Scale genesis and gravitational wave in a classically scale invariant extension of the standard model

Jisuke Kubo\textsuperscript{a} and Masatoshi Yamada\textsuperscript{b,c}

\textsuperscript{a}Institute for Theoretical Physics, Kanazawa University, Kanazawa 920-1192, Japan
\textsuperscript{b}Department of Physics, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{c}Institut für Theoretische Physik, Universität Heidelberg, Philosophenweg 16, 69120 Heidelberg, Germany

E-mail: jik@hep.s.kanazawa-u.ac.jp, m_yamada@gauge.scphys.kyoto-u.ac.jp

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Abstract. We assume that the origin of the electroweak (EW) scale is a gauge-invariant scalar-bilinear condensation in a strongly interacting non-abelian gauge sector, which is connected to the standard model via a Higgs portal coupling. The dynamical scale genesis appears as a phase transition at finite temperature, and it can produce a gravitational wave (GW) background in the early Universe. We find that the critical temperature of the scale phase transition lies above that of the EW phase transition and below few $O(100)$ GeV and it is strongly first-order. We calculate the spectrum of the GW background and find the scale phase transition is strong enough that the GW background can be observed by DECIGO.

Keywords: cosmological phase transitions, gravitational waves and CMBR polarization, dark matter theory

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1 Introduction

One of the problems of the standard model (SM) is that it cannot explain the origin of the electroweak (EW) scale. Obviously, if we start with a theory which contains a scale from the beginning, we have no chance to explain its origin. We recall that the Higgs mass term is the only term that breaks scale invariance at the classical level in the SM. So, a central question is: what is the origin of the Higgs mass term? Recently, a number of studies on a scale invariant extension of the SM have been performed, where two ideas have been considered. The one [1–52] relies on the Coleman-Weinberg (CW) potential [53], and the other one is based on non-perturbative effects: dynamical chiral symmetry breaking [54–56] is applied in [57–69] and gauge-invariant scalar-bilinear condensation [70–72] is used in [73, 74].

In the case that a scale is dynamically created, there will be a corresponding scale phase transition. Chiral phase transition is a scale phase transition in this context. If a scale phase transition is strongly first-order, the Universe undergoes a strong phase transition at a certain high temperature, thereby producing gravitational wave (GW) background that could be observed today [75–77]. There are a number of GW experiments that are on-going or planned in the near and far future (see [78, 79] and also [80–82] for instance). It is in fact this year in which LIGO [83] has observed the GW for the first time.

In this paper we focus on the second non-perturbative effect, the gauge-invariant scalar-bilinear condensation, that produces the GW background in the early Universe. The present work is a natural extension of our recent works [73, 74], because we have shown there that the scale phase transition due to the scalar-bilinear condensation is strongly first-order in a wide region of the parameter space. We have performed the analysis, using an effective field theory that describes approximately the process of the dynamical scale genesis via the scalar-bilinear condensation. Since we will be using the same effective field theory to calculate the spectrum of the corresponding GW background, we devote, for completeness,
first two sections for briefly elucidating our effective field theory method, where parallels to the Nambu-Jona-Lasinio (NJL) theory \cite{55, 56} could be read off. In \cite{73, 74} we have used the so-called self-consisting mean-field approximation \cite{84, 85} to derive the mean-field Lagrangian, we will employ the path-integral approach \cite{86, 87} (section 2) to arrive at the same Lagrangian, a procedure which may be more readable in high-energy physics society.

We will single out a benchmark point in the parameter space in section 3. The benchmark point represents the wide region of the parameter space, which is consistent with the dark matter (DM) phenomenology. (In the model we will consider there exists a DM candidate due to an unbroken flavor symmetry in the hidden sector.) In this region of the parameter space the scale phase transition is strongly first-order.

In section 4 we will calculate the spectrum of the corresponding GW background. There are three production mechanisms of GWs at a strong first-order phase transition, in which the bubble nucleation grows and the GW is produced; collisions of bubble walls $\Omega_{\text{coll}}$ \cite{88–94}, magnetohydrodynamic (MHD) turbulence $\Omega_{\text{MHD}}$ \cite{95–101} and also sound waves $\Omega_{\text{sw}}$ after the bubble wall collisions \cite{102–105}. Using the formulas given in these papers and especially in \cite{81}, we will compute these individual contributions to the GW background spectrum for a set of the benchmark parameters and find that $\Omega_{\text{coll}}$ and $\Omega_{\text{MHD}}$ are several orders of magnitude smaller than $\Omega_{\text{sw}}$. Finally we will compare our result with the sensitivity of various GW experiments. We will find that the scale phase transition caused by the scalar-bilinear condensation can be strong enough to produce the GW background that can be observed by DECIGO \cite{106–108}.

Section 5 is devoted to a summary, and in the appendix we compute the field renormalization factor.

2 The basic idea and the path-integral approach

We consider a classical scale invariant extension of the SM, which has been studied in \cite{73, 74}. The basic assumption there is that the origin of the EW scale is a scalar-bilinear condensation, which forms due a strong non-abelian gauge interaction in a hidden sector and triggers the EW symmetry breaking through a Higgs portal coupling. The hidden sector is described by an SU($N_c$) gauge theory with the scalar fields $S_a^i$ ($a = 1, \ldots, N_c, i = 1, \ldots, N_f$) in the fundamental representation of SU($N_c$). Accordingly, the total Lagrangian is given by

\[
\mathcal{L}_H = -\frac{1}{2} \text{tr} F^2 + (\partial^\mu S_i)\partial_\mu S_i - \hat{\lambda}_S (S_i^\dagger S_i)(S_j^\dagger S_j)
- \hat{\lambda}' S(S_i^\dagger S_j)(S_j^\dagger S_i) + \hat{\lambda}_{HS} (S_i^\dagger S_i) H^\dagger H - \lambda_H (H^\dagger H)^2 + \mathcal{L}'_{\text{SM}},
\]

where $D_\mu S_i = \partial_\mu S_i - ig_H G_\mu S_i$, $G_\mu$ is the matrix-valued SU($N_c$) gauge field, and the SM Higgs doublet field is denoted by $H$ (the parenthesis stands for SU($N_c$) invariant products.). The last term, $\mathcal{L}'_{\text{SM}}$, contains the SM gauge and Yukawa interactions. Note that the Higgs mass term, which is the only scale-invariance violating term in the SM, is absent in (2.1).

