Peculiarity of the oscillation stratification in sunspot penumbrae

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Abstract

Spatial distributions of the dominant oscillation frequency obtained for four sunspots show a feature shared by all the analysed levels of the solar atmosphere in these sunspots. This feature located in the inner penumbrae indicates that this region has favourable conditions for 2.5–4 mHz oscillation propagation. This agrees with the fact that the spectral composition of the oscillations at three atmospheric heights (Fe I 6173 Å, 1700 Å, and He II 304 Å) in this region are similar. There have been previous evidence of particular similarities along height of photospheric magnetic field strength, line-of-sight velocity, and temperature profile in the inner penumbra, where the internal boundary of the Evershed flow is located. The finding of the same dominant oscillation frequency at a range of altitudes from the chromosphere up to the transition region extends the height range, suggesting similarities in physical conditions.

1 Introduction

Sunspots have long been an important object for the study of oscillations and waves in the solar atmosphere. Sunspots provide a diverse range of interactions between the solar magnetic field and matter. The umbra, where the vertical magnetic field prevails, shows signatures of downward motion and weak five-minute oscillations of the whole umbra at the photospheric level (Lites, 1988; Kobanov, 1990). In the chromosphere, strong three-minute oscillations dominate in the umbra; these oscillations at first were considered as standing acoustic waves (Lites, 1988; Georgakilas, Christopoulou, and
Koutchmy, 2000). Later, these waves were shown to be moving upward (Rouppe van der Voort et al., 2003); also they were shown not to be a simple continuation of the running penumbral waves (RPW) in the horizontal direction (Kobanov, Kolobov, and Makarchik, 2006). New observations with ever-increasing spatial and temporal resolution capabilities reveal many new facts about the sunspot’s fine structure (Jess et al., 2015; Khomenko and Collados, 2015; Yuan et al., 2014; Sych and Nakariakov, 2014; Sych, 2015).

Penumbra is a complex part of a sunspot, whose understanding and modelling are largely complicated by horizontal—or rather a mixture of differently inclined—flows, steep temperature gradient, and rapid change of the magnetic field strength towards the outer boundary. Further complexity arises due to the inhomogeneity in the azimuthal direction: the bright and dark filaments are associated with different physical parameters. The magnetic field inclination has been shown to be larger in the dark filaments forming the so-called uncombed penumbra (Title et al., 1993; Solanki and Montavon, 1993; Bellot Rubio, 2003). Recently, with the help of high-resolution instruments, two components of the magnetic field were observed in a sunspot penumbra. Both components have an inclination close to 40° in the inner penumbra, and in the outer penumbra the dark-filament field grow horizontal, while the bright-filament field only reaches 60° inclination (Langhans et al., 2005).

It is necessary to analyse the oscillations in the penumbra to form a comprehensive picture of the waves propagating in sunspots. A wide range of frequencies is observed in sunspot penumbrae: Lites (1988); Brisken and Zirin (1997); Zirin and Stein (1972) registered oscillations in intensity and Doppler velocity signals. Different frequencies tend to appear at different regions of the penumbra: typically, the longer the period, the farther it is observed from the sunspot centre. Sigwarth and Mattig (1997) noted that this change in the frequency is more pronounced in the chromosphere than in the photosphere. Such a distribution is explained by the increase in the magnetic field inclination closer to the boundaries of a sunspot, and thus the decrease in the cut-off frequency (Reznikova and Shibasaki, 2012; Reznikova et al., 2012; Kobanov, Chelpanov, and Kolobov, 2013).

In a sunspot there is a phenomenon called running penumbral waves (RPW) — observed increase in brightness travelling outwards the penumbra (Beckers and Tallant, 1969; Giovanelli, 1972; Zirin and Stein, 1972). These waves span a range of frequencies from 1 to 4 mHz (Lites, 1988). Lites (1988); Brisken and Zirin (1997); Jess et al. (2013) showed that the periods
of the waves increase closer to the boundaries of a sunspot. RPWs were found at the photospheric heights as well (Musman, Nye, and Thomas, 1976; Löhner-Böttcher and Bello González, 2015). Naturally, the question was raised on the origin of RPWs: two concepts were proposed. The first one is that RPWs are real waves propagating horizontally across the penumbra (Alissandrakis, Georgakilas, and Dialetis, 1992; Tsiropoula, Alissandrakis, and Mein, 2000; Tziotziou et al., 2004, 2006); and the second concept implies them to be an apparent effect—a result of waves rising to the surface along inclined magnetic tubes, thus appearing first at the inner penumbra and then farther from the sunspot centre (Rouppe van der Voort et al., 2003; Bogdan and Judge, 2006; Kobanov, Kolobov, and Makarchik, 2006; Bloomfield, Lagg, and Solanki, 2007; Kobanov, Kolobov, and Chupin, 2008; Cho et al., 2015). The same effect is also deemed to be responsible for umbral flashes; and indeed, recently, researchers tend to consider umbral flashes and RPWs as manifestations of one phenomenon (Madsen, Tian, and DeLuca, 2015). This second explanation raises a consequent question: what is the origin of the waves responsible for the observed effect? A number of authors come to the conclusion that these waves are slow magnetoacoustic modes resulting from photospheric p-mode oscillations (Bloomfield, Lagg, and Solanki, 2007; Madsen, Tian, and DeLuca, 2015) or a broadband energy deposition process, e.g. granulation motions (Botha et al., 2011). Jess et al. (2013) studied the influence of the magnetic field inclination on the RPW. They concluded that the increase in the inclination leads to the increase in the dominant periodicity due to the dropping of the cut-off frequency. However, due to the complicated dynamical properties, reliable estimations of the RPW parameters are difficult to carry out.

