Flow-Induced Vibrations of Single and Multiple Heated Circular Cylinders: A Review

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Abstract: This study is an effort to encapsulate the fundamentals and major findings in the area of fluid-solid interaction, particularly the flow-induced vibrations (FIV). Periodic flow separation and vortex shedding stretching downstream induce dynamic fluid forces on the bluff body and results in oscillatory motion of the body. The motion is generally referred to as flow-induced vibrations. FIV is a dynamic phenomenon as the motion, or the vibration of the body is subjected to the continuously changing fluid forces. Sometimes FIV is modeled as forced vibrations to mimic the vibration response due to the fluid forces. FIV is a deep concern of engineers for the design of modern heat exchangers, particularly the shell-and-tube type, as it is the major cause for the tube failures. Effect of important parameters such as Reynolds number, spacing ratio, damping coefficient, mass ratio and reduced velocity on the vibration characteristics (such as Strouhal number, vortex shedding, vibration frequency and amplitude, etc.) is summarized. Flow over a bluff body with wakes developed has been studied widely in the past decades. Several review articles are available in the literature on the area of vortex shedding and FIV. None of them, however, discusses the cases of FIV with heat transfer. In particular systems, FIV is often coupled to heat transfer, e.g., in nuclear power plants, FIV causes wear and tear to heat exchangers, which can eventually lead to catastrophic failure. As the circular shape is the most common shape for tubes and pipes encountered in practice, this review will only focus on the FIV of circular cylinders. In this attempt, FIV of single and multiple cylinders in staggered arrangement, including tandem and side-by-side arrangement is summarized for heated and unheated cylinder(s) in the one- and two-degree of freedom. The review also synthesizes the effect of fouling on heat transfer and flow characteristics. Finally, research prospects for heated circular cylinders are also stated.

Keywords: flow-induced vibrations; vortex-induced vibration; wake-induced vibration; heated cylinders; tandem; side-by-side; staggered; fouling; surface roughness

1. Introduction

Any bluff body subjected to fluid flow experiences flow-induced vibrations due to the induced aerodynamic forces which can have catastrophic results. Flow-induced vibrations (FIV) can be very harmful especially if the vibration frequency synchronizes with one of the natural frequencies of the structure. This is of great engineering significance, as the vortex-induced forces may develop in the structures exposed to fluid flow, such as, tall stacks, bridge towers and cables, offshore structures, and pipelines. Airplanes and jets may experience unwanted noise and vibrations due to FIV which may result due to poor airfoil or rotor design. These and many other examples explain the importance of understanding FIV and its control [1]. Flow over bluff bodies and the developed wakes have been widely studied especially in the past decades. Based on the nature of excitation, FIV is classified
into two main types: vortex-induced vibrations (VIV), and wake-induced vibrations (WIV). FIV of a single cylinder is mainly due to the vortices generated from the structure. This vortex shedding induces periodic forcing on the cylinder and causes the cylinder vibration, hence it is referred to as vortex-induced vibration (VIV). For the case of multiple cylinders, FIV comprises of both, VIV and WIV. In addition to VIV, which is comparable to the single cylinder case, the wake interactions between the cylinders give rise to WIV resulting in more severe vibrations. Sarpkaya [2] defined VIV as “an inherently nonlinear, self-governed or self-regulated, multi-degree-of-freedom phenomenon. It presents unsteady flow characteristics manifested by the existence of two unsteady shear layers”.

The two types of flow-induced vibrations encountered in practice, with respect to the direction of motion, are streamwise (parallel or in-line to the flow direction) and transverse (normal to the flow direction). External flow over a circular cylinder result in periodic vortex shedding for Reynolds number \((Re)\) greater than 47, and the natural vortex shedding frequency depends on \(Re\) [3–5]. The repeating array of vortices is identified as Karman vortex street. The flow remains 2D until Reynolds number of 190 [4,6,7].

When the cylinder vibration occurs at a frequency that is near to its natural vortex shedding frequency, the shedding of vortices harmonizes with the cylinder oscillation, and this condition is known as the lock-in phenomenon [8]. FIV studies have been done experimentally using wind tunnel or water channel with digital particle image velocimetry (PIV), and computationally using numerical techniques such as direct numerical simulations (DNS), Reynolds-averaged Navier–Stokes equations (RANS), large eddy simulations (LES), detached eddy simulations (DES), and several blends of these methods. In the DNS approach, the Navier–Stokes equations are directly solved without the use of any modeling assumption. It solves the extensive range of spatial and temporal scales, as fine as the Kolmogorov length scales. Therefore, the mesh resolution and the time step size need to be very fine. This is the reason DNS technique is computationally very expensive. RANS is the most computationally friendly technique in which only the large scales are resolved and the rest of the flow with small scale flow structures is modeled using the ensemble average technique. If the flow is unsteady, then this technique is called unsteady-RANS or simply URANS. For the LES approach the mesh is more refine than RANS and only the smallest scales are filtered out through the mesh and modeled while the rest are resolved. LES is more accurate than RANS but also more computationally expensive. Mesh needs to be very fine near the surface of the body in LES, however if this criterion is relaxed and RANS is used near the wall while LES away from the wall, then this technique is called DES. It is a good compromise between solution accuracy and computational time. DNS and LES provide good information about the wake-boundary layer interaction as compared to RANS.

A particular complicated issue with FIV in tube bundles is that a small change in flow rate or mechanical design can lead to a catastrophic failure of the unit. FIVs are of key interest in the design of shell-and-tube heat exchangers and of immense practical importance. Cylindrical structures are commonly found in industrial and engineering applications such as the tubes in heat exchangers of nuclear power plants, transmission lines, drilling and production risers in petroleum production, chimney stacks, cable-stayed bridges, mooring cables, thermowells, towed cables, marine cables, and in other applications [2].

FIV is of grave significance in nuclear power plants, particularly because of its frequent occurrence and large amplitude which can be of catastrophic nature. The vibration of fuel tubes stimulated by the coolant flow in the crossflow direction was initially detected in a high flux nuclear reactor in 1948, USA. Similar event has been noted in other countries as well, including Germany, Japan, Belgium, China, Denmark, UK, Australia, and France. The world’s first nuclear reactor to generate electricity, the American reactor at EBR-I (Experimental Breeder Reactor I), suffered a partial meltdown in 1955 during a coolant flow test. FIV in heat exchangers is a serious issue as discussed above and is also reported in several studies. It has resulted in tube bundle fracture in a nuclear power plant’s heat exchanger, leaks in nuclear power plant’s steam generator, and caused abrasive wear in
recuperator [9,10]. These and other examples resulted in either shutdown of the facilities or permanent damage of the equipment. Given its importance, both designers and operators are required to have a good understanding of the engineering issues associated with FIV.

