Anomalous Josephson effect in $d$-wave superconductor junctions on TI surface

Bo Lu$^1$, Keiji Yada$^1$, A. A. Golubov$^{2,3}$, Yukio Tanaka$^{1,3}$

$^1$ Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan
$^2$ Faculty of Science and Technology and MESA+ Institute for Nanotechnology, University of Twente, 7500 AE Enschede, The Netherlands
$^3$ Moscow Institute of Physics and Technology, Dolgoprudny, Moscow 141700, Russia

(Dated: July 9, 2015)

We study Josephson effect of $d$-wave superconductor (DS)/ferromagnet insulator(FI)/DS junctions on a surface of topological insulator (TI). We calculate Josephson current $I(\varphi)$ for various orientations of the junctions where $\varphi$ is the macroscopic phase difference between two DSs. In certain configurations, we find anomalous current-phase relation $I(\varphi) = -I(-\varphi + \pi)$ with $2\pi$ periodicity. In the case where the first order Josephson coupling is absent without magnetization in FI, $I(\varphi)$ can be proportional to $\cos \varphi$. The magnitude of the obtained Josephson current is enhanced due to the zero energy states on the edge of DS on TI. Even if we introduce an $s$-wave component of pair potential in DS, we can still expect the anomalous current-phase relation in asymmetric DS junctions with $I(\varphi = 0) \neq 0$. This can be used to probe the induced $d$-wave component of pair potential on TI surface in high-$T_c$ cuprate/TI hybrid structures.

PACS numbers: 74.45.+c, 74.50.+r, 74.20.Rp

I. INTRODUCTION

Josephson effect has been a fundamental and central topic in superconductivity and contributed to determine the pairing symmetry in unconventional superconductors (CPR) of Josephson current $I(\varphi)$ between two superconductors is $I(\varphi) \sim \sin \varphi$, where $\varphi$ is the macroscopic phase difference. For $d$-wave superconductor (DS) junctions, due to the presence of Andreev bound state at the interface, exotic quantum interference effects exist. One is the non-monotonic temperature dependence of maximum Josephson current (CPR) and second is the anomalous CPR. Due to the presence of ABS, $\sin 2\varphi$ component of $I(\varphi)$ is enhanced and free energy of the junction can locate neither $\varphi = 0$ nor $\pm \pi$. Furthermore, a pure $\sin 2\varphi$ CPR is possible for $d_{x^2-y^2}$-wave superconductor junction. Thus, $d$-wave junctions have really rich current phase relation and its functionalities worth for further research.

On the other hand, 3D topological insulators (TIs) is a material with a topologically protected surface state due to the strong spin-orbit coupling. The generation of superconductivity on the surface state of TI via proximity effect has been verified by the presence of supercurrent through the Josephson junctions on TI. Josephson current in superconductor/S/ferromagnet insulator(FI)/S junctions on TI stimulates us since anomalous CPR $I(\varphi) \sim \sin(\varphi - \varphi_0)$ discussed in conventional S/Ferromagnet (F)/S junction without TI can be realized easily. It is noted that $\varphi_0$ can be tunable by magnetization.

We can imagine that more dramatic features will be expected in $d$-wave superconductor(DS)/FI/DS Josephson junctions on TI. Although there have been several works about DS/FI/DS junctions, CPR has not been clarified for general orientations of the junctions. Recent experiments have shown that induced gap function on the surface of TI which is formed on high-$T_c$ cuprate is almost isotropic. This shows that the induced pair potential has predominant $s$-wave symmetry. The possibility of inducing $d$-wave pairing on TI surface in the actual experiment is still on debate now. Therefore, besides the CPR in $d$-wave superconductor junctions, we must study the CPR in junctions with $s+d$-wave symmetry for comparison with actual experiments.

