(ω, φ)P− decays of tau leptons

A. Flores-Tlalpa and G. López Castro

Departamento de Física, Cinvestav, Apartado Postal 14-740, 07000 México, D.F., México

The τ− → (ω, φ)P−ντ decays, where P− = π−, K−, are considered within a phenomenological model with dominance of meson intermediate states. We assume SU(3) flavor symmetry to fix some of the unknown strong interaction couplings. Our predictions for the τ− → φ(π−, K−)ντ branching fractions are in good agreement with recent measurements of the BABAR and BELLE Collaborations.

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I. INTRODUCTION

Tau lepton decays into a charged pseudoscalar P− and an isoscalar vector meson V, generically denoted by τ− → VP−ντ, can occur in four possible ways:

\begin{align*}
\tau^- &\rightarrow \omega \pi^- \nu, \\
\tau^- &\rightarrow \phi \pi^- \nu, \\
\tau^- &\rightarrow \omega K^- \nu, \\
\tau^- &\rightarrow \phi K^- \nu.
\end{align*}

Owing to the quark mixing angle factors, one naively expects that processes (1) and (2) (ΔS = 0) would have larger branching fractions than decay modes involving a K− meson (ΔS = −1). However, the rich resonance structure of intermediate states combined with the high thresholds for the above processes will produce an interesting pattern worth to be investigated.

The study of τ− → VP−ντ decays is interesting for several reasons. As is well known, tau decays into several pseudoscalar mesons are dominated by the production of intermediate

*Electronic address: afflores@fis.cinvestav.mx
†Electronic address: glopez@fis.cinvestav.mx
TABLE I: Measured branching fractions of $\tau^- \to VP^- \nu_\tau$ decays.

| $VP^-$ mode | Branching fraction | Reference |
|-------------|--------------------|-----------|
| $\omega \pi^-$ | $(1.95 \pm 0.08) \times 10^{-2}$ | [7, 8] |
| $\phi \pi^-$ | $(6.05 \pm 0.71) \times 10^{-5}$ | [10], |
| | $(3.42 \pm 0.55 \pm 0.25) \times 10^{-5}$ | [11] |
| $\omega K^-$ | $(4.1 \pm 0.9) \times 10^{-4}$ | [9] |
| $\phi K^-$ | $(4.05 \pm 0.25 \pm 0.26) \times 10^{-5}$ | [10] |
| | $(3.39 \pm 0.20 \pm 0.28) \times 10^{-5}$ | [11] |

A good quantitatively description of the decay modes shown in Eqs. (1)-(4) is important to better understand the dynamics of three and four pseudoscalar mesons produced in tau lepton decays. On the other hand, the study of such decays allows a direct access to the $\langle VP | J_\mu | 0 \rangle$ hadronic matrix element in the intermediate energy regime. Since $\tau \to VP\nu$ and $B, D \to V l\nu$ decays are related by crossing, they can be useful to provide further tests of either non-relativistic [2] or relativistic [3] quark model predictions. Finally, the $\tau^- \to (\omega, \phi)\pi^-\nu$ decays are related to the $e^+e^- \to (\omega, \phi)\pi^0$ processes via isospin symmetry and their measurements can be useful to provide another test of the conserved vector current (CVC) hypothesis [4, 5, 6].

In Table I we display the experimental values for the branching ratios of $\tau^- \to VP^-\nu$ decays. The $\omega \pi^-$ final state is the most favored and its branching fraction and spectral function were the first to be measured [1, 7, 8]. Because of their smaller branching fractions, the decay modes in Eqs. (2)-(4) were measured only very recently [9, 10, 11]. Previous upper bounds on $\tau$ decays involving $\phi$ mesons were reported in Ref. [12], where the upper limits $B(\tau \to \phi\pi\nu) \leq (1.2 \sim 2.0) \times 10^{-4}$ and $B(\tau \to \phi K\nu) \leq (5.4 \sim 6.7) \times 10^{-4}$ were set at the 90% c.l. [12].

Earlier theoretical estimates for some of these decays were considered in references [4, 5, 6]. Based on the Conserved Vector Current (CVC) hypothesis and using bounds for the cross section of $e^+e^- \to \phi\pi^0$, the loosely limit $B(\tau^- \to \phi\pi^-\nu) \leq 9.0 \times 10^{-4}$ at 90% c.l. was derived in [5]. On another hand, by assuming that the form factor of this decay is dominated by the
contribution of two vector resonances ($\rho$ and $\rho'$) and that flavor SU(3) is a good symmetry, the value $B(\tau^{-} \to \phi\pi^{-}\nu) = (1.20 \pm 0.48) \times 10^{-5}$ was obtained in Ref. [6]. This prediction clearly underestimates the measured fraction of $\phi\pi^{-}$ (see Table I). Concerning the $\phi K^{-}$ modes only rough estimates are available based on phase-space and quark mixing angles considerations [10].

