Criterion for vortex acoustic lock-on in combustors with backward facing step

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Abstract. In thermoacoustic systems vortex separation and breakdown are an important source of heat release rate fluctuations. The coupling between the acoustic field and the energy released by vortex breakdown can cause instability in these systems under certain operating conditions. Non-linear simulations show that combustion instability occurs after vortex acoustic lock-on. The objective of this work is to determine the criterion for vortex acoustic lock-on and thus the criterion for combustion instability as well. The 0-to-peak amplitude of the velocity fluctuations in the system are used as a criterion to determine the transition of Helmholtz number from vortex shedding mode to acoustic mode. Helmholtz number and acoustic pressure fluctuations computed using the model are compared with experimental investigations. The phenomenological model shows a good agreement with the experiments and the criterion determined for vortex lock-on provides a better understanding of the phenomenon.

1. Introduction

In thermoacoustic systems heat release rate fluctuations excite acoustic oscillations and the acoustic oscillations in turn affect the source of heat release. According to Rayleigh’s criterion, thermoacoustic instability is encouraged if the heat release rate fluctuations are in phase with the pressure oscillations. Contemporary combustors use swirl flows, along with a backward facing step to stabilize the flame \([1]\). The problem of vortex shedding and breakdown are intrinsic to these combustors because of their geometrical characteristics. A number of experiments \([2, 3, 4]\) have shown that during stable operation of a combustor there is low amplitude broadband noise and the dominant frequency is the vortex shedding frequency. After the onset of instability, there is a high amplitude discrete tone and the dominant frequency is found to be the acoustic frequency. The non-linear driving causes vortex shedding to occur at acoustic frequency. This phenomenon is termed as vortex acoustic lock-on. The objective of this work is to obtain the criterion for vortex acoustic lock-on and investigate the possibility of using this criterion as a precursor for instability.

A reduced-order model to describe the interaction among vortex separation, chamber acoustics and combustion process was given by Matveev & Culick\([5]\). Nair & Sujith\([6]\) showed that the reduced order model can capture intermittent burst oscillations and subsequent flow acoustic lock-on observed in combustors. They postulated that the intermittency can be used as an early warning signal to an impending combustion instability.
Figure 1. Geometry of a vortex shedding combustor with a backward facing step. The figure is not drawn to scale. Vortex formation and breakdown occur at $x_{\text{sep}}$ and $x_c$ respectively.

In the previous works [5, 6], the phenomenological model was used to study only a small range of operating conditions. Even though vortex acoustic lock-on was observed in both the model and the experiments for a given system configuration, the criterion for lock-on was not postulated. Such a criterion will allow us to define the safe operating regime for these combustors beforehand. The detailed objectives of this work are as follows: to explain the phenomenon of vortex acoustic lock-on, to find the criterion for lock-on as a function of all the system parameters and to establish the validity of this criterion by comparing the model with the experimental investigations available in the literature.

2. Theory
In this section we present the theory, governing vortex shedding combustors with a backward facing step. We deviate from the previous works [5, 6] by non-dimensionalising all the equations to reduce the number of free parameters in the system. The vortex shedding process is modelled using the method described by [5]. The present thermoacoustic system is a combustor with a backward facing step as shown in figure 1. The total length of the acoustic chamber is denoted by $l_a$. The backward facing step is located at $x_{\text{sep}}$. This is the point where vortex separation occurs. The separated vortex impinges and breaks down at the point denoted by $x_c$ and the length of the cavity denoted by $l_c$. The ends of the acoustic chamber are assumed to be open.

The following scales are chosen for non-dimensionalising the equations governing the vortex shedding process and the acoustic flow field: $M = u_0/c_0$, $u' = \bar{u}'/u_0$, $p' = \bar{p}'/\gamma M p_0$, $t = c_0\tilde{t}/l_a$ and $x = \tilde{x}/l_a$. The steady state quantities are velocity ($u_0$), pressure ($p_0$) and the speed of sound ($c_0$). The other dimensional quantities are denoted with a tilde above them: velocity fluctuation ($\bar{u}'$), pressure fluctuation ($\bar{p}'$) and time ($\tilde{t}$). In case of non-dimensional quantities the tildes are dropped. The steady state vortex shedding frequency $f_{\text{st}} = M S t/d$, where $S t$ is the Strouhal number. Assuming uniform steady state density $\rho_0$, for low Mach number approximation, the linearised non-dimensional momentum and energy equations, which govern the acoustic field of this system are written as follows:

