Observer-Based Sliding Mode Control for Flexible Spacecraft With External Disturbance

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ABSTRACT
In this paper, attitude stabilization control for flexible spacecraft subject to external disturbance without angular velocity and flexible mode variables measurement is considered. First, an adaptive law is constructed to estimate the upper bound of a lumped disturbance consisting of flexible accessories and external disturbance. Based on the proposed adaptive law, an adaptive observer is established to estimate angular velocity and the lumped disturbance. Besides, for the case where flexible modal variables cannot be measured, a flexible modal variable observer-based sliding mode control law is proposed. Simulations are performed to verify the validity of the proposed control law. The simulation results show that the proposed control law can eliminate the influence of external disturbance for flexible spacecraft under the situation that angular velocity and flexible modal variables cannot be measured.

INDEX TERMS Flexible spacecraft, adaptive law, sliding mode control, attitude stabilization.

I. INTRODUCTION
Flexible spacecraft is composed of a central rigid body structure and flexible accessories [1]. Due to flexible attachments, the performance of flexible spacecraft is easily reduced by internal disturbance such as flexible vibration and inertia uncertainty. For on-orbit flexible spacecraft, the attitude control system is also influenced by external disturbance such as unknown environment disturbance [2], [3].

In order to reduce the influence of the external and internal disturbance, a feasible method is to design disturbance observer. Such an idea has been adopted in [4], [5]. In [6], a second-order observer was used to estimate the disturbance combined by external disturbance, rigid spacecraft inertia uncertainty, and actuator fault. Further, based on this disturbance observer, an adaptive finite-time attitude control law was constructed for rigid spacecraft. In [7], external and internal disturbances were treated as a lumped disturbance, and a disturbance observer was designed to estimate this lumped disturbance. With such a treatment, a dynamic attitude control law was proposed based on this designed observer to compensate the lumped disturbance. An observer was firstly designed in [8] to estimate a function of inertia uncertainties, inertial and external disturbances. Then by using the feedback linearization technique an observer based attitude control law was constructed for flexible spacecraft. The proposed attitude control law can reduce the influence of disturbances. A disturbance observer was designed in [9] to estimate external disturbance and the dynamics produced by flexible accessories. The gain of the observer was given by linear matrix inequality method. Then, a simple attitude control law was designed in the form of the sum of a linear state feedback and the estimation of the disturbance.

A reduced-order sliding mode disturbance observer was designed in [10] to estimate actuator faults, inertia uncertainties, internal and external disturbances. Then, a disturbance observer-based attitude controller was proposed by using nonsingular terminal sliding mode control technique. In this control scheme, the switching gain in the part of sliding control law was small, and thus the chattering phenomenon
can be reduced. A super-twisting disturbance observer was proposed in [11] to estimate a lumped disturbance combined by actuator faults, inertia uncertainties, internal and external disturbances and its integral. The estimation of the lumped disturbance integral was obtained by integrating the inverse velocity. With such a treatment, the nonlinear part of the system was not lumped into the disturbance, and thus the information of this part can be directly utilized.

The above method, the external and internal disturbances were lumped. In fact, they could be compensated respectively in different ways. In [12], only the disturbance caused by elastic vibration of the flexible appendages was estimated by disturbance observer. Based on this observer, a composite hierarchical controller was constructed by combining a feedforward compensator and the part for attitude stabilization. In [13], a disturbance observer was designed to estimate disturbance caused by vibration of flexible accessories, and an extended state observer was presented to estimate the other derivative-bounded disturbance such as external environmental disturbance. An anti-disturbance control law was thus designed based on the estimates of disturbances. The attitude integral was obtained by integrating the inverse velocity. For any \( x \),\( |x| = \sum_{i=1}^{n} |x_i| \in \mathbb{R}^n \), for any \( \rho = [\rho_1 \rho_2 \ldots \rho_n] \in \mathbb{R}^n \),

\[
\rho^T = [\rho_1^T \rho_2^T \ldots \rho_n^T]^T
\]

where \( p \) and \( q \) are positive odd constants with \( p < q \), \(|\rho| = [|\rho_1| \ |\rho_2| \ldots \ |\rho_n|]^T \in \mathbb{R}^n \). For any \( x = [x_1 \ x_2 \ x_3]^T \in \mathbb{R}^3 \), \( x^x \) denotes a skew-symmetric matrix given by

\[
x^x = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}.
\]

