Chapter

DNS for Turbulent Premixed Combustion

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Abstract

Most of practical combustion occurs in turbulent flows which involve strong coupling between turbulence and chemical processes. The heat release from combustion alters the fluid properties such as density and viscosity and in turns affects the turbulence. Direct numerical simulations (DNS) provides a tool for obtaining both temporally and spatially resolved data in three dimension (3D). This chapter presents a brief overview of importance of DNS in turbulent combustion, the role of turbulence and identifies different combustion modes. The mathematical formulation and numerical implementation for DNS are introduced. The second half of this chapter presents DNS results for ignition in both homogeneous and stratified mixtures. It has been found that minimum ignition energy is required to obtain successful ignition in different turbulence regimes. An increase in turbulent velocity fluctuation may leads to a misfire. Additionally the difference between growing flames and those which are quenched by turbulence have been discussed with the help of the reaction–diffusion balance analysis. Furthermore, the turbulence intensity and length scale of the mixture inhomogeneity have important influences on achieving self-sustained combustion following successful ignition events.

Keywords: premixed combustion, turbulent premixed regimes, Kolmogorov scale, flame structure, ignition

1. Introduction

Many different numerical methods have been developed for the solution of fluid flow problems. Many commercial computational fluid dynamics (CFD) codes are available and have become standard engineering tools for simulation of non-reacting flows. However, for combustion, CFD techniques are not well developed for its accuracy and robustness. The moment we introduce combustion in CFD, it invites additional complexities for stable, accurate and efficient reacting flow numerical solvers. Many standard CFD techniques are available and serve as basic methods for solving combustion problems. Reader can refer to many standard textbooks on the subject [1, 2].

Direct numerical simulations (DNS) is a CFD tool which resolves all flow features explicitly and is widely adopted in combustion research. The feasibility and challenges of DNS in tackling the problems of turbulent combustion is discussed in great detail by Cant [3]. DNS often demands a very large computational power, especially when resolving the forced ignition process of turbulent reacting flows. Despite the computation cost, DNS is highly accurate and provides an enormous
amount of detailed information in comparison to experiments because it is either extremely expensive or impossible to obtain three-dimensional temporally and spatially resolved data by experimental means.

2. Why DNS?

There are three main strategies adopted in CFD simulations of turbulent flows. These strategies are:

- Reynolds averaged Navier–Stokes (RANS)
- Large eddy simulation (LES)
- Direct numerical simulation (DNS)

The philosophies of the above mentioned strategies can be understood from Figure 1, which illustrates the grid spacing requirements for DNS, RANS, and LES in relation to the turbulent kinetic energy spectrum $E(\kappa)$. In the RANS, the grid spacing (let us say $\Delta x$) is of the order of the inverse of energy containing wave number (i.e. $\Delta x \sim 1/l$), which tells us that the whole of the turbulent kinetic energy is unresolved in the context of RANS. In the LES, the grid size is close to the filter width $\Delta$, which tells us that the physical processes are taking place for the wave number $\kappa < 1/\Delta$ are fully resolved, however, the physics at $\kappa > 1/\Delta$ are happening at subgrid level and thus remains unresolved. In the case of DNS, almost all the turbulent kinetic energy is resolved as the grid size is of the order of inverse of dissipation wave number.

Figure 2 shows the volume rendered of burnt products (blue), flame surface (red) and fresh reactants (transparent) in the computational cubic domain with comparison across three strategies. RANS only shows the statistical information of turbulence scales, whereas DNS provides time dependent instantaneous full information on turbulence scales with huge Reynolds number dependency.

In DNS turbulent fluid motion is simulated without any kind of physical approximation which indicates that you do not need any turbulence model for DNS,
all the length, time and velocity scales of turbulent flow are adequately resolved
with the help of computational grid and time step used for given simulation case. It
is important that the grid size (let us say $\Delta x$) in DNS needs to be smaller than the
smallest significant length scale of turbulence, which is the Kolmogorov length scale $\eta$. Additionally, it is also important to have DNS computational domain that con-
tains number of integral length scales so that enough number of large eddies are
available to extract meaningful statistics. DNS simulations should be carried out for
a number of integral time scales to have turbulent statistics that is independent of
initial velocity field. The time step size for DNS should be smaller than the smallest
time scale of turbulence. The computational time can be estimated by the product
of grid points and number of time steps. This makes DNS extremely computa-
tionally expensive in nature. However, for case of compressible flow, computational
time for DNS depends on Courant number, acoustic velocity, and Mach number.

