AdS Vacua, Attractor Mechanism and Generalized Geometries

based on arXiv:0810.0937 [hep-th]

Tetsuji KIMURA
Yukawa Institute for Theoretical Physics, Kyoto University
Introduction
We are looking for the origin of 4D physics

Physical information

- Particle contents and spectra
- (Broken) symmetries
- Potential, vacuum and cosmological constant
What kind of 4D models come from String Theories?

↓

What kind of Compactifications?

\[ 4 = 10 - 6 = 11 - 7 \]
B. de Wit and J. Louis, in the Proceedings “NATO Advanced Study Institute on Strings, Branes and Dualities (1997)” hep-th/9801132
Many **Abelian** Supergravities (SUGRA) in lower dimensions

Compactifications on Tori, Calabi-Yaus, etc.

Minkowski ground state, massless fields

Global $E_7$ symmetry ($4D \mathcal{N} = 8$ case)

Many **Gauged** SUGRA in lower dimensions

Compactifications on group manifolds, torsionful manifolds, etc.

Scalar potential generating masses [Moduli Stabilization]

Non-trivial cosmological constant
There are various Gauged SUGRA
which cannot be derived from String Theories
compactified on conventional geometric backgrounds
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Compactify String Theories on non-conventional geometries:
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We want to derive all Gauged SUGRA from String Theories Compactify String Theories on non-conventional geometries:

Nongeometric String Backgrounds
What is a Nongeometric String Background?

Structure group = Diffeo. $(GL(d, \mathbb{R})) + \text{Duality transf. groups}$

coming from String dualities

$GL(d, \mathbb{R}) + \text{duality transf.}$

$d$-dim. internal space $\mathcal{M}_d \simeq \text{monodrofold}$
SUGRA on Nongeometric String Backgrounds

ex.) Lower-dim. Gauged SUGRA compactified by Scherk-Schwarz mechanism

\[ [Z_a, Z_b] = f_{ab}^c Z_c + H_{abc} X^c \]

“Kaloper-Myers” algebra:
\[ [X^a, X^b] = Q_{ab}^c X^c + R_{abc} Z_c \]
\[ [X^a, Z_b] = f_{abc} X^c - Q_{ac}^b Z_c \]

Various “fluxes” are involved

N. Kaloper, R.C. Myers hep-th/9901045
J. Shelton, W. Taylor, B. Wecht hep-th/0508133, A. Dabholkar, C.M. Hull hep-th/0512005
M. Graña, R. Minasian, M. Petrini, D. Waldram arXiv:0807.4527
String Theories compactified on Nongeometric Backgrounds

↓

All(?) Gauged SUGRA

Hitchin’s Generalized Geometries to study vacua

Hull’s Doubled Formalism to find gauge symmetries

IPMU Workshop “Supersymmetry in Complex Geometry” (January 2009)
4D $\mathcal{N} = 1$ supergravity action:

$$S = \int \left( \frac{1}{2} R \ast 1 - \frac{1}{2} F^a \wedge \ast F^a - K_{\mathcal{M}\mathcal{N}} \nabla \phi^\mathcal{M} \wedge \ast \nabla \phi^{\mathcal{N}} - V \ast 1 \right)$$

$$V = e^K \left( K^{\mathcal{M}\mathcal{N}} D_{\mathcal{M}} \mathcal{W} \overline{D_{\mathcal{N}} \mathcal{W}} - 3|\mathcal{W}|^2 \right) + \frac{1}{2} |D^a|^2$$

$K$: Kähler potential

$\mathcal{W}$: superpotential

$D^a$: D-term

10D string theory provides $K$, $\mathcal{W}$, $D^a$ via compactifications: $10 = 4 + 6$
Search 4D SUSY vacua in type IIA theory compactified on generalized geometries

**Moduli stabilization**
We find SUSY AdS (or Minkowski) vacua

**Mathematical feature**
We obtain a powerful rule to evaluate vacua:
- **Discriminants** of superpotentials governing the cosmological constant

**Stringy effects**
We see \(\alpha'\) corrections in certain configurations
Contents

- Introduction
- Killing Spinors and Fluxes
- Generalized (Complex) Geometries
- Exterior Derivatives and Flux Charges
- Setup in $\mathcal{N} = 1$ Theory
- My Work: Search of SUSY AdS Vacua
- Summary and Discussions
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Decompose 10D type IIA SUSY parameters:

\[ \epsilon^1 = \epsilon_1 \otimes (\bar{a} \eta^1_{-}) + \epsilon_1^c \otimes (a \eta^1_{+}), \quad \epsilon^2 = \epsilon_2 \otimes (b \eta^2_{+}) + \epsilon_2^c \otimes (\bar{b} \eta^2_{-}) \]
Decompose 10D type IIA SUSY parameters:

\[ \epsilon^1 = \varepsilon_1 \otimes (\bar{a} \eta^1_\perp) + \varepsilon_1^c \otimes (a \eta^1_\perp), \quad \epsilon^2 = \varepsilon_2 \otimes (b \eta^2_\perp) + \varepsilon_2^c \otimes (\bar{b} \eta^2_\perp) \]

\( \delta(\text{fermions}) = 0 \) provide *Killing spinor equations* on the 6D internal space \( \mathcal{M} \):

\[
\delta \psi^A_m = \left( \partial_m + \frac{1}{4} \omega_{mab} \gamma^{ab} \right) \eta^A + (3\text{-form fluxes} \cdot \eta)^A + (\text{other fluxes} \cdot \eta)^A = 0
\]

with a pair of \( SU(3) \) invariant Weyl spinors \( \eta^1_\perp, \eta^2_\perp \):

\[
\eta^2_\perp = c_{||}(y) \eta^1_\perp + c_{\perp}(y)(v + iv')^m \gamma_m \eta^1_\perp, \quad (v - iv')^m \equiv \eta^1_\perp \gamma^m \eta^2_\perp
\]
Decompose 10D type IIA SUSY parameters:

\[
\begin{aligned}
\epsilon^1 &= \epsilon_1 \otimes (\overline{a} \eta^1_-) + \epsilon^c_1 \otimes (a \eta^1_+), \\
\epsilon^2 &= \epsilon_2 \otimes (b \eta^2_+) + \epsilon^c_2 \otimes (\overline{b} \eta^2_-)
\end{aligned}
\]

\(\delta(\text{fermions}) = 0\) provide \textit{Killing spinor equations} on the 6D internal space \(\mathcal{M}\):

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\]

with a pair of \(SU(3)\) invariant Weyl spinors \(\eta^1_+, \eta^2_+\):

\[
\eta^2_+ = c_\parallel(y) \eta^1_+ + c_\perp(y)(v + iv')^m \gamma_m \eta_1^- , \quad (v - iv')^m \equiv \eta^{1\dagger}_+ \gamma^m \eta^2_-
\]

Information of

6D \(SU(3)\) Killing spinors \(\eta^1_+, \eta^2_+\):

\[
\text{Calabi-Yau three-fold} \downarrow \quad \text{\(SU(3)\)-structure manifold with torsion} \downarrow \quad \text{Generalized Geometry}
\]
Calabi-Yau three-folds $\rightarrow$ Fluxes are highly restricted

\[
\begin{align*}
type \text{IIA} : & \quad \text{No fluxes} \\
type \text{IIB} : & \quad F_3 - \tau H \quad (\text{warped Calabi-Yau}) \\
heterotic : & \quad \text{No fluxes}
\end{align*}
\]

$SU(3)$-structure manifolds $\rightarrow$ Some components of fluxes can be interpreted as torsion

Piljin Yi, TK “Comments on heterotic flux compactifications” JHEP 0607 (2006) 030, hep-th/0605247

TK “Index theorems on torsional geometries” JHEP 0708 (2007) 048, arXiv:0704.2111

Generalized geometries $\rightarrow$ All (non)geometric fluxes can be introduced

“Complete” classification of $\mathcal{N} = 1$ SUSY solutions
10D = 4D \( (\Lambda_{\text{cosmo.}} = -|\mu|^2) \) + 6D: \[ ds^2_{10} = e^{2A(y)} g_{\mu\nu} \, dx^\mu \, dx^\nu + ds^2_6 \]

Consider polyforms \( \Phi^0_{\pm} \) on the internal space \( \mathcal{M} \) which satisfy

\[
\begin{align*}
e^{-2A+\phi}(d - H \wedge)(e^{2A-\phi}\Phi^0_+) &= -2\mu \text{Re}\Phi^-_+ \\
e^{-2A+\phi}(d - H \wedge)(e^{2A-\phi}\Phi^0_-) &= -3i \text{Im}(\mu\Phi^0_+) + dA \wedge \overline{\Phi^-_+} \\
&+ \frac{1}{16} e^\phi \left[ (|a|^2 - |b|^2)F + i(|a|^2 + |b|^2) \ast \lambda(F') \right]
\end{align*}
\]

