Vector beams generated by microlasers based on topological liquid-crystal structures

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Structured light with designable intensity, polarization and phase fields [1] is today of high relevance, with application ranging from imaging, metrology, optical trapping, ultracold atoms, classical and quantum communications and memory [2, 3, 4, 5]. Specifically, vortex [6, 7] and vector [8, 9] beams can be generated directly in the laser cavity [10, 11], however, a controllable, geometrically simple and easy to manufacture laser microcavity that generates structured light on demand, especially tailored polarization, is still an open challenge. Here we show that tunable laser vector beams can be generated from self-assembled liquid-crystal (LC) micro-structures with topological defects inside a thin Fabry-Pérot microcavity. The LC superstructure provides complex three dimensional birefringent refractive index profiles with order parameter singularities. The topology of the LC structures is transferred into the topology of the light polarization. The oriented fluorescent dye emission dipoles enable the selection of optical modes with a particular polarization, as enabled by the birefringence profile in the laser cavity. The proposed lasers have no principal limitation for realizing structured light with arbitrarily tailored intensity and polarization fields.

Structured light is mostly created from a simple Gaussian beam by passing it through a phase plate, or q-plate, or by reflecting it from a metasurface, plasmonic nanostructure, or spatial light modulator [1, 6]. LCs are especially useful in this regard, since they form fascinating self-assembled structures with artificially or naturally created defects of various topological charge. For example, vortex beams are generated by passing Gaussian light through a LC layer with a single topological defect of charge $q$, called a q-plate [12, 13] or through a nematic LC droplet which has a -1 charge topological defect in its center [14, 15]. Generation of structured light directly from a laser cavity can be advantageous.
to achieve better mode purity and higher output powers. Examples include spatial light modulators [16] or a q-plate inside a laser cavity [11] and micro-ring cavities with broken mirror symmetry [17]. There are several examples of LC microlasers which exhibit high tunability of the lasing wavelength [18, 19, 20, 21], but not much emphasis has been given to the tailoring of the shape of the output beam. In general, polarization control of the laser output has mostly been limited to cylindrical vector vortex beams, but no examples exist of laser cavities that produce arbitrary spatial polarization patterns.

In this work we demonstrate emission of laser vector beams with diverse and beyond standard intensity and polarization profiles generated from self-assembled LC structures with topological defects, confined into a Fabry-Pérot microcavity. The microcavity is made of two closely spaced dichroic mirrors with the reflective sides on the inside. The cavity is filled with LC structures with a different topology, specifically the radial nematic microdroplets with a +1 point defect, nematic disinclination line defects, torons and cholesteric fingers in chiral nematic LC, colloidal chains and micro-tori, suspended in a nematic LC. The LC is doped with a fluorescent dye, which provides optical gain when illuminated with an external nanosecond pulsed laser. Above the lasing threshold, the microcavity emits coherent light, which is sent to a camera and an imaging spectrometer (Fig. 1a and b).

We first demonstrate the emission of vector beams from a fluorescently doped radial nematic droplet suspended in an isotropic medium filling the laser microcavity (Fig. 1c and d). The spherical shape of the droplet with a higher refractive index compared to its surroundings makes the Fabry-Pérot microcavity stable. Such a cavity configuration is advantageous because the beam is always perfectly self-centered with respect to the structure of droplet. Above the lasing threshold of typically 40 nJ–100 nJ (Supplementary Section 1), laser light emission is observed as a red bright ring (Fig. 1e). By spectrally dispersing the light, we see that the emission is actually composed of several transverse laser optical modes, which repeat as different longitudinal orders (Fig. 1f). The observed modes match well with vector Bessel-Gauss (BG) modes [23] with only one polarization component. BG modes are characterized by the mode number \( l \) where the polarization axis performs \( 2l \) half turns around the center of the beam, a continuous factor \( \beta \), which determines the number of rings in the radial direction and a waist size \( w_0 \). It is worth noting that these beams are not vortex beams and do not carry orbital angular momentum. The radial intensity profile of lower order BG modes (\( \beta w_0 < 1 \)) observed in the experiments is also very similar to profiles of the well known Laguerre-Gauss beams (Supplementary Section 2). Very small droplets below \( \sim 5 \mu m \) emit lowest order modes including simple Gaussian beam. Larger droplets emit petal-like modes with higher mode numbers (Fig. 1k and Supplementary Section 3). The BG mode number \( l \) can be selected by changing the droplet size and cavity length. Modes are not always completely rotationally symmetric possibly due to a slight droplet deformation (Supplementary Section 4).

