Super-low-frequency radio waves as signals and special electromagnetic counterparts of gravitational waves (from binary mergers) having tensorial and possible nontensorial polarizations

Hao Wen

1Physics Department, Chongqing University, Chongqing 401331, China.

(Dated: March 21, 2019)

Gravitational waves (GWs, from the binary merger) interacting with ultra-strong magnetic fields of the neutron star (in the same binary system), would lead to perturbed electromagnetic waves [EMWs, in the same frequencies of these GWs, usually categorized into super-low-frequency (SLF) band for the EMWs]. Such perturbed SLF-EMWs are not only the signals, but also a new type of special EM counterparts of the GWs with very different properties compared to usual EM counterparts, e.g. gamma-ray bursts (GRBs). Here, generation of the perturbed SLF-EMWs is investigated, and the strengths of their magnetic components are estimated to be around $10^{-12}$Tesla to $10^{-17}$Tesla at the Earth for various cases, which would be already within or approaching the sensitivity range of the cutting-edge technologies of the magnetometry. Especially, for the case that the neutron star in the binary is a magnetar, the strengths of signals with level $\sim 10^{-13}$Tesla are already detectable by current highly sensitive SLF detectors and magnetometers. Waveforms of the perturbed SLF-EMWs will be modified into shapes different but related to the waveforms of the GWs, due to the amplification process during the binary mergers which could amplify the magnetic fields into $10^{12}$Tesla or even higher. Specific connection relationship between the polarizations of the perturbed SLF-EMWs and the polarizations (tensorial and possible nontensorial) of the GWs of binary mergers, are also addressed. These characteristic waveforms and particular polarizations of the perturbed SLF-EMWs will be very helpful for filtering and extracting the signals from background noise. Briefly, we propose such a potential novel way of detection for signals and special EM counterparts of the GWs from binary mergers, which would bring us some different new information of fundamental properties of the gravity and Universe that other GW detections may not provide.

PACS numbers: 04.30.-w, 04.50.-h, 04.80.Nn, 04.30.Db

Keywords:

I. INTRODUCTION

The LIGO scientific collaboration and the Virgo collaboration have so far reported 11 gravitational wave (GW) events (GW150914, GW151012, GW151226, GW170104, GW170608, GW170729, GW170809, GW170814, GW170817, GW170818, GW170823) [1–5, 7, 8] from binary black hole mergers [1–5, 7, 8] (with frequencies around 30Hz to 450Hz and dimensionless amplitudes $\sim 10^{-21}$ to $\sim 10^{-22}$ near the Earth) or from binary neutron star merger [6] [GW170817, comes with the first electromagnetic (EM) counterpart of the GWs]. These great discoveries inaugurated a new era of GW astronomy. Meanwhile, the EM counterparts which may occur in association with corresponding observable GW events, have also been massively studied based on various emission mechanisms [2, 22] due to that they can bring us crucial information with rich scientific values.

Further, in order to obtain more extensive and in-depth astronomical information, on the path forward for multi-messenger detections, it will be very expected to expand the observations in broader frequency bands (low, intermediate, high, and very high-frequency bands), through a wider variety of methods with different effects, aiming on more types of sources, to explore richer information of properties of the gravity and Universe. E.g., some interesting questions would arise: are there and how can we detect the possible nontensorial polarizations of GWs predicted by gravity theories beyond GR including those with extra-dimensions? Can we observe GWs based on different principles for seeking information that current GW detectors may not give? Can we acquire any new EM counterparts generated by other mechanisms with different characters?

In this article we address a topic that above questions would converge. A possible new way to detect the GWs from binary mergers is pro-
FIG. 1: A general frame: if one star in the binary merger is a neutron star, or even a magnetar, the binary system (i.e., neutron star-neutron star binary, or black hole-neutron star binary) could have ultra-strong magnetic fields up to $\sim 10^{11}$ Tesla. Moreover, during the amplification process, the magnetic fields would be greatly amplified and could easily reach $10^{12}$ Tesla or much higher. The GWs from such binary merger could interact with such extremely high magnetic fields, and lead to perturbed EMWs [also in the GW frequency band, and usually defined as super-low-frequency (SLF) band in context of EMW researches, around $\sim 10^1$ to $\sim 10^2$ Hz], which propagate to far field area near the Earth synchronously with the GWs due to their identical or almost identical velocities. Thus, such perturbed SLF-EMWs could be a new type of signals and special EM counterparts of GWs from binary mergers, and they would be captured by land or space based high sensitive SLF-EMW detectors, and such detections would be complementary to and cross-checked with the observations by LIGO, Virgo, KAGRA, etc for the GWs. Particularly, the perturbed SLF-EMWs have characteristic waveforms and particular polarizations which may reflect different new information about some crucial issues such as the tensorial and possible nontensorial polarizations of GWs from binary mergers.

Posed here based on the effect of EM response to GWs, which had been long studied but were usually in very high frequency bands such as over GHz ($10^9$ Hz) [28–40]. Whereas, we now apply such mechanism targeting on the GWs in the intermediate band (around $\sim 1$ to $\sim 1000$ Hz) from the binary mergers.

Specifically, as demonstrated in Fig. 1, considering one star in the binary is a neutron star (or magnetar, which usually has ultra-strong surface magnetic fields $\sim 10^{11}$ Tesla), according to the electrodynamics in curved spacetime, during the binary merger, the produced GWs could interact with such ultra-strong magnetic fields of the same source, and then lead to significant perturbed EMWs in the same frequency band [normally defined as super-low-frequency (SLF) band in context of researches for EMWs]. These perturbed SLF-EMWs propagate together with the GWs into far field area until the Earth, as a new type of signals and EM counterparts of GWs from the binary mergers. Such multi-messenger signals of GWs+SLF-EMWs could be observed via different corresponding methods, e.g., the SLF-EMWs could be captured by land or space based highly sensitive SLF detectors and magnetometers, and they would be complementary to (and cross-checked with) the observations by GW detectors of LIGO, Virgo, KAGRA and so on.

For the first step of estimation, instead of massive numerical computing, we apply a typical model [41] of surface magnetic fields of neutron stars for calculation, to try to obtain a primary but safe estimation of the order of magnitude of the signal strengths. Based on the electrodynamics equations in curved spacetime and previous works [24, 30, 31, 34, 35, 39], the perturbed SLF-EMWs are estimated for various cases including that having tensorial and possible nontensorial GWs, and the strengths of their magnetic components would be generally around $10^{-12}$ Tesla to $10^{-17}$ Tesla at the Earth, which are within or approaching the detectable windows of existing magnetometry based on atoms, superconducting quantum interference device (SQUID), spin wave interferometer, coils-antennas and so on [53, 74]. Crucially, for the case that the neutron star in the binary is a magnetar, such signals in level of $\sim 10^{-13}$ Tesla are already detectable by current highly sensitive SLF detectors and
The strong magnetic fields of binary would be further significantly amplified by the amplification process, which had been widely studied as a key feature to understand the physical behaviours during the binary merger, and it perhaps lead to the strongest magnetic fields in the Universe. Thus, the amplification process will not only result in the further stronger signals of the perturbed SLF-EMWs, but also lead to the waveforms of perturbed SLF-EMWs have an enlargement and modification in corresponding time duration (of the amplification of magnetic fields), i.e., the waveforms of perturbed SLF-EMWs would have very distinctive shapes, and should not be just linearly proportional to the waveforms of the GWs.

