Anti-de-Sitter Island-Universes from 5D Standing Waves

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Abstract

We construct simple standing wave solutions in a 5D space-time with a ghost-like scalar field. The nodes of these standing waves are ‘islands’ of 4D anti-de-Sitter space-time. In the case of increasing (decreasing) warp factor there are a finite (infinite) number of nodes and thus a finite (infinite) number of anti-de-Sitter island-universes having different gravitational and cosmological constants. This is similar to the landscape models, which postulate a large number of universes with different parameters.

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1 Introduction

The infinite extra dimension brane theories \cite{1,2,3,4} have been one of the most active areas of study in the past decade. A concise overview of these models can be found in \cite{5}. The original brane models had 5D space-times with 4D $\delta$-function sources which were positive or negative tension thin branes. Later works replaced the $\delta$-function sources with localized energy momentum tensors of finite thickness which were called thick branes. Examples of thick brane models included fluid-like matter sources \cite{6,7}, scalar field matter sources \cite{8,9}, and phantom scalar field sources \cite{10,11,12,13,14,15}. Most brane world models focused on static solutions, i.e. both metric and any fields being time independent. An exception to this are S-brane solutions \cite{16,17,18,19,20} which are used for cosmological studies.

In this paper we present a simple standing gravitational wave solution as a possible 5D brane model. The existence of this solution requires a negative cosmological constant and a phantom/ghost-like scalar field (i.e. a field with a negative sign in front of the field kinetic energy term of the Lagrangian) in the bulk. Such ghost-like scalar fields are generally
problematic since they tend to make the system unstable. To counter this criticism we will show that our model can be embedded in a 5-dimensional Weyl model of gravity. Because of the geometrical origin of the coupling of the scalar to the space-time and since the Weyl scalar does not couple with other matter fields we are able to avoid the usual problems of instability due to ghost fields.

We assume that the ghost-like field vanishes on the brane at the origin. This condition leads to a quantization of the oscillation frequency of the standing wave which in turn makes the scalar field vanish at a finite or infinite number of nodes of the standing wave depending on whether the warp factor of the metric is increasing or decreasing. By taking the matter fields to bind to the standing waves nodes one has a finite or infinite number of anti-de-Sitter island-universes, each with different parameters. In the simple model presented here the different parameters are the effective 4D gravitational constant and the 4D cosmological constant. These two 4D constants are fixed in terms of the 5D constants and the value of the warp factor at the nodes.

For the case of an increasing warp factor, and a finite number of anti-de-Sitter ‘islands’, one could address the family puzzle by taking different fermion generations as bound to different nodes and fixing the number of nodes in the bulk at three. For the case of a decreasing warp factor, and infinite number of anti-de-Sitter island-universes, one has something like the landscape picture in string theory/M theory, with its large number of universes with different parameters.

In the model presented in this paper ordinary matter fields are assumed to be bound to one of the anti-de-Sitter ‘islands’. This is an advantage over the original one-brane or two-brane models of, where the ordinary matter fields are assumed to be localized to a brane with positive or negative tension. In these models one needs to explain why these brane tensions are not observed.

There are several open questions or problems with the present 5D standing wave solution as brane world model: The first question concerns the stability of the whole 5D solution given the presence of the ghost-like scalar field. This issues of the stability of brane world models in the presence of sources which violate some or all of the usual energy conditions (e.g. negative tension branes, or ghost fields) has been known since the beginning of the study of brane world models. In section 3 we present a possible resolution to this issue by considering Weyl’s generalization of Riemannian geometry. We also offer some discussion in the conclusion section of cases where a ghost field, at least at the classical level, leads to greater stability of brane world models as compared to normal scalar fields. The second question is: “On or near the nodes are the gravitational fluctuations confined so as to give effective 4D gravity near the nodes?” In section 6 will show that near the nodes one does get effective 4D gravity in much the same way as in. The last question/problem is give a localization mechanism for the matter fields (i.e. fields with spin 0, spin 1/2, spin 1) on the brane. This localization problem has been an unresolved question for brane world models in general. Here we simply postulate that matter fields are bound to the nodes of the standing wave solutions. In section 7 we give two possible mechanisms for accomplishing this binding of matter fields to the nodes. The first mechanism involves the coupling of the quadrupole moment of the stress–energy tensor of
the matter fields to the Riemann tensor. The second mechanism involves coupling of matter fields to the ghost-like field.