In this paper we revisit this subject and provide a unified description of the four $VP^{-}$ decays shown in Eqs. (1)-(4) in the framework of a meson dominance model.

In this paper we consider the possibility that a meson dominance model with a few intermediate states can account, in a unified way, for the observed branching fractions of reactions (1)-(4). As a simplifying assumption we will rely on SU(3) flavor symmetry for the strong couplings and we will assume an ideal value $\tan \theta_{V} = 1/\sqrt{2}$ of the $\omega - \phi$ mixing angle (i.e., $\phi$ is assumed to be an almost pure $\bar{s}s$ state). On the basis of these assumptions we conclude that present data on $\tau^{-} \to VP^{-}\nu$ decays can be easily accommodated within the meson dominance model.

II. MESON DOMINANCE MODEL FOR TAU DECAYS

Thus, let us first consider the decay $\tau^{-}(p_{\tau}) \to V(p_{V})P^{-}(p_{P})\nu(p_{\nu})$, where $p_{i}$ denote the four-momenta of particle $i$. The hadronic matrix element can be decomposed in terms of four form factors ($Q = s$ or $d$) [13]:

$$
\langle VP|\bar{Q}\gamma^{\alpha}(1 - \gamma_{5})u|0\rangle = ig\epsilon^{\alpha\beta\mu\nu}\epsilon_{\beta}^{*}q_{+\mu}q_{-\nu} + f\epsilon^{*\alpha} + [a_{+}q_{+}^{\alpha} + a_{-}q_{-}^{\alpha}]\epsilon^{*} \cdot q_{+}
$$

(5)

where $\epsilon_{\beta}^{*}$ is the polarization four-vector of the outgoing vector meson ($p_{V} \cdot \epsilon^{*} = 0$), and $q_{\pm} = p_{V} \pm p_{P}$. The vector ($g$) and axial ($f$, $a_{\pm}$) form factors are functions of $s = q_{+}^{2}$ only.

If we define $\Sigma^{2} = m_{V}^{2} + m_{P}^{2}$, $\Delta^{2} = m_{V}^{2} - m_{P}^{2}$, and $\beta_{VP} = (1 - 2\Sigma^{2}/s + \Delta^{4}/s^{2})^{1/2}$, the differential decay rate can be written in the simple form:

$$
\frac{d\Gamma}{ds} = \frac{G_{F}^{2}|V_{uQ}|^{2}m_{\tau}^{2}}{128\pi^{3}}\beta_{VP}^{2} \left(1 - \frac{s}{m_{\tau}^{2}}\right)^{2} \times \left\{ \frac{1}{2}\beta_{++} + \frac{1}{2}\beta_{--} \left[ \frac{\Delta^{4}}{s^{2}} + \frac{1}{3} \left(1 + \frac{2s}{m_{\tau}^{2}}\right)\beta_{VP}^{2} \right] + \frac{\Delta^{2}}{s} Re[\beta_{+-}] + \frac{a_{+}}{m_{\tau}^{2}} \right\},
$$

(6)

where $G_{F}$ is the Fermi constant, $V_{uQ}$ is the $uQ$ entry of the Cabibbo-Kobayashi-Maskawa matrix.
FIG. 1: Intermediate virtual meson contributions to $\tau^- \to VP^-\nu$ decays

\[
\beta^+ = \frac{1}{4m_V^2} \left\{ |f|^2 + 4m_V^2 |g|^2 (s-2\Sigma^2) + |a_+|^2 s^2 \beta_{VP}^2 + 2\text{Re}[fa^*_+]2(s-3m_V^2-m_P^2) \right\}
\]
\[
\beta^- = \frac{1}{4m_V^2} \left\{ |f|^2 - 4m_V^2 s|g|^2 + |a_-|^2 s^2 \beta_{VP}^2 + 2\text{Re}[fa^*_-](s+\Delta^2) \right\}
\]
\[
\beta = \frac{1}{4m_P^2} \left\{ |f|^2 + 4m_P^2 \Delta^2 |g|^2 + (fa^*_+)^*(s+\Delta^2) + (fa^*_+)(s-3m_V^2-m_P^2) \right\}
\]
\[
\alpha = |f|^2 + s^2 |g|^2 \beta_{VP}^2 .
\]

