Fractional Decay of Matter-Wave Quantum Emitters in a Synthetic Bandgap Material

Michael Stewart, Joonhyuk Kwon, Alfonso Lanuza, and Dominik Schneble

Department of Physics and Astronomy, Stony Brook University,
Stony Brook, New York 11794-3800, USA
Harnessing atom-light interactions is central to the development of novel quantum matter [1–3] using waveguide quantum-electrodynamics (QED) [4–11]. Experiments with photonic crystals (or bandgap materials) [12–14] have observed modified spontaneous emission [4, 15–17], signatures of atom-photon bound states, bound-state mediated dipole-dipole interactions [18–20], and superradiance [21]. However such systems can be difficult to produce, tune, and control. Ultracold atoms in optical lattices can provide a flexible, well-controlled platform for the realization of waveguide-QED with matter waves [22–24]. Utilizing a system previously used to observe spontaneous emission into vacuum [23, 25], we explore here the fractional decay of matter-wave emitters coupled to the band structure of a synthetic bandgap material, realized by the periodic optical potential of a shallow optical lattice. We investigate the interplay between emitter excitation energy, vacuum coupling and bandwidth governing decay dynamics, and identify two contributing bound states of the matter-wave radiation field with disparate character. Our findings shed light on the complex role of band structure in spontaneous emission.

The band structure of a photonic crystal [12–14] can strongly modify the dynamics of a quantum emitter. While much is known about the modification of decay behavior near a single edge [26–29], including the formation of bound states [12, 13] which give rise to fractional (incomplete) decay of the emitter, the effects of coupling to an entire band have only been considered recently [30, 31]. The study of such scenarios has become possible with a recently demonstrated matter-wave platform [22, 23, 25]. Here, an excited quantum emitter is formed by a lattice well occupied by a single atom that is coupled to a continuum of atomic momentum modes through an internal-state change; the change is induced by an external coherent coupling that allows for a tunable effective vacuum coupling and excitation energy. While initial experiments [23] explored emission near a single continuum edge, the presence of an emission band with two edges can add new features. In particular, it has been proposed that, depending on the dimensionality of the system, the back-action of the band-structure on the emitter can result in multiple bound states and exotic decays [30, 31]. Here we realize the simplest case featuring one (freely tunable) band in one dimension [32], and investigate the time evolution of intraband emission, as well as the properties of the contributing bound states near the band edges.
Our experimental scheme, illustrated in Fig. 1a, follows that of [23]. We confine ultracold, optically trapped $^{87}\text{Rb}$ atoms in two relevant hyperfine ground states $|r\rangle$ ("red") and $|b\rangle$ ("blue") to a system of isolated tubes (created with a 40 recoil-energies-deep, 2D optical lattice at 1064 nm) in which the atoms can freely propagate for sufficiently short times ($\tau \ll \omega_z^{-1}$, with $\omega_z/2\pi \approx 100$ Hz along the tube axis). An additional, state-dependent lattice of wavelength $\lambda$ (depth $s_{r,b}$ in units of the corresponding recoil energy $E_r = (\hbar k)^2/2m$, where $k = 2\pi/\lambda$ is the recoil wavenumber, $\lambda \sim 790$ nm, and $m$ is the atomic mass) along the tube axis provides deep confinement ($s_r = 20$) for
the $|r\rangle$ atoms, and arbitrarily weak confinement for the $|b\rangle$ atoms. In this configuration, we first prepare a Mott insulator of $|r\rangle$ atoms and then thin it out to a sample of $\sim 30,000$ atoms with site occupation $\lesssim 0.5$ (see methods). The lattice-trapped $|r\rangle$ state is microwave coupled (strength $\Omega$, detuning $\Delta$ from the lattice-shifted atomic resonance at 6.8 GHz, see methods) to the $|b\rangle$ state which is free to propagate when $s_b = 0$. Each lattice well in a given tube thus serves as a matter-wave quantum emitter characterized by an occupational spin.

