Dynamical determination of the Kuiper Belt’s mass from motions of the inner planets of the Solar System

Lorenzo Iorio  
Viale Unità di Italia 68, 70125  
Bari, Italy  
tel./fax 0039 080 5443144  
e-mail: lorenzo.iorio@libero.it

Abstract

In this paper we dynamically determine the mass of the Kuiper Belt Objects by exploiting the latest observational determinations of the orbital motions of the inner planets of the Solar System. Our result, in units of terrestrial masses, is $0.033\pm0.115$ by modelling the Classical Kuiper Belt Objects as an ecliptic ring of finite thickness. A two-rings model yields for the Resonant Kuiper Belt Objects a value of $0.018\pm0.063$. Such figures are consistent with recent determinations obtained with ground and space-based optical techniques. Some implications for precise tests of Einsteinian and post-Einsteinian gravity are briefly discussed.

Key words: celestial mechanics—ephemerides—Kuiper belt—planets and satellites: individual (Earth, Mars, Mercury)—relativity

1 Introduction

Starting in 1992, astronomers have become aware of a vast population of small bodies orbiting the Sun beyond Neptune. There are at least 70,000 Trans-Neptunian Objects (TNOs) with diameters larger than 100 km in the 30-50 AU region. Observations show that TNOs are mostly confined within a thick band around the ecliptic, leading to the realization that they occupy a ring or belt surrounding the Sun. This ring is generally referred to as the Kuiper Belt (Edgeworth 1943; Kuiper 1951; Fernandez 1980).

Reasons of interest in the Kuiper Belt are as follows:

1 See [http://www.ifa.hawaii.edu/faculty/jewitt/kb.html](http://www.ifa.hawaii.edu/faculty/jewitt/kb.html)

2 See [http://www.ifa.hawaii.edu/faculty/jewitt/kb/gerard.html](http://www.ifa.hawaii.edu/faculty/jewitt/kb/gerard.html) for the origin of the name.
It is likely that the Kuiper Belt Objects (KBOs) are extremely primitive remnants from the early accretional phases of the solar system. The inner, dense parts of the pre-planetary disk condensed into the major planets, probably within a few millions to tens of millions of years. The outer parts were less dense, and accretion progressed slowly. Evidently, many small objects were formed.

It is widely believed that the Kuiper Belt is the source of the short-period comets. It acts as a reservoir for these bodies in the same way that the Oort Cloud acts as a reservoir for the long-period comets.

KBOs are usually not yet modelled in the data reduction softwares used for orbit determination purposes, so that they may represent a serious bias in precise tests of Einsteinian and post-Einsteinian gravity. Thus, it is important to assess their impact on planetary motions.

KBOs can be classified into three dynamical classes (Jewitt et al. 1998)

- Classical KBOs (CKBOs), following nearly circular orbits with relatively low eccentricities ($e < 0.25$) and semimajor axes $41 \text{AU} \lesssim a \lesssim 46 \text{AU}$; they constitute about 70% of the observed population.

- Resonant KBOs occupy mean motion resonances with Neptune, such as $3:2$ (the Plutinos, $a \sim 39.4 \text{AU}$) and $2:1$ ($a \sim 47.8 \text{AU}$), and amount to about 20% of the known objects.

- Scattered KBOs, which represent only about 10% of the known KBOs but have the most extreme orbits, with $a \sim 90 \text{AU}$ and $e \sim 0.6$, presumably due to a weak interaction with Neptune; we only have rather poor knowledge of them.

