SYSTEMS THEORY AND ANALYSIS OF THE
IMPLEMENTATION OF NON PHARMACEUTICAL POLICIES
FOR THE MITIGATION OF THE COVID-19 PANDEMIC

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ABSTRACT. We utilize systems theory in the study of the implementation of non pharmaceutical strategies for the mitigation of the COVID-19 pandemic. We present two models. The first one is a model of predictive control with receding horizon and discontinuous actions of unknown costs for the implementation of adaptive triggering policies during the disease. This model is based on a periodic assessment of the peak of the pandemic (and, thus, of the health care demand) utilizing the latest data about the transmission and recovery rate of the disease. Consequently, the model seems to be suitable for discontinuous, non-mechanical (i.e. human) actions with unknown effectiveness, like those applied in the case of COVID-19. Secondly, we consider a feedback control problem in order to contain the pandemic at the capacity of the NHS (National Health System). As input parameter we consider the value \( p \) that reflects the intensity-effectiveness of the measures applied and as output the predicted maximum of infected people to be treated by NHS. The feedback control regulates \( p \) so that the number of infected people is manageable. Based on this approach, we address the following questions: (a) the limits of improvement of this approach; (b) the effectiveness of this approach; (c) the time horizon and timing of the application.

1. Introduction. The recent spread of coronavirus disease 2019 (COVID-19) has become a major global health threat forcing national governments to take unprecedented measures in order to curb the disease. On account of the lack of vaccines and therapies for the novel SARS-CoV-2 virus, non pharmaceutical interventions (NPI) have become the major strategies for the mitigation of the pandemic. The first response to the disease is to limit the person-to-person physical interaction as much as possible. The aim is to slow down the spread of the pandemic and

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to reduce the peak health care demand. This is achieved by several intervention methods, e.g. case isolation at home, closing schools, universities, workplaces and by requiring people to follow social distancing etc.

The aforementioned actions are discontinuous and non-mechanical (i.e. human). Therefore, their effectiveness is difficult to predict. Furthermore, the suitable implementation, including time and severity, of the social distancing policies was not clear, at least in the beginning of the pandemic. As a consequence, significant differences in strategies followed by various countries were observed.

At the same time, research activity related to the pandemic has been triggered worldwide. On the one hand research is focused in finding suitable vaccination or pharmacies for attacking the coronavirus. On the other hand, theoretical research utilizes mathematical models and tries to shed light into several aspects of the development of the COVID-19 as well as to lay the theoretical foundation for public health interventions. To this end, mathematical modelling of the epidemic has been utilized by many investigations in order to describe the evolution of the pandemic. The so-called SIR model, although relatively simple, it is very similar to the COVID-19 outbreak and it has been adopted by a lot of researchers as the basic model for the study of the disease.

The present work has two objectives, Firstly, we propose a model of predictive control for the implementation of interventions with the characteristics of those applied in the case of COVID-19 (i.e. discontinuous, non-mechanical). This type of control consists of: (a) a dynamical model describing the evolution of the pandemic; (b) an observer estimating the states, the parameters of the model and assisting the prediction of the objective variables; (c) a cost function to be minimized and (d) a set of control actions selected and implemented by a controller mechanism.

The control works as follows. The series of control actions \( U_n \) is calculated at certain points \( t_n \) in time for the whole horizon \( H \) (usually \( t_{n+1} = t_n + T \) for some fixed \( T > 0 \)) as follows. At time \( t_n \) the observer collects the data, calculates the parameters of the model and predicts the values of the variables for the remaining part of the horizon \([t_k, H]\). The control strategy \( U_k, U_{k+1}, \ldots \) is also computed for the remaining part of the horizon \([t, H]\) so that the cost function is minimized. However, only \( U_k \) is applied. At time \( t_{k+1} = t_k + T \) the observer repeats the same procedure and a new series of control is produced whose only the first is applied at \([t_k + T, t_k + 2T]\). This is repeated for the whole horizon.

As far as the second objective is concerned, we consider an SIR model, where the so-called transmission rate of the epidemic (depending on the average number of contacts of any individual per unit time) is not constant, but it changes according to the intensity-effectiveness of social distancing policies. Then, our objective is to develop a feedback control model for controlling the implementation of NPI policies using an action control in an optimum manner without delay or overshooting.

Feedback (or closed-loop) control is an old concept which has found a plethora of exciting applications (see [5]). Roughly speaking, the output of a dynamical system can be measured and, hence, it can be compared to a critical value. Depending on the result, the controller mechanism (of some kind) modifies in turn the input variable and affects the trajectories of the states of the system. In our implementation of the feedback control, the input parameter, which affects the system, is the number \( p \) that reflects the intensity-effectiveness of the measures. The output parameter is the maximum number of the infected people which has to be contained within the capacity of the National Health System (NHS).
It should pointed out that, in the present application of feedback control to an epidemic model, the output cannot be measured but only predicted. However, this characteristic appears in many control situations. In these situations, the control designer observes the process to see which track is going to be followed. Then, it uses this information to feed back the control system.