II. PRELIMINARIES
A. FLEXIBLE SPACECRAFT ATTITUDE MODEL

The kinematics equation of a flexible spacecraft can be described by the following modified Rodrigues parameters (MRPs) ([14]):

\[
\dot{\sigma} = H(\sigma)\omega
\]

with

\[
H(\sigma) \triangleq \frac{1}{2} \left( I - \sigma^\times + \sigma \sigma^T - \frac{1 + \sigma^T \sigma}{2} I \right),
\]

where \( \omega = [\omega_1 \omega_2 \omega_3]^T \in \mathbb{R}^3 \) is the angular velocity vector; the MRPs \( \sigma = [\sigma_1 \ \sigma_2 \ \sigma_3]^T \) are defined as \( \sigma_i = q_i / (1 + q_0) \), \( i = 1, 2, 3 \) with \( \mathbf{\bar{q}} = [q_0 \ q]^T \) representing spacecraft quaternions, and \( q = [q_1 \ q_2 \ q_3]^T \); \( I \) represents an identity unit matrix with proper dimensions. In addition, the matrix \( H(\sigma) \) has the following properties [15]:

\[
\sigma^T H(\sigma)\omega = \frac{1 + \sigma^T \sigma}{4} \sigma^T \omega.
\]

Under the hypothesis of small elastic deformations, by using Euler theorem the attitude dynamic equation of a flexible spacecraft subjected to external disturbance can be described as [16]

\[
\begin{aligned}
J\ddot{\eta} + \delta^T \ddot{\eta} &= -\omega \times (J\omega + \delta^T \eta) + u + d_0 \\
\dot{\eta} + C \dot{\eta} + K \eta + \delta \omega &= 0,
\end{aligned}
\]

where \( d_0 \in \mathbb{R}^3 \) is the external disturbance; \( u \in \mathbb{R}^3 \) is the control torque; \( J \in \mathbb{R}^{3 \times 3} \) is a symmetric matrix representing the total inertia of the spacecraft combined by inertial matrix of rigid body \( J_{mb} \) and inertia matrix of flexible accessories \( \delta^T \delta \), thus

\[
J = J_{mb} + \delta^T \delta,
\]

where \( \delta \in \mathbb{R}^{3 \times N} \) is the coupling matrix between the elastic structures and rigid body; \( \eta \in \mathbb{R}^N \) is the modal coordinate vector; \( C = \text{diag} \left[ \Lambda_1 \ ... \ \Lambda_N \right] \) is the stiffness matrix, and \( K = \text{diag} \left[ 2 \xi_1 \Lambda_1 \ ... \ 2 \xi_N \Lambda_N \right] \) is the damping matrix with \( N \) the number of elastic modes considered, \( \Lambda_i, \ i = 1, \ldots, N \) the natural frequency, and \( \xi_i, \ i = 1, \ldots, N \) the associated damping. Define

\[
\psi = \dot{\eta} + \delta \omega,
\]

then (4) can be written as

\[
\begin{aligned}
\dot{\omega} &= -J_{mb}^{-1} \omega \times (J_{mb} \omega + \delta^T \psi) + J_{mb}^{-1} \delta^T (C \psi + K \eta - C \delta \omega) + J_{mb}^{-1} u + J_{mb}^{-1} d_0 \\
\dot{\eta} &= \psi - \delta \omega \\
\dot{\psi} &= -(C \psi + K \eta - C \delta \omega).
\end{aligned}
\]

For the flexible spacecraft in (5) with external disturbance \( d_0 \), the objective of this paper is to design a controller \( u \) to guarantee the MRPs \( \sigma \rightarrow 0 \) and angular velocity \( \omega \rightarrow 0 \).
B. LEMMAS

In this section, some technical lemmas are given. They will be used in the sequel sections.

Lemma 1 (Triangle Inequality): If \( 0 < n < 1 \) and \( a_i > 0, \ i = 1, 2, \ldots, m \), then
\[
\left( \sum_{i=1}^{m} a_i \right)^n \leq \sum_{i=1}^{m} a_i^n.
\]

Lemma 2 ([17], [18]): If a positive definite continuous Lyapunov function \( V(t) \) satisfies the following inequality
\[
\dot{V}(t) + c[V(t)]^\kappa \leq 0,
\]
\( V(t) \) converges to the equilibrium point in a finite time \( t_c \) depending on the initial state \( V(t_0) \) as
\[
t_c \leq \frac{[V(t_0)]^{1-\kappa}}{c(1-\kappa)},
\]
where \( c \) and \( \kappa \) are positive constants.

