On a theory of low temperature electron decoherence in disordered conductors

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Abstract. Recently von Delft (JvD) attempted to revise our theory of low temperature decoherence of electrons in disordered conductors with electron-electron interactions [D.S. Golubev and A.D. Zaikin, Phys. Rev. B 59, 9195 (1999)] and argued [Intl. J. Mod. Phys. B 22, 727 (2008)] that by “more accurate treatment of recoil effects” he was able to obtain the electron decoherence rate that vanishes at $T = 0$. In this paper we demonstrate that this JvD’s claim is in error, as it is based on ambiguous and uncontrolled manipulations violating basic principles of quantum theory, such as energy-time uncertainty relation, causality, fluctuation-dissipation theorem, detailed balance and the like. While our theory has a simple and transparent relation to the well known $P(E)$ theory (which exactly accounts for all recoil effects) JvD’s approach is inconsistent with this theory thus being unable to correctly treat even recoil effects it was explicitly aimed at.

1. Introduction

In a series of papers [1, 2, 3, 4, 5, 6] we have developed several different theoretical approaches aimed to microscopically describe weak localization effects in disordered conductors in the presence of electron-electron interactions. The main conclusion reached within all these formally independent approaches is that the electron decoherence time $\tau_\phi$ remains finite down to $T = 0$ due to the effect of electron-electron interactions. Accordingly, the weak localization correction to the conductance turns out to be smaller than its non-interacting value at all temperatures including $T = 0$. It is important to emphasize that the results obtained with the aid of completely independent techniques (cf., e.g., [2], [4] and [5, 6]) fully match in the corresponding limits and agree well with numerous available experimental data. The detailed comparison between theory and experiment can be found in Refs. [6, 7, 8].

In a recent paper [9] (see also its earlier version [10]) von Delft (JvD) attempted to revise one of our techniques [2, 3] without addressing the others [4, 5, 6]. He has re-derived our influence functional approach for interacting electrons in disordered conductors [2, 3, 8], observed that it “properly incorporates the Pauli principle” and acknowledged that we “got it completely right” which is “a significant and important achievement”. These statements are very important. After numerous unsuccessful attempts (reviewed, e.g., in Refs. [4, 8, 9]) to search for a mistake in our derivation of the influence functional [2, 3, 8] it is finally acknowledged by JvD that this mistake just does not exist. Note that previously our influence functional approach has been re-derived by Eriksen et al. [11] who independently confirmed its validity, see also the discussion [12].
Hence, one can bypass practically all steps in our derivation restricting the whole discussion to just one – purely mathematical – issue, i.e. how to correctly evaluate our path integral \[2\].

At the same time, JvD \[9, 10\] claimed that – in comparison to our work – he has achieved “a more accurate treatment of recoil effects”. According to JvD \[9\], “with this change” he could reproduce “in a remarkably simple way, the standard, generally accepted results for the decoherence rate”.

In this paper we demonstrate that JvD’s analysis of our influence functional and his statement about recoil effects “neglected” in our calculation are in error. Instead of directly evaluating our influence functional JvD effectively replaces it by a very different “influence functional for decoherence” obtained by performing uncontrolled manipulations with diagrams. As a result JvD \[9\] arrives at the expression for this functional which violates fundamental principles of quantum theory. The corresponding evidence is presented in Sec. II of the paper. In Sec. III we explicitly analyze the main drawbacks of JvD analysis which have eventually led to the above problems.

2. Some consequences of JvD’s analysis

The central result of our derivation \[2\] is a general expression for the conductivity of a disordered conductor in terms of the Feynman-Vernon influence functional for interacting electrons. The corresponding expressions are well documented in our earlier publications and are briefly summarized in Appendix A for convenience.

