Vortices and the SU(3) string tension

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Abstract

We present simulation results comparing the SU(3) heavy quark potential extracted from the full Wilson loop expectation to that extracted from the expectation of the Wilson loop fluctuation solely by elements of Z(3). The two potentials are found to coincide. This agreement is stable under multiple smoothings of the configurations which remove short distance fluctuations, and thus reflects long-distance physics. It strongly indicates that the asymptotic string tension arises from thick center vortices linking with the Wilson loop.

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Recently, strong numerical evidence has been obtained for the vortex picture of confinement \[1\], \[2\]. In these simulations the heavy quark potential for the gauge group SU(2) was found to be fully recovered solely from the Z(2) part of the fluctuation of the Wilson loop which is caused by center vortices. Furthermore, an analytical proof was recently obtained \[3\] to show that forbidding the linkage of thick vortices with a Wilson loop results into perimeter law behavior at weak coupling. In other words, the presence of such vortices linking with the loop is necessary for confinement at (arbitrarily) weak coupling.

In this paper we present simulations extending the results in \[1\] to the SU(3) gauge group. We compute the string tension extracted from the expectation of the fluctuation in the value of the Wilson loop observable by elements of Z(3). We find that it fully reproduces the asymptotic string tension extracted from the full Wilson loop. Furthermore, and most importantly, this agreement is robust under multiple smoothings of the configurations eliminating short distance fluctuations. Passing this test is indeed necessary if such a coincidence truly reflects long-distance physics. Our SU(3) results then replicate those found to hold for SU(2) \[1\]. Thus, in addition to addressing the physically realistic case \(N = 3\) among the SU(\(N\)) groups, they strongly suggest that a similar state of affairs should hold for any low \(N\).

To fix ideas let us recall that vortices are characterized by multivalued singular gauge transformations \(V(x) \in SU(N)\). The multivaluedness ambiguity lies in the center \(Z(N)\), so the transformation is singled-valued in \(SU(N)/Z(N)\). Such a \(V(x)\) becomes singular on a closed surface \(V\) of codimension 2 (i.e. a closed loop in \(d = 3\), a closed 2-dimensional sheet in \(d = 4\)) forming the topological obstruction to a single-valued choice of \(V(x)\) throughout spacetime. Vortex configurations of the gauge potentials consist of a core region enclosing \(V\), and a pure-gauge long-range tail given by \(V(x)\). The asymptotic pure-gauge part provides the topological characterization of the configurations irrespective of the detailed structure of the core.

Assume that two gauge field configurations \(A_\mu(x)\) and \(A'_\mu(x)\) differ by such a singular gauge transformation \(V(x)\), and denote the path ordered exponentials of \(A_\mu\) and \(A'_\mu\) around a loop \(C\) by \(U[C]\) and \(U'[C]\), respectively. Then \(\text{tr} U'[C] = z \text{tr} U[C]\), where \(z \neq 1\) is a nontrivial element of the center, whenever \(V\) has obstruction \(V\) linking with the loop \(C\); otherwise, \(z = 1\). Conversely, changes in the value of \(\text{tr} U[C]\) by elements of the center can be undone by singular gauge transformations on the gauge field configuration linking with the loop \(C\). This means that vortex configurations are topologically characterized by elements of \(\pi_1(SU(N)/Z(N)) = Z(N)\). Thus the fluctuation in the value of \(\text{tr} U[C]\) by elements of \(Z(N)\) expresses the changes in the number (mod \(N\)) of vortices linked with the loop over the set of configurations for which it is evaluated.

On the lattice, the surface \(V\) of codimension 2 is regulated to a coclosed set \(V\) of plaquettes (2-cells), i.e. a closed set of dual \((d - 2)\)-cells on the dual lattice: a closed loop of dual bonds (1-cells) in \(d = 3\); a closed 2-dimensional surface of dual plaquettes (2-cells) in \(d = 4\), and so on. This represents the core of a thin vortex, each plaquette in \(V\) carrying flux \(z \in Z(N)\). The probability of excitation of such a thin vortex is
suppressed by the measure at large $\beta$ with a cost proportional to the size of $\mathcal{V}$. This can be proven, at finite $N$, quite generally, and rigorously, for any reflection positive action (such as the Wilson action) by so-called ‘chessboard’ estimates.