III. MAIN RESULTS

A. ADAPTIVE ANGULAR VELOCITY AND DISTURBANCE OBSERVER DESIGN

In the design of attitude control law, angular velocity and external disturbance are needed. However, it is not easy to measure them in many cases. Thus, it is necessary to design an observer to give accurate estimates of angular velocity and external disturbance when they are not available.

From (1) and (5), the attitude kinematics equation can be transformed into a nonlinear system as follows:
\[
\begin{aligned}
\dot{\sigma} &= H(\sigma)\omega \\
\dot{\omega} &= -J_{mb}^{-1}\omega^\times \left( J_{mb}\omega + \delta^T\psi \right) + J_{mb}^{-1}\delta^T(C\psi + K\eta - C\delta\omega) + J_{mb}^{-1}u + J_{mb}^{-1}d_0 \\
y &= \sigma.
\end{aligned}
\]

Denote
\[
f = -J_{mb}^{-1}\omega^\times \left( J_{mb}\omega + \delta^T\psi \right)
\]
and
\[
d = J_{mb}^{-1}\left[ d_0 + \delta^T(C\psi + K\eta - C\delta\omega) \right],
\]
then (6) can be rewritten as
\[
\begin{aligned}
\dot{\sigma} &= H(\sigma)\omega \\
\dot{\omega} &= f + J_{mb}^{-1}u + d \\
y &= \sigma.
\end{aligned}
\]

To facilitate controller design, the following assumption for the lumped disturbance \( d \) is made.

Assumption 1: The unknown disturbance \( d \) is bounded, that is, there exists a positive constant such that \( \|d\| \leq \beta \).

For the system (7), an angular velocity and disturbance observer is designed as follows:
\[
\begin{aligned}
\dot{\hat{\omega}} &= -k\rho - \beta \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) + J_{mb}^{-1}u \\
\dot{\hat{d}} &= -k\rho - \beta \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) - f,
\end{aligned}
\]
where \( k \) is a positive constant; \( \rho = \dot{\omega} - \omega \) is the angular velocity observer error. In fact, it is difficult to obtain the exact value of the upper bound of the lumped disturbance \( d \), which brings certain difficulties to the design of the observer parameters. To solve this problem, the adaptive technique is employed to estimate the upper bound value \( \beta \). Then an adaptive angular velocity and disturbance observer is designed as follows:
\[
\begin{aligned}
\dot{\hat{\omega}} &= -k\rho - \hat{\beta} \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) + J_{mb}^{-1}u \\
\dot{\hat{d}} &= -k\rho - \hat{\beta} \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) - f,
\end{aligned}
\]
where \( \hat{\beta} \) is the adaptive value of \( \beta \) given by
\[
\dot{\hat{\beta}} = \theta \|\rho\|_1,
\]
where \( \theta \) is a positive constant to determine the adaptation rate. Denote the adaptive error of \( \beta \) as follows:
\[
\hat{\beta} = \hat{\beta} - \beta,
\]
the derivation of (10) with respect to time is calculated as
\[
\dot{\hat{\beta}} = \hat{\beta} - \beta = \hat{\beta}.
\]

Define the disturbance observer error as \( \tilde{d} = \hat{d} - d \). Then, it follows from (7) and (8) that
\[
\begin{aligned}
\dot{\tilde{d}} &= \dot{d} - \dot{d} \\
&= -k\rho - \hat{\beta} \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) - f - d \\
&= -k\rho - \hat{\beta} \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) + J_{mb}^{-1}u - \dot{\omega} \\
&= \tilde{\omega} - \dot{\omega} \\
&= \tilde{\omega}.
\end{aligned}
\]

With the previous preliminaries, the convergence property of the proposed disturbance observer is given in the following theorem.

Theorem 1: For a flexible spacecraft with external disturbance (7) satisfying Assumption 1, an adaptive angular velocity and disturbance observer is constructed in the form of (8). Then the adaptive error \( \hat{\beta} = \hat{\beta} - \beta \) is bounded, the angular velocity observer error \( \rho = \dot{\omega} - \omega \) and disturbance observer error \( \tilde{d} = d - d \) converge to zero.

