Learning to Switch Between Machines and Humans

Vahid Balazadeh Meresht\textsuperscript{1}, Abir De\textsuperscript{2}, Adish Singla\textsuperscript{3}, and Manuel Gomez-Rodriguez\textsuperscript{3}

\textsuperscript{1}Sharif University of Technology, vbalazadehmeresht@ce.sharif.edu
\textsuperscript{2}IIT Bombay, abir@cse.iitb.ac.in
\textsuperscript{3}MPI for Software Systems, \{adishs,manuelgr\}@mpi-sws.org

Abstract

Reinforcement learning algorithms have been mostly developed and evaluated under the assumption that they will operate in a fully autonomous manner—they will take all actions. However, in safety critical applications, full autonomy faces a variety of technical, societal and legal challenges, which have precluded the use of reinforcement learning policies in real-world systems. In this work, our goal is to develop algorithms that, by learning to switch control between machines and humans, allow existing reinforcement learning policies to operate under different automation levels. More specifically, we first formally define the learning to switch problem using finite horizon Markov decision processes. Then, we show that, if the human policy is known, we can find the optimal switching policy directly by solving a set of recursive equations using backwards induction. However, in practice, the human policy is often unknown. To overcome this, we develop an algorithm that uses upper confidence bounds on the human policy to find a sequence of switching policies whose total regret with respect to the optimal switching policy is sublinear. Simulation experiments on two important tasks in autonomous driving—lane keeping and obstacle avoidance—demonstrate the effectiveness of the proposed algorithms and illustrate our theoretical findings.

1 Introduction

In recent years, reinforcement learning algorithms have achieved, or even surpassed, human performance in a variety of computer games by taking decisions autonomously, without human intervention \[Mnih et al., 2015, Silver et al., 2016, 2017, Vinyals et al., 2019\]. Motivated by these successful stories, there has been a tremendous excitement on the possibility of using reinforcement learning algorithms to operate fully autonomous cyberphysical systems, especially in the context of autonomous driving. Unfortunately, a number of technical, societal and legal challenges have precluded this possibility to become so far a reality, humans are still more skilled drivers than machines, and the vast majority of work has focused on toy examples in controlled synthetic car simulator environments \[Wymann et al., 2000, Dosovitskiy et al., 2017, Talpaert et al., 2019\].

In this work, we argue that existing reinforcement learning algorithms may still enhance the operation of cyberphysical systems if deployed under lower automation levels. In other words, if we let algorithms take some of the actions and leave the remaining ones to humans, the resulting performance may be better than the performance algorithms and humans would achieve on their own \[Raghu et al., 2019a, De et al., 2020\]. However, once we depart from full automation, we need to address the following question: when should we switch control between a machine and a human? In this work, our goal is to develop algorithms that learn to optimally switch control automatically. However, to this aim, we need to address several challenges:

— Amount of human control. In each application, what is considered an appropriate and tolerable load for humans may differ \[European Parliament, 2006\]. Therefore, we would like that our algorithms provide mechanisms to control the amount of human control (or the level of automation) during a given time period.

— Number of switches. Consider two different switching patterns resulting in the same performance and amount of human control\[1\]. Then, we would like that our algorithms favor the pattern with the least

\[1\]For simplicity, we will assume that the human policy does not change due to switching.
number of switches since, every time a machine defers (takes) control to (from) a human, there is an additional cognitive load for the human [Brookhuis et al., 2001].

— Unknown human policies. The spectrum of human abilities spans a broad range [Macadam, 2003]. As a result, there is a wide variety of potential human policies. Here, we would like that our algorithms learn personalized switching policies that, over time, adapt to the particular human they are dealing with.

To tackle these challenges, we first formally define the learning to switch problem using finite horizon Markov decision processes. Under this definition, the problem reduces to finding the switching policy that provides an optimal trade off between environmental cost, the amount of human control and the number of switches. Then, we make the following contributions. We show that, if the human policy is known, we can find the optimal switching policy directly by solving a set of recursive Bellman equations using backwards induction. However, in practice, the human policy is often unknown, as discussed previously. To overcome this, we develop an algorithm that uses upper confidence bounds on the human policy to find a sequence of switching policies whose total regret with respect to the optimal switching policy is sublinear. Finally, we experiment with simulated data in two important tasks in semi-autonomous driving—lane keeping and obstacle avoidance—and demonstrate the effectiveness of the proposed algorithms as well as illustrate our theoretical finding.

Related work. There is a rapidly increasing line of work on learning to defer decisions in the machine learning literature [Bartlett and Wegkamp, 2008; Cortes et al., 2016; Geifman et al., 2018; Geifman and Ef-Yaniv, 2019; Raghu et al., 2019a; Humaswamy et al., 2018; Tschiatschek et al., 2019; Raghu et al., 2019b; Liu et al., 2019; De et al., 2020]. However, previous work has typically focused on supervised learning. More specifically, it has developed classifiers that learn to defer either by considering the defer action as an additional label value, by training an independent classifier to decide about deferred decisions, or by reducing the problem to a combinatorial optimization problem. Moreover, except for two very recent notable exceptions [Raghu et al., 2019a; De et al., 2020], they do not consider there is a human decision maker who takes a decision whenever the classifiers defer it. In contrast, we focus on reinforcement learning and develop algorithms that learn to switch control between a human and a machine policy.

Our work contributes to an extensive body of work on human-machine collaboration [Maciand and Wilson, 2012; Nikolaidis et al., 2015; Hadfield-Menell et al., 2016; Nikolaidis et al., 2017; Grover et al., 2018; Wilson and Daugherdy, 2017; Theiliou et al., 2019; Kamalaruban et al., 2019; Radanovic et al., 2019; Ghosh et al., 2020]. However, rather than developing algorithms that learn to switch control between human and machines, previous work has predominantly considered settings in which the machine and the human interact with each other. Finally, our work also relates to a recent line of work that combines deep reinforcement learning with opponent modeling to robustly switch between multiple machine policies [Everett and Roberts, 2018; Zheng et al., 2018]. However, this line of work does not consider there is a human policy neither derives theoretical guarantees on the performance of the proposed algorithms.

2 Problem Formulation

Our starting point is the following problem setting, which fits a variety of real-world applications. At each time step $t \in \{0, \ldots, T-1\}$, our (cyberphysical) system is characterized by a state $s_t \in S$, where $S$ is a finite state space, and a control switch $d_t \in D$, with $D = \{\mathbb{H}, \mathbb{M}\}$, which determines who takes an action $a_t \in A$, where $A$ is a finite action space. More specifically, the switch value $d_t$ is sampled from a (time-varying) switching policy $\pi_t(d_t | s_t, d_{t-1})$. If $d_t = \mathbb{H}$, the action $a_t$ is sampled from a human policy $p_H(a_t | s_t)$ and, if $d_t = \mathbb{M}$, it is sampled from a machine policy $p_M(a_t | s_t)$. Throughout the paper, we will assume that the machine policy $p_M$ is known. Moreover, given a state $s_t$ and an action $a_t$, the state $s_{t+1}$ is sampled from a transition probability $p(s_{t+1} | s_t, a_t)$. Finally, given a trajectory of switching patterns and states $\tau = \{(s_t, d_t)\}_{t=0}^{T-1}$ and an initial state $(s_0, d_{-1})$, we define the total cost $c(\tau | s_0, d_{-1})$ as:

$$c(\tau | s_0, d_{-1}) = \sum_{t=0}^{T-1} c(s_t, d_t) + \lambda_1 d_t \mathbb{1}[d_t = \mathbb{H}] + \lambda_2 d_t \mathbb{1}[d_t = \mathbb{M}] ,$$

where the first term $c(s_t, d_t) = \mathbb{E}_{s_t \sim p(s_t | s_t)} [c'(s_t, a_t)]$ is the expected environment cost of switch value $d_t$ at state $s_t$, $c'(s_t, a_t)$ is the environment cost of action $a_t$ at state $s_t$, the second and third terms penalize

---

2To facilitate research in this area, we will release an open-source implementation of our algorithms with the final version of the paper.
the amount of human control and number of switches, respectively, and the parameters $\lambda_1$ and $\lambda_2$ control
the trade off between the expected environmental cost, the amount of human control and the number of
switches.

Next, we characterize the above problem setting using a finite horizon Markov decision process (MDP)
$\mathcal{M} = (\mathcal{S} \times \mathcal{D}, \mathcal{D}, P_{\pi^{\text{ps}}, p_1}, C_{\pi^{\text{ps}}, p_1}, T)$, where $\mathcal{S} \times \mathcal{D}$ is an augmented state space, the set of actions $\mathcal{D}$ is
just the switch values, the transition dynamics $P_{\pi^{\text{ps}}, p_1}$ at time $t$ are given by
$$
p_{\pi^{\text{ps}}, p_1}(s_{t+1}, d_t \mid s_t, d_{t-1}) = \pi_t(d_t \mid s_t, d_{t-1})p(s_{t+1} \mid s_t, d_t),
$$
where the expectation is taken over all the trajectories induced by the switching policy given the human
policy. This is because Algorithm 1 only depends on the human policy through two expectations (lines
7 and 8) which do not depend on the switching policy but only on the machine and human policies, which are
induced, starting with $v_0$ from the above immediate costs and, while the set of actions $\mathcal{A}$ of the
machine and the human policy can be large, the action state of this Markov decision process is binary
(i.e., $H$ and $M$).

Then, our goal is to find the optimal switching policy $\pi^*$, $\pi^* = (\pi_0^*, \ldots, \pi_T^*)$ that maximizes the expected
cost, as defined by Eq. (1), i.e.,
$$
\pi^* = \arg\min_{\pi} E_{\tau \sim p_{\pi^{\text{ps}}, p_1}}[c(\tau \mid s_0, d_{-1})],
$$
where the expectation is taken over all the trajectories induced by the switching policy given the
human and machine policies $p_\text{M}$ and $p_\text{H}$.

3 Known Human Policy

In this section, we address the problem of learning to switch, as defined in Eq. (3) under the assumption
that the human policy $p_\text{H}$ is known.

