Aspiration-induced reconnection in spatial public-goods game

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Abstract – In this letter, we introduce an aspiration-induced reconnection mechanism into the spatial public-goods game. A player will reconnect to a randomly chosen player if its payoff acquired from the group centered on the neighbor does not exceed the aspiration level. We find that an intermediate aspiration level can promote cooperation best. This optimal phenomenon can be explained by a negative feedback effect, namely, intermediate aspiration level is able to result in a weak peak of reconnection, which will effectively change the downfall of cooperators and facilitate the fast spreading of cooperation. While insufficient reconnection and excessive reconnection induced by low and high aspiration levels are not conductive to such an effect. Moreover, we find that the intermediate aspiration level can lead to the heterogeneous distribution of degree, which will be beneficial to the evolution of cooperation.

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Introduction. – Cooperation is ubiquitous in biological and social systems, yet, understanding the emergence and maintenance of cooperative behavior is still a major challenge [1]. In order to resolve this puzzle, evolutionary-game theory provides a fruitful framework to address the evolution of cooperation among unrelated individuals [2,3]. For example, the prisoner’s dilemma game [4,5], the snowdrift game [4,6] and the stag-hunt game [7,8] have been intensively studied as the paradigms to investigate cooperative behavior through pairwise interactions. However, some social dilemmas involve group interactions, as can be observed by these phenomena: resource distribution and redistribution, predator inspection behavior, alarm calls, and group defense, health insurance, public transportation and environmental issues. In such cases, public-goods game rather than the game of pairwise interactions seems suited to provide reasonable explanation for the facilitation of cooperation [9–19].

In the original public-goods game consisting of \( N \) players, each player can decide whether to contribute an amount \( c \) to the common pool (cooperation) or not (defection). Whereafter, the overall contributions are multiplied by a multiplication factor \( r \), and then redistributed equally among all the players, irrespective of their initial contributions to the common pool [17]. It is obvious that selfish players are enforced upon to select defection, which can take advantage of the public goods. In the traditional game theory, players are perfectly rational, thus defection becomes the dominant strategy leading to the deterioration of cooperation, which is known as the Tragedy of the Commons [20,21].

Over the past decades, a number of theoretical and experimental evidences have been proposed and investigated to explain the emergence of cooperation. Remarkable mechanisms include kin selection [22], direct and indirect reciprocity [23,24], punishment and reward [25,26], heterogeneous activity [16], group selection [27], voluntary participation [9,10,13], spatial effects [10,28,29], image score effect [30,31], success-driven migration [32], to name but a few. Importantly, apart from the considerable attention paid to the facilitation mechanisms alone, the coevolutionary game also attracts great interest [33–45]. Since it not only reflects the evolving of strategies over time, but also characterizes the adaptive development of the network topologies or the update rules (for a further review see [33]). For instance, in [34–36] the rewiring of existing links was recognized as very beneficial to the evolution of cooperation, the growth of a network had a
positive impact on the evolution of cooperation in [41–43],
and cooperation could also be promoted when the coevolu-
tion of strategies and update rules were considered [44,45].
While in the realistic society, similar viewpoints or
phenomena of coevolution are prevalent as well. Take
some multinational corporations as examples, these corpor-
rations often extend their business to different countries
or regions to pursue their maximal profit. However, once
the profit gained from a country or region is undesirable,
they will withdraw the investment partially or entirely,
and then transfer it to other countries or regions. Inspired
by these actual phenomena and the plentiful achievements
of coevolution, a significative question appears. Namely,
if we consider an aspiration-induced reconnection in
the game, does it boost the evolution of cooperation or not?
In this letter, we propose an aspiration-induced recon-
nection mechanism to study the emergence of cooperation
in the public-goods game. In the game, if player $s$'s
payoff obtained from the group centered on one of its
neighbors does not exceed the aspiration level, the player
will cut the link with the neighbor and rewire the link
to one randomly selected player. Interestingly, by means
of Monte Carlo simulations, we find that such a simple
yet meaningful approach can promote the level of coop-
eration best under a moderate aspiration level, while it
is not conductive to facilitate cooperation for too low or
too high aspiration levels. We give an interpretation to
these observed phenomena by inspecting the process of
evolution and investigating the degree distribution of
the evolved network. Moreover, we examine the universality
of such a mechanism through the variation of game model.

