Critical temperature and Ginzburg region near a quantum critical point in two-dimensional metals

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We compute the transition temperature $T_c$ and the Ginzburg temperature $T_G$ above $T_c$ near a quantum critical point at the boundary of an ordered phase with a broken discrete symmetry in a two-dimensional metallic electron system. Our calculation is based on a renormalization group analysis of the Hertz action with a scalar order parameter. We provide analytic expressions for $T_c$ and $T_G$ as a function of the non-thermal control parameter for the quantum phase transition, including logarithmic corrections. The Ginzburg regime between $T_c$ and $T_G$ occupies a sizable part of the phase diagram.

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I. INTRODUCTION

Instabilities of the normal metallic state lead to a rich variety of quantum phase transitions in interacting electron systems. Near a quantum critical point electronic excitations are strongly scattered by order parameter fluctuations such that Fermi liquid theory breaks down. The fluctuation effects and the ensuing non-Fermi liquid behavior is particularly pronounced in two-dimensional systems. It is therefore not surprising that quantum critical fluctuations are frequently invoked as a mechanism for the enigmatic strange metal behavior of a variety of quantum phase transitions in interacting correlated electron compounds.

Theoretical works have mostly focused on the quantum critical point and its extension into the quantum critical regime at finite temperature, where quantum fluctuations are particularly important. Less attention has been paid to the Ginzburg region near the critical temperature, which is characterized by strongly interacting classical order parameter fluctuations. This is somewhat unwarranted since classical critical fluctuations also affect electronic excitations very strongly. In two dimensions they lead to a contribution of the order $T_\xi$ to the quasiparticle decay rate, where $\xi$ is the diverging correlation length.

In this paper we compute the size of the Ginzburg region above the critical temperature near a quantum critical point in two-dimensional metals. More specifically, we consider continuous quantum phase transitions associated with the spontaneous breaking of a discrete symmetry, described by an effective Hertz action for a scalar order parameter with dynamical exponents $z = 2$ or $z = 3$. We calculate the transition temperature $T_c$ as a function of the non-thermal control parameter for the quantum phase transition, as well as the Ginzburg temperature $T_G$ above $T_c$. The size of the Ginzburg region $T_G - T_c$ is determined to leading order in the distance from the quantum critical point. The dependence of the Ginzburg temperature on the control parameter was derived already by Millis. However, that study did not access the Ginzburg region between $T_G$ and $T_c$. A comprehensive analysis of all finite temperature transition and crossover lines in dimensions $d > 2$ was performed by Sachdev. For discrete symmetry breaking in two dimensions the critical temperature $T_c$ and the Ginzburg temperature below $T_c$ were recently compared within a renormalization group study which allowed to approach the finite temperature transition. However, in that work the flow equations were solved only numerically, while we now present analytic results.

The paper is organized as follows. In Section II we derive the renormalization group equations for the effective Hertz action. These are solved analytically in an approximate form in Section III. In Section IV we discuss the results for $T_c$, $T_G$ and the size of the Ginzburg region, before concluding in Section V.

II. HERTZ ACTION AND FLOW EQUATIONS

Our analysis is based on the Hertz action

$$S[\phi] = \frac{T}{2} \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \phi_{q,\omega_n} \left( \delta_0 + q^2 + \frac{\omega_n}{|q|^{2-z}} \right) \phi_{-q,\omega_n} + \frac{u_0}{d!} \int_0^T \int d\tau \int d^d x \phi^4 (x, \tau),$$

(1)

where $\phi(x, \tau)$ is a real scalar order parameter field and $\phi_{q,\omega_n}$ its momentum representation; $\omega_n = 2\pi n T$ with integer $n$ denotes the bosonic Matsubara frequencies. For the dynamical exponent $z$ we consider the cases $z = 2$, which describes density wave transitions, and $z = 3$, relevant for a nematic transition or Ising-type ferromagnetic transitions. We do not address the issue under which circumstances the Hertz action provides a faithful description of quantum criticality in two-dimensional metals.
Before embarking on the renormalization group approach, we would like to emphasize that in two dimensions \( T_c(\delta_0) \) cannot be obtained from a first order expansion in the quartic coupling \( u_0 \), even if it is weak and irrelevant at the quantum critical point. To leading order in \( u_0 \), the inverse susceptibility \( \delta \) is given by

