Atomic quantum memory for multimode frequency combs

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We demonstrate a Raman quantum memory scheme that uses several atomic ensembles to store/retrieve the multimode highly entangled state of an optical quantum frequency comb, such as the one produced by parametric down-conversion of a pump frequency comb. We analyse the efficiency and the fidelity of such a quantum memory. Results show that our proposal may be helpful to multimode information processing using the different frequency bands of an optical frequency comb.

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Quantum information is a fascinating subject, as it makes use of the deepest aspects of quantum theory, which is inherently an information theory. When the information is carried by quantum states belonging to a high-dimensional Hilbert space, for example by quantum states of highly multimode light, one expects a potential significant increase in the information capacity. In the toolbox of quantum information processing, one of the most important tools is the Quantum Memory, without which all the advantages of quantum information cannot be fully exploited. Quantum memories are actively studied, both at the theoretical and experimental level, because they constitute an important resource in long distance quantum communication.

Up to now these studies concerned mainly quantum states contained in a single pulse of single transverse mode light, i.e. single mode configurations. They have been recently extended to some kinds of multimode fields, either in the spatial or temporal domain. In the spatial domain, the quantum aspects of highly multimode light have been actively studied in the past years. In particular spatially multimode quantum memories have been designed and built. In the temporal domain, multimode quantum memories have been developed to store long pulses of different mean frequencies. Another possibility is to store different modes consisting of different time slots or different temporal shapes of short light pulses, for which efficient pulse shaping techniques exist. This is the reason why we have chosen to explore the problem of storing quantum states of "optical frequency combs": highly multimode quantum frequency combs have indeed been recently experimentally implemented, and have been shown to exhibit genuine multipartite entanglement. Such highly multimode quantum states are a promising resource for quantum information processing and measurement-based quantum computing.

An ideal optical frequency comb consists of a series of periodic phase coherent short pulses, which correspond in the frequency domain to a series of equally spaced phase-locked monochromatic components, hence the name of "combs". Typically the frequency difference between neighboring teeth $\omega_r/2\pi$ is on the order of 100 MHz to 1 GHz, whereas the teeth extend over a spectral domain of 10 nm for 100 femtosecond pulses in the visible, and 0.1 nm for 10 picosecond pulses. Let us note that the number of teeth, and therefore of spectral modes, is very large, between $10^3$ and $10^6$, so that this kind of light is likely to carry a very large number of frequency modes. Note that the memory for optical frequency combs that we consider here must not be confused with the atomic frequency comb memory.

The aim of our study is to see whether the multipartite entanglement between the frequency modes can be transferred to an atomic ensemble. This is non-trivial since 1. the bandwidth associated with the frequency combs far exceeds the typical bandwidth of atomic systems; 2. an intrinsic multimode state may not be stored/retrieved by an intrinsic single mode drive. Nevertheless we show here that by choosing a suitable set of classical drives it is still possible to map the phase correlations of the optical frequency comb into the atomic memories, and to retrieve them in the reading process.

We consider an extended ensemble of $N_a$ three-level atoms in the Λ configuration with dipole $\hat{d}$, uniformly contained in a volume of length $L$ elongated in the
FIG. 1. a. short pulses interacting with atomic ensemble; b. 3-level $\Lambda$-type atoms coupled to comb fields via perfect 2-photon resonant channels in off-resonant Raman configuration.

As known for long pulses, the signal field can be stored in the spatial dependence of the coherence between the ground state $|g\rangle$ and the metastable state $|h\rangle$ of an ensemble of atoms via a 2-photon Raman transition induced by a classical pump comb field $\hat{E}_p^{(+)}(z,t) = \alpha_p \sum m p_m e^{-i\omega_m z - \gamma t}$. In this expression $\alpha_p$ is the field amplitude and $\sum m |p_m|^2 = 1$. We assume perfect Raman resonance and a large single photon detuning $\Delta = \omega_e - \omega_s$, where $\omega_e$ is the Bohr frequency of the excited state $|e\rangle$ with linewidth $\gamma$, (see Fig. [1b]).

By destroying a photon in signal comb field and emitting in a stimulated way a photon in the other, the spectral coherence of the comb will be transferred to the $|g\rangle|h\rangle$ coherence. To describe this process, let’s introduce a collective atomic operator

$$\hat{b}(z,t) = -1 \frac{e^{-i(\omega_e - \omega_s) t}}{\sqrt{N_0}} \sum_{j=1}^{N_0} |g\rangle_j |h\rangle_j,$$  

and the single-photon field

$$|\psi(t)\rangle = \hat{b}^\dagger(z,t) |\psi_0\rangle,$$

with $|\psi_0\rangle$ the initial state of the system.

