Yukawa sector in non-supersymmetric renormalizable SO(10)

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We discuss the ordinary, non-supersymmetric SO(10) as a theory of fermion masses and mixings. We construct two minimal versions of the Yukawa sector based on $126_H$ and either $10_H$ or $120_H$. The latter case is of particular interest since it connects the absolute neutrino mass scale with the size of the atmospheric mixing angle $\theta_A$. It also relates the smallness of $V_{e3}$ with the largeness of $\theta_A$. These results are based on the analytic study of the second and third generations. Furthermore, we discuss the structure of the light Higgs and the role of the Peccei-Quinn symmetry for dark matter and the predictivity of the theory.

I. INTRODUCTION

SO(10) grand unified theory $\mathbf{10}$ is probably the best motivated candidate for the unification of the strong and electro-weak interactions. It unifies the family of fermions; it includes the SU(4)$_C$ quark-lepton symmetry $\mathbf{2}$ and the left-right (LR) symmetry $\mathbf{R}$ in the form of charge conjugation as a finite gauge symmetry; it predicts the existence of right-handed neutrinos and through the see-saw mechanism $\mathbf{R}$ offers an appealing explanation for the smallness of neutrino masses. Due to the success of supersymmetric unification, and the use of supersymmetry in controlling the gauge hierarchy, most of the attention in recent years has focused on the supersymmetric version of SO(10). However, supersymmetry may not be there. After all, it controls the Higgs gauge hierarchy, but not the cosmological constant. The long standing failure of understanding the smallness of the cosmological constant suggests that the unwelcome fine-tuning may be necessary. Our fine-tuned world can be viewed, in the landscape picture simply as a selection criterion among the large number of degenerate string vacua. Or it could be that the cosmological evolution of the universe selects a light Higgs doublet $\mathbf{10}$. If so, the main motivation behind the low-energy supersymmetry would be gone. However one could still hope that the issue might be settled by the internal consistency and the predictions of a well defined grand unified theory. In other words, it is possible that simplicity and minimality at the GUT scale requires a specific low-energy theory.

What about grand unification without supersymmetry? At first glance, one may worry about the unification of gauge couplings in this case. Certainly, in the minimal SU(5) theory, the gauge couplings do not unify without low-energy supersymmetry. What happens is the following: the colour and weak gauge couplings meet at around $10^{16}$ GeV, an ideal scale from the point of view of the proton stability and perturbativity (i.e., sufficiently below $M_{\text{Planck}}$). The problem is the U(1) coupling. Without supersymmetry it meets the SU(2)$_L$ coupling at around $10^{13}$ GeV $\mathbf{16}$; with low-energy supersymmetry the one-step unification works as is well known $\mathbf{17}$.

On the other hand, the fact that neutrinos are massive indicates strongly that SU(5) is not the right grand unified theory: it simply requires too many disconnected parameters in the Yukawa sector $\mathbf{18,19}$. The SO(10) theory is favored by the neutrino oscillation data. Most interestingly, SO(10) needs no supersymmetry for a successful unification of gauge couplings. On the contrary, the failure of ordinary SU(5) tells us that in the absence of supersymmetry there is necessarily an intermediate scale such as the left-right symmetry breaking scale $M_R$. Namely, in this case the SU(2)$_L$ and SU(3)$_C$ couplings run as in the standard model or with a tiny change depending whether or not there are additional Higgs multiplets at $M_R$ (recall that the Higgs contribution to the running is small). However, the U(1)$_L$ coupling is strongly affected by the embedding in SU(2)$_L$ above $M_R$. The large contributions of the right-handed gauge bosons makes the U(1) coupling increase much slower and helps it meet the other two couplings at the same point. The scale $M_R$ typically lies between $10^{10}$ GeV and $10^{14}$ GeV (see for example $\mathbf{10,11}$ and references therein), which fits very nicely with the see-saw mechanism. Now, having no supersymmetry implies the loss of a dark matter candidate. One may be even willing to introduce an additional symmetry in order to achieve this. In this case it should be stressed that SO(10) provides a framework for the axionic dark matter: all one needs is a Peccei-Quinn symmetry $\mathbf{U(1)_{PQ}}$ which simultaneously solves the strong CP problem.

