Mott-Anderson transition in disordered charge transfer model: insights from typical medium theory

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The Mott-Anderson transition in the disordered charge-transfer model displays several new features in comparison to what is found in the disordered single-band Hubbard model, as recently demonstrated by large-scale computational (statistical dynamical mean field theory) studies. Here we show that a much simpler typical medium theory approach (TMT-DMFT) to the same model is able to capture most qualitative and even quantitative aspects of the phase diagram, the emergence of an intermediate electronic Griffiths phase, and the critical behavior close to the metal-insulator transition. Conceptual and mathematical simplicity of the TMT-DMFT formulation thus makes it possible to gain useful new insight into the mechanism of the Mott-Anderson transition in these models.

PACS numbers: 71.27.+a, 71.10.Hf, 71.30.+h, 72.15.Rn

I. INTRODUCTION

The physical mechanism behind the metal-insulator transition (MIT) remains one of the basic science questions that still lack complete understanding, both on the conceptual and the technical levels. Early work on the subject focused on examining stability of the metallic phase with respect to weak disorder,1–3 within the framework of a quasiparticle picture and an appropriate generalization4 of Landau’s Fermi liquid theory. These approaches, while formally elegant and appealing, suffer from several conceptual shortcomings that render them of limited relevance to many real materials. Essentially, these treatments describe situations where disorder is viewed as the driving force for the metal-insulator transition, and interactions only modify the details of the critical behavior. What is implicitly assumed within this picture is that the “host” Fermi liquid is far from any interaction-induced instabilities, where strong correlation effects may destroy5 the very existence of well-defined quasiparticles. Unfortunately, recent experiments on several model systems, such as two-dimensional electron systems6 and doped semiconductors,7 have provided evidence that these strong correlation effects may very well be the dominant driving force for electron localization, and thus should be explicitly included in the theory.

Both disorder (Anderson) and correlation (Mott) mechanisms to localization can be treated on the same foot by extensions of dynamical mean field theory8 (DMFT). In the so-called statistical DMFT8 (statDMFT) strong correlations are considered in a self-consistent DMFT fashion, while disorder fluctuations are treated by a (numerically) exact computational scheme. Because it is numerically very demanding, this method has been utilized only in a handful of theoretical studies of the Mott-Anderson transition.9–13 In particular, two of us have recently used it to study the precise form of quantum criticality of the charge transfer model.14 A much simpler approach – the combination between typical-medium theory15 (TMT) and DMFT - provided the first self-consistent description of the Mott-Anderson transition, and offered some insight into its critical regime.16,17 When applied to the Hubbard model, for weak to moderate disorder TMT-DMFT found a transition closely resembling the clean Mott point, while only at stronger disorder Anderson localization modified the critical behavior.16,17 Here, we employ TMT-DMFT to solve the charge transfer model, obtaining results in surprisingly good agreement with those recently obtained by us14 within the more sophisticated statDMFT method.

Besides describing the MIT, in this work we also address the electronic Griffiths phase with non-Fermi liquid behavior, which is experimentally observed in heavy-fermion systems,18,19 as well as in doped semiconductors.20 In these systems, the susceptibility is seen to diverge in a power-law fashion in the low temperature limit, not only in the insulating phase, but also in the metallic side of the MIT.21 In a number of systems, it is the disorder that is responsible for this non-Fermi liquid behavior.5 Theoretically, this phase is “naturally” incorporated in the description given by statDMFT.22 Within the DMFT framework, it can be addressed by considering the effective model proposed in Ref. 23. By combining the latter with TMT, we are able to confirm that for the CT model a Griffiths phase is observed in the region just preceding the correlation induced MIT, as within statDMFT.14

In the present work, we consider the charge trans-
fer (CT) model because it can describe the systems of our interest better than the single-band Hubbard model. This is the case since the former gives a more realistic description of spatial charge redistribution as the MIT is approached, which is important because local correlation effects strongly depend on orbital occupation. In this context, it is interesting to note that the CT model phase diagram seems to differ from that of the single-band Hubbard, even in qualitative aspects.\textsuperscript{14} Moreover, when both DMFT and TMT-DMFT are applied to the Hubbard model, the cavity field does not fluctuate, meaning that important fluctuation effects associated with the Griffiths phase and the precise nature of quantum criticality are ignored. In contrast, when the same method is used to solve the CT model a degree of fluctuation is retained, according to the effective model cited above,\textsuperscript{23} hence even the simplified theories capture effects such as the Griffiths phase. Another advantage of considering the CT model is that its standard formulation uses the $U = \infty$ constraint for the correlated band, allowing a simpler large $N$ (slave boson) solution,\textsuperscript{24} which is not available for finite $U$ Hubbard models.

