Constraining the mass of the graviton with the planetary ephemeris INPOP

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We use the planetary ephemeris INPOP17b to constrain the mass of the graviton in the Newtonian limit. We find that the residuals for the Cassini spacecraft significantly (90% C.L.) degrades for Compton wavelengths of the graviton smaller than 1.83 × 10^{13} km, corresponding to a graviton mass bigger than 6.76 × 10^{-23} eV/c^2. This limit is comparable in magnitude to the one obtained by the LIGO-Virgo collaboration in the radiative regime. We also use this specific example to illustrate why constraints on alternative theories of gravity obtained from postfit residuals are generically overestimated.

INTRODUCTION

From a particle physics point of view, general relativity can be thought as a theory of a massless spin-2 particle — hereafter named graviton. From this perspective, it is legitimate to investigate whether or not the graviton could actually possess a mass — even if minute. Such an eventuality has been scrutinized from a theoretical point of view since the late thirties, with the pioneer work of Fierz and Pauli [1]. However, such theories are usually plagued with severe theoretical issues [2, 3] that are not easily solved [4]. The current dominant approach to massive gravity is through a specific bi-metric theory [4–6] — because it intrinsically possesses a mechanism 1 that can reduce its departure from the phenomenology of general relativity in regimes where gravitation has been severely constrained already, and because it is free from the so-called Boulware-Deser ghost [3] at the non-perturbative level [5, 6]. However, another theory of massive gravity has been proposed in the late nineties by Visser [8], where some of the theoretical issues usually plaguing massive gravity theories are evaded by postulating the existence of a prior geometry. For this theory, the Newtonian potential would become a pure Yukawa potential, such that the line element in a space-time curved by a spherical massive object at rest would read at leading order in the Newtonian regime

\[ ds^2 = \left( -1 + \frac{2GM}{c^2 r} e^{-r/\lambda_g} \right) c^2 dt^2 + \left( 1 + \frac{2GM}{c^2 r} e^{-r/\lambda_g} \right) dr^2, \] (1)

with \[ dl^2 \equiv dx^2 + dy^2 + dz^2 \] and \( \lambda_g \) the Compton wavelength of the graviton 2. Obviously, as long as \( \lambda_g \) is big enough, the gravitational phenomenology in the Newtonian regime can reduce to the one of general relativity to any given level of accuracy. In what follows, we take the phenomenological point of view that if the graviton is massive, the Newtonian potential becomes a Yukawa potential, as in the theory of Visser.

However, if the graviton is massive, its dispersion relation is modified according to \[ E^2 = p^2 c^2 + m_g^2 c^4, \] such that the speed of a gravitational waves depends on its energy (or frequency) \[ \nu_g^2/c^2 = c^2 p^2/E^2 \simeq 1 - h^2 c^2/(\lambda_g^2 E^2). \] Therefore, the waveform of gravitational waves would be modified during their propagation, while at the same time, sources of gravitational waves have been seen up to more than 490Mpc (at the 90% C.L.) [9]. As a consequence, waveform match filtering can be used to constrain the graviton mass from gravitational waves detections [10, 11]. Combining the bounds obtained with GW150914 [12], GW151226 [13] and GW170104 [9] leads to \( \lambda_g > 1.6 \times 10^{13} \) km (resp. \( m_g \leq 7.7 \times 10^{-23} \) eV/c^2) at the 90% C.L 4 [9]. It is important to keep in mind that this limit is obtained in the radiative regime, while we focus here on the Newtonian regime. Although, one could expect \( \lambda_g \) to have the same value in both regimes for most massive gravity theories, it may not be true for all massive gravity theories. Therefore, both constraints should be considered independently from an agnostic point of view.

\[ \text{Note that it is different from a fifth force, for which the Yukawa potential is only a correction to the usual Newtonian potential.} \]

\[ \text{With the definition } m_g = h/(c\lambda_g). \]

\[ \text{Assuming that the graviton mass affect the propagation only, and not the binaries dynamics.} \]
IMPORTANCE OF A GLOBAL FIT ANALYSIS

