Finite field-dependent symmetry in the Thirring model

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Received April 4, 2016; Accepted April 27, 2016; Published June 30, 2016

In this paper, we consider a $D$-dimensional massive Thirring model with $(2 < D < 4)$. We derive an extended BRST symmetry of the theory with finite field-dependent parameter. Further we compute the Jacobian of functional measure under such an extended transformation. Remarkably, we find that such a Jacobian extends the BRST exact part of the action which leads to a mapping between different gauges. We illustrate this with the help of the Lorentz and $R_\xi$ gauges. We also discuss the results in the Batalin–Vilkovisky framework.

Subject Index B05, B34, B39

1. Introduction

From earlier studies in quantum field theories (QFT), it is found that ground states showing sensitivity to the number of light fermion flavors are very important. One of the major examples of such theories is the Thirring model, a quantum field theory of fermions interacting via a conserved vector current term, described in three-dimensional space-time. The Thirring model has been studied as a candidate for the scenario of the fermion dynamical mass generation [1–3]. It is important because the fermion dynamical mass generation is the central issue of the dynamical electroweak symmetry breaking such as the technicolor [4–6] and the top quark condensate [7–10]. The role of four-fermion interaction in the context of walking technicolor [11–14] and strong extended technicolor [15–17] has also been investigated. In particular, scalar/pseudoscalar-type four-fermion interactions with the gauge interaction have played a very important role in $D = 4$ dimensions as a renormalizable model [18,19]. This model with gauge interaction is known as the gauged Nambu–Jona-Lasinio (NJL) model [20–22]. It has been observed that the phase structure of such a gauged NJL model in $D = 4$ dimensions is quite similar to that of the $D = 3$-dimensional scalar/pseudoscalar-type four-fermion theory without gauge interactions [23,24], called the Gross–Neveu model [25]. The gauged Thirring model, a natural gauge-invariant generalization of the Thirring model, has been studied in [26], where it is shown that, in the strong gauge-coupling limit, the gauged Thirring model reduces to the proposed reformulation of the Thirring model [2] as a gauge theory.

To quantize the gauge theories, the Becchi–Rouet–Stora–Tyutin (BRST) formulation [27–32] is a powerful method, which guarantees the renormalizability and unitarity of gauge theories. In [3] certain aspects of BRST quantization for the Thirring model in $(2 < D < 4)$ dimensions are discussed. This model is described in the Lorentz and $R_\xi$ gauges there. For instance, it is well known...
that, in the $R_\xi$ gauge, the Stueckelberg (also called Batalin–Fradkin) field $\theta$ is completely decoupled to the massive vector boson independently of $\xi$. This would lead to simplicity in performing numerical computations. However, for the Lorentz gauge, the Stueckelberg field $\theta$ is decoupled to the massive vector boson only for $\xi = 0$ (which refers to the Landau gauge). These two gauges have their own advantages. A mapping between these two gauges in the perspective of the Thirring model would be remarkable because if one gets a complicated expression for the calculations in one gauge, then this mapping would be very helpful. In this context, we try to achieve this goal with the help of a generalization of BRST quantization and with the Batalin–Vilkovisky (BV) description.

