Heat transfer in rotor/stator cavity

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Abstract. In the paper we analyze the results of DNS/LES of the flow with heat transfer in the rotor/stator cavity. The rotor and the outer cylinder are heated. Computations have been performed for wide range of Reynolds numbers and aspect ratios. Computations are based on the efficient pseudo-spectral Chebyshev-Fourier method. In LES we used a Lagrangian dynamic subgrid-scale model of turbulence. Analysis allowed to check the influence of the aspect ratio and Reynolds number on the statistics and the structure of the flow. We analyzed all six Reynolds stress tensor components, turbulent fluctuations, three turbulent heat fluxes and different structural parameters which can be useful for modeling purposes. The distributions of Nusselt numbers obtained for different Re and aspect ratios along disks are given. We also investigated influence of thermal Rossby number as well as distributions of temperature along heated disk on statistics. Computations have shown that turbulence is mostly concentrated in the stator boundary layer with a maximum at the junction between the stator and the outer cylinder. The results are compared to the experimental and numerical data taken from literature.

1. Introduction

The study concerns the numerical simulation of the turbulent and transitional flow with heat transfer in the rotor/stator cavity. The flow in the rotor/stator cavity is one of the simplest 3D flow cases, very well suited for studying the effect of three-dimensionality of the mean flow on turbulence structure. The flow in rotating disk system is also the topic of practical importance (among others this problem can be of great interest for the internal aerodynamics of engines). One of the first experimental investigations in sealed rotor/stator cavity was performed by Daily & Nece (1960) who have found that flow structure depends on two parameters: aspect ratio $L = (R_1 - R_0)/2h$ and Reynolds number $Re = \Omega R_1^2/\nu$ (Fig. 1). They divided flows into four regimes depending on the combination of Reynolds number and the interdisk spacing $2h$. There are two turbulent regimes (III and IV) and two laminar (I and II). The regimes I and III are characterized by merged boundary layers, II and IV, on the other hand, by separated boundary layers. Schouveiler et al. (2001) have identified two main patterns for the transition to turbulence and described those regimes as a function of Reynolds number and the interdisk spacing. They have found that in flow cases with the separated boundary layers the circular and spiral waves destabilize the stator boundary layer. When two boundary layers are merged turbulent spots or turbulent spirals appear with the increasing Re. The experimental investigations of the fully turbulent flow in rotor/stator cavity have been performed by Itoh et al. (1992) and Poncet et al. (2005). Lygren & Andersson (2001) contributed greatly to the numerical simulation of
the fully turbulent rotor/stator flow cases. The flow with the heat transfer in rotor/stator cavity was investigated numerically in Tuliszka-Sznitko et al. (2009a,b). The experimental investigation of the flow around heated single rotating disk has been performed by Elkins & Eaton (2000) delivering distributions of all six Reynolds stresses, three turbulent heat fluxes, turbulent temperature fluctuations and different correlating coefficients and structural parameters. LES of the turbulent flow around a single rotating disk has been performed by Wu & Squires (2000).

The main purpose of the present DNS/LES computations is to characterize the 3D flow with heat transfer in rotor/stator sealed cavities of different aspect ratios $L$ and curvature parameter $Rm = (R_1 + R_0)/(R_1 - R_0) = 1.8$. The non-isothermal results are discussed in the light of Elkins & Eaton (2000) experimental data obtained for the flow around a single heated rotating disk and the numerical solutions of incompressible flow in rotor/stator cavity by Lygren & Andersson (2000), Randriamampianina & Poncet (2006) and Poncet et al. (2005).

2. Geometrical and mathematical model

The flow is governed by continuity, Navier-Stokes and energy equations. The equations are written in a cylindrical coordinate system, with respect to a rotating frame of reference. To take into account the buoyancy effects induced by the involved body forces the Boussinesq approximation is used. The problem is governed by: rotational Reynolds number $Re$; Prandtl number; thermal Rossby number $B = \beta(T_2 - T_1)$, where $T_1$ and $T_2$ are temperature of the cooled stator and heated rotor; aspect ratio $L$ and curvature parameters $Rm$. The time, length and velocity are normalized as follows: $\Omega^{-1}$, $h(z = z^*/h, \bar{r} = r^*/h, \star$ denotes dimensional parameter) and $\bar{\Omega}R_1$. The dimensionless temperature is defined in the following way $\Theta = (T - T_1)/(T_2 - T_1)$. The dimensionless components of the velocity vector in radial, azimuthal and axial directions are denoted by $u$, $v$, $w$ ($u = u^*/\Omega R_1$, $v = v^*/\Omega R_1$, $w = w^*/\Omega R_1$). The no slip boundary conditions are applied to all rigid walls. The upper stationary disk is attached to the outer cylinder and the bottom rotating disk is attached to the inner cylinder. The rotating disk and the outer cylinder are heated ($\Theta = 1$) and the stator and the inner cylinder are cooled ($\Theta = 0$). The numerical solutions (DNS/LES) is based on a pseudo-spectral Chebyshev-Fourier-Galerkin approach. The time scheme is semi-implicit and the second order accurate: the second order backward differentiation formula is used for viscous diffusion terms and the Adams-Bashforth scheme for non-linear terms. In LES we used a version of the dynamic Smagorinsky eddy viscosity model proposed by Meneveau et al. (1996), in which averaging is performed over the particle pathline. The LES code for non-isothermal flow (Tulisza-Sznitko et al., 2009b) is an extended version of the DNS code by Serre & Pulicani (2001a) for incompressible flow.

