Grazing Collisions of Gravitational Shock Waves and Entropy Production in Heavy Ion Collision

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Abstract

AdS/CFT correspondence is now widely used for study of strongly coupled plasmas, such as produced in ultrarelativistic heavy ion collisions at RHIC. While properties of equilibrated plasma and small deviations from equilibrium are by now reasonably well understood, its initial formation and thermal equilibration is much more challenging issue which remains to be studied. In the dual gravity language, these problems are related to formation of bulk black holes, and trapped surfaces we study in this work is a way to estimate the properties (temperature and entropy) of such black hole. Extending the work by Gubser et al, we find numerically trapped surfaces for non-central collision of shock waves with different energies. We observe a critical impact parameter, beyond which the trapped surface does not exist: and we argue that there are experimental indications for similar critical impact parameter in real collisions. We also present a simple solvable example of shock wave collision: wall-on-wall collision. The applicability of this approach to heavy ion collision is critically discussed.

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1 Introduction

The Quark Gluon Plasma (QGP) produced in Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Lab is believed to be strongly coupled [1] as evidenced by its rapid equilibration, strong collective flows well reproduced by hydrodynamics, and strong jet quenching. Applications of AdS/CFT correspondence [2] to strongly coupled QGP has generated many fundamental results [7, 8, 9, 10], for some further results see [11] for a recent review. However this progress so far has been mostly related either to equilibrium properties of the plasma, or to kinetics/hydrodynamics close to equilibrium.

The challenging issues related with violent initial stage of the collisions, in which the QGP is formed and equilibrated, producing most of the entropy, are not yet understood. One of them worth mentioning at the beginning is the strikingly different views on equilibration held in statistical mechanics on one hand and in AdS/CFT-based dual gravity, on the other. Statistical/kinetic approaches treat equilibration and entropy production as some gradual deformation of particle distributions, from some initial non-thermal state toward the final equilibrated one. In the dual gravity setting the source of temperature and entropy are both attributed to the gravitational horizons. Those may or may not be produced in a collision: for example decreasing the collision energy or increasing the impact parameter one may reach a point at which no horizons are formed. This implies certain singularities, or a view that switching in equilibration is similar to a phase transition rather than a gradual deformation.

Formation of a black hole in a collision, which is then is falling toward the AdS center, was first considered in [18], with a spherical black hole. Janík and Peschanski [19] have proposed asymptotic (late-time) solution, corresponding to rapidity-independent (Bjorken) flow, see [20, 21] for most recent advances along this direction.

Grumiller and Romatschke [3] tried to describe the initial stage of high energy collisions, starting with the gravitational shock waves of certain type. In section 5 we will explore the formation of horizon in similar setup, but taking a different point of view: the image on the boundary must be due to the source in bulk. This will lead to a different and more consistent initial conditions, as well as subsequent evolution of matter.

A perturbative treatment of the initial conditions is discussed by Albacete, Kovchegov and Taliotis [5]. Other models of equilibration based on solutions to dynamical Einstein eqns include our previous work [22] in which a gravitationally collapsing shell of matter in AdS$_5$ space is considered. It sheds light on how formation of isotropic and homogeneous plasma may proceed through a very specific “quasiequilibrium” stage. We calculated the spectral
densities and found they deviate from their thermal counterpart by general oscillations. Another interesting solution describing isotropization of plasma was proposed by Chesler and Yaffe [23] recently.

The issue we will address in this work is formation of trapped surfaces and entropy production in the collision of two shock waves in AdS background. The work in this direction in AdS/CFT context had started with the paper by Gubser, Pufu and Yarom [4], who considered central collisions of bulk pointlike black holes. They had located the (marginally) trapped surface at the collision moment. Its area was then used as an estimate (a lower bound) of the entropy produced in heavy ion collision. In the limit of very large collision energy $E$ they found that the entropy grows as $E^{2/3}$. In section 6 we will critically discuss how realistic are these results.

In this work we extend their work in two directions. One is the obvious extension to collision of shock waves with nonzero impact parameter. We find interesting critical phenomenon, analogous to shock wave collision in Minkowski background [12, 13, 14]: beyond certain impact parameter, the trapped surface disappears and black hole formation does not happen. The other direction deals with a much simpler case of wall-on-wall collision, which was in a way overlooked before.

## 2 Shock Waves Collision and Trapped Surface

It is useful to review the main steps of [4] first. The AdS background can be written as:

$$ds^2 = L^2 \frac{-du dv + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2}$$  \hspace{1cm} (1)

where $u = t - x^3$ and $v = t + x^3$. $x^3$ is longitudinal coordinate and $x^1, x^2$ are transverse coordinates.

The shock wave moving in $+x^3$ direction is given by:

$$ds^2 = L^2 \frac{-du dv + (dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + L \frac{\Phi(x^1, x^2, z)}{z} \delta(u) du^2$$  \hspace{1cm} (2)

with $\Phi(x^1, x^2, z)$ satisfies the following equation:

$$\left( \Box - \frac{3}{L^2} \right) \Phi = 16\pi G_5 J_{uu}$$  \hspace{1cm} (3)
The 5-Dimensional source $J_{uu}$ can be arbitrary function in principle. □ is the Laplacian in the hyperbolic space $H_3$:

$$ds^2_{H_3} = L^2 \frac{(dx^1)^2 + (dx^2)^2 + dz^2}{z^2}$$  \hspace{1cm} (4)$$

The shock wave moving in $-x^3$ direction can be obtained by the substitution $u \leftrightarrow v$ to (2) and (3).

The marginally trapped surface is found from the condition of vanishing of expansion $\theta[15]$. The trapped surface is made up of two pieces: $S = S_1 \cup S_2$. $S_1(S_2)$ is associated with shock wave moving in $+x^3(-x^3)$ direction before collision. An additional condition is that the outer null normal to $S_1$ and $S_2$ must be continuous at the intersection $C = S_1 \cap S_2$ point $u = v = 0$ to avoid delta function in the expansion.

To find out $S_1$ associated with the first shock wave,

$$ds^2 = L^2 - dudv + \frac{(dx^1)^2 + (dx^2)^2 + dz^2}{z^2} + L \frac{\Phi_1(x^1, x^2, z)}{z} \delta(u) du^2$$  \hspace{1cm} (5)$$

the following coordinate transformation is made to eliminate the discontinuity in geodesics:

$$v \rightarrow v + \frac{\Phi_1}{z} \Theta(u)$$  \hspace{1cm} (6)$$

where $\Theta(u)$ is the Heaviside step function. $S_1$ is parametrized by:

$$u = 0, \ v = -\psi_1(x^1, x^2, z)$$  \hspace{1cm} (7)$$

The expansion is defined by $\theta = h^{\mu\nu} \nabla_\mu l_\nu$, with $l_\nu$ the outer null normal to $S_1$. $h^{\mu\nu}$ is the induced metric. It can be constructed from three spacelike unit vectors $w_1^\mu, w_2^\mu, w_3^\mu$, which are normal to $S_1$:

$$h^{\mu\nu} = w_1^\mu w_1^\nu + w_2^\mu w_2^\nu + w_3^\mu w_3^\nu$$  \hspace{1cm} (8)$$

The vanishing of expansion gives the equation:

$$\left(\Box - \frac{3}{L^2}\right)(\Psi_1 - \Phi_1) = 0$$  \hspace{1cm} (9)$$

4
with $\Psi_1(x^1, x^2, z) = \frac{L}{2} \psi_1(x^1, x^2, z)$.