In this paper we determine the mass of KBOs in a truly dynamical way by means of the latest observational determinations of the secular perihelion advances of the inner planets of the Solar System. The use of such celestial bodies presents many advantages. Long data sets including also many accurate radio-technical range and range-rate measurements are available; they cover many orbital revolutions, contrary to the outer planets which are more affected by KBOs like Uranus and Neptune. Indeed, mainly optical data and sparse radar-ranging measurements exist for them covering barely one period for Uranus and less than one full orbital revolution for Neptune. Moreover, for heliocentric distances of the order of just 1 AU or less many details of the true mass distribution of KBOs are not relevant and all the recently proposed analytical models converge satisfactorily to a substantially
Table 1: Observationally determined extra-precessions of the longitudes of perihelia of the inner planets, in arcseconds per century (′′ cy⁻¹), by using EPM2004 with β = γ = 1, J₂ = 2 × 10⁻⁷, from Table 3 of (Pitjeva 2005b). Both KBOs and the general relativistic gravitomagnetic force were not included in the adopted dynamical force models. The quoted uncertainties are not the mere formal, statistical errors but are realistic in the sense that they were obtained from comparison of many different solutions with different sets of parameters and observations. The correlations among such determined planetary perihelia rates are very low with a maximum of about 20% between Mercury and the Earth (Pitjeva, private communication 2005).

|         | Mercury   | Venus     | Earth     | Mars      |
|---------|-----------|-----------|-----------|-----------|
|         | −0.0036 ± 0.0050 | 0.53 ± 0.30 | −0.0002 ± 0.0004 | 0.0001 ± 0.0005 |

unified description, given the present-day accuracy in reconstructing the orbital motions in our region of the Solar System. This fact greatly simplifies the analytical calculation and allows for reliable, rather model-independent determinations of the KBOs’s mass.

2 The determined extra-rates of perihelion

The Russian astronomer E.V. Pitjeva (Institute of Applied Astronomy, Russian Academy of Sciences) recently processed almost one century of data of all types in the effort of continuously improving the EPM2004 planetary ephemerides (Pitjeva 2005a). Among other things, she also determined residual secular, i.e. averaged over one orbital revolution, rates of the longitudes of perihelion ω = Ω + ω, where ω and Ω are the argument of perihelion and the longitude of the ascending node, respectively, of the inner planets (Pitjeva 2005b) as fit-for parameters of global solutions in which she contrasted, in a least-square way, the observations (ranges, range-rates, angles like right ascension α and declination δ, etc.) to their predicted values computed with a complete suite of dynamical force models. The results are shown in Table 1. The modelled forces are

- The Newtonian N-body perturbations including also the effect of 301 largest asteroids (Krasinsky et al. 2002) and of the minor asteroid ring in the ecliptic
- The Sun’s quadrupole mass moment J₂ (Paternò et al. 1996; Pijpers 1998; Mecheri et al. 2004), set to 2 × 10⁻⁷
• The post-Newtonian gravitoelectric forces (Newhall et al. 1983) parameterized in terms of the PPN Eddington-Robertson-Schiff parameters $\gamma$ and $\beta$ (Will 1993) set to their general relativistic values $\gamma = \beta = 1$.

The un-modelled forces are

• The Newtonian force induced by KBOs

• The post-Newtonian gravitomagnetic force responsible for the Lense-Thirring effect (Lense and Thirring 1918).

Thus, the effect of KBOs is entirely accounted for by the so-obtained residual perihelia advances of Table 1.

### 3 Modelling and confrontation with data

In order to make a comparison with Table 1 a theoretical prediction for the secular precessions induced by KBOs on the planetary perihelia is required.

The action of KBOs can be treated in a perturbative way. The Gauss rate equations for $\omega$ and $\Omega$ are, for a generic orbital configuration

\[
\frac{d\omega}{dt} = -\cos i \frac{d\Omega}{dt} + \sqrt{1-e^2} \left[ -A_r \cos f + A_t \left(1 + \frac{r}{p}\right) \sin f \right],
\]

(1)

and

\[
\frac{d\Omega}{dt} = \frac{A_n}{n a \sqrt{1-e^2} \sin i} \left(\frac{r}{a}\right) \sin(\omega + f),
\]

(2)

where $i$ is the inclination of the orbit, $p = a(1-e^2)$, $n = \sqrt{GM/a^3}$ is the Keplerian mean motion, $f$ is the true anomaly counted from the pericentre, and $A_r$, $A_t$ and $A_n$ are the radial, transverse and normal components of the perturbing acceleration $\mathbf{A}$, respectively.