The rest of the paper is organized as follows. In Sections 2 and 3 we present and analyze the SIR model for the evolution of a pandemic. In Section 4 non pharmaceutical intervention methods are analyzed. Section 5 is devoted to the implementation of the predictive control model in the case of the COVID-19 pandemic. Sections 6 and 7 describe the feedback control model for the containment of the disease. In Section 8, we examine several scenarios for the application of feedback control. Finally, Section 9 concludes the article.

We close this introductory section with some comments concerning the motivation and possible future applications. The general idea of control is as follows. We have a dynamical system which contains some input and output variables that are governed by a set of relations (algebraic equations or differential equations or stochastic differential equations etc). Our purpose is to control the values of (some of) the outputs by suitably intervening in the input variables that are in our disposal. There are several ways of doing control. The selected method depends on the characteristics of the model (equations of the model, speed of the real phenomenon, etc). This general idea of control has been analyzed in several articles and has found a wide range of applications, from economics, environmental issues to education and society related topics (for instance see [1], [12], [8], [11], [9]).

In this article, the phenomenon we study, i.e. the spread of a disease, is described by a set of differential equations and it is quite slow. In this model, we wish to control one of the outputs, i.e. the maximum number of infected people. In order to study this dynamical system, we develop a model of predictive control and feedback control for the implementation of non pharmaceutical properties and the mitigation of the pandemic. More sophisticated models should also take into account other parameters, for instance the population groups and the probability to follow the physical distancing policies. Furthermore, stochastic models can also been considered, since the exact values of the input and output variables (for example the maximum number of infected people) are actually random variables and the result of any prediction is in fact a confidence interval and not a single value.

2. Susceptible-infected-recovered (SIR) model. Mathematical modelling of the evolution of a directly transmitted infectious disease can be achieved by considering a few basic disease states and studying the changes in the population of the corresponding groups of individuals. One of the simplest epidemic models is the so-called susceptible-infected-recovered (SIR) model (see for example [7], [13]). In this model we begin with some population which is assumed to remain constant, i.e. demographic changes (for instance births, deaths, ageing etc.) are ignored. Then, we consider three disease states: the susceptible state, the infected state and the recovered state. Consequently, the population is divided into three groups according to the infection status: susceptible, infected and recovered individuals. An individual is in the susceptible group if it does not have the disease at time $t$ yet, but it may be infected when contacting an already infected person. An infected individual has the disease at time $t$ and may transfer it to a susceptible individual if they come into contact with each other. Finally, a recovered individual is someone
who has either recovered or passed away from the disease at time $t$ and is no longer contagious. In this model, we assume that a recovered individual is not placed to the susceptible state anymore. Observe also that although recovery and death are completely opposite in human terms, they represent the same thing in epidemiological terms: it makes no difference whether a person is immune or dead since they no longer have impact to the spread of the disease.

We now denote by $S(t)$, $I(t)$ and $R(t)$ the number of susceptible, infected and recovered individuals respectively at the time $t$. Then, the evolution of the disease can be described by the following time-dependent dynamical system:

$$(S(t), I(t), R(t))_{t \geq 0}.$$ 

If we denote the total population size by $N$, then $S(t) + I(t) + R(t) = N$ for every $t$, and hence, one may consider only two of the three quantities in the above system.

The SIR model is very similar to the COVID-19 outbreak and it has been adopted by many studies so far.

In the traditional SIR model, we assume that each individual has on average $b$ contacts with randomly chosen others per unit time, where $b > 0$ is time-invariant. Furthermore, the probability of a person (chosen at random) being susceptible is $S(t)/N$. Consequently, each infected individual has contact with an average of $bS(t)/N$ susceptible people per unit time. Since, there are $I(t)$ infected people, it follows that the rate of newly infected people is $bS(t)I(t)/N$. At the same time, the number of susceptible people will decrease at the same rate. Thus, we reach the susceptible equation:

$$\frac{dS}{dt} = -\frac{b}{N}S(t)I(t) = -\beta S(t)I(t).$$

The number $\beta = b/N$ is called the transmission rate of the disease. Clearly, this number is affected by non pharmaceutical intervention methods that reduce the number $b$ of contacts (e.g. social distancing).

Additionally, we assume that infected individuals recover (or die) at some constant average rate $\gamma$. The time-invariant number $\gamma$ is called the recovery rate. Since there are $I(t)$ infected individuals, it follows that the number of recovery people increases with rate $\gamma I(t)$. Thus, we reach the recovered equation:

$$\frac{dR}{dt} = \gamma I(t).$$

The positive number $\gamma$ depends on the severity of the disease and for new viruses it is unknown, at least in the beginning. Clearly, it can be affected by vaccination or pharmaceutical treatments; however, pharmaceutics need usually some time before attributing satisfactory results. So, at least in the beginning the lack of pharmaceutical treatment is a realistic scenario.

