QUARK SPECTRUM ABOVE THE CRITICAL TEMPERATURE
FROM SCHWINGER-DYSON EQUATION

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We investigate a spectrum of a fermion, which we call a quark, above the critical temperature of the chiral phase transition in a gauge theory using the Schwinger-Dyson (SD) equation. The SD equation enables us to study the spectrum over a wide range of the gauge coupling. It is shown that the quark spectrum has two sharp peaks which correspond to the normal quasi-quark and the plasmino and is consistent with that obtained in the hard thermal loop approximation in the weak coupling region, while it has also two peaks but with smaller thermal masses and broader widths in the strong coupling region. Temperature-dependence of the quark spectrum is also discussed.

1. Introduction

Recent experimental researches performed at the Relativistic Heavy Ion Collider (RHIC) reveal unexpected findings. Among them, collective flow of the created matter behaves like a perfect fluid, which suggests that the quark-gluon plasma (QGP) near the phase transition is a strongly interacting system. It seems to be consistent with the recent studies in Lattice QCD, showing that the lowest charmonium states survive at temperature ($T$) higher than the critical temperature ($T_c$) of deconfinement. Theoretically, hadronic states in QGP were already predicted many years ago on the basis of symmetry arguments of the chiral phase transition. There are also many recent studies on possible hadronic bound states in QGP motivated by the RHIC experiments.

The study of quarks and gluons as well as the hadronic states is also important, because they are explicitly deconfined degrees of freedom in QGP. Rapid change of the energy density around $T_c$ obtained in Lattice QCD suggests that quarks and gluons actually come into play in thermodynamics. However, particles in medium have, in general, different spectra from those in vacuum, and thus it is quite non-trivial whether quarks and gluons keep the quasi-particle picture in the strongly coupled matter like QGP near $T_c$. It is shown in Ref. that the quark spectrum

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near $T_c$ has two massive modes taking the gluon condensate into account. In Ref. 7, dispersion relations of the quark and the gluon above $T_c$ of the deconfinement transition are obtained in quenched Lattice QCD with the maximum entropy method. They do not find any pure collective modes like the plasmino, and the thermal (pole) masses of the quasi-quark and the quasi-gluon are higher than $T$ near $T_c$. In Ref. 8, using a Brueckner-type many-body scheme together with data from the heavy quark potential in Lattice QCD, the thermal mass of quark is obtained as $m_q \sim 100$ MeV for $T = 1-2T_c$. In Ref. 9, it is shown that the quark spectrum near $T_c$ of the chiral phase transition has three peaks through a coupling with fluctuations of the chiral condensate.

In this paper, we investigate the quark spectrum near but above $T_c$ of the chiral phase transition in a gauge theory, where we call a fermion a “quark”. In a weak coupling region at finite $T$, the HTL resummation is established, and the quark spectrum is well understood at the leading order. In this study, we compute the quark spectrum in the chiral restored phase over a wide range of the coupling constant, especially in the strong coupling region, using the Schwinger-Dyson (SD) equation. The SD approach has been employed in analyses of the strong coupling gauge theory in vacuum and in the determination of the phase structure of the chiral and color superconducting transitions at finite $T$ and/or density.

The SD equation used in this paper includes HTL of the quark self-energy at the leading order, and thus reproduces the quark spectrum of HTL in the weak coupling region. In the strong coupling region, on the other hand, the SD equation incorporates nonperturbative corrections as an infinite summation of a certain kind of diagrams. In addition, it is one of advantageous points that the SD equation respects the chiral symmetry and describes the dynamical breaking of it, while it is a heavy task to respect the chiral symmetry on the lattice. Furthermore, it is straightforward to extend our formulation to finite density.

The contents of this paper are as follows. In Sec. 2, we formulate the SD equation in the imaginary time formalism with the ladder approximation. Then we perform an analytic continuation of the quark propagator to the real-time axis. We perform it numerically with a method of an integral equation. Using the retarded quark propagator obtained in this way, we investigate the quark spectrum above $T_c$, focusing on dependences on the gauge coupling and $T$, in Sec. 3.

### 2. Schwinger-Dyson Equation

The SD equation in the ladder approximation is given by

$$S^{-1}(i\omega_n, \vec{p}) - S_{\text{free}}^{-1}(i\omega_n, \vec{p}) = \frac{2T}{3} \sum_{m=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} g^2 \gamma_{\mu} S(i\omega_m, \vec{k}) \gamma_{\mu} D_{\text{free}}^{\mu\nu}(i\omega_n - i\omega_m, \vec{p} - \vec{k}),$$

where $S_{\text{free}}$ and $D_{\text{free}}^{\mu\nu}$ are the free massless fermion and free gauge boson propagator, respectively. We employ the imaginary time formalism and $\omega_n = (2n + 1)\pi T$.
Matsubara frequency for fermion. $S$ stands for the full fermion propagator which is written as

$$S(i\omega_n, \vec{p}) = \frac{1}{C(i\omega_n, p) i\omega_n \gamma_0 - A(i\omega_n, p) \vec{p} \cdot \vec{\gamma} - B(i\omega_n, p)}$$

from the rotational invariance in space and the parity invariance. The gauge coupling $g$ is not running in this study, and thus an ultraviolet cutoff comes in. We scale all the dimension-full quantities by a three-momentum cutoff.

