Asymptotically free four-fermion interactions and electroweak symmetry breaking

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Abstract

We investigate the fermions of the standard model without a Higgs scalar. Instead, we consider a non-local four-quark interaction in the tensor channel which is characterized by a single dimensionless coupling $f$. Quantization leads to a consistent perturbative expansion for small $f$. The running of $f$ is asymptotically free and therefore induces a non-perturbative scale $\Lambda_{ch}$, in analogy to the strong interactions. We argue that spontaneous electroweak symmetry breaking is triggered at a scale where $f$ grows large and find the top quark mass of the order of $\Lambda_{ch}$. We also present a first estimate of the effective Yukawa coupling of a composite Higgs scalar to the top quark, as well as the associated mass ratio between the top quark and the W boson.

1 Introduction

The large hadron collider (LHC) will soon test the mechanism of spontaneous electroweak symmetry breaking. It is widely agreed that this phenomenon is associated to the expectation value of a scalar field which transforms as a doublet with respect to the weak SU(2)-symmetry. The origin and status of this order parameter wait, however, for experimental clarification. In particular, an effective description in terms of a scalar field does not tell us if this scalar is “fundamental” in the sense that it constitutes a dynamical degree of freedom in a microscopic theory which is formulated at momentum scales much larger than the Fermi scale. Alternatively, no fundamental Higgs scalar may be present, and the order parameter rather involves an effective “composite field”. In this note we investigate the second possibility and therefore consider the fermions of the standard model without a fundamental Higgs scalar.

As long as we include only the gauge interactions of the standard model, the microscopic or classical action of such a theory does not involve any mass scale. An effective mass scale $\Lambda_{QCD}$ will only be generated by the running of the strong gauge coupling, which is asymptotically free \[1\]. Confinement removes then the gluons and quarks from
the massless spectrum. In addition, the spontaneous chiral symmetry breaking by quark-
antiquark condensates would also imply electroweak symmetry breaking by a composite order parameter, in this case the chiral condensate. In this setting all particle masses would be of the order $\Lambda_{QCD}$ or zero, such that this scenario cannot explain why the top quark mass or the $W, Z$-boson masses are much larger than 1GeV. Furthermore, all leptons would remain massless. One concludes that any realistic model needs further interactions beyond the standard model gauge interactions. Since the strong and electroweak gauge couplings remain actually quite small at the Fermi scale of electroweak symmetry breaking, they are expected to produce only some quantitative corrections to the dominant mechanism of electroweak symmetry breaking. We will therefore neglect the gauge couplings in this paper.

Our knowledge about the effective interactions between the quarks and leptons at some microscopic or “ultraviolet” scale $\Lambda_{UV}$ is very limited. Furthermore, it is not known which scale $\Lambda_{UV}$ has to be taken. Typically, one may associate $\Lambda_{UV}$ with the scale where further unification takes place, as a grand unified scale or the Planck scale for the unification with gravity. This would suggest a very high scale, $\Lambda_{UV} \geq 10^{16}$ GeV and we will have this scenario in mind for our discussion. However, much smaller values of $\Lambda_{UV}$ are also possible. In practice, we will only assume here that $\Lambda_{UV}$ is sufficiently above the Fermi scale (say $\Lambda_{UV} > 100$ TeV), such that an effective description involving only the fermions of the standard model becomes possible in the momentum range $\Lambda_{QCD} \ll |q| \ll \Lambda_{UV}$.

We will formulate our model in terms of effective fermion interactions at the scale $\Lambda_{UV}$ and restrict the discussion to a four-fermion interaction involving only the right-handed top quark and the left-handed top and bottom quarks. This is motivated by the observation that only the top quark has a mass comparable to the Fermi scale. Interactions with the other quarks and leptons are assumed to be much smaller than the top quark interactions - typically their relative suppression is reflected in the much smaller masses of the other fermions. For the discussion in this paper we omit all “light” fermions and the gauge bosons.

Since we do not know the effective degrees of freedom at the scale $\Lambda_{UV}$, the effective interaction is not necessarily local. Non-localities involving inverse powers of the exchanged momenta are typically generated by the propagators of exchanged massless particles. With the inclusion of possibly non-local interactions the limitation to an effective four-fermion interaction poses no severe restriction. Many models with additional degrees of freedom can be effectively described in this way.

Local four-fermion interactions have already been investigated earlier, for example in the models of “top quark condensation” [2]. By simple dimensional analysis a local interaction involves a coupling $\sim$ (mass)$^{-2}$. Such models therefore exhibit an explicit mass scale in the microscopic action. It is indeed possible to obtain spontaneous electroweak symmetry breaking in this way - the prototype is the Nambu-Jona-Lasinio model [3]. Without a tuning of parameters the top quark mass $m_t$ turns out, however, to be of the same order as $\Lambda_{UV}$, in contradiction to the assumed separation of scales. By a tuning of parameters it is possible to obtain $m_t \ll \Lambda_{UV}$, but the issue is now similar to the “gauge hierarchy problem” in presence of a fundamental scalar field. In order to obtain $m_t \ll \Lambda_{UV}$ the microscopic effective action must be close to an ultraviolet fixed point.
The necessity of tuning arises from a “relevant parameter” in the vicinity of the fixed point (in the sense of statistical physics for critical phenomena) which has a dimension not much smaller than one. A rather extensive search for possible ultraviolet fixed points for pointlike four-fermion interactions has been performed in [4]. Many fixed points have been found, but all show a relevant direction with substantial dimension, and therefore the need for a parameter tuning for $m_t \ll \Lambda_{UV}$.

We will therefore concentrate in this paper on non-local effective interactions. For such interactions the coupling $\sim M^{-2}$ is replaced by $f^2/q^2$, with $q^2$ the square of some appropriate exchanged momentum and $f$ a dimensionless coupling. Interactions of this type do not involve a mass scale on the level of the microscopic action - the classical action exhibits dilatation symmetry. Still, the quantum fluctuations typically induce an anomaly for the scale symmetry, associated to the running of the dimensionless coupling $f$. This may be responsible for the generation of the Fermi scale, in analogy to the “confinement scale” $\Lambda_{QCD}$ for QCD. Since the running of dimensionless couplings is only logarithmic, this offers a chance for a large natural hierarchy $\Lambda_{UV} \gg m_t$, without tuning of parameters. We will present a model of this type.

Let us first discuss the possible tensor structures for a non-local four-fermion interaction $\sim (\bar{\psi}A\psi)^2$. The basic building block is a color singlet fermion bilinear $\bar{\psi}A\psi$, where the color indices are contracted. The tensor structure with respect to the Lorentz symmetry is determined by $A$ such that $\bar{\psi}A\psi$ is a scalar or pseudoscalar, a vector or pseudovector, or a second rank antisymmetric tensor. Interactions in the vector or pseudovector channels involve bilinears with two left-handed or two right-handed fermions, $\bar{\psi}_L\gamma^\mu \psi_L$ or $\bar{\psi}_R\gamma^\mu \psi_R$. They conserve chiral flavour symmetries which act separately on $\psi_L$ and $\psi_R$ and therefore forbid mass terms for the fermions $\sim \bar{\psi}_L\psi_R$. Interactions capable of producing masses for the top quark and W/Z-bosons of comparable magnitude must therefore involve the (pseudo)scalar or tensor channels. A non-local scalar interaction $\sim (\bar{\psi}_L\psi_R)(\bar{\psi}_R\psi_L)q^{-2}$, with $q^2$ the squared momentum in the scalar exchange channel, has very similar properties as a model with a fundamental Higgs scalar which is massless at the scale $\Lambda_{UV}$. We therefore expect the usual necessity of parameter tuning if we want to achieve a small ratio $m_t/\Lambda_{UV}$. A local coupling $(\bar{\psi}_L\psi_R)(\bar{\psi}_R\psi_L)m^{-2}$ is allowed by the symmetries and will be generated by quantum fluctuations. The interesting remaining candidate is a tensor interaction, with $A \sim [\gamma^\mu, \gamma^\nu]$. For chiral tensors no local interaction in this channel is consistent with the SU(3)×SU(2)×U(1) symmetries of the standard model as well as Lorentz symmetry. We will therefore investigate a model with a non-local interaction of this type.

