What is the optimal depth for deep-unfolding architectures at deployment?

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Abstract—Recently, many iterative algorithms proposed for various applications such as compressed sensing, MIMO Detection, etc. have been unfolded and presented as deep networks; these networks are shown to produce better results than the algorithms in their iterative forms. However, deep networks are highly sensitive to the hyperparameters chosen. Especially for a deep unfolded network, using more layers may lead to redundancy and hence, excessive computation during deployment. In this work, we consider the problem of determining the optimal number of layers required for such unfolded architectures. We propose a method that treats the networks as experts and measures the relative importance of the expertise provided by layers using a variant of the popular Hedge algorithm. Based on the importance of the different layers, we determine the optimal layers required for deployment. We study the effectiveness of this method by applying it to two recent and popular deep-unfolding architectures, namely DetNet and TISTA-Net.

Index Terms—MIMO Detection, Deep Learning, Hedge algorithm, unfolding architectures, DetNet

I. INTRODUCTION

In recent times, deep learning has succeeded tremendously in solving complex data-driven problems [1]–[5]. In particular, deep learning approaches to solve detection problems have attracted attention in recent times [6]–[10]. There has been significant focus on developing a deep neural network architecture by unfolding an existing iterative algorithm [11]. In such a network, each layer represents an iteration of the algorithm whose optimal parameters are learned by the network. For example, popular iterative algorithms such as Iterative Shrinkage and Thresholding Algorithm (ISTA) and Approximate Message Passing (AMP) are unfolded into a neural network-based architecture [12], [13].

One of the well-known model-driven deep learning networks for MIMO detection is the DetNet [14]. Here, the authors unfold the iterations of a projected gradient descent into a deep neural network. They also show that the results given by DetNet are competitive when compared with the existing MIMO detectors. In the case of sparse signal recovery, OAMP-Net and TISTA-Net architectures unfold the ISTA and OAMP algorithms respectively [15], [16].

The success of deep neural networks in detection problems can be attributed to the feasibility of processing huge matrices. Therefore, we cannot discount the need for a large memory for storing and the huge computational complexity for obtaining inference, especially when we deploy an instance of the trained network in low power mobile devices or the Internet of Things-devices (IoT-devices). For such applications, deploying compact neural networks by reducing the number of layers has increasing relevance. If we determine the optimal layers during training, it is possible to reduce the memory without compromising on the performance of the network. To the best of our knowledge, no prior work in open literature focus on optimizing such model-driven deep-unfolding networks.

In this work, we determine the optimal number of layers in deep-unfolding architectures, thereby reducing the memory and computational complexities with a negligible effect on performance during the deployment. To achieve this, we first propose to employ a variant of the popular Hedge algorithm [17], namely the dHedge algorithm [18], during training to determine the relative importance of the layers. We then use this to remove the redundant layers, and hence compress the network during deployment. The number of layers that one should use has always been a subjective choice in most applications. By using our proposed method, one can train a deep-unfolding network with a large number of layers to begin with and allow dHedge to determine the required number of layers. Hence, the user need not choose the number of layers by trial and error. Though we demonstrate the utility of our results for DetNet and TISTA-Net, one can use this method to remove redundant layers in any deep-unfolding architecture. As a further addition to this method, one can also use other popular compression techniques such as pruning and quantization of weights to achieve furthermore reduction in memory consumption.

Throughout the work, $E[\cdot]$ denotes the expectation operator, $\|\cdot\|$ denotes the $L_2$ norm, $(\cdot)^T$ denotes transpose.

II. SYSTEM MODEL

For both MIMO detection and sparse signal recovery, we use the following system model:

$$y = Hx + n.$$  \hfill{(1)}

Here, $y \in \mathbb{R}^N$ is the received vector, $H \in \mathbb{R}^{N \times K}$ is the channel matrix, $n$ is the additive white Gaussian noise (AWGN) with variance $\sigma^2$. For MIMO detection architectures like DetNet, $x$ is a vector from $\mathcal{S}$, a constellation like BPSK. On the other hand, for signal recovery architectures like TISTA-Net, $x \in \mathbb{R}$. Though we have $x \in \mathbb{R}$, the work can be trivially extended to complex vectors also.
A. DetNet