2 Metric and Einstein’s equations

We study solutions of the 5-dimensional Einstein equations,

\[ R_{ab} - \frac{1}{2} g_{ab} R = \frac{8\pi G_5}{c^4} T_{ab} - \Lambda_5 g_{ab}, \]  

(1)

where \( G_5 \) and \( \Lambda_5 \) are 5D Newton and cosmological constants respectively [1, 2, 3, 4]. Lower case Latin indices \( a, b \) run over the full 5D space-time 0, 1, 2, 3, 4. The metric ansatz we use is

\[ ds^2 = e^{2a|r|} \left( dt^2 - e^u dx^2 - e^u dy^2 - e^{-2u} dz^2 \right) - dr^2, \]  

(2)

where \( a \) is a constant and the ansatz function \( u = u(t, r) \) only depends on the time \( t \) and the extra coordinate \( r \). This ansatz is a combination of the 5D warped brane world model through the \( e^{2a|r|} \) term [1, 2, 3, 4] plus an anisotropic \((t, r)\)-dependent warping of the brane coordinates, \( x, y, z \), through the terms \( e^{u(t,r)}, e^{-2u(t,r)} \). The absolute value sign around \( r \) gives a \( \delta \)-function source/brane at \( r = 0 \) exactly like the one brane models of [1, 2, 3, 4]. Since we will focus on standing wave solutions ‘to the right’, i.e. \( r > 0 \), we drop the absolute value sign in (2). We study both decreasing \((a < 0)\) and increasing \((a > 0)\) warp factors. We also find that for both increasing and decreasing warp factors the ansatz function \( u(t, r) \) has nodes – \( u(t, r) = 0 \) – at specific values of \( r \). At these nodes the space-time in (2) reduces to 4D Minkowski space-time plus a scaled, negative cosmological constant – at the nodes the space-time becomes effectively 4D anti-de-Sitter. As one moves away from these nodes there is an anisotropic stretching/shrinking along the \( x, y, z \) directions due to the metric components \( e^{u(t,r)}, e^{-2u(t,r)} \). If \( u(t, r) \) is positive (negative) as one moves away from the node in \( r \) the \( x, y \) directions will expand (shrink) as \( e^u \) while the \( z \) direction will shrink (expand) as \( e^{-2u} \). In section 7 we will use this feature of the metric to propose a possible localization mechanism for matter fields.

For the ansatz (2) we find the non-zero components of the Ricci tensor:

\[
\begin{align*}
R_{tt} &= e^{2ar} \left( -\frac{3}{2} e^{-2ar} \ddot{u}^2 + 4a^2 \right), \\
R_{xx} &= R_{yy} = e^{2ar+u} \left( \frac{1}{2} e^{-2ar} \ddot{u} - 2au' - \frac{1}{2} u'' - 4a^2 \right), \\
R_{zz} &= e^{2ar-2u} \left( -e^{-2ar} \ddot{u} + 4au' + u'' - 4a^2 \right), \\
R_{rr} &= \frac{3}{2} u'^2 - 4a^2, \\
R_{rt} &= -\frac{3}{2} \dot{u} u',
\end{align*}
\]  

(3)

where overdots mean time derivatives and primes stand for derivatives with respect to \( r \). In (3) there should be terms proportional to \( \delta(r) \) coming from the tension of the thin brane at
$r = 0$. We do not write this explicitly since we focus on $r > 0$ and have thus dropped the absolute value in the warp factor of the metric (2).

