Supersymmetric theories with explicit R-parity violation

Probir Roy
Tata Institute of Fundamental Research, Mumbai, India

• $R_p$ and motivation for explicit $R_p$ • MSSM → $R_p$ MSSM
• Bounds on $R_p$ couplings • Collider-specific phenomenology
• Cosmological implications • Summary and outlook • References

• $R_p$ and motivation for explicit $R_p$

The superfield spectrum of the Minimal Supersymmetric Standard Model $MSSM$ [1] can be written in usual notation as

$$Q_i = (U_i \mid D_i), \quad L_i = (N_i \mid E_i), \quad H_u = \left( H_u^+ \mid H_u^0 \right), \quad H_d = \left( H_d^0 \mid H_d^- \right), \quad \bar{U}_i, \bar{D}_i, \bar{E}_i$$

and its superpotential as

$$W_{MSSM} = h_{ij} Q_i \cdot H_i U_j + h_{ij} Q_i \cdot H_d \bar{D}_j + h_{ij} L_i \cdot H_d \bar{E}_j + \mu H_u \cdot H_d. \quad (1)$$

(1) has been constructed so as to conserve $R$-parity (even for particles, odd for sparticles)

$$R_p \equiv (-1)^{3B+L+2s},$$

$B, L$ and $s$ being baryon no., lepton no. and spin. This is equivalent to the conservation of matter parity which transforms the superfields as

$$(L_i, \bar{E}_i, Q_i, \bar{U}_i, \bar{D}_i) \rightarrow -(L_i, \bar{E}_i, Q_i, \bar{U}_i, \bar{D}_i),$$

$$(H_u, H_d, V_\gamma, V_Z, V_+^W, V_-^W) \rightarrow (H_u, H_d, V_\gamma, V_Z, V_+^W, V_-^W).$$

Three spectacular consequences ensue from $R_p$-conservation. First, each vertex involving sparticles has them in a pair. Second, the lightest supersymmetric particle (LSP $\tilde{\chi}_1^0$: a weakly interacting neutralino, escaping through detectors, being neutral on cosmological grounds)
is an excellent candidate for cold dark matter. Finally, each sparticle, pair-produced in a collider, would decay within the detector into particles and \( \chi^0_1 \), the latter being characterized by a hard \((E/T)\) signature.

Recall that \( R_p \)-conservation in supersymmetric theories was phenomenologically postulated \([2]\) by the need to avoid catastrophic proton decays induced by \( R_p \) vertices. However, such decays need to violate both \( B \) and \( L \). So, in place of \( R_p \), it suffices to have another discrete symmetry such as \( B \)-parity transforming the superfields as

\[
(Q_i, \bar{U}_i, \bar{D}_i) \rightarrow -(Q_i, \bar{U}_i, \bar{D}_i),
\]

\[
(L_i, E_i, H_u, H_d, V_{\gamma}, V_Z, V^+_W, V^-_W) \rightarrow (L_i, E_i, H_u, H_d, V_{\gamma}, V_Z, V^+_W, V^-_W).
\]

Analogously, one can have an \( L \)-parity instead. Since our presently scanty knowledge of high scale theories is unable \([3]\) to discriminate between various possible residual discrete symmetries at low energies, it is quite reasonable to consider \([4]\) \( R_p \) (but \( L \)- or \( B \)-conserving) interactions near weak-scale energies. In fact, supersymmetric grand unified theories have been constructed \([5]\) which have \( R_p \) interactions present at low energies. There are even stringy scenarios \([6]\) which predict them. On the other hand, a spontaneous violation of \( R_p \) requires \([7]\) an extra \( SM \)-singlet and hence a nonminimal spectrum; so we will restrict ourselves to explicit \( R_p \)-violation. The LSP, in this scenario, is neither stable nor necessarily neutral.

