Gravitino condensate in $N=1$ supergravity coupled to the $N=1$ supersymmetric Born-Infeld theory

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Abstract

The $N=1$ supersymmetric Born-Infeld theory coupled to $N=1$ supergravity in four spacetime dimensions is studied in the presence of a cosmological term with spontaneous supersymmetry breaking. The consistency is achieved by compensating a negative contribution to the cosmological term from the Born-Infeld theory by a positive contribution originating from the gravitino condensate. This leads to an identification of the Born-Infeld scale with the supersymmetry breaking scale. The dynamical formation of the gravitino condensate in supergravity is reconsidered and the induced one-loop effective potential is derived. Slow roll cosmological inflation with the gravitino condensate as the inflaton (near the maximum of the effective potential) is viable against the Planck 2018 data and can lead to the inflationary (Hubble) scale as high as $10^{12}$ GeV. Uplifting the Minkowski vacuum (after inflation) to a de Sitter vacuum (dark energy) is possible by the use of the alternative Fayet-Iliopoulos term. Some major physical consequences of our scenario to reheating are briefly discussed also.
1 Introduction

The gravitino condensate and the gravitino mass gap in $N=1$ supergravity [1] coupled to the Volkov-Akulov field [2] in four spacetime dimensions arise as the one-loop effect due to the quartic gravitino interaction coming from the gravitino contribution to the spacetime (con)torsion [3, 4]. This is similar to the Nambu-Jona-Lasinio model [5] of the dynamical generation of electron mass and the formation of Cooper pairs near the Fermi surface in superconductivity. The dynamical gravitino mass also leads to a positive contribution to the vacuum energy and, hence, the dynamical supersymmetry breaking too [6]. Given the standard (reduced) Planck mass as the only (dimensional) coupling constant, the gravitino mass gap should be of the order of Planck scale also, which prevents phenomenological applications of the gravitino condensate to physics under the Planck scale.

However, the effective scale of quantum gravity may be considerably lower than its standard value associated with the (reduced) Planck mass $M_{Pl} = 1/\sqrt{8\pi G_N} \approx 2.4 \times 10^{18}$ GeV. It may happen because the effective strength of gravity can depend upon either large or warped extra dimensions in braneworld, or the dilaton expectation value in string theory, or both these factors together [7, 8, 9]. The negative results of the Large Hadron Collider (LHC) searches for copious production of black holes imply that the low-scale gravity models may have to be replaced by the high-scale gravity (or supergravity) models, whose effective Planck scale $\tilde{M}_{Pl}$ is much higher than the TeV scale but is still under the standard scale $M_{Pl}$, i.e.

$$1 \text{ TeV} \ll \tilde{M}_{Pl} \ll M_{Pl} \ .$$

This can be of particular importance to the early Universe cosmology, where the Newtonian limit does not apply, as well as for high-energy particle physics well above the electro-weak scale.

In this scenario, supergravity may play the crucial role in the description of cosmological inflation, reheating, dark energy and dark matter, see e.g., Ref. [11] and the references therein. For instance, it is unknown which physical degrees of freedom were present during inflation, while supergravity may be the answer. Describing inflation and a positive cosmological constant (dark energy) in supergravity is non-trivial, especially when one insists on the minimalistic hidden sector. Inflation is driven by positive energy so that it breaks supersymmetry (SUSY) spontaneously. As a (model-independent) consequence, goldstino should be present during inflation in supergravity cosmology. The goldstino effective action is universal and is given by the Akulov-Volkov (AV) action up to field redefinition [12, 13]. As was demonstrated in Refs. [14, 15], the viable description of inflation and dark energy in supergravity can be achieved by employing an $N=1$ vector multiplet with its $N=1$ supersymmetric Born-Infeld (BI) action [16] in the presence of the alternative Fayet-Iliopoulos (FI) term [17, 18, 19, 20, 21] without gauging the R-symmetry. \(^2\)

In this paper we also employ an $N=1$ vector multiplet with its $N=1$ supersymmetric BI action that automatically contains the goldstino (AV) action, but we choose the gravitino condensate as the inflaton. A dynamical SUSY breaking is achieved at the very high scale with the vanishing cosmological constant. The extra (FI) mechanism of spontaneous SUSY breaking is then used to uplift a Minkowski vacuum to a de Sitter (dS) vacuum.

The BI theory has solid motivation. It is expected that Maxwell electrodynamics does not remain unchanged up to the Planck scale, because of its internal problems related to the Coulomb

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\(^1\)The effective Planck scale may also be dynamically generated [10].