As mentioned above, to solve the disordered CT model in this paper we use the TMT-DMFT method, which allows a detailed description of the system close to the MIT because of its conceptual and mathematical simplicity. According to our current results, as the interaction induced transition is approached, a fraction of sites turn into local moment, but not all of them do it. This is in contrast to the TMT-DMFT results for the Hubbard model,\textsuperscript{17} where all sites turn into local moments close to the Mott transition. The phase diagram for the CT model thus includes a disordered Mott insulating phase which is qualitatively different than the Mott insulator observed for the Hubbard model. The disorder induced MIT is also qualitatively different than for the Hubbard model. For the CT model most of the sites Anderson localize, but none of them turn into local moment as disorder increases. In the case of the Hubbard model, we have a two-fluid picture, where a fraction of the sites go through Anderson localization, while the rest of them Mott localize.\textsuperscript{17}

The paper is organized as follows. In the next section we define the model we consider and the method we use to solve it. Section III is devoted to our numerical results: we present our phase diagram, discuss the disorder (section III.A) and the interaction (section III.B) induced transitions, with special emphasis to the behavior of the physical quantities that characterize the transitions, and finally present results related to the Griffiths phase (section III.C). We end by summarizing our main conclusions.

## II. THE MODEL AND ITS SOLUTION

### A. Charge transfer model and TMT-DMFT equations

The CT model is a two band model, where one band represents conduction electrons and the other corresponds to localized or $f$-type electrons, for which the electron-electron interactions are strong. It has been used to describe various systems, including oxides\textsuperscript{25} and doped semiconductors;\textsuperscript{20} for the latter, the disordered version of the model is the relevant one, which is indeed the problem we address in this paper.

The CT model description of the Mott transition can be understood as follows: in the clean case, the insulating phase is approached as the $f$-electron energy decreases, which implies in a smaller number of conduction electrons per site; the transition itself takes place when the latter vanishes. A careful study on the regimes where this model can be used to describe the Mott transition in the clean case can be found in Ref. 26, for example.

In the disordered case, the CT model is given by the disordered Anderson lattice model supplemented by the condition that the average number of electrons on each site is equal to 1. The Hamiltonian for the Anderson lattice model is

$$
H = \sum_{ij\sigma} [(\varepsilon_j - \mu)\delta_{ij} - t] c^\dagger_{i\sigma} c_{j\sigma} + (E_f - \mu) \sum_{j\sigma} f^\dagger_{j\sigma} f_{j\sigma} + V \sum_{j\sigma} (c^\dagger_{j\sigma} f_{j\sigma} + f^\dagger_{j\sigma} c_{j\sigma}) + U \sum_j n_{fj\uparrow} n_{fj\downarrow},
$$

(1)

where $c_{j\sigma}^\dagger$ ($c_{j\sigma}$) creates (destroys) a conduction electron with spin $\sigma$ on site $j$, $f_{j\sigma}^\dagger$ and $f_{j\sigma}$ are the corresponding creation and annihilation operators for a localized $f$-electron with spin $\sigma$ on site $j$, $n_{fj\sigma} = f_{j\sigma}^\dagger f_{j\sigma}$ is the number operator for $f$-electrons, $t$ is the hopping amplitude to nearest neighbors, $E_f$ is the $f$-electron energy, $U$ is the on-site repulsion between $f$-electrons, $V$ is the hybridization between conduction and $f$-electrons, and $\mu$ is the chemical potential. Throughout this paper we use the half-bandwidth for conduction electrons as the unit of energy; the hybridization potential is chosen to be $V = 0.5$.

In eq. (1), disorder is introduced through the on-site energies $\varepsilon_j$ for conduction electrons, which follow a distribution $P(\varepsilon)$. As we want to be able to address the electronic Griffiths phase, we must deserve special attention to the disorder distribution we consider. As we mentioned before, this phase appears naturally when one treats the disordered correlated system through statDMFT,\textsuperscript{22} but this is not necessarily the case when standard DMFT is considered. In this case, it has been shown that essentially all the properties of the electronic Griffiths phase can be described if we “correctly” choose the model to study and the disorder distribution.\textsuperscript{23}

According to Ref. 23, the recipe to describe the Griffiths phase in a DMFT level is to include in the cal-
culation cavity fluctuations naturally described within statDMFT. Firstly, one has to consider a two band model as the CT model we treat here: in this case the bath seen by each impurity problem fluctuates, that is, changes from site to site [see eq. (4) below]. In addition, the disorder should be present in the on-site conduction electron energy, which necessarily follows a Gaussian distribution. This specific form of disorder generates a distribution of renormalized energies which is also Gaussian, as it is the case when statDMFT with any disorder distribution of bare energies is considered. Following these findings, in this paper we assume a Gaussian form for $P(\varepsilon)$, with zero mean and standard deviation equal to $W$, to be able to describe the Griffiths phase within TMT-DMFT.