Coupling between the flow and other parameters such as heat transfer, fluid forces, and structure elasticity and reactions can be of great importance to understand FIV mechanism and mitigate consequent damages. Heat transfer is the most common natural phenomenon. Similarly to the fact that human beings transfer heat to the surroundings in order to regulate the body temperature, most of the industrial systems and devices also need to transfer heat to the environment for their efficient performance. Heat can be transferred passively or actively. Heat exchangers are incorporated where active heat transfer is needed or where a high heat transfer rate is required. The most common examples of the use of heat exchangers are the radiator of a car, boiler of a power plant, condenser, evaporator of an air-conditioning unit, etc. FIV is commonly experienced in heated cylinders. Generally, a single heat exchanger tube can be modeled as a heated cylinder subjected to freestream fluid flow. Pin fins in electronic devices, nuclear reactor control rods, and heat exchanger tubes can all be viewed in a simplified way as a system with fluid flow over a heated cylinder, i.e., non-isothermal flow. Heat exchangers are of many types, including but not limited to, shell-and-tube type heat exchangers, plate heat exchangers, double-pipe heat exchangers, pin fin heat exchangers, etc. These types are further subdivided into several categories depending upon the type of flow, i.e., counter flow, parallel flow, or cross flow. This study will mainly focus on the shell-and-tube type heat exchangers with cross flow arrangement, as a single tube of shell-and-tube type heat exchanger can be modeled as circular cylinder subjected to crossflow. The most common type of heat exchangers are the shell-and-tube type heat exchangers, and these are widely used in industries, such as manufacturing industries, gas and steam turbine power plants, nuclear power plants, etc. Efficient working of the heat exchanger is necessary for the optimal operation of the system. Any fault in the heat exchanger is harmful not only for the system, but also for the personnel working in the vicinity of the plant. In severe cases it becomes hazardous not only for the human life but also for the environment and the neighborhood especially in the case of a nuclear power plant disaster, as observed in the incidents of Chernobyl in 1986 and Fukushima in 2011.

Due to the no slip condition at the surface of the cylinder, the first layer of fluid sticks to the surface and its velocity goes to zero and boundary layer develops. At Re > 47 vortices start to shed past the cylinder and as the shear layers get detached from the surface of the cylinder, they pull the cylinder towards them, and this results in vibrations of the cylinder. For the heated cylinder, thermal boundary also develops along with viscous boundary layer. The key to enhance heat transfer is to disrupt the thermal boundary layer. At the stagnation point of the cylinder the thermal boundary layer is the thinnest and it increases away from the stagnation point. FIV causes the position of the stagnation point to change and the point of separation of shear layers also changes on the surface of the cylinder. Both these factors disturb the thermal boundary layer and so does the thermal resistance and therefore FIV results in increase in the heat transfer. Yang et al. [11] pointed out that at the separation point thermal resistance is maximum and the local Nusselt number is minimum. Some of the studies discussing the boundary layer are [12–15]. The reader can view the work of Schlichting and Gersten [16] which is marvel in this field.

Due to the immense importance in the engineering applications FIV cannot be ignored and has been thoroughly studied especially in the past few years. A considerable volume of the literature has been made on FIV and its mitigation. In comparison to the previous reviews, this work attempts to summarize various studies and presents the important parameters affecting the FIV, focusing on the recent developments in the field. Although FIV is mostly considered hazardous, in a more recent trend, FIV can be utilized to harness energy. The ultimate objective of studying FIV is to understand, predict, and prevent FIV. Research, on the other hand, has been done to harvest energy from FIV where FIV is not suppressed but energy is harvested from it. The reader may refer to the works of
Yanfang et al. [17], Wang et al. [18], Li et al. [19], and other authors to get an idea of energy harnessing using FIV, as this topic is not discussed in this work. A number of authors have summarized the FIV phenomenon and presented design guidelines. Reviews on the FIV of a single cylinder were done by Derakhshandeh and Alam [20], Gabbai and Benaroya [21], Sarpkaya [2], Bearman [22], and Williamson and Govardhan [23]. Additionally, reviews on the flow over two cylinders subject to cross-flow were done by Sumner [24] and Zhou and Alam [15]. Weaver and Fitzpatrick [25] continued the work of Pa’idoussis [26] and reviewed FIV in tube arrays thoroughly. The authors of these reviews and other articles have done an excellent work, as summarized in the Table 1.

Table 1. Review papers on flow-induced vibrations and wake structures for circular cylinders.

| Researchers | Fixed/Free to Oscillate | Category |
|-------------|------------------------|----------|
| Pa’idoussis [26] | Free to oscillate | Tube arrays |
| Bearman [22] | Free to oscillate | Single cylinder |
| Parkinson [27] | Free to oscillate | Single cylinder |
| Weaver and Fitzpatrick [25] | Free to oscillate | Tube arrays |
| Sarpkaya [2] | Free to oscillate | Single cylinder |
| Williamson and Govardhan [23] | Free to oscillate | Single cylinder |
| Gabbai and Benaroya [21] | Free to oscillate | Single cylinder |
| Sumner [24] | Fixed cylinder | Two cylinders |
| Miwa et al. [28] | Free to oscillate | Tube arrays |
| Zhou and Alam [15] | Fixed cylinder | Two cylinders |
| Hong and Shah [29] | Free to oscillate | Single cylinder |
| Derakhshandeh and Alam [20] | Fixed cylinder | Single cylinder |
| Liu et al. [30] | Free to oscillate | Single and multiple |
| Feher and Avila [31] | Free to oscillate | Two cylinders |

Cylinders of various shapes and sizes are found in practice depending on their application. The most common type is, however, the circular cylinder, where oscillating flow separation might occur over a section of the surface of cylinder. However, for cylinder with an edge, such as a triangular, square, or rectangular cylinder, the point of separation is fixed at the edge. Alam et al. [32] worked on the effect of cylinder cross-sectional shape on the wake characteristics. They analyzed the circular and square shapes and the shapes in-between these two, i.e., with rounded edges. The effect of the corner and the rounded edge on the flow characteristics, especially how it affects the separation point and the wake bubble, was presented. Derakhshandeh and Alam [20] have reviewed different shapes of bluff body wakes which include circular, square, triangular, and rectangular shapes. They summarized the effect of cross-sectional shape of the cylinder on the flow topology in each Re regime (i.e., laminar, subcritical, critical, and supercritical). Their work provides essential information for engineers to design the pipelines depending on the Re and the application. This paper, however, only focuses on the circular cylinders as these are most commonly found in industrial processes. This review is not a collection of all the scientific/engineering advances in FIV, but one that tries to summarize the fundamentals and the major findings. Therefore, the readers are recommended to read the cited papers as well as other sources.

The initial flow conditions, such as Reynolds number, surface roughness, and turbulent intensity of the incoming flow, affect the attributes of the flow especially for a cylinder with circular cross-section, due to the dynamic nature of the flow separation. At low Reynolds number, i.e., less than 47.5, the flow is laminar and steady with symmetrical vortices behind the cylinder. It starts to get interesting when the Reynolds number increases from
47–190 (still laminar) where vortices of opposite rotations start to shed from the bottom and top side of the cylinder alternately and Karman vortex street emerges. In this range of Re, the Strouhal number increases with increasing Re. The cylinder wake evolves to three-dimensional (3D) from two-dimensional (2D) in the range of Reynolds number from 190–260 [7,33].

Although there is a lot of work done in this area and many review papers are available, new techniques have emerged which have not been discussed in detail in the previous review articles, such as two degrees of freedom (2DOF) motion of cylinder(s), effect of surface roughness on FIV and wake structures, FIV of heated cylinder(s), etc. This paper summarizes the recent trends in the FIV field of study. Considering the impact of FIV on vast number of applications, there is a dire need to understand the mechanism fully in order to avoid its occurrence and the related tube failure. Research must be done to identify the specific flow conditions where the FIV is most hazardous and to provide general engineering guidelines that can be applied to each application. Therefore, there needs to be more experimental and numerical research conducted in a more realistic way in order to mimic the actual flow conditions. Such as, considering single and multiple cylinders with heating to simulate the heat exchanger tubes and allowing the tubes to vibrate in 2DOF and taking into account their surface roughness.

