Mathematical model of the tool for reverse trimming of non-rigid products

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Abstract. The article deals with the solution of the problem of ensuring the accuracy of non-rigid parts processing. For this purpose, it is proposed to use the developed tool for backcutting. In modern conditions of competition and struggle for the consumer manufacturers are increasingly resorting to improving the finished product quality characteristics as well as to using complex configuration parts in their mechanisms in order to reduce the mating parts number and improve the individual units reliability. These trends are the reason to the machining processes intensification, the new tools, metalworking equipment and devices creation. At present one of the actual tasks requiring non-standard solutions is the non-rigid parts processing, in particular the processing of surfaces that are difficult to reach by standard metal-cutting tools and tooling, such as the inner butt ends of the levers eyelets. To solve this problem the original reverse trimming tool was designed. This tools’ mathematical model is given in the article.

1. Introduction
Processing by the tool for reverse trimming ends has a number of features [1]:
- variable cutting speed along the cutting part length;
- difficult swarf removal;
- low rigidity of the tool and the technological system as a whole. In the processing structural materials process the tool is under the influence of significant axial pressing forces $P_0$ and torque $T$.

These factors are limit the assignable cutting modes and are the main ones when considering the restrictions’ set in the optimization problem.

2. Formation of technical limitations
The tool consists of a tool head, a mandrel, hardmetal plates and a movable joint. There is also a channel for coolant supply and the possibility of bringing the cutter into working and non-working position to automate the processing process (figure 1).

An example of the workpiece is shown in figure 2.

The limit on the cutting capabilities (durability) of the tool establishes the relationship between the cutting speed $V$ and the index of machinability $V_T$ (cutting speed corresponding to a certain tool life): $V \leq V_T$. Using known dependencies [2]:

$$V = \frac{\pi D n}{1000}, \quad V_T = \frac{C_v K_v D q}{S T m S Y},$$

where $C_v$, $K_v$ are constants that taking into account the conditions of the tool in the reference and specific variants; $D$ is hole diameter, mm; $T$ is period of resistance, min; $S$ is feed, mm / Rev; $n$ -
rotational speed, min\(^{-1}\); \(q\), \(m\), \(y\) are exponents reflecting the influence of diameter, resistance and feed on the cutting speed.

![Figure 1](image1.png)

**Figure 1.** Designed tool for machining holes in non-rigid parts.

![Figure 2](image2.png)

**Figure 2.** The processed lever holes.

We bring this restriction to an explicit form [3]:

\[
nS^y \leq \frac{318D^{q-1}c_Kp}{T^m}.
\]  

(2)

The machine power limit restricts the torque that occurs during machining and affects the machine spindle:

\[
T = 10C_mS^y_mD^{q_m}K_p;
\]

\[
T_c = \frac{975 \cdot 10^3N_c\eta}{n}.
\]  

(3)

where \(C_m\) is coefficient that takes into account the influence of processing conditions; \(K_p\) is general correction coefficient that takes into account the actual processing conditions and depends on the
workpiece materials; $y_m$, $q_m$ are indicators of the variables degree; $N_c$ is power of the electric motor of the drive of the main movement, kW; $\eta$ - efficiency coefficient.

Equating the right parts of the last two dependencies and making the appropriate transformations, we get

$$nS^\gamma \leq \frac{975 \cdot 10^3 N_c \eta}{c_m D q m K_p}.$$  \hspace{1cm} (4)

The torque limitation guarantees the tool integrity in the presence of stresses in its material.

To carry out the cutting process, it is necessary that the condition is:

$$P_o = 10C_p S^\gamma D^q K_p \leq [P_c],$$  \hspace{1cm} (5)

where $P_o$, $[P_c]$ are the axial and maximum cutting force allowed by the machine feed mechanism accordingly, $N$; $C_p$, $q_p$, $y_p$, $K_p$ are reference values.

