Topology Optimization Design of Compliant Mechanism of Composite Wing Leading Edge

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Abstract. This paper introduces a topology optimization design method for flexible mechanism of composite wing leading edge based on discrete material optimization (DMO) with composite material. The DMO method is combined with the topology optimization to select the deviation of the actual displacement and the target displacement of the 10 coordinate points on the initial curve of the leading edge as the objective function, and the volume fraction of each phase material is used as the constraint to establish the composite material optimization model, which is solved by the OC optimization criterion method. The MATLAB codes are compiled to calculate the optimization problem and a distinct result image is obtained. The CATIA software is used to model the topology image in three dimensions, and the model is imported into Hyperworks software for simulation analysis. The results show that this compliant mechanism can achieve a maximum deflection of 9.1 degrees, which verifies the feasibility of the method.

1. Introduction
The research of compliant mechanism has become a hotspot of structural optimization design [1]. Compared to the traditional wings, compliant wings can automatically change the front and rear edge structures shape as the flight conditions change, thereby improving the flight performance [2]. The idea of the wing with the flexible mechanism was originally proposed by KOTA.S [3]. G. Matthew [4] used the basic structure method to optimize flexible mechanism of the wing trailing edge. Huang Jie [5] obtained a clear wing leading edge topology structural by using SIMP stiffness-density interpolation. Santer M et al [6] uses load path method to design wing leading edge structure. Ge Wenjie [7] used a combination of SIMP interpolation and genetic algorithm to obtain a stable topology result. Compared with isotropic materials, fiber-reinforced composite materials are widely used in aerospace, automotive and many other industries due to their superior material properties [8]. The fiber angles of the composite material directly determine the properties of the material and the orientation optimization has become the research focus of scholars. Callahan and Weeks [9] first used genetic algorithms to optimize the design of constant stiffness laminates. Xiu Yingjun [10] combined the neural network with genetic algorithm to optimize orientation. J.Stegmann and E.Lund [11,12] proposed a composite optimization method based on multi-phase material topology optimization principle—discrete material optimization (DMO).

2. Discrete Material Optimization (DMO) Principle
The basic idea in the Discrete Material Optimization (DMO) principle is essentially an extension of the ideas used in multiphase material structural topology optimization introduced by Sigmund [13]. This methodology can be stated as: for all elements in the structure find one distinct material from a
set of pre-defined candidate materials such that the objective function is minimized. This was first introduced as multiphase topology optimization by Sigmund who used it for designing materials with extreme thermal expansion [12]. Common to the parametrization used in [13] is that materials are assumed to be isotropic, the maximum number of phases involved is three—two distinct materials and void. The DMO formulation extends the scope of application by considering multiple phases and furthermore by assuming to be orthotropic materials.

According to the theory of composite material discrete material optimization (DMO) method, carbon fiber composite materials with 30° and 45° common fiber angles are labelled as three-phase materials with void materials together, and the interpolation scheme can be expressed as[11,12]:

\[
E_{\beta} = \sum_{\alpha} (x_{\alpha,\beta})^p D_{\alpha,\beta} \quad 0 \leq x_{\alpha,\beta} \leq 1
\]

Where \( E_{\beta} \) denotes the elastic constitutive matrix of the \( \beta \) element, \( D_{\alpha,\beta} \) denotes the elastic constitutive matrix of the \( \alpha \) th candidate material of the \( \beta \) th element, and \( m \) is the number of species of the candidate material , \( x_{\alpha,\beta} \) represents the artificial density of the candidate material of the \( \beta \) th elements, and \( p \) is the penalty index.