- \( \text{MSSM} \rightarrow R_p \text{ MSSM} \)

All renormalizable supersymmetric\(^{1}\) \( R_p \) interactions are incorporated by the addition of new terms in the superpotential

\[
W_{MSSM} \rightarrow W_{MSSM} + W_{R_p},
\]

\[
W_{R_p} = \frac{1}{2} \lambda_{[ijk]} L_i \cdot L_j \cdot \bar{E}_k + \lambda'_{ijk} L_i \cdot Q_j \cdot \bar{D}_k + \frac{1}{2} \lambda''_{[ijk]} \bar{U}_i \cdot \bar{D}_j \cdot \bar{D}_k + \epsilon_i L_i \cdot H_u,
\]

\(^1\)Supersymmetry-breaking \( R_p \) terms can occur in the Lagrangian in the form of trilinear scalar products. These have less spectacular effects than the supersymmetric \( R_p \) interactions to which we restrict ourselves.
\[ \mathcal{L}_{R_p} = \lambda_{ijL}^{[ijk]} [\bar{\nu}_i L \bar{\nu}_k R e_j L + \bar{\nu}_j L \bar{\nu}_k R \bar{\nu}_i L + \bar{\nu}_k R (\nu_i L) c e_j L - \bar{\nu}_j L \bar{\nu}_k R e_i L \\
+ \bar{e}_i L \bar{\nu}_k R \nu_j L + \bar{e}_k R (\nu_j L) c e_i L] + \lambda'_{ijk} [\bar{\nu}_i L \bar{d}_k R d_j L + \bar{d}_j L \bar{\nu}_k R \nu_i L \\
+ \bar{d}_k R (\nu_i L) c d_j L - \bar{d}_i L \bar{d}_k R \bar{u}_j L - \bar{u}_j L \bar{d}_k R \bar{e}_j L - \bar{d}_k R (e_i L) c u_j L] + \lambda''_{i[jk]} \epsilon_{\alpha\beta\gamma} \\
[\bar{u}_{iR}^* d_k R d_{jR}^* c_{jR}^* + \bar{d}_{jR}^* \bar{e}_{kR}^* c_{kR}^* + \bar{d}_{kR}^* (u_i R^* c_{iR}^* c_{jR}^* [u_j R^* c_{jR}^* c_{kR}^*] + h.c. \quad (3)]
\]

Generally, except in a specific case, we take the couplings to be real. Moreover, in analogy with the Yukawa couplings of MSSM, we anticipate a generation hierarchy in their strengths with the third generation ones expected to be the largest. Furthermore, the simultaneous presence of \( \lambda \) or \( \lambda' \) and \( \lambda'' \) type of couplings is extremely unlikely given the lack of observation of either baryon nonconserving nucleon decay or double-nucleon annihilation in nuclei. Indeed, these constraints imply [8]

\[
|\lambda'_{11k} \lambda''_{11k}| \lesssim 2 \times 10^{-27} \left( \frac{m_k}{100 \text{ GeV}} \right)^2, \quad (4a)
\]
\[
|\lambda'_{23k} \lambda''_{112}| \lesssim 10^{-14} \left( \frac{m_k}{100 \text{ GeV}} \right)^2, \quad (4b)
\]
\[
|(\lambda_{...} \text{ or } \lambda'_{...}) \lambda''_{...}| < 10^{-10} \left( \frac{m_k}{100 \text{ GeV}} \right)^2, \quad (4c)
\]

where \( m_k \) is the mass of a squark of flavor \( k \) (generic in the case of 6c) and \( \cdots \) stand for any any set of three flavor indices. Because (6) is so stringent, we will henceforth consider either \( L \)-violating and \( B \)-conserving \( \lambda, \lambda' \)-couplings or \( B \)-violating, \( L \)-conserving \( \lambda'' \)-ones.