The vanishing of expansion on $S_2$ associated with the second shock wave can be worked out similarly:

$$\left( \Box - \frac{3}{L^2} \right) (\Psi_2 - \Phi_2) = 0 \quad (10)$$

At the intersection $C = S_1 \cap S_2$, $S_1$ and $S_2$ coincide, therefore $\Psi_1(x^1, x^2, z) = \Psi_2(x^1, x^2, z) = 0$. The continuity of outer null normal can be guaranteed by $\nabla \Psi_1 \cdot \nabla \Psi_2 = 4$.

In summary, the aim of finding marginally trapped surface becomes the following unusual boundary value problem:

$$\left( \Box - \frac{3}{L^2} \right) (\Psi_1 - \Phi_1) = 0$$
$$\left( \Box - \frac{3}{L^2} \right) (\Psi_2 - \Phi_2) = 0$$
$$\Psi_1|_C = \Psi_2|_C = 0 \quad (11)$$

The boundary $C$ should be chosen to satisfy the constraint:

$$\nabla \Psi_1 \cdot \nabla \Psi_2|_C = 4 \quad (12)$$

Note (11) and (12) are written in the form of scalar equation, invariant under coordinate transformation. For central collision, the source $J_{uu}$ are identical for two shock waves. In [4], they are chosen to be

$$J_{uu} = E \delta(u) \delta(z - L) \delta(x^1) \delta(x^2) \quad (13)$$

The solution of $\Phi$ corresponds to this source give rises to the following stress tensor on the boundary field theory:

$$T_{uu} = \frac{L^2}{4\pi G_5} \lim_{z \to 0} \frac{\Phi(x^1, x^2, z)\delta(u)}{z^3} = \frac{2L^4 E}{\pi (L^2 + (x^1)^2 + (x^2)^2)^3} \delta(u) \quad (14)$$

The special source (13) preserves an $O(3)$ symmetry in $H_3$, which is manifest in the following coordinate system:
\[ ds^2_{H^3} = \frac{dr^2}{1 + r^2/L^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \]  

with the point source sitting at \( r = 0 \). We will elaborate the symmetry later in the context of non-central collision.

The \( O(3) \) symmetry helps to solve (11) analytically. The area of the trapped surface can be calculated and give a lower bound to the entropy produced in the collision of shock wave, assuming the area theorem holds in AdS background.

For non-central collision, the situation is complicated by the loss of \( O(3) \) symmetry. In Minkowski background, the problem of non-central collision of point shock waves in \( D = 4 \) was solved beautifully in [12] by conformal transformation. In \( D > 4 \), it was solved numerically in [13]. In all cases, a critical impact parameter was found, beyond which the trapped surface seized to exist.

In the next section, we will cast (11) into an integral equation, which allows us to solve (11) numerically.

### 3 Calculation of the Trapped Surface

Note (11) resembles the electrostatic problem in flat space, with \( \Psi \) being the electric potential. We are familiar with the fact that the electric potential can be expressed as an integral of surface charge density. We want to see if this can be achieved in AdS space.

Let us start with the electrostatic problem in flat space. Consider the following electrostatic problem, which is similar to (11):

\[
\begin{align*}
\nabla^2 \Psi_i(x) &= \nabla^2 \Phi_i(x) \quad (16) \\
\Psi_i(x)|_c &= 0 \quad (17) \\
\nabla \Psi_1 \cdot \nabla \Psi_2 &= 4 \quad (18)
\end{align*}
\]

where \( i = 1, 2 \), \( \nabla^2 \) is the Laplacian in flat space. \( \Psi_i \) is the electric potential corresponding to the source \( \nabla^2 \Phi_i \), placed inside an empty chamber with conducting boundary \( C \). The boundary should be chosen properly such that the constraint (18) is also satisfied.

We want to express the electric potential by an integral of the surface charge density. This can be done with the help of the free boundary Green’s function defined as the solution to:
\[ \nabla^2 G(x, x') = \delta^{(3)}(\vec{x} - \vec{x}') \] (19)

with the solution given by:

\[
G(x, x') = -\frac{1}{4\pi} \frac{1}{|\vec{x} - \vec{x}'|} \]

(20)

Take (16) multiplied by \( G(x, x') \) minus (19) multiplied by \( \Psi_i(x) \), and then integrate over the space inside \( \mathcal{C} \), we obtain:

\[
\int d^3x \left( G(x, x') \nabla^2 \Psi_i(x) - \Psi_i(x) \nabla^2 G(x, x') \right) = \int d^3xG(x, x')\nabla^2 \Phi_i(x) - \Psi_i(x') \]

(21)

\[
\int d\vec{S} \cdot \left( G(x, x') \vec{\nabla} \Psi_i(x) - \Psi_i(x) \vec{\nabla} G(x, x') \right) = \int d^3xG(x, x')\nabla^2 \Phi_i(x) - \Psi_i(x') \]

(22)

Denote \( B_i(x) = -\frac{\partial \Psi_i(x)}{\partial n} \) (the magnitude of electric field on the boundary) and note \( \Psi_i(x) \) vanishes on the boundary. With \( x' \) taken on the boundary \( \mathcal{C} \), (22) evaluates to:

\[
\int dSG(x, x')B_i(x) = \int d^3xG(x, x')\nabla^2 \Phi_i(x) \]

(23)

The constraint (18) is simply \( B_1(x)B_2(x) = 4 \). We have cast a problem in the volume into a problem on its boundary \( \mathcal{C} \). (23) is a Fredholm integral equation of the first kind.

We can use the following method to solve (16): Starting with some trial shape of \( \mathcal{C} \), we can solve (22) to obtain \( B_i(x) \) and check if (18) is satisfied. We can use iteration to tune the trial shape until (18) is satisfied.

Now we hope to apply similar method to the problem of trapped surface, the difference being the space is \( H_3 \) instead of flat.

As in case of electrostatic problem, we will keep using Green’s function in AdS, defined as the solution to the following:

\[
\left( \Box - \frac{3}{L^2} \right) G(x, x') = \frac{1}{\sqrt{g}} \delta^{(3)}(\vec{x} - \vec{x}') \]

(24)

where \( g \) is the metric of \( H_3 \).