(7)

The Feynman diagrams with the intermediate mesons that connect the weak current and the strong vertex in $\tau \to VP\nu$ decays are shown in Figure 1. Using the Feynman rules for the elements of these diagrams, we get the following expressions for the form factors ($V_j$, $A_j$ and $P_j$ denote vector, axial and pseudoscalar intermediate meson states, respectively):

\[
g = \frac{1}{2} \sum_j \frac{f_{V_j}g_{VjVP}}{D_{V_j}(s)} ,
\]
\[
f = -\frac{1}{2}(s+\Delta^2) \sum_j \frac{f_{A_j}g_{A_jVP}}{D_{A_j}(s)} ,
\]
\[ a_+ = \frac{1}{2} \sum_j f_{A_j} g_{A_j V P} \frac{D_{A_j}(s)}{D_j(s)} + 2 \sum_j f_{P_j} g_{P_j V P} \frac{D_{P_j}(s)}{D_j(s)}, \]
\[ a_- = \frac{1}{2} \sum_j f_{P_j} g_{P_j V P} \frac{D_{P_j}(s)}{D_j(s)}, \] (8)

where \( f_{M_j} \) denotes the weak coupling of the \( M_j \) intermediate meson, \( g_{M_j V P} \) is its strong coupling to the \( V P \) final state, and \( D_{M_j}(s) \equiv s - m_{M_j}^2 + i m_{M_j} \Gamma_{M_j} \), where \( m_{M_j} \) (\( \Gamma_{M_j} \)) is the mass (width) parameter of the corresponding intermediate state.

### III. STRANGENESS-CONSERVING DECAYS

The G-parity properties of the weak currents and \( V \pi^- \) system in this case, impose \( f = a_- = a_+ = 0 \). As in previous papers [4, 6], we will assume that the non-vanishing vector form factor is saturated by the exchange of two vector resonances (the \( \rho(770) \) and the \( \rho'(1523) \)). Thus we get (\( V = \omega, \phi \)):

\[ g(s) = \frac{f_{\rho} g_{\rho V \pi}}{2D_{\rho}(s)} \left\{ 1 + \alpha_{V \pi} \frac{D_{\rho}(s)}{D_{\rho'}(s)} \right\}, \] (9)

where \( \alpha_{V \pi} = f_{\rho'} g_{\rho' V \pi}/f_{\rho} g_{\rho V \pi} \) is the only free parameter of the model at this stage. We chose the \( \rho'(1523) \) state (\( m_{\rho'} = 1523 \) MeV and \( \Gamma_{\rho'} = 400 \) MeV [8, 14, 15]) as the second vector resonance, instead of the \( \rho(1450) \) [1], because its larger width allows for a better fit to data on the spectral function [8, 14, 15].

As usual [15], we can define a vector spectral function whose expression becomes very simple in this case:

\[ v(s) = \frac{s^{3/2}}{12\pi} |g(s)|^2. \] (10)

This spectral function has been measured by the ALEPH [7] and CLEO [8] Collaborations for the dominant \( \omega \pi^- \) final state. In order to fit the data on the spectral function, we use:

\( f_{\rho} = (170.0 \pm 3.4) \times 10^3 \) MeV\(^2\) and \( g_{\rho \omega \pi} = (15.2 \pm 1.9) \times 10^{-3} \) MeV\(^{-1}\) (this value is a bit larger than the average value \( g_{\rho \omega \pi} = (12.3 \pm 1.2) \times 10^{-3} \) MeV\(^{-1}\) obtained from \( \rho^\pm \rightarrow \pi^\pm \gamma, \rho^0 \rightarrow \pi^0 \gamma \) and \( \omega \rightarrow \pi^0 \gamma \) decays, but they agree within their error bars). A fit to the spectral function reported in Ref. [7] gives us \( \alpha_{\omega \pi} = -0.57 \pm 0.11 \). The data of Ref. [8] and the fitted curves of the spectral function are shown in Figure 2.
FIG. 2: Spectral function $v(s)$ of the $\omega\pi^-$ system: the best fit is represented by the solid line; the experimental data from CLEO [8] are shown with solid dots.

Using the above value of $\alpha_{\omega\pi}$ we can derive the following branching fraction by integration of Eq. (6):

$$B(\tau^- \rightarrow \omega\pi^-\nu_\tau) = (1.95 \pm 0.60)\%,$$

which is in very good agreement with the experimental value shown in Table I.