This seems to us more than sufficient motivation to carefully study ordinary non-supersymmetric SO(10). What is missing in this program is the construction of a well defined predictive theory with the realistic fermionic spectrum. This paper is devoted precisely to this task.

In particular, the search of the minimal realistic Yukawa sector is a burning question. In the absence of higher dimensional operators at least two Higgs multiplets with the corresponding Yukawa matrices are needed, otherwise there would be no mixings. The Yukawa Higgs sector can contain $\mathbf{10_H}$, $\mathbf{120_H}$ and $\mathbf{126_H}$ representations, since
$16 \times 16 = 10 + 120 + 126$.  

One version of the theory with only $10_H$ and $\mathbf{126}_H$ was studied in great detail in the case of low-energy supersymmetry [13, 14, 15]. In spite of having a small number of parameters it seems to be consistent with all the data [16, 17, 18, 19, 20]. For the type II seesaw it predicts furthermore the $1 - 3$ lepton mixing angle not far from its experimental limit: $|U_{e3}| > 0.1$ [15, 21] and it offers an interesting connection between $b - \tau$ unification and the large atmospheric mixing angle $21, 22$. Last but not least, it also predicts exact R-parity at low energies [23, 24] leading to the LSP as a candidate for the dark matter.

Thus, a first obvious possibility in ordinary SO(10) is to address the model with $10_H + \mathbf{120}_H$, and to see whether or not it can continue to be realistic.

There is another interesting alternative: $\mathbf{120}_H$ and $\mathbf{126}_H$. We limit ourselves to the analytic study, which requires ignoring the effects of the first generation. The main result of this model is the correlation of neutrino masses with the value of the atmospheric mixing angle, true both in Type I and Type II seesaw mechanisms, or in a general case when both contribute to the neutrino masses. Furthermore the large atmospheric mixing angle fits naturally with the small $V_{\text{CKM}}$.

Both cases require complexifying the Higgs fields $10_H$ or $120_H$. This in turn calls for a PQ symmetry, in any case useful for a dark matter.

At first glance it seems that the Yukawa sector in a non-supersymmetric theory does not differ from the supersymmetric version and thus there is nothing new to say. There are however several subtle differences: 1) the running of the couplings (gauge and Yukawa) is changed, so are the inputs for a numerical evaluation at $M_{\text{GUT}}$; 2) there are necessarily intermediate scales; 3) if no new symmetries are invoked, all the SO(10) representations that are not complex like $16$ or $126$ are real; 4) there are radiative corrections to the Yukawa sector, that should in principle be taken into account. All these points will be discussed below.

II. THE MINIMAL YUKAWA SECTOR

In this work, we stick to the renormalizable version of the see-saw mechanism (or alternatives using a radiatively-induced see-saw, see [25]), which makes the representation $\mathbf{126}_H$ indispensable, since it breaks the SU(2)$_R$ group and gives a see-saw neutrino mass. By itself it gives no fermionic mixing, so it does not suffice. The realistic fermionic spectrum requires adding either $10_H$ or $\mathbf{120}_H$. As promised in the Introduction, we will go carefully through both possibilities.

Before starting out, it is convenient to decompose the Higgs fields under the SU(2)$_L \times$ SU(2)$_R \times$ SU(4)$_C$ Pati-Salam (PS) group:

\begin{equation}
10 = (2, 2, 1) + (1, 1, 6)
\end{equation}
\begin{equation}
\mathbf{126} = (1, 3, 10) + (3, 1, \overline{10}) + (2, 2, 15) + (1, 1, 6)
\end{equation}
\begin{equation}
120 = (1, 3, 6) + (3, 1, 6) + (2, 2, 15) + (2, 2, 1) + (1, 1, 20)
\end{equation}