Given a finite horizon Markov decision process $\mathcal{M} = (\mathcal{S} \times \mathcal{D}, \mathcal{D}, P_{\pi^{\text{ps}}, p_1}, C_{\pi^{\text{ps}}, p_1}, T)$, as defined in
Section 2 and a switching policy $\pi = (\pi_0, \ldots, \pi_T)$, we define the optimal value function $v_t(s, d)$ for
each $t \in \{0, \ldots, T-1\}$ as
$$
v_t(s, d) = \min_{\pi_t} \mathbb{E} \left[ \sum_{t'=t}^{T-1} c_{\pi_t}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right].
$$
(4)

In the above, note that we can directly recover the objective function in Eq. (3) from $v_0(s_0, d_{-1})$. Moreover,
using Bellman’s principle of optimality, it is easy to show that $v_t(s, d)$ satisfies the following recursive
equation (refer to Appendix A.2):
$$
v_t(s, d) = \min_{\pi_t} \left\{ c_{\pi_t}(s, d) + \mathbb{E}_{d' \sim \pi_t(\cdot \mid s, d)}[v_{t+1}(s', d')] \right\},
$$
(5)
with $v_T(s, d) = 0$ for all $s \in \mathcal{S}$ and $d \in \mathcal{D}$.

Then, we can directly solve the minimization problem in Eq. (5) and express the optimal switching
policy $\pi_t^*$ in terms of the optimal value function $v_{t+1}$. More specifically, for any $s \in \mathcal{S}$ and $d \in \mathcal{D}$, define
$$
q_{t,M}(s, d) = c(s, M) + \lambda_1 \mathbb{I}(M \neq d) + \mathbb{E}_{d' \sim \pi_t(\cdot \mid s, d)}[v_{t+1}(s', M)],
$$
(6)
and
$$
q_{t,H}(s, d) = c(s, H) + \lambda_1 \mathbb{I}(H \neq d) + \mathbb{E}_{d' \sim \pi_t(\cdot \mid s, H)}[v_{t+1}(s', H)].
$$
(7)

Then, we have the following proposition (proven in Appendix A.2):

**Proposition 1.** For any $s \in \mathcal{S}$ and $d \in \mathcal{D}$, the optimal switching policy $\pi_t^*(d' = M \mid s, d) = 1$ if
$$
q_{t,M}(s, d) < q_{t,H}(s, d),
$$
and $\pi_t^*(d' = M \mid s, d) = 0$ otherwise.

Using the above result, we can find the optimal switching policy $\pi^* = (\pi_0^*, \ldots, \pi_T^*)$ using backwards
induction, starting with $v_T(s, d) = 0$ for all $s \in \mathcal{S}$ and $d \in \mathcal{D}$. Here, note that the expectations in Eqs. (6)
and (7) do not depend on the switching policy but only on the machine and human policies, which are
known. Algorithm 1 summarizes the whole procedure.

**Remarks.** In practice, to implement Algorithm 1 we only need to have access to (off-policy) historical
data about the human driver, rather than explicitly fitting a (parameterized) model for the human policy. This is because Algorithm 1 only depends on the human policy through two expectations (lines
Algorithm 1 It returns the optimal switching policy under the assumption that the human policy is known.

Require: Machine policy $p_M$, human policy $p_H$, environment cost $C = [c'(s, a)]$, parameters $\lambda_1$ and $\lambda_2$.

1: $\pi \leftarrow \text{InitializePolicy}()$
2: $v \leftarrow \text{InitializeValueFunction}()$
3: for $t = T - 1, \ldots, 0$ do
4:     for $(s, d) \in S \times D$ do
5:         $q_M \leftarrow \lambda_2 \mathbb{I}[M \neq d] + \mathbb{E}_{a \sim p_M(\cdot | s)} [c'(s, a)]$
6:         $q_H \leftarrow q_H + \mathbb{E}_{a \sim p_H(\cdot | s, d, a)}[v_{t+1}(s', M)]$
7:         $q_H \leftarrow \lambda_1 \mathbb{I}[H \neq d] + \mathbb{E}_{a \sim p_H(\cdot | s)} [c'(s, a)]$
8:         $q_{\pi} \leftarrow q_M + \mathbb{E}_{a \sim p_H(\cdot | s, d, a)}[v_{t+1}(s', H)]$
9:         if $q_{\pi} < q_H$ then
10:             $v_T(s, d) = q_M$
11:             $\pi_T(d' = M | s, d) \leftarrow 1$, $\pi_T(d' = H | s, d) \leftarrow 0$
12:         else
13:             $v_T(s, d) = q_H$
14:             $\pi_T(d' = M | s, d) \leftarrow 0$, $\pi_T(d' = H | s, d) \leftarrow 1$
15:         end if
16:     end for
17: end for
18: Return $\pi$

7-8). Therefore, we can use the historical data to compute a finite sample Monte-Carlo estimator of these expectations.

4 Unknown Human Policy

In this section, we address the problem of learning to switch, as defined in Eq. 3, in a more realistic setting in which the human policy is unknown—we do not know the particular human driver we are dealing with.

When we do not know the human policy our switching policy is dealing with, we need to trade off exploitation, i.e., minimizing the expected cost, and exploration, i.e., learning about the human policy. To this end, we look at the problem from the perspective of episodic learning and proceed as follows.

We consider $K$ independent subsequent episodes of length $L$ and denote the aggregate length of all episodes as $T = KL$. Each of these episodes corresponds to a realization of the same finite horizon Markov decision process $M = (S \times D, \pi, \mathbb{E}_{a \sim \pi(\cdot | s, d)}[c(a)], \mathbb{E}_{a \sim \pi(\cdot | s, d)}[1 L])$, where $p_M$ denotes the true human policy. However, since the true human policy $p_M^*$ is unknown to us, just before each episode $k$ starts, our goal is to find a switching policy $\pi^*$ with desirable properties in terms of total regret $R(T)$, which is given by:

$$R(T) = \sum_{k=1}^{K} \mathbb{E}_{\tau \sim P_{\pi^*}^{k}[\pi^*]} [c(\tau | s_0, d_{-1})] - \mathbb{E}_{\tau \sim P_{\pi^*}^{k}[\pi^*]} [c(\tau | s_0, d_{-1})],$$

where $\pi^*$ is the optimal switching policy under the true human policy $p_M^*$. To achieve our goal, we apply the principle of optimism in the face of uncertainty, i.e.,

$$\pi^k = \arg\min_{\pi} \mathbb{E}_{\tau \sim P_{\pi}^{k}[\pi]} [c(\tau | s_0, d_{-1})]$$

where $P^{k}_{\pi}$ is a $(|S| \times L)$-rectangular confidence set, i.e., $P^{k}_{\pi} = \Xi_{s,t} P^{k}_{\pi}$. Here, note that the confidence set is constructed using data gathered during the first $k-1$ episodes and allows for time-varying human policies $p_M(\cdot | s, t)$. However, to solve Eq. 9 we first need to explicitly define the confidence set. To this end, we first define the empirical distribution $\hat{p}^{k}_{s,t}(\cdot | s)$ of the true human policy $p_M^* (\cdot | s)$ just before episode $k$ starts as:

$$\hat{p}^{k}_{s,t}(\cdot | s) = \begin{cases} \frac{N_k(s, a)}{N_k(s)} & \text{if } N_k(s) \neq 0 \\ \frac{1}{|S|} & \text{otherwise,} \end{cases}$$
where
\[ N_k(s) = \sum_{i=1}^{k-1} \sum_{t \in [L]} \mathbb{I}(s_t = s, d_t = \mathbb{H} \text{ in episode } i), \]
\[ N_k(s, a) = \sum_{i=1}^{k-1} \sum_{t \in [L]} \mathbb{I}(s_t = s, a_t = a, d_t = \mathbb{H} \text{ in episode } i). \]

Then, similarly as in Jaksch et al. [2010], we opt for a \( L^1 \) confidence set \( \mathcal{P}_k^b(\delta) = \bigtimes_{s, t} \mathcal{P}_{k|s, t}^b(\delta) \) with 
\[ \mathcal{P}_{k|s, t}^b(\delta) = \{ p_{\mathbb{H}} : ||p_{\mathbb{H}}(\cdot | s, t) - \hat{p}_k^b(\cdot | s)||_1 \leq \beta_k(s, \delta) \}, \]
for all \( s \in S \) and \( t \in [L] \), where \( \delta \) is a given parameter and 
\[ \beta_k(s, \delta) = \sqrt{\frac{14|A| \log \left( \frac{(k-1)L|S|}{\delta} \right)}{\max\{1, N_k(s)\}}}. \]  

Moreover, for each episode \( k \), we define the optimal value function \( v_k^F(s, d) \) as 
\[ v_k^F(s, d) = \min_{\pi} \min_{p_{\mathbb{H}} \in \mathcal{P}_{k|s, t}^b(\delta)} \mathbb{E} \left[ \sum_{t'=1}^{L-1} c_{s, t'}(s_{t'}, d_{t'-1}) | s_t = s, d_{t-1} = d \right] \]
Then, we are ready to use the following key theorem (proven in Appendix A.3), which gives a solution to Eq. 2.

**Theorem 2.** For any episode \( k \), the optimal value function \( v_k^F(s, d) \) satisfies the following recursive equation:
\[ v_k^F(s, d) = \min_{\pi} \min_{p_{\mathbb{H}} \in \mathcal{P}_{k|s, t}^b(\delta)} \mathbb{E} \left[ \sum_{t'=1}^{L-1} c_{s, t'}(s_{t'}, d_{t'-1}) | s_t = s, d_{t-1} = d \right] \]
where
\[ q_k^F(\pi_{\mathbb{H}}(s, d) = \hat{c}(s, \mathbb{H}) + \lambda_1 \mathbb{I}(\mathbb{H} \neq d) + \mathbb{E}_{a \sim p_{\mathbb{H}}(\cdot | s, a)} \left[ v_{k+1}^F(s', \mathbb{H}) \right] \]
\[ q_k^F(\pi_{\mathbb{H}}(s, d) = \hat{c}(s, \mathbb{H}) + \lambda_1 + \lambda_2 \mathbb{I}(\mathbb{H} \neq d) + \mathbb{E}_{a \sim p_{\mathbb{H}}(\cdot | s, a)} \left[ v_{k+1}^F(s', \mathbb{H}) \right] \]
with \( v_k^F(s, d) = 0 \) for all \( s \in S \) and \( d \in D \). Moreover, for any \( s \in S \) and \( d \in D \), the optimal switching policy \( \pi_k^F(d' = \mathbb{H} | s, d) = 1 \) if 
\[ q_k^F(\pi_{\mathbb{H}}(s, d) < \min_{p_{\mathbb{H}} \in \mathcal{P}_{k|s, t}^b(\delta)} q_k^F(\pi_{\mathbb{H}}(s, d) \]
and \( \pi_k^F(d' = \mathbb{H} | s, d) = 0 \) otherwise.

