Variation after Angular Momentum Projection for the Study of Excited States Based on Antisymmetrized Molecular Dynamics

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Abstract

In order to study the structure of excited states we perform a variational calculation after spin parity projection (VAP) within the framework of Antisymmetrized Molecular Dynamics (AMD). The framework is proven to be a new powerful approach for the study of the various structures of excited states because it is free from model assumptions such as inert cores, existence of clusters, and the axial symmetry. By using finite range interactions with a density dependent term we reproduce well all the energy levels below 15 MeV in $^{12}$C. This is the first theoretical model that reproduces many $E2$ transition rates and $\beta$ decays to $^{12}$C successfully.
Clustering is one of the important features of the light nuclei. The Ikeda diagram predicts molecule-like structures in the excited states, according to the activation of the inter-cluster relative motion. The cluster model has been successful for the study of the molecule-like states. However the model is not suitable for describing the shell-model-like features, such as the \( j-j \) coupling and the particle-hole excitations. The model, as we see later, failed to reproduce important data such as the level spacing and the transition strength because the dissociation of clusters is not taken into account by the conventional cluster model. These shell-model-like aspects are often found in many states including the ground state. On the other hand, the shell model successfully reproduces properties of many excited levels of light nuclei (for example [2,3]), but a number of states in light nuclei have been left unsolved because it is difficult for the shell model to describe well-developed molecule-like states which may exist in low energy region.

There are very few approaches which can describe both the molecule-like and the shell-model-like phenomena. For instance, in \(^{12}\)C which has been studied for a long time [4,5,6,7], the structure of the excited 0\(^+_2\), 0\(^+_3\), and 2\(^+_2\) states is not yet clarified. Therefore, it is desirable to develop a theoretical approach that is able to deal with both aspects in a unified manner. Our aim is to systematically study how the structure of nuclei changes with increasing excitation energy.

The AMD has already proved to be a very useful theoretical approach for the structure of general light nuclei [8,9,10,11]. The AMD basis wave functions are written as Slater determinants where the spatial part of each single particle wave function is a Gaussian. An important point of AMD is that we do not need model assumptions such as inert cores and the axial symmetry. In the previous works on the ground states of light unstable nuclei [8,9,10], we applied a simplest version of AMD in which we make energy variation only after parity projection of a Slater determinant. The simplest version of AMD was found to describe the properties of ground states successfully. We also studied the change in structure along the yrast line of \(^{20}\)Ne by a cranking method of AMD [8] instead of the angular momentum projection before energy variation.

In this letter, we propose a new microscopic approach of variational calculation after parity and total angular momentum projection within the framework of antisymmetrized molecular dynamics (AMD). In order to confirm the usefulness of the method, we apply this approach for the first time to the study of excited states of \(^{12}\)C using finite range interactions.

Concerning the formulation of AMD, the reader is referred to papers [8,9,10]. An AMD wave function is a Slater determinant of Gaussian wave packets:

\[
\Phi_{AMD} = \frac{1}{\sqrt{A}} A\{\varphi_1, \varphi_2, \ldots, \varphi_A\},
\]

\[
\varphi_i = \phi z_i \chi_i \tau_i : \left\{ \begin{array}{l}
\phi z_i (r_j) \propto \exp \left[ -\nu \left( r_j - \frac{z_i}{\sqrt{\nu}} \right)^2 \right], \\
\chi_i = \left( \frac{1}{2} + \frac{Z_{4i}}{2} \right)^{\frac{1}{2}}.
\end{array} \right.
\]

where the centers of Gaussians \( z_i \)'s are complex variational parameters. \( \chi_i \) is the intrinsic spin function parametrized by \( Z_{4i} \) and \( \tau_i \) is the isospin function which is fixed to be up(proton) or down(neutron) in the present calculation. We vary the energy expectation values for the parity and total angular momentum eigenstates (VAP calculation),
\[
\frac{\langle P_{MK'}^J \Phi_{AMD}^\pm | H | P_{MK'}^J \Phi_{AMD}^\pm \rangle}{\langle P_{MK'}^J \Phi_{AMD}^\pm | P_{MK'}^J \Phi_{AMD}^\pm \rangle},
\]

where the operator of total angular momentum projection \( P_{MK'}^J \) is \( \int d\Omega D^*_{MK'}(\Omega) R(\Omega) \). The expectation values are calculated numerically by the sum of mesh points instead of integral on Euler angle \( \Omega \). We make energy variation by the use of the frictional cooling method \([9]\).

