Quantum Mechanics Without Measurements

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Abstract

Many of the conceptual problems students have in understanding quantum mechanics arise from the way probabilities are introduced in standard (textbook) quantum theory through the use of measurements. Introducing consistent microscopic probabilities in quantum theory requires setting up appropriate sample spaces taking proper account of quantum incompatibility. When this is done the Schrödinger equation can be used to calculate probabilities independent of whether a system is or is not being measured, and the results usually ascribed to wave function collapse are obtained in a less misleading way through conditional probabilities. Toy models that include measurement apparatus as part of the total quantum system make this approach accessible to students. Some comments are made about teaching this material.

I Introduction

Quantum mechanics is a difficult subject to teach, and there has been a significant effort to find out what problems students have in understanding it, and how to overcome them. In part the difficulties arise from unfamiliar mathematics: partial differential equations, complex linear algebra (or functional analysis), and probability theory. However, the greatest difficulty is surely the one encapsulated in Feynman’s well-known assertion that “Nobody understands quantum mechanics.” How are students to learn a subject that their teachers do not understand?

Feynman’s own masterful exposition of the subject is proof that physicists can, indeed, teach what we do not understand, or do not understand as well as we would like to. At the same time, what he said needs to be taken seriously; Feynman was not joking. The problems he had understanding the subject are also severe barriers to less brilliant minds, and the basic thesis of this article is that helping students overcome them, rather than sweeping them under the carpet, is well worth the effort. That is most obvious in the case of future professional physicists or electrical engineers who will need to deal with entanglement, quantum information, transport in nanocircuits, and similar subjects for which the approach found in current textbooks does not provide a helpful physical intuition. But for the sake of other students as well, we need to try and counter the quasi-magical view of the quantum world that results from trying to make sense of what one finds in current textbooks, not to
mention popular expositions written by authors who understand quantum mechanics even less than Feynman did, but are less forthright in confessing their ignorance.

At the heart of the conceptual difficulties of quantum theory is the failure of the current textbook version of the subject—often called “Copenhagen” or “standard” quantum mechanics—to introduce probabilities in quantum theory in a consistent and meaningful way. Instead, probabilities are introduced on the basis of measurements, an approach which conveniently gets around various difficulties, but leaves students with a confused idea of what quantum mechanics is all about, and the impression that understanding the subject is impossible. Instead they get the feeling that one should, to use Mermin’s memorable phrase, “shut up and calculate.” That measurements provide an unsatisfactory approach to quantum interpretation has been known for a long time in the quantum foundations community,1,2 where the “measurement problem” is widely considered both an embarrassment3 and an intractable difficulty4. More about this in Sec. II.

The consistent or decoherent histories, hereafter abbreviated to “histories,” approach to quantum theory5,6,7 allows one to introduce probabilities in a physically meaningful and mathematically consistent way without reference to measurements. Doing so requires that one confront head on the central conceptual difficulty of quantum mechanics: quantum incompatibility. This is discussed in Sec. III in terms of a spin-half particle. No interpretation of quantum mechanics can be considered satisfactory if it cannot make both mathematical (the easy part) and physical (the hard part) sense of this, the simplest of quantum systems.

Probabilistic or stochastic time development in quantum mechanics requires the notion of a quantum history, a concept which in itself is not particularly difficult, Sec. IV. Assigning probabilities without using measurements can then be done using the Born rule (not hard) and its extensions (more subtle) applied to a closed or isolated quantum system, i.e., one to which Schrödinger’s equation applies. What is going on in a real measurement process using quantum mechanical apparatus (no other kind is currently available) can then be understood by applying the fundamental probabilistic laws of quantum mechanics to the measured system and apparatus, regarded as a single quantum system. The discussion in Sec. IV attempts to communicate the essential ideas while omitting the technical machinery that is available elsewhere.5

In addition to the rules, students need simple examples which illustrate in physical terms what the formalism is all about. Section V is a brief introduction to what I call “toy models,” with application to a decaying nucleus and the subsequent detection of an alpha particle. This shows how quantum theory can be applied in principle to analyze real measurements without treating “measurement” as an axiom, and without using wave function collapse. The topic of measurements continues in Sec. VI where it is explained how and why one can interpret a Stern-Gerlach measurement as revealing a value of spin angular momentum before the particle was measured, and why the usual textbook approach, though not wrong, is seriously misleading.

All well and good, but can one teach this new understanding to students who are not as bright as Feynman? In Sec. VII I discuss my own experience, along with a few practical problems involved in introducing the new approach into the curriculum. The more difficult question of whether doing so is worthwhile is taken up in the concluding Sec. VIII.
II What Is Wrong With Measurements

The basic difficulty Feynman and everyone else has had understanding quantum theory comes from the need to introduce probabilities into the theory in a consistent way. It is well known that Einstein was opposed on aesthetic and philosophical grounds to a theory that was random at the fundamental level: God does not throw dice. However, his search for a deterministic quantum mechanics ended in failure, and at present the prospects of finding such a theory do not look hopeful. Consider the decay of radioactive nuclei, such as carbon 14. So far as we know at present, this is a purely random process: there is nothing inside a particular carbon 14 nucleus which determines whether it will decay 10 minutes from now, or in 10 years or in 10,000 years. No experiment has been able to separate any species of nucleus of this sort into a batch that will decay quickly and one that will take longer. The simplest explanation is that there is nothing in the nucleus before it decays, no “hidden variable,” that determines when it will decay. Attempts to introduce hidden variables into quantum mechanics lead both to a more complicated theory, and as shown by Bell, to mysterious long-range influences for which there is not the slightest experimental evidence. Thus it seems that most contemporary physicists have abandoned Einstein’s hope for a deterministic theory and accept the need to understand quantum mechanics as something intrinsically probabilistic or stochastic, as first proposed by Born in 1926, shortly after Schrödinger published his famous (time-dependent) equation.

But how to introduce probabilities into quantum theory? The textbook approach employs measurements, and if the textbook has been carefully written these probabilities refer to measurement outcomes, traditionally called “pointer positions” in the quantum foundations literature, and not to the microscopic events the apparatus was designed to measure. There is a very good reason for making this distinction. The naive assumption that every conceivable measurement outcome corresponds to a microscopic property leads to many paradoxes. By not talking about what is really being measured and confining the discussion to the macroscopic world, where classical physics applies to a good approximation, the paradoxes are avoided, and one has a consistent way of handling experimental results stored in macroscopic form in photographs or on magnetic disks. This “black box” approach, in which quantum wave functions and density operators are nothing but mathematical tools summarizing macroscopic preparation procedures, and used to calculate probabilities for the outcomes of macroscopic measurements, has much to recommend it in terms of overall consistency.

The trouble with the black box approach is that it provides no physical intuition about what is going on at the atomic level. Hence the physicist who wants to understand what the world is all about is no more likely to heed warnings against opening the box than are the children in one of Grimm’s fairy tales. While his chances of not getting into trouble are somewhat better than theirs, he still faces a significant probability of being eaten by the alligators inhabiting the vast swamp of inconsistent ideas and paradoxes lying inside the box, or, to change the metaphor, just beneath the surface of measurement-based interpretations of quantum mechanics. We know they are there from many decades of research in quantum foundations. If teachers and textbooks cannot bring themselves to be as frank as Feynman, they should at least consider posting warning signs! But it would be even better to get rid of the alligators by draining the swamp: by introducing probabilities for microscopic events in a fully consistent way, accompanied by an appropriate physical interpretation.
Yet more confusion is created by treatises that interpret “measurement” to mean a projective measurement of the sort introduced by von Neumann, in which a measurement is supposed to “collapse” or “reduce” the wave function of the measured system into that eigenstate of the measured (microscopic) physical variable that corresponds to the apparatus pointer position. Most real measurements on microscopic systems are not of this sort. Far more common are situations in which the measured system is destroyed in the process of measurement (e.g., a photon is absorbed), or its properties seriously altered, and the experimentalist interprets the outcome in terms of properties the measured system had before the measurement took place; e.g., the energy of an alpha particle before it entered and stopped inside a detector. Textbook quantum theory thus fails to provide the tools needed to interpret real experiments in quantum mechanical terms. In addition, wave function collapse is a concept which itself gives rise to needless conceptual headaches. It is not actually needed in quantum theory, since its real function is that of a tool for calculating conditional probabilities, and this can be done just as well by other methods which are conceptually clearer and less likely to mislead; see the example in Sec. VI.

