PLANET FORMATION WITH MIGRATION

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ABSTRACT

In the core-accretion model, gas-giant planets form solid cores that then accrete gaseous envelopes. Tidal interactions with disk gas cause a core to undergo inward type I migration in \(10^7-10^8\) yr. Cores must form faster than this to survive. Giant planets clear a gap in the disk and undergo type II migration in \(\lesssim 10^6\) yr if observed disk accretion rates apply to the disk as a whole. Type II migration times exceed typical disk lifetimes if viscous accretion occurs mainly in the surface layers of disks. Low turbulent viscosities near the midplane may allow planetesimals to form by coagulation of dust grains. The radius \(r\) of such planetesimals is unknown. If \(r < 0.5\) km, the core formation time is shorter than the type I migration timescale, and cores will survive. Migration is substantial in most cases, leading to a wide range of planetary orbits, consistent with the observed variety of extrasolar systems. When \(r \sim 100\) m and the midplane \(\alpha \sim 3 \times 10^{-5}\), giant planets similar to those in the solar system can form.

Subject headings: planetary systems: formation — planetary systems: protoplanetary disks — planets and satellites: formation — solar system: formation

1. INTRODUCTION

In the core-accretion model, gas-giant planets begin life as solid cores that grow by sweeping up small planetesimals (Inaba et al. 2003). Cores \(\gtrsim 10^{-5}\) \(M_\odot\) grow oligarchically; each radial zone in a protoplanetary disk contains a single core and many planetesimals (Kokubo & Ida 1998; Thommes et al. 2003). Perturbations from a core control the orbital distribution of nearby planetesimals that then determines the core’s growth rate. Cores larger than Mars acquire extended atmospheres of nebular gas. Passing planetesimals are slowed by gas drag as a result, increasing the chance of capture (Inaba & Ikoima 2003). When a core reaches a critical mass \(M_{\text{crit}} \sim 10\) \(M_\oplus\), a static atmosphere can no longer be supported, and the core acquires a massive gas envelope, becoming a gas-giant planet (Hubickyj et al. 2005; Ikoima et al. 2000).

Cores must grow to \(M_{\text{crit}}\) before the disk gas disperses, which typically happens in a few megayears (Haisch et al. 2001). A core of mass \(M\) sweeps up planetesimals at a rate given by

\[
\left(\frac{dM}{dt}\right)_{\text{solid}} = \frac{(2\pi \Sigma_{\text{solid}} r_H^3)}{P} P_{\text{coll}}(e, i),
\]

where \(P\) and \(r_H\) are the core’s orbital period and Hill radius, and \(\Sigma_{\text{solid}}\) is the local surface density of planetesimals (Inaba et al. 2001). The collision probability \(P_{\text{coll}}\) depends on the relative velocity \(v_{\text{rel}}\) of passing planetesimals, which is a function of their mean eccentricity \(e\) and inclination \(i\) (Inaba et al. 2001).

In the minimum-mass solar nebula (MMSN), the solid and gas surface densities are

\[
\Sigma_{\text{solid}} = \Sigma_0 n_{\text{ice}} \left(\frac{a}{5\ \text{AU}}\right)^3,
\]

\[
\Sigma_{\text{gas}} = \Sigma_0 n_{\text{gas}} \left(\frac{a}{5\ \text{AU}}\right)^3,
\]

where \(\Sigma_0 n_{\text{ice}} \approx 3\ \text{g cm}^{-2}\) and \(x \approx -3/2\) (Weidenschilling 1977).

Here, \(a\) is the orbital radius; \(\Sigma_0\) is the surface density of rocky material at 5 AU; \(n_{\text{gas}} = 200\); \(n_{\text{ice}} = 1\) inside the snow line, and \(n_{\text{ice}} > 1\) outside the snow line. Most models for the formation of Jupiter use mass-enhanced nebulae, with \(\Sigma_0 n_{\text{gas}} = 8-25\ \text{g cm}^{-2}\), in order to grow to \(M_{\text{crit}}\) before the gas disperses (Inaba et al. 2003; Alibert et al. 2005; Hubickyj et al. 2005; Thommes et al. 2005).

