On-Line Parameter Identification of a Squirrel Cage Induction Motor

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Abstract. Based on the principle of coordinate transformation, a mathematical model of induction motors in the three-phase stationary coordinate system was transformed to that in the synchronous rotating coordinate system. For motors with low-speed rotors, the multi-variable, high-order, strong-coupling and nonlinear equation of motors was further transformed to a linear equation. However, it was found difficult to solve the deduced linear equation through the method of least squares due to its high singularity. The steady-state stator resistance could be predetermined by temperature revision on the stator resistant in the motor nameplate. Identification of four motor parameters (Ls, M, Rr, Lr) was followed by the recursive least-squares method. Simulations of motor parameter estimation were carried out in Simulink of Matlab. The recursive least-squares method was used to deal with the measured data. The results demonstrate that the real-time electrical parameters (Ls, σ, Tr) can be accurately predicted. Given the assumption of Ls = Lr in the classical motor theory, the other three electrical parameters (M, Rr, Lr) can also be identified. The identification of these parameters is of great importance for the analysis, control and fault diagnosis of induction motors.

1. Introduction

Squirrel cage induction motors are of major interest due to their simplicity in structure and manufacture, low cost, resilience, small inertia, reliable performance, little maintenance and adaption in harsh environment. These advantages make it widely used in the industrial and agricultural production, transportation, national defense and military and daily life [1, 2].

The parameters of squirrel-cage induction motors are supposed to reflect a variety of motor characteristics [3]. These parameters are significantly important for the motor design and production, speed governing, complex control and diagnose after mal-function [4-6]. Therefore, it makes much sense for the exact identification of these motor parameters. Recently, the development of computers and widely application of DSP chips have made a large amount of complex calculation available in the industrial field. This gives a guarantee of on-line identification of motor parameters.

The mathematical model of the AC asynchronous motor is a high-order, nonlinear, strong-coupling and multi-variable system. A complex dynamic is inherent in this system. Numerous researches have studied the mathematical model of motors.

The electric parameters of motors can be obtained by a locked rotor test and a no-load test. In the paper [7], Stephan presented a new method for the real-time estimation of the parameters and fluxes of induction motors. Based on the simplification on the mathematical model of induction motors with
low-speed, the high-order and nonlinear equation is transformed into a linear equation. The motor parameters can then be obtained by solving the linear equation using the least-squares method. Based on the studies [7], this paper presents a simulation of real-time identification of measured data of voltage, current and speed using Matlab. The identification algorithm of motor parameter proposed in this paper is mainly used to determine whether the motor fault is occurring, rather than the accurate parameter identification of a healthy motor.

As for the motor fault, the corresponding parameters will change to varying degrees, and can be obtained by the proposed algorithm. By comparing with the earlier data, whether the motor fault is occurring and the fault degree can be identified.

2. Mathematical Models of Induction Motors

2.1. Coordinate Transformation

The coordinate transformation is not unique if no constraint condition is applied. In the analysis of a motor system, two constraint conditions are available:

1) Constant power;
2) Constant air-gap magnetomotive force.

If the transformation matrix $C_{3/2}$ from a three-phase coordinate system to a two-phase coordinate system meets: $C_{3/2}^T = C_{3/2}^{-1}$, the matrix $C_{3/2}$ is an orthogonal matrix and its power is kept constant in the transformation, i.e. the transformation is named "constant power transformation".

![Figure 1. Schematic diagram of dq0 transformation.](image)

The dq0 transformation is from a stationary A-B-C coordinate system to a rotating dq0 coordinate system. The rotating dq0 coordinate system is a combination of two phase coordinates which rotate together with the rotor and zero sequence system. If the rotor is a salient pole, the d axis (direct axis) usually coincides with the center axis of the salient pole, and the q axis (quadrature axis) is 90 electrical degrees ahead of d axis, as shown in figure 1.

The constant power transformation is adopted in this paper with the transformation matrix $C_{3s/2r}$ as shown in equation (1), where $\theta = \omega t$ ($\omega$ is the mechanical angular velocity of rotor, $n_p$ is the number of pole pairs, $\theta$ is the angle between d axis of the rotator and A phase axis of the stator.):

$$
C_{3s/2r} = \sqrt{\frac{1}{3}} \begin{bmatrix}
\cos n_p \theta & \cos (n_p \theta - \frac{2\pi}{3}) & \cos(n_p \theta + \frac{2\pi}{3}) \\
-\sin n_p \theta & -\sin (n_p \theta - \frac{2\pi}{3}) & -\sin(n_p \theta + \frac{2\pi}{3}) \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{bmatrix}
$$

(1)

Through the transformation matrix $C_{3s/2r}$, the three-phase coordinate system of the stator is transformed into the system in terms of d, q, 0 under any rotation speed in figure 1. By taking the rotation speed $\omega = 2\pi f$ ($f$ is the power frequency), the synchronous rotating reference frame of $dc$,
quadrature-axis current, voltage and flux of the stator, respectively; according to the assumption of rotor are available, the parameter identification can be conducted by simplifying the mathematical equation of motors into a linear equation under the synchronous rotating reference frame [7].

