Zero Cosmological Constant and Nonzero Dark Energy from Holographic Principle

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It is shown that the first law of thermodynamics and the holographic principle applied to a cosmic causal horizon demand the zero cosmological constant and non-zero dynamical dark energy in the form of the holographic dark energy. This dark energy has a parameter $d = 1$ and an equation of state $w_0 \simeq -0.903$ comparable to current observational data, if entropy of the horizon saturates the Bekenstein-Hawking bound.
Type Ia supernova (SN Ia) observations \[1, 2\], the Sloan Digital Sky Survey (SDSS) \[3, 6\] and cosmic microwave background observations \[7\] all indicate that the current universe is expanding at an accelerating rate. The expansion can be explained if there is a negative pressure fluid called dark energy of which pressure \(p_{DE}\) and energy density \(\rho_{DE}\) satisfy \(w_{DE} \equiv p_{DE}/\rho_{DE} < -1/3\). Being one of the most important unsolved puzzles in modern physics and cosmology, the dark energy problem consists of three sub-problems \[8\]: why it is so small, nonzero, and comparable to the critical density at the present. We also need to explain why the cosmological constant \(\Lambda\) is so small (if it is dark energy) or exactly zero. Solving this problem is not an easy task, because quantum field theory (QFT) predicts huge zero point energy that can play a role of \(\Lambda\). There are already many works on this problem \[9–11\], however, the problem seems to be far from a solution.

In this paper, it is suggested that if the holographic principle holds for a cosmic causal horizon, the cosmological constant should be exactly zero and there is holographic dark energy consistent with the recent observational data.

The holographic principle \[12\] is a conjecture claiming that all of the information in a region can be described by the physics at the boundary of the region and that the maximal number of degrees of freedom in the region is proportional to its surface area rather than the volume. More specifically, it was conjectured that the Bekenstein-Hawking Entropy

\[
S_{BH} = \frac{c^3 A}{4G}\hbar
\]

(1)
is the information bound that a region of space with a surface area \(A\) can contain \[13\].

Recently, based on the holographic principle Verlinde \[14\] and Padmanabhan \[15\] proposed a remarkable idea linking gravity to entropy, which brings out many follow-up studies \[16–27\]. Verlinde derived the Newton’s equation and the Einstein equation by assuming that energy inside a holographic screen is the equipartition energy \[14\], energy from Landauer’s principle associated with information loss at the horizon \[28\] or a kind of thermal energy corresponding to dark energy \[22\]. Here \(H = da/dt\) is the Hubble parameter with the scale factor \(a\). This dark energy, dubbed ‘quantum informational dark energy’ \[33\] or ‘entanglement dark energy’ \[28\] by the authors, is similar to the entropic dark energy based on the Verlinde’s idea \[34–37\]. It was also suggested that black hole mass and the Einstein equation itself can be derived from the relation \(dE_h = k_B T_h dS_h\), that has a quantum information theoretic origin \[29\]. Similarities between this theory and Verlinde’s theory were investigated in \[31, 38\].

In this paper, we assume that the holographic principle and the following definition of the horizon energy

\[
dE_h \equiv k_B T_h dS_h,\tag{2}
\]

hold for a cosmic causal horizon such as the cosmic event horizon or the apparent horizon. This energy could be the equipartition energy \[14\], energy from Landauer’s principle associated with information loss at the horizon \[28\] or simply the energy defined by the Clausius relation.

Inspired by the entropic \[14, 39\] or quantum information theoretic \[28, 31\] interpretation of gravity we take the holographic principle and the horizon energy in Eq. \[11\] as guiding principles for dark energy study. To do this let us first recall the cosmological constant problem. The (classical) time independent cosmological constant \(\Lambda_c\) appears in the gravity action as

\[
S = \int d^4 x \sqrt{-g} (R - 2\Lambda_c).\tag{3}
\]

