The electromagnetic amplitudes in the $J/\psi$ and $\psi(2S)$ decays into spin-1/2 baryon-antibaryon pairs

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After investigating the decays of a charmonium $\psi = J/\psi, \psi(2S)$ into a spin-1/2 baryon-antibaryon $BB$ pair, with the determination of the only parameter that gives the EM amplitude for neutral final states, in this work we focus our attention on the decays into charged baryons, whose EM amplitudes can be expressed in terms of a further parameter.

By considering the BESIII data on the $e^+e^- \rightarrow p\bar{p}$ cross section we obtain a full parametrization for the EM amplitudes and make predictions on the cross section of the decays $e^+e^- \rightarrow \Sigma^+\Sigma^-$ and $e^+e^- \rightarrow \Xi^-\Xi^+$ at the $J/\psi$ and $\psi(2S)$ masses.

I. INTRODUCTION

The decays $\psi \rightarrow BB$, being $\psi$ a vector charmonia, $\psi = J/\psi, \psi(2S)$, where $B$ is a spin-1/2 baryon of the SU(3) octet, represented by the matrix

$$B = \begin{pmatrix}
\frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\
\Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\
\Xi^- & \Xi^0 & -2\Lambda/\sqrt{6}
\end{pmatrix},$$

have been studied from decades [1, 2].

The generic decay $\psi \rightarrow BB$ can be written as the sum of three main contributions: strong, electromagnetic (EM) and mixed strong-EM, as shown in Figure 1, where, in case of perturbative QCD (pQCD), the latter is proportional to the former [3].

The three related amplitudes were studied and recently separated, for the first time, in Refs. [4, 5], for the $J/\psi$ and $\psi(2S)$, respectively.

The EM amplitudes for the neutral final states in the decays $\psi \rightarrow BB$ can be parametrized [6] in terms of only one parameter, called $D_e$, while those of the charged final states can be expressed in terms of the parameter $D_e$ and the new parameter $F_e$.

The first parameter, $D_e$, have been deeply investigated in Ref. [6], with the determination of its modulus $|D_e|$. In this work we focus our attention on the $F_e$ parameter, related to charged baryon final states, i.e., we consider the cases where $BB \in \{pp, \Sigma^+\Sigma^-, \Sigma^-\Sigma^+, \Xi^-\Xi^+\}$.

II. CROSS SECTIONS AND BRANCHING RATIOS

The electromagnetic amplitudes for the decays $\psi \rightarrow BB$ can be parametrized in terms of few parameters for the whole SU(3) octet. Following the procedure discussed in Ref. [6] the EM branching ratio of the generic decay $\psi \rightarrow \gamma^* \rightarrow BB$ can be written as [6]

$$\text{BR}_{BB} = 16\pi^2 \frac{M_B}{16\pi M_\psi \Gamma_\psi} \left| g_\psi A_{BB}(M_\psi^2) \right|^2,$$

where

$$\beta_M(q^2) = \sqrt{1 - \frac{4M_B^2}{q^2}}.$$
is the velocity of the outgoing baryon in the $B\bar{B}$ center-of-mass frame and $g_\gamma^\psi$ is the coupling constant between the $\psi$ meson and the virtual photon $\gamma^*$. The amplitudes parametrizations are reported in Table I.

**TABLE I. Parametrizations of the amplitudes of the EM decay $\psi \rightarrow \gamma^- \rightarrow B\bar{B}$ as function of the couplings $D_e$ and $F_\psi$.**

| $B\bar{B}$ | $g_\gamma^\psi A_{B\bar{B}}(M_B^2)$ |
|------------|----------------------------------|
| $\Sigma\Xi^0$ | $D_e$ |
| $\Lambda\bar{\Lambda}$ | $-D_e$ |
| $\Lambda\Xi^0 + \text{c.c.}$ | $\sqrt{3} D_e$ |
| $pp$ | $D_e + F_\psi$ |
| $n\pi$ | $-2 D_e$ |
| $\Sigma^+\Sigma^-$ | $D_e + F_\psi$ |
| $\Sigma^-\Sigma^+$ | $D_e - F_\psi$ |
| $\Xi^-\Xi^+$ | $D_e - F_\psi$ |
| $\Xi^0\Xi^0$ | $-2 D_e$ |

The moduli of the parameter $D_e$, for both $J/\psi$ and $\psi(2S)$ mesons, have been determined recently as [6]

$$|D_e|_{J/\psi} = (3.93 \pm 0.17) \times 10^{-4} \text{ GeV},$$
$$|D_e|_{\psi(2S)} = (1.25 \pm 0.07) \times 10^{-4} \text{ GeV}.$$  