The Green’s function was solved in [16, 17]. We quote the result here with \( L \) dependence restored.
\[ G(x, x') = -\frac{e^{2u}}{4\pi L \sinh u} \]
\[ \cosh u = 1 + \frac{(z - z')^2 + (\vec{x}_\perp - \vec{x}'_\perp)^2}{2zz'} \]  

where \( u \) is the invariant distance in \( H_3(\text{AdS}_3) \).

It also proves useful to note another relation:

\[
\int_M \Box f \sqrt{g} d^3x = \int_M \frac{1}{\sqrt{g}} \partial_\mu (\sqrt{g} g^{\mu\nu} \partial_\nu f) \sqrt{g} \frac{1}{2} \epsilon_{\sigma\rho\lambda} dx^\sigma \wedge dx^\rho \wedge dx^\lambda \]
\[
= \int_M d(\sqrt{g} g^{\mu\nu} \partial_\nu f \epsilon_{\mu\rho\lambda} \frac{1}{2} dx^\rho \wedge dx^\lambda) \]  

(26)

where \( \overline{dx} = g^{\mu\nu} \epsilon_{\mu\rho\lambda} \frac{1}{2} dx^\rho \wedge dx^\lambda \). \( M \) is taken to be the manifold in \( H_3 \) bounded by \( C \), the metric \( g \) refers to \( H_3 \). \( f \) is arbitrary function of \( x \).

With (26) and (25) at hand, we are ready to proceed:

\[
\begin{cases}
(\Box - \frac{3}{L^2}) \Phi_i(x) = (\Box - \frac{3}{L^2}) \Phi_i(x) \\
(\Box - \frac{3}{L^2}) G(x, x') = \frac{1}{\sqrt{g}} \delta^{(3)}(\vec{x} - \vec{x}')
\end{cases}
\]

(27)

with \( i = 1, 2 \). All the derivatives are with respect to \( x \). The first line of (27) multiplied by \( G(x, x') \) minus the second line of (27) multiplied by \( \Psi_i(x) \), then integrate over \( M \), we obtain:

\[
\int_M \left( G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \right) - \Psi_i(x) \left( \Box - \frac{3}{L^2} \right) G(x, x') \sqrt{g} d^3x = \int_M G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \sqrt{g} d^3x - \Psi_i(x') \]  

(28)

\[
\int_M \left( G(x, x') d \left( \partial_\nu \Psi_i(x) \overline{dx}' \right) - \Psi_i(x) d \left( \partial_\nu G(x, x') \overline{dx}' \right) \right) = \int_M G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \sqrt{g} d^3x - \Psi_i(x') \]  

(29)

\[
\int_M d \left( G(x, x') \partial_\nu \Psi_i(x) \overline{dx}' \right) - d \left( \Psi_i(x) \partial_\nu G(x, x') \overline{dx}' \right) = \int_M G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \sqrt{g} d^3x - \Psi_i(x') \]  

(30)

\[
\int_{\partial M} \left( G(x, x') \partial_\nu \Psi_i(x) \overline{dx}' - \Psi_i(x) \partial_\nu G(x, x') \overline{dx}' \right) = \int_M G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \sqrt{g} d^3x - \Psi_i(x') \]  

(31)
where in the last line we have used Stokes theorem on manifold $M$.

Putting $x'$ on $C$, we can simplify the above with $\Psi_i|_C = 0$:

$$\int_{\partial M} G(x, x') \partial_\nu \Psi_i(x) d\mathbf{x}' = \int_M G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \sqrt{g} d^3 x$$  \hspace{1cm} (32)

Furthermore, we have $\partial_\nu \psi dx^\nu = 0$ on $C$ since $\Psi_i|_C = 0$. On the other hand, $n_\nu dx^\nu|_C = 0$, where $n_\nu$ is the unit vector normal to the boundary $C$. Therefore, we may write:

$$\partial_\nu \Psi_i = -B_i n_\nu$$ \hspace{1cm} (33)

With the help of (33), (32) and (12) can be further simplified to:

$$-\int_{\partial M} G(x, x') B_i(x) dS = \int_M G(x, x') \left( \Box - \frac{3}{L^2} \right) \Phi_i(x) \sqrt{g} d^3 x$$ \hspace{1cm} (34)

$$B_1(x) B_2(x) = 4$$ \hspace{1cm} (35)

where $dS \equiv n_\mu dx^\mu$ is the area element.

Before proceeding to non-central collision, we would like to reproduce the 5-D result of [4] first. Working in spherical coordinates (15), the shape of $C$ is parametrized by $r = \rho_0 = \text{const}$. The simplest point shock wave corresponding to $J_{uu} = E\delta(u)\delta(z-L)\delta(x^1)\delta(x^2)$ is given by:

$$\Phi_1 = \Phi_2 = \frac{4G_5 E}{L} \frac{1 + 2(r/L)^2 - 2r/L \sqrt{1 + (r/L)^2}}{r/L}$$ \hspace{1cm} (36)

The Green’s function (25) is invariant under coordinate transformation. In spherical coordinate, it is given by:

$$G(x, x') = -\frac{1}{4\pi L} \frac{e^{2u}}{\sinh u} \cosh u = \sqrt{r'^2/L^2 + 1 - r'^2/L^2 - 1 - r/L} \left( \cos \theta \cos \theta' + \sin \theta \sin \theta' \cos (\phi - \phi') \right)$$ \hspace{1cm} (37)

In the presence of $O(3)$ symmetry, it is sufficient to show (34) holds for $\theta' = 0$, when the integral in $\phi$ is trivial. On the other hand, (35) implies $B_1 = B_2 = 2$. As a result, we only need to verify:

$$2\pi \int_0^\pi d\theta (-2) \left( \frac{\cosh u - \sinh u}{\sinh u} \right)^2 \rho_0^2 \sin \theta = \frac{(\sqrt{\rho_0^2 + 1} - \rho_0)^2}{\rho_0} (-4G_5 E) 4\pi$$

$$\cosh u = \rho_0^2 + 1 - \rho_0^2 \cos \theta$$ \hspace{1cm} (38)
It is not difficult to complete the integral in $\theta$, we finally arrive at $2G_5E = \sqrt{1 + \rho_0^2/L^2\rho_0^2}$, which is equivalent to (115) in [4].