Now we focus on the $\phi\pi$ decay channel. We will assume that flavor SU(3) is a good symmetry for the VVP couplings of the octet of vector mesons and of their radial excitations. Under this assumption, we can get $\alpha_{\phi\pi} = \alpha_{\omega\pi}$ for the relative weights of $\rho$ and $\rho'$ contributions in Eq. (9). In addition we use $g_{\rho\phi\pi} = -(1.57 \pm 0.03) \times 10^{-3}$ MeV$^{-1}$, which is obtained from the $\phi \rightarrow \rho\pi \rightarrow \pi^+\pi^-\pi^0$ decay rate [1]. Now, if we insert these parameters into Eq. (6), we get:

$$B(\tau^- \rightarrow \phi\pi^-\nu_\tau) = (3.64 \pm 0.93) \times 10^{-5},$$

which clearly favors the result reported by BABAR [11] (see Table I).
IV. COUPLING CONSTANTS FOR $\Delta S = -1$ DECAYS

The case of $\Delta S = -1$ tau decays is more difficult to deal with because the vector and axial weak currents contribute to the decay amplitude. Consequently, more independent information is needed to specify the input coupling constants. The vector form factor $g$ will be assumed to be dominated by the $K^* = K^*(892)$ and the $K^{*\prime} = K^*(1410)$ intermediate vector mesons (the coupling of the $K^{*\prime\prime} = K^*(1680)$ to the $VK^-$ system is more suppressed). The axial form factors $f, a_\pm$ will originate from the exchange of the $K^-$ pseudoscalar and the $K_1 = K_1(1270), K_1' = K_1(1400)$ axial mesons. The expressions for the form factors were given in Eq. (9), and the numerical values of coupling constants will be discussed in the following subsections.

A. Weak couplings

The value of the $K^-$ weak coupling is known with good precision, $f_K = (159.8 \pm 1.5)\text{ MeV}$, and is extracted from $K \to \mu\nu$ decays by including the effects of radiative corrections. The weak couplings of the other mesons can be extracted from the measurements of $\tau \to K\nu$ decays (where $K$ denotes either state among $K^*, K^{*\prime}, K_1, K_1'$ mesons). We get:

\[
\begin{align*}
    f_{K^*} &= (188.9 \pm 4.1) \times 10^3 \text{ MeV}^2, \\
    f_{K^{*\prime}} &= (170^{+80}_{-57}) \times 10^3 \text{ MeV}^2, \\
    f_{K_1} &= (215 \pm 25) \times 10^3 \text{ MeV}^2, \\
    f_{K_1'} &= (170 \pm 130) \times 10^3 \text{ MeV}^2.
\end{align*}
\]

The weak coupling with the largest uncertainty corresponds to the $K_1'$ meson. Since the contribution of the $K_1'$ intermediate state will provide an important contribution, a more precise measurement of the $\tau \to K_1(1400)\nu$ would be suitable to improve the accuracy of our prediction.

B. $KVK$ couplings

The determination of the strong couplings of the intermediate resonances to the $VK^-$ system in a reliable way is also a difficult task, either because such decays are not allowed by kinematics
or because there are not independent processes where their contribution can be studied. Thus we will strongly rely on SU(3) flavor symmetry to fix their values when necessary.

From the experimental value of the $\phi \to K^+K^-$ branching fraction $^{[1]}$, we get $g_{K^+\phi K^-} = (4.48 \pm 0.04)$. Now, using the SU(3) symmetry and assuming an ideal value for the $\omega - \phi$ mixing angle ($\tan \theta_V = 1/\sqrt{2}$), we get:

$$g_{K^+\omega K^-} = g_{K^+\phi K^-} \tan \theta_V = (3.17 \pm 0.03).$$

An alternative calculation of the $K\nu K$ couplings can be obtained by assuming the vector-meson dominance model of the kaon electromagnetic form factors at zero momentum transfer:

$$F_{K^+}(0) = \frac{g_{K^+\rho K^+}}{\gamma_\rho} + \frac{g_{K^+\omega K^+}}{\gamma_\omega} + \frac{g_{K^+\phi K^+}}{\gamma_\phi} = 1,$$

$$F_{K^0}(0) = \frac{g_{K^0\rho K^0}}{\gamma_\rho} + \frac{g_{K^0\omega K^0}}{\gamma_\omega} + \frac{g_{K^0\phi K^0}}{\gamma_\phi} = 0,$$