To finish the description of the CT model, we add that the condition that the average number of electrons on each site is equal to 1 can be enforced by adjusting the chemical potential and can be written as

$$\langle n_{c_j} \rangle + \langle n_{f_j} \rangle = 1,$$

(2)

where $n_{f_j} = n_{f_j\uparrow} + n_{f_j\downarrow}$ gives the number of $f$-electrons on site $j$, $n_{c_j} = n_{c_j\uparrow} + n_{c_j\downarrow}$ is the corresponding number operator for conduction electrons, with $n_{c_j\sigma} = c_j^{\dagger\sigma}c_j^{\sigma}$, and the averages are taken over the distribution $P(\varepsilon)$.

As anticipated in the Introduction, we use a combination of TMT and DMFT to solve the disordered CT model. Within this combination $^{15-17}$ the lattice problem is mapped onto an ensemble of single-impurity problems, corresponding to sites with different values of the local energy $\varepsilon_j$, each being embedded in a typical effective medium which is self-consistently calculated. In contrast to standard DMFT, $^{27}$ TMT-DMFT determines this effective medium by replacing the spectrum of the environment (“cavity”) for each site by its typical value, which is determined by the process of geometric averaging.

To be more specific, within TMT-DMFT the Hamiltonian of eq. (1) is mapped onto an ensemble of single-impurity problems, each of which is given by the following action

$$S(j) = \sum_{\sigma} \int_0^\beta d\tau \int_0^\beta d\tau' f_j^{\dagger \sigma}(\tau) [\delta(\tau - \tau') (\partial_\tau + E_f - \mu)$$

$$+ \Delta_{f_j}(\tau - \tau')] f_j^\sigma(\tau') + U \int_0^\beta d\tau n_{f_j\uparrow}(\tau)n_{f_j\downarrow}(\tau),$$

(3)

where the Fourier transform of $\Delta_{f_j}(\tau - \tau')$ satisfies

$$\Delta_{f_j}(i\omega) = \frac{V^2}{i\omega + \mu - \varepsilon_j - \lambda^2 G^{\text{typ}}_c(i\omega)}.$$  

(4)

A Bethe lattice of infinite coordination number was considered when writing the above equation.

$G^{\text{typ}}_c(i\omega)$ is the typical Green’s function for conduction electrons, which within TMT-DMFT is given by the Hilbert transform of $\rho^{\text{typ}}_c(\omega)$, the typical value of the local density-of-states (LDOS). In equations, we have

$$G^{\text{typ}}_c(i\omega) = \int_{-\infty}^{\infty} d\omega' \frac{\rho^{\text{typ}}_c(\omega')}{i\omega - \omega'},$$

(5)

where

$$\rho^{\text{typ}}_c(\omega) = \exp\{\ln \rho_{c_j}(\omega)\},$$

(6)

and

$$\rho_{c_j}(\omega) = -\pi^{-1} \text{Im } G_{c_j}(\omega)$$

(7)

is the LDOS.

The local Green’s function for conduction electrons appearing in the above equation satisfies

$$G_{c_j}(i\omega) = \frac{1}{i\omega + \mu - \varepsilon_j - \lambda^2 G^{\text{typ}}_c(i\omega) - \Phi_j(i\omega)}$$

(8)

where

$$\Phi_j(i\omega) = \frac{V^2}{i\omega + \mu - E_f - \Sigma_{f_j}(i\omega)},$$

(9)

and $\Sigma_{f_j}(i\omega)$ is the single-impurity self-energy, which is a solution of the action given in eq. (3).

By looking at eq. (6), for example, one can conclude that the problem defined by these equations corresponds to a self-consistent calculation. In other words, within TMT-DMFT the conduction electron effective medium seen by each impurity is self-consistently determined.

### B. Slave-boson impurity solver

To solve the single-impurity problems of eq. (3), we use the slave-boson (SB) technique in the $U \rightarrow \infty$ limit. $^{24,28}$ In this case, the impurity Green’s function can be written as

$$G_{f_j}(i\omega) = \frac{Z_j}{i\omega - \varepsilon_{f_j} - Z_j \Delta_{f_j}(i\omega)}$$

(10)

$$\equiv Z_j G^{\text{QP}}_{f_j}(i\omega),$$

(11)

where $Z_j$ is the local quasiparticle (QP) weight and $\varepsilon_{f_j}$ is the renormalized $f$-electron energy. These two parameters are obtained by solving the following set of equations

$$2 \int_0^\infty \frac{d\omega}{\pi} \text{Re } \Delta_{f_j}(i\omega) G_{f_j}^{\text{QP}}(i\omega) = E_f - \varepsilon_{f_j},$$

(12)

$$Z_j + 2 \int_0^\infty \frac{d\omega}{\pi} \text{Re } G_{f_j}^{\text{QP}}(i\omega) = 0.$$  

(13)

For more details on the $U \rightarrow \infty$ SB treatment we refer the reader to Ref. 29.

