The Fundamental Plane of Spiral Galaxies:
Search from Observational Data

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Abstract

The fundamental plane of spiral galaxies was searched from observational data, which can be approximately represented in observational parameters by \( L \propto V^2 R \), where \( L \), \( R \), and \( V \) are the luminosity, the linear size of galactic disk, and the rotation velocity. This plane exists at all optical bands in our samples of more than 500 spiral galaxies in total. It is more fundamental than the relationship between any two of the parameters, and can reduce the residual of the Tully–Fisher \( L-V \) relations by about 50\%. Noticed that the power index of \( V \) is doubled from that of \( R \), which implies that the total mass and the mass distribution of galaxies plays an important role in forming the fundamental plane. Involving the third parameter of galactic size has a strong physical implication on galaxy formation and the dark–mass distribution.

Key words: galaxies: fundamental parameters — galaxies: kinematics and dynamics — galaxies: spiral — galaxies: statistics

1. Introduction

The Tully–Fisher relation is a tight correlation between the internal motion and the luminosity of spiral galaxies (Tully, Fisher 1977). It can be expressed as

\[ M = \alpha \log V + \gamma, \]  

(1)

where \( M \) is the absolute magnitude, \( V \) the rotation velocity, \( \alpha \) the slope of the relation, and \( \gamma \) the zero-point. This empirical relation has been used for estimating the distances of galaxies, and hence for determining the Hubble constant (e.g. Sakai et al. 2000; Tully, Pierce 2000). It also provides a critical constraint on galaxy formation (e.g. Dalcanton et al. 1997; Mo et al. 1998; van den Bosch 2000; Avila-Reese, Firmani 2000; Firmani, Avila-Reese 2000; Koda et al. 2000b). However, the detailed physical processes concerning the origin of the Tully–Fisher relation have not yet been fully understood. This relation could be a result of the self–regulated star formation in the disks of different mass (see Silk 1997; Eisenstein, Loeb 1996), or a direct consequence of the cosmological evolution between the mass and the circular velocity (see e.g. Steinmetz, Navarro 1999; Mo et al. 1998; Mo, Mao 2000).

It was believed that the residual value deviating from the Tully–Fisher relation can pose strong constraints on scenarios of galaxy formation and evolution (see e.g. Eisenstein, Loeb 1996) which was systematically studied by, for example, Willick et al. (1997). The residuals, written as

\[ \delta = M - (\alpha \log V + \gamma), \]  

(2)

are not accountable by measurement errors. With good data at the \( R \) and \( I \) bands, the scatter of the luminosity may be partially attributed to the measurement uncertainty of 0.08 mag (Courteau 1996) or 0.04 mag (Giovanelli et al. 1997), which is further convolved with worse-determined distances. Because of the residuals, the estimates of the galaxy distance based on the Tully–Fisher relation have a typical uncertainty of 20\%.

Some efforts have been made previously to search for tighter correlations among the luminosity, rotation velocity, and disk radius for spiral galaxies. Kodaira (1989) found a much tighter correlation among these three parameters, \( L \propto VR^2 \). He pointed out that the above relation is valid for ellipticals as well. Very recently, Koda et al. (2000a) found that the \( I \)-band luminosity \( L \), \( I \)-band radius \( R \), and the rotation velocity \( V \) of spiral galaxies are distributed on the so-called scaling plane in three-dimensional parameter’s space of these quantities, which can be expressed as \( L \propto (VR)^{1.3} \). Because the galactic radius was considered (Koda et al. 2000a), the residual of the Tully–Fisher relation was significantly reduced. In fact, Courteau and Rix (1999) also realized the impor-
tance of involving the disk size to the Tully–Fisher relation, but they did not come up with such a relation. Willick (1999) explored the role of the disk scale length, and found the surface brightness dependence of the Tully–Fisher relation.