In this paper, in order to calculate DC Josephson current, we develop a formalism of Green’s function of quasi-particles on the surface of TI. Both Josephson current and local density of states (LDOS) can be calculated for general orientations of junctions. In general, the obtained current phase relation $I(\varphi)$ has a complex $\varphi$ dependence. $I(\varphi) = -I(-\varphi)$ is easily to be broken by magnetization in FI and $I(\varphi)$ can not be simply expressed by $I_0 \sin(\varphi - \varphi_0)$ with nonzero $\varphi_0$. The extreme case is $d_{x^2-y^2}$/FI/$d_{xy}$/wave-waves, where CPR becomes $\sin 2\varphi$ without magnetization due to the absence of the first order Josephson coupling. If we switch on magnetization, exotic CPR becomes possible depending on the direction of magnetization in FI:

- i)mixture of $\cos \varphi$ and $\sin(2\varphi)$ terms with $I(\varphi) = -I(-\varphi + \pi)$ and
- ii)$\sin(2\varphi - 2\varphi_0)$. The complex CPR $I(\varphi) = -I(-\varphi + \pi)$ with $2\pi$ periodicity in i) is not realized in the preexisting high-$T_c$ cuprate junctions without TI. We also calculate Josephson current where $s$-wave and $d$-wave pair potentials mix. It is found that the anomalous CPR with $I(\varphi = 0) \neq 0$ exists for junctions of asymmetric orientations even if $s$-wave component becomes dominant. This feature serves as a guide to detect the proximity induced $d$-wave component of pair potential on the surface of TI.

The outline of this paper is as follows. In section II, we present the model and derive the retarded Green’s func-
II. MODEL AND FORMULAS

As depicted in Fig. 1(a), we consider a DS/FI/DS junction on a 3D TI surface. The effective Hamiltonian for the BdG equations is given by

$$\mathcal{H} = \begin{bmatrix} h(k_x, k_y) + M & i\hat{\sigma}_y \Delta(\theta) \\ -i\hat{\sigma}_y \Delta^*(\theta) & -h^*(-k_x, -k_y) - M^* \end{bmatrix},$$  

where $h(k_x, k_y) = h(v_f(k_x \hat{\sigma}_x - k_y \hat{\sigma}_y)) - \mu(\Theta(-x) + \Theta(x-L))$. $\hat{\sigma}_{x,y,z}$ is the Pauli matrix in the spin space and $\mu$ is the chemical potential in the superconducting region. The exchange field in FI region is $M = \sum_{i=x,y,z} m_i \hat{\sigma}_i \Theta(x) \Theta(L-x)$ [32]. The pair potential is given by $\Delta_0(T) \cos(2\theta - 2\chi_1) \Theta(-x) + \Delta_0(T) \cos(2\theta - 2\chi_2) e^{-iv_F \Theta(x-L)}$, where $\varphi$ and $\theta$ are the macroscopic superconducting phase and the propagating angle, respectively. The quantity $\chi$ is taken to be the angle between the x-axis and the a-axis of d-wave superconductor on top of the TI surface. $\Delta_0(T)$ is assumed to obey the BCS relation $\Delta_0(T) = \Delta_0 \tanh(1.74 \sqrt{T_c/T - 1})$ with $\Delta_0 = 1.76k_BT_c$ and $T_c$ is the critical temperature.

To construct the retarded Green’s function, we first seek the solutions for the four types of quasiparticle injection processes: left injection for electron (hole): $\psi_1(\psi_2)$ and right injection for electron (hole): $\psi_3(\psi_4)$. Because of the translational invariance along y-axis, the wave functions $\psi_1(2) = \psi_1(2)(x) e^{ik_x y}$ in the left superconducting region can be expressed as

