Chapter 1
Shape models and physical properties of asteroids

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Abstract Despite the large amount of high quality data generated in recent space encounters with asteroids, the majority of our knowledge about these objects comes from ground based observations. Asteroids travelling in orbits that are potentially hazardous for the Earth form an especially interesting group to be studied. In order to predict their orbital evolution, it is necessary to investigate their physical properties. This paper briefly describes the data requirements and different techniques used to solve the lightcurve inversion problem. Although photometry is the most abundant type of observational data, models of asteroids can be obtained using various data types and techniques. We describe the potential of radar imaging and stellar occultation timings to be combined with disk-integrated photometry in order to reveal information about physical properties of asteroids.

1.1 Introduction

Asteroids play an important role in the formation and evolution models of the Solar System and have a direct link to life on Earth. They are connected to the delivery of water and probably also organic material to our planet and therefore are crucial for the development of life. On the other hand, some of them are considered as potentially hazardous for our future.

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News about the discovery of a new hazardous asteroid appear regularly in the media. The last popular case was the close fly-by on 30th October 2015 of asteroid 2015 TB145 (the so-called “Halloween asteroid”). However, the news coverage usually tends to be sensationalist rather than scientific, as these asteroids don’t represent an imminent risk of impact on Earth. The identification of a potential hazard comes from the forward integration of the asteroid’s motion and evolution of its orbit, and the calculation of a probability of an impact (usually less than one in a few thousand chance) with our planet over the next decades. An example of these predictions can be found in the Sentry Risk Table (http://neo.jpl.nasa.gov/risks/) maintained by NASA’s Jet Propulsion Laboratory (JPL). In order to obtain the best possible accuracy in these calculations, there are two crucial actions to be taken: (1) a very precise determination of the orbital parameters by astrometric measurements and (2) a study of the physical properties of the body.

The reason for the first action is obvious. Asteroids approaching the Earth have their orbits modified due to gravitational interactions and therefore regular astrometric measurements are required to constrain the orbital parameters. The better the orbit of an asteroid is defined, the greater will be the accuracy of our future predictions. However, for longer term predictions (i.e. several decades), second-order effects such as nongravitational forces plays a key role on the evolution of the orbit. The most important nongravitational perturbation is caused by the Yarkovsky effect ([31], [30]) which is due to radiative recoil of anisotropic thermal emission and causes asteroids to undergo a secular semimajor axis drift. The Yarkovsky acceleration depends on several physical quantities such as spin state, size, mass, shape and thermal properties [39]. On the other hand, the Yarkovsky–O’Keefe–Radzievskii–Paddack (YORP) effect slowly modifies the spin rate of asteroids with irregular shapes, which in turn affects the Yarkovsky acceleration rate. Rubincam [33] showed that YORP is strongly dependent on an asteroid’s shape, size, distance from the Sun, and orientation.

Unlike asteroid’s distance from the Sun, which can be trivially calculated with good accuracy when knowing the orbital parameters, deriving other physical properties of the asteroid, like spin state or shape, requires advanced inversion techniques and well-planned observations with a good coverage of various viewing geometries. Relative photometry is, by far, the richest source for deriving asteroid models. However, other observing techniques – such as Doppler-delay radar imaging, adaptive optics or stellar occultations – can provide valuable information of the asteroid’s shape and, more importantly, they allow for the model to be scaled.

In this paper we briefly review the importance of investigating the shapes and spin states of asteroids in the context of identifying potentially hazardous bodies. We describe the requirements of photometric data in order to solve the inversion problem, and we discuss different shape solutions. In the last chapter, we review the potential of combining lightcurves with other data types.
1.2 The importance of asteroid modelling in the assessment of asteroid impact hazard

The evolutionary processes that asteroids undergo have been traditionally explained by gravitational perturbations (e.g. gravitational pulls in close encounters with other bodies, orbital resonances) and collisions between these small bodies. The classical asteroid evolution model has been useful to explain how asteroid populations have evolved with time. Particularly interesting was efforts to understand the main source of Mars-crossing and Near Earth Object (NEO) populations, which we believe are fed with material from the main belt delivered by the effects of secular resonances ([41], [42]). However, classical models are unable to explain some of the physical characteristics observed in the NEO population. For instance, according to the classical model, the only processes able to inject these bodies into orbital resonances are asteroid collisions. Meteoroids delivered through this process should present cosmic-ray exposures (CRE) - a measure of their ages - of the order of million years [15]. However, the observed CRE for NEO population are hundreds of times higher.