where $em_V/\gamma_V$ defines the coupling of the neutral vector meson $V$ to the photon. Now, we can use the SU(3) relations between the $PVPP'$ couplings (we assume again $\tan \theta_V = 1/\sqrt{2}$):

$$g_{K^+\rho K^-} = -g_{K^0\rho K^0} = \frac{1}{2}G_{PVPP'}^8,$$

$$g_{K^+\omega K^-} = g_{K^0\omega K^0} = \frac{1}{\sqrt{2}}G_{PVPP'}^8,$$

$$g_{K^+\phi K^-} = g_{K^0\phi K^0} = \frac{1}{2}G_{PVPP'}^8.$$

Solving the set of eqs. (14)-(16), we finally get:

$$g_{K^+\omega K^-} = \frac{\gamma_\omega \gamma_\phi}{2(\gamma_\phi + \sqrt{2} \gamma_\omega)} = 2.99 \pm 0.13$$

$$g_{K^+\phi K^-} = \frac{\gamma_\omega \gamma_\phi}{\sqrt{2}(\gamma_\phi + \sqrt{2} \gamma_\omega)} = 4.24 \pm 0.19,$$

which are quite similar values to the ones computed from the $\phi \to KK$ decays (see above).

### C. $V'VP$ strong couplings

Flavor SU(3) symmetry predicts the following relations among $V'VP^-$ couplings (we assume later below $\tan \theta_V = 1/\sqrt{2}$):

$$g_{K^+\omega K^-} = \frac{1}{2\sqrt{3}}G_{VV'PJ}^8 \sin \theta_V - 2r \sqrt{2} \cos \theta_V,$$
where \( r \equiv G^0_{V'VP}/G^8_{V'VP} \) is the ratio of SU(3) singlet and octet \( V'VP \) couplings. The value of \( r \) can be obtained from the ratio of Eqs. (21,22) using the values of \( g_{\rho\omega\pi} \) and \( g_{\rho\phi\pi} \) given in the previous section. In this way we get \( r = 0.1.256 \pm 0.038 \). If we insert now this value of \( r \) into eqs. (19,20) and use the experimental value of \( g_{\rho\omega\pi} \) (see previous section) and the ideal value of the \( \omega - \phi \) mixing angle, we get:

\[
g_{K^*\phi K} = -\frac{1}{2\sqrt{3}} G^8_{V'VP} \left[ \cos \theta_V + 2r \sqrt{2} \sin \theta_V \right],
\]

(20)

\[
g_{\rho\omega\pi} = \frac{1}{\sqrt{3}} G^8_{V'VP} \left[ \sin \theta_V + \sqrt{2} r \cos \theta_V \right],
\]

(21)

\[
g_{\rho\phi\pi} = \frac{1}{\sqrt{3}} G^8_{V'VP} \left[ \cos \theta_V - \sqrt{2} r \sin \theta_V \right],
\]

(22)

D. \( AVP \) couplings

The couplings of axial-vector mesons (\( A \)) to the \( VK^- \) system are the most difficult to determine. One may attempt to compute them from the measured branching fractions of \( K_1, K'_1 \) into the \( \omega K^- \) channel (which is the only allowed by kinematics). We get from this\(^1\):

\[
g_{K_1\omega K} = -(3.17 \pm 0.46) \times 10^{-3} \text{ MeV}^{-1},
\]

(25)

\[
g_{K'_1\omega K} = (4.8 \pm 2.4) \times 10^{-4} \text{ MeV}^{-1}.
\]

(26)

However, the \( K_1\phi K \) couplings can not be obtained in this way given that \( K_1, K'_1 \rightarrow \phi K \) decays are not allowed by kinematics.

Given the poor quality of the measurements used to extract the above couplings, we can resort again to the SU(3) flavor symmetry. Since the \( (K_1, K'_1) \) physical states are a mixture of the \( (K_{1A}, K_{1B}) \) states that belong to different \( 1^3P_1 (A) \) and \( 1^1P_1 (B) \) multiplets of axial

\(^1\) In order to extract \( g_{K_1\omega K} \) we have taken the maximum values for the mass and width of \( K_1 \) that are allowed by their error bars \([1]\).
mesons, we propose as the starting point the following $AVP$ interaction Lagrangian:

\[
\mathcal{L}_{AVP} = ig^8_{AVP} f_{abc} P^a (\partial^\alpha A^b \beta (\partial_\alpha V_\beta^c - \partial_\beta V_\alpha^c) + g^8_{AVP} d_{abc} P^a (\partial^\alpha B^b \beta (\partial_\alpha V_\beta^c - \partial_\beta V_\alpha^c) \\
+ \sqrt{2} \frac{g^0_{AVP} \delta_{ab}}{3} P^a (\partial^\alpha B^b \beta (\partial_\alpha V_0^c - \partial_\beta V_0^c).
\]