The particular polarizations of the perturbed SLF-EMWs caused by tensorial and possible nontensorial polarizations of GWs will also be a very special character. In frame of GR, GWs have tensorial polarizations only (x and y modes), but generic metric theories predict up to six polarizations (including vector modes: x, y, and scalar modes: b, l)\footnote{52, 53}, and such additional polarizations relate to many important issues like the modified gravity and extra-dimensions of space. Based on current researches\footnote{54, 55}, specific relationship of how the polarizations of perturbed SLF-EMWs connect to the tensorial and nontensorial polarizations of the GWs from binary mergers, is addressed, and typical examples are presented.

Such characteristic waveforms and particular polarizations of the perturbed SLF-EMWs will be very unique features helpful for filtering and extracting the signals of perturbed SLF-EMWs from background noise.

The perturbed SLF-EMWs as a special type of EM counterparts of the GWs, sit in totally different frequency band rather than usual EM counterpart of GRBs; besides, they have another very special and important property: the GRBs are usually assumed to be generated nearly at the same time to the GWs, but actually there would be still some unknown uncertainty of their start time; differently, for the perturbed SLF-EMWs, such uncertainty could be reduced or avoided, because under the frame of EM response to GWs, they just clearly have the same start time to synchronously propagate with the GWs outward from the source of binary, and thus may provide more accurate information for those analysis based on the difference of arrival times between the EM and GW signals.

Plan of this article is as follows: In Sect\textbf{II} strengths of magnetic components of the perturbed SLF-EMWs caused by GWs of binary mergers are estimated. In Sect\textbf{III} the amplification process of magnetic fields and the modification of waveforms of the perturbed SLF-EMWs are addressed. In Sect\textbf{IV} particular polarizations of the perturbed SLF-EMWs depending on the tensorial and possible nontensorial polarizations of the GWs of binary mergers, are investigated. In Sect\textbf{V} issues of methods of detection for the perturbed SLF-EMWs are discussed. In Sect\textbf{VI} summary of results, discussion and conclusion are given.

\section{II. Strengths of Perturbed Super-Low-Frequency EMWs Caused by GWs from Binary Mergers}

In this section we estimate the strengths (at the Earth) of the perturbed SLF-EMWs caused by interaction between the GWs of binary mergers and the ultra-strong magnetic fields of neutron star (or magnetar) of the same binary system.

For the first step of estimation, instead of massive numerical computing, we apply a typical model\footnote{41} of the surface magnetic fields of neutron stars for calculation, and it can be expressed as:

\begin{align}
\mathbf{B}^{\text{surf}} &= \vec{\nabla} \times (\vec{r} \times \vec{\nabla} S), \\
S &= S(l, m) = S_l^m(r) Y_l^m(\theta, \phi), \\
Y_l^m(\theta, \phi) &= P_l^m(\cos \theta) e^{im\phi}; \quad (1)
\end{align}

In spherical coordinates with orthonormal basis of $e_r, e_\theta$ and $e_\phi$, the $\mathbf{r} = re_r$, and $\mathbf{B} = B_r e_r + B_\theta e_\theta + B_\phi e_\phi$. The $S$ is expanded in a series of spherical harmonics, and $P_l^m(\cos \theta)$ is the Legendre polynomial. For $l = 1, m = 0$, it corresponds to the dipole mode:

\begin{align}
S(1, 0) &= C \frac{\cos \theta}{r^2} \sum_{\nu=0}^{\infty} a_{\nu} \left( \frac{2M}{r} \right)^\nu, \\
a_0 &= 1, a_{\nu} = \frac{(1 + \nu)^2}{(3 + \nu)^\nu} a_{\nu-1}, \quad (\text{for } \nu \geq 1), \quad (2)
\end{align}
From Eqs. (1) to (2), the dipole component of surface magnetic field is:
\[
B_{\text{surf}}^{\text{dip}}(1, 0) = \vec{\nabla} \times (\vec{r} \times \vec{S}(1, 0))
\]
\[
= C_1 \cos \theta \frac{1}{r^3} \sum_{\nu=0}^{\infty} a_{\nu} (\frac{2M}{r})^\nu \vec{e}_r, \\
+ C_1 \sin \theta \frac{1}{r^3 h} \sum_{\nu=0}^{\infty} (\nu + 1) a_{\nu} (\frac{2M}{r})^\nu \vec{e}_\theta, \quad (3)
\]
Sum the terms in Eq. (3), a typical form of neutron star surface magnetic field in dipole mode can be obtained [see Fig. 2(a)]:
\[
B_{\text{surf}}^{\text{dip}}(1, 0)
= 2C_1 \cos \theta \frac{1}{r^3} \left[ -3r^2 \ln(1 - \frac{2M}{r}) + 2M(M + r) \right] \vec{e}_r, \\
+ C_1 \sin \theta \frac{3r^2}{r^3 h} \left[ 2M \left( \frac{M}{r} - 2M + 1 \right) + r \ln(1 - \frac{2M}{r}) \right] \vec{e}_\theta, \quad (4)
\]
the metric \(h\) is \([11]\):
\[
h = h(r) = \left( 1 - \frac{2M}{r} \right)^{-\frac{1}{2}}, M = \frac{Gm(r)}{c^2}, \quad (5)
\]
Notice that the \(h(r)\) very fast drops into 1 for \(r > 2M\), so in the calculation for the area \(r > 10m\) (typical neutron star radius) we could approximately take it as 1; \(m(r)\) is the mass function to determine the total mass enclosed within sphere of radius \(r\), and \(m(r) = \text{mass of magnetar in our case.}\)

Similarly, for \(l = 2, m = 0\), we have the quadrupole mode of surface magnetic fields [see Fig. 2(b)]:
\[
B_{\text{surf}}^{\text{quad}}(2, 0) = 3C_2 (3 \cos^2 \theta - 1) \frac{1}{r^3} \\
- 3r^2 \ln(1 - \frac{2M}{r}) + 2M(M + r) \vec{e}_r, \\
+ 3C_2 \cos \theta \sin \theta \frac{1}{r^4 h} \left[ 3r^2 \left( \frac{4M^2}{r} + M + r \right) + r^2 \ln(1 - \frac{2M}{r}) \right] \vec{e}_\theta, \quad (6)
\]
Neutron star surface magnetic fields in quadrupole mode would have comparable strength to that in dipole mode \([77]\), and the \(C_1\) and \(C_2\) are constants (with different dimensions) that have been calibrated to satisfy the typical strength of surface magnetic fields (e.g. for magnetar, \(\sim 10^{11}\)T). In dipole mode, the tangential components [i.e. \(\vec{e}_\theta\) component in Eq. (4)] have the maximum at polar angle \(\theta = \pi/2\) [Fig. 2(a)], and the radial components [i.e. \(\vec{e}_r\) component in Eq. (4)] have the maximum around \(\theta = 0\) and \(\pi\) (two poles). Differently, in quadrupole mode [Eq. (6)] the tangential components have maximum around \(\theta = \pi/4\) and \(3\pi/4\) [Fig. 2(b)], and the maximum of radial component is around \(\theta = 0, \pm\pi/2, \text{and } \pi\).

We also compare above model with more simple models (i.e. just considering the magnetic fields decay by \(\sim r^{-3}\) or \(\sim r^{-4}\), and we can see that [Fig. 2(c)] the dipole magnetic fields decay faster than the simple model of \(\sim r^{-3}\) in the near field close to the source. Actually, the decay behaviour
of magnetic fields in near field area predominately impact the generation of the perturbed SLF-EMWs (we can see that in later part of this section), so we should use such typical model instead of the simple models to obtain more safe estimations.