Here we represent the intrinsic wave function \( \Phi_{AMD} \) obtained by VAP for the lowest states with a given spin parity \( J^\pm \) as \( \Phi_0^{J^\pm} \). Higher excited states are constructed by superposing wave functions so as to orthogonalize to the lower states. That is to say that the \( n \)-th \( J^\pm \) state \( \Phi_n^{J^\pm} \) is calculated by varying the energy expectation value of the orthogonal component to the lower states;

\[
P_{MK'}^{J^\pm} \Phi_n^{J^\pm} = \sum_{k=1}^{n-1} \langle P_{MK'}^{J^\pm} \Phi_k^{J^\pm} | P_{MK'}^{J^\pm} \Phi_n^{J^\pm} \rangle P_{MK'}^{J^\pm} \Phi_k^{J^\pm}.
\]

By making VAP calculations with various sets of \( \{J^\pm, n\} \) we obtain many intrinsic states \( \{\Phi_1, \ldots, \Phi_n\} \), which approximately correspond to the \( J_n^\pm \) states. Final results are constructed by diagonalizing the Hamiltonian matrix \( \left\langle P_{MK'}^{J^\pm} \Phi_i | H | P_{MK'}^{J^\pm} \Phi_j \right\rangle \) formed from all the intrinsic states. The resonance states are treated within a bound state approximation.

In this work the adopted interactions are the central force of the modified Volkov no1 of case 3 \([12]\) which consists of the finite range two-body force and the zero range three-body repulsive force as a density dependent term, the spin-orbit force of G3RS with two range gaussians \([13]\) and the coulomb force. The Majorana parameter used here is \( m = 0.62 \), and the strength of G3RS force is \( u_1 = -u_2 = 3000 \) MeV. We chose the width parameter \( \nu \) for gaussians in Eq.\([2]\) as 0.18.

Energy levels of \( ^{12}\text{C} \) are shown in Fig.1 compared with the experimental data. The AMD calculations reproduce the energy levels very well. In the cluster model calculations the level spacing between \( 0_1^+ \) and \( 2_1^+ \) was always underestimated. For instance, in a GCM calculation \([3]\) it was 2.2 MeV which is much smaller than the experimental value 4.4MeV. In the present result the large level spacing between \( 0_1^+ \) and \( 2_1^+ \) agree with the experimental data. We think that it is because the theory describes successfully the dissociation of \( \alpha \) cluster in the ground \( 0_1^+ \) state due to the LS force. It is found that molecule-like states with well-developed 3\( \alpha \) structures construct rotational bands of \( K^\pi=3^- , 1^-, 0_3^+, 0_3^- \). Even by a \((0 + 2)h\omega\) shell-model calculation for \( p - \) shell nuclei \([4]\) they can not describe \( 0_3^+ \) and \( 2_3^+ \) states in the \( K^\pi = 0_3^+ \) band because such states contain highly excited \( n h\omega \) components in terms of the shell model.

The transition strengths are of great help to investigate the structure of excited states. The theoretical and the experimental values of \( E2 \) transition strengths are shown in Table II. In a GCM calculation \([1]\) of the 3\( \alpha \) cluster model the \( B(E2; 0_2^+ \rightarrow 2_1^+) \) was about one third of the experimental data (see Table I). The present result for \( B(E2; 0_2^+ \rightarrow 2_1^+) \) is as much as the experimental data.