\(^2\)In Refs. [22, 23, 24], the $N=1$ massive vector multiplet, unifying Starobinsky inflaton (scalaron) [25] and goldstino (photino) was used together with the BI action, the FI term, the chiral (Polonyi) multiplet representing the hidden SUSY breaking sector, and the massive gravitino as the Lightest SUSY Particle (LSP) for dark matter.
singularity and the unlimited values of electro-magnetic field. This motivated Born and Infeld [26] to propose the non-linear vacuum electrodynamics with the Lagrangian (in flat spacetime)

\[ \mathcal{L}_{\text{BI}} = -M_{\text{BI}}^4 \sqrt{-\det (\eta_{\mu\nu} + M_{\text{BI}}^{-2} F_{\mu\nu})} = -M_{\text{BI}}^4 - \frac{1}{4} F^2 + \mathcal{O}(F^4) , \tag{2} \]

where \( \eta_{\mu\nu} \) is Minkowski metric, \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and \( F^2 = F_{\mu\nu} F^{\mu\nu} \). The constant term on the right-hand-side of Eq. (2) can be ignored in flat spacetime. The BI theory has the new scale \( M_{\text{BI}} \) whose value cannot exceed the GUT scale where electro-magnetic interactions merge with strong and weak interactions. On the other hand, we need \( M_{\text{BI}} < M_{\text{Pl}} \) in order to ignore quantum gravity corrections.

The BI theory naturally emerges (i) in the bosonic part of the open superstring effective action [27], (ii) as part of Dirac-Born-Infeld (DBI) effective action of a D3-brane [28], and (iii) as part of Maxwell-Goldstone action describing partial supersymmetry breaking of \( N = 2 \) supersymmetry to \( N = 1 \) supersymmetry [29, 30]. The peculiar non-linear structure of the BI theory is responsible for its electric-magnetic (Dirac) self-duality, taming the Coulomb self-energy of a point-like electric charge, and causal wave propagation (no shock waves and no superluminal propagation) — see e.g., Refs. [39, 40] and the references therein for a review and non-abelian extensions of BI theory. All that adds more reasons for the use of the BI structure.

In a curved spacetime with metric \( g_{\mu\nu} \), the BI action is usually defined as the difference between two spacetime densities,

\[ S_{\text{BI,standard}} = M_{\text{BI}}^4 \int d^4 x \left[ \sqrt{-\det (g_{\mu\nu})} - \sqrt{-\det (g_{\mu\nu} + M_{\text{BI}}^{-2} F_{\mu\nu})} \right] , \tag{3} \]

where the first term has been added ”by hand” in order to eliminate the cosmological constant arising from the second term and in Eq. (2). In this paper we propose the gravitino condensation as the origin and the mechanism of such cancellation in the supergravity extension of the BI theory with spontaneously broken SUSY.

The \( N = 1 \) (rigid) supersymmetric extension of BI theory is also self-dual [41]. The supersymmetric BI theory coupled to \( N = 1 \) supergravity (i.e., the locally supersymmetric extension of Eq. (3)) was constructed in Ref. [42].

Our paper is organized as follows. In Sec. 2 we provide more details about our motivation and briefly consider a supersymmetrization of the BI theory (2) instead of (3) in supergravity, focusing on the presence of the cosmological constant, and relate the BI scale to the spontaneous SUSY breaking scale. The comments collected in Sec. 2 justify consistency of the whole approach in this paper. In Sec. 3 we study the dynamical gravitino condensate arising from the one-loop effective action of pure supergravity, and investigate the induced scalar potential. Slow roll inflation with the gravitino condensate playing the role of inflaton is numerically studied in Sec. 4. Uplifting the Minkowski vacuum to a de Sitter vacuum by the use of the alternative FI term is proposed in Sec. 5. Our conclusion is Sec. 6. We use the supergravity notation of Ref. [1].

## 2 Spontaneous SUSY breaking, AV and BI actions, and their coupling to supergravity

We recall that the AV Lagrangian in flat spacetime is given by [2]

\[ \mathcal{L}_{\text{AV}} = -M_{\text{susy}}^4 \det \left( \delta^\mu_{\nu} + \frac{i}{2 M_{\text{susy}}^4} \tilde{\lambda} \gamma^a \partial_b \lambda \right) = -M_{\text{susy}}^4 - \frac{i}{2} \tilde{\lambda} \gamma^a \partial_a \lambda + \mathcal{O}(\lambda^4) , \tag{4} \]

