A Note on $S$-Noetherian Domains

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Abstract. Let $D$ be an integral domain, $t$ be the so-called $t$-operation on $D$, and $S$ be a (not necessarily saturated) multiplicative subset of $D$. In this paper, we study the Nagata ring of $S$-Noetherian domains and locally $S$-Noetherian domains. We also investigate the $t$-Nagata ring of $t$-locally $S$-Noetherian domains. In fact, we show that if $S$ is an anti-archimedean subset of $D$, then $D$ is an $S$-Noetherian domain (respectively, locally $S$-Noetherian domain) if and only if the Nagata ring $D[X]_N$ is an $S$-Noetherian domain (respectively, locally $S$-Noetherian domain). We also prove that if $S$ is an anti-archimedean subset of $D$, then $D$ is a $t$-locally $S$-Noetherian domain if and only if the polynomial ring $D[X]$ is a $t$-locally $S$-Noetherian domain, if and only if the $t$-Nagata ring $D[X]_{N_v}$ is a $t$-locally $S$-Noetherian domain.

1. Introduction

1.1 Star-operations

To help readers better understanding this paper, we briefly review some definitions and notation related to star-operations. Let $D$ be an integral domain with quotient field $K$, and let $\mathcal{F}(D)$ be the set of nonzero fractional ideals of $D$. For an $I \in \mathcal{F}(D)$, set $I^{-1} := \{ x \in K \mid xI \subseteq D \}$. The mapping on $\mathcal{F}(D)$ defined by $I \mapsto I_v := (I^{-1})^{-1}$ is called the $v$-operation on $D$, and the mapping on $\mathcal{F}(D)$ defined by $I \mapsto I_t := \bigcup \{ J_v \mid J$ is a nonzero finitely generated fractional subideal of $I \}$ is called the $t$-operation on $D$; and the mapping on $\mathcal{F}(D)$ defined by $I \mapsto I_w := \{ a \in K \mid Ja \subseteq I$ for some finitely generated ideal $J$ of $D$ with $J_v = D \}$ is called the $w$-operation on $D$. It is easy to see that $I \subseteq I_w \subseteq I_t \subseteq I_v$ for all $I \in \mathcal{F}(D)$; and if an $I \in \mathcal{F}(D)$ is finitely generated, then $I_v = I_t$. An $I \in \mathcal{F}(D)$ is called a $t$-ideal (respectively, $w$-ideal) of $D$ if $I_t = I$ (respectively, $I_w = I$). A
maximal $t$-ideal means a $t$-ideal which is maximal among proper integral $t$-ideals. It is well known that a maximal $t$-ideal of $D$ always exists if $D$ is not a field. We say that $D$ is of finite character (respectively, of finite $t$-character) if each nonzero nonunit in $D$ belongs to only finitely many maximal ideals (respectively, maximal $t$-ideals) of $D$.

1.2 $S$-Noetherian domains

Let $D$ be an integral domain and $S$ a (not necessarily saturated) multiplicative subset of $D$. In [4], the authors introduced the concept of “almost finitely generated” to study Querre’s characterization of divisorial ideals in integrally closed polynomial rings. Later, the authors in [2] generalized the concept of (almost) finitely generatedness and defined a general notion of Noetherian domains. (Recall that $D$ is a Noetherian domain if it satisfies the ascending chain condition on integral ideals of $D$, or equivalently, every (prime) ideal of $D$ is finitely generated.) To do this, they first built the notion of $S$-finiteness. Let $I$ be an ideal of $D$. Then $I$ is said to be $S$-finite if there exist an element $s \in S$ and a finitely generated ideal $J$ of $D$ such that $sI \subseteq J \subseteq I$. Also, $D$ is called an $S$-Noetherian domain if each ideal of $D$ is $S$-finite. As mentioned above, the concept of $S$-Noetherian domains can be regarded as a slight generalization of that of Noetherian domains, because two notions precisely coincide when $S$ consists of units. Hence the results on $S$-Noetherian domains can recover known facts for Noetherian domains.

