Black hole space-time in dark matter halo

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Abstract. For the first time, we obtain the analytical form of a stationary black hole space-time metric in dark matter halos. First, using the relation between the rotational velocity (in the equatorial plane) and the spherically symmetric space-time metric coefficients, we obtain the space-time metric for pure dark matter. By considering the dark matter halo in spherically symmetric space-time as part of the energy-momentum tensors in the Einstein field equation, we then obtain the spherically symmetric black hole solutions with a dark matter halo. Finally, utilizing the Newman-Janis method, we further generalize to rotational black holes. As examples, we obtain the space-time metric of black holes surrounded by Cold Dark Matter and Scalar Field Dark Matter halos, respectively. Our main results regarding the interaction between black holes and dark matter halos are as follows: (i) for both dark matter models, the density profile always produces a “cusp” phenomenon at small scales; (ii) the dark matter halo increases the black hole horizon but shrinks the ergosphere, while the magnitude is small; (iii) dark matter does not change the singularity of black holes. These results are useful to study the interaction of a stationary black hole and dark matter halo system. Particularly, the “cusp” produced at the $0 \sim 1$ kpc scale would be observable in the Milky Way. Perspectives on future work regarding the applications of our results in astrophysics are also briefly discussed.

Keywords: GR black holes, gravity

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The origin of supermassive black holes in the center of galaxies is an important problem in astrophysics. A dark matter halo may be helpful for us to solve this problem in the early Universe \cite{1, 2}. For the general situation, the growth of black holes in dark matter halos over time has been analyzed through numerical simulations, but the analytical form has not yet been obtained. The reason is that when the interaction between dark matter halo and black hole is considered, the dynamics of dark matter particles are unknown. Indeed, there is no analytic solution even for stationary black holes with dark matter halos. For the Navarro-Frenk-White (NFW) dark matter model, the space-time geometry without a black hole has been obtained and generally discussed \cite{3–5}. In their work, they assume that the pure dark matter space-time geometry is “almost flat” because the dark matter density is small and no relativistic motion appears. Using these results, one could discuss several dynamical processes through geometric methods when considering a pure dark matter halo, such as the tidal disruption effect (TDE), the motion of stars in the halo, etc.. If considering both a dark matter halo and a black hole, because of the lack of a corresponding space-time metric, the above physical processes can not be studied in an analytic way.

For a supermassive black hole in the center of a galaxy, its strong gravity could enhance the dark matter density significantly, producing a so-called “spike” phenomenon \cite{6–8}. But for the NFW density profile, a “cusp” problem occurs \cite{9}, and is contrary to observations which show a relatively flat density profile. For other dark matter models, such as Scalar Field Dark Matter, Modified Newtonian Dynamics Dark Matter, and Warm Dark Matter, no “cusp” is produced at small scales. Whether the “spike” and “cusp” appear in the galactic center is unknown.

These problems inspire us to study black holes (spherically symmetric and rotational) in dark matter halos for the stationary case. Based on our results, we can investigate many
dynamical processes near the black hole and the energy density of dark matter in relativistic limits.

The paper is organized as follows. In section 2, we introduce the dark matter density profile and derive the spherically symmetric dark matter space-time metric. In section 3, we develop a general method to study spherically symmetric black hole space-time surrounded by a dark matter halo. In section 4, we generalize to rotational black holes. In section 5, we discuss the behaviour of the dark matter profile near the black hole and properties of the black hole surrounded by such halos. The summary is given in section 6.

2 Dark matter density profile and spherically symmetric dark matter space-time metric

In this section, we derive the space-time geometry for pure dark matter. We follow the method introduced by [5], in which the spherically symmetric space-time metric for pure dark matter is given by

\[ ds^2 = -f(r)dt^2 + \frac{dr^2}{g(r)} + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \] (2.1)

For a test particle in spherically symmetric space-time, its rotational velocity in the equatorial plane is determined by the metric coefficient function \( f(r) \) as

\[ V^2 = \frac{r}{\sqrt{f(r)}} \frac{d\sqrt{f(r)}}{dr} = r \frac{d\ln \sqrt{f(r)}}{dr}. \] (2.2)

