Transport properties of quasi-one-dimensional nondegenerate quantum wires formed on the surface of liquid helium in the presence of a normal magnetic field are studied using the momentum balance equation method and the memory function formalism. The interaction with both kinds of scatterers available (vapor atoms and capillary wave quanta) is considered. We show that unlike classical wires, quantum nondegenerate channels exhibit strong magnetoresistance which increases with lowering the temperature.

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I. INTRODUCTION

Electrons trapped on the free surface of liquid helium form a nondegenerate two-dimensional electron system (2DES) whose properties are complementary to the properties of the degenerate 2DES created in semiconductor structures. Considerable experimental and theoretical research has been performed on the quantum magnetotransport in such an almost pure and highly correlated 2DES (for a recent review, see Ref. 2). In the presence of a strong magnetic field applied in the normal direction to the system the electron energy spectrum is squeezed into the set of Landau levels slightly broadened due to the interaction with scatterers. For surface electrons (SE) on liquid helium, the broadening of Landau levels is extremely narrow, usually much smaller than temperature, which is the origin of the unconventional Hall effect observed in this system under different experimental conditions.3,4,5

The SE can be confined in quasi one-dimensional (1D) channels, when the helium surface is curved by capillary forces in the presence of a specially constructed dielectric substrate.6,7 The conducting channels are formed in the valleys of the helium relief because of the strong holding electric field $E_x$ applied normally to the surface. These channels can be considered as the nondegenerate version of quantum wires created in semiconductor structures. In the presence of a strong normal magnetic field, the electron states in the channels resemble the current-carrying edge states of the quantum Hall effect systems.8 There is also an interesting evidence for the self-organized current filaments in the helium microchannels.9

The confining potential affects crucially the energy spectrum of electrons subject to the magnetic field $B$, because it removes the degeneracy of Landau levels. For example, in 2DES the Landau spectrum does not depend on the orbit center coordinate $Y = -l_y^2 k_x$, and it is purely discrete $\varepsilon_{n,k_x} = \hbar \omega_c (n + 1/2)$, where $\hbar k_x$ is the electron momentum along the $x$-direction, $n = 0, 1, 2, \ldots$, $l_B = (\hbar e B/m_e c)$ is the magnetic length and $\omega_c = e B/m_e c$ is the cyclotron frequency defined in terms of the free electron mass $m_e$. In contrast, for the parabolic confining potential $U(y) = m_e \omega_0^2 y^2/2$, the electron energy spectrum under the magnetic field has a continuous term depending on the electron momentum along the channel:

$$\varepsilon_{n,k_x} = \frac{\hbar^2 k_x^2}{2 m_B} + \hbar \Omega \left( n + \frac{1}{2} \right),$$

where

$$m_B = m_e \left( 1 + \frac{\omega_c^2}{\omega_0^2} \right), \quad \Omega = \sqrt{\omega_0^2 + \omega_e^2}.$$  

The orbit center coordinate $Y_{k_x} = -(\omega_c/\Omega)^2 y_{k_x}$ (here $l_y^2 = \hbar/m_e \Omega$ is the typical electron localization length across the channel) is determined by the interplay of the magnetic field and the confining potential. Thus, the magnetic field shifts the electron wave function in the channel $\varphi_{n}(y - Y_{k_x})$ to the left or the right depending on the sign of $k_x$, as shown in Fig. [1]. It increases also the frequency of the discrete part of the electron spectrum ($\omega_0 \rightarrow \Omega$) and the effective mass of charge carriers ($m_e \rightarrow m_B \propto \Omega^2$). Such an unusual behavior of the electron effective mass is very important for channel magnetotransport, localization effects, and the polaronic transition (for a review on the last two topics, see 11).

In this work we report the theory of quantum magnetotransport in Q1D channels for highly correlated electrons, when the electron-electron collision rate $\nu_{ee}$ is much higher than the effective collision frequency $\nu$ due to available scatterers. In this regime, usually realized for SE on helium, the memory function formulation for the electron conductivity12 and the momentum balance equation approach leads to the same conductivity equation. The important advantage of these methods is that the electron conductivity along the channel can be quite generally expressed in terms of the electron dynamical structure factor (DSF) $S(q, \omega)$, which allows to track the origin of the strong dependence of the electron mobility
on the magnetic field. We find that, in contrast with classical wires, the resistivity of the nondegenerate quantum wires formed on the surface of liquid helium is strongly affected by the magnetic field. This effect depends strongly on the temperature. The results obtained here are compared with that found for the pure 2D case.

