Photo-galvanic effect in asymmetric quantum wells

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Abstract. In this work we studied the charge carriers behaviour in quantum structures where the symmetry with respect to space coordinates and time-reversal symmetry are broken simultaneously. As the models of such structures we considered finite triangular as well as finite semiparabolic quantum wells placed in external magnetic field. We have shown by numerical analysis that the energy spectra of charge carriers in such structures are anisotropic with respect to in-plane (transverse) motion $\epsilon_n(+)\neq \epsilon_n(-)$. This leads to the anisotropy of charge carriers in-plane momentum transfer which, in its turn leads to the anisotropy of photocconductivity $\sigma(+)\neq \sigma(-)$ and as it follows from our calculations, the effect though not very great, could be measurable for the magnetic field of about few T.

1. Introduction
Since the time of Emmy Noether we know how important role plays the idea of symmetry in Physics. It is interesting enough that the symmetry breaking is almost equally important. Perhaps the first person who stated it clearly was Pierre Curie: "C'est la dissymétrie qui crée le phénomène". The same idea was stressed by late Prof. B. Wybourne: "Every symmetry can be considered as a statement that a certain experiment is impossible. If the experiment is possible then the symmetry must at least be broken" [1].

Several propositions to observe a number of new effects in asymmetric quantum structures were recently discussed in scientific literature [2-4]. Thus, in Ref.[2] it was suggested that if the symmetry with respect to the space coordinates and time-reversal symmetry are broken simultaneously in the nanostructures, then some photovoltaic and magnetolectric effects have to occur in them. As an example of such structure, in Ref. [2] the asymmetric double-quantum well (QW) in an external magnetic field was considered. In this model, the space asymmetry is introduced by the $\delta$-barriers of different heights, while external magnetic field parallel to the layers provides time-invariance breaking. Later on in [5], the photogalvanic effect in an asymmetric undoped system of three GaAs/AlGaAs quantum wells was studied experimentally and the predictions by [1] concerning this effect were in general confirmed. In Ref. [6], the anisotropy of electron momentum transfer occurring in the infinite triangular quantum well in an external magnetic field due to electron-phonon interaction was considered theoretically. The models discussed in Refs [2,6], despite of their remarkable insights should be emended however, since the model of $\delta$-barriers, as wells as the infinite triangular quantum well, are not very realistic. So, the aim of this work is twofold: first, to make some revision of the model proposed in [6] and to study more realistic models of finite triangular as well as semiparabolic QWs which could be produced by means of different modern techniques such as, for instance
MBE-technology [7]; second, to analyze the anisotropy of photo-conductivity which can occur in such QW in an external magnetic field.

2. Finite triangular quantum well in an external magnetic field
To treat the two-dimensional electron gas in an external magnetic field, we start with conventional approach based on effective mass equation of the form [8]:

\[ E_c + \frac{(i\hbar \nabla + eA)^2}{2m^*} + U(z) \psi(r,z) = E \psi(r,z), \]

where \( E_c \) stands for the bottom of the semiconductor conduction band, \( A \) -vector potential, \( e \) and \( m^* \) are the electron charge and effective mass, respectively, \( r = (x,y) \) is in-plane two-dimensional vector. Let us take the gauge: \( A = (Bz,0,0) \) and assume the potential \( U(z) \) to be:

\[ U(z) = \begin{cases} U_0, & z \leq 0 \\ eEz, & 0 < z \leq d, \quad |eEd| = U_0, \end{cases} \]

where \( E \) is the electric field. This electric field arises, for instance, at the interface of a heterostructure, in this way making a QW.

