Abstract

We investigate the Lagrangian terms of scalar-gauge interactions in a weakly-coupled gauge theory beyond the ultraviolet momentum cutoff of models where the collective symmetry breaking mechanism protects the Higgs mass squared against having quadratic divergence. We also propose a five-dimensional model in which the gauge symmetry breaking is achieved by boundary conditions consistent with local gauge transformations. It is shown that the scalar-gauge operators in the two models differ in the absence of quadratic divergence.
1 Introduction

Quadratic divergence is one of the fundamental problems of the Standard Model of particle physics. The structure of quantum loop corrections is different between scalars and fermions. The self-energy of fermion is proportional to the fermion mass. By a dimensional counting, it only has logarithmic divergence at one loop. It is chiral symmetry that protects fermions from having quadratic divergence. Scalar fields do not have such a symmetry in the Standard Model. Therefore applying symmetry principle to scalar fields has been a clue to understand physics beyond the Standard Model. Supersymmetry connects fermions and bosons so that chiral symmetry is applied [1, 2]. If scalars were components of gauge fields which have gauge symmetry to prevent quadratic divergence, it is formulated in a gauge-Higgs unification scenario [3, 4, 5]. A feature of supersymmetry and gauge-Higgs scenarios is that there are many partners: superparticles and Kaluza-Klein modes. In obtaining no quadratic divergence with symmetry principle, it is not necessary to rely on symmetry for fields with different spins than scalars. If scalar fields were Nambu-Goldstone bosons in broken global symmetry, they are exactly massless and it is led to little Higgs scenarios [6, 7].

In little Higgs scenarios, if gauge and Yukawa couplings are vanishing, Higgs fields have derivative couplings and they have shift symmetry so that there is no potential. The scalar fields are described in a non-linear sigma model. The assumption that the global symmetry is explicitly broken only when two or more couplings are non-vanishing leads to a potential with at most logarithmic divergence. This collective symmetry breaking mechanism has been studied in many papers [8, 9, 10, 11, 12, 13]. In the littlest Higgs model, the resulting operator of scalar-gauge interactions is, for example,

\[ W_{1\mu}W_{\mu}^2h^\dagger h, \]  

where \( h \) is the Higgs field and \( W_{1\mu} \) and \( W_{2\mu} \) are \([SU(2)]^2\) gauge bosons. At one loop, the gauge bosons do not produce the quadratic divergence for the Higgs mass squared through the interaction (1.1), as will be explained with diagrams in Section 2. While the collective symmetry breaking mechanism requires two or more couplings, such a group as \([SU(2)]^2\) could be a subgroup of a single group. It should be clarified whether operators such as Eq. (1.1) can be derived in a renormalizable gauge theory with a single large group broken to two or more subgroups.

Recently, in Ref. [14] a weakly-coupled renormalizable ultraviolet completion of a little Higgs scenario was proposed. It was claimed that heavy modes are integrated out and that the remaining theory is a non-linear sigma model due to the Coleman-Wess-Zumino theorem [15, 16]. In the model, the Higgs mass squared receives radiative correction of the order of 100 GeV and the non-linear sigma model has the decay constant \( f \sim 1 \) TeV and the ultraviolet momentum cutoff \( \Lambda \sim 10 \) TeV. Also, in an explicit example, several effective couplings with the form (1.1) were shown. To pursue this possibility of high energy theory for no quadratic divergence as an extension of the collective symmetry breaking mechanism, it would be important to examine more couplings.

We give all the couplings of scalar-gauge interactions in a weakly-coupled gauge theory beyond the ultraviolet momentum cutoff of models where the collective symmetry breaking mechanism protects the Higgs mass squared against having quadratic divergence. As another aspect, we study a possibility of gauge symmetry breaking by boundary conditions as the first step to utilize dynamical symmetry breaking in a gauge-Higgs unification. We propose a five-dimensional model in which symmetry breaking is achieved by boundary
conditions consistent with local gauge transformations. To explain consistent boundary conditions, we give some examples of inconsistent boundary conditions. It is found that there is a difference of the structure between operators in the four-dimensional and five-dimensional models.

In the next section, the setup of a renormalizable gauge theory at higher energies than an ultraviolet momentum cutoff of models with the collective symmetry breaking mechanism and the resulting couplings are given. In Section 3, gauge symmetry breaking by boundary conditions is examined. The couplings relevant to that of the model in Section 2 are given. The character of the operators based on properties of symmetry breaking is discussed in Section 4. Conclusion is made in Section 5.

2 Emergence of operators with no quadratic divergence

We study a renormalizable gauge theory with a single group which is broken by the vacuum expectation values of scalar fields, aiming for having scalar-gauge interactions of the form

\[ g^2 W_1^\mu W_2^{\mu \dagger} h \dagger h, \quad (2.1) \]

where a gauge coupling is denoted as \( g \). For the interaction (2.1), the one-loop diagram for the Higgs mass squared is shown in Fig. 1. The divergence is at most logarithmic.

![Figure 1: The Higgs mass correction through the scalar-gauge interaction (2.1).](image)

Since Eq. (2.1) is written as

\[ g^2 \left( \frac{W_1^\mu + W_2^\mu}{\sqrt{2}} \right)^2 h \dagger h - g^2 \left( \frac{W_1^\mu - W_2^\mu}{\sqrt{2}} \right)^2 h \dagger h, \quad (2.2) \]

the absence of the quadratic divergence for the Higgs mass squared is also regarded as a cancellation as shown in Fig. 2. In little Higgs models, the collective symmetry breaking

![Figure 2: The cancellation of quadratic divergence.](image)
mechanism can produce the operator (2.1). In such a model, the self-interaction of the Higgs fields obeys a non-linear sigma model. The Lagrangian terms of expansion at lower order are

\[ |\partial_\mu h|^2 + \frac{1}{f^2} |\partial_\mu h|^2 h^\dagger h. \] (2.3)

The quadratic divergence of one-loop contribution to the kinetic term is \( \Lambda^2 / (16\pi^2 f^2) \), where \( \Lambda \) is the ultraviolet momentum cutoff. For the non-linear sigma model to be viable, the cutoff has the upper bound \( \Lambda < 4\pi f \). From Figures 1 and 2, the dominant correction to the Higgs mass squared is

\[ \frac{g^4}{16\pi^2 f^2} \log \left( \frac{\Lambda^2}{f^2} \right). \] (2.4)

If the correction is of the order of 100 GeV, the decay constant is obtained as \( f \sim 1 \text{ TeV} \) and then \( \Lambda \sim 10 \text{ TeV} \). Thus our starting theory is defined at higher scales than 10 TeV.

We choose the single gauge group as SU(5). The gauge symmetry is broken by the vacuum expectation values of two types of scalar fields. We assume that these two sectors yield separate gauge symmetry breakings

\[ \text{SU}(5) \rightarrow [\text{SU}(2) \times U(1)]^2, \quad \text{SU}(5) \rightarrow \text{SO}(5). \] (2.5)

The Higgs fields are included in the sector of the breaking SU(5)→SO(5). The scalar fields in the other sector are decoupled to the Higgs fields. The scalar fields we discuss are only in the sector of SU(5)→SO(5).

**Higgs mechanism**

Here we write down the Higgs mechanism of the symmetry breaking SU(5)→SO(5). The scalar field responsible for this breaking is a scalar 15 which transforms as a complex symmetric matrix \( S \rightarrow S^\prime = USU^T \) under SU(5). The dynamics of \( S \) is governed by a Lagrangian invariant under global SU(5)×U(1) and spacetime transformations. The SU(5)×U(1)-invariant potential is written as

\[ V = -M^2 \text{Tr} [SS^\dagger] + \lambda_1 (\text{Tr} [SS^\dagger])^2 + \lambda_2 \text{Tr} [(SS^\dagger)^2], \] (2.6)

where \( M, \lambda_1 \) and \( \lambda_2 \) are the coupling constants. The stationary condition leads to

\[ S^\dagger [(-M^2 + 2\lambda_1 \text{Tr} SS^\dagger)1_5 + 2\lambda_2 SS^\dagger] = 0. \] (2.7)

