High-precision measurement of the atomic mass of the electron

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The quest for the value of the electron’s atomic mass has been the subject of continuing efforts over the past few decades1–4. Among the seemingly fundamental constants that parameterize the Standard Model of physics5 and which are thus responsible for its predictive power, the electron mass \( m_e \) is prominent, being responsible for the structure and properties of atoms and molecules. It is closely linked to other fundamental constants, such as the Rydberg constant \( R \), and the fine-structure constant \( \alpha \) (ref. 6). However, the low mass of the electron considerably complicates its precise determination. Here we combine a very precise measurement of the magnetic moment of a single electron bound to a carbon nucleus with a state-of-the-art calculation in the framework of bound-state quantum electrodynamics. The precision of the resulting value for the atomic mass of the electron surpasses the current literature value of the Committee on Data for Science and Technology (CODATA)7 by a factor of 13. This result lays the foundation for future experimental physics experiments8,9 and precision tests of the Standard Model10–11.

Over the past few decades, the atomic mass of the electron has been determined using several Penning-trap experiments, because exploration of the scope of validity of the Standard Model requires an exceedingly precise knowledge of \( m_e \). The uniform magnetic field of these traps makes it possible to compare the cyclotron frequency of the electron with that of another ion of known atomic mass, typically carbon ions or protons. The first such direct determination dates back to 1980, when Graeff et al. made use of a Penning trap to compare the cyclotron frequencies of a cloud of electrons with that of protons, which were alternately confined in the same magnetic field, yielding a relative precision of about 0.2 parts per million (ref. 2). Since then, a number of experiments have improved the precision by about three orders of magnitude1,4,12,13. The latest version of the CODATA compilation of fundamental constants of 2010 lists a relative uncertainty of \( 4 \times 10^{-16} \), resulting from the weighted average of the most precise measurements. Given that the cyclotron frequency of the extremely light electron is subject to troublesome relativistic mass shifts if not held at the lowest possible energy, direct ultrahigh precision mass measurements are particularly delicate. To circumvent this trouble, the currently most precise measurements, including this work, pursue an indirect method that allows a previously unprecedented accuracy to be achieved.

A single electron is bound directly to the reference ion, in this case a bare carbon nucleus (Fig. 1). In this way, it becomes possible to calibrate the magnetic field \( B \) at the very place of the electron through a measurement of the cyclotron frequency

\[
\nu_{\text{cyc}} = \frac{1}{2\pi m_{\text{ion}}} B
\]

of the heavy-ion system with mass \( m_{\text{ion}} \) and charge \( q \). The cyclotron frequency of the strongly bound electron is of no further relevance, but the precession frequency of the electron spin, which depends on the electron’s magnetic moment \( \mu_e \) as follows

\[
\nu_{\text{cyc}} = \frac{2\mu_e B}{\hbar} = \frac{g_e e}{4\pi m_e} B
\]

is well defined and reveals information about the mass of the electron \( m_e \). A measurement of the ratio of these two frequencies yields \( m_e \) in units of the ion’s mass

\[
m_e = \frac{g_e e}{2q} \frac{\nu_{\text{cyc}}}{\nu_{\text{ion}}} m_{\text{ion}} \approx \frac{e}{2q} \Gamma m_{\text{ion}}
\]

where \( \Gamma \) denotes the experimentally determined ratio \( \nu_{\text{cyc}}/\nu_{\text{ion}} \). When determining \( \Gamma \) of a hydrogen-like carbon ion, which is the defining particle for the atomic mass (apart from the mass and binding energies of the missing electrons, which are sufficiently well known), the remaining unknown in equation (3) is the \( g \)-factor. Advances in quantum electrodynamics (QED) theory in recent years allow us to calculate this value with the highest precision14.