The above result readily implies that, just before each episode \( k \) starts, we can find the optimal switching policy \( \pi_k = (\pi_k^0, \ldots, \pi_k^{k-1}) \) using backwards induction, starting with \( v_L(s, d) = 0 \) for all \( s \in S \) and \( d \in D \). Moreover, similarly as in Strehl and Littman [2008], we can solve the minimization problem 
\[ \min_{p_{\mathbb{H}}(\cdot | s, t) \in \mathcal{P}_{k|s, t}^b(\delta)} q_k^F(\pi_{\mathbb{H}}(s, d) \]
analytically using the following Lemma (proven in Appendix A.4):

**Lemma 3.** Consider the following minimization problem:
\[ \min_{x} \sum_{i=1}^{m} x_i w_i \]
subject to 
\[ \sum_{i=1}^{m} |x_i - b_i| \leq d, \sum_{i} x_i = 1, \]
\[ x_i \geq 0 \forall i \in \{1, \ldots, m\}, \]
where \( d \geq 0, b_i \geq 0 \forall i \in \{1, \ldots, m\}, \sum_i b_i = 1 \text{ and } 0 \leq w_1 \leq w_2 \cdots \leq w_m. \) Then, the solution to the above minimization problem is given by:
\[ x_i^* = \begin{cases} \min\{1, b_i + \frac{d}{2}\} & \text{if } i = 1 \\ b_i & \text{if } i > 1 \text{ and } \sum_{i=1}^{l} x_i \leq 1 \\ 0 & \text{otherwise}. \end{cases} \]  

\(^3\)This choice will result into a sequence of switching policies with desirable properties in terms of total regret.
Algorithm 2 It applies the principle of optimism in the face of uncertainty to find and deploy a sequence of policies $\pi^k$.

Require: Machine policy $p_M$, environment cost $C = [c'(s, a)]$, parameters $\lambda_1$, $\lambda_2$ and $\delta$.

1: $(\{N_k(s)\}, \{N_k(s, a)\}) \leftarrow \text{InitializeCounts}()$
2: for $k = 1, \ldots, K$ do
3: $p_M^k \leftarrow \text{UpdateDistribution}((N_k(s)), \{N_k(s, a)\})$
4: $p_M^k \leftarrow \text{UpdateConfidenceSet}(p_M^k, \delta)$
5: $\pi^k \leftarrow \text{GetOptimal}(p_M^k, p_M^k, C, \lambda_1, \lambda_2)$
6: $(s_0, d_{-1}) \leftarrow \text{InitializeConditions}()$
7: for $t = 0, \ldots, L - 1$ do
8: if $\pi^k(d' = M| s_t, d_{t-1}) = 0$ then
9: $d_t \leftarrow M$
10: $a_t \sim p_M(\cdot| s_t)$
11: else
12: $d_t \leftarrow M$
13: $a_t \sim p_M(\cdot| s_t)$
14: end if
15: $N_k(s_t, a_t) \leftarrow N_k(s_t, a_t) + 1$
16: $N_k(s_t) \leftarrow N_k(s_t) + 1$
17: $s_{t+1} \sim p(\cdot| s_t, a_t)$
18: end for
19: end for
20: Return $\pi^K$

More specifically, to apply the lemma, we just need to consider $m = |\mathcal{A}|$ and, for all $i \in \{1, \ldots, m\}$, $x_i = p_M(a_i | s, t)$, $w_i = \mathbb{E}_{s' \sim p(\cdot| s, a_i)}[v_{t+1}^k(s', \mathbb{H})]$, $b_i = p_M^k(a_i | s)$ and $d = \beta(s, \delta)$. Algorithm 2 summarizes the whole procedure.

Within the algorithm, the function $\text{GetOptimal}()$ finds the optimal policy $\pi^k$ using backwards induction, similarly as in Algorithm 1; however, in contrast with Algorithm 1, it computes $q_M$ by solving a minimization problem using Lemma 3. Moreover, it is important to notice that, in lines 7–18, the switching policy $\pi^k$ is actually deployed, the machine and the true human take actions and, as a result, action data from the true human is gathered. Finally, the following theorem shows that the sequence of policies $\{\pi^k\}_{k=1}^K$ found by Algorithm 2 achieve sublinear total regret, as defined in Eq. (16) (proven in Appendix A.5):

**Theorem 4.** Assume we use Algorithm 3 to find the switching policies $\pi^k$. Then, with probability at least $1 - \delta$, it holds that

$$R(T) \leq \rho L \sqrt{|S||A||T| \log \left( \frac{|S||T|}{\delta} \right)},$$

where $\rho > 0$ is a constant.

5 Experiments

In this section, we perform a variety of simulation experiments in autonomous driving. Our goal here is to demonstrate that the switching policies found by Algorithms 1 and 2 enable the resulting cyberphysical system to successfully perform lane keeping and obstacle avoidance.

**Environment setup.** We consider three types of lane driving environments, as illustrated in Figure 1. Each type of environment requires different driving skills. For example, in the environment (a), the cyberphysical system only needs to perform lane keeping to drive through the traffic-free road. In contrast, in the environments (b–c), it needs to perform both lane keeping and obstacle avoidance to drive through heavy traffic and avoid complex obstacles such as stones. In each of these three lane environments, there are three lanes, $3 \times 10$ cells and the type of each individual cell (i.e., road, car, stone or grass) is sampled independently at random with a probability that depends on the type of environment.

We ran all experiments on a machine equipped with Intel(R) Core(TM) i7-4710HQ CPU @ 2.50GHz and 12 GB memory.
Figure 1: Three types of lane driving environments. Panel (a) shows an instance of the first type of driving environment, in which the type of each cell in the middle lane is always road while the types of each cell in the left and right lanes is road, grass or stone with probability $0.4$, $0.3$ and $0.3$, respectively. Panel (b) shows an instance of the second type of driving environment, in which the type of each cell in the middle lane is road with probability $0.6$ and car with probability $0.4$ while the types of each cell in the left and right lanes follows the same distribution as in Panel (a). Panel (c) shows an instance of the third type of driving environment, in which the type of each cell in all lanes is road, grass or car with probability $0.5$, $0.3$, and $0.2$, respectively. In all driving environments, the type of the start cell is always ‘road’ and the goal of the cyberphysical system is to drive the car from an initial state at the bottom of the lane to the top of the lane following a trajectory with the minimum cost.

The goal of the cyberphysical system is to drive the car from an initial state at the bottom of the lane to the top of the lane. At any given time $t$, we assume that whoever is in control—be it the machine or the human—can take three different actions $A = \{\text{left}, \text{straight}, \text{right}\}$. Action left steers the car to the left of the current lane, action right steers it to the right and action straight leaves the car in the current lane. If the car is already on the leftmost (rightmost) lane when taking action left (right), then the lane is randomly chosen with probability $0.5$. Irrespective of the action taken, the car always moves forward. Therefore, whenever the human policy is known, we have $T = 9$, and, whenever it is unknown, we have $L = 9$.

State space. To evaluate the switching policies found by Algorithm 1, we experiment both with a cell-based and a sensor-based state space and, to evaluate the switching policies found by Algorithm 2, we experiment only with a sensor-based state space.

— Cell-based state space: We characterize each individual lane driving environment using a different cell-based state space, where each cell within the environment represents a state. Therefore, the resulting MDP has $3 \times 10$ states. This choice of state space is transductive since it can only be used in a single environment and the resulting switching policy cannot be applied in a different environment.

— Sensor-based state space: We characterize all lane driving environments using the same sensor-based state space representation. More specifically, the state values are just the type of the current cell and the three cells the car can move into in the next time step, e.g., assume at time $t$ the car is on a road cell and, if it moves forward left, it hits a stone, if it moves forward straight, it hits a car, and, if it moves forward right, it drivers over grass, then its state value is $s_t = (\text{road, stone, car, grass})$. Moreover, if the car is on the leftmost (rightmost) lane, then we set the value of the second (fourth) dimension in $s_t$ to $\emptyset$. Therefore, the resulting MDP has $\sim 4^4$ states and, given a type of driving environment, we can compute the transition probabilities $p(s_{t+1} | s_t, a_t)$ analytically. This choice of state space representation is inductive since it can be used across different environments and the same switching policy can be applied across multiple environments.\footnote{One could think of defining a cell-based state space for a set of multiple lane driving environments, however, the resulting state space could only be used in those environments in the set.}

\footnote{Note that a switching policy that is optimal for a type (or types) of lane driving environments may be suboptimal in other...}
Algorithm 1 to find the optimal switching policies in a variety of lane driving environments using $\sigma$ with types of environments. Of equations for each individual environment while, to implement the latter, we just need to solve one set experimental setup. In our experiments, we did not find noticeable differences among the trajectories car). This happens because, in such situation, the human policy is worse control to the machine whenever there is an imminent danger (i.e., absence of traffic. Whenever there is traffic (Environments 2 and 3), the optimal switching policy gives better surprising since, in our experimental setup, the human policy is always long as the cost of human control, set by $\lambda$ and $\sigma$. Here, whenever we use the sensor-based state space, note that we obtain an $\lambda$ and $\sigma$ switching policies for several types of lane driving environments and different values of the parameters $\lambda$ and $\sigma$. Figure 2 shows the trajectories induced by the optimal policy is known. The blue and orange segments indicate machine and human control, respectively. Each line’s width is proportional to the empirical probability that the trajectory contains that transition. Each experiment is repeated 100 times.

Cost and human/machine policies. Under both state space representations, we consider a state-dependent environment cost $\hat{c}(s_t, d_t) = \hat{c}(s_t)$ that depends on the type of the cell the car is on at state $s_t$, i.e., $\hat{c}(s_t) = 0$ if the type of the current cell is road, $\hat{c}(s_t) = 2$ if it is grass, $\hat{c}(s_t) = 4$ if it is stone and $\hat{c}(s_t) = 5$ if it is car. Moreover, we consider that whoever is in control—be it the machine or the human—pick which action to take (left, straight or right) according to a noisy estimate of the environment cost of the three cells that the car can move into in the next time step. More specifically, the machine computes a noisy estimate of the cost $\hat{c}(s) = \hat{c}(s) + \epsilon_s$ of each of the three cells the car can move into, where $\epsilon_s \sim N(0, \sigma_M)$, and picks the action that moves the car to the cell with the lowest noisy estimate. The human also computes a noisy estimate of the costs $\hat{c}(s) = \hat{c}(s) + \epsilon_s$, where $\epsilon_s \sim N(0, \sigma_H)$ with $\sigma_H < \sigma_M$. However, she does not always pick the action that moves the car to the cell with the lowest noisy estimate. In particular, if there is a car in either of the three cells that she can move into, she panics and moves to the cell where the car is with probability $p_{panic}$. In other words, we assume the human driver is generally more reliable than the machine, however, when there is an imminent danger (i.e., a potential crash against another car), the machine is more reliable than the human. Throughout our experiments, if not said otherwise, we set $\sigma_H = 1$, $\sigma_M = 4$ and $p_{panic} = 0.4$. Finally, without loss of generality, we consider that only the car driven by our cyberphysical system moves in the environment.