From the above transformation, the equation of motors under the three-phase coordinate system is transformed to the synchronous rotating system in which the equation of induction motors can be rewritten as [8]:

\[
\frac{di_x}{dt} = \frac{1}{\sigma L_s} u_{sx} - \gamma i_{sx} + \frac{\beta}{r_r} \psi_{rx} + n_p \beta \psi_{ry} + n_p \omega l_{sy} \\
\frac{di_y}{dt} = \frac{1}{\sigma L_s} u_{sy} - \gamma i_{sy} + \frac{\beta}{r_r} \psi_{ry} - n_p \beta \psi_{rx} - n_p \omega l_{sx} \\
\frac{d\psi_{cx}}{dt} = M_i l_{sx} - \frac{1}{r_r} \\
\frac{d\psi_{cy}}{dt} = M_i l_{sy} - \frac{1}{r_r} \\
\frac{d\omega}{dt} = \frac{2Mn_p}{3J_L} (i_{sy}\psi_{rx} - i_{sx}\psi_{ry}) - \frac{T_L}{J}
\]

where \(T_r = \frac{L_r}{R_r}\), \(\sigma = 1 - \frac{M^2}{L_s^2 r_r}\), \(\beta = \frac{M}{\alpha L_s r_r}\), \(\gamma = \frac{R_s}{\alpha L_s r_r} + \frac{M^2 R_r}{\alpha L_s r_r^2}\); \(i_{sx}, u_{sx}, \psi_{sx}\) are the equivalent direct-axis current, voltage and flux of the stator, respectively; \(i_{sy}, u_{sy}, \psi_{sy}\) are the equivalent quadrature-axis current, voltage and flux of the stator, respectively; \(\psi_{rx}, \psi_{ry}\) are the equivalent direct and quadrature-axis flux, respectively; \(L_s, R_s\) are the stator inductance and resistance; \(L_r, R_r\) are the rotor inductance and resistance; \(M\) is the mutual inductance; \(\omega\) is the mechanical angular speed of the rotor; \(J\) is the moment of inertia; \(n_p\) is the number of pole pairs; \(T_L\) is the load torque.

2.3. Linearly Parameterized Model

Normally, it is impossible to directly measure the rotor magnetic flux. The parameter identification through equation (2) is hard to achieve [7]. If the three-phase motor currents and rotation speed of rotor are available, the parameter identification can be conducted by simplifying the mathematical equation of motors into a linear equation under the synchronous rotating reference frame [7].

According to the equation (2):

\[
\begin{pmatrix}
-\frac{di_x}{dt} - i_{sx} & n_p \omega l_{sy} & \frac{du_{sx}}{dt} & u_{sx} \\
-\frac{di_y}{dt} - i_{sy} & -n_p \omega l_{sx} & \frac{du_{sy}}{dt} & u_{sy}
\end{pmatrix}
\begin{pmatrix}
K_1 \\
K_2 \\
K_3 \\
K_4 \\
K_5
\end{pmatrix}
= \begin{pmatrix}
\frac{d^2i_x}{dt^2} - n_p \omega \frac{di_y}{dt} \\
\frac{d^2i_y}{dt^2} + n_p \omega \frac{di_x}{dt}
\end{pmatrix}
\]

Note that a nonlinear relationship can be found to exist between these five parameters. Specifically,

\[
K_1 = \frac{K_2 K_4}{K_5} + K_3
\]

\[
K_1 = \frac{R_m}{\sigma L_s} + \frac{1}{\sigma r_r}K_2 = \frac{R_m}{\sigma L_s} K_2
\]

\[
K_3 = \frac{1}{\sigma r_r}K_3
\]

\[
K_4 = \frac{1}{\sigma L_s}K_4
\]

\[
K_5 = \frac{1}{\sigma r_r L_s}
\]

Not all these five electrical parameters \((R_s, R_r, L_s, L_r, M)\) can be retrieved from the five parameters \(K_1 \sim K_5\) which determine only the four independent parameters \(R_s, L_s, \sigma, T_r\):

\[
R_s = \frac{K_2}{K_5}, L_s = \frac{K_5}{K_2}, \sigma = \frac{K_5}{K_2 K_4}, T_r = \frac{K_4}{K_5}
\]

Considering \(T_r = \frac{L_r}{r_r}\) and \(\sigma = 1 - \frac{M^2}{L_s^2 r_r}\), the terms of \(\frac{L_r}{r_r}\) and \(\frac{M^2}{L_r}\) are obtained. The deduction is based on the exclusion of any magnetic flux terms. According to the assumption of \(L_s = L_r\) in the classical motor theory, the other three parameters \((R_r, L_r, M)\) can be obtained.
Equation (3) is linear in the parameters $K_1 \sim K_5$ and does not involve the unknown rotor flux signals. This linear form of the motor model enables a direct application of a least square identification algorithm to estimate the electrical parameters of motors.

