Gravitational Anomaly and Hawking Radiation of Brane World Black Holes

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We apply Wilczek and his collaborators’ anomaly cancellation approach to the 3-dimensional Schwarzschild- and BTZ-like brane world black holes induced by the generalized $C$ metrics in the Randall-Sundrum scenario. Based on the fact that the horizon of brane world black hole will extend into the bulk spacetime, we do the calculation from the bulk generalized $C$ metrics side and show that this approach also reproduces the correct Hawking radiation for these brane world black holes. Besides, since this approach does not involve the dynamical equation, it also shows that the Hawking radiation is only a kinematic effect.

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I. INTRODUCTION

It is often instructive to study a physical phenomenon from different point of views. The Hawking radiation is one of the famous examples. After Hawking’s original derivation \cite{1}, there emerges many other approaches to recover this effect (e.g.,\cite{2,3,4,5}). For example, Christensen and Fulling use the trace anomaly to study the Hawking effect \cite{5}. They consider a conformal scalar field in a Schwarzschild black hole background and find that the anomalous trace of the renormalized stress tensor of the scalar field can reproduce the Hawking radiation under the requirement that the renormalized stress tensor is conserved covariantly. Recently, Wilczek and his collaborators propose a new approach to obtain the Hawking radiation from the viewpoint of gravitational and gauge anomalies \cite{6,7,8}. Instead of the trace anomaly, they concern the covariant anomaly and gauge anomaly in a reduced 2-dimensional spacetime, which falls into the class of quantum fields in $4k + 2$ ($k \in \mathbb{Z}$) dimensional spacetime, studied by Alvarez-Gaume and Witten \cite{9}. Wilczek \textit{et al} consider a scalar field in a stationary black hole background and show that Hawking radiation can be determined by the anomaly cancellation and regularity condition at horizon. Subsequently, this method.

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have been extended into various black hole backgrounds [10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21] and black hole-like background [22]. All of these results indicate that the gravitational anomaly has an intrinsic connection with the Hawking radiation and that this method could be applied to the more general situations.

The purpose of the present letter is to check whether the anomaly cancellation approach can be applied to the brane world black holes induced by the generalized $C$ metrics in the Randall-Sundrum (RS) scenario [23]. We mainly focus on two cases: 3-dimensional Schwarzschild- and BTZ-like black holes on RS 2-branes. Both of them are obtained from the generalized $C$ metrics. Like their 4-dimensional counterparts, these brane world black holes also have Hawking radiation, and their thermodynamics have been studied in [25]. So it is probably that the anomaly cancellation approach also applies to these situations.

Generally speaking, one may directly use this method for these brane world black holes from the brane world side. However, we would like to do it in another way, i.e., from the bulk side. The reason is that various studies show that the horizons of the brane world black holes will extend into the extra dimension, but not merely on the brane, and these brane world black holes can be described as black strings in the bulk spacetime [24, 25, 26, 27, 28]. For the two cases we consider here, the bulk is described by the generalized $C$ metric. It also contains the black hole horizon, which is just the extension of the black hole horizon on the brane [29]. So one can apply the anomaly cancellation method to the generalized $C$ metric, instead of directly applying it to the brane world black holes. For convenience, we only consider the gravitational anomaly and obtain the correct Hawking radiation for these brane world black holes. Besides, since this method does not involve the dynamical equation, i.e., the Einstein equation, it also shows that the Hawking radiation is only a kinematic effect, this is in accord with the studies about the acoustic black holes and the uniformly accelerated reference frames [30, 31, 32, 33, 34].

The letter is organized in the following way. In the next section, we will briefly review the anomaly cancellation approach proposed by Wilczek and his collaborators. In sections III and IV, we will study the Hawking radiation of the 3-dimensional Schwarzschild- and BTZ-like brane world black holes by applying this approach to the generalized $C$ metrics without and with a rotation parameter, respectively. Then we will come to the conclusions and discussions.