Before finishing the section, it is convenient to note that in terms of the two SB parameters eq. (9) can be rewritten as

$$\Phi_j(i\omega) = \frac{Z_j V^2}{i\omega - \varepsilon_{f_j}}.$$  

(14)
FIG. 1: (Color online) Phase diagram of the disordered CT model obtained within TMT-DMFT. $E_{ct} = -E_f$ is the CT energy and plays the role of the Hubbard $U$. For comparison, the inset reproduces the results obtained within statDMFT and presented in Ref. 14.

III. NUMERICAL RESULTS

Let us now present and discuss the numerical results we obtained for the CT model using TMT-DMFT. In this section, we also compare these results with those obtained by two of us within the more sophisticated statDMFT,\textsuperscript{14} which provides an exact numerical treatment of localization in the absence of interactions, and reduces to the standard DMFT treatment in the absence of disorder.\textsuperscript{9}

Fig. 1 presents our phase diagram. As we described previously,\textsuperscript{14} starting from a disordered correlated metal, a transition to a correlated Anderson insulator takes place as disorder increases; on the other hand, a disordered Mott insulating phase is observed for large values of the CT energy. The latter is defined as $E_{ct} = -E_f$ and plays the role of the interaction energy $U$ in the Hubbard model. By comparing the results in the main panel of Fig. 1 with those in the inset, we can see that in the case of the Mott-like transition TMT-DMFT predicts the phase boundary in very good agreement with statDMFT. For the Anderson transition, according to TMT-DMFT a slightly larger amount of disorder than that observed in statDMFT is necessary to drive the transition.

In the following, we look at how the order parameter and other physical quantities behave as the transitions are approached.

A. Disorder-driven transition

1. Critical behavior of the local density-of-states

Fig. 2 shows the typical LDOS for conduction electrons at the Fermi energy as the disorder-driven transition is approached for different values of the CT energy. As expected, $\rho^{hp}_c(\omega = 0)$ decreases from the clean value as $W$ increases, due to disorder induced localization effects. The typical LDOS for conduction electrons corresponds indeed to an order parameter within TMT-DMFT: its vanishing defines the critical disorder at which the MIT takes place. In the present case, for all values of $E_{ct}$, $\rho^{hp}_c(\omega = 0)$ is seen to go to zero continuously as the MIT is approached, in agreement with statDMFT results.\textsuperscript{14}

A detailed comparison with statDMFT results for $E_{ct} = 1.3$ can be seen in Fig. 3. In accordance with the phase diagram of Fig. 1, within TMT-DMFT the transition is seen at a larger $W$ value than within statDMFT. Although in the present treatment the bath fluctuates from site-to-site - note that the bath given by eq. (4) does depend on the site $j$, our results in Fig. 3 may suggest that not all the fluctuations induced by Anderson localization effects are captured by the simple TMT-DMFT treatment. Still, the behavior of the typical (and the inverse of average) LDOS is very similar in both treatments, allowing us to conclude that TMT-DMFT does give a reasonable picture of the transition. In addition, as pointed out before, since the latter is numerical and analytically simpler, it facilitates the understanding of the physics behind the problem we are looking at, as we discuss in this paper for the CT model.

In Fig. 4 the behavior of the typical LDOS is compared to that of the (arithmetic) average LDOS. It is interest-
most of the sites with the values of \( \rho \) of the average LDOS goes to zero at the same dis-
verse results \( E \) values of the LDOS for conduction electrons at the Fermi en-

FIG. 3: (Color online) Comparison between TMT-DMFT and \( \text{statDMFT} \): results are shown for the typical and 1/average values of the LDOS for conduction electrons at the Fermi energy obtained for \( E_{\text{cd}} = 1.3 \) and different values of the disorder strength, \( W \). The inset shows the typical values of the QP weight \( Z \) as a function of \( W \) for both treatments.