Here we present an ultra-precise measurement of the frequency ratio and a state-of-the-art QED calculation for the case of hydro- like \(^{12}\)C\(^{5+}\), which allows us to determine \( m_e \) with unprecedented accuracy. Exposing the electron to the binding Coulomb field of an atomic nucleus has a profound influence on the \( g \)-factor. The largest difference from the free-electron case can be deduced from a solution of the Dirac equation in the presence of the Coulomb potential of a nucleus of charge \( Z \) and an external, constant and homogeneous magnetic field: \( g_{\text{Dirac}} = \frac{2}{3} + \frac{4}{3} \sqrt{1 - (Zs)^2} \) (ref. 15). This result must be complemented by various other effects, originating mainly from QED (see Fig. 2). Many of those effects, like the one-loop self-energy and vacuum polarization terms, and the nuclear recoil contribution, are known with sufficient numerical accuracy14,16,17. The main challenge in further improving the theoretical predictions is related to the two-loop QED effect. This contribution is known only to the first few terms of its expansion in terms of \( (Zs)^n \) \( \ln(Zs)^n \). The calculation of the expansion coefficients with \( n \geq 5 \) is beyond the current state of the art, defining the overall theoretical uncertainty. However, we have been able to estimate the uncalculated higher-order contribution \( g_{\text{2L}}^{\text{higher-order}} \) and thus improve on the theoretical value with the help of our recent experimental value \( g_{\text{exp}}^{\text{Si}} \) of the \( g \)-factor of hydrogen-like silicon \((Z = 14)^{18,19}\). This contribution, which dominates the theoretical uncertainty, can be determined from the difference of the experimentally determined \( g \)-factor and the theoretical prediction, which is the sum of all known terms excluding \( g_{\text{2L}}^{\text{higher-order}} \)

\[
g_{\text{2L}}^{\text{higher-order}} (Z = 14) = g_{\text{exp}}^{\text{Si}} - g_{\text{theory}} = 2 \frac{\nu_{\text{cyc}}}{\nu_{\text{ion}}} m_e - 13 - g_{\text{Si}}^{\text{theo}}
\]

We assume an analytical form of the \( Z \)-dependence \( g_{\text{2L}}^{\text{higher-order}}(Z) \), and thus obtain an estimate of \( g_{\text{2L}}^{\text{higher-order}}(Z = 6) \) for carbon, presented in Supplementary Table II. Equation (4), together with a second formula for the \( Z \) dependence on those higher-order terms (see Supplementary equation (15)), can be solved to yield more accurate values for two variables, namely, the theoretical \( g \)-factor value for

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The triple Penning-trap setup used in this Letter. Electrodes are yellow, insulation rings are grey and magnetic field lines are red (sketch). Highly charged ions can be created in situ inside the hermetically closed cryogenic vacuum with the miniature electron-beam ion trap (EBIT), allowing for almost infinite measurement time (lower panels). The magnetic bottle in the analysis trap, used for the spin-state detection, is spatially separated from the very homogeneous field in the precision trap, which allows precise measurements of the ion’s eigenfrequencies. The lower middle panel shows one carbon, and the electron mass. The technical details of this calculation and related uncertainty are described in the Supplementary Information.

Figure 2 | The magnitude of the relevant theoretical contributions to the bound electron g-factor in $^{12}$C$^+$. The leading Dirac contribution, one- and two-loop bound-state-QED corrections, and nuclear effects (see also the Supplementary Information). Some representative Feynman diagrams$^{16}$ corresponding to the QED terms are shown. ‘Higher order’ means corrections higher than order 4 in powers of $(Zs)$.

Figure 3 | Axial dip signal of a single $^{12}$C$^+$ ion, used to determine the axial frequency. The voltage-noise density of the tank circuit, detected by the cryogenic amplifier, shows the short-cut at the axial frequency of the ion. The linewidth is only about 0.4 Hz. The inset shows a close-up of the dip feature. $V_{\text{rms}}$, root mean square voltage. The axial frequency has been offset by 670,963.26 Hz.

The key tool for our measurements is the Penning trap (Fig. 1). The homogeneous magnetic field (in our case 3.7T), which causes the precession of the spin, also forces the ion into a circular cyclotron motion and in this way confines it in the plane perpendicular to the field (the ‘radial’ plane). To retain the ion sufficiently long for a precise measurement, we add an electrostatic quadrupole potential, which yields a harmonic motion of the ion along the magnetic field lines (‘axial’) with frequency $v_±$. Simultaneously, the quadrupole generates two uncoupled harmonic eigenmotions in the radial plane: the modified cyclotron and the magnetron motion, with frequencies $v_+$ and $v_-$, respectively. The trap eigenfrequencies are connected to the free-space cyclotron frequency via the invariance relation $v_{\text{cy}} = \sqrt{v_+^2 + v_-^2}$ (ref. 20). To determine these frequencies, a superconducting tank circuit in resonance with the axial motion of the ion

serves to transform tiny currents that the ion induces by its oscillation into small yet measurable voltage signals.

