Stability of Scheduled Multi-access Communication over Quasi-static Flat Fading Channels with Random Coding and Joint Maximum Likelihood Decoding

KCV Kalyanarama Sesha Sayee, Utpal Mukherji
Dept. of Electrical Communication Engineering
Indian Institute of Science, Bangalore-560012, India
Email: sayee, utpal@ece.iisc.ernet.in

Abstract—We consider stability of scheduled multiaccess message communication with random coding and joint maximum-likelihood decoding of messages. The framework we consider here models both the random message arrivals and the subsequent reliable communication by suitably combining techniques from queueing theory and information theory. The number of messages that may be scheduled for simultaneous transmission is limited to a given maximum value, and the channels from transmitters to receiver are quasi-static, flat, and have independent fades. Requests for message transmissions are assumed to arrive according to an i.i.d. arrival process. Then, (i) we derive an outer bound to the region of message arrival rate vectors achievable by the class of stationary scheduling policies, (ii) we show for any message arrival rate vector that satisfies the outerbound, that there exists a stationary state-independent policy that results in a stable system for the corresponding message arrival process, and (iii) in the limit of large message lengths, we show that the stability region of message nat arrival rate vectors has information-theoretic capacity region interpretation.

I. INTRODUCTION

Multi-access random-coded communication with independent decoding, of messages that arrive in a Poisson process to an infinite transmitter population, and that achieve any desired value for the random coding upper bound expected message error probability, by determining message signal durations appropriately, has been considered in [1] and [2]. Recently, in [3], a generalization and extension of the model in [1] and [2] was considered and the following assertions were proved: (i) in the limit of large message alphabet size, the stability region has an interference limited information-theoretic capacity interpretation, (ii) state-independent scheduling policies achieve this asymptotic stability region, and (iii) in the asymptotic limit corresponding to immediate access, the stability region for non-idling scheduling policies is identical irrespective of received signal powers. In independent decoding, each user is decoded independently, treating all other users as interference. Since independent decoding is suboptimal, we consider in the present work joint decoding of all user signals and establish results that are similar to the results shown in [3]. Some previous work with joint decoding is reported in [4].

In this paper we consider message (packet) communication from $J \geq 2$ transmitters to a receiver over a flat bandpass AWGN channel. Requests for message transmissions at different transmitters are generated in i.i.d. processes. Messages at transmitter-$j$, $1 \leq j \leq J$, are chosen from the message alphabet $M_j$ consisting of $M_j \geq 2$ alternatives. Signals, representing messages, are to be communicated reliably; reliability is quantified by the tolerable joint message decoding error probability $p_e$. We assume that the receiver schedules messages for simultaneous transmission, i.e., the receiver can choose some numbers of messages from each of the $J$ transmitters. Due to the complexity involved in joint maximum likelihood decoding of an arbitrary number of messages, the receiver is restricted to schedule at most a finite $K \geq 1$ of messages at a time. This restriction gives rise to a set of possible schedules $S_K$, defined in Section II. The channels from transmitters to receiver are quasi-static, flat, and have independent fades. The actual communication is accomplished as follows. For a schedule $s \in S_K$ chosen by the receiver, the transmitters map their respective messages to codewords (signals) of length $N(s)$ and then transmit the signals. The length of the code word is carefully chosen so that reliable communication, quantified by $p_e$, is achieved.

The contributions in this paper are as follows. We derive an outer bound $R_{\text{out}}$ to the stability region of message arrival rate vectors $E.A = (E.A_1, E.A_2, \ldots, E.A_J)$ achievable by the class of stationary scheduling policies. Next, we propose a class of stationary policies, called "state-independent" scheduling policies and denoted by $\Omega^K$, and then characterize the stability region $R(\omega)$ of message arrival rate vectors $E.A = (E.A_1, E.A_2, \ldots, E.A_J)$ achievable by any policy $\omega \in \Omega^K$. We then go on to establish that for any message arrival processes with rate vector within the outerbound derived for stationary policies, there exists a state-independent scheduling policy $\omega$ such that the message system is stable. Finally, for a given set of average power constraints at the respective transmitters, we give information-theoretic capacity region interpretation to the stability region of message nat arrival rate vectors achievable by fixed schedules $s$. 
II. THE INFORMATION THEORETIC MODEL

