Cosmology from quantum potential in brane-anti-brane system

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Recently, some authors removed the big-bang singularity and predicted an infinite age of our universe. In this paper, we show that the same result can be obtained in string theory and M-theory; however, the shape of universe changes in different epochs. In our mechanism, first, N fundamental string decay to N D0-anti-D0-brane. Then, D0-branes join to each other, grow and and form a six-dimensional brane-antibrane system. This system is unstable, broken and present form of four dimensional universes, one anti-universe in additional to one wormhole are produced. Thus, there isn’t any big-bang in cosmology and universe is a fundamental string at the beginning. Also, total age of universe contains two parts, one in related to initial age and second which is corresponded to present age of universe \( t_{\text{tot}} = t_{\text{initial}} + t_{\text{present}} \). On the other hand, initial age of universe includes two parts, the age of fundamental string and time of transition \( t_{\text{initial}} = t_{\text{transition}} + t_{f-string} \).

We observe that only in the case of \( t_{f-string} \to \infty \), the scale factor of universe is zero and as a result, total age of universe is infinity.

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### I. INTRODUCTION

Recently, some authors obtained the second order Friedmann equations from the quantum corrected Raychaudhuri equation (QRE), and argued that these equations include two quantum correction terms, the first of which can be interpreted as cosmological constant, while the second as a radiation term in the early universe, which gets rid of the big-bang singularity and predicts an infinite age of our universe [1]. In parallel, some models have been proposed to remove big-bang singularity in Bionic system [2–5]. In this system, at the beginning, there are k black fundamental strings that transition to the BIon configuration at a special energy. At this energy, two brane and antibrane universes are produced, interact with each other through a wormhole and inflate. This wormhole gives its energy to the brane universes and causes to inflation. Shortly, the wormhole disappears, the inflation ends and a deceleration epoch begins. When the brane and antibrane universes become close together, a tachyon is produced, grows and leads the creation of a new wormhole. At this stage, this tachyonic wormhole connects the brane and antibrane universes again and leads to the late-time acceleration era of the universe [2, 4].

Now, the main question arises as to what is the relation between the model which was obtained from QRE and the one which was used BIon? We will answer this question in this paper. We also modify previous discussions in BIon by introducing D0-branes and clarify transitions in this system. In our model, at the beginning, there are N fundamental strings that excited and make a transition to N D0-anti-D0-branes. Then, by joining and growing these objects, a pair of brane-antibrane is formed. These branes interact with each other, emit D0-branes and changes to lower dimensional branes like D3-branes. The present universe is located on one of these D3-branes and radiated D0-branes form a wormhole that connects another universe with our one. This system which includes two universe-branes with one wormhole connected them, is named BIon. Ignoring evolution of universe, before formation of BIon, the scale factor is zero at the beginning. However, regarding the transition of universe from fundamental string to BIon, the scale factor is zero only in the case of \( t_{\text{universe}} \to \infty \) and the age of universe is infinite.

Maybe the question arises as to why does one choose this rather complicated model of our Universe (string to branes, to Universe, anti-universe and wormhole)? To reply this question, we can say that our model provides good reasons for the small size of Universe at the beginning and then enlarging to the current size. According to recent observations [6–8], at very early epoch, universe was very small and experienced a phase of inflation. In our model, the initial fundamental string is very tiny and then decays to little D0-branes. These branes construct two small Universe-anti-Universe-branes and one wormhole between them. This wormhole gives its energy to D3-branes and dissolves into them and causes to their expansion. Thus, this model helps us to consider evolution of Universe in string theory.

Also, one may ask that how does one know that the initial state of our Universe is a fundamental string? To answer this question, we emphasis that the origin of BIon is the fundamental string [2, 4]. Previously, it has been shown that all evolution of Universe can be described in a Bionic system [2–5]. Thus, if our Universe be a part of BIon, the initial state of it should be a fundamental string. Also, the small size of initial Universe is comparable with the size...
The outline of the paper is as follows. In section II, we will remove the Big bang singularity and show that the origin of universe is a fundamental string in a Bionic system. In section III, we will extend our calculations to M-theory and show that the age of universe is infinite. The last section is devoted to summary and conclusion.

II. REMOVING THE BIG-BANG SINGULARITY IN BION

In this section, we will propose a new model which allows to replace big-bang singularity by a fundamental string. In this model the present form of universe is created via process (fundamental string $\rightarrow$ D0 + anti-D0 $\rightarrow$ D5 + anti-D5 $\rightarrow$ D3 + anti-D3 $\rightarrow$ universe + anti-universe + wormhole). There isn’t any birth time for the fundamental string and thus, the age of universe could be infinite.

First, let us to introduce the mechanism of [1] in short terms. In this method, we calculate an explicit form for $\dot{H} = F(H)$, where $F(H)$ is a function of Hubble parameter (H) that is obtained from QRE. Using this function, we can find the age of universe:

$$\dot{H} = F(H) \rightarrow T = \frac{1}{F''(H_{\text{initial}})} \int dH \frac{1}{(H - H_{\text{initial}})^n} \rightarrow \infty$$  \hspace{1cm} (1)

where $H_{\text{initial}}$ is the hubble parameter before the present epoch of universe. As can be seen from this equation, the age of universe is infinity. To achieve this result in string theory, at first stage, we should calculate the energy density and momentums of a universe + anti-universe + wormhole system. With respect to the FRW metric,

$$ds^2_{FRW} = -dt^2 + a(t)^2(dx^2 + dy^2 + dz^2),$$  \hspace{1cm} (2)

we can calculate the energy density and pressure in one flat universe [9]:

$$\rho_{\text{uni}} = 3H^2,$$

$$p_{\text{uni}} = H^2 + 2\frac{\dot{a}}{a}$$  \hspace{1cm} (3)

Where $H = \frac{\dot{a}}{a}$ is the Hubble parameter and $a$ is the scale factor.