$$\psi_1(x) = A_1 e^{ik_x x} + a_1 A_4 e^{-ik_x x} + b_1 A_3 e^{-ik_x x},$$  

$$\psi_2(x) = A_2 e^{-ik_x x} + a_2 A_3 e^{-ik_x x} + b_2 A_4 e^{ik_x x},$$

where $k_x = \mu \cos \theta / hv_F$. Here the magnitudes of the momenta for electrons and holes are approximated to be equal since we have made the assumption of $E, \Delta \ll \mu$. The spinors are given by $A_1 = (i, e^{i\theta}, -e^{i\theta} \gamma_1, i\gamma_1)^T$, $A_2 = (ie^{i\theta} \gamma_2, -\gamma_2, 1, ie^{i\theta})^T$, $A_3 = (ie^{i\theta} - 1, \gamma_2, ie^{i\theta} \gamma_2)^T$ and $A_4 = (\gamma_1, e^{i\theta} \gamma_1, -e^{i\theta}, i)^T$ with $\gamma_1(2) = \Delta_{1(2)}/(E + \sqrt{E^2 - \Delta^2_{1(2)}})$ and $\Delta_{1(2)} = \Delta_0 \cos(2\theta \mp 2\chi_1)$. Other wave functions can be solved in a similar way. The retarded green’s function $G_r(x, x'; y, y') = \sum_{k_y} G_{k_y}(x, x') e^{ik_y (y-y')}$ can be obtained by combing all the injection processes [40]:

$$G_{k_y}(x, x') = \begin{cases} \alpha_1 \psi_1(x) \psi_1^\dagger(x') + \alpha_2 \psi_2(x) \psi_2^\dagger(x') + \alpha_3 \psi_3(x) \psi_3^\dagger(x') + \alpha_4 \psi_4(x) \psi_4^\dagger(x'), \\
\beta_1 \psi_1(x) \psi_1^\dagger(x') + \beta_2 \psi_2(x) \psi_2^\dagger(x') + \beta_3 \psi_3(x) \psi_3^\dagger(x') + \beta_4 \psi_4(x) \psi_4^\dagger(x'), \\
(x > x'), \\
\end{cases}$$

where $\psi_1(2) = \psi_1(2)$. The coefficients $\alpha_{i=1-4}$ and $\beta_{i=1-4}$ are determined by satisfying the boundary conditions for all $x, x'$ across the regions:

$$G_{k_y}(x, x) - G_{k_y}(x, x') = v_f^{-1}(i \tau_z \hat{\sigma}_y),$$

where $\tau_{x,y,z}$ is the Pauli matrix in the particle-hole space. The dc Josephson current for DS/FI/DS junction is determined by electric charge conservation rule

$$\partial_t P + \partial_x I_x + S = 0,$$

where $P = e(\Psi^\dagger \Psi + \Psi \Psi^\dagger)$, $J_x = ev_F(\Psi^\dagger \Psi - \Psi \Psi^\dagger)$ and $S = 2e \int \partial \Delta \Psi \Psi^\dagger (\Psi^\dagger \Psi - \Delta \Psi^\dagger \Psi^\dagger)$ are electric charge density, electric current and source term, respectively. After straightforward derivation following Ref. [11], we find that the total Josephson current is given by

$$I_x = \frac{ek_BT}{2h} \sum_{k_y, \omega} \frac{\text{sgn}(\omega_n) [\Delta_1 a_1(\omega_n) - \Delta_2 a_2(\omega_n)] \sqrt{\omega_n^2 + \Delta_1^2}}{\sqrt{\omega_n^2 + \Delta_1^2}} \sqrt{\omega_n^2 + \Delta_2^2}.$$  

$a_{1(2)}(\omega_n)$ is obtained by analytical continuation $E$ to $i\omega_n$, where $\omega_n$ is the Matsubara frequency $\omega_n = \pi n k_BT(2n + 1)$, $n = 0, \pm 1, \pm 2, \ldots$. Eq. (4) looks similar to the extended Furusaki-Tsunada’s formula [41] for anisotropic d-wave pair potential [42]. In addition, Eq. (4) is also applicable to the Josephson current of s+d wave pairing in which one substitutes $\Delta_1(2)$ by $\Delta_0 + \eta \Delta_0 \cos(2\theta \mp 2\chi_1)$ of which $\eta \geq 0$ is the ratio between d-wave pairing and s-wave pairing.
III, JOSEPHSON EFFECT IN DS/FI/DS JUNCTION

