Rare $B$ Decays Beyond $B \to X_s\gamma$

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Abstract

I discuss recent progress in our understanding of exclusive rare and semileptonic decays. I show the impact of HQET when combined with the predictions in the Large Energy Limit of QCD, focusing first on applications to $B \to K^*\ell^+\ell^-$. I also discuss the constraints on semileptonic form-factors that appear in HQET/LEET with the use of radiative decay data, and update these to include the effects of next-to-leading order contributions in $B \to K^*\gamma$, as well as the latest data.

1 Introduction

In the Standard Model (SM), Flavor Changing Neutral Currents (FCNCs) are forbidden at tree level. They can occur starting at one loop. For the $b \to s$ transitions, the diagrams involving the top quark dominate the short distance rate. This is essentially a consequence of non-decoupling in spontaneously broken gauge theories: heavy fermions in the loops give contributions that do not vanish as the mass increases but rather grow. Thus, FCNC processes at relatively low energies have the potential to explore high energy scales such as the weak scale or even beyond. Extensions of the SM, such as supersymmetry, technicolor, etc. would result in new contributions to FCNC loops, leading to deviations from the SM predictions. This sensitivity of FCNC processes to high energy scales, and therefore to new physics makes them of great interest. This is particularly true in $B$ decays since it is possible that the third generation is involved in electroweak symmetry breaking\footnote{For instance, this is the case in supersymmetry, where the stop plays a crucial role; as well as in topcolor models where top condensation is responsible for (at least partially) breaking electroweak symmetry.}. However, $B$ decays are affected by hadronic uncertainties that may obscure the interesting short distance processes. Such uncertainties result from the fact that hadronization is a non-perturbative problem only tractable from first principles by lattice gauge theory. The use of inclusive decay modes greatly circumvent this problems. There, the uncertainties are mostly not from hadronization (only relevant in the initial state) but from perturbative QCD. Exclusive modes, on the other hand, are largely affected by theoretical uncertainties from hadronic matrix elements of the short distance operators.

While lattice calculations progress toward greater precision and accuracy, we can ask the question: can rare $B$ decays be used as tests of the one-loop structure of the SM with our current\footnote{Invited talk given at the 9th Heavy Flavors conference, Caltech, Pasadena, Sept.10-13 2001.}
knowledge of hadronic matrix elements? The comparison with the program of electroweak precision measurements, mostly performed at the $Z$ pole, is interesting. That program relied on processes with large tree-level SM amplitudes (large SM background), but exquisite experimental precision made it possible to achieve sensitivity to one loop contributions since the accuracy of our theoretical knowledge matches this precision. On the other hand, rare $B$ decays and FCNC processes in general, start at one loop but are affected by large theoretical uncertainties. These considerations are still valid even if we are talking about new physics entering at tree level. Very large data samples of $b \to s\gamma$ and $b \to s\ell^+\ell^-$ decays will be available soon. How good a test of the SM will these be? The inclusive $B \to X_s\gamma$ decay already constrains physics beyond the SM. Even more constraints will result once $b \to s\ell^+\ell^-$ modes begin to be measured.

Inclusive modes are theoretically cleaner. The uncertainties are, in principle, from perturbative QCD and the OPE for heavy quarks. In practice, there is the added problem that cuts are needed to make contact with experiment [1]. This introduces additional uncertainties. In addition, the experimental signals require a very clean environment. Exclusive decays are easier experimentally. But, as we mentioned earlier, they are affected by large hadronic uncertainties coming as unknown form-factors. These are the kinds of things the lattice will one day compute precisely. But for now (short of using models) we have to rely on symmetries and other related tricks in order to extract the short distance physics from these modes.

