Modification of the Maxwell model for calculation of stress relaxation and creep behavior for polyester yarns

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Abstract. Textile materials, especially yarns formed from polymers such as polyester during deformation, have viscoelastic properties, namely a combination of elastic and viscous properties. Several studies have been conducted to model viscoelastic on textile materials, especially yarns. In this study, modelling is done by fitting data through experiments which are then modelled with a theory of the modification of the Maxwell model as a reference in making predictions through the data fitting method. The results of the data fitting prediction in the form of exponential curves have a regression value that approaches the experimental curve ($R^2 = 0.999$ and 0.999).

1. Introduction
Textile yarn has a blend of elastic and viscous if given a load and deformation towards the length so that it is included in the group of viscoelastic material. There have been many studies conducted to determine the viscoelastic properties of textile materials both experimentally and theoretically. Viscoelastic material manages in a book likened to a series consisting of springs as elastic material and daspot as viscous material [1-4]. This research was conducted with the aim of obtaining prediction of stress relaxation and creep behavior of polyester yarn [5] using data fitting methods and experiments with existing theory validation [6]. In modeling the modified form of the Maxwell model [1], it was chosen as a series of forms which are considered to represent the characteristics of the yarn as viscoelastic material.

2. Methods
This research begun with build a model which combain strain and dasphoot [7,8]. This model obey a Maxwell rule and we can see a model in below.
Figure 1. New Maxwell model [1,9].

\[ \sigma_{e_1} = E_1 \varepsilon_1; \sigma_{e_2} = E_2 \varepsilon_2; \sigma_{\eta_1} = \eta_1 \frac{d\varepsilon}{dt}; \sigma_{e_3} = E_3 \varepsilon_3 \]

The first step is to do the series network:

\[ \sigma_{e_1} = \sigma_{e_2} = \sigma_{\eta_1} = \sigma_{s_1} \]

Because the series network, so \( \sum \varepsilon = 0 \):

\[
\begin{align*}
\varepsilon &= \frac{\sigma_{e_1}}{E_1} + \frac{\sigma_{e_2}}{E_2} + \frac{\sigma_{\eta_1}}{\eta_1} \frac{d\varepsilon}{dt} \\
\frac{d\varepsilon}{dt} &= \frac{1}{E_1} \frac{d\sigma_{e_1}}{dt} + \frac{1}{E_2} \frac{d\sigma_{e_2}}{dt} + \frac{\sigma_{\eta_1}}{\eta_1} \\
\frac{d\sigma}{dt} &= \frac{1}{E_1 + E_2} \left[ \frac{d\varepsilon}{dt} - \frac{\sigma}{\eta_1} \right]
\end{align*}
\]

The second step is to do the parallel network:

Because the parallel network, so \( \sum \sigma = 0 \):

\[
\begin{align*}
\frac{d\sigma}{dt} &= \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \left[ \frac{d\varepsilon}{dt} - \frac{\sigma}{\eta_1} \right] + \frac{1}{E_3} \frac{d\varepsilon_3}{dt} \\
\frac{d\sigma}{dt} &= \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \left[ \frac{d\varepsilon}{dt} - \frac{\sigma - \sigma_{e_3}}{\eta_1} \right] + \frac{1}{E_3} \frac{d\varepsilon_3}{dt} \\
\frac{d\sigma}{dt} &= \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \left[ \frac{d\varepsilon}{dt} - \frac{\sigma}{\eta_1} \right] + \frac{1}{E_3} \frac{d\varepsilon_3}{dt} \\
\frac{d\sigma}{dt} &= \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \left[ \frac{d\varepsilon}{dt} - \frac{\sigma}{\eta_1} \right] + \frac{1}{E_3} \frac{d\varepsilon_3}{dt} \\
\frac{d\sigma}{dt} &= \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \left[ \frac{d\varepsilon}{dt} - \frac{\sigma}{\eta_1} \right] + \frac{1}{E_3} \frac{d\varepsilon_3}{dt} \\
\frac{d\sigma}{dt} &= \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \left[ \frac{d\varepsilon}{dt} - \frac{\sigma}{\eta_1} \right] + \frac{1}{E_3} \frac{d\varepsilon_3}{dt}
\end{align*}
\]
Stress relaxation formulation

\[ \frac{d\varepsilon}{dt} = 0 \]

\[ \frac{d\sigma}{dt} + \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \frac{\sigma}{\eta_1} = \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \frac{E_3 \varepsilon}{\eta_1} \]

