The Extragalactic Ultra-high-energy Cosmic-Ray Dipole

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Abstract

We explore the possibility that the recently detected dipole anisotropy in the arrival directions of >8 EeV ultra-high-energy cosmic-rays (UHECRs) arises due to the large-scale structure. We assume that the cosmic-ray sources follow the matter distribution and calculate the flux-weighted UHECRs’ rms dipole amplitude taking into account the diffusive transport in the intergalactic magnetic field (IGMF). We find that the flux-weighted rms dipole amplitude is ~8% before entering the Galaxy. The amplitude in the [4–8] EeV is only slightly lower ~5%. The required IGMF is of the order of 5–30 nG, and the UHECR sources must be relatively nearby, within ~300 Mpc. The absence of a statistically significant signal in the lower-energy bin can be explained if the same nuclei species dominates the composition in both energy bins and diffusion in the Galactic magnetic field reduces the dipole of these lower-rigidity particles. Photodisintegration of higher-energy UHECRs could also reduce somewhat the lower-energy dipole.

Key words: cosmic rays

1. Introduction

The origin and nature of ultra-high-energy cosmic-rays (UHECRs) are still open questions. The Pierre Auger Observatory has recently reported a significant (> 5σ) anisotropy in the arrival directions of UHECRs (Aab et al. 2017). It is a large-scale hemispherical asymmetry in the cosmic-ray arrival directions, well represented by a dipole with an amplitude $A = (6.5^{+1.3}_{-0.9})%$ pointing in the direction $(l, b) = (233^\circ, -13^\circ) \pm 10^\circ$ in Galactic coordinates. The dipole anisotropy is detected above 8 EeV. In the energy bin [4–8] EeV, the amplitude of the dipole is $A = (2.5^{+1.0}_{-0.7})%$. However, this amplitude has low statistical significance, and the data in this energy bin are compatible with isotropy (Aab et al. 2017). The median energies for these two bins are 5.0 EeV and 11.5 EeV, respectively.

We explore, here, the possibility that the observed dipole arises due to a large-scale structure (LSS) anisotropy. If the UHECR sources follow the LSS, different UHECR energies would probe different distances, as a result of the energy-dependent GZK horizon. With enough UHECR events, we would obtain a tomographic view of the LSS (Waxman et al. 1997), in a similar manner to intensity mapping (Madau et al. 1997). Following Lahav et al. (1997), we calculate the expected rms dipole in the observed cosmic-ray intensity that will arise due to fluctuations in the source distribution, assuming that those follow the LSS. This is a measure of the expected dipole and also of the fluctuations of this value. Since UHECR diffuse in the intergalactic magnetic field (IGMF) we add magnetic diffusion into the model. We expand the surface brightness of the cosmic-ray sky in spherical harmonics of order $l$, where $l = 1$ corresponds to the dipole. We express these harmonics in terms of the power spectrum $P(k)$ of the matter distribution and calculate the dipole amplitude for different values of the IGMF.

We make two approximations. First, we neglect fluctuations due to individual sources. This is justified given the large number of observed UHECRs in these energy ranges and by the absence of strong small-scale anisotropies. We are interested in the dipole that corresponds to large scales—that are dominated by the distribution as a whole. Second, we neglect nuclei photodisintegration. A quantitative analysis of this effect requires further assumptions on the specific composition, which is unknown. Instead, we present a general model and show in Section 7 that this assumption does not significantly affect the the anisotropy at the high-energy band and it decreases the anisotropy somewhat at the lower-energy range.

The structure of the Letter is as follows. We begin in Section 2 with a qualitative description of the model. Our analysis depends on the composition of the UHECRs and hence we briefly summarize in Section 3 the interpreted Auger composition. We discuss in Section 4 the notion of the cosmic-ray horizon, i.e., the fraction of the universe that would contribute to the observed anisotropy. We describe the methods in Section 5. We present the dipole that arises from the LSS in Section 6 and discuss the interpretation of the results in Section 7.

