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Type II supergravity origin of dyonic gaugings

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Dyonic gaugings of four-dimensional supergravity typically exhibit a richer vacuum structure compared to their purely electric counterparts, but their higher-dimensional origin often remains more mysterious. We consider a class of dyonic gaugings with gauge groups of the type $(SO(p, q) \times SO(p', q')) \times N$ with $N$ nilpotent. Using generalized Scherk-Schwarz reductions of exceptional field theory, we show how these four-dimensional gaugings may be consistently embedded in type II supergravity upon compactification around products of spheres and hyperboloids. As an application, we give the explicit uplift of the $N = 4$ AdS$_4$ vacuum of the theory with gauge group $(SO(6) \times SO(1,1)) \times T^{12}$ into a supersymmetric AdS$_5 \times M_3 \times S^5$ S-fold solution of IIB supergravity. The internal space $M_3$ is a squashed $S^5$ preserving an SO(4) $\subset$ SO(6) subset of its isometries.

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I. INTRODUCTION

Ungauged supergravities in four spacetime dimensions are defined up to a choice of the electric-magnetic symplectic frame. Different frames yield physically equivalent ungauged models, though described by inequivalent Lagrangians. Things change when the theory is gauged, and the corresponding gauged models, though described by inequivalent Lagrangians, are defined up to a choice of the electric-magnetic symplectic frame defining the gauge group. This freedom can be taken into account by allowing, in a fixed symplectic frame, for magnetic components of the embedding tensor defining the gauge algebra, namely by considering dyonic gaugings.1 Different initial frames will in general yield different choices of the gauge group and even gauging a same group in different frames may yield physically distinct theories.

This feature was first exploited in $N = 4$ supergravity [1]. In the maximal theory the freedom in the initial choice of symplectic frame led to the discovery of new gaugings in [2–5] and, more recently, in [6–9]. In [7,8], in particular, one-parameter families of dyonic SO$(p,q)$-gaugings were found in $N = 8$, $D = 4$ supergravity, generalizing their well-known electric counterparts [10,11]. These new models were constructed by gauging the same SO$(p,q)$ group in different frames, the choice of which is parametrized by a continuous angular parameter $\omega$. They are known as $\omega$-deformed models, where the value $\omega = 0$ corresponds to the original electric gaugings of [10,11]. The parameter $\omega$ is physical in that its value can not be offset by field redefinitions or the action of the global symmetry group $G$ of the ungauged theory, and does affect the physics of the model.

A different class of dyonic models, originally devised in [6], are based on non-semisimple groups of the form

$$\left(\text{SO}(p,q) \times \text{SO}(p',q')\right) \times N,$$

(1.1)

with $p + q + p' + q' \leq 8$ and $N$ is a subgroup generated by a nilpotent algebra whose properties are described later. These gauge groups can be defined as different contractions of the semisimple group $SO(p+p',8-p-p')$, generalizing the $CSO(p,q,r)$ gaugings of [11]. They are characterized by a $CSO(p,q,8-p-q)$ subgroup gauged by the electric vector fields and a $CSO(p',q',8-p'-q')$ gauged by the magnetic ones, with a subset of the nilpotent generators gauged by a combination of the two fields. As opposed to the $\omega$-deformed $SO(p,q)$-models, the corresponding gauged theories, also known also as dyonic $CSO(p,q,r)$ models, do not depend on a continuous parameter aside from an overall coupling constant. The only exceptions are the $SO(4)^2 \times \mathbb{R}^{16}$ gaugings and their noncompact forms, which have a one-parameter family of deformations corresponding to the ratio of gauge couplings for the two semisimple factors [9].

Dyonic gaugings feature a richer vacuum structure than their original electric counterparts. Of particular interest, for
their application to the AdS/CFT correspondence, are the anti–de Sitter vacua. In order to understand the general features of the dual three-dimensional CFT, however, a UV completion of the model within superstring or M-theory is called for. The original SO(8)-gauged maximal supergravity of [10] features a maximally supersymmetric AdS vacuum and describes a consistent truncation of eleven-dimensional supergravity compactified on a seven-sphere [12]. The CFT dual to the maximally supersymmetric vacuum of the theory is the ABJM model [13]. The electric CSO\((p,q,r)\)-gaugings, on the other hand, describe consistent truncations of eleven-dimensional theories on backgrounds in which the internal manifold has the form \(H^{p,q} \times \mathbb{R}^r\), \(H^{p,q}\) being a hyperboloid [14,15]. As for the dyonic gaugings, while the ten or eleven-dimensional origin of the \(\alpha\)-deformed SO\((p,q)\) models is as yet elusive [16,17], some progress has been made for the dyonic CSO\((p,q,r)\)-supergravities: Recently the dyonic ISO(7)-model was interpreted as a consistent truncation of massive type IIA string theory [18] on a background with topology of the form AdS\(_4 \times S^6\) [19–21]. In the present work we make further progress in this direction by defining a ten-dimensional origin for all the remaining dyonic CSO\((p,q,r)\)-models. Of special interest is the dyonic-model with gauge group \(SO(6) \times SO(1,1)\) \(\times T^{12}\), which features a characteristic \(\mathcal{N}=4\) AdS vacuum [22] of which we give a ten-dimensional description in the type IIB theory.

Exceptional field theory (ExFT) [15,23,24] has proven to be a valuable framework to study the higher-dimensional origin of \(D\)-dimensional maximal gauged theories. It provides a formulation of maximal supergravities, including the eleven and the ten-dimensional ones, which is manifestly covariant with respect to the on-shell global symmetry group of the \(D\)-dimensional model. In our analysis, we are interested in uplifting four-dimensional maximal gauged supergravities [5] so we choose to work in the \(D=4\) formulation of ExFT in which the manifest duality symmetry is the \(E_{7(7)}\) on-shell invariance of the Cremmer-Julia ungauged four-dimensional \(\mathcal{N}=8\) theory [25]. In this framework, the fields of the \(D=4\), \(\mathcal{N}=8\) supergravity are described as formally depending, in addition to the four spacetime coordinates \(x^\mu\), on 56 coordinates \(Y^M\) in the fundamental representation of \(E_{7(7)}\). This dependence is strongly restricted by the so-called "section constraints" [26,27]. Solutions to these constraints describe the eleven and ten-dimensional massless maximal supergravities written in terms of \(D=4\) fields, which only depend on specific sets of seven and six internal coordinates, respectively. In [28], a deformed version of ExFT was defined in order to describe the massive type IIA theory and its consistent truncations to \(D=4\).

\[2\]See [29] and [30] for corresponding results in the contexts of double field theory and exceptional generalized geometry, respectively.

The embedding of a gauged four-dimensional model in the eleven or ten-dimensional theories is effected through a suitable Scherk-Schwarz Ansatz [15] in which the ExFT fields depend on the internal coordinates through an \(E_{7(7)}\)-valued twist matrix \(U_M^{\mathcal{N}}(Y)\). This matrix encodes the higher dimensional fields as well as the fluxes on a certain background around which the four-dimensional fields ought to describe fluctuations. For instance, the Scherk-Schwarz Ansatz for the scalar fields of the ExFT is written in terms of the characteristic symmetric symplectic \(E_{7(7)}\)-matrix \(\mathcal{M}_{MN}(x,Y)\) as follows:

\[\mathcal{M}_{MN}(x,Y) = U_M^K(Y)U_N^L(Y)\mathcal{M}_{KL}(x),\]  

(1.2)

where \(\mathcal{M}_{KL}(x)\) describes \(D=4\) scalar fluctuations about the higher-dimensional background whose fields (metric, form-fields and fluxes) are encoded in the matrix \(U_M^{\mathcal{N}}(Y)\). If certain conditions on the twist matrix are satisfied, the dependence of the fields on the internal coordinates through \(U(Y)\) factors out in the ExFT field equations, yielding the field equations of gauged four-dimensional model in the \(x^\mu\)-dependent fields. The corresponding embedding tensor is encoded in \(U_M^{\mathcal{N}}(Y)\). The section constraints restrict the \(Y\) dependence of this matrix and thus the possible gauged models which can be described as consistent truncations of the ten- or eleven-dimensional theories.

In the present paper, the embedding of the dyonic CSO\((p,q,r)\) gaugings, with \(p+q \geq 2\), \(r \geq 2\), in the type II theories is effected by writing the twist matrix \(U(Y)\) as the product of two commuting matrices \(\breve{U}(y^i)\) and \(\hat{U}(\tilde{y}_a)\):

\[U(y^i,\tilde{y}_a) = \breve{U}(y^i)\hat{U}(\tilde{y}_a), \quad i = 1, \ldots, p + q - 1, \quad a = p + q, \ldots, 6.\]  

(1.3)

These two matrices separately define the electric \(\mathfrak{so}(p,q,r)\) and the magnetic \(\mathfrak{so}(p',q',r')\) subalgebras and the corresponding sets of coordinates \(\{y^i\}\) and \(\{\tilde{y}_a\}\) are chosen within distinct SL(8) representations satisfying a suitable condition of mutual compatibility. The total twist matrix satisfies the section constraints so that the corresponding dyonic models can be embedded either in type IIA \((p + q \text{ odd})\) or in type IIB \((p + q \text{ even})\) theories.