On the other hand, if the GWs of binary mergers contain possible nontensorial polarizations, they can be generally express as:

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & A_+ + A_b & A_x & A_y \\ 0 & -A_+ + A_b & A_x & -A_y \\ 0 & A_x & A_y & \sqrt{2} \tilde{A}_l \end{pmatrix} e^{i(k_y \cdot r - \omega t)} \tag{7}$$

the +& x, x& y, b& l respectively represent the cross- & plus- (tensor mode), x& y- (vector mode), b& l- (scalar mode) polarizations. Interaction of these GWs of binary mergers with the ultra-strong magnetic fields [background fields, Eqs. (4) and (6)] of the neutron star of the binary system, will generate +& x and B components of the perturbed SLF-EMWs for the (A& l) polarizations. Interaction of these GWs with ultra-strong magnetic fields can only interact with (A& l) GWs and cannot interact with A& b or A& l GWs.

To precisely calculate the waveform of the perturbed SLF-EMWs is a much more complicated task. However, here, as the first step, at least we can have a conservative estimation of only the order of amplitude of the signal strengths. Therefore, taking a typical example, we can calculate a simple situation shown in Fig. 3 where the binary is very close to the merger time (t = 0, defined as the time when the amplitude of GW reaches the maximum), e.g., only several millisecond before the merger time, and the distance between the two centers of stars in the binary is set to 4 · ns (ns = \( 10^4 \) meter, typical radius of neutron star). We can integrate the contributions [given by Eq. (4)], replace the \( \Delta L \) by \( dr \) of generation of the perturbed SLF-EMWs of every small accumulation distance “\( dr \)” from the \( r_0 \) (start point of the accumulation, set as 3 · ns here) until some end point of accumulation \( R_{acc} \); besides, every part of contribution of the perturbed SLF-EMWs in the “\( dr \)” will decay from the position \( r \) to \( R_0 \) (\( R_0 \) is some observer distance), so there will be a term of \( r/R_0 \) in the formula, see below. Actually, the accumulation will continue after the \( R_{acc} \), but we drop this part because it is comparatively tiny. Here we set the \( R_{acc} \) as \( r_0 + 10^5 \) meter for estimations, and the reason to chose such distance is that: when the binary is very close to the merger time, the period of binary orbiting is typically \( \sim 10^{-2} \) second, so in a very short duration of time, e.g. \( 1 \times 10^{-3} \) second (corresponding to \( 10^5 \) meter of the propagation distance of GW), the magnetic field of binary could be approximately treated as static or quasi-static for the case shown in the Fig. 3 so in this short duration and in this accumulation distance, the integral of these contributions of perturbed SLF-EMWs is valid, as a safe estimation. Therefore, together with Eqs. (4),
[7] and [8], we work out the accumulated perturbed SLF-EMWs caused by the tensorial GWs interacting with transverse surface magnetic field (tangential, or \(e_{\theta}\) component) of the dipole mode, and the strengths of their magnetic components has the form:

\[
\tilde{B}_{\text{prtd-ntsr}} \quad \text{dipole} = \int_{r_0}^{R_{\text{acc}}} \left( \frac{R_{\text{earth}} A_{\text{earth}}}{r} \right) \sin \theta C_1 \frac{3 r'^2 [2 M (\frac{M}{r'^2 - 2 M} + 1) + r' \ln(1 - \frac{2 M}{r'})]}{4 M^3} \frac{\omega}{c R_0} \frac{r}{r'^{3/2}} dr
\]

\[
= \frac{\sin \theta C_1 A_{\text{earth}} R_{\text{earth}} \omega}{4 c M^3 R_0} \left[ R_{\text{acc}}'(r_0^2 - 2 M) - r_0^2 \ln \left( 1 - \frac{2 M}{r_0^2} \right) + M \ln \left( R_{\text{acc}}'(r_0^2 - 2 M) - r_0^2 \right) \right], \quad (10)
\]

The term \(\frac{R_{\text{earth}} A_{\text{earth}}}{r}\) represents \(A(r)\) (amplitude of GW at position of \(r\)), where the \(R_{\text{earth}}\) and \(A_{\text{earth}}\) are the distance of binary to the Earth and the amplitude of GW at the Earth. The subscript “prtd” and “ntsr” of above \(\tilde{B}_{\text{prtd-ntsr}} \quad \text{dipole}\) mean “perturbed EMWs” and “caused by tensorial GWs”; the superscript “dipole” means here we include the dipole mode of surface magnetic fields for calculation. The \(\omega\) is the angular frequency. Besides, here we use the “\(r\)” for the part of magnetic field in the Eq.\((10)\), due to that the center of the neutron star is not at the center of the binary (where \(r = 0\)), but at the \(r = 2 * \text{rns}\) (also see Fig.\(3\) for our calculation, i.e., the radial coordinate of neutron star (included for calculation) is shifted into \(r' = r - 2 * \text{rns}\); also, we note the \(R_{\text{acc}}' = R_{\text{acc}} - 2 \text{rns}\), and \(r_0' = r_0 - 2 \text{rns}\), respectively. When \(R_0 > R_{\text{acc}}\), as also mentioned above, we only include the contribution of perturbed SLF-EMWs before the \(R_{\text{acc}}\) and treat the accumulated perturbed SLF-EMWs decaying spherically from the position of \(R_{\text{acc}}\) until the observation point \(R_0\), i.e., the Eq.\((10)\) is just simply \(\sim 1/R_0\) for this situation given a specific \(R_{\text{acc}}\); in this case the curves of Eq.\((10)\) can be found in Fig.\(3\) (b). Otherwise, when \(R_0 < R_{\text{acc}}\), we set \(R_{\text{acc}} = R_0\) (replace \(R_{\text{acc}}\) by \(R_0\)), so the Eq.\((10)\) turns into a more complex function of \(R_0\) [increase first and then decrease, due to the composite effect of accumulation and the decaying, the curves can be found in Fig.\(3\) (a)].

In a similar way, the strength of magnetic component of the perturbed SLF-EMWs caused by nontensorial GWs interacting with longitudinal magnetic field (radial, or \(e_r\) component) of the dipole mode, can be obtained:

\[
\tilde{B}_{\text{prtd-ntsr}} \quad \text{quad.} = \int_{r_0}^{R_{\text{acc}}} \left( \frac{R_{\text{earth}} A_{\text{earth}}}{r} \right) 2 C_1 \cos \theta \frac{1}{r'^{3/2}} \frac{3 r'^2 [2 M (\frac{4 M^2}{r'^2 - 2 M} + M + r') + r'^2 \ln(1 - \frac{2 M}{r'})]}{8 M^3} \frac{\omega}{c R_0} \frac{r}{r'^{3/2}} dr
\]

\[
= \frac{\cos \theta C_1 A_{\text{earth}} R_{\text{earth}} \omega}{4 c M^3 R_0} \left[ 2 M r'^2 - r'^2 R_{\text{acc}}^2 + 2 M \ln \frac{r_0'}{R_{\text{acc}}'} \left( R_{\text{acc}}'(r_0^2 - 2 M) - r_0^2 \right) + r_0' \ln \frac{r_0'^2 - 2 M}{r_0'} - R_{\text{acc}}' \ln \frac{R_{\text{acc}}'(R_{\text{acc}}'^2 - 2 M)}{R_{\text{acc}}^2} \right], \quad (11)
\]

