Current regulation of universal conductance fluctuations in bilayer graphene

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Abstract. We report experimental results on the universal conductance fluctuations (UCFs) in the bilayer graphene system. The UCF properties under different temperatures, magnetic fields and current bias were investigated. An anomalous current-dependent UCF was observed: the rms amplitude of the conductance fluctuations is inversely proportional to the current bias. The detailed physical mechanisms were discussed by involving the confined scattering of chiral fermions in graphene.

Graphene, a two-dimensional (2D) crystal consisting of a single graphitic atomic layer, has attracted intense attention because of its unique energy band structure and electronic properties [1]–[5]. One of the intriguing properties of graphene is its unusual chiral sensitivity occurring in the phase-coherent electronic transport. For example, the weak localization (WL) and anti-WL of graphene are dictated by the chirality-related scattering process [6]–[10]. For bilayer graphene, the trigonal warping of the electron dispersion suppresses the WL effect [11]. Another interesting effect is the universal conductance fluctuation (UCF) induced by impurity (including defect) scattering and quantum coherence. In conventional UCF, carriers can be scattered in all directions in real space, which results in conductance fluctuations of the order of the conductance quantum $e^2/h$ independent of the sample size and its disorder strength [12], provided that the phase coherent length is longer than the sample size. However, UCF in 2D graphene is still controversial. UCF in a few layer graphene has been experimentally observed to be robust over a wide range of carrier concentrations and magnetic fields [9, 10, 13, 14].
Staley et al [15] reported the suppression of conductance fluctuation near the charge neutral point in bilayer graphene. Using numerical analysis, Rycerz et al [16] found that conductance fluctuation in graphene seems to be non-universal and is much stronger than that in conventional metals. These inconsistent results for graphene demand more experimental and theoretical work on the possible mechanism of the UCF phenomenon, although the theoretical understanding of UCF in graphene has made much progress [17]. For example, the influence of confined scattering of chiral fermions on UCF has not yet been considered in the current theoretical framework. To reflect the chiral fermions in graphene, both their momentum and their pseudospin must be reversed [3, 18]. Therefore, the backscattering of charge carriers is prohibited due to the requirement of pseudospin flipping. The absence of backscattering is expected to play a significant role in conductance fluctuations in 2D graphene.

In this paper, we investigate the rms amplitude of conductance fluctuations in bilayer graphene as a function of the applied current. A model for the confinement of charge-impurity scattering is proposed to describe the observed anomalous conductance fluctuation.

The graphene samples used in our experiments were mechanically exfoliated from highly ordered pyrolytic graphite (HOPG) [1]. Exfoliated graphene flakes were transferred to the surface of a Si substrate with a 300 nm thick SiO$_2$ layer. A bilayer graphene flake was identified using Raman spectroscopy. Two-metal electrodes were then fabricated on the flake using electron beam lithography. The gate voltage ($V_g$)-dependent source–drain current ($I_{sd}$) was measured using a Keithley 4200 Semiconductor Characterization System by applying a fixed source–drain voltage ($V_{sd}$), and the Si substrate was serviced as a back-gate electrode. The conductance–magnetic field (G–B) curves of the bilayer graphene were measured by a Keithley current source/nanovoltmeter (model 6221/2182A) system under a perpendicular magnetic field and zero gate voltage.

The morphology of the two-electrode graphene device is shown in the scanning electron microscope (SEM) image (see figure 1(a)). The Raman spectrum of the graphene flake with 514 nm laser excitation is depicted in the inset of figure 1(a), which shows two strong peaks (2704.7 and 1582.1 cm$^{-1}$) with similar intensities and confirms the bilayer nature of the graphene flake [19, 20].

