The analysis of students’ combinatorial thinking skills in solving r-dynamic vertex coloring under the implementation of problem based learning

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Abstract. Learning implementation is expected to maximize the students’ combinatorial thinking skill. This research was intended to examine the students’ combinatorial thinking skill and the implementation of problem based learning to improve the students’ combinatorial thinking skill in solving the problem of r-dynamic vertex coloring. The method used was the mixed-methods that combined quantitative and qualitative research. This research involved 86 participants that were divided into two classes; experimental and control classes. There were 42 students in the control class and 44 students in the experimental class. This study showed that there was a significant difference showed by the value of independent t-test for post-test. The data analysis showed the independent sample t-test value of post-test was 0.000 ≤ 0.05, so it was significant. Thus, there was an influence on the implementation of problem based learning on improving the students’ combinatorial thinking skill in solving r-dynamic vertex coloring problem. Based on the results of the portrait phase analysis, it is known that students in the control class have relatively low combinatorial thinking and the experimental class has relatively higher combinatorial thinking can be seen from the number of indicators mastered by each student.

1. Introduction

Combinatorial thinking is the ability to consider all possible alternatives in certain situations. Combinatorics is an important topic to be taught and discussed in depth with students in the class. The first reason is that combinatorics do not require calculus prerequisites, so this topic can be taught earlier, independent of students’ mastery of calculus. Then combinatorics can also be used to train students to calculate, estimate, generalize, and think systematically[1]. Combinatorial thinking as a tool to solve problems. Combinatorial thinking is a special aspect of mathematical thinking [6].

There are five factors that influence the combinatorial thinking skill, in which each indicator has several different sub-indicators. The indicators and sub-indicators of combinatorial thinking are shown in the table 1.
Table 1. Indicators that Influence the Combinatorial Thinking Skill

| Indicator                        | Sub-Indicator                                                                 |
|----------------------------------|-------------------------------------------------------------------------------|
| Identifying several cases        | a. Identifying properties/characteristics of problems                         |
|                                  | b. Implementing several cases                                                 |
| Recognizing the pattern of all cases | a. Identifying the pattern of problem solving                                  |
|                                  | b. Expanding the pattern of problem solving obtained                           |
| Generalizing all cases           | a. Implementing mathematical symbolization                                     |
|                                  | b. Calculating the cardinality                                                 |
|                                  | c. Developing algorithm                                                        |
| Proving mathematically           | a. Conducting argumentation calculation                                       |
|                                  | b. Testing algorithm                                                           |
|                                  | c. Developing the seed                                                          |
|                                  | d. Testing the seed                                                            |
|                                  | e. Implementing inductive, deductive, and qualitative evidences                |
| Considering other combinatorial problems | a. Conducting an interpretation                                              |
|                                  | b. Proposing open-questions                                                     |
|                                  | c. Recognizing new combinatorial problems                                      |
|                                  | d. Finding the potential application                                           |

Problem Based Learning (PBL) is learning and teaching strategy that promotes the active learning by giving the process control to the students. It involves the use of unstructured open-questions to gain the learning activity [4]. PBL is a constructivism problem learning model that helps the students to think and solve the problem. Ben and Erickson emphasize that Problem Based Learning is a learning strategy that involves the students in solving the problem and integrating various concepts and skills from various science disciplines. This strategy includes collecting and uniting information and presenting the invention. Problem Based Learning (PBL) has been adopted broadly in various subjects and contexts of education to promote critical thinking and problem solving in the authentic learning situation [5].

The implementation of learning by using learning tool that is accordance with the Problem Based Learning can be used to see the students’ combinatorial thinking skill as it has been presented in the portrait phase based on the indicators of the combinatorial thinking skill. Based on the explanation above, the researcher needs to conduct a further research. Therefore, the researcher conducts a research with a title The Analysis of the Students’ Combinatorial thinking skill in Solving r-Dynamic Vertex Coloring under the Implementation of Problem Based Learning.