Rather than treating probabilities as peculiar things somehow associated with measurements, it is much better to consider them part of the fundamental laws of nature which apply to all quantum phenomena, including measurements as particular cases. Why suppose that radioactive nuclei only decay when they are being measured? In practice, physicists do not assume that. Instead, we calculate decay rates of carbon 14 without asking whether the nuclei are being measured, or the decay rates of uranium 235 at an epoch when there were no human beings around to do the measurements, or aluminum 26 in outer space, where the need to introduce measurements seems even more ludicrous. Probabilities can, indeed, be introduced as fundamental laws. But before explaining how to do it, we need to address a central conceptual issue in quantum theory, which when left unattended leads to all sorts of problems.

III Quantum Incompatibility

In classical statistical mechanics probabilities are assigned to regions in the classical phase space. For a quantum system the analog is the quantum Hilbert space. However, there is an important differences between the two which needs to be taken into account in a consistent theory of quantum probabilities. This is illustrated in Fig. 1 where (a) shows the phase space for a one-dimensional harmonic oscillator, and (b)—in schematic form, for we have replaced a complex space with a real space—the two-dimensional Hilbert space for a spin-half particle.

A physical property of a classical particle corresponds to a region in the phase space where this property is true. For example, the region inside the ellipse in Fig. 1(a) corresponds to the property that the total energy $E$ is less than $E_0$, while the lower half plane represents the property that the momentum $p$ is negative. Classical properties combined with the logical connective AND correspond to the intersection of the corresponding regions, as in a Venn diagram: $E < E_0$ AND $p < 0$ is represented by the shaded region inside the ellipse. In some cases combining two properties in this way yields a property which is always false, e.g., $E < E_0$ AND $E > 2E_0$ corresponds to the empty set. However, it is still meaningful, and the negation of a false property is a true property. Negation corresponds to the set-theoretic
Figure 1: (a) Classical phase space for harmonic oscillator. (b) Two-dimensional quantum Hilbert space

complement; thus NOT \( E < E_0 \) is the same as \( E \geq E_0 \), the region outside the ellipse in the figure.

Following von Neumann\footnote{24}, we represent a quantum property by a ray or one-dimensional subspace of the Hilbert space, i.e., the collection of all kets of the form \( \{c|\psi\rangle\} \) where \( |\psi\rangle \) is fixed and \( c \) is any complex number. Examples are shown in Fig. 1(b). More generally, a quantum property corresponds to a subspace\footnote{25} of the Hilbert space; e.g., think of a two-dimensional plane passing through the origin of a three-dimensional space. The negation of a quantum property—again we follow von Neumann—is not the set-theoretic complement of this subspace, but instead its orthogonal complement, the subspace of kets that are orthogonal to (have zero inner product with) all kets in the original subspace. Thus in Fig. 1(b) the negation of the property corresponding to \( |\psi\rangle \) is represented by the ray \( |\tilde{\psi}\rangle \) perpendicular to the \( |\psi\rangle \) ray. So far as I know, all physicists accept von Neumann’s definition of negation, which makes good physical sense. For example, in the case of a quantum harmonic oscillator, \( E < E_0 \) is naturally associated with the subspace spanned by linear combinations of energy eigenstates \( \{|n\rangle\} \) which have \( (n + \frac{1}{2})\hbar \omega < E_0 \), and its negation to the (infinite-dimensional) subspace spanned by those with \( (n + \frac{1}{2})\hbar \omega \geq E_0 \). In the case of a spin-half particle the negation of \( S_z = +\frac{1}{2} \) (in units of \( \hbar \)) is the ray corresponding to \( S_z = -\frac{1}{2} \). Since in ordinary logic either a statement or its negation is true, we conclude that in the case of a spin-half particle either \( S_z = +1/2 \) or \( S_z = -1/2 \), in agreement with the experimental result of Stern and Gerlach\footnote{26} when one interprets their experiment as they themselves did (and which, as we shall see in Sec. VI is fully justified by modern quantum mechanics), as indicating the property that the particle (in their case a silver atom) had before the measurement took place.

Note, however, a striking contrast between classical and quantum properties, Fig. 1(a) and (b). In the classical case a property and its negation fill up the entire phase space, whereas for the quantum case the rays corresponding to \( |\psi\rangle \) and its negation \( |\tilde{\psi}\rangle \) do not even begin to fill up the Hilbert space. There are plenty of other rays, such as the one associated with \( |\chi\rangle \), a ket which is neither a multiple of \( |\psi\rangle \) nor orthogonal to it. What can we say about them? In some sense this is the central conceptual difficulty of quantum mechanics, and no physical interpretation that fails to deal with it—at least no scheme based on the quantum Hilbert space along with von Neumann’s notion of negation—can hope to succeed.
Von Neumann himself was quite aware of the problem, and he and Birkhoff\textsuperscript{27} in a paper that is at least as important for the field of quantum foundations as the better known one by Einstein, Podolsky and Rosen\textsuperscript{28} proposed a solution requiring a radical modification of propositional logic. Alas, we physicists have not been able to make much use of it for understanding quantum mechanics. Perhaps we just are not bright enough, and someday robots will use it to make sense of the quantum world. But in the meantime we can make considerable progress using something much less radical.

The histories approach handles this difficulty through the concept of \textit{quantum incompatibility}, as per the following illustration. Whereas both \textit{\textquotedblright}\textit{S}$_x$ = $+1/2$\textit{\textquotedblright} and \textit{\textquotedblright}\textit{S}$_z$ = $-1/2$\textit{\textquotedblright} are meaningful statements about a spin-half particle at a particular instant of time, the logical combination \textit{\textquotedblright}\textit{S}$_x$ = $+1/2$ \textbf{AND} \textit{S}$_z$ = $-1/2$\textit{\textquotedblright} is \textit{meaningless} in the precise sense that Hilbert-space quantum mechanics can assign it no meaning. All the rays in the two-dimensional Hilbert space—at this point we need to think of the complex analog of Fig. 1(b) (points on the Bloch sphere for the reader familiar with that concept)—already have a physical interpretation, namely that the component of spin angular momentum in a particular direction in space, call it $w$, is $+1/2$, and there is none left over which could plausibly represent \textit{\textquotedblright}\textit{S}$_x$ = $+1/2$ \textbf{AND} \textit{S}$_z$ = $-1/2$\textit{\textquotedblright}.” Could this be a statement which is always false, like the classical $E < E_0$ \textbf{AND} $E > 2E_0$ considered earlier? The trouble is that the negation of a meaningful statement which is always false is one that is always true, such as $E \geq E_0$ \textbf{OR} $E \leq 2E_0$ for a classical oscillator. However, the negation of \textit{\textquotedblright}\textit{S}$_x$ = $+1/2$ \textbf{AND} \textit{S}$_z$ = $-1/2$\textit{\textquotedblright},” which is \textit{\textquotedblright}\textit{S}$_x$ = $-1/2$ \textbf{OR} \textit{S}$_z$ = $+1/2$\textit{\textquotedblright},” does not look like a good candidate for a statement that is always true, and in fact pursuing this route quickly leads to contradictory results if one employs the rules of standard logic.\textsuperscript{29} On the other hand, the negation of a meaningless statement is equally meaningless, so there is no problem as long as we agree that joining \textit{\textquotedblright}\textit{S}$_x$ = $-1/2$\textit{\textquotedblright} and \textit{\textquotedblright}\textit{S}$_z$ = $+1/2$\textit{\textquotedblright} with OR is no more sensible than joining them with AND.

