Relation between stress intensity factor of circumferential crack and adhesive thickness in bonded round bar

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Abstract. The fracture of the bonded dissimilar materials generally occurs near the interface or at the interface edge. In this study, the stress intensity factors of the circumferential crack near the interface and the circumferential interface crack are analysed by the crack tip stress method. The relation between the dimensionless stress intensity factor of the circumferential crack and the adhesive thickness is investigated. The dimensionless stress intensity factors are defined by using the singular stress value at the crack tip point in bonded round bar without crack. The dimensionless stress intensity factors based on the singular stress value become constant irrespective of the adhesive thickness. In addition, the dimensionless stress intensity factor of the circumferential crack in bonded round bar is compared with that of the edge crack in bonded plate. Both factors coincide with each other when the geometrical condition and material combinations are the same.

1. Introduction

Adhesive bonding has advantages such as bonding of different kind of materials, reduction of weight, smoothing of parts surfaces, reduction of equipment cost and workload. Therefore, it is used in various fields such as industrial and medical fields such as automobiles, airplanes and dental adhesives. With the spread of adhesive structures, the strength evaluation of adhesive bonding has become an important problem [1-11].

Numerous studies have been conducted on the adhesive strength of materials. Noda et al [5-8] have clarified that the intensity of the singular stress field (ISSF) at the interface edge is useful as a criterion for the interface strength. Furthermore, Noda et al [9,10] reported an efficient numerical method to calculate the intensity of singular stress field (ISSF) at the interface edge by using the ratio of the stress values along the interface analysed by the finite element method (FEM). Naito et al [11] have studied the effects of adhesive thickness on static joint strength of polyimide adhesive for butt jointed round bar and single lap joints from the experiments and FEM stress analysis.

The authors have studied the interface strength evaluation based on the stress intensity factors of the interface crack and the edge crack near the interface in butt joint plate [12-15]. The previous studies have shown that the stress intensity factors of small interface crack and small edge crack near the interface in butt jointed plate regularized by the corner singular stress at the crack tip position when there is no crack is constant regardless of the adhesive thickness. However, the method of describing the dimensionless stress intensity factors for three-dimensional problems has not been studied.
In this study, problems of circumferential crack close to the interface and circumferential interface crack in a butt jointed round bar will be treated. The stress intensity factors (SIFs) for circumferential cracks will be evaluated by varying the crack depth and the adhesive thickness, and the useful expression of the circumferential crack solution will be examined.

2. Analysis method and conditions
The aim of this study is to examine the strength evaluation method of bonded round bar shown in figure 1(a). In order to evaluate the intensity of the singular stress field at the interface edge, the circumferential crack near the interface and the circumferential interface crack as shown in figures 1(b) and 1(c) are assumed in this study. This is because the stress intensity factor of the small crack near the interface is strongly affected by the intensity of the singular stress field at interface edge. In figure 1, the notations ‘a’, ‘c’, ‘h’ and ‘d’ indicate the crack depth, the distance from the interface to the crack, the adhesive layer thickness and the diameter of round bar, respectively, and the red point indicates the crack tip position.

![Figure 1](image)

**Figure 1.** Bonded round bar used as the analysis model (a) without crack, (b) with circumferential crack near the interface, and (c) with the circumferential interface crack.

In this study, the stress intensity factors of circumferential crack in jointed round bar are calculated by using the crack tip stress method [12,16,17]. In the two different crack problems, the ratio of the stress values at the crack tip node obtained by FEM under the same mesh pattern is nearly equal to the ratio of the stress intensity factors. The stress intensity factors can be accurately evaluated based on the usefulness of the stress values at a crack tip calculated by FEM. The circumferential crack problems in bonded round bar are assumed as a perfect adhesive bonding and are modeled by using the four-node-quadrilateral-axisymmetric element. A very refined mesh pattern is used in the vicinity of the crack tip.