The nonzero expectation value is given by \( SS^\dagger = f_S^2 1_5 \) (for \( 10\lambda_1 + 2\lambda_2 \neq 0 \)), which is proportional to the identity matrix. This means symmetry breaking to O(5). Here the decay constant is defined as \( f_S^2 \equiv M^2 / (10\lambda_1 + 2\lambda_2) \). For the vacuum expectation value

\[ \langle S \rangle = f_S \begin{pmatrix} 1_2 \\ 1_2 \end{pmatrix}, \] (2.8)

the global SU(5)×U(1) is broken to SO(5).
The vacuum expectation value \( \langle S \rangle \) generates the masses of gauge bosons. The gauge interaction of the scalar \( S \) is included in

\[
\frac{1}{8} \text{Tr}(D_\mu S)(D^\mu S)^\dagger, \quad \text{with} \quad D_\mu S = \partial_\mu S - igA_\mu T^a S - igA_\mu S T^a T,
\]

where properties of SU(5) generators are summarized in App. A. Then the mass terms of the vacuum expectation value (2.8) are parameterized with 15 complex fields as

Next we identify scalar fields around the vacuum expectation value. Fluctuations around \( \langle S \rangle \) are a set of 5\( \times \)5 antisymmetric matrices. They form SO(5) algebra. For the neutral sector \( a = 3, 8, 15, 24 \), the mass matrix is

\[
\frac{1}{8} g^2 f_S^2 \left[ (A^1 + A^{22})^2 + (A^2 - A^{33})^2 + (A^4 + A^{13})^2 + (A^5 + A^{14})^2 \\
+ (A^6 + A^{20})^2 + (A^7 + A^{21})^2 + 2A^9 A^9 + 2A^{10} A^{10} \\
+ (A^{11} + A^{16})^2 + (A^{12} - A^{17})^2 + 2A^{18} A^{18} + 2A^{19} A^{19} \\
+ A^3 A^3 + \frac{5}{3} A^8 A^8 + \frac{7}{12} A^{15} A^{15} + \frac{3}{4} A^{24} A^{24} - \frac{\sqrt{6}}{2} A^3 A^3 \\
+ \frac{\sqrt{10}}{2} A^3 A^{24} - \frac{5\sqrt{2}}{6} A^8 A^{15} - \frac{\sqrt{30}}{6} A^8 A^{24} - \frac{\sqrt{15}}{6} A^{15} A^{24} \right].
\]

(2.10)

For the neutral sector \( a = 3, 8, 15, 24 \), the mass matrix is

\[
\frac{1}{8} g^2 f_S^2 \left( \begin{array}{ccc}
1 & 0 & -\sqrt{7} \\
0 & 5 & 5\sqrt{2} \\
-\sqrt{6} & 5\sqrt{2} & 12
\end{array} \right), \quad \text{for} \quad m_A^2 = 0, \quad \left( \begin{array}{ccc}
1 & 0 & -\sqrt{6} \\
0 & -\sqrt{6} & 12 \\
-\sqrt{10} & -\sqrt{10} & 12
\end{array} \right), \quad \text{for} \quad m_A^2 = \frac{1}{8} g^2 f_S^2.
\]

(2.11)

The eigenvalues \( m_A^2 \) and eigenvectors are given by

\[
\left( \begin{array}{c}
1 \\
0 \\
\frac{\sqrt{3}}{4}
\end{array} \right), \quad \left( \begin{array}{c}
0 \\
\frac{\sqrt{3}}{5\sqrt{6}} \\
\frac{10}{\sqrt{10}}
\end{array} \right), \quad \text{for} \quad m_A^2 = 0, \quad \left( \begin{array}{c}
1 \\
0 \\
\frac{\sqrt{3}}{12}
\end{array} \right), \quad \left( \begin{array}{c}
0 \\
\frac{\sqrt{3}}{3\sqrt{6}} \\
\frac{10}{\sqrt{10}}
\end{array} \right), \quad \text{for} \quad m_A^2 = \frac{1}{8} g^2 f_S^2.
\]

(2.12)

These generators are given in a matrix form in Eq. (A.2). The generators

\[
T_\pi \left( \begin{array}{c}
1 \\
\frac{1}{2}
\end{array} \right), \quad \text{for} \quad \frac{\sqrt{3}}{4} T_1 + \frac{\sqrt{5}}{4} T_2 + \frac{\sqrt{6}}{4} T_3 = 1
\]

(2.13)

are a set of 5\( \times \)5 antisymmetric matrices. They form SO(5) algebra.

Scalar fluctuations

Next we identify scalar fields around the vacuum expectation value. Fluctuations around the vacuum expectation value (2.8) are parameterized with 15 complex fields as

\[
S = \langle S \rangle + \tilde{S}, \quad \text{with} \quad \tilde{S} = \left( \begin{array}{c}
A \\
B \\
C \\
D \\
E
\end{array} \right), \quad \left( \begin{array}{c}
F \\
G \\
H \\
J
\end{array} \right), \quad \left( \begin{array}{c}
K \\
L \\
M
\end{array} \right), \quad \left( \begin{array}{c}
N \\
P \\
Q
\end{array} \right)
\]

(2.15)
The fluctuation can be written as

\[ \tilde{S} = iN + R, \quad \text{with} \quad N = \begin{pmatrix} \phi & h & \chi \\ h^T & K_i & h^\dagger \\ \chi^T & h^* & \phi^\dagger \end{pmatrix}, \quad R = \begin{pmatrix} \Phi & H & X \\ H^T & K_r & H^\dagger \\ X^T & H^* & \Phi^\dagger \end{pmatrix}. \tag{2.16} \]

which are composed of the two \(2\times2\) complex symmetric tensors \(\phi\) and \(\Phi\), the two complex doublet fields \(h\) and \(H\), the two \(2\times2\) Hermite tensors \(\chi\) and \(X\) and the two real fields \(K_i\) and \(K_r\). In terms of the fields in Eq. (2.15), each matrix is given as

\[ \phi = \begin{pmatrix} 2\phi_a \\ 2\phi_b \end{pmatrix}, \quad \Phi = \begin{pmatrix} 2\bar{\Phi} \\ 2\bar{\Phi} \end{pmatrix}, \quad h = \sqrt{2h} = \frac{i}{2} \begin{pmatrix} -C + L^\dagger \\ -G + M^\dagger \end{pmatrix}, \quad H = \sqrt{2H} = \frac{1}{2} \begin{pmatrix} C + L^\dagger \\ G + M^\dagger \end{pmatrix}, \quad K = 2i\bar{K}_i + 2\bar{K}_r = iK_i + K_r, \]

\[ \chi = \begin{pmatrix} \bar{D}_i \\ \bar{\chi}_B \end{pmatrix} = \frac{i}{2} \begin{pmatrix} -D + D^\dagger \\ -E + H^\dagger \end{pmatrix}, \quad X = \begin{pmatrix} \bar{E}_i \\ \bar{J}_r \end{pmatrix} = \frac{1}{2} \begin{pmatrix} D + D^\dagger \\ E + H^\dagger \end{pmatrix}. \tag{2.17, 2.18, 2.19, 2.20, 2.21, 2.22} \]

The notation of the fields with a bar over a letter such as \(\bar{h}\) stand for the canonical normalization of the kinetic terms

\[ \frac{1}{2} \text{Tr}(\partial_\mu S)(\partial^\mu S^\dagger) = \frac{1}{2} \text{Tr} \left[ (\partial_\mu \phi)(\partial^\mu \phi^\dagger) + (\partial_\mu \chi)(\partial^\mu \chi^\dagger) + (\partial_\mu \Phi)(\partial^\mu \Phi^\dagger) + (\partial_\mu \chi)(\partial^\mu \chi^\dagger) \right] + (\partial_\mu \bar{h})(\partial^\mu \bar{h}) + \frac{1}{2}(\partial_\mu \bar{K}_i)(\partial^\mu \bar{K}_i) + (\partial_\mu \bar{H})(\partial^\mu \bar{H}) + \frac{1}{2}(\partial_\mu \bar{K}_r)(\partial^\mu \bar{K}_r). \tag{2.23} \]

Substituting Eq. (2.15) into the potential (2.6) leads to

\[ V = \lambda_1 \left[ \text{Tr}(\langle S \rangle \bar{S}^\dagger + \bar{S}\langle S \rangle^\dagger) \right]^2 + \lambda_2 \text{Tr} \left[ (\langle S \rangle \bar{S}^\dagger + \bar{S}\langle S \rangle^\dagger)^2 \right]. \tag{2.24} \]

up to constant terms. Because \(\bar{S}\langle S \rangle^\dagger\) is itself quadratic terms of fields, the quadratic terms of scalar fields arise from \(\lambda_1[\text{Tr}(\langle S \rangle \bar{S}^\dagger + \bar{S}\langle S \rangle^\dagger)]^2\) and \(\lambda_2\text{Tr}[\langle S \rangle \bar{S}^\dagger + \bar{S}\langle S \rangle^\dagger]^2\). They include the operator

\[ \langle S \rangle \bar{S}^\dagger + \bar{S}\langle S \rangle^\dagger = 2f_S \begin{pmatrix} X & H & \Phi \\ H^\dagger & K_r & H^T \\ \Phi^\dagger & H^* & X^T \end{pmatrix} = 2R\langle S \rangle. \tag{2.25} \]