We assume that the SU($N_c$) gauge theory in the hidden sector is asymptotically free and gauge symmetry is unbroken for the entire energy scale, such that above a certain energy the theory is perturbative and there is no mass scale except for renormalization scale, which is present due to scale anomaly \cite{109, 110}. At a certain low energy scale (say the confinement scale) the gauge coupling $g_H$ becomes so large that the SU($N_c$) invariant scalar bilinear
dynamically forms a $U(N_f)$ invariant condensate [71, 72],

$$\langle (S_i^a S_j^a) \rangle = \left\langle \sum_{a=1}^{N_c} S_i^a S_j^a \right\rangle \propto \delta_{ij}, \quad (2.2)$$

and the mass term (constituent mass) for $S_i$ is dynamically generated. The creation of the mass term from nothing is possible only by a non-perturbative effect: scale anomaly [109, 110] cannot generate a mass term, because the $SU(N_c)$ gauge symmetry is unbroken by assumption. Of course, scale anomaly does contribute to the mass once it is generated, and how it logarithmically runs is described, for instance in [111], in a modern language of renormalization.\footnote{There exist proofs within the framework of perturbation theory that conformal anomaly does not generate mass term of scalar field in the class of massless renormalizable (not super-renormalizable) field theories. This is rigorously proven in the massless $\phi^4$ theory by Loewenstein and Zimmermann [112, 113], and the same conclusion was made in massless non-abelian gauge theories in [114]. If the regularization (e.g. cut-off regularization) breaks scale invariance, one needs a counter term for the mass of the scalar field. From this reason we will employ dimensional regularization.}

It has been intended in [73, 74] to describe the non-perturbative phenomena of condensation (2.2) approximately by using an effective theory. That is, the effective theory should describe the dynamical generation of the mass for $S_i$ via the scalar-bilinear condensation (2.2). Using the effective theory it should be also possible to approximately describe how the energy scale transfers from the hidden sector to the SM sector. Inspired by the NJL theory, which can approximately describe the dynamical chiral symmetry breaking in QCD, we have assumed that the effective Lagrangian does not contain the $SU(N_c)$ gauge fields, because they are integrated out, while it contains the “constituent” scalar fields $S_i^a$. Furthermore, since the hard breaking of scale invariance by anomaly is only logarithmic, the non-perturbative breaking may be assumed to be dominant, so that scale anomaly may be ignored in writing down an effective Lagrangian at the tree level:

$$L_{\text{eff}} = ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S (S_i^a S_i^a) (S_j^a S_j) - \lambda'_S (S_i^a S_j^a) (S_j^a S_i) + \lambda_H (S_i^a S_i) H^\dagger H - \lambda_H (H^\dagger H)^2, \quad (2.3)$$

where all $\lambda$’s are positive. This is the most general form which is consistent with the global $SU(N_c) \times U(N_f)$ symmetry and the classical scale invariance (we have suppressed the kinetic term for $H$ in (2.3) because it does not play any important role for our discussions here). That is, $L_{\text{eff}}$ has the same global symmetry as $L_H$ even at the quantum level. We emphasize that the effective Lagrangian (2.3) is not a Lagrangian for an effective theory of (2.1) after the confinement has taken place and the condensation (2.2) has appeared: it should describe the process of the condensation. Note also that the couplings $\lambda_S, \lambda'_S, \lambda_H$ in $L_H$ are not the same as $\lambda_S, \lambda'_S, \lambda_H$ in $L_{\text{eff}}$, because the latter are effective couplings which are dressed by hidden gluon contributions.

### 2.1 Path-integral formalism

In [73, 74] the self-consistent mean-field approximation [84, 85] has been applied to treat the effective Lagrangian (2.3). Here we base on the path-integral formalism to obtain the mean-field Lagrangian $L_{\text{MFA}}$. The method is known as the so-called auxiliary field method or
Hubbard-Stratonovich transformation [86, 87]. The starting path-integral with the effective Lagrangian (2.3) is given by

\[
Z = \int D\bar{S}^\dagger D\bar{S} \exp \left[ i \int d^4x \left\{ (i[\partial^\mu S_i]^\dagger \partial_\mu S_i) - \lambda_S (S_i^1 S_i) (S_j^1 S_j) - \lambda_S' (S_i^1 S_j) (S_j^1 S_i) + \lambda_H S (S_i^1 S_i) H^\dagger H - \lambda_H (H^\dagger H)^2 \right\} \right], \tag{2.4}
\]

where we focus only on the path-integral for $S_i^1$ and $S_i$. We insert the Gaussian integral

\[
1 \propto \int Df^\dagger D\phi^a \exp \left[ i \int d^4x \mathcal{L}_A \right] \quad \text{with} \quad \mathcal{L}_A = N_f (N_f \lambda_S + \lambda_S') f^2 + \frac{\lambda_S}{2} (\phi^a_0)^2 - 2\lambda_S \phi^a_0 (S_i^1 t^a_{ij} S_j), \tag{2.5}
\]

to the path-integral (2.4) and make the change of the integration variables to $f$ and $\phi^a_0$ according to

\[
f' = f - (S_i^1 S_i)/N_f, \quad \phi^a_0 = \phi^a_0 - 2(S_i^1 t^a_{ij} S_j), \tag{2.6}
\]

where $t^a \ (a = 1, \ldots, N_f^2 - 1)$ are the SU($N_f$) generators in the fundamental representation. Then the path-integral can be written as

\[
Z = \int D\bar{S}^\dagger D\bar{S} Df D\phi_0 \exp \left[ i \int d^4x \left\{ ([\partial^\mu S_i]^\dagger \partial_\mu S_i) + \lambda_H S (S_i^1 S_i) H^\dagger H - \lambda_H (H^\dagger H)^2 
+ N_f (N_f \lambda_S + \lambda_S') f^2 - 2(N_f \lambda_S + \lambda_S') f (S_i^1 S_i) + \frac{\lambda_S}{2} (\phi^a_0)^2 - 2\lambda_S \phi^a_0 (S_i^1 t^a_{ij} S_j) \right\} \right], \tag{2.7}
\]

where we have used the identity

\[
(S_i^1 t^a_{ij} S_j) (S_k^1 t^b_{kl} S_i) = S_i^1 S_j S_k^1 S_i (t^a_{ij} t^b_{kl}) = S_i^1 S_j S_k^1 S_i \times \frac{1}{2} \left( \delta_{ij} \delta_{jk} - \frac{1}{N_f} \delta_{ij} \delta_{kl} \right) = \frac{1}{2} \left( (S_i^1 S_j) (S_k^1 S_i) - \frac{1}{N_f} (S_i^1 S_i)^2 \right). \tag{2.8}
\]

Note that the Euler-Lagrange equations for the auxiliary fields $f$ and $\phi^a_0$ become $f = (S_i^1 S_i)/N_f$ and $\phi^a_0 = 2(S_i^1 t^a_{ij} S_j)$, respectively, and substituting them into (2.7), we are back to (2.4). We thus arrive at the mean-field Lagrangian

\[
\mathcal{L}_{\text{MFA}} = ([\partial^\mu S_i]^\dagger \partial_\mu S_i) - M^2 (S_i^1 S_i) - \lambda_H (H^\dagger H)^2 
+ N_f (N_f \lambda_S + \lambda_S') f^2 + \frac{\lambda_S}{2} (\phi^a_0)^2 - 2\lambda_S \phi^a_0 (S_i^1 t^a_{ij} S_j), \tag{2.9}
\]

where

\[
M^2 = 2(N_f \lambda_S + \lambda_S') f - \lambda_H S H^\dagger H. \tag{2.10}
\]

