An Option Contract with Optimized Prices in Supply Chain

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Abstract: This research considers a supply contract in a two-stage supply chain consisting of one supplier and one buyer. Market demands are stochastic, and there are two opportunities for the buyer to do demand forecast. The buyer places a firm order to the supplier with the first forecast at the beginning of production. At the same time, the buyer purchases a quantity of options. With the second forecast at the beginning of sales season, the buyer determines the execution quantity of the option. The supplier decides the option price, option execution price and wholesale price, while the buyer decides initial order quantity, option quantity and option execution quantity optimally under given prices. In this paper, we take a game theoretic approach to consider the coordination issue between the supplier and the buyer. We focus on the wholesale price that may increase supplier’s profit; however, it depends on buyer’s response to the supplier’s decision on the wholesale price.

Key Words: Supply contract, Option, Option price, Execution price, Supply chain management.

1. Introduction
Supply chain (SC) coordination with contracts is one of the methods that enable to make a win-win relationship between a buyer and a supplier. With SC contracts, the buyer and the supplier can still make decisions independently while both increase their profits. (Li and al., 1999; Tsay, 1999).

Quantity flexibility (Tsay, 1999) and buyback strategy (Choi and al., 1992) are typical examples of the SC coordination. In the buyback model, the supplier buys (back) unsold goods from the buyer at the end of the sales season sharing the buyer’s risk of dead stock. Under this guarantee, the buyer purchases goods actively and puts more effort to stimulate sales.

This research focuses on decision-making on the wholesale price, assuming that the buyer decides the optimal purchasing quantities and the supplier decides the optimal option prices.

The paper is organized as follows. Section 2 provides a brief literature review. Sections 3, 4 and 5 describe the outline of the proposed option model and the decision-making problem on the option related prices. Numerical experiments are presented in section 6. Finally, in section 7 we summarize our conclusions.

2. Literature Review
The bullwhip effect (Lee et al., 2004), bottleneck effect (Adams et al., 1988; Goldratt, 1984) and double marginalization (Spengler, 1950) are known as major root problems in supply chain. We can classify solutions for these problems into three categories: Integrated decision-making (Clark and Scarf, 1960), Information sharing (Enkawa and Cao, 1994) and SC coordination (Cachon, 2003).

If it is applicable, integrated decision-making would be the first choice. Information sharing, on the other hand, is one alternative that can be applied in case the integrated decision-making is not applicable. SC coordination is a third method that can be used, when both the aforementioned ones (i.e. integrated decision-making and information sharing) are not suitable.

SC coordination literature has developed since the late 1990s. Quantity Flexibility (Tsay, 1999), Buy-back/Return (Buchanan et al., 1998; Choi et al., 1999), Volume Discount (Anupindi and Bassok, 1998), Minimum Quantity Commitment (Bassok and Anupindi, 1997), and Option Contracts (Wang and Cao, 2006) are typical strategies for SC coordination. The objective of most of these models was to determine the optimal quantity for both the buyers and suppliers that increases their profits.

Cachon, (2003) reviewed the most important literature until the year 2003. Most of the SC coordination contract try to find a set of actions that ensures an optimal allocation of the total Supply Chain profit. To coordinate the SC, this set of actions must be a Nash equilibrium, in other words no firm can make profit from unilaterally deviating to another action. The generalized profit function in a SC are presented by the three functions below.

\[ \pi_r(q) = (p - \nu + g_r)S(q) - (c_r - \nu)q - g_r\mu - T \] (1)

\[ \pi_s(q) = g_sS(q) - c_sq - g_s\mu + T \] (2)

\[ \pi(q) = (p - \nu + g)S(q) - (c - \nu)q - g\mu \] (3)

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where $q$ is the order quantity, $\pi_c(q)$ is the retailer’s expected profit, $\pi_s(q)$ represents the supplier’s expected profit, $\pi(q)$ represents the total Supply Chain’s expected profit, $p$ is the retail price, $v$ is the retailer’s salvage value, $g_r, g_s$, are the retailer and the supplier’s goodwill penalty cost respectively while $g = g_r + g_s$. $\bar{S}(q)$ refers to the expected sales, $c_s$ is the supplier’s production cost per unit, $c_r$ is the retailer’s marginal cost per unit while $c = c_r + c_s$, $\mu$ is the expected demand and $T$ is the expected transfer price from the retailer to the supplier.