As is well known, the $\mathbf{126}_H$ provides mass terms for right-handed and left-handed neutrinos:

\begin{equation}
M_{\nu R} = \langle 1, 3, 10 \rangle Y_{126}, \quad M_{\nu L} = \langle 3, 1, \overline{10} \rangle Y_{126}
\end{equation}

which means that one has both type I and type II seesaw:

\begin{equation}
M_N = -M_{\nu R} M_{\nu R}^{-1} M_{\nu D} + M_{\nu L}
\end{equation}

In the type I case it is the large vev of $(1, 3, 10)$ that provides the masses of right-handed neutrino whereas in the type II case, the left-handed triplet provides directly light neutrino masses through a small vev [26, 27]. The disentangling of the two contributions is in general hard.

A. Model I: $\mathbf{126}_H + 10_H$

In this case the most general Yukawa interaction is (schematically)

\begin{equation}
\mathcal{L}_Y = 16_F \left( 10_H Y_{10} + \mathbf{126}_H Y_{126} \right) 16_F + h.c.
\end{equation}

where $Y_{10}$ and $Y_{126}$ are symmetric matrices in the generation space. With this one obtains relations for the Dirac masses

\begin{equation}
M_D = M_1 + M_0, \quad M_U = c_1 M_1 + c_0 M_0, \quad M_E = -3 M_1 + M_0, \quad M_{\nu D} = -3 c_1 M_1 + c_0 M_0,
\end{equation}

where we have defined

\begin{equation}
M_1 = \langle 2, 2, 15 \rangle_{126}^d Y_{126}, \quad M_0 = \langle 2, 2, 1 \rangle_{10}^d Y_{10},
\end{equation}

and

\begin{equation}
c_0 = \langle 2, 2, 1 \rangle_{10}^u, \quad c_1 = \langle 2, 2, 15 \rangle_{126}^u.
\end{equation}

These equations, together with [3] and [4], are the starting point for the analysis [28] of the fermion spectrum.
1. Who is the light Higgs?

With the minimal fine-tuning the light Higgs is in general a mixture of, among others, $(2,2,1)$ of $10_H$ and $(2,2,15)$ of $126_H$. This happens at least due to the large $(1,3,10)$ vev in the term $(126_H)^2 126_H^\dagger 10_H$.

In any case, their mixings require the breaking of $SU(4)_C$ symmetry at a scale $M_{PS}$, and it is thus controlled by the ratio $M_{PS}/M$, where $M$ corresponds to the mass of the heavy doublets. Thus, if $M \simeq M_{GUT}$, and $M_{PS} \ll M_{GUT}$, this would not work: we come to the conclusion that one needs to tune-down somewhat the mass of the heavy doublets. Thus, if the corresponding $\beta$-function coefficients $(b_1-b_3)_{(2,2,15)}$ and $(b_1-b_2)_{(2,2,15)}$ (in the usual notation) are very small. It should nevertheless be taken into account when studying unification constraints.

2. The simplest version: real $10_H$

If $10_H$ is real, then there is just one $SU(2)_L$ doublet in $(2,2,1)$ and thus $| \langle 2,2,1 \rangle_{10}^i | = | \langle 2,2,1 \rangle_{10}^d |$, namely $| c_0 | = 1$. The parameter space is thus smaller. Here we show that, in the two generation (second and third) case with real parameters, such a constraint leads to a contradiction with the data. In the physically sensible approximation $\theta_\tau = V_{tb} = 0$ we find

$$c_0 = \frac{m_\tau (m_\tau - m_b) - m_\mu (m_\mu - m_\tau)}{m_\tau m_\tau - m_\mu m_b} \approx \frac{m_\tau}{m_b},$$

(9)

clearly much bigger than 1.

This conclusion is subject to the uncertainties of the full three-generation case. Although strictly speaking this simple model cannot be ruled out yet, there is an indication that a more complicated scenario should be considered.

