Constraints on cosmological parameters from recent measurements of CMB anisotropy

S. Hancock, G. Rocha, A.N. Lasenby and C.M. Gutiérrez

1 Mullard Radio Astronomy Observatory, Cavendish Laboratory, Madingley Road, Cambridge CB3 OHE, UK
2 Instituto de Astrofísica de Canarias, 38200 La Laguna, Tenerife, Spain
3 Department of Physics, Kansas State University, Manhattan, KS 66506, USA

Accepted . Received ; in original form

ABSTRACT
A key prediction of cosmological theories for the origin and evolution of structure in the Universe is the existence of a ‘Doppler peak’ in the angular power spectrum of cosmic microwave background (CMB) fluctuations. We present new results from a study of recent CMB observations which provide the first strong evidence for the existence of a ‘Doppler Peak’ localised in both angular scale and amplitude. This first estimate of the angular position of the peak is used to place a new direct limit on the curvature of the Universe, corresponding to a density of \( \Omega = 0.7^{+0.8}_{-0.5} \), consistent with a flat Universe. Very low density ‘open’ Universe models are inconsistent with this limit unless there is a significant contribution from a cosmological constant. For a flat standard Cold Dark Matter dominated Universe we use our results in conjunction with Big Bang nucleosynthesis constraints to determine the value of the Hubble constant as \( H_0 = 30 - 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \) for baryon fractions \( \Omega_b = 0.05 \) to 0.2. For \( H_0 = 50 \text{ km s}^{-1} \text{ Mpc}^{-1} \) we find the primordial spectral index of the fluctuations to be \( n = 1.1 \pm 0.1 \), in close agreement with the inflationary prediction of \( n \approx 1.0 \).

Key words: cosmology – cosmic microwave background.

1 INTRODUCTION

Observations of the Cosmic Microwave Background (CMB) radiation provide information about epochs and physical scales that are inaccessible to conventional astronomy. In contrast to traditional methods of determining cosmological parameters, which rely on the combination of results from local observations (Ostriker & Steinhardt 1995), CMB observations provide direct measurements (Bond & Efstathiou 1987, White, Scott & Silk 1994) over cosmological scales, thereby avoiding the systematic uncertainties and biases associated with conventional techniques. The principal cosmological information is contained in the acoustic peaks (Bond & Efstathiou 1987; Hu & Sugiyama 1995; Scott, Silk & White 1993) in the power spectrum, which are generated during acoustic oscillations of the photon-baryon fluid at recombination (Efstathiou 1988). The main acoustic peak, sometimes referred to as the first Doppler peak, is a strong prediction of contemporary cosmological models with adiabatic fluctuations and is expected to occur on an angular scale \( \sim 1^\circ \). (In topological defect theories of structure formation, the first Doppler peak is expected to occur on smaller angular scales (e.g. Magueijo et al. 1996) or be of much smaller amplitude (Pen, Seljak & Turok 1997) than in inflationary theories. This is discussed further below.) The observation of this peak is thus a major goal of observational cosmology. In the case that it is not observed, this could imply either that medium-scale primordial CMB fluctuations had been wiped out by reionization (Efstathiou 1988), or perhaps that there is a fundamental flaw in our theory. On the contrary, a conclusive observation of the first peak would provide strong support for current theoretical models and the determination of its angular position would constitute a direct probe of the large scale geometry of the Universe. The angular scale \( l_p \) of the main peak reflects the size of the horizon at last scattering of the CMB photons and thus depends almost entirely (Hu & Sugiyama 1995, Kamionkowski et al. 1994) on the total density of the Universe according to \( l_p \propto 1/\sqrt{\Omega} \). In conventional inflationary theory (Guth 1981), one expects the Universe to be flat with \( \Omega = 1.0 \), which can be achieved if the total mass density is equivalent to the critical density or if there is a contribution from a cosmological constant \( \Lambda \). The height of the peak provides additional cosmological information since it is directly proportional to the fractional mass in baryons \( \Omega_b \), and also varies according to the expansion rate of the Universe as specified by the Hubble constant \( H_0 \); in general (Hu & Sugiyama 1995) for baryon fractions \( \Omega_b < 0.05 \), increasing \( H_0 \) reduces the peak height whilst the converse is true at higher baryon densities. Furthermore, by measuring the amplitude of the intermedi-
ate scale CMB fluctuations relative to those on large scales it is possible to place tight limits on the spectral slope $n$ of the initial primordial spectrum of fluctuations. The latter is predicted by inflationary theory to be approximately scale invariant, in which case $n \approx 1.0$, although (in particular versions of inflationary theory) the presence of a background of primordial gravity waves would lead to lower values of $n$, via the relation $C_l^s/C_l^a \approx 7(1-n)$, where $C_l^s/C_l^a$ is the ratio of tensor to scalar contributions to the quadrupole component of the CMB power spectrum (Crittenden et al. 1993 Steinhardt 1993). (Linkages between parameters in more general theories of inflation are discussed in Liddle (1997).) Thus, in summary, by comparing large and intermediate scale CMB observations and tracing out the Doppler peak, it is possible to directly estimate $\Omega$, $\Omega_b$, and $H_0$ and to probe inflationary theory and the existence of primordial gravity waves. Recent improvements in the quality of CMB data, in particular on the angular scales probed by the CAT and Saskatoon experiments, now make this exercise of great interest.