3. Turbulent combustion

Combustion requires fuel and oxidiser to mix at the molecular level. How this
takes place in turbulent combustion that depends on the turbulent mixing process.
The general view is that once a range of different size eddies has developed, strain
rate at the interface between the eddies enhances the mixing. During the eddy
break-up process and the formation of smaller eddies, strain rate increases and
thereby steepens the concentration gradients at the interface between reactants,
which in turn enhances their molecular diffusion rate. Molecular mixing of fuel and
oxidizer, a prerequisite of combustion, takes place at the interface between small
eddies [4]. The subject of turbulent combustion spans a broad range of disciplines
ranging from turbulent flows to combustion chemistry, which makes the analysis of
turbulent combustion a daunting task. At the heart of the challenge is the presence
of a broad range of length and time scales of the various processes governing
combustion and the degree of coupling between these processes across all scales [5].

3.1 Turbulent scales

In order to estimate whether the chemistry is fast or slow compared to turbulent
mixing, it is useful to define the time, length and velocity scales associated with
physical processes. First consider the range of length scale (eddy sizes) that one may
expect to encounter in turbulent flows. The largest length scale of turbulence is
known as the integral length scale ($L_{11}$), which is the length scale at which most of
the energetic eddies are associated. By contrast, the smallest length scale of turbu-
lenence known as Kolmogorov length scale, is determined by viscous dissipation of
turbulent kinetic energy. From the Kolmogorov’s hypothesis, the only factors influencing the behavior of the small scale motions are the overall kinetic energy dissipation rate \(\varepsilon\) and the viscosity \(\nu\). The length scale which governs these physical mechanisms is given by:

\[
\eta = \left( \frac{L^3}{\varepsilon} \right)^{1/4}
\]

(1)

This length scale is called the Kolmogorov length scale \(\eta\) and is the smallest hydrodynamic scale in turbulent flows. The kinetic energy of the flow is proportional to \(u'^2\), where \(u'\) is the velocity fluctuation. The time scale of the life time of the large eddies (commonly referred to as the large eddy turnover time) can be estimated as \(L_{11}/u'\). Therefore, the kinetic energy dissipation rate is:

\[
\varepsilon \sim \frac{u' u'}{L_{11} / u'} \sim \frac{(u')^3}{L_{11}}
\]

(2)

The ratio of the largest to smallest length scales in the turbulent flow is given by:

\[
\frac{L_{11}}{\eta} \sim \left( \frac{u' L_{11}}{\nu} \right)^{3/4} \sim Re_t^{3/4}
\]

(3)

where \(Re_t\) is the turbulent Reynolds number. Another commonly encountered length scale in turbulence is the Taylor micro-scale. This length scale does not have the same easily understood physical significance as the Kolmogorov or internal length scale but provides a convenient estimate for the fluctuating strain rate field. The Taylor micro-scale \(\lambda\) is defined through the relation:

\[
\varepsilon \sim \nu \left( \frac{\partial u'_i}{\partial x_j} \frac{\partial u'_i}{\partial x_j} \right) \sim \frac{u'^2}{\lambda^2}
\]

(4)

The large eddy turnover time \(t_e\) can be defined as \(t_e \sim L_{11}/u'\). The life time for the small eddies of turbulence can be estimated using the viscosity and the dissipation rate as \(t_\eta \sim \sqrt{\nu/\varepsilon}\). The ratio of time scales is therefore:

\[
\frac{t_e}{t_\eta} \sim \left( \frac{u' L_{11}}{\nu} \right)^{1/2} \sim Re_t^{1/2}
\]

(5)

The large scale structures in the flow are seen to have a much larger time scale (duration) than the smallest energy dissipating eddies. As the turbulent Reynolds number of the flow increases, the magnitude of the separation between both time and length scales increases. One can define computational time for DNS scales with \(Re_t^3\).