We can easily notice that maximizing the supplier’s profit may decrease the buyer’s profit and vice versa. Surely, we can maximize the total supply chain’s profit but we cannot ensure that both the supplier and the buyer can always make positive increase of profit under the integrated decision-making. This creates a need for a mechanism that ensures profit or risk sharing between the two connected players.

Actually, most of the SC coordination contracts focus on the transaction quantity. Nonetheless, there is room to improve Supply Chain profits through investigating not just the supply quantity but also supply prices. Indeed, setting a proper wholesale price along with offering options hedging against the retailer’s risk, might enhance the buyer’s purchasing quantity, and respectively increase the supplier’s profit.

In this paper, we consider not only supply quantity but also prices as decision variables. Based on the framework of option contract model (Lin and al., 2007), we analyze how the profits in supplier and buyer changes in this research.

Few papers aim to determine proper prices in option contract model. The research by Wu et al. (2002) might be one of the closest papers to ours, but it assumes that there is an extra option quantity (or production capacity) in the supply contract and only when the buyer purchases this extra quantity (deterministic precondition) will the supplier determine the optimal price for it. This is different in both the premise of modeling and decision variables from our research. In our model, we determine the optimal purchase quantity, the optimal option price and the optimal wholesale price together under the framework of Stackelberg game structure (the supplier decides his optimal prices based on his anticipation of the optimal purchase quantity of the buyer). Another relevant research is (Lariviere and Porteus, 2001). The authors studied optimal wholesale price between suppliers and buyers. They pointed out that the wholesale price depends on how the market grows. They also identified the negative correlation between the relative variability and the buyer’s price sensitivity, which allows a decentralized system to produce higher profit. In our model, we do not consider market size, however we consider forecast errors at two different times for decision making, i.e., at the time the supplier determines production quantity and at the time the buyer executes option quantity. Obviously, the latter forecast may result in higher precision due to the availability of richer and highly accurate information. Yet, the issue is that the supplier must determine all his profit-maximizing prices at the time of the first demand forecast and decide his production quantity. On the one side, if the set prices are too high, then the buyer will behave conservatively, purchase fewer amount of products, therefore fewer sales for both the buyer and the supplier. In most cases, fewer sales result in fewer profits except for luxury products. On the other side, if the set prices are too low, then the buyer will behave actively, purchase more amount of products, therefore more sales for both the buyer and the supplier. However, lager sales with low price does not guarantee higher profit.

There is trade-off between quantity and price. Every manufacturing company anticipates large sales with high price, however large sales depends on marketing power (product, price, place and promotion ···), and proper merchandising policy involving retail companies, the retailers.

In this paper, we focus on determining the wholesale price under the framework of a given option contract model, and analyze the optimal pricing policy from both the supplier’s and the buyer’s perspectives.

3. The Option Contract Model

We consider the wholesale price under a given option contract model proposed by Lin et al. (2007). The sequence of decision-making is as show in Fig. 1.

At time $t_0$, the supplier, in collaboration with the buyer, determines wholesale price first before production starts. At time $t_1$, the supplier offers the option contract prices. In response, the buyer puts a firm fixed order $Q$ and purchase amount of options $q_o$. The supplier makes decision to produce $Q + q_o$, and the production starts at the time $t_2$. From this production decision to the start of sales season, we have a time lag in which many activities must be accomplished. In the case of apparel product, this time lag is significant and the activities may include preparing materials (procurement including negotiation), cutting and sewing. At the beginning of sales season $t_4$, the buyer exercises the option quantity according to the second demand forecast update and determines the quantity of option execution $q_o$. At the end of sales season, the buyer takes the product away from shelf and the product will be transferred to the outlet with a markdown price $v_r$, which we will refer to in this paper as the inventory salvage value in the buyer. For simplicity (and to maintain channel long-term profit), we assume that the non-executed option quantity will be consumed inside the supplier company with lower price than the manufacturing cost. In other words, the salvage cost $v_s$ will bring negative impact (profit) for the supplier.

![Fig. 1 The sequence of decision-making](image-url)
3.1 The Buyer’s Problem

To determine wholesale price, we assume that the supplier knows that the buyer behaves rationally, and responds optimally to a given wholesale price, option price and option execution price.

Under this precondition, we first consider buyer’s optimal purchase quantity. We assume that this product is a seasonal special product and thus there is one chance for supplier to make decision on production quantity. We also assume that the buyer purchases products from this single supplier build upon the option contract. In other words, we consider a supply chain consisting of one supplier and one buyer in this paper.

