SU(5) × SU(5)′ Twinification and D2 Parity. Model for Composite Leptons

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Abstract

We study Grand Unified SU(5) × SU(5)′ model supplemented by D2 parity. The D2 greatly reduces the number of parameters and is important for phenomenology. The model, we present, has various novel and interesting properties. Due to specific pattern of the GUT symmetry breaking and emerged strong dynamics at low energies, the Standard Model leptons, along with right handed/sterile neutrinos, come out as composite states. The generation of the charged fermion and neutrino masses are studied within considered scenario. Moreover, the issues of gauge coupling unification and nucleon stability are investigated in details. Various phenomenological implications are also discussed.

1 Introduction

The Standard Model (SM) of electro-weak interactions has been very successful theory for decades. Triumph of this celebrated model occurred by the Higgs boson discovery [1] at CERN’s Large Hadron Collider. In spite of this success, several phenomenological and theoretical issues motivate to think of some physics beyond the SM. Due to renormalization running, the self-coupling of the SM Higgs boson becomes negative at scales near \( \sim 10^{10} \text{ GeV} \) [2], [3] (with the Higgs mass \( \sim 126 \text{ GeV} \)), causing vacuum instability. Moreover, the SM fails to accommodate atmospheric and solar neutrino data [4]. Renormalizable part of the SM interactions render neutrinos to be massless. Also, Planck scale suppressed \( d = 5 \) lepton number violating operators do not generate neutrino mass with desirable magnitude. These are already strong motivations to think about some new physics between EW and Planck scales.

Amongst various extensions of the SM, the Grand Unification (GUT) [5], [6] is leading candidate. Unifying all gauge interactions in a single group, at high energies one can deal with a single unified gauge coupling. At the same time, quantization of quark and lepton charges occurs by embedding

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all fermionic states in unified GUT multiplets. Striking prediction of the Grand Unified Theory is the baryon number violating nucleon decay. This opens prospect for probing the nature at very short distances. GUTs based on $SU(10)$ symmetry [7] (including $SU(2)_L \times SU(2)_R \times SU(4)_c$ symmetry [5] as a maximal subgroup) involve right handed neutrinos (RHN), which provide simple and elegant way for neutrino mass generation via see-saw mechanism [8]. In spite of these salient futures, GUT model building encounter numerous problems and phenomenological difficulties. With single scale breaking, i.e. with no new interactions and/or intermediate states between EW and GUT scales, GUTs (such as minimal $SU(5)$ or $SO(10)$) do not lead to successful gauge coupling unification. 

Besides this, building GUT with realistic fermion sector, understanding GUT symmetry breaking pattern and avoiding too rapid nucleon decay remain great challenge.

Motivated by these issues we consider $SU(5) \times SU(5)'$ GUT augmented with $D_2$ parity (exchange symmetry). The latter, relating two $SU(5)$ gauge factors, greatly reduces number of parameters, and at and above the GUT scale one deals with single gauge coupling. The GUTs with $SU(5) \times SU(5)'$ symmetry, considered in earlier works [9], where at least one gauge factor of the SM symmetry emerges as a diagonal subgroup, have been proven to be very successful for building models with realistic phenomenology. However, to our knowledge, the $D_2$ parity has not been applied before in such constructions. The reason could be the prejudice of remaining with extra unwanted chiral matter states in the spectrum. However, within our model due to specific construction, this does not happen and only the states of SM survive below few-TeV scale. The $D_2$ parity also plays crucial role for other phenomenology and has interesting implications. By specific pattern of the $SU(5) \times SU(5)'$ symmetry breaking and spectroscopy, the successful gauge coupling unification is obtained. Interestingly, within considered framework the SM leptons emerge as a composite states, while the quarks are fundamental objects. Lepton mass generation occurs by new mechanism, finding natural realization within presented model. Since leptons and quarks have different footing, there is no problem of their mass degeneracy (unlike to minimal SO(10) and $SU(5)$ GUTs requiring some extensions [10]). Moreover, along with composite SM leptons, model involves three families of composite SM singlet fermionic states, which may be identified with RHNs or sterile neutrinos. Thus, neutrino masses can be generated. In addition, we show that due to specific fermion pattern, the $d = 6$ nucleon decay can be adequately suppressed within considered model. The model also has various interesting properties and implications, which we also discuss.

The paper is organized as follows: In the next section, first we introduce the $SU(5) \times SU(5)' \times D_2$ GUT and discuss the symmetry breaking pattern. Then we present the spectrum of bosonic states. In Sect. 3, considering the fermion sector, we give transformation properties of the GUT matter multiplets under $D_2$ parity and build Yukawa interaction Lagrangian. The latter is responsible for generation of quark masses and CKM matrix elements. Due to specific pattern of the symmetry breaking and strong $SU(3)'$ (originating from $SU(5)'$ gauge symmetry) dynamics, the SM leptons emerge as composite objects. We present novel mechanism for composite lepton mass generation. Together with the SM leptons, three families of right handed/sterile neutrinos are composite. We also discuss the neutrino mass generation within our scenario. In Sect. 4 we give details of gauge coupling unification. The issue of nucleon stability is addressed in Sect. 5. Although the GUT scale within our model comes out to be relatively low ($\simeq 5 \cdot 10^{11}$ GeV), we show that the $d = 6$ baryon number violating operators can be adequately suppressed. This happens to be possible.

\footnote{In 2nd citation of [9], the exchange symmetry was considered, however some terms violating this symmetry have been included.}
due to specific pattern of the fermion sector we are suggesting. In Sect. 6 we summarize and discuss various phenomenological constraints and possible implications of the considered scenario. Also emphasize the model’s peculiarities and novelties, which open broad prospects for further investigations. Appendix A discusses details related to the compositeness and anomaly matching conditions. In Appendix B we give details of the gauge coupling unification. In particular, the RG equations and $b$-factors at various energy intervals are presented. The short range renormalization of baryon number violating $d = 6$ operators is also performed.

2 $SU(5) \times SU(5)' \times D_2$ Grand Unification

Let us consider theory based on $SU(5) \times SU(5)'$ gauge symmetry. Besides this symmetry, we postulate discrete parity $D_2$, which exchanges two $SU(5)$’s. Therefore, the symmetry of the model is

$$G_{GUT} = SU(5) \times SU(5)' \times D_2.$$  \hspace{1cm} (1)

As noted, action of $D_2$ interchanges the gauge fields (in adjoint representations) of $SU(5)$ and $SU(5)'$:

$$D_2 : \ (A_\mu)^a_b \rightarrow (A'_\mu)^{a'}_{b'} , \ (A'_\mu)^{a'}_{b'} \rightarrow (A_\mu)^a_b ,$$  \hspace{1cm} (2)

with $(A_\mu)^a_b = \frac{1}{2} \sum_{i=1}^{24} A_\mu^i (\lambda_i)^a_b$ and $(A'_\mu)^{a'}_{b'} = \frac{1}{2} \sum_{i'=1}^{24} A'_\mu^{i'} (\lambda^{i'})^{a'}_{b'}$ were $a, b$ and $a', b'$ denote indices of $SU(5)$ and $SU(5)'$ respectively. The $\lambda^i, \lambda^{i'}$ are corresponding Gell-Mann Matrices. Thanks to the $D_2$, at and above the GUT scale $M_G$ we have single gauge coupling

$$\alpha_5 = \alpha_{5'}.$$  \hspace{1cm} (3)

GUTs based on product groups allow to build simple models with realistic phenomenology [9], [11]. In our case, as we show below, the EW part (i.e. $SU(2)_w \times U(1)_Y$) of the SM gauge symmetry, will belong to the diagonal subgroup of $SU(5) \times SU(5)'$.

Potential and symmetry breaking

For $G_{GUT}$ symmetry breaking and building realistic phenomenology we introduce the following states

$$H \sim (5, 1) , \ \Sigma \sim (24, 1) , \ \ H' \sim (1, 5) , \ \Sigma' \sim (1, 24) , \ \Phi \sim (5, 5) ,$$  \hspace{1cm} (4)

where in brackets transformation properties under $SU(5) \times SU(5)'$ symmetry are indicated. $H$ includes SM higgs doublet $h$. Introduction of $H'$ is required by $D_2$ symmetry. By same reason, two adjoints $\Sigma$ and $\Sigma'$ (needed for GUT symmetry breaking) are introduced. The bi-fundamental state $\Phi$ will also serve for desirable symmetry breaking.

The action of $D_2$ parity on these fields is:

$$D_2 : \ H_a \mapsto H^a' , \ \Sigma^a_b \mapsto \Sigma^{a'}_{b'} , \ \Phi^a_b \mapsto (\Phi^a)^b.$$  \hspace{1cm} (5)

where we have made explicit the indices of $SU(5)$ and $SU(5)'$. With (5) and (2), one can easily make sure that, the kinetic part $|D_\mu H|^2 + |D_\mu H'|^2 + \frac{1}{2} \text{tr}(D_\mu \Sigma)^2 + \frac{1}{2} \text{tr}(D_\mu \Sigma')^2 + |D_\mu \Phi|^2$ of the scalar field Lagranjian is invariant.
The scalar potential, invariant under $G_{GUT}$ symmetry (of Eq. (1)) is:

$$V = V_{H\Sigma} + V_{H'\Sigma'} + V_{mix}^{(1)} + V_{\Phi} + V_{mix}^{(2)},$$

with

$$V_{H\Sigma} = -M_{\Sigma}^2 \text{tr}\Sigma^2 + \lambda_1 (\text{tr}\Sigma^2)^2 + \lambda_2 \text{tr}\Sigma^4 + H^\dagger (M_H^2 - h_1 \Sigma^2 + h_2 \text{tr}\Sigma^2) H + \lambda_H (H^\dagger H)^2,$$

$$V_{H'\Sigma'} = -M_{\Sigma'}^2 \text{tr}\Sigma'^2 + \lambda_1' (\text{tr}\Sigma'^2)^2 + \lambda_2' \text{tr}\Sigma'^4 + H'^\dagger (M_{H'}^2 - h_1' \Sigma'^2 + h_2' \text{tr}\Sigma'^2) H' + \lambda_{H'} (H'^\dagger H')^2,$$

$$V_{mix}^{(1)} = \lambda (\text{tr}\Sigma^2)(\text{tr}\Sigma'^2) + h \left( H^\dagger H \text{tr}\Sigma^2 + H'^\dagger H' \text{tr}\Sigma'^2 \right) + h(H^\dagger H)(H'^\dagger H'),$$

$$V_{\Phi} = -M_{\Phi}^2 \Phi^\dagger \Phi + \lambda_1 \Phi^\dagger \Phi + \lambda_2 \Phi^\dagger \Phi^\dagger \Phi,$$

$$V_{mix}^{(2)} = \mu (H^\dagger \Phi H' + H'^\dagger \Phi^\dagger H') + \frac{\lambda_{1H\Phi}}{\sqrt{25}} (\Phi^\dagger \Phi) \left( (H^\dagger H) + (H'^\dagger H') \right) + \frac{\lambda_{2H\Phi}}{\sqrt{10}} (H^\dagger \Phi^\dagger H + H'^\dagger \Phi H') + \lambda_{1\Sigma\Phi} (\Phi^\dagger \Phi)(\text{tr}\Sigma^2 + \text{tr}\Sigma'^2) - \lambda_{2\Sigma\Phi} (\Phi^\dagger \Sigma^2 \Phi + \Phi \Sigma'^2 \Phi^\dagger).$$

For making analysis simpler, we have omitted linear and cubic terms with respect to $\Sigma$ and $\Sigma'$. This simplification can be achieved by $Z_2$ discrete symmetry and will not harm anything.

The potential terms and couplings in (6), (7) allow to have desirable and self consistent pattern of symmetry breaking. First we will sketch the symmetry breaking pattern. Then we will analyze the potential and discuss the spectrum of bosonic states. We will stick on several stages of the GUT symmetry breaking. At first step, the $\Sigma$ develops the VEV $\sim M_G$ with

$$\langle \Sigma \rangle = v_\Sigma \text{Diag} (2, 2, 2, -3, -3), \quad v_\Sigma \sim M_G. \quad (8)$$

This causes the symmetry breaking

$$SU(5) \xrightarrow{\Sigma} SU(3) \times SU(2) \times U(1) \equiv G_{321}. \quad (9)$$

We select VEVs of $\Sigma'$ and $\Phi$ much smaller than $M_G$. As will turn out, phenomenologically preferred scenario is $\langle \Sigma' \rangle \sim 4 \cdot 10^6$ GeV and $\langle \Phi \rangle \sim 8 \cdot 10^4$ GeV. With

$$\langle \Sigma' \rangle = v_{\Sigma'} \text{Diag} (2, 2, 2, -3, -3), \quad (10)$$

the breaking

$$SU(5)' \xrightarrow{\Sigma'} SU(3)' \times SU(2)' \times U(1)' \equiv G_{321}' \quad (11)$$

is achieved. Last stage of the GUT breaking is done by $\langle \Phi \rangle$ with a direction

$$\langle \Phi \rangle = v_\Phi \cdot \text{Diag} (0, 0, 0, 1, 1). \quad (12)$$

This configuration of $\langle \Phi \rangle$ breaks symmetries $SU(2) \times U(1)$ (subgroup of $SU(5)$) and $SU(2)' \times U(1)'$ (subgroup of $SU(5)'$) to the diagonal symmetry group:

$$SU(2) \times U(1) \times SU(2)' \times U(1)' \xrightarrow{\langle \Phi \rangle} [SU(2) \times U(1)]_{\text{diag}}. \quad (13)$$
As we see, all VEVs preserve $SU(3)$ and $SU(3)'$ groups arising from $SU(5)$ and $SU(5)'$ respectively. However, unbroken $SU(2)_{\text{diag}}$ is coming (as superposition) partly from $SU(2) \subset SU(5)$ and partly from $SU(2)' \subset SU(5)'$. Similar applies to $U(1)_{\text{diag}}$, i.e. it is superposition of two Abelian factors: $U(1) \subset SU(5)$ and $U(1)' \subset SU(5)'$.