2 METHOD

Clear detections of CMB anisotropy have now been reported by a number of different groups including the COBE satellite (Bennett et al. 1996a Bennett et al. 1996b); ground-based switching experiments such as Tenerife (Hancock et al. in press Hancock et al. 1994), Python (Ruhl et al. 1993), South Pole (Gundersen et al. 1995); balloon mounted instruments such as ARGO (De Bernardis et al. 1994), MAX (Tanaka et al. 1995), and MSAM (Cheng et al. 1994 Cheng et al. 1995); and more recently the ground-based interferometer CAT (Scott et al. 1996). Given the difficulties inherent in observing CMB anisotropy, it is possible that some of these results are contaminated by foreground effects and it is clear that determining the form of the CMB power spectrum in order to trace out the Doppler peak requires a careful, in-depth consideration of the CMB measurements from the different experiments within a common framework. The full details including a discussion of foreground contamination are presented in Rocha et al. (in preparation) and here we present our principal findings. We consider all of the latest CMB measurements, including new results from COBE, Tenerife, MAX, Saskatoon and CAT, with the exception of the MSAM results (see below) and the MAX detection in the Mu Pegasi region which is contaminated by dust emission (Fischer et al. 1995).

The competing models for the origin and evolution of structure predict (Bond & Efstathiou 1987 Hu & Sugiyama 1993) the shape and amplitude of the CMB power spectrum and its Fourier equivalent, the autocorrelation function $C(\theta) = \langle \Delta T(m) \Delta T(m') \rangle$ where $m,m' = \cos \theta$. Expanding the intrinsic angular correlation function $C(\theta)$ in terms of spherical harmonics one obtains

$$C(\theta) = \sum_{l \geq 2} (2l+1)C_l P_l(\cos \theta)/4\pi,$$

where low order multipoles $l$ correspond to large angular scales $\theta$ and large $l$-modes are equivalent to small angles on the sky. The $C_l$s are predicted by the cosmological theories and contain all of the relevant statistical information for models described by Gaussian random fields (Bond & Efstathiou 1987). The different experiments sample different angular scales according to their window functions $W_l$ (White, Krauss & Silk 1993 White & Srednicki 1993). The window function $W_l$ specifies the relative sensitivity of an experiment to a given $l$-mode, and the observed power in CMB fluctuations as seen through a window $W_l$ is given by

$$C_{obs}(0) = \left( \frac{\Delta T_{obs}}{T} \right)^2 = \sum_{l \geq 2} (2l+1)C_l W_l/4\pi.$$

Figure 1. The window functions for the experiments listed in Table 1

Given $W_l$, then for the $C_l$’s corresponding to the theoretical model under consideration it is possible to obtain the value of $\Delta T_{obs}$ one would expect to observe using the chosen experiment. This value can then be compared to the value actually observed to test the cosmological model. Shown in Fig. 2 are the window functions for the various configurations of the experiments considered.

