Chiral flat band superconductivity from symmetry-protected three-band crossings

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We show that chiral (nearly) flat band superconductivity can develop and host novel Majorana fermions at a time reversal pair of symmetry-protected three-band crossing points. Based on symmetry analysis and mean field study, we determine and analyze the irreducible pairing channels with flat band pairings in the low-energy spin-1 fermion theory. Flat band pairing enhances superconductivity dramatically, where the critical temperature scales linearly in the interaction strength. While fully gapped flat band pairing states develop in the single-component pairing channels, we find chiral $p' \pm ip'$ flat band superconductivity in the multi-component pairing channels. 3D itinerant Majorana fermions arise at the bulk nodal points, whereas Majorana arcs appear on the surface.

I. INTRODUCTION

Chiral superconductivity has attracted much attention of modern condensed matter research in the past decades [1, 2]. Hosting finite angular momentum pairing, chiral superconductivity spontaneously breaks time reversal symmetry and manifests nontrivial topological properties. 2D chiral superconductivity exhibits an out-of-plane rotation axis, thereby manifests fully gapped quasiparticle spectrum in the bulk [3]. Such state has been studied and proposed extensively in various systems, including strontium based materials [4, 5], graphene systems [6], and fractional quantum Hall states [3, 7]. Meanwhile, 3D chiral superconductivity manifests various types of gap structures. The bulk can host either full gaps, nodal points, nodal lines, or nodal Fermi surfaces, depending on whichever band structure and symmetry are provided [8–12]. The best-known example of 3D chiral pairing state is the superfluid $^3$He-A phase [13]. Recent works have also proposed 3D chiral superconductivity in various other materials, such as heavy fermion compounds [14] and topological semimetals [9–12, 15–19].

Another mainstream of modern condensed matter research has focused on superconductivity with high ratio of critical temperature over Fermi temperature $T_c/T_F$. Two different classes of systems have been uncovered along this direction, where the comparison of bandwidth $W$ and interaction $V$ plays a crucial role. The first class is represented by the high-$T_c$ materials [20], where strong electronic interactions $V \gg W$ induce high critical temperatures. The other class manifests low-energy bands with (nearly) flat dispersion $W \ll V$. Remarkably, the critical temperature acquires a linear scaling $T_c \sim V$ in the flat band limit $W \to 0$ owing to the immense density of states [21–23]. Such dramatic enhancement has motivated intensive search for flat band superconductivity. Various 2D systems have been studied accordingly, including surfaces of gapless topological materials [21], strained graphene [22], and graphene moiré heterostructures [24, 25].

Motivated by these mainstreams of modern condensed matter research, here we consider a platform where chiral superconductivity may develop on the 3D flat bands. Our analysis addresses the pairing problem at a time reversal pair of symmetry-protected three-band crossing points. These band crossings may occur at high symmetry points in time reversal symmetric materials with certain space group symmetries [26]. Other potential platforms such as optical lattices [27, 28] have also been proposed, while experimental realization has been accomplished in the superconducting quantum circuits [29]. Notably, the $k \cdot p$ Hamiltonian at these points manifests effective spin-1 fermions. These fermions exhibit significant difference from spin-1/2 electrons in the pairing problem. Moreover, the low-energy theory manifests a middle flat band at the band crossing, hosting an immense density of states. The pairing between these flat bands can enhance superconductivity dramatically, with the critical temperature scaling linearly in the interaction strength [23].

In this paper, we further confirm that chiral superconductivity can also benefit from these flat bands. Based on symmetry analysis and mean field study, we examine the irreducible pairing channels with various valley pairings and spin-orbit coupled pairings with total angular momenta $J = 0, 1$. Our analysis focuses on the channels with flat band pairings, which raise the critical temperatures to the linear scaling of interest. Previous analysis has studied the single-component $J = 0$ pairing channels, which manifest fully gapped quasiparticle spectra [23]. Here we find chiral $p' \pm ip'$ flat band superconductivity in the multi-component $J = 1$ pairing channels. These chiral pairing states are the spin-nondegenerate analogy of the superfluid $^3$He-A phase [13]. The bulk nodal points arise and host 3D itinerant Majorana fermions [30, 31], remarkably different from the Dirac points in superfluid $^3$He-A phase. Open Majorana arcs are also uncovered on the surface accordingly [30–33]. We thus uncover a dramatically enhanced chiral flat band superconductivity where novel Majorana fermions can arise.