2. Qualitative Discussion

The idea of using UHECRs to probe the LSS was proposed by Waxman et al. (1997). If the IGMF is negligible, the UHECR horizon is the GZK distance. At energies at which the dipole is observed, this distance is almost the Hubble distance, and therefore the UHECR dipole axis should be aligned with the CMB dipole (that follows the LSS dipole matter distribution). However, both the amplitude and the direction of the UHECR dipole are not compatible with the CMB dipole.

With a significant IGMF, the UHECR horizon is the magnetic horizon defined in Section 4. This horizon is smaller than the GZK distance. It decreases with increasing IGMF and with decreasing UHECR rigidity. A smaller horizon leads to larger LSS fluctuations (e.g., Lahav et al. 1997) and thus it enhances the anisotropy. At the same time, the diffusion in the stronger IGMF weakens the anisotropy. The first effect dominates in the quasi-rectilinear regime, while the second dominates in the diffusive regime.

In addition to diffusion in the IGMF, the UHECRs, at these energies, are both deflected and scattered in the Galactic...
magnetic field (GMF). This may change both the magnitude and the direction of the dipole.

The dipole anisotropy has been observed at $>8$ EeV, but not in the [4–8] EeV energy bin. If the same species dominates both energy bins, then a stronger diffusion in the magnetic fields would lead to a lower dipole amplitude in the lower-energy bin. If different species dominate and we have a similar rigidity in both energy bins, then, unless the effective magnetic horizons are very different, we expect comparable dipole amplitudes.

3. Composition

At $>8$ EeV, where the dipole is reported, the Auger composition data (Bellido 2017) suggest that there are less than 20% protons. While this fraction is very similar for the three hadronic models used to reconstruct the composition, the exact nature of the dominant nuclei specie at these energies is still debated. All models indicate that the composition becomes heavier with increasing energy. When reconstructed using the EPOS-LHC or Sybill 2.3 models, the Auger data indicate a mixture of He and CNO elements at $[8–30]$ EeV. In these models, CNO dominate at 30 EeV, and the transition seems to arise at $\sim 15$ EeV, but this has a large uncertainty. When reconstructed using the QGSJETII 04 model, the composition seems to be dominated by He up to $\sim 30$ EeV. In the energy bin $[4–8]$ EeV, where the Auger skymap is compatible with isotropy, the composition is lighter. It is dominated by protons (at least $\sim 60\%$ in the case of QJSJETII 04) or He (when EPOS-LHC or Sybill 2.3 are used).

4. Cosmic-Ray Horizons

Above a certain threshold energy, the interactions between cosmic rays and the extragalactic photon backgrounds limit the distances that a UHECR can travel, known as the GZK distance. At around 10 EeV, the energy at which the cosmic-ray dipole anisotropy is observed, the GZK distance, $d_{\text{GZK}}$, is $\sim 1000$ Mpc for He and $\sim 2000$ Mpc for protons and CNO. At 4 EeV, the GZK distance is $\sim 2000$ Mpc for protons and the Hubble distance for He and CNO (see Figure 1).

While the lifetime of a UHECR is limited to $d_{\text{GZK}}/c$, diffusion in the IGMF limits its magnetic horizon to much less than the GZK distance. For a diffusive propagation, the horizon scale is the diffusion distance $\sim \sqrt{6D \tau_{\text{GZK}}}$, where $\tau_{\text{GZK}}$ is the GZK time, $\tau_{\text{age}}$ is the age of the source (the Hubble time if the source is always active). The diffusion coefficient, $D$, depends on the rigidity of the particles and on the strength and coherence length of the magnetic field. For a Kolmogorov turbulence, $D$ is well approximated by a fitting function taking into account both the resonant and non-resonant diffusion regimes (Globus et al. 2008):

$$D \approx 0.03 \left( \frac{\lambda_{\text{mpc}}^2 \frac{E_{\text{GeV}}}{ZB_{\text{G}}} \lambda_{\text{G}}}{ZB_{\text{G}}} \right)^{1/4} + 0.5 \left( \frac{E_{\text{GeV}}}{ZB_{\text{G}} \lambda_{\text{G}}^{1/2}} \right)^2 \text{Mpc}^2 \text{Myr}^{-1},$$

where $Z$ is the charge of the cosmic ray, $E_{\text{GeV}}$ is its energy measured in GeV, $B_{\text{G}}$ is the IGMF strength in nG, and $\lambda_{\text{mpc}}$ is its coherence length in Mpc. The diffusive approximation typically holds for $6D < d_{\text{GZK}}c$, i.e., when the diffusion