$$\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial x} = 0,$$  

$$\frac{\partial p'}{\partial t} + \frac{\partial u'}{\partial x} + \zeta p' = \frac{(\gamma - 1)}{\gamma} \frac{\beta}{M} \sum_m \Gamma_m \delta(t - t_m) \delta(x - x_c),$$

$$\frac{d\Gamma_m}{dt} = \frac{M}{2} \left[1 + u'(x_{\text{sep}}, t)\right]^2,$$

where $\beta$ is the non-dimensional heat release coefficient, $\Gamma_m$ is the circulation of the $m$-th vortex [5] and $\zeta$ is an artificially added acoustic damping term. The criterion for separation of the $m$-th vortex is $\Gamma_m \geq \Gamma_{\text{sep}}$, where $\Gamma_{\text{sep}} = [1 + u'(x_{\text{sep}}, t)]d/2S t$. A separated vortex convects
with velocity $dx_m/d\tau = M(\alpha + u'(x, t))$, where $\alpha$ is the coefficient usually chosen in the range 0.4 to 0.6 [7].

The non-dimensional partial differential Eq. (1) and Eq. (2) are reduced to a set of ordinary differential equations (ODEs) by expanding the acoustic variables in terms of basis functions using the Galerkin technique. The mode shapes chosen for decomposing $u'$ and $p'$ are

$$u'(x, t) = \sum_{n=1}^{N} \cos(k_n x) U_n(t), \quad p'(x, t) = \sum_{n=1}^{N} \sin(k_n x) P_n(t),$$  

which ensures that the open-open boundary conditions of the acoustic system are satisfied. In the above expressions, $U_n$, $P_n$ are the amplitude of the $n$-th Galerkin modes. The associated non-dimensional wave number of the $n$-th mode $k_n = n\pi$, where $n = \{1, 2, \cdots, N\}$. Substituting equations (4) in the partial differential equations (1) and (2) gives the following set of ODEs for the momentum and energy equation:

$$\dot{U}_n + k_n P_n = 0 \quad (5)$$

$$\dot{P}_n + 2\zeta_n k_n P_n - k_n U_n = 2\frac{\gamma - 1}{\gamma} \frac{\beta}{M} \sin(k_n x_c) \sum_m \Gamma_m \delta(t - t_m). \quad (6)$$

where $\zeta_n = \frac{1}{2\pi} \left( c_1 \frac{\omega_n}{\omega_1} + c_2 \sqrt{\frac{\omega_n}{\omega_p}} \right)$ is the frequency dependent damping term [5]; $c_1$ and $c_2$ are the damping coefficients which are responsible for the end loses and loses due to boundary layers respectively. We rewrite the system of coupled first order differential equations (5) and (6) as a second order differential equation in $U_n$.

$$\ddot{U}_n + 2\zeta_n k_n \dot{U}_n + k_n^2 U_n = -2k_n \frac{\gamma - 1}{\gamma} \frac{\beta}{M} \sin(k_n x_c) \sum_m \Gamma_m \delta(t - t_m), \quad (7)$$

is the second order differential equation which represents the dynamics of the thermoacoustic system presented. The dynamics is similar to that of a kicked oscillator, which behaves like a damped oscillator when the kicks are absent. The following jump conditions are obtained by integrating equation (7) in the time interval $t_m^-$ to $t_m^+$, corresponding to time instants just before and just after the $m$-th kick.

$$\begin{align*}
U_n^+ - U_n^- &= 0, \\
\dot{U}_n^+ - \dot{U}_n^- &= -2k_n \frac{\gamma - 1}{\gamma} \frac{\beta}{M} \sin(k_n x_c) \Gamma_m \end{align*} \quad (8)$$

gives the jump in pressure when a vortex impinges on the combustor wall. The strength of the kick for a fixed geometric configuration is given by $\beta \Gamma_m/M$.

