On the gravity of light

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Abstract
A solution of the old problem raised by Tolman, Ehrenfest, Podolsky and Wheeler, concerning the lack of attraction of two light pencils ‘moving parallel’, is proposed, considering that the light can be a source of nonlinear gravitational waves corresponding (from a quantum point of view) to spin-1 massless particles.

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Introduction
In this paper, we will study the repulsive behavior of gravitational interaction, in particle physics, associated with specific properties of some exact solutions of Einstein equations. The interest in repulsive gravity, or antigravity as it was usually called, goes back to the 1950s [1–3]. The general point of view was that since gravitational interaction is mediated by a spin-2 particle, it can only be attractive and thus, to obtain a repulsive behavior, some other ingredient is required. The idea was then to explore the possibility of repulsive matter–antimatter gravity, but within the old quantum field theories there was no room for such a possibility. The main arguments, reviewed in [3], were of various kinds including violation of energy conservation and disagreement with experiments of the Eötvös type due to effects of antigravity on the vacuum polarization diagrams of atoms. More recently, however, within the context of modern quantum field theories, it was proven that those arguments were no longer sufficient to exclude repulsive effects and the interest in antigravity increased again.
For example in [4] it was shown that in supergravity and string theory, due to dimensional reduction, the effective four-dimensional theory of gravity may show repulsive aspects because of the appearance of spin-1 graviphotons.

Our point of view is the following. In the usual treatment of gravitational waves only Fourier expandable solutions of the d’Alembert equation are considered; then it is possible to choose a special gauge (TT-gauge) which kills the spin-0 and spin-1 components. However, there exist (see sections 2 and 3) physically meaningful solutions [5–8] of Einstein equations which are not Fourier expandable and nevertheless whose associated energy is finite. For some of these solutions the standard analysis shows that spin-1 components cannot be killed [9, 10]; this implies that repulsive aspects of gravity are possible within pure general relativity, i.e. without involving spurious modifications. In previous works, it was shown that light is among possible sources of such spin-1 waves [8–12].

Photon–photon scattering can occur through the creation and annihilation of virtual electron–positron pairs and may even lead to collective photon phenomena. Photons also interact gravitationally but the gravitational scattering of light by light has been much less studied. Purely general relativistic treatments of electromagnetic wave interactions have been made resulting in exact solutions [13, 14], but these calculations are different from pure scattering processes and do not address the interaction at the single photon level. It is not clear, to what extent calculations of the gravitational cross section using QFT methods are consistent with classical GR. First studies go back to Tolman, Ehrenfest and Podolsky [15] and, later, to Wheeler [16] who analyzed the gravitational field of light beams and the corresponding geodesics in the linear approximation of Einstein equations. They discovered that null rays behave differently according to whether they propagate parallel or antiparallel to a steady, long, straight beam of light, but they did not provide a physical explanation of this fact. Later, Barker, Bathia and Gupta [17], following a previous analysis of Barker, Gupta and Haracz [18], analyzed in QED the photon–photon interaction through the creation and annihilation of a virtual graviton in the center–mass system; they found the interaction has eight times the ‘Newtonian’ value plus a polarization-dependent repulsive contact interaction, and also obtained the gravitational cross sections for various photon polarization states. Results of Tolman, Ehrenfest, Podolsky, Wheeler were clarified in part by Faraoni and Dumse [19], in the setting of classical pure general relativity, using an approach based on a generalization to null rays of the gravitoelectromagnetic Lorentz force of linearized gravity. They also extended the analysis to the realm of exact $pp$-wave solutions of the Einstein equations. After Barker, Bathia and Gupta, photon–photon scattering due to self-induced gravitational perturbations on a Minkowski background has been analyzed by Brodin, Eriksson and Marklund [20] solving the Einstein–Maxwell system perturbatively to third order in the field amplitudes and confirming the dependence of differential gravitational cross section on the photon polarizations.