\[
\delta = \delta_0 + a T \sum_{\omega_n} \int \frac{d^4 q}{(2\pi)^d} \frac{u_0}{\delta_0 + q^2 + \frac{\omega_n}{|q|^2}},
\]

where \( a \) is a positive constant. At finite temperature the Matsubara frequencies are discrete and the classical fluctuation contribution from \( \omega_n = 0 \) diverges logarithmically in the limit \( \delta_0 \to 0 \) in two dimensions. Trying to treat this divergence by a self-consistent equation, replacing \( \delta_0 \) by \( \delta \) under the integral, one finds that the transition temperature \( T_c \) is suppressed to zero at the critical point given by \( \delta = 0 \), irrespective of \( \delta_0 \). This behavior is reminiscent of the Mermin-Wagner theorem, which excludes spontaneous breaking of a continuous symmetry in two dimensions. However, the above first order calculation is essentially independent of the symmetry of the order parameter, and is therefore misleading at least in the case of a discrete symmetry.

We solve the problem by using flow equations which describe the renormalization of the inverse susceptibility (or "mass") \( \delta \) and the quartic coupling \( u \) due to fluctuations. The flow equations are derived from an approximate ansatz for the exact effective action \( \Gamma^\Lambda(\phi) \), that is, the generating functional for vertex functions in the presence of an infrared cutoff \( \Lambda \). The cutoff is implemented by adding a regulator term of the form \( \frac{1}{\phi^4} \) to the bare action \( S(\phi) \). The exact flow of \( \Gamma^\Lambda(\phi) \) is given by the Wetterich equation

\[
\partial_\Lambda \Gamma^\Lambda(\phi) = \frac{1}{2} \text{tr} \partial_\Lambda R^\Lambda(\phi + R^\Lambda),
\]

where \( \Gamma^{(2)}(\phi) \) is the matrix of second derivatives of \( \Gamma^\Lambda(\phi) \) with respect to \( \phi \) and the trace sums over momenta and frequencies. We approximate \( \Gamma^\Lambda(\phi) \) by an ansatz of the form Eq. \[ \text{1} \] with a renormalized mass term \( \delta \Lambda \) and a renormalized coupling \( u^\Lambda \). Inserting this ansatz in the exact flow equation for \( \Gamma^\Lambda(\phi) \) and comparing coefficients, one obtains

\[
\partial_\Lambda \delta^\Lambda = -\frac{1}{2} \delta^\Lambda T \sum_{\omega_n} \int \frac{d^d q}{(2\pi)^d} \partial_\Lambda R^\Lambda(q) \left[ \delta^\Lambda + q^2 + \frac{\omega_n}{|q|^2} + R^\Lambda(q) \right]^2,
\]

\[
\partial_\Lambda u^\Lambda = 3(u^\Lambda)^2 T \int \frac{d^4 q}{(2\pi)^d} \partial_\Lambda R^\Lambda(q) \left[ \delta^\Lambda + q^2 + \frac{\omega_n}{|q|^2} + R^\Lambda(q) \right]^3.
\]

The initial conditions for the flow are \( \delta^\Lambda_0 = \delta_0 \) and \( u^\Lambda_0 = u_0 \), where \( \Lambda_0 \) is a (fixed) ultraviolet cutoff. As a regulator we choose the Litim\[23\] function \( R^\Lambda(q) = (\Lambda^2 - q^2)^\Theta(\Lambda^2 - q^2) \), with derivative \( \partial_\Lambda R^\Lambda(q) = 2\Lambda^2 \Theta(\Lambda^2 - q^2) \), which restricts the momentum integrals in the flow equations to \( |q| \leq \Lambda \) and replaces the \( q^2 \)-term in the denominators by \( \Lambda^2 \). From now on we fix the dimensionality to \( d=2 \).