It is intriguing to know whether and how the third parameter, the galactic size, really plays some role in galaxy formation, and how the mass and luminosity are physically related to this parameter. Besides strong evidence found by Koda et al. (2000a), the linear (optical) galactic diameter has a very tight correlation to the H I mass in later galaxies (Broeils, Rhee 1997). Koda et al. (2000b) have already done a set of simulations. They found that the galactic size was involved as a key parameter in such a process that the galactic mass and angular momentum were controlled during galaxy formation. Very recently, Shen et al. (2001) considered the variations of the model parameters presented in the theoretical work of Mo et al. (1998). They also found a theoretical fundamental plane, \( L \propto V^{2.6} R^{0.5} \).

It is therefore very necessary to search the fundamental plane from observational data with the galactic radius, \( R \), as the third parameter, and see how it matches the results from simulations of galaxy formation. The primary procedure we take here is to see how the galactic size can help to reduce the residual of the Tully–Fisher relation. The best-fitting plane, expressed as

\[
M = \alpha \log V + \beta \log R + \gamma, \tag{3}
\]

was searched from observational data with three variables, \( \alpha, \beta, \) and \( \gamma \). Koda et al. (2000a) obtained \( \alpha = \beta = -3.25 \), while Kodaira (1989) gave \( \alpha = -2.5 \) and \( \beta = -5.0 \). We found that the best-fitting plane with \( \beta/\alpha \) around 0.5 for different data-sets, namely, the I-band data of Han (1992) and Palunas and Williams (2000), the R-band data of Courteau (1996, 1997), and the BVRIH-band data of calibration galaxies in Sakai et al. (2000) and Macri et al. (2000). The plane exists in different wavebands, and can reduce the residual of the Tully–Fisher relation by about 50%. This implies that about 50% of the scattering in the observed Tully–Fisher relations is dominated by the intrinsic properties of galaxies, and the other 50% by measurement uncertainties. We believe that the plane is fundamental for spiral galaxies, similar to that for elliptical galaxies (Djorgovski, Davis 1987; Dressler et al. 1987). It should provide useful constraints on models of galaxy formation (see Shen et al. 2001; Koda et al. 2000b).

2. The Fundamental Plane versus the Scaling Plane

Han (1992) presented carefully-calibrated I-band data for galaxies in clusters, including the total magnitude, \( M^\text{I}_{\text{tot}} \) and \( M^\text{I}_{23.5} \) (in units of mag), and the face-on I-band isophotal radius, \( A^\text{I}_{23.5} \) (in units of arcsec). He also gave the width of the 21 cm H I line, \( W_{20} \) (in units of \( \text{km s}^{-1} \)) corrected for the redshift and inclination. The scaling plane, \( L \propto (V R)^{1.3} \) (see figure 1 of Koda et al. 2000a), was found among these three observational parameters. Therefore, the Tully–Fisher relation between \( M_I \) and \( W_{20} \) is an oblique projection of the plane. This also explains the Freeman relation, \( L \propto R^2 \) (Freeman 1970), and the radius–velocity relation (Tully, Fisher 1977).

We used the same I-band data-set to fit equation (3). In practice, we define \( k = \beta/\alpha \) and fit equation (3) by varying \( \alpha \) and \( k \). Obviously, equation (3) becomes the original Tully–Fisher relation when \( k = 0.0 \), or it goes to the scaling plane of Koda et al. (2000a) when \( k = 1.0 \). After excluding those galaxies (1) whose recession velocities deviate by more than 1000 \( \text{km s}^{-1} \) from the mean velocity of a cluster, (2) without H I measurements and (3) which may not be a cluster member, we finally obtained 160 galaxies in the sample. Throughout this paper \( H_0 = 75 \text{ km s}^{-1}\text{Mpc}^{-1} \) is assumed. We first used the I-band total absolute magnitude data, \( M = M^\text{I}_{\text{tot}} \), together with \( V = W_{20} \) and \( R = R_{123.5} \) (in kpc), and obtained the minimum \( \chi^2 = 252.2 \) at \((\alpha, \beta, \gamma) = (-4.05 \pm 0.21, -3.00 \pm 0.14, -8.01 \pm 0.43)\), see figure 1. This minimum \( \chi^2 \) value is much smaller than the minimum \( \chi^2 = 697.6 \) from the Tully–Fisher relation \((k = 0)\). It can also be seen in figure 1 that the data scatter is much smaller when the galactic radius is involved. The best-fitting value of \( k = \beta/\alpha \) is 0.74 \pm 0.06, i.e. it is at neither \( k = 0.0 \) nor \( k = 1.0 \) (figure 1). All fitting results, including those found below, are listed in table 1. The last two columns are the \( \chi^2 \) values and the scattering of luminosity \( \delta \), both for the original Tully–Fisher relation and for the best fitting plane.