Now, making the identifications

$$SU(3) \equiv SU(3)_c, \quad SU(2)_{\text{diag}} \equiv SU(2)_w, \quad U(1)_{\text{diag}} \equiv U(1)_Y$$

(14)

and taking into account (9), (11), and (13), we can see that GUT symmetry is broken as

$$G_{GUT} \to SU(3)_c \times SU(2)_w \times U(1)_Y \times SU(3)' = G_{SM} \times SU(3)'.$$  (15)

where $G_{SM} = SU(3)_c \times SU(2)_w \times U(1)_Y$ denotes the SM gauge symmetry. Because of these, at intermediate scale $\mu = M_I(\sim \langle \Phi \rangle)$ we will have the following matching conditions for the gauge couplings:

$$\text{at } \mu = M_I : \quad \frac{1}{g_w^2} = \frac{1}{g_2^2} + \frac{1}{g_Y^2}, \quad \frac{1}{g_1^2} = \frac{1}{g_1'} + \frac{1}{g_Y^2},$$

(16)

where subscripts indicate which gauge interaction the appropriate coupling corresponds to (e.g. $g_Y$ is the coupling of $U(1)'$ symmetry, etc).

Extra $SU(3)'$ factor has important and interesting implications, which we discuss below.

As was mentioned, while $\langle \Sigma \rangle \sim M_G$, the VEVs $\langle \Phi \rangle$ and $\Sigma'$ are at intermediate scales $M_I$ and $M_I'$ respectively:

$$v_\Phi \sim M_I, \quad v_{\Sigma'} \sim M_I'$$

(17)

with the hierarchical pattern

$$M_I \ll M_I' \ll M_G.$$  (18)

Detailed analysis of the whole potential, show that there is true minima along directions (8), (10), (12) with $\langle H \rangle = \langle H' \rangle = 0$. With $\langle \Sigma \rangle \neq \langle \Sigma' \rangle$, the $D_2$ is broken spontaneously. The residual $SU(3)'$ symmetry will play important role and the hierarchical pattern of (18) will turn out to be crucial for successful unification (discussed below).

The hierarchical pattern (18), of the GUT symmetry breaking, makes it simple to minimize the potential and analyze the spectrum.

Three extremum conditions, determining $v_\Sigma, v_{\Sigma'}, v_\Phi$ along the directions (8), (10), (12) and obtained from the potential, are:

$$10(30\lambda_1 + 7\lambda_2)v_\Sigma^2 + 150\lambda v_\Sigma^2 + (10\lambda_{1\Sigma\Phi} - 3\lambda_{2\Sigma\Phi})v_\Phi^2 = 5M_\Sigma^2,$$

$$150\lambda v_\Sigma^2 + 10(30\lambda_1 + 7\lambda_2)v_\Sigma^2 + (10\lambda_{1\Sigma\Phi} - 3\lambda_{2\Sigma\Phi})v_\Phi^2 = 5M_\Sigma^2,$$

$$3(10\lambda_{1\Sigma\Phi} - 3\lambda_{2\Sigma\Phi})(v_\Sigma^2 + v_{\Sigma'}^2) + (4\lambda_{1\Phi} + 2\lambda_{2\Phi})v_\Phi^2 = M_\Phi^2.$$  (19)

Because of hierarchies (17), (18), from the 1st equation of (19), with a good approximation we obtain

$$v_\Sigma \simeq \frac{M_\Sigma}{\sqrt{2(30\lambda_1 + 7\lambda_2)}}.$$  (20)
Thus, with \((30 \lambda_1 + 7 \lambda_2) \sim 1\), we should have \(M_\Sigma \approx M_G\). On the other hand, from last two equations of (19) we derive:
\[
v_{\Sigma'}^2 \simeq \frac{M_\Sigma^2 - 30 \lambda_2 v_\Sigma^2}{2 (30 \lambda_1 + 7 \lambda_2)} , \quad v_\Phi^2 = \frac{M_\Phi^2 - 3 (10 \lambda_1 \Sigma \Phi - 3 \lambda_2 \Sigma \Phi) (v_\Sigma^2 + v_{\Sigma'}^2)}{4 \lambda_1 \Phi + 2 \lambda_2 \Phi} .
\]  
(21)

In order to obtain the scales \(M_I\) and \(M_I'\), according to (17), (18), we have to arrange (by price of tunings) \(M_\Sigma^2 - 30 \lambda_2 v_\Sigma^2 \approx M_I'\) and \(M_\Phi^2 - 3 (10 \lambda_1 \Sigma \Phi - 3 \lambda_2 \Sigma \Phi) (v_\Sigma^2 + v_{\Sigma'}^2) \approx M_I\).

**The Spectrum**

At first stage of symmetry breaking, the \((X, Y)\) gauge bosons (of \(SU(5)\)) obtain GUT scale masses. They absorb appropriate states (with quantum numbers of leptoquarks) from the adjoint scalar \(\Sigma\). The remaining physical fragments (\(\Sigma_8, \Sigma_3, \Sigma_1\)) [the \(SU(3)\) octet, \(SU(2)\) triplet and a singlet respectively] receive GUT scale masses. These states are heaviest and their mixings with other ones can be neglected. From (7), with (19) we get
\[
M_{\Sigma_8}^2 \simeq 20 \lambda_2 v_\Sigma^2 , \quad M_{\Sigma_3}^2 \simeq 80 \lambda_2 v_\Sigma^2 , \quad M_{\Sigma_1}^2 \simeq 4 M_\Sigma^2 .
\]  
(22)

Further, we will not give masses of states which are singlets under all symmetry groups. The mass squire of \(SU(3)'\) octet (from \(\Sigma'\)) is
\[
M_{\Sigma'_3'}^2 = 20 \lambda_2 v_{\Sigma'}^2 + \frac{6}{5} \lambda_2 \Sigma \Phi v_\Phi^2 .
\]  
(23)

The triplet \(\Sigma'_3\) mixes with real (CP even) \(SU(2)_w\) triplet \(\Phi_3\) (from \(\Phi\)). (Both these states are real adjoints of \(SU(2)_w\)). The appropriate mass squired couplings are:
\[
\frac{1}{2} \left( \Sigma'_3, \Phi_3 \right) \left( \begin{array}{c} 4 M_{\Sigma'_3}^2 - \frac{28}{9} \lambda_2 \Sigma \Phi v_\Phi^2 \\ 6 \sqrt{2} \lambda_2 \Sigma \Phi v_\Phi v_{\Sigma'} \\ 4 \lambda_2 \Phi v_\Phi^2 \\ 6 \sqrt{2} \lambda_2 \Sigma \Phi v_\Phi v_{\Sigma'} \\ 4 \lambda_2 \Phi v_\Phi^2 \end{array} \right) \left( \begin{array}{c} \Sigma'_3' \\ \Phi'_3 \end{array} \right) ,
\]  
(24)

where \(i = 1, 2, 3\) labels the components of \(SU(2)_w\) adjoint. The CP odd real \(SU(2)_w\) triplet from \(\Phi\) is absorbed by appropriate gauge fields after \(SU(2) \times SU(2)' \rightarrow SU(2)_w\) breaking and became genuine Goldstone mode.

By the VEVs \(v_\Sigma\) and \(v_{\Sigma'}\), the symmetry \(SU(5) \times SU(5)' \times D_2\) is broken down to \(G_{321} \times G_{321}'\) (see Eqs. (9) and (11)). Thus, between the scales \(M_I\) and \(M_{I'}\) we have this symmetry, and the \(\Phi(5, 5)\) splits into fragments
\[
\Phi(5, 5) = \Phi_{DD'} \oplus \Phi_{DT'} \oplus \Phi_{TT'} \oplus \Phi_{TD'}
\]  
(25)

with transformation properties under \(G_{321} \times G_{321}'\) given by
\[
G_{321} \times G_{321}' : \quad \Phi_{DD'} \sim \left( 1, 2, -\frac{3}{\sqrt{60}}, 1, 2', \frac{3}{\sqrt{60}} \right) , \quad \Phi_{DT'} \sim \left( 1, 2, -\frac{3}{\sqrt{60}}, 3', 1, -\frac{2}{\sqrt{60}} \right) ,
\]

\[
\Phi_{TT'} \sim \left( 3, 1, \frac{2}{\sqrt{60}}, 3', 1, -\frac{2}{\sqrt{60}} \right) , \quad \Phi_{TD'} \sim \left( 3, 1, \frac{2}{\sqrt{60}}, 1, 2', \frac{3}{\sqrt{60}} \right) .
\]  
(26)

The masses of these fragments will be dented by \(M_{DD'}, M_{DT'}, M_{TT'}\) and \(M_{TD'}\) respectively. Since the breaking \(G_{321} \times G_{321}' \rightarrow G_{SM} \times SU(3)'\) is realized by the VEV of the fragment \(\Phi_{DD'}\) at scale...
By diagonalization of (31) we get two physical states \( h \) respectively. Mass squares of these triplets are:

\[
G_{321} \times SU(3)' : \quad \Phi_{DT'} \sim \left(1, 2, -\frac{5}{\sqrt{60}}, 3'\right), \quad \Phi_{TT'} \sim (3, 1, 0, 3'), \quad \Phi_{TD'} \sim \left(3, 2, \frac{5}{\sqrt{60}}, 1\right). \quad (27)
\]

The mass squares of these fields are given by:

\[
M_{DT'}^2 = 5\lambda_{2\Sigma\Phi} v_\Sigma^2, \quad M_{TT'}^2 = 5\lambda_{2\Sigma\Phi}(v_\Sigma^2 + v_\Sigma'^2) - 2\lambda_{2\Phi} v_\Phi^2, \quad M_{TD'}^2 = 5\lambda_{2\Sigma\Phi} v_\Sigma'^2. \quad (28)
\]

With the VEVs towards directions given in (8), (10), (12), and with extremum conditions of Eq. (19), the potential’s minimum is achieved with

\[
30\lambda_1 + 7\lambda_2 > 0, \quad \lambda_2 > 0, \quad \lambda > 0,
\]

\[
10\lambda_{1\Sigma\Phi} - 3\lambda_{2\Sigma\Phi} > 0, \quad \lambda_{2\Sigma\Phi} > 0, \quad 2\lambda_{1\Phi} + \lambda_{2\Phi} > 0, \quad \lambda_{2\Phi} > 0. \quad (29)
\]

As far as the states \( H \) and \( H' \) are concerned, they are split as follows: \( H \rightarrow (D_H, T_H) \) and \( H' \rightarrow (D_{H'}, T_{H'}) \), where \( D_H, D_{H'} \) are doublets, while \( T_H \) and \( T_{H'} \) are \( SU(3)_c \) and \( SU(3)' \) triplets respectively. Mass squares of these triplets are:

\[
M_{H}^2 = M_{H'}^2 = 4h_1 v_\Sigma^2 + 30(h_2 v_\Sigma^2 + \tilde{h} v_\Sigma'^2) + 2\lambda_{1H\Phi} v_\Phi^2 / \sqrt{25},
\]

\[
M_{T_H}^2 = M_{T_{H'}}^2 = 4h_1 v_\Sigma'^2 + 30(h_2 v_\Sigma'^2 + \tilde{h} v_\Sigma^2) + 2\lambda_{1H\Phi} v_\Phi^2 / \sqrt{25}. \quad (30)
\]

The states \( D_H \) and \( D_{H'} \), under \( G_{SM} \), both have quantum numbers of the SM Higgs doublet. They mix by the VEV \( \langle \Phi \rangle \) and the mass squared matrix is given by:

\[
\begin{pmatrix}
(D_H^\dagger, D_{H'}^\dagger)
\end{pmatrix}
\begin{pmatrix}
M_{T_{H}}^2 - 5h_1 v_\Sigma^2 + \lambda_{2H\Phi} v_\Phi^2 / \sqrt{10}

\mu v_\Phi & M_{T_{H'}}^2 - 5h_1 v_\Sigma'^2 + \lambda_{2H\Phi} v_\Phi^2 / \sqrt{10}
\end{pmatrix}
\begin{pmatrix}
(D_H^\dagger, D_{H'}^\dagger)
\end{pmatrix}. \quad (31)
\]

By diagonalization of (31) we get two physical states \( h \) and \( D' \):

\[
h = \cos \theta_h D_H + \sin \theta_h D_{H'}, \quad D' = -\sin \theta_h D_H + \cos \theta_h D_{H'},
\]

\[
\tan 2\theta_h = \frac{2\mu v_\Phi}{M_{T_{H}}^2 - M_{T_{H'}}^2 - 5h_1 (v_\Sigma^2 - v_\Sigma'^2)}. \quad (32)
\]

We identify \( h \) with the SM Higgs doublet and set its mass square (by fine tuning) \( M_h^2 \sim 100 \text{ GeV}^2 \). We assume the second doublet \( D' \) to be heavy \( M_{D'}^2 \gg |M_h|^2 \). For the mixing angle \( \theta_h \) we also assume \( \theta_h \ll 1 \). Therefore, according to Eq. (32), the SM higgs mainly resides in \( D_H \) (of -plet), while \( D_{H'} \) (i.e. \( H' \)) includes light SM doublet with very suppressed weight.