Twenty years ago [10] and more recently [14], Will argued that solar system observations could be used to improve, or at least be comparable with the constraints on $\lambda_g$ obtained from the LIGO-Virgo Collaboration — assuming that the parameters $\lambda_g$ appearing in both the radiative and Newtonian limits are the same. A graviton mass would indeed lead to a modification of the perihelion advance of solar system bodies. Hence, based on current constraints on the perihelion advance of Mars derived from Mars Reconnaissance Orbiter (MRO) data, Will estimates that the graviton’s Compton wavelength should be bigger than $(1.4 - 2.7) \times 10^{24}{\text{km}}$ (resp. $m_g < (4 - 8) \times 10^{-24} \text{eV}/c^2$), depending on the specific analysis. However, making an estimation from postfit residuals cannot account for the fact that the various parameters of the ephemeris (e.g. masses, semi-major axes, Compton parameter, etc.) are all more or less correlated. (See TABLE I). For instance, while a graviton mass should induce a modification of the perihelion advances [14], the variation of many other parameters could also contribute to such a modification. As a consequence, a modification of a perihelion advance can in part be absorbed by the fit of other parameters that are correlated with the mass of the graviton $^5$. This leads to a decrease of the constraining power of the ephemeris on the graviton mass with respect to the naïve postfit estimate in [14]. As a corollary, all analyses based solely on postfit residuals tend to overestimate the constraints on alternative theories of gravity due to the lack of information on the correlations between the various parameters (see, e.g. Sec. 3 of [15]). Given the numerous constraints on various alternative theories of gravity that are deduced solely from the analyses of postfit residuals in the literature, our discussion hints to remind that such constraints are not conservative; but rather are overoptimistic estimates of the level at which a given parameter can be constrained from solar system observations. Eventually, one cannot produce conservative estimates of any parameter without going through the whole procedure of integrating the equations of motion and fitting the parameters with respect to actual observations — which is the very raison d’être of the ephemeris INPOP.

INPOP (Intégrateur Numérique Planétaire de l’Observatoire de Paris) [16] is a planetary ephemeris that is built by integrating numerically the equations of motion of the solar system, and by adjusting to solar system observations such as lunar laser ranging or space missions observations. In addition to adjusting the astronomical intrinsic parameters — like the mass of astronomical bodies — it can be used to adjust parameters that encode deviations from general relativity [17–20], such as $\lambda_g$. The latest released version of INPOP, INPOP17a [21], benefits of an improved modeling of the Earth-Moon system, as well as an update of the observational sample used for the fit [20] — especially including the latest Mars orbiter data. For this work we use an extension of INPOP17a, called INPOP17b, fitted over an extended sample of Messenger data up to the end of the mission, provided by [22].

In the present communication, our goal is to use the latest planetary ephemeris INPOP17b in order to constrain a hypothetical graviton mass directly at the level of the numerical integration of the equations of motion and the resulting adjusting procedure. By doing so, the various correlations between the system parameters are intrinsically taken into account, such that we can deliver a conservative constraint on the graviton mass from solar system observations.

MODELISATION FOR SOLAR SYSTEM PHENOMENOLOGY

Following Will [14], we develop pertubatively the potential in terms of $r/\lambda_g$, such that the line element (1) now reads

$$ds^2 = \left( -1 + \frac{2GM}{c^2 r} \left[ 1 + \frac{1}{2} \frac{r^2}{\lambda_g^2} \right] \right) c^2 dt^2 + \left( 1 + \frac{2GM}{c^2 r} \left[ 1 + \frac{1}{2} \frac{r^2}{\lambda_g^2} \right] \right) dl^2 + O(c^{-3} \lambda_g^{-2}),$$

albeit with a change of coordinate system $t \rightarrow t \sqrt{1 + \frac{GM}{c^2 \lambda_g^2}}$ and $x^i \rightarrow x^i \sqrt{1 - \frac{GM}{c^2 \lambda_g^2}}$. The change of coordinate system is meant to get rid of the non-observable constant terms that appear in the line element Eq. (1) after expanding in terms of $\lambda_g^{-1}$. Considering an N-body system, the resulting additional acceleration to incorporate in INPOP’s code is

$$\delta a^i = \frac{1}{2} \sum_p \frac{GM_p}{\lambda_g^2} \frac{x^i - x^i_p}{r} + O(\lambda_g^{-3}),$$

