Do most planetary nebulae derive from binaries? I Population synthesis model of the galactic planetary nebula population produced by single stars and binaries

Maxwell Moe\textsuperscript{1,2} & Orsola De Marco\textsuperscript{3}

\textbf{ABSTRACT}

We present a population synthesis calculation to derive the total number of planetary nebulae (PN) in the Galaxy that descend from single stars and stars in binary systems. Using the most recent literature results on galactic and stellar formation as well as stellar evolution, we predict the total number of galactic PNe with radii $< 0.9$ pc to be $(4.6 \pm 1.3) \times 10^4$. We do not claim this to be the complete population, since there can be visible PNe with radii larger than this limit. However, by taking this limit, we make our predicted population inherently comparable to the observationally-based value of Peimbert, who determined $(7200 \pm 1800)$ PNe should reside in the Galaxy today. Our prediction is discrepant with the observations at the $2.9 \sigma$ level, a disagreement which we argue is meaningful in view of our specific treatment of the uncertainty. We conclude that it is likely that only a subset of the stars thought to be capable of making a visible PN, actually do. In the second paper in this series, an argument will be presented that the bulk of the galactic PN population might be better explained if only binaries produce PNe.

The predicted PN formation rate density from single stars and binaries is $(1.1 \pm 0.5) \times 10^{-12}$ PN yr$^{-1}$ pc$^{-3}$ in the local neighborhood. This number is lower than the most recent PN birthrate density estimates $(2.1 \times 10^{-12}$ PN yr$^{-1}$ pc$^{-3}$), which are based on local PN counts and the PN distance scale, but more in line with the white dwarf birthrate densities determined by Liebert et al. ($(1.0 \pm 0.25) \times 10^{-12}$ WD yr$^{-1}$ pc$^{-3}$). The predicted PN birthrate density will be revised down, if we assume that only binaries make PNe. This revision will imply that the PN distance scale has to be revised to larger values.

\textsuperscript{1}Department of Astrophysical and Planetary Sciences and CASA, University of Colorado, 389-UCB, Boulder, CO 80309

\textsuperscript{2}REU student at the Astrophysics Department of the American Museum of Natural History

\textsuperscript{3}Astrophysics Department, American Museum of Natural History, Central Park West at 79th Street, New York, NY 10024
1. Introduction

The origin of planetary nebulae (PNe) has been the subject of a hot debate for the last thirty years (for a list of references see Soker 1997). The bone of contention has been the mechanism by which the large majority of PNe achieve axi- and point-symmetric shapes. The proposed mechanisms can be divided into two classes. The first class of mechanisms involves single stars, proposing a combination of magnetic fields and/or stellar rotation to achieve an axi-symmetric mass-loss during the Asymptotic Giant Branch (AGB) phase (García-Segura et al. 2004; Matt et al. 2004; Blackman 2004) into which the fast central star wind expands. The second class of mechanisms proposes that PNe with non-spherical morphologies must have suffered an encounter with a binary companion (whether a close encounter such as a common envelope or a lesser interaction such as gravitational focusing). Depending on the binary separation, mass ratio and other parameters the PN ejecta take on different shapes.

A simple test of which of the two classes of models is more likely to be correct is to determine the PN central star binary fraction. About 10–15% of all PN are known to harbour close binary central stars (Bond 2000). These were discovered with photometric surveys that rely on heating of one hemisphere of the companion by the primary, or by ellipsoidal variations. Since heating effects and ellipsoidal variations quickly diminish with binary separation, only binaries with periods smaller than a few days can be detected. Surveys aimed at detecting visual binaries have been carried out (Ciardullo et al. 1999) and, once again, about 10% of all observed central stars appear to have a distant companion (separations >> 100 AU). The companions of the latter group are unlikely to have played any significant role in shaping the PN, although it is not excluded that these wide binaries might harbor a third companion orbiting closer to the primary.

It is clear that until surveys are carried out that can systematically detect binaries with periods between a few days and a few years, we cannot know the actual PN binary fraction. Radial velocity surveys are ideal to fill the period gap, but their execution is particularly tricky, since relatively long observing runs are needed on moderate aperture telescopes with intermediate-to-high resolution spectrographs. Mendez (1989) reported on the results of a radial velocity survey of 28 central stars carried out at intermediate resolution. However, he only carried out one or two measurements per star, severely limiting the discovery potential of his survey (he had no conclusive evidence to prove the binarity of any stars in his sample) and precluding the determination of periods. Sorensen & Pollacco (2004) carried out a new
survey of 33 central stars, at lower resolution, but where many measurements were taken for every star. They report about 40% radial velocity variability in their sample of 33 central stars, although no periods could be determined.

More recently, De Marco et al. (2004) and Afšar & Bond (2005) carried out radial velocity surveys with resolution comparable to the Mendez (1989) and Sorensen & Pollacco (2004) studies, respectively. Their samples (11 and 19 objects, respectively) provided tantalizing evidence that a much larger fraction, possibly as high as 90%, of central stars might have companions. Periods were not detected by these surveys, leaving the possibility that in some cases the radial velocity variability might be due to stellar wind variability. From additional observations (De Marco et al., in preparation, Bond 2005) at echelle resolutions periods between 4 and 5 days were determined for both stars observed, which where drawn from the sample of De Marco et al. (2004; IC4593 and BD+33 2642). This fuels the empirically-based suggestion that the PN central star close binary fraction might indeed be very high.

Radial velocity surveys of central stars of PN are, however, far from being complete so that an observational answer to the question of the exact PN central star binary fraction is not yet in hand. We have therefore carried out a population synthesis calculation to revisit the issue of how many PNe are predicted in the Galaxy today. The empirical suggestion that most-to-all PNe are in short-period binary systems, should be supported, or at least consistent, with current theories of stellar evolution, galactic history, star formation and knowledge of related stellar populations (such as AGB stars and white dwarfs [WDs]). This suggests that population synthesis calculations which assume that single stars do make PNe should predict an overabundance of PNe compared to observations. In this paper (the first in a series), we address single star evolution and the total PN population. We analyze all the sources of error and state the issues involved with PN and WD counts and birth rates. In the second paper of this series (De Marco et al., in preparation, hereafter Paper II), we will address the issue of binarity as the main evolutionary channel for the production of PNe.

In § 2 we outline the phases of our population synthesis calculation as if it were a back of the envelope calculation. In § 3 we give a detailed explanation of our model method, as well as of our assumptions concerning the initial mass function (IMF; § 3.1), stellar lifetimes (§ 3.2), the mass of the Galaxy (§ 3.3), the star formation history (SFH; § 3.4), the age-metallicity relation (AMR; § 3.5) and the PN visibility timescales (§ 3.7). In § 4, we present our results, we explain the uncertainties, and carry out a detailed comparison with observations and past results from the literature, setting the stage for the binary synthesis (Paper II). In § 5 we compare our PN population predictions with the observed populations of the galactic bulge and galactic globular cluster (GC) system. We conclude in § 6.
2. The back-of-the-envelope calculation

The genesis of this model is the back of the envelope calculation of De Marco & Moe (2005), which we outline here because it provides an overview of the process.

In order to predict the total number of PNe that live in the Galaxy today, De Marco & Moe (2005) started with the total number of stars in the Galaxy, determined by normalizing the total galactic luminous mass \(7.5 \times 10^{10} \, M_\odot\) (Dehnen & Binney 1998; Mera et al. 1998) to the average mass of a star \(0.54 \, M_\odot\), calculated using the IMF (Kroupa et al. 1993; Chabrier 2003a). This resulted in \(1.4 \times 10^{11}\) stars, where single stars count as one, and binary stars count as two. 60% of the observed G- and late F-type main sequence stars are found to be in binaries (Duquennoy & Mayor 1991). Assuming that as the general stellar binary fraction, the total number of stellar systems (where by stellar system we mean a single star or a binary systems, i.e., we are counting binaries as one system) is \(8.8 \times 10^{10}\). Of these, we considered only single stars and primary stars in binaries with masses large enough to have evolved off the main sequence in the age of the Galaxy. To do this, we considered the Galaxy as a coeval population with an age of 8.5 Gyr (Liu & Chaboyer 2000; Zoccali et al. 2003). The turn-off mass of such a population is \(1.05 \, M_\odot\) (based on the stellar lifetime of Portinari et al. (1998)). Using the IMF, we calculated that the total number of galactic stars between \(1.05 \, M_\odot\) and \(10 \, M_\odot\) (the upper mass limit above which a star explodes as a supernova and does not make a PN; Iben 1995), is 8.4% of all stars. This results in \(7.39 \times 10^9\) systems. The mean mass of the population between \(1.05\) and \(10 \, M_\odot\) is \(1.78 \, M_\odot\) and the lifetime associated with such a stellar mass is \(1.3 \times 10^9 \, \text{yr}\) (Bressan et al. 1993; Schaller et al. 1992). Finally, we adopted 20,000 yr as the mean PN visibility time. The number of PNe in the Galaxy was then estimated by multiplying the number of stars in the prescribed mass range, by the ratio of PN visibility time to mean stellar lifetime \(\frac{7.39 \times 10^9 \times 20,000}{1.3 \times 10^9} = 113,700\) PNe).

Even accounting for a substantial uncertainty, this number is much higher than even the most optimistic observational estimates of the total number of galactic PNe. The exact number of PN in the Galaxy is hard to establish due to the difficulty of observing along the galactic plane. Parker et al. (2003) have counted 2000-2500 galactic PNe in the most extensive survey to date, but this is bound to be a lower limit because of extinction on the galactic plane would obscure a fair number of distant PNe. In § 4 we will go into greater detail of the total number of galactic PNe determined via observationally-based methods. For now, suffice it to say that our preferred observationally-based galactic PN population size is \(7200 \pm 1800\) (Peimbert 1990, 1993).

The discrepancy between the theoretically-predicted and observationally-derived galactic PN population size, prompted us to refine our prediction of the number of galactic PNe,
by using a more sophisticated method. As we will see, even after refining the calculation, the total number of PN predicted in the Galaxy is still too high compared to observations and leads one to consider the sources of such a discrepancy.

3. The Population Synthesis Method

We have constructed a stellar population synthesis code to enable us to predict the number of PNe in the Galaxy today. Although the method is analogous to the back-of-the-envelope calculation described above, all assumptions have been refined, and the calculation is carried out using time bins, instead of averages.

The main refinements can be summarized as follows: we have divided the Galaxy into its four main components, namely the spheroid, the bulge, the thick and thin disks. (Note that we never use the word halo, to avoid confusion with the dark matter halo. We use instead the term spheroid to mean the luminous halo including the globular clusters [GCs]). For each of these four components we use different mean ages as well as different IMFs, star formation rates (SFR) and metallicities. We also adopt an AMR when determining metallicity-dependent stellar lifetimes and PN visibility times. We calculate the PN visibility time using nebular kinematics and stellar evolution arguments. We also account for the fact that some red giant branch (RGB) stars undergo common envelope interactions, which reduce their envelope mass below the critical value needed to ascend the AGB. These stars are unlikely to ascend the AGB and make a PN and are therefore not counted.