II. A BRIEF REVIEW OF THE METHOD

The effective action of a massless scalar field in a $d$-dimensional static, spherically symmetric black hole background can be reduced to a 2-dimensional one for an infinite collection of 2-dimensional scalar fields in the near horizon region by using the tortoise coordinate [6]. The horizon requires a boundary condition that the outgoing modes vanish near it, thus the 2-dimensional effective fields become chiral in this region and then the gravitational anomaly will appear as [6, 35, 36]

\[
\nabla_\mu T^\mu = \frac{1}{96\pi \sqrt{-g}} \epsilon^{\beta\gamma} \partial_\beta \partial_\gamma \Gamma^\alpha_{\nu\beta},
\]

(1)
where $\Gamma_{\nu\beta}^\alpha$ is the Christoffel connection of the effective 2-dimensional metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2. \quad (2)$$

Its horizon is located at $r = r_H$, i.e., $f(r_H) = 0$, and its surface gravity is $\kappa = |f'(r_H)|/2$. For Eq.(2), the anomaly is purely timelike and can be rewritten as

$$\nabla_\mu T^\mu_\nu \equiv A_\nu \equiv \frac{1}{\sqrt{-g}} \partial_\mu N^\mu_\nu. \quad (3)$$

Eq.(11) and Eq.(2) give $N^\mu_\nu = \frac{1}{192\pi} \epsilon^{\mu\lambda} \partial_\nu \Gamma^\alpha_{\lambda\nu}$, its components are

$$N^t_t = N^r_r = 0,$$
$$N^r_t = \frac{1}{192\pi} (f^2 + f'' f),$$
$$N^t_r = \frac{-1}{192\pi} \frac{(f^2 - f'' f)}{f^2}. \quad (4)$$

The anomaly appears when the effective action of the field at quantum level fails to be diffeomorphism invariant, i.e., $\delta_\xi W = -\int d^4x \sqrt{-g} \epsilon^{\mu\nu} \nabla_\mu T^\mu_\nu \neq 0$, where $\xi^a$ is an arbitrary vector field which generates the transformation of action and $T^\mu_\nu$ is the stress tensor. If the full quantum field theory is still required to be generally covariant, namely $\delta_\epsilon W = 0$, then the flux of each outgoing partial wave, which is added to cancel the anomaly, just reproduces the Hawking radiation.

In detail, by use of two step functions $\Theta_+ = \Theta(r - r_H - \varepsilon)$ and $\Theta_- = 1 - \Theta_+$, the variation of the effective action can be written as

$$-\delta_\xi W = \int d^2x \sqrt{-g} \epsilon^{\mu\nu} \nabla_\mu \{T^\mu_\nu \Theta_- + T^\mu_\nu \epsilon^{\mu\nu} \partial_\nu \epsilon^{\mu\nu} \}$$
$$= \int d^2x \{\xi^t (\partial_r (N^t_r \Theta_-) + (T^t_r - T^t_t) \partial_r \Theta_+) + \xi^r (T^r_r - T^r_t) \partial_r \Theta_+)\}, \quad (5)$$

where $T^\mu_\nu$ and $T^\mu_\nu$ are the stress tensor in the regions $r_H < r < r_H + \varepsilon$ and $r > r_H + \varepsilon$, respectively. The integration of Eq.(4) gives rise to

$$T^t_t = -(K + Q)/f - B(r)/f - I(r)/f + T^t_t(t),$$
$$T^r_r = (K + Q)/f + B(r)/f + I(r)/f,$$
$$T^r_t = -K + C(r) = -f^2 T^t_t, \quad (6)$$

where $K$ and $Q$ are constant, while $B(r) = \int_{r_H}^r f(x) A_t(x) dx$, $I(r) = \frac{1}{2} \int_{r_H}^r T^\alpha_\alpha A_t(x) dx$ and $C(r) = \int_{r_H}^r A_t(x) dx$. Substituting Eq.(6) into Eq.(5), one gets

$$-\delta_\xi W = \int d^2x \xi^t \{\partial_r (N^t_r \Theta_-) + (N^t_r + K - K - K) \delta(r - r_H - \varepsilon)\}$$
$$+ \int d^2x \xi^r \left\{\frac{K + Q - K - Q}{f} \delta(r - r_H - \varepsilon)\right\}. \quad (7)$$
The requirement of \( \lim_{\varepsilon \to 0} \xi W = 0 \) for any \( \xi \) demands
\[
K_0 = K_\chi + \Phi,
Q_0 = Q_\chi - \Phi,
\] (8)
where
\[
\Phi = N^r_t|_{r_\mu} = \frac{k^2}{48\pi}
\] (9)
is just the flux which is needed to cancel the gravitational anomaly. Note that a beam of massless blackbody radiation moving in the radial direction at temperature \( T \) has the form \( \Phi = \frac{\pi}{12}T^2 \), thus Eq. (9) just reproduce the Hawking radiation.