Figure 1(b) presents the measured $I_{sd}$ versus $V_g$ curve, i.e. the transfer curve, with a fixed $V_{sd}$ of 1 mV at a temperature of 1.5 K. Extracted from the $I_{sd}$–$V_g$ curve, the transconductance at zero gate voltage ($g_m = (∂I_{sd}/∂V_g)|_{V_g=0}$) is $\sim 2.97$ nA V$^{-1}$. The field-effect hole mobility is given by [15]

$$
\mu_{FE} = \frac{Hg_m}{\varepsilon \varepsilon_0} \frac{L}{WV_{sd}},
$$

(1)

where $H$ is the thickness of the gate oxide layer, $\varepsilon$ is the relative dielectric constant of SiO$_2$, and $L$ and $W$ are the sample length and width, respectively. With $H = 300$ nm, $\varepsilon = 3.9$, $L = 7$ $\mu$m, and $W = 2$ $\mu$m, the hole mobility is estimated to be $\sim 903$ cm$^2$ V$^{-1}$ s$^{-1}$ at zero $V_g$. Simply taking the calculated field effect carrier mobility ($\mu_{FE}$) as the real carrier mobility ($\mu$), we can work out the 2D carrier density $n \sim 4.1 \times 10^{12}$ cm$^{-2}$ given by $n = (G/e\mu)(L/W)$, where $G$ is the conductance. Since $n$ can also be written as $n = \sigma/e\mu$ and the 2D electrical conductivity $\sigma = (e^2/2\pi^2\hbar^2)\pi k_F L$, with the Fermi wave vector of $k_F = \sqrt{2\pi n} \sim 5 \times 10^6$ cm$^{-1}$ the mean free path of the carrier is $l \sim 31$ nm. In addition, the Fermi velocity $v_F = \hbar k_F/m^*$ is about $0.8 \times 10^6$ m s$^{-1}$, where $m^* \sim 0.069 m_e$ is the effective mass of the hole in multilayer graphene [21]. In the 2D system, the diffusion constant $D$ is about $0.5 \times v_F l$ and it is $124$ cm$^2$ s$^{-1}$ for this sample.
Figure 1. (a) SEM image of a two-electrode graphene device. The electrode consists of a 50-nm-thick Au layer on top of a 10-nm-thick Ti layer. Inset: Raman spectrum of the bilayer graphene. (b) The transfer curve measured at 1.5 K with a fixed bias of 1 mV.

Therefore, the thermal diffusion length \( l_T = \sqrt{\frac{\hbar D}{k_B T}} \), where \( k_B \) is the Boltzmann constant, is estimated to be 250 nm at 1.5 K, which is much shorter than the length of the sample.

The magnetoconductance (defined as \( \Delta G(B) = G(B) - G(0) \), where \( G(B) \) is the conductance under a magnetic field and \( G(0) \) is the conductance under 0 T) of the sample is shown in figure 2(a) as a function of the magnetic field \( \Delta G - B \) measured at a temperature of 1.5 K, with a constant current 2.7 \( \mu \)A and at zero gate voltage. The phase coherence length \( l_\phi \) can be extracted by fitting the \( \Delta G - B \) curve in figure 2(a) using the following equation [6],

\[
\Delta G = \frac{W e^2}{L \pi \hbar} \left[ F \left( \frac{8\pi eB}{\hbar l_{\phi}^2} \right) - F \left( \frac{8\pi eB}{\hbar (l_{\phi}^2 + 2l_i^2)} \right) - 2F \left( \frac{8\pi eB}{\hbar (l_{\phi}^2 + l_i^2 + l_s^2)} \right) \right],
\]

where \( F(z) = \ln(z) + \psi(0.5 + z^{-1}) \), \( \psi(x) \) is the digamma function, \( l_\phi \) and \( l_i \) stand for the phase coherence length and the elastic intervalley scattering length, respectively, and \( l_s \) is defined as the combination of \( l_w \) and \( l_z \) as \( l_s^2 = l_w^2 + l_z^2 \). The fitting gives that \( l_\phi \sim 465 \text{ nm}, l_i \sim 272 \text{ nm} \) and \( l_s \sim 19.3 \text{ nm} \). The result of \( l_\phi > l_i \gg l_s \) meets the...
Figure 2. (a) $\Delta G - B$ data measured at 1.5 K and zero gate voltage with a current of 2.7 $\mu$A. The solid curve is the fitting by the WL theory. (b) UCF of a bilayer graphene as a function of increasing (black curve) and decreasing (red curve) magnetic field, at 1.5 K and zero gate voltage with a constant current of 10 nA.