- **Bounds on \( R_p \) couplings**

In deriving bounds on the absolute values of couplings, we will assume that one \( R_p \) operator acts at a time. Known upper bounds [9-14] on various \( \lambda \) and \( \lambda'' \) type of couplings, along with the physical process which is the source of each bound, are listed in Table 1. Specific references to the original derivation of these bounds may be found in the review by Bhattacharyya [10]. A similar listing has also been done, for whatever bounds [9-15] that exist on the \( \lambda' \) couplings. One remark on the bounds from \( D \bar{D} \) mixing and \( D \)-decays is in order. In SM no separate information on the unitary matrices \( U_L^u, U_L^d \), transforming \( u, d \) quarks to the mass-diagonal basis, is ever needed. Only the product combination \( U_L^u U_L^d \equiv V_{CKM} \) is constrained by experiment. Such is not the case once \( R_p \) interactions are included. The bounds above, pertaining to the \( D \)-system, have been derived by assuming \( U_L^d = 1 \).
An interesting recent development has been the derivation [16] of bounds on products of different $R_p$ couplings that are more stringent than the products of the individual bounds available. These follow if, instead of assuming the action of only one operator, we allow the possibility of two different operators acting at two different vertices in a Feynman diagram. Some of these bounds are tabulated below with the corresponding processes cited. Additional results may be found in the recent papers of Jang et al [16].

## TABLE 1

Upper bounds on $R_p$ couplings for $\tilde{m} = 100$ GeV. The numbers with (⋆) correspond to $2\sigma$ limits and those with (‡) are basis-dependent limits.

### PART A

| Coupling | Bound | Source | Coupling | Bound | Source |
|----------|-------|--------|----------|-------|--------|
| $|\lambda_{121}|$ | 0.05* | CC univ. | $|\lambda_{112}|$ | $10^{-6}$ | $NN \to K$’s |
| $|\lambda_{122}|$ | 0.05* | CC univ. | $|\lambda_{113}|$ | $10^{-5}$ | $\nu\bar{\nu}$ oscillation |
| $|\lambda_{123}|$ | 0.05* | CC univ. | $|\lambda_{123}|$ | 1.25 | Pert. unitarity |
| $|\lambda_{131}|$ | 0.06 | $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | $|\lambda_{121}|$ | 1.25 | Pert. unitarity |
| $|\lambda_{132}|$ | 0.06 | $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | $|\lambda_{123}|$ | 125 | Pert. unitarity |
| $|\lambda_{133}|$ | 0.003 | $\nu_e$-mass | $|\lambda_{223}|$ | 1.25 | Pert. unitarity |
| $|\lambda_{231}|$ | 0.06 | $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | $|\lambda_{312}|$ | $10^{-5}$ | $\nu\bar{\nu}$ oscillation |
| $|\lambda_{232}|$ | 0.06 | $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | $|\lambda_{313}|$ | $10^{-5}$ | $\nu\bar{\nu}$ oscillation |
| $|\lambda_{233}|$ | 0.06 | $\Gamma(\tau \to e\nu\bar{\nu})/\Gamma(\tau \to \mu\nu\bar{\nu})$ | $|\lambda_{323}|$ | 0.50 | $\Gamma_\tau^\beta/\Gamma_\tau^\beta$ (LEP 1) |

### PART B

| Coupling | Bound | Source | Coupling | Bound | Source | Coupling | Bound | Source |
|----------|-------|--------|----------|-------|--------|----------|-------|--------|
| $|\lambda'_{111}|$ | 0.00035 | $(\beta\beta)_{0\nu}$ | $|\lambda'_{211}|$ | 0.09 | $R_\pi$ | $|\lambda'_{311}|$ | 0.10 | $\tau^- \to \pi^- \nu_\tau$ |
| $|\lambda'_{112}|$ | 0.02* | CC univ. | $|\lambda'_{212}|$ | 0.09 | $R_\pi$ | $|\lambda'_{312}|$ | 0.10 | $\tau^- \to \pi^- \nu_\tau$ |
| $|\lambda'_{113}|$ | 0.02* | CC univ. | $|\lambda'_{213}|$ | 0.09 | $R_\pi$ | $|\lambda'_{313}|$ | 0.10 | $\tau^- \to \pi^- \nu_\tau$ |
| $|\lambda'_{121}|$ | 0.035* | APV | $|\lambda'_{221}|$ | 0.18 | $D$-decay | $|\lambda'_{321}|$ | 0.20‡ | $D^0\bar{D}^0$ mix. |
| $|\lambda'_{122}|$ | 0.02 | $\nu_e$-mass | $|\lambda'_{222}|$ | 0.18 | $D$-decay | $|\lambda'_{322}|$ | 0.20‡ | $D^0\bar{D}^0$ mix. |
| $|\lambda'_{123}|$ | 0.20‡ | $D^0 \leftrightarrow \bar{D}^0$ | $|\lambda'_{223}|$ | 0.18 | $D$-decay | $|\lambda'_{323}|$ | 0.20‡ | $D^0\bar{D}^0$ mix. |
| $|\lambda'_{131}|$ | 0.035* | APV | $|\lambda'_{231}|$ | 0.22* | $\nu_\mu$ d.i. | $|\lambda'_{331}|$ | 0.48 | $R_\tau$ (LEP) |
| $|\lambda'_{132}|$ | 0.34 | $R_\tau$ (LEP) | $|\lambda'_{232}|$ | 0.36 | $R_\mu$ | $|\lambda'_{332}|$ | 0.48 | $R_\tau$ (LEP) |
| $|\lambda'_{133}|$ | 0.0007 | $\nu_e$-mass | $|\lambda'_{233}|$ | 0.36 | $R_\mu$ | $|\lambda'_{333}|$ | 0.48 | $R_\tau$ (LEP) |
Upper bounds on some important product couplings for $\tilde{m} = 100$ GeV.