M. Graña, R. Minasian, M. Petrini, A. Tomasiello hep-th/0505212
10D = 4D ($\Lambda_{\text{cosmo.}} = -|\mu|^2$) + 6D: $\text{d}s_{10}^2 = e^{2A(y)} g_{\mu\nu} \text{d}x^\mu \text{d}x^\nu + \text{d}s_6^2$

Consider polyforms $\Phi_0^\pm$ on the internal space $\mathcal{M}$ which satisfy

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e^{-2A+\phi}(d - H \wedge)(e^{2A-\phi} \Phi_0^+) &= -2\mu \text{Re}\Phi_0^- \\
e^{-2A+\phi}(d - H \wedge)(e^{2A-\phi} \Phi_0^-) &= -3i \text{Im}(\mu \Phi_0^+) + \text{d}A \wedge \Phi_0^- \\
 & \quad + \frac{1}{16}e^\phi \left[ (|a|^2 - |b|^2)F + i(|a|^2 + |b|^2) \ast \lambda(F') \right]
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M. Graña, R. Minasian, M. Petrini, A. Tomasiello hep-th/0505212

- On Calabi-Yau ($d\Phi_0^\pm = 0$) or $SU(3)$-structure manifolds ($d\Phi_0^\pm \neq 0$) ($\eta_+^1 = \eta_+^2$):

\begin{align*}
\Phi_0^+ &= e^{-iJ}, \quad \Phi_0^- = -\Omega \\
J_{mn} &= -2i \eta_+^\dagger \gamma_{mn} \eta_+, \quad \Omega_{mnp} = -2i \eta_-^\dagger \gamma_{mnp} \eta_+
\end{align*}
Compactifications in 10D type IIA

\[10D = 4D \left( \Lambda_{\text{cosmo.}} = -|\mu|^2 \right) + 6D: \quad ds_{10}^2 = e^{2A(y)} g_{\mu\nu} \, dx^\mu \, dx^\nu + ds_{6}^2\]

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\Phi^0_+ &= e^{-iJ} \\
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J_{mn} &= -2i \eta^+_\dagger \gamma_{mn} \eta_+ \\
\Omega_{mn} &= -2i \eta^-_\dagger \gamma_{mnp} \eta_+
\end{align*}
\]

- **On $SU(3) \times SU(3)$ generalized geometries** ($\eta^1_+ \neq \eta^2_+$ at some points $y$):

\[
\begin{align*}
\Phi^0_+ &= (\overline{c}_\parallel e^{-ij} - i\overline{c}_\perp w) \wedge e^{-iv \wedge v'} \\
\Phi^-_+ &= (c_\parallel e^{-ij} + ic_\perp w) \wedge (v + iv') \\
J^A &= j \pm v \wedge v' \\
\Omega^A &= w \wedge (v \pm iv')
\end{align*}
\]
Introduce a generalized almost complex structure \( \mathcal{J} \) on \( F \oplus F^* \) s.t.

\[
\mathcal{J} : F \oplus F^* \rightarrow F \oplus F^* \\
\mathcal{J}^2 = -\mathbf{1}_{2d}
\]

\( \exists \) \( O(d,d) \) invariant metric \( L \), s.t. \( \mathcal{J}^T L \mathcal{J} = L \)

| Structure group on \( F \oplus F^* \) |
|--------------------------------------|
| \( \exists L \)                     |
| \( GL(2d) \)                        |
| \( O(d,d) \)                        |
| \( U(d/2, d/2) \times U(d/2) \)     |
| \( SU(d/2) \times SU(d/2) \)       |
Integrability is discussed by “(0, 1)” part of the complexified $F \oplus F^*$:

$$
\Pi \equiv \frac{1}{2}(1_{2d} - i\mathcal{J})
$$

$$
\Pi A = A \quad \text{where } A = v + \zeta \text{ is a section of } F \oplus F^*
$$

We call this $A$ \textit{i-eigenbundle} $L_{\mathcal{J}}$ whose dimension is $\dim L_{\mathcal{J}} = d$. 
Integrability is discussed by "(0, 1)" part of the complexified $F \oplus F^*$:

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$$

We call this $A$ i-eigenbundle $L_J$ whose dimension is $\dim L_J = d$.

Integrability condition of $J$ is

$$
\overline{\Pi}[\Pi(v + \zeta), \Pi(w + \eta)]_{\text{Courant}} = 0; \quad v, w \in \text{section of } F; \quad \zeta, \eta \in \text{section of } F^*
$$

$$
[v + \zeta, w + \eta]_{\text{Courant}} = [v, w]_{\text{Lie}} + \mathcal{L}_v\eta - \mathcal{L}_w\zeta - \frac{1}{2}d(\iota_v\eta - \iota_w\zeta) \quad \text{Courant bracket}
$$
Two examples of generalized almost complex structures:

\[ \mathcal{J}_- = \begin{pmatrix} I & 0 \\ 0 & -I^T \end{pmatrix} \quad \text{w/ } I^2 = -1_d: \text{ almost complex structure} \]

\[ \mathcal{J}_+ = \begin{pmatrix} 0 & -J^{-1} \\ J & 0 \end{pmatrix} \quad \text{w/ } J: \text{ almost symplectic form} \]

integrable \( \mathcal{J}_- \) ↔ integrable \( I \)

integrable \( \mathcal{J}_+ \) ↔ integrable \( J \)

On a usual geometry, \( J_{mn} = g_{mp} I^p_n \) is given by an \( SU(3) \) invariant (pure) spinor \( \eta_+ \) as

\[ J_{mn} = -2i \eta_+^\dagger \gamma_{mn} \eta_+ \quad \gamma^i \eta_+ = 0 \quad \gamma^\dagger \eta_+ \neq 0 \]

In a similar analogy, we want to find pure spinor(s) \( \Phi \) on generalized geometry.
On $F \oplus F^*$, we can define $\text{Cliff}(6, 6)$ algebra and $\text{Spin}(6, 6)$ spinor $\Phi$:

\[
\{\Gamma^m, \Gamma^n\} = 0 \quad \{\Gamma^m, \tilde{\Gamma}_n\} = \delta^n_m \quad \{\tilde{\Gamma}_m, \tilde{\Gamma}_n\} = 0
\]

Irreducible repr. of $\text{Spin}(6, 6)$ spinor is a Majorana-Weyl

$\rightarrow$ a generic $\text{Spin}(6, 6)$ spinor bundle $S$ splits to $S^\pm$ (Weyl)
On $F \oplus F^*$, we can define Cliff$(6, 6)$ algebra and $Spin(6, 6)$ spinor $\Phi$:

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Irreducible repr. of $Spin(6, 6)$ spinor is a Majorana-Weyl

$\rightarrow$ a generic $Spin(6, 6)$ spinor bundle $S$ splits to $S^\pm$ (Weyl)

Weyl spinor bundles $S^\pm$ are isomorphic to bundles of forms $F^*$:

$\Phi_+ \in S^+ \sim$ section of $\wedge^{\text{even}} F^*$

$\Phi_- \in S^- \sim$ section of $\wedge^{\text{odd}} F^*$

A form-valued representation of the algebra

\[
\Gamma^m = dx^m \wedge, \quad \tilde{\Gamma}_n = \nu \partial_n
\]
On $F \oplus F^*$, we can define Cliff$(6, 6)$ algebra and Spin$(6, 6)$ spinor $\Phi$:

$$\{\Gamma^m, \Gamma^n\} = 0 \quad \{\Gamma^m, \widetilde{\Gamma}_n\} = \delta^m_n \quad \{\widetilde{\Gamma}_m, \widetilde{\Gamma}_n\} = 0$$

Irreducible repr. of Spin$(6, 6)$ spinor is a Majorana-Weyl

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A form-valued representation of the algebra

$$\Gamma^m = dx^m \wedge, \quad \widetilde{\Gamma}_n = \nu \partial_n$$

IF $\Phi$ is annihilated by half numbers of the Cliff$(6, 6)$ generators:

→ $\Phi$ is called a pure spinor

cf.) $SU(3)$ invariant spinor $\eta_+$ is a pure spinor: $\gamma^i \eta_+ = 0$
Correspondence between generalized almost complex structures and pure spinors:

\[ \mathcal{J} \leftrightarrow \Phi \]
Correspondence between generalized almost complex structures and pure spinors:

\[ J \leftrightarrow \Phi \]

Then, we can rewrite the generalized almost complex structure as

\[ J_{\pm \Sigma} = \langle \text{Re}\Phi_\pm, \Gamma_{\Sigma} \text{Re}\Phi_\pm \rangle \]

w/ Mukai pairing:

- even forms: \( \langle \Psi_+, \Phi_+ \rangle = \Psi_6 \wedge \Phi_0 - \Psi_4 \wedge \Phi_2 + \Psi_2 \wedge \Phi_4 - \Psi_0 \wedge \Phi_6 \)
- odd forms: \( \langle \Psi_-, \Phi_- \rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5 \)
Correspondence between generalized almost complex structures and pure spinors:

\[ J \leftrightarrow \Phi \]