Since the problem of the gravitational interaction of two photons is still unsolved, it appears necessary to take into full account the nonlinearity of Einstein’s equations, just as in the case of gravitational waves generated by strong sources [21, 22]. Section 1 contains a short account of Einstein field equations in the linear approximation, its gravitoelectromagnetic formulation and corresponding application to the Tolman, Ehrenfest, Podolsky problem. Section 2 summarizes some geometric and physical properties (energy–momentum tensor and spin) of a family of exact solutions of Einstein equations, representing gravitational waves generated by a light beam or, more generally by massless particles. Section 3 is devoted to the motion of a massless spin-1 particle in the strong field regime. Finally, in section 4 the scheme is applied to the case of relativistic jets.
1. Weak gravitational fields

1.1. The harmonic gauge

A gravitational field $g = g_{\mu\nu}(x) \, dx^\mu \, dx^\nu$ is said to be locally weak if there exists a (harmonic) coordinate system and a region $M' \subset M$ of space-time $M$ in which the following conditions hold:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1, \quad |h_{\mu\nu,\alpha}| \ll 1.$$  \hspace{1cm} (1)

As is known, in the weak field approximations and in harmonic coordinates system, Einstein field equations reduce to the wave equation for $h_{\mu\nu}$.

The choice of the harmonic gauge plays a key role in this reduction; no other special assumption, either on the form or on the analytic properties of the perturbation $h$, is necessary.

For globally square integrable solutions of the wave-equation (that is, solutions which are square integrable on the whole of $M$), with a suitable gauge transformation preserving the harmonicity of the coordinate system and the ‘weak character’ of the field, one can always kill the ‘spin-0’ and ‘spin-1’ components of the gravitational waves. However, in the following we will meet some interesting solutions which do not belong to this class.

1.2. Gravitoelectromagnetism

A slightly different point of view, which is useful in clarifying the nature of the spin of gravitational waves is provided by the gravitoelectromagnetism, henceforth GEM (see, for example, [23]). In this scheme, one tries to exploit as much as possible the similarities between the Maxwell and the linearized Einstein equations. To make this analogy evident, it is enough to write a weak gravitational field fulfilling conditions (1) in the GEM form

$$ds^2 = \left(-1 + 2\Phi_{(g)} \right) dt^2 - 4 (A_{(g)} \cdot dx) \, dt + \left(1 + 2\Phi_{(g)} \right) \delta_{ij} \, dx^i \, dx^j,$$  \hspace{1cm} (2)

with

$$h_{00} = 2\Phi_{(g)}, \quad h_{0i} = -2A_{(g)}^i.$$  

Hereafter, the spatial part of four-vectors will be denoted in bold and the standard symbols of three-dimensional vector calculus will be adopted. In terms of $\Phi_{(g)}$ and $A_{(g)}$ the harmonic gauge condition reads

$$\frac{\partial \Phi_{(g)}}{\partial t} + \frac{1}{2} \nabla \cdot A_{(g)} = 0,$$  \hspace{1cm} (3)

and, once the gravitoelectric and gravitomagnetic fields are defined in terms of GEM potentials, as

$$E_{(g)}^\mu = -\nabla \Phi_{(g)} - \frac{1}{2} \frac{\partial A_{(g)}}{\partial t}, \quad B_{(g)}^\mu = \nabla \wedge A_{(g)},$$

one finds that the linearized Einstein equations resemble Maxwell equations. Consequently, being the dynamics fully encoded in Maxwell-like equations, the GEM formalism describes the physical effects of the vector part of the gravitational field. The situations which are usually described in this formalism are, typically, static: in fact, when this assumption is dropped, GEM gravitational waves are also possible.