On the other hand, the rotational velocity in the equatorial plane can be calculated by dark matter density profile as well. Using the empirical dark matter density profile obtained from numerical simulations, we can then obtain the space-time metric assuming \( f(r) = g(r) \). This is based on the arguments that, firstly, it is generally believed that for general vacuum black holes, such as Schwarzschild black hole, \( f(r) = g(r) \). Then given the much smaller gravitational effect of dark matter comparing to that of the black hole, it is good to set \( f(r) = g(r) \) in approximation. Secondly, from [5] and [4], we found that the difference of physical effects between \( f(r) \neq g(r) \) and \( f(r) = g(r) \) is again much smaller than the effect of dark matter on the black hole. So it is also reasonable to set \( f(r) = g(r) \). This method can be applied to all dark matter density profiles, such as Cold Dark Matter (CDM) and Scalar Field Dark Matter (SFDM).

Case I: CDM. The density profile of CDM is the NFW profile obtained from numerical simulations based on CDM and ΛCDM [10–12]. We have the following expressions for the density profile and rotational velocity

\[ \rho_{\text{NFW}}(r) = \frac{\rho_c}{\frac{r}{R_s} \left(1 + \frac{r}{R_s}\right)^2}, \quad V_{\text{NFW}}(r) = \sqrt{4\pi G \rho_c R_s^3} \left[ \frac{1}{r} \ln \left(1 + \frac{r}{R_s}\right) - \frac{r/R_s}{1 + r/R_s} \right], \] (2.3)

where \( \rho_c \) is the density of the universe at the moment when the halo collapsed and \( R_s \) is the characteristic radius. Using the relation between the rotational velocity and the
metric coefficient function (eq. (2.2)), we obtain \( f(r) \) and \( g(r) \) as
\[
f(r) = g(r) = \exp \left[ 2 \int \frac{V_{\text{FW}}^2(r)}{r} dr \right] = \left( 1 + \frac{r}{R_s} \right)^{- \frac{8 G \rho_c R_s^3}{c^2 r}}.
\] (2.4)

Case II: SFDM. This model has two density profiles which are BEC profile and finite BEC profile \([13, 14]\). In order to simplify the discussion, we focus on BEC profile. This profile corresponds to the static solution of the Klein-Gordon equation and a quadratic potential for the scalar field \( \phi \). The profile and its corresponding rotational velocity are
\[
\rho_{\text{SFDM}}(r) = \rho_c \sin(kr), \quad V_{\text{SFDM}}(r) = \sqrt{\frac{4 G \rho_c R^2}{\pi}} \left[ \frac{\sin(\pi r/R)}{\pi r/R} - \cos \left( \frac{\pi r}{R} \right) \right],
\] (2.5)
where \( k \) is determined by the Compton relationship, \( R = \pi/k \) is the radius at which the pressure and density are zero, and \( \rho_c \) is the central density. Using eq. (2.2), we obtain
\[
f(r) = g(r) = \exp \left[ - \frac{8 G \rho_c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R} \right].
\] (2.6)

3 Spherically symmetric black hole metric in dark matter halo

3.1 General method

We now consider black holes surrounded by dark matter halos. From the pure dark matter space-time (eq. (2.1)), we can obtain the corresponding energy-momentum tensors. Given that energy-momentum tensors in turn lead to many kinds of space-time metrics, we try to find the space-time metric which reduces to the Schwarzschild metric in the absence of a dark matter halo.

In General Relativity (GR), the Einstein field equation is given by
\[
R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R = \kappa^2 T^\nu_\mu.
\] (3.1)

If \( T^\nu_\mu = \text{diag}[-\rho, p_r, p, p] \), we can calculate the non-zero energy-momentum tensors for the pure dark matter space-time metric, which are given by
\[
\kappa^2 T^t_t(\text{DM}) = -\kappa^2 \rho = g(r) \left( \frac{1}{r} \frac{g'(r)}{g(r)} + \frac{1}{r^2} \right) - \frac{1}{r^2},
\]
\[
\kappa^2 T^r_r(\text{DM}) = \kappa^2 p_r = g(r) \left( \frac{1}{r^2} + \frac{1}{r} \frac{f'(r)}{f(r)} \right) - \frac{1}{r^2},
\]
\[
\kappa^2 T^\theta_\theta(\text{DM}) = \kappa^2 T^\phi_\phi(\text{DM}) = \kappa^2 p = \frac{1}{2} g(r) \left( f''(r) f(r) - f^2(r) r + \frac{1}{r} f'(r) f'(r) + \frac{1}{r} \left( f'(r) + g'(r) \right) + f'(r) g'(r) \right).
\] (3.2)