Then, we can rewrite the generalized almost complex structure as

\[ J_{\pm \Pi \Sigma} = \langle \text{Re} \Phi_{\pm}, \Gamma_{\Pi \Sigma} \text{Re} \Phi_{\pm} \rangle \]

w/ Mukai pairing:

- even forms: \[ \langle \Psi_+, \Phi_+ \rangle = \Psi_6 \wedge \Phi_0 - \Psi_4 \wedge \Phi_2 + \Psi_2 \wedge \Phi_4 - \Psi_0 \wedge \Phi_6 \]
- odd forms: \[ \langle \Psi_-, \Phi_- \rangle = \Psi_5 \wedge \Phi_1 - \Psi_3 \wedge \Phi_3 + \Psi_1 \wedge \Phi_5 \]

\[ J \] is integrable \[ \iff \] \exists vector \( v \) and one-form \( \zeta \) s.t. \( d \Phi = (v \wedge + \zeta \wedge) \Phi \)

generalized CY \[ \iff \] \exists \Phi \text{ is pure s.t. } d \Phi = 0

“twisted” GCY \[ \iff \] \exists \Phi \text{ is pure, and } H \text{ is closed s.t. } (d - H \wedge) \Phi = 0

TETSUJI KIMURA: ADS VACUA, ATTRACTOR MECHANISM AND GENERALIZED GEOMETRIES
A spinor $\Phi$ can also be mapped to a bispinor by using

$$C \equiv \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \, dx^{m_1} \wedge \cdots \wedge dx^{m_k} \quad \longleftrightarrow \quad \mathcal{C} \equiv \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \gamma^{m_1 \cdots m_k}_{\alpha\beta}$$
A spinor $\Phi$ can also be mapped to a bispinor by using

$$C \equiv \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} dx^{m_1} \wedge \cdots \wedge dx^{m_k} \quad \longleftrightarrow \quad \mathcal{C} \equiv \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \gamma^{m_1 \cdots m_k}$$

On a geometry of a single $SU(3)$-structure, the following two $SU(3,3)$ spinors:

$$\Phi_{0+} = \eta_+ \otimes \eta_+^\dagger = \frac{1}{4} \sum_{k=0}^6 \frac{1}{k!} \eta_+^\dagger \gamma^{m_1 \cdots m_k} \eta_+ \gamma^{m_1 \cdots m_k} = \frac{1}{8} e^{-iJ}$$

$$\Phi_{0-} = \eta_+ \otimes \eta_-^\dagger = \frac{1}{4} \sum_{k=0}^6 \frac{1}{k!} \eta_-^\dagger \gamma^{m_1 \cdots m_k} \eta_+ \gamma^{m_1 \cdots m_k} = -\frac{i}{8} \Omega$$

Check purity: $(\delta + iJ)_m^n \gamma_n \eta_+ \otimes \eta_-^\dagger = 0 = \eta_+ \otimes \eta_-^\dagger \gamma_n (\delta \mp iJ)_n^m$

One-to-one correspondence: $\Phi_{0-} \leftrightarrow \mathcal{J}_1, \quad \Phi_{0+} \leftrightarrow \mathcal{J}_2$
A spinor $\Phi$ can also be mapped to a bispinor by using

$$C \equiv \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \, dx^{m_1} \wedge \cdots \wedge dx^{m_k} \quad \leftrightarrow \quad \mathcal{C} \equiv \sum_k \frac{1}{k!} C^{(k)}_{m_1 \cdots m_k} \gamma^{m_1 \cdots m_k}_{\alpha \beta}$$

On a geometry of a single $SU(3)$-structure, the following two $SU(3,3)$ spinors:

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$$\Phi_{0-} = \eta_+ \otimes \eta^\dagger_- = \frac{1}{4} \sum_{k=0}^6 \frac{1}{k!} \eta^\dagger_- \gamma_{m_1 \cdots m_k} \eta_+ \gamma_{m_1 \cdots m_k} = -\frac{i}{8} \Omega$$

Check purity: $(\delta + iJ)^n_{m} \gamma_n \eta_+ \otimes \eta^\dagger_\pm = 0 = \eta_+ \otimes \eta^\dagger_\pm \gamma_n (\delta \mp iJ)^n_{m}$

One-to-one correspondence: $\Phi_{0-} \leftrightarrow \mathcal{J}_1$, $\Phi_{0+} \leftrightarrow \mathcal{J}_2$

On a generic geometry of a pair of $SU(3)$-structures defined by $(\eta^1_+, \eta^2_+)$

$$\Phi_{0+} = \eta^1_+ \otimes \eta^2_+ = \frac{1}{8} (\tilde{c}_|| e^{-ij} - i\tilde{c}_\perp w) \wedge e^{-iv \wedge v'}$$

$$|c_||^2 + |c_\perp|^2 = 1$$

$$\Phi_{0-} = \eta^1_+ \otimes \eta^2_+ = -\frac{1}{8} (c_\perp e^{-ij} + ic_|| w) \wedge (v + iv')$$
Spaces of $\Phi_\pm$ are special Kähler geometries of local type

Moduli space of $\mathcal{M}$ has Kähler potentials, prepotentials, projective coordinates

$$K_+ = -\log i \int_\mathcal{M} \langle \Phi_+, \Phi_+ \rangle = -\log i (X^A \mathcal{F}_A - X^A \overline{\mathcal{F}}_A)$$

$$K_- = -\log i \int_\mathcal{M} \langle \Phi_-, \Phi_- \rangle = -\log i (Z^I \mathcal{G}_I - Z^I \overline{\mathcal{G}}_I)$$

Expand the even/odd-forms $\Phi_\pm$ by the basis forms:

$$\Phi_+ = X^A \omega_A - \mathcal{F}_A \tilde{\omega}^A, \quad \omega_A = (1, \omega_a), \quad \tilde{\omega}^A = (\tilde{\omega}^a, \text{vol}(\mathcal{M})) : 0, 2, 4, 6\text{-forms}$$

$$\Phi_- = Z^I \alpha_I - \mathcal{G}_I \beta^I, \quad \alpha_I = (\alpha_0, \alpha_i), \quad \beta^I = (\beta^i, \beta^0) : 1, 3, 5\text{-forms}$$

$$\int_\mathcal{M} \langle \omega_A, \omega_B \rangle = 0, \quad \int_\mathcal{M} \langle \omega_A, \tilde{\omega}^B \rangle = \delta_A^B, \quad \int_\mathcal{M} \langle \alpha_I, \alpha_J \rangle = 0, \quad \int_\mathcal{M} \langle \alpha_I, \beta^J \rangle = \delta_I^J$$
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| My Work: Search of SUSY AdS Vacua |
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On generalized geometries with a single $SU(3)$-structure ($\eta_+^1 = \eta_+^2$):

\[
\begin{align*}
\text{d}_H \omega_A &= m_A^I \alpha_I - e_{IA} \beta^I \\
\text{d}_H \alpha_I &= e_{IA} \tilde{\omega}^A \\
\text{d}_H \tilde{\omega}^A &= 0 \\
\text{d}_H \beta^I &= m_A^I \tilde{\omega}^A
\end{align*}
\]

where NS three-form $H$ deforms the differential operator:

\[
\begin{align*}
\text{d}H &= 0, \\
H &= H^{fl} + \text{d}B, \\
H^{fl} &= m_0^I \alpha_I - e_{I0} \beta^I \\
\text{d}_H &\equiv \text{d} - H^{fl} \wedge
\end{align*}
\]

| background               | charges                |
|--------------------------|------------------------|
| NS three-form flux       | $e_{I0}$               |
| torsion                  | $m_0^I$                |
|                          | $e_{Ia}$               |
|                          | $m_a^I$                |
On generalized geometries with $SU(3) \times SU(3)$ structures ($\eta_+^1 \neq \eta_+^2$ at some points):

Extend to the generalized differential operator $\mathcal{D}$:

\[
\mathcal{D} \omega_A \sim m_A^I \alpha_I - e_{IA} \beta^I \\
\mathcal{D} \alpha_I \sim p_I^A \omega_A + e_{IA} \tilde{\omega}^A
\]

\[
\mathcal{D} \tilde{\omega}^A \sim -q^{IA} \alpha_I + p_I^A \beta^I \\
\mathcal{D} \beta^I \sim q^{IA} \omega_A + m_A^I \tilde{\omega}^A
\]
On generalized geometries with $SU(3) \times SU(3)$ structures ($\eta_1^1 \neq \eta_2^2$ at some points):