\[ \text{See Refs. [31, 32, 33, 34] for the extensions of BI theory to extended supersymmetry and higher dimensions.} \]
where \( \lambda(x) \) is a Majorana fermion field of spin 1/2. This fermionic field is called the goldstino because the AV action has the spontaneously broken non-linearly realized rigid SUSY under the transformations
\[
\delta \lambda = M^2_{\text{susy}} \varepsilon + \frac{i}{M^2_{\text{susy}}} (\varepsilon \gamma^a \lambda) \partial_a \lambda
\]
with the infinitesimal Majorana spinor parameter \( \varepsilon \), so that the goldstino is a Nambu-Goldstone fermion indeed. The AV theory (4) has the cosmological constant
\[
M^4_{\text{susy}}\,\text{ where } M_{\text{susy}} \text{ is the spontaneous SUSY breaking scale. A coupling of the AV action to supergravity is supposed to generate a gravitino mass via the so-called super-Higgs effect [1] when the gravitino "eats up" the goldstino and thus gets the right number of the physical degrees of freedom.}

However, there is a problem: which SUSY is going to be preserved in coupling the AV action having the rigid non-linearly realized SUSY to the supergravity action (with auxiliary fields or in superspace) having the linearly realized local SUSY? We opt for the second option because we also want to allow other couplings of the AV-extended supergravity theory to supersymmetric matter (e.g., needed for reheating) such as the Minimal SUSY Standard Model (MSSM). But it is clearly impossible to couple the AV action to supergravity in the manifestly supersymmetric way (i.e with the linearly realized SUSY) when using standard supermultiplets or unconstrained superfields because of the mismatch in the numbers of the bosonic and fermionic physical degrees of freedom.

4 We propose to embed the goldstino into a standard massless vector supermultiplet, i.e. identify the goldstino with the photino.

Then the next question arises: which action should we employ for this vector multiplet? As was already argued in Sect. 1, we take the \( N = 1 \) supersymmetric BI action because it is well motivated at very high energies and includes the goldstino AV action up to a field redefinition [12, 13].

The manifestly supersymmetric extension of the BI action (3) minimally coupled to supergravity in curved superspace of the (old-minimal) supergravity (in a superconformal gauge) with the vanishing cosmological constant is given by
\[
S_{\text{SBI}}[V] = \frac{1}{4} \left( \int d^4x d^2\theta \mathcal{E} W^2 + \text{h.c.} \right) + \frac{1}{4} M_{\text{BI}}^{-4} \int d^4x d^2\theta d^2\bar{\theta} E \frac{W^2 \bar{W}^2}{1 + \frac{1}{2} A + \sqrt{1 + A + \frac{1}{2} B^2}},
\]
where \( E \) is the chiral (curved) superspace density, \( E \) is the full (curved) superspace density, \( \mathcal{D}^a \) are the covariant spinor derivatives in superspace, \( W^a \) is the chiral gauge-invariant field strength,
\[
W_a = -\frac{1}{4} (\mathcal{D}^2 - 4 \mathcal{R}) \mathcal{D}_a V,
\]
of the gauge real scalar superfield pre-potential \( V \) describing an \( N = 1 \) vector multiplet, \( \mathcal{R} \) is the chiral (scalar curvature) supergravity superfield, \( W^2 = W^a W_a \) and \( \mathcal{D}^2 = \mathcal{D}^a \mathcal{D}_a [1] \).

Some comments are in order. The action (6) is obtained from the Bagger-Galperin action [29]
\[
S_{\text{BG}}[W, \bar{W}] = \frac{1}{4} \int d^4x d^2\theta X + \text{h.c.} , \quad X + \frac{1}{4 M_{\text{BI}}^{-4}} X \mathcal{D}^2 \bar{X} = W^2,
\]
\[\text{4}\] The manifestly supersymmetric description is, nevertheless, possible at low energies when embedding the goldstino into the constrained chiral superfield \( \bar{X} \) obeying the nilpotency condition \( \bar{X}^2 = 0 [35, 36, 37, 38] \). We avoid that goldstino superfield because it is problematic at higher energies and in quantum theory.
in terms of the constrained chiral superfield $X$ after solving the constraint in Eq. (8) and then 
minimally coupling the resulting action with the supergravity in curved superspace $[39, 19]$, where 
spacetime metric $g_{\mu\nu}$ is replaced by vierbein $e_{\mu}^{a}$ and is extended to an off-shell supermultiplet 
$(e_{\mu}^{a}, \psi_{\mu}, M, b_{\mu})$, with $\psi_{\mu}$ as the Majorana gravitino field, whereas the complex scalar $M$ and the 
real vector field $b_{\mu}$ are the auxiliary fields. $^5$

The gauge vector (photon) field $A_{\mu}$ is extended in SUSY to an off-shell (real) gauge vector 
multiplet (or the superfield) $V$ with the field components

$$V = (C, \beta, H, A_{\mu}, \lambda, D),$$

(9)

where $\lambda$ is Majorana fermion called the photino, $D$ is the auxiliary field, while the rest of the fields 
$(C, \chi, H)$ are the super-gauge degrees of freedom that are ignored in what follows.

