On the Cosmology and Symmetry of Dilaton–Axion Gravity

James E. Lidsey

Astronomy Unit, School of Mathematical Sciences, Queen Mary & Westfield, Mile End Road, LONDON, E1 4NS, U.K.

Abstract

A global $O(2, 2)$ symmetry is found in the Brans–Dicke theory of gravity when the dilaton is coupled to axion and moduli fields. The symmetry is broken if a cosmological constant is introduced. Within the class of spatially homogeneous Bianchi cosmologies, only the type I and V models respect the symmetry. Isotropic cosmological solutions are found for arbitrary spatial curvature. In the region of parameter space relevant to the pre–big bang scenario, the interplay between the scalar fields results in a bouncing cosmology.

PACS Numbers: 98.80.Cq; 04.50.+h; 11.30.-j

1Electronic mail: jel@maths.qmw.ac.uk
In the standard inflationary scenario, the early Universe becomes temporarily dominated by the self–interaction potential energy of a scalar field \[1\]. This potential plays the role of an effective cosmological constant \(\Lambda\) and results in a quasi–de Sitter expansion of the Universe. The energy stored in the false vacuum must exceed the electroweak scale if baryogenesis is to proceed after inflation has ended. However, it is well established from astrophysical observations that the current value of the cosmological constant is extremely small and corresponds to an energy of only \(\leq 0.003\) eV \[2\]. This is approximately 121 orders of magnitude smaller than the Planck scale. A problem therefore arises when one attempts to reconcile a relatively large vacuum energy in the early Universe with the small value inferred today.

Recently, an alternative inflationary picture known as ‘pre–big bang’ cosmology has been developed within the context of string theory \[3\]. In this scenario the accelerated expansion is driven by the kinetic energy of the dilaton field rather than its potential energy \[4\]. In principle, therefore, inflation may proceed without the need for a cosmological constant. The Universe evolves from the string perturbative vacuum into a regime of high curvature and strong coupling. The \(O(3,3)\) invariance (T–duality) of the four–dimensional effective action relates this inflationary phase of the Universe’s history to a corresponding decelerating branch \[5, 6\]. It is not currently known how a graceful exit from inflation might proceed, however, because the two branches are separated by a curvature singularity \[7\].

Another symmetry of string theory that has received considerable attention recently is S–duality \[8\]. This symmetry relates the strong and weak coupling regimes of the theory. Kar, Maharana and Singh have addressed the cosmological constant problem within the context of this symmetry by invoking ‘t Hooft’s ‘naturalness’ hypothesis \[9, 10\]. In short, this hypothesis states that it is natural for a physical parameter to be small if the symmetry of the system is enhanced when that parameter vanishes. Thus, the electron mass \(m_e\) is a naturally small parameter because specifying \(m_e = 0\) results in an additional chiral symmetry in the action. The field equations of the four–dimensional string effective action are invariant under S–duality if and only if \(\Lambda = 0\) \[1\]. Consequently, the symmetry of the theory is enhanced by specifying \(\Lambda = 0\) and a small value for \(\Lambda\) may be viewed as natural, in the sense of ‘t Hooft \[10\]. It is possible, therefore, that S–duality determines the value of the cosmological constant and that T–duality leads to the inflationary expansion of the Universe.

The coupling between the dilaton and graviton in the string effective action arises in the Brans–Dicke theory of gravity \[11\], where the coupling constant takes the specific value \(\omega = -1\). In view of the above developments, it is interesting to consider the symmetric nature of the Brans–Dicke theory for \(\omega \neq -1\). In this paper, we find that the theory exhibits a global \(O(2,2)\) invariance if the dilaton is appropriately coupled to other scalar fields. The symmetry is broken for non–zero \(\Lambda\). We then derive the cosmological solutions of this theory and relate them to the pre–big bang scenario.
We begin with an action of the form

\[ S = \int d^4x \sqrt{-g} e^{-\Phi} \left[ R - \omega (\nabla \Phi)^2 - \frac{1}{2} (\nabla \beta)^2 - \frac{1}{2} e^{(\gamma+1)\Phi} (\nabla \sigma)^2 - 2\Lambda \right], \tag{1} \]

where \( \Phi \) represents the dilaton field, \( \beta \) is a ‘modulus’ field, \( \sigma \) may be viewed as an axion–type field and \( \Lambda \) is assumed to be constant. The metric has signature \((-\,+,+,,+)\). The constants \( \omega \) and \( \gamma \) determine the dilaton–graviton and axion–dilaton couplings, respectively, and we assume \( \omega > -3/2 \) and \( \gamma \neq -1 \). Theory (1) is well motivated from a cosmological point of view. The modulus field may arise through the spontaneous compactification of higher dimensions and the axion field plays a dual role to that of the antisymmetric tensor potential \( B_{\mu\nu} \).