### 3.2 Premixed turbulent combustion regimes

Diagrams defining regimes of premixed turbulent combustion in terms of velocity and length scale ratio have been proposed in a number of previous analyses [6–8]. For scaling purposes it is useful to assume equal diffusivities for all reactive scalars, Schmidt number \(Sc = \nu / D\) equal to unity, where \(D\) is the mass diffusivity. Based on this one can define flame thickness as \(l_f = D / S_b\) and flame time as
$t_f = D/S_b^2$, where $S_b$ is the laminar burning velocity at given equivalence ratio. One can estimate the turbulent Reynolds number as:

$$Re_t \sim \frac{u' L_{11}}{S_b l_f}$$  \hspace{1cm} (6)

Furthermore, one can quantify the separation between chemical time scale to the Kolmogorov time scale using Karlovitz number ($Ka$) as:

$$Ka \sim \frac{t_f}{\eta} \sim \frac{l_f}{\eta^2}$$  \hspace{1cm} (7)

**Figure 3** shows the regime diagram for premixed turbulent combustion using the definition of Kolmogorov length scale. Moreover, **Figure 3** shows the typical working conditions realised in IC engines, gas turbines and counter flow regime on the regime diagram. Here the ratios $u'/S_b$ and $L_{11}/l_f$ may be expressed in terms of the two non-dimensional numbers $Re_t$ and $Ka$ as:

$$\frac{u'}{S_b} = Re_t \left(\frac{L_{11}}{l_f}\right)^{-1} = Ka^{2/3} \left(\frac{L_{11}}{l_f}\right)^{1/3}$$  \hspace{1cm} (8)

The lines $Re_t = 1$ and $Ka = 1$ represents boundaries between different regimes of premixed turbulent combustion in **Figure 3**. Other boundaries of interest are the line $u'/S_b = 1$ which separated the wrinkled flamelets from the corrugated flamelets, and the line denoted by $Ka = 100$, which separates the thin reaction zones from broken reaction zones. The line $Re_t = 1$ separates all turbulent flame regimes characterised by $Re_t > 1$ from the laminar flame regime ($Re_t < 1$), which is situated in the lower-left corner of the diagram. In the wrinkled flamelet regime, where $u' < S_b$, the turnover velocity $u'$ of even the integral eddies is not large enough to compete with the advancement of the flame front with the laminar burning velocity $S_b$. Laminar flame propagation therefore dominates over flame front corrugations by turbulence [8].

The corrugated flamelet regime is characterised by the inequalities $Re_t > 1$ and $Ka < 1$. The inequality indicates that $l_f < \eta$ (see Eq. (7)), which means that the entire
reactive-diffusive flame structure is embedded within the eddies of the size of the Kolmogorov scale, where the flow is quasi-laminar. Therefore the flame structure is not perturbed by turbulent fluctuation and retains its quasi-laminar structure [8].

The thin reaction zones regime is characterised by $Re_t > 1$ and $1 < Ka < 100$, the last inequality indicating that the smallest eddies of size $\eta$ can enter into the reactive–diffusive flame structure since $\eta < l_f$ (see Eq. (7)). These small eddies are still larger than the reaction layer thickness $l_\delta$ and can therefore not penetrate into that layer. The thickness $\delta$ of the inner layer in a premixed flame is typically one tenth of the flame thickness, such that $l_\delta$ is one tenth of the preheat zone thickness which is of the same order of magnitude as the flame thickness $l_f$.

Beyond the line $Ka = 100$ there is a regime called the broken reaction zones regime where Kolmogorov eddies are smaller than the inner layer thickness $l_\delta$. These eddies may therefore enter into the inner layer and perturb it with the consequence that chemical processes are disturbed locally owing to enhanced heat loss to the preheat zone followed by temperature decrease and the loss of radicals. When this happens the flame will extinguish and fuel and oxidizer will inter diffuse and mix at lower temperatures where combustion reaction has ceased to take place. Nevertheless, regime diagram provides a useful purpose in allowing the classification of turbulent premixed flames, different premixed turbulent combustion regimes are summarised in Table 1.

