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Thermo-Hygro-Mechanical Simulation of Cracking in Thick Restrained RC Members: Application to a 50 cm Thick Slab

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Abstract

The calculation of crack width in thick, restrained, reinforced concrete (RC) members has relevance, either at the design stage or during the assessment of existing structures. This type of structural element exhibits a complex serviceability behaviour, due to the nonlinear self-induced deformations caused by cement hydration and shrinkage, and also the interaction between primary cracks and secondary cracks which do not fully penetrate in the cross section. In this context, this paper presents a staggered thermo-hygro-mechanical (THM) analysis methodology, based on the finite element method (FEM), for calculation of the long-term development of self-induced deformations, stresses and cracks, since casting, at the macro scale. A comprehensive approach is followed, in which the mechanical material models are defined as a function of the calculated thermal and hygral fields. This analysis methodology is applied in the study of the crack formation in end restrained slab-like RC members, with a thickness of 50 cm, a parametric analysis in conducted in order to gain insight about the influence of some relevant structural and material variables.

1. Introduction

Concrete experiences important volumetric changes, even in structures not subjected to applied loads. In the first days after casting, cement hydration gives rise to temperature variations, which in turn generate temperature dependent volume changes, and also autogenous shrinkage deformations. In structures exposed to drying, additional shrinkage strains develop as internal water migrates towards the element boundaries. In real RC structures, such as slabs and walls, the deformation of some members is frequently restrained due to the connection to adjacent structural components, or by the foundations.

In restrained structures, the required area of steel reinforcement is usually governed by cracking control requirements. The following design criteria have to be fulfilled: (i) in order to avoid the formation of localized wide cracks and ensure the structure robustness, a minimum cross-sectional area of reinforcement, \( A_{min} \), has to be provided – it is intended to avoid steel yielding upon the formation of a new crack; (ii) if the crack opening has to be limited to a predefined admissible value, \( w_{adm} \), for durability, tightness or aesthetic conditions, the steel stress has to be limited accordingly.

Thus, cracking control relies on the proper calculation of steel stresses upon cracking, as well as on appropriate evaluation of crack widths. The code formulations to perform such calculations have evolved over time (Knoppik et al. 2019; Lapi et al. 2018), and there is an ongoing effort to develop improved design rules (Caldentey et al. 2018). Two important open questions can be identified, which need to be addressed by new calculation methods.

The first one, is the long-term consequence of the self-induced stresses due to the cement heat of hydration. Recent experimental researches (Turner et al. 2017) have shown that a comprehensive approach has to be followed in the analysis of restrained RC members, taking into account the development of self-induced deformations and the evolution of viscoelastic properties since casting. In the recent past, the early-age and the long-term behaviours were frequently analysed separately. It was believed that the concrete stresses due to the cement heat of hydration were significantly relaxed over time, owing to the high creep deformations of young concrete. On the contrary, the experiments of Turner et al. (2017) demonstrated that the tensile stresses developed after stabilization of the concrete temperatures remain almost to a full extent. Therefore, the effects of the heat of hydration must be properly quantified and cannot be ignored in long-term analyses. These effects are more important in thick RC members, where higher temperatures are developed in the early ages.

The second one is the nonuniformity of drying shrinkage and viscoelastic concrete properties throughout the cross-section of thick RC members, and their consequences in terms of crack formation. It is well

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known that, particularly in thick concrete members, the flow of humidity in concrete is responsible for an important spatial no uniformity of drying shrinkage deformations and drying creep properties (Barre et al. 2016; Bazant 1989). These effects, together with size effects, give rise to complex crack patterns in thick RC members, with secondary cracks that do not fully penetrate in the cross-section, in-between through cracks (Schlicke et al. 2019).

Several THM analysis methodologies have been developed or used, for serviceability analyses of restrained RC members (Gash et al. 2016; Nakamura et al. 2006; Sciumè et al. 2012; Yoneda et al. 2013). However, none of those have analysed the crack propagation in thick, restrained, RC members, since the early ages until the long-term, considering the imposed deformations due to cement hydration and shrinkage. In this context, the main objective of the present paper is the application of a THM analysis methodology for a comprehensive study of the serviceability behaviour of thick restrained RC members. Material models, previously proposed and validated, are integrated with this purpose. The study is conducted considering a concrete mix previously characterized in laboratory. Even though the present study covers only the case of quasi-constant environmental conditions (typical of controlled interior environment), it is expected to provide an original insight about the crack propagation in this type of structure and the interplay of hydration and drying effects.

The results of this study are not yet expected to be directly applied in the design practice, because the analysed structures have specific material, geometric and environment characteristics. The design should still be made in accordance with the simplified design rules prescribed by design codes. Nevertheless, these results and methodology of analysis are expected to be a contribution towards the development of new design procedures.

In this paper, section 2 describes the analysis methodology. It consists in staggered thermal, hygral and mechanical analyses, based on the FEM. The calculated temperature and relative humidity fields inside concrete provide a fundamental input for calculation of the self-induced concrete deformations. These fields also affect the viscoelastic concrete properties, which vary not only over time, but also spatially within the RC member cross-section. The tensile fracture properties of concrete, and the bond action between steel and concrete are also considered, in order to simulate the crack formation and propagation processes in end restrained members. The thermal an hygral models, as well as the procedure for estimation of self-induced deformations, are based on previously validated methodologies (Azenha et al. 2017; Faria et al. 2006). For the mechanical analysis, previously implemented numerical models (Azenha et al. 2011; Sousa et al. 2018) are further developed so that they can be used in the study of thick members. The main improvements are made in the basic creep and in the drying creep models. For the basic creep, the B3 model (Bazant and Baweja 2000) is used, with the introduction of a new correction factor, to enhance the creep function, but only in the time range of a few hours after the initial setting of concrete; the need of such correction was demonstrated by Østergaard et al. (2001). For the drying creep, the adopted model is based on the proposal of Benboudjema et al. (2005), consisting in a Kelvin element whose viscosity depends on the local relative humidity of concrete.

In section 3, an application is presented, consisting of a 50 cm thick slab-like RC member restrained at the extremities. The adopted concrete mix was previously characterized in laboratory, which provides a solid base for the definition of the material parameters.

Finally, in section 4, the results are shown and discussed. The analysis outputs confirm that the non-uniformity of the self-induced deformations throughout the slab cross-section has a strong influence in the crack propagation process. Once drying starts, a thin concrete layer, close to the exposed concrete surface, becomes micro cracked. Then, these microcracks coalesce in cracks, approximately parallel to the member cross-section, which propagate inwards from the concrete surface. Only a few of these cracks reach the member core, giving rise to through-cracks. A parametric analysis is presented, providing quantitative information about the structural effects of the amount of reinforcement, degree of restraint, member length, bond action at the steel-concrete interface and drying creep properties.

2. Analysis methodology

An uncoupled THM analysis methodology is used, based on the FEM. In the thermal and hygral models, concrete is considered as a homogenous continuum. Even though the models are three-dimensional, in the applications envisaged in the present work (slab-like structures) the fluxes of heat and humidity are unidirectional, perpendicular to the slab middle plane.