In this section we briefly touch upon the information-theoretic model of multiaccess communication, and discuss a random coding bound achievable by joint maximum likelihood decoding as derived in [5]. Let there be \( J \) independent sources of information communicating to a receiver over a memoryless Gaussian channel. Let source-\( j \)’s alphabet be defined by \( M_j \) possible message values and let \( m_j \) and \( \hat{m}_j \) denote the \( j \)th source output and its estimate at the receiver. Let \( S \) denote any non-empty subset of the set of sources \( J = \{1, 2, \ldots, J\} \) and \( P(J) \) denote the set of all non-empty subsets of the set \( J \). For a given \( S \in P(J) \), we define an error event to be of type-\( S \) if the decoded joint message \( \hat{m} = (\hat{m}_1, \hat{m}_2, \ldots, \hat{m}_J) \) and the original joint message \( m = (m_1, m_2, \ldots, m_J) \) satisfy: \( \hat{m}_j \neq m_j \) for \( j \in S \) and \( \hat{m}_j = m_j \) for \( j \in S^c \). Assuming each source is encoded independently using block encoding, let \( p_{e,S} \) be the expected probability of a type-\( S \) event over the ensemble of block codes; obviously the expected probability of error \( p_e \) is given by the upper bound on the expected joint message decoding error probability of any individual message.

For future reference, we denote the random coding upper bound and a lower bound to the expected decoding error probability of any \( J \)-source output and its estimate at the receiver. Let \( \chi(J, N) \) denote the set of all \( J \)-source outputs and \( \hat{m}_j \) copies of the original message alphabet \( M_j \); hence the product message alphabet consists of \( M_j^{\chi} \) different tuples of length \( s_j \).

Lemma 2.1: Let \( N \) be the smallest positive integer such that \( \chi(J, N) \leq p_e \). Suppose that only users in the set \( S \in P(J) \) are to be scheduled for transmission. Then, users in the set \( S \) need code words of at most \( N \) to achieve the same decoding error probability \( p_e \).

\[
\chi(S, N) \leq \chi(J, N)
\]

Lemma 2.2: For a given tolerable joint decoding error probability \( p_e \), let \( N \) be the smallest positive integer such that \( \chi(J, N) \leq p_e \). Then

(a) \( N \) can be bounded as

\[
N \geq \max_{S \in P(J)} \left[ -\ln p_e - \rho \sum_{j \in S} \ln M_j \right] E_{o,S}
\]

and

\[
N \leq \max_{S \in P(J)} \left[ -\ln \left( \frac{p_e}{\rho} \right) - \rho \sum_{j \in S} \ln M_j \right] E_{o,S}
\]

(b) for \( 1 \leq j \leq J \) and an integer \( M \geq 2 \), let \( M_j = M \) and \( \#S \) denote cardinality of the set \( S \). Then

\[
\lim_{N \to \infty} \frac{\ln M}{N} = \min_{S \in P(J)} \frac{E_{o,S}}{\rho(\#S)}
\]

In what follows we allow for the possibility of scheduling multiple messages from a user. Let \( s = (s_1, s_2, \ldots, s_J) \in \mathbb{Z}_a^J \) be a vector of non-negative integers and define the set \( S_K = \{ s : 0 \leq \sum_{j=1}^{J} s_j \leq K \} \), where \( S_K \) denotes the set of all schedules that schedule at most \( K \) messages for simultaneous transmission. We assume that, for the schedule \( s \), \( j \)th user encoder does joint encoding of \( s_j \) messages. To interpret Theorem 2.1, Lemma 2.1 and 2.2 for the schedule \( s \in S_K \), it is convenient to view the schedule \( s \) as defining new message alphabets for the sources that are product versions of their original message alphabets. For example, for source-\( j \) and for the schedule \( s \), this product message alphabet is the Cartesian product of \( s_j \) copies of the original message alphabet \( M_j \); hence the product message alphabet consists of \( M_j^{s_j} \) different tuples of length \( s_j \).

