Teasing bits of information out of the CMB energy spectrum

Jens Chluba¹* and Donghui Jeong¹†

¹ Department of Physics and Astronomy, Johns Hopkins University, Bloomberg Center 435, 3400 N. Charles St., Baltimore, MD 21218

ABSTRACT

Departures of the Cosmic Microwave Background (CMB) frequency spectrum from a blackbody – commonly referred to as spectral distortions – encode information about the thermal history of the early Universe (redshift $z \lesssim \text{few} \times 10^3$). While the signal is usually characterized as $\mu$- and $y$-type distortion, a smaller residual (non-$y$/non-$\mu$) distortion can also be created at intermediate redshifts $10^4 \leq z \lesssim 3 \times 10^5$. Here, we construct a new set of observables, $\mu_k$, that describes the principal components of this residual distortion. The principal components are orthogonal to temperature shift, $y$- and $\mu$-type distortion, and ranked by their detectability, thereby delivering a compression of all valuable information offered by the CMB spectrum. This method provides an efficient way of analyzing the spectral distortion for given experimental settings, and can be applied to a wide range of energy-release scenarios. As an illustration, we discuss the analysis of the spectral distortion signatures caused by dissipation of small-scale acoustic waves and decaying/annihilating particles for a PIXIE-type experiments. We provide forecasts for the expected measurement uncertainties of model parameters and detections limits in each case. We furthermore show that a PIXIE-type experiments can in principle distinguish dissipative energy release from particle decays for a nearly scale-invariant primordial power spectrum with small running. Future CMB spectroscopy thus offers a unique probe of physical processes in the primordial Universe.

Key words: Cosmology: cosmic microwave background – theory – observations

1 INTRODUCTION

Energy release in the early Universe causes deviations of the cosmic microwave background (CMB) frequency spectrum from a pure blackbody (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1970; Ilarionov & Sunyaev 1975ab; Danese & de Zotti 1977; Burigana et al. [1991]; Hu & Silk 1993a), which we henceforth refer to as spectral distortion (SD). Thus, far no primordial SD was found (Mather et al. 1994; Fixsen et al. 1996; Fixsen & Mather 2002; Kogut et al. 2006; Zannoni et al. 2008; Seiffert et al. 2011), but technological advances over the past quarter-century since COBE/FIRAS may soon allow much more precise (at least 3 orders of magnitudes improvement in sensitivity) characterization of the CMB spectrum (e.g., Fixsen & Mather 2002; Kogut et al. 2011). This is especially interesting because even for the standard cosmological model, several processes exist that imprint distortion signals at a level within reach of present-day technology (see Chluba & Sunyaev 2012; Sunyaev & Khatri 2013; Chluba 2013a for broader overview). PIXIE (Kogut et al. 2011) provides one very promising experimental concept for measuring these distortion signals, and more recently PRISM, an L-class satellite mission with about 10 times the spectral sensitivity of PIXIE, was put forward (PRISM Collaboration et al. 2013). These prospects motivated us to further elaborate on what could be learned from measurements of the CMB spectrum, taking another step forward towards the analysis of future distortion data.

Previous works primarily used distortions to rule out various energy-release scenarios (ERSs) on a model-by-model basis. These studies include discussion of decaying or annihilating particles (Hu & Silk 1993b; McDonald et al. 2001), the dissipation of primordial density fluctuations on small scales (Daly 1991; Barrow & Coles 1991; Hu et al. 1994a; Hu & Sugiyama 1994; Chluba et al. 2012b; Pajer & Zaldarriaga 2012; Dent et al. 2012; Ganc & Komatsu 2012; Chluba et al. 2012a; Biagetti et al. 2013), cosmic strings (Ostriker & Thompson 1987; Tashiro et al. 2008), primordial black holes (Carr et al. 2010), small-scale magnetic fields (Jedamzik et al. 2000) and some new physics examples (Lochan et al. 2012; Bull & Kamionkowski 2013; Brax et al. 2013).

Until recently, all constraints were based on simple estimates for the chemical potential, $\mu$, and Compton $y$-parameter (Zeldovich & Sunyaev 1969; Sunyaev & Zeldovich 1970). It was, however, shown that the distortion signature from different ERSs generally is not just given by a superposition of pure $\mu$- and $y$-distortion (Chluba & Sunyaev 2012; Khatri & Sunyaev 2012a, Chluba 2013b). The small residual beyond $\mu$- and $y$-distortion contains information about the time dependence of the energy-release history, which in principle can be used to directly constrain, for instance, the shape of the small-scale power spectrum, measure the lifetime of decaying...
ing relic particles, or simply to discern between different energy-release mechanisms (Chluba 2013a). In particular, Chluba (2013a) demonstrated that CMB spectrum measurement with a PIXIE-type experiment provide a sensitive probe for long-lived particles with lifetimes $\tau_k \approx 10^8 - 10^{11}$ sec. Similarly, the shape of the small-scale power spectrum can be directly probed with PIXIE’s sensitivity if the amplitude of primordial curvature perturbations exceeds $A_T \approx \text{few} \times 10^{-5}$ at wavenumber $k \approx 45 \text{Mpc}^{-1}$ (Chluba 2013a). Future CMB distortion measurements thus provide a unique avenue for studying early-universe models and particle physics.

In Chluba (2013a), model parameters (e.g., abundance and lifetime of a decaying particle) were directly translated into the SD signal (the photon intensity in different frequency channels) using a Green’s function method (Chluba 2013b), which was recently added to the cosmological thermalization code CosmoThERM (Chluba & Sunyaev 2012). Even when explicitly knowing the relation between ERS and SDs, model comparison and forecasts of uncertainties (or detection limits) are still rather involved. This is because (i) different energy-release mechanisms can cause very similar SDs, (ii) the parameters in different models are often unrelated and (iii) in general, the parameter space is non-linear especially close to the detection limit. One natural question therefore is whether the information contained by the CMB spectrum (the intensity in each frequency channel) could be further compressed and described in a model-independent way ($\mu$, $y$, plus additional distortion parameters).

The precise shape of the resulting SD directly depends on the underlying energy-release history. Model dependence is only introduced when asking which physical process caused a specific energy-release history, but this step can be separated from measuring the energy-release history itself. We thus ask, how well future CMB SDs can constrain different energy-release histories, independent of the responsible physical mechanism. For this we perform a principal component analysis (see Mortonson & Hu 2008, Pinko et al. 2012, Farhang et al. 2012, Shaw et al. 2013) for other cosmology-related applications of this method) of the residual (non-$\mu$-non-$y$) distortion signal, in order to identify spectral shapes and their associated energy-release histories that can be best-constrained by future distortion data. The amplitudes, $\nu_k$, of the signal eigenmodes then define a set of parameters that describes all information encoded by the residual distortion signal. These observables can be measured in a model-independent manner with predictable uncertainties. The mode amplitudes, by construction, are uncorrelated and the parameter dependence is linear, which greatly simplifies further analysis in this new parameter space.

The principal components depend on experimental setting (number of channels, distribution over frequency, noise in each channel and its correlations; see Sect. 2.1) as well as foregrounds and systematic effects. Here, we do not consider the effect of foreground contamination, and therefore only focus on what the minimal instrumental sensitivity should be in order to constrain or detect the signatures of different energy-injection scenarios. Generalization is straightforward, but we leave a more detailed investigation of foreground issues to future work. Along similar lines we plan on investigating the optimization of experimental settings for various ERS using the principal component analysis.

This paper is organized as follows: we start by decomposing the SD signal into temperature shift, $\mu$, $y$ and residual distortion (Sect. 2). This decomposition already depends on the experimental settings (we envision a PIXIE-like experiment), which determines the level to which different spectral shapes are distinguishable. This allows us to obtain visibility functions in redshift for the different distortion components (Fig. 2), providing a generalization of the spectral distortion visibility function $\mathcal{J}_{bb}(z)$ [see Sect. 2.3 for more details], used in earlier works to account for the suppression of distortions by the efficient thermalization process at redshift $z \gtrsim \text{few} \times 10^6$ (e.g., Burigana et al. 1991, Hu & Silk 1993a). We then construct the energy-release and signal eigenmodes (Sect. 3), and illustrate how they can be used for simple parameter estimation (Sect. 4). In Sect. 5 we demonstrate how constraints on different energy-release scenarios can be derived, with particular attention to detectability, errors, and model comparison.

2 Quasi-Orthogonal Decomposition of the Thermalization Green’s Function

The average CMB frequency spectrum, $I_{\nu}^{\text{CMB}} \equiv \text{spectral intensity}$ in units $W \text{m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ as a function of frequency $\nu$, can be broken down as follows:

$$I_{\nu}^{\text{CMB}} = B_\nu(T_0) + \Delta I_{\nu}' + \Delta I_{\nu}'' + \Delta I_{\nu}^{\text{prim}}.$$ (1)

The main theme of this paper is to develop an analysis tool for the primordial, pre-recombination distortion signal, $\Delta I_{\nu}^{\text{prim}}$, introduced by different energy-release mechanisms at early times, $z \gtrsim 10^4$ (see Sect. 2.1). Because this term is usually small compared to the other contributions to $I_{\nu}^{\text{CMB}}$, we seek a scheme to optimize the search for this signal. The first term in Eq. (1) describes the CMB blackbody part, $B_\nu(T_0) \equiv \frac{2\hbar \nu^3}{c^2} \left(\frac{\nu}{\nu_T} \right)^3$ where $T_0$ is the CMB monopole temperature $T_0 = (2.726 \pm 0.001) \text{K}$ (Fixsen et al. 1996, Fixsen 2009) and $x \equiv \hbar \nu/kT_0$. The exact value of the CMB monopole temperature, $T$, is not known down to the accuracy that can be reached by future experiments ($\Delta T = \text{few} \times 10^{-5}$). It thus has to be determined in the analysis. This is captured by the second term in Eq. (1), which is obtained by shifting a blackbody from one temperature $T_0$ to $T$,

$$\Delta I_{\nu}' = G_T(\nu) \Delta T[1 + \Delta_T] + Y_{SZ}(\nu) \Delta T^2/2 + O(\Delta T[^3]),$$ (2)

where $\Delta_T = (T - T_0)/T_0 \ll 1$. Here, we defined the spectrum of a temperature shift $G_T(\nu) = \{T \partial[B_\nu(T)]/\partial T\}_{T_0} \equiv \frac{2\hbar \nu^3}{c^2} \left(\frac{\nu}{\nu_T} \right)^3$, at lowest order in $\Delta_T$. At second order in $\Delta_T$, a correction related to the superposition of blackbodies (Zeldovich et al. 1972, Chluba & Sunyaev 2004) appears, having a spectrum that is similar to a Compton $y$-distortion, $Y_{SZ}(\nu) \equiv G_T \{x \coth(x/2) - 4\}$, also known in connection with the thermal Sunyaev-Zeldovich effect caused by galaxy clusters (Zeldovich & Sunyaev 1969).

Finally, in Eq. (1) we also added a $y$-distortion, $\Delta I_{\nu}'' = y Y_{SZ}(\nu)$, that is created at low redshifts ($z \lesssim 10^3$) but is not directly accounted for by the primordial distortion, $\Delta I_{\nu}^{\text{prim}}$. One strong source of late-time $y$-distortions stems from reionization and structure formation, giving rise to an effective $y$-parameter $y_{\text{re}} \approx 10^{-7} - 10^{-6}$ (Sunyaev & Zeldovich 1972, Hu et al. 1994b, Cen & Ostriker 1999, Miniati et al. 2000, Refregier et al. 2000, Oh et al. 2003, Zhang 2000).

