Build-up steady-state analysis of wind-driven self-excited induction generators

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Published in The Journal of Engineering; Received on 10th October 2017; Accepted on 2nd November 2017

Abstract: For the contradiction between the increasing importance of the application of induction generator in the new energy power generation such as wind energy and the relative laggard of research on its comprehensive performance, the build-up steady-state analysis is performed for the wind-driven self-excited induction generator (SEIG) system based on the unloaded transient equivalent circuits and its mathematic model of SEIG in the two-phase stationary reference frame. Thus, it obtains the build-up steady-state conditions represented by the unloaded transient mathematic model. On the basis of the result obtained for the build-up steady-state limit cycle condition, the analytic solution for the build-up steady-state operating point condition and the analytic formulas for the limit magnetising induction are further obtained. Example analysis is to test out the only set of analytic solution for the build-up steady-state condition. Thus, the analytic solution for the build-up steady-state condition produces the formulas of the torture and the power factor, which can be applicable to further analysis for the system dynamics and build-up steady-state operating performance, showing practicability and referenced value in engineering.

1 Introduction

With the increasing awareness of the energy crisis and environmental protection as well as demand for clean energy, renewable energies such as wind is drawing increasingly more concern [1, 2]. Induction generators with advantages of low cost, simple and strong structure, and easy maintenance play a significant role in grid-connected, micro-grid, and stand-alone energy conversion systems [3, 4]. Especially for remote areas and communities or small businesses, induction generators can improve the efficiency in converting wind energy and reduce the environmental burden [5]. Research on self-excited induction generators (SEIGs) in stand-alone mode can provide not only some important technological references for grid-connected study, but also reliable theoretical foundation for optimised design and comprehensive analysis and evaluation of closed-loop control systems [6–9].

Since a rigorous analytic solution of system problems can be used as the standard solution to verify the accuracy of numerical solutions, and to examine the convergence and stability of the computational programme; it is especially important for analysis of operating performance for physical objects. However, the equation of SEIG systems owns classical non-linear characteristics which is hard to be solved analytically. Instead, numerical solutions are usually solved by numerical approximation methods which are based on the steady-state equivalent circuit for only steady-state analysis [10–12], or the transient equivalent circuits for both build-up transient analysis and steady-state analysis [13, 14]. However, those computation methods above have lack of versatility because of the problems in convergence. On the other hand, some analytic solution can be also obtained by methods based on the transient equivalent circuits, which have no analysis for dynamics and power flow performance of the systems [15, 16].

The paper derives the non-linear dynamic model in terms of the state–space mathematic model based on unloaded transient equivalent circuits under the two-phase stationary reference frame. Firstly, the magnetising inductance in the generator magnetic circuit saturated state of build-up steady states is obtained by solving the build-up steady-state conditions. Then the analytic formulations of the torque and the power factor are obtained through the unique solution for the build-up steady-state conditions which is also examined through a practical example. Finally, the dynamics characteristics and the build-up steady-state operating performances are analysed further by using the analytic formulations solved above so as to provide theoretical basis and technical support for the optimal design of the SEIG control system. Furthermore, the theoretical analysis approach can be applicable to the power flow analysis and optimal control of generator set under the distributed and grid-connected modes as important references.

2 Model of wind-driven SEIG system

2.1 Topology structure

The physical system is mainly consisted of a wind turbine, a squirrel-cage induction generator (SCIG) and other necessary equipment, as illustrated in Fig. 1, where the wind turbine and the multiple gear box constitutes the driven system, the SCIG and the three-phase power capacitor parallel with it constitutes the SEIG which can be directly connected to loads or through some intermediate equipment such as a power electronic converter. The relationship between the mechanical angular frequency ωm and the rotor angular frequency ω is ω = ωm/n, in which ωm represents the number of pole pairs of the SCIG.

The requirement condition for normal operation of the SEIG driven by external wind energy is as follows:

\[ T_m = T_e + J \frac{d\omega}{dt} \]  

(1)

where \( T_e \) and \( T_m \) are the electric torque and the mechanical torque of SEIG, respectively, and \( J \) is the moment of inertia.

2.2 Build-up dynamics model of SEIG

The voltage and flux-linkage equations of squirrel-cage induction generators can be equivalently transformed from the three-phase...
stationary reference frame to the two-phase stationary reference frame by the abc/αβ transformation. Neglecting the winding slot effect, the length of the air gap can be considered even. The coefficient of the magnetising inductance is thus transformed into a time-invariant coefficient in the arbitrary reference frame. The magnetising inductance is consequently a function of only the state parameter, namely the magnetising current. However, characteristics never change with state parameters’ coordinate transformation.