Proof: Select
\[
V_1 = \frac{1}{2}\rho^T\rho + \frac{1}{2\theta}\beta^2
\]
as Lyapunov function candidate. According to (9), (10), (11) and (12), the time derivative of \( V_1 \) is given as
\[
\dot{V}_1 = \rho^T\dot{\rho} + \frac{1}{\theta}\beta\dot{\beta}
\]
\[
= \rho^T \left( -k\rho - \hat{\beta} \text{sign}(\rho) - \|f\|_1 \text{sign}(\rho) - d \right) + \frac{1}{\theta}\beta\dot{\beta}
\]
\[
= -k\rho^T\rho - \hat{\beta}\rho^T \text{sign}(\rho) - \|f\|_1\rho^T \text{sign}(\rho)
\]
\[
- \rho^Tf - \rho^Td + \hat{\beta}\|\rho\|_1
\]
\[
= -k\rho^T\rho - \hat{\beta} \left\| \rho^T \right\|_1 - \|f\|_1 \left\| \rho^T \right\|_1 - \rho^Tf
\]
\[
- \rho^Td + (\hat{\beta} - \beta)\|\rho\|_1
\]
\[
= -k\rho^T\rho - \|f\|_1 \left\| \rho^T \right\|_1 - \rho^Tf - \rho^Td - \beta\|\rho\|_1.
\]
Due to Assumption 1 and
\[
\rho^T d + \| \rho^T d \|_1 \geq \rho^T d + |\rho|^T |d| \geq 0,
\]
we can obtain
\[
-\rho^T d - \beta \| \rho \|_1 \leq \rho^T d + \| \rho^T d \|_1 - \beta \| \rho \|_1 \leq 0. \tag{14}
\]
Owing to (14) and
\[
\| f \|_1 \| \rho^T d + \rho^T f \geq |\rho|^T |f| + \rho^T f \geq 0,
\]
it is easily obtained from (13) that
\[
\dot{V}_1 \leq -k \rho^T \rho \leq 0. \tag{15}
\]
Thus, the angular velocity estimate error \( \rho \) is square integrable and \( V_1 \) is bounded, which means \( \rho \) and the adaptive error \( \bar{\rho} \) are bounded. Since
\[
-\| f \|_1 \text{sign}(\rho) - f \leq 0,
\]
and \( \rho, \bar{\rho}, \) and \( d \) are bounded, so \( \dot{\rho} \) is bounded. Then from Barbalat’s Lemma, it can be obtained that \( \dot{\rho} \) is uniformly continuous and \( \lim_{t \to \infty} \dot{\rho} = 0 \). Therefore, \( \rho \) also converges to zero. Since \( \bar{d} = \bar{d} - d = \bar{\rho} \), the disturbance observe error \( \bar{d} \) converges to zero.

In the design of the subsequent control strategy, the control law is designed by using the estimates of angular velocity and the lumped disturbance.