II. BASIC RELATIONS

A. Electron channel states

In the presence of a strong holding electric field $E_\perp$ directed along the $z$-axis the electrons in the channel are gathered near the minimum of the channel profile which we describe by the semicircular form $z(y) = R(1 - \sqrt{1 - y^2/R^2}) \approx y^2/2R$ if $y \ll R$ (here $R$ is the curvature radius of the helium surface which usually ranges from $10^{-3}$ to $10^{-4}$ cm). As a result, the potential energy of an electron across the channel can be approximated by the parabolic potential $U(y) = m_e \omega_0^2 y^2/2$, where $\omega_0^2 = eE_\perp/m_e R$. As the magnetic field $B$ is applied in the direction parallel to the holding electric field, we use the Landau gauge for the vector potential $A = (-By, 0, 0)$ to take the advantage of the translational invariance along the channel ($x$-axis). The electron motion along the $y$ direction is quantized and the electron energy spectrum has the form given by Eq. (1).

Because the curvature radius is much larger than $l_y$, we restrict our study to the model of a 2DES subject to the oscillatory confining potential $U(y)$. The electron wave function in the channel is defined as

$$\psi_{n,k_x}(x,y) = \frac{1}{\sqrt{L_x}} \exp(ik_xx) \varphi_n[y - Y_{k_x}]/l_y, \quad (3)$$

where $\varphi_n(x)$ is the Hermite functions, $L_x$ is the linear dimension of the channel and, as stated above, $Y_{k_x} = -(\omega_c/\Omega)^2/n^2 k_x$.

In general, we have to include the interaction of the electron spin with the magnetic field. This leads to the energy spectrum

$$\varepsilon_{n,k_x} = \frac{\hbar^2 k_x^2}{2m_B} + \hbar \Omega (n + \frac{1}{2}) + \hbar \omega_c \sigma,$$

where $\sigma = \pm 1/2$ is the spin projection eigenvalue. In the pure 2DES ($\Omega = \omega_c$), the inclusion of spin causes an additional degeneracy of the electron states: the energy levels with $n + 1, \sigma = 1/2$ and $n, \sigma = -1/2$ coincide. For electrons in channels, these levels are split because $\Omega > \omega_c$.

The magnetic field does not modify the nature of the electron motion along the channel, and the continuous part of the electron spectrum has the usual form $\varepsilon_{k_x} = \hbar^2 k_x^2/2m_B$. Still, the magnetic field increases strongly the effective mass $m_B$, and, for electrons with a fixed density (as it is indeed for SE on helium), it reduces the Fermi energy. At low electron densities we can consider only the lowest level with $n = 0$. Then, in the absence of the magnetic field, the Fermi momentum $k_F = \pi n_{ch}/2$ (here $n_{ch}$ is the linear electron density) and the 1D Fermi energy $\varepsilon_F \approx \varepsilon_0$, where $\varepsilon_0 = \pi^2 \hbar^2 n_{ch}^2/8m_e$. The magnetic field splits the lowest level, and for a fixed channel density, the 1D Fermi energy and the total energy of an electron at the Fermi level depend strongly on the energy parameter $\varepsilon_B = \pi^2 \hbar^2 n_{ch}^2/8m_B$ which decreases with $B$ because of the mass enhancement. The total energy at the Fermi-level can be written as

$$\varepsilon_{\text{total}} = 4\varepsilon_B + \hbar (\Omega - \omega_c)/2, \quad (4)$$

$$= \varepsilon_B + \hbar \Omega/2 + \hbar^2 \omega_c^2/16\varepsilon_B, \quad \text{if } \hbar \omega_c < 4\varepsilon_B.$$  

It should be noted that the field dependence of $\varepsilon_{\text{total}}$ is nonmonotonic: the total energy defined above increases at low fields ($\hbar \omega_c < 4\varepsilon_B$), and decreases steadily at high fields ($\hbar \omega_c > 4\varepsilon_B$). The decrease of the Fermi energy and the total energy with $B$ means that for finite temperatures and in the limit of high fields the electron channel with a fixed density eventually becomes a nondegenerate system. Because the interaction with scatterers does not involve the electron spin, we shall disregard it when calculating the scattering matrix elements.