Since the energy-momentum tensor \(T_{\mu\nu}\) for the vacuum fluctuation \(\langle T_{\mu\nu}\rangle\) is usually proportional to a spacetime metric parameter \(\langle T_{\mu\nu}\rangle\) has been regarded as a candidate for the cosmological constant and dark energy. To calculate its expectation value one usually integrates the zero point energy \(\hbar \omega/2\) for each mode of quantum fields in a flat spacetime. Thus, the energy density of the quantum vacuum is approximately given by

\[
\rho_q = \langle T_{00}\rangle \sim \int_{k_U}^{k_{UV}} \hbar \omega dk \sim k_U^4,\tag{4}
\]

where \(k_U \sim M_P\) is a UV-cutoff and \(k_I \sim 1/r\) is an IR-cutoff, and \(M_P = \sqrt{\hbar c/8\pi G}\) is the reduced Planck mass. Unfortunately, as is well known, for \(k_U \sim M_P\), the estimation gives \(\rho_q \sim M_P^4 \sim 10^{109}\text{eV}^4\) which is too large to explain the observed dark energy density \(\rho_{DE} \sim 10^{-12}\text{eV}^4\). On the other hand, if we subtract this zero point energy to calculate the renormalized vacuum energy for the universe, we usually obtain \(\rho_q \sim H^4\), which is too small compared to the observed dark energy.
It is often argued that after taking the vacuum expectation of quantum fields, the Friedmann equation

$$H^2 = \frac{8\pi G \rho_m}{3} - \frac{k_c c^2}{R} + \frac{\Lambda c^2}{3},$$

gets an additional constant contribution \(\Lambda_q = \rho_q/M_P^2 c^2 = (T_{00})/M_P^2 c^2\) from the vacuum quantum fluctuation \(\rho_q\) in Eq. (4). (Here, \(k_c\) is the spatial curvature parameter, which we will set zero for simplicity, and \(\rho_m\) is the matter energy density.) Thus, the total cosmological constant is \(\Lambda = \Lambda_c + \Lambda_q\), and the total vacuum energy density is given by

$$\rho_{\text{vac}} = M_P^2 c^2 (\Lambda_c + \Lambda_q).$$

Without a fine tuning it seems to be almost impossible for two terms to cancel each other to result in the tiny observed upper bound for the cosmological constant. This is the essence of the cosmological constant problem.

Then, in the context of QFT, from where could horizon energy \(\rho_h\) arise? Recall that \(\rho_q\) in Eq. (4) was estimated in a flat spacetime. On the other hand, for a curved spacetime, after a Bogoliubov transformation there appear excited states in addition to the vacuum. The normal-ordered quantum vacuum energy (i.e., with the subtraction of the zero point energy) in a curved spacetime with the UV and the IR cutoffs of ten has a term in the form of \(\Lambda_q\), which we will set zero for simplicity, and \(\rho_m\) is the matter energy density. Thus, the total cosmological constant is \(\Lambda = \Lambda_c + \Lambda_q\), and the total vacuum energy density is given by

$$\rho_{\text{vac}} = M_P^2 c^2 (\Lambda_c + \Lambda_q).$$

Alternatively, we can take not the bulk QFT but the holographic principle as a postulate and describe the bulk physics using only the DOF on the horizon. In this holographic context, to estimate the bulk energy density we can treat the quantum fields on the horizon as a collection of oscillators on the spherical surface with a lattice constant of order \(O(k_U^{-1})\). Then, to obtain \(\rho_h\) we have to sum the zero-point energy of the oscillators with frequency \(\omega\) on the horizon surface rather than those in the bulk. Then, this estimation results in the HDE density, because

$$\rho_h \sim \Sigma \frac{\hbar \omega}{\text{volume}} \sim \Sigma \frac{\hbar \omega}{r^3} \sim \left(\frac{r}{k_U}\right)^2 \frac{\hbar \omega}{r^3} \sim \frac{M_P^2}{r^2} \sim M_P^2 H^2,$$

where \(\Sigma\) represents a summation over the horizon oscillators with the temperature \(T_h\), and the number of oscillators are proportional to the horizon area \(\sim (r/k_U^{-1})^2\). At the last step we used the equipartition approximation \(\hbar \omega \sim k_B T_h \sim 1/r\).

This result indicates that the bulk QFT overestimates the independent DOF in the bulk and the true vacuum energy of the bulk is equal to the zero point energy of the boundary DOF, which is of order of the normal-ordered bulk vacuum.
energy in the conventional QFT. What gives the small HDE is the smallness of the number of independent DOF in the bulk. This redundancy of the bulk DOF can explain why we cannot obtain the correct dark energy density by simply calculating the zero point energy of the bulk.