The SFH of the Galaxy is split into 649 time bins. The time variable, $t$, starts at $t_0 = 0$ Gyr. Time bin boundaries are then determined by the following equation:

$$t_{i+1} = (0.1 \text{ Gyr}) \times 0.9923607^i + \sum_{j=0}^{i} t_j$$

so that $t_{650} = 13$ Gyr (adopted as the age of the oldest galactic component - see below). In this way, the resolution near time 0 Gyr is 0.1 Gyr, but near $t = 13$ Gyr it is less than 1 Myr. This level of refinement avoids large rounding errors produced by the steep gradient of stellar lifetimes, since stars more massive than $2.0 M_\odot$ have evolved off the main sequence in the past giga-year, but stars more massive than $4.0 M_\odot$ have turned off only in the past 100 Myr.

Consider bin boundaries at $t_i$ and $t_{i+1}$. The corresponding turn off masses $m_i$ and $m_{i+1}$ at the bin boundaries are calculated by determining the masses of stars that have stellar
lifetimes of \((13 \text{ Gyr} - t_i)\) and \((13 \text{ Gyr} - t_{i+1})\), respectively (note that the stellar lifetimes are computed according to metallicity-dependent stellar evolutionary tracks and the AMR; § 3.5). The relative number of stars produced in this time interval is calculated from the SFR function and this value is multiplied by the fraction of stars between \(m_i\) and \(m_{i+1}\) determined from the IMF \(\phi(m); \ § 3.1\). This results in the total number of PNe being produced today whose progenitors formed during that time interval. The fraction of PNe currently detectable is given by the ratio of the PN visibility time to the width of the time bin \((t_i - t_{i+1})\). Summing all of the bins’ contributions from 0 to 13 Gyr and all components of the Galaxy, gives the total number of PNe currently detectable in the Galaxy.

Below we give details of each phase of the calculation, including an accounting of the uncertainty.

### 3.1. The Initial Mass Function

The IMF represents the mass distribution of stars at the time of their formation. We use the IMF to determine the fraction of stars in a population that are massive enough to evolve off the main sequence during the age of that population but less massive than 8.0 \(M_\odot\) (in § 2 we used 10 \(M_\odot\) as an upper limit, but the exact value is not very important as there are relatively few stars with high mass), an upper mass limit above which stars tend to become supernovae rather than making PNe (Iben 1995). We will also use the IMF to scale the total luminous mass of the Milky Way and determine the total number of galactic stars.

Salpeter (1955) first proposed that the IMF consists of a single power law over most of the stellar mass range \((0.4M_\odot < m < 10M_\odot)\):

\[
\xi(m) = k m^{-x},
\]

where \(\xi(m)dm\) is the mass of stars in the interval \(m\) to \(m + dm\), \(x\) is the power law index (determined by Salpeter (1955) to be 1.35) and \(k\) is a normalization constant. Other authors (e.g., Kroupa et al. 1991, 1993) represent the IMF as a distribution by number instead of by mass:

\[
\phi(m) = k' m^{-\alpha},
\]

where the total number of stars in the interval \(m\) to \(m + dm\) is \(\phi(m)dm\), \(k'\) is a different constant, and \(\alpha = x + 1\), since:
\[
\int_{m_{\text{low}}}^{m_{\text{up}}} m \phi(m) \, dm = \int_{m_{\text{low}}}^{m_{\text{up}}} \xi(m) \, dm, \quad (4)
\]

where \( m_{\text{low}} \) and \( m_{\text{up}} \) are the lower and upper mass limits under consideration.

Unresolved binary stars were taken into account by Kroupa et al. (1991, 1993), who developed a three-segment power-law IMF so that random, uncorrelated pairings of stars produced the observed (e.g., Duquennoy & Mayor 1991; Fischer & Marcy 1992; Eggleton et al. 1989 and Tout 1991) frequencies and mass-ratio distributions of binaries. Kroupa (2001) showed that for the average-age galactic disk, the IMF exponent changes:

\[
\begin{align*}
\alpha_0 &= 0.3, \quad 0.01 \leq m < 0.08, \\
\alpha_1 &= 1.3, \quad 0.08 \leq m < 0.5, \\
\alpha_2 &= 2.3, \quad 0.5 \leq m.
\end{align*}
\quad (5)
\]

The decrease in the slope and eventual turnover in the sub-stellar mass regime has led some authors to adopt a log-normal mathematical representation of the IMF of stellar systems (i.e., taking into account binaries; Chabrier 2003a,b):

\[
\xi(m) = k \begin{cases} 
A \exp \left[ -\frac{(\log m - \log m_c)^2}{2\sigma^2} \right] & m \leq m_{\text{norm}}, \\
B m^{-1.3} & m > m_{\text{norm}},
\end{cases}
\quad (6)
\]

where the \( A \) and \( B \) coefficients are used to ensure continuity at the segment transitions. For the average age disk, \( m_{\text{norm}} = 1.0, m_c = 0.22 \) and \( \sigma = 0.57 \), while for the spheroid \( m_{\text{norm}} = 0.7, m_c = 0.22, \) and \( \sigma = 0.33 \).

In addition, evidence is accumulating that the IMF may be time dependent, with earlier epochs favoring a higher abundance of more massive stars (Kroupa 2001; Chabrier 2003a; Lucatello et al. 2005), so that \( \alpha \) is 0.5 higher between 0.08 \( M_\odot \) and 1.0 \( M_\odot \) for the present day IMF. In this work, we use the \( \alpha \) values from Eqs. 5, without accounting for the time dependency.

Since there is still much controversy over the IMF of the lower mass regime, as well as whether the IMF is time and environment dependent, we will consider three IMFs: the average-age disk IMF for the entire Galaxy of Kroupa (2001), that of Chabrier (2003b) and, finally, the spheroid IMF of Chabrier (2003a,b) which we will apply to the spheroid, bulge and thick disk. These three IMFs are plotted in Fig. 1, both by mass, \( \xi(m) \), and by number, \( \phi(m) \). A comparison of the PN population when using either of the two disk IMFs
will allow us to assess the error incurred because of uncertainties in the IMF prescription, while comparing the results using either a disk IMF for all galactic components or using the spheroid IMF of Chabrier (2003a,b) when dealing with star formation in older components will allow us to evaluate the uncertainty incurred because of actual changes in the IMF as a function of time and environment.

For our analysis, we will use the IMF between 0.08 $M_\odot$ and 120 $M_\odot$ to determine the total number of stars from the luminous matter in the Galaxy. The lower limit was set to a stellar value, since the galactic mass is determined from fitting the luminosity of main sequence stars and their giant descendants (see § 3.3). We can then integrate the three IMFs in Fig. 1 to obtain the median main sequence mass for galactic stars. This median mass will be different if we consider the IMF expressed by mass or number (upper and lower panels, respectively, in Fig. 1). The median mass of a star when considering the IMF by mass is 1.30 $M_\odot$, 1.93 $M_\odot$, and 1.01 $M_\odot$, for the three IMFs of Kroupa (2001) (disk), Chabrier (2003b) (disk), and Chabrier (2003a) (spheroid). The median mass of a star when considering the IMF by number is 0.2 $M_\odot$, 0.32 $M_\odot$, and 0.26 $M_\odot$, for the three IMFs, respectively. This means that half of the mass of a coeval population of main sequence stars is locked up in stars with masses greater than about 1.3 $M_\odot$ (taking the Kroupa (2001) as an example), but the typical mass of a star in that population would be about 0.24 $M_\odot$.

### 3.2. Stellar Lifetimes

The next step in determining the number of PNe in the Galaxy, is to determine how many galactic stars have had enough time to evolve to the PN phase. If the Galaxy were a coeval population, only stars more massive than the main sequence turn-off mass of the population would be available to evolve to the PN phase. Since the Galaxy is not coeval, we have to consider its SFH (§ 3.4). Before we do so, we want to determine the lifetime corresponding to each mass bin, since that, together with the age of each star (determined according to the SFH), will determine whether that star has evolved off the main sequence and into a PN.

Stellar lifetimes depend on stellar masses, but also have a weak dependence on the metallicity $Z$. We compare the lifetimes calculated from the stellar models of Schaller et al. (1992, $Z=0.02$ & $Z=0.001$) and of the Padova group: Alongi et al. (1993, $Z=0.008$), Bressan et al. (1993, $Z=0.02$), Fagotto et al. (1994a, $Z=0.0004$ & $Z=0.05$), Fagotto et al. (1994b, $Z=0.004$ & $Z=0.008$), Fagotto et al. (1994c, $Z=0.1$), and Girardi et al. (1996, $Z=0.0001$). These models considered the most up to date radiative opacities at that time (Rogers & Iglesias 1992; Iglesias et al. 1992), yielding higher accuracy. A summary of the results from
the Padova group can be found in the paper by Portinari et al. (1998). In what follows, we define the stellar lifetime, \( \tau_*(m, Z) \), to be the sum of a star’s hydrogen and core helium burning phases.

We determined a smooth function for the stellar age as a function of initial mass and metallicity by cubic spline interpolation in \( \log(m) \) and \( \log(Z/Z_\odot) \), respectively. Four isometallic functions obtained in this way are plotted\(^1\) in Fig. 2. Looking at Fig. 2, we note that the stars with the longest lifetimes tend to be those with solar metallicity. At metallicities lower than solar, the lifetimes increase with increasing metallicity, due to higher opacities, while the lifetimes of stars with metallicities greater than the Sun are limited by their supply of hydrogen fuel. Thus, even though the average metallicity of the Galaxy may be near solar metallicity, the average lifetimes of stars in the Galaxy are less than the solar lifetimes. If a star has a metallicity \( \log(Z/Z_\odot) < -2.2 \) or \( \log(Z/Z_\odot) > 0.8 \), we compute its lifetime as if it had the boundary metallicity.

For our analysis, we assume the galactic spheroid began forming 13 Gyr ago (see §3.4.1), and thus we set our time variable \( t = 13.0 \) Gyr - \( \tau_* \). The turnoff mass for a 13 Gyr population ranges from 0.74 to 0.94 \( M_\odot \), depending on metallicity.

The fraction of stars between 0.74 \( M_\odot \) and 8.0 \( M_\odot \) is 13.7%, 21.3% and 11.6%, for the Kroupa (2001), Chabrier (2003b) and Chabrier (2003a) IMFs, respectively, where we use the IMF by number, \( \phi(m) \), while between 0.94 \( M_\odot \) and 8.0 \( M_\odot \), it is 9.9%, 15.7% and 8.3%, respectively. We therefore see that if we do not consider metallicity in determining stellar lifetime, we would incur a substantial error in our determination of the number of stars available to produce today’s PN population.