Finally, by the above equations, one can deduce the infected equation describing the rate of change of the infected people. Namely, since, $S(t) + I(t) + R(t) = N$, we have

$$\frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t).$$
Consequently, the SIR model is governed by the following system of quadratic ODEs:

\[ \frac{dS}{dt} = -\beta S(t)I(t) \]  
\[ \frac{dI}{dt} = \beta S(t)I(t) - \gamma I(t). \]  
\[ \frac{dR}{dt} = \gamma I(t) \]

Clearly, the above system can be reduced to a system of two ODEs, for instance (1) and (2).

![Figure 1. Visualization of the SIR model with random $\beta$ and $\gamma$ in time $t$. As initial values, $S(0)=999$, $I(0)=1$ and $R(0)=0$ were chosen.](image)

**The maximum number of infected individuals.** Although the solution of the ODEs comprising the RIS model cannot be written explicitly, one can use several techniques in order to derive useful information about the solutions, and consequently about the spread of the disease. Hence, this relatively simple model helps to lay a theoretical foundation for public health interventions.

One of the results is that we are able to predict the peak of the disease, that is we can derive a formula for the maximum number of infected individuals $I_{\text{max}}$. We divide (1) by (2) which gives that

\[ \frac{dS}{dI} = \frac{-\beta S(t)I(t)}{\beta S(t)I(t) - \gamma I(t)}. \]

For $I(t) > 0$, the last ODE can be rewritten in the form

\[ \frac{\beta S(t)I(t) - \gamma I(t)}{\beta S(t)I(t)}dS = -dI. \]

Taking integrals in both sides implies that:

\[ \int_{t_0}^{t} \frac{\beta S(t)I(t) - \gamma I(t)}{\beta S(t)I(t)}dS = \int_{t_0}^{t} -dI, \]
or equivalently,

\[ S(t) - S(t_0) - \frac{\gamma}{\beta} \ln S(t) + \frac{\gamma}{\beta} \ln S(t_0) = I(t_0) - I(t). \]

Hence,

\[ I(t) = I(t_0) + S(t_0) - S(t) + \frac{\gamma}{\beta} \ln S(t) - \frac{\gamma}{\beta} \ln S(t_0). \]  \hspace{1cm} (4)

The maximum value of \( I(t) \) is attained when \( \frac{dI}{dt} \) equals 0. By Equation (2), this happens when \( S(t) = \frac{\gamma}{\beta} \). Thus, Equation (4), for \( t = 0 \) and \( S(t) = \frac{\gamma}{\beta} \), gives that:

\[ I_{\text{max}} = I(0) + S(0) - \frac{\gamma}{\beta} \cdot \ln \frac{\gamma}{\beta} - \frac{\gamma}{\beta} \cdot \ln S(0) \]  \hspace{1cm} (5)

\[ = I(0) + S(0) - \frac{\gamma}{\beta} + \frac{\gamma}{\beta} \cdot \ln \frac{\gamma}{\beta S(0)}. \]

The above formula provides the peak of the disease, i.e. the maximum number of infected individuals. However, the implementation of new mitigation policies and social distancing measures and the results of the present paper depend on the estimated maximum number \( I_{\text{max}}(t) \) of infected people from time \( t \) and onwards.

In order to estimate this number, we apply the SIR model at time \( t \) with initial values \( I(t), S(t), R(t) \). Hence, Equation (4) gives that

\[ I_{\text{max}}(t) = \frac{\gamma}{\beta} \cdot \ln \frac{\gamma}{\beta S(t)} - \frac{\gamma}{\beta} + I(t) + S(t). \]

However, one should observe that the above equation holds during the pandemic, i.e. when \( S(t) > \frac{\gamma}{\beta} \). If we pass this threshold, then the number of infected people is declining and thus \( I_{\text{max}}(t) = I(t) \). Hence, the estimated maximum number of infected people from time \( t \) and onwards is given by

\[ I_{\text{max}}(t) = \begin{cases} \frac{\gamma}{\beta} \cdot \ln \frac{\gamma}{\beta S(t)} - \frac{\gamma}{\beta} + I(t) + S(t), & \text{during the pandemic, i.e. } S(t) > \frac{\gamma}{\beta}; \\ I(t), & \text{after the pandemic, i.e. } S(t) < \frac{\gamma}{\beta}. \end{cases} \]

3. **Time-dependent SIR model.** Equation (5) is one of the most fundamental relations in this model of spread of a disease. This equation estimates the peak of the pandemic and, consequently, the maximum health care demand and it plays a major role in the definition of suitable policies. When the quantity \( I_{\text{max}} \) exceeds some thresholds, then suitable measures has to be taken in order to mitigate its value.

However, the assumption that \( \beta \) and \( \gamma \) are time-invariant is not a realistic scenario for most real diseases. Firstly, for relatively new diseases, e.g. COVID-19, it is difficult to make an accurate prediction of the recovery rate \( \gamma \). Most importantly, the transmission rate \( \beta \) is subject to change depending on the application of suitable policies. Furthermore, the effectiveness of non pharmaceutical intervention methods (e.g. social distancing) are difficult to estimate and it requires a substantial amount of time in order to be measured. Hence, the time evolution of the parameter \( \beta \) is also a major difficulty.

