Dynamics of the quantum Fisher information in a spin-boson model

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Abstract

The quantum Fisher information characterizes the phase sensitivity of qubits in the spin-boson model with a finite bandwidth spectrum. In contrast with Markovian reservoirs, the quantum Fisher information flows from the environments to qubits after a number of times if the bath parameter $s$ is larger than a critical value which is related to temperature. The sudden-change behavior will happen during the evolution of the quantum Fisher information of the maximal entanglement state in the non-Markovian environments. The sudden-change times can be varied with the change of the bath parameter $s$. For a very large number of entangled qubits, the sudden-change behavior of the maximal quantum Fisher information can be used to characterize the existence of the entanglement. The metrology strategy based on the quantum correlated state leads to a lower phase uncertainty when compared with the uncorrelated product state.

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1. Introduction

The Fisher information is a key quantity for extracting information about a parameter from a measurement-induced probability distribution [1–3]. In the classical realm, the Fisher information can provide a basic lower bound of the variance of any unbiased estimator due to the Cramér–Rao inequality [4]. A high value of the Fisher information represents an attainable measurement with high precision. The quantum Fisher information is referred to as the natural extension of the classic Fisher information. Among many versions of the quantum Fisher information, there is a famous definition based on the symmetric logarithmic derivatives [1, 2]. In a practical estimation, the quantum Fisher information can be used to describe the sensitivity of a quantum state with respect to a parameter, such as the frequency...
of the system [5, 6], the strength of the changing external field [7, 8], and the speed of the quantum evolution [9].

To improve the precision of parameter estimation, some research groups have put forward entanglement-enhanced metrological schemes [10–17]. It is proved that scaling with the number of entangled particles \(N\) can achieve the Heisenberg limit, proportional to \(1/N\), which overcomes the shot-noise limit (also called the standard quantum limit), proportional to \(1/\sqrt{N}\). However, the practical quantum technology unavoidably induces the decoherence and dissipation owing to the coupling of the system to the environment [18, 19]. So far, much attention has been focused on quantum metrology in the presence of Markovian or non-Markovian noise [5, 6, 20–31]. In some Markovian channels, there is a sudden change of the quantum Fisher information [21]. A local dephasing environment was also taken into account in order to exactly obtain the scaling law of the parameter estimation [6]. It was found that the non-Markovian dephasing environment enables quantum entangled states to outperform the strategy based on product states. In other realistic environments, we need to study whether this correlated-state metrology strategy is superior to the strategy based on product states.

Besides, we also want to investigate whether the sudden-change behavior of the quantum Fisher information would happen in non-Markovian environments.

The paper is organized as follows. We introduce the definition of the quantum Fisher information in section 2. In section 3, a general non-Markovian depolarizing channel is presented by a qubit coupled to a heat bath. In, section 4, the crossover between non-Markovian dynamics and Markovian ones will be manifested by the quantum Fisher information with respect to a phase parameter. The sensitivity of the phase estimation based on the \(N\)-particle maximally entangled state is also studied in contrast with the product state. Finally, a simple discussion is concluded in section 5.

2. The quantum Fisher information

In metrology, one of the basic measurements is the phase estimation with respect to a unitary transformation of a linear interferometer. The phase \(\phi\) is related to an unbiased estimator \(\hat{\phi}\) of \(\phi\). The measured state is obtained by \(\rho_\phi = e^{i\phi\hat{J}_n} \rho e^{-i\phi\hat{J}_n}\) where \(\rho\) is the input state and \(\hat{J}_n\) is a generator which describes the angular momentum operator along a direction \(\vec{n} = (n_1, n_2, n_3)\). According to the quantum Cramér–Rao theorem [1, 2], the precision of the phase parameter \(\phi\) has a lower bound limit, which is determined by

\[
\Delta \hat{\phi} \geq \frac{1}{\sqrt{v F(\rho, \hat{J}_n)}},
\]

where \(v\) is the number of the experiments and \(F(\rho, \hat{J}_n)\) denotes the quantum Fisher information, which can be defined as

\[
F(\rho, \hat{J}_n) = \text{Tr}(\rho_\phi L^2_\phi).
\]