III. QUEUEING-THEORETIC MODEL

In this section we derive a queueing-theoretic model for a \( J \) user multiaccess message communication scheme, when requests for message transmission are randomly generated. This queueing model consists of \( J \) queues, one for each source, and a single server whose service statistics depend on the state of the queues through the chosen scheduling policy.

Let maximum-likelihood decoding be used to decode the received word. Consider a fixed schedule \( s \) and suppose that
the tolerable message decoding error probability $p_e$ is given. The definition of service requirement that we consider for any source is the smallest positive integer $N(s)$ (length of the code word that each source transmits) such that $\chi(s, N(s)) \leq p_e$.

For the schedule $s$, we say that source-$j$ receives a service quantum equivalent to $s_j$ units/slot; the total service quantum then is $\sum_{j=1}^{J} s_j$ units/slot. After receiving signal transmission over $N$ channel uses, the receiver will decode the joint message (schedule). A few remarks on the definitions of service requirement and service quantum are in order. The service requirement of a message depends on the schedule of the message (schedule). A few remarks on the definitions of service requirement and service quantum are in order. The amount of service quantum available to each queue depends on the schedule.

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The stability analysis consists of characterizing the stability region $\mathcal{R}(\omega) \in \mathbb{R}_+^J$ of message arrival rate vectors $\mathbf{E}A$ for each policy $\omega$ in a class of stationary i.i.d. scheduling policies, by obtaining appropriate drift conditions for suitably defined Lyapunov functions of the state of the Markov chain. In particular, we prove that the Markov chain is $c$-regular by applying Theorem 10.3 from [6], and then show finiteness of the stationary mean number of messages in the system.

IV. A GENERAL OUTER BOUND TO THE STABILITY REGION

In this section, we derive an outer bound to the region of message arrival rate vectors $\mathbf{E}A$ for which the Markov-chain model is positive recurrent and has finite stationary mean for the number of messages, for the class of stationary scheduling policies. Later, in Section V we propose a class of stationary scheduling policies, called “state-independent” scheduling policies and denoted by $\Omega^s$, and then prove that for any message arrival processes $\{A_j: 1 \leq j \leq J\}$ with $\mathbf{E}A_j$ inside the outerbound, there exists a scheduling policy $\omega \in \Omega^s$ such that the Markov-chain model is positive recurrent and has finite stationary mean for the number of messages.

Consider message arrival processes $\{A_j: 1 \leq j \leq J\}$ and a stationary scheduling policy $\omega$ that schedules at most $K$ messages for a joint message transmission. Let $\pi_k(s)$ be a probability measure on $\mathcal{S}_K$. Define

$$\Psi_j = \sum_{\{s \in \mathcal{S}_K: s_j > 0\}} \pi_k(s) \frac{s_j}{N(s)}$$

and the set

$$\mathcal{R}_{out} = \bigcup_{\pi_k(s)} \left\{ \beta \in \mathbb{R}_+^J : \beta_j \leq \Psi_j \right\} \quad (1)$$

Theorem 4.1: Let the Markov chain $\{X_n, n \geq 0\}$ be positive recurrent and yield finite stationary mean for the number of messages in the system for the message arrival processes $\{A_j\}$ and the stationary scheduling policy $\omega$. Then $\mathbf{E}A \in \mathcal{R}_{out}$.

V. STABILITY FOR STATE-INDEPENDENT SCHEDULING POLICIES

In this section we define the class of state-independent scheduling policies $\Omega^s$, and then prove positive recurrence and finiteness of the stationary mean for the number of messages of the Markov-chain model for this class of scheduling policies. Formally, a policy in this class is defined by (i) a probability measure $\{p(s); s \in \mathcal{S}_K\}$, and (ii) the mapping $\{\omega: \mathcal{X} \times \mathcal{S}_K \rightarrow \mathcal{S}_K\}$. To implement a scheduling policy $\omega$ in $\Omega^s$, we first classify message requests at any queue based on the particular schedule $s$ to be assigned to them.

