\[ \bar{B}_s \rightarrow K \] semileptonic decay from an Omnès improved nonrelativistic quark model

C Albertus\(^1\), E Hernández\(^2\), C Hidalgo-Duque\(^3\) and J Nieves\(^3\)

\(^1\) Departamento de Física Atómica, Nuclear y Molecular e Instituto Carlos I de Física Teórica y Computacional, Universidad de Granada, Avenida de Fuentenueva s/n, E-18071 Granada, Spain
\(^2\) Departamento de Física Fundamental e IUFFyM, Universidad de Salamanca, Plaza de la Merced s/n, E-37008 Salamanca, Spain
\(^3\) Instituto de Física Corpuscular (IFIC), Centro Mixto CSIC-Universidad de Valencia, Institutos de Investigación de Paterna, Apartado 22085, E-46071 Valencia, Spain

E-mail: albertus@ugr.es, gajatee@usal.es, carloshd@ific.uv.es, jmnieves@ific.uv.es

Abstract. We study the \( f^+ \) form factor for the \( \bar{B}_s \rightarrow K^+ \ell^- \bar{\nu}_\ell \) semileptonic decay in a nonrelativistic quark model. The valence quark contribution is supplemented with a \( \bar{B}^* \)-pole term that dominates the high \( q^2 \) region. To extend the quark model predictions from its region of applicability near \( q^2_{\text{max}} = (M_{\bar{B}_s} - M_K)^2 \), we use a multiply-subtracted Omnès dispersion relation. We fit the subtraction constants to a combined input from previous light cone sum rule results in the low \( q^2 \) region and the quark model results (valence plus \( \bar{B}^* \)-pole) in the high \( q^2 \) region. From this analysis, we obtain \( \Gamma(\bar{B}_s \rightarrow K^+ \ell^- \bar{\nu}_\ell) = (5.47^{+0.54}_{-0.46}) V_{ub}^2 \times 10^{-9} \text{MeV} \), which is about 10% and 20% higher than predictions based on Lattice QCD and QCD light cone sum rules respectively.

1. Introduction

Playing a critical role in testing the consistency of the Standard Model of particle physics and, in particular, the description of CP violation, \( V_{ub} \) is still the well known element of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Any new information that can be obtained from experimentally unexplored reactions is thus relevant. This is the case of the \( B_s \rightarrow K^+ \ell^- \bar{\nu}_\ell \) semileptonic decay which is expected to be observed at LHCb and Belle and that it could be used to obtain an independent determination of \( |V_{ub}| \). In this contribution we present a study of this reaction. All the details and further results to those presented here can be found in Ref. [1].

The hadronic matrix element for the reaction can be parameterized in terms of the \( f^+ (q^2) \) and \( f^0 (q^2) \) form factors, of which only \( f^+ (q^2) \) plays a significant role for the case of a light lepton in the final state \( (l = e, \mu) \). In fact, for zero lepton masses, the differential decay width is given solely in terms of \( f^+ (q^2) \) as

\[ \frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3} |V_{ub}|^2 \frac{\lambda^{3/2}(q^2, M_{\bar{B}_s}^2, M_K^2)}{M_{\bar{B}_s}^3} |f^+(q^2)|^2 \] (1)

with \( G_F \) the Fermi decay constant and \( \lambda \) the Källen function defined as \( \lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc \).
2. Results and discussion

To obtain the $f^+(q^2)$ form factor we shall follow our earlier work in Ref. [2], where similar decays were analyzed, and then we use the quark model to evaluate the valence plus $B^*$-pole contributions to the form factors. Calculational details can be found in [1] and references therein. Results are shown in figure 1. Taking into account theoretical uncertainties, shown as a band in the figure, we obtain a reasonable description of the form factor in the high $q^2$ region, as compared to the preliminary lattice data recently reported in Ref. [4]. For high $q^2$, the $B^*$-pole term dominates but the valence contribution accounts for around 20% of the total. However, there is a large discrepancy in the low $q^2$ region between the quark model and the light cone sum rule (LCSR) results obtained in Ref. [3]. Since the latter are reliable for low $q^2$, it is clear that the non-relativistic quark model does not provide a good reproduction of the form factor in that region of $q^2$ where large relativistic effects are to be expected.