### Table 1. Summary of premixed turbulent combustion regimes.

| Combustion regimes          | Range                   |
|-----------------------------|-------------------------|
| Laminar flames              | $Re_t \leq 1$           |
| Wrinkled flamelets          | $Re_t \geq 1; u'/S_b \leq 1$ |
| Corrugated flamelets        | $Re_t \geq 1; u'/S_b > 1; Ka < 1$ |
| Thin reaction zones$^a$     | $Re_t \geq 1; 1 \leq Ka < 100$ |
| Broken reaction zones       | $Re_t \geq 1; Ka \geq 100$ |

$^a$The analyses argued that since quenching by vortices occurs only for large Karlovitz numbers, the region below the limiting value of the Karlovitz number should correspond to the flamelet regime [8].

### 4. Combustion modes: premixed and non-premixed

Generally, combustion can be divided into two categories: premixed and non-premixed combustion. Figure 4 shows a Venn diagram representing different combustion modes. Each of these categories has its advantages and disadvantages, but premixed combustion offers advantages in terms of pollutant emission because the maximum burned gas temperature can be controlled by the mixture composition. Thus fuel lean premixed combustion can potentially lead to reduction of burned gas temperature which offers reduction in thermal NOx emission [9]. In the demand to reduce harmful emissions, industrial combustors are designed to operate under fuel lean conditions and with inhomogeneous mixtures, which increasingly often leads to stratified combustion [10, 11]. Many engineering combustion systems including: lean premixed prevaporised (LPP) gas turbine combustor, afterburners, and direct-injection spark-ignition internal combustion engines, they all operate in inhomogeneous reactants mode to gain full advantages of a spatially varying mixture field [12–14].
Stratified premixed combustion combines advantages of both premixed and non-premixed combustion modes (see Figure 4). In stratified combustion a premixed flame originated from ignition source travels through mixture field of varying equivalence ratio, which may be either all lean or all rich and the flame propagation is strongly affected by local gradient of mixture field [15]. For instant, in a gasoline direct injection (GDI) spark ignition engine, the time interval between fuel being injected into the combustion chamber and the spark ignition may be too short for the mixture composition to be homogeneous at the instant of ignition, but it is long enough for most of the fuel to be mixed with air before burning. The flame kernel originated by the spark propagates through a highly inhomogeneous mixture field characterised by large fluctuations in the equivalence ratio, with the ensemble-averaged mixture composition being lean (and even beyond lean flammability limit) in some spatial regions and rich (and even beyond the rich flammability limit) in other regions. In such example inhomogeneously premixed combustion is important, as it controls majority of the total heat release, while the afterburning of the lean and rich products in the diffusion mode may be of significant importance as far as pollutant (e.g. soot formation) is concerned. By contrast, in a diesel engine, the time interval between fuel injection and autoignition is too short that only a small amount of the fuel is mixed with air before autoignition of mixture due to compression. Here, also lean and rich premixed turbulent flames coexist with diffusion flames, but contrary to GDI engine, the total heat release is mainly controlled by the non-premixed mode of burning, and such regime is called nonpremixed/premixed combustion [7, 16]. All aforementioned combustion modes (stratified, premixed/nonpremixed, and nonpremixed/premixed burning) are commonly absorbed under partially-premixed flames [7].

5. DNS results and discussions

This section includes some of the DNS results of turbulent combustion in different environments. The results are presented and subsequently discussed. All the simulations presented here are performed using a well known compressible DNS code SENGA [3]. This DNS code solves the full compressible Navier-Stokes equations on a cartesian grid. The governing equations that describe the 3D gaseous...
reacting flow consists of mass, momentum, energy and species conservation equations. The boundaries in the \(x_1\)-direction are taken to be partially non-reflecting and are specified using the Navier-Stokes characteristic boundary conditions (NSCBC) formulation \[17\], whereas the boundaries in the other directions are considered to be periodic. A 10th-order central difference scheme is used for spatial differentiation for the internal grid points, and the order of differentiation gradually reduces to a one-sided 2nd-order scheme at non-periodic boundaries \[18\]. The time advancement is carried out using a 3rd-order low-storage Runge-Kutta scheme \[19\].