The key idea of a generalization (extension) of BRST symmetry is to make the parameter of transformation finite and field dependent in a certain way, known as a finite field-dependent BRST (FFBRST) transformation [33]. The generalization, in this way, has found enormous applications in a wide variety of gauge theories [34,35] as well as in gravity theory [36]. For example, the celebrated Gribov issue [37–39] in Yang–Mills theory has been addressed in the framework of FFBRST formulation (for details, see Ref. [40–45] and the references therein). The FFBRST transformations have been emphasized in higher-form gauge theories, an important ingredient of string theories [46,47]. Further, for the superconformal Chern–Simons matter theories [48–52], aspects of the generalized BRST symmetry have also been reported in [53–56]. The validity of such generalization has been established at quantum level also [57–65] with the help of the BV formulation [66]. Recently, the FFBRST formulation has been studied in (topological) lattice sigma models [67]. A slightly different field-dependent BRST formulation has also been made, in Yang–Mills theories also [68], which involves a linear dependence on the corresponding Grassmann-odd parameter, naturally, without having recourse to any quadratic dependence. Since Ref. [68] does not deal with the case of BRST–antiBRST symmetry, any non-trivial quadratic dependence on the transformation parameters cannot occur. Moshin and Reshetnyak, in Ref. [69], incorporate systematically the case of BRST–antiBRST symmetry in Yang–Mills theories also [68], which deals with the case of a quadratic dependence on the corresponding parameters for two reasons: 1) finite BRST–antiBRST symmetry does admit a non-trivial quadratic dependence on two different Grassmann-odd parameters, 2) this dependence actually turns out to be necessary for a systematic treatment of finite BRST–antiBRST transformations. Further, the concept of finite BRST–antiBRST symmetry for the case of general gauge theories has been extended in Refs. [70,71], whereas Ref. [72] by the same authors generalizes the corresponding parameters to the case of arbitrary Grassmann-odd field-dependent parameters, as compared to the so-called “potential” form of parameters used in the previous articles [69–71].

The field-theoretic models, with fermion interactions of current–current type, are not renormalizable in $D = 4$ as the coupling constant takes the dimension of mass inverse squared. Nevertheless, it has been established that a class of $D$ ($2 < D < 4$) dimensional four-fermion models is renormalizable in the different expansion scheme [73]. In this paper, we consider a renormalizable $D$ ($2 < D < 4$) dimensional gauge non-symmetric Thirring model to discuss the various gauge connections. After the introduction of the auxiliary field, the theory still remains gauge non-invariant. Of course, the Thirring model can be rewritten into the massive vector theory with which the fermion couples minimally. First of all, we discuss the gauge-invariant version of the model, which can be quantized correctly only after breaking the local gauge invariance. This is achieved by fixing the gauges specifically. For the present case, the Lorentz and $R_\xi$ gauges are considered. We write the corresponding gauge-fixed Faddeev–Popov action. The resulting action, by summing the classical
action and the gauge-fixed action, remains invariant under the BRST symmetry. Further, we generalize the BRST symmetry, by forming the transformation parameter finite and field dependent.

The generalized BRST symmetry leaves the Faddeev–Popov action invariant. The only difference from the usual BRST symmetry is the functional measure, which is not covariant under the generalized BRST transformation. So, we compute the Jacobian for the functional measure and find that it depends, explicitly, on an infinitesimal field-dependent parameter. Then, different values of the field-dependent parameter will lead to different contributions in the generating functional. Here, we show that, for a particular value of such a parameter, the Jacobian switches the generating functional from one gauge to another gauge. We illustrate this result for a particular set of gauges, namely, the Lorentz gauge and \( R_\xi \) gauge. Further, we establish the result at quantum level, by mapping the solutions of the quantum master equation in the BV framework.

The paper is organized as follows. In Sect. 2, we discuss the BRST quantization of the Thirring model, with arbitrary as well as specific gauge choices in \( D (2 < D < 4) \) dimensions. Then, we derive the methodology for extended BRST symmetry for the Thirring model, where the Jacobian for path integral measure is computed explicitly. The connections of various gauges through this extended BRST formulation are described in Sect. 4. In Sect. 5, we establish the mapping of different gauges in the BV formulation. The last section summarizes the present investigations with future motivations.

2. Thirring model: BRST symmetry

In this section, we analyze the Thirring model in \( D (2 < D < 4) \) dimensions. The Thirring model is given by the Lagrangian density

\[
L_{\text{Th}} = \bar{\psi}^a i \gamma^\mu \partial_\mu \psi^a - m \bar{\psi}^a \psi^a - \frac{G}{2N} (\bar{\psi}^a \gamma^\mu \psi^a)^2. \tag{1}
\]

Here, \( \psi^a \) refers to a Dirac fermion with flavor index \( a \) which runs from 1 to \( N \). The gamma matrices \( \gamma_\mu (\mu = 0, 1, 2, \ldots, D - 1) \) satisfy the Clifford algebra, following \( \{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \mathbf{1} \).