II. LOW-ENERGY THEORY

In time reversal symmetric materials with space group 199, a pair of three-band crossings are stabilized at the high symmetry points $\pm P$ with momenta $P_\pm = \pm P$
III. FLAT BAND SUPERCONDUCTIVITY

We wish to study potential superconductivity in the vicinity of band crossings. In particular, we focus on the pairing states with dramatic enhancement from flat band pairings. Despite their various possible origins, the pairing states can be classified and studied based on symmetry, Fermi statistics, and topology [12, 15, 23, 37]. Here we identify the irreducible pairing channels based on a symmetry analysis, then study the flat band pairing states that can arise in these channels.

A. Irreducible pairing channels

Based on the symmetry of low-energy theory (1), the pairings can be distinguished into various irreducible pairing channels. Each channel manifests a pairing operator \( c_k^\dagger \mathcal{M} [(i\lambda^2) \gamma (c_k^\dagger)^T] \). The pairing representation \( \mathcal{M} \) is labelled by a set of good quantum numbers under the symmetry. Note that parity is not a good quantum number since there is no inversion symmetry.

Since the model manifests valley SU(2), symmetry, the valley pairings can be distinguished into singlet \( \alpha = 0 \) and triplet \( \alpha = 3, \pm \) channels. These channels manifest pairing representations \( \lambda^0, \lambda^3 / \sqrt{2} \) and \( \lambda^\pm = (\lambda^1 \pm i\lambda^2) / 2 \). Similar to the pairing of spin-1/2 states, the valley pairing is antisymmetric and symmetric in singlet \( \alpha = 0 \) and triplet \( \alpha = 3, \pm \) channels, respectively. Note that different pairing channels exhibit different pairing moments (Fig. 1) [15]. The intervalley pairings \( \alpha = 0, 3 \) manifest zero momentum and lead to Bardeen-Cooper-Schrieffer (BCS) states. Meanwhile, the intravalley pairings \( \alpha = \pm \) carry finite momenta \( \pm 2P \), thereby trigger Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) states instead [38, 39]. One may also consider the combination of FFLO states \( \alpha = 1, 2 \) in order for time reversal symmetry.

Rotation symmetry further distinguishes the pairings into different angular momentum channels. Spin-orbit coupling enforces the good quantum numbers \( (L, S, J, M_J) \), where \( L \) is orbital angular momentum, \( S \) is spin, and \( J = L + S \) is the total angular momentum with \( z \)-component \( M_J \). The pairings of spin-1 fermions are remarkably different from those of ordinary spin-1/2 electrons. With the larger single particle spin, more spin modes \( S = 0, 1, 2 \) are available under pairing. Moreover, the pairings show opposite exchange properties to the conventional ones. While spin singlet and quintet pairings \( S = 0, 2 \) are symmetric, spin triplet pairing \( S = 1 \) is antisymmetric. This important difference can lead to novel pairing states absent in spin-1/2 systems.

The combination of valley and spin-orbit coupled pairings is constrained by Fermi statistics. With valley singlet pairing \( \alpha = 0 \), the spin-orbit coupled pairing should be even. The even-\( L \) states must carry even \( S \), while the odd-\( L \) states should come with odd \( S \). When the valley pairing is triplet \( \alpha = 0, \pm \), the spin-orbit coupled pairing should be odd. This swaps the coupling between \( L \) and
Various properties of superconductivity can be uncovered ± site energies the quasiparticles come in particle-hole pairs with opposite energies ±E k , where the valley indices λ k are determined by the valley pairing channel α. The 2J + 1 irreducible representations J JM,k = \sum_{M=L,M,S|JM,J} LJM,k SSM\delta characterize the spin-orbit coupled pairings (L, S, J, M), where the additions are determined by the Clebsch-Gordan coefficients (LS; ML,MS|JM,J). The orbital representations LLM,k’s describe the 2L + 1 orbital modes (L, ML) and manifest spherical harmonics LLM,k = \sqrt{2L+1}Y LM,k. Meanwhile, the spin representations SSM\delta’s are 3 \times 3 SU(2) irreducible representations with normalization Tr(SSM\delta ST SM\delta’). They describe the 2S+1 spin pairings (S, MS) of spin-1 fermions. Constant attraction −V < 0 is assumed in each channel, which is a proper setup for our general study of potential superconductivity. The constant assumption is eligible for short-range interactions. Whether a channel is attractive depends on the mechanism inducing superconductivity.