The thermal model provides the temperature field due to the cement hydration heat, which is an input of the mechanical model: the temperature variations, multiplied by the coefficient of thermal expansion (CTE), provide estimates of the free deformations induced by temperature. The maturity of concrete is also determined as a function of the calculated temperature field, using the equivalent age concept, which is later used to account for the evolution of the mechanical properties (viscoelastic properties and autogenous shrinkage), necessary for the stress prediction.

The output of the hygral model, the relative humidity ($H$) of concrete, is also an input for the mechanical model: local drying shrinkage strains are determined as a function of the $H$ field. The drying creep model is also dependent on the local humidity of concrete.

In the mechanical model, concrete is simulated with
plane stress finite elements (FE). A total strain constitutive model with rotating cracks is used. The viscoelastic properties of concrete are simulated through a Kelvin chain model fitted to creep functions that take into account the evolution of basic creep in concrete since the initial setting until the long-term, as well as the drying creep of concrete. Concrete fracture is simulated through a bilinear strain softening model, function of the concrete fracture energy. Steel reinforcement is simulated with truss FEs, connected to the neighbouring concrete FEs through interface FE that describe the bond-slip behaviour.

2.1 Thermal model

The temperature evolution in concrete is governed by the classical thermal energy balance equation:

$$\frac{\partial}{\partial t} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right) + \dot{Q} = \rho c \dot{T}$$  \hspace{1cm} (1)

where $T$ and $\dot{T}$ are the temperature and its time derivative, $\dot{Q}$ is the heat generation rate, $k$, $c$ and $\rho$ are the thermal conductivity, the specific heat and the specific mass of concrete, and $x$, $y$ and $z$ are the spatial coordinates. The heat generation rate due to cement hydration is given by the Arrhenius-type function:

$$\dot{Q} = f(\alpha_r) A_r e^{E_a / R T}$$  \hspace{1cm} (2)

where $\alpha_r$ is the degree of heat development, $f(\alpha_r)$ is the normalized heat generation function, $A_r$ is a constant proportional to the maximum heat generation rate, $R$ is the ideal gas constant ($8.314 \text{ J mol}^{-1} \text{ K}^{-1}$) and $E_a$ is the apparent activation energy.

At the boundaries, a Neumann-type formulation is used to describe the heat flux per unit area, $q_{in}$, as a function of a lumped convection/radiation coefficient, $h_{nr}$:

$$q_{in} = h_{nr} (T - T_{env})$$  \hspace{1cm} (3)

where $T_{env}$ is the environment temperature.

A comprehensive description of the model can be found in the work of Azenha (2009) and Azenha et al. (2011).

2.2 Hygral model

The relative humidity in the concrete pores, $H_r$, is governed by the simplified moisture diffusion equation:

$$\frac{\partial H}{\partial t} = \text{div}(D_m \nabla H)$$  \hspace{1cm} (4)

where $t$ is the time and $D_m$ is the diffusion parameter which lumps the diffusivity and the moisture capacity of the material. The validity of this modelling approach was justified by Azenha et al. (2017). $D_m$ is given by the following equation, initially proposed by Bazant and Najjar (1972):

$$D_m = D_0 \left[ \frac{1 - D_0 / D_1}{1 + \left(1 - H / H^* \right)^m} \right]$$  \hspace{1cm} (5)

where $H^*$ is the relative humidity corresponding to $D_m = 0.5(D_0 + D_1)$, $m$ is a material propriety, $D_0$ is the value of $D_m$, for $H = 1$ and $D_1$ is a lower bound for the diffusion parameter. For $H = 0$, $D_m \geq D_0$, $D_m$ being closer to $D_0$ as the value of the parameter $m$ increases.

The moisture flux through the boundaries, $q_m$, is also simulated through a Neumann-type formulation:

$$q_m = h_m (H - H_{env})$$  \hspace{1cm} (6)

where $h_m$ is a moisture emissivity coefficient and $H_{env}$ is the relative humidity of the environment.

The implementation of Equations (5) and (6) in the FEM framework is described in the work of Azenha (2009).

2.3 Mechanical model

The mechanical analyses are made with resource to the FE software package DIANA, which includes an extensive library of material models for concrete and the possibility to add new constitutive models through user supplied subroutines (DIANA FEA BV 2017). The sections 2.3 (1) to (6) describe the various aspects of the adopted concrete constitutive model. It is based on a total strain formulation (DIANA FEA BV 2017), which combines: the viscoelastic behaviour of concrete between cracks (instantaneous, basic creep and drying creep deformations), stress-independent deformations (autogenous and drying shrinkage and free thermal deformations) and cracking.

A linear elastic model is adopted for steel, because the rebars yielding stress is not reached in any of the performed analyses.

(1) Instantaneous and basic creep deformations

The sum of instantaneous and basic creep deformations is modelled according to the B3 model, which is based on the solidification theory and fits, with low scatter, an extensive data bank of experimental results (Bazant and Baweja 2000). According to this model, the corresponding creep function, also known as creep compliance, is given by:

$$J(t, t') = q_i + C(t, t')$$  \hspace{1cm} (7)

where $t'$ is the concrete age when a constant unit load is applied and $q_i$ is the instantaneous strain. In the constitutive models for concrete, and throughout this paper, the time variable $t$ is equal to zero at the instant when water is added to the dry concrete components. The term $C(t, t')$ represents the basic creep compliance, given by:
\[ C_i(t,t') = q_2 Q(t,t') + q_1 \ln[1 + (t - t')^{1/2}] + q_3 \ln \frac{t'}{t} \]  

(8)

where \( q_2 \), \( q_1 \), and \( q_3 \) are empirical constitutive parameters, expressed as a function of the concrete composition and compressive strength, as given by Bazant and Baweja (2000).

Östergaard et al. (2001) demonstrated that the B3 model underestimates the concrete deformation in the first few hours after the initial setting. Those authors proposed a correction factor to increase the deformations due to stress variations in that interval of time. A similar approach is followed in the present paper, which consists in the introduction of a factor \( \beta(t') \) in the compliance function:

\[ J(t,t') = q_1 + \beta(t') C_i(t,t') \]  

(9)

The adopted \( \beta(t') \) function is given by:

\[ \beta(t') = \frac{t'}{t'^2 - t_i^2} \]  

(10)

where \( t_i \) represents the initial setting time (herein taken as the instant when both the cement heat of hydration and the stiffness start to raise rapidly). The \( \beta(t') \) value is equal to infinite when \( t' = t_i \) and decreases rapidly to \( \beta = 1 \) as the time \( t' \) increases. The rate at which this convergence to one is achieved depends on the function \( k \).

Calibration through the comparison with experimental results (Klausen et al. 2018), for tests of concrete ties restrained since casting, showed that, by using \( k = (t'/t_i)^2 \), a good agreement is reached between experiments and calculations. The resulting \( \beta(t') \) function is plotted in Fig. 1, for \( t_i = 10 \) h, showing that \( \beta(t') \) rapidly converges to 1. For \( t' > 24 \) h, \( \beta(t') \) can be taken as one. By comparison with the function used by Östergaard et al. (2001), also plotted in the figure, Equation (10) converges more rapidly to the value 1.