For the scalar field in a charged or rotational black hole, \([7, 8, 10]\), e.g., the Reissner-Nordström (RN) black hole or the Kerr-Newman black hole, there will appear gauge anomaly in addition to the gravitational anomaly. Consequently, these gauge anomalies will contribute to the black body radiation (Hawking radiation) spectrum (characterized by the distribution of particles \( I^{(\pm)} \) at given energy and chemical potentials) of the black hole as the corresponding chemical potentials
\[
I^{(\pm)} = \frac{1}{e^{\beta(E\pm\sum\mu_i)} + 1},
\] (10)
where \( \beta \) is the inverse Hawking temperature, \( E \) is the energy of the particle and \( \mu_i \) are the chemical potentials. Like the treatment in \([6, 7, 8]\), we only consider Fermi distribution.

III. HAWKING RADIATION OF SCHWARZSCHILD-LIKE BLACK HOLE ON 2-BRANE

It is well known that the generalized \( C \) metrics are a large class of degenerate vacuum (electro-vacuum) solutions for the Einstein (Einstein-Maxwell) equations with or without a cosmological constant, which may describe accelerating black holes in spacetime \([25, 37, 38]\). One of its subclasses is known as \( AdS C \) metric. Its line element can be written as
\[
ds^2 = \frac{1}{(x-y)^2}[-H(y)dt^2 + \frac{dy^2}{H(y)} + \frac{dx^2}{F(x)} + F(x)d\sigma^2]
\] (11)
with
\[
H(y) = -(\gamma + \Lambda/6) - 2ly + ky^2 - 2My^3,
F(x) = \gamma - \Lambda/6 + 2lx - kx^2 + 2Mx^3,
\] (12)
where \( k = 1, 0, -1 \), related to the topology of the horizon, \( M \) is the mass parameter of the black hole, \( l \) is the NUT parameter which is usually regarded as unphysical and can be chosen to be zero, \( \Lambda(<0) \) is the cosmological constant and \( \gamma \) is the acceleration parameter. In order to keep the Lorentz signature \((-+,+,+,+)\) in metric (11), one needs \( F(x) \geq 0 \). Generally, this spacetime has one or more event horizons, which are located at
\[
H(y) = -(\gamma + \Lambda/6) - 2ly + ky^2 - 2My^3 = 0.
\] (13)
To construct the 3-dimensional Schwarzschild-like brane world black hole from the AdS \( C \) metric, one just needs to set \( 2\gamma = -\Lambda/3 = 1 \), \( l = 0 \) and \( k = 1 \) in Eq.(11) firstly, which result in

\[
H(y) = y^2 - 2My^3, \\
X(x) = 1 - x^2 + 2Mx^3,
\]

and then make the coordinate transformations \( z = 1 - x/y \) and \( \rho = \sqrt{1 - x^2/y} \). After that, the 3-dimensional induced metric on the RS 2-brane at \( x = 0 \) is just a 3-dimensional Schwarzschild-like brane world black hole

\[
ds^2 = -(1 - \frac{2M}{\rho})dt^2 + (1 - \frac{2M}{\rho})^{-1}d\rho^2 + \rho^2d\sigma^2.
\]

As has been studied in [25], its horizon on the 2-brane is located at \( \rho = \rho_h = 2M \), and the Hawking temperature of the horizon is \( T_h = |f'(\rho)|/(4\pi)|\rho_h = 1/(8\pi M) \), which is of the same form as that of the 4-dimensional Schwarzschild black hole, and, this horizon will extend into the extra dimension \( x \), but not merely on the brane, due to the bulk/brane interaction. Namely, the horizon of the 3-dimensional brane world black hole is a higher dimensional object with spatial dimension three. Notice that this brane world black hole is a static one. Thus its horizon on the brane is a minimal surface. Besides, one can show that the bulk extension of this brane world black hole horizon is also a minimal surface, with the horizon on the brane as its boundary [29]. Furthermore, it can also be shown that such a minimal surface is just the bulk black hole horizon in AdS \( C \) spacetime. Consequently, it can be proved \( T_H = T_h \), where \( T_H \) is the Hawking temperature of the bulk black hole horizon. Therefore, one can apply the anomaly cancellation approach to the bulk black hole horizon of the AdS \( C \) metric, instead of directly applying it to the 3-dimensional Schwarzschild-like brane world black hole itself.