Numerical simulations were performed to calculate standing lasing modes inside the microcavity containing dielectric media with an arbitrarily spatially varying permittivity tensor by using a Finite-Difference Frequency-Domain (FDFD) approach (Supplementary Section 5). The calculated lasing profiles for highest Q-factor modes in a radial nematic droplet are presented in Fig. 2 showing various modes with similar standing-wave type profiles in the longitudinal direction (Fig. 2a), but with very distinct cross-section profiles (Fig. 2b-i). The modes can be grouped into two families, depending on their general radial (Fig. 2a-e) or azimuthal (Fig. 2f-i) polarization and can be explained as vector Bessel-
Figure 1: Scheme of optical setup and generation of laser vector beams. (a) Optical setup consists of a 532 nm pulsed laser for optical pumping, infrared 1064 nm laser tweezers for manipulating the LC structures, an imaging spectrometer, a camera and an objective. The sample is composed of a LC structure, which is sandwiched between two dichroic mirrors, forming a Fabry-Pérot microcavity. (b) To analyze the optical modes, the LC laser output is sent through an imaging spectrometer with the slit wide-open. The image of the lasing emission pattern is spread in wavelength and imaged on a CCD sensor. The shape of each lasing mode is retained when passing through the spectrometer and the center position of each mode on the sensor gives its wavelength. (c) Bright-field image of a radial LC droplet in the cavity. The droplet has a +1 topological point defect in the center, where the orientation of LC molecules is ill defined. A topological defect is characterized in 2D by a winding number (also called strength) \[24\], whereas in 3D it is characterized by a topological charge. The charge in 3D is measured by the surface integral of the "topological flux", which is similar to the Gaussian theorem for the flux of the electric field \[22\]. (d) The same droplet is viewed under crossed polarizers. (e) The droplet emits light when optically pumped above the threshold for lasing. (f) Spectral decomposition of the laser modes emitted from a 6.2 \(\mu\)m diameter radial nematic droplet in a 25 \(\mu\)m cavity. Three transversal modes are supported: \(BG_{2}^{\text{radial}}\), \(BG_{0}^{\text{radial}}\) and \(BG_{1}^{\text{linear}}\), which repeat as different longitudinal modes. (g) A single isolated mode, which is emitted from the droplet depicted in (c-e) with a diameter of 21 \(\mu\)m in a 25 \(\mu\)m cavity and is identified as \(BG_{8}^{\text{radial}}\). Gauss beams with an emphasized either radial or azimuthal component of the electric field (Supplementary Section 6). Each family consists of modes with different numbers of intensity maxima in the azimuthal direction, directly corresponding to the Bessel-Gauss beam mode number. In an isotropic droplet all modes have a continuous intensity ring.
Figure 2: Numerically calculated resonant eigenmodes of a laser cavity containing a radial nematic droplet. (a) The droplet is placed in the center of the microcavity, formed by two parallel, lossless and flat mirrors and assumed to be filled with an isotropic dielectric with refractive index lower than the refractive indices of the LC. 3D representation of the electric field intensity of BG$_8$ radial mode is plotted. Colors represent iso-surfaces of the normalized electric field intensity. (b-i) Electric field vectors (white arrows) and normalized electric field intensity (color map) in the plane of the electric field intensity maximum, closest to the upper mirror. Four eigenmodes with the highest Q-factors are shown for (b-e) radial polarization, and (f-i) azimuthal polarization.

Here however, due to the birefringence of the LC, the modes decouple to radial and azimuthal families.

In order to select the desired polarization in experiments we need to provide more gain to that polarization. The use of LCs is advantageous, since the emission dipole of the dissolved dye can be oriented by the LC providing more efficient gain for that particular mode. Depending on the particular dye the dipole can be aligned parallel or perpendicular to the optical axis (i.e. the director) or can even be unoriented. In LCs very complex three-dimensional orientation profiles of the gain dipoles can be achieved solely by self-assembly, which would be very difficult or even impossible to achieve in solid state lasers. With the dye oriented along the director (i.e. radial direction), the radial component of the vector beam is generated with a larger amplitude than the azimuthal one (Fig. 3a(i-iii)). Conversely, another dye with perpendicular orientation to the director (i.e. azimuthal direction), generates an azimuthal polarization (Fig. 3b(i-iii)). If the dye is randomly oriented, both polarizations are generated (Fig. 3c(i-iii)). In the latter case typically the lower order modes will be azimuthally polarized, while higher orders will have a radial polarization.

To further control the polarization of the modes, more complex self-assembled LC structures are employed. We first consider lasing from torons (25) (Fig. 3d). Torons are topologically protected LC structures, which are spontaneously formed in a layer of a cholesteric liquid crystal (CLC), with a thickness comparable to the helical pitch of the CLC. Similarly to larger nematic droplets, torons emit petal-like higher order BG modes (Fig. 3e). The polarization of the modes is not completely radial but slightly angled at $\sim 10^\circ$ (Fig. 3f-h). These modes can be described as vector modes with a more general polarization direction, which is a linear combination of both radial and azimuthal
Figure 3: Experimentally demonstrated control of the vector beam polarization. (a-c) Vector modes generated from three radial nematic droplets, with three different orientations of the emission dipoles of the dye molecules: (a) along the nematic director; (b) perpendicular to the nematic director and (c) the emission dipoles are randomly oriented. (i) The intensity of resonant vector modes in all three droplets, observed without polarizers; (ii) with a vertical polarizer and (iii) a horizontal polarizer. (d) A bright field image of several torons in a 15 µm microcavity. The central one is illuminated and pumped by an external laser and starts emitting laser light, as seen from the bright red ring. The inset shows the schematics of the director in a cross-section of a toron. Torons have a triple-twisted structure, which is smoothly embedded in the homogeneous far-field director structure of a CLC. The droplet-like shape consists of a torus-like, triple-twisted chiral nematic structure, which has a topological charge of +2. For topological charge neutrality, two hyperbolic defects of charge -1 are present at the top and bottom of the torus, facing the flat interface, denoted by black dots. Because the thickness of the cavity is smaller than the pitch of the CLC, the CLC director profile is unwound almost everywhere except in places, where torons spontaneously appear. (e) Spectral decomposition of the vortex modes emitted from a toron in a microcavity. (f) Two selected modes emitted from a toron with the polarization direction indicated. (g) Same two modes with a vertical polarizer and, (h) a horizontal polarizer. The dashed lines are aligned with the areas of peak intensity.
polarization, therefore termed hybrid modes (Supplementary Section 7). This is a consequence of the complex triple-twisted structure of a toron, with the dipole moment of the fluorescent dye following this twisted structure.