For the spatially flat Friedmann–Robertson–Walker (FRW) Universe with line element \( ds^2 = -dt^2 + e^{2\alpha(t)}dx^2 \) and scale factor \( a = e^\alpha \), the vacuum limit (\( \sigma = \beta = 0 \)) of theory (1) is invariant under the discrete scale factor duality

\[ \tilde{\alpha} = \left( \frac{2 + 3\omega}{4 + 3\omega} \right) \alpha - 2 \left( \frac{1 + \omega}{4 + 3\omega} \right) \Phi \]
\[ \tilde{\Phi} = - \left( \frac{6}{4 + 3\omega} \right) \alpha - \left( \frac{2 + 3\omega}{4 + 3\omega} \right) \Phi \tag{2} \]

if \( \omega \neq -4/3 \). This symmetry is an extension of the scale factor duality of the string effective action. The cosmological solutions are given by

\[ a^{(\pm)} \propto t^{p_{\pm}}, \quad e^\Phi \propto t^{3p_{\pm} - 1} \tag{3} \]

for vanishing \( \Lambda \), where

\[ p_{\pm} \equiv \frac{1}{4 + 3\omega} \left[ 1 + \omega \pm \left( 1 + \frac{2\omega}{3} \right)^{1/2} \right] \tag{4} \]

The (+)– and (−)– branches are related by the scale factor duality (3). When \(-4/3 < \omega < 0\), the time–reversal of the (−)–branch corresponds to an accelerated expansion, whereas the (+)–branch represents a decelerating Universe. These solutions form the basis of the pre–big bang scenario and the two branches are separated by singularities in the curvature and effective coupling at \( t = 0 \).

The terms in Eq. (1) containing the cosmological constant and modulus field are proportional to the ‘shifted’ dilaton field \( \psi \equiv 3\alpha - \Phi \) and this is invariant under the transformation (2), i.e., \( \tilde{\psi} = \psi \). Thus, the duality is respected for non–vanishing \( \Lambda \) and \( \beta \), but it is broken by the axion field. On the other hand, this field leads to a global symmetry of the theory that becomes apparent in the Einstein frame. We therefore proceed by rescaling the metric such that \( \tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \), where \( \Omega^2 \equiv e^{-\Phi} \), and redefining the dilaton field \( \varphi \equiv (3 + 2\omega)^{1/2} \Phi \). It follows that action (1) transforms to

\[ S = \int d^4x \sqrt{-\tilde{g}} \left[ \tilde{R} - \frac{1}{2} (\tilde{\nabla} \varphi)^2 - \frac{1}{2} (\tilde{\nabla} \beta)^2 - \frac{1}{2} e^{-2\lambda \varphi} (\tilde{\nabla} \sigma)^2 - 2\Lambda e^{\varphi/(3+2\omega)^{1/2}} \right], \tag{5} \]

1We drop tildes in what follows for notational simplicity.
where \( 2\lambda \equiv -(\gamma + 1)/(3 + 2\omega)^{1/2} \). The string effective action corresponds to \( \omega = -1 \) and \( \gamma = 1 \). We then define the symmetric \( 4 \times 4 \) matrix \( M \)

\[
M = \begin{pmatrix} G^{-1} & -G^{-1}B \\ BG^{-1} & G - BG^{-1}B \end{pmatrix},
\]

where \( G \) and \( B \) are the \( 2 \times 2 \) matrices

\[
G = \begin{pmatrix} e^{\lambda(\varphi + \beta)} & 0 \\ 0 & e^{\lambda(\varphi - \beta)} \end{pmatrix}, \quad B = \begin{pmatrix} 0 & \lambda \sigma \\ -\lambda \sigma & 0 \end{pmatrix}.
\]