3 Source scalar field

To the metric above we add a massless, non-self interacting scalar phantom/ghost-like field, $\phi(t, z)$ \cite{36, 37}, which obeys the Klein-Gordon equation,

$$
\frac{1}{\sqrt{g}} \partial_a (\sqrt{g} g^{ab} \partial_b \phi) = e^{-2ar} \ddot{\phi} - \dot{\phi}^2 - 4a\dot{\phi} = 0 .
$$

(4)

Here $g$ is the determinant of the 5-dimensional background space-time given by (2). The energy-momentum tensor of the phantom-like field $\phi$ is taken in the form:

$$
T_{ab} = -\partial_a \phi \partial_b \phi + \frac{1}{2} g_{ab} \partial^c \phi \partial_c \phi .
$$

(5)

Strictly speaking $\phi$ is not a phantom field as defined in \cite{38}, where the criterion for a phantom field was $p/\rho < -1$ ($p$ and $\rho$ are the pressure and energy density of the field respectively). From (5) one can obtain $p$ and $\rho$ for the field $\phi$ and since the field is non-self interacting one does not have $p/\rho < -1$.

To avoid the well-known problems with stability that occur with ghost fields we can associate the ghost-like field $\phi$ with the geometrical scalar field in a five-dimensional, integrable Weyl model. In Weyl’s model a massless scalar appears through the definition of the covariant derivative of the metric tensor,

$$
D_c g_{ab} = g_{ab} \partial_c \phi .
$$

(6)

This is a generalization of the Riemannian case where the covariant derivative of the metric is zero. The result in (6) implies that the length of a vector is altered by parallel transport. Weyl’s scalar field in (6) may imitate a massless scalar field – either an ordinary scalar or ghost-like scalar \cite{21, 22, 23, 24}. The gravitational action for Weyl’s 5D integrable model can be written as

$$
S_g = \frac{1}{16\pi G_5} \int d^5 x \sqrt{g} \left[ R - (6 - 5\xi) \partial^a \phi \partial_a \phi \right] ,
$$

(7)

where $\xi$ is an arbitrary constant which can take any value. For example, if one takes $\xi = 13/10$ in (7) then the coefficient in front of $\partial^a \phi \partial_a \phi$ becomes $-1/2$. This would give a ghost field which would lead exactly to the equation of motion (1) and the energy-momentum tensor (5) for our phantom-like scalar field. Thus we can start with a 5D Weyl model and require that we have Riemann geometry on the brane by assuming that the geometrical scalar $\phi$ is independent of brane coordinates $x^i$ and vanishes on the brane. We will show later that our solution has exactly this character – the scalar field $\phi$ vanishes at the nodes of $u(t, r)$ and is independent of $x, y, z$. Associating our scalar field with the geometrical Weyl scalar defined via (6) avoids the usual instability problems of ghost fields since the Weyl model is known to be stable for any value of $\xi$. 

4
4 The solution

For (5) one can rewrite the Einstein equations (1) in the form:

\[ R_{ab} = -\partial_a \sigma \partial_b \sigma + \frac{3}{2} g_{ab} \Lambda_5 , \]

where the gravitational constant has been absorbed via the redefinition of the scalar field,

\[ \sigma = \frac{\sqrt{8\pi G_5}}{c^2} \phi . \]

By combining (8) and (3) one can see that the “constant” terms from \( R_{\mu\nu} \) (i.e. those terms that up to some metric factor are \( \pm 4a^2 \)) can be canceled if the 5D cosmological constant is chosen as

\[ \Lambda_5 = \frac{8}{3} a^2 . \]

This is the same as the fine tuning used in standard 5D brane models [1, 2, 3, 4].

The terms from \( R_{\mu\nu} \) which are quadratic in \( u \) can be accounted for if one assumes that

\[ \sigma(t, r) = \sqrt{\frac{3}{2}} u(t, r) , \]

so that \( u(t, r) \) satisfies (4). A similar equality between the scalar field and metric ansatz function was found for the domain wall plus standing wave solutions of [39]. Since our scalar field is proportional to the metric ansatz function \( u(t, r) \) the scalar field will vanish wherever \( u(t, r) \) has nodes. This requirement, that the scalar field vanish at the zeros of \( u(t, r) \), was one of the necessary conditions pointed out in the last section in order to be able to associate our scalar field with a Weyl scalar field.