requirement of the WL theory [6, 8]. Conductance fluctuations superimposed on the background can be observed in figure 2(a). Similar fluctuations have been reported in graphene [6, 9] and ascribed to the UCF. Usually an obvious UCF phenomenon would appear if $L < l_\phi$ and $L < l_T$. However, the UCF has also been observed for $L \gg l_\phi \sim l_T$ [22], which is similar to our situation. The rms amplitude of the conductance reduced by the statistical self-averaging effect can be expressed as

$$\text{rms}[\delta G] = \frac{W e^2}{L h} \left( \frac{2W}{l_u} \right)^{1/2} \left( \frac{l_u}{L} \right)^{3/2},$$

where $l_u^{-2} = l_\phi^{-2} + l_T^{-2}$. Using the values of $l_\phi$ and $l_T$ obtained from the above discussion, we obtain $\text{rms}[\delta G] \sim 3.73 \times 10^{-3} e^2/h$, and this is consistent with the experimental value of $\text{rms}[\delta G] \sim 1.98 \times 10^{-3} e^2/h$.

Furthermore, as shown in figure 2(b), the $G - B$ curves measured at 1.5 K and a source current of 10 nA, it is found that detailed patterns can be reproduced as sweeping the magnetic field back and forth. The reproducible conductance fluctuations ($\delta G$) reflect the intrinsic
Figure 3. (a) Comparison of magnetoconductance measured at different temperatures at a fixed current of 10 nA and zero gate voltage. (b) Temperature dependence of $\text{rms}[\delta G]$ in units of $e^2/h$.

fingerprints of the sample [12]. The rms amplitude of the conductance fluctuations $\text{rms}[\delta G]$ is $0.213 e^2/h$. It is worth noting that the $\text{rms}[\delta G]$ measured at 10 nA is about two orders of magnitude larger than that at 2.7 $\mu$A. To further study the mechanism of the UCF effect, conductance fluctuations were measured at different temperatures with a fixed source current of 10 nA. Figure 3(a) demonstrates that fluctuations of the $G$–$B$ curves measured below 10 K are highly reproducible, which further confirms the intrinsic UCF nature of the sample. The amplitude of the fluctuations was, however, found to decrease with increasing temperature, due to the reduction in electron phase coherence length. The temperature-dependent rms amplitude of the conductance fluctuations is displayed in figure 3(b) with a log–log scale. The $\text{rms}[\delta G]$ shows a power-law relationship with temperatures below 10 K, that is,

$$\text{rms}[\delta G] \propto \frac{e^2}{h} T^{-\alpha}.$$  \hspace{1cm} (4)

From figure 3(b), we have $\alpha = 0.073$. The very weak temperature dependence of the amplitude of conductance fluctuations indicates that the electron–electron interaction and electron–phonon interaction may not be dominating dephasing mechanisms in graphene [12]. However, at a
higher temperature of 20 K, the conductance fluctuation lost most of the details of UCF and its amplitude was significantly reduced, which is ascribed to the electron–phonon interaction at such a high temperature.

To further understand the 2D UCF, we have measured the conductance fluctuations as a function of magnetic field at different source currents but a fixed temperature (1.5 K). To clearly display the current fluctuation $\delta G$ varying with magnetic field, the background conductance obtained from a fit to a five-order polynomial function has been subtracted. As shown in figure 4(a), the conductance fluctuation at a current of 10 nA (black curve) is much smaller than that at 1 nA (blue curve). If one rescales $\delta G$ at 10 nA (red curve) by a factor of 10, the fluctuation pattern of the scaled $\delta G$–$B$ curve is again consistent with that measured at a current of 1 nA. It can be concluded that changing the current does not affect the nature of UCF, but can drastically change the amplitude of conductance fluctuations. Additionally, the average value of the conductance shows a weak dependence on the current bias, shown in figure 4(b) (the blue plots).