A core generates spiral density waves in the gas that cause the core to undergo inward type I migration at a rate given by

\[
\left(\frac{da}{dt}\right)_{\text{I}} = -1.5\left(\frac{c_s}{v_{\text{Kep}}}\right)^2 v_{\text{Kep}},
\]

where \(v_{\text{Kep}}\) is the core’s Keplerian orbital velocity, \(c_s\) is the gas sound speed, and \(M_\star\) is the stellar mass (Tanaka et al. 2002).

In this Letter we consider only migration caused by the gaseous component of the disk. A \(10\) \(M_\odot\) core at 5 AU in a mass-enhanced nebula will migrate into its star in 10,000-30,000 years, much less than the time required to form a core in most models (Inaba et al. 2003; Alibert et al. 2005; Hubickyj et al. 2005; Thommes et al. 2003; Ida & Lin 2004). Growth and migration rates are both proportional to \(\Sigma\) (for a given disk metallicity), so this result is independent of the disk mass.

A massive planet clears an annular gap in the disk and undergoes type II migration, moving inward at the same rate that gas flows viscously toward the star:

\[
\left(\frac{da}{dt}\right)_{\text{II}} = -1.5\alpha \left(\frac{c_s}{v_{\text{Kep}}}\right)^2 v_{\text{Kep}},
\]

where \(\alpha = \nu v_{\text{Kep}}/(ac_s^2)\) and \(\nu\) is the disk viscosity (D’Angelo et al. 2002). Observed disk accretion rates imply \(\alpha \approx 0.001\) (Hueso & Guillot 2005), so a planet at 5 AU will migrate into its star in 0.5 Myr. This is less than the lifetime of most disks (Haisch et al. 2001).

2. LIVING WITH MIGRATION

The existence of gas-giant planets suggests that (1) giant-planet cores grow rapidly before type I migration moves them into the star and (2) giant planets form at locations where the
disk viscosity is lower than observed disk accretion rates would suggest.

Core growth should be fastest when planetesimals are small for two reasons. Small planetesimals experience strong gas drag, reducing  and  sufficiently for to be determined by Keplerian shear in the disk (Rafikov 2004; Chambers 2006). As a result,  is much higher than in the dispersion-dominated regime considered by most models. In addition, small planetesimals are slowed when they pass through the atmospheres of large cores, increasing the capture probability (Inaba & Ikoma 2003).

Most previous models have considered planetesimals with radii  (Inaba et al. 2003; Alibert et al. 2005; Hubickyj et al. 2005; Thommes et al. 2003). Planetesimals would have this size if they formed via gravitational instabilities (GIs) in the solid component of the disk (Wetherill 1980). However, GI requires (Garaud & Lin 2004), which probably occurs under only limited circumstances. Alternatively, planetesimals may form by pairwise coagulation which probably occurs under only limited circumstances. However, GI requires (Garaud & Lin 2004), which probably becomes small once cores grow larger than Ceres, due to collisional fragmentation (Kenyon & Bromley 1995), which is highly effective but probably confined mainly to the surface layers of a disk (Matsumura & Pudritz 2006). Near the midplane, where planets form,  is 10 g cm^{-2}, and  is 90. Core growth times are assumed to be 50% higher than in equation (1) due to collisions between neighboring cores (Chambers 2006).

Figure 1 shows the mass of a core growing at 5 AU, neglecting migration, for four values of . Here,  is 10 g cm^{-2}, solids have a density of 1.5 g cm^{-3}, and . Core growth times are shorter when is small, as expected. When  100 m, the core formation time is shorter than the type I migration timescale, shown by the dashed line. Dissipation of the nebular gas will slow type I migration, and migration will speed up core growth by increasing the supply of planetesimals (Alibert et al. 2005). Hence, cores should survive when  is somewhat larger than 100 m.

The source of viscosity in protoplanetary disks is unclear. One possibility is magnetorotational instability (Hawley et al. 1995), which is highly effective but probably confined mainly to the surface layers of a disk (Matsumura & Pudritz 2006). Near the midplane, where planets form,  is 30 times lower than in the surface layers (Turner et al. 2006), corresponding to  3 × 10^{-5}. Similar values of  can explain the size distribution of chondrules seen in meteorites (Cuzzi et al. 2001). Low  also promotes dust coagulation by reducing typical collision speeds and fragmentation. For  3 × 10^{-5}, the type II migration time for a planet at 5 AU is  15 Myr, longer than the lifetime of most planetesimals (Haisch et al. 2001).