(2) Drying creep

The drying creep, also known as Picket effect, is usually considered in numerical modelling as the excess of deformation at drying, by comparison with the sum of the shrinkage deformation at no load and the creep deformation of a similar loaded sealed specimen (Bazant 1989).

It is well accepted that this excess of deformation is, in part, justified by the concrete microcracking during the material characterization tests (Bazant and Xi 1994). In the modelling approach adopted here, this effect is taken into account by using a model to describe the softening behaviour of concrete after cracking (section 2.3 (6)), both in the mechanical analyses and in the derivation of the drying shrinkage model.

As regards the additional part of drying creep, it has been demonstrated that the creep deformations depend on the rate of humidity variation in concrete (Acker and Ulm 2001) and, for that reason, some authors identify it as stress-induced shrinkage. There is no universally accepted theory to justify and describe this dependence. According to Bazant and Chern (1985), it can be modelled as:

\[ \dot{\varepsilon}_{d,i} = \mu \frac{\dot{\theta}}{H} \sigma \]  

(11)

where \( \dot{\varepsilon}_{d,i} \) is the ratio of drying creep strain, \( \mu \) is a material parameter and \( \sigma \) is the applied stress. This equation represents a rheological model composed by a single dashpot with a viscosity \( \eta = (\mu/\theta) \).

Benchoudjema et al. (2005) showed that an improved agreement with experimental results could be reached by using a Kelvin model with a dashpot with a viscosity \( \eta_{dc} = (\theta/\theta_0) \) and a spring with a stiffness \( k_{dc} \), where \( \theta \) is a unit conversion factor (\( \theta = 1 \) s). The drying creep strain is thus obtained by solving the equation:

\[ \eta_{dc} \dot{\varepsilon}_{dc} + \frac{\dot{\theta}}{\theta_0} k_{dc} \varepsilon_{dc} = \theta \frac{\dot{\theta}}{H} \sigma \]  

(12)

Even though the model involves only two material parameters (\( \eta_{dc} \) and \( k_{dc} \)), it allows to control the kinetics of the drying creep process (by adjusting the ratio \( \eta_{dc}/k_{dc} \)) and its amplitude (through the stiffness \( k_{dc} \), the creep deformations being dependent of the humidity ratio \( H \)).

Equation (12) can be solved using the finite differences technique. The drying creep strain at time step \( i+1 \), \( \varepsilon_{d,i+1} \), can thus be determined as a function of the strain, the stress and the humidity ratio at time step \( i \) (\( \varepsilon_{d,i} \), \( \sigma_i \) and \( H_i \), respectively):

\[ \varepsilon_{d,i+1} = \varepsilon_{d,i} + \frac{\Delta t \frac{\dot{\theta}}{H_i}}{\eta_{dc}} (\sigma_i - \varepsilon_{d,i} k_{dc}) \]  

(13)

Therefore, once the humidity ratio is known (in the present study, it is an output of the hygral analysis) the drying creep compliance, \( C_i(t,t') \), can be determined using Equation (13). The compliance \( C_i(t,t') \) expresses the drying creep strain at time \( t \) due to a unit stress applied at time \( t' \). In the slab-like specimens analysed in this work, the time-histories of humidity vary over the cross-section depth. Consequently, the drying creep compliance also varies in that direction.
In order to understand the implications of such a variation (across the thickness) of the drying creep compliance, two analysis hypothesis are considered in the present work.

In the first hypothesis, labelled as “layered approach”, the creep compliance is calculated using Equation (13). Therefore, the constitutive model assigned to each FE is determined as a function of the time-history of humidity, $H$, previously calculated for that position. At the present state of knowledge, it is not possible to estimate the parameters $\eta_d$ and $k_d$, as a function of the material properties. Consequently, exploratory analyses are made, by using $\eta_d$ and $k_d$ values which lead to an average drying creep deformation (averaged over the structure cross-section), similar to one predicted by a widely accepted model for that purpose, the B3 model (Bazant and Baweja 2000) – the justification for the chosen $\eta_d$ and $k_d$ values is explained in section 3.2.

In the second hypothesis, labelled as “B3 approach”, the drying creep compliance is assumed to be independent from the local humidity in concrete, that is, independent from the position within the cross-section. The adopted model is the drying creep compliance included in the B3 model (Bazant and Baweja 2000), for prediction of the average cross-section deformation:

$$C_d(t',t_b) = q_b\left[ e^{-\eta_d t'} - e^{-\eta_d t_b} \right]^{1/3}$$  \hspace{1cm} (14)

where $t_b$ is the concrete age at the start of drying, $t_b' = \max(t',t_b)$, $\eta_d$ is the spatial average of the humidity within the cross section at time $t$ and $q_b$ is an empirical parameter.

(3) Kelvin chain

In the mechanical analyses carried out using the software DIANA, the viscoelastic behaviour of concrete is simulated through the ageing Kelvin chain model, which allows the combination of viscoelasticity and cracking (DIANA FEA BV 2017). The Kelvin chain characteristics, to be assigned to each FE, are calculated through the least squared error method, by minimizing the error between: (i) the material creep compliance, including the instantaneous, basic creep and drying creep deformations (models described in sections 2.3 (1) and 2.3 (2)); (ii) the creep function provided by the ageing Kelvin chain model.

For an accurate description of the creep compliance since the early ages (0.015 days after the initial setting time) and until the long-term (50 years), the Kelvin chain is a series association of 10 elements, the first of which is an ageing spring. The retardation time, $\tau$, for the remaining nine elements varies logarithmically ($\tau_i = 5 \times 10^{-3}$, with $\tau_i$ in seconds and $1 \leq i \leq 9$).

The ageing property of the Kelvin chain is due to the fact that the stiffness of each of the 10 spring elements vary as a function of time $t'$. Thirty $t'$ values are adopted for each spring, logarithmically spaced according to the method proposed by Pövoas (1991).

The influence of the temperature history on the development of the viscoelastic concrete properties is simulated through the equivalent age concept (Benboudjema et al. 2019). Therefore, in the Kelvin chain model, the real concrete age, $t$, is replaced by the equivalent concrete age, $t_{eq}$, given by:

$$t_{eq} = \int_0^t e^{\frac{1}{\tau_0} \left(\frac{t}{\tau_0} - 1\right)} d\tau$$  \hspace{1cm} (15)

where $T_{eq}$, if the reference temperature (20°C = 293°K) and $T(\tau)$ is the temperature history obtained after solving the thermal field problem.

(4) Drying shrinkage

One of the important issues to be analysed is the consequence, in terms of structural behaviour, of the spatial variation of drying shrinkage. Therefore, the local drying shrinkage strain, $\varepsilon_{ds}$, has to be quantified. Kwak et al. (2006) proposed a model in which such strain is expressed as a function of the local relative humidity in concrete, $H$:

$$\varepsilon_{ds} = \varepsilon_{ds,\text{ref}} \times [0.97 - 1.895(H - 0.2)^{1/2}]$$  \hspace{1cm} (16)

where $\varepsilon_{ds,\text{ref}}$ is a material parameter that represents the concrete shrinkage upon total (or almost total) drying. This procedure was experimentally validated by Azenha et al. (2017), by performing shrinkage and humidity measurements (at various depths) in concrete specimens of different sizes (10 cm, 15 cm and 20 cm thick concrete prisms). In that experimental campaign, shrinkage was also measured in 2 mm thick cement paste specimens, exposed to variable relative humidity values. Given than this drying shrinkage model provided a good agreement with the experimental results for specimens of the three thicknesses – see Azenha et al. (2017) – it is expected to provide an acceptable estimated for the 50 cm thick member, bearing in mind the objectives of the present work. It should be noted that the concrete material properties considered for the structures analysed in this paper correspond to the ones of the experimental campaign described in Azenha et al. (2017).