For almost ecliptic orbits ($i \sim 0$ deg, $\cos i \sim 1$, $\sin i \sim i$), like those of the Solar System’s major bodies, the rate equation for $\varpi$ can safely be approximated as

\[
\frac{d\varpi}{dt} \sim \sqrt{1-e^2} \left[ -A_r \cos f + A_t \left(1 + \frac{r}{p}\right) \sin f \right].
\]

(3)

Its secular rate is obtained by evaluating the right-hand-side of eq. (3) onto the unperturbed Keplerian ellipse, characterized in terms of the eccentric
anomaly $E$ by

$$
\begin{align*}
\cos f &= \frac{\cos E - e}{1 - e \cos E}, \\
\sin f &= \frac{\sin E \sqrt{1 - e^2}}{1 - e \cos E}, \\
r &= a(1 - e \cos E),
\end{align*}
$$

and averaging the result over one orbital period $P$ by means of

$$
\frac{dt}{P} = \frac{(1 - e \cos E)}{2\pi} dE.
$$

Motivated by the search for a gravitational explanation of the Pioneer anomaly (Anderson et al. 2002), various authors recently produced several analytical models for the KBOs acceleration (Anderson et al. 2002; Nieto 2005; Bertolami and Vieira 2006; de Diego et al. 2006). At heliocentric distances $\lesssim 1$ AU many details of the three-dimensional mass distribution of KBOs can be neglected, so that we can approximate it with a bi-dimensional distribution lying in the ecliptic plane. Moreover, the four models analyzed by Bertolami and Vieira (2006), i.e. two-rings, uniform disk, non-uniform disk and torus, give the same results for $r \ll 20$ AU and for various ecliptic latitudes $\beta$ close to zero. Thus, we will adopt for CKBOs the uniform disk model, which consists of a uniform, hollow thin disk lying in the ecliptic plane within distances $R_{\text{min}} = 37.8$ AU and $R_{\text{max}} = 46.2$ AU (Bernstein et al. 2004).

The Newtonian gravitational field of a thin massive ring exhibits cylindrical symmetry and has an in-plane, radial component and an out-of-plane, normal component (Owen 2003). Since we are looking for the effects induced on the longitude of the perihelion, only the radial component is relevant to us. Thus, from eq. (3)–eq. (5) the secular perihelion rate can be written as

$$
\langle \frac{d\varpi}{dt} \rangle = -\frac{1}{2\pi e} \sqrt{\frac{a(1 - e^2)}{GM}} \int_0^{2\pi} A_r(E)(\cos E - e) dE.
$$

The radial component of $A_K$ for a uniform disk is (Bertolami and Vieira 2006)

$$
A_{Kld} = K \int_0^{2\pi} \int_{R_{\text{min}}}^{R_{\text{max}}} \frac{R_K \left[ r - R_K \cos \beta \cos(\phi_K - \lambda) \right]}{\left[ R_K^2 + r^2 - 2r R_K \cos \beta \cos(\phi_K - \lambda) \right]^{3/2}} d\phi_K dR_K,
$$

where

$$
K = \frac{Gm_K}{2\pi (R_{\text{max}}^2 - R_{\text{min}}^2)},
$$
Table 2: Gravitomagnetic secular precessions of the longitudes of perihelion \( \varpi \) of Mercury, Venus, Earth and Mars in \( " \text{cy}^{-1} \). The value \((190.0 \pm 1.5) \times 10^{39} \text{ kg m}^2 \text{ s}^{-1} \) (Pijpers 1998; 2003) has been adopted for the solar proper angular momentum \( L_\odot \).

|        | Mercury | Venus | Earth | Mars |
|--------|---------|-------|-------|------|
| \( A_u \) | -0.0020 | -0.0003 | -0.0001 | -0.0003 |