To consider the evolution of universe in Bionic system, we introduce two four dimensional universes that interact with each other via a wormhole and form a binary system. In this model, our universe is located on one D-brane and connected by another universe on the anti-brane by a wormhole. The metric of wormhole is given by [10]:

$$ds^2_{\text{wormhole}} = B(r)(-dt^2 + dr^2) + C(r)d\phi^2 + D(r)du^2$$  \hspace{1cm} (4)

in which B (r), C (r) and D(r) are as a function of r only and r=V(t) is the location of the throat. The standard energy momentum on the shell of wormhole is:

$$\rho_{\text{wormhole}} = -\left(\frac{D'}{D} + \frac{C'}{C}\right)\sqrt{\Delta}$$

$$p_{\phi,\text{wormhole}} = \frac{1}{\sqrt{\Delta}}[2\dot{V} + 2\frac{B'}{B}\dot{V}^2 + \frac{B'}{B^2} + \frac{C'}{C}\Delta],$$

$$p_{\phi,\text{wormhole}} = \frac{1}{\sqrt{\Delta}}[2\dot{V} + 2\frac{B'}{B}\dot{V}^2 + \frac{B'}{B^2} + \frac{D'}{D}\Delta]$$  \hspace{1cm} (5)

where $\Delta = \frac{1}{B} + \dot{V}^2$.

The general metric of this system is:

$$ds^2_{\text{uni-wormhole}} = ds^2_{\text{uni1}} + ds^2_{\text{uni2}} + ds^2_{\text{wormhole}}$$  \hspace{1cm} (6)

According to conservation law, the energy and momentums of this system should be equal to energy and momentums of brane-anti-brane system. We should write:
\[ \rho_{\text{brane-anti-brane}} = \rho_{\text{uni1}} + \rho_{\text{uni2}} + \rho_{\text{wormhole}} \]

\[ \rho_{\text{brane-anti-brane}} = \rho_{\text{uni1}} + \rho_{\text{uni2}} + \rho_{\text{wormhole}} \]  

(7)

Now, we need to compute the contribution of the Bronic system with the four-dimensional universe energy momentum tensor. To obtain this tensor, we use of following action for [11][16]:

\[ S = -T_{Dp} \int d^{p+1}\sigma \text{STr} \left( - \det(P_{ab} E_{mn} (Q^{-1} + \delta)^{ij} E_{jn}) + \lambda F_{ab}) \det(Q^j) \right)^{1/2} \]  

(8)

where

\[ E_{mn} = G_{mn} + B_{mn}, \quad Q_j^i = \delta^i_j + i[\lambda, X^j, X^k] E_{kj} \]  

(9)

\[ \lambda = 2\pi l_s^2, \quad G_{ab} = \eta_{ab} + \partial_a X^i \partial_b X^i \text{ and } X^i \text{ are scalar fields of mass dimension. Here } a, b = 0, 1, \ldots, p \text{ are the world-volume indices of the Dp-branes, } i, j, k = p + 1, \ldots, 9 \text{ are indices of the transverse space, and } m, n \text{ are the ten-dimensional spacetime indices. Also, } T_{Dp} = \frac{1}{g_s (2\pi l_s^2)^p} \text{ is the tension of Dp-brane, } l_s \text{ is the string length and } g_s \text{ is the string coupling.} \]

We can approximately obtain a simple form for the action of Dp brane [11][14][16]:

\[ S_{Dp} = -T_{Dp} \int d^{p+1}\sigma \text{STr} \left( \Sigma_{a,b=0}^{\rho} \Sigma_{i,j=1}^{l} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \} \right) \]  

(10)

Using following rules [11][16]:

\[ \Sigma_{a,b=0}^{\rho} \Sigma_{m=1}^{m} \rightarrow \frac{1}{(2\pi l_s^2)^p} \int d^{p+1}\sigma \Sigma_{m=p+1}^{m} \Sigma_{a=0}^{\rho} \lambda = 2\pi l_s^2 \]

\[ [X^a, X^i] = i\lambda \partial_a X^i \quad [X^a, X^b] = i\lambda^2 F_{ab} \]

\[ i, j = p + 1, \ldots, 9 \quad a, b = 0, 1, \ldots, p \quad m, n = 0, 1, \ldots, 9 \]  

(11)

in action (10) and after some mathematical calculations, we get:

\[ S_{Dp} = -T_{Dp} \int d^{p+1}\sigma \text{STr} \left( \Sigma_{a,b=0}^{\rho} \Sigma_{i,j=1}^{l} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \} \right) = -\Sigma_{a=0}^{\rho} T_{D0} \int dt \text{Tr} \Sigma_{m=n}^0 [X^m, X^n]^2 = \Sigma_{a=0}^{\rho} S_{D0} \]  

(12)

where we have used of the action of D0-brane[11][16]:

\[ S_{D0} = -T_{D0} \int dt \text{Tr} \Sigma_{m=n}^0 [X^m, X^n]^2 \]  

(13)