3. The next step: complex $10_H$

If the model with real $10_H$ does fail eventually, one could simply complexify it. This of course introduces new Yukawa couplings which makes the theory less predictive. Certain predictions may remain, though, such as the automatic connection between $b - \tau$ unification and large atmospheric mixing angle in the type II seesaw. This is true independently of the number of 10 dimensional Higgs representations, since $10_H$ cannot distinguish down quarks from charged leptons. From $M_{\nu L} \propto Y_{126}$, one has $M_{\nu L} \propto M_D - M_E$.

It is a simple exercise to establish the above mentioned connection between $|m_b| \approx |m_\tau|$ and large $\theta_{atm}$; for details see [21, 22]. This fact has inspired the careful study of the analogous supersymmetric version where $m_\tau \simeq m_b$ at the GUT scale works rather well. In the non-supersymmetric theory, $b - \tau$ unification fails badly, $m_\tau \sim 2 m_b$. The realistic theory will require a Type I seesaw, or an admixture of both possibilities.

4. Axions and the dark matter of the universe

A complex $10_H$ means, as we said, an extra set of Yukawa couplings. At the same time this non-supersymmetric theory cannot account for the dark matter of the universe, since there are no cosmologically stable neutral particles and, as is well known, light neutrinos cannot too. It is then rather suggestive to profit from the complex $10_H$ and impose the $U(1)_{PQ}$ Peccei-Quinn symmetry:

$$16 \to e^{i\alpha} 16, \quad 10 \to e^{-2i\alpha} 10, \quad 126 \to e^{-2i\alpha} 126,$$  

(10)

with all other fields neutral. The Yukawa structure has the form [30] with $10_H$ now complex. This resolves the inconsistency in fermion masses and mixings discussed above, and gives the axion as a dark matter candidate as a bonus [31].

The neutrality of the other Higgs fields under $U(1)_{PQ}$ emerges from the requirement of minimality of the Higgs sector that we wish to stick to. Namely, $126_H$ is a complex representation and $10_H$ had to be complexified in order to achieve realistic fermion mass matrices and to have $U(1)_{PQ}$. It is desirable that the $U(1)_{PQ}$ be broken by a nonzero $\langle 126_H \rangle$, i.e. the scale of $SU(2)_R$ breaking and right-handed neutrino masses [31], otherwise $10_H$ would do it an give $M_{PQ} \approx M_W$, which is ruled out by experiment. Actually, astrophysical and cosmological limits prefer $M_{PQ}$ in the window $10^{10} - 10^{13}$ GeV [32].

Now, a single $126_H$ just trades the original Peccei-Quinn charge for a linear combination of $U(1)_{PQ}$, $T_{3R}$ and $B - L$ [31, 33]. Thus in order to break this combination and provide the Goldstone boson an additional Higgs multiplet is needed. One choice is to add another $126_H$ and decouple it from fermions, since it must necessarily have a different PQ charge [31]. An alternative is to use a (complex) GUT scale Higgs as considered for SU(5) by [34], with $M_{PQ} \approx M_{GUT}$, which however implies too much dark matter or some amount of fine-tuning.

Of course, the Peccei-Quinn symmetry does more than just providing the dark matter candidate: it solves the strong CP problem and predicts the vanishing $\theta$. The reader may object to worrying about the strong CP and not the Higgs mass hierarchy problem; after all, they are both problems of fine-tuning. Actually, the strong CP problem is not even a problem in the standard model, at least not in the technical sense [35]. Namely, although divergent, in the standard model $\theta$ is much smaller than the experimental limit: $\theta \ll 10^{-10}$ for any reasonable value of the cutoff $\Lambda$, e.g. $\theta \approx 10^{-19}$ for $\Lambda = M_{Planck}$. 


The physical question is really the value of $\bar{\theta}$. Peccei-Quinn symmetry fixes this arbitrary parameter of the SM. The situation with supersymmetry and the Higgs mass is opposite. Low energy SUSY helps keep Higgs mass small in perturbation theory, but fails completely in predicting it. If we do not worry about the naturalness we can do without supersymmetry. On the other hand, if we wish to predict the electron dipole moment we can do without supersymmetry. On the other hand, if we wish to predict the electron dipole moment we can do without supersymmetry.