The remainder of this paper is organized as follows. In
the second section, we will first describe the model of
the considered evolutionary game. Then we present the main
results and discussions in the third section. Finally, we will
summarize the conclusion in the fourth section.

Evolutionary game. – We consider the public-goods
game with players located on the nodes ($x$) of the spatial
network. Each player $x$ plays the game with its $k_x + 1$
neighborhoods which center on the player $x$ and its
neighbors, respectively. In such a neighborhood, it
contains a central node and all the nodes that are directly
connected to the central node. The player $x$ can follow
either a cooperator ($s_x = 1$) or defector ($s_x = 0$) strategy.
Then, each cooperator contributes a total cost $c = 1$, which
is shared equally among all the neighborhoods that it
engages. According to ref. [18], the payoff of player $x$ (with
strategy $s_x$) acquired from the neighborhood centered on
the player $y$ is given by

$$P_{x,y} = \frac{r}{k_y + 1} \sum_{i=0}^{k_y} s_i \frac{k_i + 1}{k_x + 1} - s_x$$  \tag{1}

where $k_y$ represents the neighbor number of player $y$, $i$ is
the neighbor of player $y$. Naturally, $s_i$ denotes the strategy
of player $i$, $k_i$ corresponds to its degree or neighbor
number. The total payoff of player $x$ is

$$P_x = \sum_{y \in \Lambda_x} P_{x,y},$$  \tag{2}

where $\Lambda_x$ is the neighborhood of $x$ and itself [17].

As the interaction network, we use a Newman-Watts
small-world network, where a number of long-range links
$N_{add}$ are randomly added to the two-dimensional lattice
with periodic boundary conditions [46]. Initially, each
player is designed either as a cooperator or defector with
equal probability. Players asynchronously update their
strategies in a random sequential order. Before updating
strategy, the randomly selected player $x$ evaluates the
payoff $p_{x,y}$ from the neighborhood centered on $y$ according
to eq. (1). If the payoff $p_{x,y}$ cannot satisfy the aspiration
level $E$ ($E$ is uniform for all the players), player $x$ will
remove the link with neighbor $y$, and then create a
new link to a randomly chosen non-neighbor node from
network (i.e., multiple links are prohibited.) In our model,
we assume that the local interactions with four nearest-
neighbor nodes (the von Neumann neighborhood) do not
engage in the reconnection behaviors. It is because that
these local interactions are determined by the spatial
neighborhoods, and they are reasonably assumed to be
fixed during the whole process [37]. After rewiring, player
$x$ collects its total payoff according to eq. (2), then all the
neighbors of player $x$ acquire their payoffs by means of the
same way as player $x$.

Lastly, the player $x$ will randomly select one of its
neighbors $z$ and adopts $z$'s strategy with a probability
$W(s_z \rightarrow s_x)$ depending on the payoff difference. Namely

$$W(s_z \rightarrow s_x) = \frac{1}{1 + \exp[(P_x - P_z)/K]}$$  \tag{3}

where $K$ denotes the amplitude of noise or its inverse
$(1/K)$, the so-called intensity of selection [47]. In the
limit $K \rightarrow 0$, the strategy of neighbor $y$ is always adopted
provided that $P_y > P_x$. While in the limit $K \rightarrow \infty$ all
information is lost, that is, player $x$ switches to the
strategy of neighbor $y$ by tossing a coin. Furthermore, it
should be noted that this update rule is the sequential
version of the Fermi rule, at variance with some other
works in which updates are made in parallel [17,19]. Since
the effect of $K$ on the evolution of cooperation has been
studied in detail in refs. [19,48,49], we simply set the value
of $K$ to be $K = 0.1$.