The Lagrangian can, further, be rewritten in terms of an auxiliary vector field \( A_\mu \) as

\[
L_{\text{Th}'} = \bar{\psi}^a i \gamma^\mu \left( \partial_\mu - i \sqrt{N} A_\mu \right) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} A_\mu^2, \tag{2}
\]

which coincides with (1) when we perform the equation of motion of \( A_\mu \). Here, we note that the field \( A_\mu \) is just a vector field which represents the fermionic current and does not transform as a gauge field. This Lagrangian does not have any gauge symmetry. Besides lacking the kinetic term for the Yang–Mills field, the theory given by the above Lagrangian is identical with the massive Yang–Mills theory.

The gauge-invariant version of the Lagrangian is obtained by introducing the Stueckelberg field \( \theta \) which is identified with the BF field, as shown in [74,75], as

\[
L_{\text{Th}''} = \bar{\psi}^a i \gamma^\mu \left( \partial_\mu - i \sqrt{N} A_\mu \right) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} (A_\mu - \sqrt{N} \partial_\mu \theta)^2. \tag{3}
\]

The original Thirring model can be assumed as the gauge-fixed version of this gauge-invariant Lagrangian [76], which possesses the following \( U(1) \) gauge symmetry:

\[
\delta \psi^a = (e^{i \phi} - 1) \psi^a, \tag{4}
\]

\[
\delta A_\mu = \sqrt{N} \partial_\mu \phi, \tag{5}
\]

\[
\delta \theta = \phi. \tag{6}
\]
Here, $\phi$ denotes a fictitious Nambu–Goldstone boson field, which has to be absorbed into the longitudinal component of $A_\mu$.

To quantize covariantly a gauge-invariant theory, we need to break the local gauge invariance. This removes the fictitious degrees of freedom associated with the theory, and can be achieved by restricting the gauge fields by a general gauge condition, $\Omega = \mathcal{F}[A, \theta] = 0$. This can be incorporated at the level of the Lagrangian by adding the following linearized gauge-fixing and ghost terms to the classical action,

$$\mathcal{L}_{GF+FP} = B \mathcal{F}[A, \theta] + \frac{\xi}{2} B^2 + i \tilde{C} \left( \frac{\delta \mathcal{F}[A, \theta]}{\delta A_\mu} \partial_\mu + \frac{1}{\sqrt{N}} \frac{\delta \mathcal{F}[A, \theta]}{\delta \theta} \right) C, \quad (7)$$

where $B$ is a Nakanishi–Lautrup type multiplier field and $\xi$ is an arbitrary gauge parameter.

### 2.1. Lorentz gauge

Now for a particular choice, the so-called Lorentz gauge, of $\mathcal{F}[A, \theta] = \partial_\mu A_\mu$, the above gauge-fixed Lagrangian is alleviated to

$$\mathcal{L}_{GF+FP}^L = B \partial_\mu A_\mu + \frac{\xi}{2} B^2 + i \tilde{C} \left[ \partial_\mu A_\mu \right] C. \quad (8)$$

The effective action for the Thirring model in the Lorentz gauge is given by

$$\mathcal{L}^{Th''} + \mathcal{L}_{GF+GH}^L = \bar{\psi}^a i \gamma^\mu \left[ \partial_\mu - \frac{i}{\sqrt{N}} A_\mu \right] \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} (A_\mu - \sqrt{N} \partial_\mu \theta)^2 + B \partial_\mu A_\mu$$

$$+ \frac{\xi}{2} B^2 + i \tilde{C} \partial_\mu A_\mu C. \quad (9)$$

Here, we observe that the Stueckelberg field (BF) $\theta$ is coupled to the field except for $\xi = 0$ (Landau gauge).

This action is invariant under the following BRST transformation:

$$\delta_b A_\mu (x) = - \partial_\mu C \eta,$$

$$\delta_b B = 0, \quad \delta_b C = 0,$$

$$\delta_b \tilde{C} = i B \eta, \quad \delta_b \partial_\mu A_\mu = - \frac{1}{\sqrt{N}} C \eta,$$

$$\delta_b \psi^j (x) = \frac{i}{\sqrt{N}} C \psi^j \eta, \quad (10)$$

where $\eta$ is an infinitesimal Grassmann parameter.

