An experimental study on the anisotropic and intermittent behaviour of a turbulent flow over a rough bed

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Abstract. The flow field in open channels can be classified into different zones according to the velocity distribution. To explore the anisotropic and intermittent behaviour in the turbulent flow layers, an experimental study was performed using Particle Image Velocimetry (PIV) measurements in a hydraulic flume with rough bed. Specifically, the analysis has been focused on the two-dimensional (2D) high-order structure functions applied to the velocity data. It is demonstrated that the system spans from large-scale anisotropy, which is induced by the main shear of the boundary layer, to small-scale isotropy. Moreover, it is shown that the intermittency is more pronounced in the near-bed layer, where the flow is more populated by coherent whirling structures. In essence, both anisotropy and intermittency are proved to be important phenomena for natural bed rivers, since they affect the turbulence transport characteristics of the flow.

1. Introduction

The pioneering study of Kolmogorov on turbulence is founded on the idea that the energy cascade process develops from large to small scales. At large scales, the turbulence is influenced by geometric characteristics (e.g., the pipe radius or the water depth in open-channel flows). Proceeding to small scales, in the so-called inertial range of turbulence, Kolmogorov hypothesized homogeneity and isotropy [1–3].

At a high turbulence level, the flow is supposed to be “universal” and the structure of the smallest eddies is independent from the large scale. Nevertheless, turbulence in nature is generally affected by stratification, shears and interactions with irregular boundaries [4]. In fact, these effects usually break the isotropy, particularly at large scales, leading to anisotropic energy spectra [5].

Conversely, at the small scales, turbulence is characterised by a different phenomenon, called intermittency [1], which is characterised by the increase of energy dissipation and the creation of small-scale structures. These structures produce a break in the self-similarity of turbulence and...
a clear violation of the hypothesis of monofractal turbulence [1, 3], where the \( p \)-order structure functions increase with a power law having exponent \( p/3 \) [6]. However, it is not clear yet how turbulent flows on natural beds are influenced by intermittency and anisotropy, because, so far, research has been mainly focused on the influence of boundary roughness on turbulence characteristics (e.g., Turbulent Kinetic Energy (TKE) budgets and the scales of turbulence) [7–15].

Our work is based on the investigation of the anisotropic and intermittent behaviour of turbulence on a rough bed in conjunctions with turbulent scale lengths, through laboratory experiments and Particle Image Velocimetry (PIV) measurements.

Particularly, an exhaustive statistical study was performed on the basis of the analysis of the streamwise velocity component increments and, specifically, of the \( p \)-order velocity statistics. In particular, the \( p \)-order structure function of the streamwise velocity component was examined by separating the flow depth into two layers (the outer layer and the near-bed zone), aiming at quantifying turbulence intermittency and anisotropy.

The work is organized as follows: §2 presents the experiments; §3 shows the results, and §4 discusses the main findings.

2. Experimental setup and methodology
The experimental study was conducted in a recirculating rectangular tilting flume at the Laboratorio “Grandi Modelli Idraulici”, Università della Calabria, Italy. The flume was 9.6 m-long, 0.485 m-wide and 0.5 m-deep. At 1 m downstream of the flume inlet, a recess box of 7 m in length and 12 cm in height was installed. The test section was located at 6.5 m downstream of the flume inlet.

An experimental run was performed with uniform bed sediments (i.e., with geometric standard deviation of the grain size distribution \( \sigma_g = (d_{84}/d_{16})^{0.5} < 1.5 \) [16], where \( d_{16} \) and \( d_{84} \) are the sediment sizes for which 16% and 84% by weight of sediment is finer, respectively). The bed was prepared by using coarse gravel with \( d_{50} = 17.98 \) mm. The longitudinal bottom flume slope, \( S_0 \), was fixed at 1.5\% by manoeuvring a hydraulic jack.

To damp the disturbance of the pump on the turbulence characteristics of the flow, a honeycomb was mounted at the flume inlet. The flow discharge, \( Q = 19.73 \) l/s, was measured by a V-notch weir installed in a downstream tank, located upstream of the restitution channel. At the outlet, a tailgate was installed to regulate the water depth, \( h = 0.12 \) m, at the test section within the flume; it was measured with a point gauge having an accuracy of \( \pm0.1 \) mm.

