An Analytical Model for Embedded Foundations of Building Structures Subjected to Horizontal Loads

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Abstract
This paper describes an analytical model for predicting the load–displacement relationship of structures with embedded foundations subjected to horizontal loads. The proposed analytical model uses discrete springs to represent soil resistance (passive resistance, side friction, and base friction) and considers non-linear behavior using a hyperbolic function based on tests conducted by the authors. A method of estimating the initial tangent modulus of the hyperbola is also presented. Numerical analyses are carried out using the proposed model for two horizontal loading tests on actual caissons assumed to represent the basement of a building. The analytical results are in reasonably good agreement with the measured results.

Keywords: numerical analysis; horizontal load; embedment; foundation; hyperbola

1. Introduction
For building structures with embedded foundations, embedded walls have been recognized to resist horizontal loads except in cases where liquefaction occurs in the surrounding soils (e.g., Building Center of Japan, 1995; Tamura et al., 2002). The horizontal resistance of embedded foundations is influenced by several factors, including passive resistance, side friction, and base friction. Previous studies have considered the passive resistance of embedded foundations (e.g., Zhang et al., 1998; Tokimatsu et al., 2003); however, it appears that few studies have investigated the total response of the embedded foundation while taking the side friction into account (e.g., Gazetas et al., 1987; Gadre and Dobry, 1998).

Considering the current trend in the structural design of buildings toward the limit state design, it is important even in the field of foundation design to predict the load–displacement relationship of foundations subjected to horizontal loads. In embedded foundations, it is thought that the side friction contributes to a reduction in the total horizontal displacement within an allowable level. It is therefore necessary to clarify the frictional behavior between embedded foundations and surrounding soils.

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springs. The focus of this paper is the horizontal resistance of the embedded foundation, in particular an evaluation of the soil springs associated with the embedded part. Numerous studies have considered soil springs used for piles, including a detailed description provided by the Architectural Institute of Japan (2001). Therefore, the present paper does not deal with the evaluation of soil springs for piles.

2.2 Non-linear Soil Springs

Side friction

We previously conducted a series of centrifuge modeling tests for the side friction of embedded walls in dry Toyoura sand (Watanabe et al., 2003). Fig. 2 shows a typical test result under cyclic loading. It was found that the friction and horizontal displacement curves could be approximated as a hyperbolic function, as shown in Fig. 3. A similar behavior was also obtained from horizontal loading tests of existing caissons in clayey soil (Watanabe et al., 2006; see Fig. 4). In the present study, the relationship between side friction and horizontal displacement is assumed to be a hyperbolic function:

\[ F = \frac{\delta_x}{1/k_F0 + \delta_x/F_u} \]  

where \( F \): frictional force, \( \delta_x \): horizontal displacement, \( k_F0 \): initial tangent modulus of the hyperbola, \( F_u \): ultimate frictional force.

- For cyclic loading:

\[ F = \frac{(\delta_x - \delta_u)}{1/k_F0 + \alpha/2F_u \cdot (\delta_x - \delta_u)} + F_i \]

where \( \alpha \): coefficient for the loading direction (1.0 for a positive direction and –1.0 for a negative direction), \( F_i \): frictional force at a given cyclic loading point, and \( \delta_u \): horizontal displacement at a given cyclic loading point.

Passive resistance and base friction

Based on field tests and centrifuge modeling tests for passive resistance and base friction conducted by the authors, it was found that the load–displacement relationships were also approximated by a hyperbolic function (Watanabe et al., 2001, 2006; see Fig. 4; Nagao et al., 2002, 2004). The non-linearity of each soil spring is also specified by a hyperbolic function; however, the field tests for passive resistance were conducted under monotonic loading. For passive resistance, it is thought that the behavior under cyclic loading is not specified by Equation (2). In this paper, only Equation (1) is applied for passive resistance.

Determination of parameters

In Equations (1) and (2), the initial tangent modulus and ultimate resistance are fixed values. The latter value can be determined from the shear strength of the soil or the passive earth pressure.

The initial tangent modulus of each soil spring is separately estimated based on elastic theory, as shown in Fig. 5. This approach is taken because it is thought...
that the surrounding soil is in a linearly elastic state during the initial loading stage. The approach is similar to some elastic methods (e.g., the thin layered method; Architectural Institute of Japan, 2006). Given that all of these approaches are based on the elastic theory, they contain similarities. It is advisable that the elastic approach is only applied to the initial stage of the load–displacement curve and that the evaluation of non-linearity is kept relatively simple.

For example, the procedure followed in estimating the side friction area is summarized as follows.

1) A rigid plate is embedded in an elastic half-space, as shown in Fig.6. The response of the soil in this model can be expressed by Mindlin’s solution for horizontal displacement (Mindlin, 1936).