The kinetic sector of action (5) may be written in the form

\[
(\nabla \varphi)^2 + (\nabla \beta)^2 + e^{-2\lambda \varphi} (\nabla \sigma)^2 = -\frac{1}{2\lambda^2} \text{tr} \left[ \left( G^{-1} \nabla B \right)^2 - \left( G^{-1} \nabla G \right)^2 \right]
\]

and this implies that

\[
S = \int d^4x \sqrt{-g} \left[ R + \frac{1}{8\lambda^2} \text{tr} \left( \nabla M \nabla M^{-1} \right) - 2\Lambda e^{\varphi/(3+2\omega)^{1/2}} \right].
\]

The matrix (6) satisfies the constraint \( M^T \eta M = \eta \), where

\[
\eta \equiv \begin{pmatrix} 0 & I_2 \\ I_2 & 0 \end{pmatrix}
\]

is an \( O(2, 2) \) metric in non–diagonal form and \( I_2 \) is the identity matrix in 2 dimensions. Eq. (6) is therefore an element of the group \( O(2, 2) \). Moreover, it can be verified that action (5) is invariant under the global \( O(2, 2) \) transformation

\[
\bar{M} = \Sigma M \Sigma^T, \quad \bar{g}_{\mu\nu} = g_{\mu\nu}, \quad \Sigma^T \eta \Sigma = \eta
\]

if the cosmological constant vanishes. The symmetry is broken if the cosmological constant is non–zero because the dilaton is not invariant under the action of Eq. (11). The symmetry of the theory is therefore enhanced if \( \Lambda = 0 \), so the vanishing of \( \Lambda \) is natural in this theory in the sense advocated by ’t Hooft [10]. We remark that if \( \Lambda \) is in general some function of the dilaton field, the symmetry (11) is only respected for the specific form \( \Lambda \propto e^{-\varphi/(3+2\omega)^{1/2}} \).

The symmetry (11) is related to the global \( SL(2, R) \) symmetry associated with the four–dimensional \( SL(2, R)/U(1) \) non–linear \( \sigma \)–model [14]. This becomes apparent after the introduction of the complex fields [15]

\[
T \equiv T_1 + iT_2 = \lambda \sigma + ie^{\lambda \varphi}, \quad U \equiv ie^{\lambda \beta}
\]

so that Eq. (6) takes the form

\[
M = \frac{i}{T_2 U} \begin{pmatrix} 1 & 0 & 0 & -T_1 \\ 0 & \lvert U \rvert^2 & T_1 \lvert U \rvert^2 & 0 \\ 0 & T_1 \lvert U \rvert^2 & \lvert T \rvert^2 \lvert U \rvert^2 & 0 \\ -T_1 & 0 & 0 & \lvert T \rvert^2 \end{pmatrix}.
\]
We may now consider the specific element of $O(2, 2)$ given by

$$
\Sigma = \begin{pmatrix}
  d & 0 & 0 & -c \\
  0 & d & c & 0 \\
  0 & b & a & 0 \\
  -b & 0 & 0 & a
\end{pmatrix},
$$

where $ad - bc = 1$. Eq. (11) then generates the SL$(2, R)$ transformation

$$
\bar{T} = aT + b/cT + d, \quad \bar{U} = U.
$$

This leaves the modulus invariant, but combines the dilaton and axion in a non-trivial way.

An alternative formulation of action (9) is possible when $\Lambda = 0$. We define the matrix

$$
N \equiv \begin{pmatrix}
  P^{-1} & P^{-1}Q \\
  QP^{-1} & P + QP^{-1}Q
\end{pmatrix},
$$

where $P$ and $Q$ are the $2 \times 2$ matrices

$$
P = \begin{pmatrix}
e^{\lambda \varphi} & 0 \\
0 & e^{\lambda \beta}
\end{pmatrix}, \quad Q = \begin{pmatrix}
\lambda \sigma & 0 \\
0 & 0
\end{pmatrix}.
$$

Eq. (16) satisfies the constraint $N^T J N = J$, where

$$
J \equiv \begin{pmatrix}
  0 & I_2 \\
- I_2 & 0
\end{pmatrix}
$$

and it is therefore an element of the real symplectic group $Sp(4, R)$. Moreover, the inverse of $N$ is given by $N^{-1} = -JN^T J$, since $J^2 = -I_2$. Thus, the action is given by

$$
S = \int d^4x \sqrt{-g} \left[ R + \frac{1}{4 \lambda^2} \text{tr} \left( \nabla N \nabla N^{-1} \right) \right]
$$

and the theory is invariant under the global $Sp(4, R)$ transformation

$$
\bar{N} = \Theta N \Theta^T, \quad \bar{g}_{\mu \nu} = g_{\mu \nu}, \quad \Theta^T J \Theta = J.
$$

The effect of Eq. (20) on the complex symmetric matrix $Z \equiv Q + iP = \text{diag}[T, U]$ is analogous to that of the SL$(2, R)$ transformation (13) on the complex dilaton–axion field $T$ [17]. In a sense, therefore, Eq. (20) may be viewed as a matrix–valued SL$(2, R)$ transformation.