Extend to the generalized differential operator $D$:

$$d_H = d - H^\text{fl} \wedge \rightarrow D \equiv d - H^\text{fl} \wedge - f \cdot Q \cdot -R$$

$$D\omega_A \sim m_A I \alpha_I - e_{IA} \beta_I$$

$$D\tilde{\omega}^A \sim -q^I A \alpha_I + p^I A \beta_I$$

$$D\alpha_I \sim p^I A \omega_A + e_{IA} \tilde{\omega}^A$$

$$D\beta^I \sim q^I A \omega_A + m_A I \tilde{\omega}^A$$

The internal space becomes nongeometric:

$$(f \cdot C)_{m_1 \ldots m_{k+1}} \equiv f^a \, [m_1 m_2 C]_{a \, | \, m_3 \ldots m_{k+1}}$$

(part of) structure const. in Gauged SUGRA

$$(Q \cdot C)_{m_1 \ldots m_{k-1}} \equiv Q^{ab} \, [m_1 C]_{ab \, | \, m_2 \ldots m_{k-1}}$$

T-fold

$$(R \cdot C)_{m_1 \ldots m_{k-3}} \equiv R^{abc} C_{abc m_1 \ldots m_{k-3}}$$

locally nongeometric background

Structure group = Diffeo. + duality trsf. $\rightarrow$ Hull’s Doubled formalism to study gauge symmetries
RR-fluxes on $SU(3) \times SU(3)$ generalized geometries:

$$ \begin{align*}
 G &= G^\text{fl} + DA, \quad D G = 0 \\
 G^\text{fl} &= m^A_{\text{RR}} \omega_A - e_{\text{RR}A} \tilde{\omega}^A, \quad A = \xi^I \alpha_I - \tilde{\xi}^I \beta^I \\
 \end{align*} $$

$$\downarrow$$

$$\begin{align*}
 G &\sim G^A \omega_A - \tilde{G}_A \tilde{\omega}^A \\
 G^A &\sim m^A_{\text{RR}} + \xi^I p^A_I - \tilde{\xi}^I q^A_I, \quad \tilde{G}_A \sim e_{\text{RR}A} - \xi^I e^A_I + \tilde{\xi}^I m^A_I
\end{align*}$$
## Flux charges on generalized geometry: summary

| Fluxes                  | Charges  |
|-------------------------|----------|
| NS three-form $H$       | $e_I^0$  |
| Torsion                | $m_0^I$  |
| Nongeometric fluxes    | $e_I^a$  |
| RR-fluxes              | $m_a^I$  |
| $p_{IA}$               | $q^{IA}$ |
| $e_{RRA}$              | $m_{RR}^A$ |

### Backgrounds

| Backgrounds            | Flux Charges |
|------------------------|--------------|
| Calabi-Yau             | —            |
| Calabi-Yau with $H$    | $e_I^0$      |
| $SU(3)$ geometry       | $m_0^I$      |
| $SU(3) \times SU(3)$ geometry | $e_{IA}$    |
|                        | $m_A^I$      |
|                        | $p_{IA}$     |
|                        | $q^{IA}$     |

Note: $SU(3)$ generalized geometry without RR-fluxes $\sim SU(3)$-structure manifold
All the information of the internal space is translated into the (non)geometric flux charges and the RR-flux charges.

**NEXT STEP**

Introduce the flux charges into 4D $\mathcal{N} = 1$ physics via various functionals: $K, \mathcal{W}, D^a$

$$V = e^K \left( K^{\mathcal{M}\mathcal{N}} D_\mathcal{M} \mathcal{W} \bar{D}_\mathcal{N} \bar{\mathcal{W}} - 3|\mathcal{W}|^2 \right) + \frac{1}{2} |D^a|^2$$
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Summary and Discussions
\[ N = 1 \text{ Kähler potential} \]

Functionals are given by two Kähler potentials on two Hodge-Kähler geometries of \( \Phi_\pm \):

\[
K = K_+ + 4\varphi \\
K_+ = -\log i \int_{\mathcal{M}} \langle \Phi_+, \overline{\Phi}_+ \rangle = -\log i (\overline{X^A F}_A - X^A \overline{F}_A) \\
K_- = -\log i \int_{\mathcal{M}} \langle \Phi_-, \overline{\Phi}_- \rangle = -\log i (\overline{Z^I G}_I - Z^I \overline{G}_I) \\
\int_{\mathcal{M}} \text{vol}_6 = \frac{1}{8} e^{-K_\pm} = e^{-2\varphi + 2\phi^{(10)}}
\]

Introduce \( \mathcal{C} = \sqrt{2}ab e^{-\phi^{(10)}} = 4ab e^{\frac{K_-}{2} - \varphi} \)

\[
\therefore e^{-2\varphi} = \frac{|\mathcal{C}|^2}{16 |a|^2 |b|^2} e^{-K_-} \\
= \frac{1}{8 |a|^2 |b|^2} \left[ \text{Im}(\mathcal{C} Z^I) \text{Re}(\mathcal{C} G_I) - \text{Re}(\mathcal{C} Z^I) \text{Im}(\mathcal{C} G_I) \right]
\]
4D SUSY variations yield the superpotential and the D-term:

\[
\delta \psi_\mu = \nabla_\mu \epsilon - e^{\frac{K}{2}} \mathcal{W} \gamma_\mu \epsilon^c
\]

\[
\delta \chi^A = \text{Im} F^{A}_{\mu \nu} \gamma^{\mu \nu} \epsilon + i D^A \epsilon
\]

Information of \(\mathcal{W}\) and \(D^A\) comes from 10D SUSY variations

\[\uparrow\]

Spinors \(\Phi_{\pm}\) on 6D internal geometry
\[ \mathcal{W} = \frac{i}{4ab} \left[ 4i e^{\frac{K}{2} - \varphi} \int_{\mathcal{M}} \langle \Phi_+, D\text{Im}(ab\Phi_-) \rangle + \frac{1}{\sqrt{2}} \int_{\mathcal{M}} \langle \Phi_+, G \rangle \right] \]

\[ \equiv \mathcal{W}^{\text{RR}} + U^I \mathcal{W}^Q_I + \tilde{U}_I \tilde{\mathcal{W}}^I_Q \]

\[ \mathcal{W}^{\text{RR}} = -\frac{i}{4ab} \left[ X^A e_{\text{RR}A} - \mathcal{F}_A m_{\text{RR}A} \right] \]

\[ \mathcal{W}^Q_I = \frac{i}{4ab} \left[ X^A e_{IA} + \mathcal{F}_A p^A \right], \quad \tilde{\mathcal{W}}^I_Q = -\frac{i}{4ab} \left[ X^A m_{A}^I + \mathcal{F}_A q^{IA} \right] \]

\[ U^I = \xi^I + i \text{Im}(\mathcal{C}Z^I), \quad \tilde{U}_I = \tilde{\xi}_I + i \text{Im}(\mathcal{C}\mathcal{G}_I) \]

\[ D^A = 2 e^{K_+}(K_+)^{cd} D_c X^A D_d X^B \left[ \tilde{n}^c(\sigma_x)_c^B n_B \right] \left( \mathcal{P}_B^x - \mathcal{N}_{BC} \tilde{\mathcal{P}}^{xC} \right) \]
### Field contents in $\mathcal{N} = 1$ theory

**$\mathcal{N} = 2$ multiplets:**

- **Gravity multiplet:** $g_{\mu\nu}, A^0_\mu$
- **Vector multiplets:** $A_\mu^a, t^a = b^a + iv^a$ \quad $a = 1, \ldots, b^+$
- **Hypermultiplets:** $z^i, \xi^i, \tilde{\xi}_i$ \quad $i = 1, \ldots, b^-$
- **Tensor multiplet:** $B_{\mu\nu}, \varphi, \xi^0, \tilde{\xi}_0$

---

**$\mathcal{N} = 1$ multiplets:**

- **Gravity multiplet:** $g_{\mu\nu}$
- **Vector multiplets:** $A_\mu^\hat{a}$ \quad $\hat{a} = 1, \ldots, \hat{n}_v = b^+ - n_{ch}$
- **Chiral multiplets:** $t^\hat{a} = b^\hat{a} + iv^{\hat{a}}$ \quad $\hat{a} = 1, \ldots, n_{ch}$
- **Chiral/linear multiplets:**
  - $U^I = \xi^I + i\text{Im}(CZ^I)$
  - $\tilde{U}_\hat{I} = \tilde{\xi}_{\hat{I}} + i\text{Im}(CG_{\hat{I}})$
  - $I = (\hat{I}, \hat{\hat{I}}) = 0, 1, \ldots, b^-$

*(projected out)* $B_{\mu\nu}, A^0_\mu, A_\mu^\hat{a}, t^\hat{a}, U^I, \tilde{U}_\hat{I}$

Parameters are restricted as $a = \overline{b}e^{i\theta}$ and $|a|^2 = |b|^2 = \frac{1}{2}$

---

O6 orientifold projection: $\mathcal{O} \equiv \Omega_{WS}(-1)^{F_L}\sigma$

---

T.W. Grimm hep-th/0507153
We are ready to search SUSY vacua in 4D $\mathcal{N} = 1$ theory given by $K$, $W$, $D^a$