| $\lambda_g$ | $\alpha$ a Mercur $\alpha$ Mars $\alpha$ Saturn $\alpha$ Venus $\alpha$ EMB $GM\odot$ |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1 | 0.50 | 0.49 | 0.04 | 0.39 | 0.05 | 0.66 |
| a Mercur | \ldots | 1 | 0.21 | 0.001 | 0.97 | 0.82 | 0.96 |
| a Mars | \ldots | \ldots | 1 | 0.03 | 0.29 | 0.53 | 0.06 |
| a Saturn | \ldots | \ldots | \ldots | 1 | 0.003 | 0.02 | 0.01 |
| a Venus | \ldots | \ldots | \ldots | 1 | 0.86 | 0.94 |
| a EMB | \ldots | \ldots | \ldots | 1 | 0.73 |
| $GM\odot$ | \ldots | \ldots | \ldots | \ldots | 1 |

TABLE I. Examples of correlations between various INPOP17b parameters. $\alpha$, EMB and $M\odot$ state for semi-major axes, the Earth-Moon barycenter and the mass of the Sun respectively.

$^5$ Of course, the more (different types of) data, the less correlations between the various parameters of the ephemeris.
where $M_P$ and $x^P_\nu$ are respectively the mass and the position of the gravitational source $P$. In Visser theory [8], graviton behaves like ordinary matter which has a conserved momentum-energy tensor. Moreover, the graviton term of the action does not couple with matter — including the electromagnetic field. Thus, light propagates along null geodesics in the theory of Visser. In what follows, we make the same assumption. From the null condition $ds^2 = 0$ and Eq. (2), the resulting additional Shapiro delay at the perturbative level reads

$$\delta T_{ER} = \frac{1}{2} \sum_P G M_P \frac{\lambda^g}{c^3 \lambda^2} \left[ \bar{N}_{ER} \cdot (\bar{R}_{PR} R_{PR} - \bar{R}_{PER} R_{PE}) \right] + b_P^2 \ln \left( \frac{R_{PR} + \bar{R}_{PR} \cdot \bar{N}_{ER}}{R_{PE} + \bar{R}_{PE} \cdot \bar{N}_{ER}} \right) + \mathcal{O}(e^{-3 \lambda^g}) \tag{4},$$

where $\bar{R}_{XY} = \bar{x}_Y - \bar{x}_X$, $R_{XY} = |\bar{R}_{XY}|$, $\bar{N}_{XY} = \bar{R}_{XY} / R_{XY}$ and $b_P = \sqrt{R_{PE}^2 - (\bar{R}_{PE} \cdot \bar{N}_{ER})^2}$. One can notice that the correction to the Shapiro delay scales as $(L_c / \lambda_g)^2$ with respect to the usual delay, where $L_c$ is a characteristic distance of a given geometrical configuration. Given the old acknowledged constraint from solar system observations on the graviton mass ($\lambda_g > 2.8 \times 10^{12} \text{ km}$ [10, 14, 23]), one deduces that the correction from the Yukawa potential on the Shapiro delay is negligible for past, current and forthcoming radio-science observations in the solar system.

**NUMERICAL ANALYSIS**

To model and confront the massive graviton to solar system observations, we add to the INPOP17b modeling (for the eleven objects of the solar system) the contribution of the massive graviton (Eq. (3)) and fit the new obtained ephemeris according to the procedure described in [21]. For a fixed value of $\lambda_g$, we fit the parameters of the model to the data.

We numerically compute the partial derivatives matrix of the residuals of the reference solution. We have performed an iterative fit of the INPOP17b parameters of the ephemeris, including asteroid masses, for each given values of $\lambda_g$. For each value of $\lambda_g$ we compute the standard deviation of range observations. More specifically, we exhibit residuals obtained with observations for Cassini mission, Messenger mission, and Mars Odyssey and Mex mission. Other observations are less relevant due to less accurate data and/or high correlation with

\[ \lambda_g. \] This algorithm processes for 1024 different fixed values of $\lambda_g$ between $1 \times 10^{12}$ and $8 \times 10^{13}$ km. We plot the different standard deviations with respect to $\lambda_g$ for each iteration. We remove the values of the reference solution standard deviations [7] that are listed in Table II. Indeed, in Table II, are given the $1\sigma$ standard deviations of INPOP17b residuals together with the $1\sigma$ differences obtained between INPOP17b and the Jet Propulsion Laboratory ephemeris DE436 [8] geocentric distances on the time interval of the data sample. The latter gives an idea of the internal accuracy of the reference ephemeris itself. At about the 10th iteration, the standard deviations for the three sets of residuals stop evolving — meaning that the adjustment has converged. We report the plot of the standard deviations of the last iteration (the 13th) in figure 1.