Here \( T_{D0} \) is the brane tension and \( X^m \) are transverse scalars. The origin of this D0-brane is a fundamental string. We can write:

\[ T_{F\text{-string}} = \frac{1}{2\pi l_s} \quad T_{D1} = \frac{1}{2\pi g_s l_s} \quad g_s = 1 \rightarrow T_{F\text{-string}} = T_{D1} \quad \text{and} \quad S_{D0} = S_{\text{anti-D0}} \Rightarrow \]

\[ S_{F\text{-string}} = -T_{F\text{-string}} \int d^{p+1}\sigma \text{STr} \left( \Sigma_{a,b=0}^{1} \Sigma_{i,j=2}^{l} \{ \partial_a X^i \partial_b X^i - \frac{1}{2\lambda^2} [X^i, X^j]^2 + \frac{\lambda^2}{4} (F_{ab})^2 \} \right) = -\Sigma_{a=0}^{1} T_{D0} \int dt \text{Tr} \Sigma_{m=n}^0 [X^m, X^n]^2 = \Sigma_{a=0}^{1} S_{D0} = S_{D0} + S_{\text{anti-D0}} \]  

(14)
This equation indicates that at strong coupling, fundamental string is broken and two D0-anti-D0-branes are produced. Using the action \[12\], we can propose a new mechanism for interaction of two brane-anti-brane. We can write:

\[
S_{Dp} + S_{\text{anti-}Dp} = \Sigma_{a=0}^{p} S_{D0} + \Sigma_{a=0}^{p} S_{\text{anti}D0} =
\]

\[
\Sigma_{a=0}^{p-2} S_{D0} + \Sigma_{a=0}^{p-2} S_{\text{anti}D0} + \Sigma_{a=0}^{p-2} S_{D0} =
\]

\[
S_{D(p-2)} + S_{\text{anti-}D(p-2)} + S_{D2} + S_{D0}
\]

(15)

This equation shows that each of Dp-branes in a brane-antibrane system emit two D0-branes and transits to a lower dimensional D(p-2) brane and one D2-brane is produced. For example, a system of D5-anti-D5-brane is broken and a system of D3-anti-D3-D2-brane is formed. To construct a D3-anti-D3-D2 model, we consider two D3-anti-D3-brane pairs which are placed at points \(u_1 = l_2/2\) and \(u_2 = -l_2/2\) respectively so that the separation between the brane and antibrane is \(l_2\). Here, \(l_2\) is the length of D2-brane in transverse direction respect to D3-brane. Following rules in \[11\], we can obtain the solutions for gauge fields in D3 and D2-branes \[11\] \[13\]:

\[
\lambda F_{01} \text{ in D2-brane} \rightarrow \partial_t X^1 \text{ in D3-brane}
\]

\[
F_{ab} \text{ in D3-brane} \rightarrow [X^a, X^b] \text{ in D2-brane}
\]

\[
X^i \sim \frac{l_3}{2}, \quad A^i \sim \frac{l_3}{2} \quad \text{in D2-brane}
\]

\[
X^i \sim \frac{l_2}{2}, \quad A^i \sim \frac{l_3}{2} \quad \text{in D3-brane}
\]

(16)

where \(l_2\) and \(l_3\) are coordinates of D2 and D3-branes respectively. By passing time, \(l_2\) is decreased and reduced to zero at the end of acceleration and \(l_3\) is increased. Using these rules and action \[9\], we can write the following equation for this D3- brane-anti-D3-brane+ D2-brane system:

\[
S_{D3-D2-\text{anti-D3}} = S_{D3} + S_{\text{anti-D3}} + S_{D2} + S_{D0} \simeq
\]

\[
- T_{D3} \int d^4\sigma (l_2)^4 \left( \sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2} + \sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2} \right)
\]

\[
- T_{D2} \int d^4\sigma (l_3)^4 \left( \sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2} \right)
\]

\[
- T_{D0} \int d\sigma ((l_3)^4 + (l_2)^4) \left( \sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2} \right) \simeq
\]

\[
- \int dt \frac{(l_3)^3(l_2)^2}{\pi g_s l_s^3} (\frac{2(l_2)^2}{\pi l_s} + l_3 + l_3l_2^{-2} + l_2^2l_3^{-3}) (\sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2} \simeq
\]

\[
- \int dt F_{l_2,l_3}(\sqrt{D_{l_2,l_3}})
\]

(17)

where

\[
F_{l_2,l_3} = \frac{(l_3)^3(l_2)^2}{\pi g_s l_s^3} (\frac{2(l_2)^2}{\pi l_s} + l_3 + l_3l_2^{-2} + l_2^2l_3^{-3}), \quad D_{l_2,l_3} = 1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2
\]

(18)

and prime(‘) denotes derivative respect to time (t). To obtain the explicit form of \(l_2\) and \(l_3\) in terms of time , we are using the equation of motion extracted from action \[17\]:

\[
\left( \frac{l_2'(t)}{\sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2}} \right)' = \frac{1}{\sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2}} \frac{F'_{l_2,l_3}}{F_{l_2,l_3}} (D_{l_2,l_3} - (l_2'(t))^2)
\]

(19)

\[
\left( \frac{l_3'(t)}{\sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2}} \right)' = \frac{1}{\sqrt{1 + \left( \frac{l_2'}{4} \right)^2 + \left( \frac{l_3'}{4} \right)^2}} \frac{F'_{l_2,l_3}}{F_{l_2,l_3}} (D_{l_2,l_3} - (l_2'(t))^2)
\]