Adding a negative cosmological constant to Eq. (6), as in Eq. (2), in the setup with unbroken 
(linearly realized) local SUSY is known to be impossible in supergravity, unless this cosmological 
constant is compensated by another positive contribution $[6]$. Indeed, in order to cancel the 
SUSY variation of the cosmological constant multiplied by $\sqrt{-\det(g_{\mu\nu})} = e$ due to $\delta_{\text{susy}} e_{\mu}^{a} = -i M_{\text{Pl}}^{4}(\tilde{\epsilon} \gamma^{a} \psi_{\mu})$ with the infinitesimal SUSY parameter $\epsilon(x)$, we have to add the photino-gravitino 
mixing term

$$-i e \frac{M_{\text{BI}}^{2}}{M_{\text{Pl}}} (\lambda \gamma^{\mu} \psi_{\mu})$$

(10)
to the Lagrangian, and simultaneously demand the supersymmetric variation of the photino $\lambda$ as

$$\delta_{\text{susy}} \lambda = M_{\text{BI}}^{2} \epsilon + \ldots,$$

(11)

where the dots stand for the other field-dependent terms. The identification of the photino $\lambda$ with 
the goldstino of the spontaneously broken local SUSY requires

$$M_{\text{BI}} = M_{\text{susy}}$$

(12)

already by comparison of Eqs. (5) and (11). This may not be surprising after taking into account 
that the Bagger-Galperin action (8) also possesses the second (spontaneously broken and non- 
linearly realized) SUSY whose transformation law is similar to that of Eq. (5).

The positive contribution $M_{\text{BI}}^{4}$ to the cosmological constant in supergravity comes together (by 
local SUSY) with the gravitino mass term having the mass parameter $m^{2} = \frac{1}{3} M_{\text{BI}}^{4}/M_{\text{Pl}}^{2}$, and the 
modification of the gravitino SUSY transformation law as $\delta_{\text{susy}} \psi_{\mu} = -2 M_{\text{Pl}}(D_{\mu} \epsilon + \frac{1}{2} m \gamma_{\mu}) + \ldots$.

This also implies (by local SUSY again) the presence of the goldstino mass term in the Lagrangian 
with the same mass parameter $m$ $[6]$.

The ultimate recovery of the AV action from the super-BI action is possible by identifying the 
goldstino $\lambda_{\alpha}$ with the leading field component of the superfield $W_{\alpha}$ and projecting the other fields 
out, $F_{\mu\nu}(A) = D = \psi_{\mu} = 0$ in the absence of gravity, $e_{\mu}^{a} = \delta^{a}_{\mu}$. Then the action (8) reduces to the 
AV action in Eq. (4) up to a field redefinition in the higher-order terms — see Ref. [43] for details. 
The same conclusions are supported by the superconformal tensor calculus in supergravity $[44]$. In our approach, the AV action is thus the fermionic fragment of the supersymmetric BI theory 
coupled to supergravity with the spontaneously broken SUSY at the scale $M_{\text{BI}}$. In the rest of the 
paper we concentrate on the pure supergravity sector of our theory.

$^5$The auxiliary fields of the supergravity multiplet do not play the significant role in our investigation and are 
ignored below.
3 One-loop effective action and gravitino condensate

The classical supergravity Lagrangian $\mathcal{L}_{\text{SUGRA}}$ besides the usual Einstein-Hilbert and Rarita-Schwinger terms,

$$\mathcal{L}_{\text{EH}} = -\frac{M_{\text{Pl}}^2}{2} e^R,$$

and

$$\mathcal{L}_{\text{RS}} = -\frac{1}{2} \varepsilon^{\mu\nu\lambda\rho} \bar{\psi}_\mu \gamma_5 \gamma_\nu \gamma_\lambda D_\rho \psi_\rho,$$

respectively, also has the quartic gravitino coupling,

$$\mathcal{L}_{\text{quartic}} = \frac{11}{16} \tilde{M}_{\text{Pl}}^{-2} \left[ (\bar{\psi}_\mu \psi_\mu)^2 - (\bar{\psi}_\mu \gamma_5 \gamma_\nu \psi_\nu)^2 \right] - \frac{33}{64} \tilde{M}_{\text{Pl}}^{-2} (\bar{\psi}_\mu \gamma_5 \gamma_\nu \psi_\nu)^2,$$

originating from the spacetime (con)torsion in the covariant derivative of the gravitino field in its kinetic term, in the second-order formalism for supergravity [1].