Then, the gravitoelectric and the gravitomagnetic components of the metric are given by

$$E_{(g)}^\mu = F_{\mu0}^{(g)}, \quad B_{(g)}^\mu = -\epsilon_{\mu0\alpha\beta} F_{\alpha\beta}^{(g)} / 2,$$

where

$$F_{\mu\nu}^{(g)} = \partial_{(g)} A_{(g)}^\rho - \partial_{(g)} A_{(g)}^\mu, \quad A_{(g)}^\mu = -h_{0\mu} / 2 = (-\Phi_{(g)}, A_{(g)}).$$
The first-order geodesic motion for a massive particle moving with velocity \( v^\mu = (1, v) \), \(|v| \ll 1\), in a light beam gravitational field characterized by gravitoelectric \( E^{(g)} \) and gravitomagnetic \( B^{(g)} \) fields, is determined (at first order in \(|v|\)) by the force:

\[
f^{(g)} = -E^{(g)} - 2v \wedge B^{(g)}.
\]

The first-order geodesic motion for a massless particle moving with velocity \( v^\mu = (1, v) \), \(|v| = 1\), in the light beam gravitational field, parallel(anti) to the \( z \)-axis \( (v_j = \pm \delta_{j3}) \) is slightly different:

\[
f^{(g)} = -2(E^{(g)} + v \wedge B^{(g)}).
\]

The factor 2 in front of the gravitoelectric field can be understood as resulting from two contributions, one by the light beam, which is the source of gravity, and the other by the test photon \([19]\). It turns out \([15, 19, 24]\) that for a massless particle moving parallel (antiparallel) to the light beam \( f^{(g)} = 0 \) (\( f^{(g)} \neq 0 \)).

2. Strong gravitational fields

2.1. Geometrical properties

In previous papers \([25–27, 8–10]\), a family of exact solutions \( g \) of Einstein field equations, representing the gravitational wave generated by a beam of light, has been explicitly written as

\[
g = 2f(dx^2 + dy^2) + \mu[(w(x, y) - 2q)dp^2 + 2dp dq],
\]

where \( \mu \) = \( D\Phi + B \), with \( D \) and \( B \) arbitrary constants, and \( \Phi \) is a non constant harmonic function. \( f = (\nabla\Phi)^2 \sqrt{|\mu|/\mu} \), and \( w(x, y) \) is the solution of the Euler–Darboux–Poisson equation:

\[
\Delta w + (\partial_x \ln |\mu|) \partial_x w + (\partial_y \ln |\mu|) \partial_y w = \rho,
\]

\( \Delta \) representing the Laplace operator in the \( (x, y) \)-plane and \( T_{\mu\nu} = \rho \delta_{\mu3}\delta_{\nu3} \) the energy–momentum tensor.

It is invariant for the non Abelian Lie algebra \( G_2 \) of Killing fields, generated by

\[
X = \frac{\partial}{\partial p}, \quad Y = \exp(p) \frac{\partial}{\partial q},
\]

with \([X, Y] = Y, g(Y, Y) = 0 \) and whose orthogonal distribution is integrable.

In the particular case, \( s = 1, f = 1/2 \) and \( \mu = 1 \), the above family is locally diffeomorphic \([9]\) to a subclass of Peres solutions \([5, 7]\) and, by using the transformation

\[
p = \ln |u| \quad q = uv,
\]

can be written in the form

\[
g = dx^2 + dy^2 + 2du^2 + \frac{w}{u^2} du^2,
\]

with \( \Delta w(x, y) = \rho \), and has the Lorentz invariant Kerr–Schild form:

\[
g_{\mu\nu} = \eta_{\mu\nu} + V k_{\mu} k_{\nu}, \quad k_{\mu} k^{\mu} = 0.
\]
2.2. Physical properties

2.2.1. Wave character. The wave character and the polarization of these gravitational fields have been analyzed in many ways. For example, the Zel’manov criterion [29] was used to show that these are gravitational waves and the propagation direction was determined by using the Landau–Lifshitz pseudo tensor [8, 10]. However, the algebraic Pirani criterion is easier to handle since it determines both the wave character of the solutions and the propagation direction at once. Moreover, it has been shown that, in the vacuum case, two methods agree [10]. To use this criterion, the Weyl scalars must be evaluated according to the Petrov classification [30]. In the Newmann–Penrose formulation [31] of Petrov classification, one has to choose a tetrad basis with two real null vector fields and two real spacelike (or two complex null) vector fields. Then, according to the Pirani criterion, if the metric belongs to type N of the Petrov classification, it is a gravitational wave propagating along one of the two real null vector fields.