Results and discussions. – Results of Monte Carlo
simulations presented below are obtained on a Newman-
Watts small-world network hosting $N = 2500$ players with
average degree $\langle k \rangle = 8$, namely, the number of long-range
links is $N_{add} = 2N$. (By extensive simulations, we find that
our results are robust to the network sizes and the aver-
age degrees.) In each Monte Carlo step (MCS), all players
have the chance of rewiring and updating their strategies
on average. The key quantity for characterizing the coop-
erative behavior is the fraction of cooperators $p_c$, which is
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Fig. 1: (Color online) Fraction of cooperators \( \rho_c \) in dependence on the multiplication factor \( r \) for different values of aspiration level \( E \). Note that the intermediate value of \( E \) can sustain cooperation better than the lower or larger case.

Figure 1 shows the fraction of cooperators \( \rho_c \) as a function of the multiplication factor \( r \) for different aspiration levels \( E \). One can see that, for \( E = 0.0 \) or \( E = 0.2 \), the cooperative phenomenon can be flourished even when \( r \leq 5.5 \) (especially, for the case of \( E = 0.2, \rho = 1 \) when \( r = 5.5 \)), yet for \( E = -0.2 \) or \( E = 0.4 \), cooperative phenomenon appears only for \( r \geq 6 \). These results suggest that there may exist an intermediate aspiration level \( E \), which induces the optimal cooperation. To examine the effect of \( E \) on the evolution of cooperation more precisely, we present \( \rho_c \) in dependence on the aspiration level \( E \) for different values of \( r \) in fig 2. As shown in fig. 2(a), the impact of small \( E \) on cooperation remains marginal, and thus most cooperators could not resist the exploitation against defectors or only a small fraction of cooperators could survive. However, as aspiration level \( E \) reaches an intermediate value, a remarkable increase of cooperation can be observed. By further increasing the aspiration level, the facilitation effect is deteriorated again. These results favor that the intermediate aspiration level can warrant an optimal promotion of cooperation, which is analogous to the so-called coherence resonance [50,51]. For large \( r \) (i.e., \( r \geq 7 \)), as shown in fig. 2(b), the non-monotonic phenomenon is not so distinct as the case of small \( r \). This is actually what one would expect, because, compared with small \( r \) where the sum of cooperators is very limited or zero, large \( r \) can make sure of generating more rewards for the cooperation, even though the network does not evolve.

In order to understand the nontrivial effect of aspiration level on the fraction of cooperators, we study the time courses of reconnection number \( N_r \) and the fraction of cooperators \( \rho_c \) for different values of \( E \) in fig. 3. For \( E = -0.1 \), fig. 3(a) illustrates that due to the fact that individual payoffs are always higher than aspiration level, the initial topology structure of interaction network does not change over time, namely, the number of reconnections always equals zero. At this time, cooperators on the

Fig. 2: (Color online) Fraction of cooperators \( \rho_c \) in dependence on aspiration level \( E \) for different values of multiplication factors \( r \). Note that for \( r \leq 6.5 \) (a), it can be obviously observed that there exists an intermediate aspiration level \( E \) for which the evolution of cooperation is optimal; while for \( r \geq 7 \) (b), this peak is not distinct as the above case because of the fact that large \( r \) can promote cooperation better, despite the invariance of network.

Fig. 3: (Color online) Time courses depicting the number of reconnections \( N_r \) and the fraction of cooperators \( \rho_c \) for different values of \( E \).
initial network cannot resist the exploitation from defectors, which results in the extinction of cooperation (see fig. 3(b)). Whereas for large aspiration level \((E = 0.3)\), most individuals’ payoffs are lower than the aspiration level. As a result, reconnection behavior occurs frequently and individuals will randomly connect to others as time evolves. In such case, the cooperative clusters are fragile and are easy to be destroyed by the invasion of the defectors. Naturally large aspiration level also induces the extinction of cooperation. More significantly, we observe that the immediate aspiration level \((E = 0.1)\) can induce a peak of reconnections which corresponds to the downfall of cooperators. In the very early stages of process, defection yields higher individual benefit, and the outlook for cooperators is gloomy. With the game forward, a few players’ payoffs could not exceed the aspiration level, and thus a small fraction of reconnections will appear. All the reconnections will effectively alter the evolution tide, namely, the downfall of cooperators will transform into the fast spreading of cooperation. Because, in such case, only choosing cooperation adequately can exceed the aspiration level. When cooperation becomes the dominant strategy, individual payoffs will uniformly exceed the aspiration level, which makes the possibility of reconnection vanish, namely, the number of reconnections will become zero over again. Consequently, an intermediate aspiration level can result in a peak of reconnections which promotes the level of cooperation by means of a negative feedback effect. While for low or high aspiration levels, insufficient or excessive reconnections could not provide advantageous conditions for such a feedback effect.