Compatibility and incompatibility for larger quantum systems are most conveniently discussed by considering the projectors (orthogonal projection operators) onto the subspaces corresponding to the different properties. If $P$ and $Q$ are projectors representing two subspaces, or two properties denoted by the same letters, they are compatible if and only if $PQ = QP$, in which case $PQ$ is itself a projector onto the subspace corresponding to “$P$ \textbf{AND} $Q$.” Otherwise, they are incompatible. In textbooks the term “incompatible” is employed in a similar way, but with reference to observables (physical variables represented by self-adjoint operators), and one is told that they cannot be simultaneously measured. Making reference to properties (projectors or subspaces) is both technically and conceptually simpler than referring to observables. When they are incompatible they indeed cannot be simultaneously measured, because what is meaningless cannot be measured.

Since in the classical world everything commutes, there is no exact analog of quantum incompatibility to be found in our everyday experience. However, the following analogies may help tease out some of what it does and does not mean. A photographer taking pictures of Mount Shasta can do so from a variety of different directions or perspectives: north, south, east, etc. The perspective is chosen by the photographer and has no effect on the reality represented by the mountain. The chosen perspective makes it possible to answer certain questions but not others on the basis of the resulting photograph: a view from the south will not indicate what is happening on the northern slopes. Next, replace the photographer with a classical physicist who has designed an apparatus to measure the $w$ component of angular momentum of a golf ball by an apparatus consisting of a cage initially at rest and
pivoted on low friction bearings which allow it to rotate around an axis in the $w$ direction. If it can be arranged that the moving golf ball flies into and is trapped in the center of the cage, the final rate of rotation of the cage can be converted into a value for the angular momentum of the golf ball just before it entered the cage, i.e., just before the measurement was made. The choice of orientation $w$ is made by the physicist, and this choice has no effect upon the properties of the golf ball prior to the measurement, though it does determine what he can say about those properties after the measurement is over. Finally, replace the classical physicist with a quantum physicist who measures $S_w$ for a spin-half particle using a Stern-Gerlach apparatus with the field gradient in the direction $w$. The choice of $w$ is made by the physicist and has no effect upon the properties of the spin-half particle before it is measured, a point to which we will return in Sec. VII. It does, however, determine what can be said about the earlier state of the particle on the basis of the measurement outcome.

How does the last situation differ from the first two? A photographer could arrange to have a colleague take a picture of Mount Shasta as viewed from the north at the same time as he takes one from the south, and together the photographs would provide more information than either one by itself, since the two perspectives are compatible with each other. The classical physicist could in principle make high speed photographs of the golf ball from which he could deduce the axis and rate of rotation, and thereby all components of its angular momentum, since these are compatible parts of a complete description of a macroscopic spinning body. But no corresponding possibility is available to the quantum physicist: the different components of angular momentum of a spin-half particle are incompatible, and since trying to combine one component with another yields a meaningless result, no measurement could possibly determine the two values simultaneously. And saying, “I measured $S_x = -1/2$ in this case; what would have been the result had I decided instead to measure $S_z$?” is to pose a tricky counterfactual question which easily leads to misunderstanding.30

IV Quantum Time Dependence

If Schrödinger’s (time-dependent) equation is deterministic, how is it possible to introduce in a fundamental way a stochastic or probabilistic time development in quantum theory? Born’s simple but ingenious idea19 was to use Schrödinger’s equation to calculate probabilities. The following analogy may be helpful. Classical Brownian motion of a particle modeled by a Wiener process is random: the future behavior of the particle is not determined by its present position or its past behavior. Nonetheless the probability distribution density $\rho(r,t)$ for its position as a function of time $t$ satisfied the deterministic diffusion equation

$$\frac{\partial \rho}{\partial t} = D\nabla^2 \rho. \tag{1}$$

Why cannot one think of Schrödinger’s equation in a similar way, as a deterministic equation that generates probabilities?

One can, and in fact current textbooks do use Schrödinger’s equation for this purpose, but in a half-hearted and somewhat inconsistent way. Following a tradition that goes back at least to von Neumann31 the time evolution of a quantum system is thought of as involving two distinct steps. First one solves Schrödinger’s equation to obtain a deterministic unitary time development of the wave function, which tends to be thought of intuitively as representing the “real” physical state of the microscopic quantum system. Then the system of interest
interacts with an external measuring apparatus, resulting in a random process that leads to a situation in which the measurement outcome, the only thing to which a probability can properly be applied, is somehow associated with the state of the particle after the measurement has been completed. The unsatisfactory nature of this approach using wave function collapse has already been discussed in Sec. II.

Probabilities can be introduced in a more consistent and natural way by following the route used in ordinary probability theory. There the first step is to introduce a sample space of mutually-exclusive events, one and only one of which occurs in any particular experiment. For example, if one rolls a die, the number of spots on the top face when it comes to rest will be a number between 1 and 6; if 5 occurs, 3 does not occur, etc. The quantum counterpart is a set of mutually-orthogonal subspaces of the Hilbert space whose projectors (orthogonal projection operators) form a decomposition of the identity: a collection \( \{P_j\} \) satisfying

\[
I = \sum_j P_j, \quad P_j P_k = \delta_{jk} P_j.
\]  

(2)

Note that \( P_j P_k = P_k P_j \), so the properties are compatible; otherwise it would not make sense to speak of one of them occurring rather than another; see the discussion in Sec. III. The fact that \( P_j P_k = 0 \) for \( j \neq k \) corresponds to the properties being mutually exclusive: if one occurs the other does not. That the projectors sum to the identity means that one of them will necessarily occur, or be true, at the time in question. An orthonormal basis \( \{|\phi^j\rangle\}; j = 1, 2 \ldots, \) in a finite-dimensional Hilbert space gives rise to a decomposition of the identity with \( P_j = |\phi^j\rangle\langle\phi^j| \). Note that real dice are quantum objects made up of atoms, hence describable (in principle) using a large Hilbert space, and any visibly distinct states, such as those with different numbers of spots on the top face, will correspond to mutually-orthogonal projectors. Thus (2) works for both microscopic and macroscopic systems, as one would expect, since the basic principles of quantum mechanics apply to systems of any size.

A classical probabilistic description of a random (stochastic) process also uses a sample space. In the case of a coin flipped three times it consists of the 8 mutually exclusive possibilities, here called histories, HHH, HHT, HTH, ..., TTT, where H stands for “heads” and T for “tails.” (Note that two histories are distinct elements of the sample space if they differ at any of the three times.) In the same way, in quantum mechanics histories are sequences of quantum events at a succession of times, each represented by a subspace (or its projector) of the quantum Hilbert space. (For technical reasons it is convenient to represent histories as projectors on tensor products of copies of the system’s Hilbert space.) The behavior of a real coin made up of atoms can be described in quantum terms using a suitable (large) Hilbert space, so the 8 mutually exclusive possibilities of flipping it three times in a row also form a quantum sample space or family of histories.

Sample spaces are needed to make probabilistic reasoning precise, and while the sloppy physicist’s approach that ignores this is adequate for many purposes, in quantum mechanics it leads to confusion. The first step in clearing up the conceptual difficulties which have bothered Feynman and everyone else is to introduce well-defined sample spaces for probabilities. The second step is to insist that incompatible sample spaces not be combined, for the combination will not make sense. In the histories approach this is done by a strict application of what is called the single framework rule which asserts in essence that quantum
probabilistic reasoning must be carried out using a single sample space. Given two compatible quantum sample spaces this single space is easily constructed from them by a process of refinement, whereas if they are incompatible the refinement does not exist. Combining incompatible quantum sample spaces in a way contrary to the single framework rule is at the heart of most quantum paradoxes, and identifying the point at which this happens is the key step in resolving (or, as I prefer to say, taming) such a paradox.