Among other results in [2], Anderson and Dumitrescu proved the Hilbert basis theorem for $S$-Noetherian domains, which states that if $S$ is an anti-archimedean subset of an $S$-Noetherian domain $D$, then the polynomial ring $D[X]$ is also an $S$-Noetherian domain [2, Proposition 9]. (Recall that a multiplicative subset $S$ of $D$ is anti-archimedean if $\bigcap_{n \geq 1} s^n D \cap S \neq \emptyset$ for all $s \in S$. For example, if $V$ is a valuation domain with no height-one prime ideals, then $V \setminus \{0\}$ is an anti-archimedean subset of $V$ [3, Proposition 2.1].) After the paper by Anderson and Dumitrescu, more properties of $S$-Noetherian domains have been studied further. In [14], Liu found an equivalent condition for the generalized power series ring to be an $S$-Noetherian domain. In [12], the authors studied the $S$-Noetherian properties in special pullbacks which are the so-called composite ring extensions $D + E[T^+]$ and $D + E^\times \{\leq\}$. As a continuation of [12], the same authors investigated when the amalgamated algebra along an ideal has the $S$-Noetherian property [13]. For more results, the readers can refer to [2, 12, 13, 14].

Let $P$ denote one of the properties “Noetherian” or “$S$-Noetherian”. We say that $D$ is locally $P$ (respectively, $t$-locally $P$) if $D_M$ is $P$ for all maximal ideals (respectively, maximal $t$-ideals) $M$ of $D$.

The purpose of this paper is to study the Nagata ring of $S$-Noetherian domains and locally $S$-Noetherian domains, and to investigate the $t$-Nagata ring of $t$-locally $S$-Noetherian domains. (The concepts of Nagata rings and $t$-Nagata rings will be reviewed in Section .) More precisely, we show that if $S$ is an anti-archimedean subset of $D$, then $D$ is an $S$-Noetherian domain (respectively, locally $S$-Noetherian domain) if and only if the Nagata ring $D[X]_N$ is an $S$-Noetherian domain (respec-
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tively, locally \textit{s}-Noetherian domain); a locally \textit{s}-Noetherian domain with finite character is an \textit{s}-Noetherian domain; and if \( S \) is an anti-archimedean subset of \( D \), then \( D \) is a \( t \)-locally \textit{s}-Noetherian domain if and only if the polynomial ring \( D[X] \) is a \( t \)-locally \textit{s}-Noetherian domain, if and only if the \( t \)-Nagata ring \( D[X]_{N_a} \) is a \( t \)-locally \textit{s}-Noetherian domain.

2. Main Results

We start this section with a simple result for a quotient ring of \textit{s}-Noetherian domains. This also recovers the fact that any quotient ring of a Noetherian domain is Noetherian [5, Proposition 7.3].

**Lemma 1.** Let \( D \) be an integral domain and \( S \) a (not necessarily saturated) multiplicative subset of \( D \). If \( D \) is an \textit{s}-Noetherian domain and \( T \) is a (not necessarily saturated) multiplicative subset of \( D \), then \( D_T \) is an \textit{s}-Noetherian domain.

**Proof.** Let \( A \) be an ideal of \( D_T \). Then \( A = sD_T \) for some ideal \( I \) of \( D \). Since \( D \) is an \textit{s}-Noetherian domain, there exist an element \( s \in S \) and a finitely generated ideal \( J \) of \( D \) such that \( sI \subseteq J \subseteq I \). Therefore we obtain

\[
sA = sID_T \subseteq JD_T \subseteq ID_T = A,
\]

and hence \( A \) is \textit{s}-finite. Thus \( D_T \) is an \textit{s}-Noetherian domain. \( \square \)

The next result is an \textit{s}-Noetherian version of well-known facts that a Noetherian domain is locally Noetherian; and a locally Noetherian domain with finite character is Noetherian [5, Section 7, Exercise 9].

**Theorem 2.** The following statements hold.

1. An \textit{s}-Noetherian domain is locally \textit{s}-Noetherian.

2. A locally \textit{s}-Noetherian domain with finite character is \textit{s}-Noetherian.

**Proof.** (1) This is an immediate consequence of Lemma 1.

(2) Assume that \( D \) is a locally \textit{s}-Noetherian domain which is of finite character, and let \( I \) be an ideal of \( D \). If \( I \cap S \neq \emptyset \), then for any \( s \in I \cap S \), \( sI \subseteq (s) \subseteq I \); so \( I \) is \textit{s}-finite. Next, we consider the case when \( I \) does not intersect \( S \). Choose any \( 0 \neq a \in I \). Since \( D \) has finite character, \( a \) belongs to only a finite number of maximal ideals of \( D \), say \( M_1, \ldots, M_n \). Fix an \( i \in \{1, \ldots, n \} \). Since \( D_M \) is \textit{s}-Noetherian, there exist an element \( s_i \in S \) and a finitely generated subideal \( F_i \) of \( I \) such that \( s_i ID_M \subseteq F_i D_M \). By letting \( s = s_1 \cdots s_n \) and setting \( C = (a) + F_1 + \cdots + F_n \), we obtain that \( sID_M \subseteq CD_M \). Let \( M' \) be a maximal ideal of \( D \) which is distinct from \( M_1, \ldots, M_n \). Then \( a \) is a unit in \( D_{M'} \); so \( ID_{M'} = D_{M'} = CD_{M'} \). Therefore
\[ sID_M \subseteq CD_M \] for all maximal ideals \( M \) of \( D \). Hence we have

