Localization of Bose-Fermi Mixtures in One-Dimensional Incommensurate Lattices

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Abstract. We studied the localization property of Bose-Fermi mixture systems on incommensurate optical lattices by using quantum Monte-Carlo simulations. We found a characteristic behavior of the superfluid density when changing the Bose-Bose interactions and Bose-Fermi interactions over a wide range. The superfluidity of the bosons was enhanced in the presence of the fermions by the repulsive Bose-Fermi interactions which counteracted the localization of the bosons. With attractive Bose-Fermi interactions, on the other hand, the superfluidity is also enhanced by a different mechanism when the number of the bosons is much larger than that of the fermions.

1. Introduction
Ever since the seminal papers by Fisher et al.[1] and by Giamarch et al.[2], disordered Bose systems with strong correlation, described by Bose-Hubbard models with disorder, have been one of the targets of theoretical and experimental investigation. Though the pure system is well understood, the properties of the disordered Bose-Hubbard model remain subject to debate, even at a qualitative level. Current interest is also directed towards disordered systems of ultracold atoms generated by using laser speckled patterns or additional incommensurate optical lattice potentials, called a bichromatic lattice[3, 4]. Fallani et al.[3] observed a localization transition of strongly interacting $^{87}\text{Rb}$ bosons in incommensurate lattices, which suggested the formation of a Bose glass.

It is well known that the response to disorder is different between bosons and fermions[2]. Interacting bosons in a weak random potential exhibit superfluidity if the interactions are repulsive and not too strong, while interacting fermions with repulsive interactions are localized in the presence of the randomness. With this in mind we focused on what would happen if bosons and fermions coexisted and interacted with each other in a random potential. In a previous study, we investigated the localization property of one-dimensional Bose-Fermi mixtures with repulsive Bose-Bose and Bose-Fermi interactions in a weak random potential[5]. We found that the superfluidity of the bosons was enhanced as we switched on the repulsive Bose-Fermi interactions. We surmised from the results that the enhancement of the superfluidity of the bosons and that of the Drude weight of the fermions was caused by the Bose-Fermi interactions which effectively weakened the random potential. Since this behavior was studied with a limited range of the interaction strength, more investigation was necessary to get a full view of the localization property of the mixture systems. It is also to be noted that the localization can...
be expected to occur under other types of potentials. For example, if a periodic potential is applied to the systems in addition to the optical lattice and if their periodicity is incommensurate to each other, the additional potential would work as a pseudo-random potential and cause a localization phenomenon.

In this paper we present results of quantum Monte Carlo simulations of Bose-Fermi mixtures in one-dimensional incommensurate lattices with a wide range of the Bose-Bose and Bose-Fermi interactions. In the simulations, we calculated the superfluid density of the bosons and the Drude weight of the fermions and found a transition between localized and delocalized states of the bosons when changing the Bose-Fermi interactions. This behavior depended on the particle densities and the strength of the Bose-Bose and Bose-Fermi interactions.

The paper is organized as follows. Section 2 shows the model that we used in the simulations and Sec. 3 presents the result. Conclusions are given in Sec. 4.

2. Model

We performed world-line quantum Monte Carlo (QMC) simulations[6, 7] of bosons and fermions in a one-dimensional lattice with a confinement potential and an additional periodic potential incommensurate to the lattice periodicity.

\[
H = -t_b \sum_{\langle i,j \rangle} b_i^\dagger b_j - t_f \sum_{\langle i,j \rangle} f_i^\dagger f_j + \frac{U_{bb}}{2} \sum_i n_{bi} (n_{bi} - 1) + U_{bf} \sum_i n_{bi} n_{fi} + V_c \sum_i (n_{bi} + n_{fi})(i + 1 - N/2)^2 + A \sum_i \cos(2\pi \alpha i + \delta)(n_{bi} + n_{fi})
\]

where \( b_i \) (\( b_i^\dagger \)) is the boson annihilation (creation) operator at the site \( i \), \( n_{bi} = b_i^\dagger b_i \), \( f_i \) (\( f_i^\dagger \)) is the fermion annihilation (creation) operator at the site \( i \), and \( n_{fi} = f_i^\dagger f_i \). \( U_{bb} \) denotes the boson-boson interaction and \( U_{bf} \) the boson-fermion interaction. We set \( t_b = t_f = 1 \) as energy unit. The fifth term of the Hamiltonian represents the confinement potential and the last one is a periodic potential incommensurate to the lattice.

In the simulations, the inverse temperature was fixed to \( \beta = 10 \), the number of lattice site to \( N = 200 \), the curvature parameter of the confinement potential to \( V_c = 0.001 \), the strength of the incommensurate potential to \( A = 2.0 \), and the incommensurability to \( \alpha = \frac{\sqrt{2} - 1}{2} \). The very weak confinement potential was added to circumvent a boundary condition problem that ordinary periodic boundary conditions could not be used in the presence of the incommensurate potential.

In order to see the transport property of the particles, we measured the Drude weight \( \rho_f \) for the fermions and the superfluid density \( \rho_b \) for the bosons. They are given by calculating the current-current correlation functions, \( \langle J(\omega)J(-\omega) \rangle \), in the zero-frequency limit where \( J(\omega) \) is the Fourier transform of the current operator \( J(\tau) \) of the bosons or the fermions with \( \tau \) being the imaginary time in the path integral formalism.

3. Result

Figures 1 and 2 show the superfluid density \( \rho_b \) of the bosons as binary functions of the Bose-Fermi interaction \( U_{bf} \) and the Bose-Bose interaction \( U_{bb} \). Figure 1 presents the superfluid density of 80 bosons and 40 fermions and Fig. 2 that of 80 bosons and 80 fermions. Bright areas in the figures indicate superfluid states and dark areas are localized states. When \( U_{bb} = 0 \), as previous studies showed[2], the fermions are Anderson localized whereas the bosons stay in a superfluid state as far as \( U_{bb} \) is small and turn into a Bose glass as \( U_{bb} \) becomes larger.