Definition 1. If $G = (V, E)$ is a simple graph, connected and undirected graph with the vertices set $V$ and the edge of set $E$ and $d(v)$ to degree for each $v \in V(G)$. The maximum and minimum degrees $G$ is symbolized as $\Delta(G)$ and $\delta(G)$ [2]. With $k$ color on the $G$ graph, we map $c : V(G) \Rightarrow S$, in which $|S| = k$ so that every two closed vertices have different colors. A r-dynamic with $k$ color on the graph $G$, therefore $|c(N(v))| \geq \min(r, d(v))$ to each vertex $v$ in $V(G)$ in which $N(v)$ is the environment $v$ and $c(S) = \{c(v) : v \in S\}$ for each part of $S$ [9]. The chromatic r-dynamic is written as $\chi_r(G)$ is a minimum value of $k$ and graph $G$ has r-dynamic with $k$ color.

The $k$ coloring vertex can be said as dynamic vertex coloring if for each vertex $v \in V(G)$ with $d(v) \geq 2$. The neighboring vertex have two different colors. The number of r-dynamic of the graph $G$ is notated as $\chi_r(G)$ is the minimum color $k$ on the $G$ graph. The number of color 1-Dynamic on the Graph $G$ is the color introduced as Chromatic Number and is notated as $\chi_1(G)$ and for the number of Dynamic $\geq 2$ on the graph $G$ is the introduced color as $r$-Dynamic Chromatic Number [3].
Based on the theoretical discussion formulated the hypothesis "the application of Problem Based Learning (PBL) can improve students' combinatorial thinking skills in solving the r-dynamic vertex coloring problem"

2. Research Methods

The method used in this research was a mixed method which combines the qualitative and quantitative research methods. The design used was Sequential explanatory: Quantitative data was collected first, followed by qualitative data that explained the findings from quantitative data (e.g., after assessing pragmatic competence at group-level, following up on several participants to gain understanding about their characteristics) [8] and [7,10,11] also applied this model to their research. Quantitative research analyzed the students learning outcomes after the application of problem-based learning method. Then qualitative research aimed to analyze the data from observations and interviews of selected students. This research investigated two variables, namely the implementation of problem based learning as an independent variable and students’ learning outcomes from solving the problem of the r-dynamic vertex coloring as the dependent variable.

To understand deeply about the effectiveness of Problem based learning, we continued the research by sharing observations with all students of the experimental class and selecting several students to be interviewed about their understanding process in finding a new pattern of r-dynamic vertex coloring.

The experimental design of this research was to compile two class groups, namely the experimental class and the control class, which were selected by purposive random sampling and examined by pre-test and post-test using the following design.

Information:
A : Random sampling
X : Treatment given (Independent Variable)
C : Control of treatment
O : Pre-test / Post-test (Dependent Variables observed)

In this design, there were two groups of classes in which each chosen randomly. Before the research was conducted, the two groups were given pre-test to find out their initial situation. During the research, the first class was treated (X) and the second group was not treated (C), the treated group was called as the experimental group and the untreated group was called a control group.

Then, at the end of the research, the two groups were given a post-test to see how the results were. This design was used to see the effect of the treatment (independent variable) on changes/improvements in the dependent variable that was being observed.
2.1 Population
This research was done to Mathematics Education students at the University of Jember in the odd semester of the 2018/2019 academic year. The sampling technique used was random sampling by randomly selecting two classes, the first class was the experimental class with the implementation of problem-based learning which consisted of 44 students, and the second class was class control with the application of conventional learning that consisted of 42 students.

2.2 Instrument
The instruments used in this research were tests, observation, and interview. On the figure 2 showed the combination of the research methods with research procedures consisting of three stages according to the stages in the research design, namely: preliminary study (qualitative research), analysis of combinatorial thinking skill and the application of problem-based learning (quantitative research), portrait phase (research qualitative). Explanation of research procedures was illustrated in the chart as follows.

**Figure 2.** The Model of Mixed Method
2.3 Task
Students’ combinatorial thinking skill were measured based on the indicators that had been converted into test instrument. One of the test instruments used was to discuss dynamic vertex coloring in $P_3 \bigodot S_4$ graph which consisted of dynamic vertex coloring.