The mass fraction that remains luminous (i.e., from stars that are still burning hydrogen or helium), for a population of stars that formed at time \( t \), is quantified by:

\[
\delta(t, \xi(m), \tau_*(Z)) = \frac{\int_{m_t(\tau_*(Z))}^{m_8(\tau_*(Z))} \xi(m) \, dm}{\int_{0.08}^{1.25} \xi(m) \, dm},
\]

where \( m_t(\tau_*(Z)) \) is the mass of a star with lifetime \( \tau_* = 13 - t \), i.e. it is the turnoff mass for the population. We call \( \delta \) the stellar evolution correction factor and we plot it in Fig. 3 as a function of time for three IMFs and two different metallicities. Note that if a population of stars was born as little as 1 Gyr ago at \( t = 12 \) Gyr, about 40% of that population’s mass has

\(^1\)We represent metallicity as the logarithmic fraction of the solar value, \( \log(Z/Z_\odot) \), where \( Z_\odot = 0.015 \) (Asplund et al. 2005; Basu & Antia 2004; Bahcall & Serenelli 2005).
been converted to stellar remnants. Also, only 30-50% of the original mass of a population of stars that formed at the beginning of the Galaxy remains luminous today.

3.3. The Mass of the Milky Way

In order to determine the number of galactic stars that are available to produce the PN population (i.e., those between $\sim 0.8 \, M_\odot$ and $8.0 \, M_\odot$), we use estimates of the luminous matter in the Milky Way and use the IMF to determine the number of objects. In what follows, we treat the bulge, thin disk, thick disk and spheroid separately.

3.3.1. The mass of the bulge

Kent (1992) was among the first to calculate an accurate model for the bulge, deducing that it has an axisymmetric shape. From the resulting density and luminosity distributions, the bulge was estimated to have a mass of $1.8 \times 10^{10} M_\odot$. Other models incorporating gas kinematics (Binney et al. 1991), metallicity gradients (Zhao et al. 1994), and micro-lensing events (Paczynski 1991; Zhao et al. 1995; Han & Gould 1995), indicate the bulge to have a triaxial symmetry. Luminosity fits and dynamical models of a triaxial bulge have yielded masses ranging from $1.3 \times 10^{10} M_\odot$ to $2.8 \times 10^{10} M_\odot$ (Dwek et al. 1995; Blum 1995; Zhao et al. 1996; Fux 1997, 1999; Weiner & Sellwood 1999; Sevenster et al. 1999), depending on the inclination of the major axis to the line of sight.

Recent deep near-infrared surveys have fixed the bulge orientation at $10^\circ$ - $20^\circ$ (López-Corredoira et al. 1999, 2000, 2005; Picaud & Robin 2004). With these new constraints, the mass of the outer bulge ($r > \approx 150$ pc) was calculated to be $(2.3 \pm 0.4) \times 10^{10} M_\odot$, with about a quarter of this mass contributed by stellar remnants (Robin et al. 2003; Picaud & Robin 2004, and references therein). Thus, for the present day luminous mass of the bulge we adopt $(2.0 \pm 0.4) \times 10^{10} M_\odot$. This figure accounts for the luminous mass ($1.7 \times 10^{10} M_\odot=0.75 \times 2.3 \times 10^{10} M_\odot$) in the outer bulge, plus an additional $0.3 \times 10^{10} M_\odot$ contained in the high density regions of the inner bulge (Lindqvist et al. 1992).

3.3.2. The mass of the disk

The mass of the disk is difficult to determine since it depends heavily on the observed stellar surface densities, scale height, scale length, as well as the solar position within the Galaxy. Gilmore & Reid (1983) discovered that the stellar population 1-2 kpc above and
below the galactic plane is different from that closer to the mid-plane. These thick disk stars are older, have generally lower metallicities and higher velocity dispersion than the stars of the thin disk. The thick disk has been found to account for \( \sim 10\% \) of the total luminous stellar content of the entire disk (Cabrera-Lavers et al. 2005). From models of the chemical evolution of the entire disk based on observed abundances, its mass has been estimated to be in the range \( 4 - 5 \times 10^{10} M_\odot \), with a gas fraction between \( \sim 10 \) and \( \sim 18\% \) (Prantzos & Aubert 1995; Boissier & Prantzos 1999). On the other hand, models based primarily on the stellar surface densities, scale length and scale height, find the stellar mass to be in the range \( 2.6 - 3.5 \times 10^{10} M_\odot \), with a slightly higher gas fraction, between 16 and 25\% (Dehnen & Binney 1998; Naab & Ostriker 2006; Robin et al. 2003). We therefore adopt the total luminous mass of the disk to be \( (4.0 \pm 0.7) \times 10^{10} M_\odot \), where the stellar mass of the thin disk accounts for \( (3.6 \pm 0.7) \times 10^{10} M_\odot \), while that of the thick disk makes up the remaining \( 4 \times 10^9 M_\odot \).

The value of \( 4.0 \times 10^{10} M_\odot \) was chosen as the weighted average of both the higher mass disk \( (4 - 5 \times 10^{10} M_\odot \), determined via the chemical evolution model) and the lower mass disk \( (2.6 - 3.5 \times 10^{10} M_\odot \), determined via the stellar surface density), based on the uncertainties provided by the cited papers. The lowest mass estimate of \( 2.6 \times 10^{10} M_\odot \) by Dehnen & Binney (1998) included a 25\% gas fraction, which is higher than what appears to be the more general consensus (15\%). We have therefore raised this mass estimate by a factor \( 0.85/0.75 \), to \( 3.0 \times 10^{10} M_\odot \), to account for the 15\% gas fraction that most authors argue in favor of.

### 3.3.3. The mass of the spheroid

The spheroid is the oldest population of the Milky Way and resides in a spherical region extending 10–20 kpc out from the galactic center (Reid 1996). Counting only the main sequence stars, the mass of the spheroid stars has been estimated to be in the range \( 0.3 - 3 \times 10^9 M_\odot \), based on assumptions about the local densities of such objects as well as relative counts compared to other populations (Edmunds & Phillipps 1984; Chabrier & Mera 1997; Gould et al. 1998; Robin et al. 2003). We assume the spheroid to have \( 2 \times 10^9 M_\odot \), based on the smaller errors of the more recent estimates. The exact number is, however, not of fundamental importance since the contribution of PNe from the spheroid is relatively small.
3.3.4. Errors

In order to estimate the uncertainty on the total PN population derived from determining the galactic mass we consider three distinct mass values: a low, medium and high mass Galaxy. For the three sets of models we adopt the values summarized in Table 1.

It would be more correct to use nine, instead of three, galactic mass models, since each of the three choices for the thin disk mass could be combined with each of the three values for the bulge mass. This method would give more weight to the intermediate values, rather than altering our final results. Since the total number of final models is high and since most PNe derive from the thin disc, it was decided that the slight difference in error weighting produced by the use of 9 (rather than 3) galactic mass models is not paramount.

3.4. The Star Formation History

In the back-of-the-envelope model (§ 2), we assumed that all galactic stars formed 8.5 Gyr ago. Using the IMF and the stellar lifetimes as a function of mass we determined the turn-off mass of an 8.5-Gyr old population to be 1.05 \( \text{M}_\odot \). We then determined the number of stars in that population that have left the main sequence and proceeded to the PN phase. Since the Galaxy is not a coeval population, we need to address its star formation history and use it to determine a progression of stellar ages, instead of using an approximate mean value.

3.4.1. The star formation history of the spheroid and thick disk

Stars in the galactic halo and the spheroid GCs (which we cumulatively refer to as the spheroid) began forming about 13 Gyr ago. Star formation lasted there only 4-5 Gyr, so that the spheroid has a mean age of 11.5 Gyr (Harris 1996; Liu & Chaboyer 2000; Bekki & Chiba 2001; Gratton et al. 2003; De Angeli et al. 2005). The thick disk is slightly younger with star formation beginning 1-2 Gyr after the initial formation of the spheroid and lasting for about 5 Gyr. The thick disk has a mean age of 10 Gyr (Nissen & Schuster 1991; Marquez & Schuster 1994; Fullton et al. 1996; Bensby et al. 2004).
3.4.2. The star formation history of the bulge and thin disk

The ages of the bulge and thin disk populations are more difficult to estimate, since star formation has continued there for most of the galactic history. Originally, it was believed that the bulge consisted purely of old stars (ages $> 10$ Gyr), but new evidence points to the existence of a bulge population of intermediate mass stars with ages between 5 and 10 Gyr (Holtzman et al. 1993), as well as a substantial fraction of stars near the galactic center that have formed within the past 5 Gyr (Krabbe et al. 1995; Blum et al. 1996a,b; Narayanan et al. 1996). These findings should not however tempt us to adopt a low mean age for the bulge. Ortolani et al. (1995) estimates that the intermediate mass population contributes no more than 10% to the total and that the young stars are confined to the inner $\sim 200$ pc of the bulge (Frogel 1999), where the older population still dominates by number (Rich 1999, 2001). Feltzing & Gilmore (2000) and Zoccali et al. (2003) also advocate an old age for the bulge of 10-11 Gyr, arguing that younger age estimates (9 Gyr; Gerhard & Binney 1993; Wyse et al. 1997; Sevenster 1999) are due to contamination of the bulge population by foreground stars. Since the issue remains unresolved, we adopt three distinct values for the average age of the bulge: 9, 10 and 11 Gyr.

The difficulty in measuring the average age of the thin disk arises from the fact that the star formation rate (SFR) varies with radial distance from the center. Naab & Ostriker (2006) showed that the average age of a star in the thin disk is 5.4 Gyr, while the average age of a star in the solar neighborhood is 4.0 Gyr. Observations of the scale height and age distribution of stars throughout the Galaxy as well as dynamical models of the infall of gas onto the disk have constrained the average age of the thin disk to the range 4-7 Gyr (Twarog 1980; Haywood et al. 1997a,b; Rocha-Pinto et al. 2000; Liu & Chaboyer 2000; Chang et al. 2002; Naab & Ostriker 2006). We therefore adopt three values for the average age of the thin disk: 4.5, 5.5 and 6.5 Gyr, which is the weighted mean and uncertainty of the cited papers.