To account for light within the linear radius, the absolute magnitude, \( M = M^\text{I}_{23.5} \), should probably be used. We then obtained the best-fitting parameters: \((\alpha, \beta, \gamma) = (-4.75 \pm 0.21, -2.85 \pm 0.14, -6.21 \pm 0.43)\) at \( k = 0.60 \pm 0.04 \) and \( \chi^2 = 359.4 \).

3. The Fundamental Plane in Other Bands

As a matter of fact, not many publications about the surface photometry of spiral galaxies contain information about galactic sizes. The database of Tully–Fisher calibration galaxies (Macri et al. 2000; Sakai et al. 2000), the database of Courteau (1996, 1997), and the very recently published I-band data by Palunas and Williams (2000) were used to search for the fundamental plane.

3.1. The Hubble Calibration Galaxies

Sakai et al. (2000) published the Cepheid distances and carefully-corrected absolute magnitudes at \( BVRIH \)-bands of 21 calibration galaxies for the Hubble Space Telescope (HST) key project for the distance scale. They also presented the well measured and corrected H I 20% line widths. Macri et al. (2000) further published the brightness profiles at the \( BVRI \)-bands and isophotal radii \( R_{23} \) and \( R_{23.5} \). We measured the isophotal radii \((R_{23}, R_{25}, R_{23}, R_{25}, R_{23}, \) and \( R_{25})\) from the brightness profiles. Finally, there are 17 galaxies with all parameters which we needed.

We fitted equation (3) to these observational parameters in each band. As expected, involving the radius can greatly reduce the scattering around the Tully–Fisher re-
Fig. 1. Comparison of the Tully–Fisher relation (the left side) and the best fitting plane (the right side). Data of 160 galaxies were taken from Han (1992). The typical rms errors of the absolute magnitudes, together with the displacement from the uncertainties of $W_{20}$ and $R_{23.5}$, were taken as 0.2 mag. Galaxies in different clusters were plotted by different symbols. The data distribution in 3-D space and the variation of fitting $\chi^2$ with $k$ were also plotted.

The $k$ values are around 0.5 (see lines with HST- in table 1). Notice that in these HST data-sets, those with $R_{23.5}$ and $R_{B25}$ produced the smallest residuals.

3.2. $I$-Band Data of Palunas and Williams (2000)

Very recently, Palunas and Williams (2000) conducted $I$-band photometry and Hα measurements of 74 galaxies. They presented the corrected magnitudes, $M_I^{\text{tot}}$ and radii, $R_{I23.5}$, at the 23.5 mag arcsec$^{-2}$ isophote. They showed that for Freeman type–I galaxies, i.e., galaxies with an exponential disk, the disk scale length, $r_d$, is very well–correlated with the disk size of $R_{I23.5}$, while this is not the case for Freeman type–II galaxies with a large fraction of disk which does not follow the exponential disk distribution (see their figure 2). For the Tully–Fisher relation, they measured the rotation speed from the weighted average of the rotation curve points, and defined the velocity width as twice the speed. We corrected these widths for the inclination angle and redshift effect.

As can be seen in figure 3, we found that the fitting with $R_{I23.5}$ can largely diminish the scattering around the Tully–Fisher relation. See table 1 (see lines with PW00) for the fitting results. When the disk-scale length, $r_d$, was used, the residual could be maximally reduced when $k = 0.35$.