Suppose also that the electron wave function is of the form

\[ \phi_k(x) = C \varphi(k_x,z) \exp(ik_xx + ik_yy), \]

where \( C \) is the normalizing constant and in-plane wave vector \( k = (k_x,k_y) \) Then, by means of (1)-(2) and the last formula, one obtains the following ordinary differential equation for the \( \varphi(k_x,z) \)-function:

\[ -\frac{\hbar^2}{2m^*} \frac{d^2 \varphi(k_x,z)}{dz^2} + \frac{\hbar^2 k_x^2}{2m^*} + \frac{heBk_xz}{m^*} \frac{\hbar^2 k_y^2}{2m^*} + (eBz)^2 + eEz - \epsilon \varphi(k_x,z) = 0, \]

where \( \epsilon = E_c + \epsilon(k_x) - \frac{\hbar^2 k_x^2}{2m^*} \).

Treating the infinite triangular QW and using the analytical solutions of corresponding Schrödinger equation, the author of Ref.[6] has come to the conclusion that the electron spectrum is anisotropic with respect to transverse motion, that is \( \epsilon(v_x) \neq \epsilon(-v_x) \). According to [6], the physical reason for that is this: as an electron travels at the velocity \( v_x \), the Lorentz force acts on it in the direction \((-z)\), with the result that the maximum of the electron wave function is shifted in the direction \((-z)\). If the electron velocity changes to \(-v_x\), the Lorentz force reverses direction and the maximum of the electron wave function shifts to the direction \((z)\). Hence in asymmetric potential \( U(z) \neq U(-z) \) the electron energy \( \epsilon(v_x) \neq \epsilon(-v_x) \). The claim is correct in principle and in case of finite triangular QW the physical explanation of such asymmetry is similar and even a bit simpler. It immediately follows from (3) that the energy spectrum of electrons in QW depends on the direction of electron movement along \( x \)-axis. Indeed, since the electrons moving along \( x \)-axis in opposite directions, are shifted by Lorentz force in opposite directions with respect to \( z \)-axis, they behave as if they were in QWs of different "effective" depths and hence, even without any calculations, one infers that \( \epsilon_n(k_x) \neq \epsilon_n(-k_x) \), where \( n \) is the corresponding quantum number.

Having in mind of what was said above concerning \( \epsilon_n(k_x) \neq \epsilon_n(-k_x) \), it is convenient, in what follows, to consider two equations of the form:

\[ \frac{d^2 \varphi(\zeta)}{d\zeta^2} = (\zeta - \tilde{\epsilon}_1)\varphi(\zeta), \quad \frac{d^2 \varphi(\zeta)}{d\zeta^2} = (\zeta - \tilde{\epsilon}_2)\varphi(\zeta), \]

(4)
where \( \zeta, \tilde{\epsilon}_1 \) and \( \tilde{\epsilon}_1 \) are dimensionless quantities which are equal: \( \zeta = z/z_{01} \) for the first of equations (4) and \( \zeta = z/z_{02} \) for the second one; 
\[
z_{01} = \left[ \frac{\hbar^2}{2m^*} (eE + eB\hbar k_x/m^*) \right]^{1/3}, \quad z_{02} = \left[ \frac{\hbar^2}{2m^*} (eE - eB\hbar k_x/m^*) \right]^{1/3}; \quad \tilde{\epsilon}_1 = \epsilon/\epsilon_{01}, \quad \tilde{\epsilon}_2 = \epsilon/\epsilon_{02},
\]
while
\[
\epsilon_{01} = \left[ \left( (eE + eB\hbar k_x m^{*-1})/2m^* \right) \right]^{1/3}, \quad \epsilon_{02} = \left[ \left( (eE - eB\hbar k_x m^{*-1})/2m^* \right) \right]^{1/3}.
\]

In deriving Eqs. (4) we also took into account the condition \((e^2B^2d/2m^*)/((eB\hbar k_x/m^*) + eE) \ll 1\) which plays the role of small parameter and is valid for the reasonable values of \(d\) and \(B\). For instance, at the electric field \(E = 2.12 \times 10^7\) V/m, the width of QW \(d = 15\) nm and magnetic field \(B = 1\) T this parameter is 0.063 and for the values \(E = 3.18 \times 10^7\) V/m, \(d = 10\) nm, \(B = 2\) T the above mentioned parameter is 0.085.