In the present experiment we choose a non-vanishing $s_b > 0$ such that the mode continuum acquires a band structure. The dynamics of an isolated emitter (ground and excited states $|g\rangle$ and $|e\rangle$) is then governed by the Weisskopf-Wigner-type interaction Hamiltonian

$$\hat{H} = \sum_{n,q} \hbar g_{n,q} e^{i \Delta_{n,q} t} |g\rangle \langle e| \hat{b}_{n,q}^\dagger + \text{H.c.}$$

involving Bloch bands, where $\Delta_{n,q} = \Delta - \varepsilon_{n,q}/\hbar$ is the effective detuning of the emitter (excitation energy $\hbar \Delta$) from the Bloch state $|n,q\rangle = \hat{b}_{n,q}^\dagger |0\rangle$ with energy $\varepsilon_{n,q}$ (band index $n$, quasimomentum $q$); the vacuum coupling $g_{n,q} = \gamma_{n,q} \Omega/2$ contains the Franck-Condon overlap $\gamma_{n,q} = \langle n,q|\psi_e\rangle$ with the originating harmonic-oscillator ground state $|\psi_e\rangle$ in an emitter. We choose $s_b = 2.5$ (at $\lambda = 790.4$ nm) giving a ground-band width $W_1 = \varepsilon_{1,k} - \varepsilon_{1,0} = 0.5$ $E_r \approx \hbar \times 1.8$ kHz. The band structure and corresponding Franck-Condon factors are illustrated in Fig. 1b.

We first characterize the momentum distribution of the emitted $|b\rangle$ atoms as a function of the excitation energy $\hbar \Delta$ (calibrated using lattice transfer spectroscopy, see methods). After loading the array with $|r\rangle$ atoms, we apply a rectangular microwave pulse of duration $\tau = 400 \mu s$ and Rabi frequency $\Omega = 2\pi \times 1$ kHz; following a 500$\mu$s-long rampdown of all three lattices for band mapping. The emitted $|b\rangle$ atoms are detected after time-of-flight using state-selective absorption imaging. The measured distribution, shown in Fig. 1c, reveals that emission into the ground band is much stronger than that into the first and higher bands. The suppression results from the structure of the vacuum coupling $g_{n,q}$, which for even-$n$ bands is reduced due to the approximate odd parity of the relevant Bloch states for $s_b > 0$ (see supplementary materials for a discussion of the opposite case $s_b < 0$); a further suppression of the relevant Franck-Condon factor for higher $n$ is due to the finite momentum width of $|\psi_e\rangle$ and the decrease in the density of states. In the following, we will restrict the discussion to the dynamics induced by coupling to the ground band.

We have found in earlier work that the decay dynamics in an array of emitters is generally subject to propagation, reabsorption and collisional effects [23]; however a description for
an isolated emitter should apply in the short-time limit. The matter-wave wavefunction is described by $|\psi(t)\rangle = A(t) |e; 0\rangle + \sum_q B_q(t) |g; 1, q\rangle$. The band is approximately sinusoidal, $\varepsilon(q) = -\hbar \bar{\omega} \cos(q \pi/k) + \hbar \bar{\omega}$ (denoting $\varepsilon(q) \equiv \varepsilon_{1,q}$, and $\hbar \bar{\omega} \equiv W_1/2$) and the vacuum coupling $g = \langle g_{1,q} \rangle_q \approx 0.39 \Omega$ is approximately constant over the band, with negligible coupling to other bands. Following the treatment in [32] of an isolated, initially excited emitter in one dimension, and similarly to the free space case [25], the excited-state amplitude $A(t)$ then evolves as the Fourier transform $A(t) = (i/2\pi) \int_{-\infty}^{\infty} d\omega G(\omega+i0^+) e^{i(\Delta - \omega)t}$ of the Green’s function $G(\omega) = 1/\left[\omega - \Delta - \Sigma(\omega)/\hbar\right]$, in which the interaction of the emitter with the band is captured through the self-energy $\Sigma(\omega) = (\hbar^2 g^2/2k) \int_{-k}^{k} dq/\left[\hbar \omega - \varepsilon(q)/\hbar\right] = -i\hbar g^2/\sqrt{\omega(2\bar{\omega} - \omega)}$.

Similarly, the individual quasi-momentum amplitudes evolve as $B_q(t) = (ig/2\pi) \int_{-\infty}^{\infty} d\omega G(\omega+i0^+)(e^{i\varepsilon(q)/\hbar - \omega t} - 1)/(\omega + i0^+ - \varepsilon(q)/\hbar)$.

The dynamics of the emitter and radiation are governed by the singularities of $G(\omega)$ defined by $\omega - \Delta - \Sigma(\omega)/\hbar = 0$, which can be determined using standard analytical techniques (see methods). In contrast to the single-edge case [25], there are now two bound states [32], one below the band and one in the gap above the band, as well as a nearly Markovian decay resulting from coupling to the band and additional, incoherent losses due to the divergence of the self energy at the band edges.

