Grand Unification with Anomalous $U(1)$ Symmetry and Non-abelian Horizontal Symmetry

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Abstract

Non-abelian horizontal symmetry has been considered to solve potentially SUSY flavor problem, but simple models are suffering from various problems. In this talk, we point out that (anomalous) $U(1)_A$ gauge symmetry solves all the problems in a natural way, especially, in the $E_6$ grand unified theories. Combining the GUT scenario with anomalous $U(1)_A$ gauge symmetry, in which doublet-triplet splitting and natural gauge coupling unification are realized, and realistic quark and lepton mass matrices are obtained including bi-large neutrino mixings, complete $E_6 \times SU(3)_H$ (or $E_6 \times SU(2)_H$) GUTs can be obtained, in which all the three generation quarks and leptons are unified into a single multiplet $(27,3)$ (or two multiplets $(27,2+1)$). This talk is based on Ref. [1].
1 Problems of Simple Models with Horizontal Symmetry

First of all, we recall the basic features and the problems of non-abelian horizontal symmetry,[2, 3] considering a simple model with a horizontal symmetry $U(2)_H$, under which the three generations of quarks and leptons, $\Psi_i = (\Psi_a, \Psi_3)$ ($a = 1, 2$) ($\Psi = Q, U, D, L, E, N$), transform as $2 + 1$ and the Higgs fields $H$ and $\bar{H}$ are singlets. Such a horizontal symmetry is interesting because only the Yukawa couplings for the third generation are allowed by the horizontal symmetry, that accounts for the large top Yukawa coupling, and because the $U(2)_H$ symmetric interaction $\int d^4\theta \Psi^a \Psi^*_a Z^\dagger Z$, where $Z$ has a non-vanishing vacuum expectation value (VEV) given by $\langle Z \rangle \sim \theta^2 m$, leads to the equal first and second generation sfermion masses, which may realize suppression of flavor changing neutral currents (FCNC). However, the $U(2)_H$ symmetry must be broken to obtain realistic mass hierarchical structure of quarks and leptons. Therefore, we introduce a doublet Higgs $\bar{F}^a$ and an anti-symmetric tensor $A^{ab}$, whose VEVs $|\langle \bar{F}^a \rangle| = \delta^a_2 v$ and $\langle A^{ab} \rangle = \epsilon^{ab} v$ ($\epsilon^{12} = -\epsilon^{21} = 1$) break the horizontal symmetry as

$$U(2)_H \rightarrow U(1)_H \rightarrow \text{nothing}. \quad (1)$$

The hierarchical structure of the Yukawa couplings is obtained as

$$Y_{u,d,e} \sim \begin{pmatrix} 0 & \epsilon' & 0 \\ \epsilon' & 0 & \epsilon \\ 0 & \epsilon & 1 \end{pmatrix}, \quad (2)$$

where $\epsilon \equiv V/\Lambda \gg \epsilon' \equiv v/\Lambda$. However, these $U(2)_H$ breaking VEVs lift the degeneracy of the first and second generation sfermion masses as

$$\tilde{m}_{u,d,e}^2 \sim \tilde{m}_2^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 + \epsilon^2 & \epsilon \\ 0 & \epsilon & O(1) \end{pmatrix}, \quad (3)$$

which are calculated from higher dimensional interactions, like $\int d^4\theta (\Psi^a \bar{F}^a)^\dagger \Psi^b \bar{F}^b Z^\dagger Z$, through a non-vanishing VEV $\langle \bar{F} \rangle$. These mass matrices lead to the relations

$$\frac{\tilde{m}_{2}^2 - \tilde{m}_1^2}{\tilde{m}_2^2} \sim \frac{m_{F2}}{m_{F3}}, \quad (4)$$