By inserting the space-time metric coefficient functions of CDM and SFDM models (eq. (2.4) and eq. (2.6)) into the non-zero energy-momentum tensors (eq. (3.2)), we verify that these energy-momentum tensors satisfy the Weak Energy Condition (WEC, \( \rho > 0 \)) and Strong Energy Condition (SEC, \( \rho + p_r + 2p > 0 \)). Thus, these energy-momentum tensors are physical.
In order to include a black hole, we treat dark matter as part of the energy-momentum tensor $T_{\mu}^{\nu}$. Since the Schwarzschild black hole corresponds to an energy-momentum tensor of 0, if considering such a black hole in a dark matter halo, we only need to consider the energy-momentum tensor of dark matter. The space-time metric including a black hole is thus given by

$$ds^2 = -(f(r) + F_1(r)) dt^2 + \left( g(r) + F_2(r) \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$  \hspace{1cm} (3.3)$$

We redefine the metric coefficient functions as

$$F(r) = f(r) + F_1(r),$$
$$G(r) = g(r) + F_2(r).$$  \hspace{1cm} (3.4)$$

For such a dark matter-black hole system in which dark matter is considered as part of the Einstein tensor $T_{\mu}^{\nu}$, the Einstein field equation (eq. (3.1)) becomes

$$R_{\mu}^{\nu} - \frac{1}{2} \delta_{\mu}^{\nu} R = \kappa^2 T_{\mu}^{\nu} = \kappa^2 (T_{\mu}^{\nu}(DM)).$$  \hspace{1cm} (3.5)$$

Inserting the space-time metric (eq. (3.3)) into the new Einstein field equation (eq. (3.5)), we obtain

$$\left( g(r) + F_2(r) \right) \left( \frac{1}{r^2} + \frac{1}{r} \frac{g'(r) + F_2'(r)}{g(r) + F_2(r)} \right) = g(r) \left( \frac{1}{r^2} + \frac{g'(r)}{g(r)} \right),$$
$$\left( g(r) + F_2(r) \right) \left( \frac{1}{r^2} + \frac{1}{r} \frac{f'(r) + F_2'(r)}{g(r) + F_2(r)} \right) = g(r) \left( \frac{1}{r^2} + \frac{f'(r)}{f(r)} \right),$$  \hspace{1cm} (3.6)$$

where $F_1(r)$ and $F_2(r)$ are given by

$$(rg(r)+rF_2(r))F_2'(r)+\left( rg'(r)+g(r)-r\frac{g'(r)}{g(r)} \right) F_2(r)+F_2^2(r) = rg'(r)-rg(r)g'(r),$$
$$\frac{f'(r)+F_1'(r)}{f(r)+F_1(r)} = \frac{g(r)}{g(r)+F_2(r)} \left( \frac{1}{r} + \frac{f'(r)}{f(r)} \right) \frac{1}{r}.$$  \hspace{1cm} (3.7)$$

The first equation of eq. (3.7) is a second Abel equation which is resolvable in our case to obtain the expressions of $F_1(r)$ and $F_2(r)$ as

$$F_1(r) = \exp \left[ \int g(r) - \frac{2GM}{c^2 r} \left( \frac{1}{r} + \frac{f'(r)}{f(r)} \right) dr \right] - f(r),$$
$$F_2(r) = \frac{2GM}{c^2 r}.$$  \hspace{1cm} (3.8)$$