Thick vortex configurations can be constructed by perturbing the bond variables $U_b$ in the boundary of each plaquette $p$ in $\mathcal{V}$ so as to cancel the flux $z$ on $p$, and distribute it over the neighboring plaquettes. Continuing this process by next perturbing bonds in the neighboring $p$’s one may distribute the flux over a larger region in the two directions transverse to $\mathcal{V}$. Beyond the thickness of the core, the vortex contribution reduces to the multivalued pure gauge. If $\mathcal{V}$ is extended enough, the vortex may be made thick enough, so that each plaquette receives a correspondingly tiny portion of the original flux $z$ that used to be on each $p$ in $\mathcal{V}$. Long thick vortices may therefore be introduced in $\{U_b\}$ configurations having $\text{tr}U_p \sim 1$ for all $p$ on $\Lambda$. Thus they may survive at weak coupling where the plaquette action becomes highly peaked around $\text{tr}U_p \sim 1$. Long vortices may link with a large Wilson loop anywhere over the area bounded by the loop, thus potentially disordered the loop and leading to confining behavior. Thin vortices, on the other hand, necessarily incur a cost proportional to the size of $\mathcal{V}$ as noted above, and only short ones can be expected to survive at weak coupling. These can link then only along the perimeter of a large loop generating only perimeter effects.

Hybrid vortex configurations having a thick and a thin part are also possible. At weak coupling hybrid vortices survive if the thin part is short. Such hybrid vortices formed by long thick vortices ‘punctured’ by a short (e.g. one-plaquette-long) thin part may then disorder a large Wilson loop in essentially the same way as thick vortices.

We separate out the $Z(N)$ part of the Wilson loop observable by writing $\text{arg}(\text{tr} U[C]) = \varphi[C] + \frac{2\pi}{N} n[C]$, where $-\pi/N < \varphi[C] \leq \pi/N$, and $n[C] = 0,1,\ldots,N-1$. Thus, with $\eta[C] = \exp(i\frac{2\pi}{N} n[C]) \in Z(N)$,

$$W[C] = \langle \text{tr} U[C] \rangle = \langle |\text{tr} U[C]| e^{i\varphi[C]} \eta[C] \rangle \quad (1)$$

$$= \langle |\text{tr} U[C]| \cos(\varphi[C] + \frac{2\pi}{N} n[C]) \rangle \quad (2)$$

where the last two equalities follow from the fact that the expectation is real by reflection positivity, and that it is invariant under $n[C] \rightarrow (N - n[C])$. We next define

$$W_{Z(N)}[C] = \langle \cos(\frac{2\pi}{N} n[C]) \rangle \quad (3)$$

for the expectation of the $Z(N)$ part, which, as noted above, gives the response to the fluctuation in the number (mod $N$) of vortices linking with the loop. In the following we compare the string tension extracted from the full Wilson loop $W[C]$, eq. (1), to the string tension extracted from $W_{Z(N)}[C]$, eq. (3), for $N = 3$.

3We refer to [1] for discussion and mathematical formulation.
Figure 1: The heavy quark potential at $\beta = 5.8$ on a set of $582 \times 12$ lattices extracted at time slice $T=3$.

It should be noted that $W_{Z(N)}[C]$, as defined by (3), includes the effect of all vortices, thick and thin. As discussed, only thick vortices are expected to affect the string tension of sufficiently large loops. We will explicitly address this question by employing smoothing procedures that smooth out the short-distance fluctuations.