B. Flat band pairing

When superconductivity develops in a channel, the system acquires finite condensate of pairing pairs. The pairing condensate is described by a nonzero order parameter \( \Delta(T) = \langle V \rangle \sum_k (P_{\alpha k})^T \) with \( \langle . . \rangle_T \) denoting the ensemble average at temperature T. The mean field Hamiltonian takes the Bogoliubov-de Gennes (BdG) form

\[
H = \sum_k \Psi_k^\dagger \mathcal{H}_{\text{BdG},k} \Psi_k ,
\]

where the BdG Hamiltonian reads

\[
\mathcal{H}_{\text{BdG},k} = \begin{pmatrix} \mathcal{H}_{0k} & -\bar{\Delta} \cdot \bar{J}_{Jk} \\ -\bar{J}^\dagger \cdot \bar{J}_{Jk} & -\mathcal{H}_{0k} \end{pmatrix}
\]

and \( \Psi_k^\dagger = (c^\dagger_{k\lambda+}, (c^\dagger_{k\lambda-})^T) \) is the Nambu spinor. The eigenstates are referred to as BdG quasiparticles. Due to an intrinsic particle-hole symmetry of BdG Hamiltonian, the quasiparticles come in particle-hole pairs with opposite energies ±E k , where a labels the particle-hole pair. Various properties of superconductivity can be uncovered by solving the self-consistent gap equation [23]

\[
\bar{\Delta}_{JM} = \sum_a \frac{V}{2} \sum_k \frac{\partial E^a_k}{\partial \bar{\Delta}_{JM}} \tanh \frac{E^a_k}{2T} .
\]

These include the critical temperature \( T_c \) below which superconductivity develops, and the order parameter magnitude \( |\Delta(T)| \), as well.

Our interest lies in the potential superconductivity in the vicinity of band crossings \( \mu \approx 0 \). The middle flat band is dominant in this regime, as it manifests a divergent density of states \( \nu^b(\epsilon) = \nu^b(\epsilon) \). Meanwhile, the linear bands are irrelevant since their density of states are vanishing \( \nu^b(\epsilon) \to 0 \) as \( \epsilon \to 0 \). Generically, the mechanisms inducing superconductivity only manifest a narrow attractive window \( |\epsilon - \mu| < \Lambda_\epsilon \ll v\Lambda_k \) near the Fermi level (Fig. 1). This excludes almost the whole linear bands and leaves only flat band for pairing. Whether superconductivity develops thus depends solely on the flat band pairing.

With a focus on the flat band pairing, the effective Hamiltonian \( H^0 = \sum_k (\Psi_k^0)^\dagger \mathcal{H}_{\text{BdG},k} \Psi_k^0 \) is obtained via a direct projection. The Nambu spinor is now defined \( \Psi_k^0 = (\xi_{k\lambda+}, \xi_{k\lambda-}^\dagger) \), and the BdG Hamiltonian (4) becomes

\[
\mathcal{H}_{\text{BdG},k}^0 = \begin{pmatrix} \epsilon_k^0 - \mu & -\Delta_k \\ -\Delta_k & -\epsilon_k^0 - \mu \end{pmatrix} .
\]

Note that the gap function \( \Delta_k = \Delta \cdot \bar{J}_{Jk} \) has been defined, where the pairing representation \( \bar{J}_{Jk} \) is a vector of 2J + 1 k-dependant complex scalars. The quasiparticle energies can be solved directly \( \pm E_k^0 = \pm |\epsilon_k^0 - \mu|^2 + |\Delta_k|^2 |^{1/2} \).