On the largest scales corresponding to small $l$, new COBE (Bennett et al. 1996a) and Tenerife (Hancock et al. in press) results improve the power spectrum normalisation, whilst significant gains in knowledge at high $l$ are provided by new results from the Saskatoon and CAT experiments. The full data set spans a range of 2 to $\sim 700$ in $l$, sufficient to test for the main Doppler peak out to $\Omega = 0.1$. We take the reported CMB detections and convert them to a common framework of flat bandpower results (Bond 1995c Bond 1995b) as given in Table 1. This is carried out as follows.

The CMB anisotropy measurements are converted to bandpower estimates $\Delta T_l \pm \sigma$ assuming in each case a flat spectrum of $C_l$ centred on the effective multipole $l_e$ (see below) of the window function. $l_l$ and $l_u$ represent the lower and upper points at which the window of each configuration reaches half of its peak value. In order to use the observed anisotropy levels to place constraints on the CMB power spectrum one must in general know the form of the $C_l$ under test. However, in most cases the form of $C_l$ can be represented by a flat spectrum $C_l \propto C_2/(l(l+1))$
Table 1. Details of data results used

| Experiment    | $\Delta T_l$ ($\mu$K) | $\sigma$ ($\mu$K) | $l_c$ | $l_i$ | $l_u$ | Reference         |
|---------------|-----------------------|-------------------|-------|-------|-------|-------------------|
| COBE          | 27.9                  | 2.5               | 6     | 2     | 12    | Bennett et al. 1996 |
| Tenerife      | 34.1                  | 12.5              | 20    | 13    | 31    | Hancock et al. in press |
| PYTHON        | 57.2                  | 16.4              | 91    | 50    | 107   | Rubi et al. 1996 |
| South Pole    | 39.5                  | 11.4              | 57    | 31    | 106   | Gundersen et al. 1993 |
| ARGO          | 39.1                  | 8.7               | 95    | 52    | 176   | De Bernardis et al. 1996 |
| MAX GUM       | 54.5                  | 13.6              | 145   | 78    | 263   | Tanaka et al. 1996 |
| MAX ID        | 46.3                  | 17.7              | 145   | 78    | 263   | " |
| MAX SH        | 49.1                  | 19.1              | 145   | 78    | 263   | " |
| MAX PH        | 51.8                  | 15.0              | 145   | 78    | 263   | " |
| MAX HR        | 32.7                  | 9.5               | 145   | 78    | 263   | " |
| Saskatoon1    | 49.0                  | 6.5               | 86    | 53    | 132   | Netterfield et al. 1997 |
| Saskatoon2    | 69.0                  | 6.5               | 166   | 119   | 206   | " |
| Saskatoon3    | 85.0                  | 8.9               | 236   | 190   | 274   | " |
| Saskatoon4    | 86.0                  | 11.0              | 285   | 243   | 320   | " |
| Saskatoon5    | 69.0                  | 23.5              | 348   | 304   | 401   | " |
| CAT1          | 50.8                  | 15.4              | 396   | 339   | 483   | Scott et al. 1996 |
| CAT2          | 49.0                  | 16.9              | 608   | 546   | 722   | " |