If we expand the composite field $f$ and the Higgs doublet around the vacuum values $f_0$ and $v_h$, i.e.,

\[
f = f_0 + Z^{1/2}_\sigma \sigma, \quad H = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi^1 + i \chi^2 \\ v_h + h + i \chi_0 \end{pmatrix}, \tag{2.11}
\]
and redefine $\phi_0^a$ as $\phi_0^a = Z_\phi^{1/2} \phi^a$, the mean-field Lagrangian (2.9) becomes
\[
\mathcal{L}_{\text{MFA}}' = \left( i \nabla^\mu S_{i} \nabla_{\mu} S_{i} \right) - M^2 (S_i^\dagger S_i) + N_f (N_f \lambda S + \lambda'_S) Z_\sigma \sigma^2 + \frac{\lambda_S}{2} Z_\phi \phi^a \phi^a \\
- 2(N_f \lambda S + \lambda'_S) Z_\sigma^{1/2} \sigma (S_i^\dagger S_i) - 2\lambda'_S Z_\phi^{1/2} (S_i^\dagger \phi^a S_i) \\
+ \frac{\lambda_H}{2} (S_i^\dagger S_i) h (2v_h + h) - \frac{\lambda_H}{4} h^2 (6v_h^2 + 4vh h + h^2),
\]
where $Z_\sigma$ and $Z_\phi$ are field renormalization constants of the canonical dimension two. In (2.12) we have neglected the would-be Goldstone bosons $\chi^i$ in the Higgs field and defined the constituent scalar mass squared as
\[
M_0^2 = 2(N_f \lambda S + \lambda'_S) f_0 - \frac{\lambda_H}{2} v_h^2.
\]

### 2.2 Effective potential and the mean-field vacuum

To determine the mean-field vacuum, we next derive the mean-field effective potential by integrating out the quantum fluctuations of $S$. The assumption that the bilinear condensate (2.2) is $U(N_f)$ invariant means for the composite fields that $\langle f \rangle \neq 0$ and $\langle \phi^a \rangle = 0$. Therefore, it is sufficient to consider the path-integral (2.9) with $\phi^a = 0$. Since (2.9) is quadratic in $S$, the path-integral for $S$ is Gaussian, and then we obtain
\[
Z = \int \mathcal{D}f \exp \left[ i \int d^4x \left\{ - M^2 (S_i^\dagger S_i) - \lambda_H (H^\dagger H)^2 \\
+ N_f (N_f \lambda S + \lambda'_S) f^2 + iN_cN_f \ln \det \left[ \partial^2 + M^2 \right] \right\} \right].
\]
where the fluctuation of $S$ has been integrated out around its background field $\bar{S}$. The last term in the right-hand side of (2.14) is evaluated as
\[
\ln \det \left[ \partial^2 + M^2 \right] = (VT) \frac{M^4}{2(4\pi)^2} \left( \frac{1}{\bar{\varepsilon}} - \ln(M^2) + \frac{3}{2} \right),
\]
where the dimensional regularization has been used, $VT$ is the space-time volume ($VT = \int d^4x$), and $1/\bar{\varepsilon} = 2/(d-4) - \gamma_E + \ln(4\pi)$. Then the 1PI effective action at the one-loop level is given by
\[
\Gamma[\bar{S},f,H] = VT \left[ - M^2 (S_i^\dagger S_i) - \lambda_H (H^\dagger H)^2 \\
+ N_f (N_f \lambda S + \lambda'_S) f^2 + \frac{N_cN_f}{2(4\pi)^2} M^4 \left( \frac{1}{\bar{\varepsilon}} - \ln(M^2) + \frac{3}{2} \right) \right].
\]
Note that we have not used the large-$N$ approximation to derive the effective action (2.16). Finally we obtain the effective potential
\[
V_{\text{MFA}}(\bar{S},f,H) = - \frac{\Gamma[\bar{S},f,H]}{VT} = M^2 (S_i^\dagger S_i) + \lambda_H (H^\dagger H)^2 - N_f (N_f \lambda S + \lambda'_S) f^2 + \frac{N_cN_f}{32\pi^2} M^4 \ln \frac{M^2}{\Lambda^2_H},
\]
\footnote{The field $S$ is expanded around the homogeneous background, i.e., $S \to \bar{S} + \chi$, and the path-integral of $\chi$ is performed.}
where the divergence $1/\bar{\epsilon}$ was removed by renormalization of the coupling constants in the $\overline{MS}$ scheme and $\Lambda_H = \mu e^{3/4}$ is the scale at which the quantum correction vanishes if $M = \Lambda_H$. Note here that the scale $\Lambda_H$ is generated by quantum effect in the classically scale invariant effective theory (2.3) and becomes the origin of the electroweak scale as it will be seen below.

The minima of the effective potential (2.17) can be obtained from the solution of the gap equations\(^3\)

$$0 = \frac{\partial}{\partial S_i} V_{MFA} = \frac{\partial}{\partial f} V_{MFA} = \frac{\partial}{\partial H_i} V_{MFA} \quad (l = 1, 2). \quad (2.18)$$

The first equation (2.18) yields $\langle \hat{S}_i^2 \rangle \langle M^2 \rangle = 0$, which is satisfied in the following three cases: (i) $\langle \hat{S}_i^2 \rangle \neq 0$ and $\langle M^2 \rangle = 0$; (ii) $\langle \hat{S}_i^2 \rangle = 0$ and $\langle M^2 \rangle = 0$; (iii) $\langle \hat{S}_i^2 \rangle = 0$ and $\langle M^2 \rangle \neq 0$. The case (i) corresponds to the end-point solution [118, 119] in which the effective potential has a flat direction, i.e., $V_{eff} = 0$ for $f = H = 0$. The gap equations in the case (i) imply the relation $\langle f \rangle = 2\lambda_H/Nf\lambda_HS(H^*H)$, and the effective potential at the minimum vanishes, i.e., $\langle V_{MFA} \rangle = 0$ for an arbitrary value of $\hat{S}$. In the case (ii) $\langle V_{MFA} \rangle = 0$ follows trivially. In the case (iii), using the other gap equations, we obtain

$$\langle H \rangle^2 = \frac{v^2}{2} = \frac{Nf\lambda_HS}{G} \Lambda_H^2 \exp \left( \frac{32\pi^2 \lambda_H}{NcG} - \frac{1}{2} \right), \quad \langle f \rangle = f_0 = \frac{2\lambda_H}{Nf\lambda_HS} \langle H \rangle^2, \quad (2.19)$$

$$\langle M^2 \rangle = M_0^2 = \frac{G}{Nf\lambda_HS} \langle H \rangle^2 \quad (2.20)$$

at the minimum, where $G \equiv 4Nf\lambda_H\lambda_S - Nf\lambda_H^2 + 4\lambda_H\lambda_S$. The value of the effective potential at this minimum is given by

$$\langle V_{MFA} \rangle = -\frac{NcNf}{64\pi^2} \Lambda_H^4 \exp \left( \frac{64\pi^2 \lambda_H}{NcG} - 1 \right) < 0. \quad (2.21)$$