The Born cross section $e^+ e^- \rightarrow B\bar{B}$ at the $\psi$ mass, $q^2 = M_\psi^2$, can be written in the form [6]

$$\sigma_{B\bar{B}}(M_\psi^2) = \frac{4\pi\alpha^2}{3M_\psi^2 BR_{\mu^+\mu^-}} BR_{B\bar{B}}^\gamma = \frac{|g_\gamma^\psi|^2}{16\pi M_\psi^2} \frac{\alpha^2 \beta_{M_\psi}(M_\psi^2)}{12M_\psi^2} |A_{B\bar{B}}^\gamma(M_\psi^2)|^2,$$  

where

$$BR_{\mu^+\mu^-} = \frac{|g_\gamma^\psi|^2}{16\pi M_\psi^2 \Gamma_\psi}$$

is the BR of the decay $\psi \rightarrow \mu^+\mu^-$. Concerning one of the four final states with charged baryon, the $p\bar{p}$ one, several recent data are available on the cross section $e^+ e^- \rightarrow p\bar{p}$, from the BESIII collaboration [7].

In this case the cross section of Eq. (2), using the parametrization reported in Table I becomes

$$\sigma_{p\bar{p}}(M_\psi^2) = \frac{\alpha^2 \beta_{M_\psi}(M_\psi^2)}{12M_\psi^2} BR_{\mu^+\mu^-} |D_e + F_\psi|^2,$$  

where, from PDG [8],

$$BR_{\mu^+\mu^-} = (5.961 \pm 0.033) \times 10^{-2}.$$  

**III. RESULTS**

We use the newest BESIII [7] data on the cross section of $e^+ e^- \rightarrow p\bar{p}$ to determine the modulus $|D_e + F_\psi|$ of Eq. (3). We consider the two fit functions for the cross section from Ref. [7]. The first one is based on a QCD-inspired parametrization of the form factors [9, 10]

$$\sigma_{\text{fit},1}(q^2) = \frac{A_1 (1 + 2M_\psi^2/q^2)}{(q^2)^\beta (\pi^2 + \ln^2(q^2/\Lambda_{QCD}^2))^2},$$

that includes also the logarithmic corrections, while the second is suggested by Ref. [11]

$$\sigma_{\text{fit},2}(q^2) = \frac{A_2 (1 + 2M_\psi^2/q^2)}{q^2 (1 - q^2/q_0^2)^\beta (1 + q^2/m_0^2)^2},$$

where $M_\psi$ is the proton mass, $A_1 = 72 \text{ GeV}^4$ and $A_2 = 7.7$, while $\Lambda_{QCD} = 0.52 \text{ GeV}$, $q_0^2 = 0.71 \text{ GeV}^2$ and $m_0^2 = 14.8 \text{ GeV}^2$.

In Figure II are shown the cross section data from various experiments, together with the fitting function of Eq. (4) and (5). In order to obtain the values of $|D_e + F_\psi|$, using Eq. (3), we combine the results for the cross sections of the two fitting functions of Eq. (4) and (5). The values are

$$|D_e + F_\psi|_{J/\psi} = (11.6 \pm 1.2) \times 10^{-4} \text{ GeV},$$
$$|D_e + F_\psi|_{\psi(2S)} = (3.31 \pm 0.55) \times 10^{-4} \text{ GeV},$$

and the relative phase $\rho$ between $D_e$ and $F_\psi$ as follows

$$|F_\psi| = \sqrt{|D_e + F_\psi|^2 - |D_e|^2 \sin^2(\rho) - |D_e| \cos(\rho)}.$$  

1 the final value is given by the mean of the two values, while its error is calculated as their standard deviation.
A plot of $|F_e|$ as a function of the relative phase $\rho$ is shown in Figure 3. Using the obtained values of Eq. (6), and the values for the moduli of the $D_e$ parameter of Eq. (1), we can determine the following range of values for $|F_e|$

\[
6.58 \times 10^{-4} \text{ GeV} \leq |F_e|_{J/\psi} \leq 16.85 \times 10^{-4} \text{ GeV}, \\
1.58 \times 10^{-4} \text{ GeV} \leq |F_e|_{\psi(2S)} \leq 5.18 \times 10^{-4} \text{ GeV},
\]