\( m_K \) is the KBOs’s mass, \( R_K \) and \( \phi_K \) are the polar coordinates of the disk mass element, and \( \{r, \beta, \lambda\} \) are the usual spherical ecliptic planetary coordinates. For the inner planets, with \( r \ll R_{\text{min}}, R_{\text{max}} \) and whose orbits lie almost exactly in the ecliptic plane, we can safely post:\(^3\)

\[
\left[ 1 + \left( \frac{r}{R_K} \right)^2 - 2 \left( \frac{r}{R_K} \right) \cos \beta \cos(\phi_K - \lambda) \right]^{-3/2} \sim 1 + 3 \left( \frac{r}{R_K} \right) \cos(\phi_K - \lambda).
\]

By inserting eq. (9) into eq. (7) and performing the integration we get

\[
A_u = \frac{Gm_K}{2(R_{\text{max}} + R_{\text{min}})R_{\text{max}}R_{\text{min}}} r.
\]

Note that it is positive, in agreement with the behavior of all the models considered in (Bertolami and Vieira 2006) for \( r \ll 20 \text{ AU} \). Eq. (10), with \( r = a(1 - e \cos E) \), into eq. (6) finally yields

\[
\left\langle \frac{d\varpi}{dt} \right\rangle = \frac{3}{4} \sqrt{\frac{Ga^3(1 - e^2)}{M}} \frac{m_K}{(R_{\text{max}} + R_{\text{min}})R_{\text{max}}R_{\min}}.
\]

Table 1 and eq. (11) can now be used to determine \( m_K \) in a truly dynamically, model-independent way. To this aim, let us note that, by construction, the determined extra-rates of Table 1 are not only due to KBOs, but also the Lense-Thirring field and the mismodeled part of the solar \( J_2 \), which is presently uncertain at a \( \sim 10\% \) level, contribute to them. Their nominal magnitudes are listed in Table 2 and Table 3, respectively.

Although the Lense-Thirring effect and the mismodeled part of the \( J_2 \) precessions may be neglected in our analysis, being smaller than the errors of Table 1, we prefer conservatively to cancel out, by construction, any possible impact due to them. It can be done by suitably combining the perihelia of Mercury, the Earth and Mars according to an approach followed, e.g., in

\(^3\)Recall that \( \cos \beta = \sqrt{1 - \sin^2 i \sin^2 f} \).
Table 3: Nominal values of the classical secular precessions of the longitudes of perihelion $\varpi$ of Mercury, Venus, Earth and Mars, in $'' \mathrm{cy}^{-1}$, induced by the solar quadrupolar mass moment $J_2$. The value $J_2 = 2 \times 10^{-7}$ used in (Pitjeva 2005b) has been adopted. Their mismodelled amplitudes can be obtained by assuming an uncertainty in $J_2$ of the order of $\sim 10\%$.

| Planet   | Mercury | Venus | Earth | Mars |
|----------|---------|-------|-------|------|
| $\dot{\varpi}$ | 0.0254  | 0.0026 | 0.0008 | 0.0002 |

(Iorio 2005). By converting Table 1 in $s^{-1}$, we have the CKBOs’ mass, in units of terrestrial masses

$$ m_{\mathrm{K}}^{(\text{comb})} = \frac{\dot{\varpi}_{\text{Mercury}} + c_1 \dot{\varpi}_{\text{Earth}} + c_2 \dot{\varpi}_{\text{Mars}}}{1.017 \times 10^{-15} \ \text{s}^{-1}} = 0.052, \quad (12) $$

with

$$ c_1 = -81.71, \ c_2 = 221.18. \quad (13) $$

Such coefficients, built in terms of the semimajor axes and eccentricities of the involved planets, assure that the solar Newtonian quadrupolar and post-Newtonian gravitomagnetic fields, whatever their contributions to Table 1 is, do not affect at all the recovered CKBOs’ mass. Because of the existing correlations among the determined extra-rates of perihelia, the error can be conservatively evaluated as

$$ \delta m_{\mathrm{K}}^{(\text{comb})} \leq \frac{\delta \dot{\varpi}_{\text{Mercury}} + |c_1| \delta \dot{\varpi}_{\text{Earth}} + |c_2| \delta \dot{\varpi}_{\text{Mars}}}{1.017 \times 10^{-15} \ \text{s}^{-1}} = 0.223. \quad (14) $$