We therefore conclude that the case (iii) corresponds to the absolute minimum of the effective potential (2.17) as far as $G > 0$ is satisfied. The Higgs mass at this level of approximation is calculated to be

$$m_{h0}^2 = \langle H \rangle^2 \left( \frac{16\lambda_H^2(Nf\lambda_S + \lambda_S')}{G} + \frac{NcNf\lambda_H^2}{8\pi^2} \right). \quad (2.22)$$

In the small $\lambda_HS$ limit, this mass can be expanded as

$$m_{h0}^2 = 2Nf\lambda_HS\langle f \rangle + Nf\lambda_H^2 \left( \frac{Nc}{8\pi^2} + \frac{1}{Nf\lambda_S + \lambda_S'} \right) \langle H \rangle^2 + \cdots. \quad (2.23)$$

We see that the Higgs mass is generated by the scalar-bilinear condensation $f$ and the second term in the right-hand side of (2.23) comes from the back-reaction due to the finite vacuum expectation value of the Higgs field. In other words, the first term is, due to (2.19), equal to $4\lambda_H \langle H \rangle^2$, which is the tree level expression in the SM model, so that the second term must be a correction due to the back-reaction. The correction coming from the SM sector to the Higgs mass (2.22) will be calculated below.

\(^3\)A similar potential problem has been studied in [115–119]. But they did not study the classical scale invariant case in detail, and moreover no coupling to the SM was introduced.
2.3 Corrections from the SM sector

We calculate the one-loop contribution from the SM sector to the effective potential (2.17) and evaluate the correction to the Higgs mass (2.22). The one-loop contribution to the effective potential can be calculated from

\[ V_{CM}(h) = \sum_{I=W,Z,t,h} \frac{n_I}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + m_I^2(h)) - \frac{n_t}{2} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + m_t^2(h)) + \text{c.t.}, \]  

(2.24)

where \( n_I \) (\( I = W, Z, t, h \)) is the degrees of freedom of the corresponding particle, i.e., \( n_W = 6, n_Z = 3, n_t = 12 \) and \( n_h = 1 \), and c.t. stands for the counter terms. The contributions from the would-be Goldstone bosons in the Higgs field have been neglected.

As before we use the dimensional regularization to respect scale invariance, and the counter terms are so chosen that the normalization conditions

\[ V_{CW}(h = v_h) = 0, \quad \left. \frac{dV_{CW}(h)}{dh} \right|_{h = v_h} = 0 \]  

(2.25)

with \( v_h = 246 \text{ GeV} \) are satisfied. In this way we obtain the Coleman-Weinberg potential \[53\]

\[ V_{CW}(h) = C_0(h^4 - v_h^4) + \frac{1}{64\pi^2} \left[ 6\tilde{m}_W^4 \ln(\tilde{m}_W^2/m_W^2) + 3\tilde{m}_Z^4 \ln(\tilde{m}_Z^2/m_Z^2) \right. \]
\[ \left. + \tilde{m}_H^4 \ln(\tilde{m}_H^2/m_h^2) - 12\tilde{m}_t^4 \ln(\tilde{m}_t^2/m_t^2) \right], \]  

(2.26)

where

\[ C_0 \simeq -\frac{1}{64\pi^2 v_h^2} (3m_W^4 + (3/2)m_Z^4 + (3/4)m_h^4 - 6m_t^4), \]  

(2.27)

\[ \tilde{m}_W^2 = (m_W/v_h)^2 h^2, \quad \tilde{m}_Z^2 = (m_Z/v_h)^2 h^2, \quad \tilde{m}_t^2 = (m_t/v_h)^2 h^2, \]
\[ \tilde{m}_H^2 = 3\lambda_H h^2 + \frac{\lambda_H S}{64\pi^2} \left\{ 7N_c N_f \lambda_H S h^2 - 4f N_c N_f (N_f \lambda_S + \lambda_S) \right. \]
\[ \left. - 2N_c N_f \left[ -3\lambda_H S h^2 + 4f (N_f \lambda_S + \lambda_S) \right] \ln \frac{4f (N_f \lambda_S + \lambda_S) - \lambda_H S h^2}{2\Lambda_H^2} \right\}, \]  

(2.28)

and \( m_I \) (masses given at the vacuum \( v_h = 246 \text{ GeV} \)) are

\[ m_W = 80.4 \text{ GeV}, \quad m_Z = 91.2 \text{ GeV}, \quad m_t = 174 \text{ GeV}, \quad m_h = 125 \text{ GeV}. \]  

(2.29)

We find that the Coleman-Weinberg potential (2.26) yields a one-loop correction to the Higgs mass squared (2.22)

\[ \delta m_h^2 = \left. \frac{d^2 V_{CW}}{dh^2} \right|_{h = v_h} \simeq -16C_0 v_h^2, \]  

(2.30)

which gives about \(-6\%\) correction to (2.22).

\footnote{We work in the Landau gauge.}
Figure 1. The spin-independent elastic cross section $\sigma_{SI}$ of DM off the nucleon as a function of $m_{DM}$ for $N_f = 2, N_c = 6$. The black solid line stands for the central value of the LUX upper bound [121–123] with one (green) and two (yellow) $\sigma$ bands, and the black dotted line indicates the sensitivity of XENON1T [124, 125].

3 Benchmark points and scale phase transition

Thanks to the unbroken U($N_f$) flavor symmetry the excitation field $\phi^a$ can be a DM candidate. How to evaluate its relic abundance $\Omega_{DM}^\hat{h}^2$ and its spin-independent elastic cross section off the nucleon $\sigma_{SI}$ is explained in [73], where $\hat{h}$ is the dimensionless Hubble constant today. Since we would like to perform a benchmark point analysis, let us explain the parameter space which we are interested in. To obtain the DM relic abundance and its spin-independent elastic cross section off the nucleon, the annihilation process, in which a pair of $\phi^a$ annihilates into the SM particles through the effective interaction $(\phi^a)^2 h^2$, has to be considered. The effective interaction is generated by the loop effects of $S$ as shown in figure 2, where the vertices in the diagrams are $S_i^t \epsilon_{ij} \phi^a S_j$ and $(S_i^t S_i) h^2$ in the mean-field Lagrangian (2.12). The s-channel describes the DM annihilation process, while the t-channel is used for the DM interaction with the nucleon.