The Matsubara sums in the above flow equations can be expressed in terms of polygamma functions \( \Psi_n(z) \), defined as the \( n \)-th derivative of the digamma function \( \Psi_0(z) = \Gamma'(z)/\Gamma(z) \). Explicit \( \Lambda \)-dependencies can be removed from the right hand side of the flow equations as usual by introducing rescaled dimensionless variables

\[
\tilde{q} = \frac{|q|}{\Lambda}, \quad \tilde{T} = \frac{2\pi T}{\Lambda^2}, \quad \tilde{\delta} = \frac{\delta}{\Lambda^2}, \quad \tilde{u} = \frac{T}{2\pi \Lambda^2} u^\Lambda.
\]

One then obtains

\[
\partial_\Lambda \tilde{\delta} = -2\tilde{\delta} - \frac{\tilde{u}}{2(1+\tilde{\delta})^2} - \frac{2\tilde{u}}{T^2} \int_0^1 dq' q'^{2z-3} \Psi_1[h(\tilde{\delta}, q', \tilde{T})],
\]

\[
\partial_\Lambda \tilde{u} = -2\tilde{u} + \frac{3\tilde{u}^2}{(1+\tilde{\delta})^3} - \frac{6\tilde{u}}{T^3} \int_0^1 dq' q'^{3z-5} \Psi_2[h(\tilde{\delta}, q', \tilde{T})],
\]

where \( h(\tilde{\delta}, q', \tilde{T}) = 1 + (1+\tilde{\delta})q'^{-2}/\tilde{T} \). The first term in each equation is due to the factor \( \Lambda^2 \) in the definition of the dimensionless variables, the second one captures classical fluctuations (\( \omega_n = 0 \)), and the third one quantum fluctuations (\( \omega_n \neq 0 \)).

### III. SOLUTION OF FLOW EQUATIONS

For sufficiently small but finite temperature the flow passes through two distinct regimes, which are distinguished by the size of the rescaled temperature \( \tilde{T} \). Initially one has \( \tilde{T} \ll 1 \), such that quantum fluctuations dominate, while in the final stage, for \( \tilde{T} \gg 1 \), the flow is governed by classical fluctuations. In the latter regime the third term on the right hand side of the flow equations (7) and (8) can be neglected. For \( \tilde{T} \ll 1 \) one can use the expansion of the polygamma functions for large arguments, \( \Psi_1(z) \sim z^{-1} \) and \( \Psi_2(z) \sim -z^{-2} \), to approximate the integrals in Eqs. (7) and (8) as

\[
\frac{1}{T^2} \int_0^1 dq' q'^{2z-3} \Psi_1[h(\tilde{\delta}, q', \tilde{T})] \approx \frac{1}{z(1+\tilde{\delta})^3},
\]

\[
\frac{1}{T^3} \int_0^1 dq' q'^{3z-5} \Psi_2[h(\tilde{\delta}, q', \tilde{T})] \approx \frac{1}{z(1+\tilde{\delta})^2}. \]

The cutoff scale corresponding to \( \tilde{T} = 1 \) is given by \( \Lambda_1 = (2\pi T)^{1/2} \). Following Millis\[24\] we approximate the flow by its quantum contribution with the expansion Eqs. (9) and (10) for \( \Lambda > \Lambda_1 \), and we discard the quantum terms for \( \Lambda < \Lambda_1 \).

The flow equations (7) and (8) exhibit a fixed point at \( \tilde{T} = 1 \) given by \( \Lambda_1 = (2\pi T)^{1/2} \). Following Millis\[24\] we approximate the flow by its quantum contribution with the expansion Eqs. (9) and (10) for \( \Lambda > \Lambda_1 \), and we discard the quantum terms for \( \Lambda < \Lambda_1 \).

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small during the entire flow. Above $T_c$ it remains small until the fluctuation contributions to the flow saturate at small $\Lambda$. We therefore approximate $1 + \tilde{\delta} \approx 1$ in the denominators of the flow equations, which allows us to solve them analytically. Note that within this approximation fluctuation contributions are symmetric under $\tilde{\delta} \rightarrow -\tilde{\delta}$, while in the exact flow they are larger for $\tilde{\delta} < 0$ compared to $\tilde{\delta} > 0$, which leads to a suppression of $T_c$.