This name was coined by Chluba & Sunyaev (2012), but the original derivation (accounting for the effect of Bremsstrahlung) was given in Sunyaev & Zeldovich (1970), Danese & de Zotti (1982) also included the effect of double Compton emission (Lightman 1981, Torner 1981, Chluba et al. 2007), and recent improvements to the shape of $J_{bb}(z)$ were given by Khatri & Sunyaev (2012b), using semi-analytic approximations.
et al. [2004]. The aim of this section is to find an operational decomposition of the spectral signal caused by early energy release ($z \geq 10^3$) from the non-primordial signatures such as temperature shift and late-time y-distortion.

### 2.1 Instrumental aspects

For our analysis we envision an experiment similar to PIXIE, which is based on a Fourier transform spectrometer [Kogut et al. 2011]. PIXIE covers the frequency range $\nu = 30$ GHz – 6 THz, with synthesized channels of constant frequency resolution $\Delta\nu_i = 15$ GHz, depending on the mirror stroke$^3$. The noise in each channel over the mission’s duration is $\Delta\nu_i \simeq 5 \times 10^{-26}$ W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$. We assume the noise to be constant and uncorrelated between channels (diagonal covariance matrix $C_{ij} = \Delta\nu_i^2 \delta_{ij}$), with bandpass given by top-hat functions. The SD signal we are after is important only at $\nu \simeq 30$ GHz – 1 THz, which for $\Delta\nu_i = 15$ GHz means about 65 channels. The remaining $\approx 335$ channels at $\nu$ THz are used to construct a detailed model for the dust and cosmic infrared background (CIB) component, which we assume is subtracted down to $\nu \simeq 335$ channels at 6 THz, with synchrotron and Zodiacal light (Planck Collaboration et al. 2013c) and Bremstrahlung at low frequencies becomes less efficient, while redistribution of photons over frequency by Compton scattering is still very fast. In this regime the distortion assumes the shape of a pure $\mu$-distortion, $M(\nu) = G_T (x/\beta - 1)/x$, with $\beta \equiv 3(3)/(3) \approx 1.923$. At late times ($z \leq 10^3$), even Compton scattering becomes inefficient and the distortion is very close to a pure $y$-distortion, $Y_Sz \equiv G_T [x \coth(x/2) - 4]$. At all intermediate redshifts, the Green’s functions is given by a superposition of these extreme cases with some correction, $R(v, z)$, which we call residual distortion (see Chluba 2013b) for similar discussion:

$$G_{\mu}(v, z) = \frac{G_T(v)}{4} J_T(z) + \frac{Y_{SZ}(v)}{4} J_y(z) + \alpha M(v) J_\mu(z) + R(v, z). \quad (4)$$

Here, we used the identities $\int G_T(v) dv = \int Y_{SZ}(v) dv = 4 \rho_\gamma$ and $\int M(v) dv = \rho_\gamma/\alpha$ with $\alpha = [4(2)/(3)] [3(3) - (3)/(3)]^{-1} \approx 1.401$ to re-normalize terms. The redshift-dependent function, $J_k(z)$, for $k \in \{T, y, \mu\}$, define the branching ratios of energy going into different components of the signal (see Sect. 2.3). These ratios are not unique but depend on the experimental settings, which determine the orthogonality between different spectral components. To obtain these functions, we use PIXIE-like instrumental specification (Sect. 2.4), where the CMB spectrum is sampled over some range of frequencies $v \in [v_{\text{min}}, v_{\text{max}}]$ with constant bandwidth $\Delta\nu_i$ and constant sensitivity $\Delta\nu_i$ per channel. This turns Eq. (3) into $G_{\nu}(z) = G_T(z) J_T(z)/4 + Y_{SZ}(z) J_y(z)/4 + \alpha M(z) J_\mu(z) + R(z)$, where the subscripts indicate the individual signals in the $i$th channel. Then, we can interpret $G_{\nu}(z), G_T, Y_{SZ}, M$ and $R(z)$ ($i = 1, \ldots, N$) as $N$-dimensional ($N$ = number of frequency channels) vectors.$^3$

In this vector space, the decomposition problem reduces to finding the residual distortion $R(z)$ such that it is perpendicular to the space spanned by $G_T, Y_{SZ}$, and $M$ (see Appendix A for details). Once the residual distortion is identified, we obtain all energy branching ratios, $J_k(z)$, of Eq. (4) by projecting the rest of the Green’s function on to $G_T, Y_{SZ}$ and $M$, respectively. The results are shown in Fig. 1 We also defined $J_{\mu}(z) = 1 - J_T(z) - J_y(z) - J_\mu(z)$.

![Figure 1. Energy branching ratios, $J_k(z)$ according to Eq. (4)](image)

$^3$ Excursion of the modulating (diaphragm) mirror of the Fourier transform spectrometer around the zero-point.

$^4$ At all times, the distortion has to be small compared to the CMB blackbody, since otherwise non-linear effects become important and the Green’s function approach is inapplicable.

$^5$ An alternative method is described in Khatari & Sunyaev 2012a.
which determines the amount of energy found in the residual distortion only. At redshift \( z \lesssim 4 \times 10^4 \), most of the energy release produces a y-distortion, while at \( 4 \times 10^4 \leq z \leq 1.7 \times 10^6 \) most of the energy goes into a \( \mu \)-distortion. At \( 1.7 \times 10^6 \leq z \) the thermalization process, mediated by Compton scattering, double Compton emission and Bremsstrahlung, is so efficient that practically all energy just increases the average CMB temperature.

Around \( z \approx 4 \times 10^4 \), a few percent of the energy is stored by the residual distortion, and the amplitude of this signal depends strongly on redshift (see Fig. 2). Although small in terms of energy density, the residual distortion reaches \( \approx 10\% - 20\% \) of \( \mathcal{M}(\nu) \) and \( Y_S(v) \) at high frequencies, and can even be comparable to \( \mathcal{M}(\nu) \) at \( \nu \lesssim 100 \) GHz. The fraction of energy release to the residual distortion is extremal around \( z \approx 3.8 \times 10^4 \) (see Fig. 1), while the low-frequency amplitude of the residual distortion is largest at \( z \approx 6.2 \times 10^4 \) (see Fig. 2). In Figure 2 we can also observe a small dependence of the phase of the residual distortion on the redshift of energy release. The redshift-dependent phase shift of the residual distortion provides model-independent information about the time dependence of the energy-release process, while analysis of the superposition between \( \mu \)- and y-distortion can only be interpreted in a model-dependent way.

Figure 1 also shows that \( \mu \)-distortion and temperature shift have a significant overlap around \( z \approx 10^4 \). There \( \mathcal{J}_y(z) \) exceeds unity, while \( \mathcal{J}_\mu(z) \) is negative. Similarly, for the chosen experimental setting \( \mathcal{J}_y(z) \) is negative, ensuring energy conservation. Although below \( z \approx 10^4 \) photon production becomes very weak and the thermalization of distortions to a temperature shift ceases, the shape of the distortion still projects on to \( G_T \), leading to \( \mathcal{J}_\mu(z) \neq 0 \). When thinking about the different contributions to the total distortion signal these points should be kept in mind.

Another way to define the temperature shift is to integrate the distortion over all frequencies. Scattering terms, to which the \( \mu \)- and y-distortion are related, conserve photon number density, so that any deviation from zero should be caused by contributions from a temperature shift, related to \( G_T(v) \). This approach was used by Chluba (2013b), where by construction \( \mathcal{J}_k(z) < 1 \) for \( k \in \{T, y, \mu, R\} \). In practice, i.e., with contaminations from foregrounds, this procedure may not be applicable, and simultaneous fitting of different spectral components is expected to work better. We therefore did not further follow this path.

### 2.2.1 Dependence on experimental settings

It is clear that the decomposition \( \{\mathcal{R}(\nu, z) \text{ and } \mathcal{J}_y(z)\} \) presented above depends on the chosen values for \( \{\nu_{\text{min}}, \nu_{\text{max}}, \Delta \nu_c\} \). Changing the frequency resolution has a rather small effect, while changing \( \nu_{\text{min}} \) is more important (see Fig. 3). The differences are therefore mainly driven by the way the distortion projects on to \( G_T \) and \( Y_S \) between \( \nu_{\text{min}} \) and \( \nu_{\text{max}} \) rather than how precisely the channels are distributed over this interval.

Also, so far we assumed uniform and uncorrelated noise in the different channels. In this case, the construction of the modes becomes independent of the value of \( \Delta \nu \), but more generally one has to include this into the eigenmode analysis. This can be achieved by redefining the scalar product of two frequency vectors, e.g., \( a \cdot b = \sum_{ij} a_{ij} c_{ij} b_{ij} \), where \( C_{ij} \) is the full noise covariance matrix. Similarly, signals related to foregrounds can be included when performing the decomposition of the Green’s function. These are expected to lead to a degradation of the signal towards both lower and higher frequencies, however, these aspects are beyond the scope of this paper and will be explored in another work.

### 2.3 Energy release and branching ratios

The amplitude of the SD is directly linked to the total energy that was released over the cosmic history. One way, which has been widely applied in the cosmology community, to make this connection is to use the effective \( \mu \) and y-parameter to characterize the associated distortion, \( \mu \approx 1.4 \Delta \rho_{\gamma}/\rho \|_\nu ^y \) and \( y \approx (1/4) \Delta \rho_{\gamma}/\rho \|_\nu ^\nu \) (Zeldovich & Sunyaev [1969] Sunyaev & Zeldovich [1970]). The total energy release causing distortions is \( \Delta \rho_{\gamma}/\rho \|_\nu ^{\nu, \gamma} = \Delta \rho_{\gamma}/\rho \|_\nu ^\nu + \Delta \rho_{\gamma}/\rho \|_\nu ^\gamma \), with the partial contributions, \( \Delta \rho_{\gamma}/\rho \|_\nu ^\nu \) and \( \Delta \rho_{\gamma}/\rho \|_\nu ^\gamma \), from the y- and \( \mu \)-era, respectively. In terms of the energy-release history, \( Q(\nu) \equiv d(\rho(\nu))/d \ln \nu \approx (1 + \nu) \ d(\rho(\nu))/d \nu \), the effective y- and \( \mu \)-parameters can be written as

\[
\begin{align*}
y &\approx \frac{1}{4} \int_0^\infty Q(\nu') d \ln \nu' \\
\mu &\approx 1.4 \int_0^\infty \mathcal{J}_{bb}(\nu') Q(\nu') d \ln \nu',
\end{align*}
\]

where we introduced the spectral distortion visibility function, \( \mathcal{J}_{bb}(\nu) \approx e^{-\nu/\nu_0^{\nu, \gamma}} \), with thermalization redshift \( \nu_0^{\nu, \gamma} \approx 2 \times 10^6 \) (e.g.,
see Hu & Silk [1993a]). The visibility function accounts for efficient thermalization process for redshifts \( z \geq z_0 \), at which only the average temperature of the CMB is increased and no distortion is created. In Eq. (5), the transition between the \( \mu \)- and \( y \)-era is modeled as step-function at \( z_\text{trans} \approx 5 \times 10^3 \).