The quantity $|D_e - F_e|$ as a function of $|D_e + F_e|$, $|D_e|$ and $\rho$ is given by

\[
|D_e - F_e| = \left( |D_e + F_e|^2 + 4|D_e|^2 \cos^2 \rho - 4|D_e| \cos \rho \right)^{1/2} \times \sqrt{|D_e + F_e|^2 - |D_e|^2 \sin^2 \rho}^{1/2}.
\]

and its plot is shown in Figure 4. Starting from this equation we can determine the range of values for $|D_e - F_e|$, i.e.,

\[
2.82 \times 10^{-4} \text{ GeV} \leq |D_e - F_e|_{J/\psi} \leq 20.95 \times 10^{-4} \text{ GeV}, \\
0.40 \times 10^{-4} \text{ GeV} \leq |D_e - F_e|_{\psi(2S)} \leq 6.50 \times 10^{-4} \text{ GeV},
\]

\[\text{A. Real parameters}\]

Under the hypothesis that the two parameters $D_e$ and $F_e$ are relatively real, as suggested in Ref. [1], i.e., with $\rho = 0$, we can determine the values of $|F_e|$ as

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
Quantity ($\times 10^4$) & This work [GeV] & Other work [GeV] & s.d. \\
\hline
$|D_e|_{J/\psi}$ & - & $3.93 \pm 0.17$ & - \\
$|D_e|_{\psi(2S)}$ & - & $1.25 \pm 0.07$ & - \\
$|F_e|_{J/\psi}$ & $7.6 \pm 1.2$ & $7.91 \pm 0.62$ & $\sim 0.2$ \\
$|F_e|_{\psi(2S)}$ & $2.06 \pm 0.56$ & $1.65 \pm 0.17$ & $\sim 0.7$ \\
$D_e + F_e|_{J/\psi}$ & $11.6 \pm 1.2$ & $12.43 \pm 0.65$ & $\sim 0.6$ \\
$D_e - F_e|_{J/\psi}$ & $3.31 \pm 0.55$ & $2.90 \pm 0.18$ & $\sim 0.7$ \\
$D_e - F_e|_{\psi(2S)}$ & $3.7 \pm 1.3$ & $3.39 \pm 0.65$ & $\sim 0.2$ \\
$D_e - F_e|_{\psi(2S)}$ & $0.81 \pm 0.57$ & $0.42 \pm 0.18$ & $\sim 0.7$ \\
\hline
\end{tabular}
\caption{Comparison between $D_e$ and $F_e$ related quantities, assuming that $D_e$ and $F_e$ are relatively real. In the last column are indicated the discrepancies, in standard deviations (s.d.), between the values of second and third columns.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
$B\bar{B}$ & $\sigma_{B\bar{B}}(M_{J/\psi}^2)$ [pb] & $\sigma_{B\bar{B}}(M_{\psi(2S)}^2)$ [pb] \\
\hline
$\rho\bar{\pi}^{-}$ & $11.1 \pm 2.4$ & $1.38 \pm 0.47$ \\
$\Sigma^+\Sigma^-$ & $9.0 \pm 1.9$ & $1.23 \pm 0.42$ \\
$\Sigma^+\Sigma^-$ & $0.91 \pm 0.62$ & $0.07 \pm 0.10$ \\
$\Xi^+\Xi^-$ & $0.74 \pm 0.51$ & $0.07 \pm 0.09$ \\
\hline
\end{tabular}
\caption{Cross sections computed through Eq. (2), using the values of Table I (second column) according to the parametrization of Table I.}
\end{table}

These values are in agreement with those obtained for the $J/\psi$ and for the $\psi(2S)$ in Refs. [4] and [5], respectively, where, under the hypothesis of reality, they were found to be positive.

We can calculate, using Eq. (7) with $\rho = 0$ and the values of $|D_e + F_e|$ and $|D_e|$ from Eqs. (1) and (6),

\[
|D_e - F_e|_{J/\psi} = (3.7 \pm 1.3) \times 10^{-4} \text{ GeV}, \\
|D_e - F_e|_{\psi(2S)} = (0.81 \pm 0.57) \times 10^{-4} \text{ GeV}.
\]
In Figure 5 are shown the two obtained values of $\sigma_{\Sigma^- \Sigma^+}$ and $\sigma_{\Sigma^+ \Sigma^-}$, together with the available data on the cross section from the BESIII experiment \[15\], where can be seen a good agreement with the power law decreasing trend as energy increases.

**IV. CONCLUSIONS**

We have considered the decays of a charmonium $\psi = J/\psi, \psi(2S)$ into a pair of spin-1/2 charged baryons $B$ to determine the complete parametrization of the EM amplitude using the BESIII data on the Born cross section $e^+e^- \to pp$. The EM amplitudes of the decays $\psi \to B\bar{B}$ depend on two parameters, $D_e$ and $F_e$, and are proportional to the modulus $|D_e|$, for the neutral final states, and to $|D_e \pm F_e|$ for the charged ones.