The dyonic model with \(p = 6\), \(q = 0\), \(p' = q' = 1\) mentioned earlier corresponds to a gauge group of the form \((SO(6) \times SO(1,1)) \times T^{12}\). It can be obtained from a stepwise compactification of the type IIB theory as follows. A first compactification of type IIB on AdS\(_5 \times S^5\) yields five-dimensional supergravity with gauge group SO(6) [31–33]. This model still features the SL(2,\(\mathbb{R}\)) duality symmetry of the type IIB theory, commuting with SO(6). As a last step one can perform a Scherk-Schwarz reduction down to \(D=4\), choosing a twist matrix valued in an SO(1,1) subgroup of SL(2,\(\mathbb{R}\)). The resulting model supports the above mentioned AdS\(_4\) vacuum (not at the
section II B gives a brief review of the relevant ExFT. In the main facts about the dyonic CSO decomposition of the fundamental SL(8) indices. Up to SL(8) transformations the type II embedding of the dyonic CSO Secs. II C and II D, the Scherk-Schwarz Ansätze defining the type II embedding of the dyonic CSO(p, q, r) gaugings are discussed in detail. Finally, in Sec. III, we focus on the SO(6) \times SO(1, 1) \times T^{12} gauged maximal supergravity and work out, using the general ExFT description of type IIB theory and the corresponding Scherk-Schwarz Ansatz, its uplift into the IIB theory. In particular, we give the uplift of the four-dimensional \( N = 4 \) AdS vacuum into a IIB S-fold solution. In Appendix C, we also prove that the noncompact version of this ten-dimensional geometry (i.e. before S-folding) falls in the class of Janus solutions found in \([34,35]\). We end with some concluding remarks.

II. TYPE II ORIGIN OF DYONIC GAUGINGS

A. Dyonic gaugings

Gaugings of maximal \( D = 4 \) supergravity are conveniently described by the embedding tensor formalism \([3,5,36-38]\) (for reviews, see \([39,40]\)). All the information about the gauge couplings of the theory is encoded in a tensor \( X_{MN}^P \) transforming in the 912 representation of \( E_{7(7)} \), where indices \( M, N, \ldots \) correspond to the 56 representation. In an appropriate symplectic frame an SL(8) subgroup of \( E_{7(7)} \) acts separately on electric and magnetic vectors. We are interested in non-semisimple gauge groups contained in SL(8, \( \mathbb{R} \)) of the form

\[
(SO(p, q) \times SO(p', q')) \times N, \quad (2.1)
\]

with \( N \) a nilpotent factor which becomes abelian when \( p + q + p' + q' = 8 \) \([6]\) (see also \([40]\) for a review). Its generators in the fundamental of SL(8) are triangular matrices with nonvanishing entries in the first \( p + q \) rows and last \( 8 - p - q \) columns, or in the first \( 8 - p' - q' \) columns and last \( p' + q' \) rows. These two sets of nilpotent generators overlap on a common \( (p + q)(p' + q') \)-dimensional Abelian subalgebra. This class of gaugings is described by two symmetric matrices \( \eta_{AB}, \tilde{\eta}^{AB} \) corresponding to the 36' and 36 irreps in the decomposition of the 912 under SL(8), with \( A, B, \ldots \) fundamental SL(8) indices. Up to SL(8) transformations we can write\(^3\)

\[\eta_{AB} = \text{diag}(1, \ldots, 1, -1, \ldots, -1, 0, \ldots, 0),\]

\[\tilde{\eta}^{AB} = \text{diag}(0, \ldots, 0, 1, \ldots, 1, -1, \ldots, -1), \quad (2.2)\]

such that \( \eta_{AB} \tilde{\eta}^{CB} = 0 \) in order to solve the embedding tensor quadratic constraints \([6]\). The embedding tensor then takes the form

\[
X_{AB,CD}^{\text{EF}} = \eta_{[A[C} \delta_{D]B}^{\text{EF}} - \eta_{B[C} \delta_{D]A}^{\text{EF}},
\]

\[
X^{AB} = -\tilde{\eta}_{A(E} \delta_{C}^{\text{F]}B} + \tilde{\eta}_{B(E} \delta_{C}^{\text{F]}A}, \quad (2.3)
\]

where 56 \( E_{7(7)} \) indices \( M, N, \ldots \) are decomposed into the \( 28' + 28 \) of SL(8, \( \mathbb{R} \)), described by upper and lower antisymmetric pairs of 8 indices.

Most of these gauged models are entirely specified by their gauge group embedded in SL(8), with a few notable exceptions \([9]\). When \( p + q = 7 \) we find ISO(p, q) gaugings. In this case the gauge group is entirely specified by \( \eta_{AB} \) and \( \tilde{\eta}^{AB} \) only affects the gauge connection of the \( \mathbb{R}^7 \) subgroup. A nonvanishing \( \tilde{\eta}^{AB} \) is identified with the Romans mass in a IIA uplift of the gauging \([19-21]\). Moreover, when \( p + q = p' + q' = 4 \) the relative overall normalization of \( \tilde{\eta}^{AB} \) with respect to \( \eta_{AB} \) cannot be reabsorbed in any \( E_{7(7)} \) transformation and thus determines a one-parameter family of inequivalent gaugings sharing the same gauge group.

Several of the dyonic CSO(p, q, r) models exhibit interesting vacua. Maximally symmetric vacuum solutions of the resulting gauged maximal supergravities are determined by extrema of the scalar potential \([5]\)

\[
V(\phi) = \frac{1}{672} M(\phi)^{MPQR} X^{MP} S M(\phi)^{N} X_{NM} - 7 X_{MN} X_{PQ}, \quad (2.4)
\]

where \( M(\phi)_{MN} \) is a symmetric matrix parametrizing the \( E_{7(7)}/SU(8) \) nonlinear sigma model of the scalar fields, and \( M(\phi)^{MN} \) is its inverse. The deformed ISO(7) gauging (i.e. with \( \tilde{\eta} \neq 0 \)) has several supersymmetric and non-supersymmetric AdS4 solutions \([6,22,41]\). The \( (SO(4) \times SO(2, 2)) \times T^{16} \) gauging (with equal normalizations for \( \eta_{AB} \) and \( \tilde{\eta}^{AB} \)) and the \( (SO(2) \times SO(2)) \times N_{20} \) model are part of a large class of theories exhibiting Minkowski vacuum connected through singular limits in their moduli spaces \([42]\). In this paper we will focus in particular on the \( (SO(6) \times SO(1, 1)) \times T^{12} \) gauging which is known to have an \( \tilde{N} = 4 \) AdS4 vacuum \([22]\), in addition to other unstable AdS4 solutions \([6]\).
B. Scherk-Schwarz reduction in exceptional field theory

Exceptional field theories are the manifestly duality covariant reformulations of maximal supergravities. Since our goal is a higher-dimensional embedding of four-dimensional maximal supergravities, which are obtained as gaugings [5] of the E$_{7(7)}$-invariant Cremmer-Julia theory [25], the proper framework for their higher-dimensional embedding is the E$_{7(7)}$-covariant exceptional field theory constructed in [24]. This exceptional field theory is formulated in terms of the fields of dimension 7 + 1 coordinates which are singlet under the global SL(2) symmetry and coincide with the equations of the IIA parametrization. The precise dictionary between the ExFT formulation and IIA/IIB supergravity further requires redefinitions of all the form fields originating from the higher-dimensional p forms in the usual Kaluza-Klein manner, as well as a series of dualization and nonlinear field redefinitions, cf. a [24,45].

Consistent truncations in exceptional field theory are conveniently constructed via a generalized Scherk-Schwarz reduction by the Ansatz [15]

$$\mathcal{M}_N(x,Y) = U_M^N(Y) M_{KL}(x),$$

$$\mathcal{A}^M_{\mu}(x,Y) = \rho^{-1}(Y) U^{\beta}_{\mu}(Y) B_{\mu\beta}(x),$$

$$B_{\mu\nu}(x,Y) = -2\rho^{-2}(Y)(U^{-1})_S(Y) \times \partial_M U_{R}(T^R)^S B_{\mu\nu}(x),$$

for the bosonic fields (2.7). The dependence on the internal coordinates is carried by a four-dimensional twist matrix $U^N_M$ and a scale factor $\rho(Y)$, satisfying the first order differential equations [46]

$$[U^{-1}]^P M^Q [U^{-1}]^N \partial_P U^K_N \sim 1 \rho \Theta^{a}(t_{\alpha}) N^a K,$$

$$\partial_N (U^{-1})^M_N - 3\rho^{-1} \partial_N \rho(U^{-1})^M_N \sim 2\rho \Theta^M,$$

with constant tensors $\Theta^a$ and $\partial_M$. The latter can be identified with the irreducible components of the embedding tensor of the four-dimensional gauged supergravity [5] to which the theory reduces after the generalized Scherk-Schwarz Ansatz. In particular, the notation $\{\}^a_{912}$ refers to projection onto the irreducible 912 representation of E$_{7(7)}$.  