Let us observe that ∂x and ∂y are spacelike real vector fields and ∂v is a null real vector but ∂u is not. By performing the following transformation:

\[ x \mapsto x, \quad y \mapsto y, \quad u \mapsto u, \quad v \mapsto v + w(x, y)/2u, \]

whose Jacobian is equal to 1, metric (4) becomes

\[ g = dx^2 + dy^2 + 2du\,dv + dw(x, y)\,d\ln|u|. \]

Since ∂x, ∂y, and ∂u are spacelike real vector fields and ∂v and ∂z, ∂u, and ∂x are null real vector fields, the above set of coordinates is the right one to apply for the Pirani’s criterion.

Since the only nonvanishing components of the Riemann tensor, corresponding to metric (5), are

\[ R_{ijju} = \frac{2}{u^3} \partial_j^2 w(x, y), \quad i, j = x, y \]

these gravitational fields belong to Petrov type N [29]. Then, according to the Pirani’s criterion, metric (5) does indeed represent a gravitational wave propagating along the null vector field ∂u.

It is well known that linearized gravitational waves can be characterized entirely in terms of the linearized and gauge invariant Weyl scalars. The nonvanishing Weyl scalar of a typical spin-2 gravitational wave is \( \Phi_4 \). Metric (5) also has as nonvanishing Weyl scalar \( \Phi_4 \).

2.2.2. Spin. Besides being an exact solution of Einstein equations, metric (5) is, for \( w/u^2 \ll 1 \), also a solution of linearized Einstein equations, thus representing a perturbation of the Minkowski metric \( \eta = dx^2 + dy^2 + 2du\,dv = dx^2 + dy^2 + dz^2 - dt^2 \) (with \( u = (z - t)/\sqrt{2} \) \( v = (z + t)/\sqrt{2} \)) with the perturbation, generated by a light beam or by a photon wave packet moving along the \( z \)-axis, given by

\[ h = dw(x, y)\,d\ln|z - t|, \]

whose nonvanishing components are

\[ h_{01} = -h_{13} = -\frac{w_x}{(z - t)} \quad h_{02} = -h_{23} = -\frac{w_y}{(z - t)}. \]

A transparent method to determine the spin of a gravitational wave is to look at its physical degrees of freedom, i.e. the components which contribute to the energy [32]. One should use the Landau–Lifshitz (pseudo)-tensor \( t^\mu_\nu \) which, in the asymptotically flat case, agrees with the Bondi flux at infinity [10].
It is worthy to remark that the canonical and the Landau–Lifchitz energy–momentum pseudo tensors are true tensors for Lorentz transformations. Thus, any Lorentz transformation will preserve the form of these tensors; this allows us to perform the analysis according to the Dirac procedure. A globally square integrable solution \( h_{\mu\nu} \) of the wave equation is a function of \( r = k_\mu x^\mu \) with \( k_\mu k^\mu = 0 \). With the choice \( k_\mu = (1, 0, 0, -1) \), we get for the energy density \( \rho_0 \) and the energy momentum \( p_0 \) the following result:

\[
16\pi \rho_0 = \frac{1}{4} \left( u_{11} - u_{22} \right)^2 + u_{12}^2, \quad p_0 = \rho_0
\]

where \( u_{\mu\nu} = \frac{d h_{\mu\nu}}{d r} \). Thus, the physical components which contribute to the energy density are \( h_{11} - h_{22} \) and \( h_{12} \). Following the analysis of [32], we see that they are eigenvectors of the infinitesimal rotation generator \( R \), in the plane \( x - y \), belonging to the eigenvalues \( \pm (2i) \). The components of \( h_{\mu\nu} \) which contribute to the energy thus correspond to spin-2.

In the case of the prototype of spin-1 gravitational waves (2.2.1), both the Landau–Lifchitz energy–momentum pseudo-tensor and the Bel–Robinson tensor [33–35] single out the same wave components and we have

\[
\rho_0 \sim c_1 (h_{0x,x})^2 + c_2 (h_{0y,x})^2, \quad p_0 = \rho_0
\]

where \( c_1 \) and \( c_2 \) are constants, so that the physical components of the metric are \( h_{0x} \) and \( h_{0y} \). Following the previous analysis one can see that these two components are eigenvectors of \( iR \) belonging to the eigenvalues \( \pm 1 \). In other words, metric (5), which is not pure gauge since the Riemann tensor is not vanishing, represents spin-1 gravitational waves propagating along the \( z \)-axis at light velocity.