For the aforementioned reasons, several studies adopt the time-dependent SIR model in which \( \beta \) and \( \gamma \) are not constants but change with the time. Two major problems appear in this setting: (a) to calculate the values of \( \beta(t) \) and \( \gamma(t) \); (b) to make a more accurate estimate of the maximum value of the infected people.

In order to address the first issue and develop a method for estimating the values of \( \beta(t) \) and \( \gamma(t) \), we utilize a discrete time SIR model. This is a realistic scenario,
since, for example, the data for the pandemic of COVID-19 are updated on a daily basis. More precisely, we consider a sequence \((t_k)_{k=1}^\infty\) of sampling time such that \(t_{k+1} - t_k = T\), where \(T > 0\). These are the moments when some observer measures the values of infected and recovered people (e.g. in the case of COVID-19, one has \(T = 1\)). Furthermore, the ODEs (1), (2) and (3) can be replaced by the next difference equations:

\[
\begin{align*}
\frac{S(t_k) - S(t_{k-1})}{T} &= -\beta(t_k)S(t_k)I(t_k) \quad (6) \\
\frac{I(t_k) - I(t_{k-1})}{T} &= \beta(t_k)S(t_k)I(t_k) - \gamma(t_k)I(t_k) \quad (7) \\
\frac{R(t_k) - R(t_{k-1})}{T} &= \gamma(t_k)I(t_k) \quad (8)
\end{align*}
\]

In the above equations, \(S(t_k), I(t_k), R(t_k)\) are measurable data. Using this information, we can derive \(\beta(t_k)\) and \(\gamma(t_k)\) for each sampling time \(t_k\), as follows.

- \(\gamma(t_k)\) is calculated by Equation (8), and in particular,

\[
I(t_k) \cdot \gamma(t_k) = \frac{R(t_k) - R(t_{k-1})}{I(t_k)} \iff \gamma(t_k) = \frac{R(t_k) - R(t_{k-1})}{I(t_k)}.
\]

- \(\beta(t_k)\) is calculated by Equation (6), and in particular

\[
- S(t_k) \cdot I(t_k) \cdot \beta(t_k) = \frac{S(t_k) - S(t_{k-1})}{T} \iff \beta(t_k) = \frac{S(t_{k-1}) - S(t_k)}{S(t_k) \cdot I(t_k) \cdot T}.
\]

Using the fact that \(S(t_k) + I(t_k) + R(t_k) = N\) for every \(k\), we also obtain:

\[
\beta(t_k) = \frac{(I(t_k) - I(t_{k-1})) + (R(t_k) - R(t_{k-1}))}{(N - I(t_k) - R(t_k)) \cdot I(t_k) \cdot T}.
\]

In other words, we estimate the values of the recovery and the transmission rate in various times, taking into account the latest data. Consequently, we are able to adjusting these values according to the efficiency of the measures and the recovering possibility of the population.

In order to address the second issue, i.e. to estimate the maximum value of infected people in a time-dependent SIR model, we consider for every sampling time \(t_k\), the SIR model

\[
(S(t), I(t), R(t))_{t \geq t_k}
\]

which satisfies the equations (1), (2) and (3) with transmission and recovery rates \(\beta_k = \beta(t_k)\) and \(\gamma_k = \gamma(t_k)\) and initial conditions \((S(t_k), I(t_k), R(t_k))\). For this SIR model, we can apply Equation (4) to obtain

\[
I(t) = I(t_k) + S(t_k) - S(t) + \frac{\gamma_k}{\beta_k} \ln S(t) - \frac{\gamma_k}{\beta_k} \ln S(t_k).
\]

The maximum value of \(I(t)\) is attained again when \(\frac{dI}{dt}\) equals 0 which happens when \(S(t) = \frac{\gamma_k}{\beta_k}\). Thus, the expected maximum at the \(k\)-th sampling step is given by:

\[
I^k_{\max} = I(t_k) + S(t_k) - \frac{\gamma_k}{\beta_k} + \frac{\gamma_k}{\beta_k} \cdot \ln \frac{\gamma_k}{\beta_k} - \frac{\gamma_k}{\beta_k} \cdot \ln S(t_k),
\]

or equivalently,

\[
I^k_{\max} = N - R(t_k) - \frac{\gamma_k}{\beta_k} + \frac{\gamma_k}{\beta_k} \cdot \ln \frac{\gamma_k}{\beta_k} - \frac{\gamma_k}{\beta_k} \cdot \ln(N - I(t_k) - R(t_k)).
\]
Table 1. Summary of NPI interventions considered.

| Label | Policy Description |
|-------|--------------------|
| CI | Case isolation in the home. Symptomatic cases stay at home for 7 days, reducing non-household contacts by 75% for this period. Household contacts remain unchanged. Assume 70% of household comply with the policy. |
| HQ | Voluntary home quarantine. Following identification of a symptomatic case in the household, all household members remain at home for 14 days. Household contact rates double during this quarantine period, contacts in the community reduce by 75%. Assume 50% of household comply with the policy. |
| SDO | Social distancing of those over 70 years of age. Reduce contacts by 50% in workplaces, increase household contacts by 25% and reduce other contacts by 75%. Assume 75% compliance with policy. |
| SD | Social distancing of entire population. All households reduce contact outside household, school or workplace by 75%. School contact rates unchanged, workplace contact rates reduced by 25%. Household contact rates assumed to increase by 25%. |
| PC | Closure of schools and universities. Closure of all schools, 25% of universities remain open. Household contact rates for student families increase by 50% during closure. Contacts in the community increase by 25% during closure. |

