Fatigue damage growth in notched laminates

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Abstract: The issue discussed in the current paper deals with experimental and finite element analysis of fatigue problems for notched rectangular multilayered plates. The fatigue damage growth and fracture at holes in reinforced laminates loaded in tension is presented. The analysis concerns the ±45° glass/epoxy laminated plates with several types of holes such as vertical elliptical, horizontal elliptical and circular. Also a few remarks on the static analytical solutions and fatigue live predictions are presented. Here, the initial results are only demonstrated, which are fundamental for the further analysis. The experiments shown that independently on the shape of the hole the fatigue process includes three stages, i.e. the matrix cracking, the delamination between the plies and the final failure. The first stage of the failure can be described with the help of the standard finite element codes.

1. Introduction

A great deal of research has been devoted to a study of the mechanism of fatigue, and yet there is still not a complete understanding of the phenomenon and the effect of fatigue loading on the crack propagation under the complex stress of boundary and loading conditions. The behavior and fatigue degradation of notched composite laminates under static and fatigue loading exhibits many features which usually are not observed for homogeneous materials such as metals. As the most significant features of the fatigue phenomena in composite laminates, one can mention the following:

- during fatigue loading of a notched laminate, high axial shear stresses exist near the notch root; these high shear stresses will cause the material in the vicinity of the notch to be degraded much more rapidly than throughout the rest of the laminate,

Figure 1. Fatigue failure modes in notched unidirectional glass/epoxy laminates (eight layers ±45°) - 1300000 cycles (prior to final failure) – a vertical ellipse.

Figure 2. Fatigue failure modes in notched unidirectional glass/epoxy laminates (eight layers ±45°) – 1500000 cycles (prior to final failure) – a horizontal ellipse.
frequently, the direction of crack growth in a fiber composite laminate is not perpendicular to the applied load, even for perpendicular slit notches (figure 1); it is worth to note that that effect is usually connected with the shape of the hole – Ref. [1] (compare figure 1 and 2),

- static failure modes may be completely different from the mode of failure observed during or after a program of cyclic loading,

- experiments demonstrate evidently the interaction of delamination and cracks (see Ref. [2] and figure 3).

Figure 3. Failure form (splitting) in notched unidirectional glass/epoxy laminates (eight layers ±45°) – a circular hole

Figure 4. A multilayered laminated plate with an arbitrary shaped hole under in-plane loading.

In general, the problems considered herein deal with the rectangular multilayered plates having centrally located holes and subjected to the remote tension – figure 4. Here fatigue damage growth and fracture at holes in reinforced laminates loaded in tension have been presented. In the beginning, a few remarks on the experimental results are demonstrated. Then, the possibilities of the stress estimations around holes for static tension are briefly discussed. At the end of the paper, various models used for the fatigue life predictions are studied and they are supplemented with the finite element results.

2. A short review of the fatigue life prediction methods

2.1. Static analytical solutions

For plates having circular, elliptical or irregular shaped holes under static tension (or compression), the analytical solutions can be found with the use of the Airy function, compatibility equation and complex functions. Lekhnitskii [3] used one mapping function for isotropic plates. Lu et al. [4] introduced three different mapping functions for generally anisotropic plates. The broader discussion of those problems and solutions can be found in Muc et al. [5-8].

2.2. Fatigue life prediction

Fatigue cracks appear in components after a certain number of load cycles at nominal stress levels which are often below the tensile strength of the material. The stress-life method [9-10] was the first approach used in an attempt to understand and quantify metal fatigue. This approach requires extensive experimental work and does not take into account the actual damage mechanisms. The classical S-n correlation is given by an equation of the form:

\[ \log(n) = A - B\log(\text{Seq}) + C - D; \quad R = \frac{S_{\text{max}}}{S_{\text{min}}} \]  

where \( S_{\text{max}} \) is the maximum stress, \( S_{\text{min}} \) is the minimum stress and \( n \) is the life in cycles. The quantities \( A, B, C \) and \( D \) may be viewed as material properties which have to be determined through constant amplitude stress-controlled fatigue testing. It is common practice to define the S-n diagram for constant R-value. Echtermeyer [11] gave the parameters of a standard R=-1 strain-life log-log S-n curve, for mainly [0°, 90°] lay-ups of polyester and vinylester-glass laminates, and some criteria for its use. In the case of the existence of a fatigue limit, eq. (1) is sometimes formulated as:

\[ S = S_{\infty} + B/nx \]
where $S_e$ is the fatigue or endurance limit stress – see Och [12]. Sims and Brogdon [13] used a modified power law relationship to fit various datasets as follows:

$$n = \frac{b}{(S-a+cA-y)}(1/x) \ , \ A= \frac{(1-R)}{(1+R)}$$  \hspace{1cm} (3)

and $a$, $b$, $x$, and $y$ are fitting parameters. Caprino and Giorlea [14] formulated a two-parameter S-n formulation from a strength degradation equation:

$$US - S_{max} = aS_{max}(1-R)(nc-1)$$  \hspace{1cm} (4)

where US means the ultimate strength. Epaarachchi and Clausen [15] have recently expanded this to a three-parameter S-n formulation, which takes into account the dependency of stress ratio influence on dominant fibre angle, and frequency effects:

$$US - S_{max} = aS_{max}(1-\psi)1.6-\psi |\sin\theta|S_{max}/US \ 0.6-\psi |\sin\theta| (nc-1)/fc$$  \hspace{1cm} (5)