The subscript “ntsr” of above \(\tilde{B}_{\text{prtd-ntsr}} \quad \text{quad.}\) means “caused by nontensorial GWs”. The same, the magnetic component of the perturbed SLF-EMWs caused by tensorial GWs interacting with transverse magnetic fields (\(e_{\theta}\) component) of the quadrupole mode, has the form:

\[
\tilde{B}_{\text{prtd-ntsr}} \quad \text{quad.} = \int_{r_0}^{R_{\text{acc}}} \left( \frac{R_{\text{earth}} A_{\text{earth}}}{r} \right) 3 C_2 \cos \theta \sin \theta \frac{1}{r'^{3/2}} \frac{3 r'^2 [2 M (\frac{4 M^2}{r'^2 - 2 M} + M + r') + r'^2 \ln(1 - \frac{2 M}{r'})]}{8 M^3} \frac{\omega}{c R_0} \frac{r}{r'^{3/2}} dr
\]

\[
= \frac{9 \sin \theta \cos \theta C_2 R_{\text{earth}} A_{\text{earth}} \omega}{8 c M^3 R_0} \left[ M^2 (\frac{1}{R_{\text{acc}}^2} - \frac{1}{r'^2}) + \ln \frac{r_0'^2}{R_{\text{acc}}'} \left( R_{\text{acc}}'(r_0^2 - 2 M) + \sum_{k=1}^{\infty} \frac{(2 M / R_{\text{acc}}')^k}{k^2} + \sum_{k=1}^{\infty} \frac{(2 M / r_0')^k}{k^2} \right) \right], \quad (12)
\]
The superscript “quad.” of above $\tilde{B}^{\text{quad.}}_{\text{prtbd-ntsr}}$ means the quadrupole mode of magnetic fields are included for calculation. Further, the magnetic component of the perturbed SLF-EMWs caused by nontensorial GWs interacting with longitudinal magnetic field ($\vec{e}_r$ component) of the quadrupole mode, can be given:

$$\tilde{B}^{\text{quad.}}_{\text{prtbd-ntsr}} = \int_{r_0}^{R_{\text{acc}}} \left( \frac{R_{\text{earth}}A_{\text{earth}}}{r} \right) 3C_2 (3 \cos^2 \theta - 1) \frac{1}{r^{17}} \frac{-3r'[r^2 \ln(1 - \frac{2M}{r}) + 2M(M + r')] \omega}{8M^3} \frac{r}{c R_0} dr$$

If using simple models of surface magnetic fields of the neutron stars, which just decay by $\sim r^{-n}(n=3, 4, ...)$, the strengths of magnetic components of the perturbed SLF-EMWs are:

$$\tilde{B}^{\text{simple}}_{\text{prtbd}} = \int_{r_0}^{R_{\text{acc}}} \left( \frac{R_{\text{earth}}A_{\text{earth}}}{r} \right) \frac{B_0 \cdot rns^n}{r^n} \frac{\omega}{c R_0} dr$$

$$= \frac{R_{\text{earth}}A_{\text{earth}}rns^n}{(n-1)cR_0} \frac{B_0}{R_{\text{acc}}} \left( \frac{1}{r_0^{n-1}} - \frac{1}{R_{\text{acc}}^{n-1}} \right)$$ \hspace{1cm} (14)

Above Eqs. (10) to (14) estimate the strengths of perturbed SLF-EMWs for various cases: dipole-tensorial, dipole-nontensorial, quadrupole-tensorial, quadrupole-nontensorial and simple models of magnetic fields; these results do not contain information of propagation factors or specific waveforms, and only provide estimations for the levels of strengths. Table I and Fig. I show examples of these strengths for typical parameters.

In Table I we can find that the levels of magnetic components of the perturbed SLF-EMWs are generally $10^{-12}$Tesla to $10^{-17}$Tesla at the Earth, for three type of cases: magnetar case (magnetic field = $1.0 \times 10^{11}$Tesla), amplification case (magnetic field = $1.0 \times 10^{12}$Tesla), and normal neutron star case (magnetic field = $1.0 \times 10^8$Tesla).

Importantly, for the magnetar cases, even if we do not consider the amplification process (see Sect. III, e.g., only consider the duration that the binary is closely approaching the merger time ($t = 0$), like the situation shown above in Fig. I as a conservative and safe estimation, the magnetic components of the perturbed SLF-EMWs reach $10^{-13}$Tesla at the Earth (see magnetar cases in Table I), which can already be detectable by current technologies.

Also, in Fig. I we find that the behaviours of signals among various cases are generally consistent, and the strengths based on simple models

of magnetic fields ($\sim r^{-3}, \sim r^{-4}$) are generally larger than the strengths based on the dipole or quadrupole magnetic fields, which drop more fast in the near field. Actually, even for cases with just normal neutron stars (not magnetars), the magnetic fields of the binary would be greatly amplified by 3 (or more) orders of magnitude and easily reaching $10^{16}$G ($10^{12}$Tesla) or higher by the process of magnetic fields amplification (see Sect. III). Besides, results in Table I indicate that the distance does not impact the level (at Earth) of the perturbed SLF-EMWs given the same GW amplitudes at the Earth, because larger distance of sources (binary) requires higher GW amplitudes at the sources, and then it leads to stronger generation of perturbed SLF-EMWs which whereas also need to decay for longer distance to the Earth, so their effects offset each other and thus compositively result in the irrelevance to the distance.

In short, for various cases, the signals (magnetic components of the perturbed SLF-EMWs) in levels of $\sim 10^{-12}$Tesla to $\sim 10^{-17}$Tesla are within or approaching the detectable windows of existing magnetometry [53, 54, 67, 70, 71, 72, 73, 74, 75, 76, 77].

Crucially, for the case that the neutron star in the binary is a magnetar, such signals (also as a special EM counterpart of the GWs) in level of $\sim 10^{-13}$Tesla are already detectable by current highly sensitive SLF detectors and magnetometers [55, 59, 73, 75, 76, 77, 78, 79].

III. MODIFIED WAVEFORMS OF PERTURBED SLF-EMWS DUE TO AMPLIFICATION PROCESS OF MAGNETIC FIELDS OF THE BINARY

Based on various mechanisms, the amplification process of magnetic fields of the binary mergers had
Kelvin-Helmholtz instability. Subgrid modeling was presented the magnetic fields in local high-resolution simulations[49], and similarly for other cells. Here, the cases with surface “magnetic fields of neutron star” = 1.0 × 1011, 1.0 × 1012 and 1.0 × 1016 Tesla respectively represent the magnetar cases, the cases considering the amplification of binary magnetic fields, and the cases of only normal neutron stars. The signal strengths \( \sim 10^{-12}\) Tesla to \( \sim 10^{-17}\) Tesla are within or approaching current detectable windows. Crucially, for the magnetar cases, such signals (also special EM counterparts of the GWs of binary mergers) \( \sim 10^{-13}\) Tesla are already detectable by current magnetometry.

| GW amplitude of binary sources around Earth | magnetic fields of neutron star(Tesla) | GW distance to Earth | magnetic component (Tesla) of perturbed SLF-EMWs around the Earth |
|--------------------------------------------|--------------------------------------|---------------------|---------------------------------------------------------------|
| \( 40Mpc \times 10^{-21} \)               | 1.0 × 10^{11} 1.1 \times 10^{-13} 2.0 \times 10^{-13} 1.0 \times 10^{-13} | \( 4Mpc \times 10^{-22} \) | 1.0 \times 10^{11} 1.1 \times 10^{-13} 2.0 \times 10^{-13} 1.0 \times 10^{-13} |
| \( 1.0 \times 10^{12} \)                 | 1.1 \times 10^{-12} 2.0 \times 10^{-12} 1.0 \times 10^{-12} | \( 1.0 \times 10^{8} \) 1.1 \times 10^{-16} 2.0 \times 10^{-16} 1.0 \times 10^{-16} | 1.0 \times 10^{11} 1.1 \times 10^{-14} 2.0 \times 10^{-14} 1.0 \times 10^{-14} |
| \( 1.0 \times 10^{8} \)                  | 1.1 \times 10^{-16} 2.0 \times 10^{-16} 1.0 \times 10^{-16} | \( 1.0 \times 10^{11} \) 1.1 \times 10^{-13} 2.0 \times 10^{-13} 1.0 \times 10^{-13} | 1.0 \times 10^{8} 1.1 \times 10^{-17} 2.0 \times 10^{-17} 1.0 \times 10^{-17} |

been widely studied[42, 51] as one key feature to further understand the mergers.