B. Dynamical structure factor

As stated in the Introduction, the equilibrium electron DSF $S(q, \omega)$ plays an important role in the quantum transport theory of highly correlated electrons. We
define it as

$$S(q, \omega) = \frac{1}{N_e} \int_{-\infty}^{\infty} e^{i\omega t} \langle n_q(t) n_{-q}(0) \rangle \, dt,$$

where \( n_q = \sum_r e^{-i q \cdot r} \) is the density fluctuation operator, \( q \) is the 2D wave vector and \( r \) is the 2D position of an electron. For the energy spectrum given by Eq. (1), we can evaluate the average \( \langle ... \rangle \) straightforwardly to obtain

$$S(q, \omega) = \frac{2\pi}{N_e} \sum_{n,n'} f(\varepsilon_{n,k_x}) \left[ 1 - f(\varepsilon_{n',k'_x}) \right]$$

$$\times |\langle n,k_x | e^{-i \mathbf{q} \cdot \mathbf{r}} | n', k'_x \rangle|^2 \delta(\varepsilon_{n,k_x} - \varepsilon_{n',k'_x} + \hbar \omega),$$

where \( f(\varepsilon) \) is the Fermi distribution function.

The SE on liquid helium form usually a nondegenerate system. Therefore we can disregard \( f(\varepsilon_{n',k_x}) \) as compared to 1 in Eq. (6). Then, introducing the notation

$$J_{n,n'}(q_x, q_y) = \int_{-\infty}^{\infty} e^{i q y \tau} \varphi_n(\tau) \varphi_{n'}(\tau - q_x l_y \omega_c / \Omega) / \Omega d\tau,$$

the DSF can be found in the form

$$S(q, \omega) = \hbar \left( 1 - e^{-\hbar \Omega / T} \right) \sqrt{\pi} e^{-\hbar \omega / T} \left| J_{n,n'} \right|^2$$

$$\times \exp \left\{ -\frac{\varepsilon_{q_x} - \hbar \omega - \hbar \Omega(n-n')^2}{4\varepsilon_{q_x} T} \right\}$$

with

$$|J_{n,n'}(q_x)|^2 = \frac{\min(n,n')!}{\max(n,n')!} e^{-x_q} |q_x|^{n-n'} \left| \mathcal{L}_{n-n'}^{\min(n,n')} (x_q) \right|^2,$$

and

$$x_q = \left( q_x^2 + q_y^2 \frac{\omega_c^2}{\Omega^2} \right) \frac{l_y^2}{2}.$$

Here \( \mathcal{L}_n(x) \) are the associated Laguerre polynomials.

The matrix elements \( |J_{n,n'}(q_x)|^2 \) restrict differently the wave numbers \( q_x \) and \( q_y \). For the lowest level \( n = n' = 0 \), we have

$$|J_{0,0}(q_x, q_y)|^2 = \exp \left( -\frac{q_y^2}{2} - \frac{q_y^2 \omega_c^2}{2 \Omega^2} \right).$$

In the extreme case \( B = 0 \) this equation restricts only transverse wave numbers \( q_y \) owing to the channel confining potential. For high magnetic fields \( \Omega \approx \omega_c \) both wave numbers \( q_x \) and \( q_y \) enter into these matrix elements in the same way, which is usual for the quantum magnetotransport in the 2DES. An additional restriction on \( q_x \) appears because of the factor \( \exp(-\varepsilon_{q_x} / 4T) \), but it becomes less important for strong fields when \( m_B \gg m_e \).

The channel DSF \( S(q, \omega) \) exhibit features which are typical for both the free electron gas under zero magnetic field and the 2DES in the presence of a strong perpendicular magnetic field. We first pay attention to the singularity \( S(q, \omega) \propto 1/|q_x| \) and the factor \( \exp(-\varepsilon_{q_x} / 4T) \) which are inherent for free electrons. They concern the \( q_x \)-component only, reflecting the free electron motion along the channel. In the ultra-quantum limit \( T \ll \hbar \Omega \), Eq. (8) can be approximated by the terms with \( n = 0 \). Then the channel DSF is given as a sum of Gaussian terms broadened by the width parameter \( \Gamma^* = 2\sqrt{\varepsilon_{q_x} T} \), which exhibit the resonant behavior as \( \omega - n \Omega \to 0 \). This result is similar to that obtained for the 2D Coulomb liquid under a normal magnetic field. In the limiting case \( \Gamma^* \to 0 \), the DSF is a sum of delta-functions reflecting the singular nature of the 2DES under the magnetic field \( B^\parallel \).