In this section, we show the results of Josephson current $I$ in DS/FI/DS junctions, which has been normalized to $eR_N I/\Delta_0$ where $R_N$ is the interface resistance per unit area in the normal state. To analyze the CPR further, we decompose the Josephson current into a series of different orders of Josephson coupling,

$$I(\varphi) = \sum_n I_n \sin(n\varphi) + J_n \cos(n\varphi),$$

where $n \geq 1$ is an integer. Figure 2(a) shows CPR without magnetization. In this case, the CPR is expressed as $\sum_n I_n \sin(n\varphi)$ and $J_n$ is zero. In the condition with $\chi_1 = 0$ and $\chi_2 = \pi/4$, the CPR $I(\varphi)$ becomes $\sum_n I_n \sin(n\varphi)$ ($n = 2, 4, \ldots$). The feature of this CPR is the same as that in the standard $d$-wave junctions without TI with the pair potential considered here. However, as the magnetization switches on, the CPR dramatically changes. Figure 2(b) shows that $m_y$ gives a shift of phase difference, which is similar to $\varphi_0$-junctinos realized in conventional $s$-wave superconductor/ferromagnet hybrid systems [25 31]. As the magnetization along $m_x$ or $m_z$-axis appears, the qualitative features of CPR of symmetric $d_{x^2-y^2}$/$F_1$/$d_{x^2-y^2}$ and $d_{xy}$/$F_1$/$d_{xy}$ junctions do not change as compared to the case without magnetization as shown in dotted and dash-dotted lines of Figs. 2(c) and (d). However, in the asymmetric $d_{x^2-y^2}$/$F_1$/$d_{xy}$ junction, the CPR is quite anomalous and the component proportional to $\sum_n J_n \cos(n\varphi)$ is generated. We find that $I(\varphi)$ can be expressed by $\sum_k [2k \sin(2k\varphi) + J_{2k-1} \cos(2k-1\varphi)]$ where $k \geq 1$ is an integer, and therefore, $I(\varphi)$ becomes zero at $\varphi = \pm \pi/2$ as shown in solid lines in Figs. 2(c) and (d). The present CPR is completely different from that of the standard $d_{x^2-y^2}$/$F_1$/$d_{xy}$ junction without TI. We can see that the term proportional to $J_1 \cos(\varphi)$ becomes dominant in the limit of large $m_x$ or $m_z$ in Figs. 2(e) and (f).

To explain the anomalous CPR for nonzero $m_x$ or $m_z$ in $d_{x^2-y^2}$/$F_1$/$d_{xy}$ junction on TI surface, we focus on the symmetry of this Hamiltonian. We consider the mirror reflection symmetry with respect to $xz$-plane, $M_{xz} = i\sigma_y K_{T0}$, and the time-reversal symmetry, $T = -i\sigma_y K_{T0}$, where $K$ is the complex conjugation operator. In the present system, both symmetries are broken. However, since the pair potential of $d_{x^2-y^2}$ $(d_{xy})$ is mirror even (odd) with respect to $xz$-plane, $M_{xz}$ operation produces additional phase, $I(\varphi) \rightarrow I(\varphi + \pi)$. It is also known that time reversal operation transforms $I(\varphi)$ to $-I(\varphi)$. Hence, the composition operator $T = M_{xz} T$ will give rise to $I(\varphi) \rightarrow -I(-\varphi + \pi)$. Taking into account the fact that $T$ makes the $(k_x, k_y)$ state to the $(-k_x, k_y)$ one, we can arrive at

$$TH(-i\partial_z, k_y, \varphi)T^{-1} \rightarrow H(i\partial_z, k_y, -\varphi + \pi).$$