**B. OBSERVER-BASED CONTROLLER DESIGN**

In this section, a control scheme is constructed to stabilize the attitude of flexible spacecraft by combining adaptive angular velocity and disturbance observer proposed in Section III.A with sliding mode control laws. For this aim, the sliding mode for flexible spacecraft is chosen as
\[
d = \omega + \alpha \sigma + \gamma \sigma \frac{p}{q} \dot{\sigma}, \tag{16}
\]
where \( s = [s_1 \ s_2 \ s_3]^T \in \mathbb{R}^3 \); \( \alpha \) is a positive constant; \( \gamma = \text{diag} \{ \gamma_1 \ \gamma_2 \ \gamma_3 \} \in \mathbb{R}^{3 \times 3} \); \( \gamma_i, i = 1, 2, 3 \) are positive constants; \( p \) and \( q \) are positive odd constants with \( p < q \). From (7), the derivation of the sliding mode with respect to time is calculated as
\[
\dot{s} = \dot{\omega} + \alpha \dot{\sigma} + \frac{p}{q} \gamma \sigma \frac{u_{\text{sat}}}{q} \dot{\sigma}.
\]
By setting \( \dot{s} = 0 \), the equivalent control \( u_{\text{eq}} \) is obtained as
\[
\begin{align*}
u_{\text{eq}} &= -J_{nb}^{-1}[f + d + \alpha H(\sigma) \omega + \frac{p}{q} \gamma \sigma \frac{u_{\text{sat}}}{q} H(\sigma) \omega] \\
&= \omega^T (J_{nb} \omega + \delta T \psi) - J_{nb} [d + \alpha H(\sigma) \omega] \\
&\quad + \frac{p}{q} \gamma \sigma \frac{u_{\text{sat}}}{q} H(\sigma) \omega.
\end{align*}
\]
Such an equivalent control can not be implemented since the unknown angular velocity \( \omega \) and disturbance \( d \) exist. A feasible method is to replace angular velocity and external disturbance with their estimates, which can be obtained by the adaptive angular velocity and disturbance observer proposed in the last section. Thus, the following modified equivalent control is obtained by serving \( \dot{\omega} \) as adaptive feedforward compensation of \( \omega \) and \( \bar{d} \) as adaptive feedforward compensation of \( d \), converted to
\[
u_{\text{eq}} = \dot{\omega}^T (J_{nb} \dot{\omega} + \delta T \psi) - J_{nb} [\bar{d} + \alpha H(\sigma) \dot{\omega}]
\]
\[
+ \frac{p}{q} \gamma \sigma \frac{u_{\text{sat}}}{q} H(\sigma) \dot{\omega}. \tag{18}
\]
It can be seen from (18) that the equivalent control law contains flexible modal variable \( \psi \). However, the flexible modal variable \( \psi \) is supposed to be unavailable, which brings trouble to the implementation of the control law. Next, we aim to design a modal variable observer to estimate flexible modal variables, and then replace the flexible modal variable \( \psi \) in the equivalent control law.

The flexible modal variable observer is conducted as:
\[
\begin{bmatrix}
\dot{\hat{\psi}} \\
\dot{\hat{\sigma}}
\end{bmatrix} = \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} \begin{bmatrix}
\hat{\psi} \\
\hat{\sigma}
\end{bmatrix} - \begin{bmatrix} I \\ -C \end{bmatrix} \delta \dot{\omega}
\]
\[
- P^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} \delta \dot{\omega}^T (\dot{\omega} + \alpha \sigma + \gamma \sigma \frac{p}{q}), \tag{19}
\]
where the matrix \( P \) is the solution of the following Lyapunov equation
\[
P \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} + \begin{bmatrix} 0 & I \\ -K & -C \end{bmatrix} P^T = -2Q, \tag{20}
\]
where \( Q > 0 \). With this observer, by replacing the unmeasurable flexible variable \( \psi \) with its estimate \( \hat{\psi} \) in (18), \( u_{\text{eq}} \) is conducted as:
\[
u_{\text{eq}} = \dot{\omega}^T (J_{nb} \dot{\omega} + \delta T \hat{\psi}) - J_{nb} [\bar{d} + \alpha H(\sigma) \dot{\omega}]
\]
\[
+ \frac{p}{q} \gamma \sigma \frac{u_{\text{sat}}}{q} H(\sigma) \dot{\omega}. \tag{21}
\]
The switching control \( u_{sw} \) is chosen as:
\[
u_{sw} = -K_e \text{sat}(s)
\]
\[
= -\sum_{i=1}^{n} K_{ei} \text{sat}(s_i) \tag{22}
\]
with
\[
\text{sat}(s_i) = \begin{cases} 
\text{sign}(s_i), & |s_i| \geq \phi \\
\tanh \left( \frac{3}{\phi} s_i \right), & |s_i| < \phi,
\end{cases}
\]
where \( K_e = \text{diag} \{ K_{e1} \ K_{e2} \ K_{e3} \} \in \mathbb{R}^{3 \times 3} \); \( K_{ei}, i = 1, 2, 3 \) are positive constants; \( \phi \) is a positive constant; the function \( \tanh \) is defined as follows:
\[
\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}.
\]
Now the overall control law is obtained as follows:

\[
\begin{align*}
    u &= u_{eq} + u_{sw} \\
    u_{eq} &= \hat{\omega}^T \left( J_{mb} \hat{\omega} + \delta^T \hat{\psi} \right) - J_{mb} \hat{d} + aH(\sigma)\hat{\omega} \\
    &+ \frac{p}{q} \gamma \sigma \frac{\partial}{\partial \psi} H(\sigma)\hat{\omega} \\
    u_{sw} &= -K_e \text{sat}(s).
\end{align*}
\]  

