Modelling the evolution of galaxy clustering

C.M. Baugh\textsuperscript{1}, A.J. Benson\textsuperscript{1}, S. Cole\textsuperscript{1}, C.S. Frenk\textsuperscript{1} and C.G. Lacey\textsuperscript{2}

1. University of Durham, Department of Physics, Science Laboratories, South Road, Durham DH1 3LE
2. Theoretical Astrophysics Centre, Juliane Maries Vej 30, Copenhagen, Denmark.
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\textbf{ABSTRACT}

Measurements of galaxy clustering are now becoming possible over a range of redshifts out to $z \sim 3$. We use a semi-analytic model of galaxy formation to compute the expected evolution of the galaxy correlation function with redshift. We illustrate how the degree of clustering evolution is sensitive to the details of sample selection. For a fixed apparent magnitude limit, galaxies selected at higher redshifts are located in progressively rarer dark matter haloes, compared with the general population of galaxies in place at each redshift. As a result these galaxies are highly \textit{biased} tracers of the underlying dark matter distribution and exhibit stronger clustering than the dark matter. In general, the correlation length measured in comoving units, decreases at first with increasing redshift, before increasing again at higher redshift. We show that the $\epsilon$-model often used to interpret the angular correlation function of faint galaxies gives an inadequate description of the evolution of clustering, and offers no physical insight into the clustering process. We compare our predictions with those of a simple, popular model in which a one-to-one correspondence between galaxies and dark halos is assumed. Qualitatively, this model reproduces the correct evolutionary behaviour at high redshift, but the quantitative results can be significantly in error. Our theoretical expectations are in good agreement with the high redshift clustering data of Carlberg \textit{et al.} and Postman \textit{et al.} but are higher than the measurements of Le Fèvre \textit{et al.}

\textbf{Key words:} large-scale structure of universe; galaxies:formation; galaxies:evolution.

\section{1 INTRODUCTION}

In general, the clustering properties of galaxies are expected to vary with cosmic time. The nature of this evolution depends on factors such as the values of the cosmological parameters, the identity of the dark matter and the details of the galaxy formation process. Traditionally, attempts to infer the evolution of galaxy clustering have relied on measurements of the angular correlation function of faint galaxies, $\omega(\theta)$, as a function of apparent magnitude (some recent examples include Efstathiou \textit{et al.} 1991; Roche \textit{et al.} 1993; Infante & Pritchett 1995; Hudon & Lilly 1996; Brainerd & Smail 1998; Postman \textit{et al.} 1998). The spatial correlation function, $\xi(r)$, may be inferred from the diluted signal in projection by inverting Limber’s (1954) equation given assumptions about the redshift distribution of the galaxies and the evolution of their clustering. A simple model of the evolution of the two-point spatial correlation function, the ‘$\epsilon$-model’ (Groth & Peebles 1977), has often been used to interpret this kind of data (Efstathiou \textit{et al.} 1991). Recently, it has become possible to measure the spatial correlations directly in deep spectroscopic surveys (Le Fèvre \textit{et al.} 1996; Giavalisco \textit{et al.} 1998; Carlberg \textit{et al.} 1999).

The main difficulty in interpreting measurements of galaxy clustering is the uncertain relation between the observed galaxies and the underlying mass distribution. This problem is compounded by the fact that the properties of the galaxies selected according to any simple criteria (eg a magnitude limit) are likely to vary over a long redshift baseline. Thus, the interpretation of the data requires a model for galaxy evolution. In this paper we use a semi-analytic model of galaxy formation in which the observable properties of galaxies are calculated \textit{ab initio} within a specified cosmological model. The semi-analytic model that we use is a development of the one described in a series of earlier papers (Cole \textit{et al.} 1994; Heyl \textit{et al.} 1995; Baugh, Cole & Frenk 1996a,b) and is fully discussed in Cole \textit{et al.} (in preparation). Models of this kind have been successful in accounting for many properties of the galaxy population such as the shape of the luminosity function, the distribution of colours, and the faint counts (White & Frenk 1991; Lacey \textit{et al.} 1993; Kauffmann, White & Guiderdoni 1993; Cole \textit{et al.} 1994; Somerville and Primack 1998).

Semi-analytic models have been used to investigate the clustering of galaxies (e.g. Kauffmann, Nusser & Steinmetz 1997; Kauffmann \textit{et al.} 1998, 1999). An earlier version of the
model employed here was used to predict that the ‘Lyman-break’ galaxies discovered by Steidel et al. (1996) at $z \sim 3$ should have a comoving correlation length similar to that of bright galaxies at the present day (Baugh et al. 1998; see also Davis et al. 1985). This theoretical prediction was subsequently confirmed by observations (Adelberger et al. 1998; Giavalisco et al. 1998). Baugh et al. (1998) predicted that Lyman break galaxies are hosted by the most massive and therefore the rarest dark matter haloes in place at $z \sim 3$. Such haloes are biased tracers of the underlying mass distribution and are more strongly clustered than the dark matter which, in current models of structure formation, was much more weakly clustered at $z \sim 3$ than at present.