**NEXT:** Consider two situations

- Generalized geometry with RR-flux charges:
  
  \[ e_{IA}, m_A^I, p_I^A, q^{IA}, e_{RR}, m_{RR} \]

- $SU(3)$-structure manifold:

  \[ e_{IA}, m_A^I \]
Contents

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- Summary and Discussions
\[ V = e^K \left( K^{M\bar{N}} D_M \mathcal{W} \overline{D_{\bar{N}}} \mathcal{W} - 3|\mathcal{W}|^2 \right) + \frac{1}{2} |D^a|^2 \]

\[ \equiv V_\mathcal{W} + V_D \]

Search of vacua \( \partial_p V|_* = 0 \)

- \( V_* > 0 \): de Sitter space (non-SUSY)
- \( V_* = 0 \): Minkowski space
- \( V_* < 0 \): Anti-de Sitter space
\[ V = e^K \left( K^{MN} D_M W D_N W - 3 |W|^2 \right) + \frac{1}{2} |D^a|^2 \]

\[ \equiv V_W + V_D \]

Search of vacua \( \partial_P V \big|_* = 0 \)

\[ V_* > 0 : \quad \text{de Sitter space (non-SUSY)} \]

\[ V_* = 0 : \quad \text{Minkowski space} \]

\[ V_* < 0 : \quad \text{Anti-de Sitter space} \]

\[ 0 = \partial_P V_W = e^K \left\{ K^{MN} D_P D_M W D_N W + \partial_P K^{MN} D_M W D_N W - 2 W D_P W \right\} \]

\[ 0 = \partial_P V_D \quad \rightarrow \quad D^a = 0 \]

Consider the SUSY condition \( D_P W \equiv (\partial_P + \partial_P K) W = 0 \) in various cases.
1. *Set a simple prepotential:* \( F = D_{abc} \frac{X^a X^b X^c}{X^0} \)

2. *Consider the simplest model:* single modulus \( t \) of \( \Phi_+ \) (and \( U \) of \( \Phi_- \))
Example 1: $SU(3) \times SU(3)$ generalized geometry with RR-flux charges

1. Set a simple prepotential: $\mathcal{F} = D_{abc} \frac{X^a X^b X^c}{X^0}$

2. Consider the simplest model: single modulus $t$ of $\Phi_+$ (and $U$ of $\Phi_-$)

The superpotential is reduced to

$$\mathcal{W} = \mathcal{W}^{RR} + U \mathcal{W}^Q$$

$$\mathcal{W}^{RR} = m_0^{RR} t^3 - 3 m_{RR} t^2 + e_{RR} t + e_{RR0}$$

$$\mathcal{W}^Q = p_0^{0} t^3 - 3 p_0 t^2 - e_0 t - e_{00}$$

Consider the SUSY condition:

$$D_t \mathcal{W} = 0 \quad \Rightarrow \quad 0 = D_t \mathcal{W}^{RR} + U D_t \mathcal{W}^Q$$

$$D_U \mathcal{W} = 0 \quad \Rightarrow \quad 0 = \frac{i}{\text{Im}U} \left( \mathcal{W}^{RR} + \text{Re}U \mathcal{W}^Q \right)$$

The discriminant of the superpotential $\mathcal{W}^{RR}$ (and $\mathcal{W}^Q$) governs the SUSY solutions.
Discriminant of cubic equation

Consider a cubic function and its derivative:

\[
\begin{align*}
\mathcal{W}(t) &= a t^3 + b t^2 + c t + d \\
\partial_t \mathcal{W}(t) &= 3a t^2 + 2b t + c
\end{align*}
\]

Discriminants \( \Delta(\mathcal{W}) \) and \( \Delta(\partial_t \mathcal{W}) \) are

\[
\Delta(\mathcal{W}) \equiv \Delta = -4b^3 d + b^2 c^2 - 4ac^3 + 18abcd - 27a^2 d^2
\]

\[
\Delta(\partial_t \mathcal{W}) \equiv \lambda = 4(b^2 - 3ac)
\]
$\Delta^{RR} > 0$ case: always $\lambda^{RR} > 0$, and exists a zero point: $D_t W^{RR} = 0$

\[
D_t W^{RR} |_{\ast} = 0 \\
t^{RR}_{\ast} = \frac{6 (3 m^0_{RR} e_{RR0} + m_{RR} e_{RR})}{\lambda^{RR}} - 2i \frac{\sqrt{3 \Delta^{RR}}}{\lambda^{RR}} \\
W^{RR}_{\ast} = -\frac{24 \Delta^{RR}}{\lambda^{RR}^3} \left( 36 (m_{RR})^3 + 36 (m^0_{RR})^2 e_{RR0} - 3 m_{RR} \lambda^{RR} - 4i m^0_{RR} \sqrt{3 \Delta^{RR}} \right)
\]
\( \Delta^{RR} > 0 \) case: always \( \lambda^{RR} > 0 \), and exists a zero point: \( D_t \mathcal{W}^{RR} = 0 \)

\[
D_t \mathcal{W}^{RR}|_* = 0
\]

\[
t^{RR}_* = \frac{6 (3 m_0^{RR} e_{RR0} + m_{RR} e_{RR})}{\lambda^{RR}} - 2i \frac{\sqrt{3 \Delta^{RR}}}{\lambda^{RR}}
\]

\[
\mathcal{W}^{RR}_* = -\frac{24 \Delta^{RR}}{(\lambda^{RR})^3} \left( 36 (m_{RR})^3 + 36 (m_0^{RR})^2 e_{RR0} - 3 m_{RR} \lambda^{RR} - 4i m_0^{RR} \sqrt{3 \Delta^{RR}} \right)
\]

\( \Delta^{RR} < 0 \) case: only \( \lambda^{RR} < 0 \) is physically allowed, and exists a zero point: \( \mathcal{W}^{RR} = 0 \)

\[
\mathcal{W}^{RR}_* = m_0^{RR} (t_* - e)(t_* - \alpha)(t_* - \bar{\alpha}) = 0, \quad t_* = \alpha^{RR} = \alpha_1 + i \alpha_2
\]

\[
\alpha_1 = \frac{\lambda^{RR} + F^{2/3} + 12 m_{RR} F^{1/3}}{12 m_0^{RR} F^{1/3}}
\]

\[
(\alpha_2)^2 = \frac{1}{m_0^{RR}} \left( e_{RR} - 6 m_{RR} \alpha_1 + 3 m_0^{RR} (\alpha_1)^2 \right)
\]

\[
e = -\frac{1}{m_0^{RR}} \left( -3 m_{RR} + 2 m_0^{RR} \alpha_1 \right)
\]

\[
F = 108 (m_0^{RR})^2 e_{RR0} + 12 m_0^{RR} \sqrt{-3 \Delta^{RR}} + 108 (m_{RR})^3 - 9 m_{RR} \lambda^{RR}
\]

\[
D_t \mathcal{W}^{RR}|_* = 2i m_0^{RR} (e - \alpha^{RR}) \alpha_2
\]

... Analysis of \( \mathcal{W}^Q \) is also discussed.
Three types of solutions to satisfy $0 = D_t \mathcal{W}^{\text{RR}} + U D_t \mathcal{W}^{\text{Q}}$ and $0 = \mathcal{W}^{\text{RR}} + \text{Re} U \mathcal{W}^{\text{Q}}$: 
Three types of solutions to satisfy \( 0 = D_t \mathcal{W}^{RR} + UD_t \mathcal{W}^Q \) and \( 0 = \mathcal{W}^{RR} + \text{Re}U \mathcal{W}^Q \):

**SUSY AdS vacuum:** moduli are (almost) stabilized

\[
\Delta^{RR} > 0, \quad \Delta^Q > 0; \quad D_t \mathcal{W}^{RR}|_* = 0 = D_t \mathcal{W}^Q|_*
\]

\[
t^{RR}_* = t^Q_*, \quad \text{Re}U_* = -\frac{\mathcal{W}^{RR}_*}{\mathcal{W}^Q_*}
\]

\[
V_* = -3 e^K |\mathcal{W}_*|^2 = -\frac{4}{[\text{Re}(\mathcal{C}G_0)]^2} \sqrt{\frac{\Delta^Q}{3}} \ll \mathcal{O}(1)
\]
Three types of solutions to satisfy $0 = D_t \mathcal{W}^{RR} + U D_t \mathcal{W}^{Q}$ and $0 = \mathcal{W}^{RR} + \text{Re} U \mathcal{W}^{Q}$:

**SUSY AdS vacuum: moduli are (almost) stabilized**

$\Delta^{RR} > 0, \quad \Delta^Q > 0; \quad D_t \mathcal{W}^{RR}|_* = 0 = D_t \mathcal{W}^{Q}|_*$

$t_*^{RR} = t_*^Q, \quad \text{Re} U_* = -\frac{\mathcal{W}^{RR}_*}{\mathcal{W}^{Q}_*}$

