Catastrophic mechanism analysis of deformation and instability of the layered rock slope

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Abstract. This paper is based on engineering background in the phosphate mine called Jianlinshan belonging to the Yunnan Phosphate Chemical Group limited company. In terms of the deformation and instability characteristics of stratified steep dip rock slope, the slope instability process and intrinsic controlling factor have been analyzed by using the cusp catastrophe model. The excavation open-pit slope stability has been researched with respect to the stress state of the stratified rock slope. The results show the following conclusions. (1) The increase of strata thickness can improve the flexural rigidity of rock mass, and can effectively control the occurrence of slope collapse and buckling. The risk of destabilization of the slope can be reduced by using rock-soil anchorage technology or grading bedding. (2) When the slope rock thickness is fixed, the critical slope length decreases with the increase of the dip angle, and the higher the thickness, the more obvious the decreasing trend; When the dip angle of strata is certain, the critical slope length of the slope increases with the increase of the stratum thickness, and the increasing amplitude decreases. Meanwhile, when the dip of strata is greater than 40°, critical slope length increasing amplitude caused by the thicker thickness of strata tends to coincide. The relevant research results can provide theoretical and technical guidance and advice for the open pit transportation technology of this mining area or mining area with similar conditions.

1. Introduction
The landslide disaster of Open-pit mine slope usually takes the form of abrupt change, they accumulate a lot of energy after a long period of gestation to release energy in a sudden way, its destruction degree is immeasurable, and the people's life and property will cause great loss. Disasters in the form of
mutations can lead to the sudden termination of engineering rock mass deformation process and early instability, and then to disaster prediction and prevention and treatment difficulties[1]-[6]. The evolution process of slope system is stable, unstable and stable, which is a kind of transition from stable state to another stable state, and its essence is the concrete reaction of gradual change to mutation, quantity to qualitative changes. The landslide along the bedding rock slope is also so. There are two main types of deformation and failure modes. The first is that the foot of the leading edge is empty and has obvious potential sliding surfaces, which may induce the whole bedding downward shift. The second is that the front slope of the foot is blocked, the bedding slip collapse damage. The latter is a typical failure mode of the high slope of the bedding rock mass, which is the performance of discontinuous change in the gradual failure of layered rock mass structure[7]-[13]. In this paper, Yunnan Phosphating Group Co., Ltd. layered slope as a specific research object, the use of catastrophe theory to analyze and reveal bedding slope gradient to mutation process state, is conducive to mine scientific and reasonable slope management and construction.

2. Slope Engineering Overview
Yunnan Phosphating Group Co., Ltd. Jianshan phosphate mine is a large-scale open-pit mining of phosphorus resources in China, located in Haikou town, Kunming, Yunnan Province. The rock mass in Jianshan phosphate mine is lacustrine facies sedimentary rocks, the rock mass is a layered structure, the engineering rock mass is relatively stable, the mining process in the Middle East Stope gradually forms a steep bedding rock slope, the slope is near to the east and west direction, the slope is $351^\circ \angle 46^\circ$, and the slope gradually decreases with the shape of the mountain. Before 2007, the eastern mining area adopted the "Yimianpo" mining method, the first half of 2007, the lowest elevation of mining area 2035m, slope (top elevation 2226m) The maximum height of 191 meters, slope angle 42°~46°, slope rock mass in the long-term gravity, excavation unloading and weathering, etc. A near-east-west arc crack appears on the top of the slope, the crack length is about 200 meters, the crack is wide and 15mm~500mm, the rock mass of the slope is warped, the deformation and destruction of the slope seriously affect the mine downward safe mining, the slope of the east mining area is cut down by the mine, and the development trend of slope deformation and failure is effectively suppressed, and successfully produced 6.8 million tons of phosphate ore. With the increasing of mining depth, the slope is mined 100 m to 1935m after unloading treatment. In the process of downward extension, due to the following reasons, slope in the early 2012 ~2015 gradually showed a new deformation and failure characteristics, through the east mining area slope online real-time monitoring analysis, the slope experienced a new round of "stability → instability" of the evolution process. (1) After the original unloading scheme submitted, the mine during the organization construction, because the measurement, the construction excavation error makes the unloading 2070m platform width does not meet the design request, namely 2070 platform middle (along the slope to grow 200 meters) only 2~3 meters. Therefore, under the existing mining technology conditions, the slope downward force increases, the safety reserve is reduced. (2) in the process of extending mining, the dip of floor strata is steep, which is inconsistent with the original geologic prospecting data. The dip angle of the floor strata becomes steeper, and the slope strength increases and the safety reserve decreases under the existing mining technology condition. (3) The distribution of the poor geological body of the muddy interlayer (water mica) is exposed on the slope surface after the local landslide of 2070 step slope. Compared with the surrounding rock mass, weak interlayer has the characteristics of low strength and high compressibility, although the weak interlayer occupies a very small proportion in the composition of rock mass, but its existence plays an important role in the stability of rock mass engineering. At the same time, the mine according to the development of the enterprise adjustment of the original mining design, that is, the original design 1910m closed pit, is now adjusted to 1840m closed pit, closed pit when the slope will reach 380 meters high. To sum up, the stability of slope slope is a major technical problem which restricts the development of mining area in Jianshan phosphate mine in the subsequent extended mining process. How to correctly and rationally understand the mechanism of deformation instability of rock mass in stope slope and effectively control slope stability is an urgent problem to be solved in mine.
3. Cusp catastrophe analysis of layered slope instability