We define the microscopic or classical action for a Lorentz invariant theory of massless interacting fermions by $S = S_2 + S_4$, defined in momentum space as

$$-S_2 = -\int \frac{d^4q}{(2\pi)^4} \left( \bar{t}(q)\gamma^\mu q^\mu t(q) + \bar{b}(q)\gamma^\mu q^\mu b(q) \right)$$ (1)
and

\[- S_4 = 4f^2 \int \frac{d^4 q \, d^4 p \, d^4 p'}{(2\pi)^4} \frac{P_{kl}^* (q)}{q^4} \left\{ \left[ \bar{t}(q + p)\sigma_+^k t(p) \right] \left[ \bar{t}(p')\sigma_+^l t(p' + q) \right] + \left[ \bar{t}(q + p)\sigma_-^k b(p) \right] \left[ \bar{b}(p')\sigma_-^l t(p' + q) \right] \right\}.\]

Here \( t \) and \( b \) are Dirac spinor fields describing the top and bottom quark, respectively. (The theory can be easily extended to all three generations of quarks and also to leptons.) Contracted indices are summed. The \( 3 \times 3 \) matrix \( P(q) \) involves the spacelike indices \( k, l \) and is defined by

\[ P_{kl}(q) = -(q_0^2 + q_j q_j)\delta_{kl} + 2q_k q_l - 2i\epsilon_{klj}q_0 q_j \]

It has the properties

\[ P_{kl} P_{lj}^* = q^4 \delta_{kj}, \quad P_{kl}^* (q) = P_{lk}(q). \]

The non-local character of the interaction arises from the factor \( 1/q^4 \).

In a standard spinor basis in which \( \psi = \left( \psi_L \, \psi_R \right) \), \( \bar{\psi} = \psi^\dagger \gamma^0 = (\bar{\psi}_R, \bar{\psi}_L) \), the \( 4 \times 4 \) matrices \( \sigma_+^k \) are defined in terms of the Pauli matrices \( \tau^k \),

\[ \sigma_+^k = \begin{pmatrix} \tau^k & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma_-^k = \begin{pmatrix} 0 & 0 \\ 0 & \tau^k \end{pmatrix}. \]

The fermion bilinears \( \bar{\psi}\sigma_+^k \psi \sim \bar{\psi}_R^* \sigma_+^k \psi_L \) and \( \bar{\psi}\sigma_-^k \psi \sim \bar{\psi}_L^* \sigma_-^k \psi_R \) therefore mix left- and right-handed spinors and violate the chiral symmetry that would protect the top quark from acquiring a mass. The matrices \( \sigma_\pm \) correspond to appropriate projections of the commutator \( [\gamma^\mu, \gamma^\nu] \) on left (right) handed spinors, such that eq. (2) indeed describes a tensor exchange interaction (cf. the appendix A). The Lorentz-invariance of the interaction can be checked explicitly.

Furthermore, the action (1), (2) has a global \( SU(2) \times U(1) \) symmetry - the remnant of the electroweak gauge symmetry in the limit of neglected gauge couplings. This symmetry forbids mass terms for the quarks, such that a mass term can be generated only by spontaneous symmetry breaking. Similarly, the interaction is invariant under the color symmetry \( SU(3) \), with implicitly summed color indices in the bilinears. We note that only the left-handed bottom quarks are involved in the interaction and we will therefore omit the right-handed bottom quark together with the other light quarks and the leptons. Similar to the photon-exchange description of the non-local Coulomb interaction we may obtain \( S_4 \) from the exchange of chiral tensor fields [5], according to the Feynman diagrams in fig. 1. The Lorentz symmetry of \( S_4 \) becomes more apparent in this formulation. A summary of the properties of the associated tensor fields and a proof of equivalence of a local theory with massless chiral tensor fields with our non-local fermion interaction (2) is provided in appendix A. In this paper we will not use tensor fields and concentrate on the purely fermionic action (1), (2). In particular, this avoids the delicate issues of consistency of the quantization of chiral antisymmetric tensors [6]. One can write a consistent functional integral and therefore a consistent quantum field theory based on the action (1), (2). (Anomaly cancellation involves the omitted light quarks and leptons.)
This paper is organized as follows. In sect. 2 we compute the running of the dimensionless coupling $f$ and demonstrate that it is asymptotically free. In consequence, a non-perturbative “chiral” scale $\Lambda_{ch}$ is generated where the running coupling $f$ grows large. Sect. 3 evaluates the effective interactions in the scalar and vector channels which are induced in one loop-order. We find that the induced interaction in the scalar channel $\sim f^4$ also gets large once the chiral scale is approached. As a consequence of this large interaction spontaneous electroweak symmetry breaking can be generated similar to the NJL-model [3]. We compute the resulting top quark mass $m_t$ in sect. 4, using a Schwinger-Dyson [7] or gap equation. We find $m_t \approx \Lambda_{ch}$. In sect. 5 we introduce a composite Higgs field by “partial bosonization” of the interaction in the scalar channel. We compute the running of its Yukawa coupling $h$ to the top quark. The latter is directly related to the mass ratio $m_W/m_t = g_W/(\sqrt{2}h)$, with $m_W$ and $g_W$ the mass and gauge coupling of the $W$ boson. In sect. 6 we present our conclusions.

2 Asymptotic Freedom

The dimensionless coupling constant $f$ is the only free parameter in our model. We will show in this section that the corresponding renormalized running coupling is asymptotically free in the ultraviolet. On the other side, there is a characteristic infrared scale $\Lambda_{ch}$ where the coupling $f$ grows large. This is in complete analogy to QCD, where $\Lambda_{QCD}$ corresponds to the scale where the strong gauge coupling grows large. By “dimensional transmutation” we may therefore trade $f$ for the mass scale $\Lambda_{ch}$. Besides this mass scale the model has no free dimensionless parameter. In particular, we will argue in sect. 4 that our model leads to spontaneous breaking of the $SU(2) \times U(1)$ symmetry. The Fermi scale $\varphi$ characterizing the electroweak symmetry breaking must be proportional to $\Lambda_{ch}$, $\varphi = c_w \Lambda_{ch}$, with a dimensionless proportionality coefficient $c_w$ that is, in principle, calculable without involving a further free parameter. If the model leads to a composite Higgs scalar, its mass in units of $\varphi$, $M_H/\varphi$, as well as its effective Yukawa coupling to the top quark, $h = m_t/\varphi$, are calculable quantities in our model.

In order to investigate the running of $f$, we wish to compute the effective action $\Gamma$
corresponding to the classical action (1), (2), to one-loop order. The one-loop correction reads
\[ \Delta \Gamma^{(1l)} = \frac{i}{2} \text{Tr} \ln S^{(2)}, \quad (6) \]
where the field dependent inverse propagator \( S^{(2)} \) is defined as the second functional derivative of the action with respect to the quark fields,
\[ (S^{(2)})^{cc'}_{AB,\alpha\beta}(p, p') = -\frac{\delta}{\delta \Psi^{c'}_{B\beta}(p')} \frac{\delta}{\delta \Psi^{c}_{A\alpha}(p)} (S_2 + S_4). \quad (7) \]
Here \( \Psi \) are the quark fields, with color indices \( c, c' \), flavor indices \( A, B \) (taking values \( t, b, \bar{t}, \bar{b} \)), spinor indices \( \alpha, \beta \) and momenta \( p, p' \). The trace \( \text{Tr} \) involves an integral over momentum and summation over all kinds of indices. We write \( S^{(2)} = P_0 + F \), where \( P_0 \) is the “free” part of the propagator, derived from \( S_2 \),
\[ (P_0)^{cc'}_{AB,\alpha\beta}(p, p') = [(\gamma^\mu p_\mu)_{\alpha\beta}(\delta_{AB}\delta_{BL} + \delta_{AB}\delta_{BB}) + (\gamma^\mu p_\mu)_{\beta\alpha}(\delta_{AB}\delta_{BL} + \delta_{AB}\delta_{BB})] \delta_{cc'}\delta(p - p'), \quad (8) \]
and \( \delta(p - p') = (2\pi)^4 \delta^4(p - p') \). We treat \( F \sim f^2 \) as a perturbative correction due to \( S_4 \). Then \( \Delta \Gamma^{(1l)} \) reads, up to an “infinite constant”,
\[ \Delta \Gamma^{(1l)} = \frac{i}{2} \text{Tr} \left( P_0^{-1} F \right) - \frac{i}{4} \text{Tr} \left( P_0^{-1} F P_0^{-1} F \right) + \cdots, \quad (9) \]
where the dots stand for neglected six-quark and higher interactions. We display the explicit expressions for \( F \), as well as the formal expressions for the first two terms on the r.h.s. of eq. (9), in appendix B.

The nonlocal factors \( \sigma(P^* (q)/q^4) \sigma \) appearing in the interaction (2) are attached in different ways to the fermion lines. We represent them as dashed lines in the corresponding Feynman diagrams in figs. 2-4. Our notation recalls the one-to-one correspondence with the exchange of the associated chiral tensor fields. The diagrams in fig. 2 correspond to the first term \( \sim F \) in eq. (9), while figs. 3, 4 reflect the terms \( \sim F^2 \).