Figure 4. (a) Conductance fluctuation ($\delta G$) under a current of 1 nA (blue curve) and 10 nA (black curve) at 1.5 K and zero gate voltage. Although the fluctuation under 10 nA is much smaller than that under 1 nA, the fluctuation patterns fit well with each other between the red curve (10 times amplification of the black curve) and the blue curve. (b) The rms[$\delta G$] versus current plot and current bias dependence of average conductance at 1.5 K. The red solid line is the linear fit to the experimental data.
Therefore, the transmission probability between the electrodes and the sample is sufficiently large, and should not be the origin of the large change in $\delta G$ with varying current bias.

The variation in rms[$\delta G$] with current was calculated as plotted in figure 4(b) with the log–log coordinates. In the low current regime, a power-law relationship between rms[$\delta G$] and current is observed (the solid line in figure 4(b), i.e. $\text{rms}[\delta G] \propto I^{-\beta}$, where $\beta = 0.966 \approx 1$ was obtained from the slope of the linear fit. Therefore, we can obtain

$$\text{rms}[\delta G] \propto I^{-1}. \quad (5)$$

Considering that voltage is proportional to current, we can rewrite the conductance fluctuations as a function of applied voltage, $\text{rms}[\delta G] \propto V^{-1}$. The interpretation of the source–drain current (or voltage) dependence of conductance fluctuations is complicated and still remains a puzzle. According to the Larkin and Khmel’nitskii (LK) theory [23], at low voltages, the fluctuations are expected to be enhanced with increasing bias voltage. However, in our sample, the CFs decrease in amplitude, rather than increasing with increasing $V$; this is in conflict with the LK predictions. At higher voltages, LK [23] predicted a decrease in amplitude of the CFs due to energy averaging effects for inelastic processes. The conduction carriers are accelerated by the electric field, generating phonons and thus reducing the phase coherent length. The recent theory proposed by Mirlin et al also indicates that $\text{rms}[\delta G]$ drops with $\sqrt{1/V}$ at high voltages considering the effect of dephasing due to electron–electron interaction [24]. These theories, however, are not applicable to our experimental results. Firstly, the $V_{sd} (\sim 3.5 \mu V)$ drop across the sample is much lower than the coherence energy scale $E_\phi = \hbar D/l_\phi^2 \sim 0.24 \text{ meV} \ (\hbar$ is the Planck constant) [25], which does not satisfy the premise of the theories that the voltage should be sufficiently large [23, 24]. Secondly, the $\text{rms}[\delta G]$ varies inversely with increasing voltage, contradicting the predicted decrease with $\sqrt{1/V}$. In addition, the phase coherent length seems to be stable and its variation with the source–drain current must be negligible if any. The $l_\phi$ is dependent on the characteristic intervals of magnetic field ($\Delta B \sim 0.24 \text{ T}$) for magnetoconductance fluctuation, i.e. $l_\phi^2 \sim \phi_0/\Delta B$, where $\phi_0$ is the flux quantum [15]. The phase coherent length is estimated as $l_\phi \sim 415 \text{ nm}$ and this is consistent with the value deduced from WL theory (see the discussion preceding equation (2)). It is seen in figure 4(a) that the fluctuation patterns are reproducible at different currents and $\Delta B$ does not vary with current. Therefore, the phase coherence length does not decrease with an increase in applied current, and the voltage-induced dephasing mechanism is not suitable for our results. In addition, the current-induced heating effects can also be ruled out, since the device was placed in liquid He, and the heat exchange between the sample and the condition is very good. The UCF depends weakly on the temperature (see figure 3(b)), while the temperature effect cannot result in such a significant variation in figure 4(b).