The key role in our analysis is played by the path integral \[A.2\] which defines the kernel \(J\) of the evolution operator on the Keldysh contour. It is important that the term \(S_R\) \(A.4\) contains the electron density matrix \(\rho\) which makes it necessary to keep integrals over both coordinate and momentum variables because the corresponding Hamiltonians turn out to be nonlocal. It is also important to observe that both dissipative term \(S_R\) and noise term \(S_I\) are purely real functionals. Hence, there is no way how \(iS_R\) can cancel \(S_I\) for any trajectories. Already this observation alone is sufficient to discard JvD’s analysis \[9\], see also \[13\] for more details on this point.

In order to describe weak localization in the limit \(k_F l \gg 1\) it is sufficient to employ the standard picture of time-reversed paths and evaluate the path integral \(A.2\) within the saddle point approximation. It turns out that the term \(S_R\) vanishes for any pair of time-reversed paths \[14\] while \(S_I\) grows with time at all temperatures including \(T = 0\) and determines the Cooperon decay at sufficiently long times. One can also evaluate the contribution of fluctuations around time-reversed paths \[4\] which turns out to be significant only at short times, i.e. in the perturbative (in the interaction) regime. The analysis \[4\] demonstrates that the final result can be exactly cast to the form allowing to semiclassically evaluate only those path integrals which do not contain the electron density matrix \(\rho\) while the latter was kept in its exact form in the corresponding matrix elements which cannot grow with time. This analysis was performed within the accuracy of the definition of the weak localization correction to conductivity and fully confirmed our earlier results and conclusions. Yet additional support to our results comes from a recently developed independent technique \[5, 6\] which does not employ the picture of semiclassical paths at all and will be briefly addressed towards the end of this paper.

Unfortunately these latter developments \[4, 5, 6\] were completely ignored by JvD who attempted to treat the path integral yet “somewhat more accurately” than we do. JvD’s key idea is to effectively integrate out the momentum variables in Eq. \(A.2\) which would yield the “position-only” representation of the influence functional. Eventually JvD arrived at the functional \(A.9\) which is entirely different from the correct one \(A.2\). Below we will demonstrate that the functional \(A.9\) cannot be obtained from \(A.2\) by performing momentum integrations or by any other correct means.

A quick glance already at the first JvD’s formula (Eq. (1a,b) in Ref. \[9\]) demonstrates that
this and subsequent equations violate the energy-time uncertainty relation. Indeed, JvD writes [15] that the double path integral entering his Eq. (1b) “gives the amplitude for an electron with energy $\hbar \epsilon$ to propagate from $r_{2'}$ at time $-\tau/2$ to $r_1$ at $\tau/2$ times the amplitude for it to propagate from $r_1$, at time $\tau/2$ to $r_2$ at $-\tau/2$". Thus, in JvD’s Eq. (1) and his subsequent analysis both electron energy and time are simultaneously fixed. This is a clear violation of the quantum mechanical energy-time uncertainty relation. No such amplitude can be defined in quantum mechanics.

Let us further examine the “influence functional” defined in Eqs. (2-4) of Ref. [9]. It is easy to observe that, while our action $S_R$ (A.4) is purely real, JvD’s action $\tilde{S}_R$ already contains an imaginary part. Since this part effectively adds up to the noise term $S_f$ it would imply that the equilibrium Nyquist noise spectrum “felt” by electrons in a disordered conductor would be proportional to the combination $\coth \frac{\omega}{2T} + \frac{1}{2} (\tanh \frac{\omega}{2T} - \tanh \frac{\omega}{2T})$ and not to the commonly accepted $\coth \frac{\omega}{2T}$ which also follows from our expression for $S_f$ (A.5), (A.6). Already this observation demonstrates that JvD’s “influence functional” is fundamentally different from ours. While the latter takes full account of both classical and quantum noise, JvD’s expressions (A.10-A.12) would imply “cutting out” the quantum noise for small values of $\epsilon$. As a result, according to JvD in the limit $\epsilon = 0$ and $T \to 0$ electrons would “feel” no noise at all. In this limit JvD’s $i \tilde{S}_R$ exactly cancels $S_f$, i.e. his “influence functional” is identically equal to unity for all electron paths. In other words, JvD's analysis predicts that electron-electron interactions would have no influence at all on the dc conductivity of a disordered conductor at $T = 0$. This is obviously not the case for our influence functional [2, 3, 8].