Our one-loop calculation results in an effective action
\[ \Gamma = \Gamma_2 + \Gamma_4^{(T)} + \Gamma_4^{(V)} + \Gamma_4^{(S)}, \quad (10) \]
with a kinetic term

$$\Gamma_2 = - \int \frac{d^4q}{(2\pi)^4} \left( Z_L (\bar{t}_L \gamma^\mu q_\mu t_L + \bar{b}_L \gamma^\mu q_\mu b_L) + Z_R \bar{t}_R \gamma^\mu q_\mu t_R \right)$$

(11)

and three different types of quartic interactions \( \Gamma_4 \). (Note the relative minus sign between \( \Gamma \) and the classical action \( S \) which is chosen in order to make analytic continuation to the euclidean effective action straightforward by replacing \( q_0 \to i q_0 \).) The “tensorial part”, \( \Gamma_4^{(T)} \), corresponds to the exchange of a tensor field and has the form of the classical interaction \( S_4 \). The other two parts, \( \Gamma_4^{(V)} \) and \( \Gamma_4^{(S)} \), correspond to the exchange of vectors and scalars, respectively, and will be further discussed in sect. 3.

The momentum integrals in the loop expansion (9) are logarithmically divergent, both in the ultraviolet and the infrared. We regularize our model in the ultraviolet by a suitable momentum cutoff \( \Lambda_{UV} \). In order to investigate the flow of effective couplings we also introduce an effective infrared cutoff \( k \). The effective action \( \Gamma_k \) depends then on the scale \( k \), resulting in an effective coupling \( f(k) \). Instead of the infrared scale induced by non-vanishing external momenta for the vertices (as most common for perturbative renormalization) we introduce the cutoff by modifying the quark propagators \( \frac{1}{q^2 - i\epsilon} \to \frac{1}{(q^2 + k^2)^{-1}} \). This is a procedure known from functional renormalization. Indeed, \( \Gamma_k \) may be considered as the “average action” or “flowing action” [8]. The precise implementation of the infrared cutoff is not important and does not influence the one-loop beta function for the running coupling \( f(k) \) that we will derive next.

The fermion anomalous dimensions arise from eq. (81) or fig. 2. Our computation in the purely fermionic model yields the same result as in ref. [5], where it was computed in the equivalent model with chiral tensors, namely

$$\eta_R \equiv -k \frac{\partial}{\partial k} Z_R = -\frac{3}{2\pi^2} f^2, \quad \eta_L \equiv -k \frac{\partial}{\partial k} Z_L = -\frac{3}{4\pi^2} f^2.$$  

(12)

Only the terms visualized diagramatically in fig. 3, which are \( \sim A_2 \) in eq. (82), generate the same tensor structure as the classical interaction (2). They provide the one-loop
correction to the inverse chiron propagator, i.e. to the matrix $P_{kl}$, and one obtains

$$\Gamma_4^{(T)} = -S_4(P_{kl}^* \rightarrow Z_+ P_{kl}^*).$$  \hspace{2cm} (13)

Again our fermionic computation reproduces the computation in ref. [5] with chiral tensors. The correction results in an anomalous dimension for the chiron,

$$\eta_+ \equiv -k \frac{\partial}{\partial k} Z_+ = \frac{f^2}{2\pi^2}. \hspace{2cm} (14)$$

There are no further one loop corrections in the tensor exchange channel. Among the quartic corrections shown in fig. 4, the first four terms in eq. (82) contribute to an interaction channel which is equivalent to the exchange of a vector particle. These diagrams will be evaluated in the next section. Similarly, the fifth and sixth term in eq. (82) contribute to an interaction with a tensor structure different from the classical action (2). It can be interpreted as an effective scalar exchange and will also be discussed in the next section.

The running of the renormalized coupling $f^2$ (to one-loop order) is therefore given by the anomalous dimensions of the fermions and the correction to $P_{kl}$,

$$k \frac{\partial}{\partial k} f^2 = (\eta_R + \eta_L + \eta_+) f^2 = -\frac{7}{4\pi^2} f^4. \hspace{2cm} (15)$$

This implies that the interaction is asymptotically free. The solution to the renormalization group equation (15) is

$$f^2(k) = \frac{4\pi^2}{7 \ln(k/\Lambda_{ch})}, \hspace{2cm} (16)$$

where the “chiral scale” $\Lambda_{ch}$ is the scale at which $f^2/4\pi$ becomes much larger than one. This is completely equivalent to the result of ref. [5]. The asymptotic freedom of the chiral fermion-tensor interaction is simply taken over to the non-local four-fermion interaction.

3 Induced scalar and vector interactions

In this section we evaluate the diagrams in fig. 4, representing the terms $\sim A_1$ in eq. (82). We begin with the fifth diagram with all external momenta set to zero. (The contribution of the sixth diagram is equivalent, with $t_L$ substituted by $b_L$.) The contribution to $\Gamma$ is

$$\Delta \Gamma^{(1)} = 16if^4 \int \frac{d^4q}{(2\pi)^4} \frac{P_{kl}^*(q) P_{mn}^*(q)}{q^4} \left[ \bar{\tau} m \frac{-\hat{\phi}}{q^2} \tau^l t_l \right] \left[ \bar{\tau} n \frac{\hat{\phi}}{q^2} \tau^k t_k \right].$$  \hspace{2cm} (17)

This may be rewritten in terms of Weyl spinors

$$\Delta \Gamma^{(1)} = 16if^4 \int \frac{d^4q}{(2\pi)^4} \frac{P_{kl}^*(q) P_{mn}^*(q)}{q^4} \left[ t_R^k \tau^m \frac{-\hat{\phi}}{q^2} \tau^l t_l \right] \left[ t_R^k \tau^n \frac{\hat{\phi}}{q^2} \tau^k t_L \right], \hspace{2cm} (18)$$

where now

$$\hat{\phi} = q_0 + q_i \tau^i, \quad \hat{\phi} = q_0 - q_i \tau^i. \hspace{2cm} (19)$$
Figure 4: One-loop Feynman diagrams for the four-fermion vertex which correspond to the effective exchange of scalars and vectors. The second diagram contributes twice, since we may substitute \((t_L, B^+0) \rightarrow (b_L, B^{++})\).

With the identity 

\[
P_{kl}(q) P_{mn}(q) \left[ \tau^m \bar{\tau}^l \right]_{\alpha\beta} \left[ \tau^n \bar{\tau}^k \right]_{\lambda\eta} = 5q^4 [\bar{\theta}]_{\alpha\beta} [\bar{\theta}]_{\lambda\eta} + 4q^6 \delta_{\alpha\eta} \delta_{\beta\lambda} \tag{20}\]

this simplifies to 

\[
\Delta \Gamma^{(1)} = 16i f^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^8} \left( -5[t^L_R \bar{\theta} t^R_L][t^L_R t^R_L] + 4q^2 [t^R_L] [t^R_L] \right) . \tag{21}\]

The relative minus sign is due to an exchange of Grassmann variables. Note the different color structure of the second term. The momentum integral can be evaluated by analytic continuation to Euclidean space, \(q_0 = iq_{E0}\). Introducing the infrared cutoff in the quark propagators (i.e. substituting \(q^{-4} \rightarrow (q^2 + k^2)^{-2}\), one obtains

\[
\Delta \Gamma^{(1)} = \frac{f^4}{4\pi^2 k^2} \left( 5g_{\mu\nu}[t^\mu_R \bar{\tau}^\nu_R][t^\mu_L \bar{\tau}^\nu_L] - 16 [t^R_L] [t^R_L] \right) . \tag{22}\]

Using the identities

\[
(\tau^\mu)_{\alpha\beta}(\bar{\tau}_\mu)_{\lambda\eta} = -2\delta_{\alpha\eta} \delta_{\beta\lambda} \tag{23}\]

and

\[
\delta_{cd'}\delta_{c'd} = \frac{1}{3} \delta_{cd} \delta_{c'd'} + \frac{1}{2} T^z_{cd} T^z_{c'd'} \tag{24}\]
(where $T^z$ are the eight Gell-Mann matrices) this can be further simplified to

$$\Delta \Gamma^{(1)} = \frac{f^4}{2\pi^2 k^2} [\bar{t}_R t_L] [\bar{t}_L t_R] + \frac{3f^4}{4\pi^2 k^2} [\bar{t}_R T^z d^q] [\bar{t}_L T^z d^q].$$  \hspace{1cm} (25)$$

The total contribution to $\Gamma$ from the fifth and sixth diagram in fig. 4 reads

$$\Gamma^{(S)}_4 = \Delta \Gamma^{(1)} + \Delta \Gamma^{(1)} (t_L \rightarrow b_L).$$  \hspace{1cm} (26)$$

The first term in eq. (25) is equivalent to the four-fermion interaction generated at tree level by a Yukawa interaction with a scalar field $\phi$, which has a mass $k$ and a Yukawa coupling $\bar{h}$ given by

$$\bar{h}^2 = \frac{f^4}{2\pi^2}.$$  \hspace{1cm} (27)$$

This scalar transforms as a singlet under color and a doublet with respect to the electroweak interactions. It therefore has the same quantum numbers as a (composite) Higgs doublet. The second term in eq. (25) corresponds to the exchange of a second scalar which is an octet with respect to color.