Figure 1. Thick lines: the GZK distance $d_{\text{GZK}}$ for protons (solid line), He (violet dashed line), and CNO (red dotted-dashed line). Thin lines: the diffusion distance for protons, He and CNO for 1 and 10 nG IGMF, and a field coherence length of 0.2 Mpc. The horizon is $H = \min(\sqrt{6D \tau_{\text{GZK}}}, d_{\text{GZK}})$. The shaded area indicates the energy range at which the dipole anisotropy has been observed. The median value of the energy bin (11.5 EeV) is marked by a solid line.

distance $\sqrt{6D \tau_{\text{GZK}}}$ is smaller than the GZK distance. When it takes more than the age of the universe to enter the diffusion regime, then the cosmic-ray propagation is (quasi-)rectilinear. The size of the region that contributes to the observed flux and hence to the anisotropy is thus set by the cosmic-ray horizon (Parizot 2004)

$$H = \min(\sqrt{6D \tau_{\text{GZK}}}, d_{\text{GZK}}).$$

Figure 1 depicts $d_{\text{GZK}}$ and $H$ for two different values of the IGMF $B_{6G} = 1$ and $B_{6G} = 10$. At the energies at which the dipole anisotropy has been observed ($8 < E_{\text{GeV}} < 30$; see the shaded area on Figure 1) the diffusion approximation holds for $B_{6G} \gtrsim 10$.

5. Spherical Harmonic Expansion of Background Sources

To calculate the UHECR anisotropy due to LSS, we modify the formalism introduced by Lahav et al. (1997) to include diffusion in the IGMF. We expand the surface brightness of the cosmic-ray sky in spherical harmonics:

$$I(\hat{e}) = \sum_{lm} a_{lm} Y_{lm}(\hat{e}).$$

Due of the diffusion in the IGMF, the cosmic-ray anisotropy of a shell at a radius $r$ is reduced by a factor of $\alpha_{lm}(r)$ relative to the sources’ distribution. We write the harmonic coefficients, $a_{lm}$, as

$$a_{lm} = \frac{1}{4\pi} \rho_0 \int d\Omega d\delta(r, \hat{e}) \alpha_{lm}(r) Y_{lm}^* (\hat{e}),$$

where $\delta(r, \hat{e})$ is the three-dimensional density contrast, expanded using Rayleigh expansion into plane waves (see

$^1$ Note that most of the UHECRs in this energy range have $E < 30$ EeV.
The cosmic-ray emissivity at redshift \( z = 0 \) (in erg Mpc\(^{-3}\) yr\(^{-1}\)), and \( H \) is the cosmic-ray horizon defined by Equation (2).

The Bessel function, \( j_l \), projects a wave (with a certain direction \( \hat{e} \), at a certain distance \( r \)) onto the celestial sphere, i.e., it relates the contribution of each wavenumber \( k \) along the line of sight to the pattern of anisotropies at angular scale \( \theta \approx \pi/\ell \).

As the nature of UHECR sources is unknown, it is not clear how they trace the matter distribution. Our basic assumption is that the UHECR sources trace the matter density. However, the fluctuations might be biased and thus the UHECR density fluctuations are proportional to the mass density fluctuations, \( \delta(r, \hat{e}) \), with a proportional (bias) factor \( b \) (\( b > 1 \) implies that the UHECR sources are more clustered than the mass distribution). The power spectrum \( P(k) \) is given in terms of the current mass density fluctuations \( \delta_k \) as \( \langle \delta_k \delta_{k'}^* \rangle = (2\pi)^3 P(k) \delta^{(3)}(\mathbf{k} - \mathbf{k}') \), where \( \delta^{(3)} \) is the three-dimensional delta function. Hence, assuming constant bias, the fluctuations of different multipoles are given by

\[
\langle |a_{lm}|^2 \rangle = \frac{1}{(2\pi)^3} b^2 \int dk k^2 P(k) |\Psi_{lm}(k)|^2, \tag{7}
\]

where \( \Psi_{lm}(k) \) is the window function

\[
\Psi_{lm}(k) = \frac{1}{2\pi^2} \int d\Omega j_l(kr) \alpha_{lm}(r). \tag{8}
\]

As we assume that the IGMF is purely turbulent and spatially homogeneous on large scales, \( \alpha_{lm} = \alpha_{lm}(r) \), i.e., is a function of \( r \) only. For an isotropic diffusion, which we consider here, \( \alpha_{lm} \) does not depend on \( m \). Moreover, we are interested only in the dipole anisotropy, so we will need only \( \alpha_1 \).