Our next observation is that JvD’s “influence functional” violates causality. This fact becomes obvious by inspection of analytic properties of the Fourier-transformed JvD’s function $\tilde{R}_f$ (A.11,A.12) which has poles both in the upper and lower half-planes of the $\omega$-variable, hence, implying that the electron motion should be affected by photons coming both from the past and from the future, an obvious nonsense. In contrast, any correct expression for the kernel $R(\omega, \mathbf{q})$ should have poles only in the lower half-plane of $\omega$, which is the case for our expression (A.6-A.7). Applying the least action conditions to our effective action one recovers the standard classical equation of motion for a high energy electron in a dissipative environment [16]

$$m \ddot{r} + \nabla U(r) + e^2 \int_{-\infty}^{t} dt' \nabla R(t - t', r(t) - r(t')) = 0,$$

where the time integral in a dissipative term extends from $-\infty$ to $t$ in full agreement with the causality principle. In contrast, the same equation derived from JvD’s action (A.9,A.10) would take the form

$$m \ddot{r} + \nabla U(r) + e^2 \int_{-\infty}^{+\infty} dt' \nabla \tilde{R}_f(t - t', r(t) - r(t')) = 0,$$

where the time integral already runs over all times between $-\infty$ and $+\infty$ in a direct conflict with the requirement of causality. In addition, instead of the correct kernel $R(t, r)$ (A.6) JvD’s Eq. (2) contains $\tilde{R}_f(t, r)$ (A.11) in a clear contradiction to the well established results [16]. Thus, JvD’s “influence functional” fails already on a classical level being unable to correctly describe particle’s dynamics in a dissipative environment.

In addition to the above problems JvD’s “influence functional” also violates fluctuation-dissipation theorem and detailed balance. The easiest way to observe this violation is to consider the values of $\epsilon$ large compared to temperature $T$. In that case the Pauli principle should not be important and the term $\tilde{S}_R^{\epsilon}$ should approach the action $S_R$ evaluated without the Pauli principle, e.g. for a muon moving in a metal. This situation was considered, e.g., in Ref. [17] and the corresponding expression for $S_R$ obtained there coincides with (A.4) where one should...
now set \( \rho \) equal to zero. In this case FDT is represented by the following relation

\[
I(\omega, q) = -\coth \frac{\omega}{2T} \text{Im} R(\omega, q).
\]

(3)

While FDT (3) is manifestly satisfied for our influence functional, JvD’s kernel \( \tilde{R}_\epsilon(\omega, q) \) (A.11,A.12), being substituted into Eq. (3) instead of \( R \), obviously violates this relation. As a direct consequence, JvD’s “influence functional” also violates detailed balance.

Concluding this section, we have demonstrated that there exists no environment in Nature which could be described by the “influence functional” defined in Eqs. (2-4) of Ref. [9]. Hence, this “influence functional” is unsuitable for calculations of any physical quantity including, of course, the electron decoherence rate.

3. Main problems of JvD’s analysis

We now turn to the main problems of JvD’s calculation which have eventually led to alarming consequences outlined above.

As it was already emphasized, the path integral over momentum variables (Eqs. (A.2),(A.3),(A.4)) is strongly non-Gaussian, and, hence, cannot be exactly evaluated. JvD [9] does not even attempt to directly perform the momentum integrals. Instead, he carries out a set of manipulations and formulates “a rule of thumb”, Eq. (B.91). Unfortunately this central part for the whole analysis (pp. 60 of the paper) is by far less detailed than the rest of the paper and, in fact, contains almost no equal signs (substituted in eqs. (B.91-B.93) by the sign “\( \rightarrow \)).