(24)

Then (17) can be converted to

\[
\dot{s} = f + J_{mb}^{-1} u + d + aH(\sigma)\omega + \frac{p}{q} \gamma \sigma \frac{\partial}{\partial \psi} H(\sigma)\omega + \delta^T \hat{\psi} - \left( \hat{d} + aH(\sigma)\omega \right) + J_{mb}^{-1} u_{sw} + d + aH(\sigma)\omega + \frac{p}{q} \gamma \sigma \frac{\partial}{\partial \psi} H(\sigma)\omega
\]

Then, the continuous positive definite Lyapunov function candidate:

\[
V = \frac{1}{2} s^T J_{mb} s + \frac{1}{2} \left[ e_\eta^T e_\eta + e_\psi^T e_\psi \right] P \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix}
\]

For the closed-loop system, choose the following Lyapunov function candidate:

\[
V_2 = \frac{1}{2} s^T J_{mb} s + \frac{1}{2} \left[ e_\eta^T e_\eta + e_\psi^T e_\psi \right] P \begin{bmatrix} e_\eta \\ e_\psi \end{bmatrix}
\]

From (16), (19), (20), (22) and (26), the time derivative of \( V_2 \) is eventually given as

\[
\dot{V}_2 = s^T \left( \dot{\omega}^T \delta (\hat{\psi} - \psi) + u_{sw} \right) + e_\eta^T e_\eta + e_\psi^T e_\psi \begin{bmatrix} 0 \\ -K \\ -C \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\psi} \end{bmatrix}
\]

and

\[
\begin{bmatrix} 0 \\ -K \\ -C \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\psi} \end{bmatrix} \begin{bmatrix} \eta \\ \gamma \sigma \hat{\omega} \end{bmatrix}
\]

\[
= -(\omega + a\sigma + \gamma \sigma \hat{\omega})^T \delta^T (\hat{\psi} - \psi) + s^T u_{sw} + e_\eta^T e_\eta + e_\psi^T e_\psi \begin{bmatrix} 0 \\ -K \\ -C \end{bmatrix} \begin{bmatrix} \eta \\ \gamma \sigma \hat{\omega} \end{bmatrix}
\]

\[
\leq s^T u_{sw} - \frac{p}{q} \gamma \sigma \hat{\omega} + \Delta \hat{\omega} + \alpha \sigma + \gamma \sigma \hat{\omega}.
\]

According to (23), for any value of \( s_i \), we have

\[
s_i K_e \text{sign}(s_i) = K_e |s_i| \geq 0
\]

and

\[
s_i K_e \text{tanh} \left( \frac{2\pi s_i}{\phi} \right) \geq 0.
\]

So, it can be obtained that

\[
\dot{V}_2 \leq 0.
\]

It can be seen that \( e_\eta \) and \( e_\psi \) are bounded, and the sliding mode \( s \) converges to zero by the control law (24).

When the system state is restricted to the switching surface \( s = 0 \), it can be obtained that

\[
s = \omega + a\sigma + \gamma \sigma \hat{\omega} = 0,
\]

which means

\[
\omega = -a\sigma - \gamma \sigma \hat{\omega}.
\]

(27)

Then, the continuous positive definite Lyapunov function \( V_3 \) of the sliding phase is selected as

\[
V_3 = \frac{1}{2} \sigma^T \sigma.
\]

From (3), (7) and (27), the derivative of \( V_3 \) with respect to time is given by

\[
\dot{V}_3 = \sigma^T \dot{\sigma}
\]

\[
= \sigma^T H(\sigma)\omega
\]

\[
= \frac{1}{4} + \sigma^T \sigma \tau \omega
\]

\[
= -\frac{1}{4} + \sigma^T \sigma \left( \alpha \sigma + \gamma \sigma \hat{\omega} \right)
\]

\[
= -\frac{1}{4} + \sigma^T \sigma \left( \alpha \sigma + \gamma \sigma \hat{\omega} \right)
\]

\[
+ \gamma \sigma \hat{\omega}
\]

\[
\leq -\frac{1}{4} + \gamma \sigma \hat{\omega} \min \left( \sigma_1^T \sigma_1 + \sigma_2^T \sigma_2^T + \sigma_3^T \sigma_3^T \right)
\]

\[
= -\frac{1}{4} + \gamma \sigma \hat{\omega} \min \left( \sigma_1^T \sigma_1 + \sigma_2^T \sigma_2^T + \sigma_3^T \sigma_3^T \right)
\]

\[
= -\frac{1}{4} + \gamma \sigma \hat{\omega} \min \left( \sigma_1^T \sigma_1 + \sigma_2^T \sigma_2^T + \sigma_3^T \sigma_3^T \right)
\]

\[
+ \left( \sigma_1^T \sigma_1 + \sigma_2^T \sigma_2^T + \sigma_3^T \sigma_3^T \right)
\]