Table 1 shows the elastic constants of the adhesive and the adherend used in this analysis [11]. In this study, the aluminum alloy (material 1) is used as the adherend and the polyimide (material 2) is used as the adhesive material as shown in figure 1. Dundurs’ composite parameters $\alpha$, $\beta$ and the index of corner stress singularity $\lambda$ are also indicated in table 1. Here, the Dundurs’ composite parameters $\alpha$, $\beta$ and the singularity index $\lambda$ at the interface edge can be obtained from equations (1) and (2) [10].
\[ \alpha = \frac{G_1(\kappa_2 + 1) - G_2(\kappa_1 + 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)}, \quad \beta = \frac{G_1(\kappa_2 - 1) - G_2(\kappa_1 - 1)}{G_1(\kappa_2 + 1) + G_2(\kappa_1 + 1)} \]

\[ \kappa_i = 3 - 4\nu_i \quad (i = 1, 2) \]

\[ \sin^2 \left( \frac{\pi}{2} \lambda \right) - \lambda^2 \right] \beta^2 + 2\lambda \sin^2 \left( \frac{\pi}{2} \lambda \right) - \lambda^2 \right) \alpha \beta + \lambda^2 (\lambda^2 - 1) \alpha^2 + \frac{\sin^2(\lambda\pi)}{4} = 0 \]

Table 1. Elastic constants and Dundurs’ parameters used in the analyses [11].

| Material          | \( E \) [GPa] | \( G \) [GPa] | \( \nu \) | \( \alpha \) | \( \beta \) | \( \lambda \) |
|-------------------|---------------|---------------|----------|------------|----------|----------|
| Material 1(Aluminum) | 69.6          | 26.2          | 0.33     | 0.896      | 0.214    | 0.7397   |
| Material 2(Polyimide)    | 3.77          | 1.40          | 0.342    |            |          |          |

In the analysis, we set the diameter of the round bar \( d = 12.7 \) mm, the length of the bar \( L = 38.1 \) mm, the bond line thickness \( h = 0.218 \) mm to 0.59 mm, and the tensile load \( \sigma_0 = 1 \) MPa. The two geometrical patterns of the circumferential crack near the interface in figure 1(b) are assumed \( a = 0.001 \) mm, \( c = 0.1 \) mm (\( ald = 7.87 \times 10^{-5} \), \( ald = 0.01 \)) and \( a = 0.0005 \) mm, \( c = 0.05 \) mm (\( ald = 3.94 \times 10^{-5} \), \( ald = 0.01 \)). In both patterns, the ratio between the crack depth and the distance \( a/c \) is 0.01.

3. Numerical results and discussion

3.1. Stress intensity factor of circumferential crack near the interface

In this study, the stress intensity factor of the circumferential crack near the interface in the bonded round bar with the adhesive layer shown in figure 1(b) is analysed. Figure 2 illustrates the relationship between the dimensionless stress intensity factor \( F_{I0}^{\sigma_0} \) and the bond line thickness when the bond line thickness \( h \) is changed. Here, the stress intensity factor is normalized by the remote stress \( \sigma_0 \) as shown in equation (3). From figure 2, it can be seen that the value of the dimensionless stress intensity factor \( F_{I0}^{\sigma_0} \) increases as the bond line thickness \( h \) increases in both of the two crack patterns. This means that the stress intensity factor \( K_I \) of the circumferential crack near the interface is controlled by the intensity of the singular stress field of the interface end.

\[ F_{I0}^{\sigma_0} = K_I/\sigma_0 \sqrt{\pi a} \]  

(Figure 2. Relation between \( F_{I0}^{\sigma_0} \) defined by equation (3) and adhesive thickness.)

Next, we verified the dimensionless stress intensity factor defined in equation (4). In equation (4), \( \sigma_s(a,c) \) is the singular stress value at the red point in round bar without the crack as shown in figure 1(a) calculated by FEM. Figure 3 shows the relationship with the dimensionless stress intensity factor \( F_I \) defined by equation (4) and the adhesive thickness \( h \).
\[ F_I = K_I / \sigma_y(a, c) \sqrt{\pi a} \]  \hspace{1cm} (4)

