C, P, T ARE BROKEN. WHY NOT CPT?

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Classification of effects of violation of seven symmetries – C, P, T, CP, PT, TC, and CPT – is discussed. A graphic mnemonic scheme – CPT-cube – is suggested and illustrated by simple examples.

1 Introduction

C, P, and T symmetries are known to hold at very high accuracy for electromagnetic and strong interactions. For weak interactions the 100% breaking of P and C was discovered in 1957 and served as a cornerstone of electroweak theory.

The CP violation was discovered as a tiny (milliweak) effect in the decays of neutral kaons in 1964 and recently in the decays of B mesons. But we still do not know the origin(s) of the CP violation.

T violation was explicitly proved for neutral kaons in accord with CPT invariance which remains unshaken in spite of improved precision of experimental tests and increased number of theoretical speculations.

As for gravity, it is difficult to suggest an experimental test of C, P, or T in a classical weak gravitational field, while effects of quantum gravity belong to the big bang which sets the arrow of time in the universe, or to the black holes.

In this talk I will not speak about gravity and will try to classify effects of violation of seven symmetries – C, P, T, CP, PT, TC, and CPT – in laboratory experiments.

The subject has a vast literature. I apologize for making no attempt to cover it. My aim is to suggest a simple mnemonic approach.

2 CPT-cube

Let us start by considering three orthogonal axes representing violation of C, P and T (Fig. 1).
Point 0 at the origin of coordinates corresponds to the interactions, which are C even, P even, T even, and hence CP even, PT even, TC even, and CPT even.

Point 1 corresponds to the interactions which are C odd, P even, T even, and hence CP odd, PT even, TC odd, and CPT odd.

Point 2: C even, P odd, T even, CP odd, PT odd, TC even, and CPT odd.

Point 3: C even, P even, T odd, CP even, PT odd, TC odd, and CPT odd.

As a result we have the first four vertices of the CPT-cube. Let us now consider the other four vertices (Fig. 2): three in the planes CP (point 4), PT (point 5), TC (point 6), and the last one – outside these planes (point 7).

Point 4: C odd, P odd, T even, CP even, PT odd, TC odd, and CPT even.

Point 5: C even, P odd, T odd, CP odd, PT even, TC odd, and CPT even.

Point 6: C odd, P even, T odd, CP odd, PT odd, TC even, and CPT odd.

Point 7: C odd, P odd, T odd, CP even, PT even, TC even, and CPT odd.

Thus for each of the seven transformations there are four even vertices and four odd.

For C: 0, 2, 5, 3 are even; 1, 4, 7, 6 are odd (the rear and front sides of the cube, correspondingly).

For P: 0, 3, 6, 1 are even; 2, 5, 7, 4 are odd (the left and the right sides, correspondingly).

For T: 0, 1, 4, 2 are even; 3, 6, 7, 5 are odd (the lower and upper sides; correspondingly).

For CP: 0, 4, 7, 3 are even; 1, 2, 5, 6 are odd (two vertical diagonal planes).

For PT: 0, 5, 7, 1 are even; 2, 3, 6, 4 are odd (two diagonal planes orthogonal to the page).

For TC: 0, 2, 7, 6 are even; 1, 3, 5, 4 are odd (two diagonal planes whose intersection is parallel to the page).

For CPT: 0, 4, 5, 6 are even; 1, 2, 3, 7 are odd (four diagonals of lower and upper planes).

The above classification can be easily memorized by assigning to each point the corresponding signature of (C, P, T) with + for even and - for odd: 0(+,+,+), 1(–,+,+), 2(+,–,+), 3(+,+,–), 4(–,–,+), 5(+,–,–), 6(–,+,–), 7(–,–,–).

3 C and P violation with CP conservation

Let us start with the interaction of $V - A$ weak charged currents discovered in 1957. The products $VV$ and $AA$ belong to point 0, while the product $VA$ – to point 4, as the vector current $V$ is C, P, and T odd, while the axial current $A$ is C even, P even, and T odd. The experimental manifestation of the latter is seen directly in the decay $K_1 \rightarrow 2\pi$, while of the former in the decay $K_2 \rightarrow 3\pi$, where $K_1$ and $K_2$ are correspondingly C odd and C even superpositions of $K^0$ and $ar{K}^0$:

$$K_1 = \frac{1}{\sqrt{2}}(K - \bar{K}), \quad K_2 = \frac{1}{\sqrt{2}}(K + \bar{K}).$$

As both $K_1$ and $K_2$ are P odd (pseudoscalars), $K_1$ is CP even, while $K_2$ is CP odd.