$V_* = -3 e^K |\mathcal{W}_*|^2 = \frac{4}{[\text{Re}(C G_0)]^2} \sqrt{\frac{\Delta^Q}{3}} \ll \mathcal{O}(1)$

**SUSY Minkowski vacuum: moduli are stabilized**

$\Delta^{RR} < 0, \quad \Delta^Q < 0; \quad \mathcal{W}^{RR}_* = 0 = \mathcal{W}^{Q}_*$

$\alpha^{RR} = \alpha^Q, \quad U_* = -\frac{D_t \mathcal{W}^{RR}|_*}{D_t \mathcal{W}^{Q}|_*} \neq 0$

$V_* = 0$
Three types of solutions to satisfy $0 = D_t \mathcal{W}^{RR} + U D_t \mathcal{W}^{Q}$ and $0 = \mathcal{W}^{RR} + \text{Re} U \mathcal{W}^{Q}$:

**SUSY AdS vacuum:** moduli are (almost) stabilized

$$
\begin{align*}
\Delta^{RR} &> 0, \quad \Delta^{Q} > 0; \quad D_t \mathcal{W}^{RR}|_* = 0 = D_t \mathcal{W}^{Q}|_* \\
\alpha^{RR} &= \alpha^{Q}, \quad \text{Re} U_* = -\frac{\mathcal{W}^{RR}_*}{\mathcal{W}^{Q}_*} \\
V_* &= -3 e^K |\mathcal{W}_*|^2 = -\frac{4}{[\text{Re}(CG_0)]^2} \sqrt{\frac{\Delta^Q}{3}} \ll \mathcal{O}(1)
\end{align*}
$$

**SUSY Minkowski vacuum:** moduli are stabilized

$$
\begin{align*}
\Delta^{RR} &< 0, \quad \Delta^{Q} < 0; \quad \mathcal{W}^{RR}_* = 0 = \mathcal{W}^{Q}_* \\
\alpha^{RR} &= \alpha^{Q}, \quad U_* = -\frac{D_t \mathcal{W}^{RR}_*}{D_t \mathcal{W}^{Q}_*} \neq 0 \\
V_* &= 0
\end{align*}
$$

**SUSY AdS vacua, but moduli $t$ and $U$ are not fixed:** non-stabilized point

$$
U = -\frac{D_t \mathcal{W}^{RR}(t)}{D_t \mathcal{W}^{Q}(t)}, \quad \text{Re} U = -\frac{\mathcal{W}^{RR}(t)}{\mathcal{W}^{Q}(t)}
$$
Example 2: $SU(3)$-structure manifold

1. Set $e_{RR} = 0 = m_{RR}^{A}$, $p_{I}^{A} = 0 = q^{IA}$, and single modulus $t$ of $\Phi_{+}$ (and $U$ of $\Phi_{-}$).

2. Set a deformed prepotential: $\mathcal{F} = \frac{(X^{t})^{3}}{X^{0}}$
1. Set \( e_{RRA} = 0 = m_{RR}^A \), \( p_I^A = 0 = q^{IA} \), and single modulus \( t \) of \( \Phi_+ \) (and \( U \) of \( \Phi_- \)).

2. Set a deformed prepotential: 
\[
F = \frac{(X^t)^3}{X^0} + \sum_n N_n \frac{(X^t)^{n+3}}{(X^0)^{n+1}}
\]
Example 2: $SU(3)$-structure manifold

1. Set $e_{RRA} = 0 = m_{RR}^A$, $p_I^A = 0 = q^{IA}$, and single modulus $t$ of $\Phi_+$ (and $U$ of $\Phi_-$)

2. Set a deformed prepotential: $\mathcal{F} = \frac{(X^t)^3}{X^0} + \sum_n N_n \frac{(X^t)^{n+3}}{(X^0)^{n+1}}$

Superpotential $\mathcal{W} = U \mathcal{W}^Q$ with a simple setting $N_1 \neq 0$, $N_n = 0$:

$$D_t \mathcal{W}^Q = -e_{00} + \frac{3(t - \bar{t})^2 - \partial_t P}{(t - \bar{t})^3 - P} \left( e_{00} + e_0 t \right)$$

$$P \equiv -2\left( N_1 t^4 - \bar{N}_1 \bar{t}^4 - 2N_1 t^3 \bar{t} + 2\bar{N}_1 \bar{t}^3 \right)$$

SUSY condition

$$D_t \mathcal{W} = D_U \mathcal{W} = 0$$

has a solution

$$t^Q_* = -\frac{2e_{00}}{e_0}, \quad \text{Re} U_* = 0$$

$$\mathcal{W}^Q_* = e_{00}, \quad \text{Im} N_1 < 0$$

$$V_* = -3e^K |\mathcal{W}_*|^2 = \frac{1}{[\text{Re}(CG_0)]^2} \frac{3(e_0)^4}{16(e_{00})^2 \text{Im} N_1}$$

Also heterotic string on $SU(3)$-structure manifolds with torsion which carries $\alpha'$ corrections
Summary

- Studied generalized geometries and their applications to string compactifications
- Obtained a powerful rule to discuss SUSY vacua: Discriminants
- Exhibited that $\alpha'$ corrections are included in certain configurations

Discussions

- More generic configurations
- Gauge symmetries
- Understanding the physical interpretation of nongeometric fluxes
de Sitter vacua?

In order to build (stable) de Sitter vacua perturbatively in type IIA, in addition to the usual R-R and NS-NS fluxes and O6/D6 sources, one must minimally have geometric fluxes and non-zero Romans’ mass parameter.

S.S. Haque, G. Shiu, B. Underwood, T. Van Riet arXiv:0810.5328

Romans’ mass parameter \( \sim G_0 \)

Search (meta)stable de Sitter vacua in this formulation
Thank You
Contents

- Differential Forms: Geometric Objects
- Hitchin Functional
- Killing Prepotentials
Decomposition of vector bundle on 10D spacetime:

\[ T\mathcal{M}_{9,1} = T_{3,1} \oplus F \]

\[
\begin{align*}
T_{3,1} &: \text{ a real } SO(3, 1) \text{ vector bundle} \\
F &: \text{ an } SO(6) \text{ vector bundle which admits a pair of } SU(3) \text{ structures}
\end{align*}
\]
Decompositions of spinors in 10D type IIA string

Decomposition of vector bundle on 10D spacetime:

\[ TM_{9,1} = T_{3,1} \oplus F \]

\[ \begin{align*}
T_{3,1} : & \quad \text{a real } SO(3, 1) \text{ vector bundle} \\
F : & \quad \text{an } SO(6) \text{ vector bundle which admits a pair of } SU(3) \text{ structures}
\end{align*} \]

Decomposition of Lorentz symmetry:

\[ Spin(9, 1) \to Spin(3, 1) \times Spin(6) = SL(2, \mathbb{C}) \times SU(4) \]

\[ 16 = (2, 4) \oplus (\bar{2}, \bar{4}) \quad 16 = (2, \bar{4}) \oplus (\bar{2}, 4) \]

Decomposition of supersymmetry parameters (with \( a, b \in \mathbb{C} \)):

\[ \begin{align*}
\epsilon^1_{\text{IIA}} &= \epsilon_1 \otimes (\bar{a} \eta^-_1) + \epsilon_1^c \otimes (a \eta^+_1), \\
\epsilon^2_{\text{IIA}} &= \epsilon_2 \otimes (b \eta^+_2) + \epsilon_2^c \otimes (\bar{b} \eta^-_2)
\end{align*} \]
Decompositions of spinors in 10D type IIA string

Decomposition of vector bundle on 10D spacetime:

\[ T\mathcal{M}_{9,1} = T_{3,1} \oplus F \]

\[ \begin{align*}
T_{3,1} & : \text{a real } SO(3, 1) \text{ vector bundle} \\
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Decomposition of Lorentz symmetry:

\[ Spin(9, 1) \rightarrow Spin(3, 1) \times Spin(6) = SL(2, \mathbb{C}) \times SU(4) \]

\[ 16 = (2, 4) \oplus (\bar{2}, \bar{4}) \quad 16 = (2, \bar{4}) \oplus (\bar{2}, 4) \]

Decomposition of supersymmetry parameters (with \(a, b \in \mathbb{C}\)):

\[ \begin{align*}
\epsilon_{\text{IIA}}^1 &= \varepsilon_1 \otimes (\bar{a} \eta_-^1) + \varepsilon_1^c \otimes (a \eta_+^1) , \\
\epsilon_{\text{IIA}}^2 &= \varepsilon_2 \otimes (b \eta_+^2) + \varepsilon_2^c \otimes (\bar{b} \eta_-^2) 
\end{align*} \]