where $m_{Fi}$ and $\tilde{m}_i$ are the masses of the $i$-th generation fermions and the $i$-th generation sfermions, respectively. Unfortunately, these relations of this simple model imply a problematic contribution to the $\epsilon_K$ parameter in $K$ meson mixing and the $\mu \rightarrow e\gamma$ process (problem 1). Moreover, this simple model gives the similar hierarchical Yukawa couplings for the up-quark sector, the down-quark
sector, and the lepton-sector, which are not consistent with experimental results (problem 2). In many cases of grand unified theories (GUTs), to realize the large neutrino mixing angles that have been reported in several recent experiments,[4, 5] the diagonalizing matrices for $\tilde{\Phi}$ fields of $SU(5)$, $V_l$ and $V_{dR}$, also have large mixing angles. In the cases, even if the horizontal symmetry $U(2)_H$ realizes the degeneracy of the first two generation down squarks such as

$$\Delta \tilde{m}_{dR}^2 \equiv \frac{\tilde{m}_{dR}^2 - \tilde{m}_{dL}^2}{\tilde{m}_{dL}^2} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \alpha \end{pmatrix},$$

(5)

where $\alpha$ is an $O(1)$ parameter, the mixing matrix defined [6] by

$$\delta_{dR} \equiv V_{dR}^\dagger \Delta \tilde{m}_{dR} V_{dR}, \quad \delta_{lL} \equiv V_{lL}^\dagger \Delta \tilde{m}_{lL} V_{lL}$$

(6)

have large components $(\delta_{dR})_{12}$ and $(\delta_{lL})_{12}$. It is not so natural to satisfy the constraints from $\epsilon_K$ in $K$ meson mixing

$$\sqrt{|\text{Im}(\delta_{lL})_{12}(\delta_{dR})_{12}|} \leq 2 \times 10^{-4} \left( \frac{\tilde{m}_q}{500\text{GeV}} \right)$$

(7)

$$|\text{Im}(\delta_{dR})_{12}| \leq 1.5 \times 10^{-3} \left( \frac{\tilde{m}_q}{500\text{GeV}} \right)$$

(8)

and from $\mu \to e\gamma$ process

$$|\langle \delta_{lL} \rangle_{12}| \leq 4 \times 10^{-3} \left( \frac{\tilde{m}_l}{100\text{GeV}} \right)^2,$$

(9)

at the weak scale (problem 3), even though $(\delta_{dR})_{12}$ can become fairly small.

In this talk, we show that anomalous $U(1)_A$ gauge symmetry,[7] whose anomaly is cancelled by Green-Schwarz mechanism,[8] provides a natural solution for all these problems. In a series of papers,[11, 12, 13, 14] we have emphasized that in solving various problems in GUT it is important that the VEVs are determined by the anomalous $U(1)_A$ charges as

$$\langle O_i \rangle \sim \begin{cases} \lambda^{-o_i} & o_i \leq 0 \\ 0 & o_i > 0 \end{cases},$$

(10)

where the $O_i$ are GUT gauge singlet operators with charges $o_i$, and $\lambda \equiv \langle \Theta \rangle / \Lambda \ll 1$. Here the Froggatt-Nielsen (FN)[9, 10] field $\Theta$ has an anomalous $U(1)_A$ charge of $-1$. (In this paper we choose $\Lambda \sim 2 \times 10^{16}$ GeV, which results from the natural gauge coupling unification,[14] and $\lambda \sim 0.22$.) Throughout this paper, we denote all superfields and chiral operators by uppercase letters and their anomalous $U(1)_A$ charges by the corresponding lowercase letters. When convenient, we use units in which $\Lambda = 1$. Such a vacuum structure is naturally obtained if we introduce generic interactions even for higher-dimensional operators and if
the $F$-flatness conditions determine the scale of the VEVs. And this vacuum structure plays an important role in realizing the doublet-triplet splitting\cite{11, 13} and natural gauge coupling unification\cite{14} and in avoiding unrealistic GUT relations between Yukawa matrices.\cite{11, 12} And in this talk, we stress that this vacuum structure plays an important role also in solving SUSY flavor problem with horizontal gauge symmetry.