Insights into the optimal switching policies. First, we assume the human policy is known and use Algorithm[1] to find the optimal switching policies in a variety of lane driving environments using both a cell-based and a sensor-based state space. Figure 2 shows the trajectories induced by the optimal switching policies for several types of lane driving environments and different values of the parameters $\lambda_1$ and $\lambda_2$, which control the trade off between the expected environment cost, the amount of human control and the number of switching. Here, whenever we use the sensor-based state space, note that we obtain an optimal switching policy for each type of environments, rather than a single environment. However, for ease of visualization, we show the trajectories induced by the policy in a single individual environment per environment type, picked at random. The results reveal several interesting insights. In the absence of traffic (Environment 1), the optimal switching policy gives control to the human most of the time as long as the cost of human control, set by $\lambda_1$, is not too large ($\lambda_1 = 0.6$ vs $\lambda_1 = 0.2$). However, this is not surprising since, in our experimental setup, the human policy is always better than the machine policy in absence of traffic. Whenever there is traffic (Environments 2 and 3), the optimal switching policy gives control to the machine whenever there is an imminent danger (i.e., a potential crash against another car). This happens because, in such situation, the human policy is worse than the machine policy in our experimental setup. In our experiments, we did not find noticeable differences among the trajectories based on cell-based and sensor-based state. However, to implement the former, we need to solve one set of equations for each individual environment while, to implement the latter, we just need to solve one set of environments.
of equations per type of environment.

Next, we assume the human policy is unknown and use Algorithm 2 to find a sequence of switching policies with sublinear regret in a variety of lane driving environments using a sensor-based state space. Figure 3 shows the trajectories induced by the switching policies found by our algorithm across different episodes for several values of the parameters $\lambda_1$ and $\lambda_2$. The results show that, in the latter episodes, the algorithm has learned to rely on the machine (blue segments) to drive whenever there is an imminent danger (i.e., a car). Moreover, whenever the amount of human control and number of switches is not penalized (i.e., $\lambda_1 = \lambda_2 = 0$), the algorithm switches to the human more frequently in order to reduce the environment cost.

### Quantitative performance

We first consider that the human policy is known and evaluate the performance achieved by the (optimal) switching policies found using Algorithm 1. More specifically, we investigate the influence that the quality of the human driver, as tuned by the noise variance $\sigma_H$, has on the number of switches and the amount of human control. Figure 4 summarizes the results for several types of environments and values of the parameters $\lambda_1$ and $\lambda_2$ using a sensor-based state space. As one could perhaps hoped for, we find that, if the human driver is less (more) skilled, the optimal switching policy decides to reduce (increase) the amount of human control and number of switches. Moreover, whenever the amount of human control and number of switches is penalized (i.e., $\lambda_1 > 0$, $\lambda_2 > 0$), the algorithm is stricter with the human driver and relies entirely on the machine for $\sigma_H \geq 3$.

Next, we assume we do not have any prior knowledge on the human policy and evaluate the performance achieved by the sequence of policies using Algorithm 2 under a sensor-based state space. To this aim, we compare the total regret achieved by the sequence of policies, as defined in Eq. 8, and that achieved by a greedy baseline, which just finds the optimal policy at each episode $k$ using Algorithm 1 with $\hat{p}^k_H$, as defined in Eq. 10, as human policy. Figure 5 summarizes the results for two types of environments and values of parameters $\lambda_1$ and $\lambda_2$. As expected, the sequence of policies found by Algorithm 2 achieve sublinear regret while those found by the greedy baseline, due to a lack of exploration, achieve linear regret. However, whenever the number of switches and the amount of human control is penalized (i.e., $\lambda_1 > 0$, $\lambda_2 > 0$), the human is in control less time and Algorithm 2 takes longer to accurately estimate how skilled is the human driver is dealing with. As a result, its competitive advantage with respect to the greedy algorithm only becomes apparent after 2,000 episodes.
Throughout the paper, we have assumed that the state space is discrete. It would be very interesting to lift this assumption and develop approximate value iteration methods to solve the learning to switch problem. Moreover, we have considered that the human policy does not change due to switching control, however, this assumption is often violated in practice [Wolfe et al., 2019]. Finally, it would be interesting to assess the performance of our algorithms using interventional experiments on a real-world (semi-)autonomous driving system.

6 Conclusions

In this work, we have tackled the problem of learning to switch control between machines and humans in sequential decision making. After formally defined the learning to switch problem using finite horizon MDPs, we have first shown that, if the human policy is known, the optimal switching policy can be found just by solving a set of recursive equations using backwards induction. Then, we have developed an algorithm that, without prior knowledge of the human policy, it is able to find a sequence of switching policies whose total regret is sublinear. Finally, we have performed a variety of simulation experiments on autonomous driving to show the effectiveness of our algorithms and illustrate our theoretical results.

Our work opens up many interesting avenues for future work. For example, in this work, we have assumed that the machine policy is fixed. However, there are reasons to believe that simultaneously optimizing the machine policy and the switching policy may lead to superior performance [De et al., 2020]. Throughout the paper, we have assumed that the state space is discrete. It would be very interesting to lift this assumption and develop approximate value iteration methods to solve the learning to switch problem. Moreover, we have considered that the human policy does not change due to switching control, however, this assumption is often violated in practice [Wolfe et al., 2019]. Finally, it would be interesting to assess the performance of our algorithms using interventional experiments on a real-world (semi-)autonomous driving system.
References

P. Bartlett and M. Wegkamp. Classification with a reject option using a hinge loss. *JMLR*, 2008.

K. Brookhuis, D. De Waard, and W. Janssen. Behavioural impacts of advanced driver assistance systems—an overview. *European Journal of Transport and Infrastructure Research*, 1(3), 2001.

C. Cortes, G. DeSalvo, and M. Mohri. Learning with rejection. In *ALT*, 2016.

A. De, P. Koley, N. Ganguly, and M. Gomez-Rodriguez. Regression under human assistance. In *AAAI*, 2020.

A. Dosovitskiy, G. Ros, F. Codevilla, A. Lopez, and V. Koltun. Carla: An open urban driving simulator. *arXiv preprint arXiv:1711.03938*, 2017.

European Parliament. Regulation (EC) No 561/2006. http://data.europa.eu/eli/reg/2006/561/2015-03-02, 2006.

R. Everett and S. Roberts. Learning against non-stationary agents with opponent modelling and deep reinforcement learning. In *2018 AAAI Spring Symposium Series*, 2018.

Y. Geifman and R. El-Yaniv. Selectivenet: A deep neural network with an integrated reject option. *arXiv preprint arXiv:1901.09192*, 2019.

Y. Geifman, G. Uziel, and R. El-Yaniv. Bias-reduced uncertainty estimation for deep neural classifiers. In *ICLR*, 2018.

A. Ghosh, S. Tschiautschek, H. Mahdavi, and A. Singla. Towards deployment of robust cooperative ai agents: An algorithmic framework for learning adaptive policies. In *AAMAS*, 2020.

A. Grover, M. Al-Shedivat, J. Gupta, Y. Burda, and H. Edwards. Learning policy representations in multiagent systems. In *ICML*, 2018.

D. Hadfield-Menell, S. Russell, P. Abbeel, and A. Dragan. Cooperative inverse reinforcement learning. In *NIPS*, 2016.

L. Haug, S. Tschiautschek, and A. Singla. Teaching inverse reinforcement learners via features and demonstrations. In *NeurIPS*, 2018.

T. Jaksch, R. Ortner, and P. Auer. Near-optimal regret bounds for reinforcement learning. *Journal of Machine Learning Research*, 2010.

Parameswaran Kamalaruban, Rati Devidze, Volkan Cevher, and Adish Singla. Interactive teaching algorithms for inverse reinforcement learning. In *IJCAI*, 2019.

Z. Liu, Z. Wang, P. Liang, R. Salakhutdinov, L. Morency, and M. Ueda. Deep gamblers: Learning to abstain with portfolio theory. In *NeurIPS*, 2019.

C. Macadam. Understanding and modeling the human driver. *Vehicle system dynamics*, 40(1-3):101–134, 2003.

O. Macindoe, L. Kaelbling, and T. Lozano-Pérez. Pomcop: Belief space planning for sidekicks in cooperative games. In *AIIDE*, 2012.

V. Mnih et al. Human-level control through deep reinforcement learning. *Nature*, 518(7540):529, 2015.

S. Nikolaidis, R. Ramakrishnan, K. Gu, and J. Shah. Efficient model learning from joint-action demonstrations for human-robot collaborative tasks. In *HRI*, 2015.

S. Nikolaidis, J. Forlizzi, D. Hsu, J. Shah, and S. Srinivasa. Mathematical models of adaptation in human-robot collaboration. *arXiv preprint arXiv:1707.03586*, 2017.

Ian Osband, Daniel Russo, and Benjamin Van Roy. (more) efficient reinforcement learning via posterior sampling. In *Advances in Neural Information Processing Systems*, pages 3003–3011, 2013.
Goran Radanovic, Rati Devidze, David C. Parkes, and Adish Singla. Learning to collaborate in markov decision processes. In ICML, 2019.

M. Raghu, K. Blumer, G. Corrado, J. Kleinberg, Z. Obermeyer, and S. Mullainathan. The algorithmic automation problem: Prediction, triage, and human effort. arXiv preprint arXiv:1903.12220, 2019a.

M. Raghu, K. Blumer, R. Sayres, Z. Obermeyer, B. Kleinberg, S. Mullainathan, and J. Kleinberg. Direct uncertainty prediction for medical second opinions. In ICML, 2019b.

H. Ramaswamy, A. Tewari, and S. Agarwal. Consistent algorithms for multiclass classification with an abstain option. Electronic J. of Statistics, 2018.

D. Silver et al. Mastering the game of go with deep neural networks and tree search. Nature, 529(7587):484, 2016.

D. Silver et al. Mastering the game of go without human knowledge. Nature, 550(7676):354, 2017.

A. Strehl and M. Littman. An analysis of model-based interval estimation for markov decision processes. Journal of Computer and System Sciences, 74(8):1309–1331, 2008.

V. Talpaert et al. Exploring applications of deep reinforcement learning for real-world autonomous driving systems. arXiv preprint arXiv:1901.01536, 2019.

S. Thulasidasan, T. Bhattacharya, J. Bilmes, G. Chennupati, and J. Mohd-Yusof. Combating label noise in deep learning using abstention. arXiv preprint arXiv:1905.10964, 2019.

S. Tschiatschek, A. Ghosh, L. Haug, R. Devidze, and A. Singla. Learner-aware teaching: Inverse reinforcement learning with preferences and constraints. In NeurIPS, 2019.