The physical and state parameters represent real values transformed into the stator side. $u_{αα}, u_{βα}$ are the stator voltages; $i_{αα}, i_{βα}$ are the stator currents; $u_{αβ}, u_{ββ}$ are the rotor flux linkages; and $\lambda_{αα}, \lambda_{βα}$ are the rotor flux linkages, $R_r$ and $R_s$ are the stator and rotor resistances, respectively; $L_s$ and $L_r$ are the stator and rotor leakage inductances, respectively; $M$ is the magnetising inductance between the stator and rotor windings; $C$ is the exciting capacitance; and $\omega$ is the rotor speed, which is constant. As the magnetising inductance is a function of the magnetising current, which continuously varies with time, the transient equivalent circuits are non-linear. The facts can give analytic formulas of the exciting capacitances for build-up steady states under a given rotor speed $\omega$ as

$$C_{\text{min}} = 2\alpha^2/(f_1 - f_2 + f_3),$$

$$C_{\text{max}} = 2\alpha^2/(f_1 - f_2 - f_3),$$

where

$$f_1 = (a^2 L_s + L_s^2 \sigma R_s^2)\omega^2,$$

$$f_2 = 2abR_sR_a$$

$$f_3 = (bL_s^2R_s + aL_sR_s)\omega\sqrt{M^2\omega^2 - 4aR_a}$$

$$a = L_s^2R_s + M^2R_r, \quad b = L_sR_s + L_sR_r$$

but also that of the rotor speeds for the build-up steady states under a given exciting capacitance $C$ as

$$\omega_{\text{min}} = \sqrt{(g_1 - g_2 - g_3)/(2L_s^2\sigma^2R_s^2)},$$

$$\omega_{\text{max}} = \sqrt{(g_1 - g_2 + g_3)/(2L_s^2\sigma^2R_s^2)},$$

where

$$g_1 = L_s^2\sigma(a^2 + L_s^2\sigma R_s^2)/C,$$

$$g_2 = L_s^2\sigma(b^2 + L_s^2\sigma R_s^2),$$

$$g_3 = dL_sR_s + bL_sR_r$$

$$d = L_s\sigma\sqrt{L_s^2R_s^4 + M^4/C^2 - 2L_s^2L_s\sigma R_s^2(\sigma + 1)/C}$$

as well as the solutions for $\omega$ are

$$\omega_{\text{min}} = \sqrt{(- h_{12} + \sqrt{\Delta})/(2h_{11})},$$

$$\omega_{\text{max}} = \sqrt{(- h_{12} - \sqrt{\Delta})/(2h_{11})},$$
where,
\[
\begin{align*}
h_{11} &= 2/\sigma (R_s/L_s + R_e/L_e) \\
h_{12} &= -2/\sigma^2 (\omega^2 R_s/L_s + \sigma B/L_s R_e/L_e + (R_s/L_s + R_e/L_e) + (R_s/L_s + R_e/L_e) B/L_e) \\
\Delta &= 16R_s R_e^2 \omega^2 (L_s L_e^2 + \omega^2 R_s/L_s + R_e/L_e) \\
&+ 4/\sigma^2 (R_s \sigma \omega^2 + B \sigma) (R_s/L_s + R_e/L_e) \\
&- (R_s/L_s + R_e/L_e) \sigma B/L_e (R_s/L_s + R_e/L_e)
\end{align*}
\]

The \( \Delta \) is always positive. So there exist two different real roots about \( \omega \).Then, the analytic formula for the limit value of magnetising inductance is obtained from \( M^2 \omega^2 - 4 R_s \omega I \geq 0 \) as follows:
\[
M^2 \omega^2 - 4 \left( L_s + M^2 R_s + M^2 R_e \right) \geq 0, \quad (10)
\]

Assuming the voltage components of an arbitrary steady-state operating point \( X \) are \( u_{\text{ss}}, u_{\text{sp}}, u_{\text{sc}}, u_{\text{emp}} \) is amplitude of the steady-state voltage, \( U_{\text{sc}}^2 = u_{\text{sc}}^2 + u_{\text{sp}}^2 \), that is the voltage components of build-up steady-state point \( X \) considered to be known, \( Y \) can be obtained as (11). In addition, further, the amplitudes of the capacitors currents is \( I_{\text{sc}} \), the amplitudes of the rotor and stator currents are \( I_r \) and \( I_s \), respectively, and the magnetising current is \( I_m \) are given by the analytic formulas obtained through solving the second equation of (6) using the analytic computation approach proposed in [7] as (12)
\[
Y = G^{-1} Y_0, \quad (11)
\]