(27)

The physical strange axial mesons are defined in terms of flavor SU(3) states as follows:

\[
K_1 = K_{1B} \cos \alpha - K_{1A} \sin \alpha, \quad \text{for } K_1^+, K_1^0
\]
\[
K_1' = K_{1B} \sin \alpha + K_{1A} \cos \alpha, \quad \text{for } K_1'^+, K_1'^0
\]

(28)

and

\[
\bar{K}_1 = -\bar{K}_{1B} \cos \alpha - \bar{K}_{1A} \sin \alpha, \quad \text{for } K_1^-, \bar{K}_1^0
\]
\[
\bar{K}_1' = -\bar{K}_{1B} \sin \alpha + \bar{K}_{1A} \cos \alpha, \quad \text{for } K_1'^-, \bar{K}_1'^0
\]

(29)

The determination of the $K_{1A} - K_{1B}$ mixing angle is still controversial \[16\]. According to different authors its value can be in the range $30^0 \leq \alpha \leq 60^0 \[16\]. For illustrative purposes, in the present paper we will use $\alpha = 45^0 \[1\]$. If in addition we assume a nonet symmetry for the $1^1P_1$ couplings, namely $g^8_{AVP} = g^0_{AVP}$ and the ideal value for the $\omega - \phi$ mixing angle ($\tan \theta_V = 1/\sqrt{2}$), we get the following simplified expressions for the couplings that involve the $\phi$ and $\omega$ mesons (the expressions of the couplings constants for arbitrary values of the $\alpha$ and $\theta_V$ mixing angles are given in the Appendix):

\[
\begin{align*}
 g_{K_1'^+ \omega K^-} &= g_{K_1'^+ \omega K^0} = -g_{K_1'^- \omega K^+} = -g_{K_1'^- \omega K^0} = \frac{1}{2\sqrt{2}} \Sigma^+,
 g_{K_1'^+ \omega K^-} &= g_{K_1'^+ \omega K^0} = -g_{K_1'^- \omega K^+} = -g_{K_1'^- \omega K^0} = \frac{1}{2\sqrt{2}} \Sigma^-,
 g_{K_1'^+ \phi K^-} &= g_{K_1'^+ \phi K^0} = -g_{K_1'^+ \phi K^+} = -g_{K_1'^- \phi K^0} = \frac{1}{2} \Sigma^-,
 g_{K_1'^+ \phi K^-} &= g_{K_1'^+ \phi K^0} = -g_{K_1'^+ \phi K^+} = -g_{K_1'^- \phi K^0} = \frac{1}{2} \Sigma^+,
\end{align*}
\]

(30)-(33)

where we have defined $\Sigma^\pm \equiv g^8_{AVP} \pm g^0_{AVP}$.

We can fix the values of the effective couplings $\Sigma^\pm$ by using the decay rates of $K_1, K_1'$ axial mesons in the same limit where Eqs. (30)-(32) were obtained. Using the expressions given in the Appendix and comparing with the measured rates of $K_1'^- \to K^* \pi$ decays \[1\], we obtain:
\[ \Sigma_+ = (5.50 \pm 0.27) \times 10^{-3} \text{ MeV}^{-1}. \] Similarly, from the measured rate of \( K_1^+ \rightarrow \omega K \) we get:
\[ \Sigma_- = (1.36 \pm 0.67) \times 10^{-3} \text{ MeV}^{-1}. \]
Finally, if we insert these values into Eqs. (30)-(33), we get:
\[ g_{K_1^+ \omega K^-} = -(1.94 \pm 0.10) \times 10^{-3} \text{ MeV}^{-1}, \quad (34) \]
\[ g_{K_1^- \omega K^-} = (4.8 \pm 2.4) \times 10^{-4} \text{ MeV}^{-1}, \quad (35) \]
\[ g_{K_1^- \phi K^-} = -(6.8 \pm 3.4) \times 10^{-4} \text{ MeV}^{-1}, \quad (36) \]
\[ g_{K_1^- \phi K^-} = (2.75 \pm 0.14) \times 10^{-3} \text{ MeV}^{-1}. \quad (37) \]

Observe that the \( K_1 \omega K \) coupling in Eqs. (34) and (25) have similar sizes despite the different sources used for their determination. In our calculations we will use the numerical values shown in Eqs. (34)-(37).