For a comparison with the model, we first consider the ground-band emission spectrum shown in Fig. 1c, and compare it to the calculated profiles of $|B_q(t)|^2$ for a range of $q$ from $-k$ to $k$ at fixed times $\tau = 400 \mu s$. For a more quantitative comparison, we consider the total transferred population by integrating over the quasi-momenta. The agreement of the spectrum with the prediction (Fig. 2a) is good up to an overall scaling factor of order unity. To access temporal characteristics of the decay, we monitor, for emitter energy near the center of the band, the time evolution of the excited-state amplitude for different ratios $g/\bar{\omega}$ between coupling and bandwidth. Three regimes can be distinguished, cf. Fig. 2b: irreversible decay for $g/\bar{\omega} \ll 1$ (weak coupling), damped oscillatory decay for $g/\bar{\omega} \sim 1$ (intermediate coupling), and undamped oscillations for $g/\bar{\omega} \gg 1$ (strong coupling).

A Wannier picture provides an intuitive interpretation of the model’s predictions via the quantum Zeno effect [33, 34] (cf. Fig. 2c): here, the atom coherently cycles with Rabi frequency $\Omega' = 2g$ between the strongly confining emitter well and a corresponding $|b\rangle$ well of the shallower lattice, where it is subject to tunnel escape at a rate $\sim \bar{\omega}$ that damps the coherent local evolution. In the strong-coupling limit, the bandwidth is negligible, and the
FIG. 2: a, Ground-band emission spectrum for $g/\tilde{\omega} = 0.43$ and $\tau = 400$ µs, obtained from the distributions in Fig. 1c by summing over quasimomenta. The dashed curve is the prediction of the isolated-emitter model, and the solid curve is the same prediction reduced by 40%. Deviations from the model have already set in by $\tau = 400$ µs, as can be seen in the following panel. b, Decay dynamics for $\hbar \Delta = h \times 1.8(1)$ kHz (at the band center) for (1) weak coupling with $\Omega = 2\pi \times 0.4$ kHz ($g/\tilde{\omega} = 0.18$), (2,3) intermediate coupling with $\Omega = 2\pi \times (1.0, 2.3)$ kHz ($g/\tilde{\omega} = 0.43, 1.0$), and (4) strong coupling for a reduced bandwidth ($W_1 = 0.1E_r$) with $\Omega = 2\pi \times 2.2$ kHz ($g/\tilde{\omega} = 4.9$). The dots are data taken for different hold times, averaged over at least 3 runs each, with error bars representing the standard error of the mean (S.E.M). The gray lines represent the predictions of the isolated-emitter model. c, Schematic illustrating the competition between coupling $g$ and tunnelling $\sim \tilde{\omega}$ in the shallow lattice (see text).

population trivially performs coherent Rabi oscillations between the emitter and the (now effectively single-mode) vacuum, in analogy to a simple cavity-QED scenario. A remnant of this effect persists even in the weak coupling regime for short times $t \lesssim \tilde{\omega}^{-1} \sim 0.2$ ms, when the associated Heisenberg uncertainty in energy exceeds the bandwidth such that the band is not resolved and appears as a single state. This results in decay dynamics that always start quadratically, as a Rabi oscillation.

The isolated-emitter model predicts oscillatory decay behavior generated by two bound states near the band edges, which arise as non-decaying, real poles $\omega^\pm_B$ in the Green’s function $G(\omega)$ and determine the long-term dynamics of the model [30, 32]. The bound states beat against each other with frequency $\omega^+_B - \omega^-_B$ (see methods) while the overall oscillation
amplitude decays partially due to Markovian and partially due to non-Markovian contributions arising from coupling to the band and its edges respectively. For \( g/\bar{\omega} \ll 1 \), the binding energies \( \hbar \omega_B^\pm \) converge to the corresponding band edge from above and below, i.e. \( \omega_B^+ \approx 2\bar{\omega} \) and \( \omega_B^- \approx 0 \), whereas for \( g/\bar{\omega} \gg 1 \) with negligible bandwidth, the contribution from the coupling to the band and edges is negligible, while the bound states converge to the dressed-state energies \( \hbar \omega_B^\pm = \hbar \bar{\omega} + \hbar (\Delta' \pm \sqrt{\Delta'^2 + \Omega'^2})/2 \) of the simple Rabi problem (where \( \Delta' = \Delta - \bar{\omega} \)), i.e. the cavity-QED scenario.

