On the $A$ dependence of $R = \sigma_L/\sigma_T$ and the $Q^2$ dependence of Shadowing

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Abstract

A higher–twist nuclear enhancement of $R = \sigma_L/\sigma_T$, as might be expected to arise due to fermi motion and whose magnitude is within the error-bars of recent experiment, is shown to lead to a monotonic decrease in the ratio of nuclear vs. nucleon cross–sections at small Bjorken $x$ for increasing $Q^2$. This effect at small $x$, comparable in magnitude to those reported for shadowing, is driven mainly by kinematic factors and essentially vanishes for $x > .1$. Its unusual $Q^2$ dependence rather complicates the unravelling from present data in the shadowing region the corresponding dependence in $Q^2$ of the nuclear structure functions, $F_2^A(x, Q^2)$.

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Shadowing, the observed depletion for heavy nuclei in the nuclear vs. nucleon cross-section at low Bjorken $x$ in real or virtual photon scattering, is a subject of both continuing experimental and theoretical interest. The experimental situation can roughly be separated into two categories: those at low (or zero) $Q^2$ photon virtuality, and those at moderate to large $Q^2$ values of $Q^2$. It is the data at large $Q^2$ that is of concern for this study, for which Ref. [3] is of particular interest as it is there that the double–differential cross–section $d^2\sigma/dxQ^2$ is presented for a range of $Q^2$ for a number of $x$ values in the shadowing region.

What is to be here addressed is our ability to infer from this data the correct $Q^2$ dependence of the structure functions $F_2^A(x, Q^2)$ in the shadowing region. This is important not only for hopefully distinguishing and potentially selecting different theoretical models [4], [5] for shadowing at large $Q^2$, but because it is also crucial for our understanding the results of other experiments such as Drell–Yan [6] or $J/\psi$ production [7], in which other mechanisms [8], [9] might be playing an important role. It should be emphasized at the outset that this is not an attempt to explain shadowing, for which a diverse set of models presently exist and which can be roughly seperated into those using quark and gluon [4] and those utilizing hadronic [3] degrees of freedom. Indeed, the effects here discussed all but vanishes for the smallest $Q^2$ values of Ref. [3]. Instead it is the point of this paper to demonstrate how a very simple, higher–twist nuclear effect (i.e. which vanishes with increasing $Q^2$) leads to a rather unusual $Q^2$ dependence in the double–differential cross–section, and one which would, if verified, greatly complicate our ability to infer $F_2^A(x, Q^2)$ from existing data.

To order $\alpha_s^2$, the double differential cross–section per nucleon for lepton scattering from an unpolarized nuclear target may be written as:

$$\frac{d^2\sigma}{dxQ^2} = \frac{4\pi\alpha_s^2}{Q^4} \frac{F_2(x, Q^2)}{x} \left[ 1 - \frac{Q^2}{2ME} - \frac{Q^2}{4E^2} + \frac{Q^4}{8M^2E^2x^2} \left( \frac{1 + 4M^2x^2/Q^2}{1 + R(x, Q^2)} \right) \right], \quad (1)$$

where $M = .94$GeV is the mass of a (free) nucleon, $E$ is the incident lepton’s energy, $F_2(x, Q^2)$ is the nucleon’s structure function (in a nuclear medium), and $R = \sigma_L/\sigma_T$ is the ratio of longitudinal to transverse cross–sections, which in terms of the usual functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ is given by $R(x, Q^2) = F_L(x, Q^2)/2xF_1(x, Q^2)$,
where $F_L(x, Q^2) = \left(1 + \frac{4x^2 M^2}{Q^2}\right) F_2(x, Q^2) - 2xF_1(x, Q^2)$ is the so-called longitudinal structure function. It is well known that up to target mass corrections, $R$ vanishes in the parton model for the case of spin $1/2$ partons due to helicity conservation. In perturbative QCD, $R$ acquires a non-zero value from $\alpha_s(Q^2)$ corrections to the structure functions. In addition to these corrections, $R$ is also non-zero due to an explicit higher-twist correction \cite{lo}, \cite{l1} to $F_L(x, Q^2)$:

$$F_L^{\tau=4}(x, Q^2) = \frac{4\Lambda^2}{Q^2} T_1(x), \quad (2)$$

in which $T_1(x)$ is a new nucleon matrix element. It involves transverse components of the covariant derivative acting upon quark field operators. If one ignores the fact that this is a covariant derivative (required for gauge-invariance), this new matrix element corresponds naturally to what one associates with an intrinsic transverse momentum of the quarks in the parton model. That is, one obtains in the limit that the gluon coupling in the covariant derivative $g \to 0$: \cite{l4}

$$\lim_{g \to 0} \frac{4\Lambda^2 T_1(x)}{Q^2} = \frac{4}{Q^2} \int d^2k_\perp k_\perp^2 f(x, k_\perp^2) = 4\langle k_\perp^2 \rangle/ Q^2, \quad (3)$$

where in the parton model $f(x, k_\perp^2)$ is related to $F_2(x)$ by

$$F_2^{\tau=2}(x) = x \int d^2k_\perp f(x, k_\perp^2). \quad (4)$$

Recent theoretical fits \cite{l2} to $R$ for the free nucleon indicate that such a higher-twist element is indeed required to fit the data.