DetNet is composed of $L$ layers and each layer takes $H$ and $y$ as inputs. Also, the functionality at each layer, the parameters to be optimized and the loss function used are provided in [14]. These details are repeated below for ease of reading. The architecture for the $k$th layer, where $k$ varies from 1 to $L$ is

$$
\begin{align*}
    z_k &= \rho \left( W_{1k} \begin{bmatrix} H^T y \\ \hat{x}_k \\ H^2 H x_k \end{bmatrix} + b_{1k} \right), \\
    \hat{x}_{k+1} &= \psi_k \left( W_{2k} z_k + b_{2k} \right), \\
    \hat{v}_{k+1} &= W_{3k} z_k + b_{3k}, \\
    \hat{x}_1 &= 0,
\end{align*}
$$

Here, $\rho$ is the rectified linear unit and $\psi_k(\cdot)$ is a piece-wise linear soft sign operator. The parameters that are optimized during the learning phase are:

$$
\theta = \{ W_{1k}, b_{1k}, W_{2k}, b_{2k}, W_{3k}, b_{3k}, t_k \}_{k=1}^L.
$$

To account for the problems of vanishing gradients, saturation of activation functions, etc. the loss to be minimized is defined as

$$
\begin{align*}
    l(x; \hat{x}_\theta(H, y)) &= \sum_{k=1}^{L} \log(k) \frac{||x - \hat{x}_k||^2}{||x - \hat{x}||^2},
\end{align*}
$$

where $\hat{x} = (H^T H)^{-1} H^T y$ is the standard decoder. Note that the output from all the layers are employed in computing the loss function. The final estimate is defined as $\hat{x}_\theta(y, H) = sign(\hat{x}_L)$.

B. TISTA-Net

Each layer in TISTA-Net has the following architecture [16]:

$$
\begin{align*}
    r_k &= x_k + \gamma_k W(y - H x_k), \\
    \hat{x}_{k+1} &= E \{ x_k r_k, \gamma_k \}, \hat{x}_1 = 0, \\
    v_k^2 &= \frac{||y - H x_k||^2}{\text{tr}(H^T H)} - N \sigma^2, \\
    \gamma_k &= \frac{(K + (\gamma_k^2 - 2\gamma_k) N)v_k^2 + \gamma_k^2 \sigma^2 \text{tr}(WW^T)}{K},
\end{align*}
$$

where $W$ is the pseudo inverse of $H$ and $x \in \mathbb{R}^K$ is the unknown vector. The scalar variables $\gamma_k (k = 1, ..., L)$ are the variables optimized in the training phase. TISTA-Net is trained using incremental training. In the $p$th round of the incremental training, the loss function that is minimized is $E[||\hat{x}_p - x||^2]$. In other words, only the first $p$ layers are trained at $p$th round. The final estimate of the output is $\hat{x}_L$.

III. Architecture with DHedge

To determine the optimal layers, we have to first identify the relative importance of prediction outputs $x_k$ from each layer $k$ at the end of every training epoch using the loss function. We observe that this is similar to the classical problem of deciding which expert offers the best output [17]. In a deep-unfolding architecture, we consider each network constructed using the first $k$ layers for $k = 1, ..., L$ as an expert in predicting $x$; therefore, there are $L$ experts in total. Each of these experts incurs a loss $||x - \hat{x}_k(t)||^2$ at training epoch $t$. Smaller the loss, better the prediction. Since different layers train at different paces throughout the training phase, the expertise provided by each of these networks changes over the training epochs. In other words, these experts are non-stationary in nature. In the case of DetNet, we can observe from (4) that the authors of [14] have weighed the loss from $k$th layer with $\log k$.\footnote{In case of TISTANet no such weighing ratios are discussed in [16].} Note that there is no guarantee that the fixed weighing ratios are optimal.\footnote{We use the term weighing ratios to differentiate them from the weights of the neural network.} Two questions follow naturally. The first question is whether we can dynamically update the weighing ratios at the end of each training epoch based on the loss function. The second question is, once these weighing ratios are learned, how do we determine the optimal number of layers. We answer both these questions in the subsequent subsections.