\(L_\phi\) is the symmetric logarithmic derivative with respect to the phase parameter \(\phi\) and satisfies that \(\frac{\partial \rho_\phi}{\partial \phi} = \frac{i}{2} (\rho_\phi L_\phi + L_\phi \rho_\phi)\). The operator \(L_\phi\) is mathematically expressed by the eigenvectors of the state \(\rho\). The elements of the operator are obtained as \(L_{k,j} = \langle k | \hat{J}_n | j \rangle\). To emphasize the effects of the quantum Fisher information, we set \(v = 1\) in this paper. Using the above equations, we calculate the quantum Fisher information as

\[
F(\rho, \hat{J}_n) = 2 \sum_{i \neq j} \frac{(\lambda_i - \lambda_j)^2}{\lambda_i + \lambda_j} |\langle i | \hat{J}_n | j \rangle|^2 = \vec{n} \vec{Cn},
\]

where 

\[
\vec{n} \vec{Cn}.
\]
where $|i⟩$ is the eigenvector of the density matrix $ρ$ with the corresponding eigenvalue $λ_i$, and the elements of the symmetric matrix $\hat{C}$ are expressed as $C_{ij} = \sum_{k \neq j} \frac{(\lambda_i - \lambda_k)^2}{\lambda_i + \lambda_k} \langle j|\hat{J}_k|j⟩\langle j|\hat{J}_k|i⟩$.

For a pure state, $F (\rho, \hat{F}_K) = 4(\Delta F_0)^2$. The maximum of the Fisher information together with the optimal direction was first discussed in [20]. To maximize the quantum Fisher information, we need to select the rotation along an optimal direction $\vec{n}^*$ by diagonalizing the symmetric matrix as $\hat{C}^d = \hat{O}^\dagger \hat{C} \hat{O} = \text{diag}(C_1, C_2, C_3)$. $\hat{O}$ is a transformation matrix which is composed of three orthogonal and normalization eigenvectors of $\hat{C}$. According to the result of [20], the maximum of the quantum Fisher information is written as

$$F_{\max} = 4 \max (C_1, C_2, C_3). \quad (4)$$

The optimal direction $\vec{n}^*$ is determined by the eigenvector of $\hat{C}$ with the maximal eigenvalue.

For the total state $ρ^N$ of $N$ independent qubits, the maximal mean value of the quantum Fisher information can be introduced as

$$\bar{F}_{\max} = \frac{F_{\max}}{N}. \quad (5)$$

Here, the unitary transformation related to the phase parameter $φ$ is presented as $\exp(iφ \sum_{m=1}^N \hat{F}_m)$ where $\hat{F}_m$ is the angular momentum operator for the $m$th qubit along the optimal direction $\vec{n}^*$. When the values $\bar{F}_{\max} > 1$, we have $ΔF_{QCR} < 1/\sqrt{N}$. In this condition, the ultimate estimation limit is superior to the standard quantum limit. Only if $\bar{F}_{\max} \geq N$ can the Heisenberg limit of $ΔF_{QCR} \simeq 1/N$ be attained.

### 3. The non-Markovian spin-boson model

A spin-boson system is a simple model that describes an effective two-level system coupled to a bosonic reservoir [19, 32]. The model has been extensively applied to noisy quantum dots [34], the decoherence of qubits in quantum computation [35, 36], and quantum impurities and charge transfer in donor–acceptor systems [37]. The dissipative environment in the spin-boson model can be characterized by the structured spectral function $J(ω)$ with frequency behavior of $J(ω) \propto ω^s ω_0^{1-s} \exp(-ω/ω_c)$, where $ω_c$ is the cutoff frequency of the spectrum. With the change of the bath parameter $s$, the environments vary from sub-Ohmic ones ($s < 1$) to Ohmic ($s = 1$) and super-Ohmic ones ($s > 1$). We will employ the spin-boson model to construct a general depolarizing channel.