4 Colliding Point Shock Waves at nonzero Impact Parameter

4.1 Shock Waves in Spherical Coordinate

Consider two shock waves with impact parameter $b$, given by:

$$
\left( \Box - \frac{3}{L^2} \right) \Psi_1 = -16\pi G_5 E \delta(u) \delta(z - z_0) \delta(x^1 - \frac{b}{2})\delta(x^2)
$$

$$
\left( \Box - \frac{3}{L^2} \right) \Psi_2 = -16\pi G_5 E \delta(u) \delta(z - z_0) \delta(x^1 + \frac{b}{2})\delta(x^2)
$$

The corresponding stress energy tensor associated with two shock waves are given by:

$$
T_{uu} = \frac{2L^4E}{\pi(z_0^2 + (x^1 - \frac{b}{2})^2 + (x^2)^2)^3} \delta(u)
$$

$$
T_{vv} = \frac{2L^4E}{\pi(z_0^2 + (x^1 + \frac{b}{2})^2 + (x^2)^2)^3} \delta(v)
$$

Therefore $z_0$ characterizes the size of the nucleus. We will use spherical coordinates in solving (34). In case of central collision, when $b = 0$. The shock wave center can be placed at the origin of spherical coordinates $r = 0$. This is achieved by first going to global coordinates $Y^i(i = 0, 1, 2, 3)$:

$$
Y^0 = \frac{z}{2} \left( k + \frac{L^2/k + kx^2}{z^2} \right)
$$

$$
Y^3 = \frac{z}{2} \left( -k + \frac{L^2/k - kx^2}{z^2} \right)
$$

$$
Y^1 = L \frac{x^1}{z}
$$

$$
Y^2 = L \frac{x^2}{z}
$$

The global coordinates link to spherical coordinates in the following way:

$$
Y^0 = \sqrt{r^2 + L^2}
$$

$$
Y^1 = r \cos \theta
$$

$$
Y^2 = r \sin \theta \cos \phi
$$

$$
Y^3 = r \sin \theta \sin \phi
$$
When $b = 0$, the center of the shock waves can be put at the origin if we set $k = \frac{L}{z_0}$. The possibility of moving any point to the origin reflects the maximally symmetric property of AdS space.

When $b \neq 0$, we want to place the two shock waves at opposite positions with respect to the origin, so that the boundary of trapped surface $C$ will have axial symmetry. Setting $1 + \frac{b^2}{4z_0} = \frac{L^2}{k^2 z_0}$, we have $Y^2 = Y^3 = 0$ and $Y^1 = \pm \frac{Lb}{2z_0}$. According to (41), we have the shock waves at $r = \frac{Lb}{2z_0}$, $\theta = 0$ and $\theta = \pi$. The differential equation in (34) becomes:

\[
\left( \Box - \frac{3}{L^2} \right) \Psi_i = -16\pi G_5 E \frac{L^3}{z_0^3} \frac{\sqrt{1 + r^2/L^2}}{r^2 \sin\theta} \delta(r - r_0) \delta(\theta - \theta_i) \delta(\phi) \tag{42}
\]

where $r_0 = \frac{Lb}{2z_0}$, $\theta_1 = 0$, $\theta_2 = \pi$. We observe that in spherical coordinate, the trapped surface only depends on $G_5 E \frac{L^3}{z_0^3}$ and $r_0$. Since AdS radius is a free parameter, which will not appear alone in the final result in dual field theory, we may set $z_0 = L$ without loss of generality. As a result we have $b = 2r_0$.

### 4.2 More General Shock Waves

Before proceeding to numerical study of trapped surface, we choose to take a moment to investigate the symmetries of the problem, which will help us to study more general shock waves. To see this, we prefer to work in the differential form of the problem: (11) and (12).

As we noticed before, (11) and (12) are scalar equations. $\Psi_i$ is a scalar. It is invariant under coordinate transformations: $x \to \tilde{x}$, $\Psi_i(x) \to \tilde{\Psi}_i(\tilde{x})$ the boundary remain the same $C \to \tilde{C}$, but takes a different functional form in new coordinate. As a result the third line of (11) and (12) are automatically satisfied. Suppose the transformation also preserves the form of the operator: $\Box - 3/L^2$, then $\tilde{\Psi}_i(\tilde{x})$ becomes another solution to (11) and (12). We will focus on transformations that leaves the center of the shock waves on the axis of $\theta = 0, \pi$.

To identify such a coordinate transformation, we first make a change of variable:

\[
r \sin \theta = t \\
r \cos \theta = \sqrt{L^2 + t^2 \sinh \eta}
\]

The metric of $H_3$ becomes:

\[
ds^2 = \frac{dt^2}{1 + t^2/L^2} + (L^2 + t^2) d\eta^2 + t^2 d\phi^2 \tag{43}
\]
The metric is $\eta$ independent, therefore the transformation: $\tilde{t} = t, \tilde{\phi} = \phi, \tilde{\eta} = \eta + \Delta \eta$ will not change the operator $\Box - 3/L^2$. $\tilde{t} = t$ also guarantees the center of the shock waves remain on the axis of $\theta = 0, \pi$. We have obtained the desired coordinate transformation, which is just a translation in $\eta$. It is easy to work out the corresponding transformation in spherical coordinate:

\[
\begin{align*}
\tilde{r} \sin \tilde{\theta} &= r \sin \theta = t \\
\tilde{r} \cos \tilde{\theta} &= \sqrt{L^2 + t^2} \sinh(\eta - \Delta \eta) \\
r \cos \theta &= \sqrt{L^2 + t^2} \sinh \eta
\end{align*}
\]  

One can verify explicitly (44) preserves the form of (15). (44) moves the center of the shock waves from $Y^2 = Y^3 = 0, Y^1 = \pm r_0$ to $Y^2 = Y^3 = 0, Y^1 = \pm r_0 \cosh \Delta \eta - \sqrt{L^2 + r_0^2} \sinh \Delta \eta$. This means collision of shock waves centered at $Y^2 = Y^3 = 0, Y^1 = \pm r_0 \cosh \Delta \eta - \sqrt{L^2 + r_0^2} \sinh \Delta \eta$ will generate the same entropy as those centered at $Y^2 = Y^3 = 0, Y^1 = \pm r_0$. This allows us to study the collision of more general shock waves. Let us consider the following shock waves:

\[
\begin{align*}
\left(\Box - \frac{3}{L^2}\right) \Psi_1 &= -16\pi G_5 E_u \delta(u) \delta(z - z_u) \delta(x^1 - x_u) \delta(x^2) \\
\left(\Box - \frac{3}{L^2}\right) \Psi_2 &= -16\pi G_5 E_v \delta(v) \delta(z - z_v) \delta(x^1 - x_v) \delta(x^2)
\end{align*}
\]  

In this paper, we restrict our interest in shock waves with identical invariant energy, defined by $E_u L^3 = E_v L^3 \equiv E$. This keeps the mirror symmetry of the problem intact. We will see the center of the shock waves can be placed at $Y^2 = Y^3 = 0, Y^1 = \pm r_0 \cosh \Delta \eta - \sqrt{L^2 + r_0^2} \sinh \Delta \eta$. This is equivalent to the statement that a solution can always be found to the following equations:

\[
\begin{align*}
L \frac{x_u}{z_u} &= r_0 \cosh \Delta \eta - \sqrt{L^2 + r_0^2} \sinh \Delta \eta \\
L \frac{x_v}{z_v} &= -r_0 \cosh \Delta \eta - \sqrt{L^2 + r_0^2} \sinh \Delta \eta \\
k^2 \left(1 + \frac{x_u^2}{z_u^2}\right) &= \frac{L^2}{z_u^2} \\
k^2 \left(1 + \frac{x_v^2}{z_v^2}\right) &= \frac{L^2}{z_v^2} \\
x_u - x_v &= \Delta x
\end{align*}
\]  