To obtain an $f^+(q^2)$ form factor valid for the whole $q^2$ region spanned by the decay, we adopt the scheme in Refs. [5, 6, 7], assuming a multiply subtracted Omnès functional ansatz that provides a parameterization of the form factor constrained by unitarity and analyticity properties. We take

$$
f^+(q^2) \approx \frac{1}{M_{B^*}^2 - q^2} \prod_{j=0}^{q_0} \left[ f^+(q_j^2) \left( M_{B_j}^2 - q_f^2 \right) \right]^{\alpha_j(q^2)}, \quad \alpha_j(q^2) = \prod_{j=0}^{q_0} \frac{q_j^2 - q_k^2}{q_f^2 - q_k^2}
$$

for $q^2 < s_{th} = (M_{B^*} + M_K)^2$ and where $q_0, \cdots, q_n^2 \in ] - \infty, s_{th} ]$ are the $(n+1)$ subtraction points. Note that despite the factor $\frac{1}{M_{B^*}^2 - q^2}$, the functional form is not given by a single pole. The values of $f^+(q_j^2)$ are taken as free parameters and we fix them by making a combined fit to our quark model results in the high $q^2$ region and to the LCSR results, taken from Ref. [3], in the low $q^2$ part. As in Ref. [7] we only use four subtraction points corresponding to $q_j^2 = 0, q_1^2/3, 2q_1^2/3, q_1^2$. Our final result for $f^+(q^2)$ together with its 68% confidence level band is displayed in figure 2. There, we show a comparison with different calculations using LCSR [3], LCSR+$B^*$-pole fit [8], relativistic quark model (RQM) [9], light front quark model (LFQM) [10], perturbative QCD (PQCD) [11] and the extrapolation to the physical region done in Ref. [12] of the lattice QCD (LQCD) results obtained in Ref. [4] (also shown). In the LCSR calculation in Ref. [3] the results are only given up to $q^2 = 10$ GeV$^2$, whereas in Ref. [10] no $B^*$-pole contribution is included as can be seen by the behavior of the predicted form factor in the high $q^2$ region. All other calculations include the $B^*$-pole mechanism, but with different strengths. In Ref. [9], where a RQM is used, they obtain a form factor similar to ours for high $q^2$ values. However, their approach for low and intermediate values of $q^2$ should not be as appropriate as a LCSR one,
which we include in our combined analysis. Calculations in Refs. [11] and [8] give similar results at high \(q^2\) but the one in Ref. [11] deviates from LCSR evaluations at small \(q^2\) values. The high \(q^2\) results obtained in LQCD [4, 12] are in between the results obtained in the approaches of Refs. [8, 11] and the quark model ones (both this work and the RQM calculation of Ref. [9]).

For very low \(q^2\) however, the central values of the LQCD extrapolation in Ref. [12] lie in the upper part of the LCSR band. Our combined approach should be more adequate in that region of \(q^2\) since we use LCSR data to constrain our form factor.

The differential decay width, together with its 68\% confidence level band, is displayed in figure 3. We also show the differential decay width from the calculations in Refs. [8, 9, 10, 11, 12]. For the integrated decay width we obtain

\[
\Gamma(\bar{B}_s \to K^+ \ell^- \bar{\nu}_\ell) = (5.47^{+0.54}_{-0.46}) |V_{ub}|^2 \times 10^{-9} \text{MeV} \tag{3}
\]

and a comparison with the results in other approaches is shown in table 1. The calculations in Refs. [8, 9] obtain results that are some 15\% smaller than ours. The fact that their results are so similar when compared to each other seems to be a coincidence. As seen in figure 3, their differential decay widths deviate both for small and large \(q^2\) values, but those differences
compensate in the integrated width. The result of the PQCD calculation in Ref. [11] is also similar but with a larger uncertainty, around 50%. The LFQM calculation in Ref. [10] gives a much smaller result, in part because no $B^*$-pole contribution seems to be included in that approach. The LQCD result in Ref. [12] is the one closest to ours. Its large uncertainty comes from the form factor extrapolation from high $q^2$, where the lattice points were obtained, to the low $q^2$ region. Our result is the largest although we are compatible within uncertainties with the predictions of Refs. [8, 9, 11, 12].

Table 1. Decay width in units of $|V_{ub}|^2 \times 10^{-9}$ MeV from several approaches. For the result of Ref. [8] we have propagated a 10\% uncertainty in the form factor. Results for Refs. [9, 10, 11] have been adapted from Table IV in Ref. [13].

| This work | LCSR+$B^*$-pole | RQM | LFQM | PQCD | LQCD |
|-----------|-----------------|-----|------|------|------|
| $\Gamma (|V_{ub}|^2 \times 10^{-9} \text{ MeV})$ | $5.47^{+0.34}_{-0.46}$ | $4.63^{+0.97}_{-0.88}$ | $4.50 \pm 0.45$ | $2.75 \pm 0.24$ | $4.2 \pm 2.2$ | $5.1 \pm 1.0$ |

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