| Combination | Bound | Source | Combination | Bound | Source |
|-------------|-------|--------|-------------|-------|--------|
| $|\lambda'_{1k}\lambda''_{1k}|$ | $10^{-27}$ | Proton decay | $|\lambda'_{ijk}\lambda''_{lmn}|$ | $10^{-10}$ | Proton decay |
| $|\lambda_{1j1}\lambda_{1j2}|$ | $7.10^{-7}$ | $\mu \to 3e$ | $|\lambda_{231}\lambda_{131}|$ | $7.10^{-7}$ | $\mu \to 3e$ |
| $|\text{Im} \, \lambda'_{12}\lambda''_{21}|$ | $8.10^{-12}$ | $\epsilon_K$ | $|\lambda'_{12}\lambda'_{21}|$ | $1.10^{-9}$ | $\Delta m_K$ |
| $|\lambda'_{13}\lambda''_{31}|$ | $8.10^{-8}$ | $\Delta m_B$ | $|\lambda'_{1k1}\lambda''_{2k2}|$ | $8.10^{-7}$ | $K_L \to \mu e$ |
| $|\lambda'_{k1}\lambda''_{2l1}|$ | $5.10^{-8}$ | $\mu Ti \to e Ti$ | $|\lambda'_{11j}\lambda''_{21j}|$ | $5.10^{-8}$ | $\mu Ti \to e Ti$ |
| $|\lambda''_{i32}\lambda''_{i21}|$ | $0.008$ | $\Gamma(B^{\text{ch}} \to K^{\text{neut.}} K^{\text{ch}})$ | $|\lambda''_{i31}\lambda''_{i21}|$ | $0.006$ | $\Gamma(B^{\text{ch}} \to K^{\text{neut.}} \pi^{\text{ch}})$ |

- **Collider-specific phenomenology**

  Signatures for $R_p$ interactions in collider experiments involve two aspects of the concerned processes: first, production of some sparticles and second their specific decay modes. If sparticle pair-production is utilized, as is usually the case, only MSSM vertices are involved in the production process, whereas $R_p$ shows up through the sparticle decays that follow. Single sparticle production is, however, possible with $R_p$ vertices and can be exploited sometimes.

  **Tevatron $\to$ LHC**

  In such hadron colliders the possible production and decay-chains are

  \[
  q\bar{q} \text{ or } gg \to \tilde{q}\tilde{q}^* \to qq\tilde{\chi}^0_1\tilde{\chi}^0_1,
  \]

  \[
  \tilde{\chi}^0_1 \to \ell\ell\nu \text{ or } q'\bar{q}'\ell.
  \]