\[
G = \begin{bmatrix}
-\sigma \omega_0 - R_s / L_s & M^2 \omega & M R_s & M \omega \\
M^2 \omega & -\sigma \omega_0 - R_s / L_s & M \omega & M R_s \\
M R_s & M \omega & -\sigma \omega_0 - R_s / L_s & -\omega \\
M \omega & M R_s & -\omega & -\sigma \omega_0 - R_s / L_s \\
\end{bmatrix}
\]

and
\[
\begin{align*}
I_s &= \sqrt{\left(-C_{\text{ss}} U_{\text{ss}}\right)^2 + \left(C_{\text{sp}} U_{\text{sp}}\right)^2} = C_{\text{ss}} U_{\text{ss}} \\
I_s &= \sqrt{\left(C_{\text{sc}} U_{\text{sc}}\right)^2 + \left(C_{\text{emp}} U_{\text{emp}}\right)^2} = C_{\text{sc}} U_{\text{sc}} \\
I_s &= \sqrt{\left(C_{\text{sm}} U_{\text{sm}}\right)^2 + \left(C_{\text{emp}} U_{\text{emp}}\right)^2} = C_{\text{sm}} U_{\text{sm}} \\
I_m &= U_{\text{m}} \sqrt{r_m / (d_1 + d_2)}
\end{align*} \quad (12)
\]

where
\[
\begin{align*}
n_s &= L_s^2 (\omega - \omega_0)^2 + R_s^2; \\
n_m &= (L_s - M)^2 (\omega - \omega_0)^2 + R_s^2; \\
n_m &= (L_e - M)^2 (\omega - \omega_0)^2 + R_e^2; \\
d_1 &= (R_s R_e + L_s L_e \sigma \omega (\omega - \omega_0))^2; \\
d_2 &= (L_s R_e - L_e R_s (\omega - \omega_0))^2.
\end{align*}
\]

To facilitate computations, an analytic approximation of the magnetising characteristic curve obtained experimentally is used. By experimental measurement, the magnetising curve or relationship of the magnetising inductance \( M \) represented by the magnetising current \( I_m \) is given by
\[
M = f(I_m), \quad (13)
\]

3.2 Example analysis

The example is used to test the validation of the analytic formulas obtained from the steady-state conditions, and proves that both the limit cycle and steady-state operating point conditions are essential for the steady states.

A squirrel-cage three-phase induction machine is used as an example. Based on the stability analysis above, the paper may neglect non-linearity in the magnetic unsaturated region of the unloaded induction machine. However, the analytic formulas obtained from the steady-state conditions are applicable to not only some special form of the magnetising curve, but also to all forms of the magnetising curves.

The rated values of the experimental induction machine give \( P_N = 2.2 \text{ kW, } U_m = 380 \text{ V (Y-connection), } I_N = 5 \text{ A, } f_N = 50 \text{ Hz.} \)

The rated speed for the induction motor tested as a generator is \( n_{N} = 1430 \text{ rpm, the parameters are } R_s = 3.383 \Omega, R_e = 2.973 \Omega, L_s = 0.008479 \text{ H, } L_e = 0.008479 \text{ H, and the approximation of the magnetising curve is developed and verified by experimental measurements. Two regions are defined with five breakpoints as in (14).} \)

The minimum of breakpoints is the boundary of the magnetic saturated region, the lower of which is an approximately linear magnetising region with the magnetising inductance regarded constant.

Other four breakpoints divide the non-linear magnetising curve into four more sections. To facilitate analytical computation, this study uses an analytic approximation of the magnetising curve obtained experimentally. The approximation of the magnetising inductance curve was developed in [14] and verified experimentally.

The inductance for different current regions is defined as
\[
M = \begin{cases}
0.2875, & I_m < 1.163 \\
3.500 / (I_m + 11.01), & 1.163 < I_m < 2.162 \\
3.099 / (I_m + 9.503), & 2.162 < I_m < 3.046, \quad (14) \\
2.519 / (I_m + 7.152), & 3.046 < I_m < 4.553 \\
1.527 / (I_m + 2.544), & I_m < 4.553 \\
\end{cases}
\]

Assuming \( M=0.2875 \text{ H, the rotor speed is } \omega = 314 \text{ rad/s, it is then obtained by simultaneous formulas of (12) and (13) for the limit cycle condition that the steady-state capacitances and frequency are } C_{\text{min}} = 34.4 \mu \text{F, } C_{\text{max}} = 1.866 \text{ mF, } \omega_{\text{min}} = 313.6 \text{ rad/s, } \omega_{\text{max}} = 313.6 \text{ rad/s.} \)