Because of (10) and (11), solving Einstein equations has been reduced to finding solutions to the ordinary differential equation,

\[ e^{-2ar} \ddot{u} - u'' - 4au' = 0 . \]

We want to have a standing wave solution to (12), so we use separation of variables writing

\[ u(t, r) = C \sin(\omega t) f(r) , \]

where \( C \) and \( \omega \) are constants. Because of (11) the same separation applies to \( \sigma \), but with a different constant, \( C \rightarrow \sqrt{\frac{2}{3}} C \). Equation (12) now becomes (13) is:

\[ f'' + 4af' + \omega^2 e^{-2ar} f = 0 . \]

The general solution to this equation is:

\[ f(r) = A e^{-2ar} J_2 \left( \frac{\omega}{a} e^{-ar} \right) + B e^{-2ar} N_2 \left( \frac{\omega}{a} e^{-ar} \right) , \]

where \( J_2 \) and \( N_2 \) are Bessel functions of the first and second kind, respectively.
where $A, B$ are constants and $J_2$ and $N_2$ are 2nd order Bessel functions of the first and second kind respectively. Normally the $N_2$ Bessel functions are discarded since they blow up at the origin. But here the functional dependence is $e^{-ar}$, rather than $r$, and neither $J_2$ nor $N_2$ diverges at $r = 0$.

Before moving on to the discussion of the physical meaning of the solutions in (15) we add the boundary condition that the ghost-like field should vanish at the brane, $r = 0$. To accomplish this we should take either $A = 0$ or $B = 0$, since the zeros of $J_2$ and $N_2$ do not coincide. Then we set

$$\frac{\omega}{a} = X_{2,n},$$

(16)

where $X_{2,n}$ is the $n^{th}$ zero of the 2nd order Bessel function $J_2$ or $N_2$, depending on whether one takes $A = 0$ or $B = 0$ in (15). The condition (16) quantizes the oscillation frequency, $\omega$.

5 Standing waves as Anti-de-Sitter ‘islands’

We now analyze the physical consequences for metric (2) given the solutions given by (13), and (15).

First we note that (2) in terms of the behavior of the $x, y, z$ coordinates gives different asymptotic properties for the increasing ($a > 0$) and decreasing ($a < 0$) warp factor cases. For the increasing warp factor ($a > 0$) the metric function, $u(t, r) \propto f(r)$, decreases like $e^{-2ar}$ as one moves into the bulk. As one goes to large $r \rightarrow \infty$ this factor of $e^{-2ar}$ drives $u(t, r) \rightarrow 0$ and one has an anti-de-Sitter space-time but scaled up by an overall factor $e^{2ar}$.

For the decreasing factor ($a < 0$) the function $u(t, r)$ grows like $e^{2ar}$ into the bulk therefore distances in the $x$ and $y$ directions expand like $\text{Exp}(e^{2ar})$, while in the $z$ direction they shrink like $\text{Exp}(-2e^{2ar})$.

The second observation is that $u(t, r)$ is oscillatory and has zeros at various points along the $r$-direction – whenever $f(r) = 0$. This happens when $J_2(r) = 0$ or $N_2(r) = 0$, depending on if one is considering $B = 0$ or $A = 0$. At these zero points, $r_m$, one has $u(t, r_m) = 0$ and the metric in (2) reduces to the standard 5D brane metric with an exponential warp factor [1, 2, 3, 4]. This spatial oscillatory behavior of $u(t, r)$ leads to ‘islands’ of anti-de-Sitter space-times in the bulk.