The decomposition, Eq. (5), into \( \mu \)- and \( y \)-distortion is only rough and has to be refined for the future generation of CMB experiments. Our approach described in this section provides a natural extension. By inserting Eq. (4) into Eq. (3) and integrating over all \( y \) we find that the total change of the CMB photon energy density, \( \rho_y \), caused by energy release is given by

\[
\frac{\Delta \rho_y}{\rho_y} |_{\text{int}} = \frac{\Delta \rho_y}{\rho_y} |_{\tau} + \frac{\Delta \rho_y}{\rho_y} |_{\mu} + \frac{\Delta \rho_y}{\rho_y} |_{R}
\]

\[
= 4\Delta_T + 4y + \mu + e
\]

\[
\Delta_T = \frac{1}{4} \frac{\Delta \rho_y}{\rho_y} |_{\tau} = \frac{1}{4} \int f_T(z') Q(z') d\ln z'
\]

\[
y = \frac{1}{4} \frac{\Delta \rho_y}{\rho_y} |_{\mu} = \frac{1}{4} \int f_y(z') Q(z') d\ln z'
\]

\[
\mu = \frac{1}{4} \frac{\Delta \rho_y}{\rho_y} |_{R} = \frac{1}{4} \int f_{\mu}(z') Q(z') d\ln z'
\]

\[
e = \frac{1}{4} \frac{\Delta \rho_y}{\rho_y} |_{R} = \int f_{\epsilon}(z') Q(z') d\ln z',
\]

with \( \alpha \approx 1.401 \). In addition to \( \Delta_T \approx \Delta T/T_0 \) (defining a relative temperature shift), \( y \)- and \( \mu \)-parameter, we defined \( e \) to characterize the energy stored in the residual distortion. For a given energy-release history or mechanism these numbers can be directly computed, but only \( y \), \( \mu \), and \( e \) can be used to study the energy-release mechanism. The integrals can be carried out as a simple inner product in the discretized redshift vector space, making parameter estimation very efficient.

The expressions, Eq. (6), for \( \mu \) and \( y \) are very similar to the usual formulae, Eq. (5). The main difference is that here the origin of the redshift-dependent window functions, \( f(z) \), becomes apparent, being related to the representation of the different quasi-orthogonal components to the SD. Equations (6) are thus a generalization, introducing visibility functions, or branching ratios \( f_k \), for \( k = \mu, y, \) and residual distortion, \( R(v) \), respectively. They are, however, dependent on the experimental settings (Sect. 2.2.1).

3 PRINCIPAL COMPONENT DECOMPOSITION FOR THE RESIDUAL DISTORPTION

In the previous section we showed that for a given experimental setting the Green’s function can be decomposed into quasi-orthogonal parts. The primordial distortion is then fully described by the parameters \( p = \{ \Delta_T, y, \mu \} \) and a residual distortion

\[
\Delta I_r^R = \int R(z') Q(z') d\ln z',
\]

which can be computed knowing the function \( R(z) \). To constrain the energy-release history, \( \Delta_T \), \( \Delta_T \), can be omitted, while interpretation of \( y \) and \( \mu \) only give model-dependent constraints on \( Q(z') \) [we discuss this point below]. We now ask how much can be learned about the redshift dependence of \( Q(z') \) by analyzing \( \Delta I_r^R \). Since the overall signal is only a correction to the main superposition of \( \mu \) and \( y \)-distortion signals, the experimental sensitivity has to be high or the overall energy release ought to be large. By construction, \( \Delta I_r^R \) is orthogonal to the space spanned by \( y \) and \( \mu \)-distortion. We can thus perform a simple principal component decomposition for \( \Delta I_r^R \) to get a handle on \( Q(z') \). For this we discretize the energy-release integral, Eq. (7), as a sum:

\[
\Delta I_r^R \approx \sum \hat{R}(z_a) Q_{\epsilon_a},
\]

where \( \hat{R}(z_a) = R(z_a) \Delta \ln z \) and \( Q_{\epsilon_a} = Q(z_a) \). For our computations we distributed the bins logarithmically between \( z_{\text{min}} = 10^3 \) and \( z_{\text{max}} = 5 \times 10^6 \) with log-spacing \( \Delta \ln z = 2.135 \times 10^{-2}, \) i.e., 400 grid points, using the mid-point integration rule. While only accurate at the level of \( \approx 0.1\% \), this approximation is sufficient for deriving the basis functions. When computing the SD from a given energy-release history we still explicitly carry out the full integral, Eq. (3), using Patterson quadrature rules (Patterson[1968]). The Fisher-information matrix for measuring energy-release history, \( Q_{\epsilon_a} \), from the observed residual intensities \( \Delta I_r^R \) is

\[
\mathcal{F}_{ab} = \frac{1}{\Delta I_r^R} \sum_i \frac{\partial \Delta I_r^R}{\partial Q_{\epsilon_i}} \frac{\partial \Delta I_r^R}{\partial Q_{\epsilon_j}} = \frac{1}{\Delta I_r^R} \sum_R \hat{R}(z_a) \hat{R}(z_b),
\]

where we assumed that the frequency channels, represented by index \( i \), are independent. The eigenvectors of \( \mathcal{F}_{ab} \) determine the principal components, \( E^{(k)} \), of the problem. The eigenvalues, \( \lambda_i \), furthermore determine how well one might be able to recover \( Q(z) \) for a given sensitivity \( \Delta I_r \).

The eigenmodes are vectors in discretized-redshift space, which we normalize as \( E^{(k)} = E^{(k)} = 1 \). The energy-release history, \( Q(z) \), and the residual distortion, \( \Delta I_r^R \), can then be written as

\[
Q \approx \sum_k E^{(k)} \mu_k, \quad \Delta I_r^R \approx \sum_k E^{(k)} S^{(k)} \quad S^{(k)} = \sum_a \hat{R}(z_a) E^{(k)}(\epsilon_a),
\]

where \( \mu_k \) and \( S^{(k)} \) are the amplitude and distortion signal of the \( k \)-th eigenmode, respectively. By construction, the eigenvectors, \( E^{(k)} \), span an ortho-normal basis, while all \( S^{(k)} \) only define an ortho-normal basis (generally \( S^{(1)} \cdot S^{(2)} \geq \delta_{ij} \). We furthermore defined the energy-release vector \( Q = (Q(z_0), Q(z_1), ..., Q(z_{\text{max}})) \) of \( Q(z) \) in different redshift bins and the mode amplitudes \( \mu_k = E^{(k)} \cdot Q \). The expected absolute error in the recovered mode amplitudes \( \mu_k \) is determined by \( \Delta \mu_k = 1/\sqrt{\lambda_k} \approx \Delta \mu_k \). This scaling implies that for a given frequency range and resolution the eigenvalue problem only has to be solved once. This is possible because we assume the same sensitivity in each channel, but generalization is straightforward.

3.1 Results for the eigenvectors and eigenvalues

In Fig. 4 we show the first few \( E^{(k)} \) and \( S^{(k)} \) for a PIXIE-like experiment. We defined the signs of the modes such that the mode amplitudes are positive for \( Q \propto \text{const} > 0 \). The first energy-release mode, \( E^{(1)} \), has a maximum at \( z \approx 5.3 \times 10^4 \), while higher modes show more variability, extending both towards lower and higher redshift. The corresponding distortion modes, \( S^{(k)} \), show increasing variability and decreasing overall amplitude with growing \( k \). They capture all corrections to the simple superposition of pure \( \mu \)- and \( y \)-distortion, needed to morph between these two extreme cases.

In Table 1 we summarize the projected errors for the first six mode amplitudes. The errors, \( \Delta \mu_k \), increase rapidly with mode number (this is how we order the eigenmodes), meaning that for a fixed amplitude of the distortion signal the information in the higher modes can only be accessed at higher spectral sensitivity.

Knowing the signal eigenvectors, \( S^{(k)} \), we can directly relate the mode amplitudes, \( \mu_k \), to the fractional energy, \( \epsilon \), stored by the residual distortion. It thus allows us to estimate how much information is contained by the residual distortion. Since integration over
frequency can be written as a sum over all frequency bins, with $e_k = 4 \sum S^{(i)}(\nu) \sum G_{ij}$, we have $e \approx \sum e_k \mu_k$. The first six $e_k$ are given in Table 1. The signal modes, $S^{(1)}$ and $S^{(2)}$, contribute most to the energy, while energy release into the higher modes is suppressed by an order of magnitude or more.

Even if individual mode amplitudes cannot be separated, the total energy density contained in the residual distortion might still be detectable. The error of $e$ can be found using Gaussian error propagation, $\Delta e \approx \sqrt{\sum e_k^2 \Delta \mu_k^2}$, where the numbers show, respectively, uncertainties when all modes, all but $1$, all but $2$, and all but $3$ are included. Another estimator for the residual distortion is the modulus of the residual distortion vector $|\Delta R| = \sqrt{\sum |S^{(1)}| S^{(2)} \mu_i}$. The required scalar product amplitudes are also given in Table 1. Similar to $e$, the error of $|\Delta R|$ scales like $\Delta |R| = 2 \sqrt{\sum |S^{(1)}| S^{(2)} \mu_i} \Delta \mu_i$. Both $e$ and $|\Delta R|$ can be used to estimate how much information is left in the residual when the mode hierarchy is truncated at some fixed value $k$. If the signal-to-noise ratio is larger than unity, more modes should be added.

Table 1. Forecasted 1σ errors of the first six eigenmode amplitudes, $E^{(i)}$. We also give $e_k = 4 \sum S^{(i)}(\nu) \sum G_{ij}$, and the scalar products $S^{(i)} \cdot S^{(j)}$ (in units of $10^{-18}$ W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$). The fraction of energy release to the residual distortion and its uncertainty are given by $\epsilon \approx \sum e_k \mu_k$ and $\Delta \epsilon \approx \sqrt{\sum e_k^2 \Delta \mu_k^2}$, respectively. For the mode construction we used PIXIE-settings ($|\nu_{\min} - \nu_{\max}, \Delta \nu_k| = [30, 1000, 15]$ GHz and channel sensitivity $\Delta \nu_k = 5 \times 10^{-28}$ W m$^{-2}$ Hz$^{-1}$ sr$^{-1}$). The errors roughly scale as $\Delta \epsilon \approx \Delta \nu_k / \sqrt{N_k}$.

| $k$ | $\Delta \mu_k$ | $\Delta \nu_k$ | $e_k$ | $S^{(i)} \cdot S^{(j)}$ |
|-----|---------------|---------------|------|------------------|
| 1   | $1.48 \times 10^{-7}$ | 1 | $-6.98 \times 10^{-3}$ | $1.15 \times 10^{-1}$ |
| 2   | $7.61 \times 10^{-7}$ | 5.14 | $2.12 \times 10^{-3}$ | $4.32 \times 10^{-3}$ |
| 3   | $3.61 \times 10^{-6}$ | 24.4 | $-3.71 \times 10^{-4}$ | $1.92 \times 10^{-4}$ |
| 4   | $1.74 \times 10^{-5}$ | $1.18 \times 10^2$ | $8.29 \times 10^{-5}$ | $8.29 \times 10^{-6}$ |
| 5   | $8.52 \times 10^{-5}$ | $5.76 \times 10^2$ | $-1.55 \times 10^{-5}$ | $3.45 \times 10^{-7}$ |
| 6   | $4.24 \times 10^{-4}$ | $2.86 \times 10^3$ | $2.75 \times 10^{-6}$ | $1.39 \times 10^{-8}$ |

4 PARAMETER ESTIMATION USING ENERGY-RELEASE EIGENMODES

In the previous sections, we created a set of orthogonal signal modes that can be constrained by future SD experiments and used to recover part of the energy-release history in a model-independent way. We derived a set of energy-release eigenmodes that describes the residual distortion signal that cannot be expressed as simple superposition of temperature shift, $\mu$- and $y$-distortion.