The calculated strengths of Gamov-Teller \( \beta \) decay transitions from \( ^{12}\text{N} \) to the excited states of \( ^{12}\text{C} \) are shown in Table III and compared with the experimental data. The ground state of the parent nucleus \( ^{12}\text{N} \) is obtained with a variational calculation after spin parity projection to \( 1^+ \) states. It is difficult to discuss \( \beta \) transitions to \( ^{12}\text{C}^* \) within the framework of
The strength of the $E2$ transitions of $^{12}$C.

| Transition | exp. | present | GCM [6] |
|------------|------|---------|---------|
| $B(E2; 2^+_1 \rightarrow 0^+_1)$ | 7.81±0.44 e$^2$fm$^4$ | 8.8 e$^2$fm$^4$ | 8.0 e$^2$fm$^4$ |
| $B(E2; 0^+_2 \rightarrow 2^+_1)$ | 13.4±1.8 e$^2$fm$^4$ | 19.3 e$^2$fm$^4$ | 3.5 e$^2$fm$^4$ |

the $3\alpha$ cluster model, because $\beta$ decay of nucleons in $\alpha$ clusters is forbidden. In other words, the origin of the $\beta$ decay strength is the components of the dissociation of the $\alpha$ clusters. On the other hands, even in shell-model calculations with the large model space of $(0 + 2)\hbar\omega$ the $0^+_3$ state does not appear in low energy region as mentioned above. Therefore there has been no theoretical approach which reproduce the strength of $\beta$ decay to the excited $0^+$ states. This is the first microscopic calculation that can describe a large number of the experimental data of the transition strength to the excited states of $^{12}$C. It means that in the present calculations the component of the dissociation of $\alpha$ clusters is reasonably large enough to explain the $\beta$ transition strength. This success is due to the flexibility of AMD wave functions which can describe both shell-model-like and molecule-like aspects.

Another shell-model-like aspect is found in the $1^+_1(T = 0)$ state which is considered to be a shell-model-like state and can not be described in $3\alpha$ cluster models. In this work the $\beta$ transition strength to $1^+_1$ from $^{12}$N reasonably agrees to the experimental data for the $1^+(T = 0)$ state at 12.71 MeV, though the excitation energy is overestimated (see Table II and Fig[1]).

In summary, we achieved the variation after the parity and the total angular momentum projection within the framework of antisymmetrized molecular dynamics and applied it to the excited states of $^{12}$C. The framework is a new approach for the study of the structure of excited states with freedom from any model assumptions such as inert cores and the axial symmetry. It is a practical method of the variation after spin parity projection with finite range interactions. The model can represent both clustering structures and the dissociation of clusters due to shell-model-like aspects within one framework. This microscopic model is a powerful tool which is applicable to stable as well as unstable nuclei. Many excited levels of $^{12}$C, $E2$ transition strengths and $\beta$ decays to the excited states are reproduced within one microscopic framework for the first time. We can obtain information of the structures of many excited states from the experimental data of the transition strength. It owes to the flexibility of the AMD wave function which can describe both shell-model-like and clustering aspects seen in the excited states.

The author would like to thank H. Horiuchi for many discussions and advises. She is also thankful to N. Itagaki, Y. Kondo and O. Morimatsu for helpful discussions. She is grateful to A. Ono for technical advices on numerical calculations. This work was partly supported by Research Center for Nuclear Physics (RCNP) of Osaka University and Institute of Physical and Chemical Research (RIKEN) for using computer facilities.
TABLE II. The experimental data for $\beta$ decays $^{12}$N($\beta^+$)$^{12}$C compared with the theoretical results.

| Decay to $^{12}$C* (MeV) | $J^\pi$ | $\log ft$ (exp.) | $\log ft$ (theory ($J_f^\pm$)) |
|-------------------------|--------|----------------|------------------------|
| 0                       | 0$^+$  | 4.120 ± 0.003 | 3.8 (0$^+_1$)           |
| 4.44                    | 2$^+$  | 5.149 ± 0.007 | 4.8 (2$^+_2$)           |
| 7.65                    | 0$^+$  | 4.34 ± 0.06   | 4.0 (0$^+_2$)           |
| 10.3                    | (0$^+$) | 4.36 ± 0.17   | 4.7 (0$^+_3$)           |
| 12.71                   | 1$^+$  | 3.52 ± 0.14   | 3.8 (1$^+_1$)           |

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FIG. 1. Excitation energies of the levels of $^{12}$C. In the right-hand side, VAP calculations in the AMD framework are shown and compared with the experimental data (left).