Before we jump into the technical aspect of FIV, some technical terms are defined here:

- **Excitation frequency** ($f_{ex}$): vibration frequency of a (self-excited or forced) bluff body
- **Strouhal frequency** ($f_{st}$): the frequency of vortex shedding for a body at rest
- **Vortex shedding frequency** ($f_{vs}$): frequency of vortex shedding for a body in motion (self-excited or forced)
- **Natural frequency** ($f_n$): cylinder natural frequency in vacuum

The frequency value at which the vortices are shed is called vortex shedding frequency. It is nondimensionalized with the diameter of the cylinder and free steam velocity and is referred to as Strouhal number ($St$):

$$ St = \frac{f_{st} D}{U} \tag{1} $$

The Reynolds number ($Re$) is defined as:

$$ Re = \frac{\rho UA}{\mu} \tag{2} $$

The reduced velocity ($U_r$) is defined as:

$$ U_r = \frac{U}{f_n D} \tag{3} $$

where $D$ (m) is the cylinder diameter, $U$ (m/s) is the freestream velocity, and $\mu$ (N s/m²) and $\rho$ (kg/m³) represent the dynamic viscosity and density of the fluid, respectively.

The remainder of the manuscript is organized as follows. The following section discusses the wake structures behind the cylinder and how they vary depending upon the Reynolds number. The $St$-$Re$ relation is also presented in this section, along with the dependence of oblique shedding angle on the $Re$. The wall effect for the 3D cylinder and parallel and oblique shedding is discussed in the next section. Governing equations for cylinder undergoing 1DOF and 2DOF motion are presented, followed by the parameters affecting FIV, added mass, mass and structural damping, forced vibrations, the lock-in phenomenon, and the vortex shedding modes. The later sections present the various studied covering flow over unheated and heated single and multiple cylinders in tandem, side-by-side, and staggered arrangement. The sections at the end of the manuscript present the effect of 2DOF motion and surface roughness on the wake characteristics. Lastly, conclusions and future prospects for the research in the area of flow over circular cylinders and the related FIV is presented.
2. Wake Structures

In order to understand FIV, it is vital to understand the wake structure behind the cylinder which gives rise to the vibrations. The behavior of the flow around a cylinder strongly depends on the $Re$ and a number of authors have classified various flow regimes based on $Re$. Roshko [34] defined four flow regimes: subcritical ($1 \times 10^3 < Re < 2 \times 10^5$), critical ($2 \times 10^5 < Re < 5 \times 10^5$), supercritical ($5 \times 10^5 < Re < 2 \times 10^6$), and hypercritical ($Re > 2 \times 10^6$). In subcritical regime, the vortex shedding frequency remains constant and transition to turbulence occurs in the separated shear layers. In critical regime, a decrease in the drag coefficient is experienced due to transition to turbulence with an unstable boundary layer. In supercritical regime, fluctuating vortex shedding is experienced with loss in periodicity. In hypercritical regime, fully turbulent separation of the shear layers is experienced which results in wider wake and the drag coefficient also increases. Coutanceau and Defaye [35] classified the flow over a single cylinder into ten regimes depending on the flow structures based on their visual observation (Table 2). The transition from one regime to another affects the vorticity, vortex size, separation point, and lift and drag forces [36–38]. The $Re$ is the predominant factor, affecting the flow structures. More recently, Derakhshandeh and Alam [20] classified the flow over a circular cylinder into three major regimes based on the results of Zdravkovich [39] depending upon $Re$, i.e., $Re < 300$ (laminar); $300 \leq Re < 1.4 \times 10^5$ (subcritical); and $Re \geq 1.4 \times 10^5$ (critical and supercritical). One of the reasons behind this classification is that the fluctuating lift coefficient ($C_{l f}$) acts differently in these three regions, as shown in Figure 1. So does the transition in the shear and boundary layers [39,40].

| Regime | Reynolds Number Range | Characteristics of Regime |
|--------|-----------------------|---------------------------|
| 1: Fore-aft symmetrical flow | $Re < 0.1$ | Flow surrounds the cylinder perfectly |
| 2: Fore-aft asymmetrical flow | $0.1 \leq Re \leq 4.5$ | Speed of downstream fluid is reduced |
| Regime 3 | $4.5 \leq Re \leq 35$ | Presence of symmetrical eddies |
| Regime 4 | $35 \leq Re \leq 60$ | Slight bubble asymmetry |
| Regime 5 | $60 \leq Re \leq 100$ | Shedding of alternate vortices (Karman vortex street) |
| Regime 6 | $100 \leq Re \leq 2000$ | Quasi-laminar vortices transform into turbulent vortices |
| Regime 7 | $2000 \leq Re \leq (1.5–2) \times 10^5$ | Karman vortex street emerges at steady Strouhal number of 0.2 |
| 8: Critical Regime | $(1.5–2) \times 10^5 \leq Re \leq (4–5) \times 10^5$ | Unstable boundary layer, decrease in wake width, and increase in Strouhal number |
| 9: Supercritical Regime | $(4–5) \times 10^5 \leq Re \leq 3.5 \times 10^6$ | Unstable regime and loss of wake periodicity |
| 10: Trans-critical Regime | $Re \geq 3.5 \times 10^6$ | Reappearance of the wake periodicity with greater stability |

Figure 2a,b represents the effect of increasing $Re$ on $St$ for flow over a cylinder. It is observed that vortex shedding starts at $Re$ of about 47. As $Re$ increases, $St$ increases with a decreasing rate, until it reaches a constant value of 0.2 at $Re$ of 300 (laminar regime) and stays constant in the sub-critical range from $Re = 300$ to $1.4 \times 10^5$. The vortices are three-dimensional in subcritical flow. A discontinuity is seen at $Re$ of 180 [41–44]. In the critical regime $St$ jumps abruptly from 0.2 to around 0.5 at $Re$ of $1.5 \times 10^5$, after which it decreases slightly and has random values at $Re > 6 \times 10^5$.

Many numerical studies are simplified as 2D studies, however in the 3D studies and in the experimental work, the effect of spanwise length of the cylinder is captured. The third dimension affects the wake of the cylinder. Williamson [6] in his work reviewed the shift to three-dimensionality of the flow characteristics in the near wake of a circular...
The discontinuities seen in Figure 2a are the result of the 3D effects. More detail can be found in [33,45].

In the subcritical, critical and supercritical regime, when $Re > 300$, irregular velocity fluctuations start to occur and the separated shear layers become turbulent. Vortex shedding frequency, however, still stays almost constant [46–48]. The Strouhal number stays approximately constant at 0.2 [48–51]. In the subcritical flow regime, vortices exhibit 3D characteristics [33]. Bearman [52] studied the effect of $Re$ on lift and drag forces, pressure coefficient and Strouhal number for a circular cylinder. He found that at $Re = 4.16 \times 10^5$ the mean drag coefficient was minimum where $St$ suddenly jumps to 0.46 from 0.2 (Figure 2b), and the vortex shedding frequency encompasses a wider band at $Re = 5.5 \times 10^5$.