The radiative corrections will affect obtained expressions for the masses and VEVs. However, there are enough parameters involved and one can always get considered symmetry breaking pattern and desirable spectrum. Achieving these will require some fine tunings. Without addressing here hierarchy problem and naturalness issues, we will proceed to study various properties and phenomenology of the considered scenario.
3 Fermion Sector

3.1 $D_2$ Symmetry à la $P$ Parity

We introduce three families of $(\Psi, F)$ and three families of $(\Psi', F')$

$$3 \times [\Psi(10, 1) + F(5, 1)] , \quad 3 \times [\Psi'(1, 10) + F'(1, 5)]$$

where in brackets transformation properties under $SU(5) \times SU(5)'$ gauge symmetry are indicated. Here each fermionic state is two component Weyl spinor, in $(\frac{1}{2}, 0)$ representation of the Lorentz group. The action of $D_2$ parity on these fields is determined as

$$D_2 : \quad \Psi \mapsto \overline{\Psi} \equiv (\Psi')^\dagger , \quad F \mapsto F' \equiv (F')^\dagger .$$

It is easy to verify that, with transformations in (34) and (2), the kinetic part of the Lagrangian $L_{\text{kin}}(\Psi, F, \Psi', F')$ is invariant.$^3$

We can easily write down invariant Yukawa Lagrangian

$$L_Y + L_{Y'} + L_{Y'}^{\text{mix}}$$

with

$$L_Y = \sum_{n=0} C_{\Psi \Phi}^{(n)} \left( \frac{\Sigma}{M_s} \right)^n \Psi \Psi H + \sum_{n=0} C_{\Psi F}^{(n)} \left( \frac{\Sigma}{M_s} \right)^n \Psi F H^\dagger + \text{h.c.}$$

$$L_{Y'} = \sum_{n=0} C_{\Psi' \Phi}^{(n)} \left( \frac{\Sigma'}{M_s} \right)^n \Psi' F H'^\dagger + \sum_{n=0} C_{\Psi' F'}^{(n)} \left( \frac{\Sigma'}{M_s} \right)^n \Psi' F' H'^\dagger + \text{h.c.}$$

$$L_{Y'}^{\text{mix}} = \lambda_{F' F'} F \Phi F' + \lambda_{F' F'}^\dagger F^\dagger F' + \frac{\lambda_{\Psi' \Phi'}}{M} (\Phi^\dagger)^2 \Psi' + \frac{\lambda_{\Psi' \Phi'}}{M} \Psi' \Phi^2 \Psi' ,$$

where $M_s, M$ are some cut off scales. Last two higher order operators in (38), important for phenomenology, can be generated by integrating out some heavy states with mass at or above the GUT scale. For instance, with the scalar state $\Omega$ in $(10, 10)$ representation of $SU(5) \times SU(5)'$ and $D_2$ parity: $\Omega \mapsto \Omega^\dagger$, the relevant terms (of fundamental Lagrangian) will be $\lambda_{\Psi' \Phi} \Omega \Psi \Psi' + \lambda_{\Psi' \Phi}^\dagger \Omega^\dagger \Psi' \Phi + M_{\Omega}(\Omega \Phi^2 + \Omega^\dagger (\Phi^\dagger)^2) + M_{\Omega}^2 \Omega^\dagger \Omega$. With these couplings, one can easily verify that integration of $\Omega$ generates last two operators of Eq. (38) (with $M \approx M_{\Omega}^2 / M_{\Omega}$). Being the $\Omega$ rather heavy, its only low energy implication can be emergence of these effective operators. Thus, in our further studies we will proceed with consideration of Yukawa couplings given in Eqs. (36)-(38).

With obvious identifications, let us adopt the following notations for the components from $\Psi, F$ and $\Psi', F'$ states:

$$\Psi = \{ q, u^c, e^c \} , \quad F = \{ l, d^c \} ,$$

$$\Psi' = \{ \hat{q}, \hat{u}^c, \hat{e}^c \} , \quad F' = \{ \hat{l}, \hat{d}^c \} .$$

Substituting in (36)-(38) the VEVs $\langle \Sigma \rangle, \langle \Sigma' \rangle$ and $\langle \Phi \rangle$, the relevant coupling we obtain are:

$$L_Y \to q^T Y_U u^c h + q^T Y_D d^c h^\dagger + e^c T Y_{e l} l h^\dagger .$$

$^3$The $D_2$ transformation of (34) resembles usual $P$ parity, acting between electron and positron, within QED. Unlike the QED, the states $(\Psi, F)$ and $(\Psi', F')$ transform under different gauge groups.
\( (C_{\theta\theta} q q + C_{\theta\bar{e}} u^c e^c) T_H + (C_{\bar{q}l} q l + C_{\bar{w}d} u^c d^c) T_H^\dagger + \text{h.c.} \) \hspace{1cm} (40) \\
\mathcal{L}_Y \rightarrow C^{(0)*}_{Y_Y} \frac{1}{2} \bar{q} l + \bar{u} c e^c T_H^\dagger + C^{(0)*}_{F_Y} (\bar{q} l + \bar{u} c d^c) T_H^\dagger + \text{h.c.} + \cdots \hspace{1cm} (41) \\
\mathcal{L}^{\text{mix}}_Y \rightarrow \hat{\imath}^T M_{l l} l + e^c T M_{e^c e^c} e^c + \text{h.c.} . \hspace{1cm} (42)

In (41) we have dropped out the couplings with the Higgs doublet, because, as we have assumed \( D_H \) includes SM Higgs doublet with very suppressed weight. Also, we have ignored powers of \( \langle \Sigma \rangle / M_* \) in comparison of \( \langle \Sigma \rangle / M_* \)’s exponents. As we will see, the couplings of \( h \) in (40) and terms shown in (41), (42) are responsible for fermion masses and mixing and lead to realistic phenomenology.

### 3.2 Fermion Masses and Mixings. Composite Leptons.

Let us first indicate transformation properties of all matter states, given in (39), under the unbroken \( G_{SM} \times SU(3)' = SU(3)_c \times SU(2)_w \times U(1)_Y \times SU(3)' \) gauge symmetry. Fragments from \( \Psi, F \) transform as

\[ q \sim (3, 2, -\frac{1}{\sqrt{60}}, 1) \, , \quad u^c \sim (3, 1, \frac{4}{\sqrt{60}}, 1) \, , \quad e^c \sim (1, 1, -\frac{6}{\sqrt{60}}, 1) \]

\[ l \sim (1, 2, \frac{3}{\sqrt{60}}, 1) \, , \quad d^c \sim (3, 1, -\frac{2}{\sqrt{60}}, 1) \] \hspace{1cm} (43)

while the states from \( \Psi', F' \) have the following transformation properties:

\[ \hat{q} \sim (1, 2, \frac{1}{\sqrt{60}}, 3') \, , \quad \hat{u}^c \sim (1, 1, -\frac{4}{\sqrt{60}}, 3') \, , \quad \hat{e}^c \sim (1, 1, \frac{6}{\sqrt{60}}, 1) \]

\[ \hat{l} \sim (1, 2, -\frac{3}{\sqrt{60}}, 1) \, , \quad \hat{d}^c \sim (1, 1, \frac{2}{\sqrt{60}}, 3') \] \hspace{1cm} (44)

In transformation properties of (44), by primes we have indicated triplets and anti-triplets of \( SU(3)' \). As we see, transformation properties of quark states in Eq. (43) coincide with those of the SM. Therefore, for quark masses and CKM mixing generation, first two couplings of (40) are relevant. Since in \( Y_{U,D} \) and \( Y_{e,l} \) contribute also higher dimensional operators, the \( Y_U \) is not symmetric and \( Y_D \neq Y_{e,l} \). Thus, quark Yukawa matrices can be diagonalized by bi-unitary transformations

\[ L^\dagger_u Y_U R_u = Y_U^{\text{Diag}} \, , \quad L^\dagger_d Y_D R_d = Y_D^{\text{Diag}} . \hspace{1cm} (45) \]

with these, the CKM matrix (in standard parametrization) is

\[ V_{\text{CKM}} = P_1 L^T_u L^*_d P_2 \]

with \( P_1 = \text{Diag}(e^{i\omega_1}, e^{i\omega_2}, e^{i\omega_3}) \) , \( P_2 = \text{Diag}(e^{i\rho_1}, e^{i\rho_2}, 1) \). \hspace{1cm} (46)

**Composite Leptons**

Turning to the lepton sector, we note that \( \hat{l} \) and \( \hat{e}^c \) have opposite/conjugate transformation properties with respect to \( l \) and \( e^c \) respectively. From couplings in (42) we see that these states, being vector-like, decouple acquiring masses \( M_{\hat{l}} \) and \( M_{e^c e^c} \). However, within this scenario, composite
leptons emerge. The $SU(3)'$ becomes strongly coupled and confines at scale $\Lambda' \sim \text{TeV}$ (for details see Sect. 4). Due to confinement, $SU(3)'$ singlet composite states - mesons ($M'$) or/and baryons ($B'$) - can emerge. An elegant idea, of fermion emergence through the strong dynamics as bound states of more fundamental constituents, was suggested and developed in Refs. [12]-[22]. Within our scenario, this idea finds interesting realization for the lepton states. Formation of composite states of more fundamental constituents, was suggested and developed in Refs. [12]-[22].

Let us focus on the sector of (three family) $\hat{q}, \hat{u}^c$ and $\hat{d}^c$ states, which have $SU(3)'$ strong interactions. Ignoring local EW and Yukawa interactions, the lagrangian of these states possesses global $G_f^{(6)} = SU(6)_L \times SU(6)_R \times U(1)_{B'}$ chiral symmetry. Under the $SU(6)_L$, three families of $\hat{q} = (\hat{u}, \hat{d})$ transform as sextet $6_L$, while three families of $(\hat{u}^c, \hat{d}^c) \equiv \hat{q}^c$ form sextet $6_R$ of $SU(6)_R$. The $U(1)_{B'}$ $(B')$ charges of $\hat{q}$ and $\hat{q}^c$ are respectively 1/3 and $-1/3$. Thus, transformation properties of these states under $G_f^{(6)}$ chiral symmetry are

$$G_f^{(6)} = SU(6)_L \times SU(6)_R \times U(1)_{B'}$$

(47)

where $\alpha = 1, 2, 3$ is family index. Due to the strong $SU(3)'$ attractive force, the condensates can form which will break the $G_f^{(6)}$ chiral symmetry. The breaking can occur by several steps and at each step the formed composite states should satisfy anomaly matching conditions.

In Appendix A, we give detailed account of these issues and demonstrate that within our scenario, three families of $l_0, e^c_0, \nu^c_0$ composite states:

$$\hat{q} \nu^c \hat{q} \sim l_0 = \left(\begin{array}{c}
\nu_0 \\
e_0
\end{array}\right)_\alpha,$$

(49)

emerge. In (49), for combinations $(\hat{q}\nu^c)\hat{q}$ and $(\hat{q}\nu^c)\hat{q}^c$, the spin-1/2 states are assumed with suppressed gauge and/or flavor indices. For instance, under $(\hat{q}\nu^c)\hat{q}$ we mean $e^{a'b'c'}\epsilon_{ij}(\hat{d}_{a'b'}\hat{g}_{i'j'})\hat{q}_{c'k}$, where $a', b', c' = 1, 2, 3$ are $SU(3)'$ indices and $i, j, k = 1, 2, 3$ stand for $SU(2)_w$ (or $SU(2)_L$) indices. Similar applies to the combination $(\hat{q}\nu^c)\hat{q}^c$. Thus, $(\hat{q}\nu^c)\hat{q}$ and $(\hat{q}^c\hat{q}^c)\hat{q}^c$ are singlets of $SU(3)'$. From these, taking into account Eqs. (44) and (49), it is easy to verify, that the quantum numbers of composite states under SM gauge group $G_{SM} = SU(3)_c \times SU(2)_w \times U(1)_Y$ are

$$G_{SM} : l_0 \sim (1, 2, \frac{3}{\sqrt{60}}), \quad e^c_0 \sim (1, 1, -\frac{6}{\sqrt{60}}), \quad \nu^c_0 \sim (1, 1, 0).$$

(50)

As we see, along with SM leptons we get three families of composite SM singlets fermions - $\nu^c_0$. The latter can be treated as composite right handed/sterile neutrinos in a spirit of Ref. [23]. Note that, with this composition, as was expected, the gauge anomalies also vanish (together with the chiral anomaly matching. For details see Appendix A). Interestingly, the $SU(3)'$ (originating from $SU(5)'$) triplet and anti-triplets $\hat{u}^c, \hat{d}^c$ and $\hat{q}$ play role of 'preons' constituents for the bound state leptons and RH/sterile neutrinos. Moreover, in our scheme the lepton number $L$ is related to the

---

4In case the chiral symmetry remains unbroken (at least partially) at the composite level. The models avoiding anomaly conditions were suggested in [19].
in a family space. The (obtain:

where Greek indices denote family indices and composite charged lepton states. We will use the parameterization $s$

Substituting these in $e$

M

we get

the leptons are generated:

As we see from (41), the matter of $SU(3)$

triplet scalar $T_{H'}$ with mass $M_{T_{H'}}$. Relevant four-fermion operators, emerging from the couplings of Eq. (41) and by integration of $T_{H'}$, are:

$$
\mathcal{L}_{Yeff}^{51} = \frac{C_{\Psi \Psi}^{(0)*} C_{\Psi F}^{(0)*}}{M_{H'}^2} \left[ \frac{1}{2} (\bar{q} \bar{q})(\hat{q} \hat{l}) + (\bar{u}^c \bar{e}^c)(\hat{u}^c \hat{e}^c) \right] + \text{h.c.}
$$

As we see, here appeared the combinations $(\bar{q} \bar{q})\hat{q}$ and $(\bar{u}^c \bar{e}^c)\hat{u}^c$, which according to (49) form composite charged lepton states. We will use the parameterizations

$$
\frac{1}{2} (\bar{q} \bar{q})\hat{q} = \Lambda'^c_{\alpha \beta \gamma \delta} l_0 \delta , \quad (\bar{u}^c \bar{e}^c)\hat{u}^c = \Lambda'^c_{\alpha \beta \gamma \delta} e_0 \delta
$$

where Greek indices denote family indices and $c, \bar{c}$ are dimensionless couplings - four index tensors in a family space. The $(l_0, e_0)\delta$ denote three families of composite leptons. Using (52) in (51) we obtain:

$$
\mathcal{L}_{Yeff}^{51} \rightarrow \hat{\mu} l_0 + e_0^c \hat{\mu} \hat{e}^c + \text{h.c.}
$$

with

$$
\hat{\mu}_{\delta \gamma} \equiv \frac{\Lambda'^3}{M_{H'}^2} (C_{\Psi \Psi}^{(0)*})_{\alpha \beta} (C_{\Psi F}^{(0)*})_{\gamma \delta} c_{\alpha \beta \gamma \delta} ; \quad \hat{\mu}_{\delta \gamma} \equiv \frac{\Lambda'^3}{M_{H'}^2} (C_{\Psi \Psi}^{(0)*})_{\gamma \delta} (C_{\Psi F}^{(0)*})_{\alpha \beta \gamma \delta} .
$$

At next stage, we integrate out the vector like states $\hat{l}, \hat{e}, \hat{\mu}$ which respectively receive masses $M_{\hat{l}}$ and $M_{\hat{e}}, M_{\hat{\mu}}$ through the coupling in Eq. (42). Integrating out these heavy states, from (42) and (53) we get

$$
l \simeq - \frac{1}{M_{\hat{l}}} \hat{\mu} l_0 , \quad e^{cT} \simeq - e_0^{cT} \hat{\mu} \frac{1}{M_{\hat{e}^c}} .
$$

Substituting these in $e^{cT} Y_{e \hat{l}} l h^\dagger$ coupling of Eq. (40), we see that the effective Yukawa coupling for the leptons are generated:

$$
\bar{l}_0^T Y_E e_0^c h^\dagger + \text{h.c.} \quad \text{with} \quad Y_E^T \simeq \hat{\mu} \frac{1}{M_{\hat{e}^c}} Y_{e l} \frac{1}{M_{\hat{l}}} \hat{\mu} .
$$

$U(1)_{B'}$ charge as $L = 3B'$. Therefore, 'primed baryon number' $B'$ (of the $SU(5)'$) is the origin of the lepton number.

**Charged Lepton Masses**

Now we turn to the masses of the charged leptons, which are composite within our scenario. As it turn out, their mass generation does not require additional extension. It happen via integration of the states which present in the model. As we see from (41), the matter of $SU(5)'$ couples with $SU(3)'$ triplet scalar $T_{H'}$ with mass $M_{T_{H'}}$. Relevant four-fermion operators, emerging from the couplings of Eq. (41) and by integration of $T_{H'}$, are:

$$
\mathcal{L}_{Yeff}^{51} = \frac{C_{\Psi \Psi}^{(0)*} C_{\Psi F}^{(0)*}}{M_{H'}^2} \left[ \frac{1}{2} (\bar{q} \bar{q})(\hat{q} \hat{l}) + (\bar{u}^c \bar{e}^c)(\hat{u}^c \hat{e}^c) \right] + \text{h.c.}
$$

Figure 1: Diagram responsible for generation of charged lepton effective Yukawa coupling.
The diagram corresponding to the generation of this effective Yukawa operator is shown in Fig. 1. This mechanism is novel and differs from those suggested earlier for the mass generation of composite fermions [22]. From the observed values of the Yukawa couplings we have $|\det Y_E| = \lambda_\alpha \lambda_\mu \lambda_\tau \approx 1.8 \cdot 10^{-11}$. On the other hand, natural values of the eigenvalues of $Y_{e\ell}$ can be $\sim 0.1$. Thus, $|\det Y_{e\ell}| \sim 10^{-3}$. From these and expression given in Eq. (55) we obtain

$$|\det(\tilde{\mu} \frac{1}{M_{ee}})| \cdot |\det(\frac{1}{M_{\tilde{\mu}}})| \sim 10^{-8} ,$$

-the constraint which should be satisfied by two matrices $\tilde{\mu} \frac{1}{M_{ee}}$ and $\frac{1}{M_{\tilde{\mu}}}$.

Neutrino Masses

Now we discuss the neutrino mass generation within our scenario. In order to accommodate the neutrino data [4], one can utilize SM singlet fermionic states, in order to generate either Majorana or Dirac type masses for the neutrinos. Within our model, among composite fermions we have SM singlets $\nu^c_0$ [see Eqs. (49), (50)]. Here we stick on possibility of the Dirac type neutrino masses, which can be naturally suppressed [23]. Because of compositeness, there is no direct Dirac couplings $Y_{\nu}$ of $\nu^c_0$'s with lepton doublets $l_0$. Similar to the charged lepton Yukawa couplings, we need to generate $Y_{\nu}$. For this purpose, we introduce $SU(5) \times SU(5)'$ singlet (two component) fermionic states $N$.

Assigning the $D_2$ parity transformations $N \rightarrow -N$ and taking into account (5), (34), relevant couplings, allowed by $SU(5) \times SU(5)' \times D_2$ symmetry, will be:

$$L_N = C_{FN} F N H + C_{FN}^* F^t N H^t - \frac{1}{2} N^T M_N N + \text{h.c.} \quad \text{with} \quad M_N = M_N^* .$$

These give the following interaction terms:

$$L_N \rightarrow C_{FN} t N h + C_{FN}^* \hat{d}^c N T_{H'}^1 - \frac{1}{2} N^T M_N N + \text{h.c.}$$

From these and Eq. (41), integration of $T_{H'}$ state give additional affective four-fermion operator

$$\frac{C_{\Psi F}^* C_{FN}^*}{M_{H'}^2} (\hat{u}^c \hat{d}^c)(\hat{d}^c N) + \text{h.c.}$$

By the parametrization

$$(\hat{u}^c_{\alpha} \hat{d}^c_{\beta}) \hat{d}^c_{\gamma} = N^{\alpha\beta} \hat{e}_{\alpha \beta \gamma \delta} \nu^c_{\delta} ;$$

operators in Eq. (59) are given by

$$L_N^{eff} = N \mu_{\nu} \nu^c_{\delta} + \text{h.c.} \quad \text{with} \quad (\mu_{\nu})_{\delta \delta} \equiv \frac{N^{\alpha \beta} (C_{\Psi F}^* \alpha \beta (C_{FN}^* \gamma \delta \epsilon_{\alpha \beta \gamma \delta} .$$

Subsequent integration of $N$ states, from (61) and last term of (58), gives:

$$N \approx \frac{1}{M_N} \mu_{\nu} \nu^c_{\delta} .$$

\footnote{Number of $N$ states is not limited, but for simplicity we can assume that they are not more than three.}
4 Gauge Coupling Unification

In this section we will study the gauge coupling unification within our model. We show that the symmetry breaking pattern gives possibility for successful unification. As will turn out, the $SU(3)'$ gauge interaction becomes strongly coupled at scale $\Lambda'(\sim$ few TeV). Thus, below this scale, $SU(3)'$ confines and all states (including composite ones) are $SU(3)'$ singlets. Therefore, with the masses $M_{l}^{(\alpha)}$ and $M_{e_{e}}^{(\alpha)} (\alpha = 1, 2, 3)$ of vector-like states $l, \hat{l}$ and $e, \hat{e}$ being above the scale $\Lambda'$, in the energy interval $\mu = M_{Z} - \Lambda'$ the states are just those of SM (plus possibly RHN/sterile neutrinos having no impact on gauge coupling running) and corresponding 1-loop $\beta$-function coefficients are $(b_{y}, b_{w}, b_{c}) = \left(\frac{41}{10}, \frac{-19}{6}, -7\right)$. In the energy interval $\Lambda' - M_{I}$ we have the symmetry $SU(3)_{c} \times SU(2)_{w} \times U(1)_{Y} \times SU(3)'$, and $SU(3)'$ non singlet states (i.e. $\hat{q}, \hat{u}, \hat{c}, T_{H^{c}}$ etc.) must be taken into account. As was noted in Sect. 2, we consider hierarchical breaking: $M_{I} \ll M_{I}' \ll M_{G}$ (see Eqs. (17) and (18)). This choice allows to have successful unification with confining scale $\Lambda' \sim$ few TeV. Thus, between the scales $M_{I}$ and $M_{I}'$ the symmetry is $G_{321} \times G_{321}'$ (see Eqs. (9) and (10)).

Substituting this, and expression of $l$ from Eq. (54), in first term of (58), we arrive at:

\[ l_{0}^{T} Y_{\nu}^{\mu} \nu_{0}^{\mu} h + \text{h.c.} \quad \text{with} \quad Y_{\nu} \simeq -\hat{\mu}_{l}^{T} \frac{1}{M_{I}^{2}} C_{F,N} \frac{1}{M_{N}} \mu_{\nu}. \quad (63)\]

The relevant diagram generating this effective Dirac Yukawa coupling is given in Fig. 2. With $1/\hat{\mu}_{l} \sim 10^{-2}$ and $C_{F,N} \sim M_{N} \sim 1/M_{N} \mu_{\nu} \sim 10^{-5}$ we can get the Dirac neutrino mass $M_{\nu}^{D} = Y_{\nu} \langle h(0) \rangle \sim 0.1 \text{ eV}$, which is right scale to explain neutrino anomalies. Note, using (62) in the last term of Eq. (58), we also obtain term $-\frac{1}{2} l_{0}^{T} M_{\nu}^{D} \nu_{0}^{\mu}$ with $M_{\nu}^{D} \simeq \mu_{\nu}^{2} M_{\nu}^{D}/M_{N} \mu_{\nu}$. By proper selection of the couplings $C_{F,N}$ and eigenvalues of $M_{N}$, the $M_{\nu}^{D}$ can be strongly suppressed. In this case, the neutrinos will be (quasi)Dirac. However, it is possible that some of the species of light neutrinos to be (quasi)Dirac, and some of them - Majorana’s. Detailed studies of such scenarios and their compatibilities with current experiments [24] are beyond the scope of this paper.

Figure 2: Diagram responsible for generation of effective Dirac Yukawa coupling for neutrinos.
Finally, at the GUT scale details.

With solutions (B.5), (B.6) of RG equations at corresponding energy scales, and taking into account

\[ \alpha_1^{-1}(M_I) = \alpha_1^{-1}(M_I) + \alpha_1^{-1}(M_I) , \quad \alpha_2^{-1}(M_I) = \alpha_2^{-1}(M_I) + \alpha_2^{-1}(M_I) . \]

The couplings of $G_{321}'$ gauge interactions unify and form single $SU(5)'$ coupling at scale $M_I'$:

\[ \alpha_1(M_I') = \alpha_2(M_I') = \alpha_3(M_I') = \alpha_5(M_I') . \]

Finally, at the GUT scale $M_G$, the coupling of $G_{321}$ and $SU(5)'$ unify:

\[ \alpha_1(M_G) = \alpha_2(M_G) = \alpha_3(M_G) = \alpha_5(M_G) \equiv \alpha_G . \]

With solutions (B.5), (B.6) of RG equations at corresponding energy scales, and taking into account

the boundary conditions (64)-(66), we derive:

\[
\begin{pmatrix}
(b_1^{IG} - b_2^{IG} + b_3^{IG}), & (b_1^{IG} - b_2^{IG} + b_3^{IG}), & -2\pi \\
(b_2^{IG} - b_3^{IG} + b_4^{IG}), & (b_2^{IG} - b_3^{IG} + b_4^{IG}), & -2\pi \\
(b_3^{IG} - b_4^{IG}), & (b_3^{IG} - b_4^{IG}), & 0 \\
(b_4^{IG} - b_3^{IG}), & (b_4^{IG} - b_3^{IG}), & -2\pi
\end{pmatrix}
\begin{pmatrix}
\ln \frac{M_I}{M_Z} \\
\ln \frac{M_G}{M_Z} \\
\ln \frac{M'_I}{M_L'} \\
\ln \frac{M'_G}{M'_L'}
\end{pmatrix}
= \begin{pmatrix}
2\pi(\alpha_3^{-1}(\Lambda') - \alpha_1^{-1}) + b_3^{IG} \ln \frac{\Lambda'}{M_Z} \\
2\pi(\alpha_3^{-1}(\Lambda') - \alpha_1^{-1}) + b_3^{IG} \ln \frac{\Lambda'}{M_Z} \\
-2\pi \alpha_3^{-1} \\
-2\pi \alpha_3^{-1}(\Lambda') - b_3^{IG} \ln \frac{\Lambda'}{M_Z}
\end{pmatrix},
\]

where at right hand side of this equation, the couplings $\alpha_{Y,w,e}$ are taken at scale $M_Z$. The factors $
^{b_4^{IG}} \mu^{b_4^{IG}}$, (like $b_3^{IG}$, $b_3^{IG}$ etc.) stand for effective $b$-factors corresponding to the energy interval $\mu_a - \mu_b$
and can also include 2-loop effects. All expressions and details are given in Appendix B.
Figure 3: Gauge coupling unification. $\{\Lambda', M_I, M_I', M_G\} \simeq \{1851, 8.25 \cdot 10^4, 4.16 \cdot 10^6, 4.95 \cdot 10^{11}\} \text{GeV}$ and $\alpha_G(M_G) \simeq 1/31$.