E.g., Rasio and Shapiro first pointed that the Kelvin-Helmholtz (KH) instability would significantly amplify the magnetic fields of the binary merger[12].

Recent work of general relativistic magnetohydrodynamic simulations by Ciolfi et. al[44] indicate that the amplification process can lead to magnetic fields up to \( 10^{16}\) G to \( 10^{17}\) G by the effect of the Kelvin-Helmholtz instability. Subgrid modeling was also applied[51] to find that the amplifications of up to 5 orders of magnitude are possible and the level of \( 10^{16}\) G can be easily reached.

Research of the turbulent amplification of magnetic fields in local high-resolution simulations[49], presented the magnetic fields \( \sim 10^{16}\) G throughout the merger duration of the neutron star binary. Another study[49] on the magnetic-field amplification due to the Kelvin-Helmholtz instability also found that there is an at least \( 10^3\) factor for the magnetic fields of binary neutron star mergers and it can easily reach \( 10^{15}\) G or higher.

Price and Rosswog[42] argued that the magnetic fields of neutron star in binary mergers could be amplified by several orders of magnitude, and it is highly probably much stronger than \( 2 \times 10^{15}\) G for realized cases in nature, and therefore the amplification may lead to the strongest magnetic fields in the Universe.

In brief, many previous studies generally indicated the greatly amplified magnetic fields in the amplification process during the binary merger. Obtained results of Eqs. (10) to (14) indicate that the waveform of perturbed SLF-EMWs should be linearly proportional and similar to the waveforms of GWs of binary mergers, but such magnetic field amplification will influence the interaction of the GWs with the magnetic fields, and thus result in that the waveforms of the perturbed SLF-EMWs also have an am-
FIG. 4: Strengths of magnetic components of perturbed SLF-EMWs caused by interaction between GWs from binary mergers and ultra-strong magnetic fields of neutron star in the binary system.

The curves are from Eqs. (10) to (14), and the case of “dipole, tensorial” means the dipole mode of surface magnetic fields of neutron stars and the tensorial GWs of binary mergers are included for calculation, similarly for other cases. Subfigure (a) shows examples of curves of the strengths for range from the source to the $R_{\text{acc}}$ (end point of accumulation we include for calculation). Subfigure (b) shows curves for the range from the $R_{\text{acc}}$ to very far observation point $R_0 = 40 Mpc$. Here, the surface magnetic field of neutron star is set to $1.0 \times 10^{11} T$ (for magnetar cases), and the dimensionless amplitude of GWs at the Earth is set to $1.0 \times 10^{-21}$. We can see the strengths of magnetic components of perturbed SLF-EMWs are $\sim 10^{-13}$Tesla at the Earth, which can already be captured by current technology.

IV. PARTICULAR POLARIZATIONS OF THE PERTURBED SLF-EMWS DEPENDING ON TENSORIAL AND POSSIBLE NONTENSORIAL POLARIZATIONS OF GWs FROM BINARY MERGERS

Some researches have been carried out [54, 82, 83] for additional polarizations relevant to observations by LIGO. However, more information of specific properties of possible nontensorial GWs from binary mergers, are still very expected. For the tensorial GWs from binary mergers, it is already known that the proportions of two polarizations depend on the orbital inclination $\iota$ (angle between the sight direction and the spin axis of the binary) [84]. E.g., for the “face-on” ($\cos \iota = \pm 1$) and “edge-on” ($\cos \iota = 0$) directions the GWs are circularly and linearly polarized, respectively. Excitingly, recent work [54] presents the inclination-angle dependence and relative amplitudes for GWs including nontensorial
FIG. 5: Simplified example of modification for the waveform of perturbed SLF-EMWs (at the Earth) caused by GWs from binary mergers with the amplification process of magnetic fields. Subfigure (a) is the waveform template of binary neutron star merger, and this curve is produced and modified from some materials of Ref.[78]. The subfigure (b) is an example of typical process of magnetic field amplification during binary merger (the curve is produced based on some materials of Ref.[44]), and this extreme phenomenon could amplify the magnetic fields into very high level of $\sim 10^{16}$Tesla or more. Here, the $t=0$ means the merger time where the maximum of GW amplitude appears. The subfigure (c) presents that the waveform of perturbed SLF-EMWs should be no longer linearly proportional to the waveform of the GWs of binary merger, because of the amplification process of magnetic fields of the binary (see more in Sect. III), and thus, in this example, the waveform of SLF-EMWs will be enlarged from about -10 ms to 22 ms. In other words, information of the amplification transfers into the characteristic shape of the waveform of perturbed SLF-EMWs. The subfigure (d) shows the comparison between the waveforms of the perturbed SLF-EMWs with and without the influence of the amplification process. It should be emphasized that, this figure is not the exactly predicted waveform of perturbed SLF-EMWs, but only to intuitively present the overall increasing tendency of the perturbed SLF-EMWs due to the amplification of magnetic field of the binary, based on very simplified considerations, and the precisely predicted waveforms of the perturbed SLF-EMWs need to be computed by massive numerical calculations and simulations, and such works will be carried out in separated articles as the next steps.

modes, i.e., for modes of $h_x, h_y, h_b$ and $h_l$, the inclination angle $\iota$ gives factors of $\sin 2\iota$, $\sin \iota$, $\sin^2 \iota$ and $\sin 2\iota$, respectively. Based on above knowledge, we can have a specific manner of how the polarizations of perturbed SLF-EMWs connect to the tensorial and nontensorial polarizations of GWs from binary mergers. For a simple estimation, e.g., we here focus on the influence of inclination angle $\iota$ and ignore impact by other angular parameters, and also only include the vector modes as nontensorial GWs. According to the geometrical factors given by Ref.[54]:

$$G_+ \propto \frac{5}{2} (1 + \cos^2 \iota), \quad G_\times \propto i5 \cos \iota,$$

$$G_{Vx} \propto \sqrt{\frac{525}{56}} \sin 2\iota, \quad G_{Vs} \propto \sqrt{\frac{15}{2}} \sin \iota.$$

(15)
for the $h_+, h_\times, h_\phi$ and $h_y$ GWs, the mixed GWs can be expressed as[54] (set the relative amplitudes of $h_+$ and $h_\times$ as 1 and equal to each other):

$$h = (G_+ + G_\times + A_{V_c}G_{V_c} + A_{V_c}G_{V_c})h_{GR}. \quad (16)$$

On the other hand, due to current study[39], the tensorial and nontensorial polarizations of GWs will lead to corresponding different polarizations of the perturbed SLF-EMWs, given particular types of background magnetic fields (transverse or longitudinal, to interact with the GWs). It is found[39]:

$$E_x^{(1)} \propto -h_\times B_x^{(0)} + h_+ B_y^{(0)} - h_y B_x^{(0)},$$
$$E_y^{(1)} \propto -h_+ B_x^{(0)} + h_\times B_y^{(0)} + h_x B_x^{(0)}, \quad (17)$$
i.e., the electric component $E_x^{(1)}$ (in x-direction) of perturbed SLF-EMWs can be contributed by $h_\times$ interacting with $B_x^{(0)}$ (background magnetic fields in transverse direction), and by $h_+$ interacting with $B_y^{(0)}$ (also transverse background magnetic fields), and by nontensorial $h_y$ interacting with $B_x^{(0)}$ [longitudinal background magnetic fields (which only interact with nontensorial GWs) in z direction (also the propagating direction of the GWs)][39]. With Eqs. (16) to (17) we can obtain the relationship how the polarizations of perturbed SLF-EMWs connect to the tensorial and nontensorial polarizations of GWs from binary mergers:

$$E_x^{(1)} \propto -i\mu \cos \iota B_x^{(0)}$$
$$+ \frac{5}{2} (1 + \cos^2 \iota) B_y^{(0)} - V_{c_e} \sqrt{\frac{15}{2}} \sin \iota B_z^{(0)},$$
$$E_y^{(1)} \propto -\frac{5}{2} (1 + \cos^2 \iota) B_x^{(0)}$$
$$+ i\mu \cos \iota B_y^{(0)} + V_{c_e} \sqrt{\frac{5\pi}{56}} \sin 2\iota B_z^{(0)}, \quad (18)$$
based on the above expressions we have a brief picture of some examples of polarizations of perturbed SLF-EMWs shown in Fig. 9. These figures indicate that the polarization of the EM counterparts of the perturbed SLF-EMWs obviously depends on not only the amplitudes of nontensorial GWs ($V_{c_e}$, $A_{V_c}$) and the inclination $\iota$, but also on the levels of background magnetic fields and their directions.

Therefore, if any specific polarization of the perturbed SLF-EMWs would be captured and recognized, we could reversely extrapolate the possible combination of proportions of all polarizations (including nontensorial ones) of the GWs from binary mergers. Here, only some simplified cases are presented, and further studies considering more parameters to influence the polarizations of perturbed SLF-EMWs will be carefully and detailedly addressed in other works.

V. POSSIBLE WAYS OF DETECTION FOR PERTURBED SLF-EMWS CAUSED BY GWs FROM BINARY MERGERS

The perturbed SLF-EMWs caused by GWs of binary mergers, will be in the same frequencies of such GWs, i.e., usually defined as super-low-frequency (SLF) band (mainly around $\sim 10^{14}$ Hz to $\sim 10^{15}$ Hz) in the studies of EMWs. SLF waves have very strong ability of penetration rather than EMWs in some much higher frequency bands, e.g., visible light. Hence, in the past decades, the United States, Russia and India have built some huge radio transmitters using SLF waves to communicate with the submarines. Also, SLF detections have been widely used in earthquake forecasting and mining, etc. For normal situations, requirements for receivers of SLF-EMWs are much more relaxed than that of SLF transmitters, so some small antennas or coils can be used to capture SLF signals, even by many radio amateurs.

However, attempting to detect the weak signals of magnetic components of the perturbed SLF-EMWs caused by the GWs from binary mergers, which are in level around $\sim 10^{-12}$ to $\sim 10^{-17}$ Tesla (see Sect. 11 and Table 1), ultra-sensitive installments are required. Fortunately, SLF detectors and magnetometers already with or approaching such sensitivity have appeared in recent years by rapidly developing cutting-edge methods such as those based on atoms, cesium vapor, rubidium vapor, diamond, superconducting quantum interference device (SQUID), spin wave interferometer, coils, antennas and so on.[55–70]

As addressed in section 11 for the case that the neutron star of the binary is a magnetar, the magnetic components of perturbed SLF-EMWs have strengths of $\sim 10^{-13}$ Tesla at the Earth, and thus such signals are already sit in the sensitivity range of currently existing magnetometers, see Refs. 52–54, 57, 67, 70, 72, 83 and so on. However, in order to integrate these science and technology of highly sensitive magnetometry into the frame of a novel type of GW detectors, it still involves many specific engineering and technical issues, which will be investigated in subsequent studies.

If the SLF detectors and magnetometers are placed in space, e.g. in spacecrafts like satellites or orbital space station, the noise will be largely
FIG. 6: Examples of polarizations of perturbed SLF-EMWs caused by binary merger GWs having both tensorial and nontensorial modes by different focused parameters (influence of other parameters are ignored here) based on the connection relationship of Eq. (18) in Sect. IV. The parameters (inclination angle, background magnetic fields, relative amplitudes of nontensorial GWs in vector modes) of every subfigure specifically means: $\psi, |B_x^0|, |B_y^0|, |B_z^0|, A_{V_x}, A_{V_y}$. 
reduced, and besides, the characteristic waveforms (would be predicted by massive numerical calculations) and particular polarizations of the perturbed SLF-EMWs, will carry some unique information and provide distinct features helpful to extract and distinguish the signals out of the noise, and relevant signal processing methods are similar to the methods for searching GW signals from data of LIGO, Virgo, KAGRA, etc by using matched filtering based on waveform templates.

The SLF detectors or magnetometers would have much lower cost rather than usual GW detectors and many of them are already installed on different sites on the Earth. Thus, some of them could work by current situation or after necessary updates. If a network of such observers both land and space based would be built up, their multi-detections with cross-checkings would largely improve the sensitivity and detectability, and will be well complementary to other GW detectors.

Here we only have a very brief and preliminary discussion on the possibility of the detections, and specific issues about experimental schemes, including extracting and distinguishing such signals of perturbed SLF-EMWs and relevant technique problems, should be detailedly addressed in other works.

VI. SUMMARY, DISCUSSION AND CONCLUSION

Summary:

(1) Perturbed super-low-frequency EMWs ($\sim 10^3$ to $\sim 10^2$ Hz band, caused by GWs of binary mergers interacting with ultra-strong magnetic fields of neutron stars in the same binary) as a new type of signals and special EM counterparts of GWs, have been estimated for various cases: tensorial-dipole [tensorial GWs interacting with dipole magnetic fields, Eq. (10)], nontensorial-dipole [Eq. (11)], tensorial-quadrupole [Eq. (12)] and nontensorial-quadrupole [Eq. (13)] cases. The strengths of magnetic components of these perturbed SLF-EMWs may also suggest a possible means of binary mergers. Precise predictions of such characteristic waveforms could be carried out in other separated articles by massive numerical simulations.

(2) Due to the amplification process of magnetic fields of binary mergers, which can be greatly amplified by more than 3 order of magnitude and easily reaching $10^{16}$ G or higher, the characteristic waveforms of the perturbed SLF-EMWs will be modified and have very unique shapes helpful for distinguishing and extracting such signals from background noise using method of matched filtering based on waveform templates. Inversely, detection of SLF-EMWs may also suggest a possible means to investigate the evolution (including the amplification process) of the magnetic fields during the binary mergers. Precise predictions of such characteristic waveforms could be carried out in other separated articles by massive numerical simulations.

(3) Specific relationship of how the polarizations of perturbed SLF-EMWs connect to the tensorial and possible nontensorial polarizations of GWs from binary mergers, are addressed [Eq. (18) and Fig. 6]. If such particular polarizations would be captured and recognized, we could reversely extrapolate the possible proportions of all polarizations (including nontensorial ones) of the GWs from binary mergers.

(4) The perturbed SLF-EMWs offer us a new type of EM counterparts of GWs from binary mergers. Such signals and EM counterparts have very characteristic waveforms and particular polarizations, and sit in totally different frequency band rather than usual EM counterpart of GRBs.