2 $SU(5) \times SU(2)_H$

Let us explain the basic idea of a solution for the problem 1 and 2 with an $SU(5)$ GUT model with $SU(2)_H \times U(1)_A$, though problem 3 still remains in this model. The field content is given in Table 1.

Table 1, Typical values of anomalous $U(1)_A$ charges. The half integer charges play the same role as R-parity.

|         | $\Psi_a$ | $\Psi_3$ | $T_a$ | $T_3$ | $N_a$ | $N_3$ | $H$ | $H$ | $F_a$ | $F^a$ | $S$ | $\Theta$ |
|---------|----------|----------|-------|-------|-------|-------|-----|-----|-------|-------|-----|--------|
| $SU(5)$ | 10 10 5  | 5 1 1   | 5 1   | 1 1   |       |       | 5   | 5   | 1    | 1     | 1   | 1      |
| $SU(2)_H$ | 2 1 2 1 | 1 2 1   | 1 1   | 2 1   | 1     | 1     | 2   | 2   | 1     | 1     |     |
| $U(1)_A$ | $\frac{2}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ | $\frac{1}{3}$ $\frac{1}{3}$ |

The VEV relations (10) imply $\langle \tilde{F}F_{a} \rangle \sim \lambda^{-ij} (f+\bar{f})$ which breaks $SU(2)_H$. Actually, the $F$-flatness condition of $S$ with the superpotential $W_S = \lambda^s S(1 + \lambda^f f F F)$, leads to this VEV. Without loss of generality, we can take

$$|\langle \tilde{F}^a \rangle| = |\langle F_a \rangle| \sim \delta_{a2} \lambda^{-\frac{1}{2} (f+\bar{f})},$$

(1)

using the $SU(2)_H$ gauge symmetry and its $D$-flatness condition. Then, because the $SU(2)_H \times U(1)_A$ invariant operators become

$$\lambda^{\psi+f} \Psi_a \langle \tilde{F}^a \rangle \sim \lambda^{\psi+\Delta f} \Psi_2, \quad \lambda^{\psi+f} \epsilon^{ab} \Psi_a \langle F^b \rangle \sim \lambda^{\psi-\Delta f} \Psi_1,$$

(2)

where $\Delta f \equiv \frac{1}{2} (\bar{f} - f)$, it is obvious that with the effective charges defined as $\bar{x}_1 \equiv x_3, \bar{x}_2 \equiv x + \Delta f, \bar{x}_1 \equiv x - \Delta f$ for $x = \psi, t, n$, the Yukawa matrices of the quarks and leptons $Y_{u,d,e,\nu}$ and the right-handed neutrino mass matrix $M_{\nu R}$ can be obtained as

$$\begin{align*}
(Y_u)_{ij} &\sim \lambda^{\bar{\psi}_i + \psi_j + \bar{h}}, \quad (Y_d)_{ij} \sim (Y_e^T)_{ij} \sim \lambda^{\bar{\psi}_i + \psi_j + \bar{h}} \\
(Y_{\nu})_{ij} &\sim \lambda^{\bar{\psi}_i + \psi_j + \bar{h}}, \quad (M_{\nu R})_{ij} \sim \lambda^{\bar{\psi}_i + \psi_j}
\end{align*}$$

(3)

(4)

from the generic interactions $W_{\text{fermion}} = \tilde{\Psi}^2 \lambda^h H + \bar{\psi} \bar{T} \lambda^T \bar{H} + \bar{T} \bar{N} \lambda^h H + \bar{N} \bar{N}$, where $\bar{X} \equiv \lambda^{x+f} \epsilon_{ab} X_a F_b + \lambda^{x+f} X_a F^a + \lambda^{x+3} X_3$ for $X = \Psi, T, N$. Throughout this paper, we omit $O(1)$ coefficients for simplicity. Then, the neutrino mass matrix is obtained as

$$M_{\nu} = (Y_{\nu})(M_{\nu R})^{-1}(Y_{\nu}^T)\frac{\langle H \rangle^2}{\Lambda} \sim \lambda^{\bar{\psi}_i + \psi_j + 2h} \frac{\langle H \rangle^2}{\Lambda}.$$

