The Longitudinal Dynamic Characteristics of a V/STOL Aircraft Based on Deflected Slipstream during the Hovering Phase

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Abstract. Compared with general V/STOL (vertical/short taking-off and landing) aircrafts, the V/STOL aircraft based on deflected slipstream shows extremely different dynamic characteristics during the hovering phase. In this paper, a certain new type of V/STOL aircraft based on slipstream deflection is designed as research object. According to a simplified longitudinal dynamic model for the V/STOL aircraft based on deflected slipstream, it is proved that trim flap angle and centre of gravity position are two important factors during the hovering phase. Different flap angle means different trim state while the variation of centre of gravity position could change the range of trimming. Besides, modes of aircraft during the hovering phase are drawn and the law of eigenvalues changing as the variation of trim flap angle and center of gravity position is discovered. Regarding the findings as reference, the design of V/STOL based on deflected slipstream and control law design for V/STOL based on deflected slipstream during the hovering phase can be greatly improved.

1. Introduction

V/STOL (vertical/short taking-off and landing) aircraft is a kind of special vehicle which combines the advantages of helicopter and fixed-wing Aircraft [1, 2]. In recent years, plenty of developing design schemes for V/STOL aircraft are under researching. One of those schemes is the V/STOL aircraft based on deflected slipstream. Deflected slipstream via deflecting the flap angle forces propeller slipstream changing its flow direction, then changing direction of the aerodynamic resultant force on the wing eventually[2-5], as showed in figure 1. It is believed that NACA (U.S. National Advisory Committee on Aeronautics) conduct amount of early research works for STOL aircraft and VTOL aircraft based on deflected slipstream in 1950s[3]. However, limited to technical level that time, NACA’s early VTOL testing aircraft based on deflected slipstream couldn’t accomplish hovering and transition between hovering and horizontal flying stably. In recent years, a new type of V/STOL aircraft based on deflected slipstream appears[5, 6], which has advantages of simple structure, no variable-pitch/tilt mechanism and high manoeuvrability. This paper aims at deeply analysing the longitudinal dynamic characteristics of the new type of V/STOL aircraft based on deflected slipstream. In this paper, a certain new type aircraft based on deflected slipstream is designed as research object. According to a reasonable simplified mathematical model, its trim features and longitudinal dynamic characteristics around trim point during hovering phase are mainly and deeply discussed.
2. Research object

The research object of this paper is a certain type V/STOL aircraft based on deflected slipstream, as showed in figure 2, and its aerodynamic configuration is canard configuration with strake-wing. The two electromotive propellers installed at outboard of canard, called head propellers, could provide lift force and nose-up pitching moment, besides, the two head propellers could assist aircraft in realizing lateral motion at differential rotational speed. There are another four electromotive propellers, called main propellers, installed at the wing leading edge, overcoming most gravity in hovering phase or drag in horizontal flying phase. Double slotted flaps are symmetrically installed at the inboard of wings, cooperating with main propellers realizing deflected slipstream. During the hovering phase, double slotted flaps deflect at a certain angle, causing the result force vector of propeller-wing-flap (hereinafter referred to as PWF) combination directing upward, keeping balance with the pull force of head propellers and gravity, as in figure 3. Elevons are installed at the outboard of wings. During the hovering phase, elevons angle could continuously change, improving its lateral stability and controllability. The main parameters of the aircraft are showed in table 1.
Table 1. Main parameters of the aircraft

| Parameter                        | Value                   |
|----------------------------------|-------------------------|
| Takeoff Weight                   | 5.2 kg                  |
| Longitudinal Inertia             | 0.469 kg·m²             |
| Wing Area                        | 0.480 m²                |
| Wingspan                         | 1.6 m                   |
| Aerodynamic Chord Length         | 0.3 m                   |
| Canard span                      | 0.7 m                   |

3. Mathematical model

3.1. The mathematical model of PWF combination

This paper only discusses the dynamic characteristics during the hovering phase. It is believable that airspeed of the aircraft is approximately zero during the hovering phase, and the influence of incoming flow on aerodynamic force is negligible. Thus, only the effect of deflected slipstream is considered for setting up mathematical model of PWF combination.