The $2\pi$ in $J = 0$ state is C even, P even, and CP even. Hence the decay $K^0 \rightarrow 2\pi$ is C odd, P odd, but CP even. As for the state of $3\pi$ with $J = 0$, it is P odd independent of values of relative angular momenta $l$ and $L$. The dominant state with $L = l = 0$ is C even. Therefore the corresponding decay $K^0_2 \rightarrow 3\pi$ is mainly C even, P even and CP even.

The manifestation of $VA$ (point 4) in the spin-momentum angular correlations in $\beta$-decay, decays of $\mu$ and $\tau$ leptons, semileptonic decays of mesons and nonleptonic decays of hyperons is impossible without interference with terms $VV$ and $AA$ (point 0) in the square of modulus of amplitude.

The same refers to the P violating and C violating correlations induced by neutral currents which were discovered in the 1970’s.
All the above processes are mediated by virtual $W$ and $Z$ bosons. After the discovery of these bosons in the early 1980’s a lot of experimental data has been collected on C and P violating asymmetries in their production and decay processes caused by interference of points 4 and 0.

4 CP and T violation

The discovery in 1964 of the $2\pi$ decays of the long-lived neutral kaons revealed that CP is violated. For almost three decades the effective interaction responsible for these decays was consistent with point 6: C odd, P even, T odd transition in vacuum of $K_2$ into $K_1$, described by a complex parameter $\varepsilon$:

$$K_S = K_1 + \varepsilon K_2, \quad K_L = K_2 + \varepsilon K_1.$$  \hfill (2)

The presence of $\varepsilon$ in eqs. (2) leads to the conclusion that the probability of transformation of $K$ into $\bar{K}$ during a time interval $t$ is not equal to the probability of transformation of $\bar{K}$ into $K$ during the same time interval. (The two amplitudes are different because of the presence of two different time exponents, describing the propagation of $K_L$ and $K_S$.) This prediction of violation of time reversal was experimentally confirmed only a few years ago. Another confirmation of the time reversal violation came from a CP and T violating asymmetry between $e^+e^-$ planes and $\pi^+\pi^-$ planes observed in the decays $K_L \to \pi^+\pi^-e^+e^-$. Only recently a consensus has been reached on the value of another parameter, $\varepsilon'$, describing the direct decay of $K_2$ into $2\pi$ (point 5, which is C even, P odd and T odd).

Point 5 is also responsible for the dipole electric moments of such particles as neutron and electron the search for which has up to now brought no positive evidence. (The term $\sigma E$, where $\sigma$ represents the spin of the particle, while $E$ electric field, is C even, P odd, T odd; so that the interaction is CPT invariant.)

Let us stress that at the level of the standard electroweak gauge lagrangian both points 5 and 6 for hadrons stem from the same origin, namely the phase of the CKM-matrix of charged currents of quarks.

Of special interest in connection with point 5 is the vanishingly small upper limit on the so-called $\theta$ term in QCD:

$$L_\theta = \theta G_{\mu\nu}G^{\mu\nu},$$  \hfill (3)

where $G_{\mu\nu}$ is the gluonic field tensor.

5 CP violating charge asymmetries

Both points 5 and 6 must manifest themselves in charge asymmetries. Thus from eq. (2) it follows immediately that the widths of the semileptonic decays $K_L \to e^+\nu e^-$ and $K_L \to e^-\bar{\nu}e^+$ must be different, the effect being proportional to $2Re\varepsilon$ (point 6). Such charge asymmetry was measured for both electronic and muonic channels.

Another charge asymmetry had been predicted by Okubo even before CP-violation was discovered, but is still beyond the reach of experiments. According to Okubo,

$$\frac{\Gamma(\Sigma^+ \to p\pi^0)}{\Gamma(\Sigma^+ \to n\pi^+)} \neq \frac{\Gamma(\bar{\Sigma}^- \to \bar{p}\pi^0)}{\Gamma(\bar{\Sigma}^- \to \bar{n}\pi^-)}. \hfill (4)$$

In order to show how this effect appears let me recall that for amplitudes of both S and P waves the following equalities hold

$$A(\Sigma^+ \to p\pi^0) = \sqrt{\frac{2}{3}}A_3 - \sqrt{\frac{1}{3}}A_1,$$
$$A(\Sigma^+ \to n\pi^+) = \sqrt{\frac{1}{3}}A_3 + \sqrt{\frac{2}{3}}A_1, \hfill (5)$$
where $A_3$ and $A_1$ are amplitudes for final states with isospin $T = 3/2$ and $T = 1/2$ correspondingly. Similar relations hold for antiparticles:

$$
\bar{A}(\Sigma^{-} \rightarrow \bar{p}n^0) = \sqrt{\frac{3}{2}}A_3 - \sqrt{\frac{1}{2}}A_1,
$$

$$
\bar{A}(\Sigma^{-} \rightarrow \bar{n}π^-) = \sqrt{\frac{1}{2}}A_3 + \sqrt{\frac{3}{2}}A_1,
$$

(6)

For simplicity let us consider only S wave amplitudes. In doing so we do not lose generality because S and P waves do not interfere in the expressions for partial widths.