Generally there is good agreement between the model and the observation before significant population has been emitted (which mostly coincides with the short-time regime). Deviations, including an offset and enhanced oscillations, are expected to appear when atom reabsorption by neighboring empty emitters becomes relevant [23]. Proper accounting for these effects would necessitate a multi-emitter model, possibly including collisional interactions, which is beyond the scope of this paper. That said, the observed decay dynamics displays oscillations that last to long times, in qualitative agreement with the single emitter model, and for the spectrum of Fig. 2a, taken at 400 \( \mu s \), the data only differ from the model by a constant scaling factor. For \( g/\bar{\omega} \gg 1 \), the emitters effectively decouple such that the isolated emitter model applies fully.

Remarkably, the spatial structure of a given bound state qualitatively depends on whether it is located below or above the band. Using the approach of [25], it is straightforward to show that the bound states contain evanescent matter waves of the form

\[
\psi_B^\pm(z) = \int_{-k}^{k} dq \ \phi^\pm(q) \langle z | 1, q \rangle
\]

with quasimomentum probability amplitudes \( \phi^\pm(q) = (\hbar g/2k)/[\hbar \omega_B^\pm - \epsilon(q)] \) (see methods for details). For the bound state above the band, where the energetic difference between \( \hbar \omega_B^+ \) and \( \epsilon_{1, \pm q} \) is smallest near \( q = \pm k \), this means that \( |\phi^+(q)|^2 \) possesses a double-peaked structure in quasimomentum space. In contrast, for the bound state below the band, the quasimomentum is concentrated around \( q = 0 \). To demonstrate these features, we directly detect the spatial features of two representative bound states symmetrically located on opposite sides of the band (where \( \phi^\pm(q + k) = \phi^\mp(q) \)) with emission energies \( \hbar \Delta^\pm = (1 \pm 3)\hbar \bar{\omega} \) and weak coupling \((g/\Delta^\pm)^2 \ll 1 \) (such that \( \omega_B^\pm \approx \Delta^\pm \)).

We prepare the states by slowly ramping on the coupling \( g \) using a sinusoidal ramp. The ramp duration of 2 ms is long with respect to the bound state frequencies \( \omega_B^\pm \), and no dynamics are observed for a variable hold time between 0 and 0.5 ms following the ramp. The resulting quasimomentum distributions are observed in time-of-flight after a band-map of
FIG. 3: Structure of the bound states at $h\Delta^+ = 1.0(1)E_r$ and $h\Delta^- = -0.5(1)E_r$ above a, b and below c, d the band edges. a, Calculated quasimomentum distribution of the bound state above the band at $\hbar\omega_B^+ = 4\hbar\tilde{\omega} \approx h \times 3.9(3)$ kHz and corresponding computed position space distribution, where $a_l = \lambda/2$ is the lattice spacing. b, Observed quasimomentum distribution from time-of-flight following an adiabatic (2 ms long) ramp on of the coupling up to $g/\tilde{\omega} = 0.43$. Each data-point is the average of more than 30 individual repetitions. The density plot shows the average time-of-flight picture. The gray curve is the quasimomentum distribution from a convolved with a Gaussian blur ($\sigma = 0.15k$) to accomodate finite size effects and imaging resolution. c, Quasimomentum and position distributions as in a for the bound state below the band at $\hbar\omega_B^- = -2\hbar\tilde{\omega} \approx h \times -1.9(3)$ kHz, and d, observed quasimomentum distribution, taken as in b. The gray curve is blurred with the same Gaussian as in panel b.

all optical potentials as before, c.f. Fig. 3. The observed distributions match qualitatively the predictions for $|\phi^\pm(q)|^2$ (with a small higher-band contamination, see methods), with quantitative agreement if we allow for a blurring of $0.15k$ due to finite size effects (system size $\sim 10\mu$m) and imaging resolution.

We note that the below-band bound state, cf. Fig. 3c, resembles the state found near a single edge [23] while the cutoffs at $\pm k$ lead to strong modulations in its spatial shape with the lattice periodicity, which suggests that an exact placement of distant emitters near photonic-bandgap materials can be important for the controlled engineering of coherent in-
teractions between them [3]. In the present experiment, the Franck-Condon overlap allows for a single-band approximation, but situations with higher-band couplings may be realized, for example, by changing the relative sign of the state-dependent lattice (see supplementary materials). Future work with higher-dimensional lattices and more complicated band structures should furthermore open up opportunities to study exotic emission phenomena such as analogs of chiral emission and the formation of super- and subradiant states [30, 31].