Finally we evaluate the first four diagrams of fig. 4, which generate an interaction equivalent to the exchange of a vector particle. The expression for the first diagram is

$$\Delta \Gamma^{(2)} = 8if^4 \int \frac{d^4q}{(2\pi)^4} \frac{P_{kl}(q) P_{mn}(q)}{q^4} \left[ i\sigma_-^l \frac{\vec{q}}{q^2} \sigma_+^m t \right] \left[ i\sigma_-^n \frac{\vec{q}}{q^2} \sigma_+^k t \right].$$  \hspace{1cm} (28)$$

This can again be rewritten in terms of Weyl spinors

$$\Delta \Gamma^{(2)} = 8if^4 \int \frac{d^4q}{(2\pi)^4} \frac{P_{kl}(q) P_{mn}(q)}{q^4} \left[ t^k_L \tau^m \frac{\vec{q}}{q^2} t_L \right] \left[ t^k_L \tau^m \frac{\vec{q}}{q^2} t_L \right].$$  \hspace{1cm} (29)$$

With the identities

$$P_{kl}(q) P_{mn}(q) \left[ \tau^l \frac{\vec{q}}{q^2} \tau^m \right]_{\alpha\beta} = 5q^4 \left[ \hat{\phi}_{\alpha\beta} \right]_{\lambda\eta} + 4q^6 \left[ \delta_{\alpha\beta} \delta_{\lambda\eta} - \delta_{\alpha\lambda} \delta_{\beta\eta} \right]$$  \hspace{1cm} (30)$$

and

$$\left( \bar{\tau}^\mu \right)_{\alpha\beta} (\bar{\tau}^\mu)_{\lambda\eta} = -2(\delta_{\alpha\beta} \delta_{\eta\lambda} - \delta_{\alpha\eta} \delta_{\beta\lambda})$$  \hspace{1cm} (31)$$

the expression (29) simplifies to

$$\Delta \Gamma^{(2)} = 8if^4 \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^8} \left( 5[t^k_L \hat{\phi} t_L][t^k_L \hat{\phi} t_L] - 2q^2 [t^k_L \bar{\tau}^\mu t_L][t^k_L \bar{\tau}^\mu t_L] \right).$$  \hspace{1cm} (32)$$

With an infrared cutoff $k$ in the quark propagator the momentum integral yields

$$\Delta \Gamma^{(2)} = 3f^4 \frac{1}{8\pi^2 k^2} [\bar{t}_L \gamma^\mu t_L][\bar{t}_L \gamma^\mu t_L].$$  \hspace{1cm} (33)$$

This is equivalent to the four-fermion interaction generated at tree level by the exchange of a vector field with mass $k$ and coupling $\tilde{g}$ given by

$$\tilde{g}^2 = \frac{3f^4}{8\pi^2}.$$  \hspace{1cm} (34)$$
The evaluation of the diagrams 3 and 4 is equivalent, with two or four external \( t_L \) fields substituted by \( b_L \).

The expression for the second diagram, in terms of Weyl spinors, is

\[
\Delta \Gamma^{(3)} = 16i f^4 \int \frac{d^4q}{(2\pi)^4} \frac{P_{kl}(q) P_{mn}(q)}{q^4} \left[ t_R^{T \, m} \frac{\gamma^\mu}{q^2} t_R \right] \left[ \gamma^\mu t_R^{T \, k} \right]. \tag{35}
\]

Identities similar to eqs. (30) and (31) result in

\[
\Delta \Gamma^{(3)} = \frac{3f^4}{4\pi^2 k^2} [\bar{t} R^\mu t_R] [\bar{t} R^\mu t_R]. \tag{36}
\]

The “vector exchange” interaction generated at one loop level can be summarized as

\[
\Gamma^{(V)}_4 = \frac{3f^4}{8\pi^2 k^2} \left( [\bar{t} L^\mu t_L] [\bar{t} L^\mu t_L] + 2[\bar{b} L^\mu t_L] [\bar{b} L^\mu b_L] \right) \tag{37}
\[+ [\bar{b} L^\mu b_L] [\bar{b} L^\mu b_L] + 2[\bar{t} R^\mu t_R] [\bar{t} R^\mu t_R] \right).
\]

4 Electroweak symmetry breaking

The presence of the effective Yukawa interaction in eq. (25) indicates the possibility of spontaneous symmetry breaking and an analogue of the Higgs mechanism. If we neglect for \( k \) close to \( \Lambda_{ch} \) all interactions except for the scalar singlet exchange channel in the first term of eq. (25), we may characterize the effective action by a “scalar four-fermion coupling” \( \lambda(k) \),

\[
\Gamma_k^{(S)} = \frac{\lambda}{2} \int d^4x \left[ (\bar{\psi}\psi)^2 - (\bar{\psi}\gamma^5\psi)^2 \right] = 2\lambda(\bar{\psi}_L \psi_R)(\bar{\psi}_R \psi_L) \tag{38}
\]

with

\[
\lambda(k) = \frac{f^4(k)}{4\pi^2 k^2}. \tag{39}
\]

In the limit where the momentum dependence of the scalar four-fermion interactions can be neglected this can be interpreted as an effective NJL model. Here the role of the UV cutoff in the effective NJL model is played by the scale \( k \), since the computation of \( \Gamma_k^{(S)} \) has involved only fluctuations with momenta \( q^2 > k^2 \) (due to the infrared cutoff) such that the remaining fluctuations with \( q^2 < k^2 \) still have to be included. It is well known that for \( \lambda k^2 \) exceeding a critical value the NJL model leads to spontaneous symmetry breaking.

For a rough estimate of the top quark mass \( m_t \) induced by spontaneous electroweak symmetry breaking we consider the Schwinger Dyson equation

\[
- \gamma^\mu q_\mu + m_k \gamma^5 = -\gamma^\mu q_\mu + 2i\lambda(k) \int_{p^2 < k^2} \frac{d^4 p}{(2\pi)^4} \frac{\gamma^\mu p_\mu - 6m_k \gamma^5}{p^2 + m_k^2}. \tag{40}
\]
Setting \( q = 0 \) and performing the momentum integral yields the gap equation for the top quark mass \( m_k \)

\[
m_k = \frac{3\lambda(k)}{4\pi^2} \left( k^2 - m_k^2 \ln \frac{k^2 + m_k^2}{m_k^2} \right) m_k. \tag{41}
\]

We have indicated the scale \( k \) for \( m_k \) in order to recall that the solution of the gap equation will depend on the choice of the scale \( k \) which we use as an UV cutoff for the effective NJL model. Of course, for an exact treatment the physical top quark mass should no longer depend on \( k \).

Let us next discuss the solution of eq. (41). Obviously, \( m_k = 0 \) is always one possible solution. The expression in brackets is always \( \leq k^2 \). If we write

\[
\alpha(k) = \frac{\lambda(k)k^2}{4\pi^2}, \tag{42}
\]

we see that non-zero solutions for \( m_k \) (indicating spontaneous symmetry breaking) occur for

\[
\alpha(k) > \frac{4\pi^2}{3}. \tag{43}
\]

Once the condition (43) is obeyed, one finds indeed a non-zero \( m_k \) obeying

\[
\frac{m_k^2}{k^2} \ln \left( 1 + \frac{k^2}{m_k^2} \right) = 1 - \frac{4\pi^2}{3\alpha(k)}. \tag{44}
\]

Since \( \alpha(k) \sim f^4(k) \) grows arbitrarily large as \( k \) approaches \( \Lambda_{ch} \) and \( f(k) \) diverges, the condition (43) is always fulfilled for \( k \) sufficiently close to \( \Lambda_{ch} \).