The broadening parameter of the Gaussians in Eq. (8) \( \Gamma^* = 2\sqrt{\varepsilon_{q_x} T} \) can be estimated combining two exponents proportional to \( q_x^2 \) by

$$\frac{q_x^2 l_y^2 \omega_c^2}{2 \Omega^2} + \varepsilon_{q_x} T \equiv \frac{q_x^2 l_y^2}{2},$$

where we have defined

$$l_x^2 = l_y^2 + \frac{\hbar^2}{4m_B T}.$$

Then the typical electron wave numbers \( q_x \sim \sqrt{2/l_x} \). In general \( l_x^2(B) \) is a nonmonotonous function of the magnetic field, as shown in Fig. 2, where we have used the dimensionless parameters \( T/\hbar \omega_0 \) and \( \omega_c/\omega_0 \). We shall see that this behavior affects the channel magnetotransport.

At low temperatures \( (T/\hbar \omega_0 < 0.255) \), \( l_x \) decreases steadily with \( B \), and when \( l_x \approx l_y \omega_c / \Omega \) we have \( \Gamma^* \approx 2\sqrt{T \hbar \omega_0 \omega_c} \equiv 2\epsilon_{E_{ch}} l_y \), where \( \epsilon_{E_{ch}} = \frac{q_x^2 l_y^2 \omega_c^2}{2 \Omega^2} + \varepsilon_{q_x} T \approx \frac{q_x^2 l_y^2}{2} \), and

\[ \frac{q_x^2 l_y^2 \omega_c^2}{2 \Omega^2} + \varepsilon_{q_x} T \equiv \frac{q_x^2 l_y^2}{2}, \]

where we have defined

\[ l_x^2 = l_y^2 + \frac{\hbar^2}{4m_B T}. \]
the frictional force can be quite generally expressed in terms of the electron dynamical structure factor

$$F_{\text{scat}} = \frac{N_e}{h S_A} \sum_q q_x V_q^2 Q_q^2 \times \left[ N_q^{(r)} S(q, \omega_q) + (N_q^{(r)} + 1) S(q, -\omega_q) \right]$$

where \( \omega_q = \sqrt{\alpha/\rho q^{3/2}} \) and \( N_q^{(r)} \) are the spectrum and distribution function of ripplons, \( Q_q = \hbar q/2 \rho u_q \), \( \alpha \) and \( \rho \) are the surface tension and the liquid helium mass density, respectively. In the limit of strong holding electric fields, the coupling parameter \( V_q \) does not depend on the wave number \( V_q \sim e E \). In general we have to include the polarization term \( V_q = e(E_\perp + E_q) \), where \( E_q \) has a quite complicated dependence on the 2D wave number \( q \). The channel DSF \( S(q, \omega_q) \) depends on the average velocity \( u_{av} \), so that the sum over all wave vectors \( q \) entering Eq. (12) is not zero. Usually, the determination of the relationship between \( S(q, \omega_q) \) and \( u_{av} \) is the most difficult part of the transport theory. For highly correlated electrons (\( \nu_{ee} \gg \nu_{eff} \)), there is a great simplification because in the center-of-mass reference frame moving along the channel such an electron system can be described by the equilibrium DSF \( S_0(q, \omega) \). Therefore, in the laboratory frame the frequency argument acquires the Doppler shift and \( S(q, \omega) \sim S_0(q, \omega - q_x u_{av}) \).