What was actually done by JvD with our influence functional? JvD uses it only at the very first step expanding our Eq. (A.2) in powers of \( iS_R + S_I \). In this way JvD demonstrates – in full agreement with our earlier statements [2, 3, 4, 8] – that no diagrams are missing within our influence functional approach. From this point on JvD performs his own analysis of the diagrammatic series. This analysis has no direct connection to any path integral calculation, since all Keldysh diagrams can, of course, be also recovered without path integrals. JvD’s key steps are as follows:

(i) Rewriting the Keldysh Green function for electrons as

\[
G^K(E - \omega) = [G^R(E - \omega) - G^A(E - \omega)] \tanh \frac{E - \omega}{2T}
\]

JvD neglects \( G^A \) on the forward branch of the Keldysh contour, i.e. he makes the replacement

\[
G^K(E - \omega) \rightarrow G^R(E - \omega) \tanh \frac{E - \omega}{2T},
\]

see his Eq. (B.93). Similarly, on the backward branch of the Keldysh contour JvD replaces

\[
G^K(E - \omega) \rightarrow -G^A(E - \omega) \tanh \frac{E - \omega}{2T}.
\]

This approximation is equivalent to simply dropping certain classes of diagrams. Already at this point JvD violates causality for electrons.

(ii) JvD splits the Pauli factor \( \tanh \frac{E - \omega}{2T} \) from \( G^K \) (or, better to say, from the remaining part of \( G^K \) containing \( G^R \) or \( G^A \) only) and transfers it to the photon propagators \( L^R \) on the forward or \( L^A \) on the backward branches of the Keldysh contour, i.e. he makes the following replacement

\[
G^K(E - \omega)L^{R,A}(\omega) \rightarrow \pm G^{R,A}(E - \omega) \tanh \frac{\epsilon - \omega}{2T} L^{R,A}(\omega).
\]

Note, that the energy \( \epsilon \) under \( \tanh \) is now different from \( E \). According to JvD, \( \epsilon \) is set to be constant which is not sensitive to the pre-history. At this stage JvD violates the energy-time uncertainty relation as well as causality for photons. This step in combination with the previous one is equivalent to JvD’s “rule of thumb”, Eq. (B.91), applied to the first order diagrams.
Figure 1. JvD’s “rule of thumb” illustrated for one of the third order diagrams. All three steps (1) to (3) are evident. Notice that the Golden rule combination $\coth \frac{\omega}{2T} + \tanh \frac{\omega}{2T}$ appears only after JvD’s “rule of thumb” is applied, while the initial exact expression for this diagram is by far more complicated and it does not contain this simple combination of coth and tanh.

(iii) JvD spreads his “rule of thumb” to all orders of the perturbation theory. For that purpose in addition to steps (1) and (2) JvD substitutes

$$\tanh \frac{E - \omega_1 - \omega_2 - \ldots - \omega_n}{2T} \rightarrow \tanh \frac{\epsilon - \omega_1}{2T}$$

in all the diagrams whenever more than one photon frequency under tanh is encountered. As a result, all the complicated Pauli factors are reduced to a simple and unique form $\tanh \frac{\omega}{2T}$ for all diagrams in all orders. Hence, all tanh-factors become decoupled from the electron lines and can now be absorbed in the photon Green functions. After this step electrons are not anymore sensitive to the Pauli principle. In addition, this manipulation leads to violation of FDT, since it effectively breaks down thermal equilibrium in the photon subsystem.

Summarizing, the above set of steps is equivalent to (a) dropping certain classes of diagrams and (b) replacing infinitely many remaining diagrams by completely different ones. JvD’s “rule of thumb” is also illustrated in Fig. 1.

Unfortunately no serious justification for these manipulations was offered by JvD. For instance, according to JvD [9], some contributions to the Cooperon self-energy corresponding to the so-called Hikami boxes should vanish after impurity averaging in the first order in the limit of zero frequency and wave vectors. For some unclear reasons JvD believes that this observation should be sufficient in order to perform his step (1), i.e. to disregard terms with $G^A$ and $G^R$, already before impurity averaging and in all orders of the perturbation theory.

