Numerical simulation of erosion for solid-liquid two-phase flow in u-tube

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Abstract. Study predicts erosion damage caused by solid particles impact on the inner surface of u-tube. Based on finite volume method, study uses numerical simulation of Lagrange method to discuss different inlet velocity (from 2.5 m/s to 5.5 m/s) of fluid and mass flow rate discrete phase which affect the location and ratio of erosion. The results show that: with other parameters of fluid fixed, the mean erosion rate on the pipe wall increases from \(8.72 \times 10^{-8}\) kg/m².s to \(7.3 \times 10^{-8}\) kg/m².s with a growth exponentially when the inlet velocity increases from 2.5m/s to 5.5m/s. With other parameters fixed, the facet average erosion rate increased from \(1.85 \times 10^{-8}\) kg/m².s to \(8.68 \times 10^{-8}\) kg/m².s with a liner growth. Moreover, the elbow of pipe appears obvious erosion phenomenon. Especially, due to centrifugal force, the erosion rate on lateral radius wall is significantly greater than the inner. The concentrated erosion region tends toward get wider as flow rate and mass flow rate of discrete phase increase.

1. Introduction

The system of heat exchanger with complicated mechanism has various failure modes. The erosion damage of wall includes the deformation of the pipe wall and cutting. In working conditions, the impact of solid particles and the cavitation of the fluid are the main reasons causing erosion damage, especially solid- liquid two-phase flow erosion. The particles in the fluid can not only reduce the flow rate of the fluid and efficiency of heat exchange, but also cause the leakage of fluid and other safety accidents [1]. The u-tube heat exchanger is a common type of shell-and-tube heat exchanger, which consists of u-tube and shell. The entire tube need to be extracted to clean, but it is difficult to remove dirt on the inner wall of tube.

The numerical simulation method can be used to study erosion damage directly. Shaboury [2] et al. analyzed the distribution of pressure and velocity with gas-liquid two-phase flow which predicted erosion damage in the three-way pipe. Based on the influence of impurities, some researchers began to study the two-phase flow containing solid particles. Cunkui Huang [3-4] et al. studied the theory of particle collision model on the surface of metal wall, which has a good agreement with the actual experiment. Wang guangqian [5] carried out an experimental study on the distribution of solid liquid two-phase flow velocity. Yuu [6] et al. established a two-phase turbulence model, and they used the empirical equation to give the average velocity with the motion equation of discrete particles. Danon [7] et al. proposed a turbulent model in two-phase turbulent flow based on a set of parabolic equations. In that model, the average velocity of particles was assumed to be equal to the average velocity of the fluid. In the process of numerical simulation of two-phase flow, discrete phase model is applied. The
DPM (discrete phase model) assumes that the second phase (particle phase) is relatively rare and interaction force between particles and the influence of particle volume fraction on the continuous phase are not considered [8]. This assumption asks that the volume fraction of the discrete phase is low, which is generally less than 10%, but mass rate of the particle can be greater than 10%. In the discrete phase model, the effect of continuous phase fluid on the discrete phase of solid particles is considered, while the interaction among solid particles and the influence of solid particles on the fluid is ignored.

At present, there are a few studies on erosion damage with the structure of u-tube. In this paper, the erosion model of two-phase flow in u-tube is established by finite volume method, where numerical simulation is used to investigate the erosion of solid-liquid two-phase flow with different inlet velocity.

2. Numerical simulation of solid-liquid two-phase flow

Numerical simulation has become an important method to study heat exchanger. This paper uses commercial software to simulate erosion damage on the surface of u-tube with liquid water containing solid particles (calcium carbonate).

2.1. Physical model

In order to simplify the simulation, only a u-tube is selected as the research object.