Besides, importantly, for EM counterparts such as GRBs, it is usually assumed that the EMWs and GWs are generated at the same time, but actually, there is still some unknown uncertainty (regarding the relative timing of emissions between GWs and EMWs) would impact the analysis based on the difference of arrival times between the EM and GW signals; differently, for the case of EM counterparts of the perturbed SLF-EMWs, such uncertainty would be reduced or avoided, because under the frame of electrodynamics in curved space-time, they just clearly have the same start time to synchronously propagate outward from the source of binary. Therefore, if there is some difference of arrival times between the GWs and SLF-EMWs, it would provide more accurate information underlying researches for some very important issues such as extra-dimensions of space, inflation, large-scale
structure of Universe, measurement of cosmological parameters (e.g. local Hubble constant), speed and mass of photons and gravitons, Lorentz violations in gravity, and some other crucial properties of gravity and Universe.

Discussion:

(1) Detection of the perturbed SLF-EMWs suggests a possible new way to observe the GWs from binary mergers, and such way has different effects to probe both tensorial and nontensorial polarizations of GWs, which relevant to fundamental issues of modified gravity, extra-dimensions of space and so on. The LIGO, Virgo and KAGRA, etc will form an observatory network to seek more information of polarizations (including possible additional ones) of GWs. However, detection of perturbed SLF-EMWs would reflect special connection relationship between the polarizations of the perturbed SLF-EMWs and the GWs, and thus might provide different new information relevant to the possible nontensorial modes.

(2) Detection of the perturbed SLF-EMWs could be complementary to (and cross-checked with) other GW detections by LIGO, Virgo, KAGRA and so on. It would be able to provide new information to reduce the uncertainty of the source position, and also may allow a more relaxed parameter space for detections. As the multi-messenger observations, information of GWs and EMWs can be mutual references, e.g. if other GW detectors find any GW event in prior, we could search corresponding SLF EM signals based on the data of the detected GWs. Besides, detection of the perturbed SLF-EMWs would have comparatively much lower cost, and more detectors could be built up or updated from currently existent magnetometers on the Earth or in spacecrafts to form a detector network for cross-checked multi-observations.

(3) Detection of the perturbed EMWs would find signals of GWs in broader frequency bands rather than the range of 1 – 1000 KHz, i.e., may not be limited to the SLF band, due to that the ultra-strong surface magnetic fields of neutron stars could interact to the extended frequency bands depending on the frequencies of GWs. E.g., the neutron star-neutron star binary mergers would produce GWs including frequency components over 1000 Hz.

(4) The calculation in this article can also similarly apply to the GW sources of binary inspiral, or for single spinning magnetars with asymmetric mass distribution, etc. Such continuous GWs will have different characteristic waveforms rather than the transient GWs of binary mergers, and the polarizations of corresponding perturbed SLF-EMWs could also have distinguishable features, depending on the polarizations (including both tensorial and nontensorial) of these continuous GWs and parameters such as the inclination angle. Also, for cases of continuous GWs, usually there is no common EM counterparts such as GRBs, but the special new type of EM counterpart of the perturbed SLF-EMWs would have detectable levels at the Earth given similar parameters addressed in the Sect. [1].

(5) We here propose the general frame of this possible way of detection for GWs and corresponding special EM counterparts. This frame might suggest us a multi-disciplinary direction of interesting studies involving many aspects of the GWs, astronomy, astrophysics, weak EMW detection, magnetometer, numerical GR, gravity, cosmology, extra-dimensions, etc. This proposal can be extended into extremely low frequency band of the EMWs (sometimes defined as 3Hz to 30Hz, or 3Hz to 3kHz, or also including some frequencies below 3Hz, in brief, an extension to the SLF to cover more GWs sources in broader frequency range), so we could generally call this frame as OURSELF (Observations Using Radiations in Super- and Extremely-Low Frequencies).

Conclusion:

Detection of the perturbed SLF-EMWs would be a potential novel way to observe the GWs from binary mergers having both tensorial and possible nontensorial polarizations, by already-existing and very fast developing techniques of SLF detectors and magnetometers, and such unique signals and also special EM counterparts of the GWs, would bring us some different new information of fundamental properties of the gravity and Universe that other GW detections may not provide.