In the laminar flow regime ($Re < 300$), the interaction between the shear layers, boundary layers, and the wake makes the flow behind the cylinder complex although the shape of the circular cylinder is very simple. Mostly, the wake of a circular cylinder is simplified as 2D specially in numerical studies. The wake flow in reality is 3D, much more complex than the simplified 2D wake flow [53]. It is affected by various parameters, such as aspect ratio ($AR$) of the cylinder, blockage ratio of the channel, Reynolds number, surface roughness of the cylinder, incoming flow turbulence intensity, etc. Szepessy and Bearman [54] conducted wind tunnel experiments with a circular cylinder and showed that $C_Lf$ depends on $Re$ and aspect ratio of the cylinder.

The oblique shedding angle ($\theta$) for a circular cylinder also depends on $Re$ as shown in Figure 3 (adapted from Williamson [42]). It is observed that there is an abrupt rise for $\theta$ when $Re \approx 70$, which results in irregular, weaker vortices. The oblique shedding angle increases from $2^\circ$ to $11^\circ$ at a decreasing rate from $Re$ of 47 to 65 where a jump occurs, and the oblique shedding angle attains a maximum value of $20^\circ$. Increasing $Re$ further decreases the oblique shedding angle and a constant value of $12^\circ$ is reached at $Re > 100$. The sudden jump is due to the shift in the vortex shedding frequency ($St$). Therefore, it can be concluded that the oblique shedding angle ($\theta$) and $St$ are highly sensitive to $Re$. 

Figure 1. Flow classification based on fluctuating lift coefficient. Reprinted with permission from Ref. [20]. Copyright 2019, Elsevier.
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**Figure 2.** (a): $St$ vs. $Re$ for a circular cylinder in the laminar regime. Reprinted with permission from Ref. [41]. Copyright 1996, Elsevier. (b): $St$ vs. $Re$ for a circular cylinder in subcritical, critical, and supercritical regime. Reprinted with permission from Ref. [52]. Copyright 2006, Cambridge University Press.
3. Wall Effect for 3D Cylinder

Parallel shedding and oblique shedding are observed (Figure 4) for fully developed unsteady flow over a 3D circular cylinder in the numerical work of Mittal and Sidharth [55]. They carried out DNS at $Re$ of 100. The two cell structures for oblique shedding, i.e., central cell and end cell are observed in Figure 4a. End cell region is observed near to the no-slip wall and central cell region is observed for most of the length of the cylinder. The end cell region experiences lower shedding frequency as compared to central cell region. The time histories of lift and drag coefficients ($C_L$ and $C_D$) for oblique and parallel shedding are compared in Figure 5. It can be observed that the magnitude of $C_L$ and $C_D$ decreases, and the behavior is not a mono-frequency plot.

Figure 3. Oblique shedding angle vs. Reynolds number in the laminar regime. Reprinted with permission from Ref. [42]. Copyright 2006, Cambridge University Press.

Figure 4. (a) Oblique shedding with no-slip wall vs. (b) parallel shedding with slip wall for 3D circular cylinder. Reprinted with permission from Ref. [55]. Copyright 2014, Elsevier.
4. Governing Equations for FIV

The governing equations in a simplified manner are presented in this section. For the cylinder experiencing FIV with one-degree of freedom (transverse direction only), the equation of motion is:

\[ m\ddot{y} + c\dot{y} + ky = F_L \text{ [kg-m/s}^2\text{]} \]  

(4)

where \( m \) here is the virtual mass of the system which includes the actual mass and the added mass \( \Delta m \).

\[ \ddot{y} + 2\zeta w_n\dot{y} + w_n^2 y = F_L/m \text{ [m/s}^2\text{]} \]  

(5)

where \( w_n \) is the natural frequency that can be calculated as:

\[ w_n = 2\pi f_n = \sqrt{\frac{k}{m}} \text{ [rad/s]} \]  

(6)

where \( f_n \) (Hz) is the natural frequency, \( k \) (N/m) is spring constant, \( m \) (kg) is the mass of the body, and \( c \) (N-s/m) is the damping coefficient.

Here, \( \zeta \) is the damping ratio, defined as:

\[ \zeta = \frac{c}{2\sqrt{km}} \]  

(7)

The \( f_n \) depends on \( k \) and \( m \) of the system and can be found by giving an initial displacement of the body. The frequency can be found through fast Fourier transform (FFT) of the free decay of the displacement or from a similar technique.

More generally, the transverse \( (y) \) and streamwise \( (x) \) motion of the body can be modeled using the following nondimensional equations, as given in Izadpanah et al. [56] and Nepali et al. [57]:

\[ \ddot{Y} + 4\pi F_n \zeta \dot{Y} + (2\pi F_n)^2 Y = \frac{2C_L}{\pi m^*} \]  

(8)

\[ \ddot{X} + 4\pi F_n \zeta \dot{X} + (2\pi F_n)^2 X = \frac{2C_D}{\pi m^*} \]  

(9)

where \( X, \dot{X}, \ddot{X}, Y, \dot{Y}, \) and \( \ddot{Y} \) are the dimensionless displacement, velocity, and acceleration in streamwise and transverse direction, respectively. Where \( X = x/D \), and \( Y = y/D \).

The nondimensional natural frequency is represented as:

\[ F_n = \frac{1}{U_r} \]  

(10)
The lift coefficient $C_L$ is given as:

$$C_L = \frac{2F_L}{\rho U_\infty^2 D}$$  \hspace{1cm} (11)

and the drag coefficient is given as:

$$C_D = \frac{2F_D}{\rho U_\infty^2 D}$$  \hspace{1cm} (12)

For FIV, excitation frequency is expressed as:

$$f_{ex} = \frac{1}{2\pi \sqrt{k}} \sqrt{\frac{k}{m + \Delta m}}$$  \hspace{1cm} (13)

where $\Delta m$ is the added mass.

Reduced mass or mass ratio is defined as the ratio of the mass of the cylinder to the mass of the fluid displaced. For a circular cylinder it is given as:

$$m* = \frac{m_{\text{solid}}}{m_{\text{fluid}}} = \frac{m}{\rho(\pi D^2/4)} l$$  \hspace{1cm} (14)

where $l$ is the cylinder spanwise length (for 2D cases, $l = 1.0$). The range of $m*$ is usually taken as 6–12 when the fluid is water, and its value is much higher, 250–400, when the fluid is air. The VIV is controlled by the spring constant of the system, mass of the cylinder, structural damping, fluid density, and the motion of the fluid around the body.

To simulate the vibrations of a cylinder, various techniques have been used by the researchers available in the literature. Second order Newmark- $\beta$ method have been used by [58–60]. Papaioannou et al. [61] and Han et al. [62] also used a numerical technique belonging to the Newmark family. The trapezoidal method which is a second-order implicit technique was used by Chung [63]. Galerkin FEM method was used by [64–67]. Whereas the renowned fourth-order Runge–Kutta algorithm was used by [11,13,68,69].

5. Parameters Affecting FIV

Decades of research have identified the parameters affecting VIV of cylinders, which can be summarized as: fluid density and viscosity, velocity and turbulence intensity of the incoming flow, mass, diameter, length, and roughness of the cylinder, spring constant and structural damping of the system. However, in many cases structural damping is ignored, particularly in numerical simulation, although there is no such material or structure without damping.