3.4.3. Star Formation History Modeling

Some authors have argued for epochs of enhanced star formation in the thin disk (Rocha-Pinto et al. 2000; de la Fuente Marcos & de la Fuente Marcos 2004). The errors in determining the age of the disk, however, are larger than the peaks and troughs in the SFH. We therefore approximate these SFHs by a smooth function. The two most common forms of a smooth SFH of the entire Galaxy are the exponential form (MacArthur et al. 2004):

$$\Psi(t) = k \exp \left[ -\frac{(t - t_0)}{\tau} \right]$$  \hspace{1cm} (8)
and the equation used by Sandage (1986):

$$\Psi(t) = k(t - t_0) \exp\left[\frac{-(t - t_0)^2}{2\tau^2}\right],$$  \hspace{1cm} (9)

where \(t_0\) (only in our adaptation of the equation) is the time when star formation started in a given population. The factor \(\tau\) determines the shape of the distribution of stellar ages. It has been accepted that the bulge began to form shortly after the initial spheroid collapse (Terndrup 1988; Ortolani et al. 1996; Idiart et al. 1996).

Since the spheroid is the oldest galactic component, starting to form 13 Gyr ago at \(t = 0\) and the thick disk started forming at \(t = 1.0\) Gyr, we will adopt \(t_0 = 0.5\) Gyr for the bulge. The thin disk did not begin to form until \(\sim 10\) Gyr ago (Liu & Chaboyer 2000; Sandage et al. 2003; del Peloso et al. 2005). Thus we adopt \(t_0 = 3.0\) Gyr for the thin disk. We also set \(t_{end}\) to be the time where star formation ends. Its value is 4.5 Gyr for the spheroid, 6 Gyr for the thick disc, and 13 Gyr for both the thin disc and bulge (meaning that star formation is ongoing in those two galactic components).

In Eqn. (8), \(\tau\) corresponds to the time \(t - t_0\) when an average-aged star was formed, if \(\tau\) is positive and star formation is allowed to continue indefinitely into the future (i.e., star formation does not stop at \(t_{end}\)). In Eqn. (9), \(\tau\) is the time \(t - t_0\) when the SFR peaks. The values of \(\tau\) for the 16 cases (two SFR representations for all components combined with the three average ages of the thin disc and bulge) are listed in Table 2 The values of \(t_0\), the assumed mean ages, and \(t_{end}\) of the four galactic components are also listed in the table.

The two equations each better represent a different aspect of the SFR in the Galaxy. Eq. 8 is strictly a decreasing or increasing SFR depending on the sign of \(\tau\). It is rather unphysical since the SFR should not switch on instantly. Eqn. (9) correctly addresses the initial rapid increase of stellar formation and then a decline after a certain peak. However, the decline of the SFR dictated by Eq. 9 is too rapid and does not account for the small but non-negligible population of young and intermediate age stars observed in the bulge.

Given the above considerations, we consider six SFH functions each for the bulge and thin disc and two SFH functions each for the spheroid and thick disc (i.e., we model the SFH using the two equations and the three average mean age values listed above). These models are plotted in Fig. 4 with all components normalized to unity. Notice how the SFHs of the thin disk according to Eqn. (8) can be either decreasing, uniform, or increasing due to the uncertainty in determining the average age of the thin disk.
3.5. Metallicities

Accounting for the metallicity distribution of the Galaxy, $\beta(z)$, and how it has changed over time, $\zeta(t)$, will reduce the uncertainty in the calculated stellar lifetimes (§ 3.2) and PN visibility times (§ 3.7). In Fig. 5, we display the adopted metallicity distributions expressed as a function of $Z$:

$$\beta(Z) = \frac{dN}{d(\log(Z/Z_\odot))}.$$  \hspace{1cm} (10)

Metallicity distributions, expressed in terms of $[\text{Fe/H}] = \log(\text{Fe}/\text{H}) - \log(\text{Fe}/\text{H})_\odot$, are converted to $Z$ using the following formulae (Nissen & Schuster 1991, and references therein):

$$\log(Z/Z_\odot) = \begin{cases} 
0.60[\text{Fe/H}] & \text{for } [\text{Fe/H}] \geq -1.0 \\
0.4 + [\text{Fe/H}] & \text{for } [\text{Fe/H}] < -1.0
\end{cases}.$$  \hspace{1cm} (11)

The stellar population of the spheroid has the lowest metallicities in the Galaxy ($-2.0 < [\text{Fe/H}] < -1.5$; Ryan & Norris 1991; Marquez & Schuster 1994; Carney et al. 1996; Beers et al. 2005). In this work, we use the prescription of Carney et al. (1996, their solid line in Fig. 16), who already express their distribution function in terms of $Z$ instead of $[\text{Fe/H}]$. The metallicity of the bulge peaks at $[\text{Fe/H}] \sim 0.2$ dex, (McWilliam & Rich 1994; Minniti et al. 1995; Sadler et al. 1996; Rich 1998; Ramírez et al. 2000; Feltzing & Gilmore 2000; Zoccali et al. 2003). We use the continuous $[\text{Fe/H}]$ distribution obtained by Minniti et al. (1995, their Fig. 4) and apply Eqn. (11) to convert to a $Z$ distribution. The metallicity of the thick disk ranges between $-0.6 < [\text{Fe/H}] < -0.4$, with a dispersion of $0.2$–$0.4$ dex (Nissen & Schuster 1991; Soubiran et al. 2003; Bartašiūtė et al. 2003; Schuster et al. 2005). We model the $[\text{Fe/H}]$ distribution of the thick disk as a Gaussian with a mean value of $-0.5$ and $\sigma = 0.3$ dex. The thin disk has the highest metallicities peaking near solar metallicity with a small dispersion of only $\sim 0.25$ dex (Bartašiūtė et al. 2003; Soubiran et al. 2003; Pont & Eyer 2004; Nordström et al. 2004; Haywood 2005). We use the distributions of Nordström et al. (2004, their Fig. 9), who express metallicity in terms of $Z$. Also, the metallicity distributions for the four galactic components, have been derived mainly from G dwarfs, i.e., for main sequence stars; therefore, the distribution is not affected by the stellar evolution correction factor and we do not have to apply $\delta(\phi)$ (Eq. 7).
3.5.1. The Age-Metallicity Relation

We have assumed that there is an AMR in all four components of the Galaxy, represented by a geometric equation. The average metallicity of a star formed at time $t$ is:

$$\langle \zeta \rangle(t) = \log\left(\frac{Z}{Z_{\odot}}\right) = a(t - b)^c + d,$$

with fitting parameters $a$, $b$, $c$ and $d$. Several authors (Carraro et al. 1998; Ibukiyama & Arimoto 2002; Bensby et al. 2004; Ibukiyama 2004; Haywood 2005) however, have found scattering in the AMR, with $\sim 90\%$ of stars formed at a given time scatter over about 1 dex interval in metallicity. We will therefore assume that the metallicity distribution of stars formed at any given time $t$ follows a Gaussian function:

$$\zeta(t, \log(\frac{Z}{Z_{\odot}})) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(\langle \zeta \rangle(t) - \log(\frac{Z}{Z_{\odot}}))^2}{2\sigma^2}\right],$$

where $\sigma$, the width of the AMR at any given time $t$, is chosen to be 0.25 dex for the bulge and spheroid and 0.15 dex for the thin and thick disks, since $\Delta \log(\frac{Z}{Z_{\odot}}) = 0.6\Delta[\text{Fe/H}]$ for these higher metallicity components.

The fitting parameters are determined by minimizing the equation:

$$\chi^2 = \sum (\beta_i - \sum \Psi(t_j) \times \zeta(t_j, \log(\frac{Z}{Z_{\odot}})_i))^2,$$

where $i$ is in steps of 0.1 dex from $\log(\frac{Z}{Z_{\odot}}) = -4.0$ to 2.0 dex, $j$ is in steps of 0.1 Gyr from 0 to 13 Gyr, and the SFR, $\Psi(t)$, is normalized such that $\int_0^{13 \text{ Gyr}} \Psi(t)dt = 1$.

In Fig. 6 we display the AMR. The 16 panels depict the AMR for the four galactic components, where for the thin disk and bulge two different SFR equations (Eqs. 8 and 9) have been used each with three mean ages (see Table 2). The greyscale indicates the relative distributions of metallicities for a given epoch, but does not display the variability of the stellar formation rate as a function of time.

Finally, we can write the equation to determine the scaling constant, $k$, used in the SFR Eqs. (8-9):

$$k = \frac{M}{\int_0^{13 \text{ Gyr}} \Psi(t)\delta(t, \xi, \tau_s)dt},$$

where $M$ is the mass of the Galaxy.
where $M$ is the total current “luminous” matter (§ 3.2) of the galactic component under consideration (determined in Sec. 3.3) and $\tau_*$ is the stellar lifetime weighted by the AMR.

### 3.6. Stars that do not ascend the AGB due to early binary interactions

For the present calculation, we assume that single stars and primary stars in binaries fitting the criteria outlined in § 3.1 to 3.5, are candidates to make PNe. We implicitly assume that all of these systems ascend the AGB and that only post-AGB stars can make a PN. The latter assumption is justified on the grounds that almost all of the PN central stars analyzed spectroscopically are shown to be post-AGB stars (Mendez 1989; Napiwotzki 1999).

Because of these considerations, we want to exclude from our PN progenitor sample those stars that do not ascend the AGB. There are three reasons why a star might not go through the AGB phase. (i) If its helium core mass remains smaller than 0.47 M$_\odot$ the star will not ignite core helium and will instead become a He-WD right after the RGB phase (Jimenez & MacDonald 1996). (ii) Stars that do ignite helium in the core and develop a CO core on the horizontal branch, might never ascend the AGB if their post-RGB envelope mass is too small (Dorman et al. 1993). It is difficult to estimate exactly how many stars meet this criterion, but Heber (1986) estimates that the early horizontal branch channel (the stars with low envelope masses) contribute only 2% to the total number of WDs. (iii) Finally, post-RGB stars might be prevented from ascending the AGB if they suffered a common envelope with a companion on the RGB, becoming horizontal branch short period binaries. These binaries consist of very blue horizontal branch stars (called subdwarf-B stars (Morales-Rueda et al. 2003)) with low-mass main sequence companions. Their very blue colors are due to particularly small envelope masses. If so, just as is the case for stars in point (ii) above, the primaries will not ascend the AGB.

The first group is already excluded from our counting since we impose a lower mass limit below which post-AGB stars do not make PNe (because their transition time between AGB and PN phase is too long - see § 3.7). In addition, the 0.47 M$_\odot$ helium core mass cutoff, corresponds to a progenitor mass of $\sim$0.75 M$_\odot$ (Jimenez & MacDonald 1996), which is similar to the turnoff mass of the oldest galactic components; this is like saying that the Galaxy is on the whole too young for a sizable fraction of its stars to follow this evolutionary channel.

The number of stars in the second group is very hard to estimate. It could be obtained theoretically by integrating the mass-loss rates from the start to the end of the RGB to derive the actual distribution of envelope masses for post-RGB stars and then use the Dorman
et al. (1993) models to determine how many of these stars do not ascend the AGB. We have however decided against taking this route because of the many arbitrary choices to be made along the path, because only a few of the Dorman et al. (1993) tracks completely miss the AGB, and also because Heber (1986) argue that the size of the group is small.