As for the step (2), according to JvD it is possible to neglect all energy fluctuations “if one so chooses”. This approximation “is expected to work well if the relevant physics is dominated by low frequencies”. This argument is logically inconsistent, since in the very beginning it already assumes what one needs to prove in the end. In practice, as it is argued by JvD in Sec. B6.2, one should disregard “accumulation of energy changes” in all vertex diagrams in all orders and also in the self-energy diagrams in the second and all higher orders in the interaction. Our analysis, in contrast, demonstrates that this “energy accumulation” is important and cannot be disregarded.

The combination of JvD’s steps (1) and (2) makes it impossible to fully reconstruct all contributions to the Cooperon already in the first order in the interaction. For instance, the first order non-Golden-rule terms defined, e.g., in Eqs. (70) of Ref. [3] are completely missing in JvD’s Eqs. (1-4) and his subsequent analysis.
JvD’s step (3) introduces yet one more uncontrolled approximation by neglecting all but one photon frequencies under each tanh. To support this step JvD argues that all these frequencies are smaller than $T$. It is worth stressing that in the exact diagrammatic expansion the photon frequencies are not restricted by temperature. The contribution of high frequencies is important and may even lead to ultra-violet divergencies. Only after uncontrolled approximations (1) and (2) such divergencies could disappear. However, even if one adopts these steps, it would still be inconsistent to disregard all but one photon frequency. Since all these frequencies can be of the same order, it is simply a matter of consistency of any approximation to either keep or disregard them all. In either case it would then be impossible to recover the desired combination $\coth \frac{\omega}{2T} + \tanh \frac{\epsilon - \omega}{2T}$.

The net result of all these manipulations is nothing but simple exponentiation of the first order self-energy diagrams evaluated within the approximation which effectively ignores all contributions not containing the combination $\coth \frac{\omega}{2T} + \tanh \frac{\epsilon - \omega}{2T}$. Only after that JvD goes back to path integrals and constructs the functional ((A.9). Obviously, this functional has nothing to do with our influence functional no matter whether one carries out momentum integration or not. Effectively JvD just replaces our correct influence functional by a very different object which violates causality, FDT and other fundamental principles of quantum theory (see Sec. II).

In Sec. 4 of Ref. [9] JvD incorrectly ascribes to us certain manipulations with his functional which we have never performed and could never perform simply because we do not have such a functional at all in any of our papers. The procedure discussed in Sec. 4 of Ref. [9] has, therefore, nothing to do with our saddle point analysis [2, 3, 8] and JvD’s claim that this analysis “neglects recoil effects” is highly misleading. Obviously, all recoil effects are fully included in our path integral (A.2)-(A.5). Evaluation of this non-Gaussian path integral within the saddle point approximation is a legitimate and standard mathematical procedure. It is certainly correct within its applicability range and by no means it implies neglecting recoil effects, as it was incorrectly alleged by JvD.

Finally, we would like to mention that recently we have worked out a new technique [5, 6] which is completely independent from our previous approaches [1, 2, 3, 4]. This technique does not employ the picture of quasiclassical electron paths at all and in some limits allows to obtain exact results. The method [5, 6] does not require any high frequency cuttoffs and yields regular expressions for $\tau_\phi$ which saturate to non-zero values in the limit $T \rightarrow 0$ for all types of disordered conductors. For weakly disordered conductors our results [6] match with those obtained previously in Refs. [1, 2, 3, 4] thus independently confirming the validity of those. Our work [5, 6] also allows to establish a simple and transparent relation between our results and the so-called $P(E)$-theory [18, 19] of electron tunneling in the presence of electron-electron interactions, see also our earlier paper [20]. This relation illustrates again that the fundamental physical reason for electron decoherence at zero temperature is energy exchange of interacting electrons. Regrettably, this physical phenomenon was mistreated in Ref. [9] which results are manifestly inconsistent with the $P(E)$-theory [18, 19].