(5)
Note that the effective charges $\tilde{x}_1$ can be different from $\tilde{x}_2$, though $x = x_1 = x_2$. When all the Yukawa couplings can be determined by their effective $U(1)_A$ charges, it has been understood not to be difficult to assign their charges to obtain realistic quark and lepton mass matrices.\cite{10, 11, 12} Thus the problem 2 can be solved in this scenario. It is an interesting point in theories in which Yukawa couplings are determined by $U(1)$ charges as in the above, that the unitary matrices $V_{yp}$ ($y = u, d, e, \nu$ and $P = L, R$) that diagonalize these Yukawa and mass matrices as $V_{yp}^\dagger Y_y V_{yp} = Y_y^{\text{diag}}$, the Cabibbo-Kobayashi-Maskawa matrix $V_{CKM} \equiv V_d l V_{sL}^\dagger$, and the Maki-Nakagawa-Sakata matrix $V_{MNS} \equiv V_{eL} V_{\nu_L}^\dagger$ are roughly approximated by the matrices $(V_{10})_{ij} \equiv \lambda_i\bar{\phi}_i + \lambda_j$ and $(V_{5})_{ij} \equiv \lambda_i - \lambda_j$, as $V_{10} \sim V_{uL} \sim V_{dL} \sim V_{uR} \sim V_{eR} \sim V_{CKM}$ and $V_5 \sim V_{dR} \sim V_{eL} \sim V_{\nuL} \sim V_{MNS}$.

To see how to solve the problem 1, we examine the sfermion mass-squared matrices

$$
\tilde{m}_{yf}^2 = \begin{pmatrix} \tilde{m}_{yL}^2 & A_y^\dagger \\ A_y & \tilde{m}_{yR}^2 \end{pmatrix}.
$$

(6)

In this paper, we concentrate on mass mixings through $\tilde{m}_{yf}^2$, because a reasonable assumption, for example SUSY breaking in the hidden sector, leads to an $A_y$ that is proportional to the Yukawa matrix $Y_y$.\cite{15} Roughly speaking, the sfermion mass squared matrix is given by $\tilde{m}_{yf}^2 \approx \tilde{m}^2 \text{diag}(1, 1, O(1))$, and the correction $\Delta_{yf} \equiv (\tilde{m}_{yf}^2 - \tilde{m}^2)/(\tilde{m}_{yf}^2)^2$ in the model described by Table I is approximately given by

$$
\Delta_{10} = \begin{pmatrix} \lambda_5^5 & \lambda_6 & \lambda_5^3 \\ \lambda_6 & \lambda_5^6 & \lambda_5^2 \\ \lambda_5^3 & \lambda_5^2 & \lambda_5 \end{pmatrix}, \Delta_5 = \begin{pmatrix} \lambda_5^5 & \lambda_6 & \lambda_5^3 \\ \lambda_6 & \lambda_5^6 & \lambda_5^4 \\ \lambda_5^3 & \lambda_5^4 & \lambda_5 \end{pmatrix}
$$

(7)

for 10 fields and 5 fields. Here $R_{10, 5} \sim O(1)$. For example, $(\Delta_5)_{12}$ can be derived using the interaction $\int d^4\theta \lambda^\dagger \bar{f}(TF)^\dagger (TF)Z^\dagger Z$. Note that $(\tilde{m}_{12}^2 - \tilde{m}_{31}^2)/\tilde{m}_{12}^2 \sim (m_s/m_b)^2$. The essential point here is that the hierarchy originated from the VEVs $(|F|) = |\langle F \rangle| \sim \lambda^{-2}(f + \bar{f})$ is almost cancelled by the enhancement factors $\lambda^f$ and $\lambda^\bar{f}$ in the superpotential, but not in the Kähler potential. Thus, the Yukawa hierarchy in the superpotential becomes milder, that improves the unrealistic relations (4).