The factorization of short and long distance physics takes place in the effective hamiltonian at low energies. This is obtained by integrating out the heavy fields (e.g. the $W$, the top quark, the gluino, etc.) and has the form

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb}^* V_{ts} \sum_{i=1}^{10} C_i(\mu) \, O_i(\mu)$$  \hspace{1cm} (1)

Here, the basis $\{O_i\}$ constitute a complete set of operators leading to $b \to s$ transitions, whereas the Wilson coefficient functions $C_i(\mu)$ encode the short distance information coming from integrating out the heavy degrees of freedom. For instance in the SM the $C_i(\mu)$ come from loop integrals involving the $W$ and the top quark and they will depend on their masses. If physics beyond the SM contributes to these decays it will shift the values of the Wilson coefficients with terms now depending on gluino, squark or technipion masses. The $\mu$ dependence of the Wilson coefficients is in principle canceled by that of the matrix elements of the operators. At leading order in $\alpha_s$ the operators mediating $b \to s$ transitions are

$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} b_R \, F^{\mu\nu}$$ \hspace{1cm} (2)

$$O_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_{\mu} b_L)(\bar{\ell}_\gamma \gamma_\mu \ell)$$ \hspace{1cm} (3)

$$O_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_{\mu} b_L)(\bar{\ell}_\gamma \gamma_{\mu} \gamma_5 \ell).$$ \hspace{1cm} (4)

Among these, only $O_7$ contributes to processes with the photon on-shell, such as $b \to s\gamma$. Mixing with other operators occurs due to the strong interactions. Most notably with the gluonic dipole operator $O_8 = (g/16\pi^2)m_b \bar{s}_L T^a \sigma_{\mu\nu} b_R G^{a\mu\nu}$, and especially with $O_2 = (\bar{s}_L \gamma_{\mu} c_L)(\bar{c}_L \gamma^{\mu} b_L)$, which is generated by tree-level $W$ exchange.
2 Exclusive Decays

The hadronic matrix elements can be parametrized in terms of form-factors. For the $B \to K\ell^+\ell^-$ decay the hadronic matrix elements of the operators $O_7, O_9$ and $O_{10}$ can be written as

$$
\langle K(k)|\bar{s}\sigma_\mu q^\nu b|B(p)\rangle = i\frac{f_T}{m_B+m_K}\left\{(p+k)_\mu q^2 - q_\mu (m_B^2 - m_K^2)\right\},
$$

(5)

$$
\langle K(k)|\bar{s}\gamma_\mu b|B(p)\rangle = f_+(p+k)_\mu + f_- q_\mu ,
$$

(6)

with $f_T(q^2)$, and $f_\pm (q^2)$ unknown functions of $q^2 = (p-k)^2 = m_{T+\ell-}^2$. In the SU(3) limit $f_\pm$ in (3) are the same as the form-factors entering in the semileptonic decay $B \to \pi\ell\nu$. For $B \to K^*\ell^+\ell^-$ decays of the “semileptonic” matrix elements over vector and axial vector currents are

$$
\langle K^*(k,\epsilon)|\bar{q}\gamma_\mu |B(p)\rangle = \frac{2V(q^2)}{m_B+m_V}\epsilon_{\mu\alpha\beta}\epsilon^{\nu\rho}\gamma^\alpha q^\beta
$$

(7)

$$
\langle K^*(k,\epsilon)|\bar{q}\gamma_\mu \gamma_5 |B(p)\rangle = 2iA_0(q^2)\frac{\epsilon^\nu q^\rho}{q^2}q_\mu + i(m_B+m_V)A_1(q^2)\left(\epsilon^\nu - \frac{\epsilon^\nu q^\rho}{q^2}q_\mu \right)
$$

(8)

and for the FCNC magnetic dipole operator $\sigma_{\mu\nu}$

$$
\langle K^*(k,\epsilon)|\bar{q}\gamma_{\mu}(1+\gamma_5)q^\nu b|B(p)\rangle = i2T_1(q^2)\epsilon_{\mu\alpha\beta}\epsilon^{\nu\rho}\gamma^\alpha q^\beta
$$

$$
+T_2(q^2)\left\{\epsilon^\nu (m_B^2 - m_V^2) - (\epsilon^\nu \cdot p)(p+k)_\mu \right\}
$$

$$
+T_3(q^2)(\epsilon^\nu \cdot p)\left\{q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu \right\} ,
$$

