Modelling, Simulation and Flight Test of a Model Predictive Controlled Multirotor with Heavy Slung Load

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Abstract: A controller for stable flight and precise tracking of a multirotor unmanned aerial vehicle (UAV) carrying a heavy slung load is presented within this paper. A novel mathematical model for the multi-body system is derived. Based on that model, a Model Predictive Control (MPC) scheme is designed and applied to the system. Stability and tracking ability are demonstrated through numerical simulation. The performance of the system using the MPC strategy is compared to a linear-quadratic regulator (LQR) control approach. The simulation results are then verified by real flight tests, whereby the MPC is applied to a real multirotor UAV with a heavy slung load. The system is capable of actively damping load oscillations whilst simultaneously tracking a reference trajectory.

Keywords: Unmanned Aerial Vehicle, Multirotor, Slung Load, Model Predictive Control

1. INTRODUCTION

Due to their special flight characteristics, multirotor UAVs are well suited to deliver loads even in a range of operational environments. Highly autonomous platforms may also prove to be a cost effective alternative to existing delivery methods. However, autonomous landing in complex and unknown terrain can become hazardous. Carrying suspended loads might be a suitable approach to delivery that can overcome such issues. Using suspended loads could also benefit other other applications as well, particularly in the agriculture, plant biosecurity and fire fighting sectors. For example, close proximity measurements could be taken by sensors placed underneath a multirotor flying at a safe height above the plants. Their is also the possibility of collecting gas and volatile organic compound sensor data uninfluenced by rotor downwash. The flight control strategies presented within this paper could be extended to manned operations such as those used to combat fires.

Regardless of the application or platform, it is important to ensure stable flight characteristics and precise positioning of both the vehicle and the load. The stability and tracking performance of the system is highly affected by the dynamics of the slung load. Aggressive flight manoeuvres that result in large load oscillations can cause instability. The Model Predictive Control (MPC) approach directly takes the dynamics of the coupled system into account. Constraints can be set on both the state variables and the control input. Thus, the risk of undesirable or dangerous flight conditions is minimised. Solving an optimal control problem (OCP) subject to the predicted system behavior in the future, the MPC approach also offers superior tracking of predefined, four-dimensional trajectories.

Some research has focused on controller design for manned helicopters carrying an external load. A simple anti-swing controller for a conventional helicopter with a slung load is presented by Omar [2009]. Optimal control strategies for a helicopter slung load system are designed and simulated in Oktay and Sultan [2013]. A cascaded control algorithm for a quadrotor UAV with a cable suspended load is developed by Sreenath et al. [2013]. Simulation results for two-dimensional movement of a coupled system controlled by several control algorithms including MPC approaches are presented by Trachte et al. [2014].

The contributions and the structuring of this work are as follows: A novel, three-dimensional model describing the dynamics of a coupled system including a multirotor UAV with heavy slung load is presented in section 2. The model takes the coupling into account without making simplifications in that regard. Even so, a closed-form representation of the system dynamics is obtained. Based on this model, an MPC strategy is derived. Both the basic principle of Model Predictive Control and the specific implementation are set out in section 3. For the given implementation, simulation results indicate the MPC scheme’s superior ability for tracking a predefined, time-dependent reference in comparison to a system using an LQR control approach. The simulation results are shown in section 4. Feasibility and performance of the control strategy are demonstrated through flights tests using a real indoor quadrotor setup. Both the experimental setup and the flight test results are presented in section 5.
The inertial frame of reference is denoted by $I = \{x_I, y_I, z_I\}$, where $z_I$ is the unit vector parallel to the gravity vector. The body fixed frame of reference $F = \{x_F, y_F, z_F\}$ is related to the inertial frame by the rotation matrix $R : F \to I$. The inertial frame of reference and the body fixed frame of reference are illustrated in Fig. 1. The attitude vector is denoted by $\phi = (\phi, \theta, \psi)$. The rotation matrix $R$ as stated in equation (1) is defined by the transposed composition of sequential rotations about the yaw angle $\psi$, the pitch angle $\theta$, and the roll angle $\phi$.

\[
R = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(\phi) & \sin(\phi) \\
0 & -\sin(\phi) & \cos(\phi)
\end{bmatrix} = \begin{bmatrix}
\cos(\theta) & 0 & -\sin(\theta) \\
0 & 1 & 0 \\
\sin(\theta) & 0 & \cos(\theta)
\end{bmatrix} = \begin{bmatrix}
\cos(\psi) & \sin(\psi) & 0 \\
-\sin(\psi) & \cos(\psi) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(1)