If we now turn to nuclear targets, it is clear from Eq. (1) that a nuclear dependence to $R$ could distort at any $x$ or $Q^2$ the extraction of $F_2^A(x, Q^2)$. Indeed, such a conjectured dependence \cite{l6} was in fact proposed to account for the initial discrepancies between the EMC \cite{l7} and SLAC \cite{l8} data for $x \approx .1$. Besides the ultimate experimental resolution of this discrepancy, the shift in nuclei used by \cite{l0}, $\Delta R \approx .15$, (taken to be essentially $Q^2$ independent, as would be necessary to fit the wide range of $Q^2$ available at these large values of Bjorken $x$) does not seem to be supported by later experiments \cite{l4} at SLAC.
devoted to measuring $\Delta R$. We’re now though interested in $\Delta R$ at small $x$, for which Ref. [15] is the sole source of experimental information. This data (as was that from [14]) is consistent with a value for $\Delta R = 0$, but with such large error bars that the best one should probably safely conclude is that $\Delta R < .1$. However it should be emphasized that because of the experimental difficulty in measuring this quantity, a systematic double–binning in both $x$ and $Q^2$ for $\Delta R(x, Q^2)$ in either [14] or [15] is lacking.

A simple reason to expect a nuclear dependence for $R$ is fermi motion. Returning to the heuristics of the partonic model, motion of the nucleons in the nucleus increases the average transverse momentum of the quarks inside a nucleon. By Eq. (3), such an origin for $\Delta R$ is higher–twist, and hence vanishes as $Q^2 \to \infty$. Despite this $Q^2$ dependence, it will be seen that the effect on the measured cross-section $d^2\sigma/dxdQ^2$ at given fixed $x$ actually grows with $Q^2$. This growth is a direct consequence of the kinematic factors entering Eq. (1), as is also the fact that for a given fixed $Q^2$, the effect grows for decreasing $x$.

The world’s data for $R$ for a free nucleon has been collected and analysed by Ref. [13] for which an empirical best fit has been obtained. It is given by the formula:

$$R^{fit} = \frac{b_1}{\ln(Q^2/\Lambda^2)} \left( 1 + 12 \frac{Q^2}{Q^2 + 0.125^2} + \frac{0.125^2}{Q^2 + 0.125^2 + x^2} \right) + \frac{b_2}{Q^2} + \frac{b_3}{Q^4 + 0.3^2}$$

(5)

where $\Lambda = .2$GeV, $\{b_i\} = \{.0635, .5747, -.3534\}$ and all momenta are in GeV. Note that there is a misprint in [13] for the value of the first coefficient, $b_1$: the decimal point needs to be shifted one place to the left.\footnote{This misprint might account for the apparent discrepancy between certain statements of Ref. [3] and the results reported here as to the relative importance that a nuclear dependence to $R$ generates in the differential cross–section.} Whereas the expression multiplying the inverse–log term has been chosen to fit the results from pQCD for $R$ at large $Q^2$, one should not interpret too seriously the various coefficients $\{b_i\}$ in terms of a twist expansion, as target mass corrections must first be disentangled. Such a separation was undertaken in Ref. [12]. For our purposes, Eq. (5) is used to merely generate the best information we presently have on $R$ for a free
nucleon. One caveat is that no data actually exists for $R$ for $x < .1$. This fact will be returned to later.

A higher–twist nuclear enhancement to $R^{fit}$ is conjectured. To $R^{fit}$ is added a term $\Delta R$,

$$R^A = R^{fit} + \Delta R,$$  \hspace{1cm} (6)

modelled to fall as $1/Q^2$ ala Eq. (3) with $k_\perp = .3$GeV $\approx k_{fermi}$ taken. No $x$ dependence is given this enhancement although such a dependence is likely, especially in the small $x$ region where the heuristic interpretation of the higher twist matrix element $T_1(x)$ is likely to break down [11]. In this regard, the simple connection with fermi motion is given merely as a plausibility arguement and the skeptical reader could interpret the following as simply an exercise exploring the effects that introducing such a higher–twist nuclear dependence for $\Delta R$ makes on the differential cross–section.