To explore whether a complex director configuration can be designed to yield -in principle- a vector beam with an arbitrary polarization profile we have studied various other LC structures, focusing on different topologies of the nematic material. A nematic defect with a winding number of \( \pm 1/2 \) (Fig. 4a) was assumed to be in a cylindrical region, within the laser cavity. The corresponding laser modes (for details see Supplementary Section 7), possess intensity profiles of BG modes with an odd number of maxima and a polarization profile aligned with the direction of the underlying nematic director instead of a radial or azimuthal direction (Fig. 4b), which shows that the topology of the nematic birefringent matter is directly imprinted in the topology of the electromagnetic field polarization (i.e. the mode). This can also be clearly observed in the simple case of a homogeneous director orientation which yields a linear polarization (Fig. 4c). Similarly, a \(-1\) defect generates a hyperbolically shaped polarization profile (Fig. 4d). Multiple \(+1\) and \(-1\) defects arranged into a square mesh have been also studied. In this case multiple intensity zeros are created within the beam and the direction of the polarization twists around these singularities (Fig. 4e).

By selecting the shape of the surrounding high index region which focuses the light we
can generate not only circular beam intensity profiles, but also other shapes. In order to generate Hermite-Gaussian (HG) modes cholesteric (chiral) fingers are used, which form when a toron is spontaneously elongated into a finger-like structure (Fig. 5a). Due to their elongated shape these chiral fingers break the cylindrical symmetry of a toron and now emit HG beams (Fig. 5b).

Even further, both intensity and polarization can be generated in very complex profiles, regular and irregular. In order to show an example of complex a mode, a random distribution of $+1$ and $-1$ defects with a total topological charge 0 was constructed (Fig. 5c). A laser mode with a complex intensity and polarization profile is generated in such a structure, which is conditioned by the nematic birefringent profile (Fig. 5d). To corroborate the numerical simulations, we have experimentally fabricated a LC cell with a random anchoring direction on one surface so that also the LC is randomly oriented. The polarization of the beam generated by such a structure was reconstructed by capturing multiple polarization images [27], showing complex spatial profile (Fig. 5e).

Finally, we demonstrated the ability of our lasers to switch between different transversal modes. A box structure which supports a number of modes was used for this purpose. The structure was assembled inside a microcavity from 5 µm silica colloidal particles using laser tweezers (Fig. 5f). These silica colloids have lower refractive index ($n = 1.43$) compared to both refractive indices of 5CB ($n_o = 1.54$ and $n_e = 1.71$) therefore they de-focus the light, which makes that part of the laser cavity unstable. Any laser mode that spatially overlaps with the silica colloids therefore has a very high loss and will not lase (Supplementary Section 8). The colloidal box structure with a diagonal of 50 µm allows for a number of different modes (Fig. 5g). By illuminating different parts of the interior of the box, various modes were selected. Similarly also single HG modes were selected in cholesteric fingers (Supplementary Section 9).

One advantage of LCs is the ability to manipulate the director by external stimuli and in that way also control the output of the laser. Torus shaped particles with homeotropic anchoring inside a fluorescently labelled LC were employed for this purpose. Such particles are interesting due to their topology and the fact that they have several possible configurations of the surrounding nematic director [28]. Most frequently, a topological configuration with a topological defect of winding $+1$ is created in the center of the torus (Fig. 5h). After excitation, lasing was observed from the central part of the torus in the form of radially polarized BG modes (Fig. 5i) due to the circular symmetry. By using the optical force of the laser tweezers, a $+1$ topological defect was grabbed and displaced from the center of the torus to the periphery (Fig. 5j). In this case, with the defect near the edge, the symmetry of the cavity is broken and the microcavity starts emitting HG modes (Fig. 5k).