O. Vinyals et al. Grandmaster level in starcraft ii using multi-agent reinforcement learning. Nature, pages 1–5, 2019.

H. Wilson and P. Daugherty. Collaborative intelligence: humans and ai are joining forces. Harvard Business Review, 2018.

B. Wolfe, B. Seppelt, B. Mehler, B. Reimer, and R. Rosenholtz. Rapid holistic perception and evasion of road hazards. Journal of experimental psychology: general, 2019.

B. Wymann, E. Espié, C. Guionneau, C. Dimitrakakis, R. Coulom, and A. Sumner. Torcs, the open racing car simulator. Software available at http://torcs.sourceforge.net, 4(6), 2000.

Y. Zheng, Z. Meng, J. Hao, Z. Zhang, T. Yang, and C. Fan. A deep bayesian policy reuse approach against non-stationary agents. In NeurIPS, 2018.
A Proofs

A.1 Bellman’s principle of optimality

First, we bound the optimal value function \( v_t(s, d) \) from below as follows:

\[
v_t(s, d) = \min_{\pi_t, \ldots, \pi_{T-1}} \mathbb{E} \left[ \sum_{t'=t}^{T-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

\[
= \min_{\pi_t(\cdot | s, d)} c_{\pi_t}(s, d) + \min_{\pi_{t+1}, \ldots, \pi_{T-1}} \mathbb{E}_{d' \sim \pi_t(\cdot | s, d), s' \sim p(\cdot | s, d')} \left[ \mathbb{E} \left[ \sum_{t'=t+1}^{T-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_{t+1} = s', d_t = d' \right] \right]
\]

\[
\geq \min_{\pi_t(\cdot | s, d)} c_{\pi_t}(s, d) + \mathbb{E}_{d' \sim \pi_t(\cdot | s, d), s' \sim p(\cdot | s, d')} \left[ \min_{\pi_{t+1}, \ldots, \pi_{T-1}} \mathbb{E} \left[ \sum_{t'=t+1}^{T-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_{t+1} = s', d_t = d' \right] \right]
\]

where (i) readily follows from the fact that \( \min_a \mathbb{E}[X(a)] \geq \mathbb{E}[\min_a X(a)] \).

Then, we bound the optimal value function \( v_t(s, d) \) from above as follows:

\[
v_t(s, d) = \min_{\pi_t, \ldots, \pi_{L-1}} \mathbb{E} \left[ \sum_{t'=t}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

\[
= \min_{\pi_t(\cdot | s, d)} c_{\pi_t}(s, d) + \min_{\pi_{t+1}, \ldots, \pi_{L-1}} \mathbb{E}_{d' \sim \pi_t(\cdot | s, d), s' \sim p(\cdot | s, d')} \left[ \mathbb{E} \left[ \sum_{t'=t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_{t+1} = s', d_t = d' \right] \right]
\]

\[
\leq \min_{\pi_t(\cdot | s, d)} c_{\pi_t}(s, d) + \mathbb{E}_{d' \sim \pi_t(\cdot | s, d), s' \sim p(\cdot | s, d')} [v_{t+1}(s', d')] \tag{ii}
\]

where (i) follows from the fact that

\[
\min_{\pi_{t+1}, \ldots, \pi_{L-1}} \mathbb{E}_{d' \sim \pi_{t+1}(\cdot | s, d), s' \sim p(\cdot | s, d')} \left[ \mathbb{E} \left[ \sum_{t'=t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_{t+1} = s', d_t = d' \right] \right] \leq \mathbb{E}_{d' \sim \pi_{t+1}(\cdot | s, d), s' \sim p(\cdot | s, d')} \left[ \mathbb{E} \left[ \sum_{t'=t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_{t+1} = s', d_t = d' \right] \right] \tag{iv}
\]

And, if we set \( \pi_{t+1} = \pi^{*}_{t+1}, \ldots, \pi_{L-1} = \pi^{*}_{L-1} \), where

\[
\pi^{*}_{t+1}, \ldots, \pi^{*}_{L-1} = \arg \min_{\pi_{t+1}, \ldots, \pi_{L-1}} \mathbb{E} \left[ \sum_{t'=t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_{t+1} = s', d_t = d' \right],
\]

then equality (ii) also holds. Since the upper and lower bound are the same, we can conclude that the optimal value function satisfies Eq. 5.

A.2 Proof of Proposition 1

By definition, we have that:

\[
c_{\pi_t}(s, d) = \mathbb{E}_{d' \sim \pi_t(\cdot | s, d)} \left[ \hat{c}(s, d') + \lambda_1 \mathbb{I}(d' = \mathbb{H}) + \lambda_2 \mathbb{I}(d \neq d') \right]
\]

\[
= \pi_t(d') = \mathbb{M} | s, d \cdot \hat{c}(s, \mathbb{M}) + \lambda_1 \cdot 0 + \lambda_2 \mathbb{I}(d \neq \mathbb{M})) + (1 - \pi_t(d') = \mathbb{M} | s, d) \cdot \hat{c}(s, \mathbb{H}) + \lambda_1 + \lambda_2 \mathbb{I}(d \neq \mathbb{H}) \tag{17}.
\]
Moreover, it readily follows that:
\[ \mathbb{E}_d \sim \pi_t( \cdot | s, d' ) \mathbb{E}_{s' \sim P( \cdot | s, d')} [v_{t+1}(s', d')] = \pi_t(d' = M|s, d) \cdot \mathbb{E}_{s' \sim P( \cdot | s, M)} [v_{t+1}(s', M)] + (1 - \pi_t(d' = M|s, d)) \cdot \mathbb{E}_{s' \sim P( \cdot | s, \emptyset)} [v_{t+1}(s', \emptyset)] \]
\[ \overset{(i)}{=} \pi_t(d' = M|s, d) \cdot \mathbb{E}_{a \sim \mathbb{P}(\cdot | s, s', \emptyset)} [v_{t+1}(s', M)] + (1 - \pi_t(d' = M|s, d)) \cdot \mathbb{E}_{a \sim \mathbb{P}(\cdot | s, s', \emptyset)} [v_{t+1}(s', \emptyset)] \]
\[ = \pi_t(d' = M|s, d) \cdot q_t[M|s, d] + (1 - \pi_t(d' = M|s, d)) \cdot q_t[\emptyset|s, d]. \]
Finally, it is clear that the above quantity is minimized when
\[ \pi_t(d' = M|s, d) = \begin{cases} 1 & \text{if } q_t[M|s, d] < q_t[\emptyset|s, d] \\ 0 & \text{otherwise}. \end{cases} \]
This concludes the proof.

A.3 Proof of Theorem 2

For any episode \( k \), we can show that the optimal value function \( v^k_t(s, d) \) satisfies Bellman’s principle of optimality (refer to Lemma 5 at the end of this proof), i.e.,
\[ v^k_t(s, d) = \min_{\pi_t( \cdot | s, d)} \min_{p_{\mathbb{P}}(\cdot | s, t) \in P^k_{\mathbb{P}}(\cdot | s, t)} c_{\pi_t}(s, d) + \mathbb{E}_{d' \sim \pi_t( \cdot | s, d), s' \sim P( \cdot | s, d', t)} [v^k_{t+1}(s', d')] \]
Moreover, similarly as in Eqs. 17 and 18 in the proof of Proposition 1 (Appendix A.2), we have that:
\[ c_{\pi_t}(s, d) = \pi_t(d' = M|s, d) \cdot [\tilde{c}(s, M) + \lambda_1 \mathbb{I}(d \neq M) + \mathbb{E}_{a \sim \mathbb{P}(\cdot | s, s', \emptyset)} [v_{t+1}(s', M)]] + (1 - \pi_t(d' = M|s, d)) \cdot [\tilde{c}(s, \emptyset) + \lambda_1 + \lambda_2 \mathbb{I}(d \neq \emptyset) + \mathbb{E}_{a \sim \mathbb{P}(\cdot | s, s', \emptyset)} [v_{t+1}(s', \emptyset)]] \]
(19)
Finally, it is clear that the above quantity is minimized when
\[ \pi_t(d' = M|s, d) = \begin{cases} 1 & \text{if } q^k_t[M|s, d] < q^k_t[\emptyset|s, d] \\ 0 & \text{otherwise}. \end{cases} \]
This concludes the proof.

Lemma 5 (Bellman optimality principle for unknown human policy). For any episode \( k \), the optimal
value function \( v^k_t(s, d) \), as defined in Eq. 13, satisfies the following recursive equation:

\[
v^k_t(s, d) = \min_{\pi_t(s, d)} \min_{p_{\|t|} \in P^k_{|t|, s, t}} \mathbb{E} \left[ \sum_{t' = t}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

Proof. Define \( P^k_{|t|, s, t} := \times_{s', t' \in \{\ldots, L-1\}} P^k_{|t'|, s', t'} \). Then, we proceed similarly as in Appendix A.1. First, we bound the optimal value function \( v^k_t(s, d) \) from below as follows:

\[
v^k_t(s, d) = \min_{\pi_t(s, d)} \min_{p_{\|t|} \in P^k_{|t|, s, t}} \mathbb{E} \left[ \sum_{t' = t}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

\[
= \min_{\pi_t(s, d)} \min_{p_{\|t|} \in P^k_{|t|, s, t}} c_{\pi_t}(s, d)
\]

\[
+ \min_{\pi_{t+1}, \ldots, \pi_{L-1}} \min_{p_{\|t\|, (t+1)+} \in P^k_{|t|, (t+1)+}} \mathbb{E} \left[ \sum_{t' = t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

(i) \[
\leq \min_{\pi_{t+1}, \ldots, \pi_{L-1}} \min_{p_{\|t\|, (t+1)+} \in P^k_{|t|, (t+1)+}} \mathbb{E} \left[ \sum_{t' = t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

where (i) readily follows from the fact that \( \min_a \mathbb{E}[X(a)] \geq \mathbb{E}[\min_a X(a)] \).