6. Added Mass

Whenever a body is accelerated in a fluid, there appears to be a change in its mass and that change is technically termed as the added mass. It can be negative or positive, subject to the relative direction. The effect of viscosity on the added mass was proved by Stokes [70]. It can be elaborated that “the added mass is not a concentrated mass attached to the centroid of the body. It is distributed throughout the fluid set in motion by the body. Thus, its magnitude and centroid change with time as the intensity and distribution of the kinetic energy of the fluid change with time” [2]. For cylinder undergoing FIV, added mass strongly depends on amplitude to diameter ratio ($A/D$). Added mass is a manifestation of the added inertia as the body tries to accelerate or decelerate through a fluid. The FIV is a transient phenomenon and therefore the value of added mass also fluctuates, and it can be negative or positive, as a result at certain instant the apparent mass of the cylinder might become zero. The dimensionless added mass coefficient ($C_a$) is defined as the ratio of added mass to the mass of the displaced fluid, i.e., $C_a = \Delta m/m_f$. For the numerical analysis, the effect of pressure and viscous terms is explicitly incorporated into the Navier–Stokes
equations solutions, therefore one does not need to be concerned about the value of the added mass. Vikestad et al. [71] described complete experimental procedure and the related equations to find the value of added mass coefficient for a circular cylinder. The added mass and the related added mass coefficient can be found by the following equations:

\[ \Delta m = m \left[ \left( \frac{f_n}{f_{ex}} \right)^2 - 1 \right] \]  
\[ C_a = m^* \left[ \left( \frac{f_n}{f_{ex}} \right)^2 - 1 \right] \]  

(15)  
(16)

where \( f_n \) is the cylinder natural frequency in vacuum, and \( f_{ex} \) is the vibration frequency.

Cylinders with high mass ratios are less affected by the added mass and its variations, as the lighter fluids provide less resistance to the motion of the cylinder. Heating the cylinder results in lowered values of added mass as it decreases the fluid density in the vicinity of the cylinder and hence the apparent mass ratio increases. It becomes easier for the cylinder to accelerate in lower density fluid as compared to high density fluid.

7. Mass and Structural Damping

One must be clear about the difference between the structural damping and fluid damping. The former results from the structure’s support whereas the latter result from the fluid viscosity. Structural damping is constant regardless of the fluid medium, whereas the fluid damping strongly depends on the type of fluid. For example, the fluid damping for a cylinder in water will be much higher than the one present in the air. Heating indirectly affects the effective (total) damping as the temperature change alters the fluid damping. Heating results in lowered fluid damping if the fluid is a liquid as it decreases its viscosity, however for gases the viscosity increases with the increase in temperature and so does the fluid damping. For FIV it must be noted that equivalent damping should be considered for the analysis which represents both, the structural damping as well as the fluid damping. Koopmann [72] performed experiments on elastically supported cylinder to find the damping in vacuum and in still air. His results showed that structural damping was only 15% of the damping in the still air. Sarpkaya [2] suggested that for VIV in water, the structural damping term is negligible as compared to the fluid damping, and for even dense fluids, it can be completely neglected in numerical simulations. Sarpkaya [2] and Alam [73] suggested that \( m^*\zeta \) should not be merged to form a single parameter, although it has almost become a common practice to blend these parameters into a single ‘mass-damping’ parameter \( (m^*\zeta) \). They argued very important roles are played by each \( m^* \) and \( \zeta \) in VIV. According to Khalak and Williamson [74], synchronization range is controlled mainly by \( m^* \) (when \( m^*\zeta \) is constant), however the maximum amplitude of vibration is regulated mostly by the product of \( m^*\zeta \) in \( Re = 3.5 \times 10^3 - 1 \times 10^4 \).

8. Forced Vibrations

For forced vibrations, the vibrations are imposed externally at given values of \( A \) and \( f_{ex} \). For transverse forced oscillations:

\[ Y(t) = A\sin(2\pi f_{ex}t) \]  
\[ X(t) = A\sin(2\pi f_{ex}t) \]  

(17)  
(18)

where \( Y(t) \) and \( X(t) \) are the time-dependent displacements in the transverse and streamwise directions, respectively.
9. Lock-In Phenomenon

One of the most important terms in FIV is the lock-in or synchronization where “large-amplitude vibrations occur on an elastically mounted cylinder. This phenomenon occurs at a range of $U_r$ when the vortex shedding frequency synchronizes with the structure’s natural frequency” [6]. The term lock-on is also synonymously used in the literature. Gao et al. [75] defined the lock-in region as: “the VIV amplitudes within the lock-in region must be larger than half of the maximum amplitude experienced within the whole reduced velocity range”. This approach of determining the lock-in zone was also used in a prior study [76].

The $U_r$ range where lock-in occurs for a low $m^*\zeta$ is split into two: the initial and lower branches [77]. Khalak and Williamson [78] studied VIV for a very low $m^*\zeta$ and divided the $U_r$ range into four branches: “the initial excitation, upper branch, lower branch, and desynchronization”, as indicated in Figure 6. The figure displays the data from experiments of [77] (hollow dots) and from [78] (solid dots). The cylinder was only permitted to vibrate in the transverse direction. The parameters of Feng’s study [77] using air as fluid were $m^* = 248$, $\zeta = 0.00103$, and $m^*\zeta = 0.255$, and those used by Khalak and Williamson [78] with water as fluid were $m^* = 10.1$, $\zeta = 0.0013$, and $m^*\zeta = 0.013$. In the experimental work for VIV, $U_r$ is varied by changing $Re$. For forced oscillations, $Re$ is kept constant and $f_{ex}$ is changed, which in turn varies $U_r$. This figure shows the dependence of $A/D$ on $U_r$. As $U_r$ is increased in the initial branch, $A/D$ first increases slowly and then abruptly jumps to the upper branch where the maximum amplitude is attained. In the lower branch the amplitude decreases until desynchronization is reached where $A/D$ no longer depends on $U_r$, attaining a near constant value.

![Figure 6. Amplitude response comparison for $m^* = 2.4$ (bold dots [78]) and $m^* = 248$ (hollow dots, [77]). Reprinted with permission from Ref. [78]. Copyright 1997, Elsevier.](image-url)

Parkinson [79] stated that “the vibration amplitude in the VIV regime is limited to less than $1D$ for a circular cylinder, whereas the galloping vibration amplitude of a square cylinder can increase manyfold.” In the experimental studies of Carberry [3] and Carberry [4]....
et al. [80] on vibrating cylinders, the maximum lift coefficient increased by two-folds in $Re = 2300 - 9100$.

In the lock-in region, even if the vortices are weak, cylinder can oscillate at large amplitudes because of lock-in. The self-excited and self-sustaining resonant response of the cylinder occurs at specific amplitudes and frequencies, and it is a complete dynamic phenomenon governed by the interacting shear layers, vortex shedding and the induced forces on the body. It does not strictly happen at a fixed value of amplitude and frequency due to the continuous variations in the added-mass and in the response amplitude and frequency.