(46)
(46) can be solved easily by switching to the variable \( \eta_0 = \sinh^{-1} \frac{r_0}{L} \). A solution to (46) always exists for any given \( z_u, z_v \) and \( \Delta x \). We include the corresponding \( r_0 \) here, as it is the only relevant quantity, apart from \( E \), for entropy calculation:

\[
\frac{r_0}{L} = \sqrt{\frac{(z_u - z_v)^2 + \Delta x^2}{4z_uz_v}}
\]

(47)

In summary, we have shown that for shock waves (45) satisfying \( E_u \frac{L^3}{z_u^3} = E_v \frac{L^4}{z_v^4} \equiv E \), the entropy is only a function of \( G_5E \) and (47). Note \( r_0 \) is only a function of invariant distance between the center of shock waves.

### 4.3 Numerical Solution of Trapped Surface

With all the simplification, we are ready to find the trapped surface for different impact parameter. Our procedure is as follows: Axial symmetry allows us to parametrize \( C \) by \( r = \rho(\theta) \). Integral in \( \phi \) on the LHS of (34) can be expressed in terms of elliptic integrals. (34) becomes essentially 1-D integral equation. We discretize the integral by 199 points, equally spaced in the full range of \( \theta \). The integral on the LHS of (34) is discretized accordingly, and the integral on the RHS can be expressed in terms of elementary function due to the simple form of the shock wave. We use the same sample points for \( \theta' \), bringing (34) into a matrix form:

\[
\sum_j B(\theta_j)K(\theta_j, \theta'_i) = S(\theta'_i)
\]

(48)

where the indices \( i, j = 1, \ldots, 199 \). \( K(\theta_j, \theta'_i) \) contains the Green’s function and the induced metric. \( S(\theta'_i) \) is from the RHS integral of shock wave.

A special treatment is needed for diagonal matrix element of \( K(\theta_j, \theta'_i) \) where \( \theta_j = \theta'_i \). The explicit integrand expressed in terms of elliptic integrals shows that it is logarithmically divergent in \( |\theta - \theta'| \), yet the integral is convergent. The integral in this interval, represented by the diagonal matrix element, is estimated by sampling the integrand by certain number of points in the interval. The sample integrand are used to extract the coefficients of terms \( \ln |\theta - \theta'|, 1, (\theta - \theta')\ln |\theta - \theta'| \) and \( \theta - \theta' \) by method of least squares. Those coefficients are finally used for calculation of diagonal matrix elements.

The mirror symmetry of the two shock waves implies \( B_2(\theta) = B_1(\pi - \theta) \). Therefore it is sufficient to calculate one of them. Given a trial shape of trapped surface
Table 1: critical impact parameter at different energies

| \( \frac{G_s E}{L} \) | 0.1 | 0.5 | 1   | 4   | 9   | 12  | 15  | 50  | 100 |
|-----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \frac{b}{L} \) | 0.40 | 0.86 | 1.14 | 1.90 | 2.50 | 2.74 | 2.94 | 4.28 | 5.30 |

\( r = \rho(\theta) \), which is also necessarily symmetric under \( \theta \rightarrow \pi - \theta \), we can solve for \( B(\theta) \) from (48). We then evaluate \( \Delta(\theta) = B_1(\theta)B_2(\theta) - 4 \) and tune the shape function accordingly. We repeat the process until (35) is satisfied to certain accuracy. In order to assure fast convergence, we find it very helpful to calculate the gradient of \( \rho(\theta) \). The gradient is the matrix form of the functional derivative: \( \delta \frac{\Delta[\rho(\theta)]}{\delta \rho(\theta)} \). We parametrized \( \mathcal{C} \) by:

\[
\rho(\theta) = \sum_{n=1}^{M} a_n \cos(2(n-1)\theta),
\]

where \( M \) is a truncation number. The same decomposition applies to \( \Delta(\theta) \):

\[
\Delta(\theta) = \sum_{n=1}^{M} b_n \cos(2(n-1)\theta).
\]

The gradient in this representation is given by a \( M \times M \) matrix: \( \frac{\delta \rho(\theta)}{\delta a_n} \), which again contains elliptic integrals. For a given collision energy, we can find the boundary \( \mathcal{C} \) until certain critical impact parameter is reached. The critical impact parameter is located where \( \frac{\partial \rho(\theta)}{\partial b} \) diverges[13]. Empirically, the gradient \( \frac{\delta b_n}{\delta a_n} \) converges as \( \Delta(\theta) \) reduces in the iteration, if the impact parameter is within the critical value. The gradient diverges as \( \Delta(\theta) \) reduces in the iteration, if the impact parameter lie beyond the critical value.

Fig.1 shows the shapes of trapped surface at \( \frac{G_s E}{L} = 1 \) and \( \frac{G_s E}{L} = 100 \) for different impact parameters. The shapes are represented in spherical coordinate. We observe the critical trapped surface does not scale with collision energy in spherical coordinate. As collision energy grows, the trapped surface gets elongated in the axis of mismatch and larger \( M \) is needed to reach prescribed accuracy.

We also obtained several critical impact parameters corresponding to different energies. The results are listed in Table.1.

Fig.2 shows the log-log plot of critical impact parameter versus collision energy. It suggests a simple power law within the energy range used in the numerical study. The data are fitted with \( \frac{b}{L} = \alpha \left( \frac{G_s E}{L} \right)^{\beta} \) to give:

\[
\alpha = 1.07, \quad \beta = 0.37
\]

\( b \sim E^\beta L^{1-2\beta} \), the numerical value from fitting shows the critical impact parameter grows with collision energy and nucleus size.