From Eq. (67) we can calculate $\{M_I, M_G, M_I', \alpha_G\}$ in terms of remaining inputs. For instance, phenomenologically viable scenario is obtained when $SU(3)'$ confines at scale $\Lambda' \sim 1 \text{ TeV}$. Thus, we will take $\Lambda' \sim 1 \text{ TeV}$ and $\alpha^{-1}_y(\Lambda') \ll 1$. In Table 1 we give selected input mass scales, leading to successful unification with

$$\{M_I, M_I', M_G\} \simeq \{8.25 \cdot 10^4, 4.16 \cdot 10^6, 4.95 \cdot 10^{11}\} \text{GeV}, \quad \alpha_G \simeq 1/31.$$  \hspace{1cm} (68)

The corresponding picture of gauge coupling running is given in Fig. 3. This result is obtained by solving RGs in 2-loop approximation. More details, including 1 and 2-loop RG factors at each relevant mass scales, are given in Appendix B.

### 5 Nucleon Stability

In this section we show that, although the GUT scale $M_G$ is relatively low (close to $5 \cdot 10^{11} \text{ GeV}$), the nucleon’s life time can be compatible with current experimental bounds. In achieving this, crucial role is played by lepton compositeness, because leptons have no direct couplings with $X, Y$ gauge bosons of $SU(5)$. The baryon number violating $d = 6$ operators, induced by integrating out of the $X, Y$ bosons, are

$$\frac{g_X^2}{M_X^2}(\overline{u}^c_\alpha \gamma_\mu q^i_\beta)(\overline{d}^c \gamma^\mu l^i_j)\epsilon^{abc} \epsilon_{ij},$$

$$\frac{g_X^2}{M_X^2}(\overline{u}^c_\alpha \gamma_\mu q^i_0)(\overline{e}^c \gamma^\mu q^c_\alpha)\epsilon^{abc} \epsilon_{ij},$$

where $g_X$ is the $SU(5)$ gauge coupling at scale $M_X$ (the mass of $X, Y$ states). According to Eq. (54), the states $l, e^c$ contain light leptons $l_0, e^c_0$. Using this and going to mass eigenstate basis (with Eqs. (45), (46)), from (69) we get operators:

$$O^{(e)}_{d\bar{d}} = \frac{g_X^2}{M_X^2} C_{\alpha\beta}^{(e)} \overline{u}^c_\alpha \gamma_\mu u^\beta(\overline{e}^c_\alpha \gamma^\mu d^i_\beta),$$

$$O^{(e)}_{d\bar{d}} = \frac{g_X^2}{M_X^2} C_{\alpha\beta}^{(e)} \overline{u}^c_\alpha \gamma_\mu u^\beta(\overline{e}^c_\alpha \gamma^\mu e^c_\beta),$$
\[
O^{(v)}_{d6} = \frac{g_X^2}{M_X^2} C^{(\nu)}_{\alpha \beta \gamma} (\bar{u}^\gamma \gamma_\mu d_\alpha) (\bar{d}^\beta \gamma^\mu \nu_\gamma) .
\]  

with

\[
C^{(e)}_{\alpha \beta} = (R^t_u L^*_u)_{11} (R^t_d \mu^* \frac{1}{M^* e^e} L^*_u P^*_{1VCKM})_{\alpha \beta} + (R^t_u L^*_u P^*_{1VCKM})_{1 \beta} (R^t_d \mu^* \frac{1}{M^* e^e} L^*_u)_{\alpha 1} ,
\]

\[
C^{(e)}_{\alpha \beta} = (R^t_u L^*_u)_{11} (R^t_d \frac{1}{M^*_e} \mu^* \bar{L}^*_e)_{\beta \alpha} ,
\]

\[
C^{(\nu)}_{\alpha \beta \gamma} = (R^t_u L^*_u P^*_{1VCKM})_{1 \alpha} (R^t_d \frac{1}{M^*_e} \mu^* \bar{L}^*_e)_{\beta \gamma} ,
\]

where in (70) we have suppressed the color indices. Similar to quark Yukawa matrices, the charged lepton Yukawa matrix has been diagonalized by transformation \( L^*_e Y_E R = Y_E \text{Diag} \). All fields in (70), are assumed to denote mass eigenstates. We have ignored the neutrino masses (having no relevance for the nucleon decay) and rotated the neutrino flavors \( \nu_0 \equiv L^*_e \nu \) similar to the left handed charged leptons \( e_0 = L^*_e e \).

As we will show now, with proper selection of appropriate parameters (such as \( \frac{1}{M^*_e} \mu^* \bar{L}^*_e \) and/or corresponding entries in some of unitary matrices), appearing in (71), we can adequately suppress nucleon decays within our model.\(^8\) Upon the selection of parameters, the constraint (56) must be satisfied in order to obtain observed values of charged fermion masses. Introducing the notations

\[
R^t_u L^*_u \equiv \mathbf{U} , \quad R^t_d \frac{1}{M^*_e} \mu^* \bar{L}^*_e \equiv \mathbf{L} , \quad R^t_d \mu^* \frac{1}{M^*_e} L^*_u \equiv \mathbf{R} ,
\]

the couplings in (71) can be rewritten as

\[
C^{(e)}_{\alpha \beta} = \mathbf{U}_{11} (\mathbf{R} P^*_{1VCKM})_{\alpha \beta} + (\mathbf{U} P^*_{1VCKM})_{1 \beta} (\mathbf{R})_{\alpha 1} ,
\]

\[
C^{(e)}_{\alpha \beta} = \mathbf{U}_{11} \mathbf{L}_{\beta \alpha} , \quad C^{(\nu)}_{\alpha \beta \gamma} = (\mathbf{U} P^*_{1VCKM})_{1 \alpha} \mathbf{L}_{\beta \gamma} .
\]

Since the matrices \( \mathbf{U}, \mathbf{L} \) and \( \mathbf{R} \) are not fixed yet, for their structures we will make the following selection:

\[
\mathbf{U}_{11} = 0 , \quad \mathbf{L} = \begin{pmatrix} \epsilon_1 & \epsilon_2 & \epsilon_3 \\ \times & \times & \times \\ \times & \times & \times \end{pmatrix}, \quad \mathbf{R} = \begin{pmatrix} 0 & \times & \times \\ 0 & \times & \times \\ \times & \times & \times \end{pmatrix},
\]

where \( \times \) stands for some non zero entry. With this structure, we see that for \( \alpha, \beta = 1, 2 \) we have \( C^{(e)}_{\alpha \beta} = C^{(e)}_{\beta \alpha} = 0 \), and therefore, nucleon decays with emission of the charged leptons do not take place. With one more selection we will be able to eliminate some nucleon decay modes (but not all) with neutrino emissions. We can impose one more condition, involving \( \mathbf{U}_{12} \) and \( \mathbf{U}_{13} \) entries of \( \mathbf{U} \), in such a way to have \((\mathbf{U} P^*_{1VCKM})_{11} = 0 \). The latter, in expanded form, reads:

\[
(\mathbf{U} P^*_{1VCKM})_{11} = \mathbf{U}_{12} e^{-i \omega_2} V_{cd} + \mathbf{U}_{13} e^{-i \omega_3} V_{td} = 0 , \quad \Rightarrow \mathbf{U}_{12} e^{-i \omega_2} = -\frac{V_{cd}}{V_{td}} \mathbf{U}_{13} e^{-i \omega_3}
\]

\(^8\)Importance of flavor dependence in \( d = 6 \) nucleon decay was discussed in Refs. [26], [27]. As was shown [27], in specific circumstances, within GUTs one can suppress or even completely rotate away the \( d = 6 \) nucleon decays.
and leads to $C_{127}^{(v)} = C_{117}^{(v)} = 0$. Thus, the decays $p \to \bar{v} \pi^+, n \to \bar{v} \pi^0, n \to \bar{v} \eta$ do not take place. Non-vanishing relevant $C^{(v)}$-couplings are $C_{217}^{(v)}$, which taking into account (74) and (75) are

$$C_{217}^{(v)} = (U P^*_1 V_{CKM})_{12} e_7 = \epsilon_7 U_{13} e^{-i\omega_3} \frac{V_{ts} V_{td} - V_{td} V_{cs}}{V_{cd}} \simeq \epsilon_7 U_{13} e^{-i\omega_3} \frac{s_{13} c_{13}}{V_{cd}},$$

where in last step we have used standard parametrization of the CKM matrix. Since the matrix $U$ is unitary, due to selection $U_{11} = 0$ and unitarity condition we will have $|U_{12}|^2 + |U_{13}|^2 = 1$. With this, by Eq. (75) and using central values [28] of CKM matrix elements, we obtain $|U_{12}| \simeq 0.038, |U_{13}| \simeq 1$ and $|\Delta M_{13}| = |\Delta m_{13}| \simeq 1.56 \cdot 10^{-2}$. These give $|C_{217}^{(v)}| \simeq 1.56 \cdot 10^{-2} |\epsilon_7|$. Taking into account all this, for expressions of $p \to \bar{v} K^+$ and $n \to \bar{v} K^0$ decay widths we obtain [29]:

$$\Gamma(p \to \bar{v} K^+) \simeq \Gamma(n \to \bar{v} K^0) = \frac{(m_p^2 - m_K^2)^2}{32 \pi f^2 m_p^3} \left(1 + \frac{m_p}{3 m_B} (D + 3 F)\right)^2 \left(\frac{g_X}{M_X^2} A_R |\alpha_H|\right)^2 \cdot 2.43 \cdot 10^{-4} \sum_{\gamma=1}^3 |\epsilon_\gamma|^2$$

where $|\alpha_H| = 0.012$ GeV$^3$ is a hadronic matrix element and $A_R = A_L A_S \simeq 1.48$ takes into account long ($A_L \simeq 1.25$) and short ($A_S \simeq 1.18$) distance renormalization effects (see [30] and [31] respectively. Some details of calculation of $A_S$, within our model, are given in Appendix B.1). In order to satisfy current experimental bound $\tau^{\exp}_{\mu} (p \to \bar{v} K^+) \lesssim 5.9 \cdot 10^{33}$ years [32], for $M_X \simeq 5 \cdot 10^{11}$ GeV and $\alpha_X \simeq 1/31$, we need to have $\sqrt{|\epsilon_1|^2 + |\epsilon_3|^2 + |\epsilon_3|^2} \lesssim 4.8 \cdot 10^{-6}$. This selection of parameters is fully consistent with the charged fermion masses. Note, that with (74) there is no conflict with constraint of Eq. (56). We can lower values of $|\epsilon_\gamma|$, however, there is low bound dictated from this constraint. With $|\text{Det}(\mu | - C_{\nu e})) | \cdot |\text{Det}(\mu | - C_{\nu e})) | = |\text{Det}(\mathcal{L})| \cdot |\text{Det}(\mathcal{R})| \sim 10^{-8}$ lowest value can be $|\epsilon_\gamma| \sim 10^{-8}$, obtained with $|\text{Det}(\mathcal{R})| \sim 1$. More natural would be to have $|\text{Det}(\mathcal{R})| \lesssim 10^{-2}$ which suggest $|\text{Det}(\mathcal{L})| \gtrsim 10^{-6}$, and therefore $\sqrt{|\epsilon_1|^2 + |\epsilon_3|^2 + |\epsilon_3|^2} \gtrsim 3 \cdot 10^{-6}$. This dictates an upper bound for the proton lifetime $\tau_p = \tau(p \to \bar{v} K^+) \lesssim 5 \cdot 10^{34}$ years and allows to test the model in a future [32].

Besides $X, Y$ gauge boson mediated operators, there are $d = 6$ operators generated by exchange of colored triplet scalar $T_H$. From the couplings of Eq. (40), we can see that integration of $T_H$ induces baryon number violating $\frac{1}{M_{T_H}} (qT C_{qq} q)(qT C_{ql})$ and $\frac{1}{M_{T_H}} (u^c C_{u^c e e} e^c)(u^c C_{u^c d} d^c)$ operators, which lead to the couplings $\frac{1}{M_{T_H}} (qT C_{qq} q)(qT C_{ql} \frac{1}{M_{T_H}} \mu l_0)$ and $\frac{1}{M_{T_H}} (u^c C_{u^c e e} \frac{1}{M_{T_H}} \mu^T e_0)(u^c C_{u^c d} d^c)$. Couplings $C_{ab}$ appearing in these operators are independent from Yukawa matrices and proper suppression of relevant terms are possible (similar to the case of couplings in Eq. (73)), leaving fermion masses and mixing pattern consistent with experiments. In order to make more definite statement about the nucleon lifetime, one has to study in detail the structure of fermion mass matrices. In this respect, extension with flavor symmetries is motivated framework and can play crucial role in generating the desirable Yukawa textures (guaranteeing the forms given in Eq. (74)). However, this study is beyond the scope of this paper.