3.3. $R$-Band Data of Courteau (1996, 1997)

Courteau (1996, 1997) published the kinematic (from optical spectroscopic observations) and photomatic (at $R$-band) data of 304 late-type spiral galaxies. Besides the isophotal radii, $R_{R23}$ and $R_{R25}$, he also obtained a scale length for exponential disks, $r_d$, to express the disk size. Courteau (1997) found that the peak rotational velocity, $V_{2.2}$, measured at $R = 2.15r_d$ matches the 21 cm H$\text{I}$ linewidths best, and yields the smallest Tully–Fisher residuals. Therefore, we use the velocity, $V_{2.2}$, for our discussions.
Table 1. Search for the fundamental plane from observational data.

| Sources   | Input data | Number of objects | $\alpha$ | $\beta$ | $\gamma$ | $k$ | $\chi^2_{V}/\delta^2$ | $\delta_{TV}/\delta$ (mag) |
|-----------|------------|-------------------|---------|--------|---------|----|---------------------|----------------------------|
| Han92-I-1 | $M_{V}^{tot}$ | 160               | -4.05 ± 0.21 | -3.00 ± 0.14 | -8.01 ± 0.43 | 0.74 ± 0.06 | 697.6/252.2 | 0.42/0.28 |
| Han92-I-2 | $M_{V}^{tot}$ | 160               | -4.75 ± 0.21 | -2.85 ± 0.14 | -6.21 ± 0.43 | 0.60 ± 0.04 | 760.0/359.4 | 0.44/0.30 |
| HST-I-1   | $M_{V}^{tot}$ | 17                | -5.67 ± 0.38 | -2.88 ± 0.29 | -3.92 ± 0.80 | 0.51 ± 0.06 | 124.8/17.8  | 0.32/0.15  |
| HST-I-2   | $M_{V}^{tot}$ | 17                | -6.84 ± 0.34 | -2.04 ± 0.25 | -1.63 ± 0.74 | 0.30 ± 0.04 | 124.8/56.8  | 0.32/0.24  |
| HST-R-1   | $M_{V}^{tot}$ | 17                | -5.30 ± 0.38 | -2.26 ± 0.25 | -5.37 ± 0.81 | 0.43 ± 0.06 | 113.7/29.2  | 0.32/0.17  |
| HST-R-2   | $M_{V}^{tot}$ | 17                | -6.19 ± 0.32 | -2.10 ± 0.25 | -2.75 ± 0.68 | 0.34 ± 0.04 | 113.7/42.2  | 0.32/0.21  |
| HST-V-1   | $M_{V}^{tot}$ | 17                | -4.16 ± 0.41 | -2.54 ± 0.23 | -7.86 ± 0.91 | 0.61 ± 0.08 | 144.7/23.0  | 0.35/0.16  |
| HST-V-2   | $M_{V}^{tot}$ | 17                | -5.36 ± 0.34 | -2.59 ± 0.25 | -4.04 ± 0.74 | 0.48 ± 0.06 | 144.7/32.9  | 0.48/0.19  |
| HST-B-1   | $M_{V}^{tot}$ | 17                | -3.85 ± 0.44 | -2.30 ± 0.26 | -8.62 ± 0.99 | 0.60 ± 0.10 | 124.5/47.0  | 0.41/0.25  |
| HST-B-2   | $M_{V}^{tot}$ | 17                | -4.19 ± 0.39 | -3.41 ± 0.33 | -5.69 ± 0.85 | 0.81 ± 0.10 | 124.5/18.9  | 0.41/0.18  |
| PW00-I-1  | 2(V)       | 74                | -2.56 ± 0.26 | -3.96 ± 0.24 | -11.1 ± 0.51 | 1.55 ± 0.18 | 371.8/88.5  | 0.51/0.22  |
| PW00-I-2  | 2(V)       | 74                | -5.16 ± 0.19 | -1.79 ± 0.16 | -7.88 ± 0.45 | 0.35 ± 0.03 | 371.8/252.9 | 0.51/0.38  |
| C96-R-1   | $V_{2.2}$  | 285               | -4.11 ± 0.19 | -2.23 ± 0.12 | -8.45 ± 0.41 | 0.54 ± 0.04 | 703.5/359.4 | 0.47/0.34  |
| C96-R-2   | $V_{2.2}$  | 285               | -2.25 ± 0.23 | -3.64 ± 0.16 | -11.4 ± 0.46 | 1.62 ± 0.18 | 703.5/183.3 | 0.47/0.24  |