Figure 7 shows the plot between $f_{vs}$ vs. $f_{ex}$ normalized by $f_{st}$. When $f_{vs}$ is close to $f_{st}$, i.e., on the horizontal line at $f_{vs}/f_{st} = 1$, vortex shedding is not affected by the excitation frequency. However, when $f_{vs}$ is equal to $f_{ex}$, given by the line of the slope of 1, the vortex shedding becomes synchronized with the excitation frequency and deviates from the Strouhal frequency. This trend is observed in the range of $0.5 \leq f_{ex}/f_{st} \leq 1.6$.

![Figure 7](image-url)

**Figure 7.** Normalized vortex shedding frequency vs. normalized excitation frequency for a single cylinder with $A/D = 0.05$, and $Re = 1.5 \times 10^2$. Reprinted with permission from Ref. [81]. Copyright 2004, Elsevier.

A more recent data for normalized vortex shedding frequency and normalized excitation frequency at $A/D = 0.22$ and $Re = 1500$ is shown in Figure 8.

Related to lock-in, Sarpkaya [2] has quoted that “self-excitation without lock-in is common but self-regulated lock-in without self-excitation is impossible”. Vortex shedding for a smooth stationary circular cylinder is a dynamic phenomenon and the point of separation depends on Reynolds number, and amplitude and frequency of the vibration [83,84]. Generally, for a cylinder with any cross-section, the point of separation depends on the pressure gradient, the flow conditions, roughness of the cylinder surface, shape of the cylinder, and geometrical changes across the length of the cylinder.
Figure 7. Normalized vortex shedding frequency vs. normalized excitation frequency for a single cylinder with \( A/D = 0.05 \), and \( Re = 1.5 \times 10^2 \). Reprinted with permission from Ref. [81]. Copyright 2004, Elsevier.

A more recent data for normalized vortex shedding frequency and normalized excitation frequency at \( A/D = 0.22 \) and \( Re = 1.5 \times 10^2 \) is shown in Figure 8. Figure 8. Normalized vortex shedding frequency vs. normalized excitation frequency for \( A/D = 0.22 \) and \( Re = 1.5 \times 10^2 \). Reprinted with permission from Ref. [82]. Copyright 2001, Elsevier.

Related to lock-in, Sarpkaya [2] has quoted that “self-excitation without lock-in is common but self-regulated lock-in without self-excitation is impossible”. Vortex shedding for a smooth stationary circular cylinder is a dynamic phenomenon and the point of separation depends on Reynolds number, and amplitude and frequency of the vibration \( [83,84] \). Generally, for a cylinder with any cross-section, the point of separation depends on the pressure gradient, the flow conditions, roughness of the cylinder surface, shape of the cylinder, and geometrical changes across the length of the cylinder.

From Figures 7 and 8 it can be observed that the lock-in can also happen at oscillations that are super-harmonics of the vortex shedding frequency. Moreover, the lock-in zones for the odd-number super-harmonics seem to be different from those for the even-number super-harmonics \([81,85,86]\). This depends on the nature of the vortex shedding. Moe et al. (1994) noted that oscillations close to harmonic oscillations appear for self-excited vibrations in the lock-in region.

10. Vortex Shedding Patterns/Modes

The most common types of vortex shedding patterns observed for circular cylinder(s) are shown in Figure 9 and defined as follows:

- **2S**—two single vortices shed per cycle of shedding
- **2P**—two pair of vortices shed per cycle of shedding
- **2T**—two triple vortices shed per cycle of shedding
- **P+S**—a pair of vortices with opposite signs shed at one side and single vortex sheds at the other side of the cylinder \([57]\)
- **C**—Chaotic vortex shedding where a specific pattern cannot be identified \([65]\).

The most common type of vortex shedding mode is 2S for a single cylinder. The 2P and P+S modes appear when the vibration amplitude is large while C mode emerges for a high frequency of oscillations. The relationship of these modes with the \( Re \) and \( U_r \) will be shown in the following sections.
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Figure 9. 2S, 2P and P+S vortex shedding patterns behind a circular cylinder. Reprinted with permission from Ref. [29]. Copyright 2018, Elsevier.

11. Flow over Unheated Single Cylinder
In most of the experimental and numerical studies related to FIV, the degrees of freedom have been reduced from six to one (i.e., motion in transverse direction only). In a recent trend, a second degree of freedom is permitted in the streamwise (in-line) direction. The restrictions imposed are to simplify the complex nature of the vibration which is not easy to replicate in experiments and/or in numerical simulations. Flow over a single cylinder has been studied for decades in pursuit of understanding the flow physics. The studies of flow over a single cylinder without heat transfer are usually focused on FIV and the frequency response of the structure. At high mass ratio, VIV of single cylinder consists of an initial branch and a lower branch, both corresponding to 2S vortex shedding modes [77, 87]. At low mass-damping ratio, the amplitude and the lock-in region both increase, and an upper branch appears along with the 2P mode [78, 88, 89].

Wanderley and Soares [90] modeled the VIV of spring-mounted cylinder allowed to vibrate in the transverse direction only. They carried out numerical solution of unsteady Reynolds-averaged Navier–Stokes (URANS) equations and used k–ε model to account for turbulence. Response amplitude, frequency, and lift coefficient were analyzed at $100 < \text{Re} < 1200$, and $2 < U_r < 12$. They verified that Reynolds number strongly affects VIV (Figure 10). While the synchronization regime is dependent on $Re$, the maximum response amplitude hovers around 0.55 and is not much sensitive to $Re$.

Chung [91] numerically studied transverse FIV of an elastically mounted cylinder moving near a plane wall at $Re = 100$, with the gap ratio (spacing between cylinder and the wall) $G = 0.06, 0.3$, and 31.5. To solve the URANS, Crank–Nicolson scheme was used for convection and diffusion terms. The time mean drag coefficient in this study was found to be larger than the isolated cylinder for all gap ratios in the lock-in region. Bernitsas and Raghavan [92] experimentally studied the vortex-induced vibrations on an elastically mounted cylinder. The main focus was to find the VIV dependence on the Reynolds number. The upper branch range was found to increase in the lock-in regime with increasing $Re$. In
addition, the vibration amplitude \( A/D \) increased with increasing \( Re \) in the upper branch. They observed that the effect of \( Re \) on VIV was more significant than that of \( m^* \) and \( \zeta \).

![Figure 10. Variation in vibration amplitude for different Reynolds numbers. Reprinted with permission from Ref. [90]. Copyright 2015, Elsevier.](image)

Most of the studies involve transverse vibration as it is usually the dominant component of the vibrations; however, streamwise oscillations cannot be neglected. Streamwise oscillations were first time reported by King [93] and confirmed by subsequent studies by [94,95]. Cylinder with streamwise oscillations was numerically studied by Karanth et al. [96]. They used alternating direction implicit (ADI) finite difference method to numerically solve the URANS equations. They found that the developing downstream wake causes irregular behavior of the lift coefficient. Their findings were later complemented by the work of [97–100] and several other authors. Bai [101] employed the LES and DES to investigate fixed cylinders and cylinders with forced oscillations under turbulent flows. They observed that for forced oscillating cylinder, vortex street appeared as two rows as the oscillation affected the flow field. For low oscillation frequencies, only one \( f_{os} \) appeared but as the oscillation frequency increased, a high frequency nonlinear component also appeared in the vortex shedding. Al-Mdallal et al. [102] numerically studied crossflow over a circular cylinder with streamwise oscillations at \( Re \) of 200. To solve the URANS numerically, the implicit Crank–Nicolson and Gauss–Seidel iterative schemes were used. Their results indicated the presence of symmetric and asymmetric modes of vortex shedding in the wake of the cylinder. The modes were presented and characterized by the frequency ratio. Table 3 shows selected studies for fixed and elastically-supported single cylinders.