Beside the previous-mentioned hydraulic parameters, the other characteristics of the run were the following: the mean flow velocity, \( U = Q/(Bh) = 0.34 \) m/s, where \( B \) is the flume width; the shear velocity, \( u_s = 0.033 \) m/s, which was estimated by extending linearly the double-averaged (in time and space) Reynolds shear stress per unit mass profile down to the maximum crest level; the mean water temperature, \( T = 17.7 \) °C; the water kinematic viscosity, \( \nu = 1.06 \times 10^{-6} \) m\(^2\)/s, computed as a function of \( T \) [17]; the flow Froude number, \( Fr = U/(gh)^{0.5} = 0.31 \), where \( g \) is the acceleration of gravity; the flow Reynolds number, \( Re = 4Uh/\nu = 38491 \); the shear Reynolds number \( Re_s(=u_s\epsilon/\nu = 1120) \), considering the Nikuradse equivalent sand roughness \( \epsilon = 2d_{50} \).

The velocity measurements were performed using a 2D PIV system (Trust Science Innovation-TSI). The system was composed of a 12 bit CCD camera with a 50 mm F1.8 Nikon lens, having a resolution of 2048 \( \times \) 2048 pixel\(^2\) and a frame rate of 15 Hz, and a double pulse Nd:YAG laser, having a pulse energy of 200 mJ at a wavelength of 532 mm. The sample frequency adopted in this study was equal to 15 Hz. Titanium dioxide (TiO\(_2\)), having a medium grain diameter of 3 \( \mu \)m and a relative density of 4.26, was used as seeding particles. The flow field measurements were performed along the streamwise direction at the flume centreline, capturing, for a duration of 200 s, 3000 pairs of images having an area of 163 \( \times \) 163 mm\(^2\). The interrogation area, for
Figure 1. Separation of the flow zones. Here, \( d\langle u \rangle /dz \) is the gradient of the double-averaged (in space and time) streamwise velocity component, \( \langle u \rangle \).

Each image was set equal to 16 × 16 pixel\(^2\). However, we have examined a smaller area, owing to possible distortions of images as well as problems of weakly lightened zones at the image sides. No interrogation area overlapping was considered. The PIV camera base was installed parallel to the mean bed level, so that the lens centreline was at an elevation of 5 cm from it, allowing the acquisition of the velocity data below the roughness crest.

3. Results

3.1. Second-order statistics

As reported in §1, high shears stresses tend to produce anisotropic turbulence stretching the turbulent structures, particularly at large scales [3, 18]. At small scales the anisotropy tends to reduce [1]. In this work, the aim is the evaluation of the boundary roughness on the turbulent structures. We built our study on the 2D time-averaged statistics of the 2D velocity \( \mathbf{u} = (u, w) \), \( u \) and \( w \) being the streamwise and vertical velocity components, respectively, which were computed as follows:

\[
S_p(r) = \langle |u(x + r) - u(x)|^p \rangle,
\]

where \( S_p \) is the 2D structure function of order \( p \), \( x = (x, z) \) indicates the vector of the point location, \( x \) is the streamwise component of the point location starting from the upstream side of the interrogation area, \( z \) is the vertical component of the point location starting from the maximum crest level, \( r = (r_x, r_z) \) is the 2D incremental vector and \( r \) is its magnitude \( (r_x \) and \( r_z \) are the incremental components of \( r \) in the streamwise and vertical direction, respectively).

Note that the PIV system is restricted to the streamwise and vertical directions and, consequently, the 2D statistics in Eq. 1 is a surrogate of a 3D one. Nevertheless, the analysis carried out in this study consents to speculate on both turbulence isotropy and intermittency.

The study was applied to two different flow layers (Fig. 1): we considered the first zone starting from the bed level up to the vertical elevation at which the gradient of the double-averaged (in space and time) streamwise velocity component was very small (equal to 5 s\(^{-1}\)), whereas the second zone was the remaining upper part, the so-called outer layer.