A $k$ vertex coloring is said to be dynamic vertex coloring if for each vertex $v \in V(G)$ with $d(v) \geq 2$. The neighboring vertex have two different colors. A dynamic $r$ with $k$ color on graph $G$ so that $|c(N(v))| \geq \min\{r, d(v)\}$ for each vertex $v$ in $V(G)$ where $N(v)$ is environment $v$ and $c(S) = \{c(v) : v \in S\}$ for each part of $S$. The dynamic $r$-chromatic number written with $\chi_r(G)$ was the minimum value of $k$ so that graph $G$ had dynamic $r$ with $k$-color. The graph used was a corona operating graph symbolized by $\bigodot$, the forming graph was a path graph with $n = 3$ which was operated with a star graph with $m = 4$, so the graph used was $P_3 \bigodot S_4$.

The following was the $r$-dynamic vertex coloring on $P_3 \bigodot S_4$:

The following figure was the giving of vertex notation to the graph $P_3 \bigodot S_4$:

The following figure was the coloring for $r = 1$ and $r = 2$.

\[
|c(N(x_1))| \geq \min\{r, d(x_1)\} \\
2 \geq \{1, 5\}
\]

The following was the notation for coloring at vertex $x_1$, then notation given at all vertex. On the figure beside, each vertex had a maximum of 2 neighbor colors so that the figure met the requirements of $r = 1$ and $r = 2$.

The following figure was the coloring for $r = 3$

\[
|c(N(x_1))| \geq \min\{r, d(x_1)\} \\
3 \geq \{3, 5\}
\]

On the figure beside, each vertex had a maximum of 3 neighbor colors so that the figure met the requirement of $r = 3$, $r$ increased, so that the given color also increased. Coloration was given until it reached $\Delta(G)+1$. 

Figure 3. Example of \( r \)-dynamic vertex coloring from Graph \( P_3 \circ S_4 \)

The purpose of this task was to provide a \( r \)-dynamic vertex coloring that was different from the previous pattern and gave a different color to the vertex such that each vertex had a different color.

To test the research hypothesis formulated, used independent samples t-test with a significance level of 5% or 0.05

\[ H_0 = \text{Combinatorial thinking ability of students using Problem based learning is lower than or equal to combinatorial thinking ability of students not using Problem based learning}. \]

\[ H_1 = \text{Combinatorial thinking ability of students using Problem based learning is higher than to combinatorial thinking ability of students not using Problem based learning}. \]

Information:
- If \( p \text{value} < 0.05 \) then reject \( H_0 \) and accept \( H_1 \)
- If \( p \text{value} \geq 0.05 \) then accept \( H_0 \) and reject \( H_1 \)

3. Research Finding

This research was conducted at the experimental and control classes by using two qualitative methods in order to know the students skill of combinatorial thinking. The implementation of the research was done after conducting validity and reliability tests for the instruments. After that, the experimental and control classes were given pre-test to know their initial combinatorial thinking skill.

After conducting pre-test in the control and experimental classes, the learning by implementing Problem based learning model held in the experimental class while the usual learning held in the control class where the data that would be analyzed by using spss obtained. The following was the result of the analysis by using spss application.

3.1 instrument validation

Before showing our result, we needed to conduct reliability and validity tests for the instruments of our post-test.
Table 2. The test result of the validity question

|       | No_1 | No_2 | No_3 | No_4 | Total |
|-------|------|------|------|------|-------|
|       |      |      |      |      |       |
| No_1  | Pearson Correlation | 1    | .127 | .062 | .151  | .440**  |
|       | Sig. (2-tailed)      | .412 | .690 | .329 | .003  |       |
|       | N                  | 44   | 44   | 44   | 44    |       |
| No_2  | Pearson Correlation | .127 | 1    | .227 | .063  | .510**  |
|       | Sig. (2-tailed)      | .412 | .138 | .686 | .000  |       |
|       | N                  | 44   | 44   | 44   | 44    |       |
| No_3  | Pearson Correlation | .062 | .227 | 1    | .316* | .825**  |
|       | Sig. (2-tailed)      | .690 | .138 | .036 | .000  |       |
|       | N                  | 44   | 44   | 44   | 44    |       |
| No_4  | Pearson Correlation | .151 | .063 | .316* | 1     | .578**  |
|       | Sig. (2-tailed)      | .329 | .686 | .036 | .000  |       |
|       | N                  | 44   | 44   | 44   | 44    |       |
| Total | Pearson Correlation | .440** | .510** | .825** | .578** | 1     |
|       | Sig. (2-tailed)      | .003 | .000 | .000 | .000  |       |
|       | N                  | 44   | 44   | 44   | 44    |       |