(20)
Solving above equations, we obtain:

\[ l_3 \simeq t^2(1 + e^{\frac{(t - t_{\text{end}})}{\pi l_s}})^{1/3} \]
\[ l_2 \simeq t^3(1 - e^{\frac{(t - t_{\text{end}})}{\pi l_s}})^{1/2} \]

where \( t_{\text{end}} \) is the age of universe-brane at the end of epoch. An interesting result that comes out of these equation is that the length of D2-brane in transverse dimension increases with time, turns over a maximum and then decreases and shrinks to zero at \( t_{\text{end}} \), while; D3-brane is expanded very fast. The energy momentum tensor for this system is \([2, 4, 5]\):

\[ T^{00} = F_{l_2, l_3} \sqrt{D_{l_2, l_3}}, \]
\[ T^{44} = -F_{l_2, l_3} \frac{1}{3 \sqrt{D_{l_2, l_3}}}( (l_3)^2 (l_2)^2 + \frac{(l_2')^2}{4} ) \]
\[ T^{ii} = -F_{l_2, l_3} \frac{1 + (l_3)^2}{3 \sqrt{D_{l_2, l_3}}}, \quad i = 1, 2, 3 \]

This equation indicates that with the energy-momentum tensors increases with time. This is due to the fact that with approaching two D3-branes each other, D2-brane dissolves in two universe branes and causes to expansion and and increase in values of density and momentum of universes. This higher-dimensional stress-energy tensor is related to a perfect fluid and of the form

\[ T^i_j = \text{diag}[-p, -p, -p, -p, -p, -p, -p, -p, \rho], \]

where \( \rho \) is the pressure in the extra space-like (u) dimension. Thus, this relation and also conversation low in equation \( \text{[ ]} \) help us to write:

\[ \rho_{\text{brane-anti-brane}} = \rho_{\text{uni}1} + \rho_{\text{uni}2} + \rho_{\text{wormhole}} \Rightarrow \]
\[ 6H_{\text{present}}^2 - (\frac{D'}{D} + \frac{C'}{C})\sqrt{\Delta} = \]
\[ F_{l_2, l_3} \sqrt{D_{l_2, l_3}} \]

\[ \rho_{\text{brane-anti-brane}, i} = \rho_{\text{uni}, i} + \rho_{\text{uni}2, i} + \rho_{\text{wormhole}, i} \Rightarrow \]
\[ 2H_{\text{present}}^2 + 4\frac{\text{a}_{\text{present}}}{\text{a}_{\text{present}}} + \frac{1}{\sqrt{\Delta}}[2\dot{V} + \frac{2B'}{B} \dot{V}^2 + \frac{B'}{B^2} + \frac{D'}{D} \Delta] = \]
\[ -F_{l_2, l_3} \frac{1 + (l_3)^2}{3 \sqrt{D_{l_2, l_3}}} \]

\[ \rho_{\text{brane-anti-brane}, u} = \rho_{\text{uni}1, u} + \rho_{\text{uni}2, u} + \rho_{\text{wormhole}, u} \Rightarrow \]
\[ \frac{1}{\sqrt{\Delta}}[2\dot{V} + \frac{2B'}{B} \dot{V}^2 + \frac{B'}{B^2} + \frac{C'}{C} \Delta] = \]
\[ -F_{l_2, l_3} \frac{1}{\sqrt{D_{l_2, l_3}}} ((l_3)^2 (l_2)^2 + \frac{(l_2')^2}{4}) \]

where the index present refers to present stage of universe. Assuming \( D = C \) and \( V = l_2 \), we can derive the solutions of these equations:

\[ a_{\text{present}}(t) = e^{-\int dt \ln(X^{-1})}, \quad C(t) = D = e^{-\int dt \ln(Y^{-1})}, \quad D = e^{-\int dt \ln(Z^{-1})} \]
\[ X = [t^{10}(1 + e^{\frac{(t - t_{\text{end}})}{\pi l_s}})(1 - e^{\frac{(t - t_{\text{end}})}{\pi l_s}})][t^{3/2}(1 - e^{\frac{(t - t_{\text{end}})}{\pi l_s}}) + t^{1/2}(1 + e^{\frac{(t - t_{\text{end}})}{\pi l_s}})] \times \]
\[ [1 + (2t(1 + e^{\frac{(t - t_{\text{end}})}{\pi l_s}}))^{1/3} + t^2(t - t_{\text{end}})e^{\frac{(t - t_{\text{end}})}{\pi l_s}}(1 + e^{\frac{(t - t_{\text{end}})}{\pi l_s}})^{-2/3}]^2 + \]
The universe is born at one beginning time \((t=0)\), expands in the Bionic system, then contracts and vanishes at \(t=\infty\). The parameters of wormhole and four dimensional universe are increased, turn over a maximum and then reduced to lower values and tended to zero at \(t=\infty\). This equation indicates that by passing time, the parameters of wormhole and four dimensional universe are increased, turn over a maximum and then reduced to lower values and tended to zero at \(t=\infty\). This means that universe is born at one beginning time \((t=0)\), expands in Bionic system, then contracts and vanishes at \(t=\infty\).

In this section, we will replace \(D_0\) by \(M_0\) in M-theory and show that this object has the main role in evolution of universe. In this model, first fundamental string transits to \(M_0\)-branes and anti-\(M_0\)-branes and then these objects join to each and form a \(M_5\)-anti-\(M_5\)-system. The branes in this system, emit \(M_0\)-branes, broken and transits to lower dimensional \(M_3\) and anti-\(M_3\)-brane. Coincidence with the birth of this new system, \(M_0\)-branes glued to each other and construct a wormhole. This wormhole connects two universes that are located on each \(M_3\)-brane. As a result the initial shape of \(M_3\)-branes and universes is a fundamental string. This string has no the birth and for this reason, the real age of universe is infinity.

This equation indicates that by passing time, the parameters of wormhole and four dimensional universe are increased, turn over a maximum and then reduced to lower values and tended to zero at \(t=\infty\). This means that universe is born at one beginning time \((t=0)\), expands in Bionic system, then contracts and vanishes at \(t=\infty\).