Since the supergravity action is invariant under the local SUSY, whose gauge field is $\psi_\mu$, one can choose the (physical) gauge condition $\gamma^\mu \psi_\mu = 0$, which implies $(\bar{\psi}_\mu \Sigma^{\mu\nu} \psi_\nu) = -\frac{1}{2} \bar{\psi}_\mu \gamma^\mu$, in the notation $\Sigma^{\mu\nu} = \frac{1}{4} \{\gamma^\mu, \gamma^\nu\}$, and rewrite the (non-chiral) quartic gravitino term in Eq. (6) as

$$\mathcal{L}_{\text{quartic}} = \sqrt{11} \tilde{M}_{\text{Pl}}^{-1} \rho (\bar{\psi}_\mu \Sigma^{\mu\nu} \psi_\nu) - \rho^2,$$

where the real scalar field $\rho$ has been introduced. As is clear from Eq. (16), a gravitino condensate leads to the non-vanishing Vacuum Expectation Value (VEV), $\langle \rho \rangle \equiv \rho_0 \neq 0$, whereas $\rho_0$ contributes to the gravitino mass.

The one-loop contribution to the effective potential $V_{\text{1-loop}}(\rho)$ of the scalar field $\rho$ together with its kinetic term arise after quantizing the gravitino sector and taking the Gaussian integral over $\psi_\mu$ in the gauge $\gamma^\mu \psi_\mu = 0$. This yields the one-loop contribution to the quantum effective action in the standard form

$$\Gamma_{\text{1-loop}} = -\frac{i}{2} \text{Tr} \ln \Delta(\rho),$$

where $\Delta(\rho)$ stands for the kinetic operator in the gravitino action and the interaction with gravity is ignored ($e^a_\mu = \delta^a_\mu$). The one-loop contribution to the $\rho$-scalar potential (i.e. the terms without the spacetime derivatives in Eq. (17)) was first computed in Refs. [3, 4] with the result

$$V_{\text{1-loop}} = \lim_{\mathcal{V} \to \infty} \left[ -\frac{1}{2\mathcal{V}} \sum_{n=1}^{\infty} \frac{(\sqrt{11} \tilde{M}_{\text{Pl}})^2n}{2n} \text{Tr}(P_{ab})^{2n} \right] = -\frac{4}{(2\pi)^4} \int_\Lambda d^4p \ln \left( 1 + 11 \tilde{M}_{\text{Pl}}^{-2} \rho^2 \right),$$

in terms of the standard massless gravitino propagator (in momentum space)

$$P_{ab} = -\frac{i}{2} \frac{\gamma_b \gamma^\mu P_\mu \gamma_a}{p^2},$$

the spacetime 4-volume regulator $\mathcal{V}$ and the Ultra-Violet (UV) cutoff $\Lambda$, with the trace $\text{Tr}$ acting on all variables.

The one-loop contribution (17) expanded up to the 2nd order in the spacetime derivatives also yields the $\rho$-kinetic term subject to the wave function renormalization (i.e. with the $Z$ factor), so that the initially auxiliary scalar field $\rho$ becomes dynamical with a mass $M_c$. The specific

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6We separate the quartic terms from the minimal term in Eq. (14).
calculations can be found in the literature \([3, 4, 45, 46, 47]\), though the details are not relevant for our purposes here. We take the effective potential in the form

\[
V(\rho) \equiv V_{\text{classical}}(\rho) + V_{1-\text{loop}}(\rho) = \rho^2 - \frac{4}{(2\pi)^4} \int^\Lambda d^4 p \ln \left(1 + 11\tilde{M}^{-2}_{\text{Pl}} \frac{\rho^2}{p^2}\right)
\]

(20)

that agrees with Refs. \([3, 4, 46, 47]\). The gravitino condensate is real (and, hence, is quantum stable), while the one-loop integration over gravitons can be ignored. However, our result of taking the four-dimensional integral in Eq. (20) is different from the literature, namely,

\[
V(\rho) = \rho^2 + \frac{1}{8\pi^2} \left\{ \frac{121\rho^4}{M^4_{\text{Pl}}} \ln \left(1 + \frac{\tilde{M}^2_{\text{Pl}} \Lambda^2}{11\rho^2}\right) - \frac{11\rho^2 \Lambda^2}{M^2_{\text{Pl}}} - \Lambda^4 \ln \left(1 + \frac{11\rho^2}{\tilde{M}^2_{\text{Pl}} \Lambda^2}\right) \right\}
\]