The third observation is that the physical parameters on each node are scaled by the warp factor $e^{ar}$ or some power thereof. For example, taking the 4D reduction of 5D Einstein equations [1] one finds that on the nodes of the anti-de-Sitter ‘islands’ there are effective 4D, negative cosmological constants given by

$$\Lambda_4 = e^{2ar_m} \Lambda_5.$$  

(17)

Thus the values of the effective 4D cosmological constant on the nodes is set by the scaled 5D cosmological constant. In the usual thin brane models [1, 2, 3, 4] the 5D cosmological constant is fine tuned to exactly cancel the 4D brane tension. Thus on the brane the effective 4D cosmological constant is zero. In our case at the nodes, $r_m$, there is no brane tension and so our effective 4D cosmological constant is non-zero and given by (17). For a decreasing
wrap factor (i.e. \( a < 0 \)) one has a cosmological constant which is exponentially suppressed relative to the true 5D value, \( \Lambda_5 \). Below we argue that these nodes are 4D anti-de-Sitter island universes on which matter fields are bound and on which gravity is effectively 4D.

For the case \( a > 0 \) and \( B = 0 \), there will be \((n + 1)\) such ‘islands’ (\( n \) is the number of the zero from (16)). As \( r \) runs from 0 to \( \infty \) the argument of \( J_2 \) in (15) runs from \( X_{2,n} \) to 0 giving \( n \) zeros. One additional zero comes from \( f(r \to \infty) = 0 \). The zeros will occur at \( r_m \) where \( \omega e^{-ar_m}/a = X_{2,m} \) with \( X_{2,m} \) being a zero of \( J_2 \) with \( m < n \). The case \( a > 0 \) and \( A = 0 \) works out the same with \((n + 1)\) ‘islands’ since one also has an additional zero coming from \( f(r \to \infty) \to 0 \) – in this case the divergence coming from \( N_2(0) \) is dominated by the \( e^{-2ar} \) pre-factor in (15). The \( \omega e^{-ar}/a \) dependence of \( f(r) \) has the effect of stretching out the zeros of the Bessel functions, i.e. the spatial oscillation frequency decreases with \( r \).

In the case \( a < 0 \) (with either \( A = 0 \) or \( B = 0 \)) there are an infinite number of zeros for \( u(t, r) \) due to the \( \omega e^{ar}/a \) dependence of \( f(r) \). In addition the zeros are compressed as \( r \) increases, i.e. the spatial oscillation frequency increases with \( r \). This compression \((a > 0)\) and stretching \((a < 0)\) of the location of the nodes has a connection with the effective thickness of the 4D anti-de-Sitter islands – the compression of the nodes tends to lead to ‘islands’ with a smaller effective thickness while stretching of the nodes leads to ‘islands’ with a large effective thickness. We discuss this further in section 7 where we examine localization mechanisms for matter fields to the nodes.

6 Localization of gravity

Near \( r = 0 \) in metric (2) one has a node + brane as in the 5D single brane models of [1, 2, 3, 4]. The gravitational perturbations around \( r = 0 \) will have one delta-function bound state and continuum states which start from zero mass i.e. which do not have a mass gap. Thus at the \( r = 0 \) node one has effective 4D gravity as in the original models. We now look at the nodes of \( u(r, t) \) for \( r > 0 \) to see if near these nodes gravity also becomes approximately 4D. This is done by studying the 4D perturbations of the metric near the nodes. If there is a zero mass graviton mode near the node then its exchange between particles localized on the node will lead to a Newtonian potential with corrections coming from the massive modes \((m > 0)\). Close to the nodes one has \( u(r, t) \approx 0 \) and the metric (2) takes on the usual 5D warped geometry. Near the nodes small fluctuations around this background can be written as

\[
 ds^2 \approx \left[ e^{2ar} \eta_{\mu\nu} + h_{\mu\nu}(x_\mu, r) \right] dx^\mu dx^\nu - dr^2 ,
\]

where \( x_\mu = (t, x, y, z) \) and Greek indices run over 4D space-time \(- \mu, \nu = 0, 1, 2, 3 \). The tensor, \( h_{\mu\nu} \), gives 4D perturbations around the usual 5D brane world background. Further we fix the gauge so that \( h_{\mu\nu} \) is transverse and traceless,

\[
 \partial_\mu h^\mu_\nu = h^\mu_\mu = 0 .
\]