The capacitances can produce four sets of solutions for the limit cycle condition, the computed results for the steady-state conditions are given in Table 1.

| Build-up steady-state points | \( C_{\text{min}} = 34.4 \mu \text{F} \) | \( C_{\text{max}} = 1.866 \text{ mF} \) |
|-----------------------------|-------------------------------|-------------------------------|
| \( \omega_{\text{min,max}} \) rad/s | \( \omega_{\text{min,max}} \) rad/s |
| 313.6 | 313.6 | 313.6 | 313.6 |
| 1319 | 1319 | 1319 | 1319 |

\( I_{\text{sc}}, \text{A} \) 1.1630 1.1630 1.1736 1.1736
\( I_{\text{sp}}, \text{A} \) 1.1630 1.1630 1.1736 1.1736
\( I_{\text{sc}}, \text{A} \) 1.1630 1.1630 1.1736 1.1736
\( U_{\text{sc}}, \text{V} \) 107.88 107.88 896.19 896.19

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Table 1 shows only the first set of analytic formulas just satisfies the steady-state condition, so the other sets can be neglected. Note that the magnetising current is adopted at the boundary of the magnetic saturated region, which depends on the initial conditions. The set of analytic formulas for the steady-state conditions of the unloaded SEIG with the dynamic magnetising is obtained and rotor speed is known.

With \( \omega = 314 \text{ rad/s} \), substitution of the generator parameters in (10) gives the minimum \( M_{\text{min}} = 0.03369 \) of the dynamic magnetising inductance. The amplitudes of steady-state capacitor currents, stator and rotor currents, and magnetising current \( I_{\text{sc}}, I_s, I_r \), and \( I_m \), respectively, computed by the first set of analytic formulas for the steady-state conditions with the capacitance are shown in Fig. 2. Both results computed by the analytic formulas of the stator and capacitor currents are exactly in the same range of the capacitance constrained by the steady-state conditions, (15) is the only solution of the steady-state conditions with given rotor speed.

With the rotor speed known, the analysis formulas for the steady-state conditions of the unloaded SEIG are given by

\[
\begin{align*}
I_{\text{sc}} &= \sqrt{(-C\omega U_{\text{m}})^2 + (C\omega U_{\text{m}})^2} = C\omega U_{\text{m}} \\
I_s &= \sqrt{I_{\text{m}}^2 + I_{\text{r}}^2} = U_{\text{m}}/\sqrt{n_s/(d_1 + d_2)} \\
I_r &= \sqrt{I_{\text{m}}^2 + I_{\text{r}}^2} = U_{\text{m}}/\sqrt{n_s/(d_1 + d_2)} \\
I_m &= U_{\text{m}}/\sqrt{n_s/(d_1 + d_2)}
\end{align*}
\]

where, \( h_{13}, h_{22}, \Delta, f_1, f_2, f_3, a, n_s, n_r, n_m, d_1 \) and \( d_2 \) are same as above. In the case of the initial condition \( X_0 \) is given and with two parameters of \( M, C, \omega, U_{\text{m}}, I_s, I_r \) and \( I_m \) is known; it is convenient to obtain all the other parameters using the analytic formulas (15) in theory. Note that they are more accurate in the magnetic saturated region. Generally, when the SEIG operates under safe normal conditions, all amplitudes of currents of the steady-state conditions are not too high. Therefore, amplitudes of the stator and magnetising currents are approximately equivalent.

### 3.3 Build-up steady-state torque and power factor

Since the rotor speed of the SEIG systems is assumed to be constant, the build-up steady-state torque of the systems can be analytically computed by use of (11) as follows:

\[
T_m = T_c = U_{\text{m}}^2 p_q g_1/(g_2 + g_3),
\]

where

\[
\begin{align*}
g_1 &= M^2 R_s(\omega - \omega) \\
g_2 &= L_s^2(\omega - \omega)^2(L_s^2\omega \sigma^2 + R_s^2) + R_s^2(L_s^2\omega_R^2 + R_s^2) \\
g_3 &= 2M^2 R_s R_m(\omega - \omega)
\end{align*}
\]

Furthermore, the build-up steady-state power factor \( \cos \theta_0 \) of the systems can be also analytically computed by use of (11) as follows:

\[
\cos \theta_0 = (P_a + P_r)/\sqrt{(P_a + P_r)^2 + (Q_a + Q_r - Q_c)^2}
\]

\[
= \frac{I_m^2 R_s + I_r^2 R_s}{\sqrt{(I_m^2 R_s + I_r^2 R_s)^2 + (I_m^2 R_m + I_r^2 R_m - I_m^2 R_s)^2}}
\]

where \( P_a \) and \( P_r \) represent the active power consumed by the stator and rotor windings, respectively; \( Q_a \) and \( Q_r \) represent the reactive power consumed by the stator and rotor windings, respectively; and \( Q_c \) represents the reactive power produced by the excited capacitors.