(9)

where $\epsilon_\mu$ denotes the polarization four-vector of the $K^*$. In the SU(3) limit these form-factors also describe $B \to \rho\ell\nu$, as well as $B_s \to \phi\ell^+\ell^-$ decays. The form-factors defined in eqns.(1)-(3) are not calculable in a controlled approximation with the notable exception of lattice gauge theory. We will see however, that symmetries and generally the behavior of hadrons in certain limits will allow us to gain important insight on these quantities. Notice that $T_1(0) = T_2(0)$ and $T_3$ does not contribute to the amplitude to the radiative decay into an on-shell photon, i.e. in $B \to K^*\gamma$. So now $Br(B \to K^*\gamma) \propto |T_1(0)|^2$, where the short distance is basically the same as in inclusive decays.

2.1 Heavy Quark Symmetry and $B \to K^*\ell^+\ell^-$

In the HQL $m_b \gg \Lambda_{QCD}$ the form-factors $T_i(q^2)$ corresponding to the dipole operator are not independent of the semileptonic form-factors $V(q^2), A_i(q^2)$. Instead, they obey the following

\footnote{With some caveats and exceptions we discuss later on. See also [9].}
relations \[ \frac{m_B^2 - m_V^2}{q^2} \left[ T_1(q^2) - T_2(q^2) \right] = \frac{3m_B^2 - q^2 + m_V^2}{2m_B} \frac{V(q^2)}{m_B + m_V} - \frac{m_B + m_V}{2m_B} A_1(q^2), \]

\[ T_3(q^2) = \frac{m_B^2 - q^2 + 3m_V^2}{2m_B} \frac{V(q^2)}{m_B + m_V} + \frac{m_B^2 - m_V^2}{2mbq^2} m_V A_0(q^2) \]

In terms of the symmetries of the HQET, eqns. (10) result from the Heavy Quark Spin Symmetry (HQSS) that arises in the heavy quark limit due to the decoupling of the spin of the heavy quark [2]. These relations imply, for instance, that the $B \to \rho \ell \nu$ and $B \to \rho \ell^+ \ell^-$ are described by the same form-factors. Additionally, if $SU(3)$ symmetry is assumed, these also are the $B \to K^* \ell^+ \ell^-$ form-factors.

2.2 The Large Energy Limit

We now consider the Large Energy Limit (LEL) [3] for heavy-to-light transitions into a vector meson as the ones we are studying. As a result, one recovers the HQSS form-factor relations (10), but now there are additional new relations among the form-factors defined in (7-9). These read as [3]

\[ V(q^2) = \left( 1 + \frac{m_V}{M} \right) \xi_\perp(M, E), \tag{11} \]

\[ A_1(q^2) = \frac{2E}{M + m_V} \xi_\perp(M, E), \tag{12} \]

\[ A_2(q^2) = \left( 1 + \frac{m_V}{M} \right) \left\{ \xi_\perp(M, E) - \frac{m_V}{E} \xi_\parallel(M, E) \right\}, \tag{13} \]

\[ A_0(q^2) = \left( 1 - \frac{m_V^2}{M E} \right) \xi_\parallel(M, E) + \frac{m_V}{M} \xi_\perp(M, E), \tag{14} \]

and

\[ T_1(q^2) = \xi_\perp(M, E), \tag{15} \]

\[ T_2(q^2) = \left( 1 - \frac{q^2}{M^2 - m_V^2} \right) \xi_\perp(M, E), \tag{16} \]

\[ T_3(q^2) = \xi_\perp(M, E) - \frac{m_V}{E} \left( 1 - \frac{m_V^2}{M^2} \right) \xi_\parallel(M, E). \tag{17} \]

and will receive corrections that roughly go as $(\Lambda_{QCD})/E_h$. It is apparent from eqns. (11)-(17) that, in the LEL regime, the $B \to V \ell^+ \ell^-$ decays are described by only two form-factors: $\xi_\perp$ and $\xi_\parallel$, instead of the seven apriori independent functions in the general Lorentz invariant ansatz.
of the matrix elements. Here, $\xi_{\perp}$ and $\xi_{\parallel}$ are functions of the heavy mass $M$ and the hadronic energy $E$, and refer to the transverse and longitudinal polarizations, respectively.