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* Please direct correspondence to dominik.schneble@stonybrook.edu

[1] D. E. Chang, J. S. Douglas, A. González-Tudela, C. L. Hung, and H. J. Kimble, Reviews of Modern Physics 90, 031002 (2018).
[2] J. I. Cirac and H. J. Kimble, Nature Photonics 11, 18 (2017).
[3] J. S. Douglas, H. Habibian, C. L. Hung, A. V. Gorshkov, H. J. Kimble, and D. E. Chang, Nature Photonics 9, 326 (2015).
[4] P. Lodahl, A. Floris van Driel, I. S. Nikolaev, A. Irman, K. Overgaag, D. Vanmaekelbergh, and W. L. Vos, Nature 430, 654 (2004).
[5] I.-C. Hoi, C. M. Wilson, G. Johansson, T. Palomaki, B. Peropadre, and P. Delsing, Physical Review Letters 107, 073601 (2011).
[6] D. Reitz, C. Sayrin, R. Mitsch, P. Schneeweiss, and A. Rauschenbeutel, Physical Review Letters 110, 243603 (2013).
[7] A. F. van Loo, A. Fedorov, K. Lalumière, B. C. Sanders, A. Blais, and A. Wallraff, Science 342, 1494 (2013).
[8] R. Yalla, M. Sadgrove, K. P. Nayak, and K. Hakuta, Physical Review Letters 113, 143601 (2014).
[9] J. Mlynek, A. Abdumalikov, C. Eichler, and A. Wallraff, Nature Communications 5, 5186 (2014).
[10] A. Goban, C. L. Hung, S. P. Yu, J. D. Hood, J. A. Muniz, J. H. Lee, M. J. Martin, A. C. McClung, K. S. Choi, D. E. Chang, et al., Nature Communications 5, 3808 (2014).
[11] M. Arcari, I. Söllner, A. Javadi, S. Lindskov Hansen, S. Mahmoodian, J. Liu, H. Thyrrestrup, E. H. Lee, J. D. Song, S. Stobbe, et al., Physical Review Letters 113, 093603 (2014).
[12] V. P. Bykov, Soviet Journal of Quantum Electronics 4, 861 (1975).
[13] S. John, Physical Review Letters 58, 2486 (1987).
[14] E. Yablonovitch, Physical Review Letters 58, 2059 (1987).
[15] M. D. Tocci, M. Scalora, M. J. Bloemer, J. P. Dowling, and C. M. Bowden, Physical Review A 53, 2799 (1996).
[16] U. Hoeppe, C. Wolff, J. Küchenmeister, J. Niegemann, M. Drescher, H. Benner, and K. Busch, Physical Review Letters 108, 043603 (2012).
[17] Q. Liu, H. Song, W. Wang, X. Bai, Y. Wang, B. Dong, L. Xu, and W. Han, Optics Letters 35, 2898 (2010).
[18] J. D. Hood, A. Goban, A. Asenjo-Garcia, M. Lu, S.-P. Yu, D. E. Chang, and H. J. Kimble, Proceedings of the National Academy of Sciences 113, 10507 (2016).
[19] Y. Liu and A. A. Houck, Nature Physics 13, 48 (2017).
[20] N. M. Sundaresan, R. Lundgren, G. Zhu, A. V. Gorshkov, and A. A. Houck, Physical Review X 9, 011021 (2019).
[21] A. Goban, C. L. Hung, J. D. Hood, S. P. Yu, J. A. Muniz, O. Painter, and H. J. Kimble, Physical Review Letters 115, 063601 (2015).
[22] I. de Vega, D. Porras, and J. Ignacio Cirac, Physical Review Letters 101, 260404 (2008).
[23] L. Krinner, M. Stewart, A. Pazmiño, J. Kwon, and D. Schneble, Nature 559, 589 (2018).
[24] A. González Tudela and J. I. Cirac, Nature 559, 481 (2018).
[25] M. Stewart, L. Krinner, A. Pazmiño, and D. Schneble, Physical Review A 95, 013626 (2017).
[26] K. Rzążewski, M. Lewenstein, and J. H. Eberly, Journal of Physics B 15, L661-L667 (1982).
[27] M. Lewenstein, J. Zakrzewski, and T. W. Mossberg, Physical Review A 38, 808 (1988).
[28] A. Kofman, G. Kurizki, and B. Sherman, Journal of Modern Optics 41, 353 (1994).
[29] P. Lambropoulos, G. M. Nikolopoulos, T. R. Nielsen, and S. Bay, Reports on Progress in Physics 63, 455 (2000).
[30] A. González-Tudela and J. I. Cirac, Physical Review Letters 119, 143602 (2017).
[31] A. González-Tudela and J. I. Cirac, Quantum 2, 97 (2018).
[32] A. González-Tudela and J. I. Cirac, Physical Review A 96, 043811 (2017).
[33] A. Peres, American Journal of Physics 48, 931 (1980).
[34] W. Itano, D. Heinzen, J. Bollinger, and D. Wineland, Physical Review A 41, 2295 (1990).