In conclusion, we have demonstrated that micro-lasers based on topological soft matter confined to a microcavity pave a new way towards compact and tunable microlaser sources of laser vector beams. We conjecture that it is in principle possible to create arbitrary laser beams by carefully designing appropriate 3D LC structures using reverse-engineering. The softness and fluidity of LCs as well as their adaptability to inserted objects enables the creation of complex 3D birefringent superstructures, which could be very difficult or even impossible to engineer by standard fabrication techniques such as solid-state lithography. And in turn, these complex three dimensional birefringent refractive index profiles allow for the creation of a large variety of modes with different polarization and intensity maps. The self assembly properties of topological soft matter in combination with their large susceptibility to external fields might open new pathways to novel soft
Figure 5: Various shapes of the generated laser beams and their real time control. (a) Chiral fingers and torons under crossed polarizers. (b) A typical spectral decomposition of the laser output generated by a chiral finger, consisting of $HG_{2,1}$, $HG_{1,1}$ and $HG_{4,0}$ modes. (c) Simulated intensity and polarization maps of a lasing mode from (d) a randomly oriented LC structure. (e) Intensity and reconstructed polarization direction of experimentally realized lasing in a randomly oriented LC structure. Crossed polarizer image of the same area (red box) is shown in the inset. (f) A box structure assembled from 5 µm silica colloids in a 10 µm cavity filled with LC. When micro-spheres with perpendicular anchoring of LC molecules on the surface are inserted into a NLC, point or loop defects are created. These topological defects generate structural forces of elastic origin between two particles, which enables the controlled assembly of micro-spheres into various structures [22, 26]. (g) Various modes generated from such a structure when the excitation spot is moved. (h) A toroidal colloid 50 µm in diameter in a 10 µm thick cavity with a point defect approximately in the center. (i) Such a configuration emits BG$_{radial}^{12}$ modes. The red cross shows the position of the defect. (j) The point defect is moved to one side using laser tweezers. (k) The lasing from the LC in this new geometry with a defect displaced from the center of the torus changes to linearly polarized HG modes.
matter photonic devices. Finally, topological properties of LCs, which were explored in great depth during the last decade, could open the door to an entirely new class of microlasers, based on topological soft matter.

Methods

Sample preparation. For the laser cavity planar dielectric mirrors with good transmission (>80%) at the pump wavelength (532 nm) and high reflection (∼99.9%, OD3) in the range of maximum dye emission (550 nm–650 nm) were used. The mirror spacing was controlled by using sheet spacers, typically ranging from 5 to 40 µm. In order to achieve planar or homeotropic anchoring, a 0.1% solution of PVA (polyvinyl alcohol, Sigma) in water, or 1.5% egg yolk lecithin (L-α-phosphatidylecholine, Sigma) in diethyl ether were spin coated onto the surface, respectively. For non-degenerate planar anchoring the PVA coated substrates were also rubbed in a specific direction using a velvet cloth. In most experiments 5CB LC (4-cyano-4'-pentylbiphenyl, TCI), doped with 0.1% Nile red (7-diethylenamino-3,4-benzophenoxazine-2-one, Sigma) or 1% Pyromethene 580 (1,3,5,7,8-pentamethyl-2,6-di-n-butylpyrromethene-difluoroborate complex, Exciton) was used. For random dye dipole orientation, 0.1% Rhodamine B (Sigma) was used. For perpendicular orientation 0.1% DiOC18(3) (3,3'-dioctadeylxocarboxcyanine perchlorate) in 8CB (4'-octyl-4-biphenylcarbonitile) was used. In order to achieve sufficient levels of absorption of the infrared light from the laser tweezer, D77 dye (5-[1,2,3,3a,4,8b-hexahydro-4-(4-methoxyphenyl)cyclopeanta[b]indole-7-ylmethylene]-4-oxo-2-thioxo-thiazolidin-3-ylacetic acid) in concentrations of up to 0.07% was added. Cholesteric nematic LC mixtures were obtained by adding a right-handed chiral dopant CB15 ((S)-4-cyano-4'-(2-methylbutyl)biphenyl) (∼0.6%) to achieve the desired pitch (p). Different chiral structures were observed depending on the ratio p/d, where d is the mirror spacing. Torons formed in homeotropic cells when d/p ≈ 0.7 and chiral fingers were formed at slightly larger cell thickness. Radially oriented nematic LC droplets were created by suspending dye doped 5CB and desired dyes in a 1% solution of egg lecithin in glycerol. Dispersions of colloids in the nematic LC were prepared from 10 µm–15 µm BaTiO3, 10 µm borosilicate or 5 µm silica colloids. Homeotropic anchoring on colloids was achieved by treating the particle with N,N-dimethyl-N-octadecyl-3-aminopropyl trimethoxysilyl chloride (DMOAP) silane. The torus-shaped colloids were produced by using 3D two-photon direct laser writing technique (Nanoscribe Photoconic Professional) and treated for homeotropic anchoring as described elsewhere [29]. The tori had an outer diameter of 50 µm and a thickness of 5 µm.

Optical setup. In order to observe, excite and manipulate the samples, an optical setup built around an inverted microscope was used. A Q-switched doubled Nd:YAG laser with a wavelength of 532 nm, pulse length of 1 ns, maximum pulse energy of 10 µJ (Pulselas-A-1064-500, Alpha-las) operated at repetition rates ranging from 3 to 60 Hz was employed for excitation. A dichroic beamsplitter with high reflection below 550 nm and high transmission above 550 nm was inserted into the microscope filter turret. The laser was focused by a 60 × 1.0 NA or 20 × 0.50 NA objective. By placing a Galilean beam expander and additional lenses into the optical path before the dichroic beamsplitter, the laser beam was decollimated enabling us to vary the spot size on the sample from a sub-micron diffraction limited spot up to several hundred micrometers in diameter. The light emitted by the sample was collected by the same objective and sent to either a camera or a spectrometer. The imaging spectrometer (Andor Shamrock SR-500i), with a spectral
resolution of up to 0.05 nm was used. After the input had been separated by wavelength, it was recorded by a cooled back illuminated EM-CCD camera (Andor, Newton DU970N). Optionally the laser light from the optical tweezers (1064 nm) was focused onto the sample to a diffraction limited spot by using a second dichroic beamsplitter. The beam was steered with acousto-optic deflectors and controlled via computer.