To do this, consider a massless scalar field \( \phi \) in the AdS \( C \) spacetime with the classical action

\[
S = -\frac{1}{2}\int d^4x\sqrt{-g}\phi\Box\phi \\
= -\frac{1}{2}\int d^4x\phi[-\frac{\partial_t^2}{(x-y)^2H(y)} + \partial_y(\frac{H(y)}{(x-y)^2}\partial_y \\
+\partial_x(\frac{F(x)}{(x-y)^2})\partial_x + \frac{\partial_x^2}{(x-y)^2F(x)}]\phi.
\]

Under the decomposition of \( \phi \) in some complete basis

\[
\phi = \sum_{l,m} (x-y)\hat{\phi}_{lm}(t,y)\chi_{lm}(x,\sigma),
\]

and the tortoise-like coordinate defined by

\[
dy_* = \frac{dy}{H(y)},
\]
the action reduces in the near horizon region \((y \to y_H \text{ and } H(y) \to 0)\) to

\[
S = -\frac{1}{2} \sum_{l'mm'} \int dt dx dy \, d\sigma \tilde{\phi}_{lm}(t, y_*) x_{lm}(x, \sigma) (-\partial_t^2 + \partial_{y_*}^2) \tilde{\phi}_{l'm'}(t, y_*) x_{l'm'}(x, \sigma). \tag{19}
\]

After the integration over \(x\) and \(\sigma\) with the help of

\[
\int dx d\sigma \tilde{x}_{lm} x_{lm} = \delta_{ll'} \delta_{mm'}, \tag{20}
\]

the action in the near horizon region reduces to an effective 2-dimensional one

\[
S = -\frac{1}{2} \sum_{lm} \int dt dy \, \tilde{\phi}_{lm}(t, y_*) (-\partial_t^2 + \partial_{y_*}^2) \tilde{\phi}_{lm}(t, y_*). \tag{21}
\]

This indicates that the 2-dimensional effective metric near the horizon is

\[
ds^2 = -H(y) dt^2 + \frac{dy^2}{H(y)} = -H(y)(-dt^2 + dy_*^2). \tag{22}
\]

Therefore, following the discussion of section II, from Eq. (4) and Eq. (9), the flux which needed to cancel the gravitational anomaly is

\[
\Phi = N_\xi^y \big|_{y_H} = \frac{\pi}{12} \left( \frac{H'(y_H)}{4\pi} \right)^2, \tag{23}
\]

so the Hawking temperature derived from the anomaly cancellation method is \(T = |H'(y_H)|/(4\pi)\). Note that the Killing vector which generates the bulk black hole horizon is \(\xi^a = (\partial_t)^a\). A direct calculation shows the surface gravity \(\kappa\) for the black hole horizon of the \(AdS\) metric is

\[
\kappa = \lim_{y \to y_H} \sqrt{\frac{(\xi^a \nabla_a \xi^b)(\xi^b \nabla_b \xi^c)}{-\xi^d \xi_d}} = \frac{|H'(y_H)|}{2}, \tag{24}
\]

which gives the horizon temperature by \(T = \kappa/(2\pi)\). Hence, the Hawking temperature for the black hole horizon of the \(AdS\) metric obtained from Eq.(23) is the same as the one obtained from the standard way.

Recall that the black hole horizon in the \(AdS\) metric is at \(y_H = 1/(2M)\), its Hawking temperature is just \(T_H = |H'(y_H)|/(4\pi) = 1/(8\pi M) = T_h\). This is a natural result because the 3-dimensional Schwarzschild-like brane world black hole described by Eq.(15) is just the boundary of the black hole in the bulk \(AdS\) spacetime [29]. As a result, the correct Hawking temperature for the 3-dimensional Schwarzschild-like brane world black hole is reproduced by applying the anomaly cancellation approach to the bulk \(AdS\) metric which generates it.