METHODS

Experimental procedures

Sample preparation. The experiment begins by creating an optically-trapped Bose-Einstein condensate. In order to minimize gravitational sag, the horizontal, state-independent lattices are first adiabatically ramped up in 80ms followed by the vertical state-dependent lattice (90 ms) to final depths of $40E_{r,1064\text{nm}}$, $40E_{r,1064\text{nm}}$ and $20E_{r,790.41\text{nm}}$ so that the atomic cloud sits at approximately the trap minimum potential, with a residual confinement along the $z$-direction of $\omega_z \approx 2\pi \times 100$ Hz. Here, $E_{r,\lambda}$ is the recoil energy of the lattice. This procedure creates a Mott-insulating sample deep within the Mott regime. With atoms loaded into the lattice, a variable fraction $f$ is transferred, at a bias field of 5 G, to an intermediate $|2,1\rangle$ state using a two-photon microwave and radio-frequency pulse of about 2 ms duration. The transferred atoms are removed using resonant light on the $D_2$ cycling transition ($F = 2 \rightarrow F' = 3$). After the pulse sequence (in which $f$ is adjusted between 0.6 and 0.85 to compensate for differing initial atom number), the remaining sample has about $2.7(3) \times 10^4 |r\rangle$ atoms with an average site occupation of $\langle n_i \rangle \lesssim 0.5$ in the tubes.

State-dependent lattice and atom detection. Our experimental techniques follow that of our previous work [2]. In brief, we generate the state dependent potential using $\sigma^-$ light tuned to $\lambda = 790.4 \text{ nm}$, between the $D_1$ and $D_2$ transitions of $^{87}\text{Rb}$. We detect the atoms after a 500 $\mu$s-long linear ramp-off of all optical lattice potentials to perform a band-map operation followed by 14 ms of time-of-flight (ToF) expansion. During ToF, we apply Stern-
Gerlach separation using a magnetic field gradient in order to spatially separate hyperfine states of different magnetic moment. We then perform state-selective absorption imaging in order to resolve all hyperfine states in each ground state manifold individually (used for magnetometry [3]). Images are analyzed for data extraction after using a principal component analysis routine to remove residual fringes in the images.

Determining the resonance condition. The resonance condition $\Delta = 0$ is defined with respect to the transition between the band minimum $\varepsilon_{n=1,q=0}$ and the harmonic-oscillator ground state in the QE potential (with a residual bandwidth of $1 \times 10^{-2} E_r$). We use lattice transfer spectroscopy [4] to determine the resonance condition. An optically trapped BEC of $|r\rangle$ atoms is first transferred into the $|b\rangle$ state, after which the state-dependent lattice potential is ramped on slowly. Microwave pulses of duration $\tau = 400\mu s$ are then applied at a fixed strength $\Omega = 2\pi \times 1.0$ kHz and variable frequency to transfer maximally 30% of population into the $|r\rangle$ state. The $\Delta = 0$ frequency for use in the experiment is obtained from a fit of a Rabi spectrum to the data. Systematic residual mean-field shifts are estimated to be between 150 and 270 Hz for all initial atom numbers used, based on a direct simulation of the 1D time-dependent Gross-Pitaevskii equation, and have been included in the spectrum of Fig. 2A. The resonance condition (which depends on both optical and magnetic fields) is stabilized using a post-selection magnetometry technique, yielding an uncertainty of $\sigma_E \approx 350$ Hz [2, 3].

Higher band contributions. The observed quasimomentum distributions show a small ($\lesssim 20\%$) population of atoms at higher quasimomenta ($q \approx 2.5k$). This can be attributed to a small contamination of the first excited band for $|r\rangle$ at the beginning of the measurement which is coupled to the first excited band for the $|b\rangle$ atoms due to a non-vanishing Franck-Condon overlap. These atoms are in a different region of quasimomentum space from the evanescent waves in the experimental data.