Accordingly, the gap equation (5) reduces to

\[
1 = \frac{V}{2} \sum_k \frac{|\bar{J}_{Jk}|^2}{2E_k^0} \tanh \frac{E_k^0}{2T} .
\]

We solve the gap equation at the band crossing \( \mu = 0 \). The solution shows that the characteristic energy scales of superconductivity, namely critical temperature and zero temperature order parameter, are linear in the interaction [23]

\[
T_c, |\Delta(0)| \sim V .
\]

In conventional BCS states, these scales are usually exponentially small \( T_c, |\Delta(0)| \sim \exp(-1/V \nu) \). The linear scaling here indicates that superconductivity is dramatically enhanced at three-band crossings ±P. Such enhancement is possible solely because the whole flat bands contribute immense densities of states to the pairing. When a small doping occurs \( 0 < |\mu| < \Lambda_\epsilon \), flat band pairing vanishes at the weak coupling limit \( V \to 0 \) (but conventional BCS states may develop on linear bands). This occurs since the Fermi surface is shifted away from the flat bands. Nevertheless, flat band superconductivity recovers at a small critical interaction \( V_c \sim |\mu| \), and the linear scaling (8) is resumed in the regime of stronger coupling [23].

Our analysis has assumed a leading order approximation (1) for the \( k \cdot p \) Hamiltonian at band crossings ±P. The middle band is perfectly flat under this assumption.
The energy scales $T_c$ and $|\Delta(0)|$ are again analyzed by solving the gap equation (7) [23]. Due to the finite Fermi surface, superconductivity develops immediately as the interaction is turned on [Fig. 2(b)]. In the weak coupling regime $V \ll 1$, the finite density of states only yields the conventional BCS scaling. Nevertheless, the whole flat bands involve in pairing at strong enough interactions. This enhances superconductivity dramatically and resumes the linear scaling (8). Since the band curvature $1/m$ is infinitesimal, the required interaction for linear scaling is also infinitesimal $V \sim 1/m$.

C. Superconducting channels

We now examine each irreducible pairing channel and determine those with flat band superconductivity. Our analysis focuses on the channels with the first few angular momenta $J = 0, 1, L = 0, 1, 2$, and $S = 0, 1, 2$. These channels usually serve as the leading competitors in superconductivity. The orbital modes $L = 0, 1, 2$ with respect to $\pm P$ are referred to as $s'$-, $p'$-, $d'$-wave pairings in the following discussions. It is worth noting that flat band pairing only occurs in spin singlet and quintet pairing channels. Spin triplet pairing does not support flat band pairing, since the according component in the spin representation $S_{1k}$ vanishes.

We start with the single-component $J = 0$ pairing channels. Since $J = 0$ only occurs when $L = S$, valley singlet pairing $\alpha = 0$ is necessary under Fermi statistics. Previous analysis finds flat band superconductivity in $s'$-wave spin singlet and $d'$-wave spin quintet pairing channels $(L, S, J) = (0, 0, 0)$ and $(2, 2, 0)$ [23]. The according gap functions are constant $\Delta_k \sim \Delta$ and $k_F^2 \Delta$ on the Fermi surface, leading to fully gapped quasiparticle spectra. Valley singlet pairing imposes an opposite sign between the gap functions on $FS_\pm$. This confirms the eligibility of pairing between nondegenerate flat bands under Fermi statistics.

The multi-component $J = 1$ pairing channels exhibit three-component order parameters $\Delta$'s. With an exhaustive examination, we uncover flat band superconductivity in $p'$-wave spin singlet $(1, 0, 1)$ and quintet $(1, 2, 1)$ pairing channels. Both channels exhibit valley triplet pairing. We obtain an identical form of the gap functions $\Delta_k = c \mathbf{k} \cdot \Delta$ in these channels, where $c > 0$ is a channel-dependent constant. Note that the spatial representation has been imposed to the order parameter $\Delta = (\Delta_x, \Delta_y, \Delta_z)$ for later convenience. The components are obtained from the relations $\Delta_{k\pm} = \mp(\Delta_x \mp i\Delta_y)/\sqrt{2}$ and $\Delta_0 = \Delta_z$ under the analogy $\Delta \sim j_{k}^2$. Due to the multi-component nature, each $J = 1$ pairing channel manifests
a degenerate manifold spanned by the order parameter. Whichever type of order parameter is energetically favored determines the quasiparticle spectrum.