Fig. 2. Scattering from the Tully–Fisher relation (left) and the best-fitting plane (right) for the Hubble calibration galaxies at the $HRV$ bands. Data were taken from Sakai et al. (2000) and Macri et al. (2000).
Fig. 3. Comparison of the Tully–Fisher relation (the left side) and the best-fitting plane (the right side). Data of 74 galaxies were taken from Palunas and Williams (2000). The typical rms errors of the absolute magnitudes were taken as 0.2 mag. The data distribution in 3-D space was also plotted.

Fig. 4. Comparison of the Tully–Fisher relation (the left side) and the best plane (the right side). Data of 285 galaxies were taken from Courteau (1996, 1997). The typical rms errors of the absolute magnitudes were taken as 0.3 mag. The variation in the fitting residual $\chi^2$ with the ratio $k = \beta/\alpha$ is plotted on the right side.
below. We took these parameters\textsuperscript{7}, together with the corrected total absolute magnitude, $M_{R}^{\text{tot}}$, from his combined data set (Courteau 1999, astro-ph/9903297).

As listed in table 1 (lines with C96-), using the scale-length to express the disk size gives the minimum at $k \sim 0.5$, but the residual is reduced only by a factor of 2 compared with the Tully–Fisher relation. See figure 4 for illustrations. In contrast, using the isophotal sizes of galaxies can achieve a much tighter correlation with $k > 1.0$. We discuss this point later.

4. Discussion and Conclusions

4.1. The Plane is Fundamental

We have confirmed the conclusion by Koda et al. (2000a) that the galactic radius is a fundamental parameter which should be considered to form the best-fitting plane in three-parameter space. As one can see from table 1, except for the cases in which both the optically measured velocities ($V_{2,2}$ or $V$) and the isophotal radii are used together, the $\beta$ values are always around $-2.5$, while the values of $\alpha$ are always around $-5.0$. Thus, approximately, $M = -5.0 \log V - 2.5 \log R + \gamma$. Since $M = -2.5 \log L$, we have

$$L = cV^2R.$$  

Here, $c$ is a constant related to $\gamma$, which greatly varies for different color bands or different expression of luminosities; $V^2R$ is expected from the virial theorem. The total mass, including the contributions from both the disk and the dark matter, should satisfy $M(\leq r) = [V_{\text{rot}}^2(r)r]/G$, so that the rotational velocity, $V_{\text{rot}}(r)$, at radius $r$ is related to the total mass, $M$, inside the radius, where $G$ is the gravitational constant. Therefore, our results imply that the total luminosity is proportional, on average, to the total mass inside the galactic radius. We conclude from an observational point of view that the plane represented by $L \sim V^2R$ is fundamental for spiral galaxies.

The mass of spiral galaxies is mostly in the form of dark matter, probably in the halo, while the luminosity is given mostly by the number of stars in the galaxy disk (see figure 5). Therefore, the fundamental plane derived by us implies a tighter relation between the bright mass in the disk and the dark matter in the halo; at least they have similar distribution profiles. This is coincident with the conclusion derived from the universal rotation curve by Salucci et al. (1993) and Persic et al. (1996) on the tight coupling between the dark and the luminous matter in spiral galaxies. In general, the luminosity can be expressed as $L = (m_{d}/\Upsilon)M$, where $m_{d}$ is the fraction of disk mass in the total mass, and $\Upsilon$ is the mass-to-light ratio. Our result seems to favor the constant $m_{d}$ and $\Upsilon$ values for spiral galaxies, which are often assumed in theoretical studies (see Mo et al. 1998; Shen et al. 2001). In addition, we noted that the best-fitting planes (the $k$ values) for spiral galaxies in our selected samples vary only slightly in different wavebands, which may result from the formation history of the galaxies, and can be used to constrain theories of galaxy formation.