Summarizing, globally square integrable spin-1 gravitational waves propagating on a flat background are always pure gauge. Spin-1 gravitational waves which are not globally square integrable are not pure gauge. It is always possible to write metric (5) in an apparently transverse gauge [6]; however, since these coordinates are no more harmonic this transformation is not compatible with the linearization procedure.

What truly distinguishes spin-1 from spin-2 gravitational waves is the fact that in the spin-1 case the Weyl scalar has a nontrivial dependence on the transverse coordinates \((x, y)\) due to the presence of the harmonic function. This could lead to observable effects on length scales larger than the \textit{characteristic length scale} where the harmonic function changes significantly. Indeed, the Weyl scalar enters in the geodesic deviation equation implying a nonstandard deformation of a ring of test particles breaking the invariance under the \( \pi \) rotation around the propagation direction. Eventually, one can say that there should be distinguishable effects of spin-1 waves at suitably large length scales.

It is also worthy to stress that the results of [36–38] suggest that the sources of asymptotically flat PP_waves (which have been interpreted as spin-1 gravitational waves [8, 10]) repel each other. Thus, in a field theoretical perspective (see appendix), \( pp \)-gravitons’ must have spin-1.

3. Back to the Tolman–Erhenfest–Podolsky problem

The motion of spinning particles in a gravitational field must be described by Papapetrou equations:

\[
\frac{D}{D\tau} \left( m v^\alpha + v_\sigma \frac{D S^{\sigma\alpha}}{D\tau} \right) + \frac{1}{2} R_{\mu\nu}^\alpha v^\sigma S^{\mu\nu} = 0,
\]

where \( S^{\mu\nu} \) is the \textit{angular momentum tensor} of the spinning particle and

\[
S^\alpha = \frac{1}{2} \epsilon^{\alpha\beta\rho\sigma} v_\beta S_{\rho\sigma}
\]
defines the spin four-vector of the particle. These equations have been extended to the case of massless spinning particles by Mashhoon \[28, 39\]. However, assuming that the spin is directed along the z-axis \(S = (0, 0, S_z)\), in the gravitational field represented by equation (5), Papapetrou equations for photons coincide with usual geodesic equations.

Furthermore, even in the previous strong field regime, geodesic motion can be described in gravitoelectromagnetic formalism, and by using the metric given by equation (5) we have:

\[
E^{(g)} = \frac{1}{2} (w_x, w_y, 0) u^{-2}, \quad B^{(g)} = \frac{1}{2} (w_y, -w_x, 0) u^{-2}.
\]

Thus, a massless test particle moving at velocity \(v = (v_x, v_y, v_z)\) in the neighborhood of the pencil, would experience the acceleration

\[
\mathbf{f}^{(g)} = -\frac{\nabla w}{u^2} \frac{1}{2} \left( w_x (c - v_z) \hat{i} + w_y (c - v_z) \hat{j} + (w_x v_x + w_y v_y) \hat{k} \right),
\]

where the speed of light \(c\) has been reintroduced. The velocity \(v\) of a photon is determined by the null geodesics equation

\[
v_x^2 + v_y^2 + v_z^2 = c^2 + (v_z - c)(w_x v_x + w_y v_y)/(z - ct) = 0
\]

which, for the photon moving at instant of interest parallel to the z-axis, admits the two solutions \(v_z = \pm c\).

It follows that, for a photon propagating antiparallel to the light beam, so that \(v = (0, 0, -c)\), the force \(\mathbf{f}^{(g)} = -\nabla w / u^2\) turns out to be nonvanishing, but for a photon propagating at instant of interest parallel to the light beam, so that \(v = (0, 0, c)\); it turns out that \(\mathbf{f}^{(g)} = 0\) and then there is no attraction or repulsion.