5.1 Ignition

A source term \(q'''\) has been added for accounting the heat addition to the energy transport equation:

\[
\frac{\partial \rho E}{\partial t} + \frac{\partial \rho u_k E}{\partial x_k} = - \frac{\partial u_k P}{\partial x_k} + \frac{\partial \tau_{kk}}{\partial x_k} \left[ \frac{\partial T}{\partial x_k} \right] + \dot{w}_T + q'''
\]

The specific heats at constant pressure and constant volume (i.e. \(c_p\) and \(c_v\)) are taken to be constant and same for all species for the simplified chemistry results. Therefore, the term \(\sum_{k=1}^{N} h_{ik} Y_k V_k = 0\) and not included in the Eq. (9). The source term is assumes to follow a Gaussian distribution \[20\] in the radial direction away from the centre of the igniter and can be expressed as:

\[
q'''(r) = A_q \exp \left( - \frac{r^2}{2R^2} \right)
\]

where \(r\) is the radial distance from the centre of the igniter and \(R\) is the width of the Gaussian profile. The choice of \(R\) \[21\] in the present analysis allows sufficient resolution of the temperature gradient and guarantees the rapid disappearance of any artificial effects introduced by the ignition source. The constant \(A_q\) is determined by the following volumetric integration \[21\]:

\[
\dot{Q} = \int_V q''' dV
\]

where \(\dot{Q}\) is the ignition power, which can be defined as:

\[
\dot{Q} = a_{sp} \rho_0 c_p T_0 \frac{4}{3} \pi l_f^2 \left[ H(t) - H(t - t_{sp}) \right] \frac{t_{sp}}{t_{sp}}
\]

where \(a_{sp}\) is a parameter that determined the total energy deposited by the igniter, the \(t_{sp}\) is the time duration over which the energy is deposited by the igniter, which is expressed as \(t_{sp} = b_{sp} \times t_f\), where \(b_{sp}\) is the energy duration parameter and \(t_f\) is a characteristic chemical time scale given by \(t_f = l_f / S_b\). The parameter \(b_{sp}\) varies between 0.2 and 0.4 for optimum spark duration according to experimental findings \[22\]. Here \(l_f\) is the Zel-dovich flame thickness for the stoichiometric mixture, which is defines as \(l_f = aT_0 / S_b\), where \(aT_0\) is the thermal diffusivity in the unburned reactants and \(S_b\) is the unstrained laminar burning velocity of the stoichiometric mixture. It is important to note that the details of the spark formation (momentum modification contribution, plasma formation and shock wave) remain
beyond the scope of the present analysis to keep this study computationally feasible. Parametric study of the effects of energy deposition characteristics \((a_p, b_p, R)\) on localised forced ignition and early stages of burning following successful ignition in homogeneous turbulent mixtures using 3D DNS are investigated by [23, 24].

5.2 Ignition in homogeneous mixture

5.2.1 Isosurface of temperature field

Premixed combustion can be described in terms of a composition variable known as reaction progress variable. This reaction progress variable describe the progress of the premixed reaction [25]. The active scalars which are often considered for analysing turbulent combustion are the fuel mass fraction \(Y_F\) and the reaction progress variable \(c\) [26]. The extent of the completion of chemical reaction can be quantified in terms of a reaction progress variable \(c\), defined as:

\[
c = \frac{Y_{F_u} - Y_F}{Y_{F_u} - Y_{F_b}}
\]

where \(Y_{F_u}\) is the fuel mass fractions in the unburned gas and \(Y_{F_b}\) is the fuel mass fraction in the burned gas. Both \(Y_{F_u}\) and \(Y_{F_b}\) are the function of mixture fraction. According to Eq. (13), \(c\) rises monotonically from zero in the fully unburned reactants to unity in the fully burnt products. Figure 5 shows 3D volume rendered view of non-dimensional temperature for different Karlovitz numbers ranging across different turbulent regimes. The values for non-dimensional temperature \(T\) corresponds to reaction progress variable \(c \geq 0.9\) showing the flame kernel. It is evident from Figure 5 that the isosurfaces of \(T\) remains approximately spherical during the period of energy deposition, but they become increasingly wrinkled as time progresses for all the cases. During the energy deposition period (i.e. \(t \leq t_{sp}\)) the evolution of temperature field is principally determined by the diffusion of energy deposited, but after the ignition initiated, the flame propagation controlled by the diffusion of local flame stretching mechanism. The turbulence tends to fragment the flame surface and breaks the reaction zone for higher Karlovitz number cases. Furthermore interesting observation is when \(t \gg t_{sp}\), the high temperature region has been fragmented thanks to energetic eddies of turbulence penetrating into the flame. This tendency is more prominent for \(K_d = 150\), where turbulent eddies enters into flame and start breaking the reaction zone and flame eventually extinguishes.