The Higgs portal coupling $\lambda_{HS}$ plays a triple role. First, as we see from (2.19), the smaller $\lambda_{HS}$ is, the larger $\Lambda_H$ has to become, because $|\langle H \rangle|$ is fixed at $v_h/\sqrt{2} = 246/\sqrt{2}$ GeV. Secondly, since the coupling constant for the effective interaction $(\phi^a)^2 h^2$ is proportional to $\lambda_{HS}$, the DM relic abundance decreases as $\lambda_{HS}$ increases, while $\sigma_{SI}$ increases. That is, to satisfy the experimental constraints on $\Omega_{DM}^\hat{h}^2$ [120] and $\sigma_{SI}$ [121–123] at the same time, $\lambda_{HS}$ has to lie in an interval. Equivalently, too small or too large $\Lambda_H$ is inconsistent with the DM constraints. The interval depends strongly on $N_f$, because $\Omega_{DM}^\hat{h}^2$ is proportional to $N_f^2 - 1$. This implies that the weakest constraint on $\lambda_{HS}$ is given for $N_f = 2$. Furthermore, the color degrees of freedom in the hidden sector is not completely free within our effective field theory approach. This is due to the inverse propagator $\Gamma(p^2)$ for the DM, which can have a zero for a positive $p^2$ only if $\lambda_S^2 N_c$ is large enough (the zero of $\Gamma(p^2)$ defines the DM mass). If we restrict ourselves to $\lambda_S^2 \lesssim 2$, we find that $N_c > 4$. On the other hand, the results (at least for the DM phenomenology) for $N_c = 5 - 8$ are very similar. The predicted region for $N_f = 2$ and $N_c = 6$ is shown in figure 1. As we can see from figure 1 this result could be tested by XENON1T [124, 125], whose sensitivity is indicated by the dotted line.
When discussing the scale phase transition at finite temperature and the GW background, we will consider a benchmark point in our parameter space:

\[ N_f = 2, \quad N_c = 6, \quad \lambda_S = 0.145, \quad \lambda'_S = 2.045, \quad \lambda_H = 0.15, \quad \lambda_{HS} = 0.032, \]

which yields

\[
\begin{align*}
\Lambda_H &= 0.0621 \text{ TeV}, \\
M_{DM} &= 0.856 \text{ TeV}, \\
m_h &= 0.126 \text{ TeV}, \\
\sigma_{SI} &= 5.12 \times 10^{-46} \text{ cm}^2.
\end{align*}
\]

As we can see from figure 1, most of the predicted points in the \( m_{DM} - \sigma_{SI} \) plane, except for those with smaller \( m_{DM} \), are close to the benchmark point. The third role of \( \lambda_{HS} \) will be discussed when considering phase transition at finite temperature below.

It is expected that at high temperature the thermal effects restore the electroweak symmetry and scale symmetry. We assume that even at finite temperature the mean-field approximation is still a good approximation to the original strongly interacting gauge theory (2.1). The effective potential consists of four components,

\[
V_{\text{eff}}(f, h, T) = V_{\text{MFA}}(f, h) + V_{\text{CW}}(h) + V_{\text{FT}}(f, h, T) + V_{\text{ring}}(h, T),
\]

where \( V_{\text{MFA}}(f, h) \) and \( V_{\text{CW}}(h) \) are given in (2.17) and (2.26), respectively. Since the absolute minimum of \( V_{\text{MFA}} \) is located at \( \langle S \rangle = 0 \), we suppress the \( \langle S \rangle \) dependence of \( V_{\text{MFA}} \). The main thermal effects are included in \( V_{\text{FT}}(f, h, T) \), which is

\[
V_{\text{FT}}(f, h, T) = \sum_{l=S,W,Z,h} n_l T \int \frac{d^3k}{(2\pi)^3} \ln \left( 1 + e^{\omega_l/T} \right) - n_l T \int \frac{d^3k}{(2\pi)^3} \ln \left( 1 - e^{\omega_l/T} \right)
\]

\[
= \frac{T^4}{2\pi^2} \left[ 2 N_c N_f J_B \left( \tilde{M}^2(T)/T^2 \right) + J_B (\tilde{m}_h^2(T)/T^2) + 6 J_B (\tilde{m}_W^2/T^2) + 3 J_B (\tilde{m}_Z^2/T^2) + 12 J_B (\tilde{m}_l^2/T^2) \right],
\]

where \( \omega_l = \sqrt{k^2 + m_l^2(h)} \), and the thermal masses are given by

\[
\tilde{M}^2(T) = M^2 + \frac{T^2}{6} \left( (N_c N_f + 1) \lambda_S + (N_f + N_c) \lambda'_S - \lambda_{HS} \right),
\]

\[
\tilde{m}_h^2(T) = \tilde{m}_h^2 + \frac{T^4}{12} \left( \frac{9}{4} g_2^2 + \frac{3}{4} g_1^2 + 3 y_t^2 + 6 \lambda_H - N_c N_f \lambda_{HS} \right).
\]

Here \( g_2 = 0.65 \) and \( g_1 = 0.36 \) are the SU(2)_L and U(1)_Y gauge coupling constants, respectively, while \( y_t = 1.0 \) stands for the top-Yukawa coupling constant. Further, \( M^2 \) and

...
\( \hat{m}_I (I = W, Z, t, h) \) are given in (2.10) (with \( H^\dagger H = h^2/2 \)) and (2.28), respectively. The thermal functions and their high temperature expansions are

\[
J_B(r^2) = \int_0^\infty dx x^2 \ln \left( 1 - e^{-r^2 + r^2} \right) \\
\simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12} r^2 - \frac{\pi}{6} r^3 - \frac{r^4}{32} \left[ \ln(r^2/16\pi^2) + 2\gamma_E - \frac{3}{2} \right] \quad \text{for} \ r^2 \lesssim 2, \\
J_F(r^2) = \int_0^\infty dx x^2 \ln \left( 1 + e^{-r^2 + r^2} \right) \\
\simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24} r^2 - \frac{r^4}{32} \left[ \ln(r^2/\pi^2) + 2\gamma_E - \frac{3}{2} \right] \quad \text{for} \ r^2 \lesssim 2.
\]

(3.7)

(3.8)

Although the high temperature expansions are useful, they are not suitable for large \( r > 2 \). Therefore, the following fitting functions [126] are used:\(^*5\)

\[
J_{B(F)}(r^2) \simeq e^{-T} \sum_{n=0}^{40} e^{B(F)_r} r^n.
\]

(3.9)

The contribution from the ring (daisy) diagrams of gauge boson is [128]

\[
V_{\text{ring}}(h, T) = -\frac{T}{12\pi} \left( 2a_g^{3/2} + \frac{1}{2}\sqrt{2} \left( a_g + c_g - [(a_g - c_g)^2 + 4b_g^{2}]^{1/2} \right)^{3/2} \right. \\
+ \frac{1}{2}\sqrt{2} \left( a_g + c_g + [(a_g - c_g)^2 + 4b_g^{2}]^{1/2} \right)^{3/2} \\
- \frac{1}{4} \left[ g_2^2 h^2 \right]^{3/2} - \frac{1}{8} \left[ (g_2^2 + g_1^2) h^2 \right]^{3/2} \right).
\]

(3.10)

where

\[
a_g = \frac{1}{4} g_2^2 h^2 + \frac{11}{6} g_2^2 T^2, \quad b_g = -\frac{1}{4} g_2 g_1 h^2, \quad c_g = \frac{1}{4} g_1^2 h^2 + \frac{11}{6} g_1^2 T^2.
\]

(3.11)

Note that the ring contributions from the scalar \( S \) and the Higgs field are included in (3.4).