![Geometrical structure of u-tube.](image)

**Figure 1.** Geometrical structure of u-tube.

| No. | Name of parameters           | Symbol/Unit | Value  |
|-----|------------------------------|-------------|--------|
| 1   | external diameter            | d/mm        | 10     |
| 2   | horizontal size              | l/mm        | 250    |
| 3   | external bending radius      | r/mm        | 30     |
| 4   | inner bending radius         | r/mm        | 20     |
| 5   | Roughness of pipe            | Ra/m        | \(1\times10^{-5}\) |

The size of horizontal pipe is so long enough as the fluid flows in a stable state before entering the bend. The specific model parameters are shown in Table 1. Inlet velocity is selected as 2.5 m/s, 3 m/s, 3.5 m/s, 4 m/s, 4.5 m/s, 5 m/s and 5.5 m/s. The geometric model and basic dimensions of the u-tube are shown in Figure 1. The jet velocity of solid particles is consistent with the fluid at the entrance and location are evenly distributed in the entrance section of pipeline. For the grid division of u-tube, unstructured grid is adopted, as shown in Figure 2.
The rosin-rammler model was selected as the diameter distribution of particles. The distribution of diameter is shown as Figure 3. The properties and boundary conditions of fluid materials are given in Table 2 and Table 3. The particles with a diameter range of 60–70 μm are major part.

### Table 2. Material parameters.

|                | Continuous phase | Discrete phase |
|----------------|------------------|----------------|
| material       | Water(liquid)    | calcium carbonate |
| density        | 998.23 kg/m³     | 2800 kg/m³     |
| viscosity      | 1.01×10⁻³ Pa·s   | -              |
| concentration  | -                | 0.07 kg/s      |

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### Table 3. Boundary condition.

|                | Type            | Value          |
|----------------|-----------------|----------------|
| inlet          | Velocity inlet  | \( u_{in} \)   |
| outlet         | Pressure outlet  | 0.1013 MPa     |

where a no-slip boundary condition is imposed for fluid flow, i.e., the velocity closed to the wall surface is zero:

\[
\mathbf{u} = \mathbf{v} = \mathbf{w} = 0 \quad (1)
\]

The finite volume method is used to discretize governing equations, and the discrete equations are selected in the second order windward format. The coupling of pressure and velocity is handled by using SIMPLE algorithm.

### 2.2. Model assumptions

Owing to complexity of two-phase flow, some basic assumptions are necessarily proposed to simplify calculation:

1. The influence of temperature change can be ignored.
2. The particle is assumed to be a rigid sphere, and there will be no physical and qualitative changes in the flow process.
3. When calculating the interaction among discrete phase, discrete phase and fluid, the particles are regarded as spherical.
2.3. Mathematical model:
On the surface of u-tube inlet, the discrete phase is distributed evenly in the fluid. The continuity equation of the flow of fluid inclusion in the u-tube is shown following:

\[
\frac{\partial}{\partial t}(\alpha \rho) + \nabla \cdot (\alpha \rho u) = 0
\] (2)

Where \( i = L/S \), respectively, is the liquid phase and the solid phase; \( \alpha \) is fluid volume fraction, \%; \( \rho \) is fluid density, kg/m³; \( u \) is the velocity of the fluid, m/s.

The momentum equation:

\[
\frac{\partial}{\partial t}(\alpha \beta u) + \nabla \cdot (\alpha \beta u u) = -\nabla p + \nabla \cdot \tau + \alpha \beta g + \mathcal{M}
\] (3)

\[
\tau = \alpha_1 \mu (\nabla u + \nabla u^T) + \alpha_2 (\lambda - \frac{2}{3} \mu) (\nabla \cdot u) \cdot I
\] (4)

\[
\tau_s = -p \sigma + \alpha_S \mu \sigma (\nabla u_s + \nabla u_s^T) + \alpha_3 (\lambda - \frac{2}{3} \mu) (\nabla \cdot u_s) \cdot I
\] (5)

Where \( p \) represents static pressure, Pa; \( \sigma \) represents the positive stress between solid particles, Pa; \( \mathcal{M} \) represents the momentum exchange between the two phases; \( \mu \) represents dynamic viscosity; \( \lambda \) represents the viscosity coefficient; \( I \) represents the unit momentum.