Due to the effect of deflected slipstream, the result force on the PWF combination is nonlinear and strongly coupled[4]. For simplifying complex degree of setting up mathematical model, the result force on the PWF combination is resolved into pull of four main propellers, lift and drag which generated by propeller slipstream flowing through the wing. According to momentum theory of propeller, the relational expression between lift and pull force and the relational expression between drag and pull force could be derived, as in equation(s) (1) and equation(s) (2). The equation(s) (1) and equation(s) (2) is validated by elementary experiment on the ground. These equations could ensure enough accuracy when the flap angle is large enough.

\[
L = L_0 + T \frac{C_{L_{0}} S}{A} \alpha + T \frac{C_{L_{\delta}} S}{A} \delta
\]

\[
D = D_0 + T \frac{C_{D_{0}} S}{A} \alpha + T \frac{C_{D_{\delta}} S}{A} \delta
\]

In the equation(s) (1) and equation(s) (2), \( T \) represents the pull force of electromotive propeller, \( A \) represents rotor disk area, \( S \) represents wing area, \( \kappa \) represents loss coefficient, which is used to weigh the loss causing by flow tube of slipstream constricting, \( \alpha \) represents the angle between flow direction of slipstream and cord of wing and \( \delta \) represents the angle of flaps.

3.2. The mathematical model of electromotive propeller

The relationship between rotation speed of propeller and pull force is exceedingly complex. In this paper, the effect of its complex relationship on dynamics characteristics is not the main object of discussion. Thus, relational expression is simplified as equation(s)[7] (3).

\[
T = T_{\max} \bar{n}^2
\]

In equation (3), \( T_{\max} \) represents the maximal pull force of propeller, \( \bar{n} \) represents relative rotation speed, which is a ratio of real-time rotation speed and the maximal rotation speed.

3.3. The longitudinal dynamic model of the V/STOL aircraft

Refer to the same way of setting up dynamic model for general aircraft, the new type V/STOL aircraft based on deflected slipstream is considered as rigid body, and coordinate definition in this paper is same as in normal way[8, 9]. According to 3.1 and 3.2, the 3-DOF longitudinal dynamic equations could be derived. As in equation(s) (4)

\[
\begin{align*}
m(\ddot{u} + qw) &= 2(T_{\max} + T_{\max,o}) \bar{n}^2 - 2D - mg \sin \theta \\
m(\ddot{v} - qu) &= -2L - 2T_{\max} \cdot \bar{n}^2 + mg \cos \theta \\
l \cdot \ddot{q} &= 2T_{\max} \cdot \bar{n}^2 \cdot l_h - 2L \cdot l_{\text{pwf}}
\end{align*}
\]
The expressions of $L$ and $D$ are showed in equation(s) (5)

$$
L = L_0 + \kappa C_{L_a} \alpha (S_o T_{\text{max},o} + S_r T_{\text{max},r}) \bar{n}^2 A^{-1} + \kappa C_{L_{\delta}} (S_T T_{\text{max},i} + S_o T_{\text{max},o}) \bar{n}^2 \delta_i A^{-1}
$$

$$
D = D_0 + \kappa C_{D_a} \alpha (S_o T_{\text{max},o} + S_r T_{\text{max},r}) \bar{n}^2 A^{-1} + \kappa C_{D_{\delta}} (S_T T_{\text{max},i} + S_o T_{\text{max},o}) \bar{n}^2 \delta_i A^{-1}
$$

(5)

4. Results and analysis

4.1. Trim feature at different flap angles

According to longitudinal dynamic equations (4), a significant difference contrast with general aircraft is that there are four irrelevant variables $\theta$, $\bar{n}$, $\delta_i$ and $\bar{n}$, but only three mutually independent equations are available. It means that the new type V/STOL aircraft could trim at different situation. In order to analyze its trim feature, the flap angle is selected as a parameter variable which could artificially change from $0^\circ$ to $90^\circ$, and the trim equations of this aircraft could have the only determination solution at a certain flap angle. For different flap angles, the trim equations could get different solution. The relationship between flap angle and other variables is showed in figure 4.