In order to understand FIV, first we need to understand the flow over a cylinder and the developed aerodynamic forces. Various studies have proved that for a single circular cylinder, the VIV amplitude, vibration frequency, aerodynamic forces, and vortex shedding strongly depends on the \( Re, U_r, m^* \), and \( \zeta \). It is also widely accepted that FIV is a complex phenomenon and cannot be accurately represented by 1DOF motion alone.
Table 3. Selected studies for flow over single cylinder showing the type of methodology, cylinder mounting, Reynolds number (Re), and reduced velocity (Ur). The measurements carried out in each study are also presented, where \( A/D \) = nondimensional vibration amplitude, \( f \) = vibration frequency, \( C_L \) = lift coefficient, \( C_D \) = drag coefficient, \( C_P \) = pressure coefficient, \( St \) = Strouhal number, FS = flow structure.

| Researchers                  | Methodology | Mounting                  | Re       | Ur   | Measurements     |
|------------------------------|-------------|---------------------------|----------|------|------------------|
| Bearman [52]                 | Experimental| Fixed                     | 1 × 10^3 to 7.5 × 10^3 | -     | St, C_L, C_D, C_P |
| Khalak and Williamson [78]   | Experimental| Elastically supported     | 2000–12,000 | 1–17 | A/D, f, C_P, C_L |
| Wanderly and Soares [90]     | Numerical   | Elastically supported     | 100–1200  | 2–12 | A/D, f, C_L      |
| Chung [91]                   | Numerical   | Elastically supported     | 100      | -    | A/D, C_D, C_P    |
| Raghavan and Bernitsas [92]  | Experimental| Elastically supported     | 2 × 10^3–10 × 10^4 | 3–11 | A/D, f          |
| Norberg [103,104]            | Experimental| Fixed                     | <47 and 47–2.2 × 10^7 | -    | St, C_D, C_P    |
| Kravchenko and Moin [105]    | Numerical   | Fixed                     | 3900     | -    | St, C_D, C_P    |
| Ma et al. [106]              | Numerical   | Fixed                     | 500–5000 | -    | St, C_D, C_P    |
| Park and Lee [107]           | Experimental| Fixed                     | 20,000   | -    | C_P             |
| Singh and Mittal [108]       | Numerical   | Elastically supported     | 50–500   | 4.92 | A/D, St, C_D, C_L, FS |
| Kim and Choi [109]           | Numerical   | Fixed                     | <40 and 40–3900 | -    | C_D, C_L, C_P  |
| Alam et al. [110]            | Numerical   | Forced                    | 300      | 2.77 | C_L, C_D, FS    |
| Marquart et al. [111]        | Numerical   | Fixed                     | 1 × 10^6, 1 × 10^6 | -    | C_D, FS       |
| Zhao et al. [112]            | Numerical   | Elastically supported     | 150–1000 | 2–12 | A/D, f, C_D, C_L, FS |
| Kim et al. [113]             | Experimental| Elastically supported     | 1.4 × 10^3–3.2 × 10^4 | 0.4–22 | A/D, FS       |

12. Flow over Heated Single Cylinder

Most studies with heat transfer considered were conducted on either fixed cylinders or with forced vibrations, while there are only a few on elastically mounted cylinders. Cheng [114] experimentally analyzed the effect of Reynolds number on heat transfer and flow pattern for \( 0 < Re < 4000 \), and \( 0 \leq A/D \leq 0.628 \). They found that lock-in and turbulence could enhance forced convection, with 34% increase in the heat transfer. Gau et al. [115] studied flow over a vibrating cylinder at different oscillation amplitudes and frequencies. They analyzed the effect of varying \( Re \) and vibration parameters on the heat transfer via Nusselt number. The experiment was conducted for \( 1600 < Re < 4800 \), and \( 0.016 \leq A/D \leq 0.064 \). They found that the enhancement of the heat transfer positively correlates with the excitation frequency. However, at low oscillation amplitudes or higher \( Re \), the heat transfer enhancement due to the vibration is not pronounced. Marquart et al. [111] numerically studied the flow over a cylinder with circular cross-section. They studied the effect of heating/cooling the surface of the cylinder on the aerodynamic forces. Their flow \( Re \) was \( 1 \times 10^4 \) and \( 1 \times 10^5 \). They observed that heating the cylinder increased the drag coefficient at both \( Re \). A single cylinder simultaneously undergoing FIV and heating was studied by Baratchi et al. [116]. They numerically investigated the effect of different natural frequencies on fluid forces, Nusselt number, and vortex shedding. They used the moving overset grids method to account for the cylinder oscillations and solved URANS to get the numerical solution. The study was conducted at \( Re = 200 \). They found that for a range of structural natural frequencies, the vortex shedding frequency is similar to the structural one with a slight deviation. This phenomenon is referred to as the soft lock-in. Similar work was conducted by Izadpanah et al. [36]. They used semi-implcit method for the pressure linked equations (SIMPLE) technique for pressure-velocity coupling, second order upwind scheme to discretize the convection terms in momentum and energy equations, and first order implicit scheme for discretization in time. They analyzed the effect of flow-induced vibrations on the convective heat transfer over a single cylinder. Reduced velocity and damping ratio were varied to study the variation in vortex formation, vibration amplitude, and Nusselt number. The study was conducted at \( Re = 150 \), \( Ur = 3–8 \), and \( \zeta = 0, 0.01, 0.05 \), and 0.1. They found that changes in \( Ur \) and \( \zeta \) can affect...
the heat transfer noticeably (Figure 11). It is seen in the figure that as the damping ratio is increased, the average Nusselt number of the cylinder is decreased. Moreover, for $U_r$ ranges, the Nusselt number is lower for the elastically mounted cylinder than for the fixed cylinder. Figure 12 shows the linear relation between average Nusselt number and $Re$ for $Re = 100–200$.

Figure 11. Variation in vibration amplitude and total average Nusselt number for different damping ratios and reduced velocities. Reprinted with permission from Ref. [56]. Copyright 2018, Elsevier.

Figure 12. Average Nusselt number vs. $Re$ for a stationary cylinder. Reprinted with permission from Ref. [116]. Copyright 2016, ASME International.