The position vector pointing to the centre of gravity of the vehicle is denoted by $r = (x, y, z)$. The vectors $v = (u, v, w)$ and $\omega = (p, q, r)$ are the linear and angular velocity vectors in the body fixed frame. The equations of motion of the multirotor can be expressed as

\[
\dot{r} = Rv, \\
\dot{v} = -\omega \times v + gR^Tz_F + \frac{1}{m_F} \left( R(\mathbf{f}_H + \mathbf{f}_D) - f_Tz_F \right), \\
\dot{\phi} = J\omega,
\]

(2)

(3)

(4)

where $g$ is the gravitational acceleration and $m_F$ is the mass of the multirotor. The aerodynamic drag force of the vehicle is denoted by $f_D$. The collective thrust $f_T$ of the rotors is given by

\[
f_T = \sum_{i=1}^{N_R} f_i,
\]

(5)

where $N_R$ is the number of rotors of the vehicle. As the aerodynamic drag force of the vehicle is small compared to the thrust of the multirotor, it is neglected hereafter. $\mathbf{f}_H$ is the force acting on the suspension point due to the load, which will be derived in the following subsection. The matrix $J$ converts the rates of the Euler angles to the body system. It is given by:

\[
J = \frac{1}{\cos(\theta)} \begin{bmatrix}
\cos(\theta) & \sin(\theta) & \cos(\theta) & \sin(\theta)
0 & \cos(\theta) & -\sin(\theta) & \cos(\theta)
0 & \sin(\theta) & \cos(\theta) & \sin(\theta)
\end{bmatrix}
\]

(6)

Equation (3) is denoted with respect to the body fixed frame of reference in order to comply with flight mechanics conventions. All the other equations in this section are denoted with respect to the inertial frame of reference.

### 2.2 Load dynamics

The position vector of the load with respect to the inertial frame of reference is denoted by $r_L = (x_L, y_L, z_L)$. $\mathbf{v}_L = (u_L, v_L, w_L)$ is the inertial velocity of the load denoted in inertial coordinates and $\mathbf{a}_L$ is the inertial acceleration of the load. The three-degrees-of-freedom equations of motion are given by:

\[
\dot{\mathbf{r}}_L = \mathbf{v}_L, \\
\dot{\mathbf{v}}_L = \mathbf{a}_L.
\]

(7)

(8)

The position vector of the load with respect to the suspension point $r_C$ is given by

\[
r_C = r_L - (r + \mathbf{R}r_H),
\]

(9)

where $r_H$ denotes the displacement of the suspension point with respect to the center of gravity of the vehicle. The relevant position vectors for the model described in this section are computed using the transformation matrix $\mathbf{R}$.
section are illustrated in Fig. 1. Friction in both the cord and the suspension point is neglected. The inertial acceleration of the load \( \mathbf{a}_L \) is then derived by assuming equilibrium of moments about the suspension point such that

\[ \mathbf{r}_C \times \mathbf{f}_H = 0, \]

where \( \mathbf{f}_H \) describes the force acting on the multirotor due to the load:

\[ \mathbf{f}_H = -m_L \mathbf{a}_L + m_L g \mathbf{z}_h + \mathbf{f}_{D,L}. \]

The aerodynamic drag force of the load is denoted by \( \mathbf{f}_{D,L} \):

\[ \mathbf{f}_{D,L} = \frac{1}{2} C_{D,L} \rho A_L \| \mathbf{v}_L \|_2 \mathbf{v}_L. \]

3. HIGH-LEVEL MPC APPROACH

3.1 Mathematical formulation of Model Predictive Control

This subsection deals with the mathematical formulation of model predictive control schemes in general. Therefore, no assumptions regarding linear models are made. The equations presented in this section are based on the mathematical formulation of nonlinear model predictive control by Findeisen and Allgöwer [2002] adapted to discrete time-setting. The discrete-time system to be controlled is described by a nonlinear set of difference equations

\[ x_{k+1} = f(x_k, u_k), \]

where the index \( k \in \mathbb{N}_0 \) denotes the state or control input of the system at \( k^{th} \) sampling instant. Both the state vector \( x \) and the input vector \( u \) are constrained by:

\[ u \in \mathcal{U}, \forall k, \quad x \in \mathcal{X}, \forall k. \]

In the simplest form the constraints \( \mathcal{U} \) and \( \mathcal{X} \) are given by

\[ \mathcal{U} := \{ u \in \mathbb{R}^n \mid u_{\min} \leq u \leq u_{\max} \}, \]

\[ \mathcal{X} := \{ x \in \mathbb{R}^m \mid x_{\min} \leq x \leq x_{\max} \}, \]

where \( m \) denotes the dimension of the input vector and \( n \) denotes the dimension of the state vector. \( u_{\min}, u_{\max} \) and \( x_{\min}, x_{\max} \) are predefined constant vectors. The control law is represented by the discrete-time finite horizon open-loop optimal control problem (OCP) of finding an input sequence \( \mathbf{u} \) that minimises the quadratic cost functional \( J(\cdot) \) such that:

\[ \min_{\mathbf{u}} J(x_h, \mathbf{u}, T_c, T_p). \]