But which mechanism of localization dominates the current MIT? In the case of the CT model, we have two kinds of electrons, localized or \( f \)-electrons and conduc-
tion or \( c \)-electrons. Within the SB method we consider, \( Z_j = 1 - n_{fj} \) and, for the CT model, \( (n_{c})+(n_{fj}) = 1 \). According to the above, most of the sites have \( Z_j = 1 \) (and small \( \varepsilon_{fj} \)) close to the MIT. \( n_{fj} = 0 \) for these sites, corresponding to electrons occupying \( c \)-states, which are known to localize as disorder increases. Indeed, \( \rho_{c\gamma}(\omega = 0) \sim 0 \) for these sites, as one can see in the inset of Fig. 4. Thus, as the transition is approached, most of the sites go through Anderson type of localization. In other words,

Anderson insulator has all sites with \( Z_j = 1 \). The site with \( \varepsilon = E_f = -E_{\text{cd}} \) is a special one: it corresponds to \( n_c = n_f = 0.5 \), which implies in \( Z = 0.5 \), as \( Z_j = 1 - n_{fj} \) within SB. As a consequence of the presence of the latter, close to the transition most of the sites have \( Z_j = 1 \), but some of them have \( 0.5 \leq Z_j < 1 \). Note that none form local moments (\( Z_j = 0 \)), a situation completely different than the one we will analyze in section III.B.2 below.

The typical values of \( Z \) corresponding to the results in Fig. 5(a) are compared to the \( \text{statDMFT} \) results in the inset of Fig. 3. In the region where both treatments predict the system to be metallic, although the typical LDOS is larger within TMT-DMFT than within \( \text{statDMFT} \), the typical values of \( Z \) practically coincide.

The results for the second SB parameter - the renor-
malized energy \( \varepsilon_{f} \) - are shown in Fig. 5(b). The latter is maximum for \( \varepsilon_{f} = E_f = -E_{\text{cd}} \) and is relatively small for the majority of the sites, which correspond indeed to the intermediate to large \( |\varepsilon_j| \) sites that have \( Z_j = 1 \) close to the transition.

Let us now look at the properties of the single-impurity problems into which the lattice Hamiltonian is mapped. The inset of Fig. 4 shows the LDOS for each single-

FIG. 4: (Color online) Comparison between TMT-DMFT and CPA-DMFT: results are shown for the typical and average values of the LDOS for conduction electrons at the Fermi energy obtained for \( E_{\text{cd}} = 1.3 \) and different values of the disorder strength, \( W \). The inset shows the TMT-DMFT LDOS corresponding to each single-impurity problem as a function of the on-site energy \( \varepsilon \).

ing to note that, as it is the general case for \( \text{statDMFT} \) results \(^{9,14} \) (see also Fig. 3), within TMT-DMFT the inverse of the average LDOS goes to zero at the same dis-

order at which \( \rho^{\text{stat}}_{c\gamma}(\omega = 0) \) vanishes. Fig. 4 also shows the results of standard CPA-DMFT calculation, where disorder is treated as in the Coherent Potential Approximation (CPA), being unable to describe Anderson localization effects.

2. Statistics of local quasi-particle parameters

The inset of Fig. 4 shows the LDOS for each single-

impurity of the ensemble, which is given by eq. (7) and from which the TMT-DMFT results in the main figure are calculated. The conduction electrons, whose LDOS we are analyzing, see the \( f \)-electrons through the function \( \Phi(\omega) \) [see eq. (8)], which we can identify as an effective disorder potential. According to eq. (14), \( \Phi(\omega) \) is written in terms of the two Fermi liquid parameters, \( Z \) and \( \varepsilon_f \). To understand the behavior of the LDOS close to the transition, below we present and analyze the results for these two parameters.

Fig. 5(a) shows the behavior of the QP weight \( Z_j \) as a function of the on-site energy \( \varepsilon_j \) as disorder increases for \( E_{\text{cd}} = 1.3 \) (same parameter of Fig. 3 and 4). For the smallest \( W \) considered, the values of \( Z_j \) are small (there are even sites with \( Z_j \rightarrow 0 \)); as disorder increases, the values of \( Z_j \) increase, since the system tends to have most of the sites with \( Z_j = 1 \) (we know\(^5 \) that a “pure”
in the present case it is the Anderson mechanism for localization that is responsible for driving the system as a whole through the MIT.

If we now compare the results in Fig. 5 for $W = 3.5$ and $W = 6.0$, we see that $Z_j$ and $\varepsilon_{fj}$ coincide in the range of $\varepsilon_j$ values present for both disorder strengths (the range of $\varepsilon_j$ is, of course, larger for $W = 6.0$ than for $W = 3.5$), although $\rho_{cj}(\omega = 0)$ do change in this interval, as can be seen in the inset of Fig. 4. Note, however, that the rate at which the typical DOS decrease is smaller in the region where $Z_j$ and $\varepsilon_{fj}$ coincide than it is the case for smaller $W$ values. By looking at the different quantities that determine $G_{cj}(\omega = 0)$ [see eq. (8)], the results in Fig. 5 suggest that it is the bare disorder ($\varepsilon_j$) itself, rather than the scattering coming from the $f$-electrons through $\Phi_j(\omega = 0)$, that dominates the behavior of the LDOS as the disorder driven-transition is approached within TMT-DMFT.

