Operating Fiber Networks in the Quantum Limit
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Abstract—We consider all-optical networks from a quantum perspective. We show that optimal quantum receivers allow a ~ 57% decrease in energy consumption of all-optical amplifiers. Furthermore, we prove that quantum receivers allow for a logarithmic scaling of the system capacity with the number of pulses per second, while standard Shannon-type systems are limited by the transmit power. Based on our findings we argue for a new approach to optical communication network design, wherein in-line amplifiers are operated at very low gains and in conjunction with high-spectral-bandwidth fiber and a quantum receiver, enhancing data transmission in a practically relevant quantum limit.

Index Terms—Optical amplifiers, optical design, optical devices, optical fiber communication, quantum channels, quantum communication, quantum information science, quantum networks, quantum optics.

I. INTRODUCTION

THE progressive development of fiber networks is a driver of productivity growth in modern societies [1]. As a result, network electricity consumption has been estimated as \( \approx 1.7\% \) of the global consumption, with a growth rate of \( \approx 10\% \) per year [2] in 2012. Thus, energy efficiency has become a key network design principle and one of the main motivations to switch from electrical to optical information-processing in networks, e.g., via the introduction of integrated-photonics circuits [3]. Interestingly, such circuits are also one of the major candidate platforms for quantum information processing (QIP), a field which has long promised to boost the capabilities of classical information technology [4], [5].

However, the potential advantages of QIP for classical data transmission reported so far can be described as “large gains at small practical value”, e.g., the reduction of peak signal energy for deep-space communication and the increase of transmission rate in short-distance communication [6], [7], [8], [9], [10], [11], [12].

In this work we change such perspective, proving that QIP can provide a significant simplification in the design of classical-data-transmission systems that employ optical amplification; this is thanks to a net reduction of the output signal power, until even the hypothetical removal of the amplifiers themselves. Optical amplification is a widespread system design, wherein amplifiers are placed along the path to counteract propagation losses that progressively reduce the signal’s strength, ensuring that information can be reliably decoded by the receiving side [13], [14], [15], [16]. On top of the considerable energy cost of producing high-power input signals, such amplifiers consume a relevant amount of energy, e.g., for a 1100 km-link their cost is estimated as \( \approx 13\% \) of the total energy consumption [15]; furthermore, optical amplification (either discrete or distributed) generally complicates system design [13], [14].

In this setting, we show that a current communication system employing a quantum optimal joint-detection receiver (OJDR) [17], [18], [19], [20], [21], [22], [23] can transfer the same amount of bits of a classical optimal single-symbol receiver (OSSR) by spending less energy per signal, thanks to a net reduction of amplifier output power. The difference between these two receiver designs lies in the way they manipulate the signals: the OSSR performs individual operations on the received data, which is particularly pronounced for signals with low added noise, by extracting the same amount of information from signals of lower energy with respect to an OSSR.

We start by providing a heuristic motivation for the reduction of the signal output power and the number of amplifiers, based on a striking result: in a quantum communication system employing OJDR, the strategy of turning on all the available amplifiers can be sub-optimal, contrarily to what happens for an OSSR. This property, which favours lower added noise at the cost of a smaller signal amplification, can be used to reduce the number of amplifiers and the net signal output power, while increasing the communication rate. We then refine our analysis by considering tunable amplifier gains, which measure the strength of...
the amplification process and provide a conservative estimate of its energy cost. We formulate the problem of minimizing the total energy cost with respect to the gain profile with OJDR, while attaining a larger communication rate than the OSSR. We provide an algorithm to solve this problem and employ it to predict the maximum reduction of energy consumption in a wide range of distances and numbers of amplifiers.

We show that energy savings arising from the use of an OJDR can be as large as $\approx 100\%$. However, this happens only in settings where the communication system benefits very little from the use of amplifiers in the first place. Instead, in the practically relevant setting where the amplifiers enhance the data-rate at least by a factor of 2, we identify situations where the use of an OJDR allows record energy savings up to 57\%.

To the best of our knowledge, the minimization of energy consumption of amplifiers through OJDR technology has been analyzed only in a couple of recent works, though from different perspectives: [11] reported a constant increase of the communication rate using the OJDR instead of the OSSR in an optically-amplified multi-span link; whereas [10] studied the energy-efficiency of a system employing continuous amplification in the limit where the OSSR approximates the OJDR, which fails to capture the full advantage of the OJDR. We combine both approaches to quantify the energy-efficiency advantage of the OJDR on a multi-span optically-amplified link.

