Comment on B.S. Davids et al.;
Proton-Decaying States in $^{22}$Mg and the Nucleosynthesis of $^{22}$Na in Novae

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B.S. Davids et al. make the explicit assumption that radiative widths of analog states in $^{22}$Ne and $^{22}$Mg are equal. We demonstrate that this misapplication of iso-spin symmetry leads to very wrong results. Considerations of elementary nuclear structure suggests that such an assumption can be inaccurate by a large factor (in $^{22}$Mg), as is evident from a comparison with recent measurements of radiative width in $^{22}$Mg. Estimates of radiative widths from analog transitions are common but often wrong (e.g. in $^{22}$Mg) and should not be considered a useful tool in nuclear astrophysics.

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The (ab)use of isospin symmetry in Nuclear Astrophysics to estimate radiative widths is common and in this comment we consider one such (extreme) case. B.S. Davids et al. [1] state that for the purpose of estimating the resonance strengths in $^{22}$Mg "we have assumed that they have the same radiative widths as their analog states in $^{22}$Ne". Unfortunately similar statements already appeared in the literature and for example, previously B.S. Davids et al. stated [2]: "In some cases, widths of the analog state from the mirror nucleus $^{19}$F have been measured, and we adopt these under the assumption that $\Gamma_{\gamma}(^{19}$Ne) = $\Gamma_{\gamma}(^{19}$F)". And once again B.S. Davids et al. stated [3]: "Measurements of $\Gamma_{\gamma}$ for analog states in the mirror nucleus $^{19}$F can be found for four of the states and these have been adopted under the assumption that $\Gamma_{\gamma}(^{19}$Ne) = $\Gamma_{\gamma}(^{19}$F)". Since radiative widths are essential for nucleosynthesis in astrophysical environments, we chose to examine this assumption in detail. We demonstrate that in most cases the assumption of the equality of radiative widths of analog states has no physical justification, and most certainly in the case discussed in $^{22}$Mg.

The total radiative width ($\Gamma_{\gamma}$) depends on the reduced widths [$B(E2)$ or $B(M1)$] as well as the phase space available for electromagnetic decays; i.e. the various states and the corresponding transitions available for decay with specific energies. Iso-spin symmetry does not imply that the total radiative width ($\Gamma_{\gamma}$) of analog states are equal, as assumed by B.S. Davids et al. [1][2][3]. Rather it only makes predictions of the reduced widths. Thus one must compare only the reduced widths for analog transitions and not the total radiative widths of analog states.

This point is most clear for analog states in self conjugate nuclei, where additional phase space is available via allowed $\Delta T = 1$ electromagnetic transitions. Indeed in many cases and most certainly in $^{22}$Mg we encounter electromagnetic decay with substantially different energies and thus very different phase space that leads to very different radiative widths of the analog states even if the reduced widths are equal.

Moreover, even in the limit where iso-spin is an exact symmetry, in general the reduced widths [$B(E2)$ or $B(M1)$] of only $\Delta T = \pm 1$ analog transitions are predicted to be equal (Rule 2 of Ref. [4]). We emphasize that only for E1 transitions the analog transitions are predicted to be equal also for $\Delta T = 0$ transitions (Rule 3 of Ref. [4]). And only strong or above average strength magnetic dipole (M1) transitions are expected to be approximately equal also for $\Delta T = 0$ (Rule 5 of Ref. [4]). All too often and most certainly in $^{22}$Mg we deal with $\Delta T = 0$ E2 transitions or weak $\Delta T = 0$ M1 transitions and for such transitions isospin symmetry does not predict equality of the reduced widths of analog transitions.