(21)

The logarithmic scaling of the wave function renormalization of \(\rho\) in the one-loop approximation yields the factor proportional to \(\ln \left(\frac{\Lambda^2}{\mu^2}\right)\), where \(\mu\) is the renormalization scale. Hence, the canonical (physical) scalar \(\phi\) is given by \([46]\)

\[
\phi = \text{const.} \sqrt{\ln \left(\frac{\Lambda^2}{\mu^2}\right)} \tilde{M}^{-1}_{\text{Pl}} \rho \equiv \tilde{w} M_{\text{Pl}} \sigma,
\]

(22)

where we have introduced the dimensionless (renormalization) constant \(\tilde{w}\) as the parameter. We also use the other dimensionless quantities

\[
\sigma = \tilde{M}^{-2}_{\text{Pl}} \rho, \quad \tilde{M}^{-1}_{\text{Pl}} \Lambda = \tilde{\Lambda} \quad \text{and} \quad \tilde{M}^{-1}_{\text{Pl}} M_{\text{BI}} = \alpha,
\]

(23)

which allow us to rewrite the full scalar potential as

\[
V(\sigma)\tilde{M}^{-4}_{\text{Pl}} = \sigma^2 - \frac{1}{8\pi^2} \left\{ \tilde{\Lambda}^4 \ln \left(1 + \frac{11\sigma^2}{\tilde{\Lambda}^2}\right) - 121\sigma^4 \ln \left(1 + \frac{\tilde{\Lambda}^2}{11\sigma^2}\right) + 11\sigma^2 \tilde{\Lambda}^2 \right\} + \alpha^4,
\]

(24)

where we have added the contribution of the first term on the right-hand-side of Eq. (2).

The scalar potential (24) has the double-well shape and is bounded from below, see Fig. 1, provided that

\[
\tilde{\Lambda}^2 > \frac{4\pi^2}{11} \approx 3.59, \quad \text{or} \quad \tilde{\Lambda} > \frac{2\pi}{\sqrt{11}} \approx 1.89.
\]

(25)

There is a local maximum at \(\rho = \sigma = 0\) with the positive height \(M^4_{\text{BI}}\). A similar potential near its maximum was used for describing slow-roll inflation with the inflaton field \(\phi\) \([46]\), see the next Sec. 4 for more. There are also two stable Minkowski vacua at \(\rho_c \neq 0\).

According to the previous Section, supersymmetry requires the scalar potential (24) to vanish at the minimum, i.e. \(V(\sigma_c) = 0\). In addition, according to Eq. (16), the \(\rho_c \neq 0\) determines the gravitino condensate mass

\[
m_{\phi} = \sqrt{\Pi} \rho_c / \tilde{M}_{\text{Pl}} = \sqrt{\Pi} M_{\text{Pl}} \sigma_c.
\]

(26)

The non-vanishing values of \(\rho_c\) and \(\sigma_c\) are determined by the the condition \(dV/d(\sigma^2) = 0\) that yields a transcendental equation,

\[
121\sigma_c^2 \ln \left(1 + \frac{\tilde{\Lambda}^2}{11\sigma_c^2}\right) = 11\tilde{\Lambda}^2 - 4\pi^2 > 0.
\]

(27)
The hierarchy between the inflationary scale $H_{\text{inf}}$, the BI scale $M_{\text{BI}}$, the SUSY breaking scale $M_{\text{SUSY}}$, the (super)GUT scale $M_{\text{GUT}}$, the effective gravitational scale $\tilde{M}_{\text{Pl}}$ and the Planck scale $M_{\text{Pl}}$ in our approach reads

$$H_{\text{inf.}} \ll M_{\text{BI}} = M_{\text{SUSY}} \approx M_{\text{GUT}} \approx \tilde{M}_{\text{Pl}} \ll M_{\text{Pl}},$$

(28)

where ”much less” means the 2-3 orders of magnitude ”less” (in GeV), and ”approximately” means the same order or magnitude, see the next Section for our numerical estimates. As regards the GUT scale, we take $M_{\text{GUT}} \approx \mathcal{O}(10^{15})$ GeV.

4 Gravitino condensate as inflaton

A slow-roll inflation induced by gravitino condensation in supergravity was proposed and studied by Ellis and Mavromatos in Ref. [46]. Since our induced scalar potential differs from that of Ref. [46], we reconsider this inflation in this Section, by using $\tilde{\Lambda}$ and $\tilde{w}$ as the phenomenologically adjustable parameters.