2 MODELLING GALAXY CLUSTERING

2.1 The $\epsilon$-model for the two-point galaxy correlation function

The correlation function of local, optically selected galaxies is well described by a simple power law $\xi(r) = (r_0/r)^{3+\epsilon}$, with a slope of $\gamma \approx 1.8$ and a correlation length of $r_0 \approx 5h^{-1}$Mpc (Groth & Peebles 1977) for $r < 10h^{-1}$Mpc. Groth & Peebles (1977) proposed the “$\epsilon$-model” to describe the redshift evolution of the correlation function, measured in terms of proper separation:

$$\xi(r, z) = \frac{r_0^\epsilon}{(1 + z)^{3+\epsilon}}.$$  \hfill (1)

If $\epsilon = 0$, the product of the proper number density of galaxies and the correlation function is constant and the clustering pattern is fixed in proper coordinates; this is known as stable clustering (Peebles 1980, §56). The correlation length, $x_0$, can also be written in comoving units as

$$x_0 = r_0(1 + z)^{-(3+\epsilon)/\gamma},$$  \hfill (2)

where $r_0 = x_0(z = 0)$. The clustering pattern is fixed in comoving coordinates if $\epsilon = \gamma - 3 \approx -1.2$, whilst the correlation function evolves as expected in linear perturbation theory, for $\Omega = 1$, if $\epsilon = \gamma - 1 \approx 0.8$.

2.2 The hierarchical clustering of dark matter

Significant progress has been made in our understanding of the dynamical evolution of dark matter in the universe, primarily as a result of N-body simulations (e.g. Jenkins et al. 1998). The growth of clustering in the dark matter can be approximated by a scaling formula that transforms the amplitude and scale of a linear density fluctuation into the corresponding values for a nonlinear fluctuation (Hamilton et al. 1991; Peacock & Dodds 1996).

The clustering evolution of dark matter haloes is significantly different from that of the underlying dark matter (e.g. Cole & Kaiser 1989; Mo & White 1996; Kauffmann, Nusser & Steinmetz 1997; Bagla 1998; Matarrese et al. 1997; Moscardini et al. 1998). Haloes more massive than the characteristic mass $M_\ast(z)$ initially display stronger clustering than the dark matter. The clustering of the dark matter subsequently grows faster than that of these haloes, reducing their bias. The bias is further diluted by the subsequent formation of haloes of mass $M$, when $M \leq M_\ast$. The clustering evolution of dark haloes sets the scene for understanding the clustering evolution of galaxies.

2.3 Galaxy clustering in a semi-analytic model

The expected number of galaxies per dark matter halo and its associated scatter depend on the details of the galaxy formation process (Benson et al. 1999), while the observed number per halo depends also on the selection criteria. Semi-analytic modelling provides the means to calculate how dark haloes are populated by visible galaxies and also to derive the expected galaxy correlation function. The latter requires the following four steps:

(i) Obtain the nonlinear correlation function of the dark matter distribution using the scaling formula of Peacock & Dodds (1996).

(ii) Select model galaxies according to the observational selection criteria.

(iii) Compute an effective bias parameter, $b_{\rm eff}$ (assumed to be independent of scale) using equation (4) or (5) below, weighting each halo by the number of galaxies that satisfy the observational selection criteria:

$$b_{\rm eff} = \frac{\int b(M)N_{\rm gal}(M)n(M)dM}{\int N_{\rm gal}(M)n(M)dM}.$$  \hfill (3)

where, $b(M)$ is the bias parameter of dark matter haloes of mass $M$; $N_{\rm gal}(M)$ denotes the mean number of galaxies in a halo of mass $M$ that satisfy the selection criteria; and $n(M)$ is the number density of dark matter haloes of mass $M$.

(iv) Obtain the real space correlation function of galaxies (i.e. free of distortions due to peculiar velocities) by multiplying the nonlinear dark matter correlation function by the effective bias parameter squared, $\xi_{\rm gal} = b_{\rm eff}^2\xi_{\rm DM}$.