over the width of a given experimental window, so that the bandpower is $\Delta T_l/T = \sqrt{C_{\text{obs}}(0)/I(W_l)}$, where we define $I(W_l)$ according to Bond (1995a; 1995b) as $I(W_l) = \sum_{l=0}^{\infty} (l + 0.5)W_l/(l(l + 1))$. This bandpower estimate is centred on the effective multipole $l_e = I(W_l)/I(W_l)$. In many instances experimenters now report results directly for a flat spectrum and when this is not so we have converted the quoted power in fluctuations into the equivalent flat band estimate. Each group has obtained limits on the intrinsic anisotropy level using a likelihood analysis (see e.g. Hancock et al. 1994), which incorporates uncertainties due to random errors, sampling variance (Scott, Srednicki and White 1994), and cosmic variance (Scaramella & Vittorio 1993, 1994). The errors in $\Delta T_l$ quoted in column 3 of Table 1 are at 68% confidence and have been obtained by averaging the difference in the reported 68% upper and lower limits and the best fit $\Delta T_l$ since the form of the likelihood function is in general only an approximation to a Gaussian distribution this averaging introduces a small bias into the results (Rocha et al. in preparation). With the exception of Saskatoon the errors include uncertainties in the overall calibration. There is a ±14% calibration error in the Saskatoon data, but since the Saskatoon points are not independent this will apply equally to all five points (Netterfield et al. 1997). We discuss below how this is included in the analysis. It is not possible to ascribe an error to each experiment to represent the likely degree of residual Galactic contamination present in its results, since this is not known at present. However, as emphasized above, if there is any evidence that the degree of contamination in an experimental point could be significant, we have not used that point.

Results from the MSAM experiment are not included here, because they do not provide an independent measure of the power spectrum since their angular sensitivity and sky coverage are already incorporated within the Saskatoon measurements. Netterfield et al. (1994) report good agreement between the MSAM double difference results and Saskatoon measurements, although the discrepancy with the MSAM single difference data is yet to be resolved. The window functions for the COBE and Tenerife experiments are independent at the half-power points, thus justifying their joint use even though their sky areas overlap.

The data points from Table 1 are plotted in Figure 2, in which the horizontal bars represent the range of $l$ contributing to each data point. There is a noticeable rise in the observed power spectrum at $l \simeq 200$, followed by a fall at higher $l$, tracing out a clearly defined peak in the spectrum. In the past several groups (Scott, Silk & White 1993, Kamionkowsky et al. 1994, Ratra et al. 1997) have attempted to determine the presence of a Doppler peak, but only now are the data sufficient to make a first detection and to put constraints on the closure parameter $\Omega$. As a first step, we adopt a simple three parameter model of the power spectrum, which we find adequately accounts for the properties...
of the principal Doppler peak for both standard Cold Dark Matter (CDM) models (Davis et al. 1994; Efstathiou 1981) and open Universe ($\Omega < 1$) models (Kamionkowski et al. 1994b). The functional form chosen is a modified version of that used in Scott, Silk & White (1995) — we choose the following:

$$l(l+1)C_\ell = 6C_2 \left( 1 + \frac{A_{\text{peak}}}{1 + y(l)} \right) \left( 1 + \frac{A_{\text{peak}}}{1 + y(2)} \right)$$

where $y(l) = (\log_{10} l - \log_{10}(220/\sqrt{\Omega}))/0.266$. In this representation $C_2$ specifies the power spectrum normalisation, whilst the first Doppler peak has height $A_{\text{peak}}$ above $C_2$, width $\log_{10} l = 0.266$ and for $\Omega = 1.0$ is centred at $l \approx 220$. By appropriately specifying the parameters $C_2$, $A_{\text{peak}}$ and $\Omega$ it is possible to reproduce to a good approximation the $C_\ell$ spectra corresponding to standard models of structure formation with different values of $\Omega$, $\Omega_b$ and $H_0$. Such a form will not reproduce the structure of the secondary Doppler peaks, but we have checked the model against the overall form of the $\Omega = 1$ models of Efstathiou and the open models reported in Kamionkowski et al. (1994b) and find that this form adequately reflects the properties of the main peak. This satisfies our present considerations since the current CMB data are not yet up to the task of discriminating the secondary peaks. Varying the three model parameters in equation (1) we form $C_\ell$ spectra corresponding to a range of cosmological models, which are then used in equation (2) to obtain a simulated observation for the $i$th experiment, before converting to the bandpower equivalent result $\Delta T_i[C_2, A_{\text{peak}}, \Omega(i)]$. The chi-squared for this set of parameters is given by