Theoretical considerations

Analytic structure of the self-energy and time dynamics. As discussed in the main text, the evolution of the initially excited emitter can be found in a resolvent operator approach,
resulting in an excited state amplitude of

\[ A(t) = \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega G(\omega + i0^+) e^{i(\Delta - \omega)t} \]

(1)

with Green’s function \( G(\omega) = \frac{1}{[\omega - \Delta - \Sigma(\omega)/\hbar]} \) and self energy \( \Sigma(\omega) = -i\hbar g^2/\sqrt{\omega(2\bar{\omega} - \omega)} \). Similarly, for the emitted matter-wave radiation, direct integration of the Schrödinger equation using the interaction Hamiltonian yields

\[ B_q(t) = -ig \int_0^t d\tau e^{i(\varepsilon(q)/\hbar - \Delta)\tau} A(\tau) \]

(2)

in which we swap the order of integrations, yielding

\[ B_q(t) = \frac{ig}{2\pi} \int_{-\infty}^{\infty} d\omega G(\omega + i0^+) \frac{e^{i(\varepsilon(q)/\hbar - \omega)t}}{\omega + i0^+ - \varepsilon(q)/\hbar}. \]

(3)

We note that these integrals are performed over a function of a complex variable, and so we can apply the Cauchy residue theorem to evaluate the behavior. Specifically, the singularities of \( G(\omega) \) yield exponentials with (in principle) complex frequencies. For detunings within the band, the analytic structure of \( G(\omega) \) is such that it is analytic in the complex plane except for two branch cuts of imaginary part running from 0 to \(-\infty\) and real part at the band edges and four poles, two of which are purely real, and two of which form a complex conjugate pair. We discuss these in turn.

The poles of the Green’s function are found by solving \( \omega - \Delta - \Sigma(\omega)/\hbar = 0 \). This equation is in general quartic, which we can see by inserting \( \Sigma(\omega) \) into the previous equation and expanding. The resulting equation is

\[ \omega(\omega - 2\bar{\omega})(\omega - \Delta)^2 = g^4 \]

(4)

and the roots may be found numerically for generic values of \( g, \bar{\omega}, \) and \( \Delta \). It turns out that this equation has two real roots, corresponding to two stable excitations or bound states, whose frequencies we denote by \( \omega^\pm_B \) and whose residues are \( R^\pm_B \). These bound states contribute with purely imaginary exponential terms to \( A(t) \). Likewise, there is a pair of complex conjugate roots, though only the root with negative imaginary part will reside within the integration contour, and thus only this root will contribute to \( A(t) \). This pole, having frequency \( \omega_M - i\Gamma/2 \) and residue \( R_M \), corresponds to an exponentially decaying excitation. The value of \( \Gamma \), for a large range of parameters \( g \) and \( \bar{\omega} \), corresponds to the Markovian prediction based on Fermi’s Golden Rule. Finally, the detours around the branch
cut contribute an incoherent decay, $\Phi(t)$, which tends to zero algebraically as a function of time. The full solution for $A(t)$ may schematically be written as

$$A(t) = R_B^+ e^{-i(\omega_B^+ - \Delta)t} + R_B^- e^{-i(\omega_B^- - \Delta)t} + R_M e^{i(\Delta - \omega_M - i\Gamma/2)t} + \Phi(t)$$

and it is from these equations that we plot the gray curves $|A(t)|^2$ in Fig. 2b.

We now consider the behavior of $|A(t)|^2$ in more detail. In the limit of weak coupling, in which $g \ll \bar{\omega}$, the dynamics correspond to an almost Markovian limit, in which the emitter population principally follows an exponential decay with a small oscillation superposed on top. This oscillation is in general due to a beating between the bound state frequencies, for in the limit of infinite time, $A(t \to \infty) = R_B^+ e^{-i(\omega_B^+ - \Delta)t} + R_B^- e^{-i(\omega_B^- - \Delta)t}$, and the unstable pole and branch cut contributions vanish. For intermediate couplings, this conclusion still holds, and there is thus a long-time persistent oscillation of frequency $\omega_B^+ - \omega_B^-$. For the case of detuning resonant with the band center, $\Delta = \bar{\omega}$, this may be computed as $$(\omega_B^+ - \omega_B^-)|_{\Delta=\bar{\omega}} = 2\sqrt{\bar{\omega}^2/2 + \sqrt{g^4 + \bar{\omega}^4/4}}.$$ For the limit of very strong coupling $g \gg \bar{\omega}$, the self energy can be approximated as $\Sigma(\omega) \approx \hbar g^2 / (\omega - \bar{\omega})$ which corresponds with approximating the band as a single energy level at $\bar{\omega}$. This results in a suppression of the decay arising from coupling to a band of finite width and its additional singularities while maintaining the bound state poles at frequencies $\omega_B^+ - \bar{\omega} = [(\Delta - \bar{\omega}) \pm \tilde{\Omega}] / 2$ with $\tilde{\Omega} = \sqrt{(\Delta - \bar{\omega})^2 + (2g)^2}$ the usual generalized Rabi frequency of atomic physics when measuring the detuning from the band center. The “decay” behavior can thus be interpreted as a Rabi oscillation between the emitter state and the effective single $|b\rangle$ state.