2) The soil is divided into \( n \) sections in the \( x \)- and \( z \)-directions, as shown in Fig.6. For each section, the following assumptions are made: (a) the reactions within the soil are evenly distributed throughout the section, (b) the sum of the reacting force is located at the center of each section, (c) the evaluation point of soil deformation is located at the center of each section, and (d) in the case where the evaluated deformation point corresponds to the reaction force point, the section is subdivided into four sections, with the reacting force point being located at the center of each subsection.

If the reaction force of the rigid plate under an external horizontal displacement \( \delta_x \) is \( Q_x \), then the relationship between \( \delta_x \) and \( Q_x \) can be expressed as

\[
\begin{bmatrix}
\delta_{x1} \\
\delta_{x2} \\
\vdots \\
\delta_{xn}
\end{bmatrix} = \frac{1}{G_s} \begin{bmatrix}
I_{11} & I_{12} & \cdots & I_{1n} \\
I_{21} & I_{22} & \cdots & I_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
I_{n1} & I_{n2} & \cdots & I_{nn}
\end{bmatrix}
\begin{bmatrix}
Q_{x1} \\
Q_{x2} \\
\vdots \\
Q_{xn}
\end{bmatrix}
\]

(3)

where \( I \) (\( i=1, \ldots, n \)): number of the section evaluated, \( G_s \): shear modulus of the soil, and \( I \): displacement influence factors given by Mindlin’s solution for horizontal displacement (Mindlin, 1936). Here, \( Q_{x1} = q_{x1} \times dA_1, Q_{x2} = q_{x2} \times dA_2, \) and for the \( i \)th element, \( Q_{xi} = q_{xi} \times dA_i \) (\( q_{x} \): uniform reaction stress \( dA \): area).

Equation (3) may be written as

\[
\{ \delta_x \} = \frac{1}{G_s} [I_x] \{ Q_x \}
\]

(4)

where \([I_x]\) is the soil flexibility matrix and \( \{ \delta_x \} \) and \( \{ Q_x \} \) are the vectors of horizontal displacements and reaction forces, respectively. The inverse of the soil flexibility matrix is the soil stiffness matrix \([I_x]^{-1}\), and Equation (4) can be expressed as

\[
\{ Q_x \} = G_s [I_x]^{-1} \{ \delta_x \}
\]

(5)

where \( \delta_{x1} = \delta_{x2} = \cdots = \delta_{xn} \) for a rigid plate.

Having determined \( \{ \delta_x \} \) and \( [I_x]^{-1} \), this set of linear simultaneous equations can be solved to give the nodal point values of the reaction forces \( \{ Q_x \} \).

\[
k_{F0} = F/2\delta_x
\]

(6)

where \( F = Q_{x1} + Q_{x2} + \cdots + Q_{xn} \).

The initial tangent modulus of the hyperbola for passive resistance and base friction can also be estimated by following a procedure similar to that described above. The displacement influence factors are given by Mindlin’s solution (Mindlin, 1936), Cerrutti’s solution (Cerruti, 1882), and their applications (Nagao et al., 2004; Tsuchiya et al., 1995; see Fig.5.).

3. Analyses of an Embedded Foundation

To assess the applicability of the model, numerical analyses are carried out on two horizontal loading tests on existing caissons.

3.1 Outline of Horizontal Loading Tests

Fig.7 provides an outline of the existing caissons for the loading tests and typical soil profiles. Two static horizontal loading tests were conducted on reinforced concrete caissons of the same size (about 4×5 m in section and 30 m in length), which were constructed about 40 years ago. The caissons are termed Type-C1 and Type-C2. For Type-C1, the passive resistance was reduced to measure frictional resistance between the sides of the caisson and the...
surrounding soil. Additional detailed information regarding the tests and soil conditions can be found in a previous report (Watanabe et al., 2006).

3.2 Outline of the Analytical Model

Fig. 8 shows the analytical model, which consists of beam elements for the caisson and discrete soil springs. The caisson is divided into 17 linear elements in the vertical direction. The soil springs are connected to nodes set at both ends of each beam element. Five spring types are employed, as shown in Fig. 8. For side friction, passive resistance, rotation of the caisson, and base friction, the non-linearity of the soil is taken into consideration by using the hyperbolic function described previously. For rotation of the caisson, frictional resistance in the vertical direction between the soil and the front rear of the caisson is considered as rotational resistance. In this study, the non-linearity of the rotational spring is also evaluated by the hyperbolic function. The initial tangent modulus is estimated based on Mindlin's solution for vertical displacement (Mindlin, 1936).

Non-linear analyses were performed using the bi-linear response of soil springs (Japan Road Association, 2002; Japanese Geotechnical Society, 2001) to compare the two models. For bilinear models, the value of the second inclination is fixed at $1 \times 10^{-4}$ times that of the initial inclination.

3.3 Input Parameters

The input parameters of the hyperbola are the initial tangent modulus and the ultimate resistance. For the bilinear model, the first inclination and the yield resistance are fixed.