The global symmetry associated with action (4) provides motivation for considering this theory further within a cosmological context. The class of spatially homogeneous cosmologies admits a Lie group of isometries that act transitively on the
space–like three–dimensional orbits. The anisotropy of each model is determined by the structure constants of the Lie algebra of $G_3$ and the metric on the three–surfaces may be written as

$$h_{ab}(t) = e^{2\alpha(t)} \left(e^{2\beta(t)}\right)_{ab}, \quad a, b = 1, 2, 3,$$

(21)

where $\beta_{ab} \equiv \text{diag} \left[\beta_+ + \sqrt{3}\beta_-, \beta_+ - \sqrt{3}\beta_-, -2\beta_+ \right]$ is a traceless matrix and $e^{3\alpha}$ represents the effective spatial volume of the Universe.

We only consider those models for which a Lagrangian formulation is possible. Integration over the spatial variables in action (5) then implies that

$$S = \int dt e^{3\alpha} \left[-6\dot{\alpha}^2 + 6\dot{\beta}_+^2 + 6\dot{\beta}_-^2 + \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} e^{-2\lambda \dot{\phi}^2} + e^{-2\alpha} U(\beta_{\pm}) \right]$$

(22)

for $\beta = \Lambda = 0$, where a dot denotes differentiation with respect to time, a boundary term has been neglected and the comoving volume of the Universe has been normalized to unity. The function $U(\beta_{\pm})$ determines the scalar curvature of the three–surfaces and is different for each Bianchi type.

The fields $\beta_{\pm}$ play the role of moduli and formally we may identify $d\beta^2 = 12(d\beta_+^2 + d\beta_-^2)$. After comparing Eqs. (5) and (22), we then deduce that the kinetic sector of Eq. (22) is symmetric under the global $O(2, 2)$ transformation

$${\tilde{M}} = \Sigma M \Sigma^T, \quad {\tilde{\alpha}} = \alpha, \quad {\Sigma^T} \eta \Sigma = \eta.$$  

(23)

This transformation alters the degree of anisotropy in each of the three spatial directions whilst preserving the spatial volume of the Universe. However, Eq. (22) also contains an effective potential for the moduli. Since Eq. (23) relates the fields in a non–linear fashion, this term is only invariant under such a transformation when it is independent of $\beta_{\pm}$. Thus, $U$ must be constant if the full action (22) is to be symmetric and this condition is only satisfied for the Bianchi types I and V. These models represent the anisotropic generalizations of the spatially flat and negatively curved FRW Universes, respectively. Within this context, therefore, the type I and V models are more symmetric than the other Bianchi types and it is interesting that they are both associated with isotropic FRW Universes.

We now proceed to solve the field equations for the class of FRW Universes. In general, the line element of these models is

$$ds^2 = -dt^2 + e^{2\alpha(t)} \left[\frac{dr^2}{1 - kr^2} + r^2 \left(d\theta^2 + \sin^2 \theta dv^2\right)\right],$$

(24)

where the curvature parameter takes values $-1, 0, +1$ for negatively curved, flat and positively curved models, respectively. It proves convenient to express the field equations derived from action (1) in terms of the conformally invariant time parameter.
\[\eta \equiv \int dt e^{-\alpha(t)},\] the Lorenz–Petzold variable \[X \equiv e^{2\alpha - \Phi}\] and the rescaled dilaton field \(Y \equiv -(1 + 2\omega/3)^{1/2}\Phi.\) The action (1) with \(\Lambda = 0\) then reduces to

\[S = \int d\eta X \left[ -\frac{3}{2} \frac{X'^2}{X^2} + \frac{3}{2} Y'^2 + \frac{1}{2} \beta'^2 + \frac{1}{2} \frac{e^{-\epsilon Y} \sigma'^2}{X} + 6k \right],\]  