For a qualitative investigation we first use the one loop result \( \alpha = f^4/4\pi^2 \) and replace in eq. (16) \( k \to (k^2 + c^2 m_k^2)^{1/2} \) with \( c \) of order 1. This is motivated by the effective infrared cutoff \( \sim m_k \) which stops or slows down the running of \( f^2 \). In fig. 5 we show \( m_k/\Lambda_{ch} \) as a function of \( k/\Lambda_{ch} \) for \( c = 1 \). After the onset of a nonzero \( m_k \) for \( k \approx \Lambda_{ch} \) we find first a very rapid increase until \( m_k \) settles at \( m_k = \Lambda_{ch}/c \) for \( k \to 0 \). It is obvious that \( \Lambda_{ch} \) sets the scale for the top quark mass. This coincides with the result of a two-loop Schwinger-Dyson equation in a formulation with chiral tensor fields in [5]. Indeed, since the generation of \( \alpha \) is a one-loop effect, and the gap equation (40) involves a further loop, the generation of the top quark mass consists of two nested one-loop integrals, which are equivalent to a two-loop integral. The solution of eq. (44) for \( k \to 0, m_k \neq 0 \) corresponds to an asymptotic value which is obtained from the condition \( \alpha(m_k, k \to 0) \to \infty \).

As an alternative to the ad hoc insertion of the quark mass cutoff in eq. (16) we may take into account the additional infrared cutoff due to \( m_k \) by replacing in the quark propagator \( \not{q} \to \not{q}/(q^2 + k^2 + m_k^2)^{-1} \). As a consequence, the anomalous dimensions involve a threshold function \( s(m_k^2/k^2) \),

\[
\eta_L = -\frac{3}{4\pi^2} f^2 s(m_k^2/k^2), \quad \eta_R = -\frac{3}{2\pi^2} f^2 s(m_k^2/k^2), \quad \eta_+ = \frac{1}{2\pi^2} f^2 s(m_k^2/k^2), \tag{45}
\]

given by

\[
s(m_k^2/k^2) = \frac{k^2}{k^2 + m_k^2}. \tag{46}
\]
The one loop expression for $\lambda$ (39) can be replaced by a flow equation

$$k \frac{\partial}{\partial k} \lambda = -\frac{f^4}{2\pi^2} \frac{k^2}{(k^2 + m_k^2)^2} + (\eta_L + \eta_R)\lambda.$$  \hspace{1cm} (47)

(For $m_k = 0$, $\eta_{L,R} = 0$ and constant $f$ this reproduces eq. (39).) Nonzero $m_k$ results in a threshold function for the running of $\alpha$,

$$k \frac{\partial}{\partial k} \alpha = (2 + \eta_L + \eta_R)\alpha - \frac{f^4}{2\pi^2} \tilde{s}(m_k^2/k^2),$$

$$\tilde{s}(m_k^2/k^2) = \frac{k^4}{(k^2 + m_k^2)^2}.$$  \hspace{1cm} (48)

We show the running of $f^2$ and $\alpha$ in fig. 6, for different values of $m_k^2/\Lambda_{ch}^2$. Again we stop the flow at some value of $k = \Lambda_{\text{UV}}^{\text{SDE}}$ and solve the Schwinger-Dyson equation with this UV cutoff. The value of $k$ for which the SDE yields the given $m_k^2/\Lambda_{ch}^2$ is indicated in fig. 6 by a dot. The dots in fig. 5 show the corresponding $k$ dependence of $m_k/\Lambda_{ch}$.

We are aware that our treatment of the infrared cutoff is only qualitative. While the one-loop form of the flow equation can be maintained if we interpret the flow as an approximation to the exact functional renormalization group equation for the average action \[8\], the approximation of the exact inverse quark and chiron propagators by $Z_{L,R}^q$ and $Z_+ P_{kl}(q)$ with momentum independent $Z$-factors becomes questionable in the presence of large anomalous dimensions.

5 Composite Higgs scalar

An elegant method for the description of composite particles in the context of functional renormalization uses partial bosonization [9]. It is based on the observation that an interaction of the type (38) can be described by the exchange of a composite scalar field.
Figure 6: Running of $f^2$ (red curves) and $\alpha$ (blue curves). Solid (dashed, dotted) lines correspond to $m = 0.3$ (0.6, 0.9) $\Lambda_{ch}$.

Indeed, one may add to the flowing action $\Gamma_k$ a scalar piece, with $\phi$ a complex doublet scalar field

$$
\Gamma^{(S)}_k = \int \{ Z_\phi \partial^\mu \phi^\dagger \partial_\mu \phi + \bar{m}_\phi^2 \phi^\dagger \phi + \bar{h}(\bar{\psi}_R \phi^\dagger \psi_L - \bar{\psi}_L \phi \psi_R) \} 
$$

(49)

"Integrating out" the scalar field by solving its field equation as a functional of $\psi$ and reinserting the solution into eq. (49) yields eq. (38), with $\lambda$ dependent on the squared exchanged momentum in the scalar channel

$$
\lambda(q) = \frac{\bar{h}^2}{2(Z_\phi q^2 + \bar{m}_\phi^2)}.
$$

(50)

As long as the momentum dependence of the effective scalar exchange vertex is not resolved (as in our computation where the vertex is only evaluated for $q^2 = 0$), one may take arbitrary $Z_\phi$. We will therefore replace $\Gamma^{(S)}_k$ in eq. (38) by eq. (49), and replace the flow of $\lambda$ in eq. (47) by

$$
k \frac{\partial}{\partial k} \bar{h}^2 = -\frac{f^4}{\pi^2} \tilde{s}(m_\phi^2/k^2) \bar{m}_\phi^2/k^2 + 2\bar{h}^2.
$$

(51)

At this stage our reformulation precisely reproduces the results in sect. 4. The rule for the replacement of the flow of $\lambda(q)$ by a running of the renormalized Yukawa coupling and dimensionless mass term

$$
h^2 = \frac{\bar{h}^2}{Z_\phi}, \quad \bar{m}_\phi^2 = \bar{m}_\phi^2/(Z_\phi k^2)
$$

(52)

is to adjust the flow of $h^2$ and $\bar{m}_\phi^2$ such that the flow of $\lambda(q)$ is reproduced, with

$$
\lambda(q) = \frac{h^2}{2k^2} \left( \frac{q^2}{k^2} + \bar{m}_\phi^2 \right)^{-1}.
$$

(53)

For the approximately scale invariant situation for small coupling one expects that the relative split into $q^2$-dependent and $q^2$-independent parts does not depend much on $k$. 

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Figure 7: Contribution of the Yukawa coupling to the quark anomalous dimensions. The corresponding diagram in terms of chiral tensor exchange is shown in (b). It is obtained from the box diagram in fig. 4 by contracting the $t_R$ line, in analogy to fig. 7a.

We therefore make the approximation that the flow of $\tilde{m}_\varphi^2$ receives no contribution from bosonization, such that

$$\partial_k \lambda = \partial_k (h^2/k^2)/(2\tilde{m}_\varphi^2)$$

(54)

or

$$k\partial_k h^2 - 2h^2 = 2\tilde{m}_\varphi^2 k^3 \partial_k \lambda = -\frac{f^4}{\pi^2} s(m_t^2/k^2)\tilde{m}_\varphi^2 + (\eta_L + \eta_R) h^2.$$  

(55)

The effective initial value $\tilde{m}_\varphi^2(\Lambda)$ can be computed by evaluating the momentum dependence of the four-fermion interaction in the scalar channel. At present, it remains a free parameter. In a more complete calculation one should also evaluate the diagrams in fig. 4 for non-zero external momenta and choose the flow of $h^2$ and $\tilde{m}_\varphi^2$ such that the flow of the vertex $\lambda(q)$ is well approximated by eq. (53). We expect that the resulting flow for $\tilde{m}_\varphi^2$ will be attracted to an approximate fixed point. We note that for a fixed (point) value of $\tilde{m}_\varphi^2$ the mass term $\bar{m}_\varphi^2$ decreases roughly $\sim k^2$.

The formulation in terms of a composite scalar field allows an inclusion of the scalar field fluctuations as well. This adds new diagrams, which can be viewed as a partial resummation of the box diagrams in fig. 4. In particular, the quark wave function renormalizations $Z_{L,R}$ get additional contributions from the scalar exchange diagrams in fig. 7 (with scalars represented as double lines), as well known from computations in the standard model. In consequence, the anomalous dimensions acquire a scalar contribution with opposite sign to the tensor contribution.

$$\eta_L = -\frac{3f^2}{4\pi^2} s(\tilde{m}_t^2) + \frac{h^2}{16\pi^2} s(\tilde{m}_\varphi^2, \tilde{m}_t^2),$$

$$\eta_R = -\frac{3f^2}{2\pi^2} s(\tilde{m}_t^2) + \frac{h^2}{8\pi^2} s(\tilde{m}_\varphi^2, \tilde{m}_t^2),$$

(56)

with $\tilde{m}_t^2 = m_t^2/k^2$ and

$$s(\tilde{m}_\varphi^2, \tilde{m}_t^2) = \frac{1 + \ln[(1 + \tilde{m}_t^2 + \tilde{m}_\varphi^2)/(1 + \tilde{m}_\varphi^2)]}{1 + \tilde{m}_t^2 + \tilde{m}_\varphi^2}.$$  

(57)
For $\tilde{m}_t = 0$ the scalar correction to $\eta_{L,R}$ becomes approximately $\Delta \eta_{L,R} \sim h^2/\tilde{m}_\phi^2 \sim f^4$. As long as $f^2$ remains small, the effective two-loop contribution corresponding to scalar exchange (cf. fig. 7b) remains subleading. However, for large $f^2$ the scalar contribution to $\eta_{L,R}$ may become important and modify the anomalous dimensions towards positive values. This could contribute to a final stop of the increase of $f^2$ which obeys now

$$k \frac{\partial}{\partial k} f^2 = (\eta_R + \eta_L + \eta_+) f^2,$$

with $\eta_{L,R}$ given by eq. (56).