### 2.2. $R_\xi$ gauge

For another important choice of gauge, $\mathcal{F}[A, \theta] = \partial_\mu A_\mu + \sqrt{N} \frac{\xi}{G} \theta$, the so-called $R_\xi$ gauge, the gauge-fixed Lagrangian is given by

$$\mathcal{L}_{GF+FP}^R = B \left( \partial_\mu A_\mu + \sqrt{N} \frac{\xi}{G} \theta \right) + \frac{\xi}{2} B^2 + i \tilde{C} \left[ \partial_\mu A_\mu + \frac{\xi}{G} \right] C. \quad (11)$$
Thus, the Faddeev–Popov effective action for the Thirring model in the $R_\xi$ gauge is given by

$$ L_{\text{Th}}'' + L_{\text{GF+GH}}^R = \bar{\psi}^a i \gamma^\mu \left( \partial_\mu - i \frac{1}{\sqrt{N}} A_\mu \right) \psi^a - m \bar{\psi}^a \psi^a + \frac{1}{2G} (A_\mu - \sqrt{N} \partial_\mu \theta)^2 $$

$$ + B \left( \partial_\mu A^\mu + \sqrt{N} \frac{\xi}{G} \theta \right) + \frac{\xi}{2} B^2 + i \tilde{C} \left[ \partial_\mu \partial_\mu + \frac{\xi}{G} \right] C. $$

(12)

This further reduces to

$$ L_{\text{Th}}'' + L_{\text{GF+GH}}^R = \bar{\psi}^a i \gamma^\mu (\partial_\mu - i \frac{1}{\sqrt{N}} A_\mu) \psi^a - m \bar{\psi}^a \psi^a - \frac{1}{2N} \left( \partial_\mu A^\mu \right)^2 + \frac{1}{2G} A^2 \mu $$

$$ - \frac{1}{2G^2} \theta^2 + \frac{N}{2G} (\partial_\mu \theta)^2 + i \tilde{C} \left[ \partial_\mu \partial_\mu + \frac{\xi}{G} \right] C. $$

(13)

Here, we see that the Stueckelberg field $\theta$ is completely decoupled independently of $\xi$. The effective action, in the $R_\xi$ gauge, is also invariant under the same set of BRST transformations (10).

3. Extended BRST transformation

In this section, we derive the extended BRST formulation by making the parameter finite and field dependent, known as the FFBRST transformation, at a general ground. We first define the BRST transformation of a generic field $\phi(x)$ as follows:

$$ \phi(x) \rightarrow \phi'(x) = \phi(x) + s_b \phi(x) \eta, $$

(14)

where $s_b \phi$ refers to Slavnov variation and $\eta$ is an infinitesimal anticommuting global parameter. It is well known that, under such a transformation, the path integral measure as well as the effective action remain invariant [27].

Now, we interpolate a continuous parameter ($\kappa; 0 \leq \kappa \leq 1$) through the fields $\phi(x)$ such that the $\phi(x, \kappa = 0) = \phi(x)$ is the original field and $\phi(x, \kappa = 1) = \phi'(x) = \phi(x) + s_b \phi(x) \Theta[\phi]$ is the FFBRST transformed field, where $\Theta[\phi]$ is a finite field-dependent parameter. Such an FFBRST transformation is justified by the following infinitesimal field-dependent BRST transformation [33]:

$$ \frac{d\phi(x, \kappa)}{d\kappa} = s_b \phi(\kappa) \Theta'[\phi(x, \kappa)]. $$

(15)

Now, integrating the above equation with respect to $\kappa$ from 0 to 1, we get the FFBRST transformation,

$$ \delta_b \phi(x) = \phi'(x) - \phi(x) = s_b \phi(x) \Theta[\phi(x)], $$

(16)

where the finite field-dependent parameter is given by

$$ \Theta[\phi] = \Theta'[\phi] \exp f[\phi] - 1 \over f[\phi], $$

(17)

and $f[\phi]$ is given by

$$ f[\phi] = \sum_i \int d^4 x \frac{\delta \Theta[\phi]}{\delta \phi_i(x)} s_b \phi_i(x). $$

(18)
The FFBRST transformations, with field-dependent parameter, are also a symmetry of the effective action but the cost we pay is that they are no longer nilpotent. Contrary to usual BRST symmetry, they do not leave the functional measure invariant. Eventually, the path integral measure under such a transformation changes non-trivially, leading to a local Jacobian in the functional integration. So our goal here is to compute the explicit Jacobian of the functional measure, under such a transformation.