(25)

where a prime denotes differentiation with respect to \(\eta\) and \(\epsilon \equiv (1 + \gamma)/(1 + 2\omega/3)^{1/2}.\)

The cosmological field equations are given by

\[X\beta' = p, \quad X e^{-\epsilon Y} \sigma' = q\]  

(26)

\[X'' + 4kX = 0\]  

(27)

\[(XY')' = -\frac{\epsilon q^2}{e^Y} e^Y\]  

(28)

\[3 \frac{X'^2}{X^2} - 3 Y'^2 + 12k - \beta'^2 - e^{-\epsilon Y} \sigma'^2 = 0,\]  

(29)

where \(p\) and \(q\) are arbitrary constants. The advantage of employing the Lorenz–Petzold variable is that Eq. (27) is identical to the corresponding expression for the vacuum model \((\beta = \sigma = 0)\) [22]. Its first integral may be written as

\[X'^2 + 4kX^2 = A^2 + p^2,\]  

(30)

where \(A\) is an arbitrary constant. It may then be verified by direct substitution of Eq. (30) into Eq. (29) that the latter equation is a first integral of Eq. (28). Thus, the evolution of the scale factor may be determined by solving the first–order equations (29) and (30).

The solution to Eq. (30) is given by

\[X(\tau) = \sqrt{A^2 + p^2} \frac{\tau}{1 + k\tau^2},\]  

(31)

where we have defined [22]

\[\tau = \begin{cases} \eta, & k = 0 \\ \tan\eta, & k = +1 \\ \tanh\eta, & k = -1 \end{cases}\]  

(32)

Substitution of Eq. (31) into Eq. (29) then leads to the solution

\[e^{(1+\gamma)\Phi} = \frac{q^2}{12A^2} \left[ \left( \frac{\tau}{\tau_0} \right)^n + \left( \frac{\tau}{\tau_0} \right)^{-n} \right]^2\]  

(33)

\[a^2 = (A^2 + p^2)^{1/2} \left( \frac{q^2}{12A^2} \right)^{1/(1+\gamma)} \left[ \left( \frac{\tau}{\tau_0} \right)^n + \left( \frac{\tau}{\tau_0} \right)^{-n} \right]^{2/(1+\gamma)} \frac{\tau}{1 + k\tau^2},\]  

(34)
where $\hat{A}^2 = A^2 + 2p^2/3$, $\tau_0$ is an integration constant and

$$n \equiv \frac{1}{2} \frac{1 + \gamma}{(1 + 2\omega/3)^{1/2}} \left( \frac{A^2 + 2p^2/3}{A^2 + p^2/3} \right)^{1/2}. \quad (34)$$

This solution generalizes the stiff perfect fluid ($\beta = \gamma = 0$) and string models [22, 23, 24].

We will consider the evolution of the flat ($k = 0$) model. A necessary and sufficient condition for inflation is that the second time derivative of the scale factor be positive definite, i.e., $a'' > 0$. When $p = 0$, this requires $\omega < 0$ and, indeed, the region of parameter space corresponding to $-4/3 < \omega < 0$ is relevant to the pre–big bang scenario. Although this may appear a somewhat restrictive regime, it should be emphasized that it includes the truncated string effective action and higher dimensional Einstein gravity. If the latter theory is dimensionally reduced to four dimensions with an isotropic, Ricci–flat internal space of radius $b$ and dimensionality $d$, the resultant action is given by the dilaton–graviton sector of Eq. (1) with $\Phi \propto -d \ln b$ and $\omega = -1 + 1/d$. In view of this, we will consider this range of $\omega$.

The qualitative behaviour of solution (33) is not affected by the moduli and we may therefore specify $p = 0$ without loss of generality. The sign of $n$ is important, however. We begin by considering the case $n > 0$ ($\gamma > -1$). In the limit $t \to 0$, the asymptotic form of the solution is given by the ($-$)–branch of the vacuum solution (3). On the other hand, the late time limit ($t \to \infty$) is given by the ($+$)–branch. Thus, the scale factor is initially infinitely large and the Universe undergoes an accelerated contraction to a minimum size before reexpanding to infinity. Inflationary behaviour is inevitable immediately after the bounce since the Universe must accelerate away from the point of maximum contraction. Inflation does not last indefinitely, however, since $p_+ < 1$ when $-4/3 < \omega < 0$. The effective gravitational coupling $G_{\text{eff}} \propto e^\Phi$ is also bounded from below. It is initially divergent and decreases as the Universe expands to a minimum value of $G_{\text{eff}} = (q^2/3A^2)^{1/(1+\gamma)}$ at a time $\tau_0$. It then increases indefinitely for $\eta > \tau_0$. The bound on the coupling is determined by the canonical momentum of the axion field.