The bias parameter for dark matter haloes of mass $M$ is given by (Mo & White 1996; see also Cole & Kaiser 1989):

$$b_1(M, z) = 1 + \frac{1}{\delta_c(z)D(z)} \left[ \left( \frac{\delta_c(z)}{\sigma(M)} \right)^2 - 1 \right],$$  \hfill (4)

where $D(z)$ is the linear growth factor, normalized to unity at the present day. This expression has been tested against N-body simulations (Mo & White 1996; Mo, Jing & White 1996) and works in practice down to the Lagrangian radius of the dark matter halo, $r \approx r_L$, where $r_L = (3M/4\pi\rho_0)^{1/3}$ and $\rho_0$ is the present mean density, even in the mildly nonlinear regime where $\xi(r) \sim 1$. Jing (1998) extended the comparison to higher resolution simulations and found that equation (4) systematically underpredicts the bias of haloes with $M/M_\ast \ll 1$; better agreement is obtained with the slightly modified prescription:

$$b_2(M, z) = b_1(M, z) \left( \frac{0.5}{\delta_c(z)/\sigma(M)} + 1 \right)^{(0.06-0.02n)},$$  \hfill (5)

where $n$ is the effective spectral index of the power spectrum, $d\ln P(k)/d\ln k$, at the wavenumber defined by the Lagrangian radius of the dark matter halo, $k = 2\pi/r_L$.  

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3 RESULTS

We consider the clustering of galaxies in a low density, flat cold dark matter universe, with present-day density parameter \( \Omega_0 = 0.3 \), and cosmological constant \( \Lambda_0 = 0.7 \), normalised to reproduce the observed abundance of rich clusters (\( \sigma_8 = 0.94 \); Eke, Cole & Frenk 1996). Benson et al. (1999) show that the correlation function of \( L > L_c \) galaxies in this model is in good agreement with that measured for APM Survey galaxies (Baugh 1996) over separations in the range \( 0.3h^{-1}\text{Mpc} < r < 15h^{-1}\text{Mpc} \). This agreement is not reproduced in critical density models.

The expected evolution of galaxy clustering for samples selected at magnitude limits of \( m_R = 25.5 \) and \( m_R = 23.5 \) is shown in Figs 1a and 1b respectively. The correlation functions are plotted at redshifts of \( z = 0.25 \) (solid line), \( z = 1 \) (dotted line) and \( z = 3 \) (dashed line). The corresponding dark matter correlation functions are shown in Fig. 1c. It is clear from Fig. 1 that the correlation function of galaxies changes little over the entire redshift range \( 0 < z < 3 \).

The correlation function of the mass, on the other hand, grows appreciably between \( z = 3 \) (\( x_0 = 1h^{-1}\text{Mpc} \)) and \( z = 0.25 \) (\( x_0 = 4.6h^{-1}\text{Mpc} \)). The markedly different evolutionary rates of the clustering of galaxies and mass was already noted in the first simulations of biased galaxy formation in a cold dark matter universe (Davis et al. 1985).

The non-monotonic variation of the correlation length (in comoving units) with redshift is shown in Fig. 2. The top panel displays the evolution of the comoving correlation length of galaxies with apparent magnitude brighter than \( m_R = 23.5 \) (open squares) and \( m_R = 25.5 \) (open triangles). Only points for which there are fifty or more model galaxies in the sample are plotted. The internal errors in our estimate of the correlation length are typically smaller than the symbol size, except for the point at \( z = 4.5 \) for galaxies brighter than \( m_R < 23.5 \); the uncertainty in this point makes it consistent with the correlation length obtained for the fainter sample. The correlation length typically decreases at first with increasing lookback time, reaches a minimum around redshift 1–2 and then grows again as the selected galaxies find themselves in increasingly rare haloes. Similar behaviour was seen in a simulation of the same cosmology by Kauffmann et al. (1998). The number density of galaxies in apparent magnitude selected samples falls rapidly with increasing redshift; at the same time, the bias parameter of the occupied haloes is increasing. The interplay between these effects can give rise to features in the comoving correlation.
The evolution of the comoving correlation length of galaxies is shown in Fig. 3. The lines show the predictions of the semi-analytic model, using the bias parameter given by eqn. (4) (solid line) and by eqn. (5) (dashed line.) In (a), galaxies are selected with apparent magnitude $m_R < 21.5$ and absolute magnitude $M_R < -20 - z$. The points show a preliminary analysis of clustering in the CNOC2 field survey (Carlberg et al. 1999). In (b), galaxies are selected in the observer frame $I$-band with $17.5 \leq I_{AB} \leq 22.5$. The squares show the correlation length in comoving units derived from the CFRS by LeFèvre et al. (1996) for $\Omega = 1$ (filled) and $\Omega = 0$ (open). The filled circle shows the correlation length inferred from the angular clustering over a larger area by Postman et al. (1998).