In conclusion, we have demonstrated that both our method and results [1, 2, 3] demonstrating the low temperature saturation of the electron decoherence rate in disordered conductors are valid while JvD’s analysis [9] fails to adequately address the problem and yields erroneous results and conclusions.

Acknowledgment

We are grateful to Jan von Delft for his persistent interest to our work and for his openness with which he shared with us his doubts on various points of our calculation.
Appendix A.

For reference purposes we briefly recapitulate our path integral representation for the conductance \( \sigma \) of a disordered conductor in the presence of electron-electron interactions. One finds \[2\]

\[
\sigma = \frac{e^2}{3m} \int_{t'}^{t} dt' \int dr_{Ff} dr_{Bf} \left( \nabla_{r_{Ff}} - \nabla_{r_{Bf}} \right) \bigg|_{r_{Ff} = r_{Bf}} J(t, t'; r_{Ff}, r_{Bf}; r_{Ff}, r_{Bf}) (r_{Ff} - r_{Bf}) \rho(r_{Ff}, r_{Bf})
\]

(A.1)

Here \( \rho(r_{Ff}, r_{Bf}) \) is the equilibrium single-electron density matrix, the function \( J \) is given by the path integral over the coordinates and momenta on the forward \((r_{Ff}, p_{Ff})\) and backward \((r_{Bf}, p_{Bf})\) branches of Keldysh contour,

\[
J = \int_{r_{Ff}(t')=r_{Ff}}^{r_{Ff}(t)=r_{Ff}} dr_{Ff} \int_{r_{Bf}(t')=r_{Bf}}^{r_{Bf}(t)=r_{Bf}} dr_{Bf} \int dp_{Ff} \int dp_{Bf} \times \exp \left\{ IS_0[p_{Ff}, r_{Ff}] - IS_0[p_{Bf}, r_{Bf}] - iSR[p_{Ff}, r_{Ff}, r_{Bf}, r_{Bf}] - SI[r_{Ff}, r_{Bf}] \right\}.
\]

(A.2)

The actions are defined as follows

\[
S_0[p, r] = \int_{t'}^{t} dt'' \left( \frac{p^2}{2m} - U(r) \right),
\]

(A.3)

\[
S_R = \frac{e^2}{2} \int_{t'}^{t} dt \int_{t'}^{t} dt' \left\{ R(t_1 - t_2, r_{Ff}(t_1) - r_{Ff}(t_2))(1 - 2\rho(p_{Ff}, r_{Ff})) \\
- R(t_1 - t_2, r_{Bf}(t_1) - r_{Bf}(t_2))(1 - 2\rho(p_{Bf}, r_{Bf})) + R(t_1 - t_2, r_{Ff}(t_1) - r_{Bf}(t_2)) \\
\times (1 - 2\rho(p_{Bf}, r_{Bf})) - R(t_1 - t_2, r_{Bf}(t_1) - r_{Ff}(t_2))(1 - 2\rho(p_{Ff}, r_{Ff})) \right\},
\]

(A.4)

\[
S_I = \frac{e^2}{2} \int_{t'}^{t} dt \int_{t'}^{t} dt' \left\{ I(t_1 - t_2, r_{Ff}(t_1) - r_{Ff}(t_2)) + I(t_1 - t_2, r_{Bf}(t_1) - r_{Bf}(t_2)) \\
- I(t_1 - t_2, r_{Ff}(t_1) - r_{Bf}(t_2)) - I(t_1 - t_2, r_{Bf}(t_1) - r_{Ff}(t_2)) \right\}.
\]

(A.5)

Here \( U(r) \) is the impurity potential and \( \rho(p, r) \) is the electron density matrix. With sufficient accuracy one can set \( 1 - 2\rho(p, r) \simeq \tanh[(p^2/2m + U(r) - \mu)/2T] \). The functions \( R(t, r) \) and \( I(t, r) \) read

\[
R(t, r) = \int \frac{d\omega d^3q}{(2\pi)^4} \frac{4\pi}{q^2\epsilon(\omega, q)} e^{-i\omega t + iqr},
\]