2.4. Particle collision model
It is very important to select the appropriate near-wall area model for analysis of particles with small diameter. Discrete Random Walk model [9] (Discrete Random Walk, DRW) takes the influence of boundary layer and near wall turbulence into account. Based on the DRW model, the friction coefficient of the Blasius formula can be used to obtain the near-wall shear force which can be approximated as:

\[
\tau_s = 0.5 C_f \rho U^2
\] (6)

where \( C_f = a / \text{Re}_d^a \) is the coefficient of friction. \( \text{Re}_d = \rho U / \mu \) is the Reynolds number flowing in the u-tube, \( a \) and \( m \) are the constant factors of the Reynolds number. For \( \text{Re}_d \leq 10^5 \), \( a=0.079 \) and \( m=0.25 \); For \( \text{Re}_d \geq 10^5 \), \( a=0.046 \) and \( m=0.2 \).

The drag coefficient is \( C_D \):

\[
C_D = a_1 + \frac{a_2}{\text{Re}} + \frac{a_3}{\text{Re}^2}
\] (7)

where \( a_1 \), \( a_2 \) and \( a_3 \) are constants within the corresponding Reynolds number range. The erosion rate on wall boundary is obtained by Finnie [9] formula:

\[
\text{Re}_{erosion} = \sum_{p} \frac{\text{m} C(d_p) f(\alpha) b(\nu)}{A_{face}}
\] (8)

Where, \( C(d_p) \) is the function of particle diameter;

\( \alpha \) is the impact Angle of the particle on the wall;

\( f(\alpha) \) is a function of the impact angle; \( \nu \) is the particle relative wall velocity; \( b(\nu) \) is the particle relative velocity function; \( A_{face} \) is the surface area of the wall surface.

This model describes cutting mechanism in early period according to erosion experimental phenomenon and various results which assume that particles never broken during cutting steel.

Forder [10] studied the velocity changes of the solid particle which impact steel AISI 4130, and obtained the relationship between the recovery coefficient and particle impact Angle:

\[
\varepsilon_T = 0.998 - 0.78\alpha + 0.19\alpha^2 - 0.024\alpha^3 + 0.027\alpha^4
\] (9)

\[
\varepsilon_N = 1 - 0.78\alpha + 0.84\alpha^2 - 0.21\alpha^3 + 0.028\alpha^4 - 0.22\alpha^5
\] (10)
Where, $\alpha$ is the particle impact angle; 

$\varepsilon_t, \varepsilon_n$ are the tangential and normal components of the recovery coefficient respectively. The recovery coefficient fitting polynomial is set directly in the wall boundary.

Lagrange method can be solved by differential equation of discrete phase particles trajectory. Study keeps low concentration of particles which is controlled between $0.5\% \sim 1\%$, corresponding to the mass flow rate of the solid particles calculated through formula:

$$m_p = \pi \alpha \rho_p d_{in}^2 u_{in}$$

where $m_p$ is the mass flow velocity of solid particles, kg/s; $\alpha$ is the volume concentration of solid particles, $\%$; $\rho_p$ is the density of solid particles, kg/m$^3$; $d_{in}$ is the diameter of the u-shaped tube, m; $u_{in}$ is the fluid inlet velocity, m/s.

2.5. Grid independence test

The boundary condition of the inlet velocity of 5m/s was selected to verify the independence of the mesh. It can be found that, with grid number increase, the erosion change tends to be stable. When the grid number reaches 70,000 meshes, the calculated simulation value change 0.1% with the previous value, which can be considered simulation reach the requirement of grid precision, at the same time the computational efficiency is also high.

![Figure 4. Grid independence test result.](image)

3. Result and discussion

Figure 4 shows the facet average velocity distribution of fluid in cross section with uniform interval (50mm) where inlet velocity is 5m/s. The black line represents the mean velocity in bottom tube and the red represents the top.

