Nucleosynthesis and the Mass of the $\tau$ Neutrino—Erratum

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In a recent Letter [1] we presented the first numerical treatment of the full set of Boltzmann equations for the evolution of an MeV Majorana $\tau$ neutrino in the early Universe and concluded that mass limits from Big Bang nucleosynthesis were significantly weakened compared to previous investigations.

An error in our numerical code unfortunately invalidates the results. As noticed by several people [2] one should expect a strengthening rather than a weakening of earlier mass limits based on the integrated Boltzmann equation [3,4] because our inclusion of the full neutrino spectra should reduce the annihilation of high-momentum neutrinos, thereby permitting more $\tau$ neutrinos to survive. This is indeed the case as shown in the revised Fig. 1 [5]. Our neutrino distribution functions now differ less from kinetic equilibrium than found in [1]. A typical case for $\nu_{\tau}$ is shown in Fig. 2. Deviations by a factor of 2 appear frequently when compared to a kinetic equilibrium distribution, $f_{\nu} = \exp \left( \left( \frac{E - \mu}{T} \right) + 1 \right)^{-1}$, with the same number density and $T = T_{\gamma}$, as often assumed, whereas the distribution is close to that of a kinetic distribution with $T$ and $\mu$ determined by the number- and energy densities.

The resulting $\nu_e$ and $\nu_\mu$ distributions are still significantly heated relative to the standard case with a massless $\nu_{\tau}$. For eV-mass $\nu_\mu$ or $\nu_e$ this changes the present day contribution to the cosmic density to $\Omega_\nu h^2 = \alpha m_\nu/93.03\text{eV}$ with $\alpha = 1$ for a massless $\nu_{\tau}$, and $\alpha = 1.10(1.13)$, $1.09(1.14)$, $1.03(1.05)$, $1.01(1.02)$ for $\nu_e(\nu_\mu)$ for $m_{\nu_e(\nu_\mu)} = 5, 10, 15, 20 \text{ MeV}$ ($h$ is the Hubble-parameter in units of $100\text{ km s}^{-1}\text{ Mpc}^{-1}$). Later decay of $\nu_{\tau}$ can further increase the value of $\alpha$.

Fig. 3 illustrates the consequences for Big Bang nucleosynthesis in terms of the equivalent number of massless neutrinos, $N_{\text{eq}}$, needed to give a similar production of $^4\text{He}$ at a baryonto-photon ratio $\eta = 3 \times 10^{-10}$. The revised results with and without inclusion of the change
in the $\nu_e$ number density are in fine agreement with recent results based on the integrated Boltzmann equation by Fields, Kainulainen, and Olive [6]. Including also the actual shape of the $\nu_e$ distribution to some extent compensates for the effect on the energy density, a result also found by Dolgov, Pastor, and Valle [7,8], though we disagree by several equivalent neutrino species with the total $N_{eq}$ found in [4], and to some extent also with the differential changes in $N_{eq}$ quoted in [7].

In conclusion our revised results show that no MeV $\tau$-neutrino with mass below the experimental limit of 24 MeV is permitted unless more than 4 equivalent massless neutrino flavors become allowed by future observations of the primordial element abundances.
REFERENCES

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[2] We thank Sasha Dolgov, Georg Raffelt, and Mike Turner for useful discussions, and in particular Kimmo Kainulainen for comments that led us to find the error.

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[5] The limits are still weaker than those obtained without scattering reactions, as in M. Kawasaki et al., Nucl. Phys. B419, 105 (1994).

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[7] A. D. Dolgov, S. Pastor, and J. W. F. Valle, Report no. hep-ph/9602223 (1996).

[8] In the published version of Ref. [6] the same effect will be accounted for, with very good agreement with our results (Kimmo Kainulainen, private communication).
FIGURES

FIG. 1. The relic number density of massive $\tau$ neutrinos normalized to a massless species times mass, $rm_\nu$. The solid curve is calculated using the full Boltzmann equation and all interactions. Dotted and dashed curves are adopted from [3] and [4].

FIG. 2. The distribution of $\tau$ neutrinos of mass 15 MeV at a photon temperature of 0.92 MeV. The solid curve includes all possible interactions whereas the dashed-dotted curve only includes annihilations. The dotted curve is a kinetic equilibrium distribution with the same number density as that of the solid curve and $T = T_\gamma$, whereas the dashed curve has the same number- and energy densities as the solid, with $T$ and $\mu$ adjusted accordingly.

FIG. 3. Equivalent number of massless neutrinos, shown as $\Delta N_{eq} = N_{eq} - 3$. The dashed curve is without heating of $\nu_e$, the dotted with only the electron neutrino number density changed, and the solid curve with the full distribution of $\nu_e$. 
