Its own spectrum band – a property of mechanical systems

I N Ovchinnikov
Bauman Moscow State Technical University, Moscow, Russia
E-mail: iovchin@bmstu.ru

Abstract. In experimentally and modelling, the results have been obtained, revealing the existence of the hardest mode of broadband random vibration in function of the spectral width when it impacts on the mechanical system. During broadband random vibration there are extemums of tension in the material and vibration speed during the capture of «n» original frequencies of the system (n≥2). This fact indicates on presence of the “own bands” of the mechanical system, similar to the natural frequencies, but which appears at random loading.

1. Introduction
It is known that more than 70% of malfunctions in technics are caused by a vibration, which results in a fatigue failure of parts of a construction, transported loads, also in breakages and halting of electronic devices. Obtaining of reasonable compromise between the maximum of reliability and minimum of weight is laid in the development of methods of adequate investigation of materials’ behavior in constructions with given resource and reliability, and that shows the importance of vibration trials. At present trials on broadband random vibrations (BRV) are used most often.

International Electrotechnical Commission (IEC) sets two main requirements for vibration trials on its’ result’s veracity and reproducibility in different laboratories, which are especially important in conduction of acceptance trials. Reproducibility of trials’ results cannot be reached with low veracity. Therefore, the reliability of the test results is given special attention (a new sensor deformation). It is also necessary that ranges of vibration load, which are formed, take into account the dynamics of the trials’ object [1], what is done during harmonic loadings. At present ISO series standards ignore that [2]. Long attempts to replace the random loading to an equivalent determinate loading have not been successful [3,4,5 etc]. Only in [6] the problem was solved: such a replacement is incorrect and unacceptable.

Hardest mode BRV, mentioned in [7], but determined by dynamics of trials’ object, is necessary for raising levels of reproducibility of trials’ results, standardization of modes of vibration and fatigue trials, forming modes of accelerated trials, in technological processes.

In [8] is theoretically shown than extemums of parameters of vibration loads exists in any mechanical system. They correspond to extemums of power of mechanical forces, appearing in system during broadband random vibration. In other words, there is such width of the range that the ratio (in probabilistic aspect) of amplitudes of components of oscillatory process on its own frequencies optimally. Existence of such components of oscillatory process out of original frequencies, which is usual for BRV, just lowers lifespan of the objects.

Here are presented the results of the experiment and simulation of the oscillations of the cantilever beam.
2. Experiment

Forming of hardest condition in its essence is a problem of optimization vibration loading with given power. It comes to discovering the class of effects f(x,t), which creates extreme conditions of object load. Mathematically it can be expressed by relation:

\[ \text{extr } F[y,y',y'',...,y''''(t),x,t,f(x,t)] = \max_{f(x,t) \in \Phi} y'(t,x,t) \]

where \( F \) – functional, characterizing vibrational movement of the object.

In general case the problem (1) is difficult to solve, but in practical realization of the trials there are limitations on kind of effect (power, number of harmonics), on control coordinates for object movement (dangerous intersection) and so on. That is why one can investigate particular problems:

\[
\begin{align*}
\max & y'_{x=u}, t \in T \\
\max & D_{y'_{x=u}}, t \in T \\
\min & f(x,t) t_r, t \in T, \text{where } \frac{1}{T} \int_0^T f^2 \, dt < C
\end{align*}
\]

In other words, it is necessary, by changing the range of the spectrum, to find in which one the vibration speed (2) or dispersion of vibration speed (3) in the intersection during the time t are maximal, and time before fracture of object \( t_r \) (4) is minimal for given limited power of tension.

In order to raise veracity and reliability of trials’ results a series of indestructible contactless capacitive sensors, which allow to measure almost limitless by value flexural deformations for indefinite time, was developed. Capacitive sensors, on the opposite of resistive-strain sensors, do not deform with an object, do not accumulate damage, that is why they keep their metrological characteristics constant. Capacitive sensor of deformation EED [9] has been developed in order to measure deformation of flat and cylindrical surfaces with zero gauss curvature in radial direction. It consists of metal lining 1 with cylindrical working surface, which through the layer of dielectric 2 is pushed towards (or placed close to) surface under study 3, which is the second lining of capacitor (Figure 1).