It means that $-I(-\varphi + \pi) = I(\varphi)$ will be satisfied at any $\varphi$ if we consider the junctions between a mirror even and mirror odd pair potential. In the $d_{x^2-y^2}$/$F_1$/$d_{xy}$ junction with $m_x$ or $m_z$, we can find that relation fulfills at any $\varphi$, which indicates $I(\varphi = \pm \pi/2) = 0$. Above analysis based on mirror reflection symmetry has been applied in the Josephson junctions between a singlet and triplet superconductor [14]. Now, let us look at the Josephson current at $\varphi = 0$. In the standard DS/FI/DS junctions without TI substrate, due to the spin SU(2) symmetry, the rotation or mirror reflection of the ferromagnetism does not change the CPR and one can always find $I(\varphi = 0) = 0$. However, this SU(2) symmetry is broken on TI surface due to its nature of spin-momentum locking and thus $I(\varphi = 0)$ becomes nonzero which generates exotic 2$\pi$-periodic CPR $-I(-\varphi + \pi) = I(\varphi)$.

Next, we plot the temperature dependence of the maximum Josephson current $I_c$ of DS/FI/DS junctions in the left panels of Fig. 3. For simplicity, only the $z$ component of magnetization $m_z$ is considered. We concentrate on the low temperature region.

![FIG. 2. Josephson currents as a function of $\varphi$ in the DS/FI/DS junctions. Magnetization in FI is (a) zero, (b) along $x$-axis, (c) along $y$-axis and (d) along $z$-axis. Three geometries are considered in panels (a)-(d): $d_{x^2-y^2}$/$F_1$/$d_{xy}$, $d_{x^2-y^2}$/$F_1$/$d_{x^2-y^2}$ and $d_{xy}$/$F_1$/$d_{xy}$. (e) Josephson currents in $d_{x^2-y^2}$/$F_1$/$d_{xy}$ junctions with $m_y/\mu = 0.2, 0.8$, and 1.5 and (f) those with $m_z/\mu = 0.2, 0.8$, and 1.5. Other parameters are set as $T = 0.05T_c$, $\mu = 1$, $h\nu = 1$, $\Delta = 0.01$ and $L = 1$.](image-url)
In this section, we calculate Josephson current in energy gap by high energy states. From Fig.3(a), we can see that the Josephson current changes from Kulik-Omelyanchuk (K-O) type to Ambegaokar-Baratoff (A-B) type with decreasing $\eta$. Since the spin degeneracy is lifted on the surface of TI, the obtained $I(\varphi)$ has a typical sinusoidal shape of s-wave Josephson current where the first order coupling $I_1 \sin \varphi$ plays the predominant role. Because of the s-wave component of pair potential, the mirror reflection symmetry $\hat{T}$ at $\varphi = \pm \pi/2$ is broken and thus nonzero current $I(\varphi = \pm \pi/2)$ can be expected. Also, in the presence of $m_x$ or $m_z$, we find a nonzero Josephson current at $\varphi = 0, \pi$ in such junctions. The obtained anomalous CPR in the s+d-wave Josephson junctions can be used to probe the d-wave component of the induced pair potential on TI surface. For example, one can observe the supercurrent flow without macroscopic phase difference in s+d-wave/FI/s+d-wave junctions. As seen in Fig.4 the existence of d-wave component generates a nonzero current $I(\varphi = 0)$ when one turns on either $m_x$ or $m_z$.

$T/T_c \leq 0.4$ in which the behavior of $I_c$ is highly influenced by the zero-energy states (ZESs). Therefore, we display the surface density of states at the edge of DS. It is obtained by calculating the LDOS $\rho(x,E) = -\frac{1}{\pi} \sum_{k_y} \text{Im}[G^{\tau}_{k_y,11}(x,x,E) + G^{\tau}_{k_y,22}(x,x,E)]$ at the DS/FI interface in the DS/FI/N junction as illustrated in Fig.3(b). From Fig.3(a), we can see that temperature dependence of $I_c$ in $d_{x^2-y^2}$/FI/d$_{x^2-y^2}$ junctions changes from Kulik-Omelyanchuk (K-O) type to Ambegaokar-Baratoff (A-B) type with decreasing $m_z$. However, in $d_{xy}$/FI/d$xy$ junction, this tendency is reversed when we decrease $m_z$, as shown in Fig.3(c). As seen from Figs.3(b) and (d), we can see that as LDOS at zero energy is enhanced, temperature dependence of $I_c$ is reduced to be the K-O type. In other case, it is in the A-B type. Since the spin degeneracy is lifted on the surface states of TI by spin momentum locking, we obtain the highly asymmetric Yu-Shiba-Rusinov type of LDOS in the $d_{x^2-y^2}$/FI interface. This finding is similar to that in the s-wave superconductor/FI/N junction.