6 Various Phenomenological Constraints and Implications

In this section we discuss and summarize some peculiarities, phenomenological implications of our model and constraints needed to be satisfied in order to be consistent with experiments. Also we list issues opening prospects for further investigations within presented scenario.
(i) The discovery of the Higgs boson [1], with mass \(\approx 126\) GeV, revealed that the Standard Model suffers from vacuum instability. Detailed analysis have shown [2], that due to RG, the Higgs self coupling becomes negative near the scale \(\sim 10^{10}\) GeV. Since within our model, above \(\Lambda'\) scale new states appear, this problem can be avoided. As was mentioned in Sect. 2, in our model light SM doublet \(h\) dominantly comes from \(H\)-plet. The coupling \(\lambda_H(H^+H)^2\) gives the self interaction term \(\lambda_h(h^+h)^2\) (with \(\lambda_h \approx \lambda_H\) at GUT scale). The running of \(\lambda_h\) will be given by

\[
16\pi^2 \frac{d}{dt} \lambda_h = \beta_{\lambda_h}^{SM} + \Delta \beta_{\lambda_h},
\]

where \(\beta_{\lambda_h}^{SM}\) corresponds to the SM part, while \(\Delta \beta_{\lambda_h}\) accounts for new contributions. Since the \(H\)-plet in the potential (7) has additional interaction terms, some of those couplings can help to increase \(\lambda_h\). For instance, the couplings \(\lambda_{1H\Phi}, \lambda_{2H\Phi}, \hat{h}\) etc. contribute as

\[
\Delta \beta_{\lambda_h} \approx \frac{\lambda_{1H\Phi}^2}{25} \left[ 9\theta(\mu - M_{T\Phi}) + 6\theta(\mu - M_{DT}) + 6\theta(\mu - M_{DD'}) + 4\theta(\mu - M_{D\Phi}) \right]
\]

\[
+ \frac{\lambda_{2H\Phi}^2}{10} \left[ 3\theta(\mu - M_{DD'}) + 2\theta(\mu - M_{D\Phi}) \right] + 3\hat{h}^2\theta(\mu - M_{H}) + \cdots
\]

Detailed analysis require numerical studies by solving the system of coupled RG equations (involving multiple couplings\(^9\)). While this is beyond the scope of this work, we see that due to positive contributions (see above) into the \(\beta\)-function, there is potential to prevent \(\lambda_h\) becoming negative all the way up to the Planck scale.

(ii) Since in our model leptons are composite, there will be additional contributions to their anomalous magnetic moment, given by [15]:

\[
\delta a_\alpha \sim \left( \frac{m_{e\alpha}}{\Lambda'} \right)^2.
\]

Current experimental measurements [28] of muon’s anomalous magnetic moment give \(\delta a_\mu^{exp} \approx 6 \cdot 10^{-10}\). This, having in mind possible range \(\sim (1/5 - 1)\) of undetermined pre-factor in expression of Eq. (78), constrains the scale \(\Lambda'\) from below: \(\Lambda' \lesssim (1.8 - 4.3)\) TeV. The selected value of \(\Lambda'\), within our model \((\Lambda' = 1851\) GeV), fits well with this bound.\(^10\) The value of \(\delta a_\alpha\) is more suppressed (for \(\Lambda' \approx 1.8\) TeV we get \(\delta a_\alpha \sim 10^{-13}\)) and is compatible with experiments \(\delta a_\mu^{exp} \approx 2.7 \cdot 10^{-13}\). Planned experiments [35] with reduced uncertainties will provide severe constraints and test viability of the proposed scenario.

Similarly, having flavor violating couplings at the level of constituents (i.e. in the sector of \(SU(3)'\) fermions \(\hat{q}, \hat{u}', \hat{d}'\)), the new contribution in \(e_\alpha \to e\beta\gamma\) rare decay processes will emerge. For instance, contribution in \(\mu \to e\gamma\) transition amplitude will be \(\sim \lambda_{12}\left(\frac{m_\mu}{\Lambda'}\right)^2\), where \(\lambda_{12}\) is (unknown) flavor violating coupling coming from the Yukawa sector of \(\hat{q}, \hat{u}', \hat{d}'\). This gives \(Br(\mu \to e\gamma) \sim \lambda_{12}^2\left(\frac{M_\mu}{\Lambda'}\right)^4\) and, for \(\Lambda' \approx 1.8\) TeV, the constraint \(\lambda_{12} \lesssim 4 \cdot 10^{-4}\) should be satisfied in order to be consistent with the latest experimental limit \(Br^{exp}(\mu \to e\gamma) < 5.7 \cdot 10^{-13}\) [36].

\(^9\)For methods studying stability of multi field potentials see [3], [33] and references therein.

\(^10\)In fact, this new contribution to \(a_\mu\) has potential of resolving 3-4\(\sigma\) discrepancy [28] (if it will persist in future) between theory and experiment [34].
(iii) As was mentioned in Sect. 3.2 (and will be discussed also in Appendix A), the matter sector of $SU(3)'$ symmetry (ignoring EW and Yukawa interactions) possesses $G_f^{(6)}$ chiral symmetry with sextets $6_L \sim \hat{q}_a$ and $6_R \sim \hat{q}_a'$ (see Eqs. (47) and (48)). The breaking of this chiral symmetry proceeds by several steps. At first stage, at scale $\Lambda' \approx 1.8$ TeV, the condensates $\langle 6_L 6_L T_{H'} \rangle \sim \langle 6_R 6_R T_{H'} \rangle \sim \Lambda'$ break the $G_f^{(6)}$. However, these condensates preserve SM gauge symmetry. At next stage (of chiral symmetry breaking), the condensates $\langle 6_L 6_R \rangle \equiv F_{\pi'}$ together with the Higgs VEV $\langle h \rangle \equiv v_h$, contribute to the EW symmetry breaking. The $F_{\pi'}$ denotes decay constant of (techni) $\pi'$ meson and should satisfy $v_h^2 + F_{\pi'}^2 = (246.2 \text{ GeV})^2$. With the lightest (very SM like) Higgs boson mainly residing in $h$ and with $F_{\pi'} \lesssim 0.2 v_h$, the $h$’s signal will be well compatible with LHC data [37]. Since the low energy potential would involve VEVs $\langle 6_L 6_L T_{H'} \rangle, \langle 6_R 6_R T_{H'} \rangle, F_{\pi'}$ and $v_h$, obtaining mild hierarchy $\frac{F_{\pi'}}{\Lambda} \lesssim 1/40$ will be possible by proper selection (not by severe fine tunings) of parameters from perturbative and non perturbative (effective) potentials. Situation here (i.e. symmetry breaking pattern, potential (being quite involved because of these VEVs) etc.) will differ from case obtained within QCD with $SU(n)_L \times SU(n)_R$ chiral symmetry and with $\langle n_L \times n_R \rangle$ condensate only [38]. Moreover, the hierarchy between the confinement scale and the decay constant can have some dynamical origin (see e.g. Refs. [11] [39]). Without addressing these details, our approach was rather phenomenological, with assumption $F_{\pi'}/v_h \lesssim 0.2$ and $h$ being Higgs boson (with mass $\approx 126$ GeV), such that there is allowed window for heavier $\pi'$ state such that model is compatible with current experiments [41]. Models with partially composite Higgs, in which light Higgs doublet has some ed-mixture of composite (techni-pion $\pi'$) state, with various interesting implications (including necessary constraints, limits and compatibility with LHC data) were studied in [37]. In addition, it is rather generic that, the model with composite leptons will be accompanied with excited massive leptons (lepton resonances). Current experiments have placed low bounds on masses of excited electron and muon to be heavier than $\sim 1.8$ TeV. This scale is close to the value of $\Lambda'$ we have chosen within our model, and allow to test lepton substructure [42] hopefully in not far future. More details, related to these issues, deserve separate investigations.

(iv) Since the condensate $\langle 6_L 6_R \rangle = F_{\pi'}$, by some amount, can contribute to the chiral (of $SU(3)'$ strong sector) and EW symmetry breaking, the scenario shares some properties of hybrid technicolor models with fundamental Higgs states. Moreover, together with techni-pion $\pi'$, near the $\Lambda'$ scale there will be techni meson states $\rho_T, \omega_T$, etc, with peculiar signatures [43], [44], which can be probed by collider experiments.

Finally, it would be interesting to build supersymmetric extension of the considered $SU(5) \times SU(5)' \times D_2$ GUT and study related phenomenology. These and related issues will be addressed elsewhere.

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[11] If conformal window is realized, the value of $F_{\pi'}$ can be more reduced [40].
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## A Composite Leptons and Anomaly Matching

Here we demonstrate how the composite leptons emerge within our scenario, and also discuss anomaly matching conditions. As was noted in Sect. 3.2, the sector of $\tilde{q}, \tilde{u}^c$ and $\tilde{d}^c$ states have $G_f^{(6)}$ chiral symmetry (see Eq. (47)) with the transformation properties of these states given in Eq. (48). At scale $SU(3)'$ interaction becomes strong, the $G_f^{(6)}$ symmetry breaking condensates can be formed. The chiral symmetry breaking can proceed through several steps, and at each level the formed composite states should satisfy anomaly matching conditions [14].

The bi-linear ($SU(3)'$ invariant) condensate can be $\langle 6_L \times 6_R \rangle = F_{\pi'}$, with corresponding breaking scale $F_{\pi'}$. As was shown in [38], with only fundamental states, the chiral symmetry $SU(n)_L \times SU(n)_R$ will be broken down to the diagonal $SU(n)_{L+R}$ symmetry. Since in our case $F_{\pi'}$ also contributes to EW symmetry breaking, we have a bound $F_{\pi'} \lesssim 100 \text{ GeV}$. This scale, in comparison with $\Lambda' \sim \text{few}\times \text{TeV}$, can be ignored at first stage. Moreover, in our case the light $SU(3)'$ non-singlet field content is richer and chiral symmetry breaking pattern is also different. Other $SU(3)'$ invariant condensates, including matter bi-linear, are

$$\langle 6_L 6LT_H' \rangle \quad \text{and} \quad \langle 6_R 6RT_H' \rangle \quad (A.1)$$

Note, that product of $SU(6)$ sextets give either symmetric or antisymmetric representations $(6 \times 6 = 15_A + 21_S)$, but due to $SU(3)'$ contractions, in (A.1) the antisymmetric 15-plets (i.e. $15_L$ and $15_R$) participate. The condensates (A.1) transform as $15_L$ and $15_R$ under $SU(6)_L$ and $SU(6)_R$ respectively, and therefore break these symmetries. Possible breaking channel is

$$SU(6)_L \to SU(4)_L \times SU(2)'_L \equiv G_L^{(4,2)} \quad \text{and} \quad SU(6)_R \to SU(4)_R \times SU(2)'_R \equiv G_R^{(4,2)}. \quad (A.2)$$

Indeed, with respect to $G_L^{(4,2)}$ and $G_R^{(4,2)}$ the $15_L$ and $15_R$ decompose as

$$SU(6)_L \to G_L^{(4,2)} : \quad 15_L = (1,1)_L + (6,1)_L + (4,2)_L,$n

$$SU(6)_R \to G_R^{(4,2)} : \quad 15_R = (1,1)_R + (6,1)_R + (4,2)_R, \quad (A.3)$$

and the VEVs $\langle (1,1)_L \rangle$ and $\langle (1,1)_R \rangle$ leave $G_L^{(4,2)} \times G_R^{(4,2)}$ chiral symmetry unbroken. The singlet components $\langle (1,1)_L \rangle$ and $\langle (1,1)_R \rangle$ from (A.1) are $\frac{1}{2}(\tilde{q}\tilde{q}T_{H'}) = \langle \tilde{u}\tilde{d}T_{H'} \rangle$ and $\langle \tilde{u}^c\tilde{d}^cT_{H'} \rangle$ combinations, which leave $G_{SM}$ gauge symmetry unbroken. Therefore, the values of these condensates can be $\sim \text{few}\times \text{TeV}(\sim \Lambda')$ without causing any phenomenological difficulties. Thus, as the first stage of the chiral symmetry breaking, we stick to the channel

$$G_f^{(6)} \xrightarrow{\Lambda'} G_L^{(4,2)} \times G_R^{(4,2)} \times U(1)_{B'} \quad (A.4)$$

with

$$\langle 6_L 6LT_H' \rangle = \langle \tilde{u}\tilde{d}T_{H'} \rangle \sim \Lambda', \quad \langle 6_R 6RT_H' \rangle = \langle \tilde{u}^c\tilde{d}^cT_{H'} \rangle \sim \Lambda'. \quad (A.5)$$

The $SU(6)_{L,R}$ sextets under $G_L^{(4,2)}$ are decomposed as $6_L = (4,1)_L + (1,2)_L$ and $6_R = (4,1)_R + (1,2)_R$ respectively. If composite objects are picked up as $(4',1)_{L,R} \subset [(4,1)_{L,R}]^3$ and $(1,2')_{L,R} \subset [1]$.
Then one can easily check out that the anomalies (of initial and composite states) indeed match and (4′, 1)_L,R and (1, 2′)_L,R can be identified with three families of leptons plus three states of RHN/sterile neutrinos. For demonstrating all these, it is more convenient to work in different basis. That would also make simpler to identify composite states.

