Constraints on Upper Cutoffs in the Mass Functions of Young Star Clusters

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Abstract

We test claims that the power-law mass functions of young star clusters (ages \( \lesssim \) few \( \times 10^5 \) yr) have physical upper cutoffs at \( M_\star \sim 10^5 M_\odot \). Specifically, we perform maximum likelihood fits of the Schechter function, \( \psi(M) = dN/dM \propto M^\beta \exp(-M/M_c) \), to the observed cluster masses in eight well-studied galaxies (LMC, SMC, NGC 4214, NGC 4449, M83, M51, Antennae, and NGC 3256). In most cases, we find that a wide range of cutoff masses is permitted (\( 10^5 M_\odot \lesssim M_\star < \infty \)). We find a weak detection at \( M_\star \sim 10^5 M_\odot \) in one case (M51) and strong evidence against this value in two cases. However, when we include realistic errors in cluster masses in our analysis, the constraints on \( M_\star \) become weaker and there are no significant detections (even for M51). Our data are generally consistent with much larger cutoffs, at \( M_\star \sim \) few \( \times 10^5 M_\odot \). This is the predicted cutoff from dynamical models in which old globular clusters and young clusters observed today formed by similar physical processes with similar initial mass functions.

Key words: galaxies: star clusters: general – stars: formation

Supporting material: tar.gz file

1. Introduction

One of the most important characteristics of a population of astronomical objects is its mass function, \( \psi(M) = dN/dM \). The shape of this function, and especially any distinct features, such as upper or lower cutoffs, encodes important information about the physical processes involved in the formation and subsequent evolution of the objects. For young star clusters in different galaxies, the mass function is always found, in a first approximation, to have a power-law shape, \( \psi(M) \propto M^\beta \), with an exponent close to \( \beta \approx -2 \), the range from below \( \sim 10^4 M_\odot \) to above \( \sim 10^5 M_\odot \) (e.g., Zhang & Fall 1999; Hunter et al. 2003; Fall & Chandar 2012; Krumholz et al. 2015; Linden et al. 2017). Of course, to keep the total mass of clusters in a galaxy finite, mass functions with \( \beta \approx -2 \) must have both upper and lower cutoffs. The lower cutoff likely lies near the transition between individual stars and clusters of stars at \( \sim 10^3 M_\odot \) (e.g., Krumholz 2017). The upper cutoff is the subject of this paper.

As is customary, we represent the mass function of young star clusters by the Schechter (1976) function, \( \psi(M) = (\psi_\star/M_\star) \psi_\star M^\beta \exp(-M/M_\star) \), i.e., a power law with an exponent \( \beta \) and an exponential cutoff at \( M \approx M_\star \). Fall & Zhang (2001) introduced the Schechter mass function into this field in a theoretical study of the long-term disruption of star clusters. Their models match the observed mass function of globular clusters (with peaks at \( M_\star \sim 10^5 M_\odot \)) after \( \sim 10^8 \) yr of evolution if the initial mass function has almost any shape, including a power law, and an exponential cutoff at \( M_\star \sim \) few \( \times 10^5 M_\odot \) (meaning \( 10^6 M_\odot \lesssim M_\star \lesssim 10^7 M_\odot \); see also Chandar et al. 2007; Jordán et al. 2007; McLaughlin & Fall 2008; Goudfrooij & Fall 2016).

This prompted a search for upper cutoffs in the observed mass functions of recently formed clusters. Several such studies have claimed to detect cutoffs near \( M_\star \sim 10^5 M_\odot \) (e.g., Gieles 2009; Larsen 2009; Portegies Zwart et al. 2010; Adamo et al. 2015; Messa et al. 2018), far below the cutoff predicted by the Fall & Zhang (2001) models. However, these claimed detections do not appear convincing by eye and have not been confirmed by robust statistical tests. The tests that have been performed are based on binned data and/or cumulative distributions. The purpose of this paper is to remedy this situation by performing maximum likelihood fits of the Schechter function to the mass data for young clusters in eight well-studied galaxies, including some spirals and irregulars (LMC, M83, and M51) where cutoffs at \( M_\star \sim 10^5 M_\odot \) have been claimed (e.g., Larsen 2009; Adamo et al. 2015; Messa et al. 2018).

The remainder of this paper is organized as follows. In Section 2, we describe the cluster samples, mass estimates, and mass distributions we use in this study. In Section 3, we describe the likelihood analysis we use to derive best-fit values and confidence contours for the parameters \( \beta \) and \( M_\star \) in the Schechter function. We summarize our results and discuss their implications in Section 4.