In turn, these high CRE values can be well explained when introducing nongravitational effects to the evolution model, because such effects can result in a slow delivery of material to orbital resonance zones. Specifically, the Yarkovsky effect induces a tiny force to small bodies by the reradiation of sunlight in the form of thermal energy. This force slowly changes the object’s semimajor axis, changing its orbit inwards (for objects rotating with retrograde sense) or outwards (prograde sense of rotation) with respect to the Sun. Yarkovsky effect is divided into two types of perturbations: (1) a diurnal perturbation due to the body rotation and (2) a seasonal perturbation that depends on the heliocentric longitude of the object. The acceleration $\frac{da}{dt}$ for each perturbation is given by the following equations (see [4] for further details):

$$\left(\frac{da}{dt}\right)_{\text{diurnal}} = -\frac{8\alpha}{9} \frac{\Phi}{n} F_{\text{d}}(R, l, \Theta) \cos \gamma + O(e)$$ (1.1)

$$\left(\frac{da}{dt}\right)_{\text{seasonal}} = \frac{4\alpha}{9} \frac{\Phi}{n} F_{\text{s}}(R, l, \Theta) \sin^2 \gamma + O(e)$$ (1.2)

where $\alpha$ is the albedo-factor, $\Phi$ is the radiation pressure coefficient and $\gamma$ is obliquity of the spin axis. The function $F(R, l, \Theta)$ depends on the radius of the body $R$, the penetration depth $l$, and the thermal parameter $\Theta$ (see the explicit form of this function in [39]). The total acceleration is the superposition of the diurnal and seasonal terms. Thus, the magnitude of the Yarkovsky effect depends on the object’s distance from the Sun, the spin axis orientation, and the body’s physical characteristics (i.e., size, shape, thermal properties, and rotation period).
On the other hand, another nongravitational effect called YORP, is capable of modifying the spin rates and axis orientations of asteroids. Reemitted photons apply a recoil force \( \mathbf{dF} \) normal to the surface. If the body is not perfectly symmetric, the sum of these forces produces a thermal torque (see [4] for further details):

\[
T = \int \mathbf{r} \times \mathbf{dF} \tag{1.3}
\]

where \( \mathbf{r} \) is the position vector of a surface element \( dS \).

In this case, the effect strongly depends on the body’s shape (i.e. its irregularities), and to calculate the effect it is necessary to model the body’s surface temperature distribution (see for instance [12] or [40]).

Thus, in order to include these nongravitational effects in the orbital calculations it is necessary to know in detail the physical properties of the asteroid. For NEOs the most commonly used technique to obtain the body’s size is radar ranging (see Section 4 for further details). Surface thermal properties are related to the roughness of the body surface and its regolith depth. Such properties can be derived, for instance, using infrared interferometric observations. For modelling the Yarkovsky effect, it is essential to know the asteroid’s spin state and its axis orientation. A convex representation of the body shape is usually enough to solve the lightcurve inversion problem, what is the main source of asteroid models. However, as the YORP effect is very sensitive to irregularities of the body shape, a high resolution shape model is required to calculate this effect. In this sense, shape models obtained by spacecraft in situ measurements represents the ideal case. Obviously, this kind of observations are limited to a bunch of asteroids which have been visited by space crafts, therefore generally we have to rely on remote observations. Radar echo can be useful to retrieve a complex shape model, including concavities. However, shapes obtained with this technique are not always reliable, and care should be taken when deriving results from this technique alone. Moreover, before deriving the shape from the radar echo, it is necessary to know the asteroid’s spin axis orientation. Lightcurves are a great source of information regarding the asteroid’s rotational state. As the Sun-asteroid-observer geometry changes so does the observed lightcurve. If the observations are gathered at a variety of geometries (see Section 1.3.1 for requirements) it is possible to reconstruct a shape and spin of an asteroid. In the next chapter we describe the data required to solve the inversion problem, as well as the shape representations commonly in use.