The moduli of isotopic amplitudes, as well as the final state interaction phase shifts, are the same for particle and antiparticle ($\bar{A} = A_1$, $A_3 = -A_3$, $\delta_1 = \delta_3 = \delta_3$, while the CP violating phases have opposite signs ($\Delta = -\delta_1$, $\Delta_3 = -\Delta_3$). As a result we get inequality (4) if $\delta_3 \neq \delta_1$ and $\Delta_3 \neq \Delta_1$. (The former condition is valid in the standard electroweak theory [24, 25], while the latter is known to hold from the $\pi N$-scattering experiments.)

A similar reasoning was applied by Sakharov [24] when in 1967 he addressed the problem of baryonic asymmetry of the universe. He assumed CP violation for baryon number violating processes in order to get different cross-sections for specific processes with nucleons and antinucleons. The difficulties in working out a consistent theory of baryogenesis have directed theoretical thinking towards leptogenesis caused by CP-violation in leptonic sector, including neutrino oscillations.

6 Testing CPT with antiparticles

The faith in CPT invariance is based on quantum field theory, in particular on locality of the lagrangian, its Lorentz invariance and hermiticity. QFT might be an effective approximate manifestation of a more fundamental (superstring?) theory. But, first, I am unaware of any rigorous proof that superstrings violate CPT and, second, QFT remains the only solid basis of our phenomenology. Independently of the possible origin of CPT violation, in order to confront the speculations and experiment one has to formulate the phenomenological predictions by using the language of the quantum field theory.

Most of the phenomena suggested for testing CPT belong to the point 1: they are C-odd, but PT-even. Examples are:

1. the search for mass differences of particles and corresponding antiparticles: $m_{K^0} - m_{\bar{K}^0}$, $m_{K^+} - m_{K^-}$, $m_{e^-} - m_{e^+}$, $m_{\mu^-} - m_{\mu^+}$, $m_{\nu_e} - m_{\bar{\nu}_e}$, etc.

2. the search for nonvanishing sum of magnetic moments of a particle and its antiparticle:

$$
\mu_\mu \neq -\mu_\mu, \mu_e \neq -\mu_e, \mu_\nu \neq -\mu_\bar{\nu}, \text{etc.}
$$

Especially popular nowadays are speculations on non-vanishing mass differences between neutrinos and antineutrinos [26, 27, 28, 29]. Most of them (though not all) are linked with the violation of Lorentz invariance and/or locality. The most recent references are triggered by the LSND anomaly [30]. Beware! The CPT violation in neutrino sector induces at the level of radiative corrections observable effects among charged leptons, where high precision tests of the CPT symmetry are available [31].

A CPT violating effect due to interference of points 3 and 0 would be muon polarization perpendicular to the decay planes of $K_L^0 \rightarrow \mu^+\nu_\mu\pi^-$ and $K_L^0 \rightarrow \mu^-\bar{\nu}_\mu\pi^+$, if it turns out to be the same for both decays. In that case, the correlation $s_\mu [k_\mu \times k_\pi]$ is C even, P even, but T odd. Hence it is CPT odd. However “a fake T violation” could be caused by the final state muon-pion scattering [32, 33] at point 0 with phase $\delta \sim \alpha/3$. The experimental upper limit for such polarization is 0.5% [34].
Note that the same transverse polarization in the decays $K^+ \rightarrow \mu^+ \nu\pi^0$ and $K^- \rightarrow \mu^- \bar{\nu}\pi^0$ cannot be faked by a simple final state electromagnetic scattering.

As an example of manifestation of point 7 let us consider the electric dipole moments of a particle and its antiparticle, say, $e^-$ and $e^+$. If P and T are broken, while C is conserved (point 6), the electric dipole moments are nonvanishing, however their sum must vanish, because they (similarly to charges and to the ordinary magnetic dipole moments described by point 0) must have opposite signs. This can be easily seen from the negative C parity of the photon.

If we now consider point 7, we see that it must be not only P and T odd but also C odd. This requires a term which gives not only the same absolute value, but also the same sign to the electric dipole moments of a particle and its antiparticle. If both points 7 and 6 are present, then $d_{\nu e} \neq d_{\bar{\nu} e}$. Similar inequalities would be valid for all leptons and quarks and hence hadrons.