Then the black hole space-time metric in dark matter halo is given by

$$ds^2 = - \exp \left[ \int g(r) - \frac{2GM}{c^2 r} \left( \frac{1}{r} + \frac{f'(r)}{f(r)} \right) \frac{1}{r} \right] \left( g(r)-\frac{2GM}{c^2 r} \right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2).$$  \hspace{1cm} (3.9)$$
If we do not consider a dark matter halo, i.e., \( f(r) = g(r) = 1 \), the indefinite integral will become constant and we have

\[
F_1(r) + f(r) = \exp \left[ \int \frac{g(r)}{g(r) + F_2(r)} \left( \frac{1}{r} + \frac{f'(r)}{f(r)} \right) - \frac{1}{r} dr \right] = 1 - \frac{2GM}{c^2r},
\]

\[
F_2(r) + g(r) = \frac{1}{r} \int \left[ \frac{g'(r)}{g'^2(r)} + \frac{g'(r)}{g(r)} - g'(r) + \frac{g(r)}{r} \right] r dr - \frac{2GM}{c^2r} = 1 - \frac{2GM}{c^2r}. \tag{3.10}
\]

In this case, the space-time reduces to the Schwarzschild black hole space-time. Therefore the space-time eq. (3.9) describes a Schwarzschild black hole surrounded by a dark matter halo. For any given dark matter density profile, we can obtain the corresponding space-time in this way.

### 3.2 Cold dark matter (CDM)

For a CDM dark matter halo, if \( f(r) = g(r) \), we obtain \( F_1(r) = F_2(r) = -\frac{2GM}{rc^2} \), the black hole space-time metric coefficient functions are given by

\[
F_1(r) + f(r) = F_2(r) + g(r) = \left( 1 + \frac{r}{R_s} \right)^{-\frac{8G\rho_c R^3}{c^2r}} - \frac{2GM}{rc^2}. \tag{3.11}
\]

The corresponding black hole space-time is

\[
ds^2 = - \left[ \left( 1 + \frac{r}{R_s} \right)^{-\frac{8G\rho_c R^3}{c^2r}} - \frac{2GM}{rc^2} \right] dt^2 + \left[ \left( 1 + \frac{r}{R_s} \right)^{-\frac{8G\rho_c R^3}{c^2r}} - \frac{2GM}{rc^2} \right]^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{3.12}
\]

### 3.3 Scalar field dark matter (SFDM)

For a SFDM halo, if \( f(r) = g(r) \), we obtain \( F_1(r) = F_2(r) = -\frac{2GM}{rc^2} \), the black hole space-time metric coefficient functions are given by

\[
F_1(r) + f(r) = F_2(r) + g(r) = \exp \left[ -\frac{8G\rho_c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R} \right] - \frac{2GM}{rc^2}. \tag{3.13}
\]

The corresponding black hole space-time is

\[
ds^2 = - \left[ \exp \left( -\frac{8G\rho_c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R} \right) - \frac{2GM}{rc^2} \right] dt^2 + \left[ \exp \left( -\frac{8G\rho_c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R} \right) - \frac{2GM}{rc^2} \right]^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \tag{3.14}
\]

### 4 Rotational black hole metric in dark matter halo

We now generalize the spherically symmetric black hole to a rotational black hole surrounded by dark matter halo by applying the Newman-Janis method [e.g., 16–18]. Our work directly follow the procedure in [18].
In the Newman-Janis method, $\Sigma^2 = r^2 + a^2 \cos^2 \theta$. The rotational black hole space-time metric surrounded by dark matter halo is

$$ds^2 = -\left(1 - \frac{r^2 - G(r)r^2}{\Sigma^2}\right)dt^2 + \frac{\Sigma^2}{\Delta} dr^2 + \frac{2(r^2 - G(r)r^2)a \sin^2 \theta}{\Sigma^2} d\phi dt + \Sigma^2 d\theta^2 + \frac{\sin^2 \theta}{\Sigma^2}((r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta) d\phi^2,$$

where

$$\Delta = r^2 G(r) + a^2 = r^2 G(r) + \left(\frac{J}{Mc}\right)^2,$$

$$G(r) = F(r) = f(r) + F_1(r) = g(r) + F_2(r).$$

(4.2)

Here below we present the explicit expressions of the space-time metric for the dark matter models considered in this work.