We have however, taken into account the members of the third group. We estimated the number of galactic systems that undergo a common envelope interaction on the RGB to be $\sim 10\%$ of all systems (but see Paper II for details). This number was determined from considering that 57\% of systems have a stellar companion, 25\% of which have separations $a < 600 \, R_\odot$, 68\% of which have mass ratios larger than 0.3 (Duquennoy & Mayor 1991). The separation and mass ratio requirements are due to the RGB radial expansion and tidal capture mechanisms (Soker 1996) and the minimum mass companion that will emerge from a common envelope on the RGB (Terman & Taam 1996; Sandquist et al. 1998; Yorke et al. 1995; De Marco et al. 2003).

Although these estimates are rather approximate, we feel reassured that the number of stars that do have time to evolve off the main sequence but do not ascend the AGB is small. In our calculation we reduce the number of PN progenitors by 10\% to account for this eventuality. More on this topic will be included in Paper II.

### 3.7. The PN visibility time

The number of PNe in a population depends directly on the time during which a PN is visible. In first approximation, we could use a mean PN visibility time for all PNe. Using a mean value is, however, problematic because the PN visibility is not only a function of its kinematic properties, but also of the temperature and luminosity of its central star. We therefore calculate the PN visibility time taking into account the time central stars of different masses take to heat up to ionization temperatures, as well as the time they take to fade enough that the PN cannot remain ionized. To this we add an additional criterion that no PN can remain detectable for more than 35,000 yr after its central star leaves the AGB, since its gas has by then become too dispersed.

This upper limit of 35,000 years is possibly the single most complicated issue in our counting. Changing this number directly alters the predicted galactic PN population. We stress however that the choice of 35,000 yr, while arbitrary in some respect, was made to be able to compare the predicted galactic PN population with specific observational estimates (see below). As a result, while both our prediction and the observationally-based estimates might be subject to large uncertainties, the comparison between the two has a much smaller
uncertainty, and in fact could be considered independent of the PN visibility time that is the source of so much debate.

We will only compare our predictions to observational estimates of the galactic PN population that are limited by a maximum radius of detectability of \( \sim 0.9 \) pc (see § 4.2.1). For PNe with radii larger than this threshold the surface brightness is so low that the count becomes incomplete. The PN visibility time corresponding to a radius of 0.9 pc is 35,000 yr, because the average expansion velocity of PNe is 25 km s\(^{-1}\) (Phillips 1989). Hence, our choice of this value as the maximum kinetic age of a PN.

Central star luminosities have been determined by studying Large and Small Magellanic Cloud (LMC and SMC, respectively) PNe. Values as low as \( \log(L/L_\odot) = 0.9 \) are derived, although most of them are brighter than \( \log(L/L_\odot) = 1.5 \) (Dopita et al. 1997; Jacoby & Kaler 1993). We therefore adopt \( \log(L/L_\odot) = 1.5 \), as the central star luminosity limit, below which the PN cannot remain ionized. For example, a central star descending from a 5.0-\( M_\odot \) progenitor, with quarter-solar metallicity (Z=0.004), heats and fades below \( \log(L/L_\odot) = 1.5 \) in \( \sim 25,000 \) yr (Vassiliadis & Wood 1994). Its PN visibility time would therefore be limited to \( \sim 25,000 \) yr: when the star fades, the PN, still with relatively high electron densities, will recombine quickly and disappear. It was noted by the referee that the lower luminosity limit of LMC PNe could be due to a limiting magnitude problem in the relatively far away galaxy. We note, however, that because of the relatively low maximum PN visibility time value adopted above (35,000 yr), very few central stars in our sample actually fade below \( \log(L/L_\odot) = 1.5 \). As a result, this limit has almost no effect on the predicted total galactic PN number.

Our second criterion is that no stars with an effective temperature lower than 25,000 K (\( \log(T_{\text{eff}}) < 4.4 \)) will ionize their PNe. For example, a central star from an 0.8-\( M_\odot \) progenitor takes a long 10\(^5\) yr to heat to \( \log(T_{\text{eff}}) = 4.4 \) (Schoenberner 1983) and by the time the star is hot enough, the circumstellar gas that would become the visible PN is likely to have dispersed enough that the PN will never be detected. From this criterion it follows that there is a cut-off central star mass, below which a post-AGB star cannot produce a PN. This cut-off value adds a source of uncertainty to the total number of PNe. The only evolutionary calculation for main sequence masses smaller than 1.0 \( M_\odot \) are those of Schoenberner (1983). From these we determined a cutoff central star mass for PN production of 0.90 \( M_\odot \). However, since the exact choice of this number influences the total number of PNe derived (but see § 4.1), we also carried out the calculation with two additional values: 0.85 and 0.95 \( M_\odot \). In conclusion, the PN visibility time is the time taken by a central star to evolve between \( \log(T_{\text{eff}}) = 4.4 \) and \( \log(L/L_\odot) = 1.5 \), with an upper limit of 35,000 yr, minus the time the central star takes to reach \( \log(T_{\text{eff}}) = 4.4 \).
Post-AGB evolutionary times are also metallicity dependent. Vassiliadis & Wood (1994) show that a 5.0-$M_\odot$ solar metallicity progenitor takes 70,000 years to fade below $\log(L/L_\odot) = 1.5$, nearly three times longer than a central star from a quarter solar progenitor of the same mass. The PN of the former star will therefore be visible for longer than the PN of the latter. We therefore will run two sets of models. In the first, we use the evolutionary timescales of the metallicity-independent H-burning central star tracks of Bloecker (1995) and Schoenberner (1983), while in the second, we use the metallicity-dependent timescales of the H-burning models of Vassiliadis & Wood (1994).

In Fig. 7 we show a greyscale of the PN visibility time as a function of main sequence star mass and metallicity for the Vassiliadis & Wood (1994) tracks and as a function of mass only for the Bloecker (1995) and Schoenberner (1983) tracks. These have been calculated using a linear interpolation with respect to mass and metallicity, respectively. PN lifetimes of progenitors with masses not included within the range of the evolutionary models, are given PN visibility times corresponding to the boundary values. It is clear from this figure that the Bloecker (1995) tracks predict longer visibility times for higher mass progenitors than do the Vassiliadis & Wood (1994) models. This is mostly due to the different initial-to-final mass relation derived by the two models, where the Vassiliadis & Wood (1994) tracks predict larger PN central stars masses than the Bloecker (1995) curves, in particular for higher mass progenitors. These more massive nuclei evolve faster and fade faster, so that the PN visibility times are relatively shorter. Fig. 4 of Bloecker (1995) shows that their tracks predict an initial-to-final mass relation which is closer to the observationally-based relation of Weidemann (1987). However Fig. 3 of Weidemann (2000) shows that the updated observational initial-to-final mass relation is actually closer to the determination of Vassiliadis & Wood (1994), except for progenitor masses smaller than 2 $M_\odot$, where the observational relation is in between the Bloecker (1995) and Vassiliadis & Wood (1994) predictions. Since more of the PNe visible today derive from progenitors with masses smaller than $\sim2 M_\odot$, it is appropriate to adopt both sets of tracks and average the results.

4. Results: the number and birth-rate of PNe and WDs in the Galaxy

To determine the total number of galactic PNe, we have considered three possible IMFs (§ 3.1 and Fig. 1), three values for the galactic mass (§ 3.3; Table 2), two SFH functions, three mean galactic ages (§ 3.4 and Fig. 4), three central star mass lower limits to make a PN and two PN visibility time models (§ 3.7 and Fig. 7). We therefore computed a total of 324 models and obtained 324 values for the total number of galactic PNe. The range in these values corresponds to the cumulative uncertainty contributed by these assumptions.
The mean total number of PNe in the Galaxy with radii less than 0.9 pc is \((4.6\pm1.3)\times10^4\) objects, with the thin disk, bulge, thick disk and spheroid each contributing \((3.6\pm1.1)\times10^4, 7200\pm3200, 1100\pm500\) and \(70\pm80\) PNe, respectively. The average visibility time for PNe with radii less than 0.9 pc is \(26000\pm3000\) yr. The errors given represent the 1 \(\sigma\) deviation in the values predicted by the models. Histograms of the predicted PN populations for the thin disk, bulge and entire Galaxy are plotted in Fig. 8, where we show, visually, the uncertainty in the prediction due to the range of possible assumptions (further discussed in § 4.1). Only one of our 324 models produces fewer than 20 000 PNe, this implies that there should be more than 20 000 PNe in the Galaxy from single stars and binaries at the 3\(\sigma\) level. It is interesting to notice that although the thin disk is only \(\sim 1.8\) times more massive than the bulge, it produces \(\sim 5\) times more PNe.

The galactic PN population derived by our model is lower than that derived from the back of the envelope calculation (De Marco & Moe 2005, § 2). This is mainly due to the fact that simple averages for the age of the Galaxy as well as taking lifetimes and masses of a prototypical progenitor star, instead of using bins, biased the calculation toward larger PN populations.

Quoting the PN birthrate avoids the uncertainty related to the PN visibility time. We find the birthrate of PNe in the Milky Way to be \((1.7\pm0.3)\) PN yr\(^{-1}\), with the thin disk, bulge, thick disk and spheroid each contributing \((1.2\pm0.3), (0.4\pm0.2), (0.07\pm0.03)\) and \((0.01\pm0.01)\) PN yr\(^{-1}\), respectively. Histograms of the predicted PN birthrates for the thin disk, bulge and the entire Galaxy are plotted in Fig. 9.

The central star progenitor mass distributions is plotted in Fig. 10 for thin disk, bulge and the entire Galaxy, along with the 1 \(\sigma\) errors for each bin. The mean and median progenitor masses are \((1.7\pm0.3)\) and \((1.2\pm0.2)\) M\(_{\odot}\), respectively. We can also use this distribution to check for consistency between the frequency of Type I PNe (21%; Kingsburgh & Barlow (1994)) and the frequency of their progenitors. Type I PNe are those where the N/O>0.8 because of hot bottom conversion of carbon to nitrogen. This is only thought to occur in stars with main sequence mass larger than \(4\) M\(_{\odot}\) (Becker & Iben 1980). We predict that \((6\pm4)\)% of the PN progenitors’ masses are above \(4.0\) M\(_{\odot}\), three times less than the observed Type I a frequency. The primary contribution to this error is the SFH; younger Galaxy models give higher mass progenitors while older Galaxy models give lower mass progenitors (see § 4.1). Using the initial-to-final mass relation from the Schoenberner (1983), Bloecker (1995) and Vassiliadis & Wood (1994) models we derived the central star mass distribution (Fig. 11). The mean and median central star masses are \((0.61\pm0.02)\) and \((0.57\pm0.01)\) M\(_{\odot}\), respectively.