Once a sample space or family of histories has been defined, the next task is assigning probabilities. For present purposes it suffices to consider a finite sample space, so the probabilities are a collection of nonnegative numbers, one for each history in the space, that sum to 1. Probability theory as such contains no rules for assigning these probabilities. In quantum theory Schrödinger’s equation can be used to assign probabilities to certain families of histories in a closed or isolated quantum system (no interaction with something outside the system), the situation in which Schrödinger’s equation applies. The simplest case involves only two times $t_0$ and $t_1$, a single state $|\psi_0\rangle$ at time $t_0$, and an orthonormal basis $\{|\phi_j\rangle\}$, $j = 1, 2, \ldots$, at $t_1$. If the Hamiltonian $H$ is independent of time the time evolution operator obtained by integrating Schrödinger’s equation is

$$T(t',t) = e^{-i(t'-t)H/\hbar},$$

and the Born rule then gives

$$\Pr(\phi_j | \psi_0) = |\langle \phi_j | T(t_1,t_0) | \psi_0 \rangle|^2$$

as the conditional probability of $|\phi_j\rangle$ at time $t_1$ given $|\psi_0\rangle$ at $t_0$. The fairly obvious generalization (see (2))

$$\Pr(P_j | \psi_0) = \langle \psi_0 | T(t_0,t_1) P_j T(t_1,t_0) | \psi_0 \rangle$$

of (3) is also referred to as the Born rule. (Formulas (4) and (5) apply if the Hamiltonian depends on time, but then (3) no longer gives the relationship between $T$ and $H$.)

Unlike those in quantum textbooks, the probabilities in (4) and (5) do not refer to outcomes of some external measurement, but to physical states inside the closed system described by the Hamiltonian used in (3). Born’s rule is a fundamental law of nature, on the same footing with Schrödinger’s equation and equally important. If one is interested in how a real measuring apparatus will interact with a quantum system, one should include the apparatus itself as part of the overall quantum system and then apply (4) or (5) to the combination. Examples are discussed in Secs. V and VI below. It is worth noting that $t_0$ may either precede $t_1$ or follow $t_1$. The fundamental law for quantum probabilities, and its extensions (see below), does not single out a sense of time. This important symmetry is entirely lost sight of in the measurement-based approach to quantum theory, since measurements are inherently irreversible (in the thermodynamic sense).

The right side of (4) is often written as $|\langle \phi_j | \hat{\psi}_1 \rangle|^2$, where

$$|\hat{\psi}_1\rangle = T(t_1,t_0) |\psi_0\rangle$$

is obtained from $|\psi_0\rangle$ by integrating Schrödinger’s equation from $t_0$ to $t_1$. When used in this way $|\hat{\psi}_1\rangle$, which is typically incompatible with the basis states $\{|\phi_j\rangle\}$, does not represent the physical reality of the quantum system at time $t_1$. It is instead a mathematical construct, a pre-probability used for computing probabilities. One could equally well compute them
by starting with each of the states $|\phi_j^1\rangle$ and integrating Schrödinger’s equation in the reverse direction from $t_1$ to $t_0$, making no reference whatsoever to $|\hat{\psi}_1\rangle$. For further discussion, see Sec. 9.4 of Ref. [14].

Indeed, $|\hat{\psi}_1\rangle$ could be the infamous Schrödinger cat state [37]. To discuss whether the cat is dead or alive, one should use a framework, that is to say an orthonormal basis (or, to be more practical, a decomposition of the identity) for which such concepts make sense, and then compute probabilities. Since $|\hat{\psi}_1\rangle$ is a computational tool, it requires no physical interpretation, and within the context of this framework, it cannot be given a physical interpretation, for it is incompatible with the sample space used to describe whether the cat is still alive. To be sure, one could instead adopt a different, incompatible framework or orthonormal basis that includes $|\hat{\psi}_1\rangle$ as one of its elements, in which case Born’s formula will tell us that it occurs with (conditional) probability 1. In this second framework it makes no sense to ask whether the cat is dead or alive, since the corresponding quantum properties are incompatible with $|\hat{\psi}_1\rangle$. In quantum mechanics, as in the case of Mount Shasta, certain perspectives are useful for answering certain questions, and are not useful for answering other questions. The trouble with most treatments of Schrödinger’s cat is that they attempt to discuss its morbidity while assuming that $|\hat{\psi}_1\rangle$ is its physical state, which makes no more sense than talking about $S_z$ for a spin-half particle whose $x$ component of angular momentum is $+1/2$.

For a complete stochastic description of time development of a closed quantum system it is necessary to go beyond the Born rule and provide formulas for calculating probabilities of histories involving three or more times. This extension is not trivial, as consistent probabilities can only be assigned if certain consistency conditions are satisfied. Discussing them here would lead to a somewhat lengthy detour from our main theme, and as they are treated in detail elsewhere [38], we shall move on to describe how consistent probability assignments within the context of simple models can help dissipate quantum mysteries.

V Toy Models

A major difficulty in teaching quantum mechanics is that solving the time-dependent Schrödinger equation is at best a time-consuming process, and often cannot be done in closed form. This makes it difficult for students to gain an intuitive understanding of what it involves. The advent of computer simulations with graphical output [39] is thus a welcome addition to the repertoire of teaching tools. However, these need to be supplemented by an alternative approach using toy models, which, while somewhat unrealistic, have the virtue that they can be worked out using a pencil on the back of the traditional envelope [40].

The basic idea is to discretize time so that it advances in integer steps, and the time development operator in (3) takes the form of an integer power

$$T(t', t) = T^{t' - t}$$

(7)
of some very simple unitary operator $T$, typically one representing a hopping motion of one or more particles. For example, $T = S$, where

$$S|m\rangle = |m + 1\rangle, \quad S|\bar{M}\rangle = |\bar{M}\rangle$$

(8)
is a shift operator moving a particle from a lattice site or node or “box” at site $m$, where $m$ is an integer, to the next site. The periodic boundary condition in (8) ensures that $S$
is a unitary operator on the finite-dimensional Hilbert space with orthonormal basis \(|m\rangle\), \(-M \leq m \leq M\), where \(M\) can be as large as one wants; typically much larger than the times of interest. Figure 2 shows a modification in which (8) holds except for \(m = 0\) and \(-1\), for which
\[
S|0\rangle = \alpha|0\rangle + \beta|1\rangle, \quad S|-1\rangle = -\beta^*|0\rangle + \alpha^*|1\rangle,
\]
with \(|\alpha|^2 + |\beta|^2 = 1\). One can think of this as a simple model of a decaying system: an alpha particle initially inside a nucleus at \(|m = 0\rangle\) eventually escapes to \(m = 1\) and then keeps moving. The unitary time development of an initial state \(|\psi_0\rangle = |0\rangle\) at \(t = 0\) leads to
\[
|\psi_t\rangle = T^t|\psi_0\rangle = \alpha^t|0\rangle + \beta \left[ \alpha^{t-1}|1\rangle + \alpha^{t-2}|2\rangle + \cdots + |t\rangle \right]
\]
for \(0 < t < M\). Born’s rule gives \(|\alpha|^{2t}\) for the probability that the initial state has not yet decayed. This decreases exponentially with \(t\), as one might expect.