We turn now to estimate the diffusion factor \( \alpha_1 \) and the amplitude of the dipole anisotropy \( \Delta \). The cosmic-ray surface brightness in any direction of the \( \hat{e} \) is given by \( I(\hat{e}) = I(1 + \Delta \hat{e} \cdot \hat{\mu}) \), where \( \hat{\mu} \) is the unit vector pointing in the direction of the dipole and \( I \) the average intensity.

Consider, first, a single source from which the cosmic rays propagate with a diffusion coefficient, \( D \). The flux at a given distance \( r \) from the source is given by \( F = -D \partial n / \partial r \) (Fick’s law). The observed surface brightness at a time \( t \) as a function of angle \( \theta \) to the source is \( I(\theta, r, t) = (1 + \cos \theta) I \), where \( I = n(r, t, E) c/(4\pi) \). The flux \( F \) is obtained by integrating \( I(\theta, r, t) \) over all angles (Ginzburg & Syrovatskii 1964):

\[
F = 2\pi \int_0^\pi I(1 + \cos \theta) \cos \theta \sin \theta d\theta = \frac{4\pi}{3} dI = -D \frac{\partial n}{\partial r}. \tag{9}
\]

The amplitude of the dipole anisotropy due to a single source is

\[
d = \frac{3D}{nc} \left| \frac{\partial n}{\partial r} \right| = 3\alpha_1, \tag{10}
\]

and the surface brightness \( I(\theta, r, t) = (1 + 3\alpha_1 \cos \theta) I \).

We can determine \( \alpha_1 \) from Equation (9). The density \( n \) of cosmic rays propagating from an instantaneous source occurring at \(-t \) in the past, and located at a distance \( r \), is \( n(r, t, E) = n_0(4\pi D t)^{-3/2} \exp[-r^2/(4Dt)]; \) the dipole anisotropy is \( \alpha_1 = r/(2ct) \). For a continuous source active over a time \( t_{\min} \), \( n(r, E) = n_0(4\pi D)^{-1} \text{erf}(r/\sqrt{4Dt_{\min}}) \), where \( t_{\min} = \min(\tau_{\text{GZK}}, t_{\text{age}}) \). In that case, the dipole anisotropy is

\[
\alpha_1 = \frac{D}{rc} + \frac{\sqrt{D} \exp(-r^2/4D t_{\min})}{c \sqrt{\pi t_{\min}} \text{erf}(r/\sqrt{4D t_{\min}})}. \tag{11}
\]

In the following, we consider sources that are active over the GZK time, \( t_{\min} = \tau_{\text{GZK}} \). However, the diffusion approximation is valid only when \( D < rc \) and it would be inaccurate to use Equation (11) for \( B_{\mu G} \lesssim 10 \) at the energies of interest, as already mentioned in Section 4. To take into account the quasi-linear propagation, we use the result of Harari et al. (2014). They performed numerical simulations of the propagation of cosmic rays in purely turbulent magnetic fields and derived a fitting function \( \alpha_1 \) that takes into account the transition from the diffusive to the quasi-rectilinear regime:

\[
\alpha_1 \approx \frac{D}{rc} \left[ 1 - \exp\left( -\frac{r}{D} - \frac{7}{18} \left( \frac{rc}{D^2} \right)^2 \right) \right]. \tag{12}
\]