\[1\] Another solution of \(H(y) = y^2 - 2M y^3 = 0\) is \(y_0 = 0\), which is sometimes called \(AdS\) horizon and has zero temperature from both Eq.(23) and Eq.(24).
Another subclass of the generalized $C$ metric can be expressed as \[25, 37\]

\[ds^2 = \frac{1 + a^2x^2y^2}{(x - y)^2} \left[ -\frac{H(y)}{(1 + a^2x^2y^2)^2} (dt + a x d\sigma)^2 + \frac{dy^2}{H(y)} \right.\]

\[+ \frac{dx^2}{F(x)} + \frac{F(x)}{(1 + a^2x^2y^2)^2} (d\sigma - ay^2 dt)^2] \quad (25)\]

where

\[H(y) = -\gamma - \Lambda/6 - 2ly + ky^2 - 2My^3 + (\gamma - \Lambda/6)a^2y^4,\]

\[F(x) = \gamma - \Lambda/6 + 2lx - kx^2 + 2Mx^3 - (\gamma + \Lambda/6)a^2x^4, \quad (26)\]

with $a$ the rotation parameter, $\gamma$ the acceleration parameter, $l$ the NUT parameter and $k = 1, 0, -1$. When $a = 0$, Eq.\(25\) goes back to the $AdS$ $C$ metric Eq.\(11\). This metric can be used to generate various kinds of 3-dimensional brane world black holes. Similar to their four dimensional counterparts, there are Hawking radiation and thermodynamics for these 3-dimensional brane world black holes \[25\]. In the following we take the BTZ-like brane world black hole as an example to show that the anomaly cancellation approach also applies to these brane world black holes. The discussions for other cases are almost the same.

After making the coordinate transformations

\[r = \sqrt{\frac{y^2 + \lambda x^2}{(x - y)}} \quad \text{and} \quad \rho = \sqrt{\frac{1 + kx^2 - \tilde{a}^2x^2y^2/4}{y^2 + \lambda x^2}} , \quad (27)\]

and the choice of parameters $l = M = 0$, $k = -1$, and $\gamma - \Lambda/6 = 1$, the induced 3-dimensional metric on the RS 2-brane at $x = 0$ is then the BTZ-like brane world black hole \[25, 39\]

\[ds^2 = -(k + \frac{\rho^2}{l_3^2} + \frac{\tilde{a}^2}{4\rho^2})dt^2 + (k + \frac{\rho^2}{l_3^2} + \frac{\tilde{a}^2}{4\rho^2})^{-1}d\rho^2 + \rho^2 (d\sigma - \frac{\tilde{a}}{2\rho^2} dt)^2 , \quad (28)\]

where $\tilde{a} \equiv 2a$, which is the angular momentum, and $\lambda \equiv \gamma + \Lambda/6 \equiv -1/l_3^2$, which acts as the cosmological constant on the RS 2-brane (at $x = 0$).

The horizon of this brane world black hole is located at $k + \rho^2/l_3^2 + \tilde{a}^2/(4\rho^2) = 0$, and its Hawking temperature is

\[T_h = \frac{\rho_+^2 - \rho_-^2}{2\pi \rho_+ l_3^2} = \frac{\sqrt{k^2 + \lambda \tilde{a}^2}}{\pi \tilde{a}} \left( \frac{-k - \sqrt{k^2 + \lambda \tilde{a}^2}}{2} \right)^{1/2} \quad (29)\]

with $\rho_\pm = (k \pm \sqrt{k^2 + \lambda \tilde{a}^2})/(-2\lambda)$.

Again, the horizon of this BTZ-like brane world black hole extends into the bulk. And one can show that the bulk extension of the brane world black hole horizon coincides with the black hole horizon of the bulk generalized $C$ metric (and that both of them are minimal surfaces). Consequently, one can apply the same procedure of the previous sections to check
whether the anomaly cancellation approach is applicable to the BTZ-like brane world black hole.

Near the horizon, the action for a massless scalar field $\phi$ in the generalized $C$ metric becomes

$$S = -\frac{1}{2} \lim_{y \to y_H} \int dt dx dy_s d\sigma \sqrt{-g} \phi \Box \phi$$

$$= -\frac{1}{2} \sum_{l'm'm'} \int dt dx dy_s d\sigma \tilde{\phi}_{lm} \psi_{lm} \left[ -\left( \partial_t + a y_H \partial_n \right)^2 + \partial_{y_s}^2 \right] \phi_{l'm'}, \psi_{l'm'},$$