(5) Autogenous shrinkage

The total shrinkage deformation measured in shrinkage specimens is usually decomposed into a drying shrinkage component (characterized in section 2.3 (4)) and an autogenous shrinkage component.

The applications envisaged in this work involve the use of normal strength concrete. It is well known that, in this type of material, the drying component is the most important one (Muller and Haist 2009). Nevertheless, autogenous shrinkage strains are responsible for important volume changes in the first days after casting and, consequently, they influence the crack development process. According to the fib Model Code 2010 (fib 2013), the autogenous shrinkage strain, $\varepsilon_{as}$, can be estimated as:

\begin{align*}
\varepsilon_{as} &= \varepsilon_{as,\text{ref}} \times [0.97 - 1.895(H - 0.2)^{1/2}] \\
&= 0.25 \times [0.97 - 1.895(H - 0.2)^{1/2}]
\end{align*}
\[ \varepsilon_n = -(1 - e^{-n\psi})\varepsilon_{eq} \left( \frac{0.1 f_{cm}}{6 + 0.1 f_{cm}} \right)^{2.5} \times 10^{-6} \]  \( (17) \)

where \( \alpha_n \) is a material parameter and \( f_{cm} \) is the mean cylinder compressive strength at the age of 28 days. To account for temperature effects, the concrete age \( t \) in Equation (17) is replaced by the equivalent concrete age \( t_{eq} \) provided by Equation (15).

(6) Concrete fracture

A total strain model, with rotating cracks, is used to simulate concrete tensile fracture. A smeared cracking approach is adopted, in which crack openings are simulated by crack strains, and the constitutive behaviour is expressed by continuum stress-strain relationships. This relationship is modelled according to the fib Model Code 2010 (fib 2013) proposal, which is composed by a bilinear ascending branch and a bilinear softening branch – see Fig. 2, where \( \sigma_{eq}, \varepsilon_n \) and \( \varepsilon_{cr,n} \) are, respectively, the stress, the stress-dependent strain, and the crack strain, for the direction normal to the crack. The ascending part describes the stiffness decrease due to the formation of microcracks for stresses higher than 0.9 \( f_{cm} \), where \( f_{cm} \) is the average uniaxial tensile strength of concrete. The softening part is specified as a function of the crack strain, for the direction normal to the crack. The exponent \( n \) accounts for the faster development of tensile strength by comparison with compression, is usually in the range 0.5 to 0.67 (Benboudjema et al. 2019). In this work the upper limit (0.67) is considered, because this is conservative for assessment of the cracking risk at the early ages. The coefficient \( s \) is quantified according to the fib Model Code 2010 (fib 2013) recommendations.

It will be shown in section 4.1 that, for the structures under analysis, the concrete tensile stresses generated by the heat of hydration do not reach the tensile strength. For that reason, a constant tensile strength will be used in the mechanical analyses.

(7) Bond stresses and slip at the steel-concrete interface

In the mechanical model, the nodes of each rebar FE are connected to the ‘parent’ ones of the concrete FE to which is bonded, by means of an interface FE. The constitutive model assigned to this interface FE expresses the relationship between the slip \( s \) and the bond stress \( \tau_b \) in the direction parallel to the rebar axis. The fib Model Code 2010 (fib 2013) constitutive relation is adopted for that purpose:

\[ \tau_b = \tau_{b,max} \left( \frac{s}{s_1} \right)^{0.4} \]  \( (20) \)

For good bond conditions, \( \tau_{b,max} = 2.5 \sqrt{f_{cm}} \), with \( \tau_{b,max} \) and \( f_{cm} \) in MPa, and \( s_1 = 1 \) mm. This function is valid for slip values up to 1 mm – this limit is not reached in the serviceability analyses envisaged in this work. Splitting concrete failures are not considered, because they are not expected in the service life of the analysed structures.

Unloading and reloading follows an elastic stiffness, \( D_c \). For applications without loads reversals, this assumption agrees well with the results of bond tests (fib 2000).

![Fig. 2 fib (2013) model for concrete tensile fracture, expressed as a function of the crack strain \( \varepsilon_{cr,n} \).](image-url)
3 Case study: RC restrained slab

3.1 Geometry, actions and FE model

The case study consists in a 500 mm thick slab-like structure whose axial deformation is restrained at the extremities. The structure is reinforced with two steel layers (close to the bottom and top surfaces, respectively). The concrete cover to the reinforcement is equal to 50 mm, measured with respect to the steel bar axis.

The structure is assumed to be moulded for a period of 7 days, after which the drying process is assumed to start. No additional load (not even self-weight) is considered, besides the self-induced deformations due to the cement heat of hydration and concrete shrinkage. The environmental conditions are constant during the entire period of analysis: $T_{\text{env}} = 20^\circ\text{C}$ and $H_{\text{env}} = 60\%$.

In the reference analysis, the structure length is $L = 2$ m. It was set so that at least two through-cracks could develop in the structure.

The area of steel reinforcement is $A_s = 24.94 \, \text{cm}^2/\text{m}$ (sum of top and bottom reinforcement layers). It corresponds to the area $A_{s,\text{min}}$ which, according to the Eurocode 2 (CEN 2004), is required to avoid steel yielding upon the formation of a through-crack. The code formula is, for this application:

$$A_{s,\text{min}} \sigma_s = k \, f_{\text{cm}} \, A_s$$

where $\sigma_s$ is the yield strength of the reinforcement (500 MPa in this case) and $k$ is a coefficient which takes into account, in a simplified way, the effect of the non-uniform self-equilibrating stresses. For a thickness of 50 cm, $k = 0.86$.

Besides the reinforcement area, also the rebar perimeter, $u_s$, is required to define the geometry on the interface FE-s. This was determined as a function of the reinforcement area considering a constant rebar spacing equal to 10 cm in all the analyses, this being a realistic value.

Figure 3 depicts the FE mesh discretization adopted for the mechanical analyses. A plane stress model is adopted to simulate the slab-like structure, in which no variation (of temperature, humidity or any mechanical parameter) exists throughout the direction perpendicular to the plane XY represented in the figure. Concrete is simulated with four-nodded, plane stress, quadrilateral FE-s with 25 mm edges. The FE thickness is equal to 1 m. Only half of the structure depth ($h/2$) is discretized, taking advantage of the symmetry condition.