![Figure 1. Capacitive sensor of deformations for flat and cylindrical surfaces (EED1)](image)

With the bending of an object the distance \( \delta \) between linings of the capacitor is changed and electric signal of sensor is proportional to the change of curvature and curvature deformation of an object. Area of linear characteristic of capacitive sensor in statics and dynamics is higher than it is of resistive-strain sensors. Its range covers values of deformation in the given experiment including low-cycle area of fatigue. Sensibility of the sensor is defined by curvature of measuring lining and can vary in a wide range. Two linings from two sides of an object raise sensibility by 2 times. Measuring lining does not deform with an object, and consequently its lifespan almost unlimited.

Experimental investigation of lifespan. Investigation of effect of vibration spectrum width \( \Delta f \) on vibration loading (vibration speed) and lifespan was conducted on cantilevered fixed beam in kinematic excitation (Figure 2).
Investigation of effect of vibration spectrum width $\Delta f$ on vibration and lifespan was conducted on cantilevered fixed beam in kinematic excitation (Figure 3).

The trial was conducted with random modes for following values of $\Delta f$: 10 Hz (mode №5); 30 Hz (mode №6); 100 Hz (mode №7); 300 Hz (mode №8) (Figure 3). Numbers of testing modes are given with correspondence to full series of testing modes [10]. The results of trials in harmonic mode №1 are introduced only for qualitative comparison.

Experiment on fatigue was conducted according to method, which does not simplify any random process, because replacement of random vibration loading with harmonic mode is not correct [6]. All measurements and, most importantly deformations were applied from the start of trial until the fracture of the object.

As it common for lifespan investigations the results were introduced in a form of fatigue curves – dependence of logarithm of time till the fracture on average tension in the material (Figure 4). It is shown that with the widening of load range lifespan of the material of the object is falling in the beginning and then raising.

Existence of extremum of lifespan depending on limiting frequency of spectrum of vibration loading during constant tension in the objects is clearly shown in picture 5, which has been obtained by processing results of fatigue trials. As a limiting frequency of spectrum for harmonic mode (mode №1 [10]) logarithm of first original frequency of the object $f_1$=27 Hz, and for random mode with
continuous spectrum – logarithm of sum of average spectrum frequency (first original frequency of the object) and half of spectrum width for chosen mode (Figure 3): \( \lg f_{br} = \lg \left( f_1 + \frac{1}{2} \Delta f \right) \).

During random oscillations the object was significantly being exited on the second and the third original frequencies \( (f_2=177 \text{ Hz}, f_3=475 \text{ Hz}) \).

In order to raise informational content of the results of trials “curves of the vibration loading”, which show the dependence of logarithm of time until fracture on vibration speed in sample’s dangerous cross-section (Figure 6), have been drawn first time in practice.

![Figure 6](image)

**Figure 6.** Depending \( t_{cr} \) of the samples of \( V_{AV} \) in a dangerous cross-section of the beam (“curves of the vibration loading”)

![Figure 7](image)

**Figure 7.** Dependence of material samples lifespan on width of vibration loading spectrum with constant average vibration speed.

Similar existence of extremum of lifespan depending on limiting width of spectrum \( f_{br} \), but with constant average vibration speed in dangerous intersection of the object, obtained by transforming “curves of the vibration loading”, is shown on Figure 7.

Despite the fact that curvature of graphs 5 and 7 in given coordinates is relatively small, there is no doubt in the existence of extremums, because the points on the curves correspond to high amount of fractured test objects. The veracity of the test results is high enough thanks to application more perfect methods of trials and deformation sensors.

The experimental results are agreed well with the simulation results.

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3. Simulating

With the purpose of decision of these tasks for the console fixed beam were investigated by digital simulation the dependences of average square of vibration speed meaning $<\dot{S}^2>$ of object in dangerous section at given average square meaning of tensions $<\dot{\sigma}^2>$ from width of a spectrum and also average square meaning tensions at constant average square meaning of vibration speed [11].

