset employed. The transformation codes are applied to each time series $y_t$ and described in McCracken and Ng (2020): (1) no transformation; (2) $\Delta y_{t+1}$; (3) $\Delta^2 y_{t+1}$; (4) $\log(y_{t+1})$; (5) $\Delta \log(y_{t+1})$; (6) $\Delta^2 \log(y_{t+1})$; (7) $\Delta (y_{t+1}/y_t - 1)$. ‘X’ marks the inclusion of one variable into one of the datasets.