From this equation, it is seen that the fields in the matrix \(R\) have mass terms. Thus 15 real components are massive. The number of massless components is \(30 - 15 = 15\) which is equivalent to the number of broken generator \(\text{SU}(5)\times\text{U}(1)/\text{SO}(5)\). The potential (2.24) also includes the operator

\[ \bar{S}\bar{S} = (iN + R)(-iN^\dagger + R^\dagger) = NN^\dagger + i(NR^\dagger - RN^\dagger) + RR^\dagger. \tag{2.26} \]

With a symbolic use of \(N, R\), the structure of the potential is

\[ V \sim (R + N^2 + NR + R^2)^2. \tag{2.27} \]
By power of $R$, it is classified as

$$V \sim N^4, (N^2 R, N^3 R), (R^2, NR^2, N^2 R^2), (R^3, NR^3), R^4.$$  \hfill (2.28)

For effective vertices obtained by integrating out heavy fields, lower $R$ terms $V \sim N^4, N^2 R, N^3 R, R^2$ are dominant. We obtain each term as follows: The $N^4$ term is

$$V_{N^4} = \lambda_1 \left[ \text{Tr} NN^\dagger \right]^2 + \lambda_2 \text{Tr} \left[ (NN^\dagger)^2 \right]$$

$$= \lambda_1 \left( 2 \text{Tr}(\phi\phi^\dagger) + 2 \text{Tr}(\chi^2) + K_i^2 + 4h^\dagger h \right)^2$$

$$+ \lambda_2 \left( 2 \text{Tr} \left[ (\phi\phi^\dagger + hh^\dagger + \chi^2)^2 \right] + 4(\phi h^* + hK_i + \chi h)^\dagger(\phi h^* + hK_i + \chi h) 

+ 2 \text{Tr} \left[ (\phi\chi^T + hh^T + \chi\phi)(\phi\chi^T + hh^T + \chi\phi)\dagger \right] + (2h^\dagger h + K_i^2)^2 \right).$$  \hfill (2.29)

The $N^2 R$ term is

$$V_{N^2 R} = 2\lambda_1 \text{Tr}(2R(S)^\dagger)\text{Tr}(NN^\dagger) + 2\lambda_2 \text{Tr}(2R(S)^\dagger NN^\dagger)$$

$$= 4f_S\lambda_1 \left( 2 \text{Tr}(X) + K_r \right)(2 \text{Tr}(\phi\phi^\dagger) + 2 \text{Tr}(\chi^2) + K_i^2 + 4h^\dagger h)$$

$$+ 4f_S\lambda_2 \left( 2 \text{Tr} \left[ X(\phi\phi^\dagger + hh^\dagger + \chi^2) \right] \right)$$

$$+ \text{Tr} \left[ \Phi(2\chi^T\phi^\dagger + h^*h^\dagger) \right]$$

$$+ 2H^T(\chi^T h^* + h^*K_i + \phi^\dagger h) + (2H^T(\chi^T h^* + h^*K_i + \phi^\dagger h))^\dagger$$

$$+ K_r(2h^\dagger h + K_i^2) \right).$$  \hfill (2.30)

The $N^3 R$ term is $V_{N^3 R} = 0$. The $R^2$ term is

$$V_{R^2} = \lambda_1 \left[ \text{Tr}(2R(S)) \right]^2 + \lambda_2 \text{Tr} \left[ (2R(S))^2 \right]$$

$$= 4f_S^2\lambda_1 \left( 2 \text{Tr}(X) + K_r \right)^2 + 4f_S^2\lambda_2 \left( 2 \text{Tr}(X^2) + 2 \text{Tr}(\Phi\Phi^\dagger) + 4h^\dagger H + K_i^2 \right)$$

$$= 16\lambda_1 f_S^2 \left( \sqrt{2}D_r + \sqrt{2}J_r + K_r \right)^2 + 16\lambda_2 f_S \left( D_r^2 + J_r^2 + K_r^2 \right)$$

$$+ 32\lambda_2 f_S(D\bar{A}^\dagger + \bar{B}\bar{B}^\dagger + \bar{F}\bar{F}^\dagger + \bar{H}^\dagger \bar{H} + E\bar{E}^\dagger).$$  \hfill (2.31)

By transforming the basis into the mass eigenstates via the orthogonal matrix,

$$\left( \begin{array}{c} I_1 \\ I_2 \\ I_3 \end{array} \right) \equiv \frac{1}{\sqrt{10}} \left( \begin{array}{ccc} 1 & 1 & -2\sqrt{2} \\ 1\sqrt{5} & -1\sqrt{5} & 0 \\ 2 & 2 & \sqrt{2} \end{array} \right) \left( \begin{array}{c} D_r \\ J_r \\ K_r \end{array} \right),$$  \hfill (2.32)

the mass terms are diagonalized as

$$V_{R^2} = \frac{1}{2}M_{I_1}^2 I_1^2 + \frac{1}{2}M_{I_2}^2 I_2^2 + \frac{1}{2}M_{I_3}^2 I_3^2 + 32\lambda_2 f_S(D\bar{A}^\dagger + \bar{B}\bar{B}^\dagger + \bar{F}\bar{F}^\dagger + \bar{H}^\dagger \bar{H} + E\bar{E}^\dagger).$$  \hfill (2.33)

Here the masses squared are given by

$$M_{I_1}^2 = M_{I_2}^2 = 32\lambda_2 f_S^2, \quad M_{I_3}^2 = 32(5\lambda_1 + \lambda_2)f_S^2.$$  \hfill (2.34)

**Scalar-gauge interactions**

Now we derive the couplings relevant to the Higgs mass correction through gauge boson loop. The gauge interaction is included in Eq. (2.9). In the breaking $SU(5) \rightarrow [SU(2) \times U(1)]^3$, only $W_{\mu i}$ and $B_{\mu i}$ ($i = 1, 2$) have masses below 10 TeV. As a low energy
To identify the couplings, it is convenient to write down the second and third terms in the effective theory, the coupling of $W_{\mu i}$ and $B_{\mu i}$ with scalar fields is needed. Under the group $[SU(2) \times U(1)]^2$, the charges of $S$ can be assigned as

$$Q_1^a = \begin{pmatrix} \tau^a & 0 \\ 0 & \mathbf{0}_2 \end{pmatrix}, \quad Y_1 = \frac{1}{10} \begin{pmatrix} 3 \cdot \mathbf{1}_2 & -2 \\ -2 \cdot \mathbf{1}_2 & -2 \cdot \mathbf{1}_2 \end{pmatrix}, \quad (2.35)$$

$$Q_2^a = \begin{pmatrix} 0 & \tau^a \\ \mathbf{0}_2 & -\tau^a \end{pmatrix}, \quad Y_2 = \frac{1}{10} \begin{pmatrix} 2 \cdot \mathbf{1}_2 & 2 \\ 2 & -3 \cdot \mathbf{1}_2 \end{pmatrix}. \quad (2.36)$$

To identify the couplings, it is convenient to write down the second and third terms in the covariant derivative $[2.9]$ as

$$-i g A^a T^a S - i g A^a S T^a T$$

$$= -i g (W_1^a - W_2^a) f_{\xi} \begin{pmatrix} \tau^a & 0 \\ \tau^a & \mathbf{0}_2 \end{pmatrix} - i g' (B_1 - B_2) \frac{1}{10} f_{\xi} \begin{pmatrix} \mathbf{1}_2 & -4 \\ \mathbf{1}_2 & \mathbf{1}_2 \end{pmatrix}$$

$$+ g \begin{pmatrix} W_1^a (\tau^a \phi_+ + \phi_+ \tau^a) & W_1^a \tau^a h_+ & W_1^a \tau^a \chi_+ - W_2^a \chi_- - W_2^a \tau^a \chi_+ \\ W_1^a \chi_+ \tau^a - W_2^a \tau^a \chi_+ & 0 & -W_2^a \tau^a h_+ \\ W_1^a \chi_+ \tau^a - W_2^a \tau^a \chi_+ & -W_2^a \tau^a h_+ & -W_2^a (\tau^a \phi_+ + \phi_+ \tau^a) \end{pmatrix}$$

$$+ \frac{g'}{10} \begin{pmatrix} (6 B_1 + 4 B_2) \phi_- & (B_1 + 4 B_2) h_- & (B_1 - B_2) K_i \chi_i \\ (B_1 + 4 B_2) h_+ & -4 (B_1 - B_2) (K_i - i K_i) & -4 (B_1 + B_2) \phi_+ \tau^a \\ (B_1 - B_2) \chi_+ & -4 (B_1 + B_2) \phi_+ \tau^a & -4 (B_1 + B_2) h_+ \phi_+ \tau^a \end{pmatrix}, \quad (2.37)$$

where $\phi_\pm = \phi \pm i \Phi$, $h_\pm = h \pm i H$, $\chi_- = \chi - i X$ and $g' = \sqrt{5/3} g$. The dominant gauge interactions relevant to the scalar mass correction are symbolically given by $A^2$, $A^2 N^2$ and $A^2 R$ terms at lower order of $R$. We obtain each term as follows: The $A^2$ term is

$$\frac{1}{8} \text{Tr} \left[(D_{\mu} S)(D^{\mu} S)^{\dagger}\right]_{A^2} = \frac{1}{8} g^2 f_S^2 (W_1^a - W_2^a)^2 + \frac{1}{30} g^2 f_S^2 (B_1 - B_2)^2. \quad (2.38)$$