As a generator of random excitations it was used a standard subroutine. For formation of a loading spectrum of “white noise” type the filters giving signals with a uniform spectrum on displacement $<S>$ were applied; $f_{\text{bd}}$ bottom boundary frequency of a spectrum, $f_{\text{bd}}$ - top boundary frequency of a spectrum.

The real experiment on durability was carried out to a cinematically excited beam, console fixed on shaking table, where $S(t)$ – displacement of the shaking table. Proceeding from this, the vibrating influence on a beam is proportional to acceleration $\ddot{S}(t)$.

The imitating model was under construction on the basis of the equation of bending fluctuations of a beam.

$$m_o \dddot{y}(x,t) + A \dddot{y}(x,t) + \alpha \ddot{y}(x,t) = f(x,t). \quad (5)$$

Under boundary conditions:

$$y(x) = 0, \quad \dot{y}(x) = 0 \quad \text{at} \quad x = 0;$$

$$\ddot{y}(x) = 0, \quad \dddot{y}(x) = 0 \quad \text{at} \quad x = l;$$

$y(x,t)$ – deflection of a beam;

$m_o = \rho F$ – weight of unit of length of a beam;

$F$ – area of cross section;

$l$ – length of a beam;

$A = EI$ – bending tough;

$E$ – module of elasticity of a material of a beam;

$I$ – inertial moment of section;

$\alpha$ – factor of damping;

$f(x, t)$ – external loading.

At cinematically excitation external loading is described by the following expression:

$$f(x,t) = -m_o \ddot{S}(t),$$

where $S(t)$ – displacement of a table of the vibrator (jammed end of a beam).

Let’s proceed from the equation (5) to the dimensionless form of record, having accepted $x = \epsilon \cdot l$, $\gamma = \nu \cdot l$:

$$\dddot{\nu} + \frac{EI}{m_o \epsilon^4} \dddot{\nu} + \frac{\alpha}{m_o} \ddot{\nu} = -\frac{\dddot{S}}{l} \quad (6)$$

On the basis of Galerckin method the transition from equations (6) was carried out to the system of linear differential equations.

The research of accuracy of modeling was carried out with 16 members. There was shown that it is possible to use only three members in Galerckin approximation.

Obtained system of equations was numerically integrated by a Rhounge-Kutta method at random tension $\dddot{S}(t)$ formed, as it was specified, with generator of a random signal and filters.

In result the dependences of average square tension meaning were received.
\[ <\sigma^2> = \frac{1}{T} \int_0^T \sigma^2 dt, \quad \text{where} \quad \sigma = \frac{hE}{2} \frac{\partial^2 y(x,t)}{\partial x^2} = \frac{hE}{2l} \sum_{k=1}^{\infty} f_k(t) y_k. \]

In function of boundary frequency of vibrating excitation \( f_{2bd} \) (6) (Figure 8) at constant root-mean-square meaning of vibration speed

\[ <\dot{S}^2> = \frac{1}{T} \int_0^T S^2 dt = \text{const}. \]

Dependence of average square meaning of vibration speed \( <\dot{S}^2> \) also was received from boundary frequency of entrance influence \( f_{2bd} \) at given constant meaning of average square meaning of a tension \( <\sigma^2> = \text{constant} \) (Figure 9).

The received dependences (Figure 8 and Figure 9) have extreme at boundary frequency 120 Hz \(< f_{2bd} < 160 \text{ Hz}. It is possible to accept \( f_{\text{extr}} \) about 140 Hz. The resonant frequencies for model (1) have meanings: \( f_1 = 28 \text{ Hz}, f_2 = 175 \text{ Hz}, f_3 = 485 \text{ Hz}. \)

The carried out research allows to consider, that at constant average square of a tension (vibration speed) of a beam the dependence of average square of vibration speed (tension) from boundary frequency of vibration has extremes, laying in the field of meanings between resonant frequencies of a beam, but is significantly closer to \( n\)-th own frequency.