The metallicity distribution of PN progenitors is shown in Fig. 12. Despite the large
range of metallicities of stars formed in our Galaxy, the PNe we see today are mainly confined to progenitors that had near solar metallicity. The metallicity distribution for the entire Galaxy can be fitted by a Gaussian curve with a mean of −0.07 dex and a dispersion of 0.16 dex. Again, by looking at the AMRs, all components had reached an average metallicity greater than $log(Z/Z_\odot) = -1.0$ by $t \sim 1.5$ Gyr, when the lowest mass PN progenitors formed.

Finally, by determining the PN visibility time associated with each central star mass, we computed (Fig. 13) the expected luminosity distribution of our galactic central stars. The median central star luminosity is $log(L/L_\odot) = 2.3 \pm 0.2$. The bimodal nature of the luminosity distribution (with peaks at $log(L/L_\odot) = 2.2$ and 3.5), is due to the fact that lower mass central stars (which are the most numerous) have a slow luminosity evolution right after the AGB (the 0.569-M_\odot solar metallicity central star of Vassiliadis & Wood (1994) spends 27,000 yr at approximately $log(L/L_\odot) = 3.5$), followed by a speeding up of the evolutionary time (it takes only 13,000 yr for its luminosity to drop to $log(L/L_\odot) = 2.4$), later followed by a slowing down of the evolutionary time (it takes another 60,000 yr for the stellar luminosity to drop further to $log(L/L_\odot) = 2.1$). We wonder here whether the unexplained bimodal behavior of the PN luminosity function (PNLF) of external galaxies (e.g., see the SMC PNLF of Jacoby & De Marco 2002) is due in part to this effect.

The predicted post-AGB WD birthrate for the Galaxy is $(2.4 \pm 0.5) \, \text{WD yr}^{-1}$ with the thin disk, bulge, thick disk and spheroid each contributing $(1.3 \pm 0.4), (0.7 \pm 0.3), (0.09 \pm 0.02)$ and $(0.14 \pm 0.04) \, \text{WD yr}^{-1}$, respectively. Thus, the ratio of the PN birthrate to the post-AGB WD birthrate for the Galaxy is $(0.73 \pm 0.10)$, while, for the thin disk, bulge, thick disk and spheroid it is $(0.89 \pm 0.01), (0.58 \pm 0.21), (0.71 \pm 0.22), \text{and} (0.08 \pm 0.08)$, respectively. The small ratios in the older galactic components are due to the particularly long times taken by low mass progenitors to reach $log(T_{\text{eff}}/K) = 4.4$ and the consequent low PN production rate.

### 4.1. The uncertainties

Here we analyze the main sources of uncertainty incurred because of our six assumptions. The 324 models produce 324 values of the PN population. These models can be divided into groups within which a given assumption is kept constant. For instance, there will be two groups, each containing 162 models, where one groups is calculated using the exponential equation (Eq. 8) for the SFR and the other using the Sandage equation (Eq. 9). One can then compare pairs of PN population predictions obtained by keeping all parameters constant except for the SFR equation. From these 162 pairs one can extract 162 percentage differences. The mean and standard deviation of the 162 percentage differences is reported in Table 3.
The main source of error is from the uncertainty on the age of the Galaxy, primarily that of the thin disk. The younger the thin disk, the higher the SFR at epochs when the turn-off mass was above 1.0 $M_\odot$, the longer the PN lifetimes, the more PNe are predicted. The second main contribution to the error is the uncertainty in the mass of the Galaxy. Our adopted value for the Galactic mass of $(6.2 \pm 1.1) \times 10^{10} M_\odot$ (§ 3.3), is much lower than the value of 100 billion $M_\odot$ often found in the literature (e.g., Inmanen 1966). We believe that this higher mass figure should be the number of stars in the Galaxy, not the total mass of stars in the Galaxy. Indeed, we calculate that there are $1.3 \pm 0.2 \times 10^{11}$ stars in the Galaxy, from which an average galactic star has a mass of $\sim 0.5 M_\odot$ (as can be calculated by taking the mean of any IMF). The IMF contributes the next largest error, where the Kroupa (2001) disk IMF contributes the least amount of PNe, while the Chabrier (2003b,a) disk IMF produces the most. Finally, the central star mass cut-off value below which no PN is formed, contributes the fourth largest source of uncertainty. The reason for this relatively small contribution to the uncertainty by a parameter which, after all, plays a large role at the low mass end of the IMF (where many more stars reside), is that it only plays a prominent role in old populations, which are not the main contributors to the total number of PNe in the Galaxy.

We also point out the importance of using metallicity-dependent stellar lifetimes and of using the AMR. If we had used exclusively solar-metallicity stellar lifetimes, the turn-off mass of the Galaxy would have been 0.94 $M_\odot$ and the ratio of the PN formation rate to the WD formation rate would be $\sim 0.90$ (instead of 0.73) for the entire Galaxy. However, since metallicities were actually low at the beginning of the formation of the Galaxy, and since lower metallicity stars of 1 $M_\odot$ live shorter than the same stars at higher metallicities, then the turn-off mass of the Galaxy is reduced to 0.74 $M_\odot$, which significantly reduces the ratio of the PN to WD formation rates for the older components.

4.2. Comparison with Observations

In this section we compare our prediction of the galactic PN population size, birthrates, birthrate densities and other parameters with observations or observationally-based estimates.
4.2.1. PN numbers

Having predicted the number of galactic PNe with radii smaller than 0.9 pc, we need to compare it with the observed numbers. The number of galactic PNe observed so far is \( \sim 2000-2500 \) (Parker et al. 2003), but the total number is no doubt larger, since distant PNe close to the galactic plane are systematically obscured by dust. Extrapolating the total number of PNe from the observed one, based exclusively on visibility arguments is extremely difficult. Estimates of the total galactic PN population based on local PN density are tied to the controversial distance scale. An alternative estimate of the number of galactic PNe has been obtained by Jacoby (1980) and Peimbert (1990, 1993), by using the PN luminosity function (PNLF) for several extragalactic PN populations. We adopt these two estimates for our comparison. *We do not claim that these observationally-based estimates are better than many others found in the literature, but we do claim that these estimates have very clearly defined biases. By applying the same biases to our theoretical prediction we can achieve a much more meaningful comparison, even in view of the many uncertainties.*

The method of Peimbert (1990, 1993) counts the brightest PNe in a given galaxy (estimated to be within 0.8-2.5 mag of the brightest). It then assumes that the PNLF for that galaxy is the same as the PNLF determined by Jacoby (1989), Jacoby et al. (1989) and Ciarullo et al. (1989) and extrapolates the observed number of bright PNe to 8 mag down the PNLF. This figure was adopted because it was thought to include the faintest PNe (Jacoby 1980). In this way, the total number of PNe in that galaxy is estimated. By assuming a PN visibility time of 25000 yr, the PN birthrate can then be derived \( (\text{PN yr}^{-1}) \), and using that galaxy’s bolometric luminosity, the PN birth rate per unit luminosity is calculated (called \( \dot{\xi} \)). The PN birth rate per unit luminosity, \( \dot{\xi} \), is derived in this way for several galaxies and then plotted against each galaxy’s bolometric luminosity as well as its intrinsic \( (B-V)_0 \) color index. These relationships are then used, together with our own Galaxy’s bolometric luminosity and color index, to derive \( \dot{\xi} \) for our Galaxy. This number is then converted back to an absolute number of PNe using our Galaxy’s bolometric luminosity and the same PN visibility time. Using this technique, Peimbert (1990, 1993) estimated the total number of galactic PNe to be \( 7200 \pm 1800 \). Jacoby (1980) used a very similar technique to derive a total number of \( 10000 \pm 4000 \) PNe with magnitudes brighter than 8 mag below the PNLF’s bright end cutoff.

The total galactic PN population derived in this way is technically independent of the PN visibility time adopted (since its value is first used in a multiplicative way, and then divided out). However, a dependence on the visibility time returns when extrapolating the total PN populations to 8 mag down the PNLF. Today’s deep surveys (e.g., Parker et al. 2003) have made it clear that 8 mag is not enough for a complete census. The reason why
Jacoby (1980) chose this limit was that the Abell (1966) PNe, then thought to be some of the largest and faintest, have radii as large as $\sim 0.9 \pm 0.1$ pc depending on the distance scale used (van de Steene & Zijlstra 1995; Zhang 1995; Phillips 2004, 2005). Using the fact that the mean expansion rate of a PNe is $25$ km s$^{-1}$, a maximum radius of detectability of 0.9 pc corresponds to a maximum visibility time of 35,000 years. This kinematic age is smaller than that of the largest PNe known today. However, by adopting the same limiting age, we have insured that our theoretical prediction is compatible with the total galactic PN population estimated by Peimbert (1990, 1993) and Jacoby (1980).

Calculating the weighted average assuming Gaussian statistics of the two independent measurements of $7200 \pm 1800$ and $10,000 \pm 4000$, yields $8000 \pm 2000$. This estimate is about six times smaller than our estimate and well outside the derived error bars at the 2.9σ level. Due to the unprecedented care we put in estimating all sources of uncertainty, and to the fact that when in doubt we have likely erred on the side of prudence, we confidently suggest that the difference between the predicted and observationally-estimated numbers is significant. In line with the reasoning above, it is therefore meaningless to compare our theoretical estimate with total PN numbers determined with other methods.

4.2.2. PN birthrates

We can also compare our predicted PN birthrate density with the PN birthrate densities predictions from local counts. Both predictions and observational determinations have their own distinct caveats. Our theoretical prediction of the PN birthrate density is independent of the adopted PN visibility time, but is dependent on the adopted model of galactic density (see below). The observational value depends on the as yet uncertain PN distance scale (to the fourth power!), the luminosity model of the Galaxy as well as the PN visibility time which is used to determine the birthrate from the local PN density.

To compute birthrate densities (PN yr$^{-1}$ pc$^{-3}$) from our PN birthrates (PN yr$^{-1}$) we adopt the density equations for the thin disk by Drimmel & Spergel (2001); Robin et al. (2003) and Naab & Ostriker (2006) and average the density over a region between 7.5 and 8.5 kpc from the galactic center and 0.32 kpc above and below the galactic plane, the average mass in a local cubic parsec is estimated to be $(9.0 \pm 2.0) \times 10^{-13}$ times that of the entire thin disk (where the error comes from the standard deviation on the mean of the three density equations used). Thus the local PN birthrate density (using the thin disk PN birthrate value) is $((1.2 \pm 0.3)$ PN yr$^{-1}) \times ((9.0 \pm 2.0) \times 10^{-13}$ pc$^{-3}) = (1.08 \pm 0.46) \times 10^{-12}$ yr$^{-1}$ pc$^{-3}$, which is lower, but within our quoted error, than the value of $3 \times 10^{-12}$ yr$^{-1}$ pc$^{-3}$ by Pottasch (1996) or the value of $2.1 \times 10^{-12}$ yr$^{-1}$ pc$^{-3}$ by Phillips (2002) (see also other estimates in
the summary by Phillips 1989 and Phillips 2002).

