A Single Batch Processor Scheduling Model with Additional Setup Times to Minimize Total Actual Flowtime of Parts of Multiple Items

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Abstract. The problem investigated in this paper is arranging parts for the etching process on a single batch processor in a craft company. The company produces several different metal plate types, and each type requires another processing time and setup time. The machine also needs additional setup after processing several production batches. All metal plates that have been processed will be delivered simultaneously at the expected due date. The objective is to minimize the total actual flowtime of parts of multiple items through the shop. The problem is formulated as a mixed-integer nonlinear programming model. A simple numerical example is provided to demonstrate the execution of the mathematical model to a hypothetical question.

1. Introduction

This research was conducted at a craft company with the primary base material in metal plates, one surface patterned following the final product design. The operation to be performed on the metal plates is the etching process, the immersion of the metal plates in a chemical solution with a particular concentration for a specific time. Besides the craft industry, the etching process is applied for many purposes. Various etching-based techniques have been used, among others, in micro-machines [1], for electro-extraction in aluminum foil production [2], used in the manufacture of integrated circuits [3], and chemical etching in the production of silicon [4].

The etching operation for several metal plates is carried out simultaneously in a tub, and all-metal plates will start and end the etching process simultaneously. It is not allowed to remove or insert metal plates during the etching process because it will affect the etching results’ quality. Setup time is needed before carrying out an etching operation. After several times etching operation is carried out, then the concentration of the chemical solution in the tub will decrease to require concentration checking and chemical solutions. The time needed for these two activities is referred to as additional setup time.

The problem faced by this craft company is arranging the schedule for etching with the condition that the company produces several different types of metal plates, and each type requires additional processing and setup time. All metal plates that have been processed will be sent simultaneously at an expected due date, d. The tub where the etching process is carried out a batch processor, the metal plates are a part, and some metal plates are a batch to be processed in a batch processor. The problems in this study can be described as follows. There
are \( n \) parts, consisting of \( k \) different items, that must be processed on a single batch processor. The completed piece is delivered simultaneously on the expected due date, \( d \). The processing and set up times of the part of item \( g \) are \( t_g \) and \( s_g \) respectively for \( g = 1, 2, \ldots, k \). Additional setup time \( a \) is required every time the batch processor has processed \( x \) batches. The \( s_g \) and \( a \) is independent of the batch size and batch processing order.

The batching and scheduling problem in batch processors (or batch processing machines) is quite an active topic in the literature. A batch processor can process many parts in a batch, where the processing time is the longest processing time of the parts. The number of elements in the collection cannot exceed the batch capacity. Such characteristics can be found in many types of machines, such as additive manufacturing [5], heat treatment in metalworking industries [6], and diffusion in wafer fabrication [7]. Among the most recent publications on the development of algorithms for a single-stage batch, processor scheduling is an arc-flow approach for non-identical job sizes [8] and ant colony optimization for machines with different capacities [9].

The batching and scheduling problem for a batch processor is divided into batching and scheduling phases. During the batching phase, the parts are grouped to form several batches, and then the sequence of the resulting sets is determined in the scheduling phase. We have previously developed a series of research [10–12, 15] that addressed the basic flow time concept to fit the batch scheduling problem dealt with a batch processor. Halim et al. [14] have conducted a similar study, and the difference from this study lies in the number of items from the part being processed. The model developed in [14] is single item parts, while the conditions in this craft company are multiple item parts. Therefore it is necessary to create the single item part model [14] to various item parts. Another research study has been conducted by Hidayat et al. [13] regarding scheduling on a single batch processor with multiple item parts but does not require additional set up time. Thus, the research emphasizes two basic models [13] and [14].

2. Model development

The processed parts come from several different items where each item’s processing time is additional in the multiple item scheduling model. The grouping of elements to form several batches is carried out based on the same processing time. Thus the batch processing time depends on the type of items from the parts in the set [16–21]. Equation (1) is the \( F \) model developed by Halim et al. [14] for \( n \) single item parts. It is assumed that the parts’ arrival time can be arranged just as the process is about to begin and that the completed pieces are to be delivered at an expected due date, \( d \). \( N \) is the number of batches processed, while \( t[i], s[i], a[i] \) and \( Q[i] \) are respectively processing time, typical setup, additional setup, and batch size at the i\( \text{th} \) position. Batch order is carried out backwardly, starting from the specified due date \( d \) and then moving towards time zero on a time scale. Equation (1) shows the formula of the total actual flow time.

\[
F = \sum_{i=1}^{N} \left\{ \sum_{j=1}^{i} (t[j] + s[j] + a[j]) - (s[i] + a[i]) \right\} Q[i]
\]

\[
a[i] = \begin{cases} 
  a & \forall i = N, (N - 2x), \ldots, (N - [m - 1]x) \\
  0 & \text{otherwise}
\end{cases}
\]

\( x \) denotes the number of batches between two consecutive additional setups, and \( m \) represents the different configurations during the scheduling period. The value of \( m \) equals the ratio of \( N \) to \( x \) (\( m = N/x \)), and if the result of \( N/x \) is not an integer, then \( m \) is set to be rounded up of \( N/x \).