4.2. Tully–Fisher Residuals and Galactic Radius

Note that the isophotal radii are more effective than the disk scale-length in reducing the scatter. As can be seen from table 1, the photometry-limited observational parameters, such as $R_{\text{23,5}}$ from Han (1992) and Palunas and Williams (2000), $R_{\text{23}}$ from Courteau (1996), and $R_{\text{25}}$ for the Hubble calibration galaxies, can work better. However, when the optically measured velocity is used together with the isophotal radii, the best-fitting planes are at $k > 1.0$. Two independent data-sets of Palunas and Williams (2000) and Courteau (1996,1997) show qualitatively the same result. Although we noticed the fact that the two observational parameters, log($V_{2,2}$) and log($R_{23,5}$), are measured at distinctly different radii, this may not be the reason for the flat variation of $\chi^2$ when $k > 1.0$.

The fundamental plane involving the linear galactic size can reduce the Tully–Fisher residuals by about 50%. The unified plane $L \propto (VR)^{1.3}$ that Koda et al. (2000a) found from the $I$-band data-set can also work much better than the Tully–Fisher relation, though it may not be the fundamental plane. Our results (in our table 1) are consistent with case c of theoretical simulations of Shen et al. (2001), in which they obtained the smallest scattering (see their table 1).

4.3. Other Issues

The fundamental plane does not help to improve the distance estimates of galaxies using the luminosity, because the galactic size here is a quantity having a distance dependence. In fact, during the above searches for the plane, the effect as well as uncertainty of the galactic distance was diminished for the fundamental plane, compared with that in the Tully–Fisher relation.

Based on the Tully–Fisher relation ($L \propto V^4$) and Freeman’s law ($R \propto L^{0.5}$), the fundamental plane described by equation (3) requires $\alpha$ and $\beta$ to satisfy the relation $\alpha + 2\beta = -10$. If $\alpha = \beta$ (see discussion by Zou, Han 2000), one may find that $\alpha = \beta = -10/3$ and exactly naturally becomes $L \propto (VR)^{4/3}$, identical to that obtained by Koda et al. (2000a). Whenever $\alpha \neq \beta$, the possible values of $\alpha$ and $\beta$ are in the ranges $(0, -10)$ and $(0, -5)$, respectively. The relation of Willick (1999) obtained from the $R$-band, $v_{\text{TP}} \propto L^{0.28}I_{\text{c}}^{14}$, if converted to our manner, is $L \approx V^{7/3}R^{2/3}$. As can be seen from the search results in table 1, these values slightly depend on the color band and the sample selection. It is interesting to note that these best-fitting $\alpha$ and $\beta$ values are always around the central values in their possible ranges. Both of the planes derived by Koda et al. (2000a) and by us can recover the Tully–Fisher relation and the Freeman’s law, which are just projections onto 2-D of the fundamental plane in 3-D.

In the above analyses, the bulge and disk parts of the spirals were not separately considered. However, for late-

\textsuperscript{7} Courteau (1996, 1997) used the Hubble constant of $H_0 = 70$ km s$^{-1}$Mpc$^{-1}$ to calculate the absolute size. This is fine for our discussion without further correction.
type spiral galaxies, the bulges often have rather smaller luminosities than the disks (figure 5). In our analysis, most galaxies are of the late–type, and thus the effect of the bulge component on our results is fairly small. The disk contribution to the luminosity is dominant.

4.4. Conclusions

Our search results reveal that there exists a fundamental plane for spiral galaxies, which can be expressed by \( L \sim V^2 R \). The galactic physical size should be involved to form the fundamental plane in three–dimensional space of \( \log L \), \( \log R \), and \( \log V \). The size should be preferably the linear radius at a given isophotal limit if the velocity is taken from the width of \( \text{H I} \) gas. Otherwise the scale length of optical disk can be used as \( R \) if the rotation velocity is measured from the optical spectrum. This fundamental plane can reduce the residual of the Tully–Fisher relation by an amount of about 50%, implying that only the other 50% is attributed by the measurement uncertainties. The fundamental plane exists in all optical bands. Such a fundamental plane is probably related to the mass and mass distribution of spiral galaxies, and should be used to test theoretical work concerning galaxy formation.

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