Turning now to multiple sources, the dipole anisotropy is \( \Delta = \sum_i d_i \hat{e}_i \), where \( d_i = 3\alpha_1 \). The sum of the contribution of numerous discrete sources can be approximated by integration over a continuous source density field. Applying the above formalism, the average UHECR surface brightness is \( I = a_{00}/\sqrt{4\pi} = \rho_B H/(4\pi) \), and the dipolar fluctuation in the UHECR surface brightness is

\[
\sqrt{\langle \delta I^2 \rangle} = \frac{3}{4\pi} \sqrt{\langle |a_{1m}|^2 \rangle} = \frac{3}{4\pi} \sqrt{\langle |a_1|^2 \rangle}, \tag{13}
\]

where \( a_1 = a_{1m} \), and we have used the fact that the \( a_{1m} \) coefficients are independent of \( m \). Overall, the total dipole \( \Delta \) is

\[
\Delta = \frac{3 \langle |a_1|^2 \rangle^{1/2}}{a_{00}}. \tag{14}
\]

To visualize the scales probed by the UHECR background, we show in Figure 2 the dipole window functions \( |\Psi_{-1}(k)|^2 \) for different values of the IGMF.

### 6. Quantitative Predictions for the Dipole Anisotropy

The upper panels of Figures 3 and 4 show the rms dipole amplitude, \( \Delta \), as given by Equation (14), in the energy bin [8–30] EeV for different values of the IGMF in the range \( 1 < B_{\mu G} < 100 \) nG and a field coherence length \( \lambda_{\text{Mpc}} = 0.2 \). The power spectrum \( P(k) \) is estimated using the CLASS code for a standard \( \Lambda \text{CDM} \) cosmological model (Blas et al. 2011).

The different nuclei display different anisotropies at the same energy. The total anisotropy is \( \Delta = \sum_i \Delta_i \), where \( \Delta_i \) is the abundance of nuclei specie \( i \). As the different nuclei cannot be distinguished by air shower analysis yet, we simply indicate the expectations for the different components separately.
The anisotropy increases with the magnetic field (up to \( B_{IGM} \sim 3 \) for CNO, \( B_{IGM} \sim 10 \) for He). This is due to the fact that the IGMF impacts both the size of the horizon (i.e., the distance of the more possible distant source) and the factor \( \alpha_1 \). The anisotropy is larger for a smaller horizon scale, in the case of rectilinear propagation; however, after entering the diffusion regime the anisotropy is significantly lowered by the factor \( \alpha_1 \) (since \( D \ll r_\text{c} \) in Equation (12)).

A larger IGMF coherence length decreases the horizon size and increases the diffusion (decreasing \( \alpha_1 \)), so the two changes would tend to cancel each other out. This can be seen in Figure 5, where the resulting dipole for a coherence length \( \lambda_{\text{Mpc}} = 1 \) is also shown. Note that with \( \lambda_{\text{Mpc}} = 1 \) the cosmic rays enter the diffusion regime at smaller values of \( B_{IGM} \).

In the \( >8 \) EeV energy bin, anisotropies of the order of the observed one are obtained for a wide range of IGMF values. The resulting anisotropy is significantly larger than the one that would result from the Compton–Getting effect (Kachelrieß & Serpico 2006). The observed UHECR sources are nearer and hence their fluctuations are larger.

### 7. Discussion

If the sources follow the LSS, the rms dipole moment of the UHECR background is \( \Delta \sim 8\%b \), for He and CNO nuclei at energies \( \geq 8 \) EeV, for values of the IGMF in the range \( 5 \lesssim B_{IGM} \lesssim 30 \) for He and \( 1 \lesssim B_{IGM} \lesssim 10 \) for CNO. Both ranges are for a coherence length of 0.2 Mpc. For these values, due to the diffusion in the IGMF, the horizon distance, \( H \lesssim 300 \) Mpc, is much smaller than the regular GZK distance at these energies. A source bias will increase these values. For example, if UHECR sources follow the galaxy distribution, the bias factor is \( b \sim 1.5 \) (Springob et al. 2016). It might be larger if the UHECR sources are more clustered than galaxies.

Our results are consistent with those derived by Harari et al. (2015), who found, for a mixed-composition scenario, a dipole amplitude \( \sim 10\% \) at 10 EeV for a source distribution that follows the 2MRS catalog up to \( \sim 100 \) Mpc and a 1 nG IGMF. Note, however, that for this IGMF the magnetic horizon is larger than the extend of the 2MRS catalog, 100 Mpc. Sources at larger distances would somewhat lower the anisotropy.