The cost functional is given by

\[ J(x, u; T_c, T_p) = (x_N - x_N^c)^T P (x_N - x_N^c) + \sum_{j=0}^{N-1} [(x_j - x_j^c)^T Q_x (x_j - x_j^c) + (u_j - u_j^c)^T Q_u (u_j - u_j^c)]. \]

subject to

\[ x_{j+1} = f(x_j, u_j), \quad x_0 = x_k, \]

\[ u_j \in \mathcal{U}, \quad N \leq M, \]

\[ \bar{x}_j \in \mathcal{X}, \]

where the index \( j \in \mathbb{N}_0 \) denotes the internal state vector \( \mathbf{x} \) and the internal input vector \( \mathbf{u} \) at the \( j^{th} \) prediction instant for the \( k^{th} \) sampling instant. Only the first element \( u_0 \) is implemented. \( N = T_p/\delta \) is the number of steps within the prediction horizon \( T_p \) and \( M = T_c/\delta \) is number of steps within the control horizon \( T_c \) where \( \delta \) is the prediction step size. The asterisk symbol denotes references for both state vector and input vectors respectively. \( \mathbf{P} \) is the terminal weighting matrix for deviations of the state vector at the end of the prediction horizon. In the context of this paper, it is assumed that the terminal penalty matrix equals the weighting matrix for state deviations \( \mathbf{Q}_x \). The initial value of equation (30) \( x_k \) introduces state feedback to the MPC.
Table 1. Constraints for high-level linear MPC

| Constraint subject to | Identifier | Unit | Constraint |
|-----------------------|------------|------|------------|
| Height above ground   | -z         | m    | [0, 4.0]   |
| Roll angle            | φ          | –    | [-π, π]    |
| Pitch angle           | θ          | –    | [-π, π]    |
| Vehicle attitude rates| p, q, r    | rad/s| [-π, π]    |
| Collective thrust     | f_T        | N    | [0, 14.0]  |

3.2 Implementation

To obtain a linear prediction model, equation (21) is linearised at the setpoint for steady-state hovering:

\[ x_0 = (r_0, 0, \ldots, 0, x_{L,0}, y_{L,0}, 0, 0) , \quad (33) \]

\[ u_0 = (0, f_{T,0}) . \quad (34) \]

The setpoint of the position of the vehicle \( r_0 \) is not relevant for the linearisation. The setpoint of the \( x \)-component and the \( y \)-component of the position of the load \((x_{L,0}, y_{L,0})\) is equivalent to the setpoint of the position in the \( xy \)-plane. The setpoint for the collective thrust \( f_{T,0} \) equals the gravity force of the coupled system:

\[ f_{T,0} = (m_F + m_L) g . \quad (35) \]

Whilst the reference for the input vector is constantly chosen identically to the input vector at the setpoint

\[ u^* = u_0 , \quad (36) \]

the reference for the state vector depends on the flight manoeuvre to be performed. The later yields a time-variant control law for the MPC.

The constraints for the high-level linear MPC scheme are listed in table 1. The constraint in the \( z \)-position is chosen with regard to the real flight experiment. Both the limitation of the vehicle’s roll and pitch angle and the limitation of the maximum attitude rates prevents potentially dangerous flight manoeuvres. The weighting matrices are given by:

\[ Q_x = \text{diag}(0.1, 0.1, 1, 10^{-6}, \ldots, 10^{-6}) , \quad \text{1, 1, 10^{-6}, 10^{-6}}) , \quad (37) \]

\[ Q_u = \text{diag}(10^{-2}, \ldots, 10^{-2}) . \quad (38) \]

The term \( \text{diag}(\cdot) \) refers to a matrix with the vector denoted as a parameter forming the principal diagonal of the matrix. All the other entries equal zero. Deviations in the position of the load are penalised ten times stronger than deviations in the position of the vehicle. This refers to potential real world applications where it might be desired to track the load precisely. Further controller parameters are listed in table 2.