(30)

under the decomposition $\phi = \sum_{lm} (x - y) \tilde{\phi}_{lm}(\eta, y_s) \psi_{lm}(x, \sigma)$ and the tortoise coordinate $y_s$. The coefficient $a y_H^2$ corresponds to the horizon angular velocity $\Omega_H$. One may also regard this term as $U(1)$ gauge potential $A_t = a y_H^2$ like the treatment in [7, 8]. In the near horizon region, one may make the coordinate transformation $
abla = \sigma - \Omega_H t$ and $\eta = t$,

(31)

so that

$$\partial_{\sigma} = \partial_{\varphi} \quad \text{and} \quad \partial_t = -\Omega_H \partial_{\varphi} + \partial_{\eta}.$$  

(32)

Integrating the $x$ and $\varphi$ component of Eq.(30), one gets the 2-dimensional effective action

$$S = -\frac{1}{2} \sum_{lm} \int d\eta dy_s \tilde{\phi}_l(\eta, y_s)(-\partial_{\eta}^2 + \partial_{y_s}^2) \phi_l(\eta, y_s).$$  

(33)

Eq.(33) indicates that the near horizon 2-dimensional effective metric is of the same form as Eq.(22) and requires the same flux as Eq.(23) to cancel the gravitational anomaly. Obviously, the Killing vector that generates the black hole horizon of generalized $C$ metric is $\xi^a = (\partial_t)^a + a y_H^2 (\partial_n)^a$. Thus, the surface gravity $\kappa$ is again $|H'(y_H)|/2$. Therefore, the Hawking temperature for the black hole horizon in the generalized $C$ metric is

$$T_H = \frac{|H'(y_H)|}{4\pi}.$$  

(34)

On the other hand, from Eq.(27), on the 2-brane (i.e. $x = 0$), $\rho = 1/y$ for ($y > 0$). Meanwhile, $H(y) = -\lambda + k y^2 + a^2 y^4 = 0$ gives $y^2_+ = \frac{-k \pm \sqrt{k^2 + \lambda a^2}}{a^2/2}$, and the black hole horizon is at $y = y_-$. Since one has the relation $\rho \pm = 1/y \mp$, it is easy to check that the Hawking temperature of the black hole horizon in the bulk obtained from Eq.(34) is

$$T_H = \frac{|H'(y)|}{4\pi} \bigg|_{y_-} = \frac{\sqrt{k^2 + \lambda a^2}}{\pi a} \sqrt{-k - \sqrt{k^2 + \lambda a^2}}.$$  

(35)

Clearly, one gets $T_H = T_h$, again. The reason is the same as before, i.e., the BTZ-like brane world black hole described by Eq.(28) is just the boundary of the black hole in the bulk [25, 29]. Therefore, through applying the anomaly cancellation method to the bulk generalized $C$ metric, one can get the correct Hawking temperature of the BTZ-like brane world black hole.
V. CONCLUSIONS AND DISCUSSIONS

In this letter, we showed that the anomaly cancellation method proposed by Wilczek and his collaborators can be applied to the 3-dimensional brane world black holes generated by the generalized $C$ metrics in the RS scenario. Of course, one can apply this method directly to the brane world black hole themselves. However, the brane world black holes are different from the ordinary black holes in general relativity. A significant difference is that their horizons will extend into the extra dimensions, but not merely on the brane, which means that the brane world black hole will radiate both on the brane and in the bulk, e.g., \cite{40, 41}. This property allows one to apply the anomaly cancellation approach to the bulk spacetime to reproduce the Hawking temperature of the brane world black holes. Although the 3-dimensional brane world black holes and gravitational anomaly are only taken into account here, they are easily generalized to the situations of higher dimensional brane world black holes and with additional gauge anomalies. In fact, the above calculations also hold for the generalized $C$ metrics (if one forgets the notion of brane world black holes). Meanwhile, because this method does not involve the dynamical equation, i.e., the Einstein equation, it also shows that the Hawking radiation itself is not a dynamical effect but only a kinematic one (notice that this method has been used in \cite{22} to get the Hawking radiation in acoustic black holes), which is in accord with the studies about the acoustic black holes and the uniformly accelerated reference frames \cite{30, 31, 32, 33, 34}. The previously successful works of applying this approach to various spacetime backgrounds suggest that there is a deep relationship between the anomaly and Hawking radiation of the horizon, and it is hopeful to extend this method to the more general kinds of horizons. We shall consider such problems in a later work.

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