Parameters and decision variables throughout this paper are presented below.

- $Q$: Initial firm fixed order quantity
- $q_o$: Option quantity
- $q_e$: Option execution quantity
- $p_o$: Option price
- $p_e$: Option execution price
- $p_w$: Wholesale price
- $p_r$: Retail price
- $p_c$: Production cost per unit
- $v_o$: Cost of opportunity loss
- $v_r$: Inventory salvage value in the buyer
- $v_s$: Inventory salvage value in the supplier
- $D$: Demand $D \in [L - m, l + m]$
- $m$: Predictive one-side range of demand
- $L$: Predictive average of $D$ (stochastic variable)
- $l$: Determined value of $L(l \in [K - n, K + n])$
- $n$: Predictive one-side range of average demand
- $K$: Expected value of $L$

3.2 Demand and assumptions

We assume that the supplier and buyer have rough estimation on the upper and lower limits of their product’s demand at the beginning of the production. Afterward, at the beginning of the sales season, the buyer can forecast the upper limit and lower limit with higher accuracy but still does not know the most likely value of the demand in the future. For that reason, we use uniform distribution in this paper. The uniform distribution also enables analytic modeling and mathematical treatment.

We assume that the demand follows a continuous uniform distribution with average $L$ and width is $2m$. $L$ is also assumed random. The buyer determines the exact value of $L$ at the start of the sales season from the second demand forecast. We use small character $l$ to distinguish stochastic variable $L$ and deterministic variable $l$. We assume that the average demand $L$ follows a continuous uniform distribution with an average $K$ and width $2n$.

In our model, the wholesale price is determined by negotiation between the two firms. Therefore, we assume that both the supplier and the buyer share the same information about the future demand when making their decisions. In other words, both the buyer and the supplier have common knowledge on the value of $K$ and $2n$ which is determined at the first demand forecast when both the buyer and the supplier are more interested in the distribution of the average value of market demand. We also assume that the buyer updates his forecast (i.e., second forecast) at the beginning of the sales season where he determines the values of $l$ and $m$. That is to say, determine the uniform distribution of the real demand $D \in [l - m, l + m]$. For higher accuracy of second demand forecast, we assume that the buyer has confidence on the upper limit and lower limit of the real demand. However, we do not know the exact future sales quantity during the sales season, which is one of the arguments why, in this paper, real demand follows a uniform distribution.

Constraints

Call option is the so called “right” for the buyer to purchase additional product at a predetermined future time. By using option purchasing, buyers can hedge against risk. The buyer pays extra cost for this option. Thus, the unit procurement cost of an option is more expensive than normal wholesale price. Inversely, if the unit procurement cost under the option contract (i.e., $p_o + p_e$) is smaller than the value of the normal wholesale price $p_w$, then the buyer can maximize his profit by setting $Q = 0$ and purchasing as much quantity of options as possible. Mathematically, the optimal value of the option quantity is $q_o = \infty$. Obviously, the supplier will not prefer this situation. Thus, we set the constraint on the option price shown below:

$$0 < p_w \leq p_o + p_e$$

(4)

If the cost of purchasing option $q_o$ at the beginning of production is higher than that of losing the opportunity of sales (opportunity cost), the buyer’s best response will be not to purchase any option. To preserve the buyer’s incentive to purchase option we set the following constraint.

$$p_o + p_e - p_w \leq v_o$$

(5)

If we set the opportunity cost $v_o$ equals to the difference between retail price $p_r$ and the wholesale price $p_w$, constraint (5) can be rewritten as below.

$$p_o + p_e \leq p_r$$

(6)

This means that the sum of option price and option execution price is smaller than the retailer price. This constraint ensures that the buyer can make money through purchasing option.