The restriction because of the tool strength:

$$\tau_c = \frac{1.737F}{W} = \frac{1.73 \cdot 10 C_p S^\gamma D q m K_p}{0.02d^3} \leq \frac{\sigma_p}{k},$$  \hspace{1cm} (6)

where $\tau_c$ is the total tension equal to the sum of the normal voltage by force $P_o$ and the tangent voltage by $T$, MPa; $\sigma_q$ is temporary drill material resistance to rupture, MPa; $k \approx 1,5 ... 2,0$ is factor of safety; $W$ is the tool resistance moment, mm$^3$. After the corresponding transformations we get

$$S^\gamma \leq \frac{\sigma_p 0.02 D^3}{1.73 \cdot 10 C_p D q m k K_p}.$$  \hspace{1cm} (7)

The tool rigidity affects to the processing accuracy and the axial force should not exceed the permissible axial force $[P_c]$ by the tool rigidity:

$$10C_p S^\gamma D^q K_p \leq \frac{K_y E l}{L^2},$$  \hspace{1cm} (8)

where $K_y = 2,46$ is stability coefficient, $E$ – the tool material elasticity modulus, MPa; $l = 0,039 D^4$ is the tool inertia moment, mm$^4$; $L$ is the tool overhang length.

Performing the restriction guarantees the tool integrity in conditions of possible loss of longitudinal stability.

Restrictions imposed by the machine kinematics establish the relationship between calculated values of the rotational speed $n$ and feed $S$ with the allowable the machine kinematics. These conditions can be written as the following inequalities:

$$n_{cmin} \leq n \leq n_{cmax}, S_{cmin} \leq S \leq S_{cmax},$$  \hspace{1cm} (9)

where $n_{cmin}$, $n_{cmax}$ are the minimum and maximum values of rotation frequency accordingly, $S_{cmin}$, $S_{cmax}$ are the minimum and maximum values of the tool feed accordingly.

3. The target function assignation

For a large number of production situations when the calculations use the values of the economic periods of resistance, one should choose the lowest basic time or specific processing costs as the target function $C_y$.

$$t_o = \frac{L_o}{S n} = \frac{l_1 + l_2}{S n},$$  \hspace{1cm} (10)

$$C_y = \frac{a_t}{q} \left( \frac{l_1 + l_2}{S n} \right),$$  \hspace{1cm} (11)

$$n_o = e^{x_1}, S_o = e^{x_2},$$

where $L_o$ is processing length, mm; $l$ is hole length, mm; $l_1$, $l_2$ are values of the tool cutting and tool overrun determined analytically and according to reference data: $a_1$ is cost of machine minutes of the main equipment; $q$ is cutting parameter ($q = \frac{\pi d^2}{4}$ is area of removed metal); $a_2 = a_1 t_c + \frac{a_t n_o x}{z + 1} + \frac{b}{z + 1}$ is tool costs per durability period; $a_2$ is cost of machine minute of the grinding equipment; $b$ is the cost of the new tool taking into account transport costs and sales of waste; $t_c$ is tool change time, min; $t_n$ is regrinding tool time, min; $z$ is the number of tool overflows until it is completely worn out.
4. Development of the mathematical model of the cutting process

The transformation of the original model is carried out by taking the logarithm of the constraints and the target function and obtaining the corresponding linear forms. After take the logarithm of inequality (2) we obtain:

\[ \ln n + y \ln S \leq \ln \left( \frac{31BD^{q-1}C_vK_m}{T_m} \right). \]  

(12)

Introduce notation

\[ \ln n = x_1, \ln S = x_2, \ln \left( \frac{31BD^{q-1}C_vK_m}{T_m} \right) = b_1. \]

By substitution we get a linear form of inequality

\[ x_1 + yx_2 \leq b_1. \]  

(13)

After transforming the remaining inequalities, we obtain the linear inequalities system and a linear function for the case of a drilling operation in the form

\[ \begin{cases} x_1 + yx_2 & \leq b_1 \\ x_1 + ymx_2 & \leq b_2 \\ yx_2 & \leq b_3 \\ ymx_2 & \leq b_4 \\ yp_x2 & \leq b_5 \\ x_2 & \leq b_6 \\ x_2 & \leq b_7 \\ x_1 & \leq b_8 \\ x_1 & \leq b_9 \end{cases} \]

(14)

The optimal speed and feed rates are calculated using the following equations:

\[ n_o = e^{x_1}, S_o = e^{x_2}. \]  

(15)

5. Results and conclusions

The given form of the mathematical model is the cutting process description in the case of drilling regardless of the type of machine and processing conditions. When changing the individual operations conditions, only the free terms \( b_1, \ldots, b_9 \) and the coefficients values \( y, y_m, y_p \) will be different. To determine the optimal modes using the model (14) it is necessary to find positive values \( x_1, x_2 \) at which the objective function' linear form will take the greatest value.

References

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