**. Correlation is significant at the 0.01 level (2-tailed).
*. Correlation is significant at the 0.05 level (2-tailed).

The following table showed the reliability and validity of the result. Based on the table, it could be seen that the value of $r_{count}$ from number 1 was 0.440, number 2 was 0.510, number 3 was 0.825, and number 4 was 0.578. All items resulted the value of $r_{count} > r_{table}$ with $N=44$, therefore all items were valid.

|       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|
|       | Cronbach's Alpha | .405  | N of items | 4     |       |

Based on the table 3, it could be seen that the overall values of reliability were 0.405 and $r$ (table) form the significance level of 5% with $d_k=N-1=43$, $r$(table)= 0.400. Thus, $r_{count} > r_{table}$.

This concluded that the instruments item were reliable.

3.2 Result

The initial research used was conducted on 42 students in the control class to determine the level of combinatorial thinking. 42 subjects were tested with pretest (diagram 1), in the experimental class it was found that 14% were in the level 1 category in combinatorial thinking, 34% of the students were in the level 2 category, 33% were in the level 3 category in combinatorial thinking, 19% of the students were in level 4 category on combinatorial thinking and no student reaches level 5 in combinatorial thinking.
The research was done to 44 students in the experimental class to know the level of their combinatorial thinking. 44 subjects were tested by using pre-test (chart 2), in the experimental class it was found that 9% students were in the category of level 1 in combinatorial thinking, 36% students were in the category of level 2, 32% students were in the category of level 3, 22% students were in the category of level 4, and there was no students in the category of level 5.

The data analysis used quantitative method to find out the different of the variance of problem-based learning outcomes. The data analysis was done by using SPSS application with the learning result data of pre-test. Homogeneity test was done to determine whether the variance of the data from the analyzed samples was homogeneous or not. Based on table 4, homogeneity test obtained the result of sig. 0.186. This would be significant if it was higher than 0.05 (based on the mean = 0.186>0.05), therefore, the variance of the data was homogenous.
Table 4. Independent sample pre-test

| Test of homogeneity of variance | Levene statistic | df1 | df2 | Sig. |
|---------------------------------|------------------|-----|-----|------|
| Based on mean                   | 1.537            | 9   | 27  | .186 |

The data about pre-test in the experimental and control classes had found that the variance was homogeneous. Then, independent sample t test was conducted significantly if the value of sig. was higher than 0.05. The value of Sig. (2-tailed) based on the mean=0.112>0.05). H0 was accepted, there was no different within the pre-test mean score from the control and experimental classes.

Table 5. Independent Sample t-test Pre-tests

| Levene’s Test for | t-test for Equality of Means | 95% Confidence Interval of the Difference |
|-------------------|-----------------------------|-----------------------------------------|
| Variance Assumed  | F                            | t                          | df | Mean Difference | Std. Error Difference | Lower | Upper |        |
| Equal variances   | .623                        | .432                       | 1.606 | .112            | .806                 | .308   | 2.897 |

The research was continued by conducting learning used conventional learning model followed by post-test. The research was done to 42 students in the control class to know their combinatorial thinking level after the learning. 42 subjects were tested by using post-test (Chart 3), in the control class it was found that 12% students were on the category of level 1 in combinatorial thinking, 24% students were in the category of level 2, 33% students were in the category of level 3, 31% students were in the category of level 4, and there was no students in the category of level 5.

Chart 3. The distribution of Post-test of student Combinatorial thinking skill in the Control class

The research was continued by conducting learning used problem-based learning followed by post-test. The research was done to 42 students in the control class to know their combinatorial thinking level after the learning. 42 subjects were tested by using post-test (chart 4), in the control class it was found that there was no students in the category of level 1 in combinatorial thinking, 9% students were in the category of level 2, 18% students were in the category of level 3, 31% students were in the category of level 4, and there was no students in the category of level 5.
category of level 3, 32% students were in the category of level 4, and 41% students were in the category of level 5.