B. Model II: $\mathbf{126}_H + 120_H$

Instead of $10_H$ one could use $120_H$. Since $Y_{120}$ is antisymmetric, this means only 3 new complex couplings on top of $Y_{126}$.

$120_H$ as an addition to $10_H$ was partially studied some time ago, for charged fermions only [16]. Recently it was readdressed in the context of the radiative seesaw mechanism [25]. The analytic study for the second and third generation gives $b - \tau$ unification, and small quark and large leptonic mixing angle. We recall that this model is tailor-fit for the strongly split supersymmetry with light gauginos and higgsinos and very heavy scalars [11].

The analysis here is quite similar to the case of $10_H$ and $120_H$ due to the fact that $10_H$ and $126_H$ have symmetric Yukawas. There is an important difference though, since $(2,2,1)_{10}$ is traded for $(2,2,15)_{126}$.

The Dirac mass matrices at the grand unification scale take the form

$$M_D = M_1 + M_2 \quad , \quad M_E = c_1 M_1 + c_2 M_2 \quad , \quad (11)$$

$$M_F = -3M_1 + c_3 M_2 \quad , \quad M_{ud} = -3c_1 M_1 + c_4 M_2$$

where $M_1$ and $c_1$ are defined in [47, 48], and:

$$M_2 = Y_{120} \left( \langle 2,2,1 \rangle_{120}^u + \langle 2,2,15 \rangle_{120}^d \right) ,$$

$$c_2 = \frac{\langle 2,2,1 \rangle_{120}^d \langle 2,2,15 \rangle_{120}^d + \langle 2,2,1 \rangle_{120}^u}{\langle 2,2,1 \rangle_{120}^d \langle 2,2,15 \rangle_{120}^d + \langle 2,2,1 \rangle_{120}^u} ,$$

$$c_3 = \frac{\langle 2,2,1 \rangle_{120}^d - 3\langle 2,2,15 \rangle_{120}^u}{\langle 2,2,1 \rangle_{120}^d + \langle 2,2,15 \rangle_{120}^u} ,$$

$$c_4 = \frac{\langle 2,2,1 \rangle_{120}^u - 3\langle 2,2,15 \rangle_{120}^d}{\langle 2,2,1 \rangle_{120}^u + \langle 2,2,15 \rangle_{120}^d} \quad . \quad (12)$$

The case of real $120_H$ reduces to similar constraints already encountered in the real case of model I (see also [12] for an analysis using SU(5) decomposition). For real bidoublets the definitions [12] constrain all three $c_i$ to a same order of magnitude. But this contradicts the requirements for small second generation masses of charged leptons ($c_3 \approx 3$) and of up quarks ($c_2 \approx m_t/m_b$). In other words, similarly as in model I, there is a need to complexify the Higgs fields. This is again best achieved by introducing a $U(1)_{PQ}$ global symmetry, which provides as a byproduct a dark matter candidate.

The type I seesaw contribution due to right-handed neutrinos gives the light neutrino mass matrix

$$M_N^I = -M_{ud}^T M_{ur}^{-1} M_{ud} \propto 9c_1^2 M_1 - c_2^2 M_2 M_1^{-1} M_2 \quad (13)$$

whereas the type II contribution reads

$$M_N^{II} \propto M_1 \quad . \quad (14)$$

1. Two generations case: analysis and predictions

In spite of only two Yukawa matrices, the above system of equations is rather complicated and requires painstaking numerical studies. Before plunging in this computational tedium, it is certainly useful if not indispensable to get a physical insight through analytical arguments. Following the successful approach that was adopted by us before (for the case of a supersymmetric SO(10) theory with $10_H$ and $126_H$ [21, 22]) we focus here on the study of the second and third generations, with the natural expectation that the effects of the first generation can be treated as a perturbation.