4Depending on the context, indices $\alpha, \beta, \ldots$ represent either the E$_{7(7)}$ adjoint representation or the SL(2) fundamental. This should cause no confusion.
Every solution to the system (2.10) defines a consistent truncation of exceptional field theory down to a four-dimensional gauged supergravity with all $Y$ dependence consistently factoring out from the equations. If the matrix $U_M^N$ and the scale factor $\rho(Y)$ satisfy the section constraint (2.5), the dictionary with IIA/IIB supergravity provides the explicit formulas for a geometrical uplift of the resulting four-dimensional gauging into type II supergravity. In this paper, we will construct the twist matrices $U_M^N$ that define the geometrical uplift of the dyonic gaugings defined above.

**C. Scherk-Schwarz twist matrices for dyonic gaugings**

The solutions to the consistency equations (2.10) constructed in [15] give rise to the embedding tensors associated with the gaugings of $\text{SO}(p,q)$ and $\text{CSO}(p,q,r)$ and provide a geometrical uplift of these theories via the compactification on spheres and hyperboloids. To this end, the 56 internal coordinates are decomposed in the SL(8) frame

$$\{Y^M\} = \{Y^{[AB]}, Y_{[AB]}\}, \quad A, B = 1, \ldots, 8,$$  \hspace{1cm} (2.11)

into what we will refer to as “electric” and “magnetic” coordinates. In [15], the physical coordinates are identified among the electric $Y^{[AB]}$ as $Y^a \equiv Y^{[12]}$, corresponding to the $D = 11$ solution (2.6) of the section constraint. In the SL(8) frame (2.11), the latter takes the form

$$\partial_A \otimes \partial^{BC} + \partial^{BC} \otimes \partial_A = \frac{1}{8} \delta^B_A (\partial_{CD} \otimes \partial^{CD} + \partial^{CD} \otimes \partial_{CD}),$$

$$\partial_{[AB]} \otimes \partial_{CD}] = \frac{1}{24} \epsilon_{ABCDEFGH} \partial^{EF} \otimes \partial^{GH}.$$  \hspace{1cm} (2.12)

The twist matrices $U_{A}^{B}(y^i)$ associated to sphere and hyperboloid compactifications can then be constructed within the subgroup $\text{SL}(8) \subset E_{7(7)}$.

Here, we will generalize this result to twist matrices $U \subset \text{SL}(8)$ which depend on more general subsets of coordinates (still satisfying the section constraint) and take the form of products of the solutions found in [15]. More precisely, let us consider a twist matrix of the type

$$U(y^i, \tilde{y}_a) \equiv \hat{U} (\tilde{y}_a) \hat{U}(y^i),$$

$$\rho(y^i, \tilde{y}_a) = \hat{\rho}(\tilde{y}_a) \hat{\rho}(y^i),$$  \hspace{1cm} (2.13)

where $\hat{U}$ and $\hat{U}$ separately solve the Scherk-Schwarz consistency equations, with embedding tensors denoted by $\hat{X}_MN^K$ and $\hat{X}_MN^K$, respectively. We also assume that $\hat{\rho} = \hat{\rho} = 0$. With this Ansatz, the first of the consistency equations (2.10) for $U$ reduces to

$$\hat{\rho}^{-1}[(\hat{U}^{-1} \hat{U}^{-1})_M^N \hat{X}_{NP}^Q]_{912} + \hat{\rho}^{-1} \hat{U} [\hat{X}_{MP}^Q]$$

$$\equiv \text{const} \equiv X_{MP}^Q.$$  \hspace{1cm} (2.14)

where $\hat{X}_{MN}^K$ denotes the unprojected current

$$\hat{X}_{MN}^K \equiv \hat{\rho}^{-1}(\hat{U}^{-1})_M^P(\hat{U}^{-1})_N^Q \delta_P \delta_Q \hat{U}^K_N,$$  \hspace{1cm} (2.15)

(such that $[\hat{X}_{MN}^K]_{912} = \hat{X}_{MN}^K$), and

$$\hat{U}[\hat{X}_{MP}^Q] \equiv (\hat{U}^{-1})_M^P(\hat{U}^{-1})_N^Q \hat{U}_K^N \hat{X}_M^K,$$  \hspace{1cm} (2.16)

denotes the $E_{7(7)}$-action of $\hat{U}$ on the embedding tensor $\hat{X}_{MP}^Q$. Let us further assume that the variables $y^i$ and $\tilde{y}_a$ are mutually compatible in the sense that

$$\hat{\rho}^{-1}(\hat{U}^{-1})_M^N \partial_N = \partial_M,$$

$$\hat{\rho}^{-1}(\hat{U}^{-1})_M^N \partial_N = \partial_M,$$  \hspace{1cm} (2.17)

i.e. that we have equality of the action of these differential operators on the coordinates $y^i$ and $\tilde{y}_a$, respectively. With this assumption, the lhs of equation (2.14) reduces to

$$\hat{X}_{MP}^Q + \hat{\rho}^{-1} \hat{U} [\hat{X}_{MP}^Q] = \hat{X}_{MP}^Q + \hat{X}_{MP}^Q,$$  \hspace{1cm} (2.18)

such that equation (2.14) is automatically satisfied with the resulting embedding tensor given by

$$X_{MP}^Q = \hat{X}_{MP}^Q + \hat{X}_{MP}^Q.$$  \hspace{1cm} (2.19)

We can introduce a relative coupling constant between $\hat{X}_{MP}^Q$ and $\hat{X}_{MP}^Q$ by rescaling of the $\tilde{y}_a$ vs the $y^i$ coordinates. This allows us to capture the continuous deformation parameter of the $\text{SO}(4)^2 \times T^4$ gaugings and of their noncompact forms. Finally, the second equation of (2.10) turns into

$$3 \hat{\rho}^{-1}(\hat{U}^{-1} \hat{U}^{-1})_M^N \partial_N,$$  \hspace{1cm} (2.20)

which together with (2.17) and the respective equations for $\hat{\rho}$ and $\hat{\rho}$ turns into an identity.
In the following we will consider the product Ansatz (2.13) with matrices $\hat{U}$ and $\hat{U}^{-1}$ chosen among the solutions from [15], corresponding to gauge groups $\text{SO}(p, q)$ and $\text{SO}(p', q')$, respectively. In order to satisfy the compatibility constraints (2.17) together with the section constraints (2.12), we will choose the coordinates $y^i$ among the electric and the $\bar{y}_a$ among the magnetic coordinates from (2.11). More precisely, we define coordinates $\{y^i, \bar{y}_a\}$

\[
(U^{-1})^B_A = (\hat{\rho}\hat{\rho}^{-1})^{1/2} \begin{pmatrix}
\hat{V}_i^j & 0 & 0 & \hat{\rho}^2\hat{V}_i^0 \\
0 & W_a^a & 0 & 0 \\
0 & \hat{\rho}^{-2}W_a^a & \hat{\rho}^{-2} & 0 \\
0 & \hat{\rho}^{-1}(1 + \hat{u}K(\hat{u}, \hat{v})) & 0 & \hat{\rho}^4
\end{pmatrix},
\]

which we present in the $\text{SL}(8)$ basis $\{A\} \rightarrow \{i, a, 7, 8\}$. The various blocks are given by

\[
\hat{V}_0 = \eta_{ij}y^j\hat{K}(\hat{u}, \hat{v}), \quad \hat{V}_0^0 = \eta_{ij}y^j, \\
\hat{V}_i^j = \delta_{ij} + \eta_{ik}\eta_{jl}y^k y^l\hat{K}(\hat{u}, \hat{v}), \\
W_a^a = -\eta^{ab}\bar{y}_b, \quad W_a^a = -\eta^{ab}\bar{y}_b\hat{K}(\hat{u}, \hat{v}), \\
\hat{\rho} = (1 - \hat{v})^{1/4} \equiv (1 - y^i\eta_{ij}y^j)^{1/4}, \\
\hat{\rho} = (1 - \hat{v})^{1/4} \equiv (1 - \bar{y}\rho^{-1}\bar{y}\rho_0)^{1/4}.
\]

The functions $\hat{K}(\hat{u}, \hat{v})$ and $\hat{K}(\hat{u}, \hat{v})$ are determined by first order differential equations and given explicitly in [15]. One may check explicitly that the matrix (2.22) solves the consistency equations (2.10) and gives rise to the embedding tensor (2.3) of the dyonic gaugings. We stress that it is crucial for the consistency of the construction that the coordinates $y^i$ and $\bar{y}_a$ are chosen within distinct $\text{SL}(8)$ representations in (2.11), i.e. the $y^i$ and the $\bar{y}_a$ are embedded in the electric and magnetic coordinates, respectively.

### D. Type II origin

In the previous section, we have constructed the Scherk-Schwarz twist matrices that give rise to the embedding tensor of dyonic gaugings. Since we have identified the coordinates $\{y^i, \bar{y}_a\}$ on which these matrices depend directly in the $\text{SL}(8)$ frame (2.11), it is not immediately obvious if these coordinates in the $\text{GL}(6)$ bases (2.6) correspond to a IIA or IIB solution of the section constraints. We will determine their precise higher-dimensional origin case by case according to the value of $p + q$.