\[
\begin{align*}
  sI &= \bigcap_{M \in \text{Max}(D)} sID_M \\
   &\subseteq \bigcap_{M \in \text{Max}(D)} CD_M \\
   &= C,
\end{align*}
\]

where \( \text{Max}(D) \) denotes the set of maximal ideals of \( D \) and the equalities follow from [9, Proposition 2.8(3)]. Note that \( C \) is a finitely generated subideal of \( I \). Therefore \( I \) is \( S \)-finite, and thus \( D \) is an \( S \)-Noetherian domain. \( \Box \)

Recall that an integral domain \( D \) is an \textit{almost Dedekind domain} if \( D_M \) is a Noetherian valuation domain for all maximal ideals \( M \) of \( D \).

**Remark 3.** The converse of Theorem 2(1) does not generally hold. (This also indicates that the condition being finite character in Theorem 2(2) is essential.) For example, if \( D \) is an almost Dedekind domain which is not Noetherian, then \( D \) is a locally \( S \)-Noetherian domain which is not \( S \)-Noetherian. (This is the case when \( S \) consists of units in \( D \).) For a concrete illustration, see [8, Example 42.6].

Let \( D \) be an integral domain and \( D[X] \) be the polynomial ring over \( D \). For an \( f \in D[X] \), \( c(f) \) denotes the content ideal of \( f \), i.e., the ideal of \( D \) generated by the coefficients of \( f \), and for an ideal \( I \) of \( D[X] \), \( c(I) \) stands for the ideal of \( D \) generated by the coefficients of polynomials in \( I \), i.e., \( c(I) = \sum_{f \in I} c(f) \). Let \( N = \{ f \in D[X] \mid c(f) = D \} \). Then \( N \) is a saturated multiplicative subset of \( D[X] \) and the quotient ring \( D[X]_N \) is called the \textit{Nagata ring} of \( D \). It was shown that \( D \) is a Noetherian domain if and only if \( D[X]_N \) is a Noetherian domain (cf. [1, Theorem 2.2(2)]). We now give the \( S \)-Noetherian analogue of these equivalences.

**Theorem 4.** Let \( D \) be an integral domain, \( S \) an anti-archimedean subset of \( D \), and \( N := \{ f \in D[X] \mid c(f) = D \} \). Then the following statements are equivalent.

1. \( D \) is an \( S \)-Noetherian domain.
2. \( D[X] \) is an \( S \)-Noetherian domain.
3. \( D[X]_N \) is an \( S \)-Noetherian domain.

**Proof.**

(1) \( \Rightarrow \) (2) This implication appears in [2, Proposition 9].

(2) \( \Rightarrow \) (3) This was shown in Lemma 1.

(3) \( \Rightarrow \) (1) Let \( I \) be an ideal of \( D \). Then \( ID[X]_N \) is an ideal of \( D[X]_N \). Since \( D[X]_N \) is an \( S \)-Noetherian domain, we can find an element \( s \in S \) and a finitely generated subideal \( J \) of \( ID[X] \) such that \( sID[X]_N \subseteq JD[X]_N \); so \( sID[X]_N \subseteq c(J)D[X]_N \). Let \( a \in I \). Then \( saa \in c(J)D[X] \) for some \( g \in N \); so \( sa \in c(J) \).

Hence \( sI \subseteq c(J) \). Note that \( c(J) \) is a finitely generated subideal of \( I \). Therefore \( I \) is \( S \)-finite, and thus \( D \) is an \( S \)-Noetherian domain. \( \Box \)
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Let $D$ be an integral domain and let $N_v = \{ f \in D[X] \mid c(f)_v = D\}$. Then $N_v$ is a saturated multiplicative subset of $D[X]$ and the quotient ring $D[X]_{N_v}$ is called the $t$-Nagata ring of $D$. It was shown that $D$ is $t$-locally Noetherian if and only if $D[X]$ is $t$-locally Noetherian, if and only if $D[X]_{N_v}$ is $t$-locally Noetherian [6, Theorem 1.4]. To investigate the $(t)$-Nagata ring of $(t)$-locally $S$-Noetherian domains, we need the following lemma.