Next, we bound the optimal value function \( v^k_t(s, d) \) from above as follows:

\[
v^k_t(s, d) = \min_{\pi_t(s, d)} \min_{p_{\|t|} \in P^k_{|t|, s, t}} \mathbb{E} \left[ \sum_{t' = t}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

\[
\leq \min_{\pi_t(s, d)} \min_{p_{\|t|} \in P^k_{|t|, s, t}} c_{\pi_t}(s, d)
\]

\[
+ \min_{\pi_{t+1}, \ldots, \pi_{L-1}} \min_{p_{\|t\|, (t+1)+} \in P^k_{|t|, (t+1)+}} \mathbb{E} \left[ \sum_{t' = t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

(ii) \[
\leq \min_{\pi_{t+1}, \ldots, \pi_{L-1}} \min_{p_{\|t\|, (t+1)+} \in P^k_{|t|, (t+1)+}} \mathbb{E} \left[ \sum_{t' = t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

where (ii) follows from the fact that

\[
\min_{\pi_{t+1}, \ldots, \pi_{L-1}} \min_{p_{\|t\|, (t+1)+} \in P^k_{|t|, (t+1)+}} \mathbb{E} \left[ \sum_{t' = t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

\[
\leq \mathbb{E} \left[ \sum_{t' = t+1}^{L-1} c_{\pi_{t'}}(s_{t'}, d_{t'-1}) \mid s_t = s, d_{t-1} = d \right]
\]

and, if we set \( \pi_{t+1} = \pi^*_{t+1}, \ldots, \pi_{L-1} = \pi^*_{L-1} \), then equality (ii) holds. Since the upper and lower bounds are the same, we can conclude that the optimal value function satisfies Eq. 25.
A.4 Proof of Lemma 3
Suppose there is \( \{x'_i; \sum_{i=1}^m x'_i = 1, x'_i \geq 0 \} \) such that \( \sum_{i=1}^m x'_i w_i < \sum_{i=1}^m x_i w_i \). Let \( j \in \{1, \ldots, m\} \) be the first index where \( x'_j \neq x'_j \). Then it’s clear that \( x'_j > x'_j \).

If \( j = 1 \):
\[
\sum_{i=1}^m |x'_i - b_i| = |x'_1 - b_1| + \sum_{i=2}^m |x'_i - b_i| > \frac{d}{2} + \sum_{i=2}^m b_i - x'_1 - b_1 > d
\]  
(27)

If \( j > 1 \):
\[
\sum_{i=1}^m |x'_i - b_i| = |x'_j - b_j| + \sum_{i=j+1}^m |x'_i - b_i| > \frac{d}{2} + \sum_{i=j+1}^m b_i - x'_j - b_1 = d
\]  
(28)

Both cases contradict the condition \( \sum_{i=1}^m |x'_i - b_i| \leq d \).

A.5 Proof of Theorem 4
Proof. Throughout the proof, we will assume that \( c'(s, a) + \lambda_1 + \lambda_2 \leq 1 \) for all \( s \in S \) and \( a \in A \) and we will denote

(i) \( c'(s, a) \) as the environment cost of action \( a \) at state \( s \);

(ii) \( c_{\pi|p}\mid p_s \) as the immediate cost due to switching policy \( \pi \) and human policy \( p_{\pi} \), as defined in Eq. 2;

(iii) \( p_{|s, t} \mid p_{s, t} \pi \) as the transition probability under the human policy \( p_{\pi} \) and the machine policy \( p_{\pi} \mid s, t = p_{\pi(s, t)} \);

(iv) \( p_{\pi}^{*} \) as the true human policy;

(v) \( \pi^{*} \) as the optimal switching policy, as defined in Eq. 3;

(vi) \( v^k(s, d) \) as the optimal value function, as defined in Eq. 13;

(vii) \( \pi^k \) and \( p_{\pi}^k \) as the switching policy and human policy that minimize the optimal value function \( v^k(s, d) \);

(viii) \( \bar{v}^k(s, d) \) as the value function under the policy \( \pi_k \) and the true human policy \( p_{\pi}^k \), i.e.,
\[
\bar{v}^k(s, d) = E_{\tau \sim P_{\pi_k, p_{\pi}}^{*}} \left[ \sum_{t'=0}^{L-1} c_{\pi_{t}}(s_{t'}, d_{t'-1}) \mid s_{t} = s, d_{t-1} = d \right] = c_{\pi_{s, t}}(p_{s}^{*}, p_{d}) + E_{d' \sim \pi^{*}(\cdot \mid s, d'), s' \sim p_{s}^{*}, p_{d}^{*}(\cdot \mid s, d')} [\bar{v}^k_{t+1}];
\]  
(29)

(ix) \( \Delta_k \) as the regret for the episode \( k \), i.e.,
\[
\Delta_k = E_{\tau \sim P_{\pi_k, p_{\pi}}^{*}} [c(\tau \mid s_{0}, d_{-1})] - E_{\tau \sim P_{\pi_k, p_{\pi}}^{*}} [c(\tau \mid s_{0}, d_{-1})] = \bar{v}^k(s_0, d_{-1}) - v_0(s_0, d_{-1}),
\]  
(30)

where \( v_0(s, d) \) is defined in Eq. 4.

First, we note that
\[
R(T) = \sum_{k=1}^{K} \Delta_k = \sum_{k=1}^{K} \Delta_k \mathbb{I}(p_{\pi}^{k} \in \mathcal{P}^k_{\pi}) + \sum_{k=1}^{K} \Delta_k \mathbb{I}(p_{\pi}^{k} \notin \mathcal{P}^k_{\pi}).
\]  
(31)

Next, we split our analysis into two parts. We first bound the first term \( \sum_{k=1}^{K} \Delta_k \mathbb{I}(p_{\pi}^{k} \in \mathcal{P}^k_{\pi}) \) and then bound the second term \( \sum_{k=1}^{K} \Delta_k \mathbb{I}(p_{\pi}^{k} \notin \mathcal{P}^k_{\pi}) \).

— Computing the bound on \( \sum_{k=1}^{K} \Delta_k \mathbb{I}(p_{\pi}^{k} \in \mathcal{P}^k_{\pi}) \)

First, we note that
\[
\Delta_k = v^k_0(s_0, d_{-1}) - v_0(s_0, d_{-1}) \leq v^k_0(s_0, d_{-1}) - v^k_0(s_0, d_{-1})
\]  
(32)

This is because
\[
v^k_0(s_0, d_{-1}) = \min_{\pi} \left\{ \min_{p_{\pi}} E_{(s_{t'}, d_{t'-1}) \sim \pi, p_{\pi}, p_{s, t}} \left[ \sum_{t'=0}^{L-1} c_{\pi_{t}}(s_{t'}, d_{t'-1}) \mid s_{0}, d_{-1} \right] \right\}
\leq \min_{\pi} E_{(s_{t'}, d_{t'-1}) \sim \pi, p_{\pi}, p_{s, t}} \left[ \sum_{t'=0}^{L-1} c_{\pi_{t}}(s_{t'}, d_{t'-1}) \mid s_{0}, d_{-1} \right] = v_0(s_0, d_{-1}),
\]  
(33)
where (i) holds because the true human policy $p_{H}^{*} \in \mathcal{P}_{H}^{*}$. Now, we aim to bound $v_{k}^{H}(s_{0},d_{-1})$. To this aim, we first note that

$$v_{0}^{k}(s,d) - v_{0}^{k}(s,d) = c_{\pi^{*}|p_{M}^{*},p_{H}^{*}}(s,d) + \mathbb{E}_{d' \sim \pi_{k}^{H}(\cdot|s,d),s' \sim p_{H}^{*}|p_{M}^{*},p_{H}^{*}}[\bar{v}_{k}^{H}(s',d')]$$

$$- c_{\pi^{*}|p_{H}^{*},p_{H}^{*}}(s,d) - \mathbb{E}_{d' \sim \pi_{k}^{H}(\cdot|s,d),s' \sim p_{H}^{*}|p_{H}^{*},p_{H}^{*}}[\bar{v}_{k}^{H}(s',d')]$$

$$= (1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s)}[c'(s,a)] - \mathbb{E}_{a \sim p_{M}^{*}(\cdot|s)}[c'(s,a)]$$

$$= (1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s)}[c'(s,a)] - \mathbb{E}_{a \sim p_{M}^{*}(\cdot|s)}[c'(s,a)]$$

$$= (1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s)}[c'(s,a)] - \mathbb{E}_{a \sim p_{M}^{*}(\cdot|s)}[c'(s,a)]$$

$$= (1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s)}[c'(s,a)] - \mathbb{E}_{a \sim p_{M}^{*}(\cdot|s)}[c'(s,a)]$$

$$(34)$$

where (i) follows from the definition of $c_{\pi^{*}|p_{M}^{*},p_{H}^{*}}$ and $c_{\pi^{*}|p_{H}^{*},p_{H}^{*}}$ and (ii) follows from applying conditional expectation. Next, we note that $p_{H}^{*}(\cdot|s,M) = p_{H}^{*}(\cdot|s,M,t = 0)$, because the machine policy is independent of the human policy. Hence, $\mathbb{E}_{a \sim p_{M}(\cdot|s,t = 0)}[v_{k}^{H}(s',M)] = \mathbb{E}_{a \sim p_{M}(\cdot|s,M)}[v_{k}^{H}(s',M)]$ and, by adding and subtracting $(1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s,H)}[v_{k}^{H}(s',H)]$ to Eq. (34), it follows that:

$$v_{0}^{k}(s,d) - v_{0}^{k}(s,d) = (1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s)}[c'(s,a)] - \mathbb{E}_{a \sim p_{M}^{*}(\cdot|s)}[c'(s,a)]$$

$$+ \mathbb{E}_{d' \sim \pi_{k}^{H}(\cdot|s,d),s' \sim p_{H}^{*}|p_{M}^{*},p_{H}^{*}}[\bar{v}_{k}^{H}(s',d')]$$

$$- \mathbb{E}_{d' \sim \pi_{k}^{H}(\cdot|s,d),s' \sim p_{H}^{*}|p_{H}^{*},p_{H}^{*}}[\bar{v}_{k}^{H}(s',d')]$$

$$= (1 - \pi_{H}^{k}(d' = M|s,d)) \mathbb{E}_{a \sim p_{M}(\cdot|s)}[c'(s,a)] - \mathbb{E}_{a \sim p_{M}^{*}(\cdot|s)}[c'(s,a)]$$

$$+ \mathbb{E}_{d' \sim \pi_{k}^{H}(\cdot|s,d),s' \sim p_{H}^{*}|p_{M}^{*},p_{H}^{*}}[\bar{v}_{k}^{H}(s',d')]$$

$$- \mathbb{E}_{d' \sim \pi_{k}^{H}(\cdot|s,d),s' \sim p_{H}^{*}|p_{H}^{*},p_{H}^{*}}[\bar{v}_{k}^{H}(s',d')]$$