V. STRANGENESS-CHANGING DECAYS

With the information on the coupling constants given in the previous section, the only free parameters to our disposal are the relative contributions of the vector meson contributions in the form factor \( g(s) \):
\[ \alpha_{\omega K} \equiv \frac{f_{K^*} g_{K^* \omega K}}{f_K g_{K^* \omega K}}, \quad \text{and} \quad \alpha_{\phi K} \equiv \frac{f_{K^*} g_{K^* \phi K}}{f_K g_{K^* \phi K}}. \quad (38) \]

We can further attempt the use of SU(3) symmetry to derive such couplings. Instead, we will fix the values of \( \alpha_{\omega K} \) by requiring that it reproduces the experimental branching fraction for the well measured \( \tau^- \rightarrow \omega K^- \nu \) decay (see Table I). Using this method we obtain two possible values: \( \alpha_{\omega K} = 0.54 \pm 0.38 \) and \( \alpha_{\omega K} = -0.77 \pm 0.40 \). Both are consistent with SU(3) since they have similar sizes to the value \( \alpha_{\omega \pi} = -0.57 \pm 0.11 \), which reproduces the \( \tau^- \rightarrow \omega \pi^- \nu \) decay data (see section 3).

Now, if we assume that \( \alpha_{\phi K} \approx \alpha_{\omega K} \) which is also expected on the basis of SU(3), we can predict:
\[ B(\tau^- \rightarrow K^- \nu) = \begin{cases} (2.2 \pm 2.6) \times 10^{-5}, & \text{for } \alpha_{\phi K} = 0.54 \pm 0.38, \\ (1.6 \pm 2.5) \times 10^{-5}, & \text{for } \alpha_{\phi K} = -0.77 \pm 0.40 \end{cases} \quad (39) \]
which are consistent with the experimental values measured by BABAR \cite{11} and BELLE \cite{10} Collaborations (see Table I).
Note that the large error bars quoted in Eq. (39) are dominated by the uncertainty in the $K'_1 = K_1(1400)$ weak decay constant given in Eq. (13), which was extracted from the poorly measured $\tau \to K_1(1400)\nu$ decay. This error bar can be reduced if we use a weak coupling constant obtained from a phenomenological quark model. Thus, for example, if we assume $\alpha = 45^0$, from the covariant quark model of ref. [16] we obtain $f^{\text{cqm}}_{K'_1} = (242 \pm 25) \times 10^3$ MeV$^2$. This value of the weak coupling does not affect in a sensitive way the ratio of $K'_1$ couplings extracted from $\tau^- \to \omega K^- \nu$ branching fraction, which now become: $\alpha_{\omega K^-} = 0.55 \pm 0.38$ or $\alpha_{\omega K^-} = -0.78 \pm 0.39$. Using the value of $f_{K'_1}$ obtained above, we get:

$$B^{\text{cqm}}(\tau^- \to \phi K^- \nu) = \begin{cases} 
(4.0 \pm 1.2) \times 10^{-5}, & \text{for } \alpha_{\phi K} = 0.55 \pm 0.38, \\
(3.3 \pm 1.0) \times 10^{-5}, & \text{for } \alpha_{\phi K} = -0.78 \pm 0.39
\end{cases}$$

which central values are in better agreement with experimental data of BABAR and BELLE (see Table I).

In Figure 3 we compare the invariant mass distribution of the $\phi K^-$ system, with the measurements reported by the BABAR Collaboration [11]. As we can observe, our model (with our numbers multiplied by an arbitrary scale) nicely reproduces the data on the invariant mass distribution. Although it is difficult to discriminate between values of the two-fold ambiguity in $\alpha_{\phi K}$, data seems to favor the solution with $\alpha_{\phi K} = 0.55$.

**VI. CONCLUSIONS**

The decays $\tau^- \to (\omega, \phi)P^-\nu_\tau$, where $P$ is a charged pseudoscalar meson, are studied in a phenomenological model where the form factors are dominated by the exchange of meson states with the appropriate quantum numbers. We rely on SU(3) flavor symmetry to fix the strong interaction coupling constants, and assume an ideal value for the $\omega - \phi$ mixing angle. Our predictions for the decay modes involving a $\phi$ vector meson:

$$B(\tau^- \to \phi\pi^-\nu_\tau) = (3.64 \pm 0.93) \times 10^{-5},$$

$$B(\tau^- \to \phi K^-\nu_\tau) = (4.0 \pm 1.2) \times 10^{-5},$$

are in very good agreement with measurements reported recently by the BABAR [11] and BELLE [10] Collaborations.
FIG. 3: Invariant mass distribution of the $\phi K^-$ system in tau decays. The solid (dashed) line corresponds to $\alpha_{\phi K} = 0.55$ ($\alpha_{\phi K} = -0.78$). Data points are taken from Ref. [11].