A way to make this model a useful tool for dissipating quantum mysteries is to add a toy detector, thought of as the pointer on a toy measuring apparatus, with states labeled \(n\) in Fig. 2. Let
\[
S'|n\rangle = |n + 1\rangle, \quad \text{except} \ S'|0\rangle = |0\rangle \quad \text{and} \quad S'|-1\rangle = |1\rangle,
\]
be the corresponding shift operator, and again assume a periodic boundary condition \(S'|N\rangle = |-N\rangle\). The total time development operator on the tensor product of the particle and pointer Hilbert spaces is
\[
T = (S \otimes I)R(I \otimes S'),
\]
where \(R(|m\rangle \otimes |n\rangle) = |m\rangle \otimes |n\rangle\) is the identity \(I \otimes I\) except for
\[
R(|2\rangle \otimes |0\rangle) = |2\rangle \otimes |1\rangle, \quad R(|2\rangle \otimes |1\rangle) = |2\rangle \otimes |0\rangle.
\]
If the detector pointer is initially at \(n = 0\) in its “ready” state and the particle arrives at \(m = 2\), the effect of \(T\) is to kick the pointer to \(n = 1\), after which it continues moving. At the same time the particle continues on to \(m = 3\), as it would have done in the absence of the detector.

Unitary time development of an initial state \(|\Psi_0\rangle = |m = 0\rangle \otimes |n = 0\rangle\) to a time \(t \geq 3\) results in
\[
|\Psi_t\rangle = T^t|\Psi_0\rangle = \left[ \alpha^t|0\rangle + \beta \alpha^{t-1}|1\rangle + \beta \alpha^{t-2}|2\rangle \right] \otimes |0\rangle \\
+ \beta \left[ \alpha^{t-3}|3\rangle \otimes |1\rangle + \alpha^{t-4}|4\rangle \otimes |2\rangle + \cdots + |t\rangle \otimes |t - 2\rangle \right].
\]
Notice that in this expression the detector is in a superposition of different pointer positions, so we have the toy analog of a Schrödinger cat—a Schrödinger kitten? A useful physical interpretation is obtained by using the Born rule (14) at time \( t_1 = t \), with the orthonormal basis \( \{ |m \rangle \otimes |n \rangle \} \), i.e., both particle and pointer are at definite locations. If we think of \( |\Psi_t \rangle \) as a pre-probability, the analog of \( |\hat{\psi}_1 \rangle \) in (14), then the probability that the particle is at \( m \) and the pointer at \( n \) at time \( t \) is just the absolute square of the corresponding coefficient on the right side of (14). This joint probability distribution \( \text{Pr}(m, n) \) has exactly the same properties and the same physical interpretation as in ordinary probability theory. In particular, we can use it to compute the conditional probabilities \( \text{Pr}(m | n) \), and from them deduce that if the pointer is at \( n = 0 \), then \( m \leq 2 \), i.e., the alpha particle is still in the nucleus or on its way to the detector; whereas if the pointer is at some \( n > 0 \), the particle is at the location \( m = n + 2 \), as one would expect if the particle triggered the detector while hopping from \( m = 2 \) to \( 3 \). Quantum mechanics does not say which of these mutually exclusive and physically reasonable possibilities is actually the case, but only provides probabilities.

This simple example, in which the measuring device is part of the total quantum system, is useful in countering a number of misleading ideas that students unfortunately pick up while taking elementary (and more advanced) quantum courses: that particles (and pointers) can be in two places at the same time, that quantum mechanics necessarily leaves everything in a fog, that there are magical long-range influences, etc. Note in particular how wave function collapse is not needed when probabilities are introduced in a consistent way into quantum theory. Removing wave function collapse from textbooks and replacing it with conditional probabilities would be a significant step towards improving students’ understanding of quantum mechanics.

The preceding discussion might tempt one to conclude that if at \( t = 5 \) the pointer is at \( n = 1 \), then at \( t = 2 \) the particle was at \( m = 1 \). This conclusion is correct, but cannot be justified on the basis of the Born rule alone, as it involves probabilistic reasoning applied to a closed quantum system at 3 different times: the initial state at \( t = 0 \), the pointer position at \( t = 5 \), and the particle position at \( t = 3 \). One must use an appropriate extension of the Born rule and check for consistency.41 We will give another example in the next section of how measurement outcomes can be used to infer properties of a measured system before the measurement took place.

VI Measurements Reconsidered

Measurement apparatus is essential for experiments exploring the quantum properties of microscopic systems, for it amplifies very small effects and makes them visible or audible or otherwise evident in macroscopic effects accessible to human beings. Thus it is very important to understand how the apparatus works, and how its macroscopic output is related to the microscopic input. Does the process introduce noise, and if so how much? Is the output influenced by extraneous effects? These questions can be studied to some extent by carrying out experimental tests. But an important theoretical component goes into such analyses, and in this respect measurement-based quantum mechanics as found in the textbooks is inadequate. It is hard to analyze real measurements when the very concept of a measurement is considered as axiomatic, and thus unanalyzable in quantum terms. Introducing probabilities in a consistent way makes it possible, in principle, to analyze real
apparatus in a completely quantum mechanical way. Future experimentalists, and theorists who give them advice, need to know that a consistent approach to these questions exists, that it does not depend upon dubious ideas like wave function collapse, and that it supports many of the general intuitions which experimental physicists have about measuring apparatus, such as the fact that if there is a collimator between source and particle detector, then on its way to the detector the particle has to pass through the hole in the collimator. At the same time it places limits on that intuition, and indicates places at which it will break down and caution needs to be observed. This article is not the place to go into details, but the most essential ideas can be explained in terms of a simple example, a somewhat idealized and modernized version of the famous Stern-Gerlach measurement. This will show how the measurement-based approach of textbooks can be unhelpful and misleading even when it is in some respects correct, and how to replace it with something more useful.

Figure 3: Stern-Gerlach apparatus separating particles into $S_z = \pm 1/2$ beams, which are then detected.

Figure 3 shows the well-known schematic diagram: a stream of spin-half particles enter on the left and are separated into two outgoing beams: the upper one corresponding to $S_z = +1/2$ and the lower to $S_z = -1/2$, for a magnetic field gradient in the z direction. That is, if at time $t_1$ just before entering the apparatus the spin state is $S_z = +1/2$, the particle will emerge in the upper beam, and can be detected by the upper detector. Similarly, if $S_z = -1/2$ the particle will emerge in the lower beam and be detected there. We suppose that the magnetic field is negligible at and to the left of $t_1$ in Fig. 3.

What will happen if the particle is prepared, via some previous apparatus, so that it is in a state with $S_x = +1/2$ at a time $t_0 < t_1$? Since $S_x = +1/2$ is a linear superposition of the $S_z = +1/2$ and $S_z = -1/2$ states with equal amplitude, the standard (correct) answer is that it will be detected with probability 1/2 in the upper and probability 1/2 in the lower beam. Suppose it has been detected by the upper detector, as indicated by a pointer on that device, at time $t_3$. Was the particle in the upper beam at time $t_2$, after leaving the field gradient but before detection? Experimental physicists will tend to answer that it was, for otherwise they will have difficulty designing equipment, thinking about errors, etc. Theoretical physicists trained in the usual textbook approach may disagree, for they think of the original spin superposition as developing unitarily into a superposition of two wave packets, one traveling upwards and one downwards after the atom leaves the field gradient. (Let us assume the vacuum is good enough that decoherence from collisions does not complicate matters.) And what can one say about the spin state of a particle at the earlier time $t_1$ if it is later detected by the upper detector?

All of these questions have reasonable answers if one abandons the measurement approach and instead introduces microscopic quantum probabilities on appropriate sample spaces, that
is, consistent families of quantum histories. The key issue is the choice of sample space, for in a situation of this sort there are several incompatible alternatives. We will consider various possibilities, always assuming as given data an initial $S_x = +1/2$ spin state and detectors in the ready state at time $t_0$, and that at time $t_3$ it is the upper detector that has been triggered by the arrival of the particle. Note that the detectors are here thought of as part of a large closed quantum system that also includes the particle.