Using the thermal effective potential (3.3), it is possible to consider phase transitions at finite temperature: there exist two phase transitions in this model, the scale and EW phase transitions (whose critical temperatures are denoted by \( T_S \) and \( T_{EW} \), respectively). The scale phase transition in hidden sector can be strongly first-order for a wide range in the parameter space [74]. In the case with \( N_f = 1 \) (no DM) we see that both phase transitions can be strongly first-order and occur at the same temperature, \( T_S = T_{EW} \). On the other hand, if DM is consistently included (\( N_f > 1 \)), the EW phase transition becomes weak, although the scale phase transition is still strongly first-order, and \( T_{EW} < T_S \). The crucial difference between the two cases comes from the value of the Higgs portal coupling constant \( \lambda_{HS} \), which controls the strength of the connection between the SM and hidden sector. As discussed in section 3, smaller \( \lambda_{HS} \) means larger \( \Lambda_H \), which in turn implies larger \( T_S \), while \( T_{EW} \) stays around \( O(100) \) GeV. This is the third role of \( \lambda_{HS} \). Therefore, within the minimal model with a DM candidate we are considering here, the scale phase transition occurs at a higher temperature than the EW phase transition and is strongly first-order. Since we are

\(^*5\)The validness of the approximations for the thermal functions is discussed in [127].
especially interested in the possibility to explain the origin of the EW scale and DM at the same time, we focus in the following discussions on the model with $N_f = 2$, in which, as explained in section 3, the weakest constraint on $\lambda_{HS}$ has to be satisfied.

The EW phase transition is shown in figure 3 for the set of the benchmark parameters (3.1). We see from figure 3 that a weak transition appears around $T_{EW} \simeq 0.161$ TeV.

Figure 4 (left) presents $\langle f \rangle^{1/2}/T$ as a function of $T$, while figure 4 (right) shows the effective potential $V_{\text{eff}}$ given in (3.3) with $h = 0$ at $T = T_S = 0.323$ TeV as a function of $f^{1/2}$, showing that the scale phase transition is strongly first-order.

4 Gravitational wave from scale phase transition

As we have seen in the previous section, the scale phase transition occurs at $T \lesssim$ few hundred GeV. This is larger than $T_{EW}$, and moreover the phase transition is strongly first-order.\footnote{The GW background produced by a strong EW phase transition has been studied in [129–139]. Similar studies have been made in [140, 141].} This strong first-order phase transition produces the EW scale and at the same time dark matter.
The characteristics of a first-order phase transition in discussing the GW background spectrum are the duration of the phase transition and the latent heat released. The duration time has to be sufficiently short compared with the expansion of the Universe, and it is clear that the more latent heat is released, the larger $\Omega_{\text{GW}}$ can become. In the following two subsections we will discuss these issues.

4.1 Latent heat

Given the effective potential $V_{\text{eff}}$ in (3.3), it is straightforward to compute the latent heat $\epsilon(T)$ at $T < T_S$:

$$\epsilon(T) = -V_{\text{eff}}(f_B(T), T) + T \frac{\partial V_{\text{eff}}(f_B(T), T)}{\partial T},$$

where $f_B(T)$ is $\langle f \rangle$ at $T$, and we have set $h$ equal to zero, because $\langle h \rangle = 0$ for $T > T_{EW}$. The ratio of the released vacuum density $\epsilon(T_t)$ to the radiation energy density is

$$\alpha = \frac{\epsilon(T_t)}{\rho_{\text{rad}}},$$

which is one of the basic parameters entering into $\Omega_{\text{GW}}$, where $T_t$ (defined in (4.8)) is the temperature just below $T_S$, and $\rho_{\text{rad}} = g_*(T_t) \pi^2 T_t^4 / 30$ with $g_* = 106.75$.

Since the latent heat in the lattice SU(3)$_c$ gauge theory has been calculated [143], it may be worthwhile to compare our result with that of [143]. To this end, we calculate the latent heat in the mean-field approximation with $N_c = 3, N_f = 1$ and $\lambda_{HS} = \lambda_H = 0$. In this case the dynamical scale symmetry breaking (at $T = 0$) occurs for $\lambda_S + \lambda'_S \geq 2.8$, and therefore, we calculate the latent heat $\epsilon(T)$ just below the critical temperature for $\lambda_S + \lambda'_S = 3, 4$ and 5, respectively. We find:

$$\frac{\epsilon(T_t)}{T_t^4} = \begin{cases} 0.70 & \text{for } \lambda_S + \lambda'_S = 3 \\ 0.55 & \text{for } \lambda_S + \lambda'_S = 4 \\ 0.43 & \text{for } \lambda_S + \lambda'_S = 5 \end{cases}.$$

These results should be compared with $\epsilon(T_t)/T_t^4 = 0.75 \pm 0.17$ of [143]. Though this lattice result is obtained in the theory without matter field, we see that the values (4.3) are comparable in size to the lattice result. This is a good news, because the scale phase transition and the deconfinement phase transition appear at the same time, and the latent heat is proportional to the change of entropy which we do not expect to change a lot if one scalar field is included or not.
4.2 Duration time

To estimate the duration time of a first-order phase transition, we have to consider the underlying physical process, the tunneling process from the false vacuum to the true vacuum, because the duration time is the inverse of the decay rate of the false vacuum. The decay (the tunneling probability) per unit time per unit volume $\Gamma(t)$ can be written as

$$\Gamma(t) \sim e^{-S_E(t)},$$

(4.4)

where $S_E$ is the Euclidean action in the full theory (2.1). At finite temperature $T$ the theory is equivalent to the Euclidean theory, which is periodic in the Euclidean time with the period of $T^{-1}$. Above a certain high temperature the typical size of the bubbles generated by the phase transition may become much larger than the period $T^{-1}$ [142]. Then we may assume [142] that $S_E = S_3/T$, where $S_3$ is the corresponding three-dimensional action.