IV. CHIRAL GROUND STATES WITH MAJORANA FERMIONS

We have found flat band superconductivity in $J = 1$ $p'$-wave spin singlet and quintet pairing channels. These multi-component pairing channels manifest degenerate manifolds of order parameters. A natural question arises as which type of order parameter dominates when superconductivity develops. Such problem can be solved through a Ginzburg-Landau analysis, where free energy minimization determines the energetically favored ground state. The according quasiparticle spectrum may host novel characteristics which are absent in single-component pairing channels.

A. Ginzburg-Landau analysis

We first conduct the Ginzburg-Landau analysis of free energy. The mean field free energy is derived from a coherent path integral calculation $f^0 = |\Delta|^2/V - \text{Tr} \ln(G^0)^{-1}$ [23]. Here $(G^0)^{-1}_{ab} = i \omega_n - \Pi_{ab}^{(0)}$ is the inverse Gor’kov Green’s function with fermionic Matsubara frequency $\omega_n = (2n + 1)\pi T$. The trace denotes a frequency-momentum summation $T \sum_n$ where the momentum summation has been rewritten as a continuous integral $(1/V) \sum_k \to \int d \mathbf{k}$. Near the critical temperature, the free energy can be expanded with infinitesimal order parameter [40]. An expansion up to quartic order gives the Ginzburg-Landau free energy

$$f = r|\Delta|^2 + u|\Delta|^4 - \frac{u}{3} |\Delta|^2 |\Delta|^2.$$  \hspace{1cm} (9)

The prefactor of quadratic term takes the form $r = 1/V + (c^2/3)\text{Tr}(k^2 G_+ G_-)$ with $G_{\pm,\mathbf{k}} = (i \omega_n + c^2 k^2)^{-1}$. In accordance with the onset of superconductivity, the prefactor turns negative $r < 0$ and triggers nonzero order parameter below $T_c$. The energetically favored ground state is determined by the quartic terms. At quartic order, a positive prefactor $u = (c^4/10)\text{Tr}(k^4 G_+^2 G_-^2)$ of isotropic term $|\Delta|^4$ stabilizes the free energy. An additional anisotropy also presents and manifests the subsidiary order $\Delta_{1\mathbf{k}} \Delta$ [12]. The expression is derived from the identity $|\Delta_{1\mathbf{k}} |^2 = | -i \Delta \times \Delta |^2$, where the relations $I_{1\pm} = \frac{1}{2}(I_x \pm i I_y)/\sqrt{2}, I_{10} = I_z$ and the spatial representations $(I_a)_{bc} = -i \epsilon_{abc}$ are utilized. Subsidiary order $\Delta_{1\mathbf{k}} \Delta$ captures the ‘magnetic dipole moment’ of superconductivity, thereby determines whether time reversal symmetry is broken. Our analysis finds a negative prefactor $-u/3$ for the anisotropy. This implies the preference of ground states with finite dipole moment $|\Delta_{1\mathbf{k}} | \neq 0$. According to this feature, we conclude that the energetically favored ground states are the chiral $p' \pm ip'$ pairing states $M_f = \pm 1$ which manifest time reversal symmetry breaking.

B. Bulk and surface Majoranas

Chiral pairing states host novel features both in the bulk and on the surface. To uncover these features, we turn to the BdG Hamiltonian (6) and study the quasiparticle spectrum. Consider the chiral $p' + ip'$ pairing state with order parameter $(\Delta_1, \Delta_0, \Delta_{-1}) = (\Delta > 0, 0, 0)$ and according gap function

$$\Delta_k = -c\Delta \sqrt{2}(k_x + ik_y).$$ \hspace{1cm} (10)

The rotation axis has been set as $\hat{z}$ in this representation. For the low-energy theory (1) with full rotation symmetry, $\hat{z}$ may point in arbitrary direction. However, the projection of practical interaction usually involves anisotropy along $\pm \mathbf{P}$, thereby fixes the axis as $\hat{z} = \mathbf{P}/|\mathbf{P}|$. The quasiparticle spectrum is analogous to the one in superfluid $^3$He-A phase [13]. Since the gap function (10) vanishes at the north and south poles $K_k = \pm k_2 \hat{z}$ on the Fermi surface, a pair of nodal points appear at $K_{\pm}$ in the quasiparticle spectrum (Fig. 3). Notably, the flat bands at three-band crossing points do not host spin degeneracy as $^3$He does. While the spin-degenerate nodal points in superfluid $^3$He-A phase manifest low-energy Dirac quasiparticles, non-Dirac-type quasiparticles are expected at the nodal points herein.