To finish this subsection, the situation described here can be compared to that observed within TMT-DMFT for the Hubbard model, where close to the transition the sites have either $Z_j = 1$ or $Z_j = 0$, corresponding to electrons going through Anderson or Mott localization, respectively. (Note that the dependence of the LDOS for conduction electrons on $Z$ is different in the two models considered.) In the current case, we do not have sites going through Mott localization ($Z_j = 0$): according to Fig. 5(a), the majority of the them have $Z_j = 1$, while a finite fraction has $0.5 \leq Z_j < 1$. For the latter $n_{fj} \neq 0$, that is, the occupation of strongly correlated $f$-electrons is different from zero for these sites. Thus, although we do not have Mott localized sites, because of the presence of the $0.5 \leq Z < 1$ sites, the current situation is also different than that in the non-interacting limit (where all sites have $Z_j = 1$), and a correlated Anderson insulator is present in the CT model phase diagram.

After discussing the results for the disorder-induced transition, in the next section we focus on the Mott-like transition.

**B. Interaction-driven transition**

1. **Critical behavior of the local density-of-states**

Fig. 6 shows the typical LDOS for conduction electrons at the Fermi energy as the Mott-like transition is approached for intermediate values of disorder. A non-monotonic behavior is observed, implying in an initial increase of the system “conductivity” when $E_{ct}$ increases, which suggests that the disorder potential is screened by the correlation effects considered to exist between $f$-electrons. This non-monotonic behavior is in agreement with the statDMFT results we presented recently. Indeed, in the current case the screening is stronger than within statDMFT - see, for example, the detailed comparison between TMT-DMFT and statDMFT presented in Fig. 7 for $W = 1.5$. Although strong, here the screening is not perfect and $\rho_{ct}$ does not reach the value corresponding to the clean limit, as it is the case for the Hubbard model within DMFT and TMT-DMFT.

Probably as a consequence of the strong disorder screening discussed above, the typical DOS within TMT-DMFT is seen to present a jump at the transition (see Fig. 6 and 7). This is in disagreement with statDMFT for the CT model, which predicts that the order parameter vanishes continuously as the transition is approached. Note, though, that according to Fig. 7 a good agree-
ment is observed between the two calculations concerning the overall behavior of the typical and inverse of average LDOS, as well as the $E_{ct}$ value at which the transition takes place (see also Fig. 1). Although the current results suggest that TMT-DMFT does not completely describe Anderson localization effects, which were shown to be responsible for the critical behavior also in the vicinity of the Mott-like transition, we can say that it does give a reasonable picture of it.

To complete the discussion on the LDOS results, in Fig. 8 we compare the typical and average values of the LDOS obtained within TMT-DMFT with those valid within standard DMFT for $W = 1.5$ (same parameter as Fig. 7). As it is the case for the disorder-induced transition (see Fig. 4), here the inverse of the average LDOS within TMT-DMFT is seen to vanish together with the typical LDOS. Also, standard DMFT average LDOS remains finite at the critical $E_{ct}$ predicted by TMT-DMFT.

2. Statistics of local quasi-particle parameters

As we did in the previous subsection, we now look at the properties of the single-impurity problems, with the goal of understanding which sites of the ensemble dominate the behavior of the LDOS in the critical region. Fig. 9(a) shows the QP weight $Z_j$ for each single-impurity problem of the ensemble, for fixed disorder ($W = 1.5$), as the Mott-like transition is approached. As we can see, as $E_{ct}$ increases, the large $\varepsilon_j$ sites start to have $Z_j = 0$; as $E_{ct}$ increases even further, more sites present $Z_j = 0$, while the region of sites with $Z_j \neq 0$ ($0 < Z_j < 1$ indeed) shrinks to the left of the figure. Very close to the transition all sites with positive $\varepsilon_j$, as well as few with $\varepsilon_j \lesssim 0$, have $Z_j = 0$. Correspondingly, the typical value of $Z$ decreases as the transition is approached, in very good agreement with statDMFT, as can be seen in the inset of Fig. 7. Regarding the renormalized energy, which is shown in Fig. 9(b), we can see that the sites that form local moments ($Z_j = 0$) close to the transition are completely screened ($\varepsilon_{fj} = 0$). For the rest of the sites, $\varepsilon_{fj}$
exists sites with $0 < Z_j < 1$. Because of the presence of the $Z_j \neq 0$ sites, the insulating phase we observe here corresponds to a disordered Mott insulator.