In the basis where $M_1$ is diagonal, real and non-negative:

$$M_1 \propto \begin{pmatrix} \sin^2 \theta & 0 \\ 0 & \cos^2 \theta \end{pmatrix} \quad (15)$$

the most general charged fermion matrix can be written as:

$$M_f = \mu_f \begin{pmatrix} \sin^2 \theta & i(\sin \theta \cos \theta + \epsilon_f) \\ -i(\sin \theta \cos \theta + \epsilon_f) & \cos^2 \theta \end{pmatrix} , \quad (16)$$

where $f = D, U, E$ stands for charged fermions and $\epsilon_f$ vanishes for negligible second generation masses. In other words $|\epsilon_f| \propto m_2^f/m_1^f$ (see below). Furthermore the real parameter $\mu_f$ sets the third generation mass scale, made explicit below.

We determine next the unitary matrices $L_f$ and $R_f$, which diagonalize $M_f$ in the physically relevant approximation of small $|\epsilon_f|$. We obtain

$$M = R_f^\dagger \text{diag}\{-\mu_f \epsilon_f \sin 2\theta, \mu_f (1+\epsilon_f \sin 2\theta)\} \cdot L_f + O(|\epsilon_f^2|) \quad (17)$$

where

$$L_f = \begin{pmatrix} 1 & -i \cos 2\theta \tau_f \\ -i \cos 2\theta \epsilon_f & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & -i \sin \theta \\ -i \sin \theta & \cos \theta \end{pmatrix} \quad (18)$$
and $R_I$ is given by the same expression with $i \to -i$. From eq. (17), at the leading order in $|\epsilon|$ we get:

$$|\mu_f| = \frac{m_f^2}{m_2^2},$$

$$\sin 2\theta_f = \frac{m_f^2/m_2^2}{1 - \sin^2 2\theta_A/2} + \mathcal{O}(|\epsilon|).$$

The phases of the three $\epsilon_f$ parameters are not determined, while the meaning of the angle $\theta$ will be clear in a moment. Now we are in the position to state the three predictions of this theory regarding 1) neutrino masses, 2) the relation between bottom and tau masses, and 3) the quark mixing angle $V_{cb}$.

1) We begin with the predictions regarding neutrino masses. By using eqs. (18) and (19) and an explicit form of the $2 \times 2$ matrices one concludes that type I and type II seesaw lead to the same structure

$$M_N^I \propto M_N^{II} \propto M_1$$

(21)

In the selected basis the neutrino mass matrix is diagonal. We see that the angle $\theta$ has to be identified with the leptonic (atmospheric) mixing angle $\theta_A$ up to terms of the order of $|\epsilon_L| \approx m_\mu/m_\tau$. For the neutrino masses we obtain from (18)

$$m_3^2 - m_2^2 = \frac{\cos 2\theta_A}{1 - \sin^2 2\theta_A/2} + \mathcal{O}(|\epsilon|)$$

(22)

This equation points to an intriguing correlation: the degeneracy of neutrino masses is measured by the max-

imality of the atmospheric mixing angle. What about numerical predictions? Clearly, without including the ef-

fects of the first generation and the impact of the running from the GUT to the weak scale, no precise determina-

tion can be made. It may be illustrative though to give an estimate in case this formula were to remain approx-

imately valid. The value of $m_2$ could not then be too small: e.g., with the value $\Delta m_3^2 = |m_3^2 - m_2^2| \approx 2.5 \cdot 10^{-3}$ eV$^2$ and the $99\%$ CL limit $\theta_A = 45^\circ \pm 9^\circ$ from [43] one would get $m_2 > 30$ meV. On the other hand, there is an upper limit from cosmology and neutrinoless double beta decay, which (depending on the selected data and analysis) varies from 0.14 eV to 0.5 eV, see again [43]. Clearly, the larger the limit, the closer one can be to $\theta_A = 45^\circ$. The analysis in 3) below suggests though that $\theta_A$ should be as far as possible from the maximal value, i.e. that neutrinos should be as hierarchical as allowed by (22).