Thus, the lack of attraction found by Tolman, Ehrenfest, Podolsky (later also analyzed by Wheeler, Faraoni and Dumse) comes out also from the analysis of the geodesical motion of a massless spin-1 test particle in the strong gravitational field of the light.

A relevant progress could be obtained determining the gravitational field generated by two photons, each one generating spin-1 gravitational waves, but this is not an easy task. However, since helicity seems to play for photons the same role that charge plays for charged particles, according to quantum field theory (QFT) two photons with the same helicity should repel one another (see appendix).

In order to provide some numerical estimates of the gravitational interaction we are discussing, in the next section we will consider some specific sources.

4. Relativistic jets

Relativistic jets are extremely powerful jets of plasma which emerge from presumed massive objects at the centers of some active radio galaxies and quasars. Their lengths can reach several thousand or even hundreds of thousands of light years. Among different types of astrophysical jets, the most energetic ones are potential candidates to give rise to the emission of gravitational waves. For example, highly relativistic jets should be associated with some sources of gamma ray bursts (GRBs) \[40\]. The impact of an ultra relativistic jet over the spacetime metric can be studied starting from the extreme situation where the velocity of particles in the beam is assumed to be equal to the velocity of light. The jet is then represented by a beam of null particles. We shall take as our general macroscopic expression for the energy–momentum tensor corresponding to a flow of radiation of a null electromagnetic field propagating along the z-axis

\[
T_{00} = \frac{\rho}{z - ct}, \quad T_{03} = T_{30} = -\frac{\rho}{z - ct}, \quad T_{33} = \frac{\rho}{z - ct}
\]

\[5\] It is worthy to remark that \(E^{(g)}\) and \(B^{(g)}\) are orthogonal to each other, \(E^{(g)} \cdot B^{(g)} = 0\), and to the propagation direction. Since \(E^{(g)}^2 = B^{(g)}^2\), we have then a perfect analogy with the case of a null electromagnetic field.
where $\rho$ represents the amplitude of the field, i.e. the density of radiant energy at point of interest. They are just the components in the coordinates $t, x, y, z$ of the energy–momentum tensor $T = \rho du^2$ of section 2.

Introducing back the standard coupling constant of the Einstein tensor with the matter energy–momentum tensor, from section 2 we have

$$\Delta w(x, y) = \frac{8\pi G}{c^4} \rho.$$  \hspace{1cm} (7)

We assume that the energy density is a constant $\rho_0$ within a radius $0 \leq r = \sqrt{x^2 + y^2} \leq r_0$ and vanishes outside.

Thus, the source represents a cylindrical beam of null particles with width $r_0$ and constitutes a simple generalization of a single null particle.

The cylindrical symmetry implies that $w(x, y)$ will depend only on distance $r$ from the beam. A solution $w(r)$ of Poisson equation (7) satisfying the continuity condition at $r = r_0$ can be easily written as

$$w(r) = \begin{cases} \frac{4\pi G}{c^4} \rho_0 r^2 & \text{for } r \leq r_0 \\ \frac{8\pi G}{c^4} \rho_0 r_0^2 \left[ \ln \left( \frac{r}{r_0} \right) + \frac{1}{2} \right] & \text{for } r > r_0, \end{cases}$$ \hspace{1cm} (8)

so that a photon moving antiparallel and external to the beam will experience at the spacetime point $(t, x, y, z)$ a transversal gravitational attraction expressed by

$$f(g) (t, x, y, z) = -\frac{16\pi G}{c^4} \rho_0 r_0^2 \frac{r}{r^2 (z - ct)^2},$$ \hspace{1cm} (9)

where the speed of light $c$ has been reintroduced and the retardation is automatically accounted for. As a consequence of spin-1 of our wave and of QFT a photon moving parallel and external to the beam will experience at the spacetime point $(t, x, y, z)$ a transversal gravitational repulsion given by

$$f(g) (t, x, y, z) = \frac{16\pi G}{c^4} \rho_0 r_0^2 \frac{r}{r^2 (z - ct)^2}.$$ \hspace{1cm} (10)