$$(35)$$
where (i) follows from the fact that
\[
\mathbb{E}_{d' \sim \pi^0_\mathbb{H}(d' | s, d'), s' \sim p_{\mathbb{H}_0} \circ \nu_{d'}} [e_1^k(s', d') - e_1^k(s', d')]
\]
\[
= \pi^0_\mathbb{H}(d' = \mathbb{H} | s, d) \left[ \mathbb{E}_{s' \sim p_{\mathbb{H}_0} \circ \nu_{d'}} [e_1^k(s', \mathbb{H})] - \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{H}| [e_1^k(s', \mathbb{H})]) \right]
+ (1 - \pi^0_\mathbb{H}(d' = \mathbb{H} | s, d)) \left[ \mathbb{E}_{s' \sim p_{\mathbb{H}_0} \circ \nu_{d'}} (|s, \mathbb{H}| [e_1^k(s', \mathbb{H})]) - \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{H}| [e_1^k(s', \mathbb{H})]) \right]
\]
and (ii) follows from the fact that
\[
\mathbb{E}_{d' \sim \pi^0_\mathbb{H}(d' | s, d)} (|d' = \mathbb{H}|) = P(d' = \mathbb{H}) = 1 - \pi^0_\mathbb{H}(d' = \mathbb{M} | s, d).
\]
Now, we can bound the term
\[
\mathbb{E}_{a \sim p_\mathbb{H}_0(s)} [c'(s, a)] - \mathbb{E}_{a \sim p_\mathbb{D}(s, t=0)} [c'(s, a)] + \mathbb{E}_{s' \sim p_{\mathbb{M}, \rho_{\mathbb{D}}}(s, \mathbb{M})} [e_1^k(s', \mathbb{H})] - \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{D}, t=0) [e_1^k(s', \mathbb{H})]
\]
as follows:
\[
\mathbb{E}_{a \sim p_\mathbb{H}_0(s)} [c'(s, a)] - \mathbb{E}_{a \sim p_\mathbb{D}(s, t=0)} [c'(s, a)] + \mathbb{E}_{s' \sim p_{\mathbb{M}, \rho_{\mathbb{D}}}(s, \mathbb{M})} [e_1^k(s', \mathbb{H})] - \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{D}, t=0) [e_1^k(s', \mathbb{H})]
\]
\[
= \mathbb{E}_{a \sim p_\mathbb{H}_0(s)} [c'(s, a)] + \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{D}) [e_1^k(s', \mathbb{H})] - \mathbb{E}_{a \sim p_\mathbb{D}(s, t=0)} [c'(s, a)] + \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{D}) [e_1^k(s', \mathbb{H})]
\]
\[
\leq \min \left[ L, \sum_{a \in A} p_\mathbb{H}_0(a | s) - p_\mathbb{D}(a | s, t = 0) \right] L \min \{1, \beta_k(s, \delta)\}
\]
where (i) follows from the fact that \(c'(s, a) + \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, a) [e_1^k(s', \mathbb{H})] \leq L\) since, by assumption, \(c'(s, a) + \lambda_1 + \lambda_2 \leq 1\) for all \(s \in \mathcal{S}\) and \(a \in \mathcal{A}\) and (ii) follows from the fact that, by assumption, both \(p_\mathbb{H}_0\) and \(p_\mathbb{D}\) lie in the confidence set \(P_\mathbb{H}\). Then, if we combine Eq. 38 in Eq. 33 we have that, for all \(s \in \mathcal{S}\) and \(d' \in \{\mathbb{H}, \mathbb{M}\}\), it holds that
\[
\mathbb{E}_{d' \sim \pi^0_\mathbb{H}(d' | s, d)} (|d' = \mathbb{H}|) \mathbb{E}_{s' \sim p_{\mathbb{M}, \rho_{\mathbb{D}}}(s, \mathbb{M})} [e_1^k(s', \mathbb{H})] - \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, \mathbb{D}, t=0) [e_1^k(s', \mathbb{H})]
\]
\[
= \min \left[ L, \sum_{a \in A} p_\mathbb{H}_0(a | s) - p_\mathbb{D}(a | s, t = 0) \right] \mathbb{E}_{d' \sim \pi^0_\mathbb{H}(d' | s, d), s' \sim p_{\mathbb{M}, \rho_{\mathbb{D}}}(s, d')} [e_1^k(s', d') - e_1^k(s', d')]
\]
Similarly, we can show that, for all \(s \in \mathcal{S}\) and \(d' \in \{\mathbb{H}, \mathbb{M}\}\), it holds that
\[
\mathbb{E}_{d' \sim \pi^0_\mathbb{H}(d' | s, d)} (|d' = \mathbb{H}|) \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, d', t=0) [e_1^k(s', \mathbb{H})] - \mathbb{E}_{s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, d', t=0) [e_1^k(s', \mathbb{H})]
\]
\[
= \min \left[ L, \sum_{a \in A} p_\mathbb{H}_0(a | s) - p_\mathbb{D}(a | s, t = 0) \right] \mathbb{E}_{d' \sim \pi^0_\mathbb{H}(d' | s, d), s' \sim p_{\mathbb{D}} \circ \nu_{d'}} (|s, d') [e_1^k(s', d') - e_1^k(s', d')]
\]
Hence, we can show by induction that:
\[
\mathbb{E}_{d \sim \pi^0_{\mathbb{H}_0}} (s_0, d) - \mathbb{E}_{d \sim \pi^0_{\mathbb{D}}} (s_0, d) \leq L \sum_{t=0}^{T-1} \mathbb{E}_{d \sim \pi^0_{\mathbb{H}_0}} (s_t, d) \min \{1, \beta_k(s_t, \delta)\} |s_0, d_0)
\]
where the expectation is taken over the MDP with switching policy \(\pi^k\) under true human policy \(p_\mathbb{H}_0\).

As one may expect, we only have regret when the optimistic switching policy \((i.e., \pi^k)\) chooses human \(i.e., d_t = \mathbb{H}\) and observing more human actions makes \(\delta_k(s, \delta)\) smaller. Hence, when \(p_\mathbb{H}_0 \in P_{\mathbb{H}_0}\), we can bound the total regret as follows:
\[
\sum_{k=1}^{K} \Delta_k \mathbb{E}_{d' \sim \pi^0_{\mathbb{H}_0} [d' \in P_{\mathbb{H}_0}]} \leq \sum_{k=1}^{K} L \sum_{t=0}^{T-1} \mathbb{E}_{d \sim \pi^0_{\mathbb{H}_0}} (|d_t = \mathbb{H}|) \min \{1, \beta_k(s_t, \delta)\} |s_0, d_0)
\]
Finally, since \(c'(\cdot, \cdot) + \lambda_1 + \lambda_2 < 1\), the worst-case regret is bounded by \(T\). Therefore, we have that:
\[
\sum_{k=1}^{K} \Delta_k \mathbb{E}_{d' \sim \pi^0_{\mathbb{H}_0} [d' \in P_{\mathbb{H}_0}]} \leq \sum_{k=1}^{K} L \sum_{t=0}^{T-1} \mathbb{E}_{d \sim \pi^0_{\mathbb{H}_0}} (|d_t = \mathbb{H}|) \min \{1, \beta_k(s_t, \delta)\} |s_0, d_0)
\]
\[
\leq 12L \sqrt{\mathcal{S}|\mathcal{A}|T \log \left( \frac{|S|T}{\delta} \right)}
\]
where (i) follows from Lemma 6 which is given at the end of this proof.

— Computing the bound on \(\sum_{k=1}^{K} \Delta_k \mathbb{I}(p^*_k \not\in P^k_{\mathbb{H}_0})\)

Here, we use a similar approach to [Jaksch et al. 2010]. First, we note that
\[
\sum_{k=1}^{K} \Delta_k \mathbb{I}(p^*_k \not\in P^k_{\mathbb{H}_0}) = \sum_{k=1}^{K} \Delta_k \mathbb{I}(p^*_k \not\in P^k_{\mathbb{H}_0}) + \sum_{k=1}^{K} \Delta_k \mathbb{I}(p^*_k \not\in P^k_{\mathbb{H}_0})
\]
Next, our goal is to show that second term of the RHS of above equation vanishes with high probability. If we succeed, then it holds that, with high probability, \( \sum_{k=1}^{K} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) \) equals the first term of the RHS and then we will be done because

\[
\sum_{k=1}^{\lfloor \frac{\sqrt{T}}{L} \rfloor} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) \leq \sum_{k=1}^{\lfloor \frac{\sqrt{T}}{L} \rfloor} \Delta_k \leq \left\lfloor \frac{\sqrt{T}}{L} \right\rfloor L = \sqrt{T},
\]

where (i) follows from the fact that \( \Delta_k \leq L \) since \( c'(s, a) + \lambda_1 + \lambda_2 \leq 1 \) for all \( s \in \mathcal{S} \) and \( a \in \mathcal{A} \).

To prove that \( \sum_{k=1}^{K} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) = 0 \) with high probability, we proceed as follows. From Lemma 7, which is given at the end of this proof, we have

\[
\Pr(p^*_s \not\in \mathcal{P}^k_{\Pi}) \leq \delta \frac{t}{t_k},
\]

where \( t_k = (k - 1)L \) is the start time of episode \( k \). Therefore, if wollos that

\[
\Pr \left( \sum_{k=1}^{K} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) = 0 \right) = \Pr \left( \forall k : \left\lfloor \frac{\sqrt{T}}{L} \right\rfloor + 1 \leq k \leq K = \frac{T}{L} ; p^*_s \in \mathcal{P}^k_{\Pi} \right)
\]

\[
= 1 - \Pr \left( \exists k : \left\lfloor \frac{\sqrt{T}}{L} \right\rfloor + 1 \leq k \leq \frac{T}{L} ; p^*_s \not\in \mathcal{P}^k_{\Pi} \right)
\]

\[
\geq 1 - \sum_{k=\lfloor \frac{\sqrt{T}}{L} \rfloor + 1} t_k \Pr(p^*_s \not\in \mathcal{P}^k_{\Pi})
\]

\[
\geq 1 - \sum_{k=\lfloor \frac{\sqrt{T}}{L} \rfloor + 1} t_k \delta \frac{t}{t_k}
\]

\[
\geq 1 - \sum_{t_k=\sqrt{T}}^T t_k \delta \frac{t}{t_k} \geq 1 - \int_{\sqrt{T}}^T \frac{\delta}{\sqrt{t}} \, dt \geq \frac{\delta}{5T^{3/4}}.
\]

where (i) follows from a union bound, (ii) follows from Eq. 46 and (iii) holds using that \( t_k = (k - 1)L \).