Now, we add contributions of initial stages which fundamental string transits to BIon. To this end, we define a new scale factor \(a_{tot} = a_{initial} + a_{present}\) which \(a_{initial}\) can be obtained from following equation:

\[
\rho_{\text{fundamental string}} = \rho_{D_0} + \rho_{\text{anti-}D_0} = \rho_{\text{uni}1} + \rho_{\text{uni}2} \Rightarrow

6H^2_{\text{initial}} = 2T_{D_0} \int dt Tr(X^9_{m=0}[X^m, X^n]^2) \simeq \frac{5}{6} (1 + e^{(t - t_{\text{end}}) / \pi t_{\text{end}}}) + t^4 (t - t_{\text{end}})^2 \rightarrow

a_{initial}(t) = A e^{- \frac{t}{t_{f - \text{string}}}} \int_{t_{f - \text{string}}}^{t} dt \frac{5}{6} (1 + e^{(t - t_{\text{end}}) / \pi t_{\text{end}}}) + t^4 (t - t_{\text{end}})^2
\]

where, \(t_{f - \text{string}}\) is the age of the fundamental string and we have used of this fact that \(X^2 = l_2\) and \(X^3 = l_3\). When \((t_{f - \text{string}} \rightarrow \infty)\), the initial scale factor is zero. Thus, total age of universe contains two parts, one in related to initial age and second which is corresponded to present age of universe \((t_{\text{total}} = t_{\text{initial}} + t_{\text{present}})\). Also, initial age of universe includes the age of fundamental string and time of transition \((t_{\text{initial}} = t_{\text{transition}} + t_{f - \text{string}})\). As a result of \((t_{f - \text{string}} \rightarrow \infty)\), total age of universe is infinity.

### III. REMOVING THE BIG BANG IN M-THEORY

In this section, we will replace \(D_0\) by \(M_0\) in M-theory and show that this object has the main role in evolution of universe. In this model, first fundamental string transits to \(M_0\)-branes and anti-\(M_0\)-branes and then these objects join to each and form a \(M_5\)-anti-\(M_5\)-system. The branes in this system, emit \(M_0\)-branes, broken and transits to lower dimensional \(M_3\) and anti-\(M_3\)-brane. Coincidence with the birth of this new system, \(M_0\)-branes glued to each other and construct a wormhole. This wormhole connects two universes that are located on each \(M_3\)-brane. As a result the initial shape of \(M_3\)-branes and universes is a fundamental string. This string has no the birth and for this reason, the real age of universe is infinity.

Let us to begin with the Born-Infeld action for \(M_0\)-brane by replacing two dimensional Nambu-Poisson bracket [17][20] for \(Dp\)-branes by three one in action and using the \(\text{Li-3}\)-algebra:

\[
S_{M_0} = T_{M_0} \int dt Tr(\Sigma^1_{M,N,L=0} ([X^M, X^N, X^L], [X^M, X^N, X^L]))
\]

where \(X^M = X^M_\alpha T^\alpha\) and

\[
[T^\alpha, T^\beta, T^\gamma] = f^{\alpha\beta\gamma} T^\eta
\]
where \(X^M(i=1,3,\ldots,10)\) are transverse scalars to M0-brane. By compactifying M-theory on a circle of radius R, this action will be made a transition to ten dimensional action for D0-brane. To show this, we use the method in \[20\] and define \(X^{10} = \frac{R}{l_p}\), where \(l_p\) is the Planck length. We have:

\[
S_{M0} = -T_{M0} \int dt Tr \left( \Sigma_{M,N,L}^{10} ([X^M, X^N, X^L], [X^M, X^N, X^L]) \right) =
- T_{M0} \int dt Tr \left( \Sigma_{M,N,L,F,G=0}^{10} \xi^{MNLD} \xi^{D}_{EF} X^M X^N X^L X^E X^F X^G \right) =
-6 T_{M0} \int dt Tr \left( \Sigma_{M,N,E,F=0}^{9} \xi^{MN10D} \xi^{D}_{EF10} X^M X^N X^{10} X^E X^F X^{10} \right) =
6 T_{M0} \int dt \left( \Sigma_{M,N,E,F,G=0,\neq 10}^{9} \xi^{MNLD} \xi^{D}_{EF} X^M X^N X^L X^E X^F X^G \right) =
-6 T_{M0} \left( \frac{R^2}{l_p^2} \right) \int dt Tr \left( \Sigma_{M,N=E,F=0}^{9} \xi^{MN10D} \xi^{D}_{EF10} X^M X^N X^L X^E X^F \right) =
6 T_{M0} \int dt \left( \Sigma_{M,N,E,F,G=0,\neq 10}^{9} \xi^{MNLD} \xi^{D}_{EF} X^M X^N X^L X^E X^F X^G \right) =
S_{D0} - 6 T_{M0} \int dt \left( \Sigma_{M,N,L,E,F,G=0,\neq 10}^{9} \xi^{MNLD} \xi^{D}_{EF} X^M X^N X^L X^E X^F X^G \right) =
S_{D0} + V_{Extra,1}
\]

where \(T_{M0/D0}\) is tension of brane and \(V_{Extra,1} = -6 T_{M0} \int dt \left( \Sigma_{M,N,L,E,F,G=0}^{9} \xi^{MNLD} \xi^{D}_{EF} X^M X^N X^L X^E X^F X^G \right)\). We define \(T_{D0} = 6 T_{M0} \left( \frac{R^2}{l_p^2} \right) = \frac{1}{g_s l_s}\), where \(g_s\) and \(l_s\) are the string coupling and string length respectively. Thus, the actions in string theory and M-theory are completely related and all results in string theory can be generalized to M-theory.