In the momentum-balance equation method\(^{18-19} \), \( S_0(q, \omega - q_x u_{av}) \) is usually expanded in powers of \( q_x u_{av} \). Instead, the sum of Eq. (12) can be rearranged and presented in a more convenient form

$$F_{\text{scat}} = \frac{N_e}{h S_A} \sum_q q_x V_q^2 Q_q^2 N_q^{(r)} \times \left[ 1 - e^{\hbar q x u_{av}/T} \right] S_0(q, \omega_q - q_x u_{av}), \quad (13)$$

where we have used the properties of the equilibrium DSF \( S_0(-q, \omega) = S_0(q, \omega) \), and \( S_0(q, -\omega) = \exp(-\hbar \omega/T) S_0(q, \omega) \), which can be seen from Eq. (5). This representation is convenient because the linear theory result can be obtained by disregarding the Doppler correction \( q_x u_{av} \) in the argument of the DSF leading to

$$m_e \nu_{eff}(B) = \frac{1}{S_A T} \sum_q V_q^2 Q_q^2 q_x N_q^{(r)} S_0(q, \omega_q). \quad (14)$$

It is worthy of remark that the effective mass of electrons in the channel \( m_e \) enters into the frictional force only by means of the channel DSF (the free electron mass \( m_e \) in the left-hand part of Eq. (14) is chosen as a convenient proportionality factor). In contrast to the isotropic 2D case, the DSF of the electron channel \( S_0(q, \omega_q) \) depends strongly on the direction of the wave vector \( q \).

The same expression for the effective collision frequency could be obtained by means of the memory function formalism introduced by Götze and Wölfle\(^{20} \). In this approach the quantum conductivity equation looks like

\[ J_x \equiv e N_e u_{av} = N_e e \mu E_{\perp}, \quad \mu = e/(m_e \nu_{eff}), \quad (11) \]

where \( \mu \) is the channel mobility. It is well known that for classical thin wires \( \nu_{eff}(B) \) does not depend on \( B \) (it coincides with the conventional collision rate) and the theory gives zero magnetoresistance. In the following we shall see that there is a strong magnetoresistance for nondegenerate quantum wires.

In order to find \( F_{\text{scat}} \) we can calculate the momentum absorbed by scatterers per unit time \( F_{\text{scat}} = -\dot{P} \). For SE on helium, the only scatterers are helium vapor atoms and capillary wave quanta (ripplons). At low temperatures \( T \lesssim 0.5 \) K the electrons are scattered predominantly by ripplons, because the vapor atom density decreases with cooling at an exponential rate. In this case the interaction Hamiltonian is proportional to the electron density fluctuation operator \( n_q \) and given by

\[ H_{\text{int}} = \frac{1}{\sqrt{S_A}} \sum_q V_q S_q n_{-q}, \]

where \( S_A \) is the surface area, \( V_q \) is the electron-ripplon coupling, and \( S_q \) is the Fourier-component of the surface-displacement operator \( \xi(r) \). Then, following Refs. 2, 15, the fricitonal force acting on an electron at \( y = \sqrt{2T/m_0} \) due to the confining potential. It is interesting that in this limiting case, the temperature and field dependencies of the broadening of the channel DSF \( \Gamma \propto \sqrt{T/E} \) are the same as that induced by strong internal forces in the 2D Coulomb liquid. Therefore, the usual many-electron effects on the channel magnetotransport can be disregarded, if the average internal field of the fluctuational origin \( E_1 \approx 3/\sqrt{T/n_e^{1/2}} \) is smaller than the channel field \( E_{\text{ch}} \approx \omega_0 \sqrt{2m_e T/e} \).

C. Channel magnetotransport

In this work we primarily consider the DC magnetotransport, which means that the frequency of the driving electric field is zero or negligibly low. Therefore, the current across the channel is assumed to be zero \( (J_y = 0 \) and the Lorentz force is balanced by the field of the confining potential). Then for highly correlated electrons, the electrical current along the wire \( J_x \) can be easily found by balancing the force of the driving electric field \( N_e e E_x \) and the frictional force \( F_{\text{scat}} \) on the electron system due to the scatterers. In the linear transport regime the absolute value of the frictional force is proportional to the current or the average electron velocity in the channel \( u_{av} \), and can be written as \( F_{\text{scat}} = -N_e m_e \nu_{eff}(B) u_{av} \).