**Numerical simulations** The Finite-Difference Frequency-Domain (FDFD) method was used in numerical simulations [30]. Maxwell curl equations were written in the frequency domain and a matrix form of the equations was used. The eigenproblem for the magnetic field was formulated as:

$$\mu^{-1} \nabla \times \varepsilon^{-1} \nabla \times \mathbf{H} = \omega^2 \mathbf{H}$$

(1)

The eigenvalue represents the frequency \(\omega\) of the optical mode. The electric field \(\mathbf{E}\) in every point of the cavity was calculated from the (nodal) eigenvector \(\mathbf{H}\). Information about the LC configuration between the mirrors of the FP microcavity was included in the electric permittivity matrix \(\varepsilon\). Numerical results presented in this article were calculated by custom written code in the MATLAB R2019a environment and ran on Intel Xeon nodes with 190GB RAM. Perfectly matched layers (PML) were used to truncate the domain and simulate infinite boundary conditions in transverse directions. Perfect electric conductor boundary conditions were used on top and bottom boundaries to reproduce the effects of perfect mirrors. Quality factors were calculated directly from the real and imaginary part of \(\omega\) and served as a tool to extract the localized modes and classify them. For more see Supplementary Section 5 on Simulations.

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**Author Contributions**

M.H. designed the study. M.P. performed experiments and analyzed the experimental data. U.M. performed the numerical simulations. M.R., I.M. and M.H. supervised the project. All authors interpreted the results and prepared the manuscript.

**Competing Interests**

The authors declare no competing interests.
Supplementary Information

1 Laser threshold

The output intensity from a liquid crystal (LC) filled Fabry-Pérot cavity 12 µm in thickness was measured as the pump pulse energy was increased up to 150 nJ (Supplementary Fig. 1). The excitation beam was tightly focused to a diffraction limited spot. Below the lasing threshold no fluorescent light is observed due to the high reflection of the dichroic mirrors. After the threshold the output starts to increase rapidly.

![Graph showing the input-output characteristics of a droplet laser.](image)

Supplementary Figure 1: Input-output characteristics of a droplet laser.
2 Comparison between Laguerre-Gauss (LG) and Bessel-Gaussian (BG) representation

Such beams can be described by using several different approaches, either as a linear combination of circularly polarized scalar Laguerre-Gauss modes carrying the opposite orbital angular momentum, also known as Poincare beams [8], or as solutions of vector Helmholtz equation written in the Laguerre-Gauss basis (cylindrical or vector Laguerre-Gauss beams) [31], Bessel-Gauss basis (vector Bessel-Gauss beams) [23] or even as a product of both (vector Laguerre-Bessel-Gauss beams) [32]. All approaches lead to a family of solutions - vector beams - with mode numbers $l$ where the polarization axis performs $2l$ half turns around the center of the beam.

Both experimentally measured and simulated modes in this work can be best described in terms of BG vector modes. BG modes are characterized by the number of azimuthal maxima by the Bessel index $l$. The other parameter governing the shape of a BG beam is the product of the Gaussian waist size $w_0$ and the constant within the Bessel function $\beta$, which determines the number of oscillations visible within the Gaussian envelope in the radial direction.

Vector beams which we observe in the experiments can also be formulated in a Laguerre-Gauss (LG) basis as a coaxial superposition of LG beams with orthogonal circular polarization states [11]. Namely, a vector beam where the polarization axis performs $2l$ half turns around the center of the beam and can be expressed as

$$E_{p,\pm l} = LG_{p,l}e_R + LG_{p,-l}e_L$$

where $LG_{p,l}$ includes the phase and amplitude profile of a Laguerre-Gaussian beam and $e_{R,L}$ are right and left-handed circular polarization states [33].

This description in terms of LG or BG modes is verified by looking at the beam radial profiles in Fig. 3a. Both LG and BG solutions perfectly describe the experimental intensity profile (Supplementary Fig. 2).

Supplementary Figure 2: Radial intensity profile of A mode fitted with a Laguerre-Gaussian (red) and Bessel-Gaussian (green) intensity distribution. The mode can be identified as LG$_{0,1}$. 

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3 Quantification of mode numbers for droplets of different sizes

Droplets larger than approximately 5 \( \mu m \) emit BG modes with higher \( l \) mode numbers, which are visible as petal-like intensity profiles (Supplementary Fig. 3a). These modes are also radially (Supplementary Fig. 3b and c) or azimuthally polarized.

![Supplementary Figure 3](image)

Supplementary Figure 3: (a) Laser modes from a droplet with the diameter of 20 \( \mu m \) in diameter in a 25 \( \mu m \) cavity. (b) One of the modes with an analyzer in the horizontal and (c) vertical direction.

In order to quantify the relationship between a droplet’s size and its modes, we analyzed the spectra of 50 droplets in two cavities with two different thicknesses of 30 and 50 \( \mu m \). In the case of the 30 \( \mu m \) thick cavity only droplets larger than 13 \( \mu m \) exhibited lasing since droplets smaller than this limit focus the light more than necessary, resulting in an unstable resonator. The limit in the 50 \( \mu m \) cavity is 17 \( \mu m \). In this cavity the mode indexes of droplets larger than approximately 35 \( \mu m \) are too high to determine accurately. Since one droplet can generate modes with several different Bessel indices \( l \), the mean mode number of each droplet was calculated by averaging \( l \) over a set of transverse modes and the range of \( l \) is represented by the error bars (Supplementary Fig. 4).