The above equation is multiplied by \( \frac{(E_2 + E_1)\eta_1}{E_2 E_1} \), so:

\[ \frac{(E_2 + E_1)\eta_1}{E_2 E_1} \frac{d\sigma}{dt} + \sigma = E_3 \varepsilon \]

\[ \frac{(E_2 + E_1)\eta_1}{E_2 E_1} \frac{d\sigma}{dt} = E_3 \varepsilon - \sigma \]

\[ \frac{d\sigma}{dt} = \frac{(E_3 \varepsilon - \sigma) E_2 E_1}{(E_2 + E_1)\eta_1} \]

This is a Differential Equation of the one order, where:

\[ p(t) = \frac{E_1 E_2}{(E_2 + E_1)\eta_1} \]

\[ g(t) = \frac{E_1 E_2 E_3 \varepsilon}{(E_2 + E_1)\eta_1} \operatorname{dan} \int p(t) dt = \frac{E_1 E_2}{(E_2 + E_1)\eta_1} t \]

\[ \sigma = e^{-\frac{E_1 E_2}{(E_2 + E_1)\eta_1} t} \left[ \int \frac{E_1 E_2 E_3 \varepsilon}{(E_2 + E_1)\eta_1} \frac{E_1 E_2}{e^{(E_2 + E_1)\eta_1} t} dt + C \right] \]

\[ \sigma = \frac{E_1 E_2}{(E_2 + E_1)\eta_1} t \left[ \frac{E_1 E_2 E_3 \varepsilon}{(E_2 + E_1)\eta_1} \frac{E_1 E_2}{(E_2 + E_1)\eta_1} e^{(E_2 + E_1)\eta_1 t} + C \right] \]

\[ \sigma = E_3 \varepsilon + C e^{-\frac{E_1 E_2}{(E_2 + E_1)\eta_1} t} \]

2.1 Equation of stress relaxation:

\[ \sigma = E_3 \varepsilon + \sigma_0 e^{-\frac{E}{2\eta} t} \]  

\[ \frac{d\sigma}{dt} + \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \frac{\sigma}{\eta_1} = \left[ \frac{E_2 E_1}{E_2 + E_1} \right] \frac{E_3 \varepsilon}{\eta_1} + \left[ \frac{1}{E_3} + \frac{E_1 E_2}{(E_2 + E_1)} \right] \frac{d\varepsilon}{dt} \]

\[ E_2 = E_1 = E_3 = E \]
\[
\frac{d\sigma}{dt} + \frac{E^2}{2E\eta_1} \sigma = \frac{E^2}{2E} \frac{\sigma}{\eta_1} + \frac{1}{E + \frac{E^2}{2E}} \frac{d\varepsilon}{dt}
\]

\[
\frac{d\sigma}{dt} + \frac{E^2}{2\eta_1} \varepsilon = \frac{E^2}{2} \frac{\varepsilon}{\eta_1} + \frac{1}{E^2 + \frac{E^2}{2}} \frac{d\varepsilon}{dt}
\]

\[
\frac{d\sigma}{dt} + \frac{E^2}{2\eta_1} \frac{\sigma}{\eta_1} = \frac{E^2}{2} \frac{\varepsilon}{\eta_1} + \frac{2 + E^2}{E^2} \frac{d\varepsilon}{dt}
\]

(multiplied by \(\frac{\eta_1}{E}\)), so we’ve got:

\[
\frac{\eta_1}{E} \frac{d\sigma}{dt} + \frac{\sigma}{2} = \frac{E\varepsilon}{2} + \frac{\eta_1}{\eta_1} \frac{2 + E^2}{E^2} \frac{d\varepsilon}{dt}
\]

\[
\frac{d\sigma}{dt} = 0 , \text{ so}
\]

\[
\frac{\sigma}{2} = \frac{E\varepsilon}{2} + \frac{\eta_1}{\eta_1} \frac{2 + E^2}{E^2} \frac{d\varepsilon}{dt}
\]

\[
\sigma = E\varepsilon + \frac{\eta_1}{\eta_1} \frac{2 + E^2}{E^2} \frac{d\varepsilon}{dt}
\]

\[
\sigma - E\varepsilon = \frac{\eta_1}{\eta_1} \frac{2 + E^2}{E^2} \frac{d\varepsilon}{dt}
\]