In this study, the batch size and batch processing sequence will be found to minimize the objective function \( F \). Equation (1) shows that the number of part items does not affect \( F \). Thus,
equation (1) applies to the condition of multiple items part. In the numerous items condition, the processing time and batch set up $b_i$ are $t_i = \sum_{g=1}^{k} z_{ig} t_g$ and $s_i = \sum_{g=1}^{k} z_{ig} s_g$. The binary variable, $z_{ig}$ will have a value of 1 if the parts in batch $b_i$ are item $g$, or 0 if it is not item $g$. The number of batches in which part of item $g$ must be equal to the sum of the binary $z_{ig}$, that is $N_g = \sum_{i=1}^{N} z_{ig}$. The request for part of item $g$ must be equal to the sum of the multiplication of the binary number $z_{ig}$ with the batch size $b_i$ namely $n_g = \sum_{i=1}^{N} Q_i z_{ig}$ for $g = 1,2, ..., k$ and $i = 1, 2, ..., N$.

The time needed to process $N$ batches is equal to $\sum_{i=1}^{N} (t_i + s_i + a_i) - (s_N + a_N)$ . To fulfill the condition that the processing of the entire batch on the production floor does not exceed the common due date and does not overtake the point $t = 0$, then $\sum_{i=1}^{N} (t_i + s_i + a_i) - (s_N + a_N) \leq d$. To fulfill the condition that all processed parts are equal to the total demand, then $N_g = \sum_{i=1}^{N} z_{ig}$ and $n_g = \sum_{i=1}^{N} Q_i z_{ig}$ . In the backward scheduling approach, the first position batch is the batch that will be processed last. The final batch processing has to be finished at the deadline therefore $B_1 + t_1 = d$.

3. Problem formulation
The problem can be formulated as a Mixed Integer Non-Linear Programming (MINLP) problem as follows.

Minimize

$$F = \sum_{i=1}^{N} \left\{ \sum_{j=1}^{i} (t_j + s_j + a_j) - (s_i + a_i) \right\} Q_i$$

subject to:

$$t_i = \sum_{g=1}^{k} z_{ig} t_g \quad \forall i = 1,2,\ldots,N$$

$$s_i = \sum_{g=1}^{k} z_{ig} s_g \quad \forall i = 1,2,\ldots,N$$

$$N_g = \sum_{i=1}^{N} z_{ig} \quad \forall g = 1,2,\ldots,k$$

$$n_g = \sum_{i=1}^{N} Q_i z_{ig} \quad \forall g = 1,2,\ldots,k$$

$$\sum_{i=1}^{N} (t_i + s_i + a_i) - (s_N + a_N) \leq d$$

$$B_1 + t_1 = d$$

$$0 < Q_i \leq c \quad \forall i = 1,2,\ldots,N$$

$$N \geq k$$

$$z_{ig} \in 0,1 \quad \forall g = 1,2,\ldots,k; \forall i = 1,2,\ldots,N$$

$$a_{[i]} = \begin{cases} a & \forall i = N, (N - 2x), \ldots, (N - [m - 1]x) \\ 0 & \text{otherwise} \end{cases}$$
Constraint (2) states that the batch processing time $b_i$ depends on the type of item from the part that is in the $b_i$ batch. This also applies to the setup time of constraint (3). Constraint (4) ensures that the number of batches that are part of item $g$ equals the sum of the binary variable for item $g$. Constraint (5) provides that the part number of item $g$ processed is the same as the part number of item $g$ requested by the customer. Constraint (6) ensures that the entire batch is processed within the available timeframe, i.e., does not exceed the deadline. Constraint (7) is to ensure that batch processing of $b_i$ is completed on the due date. Constraint (8) represents the upper and lower limits for the batch size. Constraint (9) states that the total batch formed is greater than or equal to the number of types of items. Constraint (10) defines a binary variable $z_{ig}$ to represent the item type of the part in the batch $b_i$. Constraint (11) states that a batch can accept parts from only one item type. Constraint (12) represents the value added to the setup time of the batch processor.

4. Numerical example

In an etching facility, three-part items (types) are to be processed in batches. Table 1 shows the number of parts, the processing time, and each item’s setup time. Each set can only accept a maximum of 10 parts. In addition to each batch setup, an additional configuration of 4 time-unit is performed for every three consecutive batches, starting from the first processed batch. The due date for all parts is 100.

| Part | Quantity Ordered | Processing time | Setup time |
|------|------------------|----------------|------------|
| 1    | 16               | 8              | 4          |
| 2    | 8                | 7              | 3          |
| 3    | 7                | 12             | 5          |

The MINLP model is implemented and executed in LINGO software. The optimal global solution for the numerical example can be found in a few seconds. Table 2 lists the part item, quantity, and the start time of each batch. Figure 1 shows the Gantt chart. The additional setups, shown in red color in the chart, are performed before processing batch four and batch 1. Note that the batch numbering is backward, which means that batch four is processed first, and batch 1 is the closest batch to the due date.

| Batch | Part | Quantity | Start time ($B_i$) |
|-------|------|----------|--------------------|
| 1     | 1    | 10       | 92                 |
| 2     | 2    | 8        | 77                 |
| 3     | 1    | 6        | 66                 |
| 4     | 3    | 7        | 50                 |

Figure 1: Gantt chart of the optimal solution
5. Concluding remarks
This paper has outlined the initial development of a model intended to solve a practical problem in a craft company’s etching process. This paper’s main contribution is incorporating the multiple-item condition, which is more likely to be found in practice, to a batching and backward scheduling model of a batch processing machine. Consideration of additional setups and the regular batch setups are also included in the model. Through the numerical example, the correctness of the model can be verified. Our future work is directed to develop heuristics or metaheuristics procedures that can find the solutions efficiently.

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