The heavy quark potential was extracted following [4]. Both on and off axis loops were computed, and, for loop $C$ of space extension $R$ and time extension $T$, the potential obtained from

$$V(R, T) = -\ln \frac{W(R, T+1)}{W(R, T)}.$$  (4)

The heavy quark potential is formally obtained by the large $T$ limit. Similarly, we define the potential extracted from the $Z(3)$ fluctuation expectation by replacing $W[C]$ in (4) by $W_{Z(3)}[C]$. In the following the potentials are always displayed only after they have reached a good plateau, which, at our values of $\beta$, already happens at time extension of a few lattice spacings. We worked with the Wilson action at lattice spacings $a = 0.15$ fm and $a = 0.10$ fm for $\beta = 5.8$ and $\beta = 6.0$, respectively. This is computed from the string tension assuming that its physical value is 440MeV.

The results are presented in Figs. 1 and 2. The agreement between the potential extracted from the full Wilson loop and that from the $Z(3)$ fluctuation expectation (3) is striking. Note that it includes also the short-distance regime. This is because, as noted above, (3) counts both thick and thin vortices, and the thin ones are clearly important.
at short distances (narrow loops). At longer distances, however, only sufficiently thick vortices can be expected to contribute to the string tension.

To test this further we performed local smoothing on our configurations which removes short distance fluctuations but preserves the long distance physical features. If the string tension is really fully reproduced by the vortex fluctuations, the agreement seen in Figs. 1, 2 should persist at long distances when the potential is measured on the smoothed configurations. This is in fact a very stringent test which appears to be routinely failed by several other recent attempts to isolate excitations responsible for long-distance physics. We used the smoothing procedure of Ref. 5 applied here to SU(3). Results for the potentials on twice smoothed configurations are given in Figs. 3 and 4.

We see that the potentials extracted from $W[C]$ and $W_{Z(3)}[C]$ now disagree over short distances, but then again merge together with no discernible difference at distances $R/a > 3$. This is as expected: smoothing destroys thin vortices but leaves vortices thicker than the smoothing scale unaffected.

Performing further smoothing steps extends the distance scale over which fluctu-

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4For all size Wilson loops thin vortices contribute significantly to the length piece, i.e. the constant term in the potential.

5This utilizes the type of smoothing, sometimes refered to as APE smearing, first described in 6.
Figure 3: The heavy quark potential at $\beta = 5.8$ on a set of $582 \times 8^3 \times 12$ lattices extracted at time slice $T=4$ on 2 times smoothed configurations.

...ations are smoothed, thus progressively destroying vortices of larger sizes; but the asymptotic string tension should not be affected, since, for sufficiently large loops, there is a scale beyond which linked vortices are not affected. This is rather strikingly illustrated by comparing Fig. 4 to Fig. 5 which displays the potentials resulting on six times smoothed configurations. Alternatively, for a fixed amount of smoothing, increasing the time extension $T$ should enlarge the range of agreement of the full and $Z(3)$ potentials to include shorter distances. Indeed, as $T$ increases, more linked vortices can now survive the smoothing by being allowed to thicken in the enlarged time direction. This is seen in Fig. 6 compared to Fig. 5. A smooth evolution extrapolating between these two figures is found as $T$ is increased from 4 to 7.

In conclusion, our numerical simulations show the $SU(3)$ string tension to be fully reproduced by the expectation of the $Z(3)$ fluctuation of the Wilson loop observable. The result is stable under multiple smoothings of the configurations removing short distance fluctuations. This indicates that it represents an actual long-distance physical feature. More generally, all our findings appear consistent with a physical picture of locally smooth extended thick vortices occurring over all long scales, and giving rise to the full asymptotic string tension.
Figure 4: The heavy quark potential at $\beta = 6.0$ on a set of 112 $12^3 \times 16$ lattices extracted at time slice $T=4$ from 2 times smoothed lattices.

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Figure 5: The heavy quark potential at $\beta = 6.0$ on a set of 112 $12^3 \times 16$ lattices extracted at time slice $T=4$ from 6 times smoothed lattices.

Figure 6: Same as Fig. 5 except at $T=7$. 

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