#### 1. $p + q = 6$

In this case, the coordinates (2.21) are given by $\{Y^{18}, Y^{28}, Y^{38}, Y^{48}, Y^{58}, Y_{67}\}$. Comparing this set to the section constraint (2.12), it follows that fields can depend on none of the other 50 internal coordinates without violating the section constraint. We conclude that exceptional field theory on this set of coordinates is equivalent to IIB supergravity. More specifically, we can identify the $\text{SL}(2)_{\text{IIB}}$ under which these coordinates are singlets as the subgroup of $\text{SL}(8)$ whose generators are given by

\[
\text{SL}(2)_{\text{IIB}} = \langle T_6^7, T_7^6, T_7^7 - T_6^6 \rangle,
\]

where $E_{(77)}$ generators are defined in Appendix B. The $\text{GL}(1)_{\text{IIB}} C \text{GL}_{\text{IIB}}$ which provides the geometric grading of coordinates (2.6) and fields is generated by

\[
\text{GL}(1)_{\text{IIB}} = \left\langle T_8^{8} - \frac{1}{2}(T_6^6 + T_7^7) \right\rangle.
\]

Indeed, evaluating the charges of the various coordinates under this $\text{GL}(1)_{\text{IIB}}$, we find

\[
\{Y^{18}, Y_{67}\} = -4, \quad \{Y^{a8}, Y_{ia}\} = -2, \\
\{Y^{ij}, Y_{ij}\} = 0, \quad ..., \quad \{Y^{ij}, Y_{ij}\} = 0, \quad ....
\]

thus reproducing the IIB charges of (2.6).

#### 2. $p + q = 5$

In this case, the coordinates (2.21) are given by $\{Y^{18}, Y^{28}, Y^{38}, Y^{48}, Y_{57}, Y_{67}\}$. It is straightforward to verify
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5. \( p + q = 2 \)

In this case, the coordinates are given by \( \{ Y^{18}, Y_{27}, Y_{37}, Y_{47}, Y_{57}, Y_{67} \} \). Upon flipping \( Y^{AB} \leftrightarrow Y_{AB} \), this choice maps into the case of \( p + q = 6 \) above, it thus corresponds to a IIB embedding of the theory.

III. UPLIFT OF THE \( \mathcal{N} = 4 \) AdS4 VACUUM

A. \( \mathcal{N} = 4 \) AdS4 vacuum in \( D = 4 \) supergravity

The (SO(6) \( \times \) SO(1, 1)) \( \ltimes T^{12} \) gauged maximal supergravity admits an \( \mathcal{N} = 4 \) AdS4 vacuum preserving SO(4) gauge symmetry [22]. This solution is part of a one-parameter family of \( \mathcal{N} = 4 \) AdS vacua belonging to inequivalent gauged maximal supergravities but exhibiting similar physical properties. The other elements of this family of solutions are vacua of the \( \omega \)-deformed SO(7, 1) supergravities, whose higher-dimensional origin is unknown. However, at a singular point in the parameter space of the family the gauging degenerates into (SO(6) \( \times \) SO(1, 1)) \( \ltimes T^{12} \), for which we can now provide an uplift to type IIB supergravity.

Using (2.22) for \( p = 6, q = 0 \) and \( p' = q' = 1 \), the Scherk-Schwarz \textit{Ansatz} (2.9) describes the consistent truncation of type IIB supergravity to (SO(6) \( \times \) SO(1, 1)) \( \ltimes T^{12} \) gauged maximal \( D = 4 \) supergravity described by the embedding tensor (2.3) with

\[
\eta_{AB} = \text{diag}(-1, \ldots, -1, 0, 0, -1),
\]

\[
\tilde{\eta}^{AB} = \text{diag}(0, \ldots, 0, -1, 1, 0).
\]

In order to uplift the \( \mathcal{N} = 4 \) AdS4 solution of [22] we will need to reproduce the vacuum extremizing the scalar potential (2.4) in terms of the scalar matrix

\[
M_{MN} = (LL^T)_{MN} = \begin{pmatrix}
M_{AB,CD} & M_{AB}^{CD} \\
M_{AB}^{CD} & M_{AB,CD}
\end{pmatrix},
\]

expanded in the SL(8) basis (2.11). Here \( L \) is a coset representative for \( E_{7(7)}/SU(8) \). The \( \mathcal{N} = 4 \) AdS4 vacuum is located in an SO(4) \( \subset \) SO(6) invariant subspace of the scalar manifold, which turns out to be a GL(3)/SO(3) subcoset space generated by [8]

\[
t_1 = \frac{1}{12\sqrt{2}} (T_1 + T_2 + T_3 - T_4 - T_5 - T_6),
\]

\[
t_2 = \frac{1}{24\sqrt{2}} (T_1 + T_2 + T_3 + T_4 + T_5 + T_6 - 3T_8),
\]

\[
t_3 = \frac{1}{4\sqrt{6}} (T_6 - T_7).
\]
INVERSO, SAMTLEBEN, and TRIGIANTE

\[ t_4 = \frac{1}{4\sqrt{6}}(T_6^7 + T_7^6), \quad t_5 = \frac{1}{4\sqrt{3}}T_{1236}. \quad t_6 = -\frac{1}{4\sqrt{3}}T_{1237}. \]

(3.6)

We normalize these generators so that \( \text{Tr}\text{sgn}(t_4 t_5^T) = 1 \) in the fundamental of \( E_7(7) \).

Actually only some of the fields associated with these generators acquire a nontrivial value at the fundamental of \( E_7(7) \). We normalize these generators so that \( \text{Tr} \text{sgn}(M) = 1 \) in the fundamental of \( E_7(7) \).

\[ M \]

The explicit form of the scalar matrix \( \text{M}_{MN} \) at the vacuum in Appendix A.

There are, of course, flat directions of the solution (3.7) associated with the broken gauge symmetries. The flat direction associated with the broken SO(1, 1) will be relevant in the following. We give the explicit form of the scalar matrix \( \text{M}_{MN} \) at the vacuum in Appendix A.

\[ \text{M}_{MN} = \begin{pmatrix}
\text{M}_m^m & \text{M}_m^m\beta \\
\text{M}_m^{m'\beta} & \text{M}_m^{m'\beta} \\
\text{M}_m^{m'p} & \text{M}_m^{m'p} \\
\text{M}_m^{m'p}\beta & \text{M}_m^{m'p}\beta \\
\end{pmatrix}. \]

(3.11)

The explicit form of these blocks is read off from expanding the exponential series (2.8) and (after proper normalization) gives rise to the following identification of the IIB fields

\[ \text{M}^{mn} = G^{-1/2}G^{mn}, \]

\[ \text{M}^{m\alpha} = \frac{1}{\sqrt{2}} G^{-1/2} G^{mk} b_{k\alpha} b_{\alpha}, \]

\[ \text{M}_{m\alpha n\beta} = \frac{1}{2} G^{mn} m_{\alpha n} + \frac{1}{2} G^{1/2} G^{kl} b_{mk} b_{nl} \delta_{\alpha\beta}, \]

\[ \text{M}^{m\alpha n\beta} = \text{M}^{m\alpha n\beta} + \text{M}^{m\alpha n\beta} \delta_{\alpha\beta} + \text{M}^{m\alpha n\beta} \delta_{\alpha\beta}. \]

(3.12)

For the uplift formulas, we need to evaluate the lhs of these expressions via the Scherk-Schwarz Ansatz (2.9),

\[ \text{M}_{MN}(x, y) = U_M^M(y) U_N^N(y) \text{M}_{MN}(x), \]

with the SL(8) valued twist matrix \( U \) from (2.22). In order to reconcile the GL(6) \( \times \) SL(2) decomposition of (3.11) with the SL(8) form of the twist matrix, we have to break both groups down to their common SL(5) \( \times \) SL(2). For the coordinates (3.10), this implies

\[ t_1 \rightarrow -t_1, \quad t_{5,6} \rightarrow -t_{5,6}. \]

(3.9)

B. Uplift formulas from generalized Scherk-Schwarz reduction

The explicit uplift formulas that provide the embedding of the four-dimensional gauging into the IIB theory are straightforwardly obtained by combining the Scherk-Schwarz Ansatz (2.9) with the dictionary between the IIB theory and \( E_7(7) \). We will take both negative in the coset representative give different instances of equivalent vacua. We will take both negative in the coset representative give different instances of equivalent vacua. We will take both negative in the coset representative give different instances of equivalent vacua.

\[ \{Y^M\} \rightarrow \{\bar{Y}^m, \bar{Y}_{ma}, \bar{Y}^{kmn}, \bar{Y}^{mn}, \bar{Y}_m\}, \]

(3.10)
Upon splitting \( \{m\} \rightarrow \{i, 6\} \). Similarly, for the SL(8) coordinates (2.11), and in accordance with (2.27), we use the split of SL(8) indices,

\[
\{A\} \rightarrow \{i, a\}, \quad \text{with} \quad \hat{a} = (6, 7),
\]

in order to decompose the \( \{Y_{AB}, Y_{[AB]}\} \). We may then identify the coordinates (3.14) among the SL(8) coordinates (2.11) as

\[
\begin{align*}
\{\hat{Y}_i, \hat{Y}_6, \hat{Y}_{i\alpha}, \hat{Y}_{6\alpha}, \hat{Y}^{ij}, \hat{Y}^{i6}, \hat{Y}_{i\alpha}, \hat{Y}_6\} &= \{Y^{i8}, Y_{i8}, e_{ab}Y^{28}, e^{i8j}Y_{i\alpha}, \hat{Y}^{ij}, Y^{i8}, e_{ab}Y_{ib}, Y_{ib}, Y^{67}\}, \\
&= \{Y^{i8}, Y_{i8}, e_{ab}Y^{28}, e^{i8j}Y_{i\alpha}, \hat{Y}^{ij}, Y^{i8}, e_{ab}Y_{ib}, Y_{ib}, Y^{67}\},
\end{align*}
\]

where according to (2.25) we identify \( \{\hat{a}\} = \{6, 7\} \) from (3.15) with the SL(2) doublet indices \( \{a\} = \{1, 2\} \).