ACKNOWLEDGMENTS

This work is supported by National Natural Science Foundation of China (Grant No.11605015, No.11375279, No.11873001, No.11847301), Funda-
B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 061102 (2016).
[2] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 241103 (2016).
[3] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 118, 221101 (2017).
[4] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), The Astrophysical Journal Letters 851, L35 (2017).
[5] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119, 141101 (2017).
[6] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 119, 161101 (2017).
[7] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), arXiv:1811.12907 [astro-ph.HE].
[8] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration). Phys. Rev. X 6, 041015 (2016).
[9] V. Connaughton, E. Burns, A. Goldstein, et al., The Astrophysical Journal Letters 826, L6 (2016).
[10] J. Abadie et al., Astronomy & Astrophysics 539, A124 (2012).
[11] Z. Bagoly et al., Astronomy & Astrophysics 593, L10 (2016).
[12] J. Greiner, J. M. Burgess, V. Savchenko, and H.-F. Yu, The Astrophysical Journal Letters 827, L38 (2016).
[13] S. J. Smartt et al., Nature 551, 75 (2017).
[14] B. P. Abbott et al., The Astrophysical Journal Supplement Series 225, 8 (2016).
[15] P. S. Cowperthwaite, E. Berger, M. Soares-Santos, et al., The Astrophysical Journal Letters 826, L29 (2016).
[16] H. Yu, B.-M. Gu, F. P. Huang, Y.-Q. Wang, X.-H. Meng, and Y.-X. Liu, Journal of Cosmology and Astroparticle Physics 2017, 039 (2017).
[17] S. Nissanke, M. Kasiwil, and A. Georgiev, The Astrophysical Journal 767, 124 (2013).
[18] B. Kocsis, Z. Haiman, and K. Menou, The Astrophysical Journal 684, 870 (2008).
[19] C. Palenzuela, L. Lehmer, and S. Yoshida, Phys. Rev. D 81, 084007 (2010).
[20] D. Lazzati, A. Deich, B. J. Morsony, and J. C. Workman, Monthly Notices of the Royal Astronomical Society 471, 1652 (2017).
[21] G. P. Lamb and S. Kobayashi, Monthly Notices of the Royal Astronomical Society 472, 4953 (2017).
[22] R. Takahashi, The Astrophysical Journal 835, 103 (2017).
[23] V. B. Braginsky, L. P. Grishchuk, A. G. Doroshkevich, Y. B. Zeldovich, I. D. Novikov, and M. V. Sazhin, Zh. Eksp. Teor. Fiz 65, 1729 (1973).
[24] D. Boccaletti, V. De Sabbata, P. Fortint, and C. Guadni, Nuovo Cim. B 70, 129 (1970).
[25] W. K. DeLogi and A. R. Mickelson, Phys. Rev. D 16, 2915 (1977).
[26] P. Chen, Stanford Linear Accelerator Center Report(SLAC-PUB-6666), 379 (1994).
[27] H. N. Long, D. V. Soa, and T. A. Tuan, Physics Letters A 186, 382 (1994).
[28] F. Y. Li, M. X. Tang, J. Luo, and Y. C. Li, Phys. Rev. D 62, 044018 (2000).
[29] F. Y. Li, M. X. Tang, and D. P. Shi, Phys. Rev. D 67, 104008 (2003).
[30] F. Y. Li, R. M. L. Baker, Jr., Z. Y. Fang, G. V. Stephenson, and Z. Y. Chen, Eur. Phys. J. C 56, 407 (2008).
[31] F. Y. Li, N. Yang, Z. Y. Fang, R. M. L. Baker, G. V. Stephenson, and H. Wen, Phys. Rev. D 80, 064013 (2009).
[32] F. Y. Li, H. Wen, and Z. Y. Fang, Chinese Physics B 22, 124042 (2013).
[33] J. Li, K. Lin, F. Li, and Y. Zhong, General Relativity and Gravitation 43, 2209 (2011).
[34] H. Wen, F. Y. Li, and Z. Y. Fang, Phys. Rev. D 89, 104025 (2014).
[35] H. Wen, F. Y. Li, Z. Y. Fang, and A. Beckwith, The European Physical Journal C 74, 2998 (2014).
[36] F. Y. Li, H. Wen, Z. Y. Fang, L. F. Wei, Y. W. Wang, and M. Zhang, Nuclear Physics B 911, 500 (2016).
[37] H. Wen, F.-Y. Li, J. Li, Z.-Y. Fang, and A. Beckwith, Chinese Physics C 41, 125101 (2017).
[38] H. Zheng, L. F. Wei, H. Wen, and F. Y. Li, Phys. Rev. D 98, 064028 (2018).
[39] F.-Y. Li, H. Wen, Z.-Y. Fang, D. Li, and T.-J. Zhang, arXiv:1712.00766 [gr-qc] (2018).
[40] F. Zhang, Phys. Rev. D 94, 024048 (2016).
[41] K.-H. Räder, H. Fuchs, U. Gerbert, M. Rheinhardt, and T. Zannias, Phys. Rev. D 64, 083008 (2001).
[42] F. A. Rasio and S. L. Shapiro, Classical and Quantum Gravity 16, R1 (1999).
[43] D. J. Price and S. Rosswood, Science 312, 719 (2006).
[44] R. Ciolfi, W. Kastaun, B. Giacomazzo, A. Endrizzi, D. M. Siegel, and R. Perna, Phys. Rev. D 95, 063016 (2017).
[45] K. Kiuchi, K. Kyutoku, Y. Sekiguchi, M. Shibata, and T. Wada, Phys. Rev. D 90, 041502 (2014).
[46] K. Kiuchi, P. Cerdá-Durán, K. Kyutoku, Y. Sekiguchi, and M. Shibata, Phys. Rev. D 92, 124034 (2015).
[47] K. Kiuchi, K. Kyutoku, Y. Sekiguchi, and M. Shibata, Phys. Rev. D 97, 124039 (2018).
[48] K. Dionysopoulou, D. Alic, and L. Rezzolla, Phys. Rev. D 92, 084064 (2015).
[49] J. Zrake and A. I. MacFadyen, The Astrophysical Journal Letters 769, L29 (2013).
[50] A. Endrizzi, R. Ciolfi, B. Giacomazzo, W. Kastaun, and T. Kawamura, Classical and Quantum Gravity 33, 164001 (2016).
[51] B. Giacomazzo, J. Zrake, P. C. Duffell, A. I. MacFadyen, and R. Perna, The Astrophysical Journal 809, 39 (2015).
[52] D. M. Eardley, D. L. Lee, A. P. Lightman, R. V. Wagoner, and C. M. Will, Phys. Rev. Lett. 30, 884 (1973).
[53] D. M. Eardley, D. L. Lee, and A. P. Lightman, Phys. Rev. D 8, 3308 (1973).
[54] H. Takeda, A. Nishizawa, Y. Michimura, K. Nagano, K. Komori, M. Ando, and K. Hayama, Phys. Rev. D 98, 022008 (2018).
[55] http://physics.princeton.edu/romalis/magnetometer/.
[56] J. M. Taylor, P. Cappellaro, L. Childress, L. Jiang, D. Budker, P. R. Hemmer, A. Yacoby, R. Walsworth, and M. D. Lukin, Nature Physics 4, 810 (2008).
[57] D. Budker and M. Romalis, Nature Physics 3, 227 (2007).
[58] I. K. Kominis, T. W. Kornack, J. C. Allred, and M. V. Romalis, Nature 422, 596 (2003).
[59] J. C. Allred, R. N. Lyman, T. W. Kornack, and M. V. Romalis, Phys. Rev. Lett. 89, 130801 (2002).
[60] D. Drung, C. Abmann, J. Beyer, A. Kirste, M. Peters, F. Ruede, and T. Schurig, IEEE Transactions on Applied Superconductivity 17, 699 (2007).
[61] H. B. Dang, A. C. Maloof, and M. V. Romalis, Applied Physics Letters 97, 151110 (2010).
[62] S.-K. Lee, K. L. Sauer, S. J. Seltzer, O. Alem, and M. V. Romalis, Applied Physics Letters 89, 214106 (2006).
[63] D. Sheng, S. Li, N. Dural, and M. V. Romalis, Phys. Rev. Lett. 110, 160802 (2013).
[64] S. Groeger, G. Bison, J.-L. Schenker, R. Wynands, and A. Weis, The European Physical Journal D - Atomic, Molecular, Optical and Plasma Physics 38, 239 (2006).
[65] R. Mhaskar, S. Knappe, and J. Kitching, Applied Physics Letters 101, 241105 (2012).
[66] Y. Ito, H. Ohnishi, K. Kamada, and T. Kobayashi, IEEE Transactions on Magnetics 47, 3550 (2011).
[67] R. J. Cooper, D. W. Prescott, P. Matz, K. L. Sauer, N. Dural, M. V. Romalis, E. L. Foley, T. W. Kornack, M. Monti, and J. Okamitsu, Phys. Rev. Applied 6, 064014 (2016).
[68] E. Breschi, Z. D. Gruji?, P. Knowles, and A. Weis, Applied Physics Letters 104, 023501 (2014).
[69] R. J. France, T. D. Clark, and H. Francke, Review of Scientific Instruments 74, 3735 (2003).
[70] I. Savukov, T. Karaulanov, and M. G. Boshier, Applied Physics Letters 104, 023504 (2014).
[71] C. Granata, A. Vettolieri, and M. Russo, Applied Physics Letters 91, 122500 (2007).
[72] Z. D. Gruji?, P. A. Koss, G. Bison, and A. Weis, The European Physical Journal D 69, 135 (2015).
[73] V. G. Lucivero, P. Anielski, W. Gawlik, and M. W. Mitchell, Review of Scientific Instruments 85, 113018 (2014).
[74] O. Alem, K. L. Sauer, and M. V. Romalis, Phys. Rev. A 87, 013413 (2013).
[75] A. Kulak, J. Kubisz, S. Klucjasz, A. Michalec, J. Młynarczyk, Z. Nieckarz, M. Ostrowski, and S. Zieba, Radio Science 49, 361 (2014).
[76] M. Balynsky, D. Gutierrez, H. Chiang, A. Kozhevenkov, G. Dudko, Y. Filimonov, A. A. Balandin, and A. Khitun, Scientific Reports , 11539 (2017).
[77] C. Thompson, The Astrophysical Journal 688, 1258 (2008).
[78] W. Kastaun, R. Ciolfi, A. Endrizzi, and B. Giacomazzo, Phys. Rev. D 96, 043019 (2017).
[79] T. Kawamura, B. Giacomazzo, W. Kastaun, R. Ciolfi, A. Endrizzi, L. Baiotti, and R. Perna, Phys. Rev. D 94, 064012 (2016).
[80] Z. Cao, Phys. Rev. D 91, 044033 (2015).
[81] B. Sun, Z. Cao, Y. Wang, and H.-C. Yeh, Phys. Rev. D 92, 044034 (2015).
[82] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 120, 031104 (2018).
[83] M. Isi and A. J. Weinstein, arXiv:1710.03794 [gr-qc] (2017).
[84] B. P. Abbott et al. (LIGO Scientific Collaboration and Virgo Collaboration), Phys. Rev. Lett. 116, 241102 (2016).
[85] M. E. Limes, D. Sheng, and M. V. Romalis, Phys. Rev. Lett. 120, 033401 (2018).