The $A^2 N^2$ term is

$$\frac{1}{8} \text{Tr} \left[(D_{\mu} S)(D^{\mu} S)^{\dagger}\right]_{A^2 N^2}$$

$$= \frac{1}{16} g^2 (W_1^a + W_2^a) \text{Tr} \left[\phi \phi^\dagger\right] + \frac{1}{16} g^2 (W_1^a W_1^b + W_2^a W_2^b) \text{Tr} \left[\tau^a \phi \tau^b \phi^\dagger\right]$$

$$+ \frac{1}{2} g^2 (W_1^a + W_2^a) \text{Tr} \chi^2$$

$$+ \frac{1}{2} g^2 (W_1^a + W_2^a) \text{Tr} \chi$$

$$+ \frac{1}{200} g^2 (13 B_1^2 + 24 B_1 B_2 + 13 B_2^2) \text{Tr} \phi \phi^\dagger$$

$$+ \frac{1}{200} g^2 (17 B_1^2 + 16 B_1 B_2 + 17 B_2^2) \text{Tr} h \text{h}^\dagger h$$

$$+ \frac{1}{200} g^2 (B_1 - B_2)^2 \text{Tr} \chi^2$$

$$+ \frac{1}{280} g^2 (B_1 - B_2)^2 \text{Tr} \chi$$

$$+ \frac{1}{280} g^2 (B_1 - B_2)^2 \text{Tr} \chi$$

$$+ \frac{1}{200} g^2 (B_1 - B_2)^2 \text{Tr} \chi \text{h}^\dagger \text{h}$$

$$+ \frac{1}{200} g^2 (B_1 - B_2)^2 \text{Tr} \chi \text{h} \text{h}^\dagger \text{h}.$$

(2.39)

The $A^2 R$ term is

$$\frac{1}{8} \text{Tr} \left[(D_{\mu} S)(D^{\mu} S)^{\dagger}\right]_{A^2 R}$$

$$= \frac{1}{8} g^2 f_S (W_1^a - W_2^a)^2 \text{Tr} [X] + \frac{1}{200} g^2 f_S (B_1 - B_2) (W_1^a - W_2^a) \text{Tr} [\tau^a X]$$

$$+ \frac{1}{200} g^2 f_S (B_1 - B_2)^2 \text{Tr} [X] + \frac{1}{280} g^2 f_S (B_1 - B_2)^2 \text{K} \text{r}. \quad (2.40)$$
The equations (2.30), (2.33), (2.23), (2.39) and (2.40) are all the vertices needed for the scalar mass corrections via gauge boson loop at lower order of $R$. From Eq. (2.39), the tree vertex is given by the contact term

$$\frac{1}{8} \text{Tr} [(D_\mu S)(D^\mu S)\dagger]_{A^2 N^2} = \frac{1}{16} g^2 (W_1^a W_1^a + W_2^a W_2^a) (\text{Tr} [\phi \phi^\dagger] + h^\dagger h + \text{Tr} [\chi^2])$$

$$+ \frac{1}{200} g^2 (13 B_1^2 + 24 B_1 B_2 + 13 B_2^2) \text{Tr} [\phi \phi^\dagger]$$

$$+ \frac{1}{400} g^2 (17 B_1^2 + 16 B_1 B_2 + 17 B_2^2) h^\dagger h$$

$$+ \frac{1}{400} g^2 (B_1 - B_2)^2 \text{Tr} [\chi^2] + \frac{1}{30} g^2 (B_1 - B_2)^2 K_i^2.$$  

(2.41)

The effective $A^2 N^2$ vertex is made through integration of the heavy field $R$ with the vertices $V_{N^2 R}$ and $V_{A^2 R}$. From Eq. (2.30), the corresponding Lagrangian term for $N^2 R$ vertex is

$$- V_{N^2 R} = - 4 f_S \lambda_1 (2 \text{Tr} (X) + K_r) (2 \text{Tr} (\phi \phi^\dagger) + 2 \text{Tr} (\chi^2) + K_r^2 + 4 h^\dagger h)$$

$$- 4 f_S \lambda_2 K_r (2 h^\dagger h + K_r^2) - 4 f_S \lambda_2 \text{Tr} (X) \text{Tr} [\phi \phi^\dagger + h^\dagger h + \chi^2],$$

(2.42)

where we have used a decomposition $X = \frac{1}{2} \text{Tr} (X) \mathbf{1}_2 + (X - \frac{1}{2} \text{Tr} (X)) \mathbf{1}_2$. From Eq. (2.42), it is seen that the heavy fields $\text{Tr} (X) = 2 (I_1 + I_2)/\sqrt{5}$ and $K_r = 2 (-2 I_1 + I_2)/\sqrt{5}$ make contributions to the effective vertex. The contributing Lagrangian term of the $A^2 R$ vertex is

$$\frac{1}{8} \text{Tr} [(D_\mu S)(D^\mu S)\dagger]_{A^2 R} = \frac{1}{8} g^2 f_S (W_1^a W_1^a - W_2^a W_2^a)^2 \text{Tr} (X) + \frac{1}{200} g^2 f_S (B_1 - B_2)^2 (\text{Tr} (X) + 8 K_r).$$

(2.43)

From Eqs. (2.34), (2.42) and (2.43), the $A^2 N^2$ term by integral of the field $R$ becomes

$$- V_{A^2 N^2 \text{Int}} = - \frac{1}{16} g^2 (W_1^a W_1^a - W_2^a W_2^a)^2 \left[ \text{Tr} (\phi \phi^\dagger) + \text{Tr} (\chi^2) + h^\dagger h \right]$$

$$- \frac{1}{16} g^2 (B_1 - B_2)^2 \left[ \text{Tr} (\phi \phi^\dagger) + \text{Tr} (\chi^2) \right]$$

$$- \frac{1}{30} g^2 (B_1 - B_2)^2 K_i^2 - \frac{17}{400} g^2 (B_1 - B_2)^2 h^\dagger h.$$  

(2.44)

Therefore we obtain the Lagrangian term of effective vertex $V_{A^2 N^2 \text{Eff}}$ as

$$- V_{A^2 N^2 \text{Eff}} = \frac{1}{8} \text{Tr} [(D_\mu S)(D^\mu S)\dagger]_{A^2 N^2} + (- V_{A^2 N^2 \text{Int}})$$

$$= (\frac{1}{4} g^2 W_1^a W_1^a + \frac{1}{4} g^2 B_1 B_2) h^\dagger h + \frac{1}{4} g^2 W_1^a W_1^a (\text{Tr} [\phi \phi^\dagger] + \text{Tr} [\chi^2])$$

$$+ \frac{1}{400} g^2 (B_1 + B_2^2 + 49 B_1 B_2) \text{Tr} [\phi \phi^\dagger] - \frac{3}{30} g^2 (B_1 - B_2)^2 \text{Tr} (\chi^2).$$  

(2.45)

As expected, $h^\dagger h$ does not have $W_1^a W_1^a, W_2^a W_2^a, B_1^2$ and $B_2^2$ terms. Thus for the mass of the Higgs field $h$, there is no quadratic divergence from gauge boson loop. After log divergence is regularized, the Higgs boson mass squared changes the value with quantum loop effects. The value itself depends on its couplings with the gauge bosons $W_1, W_2, B_1$ and $B_2$. On the other hand, $\text{Tr} (\phi \phi^\dagger), \text{Tr} (\chi^2)$ have interacting terms with $B_1^2$ and $B_2^2$. Due to these $U(1)$ factors, the masses of $\phi$ and $\chi$ receive quadratic divergence. The coupling for $\chi$ has a negative sign. If theory is arranged such that the mass scale of heavy fields is stabilized, it should be traced carefully whether corrections to scalar masses squared have a positive sign.
3 Operators in breaking by boundary conditions

We have found the effective couplings in the model with gauge symmetry broken by the vacuum expectation value of the complex symmetric tensor $S$. Instead of such a vacuum expectation value, we examine the case in which the two breaking sectors in higher energy scales are spatially separated. At $y = 0$, SU(5) is broken to $[SU(2) \times U(1)]^2$ and at $y = \pi R$, SU(5) is broken to SO(5). Here $y$ is the coordinate of the fifth dimension and $R$ is the compactification radius. We assume that the five-dimensional spacetime is flat.