Next we separate the tensor perturbation into a 1D and 4D part as

\[
 h_{\mu\nu}(x_\mu, r) = \psi(r) h^{(4)}_{\mu\nu}(x_\mu) ,
\]
with $h_{\mu\nu}^{(4)}(x^\mu)$ satisfying
\[ \Box^{(4)} h_{\mu\nu}^{(4)}(x^\mu) = p^2 = m^2. \] (21)

Putting all this together yields the following equation for the 1D part of the perturbations
\[ \psi'' - 4a^2 \psi - m^2 e^{-2ar} \psi = 0. \] (22)

In order for gravity to be effectively 4D near the nodes (22) should have a zero mode solution $-m = 0$. It is easy to see that it does have a zero mode given by,
\[ \psi_0(r) = e^{2ar}, \] (23)
up to a normalization constant. Equation (22) also has $m \neq 0$ solutions which are similar to (15) i.e. combinations of second order Bessel functions of the first and second kind, $J_2, N_2, \cdots$. The exchange of the zero mode, $m = 0$, between massive particles fixed on the node, will lead to a $1/r$ Newtonian potential while the exchange of the $m \neq 0$ modes will lead to corrections which go as $1/(a^2 r^2)$ [1, 2, 3, 4, 5].

The effective 4D Newton’s constant on a particular node can be obtained using the above results. Let us focus on one particular node, $r_m$. The massless gravitational perturbation is
\[ h_{\mu\nu} = e^{2ar} h_{\mu\nu}^{(4)}(x^\mu), \] (24)
where $h_{\mu\nu}^{(4)}(x^\mu)$ is the 4D part. The effective 4D Newton’s constant can be obtained by inserting (24) into the 5D gravitational action and looking at the quadratic part
\[ S_g = \frac{1}{16\pi G_5} \int_{r_m - d}^{r_m + d} dr \int d^4 x (\partial h)^2 \]
\[ = \frac{1}{16\pi G_5} \int_{r_m - d}^{r_m + d} dr e^{2ar} \int d^4 x (\partial h_{\mu\nu})^2 \]
\[ = \frac{e^{2ar} \sinh(2ad)}{16\pi G_5 a} \int d^4 x (\partial h_{\mu\nu})^2, \] (25)
where $2d$ is the width around the node, $r_m$, within which the matter particles are assumed to be bound. The index $m$ satisfies $m \leq n$ where for $m = n$ one is located at the $r = 0$ brane. Using the last expression in (25) the effective 4D Newton’s constant can be written as
\[ G_4 = \frac{aG_5}{e^{2ar_m} \sinh(2ad)} \approx \frac{G_5}{2d e^{2ar_m}}. \] (26)

The last expression assumes a small thickness i.e. $d \ll 1$, or $ad \ll 1$. In the case when $a > 0$ one can exponentially suppress the 4D Newton’s constant, $G_4$, relative to the 5D Newton’s constant, $G_5$. 

8
Localization of matter

So far in this paper we have assumed some mechanism for binding matter fields to the nodes of the standing waves. Now we want to give two possible localization mechanisms.

The first mechanism is borrowed from condensed matter physics. It is known that standing electromagnetic waves, so called optical lattices, can provide trapping of various particles by scattering and dipole forces [40, 41]. In [42, 43] localization was demonstrated through quadrupole forces as well. It is also known that the motion of test particles in the field of a gravitational wave is similar to the motion of charged particles in the field of an electromagnetic wave [44]. Thus standing gravitational waves could also provide confinement of matter via quadrupole forces. As an example let us consider the equations of motion of the system of spinless particles in the quadrupole approximation [45, 46, 47],

\[ \frac{dp^\mu}{ds} = F^\mu_{\text{quad}} = -\frac{1}{6} J^{\alpha\beta\gamma\delta} D^\mu R_{\alpha\beta\gamma\delta}, \]