The above equation provides an estimation for the maximum of the disease taking into account the latest data for the infected and recovered people and updating the values of the transmission and recovery rates according to this data. As a matter of fact, the above described method produces a sequence \((I_{\text{max}}^k)_{k=0}^\infty\) of maxima, where \(I_{\text{max}}^k\) is the expected maximum at the \(k\)-th step (sampling time). Finally, we observe that the value \(I_{\text{max}}^k\) depends only on measurable data of the disease, namely the number of infected and recovered people at times \(t_k\) and \(t_{k+1}\), i.e.

\[
I_{\text{max}}^k = F(I(t_k), I(t_{k+1}), R(t_k), R(t_{k+1})).
\]

4. **Adaptive triggering policies.** During a pandemic adaptive triggering policies need to be made in order to control the outbreak [10]. In this paper only non-pharmaceutical intervention (NPI) methods are discussed. Such methods are physical distancing, sample testing, tracing contacts and quarantines. Such policies are presented in Table 1 as described by Neil Ferguson et al. [4].

The purpose of these monitoring measures is to start flattening the curve of \(I(t)\). According to the SIR model, \(\beta\) parameter is responsible for the transmission rate. Figure 2 shows clearly the effect of adaptive triggering monitoring for different policy scenarios for UK [4].

It follows that the controller mechanism of each country has at its disposal a sequence of NPI policies

\[
P_1, P_2, \ldots, P_r
\]

that can be triggered according to the evolution of the pandemic, the model predictions for disease peak and the capacity of health care system. These policies start from the lightest ones and proceed towards the more intense. The corresponding measures aim at reducing the transmission rate \(\beta\) and thus flattening the curve of \(I(t)\). According to the table 1, each policy is accompanied with a prediction of the reduction rate in the number of contacts and the compliance of the population with the instructions. Hence, for every policy \(P_i\), the controller mechanism has a prediction of the transmission rate \(\beta_i\) and the following sequence

\[
\beta_1 > \beta_2 > \ldots > \beta_r.
\]
is defined. Next, we appeal to Equation (11), where we replace \( \beta_k \) with the aforementioned estimations \( \tilde{\beta}_i \) of the transmission rate. In this way, one obtains the estimated peak \( \tilde{I}_{\text{max}} \) of the pandemic after the implementation of the policy \( P_i \). Hence, the next sequence is also defined

\[
\tilde{I}_{1\text{max}} > \tilde{I}_{2\text{max}} > \ldots > \tilde{I}_{r\text{max}}.
\]

It should be pointing out that the above values are not time invariant. Since they follow from Equation (11), these values depend on the sampling time \( t_k \) and, hence, they have to be calculated at every step of the control.

In order to decide the suitable NPI policy, the controller mechanism should also take into account the capacity of the health care system, which we denote by \( I_c \). It should be pointed out that the control system does not exhaust the capacity \( I_c \). Hence, in real situations some portion of \( I_c \) is usually considered. It is expected that the toughest policy \( P_r \) (for instance, maximum possible isolation at home) will reduce the estimate peak \( \tilde{I}_{r\text{max}} \) below the health care capacity \( I_c \). Therefore, there is a number of (possibly extreme) policies \( P_\lambda, P_{\lambda+1}, \ldots, P_r \) such that \( \tilde{I}_{j\text{max}} \leq I_c \) for every \( j = \lambda, \ldots, r \).

5. Predictive control for the mitigation of the negative effects of the COVID-19 pandemic. In this section, we apply the method of control described in the introduction to the case of COVID-19 pandemic. As we have commented earlier (see Section 1), the strategy for the mitigation of the pandemic has certain characteristics.

1. The control actions are discontinuous and of unknown effectiveness.
2. The observe mechanism is subjective to human error.
3. The series of control actions cannot be calculated for the whole length of the remaining horizon but only for the current time period.
4. The application of the control actions is non mechanical (i.e. human) and its effectiveness is prone to statistical or other errors.

The above characteristics indicate that a suitable type of control is the proposed one. This control works as follows.

1. We start with the dynamical system \((S(t), I(t), R(t))_{t \geq 0}\) describing the evolution of the COVID-19 pandemic. We fix the horizon \(H\), which in the case of disease of this type is estimated in a few months. We also consider a sequence of sampling time \((t_k)\). These are the moments when one measures the number of infected and recovered people. In the case of COVID-19 this happens on a daily basis.

2. At every step \(k\), the observer measures \(S(t_k), I(t_k), R(t_k)\) and use them to estimate the parameters \(\beta(t_k), \gamma(t_k)\) (i.e. the transmission and recovery rates at the time \(t_k\)). By Equation (12), \(I_{max}^k\) can be predicted for the remaining part of the horizon \([t_k, H]\). Furthermore, the values \(\tilde{I}_{1_{max}} > \tilde{I}_{2_{max}} > \ldots > \tilde{I}_{r_{max}}\) are calculated as described in Section 4.

