Coupling Analysis of Multi-Physical Field of High-Speed Motorized Spindle Based on Fractal Contact Theory

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Abstract. In order to improve the modeling accuracy of the motorized spindle, a comprehensive thermal contact resistance (TCR) modeling method was proposed. The fractal theory was introduced to calculate the joint thermal contact resistance. The W-M function and the M-B fractal model were established to characterize the microscopic contour of the rough surface. The bulk thermal resistance, constrained thermal resistance, micro-gap thermal resistance and overall thermal contact resistance of the rough surface of different materials were discussed. The finite element method (FEM) was used to establish the multi-physics coupling analysis model of the motorized spindle, adding the theoretical calculation of thermal contact resistance in the joint surface of the motorized spindle. The temperature rise and deformation variation of the motorized spindle were simulated under the condition of thermal contact resistance. The simulation results demonstrate the influence of thermal contact resistance on the temperature and thermal deformation of the motorized spindle. The results show that when the thermal resistance of the high-speed motorized spindle is considered, the temperature rises and the deformation increases, and the accuracy of the model is more accurate, which is closer to the actual processing.

1. Introduction

As a key functional part of the machine tool, the high-speed motorized spindle plays an important role in machining accuracy. The thermal error caused by the time variation of the spindle temperature is mainly affected by the machining accuracy of the machine tool. A lot of research shows that the thermal error is the main error source inside the machine tool. In the existing thermal error theory research, most of the effects of thermal contact resistance between rough solid surfaces are neglected, and the results cannot reflect the actual engineering application. Ma [1] analyzed the transfer relationship between multi-variables in the bearing balance equation system, and proposed a Newton-Raphson algorithm with contact angle as iteration variable. The Weierstrass-Mandelbrot (W-M) function in fractal geometry was used to characterize the rough surface morphology of the bearing, and the asperities contact deformation model was established to calculate the contact parameters in the thermal contact resistance model. Zhang [2] used a three-dimensional W-M function to simulate a rough surface, and considered three deformation states, namely complete plastic deformation, elastoplastic deformation and elastic deformation, and established a thermal contact conductance considering the bulk thermal resistance and constrained thermal resistance model. Liu [3] proposed a
closed-loop iterative modeling method for thermal characteristics, considering the influence of thermal contact resistance on the spindle system, analyzed the main heat sources of the spindle, simulated and calculated the boundary conditions of the spindle, and proposed a new geometric mechanical model of thermal contact resistance. Cui [4] proposed the temperature distribution and thermal deformation modeling of the motorized spindle system and the thermal-structural coupling simulation analysis method. The relevant thermal contact resistance of the machine tool was obtained by experiment and theory, and the validity of the method was verified. Zhao [5] proposed a three-dimensional fractal theory based on thermal contact conductance model. The effects of different fractal dimensions and roughness parameters on thermal contact conductance were compared. The results show that the deformation of micro-convex body and gap thermal conductance should not be neglected. In order to get a more accurate model, considering the influence of the thermal contact resistance on the heat transfer of the spindle joint surface, a comprehensive thermal contact resistance prediction model is established in this paper. In general, the thermal contact resistance is a very necessary and important factor in the thermal analysis of spindle.

2. Fractal contact theory

2.1 Microscopic morphology of joint surface

The Weierstrass-Mandelbrot function [6] (W-M function) is often used to characterize the micro-contour of rough surface topography. On this basis, a fractal model of elastic-plastic contact of a single asperities body on rough surface, namely M-B model [7] is established. Its two-dimensional expression is as follows:

\[ z(x) = G^{D-1} \sum_{n=1}^{\infty} f(n^{D-2}) n \cos(2\pi n x) \]

(1)

\[ 1 < D < 2; \gamma > 1 \]

Where, \( z(x) \) is a two-dimensional rough surface contour height function; \( x \) is the displacement coordinate of the rough surface contour; \( D \) is the contour fractal dimension; \( G \) is the contour fractal roughness parameter; \( f \) is the ordinal corresponding to the lowest cut-off frequency; \( n \) is the spatial frequency ordinal; \( \gamma \) is greater than 1 Constant, usually taken \( \gamma = 1.5 \).

As shown in Figure 1. For the two-dimensional microscopic topography of the rough surface, it can be clearly seen that the complexity of the curve increases with the increase of the fractal dimension \( D \). The larger the \( D \), the more complex the contour structure and the richer contour details.

![Figure 1. Two-dimensional topography of random rough surface.](image-url)
Considering the elastic deformation, plastic deformation and elastoplastic deformation of asperities, an M-B modified model based on three-dimensional W-M function is proposed [8]:

\[
2 < D < 3, \gamma > 1
\]

Where, \( z(x, y) \) is a three-dimensional rough surface contour height function; \( M \) is the overlap number of the surface; \( \phi_{m,n} \) is a random phase, and \( \phi_{m,n} \in [0, 2\pi] \); \( n_{\text{max}} \) is a frequency ordinal, and \( n_{\text{max}} = \text{int}[\log(L/L_{L})/\log\gamma] \); \( L \) is the sampling interval.