To study the low-energy quasiparticles at the nodal points, we expand the BdG Hamiltonian (6) in the vicinity of $K_{\pm}$ on $FS_{\pm}$. This yields a low-energy model $H^K = \sum_q \Psi_q \tilde{H}_q \Psi_q$ of the four-component fermion $\Psi_q = (c_{q\lambda \pm}, c_{q\lambda -}, c_{-q\lambda +}, c_{-q\lambda -})$ with $c_{q\lambda \pm} = c_{q\lambda \pm}(\pm K)\lambda$ [31]. The Hamiltonian

$$\mathcal{H} = -i \mathbf{q}\cdot \sigma \cdot \varepsilon \left( q_x \sigma^x - q_y \sigma^y \right) \mathbf{\sigma}$$ \hspace{1cm} (11)

FIG. 3. When the chiral $p' \pm ip'$ BCS state $\alpha = 3$ develops, the Fermi surfaces $FS_{\pm}$ are gapped out except at the bulk Majorana points $K_{\pm}$. Monopole charges $q^z = \pm 1/2$ are carried by $K_u$’s, respectively. These bulk Majorana points bring about Majorana arcs in the surface Brillouin zine (green lines). The FFLO state $\alpha = \pm$ only host bulk Majorana points on one Fermi surface $FS_{\pm}$ and an according surface Majorana arc.
exhibits linear dispersions $\pm E_\mathbf{q} = \pm \sqrt{\nu_0^2 (q_x^2 + q_y^2) + \nu_F^2 q_z^2}^{1/2}$ in the vicinity of $\mathbf{K}_\pm$ on FS$_{\mathbf{K}_\pm}$. Here $v_F = k_F/m$ and $\nu_\Delta = c\Delta/\sqrt{2}$ are the effective velocities along and perpendicular to $\hat{z}$, respectively. The Pauli matrices are defined so that $\sigma^\pm = \pm 1$ label $\mathbf{K}_\pm$ on FS$_{\mathbf{K}_\pm}$ and $\sigma^z = \pm 1$ denote particle-hole components. Remarkably, the four component fermion is invariant under particle-hole transformation $\Psi^\dagger_\mathbf{q} = (\sigma^z \Psi_{-\mathbf{q}})^T$. This indicates the equivalence between any particle and its antiparticle in the low-energy model. The low-energy quasiparticles are thus identified as Majorana fermions [31], which differ from the Dirac quasiparticles in superfluid $^3$He-A phase [13]. Note that the Hamiltonian (11) gives the real Majorana equation in quantum field theory (with anisotropic velocity) [41] when a ‘real’ representation $\Psi^\dagger_\mathbf{q} = \Psi_{-\mathbf{q}}$ is adopted. The Majorana feature is generic for spin-nondegenerate nodal points in 3D chiral superconductivity [12, 30–33]. Unlike the Majorana bound states in 1D and 2D chiral superconductivity [3], the Majorana fermions herein are itinerant in the bulk of 3D chiral superconductivity.

Analogous to the Weyl points in Weyl semimetal [42], the bulk Majorana points carry nontrivial monopole charges. From the low-energy Hamiltonian (11), we obtain opposite monopole charges $q^\pm = \pm 1/2$ at the Majorana points $\mathbf{K}_\pm$. The numbers of Majorana points are different for different valley triplet pairings. In the BCS state $\alpha = 3$, four Majorana points $\mathbf{K}_\pm$ on FS$_{\mathbf{K}_\pm}$ are present (Fig. 3). Valley triplet pairing imposes the same monopole charge $q^+ = 1/2$ at $\mathbf{K}_+$’s on FS$_{\mathbf{K}_\pm}$. The other points $\mathbf{K}_-$’s carry an opposite monopole charge $q^- = -1/2$ to the one carried by $\mathbf{K}_+$’s. The net vorticity on each Fermi surface FS$_{\mathbf{K}_\pm}$ is zero, as the flat band pairing does not exhibit nontrivial monopole structure. Such configuration differs from the monopole superconductivity in inversion symmetric Weyl semimetal [15, 18]. The later manifests nonzero net vorticity on each Fermi surface due to nontrivial pairing monopole structure. On the other hand, the FFLO states $\alpha = \pm 1$ manifest two Majorana points $\mathbf{K}_\pm$ with opposite monopole charges on a single Fermi surface FS$_{\mathbf{K}_\pm}$.