Isospin breaking Coulomb interaction plays a major role in "violating" these rules [4]. For example, the energy of analog transitions can be affected by a Coulomb (iso-spin breaking) correction which is significant for quasi bound states (the Thomas-Ehren shift [6]). Indeed it was shown that such a Coulomb shift for the $1/2^+ \rightarrow 1/2^-$ analog transition in $^{13}$N amounts to a large fraction (25%) of the transition energy. Barker [7] evaluated the Coulomb correction for the quasi bound state in $^{13}$N, to yield a $B(E1)$ which is more than a factor of 3 different than its analog transition in $^{13}$C (involving bound states). The $\Delta T = 0$ $B(E1)$ in the $^{13}$C $-^{13}$N iso-doublet are measured to be large fraction of the single particle Weisskopf Unit estimate, yet even for these strong analog transitions we observe large variations of the reduced widths.

In the case of weak transitions (as compared to single
particle Weisskopf Unit estimate), the situation becomes even more confused. As pointed out by John Millener 3, for all multipolarities (except the purely isovector E1), the matrix elements contain both isoscalar and isovector contributions so that the reduced transition probabilities are of the form $(m_0 + m_1)^2$ and $(m_0 - m_1)^2$ for the mirror transitions. For strong E2 transitions, $m_0$ dominates. For strong M1 transitions, $m_1$ dominates. For weaker transitions $m_0$ and $m_1$ are quite possibly comparable in magnitude, in which case the reduced transition probabilities could be very different for the mirror transitions. In addition cancellation are common for weak transitions which makes the theoretical situation very confusing, most notably for weak E1 transition. Estimates of the reduced widths of weak transitions may differ by several orders of magnitudes, since theory is not able to fine tune cancellation or the prediction of weak (E1) transitions.

High lying low spin states (e.g. high lying $0^+, 2^+$ states) that are most relevant for stellar burning, are well known to have complicated nuclear structure with admixture from several configurations. As such weak transitions originating from high lying low spin states are particularly hard to predict.

At last we note that bound states (e.g. the $0^+_2$ in $^{22}\text{Ne}$) and quasi bound states (e.g. the $0^+_1$ in $^{22}\text{Mg}$) are expected 4, 7 to have wave function with different radial dependence which is expected to result considerable alteration of the width of mainly electric (but not magnetic) dipole transitions 4, 7.

Based on these general comments one may not assume that the reduced widths of the analog transitions, and most certainly not the total radiative widths of analog states in $^{22}\text{Ne}$ and $^{22}\text{Mg}$ are equal, in sharp contrast to the assumption made by B.S. Davids et al. 4.

As an explicit example we now consider the assumption 4, 7, that the radiative width of the $0^+_2$ state at 5.962 MeV and the tentative $1^-$ states at 6.046 MeV in $^{22}\text{Mg}$ are equal to the radiative width of the analog $0^+_2$ state at 6.235 MeV and the analog $1^-$ state at 6.689 MeV in $^{22}\text{Ne}$. B.S. Davids et al. 4 use the measured radiative widths of these states in $^{22}\text{Ne}$ 5, 8, 9 to deduce the radiative widths in $^{22}\text{Mg}$.

We first note that no measured E1 transitions are involved in the decay of the $0^+_2$ in $^{22}\text{Ne}$, hence even in the limit where iso-spin is an exact symmetry we do not expect the analog transitions in $^{22}\text{Mg}$ to be equal as these are manifestly $\Delta T = 0$ transitions. Using the same data we extract in $^{22}\text{Ne}$ 5, 8, 9 $B(M1^+; 1^+ \rightarrow 0^+_2) = 0.04$ Wu. We conservatively estimate that the non-observed transition $B(E2^+; 2^+ \rightarrow 0^+_2)$ could be as small as 0.001 Wu. This small B(E2) observed in $^{22}\text{Ne}$ may arise from cancellations or a structure that is different than the ground state band in $^{22}\text{Ne}$. In any case the analog E2 transition in $^{22}\text{Mg}$ can not be assumed to have the exact same strength and be as small.