Unfortunately, these results cannot be compared directly with the observations as the UHECRs have to traverse the GMF on their way to Earth. The GMF impacts both the amplitude and the direction of the observed dipole (Harari et al. 2010). The GMF has a complex structure (Jansson & Farrar 2012), and its influence will depend on the direction of the extragalactic dipole. Hence, it cannot be studied in a statistical manner. Using the Jansson & Farrar (2012) GMF model, Aab et al. (2017) found that the initial dipole direction (before entering the Galaxy) is in good agreement with the direction of the flux-weighted dipole from the 2MRS galaxy catalog, i.e.,

![Figure 2](image-url)

*Figure 2.* Dipole window functions \( |\psi_{-1}(k)|^2 \), where \( \langle |\psi_{-1}(k)|^2 \rangle \propto \langle |dk^2 P(k)|\psi_{l}(k)|^2 \rangle \) as given by Equation (8) in the text, calculated here for He at 11.5 EeV for \( B_{IGM} = 0, 1, \) and 10. The functions weigh the contribution from each plane wave \( k \) to the dipole \( l = 1 \). The dashed line represents \( k^3 P(k) \) for a standard \( \Lambda \) CDM cosmological model. The vertical scaling is arbitrary.

![Figure 3](image-url)

*Figure 3.* Upper panel: the range of amplitudes of the dipole \( \Delta \) (for \( b = 1 \)), as defined in Equation (14), for different energies in the range \([8–30]\) EeV for He (violet) and CNO (red). The lower limit of the shaded area corresponds to 8 EeV, the upper limit to 30 EeV. The solid line indicates the flux-weighted dipole. The black dashed–dotted line indicates the observed amplitude 6.5% and the gray shaded area its uncertainty. Lower panel: the magnetic horizon, \( H \), as given by Equation (2). In the case of He, the cosmic-ray horizon at 30 EeV (lower limit of the shaded area) is significantly reduced by the GZK effect.
$(l, b) = (251^\circ \pm 12^\circ, 37^\circ \pm 10^\circ)$ for a sample of galaxies within $\sim 100$ Mpc (Erdoğdu et al. 2006). If the UHECR sources are within 100 Mpc and the UHECR dipole is He-dominated, this would imply an IGMF strength $B_{10nG} \sim 0.1$.

Our estimates of the extragalactic rms dipole are $\sim 5\%$ in the $[4–8]E_{\text{EeV}}$ range both for protons and He (Figure 4). The observed anisotropy level in the energy bin $[4–8]E_{\text{EeV}}$ is significantly lower. If both energy ranges are dominated by the same species (i.e., He), then this discrepancy can be explained by the propagation in the GMF.

Another effect that lowers the dipole anisotropy at the $[4–8]E_{\text{EeV}}$ band is the photodisintegration of higher-energy nuclei emitted at sources with $d > H$, which we have ignored so far. If the GZK horizon of the daughter nuclei is larger, they will reach Earth and contribute to the lower-energy bin. As they come from large distances, they would have a lower anisotropy. Consider, for example, a 12 EeV C whose daughters are He at 4 EeV. The corresponding GZK distances are $\sim 950$ Mpc and $\sim 4000$ Mpc and the magnetic horizons for a 1 nG field are 430 and 885 Mpc. C emitted from sources between the two horizons will photodisintegrate and contribute to He at 4 EeV. The rms dipole of these He particles is $\sim 1\%$ in the quasi-linear regime and $\sim 0.1\%$ in the diffusive regime. This can be quantified for a given composition and could significantly reduce the lower band extragalactic dipole. Similarly, photodisintegration of He emitted from sources at large distances produces lower-energy protons. This lowers the anisotropy and at the same time softens the proton spectrum in the lower-energy range, as observed by Apel et al. (2013).

Finally, we note that because of different rigidities and different GZK distances, a direct prediction of this model is that different species will show different levels of anisotropy, corresponding to their different rigidities. At present, it is impossible to carry out this analysis. But one can hope that this prediction will be tested in the future.

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