### 1.3 Models based on photometry

Deriving asteroid’s spin state and shape is a necessity in order to model the non-gravitational forces. To that end, photometry is by far the most fruitful observing
technique. The classical photometric observations of asteroids (henceforth "dense lightcurves") that have been collected during the last decades, are the main source of our knowledge about asteroids and their physical parameters. However, gathering enough photometric data to derive a model is an arduous task, which requires good planning and, often, a collaboration between several observers. When the collected data fulfils the requirements, an inversion technique can be applied to obtain a model of an asteroid. Such model includes asteroid’s rotational state as well as an approximation of the shape of the body. In this sense, different shape representations can be used (e.g. triaxial ellipsoid, convex or nonconvex figures). In this chapter we summarize the modelling process, from the obtention of data, to finding the solution of the inversion problem.

1.3.1 Requirements for the lightcurves

Lightcurves can be obtained by comparing the apparent brightness of an asteroid with that of comparison stars (relative photometry), or with that of photometric standard stars (absolute photometry). It might seem that performing absolute photometric measurements should always be preferred. However, absolute photometry limits observable targets to bright asteroids, due to the use of filters, not to mention the requirement of excellent weather conditions. These constraints are of special importance when observing asteroids with small amplitude range (e.g. below 0.1 mag), as uncertainties of absolute magnitude measurements can be of a comparable range.

The usual lightcurve format for one apparition (the period during which the asteroid is observable from the Earth) is basically a series of photometric measurements collected during several observing nights (e.g. Fig. 1.1), with a 0.05 magnitude precision at worst. Ideally, a lightcurve should contain at least 50 well placed data points with a precision better than 0.01 magnitude. A general practice is to do continuous asteroid exposures 1 to 5 minutes long, depending on the object brightness. The field of view (FOV) should be large enough to include three comparison stars of brightness similar to that of an asteroid and, preferably, also of similar colour. When the lightcurve is complete, the rotational period of the asteroid is well covered but the quantity of information on the body shape is limited, as the viewing geometry of the asteroid is almost constant during observations. Consequently, to obtain a unique spin and shape solution, we need a set of dense lightcurves observed at a large span of viewing geometries (i.e. well-spread ecliptic longitudes and a substantial span of phase angles). This observational constraint makes this technique highly time consuming, what is significantly limiting the number of objects for which we have enough dense lightcurves to derive a complex shape of the body.

The quality of photometric observations is related to signal-to-noise ratio (SNR), which is a statistical term that defines the ratio between the useful signal (photons from the object) versus the total signal received (photons from the object, sky background, inherent noise in the chip, etc). The larger this number, the more signal
An ideal SNR value would be above 100, which means that the noise is about 1 per cent of the total signal, or in other words, that the measurements are of about 0.01 mag precision. In practice one can still get good results when the SNR drops to 50 or 30. Getting the necessary SNR depends on many factors: size of the telescope, type of CCD camera, whether or not filters are used, the sky background brightness, how fast the asteroid is moving, the quality of dark and flat frames, etc.

### 1.3.2 Shape models

Long term variations in apparent brightness of an asteroid depend mainly on its distance to the Sun and to the observer, and the angle between their pointing vectors (the so-called phase angle). However, any asteroid with non-spherical shape (practically each asteroid) has also shorter cyclical variations due to its rotation. The lightcurve characteristics (e.g. amplitude, period, shape) depends on the asteroid’s spin state, but also on its shape. If observed in equatorial view, an elongated body will produce a lightcurve with large amplitude, while a nearly spherical object will present a lightcurve with low amplitude. However, if the observation is taken in a pole-on viewing geometry, its lightcurve will be almost flat, no matter the body’s shape.

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**Fig. 1.1** Composite lightcurve of asteroid (1572) Posnania observed with a 0.4m Newtonian telescope at the Borowiec observatory.
In order to reproduce such variations, the inversion method has to include a recreation of the real asteroid’s shape. A 3-axis ellipsoid shape can be a fairly good approximation to solve the inversion problem for the majority of cases ([10], [26], [37]). Such ellipsoid can be defined as the region bound by a surface given by the equation:

\[(x/a)^2 + (y/b)^2 + (z/c)^2 = 1\]  \hspace{1cm} (1.4)

where \(a\), \(b\) and \(c\) are the semi-axes and satisfy the condition \(a \geq b \geq c\).