A. Determining weighing ratios by dHedge

To determine the correct weighing ratios, we need a suitable weight-update algorithm, one that initializes the weighing ratios of the experts and updates these ratios at every training epoch. The algorithm should update the ratios based on the feedback obtained on the experts’ performance, i.e., penalize it for poor performance and reward it otherwise. In our case, this can be measured by means of the loss function $||x - \hat{x}_k(t)||^2$ at each layer, for every training epoch $t$. The Hedge algorithm is a well-known algorithm used for stationary experts [17]. In our specific problem, we define the $k$th expert as the network up to and including $k$ layers and use the output at the $k$th layer to obtain the $k$th expert prediction of $x$. To account for the non-stationary nature of these experts, we use discounted Hedge (dHedge), a modified version of the Hedge algorithm, which can handle the evolution of experts over time [18].

In the dHedge algorithm, we assign weighing ratios $w_k[1]$ for $k = 1, ..., L$ for the $L$ experts at the first epoch. After a round of prediction by all experts, if the expert $k$ incurs a loss of $l_k[t]$ after the $t$th time-step, we update the weighing ratios, $w_k[t+1] = w_k[t] \gamma \beta l_k[t]$, \hspace{1cm} (6)

where $\beta$ and $\gamma$ are the hedge parameter and the discount factor respectively. These are problem dependent tunable parameters.

In the case of DetNet, let $w[t] = [w_1[t], w_2[t], ..., w_L[t]]$ be the weighing ratio for each layer at the $t$th training epoch. The initialized weighing ratios are $w_k[1] = 1/L \forall k = 1, 2, ..., L$. Our aim is to minimize the following loss function:

$$
\begin{align*}
    l(x; \hat{x}(H, y; \theta)) &= \sum_{k=1}^{L} w_k[t] \frac{||x - \hat{x}_k||^2}{||x - \hat{x}||^2},
\end{align*}
$$

After setting $w[0]$, we perform an iteration of backpropagation to train the parameters $\theta$ of DetNet. The weighing ratios for each layer at $t$th training epoch are updated as,

$$
\begin{align*}
    w_k[t+1] = w_k[t] \gamma |x - \hat{x}_k(t)||^2, \hspace{1cm} (8)
\end{align*}
$$
After the update, we normalize the weighing ratios before the next training epoch. Note that the major change we have made to the original DetNet architecture is the change in the weighing ratio from $\log k$ to $w[k]$, which we update after every training epoch based on (8). Similarly, for TISTA-Net, we minimize the following loss function:

$$l(x, \hat{x}(H, y; \gamma)) = \sum_{k=1}^{L} w_k[t]||\hat{x}_k - x||^2.$$  

(9)

\subsection*{B. Determining optimal number of layers}

Although we use the dHedge algorithm to account for the non-stationary nature of the experts during training, we note that the weights of the neural network converge at the end of training, thereby resulting in stationary experts. Therefore, we assume that the weighing ratios after training converge to $\hat{w}_k$, $k = 1, \ldots, L$. These learned weighing ratios provide an average measure of the relative importance of the network up to $k$th layer in predicting the output. Since the expert with the least average loss has the maximum weighing ratio at the end of the training, the average loss will be minimum if we predict the output using this network during deployment. Also, we can eliminate any layer beyond this without any loss in performance, i.e., we can eliminate all layers $l > M$, for

$$M = \max_{k \in \{1, L\}} |\hat{w}_k - \hat{w}_L| \geq \epsilon.$$  

(10)

In case $M = L$, i.e., the final layer gives the least loss, we cannot eliminate any layer without some loss in performance. However, we can still remove some layers by observing how the weighing ratios differ from one another. For example, if we determine an $M$ such that the weighing ratios $\hat{w}_k$ for $k > M$ are nearly equal to $\hat{w}_M$, (i.e., $|\hat{w}_L - \hat{w}_M| < \epsilon$ for some tolerable error $\epsilon$), we can still afford to eliminate the last $L-M$ layers and suffer only a negligible loss in performance. In other words, if the weighing ratios beyond the $M$th layer are all only $\epsilon$ away from $\hat{w}_L$ then the final $L-M$ networks are nearly equal experts in predicting $x$. Hence, one can truncate the network to the first $M$ layers with negligible loss in the performance. However, determining the right trade-off between $M$ and the loss in performance depends on the evolution of weighing ratios over the layers. The entire heuristic algorithm is presented in Algorithm 1. In the subsequent section, for DetNet and TISTA-Net, we have shown that we can significantly reduce the number of layers required without suffering any loss in performance.