As explained above, nothing can be learned from the change in the value of the CMB temperature caused by energy release. Thus, the useful part of the primordial signal is determined by the parameters $p_{\text{opt}} = \{
u, \mu, \mu_k\}$. The number of residual modes, $\mu_k$, that can be constrained depends on the typical amplitude of the distortion and instrumental aspects. To the primordial signal, we need to add $y_{\text{re}}$ to describe the late-time $y$-distortion, and $\Delta \nu$ to parametrize the uncertainty in the exact value of the CMB monopole. The total distortion signal therefore takes the form

$$\Delta I = \Delta I^T + \Delta I^\nu + \Delta I^\mu + \Delta I^y,$$

where $G_{i,j}$, $Y_{i,j}$, and $M_i$ are the average signals of $G_T$, $Y_{SZ}$ and $M$ over the $i$th channel. The dependence of $\Delta I$ on $\Delta \nu$ is quadratic, but since $\Delta \nu \ll 1$, the problem remains quasi-linear, with the second-order term leading to a negligible correction to the covariance matrix, once expanded around the best-fitting value for $\Delta \nu$. For estimates one can thus set $\Delta I^\nu = G_{i,j} \Delta \nu$ without loss of generality. This defines the parameter set $p = \{\Delta \nu, y^*, \mu_k\}$, where $y^* = y_{\text{re}} + y$. Note that because of the low-$z$ contribution, it is hard to disentangle the primordial components of $\Delta \nu$ and $y^*$. The primordial energy release, therefore, is best constrained with $\mu$ and the $\mu_k$.

4.1 Errors of $\Delta \nu$, $y^*$ and $\mu$

As a first step, we estimate the errors on the values of $\Delta \nu$, $y^*$ and $\mu$ assuming PIXIE-like settings. The relevant projections to construct
We decomposed the signal explicitly, but included all contributions to the distortion. We then added a temperature shift with \( \Delta T = 1.2 \times 10^{-4} \) and a y-distortion with \( y_{\text{re}} = 4 \times 10^{-7} \) to the input signal, and analyzed it using the model, Eq. (11), with only \( \mu_1 \) included. Figure 5 shows the results of this analysis. All the recovered values and errors agree with the predictions. We can furthermore see that \( \mu_1 \) does not correlate to any of the standard parameters \( \rho_\gamma = (\Delta \gamma^* y^*, \mu) \), as ensured by construction. The standard parameters are slightly correlated with each other, since in the analysis we used \( G_{\gamma T}, Y_{SZ} \) and \( M_i \) which themselves are not orthogonal. Alternatively, one could use the orthogonal basis \( G_{\gamma T}, Y_{SZ} \) and \( M_i \) (see Appendix A), but since the interpretation of the results is fairly simple we preferred to keep the well-known parametrization. We confirmed that adding more distortion eigenmodes to the estimation problem does not alter any of the constraints on the other parameters. This demonstrates that the eigenmodes constructed above can be directly used for model-independent estimations and compression of the useful information provided by the CMB spectrum.

### 4.3 Partial recovery of the energy-release history

The energy-release eigenmodes define an ortho-normal basis to describe the energy-release history over the considered redshift range. In the limit of extremely high sensitivity and very fine spectral coverage (all modes can be measured) a complete reconstruction of the input history would be possible. Since realistically only a finite number of energy-release eigenmodes (2 or 3 really) might be measured, this means that a partial but model-independent reconstruction of the input energy-release history can be derived.

Considering the simple example, \( Q = 5 \times 10^{-8} \), in Fig. 6 we show the comparison of input history and the corresponding reconstruction if one, three or five modes can be measured. Clearly, the SD signal can only probe energy release around \( z \approx 5 \times 10^4 \), providing the means to obtain a wiggle recovery of the input history. The SD signal created by an energy-release history that is constant, or has the other shapes is virtually indistinguishable from the observational point of view, because the energy release from the oscillatory parts does not leave any significant traces. Still, the trajectories of
energy-release histories from different scenarios are directly constrained once the set of $\mu_k$ is known. This is one of the interesting model-independent ways of interpreting CMB SD results.

4.4 Overall picture and how to apply the eigenmodes

We now have all the pieces together to explain how to interpret and use the eigenmode decomposition presented above. Given the distortion data, $\Delta T^i$ (we assume that foregrounds have been removed perfectly), in different frequency channels we can estimate the spectral model parameters $p_m = \{A_T, y', \mu, \mu_i\}$. Using the signal eigenvectors, $S^{(k)}$, we can directly obtain the mode amplitudes by $\mu_k \approx \sum_k \Delta T^i S^{(k)}(i)/S^{(k)}(i)$. Similarly, we can compute $y'$ and $\mu$ as simple scalar products of the data vector with $F_{SZ}$ and $M_\nu$. The errors can be deduced using Table 1 and Sect. 4.1. At this point, we have compressed all the useful information contained by the CMB spectrum into a few numbers, $p_m$. The number of operations needed to compute the SD from a given ERS also roughly reduces by a factor of $n \approx (m + 2)/N_{\text{freq}}$, where $m$ is the included number of eigenmodes and $N_{\text{freq}}$ the number of channels. For PIXIE, this means $n^{-1} \approx 15-20$ times improvement of the performance, when using the signal eigenvectors for parameter estimation.

The $\mu$-parameter provides an integral constraint on the energy release with redshift-dependent weighting function, $f_\mu(z)$ (see Fig. 1). Many energy-release histories can give rise to exactly the same value of $\mu$. Still any specific scenario has to reproduce this number, although an interpretation becomes model dependent at this point. Similarly, the recovered $y$-parameter can only be interpreted in a model-dependent way. Since only the combination $y' = y_{\text{re}} + y$ can be constrained, the model-dependent step allows us to deduce an estimate for $y_{\text{re}}$, but otherwise does not help constraining the energy-release history except $y_{\text{re}}$ is known (precisely) by another method. Conversely, $y_{\text{re}}$ remains uncertain, since a large contribution to $y'$ could be caused by pre-recombination energy release. This compromises our ability to learn about reionization and structure formation by studying the average CMB spectrum.

On the other hand, the recovered eigenmode amplitudes $\mu_k$ allow us to constrain the energy-release history, $Q(z)$, in a model-independent way (Sect. 4.3 and Fig. 6). Since we can only expect the first few modes to be measured, from Fig. 4 it is clear that one is most sensitive to energy release around $z \approx 5 \times 10^4$. ERSs with little activity during that epoch will project weakly on to $\mu_k$. Different ERSs are furthermore expected to have specific eigenspectra, $\mu_k$, which in principle allows distinguishing them and constraining their specific model parameters. Computing the eigenspectra as a function of parameters can thus be used to quickly explore degeneracies between models. It is also clear that for ERSs with $m$ parameters, at least $m$ distortion parameters (excluding $y$) have to be observable. To distinguish between different types of models generally one additional parameter has to be measured and the eigenspectra of the scenarios have to be sufficiently orthogonal with respect to the experimental sensitivity. We find that even for optimistic setting typically no more than the first three eigenmodes plus $\mu$ can be measured, so that in the foreseeable future energy-release models with more than 4 parameters cannot be constrained without providing additional information.

5 CONSTRAINTS ON DIFFERENT SCENARIOS AND MODEL COMPARISON

The signal decomposition and residual eigenmodes developed in the previous sections provide new insight into the primordial energy-release analysis. This is because we collapse the multi-frequency data (order $\approx 100$ numbers) to lower dimensions, with only a handful number of parameters required to describe the distortion signal. In this section, we shall present a few illustrative examples to illustrate how to use the signal eigenvmodes in the analysis. In particular, we consider three different classes of early ERSs: dissipation of acoustic modes, particle annihilation and decaying particles. We summarize the parameters and eigenspectra for some examples in Table 2.

In the following we precede in a step-by-step manner: we first give details about the parametrizations of the different ERSs (Sect. 5.1). In Sect. 5.2 we illustrate the general dependence of the distortion signals on the model parameters, while in Sect. 5.3 we discuss future detection limits for $\mu$ and $\mu_i$. We close our analysis in Sect. 5.4 by providing details about direct model comparisons.

5.1 Parametrization of the energy-release scenarios

The two cases for the dissipation of small-scale acoustic modes presented in Table 2 are computed according to Chluba et al. (2012b), using the standard parametrization of the primordial curvature power spectrum, $P_c(k) \equiv A_c (k/k_0)^{n-11} \sqrt{\Omega_\text{CMB}}(k_0)$, with the pivot-scale $k_0 = 0.05 \text{ Mpc}^{-1}$. The associated SD is thus a family of three parameters ($A_c, n_s, n_{\text{run}}$), with heating rate defined by (cf. Chluba et al. 2012b, Chluba & Grin 2013)

$$\frac{d(Q/\rho)}{dz}\bigg|_{\text{run}} \approx 2D^2 \int_{\text{run}}^{\infty} P_c(k) \partial_i e^{-2\gamma_i/k_0} \, d\ln k,$$  

(13)

where $k_0(z)$ is the dissipation scales, $k_{\text{cut}} \approx 1 \text{ Mpc}^{-1}$ denotes the k-space cut-off scale and $D^2 \approx 0.81$ is the heating efficiency for adiabatic modes (assuming the standard value for the effective number of relativistic neutrino species $N_{\text{eff}} = 3.046$). The distortion depends on the type of initial conditions (adiabatic versus isocurvature); however, as shown by Chluba & Grin (2013), the differences can be captured by redefining the heating efficiency, the spectral index and its running. Thus, a discussion of the SD caused by adiabatic modes sweeps the whole parameter space. For a scale-invariant power spectrum $d(Q/\rho)/dz_{\text{run}} \propto z^{-1}$ so that $Q_{\text{run}} \propto \text{const.}$

The two annihilation scenarios given in Table 2 are for s-wave and p-wave annihilation cross-section with redshift dependence $(\sigma v) = \text{const}$ and $(\sigma v) \propto (1 + z)$, respectively. The heating rate can be parametrized as (see also Chluba & Sunyaev 2012)

$$\frac{d(Q/\rho)}{dz}_{\text{ann}} \approx f_{\text{ann}} \frac{N_{\text{ann}}(z)(1 + z)^{\lambda+1}}{H(z)\rho(z)},$$  

(14)

where $\lambda = 0$ for s-wave and $\lambda = 1$ for p-wave annihilation. Furthermore, $N_{\text{ann}}(z) \approx 1.9 \times 10^{-7}(1 + z)^3 \text{ cm}^{-3}$ denotes the number density of hydrogen nuclei, and $H(z) \approx 2.1 \times 10^{-6}(1 + z)^2 \text{ sec}^{-1}$ is the Hubble rate, assuming radiation domination. Thermally produced dark matter particles are expected to have s-wave annihilation cross-section with possible amplification due to Sommerfeld-enhancement (e.g., see Hamann & Tran 2011). The p-wave sce-
Table 2. Eigenspectra for different energy-release scenarios. The mode amplitudes were scaled by the variable $A$, as indicated. An asterisk (*) indicates that the parameter can be detected at more than 1σ with PIXIE-like sensitivity, while a dagger (†) shows that 5 times the sensitivity is required for a 1σ detection. The last few rows are $\rho_1 = [\mu_1/\Delta \mu_1]/[\mu/\Delta \mu]$, which give a representation that shows how the difficulty of a measurement relative to $\mu$ increases. Also, by comparing the numbers between models one can directly estimate how hard it is to distinguish them experimentally.