Finally, we should remark that the predicted galactic PN population size (and PN birthrate density) that derive from binary interactions only (Paper II), will obviously be smaller than we predict assuming that also single stars make PNe. While the total PN population size will then be more in line with what is estimated from observations, the smaller PN birthrate densities will be at odds with the estimates based on local PN counts. The only way to reconcile our predictions of the birthrate densities with observationally-based counts will then be to admit that the galactic distance scale should be revised to overall larger distances (which will reduce the local observed PN density). This discussion will be expanded in Paper II.

4.2.3. PN central star masses

We can also compare our theoretically-predicted mean central star mass and mass distribution to those from observed samples.

Galactic PN central star masses are derived in two ways. The first (photometric) method uses central star apparent magnitudes, reddenings, effective temperatures and distances to locate the central star on the Hertzsprung-Russell diagram. Masses are then derived by comparing these positions to those of theoretical evolutionary tracks. The second (spectroscopic) method uses stellar atmosphere models to fit the central star spectra and derive effective temperatures and gravities. The masses are derived by comparing these two parameters (which are distance independent) with theoretical evolutionary tracks on the log $g$–$T_{\text{eff}}$ plane.

From a sample of 76 galactic central stars, Gorny et al. (1997) derive central star masses using the photometric method. Their mean mass is $0.624 ~M_\odot$. The combined samples (24 central stars) of Rauch et al. (1998), Rauch et al. (1999) and Napiwotzki (1999), who use the spectroscopic method, yield a mean mass of $0.594 ~M_\odot$. Only three objects are common to the two samples and the mass estimates are quite discrepant (NGC 6720: $M = 0.611$ vs. $0.56 ~M_\odot$; NGC 7094: $M = 0.583$ vs. $0.87 ~M_\odot$; NGC 3587: $M = 0.854$ vs. $0.55 ~M_\odot$ for the former and latter methods, respectively), revealing the uncertainties inherent to this mass determinations. Our predicted mean central star mass, $(0.61 \pm 0.02) ~M_\odot$, falls in between the values obtained via the photometric and spectroscopic methods. The minimum central stars mass in the observed galactic samples ($0.56 ~M_\odot$) is the same as our prediction, supporting the argument that there is a minimum central star mass to make a visible PN. The mass distribution of the photometrically-derived sample is broader than ours (Fig. 14; we do not
pay much attention to the apparent bi-modality of the histogram since that could be due to sampling), while that of the spectroscopically-derived sample is more similar to ours.

PNe in the LMC and SMC do not suffer from the distance uncertainties present for galactic PNe and the photometric method can be applied with fewer caveats. Barlow (1989) adopts a mean central stars mass for both galaxies of $(0.586\pm0.018)\,M_\odot$ from analyses by Monk et al. (1989) and Aller et al. (1987) of a samples of 9 and 12 LMC and SMC PNe, respectively. Using the *Hubble Space Telescope* to observe a sample of LMC and SMC PNe, Villaver et al. (2003) (LMC) and Villaver et al. (2004) (SMC) find mean central stars masses of 0.65 and 0.63 $M_\odot$, respectively (sample sizes of 16 and 12, respectively). Adding measurements by Jacoby & Kaler (1993) and Dopita et al. (1997) to the LMC sample brings the LMC central star mass mean up to 0.67 $M_\odot$. Adding measurements by Jacoby & Kaler (1993) and Liu et al. (1995) to the SMC sample brings the mean central stars mass up to 0.64 $M_\odot$ (the central star of PN J 18 is measured to be 0.63 $M_\odot$ by Villaver et al. (2004), but 0.56 $M_\odot$ by Jacoby & Kaler (1993), once again showing considerable spread). No matter what mean is adopted, and making no distinction between the LMC and the SMC (whose central star mass distributions are very similar), we see that the mean PN central star mass in these two galaxies is larger than in our Galaxy. This is likely due to the fact that both these galaxies are currently star-forming (Gallagher et al. 1996; Harris & Zaritsky 2004; Zaritsky & Harris 2004), making the average ages of these two systems younger and their central stars relatively more massive. From Fig.14 we notice, however, that the minimum central star mass encountered in these samples is very similar to the predicted one.

An obvious difference between our predicted mass distribution and those of observed samples (including that of the WDs, discussed in § 4.2.4), is the relatively larger number of more massive central stars in the observed samples. This might be due to the presence of mergers, which push stars into larger mass bins. This is also the conclusion of Liebert et al. (2005) regarding the WD sample (§ 4.2.4).

### 4.2.4. Birthrates and mass distribution of WDs and the birthrates of AGB stars

Liebert et al. (2005) determined new values for the WD mass distribution and birthrates from observed samples. The WD mass distribution (Fig. 14, bottom panel) has three peaks. They are at 0.565 $M_\odot$ (FWHM=0.188), 0.403 $M_\odot$ (FWHM=0.055) and 0.780 $M_\odot$ (FWHM=0.255). The contributions to the three peaks are 76%, 8% and 16%, respectively. Liebert et al. (2005) interpret the low mass peak as those WD that derive from post-RGB stars (i.e., that never ascended the AGB), the main peak as the post-AGB WDs and the high mass peak as those WDs that derive from mergers. If we take the entire sample of Liebert
et al. (2005) of 347 WDs and eliminate the 33 stars with masses smaller than 0.47 M\(_\odot\) (these WDs must evolve from stars that never ascended the AGB since masses this low do not ignite core helium; Jimenez & MacDonald 1996; F. Herwig, priv. comm.), we have a sample of 314 WDs which presumably ascended the AGB. Of these 314 WDs, 242 have masses larger than 0.56 M\(_\odot\) and could be the WDs which are massive enough to have gone through a PN phase. The ratio of the post-PN WDs (242 objects) to the total post-AGB WDs (314 objects) is 0.77. This means that 77% of all post-AGB WDs have potentially gone through a PN phase, based on core mass arguments alone (but only 70% of the entire WD population has gone through a PN phase). This percentage (77%) compares reasonably well with our ab-initio prediction of (73 ± 10)%.

The WD recent birthrates density derived by Liebert et al. (2005) is \((1 \pm 0.25) \times 10^{-12}\) WD yr\(^{-1}\) pc\(^{-3}\). Our predicted local (i.e., thin disk) WD formation rate density, \((1.3 \pm 0.4)\) WD yr\(^{-1}\) times \((9.0 \pm 2.0) \times 10^{-13}\) pc\(^{-3}\) = \((1.2 \pm 0.4)\) \times 10\(^{-12}\) WD yr\(^{-1}\) pc\(^{-3}\), is close to the value of Liebert et al. (2005).

It is worth at this point to note that Liebert et al. (2005) compare their WD birthrate density to the PN birthrate density of Phillips (2002, \(2.1 \times 10^{-12}\) WD yr\(^{-1}\) pc\(^{-3}\)) and of Pottasch (1996, \(3 \times 10^{-12}\) WD yr\(^{-1}\) pc\(^{-3}\)). The PN/WD birth rate density ratio is thus 2.1–3. Liebert et al. (2005) argue that this ratio should be less than unity in view of the fact that many WDs did not go through a PN phase (i.e., those that never ascended the AGB as well as those that never developed a bright PN because of small central star mass leading to slow post-AGB evolutionary times). They conclude that it is unclear whether this comparison is telling us something, or whether the PN birthrate densities are too unreliable a number to be used (the WD birthrate densities is a much more accurate estimate). Our predicted PN birthrate density ((\(1.1 \pm 0.5\) \times 10\(^{-12}\) PN yr\(^{-1}\) pc\(^{-3}\)) being lower than the observationally-based estimates of Phillips (2002) or Pottasch (1996), compares more favorably with the WD’s.

Phillips (1989) himself addresses the problem outlined above, concluding that smaller PN birthrate densities would be more in line with what we know from WDs. However, he does express the concern that many higher estimates for the PN birthrate densities are more in line with other evidence and that lower estimates would not be readily explained. We point out that if our lower predictions of the PN birthrate density were to be accepted, we would have to accept a longer galactic distance scale to derive overall lower local PN densities. This would be even more the case if we assumed (as we do in Paper II) that only binaries make PNe, thereby predicting even lower birthrate densities.

Finally, we turn our attention to the AGB population and seek further constraints of our predictions and consistency with past results. 90% (314/357) of the entire WD population
has evolved through the AGB phase and should therefore be the direct progeny of the AGB population \textit{whether or not} it went through a PN phase. The birthrates of AGB stars should be the same as those of post-AGB WDs and larger than those of PNe by the same factor. Ortiz & Maciel (1996) argue that the AGB star birthrates are between 0.8 and 2.7 AGB stars yr$^{-1}$, depending on what evolutionary model one adopts. Our predicted post-AGB WD birthrate of 2.4 WDs yr$^{-1}$ (§ 4) is within the observed AGB birthrate range, lending further (albeit week) support to our calculation.

5. The number of PNe in the bulge and in globular clusters

Another useful comparison can be carried out with the PN populations of two regions of the Galaxy that are reasonably well defined: the bulge and the GC system.

5.1. PNe in the bulge

Jacoby & Van de Steene (2004) predict 250 PNe should reside in the inner 4×4 deg of the Galaxy. This result is from their survey, corrected for incompleteness by assuming a scaled bulge luminosity and using the bright end of the PNLF. If we assume the bulge within 2 deg (300 pc) of the center to have a mass of $2 - 3 \times 10^9$ M$_\odot$ (Lindqvist et al. 1992) and using our average-mass model of the bulge ($2.0 \times 10^{10}$ M$_\odot$), we can scale the total number of observationally-estimated bulge PNe to the total bulge mass, obtaining 1700 – 2500 PNe from single and stars and binaries. Using the average-mass bulge model, we predict $(7200 \pm 3000)$ PNe, which is 3–5 times the amount estimated from observations. The over-abundance of predicted vs. “observed” PNe is in line with the comparison carried out for the entire Galaxy.

5.2. PNe in globular clusters

For the GCs the situation is more complicated. The number of PNe contributed by the spheroid can be used to predict the number of PNe in GCs, assuming they have similar star formation histories. There are about 150 GCs with an average of 300,000 stars each$^2$. We assume that all GCs have the same mean age of 11.5 Gyr (Rakos & Schombert 2005), which

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$^2$This number is an extrapolation from GC luminosities and luminosity functions (data from Harris (1996)). It is likely to be accurate to within a factor of 2-3; see also later.
also corresponds to the peak period of star formation in the spheroid (since the spheroid has an average metallicity of \( \log\left(\frac{Z}{Z_\odot}\right) = -1.5 \) and an average turn-off mass of 0.85 M\(_\odot\)).