Let us now make the uplift formulas explicit. Combining (3.12) with (3.13) and the form of the twist matrix (2.22), we obtain

\[
G^{-1/2}G^{ij} = M^{ij} = 2M^{88,88} = 2(U^{-1})^{ij}_{kl}\eta^{kl}M^{kl, mn}(x) = 2\rho^2k_{ij}k_{mn}^\prime M^{kn, mn}(x),
\]

with \( \rho^2 \equiv (y_0^1)^2 + (y_0^2)^2 + (y_0^3)^2 \leq 1 \) and

\[
\Delta = ((1 + 2r^2)(3 - 2r^2))^{-1/4}.
\]

In a similar way, we may obtain the uplift formulas for the remaining IIB fields from (3.12). For the two-form, we find that its only nonvanishing components are given by

\[
b_{ij}^a = 2G^{1/2}G_{ik}e^{ab}M^{88,88} = 4G^{1/2}G_{ik}\epsilon^{ab}(U^{-1})^{ij}_{kl}\epsilon_{mn}M^{mn, mn} = -2\Delta G_{ik}k_{ij}^\prime\partial_j\gamma^m\epsilon^{cd}A_{j}^{\prime\prime}M^{mn, mn},
\]

where \( M^{kl, mn}(x) \) refers to part of the lower right block of the \( E_{11} \) matrix (3.2), and we have expressed the relevant components of the twist matrix \( U \) in terms of the Killing vectors on the round five-sphere

\[
\begin{align*}
\mathcal{K}_{mn}^i &= \hat{G}^{ij}\partial_jY_{mn}, \\
\hat{G}^{ij} &= \delta^{ij} - y_iy_j.
\end{align*}
\]

Similar calculation determines the remaining components of the internal six-dimensional metric, such that together we find

\[
\begin{align*}
G^{ij} &= 2\Delta k_{kl}k_{mn}^\prime M^{kl, mn}(x), \\
G^{ij} &= 2\Delta \rho^2k_{ij}k_{mn}^\prime M^{kl, mn}(x), \\
G^{ij} &= 2\Delta \rho^4k_{ij}k_{mn}^\prime M^{kl, mn}(x),
\end{align*}
\]

with \( \rho \) from (2.24) and the scale factor \( \Delta \) defined by

\[
\Delta = \rho^2(\det G)^{1/2}.
\]

While (3.19) represent the uplift formulas for generic solutions of the four-dimensional theory, in the vacuum (3.7) we are interested in lifting, the matrix \( M_{MN}(x) \) is constant, and these formulas further reduce to

\[
\begin{align*}
&i, j \in \{1, 2, 3\} \\
&i, j \in \{4, 5, 6\} \\
&i \in \{1, 2, 3\}, j \in \{4, 5, 6\}
\end{align*}
\]

where as above we identify \( \{\hat{a}\} = \{6, 7\} \) from (3.15) with the SL(2) doublet indices \( \{a\} = \{1, 2\} \), and \( M^{kl, mn} \) is given in (A2). The SL(2) matrix \( A_{\hat{a}}^{\hat{a}\prime} \) is read off as

\[
A_{\hat{a}}^{\hat{a}\prime} = \begin{pmatrix}
\rho^2 \\
\tilde{y}_6 \\
\rho^{-2}(1 + \tilde{y}_6^2)
\end{pmatrix} = \begin{pmatrix}
\sqrt{1 + \tilde{y}_6^2} & \tilde{y}_6 \\
\tilde{y}_6 & \sqrt{1 + \tilde{y}_6^2}
\end{pmatrix},
\]

from the (6, 7) block of (2.22), using that \( \hat{K} = 1 \) in this case.
Next, the IIB dilaton/axion matrix is obtained from (3.12) as
\[
m_{\alpha\beta} = \frac{1}{3} G(M^{mn} M_{mn,\alpha\beta} - 4 M^{a} M_{ak} M_{n\beta}),
\]
which when put together with (3.13) in our vacuum yields
\[
m_{\alpha\beta} = \frac{2}{3} \Delta^2 \mathcal{Y}_\alpha \mathcal{Y}_\beta \sum m_{a\alpha b} A_a (\epsilon A_b) \epsilon_{\alpha\beta},
\]
with the matrix \( A_a^\alpha \) from (3.24) and
\[
S^{mn,ab} = \frac{1}{2} M^{mn,ab} + \epsilon^{abc} b^{d} (M^{dln} + M^{dmn} M_{ln}).
\]
With the explicit values (A1)–(A6) of \( M_{MN} \) in our vacuum, this expression reduces to
\[
m_{\alpha\beta} = (\Delta^{-1} M_{c^T})_{\alpha\beta},
M_{ab} = \Delta^2 \sqrt{3} \left( \begin{array}{cc} 3 + 2r^2 & -4r^2 \\ -4r^2 & 3 + 2r^2 \end{array} \right),
\]
and it comes as a nontrivial consistency check, that with the expression (3.22) for the scale factor \( \Delta \), this matrix indeed has determinant 1.

Finally, the expression for the only nonvanishing components of the IIB four-form follows from
\[
\mathcal{M}^{ijkl} = \frac{1}{2} \mathcal{M}^{8,8'} \epsilon_{ijkl} \epsilon_{88'} = \frac{1}{2} \rho_{ijkl} K_{88'} (U^{-1})_{mn} \epsilon_{88'} M^{mn},
= \frac{1}{2 \Delta} \rho^2 G^{ij} \hat{C}_{ijkl} = \frac{1}{2} \rho^2 \hat{C}^{ij} \epsilon_{ijkl} \epsilon_{88'},
= \frac{1}{2 \Delta} \rho^2 G^{ij} \hat{C}_{ijkl} = \frac{1}{2} \rho^2 \hat{C}^{ij} \epsilon_{ijkl} \epsilon_{88'},
\]
with \( \hat{C}_{ijkl} \) defined as giving rise to the S\(^5\) background flux:
\[
5 \partial_\ell \hat{C}_{ijkl} = \partial_\ell \hat{C}_{ijkl} = \hat{\partial}_{ijkl} \epsilon_{88'},
\]
Together, the expression for the IIB four-form is given as
\[
c_{ijkl} = \hat{C}_{ijkl} + \frac{1}{4} \Delta \mathcal{K}^{ij} m^{ij} \mathcal{K}_{ijkl} \hat{\nabla}^j \epsilon_{ijkl} M^{mn,nn},
\]
We have thus obtained all the nonvanishing IIB fields as functions of the S\(^5\) Killing vectors and sphere harmonics. Let us note that the expansion (2.8) also carries some components \( b_\alpha \equiv \epsilon^{klmnpq} \epsilon_{klmnpq} a \) of the dual six-form of the IIB theory which, however, vanish identically in our vacuum.