| Shape  | Dissipation | Dissipation | Annihilation | Annihilation | Decay | Decay | Decay |
|--------|-------------|-------------|--------------|--------------|-------|-------|-------|
| parameters | $n_s = 1$ | $n_s = 0.96$ | $n_{\text{run}} = 0$ | $n_{\text{run}} = -0.02$ | $(\sigma \nu) = \text{const}$ (s-wave) | $(\sigma \nu) = (1 + z)$ | $(\sigma \nu) = \text{rel.}$ | $\xi_X = 2 \times 10^4$ | $\xi_X = 5 \times 10^4$ | $\xi_X = 10^5$ |
| $A$ | $\Delta A_{\text{d}} = 2 \times 10^{-9}$ | $\Delta A_{\text{d}} = 2 \times 10^{-9}$ | $\Delta A_{\text{p}} = 2 \times 10^{-9}$ | $\Delta A_{\text{p}} = 2 \times 10^{-9}$ | $\Delta f_{\text{d}}/f_{\text{d}} \Gamma_{\text{d}}$ | $\Delta f_{\text{p}}/f_{\text{p}} \Gamma_{\text{p}}$ | $\Delta f_{\text{p}}/f_{\text{p}} \Gamma_{\text{s}}$ |
| $y/A$ | $4.7 \times 10^{-9}$ | $4.7 \times 10^{-9}$ | $5.1 \times 10^{-9}$ | $5.1 \times 10^{-9}$ | $4.8 \times 10^{-9}$ | $4.8 \times 10^{-9}$ | $4.8 \times 10^{-9}$ | $1.4 \times 10^{-7}$ | $1.4 \times 10^{-7}$ | $1.4 \times 10^{-7}$ |
| $µ/A$ | $3.1 \times 10^{-8}$ | $3.1 \times 10^{-8}$ | $3.9 \times 10^{-9}$ | $3.9 \times 10^{-9}$ | $8.3 \times 10^{-8}$ | $8.3 \times 10^{-8}$ | $8.3 \times 10^{-8}$ | $2.2 \times 10^{-7}$ | $2.2 \times 10^{-7}$ | $2.2 \times 10^{-7}$ |
| $\mu_1/A$ | $5.4 \times 10^{-8}$ | $5.4 \times 10^{-8}$ | $2.9 \times 10^{-8}$ | $2.9 \times 10^{-8}$ | $6.8 \times 10^{-9}$ | $6.8 \times 10^{-9}$ | $6.8 \times 10^{-9}$ | $1.5 \times 10^{-7}$ | $1.5 \times 10^{-7}$ | $1.5 \times 10^{-7}$ |
| $\mu_2/A$ | $1.0 \times 10^{-9}$ | $1.0 \times 10^{-9}$ | $-5.1 \times 10^{-9}$ | $-5.1 \times 10^{-9}$ | $2.6 \times 10^{-10}$ | $2.6 \times 10^{-10}$ | $2.6 \times 10^{-10}$ | $-1.7 \times 10^{-7}$ | $-1.7 \times 10^{-7}$ | $-1.7 \times 10^{-7}$ |
| $\mu_3/A$ | $3.5 \times 10^{-9}$ | $3.5 \times 10^{-9}$ | $1.9 \times 10^{-9}$ | $1.9 \times 10^{-9}$ | $4.3 \times 10^{-9}$ | $4.3 \times 10^{-9}$ | $4.3 \times 10^{-9}$ | $1.6 \times 10^{-7}$ | $1.6 \times 10^{-7}$ | $1.6 \times 10^{-7}$ |
| $\mu_4/A$ | $2.2 \times 10^{-9}$ | $2.2 \times 10^{-9}$ | $-5.1 \times 10^{-9}$ | $-5.1 \times 10^{-9}$ | $4.3 \times 10^{-9}$ | $4.3 \times 10^{-9}$ | $4.3 \times 10^{-9}$ | $-1.3 \times 10^{-6}$ | $-1.3 \times 10^{-6}$ | $-1.3 \times 10^{-6}$ |
| $\rho_1$ | $3.9 \sigma_*$ | $2.9 \sigma_*$ | $0.43 \sigma_*$ | $0.43 \sigma_*$ | $0.17 \sigma_*$ | $0.17 \sigma_*$ | $0.17 \sigma_*$ | $117 \sigma_*$ | $70.6 \sigma_*$ | $24.7 \sigma_*$ |
| $\rho_2$ | $2.3 \sigma_*$ | $2.3 \sigma_*$ | $0.29 \sigma_*$ | $0.29 \sigma_*$ | $6.1 \sigma_*$ | $6.1 \sigma_*$ | $6.1 \sigma_*$ | $16.6 \sigma_*$ | $51.6 \sigma_*$ | $73.8 \sigma_*$ |

5.2 Shape of the distortion signal

5.2.1 Annihilating particles

We start with the annihilation scenarios, for which the distortion has a fixed shape and only the overall amplitude changes, depending on the annihilation efficiency, $\Gamma_{\text{ann}}$. The residual distortion signals are illustrated in Fig. [7](upper panel). We scaled the total distortion such that in both cases $\mu = 10^{-8}$. This emphasizes the differences in the shape of the distortion rather than its overall amplitude. The residual distortion is significantly smaller for the p-wave scenario, showing that most of the energy is released during the $\mu$-era ($\gamma, \mu_1 < \mu$, see Table 2). The small difference in the phase and amplitude of the residual distortion relative to $\mu$ in principle allows discerning the s- and p-wave cases, however, a detection of $\mu$ is required to break the degeneracy. The values of $\gamma$, $\mu$, and $\mu_2$ given in Table 2 fully specify the shape of the distortion for s- and p-wave annihilation scenarios and all other cases can be obtained by rescaling the overall amplitude appropriately.

5.2.2 Dissipation of small-scale acoustic modes

The central panel of Fig. [7] illustrates the $n_s$-dependence of the residual distortion for the dissipation scenario. For different values of $n_s$, mainly the amplitude of the distortion changes, while the shape and phase of the residual distortion is only mildly affected. For $n_s > 1$, relative to the scale-invariant case more energy is released at earlier times. This increase the value of $\mu$ relative to the $\mu_s$, implying that the amplitude of the residual distortion decreases. By measuring $\mu$ and $\mu_1$ one can thus constrain $A_\zeta$ and $n_{\text{run}}$ independently. However, when allowing $n_{\text{run}}$ to vary, also $\mu_2$ (which is significantly harder to access) is required to distinguish these cases. Running again dominantly affects the amplitude of the residual distortion, while changes in the phase of the signal are weaker.
We can represent the dependence of the distortion on the parameters by specifying the amplitude of $\mu(A_1, nS, n_{\text{run}})$ and the ratios $\mu_i/\mu$, which are only function of $p = (nS, n_{\text{run}})$. To also rank the variables in terms of the level of difficulty that is met to measure them, we furthermore weight them by their 1σ-errors. This defines the new variable

$$\rho_k = \frac{\mu_k/\Delta \mu_k}{\mu/\Delta \mu}. \quad (16)$$

and the distortion parameter set $p = (y, \mu, \rho_k)$. To give an example, having $p_\delta \equiv 1$ means the value of $\mu_1$ is as hard to measure as $\mu$, while $p_\delta < 1$ means it is $\rho_\delta^{-1}$ times harder. If $\mu$ is observable with significance, $s_\mu > 1$, then $p_\delta < 1$ only implies nondetections of $\mu_k$ if also $\rho_k s_\mu < 1$. The real advantage of this variable is that it parametrizes the shape of the energy-release history without depending on the overall amplitude. Its error is simply $\Delta \mu_k \approx (1 + \rho_k^2)^{1/2} \Delta \mu/\mu \approx \Delta \mu/\mu$, where in the last step we assumed $|\rho_k| \ll 1$. Especially for model comparisons, this parameterization is very useful (see Sect. 5.3).

In Figure 8 we show the dependence of the first three $\rho_k$ on $nS$ and $n_{\text{run}}$. We emphasize that in the considered range of parameters for fixed $n_{\text{run}}$ the vector $p = (\rho_1, \rho_2, \rho_3)$ is uniquely linked to $nS$. The different curves have, however, very similar shapes when varying $n_{\text{run}}$. The equivalent shift in $nS$ for each $\rho_k$ differs slightly and also depends on $nS$, so that sensitivity to $p = (A_c, nS, n_{\text{run}})$ can be expected. Both $\rho_1$ and $\rho_3$ vary rather slowly, while $\rho_2$ changes sign around $nS \approx 1$. This indicates that if $\mu$, $\rho_1$, and $\rho_2$ are measurable, most sensitive constraints on $p = (A_c, nS, n_{\text{run}})$ are expected around $nS \approx 1$. However, since in the considered range $p_2 \approx 10^{-1} - 10^{-2}$, it is already clear that pretty high precision for the measurement of $\mu$ is needed (see Fig. 11). We can furthermore see that around $nS \approx 1.2 - 1.4$, the dependence of $p_2$ on $n_{\text{run}}$ is rather weak, and degenerate with $nS$. This indicates that high sensitivity is required to discern different cases in this regime.

5.2.3 Decaying relic particles

The lower panel of Fig. 7 illustrates the dependence of the distortion signal caused by a decaying particle on its lifetime. Shorter lifetime means most energy is released at earlier times so that the distortion is closer to a pure $\mu$-distortion with a smaller residual distortion. Increasing the lifetime (lowering $\tau_{S}$), the overall amplitude of the residual distortion increases and shows a small phase shift...
5.3 Detectability of the distortion signal

5.3.1 Annihilating particles

The signal caused by the s-wave annihilation scenario given in Table 2 is undetectable with a PIXIE-like experiment, but could be detected at $\approx 3\sigma$ with PRISM. The distortion signal depends on only one free parameter, $f_{\text{anns}}$, for which we chose a value that is close to the upper 1σ bound derived from current CMB anisotropy measurements (Galli et al. 2009; Hütsi et al. 2009; Slatyer et al. 2009; Hütsi et al. 2011; Planck Collaboration et al. 2013d). The spectral sensitivity needs to be increased $\sim 4$ times over PIXIE to detect the s-wave distortion signature, while a factor of $\approx 22$ improvement is needed to recover the first distortion eigenmode, $\mu_{1}$.

The considered p-wave scenario illustrates how the eigenspectra change when the redshift scaling of the energy-release rate is modified. Since most of the energy is liberated at early times, the distortion signal is dominated by $\mu$ (see Sect. 5.4). With a PIXIE-like experiment the first distortion eigenmode remains undetectable and even PRISM will not suffice to measure this number values. Again, the distortion is just determined by $f_{\text{annp}}$, but since the eigenspectrum differs from the one of the s-wave scenario, by measuring the first eigenmode these are distinguishable. For $f_{\text{anns}} \approx 2 \times 10^{-23} \text{eV sec}^{-1}$ and $f_{\text{annp}} \approx 4.8 \times 10^{-26} \text{eV sec}^{-1}$, s- and p-wave scenarios both give rise to $\mu \approx 4 \times 10^{-7}$. In this case, $\mu_{1} \approx 9.7 \times 10^{-10}$ for the p-wave, and $\mu_{1} \approx 6.4 \times 10^{-9}$ for the s-wave case. Thus, by increasing the sensitivity $\approx 22$ times over PIXIE, the s- and p-wave scenarios in principle become distinguishable ($\mu_{1}$ from the s-wave scenario would be detected at 1σ, while for a p-wave scenario $\mu_{1}$ should be consistent with zero). These findings are in good agreement with those of Chluba (2013a), where an MCMC analysis was used.