This simplification leads to new relations among the form-factors. For instance, the ratio of the vector form-factor $V$ to the axial-vector form-factor $A_1$,

$$R_V(q^2) \equiv \frac{V(q^2)}{A_1(q^2)} = \frac{(m_B + m_V)^2}{2E_Vm_B},$$

(18)

is independent of any of these unknown, non-perturbative functions $\xi_{\perp,\parallel}$ and is determined by purely kinematical factors. Here, $E_V = (m_B^2 + m_V^2 - q^2)/(2m_B)$ denotes the energy of the final light vector meson. A similar relation holds for $T_1$ and $T_2$, since they both are also proportional to the “transverse” form-factor $\xi_{\perp}$. As we will see below, these predictions have important consequences for observables at large recoil energies ($\text{low } q^2$).

### 2.3 Corrections to LEET

The predictions obtained in the LEL receive corrections from several sources. In the past year or so some of them have been extensively addressed in the literature. Here is a quick review.

**Hard Corrections:** These corrections result from the exchange of hard gluons. There are two kinds of them: factorizable and non-factorizable gluon exchange. The first kind corresponds to either the renormalization of the heavy-light currents which already appears in Ref. [2], or hard gluon exchange with the spectator quark. The latter can be computed in the Brodsky-Lepage formalism for exclusive processes at large momentum transfers [7]. This was done in Ref. [8], where it was found that the form-factor relations in (11)-(17) receive $\alpha_s$ corrections that are typically 10% or smaller. An interesting case is the ratio of vector to axial-vector form-factors in eqn. (18), which receives no $\alpha_s$ corrections to leading order in the $1/E$ expansion. This has an interesting explanation and we will come back to this point below.

The second kind of hard gluon corrections are the so-called non-factorizable ones [9,10]. These are mediated by diagrams where the gluon exchanged with the spectator comes from either the insertion of the operator $O_8$, or from the insertion of four-fermion operators $O_1 - O_6$ with the gluon attached to the quark loop. These corrections cannot be absorbed by form-factors or renormalization of the currents. Thus they are genuinely distinct hard corrections that do not occur in inclusive decays. In Refs. [9,10] the effects of all these contributions are computed and are rather large in $B \to K^*\gamma$. The effective “exclusive” Wilson coefficient $C_7^{\text{excl}}$ is shifted by roughly 30%, whereas the effect in $C_9$ is smaller. The authors go on to compute the exclusive rate by making use of light-cone QCD sum rules for the form-factors, resulting in a rate which is approximately a factor of two larger than the experimental rate. This may be signaling a problem with the form-factor calculation rather than one with the non-factorizable hard gluons.

**Collinear Gluons:** In Ref. [11] an effective field theory including collinear quarks and gluons is developed. This theory includes LEET but, unlike LEET, it has the correct infrared behavior.
Sudakov logarithms are accounted for in this treatment. Concerning its application to exclusive decays, this “complete” LEET does not modify the form-factor relations implied by eqns.[11,17]. A main ingredient of LEET is preserved when the interaction with collinear gluons is taken into account: energetic quarks still are two component spinors in the LEL. Thus the complete LEET including collinear gluons confirms that the LEET relations are valid in the large energy limit of QCD.

Power Corrections: Just as in HQET the $1/m_Q$ corrections are potentially a very important source of uncertainty, the LEL predictions are affected by $1/E$ corrections. Unlike in HQET, it is not clear that LEET is a framework where the $1/E$ corrections can be estimated. Even when collinear gluons are incorporated, it is difficult to separate degrees of freedom to be integrated out so that an effective field theory can emerge. Perhaps, we can still estimate the size of these corrections with this caveat in mind. More work is needed, and certainly once experimental tests of LEET predictions become available we hope to understand more about them.