We next turn to the scalar contributions to the running of $\tilde{m}_\phi^2$ and $Z_\phi$. The fermion loop correction to the inverse scalar propagator, as depicted in fig. 8, results in the standard result for the anomalous dimension of the scalar field,

$$k \frac{\partial}{\partial k} Z_\phi = -\frac{3}{8\pi^2} \tilde{h}^2 s(\tilde{m}_t^2), \quad \eta_\phi = -k \frac{\partial}{\partial k} \ln Z_\phi = \frac{3}{8\pi^2} \tilde{h}^2 s(\tilde{m}_t^2).$$

(59)

Due to the Yukawa coupling $\tilde{h}^2$ a positive non-vanishing $Z_\phi$ is generated, even if it is absent at some microscopic scale. This produces a pole in the scalar propagator for $q^2 = -m_\phi^2 = -\tilde{m}_\phi^2/Z_\phi$, such that the scalar behaves as a dynamical particle. The contribution of the quark loop shown in fig. 8 to the flow of the scalar mass term is $\sim h^2$. Within functional renormalization it has been investigated in [10] and one finds with our cutoff in the fermion propagator

$$k \frac{\partial}{\partial k} \tilde{m}_\phi^2 = \frac{3}{2\pi^2} h^2 \tilde{s}(\tilde{m}_t^2),$$

$$\tilde{s}(\tilde{m}_t^2) = \ln \frac{\tilde{\Lambda}^2}{k^2} - 1 - \ln(1 + \tilde{m}_t^2).$$

(60)

The momentum integral for the contribution of fig. 8 to the flow of $\tilde{m}_\phi^2$ has been cut at some effective scale $\tilde{\Lambda}$. Here $\tilde{\Lambda}$ is a characteristic scale below which the description of the flow in terms of scalar fluctuations becomes a reasonable approximation. This should be somewhat above $\Lambda_{ch}$, but the precise value remains somewhat arbitrary without additional computations. (In any case the use of an improved infrared cutoff within functional renormalization would remove this spurious dependence on a scale.)
For sufficiently large $h^2$ the flow (60) drives $\tilde{m}_\varphi^2$ to negative values, indicating the onset of spontaneous symmetry breaking. This is the same physics that is responsible for the nontrivial solutions of the Schwinger-Dyson equation for $m_t$ in the preceding section. Indeed, for a qualitative picture we can take for $k < k_0$ a constant $\tilde{h}^2$ and $\hat{s}$, and neglect $\eta_\varphi$ as well as the contribution from bosonization. This replaces eq. (60) by

$$k \frac{\partial}{\partial k} \tilde{m}_\varphi^2 = \frac{3\hat{s}\tilde{h}^2}{2\pi^2 k^2}. \quad (61)$$

For the solution one finds the critical value $\tilde{h}_c^2 = (4\pi^2/3\hat{s})\tilde{m}_\varphi^2(k_0)/k_0^2$ for which $\tilde{m}_\varphi^2$ reaches zero for $k \to 0$. Inserting $\hat{s} = \frac{1}{2}$ and using

$$\alpha = \frac{\tilde{h}_c^2 k_0^2}{2\tilde{m}_\varphi^2(k_0)}, \quad (62)$$

this indeed corresponds to $\alpha_c = 4\pi^2/3$. The vacuum expectation value $\varphi_0 = Z^{1/2}_\varphi \tilde{\varphi}_0$, with $\tilde{\varphi}_0$ the location of the minimum of the scalar effective potential, differs from zero if $\tilde{m}_\varphi^2$ gets negative. For the computation of its value, which should be $\varphi_0 = 175$ GeV in a realistic model, one further needs to compute the quartic scalar self interaction $\lambda_\varphi$. We can adjust the value of $\Lambda_{ch}$ (or the ultraviolet value $f^2(\Lambda_{UV})$) such that the Fermi scale $\varphi_0$ has the correct value.

A particularly interesting quantity is the renormalized Yukawa coupling $h$ (52). Its value for $k = 0$ determines the top quark mass in terms of the Fermi scale

$$m_t = h(k = 0)\varphi_0 = h_t \varphi_0. \quad (63)$$

In other words, the knowledge of $h_t = h(k = 0)$ is equivalent to a determination of the mass ratio $m_t/m_W$, where we use $m_W = (g_W/\sqrt{2})\varphi_0$, with $g_W$ the known weak gauge coupling ($g_W^2/4\pi \approx 0.033$). The observational value is $h_t = 0.98$. While the scale $\varphi_0$ is set by dimensional transmutation and therefore a free parameter, a computation of $h_t$ is equivalent to a parameter-free “pre”-diction for the mass ratio $m_t/m_W$.

In our approximation we can infer the flow equation for $h^2$ from eq. (51),

$$k \frac{\partial}{\partial k} h^2 = (2 + \eta_L + \eta_R + \eta_\varphi) h^2 - \frac{f^4}{\pi^2} \tilde{m}_\varphi^2 \hat{s}(\tilde{m}_t^2) \quad (64)$$

We have solved the flow equations numerically until the onset of spontaneous symmetry breaking at $k_{SSB}$ where $\tilde{m}_\varphi^2(k_{SSB}) = 0$. For $k > k_{SSB}$ one has $\tilde{m}_t = 0$ such that many threshold functions equal one. We display the running of $h$ and $f$ in fig. 9. For $k < k_{SSB}$ one has to continue the flow in the regime with spontaneous symmetry breaking and non-zero $\varphi_0(k)$, as well known from many studies of functional flow equations [10] [11]. We will not do so here.

In the present approximation to the flow equations we observe an increase of $f$ and $h$ to very large values as $m_\varphi^2$ approaches zero. We do not expect our approximations to
Figure 9: Running of $f$ (red curves), $h$ (blue curves in the left picture) and $\tilde{m}_\varphi^2$ (picture on the right). At large values of $k$ we started with $\tilde{m}_\varphi^2 = 0.1$ (solid lines) and $\tilde{m}_\varphi^2 = 0.3$ (dashed lines).

remain valid for large couplings, even though the one-loop form of the functional flow equations is exact \[8\]. One of the main shortcomings is the inaccurate truncation of the general form of momentum dependence for the fermion propagators and non-local interactions in a region where the anomalous dimensions $\eta_{L,R,+}$ are of the order one. For example, the quartic interactions for the bare quarks in the tensor channel replaces in our approximation $P_{kl}/q^4 \to P_{kl}/(Z_+q^4)$ in eq. (2). We may consider an effective momentum dependence of $Z_+$ given by the anomalous dimension $\eta_+(q^2)$ evaluated for $k = \sqrt{q^2}$, i.e. $Z_+ \sim (q^2)^{-2\eta_+(q^2)}$ - this would indeed be a valid approximation for small $|\eta_+| \ll 1$ and $q^2 \gg k^2$. However, for $\eta_+ = 1$ the momentum dependence of $Z_+$ would effectively cancel the nonlocality $\sim 1/q^2$ of the interaction. For a quasi-local interaction in the tensor channel we expect strong modifications of the flow of $f$ - for example, it could reach a fixed point $f^*$, replacing in the interaction $f^2/q^2 \to f^*/(c\Lambda_{ch}^2)$. (In the language of chiral tensor fields this would correspond to the generation of a non-local mass for the “chirons” \[6\].)