![Figure 4](image1.png)

**Figure 4.** The schematic diagram of relationship between flap angle and other variables.

The relationship between flap angle and trim pitch angle is showed in figure 4. (a). From this figure, the trim pitch angle almost linearly descends with the increase of flap angle. The maximal trim pitch angle is near $90^\circ$ and the aircraft can hovering like a tail sitter VTOL aircraft at the maximal trim pitch angle. The minimal trim pitch angle is near $55^\circ$. The relationship between flap angle and relative rotation speed and flap angle is showed in figure 4. (b). According to this picture, as flap angle increasing, the relative rotation speed of main propeller increases slightly, from 0.78 to 0.81, while the rotation speed of head propeller increases more violently, from 0.08 to 0.42. It seems that as trim pitch angle descending, the head propeller takes more important role in overcoming gravity during hovering phase. And the reason of relative rotation speed of main propeller increasing might owe to the drag of flap increasing as its angle increasing.

4.2. Trim feature at different positions of center of gravity

The relationship between the position of center of gravity and the position of aerodynamic focus is crucial for fixed-wing airplane and V/STOL aircraft in horizontal flying phase. It is worth discussing that the effect of center of gravity position changing on the new type V/STOL based on deflected slipstream. The result is showed in figure 5 and figure 6.
Figure 5. The schematic diagram of relationship between flap angle and pitch angle at different c.g..

Figure 6. The schematic diagram of relationship between flap angle and relative rotation speed of main propeller and head propeller at different c.g.

In the figure 5 and figure 6, c.g. represents the ratio between the length of position of center of gravity to action point of result force of PWF combination and the length of aircraft nose to action point of result force of PWF combination. For being limited by the maximal pull force of head propeller, the shadow area in the figure 5 and figure 6 represent areas that cannot be trimmed. It is obvious that increasing maximal pull force of head propeller could reduce shaded area in the figure 5 and figure 6. The smaller shaded area means the wider range for trimming in hovering phase.

From figure 5, it indicates that as c.g. increasing, position of center of gravity moving ahead, the range of trim pitch angle is wider. At c.g.=0.4, the trim pitch angle is range from approximate 42° to 90°. However, according to figure 6 (b) as c.g. increasing, the load on the head propeller seems heavier.

4.3. Dynamic characteristics at different flap angles
In order to analyze the dynamic characteristics of the V/STOL aircraft during the hovering phase, it is necessary to linearize the longitudinal dynamic equations around trim point. The result of linearization of equation(s) (4) is equation(s) (6).

\[
\begin{align*}
\dot{\mu} &= 4(T_{\text{max},o} + T_{\text{max},i})\bar{n}\Delta\bar{n} - 2\frac{\partial D}{\partial \delta_i} \Delta\delta_i - 4\frac{D}{V_2}u - 2\frac{\partial D}{\partial \alpha'} \frac{w}{V_2} - 2\frac{\partial D}{\partial \bar{n}} \Delta\bar{n} - 2\frac{\partial D}{\partial \bar{q}} \Delta\bar{q} - mg \cos(\theta) \Delta \theta \\
\dot{\bar{w}} &= -2\frac{\partial L}{\partial \delta_i} \Delta\delta_i - 4\frac{L}{V_2}u + 2\frac{\partial L}{\partial \delta_i} \frac{w}{V_2} - 2\frac{\partial L}{\partial \bar{n}} \Delta\bar{n} - 2\frac{\partial L}{\partial \bar{q}} \Delta\bar{q} + 4T_{\text{max},o} \bar{n}_o \Delta\bar{n}_o - mg \sin(\theta) \Delta \theta \\
I_\gamma \dot{\gamma} &= 4T_{\text{max},o} \bar{n}_o \Delta\bar{n}_o - l_{\text{pot}} (2\frac{\partial L}{\partial \delta_i} \Delta\delta_i + 4\frac{L}{V_2}u + 2\frac{\partial L}{\partial \delta_i} \frac{w}{V_2} + 2\frac{\partial L}{\partial \bar{n}} \Delta\bar{n} + 2\frac{\partial L}{\partial \bar{q}} \Delta\bar{q})
\end{align*}
\]