Unfortunately, as discussed in the previous section, because the neutrino mixing angles are large and because $R_5 \sim O(1)$, the suppression of FCNC processes may not be sufficient (problem 3). The various FCNC processes constrain the mixing matrices defined by $\delta_{yp} \equiv V_{yp}^\dagger \Delta_{yp} V_{yp}$.\cite{6} In the model in Table I, the mixing matrices are approximated as

$$
\delta_{10} = \begin{pmatrix} \lambda_5^5 & \lambda_6 & \lambda_5^3 \\ \lambda_6 & \lambda_5^6 & \lambda_5^2 \\ \lambda_5^3 & \lambda_5^2 & \lambda_5 \end{pmatrix}, \delta_5 = R_{10} \begin{pmatrix} \lambda_5^3 & \lambda_6 & \lambda_5^3 \\ \lambda_6 & \lambda_5^4 & \lambda_5^5 \\ \lambda_5^3 & \lambda_5^4 & \lambda_5 \end{pmatrix}
$$

(8)

at the GUT scale. The constraints at the weak scale from $\epsilon_K$ in $K$ meson mixing (8) requires scalar quark masses larger than 1 TeV, because in this model
\[ \sqrt{|(\delta_{L})_{12}(\delta_{R})_{12}|} \sim \lambda^4(\eta_q)^{-1} \text{ and } |(\delta_{L})_{12}| \sim \lambda^2(\eta_q)^{-1}, \]

where we take a renormalization factor \( \eta_q \sim 6.1 \). And the constraint from the \( \mu \rightarrow e\gamma \) process (9) requires scalar lepton masses larger than 300 GeV, because \( |(\delta_{L})_{12}| \sim \lambda^2 \) in this model.

3  \( E_6 \times SU(2)_H \)

It is obvious that if all the three generation \( \bar{5} \) scalar fermions have degenerate masses, the problem 3 can be solved. In this section, we show that in \( E_6 \) GUT, such scalar fermion mass structure can be obtained in a natural way. First, note that under \( E_6 \supset SO(10) \supset SU(5) \), the fundamental representation \( 27 \) is divided as

\[ 27 \rightarrow 16[10 + \bar{5} + 1] + 10[\bar{5}' + 5] + 1[1]. \] (1)

To break \( E_6 \) into \( SU(5) \), two pairs of \( 27 \) and \( \bar{27} \) are introduced. (And an adjoint Higgs \( A(78) \) is needed to break \( SU(5) \) into the standard model gauge group, but here the Higgs does not play an important role and we do not address the Higgs.) The VEVs \( |\langle \Phi \rangle| = |\langle \bar{\Phi} \rangle| \sim \lambda^{-\frac{1}{2}}(\phi + \bar{\phi}) \) break \( E_6 \) into \( SO(10) \), which is broken into \( SU(5) \) by the VEVs \( |\langle C \rangle| = |\langle \bar{C} \rangle| \sim \lambda^{-\frac{1}{2}}(c + \bar{c}) \). And as matter fields, three fundamental representation fields \( \Psi_i(27) \) \( (i = 1, 2, 3) \) are introduced, which include \( 3 \times 5 + 6 \times \bar{5} \) of \( SU(5) \). Note that only three of the six \( \bar{5} \) become massless, which are determined by the \( 3 \times 6 \) mass matrix obtained from the interactions \( W = \lambda \psi_i \psi_j + \phi \psi_i \Psi_j \Phi + \lambda \psi_i \psi_j + c \Psi_i \Psi_j C \). It is essential that because the third generation fields have larger Yukawa couplings than the first and second generation fields \( (\psi_3 < \psi_1, \psi_2) \), the third generation fields \( \bar{5}_3 \) and \( \bar{5}'_3 \) have larger masses in the \( 3 \times 6 \) mass matrix than the first and second generation fields \( \bar{5}_a \) and \( \bar{5}'_a \) \( (a = 1, 2) \), respectively. Therefore, it is natural that these three massless \( \bar{5} \) fields come from the first and second generation fields, \( \Psi_1 \) and \( \Psi_2 \), as discussed in Ref. [12]. If the first two multiplets become the doublet \( \Psi(27, 2) \) under \( SU(2)_H \) in this \( E_6 \) GUT, then it is obvious that the sfermion masses for these three massless modes \( \bar{5} \) are equal at leading order, because the massless modes are originated from a single multiplet \( (27, 2) \). This sfermion mass structure is nothing but what is required to solve the problem 3.