5.2.2 Reaction-diffusion balance analysis

It is extremely important that heat release due to chemical reaction should overcome the heat transfer from the hot gas kernel in order to obtain self-sustained combustion following successful ignition. The transport equation of the reaction progress variable \(c\) in the context of turbulent premixed flames is:

\[
\frac{\partial pc}{\partial t} + \frac{\partial \rho u_j c}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \frac{\partial D c}{\partial x_j} \right) + \dot{w}_c
\]

where \(\dot{w}_c\) is the reaction progress variable reaction rate.
It is important to examine the reaction–diffusion balance in order to understand the difference in flame kernels which are growing in an unperturbed manner and those which are fragmented and about to be quenched by turbulence. In that respect, following terms are defined to explain the reaction–diffusion balance within the flame:

\[
\begin{align*}
\text{reaction term} &: \quad \dot{\omega}_c, \\
\text{molecular diffusion term} &: \quad \nabla \cdot (\rho D \nabla c), \\
\text{flame normal diffusion rate term} &: \quad \vec{N} \cdot \nabla \left( \rho D \vec{N} \cdot \nabla c \right), \\
\text{tangential diffusion rate term} &: \quad -\rho D \nabla \cdot \vec{N} |\nabla c|.
\end{align*}
\]

where \(\vec{N}\) is the flame normal vector, which can be defined as:

\[
\vec{N} = -\frac{\nabla c}{|\nabla c|}.
\]

Figure 5. Volume rendered view of non-dimensional temperature field in domain \(51l_f \times 51l_f \times 51l_f\) for different Karlovitz numbers (Ka) in different turbulent regimes.
It can be seen from Figure 6 that the term $\dot{\omega}_c$ remains negligible in the unburned side and increases sharply towards the burned side before decreasing to zero in the fully burned products. The magnitude of this term decreases with time, once the igniter is switched off, which is principally due to the decrease in fuel reaction rate magnitude with time. Therefore, it can be seen that maximum reaction takes place in the region $0.6 \leq c \leq 0.99$. The term $\nabla \cdot (\rho D \nabla c)$ shows opposite behaviour to the term $\dot{\omega}_c$, suggesting reactants are diffusing prior to approaching the flame kernel and term remains negative in burned side. Additionally, the term $\dot{\omega}_c + \nabla \cdot (\rho D \nabla c)$ suggests a self-sustained propagation of flame kernel. The term $\nabla \left( \rho D \nabla \nabla c \right)$ remains positive in unburned side but becomes negative towards burned side. The term $-\rho D \nabla \cdot \nabla c$ remains negative for the flame brush. In the flame normal vector, $N_j$ is the $j$th component of flame normal which can be defined in terms of flame curvature [27]:

$$\kappa_m = \frac{1}{2} \nabla \cdot \nabla c = \frac{\kappa_1 + \kappa_2}{2} \sim \frac{1}{R_{fk}}$$

where $\kappa_1$ and $\kappa_2$ are the two principle curvatures of the concerned flame surface, $\kappa_m$ is the arithmetic mean of these two principle curvatures and $R_{fk}$ is the radius of a flame kernel. As the flame kernel increases in size (i.e. $R_{fk}$ increases), the probability of high values of $\kappa_m$ decreases (see Eq. (20)). Furthermore according to Eq. (20) positively curved location (i.e. $\kappa_m>0$) are convex towards the reactant. The flame is initiated as a spherical kernel from the localised forced ignition, which has a positive mean curvature. In this respect the term $-\rho D \nabla \cdot \nabla c$ will attain negative values in the case of expanding flame kernel (i.e. self-sustained propagating flame kernel). The growth of the kernel leads to a decrease in the magnitude of the negative contribution of the term $-\rho D \nabla \cdot \nabla c$, whereas the term $-\rho D \nabla \cdot \nabla c$ is expected to assume large negative values in the kernels which are quenching. This findings are in good agreement with previous experimental studies [28, 29].