The proportionality factor \( \nu_{eff}(B) \) is called the effective collision frequency, which can be found by evaluating the momentum loss of the electron system per unit time. Then the current along the channel is given by

\[ J_x \equiv e N_e u_{av} = N_e e \mu E_{\perp}, \quad \mu = e/(m_e \nu_{eff}), \quad (11) \]
an extension of the classical Drude formula, in which the imaginary part of the conductivity relaxation kernel [the memory function $M(\omega)$] plays the role of the effective collision frequency. It should be noted that the approximation for the memory function frequently used in quantum transport equations is actually a high-frequency approximation ($\omega \gg \nu$), even though it usually gives correct results in the whole frequency range. Platzman et al. were the first to apply this approach for the analysis of electron transport, and the effective collision frequency as it is in the semiclassical kinetic equation method.

If the temperature is relatively high ($T \gtrsim 0.7$ K), we have to consider the possibility of electron scattering by helium vapor atoms. Even though the helium vapor atoms represent a sort of impurity-like scatterers, we can disregard quantum localization effects because the electron-electron collision rate is extremely high ($\nu_{ee} \gg \nu$). In the quasi-elastic approximation the above described treatment leads to the following correction to the effective collision frequency induced by vapor atoms:

$$m_e \nu_{\text{eff}}^{(a)}(B) = \frac{3n_a V_a^2 \gamma}{16 T S_A} \sum_q q_x^2 S_0(q, 0),$$

where $n_a$ is the density of vapor atoms, $\gamma$ is the parameter describing the wave function of SE states $\psi_1(z) \propto z \exp(-\gamma z)$, and $V_a$ describes the interaction of a free electron with a single helium atom $V(R - R_a) = V_a \delta(R - R_a)$. Even though the interaction parameter is usually written in the form $V_a = 2\pi h^2 s_0/m_e$ containing the scattering length $s_0$ and $m_e$, the effective mass of electrons in the channel $m_B$ appears only in the channel DSF.

$$m_e \nu_{\text{eff}}^{(a)}(B) = \frac{2 n_a V_a^2 \gamma}{(2\pi)^{3/2} \alpha T^{1/2}} \int_0^\infty dq_x q_x e^{-q_x^2 / 2} \times \int_0^\infty dq_y \left[ V_q^2 / (q_x^2 + q_y^2) \right] e^{-q_y^2 / 2}.$$

We point out that, in contrast with the pure 2D case where $\nu_{\text{eff}}$ and $\sigma_{xx}$ are proportional to $1/\sqrt{T}$, the effective collision frequency [Eq. (17)] is finite in the formal limit $T \to 0$ because $l_\perp^2$ contains the temperature dependent term for $q_x \gg \nu_{ee}$ in the whole frequency range. Platzman et al. [21] results in the quasi-elastic approximation for the memory function frequently used in quantum transport equations is actually a high-frequency approximation ($\omega \gg \nu$), even though it usually gives correct results in the whole frequency range. Platzman et al. [21] results in the quasi-elastic approximation for the memory function frequently used in quantum transport equations is actually a high-frequency approximation ($\omega \gg \nu$).

III. RESULTS AND DISCUSSION

It is instructive to consider the ultra-quantum limit $\hbar \omega \gg T$. In this case, the electron DSF can be approximated by the term with $n = n' = 0$:

$$S(q, 0) \simeq \hbar \sqrt{\frac{\pi}{\varepsilon_{q, T}}} \exp \left( -\frac{q_y^2 l_y^2}{2} - \frac{q_x^2 l_x^2}{2} \right),$$

where the parameter $l_x$ was defined in Eq. (14). We consider only the DC case, and disregard the frequency argument because $\hbar \omega / T$ is quite small and the relevant parameter $\hbar \omega / \Gamma$ is small when $B$ is not too high. In this approximation Eq. (14) turns out to be

$$m_e \nu_{\text{eff}}^{(a)}(B) = \frac{2 n_a V_a^2 \gamma}{(2\pi)^{3/2} \alpha T^{1/2}} \int_0^\infty dq_x q_x e^{-q_x^2 / 2} \times \int_0^\infty dq_y \left[ V_q^2 / (q_x^2 + q_y^2) \right] e^{-q_y^2 / 2}.$$