3.1 Consistency of boundary conditions

The gauge symmetry breaking of SU(5) to SO(5) by boundary conditions at $y = \pi R$ is obtained by imposing Neumann boundary conditions at $y = \pi R$ only on fields for the generators in Eq. (2.13). There are no scalar fields required for this gauge symmetry breaking. On the other hand, extra-dimensional components of bulk gauge bosons can have nonzero values as four-dimensional scalar fields. In such a situation, the consistency of boundary conditions with gauge transformations is nontrivial. In this subsection, we give two examples of inconsistent boundary conditions for distinct patterns of gauge symmetry breaking. Then we check the consistency of the gauge symmetry breaking of SU(5) to SO(5).

A model of SU($N$) → SU($N_1$) × U(1)

We consider a gauge symmetry breaking of SU($N$),

$$SU(N) \rightarrow SU(N_1) \times SU(N - N_1) \times U(1) \rightarrow SU(N_1) \times U(1).$$

(3.1)

Here we assume the following boundary conditions [17] as shown in Table 1: Neumann (N) for $A^a_\mu(x, y)$, Dirichlet (D) for $A^a_\mu(x, y)$, D for $A^a_\mu(x, y)$ and N for $A^a_\mu(x, y)$ at $y = 0$; D for $A^a_\mu(x, y)$, N for $A^a_\mu(x, y)$, N for $A^a_\mu(x, y)$ and D for $A^a_\mu(x, y)$ at $y = \pi R$, where $a$ and $\hat{a}$ indicate the indices of SU($N - N_1$) and SU($N$)/[SU($N_1$) × SU($N - N_1$) × U(1)], respectively. We omit the boundary conditions for the groups SU($N_1$) and U(1). The boundary conditions for gauge transformation functions are the same as that of the four-dimensional gauge bosons: N for $\epsilon^a(x, y)$ and D for $\epsilon^{\hat{a}}(x, y)$ at $y = 0$; D for $\epsilon^a(x, y)$ and N for $\epsilon^{\hat{a}}(x, y)$ at $y = \pi R$.

| Table 1: The boundary conditions for SU($N$) → SU($N_1$) × U(1). |
|-----------------|-----------------|-----------------|
| $y = 0$         | $y = \pi R$     |
| $A^a_\mu$, $A^a_\mu$, $A^a_\mu$, $A^a_\mu$ | $A^a_\mu$, $A^a_\mu$, $A^a_\mu$, $A^a_\mu$ |
| N, D, N, D      | D, N, N, D      |

For each color, the gauge transformation laws are given by

$$\delta A^a_M = \partial_M \epsilon^a + gf^{abc} A^b_M \epsilon^c + gf^{\hat{a}\hat{b}\hat{c}} A^{\hat{b}}_M \epsilon^{\hat{c}},$$

(3.2)

$$\delta A^{\hat{a}}_M = \partial_M \epsilon^{\hat{a}} + gf^{\hat{a}\hat{b}\hat{c}} A^{\hat{b}}_M \epsilon^{\hat{c}} + gf^{abc} A^b_M \epsilon^c,$$

(3.3)

where $M = (\mu, y)$. The boundary conditions of the left-hand and right-hand sides for the gauge transformations (3.2) and (3.3) are tabulated in Table 2. From Table 2, the
boundary conditions at $y = 0$ are consistent with the gauge transformations. It is seen that the other boundary conditions are inconsistent because for example the NN term for $A_\mu^a$ does not obey Dirichlet condition, $(A_\mu^b \epsilon^c)|_{y=\pi R} \neq 0$ and the ND term for $A_y^a$ does not obey Neumann condition, $\partial_y (A_y^b \epsilon^c)|_{y=\pi R} = (A_y^b \partial_y \epsilon^c)|_{y=\pi R} \neq 0$.

A left-right symmetric model

As another example, we discuss a left-right symmetric model with the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ broken as

$$SU(2)_L \times SU(2)_R \times U(1) \rightarrow SU(2)_D \times U(1), \quad \text{at } y = 0,$$

$$SU(2)_L \times SU(2)_R \times U(1) \rightarrow SU(2)_L \times U(1)_Y, \quad \text{at } y = \pi R.$$  

The gauge bosons of $SU(2)_L$, $SU(2)_R$ and $U(1)$ are denoted as $A_{L,\mu}^a$, $A_{R,\mu}^a$ and $B_\mu$, respectively. The gauge coupling constants are identical for $SU(2)_L$ and $SU(2)_R$ and it is denoted as $g$. The gauge coupling constant of $U(1)$ is denoted as $g'$. The boundary conditions are given in Table 3.

Table 2: The boundary conditions for the terms of the transformation laws.

| LHS | RHS | LHS | RHS |
|-----|-----|-----|-----|
| $A_\mu^a$ | N | N + NN + DD | D | D + DD + NN |
| $A_y^a$ | D | D + DN + ND | N | N + ND + DN |
| $A_{L,\mu}^a$ | D | D + ND + DN | N | N + DN + ND |
| $A_{y}^a$ | N | N + DD + NN | D | D + NN + DD |

Table 3: The boundary conditions for the left-right symmetry model.

| y = 0 | y = $\pi R$ |
|-------|-------------|
| $A_{L+R,\mu}^a$ | $A_{L-R,\mu}^a$ | $B_\mu$ | $A_{L,\mu}^a$ | $A_{R,\mu}^{1\mu}$ | $A_{R,\mu}^3$ | $A_{Y,\mu}^3$ |
| N | D | N | N | D | N | N |

Here we have defined

$$\begin{pmatrix} A_{L+R}^a \\ A_{L-R}^a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} A_L^a \\ A_R^a \end{pmatrix}, \quad \begin{pmatrix} A_R^3 \\ A_Y \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} A_R^3 \\ B \end{pmatrix}$$  

(3.6)

where the vector index $M$ has been omitted. The gauge transformation functions $\epsilon_{L}^a$, $\epsilon_{R}^a$ and $\epsilon_B$ have the same boundary conditions as the gauge bosons with the definition

$$\begin{pmatrix} \epsilon_{L+R}^a \\ \epsilon_{L-R}^a \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \epsilon_L^a \\ \epsilon_R^a \end{pmatrix}, \quad \begin{pmatrix} \epsilon_R^3 \\ \epsilon_Y \end{pmatrix} = \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & -g' \\ g' & g \end{pmatrix} \begin{pmatrix} \epsilon_R^3 \\ \epsilon_B \end{pmatrix}. $$  

(3.7)

The extra-dimensional components have the opposite boundary conditions to the four-dimensional components. The gauge transformation laws are written in terms of N and D fields at $y = 0$ as

$$\delta A_{L+R,M}^a = \partial_M \epsilon_{L+R}^a + gf^{abc} A_{L+R,M}^b \epsilon_{L+R}^c + gf^{abc} A_{L-R,M}^b \epsilon_{L-R}^c,$$

$$\delta A_{L-R,M}^a = \partial_M \epsilon_{L-R}^a + gf^{abc} A_{L+R,M}^b \epsilon_{L-R}^c + gf^{abc} A_{L-R,M}^b \epsilon_{L+R}^c,$$

$$\delta B_M = \partial_M \epsilon_B,$$

(3.8) (3.9) (3.10)
and in terms of N and D fields at $y = \pi R$ as

$$
\delta A_{LM}^a = \partial_M \epsilon_L^a + g f^{abc} A_{LM}^b \epsilon_L^c, \\
\delta A_{RM}^1 = \partial_M \epsilon_R^1 + \frac{g^2}{\sqrt{g^2 + g'^2}} (A_{RM}^2 \epsilon_R^2 - \bar{A}_{RM}^3 \epsilon_R^3) - \frac{g g'}{\sqrt{g^2 + g'^2}} (A_{RM}^2 \epsilon_Y - A_{RM} \epsilon_Y^2), \\
\delta A_{RM}^2 = \partial_M \epsilon_R^2 - \frac{g^2}{\sqrt{g^2 + g'^2}} (A_{RM}^2 \epsilon_R^2 - \bar{A}_{RM}^3 \epsilon_R^3) + \frac{g g'}{\sqrt{g^2 + g'^2}} (A_{RM}^2 \epsilon_Y - A_{RM} \epsilon_Y^2), \\
\delta \bar{A}_{RM}^3 = \partial_M \epsilon_R^3 + \frac{g^2}{\sqrt{g^2 + g'^2}} (A_{RM}^2 \epsilon_R^2 - A_{RM}^2 \epsilon_R^1), \\
\delta A_{YM} = \partial_M \epsilon_Y + \frac{g g'}{\sqrt{g^2 + g'^2}} (A_{YM}^1 \epsilon_Y^2 - A_{YM}^2 \epsilon_Y^1).
$$