Most of the asteroids observed at low phase angles show two maxima and two minima per rotational cycle. Such a lightcurve can be explained considering an ellipsoidal shape rotating with a given sidereal period \(P\) about its spin axis (which orientation is described by its north pole position in the ecliptic coordinates \(\lambda\), \(\beta\)) which usually is an axis of the biggest moment of inertia. The shape of the ellipsoid is then defined by two parameters, namely, the ratios of the lengths of the principal axes (\(\frac{a}{b}\) and \(\frac{b}{c}\)). A model based on such a representation of shape is completed with an initial rotation angle \(\phi_0\) and the sense of rotation of the body (prograde or retrograde, defined by a sign of \(\beta\)). Using these parameters, it is possible to explain the variation in brightness of an asteroid, not only due to a rotation itself, but also due to the changes of the viewing geometry for the Sun–observer–asteroid system. Analytically, the brightness of the asteroid at a given time \(t\), is proportional to the surface area seen from a given reference frame (cross-section of the asteroid presented to the observer). The cross-section can be calculated using the following equation:

\[S = \pi abc \left(\frac{\cos^2 \phi \sin^2 \gamma}{a^2} + \frac{\sin^2 \phi \sin^2 \gamma}{b^2} + \frac{\cos^2 \gamma}{c^2}\right)\]  \hspace{1cm} (1.5)

where \(\phi\) is the asteroid’s rotation angle and \(\gamma\) is the aspect angle (the angle between the rotation axis and the asteroid–observer line of sight). As we change the rotation angle \(\phi\), so does the cross-section observed, thus we obtain a sinusoidal variation of brightness.

Nevertheless, some lightcurve shapes cannot be explained by the use of a simple triaxial ellipsoid model. Asteroids with complex shapes can produce lightcurves with 3 or more maxima per cycle. In the majority of cases these asteroids are modelled using a convex representation of their real shapes ([19], [20]), which despite being a first-order approximation of the real shape of the body, have been proven to be good enough to fit the lightcurves and to derive asteroid’s main physical parameters. In short, this method attempts to fit a set of parameters namely:

- A convex shape represented as a collection of triangular facets
- Sidereal rotation period
- Pole direction
- Albedo-dependent coefficient for Lommel-Seeliger and Lambert scattering laws
The standard solution of the inversion problem involves minimizing the residuals between disk-integrated photometric data and synthetic brightness generated by the model. The process relies on the Minkovski minimization stability of convex bodies ([22]) which makes the method not very sensitive to random noise in data. This inversion technique has been used by several authors during the last decade (e.g. [13], [24], [25]) resulting in around a hundred of convex asteroid models based on dense lightcurves.

However, from direct images of asteroids obtained by radar, adaptive optics or during space missions like NEAR Shoemaker or Hayabusa, we know that the real shapes of asteroids are not convex, but generally are full of concavities. In order to obtain a more accurate (realistic) shape model, alternative methods have been proposed. For instance, Bartczak et al. [1] recently developed a new inversion method called SAGE (Shaping Asteroids with Genetic Evolution) capable to derive non-convex shape models for single and binary asteroids relying on their disk-integrated photometric measurements. In this case, the optimization problem is tackled by a genetic algorithm, which randomly mutates the model parameters and selects the best trial solutions until the evolution stabilizes. These models confirm the pole directions and rotation periods derived with previous methods, and additionally highly detailed description of the asteroids’ shape allows more accurate determinations of their physical properties, like the volume and in turn, density.

In all the cases, the inversion techniques generally relies on relative photometry, so the resulting models are also relative in terms of dimensions. In order to scale them, we need an absolute measurement of the asteroid size. This can be obtained from other observation techniques like the time chords recorded during a stellar occultation by an asteroid, or direct imaging techniques, like radar (see Section 1.4).

Several approaches to the multi-data inversion problem have been developed during the last years. For instance, the KOALA (Knitted Occultation, Adaptive optics and Lightcurve Analysis [6]) algorithm solutions are based on lightcurves and AO silhouette contours, while ADAM (All-Data Asteroid Modeling [38]) is a collection of functions from which one can tailor an inversion procedure for multiple data sources including direct imaging, radar and interferometry.

For all the methods described, the main constraint for enlarging the number of derived models is the availability of good-quality photometric data fulfilling the requirements described in Section 1.2. The organization of observing campaigns, can potentially generate enough dense photometric data to derive a few tens of new models per year.