3.1. Jacobian for field-dependent BRST transformation

To compute the Jacobian for the path integral measure under the FFBRST transformation with an arbitrary parameter $\Theta_1$, we first define the generating functional for the Thirring model described by an effective action $S_{\text{Th}}^{\text{FP}}[\phi]$ as follows,

$$Z[0] = \int \mathcal{D}\phi \ e^{iS_{\text{Th}}^{\text{FP}}[\phi]}, \quad (19)$$

where $\mathcal{D}\phi$ refers to the full functional measure. Furthermore, we write the functional measure under the action of the FFBRST transformation as follows [33]:

$$\mathcal{D}\phi = J(\kappa) \mathcal{D}\phi(\kappa)$$

where

$$J(\kappa) = J(\kappa + d\kappa)^{-1}.$$

This reduces to

$$\frac{d \ln J[\phi]}{d\kappa} = -\int d^Dx \sum_\phi \pm s_b \phi(x, \kappa) \frac{\delta \Theta'[\phi(x, \kappa)]}{\delta \phi(x, \kappa)}.$$

To attain the expression for the finite Jacobian from the (above) infinitesimal one, we integrate it over $\kappa$ with limits from 0 to 1. This leads to a logarithmic series,

$$\ln J[\phi] = -\int_0^1 d\kappa \int d^Dx \sum_\phi \pm s_b \phi(x, \kappa) \frac{\delta \Theta'[\phi(x, \kappa)]}{\delta \phi(x, \kappa)},$$

Now, exponentiating the above relation leads to the expression for the Jacobian for the functional measure, under an FFBRST transformation with an arbitrary field-dependent parameter $\Theta'$, as follows:

$$J[\phi] = \exp \left( -\int d^Dx \sum_\phi \pm s_b \phi(x) \frac{\delta \Theta'[\phi(x)]}{\delta \phi(x)} \right).$$
Here, we see that the Jacobian (25) extends the Faddeev–Popov action (within the functional integral) of the theory, given in (19), as follows:

\[
\int D\phi' e^{iS_{FP}[\phi']} = \int J[\phi] D\phi e^{iS_{FP}[\phi]} = \int D\phi e^{i\left(\mathcal{S}_{FP}[\phi] + i \int d^Dx \left( \sum_\phi \pm \phi \frac{\Theta_1}{\sqrt{N}} \right) \right)}.
\]

Notice that the Jacobian changes only the exact part of the BRST action, so the dynamics of the theory does not change as the BRST exact part of a BRST invariant function does not alter the dynamics of the theory at cohomological level.

### 4. Connection of Lorentz gauge to \( R_\xi \) gauge

In this section, we illustrate the results of Sect. 3 with a specific example. Following the methodology discussed above, we first construct the FFBRST transformation for the Thirring model,

\[
\begin{align*}
\delta_b A_\mu(x) &= -\partial_\mu \Theta[\phi], \\
\delta_b B &= 0, \quad \delta_b C = 0, \\
\delta_b \bar{C} &= iB \Theta[\phi], \\
\delta_b \theta &= -\frac{1}{\sqrt{N}} C \Theta[\phi], \\
\delta_b \psi^j(x) &= \frac{i}{\sqrt{N}} C \psi^j \Theta[\phi],
\end{align*}
\]

where \( \Theta[\phi] \) is an arbitrary finite field-dependent parameter satisfying \( \Theta^2 = 0 \). Now, we construct a specific \( \Theta' \), to see the effect of the FFBRST transformation in the Thirring model. This is given by

\[
\Theta'[\phi] = \int d^Dx \left[ \bar{C} \sqrt{N} \frac{\xi}{G} \theta \right].
\]

For this choice of parameter, we calculate the Jacobian for the functional measure

\[
J[\phi] = \exp \left( i \int d^Dx \left[ B \sqrt{N} \frac{\xi}{G} \theta + iC \sqrt{N} \frac{\xi}{G} \right] \right),
\]

where (25) is utilized.