We have previously obtained the value of $|D_e|$ at the $J/\psi$ and $\psi(2S)$ masses \[6\], showing that the PDG value of the branching ratio for the so-considered purely EM decay $\psi \to \Lambda \Sigma^0 + c.c.$ is too large under this assumption. In this work we have completed the determination of the moduli of all EM amplitudes for the decays $\psi \to B\bar{B}$, with the values of the parameter reported in Table \[11\]. The results agree within maximum 0.7 standard deviations with the predictions made in other works, where have been taken into account the full, strong, EM and mixed strong-EM amplitudes, of the decays $J/\psi \to B\bar{B}$ \[4\] and $\psi(2S) \to B\bar{B}$ \[3\].

Using the results reported in Table \[11\] we have predicted the values of the cross sections $e^+e^- \to \Sigma^+\Sigma^-, e^+e^- \to \Sigma^-\Sigma^+$ and $e^+e^- \to \Xi^-\Xi^+$, at the $J/\psi$ and $\psi(2S)$ masses, the values are reported in Table \[11\]. In particular, the values of the predicted cross sections for the $\Sigma^+\Sigma^-$ and $\Sigma^-\Sigma^+$ final states are compared with those obtained by the BESIII collaboration at lower energies, as shown in Figure 5 where the typical decreasing trend can be observed.

These values are reported, together with other quantities, in Table \[11\] where they are also compared with the values obtained in other works, all results are compatible within maximum 0.7 standard deviations. Using the obtained values for $|D_e - F_e|$ of Eq. \[8\] we can calculate the values of the cross sections of

[1] L. Kopke and N. Wermes, Phys. Rept. 174 (1989), 67 doi:10.1016/0370-1573(89)90074-4
[2] M. Claudson, S. L. Glashow and M. B. Wise, Phys. Rev. D 25 (1982), 1345 doi:10.1103/PhysRevD.25.1345
[3] J. G. Korner, Z. Phys. C 33 (1987), 529 doi:10.1007/BF01548265
[4] R. Baldini Ferroli, A. Mangoni, S. Pacetti and K. Zhu, Phys. Lett. B 799 (2019), 135041 doi:10.1016/j.physletb.2019.135041 [arXiv:1905.01069 [hep-ph]].
[5] R. B. Ferroli, A. Mangoni, S. Pacetti and K. Zhu, Phys. Rev. D 103 (2021), 016005 doi:10.1103/PhysRevD.103.016005
[6] R. B. Ferroli, A. Mangoni and S. Pacetti, Eur. Phys. J. C 80 (2020) no.9, 903 doi:10.1140/epjc/s10052-020-08474-x [arXiv:2007.12380 [hep-ph]].
[7] M. Ablikim et al. [BESIII], Phys. Rev. D 99 (2019) no.9, 092002 doi:10.1103/PhysRevD.99.092002 [arXiv:1902.00665 [hep-ex]].
[8] P. A. Zyla et al. [Particle Data Group], PTEP 2020 (2020) no.8, 083C01 doi:10.1093/ptep/ptaa104
[9] A. Bianconi and E. Tomasi-Gustafsson, Phys. Rev. C 93 (2016) no.3, 035201 doi:10.1103/PhysRevC.93.035201 [arXiv:1510.06338 [nucl-th]].
[10] D. V. Shirkov and I. L. Solovtsov, Phys. Rev. Lett. 79 (1997), 1209-1212 doi:10.1103/PhysRevLett.79.1209 [arXiv:hep-ph/9704333 [hep-ph]].

[11] E. Tomasi-Gustafsson and M. P. Rekalo, Phys. Lett. B 504 (2001), 291-295 doi:10.1016/S0370-2693(01)00312-4.

[12] M. Ablikim et al. [BESIII], Phys. Rev. D 91 (2015) no.11, 112004 doi:10.1103/PhysRevD.91.112004 [arXiv:1504.02680 [hep-ex]].

[13] J. P. Lees et al. [BaBar], Phys. Rev. D 87 (2013) no.9, 092005 doi:10.1103/PhysRevD.87.092005 [arXiv:1302.0055 [hep-ex]].

[14] B. Aubert et al. [BABAR], Phys. Rev. D 73 (2006), 012005 doi:10.1103/PhysRevD.73.012005 [arXiv:hep-ex/0512023 [hep-ex]].

[15] M. Ablikim et al. [BESIII], [arXiv:2009.01404 [hep-ex]].