The parameters $\psi$, $\theta$, and $f$ quantify the R-value dependence, dominant fibre angle, and frequency, resp. In general, the above methods contain an assumed form of R-value dependency and should only be used with care where this dependency is not experimentally verified. In most cases, the mean S-n curve is used to describe fatigue data. However, for certification purposes it may be worthwhile to describe the S-n data regarding percentiles other than this 50% survivability line – see, e.g., Och [12]. The next group of fatigue life prediction is directly connected with the S-n method although fatigue life is estimated with the use of the fatigue failure criteria or equivalently damage parameter or parameters (in general denoted by the symbol d) [16]. The fatigue life is established with the use of the failure criteria (e.g., in the form of the Tsai-Wu criterion):

$$F_{ij}S_j+S_j=1, \ i,j=1,2,6$$  \hspace{1cm} (6)

however, the coefficients are defined with the use of the three S-n curves. The application of the above method is presented by Muc et al. [17]. In view of the above concepts dealing with the fatigue damage analysis it is worth to mention Reifsnider’s approach—the remaining (residual) strength of a “critical element” governs the life of the entire structure [18]. When composing his construction from individual layers, the designer would like to have design rules that are based on strains rather than stresses. Strains are equal for all layers (in the structural reference frame of a unidirectional glass/epoxy composite. The mechanical properties of the composite are given in table 1.

3. Materials and methods

The samples applied in the fatigue tension experiments were made of 8-layer of unidirectional glass/epoxy composite. The mechanical properties of the composite are given in table 1.
### Table 1. Mechanical properties of unidirectional glass/epoxy composite

| Material                  | Fiber orientation | $E_1$ [GPa] | $E_2$ [GPa] | $G_{12}$ [GPa] | $\nu_{12}$ |
|---------------------------|-------------------|-------------|-------------|----------------|------------|
| Unidirectional glass/epoxy| $\pm 45^\circ$    | 46.4        | 14.9        | 5.2            | 0.27       |

All specimens were made in autoclave under the pressure 0.4MPa and cured at the temperature 135ºC. The dimensions of the laminated plates are shown in figure 4. The total thickness of the laminate was equal to $t=2.2$ mm and the width and the height were equal to $L_x=L_y=250$ mm. The diameter of the circular hole was equal to 50mm, whereas the semi-axes of the ellipses were following: vertical ellipse $a=17.5$ mm, $b=35.7$ mm; horizontal ellipse $a=35.7$ mm, $b=17.5$ mm. The experimental fatigue tests were performed on the MTS Landmark 370 servo hydraulic testing machine with a FlexTest 40 controller and the MTS 793 System Software. The experimental settings of fatigue tests are presented in table 2.

### Table 2. Fatigue tests settings

| Hole                  | Circular | Elliptical horizontal | Elliptical vertical |
|-----------------------|----------|-----------------------|---------------------|
| The average force $P_m$ [kN] | 44       | 38                    | 50                  |
| Amplitude $(P_{max}-P_m)$ [kN] | 4        | 4                     | 4                   |
| Frequency $f$ [Hz]     | 15       | 30                    | 30                  |

4. Results and discussion

The experiments have been carried out for three shapes of cut-outs: circular, horizontal elliptical and vertical elliptical – figure 4 and table 2. As it may be observed the whole process of fatigue failure is identical for all shapes of the holes (figure 5). In general, it is composed of three parts:

- the first stage I is associated with the initiation of the crack (at four points around the hole) and further (rather rapid) development of the stiffness degradation due to the matrix cracking – see figure 1 and 2;
- the second stage II is connected with the gradual (almost linear) increase of the strain that is caused by the delamination failure of the plies – figure 3;
- the last (third) III stage is connected with a rapid increase of strains and the final fatigue failure – see figure 6.

![Figure 5. The process of fatigue failure](image1)

![Figure 6. The final fatigue failure mode of the plate with the circular cutout.](image2)

![Figure 7. The propagation of fatigue failure.](image3)
With the use of the FE modeling or the analytical approach, it is possible to capture the initiation of the first-ply-failure of the notched plate using, e.g., the Hashin or the Tsai-Wu criterion. In the stage I the matrix degradation can also be evaluated with the aid of the FE codes - the representative results of such an analysis are demonstrated in figure 7. However, using the standard FE codes, it is impossible to detect the final fatigue failure of the notched laminates since the classical codes do not incorporate the evaluation and the final failure of the mixed modes that are connected with the simultaneous matrix cracking and delamination modes – see figure 5 and the discussion in Ref. [2]. Briefly speaking, the variety of fracture criteria (Paris, Wallker, Collpriest, Newmann etc. – see Ref. [18]) cannot be applied herein.

5. Summary
In the paper, the problems connected with the fatigue failure process of rectangular plates made of eight plies oriented at ±45°, subjected to axial tension and having circular or two elliptical (vertical and horizontal) holes is discussed from both experimental and theoretical point of view. It is necessary to emphasize that the initial results are only demonstrated which will be fundamental for the further analysis. The experiments show that independently on the shape of the hole the fatigue process consists of three stages, i.e. the matrix cracking, the delamination between the plies and the final (very rapid) failure – fibres cracking and debonding. The first stage of the failure can be described with the use of the standard FE codes. The consistent characterization of the next stages of failure requires further investigations. The possible theoretical approaches and methods of the solution of the latter problem are also discussed herein.

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