Hence, with probability at least 1 - \( \frac{\delta}{5T^{3/4}} \), we have that

\[
\sum_{k=\lfloor \frac{\sqrt{T}}{L} \rfloor + 1}^{K} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) = 0
\]

If we combine the above equation and Eq. 45, we can conclude that, with probability at least 1 - \( \frac{\delta}{5T^{3/4}} \), we have that

\[
\sum_{k=1}^{K} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) \leq \sqrt{T}
\]

Next, if we combine Eqs. 43 and 53, we have that

\[
R(T) = \sum_{k=1}^{K} \Delta_k I(p^*_s \in \mathcal{P}^k_{\Pi}) + \sum_{k=1}^{K} \Delta_k I(p^*_s \not\in \mathcal{P}^k_{\Pi}) < 12L \sqrt{|\mathcal{S}| |\mathcal{A}| T \log \left( \frac{|\mathcal{S}| T}{\delta} \right)} + \sqrt{T}
\]

\[
< 13L \sqrt{|\mathcal{S}| |\mathcal{A}| T \log \left( \frac{|\mathcal{S}| T}{\delta} \right)}
\]

Finally, since \( \sum_{T=1}^{\infty} \frac{\delta}{5T^{3/4}} \leq \delta \), with probability 1 - \( \delta \), we have that \( R(T) < 13L \sqrt{|\mathcal{S}| |\mathcal{A}| T \log \left( \frac{|\mathcal{S}| T}{\delta} \right)} \).

This concludes the proof.
Lemma 6. It holds that
\[
\min \left\{ T, \sum_{k=1}^{K} L \cdot \mathbb{E} \left[ \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right] \right\} \leq 12L \sqrt{|S| |A| T \log \left( \frac{|S| T}{\delta} \right)}.
\]

Proof. The proof is adapted from Osband et al. [2013]. We first note that
\[
L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right] = L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right] + L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right]
\]

Then, we bound the first term of the above equation
\[
L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right] \leq L \cdot \mathbb{E} \left[ \sum_{i \in S} \text{# of times } s \text{ is visited while } d = \mathbb{H} \text{ and } N_k(s) \leq L \mid d_{-1}, s_0 \right]
\]

To bound the second term, we first define \( n_{t_k}(s) \) as the number of times \( s \) has been visited in the first \( \tau \) steps across episodes, i.e., if we are at the \( t_k \)th time step in the episode \( k \), then \( \tau = t_k + t \), where \( t_k = (k - 1)L \), and note that
\[
n_{t_k+t}(s) \leq N_k(s) + t
\]

because we will visit state \( s \) with human control at most \( t \in \{0, \ldots, L - 1\} \) times within episode \( k \). Now, if \( N_k(s) > L \), we have that
\[
n_{t_k+t}(s) + 1 \leq N_k(s) + t + 1 \leq N_k(s) + L \leq 2N_k(s)
\]

Hence we have,
\[
\mathbb{I}(d_t = \mathbb{H}) \min \{1, N_k(s_t)\} \leq \mathbb{I}(d_t = \mathbb{H}) \min \{1, N_k(s_t)\} \leq \frac{2}{n_{t_k+t}(s) + 1}
\]

Then, using the above equation, we can bound the second term in Eq. \[57\]
\[
L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right] \leq L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = \mathbb{H}) \min \{1, \beta_k(s_t, \delta)\} \mid d_{-1}, s_0 \right]
\]

where (i) follows from the definition of \( \beta_k(s_t, \delta) \), (ii) follows from Eq. \[60\] and (iii) follows from the fact that
\[
\sqrt{28|A| \log \left( \frac{|S| T}{\delta} \right)} \leq \sqrt{28|A| \log \left( \frac{|S| T}{\delta} \right)},
\]

20
using that \( t_k \leq T \). Next, we can further bound \( \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \frac{1}{n_{t_k+1}(s_t) + 1} \right] \) as follows:

\[
\mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \frac{1}{n_{t_k+1}(s_t) + 1} \right] = \mathbb{E} \left[ \sum_{t=0}^{T} \frac{1}{n_{t}(s_t) + 1} \right] \leq \mathbb{E} \left[ \sum_{s \in \mathcal{S}} \sum_{\nu=0}^{N_{T+1}(s)} \sqrt{\frac{1}{\nu + 1}} \right] = \sum_{s \in \mathcal{S}} \mathbb{E} \left[ \sum_{\nu=0}^{N_{T+1}(s)} \sqrt{\frac{1}{\nu + 1}} \right] \leq \sum_{s \in \mathcal{S}} \mathbb{E} \left[ \int_{0}^{N_{T+1}(s)} \frac{T}{x} dx \right] \leq \sum_{s \in \mathcal{S}} \mathbb{E} \left[ 2 \sqrt{N_{T+1}(s)} \right] \leq \mathbb{E} \left[ 2 \sqrt{\mathcal{S} | \mathcal{A}| T} \right].
\]

where (i) follows from summing over states instead of time and from the fact that we visit each state \( s \) exactly \( N_{T+1}(s) \) times after \( K \) episodes, (ii) follows from Jensen’s inequality and (iii) follows from the fact that \( \sum_{s \in \mathcal{S}} N_{T+1}(s) = T \). Next, we combine Eqs (61) and (62) to obtain

\[
L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = H) \mathbb{I}(N_k(s_t) > L) \beta_k(s_t, \delta) \right] \leq L \cdot 28|\mathcal{A}| \log \left( \frac{|\mathcal{S}|T}{\delta} \right) \times 2 \sqrt{|\mathcal{S}|T} = \sqrt{12L} \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)}.
\]

Further, we plug in Eqs. (61) and (62) in Eq.(60)

\[
L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = h) \min\{1, \beta_k(s_t, \delta)\} \right] \leq 2L^2 |\mathcal{S}| + \sqrt{12L} \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)}.
\]

Thus,

\[
\min \left\{ T, L \cdot \mathbb{E} \left[ \sum_{k=1}^{K} \sum_{t=0}^{L-1} \mathbb{I}(d_t = h) \min\{1, \beta_k(s_t, \delta)\} \right] \right\} \leq \min \left\{ T, 2L^2 |\mathcal{S}| + \sqrt{12L} \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)} \right\}
\]

Moreover, if \( T \leq 2L^2 |\mathcal{S}||\mathcal{A}| \log \left( \frac{|\mathcal{S}|T}{\delta} \right) \),

\[
T^2 \leq 2L^2 |\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right) \quad \Rightarrow \quad T \leq \sqrt{2L} \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)}
\]

and if \( T > 2L^2 |\mathcal{S}||\mathcal{A}| \log \left( \frac{|\mathcal{S}|T}{\delta} \right) \),

\[
2L^2 |\mathcal{S}| < \sqrt{2L^2 |\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)} \leq \sqrt{2L} \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)}.
\]

Thus, the minimum in Eq. (65) is less than

\[
(\sqrt{2} + \sqrt{12}) L \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)} < 12L \sqrt{|\mathcal{S}||\mathcal{A}|T \log \left( \frac{|\mathcal{S}|T}{\delta} \right)}
\]

This concludes the proof. \( \square \)

**Lemma 7.** For each episode \( k > 1 \), the true human policy \( p_h^k \) lies in the confidence set \( \mathcal{P}_H^k \) with probability at least \( 1 - \frac{\delta}{T_k} \), where \( t_k = (k - 1)L \), is the beginning time of episode \( k \).
Proof. We adapt the proof from Lemma 17 in Jaksch et al. [2010]. We note that,
\[
\Pr(p^*_H \notin P_H^k) \overset{(i)}{\leq} \Pr \left( \bigcup_{s \in S} \| p^*_H (\cdot \mid s) - \hat{p}^k_H (\cdot \mid s) \|_1 \geq \beta_k (s, \delta) \right) 
\]
(68)
\[
\overset{(i)}{\leq} \sum_{s \in S} \Pr \left( \| p^*_H (\cdot \mid s) - \hat{p}^k_H (\cdot \mid s) \|_1 \geq \sqrt{\frac{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)}{\max \{1, N_k (s) \}}} \right)
\]
(69)
\[
\overset{(iii)}{\leq} \sum_{s \in S} \sum_{n=0}^{t_k} \Pr \left( \| p^*_H (\cdot \mid s) - \hat{p}^k_H (\cdot \mid s) \|_1 \geq \sqrt{\frac{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)}{n}} \right)
\]
(70)
where (i) follows from the definition of the confidence set, i.e., the true human policy does not lie in the confidence set if there is at least one state \( s \) in which \( \| p^*_H (\cdot \mid s) - \hat{p}^k_H (\cdot \mid s) \|_1 \geq \beta_k (s, \delta) \), (ii) follows from the definition of \( \beta_k (s, \delta) \) and a union bound over all \( s \in S \) and (iii) follows from a union bound over all possible values of \( N_k (s) \). Now, recall that, for \( N_k (s) = 0 \), we had defined the empirical distribution \( \hat{p}^k_H (\cdot \mid s) = \frac{1}{|A|} \). So, we split the sum into \( n = 0 \) and \( n > 0 \):
\[
\sum_{s \in S} \sum_{n=0}^{t_k} \Pr \left( \| p^*_H (\cdot \mid s) - \hat{p}^k_H (\cdot \mid s) \|_1 \geq \sqrt{\frac{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)}{n}} \right)
\]
\[
= \sum_{s \in S} \sum_{n=0}^{t_k} \Pr \left( \| p^*_H (\cdot \mid s) - \hat{p}^k_H (\cdot \mid s) \|_1 \geq \sqrt{\frac{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)}{n}} \right)
\]
\[
+ \sum_{s \in S} \Pr \left( \| \hat{p}^k_H (\cdot \mid s) - \frac{1}{|A|} \|_1 \geq \sqrt{\frac{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)}{n}} \right)
\]
(71)
\[
\overset{(i)}{\leq} t_k |S| |A| \exp \left( -7 |A| \log \left( \frac{|S| t_k}{\delta} \right) \right) \leq \frac{\delta}{\beta_k} .
\]
(72)
where (i) follows from the fact that \( \| p^*_H (\cdot \mid s) - \frac{1}{|A|} \|_1 < \sqrt{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)} \) for any non trivial MDP. More specifically,
\[
|A| \geq 1, |S| \geq 2, \delta < 1, t_k > 1 \implies \sqrt{14 |A| \log \left( \frac{|S| t_k}{\delta} \right)} > \sqrt{14 \log (2)} > 2,
\]
\[
\left\| p^*_H (\cdot \mid s) - \frac{1}{|A|} \right\|_1 \leq \sum_{a \in A} \left( p^*_H (a \mid s) + \frac{1}{|A|} \right) \leq 2,
\]
(73)
and (ii) follows from the fact that, after observing \( n \) samples, the \( L^1 \)-deviation of the true distribution \( p^* \) from the empirical one \( \hat{p} \) over \( k \) events is bounded by:
\[
\Pr (\| p^*(\cdot) - \hat{p}(\cdot) \|_1 \geq \epsilon) \leq 2^k \exp \left( -n \frac{\epsilon^2}{2} \right)
\]
(74)