A core opens a gap in the disk when  , where  is the mass needed to open a gap in a nonviscous disk, given by

\[
M_{\text{vis}} = \frac{M_j}{3} \left( \frac{c_s}{v_{\text{Kep}}} \right)^3 \min \left[ 5.2 Q^{-5/7}, 3.8 \left( \frac{c_s}{Q v_{\text{Kep}}} \right)^{5/13} \right] \tag{5}
\]

\[
M_{\text{vis}} = \frac{M_j}{3} \left( \frac{c_s}{v_{\text{Kep}}} \right)^3 \min \left[ 5.2 Q^{-5/7}, 3.8 \left( \frac{c_s}{Q v_{\text{Kep}}} \right)^{5/13} \right] \tag{5}
\]

Lin & Papaloizou 1986). In the MMSN with  ,  is 15 at 5 AU. Numerical simulations show that gas flows onto a core even when a gap exists (D’Angelo et al. 2002), allowing a gas-giant planet to form. When , the maximum envelope growth rate is

\[
\frac{dM}{dt}_{\text{gas, max}} = \frac{2 \pi a \Sigma_{\text{gas}}}{R_{\text{Kep}}} \left( \frac{da}{dt} \right)_{\text{II}} \times \left[ 0.04 + 1.66 \left( \frac{M_i}{M_j} \right)^{1/3} \exp \left( -\frac{2 M}{3 M_j} \right) \right], \tag{7}
\]

where  is the mass of Jupiter (Alibert et al. 2005; Veras & Armitage 2004). In the MMSN with  , a 10 core at 5 AU will grow to  in 4 Myr according to equation (7).

3. SIMULATIONS OF GROWTH WITH MIGRATION

We now examine the growth and migration of giant planets using a more detailed model. We consider a disk with 5 AU 50 AU, containing 640 cores, each of  , and a population of planetesimals. Cores sweep up planetesimals following equation (1). At each radial location,  and  vary due to gas drag and perturbations from the cores, calculated independently following Inaba et al. (2001) and Ohtsuki et al. (2002). Cores are assumed to have circular, coplanar orbits due to dynamical friction with planetesimals and tidal interaction with the gas. Neighboring cores merge when their orbital separation is 8 , maintaining the typical spacing of (10–12) in N-body simulations of oligarchic growth (Kokubo & Ida 1998). The capture cross section of a core’s atmosphere is calculated following Inaba & Ikoma (2003). Cores undergo migration according to equations (3) and (4). The maximum
type II migration rate is determined by the rate of angular momentum transport through the disk (Ida & Lin 2004). The core gap-opening mass is determined by equations (5) and (6).

The critical mass for a core to begin accreting a gaseous envelope is

$$M_{\text{crit}} = M_c \left( \frac{M}{10^{-6} M_{\odot}} \right)^{1/4} \left( \frac{\kappa}{1 \text{ cm}^2 \text{ g}^{-1}} \right)^{1/4},$$

where $\kappa$ is the opacity of the core’s atmosphere and $M$ is the rate at which the core sweeps up planetesimals (Inaba et al. 2003; Ikoma et al. 2000; Ida & Lin 2004). Here, we assume $\kappa = 0.05 \text{ cm}^2 \text{ g}^{-1}$ and $M_c = 20 M_{\oplus}$. When $M > M_{\text{crit}}$, the envelope growth rate is

$$\frac{dM}{dt}_{\text{gas}} \approx \left( \frac{M}{M_{\oplus}} \right)^{3/2} \left( \frac{\kappa}{1 \text{ cm}^2 \text{ g}^{-1}} \right)^{1/2} \frac{M}{10^{-9} \text{ yr}}$$

(Ikoma et al. 2000; Ida & Lin 2004), where $y = -\frac{1}{2}$ is an empirical fit based on simulations by Hubickyj et al. (2005).