Clearly, the bright area is larger in Fig. 1 than in Fig. 2. More interestingly, the bright area in Fig. 1 has several branches, which produce a qualitative difference between the two figures.
in particular for $U_{bb} > 6.0$. Let us look at both figures with $U_{bb}$ fixed to a relatively large value, e.g. 8.0, and trace the figures vertically along the $U_{bf}$ axis. In Fig. 1 we see two bright areas at around $U_{bf} = +3.0$ and $6.0$ but we see only one bright area in Fig. 2 at around $U_{bf} = +4.0$. This is more obvious in Fig. 3.

**Figure 1.** Superfluid density of 80 bosons with 40 fermions.

**Figure 2.** Superfluid density of 80 bosons with 80 fermions.

**Figure 3.** Superfluid density and Drude weight with different Bose-Bose interaction strengths.

**Figure 4.** Superfluid density and Drude weight with different number ratios of the bosons and the fermions.

Figure 3 shows a cross-section view of Fig. 1 at $U_{bb} = 2.0$ and 8.0 presenting the superfluid density of the bosons and the Drude weight of the fermions. When $U_{bb} = 2.0$, the superfluid density, closed circles, and the Drude weight, open circles, have a single peak as a function of $U_{bf}$ indicating the delocalization of the bosons and the fermions due to the Bose-Fermi interactions as explained in Introduction. However when $U_{bb} = 8.0$, the superfluid density, closed triangles, and the Drude weight, open triangles, behave quite differently from each other. Namely, the superfluid density has two peaks at around $U_{bf} = 2.0$ and 6.0, while the Drude weight has a single peak at around $U_{bf} = 2.0$.

We also see in Figs. 1 and 2 that the ratio of the boson density and the fermion density plays an important role. Figure 4 shows the superfluid density of the bosons and the Drude weight of the fermions with different number ratios of the bosons and the fermions. In this figure the superfluid density at the ratio 0.5, closed circles, has a double peak structure while the superfluid density at the ratio 1.0, open circles, shows only one peak.

The peaks are the indication of the delocalization. Then why is there a single peak in some cases and two peaks in other cases? The peaks on the positive $U_{bf}$ side are a manifestation of the delocalization due to the Bose-Fermi interactions, which make it energetically unfavorable for
the bosons and the fermions to stay in the same deep-potential sites and keep them staying away from each other. Namely a fermion approaching to a lattice site where bosons are localized could kick them out through the Bose-Fermi interactions. This is the delocalization effect of positive $U_{bf}$. This explanation cannot be applied to the systems with negative $U_{bf}$ since the fermion can stay with the bosons on the same site in a stable manner if the Bose-Fermi interactions are attractive. Therefore the peaks on the negative $U_{bf}$ side should be the delocalization caused by a different mechanism, which we explain below.

Let us think about the fermions at first. They can be localized in the deep random potential wells. Then we introduce the bosons to the fermion system. Suppose the Bose-Fermi interactions are attractive ($U_{bf} < 0$) but their strength is smaller than that of the Bose-Bose interactions ($|U_{bf}| < U_{bb}$). Also suppose the number of the bosons is larger than that of the fermions. In this case each localized fermion would attract the bosons nearby but could not stay on the same site with multiple bosons because the bosons would stay away from each other through large $U_{bb}$. Therefore, some bosons are trapped by the fermions and some are not. (Remember that we have more bosons than fermions in the system.) In other words, some bosons are localized and some are not. Of course more and more bosons would be localized as $U_{bf}(< 0)$ gets stronger. The situation could be different if $U_{bf}$ is strong enough and its strength becomes comparable to $U_{bb}$, however. Let us focus on the bosons which are not trapped by the localized fermions. These bosons would feel a barrier of $U_{bb}$ from each of the fermions localized together with other bosons. However, if the strength of $U_{bf}$ is comparable to that of $U_{bb}$, the barrier would be removed and the bosons can move around in larger areas, which enhances their superfluidity. This explains the peaks on the negative $U_{bf}$ side. When we have stronger $U_{bf}$, even stronger than $U_{bb}$, each fermion can be localized together with multiple bosons and the localization of the bosons would be strengthened.

This scenario works not only for the systems in the incommensurate potentials but also for the systems in a random potential. We numerically confirmed that qualitatively similar results could be obtained from the Bose-Fermi mixture systems in random potentials.

4. Conclusions
We performed the QMC simulations of the one-dimensional Bose-Fermi mixtures in the incommensurate lattices to investigate their localization and delocalization properties. The calculation was conducted over a wide range of the Bose-Bose interactions and the Bose-Fermi interactions with different number ratios of the bosons and the fermions. When the fermion density is much smaller than the boson one, the superfluid density of the bosons is enhanced on both positive and negative $U_{bf}$ sides. The mechanism of the enhancement is different between when the Bose-Fermi interactions are attractive and when they are repulsive.

5. References
[1] Fisher M P A et al., 1989 Phys. Phys. Rev. B 40, 546.
[2] Giamarchi T and Schulz H J, 1988 Phys. Rev. B 37, 325.
[3] Fallani L et al., 2007 Phys. Rev. Lett. 98, 130404.
[4] Billy J et al., 2008 Nature 453, 891.
[5] Mori H and Ikeda S, 2009 J. Phys. Conf. Ser. 150, 032063.
[6] Hirsh J E et al., 1982 Phys. Rev. B 20, 5055.
[7] Batrouni G G and Scalettar R T 1992 Phys. Rev. B 46, 9051.