If we only use the Mars perihelion, whose Lense-Thirring and mismodelled $J_2$ precessions are negligible by one order of magnitude, we get

$$ m_{\mathrm{K}}^{(\text{Mars})} = 0.026 \pm 0.134. \quad (15) $$

A weighted mean of such two measurements yields

$$ m_{\mathrm{K}}^{(\text{weighted})} = 0.033 \pm 0.115. \quad (16) $$

In regard to the Resonant KBOs, we can adopt the two-rings model with $R_1 = 39.4 \ \text{AU}$ and $R_2 = 49.8 \ \text{AU}$. The radial acceleration is (Bertolami and Vieira 2006)

$$ A_{2R} = -\frac{G m_{\mathrm{K}}}{2\pi(R_1 + R_2)} \int_0^{2\pi} \sum_{i=1}^{2} \frac{R_i [r - R_i \cos \beta \cos(\phi_{\mathrm{K}} - \lambda)]}{[r^2 + R_i^2 - 2r R_i \cos \beta \cos(\phi_{\mathrm{K}} - \lambda)]^{3/2}} d\phi_{\mathrm{K}}. \quad (17) $$
By using the same approximation of eq. (9), we obtain $m_{K}^{(\text{comb})} = 0.029 \pm 0.123$ and $m_{K}^{(\text{Mars})} = 0.015 \pm 0.073$, with $m_{K}^{(\text{weighted})} = 0.018 \pm 0.063$. Note that such figures are smaller than those for CKBOs, consistent with the existing estimates for the population of Resonant KBOs.

At this point, we can a posteriori justify the use of the approximation of eq. (9) for both the models used: indeed, by using our values for $m_{K}$ it is possible to show that additional terms in the expansion of eq. (9) yield precessions far too small to be detected with the present-day accuracy.

4 Discussion and conclusions

In this paper we dynamically determined the mass of KBOs from an analysis of the recently determined secular perihelion advances of the rocky planets of the Solar System. We modelled CKBOs as a uniform thick ring; such a simple model is justified by the fact that at heliocentric distances of about 1 AU many details of the true three-dimensional KBOs mass distribution can be neglected; indeed, many different, more or less complicated analytical models manifest substantially the same behavior at distances much less than 20 AU. For CKBOs we obtained a mass of $0.033 \pm 0.115$, in units of terrestrial masses. A two-rings model for the Resonant KBOs yields a mass of $0.018 \pm 0.063$.

Such figures are consistent with those by

- Bernstein et al. (2004), who give a nominal mass of 0.010 for CKBOs and 0.021 for their Excited class including some Plutinos and Scattered KBOs. They used the ACS camera of the Hubble Space Telescope

- Gladman et al. (2001) yielding a mass of $0.04 - 0.1$ for all the TNOs in the range 30-50 AU, excluding the Scattered KBOs. The adopted technique was deep imaging on the Canada-France-Hawaii Telescope and the ESO Very Large Telescope UT1

- Trujillo et al. (2001) giving for CKBOs a value of 0.030, from a wide-field survey with the CCD Mosaic camera of the Canada-France-Hawaii Telescope.

The upper limit of 0.3 terrestrial masses obtained by Backman et al. (1995) from far-IR emission measurements is, instead, ruled out.