2. Cluster Mass Functions

We re-examine the mass functions of the cluster populations in eight galaxies from our previous studies (Chandar et al. 2015, 2017). The selection and photometry of clusters are based on UBVI\( \alpha \) images taken with the Hubble Space Telescope (HST) for NGC 4214 (Chandar et al. 2017), NGC 4449 (Rangelov et al. 2011), M83 (B. Whitmore et al. 2018, in preparation), M51 (Chandar et al. 2016), the Antennae (Whitmore et al. 2010), and NGC 3256 (Mulia et al. 2016), and UBV images taken with the Michigan Curtis Schmidt telescope for the Large and Small Magellanic Clouds (LMC and SMC; Hunter et al. 2003). Clusters were selected to be compact, but no attempt was made to distinguish bound from unbound clusters based on their appearance. The number of clusters in each galaxy varies from a few hundred (e.g., NGC 4449 and NGC 4214) to many thousands (e.g., M83, M51, the Antennae). The mass and age of each cluster were estimated by comparing the observed shape of the spectral energy distribution with predictions from the Bruzual & Charlot (2003) stellar population
Figure 1. Cluster mass functions with equal logarithmic bins in three age intervals, \( \tau < 10 \) Myr (circles), \( \tau = 10-100 \) Myr (triangles), and \( \tau = 100-400 \) Myr (squares) for the eight galaxies in our sample (as indicated). The straight lines show the best-fit power laws to each distribution, \( \psi(M) \propto M^\beta \) (reproduced from Figure 4 of Chandar et al. 2017). These binned mass functions are for visualization purposes only. All Schechter parameters are determined from the unbinned mass estimates of individual clusters using the maximum likelihood analysis described in Section 3.

The major uncertainty in the mass estimates of young clusters comes from uncertainty in the ages, through the age-dependent mass-to-light ratios from the stellar population models. Clusters with ages \( \tau \lesssim 10 \) Myr and \( \tau \approx 100-400 \) Myr have typical uncertainties of \( \sim 0.3 \) in \( \log M \), corresponding to a factor of \( \sim 2 \) in \( M \) (e.g., Hunter et al. 2003; Fall et al. 2005; deGrijs & Anders 2006; Chandar et al. 2010). The uncertainties may be larger for clusters with ages in the interval \( \tau = 10-100 \) Myr, where the stellar population models show loops in the color–color space, potentially leading to non-unique ages and hence mass estimates. Errors in the distance or assumed stellar IMF will not affect the shape of the cluster mass function, although they will affect its normalization. Stochastic fluctuations in the luminosities and colors of clusters may affect determinations of the mass function below \( \sim 3 \times 10^3 M_\odot \), but not at the higher masses of interest here (Fouesneau et al. 2012; Krumholz et al. 2015; Krumholz & Ting 2018).

As our previous studies, we divide the clusters into three age intervals: \( <10 \) Myr, \( 10-100 \) Myr, and \( 100-400 \) Myr (Chandar et al. 2015, 2017). The oldest age interval of \( 100-400 \) Myr is best suited to characterizing the cluster mass functions, because it is well populated, has reliable mass estimates, and has uniform completeness. In contrast, the middle age interval (\( 10-100 \) Myr) has uncertain and non-uniform mass estimates, and the youngest age interval (\( <10 \) Myr) has potential incompleteness due to dust obscuration and crowding (Chandar et al. 2014).

Figure 1 shows the binned mass functions of the cluster populations in our eight galaxies in the three age intervals: \( <10, 10-100, \) and \( 100-400 \) Myr (reproduced from Chandar et al. 2017). All of these mass functions are well represented by power laws, \( \psi(M) \propto M^\beta \) with \( \beta \approx -2 \), and have no obvious bends or breaks. Thus, any upper cutoff must occur near or beyond the maximum observed mass \( M_{\text{max}} \) in each sample of clusters. This circumstance raises the question of whether \( M_{\text{max}} \) is determined by a physical cutoff, as in the Schechter function, or by a statistical cutoff linked to the sample size.