  Whether $\tilde{\chi}^0_1$ decays largely into $\ell^+\ell^-\nu$ or two jets and a lepton depends on which of the operators $L_i \cdot L_j \bar{E}_k$ and $L_i \cdot Q_j \bar{D}_k$ in (2b) is dominant. These $\ell\ell\ell\ell$ or $jjjj\ell\ell$ signals constitute very characteristic signatures [17] of $R_p$. Another process involving $R_p$ interactions in both production and decay is

  \[
  g + d \to \tilde{u}_j + \ell^-,
  \]

  \[
  \tilde{u}_j \to d_k + \ell,
  \]
leading to a $\ell\bar{\ell}j$ final state. The lower limits of $O(250)$ GeV, derived from Tevatron data on squark and gluino masses in MSSM, get diluted to about $O(100 - 130)$ GeV in $R_p$ MSSM. Another important point is that $\lambda'$-couplings make additional contributions to top semileptonic decay $t \to b\ell^+\nu$, leading to a violation of $e - \mu$ universality.

**HERA**

Resonant squark production in deep inelastic $ep$ scattering is possible via $R_p$ interactions, e.g.

\[
e^+d \to \bar{u} \to (e^+d_j, \bar{e}u_j, \bar{u}u_j, \bar{e}d_j), \\
e^-u \to \bar{d} \to (\bar{e}u_j, \bar{d}u_j, \bar{u}d_j), \\
\bar{\chi}_1^0 \to (\ell^0 + \nu)jj\text{ or } jj\ell, \\
\bar{\chi}_1^\pm \to (\ell^\pm + \nu)jj\text{ or } jj\ell,
\]

leading to a variety of final states containing multileptons + $E_T$ or multileptons + jets. For these signals to be observable, assuming that the squark masses are within the energy reach, one will need the relevant $\lambda'$ couplings to be between $[18] O(10^{-2})$ and $O(10^{-1})$ in magnitude. A point to note is that these are precisely the couplings that are relevant to atomic parity violation (cf. Table 1).

**LEP 2 and NLC**

In $e^+e^-$ colliders one can look for LSP pair-production followed by $R_p$ decays:

\[
e^+e^- \to \bar{\chi}_1^0\chi_1^0 \\
\bar{\chi}_1^0 \to \ell^+\ell^-\nu \text{ or } jj\ell.
\]

Thus one can again have quadrilepton + $E_T$ or dilepton + multijet final states. Particularly interesting [14] are final states containing an isolated likesign ditau with multijets. These signals can be visible even if the concerned $\lambda$ or $\lambda'$ couplings are as low as $O(10^{-4})$.

**Cosmological implications**

**Baryogenesis**

Any baryon asymmetry, generated at temperatures pertaining to the GUT scale, is in danger of being washed out by sphaleron-induced processes if there are additional $B-L$ interactions at the electroweak scale. The requirement of the preservation of primordial baryon asymmetry was alleged to impose strong ($< 10^{-7}$) constraints on the strengths of $R_p$ interactions which violate $B-L$. However, the conservation of $\frac{1}{3}B-L_i$, where $i$ is any $B-L$ one generation, has been shown [19] to suffice for this purpose – thereby leaving most of the $R_p$ couplings cosmologically unconstrained.
LSP decay

The simplest possible decay mode of the lightest neutralino is $\tilde{\chi}_1^0 \rightarrow \bar{e}u\bar{d}$ with the rate given by Dawson’s formula [20] $\tau_{\tilde{\chi}_1}^{-1} = (128\pi^2)^{-1}3\alpha|\lambda'_{121}|^2 M_{\tilde{\chi}_1}^5 m_{\tilde{e}}^4$. The requirement of the LSP decaying within a detector can be quantified as $c_{\gamma L}\tau_{\tilde{\chi}_1} < 1 m$ and leads to the constraint

$$|\lambda'_{121}| > 1.4 \times 10^{-6} \sqrt{\gamma_L} \left(\frac{m_{\tilde{e}}}{200 \text{ GeV}}\right)^2 \left(\frac{100 \text{ GeV}}{M_{\tilde{\chi}_1}}\right)^{5/2},$$

where $\gamma_L$ is the Lorentz boost factor. On the other hand, from the experimental absence of a long-lived relic LSP whose decay would lead to detectable upward going muons, one can infer [21] that $1 s < \tau_{\tilde{\chi}_1} < 10^{17}$ yrs leading to the forbidden interval $10^{-10} \lesssim |\lambda, \lambda', \lambda''| \lesssim 10^{-22}$. Note that $\tau_{\tilde{\chi}_1} > 10^{17}$ yrs would make $R_p$ MSSM indistinguishable from MSSM. Furthermore, the interval $10^{-8}s < \tau_{\tilde{\chi}_1} < 1s$ is practically out of observational reach. Of course, with an unstable LSP, one needs another candidate particle (axion?) for cold dark matter.