For cores that have opened a gap, the maximum gas accretion rate is given by equation (7). Initially, $\Sigma_{\text{solid}}$ and $\Sigma_{\text{gas}}$ are given by equation (2) with $x = -\frac{1}{2}$, modified by a factor of $\exp(-a/25 \text{ AU})$. This profile is shallower than the MMSN to allow for migration and planetesimal drift due to gas drag. The disk is gravitationally stable everywhere such that $Q > 2$. Initially, $\Sigma_{\text{gas}}/\Sigma_{\text{solid}} = 90$, and gas disperses exponentially over time with a time constant of 1 Myr. Solid material has a density of 1.5 g cm$^{-3}$, the stellar mass is $1 M_{\odot}$, and $r$ and $\alpha$ are assumed to be independent of $a$ and $t$. Simulations last for 10 million years.

Figure 2 shows the evolution of three cores in a simulation with $r = 100$, $\alpha = 3 \times 10^{-5}$, and $\Sigma_{\text{solid}} = 6 \text{ g cm}^{-2}$ at 5 AU. The upper panel shows the cores’ orbital evolution, with arrows showing when each core opens a gap. The core at 5 AU grows rapidly and begins noticeable type I migration after ~30,000 years. At 50,000 years, the core opens a gap and type II migration begins. However, inward migration is offset by the angular momentum gained by absorbing smaller bodies that migrate into the core’s vicinity. These collisions can be seen as jumps in the core’s mass, shown in the lower panel of Figure 2. After 40,000 years, the core starts to acquire an envelope. Envelope growth slows over time as $\Sigma_{\text{gas}}$ decreases and $M$ increases, reducing accretion across the gap. A second core at 20 AU undergoes rapid type I migration and then opens a gap when $a \sim 9$ AU. This core acquires an envelope, but its growth lags behind the body at 5 AU. The core moves inward by ~1 AU due to type II migration, but migration slows when $\Sigma_{\text{gas}}$ becomes small. A third core at 23 AU grows and undergoes type I migration, but remains too small to acquire an envelope before the gas disperses. These three planets are close analogs of Jupiter, Saturn, and Uranus, respectively.

Figure 3 shows the outcome of three simulations with different disk masses. Each row of circles shows the surviving planets, with circle radius $\propto M^{0.3}$. The black and gray segments show the solid and gas mass fractions, respectively. The numbers to the left of the circles indicate $\Sigma_{\text{solid}}$ at 5 AU. The last row of circles shows the giant planets of the solar system, where solid mass fractions include elements heavier than helium in each planet’s envelope. More massive disks lead to larger final planets and giant planets to form farther from the star. The model predicts that gas-giant planets will form and survive migration provided that $\alpha \leq 3 \times 10^{-4}$ and $r \leq 0.5$ km. Most simulations generate radially ordered systems: one or two gas-giant planets with $3 \text{ AU} < a < 20 \text{ AU}$, followed by one or two large cores containing little gas, and finally a disk of sub–Earth-mass objects, akin to the Kuiper Belt.
Gas-giant planets produced in the model have solid-to-gas ratios similar to Saturn but higher than Jupiter. High solid fractions arise when the innermost planet absorbs other cores as they migrate inward. In the model, closely spaced cores always coalesce. Jupiter’s low solid-to-gas ratio suggests that it gravitationally scattered the nearby cores rather than absorbing them. In most simulations, several cores cross the disk’s inner edge due to type I migration. Presumably such objects would be lost by falling into the star. These bodies contain only a few Earth masses and migrate rapidly, so their dynamical effect on nascent terrestrial planets is probably not severe. No planets are lost via type II migration unless \( \alpha \geq 10^{-4} \). A gas giant migrating through the inner solar system would remove most of the solid material present. Terrestrial planets that formed subsequently would contain mostly ice-rich material from the outer disk (Raymond et al. 2006). The rocky compositions of the inner planets in the solar system suggest that no giant planets were lost this way and that \( \alpha \) was small.

If \( \alpha \sim 10^{-4} \) at the disk midplane, type II migration times will be comparable to disk lifetimes. In a flared disk, the migration rate is roughly independent of \( a \) (see eq. [4]). Hence, giant planets should be common everywhere between their formation location and the inner edge of the disk. The model predicts that this is the case. Figure 4 shows the planets produced in 48 simulations with 100 m \( \leq r \leq 500 \) m and \( 3 \times 10^{-3} \leq \alpha \leq 3 \times 10^{-4} \). Giant planets are abundant at all distances from the inner edge of the disk out to 20 AU. This is in accord with the observed distribution of extrasolar planetary orbits (Marcy et al. 2005).

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