For a realistic model of electroweak interactions the increase of the Yukawa coupling should stop or be substantially slowed down in the vicinity of its final value at $k = 0$, say for $1 \lesssim h \lesssim 2$. For the corresponding region in the scale $k$, $1.1 < k/\Lambda_{ch} < 1.4$ we find values $0.8 < f^2/(2\pi^2) = \eta_+ < 2.4$. (We use $\tilde{m}_\varphi^2(\Lambda) = 0.1$ for the quantitative estimates.) In view of our discussion, it seems not unreasonable that the non-perturbative infrared physics stops the further increase of $f$ and $h$ in this range. If this happens, the ratio $m_t/m_W$ may come out in a reasonable range. A typical value of the scale for non-perturbative physics where the increase of $f$ and $h$ stops, may be $k_{np} = 1.3\Lambda_{ch}$. At this scale the quantity $\alpha$ (cf. eq. (62) with $k_0 = k_{np}$) has reached a value $\alpha(k_{np}) \approx 10$, not too far from the critical value in the Schwinger-Dyson approach. It is well conceivable that the top quark mass is substantially below the scale $k_{np}$, such that the region of strong interactions may correspond to the multi-TeV-scales and not disturb too much the LEP precision tests of the electroweak theory.

Our computation of the flow of $f$ and the Yukawa coupling $h$ has further substantial
quantitative uncertainties. For example, we have neglected effects from the interactions in sect. 3 that correspond to the exchange of color-octet scalars or vectors. This may be motivated for the region of $k$ where $m_{\phi}^2$ is already small, since a resonance type behavior and spontaneous symmetry breaking is only expected in the scalar singlet sector. On the other hand, these contributions may still play a role in the interesting region where the increase of $f$ may stop.

6 Conclusions

We conclude that models for quarks and leptons with a non-local four-fermion interaction in the tensor channel appear to be promising candidates for an understanding of the electroweak symmetry breaking. No fundamental Higgs scalar is needed. The theory is asymptotically free and generates an exponentially small “chiral scale” $\Lambda_{ch}$ where the dimensionless coupling $f$ grows large. Furthermore, a strong interaction in the scalar channel is generated at scales where $f$ is large. A solution of Schwinger-Dyson equations suggests that this interaction triggers the spontaneous breaking of the electroweak symmetry at a scale determined by $\Lambda_{ch}$. This would solve the gauge hierarchy problem.

We have also introduced a composite Higgs scalar and investigated the flow of its mass. We find indeed spontaneous symmetry breaking with a Fermi scale somewhat below the chiral scale. Our first attempt of an estimate of the Yukawa coupling of the top quark is encouraging, yielding a reasonable range for the ratio $m_t/m_W$.

The model has also other interesting features. It was shown [5] that the flavor and CP-violation is completely described by the CKM matrix [12]. Masses of the light quarks and leptons arise from appropriate four-fermion couplings [13]. First phenomenological constraints from LEP precision tests and the anomalous magnetic moment of the muon have been computed [5].

At the present stage the understanding of the strong interactions around the scale $\Lambda_{ch}$, which would be a few TeV in a realistic model, is still in its infancy. For this reason our estimate of $m_t/m_W$ is only very crude. Nevertheless, no free parameter enters in the determination of this ratio in our model. If the strong interactions can be understood quantitatively, our model leads to a unique “pre”-diction of $m_t/m_W$. It would also predict the masses and interactions of the composite scalar fields.
Appendix A: Non-local four-fermion interaction and chiral tensor fields

In this appendix we show the equivalence of the microscopic action with a theory of chiral tensor fields [5, 6, 14, 15, 16]. This is similar to the equivalence of quantum electrodynamics to a theory with a non-local four-fermion interaction, generated by integrating out the photon. We may start from the non-local four-fermion interaction and obtain the chiral tensor theory by a Hubbard-Stratonovich transformation. We proceed here in the opposite way, starting with a chiral tensor theory and integrating out the chiral tensors.

Starting point is the theory of chiral antisymmetric tensor fields investigated in ref. [5]. We consider a complex antisymmetric tensor field $\beta_{\mu \nu} = -\beta_{\nu \mu}$ which is a doublet of weak isospin and carries hypercharge $Y = 1$. The field can be decomposed into two parts which correspond to irreducible representations of the Lorentz group. The two parts are the “chiral” components of $\beta_{\mu \nu}$:

$$\beta_{\mu \nu}^\pm = \frac{1}{2} \beta_{\mu \nu} \pm \frac{i}{4} \epsilon_{\mu \nu \rho \sigma} \beta_{\rho \sigma}.$$  \hspace{1cm} (65)

These components can be written as $4 \times 4$ matrices acting in the space of Dirac spinors via

$$\beta_{\pm} = \frac{1}{2} \beta_{\mu \nu} \sigma^{\mu \nu},$$  \hspace{1cm} (66)

where $\sigma^{\mu \nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$. The matrix $\beta_+$ ($\beta_-$) acts only on left-handed (right-handed) fermions, i.e.

$$\beta_{\pm} = \beta_{\pm} \frac{1 \pm \gamma^5}{2}.$$  \hspace{1cm} (67)

One may introduce an interaction between the fermions and the chiral tensors:

$$- L_{ch} = \bar{u}_R F_U \beta_+ q_L - \bar{q}_L F_U^\dagger \beta_+ u_R + \bar{d}_R F_D \beta_- q_L - \bar{q}_L F_D^\dagger \beta_- d_R + \bar{e}_R F_L \beta_0 l_L - \bar{l}_L F_L^\dagger \beta_- e_R.$$  \hspace{1cm} (68)

Here the chiral couplings $F_{U,D,L}$ are $3 \times 3$ matrices in generation space, $q_L$ are the left-handed quark doublets, $u_R$ ($d_R$) are the right-handed up-type (down-type) quarks, $l_L$ are the left-handed lepton doublets, $e_R$ are the right-handed electron-type leptons, and we defined

$$\beta_{\pm} = \frac{1}{2} (\beta_{\mu \nu}^\pm)^\ast \sigma^{\mu \nu} = \beta_{\pm} \frac{1 \mp \gamma^5}{2}, \quad \bar{\beta}_+ = -i \beta_+^T \tau_2, \quad \bar{\beta}_- = - \tau_2 \bar{\beta}_+,$$  \hspace{1cm} (69)

where the transposition $\beta^T$ and the Pauli matrix $\tau_2$ act in weak isospin space, i.e. on the two components of the weak doublet $\beta_{\mu \nu}^\pm$.

The fields $\beta_{\mu \nu}^\pm$ can be represented by three-vectors $B_{k}^\pm$,

$$\beta_{jk}^+ = \epsilon_{jkl} B_{l}^+, \quad \beta_{0k}^+ = i B_{k}^+ , \quad \beta_{jk}^- = \epsilon_{jkl} B_{l}^-, \quad \beta_{0k}^- = -i B_{k}^- .$$  \hspace{1cm} (70)

Rewriting the kinetic term

$$- L_{kin} = \frac{1}{4} \left( (\partial^\rho \beta^{\mu \nu})^\ast \partial_\rho \beta_{\mu \nu} - 4 (\partial_\mu \beta^{\mu \nu})^\ast \partial_\rho \beta^{\rho \nu} \right)$$  \hspace{1cm} (71)
in terms of the $B$-fields and in Fourier space gives

$$-S_{\text{kin}} = \int \frac{d^4q}{(2\pi)^4} \left[ (B^+_k)^*(q)P_{kl}(q)B^+_l(q) + (B^-_k)^*(q)P^*_{kl}(q)B^-_l(q) \right]. \quad (72)$$

The propagator

$$P_{kl} = -(q^2_0 + q_jq_j)\delta_{kl} + 2q_kq_l - 2i\epsilon_{klj}q_0q_j \quad (73)$$

has the properties

$$P_{kl}P^*_{lj} = q^4\delta_{kj}, \quad P^*_{kl}(q) = P_{lk}(q). \quad (74)$$

In the following we ignore for simplicity all couplings except $f \equiv f_t$, i.e. we assume

$$F_U = \begin{pmatrix} f & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad F_D = F_L = 0. \quad (75)$$

In this case, only the $B^+$ fields couple to top quarks $t$ and left-handed bottom quarks $b$. All other fields are free if we ignore gauge couplings. We denote the two weak isospin components of $B^+$ as $B^{++}$ and $B^{+0}$, since they correspond to electric charge +1 and 0 after electroweak symmetry breaking. The action for the chiral interactions is then

$$-S_{\text{ch}} = 2f \int d^4x \left[ -B^{+0}_k \bar{t}\sigma^k_+ t + B^{++}_k \bar{t}\sigma^k_+ b + (B^{+0}_k)^* \bar{t}\sigma^k_- t - (B^{++}_k)^* \bar{b}\sigma^k_- t \right], \quad (76)$$

In a spinor basis in which

$$\gamma^0 = -i \begin{pmatrix} 0 & 12 \\ 12 & 0 \end{pmatrix}, \quad \gamma^i = -i \begin{pmatrix} 0 & \tau^i \\ -\tau^i & 0 \end{pmatrix}, \quad \gamma^5 = -i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} 12 & 0 \\ 0 & -12 \end{pmatrix} \quad (77)$$

the matrices $\sigma^k_\pm$ are defined in terms of the Pauli matrices $\tau^k$ as

$$\sigma^k_+ = \begin{pmatrix} \tau^k & 0 \\ 0 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 \\ 0 & \tau^k \end{pmatrix}. \quad (78)$$