The existence of such parameter enables to neglect the term \(\sim B^2\) in the Eqs.(4).

In Fig.1 an electron energy in the ground state of finite triangular QW is plotted versus \(|k_x|\); it is clearly seen that definitely, starting from some values of \(|k_x|\), \(\epsilon (+k_x) \neq \epsilon (-k_x)\). In the same picture the second derivatives of energy \(\epsilon (\pm k_x)\) with respect to \(k_x\) are also shown; notice, that they are represented by straight lines up to the \(k_x = k_F\), where \(k_F\) is the Fermi wave vector.

![Figure 1. Electron energy in the ground state of finite triangular QW and its second derivative versus \(|k_x|\)](image)

3. Anisotropy of the photoconductivity

The anisotropy of energy spectrum \(\epsilon_n (+k_x) \neq \epsilon_n (-k_x)\) suggests that the anisotropy of electron momentum transfer could occur under light absorption within this QW and in an external magnetic field. Indeed, suppose that the light linearly polarized along \(x\)-axis with the wave vector \(k_{\text{phot}} = (0, 0, k_z, \text{phot})\) is incident on the semiconductor structure containing finite triangular QW (Fig.2). That the second derivatives of \(\epsilon (\pm k_x)\) with respect to electron wave number \(k_x\) are the straight lines withing the wide range of \(k_x\)-values, enables to characterize the electron dynamics by means of ‘renormalized’ effective masses (see below), which are different for \((-x)\) and \((+x)\).
Then, despite the fact that absorbed photons supply the electrons with \( +k_x \) and \( -k_x \), the *same momentum*, the electrons’ momenta corresponding to \( \langle -x \rangle \) and \( \langle +x \rangle \) are different, because the renormalized effective masses corresponding to \( +k_x \) and \( -k_x \) are different. Defining the renormalized effective masses as

\[
\tilde{m}_n^*(\pm k_x) = \frac{\hbar^2}{d^2 \epsilon_n(\pm k_x)}
\]

and calculating them numerically, we can see that \( \tilde{m}_n^*(+k_x) \neq \tilde{m}_n^*(-k_x) \). We term these effective masses ’renormalized’ and denote them by tilde, because they depend on magnetic field \( B \) as well as an electric field \( E \).

**Figure 2.** Schematic representation of the structure containing triangular QW and the optical transitions

Proceed now to the analysis of anisotropy of photoconductivity, induced by the polarized light beam incident on the structure, as it is shown in Fig. 2. Here two scenarios are possible. The first one corresponds to optical inter-subband transitions between the states of two triangular QWs, one for the electrons in conduction band and another one for the holes in valence band. The second scenario corresponds to optical transitions between valence band and the states of electrons in triangular QW in the conduction band (see Fig. 2).

Using the standard approach (see, for instance [9]), one gets the general expression for the real part of the photoconductivity which we denote by \( \sigma \):

\[
\sigma = \frac{\pi e^2}{m^2 \omega} \frac{2}{\Omega} \sum_{i,j} | \langle j|e \cdot \hat{p}|i \rangle |^2 [f(E_i) - f(E_j)] \delta (E_j - E_i - \bar{\hbar} \omega),
\]

where, since the electromagnetic wave is polarized along \( z \), \( e = (0, 0, 1) \) and \( e \cdot \hat{p} = -i\hbar \partial / \partial z \). The factor of 2 in front of the summation is for spin, \( f(E_i), f(E_j) \) are the corresponding Fermi-factors and \( \Omega \) stands for the volume of the system. Then doing in a similar way as in Ref.[9], after some manipulations which include the summation of Dirac-comb, one gets the next expression for the inter-subband transitions in triangular QWs:

\[
\sigma^\pm = \frac{\pi e^2}{m^2 \omega} \sum_{n,m} |e \cdot \mathbf{p}_{n,m}|^2 |\langle cn|vm \rangle|^2 \left( m_e \tilde{m}^*(\pm) / \pi \hbar^2 \right) \Theta [\hbar \omega - (E_g + \epsilon_{cn} - \epsilon_{vm})]. \tag{5}
\]
Here
\[ e \cdot \mathbf{p}_{cn,vm} \langle cn|vm \rangle = e \cdot \mathbf{p}_{cn,vm} \int \varphi_{cn}^*(z) \varphi_{vm} dz \approx \langle cn|e \cdot \hat{p}|vm \rangle, \]
where 'c' and 'v' stand for the conduction and valence bands, \( n \) and \( m \) enumerate the bound states within the QWs and \( \mathbf{k} \) stands for the in-plane (transverse) wave vector, while the matrix element \( e \cdot \mathbf{p}_{cn,vm} \) depends on the nature of Bloch functions and on the polarization \( \mathbf{e} \); \( \Theta(x) \) is the step function and the \( \pm \)-superscripts of \( \sigma \) and \( \tilde{m}_{cn,vm} \) correspond to \( +k_x \) and \( -k_x \), respectively.

The main difference between formula (6) and the analogous formula from Ref.[9], is that here, instead of reduced effective mass \( m^*_{cv} \) we have \( \tilde{m}_{cn,vm}^\pm \), which are defined as follows:

\[ (\tilde{m}_{cn,vm}^+)^{-1} = (\tilde{m}_{cn}^+)^{-1} + (\tilde{m}_{vm}^+)^{-1}, \]
for the first scenario and
\[ (\tilde{m}_{cn,vm}^-)^{-1} = (\tilde{m}_{cn}^+)^{-1} + (m_h^b)^{-1}, \]
for the second one. Here \( \tilde{m}_{cn}^\pm \) and \( \tilde{m}_{vm}^\pm \) are the renormalized effective masses of charge carriers in the QWs of conduction and valence bands, respectively, while \( m_h^b \) is the usual hole effective mass.

The results of our calculations corresponding to the first scenario are presented in Fig.3 as the \( \Delta\sigma_1(B)/\sigma_1(0) \)-curves plotted versus \( B \) for different \( d \), that is the different QW-widths.

Here \( \Delta \sigma = \sigma^+ - \sigma^- \) and \( \sigma(0) \) is the usual photoconductivity at zero magnetic field. The results for optical transitions between valence band and the states of electrons in triangular QW in the conduction band are very similar to that ones shown in Fig.3.

4. Finite semiparabolic quantum well in an external magnetic field
Let us notice that the galvano-magnetic effect described above is universal in the sense that it could be observed in QW of different shapes, if only the QW-potential is asymmetric with respect to the space-coordinates inversion. Indeed, the fact that \( \epsilon_n(+-k_x) \neq \epsilon_n(-k_x) \) means the existence of charge current along \( x \)-axis, whose density in some quantum state \( n \) can be defined as:
\[ j_x = -\frac{e}{\hbar} \frac{1}{dk_x} \frac{1}{dh} \frac{d\epsilon_n(+-k_x)}{dk_x} - \frac{1}{\hbar} \frac{d\epsilon_n(-k_x)}{dk_x}. \]
Remember however, that after integration over all populated states with taking into account the equilibrium distribution function, the total current along \( x \)-axis has to become zero. The non-zero current is possible, only if there is some external energy source which gets the system out of equilibrium and in the example considered in Sec.3 it was the source of light.