Subsequently, it remains of interest to examine the degree distributions of the evolved networks for different aspiration levels. It has been known that the heterogeneity of host network plays an important role in the substantial promotion of cooperation. If the hub node takes the cooperation (defection) strategy, its strategy will become an example to be imitated by its neighborhood, which increases (decreases) cooperators’ (defectors’) payoffs and results in the great promotion of cooperation \([4,17,18]\). Hence, what we would expect is to present a highly heterogeneous distribution as well. Figure 4 demonstrates clearly the degree distributions in the final steady states. According to the time courses of \(N_r\) in fig. 3(a), we observe that the network does not evolve with time for \(E = 0.1\), so the degree distribution obeys norm distribution with \(\langle k \rangle = 8\) (see fig. 4). While for \(E = 0.3\), long-range links which are generated randomly and excessively make the degree distribution conform to the norm distribution likewise. Interestingly, for \(E = 0.1\), the degree distribution deviates from the norm distribution and the heterogeneity of network is formed (see also the inset of fig. 4 by using a log-log representation), which is consistent with our expectation. The highly heterogeneous distribution induced by the intermediate aspiration level is crucial for the optimal promotion phenomenon of cooperation presented in fig. 2. Therefore, the promotion of cooperation partly attributes to the potential heterogeneous states within the network.

In the above evolutionary game, we assume that each cooperator contributes the same total cost \(c = 1\), which is then shared equally among all the neighborhoods that it engages in, namely, the individual contribution is independent of the number of its social ties. Whereas, in the opposite limit considered in ref. \([18]\), every cooperator separately contributes a cost \(c = 1\) to each neighborhood that it engages. In this case, the total contribution of each
cooperator $x$ is equal to $k_x + 1$. Similarly as in eq. (1), the payoff of player $x$ (with strategy $s_x$) obtained from the neighborhood centered on the player $y$ is given by

$$p_{x,y} = \frac{r}{k_y + 1} \sum_{i=0}^{k_y} s_i - s_x. \quad (4)$$

Interestingly, as shown in fig. 5, there also exists an intermediate aspiration level, leading to the highest cooperation level when $r$ is fixed. Thus, the aspiration-induced reconnection mechanism is robust for promoting cooperation, regardless of the total contribution of cooperators.

Summary. — In summary, we have studied the effect of the aspiration-induced reconnection on cooperation in spatial public-goods game. In the game, if player’s payoff acquired from the group centered on the neighbor does not exceed aspiration level, it will remove the link with the neighbor to a randomly selected player. Through scientific simulations, we have shown that, irrespective of the total contribution of each cooperator, there exists an intermediate aspiration level, for which the promotion of cooperation is optimal. This kind of phenomenon can be explained by means of a negative feedback effect. In the early stages of the evolution process, though cooperators were decimated by defectors, the peak of reconnection induced by the intermediate aspiration level will change the downfall of cooperation and facilitate the fast spreading of cooperation. While for small or large aspiration levels, non-reconnection or excessive reconnection do not provide any possibility for the emergence of such a feedback effect. Moreover, we have analyzed the degree distributions of the network. Of particular interest is the fact that the heterogeneous degree distribution induced by the intermediate aspiration level, warrants a potent promotion of cooperation. Since the phenomena related with aspiration-induced reconnection are abundant in our society, we hope that our results can offer a new insight into understanding these phenomena.

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