It is clearly evident that larger droplets produce higher order modes. Smaller droplets cannot produce arbitrarily high modes since, at a certain point, a sizable part of the transverse intensity profile exceeds the droplet’s radius. The droplet therefore cannot focus the entire mode, resulting in mode-dependent diffraction loss towards the droplet edge.

It is also evident that larger droplets produce a wider array of modes. This can be explained by the same mechanism as before. The effective beam diameter of higher order modes is approximately proportional to \( l^{0.6} \), so higher order modes have more similar effective radii than lower orders. We must also consider the effect of spherical aberration since we are dealing with waist sizes comparable to a droplet’s radius. In the case of larger droplets in thicker cavities, modes with the product \( \beta w_0 \ll 1 \) and Bessel indices which can be as large as \( l = 20 \), are prevalent. This is a direct consequence of the spherical aberration of the beam inside the cavity. Namely, the modes with higher \( \beta w_0 \) have the intensity profile spanning from the center to the edge. However, due to spherical
aberration the light at the center is not focused to the same focal distance as the light at the edge. Therefore, the droplet can form a stable resonator only for modes with a localized intensity profile in the radial direction, that is in the shape of a ring, so with small $\beta w_0$.

Since the topological defect in the center of a droplet scatters light, the mode $BG_1$, with an intensity maximum at the center, rarely appears in larger cavities.

Supplementary Figure 4: Average Bessel mode index $l$ versus the droplet radius in two different cells with thicknesses of 30 and 50 $\mu$m. The error bars represent the standard deviation of the average mode number $l$ and the shaded areas correspond to a $\sigma$ confidence interval of the fit.
4 Laser modes of a droplet deformed into a spheroid

The effects of a small deformation of a droplet into a prolate/oblate spheroid with the same volume and a radial director profile on its optical modes were analyzed numerically. A deformed droplet was approximated by a spheroid with the length of the third axis $R_3 = R(1 + D)$ and the lengths of perpendicular axes $R_{1,2} = R(1 - D/2)$, where $D$ is the deformation. Results for two selected modes of a nondeformed droplet are shown in Supplementary Fig. 5. When the third axis lays in the plane parallel to the mirrors (XY plane), the symmetry is broken by a deformation, which is reflected in the intensity patterns of the modes (Supplementary Fig. 5a). Conversely, when the third axis is perpendicular to the mirrors (z-direction), no symmetries are broken and the intensity patterns, except for a slight change in their outer radii, remain visually unchanged (Supplementary Fig. 5b).

In experiments the deformation of the droplet parallel to the mirrors was measured from the microscopy image, yielding $D = 0.025$. The corresponding experimental intensity profile (Fig. 3a) quantitatively matches the simulations at a similar deformation.

Supplementary Figure 5: Two selected modes of a spheroid shaped droplet with the third axis (a) parallel and (b) perpendicular to the mirrors. Negative sign of the deformation represents an oblate and positive sign a prolate spheroid. The colors represent the normalized electric field intensity in the XY plane (parallel to the mirrors).
5 Simulations

The numerical simulations were performed with a custom written code in Matlab R2019a and ran on Intel Xeon nodes with 190GB RAM. A fixed number of eigenmodes around the desired wavelength $\lambda$ was calculated in each run, where $\lambda$ in the material was kept large enough to obey the standard rule of thumb determining the grid spacing of at least $\lambda/10$. Perfectly matched layers (PML) with the thickness larger than $\lambda/2$ were used to truncate the domain and simulate infinite boundary conditions in transverse directions. Perfect electric conductor (PEC) boundary conditions were used as top and bottom boundaries to reproduce the effects of perfect mirrors.

In the analysis of a droplet, only one octant with a size of $55 \times 55 \times 50$ pixels was simulated due to the symmetry of the lasing cavity. Three different types of octants with PEC boundary condition on the surfaces in the vertical direction were used. Two of the lateral boundary surfaces included PML boundary conditions, representing the outer boundary of the lasing cavity. Other two lateral boundary conditions were either both set to PEC/PMC or one to PEC and the other to PMC symmetry boundary condition. Such a selection of boundary conditions covers all possible symmetries of eigenmodes in the lateral direction.

The director field of the liquid crystal inside the cavity was determined by an analytical formula. In the case of a radial droplet, a field with a $+1$ defect was used. The radius of the droplet was set to 40 pixels, which is 80% of the Fabry-Perot layer thickness. The refractive indices of the liquid crystal and the surrounding isotropic material were set to the same values as used in experiments, namely $n_o = 1.54$, $n_e = 1.71$ and $n_{out} = 1.47$.