7 CPT and hermiticity

What would the discovery of circular polarization of a photon from the decay $\pi^0 \rightarrow \gamma\gamma$ (or $\eta^0 \rightarrow \gamma\gamma$) mean? One can easily see that this would be a signal that CPT is broken. Indeed, C is conserved in this decay, while the product $sk$ is P odd, but T even. (Here $s$ and $k$ are photon’s spin and momentum respectively; $s$ is P even, $k$ is P odd, while both are T odd. Similar considerations apply to the longitudinal polarizations of muons in the decay $\eta^0 \rightarrow \mu^+\mu^-$ observed at the level of $6 \times 10^{-6}$. The correlation $s_\mu k_\mu$ is P odd and T even. On the other hand it must be C even, because $\eta^0$ is C even as well as the pair of muons both in the scalar and pseudoscalar states. Thus observation of the $s_\mu k_\mu$ correlation would signal the CPT violation.)

Let us consider the two terms in the lagrangian the interference of which can lead to the correlation $sk$ in the decay $\pi^0 \rightarrow \gamma\gamma$. First of them is a scalar $g\phi F_{\mu\nu} F^{\rho\sigma} \epsilon^{\mu\nu\rho\sigma}$, the second one is a pseudoscalar, $h\phi F_{\mu\nu} F_{\mu\nu}$, where $\phi$ is the pseudoscalar field of the pion, which is C even, P odd and T odd. Due to hermiticity of the lagrangian both $g$ and $h$ must be real. However the degree of circular polarization is proportional to $g^*h - gh^*$. Hence for a hermitian lagrangian it should vanish at the tree approximation. (It would appear due to the difference of the absorptive parts of Fig. 3 in scalar and pseudoscalar amplitudes.) If the degree of polarization turns out to be different from that predicted by Fig. 3, e.g. much larger, we have to break hermiticity.

Let us consider first the case when $\text{Im}g = 0$, $\text{Re}h = 0$. In that case the term proportional to $g$ belongs to point 0, while the term proportional to $h$ belongs to point 2 because it is C even, P odd, T even, and hence CPT odd.

In the case when $\text{Re}g = 0$, $\text{Im}h = 0$ the $g$-term is C even, P even, T odd and hence CPT odd (point 3), while the $h$-term is C even, P odd, T odd, and hence CPT even (point 5). Interference of points 3 and 5 (as well as that of points 2 and 0) gives P-violating and CPT-violating correlation $sk$.

In the general case, when both $g$ and $h$ are complex,

$$g^*h - gh^* = 2i\text{Re}g \cdot \text{Im}h - 2i\text{Im}g \cdot \text{Re}h$$

Here the first term on the r.h.s. represents interference of points 2 and 0, while the second - of points 3 and 5.

As was noticed by M.B. Voloshin (private communication), the graphs of Fig. 4 with complex $g$ give complex magnetic moment of the proton. Similar graphs with complex $h$ give complex electric dipole moment. Interaction of these moments with corresponding external fields $B$ and $E$ would give complex energy. Complex energy looks rather exotic, but it is not more exotic than the violation of unitarity of $S$-matrix, or in other words, the non-conservation of probability.
(Note that a complex energy might correspond to a tunneling of particle through a wormhole to another vacuum.)

To my knowledge, nobody has tried to measure the helicity of photons from $\pi^0 \rightarrow \gamma\gamma$. V.L. Telegdi (private communication) noticed that the search for the circular polarization of photons from the decay of parapositronium is much easier than from the decay of pions. (At low energy the Compton scattering is dominant, hence the magnetized iron is a more effective detector of helicity of low energy photons.)

When thinking about possible mechanism of CPT-violation in the two photon decay of positronium one sees that circular polarization of photons might be produced by imaginary electric dipole moment of electron (see Figs. 5b and 5c) via interference with the standard QED diagram (Fig. 5a).

Unlike the case of particle-antiparticle mass differences the above examples of CPT-violating polarizations can be formulated in a Lorentz-invariant way. In the lagrangian language the CPT-violating terms have the wrong phase which violates hermiticity. The antihermitian terms in lagrangian break unitarity of $S$-matrix. But testing unitarity directly might be more difficult than searching for CPT violating correlations discussed above.

8 Conclusions

It is time now to answer the question in the title of this talk: Why not CPT?

Of course, whatever could be measured should be measured and whatever could be tested should be tested. There should be no reservations: such a fundamental symmetry as CPT should be tested.

However one should keep in mind that unlike breaking of C, P, T, CP, PT, and TC, the breaking of CPT is non-compatible with the standard quantum field theory, the only basis for a self-consistent phenomenological description of any process, which we know up to now. Therefore the chances that CPT breaking would be discovered are vanishingly small.

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Fig. 4a

Fig. 4b

Fig. 5a

Fig. 5b

Fig. 5c
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