On the other hand, during the last years, some observatories around the world have conducted sky surveys mainly focused on detecting new NEAs or to improve their orbital parameter (e.g. USNO in Flagstaff, or Catalina Sky Survey). As a by-product of these astrometric survey programs a vast amount of sparse-in-time photometric measurements for tens of thousands of asteroids have been retrieved. For each object some tens, or often hundreds of discrete observations were collected for different geometries and illuminations. Combining these datasets with dense lightcurves allowed some authors to increase the modelled population of asteroids from 100 (classical photometry) to 400 (combination of classical and
sparse photometric data), using a modified version of the convex lightcurve inversion method (e.g. [17], [18]). The resulting models resemble the ones obtained with dense lightcurves (are equivalent in terms of spin solution) but the shape model is usually a low-resolution, “angular” convex shape, due to the limited quality of the data.

In turn, Gaia observations will generate a similar sparse set of photometric measurements during its 5 years operation. But the data improvement will be considerable, both in terms of quantity (observations are expected for $\sim$300,000 asteroids, [29]) and quality (the photometric accuracy is estimated to be $\sim$0.01 mag for asteroids up to 18 magnitude, and $\sim$0.03 mag up to 20 magnitude [7], [8]). As a result of this enormous amount of new data, asteroid models for at least 10,000 objects are expected. This means an improvement of two order of magnitudes from our current knowledge level.

However, on average Gaia will observe each asteroid 50-70 times during 5 years. Despite of the high data quality, this number of measurements is not enough to constrain a complex model shape by its own. Moreover, processing such enormous amount of data would be highly CPU demanding. For these reasons, the inversion method chosen for inverting Gaia photometric data of asteroids is a low CPU demanding method: a triaxial ellipsoid model, which brightness can be analytically calculated as a function of the asteroid–Gaia–Sun position vectors and the Lommel-Seeliger law [9]. This method, while simple, has been proved to be effective even when inverting synthetic data generated with nonconvex shapes [34]. The results coming out from Gaia are expected to have a direct impact on the Solar System formation theories, as a statistically large sample of objects with known properties may reveal physical effects which play an important role for the whole population.

It is worth noting that even such precise data will provide models not completely free from various biases, or selection effects, that favour i.e. elongated targets with extreme obliquities of spin axes [34]. It is important for ground-based studies to focus on those targets that will not be fully covered by studies based on data from Gaia or other future surveys.

### 1.3.3 Models of binary asteroids

One particularly interesting case are the asteroids with satellites. Such systems are specially appreciated by the Solar System researchers as they give a unique opportunity to derive the mass of the components directly from the third Kepler’s law. For this reason, they are invaluable targets for studies on internal structure and composition.

The synchronous binary systems have been extensively studied and modelled (e.g. in [27], [28] and [21]). Recently a new algorithm capable to generate model solutions for binary asteroids has been developed using a nonconvex shape representation of the components [1]. As the model is able to reproduce body concavities, the relative volume obtained for the components is more accurate than for the pre-
vious models, which were based in Roche ellipsoids \[11\], having a direct impact on the density calculation.

We currently know of more than a hundred binary asteroids in the main belt, and about three hundred in total if binary NEOs and TNOs are included. The majority of them have been discovered by recording their mutual events in a classical dense lightcurve. Resolved imaging such as the ones obtained from radar or adaptive optics have allowed to confirm or, in a few cases, discover such objects. The number of asteroids with known satellites is expected to be increased significantly due to the huge amount of data expected from surveys like \textit{Gaia}. To that end, it is necessary to develop automated strategies to find binary candidates in such large datasets.

![Fig. 1.2 Two different spatial views of the nonconvex model for 90 Antiope binary system shown at equatorial viewing (on the left), and the pole-on view on the right (model from Bartczak et al. \[1\]).](image)

It is thought that NEO population can contain a high number of binary (or multiple) asteroids (see for instance Pravec & Harris \[32\]). One possible explanation for a high rate of multiplicity among this population could be the catastrophic disruption of rubble piles due to YORP spin up. Thus, inversion techniques capable of deriving models of binary asteroids can help us to better understand their formation processes and the physics of nongravitational effects.