Numerical simulations have been performed on meshes with the maximum of collocation points 10 millions. To check the accuracy of the mathematical description of the flow and heat transfer near the disks we analyzed radial distributions of the axial wall coordinate $(z^+)_m = z^*/h r^*/\nu$ where $u_r = [\nu^2((\partial u^*/\partial z^*)^2 + (\partial v^*/\partial z^*)^2)]^{1/4}$ and $z^*_m$ is the smallest cell in the axial direction. Fig. 2 presents the results obtained for $L = 25$, $Rm = 1.8$, $Re = 180000$. We observe that $(z^+)_m$ increases with the radius $\bar{r}$ reaching the maximum near the outer cylinder. In our simulation we used condition $(z^+)_m < 1$ as a criterion for a precise description in the near wall area (Séverac et al., 2007b). We can see that this condition is fully satisfied in the considered flow case.

3. Three dimensional mean flow

Boundary layers in the rotor/stator cavity are strongly three dimensional. Characteristic feature which differentiate 3DTBL from 2DTBL is that the shear stress vector is misaligned from the mean strain vector, that has been observed in our computations. Another characteristic feature
of 3DTBL is the reduction of the Townsend parameter below the limit value 0.15 typical for 2DTBL. The Townsend parameter is defined as the ratio of the shear stress vector magnitude to twice of the turbulent kinetic energy $A_1 = (\overline{u'w'^2} + \overline{v'w'^2})^{1/2}/k$. In Fig. 3 the axial profiles of $A_1$ obtained in the middle section of different cavities ($L = 5, 9$ and $25$) and for different Reynolds numbers are presented. The reduction confirms turbulent three dimensional character of the stator boundary layer.

For smaller $L$ the flow is of typical Batchelor type: the flow consists of two boundary layers separated by inviscid core (the inviscid central core is gradually shrinking with increasing $L$). Fig. 4 shows polar plots of the mean radial and azimuthal velocity vector components obtained in the middle section of cavities. The polar profiles of the stator boundary layers resemble a characteristic triangular form, typical for 3DTBL (Fig. 4). In the middle section rotor profile is closer to the laminar one. Only the outer part of the rotor boundary layer is turbulent. The mean tangential velocity profile can be described by the conventional logarithmic law with constants $\kappa = 0.41$ and $C = 0.5$ (Fig. 5). The mean temperature also satisfies the traditional thermal law (we have analyzed influence of the Prandtl number). The exemplary iso-surfaces of the temperature are presented in Fig. 6 a and b ($L = 15$, $Re = 170000$ and $L = 1$, $Re = 90000$). We can see that the flow is pumped along heated bottom disk ($\Theta = 1$) towards the outer cylinder and recirculates along the cooled stator ($\Theta = 0$). To check the influence of the distribution of
the temperature along rotor, for some flow cases computations have been performed for linear distribution of $\Theta$ along the disk (Fig. 10 and Fig. 11). We also analyzed the influence of the thermal Rossby number $B$ on statistics.