By using cusp catastrophe theory to analyze the deformation and failure evolution process of bedding rock slope, the mechanical model is described as follows: ① the bedding rock slope studied in this paper, the rock slope is obstructed, the slope is not free, the slip-bending deformation is destroyed. ② because bedding rock slope strata length and width direction are much larger than its thickness, and the bending deflection of strata is less than the thickness of strata, the rock strata can be equivalent to elastic thin plate. ③ thin plate structure has a certain bearing capacity after buckling, and its destructive load ratio is large, but the bedding rock slope is cut by jointed fissure, which makes the rock slope bend and deformation, and in the outside force and creep, soon loses the structure selfstabilizing ability, which leads to the instability of the slope, Therefore, the performance of rock mass structure after buckling is not considered. ④ When the bedding rock slope is damaged by bending and deformation, surface one or several layers of a part of the outer normal direction of the plane bulge, but not to the internal normal direction of depression, therefore, the bottom of the deformation is a rigid unilateral constraints relative to the bending level, the constraint is driven by force, without considering the constraints of deformation, do not work during the bending process.

In the precision range of engineering requirements, when the plate thickness is less than 1/5 of width, can be calculated according to the sheet. On the basis of predecessors’ research, this paper simplifies the layered rock mass of the slope into the model of the thin plate under Fig.1 and analyzes its stability. The surface equidistant from the two surfaces of the thin plate is called the middle plane, and the middle plane of the thin plate is plane in the natural state without force. In order to simplify the calculation and analysis, it is assumed that the thin plate is of equal thickness, the thickness is \( t \), the length in the \( x \) direction is \( L \), the width along the \( y \) axis is \( b \), and the elastic modulus of the stratum sheet is \( E \). As shown in Fig.1 below, under the action of external force and deadweight, the surface layer of bedding rock slope (rock stratum with \( T \)-thickness) has slip bending, and the whole surface rock mass can be divided into two parts, namely the AB segment of length \( a \), and the BC segment with length as \( L-a \), the AB segment is separated from the bottom layer, and the bulge is outward; The BC segment is a slip along the bottom surface. Among them, \( \alpha \) is the dip of strata, the internal friction angle and cohesion between strata are \( \varphi \) and \( C \) respectively, and the deadweight of the curved part is \( g \), \( p \) as the thrust of the upper sliding part on the lower part, which is the main force of the lower rock plate producing bending deformation. According to the literatures, the constraint of two bearing sides of thin plate can be abstracted as simple branch or fixed constraint, when both ends and roof and bottom plate of the rock layer are well consolidated, it can be regarded as the fixed-support constraint, while the consolidation is poor, it can be regarded as a simple support constraint. Under the combined action of longitudinal and transverse forces, the bedding slopes are constantly changing from one equilibrium state to another equilibrium state. We regard the change process as quasi-static motion process. According to the action of each part of the slope system and the change of its deformation potential energy, the potential energy function in the process of system change is established. Determine the mechanical conditions of the system instability by giving the set of all critical points of the potential energy function.