The important feature of the oscillations discussed above is their relation to the Evershed flow. Kobanov and Makarchik (2004) found three ranges of oscillations that most likely connect the direct Evershed flow and the inverse—the so-called St. John’s—flow. The 20–35-minute oscillations have the most consistent phase difference between the photospheric and chromospheric heights.

In our previous works (Kobanov, Kolobov, and Chelpanov, 2015; Kolobov et al., 2015), we revealed that the dominant frequency spatial distribution shows a peculiarity in sunspots’ penumbra. We constructed plots showing the dominant frequencies averaged in the azimuthal direction as a function of the distance to sunspot’s barycentre, and these plots converged in the inner penumbra. This feature corresponds to the 3–4 mHz frequency range, which
indicates that five-minute oscillations dominate above the inner penumbra at all the heights from the photosphere to the transition region. One can assume that in this ring-shaped region above the inner penumbra, favourable physical conditions for five-minute wave propagation exist.

2 Methods

For this study we used full-disk narrow-band images provided by the Atmospheric Imaging Assembly (AIA: Lemen et al., 2012) onboard the Solar Dynamics Observatory (SDO). Of all the spectral bands used in the observations we chose four: 1700 Å, He\textsc{ii} 304 Å, Fe\textsc{ix} 171 Å obtained by AIA, and Fe\textsc{i} 6173 Å obtained by Helioseismic and Magnetic Imager (HMI: Scherrer et al., 2012). These lines cover the height range from the deep photosphere to the corona.

The AIA data have time resolution 12 and 24 s, depending on the spectral band. Each pixel of these data corresponds to 0.6”. The HMI data have a 45 s time resolution and 0.5” detector pixel size. Fe\textsc{i} 6173 Å line is formed in the photosphere at a height of 200 km (Beckers and Tallant, 1969). Also, the Doppler velocity maps and magnetograms are available in this line.

In the analysis we used four sunspots in NOAA 11311, 11479, 11711, and 12149 (Table 1). In all four cases the spots had roughly regular round shapes and located close to the disk centre, and thus the effects of the foreshortening and asymmetry were minimal. The data series lengths ranged from 4 to 8 hours, and no flares were registered during these observations.

After removing trends from the time series, the power spectra were calculated by Fast Fourier Transform (FFT) using the standard routine of

| No. | NOAA | Date       | Disc location | T\textsubscript{start} (UTC) hh:mm | T\textsubscript{end} (UTC) hh:mm |
|-----|------|------------|---------------|-----------------------------------|---------------------------------|
| 1   | 11311| 6 Oct 2011 | 11°S 08°E     | 03:21                             | 04:45                           |
| 2   | 11479| 16 May 2012 | 15°N 11°E    | 00:07                             | 07:59                           |
| 3   | 11711| 6 Apr 2013  | 17°S 03°W    | 00:00                             | 04:20                           |
| 4   | 12149| 27 Aug 2014 | 10°N 02°W    | 06:00                             | 10:00                           |
the Interactive Data Language (IDL). Based on these spectra, we constructed the dominant frequency distributions within the analysed regions. To this end, the spectrum from each pixel was smoothed using convolution with a 1 mHz window, and the maximum frequency value of the resulting function was considered as the dominant frequency within the given pixel.

3 Results and Discussion

Oscillations of different frequencies are observed in sunspots, and within a sunspot they are distributed non-uniformly. First, different frequency bands occupy different regions of a sunspot. High-frequency oscillations tend to be located within the umbra boundaries, while lower frequencies form circles, whose radii increase with the decreasing frequency. Such distributions are believed to be related to the magnetic field configuration, namely, magnetic field inclination angle: the location of high frequencies coincides with that of the vertical magnetic field, while low frequencies are located at the high-inclination outer penumbra (Reznikova and Shibasaki, 2012; Kobanov, Chelpanov, and Kolobov, 2013). Second, the circles of oscillation distributions grow with the height. Such a pattern, again, is deemed to be related to the magnetic field configuration: waves propagating along magnetic field lines inclined from the spot centre approach the spot’s outer boundaries at each consecutive height level (Kobanov, Chelpanov, and Kolobov, 2013).