2.4 Forward-Backward Lepton Asymmetry

The forward-backward asymmetry for leptons as a function of the dilepton mass squared $m_{\ell\ell}^2 = q^2$ is defined as

$$A_{FB}(q^2) = \int_0^1 \frac{d^2\Gamma}{dx dq^2} dx - \int_{-1}^0 \frac{d^2\Gamma}{dx dq^2} dx,$$

(19)

where $x \equiv \cos \theta$, and $\theta$ is the angle between the $\ell^-$ and the $\bar{B}^0$ in the dilepton center-of-mass frame.\footnote{In Ref. [12] it is erroneously stated that $\theta$ is defined with respect to $\ell^+$. But the expressions there correspond to the current definition. This explains the sign difference with respect to Ref. [10].}

The asymmetry is proportional to the Wilson coefficient $C_{10}$ and vanishes with it. Furthermore, it is proportional to a combination of $C_{9}^{\text{eff.}}$ and $C_7^{\text{eff.}}$ such that it has a zero in the physical region if the following condition is satisfied \[\text{Re}[C_{9}^{\text{eff.}}] = -2m_B^2 q_0^2 C_7^{\text{eff.}}\] \[\{T_1 V (m_B + m_{K^*}) - (m_B - m_{K^*}) \frac{T_2}{A_1}\} \]

(20)

where $q_0^2$ is the position of the $A_{FB}$ zero and all $q^2$-dependent quantities\footnote{Note that the sign difference in eqn.(20) with respect to a similar expression in Ref.[13] is due to a sign in the definition of $V$.} are evaluated at $q_0^2$. In inclusive $B \to X_s\ell^+\ell^-$, the zero of the asymmetry implies the relation:

$$Re[C_{9}^{\text{eff.}}] = -2(m_B^2/q_0^2) C_7^{\text{eff.}}.$$\[\text{Thus, in principle the condition in (20) is affected by the presence of the hadronic form-factors making it a priori a more uncertain relation. However, by making use of the HQSS relations (11) for $T_1(q^2)$ and $T_2(q^2)$, the form-factor eqn. (21) simplifies to}$$

\[\frac{Re[C_{9}^{\text{eff.}}]}{C_7^{\text{eff.}}} = \frac{m_b}{q_0^2} \left\{ \frac{2m_B k^2}{(m_B + m_{K^*})^2} R_V + \frac{(m_B + m_{K^*})^2}{2m_B R_V} + 2(m_B - E_{K^*}) \right\} \]

(21)
where $R_V$ is the ratio of vector to axial-vector form-factors defined in (18) and evaluated at $q_0^2$. Thus, the determination of the zero of $A_{FB}(q^2)$ in $B \to K^*\ell^+\ell^-$ gives a relation between the short distance Wilson coefficients $C_9^{\text{eff.}}$ and $C_7^{\text{eff.}}$, where the only uncertainty from form-factors is in the ratio $R_V$. As mentioned earlier, since (21) was derived using the heavy quark spin symmetry, it is expected to receive small corrections. In principle, information on $R_V(q^2)$ could be extracted from the $B \to \rho\ell\nu$ decay. But it turns out that this may not be necessary. If we plot the asymmetry vs. the dilepton mass in a variety of models we note that the asymmetry is agreed upon with exceptional accuracy! In Ref. [12] it was noted that this feature must emerge from a “factorization” of the soft physics in such a way that

$$ V(q^2) \simeq A_{\text{kin.}} \times F_{\text{soft}}, \quad A_1(q^2) \simeq B_{\text{kin.}} \times F_{\text{soft}}, $$

so that the soft physics cancels in the ratio $R_V$. It was first recognized in Ref. [13] that this is precisely what the LEET predicts: $R_V(q^2) = (m_B + m_{K^*})^2/(2m_BE_{K^*})$ as extracted from eqns. (11) and (12). Then, the condition for the vanishing of $A_{FB}(q^2)$ reads now

$$ \Re[C_9^{\text{eff.}}] = -2 \frac{m_B^2}{q_0^2} C_9^{\text{eff.}} \left(1 - \frac{m_{K^*}^2}{2m_BE}ight). $$