The Hamiltonian of the spin-boson model can be described as

$$H = H_0 + H_E + H_I, \quad (6)$$

where $H_0 = \frac{ω_0}{2} \hat{σ}_z$ is a local Hamiltonian of the system with the tunneling frequency of $ω_0$, and $H_E = \sum_k \hat{b}_k^\dagger \hat{b}_k$ denotes the Hamiltonian of the environment including all degrees of freedom. The interaction Hamiltonian between the spin and the environment is expressed as $H_I = \sum_k g_k \hat{σ}_+ \hat{b}_k + \hat{σ}_- \hat{b}_k^\dagger$, where $g_k$ is the coupling strength of the $k$th mode field, $\hat{b}_k$ and $\hat{b}_k^\dagger$ represent the annihilation and creation operator, respectively. Using a transformation $\hat{U}$, we can diagonalize the system Hamiltonian into $H_0^{\text{eff}} = \hat{U}^\dagger H_0 \hat{U} = \frac{ω_0}{2} \hat{σ}_z$.

The spin operators are also mapped into the new ones, i.e., $\hat{σ}_j = \hat{U}^\dagger \hat{σ}_j \hat{U}$, ($j = z, ±$). We can obtain the time-convolutionless master equation of the density matrix of the spin in the interaction picture as

$$\frac{dρ(t)}{dt} = -i[H_0^{\text{eff}}, ρ(t)] + \hat{L}[ρ(t)]. \quad (7)$$
where $\rho(t)$ is the density matrix of the system after the unitary transformation. The Lindblad superoperator $\hat{L}$ can be written as

$$\hat{L}[\rho(t)] = \sum_{m=\pm} \gamma_m(t) \left[ \hat{\sigma}_m \rho(t) \hat{\sigma}_m^+ - \frac{1}{2} \{ \hat{\sigma}_m^+ \hat{\sigma}_m, \rho(t) \} \right].$$

We should note that the master equation is obtained in the secular approximation [19] where the high-frequency oscillating terms are neglected. The secular approximation is reasonable under the assumption of the weak coupling between the system and the environment. The time-dependent decay parameters at a finite temperature $T$ are expressed as

$$\gamma_\pm(t, T) = \frac{1}{2} \int J(\omega) \coth \left( \frac{\hbar \omega}{2k_B T} \right) \sin \omega t \frac{\omega}{\omega \pm \omega_0} d\omega,$$

$$\gamma_\mp(t, T) = \frac{1}{2} \int J(\omega) \left[ (n_T + 1) \sin(\omega \pm \omega_0) t + n_T \sin(\omega \mp \omega_0) t \right] \frac{\omega}{\omega \pm \omega_0} d\omega. \quad (9)$$

The mean number of the field is $n_T = [\exp(\hbar \omega/k_B T) - 1]^{-1}$. The analytical expression of $\rho(t)$ can be written in the Hilbert space spanned by $|1\rangle, |0\rangle$ as

$$\rho(t) = \begin{pmatrix} (a-b)\rho_{11}(0) + b & c\rho_{10}(0) \\ c^*\rho_{01}(0) & (1-a) + (a-b)\rho_{00}(0) \end{pmatrix}, \quad (10)$$

where $\rho_{ij}(0)$ is the element of the density matrix $\rho(0)$ at the initial time and $|1(0)\rangle$ denotes the eigenvector of the new operator $\hat{\sigma}_z$ with the corresponding eigenvalue $\pm 1$. The decoherence parameters are obtained as

$$a = \frac{1}{2} \left[ 1 + \delta e^{-\Gamma} \right],$$

$$b = \frac{1}{2} \left[ 1 + \delta e^{-\Gamma} \right],$$

$$c = e^{-\Lambda - \omega T}, \quad (11)$$

where $\Gamma(t) = \int_0^t [\gamma_+(t') + \gamma_-(t')] dt'$, $\Lambda(t) = \frac{1}{2} \Gamma(t) + 2 \int_0^t \gamma_\pm(t') dt'$ and $\delta(t) = \int_0^t e^{\Gamma(t')} [\gamma_+(t') - \gamma_-(t')] dt'$. If $a + b = 1$ and $a - b = c$, the dynamical map will be reduced to a depolarizing channel where an initial state $\rho(0)$ will evolve into a mixture of $\rho(0)$ and maximally mixed state $I$.