C. The supersymmetric IIB AdS\(_4\) \( \times M_S \times S^1 \) solution

In this section, we calculate the field strengths and present the IIB solution in its most compact form. The vacuum (3.7) of the four-dimensional theory preserves \( \mathcal{N} = 4 \) supersymmetry and, accordingly, a global \( SO(4) = SO(3) \times SO(3) \) symmetry that shows up as the internal isometry group of the IIB solution. In order to make these isometries manifest, we split the S\(^5\) sphere harmonics into
\[
\{ \mathcal{Y}_\alpha \} = \{ \mathcal{Y}^p, \mathcal{Z}^p \equiv \mathcal{Y}^{p+3} \}, \quad p = 1, 2, 3,
\]
\[
\mathcal{Y}^p \mathcal{Y}^p = 1 - \mathcal{Z}^p \mathcal{Z}^p = r^2.
\]
In terms of these harmonics, the ten-dimensional IIB metric is given by
\[
ds^2 = \Delta^3 \left( (3 - 2r^2) \delta^{pq} + 8 \mathcal{Y}^p \mathcal{Y}^q \right) d\mathcal{Y}^p d\mathcal{Y}^q
+ \Delta^3 \left( 1 + 2r^2 \right) d\mathcal{Z}^p d\mathcal{Z}^p
+ \Delta^{-1} \left( d\eta d\eta + \frac{1}{2} d s^2_{AdS_4} \right),
\]
with the warp factor given by (3.22) as
\[
\Delta = \left( (1 + 2r^2)(3 - 2r^2) \right)^{-1/4},
\]
and the AdS\(_4\) radius fixed to \( r_{AdS} = 1 \). With respect to the previous sections, we have also changed coordinates \( \tilde{y}_a = \sinh \eta \) along the S\(^1\) direction. The internal five-dimensional space is a deformation of the round metric on S\(^5\) which preserves an SO(3) \( \times \) SO(3) \( \subset \) SO(6) of the isometry group. Indeed, the harmonics \( \mathcal{Y}^p, \mathcal{Z}^p \) can be regarded as embedding coordinates for two S\(^2\)'s spheres of radii \( r \) and \( \sqrt{1 - r^2} \), respectively. The S\(^5\) geometry is parametrized in terms of these two spheres fibered over the interval \( r \in (0, 1) \), and at the points \( r = 0, 1 \) one of the S\(^2\)’s shrinks smoothly to zero size. Denoting \( d s^2_{AdS_4} \) the round metrics of unit radius on the S\(^2\)’s, an explicit expression for (3.33) is
\[
ds^2 = \Delta^3 (3 - 2r^2) r^2 d S^2_{AdS_4} + \Delta^3 (1 + 2r^2)(1 - r^2) d\Omega^2_{AdS_4}
+ \Delta^{-1} \left( d\eta^2 + \frac{dr^2}{1 - r^2} + \frac{1}{2} d s^2_{AdS_4} \right),
\]
The SL(2) matrix of IIB supergravity,
\[
m_{\alpha\beta} = \frac{1}{\text{Im} \tau} \begin{pmatrix} |\tau|^2 & -\text{Re} \tau \\ -\text{Re} \tau & 1 \end{pmatrix}, \quad \tau = C_0 + i e^{-\phi},
\]
describing the dilaton and axion is given by (3.28) as
\[
m_{\alpha\beta} = (\Delta^{-1} M_{c^T})_{\alpha\beta},
\]
as a product of the SL(2) matrices.
The periodicity in $\eta$ is restricted if we require that the resulting monodromy belongs to $SL(2, \mathbb{Z})$. For instance, to obtain the representatives of the infinite sequence of hyperbolic $SL(2, \mathbb{Z})$ conjugacy classes (see e.g. [47])

$$\mathfrak{M}(n) = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix}, \quad n \in \mathbb{N}, \quad n \geq 3. \quad (3.43)$$

we must set $T = \log \frac{1}{2} (n + \sqrt{n^2 - 4})$ and redefine $A(\eta)$ in all expressions (including the Scherk–Schwarz matrix (2.22) as follows:

$$A(\eta) \rightarrow A(\eta) g, \quad g = \begin{pmatrix} (n^2 - 4)^{1/4} & 0 \\ \sqrt{2(n^2 - 4)^{1/4}} & \sqrt{\frac{n}{2}} \end{pmatrix}. \quad (3.44)$$

This results in the monodromy matching (3.43):

$$\mathfrak{M}_{S^1} \rightarrow g^{-1}\mathfrak{M}_{S^1} g = \mathfrak{M}(n). \quad (3.45)$$

Notice that this redefinition does not affect the embedding tensor resulting from Scherk–Schwarz-reduction. Indeed, the $D = 4$ gauged supergravity obtained upon truncation is blind to the choice of $SL(2, \mathbb{Z})$ conjugacy class of the monodromy.

Interestingly, the fact that $\mathfrak{M}_{S^1}$ is in the hyperbolic conjugacy class of $SL(2)$ also means that we can find a global parametrization of the $SL(2)/SO(2)$ axio-dilaton coset representatives such that no compensating local $SO(2)$ transformation on the IIB fermions is induced by the action of $\mathfrak{M}_{S^1}$. The standard parametrization of $m_{ab}$ in (3.36) can be obtained for instance from the $SL(2)/SO(2)$ coset representative $\ell^t(C_0, \Phi)$ as

$$m_{ab} = (\ell^t \ell^t)^{ab}, \quad \ell^t(C_0, \Phi) = \begin{pmatrix} e^{-\Phi/2} & -e^{\Phi/2}C_0 \\ 0 & e^{\Phi/2} \end{pmatrix}. \quad (3.46)$$

while in order to avoid $SO(2)$ compensating transformations under $\mathfrak{M}_{S^1}$ we may for instance change parametrization to

$$m_{ab} = (\ell^t \ell^t)^{ab}, \quad \ell \rightarrow \ell^t = g^{-1}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \ell(\chi_0, \phi), \quad (3.47)$$

where an expression for the axio-dilaton in terms of $\chi_0, \phi$ can be easily constructed. This is still a global choice of coset representative and $SO(2)$ gauge, and now $\mathfrak{M}_{S^1}$ acts as

\[\text{Note that } g \text{ is not unique, it can be redefined by } g \rightarrow \exp(\zeta \log \mathfrak{M}_{S^1})g \text{ for any } \zeta. \text{ Also note that conjugacy classes with } n < -3 \text{ cannot be obtained from our initial monodromy matrix.}\]
a shift of the field $\phi$ without inducing local SO(2) transformations.

The importance of this observation lies in the fact that under a Scherk-Schwarz reduction of ExFT fermions behave as scalar densities, at least for a certain SU(8) gauge choice [15]. Hence, their dependence on internal coordinates is entirely encoded in the function $\rho$ of (2.13). This also applies to the uplift of the gauged supergravity residual Killing spinors at the vacuum. Because we can find an SO(2) gauge such that the $S^1$ monodromy does not require compensating gauge transformations on the fermions, we can conclude that the $N = 4$ Killing spinors of the AdS$_5$ solution described above uplift to globally well-defined Killing spinors of type IIB. This means that the S-fold solution above preserves 16 supercharges which are single-valued and, in fact, $\eta$ independent at least in an appropriate gauge.

As an aside, it is interesting to note that choosing a different $A(\eta)$ taking values in the SO(2) or $\mathbb{R}$ subgroups of SL(2) one arrives at an S-fold interpretation of the reduction Ansatz (2.22) for the $(SO(6) \times SO(2)) \ltimes T^{12}$ and $(SO(6) \times \mathbb{R}) \ltimes T^{12}$ gaugings, respectively. The $(SO(6) \times \mathbb{R}) \ltimes T^{12}$ case has a second interesting interpretation: the $\mathbb{R}$ valued $A(\eta)$ matrix can be interpreted as inducing $F_1 = dC_0$ flux along $S^1$, while $S^5$ is supported by $F_5$. If we $T$-dualize along $\eta$, $F_1$ goes into the Romans mass $F_0$ and $F_5$ goes into $F_6$ filling $S^5 \times S^1$. The reduction Ansatz can then be reinterpreted as type IIA on $S^5 \times S^1$ with $F_6$ and $F_0$ flux, where $A(\eta) \in \mathbb{R}$ generates the Romans mass in terms of a linear dependence of $C_1$ on the winding coordinate $\eta = Y_{67}$ (the physical coordinate would be $y^{68}$). This is analogous to [29] and in fact the $A(\eta)$ part of such an Ansatz matches one of the nongeometric twist-matrices that generate the Romans mass provided in [28]. One can alternatively implement the Romans mass directly in ten dimensions in terms of a deformation of the exceptional field theory/generalized geometry [28,30], and use the $SO(6,0,2)$ Ansatz based on $(\hat{p}, \hat{U})$ alone to implement a geometric reduction of massive IIA to $(SO(6) \times \mathbb{R}) \ltimes T^{12}$ gauged supergravity.

**IV. DISCUSSION**

In this paper, we have constructed the twist matrices that define the consistent truncation of $E_{7(7)}$ exceptional field theory down to the $D = 4$ dyonic gauging with gauge group $(SO(p,q) \times SO(p',q')) \ltimes N$. The twist matrix satisfies the section constraints so that the corresponding dyonic models can be embedded either in type IIA ($p + q$ odd) or in type IIB ($p + q$ even) theories. Using the dictionary between exceptional field theory and IIB supergravity, we have worked out the explicit uplift formulas for the $(SO(6) \times SO(1,1)) \ltimes T^{12}$ gauging and given the uplift of the four-dimensional $AdS_4$ $N = 4$ vacuum [22] into a supersymmetric $AdS_4 \times M_5 \times S^1$ S-fold solution of IIB supergravity. The internal space $M_5$ is a deformation of the round sphere preserving an SO(4) $\subset$ SO(6) subset of its isometries.