Fig. 2 reveals the 2D second-order statistics contour lines \( (p = 2) \) \( S_2 \) as a function of the incremental spatial components \( r_x \) and \( r_z \), for zones 1 and 2. At the smallest scales, the contour lines have a quasi-circular shape, suggesting that no significant variations in velocity structures are presented and confirming, hence, the Kolmogorov hypothesis [19–21].
Figure 2. Contour lines of 2D second-order statistics \( S_2(r) \) for (a) Zone 1, and (b) Zone 2.

Figure 3. Anisotropy angle \( \theta \) for each zone. The solid black line identifies the \( \theta \) isotropic condition.

scale increases, the contour lines become elongated in the streamwise direction. Therefore, the isotropic behaviour of the statistics is verified at low \( r_x \) and \( r_z \) [22].

In essence, the isotropy close to the bed roughness (Zone 1) was present only for the smallest scales. Moving from Zone 1 to Zone 2, the isotropic condition gradually extended to higher scales. This is probably due to the stronger influence of the bed roughness in Zone 1 than in Zone 2.

Isotropy is achieved when \( S_2(r,0) = S_2(0,r) \), i.e. when the angle \( \theta(r) = \arctan \sqrt{2S_2(r,0)/S_2(0,r)} \approx 54.74^\circ \) (solid black lines in Fig. 3). Specifically, for \( r \in [-0.04; 0.04] \), the anisotropy angle of the Zone 2 fluctuates with respect to the isotropic value \( \theta = 54.74^\circ \). Conversely, when \( r \) increases further, the angle values deviate from the solid black line. As regards Zone 1, owing to the effect of the bed roughness, the angles \( \theta \) assume values distant from the one denoting isotropic behaviour.

3.2. High-order statistics of turbulence

Further information on turbulence statistics can be obtained from the study of the high-order structure functions. Starting from the increment of velocity \( S_1(r) = \langle |u(x + r) - u(x)| \rangle \), the scale invariance can be computed as follows:

\[
S_1(r) = \lambda^{-H} S_1(\lambda r),
\]
where $\lambda$ is the scale factor and $H$ is the Hurst coefficient. Therefore, Eq. 2 can be generalized as follows:

$$S_p(r) = \lambda^{-\xi_p} S_p(\lambda r),$$

where $\xi_p$ is the scaling exponent. In the simple scale behaviour, $\xi_p = pH$, whereas, for a multi-scaling, $\xi_p$ is a non-linear function of order $p$. In the scaling range, $S_p(r)$ can be described by the following expression:

$$S_p(r) \sim r^{\xi_p}.$$  (4)

The Kolmogorov [19] hypothesis predicts that, in the so-called inertial range, the $p$-order structure function scales with the exponent $p/3$ (i.e., the Hurst coefficient is $H = 1/3$)[3]. In the presence of the intermittency phenomenon, a divergence from the Kolmogorov hypothesis is present, and different Hurst coefficients are observed [23].

To investigate the intermittency phenomenon in each flow zone, we computed the high-order generalized structure functions. The $p$-order statistics $S_p$, with $p = 1, 2, 3$ and 4, as a function of $r$ for all the zones are reported in Fig. 4. Three different scale ranges are present: scaling, transition and saturation range. The scaling region occurs at small spatial increments. On the contrary, the saturation region is attested at large spatial increments, where $S_p$ assumes a quasi-constant value. Between the scaling and the saturation ranges, the transition range is established.

At small spatial increments (i.e., in the well-known inertial range), the scaling behaviour can be recognised by plotting the scaling exponent $\xi_p$ as a function of the order $p$. The $\xi_p$ exponents were computed using a power function to the scaling zone. In Fig. 5 the $\xi_p$ trend is evidently visible for the different water depth zones. Moving from Zone 1 to Zone 2, the quasi-linear tendencies are progressively abandoned, indicating a multi-scale behaviour close to the rough boundary and self-similarity in the rest of the flow field. As evident from this consideration, Zone 1, with its deviation from $\xi_p = p/3$, is always more intermittent [19, 20].