Owing to the uniformity of the structure and the self-induced deformations along direction X, the concrete stresses before cracking are also uniform along that direction. Therefore, a weaker spot, which consists of four FE-s with a 5% lower tensile strength, was adopted as shown in Fig. 3, to facilitate the crack localization process. Preliminary analyses revealed that the results are almost insensitive to the location of this weaker spot. Moreover, in various analyses the location of through-cracks do not coincide with this weaker spot.

The external restraint in the X direction was simulated by fixed supports in the leftmost extremity (cross-section A in Fig. 3) and elastic springs in the rightmost one (B). A tying condition is additionally prescribed at the latter, imposing that all the nodes have the same displacement in the X direction (plane section condition).

The degree of restraint, $R$, is defined as the ratio between the restrained deformation and the free concrete deformation. Given that the supports elasticity remains constant throughout the analysis, the degree of restraint varies over time. This replicates the conditions found in real structural elements which, upon casting, become connected to other pre-existing structural components inducing the restraint. The stiffness of the pre-existing element has a much lower evolution than the stiffness of the newly cast one. In the first hours after the initial setting, when the concrete stiffness is low, the $R$ value is close to 1. Then, as the stiffness of the newly cast element increases, the degree of restraint decreases.

The spring stiffnesses, $k_i$, considered in the reference analysis, correspond to a degree of restraint $R_{28} = 2/3$, with $R_{28}$ being the ratio between the restrained deformation and the free concrete deformation, for a concrete stiffness given by the modulus of concrete at the age of 28 days, $E_c$. The stiffness $k_i$ is thus given by:

$$k_{\text{total}} = \sum_{i=1}^{n} k_i = \frac{R_{28} \times E_c \times A}{1 - R_{28} \times L}$$

where $A$ is the cross-sectional area of concrete, $n_i$ is the number of springs and $k_{\text{total}} = n_i \times k_i$.

The FE meshes for the thermal and hygral analyses are equal to that in Fig. 3, in terms of discretization along the Y direction. Given that the fluxes of tempera-
ture and humidity are unidirectional, these models are composed by a single column of FEs. Contact with the exterior environment occurs in a single Neumann boundary condition, corresponding to the top concrete surface in Fig. 3.

### 3.2 Material and boundary properties

The concrete material adopted in the analysis corresponds to the mix previously characterized in an extensive laboratorial campaign described in Azenha et al. (2017). The mix includes 280 kg/m$^3$ of CEM II 42.5R, 40 kg/m$^3$ of fly ash, 143 kg/m$^3$ of water, 6 kg/m$^3$ of water reducing admixture and granitic aggregates. This is a feasible concrete composition for applications with normal strength requirements. The material and boundary properties for the thermal and hygral analyses, whose determination is also explained in Azenha et al. (2017), are shown in Table 1. They are based on dedicated laboratorial tests, previous experience and validation (comparison between the calculated fields of temperature and humidity and the measured temperature and humidity values).

The material parameters for the constitutive models described in section 2.3 (mechanical model) are shown in Table 2. They were quantified according to the recommendations of the reference documents cited in section 2.3, taking into account the concrete composition and a cylinder average compressive strength of 38 MPa at the age of 28 days.

The concrete shrinkage upon total drying, $\varepsilon_{dc,t}$, is based on the test results of Azenha et al. (2017). This value is also used in the quantification of the parameter $q_3$ for the drying creep model (B3 approach explained in section 2.3 (2)).

Regarding the simulation of drying creep through the layered approach, as described in section 2.3 (2), at the present state of knowledge the parameters $\eta_{dc}$ and $k_{dc}$ cannot be directly quantified as a function of the material properties. Nevertheless, it is important to understand the structural implications of the spatial variations of drying creep. For that reason, an exploratory quantification of $k_{dc}$ and $\eta_{dc}$ was made. These parameters were set so that they lead to a spatially averaged drying creep deformation in each instant of time, $\bar{\varepsilon}_{dc}(t)$, comparable to the deformation provided by the B3 drying creep model.

Thus, for determining the $k_{dc}$ and $\eta_{dc}$ values, a simple computer code had to be implemented to calculate the $\bar{\varepsilon}_{dc}(t)$ value. This code simulates a drying creep test of a concrete specimen having the same cross-section as the member under analysis (500 mm thick, with two exposed surfaces and a unidirectional flow of humidity). The calculation is based on the assumption that plane sections remain plane. The cross-section is divided into 25 mm thick layers. The stress-strain relationship, in each layer, provided by the hygral model. At the time $t'$, a unitary average stress is applied to the cross-section and kept constant. An incremental analysis procedure is used, in which the time scale is decomposed into time increments. At each time increment, the problem unknowns are: the increment of deformation (equal in all the layers because the cross section remains plane); the stress installed in each layer (the average over the cross-section being equal to the unitary stress). The solution is found iteratively.

Figure 4 represents, by dashed lines, the calculated

| Analysis   | Property     | Value                      |
|------------|--------------|----------------------------|
| Thermal    | $k$          | 2.6 W m$^{-1}$ K$^{-1}$    |
|            | $\rho_c$     | 2400 kg m$^{-3}$ K$^{-1}$  |
|            | $A_p$        | 2.645×10$^{-5}$ s$^{-1}$   |
|            | $E_a$        | 38.38 kJ mol$^{-1}$        |
|            | $f (\alpha_t)$ | See Azenha et al. (2017)  |
|            | $h_v$ before demoulding | 5 W m$^{-2}$ K$^{-1}$   |
|            | $h_v$ after demoulding | 10 W m$^{-2}$ K$^{-1}$   |
| Hygral     | $D_l$        | 3.08×10$^{-10}$ m$^2$ s$^{-1}$ |
|            | $\theta_l$   | 0.0967 $D_l$               |
|            | $H_t$        | 0.8                        |
|            | $m$          | 2                         |
|            | $h_m$        | 4.81×10$^{-8}$ m s$^{-1}$  |

| Table 1 Material and boundary properties for the thermal and hygral analyses. |

| Aspect                          | Property     | Value                      |
|---------------------------------|--------------|----------------------------|
| Instantaneous deformation and basic creep | $q_1$      | 20.56×10$^{-6}$ MPa$^{-1}$ |
|                                 | $q_2$      | 117.46×10$^{-6}$ MPa$^{-1}$ |
|                                 | $q_3$      | 2.32×10$^{-6}$ MPa$^{-1}$  |
|                                 | $q_4$      | 5.31×10$^{-6}$ MPa$^{-1}$  |
|                                 | $t_r$      | 10 hours                   |
| Drying creep (layered approach) | $k_{dc}$    | 2.2 GPa                    |
|                                 | $\eta_{dc}$ | 2.2×10$^{3}$ MPa s$^{-1}$  |
| Drying creep (B3 approach)      | $q_1$      | 457×10$^{-6}$ MPa$^{-1}$   |
| Drying shrinkage                | $\varepsilon_{dc,t}$ | 539 $\mu$               |
| autogenous shrinkage            | $\alpha_{as}$ | 600                        |
| Fracture                        | $f_{cr}$    | 2.9 MPa                    |
|                                 | $G_f$       | 0.14 N mm$^{-1}$           |
|                                 | $h$         | 25 mm                      |
|                                 | $\delta_l$  | 1 mm                       |
| Bond-slip                       | $\tau_{s,max}$ | 2.5$\sqrt{f_{cm}}$      |
|                                 | $D_i$       | 10$^4$ N mm$^{-3}$         |
| Other                           | CTE         | 10$^{-1}$ ºC$^{-1}$        |
|                                 | $v$         | 0.2                        |
|                                 | $E_s$       | 200 MPa                    |
The variable $\varepsilon_{ct}(t)$, for 5 different loading ages (14 days, 90 days, 1 year, 3 years and 10 years), considering the values shown in Table 2 for the material parameters $k_a$ and $\eta_a$. The variable $\varepsilon_{ct}(t)$ can also be seen as a drying creep compliance that gives the relationship between the average stress and the average deformation of the cross-section. In Fig. 4, the results provided by the B3 model for drying creep are also plotted, for the same load ages.