There are various complications when computing $S_3$ in the effective theory in the mean-field approximation, where they are related with each other. First, the canonical dimension of the mean field $f$ is two. This itself is not a big problem, and we redefine $f$ as

$$f = \gamma \chi^2,$$

(4.5)

where $\gamma$ is dimensionless so that the canonical dimension of $\chi$ is one. Second, the kinetic term for $f$ is absent at the tree-level and is generated in the one-loop order. Consequently, the kinetic term of $f$ in the action involves the field renormalization factor $Z$. Since the effective potential is computed at zero external momenta, $Z$ may be computed also at zero external momenta, which is done in the appendix. As we can see from (A.8), $Z$ depends on $f$ as well as on $T$. However, we have a problem here, because how to include $Z$ in the kinetic term is not unique: $Z^{-1} \langle \partial_i f \partial_i f \rangle$ and $(\partial_i Z^{-1/2} f)(\partial_i Z^{-1/2} f)$, for instance, give different equations of motion for $f$. The third complication is most serious: in principle we have to compute $S_E$ in the full theory (2.1). That is, instantons have to be taken into account. Therefore, we expect that the action $S_3/T$ computed in the effective theory in the mean-field approximation cannot be a good approximation to the full quantity, because the effective theory does not know about instantons and confinement. So, we do not trust $S_3/T$ obtained in the effective theory, although we believe that the effective potential $V_{\text{eff}}$ is a good approximation (in fact the latent heat computed from $V_{\text{eff}}$ gives reasonable values compared with the lattice result [143]). That is, we do not trust the kinetic term obtained in the effective theory (which is anyhow ambiguous), and instead we make an Ansatz for the kinetic term. The simplest assumption is that the redefinition (4.5) gives a correct, canonically normalized kinetic term for $\chi$, where $\gamma$ should be regarded as a free parameter constant independent of $\chi$ and $T$.\footnote{As we see from (A.6), $Z$ at $T = 0$, obtained in the one-loop order in the effective theory, is $16\pi^2 12 N_f (N_f \lambda_\Sigma + \lambda_\Sigma^2) f \sim 8 \times 10^2 f$ for the benchmark parameters (3.1). Then $Z^{-1} \partial_i f \partial_i f = \partial_i \chi \partial_i \chi$ if $\gamma = 4\pi^2 12 N_f (N_f \lambda_\Sigma + \lambda_\Sigma^2) \sim 2 \times 10^2$. This implies that, although $\langle f^{1/2} \rangle / T_S \sim 1$ for the benchmark point (3.1), $\langle \chi \rangle / T_S \sim 0.02$ for $\chi$. So, in terms of $\chi$ the phase transition would no longer be a strong phase transition. This is also a reason why we do not trust the kinetic term for $f$ obtained in the effective theory.}

That is, we make an Ansatz for $S_3$:

$$S_3(T) = \int d^3 x \left[ \frac{1}{2} (\nabla_i \chi)^2 + V_{\text{eff}}(\gamma \chi^2, T) \right],$$

(4.6)

where the $h$ dependence of the effective potential is suppressed (because $\langle h \rangle = 0$ for $T > T_{\text{EW}}$), and it is normalized as $V_{\text{eff}}(0, T) = 0$. At high enough temperatures we may assume [142] that, not only $S_E = S_3/T$ is satisfied, but also the classical solution to the field
equation is $O(3)$ symmetric: $\chi$ depends only on $r = (x_1^2 + x_2^2 + x_3^2)^{1/2}$ and satisfies

$$\frac{d^2 \chi}{dr^2} + \frac{2}{r} \frac{d \chi}{dr} = \frac{dV_{\text{eff}}(\gamma \chi^2, T)}{d\chi}. \quad (4.7)$$

To compute the tunnel provability at $T < T_S$ from the false vacuum with $\chi = 0$ to the true vacuum with $\chi = \chi_B \neq 0$, we look for the classical solution, the so-called bounce solution, that satisfies the boundary conditions, $\chi(r = \infty) = 0$. The initial value $\chi(0)$ should be chosen slightly smaller than $\chi_B$, such that $\chi(r = \infty) = 0$ is satisfied. Then we insert the solution into (4.6) to compute $S_3(T)/T$.

At the temperature $T_t$ (or at the time $t_t$) the vacuum is overwhelmed by the bubble of the broken phase. Since the Universe is expanding, it is the time at which the tunneling provability per Hubble time per Hubble volume becomes nearly one, because after each tunneling process we have one bubble nucleation. This defines the transition time $t_t$ and also the transition temperature $T_t$:

$$\frac{\Gamma(t)}{H(t)^4} \bigg|_{t=t_t} \simeq 1, \quad (4.8)$$

where $H(t)$ is the Hubble parameter at $t$. This condition is rewritten as [131]

$$S_E(t_t) = \frac{S_3(T_t)}{T_t} \simeq 140–150. \quad (4.9)$$

Since the transition time $t_t$ (or the transition temperature $T_t$) is now defined, we can compute the duration of the phase transition. To this end, we expand the action around $t_t$:

$$S_E(t) = S_E(t_t) - \beta \Delta t + O((\Delta t)^2), \quad (4.10)$$

where $\Delta t = (t - t_t) > 0$. Then the tunneling per unit time per volume can be approximated as

$$\Gamma(t) \sim e^{\beta \Delta t}. \quad (4.11)$$

Therefore, $\beta^{-1}$ is the duration time and can be computed from

$$\beta = -\frac{dS_E}{dt} \bigg|_{t=t_t} = \frac{1}{\Gamma} \frac{dT}{dt} \bigg|_{t=t_t} = H_t T_t \frac{d}{dT} \left( \frac{S_3(T)}{T} \right) \bigg|_{T=T_t}, \quad (4.12)$$

where $dT/dt = -HT$ is used and $H_t$ is $H(t_t)$.

This $\tilde{\beta}$ and $\alpha$ in (4.2) are the parameters which determine the characteristics of the first-order phase transition and enter into the GW background spectrum. Their values for the set of the benchmark parameters (3.1) with $\gamma = 0.5, 1$ and 2 are shown in table 1.

### 4.3 GW background spectrum

It is known at present that there are three production mechanisms of GWs at a strong first-order phase transition: the bubble nucleation in a strong first-order phase transition grows, leading to the contributions to $\Omega_{\text{GW}}$ from collisions of bubble walls $\Omega_{\text{coll}} [88–94]$, magnetohydrodynamic turbulence $\Omega_{\text{MHD}} [95–101]$ and also sound waves $\Omega_{\text{sw}}$ after the bubble wall collisions [102–105]. Then the total GW background spectrum is given by

$$\Omega_{\text{GW}}(\nu) \hat{h}^2 = [\Omega_{\text{coll}}(\nu) + \Omega_{\text{MHD}}(\nu) + \Omega_{\text{sw}}(\nu)] \hat{h}^2, \quad (4.13)$$
Table 1. Relevant quantities for the GW background spectrum for the set of the benchmark parameters (3.1). The quantities $\alpha$, $\tilde{\beta}$ and $\gamma$ are defined in (4.2), (4.12) and (4.5), respectively.

| $\gamma$ | $T_t$ [TeV] | $S_3(T_t)/T_t$ | $\alpha$ | $\tilde{\beta}$ | $\tilde{\Omega}_{sw}\hat{h}^2$ | $\tilde{\nu}_{sw}$ [Hz] |
|----------|--------------|----------------|---------|----------------|------------------|------------------|
| 0.5      | 0.300        | 149            | 0.070   | $3.7 \times 10^3$ | $1.9 \times 10^{-13}$ | 0.37             |
| 1.0      | 0.311        | 145            | 0.062   | $7.0 \times 10^3$ | $7.4 \times 10^{-14}$ | 0.73             |
| 2.0      | 0.316        | 146            | 0.059   | $13 \times 10^3$ | $3.4 \times 10^{-14}$ | 1.4              |

\[
\tilde{\Omega}_{sw}\hat{h}^2 = 2.65 \times 10^{-6} v_b \tilde{\beta}^{-1} \left( \frac{\kappa \alpha}{1 + \alpha} \right)^2 \left( \frac{100}{g_s} \right)^{1/3}
\]
with the peak frequency $\tilde{\nu}_{sw}$ given by

$$
\tilde{\nu}_{sw} = 1.9 \times 10^{-5} \text{Hz} \times \frac{\tilde{\beta}}{v_b} \left( \frac{T_\ast}{100 \text{GeV}} \right) \left( \frac{g_*}{100} \right)^{1/6}.
$$