**Figure 1**

The evolution of the scale factor and effective coupling is shown in Figure 1 and is comparable to that found in the specific models of Refs. [24, 25]. Our analysis shows that such behaviour does not depend too sensitively on the coupling between the dilaton and axion fields and it may therefore be quite generic in theories of this type. The curvature invariants diverge at $t = 0$ and there is an infinite proper distance between two given points in space–time. Thus, the Universe is singular at this point even though it has infinite size. A singularity of this type has been termed an ‘anti–big bang’ singularity [25], since it is in contrast to the conventional big bang singularity where the proper distance between two points is zero.
The high and low curvature regimes are related by the scale factor duality (2) and the vacuum solutions are recovered in these limits because the kinetic energy of the dilaton dominates the dynamics. The vacuum Universe (3) undergoes monotonic expansion or contraction and the role of the axion is to induce a bounce between the contracting and expanding branches [24]. Thus, a classical transition between the two branches is possible when an axion field is present.

For completeness, we remark that the role of the vacuum branches (3) is interchanged when $n < 0$ ($\gamma < -1$). The (+)–branch applies as $t \to 0$ and the (−)–branch as $t \to \infty$. Thus, the Universe has zero spatial volume initially and expands out of the singularity. There is now an upper bound on the maximum size attained by the Universe, however, and it recollapses after a finite proper time. Although this solution does not appear to be physically relevant, it would be interesting to investigate whether perfect fluid matter sources can prevent the recollapse from proceeding. Similar qualitative behaviour is expected in the vicinity of the singularity since the energy density of the dilaton field dominates ordinary matter at early times [23]. However, the Universe would become dominated by the fluid sources at later times.

In conclusion, we have identified a global $O(2,2)$ symmetry in the Brans–Dicke theory of gravity that exists when the dilaton is coupled to moduli and axion fields. The symmetry is an extension to the Brans–Dicke theory of the $SL(2,\mathbb{R})$ invariance of the truncated string effective action. The symmetry is not respected when a cosmological constant is present and a vanishingly small $\Lambda$ in this theory is therefore consistent with 't Hooft's naturalness hypothesis. When the modulus field is related to the anisotropy parameters of the spatially homogeneous Universes, the symmetry is only respected for the Bianchi types I and V. Cosmological solutions were found for arbitrary spatial curvature and the flat model was discussed within the context of the pre–big bang scenario. The axion field induces a classical transition between accelerating and decelerating phases by causing the Universe to bounce.

Acknowledgments The author is supported by the Particle Physics and Astronomy Research Council (PPARC), UK. We thank D. Wands for helpful discussions.

References

1. A. A. Starobinsky, Phys. Lett. 91B, 99 (1980); A. H. Guth, Phys. Rev. D23,
347 (1981); K. Sato, Mon. Not. R. astron. Soc. 195, 467 (1981); A. D. Linde, Phys. Lett. 108B, 389 (1982); S. W. Hawking and I. G. Moss, Phys. Lett. 110B, 35 (1982); A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982).

2. For recent reviews see, e.g., J. P. Ostriker and P. J. Steinhardt, Nat. 377, 600 (1995); L. M. Krauss and M. S. Turner, Gen. Rel. Grav. 27, 1137 (1995); G. Efstathiou, “An Anthropic Argument for a Cosmological Constant”, 1996 (unpublished).

3. M. Gasperini and G. Veneziano, Astropart. Phys. 1, 317 (1993); Mod. Phys. Lett. A8, 3701 (1993); Phys. Rev. D50, 2519 (1994).

4. J. J. Levin, Phys. Rev. D51, 462 (1995).

5. G. Veneziano, Phys. Lett. 265B, 287 (1991); K. A. Meissner and G. Veneziano, Phys. Lett. 267B, 33 (1991); Mod. Phys. Lett. A6, 3397 (1991); M. Gasperini, J. Maharana, and G. Veneziano, Phys. Lett. 272B 277 (1991).