Finally, we provide a theoretical justification for a novel approach to future data transmission network design and detail the first implementable steps to its realization. To this purpose, we point out the strikingly different growth of data transmission capacities of both OSSR and OJDR system-design approaches: under any power limit and any thermal noise level, measured in photons per second, the link capacity grows unbounded with a logarithmic dependence on the baud-rate when an OJDR is used as the receiver. In sharp contrast, the link capacity is upper-bounded in terms of the transmit power when an OSSR is used. Our technological hypotheses can be tested with novel system design concepts resting on the works of Guha et al. [8], [19] and follow-up works of Rosati et al. [9] and Klimek et al. [24]. For the optical transmission medium, we suggest hollow-core fiber [25], [26].

II. AMPLIFIED FIBER CAPACITY WITH CLASSICAL AND QUANTUM DECODERS

A. Communication System Model

Consider a multi-span transmission line comprising an optical-fiber link of length $L$ and $K$ optical-amplifier modules placed at equally spaced intervals of length $L/K$\(^1\) (see Fig. 1). Each of these components is modelled by a quantum bosonic Gaussian channel, taking as input a quantum state of the electromagnetic field. For such channels, the maximum information transmission rate can be attained by encoding classical information into sequences of optical coherent states [27], [28], [29] and

\[^1\]The techniques we develop can be applied to amplifiers with arbitrary spacing. Since this does not mark any qualitative change in our results, we restrict to equal spacing for simplicity.

performing a collective quantum measurement of the received sequences, to recover the classical message [17], [18], [19], [20], [21], [22], [23], [30].

For our purposes, the action of each transmission-line component can be described in terms of input-output relations for the mean energy of the field comprised of signal and noise \(^2\):

1) transmission on an optical-fiber segment is modelled by a pure-loss channel,

$$n \mapsto \eta \cdot n,$$

with $\eta := \exp\{-\alpha L\}$ and $\alpha \geq 0$ a system-specific coefficient;

2) optical amplification is modelled by a quantum-limited amplifier channel,

$$n \mapsto G \cdot n + G - 1,$$

with $G \geq 1$ the amplifier gain and $G - 1$ the minimum photon-noise addition allowed by quantum mechanics.

The action of the entire transmission line is then obtained by repeatedly applying these two channels: at the end of the $i$-th segment, the input signal (plus noise) to the next segment has

![Fig. 1. Depiction of the classical (top) and quantum-enhanced (bottom) communication systems studied: the optical-fiber link is divided into $K$ segments (blue lines) interleaved by optical amplifiers (yellow triangles). Classical information is encoded in a quantum state and sent on the transmission link. Classical system (top): the transmission on each segment $i$ is modelled by a pure-loss bosonic channel of attenuation $\eta$ followed by a quantum-limited bosonic amplifier channel of maximum gain $G_{\text{max}}$. The last segment does not benefit from amplification and it is followed by a classical single-symbol receiver (OSSR) that extracts the classical message by measuring one symbol at a time. Quantum-enhanced system (bottom): the transmission on each segment is still characterized by a pure-loss channel of attenuation $\eta$, followed by an amplifier channel. However, in this case the amplifiers have non-maximum gains $G_i \leq G_{\text{max}}$, hence they add less noise to the signal and spend less pump energy. Crucially, the OSSR is substituted by a quantum multi-symbol receiver (OJDR) that is able to extract more information from lower-energy signals with respect to the former. A combination of smaller gains and better detection thus enables to spare energy while attaining the same communication rate. In particular, a pretty good choice consists in taking a decreasing sequence of gains, $G_{\text{max}} \geq G_1 \geq G_2 \geq \cdots \geq G_{K-1}$ (here depicted with lighter colours along the line). A sub-optimal approximation of the OJDR is possible with linear optics [8], [9], [24].]

Throughout the main text, except in Results III-C, we set $\hbar \omega = 1$ for some fixed signal frequency $\omega$, effectively equating the mean energy of the field with its mean number of photons.
energy $\tau_i \cdot n + \nu_i$, with

$$\tau_i = G_i \cdot \eta \cdot \tau_{i-1}, \quad \nu_i = G_i \cdot \eta \cdot \nu_{i-1} + G_i - 1,$$

and initial values $\tau_0 = 1, \nu_0 = 0$. The resulting coefficients for the entire transmission line are $\tau := \tau_K, \nu := \nu_K$.