Chart 4. The distribution of Post-test of student Combinatorial thinking skill in the experiment class

After that, it was followed by normality test. This test was done to determine whether the distribution of the data was normal or not. The distribution of the data would be said as significant if the value was higher or the same with 0.05. Based on Table 6, it was shown that the significance value from the experimental class was $0.200 \geq 0.05$ and the control class was $0.200 \geq 0.05$. Thus, this data from both classes were normally distributed.

| Test of Normality | Kolmogorov-Smirnova | Shapiro-Wilk |
|-------------------|---------------------|--------------|
|                   | Statistic | Df | Sig. | Statistic | Df | Sig. |
| Experiment        | .101      | 42 | .200 | .982      | 42 | .753 |
| Control           | .106      | 42 | .200 | .979      | 42 | .635 |

The data of the implementation of post-test in the experimental and control classes had found that it was normally distributed. Then, independent sample t test was conducted significantly if the value based on Table 7, it showed that Sig. (2-tailed) based on mean $= 0.000 < 0.05$ $H_0$ was rejected, there was a different of post-test mean score of the control and experimental classes.

| Table 7. Independent Sampel t-ttes Post-tes |
|--------------------------------------------|
| Levene's Test for Equality of Variances | t-test for Equality of Means |
|------------------------------------------|-----------------------------|
| F | Sig. | t | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
|---|------|---|----|-----------------|-----------------|---------------------|------------------------------------------|
| Value Equal variances assumed | .614 | .436 | - | 84 | .000 | -4.418 | .961 | -6.328 | -2.507 |

Precentage of Post-test Combinatorial Thinking Skills Experiment class

| Level 1 | Level 2 | Level 3 | Level 4 | Level 5 |
|---------|---------|---------|---------|---------|
| 0%      | 9%      | 18%     | 32%     | 41%     |

Percentage of Post-test Combinatorial Thinking Skills Experiment class

| Precentage of Post-test Combinatorial Thinking Skills Experiment class |
|-----------------------------------------------------------------------|
| Investigating some cases | Recognizing a pattern of all cases | Generalizing for all cases | Proving mathematically | conjecturing another combinatorial problems |
| I1 | I2 | R1 | R2 | G1 | G2 | G3 | P1 | P2 | P3 | P4 | P5 | C1 | C2 | C3 | C4 |
| 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 4  | 7  | 8  | 5  | 9  | 9  | 9  | 6  | 5  | 9  | 13 | 5  | 6  | 9  | 13 | 6  |
| 8  | 9  | 9  | 8  | 7  | 6  | 12 | 8  | 3  | 8  | 10 | 9  | 13 | 6  | 7  | 8  |
| 14 | 10 | 10 | 10 | 14 | 17 | 15 | 17 | 11 | 16 | 11 | 14 | 12 | 10 | 11 | 12 |
| 18 | 18 | 17 | 21 | 16 | 15 | 9  | 12 | 15 | 20 | 12 | 17 | 14 | 21 | 15 | 17 |
The result of independent sample t-test based on table 7 obtained the sig value. (2-tailed) 0.000 < 0.05 then $H_0$ is rejected and $H_1$ is accepted. It was concluded that the results of the post test between the control class and the experimental class had significant differences after the implementation of learning with problem based learning. The experimental class has an average of 45.73 while the control class has an average of 41.31 meaning that the average results of combinatorial thinking skills of experimental class students are higher than the average results of combinatorial thinking skills of control class students and showed that problem based learning had a greater influence on students' combinatorial thinking abilities significantly.

The distribution of the observation result from problem-based learning conducted in the experimental and control classes with 44 students. Based on Chart 5, it was found that 49% students were very active in the learning, 30% students were active, 7% students were quite active, 9% students were not active and 5% students were very inactive. Thus, PBL could affect the students work in solving problem of $r$-dynamic vertex coloring.