where \( p^\mu \) is the total momentum of the matter field/particle and \( J^{\alpha\beta\gamma\delta} \) is the quadrupole moment of the stress-energy tensor for the matter field/particle. The oscillating metric due to gravitational waves should induce a quadrupole moment in the matter fields. If the induced quadrupole moment is out of phase with the gravitational wave the system energy increases in comparison with the resonant case and the fields/particles will feel a quadrupole force, \( F^\mu_{\text{quad}} \), which ejects them out of the high curvature region i.e. it would localize them at the nodes. This gravitational binding mechanism would be effective for all types of matter fields since gravity couples to all forms of energy-momentum. Since the matter fields are ejected from the high curvature region and toward the nodes the width of the anti-de-Sitter ‘islands’ (i.e. the thickness of the branes) depends on how rapidly the ansatz function \( u(t,r) \propto f(r) \) from (13) (15) changes from zero. The width of some anti-de-Sitter ‘island’ will depend in the derivative of \( f(r) \) at the node – \( f'(r_m) \). The larger \( f'(r_m) \) is the smaller the width of the anti-de-Sitter ‘island’. Now \( f'(r_m) \) depends on the size of either \( A \) or \( B \) and as well as \( a, \omega \) and \( r_m \). For some choice of these parameters some nodes might not be phenomenologically acceptable since the brane thickness might be too large. Also in general for the case when \( a > 0 \) there will be fewer phenomenologically acceptable branes since in this situation the nodes are stretched out, as discussed in section 5, and this tends to decrease \( f'(r_m) \) as \( r \) increases. On the other hand when \( a < 0 \) there is an increase of \( f'(r_m) \) as \( r \) increases which leads to more nodes which have a phenomenologically acceptable thickness.

The second mechanism would be to propose some coupling between the ghost-like field, \( \sigma \), and matter fields of the form \( \sigma AA \), where \( A \) is a scalar, spinor, vector or tensor field. For a normal scalar field such a coupling leads to an attractive force, while for a ghost-like field it leads to repulsion of the matter fields from regions with non-zero \( \sigma \). This would force the fields \( A \) to congregate at regions were \( \sigma \) vanishes, i.e. the nodes of the standing waves. Note that coupling ordinary matter fields to ghost fields is problematic since in general it leads to instabilities. However, as mentioned in the Sec. 3, our massless ghost-like field can be associated with the geometrical scalar of integrable Weyl models.

Both binding mechanisms have open questions (e.g. “Does the gravitational standing wave induce a quadrupole moment of the matter field stress-energy tensor and if so what is its
A simple standing wave solution for 5D spacetime with a ghost-like scalar field plus 5D negative cosmological constant was presented. The requirement that the ghost-like field vanish on the brane quantized the standing wave oscillation frequency in terms of the zeros of the 2nd order Bessel functions. Thus the ghost-like field is not observable on the brane, at \( r = 0 \), nor at any of the nodes, \( r_m \), of the standing wave. This is similar to the static brane model [10, 11, 12], where the phantom/ghost field only existed in the bulk.

For the case of increasing (decreasing) warp factors the ghost-like field, \( \sigma(t, r) \), and the metric function, \( u(t, r) \), vanish at a finite (infinite) number of places in the bulk forming 'islands' of 4D anti-de-Sitter space-time. By assuming that the matter fields are bound to these nodes we arrive at a model where each node is an anti-de-Sitter island-universe with different parameters, i.e. an effective 4D Newton's constant from (26) and an effective 4D cosmological constant from (17) scaled by the value \( e^{2\pi r_m} \) at the particular node. One could address the hierarchy problem and the small size of the cosmological constant by taking our universe as one of these Bessel nodes, where the 5D Newton and cosmological constants are appropriately scaled to their effective 4D values. A problem is that the scalings go in opposite directions. For example, with \( a > 0 \) (17) gives a 4D cosmological constant which is exponentially larger than the 5D cosmological constant, while (26) gives a 4D Newton's constant which is exponentially smaller than the 5D Newton's constant. For \( a < 0 \) the effective 4D Newton's constant and the effective 4D cosmological constant have opposite scaling to the \( a > 0 \) case.