### 1.4 Models from various data types

Models derived from disk-integrated relative photometry can be combined with other data types in order to derive additional physical properties of asteroids. For instance, radar echo is a very powerful technique to study the NEO population, while stellar occultation is an affordable technique to obtain sizes of main belt (or even trans-neptunian!) objects and discover satellites. In this chapter we briefly de-
scribe both techniques and we outline a method to combine them with lightcurves. Finally, it is worth noting that this is not intended to be an extensive and comprehensive review of the data types that can complement photometric observations. Other techniques like adaptive optics, spectroscopy, thermal infrared observations or polarimetry can be also combined with photometry to investigate asteroids. Here we provide the description of joining lightcurve data with radar imaging and with stellar occultation timings.

### 1.4.1 Photometry and radar

An Asteroid radar image is a reconstruction of a radio signal sent from the Earth and reflected by body’s surface. For this reason, this technique is best working for objects approaching the Earth, such as NEOs. One dimension of such image comes from time delay, as the signal has to travel different distances depending on which a part of asteroid’s surface it is reflected from. Second dimension is directly associated with body’s rotation. Received echo’s frequency is shifted with respect to incident ray due to Doppler effect and depends on radial velocity of a surface element which increases as we move away from asteroid’s rotation axis. Range of frequency shift depends on asteroid’s rotation period and aspect (angle between rotation axis and direction to the observer). As a result one can produce range-Doppler image where each pixel value corresponds to echo power at certain distance and radial velocity.

Radar techniques can only be used when precise astrometry is available. Importance of knowing body’s orbit accurately, especially in case of NEOs, cannot be overestimated.

Radar imaging is a rare situation in astronomy where we conduct an actual experiment by controlling the signal that gets reflected from a target body. It becomes possible to probe asteroids surface features comparable in size with signal wavelength or even have a glance at sub-surface properties. Asteroid’s shape is represented on images derived from radar observations in a form of a blend of top and bottom view of asteroid (in respect to line of sight). By examining radar images astronomers can determine large scale surface features such as big concavities or adjudge whether an object has satellites. Body size constrains can also be obtained.

Radar images are a very rich source of information and are used to create accurate three dimensional asteroid models that consist of asteroid’s shape, spin axis orientation and rotation period. It can be done by using radar data only or by combining them with photometric data. Shape program \[23, 16\] is broadly, if not the only, such algorithm in use. It is an iterative method that starts with triaxial ellipsoid and by gradually changing initial shape and spin parameters arrives at a final model. Both radar and optical scattering laws are assumed prior to modelling process and stay fixed.

In each iteration a simulated asteroid image is created and compared with the observations. If radar data is used alongside photometric data, both images are computed separately from the same model. Every point on synthetic lightcurve is a cal-
culated amount of light reflected by model’s surface (Fig. 1.3, Fig. 1.4). Similarly, a radar range-Doppler image is created. Then a $\chi^2$ is determined by minimizing the differences between synthetic and observational measurements. Data is weighted depending on the type and quality, and additional penalty functions defined by user are taken into consideration. This approach steers modelling process into global minimum. Final model has to fit both radar and photometric observations.

The shape of an asteroid is derived in three stages and best fitting model from previous stage becomes initial model of the next. The first stage uses only triaxial ellipsoid as a shape model; in the second stage, model is represented by spherical harmonics to be then transformed into polyhedral model described by vertices and triangular surface elements in the last stage. Concavities are allowed only at the last stage as they are difficult to represent using spherical harmonics (especially those of a low degree).

The described method is not fully automatic, meaning that it needs considerable human interference. Modelling is initialized with different values for parameters that are being further changed during the process to help the algorithm to come up with the model that reflects all important features present in observational data. This has it’s reflection in the low number of modelled asteroids based on radar observations.
1.4.2 Photometry and stellar occultations

Stellar occultation is an interesting technique of asteroid imaging. The idea is to measure asteroid’s shadow cast on Earth’s surface while it passes in front of a star. Observers are set on shadow path and each of them notes a time of the beginning and end of occultation (e.g. Fig. 1.5).