In the action $-S_{\text{ch}}$, a summation over quark color is understood. The classical field equations are obtained by varying $S_B \equiv S_{\text{kin}} + S_{\text{ch}}$ with respect to the $B^+$ fields. They determine these fields as functionals of the quark fields. In momentum space, these relations are

$$B^{+0}_k(q) = -2f \frac{P^*_{kl}(q)}{q^4} \int \frac{d^4k}{(2\pi)^4} \bar{t}(k)\sigma^-_t(k + q), \quad (79)$$

$$(B^{+0}_k)^*(q) = 2f \frac{P_{kl}(q)}{q^4} \int \frac{d^4k}{(2\pi)^4} \bar{t}(k)\sigma^+_t(k - q),$$

$$B^{++}_k(q) = 2f \frac{P^*_{kl}(q)}{q^4} \int \frac{d^4k}{(2\pi)^4} \bar{b}(k)\sigma^-_b(k + q),$$

$$(B^{++}_k)^*(q) = -2f \frac{P_{kl}(q)}{q^4} \int \frac{d^4k}{(2\pi)^4} \bar{t}(k)\sigma^+_b(k - q).$$
We then insert these relations into $S_B$ and obtain the action $S_4$ of the non-local four-fermion interaction given in eq. (22). In a functional integral formulation this procedure is equivalent to performing the Gaussian integral over the $B^+$ fields.

**Appendix B: One loop expressions for the effective action**

In this appendix we evaluate the interaction contribution $F$ to the inverse propagator $S^{(2)}$ in eq. (7) for the different quark types separately, $(S^{(2)} = P_0 + F)$. One obtains

$$F^{cc'}_{lt,\alpha\beta}(p, p') = 4f^2 \int \frac{d^4q}{(2\pi)^4} \bar{t}_\gamma(q + p') t_\gamma(q + p) \left[ \sigma^k_{\alpha\gamma} \sigma^l_{-\gamma\beta} + \sigma^l_{+\alpha\gamma} \sigma^k_{-\gamma\beta} \right] \left( -\frac{P_{kl}(q)}{q^4} \right)$$

$$+ \delta_{cc'} \bar{t}_\gamma(q + p') t_\gamma(q + p) \left[ \sigma^k_{+\alpha\beta} \sigma^l_{-\gamma\eta} + \sigma^l_{+\alpha\beta} \sigma^k_{-\gamma\eta} \right] \frac{P_{kl}(p' - p)}{(p' - p)^4}$$

$$+ \bar{b}_\gamma(q + p') b_\gamma(q + p) \sigma^k_{+\alpha\gamma} \sigma^l_{+\gamma\beta} \left( -\frac{P_{kl}(q)}{q^4} \right) \right\},$$

$$F^{cc'}_{bb,\alpha\beta}(p, p') = -4f^2 \int \frac{d^4q}{(2\pi)^4} \bar{t}_\gamma(q + p') t_\gamma(q + p) \sigma^l_{-\alpha\beta} \sigma^k_{+\alpha\beta} \frac{P_{kl}(q)}{q^4},$$

$$F^{cc'}_{tb,\alpha\beta}(p, p') = 4f^2 \int \frac{d^4q}{(2\pi)^4} \delta_{cc'} \left[ \bar{b}(q + p') \sigma^l_{-\beta'} t(q + p) \right] \sigma^k_{+\alpha\beta} \frac{P_{kl}(p' - p)}{(p' - p)^4},$$

$$F^{cc'}_{tt,\alpha\beta}(p, p') = 4f^2 \int \frac{d^4q}{(2\pi)^4} \bar{t}(q + p') \sigma^l_{-\beta'} b(q + p) \left[ \sigma^k_{+\alpha\beta} \sigma^l_{-\alpha\gamma} - \sigma^k_{+\alpha\beta} \sigma^l_{-\beta\gamma} \right] \frac{P_{kl}(q)}{q^4},$$

$$F^{cc'}_{tt,\alpha\beta}(p, p') = 4f^2 \int \frac{d^4q}{(2\pi)^4} \bar{t}_\gamma(p' - q) t_\gamma(p + q) \left[ \sigma^k_{+\gamma\beta} \sigma^l_{-\eta\alpha} - \sigma^k_{+\gamma\beta} \sigma^l_{-\eta\alpha} \right] \frac{P_{kl}(q)}{q^4},$$

$$F^{cc'}_{tb,\alpha\beta}(p, p') = -4f^2 \int \frac{d^4q}{(2\pi)^4} \bar{t}_\gamma(p' - q) t_\gamma(p + q) \left[ \sigma^k_{+\alpha\gamma} \sigma^l_{-\eta\beta} - \sigma^k_{+\alpha\gamma} \sigma^l_{-\eta\beta} \right] \frac{P_{kl}(q)}{q^4},$$

$$F^{cc'}_{tt,\alpha\beta}(p, p') = 4f^2 \int \frac{d^4q}{(2\pi)^4} \bar{t}_\gamma(p' - q) t_\gamma(p + q) \sigma^k_{+\gamma\beta} \sigma^l_{-\gamma\eta} \frac{P_{kl}(q)}{q^4},$$

$$F^{cc'}_{BB,\alpha\beta}(p, p') = -F^{cc'}_{AB,\beta\alpha}(p', p). \quad (80)$$

Inserting this into the expression (9), we find an anomalous dimension term for $t$ and for the left-handed $b$

$$\frac{i}{2} \text{Tr} \left( \frac{1}{P_0} F \right) = 4if^2 \int \frac{d^4pd^4q}{(2\pi)^8} \frac{P^{cc'}_{kl}(q - p)}{(q - p)^4} \left\{ \bar{t}(q) \sigma^l_{-\gamma\mu} \sigma^k_{+\gamma\nu} t(q) \right\}$$

$$+ 2\bar{t}(q) \sigma^k_{+\gamma\mu} \sigma^l_{-\gamma\nu} t(q) + \bar{b}(q) \sigma^k_{+\gamma\mu} \sigma^l_{+\gamma\nu} b(q) \right\}. \quad (81)$$
The term $\sim F^2$ in eq. (9) produces an effective four fermion interaction

$$
- \frac{i}{4} \text{Tr} \left( \frac{1}{P_0} F \frac{1}{P_0} F \right) = 8i f^4 \int \frac{d^4 p \, d^4 p' \, d^4 q \, d^4 q'}{(2\pi)^4 \, p \cdot p'} \, \frac{P_{ij}^a(q) \, P_{mn}^a(q')}{q^4} \, A_1
$$

\[ -6 \frac{P^a_{ij}(p-p') \, P^a_{mn}(p-p')}{(p-p')^4} \, \text{tr}(\gamma^\mu \sigma_+^{k} \gamma^\nu \sigma_-^{n}) A_2 \} \] ,

with

$$
A_1 = \left( \bar{t}(q' + p) \sigma_-^n \gamma^\mu \sigma_+^k t(q + p) \right) \left( \bar{t}(q + p') \sigma_-^l \gamma^\nu \sigma_+^m t(q' + p') \right) + 2 \left( \bar{t}(q' + p) \sigma_+^n \gamma^\mu \sigma_-^k t(q + p) \right) \left( \bar{t}(q + p') \sigma_-^l \gamma^\nu \sigma_+^m t(q' + p') \right) + 2 \left( \bar{b}(q' + p) \sigma_-^n \gamma^\mu \sigma_+^k b(q + p) \right) \left( \bar{t}(q + p') \sigma_-^l \gamma^\nu \sigma_+^m b(q' + p') \right) + 2 \left( \bar{t}(p + q) \sigma_-^l \gamma^\nu \sigma_+^n t(p + q') \right) \left( \bar{t}(p' - q) \sigma_-^l \gamma^\nu \sigma_+^m t(p' - q') \right) + 2 \left( \bar{t}(p + q) \sigma_+^l \gamma^\nu \sigma_-^n t(p + q') \right) \left( \bar{b}(p' - q) \sigma_-^l \gamma^\nu \sigma_+^m b(p' - q') \right)
$$

and

$$
A_2 = \left( \bar{t}(q + p') \sigma_-^l t(q + p) \right) \left( \bar{t}(q' + p) \sigma_+^n t(q' + p') \right) \left( \bar{b}(q + p') \sigma_-^l t(q + p) \right) \left( \bar{t}(q' + p) \sigma_+^n b(q' + p') \right).
$$

Color indices are suppressed, since all pairs of fermions in large brackets are color singlets.
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