Communication performance is quantified via the spectral efficiency (SE), i.e., the maximum number of bits transmittable per unit time and frequency, which in our narrowband case coincides with the channel capacity. Crucially, depending on the receiver’s capabilities, this quantity can take different values; the Shannon SE [33], attainable by performing heterodyne detection on each received signal,

$$S_{sh}(n, G^K) := \log \left(1 + \frac{\tau \cdot n}{1 + \nu} \right),$$

and the Holevo SE [30], [34], attainable by performing a collective quantum measurement (i.e., the OJDR for this channel) on the received sequences,

$$S_{ho}(n, G^K) := g(\tau \cdot n + \nu) - g(\nu),$$

where $g(x) := (x + 1) \log(x + 1) - x \log x$. Here and in the rest of the article we use the notation $G^K = (G_1, \ldots, G_K)$.

We stress that the realization of a collective measurement that attains (5) is still an open problem in quantum information theory, with several theoretical proposals that we refer to as OJDR [17], [18], [19], [20], [21], [22], [23]. Specifically, the all-optical Hadamard receiver can approximate the OJDR for low received signal energy [8], [9] and no noise, employing only passive optical elements with minimal expected energy cost; however, it is unknown whether a broader class of optical receivers can implement the OJDR in general, as it might require more complex quantum optical components that make use of quantum coherence to analyse the signals [35], [36], [37], [38], [39], [40], in which case the receiver energy consumption might be non-negligible. Furthermore, observe that (5) is valid independently of the specific system design chosen for implementing collective detection (OJDR), and at the theoretical level of our analysis there is no difference between the different possible implementations.

In Fig. 2 we show the two exemplary realizations which form the basis of discussion in this manuscript. In both cases joint detection is used for communication on the C-band, but different baud-rates are used.

On the other hand, (4) is attained with an explicit receiver design, which coincides with the OSSR for this channel in the practically relevant regime $\tau \cdot n \gtrsim 2$ [38], [41]. For completeness, in Appendix D (supplementary material) we study the performance of the homodyne receiver as well, which coincides with the OSSR for $\tau \cdot n \gtrsim 2$, observing an energy-advantage of OJDR also in this case.

**B. Maximizing Spectral Efficiency: More Data With Less Amplification**

We are now interested in maximizing the SE’s (4,5) with respect to the amplifier gains. Following the approach taken in [11], we assume that an energy constraint is imposed on the entire communication link, such that the energy of the field at any point along the link stays below a threshold:

$$\tau_i \cdot n + \nu_i \leq n_{max} \forall i = 1, \ldots, K.$$  

The value of $n_{max}$ can be determined by practical constraints; in particular we will focus on the case where the sender produces signals of maximum energy, i.e., $n = n_{max}$. The maximum amplifier gain compatible with this constraint is

$$G_{max} := \frac{1 + n}{1 + \eta \cdot n},$$

which, for large photon number per pulse ($n \gg 1$), is approximately equal to $\eta^{-1}$. In the following we study the problem of spectral-efficiency-optimal gain selection (SEGS):

$$S_{opt}(n) := \max_{G^K \in \mathbb{R}^K} S(n, G^K)$$

subject to $G_K = 1, 1 \leq G_i \leq G_{max} \forall i,$

$$\tau_i \cdot n + \nu_i \leq n_{max} \forall i = 1, \ldots, K,$$

where $S$ is either the Shannon or Holevo SE and the corresponding problems are called Shannon-SEGS and Holevo-SEGS. Let us note that the last amplifier can always be turned off, i.e., $G_K = 1$. Indeed, this amplifier simply constitutes an additional channel acting before signal detection; hence, by the data-processing inequality, it cannot increase the overall spectral efficiency.

For the Shannon-SEGS, the optimal amplification strategy is simply to turn on all the remaining amplifiers at the maximum value of the gain allowed by the energy constraint (6): $G_i = G_{max}$ for all $i < K$. This follows from the fact that the Shannon SE (4) is a monotonically increasing function of $G_i$ for all $i < K$ (see Appendix A, supplementary material). The resulting optimal Shannon SE is

$$S_{sh}^{opt}(n) = \log \left(1 + \frac{\tau_{max} \cdot n}{1 + (\eta - \tau_{max}) \cdot n} \right),$$

$$\tau_{max} = \frac{\eta \cdot G_{max}^{K-1}}{1},$$

3Throughout the article the logarithms are taken in base 2.
On the other hand, the Holevo-SEGS amplification strategy can be considerably different, due to the presence of a quantum OJDR. In this case, the optimization depends non-trivially on the system parameters \( \eta, K \) and on the input signal energy \( n \). Still, it can be shown that the optimal sequence of gains is non-increasing with \( i \); indeed, if two gains are in increasing order, switching them always decreases the noise \( \nu \) in the overall channel without changing its amplification coefficient \( \tau \) (see Appendix A, supplementary material). This property, together with the well-known fact that the Holevo SE is never smaller than the Shannon SE \cite{30,34} leads us to the conclusion that

\[
S_{\text{ho}}^\text{op}(n) \geq S_{\text{ho}}(n,(G_{\text{max}}, \ldots, G_{\text{max}}, 1)) \geq S_{\text{sh}}^\text{op}(n). \tag{11}
\]

Moreover, in Appendix A (supplementary material) we prove that the first inequality can be strict, i.e., there exists a whole range of system parameter values for which the choice of maximum gains results in a sub-optimal value of \( S_{\text{ho}} \) and, in fact, a certain number of amplifiers can even be turned off.