Structure of the bound states. By using the method of Laplace transforms and assuming a stable bound-state pole, we can find the composition of the bound state. Specifically, applying equations (37-38) and (40-46) from [5], we find a bound state of the form

$$|\psi_B^\pm\rangle = |e; 0\rangle + \frac{g}{2k} \int_{-k}^k dq \frac{|g; 1, q\rangle}{\omega_B^\pm - \omega(q)}$$

(up to an omitted normalization factor) where we have assumed a uniform Franck-Condon overlap of all quasimomentum states with the initial emitter state. In order to learn about the spatial shape of the blue (emitted) component, we compute numerically (by exact diagonalization) the Bloch waves, $\psi_q(z)$, corresponding to the band structure of interest with small quasimomentum spacing and add the results according to the defining equation
\[ \psi_B(z) = g \sum_{q=0}^{k} \psi_q(z)/[\omega_B^+ - \omega(q)]. \] By taking the product of \( \langle q \rangle \) with \( |\psi_B\rangle \) we obtain the probability amplitudes for the quasimomenta which compose the state,

\[ \phi^\pm(q) = \frac{g}{2k} \frac{1}{\omega_B^+ - \omega(q)}. \]  

Combined, these give a picture of a bound state which couples most strongly to the \( q = 0 \) quasimomentum state for \( \omega_B^- \) below the band and, exotically, a bound state whose strongest contributions are \( q = \pm 1 \) when \( \omega_B^+ \) is above the band. These differences manifest themselves in a stronger modulation with a slight revival in the spatial structure of the bound state at the first neighboring well for the above-band bound state.

* Please direct correspondence to dominik.schneble@stonybrook.edu

[1] D. Pertot, D. Greif, S. Albert, B. Gadway, and D. Schneble, New Journal of Physics 42, 215305 (2009).

[2] L. Krinner, M. Stewart, A. Pazmiño, J. Kwon, and D. Schneble, Nature 559, 589 (2018).

[3] L. Krinner, M. Stewart, A. Pazmiño, and D. Schneble, Review of Scientific Instruments 89, 013108 (2018).

[4] J. Reeves, L. Krinner, M. Stewart, A. Pazmiño, and D. Schneble, Physical Review A 92, 023628 (2015).

[5] M. Stewart, L. Krinner, A. Pazmiño, and D. Schneble, Physical Review A 95, 013626 (2017).

SUPPLEMENTARY MATERIALS

Tunability of state-dependent potentials. It is possible to realize the case \( s_b < 0 \) by shifting the lattice wavelength in the opposite direction from the tune-out point. This results in Franck-Condon factors that are appreciable for excited bands \( n \geq 2 \), c.f. Fig. 4a, and in observed emission profiles with appreciable contributions from both the ground and first exited bands, cf. Fig. 4b.
FIG. 4: a, Franck-Condon factor $\gamma_{n,q}$ and observed emission profile b, for $s_b = -2.6$ at $\lambda = 789.8$ nm. The strongest emission signal occurs in the first excited band; the relatively strong percentage of atoms within the first band gap results from the strong coupling to the first excited band, giving rise to non-Markovian emission in the band gap.

Theoretical computation of $B_q(t)$. In order to calculate $|B_q(t)|^2$ for $t = 400\mu s$, we use Eq. (3) for the first Brillouin zone, and set it equal to zero outside this zone, c.f. Fig. 5. This is an artifact of the band-mapping procedure arranging the quasimomenta in an extended-zone scheme and our model having only one band. In order to make a comparison with the experimental data of Fig. 1c, we apply a Gaussian blur of $\sigma_E = 0.1E_r$ in the energy axis and $\sigma_q = 0.15k$ in the momentum axis to account for magnetic field and finite size uncertainty.

FIG. 5: Computed $|B_q(t)|^2$ for $-k \leq q \leq k$. The single band model predicts identically zero contribution outside of the first Brillouin zone.