For each message arrival at queue-$j$, a schedule $s \in \{s \in \mathcal{S}_K: s_j > 0\}$ is chosen randomly with the fixed probability measure defined later in [4] and the message is further classified by assigning the class-$\langle j, s \rangle$ to it. With this classification a message of class-$\langle j, s \rangle$ will be scheduled to transmit only when the schedule $s$ gets chosen for transmission. One consequence of class sub-classification is that messages of class-$\langle j, s \rangle$ will be required to use code words of length $N(s)$ for transmission, i.e., service requirement gets fixed. We first fix a scheduling policy $\omega = p(s)$ and then, in each time slot, a schedule $s$ is chosen from the set $\mathcal{S}_K$, independent of the state $\alpha$, with probability $p(s)$. We constrain the operation of the system by requiring that there can be at most one on-going transmission for any given schedule. For any policy $\omega \in \Omega^s$, the queuing model consists of a number of message queues, one for each class-$\langle j, s \rangle$. To define the state of the system, we keep track of the following information about each message class: for the message class-$\langle j, s \rangle$, let $n_{js}(\alpha)$ denote the number of fresh 3 messages, $x_{js}$ the number of messages that are }

Suppose that schedule $s$ is chosen in state $\alpha$ with probability $p(s)$; then the actual schedule that gets implemented is $s' = \omega(\alpha, s) \in \mathcal{S}_K$ and is defined as follows. For $1 \leq j \leq J$, $s'_j = 0$ if $s_j = 0$. Let, for at least one message class-$\langle j, s \rangle$ associated with the schedule $s$, $x_{js} \neq 0$. Then $s'_j = x_{js}$. Otherwise $s'_j = \min\{n_{js}(\alpha), s_j\}$.

A joint message for which at least one time-slot of transmission is complete, and transmission for at least one more time-slot remains to be completed.

1A joint message that is fresh if that message has not yet been scheduled for the first time, i.e., first code symbol of the corresponding code word is yet to be transmitted.
Define \( \mu_j = (\mu_{js}, s \in S_K : s_j > 0) \) be a splitting probability vector defined by
\[
\mu_{js} = \frac{p(s) n_j}{N(s)} \sum_{s' \in S_K : s'_j > 0} \frac{p(s') s_j}{N(s')},
\]

Then, given that a message arrives at queue-\( j \), \( \mu_{js} \) is the probability that the message request is assigned schedule \( s \).

The sufficient condition for -regularity of the Markov-chain \( \{X_n, n \geq 0\} \) stated in Lemma 5.1 and the sufficient condition for transience stated in Theorem 5.1 together give the exact characterization of the stability region, as stated in the following theorem.

**Theorem 5.2:** For the scheduling policy \( \omega \), the Markov chain \( \{X_n, n \geq 0\} \) is

(a) positive recurrent and yields finite stationary mean for the number of messages, if, for each queue-\( j \),
\[
E A_j < \sum_{s \in S_K : s_j > 0} \frac{p(s) s_j}{N(s)}.
\]

(b) transient if, for at least one message class-\( (j, s) \),
\[
E A_j > \frac{p(s) s_j}{N(s)}.
\]

Define
\[
\psi_j = \sum_{s \in S_K : s_j > 0} \frac{p(s) s_j}{N(s)}
\]

and the set
\[
\mathcal{R}(\Omega^K) = \bigcup_{p(s) \in \Omega^K} \{ \beta \in \mathbb{R}^J_+ : \beta_j < \psi_j \}
\]

**Corollary 5.1:** For any given message arrival rate vector \( EA \in \mathcal{R}(\Omega^K) \) there exists a scheduling policy \( p(s) \in \Omega^K \) such that the Markov chain is positive recurrent and yields finite stationary mean for the number of messages of each class.