(28)
where $\gamma_{\min} = \min\{\gamma_1, \gamma_2, \gamma_3\} > 0$. By Lemma 1, we have

$$(2V_3)^{\frac{p+q}{p+q}} = \left(\sigma_1^2 + \sigma_2^2 + \sigma_3^2\right)^{\frac{p+q}{p+q}} \leq \left(\sigma_1^2\right)^{\frac{p+q}{p+q}} + \left(\sigma_2^2\right)^{\frac{p+q}{p+q}} + \left(\sigma_3^2\right)^{\frac{p+q}{p+q}}.$$ 

With this, it follows from (28) that

$$\dot{V}_3 \leq -2\frac{p+q}{p+q} 1 + \sigma^T \sigma \frac{p+q}{p+q} \gamma_{\min} V_3^{\frac{p+q}{p+q}}.$$

Hence it can be concluded that

$$\dot{V}_3 + 2\frac{p+q}{p+q} 1 + \sigma^T \sigma \frac{p+q}{p+q} \gamma_{\min} V_3^{\frac{p+q}{p+q}} \leq 0.$$ 

According to Lemma 2, the MRPs can converge to zero in finite time. From (27), the angular velocity $\omega$ also converges to zero in finite time.

From the above theorem, it can be seen that the observer errors $\epsilon_\psi$ and $\epsilon_\eta$ of the flexible modal variables are bounded. Besides, the state of the system $\sigma$ and $\omega$ can be driven into the sliding surface $s = 0$ by the control law (24), then they can both converge to zero in finite time.

**IV. SIMULATION AND ANALYSIS**

Numerical simulations on flexible spacecraft are conducted in this section to verify the effectiveness of the proposed approach. The physical parameters of the flexible spacecraft in the simulation according to [19] are given in Table 1. The initial values of angular velocity, estimate of angular velocity, the MRPs and flexible modal variables are given in Table 2. The parameters of the proposed observers and control law are shown in Table 3.

The simulation results of the observer-based control law are shown in Figure 1. The control torques $u_i, i = 1, 2, 3$ are shown in Fig.1(a). It can be obtained that control law in (24)
It can be observed that the attitude angular velocity and MRPs converge to zero under the observer-based control law. The behavior of the sliding surface $s_i, i = 1, 2, 3$ are given in Fig.1(d), which indicates that the selected sliding surface is valid.

The simulation results of the observer (8) and observer (19) are shown in Figure 2. The estimate errors of angular velocity $\hat{\rho}_i, i = 1, 2, 3$ are shown in Fig.2(a) and the estimate errors of the lumped disturbance $\hat{d}_i, i = 1, 2, 3$ are shown in Fig.2(b). It can be seen that all the errors converge to zero which means angular velocity $\omega$ and the lumped disturbance $d$ can be estimated accurately by their estimates. In addition, the estimate errors of flexible modal $e_{\psi i}, i = 1, 2, 3, 4$ and $e_{\eta i}, i = 1, 2, 3, 4$ are shown in Fig.2(c) and Fig.2(d). It is clear that the estimate errors of the flexible modal variables are really small, that is, the flexible modal variables can accurately estimated by the proposed observer in (19).

V. CONCLUSION

A control method based on observers is proposed for attitude stabilization of flexible spacecraft subjected to external disturbance. In the designed adaptive angular velocity and disturbance observer, flexible accessories and unmeasurable
external disturbance are regarded as a lumped disturbance. Then an adaptive scheme is constructed to estimate the upper bound of the lumped disturbance, and the adaptive value is used to give accurate estimates of angular velocity and the lumped disturbance. Also, when the flexible modal variables are not measurable, a flexible modal variable observer-based sliding mode control law is presented. The designed control law can eliminate the effects of the external disturbance on flexible spacecraft.

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