Furthermore, if the inventory salvage value in the buyer ($v_r$) is higher than the wholesale price ($p_w$), the buyer may put extraordinarily big firm fixed orders since, in this case, he never loses money with dead stock. In order to prevent such unrealistic situation, we introduce the following constraint.

$$v_r \leq p_w$$

(7)

To avoid the trivial case where the retailer always exercises all the purchased options, we assume also that

$$p_e > v_r$$

(8)
3.3 The Buyer’s Expected Profit

The buyer’s expected profit $G_B$ can be obtained by subtracting costs from gross income. In this paper, we define the gross income as the sum of the expected sales income of the fresh products and the product salvage income (discounted product to outlet shop). The first two terms in equation (6) represent it. Costs include expected purchasing costs of the initial firm fixed order, option quantity, option execution cost, and opportunity loss.

$$G_B = p_r E[\min(D, Q + q_e)] + v_c E[\max(0, Q + q_e - D)]$$

$$-p_w Q - p_o q_o - p_v E[\min(q_o, \max(0, D - Q))]$$

$$-v_o E[\max(0, D - Q - q_c)] \quad (9)$$

As can be seen in equation (9), there are three decision variables $Q$, $q_o$, and $q_e$ for buyer and three decision variables $p_o$, $p_v$, and $p_w$ for supplier. The buyer maximizes his profit $G_B$ determining $Q$, $q_o$, and $q_e$ optimally. We use backward decision-making process to derive optimal $Q$, $q_o$, and $q_e$. Readers can also refer Wang and Tsao (2006) and Lin, Wang, and Cao (2007) for derivation process.

3.4 The Buyer’s Optimal Option Execution Quantity

At $t_2$ the start of sales season, the buyer determines $q_e^*$ that maximizes $G_B$ in the following way:

$$q_e^* = \begin{cases} 
q_o & \text{if } l + z - Q \leq 0 \\
q_o & \text{if } l + z - Q \leq q_o - z \\
0 & \text{if } l + z - Q \leq [K - n, Q - z] 
\end{cases} \quad (10)$$

Where,

$$z = m - \frac{2n(p_o - v_r)}{p_r + v_o - v_r}.$$

As given in equation (10), the optimal option execution quantity $q_e^*$ is equal to zero if the upper limit of real demand is smaller than a certain quantity (i.e. $Q - z$). Conversely, the buyer will execute full quantity of the purchased option $q_o$, if the lower limit of real demand is larger than a certain quantity (i.e. $Q + q_o - z$). These values are determined by differentiating the objective function classifying possible range overlapping of the two-demand forecasting. Readers may refer to Wang and Cao (2006) and Lin et al. (2007) for detail of derivation.

The Buyer’s Optimal Initial Fixed Order Quantity and Option Purchase Quantity Since $q_e^*$ is the function of $Q$ and $q_o$, having the optimal $q_e^*$, the optimal $Q^*$ and $q_o^*$ that maximize $G_B$ could be determined by substituting $q_e^*$ to the equation of $G_B$[17]:

$$Q^* = K + m + n - \frac{2n(p_w + p_o - v_r)}{p_r - v_r} - \frac{m(p_e - v_r)}{p_r + v_o - v_r} \quad (11)$$

$$q_o^* = \frac{2n(p_w + p_o - v_r)}{p_r - v_r} - \frac{2np_o}{p_r + v_o - p_e} - m \quad (12)$$

As can be seen from equation (11) and (12), the value of $Q^*$ is positive under the constraints we set in this paper, but $q_o^*$ could be negative. A negative $q_o^*$ indicates that the supplier buys-back at price $p_e$ a part of the goods of the firm fixed order $Q^*$ that was sold to the buyer at price $p_w$. Since we set that $p_e$ is not smaller than $p_w$, the negative $q_o^*$ encourages the buyer to purchase unnecessarily big number of options and the supplier takes 100 percent risk of dead stock. To avoid such situation, in this research, we assume that the supplier does not buyback any products from the buyer. This is expressed by the constraint (13).

$$0 \leq \frac{2n(p_w + p_o - v_r)}{p_e - v_r} - \frac{2np_o}{p_r + v_o - p_e} - m \quad (13)$$

4. The Supplier’s Problem

4.1 The Supplier’s Expected Profit and Constraints

Now we consider the problem of maximizing the expected profit of the supplier $G_s$. The supplier’s profit can be obtained by subtracting costs of production $p_e(Q + q_o)$ from incomes of selling, the sum of initial firm fixed order $Q \times p_w$, options $q_o^* p_o$, execution options $q_e^* p_e$, and inventory salvage value $v_o(q_o^* q_e^*)$. As mentioned earlier, the supplier will act as a Stackelberg leader. Indeed, the supplier’s problem is to maximize his expected profit under the assumption that he knows the retailer’s best response ($Q^*, q_o^*, q_e^*$). Thus the supplier’s problem is:

Maximize

$$G_s(p_o, p_e) = Q_w^o + q_o^o p_o + q_e^o p_e + v_o(q_o^* q_e^*) - p_e(Q^* q_o^*) \quad (14)$$

Subject to

$$Q^* = K + m + n - \frac{2n(p_w + p_o - v_r)}{p_r - v_r} - \frac{m(p_e - v_r)}{p_r + v_o - v_r} \quad (15)$$

$$q_o^* = \frac{2n(p_w + p_o - v_r)}{p_r - v_r} - \frac{2np_o}{p_r + v_o - p_e} - m \quad (16)$$

$$q_e^* = \begin{cases} 
q_o & \text{if } l + z - Q \leq 0 \\
q_o & \text{if } l + z - Q \leq q_o - z \\
0 & \text{if } l + z - Q \leq [K - n, Q - z] 
\end{cases} \quad (17)$$

$$p_o + p_e - p_w \leq v_o \quad (18)$$

$$p_o + p_e \geq p_w > 0 \quad (19)$$

$$p_o + p_e \leq p_r \quad (20)$$

$$v_o \leq p_w \quad (21)$$

$$p_e > v_r \quad (22)$$

Our purpose is to determine $p_o$ and $p_e$ optimally under the constraints given by equations (4) to (8) and equation (13). The optimal $p_o$ and $p_e$ are denoted by $p_o^*$ and $p_e^*$, respectively.
4.2 The Optimal Option Price and Option Execution Price

To solve this problem, we differentiate $G_S$ successively with respect to $p_o$ and $p_e$.

$$
p_w = \frac{\partial Q^*}{\partial p_w} + \frac{\partial q_o^*}{\partial p_w} + q_o^* + \frac{\partial q_e^*}{\partial p_w} p_e + v_s \left( \frac{\partial q_e^*}{\partial p_w} \right)
$$

$$
- \frac{\partial q_o^*}{\partial p_w} - p_c \frac{\partial q_e^*}{\partial p_w} - \frac{\partial q_e^*}{\partial p_w} \frac{\partial q_e^*}{\partial p_w} = 0
$$

(23)

$$
p_w = \frac{\partial Q^*}{\partial p_e} + \frac{\partial q_o^*}{\partial p_e} p_o + q_e^* + \frac{\partial q_e^*}{\partial p_e} p_e + v_s \left( \frac{\partial q_e^*}{\partial p_e} \right)
$$

$$
- \frac{\partial q_o^*}{\partial p_e} - p_c \frac{\partial q_e^*}{\partial p_e} - \frac{\partial q_e^*}{\partial p_e} \frac{\partial q_e^*}{\partial p_e} = 0
$$

(24)

Since the $Q^*$, $q_o^*$ and partial differentials $\partial Q^*/\partial p_w$, $\partial q_o^*/\partial p_w$, $\partial Q^*/\partial p_e$, $\partial q_e^*/\partial p_e$, $\partial q_e^*/\partial p_e$ are functions of $p_o$ and $p_e$, we substitute them into Eqs. (23) and (24) and obtain Eqs. (25) and (26).

$$
mp_e^4 + \alpha_1 p_e^3 + \alpha_2 p_e^2 + \alpha_3 p_e + \alpha_4 p_e p_o + \alpha_5 p_o + \alpha_6 = 0
$$

(25)

$$
4m^2 p_e^7 + \beta_1 p_e^6 + \beta_2 p_e^5 + \beta_3 p_e^4 + \beta_4 p_e^3 +
\beta_5 p_e^2 + \beta_6 p_e + \beta_7 p_o^2 + \beta_8 p_o + \beta_9 = 0
$$

(26)

5. Optimization of the Wholesale Price

We consider the problem of wholesale price. At time $t_o$, wholesale price $p_w$ is determined by negotiation (game) between the supplier and the buyer. The supplier and the buyer are cooperative and willing to maximize whole supply chain profit. In other words, we do not consider benefit-sharing methods in this paper. This time, we consider there is only one decision variable, i.e., wholesale price $p_w$. To find extreme point, we differentiate the supplier’s profit function and the buyer’s profit function with respect to the wholesale price. Setting two differentiation functions to be zero, we obtain below system functions.

$$
\frac{\partial G_B(p_w, p_o, p_e, q_o^*, q_e^*)}{\partial p_w} = 0
$$

(27)

$$
\frac{\partial G_S(p_w, p_o, p_e, q_o^*, q_e^*)}{\partial p_w} = 0
$$

(28)

To solve this problem, we use the software MATHEMATICA. For a given set of parameters values and initial value of the wholesale price, optimal option prices are determined. Then, the optimal wholesale price is determined solving equation (27)(28), and the optimal value leads back to the option price and the option execution prices determination process. Finally, we print out all decision values for the supplier and the buyer including option price, option execution price, firm fixed initial order, option quantity, option execution quantity and the wholesale price.