5.3 Ignition in stratified mixture

Premixed combustion offers an option of controlling flame temperature and reducing pollutant emission such as nitrogen oxides (NOx) but, in practice, perfect mixing is often difficult to achieve and thus combustion in many engineering applications takes place in turbulent stratified mixtures. Many previous findings
[30, 31] shows that the flame propagation statistics are strongly influenced by the local equivalence ratio gradient. The length scale of mixture inhomogeneity is taken as the Taylor micro-scale of the equivalence ratio $l_{\phi}$ and is defined as [32]:

$$l_{\phi} = \sqrt{\frac{6 \langle (\phi - \langle \phi \rangle)^2 \rangle}{\langle \nabla (\phi - \langle \phi \rangle) \cdot \nabla (\phi - \langle \phi \rangle) \rangle}}$$

(21)

The equivalence ratio $\phi$ variation is initialised using a random distribution of $\phi$ following a Bi-modal distribution for specified values of the mean global equivalence ratio $\langle \phi \rangle$. In practical, when fuel is introduced in the liquid phase, the probability density function (PDF) of the equivalence ratio distribution is likely to be Bi-modal as a result of localised liquid fuel evaporation during the early stage of mixing. The fuel-air mixture is likely to be fuel rich close to the evaporation sites and the mixture is expected to be fuel-lean far away from the droplets. The initial mixture distribution for $\langle \phi \rangle = 1$ and $\phi' = 0.4$ with different values of $l_{\phi}/l_f$ are shown in Figure 7, which indicated that the clouds of mixture inhomogeneities increase in size with increasing $l_{\phi}/l_f$.

5.3.1 Mode of combustion

The role of the reaction progress variable $c$, in the turbulent stratified mixtures has been discussed in detail by Bray et al. [33]. In the context of stratified combustion, the reaction progress variable can be defined in the following manner [21, 25]:

$$c = \frac{\xi Y_{F\infty} - Y_F}{\xi Y_{F\infty} - \max [0, \frac{\xi - \xi_{st}}{1 - \xi_{st}}] Y_{F\infty}}$$

(22)

where $\xi$ is the conserved scalar and can be defined as:

$$\xi = \frac{Y_F - \frac{Y_O}{s} + \frac{Y_{O\infty}}{s}}{Y_{F\infty} + \frac{Y_{O\infty}}{s}}$$

(23)

where $s$ is the mass of the oxidiser consumed per unit mass of fuel consumption, $Y_F$ and $Y_O$ are local fuel and oxidiser mass fractions respectively, $Y_{F\infty}$ is the fuel mass fraction in pure fuel stream and $Y_{O\infty}$ is the oxidiser mass fraction in air.
In order to understand the flame structure originating from localised forced ignition (e.g. spark or laser), the Takeno flame index \([34, 35]\) can be used to identify the local combustion mode:

\[
I_c = \frac{\nabla Y_F}{|\nabla Y_F|} : \frac{\nabla Y_O}{|\nabla Y_O|}
\]  

(24)

Based on Eq. (24), the Takeno flame index obtains positive value in premixed mode of combustion and negative value in non-premixed mode of combustion. The volume rendered views of the region corresponding to \(0.01 \leq c \leq 0.99\) are shown in Figure 8 for selected cases. It is evident from Figure 8 that the reaction takes place predominantly in the premixed mode (i.e. \(I_c > 0\)) but some pockets of \(I_c < 0\) indicate the possibility of finding local pockets of non-premixed combustion. The probability of finding \(I_c < 0\) decreases with increasing time due to mixing process. The \(I_c\) predominantly assumes positive values and major portion of overall heat release arises due to the premixed mode of combustion in all cases. Moreover, the percentage of heat release arising from the non-premixed mode of combustion decreases with increasing value of \(u'\) as a result of improved mixing.