**Lemma 5.** Let $D$ be a quasi-local domain with unique maximal ideal $M$, $S$ a (not necessarily saturated) multiplicative subset of $D$, and $I$ an ideal of $D$. Then $I$ is $S$-finite if and only if $ID[X]_{MD[X]}$ is $S$-finite.

**Proof.** If $I$ is $S$-finite, then there exist an element $s \in S$ and a finitely generated subideal $J$ of $I$ such that $sI \subseteq J$; so we obtain

$$sID[X]_{MD[X]} \subseteq JD[X]_{MD[X]} \subseteq ID[X]_{MD[X]}.$$  

Thus $ID[X]_{MD[X]}$ is $S$-finite. Conversely, if $ID[X]_{MD[X]}$ is $S$-finite, then there exist suitable elements $s \in S$ and $f_1, \ldots, f_n \in ID[X]$ such that $sID[X]_{MD[X]} \subseteq (f_1, \ldots, f_n)D[X]_{MD[X]}$; so we obtain

$$sID[X]_{MD[X]} \subseteq (c(f_1) + \cdots + c(f_n))D[X]_{MD[X]}.$$  

Note that $JD[X]_{MD[X]} \cap D = J$ for all ideals $J$ of $D$, because $D$ is quasi-local. Hence we obtain

$$sI = sID[X]_{MD[X]} \cap D \leq (c(f_1) + \cdots + c(f_n))D[X]_{MD[X]} \cap D = c(f_1) + \cdots + c(f_n).$$

Note that $c(f_1) + \cdots + c(f_n)$ is a finitely generated subideal of $I$. Thus $I$ is $S$-finite. $\square$

We are ready to study the polynomial extension and the $t$-Nagata ring of $t$-locally $S$-Noetherian domains.

**Theorem 6.** Let $D$ be an integral domain, $S$ an anti-archimedean subset of $D$, and $N_v := \{ f \in D[X] \mid c(f)_v = D\}$. Then the following statements are equivalent.

1. $D$ is a $t$-locally $S$-Noetherian domain.
2. $D[X]$ is a $t$-locally $S$-Noetherian domain.
3. $D[X]_{N_v}$ is a locally $S$-Noetherian domain.
4. $D[X]_{N_v}$ is a $t$-locally $S$-Noetherian domain.

**Proof.** (1) $\Rightarrow$ (2) Let $M$ be a maximal $t$-ideal of $D[X]$ and let $K$ be the quotient field of $D$. If $M \cap D = (0)$, then $D[X]_M$ is a quotient ring of $K[X]$; so $D[X]_M$ is a principal ideal domain. Hence $D[X]_M$ is an $S$-Noetherian domain. Next, we assume that $M \cap D \neq (0)$, and let $P = M \cap D$. Then $M = PD[X]$ and $P$ is a
maximal $t$-ideal of $D$ [7, Proposition 2.2]. Since $D$ is $t$-locally $S$-Noetherian, $D_P$ is $S$-Noetherian. Also, since $S$ is an anti-archimedean subset of $D_P$, $D_P[X]$ is $S$-Noetherian [2, Proposition 9]; so by Lemma 1, $D_P[X]_{|PD_P[X]}$ is $S$-Noetherian. Note that $D[X]_M = D_P[X]_{|PD_P[X]}$; so $D[X]_M$ is an $S$-Noetherian domain. From both cases, we conclude that $D[X]$ is a $t$-locally $S$-Noetherian domain.

(2) $\Rightarrow$ (3) Let $Q$ be a maximal ideal of $D[X]_{|N_0}$. Then $Q = MD[X]_{|N_0}$ for some maximal $t$-ideal $M$ of $D$ [9, Proposition 2.1(2)]. Note that $(D[X]_{|N_0})_Q = (D[X]_{|N_0})_{MD[X]_{|N_0}} = D[X]_{MD[X]}$ and $MD[X]$ is a maximal $t$-ideal of $D[X]$ [7, Proposition 2.2]. Since $D[X]$ is $t$-locally $S$-Noetherian, $D[X]_{|MD[X]}$; and hence $(D[X]_{|N_0})_Q$ is $S$-Noetherian. Thus $D[X]_{|N_0}$ is a locally $S$-Noetherian domain.

(3) $\Leftrightarrow$ (4) This equivalence follows directly from the fact that the set of maximal $t$-ideals of $D[X]_{|N_0}$ is precisely the same as that of maximal ideals of $D[X]_{|N_0}$ (cf. [9, Propositions 2.1(2) and 2.2(3)]).

We next study locally $S$-Noetherian domains in terms of the Nagata ring.

**Theorem 7.** Let $D$ be an integral domain, $S$ an anti-archimedean subset of $D$, and $N := \{f \in D[X] \mid c(f) = D\}$. Then the following statements are equivalent.