The MATLAB software is used to simulate the generation of three-dimensional fractal rough surface and contour map. As shown in Figure 2, the larger the \( G \) value of the contour fractal roughness parameter, the larger the surface contour amplitude and the rougher the contact surface.

![Figure 2. Three-dimensional shape and contour map of random rough surface.](image)

2.2 Jointing surface thermal contact resistance mechanism

Thermal contact resistance plays an important role in heat transfer and has a large impact on the spindle system. Ignoring the thermal contact resistance results in inaccurate temperature distribution or thermal deformation calculations. As shown in Figure 3, when the two solid rough surfaces are in contact with each other, asperities of different sizes are formed. When heat flows through the solid surface, the heat flux density shrinks at these discrete points. In addition, the rougher the contact surface, the smaller the contact area, and the more severe the heat flux density shrinks. Therefore, the heat flux is limited by the thermal contact resistance of the solid, resulting in a sharp drop in temperature at the solid contact surface [9].

![Figure 3. Formation mechanism of contact surface thermal resistance.](image)
The rough surface contact between the main shaft and the bearing is calculated as follows according to the thermal contact resistance [10]:

$$ R = \frac{L_g}{A \left[ 2A^*_r \lambda_1 + \lambda_f \left( 1 - A^*_r \right) \left( \lambda_1 + \lambda_f \right) \right]} $$

(3)

Where, $L_g$ is the thickness of the joint surface void; $\lambda_1$, $\lambda_2$, $\lambda_f$ respectively, the joint surface material and the intermediate medium conduction coefficient; A is the nominal contact area; $A^*_r$ is the actual contact area.

When the fractal dimension of the rough surface $1 < D < 1.5$,

$$ A^*_r = \left( \frac{3\sqrt{2\pi}}{4} \right)^{2/(3-D)} \left( \frac{L_n}{G} \right)^{(2D-2)/(3-D)} g_4(D) \left( \frac{F}{E} \right)^{2/(3-D)} $$

(4)

When the fractal dimension of the rough surface $1.5 < D < 2$,

$$ A^*_r = \left\{ \begin{array}{l}
\frac{4}{3\sqrt{2\pi}} \left( \frac{G}{L_n} \right)^{(D-1)/2} \left[ -g_1(D) \left( \frac{2 \alpha}{L_c^*} \right)^{(3-D)/2} + \frac{H}{E} g_2(D) \left( \frac{2 \alpha}{L_c^*} \right)^{(2-D)/2} \right] \right. \\
\left. g_3(D) \left( \frac{F}{E} \right)^{2/(3-D)} \right\}^2
$$

(5)

Where

$$ g_1(D) = \left[ \frac{2-D}{D} \right]^{D/2} \frac{D}{3-2D} $$

$$ g_2(D) = \left[ \frac{D}{2-D} \right]^{(2-D)/D} $$

$$ g_3(D) = \frac{D}{2^{D/2}} \psi^{-1/2} $$

$$ g_4(D) = \left[ g_1(D) g_3(D) \psi^{-1/2} \right]^{D/(3-D)} \left[ \frac{D}{4-2D} \right]^{(3-D)/(3-D)} $$

$$ \psi = \left[ \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right]^{-1} $$

Where $\alpha$ is the critical micro contact area; $H$ is the hardness of the material; $L_n$ is the upper limit of the length of the fractal domain; $F$ is the pressure; $E$ is the equivalent elastic modulus,

$$ E = \left( \frac{1-v_1^2}{E_1} + \frac{1-v_2^2}{E_2} \right)^{-1} $$

$E_1$, $E_2$, $v_1$, $v_2$ are the Elastic modulus and Poisson's ratio of the two materials, respectively.

3. Thermal contact resistance model discussion

In this paper, four materials of Stainless steel, HT300, Copper and Aluminium alloy are taken as examples. The physical properties of the materials are shown in Table 1. Under normal temperature and pressure.

Table 1. Physical properties of four different materials.

| Material       | Elastic modulus $E$ / MPa | Thermal conductivity $kW/(m \cdot K)$ | Poisson's ratio $\nu$ | Hardness coefficient $H$ / MPa |
|----------------|---------------------------|--------------------------------------|-----------------------|-------------------------------|
| Stainless steel| $1.9 \times 10^5$         | 16.2                                 | 0.29                  | 3800                          |
| HT300          | $1.2 \times 10^5$         | 45                                   | 0.25                  | 2300                          |
| Copper         | $1.2 \times 10^5$         | 398                                  | 0.35                  | 1380                          |
| Aluminium alloy| $1.9 \times 10^5$         | 154                                  | 0.33                  | 1400                          |