We can see that only the increase of the effective mass $m_B = m_e \Omega^2 / \omega_0^2$ cannot explain the whole magnetic field dependence of the electron mobility. The field dependence of the effective collision frequency is determined by the interplay of the mass enhancement and the field dependencies of $l_x(B)$ and $l_y(B)$. For example, an additional increase of the channel magnetoresistance appears because of the factor $1/l_\perp^2 \simeq \Omega$ which comes from the scattering matrix elements. Recalling the definitions of $m_B$ and $l_\perp^2$ given above we can find that the low temperature limit $\nu_{\text{eff}}^{(a)}(B) = \nu_{\text{eff}}^{(a)}(B)$ increases with $B$ as

$$\nu^{(a)}(B) = \frac{\omega_0^2}{2\alpha h} \left( \frac{eE_\perp}{\omega_0} \right)^2.$$

For zero magnetic field, $\nu^{(a)}(B)$ tends to $\nu_0 = (eE_\perp)^2 / (2\alpha h)$. Under the condition $4T \Omega / \omega_0^2 \ll 1$, Eq. (15) gives the following asymptote for the channel mobility

$$\mu \simeq \frac{2\alpha h}{m_e e E_\perp^2 \left( \frac{\omega_0}{\Omega} \right)} \left( 1 + \frac{4}{\pi} \left( \frac{T \Omega}{\hbar \omega_0} \right)^{1/2} \right).$$
electron velocity which decreases strongly with number of multiple scattering events is limited by the increases due to the multiple electron scattering. The quasi-2DES. Physically, the effective collision frequency at high fields ($B \to \infty$) the condition $4T\Omega/\hbar\omega_0^2 \ll 1$ breaks down and we have to use the more general expression given in Eq. (18). For different temperatures the magnetic field dependence of this expression is shown in Fig. 4 using the normalized units $T/\hbar\omega_0$ and $\omega_c/\omega_0$. We can see that even at relatively low temperatures ($T/\hbar\omega_0 = 0.1$), $v_{\text{eff}}(B)/v_0$ deviates strongly from the zero-temperature asymptote. It is remarkable that, for $T/\hbar\omega_0 > 0.84$, we observe a negative magneto resistance in the region of low fields. The origin of this unusual behavior is the nonmonotonous field dependence of $I_2^2(B)$ discussed above and shown in Fig. 2. Still, at such high temperatures, we cannot neglect the terms with $n, n' > 0$ in the equation for the channel DSF.

Figure 4 shows how the magnetic field affects the temperature dependence of the resistivity of the electrons in the channel. For zero magnetic field (line 0), the temperature dependence is relatively weak. The magnetic field makes the temperature dependence sharper in the low-temperature range (lines representing $\omega_c/\omega_0 = 1, 2, 3$), which is consistent with the singular nature of the 2DES subject to the normal magnetic field because at high fields ($B \to \infty$) the channel becomes effectively a quasi-2DES. Physically, the effective collision frequency increases due to the multiple electron scattering. The number of multiple scattering events is limited by the electron velocity which decreases strongly with $B$.

At high fields and low temperatures the effective broadening $\Gamma^* \sim \sqrt{2\epsilon E_{\text{ch}}l_B}$ of the channel DSF decreases as $\Gamma^* \propto \sqrt{T/B}$. In this regime the electron channel becomes similar to the 2DES in which the broadening parameter $\Gamma_{n,n'}$ of the DSF is determined by the interaction with scatterers. In the self-consistent Born approximation $\Gamma_{n,n'} \sim \sqrt{(\Gamma_n^2 + \Gamma_{n'}^2)/2}$, where $\Gamma_n$ is the collision broadening of the Landau levels. The collision broadening usually increases with $B$. Therefore, the above results are valid under the condition $\sqrt{2\epsilon E_{\text{ch}}l_B} \sim \Gamma_0$. For the electron-ripplon interaction, the collision broadening has the same temperature dependence ($\Gamma_0 \propto \sqrt{T}$), and the breakdown of the channel magnetotransport equations occurs when the magnetic field is increased beyond the value given by the above condition.