Here, we observe that the Jacobian contributes to the unphysical (BRST exact) part of the action. This Jacobian modifies the expression of the generating functional in the Lorentz gauge as follows:

\[
\int D\phi' e^{i \int d^Dx (\mathcal{L}_G + \mathcal{L}_{GF+GH})[\phi']} \xrightarrow{\text{FFBRST}} \int D\phi e^{i \int d^Dx (\mathcal{L}_G + \mathcal{L}_{GF+GH})[\phi]},
\]

where the final expression is nothing but the generating functional in the \( R_\xi \) gauge. Such a modification does not alter the theory because the extra pieces, due to the Jacobian, are attributed to the BRST exact part of the action. Though we have shown the connection of two specific gauges, the results are valid for any arbitrary pair of gauges. Suppose we choose a parameter \( \Theta'[A, \theta, \bar{C}] = \int d^Dx \left[ \bar{C} \left( \mathcal{F}_1[A, \theta] - \mathcal{F}_2[A, \theta] \right) \right] \), where \( \mathcal{F}_1 \) and \( \mathcal{F}_2 \) are two arbitrary gauges, then the Jacobian will map the generating functional corresponding to these two gauges. Thus, we see that the two well-studied gauges of the Thirring model are related to each other. This is shown with the help of an extended BRST transformation, with a particular parameter of transformation.
5. BV formulation and FFBRST symmetry

In the BV formulation, the generating functional of the Thirring model (in the Lorentz gauge), by introducing antifields $\phi^*$ corresponding to the fields $\phi(\equiv A_\mu, \bar{C}, C, B, \theta)$ with opposite statistics, is given by

$$Z_L = \int \mathcal{D}\phi \ e^{i \int d^4x (L_{Th'} + L_{GF+GH}[\phi, \phi^*])}. \quad (31)$$

This can, further, be written in compact form as

$$Z_L = \int \mathcal{D}\phi \ e^{iW_{\Psi_L}[\phi, \phi^*]}, \quad (32)$$

where $W_{\Psi_L}[\phi, \phi^*]$ is an extended quantum action in the Lorentz gauge. The generating functional does not depend on the choice of gauge-fixing fermion [66]. The extended quantum action for the Thirring model, $W_{\Psi_L}[\phi, \phi^*]$, satisfies the following mathematically rich relation, called the quantum master equation [27]:

$$\Delta e^{iW_{\Psi_L}[\phi, \phi^*]} = 0 \quad \text{with} \quad \Delta \equiv \frac{\partial_r}{\partial\phi} \frac{\partial_r}{\partial\phi^*} (-1)^{\epsilon+1}. \quad (33)$$

The antifields, which get identified with the gauge-fixing fermion in the Lorentz gauge $\Psi_L = -i \bar{C} \left( \partial_\mu A_\mu + \frac{\xi}{2} B \right)$, are

$$A^*_\mu = \frac{\delta \Psi_L}{\delta A^\mu} = i \partial_\mu \bar{C}, \quad \bar{C}^* = \frac{\delta \Psi_L}{\delta \bar{C}} = -i \left( \partial_\mu A^\mu + \frac{\xi}{2} B \right), \quad C^* = \frac{\delta \Psi_L}{\delta C} = 0, \quad \theta^* = \frac{\delta \Psi_L}{\delta \theta} = 0. \quad (34)$$