A first consistent family $\mathcal{F}_a$ can be represented, using the notation employed in Ref. 14, in the form

$$\mathcal{F}_a : \quad x^+ \odot I \odot \left\{ \begin{array}{l}
u \odot U \\
l \odot L \end{array} \right\}$$

(15)

Here each letter represents a projector in a history associated with the four successive times $t_0 < t_1 < t_2 < t_3$ indicated on the lower line, and the $\odot$ symbols can for present purposes be thought of as commas separating the projectors at successive times. In particular, $x^+$ at the time $t_0$ means $S_x = +1/2$, the identity $I$ at $t_1$ indicates that no information is being provided about the state of the particle at this time (in contrast to (18) and (20) below), $u$ and $l$ at $t_2$ signify that the particle is in the upper and lower path, respectively, while $U$ and $L$ are projectors corresponding to the upper and lower detector, respectively, having detected the particle at $t_3$. One can think of (15) as a shorthand for two histories, $x^+ \otimes I \otimes u \otimes U$ and $x^+ \otimes I \otimes l \otimes L$, with the curly brace indicating that they are identical up to the time $t_1$. The extended Born rule assigns a probability of 1/2 to each of the two histories in (15). The conditional probabilities

$$\Pr(u_2 \mid U_3) = 1, \quad \Pr(l_2 \mid U_3) = 0,$$

(16)

where the subscripts refer to times $t_2$ and $t_3$, follow at once from the fact that there is only one history in $\mathcal{F}_a$ for which the upper detector triggers, and in that history $U$ is preceded by $u$, not $l$. What (16) tells us is that if the upper detector triggers, one can be certain that at the earlier time $t_2$ the particle was following the upper and not the lower path. So the experimentalist is right.

But there is also a second consistent family

$$\mathcal{F}_b : \quad x^+ \odot I \odot c \odot \left\{ \begin{array}{l}U \\
L \end{array} \right\}$$

(17)

where the times are the same as in (15). Here $c$ at $t_2$ is a projector onto the coherent superposition of states that evolve from the initial state with $S_x = +1/2$, and since it is found in both histories, it occurs with probability 1, just as the theoretician supposed. Since both $\mathcal{F}_a$ and $\mathcal{F}_b$ are consistent families, the conclusions of a probabilistic analysis applied using just one of them while disregarding the other will be correct. However, the families are incompatible, and so these conclusions cannot be combined. One cannot say that at time $t_2$ the particle is both in a superposition state $c$ AND that it is moving on the upper trajectory $u$, for that would be meaningless in the same way that “$S_x = +1/2$ AND $S_z = +1/2$” makes no sense. Note that incompatibility, the fact that the two families cannot be combined, does not mean that one is “wrong” and the other is “right.” Seeking some law of nature which “chooses” one rather than the other is to misunderstand the nature of quantum descriptions. It is the physicist who chooses which description to use, depending upon the sort of question.
he is asking, while noting that only descriptions compatible with the desired information will be useful for this purpose. Remember Mount Shasta.

Thus far we have said nothing about the spin state of the particle at the time \( t_1 \) when it is just about to enter the field gradient; both \( \mathcal{F}_a \) and \( \mathcal{F}_b \) contain a noncommittal \( I \) at \( t_1 \). It is again useful to consider two different consistent families. In

\[
\mathcal{F}_c : \quad x^+ \circ \left\{ z^+ \circ u \circ U \atop z^- \circ l \circ L \right\}
\]

(18)

one can talk about \( S_z \) at \( t_1 \): the projectors \( z^+ \) and \( z^- \) correspond to \( S_z = \pm 1/2 \). Once again there is only one history that terminates in \( U \), and therefore

\[
\Pr(z_1^+ | U_3) = 1, \quad \Pr(z_1^- | U_3) = 0.
\]

(19)

That is, one can be sure that if the upper detector detected the particle, \( S_z \) had the value \(+1/2\), not \(-1/2\), at the earlier time \( t_1 \). This is what one would expect if the total apparatus, which consists of field gradient followed by detectors, functions as designed, as a device to measure the \( z \) component of the spin of a spin-half particle.

In the second consistent family

\[
\mathcal{F}_d : \quad x^+ \circ x^+ \circ I \circ \left\{ \atop U \right\}
\]

(20)

it is \( S_x \) that makes sense at \( t_1 \), and \( S_x = +1/2 \) occurs with probability 1. This family is of no use in deciding whether the measuring apparatus is functioning properly, since that question makes reference to \( S_z \) at \( t_1 \) and not \( S_x \), but it could provide a check on whether the region traversed by the particle during the interval from \( t_0 \) to \( t_1 \) was free of magnetic fields, as we have supposed. Of course, \( \mathcal{F}_d \) is incompatible with \( \mathcal{F}_c \), so it makes no sense to combine the probability 1 inferences obtained by using them separately. (Incidentally, in \( \mathcal{F}_d \) one could replace the \( I \) at time \( t_2 \) with the pair \( u \) and \( l \), as in (18). The result would be a family \( \mathcal{F}_d' \) which would serve equally well for the matters we have been discussing. Likewise, in \( \mathcal{F}_c \) one could replace \( u \) and \( l \) at \( t_2 \) with \( I \).)

The following conceptual difficulty can arise when using the family \( \mathcal{F}_c \). How can it be that \( S_x = +1/2 \) at \( t_0 \) (as an initial datum) and \( S_x = +1/2 \) at \( t_1 \) (with probability 1) if there is no magnetic field acting on the particle during the time interval between \( t_0 \) and \( t_1 \), and thus no torque which could have caused the spin direction to precess from \(+x\) to \(+z\)? This problem arises from a misleading mental picture of a spin-half particle in the state \( S_x = +1/2 \). One tends to think of it as a little gyroscope with its axis of rotation lined up precisely along the \(+x\) axis, and if at a later time the gyroscope axis is in the \(+z\) direction, this must have come about through the application of a torque. But a gyroscope has \( y \) and \( z \) components of angular momentum equal to 0 if its axis is in the \( x \) direction, whereas for a spin-half particle these other components are undefined when \( S_x = +1/2 \). A better, less misleading image is to think of \( S_x = +1/2 \) as resembling a gyroscope with its axis in a random direction, i.e., random \( y \) and \( z \) components of angular momentum, subject only to the constraint that the \( x \) component is fixed. Then even if the gyroscope is not subject to a torque, there is no reason why its \( x \) component of angular momentum cannot be positive at one time and its \( z \) component positive at a later time. Classical images of some sort are probably essential in quantum physics, since they help us organize intuitive knowledge, and
they always mislead to some extent. But some mislead less than others, as shown by this example.

One can continue the discussion of the Stern-Gerlach experiment using additional families of histories which combine information about a spin component at $t_1$ with information about a position or superposition of positions at $t_2$, but the preceding suffices for making the main points. Families $\mathcal{F}_b$ and $\mathcal{F}_d$ correspond in a rough sense to the viewpoint of von Neumann and the typical textbook, in which unitary time development persists up until the last instant before the final measurement, meaning the amplification of a microscopic signal to a macroscopic level, takes place. Thus they show that the textbook approach makes a certain amount of sense. However, the conclusions we reached using $\mathcal{F}_b$ and $\mathcal{F}_d$ are based on the systematic use of fundamental principles of quantum dynamics applied to a closed quantum system, not on anything specific to a measurement, and standard probabilistic reasoning, not guesswork or arm waving.