As it is well known (and in our case turns out more useful), one can describe the SU(6) symmetry (and its representations as well) by its special subgroup (‘S-subgroup’ [45]) SU(3)_f × SU(2) ⊂ SU(6). In our case:

\[
SU(6)_L \supset SU(3)_f L \otimes SU(2)_L , \quad SU(6)_R \supset SU(3)_f R \otimes SU(2)_R .
\]

Under these S-subgroups sextets decompose as:\(^{12}\)

\[
\hat{q}(6)_L = \hat{q}(3, 2)_L , \quad \hat{q}_c(6)_R = \hat{q}_c(3, 2)_R .
\]

In these decompositions, \(\hat{q}\) and \(\hat{q}_c\) can be written as matrices

\[
\hat{q} = \begin{pmatrix} \hat{u} & \hat{c} & \hat{t} \\ \hat{d} & \hat{s} & \hat{b} \end{pmatrix}_{SU(2)_L} \quad \uparrow , \quad \hat{q}_c = \begin{pmatrix} \hat{u}_c & \hat{c}_c & \hat{t}_c \\ \hat{d}_c & \hat{s}_c & \hat{b}_c \end{pmatrix}_{SU(2)_R} \quad \uparrow
\]

where schematically actions of SU(3) and SU(2) rotations are depicted. Therefore transformation properties under the chiral group

\[
G_f^{(3,2)} = SU(3)_f L \otimes SU(2)_L \times SU(3)_f R \otimes SU(2)_R \times U(1)_{B'}
\]

are:

\[
G_f^{(3,2)} : \quad \hat{q} \sim \left( 3_{fL}, 2_L, 1, 1, \frac{1}{3} \right) , \quad \hat{q}_c \sim \left( 1, 1, 3_{fR}, 2_R, -\frac{1}{3} \right) .
\]

Relevant anomalies which does not vanish are:

\[
A \left( [SU(3)_f L]^2 \cdot U(1)_{B'} \right) = -A \left( [SU(3)_f R]^2 \cdot U(1)_{B'} \right) = 1 ,
\]

\[
A \left( [SU(2)_L]^2 \cdot U(1)_{B'} \right) = -A \left( [SU(2)_R]^2 \cdot U(1)_{B'} \right) = \frac{3}{2} .
\]

Anomaly matching condition can be satisfied with the symmetries SU(3)_f L and SU(3)_f R are spontaneously broken down to SU(2)_f L and SU(2)_f R respectively. (This happens by condensates (A.5) discussed above.) Thus, the chiral symmetry \(G_f^{(3,2)}\) is broken down to \(G_f^{(2,2)}\), where

\[
G_f^{(2,2)} = SU(2)_f L \otimes SU(2)_L \times SU(2)_f R \otimes SU(2)_R \times U(1)_{B'} .
\]

This breaking is realized, for instance, by the condensates \(\langle \hat{u}_3 \hat{d}_3 T^b_{H'} \rangle\) and \(\langle \hat{u}_3 \hat{d}_3 T^{b}_{H'} \rangle\). Note that with SU(3)_f L → SU(2)_f L and SU(3)_f R → SU(2)_f R we will have decompositions 3_{fL} = 2_{fL} + 1_{fL}

\(^{12}\)Similar to description of three flavor QCD with \((u, d, s)\) spin-1/2 states, either by sextet of SU(6) or by (3, 2) of SU(3)_f × SU(2)_s - the Wigner-Weyl realization of the SU(6) chiral symmetry. Here, however, SU(2)_s stands for spin group and SU(3)_f for the flavor. In our case of Eq. (A.6), SU(2) factors act like isospin rotations relating \(\hat{u}_\alpha\) and \(\hat{d}_\alpha\), and \(\hat{u}_\alpha^*\) and \(\hat{d}_\alpha^*\) respectively (\(\alpha = 1, 2, 3\)).
and $3_{f_R} = 2_{f_R} + 1_{f_R}$. At composite level, the spin-1/2 and $SU(3)'$ singlet combinations $(\hat{q}\hat{q})\hat{q}$ and $(\hat{q}^c\hat{q}^c)\hat{q}^c$ picked up as $[2'_{f_L} + 1'_{f_L}]$ from $[2_{f_L} + 1_{f_L}]^3$ and $[2'_{f_R} + 1'_{f_R}]$ from $[2_{f_R} + 1_{f_R}]^3$. Thus, transformations of $(\hat{q}\hat{q})\hat{q}$ and $(\hat{q}^c\hat{q}^c)\hat{q}^c$ composites under $G_f^{(2,2)}$ are:

\[
G_f^{(2,2)} : (\hat{q}\hat{q})\hat{q} \sim ([2_{f_L} + 1_{f_L}], \ 2_L, \ 1, \ 1), \quad (\hat{q}^c\hat{q}^c)\hat{q}^c \sim (1, \ 1, \ [2_{f_R} + 1_{f_R}], \ 2_R, \ -1). \quad (A.13)
\]

These representations will have anomalies which precisely match with those given in Eq. (A.11). Thus we have three families of $l_0, c_{0}^c, \nu_0^c$ composite states represented in Eq. (49), with transformation properties under $G_{SM}$ given in Eq. (50).

**B RG Equations and $b$-Factors**

In this Appendix we discuss details of gauge coupling unification and present one and two-loop RG coefficients at each relevant energy scales. At the end we calculate short range renormalization factors $A_S^i$ and $A_S^{c_i}$ for baryon number violating $d = 6$ operators within our model.

The two loop RG equation, for gauge coupling $\alpha_i$, has the form [46]:

\[
\frac{d}{d \ln \mu} \alpha_i^{-1} = -\frac{b_i}{2\pi} - \frac{1}{8\pi^2} \sum_j b_{ij} \alpha_j + \frac{1}{32\pi^3} \sum_j a_{ij} \lambda_j^2 \lambda_j
\]

(B.1)

$b_i$ and $b_{ij}$ account for 1 and 2-loop gauge contributions respectively, and $c_i^f$ for two loop correction via Yukawa coupling $\lambda_f$. For consistency, it is enough to consider the Yukawa coupling RG at 1-loop approximation:

\[
16\pi^2 \frac{d}{d \ln \mu} \lambda_f = c_f \lambda_f^3 + \lambda_f \left( \sum_j d_{j}^f \lambda_j^2 - 4\pi \sum_i c_i^j \alpha_i \right).
\]

(B.2)

RG factors can be calculated using general formulae [46]. Since at different energy scales different states appear, these factors also change with energy. For instance, at scale $\mu$, the $b_i$ and $b_{ij}$ can be written as $b_i(\mu) = \sum_a \theta(\mu - M_a) b_i^a$, $b_{ij}(\mu) = \sum_a \theta(\mu - M_a) b_{ij}^a$, where $a$ stands for the state with mass $M_a$ and step function $\theta(x) = 0$ for $x \leq 0$, and $\theta(x) = 1$ for $x > 0$.

Integration of (B.1), in energy interval $\mu_1 - \mu_2$, gives

\[
\alpha_i^{-1}(\mu_2) = \alpha_i^{-1}(\mu_1) - \frac{b_i^{\mu_1\mu_2}}{2\pi} \ln \frac{\mu_2}{\mu_1},
\]

(B.3)

where an effective $b_i^{\mu_1\mu_2}$ factor is given by

\[
b_i^{\mu_1\mu_2} = \left( \sum_a \theta(\mu_2 - M_a) b_i^a \ln \frac{\mu_2}{M_a} + \frac{1}{4\pi} \sum_a \int_{\mu_1}^{\mu_2} \theta(\mu - M_a) b_{ij}^a \alpha_j d \ln \mu - \frac{1}{8\pi^2} \int_{\mu_1}^{\mu_2} c_i^j \lambda_j^2 d \ln \mu \right) \frac{1}{\ln \frac{\mu_2}{\mu_1}}
\]

(B.4)

Under combination $(\hat{q}\hat{q})\hat{q}$ (suppressed gauge/chiral indices) we mean $e^{a' b' c'} e_{ij} (\hat{q}_{a' j} \hat{q}_{b' k}) \hat{q}_{c' i}$, where $a', b', c' = 1, 2, 3$ are $SU(3)'$ indices and $i, j, k = 1, 2$ stand for $SU(2)_L/SU(2)_W$ indices. Similar is applied to the combination $(\hat{q}^c\hat{q}^c)\hat{q}^c$. 

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13 Under combination $(\hat{q}\hat{q})\hat{q}$ (suppressed gauge/chiral indices) we mean $e^{a' b' c'} e_{ij} (\hat{q}_{a' j} \hat{q}_{b' k}) \hat{q}_{c' i}$, where $a', b', c' = 1, 2, 3$ are $SU(3)'$ indices and $i, j, k = 1, 2$ stand for $SU(2)_L/SU(2)_W$ indices. Similar is applied to the combination $(\hat{q}^c\hat{q}^c)\hat{q}^c$. 

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The second and third terms in (B.4) can be evaluated iteratively [47]. Although Eqs. in (B.1) can be solved numerically (which we do perform for obtaining final results), expressions (B.3), (B.4) are useful for understanding how unification works.

In the energy interval $M_Z - \Lambda'$ we have just SM, while between $\Lambda'$ and $M_I$ scales we have $G_{SM} \times SU(3)'$ gauge interactions plus additional states. Applying (B.3) for the couplings $\alpha_Y, \alpha_w, \alpha_c$ and $\alpha_{3'}$ we will have:

$$
\alpha^{-1}_i(M_I) = \alpha^{-1}_i(M_Z) - \frac{b^{*I}_i}{2\pi} \ln \frac{M_I}{M_Z}, \quad i = Y, w, c,
$$

$$
\alpha^{-1}_{3'}(M_I) = \alpha^{-1}_{3'}(\Lambda') - \frac{b^{*II}_{3'}}{2\pi} \ln \frac{M_I}{\Lambda'},
$$

where $b^{*I}_i, b^{*II}_{3'}$ can be calculated via (B.4) having appropriate RG factors.

Above the scale $M_I$ we have gauge interactions $G_{321}$ going all the way up to the GUT scale. The $G_{321}'$ gauge symmetry appears between scales $M_I$ and $M_I'$, while $SU(5)'$ appears above the $M_I'$ scale. Therefore, we will have

$$
\alpha^{-1}_i(M_G) = \alpha^{-1}_i(M_I) - \frac{b^{G}_i}{2\pi} \ln \frac{M_G}{M_I}, \quad i = 1, 2, 3,
$$

$$
\alpha^{-1}_{i'}(M_I') = \alpha^{-1}_i(M_I) - \frac{b^{I'}_{i'}}{2\pi} \ln \frac{M_I'}{M_I}, \quad i' = 1', 2', 3',
$$

$$
\alpha^{-1}_{3'}(M_G) = \alpha^{-1}_{3'}(M_I') - \frac{b^{G}_{3'}}{2\pi} \ln \frac{M_G}{M_I'}.
$$

From (B.5), (B.6) and taking into account the boundary conditions (64)-(66), we arrive at relations given in Eq. (67). The four equations in (67) allow to determine $M_I, M_I', M_G$ and $\alpha_G$ in terms of other input mass scales. The latter must be selected in such a way as to get successful unification. This has been done numerically and results are given in Table 1, Eq. (68) and Fig. 3.

Now we present all RG $b$-factors needed for writing down RG equations. In the energy interval $\mu = M_Z - \Lambda'$ the RG factors are just those of SM:

$$
\mu = M_Z - \Lambda' : \quad b_i = \left( \frac{41}{10}, -\frac{19}{6}, -7 \right), \quad b_{ij} = \left( \frac{199}{9}, \frac{37}{10}, \frac{27}{8}, \frac{44}{5}, \frac{22}{7}, -26 \right), \quad (i = Y, w, c).
$$

In the energy interval $\Lambda' - M_I$ we have the symmetry $SU(3)_c \times SU(2)_W \times U(1)_Y \times SU(3)'$. Also, instead of composite leptons we have three families of $SU(3)'$ triplets $\hat{q}, \hat{u}^c, \hat{d}^c$, and vector-like states $(\hat{l}, \hat{\bar{l}})_{\alpha}$ and $(\hat{e}^c, \hat{\bar{e}}^c)_{\alpha}$ ($\alpha = 1, 2, 3$) with masses $M^{(\alpha)}_{l_{\bar{l}}}$ and $M^{(\alpha)}_{e_{\bar{e}}}$ respectively. Moreover, some fragments of $\Phi(5, 5)$ (see Eq. (25)) and $\Sigma_{3'}$ (of $\Sigma'$) can appear below $M_I$. Thus, the corresponding $b$-factors in this energy interval are given by:

$$
\mu = \Lambda' - M_I : \quad b_Y = \frac{9}{2} + \frac{1}{15} \theta(\mu - M_{l_{\bar{l}}}),
$$

$$
\frac{2}{5} \sum_{\alpha=1}^{3} \theta(\mu - M^{(\alpha)}_{l_{\bar{l}}}) + \frac{4}{5} \sum_{\alpha=1}^{3} \theta(\mu - M^{(\alpha)}_{e_{\bar{e}}}) + \frac{5}{6} \theta(\mu - M_{DT}) + \frac{5}{6} \theta(\mu - M_{TT})
$$