Figure 2 shows the observed maximum mass (\( M_{\text{max}} \)) plotted against the observed total mass (\( M_{\text{tot}} \)) in clusters more massive than \( 10^4 M_\odot \) (a proxy for sample size) in each of the three age intervals and eight galaxies. We have also plotted the predicted statistical relations between \( M_{\text{max}} \) and \( M_{\text{tot}} \) for a

\[ \text{Observed correlation between maximum cluster mass } M_{\text{max}} \text{ and the total mass in clusters } M_{\text{tot}} \text{ (with } M > 10^4 M_\odot \) for the eight galaxies in our sample. Triangles represent \( <10 \) Myr age bins, squares represent \( 10-100 \) Myr bins, and circles represent \( 100-400 \) Myr bins. Arrows indicate age bins with only lower limits on \( M_{\text{tot}} \) since their incompleteness limits are above \( 10^4 M_\odot \). Uncertainties of 0.3 are shown in the bottom right: a typical value for \( M_{\text{max}} \) and an upper limit for \( M_{\text{tot}} \). The dotted and dotted-dashed red lines show the predicted statistical relations, respectively, for a pure power-law mass function with \( \beta = -2 \) and a Schechter mass function with \( \beta = -2 \) and \( M_\alpha = 10^4 M_\odot \). The gray bands show the corresponding 95% confidence regions as derived by bootstrap sampling. Evidently, most of the data points are consistent with sampling from a pure power-law mass function (with \( M_\alpha \rightarrow \infty \)).

...models, assuming a Chabrier (2003) stellar initial mass function (IMF) and a Milky Way extinction curve (Fitzpatrick 1999). Details of the observations, data reduction, and the cluster catalogs can be found in Chandar et al. (2017) and the references therein.  

\[ \text{As in our previous studies, we divide the clusters into three age intervals: } <10 \text{ Myr, } 10-100 \text{ Myr, and } 100-400 \text{ Myr (Chandar et al. 2015, 2017). The oldest age interval of } 100-400 \text{ Myr is best suited to characterizing the cluster mass functions, because it is well populated, has reliable mass estimates, and has uniform completeness. In contrast, the middle age interval (10-100 Myr) has uncertain and non-unique mass estimates, as mentioned above, while the youngest age interval (<10 Myr) has potential incompleteness due to dust obscuration and crowding (Chandar et al. 2014).} \]
pure power law with $\beta = -2$ and a Schechter function with $\beta = -2$ and $M_\ast = 10^5 M_\odot$ (red dotted and dotted-dashed lines, respectively). These were computed from the requirement that the expected number of clusters more massive than $M_{\text{max}}$ be unity. For the pure power law with $\beta = -2$, the $M_{\text{max}}$-$M_{\text{tot}}$ relation takes the particularly simple form

$$M_{\text{tot}} = M_{\text{max}} \times \ln \left( \frac{M_{\text{max}}}{10^4 M_\odot} \right).$$

(1)

Using the methodology in Chapter 5.3 of Bevington & Robinson (2003), we generate mock cluster catalogs from a power-law mass function with $\beta = -2$ and a Schechter function with $\beta = -2$ and $M_\ast = 10^5 M_\odot$. Drawing 1000 random realizations to match key values of $M_{\text{tot}}$, we plot connected gray bands in Figure 2 showing the region encompassing 95% of the samples. Overall, the observed correlation between $M_{\text{max}}$ and $M_{\text{tot}}$ for our sample appears to follow more closely the predicted relation for a pure power law than that for a Schechter function with $M_\ast \sim 10^5 M_\odot$.

3. Maximum Likelihood Fits

We now determine the best-fit values and confidence intervals of the parameters $\beta$ and $M_\ast$ in the Schechter function by the method of maximum likelihood. This method has the advantages of not requiring binned data (where weak features at the ends of the distribution may be hidden) or cumulative distributions (where the data points are not independent of one
Figure 4. Likelihood fits of the Schechter parameters $\beta$ and $M_*$ to the masses of clusters in the three age intervals, <10 Myr (left), 10–100 Myr (center), and 100–400 Myr (right), in the four galaxies: M83, M51, the Antennae, and NGC 3256. The dashed lines show the best-fit values of $\beta$ and $M_*$, while the boundaries of the shaded regions show the $1\sigma$, $2\sigma$, and $3\sigma$ confidence contours.