$$\chi^2(C_2, A_{\text{peak}}, \Omega) = \sum_{i=1}^{nd} \frac{(\Delta T_{i,\text{obs}}(i) - \Delta T_i[C_2, A_{\text{peak}}, \Omega(i)])^2}{\sigma_i^2},$$

where $nd$ is the number of data points in Table 1 and the relative likelihood function is formed according to $L(C_2, A_{\text{peak}}, \Omega) \propto \exp(-\chi^2(C_2, A_{\text{peak}}, \Omega)/2)$. We vary the power spectrum normalisation $C_2$ within the 95% limits for the COBE 4-year data (Bennett et al. 1994) and consider $A_{\text{peak}}$ in the range 0 to 30 and values of the density parameter up to $\Omega = 5$. The data included in the fit are those from Table 1. Because of the ±14% calibration error in the Saskatoon data, the likelihood function is evaluated for three cases: (i) that the calibration is correct, (ii) the calibration is the lowest allowed value and (iii) the calibration is the maximum allowed value. In each case the likelihood function is marginalised over $C_2$ before calculating limits on the remaining two parameters according to Bayesian integration with a uniform prior.

3 RESULTS AND DISCUSSION

In Fig. 3 the likelihood function obtained from fitting the model $C_\ell$ spectra to the data of Table 1 is shown plotted as a function of the amplitude and position of the Doppler peak. The position is parameterized via the value of $\Omega$, assuming that the cosmological constant is zero. The highly peaked nature of the likelihood function in Fig. 3 is good evidence for the presence of a Doppler peak localised in both position ($\Omega$) and amplitude. In Fig. 4 we show the 1-D marginal likelihood curve for $\Omega$, obtained by marginalising the likelihood function over $C_2$ and the peak amplitude, $A_{\text{peak}}$. The best fit value of $\Omega$ is 0.7 with an allowed 68% range of $0.2 \leq \Omega \leq 1.5$.

In Figure 2 the best fit model, represented by the solid line, is shown compared to the data points, assuming no error in the calibration of the Saskatoon observations. The chi-squared per degree of freedom for this model is 0.9, implying a good fit to the data. The peak lies at $l = 263^{+139}_{-149}$.
Constraints on cosmological parameters

Figure 5. The data points from Table 1 compared to the exact forms of the \( C_l \) for an \( \Omega = 1 \), \( \Omega_b = 0.10 \), \( H_0 = 45 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) standard CDM model (bold line), an \( \Omega = 0.3 \), \( \Omega_b = 0.03 \), \( H_0 = 50 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) open model (dashed line), a flat \( \Omega = 0.3 \), \( \Omega_b = 0.05 \), \( H_0 = 50 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) model (dot-dash line) and an example cosmic string model (dotted line) (Magueijo et al. 1996). The analytic approximation to the true \( C_l \) such as we use here, is a useful general tool, but as a detailed check we have also applied the chi-squared goodness of fit to actual COBE normalised \( C_l \) models. In Fig. 5 the data are compared to exact forms of the \( C_l \) for a standard flat CDM model, an open CDM model, a \( \Lambda \) dominated model and a cosmic string model (Magueijo et al. 1996). Allowing the model normalisation to vary within the two sigma COBE limit we find that the standard CDM model, non-zero \( \Lambda \) model and the string model all offer acceptable \( \left( P(\chi^2) \geq 0.05 \right) \) chi-squared fits, whilst the probability of the open model fitting is \( P(\chi^2) < 0.01 \). Note that the result for the open model (\( \Omega = 0.3 \)) using the true \( C_l \) differs from the corresponding result for the analytic form of the \( C_l \): whilst in the latter case this model is still in the allowed range of the marginal distribution of \( \Omega \), in the former case it is already excluded by the data. Considering a range of CDM models with varying \( \Omega \), in order to find the lowest \( \Omega \) compatible with the observations, we have considered exact models with \( H_0 = 50 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) and \( \Omega_b = 0.03 \) for \( \Omega = 1 - 0.5 \) [Kamionkowski et al. 1994b]. We find that \( \Omega = 0.5 \) is allowed, \( \Omega = 0.3 \) and below are completely ruled out (95% confidence) and \( \Omega = 0.4 \) is excluded unless all of the Saskatoon points have the minimum allowed calibration.