Summary and outlook

We conclude by emphasizing four main points. (1) There is presently no credible theoretical objection against the presence of $R_p$ interactions near the weak scale. (2) Though many of the $R_p$ couplings have been constrained in Tables 1-3, several (e.g. $\lambda'_{322}, \lambda'_{333}$ etc.) are totally unconstrained. (3) Vigorous searches are in progress and detection strategies are under formulation for collider signatures of $R_p$ in terms of excess lepton-jet combinations at HERA, violation of $e-m$ universality in top semileptonic decay at the Tevatron and isolated ditaus and jets in LEP 2 and NLC. (4) It is highly desirable to find some way of discriminating between different CDM candidates such as a neutralino and an axion.

I compliment the P$^4$ workshop organizers for a job well-done.

References

[1] H.E. Haber and G.L. Kane, Phys. Rep. 117 (1975) 75.
[2] G. Farrar and P. Fayet, Phys. Lett. B76 (1978) 575.
[3] L.E. Ibanez and G.G. Ross, Phys. Lett. B260 (1991); Nucl. Phys. B368 (1992) 3.
T. Banks and M. Dine, Phys. Rev. D45 (1992) 1424.
[4] L.J. Hall and M. Suzuki, Nucl. Phys. B231 (1984) 419.
[5] D. Brahm and L.J. Hall, Phys. Rev. D40 (1989) 2449. K. Tamvakis, Phys. Lett. B383 (1996) 207. G.F. Giudice and R. Rattazzi, hep-ph/9704339.
[6] M.C. Bento, L.J. Hall and G.G. Ross, Nucl. Phys. B292 (1987) 400.
[7] C.S. Aulakh and R.N. Mohapatra, Phys. Lett. 119B (1982) 136. A. Santamaria and J.W.F. Valle, Phys. Lett. 195B (1987) 423.
[8] R. Hempfling, Nucl. Phys. B478 (1996) 3. B. Mukhopadhyay and S. Roy, Phys. Rev. D55 (1997) 7020.

[9] J.L. Goity and M. Sher, Phys. Lett. B346 (1995) 69. A.Yu. Smirnov and F. Vissani, Phys. Lett. B380 (1996) 317.

[10] V. Barger, G.F. Giudice and T. Han, Phys. Rev. D40 (1989) 2987. G. Bhattacharyya, hep-ph/9709396. H. Dreiner, hep-ph/9707435.

[11] B. Brahmachari and P. Roy, Phys. Rev. D50 (1994) 2987. err. ibid. D51 (1995) 193.

[12] D. Chang and W-K. Keung, Phys. Lett. B389 (1996) 294.

[13] G. Bhattacharyya, D. Choudhury and K. Sridhar, Phys. Lett. B355 (1995) 193.

[14] R.M. Godbole, P. Roy and X. Tata, Nucl. Phys. B401 (1993) 67.

[15] M. Hirsch, H.V. Klapdor-Kleingrothaus and S.G. Kovalenko, Phys. Rev. Lett. 75 (1995) 2276. K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. 75 (1995) 2276.

[16] D. Choudhury and P. Roy, Phys. Lett. B378 (1996) 153. J-H. Jang, J.K. Kim and J.S. Lee, Phys. Rev. D55 (1997) 7296.

[17] H. Dreiner and G.G. Ross, Nucl. Phys. B365 (1991) 597. D.P. Roy, Phys. Lett. B128 (1992) 270.

[18] D. Choudhury and S. Raychaudhuri, Phys. Lett. B401 (1997) 54.

[19] H. Dreiner and G.G. Ross, Nucl. Phys. B410 (1993) 183.

[20] S. Dawson, Nucl. Phys. B261 (1985) 297.

[21] E.A. Baltz and P. Gondolo, hep-ph/9704411.