Thus, in the LEL the position of the zero of $A_{FB}(q^2)$ in $B \to K^*\ell^+\ell^-$ is predicted in terms of the short distance Wilson coefficients $C_9^{\text{eff.}}$ and $C_7^{\text{eff.}}$. The hard gluon corrections discussed above would shift the position of the zero by a calculable amount given in Ref. [14]. There it is pointed out that non-factorizable contributions change the value of $q_0^2$ from what it would be obtained if the “inclusive” Wilson coefficients are used by an amount around $(20 - 30)\%$. This is roughly $q_0^2 = (4.2 \pm 0.6)$ GeV$^2$. The plot in Fig.1 takes all these corrections into account.
2.5 Semileptonic Form-factors in the Large Energy Limit

We finally discuss another application of LEET. In Ref. [14] the LEET prediction for $R_V$ in eqn. (18) was used in combination with the $B \to K^{*}\gamma$ and $B \to X_s \gamma$ data, and Heavy Quark Spin Symmetry in order to constrain the semileptonic form-factors $V$ and $A_1$ at $q^2 = 0$. Here we update this analysis by incorporating the factorizable and non-factorizable hard corrections from Refs. [9,10].

Since the NLO corrections in the exclusive mode are not canceled by the ones in the inclusive decay, we will not normalize the exclusive branching ratio to the inclusive one. Instead we make use of the exclusive data only in order to extract the $B \to K^{*}\gamma$ form-factor. The branching ratio reads

$$Br(B \to K^{*}\gamma) = \frac{\alpha G_F^2}{32\pi^2} |V_{cb}V_{cs}^*|^2 m_B^2 m_{K^*}^2 (1 - \frac{m_{K^*}^2}{m_B^2})^3 |A_7|^2 |T_1(0)|^2,$$

(24)

where the form-factor $T_1(q^2)$ was defined in [9], and $A_7$ is the NLO effective Wilson coefficient for the exclusive mode, which differs from the one entering in inclusive decays. We use the running mass $m_b(m_b) = (4.2 \pm 0.2)$ GeV, and $|V_{cb}V_{cs}^*| = 0.04$. The value of $A_7$ was computed in Ref. [9] to be $A_7 = -0.4072 - 0.00256$. We make use of the world average for the neutral mode $\Br(B^0 \to K^{*0}\gamma) = (4.56 \pm 0.37) \times 10^{-5}$. We obtain $T_1(0) = 0.31 \pm 0.02$, where the error mainly reflects the experimental uncertainty, the uncertainty in the $b$ quark mass and the scale dependence in $A_7$. This result assumes the SM for the calculation of the short distance coefficient $A_7$. However, this is not a very strong assumption given that the agreement of the inclusive calculation with the $B \to X_s \gamma$, and the fact that the new physics is likely to affect this rate and the exclusive one similarly. Armed with this extracted value of $T_1(0)$, now we can go to the HQSS relations and turn them into a relation between $V(0)$ and $A_1(0)$. This is

$$A_1(0) = \frac{2m_B}{m_B + m_{K^*}} T_1(0) - \frac{m_B - m_{K^*}}{m_B + m_{K^*}} V(0).$$

(25)

Eqn. (25) results in a constraint in the $(V(0), A_1(0))$ plane, which in Fig. 2 corresponds to the band descending from left to right. In addition, the expression (18) for $R_V$ gives another constraint corresponding to a straight line going through the origin. This LEET prediction will surely receive corrections at next to leading order in the $1/E$ and $1/m_b$ expansions. These corrections have not been computed and we simply guess they are of order $m_{K^*}/m_B \approx 0.17$, giving the cone in Fig. 2. We should notice, however, that the LEET prediction for $R_V$ does not receive hard gluon or collinear gluon corrections. We will come back to this point. The ellipses in Fig. 2 correspond to 68% and 95% C.L. intervals. The fit gives

$$V(0) = 0.36 \pm 0.04,$$

$$A_1(0) = 0.27 \pm 0.03.$$

(26)

The results in eqn. (26) differ from Ref. [14] where the NLO corrections had not been taken into account. These lift the value of the Wilson coefficient by roughly 30%, resulting in a lower value for $T_1(0)$, and through the fit in Fig. 2, in lower values for both $V(0)$ and $A_1(0)$. Also,
here we use the most recent average of the neutral mode resulting in a thinner band than the one in Ref. [14].