The area of the trapped surface(twice the area of \( \mathcal{C} \)) sets a lower bound of the entropy produced, given as follows:
Figure 1: (left) The shapes of $C$ (the trapped surface at $u = v = 0$) at $\frac{G_5 E}{L^2} = 1$. The impact parameters used in the plot are $0.4L$, $0.6L$, $0.8L$, $1.0L$, $1.1L$, $1.14L$ from the outer to the inner. The innermost shape being the critical trapped surface. (right) The shapes of $C$ (the trapped surface at $u = v = 0$) at $\frac{G_5 E}{L^2} = 100$. The impact parameters used in the plot are $1.0L$, $2.0L$, $3.0L$, $4.0L$, $5.0L$, $5.3L$ from the outer to the inner. The innermost shape being the critical trapped surface. As collision energy grows, the trapped surface gets elongated in the axis of mismatch.

$$S_{trapped} = \frac{2A}{4G_5} = \frac{1}{2G_5} \int \sqrt{f} d^3x$$  \hspace{1cm} (50)$$

where $A$ is the area of the boundary $C$. The prefactor is

$$\frac{L^3}{G_5} = \frac{2N^2}{\pi}$$  \hspace{1cm} (51)$$

We plot the lower bound of entropy in the dual field theory for energy $\frac{G_5 E}{L^2} = 100$ in Fig.3

5 Wall-on-wall collisions

In this section we address a simpler form of the shock waves, called wall-on-wall in [18], in which there is no dependence on two transverse coordinates. Grumiller and Romatschke [3] have also discussed it, using gravitational shock waves. The problem with their approach is (power) growing amplitude of the shock as a function of holographic coordinate $z$. If so, collision dynamics resembles the atmospheric turbulence in the sense that the largest perturbation is at the largest $z$ – namely in the infrared modes – cascading down toward
higher momenta (UV). First of all this is physically different from the heavy ion collisions, in which case the initial wave function have partons well localized near the so called “saturation scale”, from which the equilibration domain (trapped surface) propagates both into small $z$ (UV) and large $z$ (IR) directions. Second, we think it is also inconsistent: a function growing with $z$ cannot be considered as a small perturbation to the background metric, which is decreasing at large $z$ as $1/z^2$. One may think that gravity near the AdS center $z = \infty$ should never be touched, as this is the original positions of the $D_3$ branes which gave the basis to AdS/CFT correspondence in the first place.

Our choice of the initial conditions, describing colliding walls with fixed parton density and thus fixed saturation scale is given by the following source

$$\left( \Box - \frac{3}{L^2} \right) \Phi(z) = -16\pi \frac{G_5 E}{L^2} \delta(z - z_0)$$

The corresponding solution to Einstein eqn subject to the boundary condition $\Psi(z) \to 0$ as $z \to 0$ is easily obtained:
\[ \Phi(z) = \begin{cases} 
4\pi G E \frac{z^3}{z_0} & z < z_0 \\
4\pi G E \frac{1}{z} & z > z_0 
\end{cases} \]  
(53)

Note that it decreases in both direction from the original scale \( z_0 \): therefore (as we will see shortly) the trapped surface has finite extensions in both directions from it.

The corresponding stress energy tensor on the boundary (as seen by an observer living in dual gauge theory) is

\[ T_{uu} = \frac{EL^2}{z_0} \delta(u) \]  
(54)

The stress energy tensor is the same as that used in [3, 5], and our solution converges well as \( z \to 0 \). We choose to collide states with different energy therefore we fix \( z_0 \), but use different \( E \). Applying the general discussion of shock wave in Sec.2 and noting the trapped surface only depends on \( z \), we obtain:

\[ z^2 \Psi_1'' - z \Psi_1' - 3\Psi_1 = -16\pi G E_1 \delta(z - z_0) \]  
(55)

\[ \Psi_1(z_a) = \Psi_1(z_b) = 0 \]  
(56)

\[ \Psi_2(z_a) = \Psi_2(z_b) = 0 \]  
(57)

with \( i = 1, 2 \). \( \Psi_1 \) and \( \Psi_2 \) are shape functions corresponding to two shock waves. The trapped region at \( u = v = 0 \) is limited by the interval \( z_a < z < z_b \). The constraint (35) takes a simple form:

\[ \Psi_1(z_a)\Psi_2(z_a) \frac{z_a^2}{L^2} = 4 \]

\[ \Psi_1(z_b)\Psi_2(z_b) \frac{z_b^2}{L^2} = 4 \]  
(58)

(55) is easily solved to give:

\[ \Psi_1(z) = \begin{cases} 
C \left( \left( \frac{z}{z_a} \right)^3 - \frac{z_a}{z} \right) & z < z_0 \\
D \left( \left( \frac{z}{z_b} \right)^3 - \frac{z_b}{z} \right) & z > z_0 
\end{cases} \]

\[ C = -4\pi G E_i \frac{\left( \frac{z_a}{z_b} \right)^3 - \frac{z_a}{z_b}}{\left( \frac{z_b}{z_a} \right)^3} \]
\[ D = -4\pi G_5 E_i \left( \frac{z_a}{z_0} \right)^3 \left( \frac{z_b}{z_0} \right)^3 + \frac{z_a}{z_0} - \left( \frac{z_b}{z_0} \right)^3 z_a \] (59)

Plugging (59) in (58), we obtain:

\[ z_a + z_b = \frac{8\pi G_5 \sqrt{E_1 E_2}}{L} \] (60)

\[ \frac{(z_a + z_b)^2 - 3z_a z_b}{(z_a z_b)^3} = \frac{L^3}{z_0^4} \] (61)

Note \( E_1, E_2 \) appear only in the combination \( \sqrt{E_1 E_2} \). This is consistent with the picture that only the center of mass contributes to the entropy. Recall the the center of mass of two massless particles with energy \( E_1, E_2 \) is \( 2\sqrt{E_1 E_2} \). The resulting cubic eqn (60) can be solved by Cardano formula, but the explicit solution is not illustrative and is not showed here. The entropy is given by:

\[ S = \frac{2A}{4G_5} = \int \sqrt{g} dz d^2 x_\perp \]

\[ s \equiv \frac{S}{\int d^2 x_\perp} = \frac{L^3}{4G_5} \left( \frac{1}{z_a^2} - \frac{1}{z_b^2} \right) \] (62)

The leading behavior of entropy per transverse area \( s \) in energy is extracted:

\[ s \sim \frac{4L^2}{z_0^4} (\pi G_5 \sqrt{E_1 E_2 z_0^2})^\frac{2}{3} \] (63)

The power 2/3 is the same as point shock wave obtained in [4]. There is also an obvious lower bound of the energy for the formation of trapped surface:

\[ 4\pi G_5 E \geq z_0 \] (64)

The equality is reached at \( z_a = z_b \), when the \( \mathcal{C} \) has vanishing volume. For general energy, \( s \) is evaluated as a function of effective colliding energy \( E = \sqrt{E_1 E_2} \). We again set \( z_0 = L \). Fig.4 shows the entropy as a function of effective colliding energy.

6 Matching heavy ion collisions to those of gravitational shock waves

As the reader who came to this point of the paper knows, it was up to this point a methodical work devoted to solving well-posed mathematical problems set in the AdS/CFT dual-gravity
Figure 4: The scaled entropy per transverse area \( \frac{2G_s s}{L^2} \) (the area of \( \mathcal{C} \) per transverse area) as a function of scaled effective colliding energy \( G_5 E/L^2 \), where \( \frac{L^3}{G_5} = \frac{2N_c^2}{\pi^2} \).

framework. However now, near the end of this work, we would like to address a wider issues of applicability limits of such approach, as well as the best strategy to use it for practical problems.