Of course, the breakings of the \( E_6 \) and the horizontal symmetry \( SU(2)_H \) lift the degeneracy. To estimate the corrections, we fix a model. If we adopt the anomalous \( U(1)_A \) charges as \( (f, \bar{f}) = (-2, -3) \) and \( (\psi, \psi_3, \phi, \bar{\phi}, c, \bar{c}) = (5, 2, -4, 2, -5, -2) \) (noting that odd R-parity is required for the matter fields \( \Psi \) and \( \Psi_3 \)), the massless modes become \( (\bar{5}_1, \bar{5}_2, \bar{5}'_1 + \lambda \Delta \bar{5}_3), \) where \( \Delta = \psi_1 - \psi_3 + \frac{1}{2}(\phi - \bar{\phi} - c - \bar{c}) = 2. \) As discussed in Ref. [13], the massless mode \( \bar{5}'_1 + \lambda \Delta \bar{5}_3 \) has Yukawa couplings

\(^{1}\)The renormalization factor is strongly dependent on the ratio of the gaugino mass to the scalar fermion mass and the model below the GUT scale. If the model is MSSM and the ratio at the GUT scale is 1, then \( \eta_q = 6 \sim 7. \)
only through the mixing with $\tilde{5}_3$, because the $\tilde{5}'$ fields have no direct Yukawa couplings with the Higgs fields $H$ and $\tilde{H}$, which are included in $10_\Phi$ of $SO(10)$ in many cases. Then the structure of the quark and lepton Yukawa matrices becomes the same as that found in the previous $SU(5)$ model. As discussed above, all the sfermion masses for $\tilde{5}$ become equal at the leading order in this model. The correction to the sfermion masses $\delta \tilde{m}_\tilde{5}$ can be approximated from the higher dimensional interactions as

$$\frac{\delta \tilde{m}^2_\tilde{5}}{m^2} \sim \begin{pmatrix} \lambda^5 & \lambda^6 & \lambda^{5.5} \\ \lambda^6 & \lambda^5 & \lambda^{4.5} \\ \lambda^{5.5} & \lambda^{4.5} & \lambda^2 \end{pmatrix},$$

which leads to the same $\delta_5$ as that in Eq. (8) if we take $R_5 = \lambda^2$. This decreases the lower limit of the scalar quark mass to an acceptable level, 250 GeV. Note that $R_5$ is determined by the $E_6$ breaking scale, $\langle \Phi \Phi \rangle \sim \lambda^{-(\phi+\phi')}$, because the $E_6$ breaking VEVs lift the degeneracy through the interaction $\int d^4 \theta \bar{\Psi} \hat{\Psi}\Phi \Phi Z^\dagger Z$. In the model in Table 1, $R_5 = (\delta \tilde{m}^2_\tilde{5}/m^2)_{33} \sim \langle \Phi \Phi \rangle \sim \lambda^{-(\phi+\phi')} \sim \lambda^2$, but we can make various models with different $R_5$ by taking various charges $\phi + \phi'$.