2.1. Procedure to compute the fluctuations $u'$ and $p'$

First, we set the geometric parameters $\{x_{sep}, x_c, d\}$ and the flow parameters $\{M, c_1, c_2, \alpha, \beta, St\}$. Next, we set the size of the time step $d\tau$, the number of time steps and the number of Galerkin modes $N$. Then, we solve for $U_n$ and $\dot{U}_n$ from equations (5) and (7) using fourth order Runge-Kutta method. The above computations are solved in the absence of kicking ($t \neq t_m$) so the right hand side of equation (7) is set to zero. After that, we compute the circulation at $x_{sep}$, $\Gamma_m$ and the critical circulation $\Gamma_{sep}$. If $\Gamma_m \geq \Gamma_{sep}$ the $m$-th vortex is shed and the circulation of the $(m+1)$-th vortex is computed in the next time step, else the circulation is incremented. All separated vortices in the cavity are convected for time step $d\tau$. If a vortex impinges at $x_c$, the jump conditions given by equation (8) are applied. Thus, the non-periodic Dirac delta function on the right hand side of equation (7) is solved. The same is repeated for desired number of time steps. Finally, the fluctuations at any location can be computed using equations (4).
Figure 2. For $M = 0.03$: (a) Helmholtz number as a function of $\beta$, $He = 0.5$ corresponds to the acoustic mode. (b) Total number of vortices shed in 200 cycles as a function of $\beta$. (c) $p_{RMS}$ as a function of $\beta$ (computed after discarding the first 100 cycles), the vortex acoustic lock-on correlates with the sharp increase in the number of vortices shed and the $p_{RMS}$. (d) The 0-to-peak amplitude of $u'$ as a function of $\beta$. As $\beta$ is increased three regimes are noted, which are explained in detail in section 3.1.

3. Results

The geometrical parameters are set for all the case studies as follows $\{x_{sep} = 0.3, x_c = 0.45, d = 0.01\}$. The damping coefficients $c_1 = 0.315, c_2 = 0.015[5]$, the vortex convection coefficient $\alpha = 0.4$ [7] and Strouhal number $St = 0.1$ are also fixed for all case studies. The system is studied for various values of $M$ and $\beta$. For the non-linear simulations the size of the time step is set to be $dt = 10^{-4}$ and the number of Galerkin modes is chosen as $N_{modes} = 32$. The number of modes chosen here is higher than the number chosen in earlier investigations [5, 6]. The quantities of interest, $p'$ and $u'$ are computed at the location $x_{sep}$ for all the case studies. The dominant frequency in the Fourier transform of $p'$ gives the Helmholtz number $He$. In this study values of $He$ close to $\{0.5, 1, 1.5, \cdots\}$ correspond to the acoustic mode. If $He = mf_{s0}$, $m = \{1, 2, 3, \cdots\}$, the dominant mode is the vortex shedding mode.

3.1. Case study #1: Phenomenon of vortex lock-on

In this case study we explain the phenomenon of vortex lock-on using by varying $\beta$ for a fixed $M = 0.03$. For the chosen system configuration the steady state frequency $f_{s0} = 0.3$. Figure 2 shows the computed values of Helmholtz number ($He$), number of vortices shed, root mean square value of the pressure fluctuations ($p_{RMS}$) and the 0-to-peak amplitude of $u'$ as a function of $\beta$. In general the following sequence of events are observed for increasing values of $\beta$. In regime I, the $He = 0.6 = 2f_{s0}$, therefore the dominant mode is the vortex shedding mode. Next, in regime II, starting at $\beta = 0.55$, the number of vortices shed increases by more than 100% and $He = 0.5$ corresponds to the acoustic mode. Finally, in regime III, starting at $\beta = 0.85$, the number of the vortices shed increases 5-fold compared to the steady state value and there is a sharp increase in $p_{RMS}$ indicating combustion instability. We also find that the vortex acoustic lock-on occurs (at $\beta = 0.55$) well ahead of the combustion instability (at $\beta = 0.85$). The same is true for most of other Mach numbers in the range $0.01 \leq M \leq 0.1$ for which the study was performed.
3.2. Case study #2: Criterion for vortex lock-on
In this case study we find the criterion for lock-on as a function of the system parameters. In figure 2d, we find that the phenomenon of vortex lock-on correlates with the 0-to-peak amplitude of $|u'| > 1$ in the first 200 time cycles. The validity of this criterion can be tested by forcing the system using the function $u'(x_{sep}) = A \sin \omega t$. For $A \geq 1$, we expect the system change its dynamics. Figure 3 shows the mean and standard deviation of the time period between the kicks $\tau$ as a function of $A$ for three different values of $\omega$. For $M = 0.03$, $f_{\omega 0} = 0.3$. Figure 3a shows the results for forcing frequency $\omega = 2\pi(0.4)$. For small $A$, vortex break down occurs at $\tau_{\omega 0} = 1/f_{\omega 0}$. Then it locks on to the forcing frequency of 0.4, which corresponds to $\tau = 2.5$ cycles. The 1σ standard deviation in $\tau$ is noticeably small. Around $A = 1$, the vortex breakdown becomes aperiodic and 1σ standard deviation in $\tau$ becomes large. Similar results are observed for $\omega = 2\pi(0.5)$ and $\omega = 2\pi(0.6)$. This change is dynamics correlates with vortex acoustic lock-on. Therefore, for a given system configuration the value of $\beta$ for which 0-to-peak amplitude of $|u'(x_{sep})| > 1$, is designated as $\beta_{lock-on}$. For all the non-linear simulations performed, we observed that the vortex lock-on occurs at $\beta_{lock-on} = 0.2\beta_{lock-on}$.