Using any of the three IMFs from § 3.1, the average mass of a star in a GC is \( \approx 0.45 \) M\(_\odot\). This means that the total mass of the galactic GC system is \( 2.0 \times 10^7 \) M\(_\odot\) (0.45 x 300 000 x 150; cf. this number with the value \( 2 \times 10^7 \) M\(_\odot\) of Pena et al. (1995)). Scaling the number of spheroid PNe (70 ± 80) by the ratio of the spheroid and GC system masses, the predicted number of PNe in GCs is 0–1.5 PNe (\( (70 \pm 80) \times (2.0 \times 10^7) / (2 \times 10^9) \); where \( 2 \times 10^9 \) M\(_\odot\) is the mass of the spheroid; § 3.3.3). The main reason for the large uncertainty, is the uncertainty in the mass cut-off value for PN production (§ 3.7), which primarily affects old populations. In fact, when the cutoff mass is taken to be 0.95 M\(_\odot\) (the largest of the three values we use in § 3.7), no GC PN is produced in any of the models.

The total number of PNe found in the galactic GC system is four (Jacoby et al. 1997). Jacoby et al. (1997) determined that, by applying the fuel consumption method of Renzini & Buzzoni (1986), 16 PNe are expected to reside in the Milky Way GC system. However, they conceded that this method does not account for the fact that low mass central stars evolve too slowly to ionize the PN before the circumstellar gas is dispersed (§ 3.7). If a central star cut-off mass is assumed, no PN might be expected to reside in GCs at all, in line with our prediction and at odds with the observations.

Looking at the characteristics of the four GC PNe we find that two of them have a very high central star mass (K648 has a derived central star mass of 0.62 M\(_\odot\), while IRAS 18333-2357 has a central stars mass of 0.75 M\(_\odot\); Alves et al. 2000; Harrington & Paltoglou 1993). These masses correspond to main sequence masses (2.3 and 3.6 M\(_\odot\), respectively; Weidemann 2000), that are up to three times the cluster turn-off mass, implying some kind of merger formed the progenitors of these central stars. This could be a common envelope merger or a merger that occurred during the main sequence and that temporarily generated a blue straggler star, which later evolved into a PN (but see Ciardullo et al. (2005) for a connection between bright PNe and blue stragglers). One of the four GC PNe (IRAS!18333-2357) is hydrogen-deficient, a characteristics displayed by only 5 galactic PNe (Harrington 1996), making this a very rare phenomenon. Hydrogen-deficient PNe are attributed to the action of a last helium thermal pulse that ejects and ingests the remaining hydrogen envelope of a central star (Iben et al. 1983; Herwig 2000). However, others (e.g., De Marco & Soker 2002) have argued that binary mergers might be responsible for these objects. While the jury is still out on hydrogen-deficient PNe, we might speculate that the high incidence of H-deficient PNe in the GC population (although we are dealing with very low number statistics), is in line with the idea that these objects derive from merged stars.

One could therefore argue that it is possible that none of the observed GC PNe is
a “regular” PN and that all of them are nebulosities deriving from some kind of binary interaction, favored in the crowded environments of GCs, because of the high frequency of orbit-reducing stellar exchanges (Hurley & Shara 2003). This possibility is in line with the finding by Jacoby et al. (1997) that the presence of a PN in a GC appears to correlate with the number of X-ray sources, which are in turn associated to low mass X-ray binaries.

The issue of the presence of PN in GCs is not a simple one and cannot be used as a model constraint. On the other hand, if indeed GC PNe are merger products, the well determined distances to GCs will make these four PNe ideal for case studies of this elusive class of objects.

6. Conclusion

In this paper we have calculated the number of PNe expected to reside in the Galaxy, if PNe derive from single stars as well as from stars in binary systems. The resulting value, \((4.6 \pm 1.3) \times 10^4\) objects with radius smaller than 0.9 pc, is six times higher than the observationally-based estimate of 7200\(\pm 1800\) objects with the same radius constraints (discrepant at the 2.9\(\sigma\) level). Although it could be argued that the actual galactic PN population is larger, we stress that our predicted value is not absolute (since it is constrained by a maximum PNe radius), but is rather designed to be compared to the determination of Peimbert (1993) and as such the comparison is much more reliable than the individual figures.

We argue that the discrepancy is due to the fact that not all stars otherwise fit to make a PN, actually do make a PN. In Paper II we will calculate the galactic PN population deriving from binary interactions and show that, based on population synthesis arguments alone, it is more likely that most PNe could derive from such interaction rather than being produced by single stars.

We also compared our predicted PN birthrate density with that derived observationally from counts of local PNe. Our prediction \(((1.1 \pm 0.5)\times 10^{-12} \text{ PN yr}^{-1} \text{ pc}^{-3})\) is lower than most observationally-based estimates \((2.1 \times 10^{-12} \text{ PN yr}^{-1} \text{ pc}^{-3}; \text{Phillips 2002})^3\), although it is within the quoted uncertainty. Our estimate is similar to the more reliable observational estimate of the WD birthrate density \((1.0 \times 10^{-12} \text{ WD yr}^{-1} \text{ pc}^{-3}; \text{Liebert et al. (2005)})\). The

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^3It will strike the reader that from observational PN birthrate density values that are generally larger than our prediction, total galactic PN populations are derived that are lower than our estimate (see for instance Phillips (2002)). This is due to the very different methods to derive the birthrate densities in our theoretical derivation and in the observationally-based derivations found in the literature.
PN birthrate density from binary interaction we will derive in Paper II will clearly be lower. A discussion of the implication of this discrepancy is left to that paper.

We predict that only 73% of post-AGB WDs went through the PN phase. This is in line with what we know of post-AGB evolutionary timescales (namely, that central stars with mass <0.56 M⊙ do not make a PN), the observed mass distributions of central stars (minimum central stars mass ~0.56 M⊙) and the mass distribution of WDs (which indicates that only 77% of post-AGB WDs have masses large enough to go through a PN phase.).

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Fig. 1.— The three IMFs (normalized) that will be considered in our analysis. Top panel is the relative mass of stars per mass interval while bottom panel is the number of stars per mass interval.
Fig. 2.— Five isometallic lifetimes of stars as a function of mass. The horizontal line in the top left at 13 Gyr represents the range of masses with lifetimes equal to the age of the universe.
Fig. 3.— The mass fraction of stars $\delta$ that were formed at time $t$ and remain luminous today for varying IMFs and metallicities.
Table 1. Mass estimates for the four galactic components.

| Component   | Masses ($10^9$ M$_\odot$) |
|-------------|----------------------------|
|             | low | medium | high |
| Thin Disk   | 29  | 36     | 43   |
| Thick Disk  | 4   | 4      | 4    |
| Bulge       | 16  | 20     | 24   |
| Spheroid    | 2   | 2      | 2    |
| Total       | 51  | 62     | 73   |

Fig. 4.— Star formation history of the four components of the Galaxy assuming three different mean ages (called “Young”, “Average” and “Old” in Table 2) and two different SFR functions for the thin disk and bulge (the “Exponential” and the “Sandage” function, Eqs. 8 and 9, respectively).
Fig. 5.— Adopted metallicity distributions for the different parts of the Galaxy.

Fig. 6.— The age-metallicity relation for the four galactic components. For the thin disk and the bulge the relation is shown for the exponential (marked “Exp” in the panels’ titles) and Sandage (marked “San” in the panels’ titles) forms of the equation (Eqns. 8 and 9, respectively), as well as for the Old, Average and Young ages of the populations (see Table 2).
Fig. 7.— Greyscale of the PN visibility time as a function of mass and metallicity (top panel from the calculation of Vassiliadis & Wood (1994)) and mass only (bottom panel, from the calculation of Bloeker (1995)).

Table 2. Parameters used in the star formation rate equations, 8 and 9.

| Component        | $t_0$ | $t_{\text{end}}$ | Mean Age | $\tau$(Eq.8) | $\tau$(Eq.9) |
|------------------|-------|-------------------|----------|---------------|---------------|
| Spheroid         | 0.0   | 4.5               | 11.5     | 2.09          | 1.25          |
| Thick Disc       | 1.0   | 6.0               | 10.0     | 4.06          | 1.62          |
| Bulge - Young    | 0.5   | 13.0              | 9.0      | 4.14          | 2.79          |
| Bulge - Average  | 0.5   | 13.0              | 10.0     | 2.60          | 1.99          |
| Bulge - Old      | 0.5   | 13.0              | 11.0     | 1.50          | 1.20          |
| Thin Disc - Young| 3.0   | 13.0              | 4.5      | -16.6         | 5.33          |
| Thin Disc - Average| 3.0  | 13.0              | 5.5      | 16.6          | 3.74          |
| Thin Disc - Old  | 3.0   | 13.0              | 6.5      | 5.25          | 2.80          |
Fig. 8.— Histogram of the PN population sizes predicted by different models. The bin sizes are 2500, 500 and 2500 for the bulge, thin disk and total respectively.

Table 3. Percentage errors incurred from each step.

| Parameter   | % Error | $\sigma_{\%Error}$ |
|-------------|---------|---------------------|
| SFH Age     | 22%     | 4%                  |
| Mass        | 19%     | 0.2%                |
| IMF         | 17%     | 0.5%                |
| Cutoff Mass | 9%      | 4%                  |
| SFH Type    | 5%      | 2%                  |
| PN Visibility | 1%    | 1%                  |
Fig. 9.— Histogram of the PN birthrates predicted by different models. The bin sizes are 0.05, 0.1 and 0.15 PN yr\(^{-1}\) for the bulge, thin disk and total respectively.
Fig. 10.— PN central star progenitor mass distribution. Error bars are for the total population.

Fig. 11.— PN central star mass distribution. Error bars are for the total population.
Fig. 12.— PN central star progenitor metallicity distribution. Error bars are for the total population.

Fig. 13.— Central star luminosity function of PN.
Fig. 14.— Histograms of central stars masses. From top to bottom we compare: our population synthesis results (see also Fig. 11), the galactic sample of Gorny et al. (1997) (photometric method), the galactic sample of Rauch et al. (1998), Rauch et al. (1999) and Napiwotzki (1999) (spectroscopic method), the combined LMC and SMC samples of Villaver et al. (2003), Villaver et al. (2004), Jacoby & Kaler (1993), Dopita et al. (1997), and Liu et al. (1995) and, finally, the WD sample of Liebert et al. (2005). The vertical line marks central star mass $0.56 \, M_\odot$. 