Gubser et al [4] have applied the gravitational collision scenario literally, selecting initial conditions at time long before nuclei collide. More specifically, they have (i) tuned the scale \( L \) or \( z_0 \) of the bulk colliding object to the size of the nucleus \( R \) and (ii) have used the realistic CM gamma factor of the colliding nuclei \( E/m = \gamma \sim 100 \). The result of such choice is a completely unrealistic fireball produced, in spite of a reasonable entropy. Indeed, the size of the trapped surface [4] is huge, about 300 fm, which is very large compared to colliding nuclei. In real heavy ion collisions the produced fireball has the same size as the nuclei, with the radius about 6 fm. The initial temperature – as estimated by \( z_{\text{min}} \sim 1/\pi T_i \) where \( z_{\text{min}} \) is the minimal distance of the trapped surface to the AdS boundary – is however way too high. So, what went wrong with this straightforward approach?

The answer to this question is in fact well known: initial formation of the partonic wave function, describing nuclei at the collision moment, can not be adequately described by the dual gravity. We know from experiment that growing partonic density makes hadrons and nuclei blacker and of larger size, as the collision energy grows. This is usually described by a Pomeron fit in which cross section \( \sim s^{\alpha(t)-1} \). Although qualitatively similar to what happens in gravitational collisions, this growth is very compared to that predicted in dual gravity. Indeed, the observed Pomeron intercept \( \alpha(t = 0) - 1 \sim 0.1 \) while in the AdS/CFT world the Pomeron intercept \( \alpha(t = 0) - 1 = 1[24] \). Thus the effective size of objects in gravitational collision grows with energy with an exponent ten times that in the real QCD. In view of this, one should clearly give up the idea to tune the scale \( L \) or \( z_0 \) of the bulk colliding
object to the size of the nucleus, and tune it perhaps to the parton density ("saturation scale" \(Q_s\) in the "color glass" models) of the corresponding nuclear wave function.

More generally, we are dealing with a complicated problem in QCD, in which the effective coupling runs, from higher scale to lower as the collisions progress from initial violent partonic stage toward equilibration, expansion and cooling. So in principle, it would be logic to switch – as smoothly as possible – from the weak-coupling based methods (such as classical Yang-Mills) to strong coupling ones (such as AdS/CFT) at certain proper time \(\tau_{\text{switch}}\) appropriately chosen by the evolution of the coupling \#1.

Therefore one should not try to tune the parameters of the gravitational collision model neither to initial nuclei, at \(\tau = -\infty\), nor to "decoherent" partons at the collision moment, at \(\tau = 0\), but at the later time \(\tau_{\text{switch}}\). #2 Although we at the moment do not understand the evolution of appropriate coupling quantitatively enough, one may always treat it as a parameter. The practical utility of the AdS/CFT approach at later time \(\tau > \tau_{\text{switch}}\) still remains significant: namely one can use much more fundamental dual gravity description instead of its near-equilibrium approximation, the hydrodynamics, currently used.

7 Are there critical impact parameters in heavy ion collisions?

Summarizing our findings in one sentence, those is the existence of the discontinuity in grazing gravitational collisions in the AdS space. As one smoothly increases the impact parameter \(b\), the trapped surface and black hole formation disappear suddenly, at certain critical impact parameter \(b_c(E)\) depending the collision energy \(E\). The reason for this seems quite general: increasing \(b\) one increases the angular momentum of the system while at the same time decrease the mass which can be stopped, and at some moment – as one knows from Kerr solution for rotating black holes – black hole formation becomes impossible.

Suppose the AdS gravitational shock waves can describe the strongly coupled plasma in heavy ion collisions: then one would expect similar behavior in heavy ion experiments. We have looked at the data and found that indeed there are experimental indications that relatively rapid switch of the underlying dynamics at some \(b_c(E)\) seem to exist.

#1 The so called AdS/QCD approach (see e.g.\cite{31, 30}) tries to incorporate the running coupling into the gravitational framework. A particularly simple example of that is a jump of the coupling at certain "domain wall" scale proposed in \cite{33}.

#2 It is proposed in \cite{5} that one may choose to collide some special unphysical shock waves
Figure 5: (left) PHOBOS data [25] on integrated number of charged particles, scaled by $N_{\text{part}}/2$, in p+p, d+Au, Cu+Cu and Au+Au collisions as a function of centrality. The uncertainty of Npart has been included in the error bars. (right) The height of the ridge as a function of the number of mean binary collisions per nucleon. The data are from STAR collaboration [27, 28] at two collision energies shown in the figure.

The most straightforward observable is entropy, related to the particle multiplicity versus the impact parameter. In Fig.5(left) from [25] we show some data plotted as a function of the number of participants $N_{\text{part}}$. The right end of the figure corresponds to all nucleons participating, or central collisions: toward the left end are peripheral collisions. There are indeed two values of multiplicity per participant observed, one for small systems, pp and dAu collisions (stars and crosses) and one for “large” systems, CuCu and AuAu (circles and squares). There must be a transition between them somewhere, but, unfortunately, the experimental multiplicity measurements for “grazing” collisions are not available yet #3. So, unfortunately, we do not yet know how exactly transition from one regime to another happens and what is $b_c(E)$, if it can be defined.

However some other observables associated with collective flows of excited matter do show rapid changes at certain $b_c(E)$ seems to be there. Some evidence for that were seen in the elliptic flow measurements, as deviations from the hydrodynamical predictions for very peripheral collisions. Even more clearly those are seen in the centrality dependence of the so called “ridge” phenomenon (see its relation to flow in [26, 28]) which we show in Fig.5(right).

Admittedly, these rapid change of the dynamics have not been systematically studied

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#3 Small multiplicity collisions are detected for all systems, but their accurate separation from beam-residual gas collisions has not yet been systematically resolved.
yet, neither experimentally nor theoretically. The naive explanation often given to it attribute the change to the fact that it happens when overlap system gets “too small” in terms of participating nucleons $N_p$, causing large enough fluctuations $O(1/\sqrt{N_p})$. However, if this would be the reason, one would expect this jump to be dependent on $N_p$ and independent on the collision energy. Furthermore, the gravitational collisions do not have any discrete elements at all, while predicting $b_c(E)$ growing with $E$, as observed in Fig.5. We therefore suggest that angular momentum may also be important: this issue clearly deserves to be studied further.

8 Conclusions

In this work we have developed a method to solve for the shape of the trapped surface based on an analogy to electrostatic problem in flat space: its main idea is to proceed from differential to integral form of the equation. We used the method to obtain the shape of trapped surface at different impact parameters and collision energy. We observe a critical impact parameter within the range of energy we explored. The phenomenon is analogous to the the critical behavior found in flat space[12, 13, 14], the difference being the critical trapped surface depends both on the collision energy and the nucleus size. We found the dependence is approximately given by a power law. Furthermore, the shape of the critical trapped surface gets elongated in spherical coordinate as the collision energy grows. We also discussed in the preceding subsection that grazing heavy ion collisions also seem to suggest a rapid switch to another dynamics, without equilibration. The exact cause of this jump is to be clarified in further studies.