Fig. 1 plots the resulting ratio of nuclear to nucleon cross–sections, Eq. 1, such a $\Delta R$ generates for various values of Bjorken $x$ ($\{x_i\} = \{.0125, .025, .05, .1\}$) assuming no change in the nucleon to nuclear structure functions, i.e. $F_2^A = F_2^N$. The kinematics is that of Ref. [3] ($E = 200$GeV) for which the curves cutoff for each $x_i$ at the kinematic upper limits of the experiment. From top to bottom the curves correspond to decreasing $x_i$. Note the trend with $Q^2$ for each curve. While the plots in Fig. 1 are all consistently less than the shadowing actually seen, especially at small $Q^2$ (hence the initial statements that this is not an explanation of shadowing), for the larger $Q^2$ values the suppressions are large enough to play an observable role (e.g. the shadowing for Calcium at $x = .025$ is roughly 5% [3]). Observe most importantly that the effect at fixed $Q^2$ grows for decreasing $x$. This is dramatically demonstrated by the plot for $x = .0125$, where though it is admittedly a point for which both $R^{fit}$ and $\Delta R$ might be most seriously doubted. It has been included nevertheless for emphasis. In Fig. 4 the solid line is the $x$ independent nuclear–enhancement, $\Delta R$, added to $R^{fit}(x, Q^2)$ of Eq. (3). For reference is also plotted $R^{fit}(x, Q^2)$ for two illustrative values of Bjorken $x$. The dashed line is $R^{fit}(x, Q^2)$ for $x = .1$ and the dotted line is the (interpolated) value for $x = .025$. 

5
The results in Fig. 1 are significantly more sensitive to the assumed size of $\Delta R$ in the shadowing region than they are to $R$ itself. For example, arbitrarily multiplying $R^{fit}$ by a factor of two makes a change of only about .5% and 1% in the ratio of cross-sections for $x = .025$ and $x = .0125$ respectively. Since $R$ for the nucleon is unknown in the shadowing region, this insensitivity is relevant. On the other hand, doubling (or halving) $\Delta R$ effectively doubles (or halves) the effect at large $Q^2$ for these same $x$ values. As for the actual magnitude of $\Delta R$ being introduced, for $Q^2 > 5\text{GeV}^2$, $\Delta R < .065$ and would appear to be small enough to fit within the uncertainties (at any Bjorken $x$) of experiment [14], [15]. Indeed, for $Q^2 = 9\text{GeV}^2$, where the effect in Fig. 1 for $x = .025$ is largest, $\Delta R < .04$, well within experimental error bars. For smaller values of $Q^2$ however, the $x$ independent value of $\Delta R$ is in apparent contradiction with the measured values [14] at $x > .1$. While this fact only further emphasizes the result contained in Fig. 1 that a nuclear dependence for $R$ yields negligible effects on the measured cross-section outside of the shadowing region, theoretically one notes that for the smaller $Q^2$ values, the magnitude of the $\Delta R$ being introduced appears less and less as a perturbation to $R(x, Q^2)$. For example, at $Q^2 = 2\text{GeV}^2$, the used value for $\Delta R = .18$ would be over a 60% correction to $R(x, Q^2)$ at $x = .1$. In such circumstances corrections of order $Q^{-4}$ cannot be justifiably ignored, and indeed a twist expansion may not even converge [19]. For the future experiments at CEBAF ($E = 6\text{GeV}$) that will measure $\Delta R(x, Q^2)$, this last consideration suggests that only at the very highest $Q^2$ there accessible might a twist–expansion in fact appear. For of course the main focus of this study, the shadowing region, a machine with much higher energy (and presumably an electron beam for higher intensities in order to improve upon the CERN data, as in e.g the proposed 50GeV SLAC upgrade) is neccessary for probing $\Delta R(x, Q^2)$.

In conclusion, the $Q^2$ dependence of Fig. 1 is rather startling. In a counter–intuitive fashion, an intrinsically higher–twist effect for $\Delta R$ is of growing importance with increasing $Q^2$ in the observed rate. Examining Eq. (1) this apparently surprising dependence with $Q^2$ occurs because near the ends of phase–space, $1/xE$ enters as an important new scale where the parameter $y = Q^2/2MxE$ (the fractional energy loss of the electron) is reaching
its kinematic upper bound: $y \leq 1$. In support of this effect, a close scrutiny of the data of Ref. [3] does indicate an apparent systematic increase with $Q^2$ of the shadowing observed for most of the values of Bjorken $x < .1$. While a nuclear enhancement of $R$ would naturally account for such a trend in the data, such an enhancement would also, depending on its size for any particular $x$ in the shadowing region, act to mask the true $Q^2$ dependence of $F_2^A(x, Q^2)$. Without therefore better knowledge of $\Delta R(x, Q^2)$ in the shadowing region, the extrapolations of present data to either yet higher $Q^2$ or other processes must be considered problematic.

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FIGURES

FIG. 1. The ratio of nuclear to nucleon differential cross-sections generated by a \( \Delta R \propto 1/Q^2 \), as a function of \( Q^2 \) for given \( x \). From bottom to top, the curves correspond to \( \{x_i\} = \{0.0125, 0.025, 0.05, 0.1\} \).

FIG. 2. In solid line, the value of the nuclear enhancement \( \Delta R \) being introduced. For reference, also the values of \( R^{fit}(x = 0.1) \) in dashes, and \( R^{fit}(x = 0.025) \) in dots.