The current situation is different than that observed for the Hubbard model within TMT-DMFT. In the latter all sites turn to local moments as the transition is approached, in contrast to the present case where there exist sites with $0 < Z_j < 1$. Because of the presence of the $Z_j \neq 0$ sites, the insulating phase which we observe here corresponds to a disordered Mott insulator.

Besides giving a good description of the MIT, TMT-DMFT is also able to describe the emergence of a Griffiths phase inside the disordered metallic region. The latter is possible by considering a Gaussian distribution of the on-site energy, as suggested in Ref. 23 and summarized by us in Section II.

To study the Griffiths phase, we focus on the behavior of $Z$ for small disorder. In addition, instead of looking at its behavior as a function of $\varepsilon$, as we did above, we look at the evolution of its distribution, $P(Z)$. In Fig. 10 we have the results for fixed CT energy, $E_{ct} = 1.3$. As disorder increases, the distribution moves to smaller values of $Z$. More importantly, it develops a tail that follows a power-law of the form

$$P(Z) \sim Z^{-\alpha},$$

which is better visualized in Fig. 11. The exponent $\alpha$ is found by fitting the numerical data to the above equation; the values obtained in the present case of $E_{ct} = 1.3$ are shown in the inset of the figure. As we can see, $\alpha$ is a continuous function of $W$, becoming smaller than 1 for $W \sim 0.3$ in the current case. As a consequence of $P(Z)$ following a power-law with $\alpha < 1$ (in some range of $W$), the system susceptibility and specific heat divided by the temperature $T$ diverge in the low $T$ limit (see Ref. 22 for a detailed discussion on this). This characterizes a Griffiths phase with non-Fermi liquid behavior.

According to the results in Fig. 10, as disorder increases even further, $P(Z)$ moves to larger values of $Z$ and the low $Z$ tail disappears. To precisely determine at which disorder the Griffiths phase terminates for
For $E_{ct} = 1.3$, one has to explore it in more detail, for example by performing the current analysis as a function of $E_{ct}$ for different, fixed $W$. This is illustrated below for one fixed value of $W$.

Fig. 12 shows the distribution of $Z$ (main panel) and corresponding $\alpha$ (inset of the figure) for $W = 1.5$. As $E_{ct}$ increases, $P(Z)$ moves to smaller values of $Z$. In the present case, $\alpha$ becomes smaller than 1 and the system enters the Griffiths phase for $E_{ct} \sim 1.3$. Differently than the previous case, here $P(Z)$ moves to even smaller values of $Z$, with $\alpha$ decreasing to zero, as the Mott-like transition is approached. Note that the $E_{ct}$ we have just found for the onset of the Griffiths phase for $W = 1.5$ corresponds to the $E_{ct}$ analyzed in Fig. 10 and Fig. 11; we can thus conclude that for $E_{ct} = 1.3$ the Griffiths phase is observed between $W \sim 0.3$ and $W \sim 1.5$.

The results in these three figures indicate that within TMT-DMFT the range of $W$ and $E_{ct}$ for which $\alpha < 1$ corresponds to the existence of a Griffiths phase in the region just preceding the Mott transition. This region is signalized in the phase diagram of Fig. 1 and is in accordance with $statDMFT$ results for the same model (see Ref. 14 and also the inset of Fig. 1). A similar behavior has also been observed within $statDMFT$ for the two-dimensional Hubbard model.\(^\text{10}\)

To finish, in the insets of Fig. 11 and Fig. 12, we compare the results obtained for $\alpha$ using TMT-DMFT (corresponding to $P(Z)$ in the respective main panels) and standard DMFT. A Gaussian distribution of disorder is used in the two calculations. In both figures, in the range of parameters shown, a very good agreement is seen between the two treatments considered. Note, however, that standard DMFT agrees well with TMT-DMFT concerning the onset of the Griffiths phase, but not its extension, as the former does not give a good prediction for the critical $E_{ct}$ and $W$ values at which the transitions take place, as previously showed in this paper.

### IV. CONCLUSIONS

In this paper we solved the disordered charge-transfer model (CT) by using an extension of dynamical mean field theory able to describe Anderson localization effects. In general, our results compare surprisingly well with those previously obtained by two of us using the $statDMFT$ treatment.\(^\text{14}\) The current calculation is simpler than the latter, allowing us to better characterize the system when the metal-insulator transition is approached. Our results show, in particular, that as the interaction induced transition is approached, a fraction of sites turn into local moment, but not all of them do; this means that the corresponding insulating phase is a disordered Mott insulator. In the case of the transition due to disorder, most of the sites Anderson localize; some of the correlated sites, though, remains occupied, corresponding to the presence of a correlated Anderson insulator in the phase diagram of the CT model.