4. Dynamical crossover and phase estimation

In these general depolarizing environments, the quantum Fisher information for time-dependent mixed state $\rho(t)$ can be analytically obtained as follows. We may use the Bloch vector $\vec{B}(t)$ to simplify the description of the density matrix of a single qubit as

$$\rho(t) = \frac{\hat{I} + \vec{\sigma} \cdot \vec{B}(t)}{2},$$

where the three components of the Bloch vector are written as

$$B_1(t) = e^{-\Lambda} \sin \theta \cos(\omega_0 t + \varphi),$$

$$B_2(t) = e^{-\Lambda} \sin \theta \sin(\omega_0 t + \varphi),$$

$$B_3(t) = e^{-\Gamma} (\cos \theta + \varphi). \quad (13)$$

Here, the Bloch vector can also be expressed as $\vec{B}(t) = |\vec{B}(t)| (\sin \alpha \cos \beta, \sin \alpha \sin \beta, \cos \alpha)$. These time-dependent angle parameters satisfy the initial conditions of $\alpha(0) = \theta, \beta(0) = \varphi, \varphi \in [0, 2\pi]$. The maximal quantum Fisher information is obtained as

$$F_{\text{max}}(t) = e^{-2\Lambda} \sin^2 \theta + e^{-2\Gamma} (\cos \theta + \delta)^2. \quad (14)$$
Figure 1. (a) The flow of the average of the maximal quantum Fisher information is plotted as functions of the bath parameter $s$ and scaled time $\omega_c t$ if $\frac{\kappa B}{\hbar} = 0.01$ and $\omega_c = 100\omega_0$. (b) The dynamical crossover related to the bath parameter $s$ is shown. The flow of the quantum Fisher information $\frac{dF_{A_{\text{max}}}}{dt} > 0$ in the region NM while $\frac{dF_{A_{\text{max}}}}{dt} < 0$ in the region M.

To obtain the maximal quantum Fisher information, we can apply the two different kinds of optimal rotations where the directions are expressed as

$$\vec{n}_\parallel = (\sin \beta, \cos \beta, 0),$$
$$\vec{n}_\perp = (0, 0, 1).$$

With regard to all possible initial states, we use the average value of $F_{\text{max}}$ which is defined as

$$F_{\text{max}}^A = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{\pi} F_{\text{max}} \sin \theta \, d\theta \, d\varphi.$$  

(16)

In fact, the non-monotonic behaviors of the quantum Fisher information can be used to define and quantify the non-Markovianity of quantum dynamics [33]. The flow of the maximal quantum Fisher information for a qubit at a finite temperature is shown in figure 1(a). From figure 1(a), we can see that the large negative values of $\frac{dF_{A_{\text{max}}}}{dt}$ occur if the bath parameter is less than a temperature-dependent critical value, i.e., $s < s_c$. This also means that the quantum Fisher information is decreased with time. In the condition of $s < s_c$, which describes the Markovian environment, the quantum information always flows from the system to the environment. When $s > s_c$, the non-Markovianity of the reservoirs occurs because of the small positive values of $\frac{dF_{A_{\text{max}}}}{dt} > 0$. For the temperature $\frac{\kappa B}{\hbar} = 0.01$, the critical value of the bath parameter is about $s_c \approx 2.4$. 