According to the solution of the superposition of the rectangular thin plate in elasticity, the expression of the deflection of the simple supported thin plate is given as follows:

\[
\omega = f \sin \frac{\pi x}{a} \sin \frac{\pi y}{b}
\]
The \( f \) in the formula is the maximum value of the deflection of the sheet structure, \( f = \omega \sin \left( \frac{\pi}{2} \right) a \frac{b}{2} = \omega_{\max} \). The formula (1) satisfies the boundary condition of the thin plate, namely:

\[
\begin{align*}
\omega_{x=0} &= 0 \\
\frac{\partial^2 \omega}{\partial x^2} \bigg|_{y=0} &= 0 \\
\omega_{x=a} &= 0 \\
\frac{\partial^2 \omega}{\partial x^2} \bigg|_{y=a} &= 0 \\
\omega_{y=0} &= 0 \\
\frac{\partial^2 \omega}{\partial y^2} \bigg|_{x=0} &= 0 \\
\omega_{y=b} &= 0 \\
\frac{\partial^2 \omega}{\partial y^2} \bigg|_{x=b} &= 0
\end{align*}
\]

4. Analysis of potential function of thin plate system in strata

The characteristics of bedding rock slope excavation are characterized by its continuous evolution from one equilibrium state to another in the longitudinal and transverse forces, and the process of slope excavation stability is considered as quasi static motion due to the continual changes of various loads in the course of excavation. According to the changes of the forces acting on the slope system and their own deformation potential energy, the potential energy function in the system change process is established, and the mechanical conditions of the system instability are determined by giving out all the critical points of the potential function.

The total potential energy \( \mathcal{V} \) of the system can be expressed as the combination of the elastic strain
energy $U$ and the load potential energy of the thin plate structure:

$$V = U + \sum_{i=1}^{n} P_i \delta_i$$  \hspace{1cm} (3)

In the formula: $P_i$ for the thin plate structure of the load, $\delta_i$ for its corresponding displacement, $n$ is the number of loads.

According to the stress characteristics of the thin plate structure, the upper sliding rock mass exerts a thrust force on the lower bending rock mass in the bending deformation process, and the $W_1$ of the lower bending rock mass is divided into negative work $W_2$ along the normal direction. So:

$$V = U - W_1 + W_2$$  \hspace{1cm} (4)

4.1. Bending deformation of thin plate of strata

The bending problem of thin plate structure can be regarded as a small deflection problem of thin plates because the deflection of the thin plate is much smaller than the thickness of the rock plate. Based on the basic assumption of elasticity to the deformation and stress of elastic thin plates, the deformation components are not counted $\varepsilon_x$, $\gamma_{xy}$, $\gamma_{xy}$, $\gamma_{xy}$, $\tau_{xy}$, $\alpha_x$. The stress state of the thin plate bending problem is approximated as being in the plane stress state, so the expression of the deformation potential energy of the thin plate structure is simplified as follows:

$$U = \frac{1}{2} \iiint \left( \sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \tau_{xy} \gamma_{xy} \right) dx dy dz$$  \hspace{1cm} (5)

In-style: $dx dy dz$ is a micro-element which is removed from the plate structure after being subjected to the load deformation.

According to the differential equation of the elastic surface, the stress and strain components in the upper form are represented by deflection $\omega$:

\[
\sigma_x = -\frac{Ez}{1 - \mu^2} \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right)
\]

\[
\sigma_y = -\frac{Ez}{1 - \mu^2} \left( \frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2} \right)
\]

\[
\tau_{xy} = -\frac{Ez}{1 + \mu \frac{\partial^2 \omega}{\partial x \partial y}} \gamma_{xy} = -z \frac{\partial^2 \omega}{\partial x \partial y}
\]

Formula (5) with the type (6) After finishing, the formula (7):

$$U = \frac{E}{2(1 - \mu^2)} \iiint z^2 \left( (\nabla^2 \omega)^2 - 2(1 - \mu) \left( \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \left( \frac{\partial^2 \omega}{\partial x \partial y} \right)^2 \right) \right) dx dy dz$$  \hspace{1cm} (7)