**Chart 5.** The observation result distribution of all subject in the experimental class

| Precentage of activity Criteria | Very Active | Active | Hesitate | Inactive | Very Inactive |
|--------------------------------|-------------|--------|----------|----------|--------------|
| **Very Active**                | 21          | 24     | 23       | 20       | 19           |
| **Active**                     | 13          | 13     | 10       | 12       | 12           |
| **Hesitate**                   | 3           | 3      | 5        | 6        | 7            |
| **Inactive**                   | 4           | 1      | 4        | 3        | 3            |
| **Very Inactive**              | 2           | 2      | 1        | 2        | 2            |

### 3.3 Portrait phase

Portrait phase was taken to draw the processes, we used portrait phase, the portrait phase was brought to draw the processes from combinatorial thinking skill, we had selected six objects from experimental and control groups, but in this research we only described three subjects as the illustrations. The interview was carried out on selected subjects to find out the thinking process in completing the $r$-dynamic vertex coloring.

**Figure 4.** Graph of the subject 1
The analysis of the works was done to know the process of students’ worksheet completion in producing the final result, to strengthen the solution of this students’ worksheet, the result of the work sheet related to the interview and the data of observation.

The analysis of subject 1 (figure 4) r-dynamic coloring on graph $P_n \Theta S_m$ with $n = 3$ dan $m = 4$ (expanding 4) and verify the truth of color from the suitable vertex and met the requirement for graph coloring in dynamic 1 and 2.

Subject 1 has reached level 3 of combinatorial thinking skills, based on the results of the analysis of work subject 1 has been able to understand simple coloring in a graph then subject 1 has been able to apply mathematical symbolization shown by being able to calculate cardinality and can develop a method of giving coloring that is prioritized by giving coloring at the vertex that has the greatest degree. This ability is clearly explained by the results of the interview below.

The interview result of subject 1:
Researcher: can you understand the vertex coloring of graph?
Student: Yes, I understand the vertex coloring of graphs
Researcher: can you understand the coloring of graph if the graph given is different?
Student: Yes, I can understand the coloring on different graphs
Researcher: can you provide the dynamic vertex coloring of the graph?
Student: Yes, I can give the dynamic vertex coloring according to the conditions specified
Researcher: can you provide the dynamic vertex color staining on an addition (expand) graph?
Student: no, I can only give coloring to this graph, if I expand it I can't do it.
Researcher: can you apply the mathematical symbol in the completion of the given coloring?
Student: yes, I can provide coloring along with mathematical symbols even though I still have to go back and forth or look back at the symbolic information provided.
Researcher: can you calculate the cardinality of the graph given?
Student: yes, I can calculate the cardinality of the graph given
Researcher: can you develop an algorithm?
Student: yes, I can make a simple way to give coloring to a graph
Researcher: can you calculate and test the algorithm that you made?
Student: no, I can only use it without being able to test it
Researcher: can you develop a bijection?
Student: no, I can't develop it
Researcher: can you propose open problems regarding the coloring problems you are working on?
Student: I can't do it, I can only do the coloring given.

Figure 5. Phase potrait subject 1
Subject 2 has reached level 4 of combinatorial thinking skills, based on the results of the analysis of work subject 2 managed to develop algorithms, calculate and test algorithms, subject 2 can also apply inductive, deductive and qualitative evidence. This ability is clearly explained by the results of the interview below.

The interview result of subject 2
Researcher  : are you able to understand the vertex coloring of graphs?
Student     : yes, I can understand it
Researcher  : can you do coloring on the other graphs?
Student     : yes, I can give coloring to a graph that is different from the one given
Researcher  : can you provide dynamic vertex coloring on a given graph?
Student     : yes, I can provide dynamic vertex coloring
Researcher  : can you do it if it's expanded?
Student     : yes, I can do it
Researcher  : can you apply mathematical symbolization and calculate the cardinality of the graph given?
Student     : yes, I can use it, but it's still not too smooth because I still have to look back at the information, and I can also calculate cardinality but it takes a long time
Researcher  : can you develop a bijection?
Student     : Yes, I can develop a wisdom and test it, but I don't know the truth
Researchers : can you apply inductive, deductive and qualitative evidence?
Student     : yes, I can apply it
Researcher  : can you interpret and propose an open problem?
Student     : no, I can't
Researchers : can you find out new combinatorial problems and find potential applications?
Student     : I can't, I can only do the graph that I'm working on.
Subject 3 has reached level 5 of combinatorial thinking skills, based on the results of the analysis of work subject 3 managed to develop algorithms, calculate and test algorithms, subject 3 can also apply inductive, deductive and qualitative evidence. This ability is clearly explained by the results of the interview below.