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The dynamics of the Fisher information is determined by the properties of the decaying rates in the time-dependent master equation. To give this physical explanation, we analytically deduce the decaying rate in the zero temperature $T = 0$. The time-dependent decaying parameter is obtained as $\gamma_r(t, T = 0) = \omega_0 [1 + (\omega_0 t)^2]^{-\nu/2} \Gamma' [s] \sin [s \arctan (\omega_0 t)]$ where $\Gamma' [s]$ is the Euler Gamma function. It is found that $\gamma_r(t, T = 0)$ can take temporarily negative values if the bath parameter $s > s_c = 2$. According to the results of [19], the negative decaying rates denote the existence of the memory effects from the environment. These memory effects can give rise to the increase of the Fisher information of the qubit.

In figure 1(b), the crossover between the Markovian dynamics and the non-Markovian ones is clearly determined by the flow of the quantum Fisher information. If the bath parameter $s < s_c$, there is no dynamical crossover. For the non-Markovian environments of $s > s_c$, the values of $\frac{\partial F_{\max}}{\partial t}$ > 0 will occur after a certain time $\omega_0 \tau$. It means that the information can flow from the environment to the system after the times $t > \tau$. This non-Markovian dynamical behavior is represented by the region of NM. Before the time $\omega_0 \tau$, the values of $F_{\max}$ in the region of M are monotonically decreased, which is referred to as the Markovian dynamics.

We want to know if the metrology strategy based on the quantum correlated states will have an advantage in improving the sensitivity of the phase estimation in the practical environment. The $N$-particle maximally entangled state is chosen to be the input state as

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (\Pi_{j=1}^{N} |1\rangle_j + \Pi_{j=1}^{N} |0\rangle_j).$$

(17)

We assume that $N$ qubits are subject to the local environments independently. The total density matrix is obtained as

$$\rho_N(t) = \frac{1}{2} \left[ \Pi_{j=1}^{N} \hat{\epsilon}_j(|1\rangle_j\langle 1|) + \Pi_{j=1}^{N} \hat{\epsilon}_j(|0\rangle_j\langle 0|) \right].$$

(18)

According to the time-dependent density matrix written by equation (10), the dynamical map $\hat{\epsilon}_j$ for the $i$th qubit can be expressed as

$$\hat{\epsilon}_j(|1\rangle_j\langle 1|) = a|1\rangle_j\langle 1| + (1 - a)|0\rangle_j\langle 0|,$$

$$\hat{\epsilon}_j(|1\rangle_j\langle 0|) = c|1\rangle_j\langle 0|,$$

$$\hat{\epsilon}_j(|0\rangle_j\langle 1|) = c^*|0\rangle_j\langle 1|,$$

$$\hat{\epsilon}_j(|0\rangle_j\langle 0|) = b|1\rangle_j\langle 1| + (1 - b)|0\rangle_j\langle 0|.$$

(19)

Figure 2 shows the evolutions of the phase sensitivity described by the maximal mean values of the quantum Fisher information in a non-Markovian depolarizing environment. When the number of the experiments $\nu = 1$, the phase sensitivity can be given as $\Delta \phi \sim \frac{1}{F_{\max}}$ according to the quantum Cramer–Rao theorem. It is clearly seen that the variances of the phase parameter based on quantum correlated states are always smaller than those gained by the product state. Here, we choose the product state as $|\Psi_p\rangle = \Pi_{j=1}^{N} \frac{1}{\sqrt{2}} (|1\rangle_j + |0\rangle_j)$. In the correlated-state metrology strategy, there exists a critical time $\tau_c$ where $F_{\max} = 1$. Before the critical time, we can obtain the ultimate sensitivity limit which is better than the standard quantum limit because of the high values $F_{\max} > 1$. However, the product-state metrology strategy cannot achieve the standard quantum limit owing to $F_{\max} < 1$. The maximal value of the variance arrives at the crossover time $t = \tau$. When $t > \tau$, the memory effects from the non-Markovian depolarizing environments can give rise to a decrease of the phase sensitivity. After a long time, the sensitivity caused by using the quantum correlated state is nearly equivalent to that caused by using the product state.
The sudden-change behavior in the dynamics can be shown in the inset of figure 2. Such behavior results from the competition of the two different kinds of the optimal SU(2) rotations in either the $x$–$y$ plane or along $z$ direction. Prior to the sudden-change time, the small variance of the phase parameter can be obtained by the $x$–$y$ plane rotation along $\vec{n}_0 \parallel$, while the optimal values of $(\Delta \phi)^2$ are achieved by the $z$ direction rotation along $\vec{n}_0 \perp$ at the time $t > t_c$. No sudden-change behavior happens in the case of the product state because the value of $\bar{F}_{\text{max}}$ is the same with respect to the two kinds of optimal rotation directions.