In equation(s) (6), \(\Delta\bar{n}, \Delta\bar{h}_o\) and \(\Delta\delta_i\) are control variables, and \(u, w, \theta\) and \(q\) are state variables. According to the trim result of 4.1, the eigenvalues could be calculated. The result is showed in table 2.

**Table 2.** Eigenvalues at different flap angles.

| flap angle (°) | Eigenvalue \(\lambda_1\) | Eigenvalue \(\lambda_2\) | Eigenvalue \(\lambda_3\) | Eigenvalue \(\lambda_4\) |
|---------------|--------------------------|--------------------------|--------------------------|--------------------------|
| 0             | -1.490 + 0.7845i         | -1.490 - 0.7845i         | 0.0111                   | 0.9514                   |
| 5             | -1.515 + 0.7596i         | -1.515 - 0.7596i         | 0.0072                   | 0.9480                   |
| 10            | -1.541 + 0.7329i         | -1.541 - 0.7329i         | 0.0003                   | 0.9448                   |
| 15            | -1.567 + 0.7042i         | -1.567 - 0.7042i         | -0.0007                  | 0.9419                   |
| 20            | -1.594 + 0.6732i         | -1.594 - 0.6732i         | -0.0046                  | 0.9392                   |
| 25            | -1.620 + 0.6397i         | -1.620 - 0.6397i         | -0.0086                  | 0.9368                   |
| 30            | -1.647 + 0.6033i         | -1.647 - 0.6033i         | -0.0126                  | 0.9345                   |
| 35            | -1.673 + 0.5635i         | -1.673 - 0.5635i         | -0.0165                  | 0.9325                   |
| 40            | -1.700 + 0.5196i         | -1.700 - 0.5196i         | -0.0204                  | 0.9305                   |
| 45            | -1.726 + 0.4707i         | -1.726 - 0.4707i         | -0.0241                  | 0.9287                   |
| 50            | -1.752 + 0.4152i         | -1.752 - 0.4152i         | -0.0277                  | 0.9269                   |
| 55            | -1.778 + 0.3501i         | -1.778 - 0.3501i         | -0.0312                  | 0.9252                   |
| 60            | -1.803 + 0.2691i         | -1.803 - 0.2691i         | -0.0345                  | 0.9235                   |
| 65            | -1.827 + 0.1486i         | -1.827 - 0.1486i         | -0.0375                  | 0.9217                   |
| 70            | -2.019                   | -1.683                   | -0.0403                  | 0.9199                   |
| 75            | -2.154                   | -1.595                   | -0.0429                  | 0.9180                   |
| 80            | -2.254                   | -1.539                   | -0.0452                  | 0.9160                   |
| 85            | -2.338                   | -1.498                   | -0.0472                  | 0.9138                   |
| 90            | -2.412                   | -1.465                   | -0.0489                  | 0.9116                   |

Table 2 shows that at least a positive real number eigenvalue exists. Thus, it is not a stable dynamics system. Among the four Eigenvalues, there are a pair of conjugate negative eigenvalues. The absolute values of real part of conjugate negative eigenvalues is largest among the four eigenvalues. As the increasing of flap angle, the absolute values of imaginary part of conjugate negative eigenvalues descend and degenerate into two negative real numbers eventually. One of the four eigenvalues is a small positive real number. As the increasing of flap angle, it changes from positive eigenvalue to negative eigenvalue. The last eigenvalue is a positive root.