3. Objective function of wing leading edge

The deviation between the actual displacement of the 10 discrete points on the leading edge of the selected wing and the target displacement is the objective function, and the mathematical model is established by taking the volume fraction of each candidate material as the constraint. The expression is:

\[
\text{min} : f(x) = \sum_{i} o_i \left( u_{i,j} - u'_{i,j} \right)^2 + \left( u_{i,j} - u'_{i,j} \right)^2
\]

\[
\text{s.t.} : \sum_{\beta=1}^{N} x_{\alpha,\beta} v_{\alpha}^* \leq V_{\alpha}^*,
\]

\[
0 < x_{\alpha,\beta} < 1
\]

The sensitivity of objective function is expressed as:

\[
\frac{\partial f(x)}{\partial x_{\alpha,\beta}} = \sum_{i} o_i \left\{ \left( u_{i,j} - u'_{i,j} \right)^2 + \left( u_{i,j} - u'_{i,j} \right)^2 \right\}^{-1/2} \cdot

\left[ \left( u_{i,j} - u'_{i,j} \right), \frac{\partial u_{i,j}}{\partial x_{\alpha,\beta}} + \left( u_{i,j} - u'_{i,j} \right)^2 \frac{\partial u_{i,j}}{\partial \rho_{\alpha,\beta}} \right]\right]
\]

Where: \( n \) is the number of selected displacement output points, \( u_{i,j}, u'_{i,j}, u_{i,j}, u'_{i,j} \) are the x and y direction of the actual displacement and target displacement of \( i \)-th output point, respectively; and \( o_i \) is the weighting factor of \( i \)-th output point The value ranges from \( 0 \leq o_i \leq 1 \), and the sum of the weight factors is 1, that is, \( \sum_{i} o_i = 1 \). \( x \) is the design variable vector, \( x_{\alpha,\beta} \) is the density of the material phase \( \alpha \) in element \( \beta \), \( N \) is the total number of discrete elements, \( K \) is the total stiffness matrix, \( F \) is the external force vector, \( L \) is the adjoint load vector, \( U \) is the displacement vector, \( \tilde{U} \) is the accompanying displacement vector, \( V_{\alpha}^* \) is the target volume of material phase \( \alpha \). In any element of the design domain, the sum of the density and volume of the alternative phase material is 1, respectively.

According to the finite element theory, the stiffness matrix of the \( i \)-th unit is expressed as:
\[ k_e = \int_{\Omega} B^T D B d\Omega \]  
\[ k_e = \int_{\Omega} B^T \left( \sum_{i=1}^{n} (\epsilon_{i,\beta})^T D_{i,\beta} \right) B d\Omega \]  
\[ K = \sum_{\beta} \int_{\Omega} B^T \left( \sum_{i=1}^{n} (\epsilon_{i,\beta})^T D_{i,\beta} \right) B d\Omega \]  
\[ u_e = \sum_{\beta} \hat{u}_\beta^T k_{e,\beta} u_{\beta} \]  
\[ \frac{\partial U_e}{\partial x_{e,\beta}} = -u_{\beta}^T \frac{\partial k_{e,\beta}}{\partial x_{e,\beta}} u_e \]  
\[ \text{s.t.} \]  
\[ B_{\beta} = -\frac{\partial f(x)}{\partial x_{e,\beta}} \left( \sum_{i=1}^{N} h_{i} x_{i,\beta} \right) \left( \sum_{i=1}^{N} h_{i} x_{i,\beta} \right) \]  
\[ m_\beta \text{ is the moving limit constant, which is taken as 0.2, } \eta \text{ is the damping factor which is 0.4. } \lambda \text{ is the lagrangian operator.} \]

In order to avoid the formation of checkerboard patterns, mesh-dependency filter works described in [15] is introduced. The mesh-dependency filter function is expressed as:

\[ \frac{\partial f(x)}{\partial x_{e,\beta}} = \frac{1}{\gamma \cdot x_{e,\beta}} \sum_{d \in N_e} H_{d} x_{e,d} \frac{\partial f(x)}{\partial x_{e,\beta}} \]  

Where: \( N_e \) is the number of all units with the center distance \( \Delta (e, d) \) of the unit \( e \) being smaller than the filter radius \( r_{\text{min}} \). \( \gamma \) takes 10\(^{-3}\). \( H_{d} \) is the weight coefficient, and the expression is:

\[ H_{d} = \max(0, r_{\text{min}} - \Delta(e, d)) \]  

The flow chart of topology optimization design is shown in Figure 1.
4. Optimization settings

4.1. Topology optimization design of flexible wing leading edge structure

The NACA2418 airfoil leading edge is chosen as the research object. The initial design domain is shown in Figure 2, the gray area is the design field and the white area is the non-design field. Coordinates, target displacement and weight factors of the output points are listed in table 1.