Set \( SU(3) \) invariant spinor \( \eta_+^A \) s.t. \( \nabla^{(T)} \eta_+^A = 0 \) \((A = 1, 2)\)

a pair of \( SU(3) \) on \( F \ (\eta_+^1, \eta_+^2) \) \( \iff \) a single \( SU(3) \) on \( F \ (\eta_+ = \eta_+^1 = \eta_+^2 = \eta_+) \)
Requirement that we have a pair of $SU(3)$ structures means there is a sub-supermanifold

$$\mathcal{N}^{9,1|4+4} \subset \mathcal{M}^{9,1|16+16}$$

\[
\begin{pmatrix}
(9,1): \text{ bosonic degrees} \\
4+4: \text{ eight Grassmann variables as spinors of } Spin(3,1) \text{ and singlet of } SU(3)\text{s}
\end{pmatrix}
\]

Equivalence such as

- eight SUSY theory reformulation of type II strings
- a pair of $SU(3)$ structures on vector bundle $F$
- $SU(3) \times SU(3)$ structures on extended $F \oplus F^*$
Geometric objects

- with a single SU(3):
  - a real two-form: $J_{mn} = \mp 2i \eta_\pm^\dagger \gamma_{mn} \eta_\pm$
  - a complex three-form: $\Omega_{mnp} = -2i \eta_-^\dagger \gamma_{mnp} \eta_+$
with a single $SU(3)$:

| Geometric object | Equation |
|------------------|----------|
| a real two-form   | $J_{mn} = \mp 2i \eta_\pm^\dagger \gamma_{mn} \eta_\pm$ |
| a complex three-form | $\Omega_{mnp} = -2i \eta_\mp^\dagger \gamma_{mnp} \eta_+$ |

with a pair of $SU(3)$:

| Geometric object | Equation |
|------------------|----------|
| two real vectors | $(v - iv')^m = \eta_+^{1\dagger} \gamma^m \eta_-^2$ |
| $(J^A, \Omega^A)$ | $J^1 = j + v \wedge v', \quad \Omega^1 = w \wedge (v + iv')$ |
|                  | $J^2 = j - v \wedge v', \quad \Omega^2 = w \wedge (v - iv')$ |
|                  | $(j, w)$: locally $SU(2)$-invariant two-forms |
with a single $SU(3)$:

| Geometric objects |
|-------------------|
| a real two-form    |
| $J_{mn} = \mp 2i \eta^\dagger_\pm \gamma_{mn} \eta_\pm$ |
| a complex three-form |
| $\Omega_{mnp} = -2i \eta^\dagger_- \gamma_{mnp} \eta_+$ |

with a pair of $SU(3)$:

| Geometric objects |
|-------------------|
| two real vectors  |
| $(v - i v')^m = \eta^1_+ \gamma^m \eta_2^-$ |
| $(J^A, \Omega^A)$ |
| $J^1 = j + v \wedge v'$, $\Omega^1 = w \wedge (v + i v')$ |
| $J^2 = j - v \wedge v'$, $\Omega^2 = w \wedge (v - i v')$ |

$(j, w)$: locally $SU(2)$-invariant two-forms

\[
\eta^2_+ = c_\parallel \eta^1_+ + c_\perp (v + i v')^m \gamma^m \eta^1_-,
\quad |c_\parallel|^2 + |c_\perp|^2 = 1
\]

If $\eta^1_+ \neq \eta^2_+$ globally: a single $SU(2)$ w/ $(j, w, v, v')$
If $\eta^1_+ = \eta^2_+$ globally: a single $SU(3)$ w/ $(J, \Omega)$

a pair of $SU(3)$ on $F \sim SU(3) \times SU(3)$ on $F \oplus F^*$
Information from Killing spinor eqs. with torsion $\nabla^{(T)} \eta_{\pm} = 0$ ($^3$complex Weyl $\eta_{\pm}$)

- Invariant $p$-forms on $SU(3)$-structure manifold:
  
  A real two-form
  \[ J_{mn} = \mp 2i \eta_{\pm}^\dagger \gamma_{mn} \eta_{\pm} \]

  A holomorphic three-form
  \[ \Omega_{mnp} = -2i \eta_{\mp}^\dagger \gamma_{mnp} \eta_{+} \]

  \[
d J = \frac{3}{2} \text{Im}(\overline{W}_1 \Omega) + W_4 \wedge J + W_3 \]

  \[
d \Omega = W_1 J \wedge J + W_2 \wedge J + \overline{W}_5 \wedge \Omega \]

- Five classes of (intrinsic) torsion are given as

| components | interpretations | $SU(3)$-representations |
|------------|----------------|-------------------------|
| $W_1$      | $J \wedge d\Omega$ or $\Omega \wedge dJ$ | $1 \oplus 1$ |
| $W_2$      | $(d\Omega)^{2,2}_0$ | $8 \oplus 8$ |
| $W_3$      | $(dJ)^{2,1}_0 + (dJ)^{1,2}_0$ | $6 \oplus 6$ |
| $W_4$      | $J \wedge dJ$ | $3 \oplus \overline{3}$ |
| $W_5$      | $(d\Omega)^{3,1}$ | $3 \oplus \overline{3}$ |
Classification of $SU(3)$-structure manifolds:

| Complex          | Hermitian                        | $\mathcal{W}_1 = \mathcal{W}_2 = 0$ |
|------------------|----------------------------------|--------------------------------------|
| Balanced         | $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = 0$ |
| Special Hermitian| $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$ |
| Kähler           | $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = 0$ |
| Calabi-Yau       | $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$ |
| Conformally Calabi-Yau | $\mathcal{W}_1 = \mathcal{W}_2 = \mathcal{W}_3 = 3\mathcal{W}_4 + 2\mathcal{W}_5 = 0$ |
| Almost Complex   | Symplectic                        | $\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$ |
| Nearly Kähler    | $\mathcal{W}_2 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$ |
| Almost Kähler    | $\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$ |
| Quasi Kähler     | $\mathcal{W}_3 = \mathcal{W}_4 = \mathcal{W}_5 = 0$ |
| Semi Kähler      | $\mathcal{W}_4 = \mathcal{W}_5 = 0$ |
| Half-flat        | Im$\mathcal{W}_1 =$ Im$\mathcal{W}_2 = \mathcal{W}_4 = \mathcal{W}_5 = 0$ |
Contents

- Differential Forms: Geometric Objects
- Hitchin Functional
- Killing Prepotentials
Start with a real form \( \chi_f \in \wedge^{\text{even/odd}} F^* \) (associated with a real \( Spin(6, 6) \) spinor \( \chi_s \))

Regard \( \chi_f \) as a stable form satisfying

\[
q(\chi_f) = -\frac{1}{4} \langle \chi_f, \Gamma_{\Pi\Sigma} \chi_f \rangle \langle \chi_f, \Gamma_{\Pi\Sigma} \chi_f \rangle \in \wedge^6 F^* \otimes \wedge^6 F^*
\]

\[
U = \{ \chi_f \in \wedge^{\text{even/odd}} F^* \mid q(\chi_f) < 0 \}\]
Start with a real form $\chi_f \in \wedge^{\text{even/odd}} F^*$ (associated with a real $Spin(6,6)$ spinor $\chi_s$)

Regard $\chi_f$ as a stable form satisfying

$$ q(\chi_f) = -\frac{1}{4} \langle \chi_f, \Gamma \Pi \Sigma \chi_f \rangle \langle \chi_f, \Gamma \Pi \Sigma \chi_f \rangle \in \wedge^6 F^* \otimes \wedge^6 F^* $$

$$ U = \{ \chi_f \in \wedge^{\text{even/odd}} F^* \mid q(\chi_f) < 0 \} $$

Define a Hitchin function

$$ H(\chi_f) \equiv \sqrt{-\frac{1}{3} q(\chi_f)} \in \wedge^6 F^* $$

which gives an integrable complex structure on $U$
Start with a real form $\chi_f \in \wedge^{\text{even/odd}} F^*$ (associated with a real $Spin(6,6)$ spinor $\chi_s$)

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Define a Hitchin function

$$H(\chi_f) \equiv \sqrt{-\frac{1}{3} q(\chi_f)} \in \wedge^6 F^*$$

which gives an integrable complex structure on $U$

Then we can get another real form $\hat{\chi}_f$ and a complex form $\Phi_f$ by Mukai pairing

$$\langle \hat{\chi}_f, \chi_f \rangle = -dH(\chi_f) \quad \text{i.e.,} \quad \hat{\chi}_f = -\frac{\partial H(\chi_f)}{\partial \chi_f}$$

$$\rightarrow \quad \Phi_f \equiv \frac{1}{2} (\chi_f + i \hat{\chi}_f) \quad H(\Phi_f) = i \langle \Phi_f, \overline{\Phi}_f \rangle$$

Hitchin showed: $\Phi_f$ is a (form corresponding to) pure spinor!