4. Turbulent field

In Fig. 7 we present the axial distributions of two Reynolds stress tensor components in the stator boundary layer as a function of the wall coordinate $z^+ = z^* u_\tau / \nu$. To compare the level of turbulence with Lygren & Andersson (2001) and with Randriamampianina & Poncet (2006) the Reynolds stress tensor components have been normalized by a wall friction $\tau_w$. We obtained the same level of turbulence as Randriamampianina & Poncet (2006) who performed computations for annular rotor/stator of aspect ratio $L=18.32$. In Fig. 7 we observe that more intensive cooling of the stator ($B=0.3$) causes the decrease in Reynolds stress tensor components and
turbulent kinetic energy (Fig. 8b). We can also see that $(\overline{v'v'}/\tau_w)^{0.5}$ is about 1.8 times larger than $(\overline{u'v'}/\tau_w)^{0.5}$ showing significant anisotropy. The distributions of the magnitude of the turbulent $(\overline{u'w'^2} + \overline{v'w'^2})^{0.5}$ and total $|[(\nu \partial v^*/\partial z^* - \overline{v'w'^2})^2 + (\nu \partial u^*/\partial z^* - \overline{u'w'^2})^2]|^{0.5}$ shear stress vector, normalized by $\tau_w$, in plane parallel to the disk in terms of the wall coordinate are displayed in Fig. 8a. Computations are obtained for different aspect ratios L and Reynolds numbers. We observe that the magnitude of the dimensionless turbulent shear stress vector $\tau_{turb}$ reaches maximum at about $z^+ \sim 20$ for all considered flow cases. The magnitude of the dimensionless total shear stress vector $\tau_r$ decreases gradually from value of 1 at the disk. The results are in agreement with the results of Randriamampianina & Poncet (2006) obtained for annular rotor stator cavity of $L = 18.32$, $Re = 95000$. The distributions of turbulence are also visible in the Fig. 9 where the iso-surfaces of the axial velocity components are analyzed ($L = 15$, $Rm = 1.8$, $Re = 170000$). The maximum of turbulence occurs in the vicinity of the outer cylinder.

Figure 5. Mean azimuthal velocity component in terms of wall coordinate. $Rm=1.8$, $B=0.1$.

Figure 6. Iso-surfaces of temperature, a) $L = 15$, $Rm = 1.8$, $Re = 170000$, $B = 0.1$, b) $L = 1$, $Rm = 1.8$, $Re = 90000$. 
(in both rotor and stator boundary layers). Axial distributions of the temperature fluctuation \( \sqrt{T'} \) normalized by the friction temperature \( T_d = -\lambda(\partial T/\partial z)/\rho c_p u_\tau \) obtained for different \( L \) are plotted in Fig. 10. We observe that the maximum \( \sqrt{T'}/T_d \) occurs at \( z^+ \sim 15 \). We have found that the maximum value of temperature fluctuations \( (\sqrt{T'}/T_d)_{max} \) ranges from 2.28 to 2.45 depending on aspect ratio \( L \). These results are in agreement with the experimental data achieved by Elkins \& Eaton (2000) for a single heated rotating disk who obtained values from 2.0 to 2.6. In Fig. 10 there is also a comparison of the axial profiles \( \sqrt{T'}/T_d \) obtained for constant dimensionless temperature along rotor \( \Theta = 1 \) with results achieved for linear distribution of temperature along rotor \( (L = 5, Re = 195000) \). From Fig. 10 we can see that differences between profiles are not significant.

\[ Rm = 1.8, B = 0.1 \]

\[ L = 25, Re = 230000 \]
\[ L = 11, Re = 200000 \]
\[ L = 8, Re = 200000 \]
\[ L = 5, Re = 195000 \]
\[ L = 8, Re = 200000, B = 0.3 \]

\[ a) \left( \frac{\nu' v'}{\tau_w} \right)^{0.5}, \quad b) \left( \frac{\nu' w'}{\tau_w} \right)^{0.5}. \]

**Figure 7.** Reynolds stresses tensor components profiles: a) \( (u'v'/\tau_w)^{0.5} \), b) \( (w'w'/\tau_w)^{0.5} \). Middle section of the stator boundary layer. \( Rm = 1.8, B = 0.1 \).