There are two distinctly different cases for the standing wave solutions – increasing warp factor \((a > 0)\) and decreasing warp factor \((a < 0)\) – corresponding to a finite, or infinite number of anti-de-Sitter ‘islands’ respectively. The finite case could be applied to the generation problem by fixing the model to have three ‘islands’ with different fermion generations bound to different nodes. This is different from brane world models of the generation puzzle like [48, 49, 50], where the generations were associated with different zero modes bound to a single brane. For the infinite branes case one has a simple version of the landscape picture coming from string/M theory, where one has a large number of anti-de-Sitter island-universes each having different parameters e.g. effective 4D gravitational and 4D cosmological constants.

The present models has the same kind of warped geometry as the usual brane models [1, 2, 3, 4]. In distinction from these models the “branes” (i.e. the nodes of \( u(r, t) \), excluding the one at \( r = 0 \)) of the present standing wave background solution do not have a brane tension but are simply 4D anti-de-Sitter space-times. This is an advantage since for the models of [1, 2, 3, 4] one should explain why the \( \delta \)-source brane tension is not observed. The disadvantage of the present model is the need for a ghost-like field. The need for the ghost-like field is ameliorated by the fact that it vanishes on the \( r = 0 \) brane and on all the
Bessel node anti-de-Sitter ‘islands’.

The issue of instability due to an unusual matter source is also a problem for the original two-brane model which has a negative tension brane. If the negative tension brane is free to oscillate this results in arbitrarily large negative energy modes making the system unstable [28]. A similar problem occurs in the present model if we take our scalar field as an ordinary ghost field. However, if we associate our ghost field with the geometrical scalar of a 5D integrable Weyl model [21, 22, 23, 24] this alleviates some of the instability problems since the scalar field comes from the metric via (6). In addition for Weyl models the ghost-like field does not have the standard couplings with the brane energy-momentum and thus does not cause instabilities.

Additionally previous work with standard ghost fields shows – that at least at the classical level – bulk ghost fields may in fact be better at stabilizing brane world models as compared to bulk regular scalar fields. In [51] bulk ghost fields in 5D are shown to lead to radion stabilization for models with positive tension branes. Essentially the bulk ghost field replaces the negative tension brane. Further it was shown that a bulk scalar field whose sign can vary between regular (positive) and ghost (negative) can be used to model both positive and negative tension branes. Also, there are certain configurations of the bulk ghost fields of reference [51] which lead to stronger localization of gravity than the original brane models. In references [52, 53] it is shown that the two brane model of [1, 2, 3, 4] is stabilized by a ghost/tachyon bulk field, while for a regular bulk scalar field the two brane model is unstable. While the stability issue is crucial for all brane world models it has not yet found a complete solution even for the original models of [1, 2, 3, 4]. Here, as in [10, 11, 12, 51] we simply use the ghost scalar to build a brane world model from a 5D standing wave, leaving the generally unresolved question of stability of the solution for a future work. As a final note in [54, 55, 56] it is shown how to construct a consistent field theory with tachyons in the context of D-branes.

Although throughout this paper we have assumed some mechanism for binding matter fields/particles to the anti-de-Sitter ‘islands’ in section 7 we put forward two possible mechanisms for accomplishing this binding. The first localization mechanism was based on the quadrupole force in (27) which would eject particles from the high curvature regions and thus drive them to the nodes. One might give a heuristic description of this binding mechanism by noting that as one moves away from the nodes where \( u(t, r) = 0 \) that the brane coordinates \( x, y, z \) are distorted in an anisotropic way – the \( x, y \) coordinates will be stretched/compressed while the \( z \) coordinate will be compressed/stretched. This anisotropy causes a force, via (27), which tends to drive the matter fields toward the nodes. The second mechanism also involved ejecting the matter fields from the anti-nodes and towards the nodes, but in this case the ejection mechanism was accomplished by coupling the matter fields to the ghost-like scalar. This would give rise to a repulsive force which would drive the matter fields toward regions where the ghost-like scalar field was small i.e. toward the nodes. For both mechanisms the thickness of the brane was related to how rapidly the metric and scalar field ansatz functions changed near the nodes i.e. to have a thinner brane one should make \( f'(r_m) \) larger. In general the case with \( a < 0 \) would have thinner branes.

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