It is surprising that in $E_6$ GUT with anomalous $U(1)_A$ symmetry the horizontal gauge group can be extended into $SU(3)_H$. In the models, the three generations of quarks and leptons can be unified into a single multiplet, $\Psi(27,3)$. Supposing that the horizontal gauge symmetry $SU(3)_H$ is broken by the VEVs of two pairs of Higgs fields $F_i(1,3)$ and $\bar{F}_i(1,\bar{3})$ ($i = 2,3$) as

$$| \langle F_{ia} \rangle | = | \langle \bar{F}_{ia} \rangle | \sim \delta^a \lambda^{-\frac{1}{2}(f_i + \bar{f}_i)},$$

the effective charges can be defined from the relations

$$\lambda^{\psi + f_i} \Psi_a \langle F_{ia} \rangle \sim \lambda^{\psi + \frac{1}{2}(f_i - \bar{f}_i)} \Psi_i (i = 2,3),$$

$$\lambda^{\psi + f_2 + f_3 + \epsilon} \epsilon^{abc} \Psi_a \langle F_{2b} F_{3c} \rangle \sim \lambda^{\psi - \frac{1}{2}(f_2 - f_3 - f_3 - f_3)} \Psi_1,$$

as

$$\tilde{\psi}_i \equiv \psi + \frac{1}{2}(f_i - \bar{f}_i), \quad \tilde{\psi}_1 \equiv \psi - \frac{1}{2}(f_2 - f_2 + f_3 - f_3).$$

Note that in order to realize $O(1)$ top Yukawa coupling, $SU(3)_H$ must be broken at the cutoff scale, namely, $f_3 + \bar{f}_3 = 0$. To obtain the same mass matrices of quarks and leptons as in the previous $E_6 \times SU(2)_H$ model, the effective charges must be taken as $(\tilde{\psi}_1, \tilde{\psi}_2, \tilde{\psi}_1) = (11/2, 9/2, 2)$. For example, a set of charges $(f_3, \bar{f}_3, f_2, \bar{f}_2) = (2, -2, -3, -2)$ and $\psi = 4$ is satisfied with the above conditions. [The model obtained by choosing $(f_3, \bar{f}_3, f_2, \bar{f}_2) = (2, -3, -4, -3), \psi = 13/2$, and $(\phi, \tilde{\phi}, c, \bar{c}) = (-7, 3, -8, 0)$ may be more interesting, because mass matrices for quarks and leptons that are essentially the same as those in Ref. [12] are obtained if we set $\lambda^{1.5} = 0.22$.]
For both models $E_6 \times SU(2)_H$ and $E_6 \times SU(3)_H$, if we add a Higgs sector that breaks $E_6$ into the gauge group of the standard model, as in Ref. [13], then we can obtain complete $E_6 \times SU(2)_H$ and $E_6 \times SU(3)_H$ GUT, in which the degeneracy of the sfermion masses for 5 fields is naturally obtained. As discussed in Refs. [12]–[14], these models yield not only realistic quark and lepton mass matrices but also doublet-triplet splitting and natural gauge coupling unification.

Because the $SU(3)_H$ symmetry must be broken at the cutoff scale to realize $O(1)$ top Yukawa coupling, the degeneracy of sfermion masses between the third generation fields $\Psi_3$ and the first and second generation fields $\Psi_a (a = 1, 2)$ is not guaranteed. Therefore, the $E_6 \times SU(3)_H$ GUT gives the same predictions for the structure of sfermion masses as $E_6 \times SU(2)_H$ GUT. Roughly speaking, all the sfermion fields have nearly equal masses, except the third generation fields included in $10$ of $SU(5)$. It must be an interesting subject to study the predictions on FCNC processes (for example, $B$-physics[16]) from such a special structure of sfermion masses. More precisely, this degeneracy is lifted by $D$-term contributions of $SU(3)_H$ and $E_6$. Though the contributions are strongly dependent on the concrete models for SUSY breaking and on GUT models and and some of them must be small in order to suppress the FCNC processes, it is important to test these GUT models with precisely measured masses of sfermions, as discussed in Ref. [17].

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