4. Discussion and conclusions

In our study, we presented an experimental campaign based on 2D PIV measurements in a laboratory flume with rough bed in no-motion conditions. We proposed our methodology on the study of the velocity increments and, specifically, of the $p$-order velocity statistics. Particularly, we computed the $p$-order 2D structure functions of the streamwise-vertical velocity components, by dividing the flow depth into two zones (the near-bed zone and the upper layer). We computed the anisotropy level and we analysed the intermittency of the inertial range.
Figure 5. The $\xi_p$ exponent as a function of structure function order $p$ for Zone 1 (full blue circles) and Zone 2 (open red circles).

This work demonstrates that the turbulence anisotropy influences the boundary layers at large scales. This impact is more evident in the near-bed zone, where the second-order velocity statistics are more stretched. Nevertheless, the turbulence tends to isotropic conditions at smaller scales, as typically supposed in fluids [24, 25]. We quantified the anisotropy degree using the anisotropy angle $\theta$. The analysis of $\theta$ shows that: (1) smaller scales are always more than the large scale in isotropic condition, and (2) anisotropy is present in the roughness layer. Finally, the high-order multi-fractal evaluation built on the $p$-order velocity statistics shows that in the near-bed layer additional intermittency, induced by singular vortices, occurs, causing the break of the Kolmogorov self-similarity hypothesis.

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References
[1] Kolmogorov A N 1962 Journal of Fluid Mechanics 13 82–85
[2] Monin A and Yaglom A 1971 Statistical fluid mechanics: mechanics of turbulence, vol 1 (Lumley JL, ed)
[3] Frisch U 1995 Turbulence: the legacy of AN Kolmogorov (Cambridge University Press)
[4] Smyth W D and Moum J N 2000 Physics of Fluids 12 1327–1342
[5] Del Alamo J C and Jiménez J 2003 Physics of Fluids 15 L41–L44
[6] Benzi R, Paladin G, Parisi G and Vulpiani A 1984 Journal of Physics A: Mathematical and General 17 3521
[7] Dittrich A and Koll K 1997 International Journal of Sediment Research 12 21–33
[8] Kironoto B, Graf W H, Song T and Lemmin U 1994 Proceedings of the Institution of Civil Engineers. Water, Maritime and Energy 106 333–344
[9] Nikora V, Goring D, McEwan I and Griffiths G 2001 Journal of Hydraulic Engineering 127 123–133
[10] Smith J D and McLean S 1977 Journal of Geophysical research 82 1735–1746
[11] Giménez-Curto L A and Lera M A C 1996 Journal of Geophysical Research: Oceans 101 20745–20758
[12] Nikora V, McEwan I, McLean S, Coleman S, Pokrajac D and Walters R 2007 Journal of Hydraulic Engineering 133 873–883
[13] Aberle J, Koll K and Dittrich A 2008 Acta Geophysica 56 584–600
[14] Padhi E, Penna N, Dey S and Gaudio R 2018 Physics of Fluids 30 125106
[15] Coscarella F, Penna N, Servidio S and Gaudio R 2020 Physics of Fluids 32 115127
[16] Dey S 2014 Fluvial Hydrodynamics (Springer)
[17] Julien P Y 1998 Erosion and Sedimentation (Cambridge University Press)
[18] Batchelor C K and Batchelor G 2000 An introduction to fluid dynamics (Cambridge University Press)
[19] Kolmogorov A N 1941 The local structure of turbulence in incompressible viscous fluid for very large Reynolds numbers Dokl. Akad. Nauk SSSR vol 30 (JSTOR) pp 301–305
[20] Kolmogorov A N 1941 Dissipation of energy in locally isotropic turbulence *Dokl. Akad. Nauk SSSR* vol 32 (JSTOR) pp 16–18
[21] Monin A S and Yaglom A M 2013 *Statistical fluid mechanics, volume II: mechanics of turbulence* vol 2 (Courier Corporation)
[22] Pope S B 2001 *Turbulent flows* (IOP Publishing)
[23] Toschi F, Amati G, Succi S, Benzi R and Piva R 1999 *Physical review letters* 82 5044
[24] Biferale L, Boffetta G, Celani A and Toschi F 1999 *Physica D: Nonlinear Phenomena* 127 187–197
[25] Procaccia I, Benzi R and Biferale L 2008 On intermittency in shell models and in turbulent flows *IUTAM Symposium on Computational Physics and New Perspectives in Turbulence* (Springer) pp 35–45