(4.16)

Since we are interested in order-of-magnitude estimates of the GW background spectrum, we consider the case that the wall velocity is the sound velocity $c_s = 0.577$. In this case the efficiency factor $\kappa$ becomes [102–105]:

$$
\kappa = \frac{\alpha^{2/5}}{0.017 + (0.997 + \alpha)^{2/5}}.
$$

(4.17)

The result for the set of the benchmark parameters (3.1) with $\gamma = 0.5, 1$ and 2 is shown in table I, and the model prediction of the GW background spectrum is compared with the experimental sensitivity of present and future experiments in figure 5. As we can see from figure 5, the GW background produced by the scale phase transition in the early Universe can be observed by DECIGO, where $\gamma$ is introduced in (4.5) and its double meaning is explained there.

5 Summary

In this paper we have considered a non-perturbative effect, the gauge-invariant scalar-bilinear condensation, that generates a scale in a strongly interacting gauge sector, where the scale is transmitted to the SM sector via a Higgs portal coupling $\lambda_{HS}$. The dynamical scale genesis appears as a phase transition at finite temperature, and it can produce the GW background in an early Universe. Since $\lambda_{HS}$ controls the strength of the connection between the SM and hidden sector, smaller $\lambda_{HS}$ means larger $\Lambda_H$, which in turn implies larger critical temperature $T_S$ for the scale phase transition. Therefore, $T_S \gg T_{EW} \sim O(100) \text{GeV}$ for smaller $\lambda_{HS}$. The coupling $\lambda_{HS}$, on the other hand, cannot be too small, because it is constrained from below by the DM relic abundance, implying that $T_S < \text{few hundreds GeV}$. Interestingly, in this interval of $\lambda_{HS}$, where we obtain consistent values of the DM relic abundance, the scale phase transition is strongly first-order.

We have calculated the spectrum of the GW background, using the effective field theory, which has been developed in [73]. Our intention in the present paper has not been to describe in detail the process of the generation of the GW background. We instead have applied the formulas given in [81] to compute the GW background spectrum. We have found that the contributions to the GW spectrum from the collisions of bubble walls and also from the magnetohydrodynamic turbulence are negligibly small compared with the sound wave contribution. We have found that the peak frequency $\tilde{\nu}_{sw}$ of the GW background is $O(10^{-1}) \sim O(1) \text{ Hz}$ with the peak relic energy density $\Omega_{sw} h^2 = O(10^{-14}) \sim O(10^{-13})$. Therefore, the scale phase transition caused by the scalar-bilinear condensation is strong enough that the corresponding GW background can be observed by DECIGO [106–108] in future.

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A Field renormalization factor

We derive the kinetic term of the composite field at the one-loop level. The two-point function is given by

\[ \Pi(\omega, \vec{p}, T, M) = T \sum_{\lambda} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\omega_n - \omega_l)^2 + (\vec{p} - \vec{k})^2 + M^2 (\omega_l^2 + \vec{k}^2 + M^2)} \]

where \( M^2 \) is (2.10) with \( \lambda_{HS} = 0 \). Although at finite temperature the kinetic term between the time-direction momentum mode and spacial-direction one is anisotropic, hence the field renormalization factors should be defined as

\[ Z^{-1}(\lambda) \equiv \frac{d\Gamma}{d\omega_n^2} \bigg|_{p=0}, \quad Z^{-1}(\parallel) \equiv \frac{d\Gamma}{d\vec{p}^2} \bigg|_{p=0} \]

we hereafter assume that \( Z^{-1}_\perp \approx Z^{-1}_\parallel \) and write them as simply \( Z^{-1} \).

Expanding the integrant of (A.1) into the polynomial of \( \vec{p} \) around \( \omega_n = \vec{p} = 0 \),

\[ \frac{1}{\omega_n^2 - (\vec{p} - \vec{k})^2 + M^2 (\omega_l^2 + \vec{k}^2 + M^2)} = \frac{1}{\omega_l^2 + \vec{k}^2 + M^2} - \frac{\vec{p}^2 - 2\vec{p} \cdot \vec{k}}{(\omega_l^2 + \vec{k}^2 + M^2)^2} + \frac{4(\vec{k} \cdot \vec{p})^2}{(\omega_l^2 + \vec{k}^2 + M^2)^3} + \cdots, \]

and replacing \( \vec{p} \cdot \vec{k} \) and its squared by 0 and \( \vec{k}^2 \vec{p}^2 / 3 \), respectively, we obtain

\[ Z^{-1}(\lambda) = T \sum_{\lambda} \int \frac{d^3k}{(2\pi)^3} \left( \frac{1}{(\omega_n^2 + \vec{k}^2 + M^2)^3} - \frac{4\vec{k}^2}{3(\omega_n^2 + \vec{k}^2 + M^2)^4} \right). \]

Using the standard calculation method at finite temperature, we can separate this into the zero temperature mode and the finite one:

\[ Z^{-1}(\lambda) = Z_{T=0}^{-1}(\lambda) + Z_{T\neq0}^{-1}(\lambda), \]

with

\[ Z_{T=0}^{-1}(\lambda) = \int \frac{d^3k}{(2\pi)^3} \left( \frac{1}{(k^2 + M^2)^3} - \frac{k^2}{(k^2 + M^2)^4} \right) = \frac{1}{16\pi^2} \frac{1}{6M^2}, \]

and

\[ Z_{T\neq0}^{-1}(\lambda) = \int \frac{d^3k}{(2\pi)^3} \left( \frac{3}{8} \frac{1}{(k^2 + M^2)^{5/2}} + \frac{5}{12} \frac{1}{(k^2 + M^2)^{7/2}} \right) \frac{1}{e^{\sqrt{k^2 + M^2}/T} - 1} \]

\[ = \frac{1}{16\pi^2 T^2} \left( 3g_{3/5}(M/T) + \frac{10}{3} g_{5/7}(M/T) \right). \]

The thermal function is defined as

\[ g_{n/m}(y) = \int_0^\infty dx \frac{x^{n-1}}{(x^2 + y^2)^{m/2}} \left( e^{\sqrt{x^2 + y^2}} - 1 \right)^{-1}. \]

More precisely, the field renormalization factor should be defined at on-shell external momentum. However, we calculate it at \( p = 0 \) for simplicity.
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