A slow roll is possible near the maximum of the scalar potential (24). Since the height of the potential at the maximum is related to the inflationary Hubble scale $H_{\text{inf.}}$ by Friedmann equation,

$$V_{\text{max.}} = 3M_{\text{Pl}}^2 H_{\text{inf.}}^2,$$

(29)

the value of $H_{\text{inf.}}/M_{\text{Pl}}$ is suppressed by the factor $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}})^2$. On the other hand, the inflationary Hubble scale is related to the Cosmic Microwave Background (CMB) tensor-to-scalar ratio $r$ as

$$\frac{H_{\text{inf.}}}{\tilde{M}_{\text{Pl}}} = 1.06 \cdot 10^{-4} \sqrt{r}.$$

(30)
In turn, the $r$ is restricted by Planck (2018) measurements \[49\] as $r < 0.064$ (with 95\% CL), which implies $H_{\text{inf.}} < 6 \cdot 10^{13}$ GeV. Therefore, the ratio $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}})$ should be of the order $10^{-2} \div 10^{-3} \ll 1$ for viable inflation. This justifies our setup in the Introduction (Sec. 1). We define the dimensionless parameter $\gamma$ as $(\tilde{M}_{\text{Pl}}/M_{\text{Pl}})^2 \equiv 10^{-3}/\gamma$, where $\gamma$ is of the order one.

In our numerical calculations, we have chosen the cutoff scale $\tilde{\Lambda} = 3$, so that the restriction (25) is satisfied. Then Eqs. (24) and (27) imply
\[
V_{\text{max}} \tilde{M}_{\text{Pl}}^{-4} = 0.245 \quad \text{and} \quad \sigma_{\text{cr.}} = \tilde{w}^{-1}(\phi_{\text{cr.}}/M_{\text{Pl}}) = 0.722 .
\]
(31)
In turn, this implies
\[
m_{3/2} = 2.39 \tilde{M}_{\text{Pl}} \quad \text{and} \quad m_{\text{cond.}} = \sqrt{8/11} m_{3/2} = 2.038 \tilde{M}_{\text{Pl}} .
\]
(32)

We numerically studied the running of the slow inflationary parameters $\varepsilon = \frac{1}{2}M_{\text{Pl}}^2(V''/V)^2$ and $\eta = M_{\text{Pl}}^2(V''/V)$ with respect to the inflaton field $\phi$ for the values of the parameter $\gamma$ as 0.1, 0.5 and 1, and found that $\varepsilon$ is always under $\mathcal{O}(10^{-4})$ so that it can be ignored within the errors of Planck 2018 data. Then the value of the scalar index $n_s = 1 - 6\varepsilon + 2\eta = 0.9649 \pm 0.0042$ (with 68\% CL) [49] can be reached with $\eta = -0.0177$ at the horizon crossing by using the parameter $\tilde{w}$ of the order one. There are no additional constraints on the parameters $\gamma$ and $\tilde{w}$ from demanding the e-folding number
\[
N_e = -\frac{1}{M_{\text{Pl}}^2} \int_{\phi_{\text{init.}}}^{\phi_{\text{end.}}} \frac{V}{V'} d\phi
\]
(33)
to be between 50 and 60, as is desired for viable inflation, when assigning the inflaton field $\phi/M_{\text{Pl}}$ to run somewhere between 0 and 5 during inflation. The running of the slow-roll parameter $\eta$ is displayed in Fig. 2.
In summary, our results qualitatively agree with those of Ref. [46], but quantitatively allow considerably higher values of $\varepsilon$ and $r$ up to the order $O(10^{-4})$ contrary to $O(10^{-8})$ of Ref. [46], with the Planckian values of the inflaton $\phi$ during inflation, contrary to its sub-Planckian values of $O(10^{-3}) M_{Pl}$ in Ref. [46]. Hence, the inflationary scale $H_{\text{inf}}$ can be as high as $10^{12}$ GeV versus $10^{10}$ GeV in Ref. [46].

5 Adding the FI term

In order to uplift the Minkowski vacuum to a de Sitter vacuum (dark energy) in our approach, we need an extra tool of spontaneous SUSY breaking. In the BI theory (without chiral matter) coupled to supergravity such tool can be provided by the (new) FI terms [18, 19, 20, 21] that do not require the gauged R-symmetry, unlike the standard FI term [17] whose extension to supergravity is severely restricted [48].