The ultimate and yield resistances for each node are evaluated using the conventional method (Japan Road Association, 2002), based on the shear strength and the SPT N-value (Table 1.).

The initial tangent modulus of the hyperbola is estimated using the method described above, based on elastic theory. In this estimation, the soil is divided into 25 cm grid spans in each direction for each side. To estimate the initial tangent modulus at each node, the reaction forces in Equation (6) are chosen from the calculated nodal point values of the reaction force.

Considering the strain level of the soil, the initial
shear modulus $G_0$ obtained from PS logging should be adopted as the value of $G_s$ in Equation (5). In the conventional method (the bilinear model), various coefficients of the model are estimated based on Young's modulus of the soil $E_m$ obtained from pressure-meter tests (Japan Road Association, 2002). The analysis case and the input parameters for each soil layer are summarized in Table 2. For $G_o$ (using the proposed method) and $E_m$ (using the conventional method), the upper and lower values are adopted as input data in those soil layers for which the soil parameters vary widely within the layer.

The properties of the caisson body are listed in Table 3. The compressive strength and Young's modulus of the concrete are the mean values of the results of compressive tests conducted on the core samples.

### 3.4 Analytical Results and Discussion

Non-linear analyses of the Type-C1 (reduced passive resistance) and Type-C2 caissons were performed using the incremental approach.

**Horizontal load–displacement relationship**

Comparisons between predicted and measured horizontal load–displacement relationships are shown in Figs.9. and 10. for Type-C1 and Type-C2, respectively. In the figures, "Measured" indicates test results, "Proposed" indicates the results of the proposed analysis using hyperbolic soil springs, and "Conventional" indicates the results obtained via conventional analysis using bi-linear soil springs. "Yielding Load" was evaluated based on the results of horizontal loading tests (Watanabe et al., 2006). The objective of the analyses is to examine the applicability of the hyperbolic soil springs. The tests reveal that the bending behavior of the caisson body is the predominant behavior once the horizontal load exceeds the yielding load. Accordingly, the plots in Figs. 9. and 10. mainly show the results obtained over the range of loads for which the caisson body behaves elastically.

Fig. 9. reveals that the predicted behavior is similar to the measured behavior, although the proposed curves lie below the measured curve. It is also clear that the curves obtained using the conventional method tend to underestimate the measured value and show wide variation. The broken line represents the side friction calculated from the sum of the response of the side springs in the case of H-Uc. The side friction carried the majority of the horizontal load.

Fig. 10. shows that similar trends are also observed for Type-C2 (i.e., passive resistance not reduced); however, the degree of underestimation and difference appears to be larger in the conventional method. The figure shows that about 50% of the loads are carried by side friction.

**Distribution of horizontal displacement**

The distributions of the horizontal displacement of Type-C1 and Type-C2 are shown in Figs. 11. and 12., respectively, in which the measured results are obtained from inclinometers. Figures 11 and 12 are then plotted at horizontal loads of about 1.8 and 1.4 MN.

The figures reveal that the shapes of the proposed and measured profiles are in reasonable agreement, but that the results of the conventional method are overestimated. The horizontal load–displacement curves predicted by the proposed method lie below the measured ones, especially at lower load levels. Moreover, it is considered that the initial stiffness of the soil springs might be underestimated. In estimating the initial tangent modulus of the soil springs, the distribution of displacement was assumed to be constant throughout the embedded length; this assumption is considered to be one of the main reasons for the underestimation. Additional analyses (see Table 2., Case H-Ud) were performed to clarify the effects of the assumption. In the analyses, the distribution of the displacement in Equation (6) was assumed to decrease linearly from the top to tip of the embedded length, where the value of the tip was zero. These findings are
also reflected in Figs. 9. and 10., respectively. These results show larger values than the Case H-Uc, and are in agreement with the measured results.

Although there may be scope for further study, the results of the current study demonstrate that the predicted results are in reasonable agreement with the measured results, thereby confirming the validity of the analytical model.

4. Conclusion

This paper presents an analytical method for predicting the horizontal load-displacement relationship of buildings with embedment. A simplified analytical method was employed, with beam elements used to represent embedded foundations and discrete springs used to represent soil.

This paper focused on an evaluation of the non-linearity of the soil springs. The non-linearity was specified using a hyperbolic function on the basis of a series of centrifuge tests and the results of field tests conducted by the authors. In the hyperbolic function, the estimation of the initial tangent modulus is important. The authors presented estimating procedures based on elastic theory. It was also proposed that the initial shear modulus of soil $G_0$ obtained from PS logging was suitable for use as the value of shear modulus of soil $G_s$ in estimating the initial tangent modulus.

To examine the applicability of the proposed model, numerical analyses were carried out on two horizontal loading tests on actual existing caissons assumed as the basement of a building. The caisson behaviors obtained from the analyses were in reasonable agreement with the measured results, thereby confirming the validity of the analytical model.

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