The boundary conditions of the left- and right-hand sides are shown in Table 4.

| $y = 0$ | $y = \pi R$ |
|--------|-------------|
| LHS    | RHS         |
| $A_{LM+R}^a$ | N N + NN + DD |
| $A_{LM+Ry}$  | D D + DN + ND |
| $A_{LM-R}^a$ | D D + ND + DN |
| $A_{LM-Ry}$  | N N + DD + NN |
| $B_\mu$      | N N          |
| $B_y$         | D D          |
| $A_{LM}^a$    | N N + NN     |
| $A_{LM}^y$    | D D + DN     |
| $A_{LM}^2$    | D D + DD + DN |
| $A_{LM}^3$    | N N + ND + NN |
| $A_{LM}^4$    | D D          |
| $A_{LM}^5$    | N N + ND     |
| $A_{LM}^6$    | N N + DD     |
| $A_{LM}^7$    | D D          |
| $A_{LM}^8$    | N N + ND     |
| $A_{LM}^9$    | N N + DD     |
| $A_{LM}^{10}$ | D D          |

From Table 4 the boundary conditions at $y = 0$ are consistent with the gauge transformations. The inconsistency arises from the ND terms of $A_{LM}^1$ and $\bar{A}_{LM}^3$ because the ND term does not obey Neumann condition.

**Consistent boundary conditions for SU(5) → SU(2) × U(1)**

We have seen the two examples which have the inconsistency between gauge transformation laws and boundary conditions. It is needed to examine the consistency of possible boundary conditions to yield gauge symmetry breaking $SU(5) \rightarrow [SU(2) \times U(1)]^2$ at $y = 0$ and $SU(5) \rightarrow SO(5)$ at $y = \pi R$. In order to realize the symmetry breaking above, we assign Neumann condition for the gauge bosons of the generators $T_1, T_2, T_3, T_8, T_{15}, T_{22}, T_{23}, T_{24}$ at $y = 0$ and $T_1, \cdots, T_{10}$ given in Eq. (2.13) at $y = \pi R$ and Dirichlet condition for the other gauge bosons. Only the fields with Neumann condition at both boundaries have zero modes. The generators of zero modes are

$$
T_7, T_{17}, T_{17}, T_{17}.
$$

These generators form $SU(2) \times U(1)$ algebra. The gauge transformation laws are written as

$$
\delta A_M^a = \partial_M \epsilon_a + g f^{abc} A_M^b \epsilon^c, \\
\delta A_M^\dot{a} = \partial_M \epsilon^\dot{a} + g f^{\dot{a}bc} A_M^b \epsilon^c + g f^{\dot{a}\dot{b}c} A_M^\dot{b} \epsilon^c.
$$
where \( a \) and \( \hat{a} \) indicate the generators of the subgroup and the coset, respectively. At \( y = 0 \), \( a \) and \( \hat{a} \) represent the indices for \([SU(2) \times U(1)]^2\) and \([SU(5)/SU(2) \times U(1)]^2\), respectively. At \( y = \pi R \), \( a \) and \( \hat{a} \) represent the indices for \( SO(5) \) and \( SU(5)/SO(5) \), respectively. The boundary conditions at \( y = 0 \) and \( y = \pi R \) are collectively written with the \( a \) and \( \hat{a} \). The left- and right-hand sides in the transformation laws have the boundary conditions shown in Table \( 5 \). From Table \( 5 \), it is found that all the boundary conditions are consistent with the gauge transformations. The result of the consistency in this case is also seen from the fact that Neumann and Dirichlet conditions imposed at each boundary can be assigned by the automorphism of orbifolds.

### 3.2 Interactions of scalar fields with gauge bosons

In the gauge symmetry breaking by boundary conditions, there exist zero-mode scalar fields. From the boundary conditions given above Eq. (3.16), \( A_y \) has Neumann condition for the 16 generators \( T_4, \ldots, T_7, T_9, \ldots, T_{14}, T_{16}, \ldots, T_{21} \) at \( y = 0 \) and the 14 generators \( T_{17}, \ldots, T_{25}, T_9, T_{10}, T_{18}, T_{19} \) at \( y = \pi R \). The common 10 generators \( T_{13}, \ldots, T_{15}, T_9, T_{10}, T_{18}, T_{19} \) correspond to zero modes of scalar fields.

To identify the couplings of the scalar fields with gauge bosons, we focus on the Lagrangian term

\[
-\frac{1}{2}g^2 f^{abc} f^{def} A^a_\mu A^b_\nu A^{\mu \nu \rho} A^c_\rho, \tag{3.19}
\]

Keeping the zero modes of gauge bosons \( A^I_\mu, A^7_\mu, A^\tau_\mu, A^\mu_{10}, A^9_\mu, A^{10}_y, A^{18}_y, A_{19} \), Eq. (3.19) become

\[
-\frac{g^2}{16} \left( (A^T_\mu)^2 + (A^T_\mu)^2 \right) \left( (A^T_\nu)^2 + (A^T_\nu)^2 + (A^T_\nu)^2 + (A^T_\nu)^2 + 4\{ (A^T_\nu)^2 + (A^T_\nu)^2 \} \right)
\]

\[
-\frac{g^2}{4} \left( -\frac{1}{2} A^3_\mu + \sqrt{2} A^8_\mu \right)^2 + \left( -\sqrt{2} A^8_\mu + \frac{\sqrt{2}}{2} A^5_\mu + \frac{\sqrt{6}}{4} A^2_\mu \right)^2 \right) \left( (A^T_\nu)^2 + (A^T_\nu)^2 \right)
\]

\[
-\frac{g^2}{4} \left( -\frac{1}{2} A^3_\mu + \sqrt{3} A^8_\mu + \frac{\sqrt{2}}{3} A^5_\mu \right)^2 \left( (A^T_\nu)^2 + (A^T_\nu)^2 \right)
\]

\[
-\frac{g^2}{4} \left( -\frac{1}{2} A^3_\mu + \frac{\sqrt{2}}{2} A^8_\mu + \frac{\sqrt{6}}{3} A^5_\mu \right)^2 \left( (A^T_\nu)^2 + (A^T_\nu)^2 \right)
\]

\[
-\frac{\sqrt{2}g^2}{8} \left( A^T_\mu A^{10}_\mu (A^T_\nu A^T_\nu A^T_\nu + A^T_\mu A^T_\mu) + A^7_\mu A^{10}_\mu (A^T_\nu A^T_\nu - A^T_\nu A^T_\nu) \right)
\]

\[
-\frac{g^2}{8} \left( (A^T_\mu)^2 + (A^T_\mu)^2 \right) \left( (A^9_\nu)^2 + (A^{10}_\nu)^2 + (A^{18}_\nu)^2 + (A^{19}_\nu)^2 \right)
\]

\[
-\frac{g^2}{4} \left( (A^T_\mu)^2 - (A^T_\mu)^2 \right) (A^9_\nu A^{18}_\nu + A^{10}_\nu A^{19}_\nu) - \frac{g^2}{2} A^T_\mu A^T_\mu - \frac{g^2}{2} A^T_\mu A^T_\mu \right). \tag{3.20}
\]

| LHS | RHS |
|-----|-----|
| \( A^a_\mu \) | \( N \) |
| \( A^a_\mu \) | \( N + NN + DD \) |
| \( A^a_\mu \) | \( D \) |
| \( A^a_\mu \) | \( D + DN + ND \) |
| \( A^a_\mu \) | \( D \) |
| \( A^a_\mu \) | \( D + DN + ND \) |
| \( A^a_\mu \) | \( N \) |
| \( A^a_\mu \) | \( N + NN + DD \) |
Here

\[
\begin{pmatrix}
A_\mu^3 \\
A_\mu^8 \\
A_\mu^{15} \\
A_\mu^{24}
\end{pmatrix}
= \frac{4\sqrt{2}}{20 + 3\sqrt{10}} \begin{pmatrix}
4 & 5 \left(1 - \sqrt{11}\right) & 5 & 3\sqrt{11} \\
0 & 13\sqrt{12} + 3\sqrt{30} & 0 & 5\sqrt{12} + 13\sqrt{15} \\
\sqrt{6} & 5\sqrt{3} & -\sqrt{6} & -\sqrt{3} \\
-\sqrt{10} & 5/2 + \sqrt{10} & \sqrt{10} & -\sqrt{2} - \sqrt{5}/2
\end{pmatrix}
\begin{pmatrix}
A_\mu^{14} \\
A_\mu^{19} \\
A_\mu^{15} \\
A_\mu^{20}
\end{pmatrix},
\tag{3.21}
\]