The (abelian) gauge vector multiplet superfield $V$ can be decomposed into a sum of the reduced gauge superfield $\mathcal{V}$ including the gauge field $A_{\mu}$, and the nilpotent gauge-invariant goldstino superfield $\mathcal{G}$ that contains only goldstino $\lambda$ and the auxiliary field $D$ [19],

$$V = \mathcal{V} + \mathcal{G} , \quad \mathcal{G}^2 = 0 .$$

The simplest examples of the goldstino superfield are given by [18, 19]

$$\mathcal{G}_1 = -4 \frac{W^2 \bar{W}^2}{D^2 W^2 \bar{D}^2 \bar{W}^2} (\mathcal{D} \mathcal{W})$$

and

$$\mathcal{G}_2 = -4 \frac{W^2 \bar{W}^2}{(\mathcal{D} \mathcal{W})^3} ,$$

respectively, in terms of the standard $N = 1$ gauge superfield strength

$$W_\alpha = -\frac{1}{4} (\bar{D}^2 - 4 \mathcal{R}) D_\alpha V ,$$

where $\mathcal{R}$ is the chiral scalar curvature superfield. The $W_\alpha$ obeys Bianchi identities

$$\bar{D}_{\beta} W_\alpha = 0 \quad \text{and} \quad D_\alpha \bar{W}^\alpha \equiv \bar{D} \mathcal{W} = D^\alpha W_\alpha \equiv \mathcal{D} \mathcal{W} .$$

The field components are given by $W_\alpha| = \lambda_\alpha$, $\mathcal{D} \mathcal{W} = -2D$, and $D_\alpha W_{\beta}| = i (\sigma^{ab})_{\alpha \beta} F_{ab} + \ldots$. The difference between the superfields $\mathcal{G}_1$ and $\mathcal{G}_2$ is only in the gauge sector, while it is not essential for our purposes here.

The extra FI term with the coupling constant $\xi \neq 0$ is given by

$$S_{\text{FI}} = \xi \int d^4 x d^4 \theta E \mathcal{G} ,$$

where $E$ is the supervielbein (super)determinant [1]. This FI term is manifestly SUSY- and gauge-invariant, does not include the higher spacetime derivatives of the field components, but leads to the inverse powers of the auxiliary field $D$ (up to the forth order) in the non-scalar sector of the
Integrating out the auxiliary field $D$ leads to the positive contribution to the cosmological constant

$$V_\xi = \frac{1}{2} \xi^2 > 0.$$  

(40)

Matching $V_\xi$ with the observed cosmological constant allows us to include a viable description of the dark energy into our approach.

It is worth mentioning that the alternative FI term (39) breaks the second (nonlinearly realized) SUSY of the super-BI action. The nilpotent Goldstino superfield $G$ introduced above is composed of the usual (standard) superfields and, hence, is very different from the intrinsically nilpotent goldstino superfield introduced in Refs. [35, 36, 37, 38].

6 Conclusion

The gravitino condensate can be considered as a viable candidate for inflaton in supergravity, when assuming the effective (quantum) gravity scale to be close to the (super)GUT scale that is also close to the SUSY breaking scale in our approach, with all scales close to $10^{15}$ GeV. Actually, in this scenario we have the hyper-GUT where all fundamental interactions merge, including gravity. At the same time, it is the weak point of our calculations, because we ignored (other) quantum gravity corrections.

The inflationary (Hubble) scale is well below the GUT scale, and can be as large as $10^{12}$ GeV. The gravitino mass is above the inflationary scale, so that there is no gravitino overproduction problem in the early Universe. The constraints from proton decay and the Big Bang Nucleosynthesis (BBN) are very weak because of high-scale SUSY. Then SUSY is not a solution to the hierarchy problem with respect to the electro-weak scale. This is similar to the setup studied in Refs. [50, 51]. Our scenario is consistent with the known Higgs mass of about 125 GeV after taking into account the extreme possible values of the gaugino mixing parameter $\tan \beta$ in the context of SUSY extensions of the Standard Model [52].

As regards reheating after inflation, the inflaton (gravitino condensate) field decays into other matter and radiation, which is highly model-dependent, as usual. Unlike Ref. [24], the inflaton as the gravitino condensate cannot decay into gravitinos because Eq. (32) leads to the kinematical constraint $2m_{3/2} > m_{\text{cond}}$. It also implies that gravitino cannot be a dark matter particle in this scenario. A detailed study of reheating requires the knowledge of couplings of gravitino and gravitino condensate to the Standard Model particles, which is beyond the scope of this paper.

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\footnote{The limit $\xi \to 0$ does not lead to a well defined theory, so that $\langle D \rangle = \xi$ must be non-vanishing.}
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