In the quantum regime ($\Lambda > \Lambda_1$) the approximate flow equations have the form

$$\Lambda \partial_\Lambda \tilde{\delta} = -2\tilde{\delta} - \frac{2}{zT} \tilde{u}, \quad (11)$$

$$\Lambda \partial_\Lambda \tilde{u} = -2\tilde{u} + \frac{6}{zT} \tilde{u}^2. \quad (12)$$

Recall that $\tilde{T} = 2\pi T / \Lambda^z$ is also a flowing quantity. The explicit solution for $z=3$ reads

$$\Lambda^2 \tilde{u}^A = \frac{\pi T}{C_3 - \Lambda}, \quad (13)$$

$$\Lambda^2 \tilde{\delta}^A = C'_3 + f(C_3, \Lambda), \quad (14)$$

where

$$f(x, \Lambda) = \frac{1}{6} \Lambda^2 + \frac{1}{3} x \Lambda + \frac{1}{3} x^2 \ln \left(1 - \frac{\Lambda}{x}\right). \quad (15)$$

The integration constants $C_3$ and $C'_3$ are determined by the initial conditions at $\Lambda_0$ as $C_3 = \pi T \Lambda_0^{-2} \tilde{u}_0^{-1} + \Lambda_0$ and $C'_3 = \delta_0 - f(C_3, \Lambda_0)$. The solution for $z = 2$ is given by

$$\Lambda^2 \tilde{u}^A = \frac{\frac{2}{3} \pi T}{C_2 - \ln \Lambda}, \quad (16)$$

$$\Lambda^2 \tilde{\delta}^A = C'_2 + \frac{1}{3} e^{2C_2} \text{Ei}(2 \ln \Lambda - 2C_2), \quad (17)$$

where $\text{Ei}(x) = \int_{-\infty}^{x} dt \, e^t / t$. The integration constants $C_2$ and $C'_2$ are determined by the initial conditions: $C_2 = \frac{2}{3} \pi T \Lambda_0^{-2} \tilde{u}_0^{-1} + \ln \Lambda_0$ and $C'_2 = \delta_0 - \frac{1}{3} e^{2C_2} \text{Ei}(2 \ln \Lambda_0 - 2C_2)$.

In the classical regime ($\Lambda < \Lambda_1$) the approximate flow equations read

$$\Lambda \partial_\Lambda \tilde{\delta} = -2\tilde{\delta} - \frac{1}{2} \tilde{u}, \quad (18)$$

$$\Lambda \partial_\Lambda \tilde{u} = -2\tilde{u} + 3\tilde{u}^2. \quad (19)$$

The explicit solution has the form

$$\tilde{u}^A = \frac{1}{CA^2 + \frac{3}{2}}, \quad (20)$$

$$\Lambda^2 \tilde{\delta}^A = C' - \frac{1}{4C} \ln \left(\Lambda^2 + \frac{3}{2C}\right). \quad (21)$$

The integration constants $C$ and $C'$ are determined by the boundary conditions $\tilde{u}^{A_1} = \tilde{u}_1$ and $\tilde{\delta}^{A_1} = \delta_1$ at the scale $\Lambda_1$, yielding $C = \Lambda_1^{-2} (\tilde{u}_1^{-1} - \frac{3}{2})$ and $C' = \Lambda_1^2 \delta_1 + \frac{1}{4C} \ln (\Lambda_1^2 + \frac{3}{2C})$.

At $T=0$ one has $\Lambda_1 = 0$ and the flow can be obtained by taking the zero temperature limit of the solution in the quantum regime. For $z = 3$ the unscaled quartic coupling $u^A$ saturates at the finite value

$$u'_0 = \frac{u_0}{1 + \frac{1}{2\pi^2} u_0 \Lambda_0} = \frac{2\pi^2}{C_3}, \quad (22)$$

for $\Lambda \rightarrow 0$. Note that the rescaled variable $\tilde{u}$ vanishes at $T = 0$. For a generic choice of $\delta_0$ the inverse susceptibility $\delta^A$ scales to a finite value near $\delta_0$. At the quantum critical point,

$$\delta_0 = \delta_0^{qc} = f (\frac{2\pi^2}{u'_0} \Lambda_0) = -\frac{\Lambda_0^3}{18\pi^2} u'_0 + O(u'_0^2), \quad (23)$$

the inverse susceptibility scales to zero for $\Lambda \rightarrow 0$.