Figure 1 shows spatial distributions of the dominant frequencies based on FFT power spectra. The distributions are reconstructed from the SDO data. They follow the two aforementioned features of the oscillation behaviour in sunspots. Based on these distributions we plotted profiles of dominant frequencies as functions of distance from a sunspot centre $r$ presented in Figure 2. These profiles were plotted for three height levels—from the photosphere to the transition region—in the four sunspots.

These plots behave as we expected—high-frequency range in the centre and gradual decrease towards the sunspot boundaries. The interesting feature in all the studied sunspots that caught our attention is the convergence of all the profiles in the inner penumbrae (see Figure 2). In case of NOAA 11711 the profiles intersect in this region. Figure 3 shows intensity oscillation power spectra azimuthally averaged over the region marked in Figure 2. The panels for NOAA 12149 show that in the transition region
Figure 1. Dominant frequency distributions in sunspots at different height levels, from the photosphere to the corona. The black lines show the inner and outer penumbra boundaries as seen in the 1700 Å band AIA images.
Figure 2. Radial distributions of the dominant frequencies at three heights in sunspots as functions of the distance from the barycentre. The vertical dashed lines mark the penumbra boundaries. The region of interest is marked with grey area, which corresponds to the averaging over the circular-shaped domain in the penumbra.

(He \textsc{ii} 304 Å) the highest peaks are shifted to the higher frequencies. This can be explained by the inhomogeneity of the penumbra or by the fact that sunspot has not purely circular shape. There are five-minute oscillations in the narrow region of the penumbra that dominate at all the heights (Figure 3). We consider this region to be a channel transporting five-minute oscillations from the photosphere up through the chromosphere.

The interesting behaviour in this penumbra region motivated a more detailed study of the distributions of several parameters there, including those found in earlier works by other researchers.

Figure 4 shows the azimuthally averaged magnetic field inclination at the He \textsc{ii} 304 Å line formation level estimated from the dominant frequency distributions. The details of the estimation procedure can be found in Kobanov, Kolobov, and Chelpanov (2015). The inner penumbra in these distributions is characterized by the steepest inclination angle gradient; the inclination angle there being 60–65°.

Various signatures indicating peculiar properties of the inner penumabra of sunspots have been found previously by other authors. Identical field strength was found at three photospheric heights (deep photosphere, log $\tau_{500} = 0$; mid photosphere, log $\tau_{500} = -1.5$; and top of the photosphere, log $\tau_{500} = -3$) by Borrero and Ichimoto (2011). At about the same distance from the sunspot centre, the magnetic field profiles of the three photospheric levels show the same value, and in the outer penumbra the order of the profiles is reversed (Figure 5). Bellot Rubio, Schlichenmaier, and Tritschler (2006), see Figure 3 therein, noted a hump in the inner penumbra in the azimuthally averaged temperatures at all the photospheric heights.
The amplitude of the hump was found to decrease with height. The authors suggested that these temperature enhancements could be due to hot penumbral tubes, by which plasma emerges from the subphotospheric layers. Based on these two works, one can conclude that the unique properties of the inner penumbrae seem to originate in deeper levels than that we study here.

These tubes are probably related to the photospheric and chromospheric Evershed flow, which peaks at the outer penumbra boundary and sharply terminates at the same region of the inner penumbra (see Figure 8 in Bellot Rubio, Schlichenmaier, and Tritschler (2006), and Figure 4 in Georgakilas et al. (2003)). Also, Bellot Rubio, Schlichenmaier, and Tritschler (2006) showed that the microturbulence velocity rapidly drops to zero in the inner penumbra.

Today, we lack a comprehensive explanation for this phenomenon. Probably, the key for understanding such a behaviour of the distributions is a model that describes penumbra magnetic field as a series of two types of interlocking-comb filaments (Weiss et al., 2004). The first one having a large inclination angle are located at low heights and dive beneath the photosphere.
Figure 4. Estimation of the magnetic field inclination at the He\textsuperscript{II} 304 Å line height based on the dominant frequency distributions. The region of interest is marked with grey area.

at the outer penumbra boundary. These filaments are associated with the Evershed flow. The second filament type is closer to vertical in orientation. Their magnetic field lines rise high in the atmosphere and either return to the surface far from the spot or form an open field line.

As follows from the aforesaid, interesting peculiarities are observed in the behaviour of a number of physical parameters in the sunspots’ inner penumbra. In this paper, we hope to draw attention to this problem, solving which requires widening the height range of data analysis and modelling.

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Figure 5. Magnetic field vertical component averaged over azimuth in sunspot NOAA 10923 in the deep photosphere at the continuum level, in the mid-photosphere, and in the upper photosphere. The vertical lines mark the penumbra boundaries. The region of interest is marked with grey area (courtesy of Juan M. Borrero, see (Borrero and Ichimoto, 2011))
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