From (1) and (4), we note \( 4 \) that \( \mathcal{R}(\Omega^K) = \mathcal{R}_{out}^\circ \). This observation essentially states that, if a stationary scheduling policy is stable for the message arrival processes \( \{A_j\} \), then there exists a state-independent scheduling policy which makes the Markov-chain stable for the same message arrival processes \( \{A_j\} \).

**Proof:** Suppose that, for some stationary scheduling policy, the Markov-chain model \( \{X_n, n \geq 0\} \) is stable for the message arrival processes \( \{A_j\}; 1 \leq j \leq J \). Let \( \pi_K(s) \) be the induced stationary probability distribution on the set of schedules \( S_K \). Let \( \pi_K(0) > 0 \) be the stationary probability that no schedule is served in a time-slot. In the steady state, let \( E A_j \) be the rate at which joint messages of composition \( s \) finish service requirement. Then \( E A_j, N(s) = \pi_K(s) s_j \).

Let us define a new probability distribution \( \{p(s), S_K\} \) as follows: for any non-empty schedule \( s \in S_K \), define \( 4 \) Interior of the set \( A \) is denoted by \( A^\circ \).
Asymptotic stable region of message rate vectors.

VI. INFORMATION-THEORETIC CAPACITY INTERPRETATION

In this section we give information-theoretic capacity interpretation to the stability region of message rate vectors. A formal statement of this interpretation is made in Theorem 6.1. Let \( \hat{A}_j = A_j \ln M_j \) denote the rate of message class-\( j \) for all \( j \geq 1 \). Then, for any rate vector \( \rho = (\rho_1, \rho_2, \ldots, \rho_J) \), the state-independent policy \( \omega = (\hat{A}_j, 1 \leq j \leq J) \) makes the Markov-chain stable.

Proof: We first show that \( C^o(P, \sigma^2) \subset \bigcup_{K \geq 1} \bigcup_{\{s \in S_K\}} \mathcal{R}(s) \). Let \( r = (r_1, r_2, \ldots, r_J) \in C^o(P, \sigma^2) \). There exists an arbitrarily small \( \epsilon > 0 \) such that \( r + \epsilon = (r_1 + \epsilon, r_2 + \epsilon, \ldots, r_J + \epsilon) \in C^o(P, \sigma^2) \). Consider a schedule \( s \) such that, for each \( j \),

\[
\frac{r_j}{s_j} \neq \frac{r_j + \epsilon}{s_j + \epsilon}, \quad \text{for } j \neq i.
\]

Now, with \( s_j \) chosen as suggested, it can be shown that the asymptotic coding rate for message class-\( j \) is \( \mathcal{R}_j(s) + \epsilon \). Since \( \lim_{k \to \infty} s_k \rho_k \) for each \( \rho \in \mathcal{P}(J) \), we see from equation (6) that for every \( S \in \mathcal{P}(J) \),

\[
\frac{s_j}{s_k} \left( 1 + \frac{\sum_{k \in S_k} \rho_k}{\sigma^2} \right) > \frac{r_j}{s_j} + \epsilon.
\]

That is, \( s_j \) and since for some \( K \geq 1 \), we have \( s \in S_K \) which is true for all \( S \). Let \( R \in \mathcal{U}_{K \geq 1} \mathcal{R}(s) \). Then, for some schedule \( s \), \( R \in \mathcal{R}(s) \). Since the set \( C^o(P, \sigma^2) \) is characterized by \( 2^J - 1 \) constraints, we show that \( R \) satisfies all those \( 2^J - 1 \) constraints. Let \( S \) be a non-empty subset of the set \( \{1, 2, \ldots, J\} \). For each \( j \in S \), we have

\[
R_j < \frac{s_j}{\sum_{k \in S_k} s_k} \left( 1 + \frac{\sum_{k \in S_k} \rho_k}{\sigma^2} \right)
\]

Then \( \lim_{k \to \infty} s_k \rho_k \) for each \( \rho \in \mathcal{P}(J) \). Since this is true for any non-empty subset \( S \), we conclude that \( R \in C^o(P, \sigma^2) \).

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