Before compactification of the $\eta$ direction, the solution we construct in section III C has the same topology as $AdS_4 \times S^5$. The parametrization we give is in the form of a warped product $AdS_4 \times S^2 \times S^2 \times \Sigma$, where $\Sigma$ is an infinite strip parametrized by $\eta$ and $r$. At the boundary of the strip ($r = 0, 1$) one of the two $S^2$ smoothly shrinks to zero size, reproducing the $S^5$ topology. Another important observation is that with a constant SL(2, $\mathbb{R}$) rotation the axion can be set to vanish, while the dilaton runs along the $\eta$ and $r$ directions as

$$e^{\alpha} = \frac{e^{-2\eta}}{\sqrt{3}} \left(\frac{3 - 2r^2}{1 + 2r^2}\right)^{1/2}.$$ (4.1)

This strongly suggests that our solution be part of the class of Janus solutions with 16 supercharges of [34,35]. This is indeed proven in Appendix C. More specifically, it corresponds to a smooth solution without NS5 or D5 sources, with the dilaton varying from $-\infty$ to $+\infty$ along the infinite stripe. This differs from the regular Janus solution of [34,35], where the dilaton varies between finite boundary values. Janus configurations and their relation with interface $N = 4$ super Yang-Mills have been largely studied in the literature [48–51]. It would be interesting to understand whether the S-fold compactified $AdS_4$ solution we find upon imposing periodicity in $\eta$ is also part of other constructions relating supersymmetric Janus solutions to three-dimensional $N = 4$ conformal field theories [52–55]. In fact, imposing periodicity in $\eta$ corresponds to compactifying the infinite strip $\Sigma$ to a finite cylinder, which seems analogous to the construction in [54].

There has also been some recent activity on S-folds in the context of $D = 4$ $N = 3$ conformal field theories [56–58]. In those cases a generalization of the $O3$ orientifold projections is introduced, that acts with a $Z_k \subset SL(2,\mathbb{Z})$ on the type IIB fields and on the stack of D3 branes defining the CFT ($k = 2, 3, 4, 6$). No dimensional reduction is performed, and the theories obtained from D3 branes on top of such background are either $N = 4$ or genuinely $N = 3$. Only the elliptic subgroups of SL(2, $\mathbb{Z}$) are used in that case, as there must be a fixed valued of the complex coupling $\tau$, so that the projection is by a symmetry of the original theory.

A distinguished property of our solution is that it arises from a consistent truncation of type IIB supergravity to $D = 4$, $(SO(6) \times SO(1,1)) \ltimes T^{12}$ gauged maximal supergravity. Thanks to the Scherk-Schwarz Ansatz (2.9), we have access to the full configuration space of the consistent truncation, which is part of the configuration space of IIB supergravity, also away from the solution with 16 supercharges. In the holographic context, this gives access also on the field theory side to a consistent truncation to a subset of operators. On the gravity side, this can be used to generate
other interesting solutions. For instance, other vacua of the
gauged supergravity may have \( \mathcal{N} < 3 \) supersymmetry\(^6\) and
lift to less supersymmetric Janus solutions and their compa-
tications. All types of solutions of this gauged supergravity
(domain walls, black holes, etc.) now also admit a type IIB
embedding. It would thus be very interesting to
further clarify the relation of \((\text{SO}(6) \times \text{SO}(1,1)) \times \mathbb{T}^{12}\)
gauged maximal supergravity to Janus solutions with
\((\text{SL}(2)\text{ duality twists}), \) and thus their relation to interface
\( \mathcal{N} = 4 \) super Yang–Mills and \( \mathcal{N} = 4, \) \( D = 3 \) conformal
field theories.

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**Appendix A: THE \( \mathcal{N} = 4 \) VACUUM
IN MODULI SPACE**

The nonvanishing entries of \( M_{MN} \) at the vacuum (3.7) are

\[
M_{ij}^{kl} = M_{ij}^{kl} = \begin{cases}
3\delta_{ij}^k\delta_{ij}^l & i, j = 1, 2, 3 \\
3\delta_{ij}^k\delta_{ij}^l & i, j = 4, 5, 8 \\
\delta_{ij}^k\delta_{ij}^l & \text{otherwise}
\end{cases}
\]

(A1)

\[
M_{ij}^{kl} = \begin{cases}
-3^{-1/4}e_{ij}^{jk} & i, j, k = 1, 2, 3 (e^{123} = +1) \\
(-)^{i+1}3^{-1/4}e_{ij}^{jk} & i, j, k = 4, 5, 8 (e^{458} = +1)
\end{cases}
\]

(A2)

\[
M_{ij}^{kl} = \begin{cases}
-3^{-1/4}e_{ij}^{jk} & i, j, k = 1, 2, 3 (e^{123} = +1) \\
(-)^{i+1}3^{-1/4}e_{ij}^{jk} & i, j, k = 4, 5, 8 (e^{458} = +1)
\end{cases}
\]

(A3)

\[
M_{ij}^{ab} = \begin{cases}
\sqrt{\frac{2}{3}} \delta_{ij}^{ab} & i, j = 1, 2, 3 \\
\frac{1}{2\sqrt{3}} \delta_{ij}^{ab} - 4\sigma_{ijab} & i, j = 4, 5, 8
\end{cases}
\]

(A4)

\[
M_{ij}^{ab} = \begin{cases}
\frac{1}{2\sqrt{3}} \delta_{ij}^{ab} & i, j = 1, 2, 3 \\
\frac{\Delta}{\Delta} \delta_{ij}^{ab} & i, j = 4, 5, 8
\end{cases}
\]

(A5)

\[
M_{ij}^{g7} = M_{ij}^{g7} = \delta_{ij} = 1/2.
\]

(A6)

\(^6\)Some unstable vacua are known [6].

**Appendix B: E\(_7(7)\) GENERATORS**

\[
[T_A]_{[C} D] = 4\delta_{[C}^i \delta_{D]}^j - \frac{1}{2} \delta_{[C}^{i}\delta_{D]}^{j}, \quad \text{(B1)}
\]

\[
[T_A]_M^N = \left( \begin{array}{c}
2\delta_{[C}^{[E} [T_A]_{D]}^{F]} \\
-2\delta_{[C}^{[E} [T_A]_{D]}^{F]}
\end{array} \right), \quad \text{(B2)}
\]

\[
[T_{ABC}]_M^N = \left( \begin{array}{c}
4\delta_{EFGH}^{[E}^{[F} \delta_{GHI]}^{[C} \delta_{D]}^{F]}
\end{array} \right). \quad \text{(B3)}
\]

**Appendix C: RELATION TO THE \( \mathcal{N} = 4 \) JANUS SOLUTION**

In this appendix, we show that the solution discussed in
Section III C, upon suitable redefinitions and an S-duality
rotation, coincides with the \( \mathcal{N} = 4 \) supersymmetric Janus
solution of [53].

Let us define the \( S^2 \times S^2 \) sphere harmonics as

\[
\gamma_1^\mu \equiv \frac{1}{r} \gamma_1^\mu, \quad \gamma_2^\mu \equiv \frac{1}{\sqrt{1 - r^2}} \gamma_2^\mu.
\]

(C1)

such that \( \gamma_1^\mu \gamma_1^\mu = 1 = \gamma_2^\mu \gamma_2^\mu \). Then,

\[
d\gamma_1^\mu = rd\gamma_1^\mu + \gamma_1^\mu dr,
\]

\[
d\gamma_2^\mu = \sqrt{1 - r^2} d\gamma_2^\mu - \frac{r}{\sqrt{1 - r^2}} \gamma_2^\mu dr.
\]

(C2)

Let us also set

\[
r = \sin x,
\]

(C3)

with \( 0 \leq x \leq \pi/2 \). We shall define on the surface \( \Sigma \)
parametrized by \( \eta, x \) the complex coordinate \( z = \eta - ix \),
with \( \text{Im} z = x \in [0, \frac{\pi}{2}] \). Upon these redefinitions, the ten-
dimensional IIB metric (3.33) has the form

\[
ds^2 = \Delta^1 \sin^2 x (1 + 2\cos^2 x) d\gamma_1^\mu d\gamma_1^\mu
\]

\[
+ \Delta^3 (1 + 2\sin^2 x) \cos^2 x d\gamma_2^\mu d\gamma_2^\mu
\]

\[
+ \Delta^{-1} dxd\eta + d\eta d\eta + \frac{1}{2} \Delta^{-1} ds^2_{\text{AdS}_4},
\]

(C4)

with the warp factor given by

\[
\Delta = (1 + 2\sin^2 x)(1 + 2\cos^2 x)^{-1/4},
\]

(C5)

and the \( \text{AdS}_4 \) radius fixed to \( r_{\text{AdS}} = 1 \).