Case I: CDM

$$ds^2 = -\left(1 - \frac{r^2 + 2GMr}{c^2} - r^2 \left[1 + \frac{\rho}{R_s}\right] \frac{8G\rho c R^2}{c^2 r^2} \right) dt^2 + \frac{\Sigma^2}{\Delta} dr^2 + \Sigma^2 d\theta^2 + \frac{\sin^2 \theta}{\Sigma^2} ((r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta) d\phi^2$$

$$+ \frac{2}{\Sigma^2} \left( \frac{r^2 + 2GMr - \rho}{c^2} - 2GMr \right) a \sin^2 \theta d\phi dt,$$

$$\Delta = r^2 \left[1 + \frac{\rho}{R_s}\right] \frac{8G\rho c R^2}{c^2 r^2} - 2GMr + a^2.$$

(4.3)

Case II: SFDM

$$ds^2 = -\left(1 - \frac{r^2 + 2GMr}{c^2} - r^2 \exp \left[\frac{-8\rho c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R}\right]\right) dt^2 + \frac{\Sigma^2}{\Delta} dr^2 + \Sigma^2 d\theta^2$$

$$+ \frac{2}{\Sigma^2} \left( \frac{r^2 + 2GMr}{c^2} - r^2 \exp \left[-\frac{8\rho c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R}\right]\right) a \sin^2 \theta d\phi dt + \frac{\sin^2 \theta}{\Sigma^2} ((r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta) d\phi^2,$$

$$\Delta = r^2 \exp \left[-\frac{8\rho c R^2}{\pi} \frac{\sin(\pi r/R)}{\pi r/R}\right] - 2GMr + a^2.$$

(4.4)

Eq. (4.3) and eq. (4.4) represent the space-time metrics of rotational black holes surrounded by a CDM halo and a SFDM halo, respectively. In the absence of dark matter, they reduce to the Kerr black hole space-time. In addition, we have calculated the energy-momentum tensors of eq. (4.3) and eq. (4.4) and verified that they can reduce to the case of spherical symmetry (eq. (3.12) and eq. (3.14)). Therefore, our rotational black hole metrics (eq. (4.3) and eq. (4.4)) satisfy the Einstein field equation. These results would be useful to study dark matter near Kerr black holes, especially for high spin black holes.
5 Discussion

We now discuss the behaviour of dark matter profiles and properties of black holes for the rotational black hole space-time metrics surrounded by dark matter halos that we obtained in section 4. Our dark matter halo parameters come from the Low Surface Brightness (LSB) galaxy ESO1200211 presented in [19] and [20]. We adopted, for CDM, $\rho_c = 2.45 \times 10^{-3} M_\odot/\text{pc}^3$, $R_s = 5.7 \text{kpc}$ and for SFDM, $\rho_c = 13.66 \times 10^{-3} M_\odot/\text{pc}^3$, $R = 2.92 \text{kpc}$.

5.1 The cusp phenomenon

The NFW density profile is obtained in numerical simulations of CDM and ΛCDM. When the distance $r$ from the black hole is below 1–2 kpc, this profile exhibits the “cusp” phenomenon [9]. But for the SFDM density profile, no “cusp” phenomenon appears and the density is close to be constant at small distances [20]. In this work, we investigate the dark matter density profile by taking into account the black hole.

From the Einstein field equation (eq. (3.5)), the energy density $\rho$ in a dark matter halo is given by

$$\kappa^2 \rho = \frac{1}{r^2} - G(r) \left( \frac{1}{r} \frac{G'(r)}{G(r)} + \frac{1}{r^2} \right). \tag{5.1}$$

Because the velocity of dark matter particles is much smaller than the speed of light, the energy density of dark matter is approximately the mass density. On the other hand, the density profile is usually observed at large distances from the black hole, so that the black hole spin $a$ then can be approximated as zero. Here below we show the explicit expressions for the dark matter models considered in this work.