\section*{IV. Numerical Results}

In this section, we provide numerical results for DetNet and TISTA-Net modified with the dHedge algorithm. Each element of $H$ is sampled from $\mathcal{N}(0, 1)$. We train both the networks using Adam Optimizer [19]. For DetNet, we draw $x$ from a BPSK constellation. We generate 200000 mini-batches of size 5000 for each SNR value. For the optimizer, the learning rate decays exponentially starting with an initial learning rate of $10^{-4}$, a decay factor of 0.97 and decay step-size of 1000. The values of $\beta$ and $\gamma$ of the dHedge algorithm are set to 0.677 and 0.409 respectively, using the hyperparameter tuning tool Hyperopt [20]. In the case of TISTA-Net, we sample each component of the sparse signal $x$ from the Bernoulli-Gaussian distribution with $p = 0.1$ and $\alpha^2 = 1$. Here, we generate 200 mini-batches of size 1000. The learning rate of the optimizer is $4.0 \times 10^{-2}$. The values of $\beta$ and $\gamma$ for the dHedge algorithm are 0.7 and 0.2, respectively.

![Weighing ratios vs Layers for DetNet](image)

\textbf{Fig. 1:} Weighing ratios vs Layers for DetNet

In Fig. 1, we plot the weighing ratios learned by dHedge over the layers. We also plot the normalized logarithmic weights originally proposed in [14] for comparison. We can observe that for all the three cases of $L$, the weighing ratios increase monotonically over the layers. However, the increase is less pronounced in the final layers. The difference in the weighing ratio between the 120th and the 55th layer is in the order of $7 \times 10^{-5}$. This implies that the loss measured at the 55th layer will only be marginally greater than the loss measured at the 120th. We can verify this by the BER curves in Fig. 2. For all these networks, for $\epsilon = 7 \times 10^{-5}$, we can choose $M = 55$, and eliminate the final $L - 55$ layers. Also, the loss in the performance for both these networks is minimal, when compared with the original DetNet. We also obtain some savings in memory usage.

\begin{algorithm}
1: \textbf{Input:} System parameters: $L$, $T$ training batches of $(x, H, y)$, network parameters $\theta$
2: \textbf{Input:} dHedge parameters: $\beta$, $\gamma$, tolerable error $\epsilon$
3: Initialize weighing ratios $w_1, w_2, \ldots, w_L$
4: for $t = 1$ to $T$ do
5: \hspace{1em} Forward propagation
6: \hspace{1em} Calculate loss $\sum_{k=1}^{L} w_k[t] ||x - \hat{x}_k(t)||^2$ \hspace{1em} $||x - \hat{x}||^2$
7: \hspace{1em} Back propagation
8: \hspace{1em} Weight update: $w_k[t+1] = w_k[t] \gamma \beta ||x - \hat{x}_k(t)||^2$
9: \hspace{1em} end for
10: \textbf{Output:} Final weighing ratios $\hat{w}_1, \ldots, \hat{w}_L$
11: if $M = \max_{k \in \{1, L\}} |\hat{w}_k - \hat{w}_L| \geq \epsilon$ then
12: \hspace{1em} Remove the final $L - M$ layers
13: \textbf{end if}
\end{algorithm}
Since it gives a principled approach for reducing the depth of the network without loss of performance.

**References**

[1] G. Hinton, L. Deng, D. Yu, G. E. Dahl, A. Mohamed, N. Jaitly, A. Senior, V. Vanhoucke, P. Nguyen, T. N. Sainath, and B. Kingsbury, “Deep neural networks for acoustic modeling in speech recognition: The shared views of four research groups,” *IEEE Signal Processing Magazine*, vol. 29, no. 6, pp. 82–97, Nov 2012.

[2] A. Krizhevsky, I. Sutskever, and G. E. Hinton, “Imagenet classification with deep convolutional neural networks,” in *Advances in Neural Information Processing Systems* 25, F. Pereira, C. J. C. Burges, L. Bottou, and K. Q. Weinberger, Eds. Curran Associates, Inc., 2012, pp. 1097–1105.