Naturally, the two modelling approaches (layered and B3) lead to significant differences, because the model formulations and objectives are very different. In the results of the layered approach, the strains $\varepsilon_{ct}(t)$ calculated for concrete ages lower than $\sim$120 days are negligible. This is because, in the first weeks, the humidity ratio $H$ in the interior part of the concrete member is close to zero (the calculated humidity fields will be shown in section 4.1). Given that the model viscosity is inversely proportional to $H$, the interior part becomes very stiff, in terms of drying creep deformations. For this reason, for loads applied at 14 days, Fig. 4 shows a big time delay when the layered-approach deformations start to rise visibly.

There are important differences, between the B3 model and the layered approach results, also in the very long-term strain variation. As the time increases beyond $\sim$3000 days, $H$ become progressively very small and, consequently, the strain variations $\varepsilon_{ct}(t)$ provided by the layered approach progressively tend to zero.

Despite these differences, it is worth assessing the structural implications of considering the layered approach in the mechanical analyses, because it reproduces, in an approximate way, the different drying creep behaviours over the cross-section depth, and their dependence with respect to the time ratio of the concrete relative humidity. These are the most important differences between the layered approach and the B3 model, not the differences in the average section deformations, visible in Fig. 4. The differences visible in Fig. 4 are not as significant because the total creep compliance is the sum of instantaneous, basic and drying components, this sum being much higher than the drying component.

In the future, further research is needed to enhance the knowledge about constitutive models for drying creep.

For the layered approach, a different Kelvin chain is assigned to each of the 10 rows of FEs visible in Fig. 3. Table 2 also shows the values for additional properties needed in the mechanical analyses: CTE is the coefficient of thermal expansion which, for simplification, was considered constant over time; $\nu$ is the coefficient of Poisson for uncracked concrete and $E_s$ is the steel modulus of elasticity.

Given that the constitutive model for concrete is based on a total strain formulation, after cracking the $\nu$ value is progressively reduced until zero. Stretching of a cracked direction does no longer lead to contraction of the perpendicular directions. In the implemented formulation (DIANA FEA BV 2017) the $\nu$ value decreases after cracking in the same pace as the secant modulus.

The initial setting time is estimated as $t_s = 10$ h after the addition of water to the dry concrete components. This is the starting time for the temperature variations due to the cement heat of hydration. The variable $t_s$ is also used in Equation (10).

The starting time for the mechanical analysis is set as 0.015 days after the instant specified as the initial setting time. This delay is required so that the structure has some stiffness in the first time increment of the mechanical calculation.

The time scale was finely discretized using a total of 1164 time increments, whose size increases progressively: 300 increments until the 7 days, 320 until the 8 days, 402 until the 90 days, 500 until the 1 year of analysis, 600 until the 3 years and 1164 until the 50 years.

The Secant BFGS iterative method (DIANA FEA BV 2017) was used in the mechanical analyses, being the convergence criterion based on the internal energy, with a convergence tolerance equal to $10^{-4}$.

3.3 Parametric analysis

Table 3 shows the six variable parameters considered in this study. The values for the reference analysis (labelled with REF), with the property values described in sections 3.1 and 3.2 are summarized, as well as the variant values for the parametric analysis. The label used for each parameter is also shown.

For the cross-sectional area of reinforcement, the value $A = 42.4$ cm$^2$/m is the amount which, according to the results of the THM analysis framework, leads to a maximum crack opening of 0.30 mm (a typical value for the admissible crack opening in RC structures). This amount of reinforcement was quantified iteratively, by repeating the mechanical analysis for different $A$, val-
ues, until the objective crack opening was reached.

For the degree of restraint, \( R = 1 \) corresponds to the full restraint. In this case rigid supports were used in the cross-section B, instead of the springs indicated in Fig. 3.

For the length of the RC tie, the variant value \( L = 4 \) m was used to evaluate the implications of having a larger number of through cracks formed in the structure.

The variant bond-slip law corresponds to the formulation proposed by the fib Model Code 2010 (fib 2013) for other bond conditions (OBC), significantly poorer than the good bond conditions considered in the REF analysis. This variant law corresponds to a model with significantly lower bond stresses, in this case 39.5% of the stress given by the reference law for the same slip value.

The effect of the elastic stiffness of the bond-slip model was also evaluated. In the REF analysis, a high stiffness (\( D_s = 10000 \) MPa/mm) was adopted so that, in the ascending branch of the \( \tau_s - s \) relationship, the elastic limit is reached for very small slip values, and the behaviour becomes governed by Equation (20). However, in the used software package (DIANA FEA BV 2017) the elastic stiffness \( D_s \) is the parameter which also defines the unloading and reloading response of the bond model. Therefore, in the variant analysis, the parameter \( D_s \) takes the value recommended by the fib Model Code 2010 (fib 2010) for unloading and reloading (\( D_s = 244 \) MPa/mm). Given the great difference between these two \( D_s \) values, the best option would be a different material model, with unloading and reloading paths governed by a stiffness value different from the elastic one. The implementation of this material model is recommended as a further development.

The last variable parameter is the modelling approach used to simulate drying creep. The layered approach is used in the REF analysis. The B3 model is used as a variant.

## 4 Results and discussion

Firstly, the results are presented for the reference analysis. At this stage, some conclusions are also drawn about the structural implications of the adopted modelling approach to simulate drying creep.

Then, the parametric analyses are discussed showing the time evolution of some results which can be seen as indicators of the long-term structural performance: crack opening and steel stress at the most critical position, which coincides with the first through crack.

### 4.1 Reference analysis

Figure 5 shows the results of the thermal and hygral

![Fig. 5 Results of the thermal and hygral analyses.](image-url)
analyses. The temperature raise due to the cement heat of hydration reaches 21°C in the inner points and 17°C in the exposed surfaces. These peak values are reached approximately one day after the initial setting time of concrete. In the decreasing part of the temperature evolution, a discontinuity is observed at the age of 7 days, corresponding to the demoulding and modification of the convection and radiation effects at the boundary. The maximum temperature difference over the cross-section depth is not very significant (4°C).

On the contrary, the humidity field has important gradients across the depth, particularly in the first weeks after the surface is exposed to drying. Once drying starts, 7 days after casting, the humidity in the outermost concrete layers rapidly evolves to values close to the humidity of the environment. In the interior of the RC member the evolution is very slow, and one year after casting it is still higher than 95% – see the calculated humidity profiles in Fig. 5 for the ages of 14, 28 and 90 days.