In order to predict a shadow path and allow observers to choose observing sites properly, an asteroid’s ephemeris as well as an occulted star position have to be very precise. Only if the ephemeris and star position are known with high accuracy will occultation time measurements result in good coverage of the body’s shape. Nowadays, this represents a constraint as the availability of precise star astrometry is limited to the Hipparcos catalogue. Publication of Gaia catalogue will greatly improve occultation events predictions; accuracy that is now available for main belt asteroids 100 km in diameter, will be achievable for 15 km asteroids [35]. On the other hand, this technique is not appropriate to NEOs, as their fast apparent movement against the background of distant stars constrict the possibility of recording the event.

Despite this fact, this technique has been included in this paper, as it allows for investigations of internal structure of main belt objects. Keeping in mind that they
are the source of asteroids with Earth-crossing orbits, these studies also increases our knowledge about NEOs composition.

Choosing observational sites is crucial to successful occultation time measurements. Ideally, observers will cover the path of the shadow evenly and densely. Shadow path prediction however is directly dependent on the knowledge of ephemeris of the observed Solar System body and an estimation of the body’s size. With more precise stellar catalogues, like the one ESA *Gaia* mission will produce, more occultation events will be predicted with better precision, even for small bodies.

Occultation observations are mainly carried out by amateur astronomers. Observing groups have to be mobile in order to cover the right area on the ground; fortunately stellar occultation events can be observed with small telescopes if occulted star is a bright one. Systems like GPS are of great help when it comes to establishing the observer’s location and time essential to valid measurements.

![Fig. 1.5](image)

**Fig. 1.5** Best fit of a SAGE nonconvex model of (9) Metis to the stellar occultation chords obtained during the 2008 occultation.
Every chord (line along the shadow path) with marked beginning and end of the occultation event provides two points on the plane of sky. Given many points (five at minimum, [14]) an ellipse can be fit to match these points. This is a good estimate of shape for large bodies, e.g. planets and TNOs objects; in case of smaller bodies this method provides rough estimate only as asteroids take a wide range of convex shapes that are irregular in general.

Stellar occultation remains the best available method for determining body’s radius its resolution being at the level of kilometres. Given dense chord distribution the resulting body’s shape envelope is very precise thus fit for verifying models obtained by inversion techniques against reality.

Models can be enriched with additional information, the size of the body being the most valuable one as it gives the model proper dimensions and allows density and albedo estimations, as was done in case of (90) Antiope (see for instance Bartczak et al. [1]). The described method is also sufficient to determine whether an asteroid is a binary system.

Analysis of lightcurve profiles captured during occultation event (immersion and emersion at the beginning and end of occultation) can tell us about the presence of an atmosphere [14]. Moreover, rings around bodies can by detected and studied, like in the case for giant planets in Solar System. It is possible to detect rings around smaller objects [5] but it requires extremely high precision timings. Still it shows the power of the method, where some features could not be discovered or that precisely measured using other observing techniques.

1.5 Conclusions

We have shown that asteroid modelling is an effort that needs to be undertaken in order to study the nongravitational forces affecting these small bodies. Disk-integrated photometry is the main technique used to derive spin states and shape models. However, for solving the inversion problem, lightcurves need to fulfil certain requirements in terms of quality and viewing geometries.

A substantial part of this paper has been devoted to describe how to gather such data and what are the inversion techniques commonly applied. The first and simplest solution – which is, however, still useful in some cases due to specific situations, such as the the analysis of the huge amount of data generated by the Gaia mission – is a triaxial ellipsoid representation of the body shape, for which synthetic brightness can be evaluated analytically, making it ideal for problems with high CPU demand. The so-called lightcurve inversion method, which solution consist of a convex shape model of the asteroid and its spin state, is a worthwhile technique when we are specifically interested in the study of the spin rate and shape outline of the body. However, other techniques producing more detailed shape solutions (i.e. nonconvex shapes) are necessary when investigating further physical properties. In particular, modelling of nongravitational effects acting on asteroids is extremely sensitive to the used shape representation. Some inversion methods are also able to derive shape
models of binary asteroids. We have shown the example of SAGE, which is a technique capable of deriving nonconvex models for synchronous binary asteroids from relative photometry only.

It is also possible to combine other observing techniques to derive additional physical properties of asteroids. In chapter four, we have described two techniques – radar imaging and stellar occultation timings – which are mainly used to scale the model in size and, in some cases, derive additional information. This includes the study of fine details in the shape or the discovery of satellites. In addition, we have briefly described some procedures to combine these observations with photometry in the modeling process.

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