When simulating quasi 2D defects presented in Fig. 4 of the main text no symmetry boundary conditions were used. A computational domain with a size of $80 \times 80 \times 30$ pixels was entirely filled with a liquid crystal containing the desired topological defects. Identical refractive indices as in the previous case were used.
6 Bessel-Gauss beams

Bessel-Gauss (BG) beams are solutions of the paraxial vector wave equation. Following [23] they can be expressed as:

\[ \mathbf{E}(r, \phi, z, t) = \mathbf{F}(r, \phi, z) \exp(i(kz - \omega t)) \]  
\[ \mathbf{F}(r, \phi, z) = A(r, z) \left( F_r(\phi)e_r + F_\phi(\phi)e_\phi \right) \]  
\[ F_r(\phi) = (a_lJ_{l-1}(u) + b_lJ_{l+1}(u)) \cos(l\phi + \phi_0) \]  
\[ F_\phi(\phi) = (a_lJ_{l-1}(u) - b_lJ_{l+1}(u)) \sin(l\phi + \phi_0) \]  
\[ u = \frac{\beta r}{1 + iz/z_0} \]  
\[ A(r, z) = \frac{1}{1 + iz/z_0} \exp\left(\frac{-r^2/w_0^2}{1 + iz/z_0}\right) \exp\left(\frac{-\beta^2 z/(2k)}{1 + iz/z_0}\right) \]  

where \( J_l \) is the \( l \)-th order Bessel function of the first kind and \( a_l, b_l \) and \( \phi_0 \) are free parameters. \( k = 2\pi/\lambda \) is the wavevector, \( z_0 \) and \( w_0 \) are the Rayleigh range and beam waist of the elementary Gaussian beam, which is the solution at \( \beta = 0 \) and \( l = 1 \). The ratio between \( w_0 \) and \( 1/\beta \) determines the number of visible rings. If \( \beta w_0 < 1 \) only the first maximum of the \( J_{l-1} \) Bessel function in the radial direction is visible and the outer maxima are suppressed by the Gaussian envelope. If \( \beta w_0 > 1 \) multiple rings are visible [35]. Selected beam profiles for \( l = 0, 1, 2, 3 \) and \( \beta w_0 = 1 \) and \( \beta w_0 = 7 \) are shown in Supplementary Fig. 6a,b(i).

Modes of Bessel-Gauss form emerge from a FP resonator containing an optically isotropic spherical droplet or particle [24]. However, if the droplet is birefringent - in our case with a radial profile of the optical axis/director - the optical modes effectively separate into radial (Supplementary Fig. 6a(ii-iv)) and axial (Supplementary Fig. 6b(ii-iv)) contributions. The obtained modes can be characterized by considering a linear combination of components \( \gamma F_r e_r + \delta F_\phi e_\phi \) where \( \gamma \neq \delta \) (results for \( \gamma = 1 \) and \( \delta = 0.5 \) and vice-versa are shown in Supplementary Fig. 6a,b(ii) respectively). By comparing analytical and simulated phase profiles the value of \( \beta w_0 \) can be determined. Simulated radial modes can be described by selecting \( \beta w_0 = 1 \) and axial by selecting \( \beta w_0 = 7 \).

In actual resonators with soft birefringent optical materials, indicated BG modes with \( a_l \neq b_l \) are emitted (Supplementary Fig. 6a,b(iii,iv)). In the modes obtained from the numerical simulations the position of the outer intensity maximum coincides well with the radius of the droplet, as seen in Supplementary Fig. 6a(iv), meaning that \( \beta w_0 \) and/or \( w_0 \) vary with \( l \).
Supplementary Figure 6: Comparison of normalized electric field intensities (color code) and polarizations (white arrows) of Bessel-Gauss modes, their components and simulation results. (a) (i) Bessel-Gauss modes with different values of $l$ and $\beta w_0 = 1$. (ii) Bessel-Gauss modes with the emphasized radial component. $l = 0$ mode with radial polarization is obtained by taking $\phi_0 = \pi/2$. (iii) Linear combination of radial and axial components with $a_l \neq b_l$ that is the most comparable with the numerical results in (iv). $b_l$ is set to 1 for every $l$ and $a_1 = 1$, $a_2 = 10^{-3}$, $a_3 = a_4 = 0$. (b) (i) Bessel-Gauss modes with different values of $l$ and $\beta w_0 = 7$. (ii) Bessel-Gauss modes with the emphasized axial component. (iii) Linear combination of radial and axial components with $a_l \neq b_l$ that is the most comparable with the numerical results in (iv). $b_l = 1$ and $a_1 = 0.4$ for every $l$. 
7 Laser modes of 2D topological defects

In order to numerically study the effects of a different topology we assumed that the Fabry-Perot cavity contains topological defect lines, spanning from one cavity mirror to the other. Inside the certain radius around the core of the defect (here 12 pixels) the optical axis (nematic director) was located in the plane parallel to the mirrors. Outside that radius the director was gradually rotated in the direction normal to the mirrors (Supplementary Fig. 7b,c). Such a setup ensured the localization of the modes around the defect. The same can be achieved by using a surrounding isotropic material with a refractive index lower than the indices of the liquid crystal, as done in the case of the radial nematic droplet. It is noteworthy that the director manipulation as described here, will only localize modes polarized in the direction of the director field, as the perpendicular polarization sees no refractive index contrast.