The humidity variation is directly related to the evolution of local drying strain. Therefore, high shrinkage deformations will occur initially close to the surface, with strong implications in the stress development.

As regards the mechanical analyses, it is important to discuss, firstly, the self-induced stresses which would develop in concrete if there was no cracking. Figure 6 shows the calculated stresses, at the surface and centre of the cross section, due to cement hydration only, that is, assuming that the loss of humidity is prevented and consequently both drying shrinkage and drying creep are null. Important tensile stresses arise in the first days (at 7 days, they are equal to 1.9 and 1.5 MPa, at the interior and surface FEs, respectively). The long-term relaxation of these stresses is very moderate. This is in agreement with the experimental evidence reported by Turner et al. (2017).

The calculated stresses depicted in Fig. 7 include drying effects. Once drying starts, the stresses at the FEs closest to the surface rise rapidly and reach the tensile strength. The tensile strength evolution is plotted in the same figure, considering the temperature effect in the determination of the equivalent concrete age (owing to the small temperature gradients over the cross-section depth, only one $f_{cm}(t_{eq})$ curve is plotted, considering the spatially averaged temperatures). Here one can see that, if the tensile strength is assumed to be constant (and equal to the reference value at 28 days), the instant when the tensile strength is reached is practically unaltered. Therefore, a constant tensile strength, equal to $f_{cm,28}$, will be considered in the fracture model.

To analyse the long-term self-induced stresses, due to hydration and drying, Fig. 8 depicts the stress diagrams, for five instants, and for the two drying creep modelling hypotheses. Upon the exposure to drying, the tensile stresses increase rapidly in the outer concrete layers, due to the drying shrinkage effects. At 14 days, the tensile stress at the outer FEs is 5.3 or 6.7 MPa, depending on the modelling approach for drying creep. The layered approach leads to the lower tensile stress, because the rapid development of shrinkage deformations in the outer layer is accompanied by high drying creep deformations. These stress values (5.3 and 6.7 MPa) are the average stresses in the FE, which are approximately equal to the stresses at the centre of the FE (12.5 mm apart from the concrete surface).

The stress diagrams at 90 days show that, during the first months, the stresses in the interior remain almost equal to the ones caused by the heat of hydration and autogenous shrinkage. In the long term, as shown in the diagrams for the ages of 3 and 50 years, the stresses increase in the interior, as the humidity decrease in that location. In the very long term (50 years), the tensile stresses are significantly higher in the interior, because there shrinkage deformations occur at an age when the concrete is very mature and stiff.
In Fig. 8 it is also interesting to see that the results provided by the two approaches to model drying creep are not very much different. Therefore, the uncertainty associated to the quantification of the parameters $\eta$ and $k_a$ of the layered approach is expected not to have important implications in the results of the mechanical analyses.

Figure 9 reproduces the crack propagation calculated using the layered approach for drying creep, considering now the effects of concrete fracture and bond-slip action. Soon after the start of drying, the entire row of FEs closest to the surface becomes micro cracked. As drying continues, these micro cracks coalesce into a few number of localized cracks, which propagate in depth. The deformed mesh at 0.46 years shows this shape (Fig. 9), with 9 propagating cracks. The depth reached by each partial crack is variable. Their spacing (0.22 m on average) is quite regular throughout the RC tie length. The maximum steel stress at the location of each of these cracks is below 60 MPa (Fig. 9). As the local shrinkage strain increases in the most interior concrete, one of those cracks fully propagates, forming a through crack. That occurs soon after the 0.46 years. These through cracks are usually named as primary cracks, whereas the
ones that do not fully penetrate in the cross-section are named as secondary cracks (Schliche et al. 2019).

Figure 9 also shows the deformed mesh immediately after the formation of the second through crack, at 2.68 years. The partial cracks remain open and play an important role in limiting the width of the through crack. At the end of the analysis (50 years), a total of three through cracks are formed.

The time variation of the support reaction (equal to the axial force in the RC tie), as well as the steel stress and the crack width at the position of the first through crack are shown in Figs. 10 and 11. The support reaction corresponds to the discretized structure, whose thickness is 0.25 m, half the full thickness of 0.50 m, owing to the symmetry condition. Figure 10 shows that the highest support reaction is reached upon the formation of the first through crack. The second and third through cracks are formed at progressively lower values of the support reaction. This is because they are formed at later ages, where the progression of drying shrinkage induces higher tensile stresses in the concrete at the position of the new through cracks and, consequently, a lower axial force is necessary to fully open the concrete sections at those locations.

The crack width is quantified as the integral of the crack strains $\varepsilon_{cr}$ over the length of the FE where the trough crack is formed. Figure 9 shows that the crack width varies along the cross-section depth. At very-long term, the crack width $w$ is higher at the core, by comparison with the surface. The beneficial effect of the secondary cracks (in terms of reduction of the primary cracks’ width) does not exist in the central layer of the RC tie. The $w$ value shown in Fig. 11 is determined for the FE immediately above the reinforcement.

![Fig. 10 Axial force in the RC tie, for three time scales: a) 0 to 14 days; b) 0 to 5 years; c) 5 to 50 years.](image1)

![Fig. 11 Results of the parametric analysis: a) steel stress at the position of the first through crack; b) opening of the first through crack.](image2)
In the steel stress diagram presented in Fig. 11, it is also interesting to see that the peak stress is reached immediately before the formation of the second through crack. In fact, the occurrence of the first through crack is accompanied by a rapid increase of the steel stress and crack width. As the time progresses and the drying shrinkage increases, both the steel stress and the crack width keep growing, reaching a maximum upon the formation of the second through crack. At that instant, there is a sudden decrease of the axial force in the restrained structure. The steel stress at the crack also drops. The drop in the crack width is very moderate, owing to the bond behaviour upon unloading (unloading and reloading are governed by the elastic stiffness). In the steel stress diagrams plotted in Fig. 9, two peaks are observed, at 50 years, in the vicinity of the first through crack, whereas a single peak exists when the highest steel stress is reached (at 2.68 years). The two peaks are a consequence of the bond-stress-slip relationship upon unloading, after the formation of a new through crack.

4.2 Parametric analysis
In Figs. 10 and 11, the results of the reference analysis are compared with the calculations in which one of the parameters is varied (identified through the label shown in Table 3).

Figure 10 shows the evolution of the restraint force (half of the total force applied to the 500 mm thick element, because only that half is discretized). Among the considered variable parameters, only the degree of restraint, R, causes perceptible variations in the restraint force in the first 14 days. The remaining parameters affect, essentially, the behaviour after cracking. The highest restraint force is reached during the first month, in all of the analyses. In fact, even though the self-induced deformations progress significantly throughout time, the superficial cracking (which starts soon after the 7 days, as shown in section 4.1) progressively reduces the structure stiffness, and limits the restraint force increase. In a scenario of uniform stresses throughout the cross-section, the concrete cracking force at 28 days would be \( f_{\text{cr}} A_c = 2900 \times 0.25 = 725 \) kN. The highest tensile force reached is 620 kN (for \( R = 1 \)).