2.4. Least-Squares Method

The equation (3) can be rewritten as [7]:

$$\omega^T(n) * K = y(n)$$  \hspace{1cm} (7)

where $n$ is the number of samples; $K$ denotes vector of the five unknown parameters; $\omega^T$ is the data matrix; $y$ is the measured vector.

An exact unique solution of this equation can be obtained by iterations. However, several factors contribute to errors which make the solution only approximately valid in practice, such as the errors from $\frac{d\omega}{dt} \approx 0$, quantitative error due to sampling of voltage and current, error from differential operation and environmental noise interference. Therefore, the obtained model of the induction motor is only an approximate representation of the real system.

The cost function of the least square method is generally defined as:

$$J = \sum_{i=1}^{N} e^2(i)$$ \hspace{1cm} (8)

where $e(i) = y(i) - \omega^T K$.

Solving this expression for $K^*$ yields the least-squares solution:

$$K^* = \left[\sum_{i=1}^{N} \omega(i) \omega^T(i)\right]^{-1} \left[\sum_{i=1}^{N} \omega(i)y(i)\right]$$ \hspace{1cm} (9)

The standard least square method equation (9) can be simplified as: $K^* = P^{-1}(i)R(i)$.

Then, $P(i)$ and $R(i)$ are determined at each time step according to:

$$P(i) = P(i-1) + \omega(i)\omega^T(i)$$ \hspace{1cm} (10.1)

$$R(i) = R(i-1) + \omega(i)y(i)$$ \hspace{1cm} (10.2)

3. Problems in the Numerical Analysis

Table 1. Motor parameters

| Parameter                  | Value            | Parameter                  | Value            |
|----------------------------|------------------|----------------------------|------------------|
| Rated output power         | 7.5kW            | Stator resistance          | Rs=0.7834Ω      |
| Rated Voltage              | 380V             | Rotor equivalent-resistance| Rr=0.7402Ω      |
| Rated Current              | 15.4A            | Leakage inductance of stator and rotor | L1s=L1r=0.00304 5H |
| Number of pole pairs       | 2                | Mutual inductance between stator and rotor | Lm=0.1241H      |
| Number of stator slots     | 36               | Moment of inertia after conversion | J=0.0343kg·m    |
| Number of Rotor bars       | 32               |                            |                  |

The assumption that is known can be incorporated in the algorithm by lettering $R_s = 0$ in the five parameters $K_1 \sim K_5$ and replacing $u_s$ by $u_s - R_s i_s$. And then, the new parameters become:

$$K'_1 = \frac{1}{\sigma_{T'_r}}, \quad K'_2 = 0, \quad K'_3 = \frac{1}{\sigma_{T'_r}}, \quad K'_4 = \frac{1}{\sigma_s}, \quad K'_5 = \frac{1}{\tau_{L_s}}$$ \hspace{1cm} (11)

In this way, $K'_2 = 0, K'_4 = K'_5$ are obtained. There are only three parameters to be determined. The equation (3) can be further simplified as:

$$\begin{pmatrix}
-\frac{di_{sx}}{dt} + n_p \omega i_{sy} & \frac{du_{sx}}{dt} & u_{sx} - R_s i_{sx} \\
-\frac{di_{sy}}{dt} - n_p \omega i_{sx} & \frac{du_{sy}}{dt} & u_{sy} - R_s i_{sy}
\end{pmatrix}
\begin{pmatrix}
K'_1 \\
K'_3 \\
K'_5
\end{pmatrix}
= \begin{pmatrix}
\frac{d^2i_{sx}}{dt^2} - n_p \omega \frac{di_{sx}}{dt} \\
\frac{d^2i_{sy}}{dt^2} + n_p \omega \frac{di_{sy}}{dt}
\end{pmatrix}$$ \hspace{1cm} (12)
Where the least square method and recursive algorithm can be used to determine the three unknown parameters. Additionally, the nonlinear relationship is automatically enforced by the fact that $K'_3 = K'_5$ can be directly accounted for in equation (12).

The electrical parameters of motors can be derived as follows:

$$L_s = \frac{K'_3}{K'_5} \sigma = \frac{K'_1}{K'_5 k'_4}, \quad T_r = \frac{K'_4}{K'_5}$$

(13)

4. Numerical Simulation

4.1. Simulation Model in Matlab

This paper will directly identify the parameters on the motor model in the module library for power system of Simulink tools. The motor used in the simulation experiment of this paper is the same as the laboratory prototype Y132M-4 in the laboratory, and the motor parameters are provided by the manufacturer as shown in table 1. The motor model with international units system is filled with the above motor parameters.