![Figure 8](image1.png)

(a) \(u'/S_b = 0\)  
(b) \(u'/S_b = 4.0\)  
(c) \(u'/S_b = 6.0\)

\(I_c > 0\) (■); \(I_c < 0\) (□)

Figure 8. 
Volumetric rendering of the region corresponding to \(0.01 \leq c \leq 0.99\) in domain \(33l_f \times 33l_f \times 33l_f\) coloured with local values of \(I_c\) for different values of \(u'/S_b\) and mixture inhomogeneity \(l_\phi/l_f = 8.3\).

![Figure 9](image2.png)

(a) \(u'/S_b = 0\)  
(b) \(u'/S_b = 4.0\)

\(l_\phi/l_f = 2.1\) (■); \(l_\phi/l_f = 5.5\) (□); \(l_\phi/l_f = 8.3\) (■)

Figure 9. 
Temporal evolution of mean burnt gas mass with \(\phi' = 0.2\) and \(l_\phi/l_f = 5.5\) with standard deviation due to different realisation of initial conditions shown in the form of bars.
5.3.2 Extent of burning

The extent of burning can be characterised by the mass of burned gas $m_b$ with $c \geq 0.9$ [21]. The temporal evolution of burnt gas mass normalised by the mass of an unburned gas sphere with a radius equal to $l_f$ for $l_{\phi}/l_f = 5.5$ are shown in Figure 9. It is important to note that $\int_{V} \rho c \, dV$ provides the measure of total burned gas mass within the computational domain. Figure 9 shows that an increase in $u'$ leads to reduction in burned gas mass for all cases. An increase in $u'$ leads to an increase in eddy diffusivity $D_t \sim u'L_{11}$ for a given high values of turbulence intensity. For self-sustained flame propagation following successful ignition, the heat release from the combustion must overcome the heat loss from hot gas kernel. Due to heat transfer from hot gas kernel, the probability of $c \geq 0.8$ also decreases and at such point the heat loss overcomes chemical heat release, the hot gas kernel shrinks and eventually leads to flame extinction. The detrimental effect of $u'$ on the extent of burning is consistent with previous findings [36]. The influence of $l_{\phi}/l_f$ on the extent of burning is found to be non-monotonic and dependent on $\phi'$. For high values of $l_{\phi}/l_f$ the clouds of mixture stratification are relatively big and as a result of this, there is a possibility that igniter has encountered a rich region (highly flammable) of mixture which leads to higher burning rate. It is well known that combustion succeeds only for selected realisation of initial distribution of $\phi$ and this aspect is particular interest in the cylinder of IC engines due to cycle-to-cycle variation [20]. The temporal evolution of mean and standard deviations of burnt gas mass for all realisations are shown in Figure 9 to demonstrate the probabilistic nature of the localised ignition of stratified mixtures. Furthermore, the large variation of error bar for high values of $l_{\phi}/l_f$ suggests that it is possible to obtain large clouds of both highly flammable and weakly flammable mixtures at the igniter location which leads to large variation of burnt gas mass.

The above discussion suggests that turbulent intensity $u'$ and length scale of mixture inhomogeneity $l_\phi$ have important influences on achieving self-sustained combustion following successful ignition event.

6. Epilogue

Theories and results presented in this chapter suggests that turbulent premixed combustion is a complex and difficult subject, but very rich in the physics. With recent advances in computational capability, the application of DNS will become possible for higher values of turbulent Reynolds number and complex flow configurations. DNS provides highly accurate and detailed 3D information in comparison to experiments because it is extremely expensive or impossible to obtain 3D temporally and spatially resolved data by experimental means. However, with current advances in laser technology, it is possible to have simultaneous planer laser-induced measurement of turbulent concentration and velocity fields. Once this experimental data becomes available, it will be used for validation of DNS results. Moreover, successful ignition often leads to momentum modification contribution, plasma formation, and shock waves, which remains beyond the scope of the present DNS analysis. Additionally, detailed chemical mechanism involving large number of intermediate species which lead to back-diffusion of light radical and post diffusion flames are necessary to gain further fundamental understanding of turbulent combustion processes.
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Conflict of interest

The authors declare no conflict of interest.

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