At zero $T$, the Landau gauge is often chosen in the SD equation with the ladder approximation, because it satisfies the Ward-Takahashi identity in the case of QED. At finite $T$, on the other hand, any constant gauge-fixing parameter including the Landau gauge does not satisfy the Ward-Takahashi identity. We do not pursue this issue here and adopt the Feynman gauge in the following, which makes the analytic continuation simple.

To study the quark spectrum, we need the analytic continuation of the solutions of Eq.(1) obtained on the imaginary axis in the complex energy plane to the real axis. In this study, following a method proposed by Ref. 14, the analytic continuation is done numerically. The retarded quark Green function $S_R(p_0, \vec{p})$ is then obtained by solving a following integral equation:

$$S_R^{-1}(p_0, \vec{p}) = S_R^{-1\text{free}}(p_0, \vec{p})$$

$$S_R(p_0, \vec{p}) = \frac{4}{3} \int \frac{d^3 k}{(2\pi)^3} \int_{-\infty}^{\infty} dz g^2 T \frac{\gamma_0 + \vec{\gamma} \cdot \vec{p}}{p_0 - z - i\omega_m} \gamma_\nu \rho_\mu^\nu_B(z, \vec{p} - \vec{k})$$

$$S_R(p_0, \vec{p}) = \frac{4}{3} \int \frac{d^3 k}{(2\pi)^3} \int_{-\infty}^{\infty} dz g^2 \gamma_\mu S_R(p_0 - z, \vec{k}) \gamma_\nu \rho_\mu^\nu_B(z, \vec{p} - \vec{k}) \frac{1}{2} \left[ \tanh \frac{p_0 - z}{2T} + \coth \frac{z}{2T} \right]$$

where $\rho_\mu^\nu_B$ stands for the spectral function for the gauge boson, and $S$ is the solution of Eq.(1). This method is known as a reliable way of the analytic continuation in condensed matter physics[14] and suitable for the SD approach because the SD equation Eq.(1) is also an integral equation which is similar to Eq.(3).

3. Quark spectrum

The quark spectral function is given by

$$\rho_\pm(p_0, p) = -\frac{1}{\pi} \text{ImTr} \left[ \frac{\gamma_0 \pm \vec{\gamma} \cdot \vec{p}}{2} S_R \right]$$

where $\rho_+(\rho_-)$ denotes the quark (anti-quark) spectrum at zero $T$, and in addition, the anti-plasmino (plasmino) spectrum, if they exist at finite $T$. We show only $\rho_+$ in the following, because there is a relation of $\rho_-(p_0, p) = \rho_+(p_0, -p)$. We first investigate the quark spectrum in the weak coupling region. The spectral function $\rho_+$ for $p = 0$, $g = 0.1$, and $T/\Lambda = 0.3216$ is shown in Fig.1. We see that $\rho_+$ has two sharp peaks: One is in the positive energy region and corresponds to
the normal quasi-quark, and the other in the negative region to the anti-plasmino. From the relation between \( \rho_+ \) and \( \rho_- \) mentioned above, the quasi-antiquark and the plasmino also appear. Positions of these peaks with the positive energy for finite momentum are shown in Fig. 2, which give approximately dispersion relations of the modes. We see that the dispersion relations obtained from the SD equation are in good agreement with those in the HTL approximation which is valid in the weak coupling limit. With the narrow widths of the peaks, this means that the spectrum from the SD equation reproduce that in the HTL approximation correctly in the weak coupling region.

For the strong coupling, we plot the coupling dependence of the spectral functions \( \rho_+ \) for \( T/\Lambda = 0.3216 \) in Fig. 3. There also exist two peaks but with broad widths, showing that the normal quasi-quark and the plasmino appear even in the strong coupling region. We see that as the coupling is larger, the widths are broader. It could be understood from the fact that probability of gluon emission and absorption from a quark increases with the coupling. In fact, these peaks obtained at the one loop order without the HTL approximation are already broad in the strong coupling region. In an extremely strong coupling, e.g. \( g = 6 \), the peaks are so broad for low momentum that the quasi-particle picture is no longer valid.

As momentum is higher, the peak of the plasmino is smaller, which is consistent with the momentum-dependence of the strength of the plasmino in the HTL approximation. On the other hand, the peak of the normal quasi-quark is sharper for higher momentum. This is because thermal effects get smaller as momentum increases, and thus the spectrum approaches that at zero \( T \).

Positions of the peaks for \( p = 0 \) give approximately the thermal masses of the modes. In Fig. 4 we plot the coupling dependence of the thermal mass. We note that the thermal masses of the normal quasi-quark and the plasmino are same in the chiral limit at zero density. In Fig. 4 we also plot the thermal mass obtained at the one-loop order with and without the HTL approximation. This shows that the thermal mass in the weak coupling is proportional to the coupling \( g \) and coincides
with the one obtained in the HTL approximation. In the strong coupling region, on the other hand, the thermal mass is almost independent of the coupling \( g \) and expressed as \( M \sim T \). We have checked that this property is independent of \( T \). As a result, the thermal mass is smaller than the one obtained in the HTL approximation and at the one-loop order without the HTL approximation.

Finally, we mention the \( T \)-dependence of the quark spectrum in the strong coupling region. We see that the thermal mass is proportional to \( T \), which means that it is determined only by the infrared dynamics characterized by \( T \). The widths of the peaks are broader as \( T \) increases, because damping effects resulting from the interaction with thermal particles are more significant for higher \( T \).

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