An important point to have in mind is that Mars data constraints the global fit of parameters to observations — thanks to the important weight in the fit given to the Mars Odyssey and Mex missions accurate data. So, while in Will’s analysis [14], the high quality of Martian data is expected to allow the best constraints on $\lambda_g$, this high quality actually helps to better adjust the whole set of parameters — but they do not significantly constrain $\lambda_g$. Alternatively, given the fact that Saturn’s semi-major axes is less correlated to $\lambda_g$ (see Table I), it is not surprising to see in figure 1 that Saturn positions

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6 However, note that the scaling of the correction to the Shapiro delay illustrates the breakdown of the $\lambda_g^{-1}$ development in cases where the characteristic distances involved are large with respect to the Compton wavelength — as it should be expected.

7 i.e. for $\lambda_g = 0$

8 Which is based on DE430 ([24]).
deduced from the Cassini observations are actually the most constraining on $\lambda_g$.

After 12 iterations, only residuals deduced from Saturn positions obtained with Cassini show important deviations at a high value of $\lambda_g$. On the other hand, all the values of standard deviation of Mars data are below 1.5 m higher than the reference value. Around $\lambda_g = 1.5 \times 10^{13} km$, Messenger data goes a little above 3 m higher than the reference value, but decrease then as $\lambda_g$ decreases, while Mars standard deviation does exactly the opposite — indicating a compensating mechanism between the two sets of standard deviations, whose controlling parameters are indeed highly correlated. Of course, the compensating mechanism actually is across the whole set of residuals and depends on both the weights attributed to the different data and to the correlations between various parameters. In the end, only residuals deduced from Cassini show a significant (and monotonic) increase as $\lambda_g$ decreases, as one can see in Fig. 1.

### EVALUATION OF THE SIGNIFICANCE OF THE RESIDUALS DETERIORATION

To give a confidence interval for $\lambda_g$, we proceed as follows. Residuals are computed for each value of $\lambda_g$; at the same dates that for the reference solution INPOP17b — therefore, for the same number of observations (see Tab. 1). We perform a Pearson $[25]$ $\chi^2$ test between both residuals in order to look at the probability that they were both built from the same distribution. To compute $\chi^2$ we proceed as follows. We build an optimal histogram with the Cassini residuals of INPOP17b using the method described in [26], assuming the gaussianity of the distribution of the residuals. It consists in determining the optimal bins in which are counted the residuals to build the histogram. Then, using the same bins, we build an histogram for the Cassini residuals obtained by the solution to be tested with a given value of $\lambda_g$. Note that the first bin left-borned is $-\infty$ and the last bin right-borned is $+\infty$. Let $(C_i)_i$ be the bins in which are counted the values of the residuals and $N_i^F, N_i^G$ be the number of residuals of INPOP17b and the solution to be tested, respectively, counted in bin number $i$. One can then compute

$$\chi^2(\lambda_g) = \sum_{i=1}^{n} \frac{(N_i^G - N_i^F)^2}{N_i^G}$$

For our Cassini data, it occurs that the optimal binning gives 10 bins. As a result, this $\chi^2$ follows a $\chi^2$ law with 10 degrees of freedom. If the computed $\chi^2$ is then greater than its quantile for a given confidence probability $p$, we can say that the distribution of the residuals obtained for $\lambda_g$ is different from the residuals obtained by the reference solution with a probability $p$. This test can be done for both a positive detection of a physical effect and a rejection of the existence of a physical effect. If the computed $\chi^2(\lambda_g)$ becomes then greater than its critical value for a probability $p$, one has to check if residuals are smaller or bigger than those obtained by the reference solution. In the first case (smaller – or better – residuals), it means that the added effect increases significantly the quality of the residuals and is probably (with a probability $p$) a true physical effect. On the contrary, in the second case (bigger – or degraded – residuals), it means that the added effect is probably physically false.

In our work, the critical increasing of $\chi^2(\lambda_g)$ corresponds to a degradation of the residuals (compare figures 1 and 2). Then the massive graviton can be rejected for high enough values of the mass (or low enough values of $\lambda_g$).