3.3. Case study #3: Comparison with experiments
In this case study we validate the criterion obtained in section 3.2 by comparing the phenomenological model with experimental investigations available in the literature. Chakravarthy et. al.,[3] showed the transition of $He$ from vortex shedding mode to acoustic mode for a number of system configurations and found that this transition is marked by non-linearity in the $p_{RMS}$ variations. In current study we reproduce the experimentally observed transitions using the model described in section 2. The experimental setup of [3] is as follows. The non-dimensionalised geometrical parameters of the configuration are $\{x_{sep} = 0.3, x_c = 0.45, d = 0.0224\}$. The air inlet velocity ranges from 3 ms$^{-1}$ to 30 ms$^{-1}$, which corresponds to 0.004 $\leq M \leq 0.04$. The speed of sound is assumed to be 750 ms$^{-1}$. In the experimental investigations of [3], the fuel flow rate is fixed as $\dot{m}_f = 229mg/s$. This information is not sufficient to determine $\beta$. Therefore, we perform all simulations for $\beta_{lock-on}$, which is a lower limit for combustion instability, according to our criterion.

Figures 4a & 4b show $He$ and $p_{RMS}$ as a function of $M$ for both the experimental...
investigations of [3] and the current study. In case of $He$ there is a good agreement between the theory and the experiments for $0.017 < M < 0.037$. In both the experiments and non-linear simulations, we observe that $He$ is close to the acoustic mode for $0.017 < M < 0.025$ and $He$ is a multiple of the vortex shedding mode for $0.025 < M < 0.037$. The $p_{RMS}$ given by [3] in units of $N m^{-2}$ is non-dimensionalised using the factor $1/\gamma M p_0$, where $\gamma = 1.4$ and $p_0 = 101325 Pa$. There is a good match in the trend between the theoretical and experimental values of $p_{RMS}$ for all the $M$. The mismatch in the range $0.03 < M < 0.033$ can be attributed to the insufficient knowledge of all the free parameters. The role of other hydrodynamic phenomenon in the discrepancy can also be not ruled out.

4. Conclusion
Using a phenomenological model, we empirically find that the criterion for vortex acoustic lock-on highly correlates with the 0-to-peak amplitude of velocity fluctuations. The non-linear simulations show that the vortex acoustic lock-on occurs when the 0-to-peak velocity fluctuations are close to $u' > 1$. The same criterion is validated by externally forcing the system. We find that the system changes its dynamics when the amplitude of forcing is greater than one. After lock-on, vortex shedding occurs at the acoustic frequency and the low amplitude broadband noise becomes a high amplitude discrete tone, which is a characteristic of combustion instability. We use this criteria for vortex acoustic lock-on to predict the Helmholtz number ($He$) and the RMS fluctuations in pressure ($p_{RMS}$) during combustion instability. The predictions are compared with the available experimental results. A good agreement is observed in the trend of $He$ and $p_{RMS}$.

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