### 5. Structural parameters

Following Elkins \& Eaton (2000) we analyze distribution of the following structural parameters: Townsend parameter \( A_1 \), \( R_{w\Theta} = (\nu'\Theta'^2 + v'\Theta'^2)^{1/2}/(\Theta'^2)^{1/2}(\nu'^2 + v'^2)^{1/2} \), \( R_{w\Theta} = w'\Theta'/(\Theta'^2)^{1/2}(w'^2)^{1/2}, (w'w'^2 + v'\Theta'^2)^{1/2}/w'w' \) and turbulent Prandtl number. These structural parameters (important for modeling purposes) seem to be less sensitive to boundary conditions than Reynolds stress tensor components and heat fluxes. Both \( R_{w\Theta} \) and \( R_{w\Theta} \) are constant in 2DTBL, whereas in 3DTBL they are a function of \( z \). For example, in 2DTBL \( R_{w\Theta} = 0.4 - 0.45 \) whereas in our paper \( R_{w\Theta} \) starts from a value between 0.3-0.35 near the disk to reach the maximum 0.45-0.5 at \( z^+ \sim 12 \). After reaching the maximum \( R_{w\Theta} \) decreases gradually. We observe an agreement with results obtained by Elkins \& Eaton (2000) for a single heated rotating disk. The coefficient \( (w'w'^2 + v'\Theta'^2)^{1/2}/w'w' \) is a measure of the coherence of the turbulent structures. The behavior of this parameter depends strongly on the boundary conditions. In 2DTBL (Klebanoff, 1955) it starts from value about 5 near the wall and decreases to value 2 at the edge of the boundary layer. In Elkins \& Eaton (2000) experiment \( (w'w'^2 + v'\Theta'^2)^{1/2}/w'w' \) starts from 4.5 near the disk to reach maximum \( \sim 6 \) at the edge of the boundary layer. In the present computations we have received very large values near the disk, then it decreases rapidly to value about 2 at the edge of boundary layer. The turbulent Prandtl number is defined as the ratio of the eddy diffusivity for momentum to the eddy diffusivity for heat \( Pr_t = (-\nu'w'/\partial v/\partial z)/(\Theta'w'/\partial \Theta/\partial z) \). In many 2DTBL \( Pr_t \) starts slightly above 1 near the wall, then decreases to value about 0.8 at the edge. Elkins \& Eaton (2000) showed that \( Pr_t \) decreases from 0.9-1.1 near the disk to the value 0.6 in the outer part of boundary layer. Our
results are in agreement with Elkins & Eaton (2000) data as well as with 2DTBL of Wróblewski & Eibeck (1990) and Gibson & Verriopoulos (1984). A more detailed analysis of turbulent Prandtl number can be found in Tuliszka-Sznitko & Majchrowski (2010).

6. Distributions of Nusselt number

Convective heat transfer over the surface of a rotating disk in an open rotor-stator system was studied by Pellé & Harmand (2007). Heat transfer on a single rotating disk has been investigated among others by Dorfman (1963). The local Nusselt number is defined in the following manner:
Figure 10. Axial profiles of the temperature distributions normalized by friction temperature $\sqrt{T'_{d}^2}/T_d$.

Figure 11. The axial profiles of correlation coefficient $R_{\omega\Theta}$ obtained in the middle section of stator boundary layer. $Rm = 1.8$, $B = 0.1$.

$Nu = \alpha r^*/\lambda$. Fig. 12 a and b show distributions of the local Nusselt numbers obtained in the present paper in terms of dimensionless radius along stator and rotor. Results are obtained for cavities of the aspect ratio $L = 15$ and $25$ and for different Reynolds numbers. We can see that in the stator boundary layer the linear dependence between $Nu$ and radius is up to $\sim 67$ ($L = 25$) and $\sim 39$ ($L = 15$). Then, curves change their slopes and exhibit an enhancement in the heat transfer. We observe an increase of the $Nu$ along with the increasing Reynolds number. We can see that the Nusselt numbers on rotor are lower than on stator. The distribution of the local Nusselt number reflects the flow structure. From the analysis in previous sections we know that for all considered $Re$, the rotor boundary layer is turbulent only in the vicinity of the outer cylinder. Additionally, we observe concentration of turbulence kinetic energy near cylinders, which intensifies the heat transfer in these areas.

7. Conclusions

In the paper new data have been provided for momentum and thermal transport in a sealed rotor/stator cavity. DNS/LES have been performed to describe the turbulent and transitional
flow with heat transfer in annular sealed rotor/stator cavities of the aspect ratio from the range 1.0-25.0. In our computations stator (upper disk) was cooled and rotor (bottom disk) was heated. Computations have been mostly performed for the thermal Rossby number $B = 0.1$, however, some tests have been performed for higher $B = 0.3$. The influence of thermal Rossby number on structure of the flow is small. The flow belongs to the Batchelor family i.e. the flow consists of two boundary layers and inviscid core, however, with the increasing $L$ area of inviscid core is shrinking. The polar plots of radial and azimuthal velocity components are between typical turbulent flow and the laminar solution. We have analyzed all six Reynolds stress tensor components, turbulent fluctuations and three turbulent heat fluxes. Structure parameters are not constant across boundary layer as it takes place in 2DTBL. The results have been discussed in the light of the experimental data of Elkins & Eaton (2000) and Randriamampianina & Ponct (2006). The investigations have shown that distributions of the local Nusselt number reflect the flow structure. We have observed a rapid enhancement in the heat transfer near the outer cylinder where the level of turbulence is the highest.

Figure 12. Distributions of the local Nusselt numbers in terms of dimensional radius. a) $L = 25$, b) $L = 15$, $Rm = 1.8$, $B = 0.1$. 
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