We compare our findings for $V(0)$ and $A_1(0)$ with several model predictions. For illustration, we take the Bauer-Stech-Wirbel (BSW) model from Ref. [16] (cross), the modified version of the Isgur-Scora-Gnstein-Wise (ISGW2) model from Ref. [17] (diamond), a recent relativistic constituent quark model prediction by Melikhov and Stech (MS) [18] (star), a recent calculation in the Light Cone QCD Sum Rule (LCSR) formalism of Ref. [19] (diagonal cross) and the prediction by Ligeti and Wise (LW) from Ref. [20] (square). As it can be seen from Fig. 2, the latter is even more excluded now. As discussed in more detail in Ref. [14], this is likely to be due largely to the use of a monopole form-factor to extrapolate from the charm data to the maximum recoil in a $B$ decay. Corrections to the heavy quark flavor symmetry, although large, are unlikely to be the sole explanation for such a discrepancy. A new finding in this update is the apparent exclusion of the MS and LCSR predictions for $V(0)$ and $A_1(0)$. These models, were in agreement with the fit in Ref. [14]. Of course, this exclusion refers to the predicted central values for the models. In some cases the predictions have large errors and modifications in the calculations may bring them into agreement with the fit.

A potentially large isospin violation splitting the neutral and charged modes was found in Ref. [20] in the context of QCD factorization. This would shift the value of $A_7$ in the neutral mode by a few percent, resulting in a similarly small shift in the value of $T_1(0)$ extracted from eqn. (24).

Finally, we comment on the leading order LEET prediction for $R_V$, eqn. (18). Making use of the HQSS relations (10), the transverse helicity amplitudes for a generic $B^- \rightarrow V^- \ell \ell'$ transition can be written as

$$H_\pm = F \left( V \mp \frac{(m_B + m_V)^2}{2m_B k_V} A_1 \right),$$

(27)

where $F$ is a factor depending on the mode under consideration (e.g. Wilson coefficients, coupling constants, etc...) and $k_V$ is the momentum of the vector meson. Thus, we see from the form of $R_V$ in the large energy limit, that the “+” helicity vanishes $H_+ = 0$ in the LEL regime, up to residual terms of order $m_V^2/2E_V^2$. This is not a surprise: in the limit of an infinitely heavy quark decaying into a light quark, the helicity of the latter is “inherited” by the final vector meson. In the SM, the $(V - A)$ structure in semileptonic decays is reflected in the dominance of the $H_-$ transverse helicity. On the other hand, the amplitude to flip the helicity of the fast outgoing light quark is suppressed by $1/E_h$. Quark models tend to have this concept built in, which explains the agreement on the position of the zero in Fig. 1. This is also the reason why $\alpha_s$ corrections from hard gluon exchange between the spectator quark and the fast light quark do not affect eqn. (18): they are not helicity-changing at leading order in $1/E_h$. The same is true for the ratio of $T_1$ and $T_2$.

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Figure 2: Constraints on the semileptonic form-factors \( V(0) \) and \( A_1(0) \) from \( B^0 \rightarrow K^{*0}\gamma \) data plus HQSS (thicker band) together with the relation from the LEL (cone). The ellipses correspond to 68% and 95% confidence level intervals. Central values of model predictions are also shown and correspond to BSW \[16\] (vertical cross), ISGW2 \[17\] (diamond), MS \[18\] (star), LCSR \[13\] (diagonal cross) and LW \[19\] (square), respectively. This updates Ref. \[14\] to include hard corrections as computed in Ref. \[9,10\], as well as the most recent data.

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