The evolution of the galaxy clustering length is very similar to that predicted by the semi-analytic model. However, quantitatively, the simple model seriously underpredicts the correlation length at low redshifts, where haloes that host more than one galaxy make a significant contribution to the clustering. At high redshifts, where one might at first expect the simple model to be more appropriate, the clustering length can be up to a factor of two greater than in the semi-analytic model, when the effects of dust are included. In the semi-analytic model, we find that the mean UV luminosity of the central galaxy in a dark matter halo increases with halo mass, though there is scatter in this relationship. This means that there is not a sharp transition in mass between haloes that never contain galaxies bright enough to be included in the sample and haloes for which the mean galaxy occupation number is unity. In addition at high redshift, the host haloes are more massive than $M_\ast$, so their abundance falls exponentially with increasing mass. The interplay between these effects means that haloes for which the mean galaxy occupation number is less than unity can make a significant contribution to the clustering amplitude. Assuming that there is exactly one galaxy per halo and that the dark matter halo abundance matches a given galaxy abundance, results in too large a dark halo mass being assigned to each galaxy; consequently the “one-galaxy-per-halo” model overestimates the correlation strength.

We compare the predicted clustering of galaxies in our model with observational data out to $z \sim 1$ in Fig. 3. In the top panel, the data points come from a preliminary analysis of clustering in the CNOC2 field survey by Carlberg et al. (1999). These authors consider galaxies brighter than the spectroscopic limit of their survey, $m_R = 21.5$, and use the measured redshift of each galaxy to estimate an absolute magnitude, $M$. Galaxies brighter than $M = -20 - z$ are then selected, where the redshift dependence represents an approximate correction for band shifting and evolution. We apply the same selection criteria to galaxies in our semi-analytic model. The solid line in Fig. 3 shows the predicted correlation length obtained using equation (4), while the dashed line shows the predictions using the modified expression for the bias given by equation (5). Fig. 3(b) shows the model predictions for galaxies with observer frame $I$-band magnitudes in the range $17.5 \leq I_{AB} \leq 22.5$. The squares show the comoving correlation lengths estimated from the

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**Figure 3.** The evolution of the comoving correlation length of galaxies out to $z \sim 1$. The lines show the predictions of the semi-analytic model, using the bias parameter given by eqn. (4) (solid line) and by eqn. (5) (dashed line.) In (a), galaxies are selected with apparent magnitude $m_R < 21.5$ and absolute magnitude $M_R < -20 - z$. The points show a preliminary analysis of clustering in the CNOC2 field survey (Carlberg et al. 1999). In (b), galaxies are selected in the observer frame $I$-band with $17.5 \leq I_{AB} \leq 22.5$. The squares show the correlation length in comoving units derived from the CFRS by LeFèvre et al. (1996) for $\Omega = 1$ (filled) and $\Omega = 0$ (open). The filled circle shows the correlation length inferred from the angular clustering over a larger area by Postman et al. (1998).
Canada-France Redshift Survey (CFRS) by LeFèvre et al. (1996), for two different assumptions for the underlying cosmology. The solid circle comes from Postman et al. (1998) who analysed angular correlations over a much larger solid angle, applying a similar magnitude selection to that in the CFRS. The correlation length they obtain is a factor of 2 larger than the CFRS value. Postman et al. argue that the CFRS fields are too small and that sampling fluctuations of this magnitude are expected in fields of this size. Our model predictions agree very well with the results of Postman et al.

4 DISCUSSION AND CONCLUSIONS

We have shown that the application of simple selection criteria (e.g. an apparent magnitude limit) can lead to a non-monotonic dependence of clustering strength on redshift. The often used “one-galaxy-per-halo” model predicts clustering evolution that is qualitatively similar to that predicted by the semi-analytic model. However this simple model can give results that are quite wrong in detail. The relatively complex behaviour of the galaxy clustering strength with redshift is not reproduced by the $\epsilon$-model which, furthermore, offers no insight into the physical processes that drive the evolution of clustering.

Our results and those of Benson et al. (1999) indicate that a cold dark matter model that agrees well with galaxy clustering in the local universe also agrees well with clustering data at high redshift. In particular our $\Omega_0 = 0.3$, $A_0 = 0.7$, $\sigma_8 = 0.94$ cold dark matter model matches the APM correlation function at low redshift (Benson et al. 1999), the CNOC2 (Carlberg et al. 1999) and Postman et al. (1998) data at $z \leq 0.5$ and the Lyman-break galaxy data (Adelberger et al. 1998) at $z \approx 3$ (Baugh et al. 1998; Governato et al. 1998).

There is little doubt that forthcoming large-area imaging surveys and catalogues of galaxy redshifts will lead to an explosion in the quantity of data bearing on the evolution of galaxy clustering. It is unclear whether these data will constrain cosmological parameters or the nature of galaxy evolution. In either case, the modelling techniques used in this paper, and those used in related studies (Governato et al. 1998; Benson et al. 1999; Kauffmann et al. 1998, 1999), will play an important role in the interpretation of the data.

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