We have also considered a more complete set of open models, for which partial results can be given here. (Results over a full set of parameters will be given in Rocha et al., in preparation.) The grid considered has \( h \) values of 0.3, 0.5, 0.7 and 0.8 (where \( h = H_0/(100 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}) \)), and baryon density \( \Omega_b \) values of 0.01, 0.03, 0.06, together with \( \Omega_b = 0.0125 \, h^{-2} \) and 0.024\( h^{-2} \) for each of the above values of \( h \). The \( \Omega \) range considered is 0.1 to 1.0 in steps of 0.1. (These models were kindly provided by N. Sugiyama).

The unavailability of exact models for \( \Omega > 1 \) limits some of the statistical conclusions we can draw here, but the results are still of interest. Assuming case (i) for the calibration and allowing the model normalisation to vary within the two sigma COBE limit we find that the best fit model has \( \Omega = 0.7 \), \( H_0 = 50 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) and \( \Omega_b = 0.096 \). This best fit value of \( \Omega \) gives good agreement with the results obtained using the analytic approximation. Marginalizing over the other parameters, we obtain an allowed 68% range for \( \Omega \) of 0.5 \( \leq \Omega \leq 1.0 \). (The upper limit of 1 is due to the cutoff in the range of models considered.)

The situation for models in which structure formation is initiated by cosmic strings (Magueijo et al. 1996a; Seljak & Turok 1997) is now more complex, since some of the predictions for the power spectra for strings have recently changed. Previous calculations for the cosmic strings model (Magueijo et al. 1996a) and low \( \Omega \) CDM models both have the first Doppler peak occurring in roughly the same position in \( l \), so it might be thought surprising that only the latter are eliminated by the current data. This is traceable to the form of the low \( \Omega \) power spectra in the range \( l \approx 10-100 \), where in order to match the COBE normalisation at low \( l \), the models are forced to have values which are too low compared to the data over the intermediate angular scale range. It is likely, however, that these string models will be eliminated if more accurate data on the CAT range of angular scales confirms the existing CAT results (Scott et al. 1994). More recent calculations of topological defect theories indicate that the Doppler Peak is strongly suppressed (Pen, Seljak & Turok 1997) and these predictions are likely to be ruled out by the current data.

In order to set constraints on the Hubble constant in a flat universe, in Fig. 6 we have considered COBE normalised standard CDM models (provided by G. Efstathiou) for a range of \( \Omega \) and \( H_0 \). All these models have \( \Omega = 1 \), \( \Lambda = 0 \), \( n = 1 \) and zero tensor (gravitational wave) component. They are thus somewhat specialized, and it is well-known that they do not provide good fits for the matter power spectrum on smaller scales. However, we believe our results for \( H_0 \) are still of interest in indicating the type of constraints that will be available in the future, when the increased quality of the CMB data will allow more parameters to be fitted simultaneously. Our method is as follows: a dot is placed in the appropriate place in the parameter space if the exact power spectrum corresponding to these parameters gives a fit to the data in Table 1 with an acceptable \( \chi^2 \) value (\( P(\chi^2) \geq 0.05 \)). A blank is left at that position if not. Overlying these power spectrum constraints is the limit 0.009 \( \leq \Omega_b h^2 \leq 0.02 \) provided by nucleosynthesis.
of the light elements (Copi, Schramm & Turner 1995). As shown, the models offering an acceptable chi-squared fit to the CMB power spectrum, whilst simultaneously satisfying nucleosynthesis constraints, encompass $0.09 < \Omega_b h^2 < 0.02$ (Copi, Schramm & Turner 1995) imposed by nucleosynthesis gives the allowed models lying between the solid curves. Models with $H_0 > 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ are not allowed by the combined constraint.