On the other hand, keeping the gauge bosons for Neumann condition at \( y = 0 \), \( A_\mu^1, A_\mu^2, A_\mu^3, A_\mu^8, A_\mu^{15}, A_\mu^{22}, A_\mu^{23}, A_\mu^{24} \), Eq. (3.19) becomes

\[
\begin{align*}
-\frac{g^2}{16} ((A_\mu^3)^2 + (A_\mu^2)^2 + (A_\mu^{22})^2 + (A_\mu^{23})^2) & ((A_y^{13})^2 + (A_y^{14})^2 + (A_y^{15})^2 + (A_y^{16})^2) \\
-\frac{g^2}{8} ((A_\mu^1 - A_\mu^{22})^2 + (A_\mu^2 + A_\mu^{23})^2) & ((A_y^{17})^2 + (A_y^{18})^2) \\
-\frac{g^2}{2} \left( \frac{1}{8} (A_\mu^3 + \sqrt{3} A_\mu^8)^2 + \frac{1}{6} (A_\mu^8 - \sqrt{2} A_\mu^{15})^2 \right) & ((A_y^{19})^2 + (A_y^{20})^2) \\
-\frac{g^2}{2} \left( \frac{1}{8} (A_\mu^3 - \sqrt{3} A_\mu^8)^2 + \frac{1}{6} (A_\mu^8 + \sqrt{2} A_\mu^{15} - \sqrt{30} A_\mu^{24})^2 \right) & ((A_y^{15})^2 + (A_y^{26})^2) \\
-\frac{g^2}{8} \left( (A_\mu^3 - \frac{1}{\sqrt{2}} A_\mu^8 - \frac{1}{\sqrt{6}} A_\mu^{15})^2 + (A_\mu^3 + \frac{1}{\sqrt{2}} A_\mu^8 + \frac{1}{\sqrt{6}} A_\mu^{15} + \frac{\sqrt{10}}{2} A_\mu^{24})^2 \right) & ((A_y^{17})^2 + (A_y^{18})^2) \\
-\frac{g^2}{8} \left( (A_\mu^1 - \frac{1}{\sqrt{3}} A_\mu^8 - \frac{5}{4\sqrt{2}} A_\mu^{15} - \frac{\sqrt{30}}{8} A_\mu^{24}) \right) & \left( A_\mu^{22}(A_y^{13} A_y^{15} + A_y^{14} A_y^{16}) + A_\mu^{23}(A_y^{13} A_y^{16} - A_y^{14} A_y^{15}) \right) \\
-\frac{g^2}{8} ((A_\mu^1)^2 + (A_\mu^2)^2 + (A_\mu^{22})^2 + (A_\mu^{23})^2) & ((A_y^{19})^2 + (A_y^{20})^2) \\
-\frac{g^2}{2} (A_\mu^3 + \frac{1}{\sqrt{3}} A_\mu^8 + \frac{4}{\sqrt{6}} A_\mu^{15})^2 & ((A_y^{19})^2 + (A_y^{10})^2) \\
-\frac{g^2}{8} (A_\mu^3 - \frac{1}{\sqrt{3}} A_\mu^8 - \frac{1}{\sqrt{6}} A_\mu^{15} - \frac{\sqrt{10}}{2} A_\mu^{24})^2 & ((A_y^{18})^2 + (A_y^{19})^2) \\
+ \frac{g^2}{2} ((A_\mu^1 A_y^{10} + A_\mu^2 A_y^{18} + A_\mu^{22} A_y^{19}) + (A_\mu^1 A_y^{10} - A_\mu^2 A_y^{9})(A_\mu^{22} A_y^{19} - A_\mu^{23} A_y^{18})) & .
\tag{3.22}
\end{align*}
\]

In Eqs. (3.20) and (3.22), all the scalar fields are coupled to gauge bosons of the form \( A_\mu^a A_\mu^{au} \) with the same sign. This means that their scalar fields do not have the couplings given in Eqs. (2.1) and (2.2). The scalar fields are a component of higher-dimensional gauge bosons. If Kaluza-Klein modes are included infinitely, higher-dimensional gauge symmetry may work as a symmetry principle. However, the collective symmetry breaking mechanism would be a distinct property. In a five-dimensional model, the breaking \( SU(5) \to SO(5) \) where the Higgs boson resides could be driven by the vacuum expectation values of scalar fields. For such a breaking, it can be seen that the expected cancellation takes place in a parallel way with the four-dimensional model without relying on a summation over the contributions of Kaluza-Klein modes. Here the collective symmetry breaking mechanism works. In the case of the breaking by boundary conditions, one may wonder whether the cancellation occurs between zero mode and one of any Kaluza-Klein mode without invoking an infinite number of Kaluza-Klein modes. The analysis here has been given for fields dependent on the five-dimensional coordinates. It can be seen that the couplings for
any Kaluza-Klein mode of gauge bosons are the same sign as that of the zero mode. This means that the expected cancellation does not occur. Even though higher-dimensional gauge symmetry is employed to cancel the quadratic divergence, one would need to take into account contributions such as a three-point coupling with a gauge boson and two scalar fields, which are not relevant to the collective symmetry breaking mechanism.

We have examined interactions of zero-mode gauge bosons with scalar fields. In the five-dimensional setup, there are no three- and four-point scalar couplings because of $F_{yy} = 0$. Therefore heavy scalar fields do not contribute to changing the scalar-gauge interactions at tree level.

4 Properties of symmetry breaking and operators in the two models

As we have derived in Eqs. (2.45), (3.20) and (3.22), the operators have different forms depending on the source of symmetry breaking which is vacuum expectation values or boundary conditions. In this section, we discuss why the difference of the two models is induced, by comparing the meaning of $SU(5) \rightarrow [SU(2) \times U(1)]^2$.

In the model with scalars having vacuum expectation values, the scalar-gauge interactions (2.45) involve gauge bosons of $[SU(2) \times U(1)]^2$ in breaking

$$SU(5) \rightarrow [SU(2) \times U(1)]^2.$$  \hfill (4.1)

Two of the group $[SU(2) \times U(1)]$ have identical gauge coupling constants because they are subgroups of a single group. We cannot switch off only one of the gauge couplings of two $[SU(2) \times U(1)]$. Instead we consider the breaking into a single $[SU(2) \times U(1)]$ as

$$SU(5) \rightarrow SU(2) \times U(1),$$  \hfill (4.2)

with appropriate choices of scalar potentials independently. In the breaking into a single $SU(2) \times U(1)$, there exists global $SU(3)$. The corresponding Nambu-Goldstone bosons have no potential. The generation of potential is made when two of $[SU(2) \times U(1)]$ are taken into account as in Eq. (4.1). That the potential requires two of groups leads to the absence of quadratic divergence. This type of discussion to consider a single $[SU(2) \times U(1)]$ as a variation was given in Ref. [14]. Because the operation of switch-off is not necessarily equivalent to the direct breaking (4.2), it is nontrivial whether resulting scalar-gauge couplings have the form (2.1). We have checked the emergence of the form (2.1) by deriving all the scalar-gauge couplings.