In addition, according to our current TMT-DMFT results, the inverse of the arithmetic local DOS is seen to vanish precisely at the disorder or interaction value at which the typical local DOS goes to zero, which indeed determines where the transition takes place. Exactly the same behavior is observed within $statDMFT$,\(^\text{9,14}\) but an explanation for it is yet not known. The fact that the current treatment, which is analytical and numerically simpler than $statDMFT$, does show this behavior opens the possibility of understanding it, which is left as a direction of work to follow in the future.

This work was supported by CAPES (W.S.O.), CNPq and FAPEMIG (M.C.O.A.) and NSF grant DMR-1005751 (V.D.).

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1. A. M. Finkel’stein, Zh. Eksp. Teor. Fiz. 86, 367 (1984), [Sov. Phys. JETP 59, 212 (1983)].
2. A. M. Finkel’stein, Z. Phys. B 56, 189 (1984).
3. C. Castellani, C. DiCastro, P. A. Lee, and M. Ma, Phys. Rev. B 30, 527 (1984).
4. C. Castellani, G. Kotliar, and P. A. Lee, Phys. Rev. Lett. 59, 323 (1987).
5. E. Miranda and V. Dobrosavljevic, Reports on Progress in Physics 68, 2337 (2005).
6. E. Abrahams, S. V. Kravchenko, and M. P. Sarachik, Rev. Mod. Phys. 73, 251 (2001).
7. P. Dai, Y. Zhang, and M. P. Sarachik, Phys. Rev. B 45,
| Reference                                                                 | Journal                                                                 |
|--------------------------------------------------------------------------|-------------------------------------------------------------------------|
| 8 A. Georges, G. Kotliar, W. Krauth, and M. J. Rozenberg,                | Rev. Mod. Phys. 68, 13 (1996).                                          |
| 9 V. Dobrosavljević and G. Kotliar, Phys. Rev. Lett. 78,                  | 3943 (1997).                                                            |
| 10 E. C. Andrade, E. Miranda, and V. Dobrosavljevic, Phys. Rev. Lett.    | 102, 206403 (2009).                                                     |
| 11 E. C. Andrade, E. Miranda, and V. Dobrosavljevic, Phys. Rev. Lett.    | 104, 236401 (2010).                                                     |
| 12 D. Semmler, J. Wernsdorfer, U. Bissbort, K. Byczuk, and W. Hofstetter,| Phys. Rev. B 82, 235115 (2010).                                         |
| 13 D. Semmler, K. Byczuk, and W. Hofstetter, Phys. Rev. B 84, 115113     | (2011).                                                                |
| 14 M. C. O. Aguiar and V. Dobrosavljević, Phys. Rev. Lett. 110,           | 066401 (2013).                                                          |
| 15 V. Dobrosavljević, A. A. Pastor, and B. K. Nikolić, Europhys. Lett.   | 62, 76 (2003).                                                          |
| 16 K. Byczuk, W. Hofstetter, and D. Vollhardt, Phys. Rev. Lett. 94,       | 056404 (2005).                                                          |
| 17 M. C. O. Aguiar, V. Dobrosavljevic, E. Abrahams, and G. Kotliar,      | Phys. Rev. Lett. 102, 156402 (2009).                                    |
| 18 G. R. Stewart, Rev. Mod. Phys. 56, 755 (1984).                        |                                                                         |
| 19 G. R. Stewart, Rev. Mod. Phys. 73, 797 (2001).                        |                                                                         |
| 20 B. Shklovskii and A. Efros, *Electronic properties of doped semiconductors* (Springer-Verlag, Berlin, 1984). |
| 21 M. J. Hirsch, D. F. Holcomb, R. N. Bhatt, and M. A. Paalanen, Phys. Rev. Lett. 68, 1418 (1992). |
| 22 M. C. O. Aguiar, E. Miranda, and V. Dobrosavljević, Phys. Rev. B 68, 125104 (2003). |
| 23 D. Tanasković, E. Miranda, and V. Dobrosavljević, Phys. Rev. B 70, 205108 (2004). |
| 24 P. Coleman, Phys. Rev. B 35, 5072 (1987).                             |                                                                         |
| 25 J. Zaanen, G. A. Sawatzky, and J. W. Allen, Phys. Rev. Lett. 55,       | 418 (1985).                                                            |
| 26 G. Sordi, A. Amaricci, and M. J. Rozenberg, Phys. Rev. B 80, 035129 (2009). |
| 27 D. Tanaskovic, V. Dobrosavljevic, E. Abrahams, and G. Kotliar, Phys. Rev. Lett. 91, 066603 (2003). |
| 28 N. Read and D. M. Newns, J. Phys. C 16, 3273 (1983).                   |                                                                         |
| 29 E. Miranda, V. Dobrosavljević, and G. Kotliar, J. Phys.: Condens. Matter 8, 9871 (1996). |