For jets which start with a small opening angle $\theta_0 \leq 10^{-3} - 10^{-4}$ [40], it can be assumed that the width of the beam remains constant during the first stage of the jet expansion [41] and, for a beam-length $L = c\tau \sim 10^6 - 10^7$ km (a typical jet lasts $\tau \sim 100$ s), will be of the order $r_0 = L\theta_0 \sim 10^2 - 10^3$ km. The energy is of the order $E = \sim 10^{44} - 10^{45}$ J, so that $\rho_0 = E/L \sim 10^{37} - 10^{39}$ J m$^{-1}$.

Replacing these values in equation (10) and taking $G/c^4 \sim 10^{-44}$ N m$^{-1}$, we obtain for the transversal acceleration per unit length

$$a(g) (t, x, y, z) = \left| f(g) (t, x, y, z) \right| = \frac{10^{-5}}{r^2(z^2 - ct)} \text{ cm}^{-1},$$

where $r = \sqrt{x^2 + y^2}$ and $z$ are the distances, expressed in cm, between the source and the point of interest and $t$ the observation time.

5. Conclusions

Repeating the above calculations for a laser beam in an interferometer of LIGO or VIRGO type, in the formula above we would get a factor of $10^{-50}$ instead of $10^{-5}$. Then, the repulsion (as well as the attraction) turns out to be very weak. However, it could play a relevant role at the cosmic scale and could give not trivial contributions to the dark energy.
At this point, together with gravitons (spin-2), one could postulate the existence of graviphotons (spin-1) and of graviscalar (spin-0) too. Through coupling to fermions, they might give forces depending on the barion number. These fields might give [42] two (or more) Yukawa-type terms of different signs, corresponding to repulsive graviphoton exchange and attractive graviscalar exchange (range $\gg 200$ m). However, much more work must be done for a better understanding of the role played by the gravitational field of the electromagnetic radiation and/or of null particles beams in the evolution of the universe.

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Appendix

It is known from quantum field theory (QFT) that a consequence of spin-1 messengers is that particles with the same orientation repel and particles with opposite orientation attract. Indeed, the path integral formalism describing a massive vector field theory $A_\mu$ makes use of the partition function which can be represented by using the Feynman path integral

$$Z(J) = \int DA \exp \left[ \frac{i}{\hbar} S(A, J) \right],$$

where

$$S(A, J) = \int d^4x \left( \left( \partial^2 + m^2 \right) g^{\mu\nu} - \partial^\mu \partial^\nu \right) A_\nu + J^\mu A_\mu$$

is the classical action.

We also have (see for instance [24])

$$Z(J) = \exp \left[ \frac{i}{\hbar} W(J) \right],$$

with

$$W(J) = -\frac{1}{2} \int d^4x d^4y J^\mu(x) D_{\mu\nu}(x - y) J^\nu(y),$$

where $D_{\mu\nu}(x)$ is the Green function defined by

$$\left( \left( \partial^2 + m^2 \right) g^{\mu\nu} - \partial^\mu \partial^\nu \right) D_{\mu\nu}(x) = \delta^{(4)}(x).$$

Taking the Fourier transform, we get

$$W(J) = -\frac{1}{2} \int d^4k J^{\mu\nu}(k) D_{\mu\nu}(k) J^{\nu}(k),$$

where

$$D_{\mu\nu}(k) = -\frac{g_{\mu\nu} + k_\mu k_\nu/m^2}{k^2 - m^2}$$

is called the propagator for the massive vector field $A_\mu$.