**IV. JOSEPHSON EFFECT WITH s+d-WAVE PAIRING**

Recent experiments have shown that the induced energy gap by high $T_c$ cuprate on the surface of TI is almost isotropic. It is interesting to clarify the role of the induced s-wave pair potential on DS/FI/DS junctions. In this section, we calculate Josephson current in $s+d$-wave/FI/$s+d$-wave junctions on TI. The pair potential is $\Delta = \Delta_0 + \eta \Delta_0 \cos(2\theta - 2\chi)$ with $\chi = 0(\pi/4)$ on the left(right) side. The ratio $\eta$ is chosen to be 0.5 so that the system is s-wave dominant and fully gapped. The obtained $I(\varphi)$ has a typical sinusoidal shape of s-wave Josephson current where the first order coupling $I_1 \sin \varphi$ plays the predominant role. Because of the s-wave component of pair potential, the mirror reflection symmetry $\hat{T}$ at $\varphi = \pm \pi/2$ is broken and thus nonzero current $I(\varphi = \pm \pi/2)$ can be expected. Also, in the presence of $m_x$ or $m_z$, we find a nonzero Josephson current at $\varphi = 0, \pi$ in such junctions. The obtained anomalous CPR in the s+d-wave Josephson junctions can be used to probe the d-wave component of the induced pair potential on TI surface. For example, one can observe the supercurrent flow without macroscopic phase difference in s+d-wave/FI/s+d-wave junctions. As seen in Fig.4 the existence of d-wave component generates a nonzero current $I(\varphi = 0)$ when one turns on either $m_x$ or $m_z$.

**V. CONCLUSION**

In summary, we have theoretically studied the Josephson effect in d-wave superconductor-ferromagnet insulator (FI) hybrids on the surface of TI. Depending on the orientation of the magnetization in FI, the exotic current-phase relation which violates $I(\varphi) \neq -I(-\varphi)$ have been obtained in two different ways: (i) through a simple phase shift and (ii) mixture of cos $\varphi$ term into the original CPR. The latter case can generate the exotic current-phase relation $I(\varphi) = -I(-\varphi + \pi)$ with $2\pi$ periodicity. We show that the Josephson current is enhanced due to the zero energy states on the edge of d-wave superconductor. For comparison with actual experiments, we calculate the Josephson current when both s- and d-wave pair potentials exist. The anomalous current-phase relation is also found which provides a way to probe the fingerprint of d-wave pair potential in high-$T_c$ cuprate/TI heterostructures. Our preliminary theoretical investigation has practical significance for controlling the Josephson current and designing new functional devices.
ACKNOWLEDGEMENTS

We thank Y. Asano, M. Sato and P. Burset for valuable discussions. This work was supported in part by Grants-in-Aid for Scientific Research from the Ministry of Education, Culture, Sports, Science and Technology of Japan (Topological Quantum Material No.15H05853) and by the Ministry of Education and Science of the Russian Federation Grant No.14Y.26.31.0007.