For $z = 2$ the quartic coupling $u^A$ vanishes logarithmically for $\Lambda \rightarrow 0$. The inverse susceptibility remains generically finite, except at the quantum critical point given by

$$\delta_0^{qc} = \frac{\Lambda_0^2}{3} \text{exp} \left(\frac{8\pi^2}{3u_0} \text{Ei} \left(-\frac{8\pi^2}{3u_0}\right)\right). \quad (24)$$

IV. RESULTS FOR $T_c$ AND $T_G$

The phase transition line in the $(\delta_0, T)$ phase diagram is determined by the condition $\delta^A \rightarrow 0$ for $\Lambda \rightarrow 0$. Using the solution for $\delta^A$ in the classical regime, Eq. (21), this yields a condition on the integration constants $C$ and $C'$, namely $C'' = \frac{1}{\pi} \ln \frac{\Lambda}{C'}$. The constants $C$ and $C'$ can be expressed in terms of the bare variables $\delta_0$ and $u_0$ by matching the initial condition for the classical flow at $\Lambda_1$ to the solution of the flow in the quantum regime.

For $z = 3$, one obtains

$$\delta_0^{qc} - \delta_0^{qc} = -f \left(\frac{2\pi^2}{u'_0} \Lambda_0\right) \text{Ei} \left(2\ln \Lambda - 2C_2\right), \quad (25)$$

where $C(T) = \frac{2\pi}{u'_0} \frac{\Lambda_0}{\ln \Lambda_0}$. Expanding for small temperatures $T$ yields

$$\delta_0^{qc} - \delta_0^{qc} = \frac{u'_0}{24\pi} T \ln T_0 + \frac{u'_0}{9\pi} T + O \left(T^{4/3} \ln T\right), \quad (26)$$

with $T_0 = \frac{8(2\pi)^3}{27u_0^3}$. Note that dependencies on the ultraviolet cutoff $\Lambda_0$ are absorbed in $u'_0$ and $\delta_0^{qc}$. Inverting Eq. (25) to leading order in $T$ yields

$$T_c(\delta_0) = \frac{24\pi}{u'_0} \frac{\delta_0^{qc} - \delta_0}{\ln \left(\frac{\Lambda_0}{\delta_0^{qc} - \delta_0}\right)}, \quad (27)$$

with $A_0 = \frac{u'_0}{24\pi} T_0$. For $z = 2$, we find

$$\delta_0^{qc} - \delta_0^{qc} = -\frac{1}{3} e^{2C_2} \text{Ei} \left(2\ln T - 2C_2\right) \ln \left(\frac{1 + \frac{1}{2\pi^2} C(T)}{C(T)}\right). \quad (28)$$
T line looks almost linear. ... corrections are hardly visible, such that the critical line as obtained from Eq. (25) for $z = 2$ in the Gaussian regime with mean-field exponents. On the other hand, if $\tilde{u}$ has reached a value close to its fixed point at $\Lambda = \Lambda_G$, it affects the flow of $\delta$ substantially leading to non-Gaussian scaling. There is no unique choice of $x$ quantifying the “closeness” to the fixed point. This reflects the fact that the Ginzburg line marks a crossover regime and not a sharp transition.

The leading low-$T$ behavior of $\delta_0^G - \delta_0^c$ computed from the Ginzburg criterion described above turns out to be the same as that for $\delta_0^G - \delta_0^c$, with the same prefactor, irrespective of the choice of $x$. However, differences appear in the first subleading term. For $z = 3$, one obtains

$$\delta_0^G - \delta_0^c = -\frac{u_0}{8\pi} \ln(x) T,$$

and for $z = 2$,

$$\delta_0^G - \delta_0^c = -\frac{\pi}{3} \ln(x) \frac{T}{\ln(T/T_c)},$$

at low temperatures. We have used the fixed point value $\tilde{u}^* = 2/3$ as deduced from Eq. (15). Note that the terms on the right hand sides are positive. Solving for $T_G - T_c$ as a function of $\delta_0^G - \delta_0^c$, one finds that $(T_G - T_c)/T_c$ is of order $[\ln(\delta_0^G - \delta_0^c)]^{-1}$ for $z = 3$, and of order $[\ln(\delta_0^G - \delta_0^c)]^{-1}$ for $z = 2$. Hence, the size of the Ginzburg region $T_G - T_c$ is practically of the order $T_c$ near the quantum critical point. By contrast, in three dimensions it is of order $T_c^2$. The results for $T_G$ are plotted in Fig. 1 for the same choice of parameters as above. We can see a substantial Ginzburg regime opening between $T_G$ and $T_c$. The $T_c$- and $T_G$-lines merge when the quantum critical point is approached, $\delta_0 \to \delta_0^c$, since that critical point is Gaussian.