### RESULTS AND DISCUSSION

In figure 2 we plot $\chi^2$ as a function of $\lambda_g$. In this plot, we give two values of quantiles associated to two probabilities of significance, $p = 90\%$ and $p = 99, 9999999\%$, which correspond to critical values of $\chi^2$ equal to 15.99 and 62.94 respectively for a 10 degrees of freedom $\chi^2$ distribution. We obtain respectively $\lambda_g > 1.83 \times 10^{13}$ km (resp. $m_g < 6.76 \times 10^{-23}$ eV/c$^2$) and $\lambda_g > 1.66 \times 10^{13}$ km (resp. $m_g < 7.45 \times 10^{-23}$ eV/c$^2$). They correspond to standard deviations of 34.0 m and 42.1 m respectively.

These results are shown in Fig. 2. We also provide a zoom of the main figure in order to show that the $\chi^2$ is not monotonic for small differences of $\lambda_g$. However,
FIG. 2. Plot of $\chi^2(\lambda_g)$ and the constraints deduced for $\lambda_g$. The probabilities $p = 90\%$ and $p = 99.9999999\%$ correspond to critical values of $\chi^2$ equal to respectively 15.99 and 62.94 respectively.

if a given limit is crossed several times, our algorithm automatically takes the most conservative value in the discrete set of $\lambda_g$, as can be seen in Fig. 2.

We note that the constraint given by our $\chi^2$ method corresponds to an increase of Cassini standard deviation comparable to the standard deviation of the differences between INPOP17b and DE436 (Table II) : respectively $\Delta \sigma = +10.0 \text{ m}$ and $\delta \sigma = 11.7 \text{ m}$. This results suggests that our constraint on $\lambda_g$ is robust.

In [14], Will uses uncertainties from postfit residuals of several ephemerides for which the parameter spaces have (not always) been extended in order to encompass some alternative theory parameters: the two first post-Newtonian parameters $\gamma$ and/or $\beta$, Nordtvedt parameter $\eta = 4\beta - \gamma - 3$ implied in the violation of the strong equivalence principle in metric theories, and/or also a variation of the gravitation constant via a sun mass parameter variation $\mathrm{d}\ln (GM_\odot)/\mathrm{d}t$ — but not a graviton mass [18, 19, 27–31]. While we already explained why using postfit residuals in order to constrain a parameter that was not considered in the initial fit of the ephemerides is conceptually problematic, we can use the fact that Will used the results of three previous versions of the INPOP ephemerides [18, 19, 28], among others, in order to illustrate this further. All the bounds found in [14] from postfit residuals of older versions of INPOP are superior — between a factor 1.4 and 2.0 — to our current 90\% C.L. bound, which was however obtained with a newer and better ephemeris solution that uses more data. This shows that the bounds from the analyses of the postfit residuals obtained in [14] overestimate the actual bound that can be inferred from the latest data — although a complete comparison between the bounds is difficult given the fact that the various analyses use different methods in order to constrain their (different) non-general relativity parameters. However, the deduction that analyses based on postfit residuals overestimate constraints is consistent with previous studies in other frameworks of alternative theories of gravity (see, e.g., Sec. 3 in [15]). Given that it is also a legitimate expectation from a theoretical data analysis point of view, we encourage the community to deal with constraints from postfit analysis with great care, and not to consider those constraints as being conservative.

CONCLUSION

In the present manuscript, we deliver the first conservative estimate of the graviton mass from an actual fit of a combination of solar system data, using a criterion based on a state of the art solar system ephemerides: INPOP17b. The bound reads $\lambda_g > 1.83 \times 10^{13} \text{ km}$ (resp. $m_g < 6.76 \times 10^{-23} \text{ eV}/c^2$) with a confidence of 90\% and $\lambda_g > 1.66 \times 10^{13} \text{ km}$ (resp. $m_g < 7.45 \times 10^{-23} \text{ eV}/c^2$) with a confidence of 99.9999999\%.

Finally, let us note that it is a surprise that our 90\% C.L. bound is, by coincidence, comparable in magnitude to the one obtained by the LIGO-Virgo collaboration in the radiative regime [9]. Indeed, not only do the two bounds rely on totally different types of observation — gravitational waves versus radioscience in the solar system — but they also probe different aspects of the massive graviton phenomenology — radiative versus Keplerian.

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