We also studies wall-to-wall collision of shock waves as a simple version of the problem. The wall is sourced by a delta function at certain initial scale $z_0$. We believe it is more reasonable initial conditions than those used by Grumiller and Romatschke [3], to be used in future following their method to study the initial stage.

The applicability and limitation of this approach is discussed. We claim it is more realistic to adopt partonic picture in initial stage and only switch to effective gravity treatment at some time after collision, when the coupling becomes strong enough. However, we argue that the observed critical phenomenon is still relevant for heavy ion collisions, where there also seems to be rapid change of collision regime as a function of impact parameter.

Finally we would like to mention very recent work by Alvarez-Gaume et al[29] who discussed another extension of the problem. They considered central collision of shock
waves sourced by certain nontrivial matter distribution in the transverse space. They in particular discuss critical phenomenon occurring as the shock wave reaches some diluteness limit and the formation of the trapped surface is no longer possible. It would obviously be interesting to study how the two forms of critical phenomenon are related.

*Note added in version 2.* Horatiu Nastase had informed us about his early work [35] in which he also addressed non-central gravitational collisions, see also summary in [34]. His estimate of the critical impact parameter $b_c(E) \sim E^{1/6}$ at large $E$ is quite different from our power, perhaps because we are not working at asymptotically large $E$. We were also provided by (so far not posted) work by the Princeton group [36], who compared our numerical results for noncentral collisions with their analytical formulae and observed good agreement.

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**References**

[1] E.V.Shuryak, Prog. Part. Nucl. Phys. 53, 273 (2004) [ hep-ph/0312227]. E.V.Shuryak and I. Zahed, hep-ph/0307267, Phys. Rev. C 70, 021901 (2004) Phys. Rev. D69 (2004) 014011. [ hep-th/0308073].

[2] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200].

[3] D. Grumiller and P. Romatschke, JHEP 0808, 027 (2008) [arXiv:0803.3226 [hep-th]].

[4] S. S. Gubser, S. S. Pufu and A. Yarom, Phys. Rev. D 78, 066014 (2008) [arXiv:0805.1551 [hep-th]].

[5] J. L. Albacete, Y. V. Kovchegov and A. Taliotis, JHEP 0807, 100 (2008) [arXiv:0805.2927 [hep-th]].

[6] E.Witten, Adv.Theor.Math.Phys.2,235 (1998), hep-th/9802150.

[7] S.S.Gubser, I.R.Klebanov and A.A. Tseytlin, Nucl. Phys. B534 (1998) 202
G. Policastro, D. T. Son and A. O. Starinets, Phys. Rev. Lett. 87 (2001) 081601.

J. Casalderrey-Solana and D. Teaney, hep-ph/0605199.

S.-J. Sin and I. Zahed, Phys. Lett. B608 (2005) 265–273, hep-th/0407215.
H. Liu, K. Rajagopal, and U. A. Wiedemann, hep-ph/0605178.
C. P. Herzog, A. Karch, P. Kovtun, C. Kozcaz, and L. G. Yaffe, hep-th/0605158.
S. S. Gubser, A. Buchel, hep-th/0605178. hep-th/0605182.
S.-J. Sin and I. Zahed, hep-ph/0606049.

E. Shuryak, Prog. Part. Nucl. Phys. 62, 48 (2009) [arXiv:0807.3033 [hep-ph]].

D. M. Eardley and S. B. Giddings, Phys. Rev. D 66, 044011 (2002) [arXiv:gr-qc/0201034].

H. Yoshino and Y. Nambu, Phys. Rev. D 67, 024009 (2003) [arXiv:gr-qc/0209003].

E. Kohlprath and G. Veneziano, JHEP 0206, 057 (2002) [arXiv:gr-qc/0203093].

S. W. Hawking and R. Penrose, Proc. Roy. Soc. Lond. A 314, 529 (1970).

D. E. Berenstein, R. Corrado, W. Fischler and J. M. Maldacena, “The operator product
expansion for Wilson loops and surfaces in the large Phys. Rev. D 59, 105023 (1999)
arXiv:hep-th/9809188.

U. H. Danielsson, E. Keski-Vakkuri and M. Kruczenski, JHEP 9901, 002 (1999)
arXiv:hep-th/9812007.

E. Shuryak, S. J. Sin and I. Zahed, arXiv:hep-th/0511199.

R. A. Janik and R. Peschanski, Phys. Rev. D 73, 045013 (2006) arXiv:hep-th/0512162,
arXiv:hep-th/0606149.

S. Nakamura and S. J. Sin, arXiv:hep-th/0607123. Phys. Lett. B 608, 258 (2005)
arXiv:hep-th/0310031].

M. P. Heller, R. A. Janik and R. Peschanski, Acta Phys. Polon. B 39, 3183 (2008)
[arXiv:0811.3113 [hep-th]].

S. Lin and E. Shuryak, Phys. Rev. D 78, 125018 (2008) [arXiv:0808.0910 [hep-th]].

P. M. Chesler and L. G. Yaffe, arXiv:0812.2053 [hep-th].

24
[24] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88, 031601 (2002) [arXiv:hep-th/0109174].

[25] G. I. Veres et al. [PHOBOS Collaboration], arXiv:0806.2803 [nucl-ex].

[26] E. V. Shuryak, Phys. Rev. C 76, 047901 (2007) [arXiv:0706.3531 [nucl-th]].

[27] M. Daugherity (for the STAR coll.), Anomalous centrality variation..., QM08, J.Phys.G.Nucl/Part.Phys. 35 (2008) 104090

[28] A. Dumitru, F. Gelis, L. McLerran and R. Venugopalan, Nucl. Phys. A 810, 91 (2008) [arXiv:0804.3858 [hep-ph]].

[29] L. Alvarez-Gaume, C. Gomez, A. S. Vera, A. Tavanfar and M. A. Vazquez-Mozo, arXiv:0811.3969 [hep-th].

[30] U. Gursoy, E. Kiritsis and F. Nitti, JHEP 0802, 019 (2008) [arXiv:0707.1349 [hep-th]].

[31] U. Gursoy and E. Kiritsis, JHEP 0802, 032 (2008) [arXiv:0707.1324 [hep-th]].

[32] G. I. Veres et al. [PHOBOS Collaboration], arXiv:0806.2803 [nucl-ex].

[33] E. Shuryak, arXiv:0711.0004 [hep-ph].

[34] H. Nastase, Prog. Theor. Phys. Suppl. 174, 274 (2008) [arXiv:0805.3579 [hep-th]].

[35] K. Kang and H. Nastase, Phys. Rev. D 72, 106003 (2005) [arXiv:hep-th/0410173].

[36] S. S. Gubser, S. S. Pufu and A. Yarom, “Off-center collisions in AdS5 with applications to multiplicity estimates in heavy-ion collisions”