Similar to Dp-branes, different Mp-branes can be built from M0-brane by using the following rules \[17,20\]:

\[
\langle [X^a, X^b, X^i], [X^a, X^b, X^i] \rangle = \frac{1}{2} \varepsilon^{abcd} \varepsilon^{ab} (\partial_a X^i) (\partial_a X^i) \langle (T^a, T^b) = \frac{1}{2} (\partial_a X^i, \partial_a X^i)
\]

\[
\langle [X^a, X^b, X^c], [X^a, X^b, X^c] \rangle = \left( F_{abc}^{\alpha\beta\gamma} \right) \left( F_{abc}^{\alpha\beta\gamma} \right) \langle (T^a, T^b, T^c) \rangle = (F_{abc}^{\alpha\beta\gamma})(F_{abc}^{\alpha\beta\gamma}) \delta^{\alpha\beta\gamma} \langle (T^a, T^b, T^c) \rangle = (F_{abc}^{\alpha\beta\gamma})(F_{abc}^{\alpha\beta\gamma})
\]

\[
\Sigma_m \rightarrow \frac{1}{(2\pi)^p} \int d^{p+1} \sigma \sum_{m-p-1}^{j} \int \left( F_{abc} = \partial_a A_{bc} - \partial_b A_{ac} + \partial_c A_{ab} \right)
\]

where

\[
F_{abc} = \partial_a A_{bc} - \partial_b A_{ac} + \partial_c A_{ab}
\]

and \(A_{ab}\) is 2-form gauge field. Replacing commutation relations by derivatives and fields of equations \[30\] in action \[27\], we can obtain the relevant action for Mp-brane

\[
S_{Mp} = \Sigma_{a=0}^{p} - \Sigma_{a=0}^{p} T_{M0} \int dt Tr \left( \Sigma_{m=0}^{9} ([X^a, X^b, X^c], [X^a, X^b, X^c]) \right) =
\]
\[-T_{M_p} \int d^{p+1} \sigma Tr\left( \Sigma^p_{a,b,c=0} \Sigma^{i,k=1}_{i,j,k=p+1} \left\{ \langle \partial_a X^i, \partial_a X^i \rangle - \frac{1}{4} \langle [X^i, X^j, X^k], [X^i, X^j, X^k] \rangle \right\} + \frac{1}{6} \langle F_{abc}, F_{abc} \rangle \right) \]

This is consistent with the earlier studies done on the Mp-branes. This mechanism can be applied for deriving action for other Mp-branes, and so it can be used for obtaining suitable action for Dp-branes by compactifying M-branes on circle. Now, we can answer to question that what is the origin of M0-branes. In previous section, we show that one fundamental string decays to D0-anti-D0-branes. On the hand, we obtain the relation between D0brane and M0-brane in equation (29). Thus, using equations (14 and 29) we can write:

\[S_{F-string} = S_{D0} + S_{anti-D0} = S_{M0} + S_{anti-M0} + 2V(extra, 1)\]

This equation indicates that Fundamental strings can decay to a pair of M0-anti-M0-brane and some extra energy is produced. In fact, the origin of all objects are fundamental strings.

Similar to previous section, we can propose a new mechanism for interaction of two brane-anri-brane. We can write:

\[S_{M_p} + S_{anti-M_p} = \Sigma^p_{a=0} S_{M0} + \Sigma^p_{a=0} S_{M0} =
\]

\[\Sigma^p_{a=0} S_{M0} + \Sigma^p_{a=0} S_{M0} + \Sigma^2_{a=0} S_{M0} + S_{M0} =
\]

\[S_{M(p-2)} + S_{anti-D(M-2)} + S_{M2} + S_{M0}\]

This equation shows that a system of Mp-anti-Mp-brane is broken and two lower dimensional branes and one M2-brane are formed. For example, two M5-branes are broken and two M3-branes and a M2 are produced.

Similar to previous section, the general form of Born-Infeld action for Mp-brane [17, 20] can be obtain by replacing two-form commutative brackets with three one :

\[S = -T_{M_p} \int d^{p+1} \sigma STr \left( -det(P_{abc}) \left[ \langle E_{mnl}, E_{mik} \rangle (Q^{-1} + \delta)^{jk} E_{kin} \right] + \lambda F_{abc} \right) \left( Q_{ij,k} \right)^{1/2} \]

where

\[E^\alpha_{mnl} = G^\alpha_{mnl} + B^\alpha_{mnl} \quad Q^i_{j,k} = \delta^i_{j,k} + i \lambda \left[ X^j T^\alpha, X^k T^\beta, X^l T^\gamma \right] E^\alpha_{k,j,l} \]

\[\lambda = 2\pi l_2^2, \quad G_{mnl} = \eta_{mnl} + \partial_m X^i \partial_n X^i \partial_l X^i \quad \text{and} \quad X^i \text{ are scalar fields of mass dimension.} \]