We thus conclude that, in general, starting from a fully-amplified link with OSSR, the introduction of an OJDR allows to decrease amplifier gains while increasing the spectral efficiency. In turn, since smaller gains imply a smaller energy consumption to operate the amplifiers, the OJDR allows to spare a certain amount of energy. This motivates taking a comparative look at energy-optimal gain selection for OJDR versus OSSR.

III. REDUCING ENERGY CONSUMPTION WITH A QUANTUM DECODER

A. Minimizing Amplifier Energy Consumption: Problem Definition

Every amplifier needs to use up at least the amount of energy that it adds to the incoming field, hence the total amplification energy consumption is at least

\[
E(n,G^K) = \sum_{i=1}^{K} (G_i - 1) (\eta (\tau_{i-1}n + \nu_{i-1}) + 1). \tag{12}
\]

A fully-amplified link with OSSR attains SE \( S_{\text{sh}}^\text{op}(n) \) with an energy consumption of \( E_{\text{sh}} := (K-1)(1-\eta)n \). Using the former as a baseline, we can then define the following energy-optimal gain selection (EGS) problem for a transmission link with OJDR:

\[
E_{\text{reg}} := \min_{G^K \in \mathbb{R}^K} E(n,G^K)
\]

subject to :

\[
G_K = 1, 1 \leq G_i \leq G_{\text{max}} \forall i, \quad S_{\text{ho}}(n,G^K) \geq S_{\text{sh}}^\text{op}(n). \tag{13}
\]

Importantly, note that the energy needed to provide a given gain \( G_i \) depends on all previous gains \( G_j < i \), and on the attenuation per segment \( \eta \). In particular, as the noise added by previous amplifiers increases, so does the energy cost for providing a certain gain. This implies that monotonically decreasing gains are not necessarily optimal for EGS.

We therefore also consider a relaxation of EGS (REGS):

\[
E_{\text{regs}} := \min_{G^K \in \mathbb{R}^K} E'(n,G^K)
\]

subject to :

\[
G_K = 1, 1 \leq G_i \leq G_{\text{max}} \forall i, \quad S_{\text{ho}}(n,G^K) \geq S_{\text{sh}}^\text{op}(n), \tag{14}
\]

Here the energy cost for each amplifier is overestimated as the maximum one, via \( \alpha \), i.e.,

\[
E'(n,G^K) = \sum_{i=1}^{K} (G_i - 1) (\eta \cdot n + 1), \tag{15}
\]

and it does not depend on the previous gains. Therefore, in this case, it is clear that non-increasing gains can always attain the optimum of REGS and, at the same time, maximize the SE, as discussed in Section II-B.

While non-increasing gains already provide reductions in energy consumption when using an OJDR, the subtle differences between problems EGS and REGS show that optimum gain selection for quantum communication networks is an important nontrivial aspect of quantum network design by itself.

B. Minimizing Amplifier Energy Consumption: Practical Considerations

The problem definition and initial study provided in this article are limited in scope, since many practical aspects would inevitably increase the model’s complexity. Among these are the energy consumption of the transceivers as well as effects arising from gain-flattening, inter-symbol interference, four-wave mixing, cross-phase modulation. Nevertheless, some aspects are easily being accounted for by simple modifications. These include, e.g., Raman amplification and losses in pump energy. We detail here how the latter can be incorporated by assuming a pump transmissivity of \( \exp\{-\beta_\text{p} \cdot L\} \) per span and bi-directional pumping; the energy consumption formula is thus updated to

\[
E''(n,G^K) = \sum_{i=1}^{K} e^{\beta L \min[i,K+1-i]} E_i(n,G^K) \tag{16}
\]

where we set \( E_i(n,G^K) := (G_i - 1)(\eta (\tau_{i-1}n + \nu_{i-1}) + 1) \) as the baseline pump energy cost for the \( i \)-th amplifier (see \( (12) \)), and we recall that \( \eta = \exp\{-\alpha \cdot L\} \). Another example that is within scope of our analysis are reduced maximum gains.

In the next section, we study the role of these effects via numerical results (see Figs. 5 and 6), concluding that they increase the energy savings.