Comparing to the notation of [53], in which the metric is
written as

\[
ds^2 = f_1^2 ds^2_{\text{AdS}_4} + f_1^2 ds^2 + f_2^2 ds^2 + 4\rho^2 dzd\bar{z}.
\]

(C6)
we can make the following identifications:

\[ f_4^\alpha = \frac{1}{16} \Delta^{-1} = \frac{1}{16} (1 + 2 \sin^2 x)(1 + 2 \cos^2 x), \]

\[ f_2^\alpha = \Delta^3 \sin^2 x(1 + 2 \cos^2 x), \]

\[ H_3^\alpha = \Delta^3 (1 + 2 \sin^2 x) \cos^2 x, \]

\[ 4 \rho^2 = \Delta^{-1}. \quad (C7) \]

As explained in Sec. III D, in order to match our solution with that of [53], the following S-duality transformation has to be performed on the SL(2)-covariant fields:

\[ m_{\mu
u} \rightarrow m'_{\sigma\tau} = \kappa^a S_{a\mu\nu} m_{\sigma\tau} = \begin{pmatrix} e^{-\Phi} & 0 \\ 0 & e^{\Phi} \end{pmatrix}, \]

\[ H_3^a \rightarrow H_3'^a = S^{-1}_{a\mu\nu} H_3^{\mu\nu}, \quad (C8) \]

where

\[ S_{a\mu\nu} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \quad (C9) \]

We then find

\[ m_{\mu\nu} = \begin{pmatrix} \sqrt{3} e^{2\eta} \frac{(1 + 2 \sin^2 x)^{1/2}}{(1 + 2 \cos^2 x)^{1/2}} & 0 \\ 0 & \frac{1}{\sqrt{3}} e^{-2\eta} \frac{(1 + 2 \cos^2 x)^{1/2}}{(1 + 2 \sin^2 x)^{1/2}} \end{pmatrix}, \quad (C10) \]

from which we read off

\[ e^{-2\Phi} = 3 e^{\eta} \frac{(1 + 2 \sin^2 x)}{(1 + 2 \cos^2 x)}. \quad (C11) \]

The three-form field strengths take the form

\[ H_3^{\mu+} = \sqrt{23-1/4} e^{-\eta} \sin^2 x \]

\[ \left( \begin{array}{c} e_{\mu
u} \gamma_0 d\gamma_1 \wedge d\gamma_2 \\ e_{\mu
u} \gamma_0 \gamma_1 d\gamma_1 \wedge d\gamma_2 \\ \end{array} \right) \]

\[ \wedge \left( \begin{array}{c} 3 + 2 \sin^2 x \\ 3 + 2 \sin^2 x \\ \end{array} \right) \cos x \sin x dx \sin x d\eta \]

\[ = \omega_{S_1} \wedge db^1 \]

\[ H_3^{\mu-} = \sqrt{23-1/4} e^{\eta} \cos^2 x \]

\[ \left( \begin{array}{c} e_{\mu
u} \gamma_0 d\gamma_1 \wedge d\gamma_2 \\ e_{\mu
u} \gamma_0 \gamma_1 d\gamma_1 \wedge d\gamma_2 \\ \end{array} \right) \]

\[ \wedge \left( \begin{array}{c} 3 + 2 \cos^2 x \\ 3 + 2 \cos^2 x \\ \end{array} \right) \cos x \sin x dx \cos x d\eta \]

\[ = \omega_{S_2} \wedge db^2, \quad (C12) \]

where

\[ b^1 = \frac{2 \sqrt{23-1/4} e^{-\eta} \sin^3 x}{1 + 2 \sin^2 x}, \quad b^2 = -\frac{2 \sqrt{23-1/4} e^{\eta} \cos^3 x}{1 + 2 \cos^2 x}. \quad (C13) \]

Finally, the self-dual IIB five-form field strength is given by

\[ H_5 = \frac{9}{4} \Delta^2 \sin^2 x \cos^2 x \gamma_2 \gamma_0 \gamma_1 d\gamma_1 \wedge d\gamma_2 \wedge d\gamma_3 \wedge d\gamma_4 \]

\[ \wedge \left( dx + \frac{4}{3} \sin x \cos x d\eta \right) \]

\[ - \frac{1}{16 \cdot 4} \sqrt{|g|} \epsilon_{\mu
u\rho\sigma} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \]

\[ \wedge \left( d\eta - \frac{4}{3} \sin x \cos x dx \right) \]

\[ = \frac{3}{2} \Delta^4 \sin^2 x \cos^2 x \omega_{S_1} \wedge \omega_{S_2} \wedge \left( dx + \frac{4}{3} \sin x \cos x d\eta \right) \]

\[ - \frac{3}{2 \cdot 4} \omega_{0123} \wedge \left( d\eta - \frac{4}{3} \sin x \cos x dx \right), \quad (C14) \]

where we used the property

\[ \gamma_1 \gamma_0 \gamma_1 d\gamma_1 \wedge d\gamma_2 = \frac{1}{3} e^{3q \eta} \omega_{S_1}. \quad (C15) \]

To compare the solution to the Janus one, it is useful to write \( H_5 \) in the form

\[ H_5 = f_4^1 f_2^2 \omega_{S_1} \wedge \omega_{S_2} \wedge (\ast_2 \gamma_2) - f_4^1 \omega_{0123} \wedge \gamma_2, \quad (C16) \]

where \( \ast_2 \) is the Hodge duality operation on the disk spanned by \( \eta \) and \( x \), and

\[ f_4^1 \gamma_2 = d j_1, \quad j_1 = \frac{1}{8} (3 \eta + \cos(2x)), \quad (C17) \]

so that

\[ f_4^1 \gamma_2 = \frac{3}{8} \left( d\eta - \frac{4}{3} \cos x \sin x dx \right), \quad (C18) \]

and

\[ f_4^1 f_2^2 \ast_2 \gamma_2 = \frac{3}{2} \Delta^4 \sin^2 x \cos^2 x \left( dx + \frac{4}{3} \cos x \sin x d\eta \right) \quad (C19) \]

1. **Reconstruct the solution from the harmonic functions \( A_1, A_2 \)**

To show the matching of the above solution with that in [53], we need to prove that all the functions describing it can be expressed, through appropriate relations given in the same reference, in terms of only two harmonic functions...
in terms of which we define the harmonic functions \( h_1, h_2 \) and their duals \( \tilde{h}_1, \tilde{h}_2 \):

\[
\begin{align*}
h_1 &= -i(A_1 - \tilde{A}_1) = -\frac{3^{1/4}}{2\sqrt{2}} e^\eta \sin x, \\
h_2 &= A_2 + \tilde{A}_2 = -\frac{3^{-1/4}}{2\sqrt{2}} e^{-\eta} \cos x, \\
\tilde{h}_1 &= A_1 + \tilde{A}_1 = \frac{3^{1/4}}{2\sqrt{2}} e^\eta \cos x, \\
\tilde{h}_2 &= i(A_2 - \tilde{A}_2) = \frac{3^{-1/4}}{2\sqrt{2}} e^{-\eta} \sin x.
\end{align*}
\]

It is then straightforward to show that the functions entering the solution satisfy the following relations characterizing the solution of [53]:

\[
\begin{align*}
W &= \partial \tilde{h}_1 \tilde{h}_2 + \tilde{h}_1 \partial h_2 = -\frac{1}{8} \sin x \cos x, \\
N_1 &= 2h_1 h_2 |\partial h_1|^2 - \tilde{h}_1^2 W = \frac{\sqrt{3}}{128} e^{2\eta} \sin x \cos x(1 + 2\sin^2 x), \\
N_2 &= 2h_1 h_2 |\partial h_2|^2 - \tilde{h}_2^2 W = \frac{1}{\sqrt{3128}} e^{-2\eta} \sin x \cos x(1 + 2\cos^2 x), \\
f_1^4 &= 16N_1 N_2 W^2 = \frac{1}{16} \Delta^4, \\
(4\rho^2)^4 &= 256 N_1 N_2 W^2 h_1^4 h_2^4 = \Delta^4.
\end{align*}
\]

We also find that the two functions \( b_1, b_2 \) entering the expression of the three-form field strengths are related to the above functions as prescribed in [53]:

\[
\begin{align*}
b_1 &= 2\tilde{h}_1 \frac{h_1}{N_1} h_2 (\partial h_1 \tilde{h}_2 - \tilde{h}_1 \partial h_2) + 2\tilde{h}_2 \\
&= 2\sqrt{2} \cdot 3^{-1/4} e^{-\eta} \sin^3 x, \\
b_2 &= 2\tilde{h}_2 \frac{h_2}{N_2} h_1 (\partial h_1 \tilde{h}_2 - \tilde{h}_1 \partial h_2) - 2\tilde{h}_1 \\
&= -2\sqrt{2} \cdot 3^{1/4} e^\eta \cos^3 x.
\end{align*}
\]

Similarly, just as in the Janus solution, the function \( j_i \) entering the five-form field strength can be expressed as

\[
\begin{align*}
j_1 &= 3(\mathcal{C} + \tilde{\mathcal{C}} - \mathcal{D}) + i\frac{h_1}{W} (\partial h_1 \tilde{h}_2 - \tilde{h}_1 \partial h_2), \\
\end{align*}
\]

where \( \mathcal{C} \) satisfies the relation \( \partial \mathcal{C} = A_1 \partial A_2 - A_2 \partial A_1 \) and is given by \( \mathcal{C} = \frac{\tilde{h}_1}{10} \) while \( \mathcal{D} \) reads

\[
\mathcal{D} = \tilde{A}_1 A_2 + A_2 \tilde{A}_2 = -\frac{1}{10} \cos(2x).
\]

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