[1] C. C. Tsuei and J. R. Kirtley, Rev. Mod. Phys. 72, 969 (2000).
[2] S. Kashiwaya and Y. Tanaka, Rep. Prog. Phys. 63, 1641 (2000).
[3] A. A. Golubov, M. Yu. Kupriyanov, and E. Il’ichev, Rev. Mod. Phys. 76, 411 (2004).
[4] C. R. Hu, Phys. Rev. Lett. 72 1526 (1994).
[5] S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima, and K. Kajimura, Phys. Rev. B 51 1350 (1995).
[6] Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. 74 3451 (1995).
[7] Y. Tanaka and S. Kashiwaya, Phys. Rev. B 53, R11957 (1996).
[8] Y. S. Barash, H. Burkhardt, and D. Rainer, Phys. Rev. Lett. 77,4070 (1996).
[9] Y. Tanaka and S. Kashiwaya, Phys. Rev. B 56,892 (1997).
[10] E. Il’ichev, M. Grajcar, R. Hlubina, R. P. J. Lisselstijn, H. E. Hoening, H.-G. Meyer, A. Golubov, M. H. S. Amin, A. M. Zagoskin, A. N. Omelyanchouk, and M. Yu. Kupriyanov, Phys. Rev. Lett. 86, 5369 (2001).
[11] G. Testa, E. Sarnelli, A. Monaco, E. Esposito, M. Ejnaes, D.-J. Kang, S. H. Mennema, E. J. Tarte, and M. G. Blamire, Phys. Rev. B 71, 134520 (2005).
[12] L. Fu and C. L. Kane, Phys. Rev. B 76, 045302 (2007).
[13] H. Zhang, C. X. Liu, X. L. Qi, X. Dai, Z. Fang, and S. C. Zhang, Nature Phys. 5, 438 (2009).
[14] D. Hsieh, D. Qian, L. Wray, Y. Xia, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature (London) 452, 970 (2008).
[15] D. Hsieh, Y. Xia, L. Wray, D. Qian, A. Pal, J. H. Dil, J. Osterwalder, F. Meier, G. Bihlmayer, C. L. Kane, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Science 323, 919 (2009).
[16] Y. Xia, D. Qian, D. Hsieh, L. Wray, A. Pal, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Nature Phys. 5, 398 (2009).
[17] Y. L. Chen, J. G. Analytis, J.-H. Chu, Z. K. Liu, S.-K. Mo, X. L. Qi, H. J. Zhang, D. H. Lu, X. Dai, Z. Fang, S. C. Zhang, I. R. Fisher, Z. Hussain, and Z.-X. Shen, Science 325, 178 (2009).
[18] D. Hsieh, Y. Xia, D. Qian, L. Wray, F. Meier, J. H. Dil, J. Osterwalder, L. Pattthay, A. V. Fedorov, H. Lin, A. Bansil, D. Grauer, Y. S. Hor, R. J. Cava, and M. Z. Hasan, Phys. Rev. Lett. 103, 146401 (2009).
[19] Z. Ren, A. A. Taskin, S. Sasaki, K. Segawa, and Y. Ando, Phys. Rev. B 82, 241306 (2010).
[20] B. Sacepe, J. B. Oostinga, J. Li, A. Ubaldini, N. J. G. Couto, E. Giannini, A. F. Morpurgo, Nat. Commun. 2, 575 (2011).
[21] D. M. Zhang, J. Wang, A. M. DaSilva, J. S. Lee, H. R. Gutierrez, M. H. W. Chan, J. Jain and N. Samarth, Phys. Rev. B 84, 165120 (2011).
[22] M. Veldhorst, M. Snelder, M. Hoek, T. Gang, V. K. Guduru, X. L. Wang, U. Zeitler, W. G. van der Wiel, A. A. Golubov, H. Hilgenkamp and A. Brinkman, Nature Mat. 11, 417 (2012).
[23] J. R. Williams, A. J. Bestwick, P. Gallagher, S. S. Hong, Y. Cui, A. S. Bleich, J. G. Analytis, I. R. Fisher, and D. Goldhaber-Gordon, Phys. Rev. Lett. 105, 056803 (2012).