6. Numerical Experiments

To evaluate the proposed method, we compare the expected profits of the proposed model to those of the previous research (Lin et al., 2007) and the Newsvendor Problem (NVP). In the previous research, $Q^*, q_o^*$ and $q_e^*$ are optimized under a given set of contract prices, i.e., $p_o = 10$, $p_e = 120$, $p_w = 100$ (Lin et al. 2007).

6.1 Numerical Experiments and Results

Table 1 illustrates parameter value setting satisfying the constraints introduced in section 3.

| Parameter setting | $p_o$ | $p_e$ | $v_o$ | $v_e$ | $m$ | $n$ | $K$ |
|-------------------|-------|-------|-------|-------|-----|-----|-----|
|                   | 200   | 40    | 60    | 40    | 30  | 150 | 200 |

Commercial software MATHEMATICA is used in the numerical examination dealing with the complicated integration that contains joint distribution. We deduce the optimal solution out from the set of generated solutions by excluding the irrelevant results that refer to unrealistic cases or that violate the imposed constraints.

$$(p_w^*, p_o^*, p_e^*) = (59.9, 17.28, 44.23).$$

(29)

The supplier offers the prices above before the beginning of the production season, the buyer responds by ordering $Q^*$, $q_o^*$ and then $q_e^*$. The buyer’s expected profit values in the three compared cases are calculated according to these order quantities. Table 2 displays the expected profits of three different models: the NVP, the previous research (Lin et al. 2007) and the proposed model in this paper. $G_B + G_S$ in the last column of Table 2 refers to the SC’s profit function.

Based on the results presented in Table 2, we can notice that under the same parameter settings (presented in Table 1), the expected SC’s profit from using our proposed model (157662) is 1.85% higher than that gained using the previous model (154796). Furthermore, our model ensures 3.25% higher SC’s profit than that gained by a NVP. This indicates that the optimization of the contract prices can improve the total supply chain performance. We also notice that the expected buyer profit in our proposed model is remarkably higher (49.73%) compared to the expected buyer profit in the previous model. This obviously will induce the buyer to purchase optimally his firm fixed quantity, option quantity, and option execution quantity.

Conversely, this may decrease the supplier’s profit compared to higher wholesale price setting. In fact, we can spot that the wholesale price $p_w^*$ in our proposed model presents just 60% of that in the other model. Compared to the previous model, despite proposing a higher option price ($p_o^* = 17.28 > 10$) our contract proposes a smaller option execution price ($p_e^* = 44.23 < 120$). In addition, the difference between the wholesale price and option cost, i.e., $p_w^* - (p_o^* + p_e^*)$, also decreased by approximately 60% compared to the previous model. This gives incentive to the buyer to increase his total purchase quantity $Q^* + q_o^*$, particularly his option quantity $q_e^*$. The increase of the
quantity $q^*_u$ is mainly due to the decrease of the option price $p^*_o + p^*_c$. Actually, the major finding of this research is that, under an option contract, the buyer’s purchasing behavior can be altered by the supplier’s optimal pricing behavior. 

Surely, we should consider proper benefit sharing between the supplier and the buyer. If the supplier and the buyer are two independent companies that consider preserving their own profit, then the supplier will not set such low prices that might result in negative profit. Proper benefit sharing is essential for the proposed models in practice.

### 6.2 Discussion

As stated in 3.1.1, the buyer has two demand forecasts opportunities. The first forecast generates the range of the average of demand with width $2n$. The larger $2n$ is, the larger the uncertainty of the average demand is. Besides, the awareness of the larger uncertainty along with risk aversion results in larger error in demand forecasting. Therefore, the value of $n$ can be considered as a parameter that represents forecast error (range of system error).

At the start of sales season, we assume that the average demand value is determined with high accuracy, that we call the deterministic value $l \in [K - m, K + m]$. Nevertheless, we still do not know the real demand during the sales season. Thus, we introduced the demand fluctuation range $2m$ to exhibit this uncertainty during the sales season. Therefore, the $m$ could be considered as a parameter that represents uncertainty of demand. We carry out experiments by setting $m$ and $n$ to the two levels respectively.