Figure 6. The COBE normalised CDM models with acceptable ($P(\chi^2) \geq 0.05$) chi-squared fits to the CMB data (assuming the nominal Saskatoon calibration) are plotted as dots in the $\Omega_b, H_0$ space. Overlying the constraint of $0.09 < \Omega_b h^2 < 0.02$ (Copi, Schramm & Turner 1995) imposed by nucleosynthesis gives the allowed models lying between the solid curves. Models with $H_0 > 50 \text{ km s}^{-1} \text{ Mpc}^{-1}$ are not allowed by the combined constraint.

and $n$ give 68% confidence intervals of $30 \text{ km s}^{-1} \text{ Mpc}^{-1} \leq H_0 \leq 55 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $0.85 \leq n \leq 1.18$ respectively.

4 CONCLUSIONS

Our current results provide good evidence for the Doppler peak, verifying a crucial prediction of cosmological models and providing an interesting new measurement of fundamental cosmological parameters. In Rocha et al. (in preparation), a detailed comparison of the CMB data is made with the theoretical power spectra predicted by a range of flat, tilted, reionized, open models and models with non-zero cosmological constant. The existence of the Doppler peak has important consequences for the future of CMB astronomy, implying that our basic theory is correct and that improving our constraints on cosmological parameters is simply a matter of improved instrumental sensitivity and ability to separate out foregrounds. New instruments such as VSA (Lasenby & Hancock 1995), MAP and the proposed Planck Surveyor satellite (Mandolesi et al. 1995) will provide this improved sensitivity and should delimit $\Omega$ and other parameters with unprecedented precision.

ACKNOWLEDGEMENTS

We wish to thank all the members of the CAT and Tenerife teams for their help and assistance in this work. We thank G. Efstathiou and N. Sugiyama for access to their theoretical power spectra and B. Netterfield for supplying the Saskatoon window functions. S. Hancock wishes to acknowledge a Research Fellowship at St. John’s College, Cambridge, U.K. G. Rocha wishes to acknowledge a JNICT Studentship from Portugal and a NSF grant EPS-9550487 with matching support from the state of Kansas and from a K*STAR First award.

REFERENCES

Bennett C.L. et al., 1996, ApJ., 464, L1
Bond J.R., 1995a, “Cosmology and Large Scale Structure” ed. Schaeffer, R. Elsevier Science Publishers, Netherlands, Proc. Les Houches School, Session LX, August 1993
Bond, J.R., Astrophys. Lett. and Comm., 1995b, 32, 63
Bond, J.R., Efstathiou, G.P., 1987, MNRAS, 226, 655
Cheng E.S. et al., 1994, ApJ., 422, L37
Cheng E.S. et al., 1996, ApJ., 456, L71
Copi C.J., Schramm D.N., Turner M.S., 1995, Science, 267, 192
Crittenden, R., Bond, J.R., Davis, R.L., Efstathiou, G., Steinhardt, P.J., 1993, Phys. Rev. Lett., 71, 324
Davis, M., Efstathiou, G., Frenk, C.S., White, S.D.M, 1992, Nature, 356, 489
De Bernardis P. et al., 1994, ApJ., 422, L33
Efstathiou, G.P., 1989, in “Physics of the Early Universe”, proceedings of the thirty-sixth Scottish Universities Summer School in physics 1989, p361, eds. Peacock, J.A., Heavens, A.F., Davies, A.T.
Fischer, M.L. et al., 1995, ApJ., 444, 226
Freedman, W.L. et al., 1994, Nature, 371, 757
Gundersen, J.O et al., 1995, ApJ., 433 L57
Guth, A.H., 1981, Phys. Rev. D, 23, 347