The result, received by modeling, will well be coordinated with the fatigue experiment [10,11], in which vibration was formed with 1/3- octave filters with average frequency \( f_{av} = f_1 \) and width of a spectrum \( \Delta f = 10, 30, 100 \) and 300 Hz.

Extreme meanings \( \dot{S}^2 \) and \( \sigma^2 \) are received for a mode with \( \Delta f = 100 \text{ Hz}, \) boundary frequency \( f_{bd} \) which with account flatness of characteristic of the filter is close to the meaning \( f_{2bd} \), appropriate calculated extremes. On the mode with \( \Delta f = 100 \text{ Hz} \) the time before destruction of a beam was the least, i.e. the specified mode has appeared the hardest one.

At the first stages of revealing extremes in the described dependences it not so important the accuracy of concurrence of experimental and calculated results, how the fact of confirmation both in experiment, and in account of the existence of effective width of a spectrum, which correspond extreme parameters of loading.

Modeling of influence of a broadband random spectrum on the console fixed beam by the described method was carried out in a range to sixteen first own frequencies of a beam. At capture by a loading spectrum of every one following own frequency of a beam vibration speed and tension in dangerous section accepted the extreme meanings. Appropriate width of a spectrum was named as an "own strip of a spectrum".

![Figure 8](image1.png)  
**Figure 8.** Extreme of root-mean-square meaning of a tension in function of boundary frequency of vibrating action

![Figure 9](image2.png)  
**Figure 9.** Extreme of root-mean-square meaning of vibration speed in function of boundary frequency
The received result can be treated as existence in mechanical systems "of own strips of a spectrum" (pass bands), in sense sensitivity to vibration, similar to own frequencies, but shown at broadband random vibration. The received result is similar submitted in [8], where the theory of dynamic systems was used.

4. Discussion
The modeling of impact of broadband random spectrum on cantilevered beam by described method was held in a range of first sixteen natural frequencies of the beam; the results are similar to those shown in Figure 8 and Figure 9 and for the first three natural frequencies are illustrated in Figure 10a, 10b, 10c.

In Figure 10a the amplitude frequency response of cantilever beam in the area of first three natural frequencies is given. The Figure 10b presents by the three spectrum of equal power slightly different by the width and, of course, amplitude very intensively exciting the beam in the area of first two and first three natural frequencies generated by the vibrator control system in place where the beam is fixed. The Figure10c shows the changes of average voltage and average vibration velocity in the rigid fixing of the beam depending on the width of the spectrum of external influence. At some width of this spectrum (for clarity, this is average spectrum, Figure 10b) the vibration velocity has a maximum and voltage – a minimum. Such a picture repeats with the coverage by the spectrum of external vibration exposure of the first two, three, etc. natural frequencies of the beam. The curves in Figure 10c are in fact a kind of analog amplitude frequency response of the beam, which characterize her reaction on changing by the width the random vibration loading.

The obtained result can be interpreted as the existence of “own spectrum bands” (passbands) of mechanical systems, in terms of sensitivity to vibration exposure similar to natural frequencies, but manifested in broadband random vibration loading.

Figure 10a. Amplitude frequency response of testing object

Figure 10b. The spectral density of external Influence
5. Conclusions
1. The existence of extremums in the dependencies of lifespan, tension and vibration speed, which are invariant toward the width of the vibration loading spectrum, has been noticed first time. The width of the spectrum of the continuous vibrational load corresponding to these extremes was called the previously effective spectral width [11,12, 13] (Figure 10b, 10c), and after [14] - the "own bands of the spectrum": $\Delta f_n^{\text{own}}$, where $n \geq 2$, determines the number of natural frequencies of the object that are taken into account when vibrating.
2. The hardest BRV condition of mechanical system is defined by its characteristics, similar to natural frequencies of the system.
3. Knowledge about the hardest BRV condition is useful not only for conducting accelerated trials but also for standardization of trials, for evaluation of level of danger of operating vibrations, for technological processes, acoustics and probably for resistivity to earthquakes.
4. The developing of the automated system for definition of hardest mode of vibration loading is necessary, which would determine the own spectrum bands of mechanical system, as determining the natural frequencies of this system.

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