On the other hand, $\mathcal{F}_a$ and $\mathcal{F}_c$ provide the sort of information needed by someone designing a quantum measuring apparatus, or analyzing how it functions. The key point is that such an analysis in quantum terms is only possible if the relevant properties of the measured system at a time before the measurement takes place are part of the quantum description. This is not so in the usual textbook approach, which is defective not in that it is wrong—as we have seen, it can be justified to some extent by using families like $\mathcal{F}_b$ and $\mathcal{F}_d$—but in that equally valid alternatives for discussing quantum time development are never mentioned, and the student is left with the incorrect idea that quantum measurements really do not measure anything, they just cause the great smoky dragon to collapse.

VII Practical Considerations

For a period of ten years I have been teaching various advanced undergraduate and beginning graduate quantum mechanics courses, and courses in quantum information, using the new perspective in which quantum mechanics is based on probabilistic laws of universal validity, with measurements being only one of the applications. The reaction of students has generally been positive, though there are always signs of shock when I tell them that by the end of the course, and provided they do their homework, they will understand (some aspects of) quantum mechanics better than Feynman did. Homework and examinations results indicate that they understand this material about as well, or as badly, as other topics in such courses, but there have been no follow-up studies to see what they have retained a year later.

How long does it take to present the new ideas? Longer than the material they replace, but not enormously so. Courses at the advanced undergraduate and beginning graduate level typically devote a certain amount of time to introducing fundamental quantum concepts; defining a quantum Hilbert space, Dirac notation, tensor products; introducing Schrödinger’s equation and a probabilistic interpretation of the formalism; and examples illustrating all of these. Before moving on to angular momentum, the hydrogen atom, scattering, and so forth. It is in the first part that changes are most needed, and my experience suggests that the revised version requires about six weeks total (of a fourteen week semester) in an introductory graduate course; perhaps one or two more than if one follows the older approach. The new material includes a proper discussion of quantum incompatibility; histories and consistency
conditions; the toy models needed to provide illustrations; and a one hour introduction to probability theory for students who have not yet had a course in that subject. Along the way the students learn how to deal with the double slit paradox, or the equivalent using a Mach-Zehnder interferometer, in a reasonable way. Resolving the Einstein-Podolsky-Rosen problem without invoking long range influences requires less than one additional class period if the foundations have been properly laid. There is no need to consider Bell’s inequality, though this can serve as a useful illustration of what goes wrong when one tries to import classical ideas into the quantum world.

What gives the students the most difficulty? Quantum incompatibility. The problems they face are analogous to those encountered when first studying relativity, only worse: habits of classical reasoning lie closer to the soul of the apprentice physicist than does the notion of temporal simultaneity. However, just as students are capable of learning that putting \(x\) to the left of \(p\) in quantum theory does not yield anything like the classical \(xp\), they can also learn its logical counterpart, especially if one starts with the simple case of spin half. Next in order of (decreasing) difficulty come consistency relations. Followed by probability theory in the case of students who have never been exposed to its formal structure, nor dealt with simple stochastic processes. Fortunately, in an introductory quantum course one can get by with finite sample spaces and finite-dimensional Hilbert spaces, with only some talk about their infinite counterparts, so the formal mathematics is not very difficult.

A different kind of conceptual barrier can be present, especially for graduate students who in previous courses taught by respected teachers have learned the measurement-based approach to quantum mechanics with wave function collapse, etc., while never becoming aware of its many shortcomings and inconsistencies. It is then hard to persuade them to pay serious attention to something which appears contrary to what they think of as quantum orthodoxy. Another objection that is raised, again primarily by graduate students, is that they are being required to learn esoteric material about quantum foundations, rather than how to do calculations that will aid them in passing exams and preparing for research. The fact that students are often hesitant to express these reservations openly to the teacher makes it harder to deal with them. When countering prejudices of this type I think it not inappropriate to point out that Feynman, who knew how to do calculations better than most of us, was quite forthright in admitting that he did not understand quantum mechanics as formulated in the traditional way, and that anecdotal evidence suggests he was impressed by the new ideas when he first heard them shortly before his untimely death.

What material is available for a course taught from the new point of view? No textbook, so far as I know, has incorporated the new ideas. There are two monographs by Omnès, of which the second is simpler. My own book is simpler still, and can be used as a supplement to a regular textbook. The most crucial chapters are available on the Internet, along with a small number of exercises.

VIII Conclusion

I have argued that the treatment of quantum probabilities found in textbooks, where they are introduced in connection with measurement outcomes, is a major source of conceptual difficulties for students trying to learn the subject. And that modern developments in our understanding of quantum mechanics make it possible to do a much better job, through
a systematic and coherent introduction of microscopic probabilities as a fundamental part of the theory. Measurements can then be understood as particular examples of quantum processes, not as something fundamentally different, and can be shown to reveal something about the measured system before the measurement took place. Wave function collapse can be assigned to the trash can of outmoded ideas, replaced by a consistent use of conditional probabilities. Furthermore, such an approach is not beyond the grasp of students, especially when explained with the aid of toy models that allow them to understand the fundamentals of quantum dynamics without becoming entangled in the technical difficulties of solving Schrödinger’s equation. As a consequence, students can now begin to understand those aspects of quantum mechanics that Feynman found so difficult.

Even the reader who thinks these arguments have merit may well ask, and properly so, whether it is really worthwhile replacing the traditional approach embodied in standard textbooks with something newer. Has not the older approach, whatever its flaws, allowed several generations of physicists to carry out excellent research? Have not the textbooks been written by authors with considerable pedagogical skill? Indeed, are the conceptual gaps, which even textbook writers themselves have sometimes acknowledged, all that serious? Do not good physicists, whether engaged in theory or experiment, eventually develop the sort of intuition which allows them to work around deficiencies in their courses? Do not our present courses at least teach students how to calculate things in agreement with experiment?

While sympathetic with such concerns, I must ask: Is it our primary goal to impart calculational skills to our students? No doubt this is one of the things we aim to do. The engineering student who can successfully apply the formula numbered 37 in his freshman mechanics text to a physics problem will succeed later, we hope, in applying the right formula from the appropriate engineering handbook to some design problem. The difficulty comes in situations in which formula 37 is no longer applicable, or perhaps one is not sure whether it applies, or maybe it is necessary to make some approximations, and good judgment is needed as to whether these are appropriate, etc. There are lots of reasons why when we teach classical mechanics we want our students not only to know the formulas in the blue boxes, but to imbed them in a real understanding of the deeper principles of the subject. If this is so, should our goal in the case of quantum mechanics be different?

To be sure, in any discipline of physics one eventually arrives at principles which in our present state of knowledge cannot be explained in terms of anything more fundamental. At that stage we have to stop and recognize that there is a limitation to our understanding, there are things that simply have to be accepted on faith, hopefully supported by the fact that they have been shown to work in a large number of circumstances. Especially when we have a consistent and coherent framework for some subject there is no reason to apologize, even when we know it is at best an approximation to the real world. Classical electricity and magnetism has this character. It is approximate (the real world is quantized) and there are always some loose ends to be understood better, but overall it is satisfactory, and we teach it with confidence to our students.

The situation in quantum mechanics, as reflected in current textbooks, is very different. A significant contribution of decades of research in quantum foundations has been to remind the community that quantum mechanics as traditionally taught contains all sorts of unresolved problems and paradoxes that cast serious doubt on its coherence as an intellectual discipline. These issues were often ignored in older textbooks, but newer ones feel obliged to devote at least a few pages to Einstein-Podolsky-Rosen and similar things. This acknowledges, in an
indirect way, that the system being taught has serious flaws, and in this respect textbook writers are at last catching up to what Feynman was saying in 1964. Is this flawed approach what we want to pass on to our students, or should we aim for something better?

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References

[1] D. F. Styer “Common misconceptions regarding quantum mechanics,” Am. J. Phys. 64, 31-34 (1996); C. Singh, “Student understanding of quantum mechanics,” Am. J. Phys. 69, 885-895 (2001); R. Müller and H. Wiesner “Teaching quantum mechanics on an introductory level” Am. J. Phys. 70 200-209 (2002).