© 1996 RAS, MNRAS
Constraints on cosmological parameters

Hancock, S., Gutierrez, C.M., Davies, R.D., Lasenby, A.N., Rocha, G., Rebolo, R., Watson, R.A., and Tegmark M., 1997, MNRAS, in press
Hancock, S., Davies, R.D., Lasenby, A.N., De La Cruz, C.M.G., Watson, R.A., Rebolo, R., Beckman, J.E., 1994, Nature, 367, 333
Hu W., Sugiyama N., 1995, ApJ., 444, 489
Kamionkowski, M., Spergel, D.N., Sugiyama, N., 1994a, ApJ., 426, L57
Kamionkowski, M., Ratra, B., Spergel, D.N., and Sugiyama, N., 1994b, ApJ., 434, L1
Kennicutt, R.C., Freedman, W.L., Mould, J.R., 1995, ApJ., 110, 1476
Lasenby, A.N., Hancock, S., 1995, Proc. of: “Current Topics in Astroparticle Physics: The Early Universe”, p327, eds. Sanchez, N., Zichichi, A., Kluwer
Lasenby, A.N., Jones M.E., 1997, Proc. of: “The Extragalactic distance Scale”, p76, eds Livio N., Donahue M., Panagia N., CUP
Liddle, A.R., 1997, Proceedings of ‘From Quantum Fluctuations to Cosmological Structures’, Casablanca, Morocco, December 1996, in press (astro-ph/9612093)
Liddle, A.R., Lyth, D.H., 1992, Phys. Letters, B291, 391
Lineweaver, C.H., Barbosa, D., in press (astro-ph/9612146)
Magueijo, J., Albrecht, A., Coulson, D., Ferreira, P., 1996, Phys. Rev. Lett., 76, 2617
Mandels, N. et al., 1995, Planetary and Space Science, 43, 1459
Netterfield, C.B., Devlin, M.J., Jarosik, N., Page L., and Wollack, E.J., 1997, ApJ., 474, 47
Ostriker, J.P., Steinhardt, P.J., 1995, Nature, 377, 600
Pen, U.-L., Seljak, U., Turok. N, 1997, preprint (astro-ph/9704165)
Pierce, M.J., Welch, D.L., Mcclure, R.D., Van Den Bergh, S., Racine, R., Stetson, P.B., 1994, Nature, 371, 385
Ratra, B., Sugiyama, N., Banday, A.J., and Gorsky, K.M., 1997, ApJ., 481
Rocha, G., Hancock, S., Lasenby, A.N., Gutierrez, C.M. in preparation.
Ruhl, J.E., Dragovan, M., Platt, S.R., Kovac, J., Novak, G., 1995, ApJ., 453, L1
Scaramella R., Vittorio N., 1990, ApJ., 353, 372
Scaramella R., Vittorio N., 1993, ApJ., 411, 1
Scott D., Srednicki M., White M., 1994, ApJ., 241, L5
Scott D., Silk, J., White, M., 1995, Science, 268, 5212
Scott, P.F. et al., 1996, ApJ., 461, L1
Seljak, U., Zaldarriaga, M., 1996, ApJ., 469, 437
Smoot, G.F. et al., 1992, ApJ., 396, L1
Steinhardt, P., 1993, Proc. of the Yamada Conference XXXVII “Evolution of the Universe and its Observational Quest”, p159, ed. Sato, K.
Tanaka, S.T. et al., 1996, ApJ., 468, L81
White M., Srednicki M., 1995, ApJ., 443, 6
White M., Krauss L.M., Silk J., 1993, ApJ., 418, 535
White, M., Scott, D., Silk, J., 1994, Ann. Rev. Astron. Astrophys., 32, 319