A PIXIE-type experiment could place independent 1σ-limits of $f_{\text{anns}} \approx 6.9 \times 10^{-23} \text{eV sec}^{-1}$ and $f_{\text{annp}} \approx 1.6 \times 10^{-28} \text{eV sec}^{-1}$ on the annihilation efficiency, with practically all the information coming from $\mu$ itself (see also Chluba 2013a). Using the parametrization according to the recent Planck papers (Planck Collaboration et al. 2013d), this implies $p_{\text{ann}} \approx 9.2 \times 10^{-6} \text{m}^{2} \text{kg}^{-1} \text{s}^{-2} (95\% \text{c.l.})$, which is several times weaker than the current CMB anisotropy limit obtained with Planck ($p_{\text{ann}} < 3.1 \times 10^{-6} \text{m}^{2} \text{kg}^{-1} \text{s}^{-2}$). Uncertainties in the modeling of the energy-deposition rates indicate that this limit is in fact slightly weaker, but still once the full polarization data from Planck is included, one does expect an improvement of this bound to $p_{\text{ann}} < 1.7 \times 10^{-7} \text{m}^{2} \text{kg}^{-1} \text{s}^{-2}$ (Galli et al. 2013). Thus, only an increase of the spectral sensitivity by a factor of $\approx 50$ over PIXIE could make future CMB distortion measurements a competitive probe for annihilating dark matter particles, although one should emphasize that SD would still give an independent measurement, suffering from very different systematics. In the future, PRISM might allow direct detection of a dark matter annihilation signature, if $f_{\text{anns}} \gtrsim 4.6 \times 10^{-17} \text{m}^{2} \text{kg}^{-1} \text{s}^{-2}$.

5.3.2 Dissipation of small-scale acoustic modes

From measurements of the CMB anisotropies at large scales we have $A_{L} \approx 2.2 \times 10^{-9}$, $n_{S} \approx 0.96$ and $n_{\text{run}} \approx -0.02$ (Planck Collaboration et al. 2013d). Using these values and extrapolating all the way to wavenumber $k \approx \times 10^{4} \text{Mpc}^{-1}$, we obtain the distortion parameters given in Table 2. For comparison, we also show the case with no running and scale-invariant power spectrum. For these two dissipation scenarios the $\gamma$-parameter will contribute at a few $\sigma$-level to $y = y_{\text{run}} + y$ for a PIXIE-like experiment, while no information can be extracted from the residual distortion (none of the $\mu_{i}$ can be detected). For a scale-invariant power spectrum also a non-vanishing $\mu$-parameter could be found ($\approx 2.3\sigma$) with a PIXIE-like experiment (see also Chluba et al. 2012b). For PRISM, a more than $20\sigma$ detection of $\mu$ for a scale-invariant power spectrum should be feasible, while for $A_{L} \approx 2.2 \times 10^{-9}$, $n_{S} \approx 0.96$ and $n_{\text{run}} \approx 0$ we expect $\approx 17\sigma$ detection of $\mu$.

Since the $\gamma$-parameter is degenerate with $y_{\text{run}}$, only $\mu$ can be used to place constraints in these cases, however, the degeneracy among model parameters is very large. For example, the small difference in the value of $\mu$ for the two considered cases can be compensated by adjusting $A_{L}$ at small scales. Increasing the sensitivity 10 times over PIXIE will allow an additional detection of the first eigenmode ($\approx 3.7\sigma$ and $\approx 2.0\sigma$ for the two dissipation scenarios given Table 2, respectively). In this case, the parameter degeneracies ($A_{L}$, $n_{S}$, and $n_{\text{run}}$) can be partially broken (two numbers, $\mu$ and $\mu_{1}$, are used to limit three variables). Improvement by another factor of 10 allows marginal detections of the second mode amplitude, if $f_{\text{anns}} \approx 1.5 \times 10^{-17} \text{eV kg m}^{-3}$. 

Figure 9. Dependence of $\rho_{2}$ on the lifetime of the decaying particle. We indicated parts of the curves that are negative.
two power spectrum parameters. To determine all
PIXIE
ing
that the value of
k
is required for a detection. For
For
thermore needs
Around
results of this exercise are shown in Fig. 10 for PIXIE settings.

Figure 10. Forecasted constraints on \( A_s, n_S \) and \( n_{\text{run}} \). The case labeled Planck+WP+highL uses the published covariance matrix of Planck with in-
clusion of WMAP polarization data and the high \( \ell \) data from ACT and SPT. The case labeled PRISM is based on estimates given in [Andre et al. (2013)] for the PRISM imager and spectrometer part. The upper panel shows the 2D contours and marginalized distributions for \( A_s, n_S \) and \( n_{\text{run}} \), while the lower panel illustrates the expected improvement (decrease) in the measure-
ment uncertainty of the PRISM imager over Planck (horizontal lines) and the additional gain when adding the PRISM SD data. Note that the PRISM spectrometer is about one order of magnitude more sensitive than PIXIE.

but to truly constrain the shape of the small-scale power spectrum (assuming the standard parametrization) using SD data alone an overall factor \( \geq 200 \) over PIXIE will be necessary, making this application of SDs rather futuristic (see also Chluba (2013a)).

These simple estimates indicate that SD alone only provide competitive constraints on \( A_s, n_S \) and \( n_{\text{run}} \) for much higher spectral sensitivity; however, SD data can help to slightly improve the constraint on \( n_{\text{run}} \) when combined with future CMB anisotropy measure-
ments (see [Powell (2012)] [Khatri & Sunyaev (2013)] for similar discussion). This is simply because both \( A_s \) and \( n_S \) can be tightly constrained with the CMB anisotropy measurement, while the long
lever arm added with SD measurements improves the sensitivity to running of the power spectrum. We illustrate this in Fig. 10

Figure 11. 1\( \sigma \)-detection limits for \( \mu, \mu_1, \mu_2 \) and \( \mu_3 \) caused by dissipation of small-scale acoustic modes for PIXIE-like settings. We used the standard parametrization for the power spectrum with amplitude, \( A_s \), spectral index, \( n_S \), and running \( n_{\text{run}} \) around pivot scale \( k_0 = 45 \text{ Mpc}^{-1} \). The heavy lines are for \( n_{\text{run}} = 0 \), while all other lines are for \( n_{\text{run}} = (0.1, 0.1) \) in each group. For reference we marked the case \( n_{\text{run}} = 0.1 \).

5.3.3 Dissipation of small-scale acoustic modes: generalization

The above statements assume that the three-parameter Ansatz for the primordial curvature power spectrum holds for more than six to seven decades in scales. Strictly speaking, the exact shape and amplitude of the small-scale power spectrum are unknown and a large range of viable early-universe models (e.g., Salopek et al. (1989) Starobinski (1992) Ivanov et al. (1994) Randall et al. (1996) Stewart (1997) Copeland et al. (1998) Starobinski (1998) Chung et al. (2000) Hunt & Sarkar (2007) Joy et al. (2008) Barnaby et al. (2009) Barnab y (2010) Ben-Dayan & Brustein (2010) Achucarro et al. (2011) Cespedes et al. (2012)) producing enhanced small-scale power exist (see, Chluba et al. (2012a) for more examples and simple SD constraints). Observationally, the amplitude of the primordial small-

scale power spectrum is limited to \( A_s \leq 10^{-7} - 10^{-6} \) at wavenumber 3 Mpc\(^{-1} \lesssim k \lesssim \) few \( \times 10^3 \text{ Mpc}^{-1} \) (the range that is of most interest for CMB distortions) using ultra compact mini haloes [Bringmann et al. (2012)] [Scott et al. (2012)]. Although slightly model-independent, this still leaves a lot of room for non-standard dissipation scenarios, with enhanced small-scale power.

To study how well the small-scale power spectrum might be constrained by future SD measurements, it is convenient to con-
sider the shape and amplitude of the curvature power spectrum at

© 0000 RAS, MNRAS 000, 000–000
3 Mpc$^{-1} \leq k \leq \text{ few } \times 10^{4} \text{ Mpc}^{-1}$ independent of the large-scale power spectrum. We therefore change the question as follows: by shifting the pivot scale to $k_0 = 45 \text{ Mpc}^{-1}$ (corresponding to heating around $z_{\text{heating}} \approx 4.5 \times 10^{4}[k/10^{2} \text{ Mpc}^{-1}]^{-1/3} \approx 5.7 \times 10^{3}$) and using the standard parametrization for the power spectrum, how large does the power spectrum amplitude, $A_{k}(k_0 = 45 \text{ Mpc}^{-1})$, have to be to obtain a 1$\sigma$-detection of $\mu$, $\mu_1$, and $\mu_2$, respectively? The results of this exercise are shown in Fig. 11 for PIXIE settings. Around $n_S = 1$, a detection of $\mu$ is possible for $A_{k} \geq 10^{-9}$, while $A_{k} \geq 6 \times 10^{-9}$ is necessary to also have a detection of $\mu_1$. In this case, two of the three model parameters can in principle be constrained independently. To also access information from $\mu_2$ and $\mu_3$ one furthermore needs $A_{k} \geq 10^{-7}$. In this case, we could expect to break the degeneracy between all three parameters with a PIXIE-type experiment.

The detection limits depend both on the value of $n_S$ and $n_{\text{run}}$. For $n_{\text{run}} < 0$, in total less energy is released so that larger $A_k$ is required for a detection. For $n_{\text{run}} > 1$, more power is found at $k > 45 \text{ Mpc}^{-1}$, so that more energy is released in the $\mu$-era. Consequently, the $\mu$-distortion can be detected for lower $A_{k}$. Similarly, when increasing $n_S$, less energy is released around $z \approx 5 \times 10^{4}$, so that the value of $\mu_1$ decreases. Thus, larger $A_{k}$ is required to warrant a detection of $\mu_1$.

The above statements can be phrased in another way. Assuming $A_{k} \approx 10^{-9}$ and $n_{S} \approx 1$, at least a factor of 5 improvement over PIXIE sensitivity is needed to allow constraining combinations of two power spectrum parameters. To determine all $p = \{A_{k}, n_{S}, n_{\text{run}}\}$ independently an overall factor of $\geq 200$ improvement over PIXIE sensitivity is required, although in this (very conservative) case the corresponding constraints would still not be competitive with those obtained using large-scale CMB anisotropy measurements.

We can also ask the question of how well the power spectrum parameters can be constrained for different cases. If only $\mu$ is available, then the corresponding constraints on small-scale power spectrum parameters remain rather weak, but could still be used to limit the parameters space (e.g., Chluba et al. 2012). If $\mu$ and $\mu_1$ can be accessed, we can limit the overall amplitude of the power spectrum for given pairs of $n_S$ and $n_{\text{run}}$. This can be seen from the upper panel of Fig. 12 where we illustrate the possible parameter space of $\mu_1, \mu_2 \propto \mu_1/\mu$ and $\mu_2 \propto \mu_2/\mu$ in some range of $n_S$ and $n_{\text{run}}$. For the considered sensitivity and fiducial value of $A_{k}$, the errors on $\mu$ and $\rho_1$ are very small, but since $A_{k}$ can be adjusted without affecting $\rho_1$, the measurement is not independent of $n_S$ and $n_{\text{run}}$.