C. Minimizing Amplifier Energy Consumption: Numerical Results

Using \( \alpha, L \) and the number \( K \) of segments as an input, we can now optimize the gain profile to solve problems 13 and 14 for concrete cases that are matched to the technical reality of optical fiber networks. In Fig. 3 we display an overview of our findings when the system is transmitting over a number \( f = 144 \) individual channels, each transmitting \( b = 50 \cdot 10^9 \) pulses per second (or equivalently, using a baud-rate of 50 Gbd) in the

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Fig. 3. Displayed is the percentage of energy saved at the in-line amplifiers of links operating an OJDR relative to the same link with OSSR, as a function of link length and number of amplifiers. The top (bottom) image shows energy savings calculated according to problem EGS (REGS). The photon number is set to $n = 10^5$, corresponding to a maximum signal power of $100 \text{ mW}$ spread over 144 channels, each operating at $50 \text{ Gb/s}$. SE is constant along black lines. The gain $AE$ achieved from the use of amplifiers is constant along white lines. Interesting areas are those to the right of the second white line (at least two-fold gain from amplification). The link capacity in TBit/s can be read off the drawing by multiplying the number indicating the spectral efficiency by $10$. We included explicit energy saving percentages at some points of practical interest in the upper plot.

C-band around the standard telecom wavelength $\lambda = 1550 \text{ nm}$ with a transmission power of $100 \text{ mW}$; for the quantum receiver, we assume that quantum joint detection takes place at the end of each individual channel independently. Using the Planck formula

$$e_p = \frac{h \cdot c}{1550 \text{ nm}},$$

(17)

with the wavelength $\lambda = 1550 \text{ nm}$ and speed of light $c = 2 \cdot 10^8 \text{ m/s}$, the energy of a single photon is calculated and translated into a number of approximately $P = 10^{18}$ photons per second. We convert this number into a photon number $n$ per pulse via

$$n = P/(f \cdot b),$$

(18)

arriving at a value of $n = 10^5$ photons per pulse in our particular base scenario. For the attenuation coefficient, we use the value $\alpha = 0.05 \text{ km}^{-1}$. Similar results for different system parameters are summarized in Fig. 4. The algorithm employed to calculate the energy savings is explained in Appendix C (supplementary material).

1) Results: Fig. 3 shows that the use of OJDR technology allows for energy savings for all combinations of distance and number of amplifiers. We plot the percentage of energy spent by OJDR with respect to a fully-amplified link with OSSR, i.e., $E_{\text{egs,reg}}/E_{\text{sh}}$, as a function of the link length and the number of segments. Furthermore, we consider two operational criteria to evaluate the practical relevance of our findings, aimed at identifying when the classical transmission system benefits the most from amplification:

i) the Shannon SE enhancement attainable by full amplification compared to no amplification, i.e. the ratio $AE := S_{\text{op}}(n)/S_{\text{sh}}(n, \{1\}_{i=1}^K)$ (constant along white lines in Fig. 3);

ii) the Shannon SE achievable by full amplification $S_{\text{sh}}(n)$ (constant along black lines in Fig. 3).

The record savings for a photon number of $n = 10^5$ along the white line where $AE = 2$ in Fig. 3 amount to 57% and occur at a total link length of $164 \text{ km}$ when the link is using one amplifier placed at a distance of $82 \text{ km}$ from the sender. For this configuration, the SE reached with amplification is $\approx 9.7 \text{ bits/s/Hz}$
and that without amplification is \( \approx 4.8 \text{bits/s/Hz} \) when an OSSR is used and \( \approx 6.2 \text{bits/s/Hz} \) when an OJDR is employed.

A collective study over different photon numbers per pulse \( n = 10^4, 10^2 \) and \( 10^8 \), modelling the different possible configurations of transmission power, baud-rate and number of channels is depicted in Fig. 4. It displays the increasing advantage of the OJDR over the OSSR when the number of photons per pulse is reduced.

We note further that even larger energy savings appear in situations of lesser practical relevance for commercial optical-fiber networks, which have been previously studied in terms of SE [6], [7], [11] and in terms of energy-efficiency in the limit where OSSR approximates OJDR [10] (bottom-right and upper-left regions of Fig. 3). Indeed, at least 95% of the energy can be saved by the OJDR in the regimes of long distance, though with extremely low achievable SE, and short distance, though with extremely low enhancement compared to the unamplified case.

While our observations motivate the use of the OJDR, it may be argued that the theoretically observed enhancements brought about by the new technology will likely vanish in a real-world implementation which has to account for many practical imperfections. That such argument is not necessarily correct can be seen by studying energy savings obtained in the presence of pump-energy loss (16). When this formula is applied to calculate energy consumption, one can actually observe even higher energy savings, as depicted in Fig. 5.