On the one hand, the occurrence of the sudden-change behavior of the maximal quantum Fisher information is determined by the optimal selection of the rotation operations. On the other hand, $F_{\text{max}} > 1$ before the critical times is considered as sufficient for the existence of the entanglement [17]. After the time $t > t_c$, the entanglement of the open quantum systems is nearly decreased to zero. Therefore, it is of interest to study the relationship between the critical times and the sudden-changing times. Through the numerical calculation, we find that the vanishing of the entanglement is always earlier than the occurrence of the sudden-change behavior. The difference between the sudden-changing time and the critical time is plotted with the increase of the number of qubits in figure 3. The scaling property is obtained as $\omega_c (t_s - t_c) \sim N^{-0.75}$ on the condition of $2\omega_c^2 \frac{t_c}{\hbar} = 0.01$, $\omega_c = 10\omega_0$, $s = 3$ and the number of qubits is $N = 5$. When $N \to \infty$, the sudden-changing times are infinitely close to the critical times. For a very large number of entangled qubits, the sudden-change behavior of the maximal quantum Fisher information can be used to characterize the existence of the entanglement of open systems.

The impacts of the reservoirs on the sudden-change dynamics of the phase sensitivity are shown in figure 4. It is clearly seen that the sudden-change behavior can occur in both Markovian ($s < s_c$) environments and non-Markovian ($s > s_c$) ones. For the cases of the smaller values of bath parameter $s$, a lower variance of the phase parameter will be obtained. The values of the sudden-changing time $t_s$ are decreased with the increase of the bath parameter. When $t < t_s$, a better resolution limit is achieved in the sub-Ohmic heat bath where the higher values of the maximal quantum Fisher information can be kept.
Figure 3. The scaling property of the critical time $t_c$ and the sudden-change time $t_s$ is plotted in a logarithmic scale if $\frac{\kappa}{\hbar} = 0.01$, $\omega_c = 10\omega_0$ and $s = 3$. The squares denotes the numerical result and the dashed line represents the fit result. On this condition, $\omega_c (t_s - t_c) \sim N^{-0.75}$.

Figure 4. For the strategy based on the maximally entangled state, the sudden-change behavior can be shown with the change of the bath parameter $s$ if $\frac{\kappa}{\hbar} = 0.01$, $\omega_c = 10\omega_0$ and $N = 5$.

5. Discussion

We employ the spin-boson system to construct the practical non-Markovian depolarizing channel. The evolution of the quantum Fisher information is applied to discriminate between the Markovian dynamics and the non-Markovian ones. There exists the temperature-dependent critical bath parameter $s_c$. Only if $s > s_c$ will the crossover between the non-Markovian decoherence and the Markovian one exist. The sensitivity of the phase estimation was studied in both the correlated-state metrology strategy and the product-state metrology strategy. It is found that the quantum correlated states can be used to improve the phase sensitivity. During the evolution, we have the critical time and the sudden-change time. Before the critical time, the resolution limit is intermediately between the standard quantum limit (or shot-noise limit) and the Heisenberg limit. The sudden-change behavior will occur because of the competition of two different optimal rotations. The change of the reservoir parameter $s$ can lead to the variation of the sudden-changing times. Moreover, for the large-$N$ cases, the sudden-change times are approximately equal to the critical times. For large-$N$ entangled states, we can obtain
the maximal quantum Fisher information using the optimal SU(2) rotation in the $x$–$y$ plane, which denotes the existence of the entanglement of open systems.

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