Similarly, one can also directly calculate the horizon energy $E_h$ as the vacuum energy of the universe without using QFT. Let us consider a causal cosmic horizon with a radius $r$, having generic holographic entropy

$$S_h = \frac{\eta c^2 \pi^2}{G h},$$

and temperature

$$T_h = \frac{\epsilon h c}{k_B r},$$

with constants $\eta$ and $\epsilon$ (See Fig. 1). For the Bekenstein entropy $\eta = \pi$, and the Hawking-Gibbons temperature $\epsilon = 1/2\pi$. By integrating $dE_h$ on the isothermal surface $\Sigma$ of the causal horizon with Eqs. (9) and (10), we obtain the horizon energy

$$E_h = \int_{\Sigma} dE_h = k_B T_h \int_{\Sigma} dS_h = \frac{\eta c^4 r}{G}.$$  

Then, the energy density due to $E_h$ is given by

$$\rho_h = \frac{3E_h}{4\pi r^3} = \frac{6\eta c^3 M_p^2}{\hbar r^2} = \frac{3d^2 c^3 M_p^2}{\hbar r^2},$$

which has the form of the holographic dark energy [41]. This $\rho_h$ seems to correspond to the estimation in Eq. (8). This kind of dark energy was also derived in terms of entanglement energy [28] and quantum entanglement force [38]. From the above equation we immediately obtain a formula for the constant

$$d = \sqrt{2\eta \rho},$$

that is the important parameter determining the nature of HDE. If $S_h$ saturates the Bekenstein bound and $T_h$ is the Hawking-Gibbons temperature $h c/2\pi k_B r$, then $\eta = 1/2$ and $d = 1$. Thus, the holographic principle applied to a cosmological horizon naturally leads to HDE with $d = 1$ [38], which is favored by observations and theories [12] [43].

Let us turn to the cosmological constant problem. From the holographic viewpoint, it is simple to see why the cosmological constant $\Lambda$ should be zero. If we accept the holographic principle applied to the cosmic horizon and the definition of the horizon energy (Eq. (11)), the bulk vacuum energy density $\rho_{vac}$ in Eq. (6) should be smaller than the horizon energy density $\rho_h$ in Eq. (12). One may assume that the holographic principle holds only up to some large radius $r_c$, in this case from Eq. (9) and Eq. (12) this radius should be $r_c = \sqrt{\frac{2\eta \rho}{\Lambda}}$, where we have set $h = 1 = c$. However, there is no obvious reason that the holographic principle should be broken at large $r$. If the holographic principle holds for arbitrary large $r$, the vacuum energy $E_\Lambda$ proportional to $\Lambda r^3$ is problematic. It clearly violates the holographic principle for large $r$, because we know that, according to the principle and the first law of thermodynamics with $T_h \propto 1/r$, the total horizon energy $E_h$ is proportional to $r$ from $\rho_h \sim 1/r^2$. For $r > O(r_c)$, $E_\Lambda > E_h$ and the holographic principle can be violated. Thus, the principle holds true for arbitrary $r$ only if the cosmological constant $\Lambda$ is zero. Thus, we can say that the holographic principle demands that the cosmological constant is zero (i.e., $\omega_{DE} \neq -1$). (We excluded an implausible case that $\Lambda_c$ and a constant part of the quantum contribution miraculously cancel each other to result in $\rho_{vac} \sim 1/r^2$.)

Note that this solution to the cosmological constant problem is more than a simple transformation of one problem into another form, because the formalism we used here is based not on the conventional QFT but on the holographic principle that allows a reformulation of gravity and QFT as recently suggested. Like Jacobson’s model or Verlinde’s model, our approach takes the holographic principle and thermodynamic relations as starting postulates and there is possibility that these approaches could open a new route to a unification of gravity and quantum mechanics.

Furthermore, the solution suggested in this paper can overcome some difficulties often encountered by conventional approaches based on infrared or ultraviolet modifications of gravity, adjusting initial conditions, or dynamical attractor mechanisms (See [47] for example.). They failed to explain both of the early small universe and current large universe and why the QFT vacuum loops or cosmological phase transitions did not curl up the universe. Let us discuss these facts in detail.