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Appendix

There are two octets of axial mesons corresponding to the $1^3P_1$ (denoted by $A$) and $1^1P_1$ (denoted by $B$) quantum number configurations. The $A$ ($B$) octet is composed of one isotriplet $a_1$ ($b_1$), two isodoublets $K_{1A}$ ($K_{1B}$) and one isosinglet $f_1^8$ ($h_1^8$) states. The axial-vector-pseudoscalar interaction is governed by the Lagrangian shown in Eq. (27), with physical strange mesons $K_1(1270)$ and $K_1(1400)$ defined in Eqs. (28)-(29).
From the interaction Lagrangian (27) we can derive the following strong coupling constants that involve the $K_1, K_1'$ axial mesons and the $\omega, \phi$ vector mesons of our interest:

\[
g_{K_1^+\omega K^-} = g_{K_1^0\omega K^0} = -g_{K_1^-\omega K^+} = -g_{K_1^0\omega K^0} = \frac{\sqrt{3}}{2} \left[ g_{AVP}^8 \sin \theta_V - \frac{1}{3} g_{BVP}^8 \cos \alpha (\sin \theta_V - 2 \sqrt{2} r_B \cos \theta_V) \right],
\]

\[
g_{K_1^+\omega K^-} = g_{K_1^0\omega K^0} = -g_{K_1^-\omega K^+} = -g_{K_1^0\omega K^0} = -\frac{\sqrt{3}}{2} \left[ g_{AVP}^8 \cos \alpha \sin \theta_V + \frac{1}{3} g_{BVP}^8 \sin \alpha (\sin \theta_V - 2 \sqrt{2} r_B \cos \theta_V) \right],
\]

\[
g_{K_1^+\phi K^-} = g_{K_1^0\phi K^0} = -g_{K_1^-\phi K^+} = -g_{K_1^0\phi K^0} = \frac{\sqrt{3}}{2} \left[ g_{AVP}^8 \sin \alpha \cos \theta_V - \frac{1}{3} g_{BVP}^8 \cos \alpha (\cos \theta_V + 2 \sqrt{2} r_B \sin \theta_V) \right],
\]

\[
g_{K_1^+\phi K^-} = g_{K_1^0\phi K^0} = -g_{K_1^-\phi K^+} = -g_{K_1^0\phi K^0} = -\frac{\sqrt{3}}{2} \left[ g_{AVP}^8 \cos \alpha \cos \theta_V + \frac{1}{3} g_{BVP}^8 \sin \alpha (\cos \theta_V + 2 \sqrt{2} r_B \sin \theta_V) \right],
\]

where we have defined the ratio of singlet and octet couplings $r_B \equiv g_{BVP}^0/g_{BVP}^8$. In the above expressions, $\theta_V$ (respectively $\alpha$) denotes the $\omega - \phi$ $(K_1 - K_1')$ mixing angle.

Other useful couplings involving the $K_1$ and $K_1'$ axial mesons are:

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = \frac{1}{2} (g_{AVP}^8 \sin \alpha + g_{BVP}^8 \cos \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = \frac{1}{\sqrt{2}} (g_{AVP}^8 \sin \alpha + g_{BVP}^8 \cos \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = -\frac{1}{2} (g_{AVP}^8 \cos \alpha - g_{BVP}^8 \sin \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = -\frac{1}{\sqrt{2}} (g_{AVP}^8 \cos \alpha - g_{BVP}^8 \sin \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = \frac{1}{2} (g_{AVP}^8 \sin \alpha - g_{BVP}^8 \cos \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = -\frac{1}{\sqrt{2}} (g_{AVP}^8 \sin \alpha - g_{BVP}^8 \cos \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = \frac{1}{2} (g_{AVP}^8 \cos \alpha + g_{BVP}^8 \sin \alpha),
\]

\[
g_{K_1^+\rho K^-} = g_{K_1^0\rho K^0} = -g_{K_1^-\rho K^+} = -g_{K_1^0\rho K^0} = \frac{1}{\sqrt{2}} (g_{AVP}^8 \cos \alpha + g_{BVP}^8 \sin \alpha).
\]

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