3. The value \(I_{max}^k\) is compared with the public health system capacity \(I_c\) and the specific thresholds \(\tilde{I}_{1_{max}} > \tilde{I}_{2_{max}} > \ldots > \tilde{I}_{r_{max}}\). Depending on the position of \(I_{max}^k\) in the sequence \(\tilde{I}_{1_{max}} > \tilde{I}_{2_{max}} > \ldots > \tilde{I}_{r_{max}}\), the controller mechanism decides and implements the new control action.

This action is implemented only for the interval \([t_k, t_{k+1}]\). At \([t_{k+1}, t_{k+2}]\) this procedure is repeated and so on until the end of the horizon \(H\).

6. Social distancing SIR model. We now pass to the second objective of the paper, i.e. to develop a feedback control model for the implementation of non pharmaceutical policies for the mitigation of the pandemic. Firstly, we need to modify the SIR model. As it was commented earlier, the assumption that \(b\) (and thus \(\beta\)) is constant is relatively simple and not in accordance with realistic scenarios. Therefore, it cannot capture the trend of the disease and for this reason, in several studies this parameter has been replaced by some more sophisticated time-dependent parameter.

Since this work is primarily aimed at the implementation of social distancing policies, we replace the parameter \(\beta\) with \(\beta = \beta_0(1 - p)\), where:

- \(\beta_0\) is the transmission rate when no mitigation policy has been implemented,
- \(p = p(t)\) is a parameter that reflects the effectiveness of the policies to the mitigation of the transmission rate of the disease.

It is clear that for \(p = 0\) (i.e. no policy is applied), we have transmission rate \(\beta = \beta_0\). On the other hand, when \(p = 1\), then \(\beta = 0\). Therefore, the value \(p = 1\) corresponds to the (rather unrealistic) scenario of total isolation. Consequently, the bigger the \(p\) is, the more intense the mitigation policies are and vice versa.

The SD-SIR (Social Distancing SIR) model is governed by the differential equations (1), (2) and (3) where we replace \(\beta\) with \(\beta_0(1 - p)\). However, we also need a further equation describing the evolution of \(p\) in time. This equation is determined as follows. One of the objectives of the mitigation policies is to restrict the pandemic below the capacity of the NHS (National Health System). Let \(I_c\) denote the capacity of the NHS. Then the need of the severity of the measures should be analogous to the difference \(I_{max}(t) - I_c\), where \(I_{max}(t)\) denoted the estimated (at time
(t) maximum number of infected people. Thus, we have the differential equation:

$$\frac{dp}{dt} = \lambda \cdot (I_{\text{max}}(t) - I_c)$$  \hspace{1cm} (13)$$

where $\lambda$ is a positive constant.

Since, the COVID-19 data are updated on a daily basis, we may revise the above differential equations into discrete time difference equations:

$$S(t + \Delta t) = S(t) - \Delta t \cdot \beta_0 (1 - p) S(t) I(t)$$ \hspace{1cm} (14)$$

$$I(t + \Delta t) = I(t) + \Delta t \cdot \beta_0 (1 - p) S(t) I(t) - \Delta t \cdot \gamma I(t)$$ \hspace{1cm} (15)$$

$$R(t + \Delta t) = R(t) + \Delta t \cdot \gamma I(t)$$ \hspace{1cm} (16)$$

$$p(t + \Delta t) = p(t) + \Delta t \cdot \lambda \cdot (I_{\text{max}}(t) - I_c).$$ \hspace{1cm} (17)$$

7. Feedback control. Feedback control problems involve three type of variables: the inputs, outputs and state variables. The inputs are variables that are in our disposal and we usually can control, whereas the outputs are those variables that we wish to control or regulate, they are measured or predicted, and depend upon the inputs. Finally, the state variables are necessary for describing the dynamic behaviour of the system and the interconnection between the inputs and the outputs. All these variables are related with each other in terms of differential and algebraic equations, which constitute an ideal model of the dynamic behaviour of the system.

In feedback control an objective target has to be met. This objective target usually involves achieving a desirable behaviour of the output variables through a suitable choice of the input variables. In this framework, various variables are measured, the realization of the target is valued, and the input variables are modified or corrected accordingly in order for the target to be achieved.

The feedback control for the implementation of social distancing policies in the case of the pandemic, has the following characteristics. The input parameter is the value of $p$ which reflects the effectiveness of the applied policies. The output variable is the estimated maximum number of infected individuals $I_{\text{max}}(t)$ at time $t$. As state variables are considered the number of $I(t)$, $S(t)$ and $R(t)$ of the SIR model.