Table 3 shows the results, based on which we may derive the following findings.

In the case where the accuracy of the first demand forecast is low (i.e. $n$ is large) and for the same value of $m$, the expected SC profit $(G_B + G_S)$ increases with $n$. This trend exists in both the proposed model and the previous research model. In this case, the buyer tries to decrease the fixed order quantity, while increasing the option quantity. Besides, the increase of the option purchase quantity gives merits to the buyer such as hedging inventory risk since the second demand forecast is relatively more accurate than the first demand forecast.

When uncertainty of demand is low (i.e. $m$ value is small) and for the same value of $n$, the expected profit of the SC $(G_B + G_S)$ increases when $m$ decreases. The reason obviously comes from higher accuracy of the second demand forecast that is described by the smaller value of the $m$.

When the prices are optimized, the sensitivity to the accuracy of the first demand forecast decreases. In spite of (1), the difference of the expected profit between larger $n$ and smaller $n$ in the proposed model is small compared to that in the previous research. For example, if we set $m = 150$, the difference between the expected total SC profits for $n = 200$ and for $n = 250$ is 523 while that in the previous research is 758.

The profit of the SC in the proposed model is always bigger than that in the previous research, while each party determines the prices and order quantity independently to maximize own profits.

### 7. Conclusions

This research proposed a model to determine the wholesale price, the optimal option price and option execution price. We performed numerical experiments and the results revealed that expected profit of the SC in the proposed model was increased compared to the previous research, which can justify the effectiveness of the proposed model.

We further analyzed some cases changing parameters. Letting $n$ and $m$ be parameters that represent accuracy of the first demand forecast and uncertainty of demand respectively, we observed the difference between smaller $n$ and larger $n$, where the smaller $n$ represents higher accuracy of the first demand forecast. From the numerical calculation of the expected SC profit, we witnessed that the profit in both the previous research and the proposed model would increase when the accuracy of the first demand forecast increases.

Finally, we increased the total SC profit using computational effort; however, we could not obtain a closed form of the optimal wholesale price, option price and the option execution price. Solving the seven ordered two implicit system functions is the core problem for obtaining the closed form of the optimal prices. We can tackle that

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**Table 2** Expected profits in the three models

| Model    | $p^*_n$ | $p^*_o$ | $p^*_c$ | $Q^*$ | $q^*_o$ | $q^*_c$ | $G_B$ | $G_S$ | $G_B + G_S$ |
|----------|---------|---------|---------|-------|---------|---------|-------|-------|-------------|
| Proposed | 59.9    | 17.28   | 44.23   | 985   | 169     | 1154    | 133478| 24184| 157662      |
| Previous | 100     | -       | -       | 1019  | 83      | 1102    | 89144 | 65652| 154796      |
| NVP      | 100     | -       | -       | 1067  | -       | 1067    | 86698 | 64000| 152698      |

**Table 3** Relationship between the expected profit and the different values of $m, n$

| Model         | $m$ | $n$ | $p^*_o$ | $p^*_c$ | $Q^*$ | $q^*_o$ | $q^*_c$ | $G_B$ | $G_S$ | $G_B + G_S$ |
|---------------|-----|-----|---------|---------|-------|---------|---------|-------|-------|-------------|
| Proposed model| 150 | 200 | 60      | 17      | 44    | 985     | 169     | 1154  | 133478| 24184      |
|               | 100 | 200 | 71      | 20      | 56    | 967     | 177     | 1144  | 122056| 37038      |
|               | 150 | 250 | 67      | 20      | 50    | 964     | 218     | 1182  | 124679| 33506      |
|               | 100 | 250 | 75      | 19      | 65    | 958     | 221     | 1179  | 116000| 42834      |
| Previous Research| 150 | 200 | 100     | 10      | 120   | 1019    | 83      | 1102  | 89144 | 65652      |
|               | 100 | 200 | 100     | 10      | 120   | 990     | 133     | 1123  | 90935 | 66571      |
|               | 150 | 250 | 100     | 10      | 120   | 1002    | 142     | 1144  | 87791 | 67763      |
|               | 100 | 250 | 100     | 10      | 120   | 974     | 192     | 1166  | 89523 | 68655      |

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with further research.

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