It is important to note that all the previous estimates are based on various assumptions about, e.g., albedo and density of the KBOs for which
Table 4: Nominal values of the classical secular precessions of the longitudes of perihelion $\varpi$ of Mercury, Venus, Earth and Mars, in " cy$^{-1}$, induced by CKBOs. For $m_K$ the value of eq. (12) has been used.

|       | Mercury | Venus | Earth | Mars |
|-------|---------|-------|-------|------|
| Value | 0.00002 ± 0.0001 | 0.00006 ± 0.0003 | 0.0001 ± 0.0004 | 0.0002 ± 0.0008 |

very large uncertainties still exist, so that the authors of the previously cited works decided to release no errors of their mass estimations.

The impact of KBOs on the inner planets of the Solar System lies at the edge of the present-day accuracy. Its induced effects, shown in Table 4 for a particular value of the CKBOs’ mass, should not be neglected in, e.g., precision tests of gravity because, especially for the Earth and Mars, they are of the same order of magnitude, or even larger, than some Einsteinian (see Table 2) and post-Einsteinian features of motion which recently attracted much attention in view of a possible detection in the near future. If not accounted for in the dynamical force models of the orbit data reduction softwares, KBOs may bias the recovery of such effects when the required precision level will be finally attained. In the case of the measurement of the Lense-Thirring effect by only using the perihelion of Mercury (Iorio 2005a), the impact of KBOs is negligible. It is not so if a combination of the perihelia of Mercury and the Earth is used in order to cancel out the mismodelling of $J_2$ (Iorio 2005a). Thanks to the availability of a simple and reliable formula as that of eq. (11) for the inner planets, such a problem could be circumvented by setting a suitable three-elements combination allowing for de-coupling the Lense-Thirring effect from $J_2$ and KBOs as well.

Incidentally, let us note that our results further enforce the conclusion that the Pioneer anomaly cannot be due to KBOs (Anderson et al. 2002; Nieto 2005; Bertolami and Vieira 2006).

References

[1] Anderson, J.D., Laing, P.A., Lau, E.L., Liu, A.S., Nieto, M.M., and Turyshev, S.G., Study of the anomalous acceleration of Pioneer 10 and 11, Phys. Rev. D, 65, 082004, 2002.

4In the context of the multidimensional model by Dvali, Gabadadze and Porrati (Dvali et al. 2000), secular perihelion precessions of the order of 0.0005 " cy$^{-1}$ are predicted for the Solar System planets (Lue and Starkman 2003; Iorio 2005b).
[2] Backman, G.E., Dasgupta, A., and Stencel, R.E., Model of a Kuiper Belt Small Grain Population and Resulting Far-Infrared Emission, *Astroph. J.*, 450, L35-L38, 1995.

[3] Bernstein, G.M., Trilling, D.E., Allen, R.L., Brown, M.E., Holman, M., and Malhotra, R., The size distribution of trans-Neptunian bodies, *Astron. J.*, 128, 1364-1390, 2004.

[4] Bertolami, O., and Vieira, P., Pioneer anomaly and the Kuiper Belt mass distribution, *Class. Quantum Grav.*, 23, 4625-4635, 2006.

[5] de Diego, J.A., Núñez, D., and Zavala, J., Pioneer anomaly? Gravitational pull due to the Kuiper belt, *Int. J. Mod. Phys. D*, 15, 533-544, 2006.

[6] Dvali, G., Gabadadze, G., and Porrati, M., 4D Gravity on a Brane in 5D Minkowski Space, *Phys. Lett. B*, 485, 208-214, 2000.

[7] Edgeworth, K. E., The evolution of our planetary system, *J. Brit. Astron. Assoc.*, 53, 181-188, 1943.

[8] Fernandez, J., On the existence of a comet belt beyond Neptune, *Mon. Not. Roy. Astron. Soc.*, 192, 481-491, 1980.

[9] Gladman, B., Kavelaars, J.J., Petit, J.-M., Morbidelli, A., Holman, M.J., Loredo, T., The Structure of the Kuiper Belt: Size Distribution and Radial Extent, *Astron. J.*, 122, 1051-1066, 2001.