The interaction with the tank circuit also results in a weak damping, which brings the ion into thermal equilibrium with the resonator. Electronic noise feedback techniques allow further cooling, until the effective temperature is well below that of the environment (4.2 K), greatly reducing systematic errors. At thermal equilibrium, the ion exactly cancels the thermal noise of the tank circuit, leaving a characteristic ‘dip’ in the detected spectrum (Fig. 3). A fit with a well-known lineshape directly reveals the axial frequency of the ion to sufficient precision. The remaining eigenfrequencies, which do not couple to the resonator directly, must be detected via mode-coupling to the axial motion. The recent development of the ‘pulse and amplify’ technique$^{21}$ has enabled us to perform phase-sensitive measurements of several hundreds of spin-flips, detected via the continuous Stern–Gerlach effect. The axial frequency has been offset by 412,223 Hz. The exemplary g-factor resonance in the lower right panel, containing about 500 events, shows the detected fractional spin-flip rate as a function of the frequency ratio $\Gamma = v_1/v_\nu$, in the precision trap with a relative full width at half maximum (FWHM) of 0.7 p.p.b. Several such resonances have been recorded under different conditions in order to check the model of systematics. A detailed discussion can be found in the Supplementary Information.

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of the modified cyclotron frequency at very low energies below the detection threshold of the image-current amplifier, a significant improvement on the established ‘pulse and probe’ technique2. Combined with the axial and magnetron frequency information from dip fits, the invariance relation allows us to calculate the free-space cyclotron frequency, which is a measure of the magnetic field at the ion’s location.

The Larmor precession frequency, nominally 105 GHz in our case, cannot be detected directly with the image current detector. Instead, the Zeeman splitting of the bound electron’s spin is probed with a microwave excitation. The key requirement for this is the ability to detect the spin state with the continuous Stern–Gerlach effect23. To this end, a strong magnetic field inhomogeneity, is generated by an electrode made from ferromagnetic material. In our setup the quadratic portion of this bottle-shaped field amounts to $B_2 = 10^4 \text{ T m}^{-2}$ (ref. 24).

In this inhomogeneous field, the magnetic moment couples to the axial motion and causes a small, spin-dependent frequency difference. Provided all other influences on the axial frequency, notably the ion’s energy and the voltages applied to the trap, can be sufficiently well controlled, the determination of the axial frequency of the ion becomes a quantum non-demolition measurement of the electron’s spin. We use a double-trap setup to spatially separate the spin analysis in the inhomogeneous field of the ‘analysis trap’ (AT) and the high-precision eigenfrequency measurement in the ‘precision trap’ (PT) (Fig. 1). During the experiment, the ion is adiabatically shuttled between these two traps. After determining the initial spin-state in the AT, the ion is transported to the PT, where a microwave excitation at a random frequency offset with respect to the expected Larmor frequency probes the Zeeman splitting at the same time as the ‘pulse and amplify’ measurement of the cyclotron frequency is performed, which suppresses fluctuations of the magnetic field. The axial frequency, which is basically independent of the magnetic field, is measured before and after the ‘pulse and amplify’ cycle and interpolated. After transporting the ion back to the AT, an analysis of the spin state allows us to detect a possible successful spin-flip in the PT. By repeating this process (see Supplementary Fig. 1) several hundred times it becomes possible to map the probability of spin-flips in the homogeneous magnetic field of the PT as a function of the frequency ratio $\beta$ (right panel of Fig. 1).