A similar effect occurs when, instead of the maximum allowed gains, a reduced value \( G' := (1 + G_{\text{max}})/2 \) is used, where \( G_{\text{max}} \) is calculated according to (7). Such a reduced gain leads to higher energy savings, as depicted in Fig. 6.

The qualitative difference between our approximate solutions to problems EGS and REGS are shown in Fig. 3 to be quite small (on average over the data presented in Fig. 3 it amounts to \( \approx 5\% \), with a maximum difference in the energy savings of \( \approx 23\% \), implying that a simple decreasing-gain profile already provides near-optimal energy savings for a large number of configurations.

IV. COMMUNICATION IN THE QUANTUM LIMIT

We have so far quantified the possibility of energy savings in amplifiers based on the use of the OJDR, with a main focus on the current system parameters.

To point out the timeliness and usefulness of quantum receiver technology also with regards to future system architectures, we consider here in addition the limit of high baud rates.

The key idea of this section follows the reasoning of operating a communication system in a parameter regime which is chosen in an attempt to harness the superior performance of quantum data transmission techniques. When a power constraint is imposed on a communication link with a large enough spectral bandwidth, high-baud-rate transmission systems naturally operate on a low photon number per pulse. The technological feasibility of such a system must then be answered based on the imposed bandwidth limitations and the available processing speed of the receiver in conjunction with the expected added value arising from the novel system design. The following arguments are separated from each other based on the dependence of the noise term \( \nu \) on the baud-rate.

A. Constant Gains

The interesting observation one can make based on formulas (4) and (5) is that, for a fixed maximum number \( N \) of photons per second, the number of photons per pulse (at the transmitter) satisfies \( n = N/b \) where \( b \) is the number of pulses per second (also called baud-rate). Thus, at a given attenuation \( \tau \) and noise spectral density \( \nu \), expressed in noise photons per second and Hertz, the number of bits per second that can be transmitted using an OSSR saturates for high baud-rates:

\[
\lim_{b \to \infty} \log \left( 1 + \frac{\tau \cdot N}{1 + \nu} \right) \cdot b = \frac{N \cdot \tau}{(1 + \nu) \ln(2)} \leq \frac{N \cdot \tau}{\ln(2)}
\]

whereas the corresponding quantity for an OJDR diverges at zero noise:

\[
\lim_{b \to \infty} \left( g \left( \frac{N}{\tau} + \nu \right) - g(\nu) \right) \cdot b = N \cdot \tau \cdot \log \left( 1 + \frac{1}{\tau} \right)
\]

In (20) clearly shows that the capacity of the OJDR will be limited in any practical system design if only the smallest amount of thermal background noise is present. However, note that in the context of the present work, a noise spectral density \( \nu \) is motivated solely through the use of amplifiers; other possible
noise contributions are not investigated. To nonetheless arrive at a constant noise spectral density $\nu$ in this context one can, instead of the constraint (6), assume that $G = \eta^{-1}$ is set for every amplifier except the $K$-th. Then $\nu = 1 - \eta^{-1}$ and the expressions (19,20) above are valid for OJDR and OSSR when the limit $b \to \infty$ is taken.

### B. Baud-Rate-Dependent Gains

At variance with the above observations, in the present work we study the system under the energy constraint (6) which leads to the maximum gain being calculated via (7). In this setting, when in addition high baud-rates are employed, the photon number per pulse $N/b$ approaches zero as $b \to \infty$. A Taylor series expansion then reveals that the maximum gain scales as $1 + (1 - \eta)N/b$, and therefore the noise $\nu = G - 1$ that is contributed by an amplifier stage approaches zero as $b \to \infty$. The noise as a function of $b$ can then be written as $\nu(b) = \frac{N}{b}(\eta - \eta^K) =: \nu/b$.

If thermal noise contributions are ignored, the limiting expression for the capacity of the OJDR under growing baud-rates is obtained by noting that, for any $x > 0$, we have

$$\lim_{b \to \infty} b \cdot \log(1 + \frac{b}{x}) = x/\ln(2)$$

and

$$\lim_{b \to \infty} \log(1 + \frac{x}{b}) = -\log(1 + \frac{x}{y}) = \log(\frac{y}{x})$$

we know that for each choice of $(N, \tau, \nu')$ there exists a sequence $(\epsilon_b)_{b \in \mathbb{R}}$ and a constant $\epsilon = \epsilon(N, \tau, \nu')$ such that $\epsilon_b \to \epsilon$ and

$$\lim_{b \to \infty} \frac{b}{\ln(2)} \cdot \log(1 + \frac{b}{x}) = x/\ln(2) + \log(x/\ln(2)) + \epsilon_b. \quad (22)$$