2) The second prediction regards the ratio of tau and bottom mass at the GUT scale:

$$\frac{m_\tau}{m_b} = 3 + 3\sin 2\theta_A \Re[\epsilon_L - \epsilon_D] + \mathcal{O}(|\epsilon^2|)$$

(23)

At first glance, this appears to kill the model; after all, the extrapolation in the standard model leads to expect $m_\tau \approx 2m_b$. However, it is not possible to exclude that several effects modify this conclusion and avoid a flat contradiction with data (although we would in any case expect that $m_b$ comes out as small as possible). In particular, we note that with a suitable choice of phases the corrections order $\epsilon$ can amount to a $10\%$ reduction, that the large Dirac Yukawa coupling can produce a $10$ or $20\%$ effect, similar to what one can estimate for the change due to the full three flavor analysis.

3) Last but not least there is an important relation between the quark mixing $V_{cb}$ and the atmospheric mixing angle. Eq. (18) shows that the main part of the (unphysical) up- and down-quark rotations are the same; thus, the quark mixing is found to be:

$$|V_{cb}| = |\Re \xi - i \cos 2\theta_A \Im \xi| + \mathcal{O}(|\epsilon^2|)$$

(24)

where $\xi = \cos 2\theta_A (\epsilon_D - \epsilon_U)$. This equation demonstrates the successful coexistence of small and large mixing an-

gles. In order for it to work quantitatively, $|\cos 2\theta_A|$ should be as large as possible, i.e. $\theta_A$ should be as far as possible from the maximal value 45$^\circ$. Strictly speak-
ing, even this would not be sufficient if this prediction is taken at its face value. However the neglected effects from the first generation and the loops prevent us from sentenceing this prediction and this model.

The analysis we presented above could be in principle changed by the two loop corrections in the Yukawa sector [41]. In the model I this consists only in the renormalization of the original couplings, while in model II it could generate an effective coupling of the fermions to a one-index object (10$_H^{eff}$) such as for example

$$16_F \frac{210_H 126_H}{M} 16_F, \ 16_F \frac{45_H 120_H}{M} 16_F, \ ...$$

(25)

Such terms, even if present, are negligible in the present study of the two generation case, but they should be taken into account in the full analysis of the three generations.

III. SYMMETRY BREAKING PATTERNS AND NEUTRINO MASS

As argued already in the introduction, SO(10) GUT works perfectly well without invoking supersymmetry. It is true that supersymmetry leads naturally to the uni-

fication of gauge couplings, but the same effect can be equally achieved with left-right symmetry as an interme-

diate scale. This is precisely what happens in SO(10). In the over-constrained models discussed in this paper, the Dirac neutrino Yukawa couplings are not arbitrary. Thus one must make sure that the pattern of intermediate mass scale is consistent with a see-saw mechanism for neutrino masses. More precisely, the $B - L$-breaking scale responsible for right-handed neutrino masses cannot be too low. On the other hand, this scale, strictly speaking, cannot be predicted by the renormalization group
study of the unification constraints. The problem is that the right-handed neutrinos and the Higgs scalar responsible for $B - L$ breaking are Standard Model singlets, and thus have almost no impact (zero impact at one-loop) on the running. Fortunately, we know that the $B - L$ breaking scale must be below SU(5) breaking, since the couplings do not unify in the Standard Model. Better to say, $M_{B-L} \leq M_R$, the scale of SU(2)$_R$ breaking, and hence one must make sure that $M_R$ is large enough. This, together with proton decay constraints, will allow us to select between a large number of possible patterns of symmetry breaking.

Our task is simplified by the exhaustive study of symmetry breaking in the literature, in particular the careful two-loop level calculations of Ref. [10]. Recall, though, that the $(2,2,15)$ field must lie below the GUT scale as discussed in sect. II A 1 and although its impact on the running is very tiny, it must be included.

The lower limit on $M_R$ stems from the heaviest neutrino mass

$$m_\nu \geq \frac{m_r^2}{M_R},$$

which gives $M_R \geq 10^{13}$ GeV or so. One can now turn to the useful table of Ref. [10], where the most general patterns of SO(10) symmetry breaking with two intermediate scales consistent with proton decay limits are presented. (Notice that the models with subscript 'b' in the table utilize $16_H$ in place of $\overline{126}_H$ to break the SU(2)$_R$ symmetry, and thus are not relevant for our discussion.)