In this paper, the bulk thermal resistance, constrained thermal resistance and micro-gap thermal resistance of the rough surface of different materials are discussed. The thermal resistance of the four materials under certain fractal dimension $D$ and roughness parameter $G$ is shown in Figure 4, which
shows that under the same load. The materials have different thermal contact resistance. With the increase of load, the bulk thermal resistance, constrained thermal resistance and micro-gap thermal resistance and thermal contact resistance of the four materials are linearly decreasing. The thermal contact resistance of stainless steel material is greater than that of the other three materials, and that of copper material is the smallest. Due to the influence of physical properties of the material on the thermal resistance of the contact, the material with higher hardness has a larger contact gap as the load increases, so that the thermal contact resistance is large. On the contrary, the material with smaller hardness has smaller contact gap and smaller change of thermal contact resistance.

Figure 4. Relationship between thermal resistance and load of four materials.

4. Simulation analysis of motorized spindle

The finite element method is used to simulate and analyze the motorized spindle system. Adding the material properties of the motorized spindle system is the key to analyze the thermal characteristics. After importing the model, the material properties of the motorized spindle are set as shown in Table 2. The boundary conditions are set in the finite element modeling, and when the thermal contact resistance is considered, the calculated value is manually added to the contact surface of the spindle unit. Modeling and simulation are carried out without considering contact thermal resistance and contact thermal resistance respectively.

| Parts         | Density Kg/m³ | Thermal conductivity W/m·k | Specific heat J/Kg·k | Modulus of elasticity GPa | Poisson’s ratio | Linear expansion coefficient |
|---------------|---------------|----------------------------|----------------------|---------------------------|----------------|-----------------------------|
| Spindle       | 7800          | 45                         | 400                  | 210                       | 0.23           | 1.20×10⁻³                   |
| Bearing       | 7850          | 40                         | 400                  | 206                       | 0.30           | 1.25×10⁻⁶                   |
| Bearing block | 7800          | 45                         | 480                  | 120                       | 0.21           | 1.11×10⁻⁵                   |
| Rolling element | 3200        | 35                         | 850                  | 314                       | 0.26           | 3.20×10⁻⁶                   |
| Box body      | 7300          | 45                         | 480                  | 120                       | 0.25           | 1.20×10⁻⁵                   |

The thermal-structural coupling analysis of high-speed motorized spindle system is carried out. The model is imported into the software to divide the CFD hexahedral mesh and add boundary conditions to solve the temperature field and thermal deformation changes when the thermal contact resistance is taken into account and the thermal contact resistance is not taken into account. As can be seen from Figure 5-6. The maximum temperature of the motorized spindle is 34.212°C without considering the thermal contact resistance of the joint surface. The temperature of the stator is lower than that of the rotor, which is due to the circulating cooling sleeve outside the stator, which can reduce the
temperature in real time. Considering the thermal contact resistance of the interface, the maximum temperatures are 37.64 °C, respectively. The average temperature increased by 3.428 °C.

Figure 5. Without considering the joint surface thermal contact resistance temperature field cloud map.

Figure 6. Considering the joint surface thermal contact resistance temperature field cloud map.

As can be seen from Figure 7-8. The maximum axial thermal deformation of the spindle is 0.01554 mm without considering the thermal contact resistance of the joint surface, and 0.020364 mm with considering the thermal contact resistance of the joint surface. The maximum axial thermal deformation of the spindle with considering the thermal contact resistance of the joint surface is 31% higher than that without considering the thermal contact resistance of the joint surface. Therefore, considering the thermal contact resistance, the spindle temperature rises rapidly and the thermal deformation increases, which is closer to the actual temperature and deformation in the process of processing.

Figure 7. Without considering the joint surface thermal contact resistance heat deformation cloud map.

Figure 8. Considering the joint surface thermal contact resistance heat deformation cloud map.
5. Conclusion

Traditionally, the influence of thermal contact resistance on temperature and thermal deformation is often neglected in the thermal analysis and modeling of spindle, which leads to inaccurate modeling. In practice, considering the thermal contact resistance caused by asperities of different sizes between rough contact surfaces of high-speed motorized spindle, the simulation results of motorized spindle are closer to the actual processing situation. The specific conclusions are as follows:

A comprehensive thermal contact resistance analysis model was established, and the influence of thermal contact resistance on spindle temperature field and thermal deformation was analyzed. The simulation results of multi-physical field coupling of the motorized spindle system show that the temperature value of the thermal contact resistance of the spindle considering the joint surface is 3.42°C higher than that without considering the joint surface, and the thermal deformation value is increased by 31%. Therefore, the thermal contact resistance must be fully considered in the thermal analysis of the spindle.

Acknowledgments

The authors gratefully acknowledge the National Natural Science Foundation of China (No.51805012), National Science and Technology Major Project (No.2018ZX04032002) and Beijing Science and Technology Plan Project (No.K2001011201801).

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