For electron scattering by vapor atoms in the ultracold conditions, Eq. (18) can be evaluated analytically

$$m_e v_{\text{eff}}^{(n)}(B) = \frac{3n_a V_a^{2} m_B^{1/2}}{16\pi T^{3/2} l_y l_z^2}. \tag{22}$$

Using the dimensionless units $\Delta_c = \omega_c/\omega_0$ and $\tau = T/\hbar\omega_0$, the corresponding mobility of the electrons in the channel can be written as

$$\mu_a(B) = \frac{1 + 4\tau \Delta_c^2 / \sqrt{1 + \Delta_c^2}}{(1 + \Delta_c^2)^{7/4}}. \tag{23}$$

In this case the negative magneto resistance originating from the field dependence of $I_2^2(B)$ becomes more prominent for such short-range impurity-like scatterers, as shown in Fig. 5. It should be noted that the negative magneto resistance of the electrons occupying the lowest level ($n = 0$) resembles that observed for quantum localization effects in nondegenerate 2DES.
the condition $\nu_{ee} \ll \nu$, although in the transport regime $\nu_{ee} \gg \nu$ considered here quantum localization effects can be disregarded. However if the parameter $\tau$ is not small, the other terms of the DSF ($n, n' > 0$) are important and the system behaves like the 2DES exhibiting positive magnetoresistance at $\nu_{ee} \gtrsim \nu$. In this case we have to perform numerical calculations.

Typical field dependencies of the channel mobility evaluated numerically using Eqs. (14) and (15) for different $n_{max} = n'_{max}$ which restrict the sums over $n$ and $n'$ are depicted in Fig. 5. For $\hbar \omega_0/T = 0.46$, the electrons predominantly occupy the high energy levels. We observe clearly the negative magnetoresistance only when $n_{max} = 0$ and $n_{max} = 1$. The inclusion of high levels suppresses the negative magnetoresistance of the electrons in the channel, and in the limit $n_{max} \gg 1$ the electron channel shows only a positive magnetoresistance because, as expected, it becomes effectively a quasi-2DES.

For fixed values of the magnetic field, typical temperature dependencies of the channel mobility evaluated numerically including high levels ($n, n' \gg 1$) are shown in Fig. 6. The solid lines of the figure represent the contributions from both scattering mechanisms which interplay at $T \approx 1$ K, while the dotted and dashed lines show the separate contributions from scattering by vapor atoms and ripplons correspondingly. For $T < 1$ K electrons are predominantly scattered by ripplons. In this regime at high magnetic fields, there is a range where the electron mobility $\mu \propto \sqrt{T}$. Nevertheless, at lower temperatures $\mu$ approaches a finite value which decreases strongly with $B$.

It should be noted that in the limit of high $B$, the inelastic effect of the electron-ripplon interaction cannot be disregarded. In this case the expression in the integrand appearing in Eq. (17) contains an additional exponential factor $\exp[-(\hbar \omega_q/\Gamma^*)^2]$ because of the finite frequency argument of the electron DSF $S_0(q, \omega_q)$. Typical ripplon wave numbers and frequencies involved in scattering events increase strongly with $B$, while the effective broadening of the DSF of the electron channel $\Gamma^* = 2\sqrt{\varepsilon_0 T}$ decreases (it decreases also by lowering $T$). Eventually, the energy exchange $\hbar \omega_q$ at a collision becomes comparable with $\Gamma^*$ and the effective collision frequency decreases with cooling the system.

For the pure 2DES realized on the surface of liquid
helium such an inelastic effect was discussed in Ref. 25. It was shown that for \( T > 0.1 \) K, the decrease of \( \nu_{\text{eff}} \) due to inelastic effects becomes important only if \( B > 2 \) T (actually it was observed at \( B = 6.4 \) T). In this paper we consider lower magnetic fields to keep \( \omega_c \) comparable with the confinement frequency \( \omega_0 \). Additionally, \( \Gamma^* \) is assumed to be larger than the collision broadening of the Landau levels. In this range inelastic effects on the channel magnetotransport can be disregarded.

In conclusion, we have shown that nondegenerate quantum wires may constitute a remarkable laboratory for testing the quantum transport theory. In contrast with classical wires, they exhibit strong magnetoresistance (the channel mobility decreases with the field intensity) which is a result of the interplay between the mass-enhancement of channel carriers \( m_B \) and the magnetic-field-induced increase of the momenta exchange (\( h q_x \) and \( h q_y \)) in scattering events. For highly correlated electrons (\( \nu_{ee} \gg \nu \)), we have obtained the general relation between the channel mobility and the dynamical structure factor of such anisotropic electron system. Evaluations performed for particular cases indicate that the effect of a normal magnetic field on the channel mobility is very strong for conducting channels formed on the surface of liquid helium under usual experimental conditions.

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