Similarly, the generating functional for the Thirring model in the $R_\xi$ gauge is defined, compactly, as

$$Z_R = \int \mathcal{D}\phi \ e^{i \int d^4x (L_{Th'} + L_{GF+GH}[\phi, \phi^*])},$$

$$= \int \mathcal{D}\phi \ e^{iW_{\Psi_R}[\phi, \phi^*]}. \quad (35)$$

The following expressions for the antifields, in the case of the $R_\xi$ gauge, are obtained:

$$A^*_\mu = \frac{\delta \Psi_R}{\delta A^\mu} = i \partial_\mu \bar{C}, \quad \bar{C}^* = \frac{\delta \Psi_R}{\delta \bar{C}} = -i \left( \partial_\mu A^\mu + \sqrt{N} \frac{\xi}{G} \theta + \frac{\xi}{2} B \right), \quad C^* = \frac{\delta \Psi_R}{\delta C} = 0, \quad \theta^* = \frac{\delta \Psi_R}{\delta \theta} = -i \sqrt{N} \frac{\xi}{G} \bar{C}, \quad (36)$$

where $\Psi_R = -i \bar{C} \left( \partial_\mu A^\mu + \sqrt{N} \frac{\xi}{G} \theta + \frac{\xi}{2} B \right)$ is utilized. To connect the Lorentz and $R_\xi$ gauges in the BV formulation, we construct the following infinitesimal field-dependent parameter $\Theta'[\phi]$:

$$\Theta'[\phi] = i \int d^Dy \left[ \bar{C} \bar{C}^* - \bar{C} \bar{C}^* \right]. \quad (37)$$
The Jacobian of the path integral measure in the generating functional, for this parameter, is computed utilizing relation (25). The resulting Jacobian factor changes the quantum action as

\[ W^{\text{FFBRST}}_{\Psi_L}[\phi, \phi^*] \rightarrow W^{\text{FFBRST}}_{\Psi_R}[\phi, \phi^*]. \]  

(38)

This reflects the validity of the result at quantum levels also. Hence, we conclude that the FFBRST transformations connect two different solutions of the quantum master equation of the Thirringer model.

6. Conclusion

From the effective potential points of view, the existence of the second-order phase transition associated with the spontaneous breakdown of the chiral symmetry in the \( D (2 < D < 4) \) dimensional Thirringer model has been analyzed, and the explicit critical number of flavors has been derived as a function of the four-fermion coupling constant. The Thirringer model as a gauge theory, by introducing the Stueckelberg field as a BF field, is well studied. In this context, without gauge interactions the Thirringer model is identified with the gauge-fixed version of a gauge theory and has the well-known BRST symmetry even after the gauge-fixing.

In this paper, we have considered the gauge-invariant as well as renormalizable Thirringer model in \( D (2 < D < 4) \) dimensions. Since the gauge invariance possesses the unphysical degrees of freedom, according to standard quantization procedure we have to remove them by breaking the local gauge invariance. This can be achieved by fixing an appropriate gauge. We have discussed the well-studied Lorentz and \( R_\xi \) gauges in this context. The remarkable properties of these gauges in the Thirringer model are that, in the \( R_\xi \) gauge, the Stueckelberg (BF) field \( \theta \) is completely decoupled to the massive vector boson independently of \( \xi \), which makes the computations simple. However, in the Lorentz gauge, the Stueckelberg field \( \theta \) is decoupled to the massive vector boson only if \( \xi = 0 \). In this sense, the \( R_\xi \) gauge is more acceptable for the model. To map these gauges, we have extended the BRST symmetry by making the transformation parameter finite and field dependent. Under such a transformation, the path integral measure is not unchanged, rather it changes in a non-trivial way. The Jacobian of the path integral measure depends, explicitly, on a field-dependent parameter. We compute the Jacobian for an arbitrary parameter, to be valid at a general ground. However, for a specific choice of parameter, we have illustrated that the Jacobian connects the Lorentz gauge to the \( R_\xi \) gauge. Though we have established a connection for a particular set of gauges, this formalism would be valid for connecting any two sets of gauges. We have computed the extended quantum action as well as the quantum master equation, utilizing the BV formulation. Further, we have shown a mapping between the two different solutions of the quantum master equation with the help of the FFBRST transformation. The analysis of the Thirringer model as the gauged non-linear sigma model is given from the viewpoint of the constrained system, which implies that the present investigation might be useful from the perspective of the non-linear sigma model.

Funding

Open Access funding: SCOAP$^3$.

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