6. J. Maharana and J. H. Schwarz, Nucl. Phys. B390, 3 (1993).

7. R. Brustein and G. Veneziano, Phys. Lett. 329B, 429 (1994); N. Kaloper, R. Madden, and K. A. Olive, Nucl. Phys. B452, 677 (1995); Phys. Lett. 371B, 34 (1996); R. Easther, K. Maeda, and D. Wands, Phys. Rev. D53, 4247 (1996); M. Gasperini, J. Maharana, and G. Veneziano, Nucl. Phys. B472, 349 (1996). J. E. Lidsey, “Inflationary and Deflationary Branches in Extended Pre–Big Bang Cosmology”, 1996 [gr-qc/9605017].

8. A. Font, L. Ibanez, D. Lüst, and F. Quevedo, Phys. Lett. 249B, 35 (1990); S. J. Rey, Phys. Rev. D43, 526 (1991); A. Sen, Nucl. Phys. B404, 109 (1993); Phys. Lett. 303B, 22 (1993); Int. J. Mod. Phys. A8, 2023 (1993); Int. J. Mod. Phys. A9, 3707 (1994); J. H. Schwarz and A. Sen, Nucl. Phys. B411, 35 (1994).

9. S. Kar, J. Maharana, and H. Singh, Phys. Lett. 374B, 43 (1996).

10. G. ’t Hooft, Under the Spell of Gauge Principle (World Scientific, Singapore, 1994).

11. C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961).

12. J. E. Lidsey, Phys. Rev. D52, R5407 (1995).

13. J. O’Hanlon and B. O. J. Tupper, Il Nuovo Cimento 7, 305 (1972).
14. E. Cremmer, J. Scherk, and S. Ferrara, Phys. Lett. **74B**, 61 (1978); E. Cremmer and J. Scherk, Phys. Lett. **74**, 341 (1978); E. Cremmer and B. Julia, Nucl. Phys. **B159**, 141 (1979).

15. R. Dijkgraaf, E. Verlinde and H. Verlinde, in Proc. Copenhagen Conf., Perspectives in String Theory, eds. P. Di Vecchia and J. L. Petersen (World Scientific, Singapore, 1988).

16. D. Bailin, A. Love, W. A. Sabra, and S. Thomas, Phys. Lett. **320B**, 21 (1994); W. A. Sabra, “Spacetime Duality and SU(n,1)/SU(n)×U(1) Cosets of Orbifold Compactification”, 1996 ([hep-th/9603085](https://arxiv.org/abs/hep-th/9603085)).

17. D. V. Gal’tsov and O. V. Kechkin, Phys. Lett. **361B**, 52 (1995); D. V. Gal’tsov, in Quantum Field Theory under the Influence of External Conditions”, ed. M. Bordag (Proc. of the International Workshop, Leipzig, Germany 1995) ([hep-th/9606041](https://arxiv.org/abs/hep-th/9606041)).

18. R. M. Wald, General Relativity (University of Chicago Press, Chicago, 1984).

19. S. Capozziello, G. Marmo, C. Rubano, and P. Scudellaro, “Noether Symmetries in Bianchi Universes”, 1996 ([gr-qc/9606050](https://arxiv.org/abs/gr-qc/9606050)).

20. R. M. Wald, Phys. Rev. **D28**, 2118 (1983).

21. D. Lorenz–Petzold, Astrophys. Space. Sci. **98**, 101 (1984).

22. J. P. Mimoso and D. Wands, Phys. Rev. **D51** 477 (1995).

23. L. E. Gurevich, A. M. Finkelstein, and V. A. Ruban, Astrophys. Space Sci. **22**, 231 (1973).

24. E. J. Copeland, A. Lahiri, and D. Wands, Phys. Rev. **D50**, 4868 (1994).

25. F. G. Alvarenga and J. C. Fabris, Gen. Rel. Grav. **28**, 645 (1996).
Figure Caption

Figure 1: (a) At early times the dilaton dominates the axion and the Universe undergoes an accelerated contraction from an initially singular state. The axion induces a bounce and the Universe reexpands after a finite proper time into a decelerating phase. The high and low curvature limits of the solution are determined by the ($-$)– and ($+$)–branches of the vacuum solution and are related by the scale factor duality of the theory. (b) The effective gravitational coupling is initially divergent, but becomes progressively weaker until a lower bound is attained. The bound is determined by the momentum associated with the axion. The coupling then increases monotonically with time.