If in addition $\mu_2$ can be constrained, then the degeneracy can be broken. For PIXIE-settings and $n_S \approx 0.96$, this is only conceivable if the amplitude of the small-scale power spectrum is $A_{k} \geq 10^{-7} - 10^{-6}$ (see Fig. 11). As the lower panel of Fig. 12 indicates, the relative dependence on $n_{\text{run}}$ seems rather similar in all parts of parameter space: although the absolute distance between the lines varies relative to the error bars they seem rather constant. To show this more explicitly, from $\mu, \mu_1,$ and $\mu_2$ we compute the expected 1$\sigma$-errors on $A_{k}(k_0 = 45 \text{ Mpc}^{-1}), n_S$, and $n_{\text{run}}$ around the fiducial value using the Fisher information matrix, $F_{ij} = \Delta \mu^2 \partial_{\mu} \partial_{\mu}, \Delta \rho_1^2 \partial_{\rho_1} \partial_{\rho_1}, \Delta \rho_2^2 \partial_{\rho_2} \partial_{\rho_2}, \rho_1 \equiv \{A_{k, n_S, n_{\text{run}}\}}$. Figure 13 shows the corresponding forecasts assuming PIXIE-setting but with 5 times its sensitivity. If only $p = \{A_{k}, n_S\}$ are estimated for fixed $n_{\text{run}}$, the errors of $A_{k}$ and $n_{\text{run}}$ are only a few percent. When also trying to constrain $n_{\text{run}}$, we see that the uncertainties in the values of $A_{k}$ and $n_S$ increase by about one order of magnitude, with an absolute error $\Delta n_{\text{run}} \approx 0.07$ rather independent of $n_S$.

Little information is added when also $\mu_2$ can be measured (we find small differences in the constraints for small $n_S$ when $n_{\text{run}}$ is varied), although for model comparison $\mu_2$ could become important. Also, for power spectra that result in $\mu_2 \approx 0$, the detection limit of $\mu_3$ is much lower (see Fig. 11), so that the combination of $\mu_2$ consistent with zero but $\mu_3 > 0$ provides a useful confirmation of the dissipation scenario.

We can also use the results of Figure 13 to estimate the expected uncertainties for other cases. Adjusting the spectral sensitivity by a factor $f = \Delta L_i/[10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}]$, all curves can be rescaled by this factor to obtain the new estimates for the errors. Similarly, if $A_{k}(k_0 = 45 \text{ Mpc}^{-1})$ differs by $f_L = A_{k}/5 \times 10^{-8}$, we have to rescale the error estimates by $f_L^{-1}$. We checked the pre-
5 times PIXIE sensitivity
Reference $A^*_v = 5 \times 10^4$

![Graph](image)

Figure 13. Expected uncertainties of $A^*_v(k_0 = 45 \text{ Mpc}^{-1})$, $n_s$, and $n_{	ext{run}}$ using measurements of $\mu$, $\mu_1$, and $\mu_2$. We assumed 5 times the sensitivity of PIXIE and $A^*_v = 5 \times 10^{-8}$ as reference value (other cases can be estimated by simple rescaling). For the upper panel we also varied $n_{	ext{run}}$ as indicated, while in the lower panel it was fixed to $n_{	ext{run}} = 0$.

5.3.4 Decaying relic particles

The distortion signals for the three decaying particle scenarios presented in Table 2 will all be detectable with a PIXIE-like experiment. More generally, Fig. 14 shows the 1-$\sigma$ detection limits for $\mu$, $\mu_1$, $\mu_2$, and $\mu_3$, as a function of the particle lifetime. CMB SDs are sensitive to decaying particles with $\epsilon_X = f_X/z_X$ as low as $\sim 10^{-5}$ eV for particle lifetimes $10^7$ sec $\leq t_X \leq 10^{10}$ sec. For PRISM the detection limit will be as low as $\epsilon_X \approx 10^{-9}$ eV in this range. To directly constrain $t_X$, at least a measurement of $\mu_1$ is needed. At PIXIE sensitivity this means that the lifetime of particles with $2 \times 10^9$ sec $\leq t_X \leq 6 \times 10^{10}$ sec for $\epsilon_X \gtrsim 0.1$ eV and $3 \times 10^9$ sec $\leq t_X \leq 10^{12}$ sec for $\epsilon_X \gtrsim 1$ eV will be directly measurable. Most of this parameter space is completely unconstrained [see upper limit from measurements of the primordial $^3\text{He}/D$ abundance ratio (1-$\sigma$-level, adapted from Fig. 42 of Kawasaki et al. 2005)].

![Graph](image)

Figure 14. Detectability of $\mu$, $\mu_1$, $\mu_2$, and $\mu_3$. The upper panel shows the limits for $\epsilon_X = f_X/z_X$, while the lower panel uses the standard yield variable, $E_{\text{vis}} Y_X$ (cp. Kawasaki et al. 2005). For a given particle lifetime, we compute the required value of $\epsilon_X$ for which a 1-$\sigma$ detection of the corresponding variable is possible with PIXIE. The violet shaded area is excluded by measurements of the primordial $^3\text{He}/D$ abundance ratio (1-$\sigma$-level, adapted from Fig. 42 of Kawasaki et al. 2005).
Figure 15. Parameter range of $\mu$, $\rho_1$, and $\rho_2$ for decaying particle scenarios. We assumed PIXIE settings and sensitivity, and $\epsilon_\chi = f_\chi/\chi = 1$ eV (i.e., $A \equiv \epsilon_\chi/1$ eV). The blue symbols with error bars are for $\chi = 1$ eV and $\rho_1 - \rho_2$ plane can be used to further improve this measurement, but also for model comparison.

Figure 16. Relative error for determination of $\epsilon_\chi = f_\chi/\chi$ and $\chi$ using measurements of $\mu$ to $\rho_2$. We assumed 5 times the sensitivity of PIXIE and $\epsilon_\chi = 1$ eV as reference value (other cases can be estimated by simple rescaling). The corresponding error in the particle lifetime is $\Delta \Lambda/\Lambda \approx 2 \Delta \chi/\chi$. 

widen the range over which the particle lifetime can be directly constrained.

To illustrate this aspect even further, we can again study the $\mu - \rho_1$-parameter space covered by decaying particles. The projections into the $\mu - \rho_1$ and $\rho_1 - \rho_2$-plane are shown in Fig. 15 for decay efficiency $\epsilon_\chi = 1$ eV and PIXIE settings. Varying $\epsilon_\chi$ would move the $\mu - \rho_1$ trajectory left or right, as indicated in the upper panel of Fig. 15. Furthermore, all error bars of $\rho_1$ would have to be rescaled by $f \equiv [\epsilon_\chi/1 \text{ eV}]^{-1}$ under this transformation. Measuring $\mu$ and $\rho_1$ is in principle sufficient for independent determination of $\epsilon_\chi$ and the particle lifetime, $t_\chi \approx [4.9 \times 10^7/(1 + \chi)]^2$ sec, with most sensitivity around $\chi \approx 5 \times 10^4$ $t_\chi \approx 2 \times 10^6 - 10^{10}$ sec for the shown scenario. For shorter lifetime, the SD signal is very close to a pure $\mu$-distortion, with little information in the residual ($\rho_1$ and $\rho_2$ are both very small and also show very little variation with redshift). Similarly, for longer lifetimes the particle signature is close to a $y$-distortion. In both cases the sensitivity to the lifetime is very weak and only an overall integral constraint can be derived, with large degeneracy between $\epsilon_\chi$ and $\chi$ (see discussion in Chluba 2013a).

We can again estimate the expected 1$\sigma$-errors on $\epsilon_\chi$ and $\chi$ around the fiducial value using the Fisher information matrix, $F = \partial^2 \ln L / \partial \mu^2 \partial \rho_2$, $\mu + \sum_i \Delta \mu_i \partial \rho_2$, with parameters $p \equiv (\epsilon_\chi, \chi)$. In Fig. 16 we show the corresponding Fisher-forecasts assuming PIXIE-setting but with 5 times its sensitivity. We included information from $\mu$, $\rho_1$ and $\rho_2$, because adding $\mu_2$ did not change the forecast significantly. For $1.7 \times 10^4 \leq \chi \leq 3.5 \times 10^4$ (2 $\times 10^6$ sec $\leq t_\chi \leq 8.3 \times 10^{10}$ sec), the particle lifetime can be constrained to better than $\pm 20\%$ and $\epsilon_\chi$ can be measured with uncertainty $\leq 10\%$. These findings are in good agreement with those of Chluba (2013a), where direct MCMC simulations were performed. CMB SD are thus a powerful probe of early-universe particle physics, providing tight constraints that are independent and complementary to those derived from light element abundances (e.g., Kawasaki et al. 2005, Kohri & Takahashi 2010, Pospelov & Pradler 2010).

We emphasize that the CMB spectrum can be utilized to directly probe the particle lifetime, a measurement that cannot be obtained by other means. CMB SDs furthermore provide a calorimetric constraint, which is sensitive to the total heat that is generated in the decay process. For very light relic particles (mass smaller than a few MeV), measurements of light element abundances will not allow placing constraints, while the CMB spectrum should still be sensitive, assuming that the particle is abundant enough.

We also mention, that the Fisher estimates become crude, once the error reaches much more than $\approx 15\% - 20\%$. In this case, the likelihood becomes non-Gaussian and non-linear dependences are important. We also find that the solutions can be multi modal, with regions of low probability far away from the fiducial value. This means that MCMC sampling has to be performed in several steps, using wide priors to find regions of interest, followed by re-simulations around different maximum likelihood points. In this case, we refer to the methods developed in Chluba (2013a).
nario with \((n_S, n_{\text{ran}}) = (1, 1)\) has a \(\rho\)-vector \(\rho_{\text{diss}} \approx (0.161, 5.86 \times 10^{-3}, 4.31 \times 10^{-3})\), while for the \(s\)-wave annihilation scenarios we find \(\rho_{\text{ann.s}} \approx (0.159, 1.18 \times 10^{-3}, 4.17 \times 10^{-3})\). Comparing the entries of these vectors indicates that the two cases are quasi-degenerate. The small differences stem from the late-time behavior of \(Q(z)\) at \(z \lesssim 10\), but very high precision is indeed needed to discern them. In addition, by slightly adjusting \(n_S\) to \(\approx 1.01\) one can align these two \(\rho\)-vectors nearly perfectly. For the \(p\)-wave scenario, we find \(\rho_{\text{ann.p}} \approx (2.33 \times 10^{-2}, 4.37 \times 10^{-3}, 1.38 \times 10^{-3})\), which clearly is different from the \(s\)-wave annihilation and scale-invariant dissipation scenarios. However, adjusting \(n_S \approx 1.67\) practically aligns \(\rho_{\text{diss}}\) with \(\rho_{\text{ann.p}}\). This is expected since for \(n_{\text{ran}} = 0\) one has \(Q_{\text{ac}} \propto (1 + z)^{3(n_S-1)/2}\) (e.g., Chluba et al. 2012b), which becomes \(Q_{\text{ac}} \propto (1 + z)\) for \(n_S \approx 5/3\). Similarly, we find \(\rho_{\text{diss}} \approx (0.235, -8.07 \times 10^{-3}, 6.25 \times 10^{-3})\) for \((n_S, n_{\text{ran}}) = (0.96, -0.02)\), which is distinguishable from the \(s\)- and \(p\)-wave annihilation scenario, if \(\rho_1\) could be measured with \(\approx 10\%\) precision. However, as soon as the dissipation model parameters are varied, degeneracies reappear, unless even higher experimental precision is achieved. Therefore, \(s\)-wave annihilation scenarios and quasi-scale-invariant dissipation scenarios with small running are observationally hard to distinguish using CMB SD data. Similarly, \(p\)-wave and dissipation scenarios with \(n_S \approx 1.67\) and small running are degenerate. For values of \(n_S \neq (1, 1.67)\) the degeneracies with annihilation scenarios are less severe and at high spectral sensitivity they in principle can be discerned.