$Z$ is integral, from $-t/2$ to $t/2$, and the bending stiffness of thin plate is applied $D = \frac{Et^3}{12(1 - \mu^2)}$. Get the formula (8):

$$U = \frac{1}{2} \iint D \left( (\nabla^2 \omega)^2 - 2(1 - \mu) \left( \frac{\partial^2 \omega}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} - \left( \frac{\partial^2 \omega}{\partial x \partial y} \right)^2 \right) \right) dx dy$$  \hspace{1cm} (8)

In-style: $\nabla^2$ is a Laplace operator, namely:

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$  \hspace{1cm} (9)

Because the thin slab of rock stratum is simply supported by four edges, the method, the formula (9) is simplified to obtain the formula (10):

$$U = \frac{D}{2} \iint (\nabla^2 \omega)^2 dx dy$$  \hspace{1cm} (10)

The formula (1) is carried in the type (10) and is:
\[ U = \frac{D}{2} \left[ \int_{a}^{b} \int_{0}^{\pi} \left( \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} \right) f^2 \sin^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} \, dx \, dy \right] - D(1-\mu) \cdot \]

\[ \int_{0}^{\pi} \int_{a}^{b} \frac{\pi^4}{a^2 b^2} f^2 (\sin^2 \frac{\pi x}{a} \sin^2 \frac{\pi y}{b} - \cos^2 \frac{\pi x}{a} \cos^2 \frac{\pi y}{b}) \, dx \, dy \]

\[ = \frac{abD}{8} \left( \frac{\pi^2}{a^2} + \frac{\pi^2}{b^2} \right)^2 f^2 \]

4.2. The work \( W_1 \) the axial force of the thin plate of strata

The thrust \( p \) produced by the upper sliding rock mass on the lower bending rock mass is:

\[ p = (L-a)[\nu \sin \alpha - \cos \alpha \tan \varphi] \tag{11} \]

The normal stress of a sheet of rock is \( \sigma_r \):

\[ \sigma_r = \frac{P}{bl} + \gamma (a-x) \sin \alpha \tag{12} \]

4.3. \( W_2 \) of transverse force of thin slab

The transverse force is the partial force of the thin plate of the bending section along the vertical plate, and the negative work is as follows:

\[ W_2 = \int \int \mathcal{P} \cos \alpha \, dx \, dy \tag{15} \]

Make \( q = \pi \cos \alpha \), The integral of the formula (15) is:

\[ W_2 = \int_{a}^{b} \int_{0}^{\pi} q \sin \alpha \, dx \, dy = \left[ \int_{0}^{\pi} \frac{\pi y}{b} \, dy \right] \int_{a}^{b} q \sin \frac{\pi x}{a} \, dx = \frac{4abq}{\pi^2} \]

The total potential energy function of the model system can be expressed as formula (16) by the Taylor series expansion of the integrand in the upper formula and the integral after truncation.

\[ V = \frac{D}{2} \left[ \int \int \left( \frac{\partial^2 \omega_x}{\partial x^2} + \frac{\partial^2 \omega_y}{\partial y^2} \right)^2 \, dx \, dy \right] - \frac{1}{2} \int \int \sigma_r \frac{\partial^2 \omega}{\partial x^2} \, dx \, dy + \frac{4abq}{\pi^2} f \tag{16} \]

The formula (16) is substituted as a variable and transformed into a standard form of a cusp mutation type. Therefore, the order:

\[ x = \left[ \frac{64}{3D\pi^6 ab \left( \frac{1}{a^2} + \frac{1}{b^2} \right)} \right] \frac{1}{f} \tag{17} \]

\[ u = \left[ \frac{D\pi^4}{8ab} \left( \frac{b}{a} + \frac{a}{b} \right)^2 \frac{\pi^2 b}{8a} \right] \left[ \frac{64}{3D\pi^6 ab \left( \frac{1}{a^2} + \frac{1}{b^2} \right)} \right] \frac{1}{f} \tag{18} \]
The formula (18) and the formula (19) are brought into the sudden jump mutation mode, and the cross-set equation can be scored:

$$v = \frac{4abq}{\pi^2} \left[ \frac{64}{3D\pi^4 ab(\frac{1}{a^2} + \frac{1}{b^2})} \right]^2$$