A simple calculation [24] of the potential energy between like charges gives

$$U \sim \exp \left( \frac{-mr}{4\pi r} \right),$$

so that $dU/dr < 0$ and the force between like charges turns out to be repulsive, as we already know from electrodynamics.
References

[1] Morrison P and Gold T 1958 Essays on Gravity (New Boston, NH: Gravity Research Foundation) pp 45–50
[2] Morrison P 1958 Ann. J. Phys. 26 358
[3] Nieto M M and Goldman T 1991 Phys. Rep. 205 221
[4] Fabbrichesi M and Roland K 1992 Nucl. Phys. B 388 539
[5] Peres A 1959 Phys. Rev. Lett. 3 571
[6] Stephani H 1996 General Relativity: An Introduction to the Theory of the Gravitational Field (Cambridge: Cambridge University Press)
[7] Stephani H, Kramer D, MacCallum M, Honselaers C and Herlt E 2003 Exact Solutions of Einstein Field Equations (Cambridge: Cambridge University Press)
[8] Canfora F, Vilasi G and Vitale P 2002 Phys. Lett. B 545 373
[9] Canfora F and Vilasi G 2004 Phys. Lett. B 585 193
[10] Canfora F, Vilasi G and Vitale P 2004 Int. J. Mod. Phys. B 18 527
[11] Canfora F, Parisi L and Vilasi G 2007 Theor. Math. Phys. 152 1069
[12] Vilasi G 2007 J. Phys.: Conf. Ser. 87 012017
[13] Ferrari V, Pendenza P and Veneziano G 1988 Gen. Rel. Grav. 20 1185
[14] Ferrari V and Ibanez J 1989 Phys. Lett. A 141 233
[15] Tolman R, Ehrenfest P and Podolsky B 1931 Phys. Rev. 37 602
[16] Wheeler J 1955 Phys. Rev. 97 511
[17] Barker B, Bhatia M and Gupta S 1967 Phys. Rev. 158 1408
[18] Barker B, Gupta S and Haracz R 1966 Phys. Rev. 149 1027
[19] Faraoni V and Dumse R M 1999 Gen. Rel. Grav. 31 9
[20] Brodin G, Eriksson D and Maklund M 2006 Phys. Rev. D 74 124028
[21] Christodoulou D 1991 Phys. Rev. Lett. 67 1486
[22] Thorne K 1992 Phys. Rev. D 45 520
[23] Mashhoon B 2003 Gravitoelectromagnetism: a brief review arXiv:gr-qc/0311030v2
[24] Zee A 2003 Quantum Field Theory in a Nutshell (Princeton, NJ: Princeton University Press)
[25] Sparano G, Vilasi G and Vinogradov A 2001 Phys. Lett. B 513 142
[26] Sparano G, Vilasi G and Vinogradov A 2002 Diff. Geom. Appl. 16 95
[27] Sparano G, Vilasi G and Vinogradov A 2002 Diff. Geom. Appl. 17 15
[28] Mashhoon B 1975 Ann. Phys. 99 254
[29] Zakharov V 1973 Gravitational Waves in Einstein’s Theory (New York: Halsted Press)
[30] Petrov A 1969 Einstein Spaces (New York: Pergamon)
[31] Penrose R 1960 Ann. Phys. 10 171
[32] Dirac P A M 1975 General Theory of Relativity (New York: Wiley)
[33] Bel L 1958 C. R. Acad. Sci., Colon. 247 1094
[34] Bel L 1958 C. R. Acad. Sci., Colon. 248 1297
[35] Robinson I 1997 Class. Quantum Grav. 14 A331
[36] Aichelburg A and Sexl R 1971 Gen. Rel. Grav. 2 303
[37] Felber F S 2008 Exact ‘antigravity-field’ solutions of Einstein’s equation arxiv.org/abs/0803.2864
[38] Felber F S 2010 Dipole gravity waves from unbound quadrupoles arxiv.org/abs/1002.0351
[39] van Holten J W 2008 The gravitational field of a light wave arXiv:0808.0997v1
[40] Bini D, Cherubini C, Geralico A and Jantzen T 2006 Int. J. Mod. Phys. D 15 737
[41] Piran T 2004 Rev. Mod. Phys. 76 1143
[42] Piran T 2000 Phys. Rep. 333 529–53
[43] Mezard P 1999 Prog. Theor. Phys. Suppl. 136 300–20
[44] de Rey Noto E C, de Araujo J C N and Aguiar O D 2003 Class. Quantum Grav. 20 1479–88
[45] Stacey F, Tuck G and Moore G 1987 Phys. Rev. D 36 2374