Results are shown in Supplementary Fig. 7. The $+1$ defect shows similar modes as in the case of a radial droplet (Supplementary Fig. 7a(i)). Similar intensity patterns were found when the $-1$ defect was used, however with different polarizations imprinting the director field of the structure (Supplementary Fig. 7a(ii)). In the case of a half-integer $+1/2$ defect, the intensity patterns with an odd number of intensity maxima emerged, but once again with a polarization resembling the underlying director field (Supplementary Fig. 7a(iii,iv)). The modes emergent around topological nematic defects can be effectively

Supplementary Figure 7: Normalized intensity (color code) and polarization (white arrows) of optical modes of defect lines with different winding numbers. (a) Simulated modes for a (i) $+1$, (ii) $-1$ and (iii,iv) $+1/2$ winding number and analytically calculated products of BG modes with a $P_{1/2}$ polarizer for (v) $\phi_0 = 0$ and (vi) $\phi_0 = \pi/2$. The value of $\beta w_0$ was set to 7 for $l = 1$ and $l = 2$ and kept at 1 for $l = 3$ and $l = 4$ analytical modes to match the numerical results. (b,c) XY (b) and XZ (c) plane cross section of a director field for a defect line with a $+1/2$ winding number.
described by a product of a Bessel-Gauss beam $E_{BG}$, defined in Eq.3 and a polarizer with the optical axis in the direction of the director field around the defect $P$ as:

$$E_{pol} = (E_{BG} \cdot P)P$$

(9)

Calculated results for a $+1/2$ defect are shown in Supplementary Fig.7a(v,iv), where the $+1/2$ polarizer field is written as $P_{1/2}(r,\phi) = \cos(-\frac{1}{2}\phi)e_r + \sin(-\frac{1}{2}\phi)e_{\phi}$.

In a similar manner the modes which emerge from a toron and are shown in Fig. 3d-h of the main text can be explained with the use of a polarizer $P_{toron}(r,\phi) = \cos(\Phi_0)e_r + \sin(\Phi_0)e_{\phi}$. Calculated results for $l = 7, l = 11$ and $\Phi_0 = 20^\circ$ are shown in Supplementary Fig. 8.

Supplementary Figure 8: Comparison of analytically calculated and experimental modes of a toron. (a,b) Normalized intensity (color code) and polarization (white arrows) of analytically calculated products of (a) $l = 7$ and (b) $l = 11$ BG mode with a polarizer $P_{toron}$ with $\Phi_0 = 20^\circ$. (c) Experimentally observed laser modes from a toron.
8 Lasing in nematic colloids

A dispersion of colloids in a dye doped nematic liquid crystal was introduced into the cavity with planar anchoring. The colloids had homeotropic anchoring. Two types of colloids were used, high refractive index colloids (BaTiO$_3$) and low refractive index colloids (borosilicate glass or silica). The BaTiO$_3$ has a significantly higher refractive index ($n = 1.95$) than the two refractive indices of 5CB ($n_o = 1.54$ and $n_e = 1.71$). In this case, the colloids focused the light, thus making a stable laser cavity. Even though there was no dye inside the colloids, the dye doped liquid crystal above and below the colloid provided enough gain to enable lasing. The generated mode was always the fundamental Gaussian mode (Supplementary Fig. 9).

On the other hand, borosilicate glass ($n = 1.52$) and silica colloids ($n = 1.43$) have a lower refractive index than both refractive indices of 5CB. Therefore these colloids act as a lens with a negative focal length and defocus the light, which makes the laser cavity unstable. Any laser mode that spatially overlaps with the silica colloids will have a very high loss and consequently will not lase. Therefore, if an area of the cavity containing the colloids is illuminated, the lasing will only appear where there are no colloids (Supplementary Fig. 9b). Although an empty cavity is not stable, the lasing still appears at high enough pump energies. The lasing threshold of an empty cavity (100 - 150 nJ) is typically approximately two times larger compared to the case where a lensing element makes the cavity stable. In the empty spaces between the colloids, such as gaps, well defined transverse modes start to appear the size of which corresponds to the gap width (Supplementary Fig. 9c). If the empty space is very small, only the fundamental Gaussian mode will appear (Supplementary Fig. 9d).

Supplementary Figure 9: (a) High refractive index colloids in a planar LC cell. The illumination area is shown by the circle and the laser output is visible in red. A mode close to a fundamental Gaussian is generated. The lasing colloid is 12 µm in diameter inside a 20 µm cavity. (b) Low refractive index 10 µm borosilicate beads in a similar 15 µm cell. The laser output is generated only where there are no colloids. (c) In some areas more well-defined modes start to appear. (d) With high confinement a simple mode with only one central maximum is generated.
9 Selection of modes in chiral fingers

Although the chiral fingers are very closely related to torons, a very different set of modes was recorded in this case due to their elongated shape. All the generated laser modes are Hermite-Gaussian, but they also exhibit certain irregularities in the direction parallel to the elongated shape of the finger. Namely, they are stretched out in the parallel direction, as the chiral structure acts as a waveguide. The described irregularities differ between fingers.

In the case of a tightly focused excitation beam, it is possible to switch between different HG modes, Supplementary Fig. 10. Here we demonstrate how to switch from the most basic $\text{HG}_{0,0}$ up to $\text{HG}_{3,4}$ simply by moving the excitation laser. Generally, lower order modes are achieved when the pump laser is focused near the far end and center of the structure, and higher order modes are observed otherwise. With this kind of illumination almost always only a single transversal mode can be excited. Similar modes are also expected in the case of smectic A liquid crystal optical fibers [37].

![Supplementary Figure 10: (d) Optical images of a chiral finger lasing, depending on the exact position of the pump laser (its approximate location is marked with the circle in the center of each image). The structure is outlined in white and the scale bar represents 10 µm.](image-url)
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