Placing two M3-branes at points \(u_1 = l_2/2\) and \(u_2 = -l_2/2\) and one M2-brane between them, we can construct M3-anti-M3-M2-system. Here, \(l_2\) is the length of M2-brane in transverse direction respect to M3-brane. Generalizing rules in [16] to M-theory, we can achieve to the solutions for gauge fields in M3 and M2-branes :

\[\lambda F_{011} \text{ in M2-brane} \rightarrow \partial_i X^1 \text{ in M3-brane} \]

\[F_{abc} \text{ in M3-brane} \rightarrow [X^a, X^b, X^c] \text{ in M2-brane} \Rightarrow \]

\[X^i \sim \frac{l_3}{2}, \quad A^i \sim \frac{l_2}{2} \quad \text{in M2-brane} \]

\[X^i \sim \frac{l_2}{2}, \quad A^i \sim \frac{l_3}{2} \quad \text{in M3-brane} \]

where \(l_2\) and \(l_3\) are coordinates of M2 and M3-branes respectively. Using this equation and equation[40], we can estimate the action for the case of a M3-anti-M3-brane pair with lengths \(l_3\) with a M2-brane with length \(l_2\) between them,

\[S_{M3-M2-anti-M3} = S_{M3} + S_{anti-M3} + S_{M2} + S_{M0} \simeq \]
The energy momentum tensor for this system is:

\[-T_{M2} \int d^3 \sigma (l_{2})^6 \left( \frac{(l'_2)^2}{4} + \frac{(l''_2)^2}{6} + \frac{(l'''_2)^2}{6} \right) \]

\[-T_{M3} \int d^4 \sigma (l_{3})^6 \left( \frac{(l'_3)^2}{4} + \frac{(l''_3)^2}{6} + \frac{(l'''_3)^2}{6} \right) \]

\[-T_{M0} \int d\sigma ((l_{2})^6 + (l_{3})^6) \sqrt{1 + \frac{(l'_2)^2}{4} + \frac{(l''_2)^2}{6} + \frac{(l'''_2)^2}{6}} \approx \]

\[-\int dt \frac{(l_{3})^5 (l_{2})^4}{\pi \gamma s_6} \left( \frac{2(l_{2})^4}{\pi l_{2}^4} + l_{3}^2 + (l_{2})^{-4}(l_{3}) + (l_{3})^{-5}(l_{2})^2 \right) \sqrt{1 + \frac{(l'_2)^2}{4} + \frac{(l''_2)^2}{6} + \frac{(l'''_2)^2}{6}} \right) \approx \]

\[-\int dt \tilde{F}_{l_{2}, l_{3}} (\tilde{D}_{l_{2}, l_{3}}) \]

where

\[
\tilde{F}_{l_{2}, l_{3}} = \frac{(l_{3})^5 (l_{2})^4}{\pi \gamma s_6} \left( \frac{2(l_{2})^4}{\pi l_{2}^4} + l_{3}^2 + (l_{2})^{-4}(l_{3}) + (l_{3})^{-5}(l_{2})^2 \right) \]

\[
\tilde{D}_{l_{2}, l_{3}} = 1 + \frac{(l'_2)^2}{4} + \frac{(l''_2)^2}{6} + \frac{(l'''_2)^2}{6} \]

(39)

and prime(\(')\) denotes derivative respect to time (t). Now, we can extract the equation of motion from action (38):

\[
\left( \frac{l'_2(t)}{\sqrt{1 + \frac{(l'_2)^2}{4} + \frac{(l''_2)^2}{6} + \frac{(l'''_2)^2}{6}}} \right) = \frac{1}{\sqrt{1 + \frac{(l'_2)^2}{4} + \frac{(l''_2)^2}{6} + \frac{(l'''_2)^2}{6}}} \left[ \frac{\tilde{F}_{l_{2}, l_{3}}}{\tilde{D}_{l_{2}, l_{3}}} (\tilde{D}_{l_{2}, l_{3}} - (l'_2(t))^2) \right] \]

(40)

\[
\left( \frac{l'_3(t)}{\sqrt{1 + \frac{(l'_3)^2}{4} + \frac{(l''_3)^2}{6} + \frac{(l'''_3)^2}{6}}} \right) = \frac{1}{\sqrt{1 + \frac{(l'_3)^2}{4} + \frac{(l''_3)^2}{6} + \frac{(l'''_3)^2}{6}}} \left[ \frac{\tilde{F}_{l_{2}, l_{3}}}{\tilde{D}_{l_{2}, l_{3}}} (\tilde{D}_{l_{2}, l_{3}} - (l'_3(t))^2) \right] \]

(41)

The solutions of these equations are:

\[
l_{3} \simeq t^{4} \exp(- \left( 1 - \frac{t}{t_{\text{end}}} \right)^2 \gamma \pi s_{6}^{-2}) \]

\[
l_{2} \simeq t^{5} \exp(- \left( 1 - \frac{t}{t_{\text{end}}} \right)^2 \gamma \pi s_{6}^{-2}) \]

(42)

where \(t_{\text{end}}\) is the age of universe-brane at the end of the present epoch. Comparing these solutions with results of equation (21), we find that the value of \(l_{2}\) in M-Theory decreases faster than than relevant value in string theory and M2-brane is less stable than D2. The energy momentum tensor for this system is:

\[
T^{00} = \tilde{F}_{l_{2}, l_{3}} \left( \sqrt{\tilde{D}_{l_{2}, l_{3}}} \right), \]

\[
T^{44} = -\tilde{F}_{l_{2}, l_{3}} \frac{1}{\sqrt{\tilde{D}_{l_{2}, l_{3}}}} \left( (l_{3})^2 (l_{2})^2 + \frac{(l'_2)^2}{4} \right) \]

\[
T^{ii} = -\tilde{F}_{l_{2}, l_{3}} \frac{1}{3 \sqrt{\tilde{D}_{l_{2}, l_{3}}}} (l_{3})^2, i = 1, 2, 3 \]

(43)
Obviously, the energy-momentum tensors in M-Theory increase faster than those in string theory. This is because due to this fact that M2-brane dissolves quickly in M3-branes and causes their expansion. Similar to string theory, we can write the conservation law in M-Theory.