The model is essential for the estimation of variables that cannot be observed but are projected on time and also for the accurate calculation of the input correction. The aim of this particular problem is $I_{\text{max}}(t)$ not to exceed or to be close to $I_c$ of the NHS. During the time that the COVID-19 pandemic is active, to achieve such a goal, $I_{\text{max}}(t)$ has to be regulated through a gradual enforcement of NPM, as these are quantified through the variable $p$. The control designer has to select a profile for $p$ so that $I_{\text{max}}(t) - I_c$ is minimized. In this problem the profile depends on two factors: (a) the parameter $\lambda$, (b) how long the feedback is exercised.

The control feedback loop in the case of SD-SIR model is represented in Figure 3 and involves the following steps.

1. The states of the SD-SIR model (with a given $p$) are estimated either continuously or discrete, either by observing or by integrating the model in real data.
2. $I_{\text{max}}(t)$ is predicted and compared each time to $I_c$.
3. The input $p$ is updated via the differential equation $\frac{dp}{dt} = \lambda \cdot (I_{\text{max}}(t) - I_c)$.

8. Implementation of the feedback control. In this section, we apply the feedback control in three different scenarios.
Scenario 1: Uncontrolled. In the uncontrolled problem the different types of measures are not applied, i.e. $p = 0$. Hence, the pandemic evolves without mitigation policies. Figure 4 shows the development of the number $I(t)$ of infected people with respect to this scenario. We observe that the peak occurs in $T = 4$ time units during the outbreak and that the maximum number of infected people is approximated 42% of the total population.

Scenario 2: Applying feedback control and mitigation policies from the beginning. We now simulate the case where mitigation policies and the feedback
control are applied from the beginning of the pandemic. We examine two different profiles. Firstly, we consider $\lambda = 0.0025$. Figure 5 presents the evolution of the number of infected people as well as the evolution of parameter $p$, which represents the effectiveness of measures. We observe that the maximum number of infected people of the pandemic drops from 42% to 30% of the total population, which is translated into 25% improvement. Furthermore, it can be seen that the mitigation measures increase linearly from $p = 0$ to $p = 0.5$ at $T = 7.5$ time units until the end of the pandemic.

![Figure 5](image1.png)

**Figure 5.** The evolution of the number $I(t)$ of infected people (on the left) and of the parameter $p$ (on the right) in the case where a feedback control scenario with $\lambda = 0.0025$ is applied.

For the second profile we consider $\lambda = 0.002$. Thus, the value of $\lambda$ is smaller than its value in the aforementioned scenario. Smaller $\lambda$ means that mitigation policies are not applied so intensively and on the unit time the restrictive measures are more relaxed. Hence, the parameter $p$ increases from $p = 0$ to $p = 0.38$ at $T = 7.3$ time units until the end of the pandemic (see Figure 6). However, as it was expected, the maximum number of infected people drops only to 34% of the total population, that is we have an improvement of only 15% (see Figure 6).

![Figure 6](image2.png)

**Figure 6.** The evolution of the number $I(t)$ of infected people (on the left) and of the parameter $p$ (on the right) in the case where a feedback control scenario $\lambda = 0.002$ is applied.

Scenario 3: Applying feedback control and mitigation policies with delay. We now examine the case where any policies and feedback control are not applied from the beginning of the pandemic but with a delay of $T = 2$ unit times. We
choose the profile $\lambda = 0.002$ for the feedback control. The relevant diagrams of $I(t)$ and $p$ are presented in Figure 7. We observe that the maximum number of infected people is approximately 40% of the total population. Consequently, in this setting the improvement is rather tiny and insignificant compared to the uncontrolled case. Thus, in accordance with other studies (see for example [2]) it follows that it is preferable to apply the social distancing measures from the beginning of the pandemic rather than later on.

**Figure 7.** The evolution of the number $I(t)$ of infected people (on the left) and of the parameter $p$ (on the right) in the case where a feedback control scenario $\lambda = 0.002$ is applied with a delay of $T = 2$ unit times.

9. **Conclusions.** Utilizing systems theory, the present article contributes to the investigation about the spread of the coronavirus disease and the implementation of non-pharmaceutical strategies aiming at the mitigation of the pandemic. We firstly proposed a model of predictive control for the implementation of social distancing measures. This model utilizes the latest trend of the disease in order to estimate the recovery and transmission rates. With this information, the maximum number of infected individuals is estimated. According to the place of this number in a series of thresholds, the suitable control actions is decided and implemented during the next time interval. The procedure is repeated periodically until the end of the horizon.

Subsequently, we considered a feedback control problem in order to contain the pandemic to the capacity of the health care system. As input parameter, we set the parameter $p$ that reflects the intensity of the applied social distancing policies. The output parameter is the maximum number of infected people, to be treated by the NHS. The critical value of the output is the capacity of national health system. By examining several scenarios, we observed that the early implementation of non-pharmaceutical policies can have significant impact on the mitigation of the pandemic. On the contrary, the delayed implementation of the measures restrict their effectiveness and, hence, the effect they bring to the peak of the pandemic is very small. Consequently, it is preferable to apply the social distancing policies from the beginning of the pandemic rather than later on.
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Appendix A. The uncertainty of measurements regarding the COVID-19 throughout the different organizations and universities. The current measurements of the people infected by the COVID-19 pandemic are subject to statistical errors. For example, some part of the real cases are possible to remain undetectable due to mild symptoms. Additionally, not only statistical or human errors occur, but also different methods for calculating the confirmed cases and the cases of the diseased about the COVID-19 take place around the world. As a result, many data banks for the same day and country present different cases (in Figures 10, 8 and 9 below such an issue is presented). Furthermore, countries or organizations may modify their measurements methods during the disease leading to amendment data.