The above limit on $M_R$ immediately rules out a number of the remaining possibilities; the most promising candidates are those with an intermediate SU(2)$_L \times$ SU(2)$_R \times$ SU(4)$_C \times$ P symmetry breaking scale (that is, PS group with unbroken parity). This is the case in which the breaking at the large scale is achieved by a Pati-Salam parity even singlet, for example contained in $54_{H}$. In the searching for a realistic symmetry breaking pattern one does not need to stick to the global minimum of the potential as in [11], but a local metastable minimum with a long enough lifetime will do the job as well. It has to be stressed however, that a big uncertainty is implicit in all models with complicated or unspecified Higgs sector, due to possibly large and uncontrolled threshold corrections [12].

In any case, the nature of the GUT Higgs and the pattern of symmetry breaking will also enter into the fitting of fermion masses, since they determine the decomposition of the light (fine-tuned) Higgs doublet (e.g., they provide relations among the parameters $c_1$, $c_2$, $c_3$, $c_4$ in eq. [12]). This point is often overlooked but it is essential in the final test of the theory. At this point, for us it is reassuring that both the pattern of symmetry breaking and the nature of Yukawa interactions allow for a possibly realistic, predictive minimal model of non-supersymmetric SO(10).

IV. SUMMARY AND OUTLOOK

The recent years have witnessed an in-depth study of supersymmetric SO(10) grand unification based on the renormalizable see-saw mechanism. What has emerged is a possibly realistic picture for the unification of matter and forces with a predictive pattern for neutrino masses and mixings. The crucial point is that the SO(10) symmetry may be sufficient by itself, without the need for any additional physics. While the theory has a number of appealing features, it may be killed by its main ingredient: there may not be low-energy supersymmetry. It may be partially or completely broken. A nice example of partial breaking is the so-called split supersymmetry with light higgsinos and gauginos and heavy scalars. This picture allows for the interesting possibility of a radiative see-saw mechanism for neutrino masses, and another simple predictive version of the SO(10) theory.

Since we know nothing about the existence of supersymmetry or the nature of its breaking, it is mandatory to study the non-supersymmetric version, as a part of the search for the SO(10) GUT. This was the scope of our paper. We have identified two potentially realistic, predictive Yukawa structures for the case of the renormalizable see-saw mechanism, based on a $\mathbf{126}_H$. This choice is motivated by the fact that the alternative radiative see-saw seems to favor split supersymmetry [25]. We have focused on the renormalizable version simply in order to be predictive, without invoking unknown physics.

The models require adding $\mathbf{10}_H$ or $\mathbf{120}_H$ fields. The latter is particularly interesting, due to the small number of Yukawa couplings. Both models seem to require adding U(1)$_{PQ}$. While this may be appealing since it provides the axion as a dark matter candidate, it is against the spirit of sticking to pure grand unification.

A number of issues must be addressed in order to construct a fully realistic theory. The first task, as we repeatedly argued, is a complete three generation numerical study. This also includes the construction of the minimal GUT Higgs sector and the study of its impact on the fermion masses and mixings. For a successful model, if any, one must study in turn the proton decay predictions, and in particular, the branching ratios that are calculable in the over-constrained theories discussed here. This is a less urgent (but equally important) task simply due to the lack of experimental data. Beside proton decay, the other generic feature of grand unification is the existence of magnetic monopoles which brings along the so-called monopole problem due to the over-production in the early universe. While there is always the possibility of the inflation solution, it is worth recalling that in non-supersymmetric theories there are other interesting ways out of this impasse. These are for example the symmetry non-restoration at high temperature [10] and the possibility that unstable domain walls sweep the monopole away [15]. In principle, either of these alternative solutions can provide further constraints on the parameters of the theory. Yet another constraint comes from leptoge-
ness, which finds its natural role in SO(10) with seesaw mechanism.

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