As shown in figure 3, it is found that the dimensionless stress intensity factor \( F_I \) becomes constant at \( F_I = 1.12 \) regardless of the adhesive layer thickness \( h \). For readers convenience, \( \sigma_y(a, c) \) analysed directly by FEM directly are also shown in figure 3. The values of \( \sigma_y(a, c) \) increase as the adhesive thickness \( h \) increases. Therefore, the stress intensity factor of the circumferential small crack near interface can be expressed by the following equation (5).

\[ K_I = 1.12 \sigma_y(a, c) \sqrt{\pi a} \]  \hspace{1cm} (5)

![Figure 3. Relation between \( F_I \) defined by equation (4) and adhesive thickness.](image)

In this crack model, the crack depth and position is important. The previous studies [14,15] have reported that the dimensionless stress intensity factor \( F_I \) of edge crack in butt joint plate becomes constant when the ratio between the crack length and the crack distance \( a/c < 0.01 \). In this study, the circumferential cracks are set within this range.

3.2. Stress intensity factor of circumferential interface crack

Next, the problem of circumferential interface crack shown in figure 1(c) is analysed, and the same examination is conducted on the dimensionless stress intensity factor. The stress intensity factor of the interface crack is defined as the equation (6). In the definition (6), \( \epsilon \) expresses the oscillation singularity index of interface tip. As shown in equation (7), by using the singular stress \( \sigma_\epsilon(a) \) at the crack tip position in the bonded round bar without the crack, the dimensionless stress intensity factor in the three-dimensional bonded round bar is calculated. Figures 4 and 5 show the dimensionless stress intensity factors \( F_I \) and \( F_{II} \) when the adhesive layer thickness is changed. In figures 4 and 5, red triangles indicate the results of \( a=0.001 \) mm and the blue circles show the results of \( a=0.0005 \) mm. The numerical values above and below the marks are stress values \( \sigma_y(a) \) analysed directly by FEM when there is no crack.

Since the values of \( \sigma_y(a) \) are changed as the adhesive thickness increases, \( F_I \) and \( F_{II} \) for the circumferential interface crack in bonded round bar are almost constant, \( F_I = 1.25 \) and \( F_{II} = -0.41 \), regardless of the adhesive layer thickness \( h \). In this analysis, the sign of \( F_{II} \) means the crack tip displacement in tangential direction of crack surface.

\[ \sigma_y + i\tau_{xy} = \frac{K_I + iK_{II}}{\sqrt{2\pi r}} \left( \frac{r}{2a} \right)^{\xi e} \]  \hspace{1cm} (6)
\[ F_I = \frac{K_I}{\sigma_y(a)\sqrt{\pi a}}, \quad F_{II} = \frac{K_{II}}{\sigma_y(a)\sqrt{\pi a}} \]  

(7)

**Figure 4.** Relation between \( F_I \) defined by equation (7) and adhesive thickness.

**Figure 5.** Relation between \( F_{II} \) defined by equation (7) and adhesive thickness.

### 3.3. Comparison of dimensionless stress intensity factor between 2D and 3D models

Finally, we compare the dimensionless stress intensity factors calculated by two dimensional and three dimensional models as shown in figure 6. Table 2 shows the dimensionless stress intensity factors defined by equation (4) of the circumferential crack and the double edge crack close to the interface. Table 3 also indicates the dimensionless stress intensity factors defined by equation (7) of the circumferential interface crack and the double edge interface crack. In tables 2 and 3, \( F^{3D} \) stands for the dimensionless stress intensity factor of circumferential crack in bonded round bar and \( F^{2D} \) stands for that of edge crack in bonded plate. Two dimensional model is analysed by using the plane strain finite element. As shown in tables 2 and 3, both factors coincide with each other when the geometrical condition and material combinations are the same. Since the singular stresses \( \sigma_y(a,c) \) and \( \sigma_y(a) \) in the equations (4) and (7) differ in the two and three dimensional models, the stress intensity factors are naturally different.
Figure 6. Comparison of circumference crack model and edge crack model. (a), (b) the crack near the interface, (c), (d) the interface crack.