Table 1

| Galaxy     | <10 Myr | 10–100 Myr | 100–400 Myr |
|------------|---------|------------|-------------|
| LMC        | $1.42 [1.15, 1.65]$ | $1.57 [0.85, 2.15]$ | $1.65 [1.25, 2.00]$ |
| SMC        | $1.88 [1.40, 2.30]$ | $1.51 [0.00, 2.70]$ | $0.95 [0.00, 2.40]$ |
| NGC 4214   | $2.22 [1.45, 2.85]$ | $0.24 [0.00, 2.00]$ | $1.67 [1.25, 2.05]$ |
| NGC 4449   | $0.55 [0.00, 1.55]$ | $1.93 [0.00, 3.00]$ | $2.00 [1.50, 2.55]$ |
| M83        | $1.90 [1.55, 2.20]$ | $1.43 [1.15, 1.70]$ | $2.38 [1.85, 2.80]$ |
| M51        | $1.82 [1.65, 2.00]$ | $1.58 [1.25, 1.90]$ | $1.85 [1.60, 2.10]$ |
| Antennae   | $2.16 [2.05, 2.25]$ | $1.81 [1.65, 1.95]$ | $2.29 [2.15, 2.40]$ |
| NGC 3256   | $1.65 [1.10, 2.10]$ | $0.92 [0.30, 1.50]$ | $1.72 [1.25, 1.95]$ |
We follow the procedure described in detail in Chapter 15.2 of Mo et al. (2010) for fitting a Schechter function to discrete luminosity or mass data. Specifically, we compute the likelihood

\[ L(\beta, M_{\ast}) = \prod_{i} P_{i} \]

as a function of \( \beta \) and \( M_{\ast} \), where the probability \( P_{i} \) for each cluster is given by

\[ P_{i} = \frac{\psi(M_{i})}{\int_{M_{\min}}^{M_{\max}} \psi(M) dM}, \]

and the product is over all clusters in the sample in question. We adopt the \( M_{\min} \) values listed in Table 4 of Chandar et al. (2017), which lie entirely above the completeness limit of each sample, and we set \( M_{\max} = 10^{5} M_{\odot} \) in all cases. We use the Nelder & Mead (1965) algorithm to find the maximum likelihood \( L_{\max} \) and the standard formula

\[ \ln L(\beta, M_{\ast}) = \ln L_{\max} - \frac{1}{2} \chi_{p}^{2}(k), \]

where \( \chi_{p}^{2}(k) \) is the chi-squared distribution with \( k \) degrees of freedom at \( p \) confidence level (Mo et al. 2010) to derive the corresponding confidence contours. We have checked all our results using a Markov chain Monte Carlo (MCMC) routine and find similar contours whenever these close around best-fit values of \( \beta \) and \( M_{\ast} \). For the cases of contours that extend to the right edge, the corresponding contours derived from the MCMC routine are slightly smaller than those derived from Equation (3), leading to tighter lower limits on \( M_{\ast} \).
Figures 3 and 4 show the best-fit values of $\beta$ and $M_*$ (dashed lines), and the 1$\sigma$, 2$\sigma$, and 3$\sigma$ confidence contours (shaded regions) derived from Equation (3) for each of the three age intervals, <10 Myr (left panels), 10–100 Myr (middle panels), and 100–400 Myr (right panels), and for each galaxy in our sample. The best-fit values of $\beta$ and $M_*$ and their 2$\sigma$ uncertainties are listed in Table 1. The resulting shapes of the confidence contours partly reflect correlations between the exponent $\beta$ and cutoff $M_*$, since there is a trade-off between steeper $\beta$ and larger $M_*$, and vice versa.

Thus far, we have neglected uncertainties in the mass estimates of the clusters. The uncertainties in the Schechter parameters $\beta$ and $M_*$ shown in Figures 3 and 4 and listed in Table 1 are, therefore, actually lower limits to the true uncertainties. To determine how much uncertainties in the mass estimates affect the fitted parameters, we convolve the Schechter mass function with a log-normal error distribution (Gaussian in log $M$), with a standard deviation of $\sigma$ (log $M$) (see Efstathiou et al. 1988; Ratcliffe et al. 1998). We adopt $\sigma$(log $M$) = 0 (as before), 0.15, and 0.30 (the typical uncertainty in individual mass estimates discussed in Section 2). In all cases, we find that the best-fit value of $M_*$ increases, and its statistical significance decreases with increasing $\sigma$(log $M$). As an example, we show for the results for the 100–400 Myr age bin in M51 in Figure 5. Evidently, the possible detection of an upper cutoff at $M_* \sim 10^4 M_\odot$ for $\sigma$(log $M$) = 0 disappears for $\sigma$(log $M$) = 0.3.

Next, we check whether the fitted parameters $\beta$ and $M_*$ are robust with respect to different cluster catalogs by repeating our analysis with the Silva-Villa et al. (2014) catalog of M83 clusters and the Legacy Extragalactic Ultraviolet Survey (LEGUS) catalog of M51 clusters (see Massa et al. 2018). The results of these tests are shown in Figure 6. Compared to the confidence contours for our cluster catalogs, those for the other catalogs are generally similar or less restrictive. In particular, the 2$\sigma$ confidence contour is unbounded for the 100–400 Myr clusters in the LEGUS catalog of M51 clusters.

