A Simple Stability Analysis Method for a Period-1 Solution in a Forced Self-excited System with Stick-Slip Vibration

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Abstract

We propose a simple stability analysis method for a forced self-excited system with stick-slip vibration. First, we show the physical model. Next, we sample time date in switching events and define the Poincaré map. Then, by using the Jacobian matrix and the characteristic equation, we discuss the stability analysis algorithm. Finally, we numerically verify the effectiveness of our proposed method.

1. Introduction

Forced self-excited systems with stick-slip vibration are often observed in the mechanical field. Typical examples are dynamic absorbers [1], rail wheels [2], disk brakes [3] and seismic isolation systems [4]. In these systems, we observe various bifurcation phenomena upon varying the angular velocity of the external force. Many researchers have analyzed the stability in such systems in the past few [5, 6]. In particular, a stability analysis method via numerical analysis has been proposed [7]. However, its driving calculation is messy due to its complicated derivation process of the Jacobian matrix. Generally, we sample waveform data in every period of the external force to derive the Poincaré map. Then, we differentiate the Poincaré map with respect to the state variable. On the other hand, we consider that we can develop a simpler stability analysis method by sampling time data in switching events in forced self-excited systems with stick-slip vibration, because this approach enable the stability analysis to of a low-dimensional problem. However, there have been no reports of this kind of approach based on numerical analysis to our knowledge.

This paper addresses the first step to developing a simple stability analysis method for forced self-excited systems with stick-slip vibration by focusing on a local sections and a time. First, we give a physical model of the single-degree-of-freedom system in [7]. Next, we define the Poincaré map by focusing on the local sections and the time. Furthermore, we derive the Jacobian matrix and the characteristic equation used for stability analysis. Finally, we confirm that our method is effective for local bifurcations.

2. System and Its Behavior

Figure 1 shows the physical model of a forced self-excited system with stick-slip vibration, which has a single degree of freedom. The following notation is used: mass $m$, spring constant $k$, displacement of mass $u(t)$, gravity $g$, normal force $P$, excitation $F_0 \cos(\omega t)$, belt speed $V$, and relative velocity $V_r = V - \dot{u}$. The equation of motion is described as follows:

$$m\ddot{u}(t) - f_0(V_r) + ku(t) = F_0 \cos(\omega t)$$ (1)

During the slip mode ($V_r \neq 0$), the friction force can be determined via the friction coefficient $\mu(V_r)$. The friction force is described as follows:

$$f_0(V_r) = \mu(V_r)m \left(g + \frac{P}{m}\right)$$ (2)

![Figure 1: Physical model](image-url)
where $\mu(V_t)$ is given by

$$
\mu(V_t) = \begin{cases} 
\gamma_0 - \gamma_1 V_t + \gamma_3 V_t^3, & V_t > 0 \\
[-\gamma_0, \gamma_0], & V_t = 0 \\
-\gamma_0 - \gamma_1 V_t + \gamma_3 V_t^3, & V_t < 0 
\end{cases}
$$

where $\gamma_0$, $\gamma_1$ and $\gamma_3$ are positive constants. In particular, $\gamma_0$ is the maximum static friction force. We use the same dimensionless values as those in [7] as follows:

$$
\omega_n^2 = k/m, \quad \tau = \omega_n t, \quad x = x_0/V, \quad v = \omega/V,
$$

$$
F = F_0 \omega_n/(kV), \quad A_0 = \gamma_0 c/(V \omega_n), \quad A_1 = \gamma_1 c/\omega_n,
$$

$$
A_3 = \gamma_3 V^2 c/\omega_n, \quad V = 1 - x, \quad c = g + P/m
$$

We also define the initial value of the displacement and the velocity on each local section as follows:

$$
\begin{cases} 
x_0 = \varphi(0; \tau_0, x_0, \dot{x}_0, A) = A_0 + F \cos(\nu \tau_0) \\
\dot{x}_0 = \phi(0; \tau_0, x_0, \dot{x}_0, A) = 1
\end{cases}
$$

$\tau_0$ is the time when the stick mode is connected to the slip mode on the local section $\Pi_0$. $\tau_1$ is the time when the slip mode is connected to the stick mode on the local sections $\Pi_0$ and $\Pi_1$.

3. Stability Analysis Method

We use the Poincaré map for the stability analysis. The Poincaré map $T_i$ from the initial value to $\Pi_i$ can be described as follows:

$$
T_i : \quad \Pi_i \rightarrow \Pi_i
$$

$$
\tau_0 \mapsto \tau_i
$$

$$
x_0 \mapsto x_i = \varphi(\tau_i; \tau_0, x_0, \dot{x}_0, A)
$$

$$
\dot{x}_0 \mapsto \dot{x}_i = \dot{\varphi}(\tau_i; \tau_0, x_0, \dot{x}_0, A)
$$

where $\tau_i$ denotes the time when the state variable reaches the local section $\Pi_i$. Here, using the projection $h_i$:

$$
h_i : \Pi_i \rightarrow \Sigma_i = \{\tau, x \in \mathbb{R}^2\}, \quad \begin{bmatrix} \tau_i \\ x_i \end{bmatrix} \mapsto \begin{bmatrix} \tau_i \\ x_i \end{bmatrix}
$$

the Poincaré map $T_1$ on the local section $\Pi_1$ is described as follows:

$$
T_1 : \quad \Sigma_i \rightarrow \Pi_1
$$

$$
\tau_i \mapsto \tau_1
$$

$$
x_i \mapsto x_1 = \varphi(\tau_i; \tau_1, x_1, \dot{x}_1, A)
$$

where $\tau_1$ denotes the time when the state variable reaches the local section $\Pi_1$. We also use the projection $h_1$ and inverse $h_1^{-1}$:

$$
h_1 : \Pi_1 \rightarrow \Sigma_1 = \{\tau \in \mathbb{R}\}, \quad \begin{bmatrix} \tau_i \\ x_i \end{bmatrix} \mapsto X
$$

$$
h_1^{-1} : \Sigma_1 \rightarrow \Pi_1, \quad X \mapsto \begin{bmatrix} \tau_1 \\ x_1 \end{bmatrix}
$$

Consequently, the Poincaré map $T$ in the local coordinates can be defined as a one-dimensional mapping as follows:

$$
T : \quad \Sigma_1 \rightarrow \Sigma_1,
$$

$$
X_0 \mapsto X_1 = h_1 \circ T_1 \circ h_1 \circ T_1 \circ h_1^{-1}(X_0).
$$

Then, $X$ and $T(X)$ are given by

$$
X = \frac{v}{2\pi} \tau_0
$$

$$
T(X) = \frac{v}{2\pi} \tau_1 \mod 1
$$
Therefore, from Eq. (15), we can conclude that the proposed method has a simpler algorithm than that in [7]. The dynamics of the system can be explained by investigating the sequence $X_n$ obtained from the iteration

$$X_{n+1} = T(X_n), \quad X_n \in I$$

(17)

We calculate the derivative of $T$ to analyze the stability. Let $DT(X)$ be the differential coefficient of $T(X)$ on $X$. $DT(X)$ is calculated using the following relatively simple form:

$$DT(X) = \frac{\partial T(X)}{\partial X} = \frac{\partial \tau_1}{\partial \tau_0} \frac{dx_1}{d\tau_1} + \frac{dx_0}{d\tau_0}$$

(18)

We can analyze the stability of the fixed point by calculating $DT(X)$ using Eq. (18) with a numerical integration method.

4. Application Result

In this paper, we fix the parameters to $A_0 = 1.50$, $A_1 = 1.50$, $A_3 = 0.45$ and $F = 0.40$. Figure 3 shows the behavior of the system for various values of $\nu$. We detect several periodic solutions and non-periodic solutions in the system. For example, a period-1 solution bifurcates to a period-2 solution via period-doubling bifurcation (see Figs. 3(e) and 3(f)). Subsequently, the period-2 solution may undergo grazing bifurcation and a new period-1 solution is generated as shown in Fig. 3(d). Thus, it may be concluded that there are not only period-doubling bifurcation but also other types of bifurcations leading to non-periodic solutions. Because here we study a stability analysis method applicable to local bifurcation with a period-1 solution, we apply the proposed method to the period-doubling bifurcation observed in Figs. 3(e) and 3(f). Then, we calculate $DT(X)$ using Eq. (18). Table 1 shows the analytical results. Consequently, we conclude that the period-doubling bifurcation occurs at $\nu = 0.57778$. After that the behavior becomes unstable and a period-2 solution is generated. In Fig. 4, we show the one-parameter bifurcation diagram with a function of the parameter $\nu$ in order to confirm the validity of our proposed method. Similarly, the period-doubling bifurcation is observed at $\nu = 0.57778$ in this figure. From this result, the validity of our proposed method was confirmed.

5. Conclusion

We proposed a simple stability analysis method for a forced self-excited system with stick-slip vibration. First, we showed the physical model and then we defined the Poincaré map. On the basis of the Poincaré map, we derived the Jacobian matrix in a simple form. Finally, we analyzed a period-doubling bifurcation by using the proposed method and confirmed its validity. In future work, we will analyze other bifurcation phenomena with period-$m$ solutions by improving the proposed method. Furthermore, we consider that qualitative property of forced self-excited systems with stick-slip vibration is the same as that of the Rayleigh-type oscillators studied in [8-11]. We will confirm this hypothesis from both mathematical and experimental viewpoints.

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(a) Non-periodic solution ($\nu = 0.50$)

(b) Period-2 solution ($\nu = 0.515$)

(c) Non-periodic solution ($\nu = 0.52$)

(d) Period-1 solution ($\nu = 0.53$)

(e) Period-2 solution ($\nu = 0.57$)

(f) Period-1 solution ($\nu = 0.58$)

Figure 3: Numerical simulation of waveform

Figure 4: One-parameter bifurcation diagram

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