Thus for every choice of $N, \tau$ and $\nu'$ the dependence of the channel capacity on the baud-rate is effectively logarithmic and grows unbounded as $\tau \cdot N \cdot \log(\frac{b}{\tau \cdot N \cdot \nu'})$ when using an OJDR. In sharp contrast, the capacity of the same channel using an OSSR is limited by $\frac{b}{\tau \cdot N \cdot \nu'}$. From a theoretical perspective, our analysis suggests that a scaling of capacity with baud-rate is possible even in the presence of amplifiers, when quantum information processing technology is used. This new possibility is of high relevance for future integrated classical- and quantum networks, which might need to transmit classical messages on the same shared medium as quantum services. For transmission and maintenance of entangled states, noise is a challenging problem. Thus for integrated networks, where noise needs to be avoided at all costs, it is important to observe that there exist quantum data transmission methods which avoid the noise induced by amplification in data transmission, so that the resulting networks can more easily be used as physical carriers for quantum networking tasks.

### C. Discussion and Example Parameters

As one can see from the above two examples, it is in principle possible to have amplified fiber links where capacity scales in a logarithmic way with the baud-rate.

In today’s single-mode silicon fiber links, transmission inside the C-band would however allow for a maximum baud rate in the order of (only) $b \approx 3.7$ TeraBaud. At this baud rate, the number of bits per second for transmission over a cable of length $L = 185$ km at $1$ mW is $\approx 1$ TeraBit/s with an OSSR and $\approx 4.3$ TeraBit/s with an OJDR - more than a 4-fold increase in data transmission rate, which is mainly limited by the carrier frequency and the spectral bandwidth limitations imposed by the physical properties of optical fiber in the C-band, plus potentially the processing speed of electronic gates in the receiver. For both problems, solutions are in principle possible: hollow-core
fibers can be better tailored to the transmission of ultra-short pulses [25], [26], [42], and optical processing as a physical-layer technique can utilize the principles of quantum-mechanical detection based on newly developed integrated optics techniques.

Spreading the signal energy for data transmission over a wide spectral range might in addition open up a way for co-existence of data transmission with other services of future quantum networks. Our analysis thus suggests an exciting new venue for quantum communication network development.

While above considerations provide interesting hints for the development of future networks, their implication in parameter regions close to those of existing fiber networks is somewhat limited: Assume a communication link of a length $L = 300 \text{ km}$ with attenuation coefficient $\alpha = 0.05$, using a total power of $100 \text{ mW}$ ($10^{18}$ photons per second). Assume this energy is spread equally over 144 ITU channels in the C-band, at a baud-rate of 50 Gbd. Numerical evaluation of the capacity formulas for the OSSR and the OJDR using amplifier gain $G_{\text{max}}$ calculated using $n = 0.2 \cdot 10^{-10} \cdot 10^{18}$ and $\eta = \exp(-0.05 \cdot 150))$ yields 48.3 TBit/s for the former, and 52.6 TBit/s for the latter. Removing the amplifier yields capacities of 0.4 TBit/s and 2.0 TBit/s. This numerical comparison shows clearly the advantage of using the amplifier. The difference between the amplification scenarios discussed in Sections IV-A and IV-B is minute here, since the respective amplifier gains differ from each other in the order of one percent only.

However, scenarios where amplification is hard to achieve are not covered by the above analysis. This applies for example to cases where hollow-core fibers are used. Indeed, the question whether the development of amplifiers for hollow-core fibers will lead to results as satisfactory as those for standard silicon fiber is open [43]. Instead, recent advances [44] let us hope for ranges of 330 nm-wide transmission windows around 1550 nm. Such wide transmission windows can be reflected in our numerical analysis by changing the baud-rate of the carriers by a factor of 10. If no amplifiers are used, then the capacity of an OSSR on a 200 km transmission line becomes 0.65 TBit/s and the capacity of the OJDR reaches 3.97 TBit/s. Since hollow-core fiber offers a reduced latency, the OJDR technology might be attractive for the realization of use cases operating at the boundary of the possible in future networks.