The presence of bulk Majorana points generically leads to Majorana arcs in the surface Brillouin zone [30–33]. The configuration of these arcs depends on the surface of interest. Here we choose a surface parallel to the $xz$-plane, where $p_x$ and $p_z$ form the surface Brillouin zone. The surface zero mode at $p_z$ corresponds to the edge mode in the effective bulk 2D system $H_{p_x,p_z}$ at $p_z$ [33, 42]. When $p_z$ lies between two Majorana points on the same Fermi surface $|p_z - (\pm P_2)| < k_F$, the 2D band encloses an odd number of Majorana points. Nontrivial Chern number $C = \pm 1$ is manifested accordingly, leading to a topologically protected chiral edge mode at $p_z = 0$. This edge mode is of Majorana type due to the bulk BdG structure. As $p_z$ goes into the rest region $|p_z - (\pm P_2)| > k_F$, the 2D band encloses pairs of Majorana points with opposite monopole charges. No topologically protected edge mode exist in this case. From these inscriptions, we conclude that Majorana bound states exist on the surface as open arcs. Each Majorana arc connects the projections of Majorana points $\mathbf{K}_\pm$ from the same Fermi surface (Fig. 3). The surface Majorana arcs may be probed experimentally by, for example, angle-resolved photoemission spectroscopy (ARPES), which is powerful in probing surface spectrum [33].

V. DISCUSSION

We have studied the spin-1 fermion pairing states at a time reversal pair of symmetry-protected three-band crossing points. Based on symmetry analysis and mean field study, we have exhaustively examined irreducible pairing channels with valley singlet, triplet and spin-orbit coupled $J = 0, 1$ pairings. We have focused particularly on the channels with flat band pairings, where superconductivity can be dramatically enhanced. Such enhancement leads to a linear scaling of critical temperature in the interaction strength. While $J = 0$ flat band pairing states exhibit full bulk gaps, we have uncovered $J = 1$ chiral $p'$ ± $ip'$ flat band superconductivity with bulk topological nodal points. The spin-nondegenerate nodal points host 3D itinerant Majorana fermions as low-energy quasiparticles. Meanwhile, open Majorana arcs arise on the surface and connect the projections of bulk Majorana points.

Further investigations from this work are still open, as we briefly discuss below. Our work has analyzed the irreducible pairing channels on an equal footing, without addressing the issue of whichever channel is leading. We have also examined each channel independently, while interchannel intertwintement may occur in practice. The projection of practical interaction on low-energy theory may introduce explicit interchannel coupling, as well. Additionally, our analysis has adopted the full rotation symmetry of low-energy theory, while a reduction down to lattice group symmetry may alter the pairing states. Furthermore, the cutoff of the ‘flat band’ regime has been left as undetermined in our analysis, where flat band enhancement may reduce and multiband pairing may arise. All of the above issues depend strongly on the details in the systems of interest. According analyses would provide useful information for the study of practical systems, which are left as future work. On the other hand, we have not addressed the fate of the two Fermi arcs originating from the nontrivial linear bands. These Fermi arcs have no relevance for the flat band superconductivity. However, as the much weaker pairing state develops on the doped linear bands, the behavior of these Fermi arcs and the connected additional (off-Fermi level) Weyl points may be altered. How these effects depend on different types of linear band pairing states may be interesting topic for future work. Finally, our analysis has focused on superconductivity without addressing the other instability. The study of the other potential flat band instability would be an interesting problem for future work.
Our work raises the interesting issue that dramatically enhanced chiral superconductivity can develop on 3D flat bands and host novel Majorana fermions. The information herein may be beneficial to the experiments on practical materials, and also to the theoretical study of novel superconductivity.

Note added. During the finalizing process of this manuscript, I learned about an independent study of superconductivity in systems with three-band crossings by Sim, Park, and Lee [43]. While their work studies the valley triplet $s'$-wave spin triplet pairing channel at a broad range of doping, the analysis in this manuscript finds and focuses on other pairing channels with flat band pairing, which would support much stronger superconductivity in the vicinity of band crossings.

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