(1) $D$ is a locally $S$-Noetherian domain.

(2) $D[X]_N$ is a locally $S$-Noetherian domain.

**Proof.** (1) $\Rightarrow$ (2) Let $Q$ be a maximal ideal of $D[X]_N$. Then $Q = MD[X]_N$ for some maximal ideal $M$ of $D$ [9, Proposition 2.1(2)]. Since $D$ is locally $S$-Noetherian, $D_M$ is $S$-Noetherian. Also, since $S$ is an anti-archimedean subset of $D_M$, $D_M[X]$ is $S$-Noetherian [2, Proposition 9]. Hence by Lemma 1, $D_M[X]_{|MD_M[X]}$ is an $S$-Noetherian domain. Note that $(D[X]_N)_Q = D[X]_{MD[X]} = D_M[X]_{MD_M[X]}$; so $(D[X]_N)_Q$ is $S$-Noetherian. Thus $D[X]_N$ is a locally $S$-Noetherian domain.

(2) $\Rightarrow$ (1) Let $M$ be a maximal ideal of $D$. Then $MD[X]_N$ is a maximal ideal of $D[X]_N$ [9, Proposition 2.1(2)]. Since $D[X]_N$ is locally $S$-Noetherian, $(D[X]_N)_{MD[X]_N} = MD[X]_{MD_M[X]}$; so $(D[X]_N)_{MD[X]_N}$ is $S$-Noetherian. Note that $(D[X]_N)_{MD[X]_N} = D[X]_{MD[X]} = D_M[X]_{MD_M[X]}$; so $D_M[X]_{MD_M[X]}$ is $S$-Noetherian. Let $I$ be an ideal of $D_M$. Then $ID_M[X]_{|MD_M[X]}$ is $S$-finite. Since $D_M$ is quasi-local, $I$ is $S$-finite by Lemma 5. Hence $D_M$ is $S$-Noetherian, and thus $D$ is a locally $S$-Noetherian domain.

We are closing this article by comparing our results with recent researches related to $S$-Noetherian domains. In [11], the authors defined an integral domain $D$ to be an $S$-strong Mori domain ($S$-SM-domain) if for each nonzero ideal $I$ of $D$, there exist an element $s \in S$ and a finitely generated ideal $J$ of $D$ such that
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$sI \subseteq J_w \subseteq I_w$. This concept generalizes the notions of both $S$-Noetherian domains and strong Mori domains. (Recall from [15, Definition 4] that $D$ is a **strong Mori domain** (SM-domain) if it satisfies the ascending chain condition on integral $w$-ideals of $D$, or equivalently, for each (prime) $w$-ideal $I$ of $D$, $I = J_w$ for some finitely generated ideal $J$ of $D$ [15, Theorem 4.3].) It was shown that if $D$ is a $t$-locally $S$-Noetherian domain with finite $t$-character, then $D$ is an $S$-SM-domain [11, Proposition 2.1(2)]; and that if $S$ is an anti-archimedean subset of $D$, then $D$ is an $S$-SM-domain if and only if $D[X]_{N_v}$ is an $S$-SM-domain [11, Theorem 2.10].

**Lemma 8.** Let $D$ be an integral domain, $N := \{f \in D[X] \mid c(f) = D\}$, and $N_v := \{f \in D[X] \mid c(f)_v = D\}$. Then the following assertions hold.

1. $D$ is of finite character if and only if $D[X]_N$ is of finite character.
2. $D$ is of finite $t$-character if and only if $D[X]_{N_v}$ is of finite character.

**Proof.** The equivalence is an immediate consequence of the fact that $\{MD[X]_N \mid M$ is a maximal ideal of $D\}$ (respectively, $\{MD[X]_{N_v} \mid M$ is a maximal $t$-ideal of $D\}$) is the set of maximal ideals of $D[X]_N$ (respectively, $D[X]_{N_v}$) [9, Proposition 2.1(2)].

By Theorems 6 and 7 and Lemma 8, we obtain

**Corollary 9.** Let $D$ be an integral domain, $S$ an anti-archimedean subset of $D$, $N := \{f \in D[X] \mid c(f) = D\}$, and $N_v := \{f \in D[X] \mid c(f)_v = D\}$. Then the following assertions hold.

1. $D$ is a locally $S$-Noetherian domain with finite character if and only if $D[X]_N$ is a locally $S$-Noetherian domain with finite character.
2. $D$ is a $t$-locally $S$-Noetherian domain with finite $t$-character if and only if $D[X]_{N_v}$ is a locally $S$-Noetherian domain with finite character.

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