Table 2. Comparison of $F_1$ between two and three dimensional models (Bonded round bar and bonded plate with crack near the interface).

| $a$=0.001[mm], $c$=0.1[mm] | $h$ [mm] | 0.218 | 0.294 | 0.406 | 0.519 | 0.59 |
|------------------------------|----------|-------|-------|-------|-------|------|
| $F_1^{3D}$ (Bonded Round Bar) |          | 1.1235| 1.1235| 1.1237| 1.1238| 1.1238|
| $F_1^{2D}$ (Bonded Plate)   |          | 1.1232| 1.1234| 1.1236| 1.1237| 1.1237|

| $a$=0.0005[mm], $c$=0.05[mm] | $h$ [mm] | 0.218 | 0.294 | 0.406 | 0.519 | 0.59 |
|------------------------------|----------|-------|-------|-------|-------|------|
| $F_1^{3D}$ (Bonded Round Bar) |          | 1.1237| 1.1238| 1.1239| 1.1239| 1.1239|
| $F_1^{2D}$ (Bonded Plate)   |          | 1.1236| 1.1237| 1.1238| 1.1215| 1.1239|
Table 3. Comparison of $F_I$, $F_{II}$ between two and three dimensional models (Bonded round bar and bonded plate with interface crack).

| $h$ [mm] | $a=0.001$ [mm] |  |  |  |  |
|----------|-----------------|------------------|------------------|------------------|------------------|
| $F_{I\text{3D}}$ (Bonded Round Bar) | 1.2559 | 1.2552 | 1.2586 | 1.2581 | 1.2575 |
| $F_{I\text{2D}}$ (Bonded Plate) | 1.2557 | 1.2550 | 1.2586 | 1.2581 | 1.2574 |
| $F_{II\text{3D}}$ (Bonded Round Bar) | -0.4103 | -0.4094 | -0.4102 | -0.4098 | -0.4096 |
| $F_{II\text{2D}}$ (Bonded Plate) | -0.4103 | -0.4095 | -0.4103 | -0.4099 | -0.4097 |

| $h$ [mm] | $a=0.0005$ [mm] |  |  |  |  |
|----------|-----------------|------------------|------------------|------------------|------------------|
| $F_{I\text{3D}}$ (Bonded Round Bar) | 1.2526 | 1.2524 | 1.2520 | 1.2518 | 1.2517 |
| $F_{I\text{2D}}$ (Bonded Plate) | 1.2526 | 1.2540 | 1.2519 | 1.2517 | 1.2516 |
| $F_{II\text{3D}}$ (Bonded Round Bar) | -0.4102 | -0.4087 | -0.4085 | -0.4084 | -0.4084 |
| $F_{II\text{2D}}$ (Bonded Plate) | -0.4103 | -0.4066 | -0.4086 | -0.4085 | -0.4084 |

By using the normalized factors, the stress intensity factors of circumferential crack can be easily obtained. Then, the stress intensity factor of the crack can be applied to strength evaluation of the butt joint material instead of the intensity of singular stress field. In [6,15], the strength evaluation of the butt joint plate has been studied by using the stress intensity factor of fictitious crack.

4. Conclusion

In this study, the stress intensity factors of the circumferential crack near the interface and the circumferential interface crack were analysed by the crack tip stress method. The relation between the dimensionless stress intensity factor of the circumferential crack and the adhesive thickness was investigated. In addition, the dimensionless stress intensity factor of the circumferential crack in bonded round bar was compared with that of the edge crack in bonded plate. The conclusions can be summarized as follows.

- The stress intensity factor of a circumferential crack near the interface in bonded round bar with adhesive layer can be expressed as $K_I = 1.12\sigma_y(a,c)\sqrt{\pi a}$ when the crack depth is small enough. In this expression, $\sigma_y(a,c)$ is the singular stress in the y-direction at the crack tip position in bonded round bar without crack.

- The dimensionless stress intensity factors $F_I$ and $F_{II}$ of the circumferential interface crack in bonded round bar become constant value regardless of the adhesive layer thickness $h$. In this problem, the dimensionless factors are based on the singular stress $\sigma_y(a)$ at the crack tip point in the bonded round bar without the interface crack.

- The dimensionless stress intensity factors of the circumferential crack model in bonded round bar were compared with that of the edge crack model in bonded plate. Both factors coincide with each other when the geometrical condition and material combinations are the same.
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