\[
\frac{d\varepsilon}{dt} = \frac{\sigma - E\varepsilon}{\eta_1 \left[ \frac{2 + E^2}{E^2} \right]}
\]

\[
\frac{d\varepsilon}{dt} = \frac{\sigma}{\eta_1 \left[ \frac{2 + E^2}{E^2} \right]} \left( \frac{1 - \frac{E\varepsilon}{\sigma}}{\sigma} \right)
\]

\[
\int \left[ \left( \frac{1 - \frac{E\varepsilon}{\sigma}}{\frac{E\varepsilon}{\sigma}} \right) \right] \frac{d\varepsilon}{dt} = \frac{\sigma}{\eta_1 \left[ \frac{2 + E^2}{E^2} \right]} \int dt
\]

\[
-\frac{\sigma}{E} \int \frac{d\left( 1 - \frac{E\varepsilon}{\sigma} \right)}{\left( 1 - \frac{E\varepsilon}{\sigma} \right)} = \frac{\sigma}{\eta_1 \left[ \frac{2 + E^2}{E^2} \right]} \int dt
\]

\[
-\frac{\sigma}{E} \ln \left( 1 - \frac{E\varepsilon}{\sigma} \right) = \frac{\sigma}{\eta_1 \left[ \frac{2 + E^2}{E^2} \right]} t
\]

\[
\frac{\sigma}{E} \ln \left( 1 - \frac{E\varepsilon}{\sigma} \right) = -\frac{\sigma}{\eta_1 \left[ \frac{2 + E^2}{E^2} \right]} t
\]

\[
\ln \left( 1 - \frac{E\varepsilon}{\sigma} \right) = -\frac{E}{\eta_1 \left[ \frac{2}{E^2} + 1 \right]} t
\]
\[ \left( 1 - \frac{E\varepsilon}{\sigma} \right) = e^{-\frac{E}{\eta_1 \left( \frac{E^2}{\sigma^2} + 1 \right) t}} \]

2.2 Creep behaviour equation:

\[ \varepsilon = \frac{\sigma}{E} \left( 1 - e^{-\frac{E}{\eta_1 \left( \frac{E^2}{\sigma^2} + 1 \right) t} } \right) \]  

The implementation and modelling as well as the prediction of the viscoelastic properties of the textile [10-12] sector is to determine the properties of viscoelastic materials in fibers, threads or fabrics for example in 100% wool viz yarn which has 36 tex yarn numbers [13-16]. The results of the data for stress relaxation (constant strain) and creep behavior (constant stress) are as follows [17-20]:

**Table 1.** Stress relaxation calculation.

| Time (10^3 Second) | Stress (cN/text) |
|--------------------|------------------|
| 0                  | 6                |
| 0.25               | 5.2              |
| 1                  | 5.1              |
| 1.5                | 5                |
| 3.5                | 4.8              |

**Table 2.** Creep behaviour calculation.

| Time (10^3 second) | Strain (%) |
|--------------------|------------|
| 0                  | 0          |
| 0.25               | 10         |
| 1                  | 12         |
| 1.5                | 12.2       |
| 3.5                | 13         |

The results of equations (1) and (2) can be presented in figures 2 and 3:

**Figure 2.** Stress relaxation vs time.
After the data fitting process is done, it produces $R^2 = 0.99$ for stress relaxation versus time curves.

![Figure 3. Creep Behaviour vs Time.](image)

After doing the fitting process the data produces a value of $R^2 = 0.990$ for the creep behavior versus time curve.

3. Results and discussion
In general, the results of predictions of both stress relaxation and creep behavior equations after being tested for correlation both have close values. In general, stress relaxation and creep behavior can be predicted using equations (1) and (2).

From the presentation (1) and (2) each obtained the elastic modulus ($E$) and viscous ($\eta$) values that are not exactly the same between stress relaxation and creep behavior conditions. After reviewing it, the authors suspect that this is due to the limitation of the value of $E$ which is considered $E_1 = E_2 = E_3 = E$, which could be the actual condition of the thread not like that. However, to predict and model stress relaxation and creep behavior equations (1) and (2) can be used.

4. Conclusion
From this research, it can be concluded that simple modeling using data fitting method can be used to predict stress relaxation and creep behavior of polyester yarn as viscoelastic material. The modeling obtained for the stress behavior is in equation (1) and modeling for creep behavior is found in equation (2).

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