In the model with boundary conditions, the breaking (4.2),

$$SU(5) \rightarrow SU(2) \times U(1)$$  \hfill (4.3)

is achieved by boundary conditions consistently as an overlapping at two boundaries. However, the breaking (4.3) at a single boundary is forbidden by consistency with local gauge transformation as follows. The breaking (4.3) needs Dirichlet condition for diagonal blocks with respect to gauge bosons in a matrix form. In Section 3.1 in a model of $SU(N) \rightarrow SU(N_1) \times U(1)$ we have seen that D for $A^a_\mu(x, y)$, N for $A^a_\mu(x, y)$, N for $A^a_\mu(x, y)$ and D for $A^a_\mu(x, y)$ at a single boundary (in Section 3.1 at $y = \pi R$) is an inconsistent boundary condition. In addition, we see that the boundary condition D for $A^a_\mu(x, y)$, N
Table 6: Inconsistent boundary terms of the transformation laws with Dirichlet condition for $A^\mu_\alpha(x, y)$ and $A^\mu_\hat{\alpha}(x, y)$.

| LHS | RHS |
|-----|-----|
| $A^\alpha_y$ | N + ND + ND |

for $A^\alpha_y(x, y)$, D for $A^\alpha_\mu(x, y)$ and N for $A^\alpha_\hat{\alpha}(x, y)$ is also not compatible with local gauge transformation. In the same indication as in Table 2, an inconsistent boundary condition is shown in Table 6. Therefore D for $A^\alpha_\mu(x, y)$ is not allowed. In other words, as a variation of the breaking $SU(5) \to [SU(2) \times U(1)]^2$ for switching off a single $SU(2) \times U(1)$, the breaking $SU(5) \to SU(2) \times U(1)$ at one boundary does not exist. In the model with boundary conditions, the generation of potential for $[SU(2) \times U(1)]^2$ is regarded as the generation of the potential just for a subgroup with a single gauge coupling constant. This suggests that the collective breaking does not work. On the other hand, the pattern of the symmetry breaking group itself is identical to that of the model with scalars having vacuum expectation values. Like the model of vacuum expectation values, there might be a possibility that the scalar potential could be accidentally prevented from having quadratic divergence. We have shown that it does not occur by deriving all the relevant scalar-gauge operators.

5 Conclusion

We have derived all the couplings of scalar-gauge interactions relevant to scalar mass corrections via gauge boson loop in high energy models. We have found that while the Higgs fields are protected with the coupling of the form (1.1) from having quadratic divergence, the other fields receive quadratic divergence through loop of the U(1) gauge boson. As another aspect, we have considered a possibility of gauge symmetry breaking by boundary conditions as the first step to utilize dynamical symmetry breaking in a gauge-Higgs unification. In our assignment for boundary conditions, the same gauge symmetry breaking as in the vacuum expectation value is produced in a consistent way with local gauge transformations. In this gauge symmetry breaking, we have presented the couplings between zero-mode gauge bosons and scalar fields. It has been found that there is a difference of the structure between the operators in the two models of vacuum expectation values or boundary conditions.

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A  SU(5) generators and structure constants

The generators of SU(5) transformations, $\lambda_a$ ($a = 1, \cdots 24$) are

$$
\begin{align*}
\lambda_1 &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_2 &= \begin{pmatrix} 1 & & & & \\ & i & & & \\ & & \ddots & & \\ & & & i & \\ & & & & 1 \end{pmatrix}, & \lambda_3 &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & i & \\ & & & & 1 \end{pmatrix}, & \lambda_4 &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, \\
\lambda_5 &= \begin{pmatrix} i & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, & \lambda_6 &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_7 &= \begin{pmatrix} i & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, \\
\lambda_8 &= \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}, & \lambda_9 &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{10} &= \begin{pmatrix} i & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, \\
\lambda_{11} &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, & \lambda_{12} &= \begin{pmatrix} 1 & & & & \\ & i & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, & \lambda_{13} &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{14} &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, \\
\lambda_{15} &= \frac{1}{\sqrt{6}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -3 \end{pmatrix}, & \lambda_{16} &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{17} &= \begin{pmatrix} i & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & i \end{pmatrix}, \\
\lambda_{18} &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{19} &= \begin{pmatrix} 1 & & & & \\ & i & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{20} &= \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{21} &= \begin{pmatrix} 1 & & & & \\ & i & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, \\
\lambda_{22} &= \begin{pmatrix} 1 & & & & \\ & i & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{23} &= \begin{pmatrix} 1 & & & & \\ & i & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}, & \lambda_{24} &= \frac{1}{\sqrt{10}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -4 \end{pmatrix},
\end{align*}
$$

where dots indicate 0. Here $T_a = \frac{1}{2} \lambda_a$. The $\lambda_a$ obey the following commutation relationship:

$$
[\lambda_a, \lambda_b] \equiv \lambda_a \lambda_b - \lambda_b \lambda_a = 2i f_{abc} \lambda_c.
$$  \tag{A.1}

The $f_{abc}$ are odd under the permutation of any pair of indices. The nonzero values are tabulated in Table 7.

The generators (2.13) are given in a matrix form for $\lambda_a$ as

$$
\begin{align*}
\lambda_1 &= \lambda_1 - \lambda_{22} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}, & \lambda_2 &= \lambda_2 + \lambda_{23} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & & & & \\ & i & & & \\ & & \ddots & & \\ & & & i & \\ & & & & i \end{pmatrix}, \\
\lambda_3 &= \lambda_3 - \lambda_{13} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}, & \lambda_4 &= \lambda_4 - \lambda_{14} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & & & & \\ & i & & & \\ & & \ddots & & \\ & & & i & \\ & & & & i \end{pmatrix}, \\
\lambda_5 &= \lambda_5 - \lambda_{20} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}, & \lambda_6 &= \lambda_6 - \lambda_{21} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & & & & \\ & i & & & \\ & & \ddots & & \\ & & & i & \\ & & & & i \end{pmatrix}, \\
\lambda_7 &= \lambda_7 - \lambda_{16} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \\ & & & & -1 \end{pmatrix}, & \lambda_8 &= \lambda_8 - \lambda_{17} \sqrt{2} = \frac{1}{\sqrt{2}} \begin{pmatrix} i & & & & \\ & i & & & \\ & & \ddots & & \\ & & & i & \\ & & & & i \end{pmatrix},
\end{align*}
$$

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The other generators of SU(5) are written as

\[
\lambda_\overline{5} = \frac{1}{\sqrt{2}} (\lambda_3 + \frac{\sqrt{6}}{4} \lambda_{15} - \frac{\sqrt{10}}{4} \lambda_{24}) = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 & -1 & -1 & 1
\end{pmatrix},
\]

\[
\lambda_{\overline{10}} = \frac{1}{\sqrt{2}} (\frac{\sqrt{3}}{3} \lambda_8 + \frac{5\sqrt{6}}{12} \lambda_{15} + \frac{\sqrt{10}}{4} \lambda_{24}) = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 1 & 1 & 1
\end{pmatrix}, \quad \text{(A.2)}
\]

The other generators of SU(5) are written as

\[
\lambda_{11} = \frac{\lambda_1 + \lambda_{22}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 1 & 1 & 1
\end{pmatrix}, \quad \lambda_{12} = \frac{\lambda_2 - \lambda_{23}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 & -1 & -1 & 1
\end{pmatrix},
\]

\[
\lambda_{13} = \frac{\lambda_4 + \lambda_{13}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 1 & 1 & 1
\end{pmatrix}, \quad \lambda_{14} = \frac{\lambda_5 + \lambda_{14}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 & -1 & -1 & 1
\end{pmatrix},
\]

\[
\lambda_{15} = \frac{\lambda_6 + \lambda_{20}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 1 & 1 & 1
\end{pmatrix}, \quad \lambda_{16} = \frac{\lambda_7 + \lambda_{21}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 & -1 & -1 & 1
\end{pmatrix},
\]

\[
\lambda_{17} = \frac{\lambda_{11} + \lambda_{16}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 1 & 1 & 1
\end{pmatrix}, \quad \lambda_{18} = \frac{\lambda_{12} + \lambda_{17}}{\sqrt{2}} = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 & -1 & -1 & 1
\end{pmatrix},
\]

\[
\lambda_{19} = \frac{1}{\sqrt{2}} (\lambda_3 - \frac{\sqrt{6}}{4} \lambda_{15} + \frac{\sqrt{10}}{4} \lambda_{24}) = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & -1 & -1 & -1 & 1
\end{pmatrix},
\]

Table 7: The \(f_{abc}\). There are 66 independent nonzero structure constants.
\[ \lambda_{20} = \frac{1}{\sqrt{10}} \left( \frac{5 \sqrt{3}}{3} \lambda_8 - \frac{5 \sqrt{3}}{12} \lambda_{15} - \frac{\sqrt{10}}{4} \lambda_{24} \right) = \frac{1}{\sqrt{10}} \begin{pmatrix} \lambda_9 \\ \lambda_{10} \\ \lambda_{18} \\ \lambda_{19} \end{pmatrix}, \]

(A.3)

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