In order to verify the conjecture concerning the universality of the effect, proceed to the semiparabolic QW. In case of semiparabolic QW, the Eq.(3) reads:
\[ \left( \frac{\hat{p}_x^2}{2m^*} + \frac{(eBz + \hbar k_x)^2}{2m^*} + \alpha z^2 - \varepsilon \right) \varphi(k_x, z) = 0. \]

Here \( 0 \leq z \leq d \), \( d \) is the QW-width while the coefficient \( \alpha \) can be defined as follows:
\[ \alpha = \frac{1}{2} m^* \omega_0^2 = U_0/d^2 \quad \text{and} \quad \omega_0^2 = 2U_0/d^2 m^*, \]
where \( U_0 \) is the QW-depth. Introduce also two additional auxiliary parameters, \( \omega_c = |eB|m^*, \omega_0^2 = \omega_c^2 + \omega_0^2 \); then instead of Eq.(6) we arrive at the following effective mass equation:
\[ \left( \frac{\hat{p}_x^2}{2m^*} + \frac{1}{2} m^* \omega_0^2 \right) (z + \frac{\omega_c^2}{\omega_0^2} z_k^2 - \varepsilon_1) \varphi(k_x, z) = 0, \]
where \( \varepsilon_1 = \varepsilon - \frac{1}{2}m^*(\omega_0^2\omega_c^2/\omega_{0c}^2)z_k^2 \), \( z_k = \hbar k_x/|e|B \). Introducing new variable \( \zeta = \sqrt{m^*\omega_{0c}/\hbar z} \), one gets the next equation to be solved numerically:

\[
\frac{d^2}{d\zeta^2} \left( \zeta + \frac{\omega_c^2}{\omega_{0c}^2} \zeta_k^2 \right)^2 + \tilde{\varepsilon}(\zeta, \zeta_k) = 0.
\]

Here \( \zeta_k = \sqrt{m^*\omega_{0c}/\hbar z} \), \( \tilde{\varepsilon} = 2\varepsilon_1/\hbar\omega_{0c} \), and \( 0 \leq \zeta \leq \zeta_0 = \sqrt{m^*\omega_{0c}/\hbar d} \). The results of numerical solution of the last equation at the same boundary conditions as in case of triangular QW and some values of magnetic field \( B \) are shown in Table 2. It is worthy to mention that unlike the previous case of finite triangular QW, here we do not use any small parameter and the term \( \sim B^2 \) is included.

The dispersion curve for the subband \( n = 1 \) is shown in Fig.4. while the results of our calculations for the anisotropy of the photoconductivity are presented in Fig.5.

It is clearly seen from Figs. 3, 5 that the anisotropy of the photoconductivity should occur in such asymmetric nanostructures in an external magnetic field and that the effect should be measurable, since \( \Delta\sigma/\sigma(0) \)-ratio is about 0.016 at the moderate magnetic field of about \( \sim 5T \).

5. Conclusion

The results of the work can be summarized as follows. We studied the charge carriers behaviour in a quantum structure where the symmetry with respect to space coordinates and time-
reversal symmetry are broken simultaneously. As the models of such structure we considered the finite triangular, as well as semiparabolic quantum wells placed in external magnetic field. We have shown, by numerical analysis of corresponding Schrödinger equation, that the energy spectrum of charge carriers is anisotropic with respect to in-plane (transverse) motion $\epsilon_n(+k_x) \neq \epsilon_n(-k_x)$. This leads to the anisotropy of momentum transfer under the carriers interaction with polarized light, which can be very naturally explained by introducing the concept of charge carriers ‘renormalized’ effective mass. Indeed, despite the fact that absorbed photons supply to the charge carriers (electrons, for definiteness) with $+k_x$ and $-k_x$ the same momentum, their momenta corresponding to $\langle -x \rangle$ and $\langle +x \rangle$ are different. The anisotropy of momentum transfer leads to the interesting galvano-magnetic effect, anisotropy of photoconductivity $\sigma(+k_x) \neq \sigma(-k_x)$ and, as it follows from our calculations, the effect though not very great, could be measurable for the magnetic field of about few $T$. 

**Figure 4.** The ground-state electron energy versus $|k_x|$ in case of semiparabolic QW

**Figure 5.** The anisotropy of the photoconductivity
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