For comparison of dissipation and decaying particle scenarios, let us consider the general case with all model parameters varying. As mentioned above, if only \(\mu\) is measurable, no distinction can be made, unless priors are used (e.g., we assume that the primordial small-scale power spectrum is determined by extrapolation from large scales but find \(\mu \approx 10^{-7}\), which cannot be explained in this case). Assuming that \(\mu\) and \(\mu_1\) are measurable, by comparing the upper panels of Figs. 12 and 15 it is evident that due to freedom in the overall amplitude (\(A_e\) and \(\epsilon_X\) can be re-scaled), dissipation and decaying particle scenarios again cannot be distinguish in a model-independent way (moving the curves left and right one can make then coincide).

The situation changes when also \(\mu_2\) can be measured. Figure 17 shows the trajectories of dissipation and decaying particle scenarios in the \(\rho_1 - \rho_2\)-plane for two spectral sensitivities. Assuming that the small-scale power spectrum is quasi-scale-invariant with small running a PIXIE-type experiment will already be able to directly distinguish this from a decaying particle scenario. Allowing large negative running does increase the degeneracy and higher spectral sensitivity is needed to discern these cases. The lower panel illustrates the improvement for 5 times the sensitivity of PIXIE. Clearly, measurements of \(\mu_1\) and \(\mu_2\) allow discerning dissipation and decaying particle scenarios over a wide range of the parameter space, with degeneracies appearing for large negative running \((n_{\text{ran}} \ll -0.6)\) and in the limit of large and small spectral index.

We mention, however, that when allowing more complex shapes of the small-scale power spectrum, e.g., with bumps caused by particle production during inflation (Chung et al. 2000; Barnaby & Huang 2009; Barnaby 2010), closer resemblance of the energy-release history with the one of a decaying particle can be achieved. In this case, a distinction of the two scenarios will be more challenging. Also, a combination of decaying particle and dissipation scenarios could be possible but would be hard to distinguish from the single scenarios. Nevertheless, CMB SD measurements provide a unique way to study different ERSs allowing direct model comparisons and distinction in certain situations.

---

Figure 17. Model comparison for dissipation and decaying particle scenarios in the \(\rho_1 - \rho_2\) plane. We assumed \(A_e(k_0 = 45\ \text{Mpc}^{-1}) = 5 \times 10^{-3}\) and \(\epsilon_X \approx 1\) eV. The upper panel is for PIXIE sensitivity, while the lower is for 5 times higher sensitivity.

## 5.4 Comparing models using distortion eigenmodes

In the previous section, we presented parameter estimation cases using eigenmodes in a model-by-model basis. Furthermore, since each model has rather unique predictions for the observable \(\mu_i\)s, the eigenmode analysis opens a new possibility of distinguishing different ERSs. In this section, we shall illustrate this point by some solid examples. First, let us assume that the time dependence of the energy release is fixed. In that case, the shape of eigenspectrum does not change and only the overall amplitude is free, and we can directly use the examples given in Table 2. If only \(\mu\) can be constrained then different models cannot be distinguished unless some other constraint can be invoked. For example, finding \(\mu \approx 10^{-7}\) is unlikely to be caused by \(s\)-wave annihilation, which is bound to much smaller annihilation efficiencies by CMB anisotropy measurements. It could, however, be caused by a decaying relic particle or the dissipation of small-scale perturbations.

Once some of the \(\mu_i\) which directly probe the time dependence of the energy-release history, can be determined with signal-to-noise \(S/N > 1\), one can in principle distinguish between different scenarios. For instance, from Table 2 the dissipation sce-
6 CONCLUSIONS

In this work, we derived a decomposition of the CMB SD signal into temperature shift, $\gamma$, $\mu$ and residual distortion. The residual distortion was defined to be orthogonal to the temperature shift, $\gamma$- and $\mu$-distortion, taking experimental settings into account. Using this decomposition, we can explicitly show how much energy, at a given instance, is transferred to the various components of the CMB spectrum (Fig. 1). The $\gamma$-distortion part of the CMB spectrum cannot be used in a model-independent way to learn about the primordial energy-release (occurring at $z \gtrsim 10^4$), since it is degenerate with $\gamma$-distortions introduced at later times, by reionization and the formation of structures. The $\mu$-distortion component only provides a measure for the overall (integrated) energy release at $z \approx 10^3 \text{to} 10^4$, which can again only be interpreted in a model-dependent way. Adding the information in the residual distortion allows us to directly constrain the time dependence of the energy-release history, and thus provides a way to discern different scenarios (see also, Chluba & Sunyaev 2012; Chluba 2013a).

We took a step forward towards the analysis of future CMB SD data. The information contained in the residual distortion can be compressed into a few numbers. This compression is achieved by performing a principal component analysis to determine residual-distortion and energy-release eigenmodes (see Sect. 3). It introduces a new set of distortion parameters, $\mu$, which parametrize the shape of the residual distortion. We demonstrated that the eigenmodes can be used to simplify the analysis of future distortion data, providing a model-independent way to extract all useful information from the average CMB spectrum. Using this method we discussed annihilating and decaying particle scenarios, as well as energy release caused by the dissipation of small-scale acoustic modes (corresponding to wave numbers $1 \text{Mpc}^{-1} \lesssim k \lesssim 10 \text{Mpc}^{-1}$) for different experimental sensitivities. We showed that future CMB SD measurements will allow direct detection of s-wave annihilation signals if the annihilation efficiency is $\rho_{\text{ann}} \gtrsim 4.6 \times 10^{-7} \text{Mpc}^{-1}$ for different experimental sensitivities. These conclusions are in good agreement with those of Chluba & Sunyaev (2012). For decaying particle models with $2 \times 10^9 \text{sec} \lesssim t_x \lesssim 8.3 \times 10^{10} \text{sec}$ and total energy release $\Delta \rho_0/\rho_0 \approx 6.4 \times 10^{-7}$, the particle lifetime can be constrained to be better than $\lesssim 20\%$ and $c_0$ could be measured with uncertainty $\lesssim 10\%$ using a $\text{PIXIE}$-type experiment with 5 times its sensitivity (see Fig. 15 for details). These findings are in good agreement with those of Chluba (2013a), where direct MCMC simulations were performed. CMB SD are thus a powerful probe of early-universe particle physics, providing tight limits that are independent and complementary to those derived from light element abundances (e.g., Kawasaki et al. 2005; Kohri & Takahashi 2010; Pospelov & Pradler 2010) and the CMB anisotropies (Chen & Kamionkowski 2004; Zhang et al. 2007; Giesen et al. 2012).

Finally, we demonstrated how the eigenmode decomposition of the residual distortion can be used for direct model comparison. The dissipation caused by a quasi-scale-invariant power spectrum gives rise to a distortion signature that is degenerate with a s-wave distortion scenario (see also, Chluba & Sunyaev 2012; Chluba 2013a). However, a combination of future CMB anisotropy constraints with CMB SD measurements might provide the means to disentangle these cases. In particular, a detection of an annihilating particle signature with CMB SD measurements could be independently confirmed using CMB SDs. Furthermore, decaying particle scenarios have distortion eigenspectra that are distinct from the one caused by the dissipation of small-scale acoustic modes, if the power spectrum is neither too blue nor too red and does not show too much negative running (see Fig. 17). This again demonstrates the potential of future CMB distortion measurements and we look forward to extending our method to include more realistic instrumental effects and foregrounds. The principal component decomposition can furthermore be used to determine the optimal experimental settings for the detection of different SD signatures, another application we plan for the future.

ACKNOWLEDGMENTS

The authors specially thank Yacine Ali-Haïmoud and the referee for insightful comments and suggestions. JC furthermore thanks Kazunori Kohri and Josef Pradler for useful discussions about particle physics scenarios. The authors also thank Rishi Khatri and Rashid Sunyaev for comments on the manuscript, and Silvia Galli for providing simulated covariance matrices for PRISMI. Use of the GPC supercomputer at the SciNet HPC Consortium is acknowledged. SciNet is funded by: the Canada Foundation for Innovation under the auspices of Compute Canada; the Government of Ontario; Ontario Research Fund – Research Excellence; and the University of Toronto. This work was supported by DoE SC-0008108 and NASA NNX12AE86G.

APPENDIX A: ORTHOGONAL BASIS

To define the residual distortion, $R(z)$, that is perpendicular to the space spanned by $G_T$, $Y_{SZ}$ and $M$, we simply follow the Gram-Schmidt orthogonalization procedure. Aligning one axis with $Y_{SZ}$, this is given by the orthonormal basis $e_y = Y_{SZ}/|Y_{SZ}|$, $e_\mu = M_L/|M_L|$, and $e_T = G_{T,L}/|G_{T,L}|$, where $M_L = M_y M_M$, $G_T = G_y M_T$ and $G_{T,L} = G_y M_T G_{y,L}$. With $a = \sum b_i$, the required projections are $R = e_y \cdot M$, $G = e_x \cdot G_T$, and $G_L = e_x \cdot G_{T,L}$. Assuming $\text{PIXIE}$-type settings ($|Y_{\text{main} - \text{max}}|, \Delta \Lambda_{\text{y}} = [30, 1000, 15] \text{GHz}$), we find $|Y_{SZ}|, |M_L|, |G_{T,L}|\approx [73.3, 7.99, 21.4] \times 10^{-18} \text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$ and $\{M_y, G_y, G_{y,L}\} = [7.66, 16.8, 41.5] \times 10^{-18} \text{W m}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$. From Eq. (2) we furthermore obtain

\begin{align}
R(z) &= G_{y,0}(z) - G_T J_T(z)/4 - Y_{SZ} J_y(z)/4 - a M J_M(z) \\
J_T(z) &= 4 e_y \cdot G_y J_M(z)/|M_L| \\
J_y(z) &= 4 e_y \cdot G_y (z) - a M J_M(z) - G_T J_T(z)/4 |Y_{SZ}|
\end{align}

where we also introduced $J_M(z)$, which determines the amount of energy found in the residual distortion. All $J_i(z)$ are illustrated in Fig. [1].

REFERENCES

Abazajian K. N. et al., 2013, ArXiv:1309.5381
Achúcarro A., Gong J.-O., Hardeman S., Palma G. A., Patil S. P., 2011, JCAP, 1, 30
André P. et al., 2013, ArXiv:1310.1554
Barnaby N., 2010, Phys.Rev.D, 82, 106009
Barnaby N., Huang Z., 2009, Phys.Rev.D, 80, 126018
Barnaby N., Huang Z., Kofman L., Pogosyan D., 2009, Phys.Rev.D, 80, 043501
Barrow J. D., Coles P., 1991, MNRAS, 248, 52

© 0000 RAS, MNRAS 000, 000–000