\[
\rho = \rho_{\text{Uni1}} + \rho_{\text{Uni2}} + \rho_{\text{wormhole}} = \rho_{\text{M2-M3}}
\]
\[
p_i, \text{tot} = p_i, \text{Uni1} + p_i, \text{Uni2} + p_i, \text{wormhole} = p_i, \text{M2-M3}, \quad i=1,2,3
\]
\[
p_{a,\text{tot}} = p_{a,\text{wormhole}} = p_{2,\text{M2-M3}}
\]

(44)

Substituting equations 3, 5 and using (43) and relation (23) in (44), we derive the following relations:

\[
6H^2_{\text{present}} - \left(\frac{D'}{D} + \frac{C'}{C}\right) \sqrt{\Delta} = \tilde{F}_{l_2,l_3} \sqrt{\tilde{D}_{l_2,l_3}}
\]
\[
2H^2_{\text{present}} + 4 \frac{\rho_{\text{present}}}{a_{\text{present}}} + \frac{1}{\sqrt{\Delta}} [2\tilde{V} + 2\frac{B'}{B} \tilde{V}^2 + \frac{B'}{B^2} + \frac{D'}{D} \Delta] =
\]
\[
- \tilde{F}_{l_2,l_3} \frac{1}{3} \left(\frac{(l_2)^2}{\tilde{D}_{l_2,l_3}} + \frac{(l_2)^2}{4}\right)
\]

(45)

Assuming \(D = C\) and \(V = l_2\), we can derive the solutions of these equations:

\[
a_{\text{present}}(t) = e^{-\int dt \ln(W^{-1})}, \quad C(t) = D = e^{-\int dt \ln(E^{-1})}, \quad B = e^{-\int dt \ln(R^{-1})}
\]

\[
W = [t^{13} \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{t-t_{\text{end}}}) \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{t-t_{\text{end}}})][t^6 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})]
\]
\[
t^7 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})[1 + 4t^3 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})] + 2t^4 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})
\]
\[
exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})^2 + 5t^4 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})}) + t^5 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})
\]
\[
exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})^2]^{1/4}
\]

\[
E = [t^6 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})}) + t^7 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})} + e^{(t-t_{\text{end}})^2}) \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})]
\]
\[
[t^{13} \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})}) \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})][t^6 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})]
\]
\[
t^7 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})[1 + 4t^3 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})] + 2t^4 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})
\]
\[
exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})^2 + 5t^4 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})}) + t^5 \exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})
\]
\[
exp(-\frac{1 - e^{(t-t_{\text{end}})^2}}{(t-t_{\text{end}})})^2]^{-1/2}
\]
As can be seen from this equation, the parameters of wormhole and scale factor of universe are zero at t=0, increase with time, turn over a maximum and reduce to zero at the end. From this point of view, we will have a birth time for universe. However, regarding initial transitions, we can show that universe has no beginning. Similar to string theory, we define a new scale factor $a_{\text{tot}} = a_{\text{initial}} + \theta_{\text{present}}$ which $a_{\text{initial}}$ can be obtained from following equation:

$$
\rho_{\text{fundamental string}} = \rho_{3M0} + \rho_{3-\text{anti-M0}} = \rho_{\text{uni1}} + \rho_{\text{uni2}} \Rightarrow
$$

$$
6H^2_{\text{initial}} = 2T_{M0} \int dt Tr(S^{10}_{M,N,L=0}(X^M, X^N, X^L), [X^M, X^N, X^L]) \simeq
$$

$$
t^g \int \frac{(t-t_{\text{end}})^8}{8} + \frac{(t-t_{\text{end}})^2}{2} e^{-\frac{(t-t_{\text{end}})^2}{\pi t^2}} \rightarrow
$$

$$
a_{\text{initial}}(t) = A \exp(-\int_{t_f-\text{string}}^t \frac{dt}{g} \exp(-\frac{2(1-e^{-\frac{(t-t_{\text{end}})^2}{\pi t^2}})}{(t-t_{\text{end}})^2} + \frac{(t-t_{\text{end}})^8}{8} + \frac{(t-t_{\text{end}})^2}{2} e^{-\frac{(t-t_{\text{end}})^2}{\pi t^2}}) (47)
$$

Here, $t_{f-\text{string}}$ is the age of the fundamental string and $X^2 = l_2$ and $X^3 = l_3$. An interesting result that comes out of this equation is that only in the case of $(t_{f-\text{string}} \rightarrow \infty)$, the initial scale factor is zero. This means that the age of fundamental string is infinity and since this string is the initial state of present universe, we conclude that the total age of universe is infinity.

**IV. SUMMARY AND DISCUSSION**

In this paper, we have reconsidered the results of [1] in string theory and M-theory. We have shown that there is no big bang for our universe in string theory and the age of universe is infinity which is in agreement with predictions of [1]. We have discussed that universe was a fundamental string at the beginning, this string decayed to N D0 and anti-D0-brane. These branes joined to each other and formed a system of D5-anti-D5-brane. The brane and antibrane interact with each other with exchanging D0-branes and transit to lower dimensional D3 and anti-D3-brane. Our universe is located on one of D3-branes and emitted D0-branes constructed a wormhole. This wormhole connects two universes and causes to evolution of scale factor of present form of universe. We have observed that this scale factor is zero at the birth of D3-branes. Then, we define a new scale factor in related to initial states of universe. This scale factor is zero only in the case that only in the case of $(t_{f-\text{string}} \rightarrow \infty)$ where $t_{f-\text{string}}$ is the age of fundamental string. This means that the age of fundamental string is infinity and since this string is the initial state of present universe, we conclude that the total age of universe is infinity.

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