(19)

The formula (18) and the formula (19) are brought into the sudden jump mutation mode, and the cross-set equation can be scored:

$$4 \cdot \left[ \frac{D\pi^4 \left( b + \frac{a}{b} \right)^2 - \frac{\pi^2 b^2}{8a} p}{8ab \left( \frac{a}{b} + b \right)^2} \right]^3 + \frac{4abq}{\pi^2} \left[ \frac{64}{3D\pi^4 ab(\frac{1}{a^2} + \frac{1}{b^2})} \right]^2 = 0$$

(20)

It is known from cusp mutation that when the axial force $P$ and transverse force $Q$ satisfy the upper form, the structure of the thin slab is unstable and the mining may jump from one equilibrium state to another.

5. Sufficient necessary mechanical conditions for slope instability

(1) The necessary mechanical conditions for destabilization of bedding rock slopes; From the foregoing analysis, it can be seen that when the bifurcation set $\nabla \leq 0$, that is, the $u \leq 0$, the system may cross the fork set, therefore, the necessary conditions for the abrupt change of the bedding rock slope system by the formula (20) are:

$$\frac{D\pi^4 \left( b + \frac{a}{b} \right)^2 - \frac{\pi^2 b^2}{8a} p}{8ab \left( \frac{a}{b} + b \right)^2} \leq 0$$

namely:

$$p \geq \frac{D\pi^4 \left( b + \frac{a}{b} \right)^2 - \frac{\pi^2 b^2}{8a} p}{b^2 \left( \frac{a}{b} + b \right)^2} = \frac{Et'\pi^2}{12b(1-\mu^2)} \left( \frac{b}{a} + \frac{a}{b} \right)^2$$

(21)

(2) The sufficient mechanical conditions for the destabilization of bedding rock slopes; bifurcation set $\nabla \leq 0$, the system appears abrupt instability, at this time the corresponding equation:

$$4 \cdot \left[ \frac{D\pi^4 \left( b + \frac{a}{b} \right)^2 - \frac{\pi^2 b^2}{8a} p}{8ab \left( \frac{a}{b} + b \right)^2} \right]^3 + \frac{4abq}{\pi^2} \left[ \frac{64}{3D\pi^4 ab(\frac{1}{a^2} + \frac{1}{b^2})} \right]^2 = 0$$

namely:

$$p = \frac{Et'\pi^2 a^2}{32(1-\mu^2)} \left( \frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{\frac{27}{4} \left( \frac{4abq}{\pi^2} \right)^2 + \frac{Et'\pi^2}{96b^2(1-\mu^2)} \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2}$$

(22)

(3) The mechanical condition for the instability of rock slopes; the formula (21) and the formula (22) are available (23):

$$p = \frac{Et'\pi^2 a^2}{32(1-\mu^2)} \left( \frac{1}{a^2} + \frac{1}{b^2} \right) \sqrt{\frac{27}{4} \left( \frac{4abq}{\pi^2} \right)^2 + \frac{Et'\pi^2}{96b^2(1-\mu^2)} \left( \frac{1}{a^2} + \frac{1}{b^2} \right)^2}$$

and, $p > \frac{D\pi^4 \left( b + \frac{a}{b} \right)^2}{b^2 \left( \frac{a}{b} + b \right)^2} = \frac{Et'\pi^2}{12b(1-\mu^2)} \left( \frac{b}{a} + \frac{a}{b} \right)^2$

It is known from the above formula that the necessary conditions for the abrupt change of bedding rock slope depend on the geometrical size of the slope structure and the physical and mechanical properties of the rock mass. When the geometry size of the slope is determined, the physical and mechanical properties of the rock mass play a key role. When the modulus of rock mass is larger, the bigger the flexural stiffness of rock mass, the more the axial thrust $p$ is, the more difficult the landslide damage is, on the contrary, when the flexural rigidity of rock mass is small, that is, when the rock mass belongs to soft rock, the stable state of the slope is prone to abrupt change to instability.
6. Analysis of slope failure stability