4. Discussion and Conclusions

The main results of this paper are displayed graphically in Figures 3 and 4. Before discussing these results in detail, we offer a few general remarks. Ideally, we should find consistent estimates of, or limits on, the Schechter parameters $\beta$ and $M_*$ in all three age intervals, because we do not expect the physics of cluster formation to change significantly over the relatively short period spanned by our data ($4 \times 10^8$ yr, i.e., $\sim$3% of the Hubble age). Thus, it would be physically implausible for $M_*$ to increase with age, although it could, in principle, decrease as a result of the preferential disruption of the most massive clusters. Nevertheless, systematic errors potentially affect mass estimates and sample completeness differently in the three age intervals. The upper cutoff $M_*$ is particularly sensitive to the presence or absence of only a few clusters and any errors in their masses.

As we have already noted, sample completeness is likely lowest in the youngest age interval (<10 Myr), due to dust obscuration and crowding, while systematic errors and non-uniqueness in mass estimates are likely highest in the middle age interval (10–100 Myr) due to loops in the color tracks of stellar population models. Moreover, the middle age interval also tends to have the smallest number of clusters and thus the largest sampling errors. This leaves the oldest age interval (100–400 Myr) as the most reliable one for determining the parameters $\beta$ and $M_*$ in the Schechter mass function. This age interval is well populated with clusters, has a higher degree of completeness, and has more reliable mass estimates.

With these remarks in mind, we now group the results shown in Figures 3 and 4 into three broad categories based largely on our likelihood analysis in the oldest age interval. The first category, which includes the LMC, SMC, NGC 4214, and M83, shows no evidence for a cutoff. For these galaxies, a wide range of $M_*$ is allowed by the long horizontal confidence contours that start below $10^5 M_\odot$ and continue without closing to the right edge of the diagrams at $10^5 M_\odot$. This means that the cluster masses are consistent with being drawn from a pure power law, but that an upper cutoff (over this mass range of $M_*$) cannot be ruled out. We note that the large allowable ranges in $M_*$ for the LMC, SMC, and NGC 4214 are driven, in part, by the relatively small numbers of clusters in these galaxies.

A second category, which includes NGC 4449, the Antennae, and NGC 3256, shows evidence against an upper mass cutoff near $M_* \sim 10^5 M_\odot$. While the confidence contours for these galaxies also remain unbounded up to the maximum adopted value of $10^5 M_\odot$, they do not extend down to $10^5 M_\odot$. The youngest age interval in NGC 4449 has closed contours that suggest a value of $M_* \sim 10^4 M_\odot$, but this is inconsistent with the contours for the oldest age interval. This galaxy, in particular, violates the physical principle noted above that $M_*$ should not increase rapidly with age.

M51 is the only galaxy in our sample that shows evidence, at the $\sim$3$\sigma$ level, for a cutoff near $M_* \sim 10^4 M_\odot$, when no uncertainties on cluster mass estimates are included. However, as shown in Figure 5, when realistic errors in mass estimates ($\sim$0.3 log $M$) are included, no statistically significant cutoff near $10^5 M_\odot$ is found in M51. It is worth noting that this cutoff could also be explained if only a few clusters were missing from the catalogs as a result of dust obscuration and/or crowding. Furthermore, in a sample of eight galaxies, there is a nonnegligible probability (34/2.4%) that an upper cutoff will be detected with marginal significance ($2\sigma/3\sigma$) in one of them (e.g., M51), even if the underlying mass function of clusters is a pure power law.

In conclusion, there are four galaxies in our sample (LMC, SMC, NGC 4214, and M83) for which a wide range of cutoff mass is permitted ($10^5 M_\odot \lesssim M_* \lesssim \infty$); one galaxy (NGC 4449) for which our analysis gives an unphysical result; and two galaxies (Antennae and NGC 3256) for which an upper mass cutoff at $10^5 M_\odot$ is excluded. Only for M51 is there a possible detection at $M_* \sim 10^5 M_\odot$, but even this becomes insignificant when we include realistic errors in cluster masses in our analysis. On the other hand, much higher cutoffs, at $M_* \sim$ few $\times 10^6 M_\odot$, are consistent with our likelihood analysis in nearly all cases. The higher cutoffs are needed to reconcile the mass functions of young clusters observed today with those of old globular clusters, assuming they formed by similar physical processes with similar initial mass functions, as in the Fall & Zhang (2001) models.

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