4.2. Identification Results Using General Least Squares

![Figure 2](image1)

Figure 2. Identification results of $k'_3, k'_4, k'_5$.

![Figure 3](image2)

Figure 3. Identification results of electrical parameters $L_s, L_m, R_F, L_F$. 
The motor parameters are input into the simulation model, and the three-phase voltage, current and rotation speed are measured every 0.0005s. According to the references [9-11], the mathematics method, i.e. the least-squares method in section 2.4, is used to compile the identification program of squirrel cage asynchronous motor, which is saved as the default Matlab M-file. The M-file and the data measured in section 4.1 are adopted into the Matlab command window to identify the motor parameters. The identification results of $k_3', k_4', k_5'$ in equation(12) are shown in figure 2.

In figure 2(a), (b), (c), the straight lines represent the theoretical value of $k_3', k_4', k_5'$, and the curves are the corresponding identification results. The identification results in the 1000th and 2000th steps are as following:

The 1000th step: $k_3' = 119.5$, $k_4' = 163.60$, $k_5' = 875$;

The 2000th step: $k_3' = 119.66$, $k_4' = 163.63$, $k_5' = 909.69$.

From the trend of the three curves, the identification results are closer to the theoretical values along with the increase of the identification number.

The identification results of $L_s$, $\sigma$, $T_r$ can be calculated by equation(13). According to the classical theory of motors, $L_s$ P.U.value can be generally considered to be equal with the $L_r$ P.U.value, i.e. $L_s = L_r$. Therefore, $L_s$, $L_m$, $R_r$, $L_r$ are obtained, and their identification results are shown in figure 3.

In figure 3(a), (b), (c), (d), the straight lines represent the theoretical value of $L_s$, $L_m$, $R_r$, $L_r$ just like Figure 2, and the curves are the corresponding identification results. The variation trend of the four electrical parameters is almost the same as that of $k$ in figure 2, i.e., the identification results are closer to the theoretical values along with the increase of the identification number. The identification errors of $L_s$, $L_m$, $R_r$, $L_r$ in the 2000th step are calculated respectively: $\Delta L_s\% = 3.5\%$, $\Delta M\% = 3.71\%$, $\Delta R_r\% = 1.66\%$, $\Delta L_r\% = 3.5\%$.

In this case, the errors mainly come from the approximation of equation and the differential calculation of voltage and current. The errors are confined within 5% which can meet the requirement of industrial application. The identification results are very close to the actual motor parameters, which can validate that the identification algorithm of motor parameter, by using the least-squares method, proposed in this paper is feasible. Noted that this algorithm is mainly used to obtain the parameters of the motor fault.

4.3. Parameter Identification of Motor Fault with Rotor Asymmetries

![Figure 4](image-url)
In the classical fault diagnosis methods, the motor fault was judged from the characteristics of the voltage, current, rotation speed or vibration signals. However, this paper attempts to identify the motor fault in another way. Several parameters of the motor are real-time monitored based on the parameter identification method. According to the variation of those parameters, the motor fault with rotor asymmetries will be identified.

In the simulation of the motor with the rotor asymmetries, the parameters shown in section 4.1 are still adopted. The identification results, based on the least-squares method, are shown in figure 4.

When comparing with the healthy motor in figure 3, figure 4 shows that there is obvious periodic fluctuant for the identified parameters of $L_s, L_m, R_r, L_r$, and means that the motor fault with rotor asymmetries happens.

5. Conclusions
This paper presents the transformation of a high-order, nonlinear, strong-coupling and multi-variable mathematical equation of motors to a linear equation. It is impossible to solve the equation by the least square method due to the serious singularity of its data matrix. To solve this problem, the stator resistance is predetermined and the other four parameters $(L_s, M, R_r, L_r)$ are identified by the iterative least-squares method. The simulation analysis is conducted in the Simulink environment of Matlab software. The results show that the proposed method is capable of accurately identifying the parameters of $L_s$, $\sigma$, $T_r$. Based on the assumption of $L_s = L_r$ in the classical motor theory, the other three parameters of $M, R_r, L_r$ can also be identified. Without the need for measuring the flux signals, the proposed method fits well for the filed implementation. This method can be well applied to the parameter identification of the motor fault with rotor asymmetries. This paper puts forward a new theoretical method to identify whether the motor fault is occurring and what the fault degree is, and builds the theoretical foundation for an experimental verification of the future. The experiment will provide the corresponding parameters with the help of instrument test, and the theoretical method of this paper will deal with these parameters to identify the motor fault.

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