Due to the chiral nature of carriers in graphene and the absence of backscattering, the observed anomalous current dependence of $\text{rms}[\delta G]$ can be explained by considering the scattering confinement in graphene. The classic UCF theory indicates that the fluctuations have essentially the same amplitude of $\sim e^2/\hbar$, which is caused by interference of the electronic wave along different scattering paths [12]. In a normal disordered system, there are a large number of paths for electrons to scatter with impurities. However, in graphene, the lack of backscattering will strongly suppress the number of possible scattering paths and directions, which will play a significant role in 2D UCF. It is well known that the conductance $G = Ne^2/\tau/m^*L^2$, where $N$ is the number of carriers, $\tau$ represents the relaxation time and $m^*$ is the effective mass of the carrier. Usually, we have $\tau^{-1} = \tau_{c-\tau}^{-1} + \tau_{c-p}^{-1} + \tau_{c-c}^{-1}$, where $\tau_{c-\tau}$, $\tau_{c-p}$

New Journal of Physics 12 (2010) 083016 (http://www.njp.org/)
and $\tau_{c-c}$ represent the relaxation times of the scattering events of carriers with the impurities, the phonons and other carriers, respectively. At very low temperatures, the main contribution of conductance fluctuation comes from the scattering of impurities, that is, $\delta G \approx \delta G_{c-i}$. The conductance contributed by carrier–impurity scattering can be expressed as

$$G_{c-i} = \frac{N_{c-i} e^2 \tau_{c-i}}{m^* L^2},$$

where $N_{c-i}$ is the effective number of carriers participating in electron–impurity scattering.

In a bulk system, the scattered carriers by impurities can take a large number of possible paths along different directions in real space. Unlike the 3D bulk system, the confinement in the normal dimension of the graphene layer will strongly suppress the number of scattering paths of carriers. The absence of backscattering in graphene will further decrease the possibility of carrier–impurity scattering ($p_{c-i}$) and enhance the carrier mean free path. The mutual impulsion of carriers with the same charge (electrons or holes) also restricts the carrier–impurity scattering. For example, adjacent to a given scattering center, the probability of two or more electrons scattering with the same scattering center is negligible within the electron–electron shielding length during the characteristic electron–scatter interaction time $\tau_{int}$, which is the time span for one electron entering the scattering area ($\pi r^2$ centered at the scatter and $r$ is the screening length) to leave this area. Due to the 2D nature, the absence of backscattering as well as the shielding effect of carrier–impurity scattering, the confined scattering in graphene results in the effective number of carriers participating in the electron–impurity scattering ($N_{c-i}$ in equation (6)) being smaller than the total number of carriers $N$. And the probability of carrier–impurity scattering $p_{c-i}$ is proportional to the transit time $\tau_t$ of a carrier transported through the graphene from one source to the drain. Therefore, we can express the effective number of carriers participating in the carrier–impurity scattering, $N_{c-i}$, in equation (7) as

$$N_{c-i} = N p_{c-i} \propto N \tau_t,$$

where $\tau_t = L^2/\mu V_{sd}$ is the transit time of the carrier and is about 15 $\mu$s for 10 nA bias current, $\mu$ is the carrier mobility that is similar to the field effect mobility $\mu_{FE}$ in quantity in graphene [26] and $V_{sd}$ is the voltage drop across the sample. The conductance due to electron–impurity scattering can be rewritten as

$$G_{c-i} \propto \frac{e^2 \tau_{c-i} N L^2}{m^* L^2} \mu V \propto V^{-1}. $$

It is clear that the conductance fluctuation is inversely proportional to the source current, that is, $\delta G \approx \delta G_{c-i} \propto I^{-1}$, which is the relationship found in our experiments.