(A.6)

\[
I(t, r) = \int \frac{d\omega d^3q}{(2\pi)^4} \text{Im} \left( \frac{-4\pi}{q^2\epsilon(\omega, q)} \right) \coth \left( \frac{\omega}{2T} \right) e^{-i\omega t + iqr},
\]

where \( \epsilon(\omega, k) \) is the dielectric function

\[
\epsilon(\omega, q) = 1 + \frac{4\pi\sigma_D}{-i\omega + Dq^2},
\]

(A.7)

\( \sigma_D \) is the Drude conductivity and \( D \) is the diffusion coefficient.
For comparison, we also present the results obtained by JvD [9], see Eqs. (1-4) of that paper. According to JvD the conductivity should read

\[ \tilde{\sigma} = \frac{2e^2}{3m^2} \int dx_2 \left( \nabla_{r_1} - \nabla_{r_1'} \right) \left. \left( \nabla_{r_2} - \nabla_{r_2'} \right) \right|_{r_1=r_1', \, r_2=r_2', \, x_2} \times \frac{d\varepsilon}{4T \cosh^2 \varepsilon/2T} \int_0^\infty d\tau \tilde{P}^\varepsilon(\tau; r_1, r_2; r_1', r_2), \quad (A.8) \]

where

\[ \tilde{P}^\varepsilon(\tau; r_1, r_2; r_1', r_2) = \int_{r_1}^{r_1(\tau/2)} D r_F \int_{r_1(\tau/2)}^{r_1'} D r_{F'} \int_{r_1}^{r_1(\tau/2)} D r_B \int_{r_1(\tau/2)}^{r_1'} D r_{B'}, \quad (A.9) \]

\[ \tilde{S}_0[\varepsilon] = \int_{-\tau/2}^{\tau/2} d\tau' \left( m\varepsilon^2/2 - U(\varepsilon) \right) \]

\[ \tilde{S}_R[\varepsilon] = \frac{e^2}{2} \int_0^\tau dt_1 \int_0^{t_1} dt_2 \left( \tilde{R}_e(t_1 - t_2, r_F(t_1) - r_F(t_2)) + \tilde{R}_e(t_1 - t_2, r_B(t_1) - r_B(t_2)) \right) \]

\[ - \tilde{R}_e(t_1 - t_2, r_F(t_1) - r_B(t_2)) - \tilde{R}_e(t_1 - t_2, r_B(t_1) - r_F(t_2)) \], \quad (A.10) \]

where \( \tilde{R}_e(t, r) = \tilde{R}_e^{Re}(t, r) + i \tilde{R}_e^{Im}(t, r) \),

\[ \tilde{R}_e^{Re}(t, r) = \frac{1}{2} \int \frac{d\omega dq}{(2\pi)^4} \text{Re} \left[ R(\omega, q) e^{-i\omega t + iqr} T_+(\varepsilon, \omega) \right], \quad (A.11) \]

\[ \tilde{R}_e^{Im}(t, r) = \frac{1}{2} \int \frac{d\omega dq}{(2\pi)^4} \text{Im} \left[ R(\omega, q) e^{-i\omega t + iqr} T_-(\varepsilon, \omega) \right], \quad (A.12) \]

\[ T_\pm(\varepsilon, \omega) = \tan \frac{\omega}{2} \mp \tan \frac{\omega}{2} \] and \( R(\omega, q) = \int dt dr R(t, r) e^{i\omega t - iqr} \).

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[14] This property is general for the models in question and has nothing to do with the presence or absence of \( \rho \) in \( \tilde{S}_R \).
[15] At this point we cite Ref. [10]. Reacting to our earlier critique [13], in the published version [9] JvD writes “average energy” instead of “energy”. Unfortunately this change in wording as well as additional explanations in App. C4 of Ref. [9] cannot help to repair the problem. In Sec. III we explicitly demonstrate the point in JvD’s analysis where energy-time uncertainty violation emerges due to uncontrolled approximations.
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