N.J. Hitchin math/0010054, math/0107101, math/0209099
Consider the space of pure spinors $\Phi$ ...

\[
\begin{array}{ccc}
\text{Mukai pairing } \langle *, * \rangle & \longrightarrow & \text{symplectic structure} \\
\text{Hitchin function } H(*) & \longrightarrow & \text{complex structure}
\end{array}
\]

The space of pure spinor is Kähler
Consider the space of pure spinors $\Phi$ ...

| Mukai pairing $\langle *, * \rangle$ | $\longrightarrow$ symplectic structure |
| Hitchin function $H(\ast)$ | $\longrightarrow$ complex structure |

The space of pure spinor is Kähler

Compatible with $\Phi \rightarrow \lambda \Phi$ w/ $\lambda \in \mathbb{C}^*$

$\rightarrow$ The space becomes a local special Kähler geometry with Kähler potential $K$:

$$\exp(-K) = H(\Phi) = i\langle \Phi, \Phi \rangle = i(X^A \mathcal{F}_A - X^A \overline{\mathcal{F}}_A) \in \wedge^6 F^*$$

$X^A$: holomorphic projective coordinates

$\mathcal{F}_A$: derivative of prepotential $\mathcal{F}$ ($\mathcal{F}_A = \partial \mathcal{F} / \partial X^A$)
Contents

- Differential Forms: Geometric Objects
- Hitchin Functional
- Killing Prepotentials
10D spinors in type IIA in Einstein frame

\[
\delta \Psi^A_M = \nabla_M \epsilon^A - \frac{1}{96} e^{-\phi} \left( \Gamma^{PQR}_M H_{PQR} - 9 \Gamma^{PQ} H_{MPQ} \right) \Gamma_{(11)} \epsilon^A \\
- \sum_{n=0,2,4,6,8} \frac{1}{64n!} e^{\frac{5-n}{4} \phi} \left[ (n - 1) \Gamma^N_{M1} \cdots N_n - n(9 - n) \delta^N_{M1} \Gamma^{N2\cdots N_n} \right] F_{N1\cdots N_n} (\Gamma_{(11)})^{n/2} (\sigma^1 \epsilon)^A
\]

\[
\delta \Psi^A_M = 0 \quad \text{with} \quad \begin{cases} 
\delta \psi_{A\mu} = 0 & \rightarrow \text{superpotential } \mathcal{W} \\
\delta \psi^A_m = 0 & \rightarrow \text{Kähler potential } K
\end{cases}
\]
Killing prepotential

See the SUSY variation of 4D $\mathcal{N} = 2$ gravitinos:

$$
\delta \psi_{A\mu} = \nabla_{\mu} \varepsilon_{A} - S_{AB} \gamma_{\mu} \varepsilon^{B} + \ldots
$$

$$
S_{AB} = \frac{i}{2} e^{\frac{K_{\pm}}{2}} \left( \begin{array}{cc}
\mathcal{P}^1 - i\mathcal{P}^2 & -\mathcal{P}^3 \\
-\mathcal{P}^3 & -\mathcal{P}^1 - i\mathcal{P}^2
\end{array} \right)_{AB}
$$

The components are also written by $\Phi_{\pm}$:

$$
\mathcal{P}^1 - i\mathcal{P}^2 = 2 e^{\frac{K_{-}}{2} + \varphi} \int_{\mathcal{M}} \langle \Phi_{+}, D\Phi_{-} \rangle, \quad \mathcal{P}^1 + i\mathcal{P}^2 = 2 e^{\frac{K_{-}}{2} + \varphi} \int_{\mathcal{M}} \langle \Phi_{+}, D\Phi_{-} \rangle
$$

$$
\mathcal{P}^3 = -\frac{1}{\sqrt{2}} e^{2\varphi} \int_{\mathcal{M}} \langle \Phi_{+}, G \rangle
$$

Note: $\hat{\Psi}_{A\mu} = \Psi_{A\mu} + \frac{1}{2} \Gamma_{\mu}^{m} \Psi_{m}^{A} = \psi_{A\mu\pm} \otimes \eta_{+} + \psi_{A\mu\mp} \otimes \eta_{-} + \ldots$
4D $\mathcal{N} = 1$ fermions given by the SUSY truncation from 4D $\mathcal{N} = 2$ system:

**SUSY parameter:**

$$\varepsilon \equiv \bar{n}^A \varepsilon_A = a \varepsilon_1 + b \varepsilon_2$$

**gravitino:**

$$\psi_{\mu} \equiv \bar{n}^A \psi_{A\mu} = a \psi_{1\mu} + b \psi_{2\mu}, \quad \bar{\psi}_{\mu} \equiv (b \psi_{1\mu} - \bar{a} \psi_{2\mu})$$

**gauginos:**

$$\chi^A \equiv -2 e^{\frac{K_+}{2}} D_b X^A (\bar{n}^C \varepsilon_C \varepsilon^b)$$

where

$$\bar{n}^A = (a, b), \quad \varepsilon_{AB} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$
SUSY variations yield the superpotential and the D-term:

\[ \delta \psi_\mu = \nabla_\mu \varepsilon - \bar{n}^A S_{AB} \gamma_\mu \varepsilon^c \equiv \nabla_\mu \varepsilon - e^{K_2} \mathcal{W} \gamma_\mu \varepsilon^c \]

\[ \delta \tilde{\psi}_\mu = 0 \]

\[ \delta \chi^A = \text{Im} F_{\mu \nu}^A \gamma^{\mu \nu} \varepsilon + i D^A \varepsilon \]

\[ \mathcal{W} = \frac{i}{4\bar{a}b} \left[ 4i e^{K_2 - \Phi} \int_{\mathcal{M}} \langle \Phi_+, \mathcal{D} \text{Im}(ab\Phi_-) \rangle + \frac{1}{\sqrt{2}} \int_{\mathcal{M}} \langle \Phi_+, G \rangle \right] \]

\[ \equiv \mathcal{W}^{RR} + U^I \mathcal{W}_I^Q + \tilde{U}_I \tilde{W}_I^Q \]

\[ \mathcal{W}^{RR} = -\frac{i}{4\bar{a}b} \left[ X^A e_{RRA} - \mathcal{F}_A m_{RR}^A \right] \]

\[ \mathcal{W}_I^Q = \frac{i}{4\bar{a}b} \left[ X^A e_{IA} + \mathcal{F}_A p^A \right], \quad \tilde{W}_Q^I = -\frac{i}{4\bar{a}b} \left[ X^A m_A^I + \mathcal{F}_A q^I \right] \]

\[ D^A = 2e^{K_+} (K_+)^{cd} D_c X^A \overline{D_d X^B} \left[ \bar{n}^C (\sigma_x) c^B n_B \right] \left( P_{BC}^{xc} - N_{BC} \tilde{P}^{xc} \right) \]
(Lower dimensional) supergravity related to this topic

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L. Andrianopoli, M. Bertolini, A. Ceresole, R. D’Auria, S. Ferrara, P. Fré, T. Magri hep-th/9605032 P. Fré hep-th/9512043
N. Kaloper, R.C. Myers hep-th/9901045
E. Bergshoeff, R. Kallosh, T. Ortin, D. Roest, A. Van Proeyen hep-th/0103233
M.B. Schulz hep-th/0406001 S. Gurrieri hep-th/0408044 T.W. Grimm hep-th/0507153
B. de Wit, H. Samtleben, M. Trigiante hep-th/0507289

EOM, SUSY, and Bianchi identities on generalized geometry

M. Graña, R. Minasian, M. Petrini, A. Tomasiello hep-th/0407249 hep-th/0505212
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D. Cassani, A. Bilal arXiv:0707.3125 D. Cassani arXiv:0804.0595
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A.K. Kashani-Poor, R. Minasian hep-th/0611106 A. Tomasiello arXiv:0704.2613 B.y. Hou, S. Hu, Y.h. Yang arXiv:0806.3393
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D. Lüst, D. Tsimpis hep-th/0412250
C. Kounnas, D. Lüst, P.M. Petropoulos, D. Tsimpis arXiv:0707.4270 P. Koerber, D. Lüst, D. Tsimpis arXiv:0804.0614
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M. Graña, R. Minasian, M. Petrini, A. Tomasiello hep-th/0609124
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M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell, A. Westerberg hep-th/9611159
E. Bergshoef, P.K. Townsend hep-th/9611173

Mathematics

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M. Gualtieri math/0401221

Doubled formalism

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C.M. Hull hep-th/0406102 hep-th/0605149 hep-th/0701203 C.M. Hull, R.A. Reid-Edwards hep-th/0503114 arXiv:0711.4818
J. Shelton, W. Taylor, B. Wecht hep-th/0508133 A. Dabholkar, C.M. Hull hep-th/0512005
A. Lawrence, M.B. Schulz, B. Wecht hep-th/0602025
G. Dall’Agata, S. Ferrara hep-th/0502066
G. Dall’Agata, M. Prezas, H. Samtleben, M. Trigiante arXiv:0712.1026 G. Dall’Agata, N. Prezas arXiv:0806.2003
C.M. Hull, R.A. Reid-Edwards arXiv:0902.4032
C. Albertsson, R.A. Reid-Edwards, TK arXiv:0806.1783

and more...