**Figure 1.** The process of topology optimization.

**Figure 2.** Initial design area.

| Number | Coordinates | Target Displacement | Weight Factor |
|--------|-------------|---------------------|--------------|
| X/mm   | Y/mm        | ΔX/mm | ΔY/mm |                  |
| 1      | 220         | 16    | -15.23 | 9.95              | 0.04          |
| 2      | 180         | 33    | -11.36 | 23.38             | 0.08          |
| 3      | 120         | 55    | -7.31  | 30.78             | 0.11          |
| 4      | 60          | 90    | -0.53  | 38.17             | 0.15          |
| 5      | 10          | 140   | 9.82   | 43.69             | 0.2           |
| 6      | 15          | 180   | 15.80  | 38.17             | 0.16          |
| 7      | 75          | 207   | 20.38  | 29.70             | 0.12          |
| 8      | 150         | 211   | 21.45  | 21.41             | 0.09          |
| 9      | 210         | 219   | 18.07  | 5.74              | 0.05          |

**Table 1.** Output Point Coordinates, Target Displacement and Weight Factor.
The design domains are all meshed by 250×200 rectangular finite element. The design domain boundary is defined by the airfoil function. The candidate materials used are Carbon fiber reinforced composites with the orthotropic properties $E_L=54\text{GPa}$, $E_T=8.6\text{GPa}$, $v_{LT}=0.3$, $G_{LT}=9\text{GPa}$ oriented at $30^\circ$, $45^\circ$. The volume fraction constraints is chosen of $0.16:0.14:0.7$. $F=150\text{N}$, $p=3$ are employed.

4.2. Optimization results and simulation analysis
The optimization results of the wing leading edge structure with multiphase by MATLAB are shown in Figure 3. As shown in Figure 4, the black area composite fiber has a $30^\circ$ angle and the blue area composite fiber has a $45^\circ$ angle.

According to the structure diagram of Figure 4, the topology optimization structure is modeled by CATIA software, as shown in Figure. 4.

![Figure 3. Results of optimization by MATLAB.](image1)

![Figure 4. Geometric reconstruction by CATIA.](image2)

The 3D model is imported into hyperworks software for simulation analysis. The results are shown in Figure 5-6.

![Figure 5. Displacement distributions.](image3)

![Figure 6. Stress distributions.](image4)

As is shown above, the simulation results shows that topology optimization structure of the leading edge can achieve continuous deformation with a maximum deflection angle of 9.1 degrees. The maximum stress of the unit is 78.45MPa, which is smaller than the yield strength of the carbon fiber composite, which meets the strength requirements.

5. Conclusion
This paper proposed a topology optimization method for compliant airfoil leading edge with multiphase composite by using discrete material optimization (DMO). The topology optimization model is obtained to minimize the least square error (LSE) between deformed curve and desired aerodynamics shape by using DMO material interpolation scheme. In the present paper DMO is introduced. The DMO method is derived from multiphase material optimization and the element stiffness is computed from a weighted sum of candidate materials. The aim of optimization is to select the material that minimizes the target the most from the set of candidate materials. The candidate materials can be either isotropic or orthotropic with a given fiber angle. So far, the major drawback of DMO method is the large number of design variables, which increases the computational cost. In this paper, the OC method is used to solve the topology optimization problem because of the less quantity of finite element meshes and design variables. However, the optimization algorithm MMA is the best choice when solving topology optimization problems with a large number of design variables.
A distinct result of two angles composite materials is obtained by using OC methods and sensitivity filtering approaches to solve the topology optimization problem and ensure the existence of solutions. The simulation result obtained by Hyperworks shows that the leading wing compliant mechanism can achieve a maximum deflection of 9.1°, which verifies the feasibility of the method.

6. References
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