After the peak is reached, the restraint force decreases progressively and, for all of the analyses, the first through crack arises at an age close to 6 months (Fig. 10b).

The variation of the degree of restraint (\( R_{28} = 2/3 \) or full restraint) did not have a significant influence on the mechanical results after cracking, as shown in Figs. 10 and 11. This fact can be theoretically understood by observing the deformed mesh, in Fig. 9, for the age of 50 years (results for the analysis with \( R_{28} = 2/3 \)). This figure shows that, after cracking, the overall shortening of the RC tie is close to zero, that is, the tie behaviour is close to the behaviour under full restraint. This fact is justified by the very important stiffness decrease in the RC tie that occurs after the formation of through cracks. When the tie stiffness decreases, the actual degree of restraint increases, because the stiffness of the restraining elements (herein simulated by the spring stiffnesses \( k_i \)) is not modified.

After the formation of the first through crack, the variable parameters that most affect the structural response are the bond-slip curve (labelled with OBC) and the cross-sectional area of reinforcement (labelled with \( A_r = 42.2 \)). These are the only parameters (among the ones under evaluation) which significantly affect the calculated crack openings (Fig. 11b) and maximum steel stresses (Fig. 11a).

The calculated crack openings, w, at long-term (50 years) are approximately inversely proportional to the bond stress in the ascending branch of the bond-slip law. As shown in section 3.3, the OBC model prescribes bond-stress which are 60% lower than the ones for good bond conditions. The long-term w values for these bond scenarios are 0.65 mm and 0.39 mm. The corresponding increase of \( w \) is thus 67% (0.65 / 0.39 = 1.67).

Figure 10b shows that, for poorer bond conditions, after the onset of the first through crack the restraint force increases more slowly. This is a consequence of the higher stiffness decrease associated to the occurrence of a new crack, for poorer bond conditions. Therefore, in the OBC calculation, the second through crack is formed later (at 6.3 years) than in the REF analysis (at 2.7 years). Also, for poorer bond conditions, only two cracks exist at the end of the analysis.

As expected, higher areas of reinforcement ensure lower long-term crack-widths. However, the relative decrease of \( w \) is lower than the relative increase of \( A_r \). When \( A_r \) increases from 24.9 to 42.4 cm²/m, the maximum \( w \) value decreases from 0.43 mm to 0.30 mm. In this case, the number of through cracks formed at the end of the analysis is the same (three), for both \( A_r \) values.

As regards the length of the analysed member, L, the main consequence of this parameter is felt in the interval of time between the formation of through cracks. When the length doubles (from 2 m in the REF analysis to \( L = 4 \) m), that interval of time is roughly halved. This is expected since the tie length disturbed by the formation of a new through crack is approximately the same independently from the total RC tie length (if the \( A_r \), value and bond characteristics are unchanged), and consequently, in the longer RC tie, the restraint force will evolve more rapidly after a new crack is formed. The restraint force at the end of the analyses labelled as REF and \( L = 4 \) is not equal, because the total number of through cracks per unit length is not exactly the same (three cracks for \( L = 2 \) m and five cracks for \( L = 4 \) m). It is important to note that the exact position of each secondary and through crack is not repeated, even for very small variations in the input parameters, so the final numerical outputs are subjected to some variance too. Preliminary sensitivity analyses were made to
evaluate this variation, by changing the position of the weaker spot mentioned in Fig. 3. The results are not shown here for the sake of concision, but it can be said that the maximum variation obtained in the calculated crack openings was ~5%, which is acceptable for this type of analyses.

The influence of the elastic stiffness, $D_s$, of the bond-slip model was evaluated for both RC tie lengths ($L$ equal to 2 and 4 m). The main consequence of a lower $D_s$ is felt in the rapid decrease of the $w$ value, in pre-existing cracks, when a new through crack is formed (see, in Fig. 11, the sudden decrease of $w$ upon the formation of the second through crack, at ~3 years). Despite this fact, changing the $D_s$ value had no significant influence on the final crack openings, as shown in Fig. 11b. However, it did have some relevance in the final restraint force, as can be seen in Fig. 10c. In the RC ties with $L = 2$ m and $D_s = 244$ MPa/mm, only two through cracks were formed (the third crack was in the imminence of being formed, but the drying shrinkage evolution stabilized before). Accordingly, in this analysis the stiffness decrease due to cracking is lower than in the one with $D_s = 10000$ MPa/mm, in which three through cracks arose, causing a lower restraint force.

As a further development, it is worth implementing a different constitutive model to simulate the bond action, in which the unloading and reloading behaviour is independent from the elastic stiffness. Nevertheless, this issue is much less important than having a good characterization of the bond-slip law for monotonic loading, as demonstrated by the differences in the calculated $w$ values in the REF and OBC analyses.

As regards the influence of the modelling approach to simulate drying creep, it is interesting to see that the results (restraint force, steel stress and crack opening) obtained in the REF analysis (using the layered approach) are very similar to the results of the analysis using the B3 approach. This absence of important differences is justified by: the similarity in the total creep compliances (instantaneous plus basic and drying components) provided by the layered approach and the B3 model, in terms of average deformations of the cross section; the fact that the concrete stresses due to hydration and drying; simulates the ageing viscoelastic concrete response since the initial setting until the very long term; and includes the effects of concrete fracture and bond action at the steel-concrete interface.

For simulation of basic creep effects, a modifying factor was introduced in the B3 creep model, to account for the increased deformability in the first hours after the initial setting of the concrete. The calculated concrete stresses due to cement hydration (heat of hydration and autogenous shrinkage) are in line with the experimental results of new tests reported in the literature.

The adopted approach for simulation of drying creep is based on existing rheological models and takes into account the local relative humidity in concrete (which varies over the cross-section depth and along time). An exploratory approach was introduced to calibrate the two material parameters needed for the drying creep model.

The THM analysis framework is applied in the analysis of a restrained slab-like structure. Previous experimental campaigns for characterization of the material properties (namely the thermal, humidity diffusion and shrinkage properties) provided a solid background for the analysis of the structural response.

The numerical analyses undertaken using the DIANA software, and grounded on the FEM method, showed that the crack propagation sequence is in agreement with the evidences of test results reported in the literature. A parametric analysis was conducted in order to evaluate the effects of geometrical and material parameters. Among the parameters considered, the ones with greater influence in the structural behaviour are the bond-slip law and the amount of reinforcement. This fact indicates that the characterization of the bond action between steel and concrete is of paramount importance for a good estimation of the long-term crack widths.

The results provided by different approaches do model drying creep were not significantly different. Therefore, the uncertainty associated to the quantification of the material parameters of the local drying creep model is expected not to have important implications in the results of the mechanical analyses.

The results of this study are not expected to be directly applied in the design practice, because the analysed structures are made with a specific concrete mix (previously characterized in laboratory), a specific geometry, and constant environment characteristics. Further developments should be made in the future, to study structures exposed to exterior environment. Nev-
ertheless, these results and methodology of analysis are expected to be a contribution towards the development of new design procedures, by providing an original insight about the crack propagation in thick, end restrained structures and the interplay of hydration and drying effects.

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