[3] J. Devlin, R. Zhemb, Z. Huang, T. Larson, R. Schwartz, and J. Mahkoul, “Fast and robust neural network joint models for statistical machine translation,” in *Proceedings of the 52nd Annual Meeting of the Association for Computational Linguistics (Volume 1: Long Papers)*. Baltimore, Maryland: Association for Computational Linguistics, Jun. 2014, pp. 1370–1380.

[4] Y. Lecun, L. Bottou, Y. Bengio, and P. Haffner, “Gradient-based learning applied to document recognition,” *Proceedings of the IEEE*, vol. 86, no. 11, pp. 2278–2324, Nov 1998.

[5] V. Raj and S. Kalyani, “Backpropagating through the air: Deep learning at physical layer without channel models,” *IEEE Communications Letters*, vol. 22, no. 11, pp. 2278–2281, Nov 2018.

[6] T. J. O’Shea, T. Erpek, and T. C. Clancy, “Deep learning based MIMO communications,” *CoRR*, vol. abs/1707.07980, 2017. [Online]. Available: http://arxiv.org/abs/1707.07980

[7] N. Farsad and A. J. Goldsmith, “Detection algorithms for communication systems using deep learning,” *CoRR*, vol. abs/1705.08044, 2017. [Online]. Available: http://arxiv.org/abs/1705.08044

[8] S. Drenner, S. Cammerer, J. Hoydis, and S. t. Brink, “Deep learning based communication over the air,” *IEEE Journal of Selected Topics in Signal Processing*, vol. 12, no. 1, pp. 132–143, Feb 2018.

[9] M. Mohammadkarimi, M. Mehrabi, M. Ardakani, and Y. Jing, “Deep learning-based sphere decoding,” *IEEE Transactions on Wireless Communications*, vol. 18, no. 9, pp. 4368–4378, Sep. 2019.

[10] X. Jin and H. Kim, “Parallel deep learning detection network in the mimo channel,” *IEEE Communications Letters*, vol. 24, no. 1, pp. 126–130, Jan 2020.

[11] J. R. Hershey, J. L. Roux, and F. Weninger, “Deep unfolding: Model-based inspiration of novel deep architectures,” *CoRR*, vol. abs/1409.2574, 2014. [Online]. Available: http://arxiv.org/abs/1409.2574

[12] K. Gregor and Y. LeCun, “Learning fast approximations of sparse coding,” in *Proceedings of the 27th International Conference on International Conference on Machine Learning*, ser. ICML’10. USA: Omnipress, 2010, pp. 399–406. [Online]. Available: http://dl.acm.org/citation.cfm?id=3104322.3104374

[13] M. Borgerding and P. Schniter, “Onsager-corrected deep learning for sparse linear inverse problems,” *CoRR*, vol. abs/1607.05966, 2016. [Online]. Available: http://arxiv.org/abs/1607.05966

[14] N. Samuel, T. Diskin, and A. Wiesel, “Deep mimo detection,” in *2017 IEEE 18th International Workshop on Signal Processing Advances in Wireless Communications (SPAWC)*, July 2017, pp. 1–5.

[15] H. He, C. Wen, S. Jin, and G. Y. Li, “A model-driven deep learning network for mimo detection,” in *2018 IEEE Global Conference on Signal and Information Processing (GlobalSIP)*, Nov 2018, pp. 584–588.

[16] D. Ito, S. Takabe, and T. Wadayama, “Trainable isfa for sparse signal recovery,” *IEEE Transactions on Signal Processing*, vol. 67, no. 12, pp. 3113–3125, June 2019.

[17] Y. Freund and R. E. Schapire, “A decision-theoretic generalization of on-line learning and an application to boosting,” *Journal of computer and system sciences*, vol. 55, no. 1, pp. 119–139, 1997.

[18] V. Raj and S. Kalyani, “An aggregating strategy for shifting experts in discrete sequence prediction,” *arXiv preprint arXiv:1708.01744*, 2017.

[19] D. P. Kingma and J. Ba, “Adam: A method for stochastic optimization,” *arXiv preprint arXiv:1412.6980*, 2014.

[20] J. Bergstra, D. Yamins, and D. D. Cox, “Making a science of model search: Hyperparameter optimization in hundreds of dimensions for vision architectures,” 2013.