First, our approach based on the holographic principle suggests that the energy density from the vacuum loop energy for a quantum field with a UV cutoff energy scale $M$ is $\rho_M = O(M^2 H^2) \ll O(M_p^2 H^2)$ not of $O(M^4)$. Since the
Friedmann equation is $\rho_{\text{tot}} = 3H^2M_p^2$, the vacuum loop energy is not a dominant contribution to $\rho_{\text{tot}}$ unless $M \simeq M_P$. Second, our approach could also avoid the issue related to the cosmological phase transitions. For example, consider a phase transition of the scalar field $\phi$ with a thermal effective potential $V(\phi, T)$, showing the transition at the temperature $T = T_c$. Then, in conventional approaches even if $\rho_\Lambda$ was set to be 0 before the transition, during the phase transition the potential could generate temporally the energy difference between the false vacuum and the true vacuum, which is of $-O(T_c^4) = -O(M_p^2H^2)$, where $H$ is the Hubble parameter at the transition. This energy difference could act as a negative cosmological constant and make the universe rapidly collapse. However, in our theory there is always positive $O(M_p^2H^2)$ dark energy that could cancel the negative energy term and prevent the collapse. Therefore, in our theory, the zero cosmological constant does not make a problem during the cosmological evolution. Third, unlike the dynamical attractor theories, our theory does not directly rely on the contributions of collapse. Therefore, in our theory, the zero cosmological constant does not make a problem during the cosmological evolution.

Following [38] and [37] one can obtain an entropic force for the dark energy

$$F_h \equiv \frac{dE_h}{dr} = \frac{c^4\eta e}{G},$$

which could be also identified as a ‘quantum entanglement force’ as in [38], if $S_h$ is the entanglement entropy.

From $\rho_{\text{DE}} = \rho_h$ and a cosmological energy-momentum conservation equation, one can obtain an effective dark energy pressure [41] in the bulk

$$p_{\text{DE}} = \frac{d(a^3\rho_h(r))}{-3a^2da},$$

from which one can derive the equation of state.

![Graph](image)

FIG. 2. (Color online) Theoretical evolution of the dark energy equation of state (the blue thick line) $w_{\text{DE}}$ versus the redshift $z$ compared to the observational constraints (Data extracted from Fig. 2 in [42]). The green thin line represents the best fit. The dashed lines and the dotted lines shows 1σ and 2σ errors, respectively.

To compare predictions of our theory with current observational data, at this point we need to choose a horizon among various cosmological horizons such as an apparent horizon, a Hubble horizon, and a future event horizon. If there is no interaction term, only the event horizon can result in the accelerating universe [41]. Thus, from now on we assume the simplest case that the causal horizon is the cosmic event horizon. For this case one can find the equation of state for holographic dark energy as a function of the redshift $z$ as in Ref. [41]:

$$\omega_{\text{DE}} = \left(1 + \frac{2\sqrt{\Omega_\Lambda}}{d}\right) \left(1 + z\sqrt{\Omega_\Lambda}(1 - \Omega_\Lambda)\right),$$

where the current dark energy density parameter $\Omega_\Lambda \simeq 0.73$. Fig. 1 compares this prediction with $d = 1$ to the observational data obtained from the 182 gold SN Ia data, the baryon acoustic oscillation, SDSS, and the 3-year Wilkinson Microwave Anisotropy Probe (WMAP) data. One can also find that the equation of state [3, 41]

$$w_0 = -\frac{1}{3} - \frac{2\sqrt{\Omega_\Lambda}}{d},$$

and its change rate at the present $w_1$ with $w_{\text{DE}}(a) \simeq w_0 + w_1(1 - a)$.

For $d = 1$ these equations give $w_0 = -0.903$ and $w_1 = 0.104$. According to WMAP 5-year data [49], $w_0 = -1.04 \pm 0.13$ and $w_1 = 0.11 \pm 0.7$. WMAP 7-year data with the baryon acoustic oscillation, SN Ia, and the Hubble constant yields $w_0 = -0.93 \pm 0.13$ and $w_1 = -0.41 \pm 0.71$ [2].
If we use an entanglement entropy calculated in \[38\] for \(S_h\), one can obtain \(d\) slightly different from 1. It is also straightforward to study the cases with other horizons such as apparent horizons or Hubble horizons. We saw that the predictions of our theory well agree with the recent observational data. Note that although the cosmological constant is most favored by the cosmological observations, the observational data still allow dynamical dark energy models.

It was also shown that holographic dark energy models with an inflation with a number of e-folds \(N_e \approx 65\) can solve the cosmic coincidence problem \[41\, 50\] thanks to a rapid expansion of the event horizon during the inflation.

I summarize how the holographic principle and the horizon energy can solve the dark energy problem. In this theory the dark energy density is small due to the holographic principle, comparable to the critical density due to \(O(1/H)\) horizon size, and non-zero due to the quantum fluctuation. The holographic principle and the first law of thermodynamics also demand that the cosmological constant is zero, because the nonzero time independent cosmological constant is inconsistent with them.

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