The top elevation of the open-pit slope of Jianshan phosphate mine is 2226m, at present, the minimum elevation is 1940m, the slope is 286m, and the rock slope is mesiucun layer of sand dolomite, the average stratum thickness is 46.7m, the interval is average 0.3m, the stratum is easy to produce sliding part of the slope, and the lower portion is the thick layer fine-grained dolomite of the shadow group, the formation thickness is greater than 460m, and the stable stratum. The elastic modulus of layered sandy dolomite is $E = 3.6$ gpa, $\gamma = 2.7$ g/cm$^3$, Poisson’s ratio $\mu = 0.22$ shear strength is $C = 0.12$ mpa and $\phi = 30^\circ$, and the angle of slope is basically the same as that of strata, it is 46°. During the period of production and construction, the slope of Open-pit mine has been sliding unstable, therefore, based on the analysis results of the failure mechanism of layered slope, the open sea m slope is equivalent to the rock plate structure and the rock beam structure respectively, and the stability of different mining stages of the slope is analyzed.

6.1. Theoretical analysis based on buckling failure of rock plates

The critical stress of buckling of rock-plate structures $\sigma_{cr}$ can be expressed as:

$$\sigma_{cr} = \frac{E \pi^2}{12a^4(1-\mu^2)} \left(1 + \frac{a^2}{b^2}\right)$$  

(24)

The value of the downward stress at the maximum deflection of the rock plate is type (25):

$$\sigma = \left[\gamma(\sin \alpha - \cos \alpha \tan \varphi) - c\right](L - a) + \frac{1}{2}\gamma a \sin \alpha$$

(25)

Therefore, when the falling stress of the rock-plate structure is greater than the critical stress, the structure is in a failure state, that is, the layered slope which is simplified as the rock-plate structure is in the unstable state, so the criterion for the instability of the layered slope is given, see formula (26).

$$\frac{\sigma}{\sigma_{cr}} < 1 \quad \text{Instability}$$

$$\frac{\sigma}{\sigma_{cr}} = 1 \quad \text{Limit equilibrium}$$

$$\frac{\sigma}{\sigma_{cr}} > 1 \quad \text{Stability}$$

(26)

When mining to the 2035m level in Jianshan open-pit mine, the slope is 191m high, the slope length is $L = 256.52$m, the slope top is cracked along the direction of the slope, the crack length is about 200 meters, the rock mass in the lower slope appears warping deformation, so the formula (26) is taken into the physical and mechanical parameters of the related rock mass, and the $b = 200$ is solved $a = 65.41$ respectively, the obtained to $a$ value and $b$ value with the type (24) and (26), can be $\eta = 1.02 \approx 1$, the slope is in the ultimate equilibrium state, this result is in line with the field investigation.

Jianshan phosphate mine from 2035m to extend downward mining, first of all, the slope was unloaded, that is, 2070m level above the use of 30m to the steps, a total of 5 steps, the lower part of the 2070m Yimianpo ore mining, when mining to the 1940m level, slope high 130m, slope length $L = 180.72$m, 2070m platform landslide, where the length of slippage along the 2070m platform is about 200m, so taking $b = 200$, according to the formula (26), the solution to $a = 87.62$, can be $\eta = 0.928 < 1$, the slope is in an unstable state, the results of the analysis and the actual site damage characteristics.

From the analysis process of slope excavation stability, based on the on-line dynamic monitoring of excavation slope, when obtaining the numerical value of buckling deformation of slope rock mass along inclination direction and direction, we can analyze the state of slope according to the principle of formula (25) and get the corresponding evaluation result.

6.2. Theoretical analysis based on buckling failure of rock beams

The slab-crack medium is taken as the unit length along the width direction, the rock plate is simplified to the rock beam, and the instability mechanism of the layered slope is analyzed by the cusp catastrophe model. The total length of the rock beam in the $x$-axis direction is $L$, the length of the bending section
of the rock beam is $\alpha$. The elastic modulus $E$, the heavy $\gamma$, the thickness $h$, the dip $\alpha$, the cohesion force $C$, the internal friction angle $\phi$ of the rock mass. The deflection equation of taking beam is $\omega = f \sin \frac{\pi x}{a}$, similar to the stress analysis of rock plate, its energy balance expression is shown in the formula (27).