The trim state at 60° flap angle is selected as an example to research characteristics of mode of the V/STOL aircraft based on deflected slipstream during the hovering phase. The system has a pair of conjugate negative eigenvalues, a small negative real number eigenvalue and a positive number eigenvalue. Thus, it is undoubtably that the dynamics system has three different longitudinal motion modes. The corresponding eigenvectors are showed in table 3. The motion modal excitation response curve is showed in figure 7.
Table 3. Eigenvectors at 60° flap angle.

| eigenvalue | eigenvector |
|------------|-------------|
| $\lambda_1, \lambda_2$ | $[-0.6565 \pm 0.0609i -0.7305 -0.1527 \pm 0.0264i 0.0812 \mp 0.0268i]^T$ |
| $\lambda_3$ | $[0.9537 -0.3007 0 -0.0011]^T$ |
| $\lambda_4$ | $[0.1923 -0.8992 0.2667 0.2888]^T$ |

Figure 7. Motion modal excitation response curve at 60° flap angle.

According to table 3 and figure 7, the first motion modal matched with the pair of conjugate negative eigenvalues is all state variables converging to zero rapidly. The second motion modal matched with the small negative eigenvalue is state variables $u$ and $w$ converging to zero slowly and state variables $q$ and $\theta$ almost unchanged. Thus, the major motion variables are $u$ and $w$. The third motion modal matched with the positive eigenvalue is all state variables diverging to infinity.

4.4. Eigenvalues changing at different positions of center of gravity

The law of eigenvalues changing as positions of center of gravity moving is worth analyzing. Eigenvalues changing curves as position of center of gravity moving at 60° flap angle are showed in figure 8.
Figure 8. Eigenvalues changing curves as position of center of gravity moving.

In figure 8, the arrow direction presents the changing direction as position of center of gravity moving ahead. It indicates that all eigenvalues are moving close to imaginary axis. Because of positive eigenvalue decreasing, the dynamics system is close to a stable system. The absolute values of imaginary part of conjugate negative eigenvalues tend to raise from 0.24 to 0.52 at first, and then decreasing to near zero rapidly.

5. Conclusion
In this paper, the longitudinal dynamic characteristics of a certain type of V/STOL aircraft based on deflected slipstream are mainly and deeply discussed. According to a simplified longitudinal dynamic model, following five conclusions are drawn:

1). Compared with general V/STOL aircraft, the new type V/STOL aircraft based on deflected slipstream could trim at innumerable state. Flap angle is selected as a parameter variable to analyse the trim range of the new type V/STOL. As flap angle increasing from 0° to 90°, the trim pitch angle descends from near 90° to near 55°.

2). Position of centre of gravity is an important factor of the range of trimming. Appropriately moving position of centre of gravity ahead could extend the range of trim. For example, at c.g.=0.4, the minimal trim pitch angle decreases to 42°. However, limited by the maximal pull force of head propeller, if c.g. exceed 0.4, the minimal trim pitch angle increases and the range of trim shrinks.

3). The dynamics system has four different eigenvalues. Two of them are a pair of conjugate negative eigenvalues when flap angle is less than 70°. when flap angle exceeds 70°, conjugate negative eigenvalues degenerate into two negative real numbers. One of the remaining two eigenvalues is a small real number changing from positive to negative as flap angle increasing. The last one is a positive real number. It indicates the dynamics system is instable.

4). Motion modal excitation response curve and eigenvectors show its motion mode characteristic during hovering phase. In the motion modal matched with conjugate negative eigenvalues, all state variables converge to zero rapidly. In the motion modal matched with small negative eigenvalue, u and w are the main motion variables and converge to zero extremely slowly while state variables q and θ almost unchanged. In the last motion modal matched with positive eigenvalue, all state variables diverge to infinity rapidly.

5). Position of centre of gravity is also an important factor for dynamic characteristics. As position of centre of gravity moving ahead the positive eigenvalue moves close to imaginary axis. Thus, the dynamics system is close to be a stable system.

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