Table 2. High-level controller parameters

| Parameter | Identifier | Unit | Value |
|-----------|------------|------|-------|
| Update rate | \( \frac{1}{T_c} \) | Hz | 80.000 |
| Prediction horizon | \( T_p \) | s | 5.0000 |
| Prediction step size | \( \delta \) | s | 0.1000 |
| Control horizon | \( T_c \) | s | 5.0000 |

4. Simulation

4.1 Simulation Environment

The simulation is set up using MATLAB and Simulink. The nonlinear dynamics given by equation (22) are used for the simulation model. The physical parameters for both the linear prediction model and the nonlinear simulation model are listed in table 3. The ACADO Toolkit is integrated to solve the OCP emerging from the MPC approach (cf. Houska et al. [2011]).

A linear-quadratic regulator (LQR) control approach is implemented to compare the performance of the MPC scheme to a more classical optimal control strategy. The constant feedback matrix is obtained by executing the MATLAB built-in \texttt{lqr}() command with the same linear model state-space representation as used for the MPC prediction model. Hence, no integral action is superinduced. The weighting matrices are chosen identically to the ones applied to the MPC.

4.2 Simulation Results

The simulation of an exemplary test case of tracking a predefined trajectory is demonstrated within this paper. The reference state vector \( x^* \) contains non-zero elements only where the trajectory of the vehicle and the load is influenced directly. These elements are \( r^* = (x^*, y^*, z^*) \). The reference for the load’s position in the \( x \) and \( y \)-components is equal to the reference for the vehicle’s position. The reference trajectory is a figure-eight curve in the \( x_2y_2 \)-plane. The values in meters are given by:

\[ r^* = (2 \sin(\omega t), 2 \sin(\omega t) \cos(\omega t), -1.5) . \quad (39) \]

The frequency is ramped up from zero to \( \omega = 0.4 \text{ rad/s} \) within the first four seconds of the simulation. The predefined reference states spanning the prediction horizon are

1 Measured maximum for the DJI F330 with 3s lithium polymer accumulator
Fig. 2. Simulation, LQR: Figure-eight curve tracking

Fig. 3. Simulation, MPC: Figure-eight curve tracking

5. FLIGHT TEST

5.1 Flight Test Environment

The ARCAA Indoor Flying Laboratory is set up to design and verify control algorithms for UAVs. The global position and attitude are provided by an infrared motion capture system via Ethernet. The velocity of the vehicle and the load is obtained by numerical differentiation. A nonlinear state estimator as presented in Lupashin et al. [2014] is applied in order to both reject corrupted measurements and account for system latency. The user code runs on a desktop computer. The commands are transmitted wirelessly to the vehicle. The core component is a software module which is solely implemented in Simulink. Onboard the vehicle, a \( \mu \)-controller decodes the radio signals and converts the serial input to a pulse-width modulated output, which is fed into a low-level PID controller. The physical parameters of the real system are identical with those of the simulated system listed in table 3. For the flight experiment, an integral gain acting in parallel to the MPC penalises deviations in the \( z \)-direction in order to compensate for thrust miscalibration.
5.2 Flight Test Results

A test flight using the same reference trajectory as defined in subsection 4.2 is presented here. The whole flight from take off to landing is shown in Fig. 6. The data is captured by infrared cameras at a sampling rate of 200 Hz. A moving average low-pass filter is applied over ten consecutive measuring points. As the weighting matrices are chosen equal to those used in the simulation, the controller aims at keeping the position of the load aligned with the reference rather than the position of the vehicle. Similar to simulation, the impact of the stronger penalisation for deviations in the load’s position can be seen in Subfig. 4a on the first leg of the figure-eight curve (corresponding to positive values for $x$). The overshooting of the load’s $x$-position is kept below 60 mm. On the second leg the trajectory reaches the border of the camera coverage. The spurious measurement results in a less accurate tracking performance. The system lags in time by under half a second, whereby the load’s position lags in time more than the vehicle’s position (cf. Subfig. 4b). The authors ascribe this fact to undervaluing the aerodynamic drag force of the load in the prediction model (cf. table 3). Nonetheless, accurate four-dimensional tracking performance of the MPC approach shown in simulation is verified by the flight test results.

6. CONCLUSION

In this work, a mathematical model for the coupled system dynamics of a multirotor UAV with heavy slung load has been derived. Based on this model, a Model Predictive Control scheme has been set up. Simulation results for the test case of tracking a figure-eight curve have been discussed. For the given implementation, the MPC scheme’s superior ability for tracking a predefined, time-dependent reference in comparison to a system using an LQR control approach has been shown. The feasibility of the predictive control approach has been demonstrated in flight tests.

For outdoor use of a multirotor UAV with heavy slung load implementing an MPC scheme as presented within this paper, one needs to become independent from using a motion capture system. A setup for visual load detection using an onboard camera is presented by Zürn et al. [2016].

The feasibility of running a specially tailored MPC on an onboard computer of a small UAV is demonstrated by Joos and Fichter [2011].

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