$$\frac{1}{2} \int_0^a P(\omega')^2 \, dx + \frac{1}{2} q \sin \alpha \int_0^a (\alpha - x)(\omega')^2 \, dx = \frac{1}{2} \int_0^a EI(\omega')^2 \, dx + \int_0^a q \omega \cos \alpha \, dx$$  \hspace{1cm} (27)

Type, $P = [\gamma (\sin \alpha - \cos \alpha \tan \phi) - C]_0$, $q = \gamma$. To put $\omega' = f \frac{\pi x}{a}$; $\omega^* = -f \frac{\pi^2}{a^2} \sin \frac{\pi x}{a}$ into the formula:

$$P = \frac{4EI\pi^2}{a^2} - \frac{1}{2} q \sin \alpha + \frac{4qa^2 \cos \alpha}{f\pi}$$

When the structure breaks down, the maximum deflection $f$ tends to infinity, thus the critical load $P_{cr}$ can be obtained.

$$P_{cr} = \frac{4EI\pi^2}{a^2} - \frac{1}{2} q \sin \alpha$$  \hspace{1cm} (28)

When the bending length of the rock beam $a = L_1$, that is, there is no friction and cohesion between the strata, and the action of the downward section $P = 0$. If the structure reaches the limit equilibrium state, there is $P_{cr} = 0$, therefore, the critical slope length corresponding to the limit equilibrium state of the slope:

$$l_{cr} = \left[ \frac{8\pi^2 EI}{q \sin \alpha} \right]^{\frac{1}{3}} = \left[ \frac{2\pi^2 Eh^2}{3\gamma \sin \alpha} \right]^{\frac{1}{3}}$$  \hspace{1cm} (29)

Using $l_{cr}$ as the basis for evaluating slope stability, the stability coefficient is defined as:

$$\eta = \frac{l_{cr}}{l}$$  \hspace{1cm} (30)

Combined with the above analysis process, the slope stability is calculated according to the formula (28) and the formula (29) when the slope is excavated to 2035m elevation:

$$l_{cr} = \left[ \frac{8\pi^2 EI}{q \sin \alpha} \right]^{\frac{1}{3}} = \left[ \frac{8 \times 9.8596 \times 64680}{0.81 \times 0.71933} \right]^{\frac{1}{3}} = 206.24 m$$

$$l_{cr} / l = 206.24 / 256.52 = 0.804;$$

The slope is in an unstable state.

The slope stability is calculated when the slope is excavated to 1940m elevation:

$$l_{cr} = \left[ \frac{8\pi^2 EI}{q \sin \alpha} \right]^{\frac{1}{3}} = \left[ \frac{8 \times 9.8596 \times 9720}{0.81 \times 0.71933} \right]^{\frac{1}{3}} = 109.58 m$$

$$l_{cr} / l = 109.58 / 180.72 = 0.606;$$

The slope is in an unstable state.

7. Conclusion

Based on the characteristics of deformation and instability of steep-dip layered slopes, the evolutionary process and internal control factors of slope instability are analyzed by using cusp-point model, and the stability of open-pit slope excavation is studied with the characteristics of stress analysis of layered rock mass:

1. The steep dip rock slope of the bedding layer is the rock plate model, based on catastrophe theory, the mechanical mechanism of slope instability is studied, the expression of total potential energy function of layered rock mass system is deduced, the cusp catastrophe model of this system is established, and the bifurcation set which is unstable in horizontal ground stress and vertical force control space is
given. The process of state mutation caused by the gradient of the force is analyzed.

(2) The buckling deformation of rock and rock beams is analyzed by force analysis, in this paper, the stability of open pit slope excavation is calculated, and it is pointed out that based on the shortcoming of rock beam analysis method, the critical slope length calculated by this method is regarded as all warping, ignoring the effect of interlayer strength, and the result is conservative.

(3) Through the study of the relationship between the inclination of strata and the variation of strata thickness and the critical slope length of slopes, it is shown that the increase of strata thickness can improve the flexural rigidity of rock mass, and can effectively control the occurrence of slope collapse and buckling. Therefore, it is pointed out that adopting rock-soil anchorage technology or grading-bedding clearing can reduce the risk of instability of such slopes.

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