It can be seen that flow rate of the fluid increases at the entrance gradually from Figure 5. Owning to the effect of centrifugal force and gravity force, velocity on the lateral bending wall is slightly greater than the inner, and fluid velocity on the upper side of the outlet of the pipe is higher than that of the lower lateral. As the inlet velocity increases, fluid flow rate distribution increases, but the change trend of fluid velocity in the u-tube is unchanged roughly.

Figure 6 shows distribution of facet average static pressure on the center section on different location of u-tube with inlet velocity of 5m/s. In the inlet section of the pipe, the pressure of fluid
decreases homogenously. The results of simulation show that pressure is relatively smaller in the bending tube close to external bending than the lateral. The trend of fluid pressure with different velocity is the same. As the velocity increases, the fluid dynamic pressure increases gradually.

Figure 7 and 8 show the erosion condition of the fluid on the pipe wall. It can be seen that the fluid has the most serious erosion condition on the u-tube bend. Since fluid flows from the bottom of the pipe, the bottom part of the bending tube is the most serious.

The erosion damage on the lateral bending is obviously greater than the inner with effect of centrifugal force. With the fluid velocity increases, the erosion condition becomes more serious.

Figure 9 shows the growth trend of the inlet velocity from 2.5 m/s to 5.5 m/s on the maximum and mean erosion rate of the pipe wall, which is almost exponential distribution and matches the given experimental data.

The concentration of discrete phase is also an important factor which influences erosion rate. The study selects uniform distribution of mass flow rate (50 mg/L, 100 mg/L, 150 mg/L, 200 mg/L, 250 mg/L, 300 mg/L and 350 mg/L) with inlet velocity and other variables fixed. The trend of facet average presents a linear increase, as shown as Figure 10.

In the real situation, the maximum erosion rate may reach a stable value when velocity increases. Stable value correlates with wall material properties, solid properties, size and so on.

![Figure 5. Distribution of velocity.](image)

![Figure 6. Distribution of static pressure.](image)

![Figure 7. The distribution of erosion.](image)

![Figure 8. The distribution of erosion.](image)
4. Conclusions

Different physical model may generate different results. The study with numerical simulation found that erosion condition in u-tube mainly occurs on the surface of elbow. Lagrange is a method with high accuracy that can predict flow of discrete phase (calcium carbonate). The numerical simulation of two phase flow erosion with different inlet velocity can draw the following conclusion:

(1) With velocity increased, erosion rate increased gradually when solid particles (calcium carbonate) impact surface of the u-tube. The most serious are mainly distributed on elbow surface. The results show that: with other parameters of fluid fixed, the mean erosion rate on the pipe wall increases from $9.28.72 \times 10^{-8}$ kg/m$^2$·s to $8.27.3 \times 10^{-8}$ kg/m$^2$·s when the inlet velocity increases from 2.5m/s to 5.5m/s. With other parameters fixed, the facet average erosion rate increased from $82.1.85 \times 10^{-8}$ kg/m$^2$·s to $82.8.68 \times 10^{-8}$ kg/m$^2$·s with inlet concentration of particle from 50 mg/L to 350 mg/L. Moreover, velocity has a more significant influence than the concentration of particle.

(2) With velocity of fluid and concentration of particle increased, location of the most serious erosion changes to bottom of tube from top gradually. When inlet velocity is lower than 3m/s, erosion...
location appeared at the bottom of elbow. While flow rate is higher than 3 m/s, due to the fluid velocity is high, the concentrated location of erosion appears on the top of tube. The velocity affects not only wall erosion rate, but also affect the location.

(3) For the manufacture and maintenance of u-tube heat exchanger equipment, considering the solid-liquid two-phase flow, it will be appropriate to increase the thickness of the material where abrasion is more concentrated to prevent the leakage of medium. Or in working situation, it can be monitored on the concentrated erosion area to maintain safe operation.

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