[2] L. Bao and E. F. Redish “Understanding probabilistic interpretations of physical systems: A prerequisite to learning quantum physics” Am. J. Phys. 70 210-217 (2002).

[3] C. Singh, M. Belloni, and W. Christian, Phys. Today 59, No. 8, 43 (Aug. 2006).

[4] R. Feynman, The Character of Physical Law, (M. I. T. Press, 1965), p. 129.

[5] R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman Lectures on Physics, (Addison-Wesley, 1963), Vol. III.

[6] N. D. Mermin, “Could Feynman have said this?” Phys. Today 57, No. 5, 10-11 (May, 2004).

[7] A valuable collection of reprints and translations of articles dealing with quantum foundations and the measurement problem is found in J. A. Wheeler and W. H. Zurek, editors, Quantum Theory and Measurement (Princeton University Press, 1983).

[8] A good introduction to issues in quantum foundations is provided by the articles by Bell reprinted in (a) J. S. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge University Press, Cambridge, 1987), and in (b) M. Bell, K. Gottfried and M. Veltman, John S. Bell on the Foundations of Quantum Mechanics (World Scientific, Singapore, 2001).

[9] J. S. Bell, “Against measurement”, Phys. World 3, 33-40 (1990), reprinted in Ref. 8 (b), pp. 208-215; also in Sixty-Two Years of Uncertainty, edited by A. I. Miller (Plenum Press, New York, 1990), pp. 17-31.

[10] See the very thorough analysis in P. Mittelstaedt, The Interpretation of Quantum Mechanics and the Measurement Process (Cambridge University Press, Cambridge, 1998).
[11] R. B. Griffiths, “Consistent histories and the interpretation of quantum mechanics,” J. Stat. Phys. 36, 219 (1984); “Consistent Histories and Quantum Reasoning,” Phys. Rev. A 54, 2759 (1996); “Choice of consistent family, and quantum incompatibility,” Phys. Rev. A 57, 1604 (1998).

[12] R. Omnès, “Logical reformulation of quantum mechanics I. Foundations,” J. Stat. Phys. 53, 893 (1988); “Consistent interpretations of quantum mechanics,” Rev. Mod. Phys. 64, 339 (1992); The Interpretation of Quantum Mechanics (Princeton University Press, Princeton, 1994); Understanding Quantum Mechanics (Princeton University Press, Princeton, 1999).

[13] M. Gell-Mann and J. B. Hartle, “Quantum Mechanics in the Light of Quantum Cosmology” in Complexity, Entropy, and the Physics of Information, edited by W. Zurek (Addison Wesley, Reading, 1990), p. 425; “Classical equations for quantum systems,” Phys. Rev. D 47, 3345 (1993).

[14] R. B. Griffiths, Consistent Quantum Theory (Cambridge University Press, 2002) and http://quantum.phys.cmu.edu.

[15] Chs. 8 to 11 of Ref. [14].

[16] Different isomers decay at different rates, but that can be understood on the basis of different internal quantum states. Given a particular unstable isomer, we have no way to separate individual nuclei on the basis of their future longevity.

[17] J S Bell, “On the Einstein Podolsky Rosen paradox,” Physics 1, 195-200 (1964), reprinted in Ref. [8(a)] pp. 14-21, and (b), pp. 7-12.

[18] Believers in these influences often concede that they cannot be used to transmit information, which is the same as saying that no experimental test will ever reveal their presence. In fact, a proper quantum analysis using consistent microscopic probabilities shows that such influences are not needed when one uses conditional probabilities to analyze the situations in which they supposedly arise. See the discussion in Chs. 23 and 24 of Ref. [14].

[19] M. Born, “Zur Quantenmechanik der Stoßvorgänge,” Z. Phys. 37 863-867 (1926). English translation in Ref. [7] pp. 52-55.

[20] See, for example, M. Redhead, Incompleteness, Nonlocality, and Realism (Clarendon Press, Oxford, 1987).

[21] For a clear introduction to this way of thinking, see W. M. de Muynck, Foundations of Quantum Mechanics, an Empiricist Approach, (Kluwer, 2002).

[22] See Chs. 20 through 25 of Ref. [14] for a number of examples of how paradoxes can be dealt with by a consistent use of probabilities.

[23] J. von Neumann, Mathematische Grundlagen der Quantenmechanik (Springer, 1932); English translation: Mathematical Foundations of Quantum Mechanics (Princeton University Press, 1955). His theory of the measuring process is in Ch. VI.
[24] Ch. III Sec. 5 in Ref. 23.

[25] For an infinite-dimensional Hilbert space one requires that the subspace be closed.

[26] W. Gerlach and O. Stern “Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld,” Z. Phys. 9 349-352 (1922).

[27] G. Birkhoff and J. von Neumann, “The logic of quantum mechanics,” Annals of Math. 37, 823 (1936); John von Neumann Collected Works, edited by A. H. Taub (Macmillan, New York, 1962), Vol. IV, p. 105.

[28] A. Einstein, B. Podolsky and N. Rosen, “Can quantum-mechanical description of physical reality be considered complete?” Phys. Rev. 47 777 (1935).

[29] Sec. 4.6 of Ref. 14.

[30] Counterfactual reasoning of the sort “If it had been the case that...then...” is a particularly dangerous alligator. See Ch. 19 of Ref. 14 for an explanation of how to do some forms of counterfactual reasoning in a consistent way in the quantum context, and later chapters for examples of what happens when one does it inconsistently.

[31] Ch. V Sec. 1 in Ref. 23.

[32] For example, W. Feller, An Introduction to Probability Theory and Its Applications, Vol. 1, 3d ed. (John Wiley & Sons, New York, 1968); M. H. DeGroot and M. J. Schervish, Probability and Statistics (Addison-Wesley, 2002).

[33] Sec. 8.3 of Ref. 14.

[34] Sec. 16.1 of Ref. 14.

[35] Sec. 5.3 of Ref. 14.

[36] In the terminology of Sec. 9.4 of Ref. 14.

[37] E. Schrödinger, “Die gegenwärtige Situation in der Quantenmechanik,” Naturwissenschaften 23, 807-812, 823-828, 844-849 (1935). English translation in Ref. 7 pp. 152-167. The cat makes its appearance in Sec. 5.

[38] Chs. 10 and 11 of Ref. 14.

[39] See references in Ref. 3.

[40] Numerous examples are given in Ref. 14.

[41] The process is not difficult, and is carried out for various toy models in Chs. 12 and 13 of Ref. 14.

[42] The symbol $\otimes$ is used in Ref. 14 as a special form of tensor product symbol $\otimes$; see the discussion in Sec. 8.3.
[43] The projector at $t_0$ should include, along with the spin state of the particle, its initial spatial wave function and the initial (untriggered) state of the two detectors, but as these are exactly the same in all the histories we shall consider, and irrelevant to our discussion, there is no harm in omitting them from the notation used in (15) and later.

[44] Ch. 10 of Ref. 14.

[45] J. A. Wheeler, “On recognizing ‘law without law’,” Am. J. Phys. 51, 398 (1983).

[46] I find it best when introducing the subject to use the approach in Sec. 11.6 of Ref. 14 rather than the general formulation of Ch. 10; the former suffices for most purposes.

[47] See Ref. 2 for some of the difficulties students have with probabilities.

[48] M. Gell-Mann and J. Hartle, letter to Phys. Today 52, No. 2, 11 (Feb. 1999).

[49] Anyone interested in such a project is welcome to contact me.

[50] F. Laloë “Do we really understand quantum mechanics? Strange correlations, paradoxes, and theorems,” Am. J. Phys. 69, 655-701 (2001).