The interview result of subject 3
Researcher : were you able to understand vertex coloring on graph?
Student : Yes, I understood vertex coloring on graph
Researcher : did you understand vertex coloring on graph to the other graph?
Student : yes, I understood the coloring on the other graph with the same coloring condition
Researcher : could you understand the coloring pattern of the graph given?
Student : Yes, I could understand the coloring pattern on the graph given, what I did was determining the minimum color on the graph.

Figure 7. Phase portrait subject 2

Figure 8. Graph of the subject 3
Researcher: Could the pattern be used on a graph with the same conditions?
Student: The coloring pattern could only be used for certain graphs.
Researcher: Could you write down the results of your work in mathematical form and determine the cardinality?
Student: Yes, I could write it in mathematical symbols and determine its cardinality.
Researcher: Could you develop an algorithm?
Student: No, I couldn’t make an algorithm.
Researcher: Could you test an algorithm?
Student: No, I only gave the coloring pattern according to my thought and only on certain graphs.
Researcher: Could you understand the problem of coloring on different graphs with the graph that you did coloring to its dynamic vertex?
Student: Yes, I could understand the problem of coloring on other graphs.

Figure 9 showed the thinking process of subject 3 in understanding the completion of the worksheet in the form of generalization. From step 1 to step 3b, thinking straight according to what was explained in step 3b jumped to 4a then to 4b, but in 4b, subject 1 went back to 2a. In step 3a, he jumped to 4e and then returned to 4a, continued to 4d, 4e, 5a, 5b, 5c, the thinking process was back to 4a then continued until it reached 5d.

![Figure 9. Phase portrait subject 3](image)

Figure 10 was the combination of three subjects that had been studied carefully, from the images that were processed as the students’ thinking processes in global; the different combinatorial thinking skill created the diversity in solving the students’ problems.

![Figure 10. Phase portrait of combination](image)

4. Discussion
The findings showed that the students were required to use their combinatorial thinking and find a systematic to ensure that the possibilities had been discussed. In his point of view, combinatorial thinking referred to a special aspect of mathematical thinking [6]. It is in line with
this research which was intended to analyze the combinatorial thinking skill and the implementation of problem-based learning in maximizing the combinatorial thinking skill.

The control class found that 12% students were in the category of level 1 in combinatorial thinking, 24% students were in the category of level 2, 33% students were in the category of level 3, and 31% were in level 4, while there was no student who reached level 5 on combinatorial thinking. In the experimental class, it was found that there was no students in level 1 on combinatorial thinking, 9% students were in the category of level 2, 18% students were in the category of level 3, 32% students were in level 4, and 41% students were in level 5 on combinatorial thinking. The results of the analysis of independent sample test showed that the students' learning outcomes at the pre-test stage were not different and at the post-test stage, the different analysis test showed significant value of \( p \leq 0.05 \) which meant that post-learning had different results.

Portrait Phase is a picture of students' thinking flow in solving a problem. In this study the portrait of the student phase is based on the flow of students' ability to combinatorial thinking in completing the study of r-Dynamic Vertex Coloring based on Problem based learning. Combinatorial thinking is outlined in the student phase portrait with 3 students found subject 1 is at level 3 in combinatorial thinking, subject 2 is at level 4 in combinatorial thinking and subject 3 is at level 5 in combinatorics.

5. Conclusion
Based on this research, the implementation of PBL had a significant effect on the students’ combinatorial thinking skill in the experimental group. The students in the experimental group showed their combinatorial thinking skill compared to the control group. The result showed that the enhancement in the students’ learning outcomes and combinatorial thinking skill were seen from the post-test. The scores of the experimental group were far better as it was supported by problem-based learning (PBL) to improve student combinatorial thinking.

Based on the results of the portrait phase analysis, it is known that students in the control class have relatively low combinatorial thinking and the experimental class has relatively higher combinatorial thinking can be seen from the number of indicators mastered by each student. This research is only carried out on a small scope of advice for future researchers to implement it in a larger scope.

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