If the number of photons in the input signal field is much less than the number of atoms, and all atoms are initially in the ground state, the coupled atom-field equations are

$$\partial_t \hat{a}(z,t) = -\frac{i}{\hbar} [\hat{H}_{\text{atom}} + \hat{H}_{\text{comb}}, \hat{a}(z,t)] + \frac{1}{\sqrt{N_0}} \sum_{j=1}^{N_0} |g\rangle_j \langle h|^2, \langle h|^z \hat{a}(z,t) \rangle = 0,$$

where $z$ labeling all the atoms in a layer of infinitesimal thickness $dz$ at $z$, (see Fig. [1a]). The vanishing equal-time bosonic commutation relation of collective operators of slices at $z$ and $z'$ obeys $[\hat{b}(z,t), \hat{b}^\dagger(z',t)] \neq 0$. The number of photons in the input signal field is much less than the number of atoms, and all atoms are initially in the ground state, the coupled atom-field equations are

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pulse case \cite{23}, the actual interaction time during the comb storage is much shorter than the pulse duration $T$, which counter-intuitively but advantageously avoids the constraint for the induced decay rate $\gamma_s$ being smaller than $1/T$.

Let’s now turn to the reading process, in which another pump field $p_{\text{read}}$ is applied to the atomic medium. The signal field annihilation operator column vector at $z = 1$ reads

$$s_{\text{out}}(t) = -p_{\text{read}} e^{-\gamma_s t} \sqrt{\frac{\alpha}{T}} \int_0^T dy J_0(2\sqrt{\frac{1-y}{T}}) \hat{b}(y, T).$$

Taking $p_{\text{read}} = p$ (reading beam identical to writing beam), the input-output relation in the Fourier frequency domain turns out to be

$$\hat{s}_{\text{out}}(\omega) = -K_\omega e^{i\omega T} \hat{p} \hat{p}^H \hat{s}_{\text{in}}(\omega),$$

$$K_\omega = 1 - e^{-\frac{\gamma_s T}{\pi \omega + 2\pi}},$$

where the underlines indicate Fourier transforms. The conclusion is that one can indeed retrieve at the output a copy of the input field, but with the constraint that the retrieved light is in the single mode $p$ because of the projector $\hat{p} \hat{p}^H$. At large optical depth $d$, $K_\omega \to 1$, therefore $\hat{s}_{\text{out}}(\omega) \to -e^{i\omega T} \hat{p} \hat{p}^H \hat{s}_{\text{in}}(\omega)$. This represents the first result of this paper: one is able to store and retrieve a squeezed frequency comb of any modal shape by choosing the proper pump field shape.

Let us now consider the problem of storing and retrieving a multimode frequency comb, more precisely, as in the experimental situation \cite{10, 12, 13}, a quantum state consisting of $M$ uncorrelated squeezed states defined in orthonormal specific supermodes $|\psi_m^{\text{in}}\rangle_{k=1}^M$. For this purpose, we will use $M$ atomic ensembles interacting in a row with the same signal comb, each one being pumped by a different pump field mode $|p_k\rangle_{k=1}^M$, which span the same sub-Hilbert space as the one by $|\psi_k\rangle_{k=1}^M$.

$$\sum_{m=1}^M \psi_m \psi_m^H = \sum_{m=1}^M p_m p_m^H = I_M,$$

where $I_M$ is the identity operator acting on the sub-Hilbert space, (see Fig 2). The signal field and the pump comb fields are assumed to be in orthogonal polarizations, so that they can be mixed and separated by polarizing beam splitters. Owing to the orthogonality between different pump modes, the total input-output relation is a generalization of Eq. (6a),

$$\hat{s}_{\text{out}}^{\text{in}}(\omega) = -K_\omega e^{i\omega T} \sum_{k=1}^M p_k p_k^H \hat{s}_{\text{in}}^{\text{in}}(\omega)$$

$$= -K_\omega e^{i\omega T} \hat{I}_M \hat{s}_{\text{in}}^{\text{in}}(\omega).$$

Therefore, the covariance matrices $C^{\text{in}}(\omega)$ of the write-in beam and the read-out beam yields

$$\hat{\varepsilon}_{2M} - C_{\text{out}}(\omega) = |K_\omega|^2 |\hat{\varepsilon}_{2M} - C_{\text{in}}(\omega)|.$$