### D. Realization

Finally we observe that, though the realization of a full-fledged OJDR is still an open problem, the Hadamard receiver [8], [9], [19], [24], realizable with integrated photonics, offers the opportunity to observe the high-baud-rate advantage enabled by a quantum receiver in a realistic setting. In the above setting of communicating in the C-band over 185 km at 1 mW, Hadamard codes with orders 4, 8, 16, 32 would offer respective transmission rates of 1.3799 TBit/s, 1.9191 TBit/s, 2.18987 TBit/s, 2.0744 TBit/s. These rates can be calculated in the zero-noise case as

$$\frac{b}{k} \left(1 - \exp\left(-\frac{k\tau n}{b}\right)\right) \log k,$$

(23)

using for example Ref. [8].

### V. Discussion

We have shown that the introduction of a quantum detection method, the OJDR, can reduce the energy cost of a fiber-optical communication line by up to $\approx 57\%$, in a practically relevant parameter regime where optical amplifiers enable to maintain a large communication rate.

This determines, for the first time to our knowledge, a striking energy advantage of quantum vs. classical detection for the transmission of classical information on optical fiber. Our results highlight the relevance of QIP in communication beyond the traditional settings of secure-communication and entanglement-transmission, opening up a third research direction with potentially closer-term applications, which is fully compliant with the vision of all-optical networking [45].

It is well-known that the performance of quantum receivers approaches that of their classical counterparts under high noise. Resulting from this fact is a perception that quantum receiver technology might not be relevant in practical applications, where noise cannot be avoided. Our approach inverts the argument: by reducing the amplifier gains along the line we reduce the noise resulting from spontaneous emission, thereby moving towards the regime where the quantum receiver is superior while at the same time reducing the energy consumption of the amplifiers.

Along this line of argument, we have shown that even a complete removal of amplifiers is possible, given enough spectral bandwidth supporting high baud-rates. We have described the respective scaling laws of the data transmission rates with the baud-rate for both OJDR and OSSR technology.

We stress that our analysis can be refined by employing more complex models of energy consumption, including nonlinearities and the energy transfer towards the amplifiers. In particular, we included two possible practical issues with the amplifiers, i.e., pump energy loss and reduced maximum gain, to discover that even higher energy savings can be obtained (see Figs. 3 and 5).

Our results renew the urgency of devising an explicit optical receiver design that can approximate the OJDR at all signal energies and set the stage for a redesign of established data transmission technology based on a use of quantum-mechanical principles.

In what follows, we list implications of our results which are relevant to other research domains in networking. Namely, OJDR technology can:

1. **Provide advantages already today:** these come in terms of energy savings at amplifiers.
2. **Increase the length of non-amplified links:** If new types of optical fiber with more spectral bandwidth were deployed, the observed initial savings could eventually translate into a complete removal of the amplifiers, without any sacrifice on the data rates.
3. **Lay the ground for quantum network development:** A removal of amplifiers decreases the overall noise in the channel, and we can expect this to imply an increase of the quantum-data-transmission capacity. For example, consider the most basic instance of our communication line (Fig. 1) with $K = 2$ optical-fiber links of loss $\eta$, separated by a quantum-limited amplifier of gain $G$. The
overall channel, determined by (3), can be described as a thermal-attenuator bosonic channel with attenuation coefficient $G\tau^2$ and extra-noise coefficient $\frac{T(\tau+2)(G-1)}{2(1+G\tau^2)}$, in the notation of [46, Eq. (11)]. Both the upper and lower bounds [46, Eqs. (18,40)] appear to be decreasing as a function of the gain and hence one can expect that quantum communication benefits from the complete removal of amplifiers along the line.\(^4\)

4) **Simplify network deployment and maintenance:** The use of this technology in conjunction with high baud-rates suggests the possibility of reducing the number of amplifiers, thus potentially simplifying deployment and operation of data networks.

5) **Offer a way to evade noise induced by the Kerr effect:** A consequence of our work is the emergence of transmission in the quantum limit as a new design option for fiber-optic networks. For such networks, the non-linear Shannon limit resulting among others from the Kerr effect imposes severe practical boundary conditions [50]. Optimization under these boundary conditions is an active research field [51]. The Kerr effect itself induces a specific type of noise to the system which depends on the signal energy [52] and thus eventually prevents a growth of the system capacity with the signal energy. Our approach does instead advertise communication in a low-energy regime where the Kerr effect has little impact, and where the capacity loss incurred from the lower energy is compensated for by high baud-rates. This approach is in line with the historic trend of pushing baud-rates to higher and even higher numbers, which is documented e.g. in [53].

We stress that our analysis rests on the established formulas for Shannon [33] and Holey [34] capacity, which take into account only attenuation and thermal noise. Any further effects arising from a use of ultra-short pulses are beyond the scope of our analysis. Thus our work justifies a more in-depth analysis of the above statement should rely on the study of the joint capacity region for classical and quantum communication benefits from the complete removal of amplifiers along the line.\(^4\)

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