### 4 Build-up steady-state analysis for SEIGs

Figs. 3 and 4 illustrate the mechanical torque and the power factor of build-up steady states with capacitance under the conditions of the given rotor speed, respectively. To facilitate general comparison and analysis of comprehensive performances of the build-up steady-state processes, Figs. 3 and 4 provide all the computed results in three different operating occasions with rotor speed values of 282.6, 251.2 and 219.8 rad/s.

Fig. 3 shows the comparisons of the build-up steady-state torque with capacitance under three different rotor speeds. The generator never works until the capacitance is sufficient high, with its

Fig. 3 Mechanical torque of unloaded steady states with capacitance

Fig. 4 Power factor of unloaded steady states with capacitance
electromagnetic torque and equally mechanical torque zero ideally. Once capacitance is higher than the critical value depended on the rotor speed value, the build-up steady-state mechanical torque and its ratio to capacitance increases with the increasing capacitance. If rotor speed varies, the permitted capacitance of the generator thus varies theoretically. Also, the higher the rotor speed, the smaller the permitted various range of the capacitance, which is depended on the magnetic circuit saturation characteristics. These facts demonstrate that the mechanical torque and its ratio to capacitance increase with the increasing rotor speed for the same capacitance. The maximum of theoretical excited capacitances is solved by the simultaneous of analytic formulas of the critical magnetising inductance (10) and analytic formulas of the build-up steady-state operating points (15). In the matter of fact, the permitted magnetic circuit saturated grade of the generator normal operation is too limited to be over saturated. Therefore, the real allowable range of capacitance change is much smaller; it is usually <150 in this paper. In one word, the mechanical torque is directly related with the relative distance between build-up steady-state operating points and the equilibrium point of the system. The larger the distance, the more deep the saturated grade of the generator magnetic circuit, the smaller the build-up steady-state magnetising inductance, the larger the mechanical torque, and the higher the requirement for the drive system when the mechanical power is constant. Thus, these results can provide theoretical bases and technical references for the optimised control design for the wind-driven SEIG systems.

Fig. 4 shows the comparisons of the build-up steady-state power factor with capacitance under three various rotor speeds. When the capacitance is larger than the critical value the generator start working with the maximum power factor. For the same rotor speed, the power factor presents monotonically decreasing with the capacitance. For the same capacitance, the higher the rotor speed, the smaller the build-up steady-state power factor, and the faster the power factor decreases. These facts demonstrate that the build-up steady-state power factor change of the SEIG system with constant rotor speed characterises the change of its load carrying capacity, which monotonically increases with the increasing capacitance. For the same capacitance, the higher the build-up steady-state rotor speed of the SEIG system, the larger its load carrying capacity. For the build-up steady-state SEIG system with the same power factor, the higher the rotor speed, and the smaller the requirement for excited capacitance. Thus, further verifying the correctness of holistic performances analysis for the build-up steady-state SEIG in the view of system macroscopic energy, which provides researches of loaded steady-state power flow analysis and optimised control with the important theoretical foundation and method preparations.

5 Conclusion
Based on the transient equivalent circuits in the two-phase stationary reference frame, this paper carried a thorough analysis for the build-up steady states of wind-driven SEIG systems. Firstly, analytic formulas of the critical magnetising inductance and operating points of the build-up steady states were obtained through solving the equations of the non-linear dynamics model of SEIG systems. Then, the only set of analytic formulas for the build-up steady-state conditions was obtained through testing by example analysis, based on which the analytic formulas of the build-up steady-state mechanical torque and power factor, respectively. Finally, we performed detailed analysis on the system dynamic characteristics and build-up steady-state operating performances. The approach to the build-up steady states of the driven SEIG systems and the results concluded can provide not only reliable theoretical foundation for loaded steady-state operating performance and power flow analysis as well as corresponding optimised control research, but also technical references for optimised design for various renewable energy generation such as wind, small hydro etc. so as to improve the utilisation of renewable energies and the operating economy and reliability of new energy power systems.