23
b_w = -\frac{7}{6} + \frac{2}{3} \sum_{a=1}^{3} \theta(\mu - M_0^{(a)}) + \frac{11}{2} \theta(\mu - M_{DT}) - \frac{1}{2} \theta(\mu - M_{TD}),
\nonumber
b_c = -\frac{1}{3} \theta(\mu - M_{TD}) + \frac{1}{2} \theta(\mu - M_{TT}),
\nonumber
b_y = -\frac{7}{6} \theta(\mu - M_{T_D}) + \frac{11}{2} \theta(\mu - M_{TT}) + \frac{1}{2} \theta(\mu - M_{D}),
\nonumber
(B.8)
\mu = \Lambda' - M_I: b_{ij} = \begin{pmatrix}
\frac{13709}{9} & \frac{44}{5} & \frac{44}{5} \\
\frac{3}{5} & \frac{12}{3} & \frac{12}{3} \\
\frac{11}{10} & 0 & 0
\end{pmatrix} + \sum_{a} \theta(\mu - M_0) b_{ij}^{a}, \quad (i, j = Y, w, c, 3') \quad \text{with:}
\nonumber
b_{ij}^{T_W} = \begin{pmatrix}
\frac{4}{75} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\frac{2}{15} & 0 & \frac{11}{15}
\end{pmatrix}, \quad b_{ij}^{D_T} = \begin{pmatrix}
\frac{25}{6} & \frac{15}{2} & 0 & 0 \\
\frac{25}{6} & \frac{15}{2} & 0 & 8 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad b_{ij}^{T_T} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 11 \\
0 & 0 & 0 & 8 \\
0 & 0 & 0 & 0
\end{pmatrix}, \quad b_{ij}^{T_{D_T}} = \begin{pmatrix}
\frac{25}{6} & \frac{15}{2} & 0 & 2 \\
\frac{25}{6} & \frac{15}{2} & 0 & 8 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix},
(B.9)

Between the scales \(M_I\) and \(M_I'\) the symmetry is \(G_{321} \times G_{321}'\), and all matter states are massless. Also, above the scale \(M_I\), we should include the states \(T_{H'}\) and \(\Phi_{DD'}\) as masses, and remaining fragments above their mass thresholds. Since \(G_{321}\) goes all the way up to the \(M_G\), its 1-loop b-factors can be determined in the interval \(M_I - M_G\) and are given by:
\mu = M_I - M_G: b_1 = \frac{43}{10} \theta(\mu - M_{DT}) + \frac{3}{10} \theta(\mu - M_{TT}) + \frac{1}{2} \theta(\mu - M_{TD}),
\nonumber
b_2 = -\frac{17}{6} + \frac{1}{2} \theta(\mu - M_{DT}),
\nonumber
b_3 = -\frac{7}{6} \theta(\mu - M_{TT}) + \frac{1}{2} \theta(\mu - M_{TD}).
(B.10)
The gauge group \(G_{321}'\) appears in the interval \(M_I - M_I'\) and corresponding 1-loop b-factors are:
\mu = M_I - M_I' : b_{1'} = \frac{64}{15} \theta(\mu - M_D) + \frac{2}{15} \theta(\mu - M_{DT}) + \frac{1}{5} \theta(\mu - M_{TD})
\nonumber
+ \frac{3}{10} \theta(\mu - M_{DD}) - \frac{55}{3} \theta(\mu - M_{X}),
\nonumber
b_{2'} = -3 + \frac{1}{6} \theta(\mu - M_D) + \frac{1}{2} \theta(\mu - M_{DD}) - 11 \theta(\mu - M_{X}),
\nonumber
b_{3'} = -\frac{41}{6} + \frac{1}{2} \theta(\mu - M_{TT}) + \frac{1}{3} \theta(\mu - M_{D_T}),
(B.11)

where terms with \(\theta(\mu - M_{X})\) account for the threshold of \((X', Y')\) gauge bosons of \(SU(5)'\), in case their mass \(M_{X'}\) lie slightly below the \(M_I'\) scale. We will take this effect into account at 1-loop.
level. The 2-loop $b_{ij}$ factors of $G_{321} \times G_{321}'$ form $6 \times 6$ matrices and are determined in the interval $M_I - M'_I$:

$$
\mu = M_I - M'_I : \ b_{ij} = (b^f + b^h + b^g + b^{T_H'} + b^{D'D'})_{ij} + \sum_a \theta(\mu - M_a) b^a_{ij}, \quad (i, j = 1, 2, 3, 1', 2', 3')
$$

with:

$$
b^f_{ij} = 3 \begin{pmatrix}
\frac{1}{15} & \frac{3}{5} & \frac{44}{15} & 0 & 0 & 0 \\
\frac{1}{15} & \frac{3}{5} & \frac{44}{15} & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\
0 & 0 & 0 & \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\
0 & 0 & 0 & \frac{19}{15} & \frac{3}{5} & \frac{44}{15} \\
0 & 0 & 0 & \frac{19}{15} & \frac{3}{5} & \frac{44}{15}
\end{pmatrix}, \ b^h_{ij} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
b^{T_H'}_{ij} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \ b^{g_{ij}} = \begin{pmatrix}
9 & 9 & 9 & 9 & 0 & 0 \\
9 & 9 & 9 & 9 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
b^{D'D'}_{ij} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \ b^{D'T'}_{ij} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
b^g_{ij} = \text{Diag} \left( 0, -\frac{136}{3}, -102, 0, -\frac{136}{3}, -102 \right), \ b^g_{ij}^{\Sigma'} = \text{Diag} \left( 0, 0, 0, 0, 0, 21 \right).
$$

Since at and above the scale $M'_I$ the $G_{321}'$ is embedded in $SU(5)'$, we will deal with $b$-factors of $G_{321} \times SU(5)'$ symmetry. 1-loop $b$-factors of $G_{321}$ are given in (B.10). At energies corresponding to unbroken $SU(5)'$, the fragments $(\Phi_{D'D'}, \Phi_{D'T'})$ form unified $(2, 5) \equiv \Phi_{D5'}$-plet of $G_{321} \times SU(5)'$. Similarly: $(T_H', D') \subset H'$. Above the scale $M'_I$, these states (together with all fragments of the $\Sigma'$-plet) should be included as massless states. Thus, 1-loop $b$-factor of $SU(5)'$ is given as:

$$
\mu = M_I' - M_G : \ b_{ij}' = -13 + \frac{1}{2} \theta(\mu - M_T5'),
$$

where $M_{T5'} = \text{max}(M_{T'T}, M_{T'D'})$ denotes mass of $(3, 5)$-plet, which includes $\Phi_{T'T}$ and $\Phi_{T'D'}$ states: $(\Phi_{T'T}, \Phi_{T'D'}) \subset \Phi_{T5'}$. The 2-loop $b_{ij}$ factors, above the scale $M'_I$, form $4 \times 4$ matrices and are:

$$
\mu = M_I' - M_G : \ b_{ij} = (b^f + b^h + b^g + b^{T_H'} + b^{D'D'})_{ij} + \theta(\mu - M_{T5'}) b^g_{ij}^{T5'}, \quad (i, j = 1, 2, 3, 5')
$$

with:

$$
b^f_{ij} = 3 \begin{pmatrix}
\frac{1}{15} & \frac{3}{5} & \frac{44}{15} & 0 \\
\frac{1}{15} & \frac{3}{5} & \frac{44}{15} & 0 \\
0 & 0 & 0 & \frac{19}{15} \\
0 & 0 & 0 & \frac{19}{15}
\end{pmatrix}, \ b^h_{ij} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}, \ b^g_{ij} = \text{Diag} \left( 0, -\frac{136}{3}, -102, -\frac{850}{3} \right).
$$
\[ b_{ij}^{H'} = \frac{97}{15} \delta_{55} \delta_{55}, \quad b_{ij}^{\Sigma} = \frac{175}{3} \delta_{55} \delta_{55}, \quad b_{ij}^{D_{55}} = \begin{pmatrix} 9 & 9 & 0 & 72 \ 3 & 2 & 10 & 0 \ 0 & 0 & 0 & 24 \ 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 0 \ \frac{3}{5} & 3 & 0 & 194 \ 5 \ 0 & 8 & 24 \ 5 \ 0 \ \end{pmatrix}, \quad b_{ij}^{T_{55}} = \begin{pmatrix} 4 & 0 & 16 & 48 \ 0 & 0 & 0 & 0 \ 2 & 0 & 55 & 24 \ 3 & 0 & 97 & 5 \ \end{pmatrix}. \quad (B.14) \]

As far as the Yukawa coupling involving RG factors, \( a_t^i, c_f, d_f^{ij} \) and \( c_f^i \) (see Eqs. (B.1) and (B.2)), are concerned, within our model only top and 'mirror-top' Yukawa couplings are large. All other Yukawa interactions are small and can be ignored. Thus, the Yukawa terms \( \lambda_t q_t c h, (\lambda_t \tilde{t} b + \lambda_{t \tilde{t} e} \tilde{e} c \tilde{c}) T_{H'} \) and \( \lambda_t \tilde{q}_3 \tilde{e} D' \) are relevant. All these four couplings unify at \( M_G \) due to gauge symmetry and \( D_2 \) parity. For the top Yukawa involved RG factors, in the energy interval \( M_Z - M_I \), we have:

\[ a_t^i = \left( \frac{17}{10}, \frac{3}{2}, \frac{9}{2} \right), \quad c_t^i = \left( \frac{17}{20}, \frac{9}{4}, \frac{8}{3} \right), \quad (i = Y, w, c), \quad c_t = \frac{9}{2}, \quad d_t^{ij} = 0. \quad (B.15) \]

In energy interval \( M_I - M_G \), with replacement of the indices \( (Y, w, c) \to (1, 2, 3) \), the corresponding RG factors will be same. Since the mass of the state \( D' \) is \( \sim M_I' \), the RG with \( \lambda_t \) will be relevant above the scale \( M_I' \). Within our model, \( M_{T_{H'}} \sim \Lambda' \) and in RG, the couplings \( \lambda_{t \tilde{b}} \) and \( \lambda_{t \tilde{e} c} \) will be relevant above the scale \( \Lambda' \). Between the scales \( \Lambda' \) and \( M_I \) the mirror matter has EW and \( SU(3)' \) interactions. Therefore, we have

\[ \mu = \Lambda' - M_I' : \quad (a_Y, a_w, a_3)_{t b} = \left( \frac{1}{15}, 2, \frac{4}{3} \right), \quad (a_Y, a_w, a_3)_{t e} = \theta(\mu - M_{e e}) \left( \frac{13}{15}, 0, \frac{1}{3} \right), \]

\[ (c_Y, c_w, c_3)_{t b} = \left( \frac{1}{10}, \frac{9}{2}, \frac{8}{3} \right), \quad (c_Y, c_w, c_3)_{t e} = \theta(\mu - M_{e e}) \left( \frac{13}{5}, 0, 4 \right), \]

\[ c_{t b} = 4, \quad d_{t b}^{e e} = \theta(\mu - M_{e e}), \quad c_{t e} = 3\theta(\mu - M_{e e}), \quad d_{t e}^{e e} = 2\theta(\mu - M_{e e}). \quad (B.16) \]

Between \( M_I \) and \( M_I' \) scales, with replacements \( (Y, w) \to (1', 2') \), the corresponding factors will be same. At and above the scale \( M_I \), the \( G_{321}' \) is unified in \( SU(5)' \) group, \( D' \) should be included in RG, and three Yukawas unify \( \lambda_{t b} = \lambda_{t \tilde{e} c} = \lambda_t \). Thus, dealing with \( \lambda_t \), we will have

\[ \mu = M_I' - M_G : \quad a_{3 i} = \frac{9}{2}, \quad c_t = 9, \quad c_t^i = \frac{108}{5}, \quad d_t^{ij} = 0. \quad (B.17) \]

### B.1 Short Range RG Factors for \( d = 6 \) Operators

The baryon number violating \( d = 6 \) operators of Eq. (70) involve couplings \( C^{(e)} \) and \( C^{(t)} \) respectively. These couplings run and in nucleon decay amplitudes the short range RG factors

\[ A_S^{(e)} = \frac{C^{(e)}(M_Z)}{C^{(e)}(M_X)} , \quad A_S^{(t)} = \frac{C^{(t)}(M_Z)}{C^{(t)}(M_X)} \quad (B.18) \]

emerge. These factors, having SM gauge interactions and states below the GUT scale, have been calculated in [31]. Within our model, calculation can be done similarly. The RG equations for \( C^{(t)} \) and \( C^{(e)} \), in 1-loop approximation, are given by:

\[ 4\pi \frac{d}{dt} C^{(t)} = -C^{(t)} \left[ \theta(M_I - \mu) \left( \frac{23}{20} \alpha_Y + \frac{9}{4} \alpha_w \right) + 2\alpha_e + \theta(\mu - M_I) \left( \frac{23}{20} \alpha_1 + \frac{9}{4} \alpha_2 \right) \right], \]

\[ 4\pi \frac{d}{dt} C^{(e)} = -C^{(e)} \left[ \theta(M_I - \mu) \left( \frac{11}{20} \alpha_Y + \frac{9}{4} \alpha_w \right) + 2\alpha_e + \theta(\mu - M_I) \left( \frac{11}{20} \alpha_1 + \frac{9}{4} \alpha_2 \right) \right]. \quad (B.19) \]
Having numerical solutions for the gauge couplings, also Eqs. in (B.19) can be integrated. Doing so and taking into account (B.18), within our model we obtain $A_{L}^{S} = 1.18$ and $A_{c}^{S} = 1.17$.

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