Case I: CDM

$$\kappa^2 \rho = \frac{1}{r^2} \left( 1 - \left[ 1 + \frac{r}{R_s} \right]^{-\frac{8\pi G \rho_c R_s^3}{c^2 r}} \right) - \frac{1}{r} \left[ 1 + \frac{r}{R_s} \right]^{-\frac{8\pi G \rho_c R_s^3}{c^2 r}} \left[ \frac{8\pi G \rho_c R_s^3}{c^2 r^2} \ln \left( 1 + \frac{r}{R_s} \right) - \frac{8\pi G \rho_c R_s^3}{c^2 r(r+R_s)} \right]. \tag{5.2}$$

Case II: SFDM

$$\kappa^2 \rho = \frac{1}{r^2} \left( 1 - \exp \left( - \frac{8G\rho_c R^2 \sin(\pi r/R)}{\pi} \right) \right) - \frac{1}{r} \frac{8G\rho_c R^2}{\pi} \exp \left( - \frac{8G\rho_c R^2 \sin(\pi r/R)}{\pi r} \right) \left[ \frac{1}{r} \cos \left( \frac{\pi r}{R} \right) - \frac{R}{r^2} \sin \left( \frac{\pi r}{R} \right) \right]. \tag{5.3}$$

From eq. (5.2) and eq. (5.3), it is clear that the energy density $\rho$ reaches infinity when the distance $r$ is close to 0. Figure 1 shows the profiles of CDM and SFDM models, with and without considering the black hole. We find that when the black hole is taken into account, both models show a “cusp” and the energy density $\rho$ is significantly enhanced close to the black hole, especially for the SFDM profile. For the CDM model, the “cusp” still exists even if we do not consider the black hole, while for the SFDM model, there is no “cusp” when the black hole is neglected. This is an interesting finding for dark matter density profiles.
5.2 Black hole properties in dark matter halos

Based on the space-time metrics (eq. (4.3), (4.4)), we now qualitatively discuss how the dark matter halo changes the Kerr black hole properties, including the horizon, ergosphere and singularity.

Horizon: like Kerr space-time, the rotational black hole surrounded by a dark matter halo has two horizons, i.e., the inner horizon and the event horizon. Because dark matter does not make black hole produce a new horizon, the horizon is defined by $\Delta = 0$. Through numerical calculations, we find that the dark matter halo makes the horizon increase with a very small difference of a factor of $10^{-7}$. Comparing the two kinds of dark matter models considered in this work, we find that the change of horizon is larger for CDM and smaller for SFDM. The size of two horizons depends on the parameters of dark matter halo, i.e., the scale density $\rho_c$ and characteristic radius $R_c$ (or the scalar curvature).

Ergosphere: the ergosphere exists between the event horizon and the inner static limit surface, which is defined by $g_{tt} = 0$. Through numerical calculations, we obtain the following results. Firstly, the dark matter decreases the size of the ergosphere with a very small difference of a factor of $10^{-7}$, thus the energy extraction (Penrose Process) of the black hole will decrease accordingly. Secondly, different dark matter models have different effects on the ergosphere; the change of the ergosphere is larger for CDM and smaller for SFDM.

Singularity: through calculating the scalar curvature, we find that these black hole space-time are singular at $\Sigma^2 = r^2 + a^2 \cos^2 \theta = 0$ which represents a ring in Boyer-Lindquist coordinates. This implies that dark matter cannot change the singularity of the black hole. However, our results are based on $f(r) = g(r)$.

6 Summary

In this work, we obtain for the first time the analytical form of a stationary space-time with a black hole in a dark matter halo. We start from the pure dark matter space-time metric and then derive the one for spherically symmetric black holes surrounded by dark matter halos. Finally we generalize to rotational black holes. Qualitative analysis, using LSB galaxy data, of the behaviour of dark matter density profiles and properties of the black hole for the CDM and SFDM halos considered in this work show that both dark matter density profiles near
the black hole produce the “cusp” phenomenon, and both cause the black hole horizon to increase and the ergosphere to decrease, though the magnitude is small. On the other hand, the dark matter halo does not change the singularity of the black hole.

Our results are useful to study the interaction of black hole and dark matter halo in stationary situation. Particularly, the black hole can enhance the dark matter profile significantly at small but observable distances through the “cusp” phenomenon, which could be tested with observations of the Milky Way.

In the future work, we will try to investigate applications of our findings in astrophysics, such as TDE by black holes surrounded by dark matter, black hole shadows in dark matter halos, dynamical processes of black hole affected by dark matter, etc.

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