This situation creates some confusion about the clarity of data and the reality of the epidemic. Inevitably, the estimated peak of the disease is affected and, consequently, the decision about the most suitable policy.

However, the present work focuses on the adaptive triggering policies with multiple short term samplings throughout the population for the betterment of the policy taking. The proposed methodology has the advantage that it can be easily adapted to any new data or measurement method: If this method is changed at the time \( t_k \), then from the sampling time \( t_{k+1} \) one starts to use the new data in order to calculate the peak \( I_{k+1}^{\text{max}} \) using Equation (12).

Figure 8. According to WHO there are 4,761,559 confirmed cases. Source: WHO (2020) [17].

Appendix B. The \( b \) parameter. The parameter \( b \), which shows the average contacts of an individual with others per unit time, plays an prominent role in the SIR model, since it determines the transmission rate \( \beta \). In the classical SIR model, it is considered to be time invariant. However, a time dependent parameter \( b \) is much better to capture the disease spread and to predict the future trend.

Due to the gravity of this parameter, there are several ways to estimate it.

(a) By using the least square fitting method to actual data [14].

(b) By using the discrete time time-dependent SIR model with the assumptions that at the beginning of a spread the number of infected persons is very low,
so we can assume that \( S(t) \approx N [3] \). Then, Equation (10) takes the following form:

\[
b(t) = \frac{[I(t + 1) - I(t)] + [R(t + 1) - R(t)]}{I(t)}
\]  

(18)

(c) More sophisticated models divide the population into age classes, indexed by \( \alpha \), to allow for variable transition rates dependent upon age. In this case, one has:

\[
b_\alpha(t) = R_0 \zeta_\alpha M(t)(1 + \varepsilon \cos(2\pi(t - t_{max}))) / t_i
\]

(19)

where \( \zeta_\alpha \) is the degree to which particular age groups are isolated from the rest of the population, \( M(t) \) is the time course of mitigation measures, \( \varepsilon \) is
Table 2. Sample of twenty-one countries from the 267 countries and big cities. For China the city of Beijing was chosen. The $b$ parameter was estimated by using Equation (18).

| COUNTRY    | $b$ parameter |
|------------|---------------|
| SWISS      | 0.257158556   |
| BRAZIL     | 0.233817403   |
| ITALY      | 0.203539698   |
| NORWAY     | 0.197869991   |
| SPAIN      | 0.19306215    |
| EGYPT      | 0.19012821    |
| USA        | 0.189437334   |
| PAKISTAN   | 0.184307553   |
| BELGIUM    | 0.178032037   |
| ETHIOPIA   | 0.167117704   |
| RUSSIA     | 0.160845659   |
| FRANCE     | 0.153958202   |
| UK         | 0.153852808   |
| GERMANY    | 0.153678417   |
| SOUTH KOREA| 0.148665135   |
| SWEDEN     | 0.141608744   |
| BULGARIA   | 0.141558434   |
| CYPRUS     | 0.132396806   |
| GREECE     | 0.120185025   |
| ALBANIA    | 0.109905162   |
| CHINA      | 0.053298278   |

the amplitude of seasonal variation in transmissibility, and $t_{\text{max}}$ is the time of the year of peak transmission [16].

B.1. Range of values $b$ for several countries. Following the second method (see [3]) we can easily estimate the value of $b$ for different countries and find out what is the range of the particular parameter for our sample of twenty-one countries. The data for the calculations were taken from the latest update database of OCHA Services regarding - NOVEL CORONAVIRUS (COVID-19) DATA [15]. By Equation (18) the results of Table 2 are produced. This table shows the average of $b$ values for the whole progress of the outbreak in our sample. The choice of countries was random in order to find a representing value.

Throughout our research it is shown that the transmission rate $b(t)$ follows a declined trend after the 60th day and drives towards zero.

Appendix C. Visual representation of $I_{\text{max}}$. By applying Equation (12) to the above data we will simulate the predicted $I_{\text{max}}$ in time $t$. For the techniques
presented in this paper, the prediction of the maximum infected people for time $t$ is necessary. It is also necessary the short term data gathering from the governments in order to apply the corresponding policies.

Figure 13 shows the evolution of $I_{\text{max}}$ in time $t$. The red line is the best $2^{nd}$ degree polynomial fit line. The negative trend is expected because $I_{\text{max}}$ is the prediction of infected people at each stage. So by applying each policy to prevent the spread of a disease the expectation of infected people $I_{\text{max}}$ decreases.
Figure 13. Daily evolution of the average $I_{\text{max}}$ and the $I(t)$ from 4 countries as an indicative trend, starting the 22/1/2020 and finishes the 8/5/2020. The blue line represent the $I(t)$ and the red the $I_{\text{max}}$. 