[10] Iorio, L., First preliminary evidence of the general relativistic gravitomagnetic field of the Sun and new constraints on a Yukawa-like fifth force, [http://arxiv.org/abs/gr-qc/0507041](http://arxiv.org/abs/gr-qc/0507041) 2005a.

[11] Iorio, L., On the effects of the Dvali-Gabadadze-Porrati braneworld gravity on the orbital motion of a test particle, *Class. Quantum Grav.*, 22, 5271-5281, 2005b.

[12] Jewitt, D.C., Luu, J.X., and Trujillo, C.A., Large Kuiper Belt Objects: The Mauna Kea 8K CCD Survey, *Astron. J.*, 115, 2125-2135, 1998.

[13] Krasinsky, G.A., Pitjeva, E.V., Vasiljev, M.V., and Yagudina, E.I., Hidden Mass in the Asteroid Belt, *Icarus*, 158, 98-105, 2002.
[14] Kuiper, G.P., On the Origin of the Solar System, in: Hynek, J.A. (ed.), Proceedings of a topical symposium, commemorating the 50th anniversary of the Yerkes Observatory and half a century of progress in astrophysics, (McGraw-Hill, New York), 1951, pp. 357-424.

[15] Lense, J., and H. Thirring, Über den Einfluss der Eigenrotation der Zentralkörper auf die Bewegung der Planeten und Monde nach der Einsteinschen Gravitationstheorie, Phys. Z., 19, 156-163, 1918, translated and discussed by Mashhoon, B., Hehl, F.W., and Theiss, D.S., On the Gravitational Effects of Rotating Masses: The Thirring-Lense Papers, Gen. Rel. Grav., 16, 711-750, 1984. Reprinted in: Ruffini, R.J., and Sigismondi, C. (eds.), Nonlinear Gravitodynamics, (World Scientific, Singapore), 2003. pp. 349-388.

[16] Lue, A., and Starkmann, G., Gravitational Leakage into Extra Dimensions Probing Dark Energy Using Local Gravity, Phys. Rev. D, 67, 064002, 2003.

[17] Mecheri, R., Abdelatif, T., Irbah, A., Provost, J., and Berthomieu, G., New values of gravitational moments J2 and J4 deduced from helioseismology, Sol. Phys., 222, 191-197, 2004.

[18] Newhall, X.X., Standish, E.M., and Williams, J.G., Astron. Astrophys., DE 102 - A numerically integrated ephemeris of the moon and planets spanning forty-four centuries, 125, 150-167, 1983.

[19] Nieto, M.M., Analytical gravitational force calculations for models of the Kuiper-Belt, with applications to the Pioneer anomaly, Phys. Rev. D, 72, 083004, 2005.

[20] Owen, G.E., Introduction to Electromagnetic Theory, (Dover, New York), 2003.

[21] Paternò, L., Sofia, S., and Di Mauro, M.P., The rotation of the Sun’s core, Astron. Astrophys., 314, 940-946, 1996.

[22] Pijpers, F.P., Helioseismic determination of the solar gravitational quadrupole moment, Mon. Not. Roy. Astron. Soc., 297, L76-L80, 1998.

[23] Pijpers, F.P., Astroseismic determination of stellar angular momentum, Astron. Astrophys., 402, 683-692, 2003.
[24] Pitjeva, E.V., High-Precision Ephemerides of Planets-EPM and Determination of Some Astronomical Constants, *Sol. Sys. Res.*, **39**, 176-186, 2005a.

[25] Pitjeva, E.V., Relativistic Effects and Solar Oblateness from Radar Observations of Planets and Spacecraft, *Astron. Lett.*, **31**, 340-349, 2005b.

[26] Trujillo, C.A., Jewitt, D.C., and Luu, J.X., Properties of the Trans-Neptunian Belt: Statistics From the Canada-France-Hawaii Telescope Survey, *Astron.J.*, **122**, 457-473, 2001.

[27] Will, C M., *Theory and Experiment in Gravitational Physics, 2nd edition*, (Cambridge University Press, Cambridge), 1993.