If on the other hand one makes the reasonable assumption that the analog transition $B(E2^+; 2^+ \rightarrow 0^+_2)$ in $^{22}\text{Mg}$ is of average strength of 1 Wu with an upper limit of 10 Wu. (as one may deduce from measured $B(E2)$s for the $0^+_3$ in $^{18}\text{O}, ^{19}\text{Ne}, ^{20}\text{Ne}$ 10), it will be the dominant electromagnetic decay mode with the largest electromagnetic branching ratio, due to its large transition energy. Such a transition will determine the total radiative width of the $0^+_2$ state in $^{22}\text{Mg}$ to be very different than its analog $0^+_2$ in $^{22}\text{Ne}$. In such a case the estimated $B(E2)$ will differ by a factor of 1000 with an upper limit of 10,000.

Preliminary estimate of the $B(E2^+; 2^+ \rightarrow 0^+_2)$ in $^{22}\text{Mg}$ 13, with the hint that this transition is indeed the dominant decay of the $0^+_2$ in $^{22}\text{Mg}$ 13, yields approximately 0.3 Wu., and in this case B.S. Davids is in fact found to be wrong by a factor of approximately 300.

B.S. Davids et al. assume the analog of the 6.046 MeV state is the $1^-$ at 6.689 MeV in $^{22}\text{Ne}$, but it is quite possible that its analog is the $3^-$ state at 5.911 MeV in $^{22}\text{Ne}$. In either case we observe for these states in $^{22}\text{Ne}$ very weak B(E1) transitions ranging between 0.008 and 0.0002 Wu. For such weak E1s isospin symmetry can not guarantee the equality of the reduced widths of the analog transition in $^{22}\text{Mg}$.

The cancellation discussed in this paper is reminiscent of the text book case of cancellation that leads to the anomalous long beta-decay lifetime of $^{14}\text{C}$ with log ft = 9.04 11. This small beta-decay matrix element is a million times retarded as compared to similar Gamow-Teller transitions in light nuclei with an average log ft = 3.5 12. The beta-decay matrix element in $^{14}\text{C}$ is a factor of 100 smaller than its analog in $^{14}\text{O}$ (log ft = 7.2), which emphasizes the fine tuning of matrix elements of analog states that involve cancellation.

Recent measurement of radiative widths in $^{22}\text{Mg}$ 13 vividly illustrate how non useful are these very crude and mostly wrong guesses of radiative widths based on misapplication of isospin symmetry 1, 2, 3. The measured central value of $\omega\gamma$ for the 5.962 MeV state differs by a factor of 4.3 and for the 6.046 MeV state it differs by 16.4. A comparison of Fig. 4 of B.S. Davids et al. 4 and Fig. 15 of Ref. 13 demonstrates that the burning rates at higher temperatures are very different than proposed or would be calculated based on the crude guesses of B.S. Davids et al.. Specifically the contribution of the 6.046 MeV state was considered by B.S. Davids et al. to be negligible and it was not included in the plot of contributions from “most important states” 4. In fact this state contributes considerably more than the 5.962 MeV state shown in Fig. 4 4, and at high temperature its contribution competes with that of the 5.714 MeV that was considered by B.S. Davids et al. to be the only significant state. We also note that B.S. Davids restricted their calculation of nuclear burning to lower temperatures. Figure 15 of Ref. 13 is the correct one as it includes measured values of $\omega\gamma$ for states in $^{22}\text{Mg}$, including the contribution of the 6.046 MeV state. This figure also includes the entire temperature range of interest for novae and x-ray bursts, unlike Fig. 4 of B.S. Davids et al. 4.
The measurement of resonance strengths in $^{22}\text{Mg}$ underlines the "danger" of (ab)using isospin symmetry to extract electromagnetic properties of analog transitions. Such practices are all too common in the field of Nuclear Astrophysics. The case discussed in $^{22}\text{Mg}$ leads to conclusions that are off by large factors, large enough to make a difference even in Astrophysics.

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