[24] M. Snelder, C. G. Molenaar, Y. Pan, D. Wu, Y. K. Huang, A. de Visser, A. A. Golubov, W. G. van der Wiel, H. Hilgenkamp, M. S. Golden, A. Brinkman, Supercond. Sci. Technol. 27, 104001 (2014).
[25] R. Grein, M. Eschrig, G. Metalidis, and G. Schon, Phys. Rev. Lett. 102, 227005(2009).
[26] M. Eschrig, T. Lofwander, T. Champel, J. C. Cuevas, J. Kopu and G. Schon, J. Low Temp. Phys. 147, 457 (2007).
[27] V. Braude and Y. V. Nazarov, Phys. Rev. Lett. 98, 077003 (2007).
[28] Y. Asano, Y. Sawa, Y. Tanaka, and A. A. Golubov, Phys. Rev. B 76, 224525 (2007).
[29] M. Eschrig and T. Lofwander, Nat. Phys. 4, 138 (2008).
[30] A. Buzdin, Phys. Rev. Lett. 101, 107005 (2008).
[31] F. Konschelle and A. Buzdin, Phys. Rev. Lett. 102, 017001 (2009).
[32] Y. Tanaka, T. Yokoyama, and N. Nagaosa, Phys. Rev. Lett. 103, 107002 (2009). In this paper, the momentum spin locking term in Hamiltonian is chosen to be $h(k_x, k_y) = v_J(k_x \sigma_x + k_y \sigma_y)$ and magnetization along $x$-axis only shift the phase difference.
[33] J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbø, and N. Nagaosa, Phys. Rev. Lett. 104, 067001 (2010).
[34] J. Linder, Y. Tanaka, T. Yokoyama, A. Sudbo, and N. Nagaosa, Phys. Rev. B 81, 184525 (2010).
[35] P. Lucignano, A. Mezzacapo, F. Tafuri, and A. Tagliazucchi, Phys. Rev. B 86, 144513 (2012).
[36] E. Wang, H. Ding, A. V. Fedorov, W. Yao, Z. Li, Y.-F. Lv, K. Zhao, L.-G. Zhang, Z. Xu, J. Schneeloch, R. Zhong, S.-H. Ji, L. Wang, K. He, X. Ma, G. Gu, H. Yao, Q.-K. Xue, X. Chen, and S. Zhou, Nat. Phys. 9, 621 (2013).
[37] Z. X. Li, C. Chan, H. Yao, Phys. Rev. B 91, 235435 (2015).
[38] S. Y. Xu, C. Liu, A. Richardella, I. Belopolski, N. Alidoust, M. Neupane, G. Bian, N. Samarth, and M. Z. Hasan, Phys. Rev. B 90, 085128 (2014).
[39] K. Lee,A. Vaezi,M. H. Fischer, and E. A. Kim1, Phys. Rev. B 90, 214510 (2014).
[40] W. L. McMillan, Phys. Rev. 175, 559 (1968).
[41] A. Furusaki and M. Tsukada, Solid State Commun. 78, 299 (1991).
[42] Y. Tanaka and S. Kashiwaya, Phys. Rev. B 53,11957 (1996).
[43] C. K. Lu and S. K. Yip, Phys. Rev. B 80, 024504 (2009).
[44] In cuprate junctions where the geometry is the same as Fig.1(a), the orientation of ferromagnetism plays no role in CPR due to the spin SU(2) symmetry. Therefore in the case $\varphi = 0$, the system and its time reversal copy have the same Josephson current, which implies $I(\varphi = 0) = 0$.
[45] I. O. Kulik and A. N. Omelyanchuk, Fiz. Nizk. Temp. 4, 296 (1978)[Sov. J. Low Temp. Phys. 4, 142 (1978)].
[46] V. Ambegaokar and A. Baratoff, Phys. Rev. Lett. 10, 486 (1963); 11, 104 (1963).
[47] L. Yu, Acta Phys. Sin. 21, 75 (1965).
[48] H. Shiba, Prog. Theor. Phys. 40, 435 (1968).
[49] A. I. Rusinov, Sov. Phys. JETP Lett. 29, 1101 (1969).
[50] B. Lu, P. Burset, K. Yada, and Y. Tanaka, arXiv preprint, arXiv:1504.06208 (2015).