It is also worth noting that the average conductance slightly increases with increasing the current bias (the blue line in figure 4(b)). Since the probability of carrier–impurity scattering decreases with increasing bias, the carriers tend to be transported through the graphene with less carrier–impurity scattering at high bias, which enhances the total conductance. Since the carrier-impurity scattering in graphene contributes to a residual resistance that should be much smaller than the total resistance, the average conductance is almost unaffected by the carrier-impurity scattering mechanism and therefore has a weak dependence on the bias.

In summary, we have investigated the UCF in the bilayer graphene system. An inverse relationship between the rms of conductance fluctuations and the applied source current was observed. A model for scattering confinement in graphene was proposed to explain the unique UCF properties we measured. Our results and discussion may be helpful to understand the nature of electron–impurity scattering in graphene.

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References

[1] Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Nature 438 197
[2] Zhang Y B, Tan Y W, Stormer H L and Kim P 2005 Nature 438 201
[3] Katsnelson M I, Novoselov K S and Geim A K 2006 Nat. Phys. 2 620
[4] Zhang Y B, Tang T T, Girit C, Hao Z, Martin M C, Zettl A, Crommie M F, Shen Y R and Wang F 2009 Nature 459 820
[5] Miller D L, Kubista K D, Rutter G M, Ruan M, de Heer W A, First P N and Stroscio J A 2009 Science 324 924
[6] Ki D K, Jeong D C, Choi J H and Lee H J 2008 Phys. Rev. B 78 125409
[7] Wu X S, Li X B, Song Z M, Berger C and de Heer W A 2007 Phys. Rev. Lett. 98 136801
[8] Tikhenenko F V, Horsell D W, Gorbachev R V and Savchenko A K 2008 Phys. Rev. Lett. 100 056802
[9] Morozov S V, Novoselov K S, Katsnelson M I, Schedin F, Ponomarenko L A, Jiang D and Geim A K 2006 Phys. Rev. Lett. 97 016801
[10] Gorbachev R V, Tikhenenko F V, Mayorov A S, Horsell D W and Savchenko A K 2007 Phys. Rev. Lett. 98 176805
[11] Kechedzhi K, Fal’ko V I, McCann E and Altshuler B L 2007 Phys. Rev. Lett. 98 176806
[12] Lee P A, Stone A D and Fukuyama H 1987 Phys. Rev. B 35 1039
[13] Berger C et al 2006 Science 312 1191
[14] Heersche H B, Jarillo-Herrero P, Oostinga J B, Vandersypen L M K and Morpurgo A F 2007 Nature 446 56
[15] Staley N E, Puls C P and Liu Y 2008 Phys. Rev. B 77 155429
[16] Rycerz A, Tworzydlo J and Beenakker C W J 2007 Europhys. Lett. 79 57003
[17] Kharitonov M Y and Efetov K B 2008 Phys. Rev. B 78 033404
[18] Brihuega I, Mallet P, Bena C, Bose S, Michaelis C, Vitali L, Varchon F, Magsud L, Kern K and Veuillen J Y 2008 Phys. Rev. Lett. 101 206802
[19] Das A, Chakraborty B, Piscanec S, Pisana S, Sood A K and Ferrari A C 2009 Phys. Rev. B 79 155417
[20] Malard L M, Pimenta M A, Dresselhaus G and Dresselhaus M S 2009 Phys. Rep. 473 51
[21] Li G and Andrei E Y 2007 Nat. Phys. 3 623–7
[22] Beenakker C W J and van Houten H 1988 Phys. Rev. B 37 6544
[23] Larkin A I and Khmel’nitskii D E 1986 Sov. Phys.—JETP 64 1075
[24] Ludwig T, Blanter Y M and Mirlin A D 2004 Phys. Rev. B 70 235315
[25] Ambegaokar V and Eckern U 1990 Phys. Rev. Lett. 65 381–4
[26] Lin Y-M, Dimitrakopoulos C, Jenkins K A, Farmer D B, Chiu H-Y, Grill A and Avouris P 2010 Science 327 662