The dominant systematic uncertainty arises from the self-interaction mediated by image charges and currents in the trap electrodes (see Table 1). In contrast to the free electron case, the retardation of the field and the resultant damping through a coupling to modes of the trap acting as a cavity is negligible owing to the very much higher cyclotron wavelength. However, instead, the influence of the intermediate Coulomb interaction—that is, image charges—is enhanced. Even though the resultant shift can be readily calculated, finite machining accuracies and the imperfect knowledge of the ion’s geometric position impose a relative uncertainty of $\delta (\nu_{\text{cycl}})/\nu_{\text{cycl}} = 1.5 \times 10^{-11}$.

The extrapolated frequency ratio $\beta_{\text{cycl}} = \Gamma(E_0 = 0)$, corrected for all systematic shifts (Table 1), yields the final value $\beta_{\text{cycl}} = 4376.21505089 (11)(7)$, with the statistical and systematic uncertainties, respectively, given in parentheses.

The theoretical prediction of the $g$-factor presented here (see Supplementary Table II) permits the calculation of the mass of the electron in units of the ion’s mass. By correcting for the mass of the missing electrons and their respective atomic binding energies35, we can finally calculate $m_e$ in atomic mass units:

$$m_e = 0.000548579909067(14)(9)(2) (5)$$

The first two errors are the statistical and systematic uncertainties of the measurement, and the third error represents the uncertainties of the theoretical prediction of the $g$-factor and the electron binding energies. The theoretical result for the $g$-factor, with corrections obtained from the experimentally determined value for hydrogen-like $^{28}\text{Si}^{13+}$ (ref. 18), implicitly assumes the correctness of QED. However, the thus-far-untested higher-order contribution determined in this work scales with $(Z \alpha)^3$ and thus contributes less than $10^{-11}$ in relative terms for the $^{12}\text{C}^+$ system.

The relative precision of $3 \times 10^{-11}$ for $m_e$ obtained in this work surpasses that of the current CODATA7 averaged literature value by a factor of 13 and the previous best measurement7 by a factor of 17 (see Fig. 4). Furthermore, our result gives the electron–proton mass ratio with a relative precision of 94 parts per trillion, solely limited by the uncertainty (in parentheses) of the proton mass value

$$m_p/m_e = 1836.15267377(17) (6)$$

The main limitations seen in our work are the uncertainty resulting from the ion’s self-interaction with its own image-charge in the trap electrodes and the temperature of the ion in connection with the temporal stability of the magnetic field.

Our result sets the stage for future ultrahigh precision tests of the Standard Model at low energies. One example is the determination of the fine-structure constant $\alpha$ via a measurement of the recoil momentum exerted on an atom upon absorption of a photon26. The electron atomic
mass presented in this Letter, combined with the Rydberg constant\(^6\), the atomic mass of rubidium\(^27\) and an atom interferometric measurement of \(\hbar/m_0\) (ref. 26), yields a value for \(\alpha\). By inserting this value into the Kinoshita theory\(^10\) for the \(g\)-factor of the free electron, it is possible to test the Standard Model through the Gabrielse experiment\(^9\) and, among other things, to observe the unification of the electro-weak interaction at low energies and probe the existence of light dark-matter particles\(^11\). Furthermore, the new value for \(m_e\) paves the way to probing the validity of QED at much higher field strengths through \(g\)-factor determinations in heavy, charged ions\(^7,8,15\).