This relation constitutes the main result of this letter: the two terms related to identity matrix $\hat{\varepsilon}_{2M}$ in $\varepsilon_{2M}$ represents the contributions from vacuum inputs of atoms and light, as well as from the fluctuations of Langevin noises; while $|K_\omega|^2 C_{\text{in}}(\omega)$ shows the amount of squeezing and anti-squeezing remaining at the read-out. The scaling factor $|K_\omega|^2$ is insensitive to Fourier frequency $\omega$ when $|\omega| \ll \gamma_s$. Let us take $\gamma_s/\Delta \approx 10^{-4}$, an induced decay rate $\gamma_s \approx 50$ kHz, an averaged power of the pump comb field of 100 $\mu$W and an atomic coherence lifetime of 0.1s \cite{24}. The off-resonant Raman memory model is still valid with such a high pump power \cite{25}, for example, in the case of the $^{87}$Rb atoms, $|\Omega_p| \approx 0.07 \Delta \ll \Delta$. When the pulse duration $T$ is much greater than $\gamma_s^{-1}$ but still smaller than the atomic coherence time, for example $T = 10$ ms, all the spectral components of the slowly varying envelope of the pulse lay within $|\omega| \ll \gamma_s$, thus the scaling factor $|K_\omega|^2$ can be replaced by its value at $\omega = 0$. This is what we will do in the remaining part of the paper.

The positive eigenvalues of $\hat{\varepsilon}_{2M} - C$ reveal the squeezing in a Gaussian state \cite{21}. Under mode basis $\{|\psi_k\rangle_{k=1}^M$, the covariance matrix $C$ turns out to be $\Theta_{k=1}^M C^{\psi_k}$, which is diagonal with $C^{\psi_k}$ being the block on mode $\psi_k$. Therefore, the relation of the amount of the squeezing between read-out and write-in state is determined by $K_\omega^2$, according to Eq. (9), leads to the memory efficiency \cite{18} of any mode $\psi_m$

$$\eta = K_\omega^2 = (1 - e^{-d})^2$$

close to 1 even for moderate values of the on-resonance optical depth. The amount of squeezing $\zeta$ in the retrieved
served in the write/read process when $\eta$ is close to 1. The product of field quadrature variances of $m$-th supermode, $\det(C_n^{\text{out}}) = 1 + \eta(1-\eta)[\text{Tr}(C_n^{\text{out}} - \text{diag}(1, -1))^2$, attains the Heisenberg limit 1 only at large optical depth, meaning that the purity of the read-out state in mode $\psi_m$ slightly degraded, in the same proportion as the loss of squeezing. Finally the fidelity $F = (\text{Tr}(\rho_m^{\text{out}}))^{1/2}$ for squeezed vacuum states on mode $\psi_m$ is

$$F_m = 2[4 + (1-\eta^2)\text{Tr}(C_n^{\text{in}} - \mathbb{I}_2)^{-1/2}. \quad (12)$$

It depends not only on the optical depth but also on the variances of the initial signal comb field. Ideally, when the optical depth is large, or the initial state is a multimode coherent state, the fidelity is close to 1.

To illustrate the protocol in action we consider a set of memories working with an intrinsic $M$-mode pure squeezed vacuum state that is generated by a SPOPO device below but close to threshold [26], and the squeezing data in each supermodes of different states see top graph in Fig. 3. The purity, squeezing, and fidelity of each supermode of different retrieval states at optical depth 4, are shown in the remaining 3 graphs in Fig. 3. To have better purity, squeezing and higher fidelity of the retrieved light, one can choose ensembles with larger optical depth, for example, $d \approx 20$ [18], 1800 [7].

Any pure multimode Gaussian state can be reduced by a mode transformation as a tensor product of squeezed vacuum states on different orthogonal modes [21, 27]. Therefore, the previous scheme for storing several uncorrelated squeezed vacuum states in different modes is also a scheme for storing multimode entanglement in another mode basis. We thus have the ability to store multimode entanglement by using this scheme.

In this work, we have shown a possible way to store the multipartite entanglement in quantum optical frequency comb state, by using a set of memories driven with different classical comb light beams. We also showed that the purity, fidelity and efficiency are very close to 1 at large, but experimentally feasible optical depth, and that the crucial element to store and retrieve a multimode state is the optical depth. We have also shown that it is possible to manipulate a large set of entangled modes in signal light by using a series of independent atomic ensembles.

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