V. CONCLUSIONS

We have derived analytic expressions for the transition temperature $T_c$ and the Ginzburg temperature $T_G$ above $T_c$ as a function of the non-thermal control parameter $\delta_0$ near a quantum critical point with a scalar (Ising universality class) order parameter in a two dimensional metal. The calculations are based on flow equations derived from a perturbative renormalization group for the Hertz model. The renormalization of the quartic coupling is crucial to avoid an artificial suppression of $T_c$ to zero in two dimensions. Both $T_c$ and $T_G$ are essentially proportional to $\delta_0 - \delta_0^c$, with logarithmic corrections depending on the dynamical exponent $z$. For
we confirm the results by Millis.\textsuperscript{9} For $T_c$, we obtain the same logarithmic corrections as for $T_G$, in agreement with earlier evidence from a numerical solution of flow equations for the symmetry broken phase.\textsuperscript{11} Nevertheless, the size of the Ginzburg region $T_G - T_c$, which has been calculated analytically for the first time in this paper, is practically proportional to the distance to the quantum critical point, $\delta_0 - \delta_{q}\textsuperscript{c}$. Hence, the Ginzburg region with its large non-Gaussian classical fluctuations covers a substantial part of the phase diagram near a continuous quantum phase transition in two dimensional metals. Electronic excitations are strongly scattered by order parameter fluctuations in that region, which can lead to enhanced decay rates, pseudogaps, and other unconventional electronic properties.

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\textsuperscript{1} M. Vojta, Rep. Prog. Phys. \textbf{66}, 2069 (2003).

\textsuperscript{2} H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle, Rev. Mod. Phys. \textbf{79}, 1015 (2007).

\textsuperscript{3} For a discussion of the Ginzburg criterion, see, for example, N. Goldenfeld, *Lectures on phase transitions and the renormalization group* (Addison Wesley, Reading, 1992).

\textsuperscript{4} Y. M. Vilk and A.-M.S. Tremblay, J. Phys. I (France) \textbf{7}, 1309 (1997).

\textsuperscript{5} A. Abanov, A. V. Chubukov, and J. Schmalian, Adv. Phys. \textbf{52}, 119 (2003).

\textsuperscript{6} A. A. Katanin, A.P. Kampf, and V.Yu. Irkhin, Phys. Rev. B \textbf{71}, 085105 (2005); A. A. Katanin, Phys. Rev. B \textbf{72}, 035111 (2005).

\textsuperscript{7} L. Dell’Anna and W. Metzner, Phys. Rev. B \textbf{73}, 045127 (2006).

\textsuperscript{8} J. A. Hertz, Phys. Rev. B \textbf{14}, 1165 (1976).

\textsuperscript{9} A. J. Millis, Phys. Rev. B \textbf{48}, 7183 (1993).

\textsuperscript{10} S. Sachdev, Phys. Rev. B \textbf{55}, 142 (1997).

\textsuperscript{11} P. Jakubczyk, P. Strack, A. A. Katanin, and W. Metzner, Phys. Rev. B \textbf{77}, 195120 (2008).

\textsuperscript{12} For a discussion of this point, see, for example, Ar. Abanov and A. V. Chubukov, Phys. Rev. Lett. \textbf{93}, 255702 (2004); D. Belitz, T. R. Kirkpatrick, and T. Vojta, Rev. Mod. Phys. \textbf{77}, 579 (2005); H. v. Löhneysen, A. Rosch, M. Vojta, and P. Wölfle, Rev. Mod. Phys. \textbf{79}, 1015 (2007); M. A. Metlitski and S. Sachdev, Phys. Rev. B \textbf{82}, 075127 (2010), *ibid* \textbf{82}, 075128 (2010).

\textsuperscript{13} J. Berges, N. Tetradis, and C. Wetterich, Phys. Rep. \textbf{363}, 223 (2002).

\textsuperscript{14} C. Wetterich, Phys. Lett. B \textbf{301}, 90 (1993).

\textsuperscript{15} The derivation is analogous to the derivation of the flow equations in the symmetry-broken state in Ref.\textsuperscript{11}, and simplified due to the absence of Z-factors.

\textsuperscript{16} D.F. Litim, Phys. Rev. D \textbf{64}, 105007 (2001).