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1. Farnham, D. L., Van Dyck, R. S., Jr & Schwinberg, P. B. Determination of the electron’s atomic mass and the proton/electron mass ratio. Phys. Rev. Lett. 75, 3598–3601 (1995).
2. Gräff, G., Kalinowsky, H. & Traut, J. A direct determination of the proton electron mass ratio. Z. Phys. A 287, 35–39 (1980).
3. Beier, T. et al. New determination of the electron’s mass. Phys. Rev. Lett. 88, 011603 (2001).
4. Hori, M. et al. Two-photon laser spectroscopy of antiprotonic helium and the antiproton-to-electron mass ratio. Nature 475, 484–488 (2011).
5. Cottingham, W. N. & Greenwood, D. A. An Introduction to the Standard Model of Particle Physics (Cambridge Univ. Press, 2007).
6. Mohr, P. J., Taylor, B. N. & Newell, D. B. CODATA recommended values of the fundamental physical constants: 2010. Rev. Mod. Phys. 84, 1527–1605 (2012).
7. Shabaev, V. M. et al. \(g\)-Factor of high-Z lithium-like ions. Phys. Rev. A 65, 062104 (2002).
8. Hanneke, D., Fogwell, S. & Gabrielse, G. New measurement of the electron magnetic moment and the fine structure constant. Phys. Rev. Lett. 100, 120801 (2008).
9. Aoyama, T., Hayakawa, M., Kinoshita, T. & Nio, M. Tenth-order QED contribution to the electron \(g\) – 2 and an improved value of the fine structure constant. Phys. Rev. Lett. 109, 111807 (2012).
10. Boehm, C. & Silk, J. A new test for dark matter particles of low mass. Phys. Lett. B 661, 287–289 (2008).
11. Häffner, H. et al. High-accuracy measurement of the magnetic moment anomaly of the electron. Phys. Rev. Lett. 85, 5308–5311 (2000).
12. Verdu, J. et al. Electronic \(g\) factor of hydrogenlike oxygen \(^{16}\)O\(^7\)\(^+\). Phys. Rev. Lett. 92, 093002 (2004).
13. Verdu, J. et al. Quark-hadron duality and the \(g\)-factor of hydrogenlike oxygen \(^{16}\)O\(^7\)\(^+\). Phys. Rev. Lett. 97, 073001 (2006).
14. Pachucki, K., Czarnecki, A., Jentschura, U. D. & Yerokhin, V. A. Complete two-loop correction to the bound-electron \(g\) factor. Phys. Rev. A 72, 022108 (2005).
15. Breit, G. The magnetic moment of the electron. Nature 122, 649 (1929).
16. Breit, G. The magnetic moment of the electron. Phys. Rev. 339, 79–213 (2000).
17. Yerokhin, V. A., Indelicato, P. & Shabaev, V. M. Evaluation of the self-energy correction to the \(g\) factor of S states in H-like ions. Phys. Rev. A 69, 052503 (2004).
18. Sturm, S. et al. \(g\)-factor measurement of hydrogenlike \(^{28}\)Si\(^{13}\)\(^+\) as a challenge to QED calculations. Phys. Rev. A 87 (3), 030501 (2013).
19. Sturm, S. et al. \(g\) Factor of hydrogenlike \(^{28}\)Si\(^{13}\). Phys. Rev. Lett. 107, 023002 (2011).
20. Gabrielse, G. Why is sideband mass spectrometry possible with ions in a Penning trap? Phys. Rev. Lett. 102, 172501 (2009).
21. Sturm, S., Wagner, A., Schabinger, B. & Blaum, K. Phase-sensitive cyclotron frequency measurements at ultralow energies. Phys. Rev. Lett. 107, 143003 (2011).
22. Cornell, E. A. et al. Single-ion cyclotron resonance measurement of M(CO\(^+\))/M(N\(_2\)\(^+\)). Phys. Rev. Lett. 63, 1674–1677 (1989).
23. Dehmelt, H. Continuous Stern-Gerlach effect: principle and idealized apparatus. Proc. Natl Acad. Sci. USA 83, 2291–2294 (1986).
24. Brown, L. S. & Gabrielse, G. Geonium theory: physics of a single electron or ion in a Penning trap. Rev. Mod. Phys. 58, 233–311 (1986).
25. Kramida, A. Atomic Energy Levels and Spectra Bibliographic Database (version 2.0). http://physics.nist.gov/PhysRefData/ASD/ionEnergy.html (Physical Measurement Laboratory, Quantum Measurement Division, NIST).
26. Bouchendira, R., Cladé, P., Gouellati-Khelifa, S., Nez, F. & Biraben, F. New determination of the fine structure constant and test of the quantum electrodynamics. Phys. Rev. Lett. 106, 080801 (2011).
27. Mount, B. J., Redshaw, M. & Myers, E. G. Atomic masses of \(^6\)Li, \(^{23}\)Na, \(^{39,41}\)K, \(^{85,87}\)Rb and \(^{133}\)Cs. Phys. Rev. A 82, 042513 (2010).

Supplementary Information is available in the online version of the paper.

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