To calculate the Nusselt number, there are several relations available in the literature, some of which are represented here:
Knudsen and Katz [117] correlation:

\[ Nu = 0.683Re^{0.466}Pr^{1/3} \]  \hspace{1cm} (19)

Zukauskas [118] correlation:

\[ Nu = 0.51Re^{0.5} \]  \hspace{1cm} (20)

Churchill and Bernstein [119] correlation:

\[ Nu = 0.3 + 0.62Re^{1/2}Pr^{1/3} \left[ 1 + \left( \frac{Re}{282000} \right)^{5/8} \right]^{4/5} \left[ 1 + \left( \frac{0.4}{Pr} \right)^{2/3} \right]^{1/4} ; \text{ for } RePr \geq 0.2 \]  \hspace{1cm} (21)

For a single heated cylinder under FIV, Cheng et al. [114] proposed a relationship between \( Nu \) and \( Re \) with air as the fluid as:

\[ Nu = 0.658Re^{0.475} \]  \hspace{1cm} (22)

Wang and Travniecek [120] experimentally examined the heat transfer between the heated cylinder and the laminar air flow. Their goal was to find the reference temperature for computation of kinematic viscosity needed to understand the convective heat transfer phenomenon. Their work was continued by Baranyi et al. [121] for various temperatures and Reynolds numbers. Nakamura and Igarashi [122] experimentally studied the unsteady heat transfer between the flowing air and heated cylinder. In their findings, the alternating shear layers, that got detached from the cylinder surface, gave rise to the alternating vortices in the wake of the cylinder. Salimipour [123] performed numerical analysis for flow over a cylinder with convective heat transfer. The second order schemes were used to discretize the momentum and energy equations and modified strongly implicit (MSI) procedure (developed by Schneider and Zedan [124]) was used to solve the pressure equation. He attempted to suppress FIV by weakening vortex shedding and decreasing drag. Witte et al. [125] numerically studied the effect of heat transfer for a cylinder at subcritical flow at a Reynolds number of 3900. They used DNS and LES techniques and simulated 2D flow for pulsating laminar flow at \( Re = 0.4, 4.0, \) and 40. Joge et al. [126] performed LES on a heated circular cylinder at \( Re \) of 3900. Large differences in temperature, 25, 100, 200, and 300 °C between the cylinder and the fluid flow, were considered, and the effect on the flow and thermal characteristics in the wake of the cylinder was studied. They observed that with the increase in temperature the flow dynamics vary non-linearly. The effect of temperature was not significant until a temperature difference of 100 °C in their study. The transverse component was found to be dominant over the streamwise components of shear stress fluctuations, which caused enhancement in the local mixing of the fluid and therefore increase in the rate of heat transfer.

Baughn et al. [127] experimentally measured the heat transfer between a hot cylinder and a cold fluid using a constant heat flux at the cylinder wall. They carried out the analysis for single cylinder and for tandem cylinders at two values of Reynolds number in the subcritical range. They highlighted the development of Von Karman vortex street behind the cylinder and noticed that the strength of the vortices is high in the near wake region of the cylinder while it decreases slowly in the streamwise direction. Matsumura and Antonia [128] and Antonia et al. [129] studied experimentally the change in heat transfer and momentum due to the vortices and evaluated various parameters in detail: fluctuations in temperature, velocity relations and the Prandtl number. They indicated that in the far wake region, the heat transfer by vortices is more significant than that by momentum. The heat transfer and flow characteristics for different arrangements of tubes were studied experimentally by Buyruk [130]. His aim was to determine the fluctuations in the pressure coefficient and the Nusselt number. It was concluded that the unsteady heat transfer influenced the flow characteristics near the surface of the cylinder. Kumar
Ali et al. [132] performed forced streamwise oscillations of a heated circular cylinder at frequency ratios \( f_{ex}/f_{st} \) of 0.1, 1 and 2. They performed DNS at \( Re \) of 100. They found that the forcing function interacted with the Strouhal frequency of the system, for all values of forcing frequency. The complex spectrum was observed, where two distinct frequencies appeared corresponding to the sum and the difference between the Strouhal and the forcing function (Figure 13a). They observed that when the forcing frequency was twice the Strouhal frequency, lift coefficient increased drastically representing lock-in. In comparison to the stationary unheated cylinder, heating the cylinder at a temperature 300 K above the incoming flow, lift coefficient decreased by half and natural frequency by 16%, whereas drag showed slight increase. They concluded that heating of cylinder was equivalent to running the flow at low \( Re \). They also noted that the condition of lock-on at a frequency ratio of 2 only holds true for isothermal cylinder and not for the heated cylinder. Figure 13a shows the emergence of two additional frequencies (sum and difference of the forcing and Strouhal frequency) for the forced oscillating cylinder. Figure 13b shows a decrease in the vortex shedding frequency at all the frequency ratios due to the addition of heat to the cylinder wall. Figure 14a shows the behavior of rms of lift coefficient and average drag coefficient for isothermal cylinder under influence of forced vibrations. From the figure, it is observed that increasing the forcing frequency increases both \( C_D \) and \( C_L \) with a huge jump at \( f_{ex}/f_{st} \) of 2 which is due to the lock-on phenomena. The effect of heating the cylinder on the rms of lift coefficient and average drag coefficient is displayed in Figure 14b. The rms of lift coefficient decreases, whereas the average drag coefficient increases with the increase in the temperature difference. The abnormal value at temperature difference of 0 K are due to the lock-on condition. Table 4 summarizes the selected studies conducted for heated circular cylinder.

![Figure 13. Spectral analysis for (a) isothermal cylinder, and (b) heated cylinder with \( \Delta T = 300 \) K at frequency ratios of 0, 0.5, 1 and 2. Reprinted with permission from Ref. [132]. Copyright 2021, ASME International.](image_url)
Figure 13. Spectral analysis for (a) isothermal cylinder, and (b) temperature difference between the cylinder wall and the incoming fluid. Reprinted with permission from Ref. [132]. Copyright 2021, ASME International.

Table 4. Selected studies for flow over heated single circular cylinder showing the type of methodology, cylinder mounting, and Reynolds number ($Re$). The measurements carried out in each study are also presented, where $A/D =$ nondimensional vibration amplitude, $C_L =$ lift coefficient, $C_D =$ drag coefficient, $St =$ Strouhal number, $Nu =$ Nusselt number, $h =$ convective heat transfer coefficient, $FS =$ flow structure.

| Researchers             | Methodology | Mounting            | $Re$    | Measurements                              |
|-------------------------|-------------|---------------------|---------|-------------------------------------------|
| Abdelhamid et al. [5]   | Numerical   | Fixed               | 40–180  | $Nu$, $C_D$, $C_L$, flow field            |
| Alam et al. [32]        | Numerical   | Fixed               | 150     | $Nu$, $C_D$, $C_L$, $FS$, flow field      |
| Izadpanah et al. [56]   | Numerical   | Elastically supported | 150    | $Nu$, $A/D$, $FS$                        |
| Cheng et al. [114]      | Experimental| Forced transverse vibration | 0–4000 | $Nu$, $FS$                               |
| Gau et al. [115]        | Experimental| Forced transverse vibration | 1600–4800 | $Nu$, $FS$                           |
| Baratchi et al. [116]   | Numerical   | Elastically supported | 200   | $Nu$, $C_D$, $St$, $FS$                  |
| Nakamura and Igarashi [122] | Experimental | Fixed             | 3000–15,000 | $Nu$, $FS$                        |
| Kumar and Jayavel [131] | Numerical   | Fixed               | 50–1000 | $Nu$, $C_D$, $St$, $FS$                 |
| Ali et al. [132]        | Numerical   | Forced streamwise vibration | 100  | $C_D$, $C_L$, $St$                        |
| Zafar and Alam [133]    | Numerical   | Fixed               | 150     | $Nu$, $C_D$, $C_L$, $FS$, flow field     |
| Ali et al. [134]        | Numerical   | Forced transverse vibration | 100  | $C_D$, $C_L$, $St$                        |
| Homsi et al. [135]      | Numerical   | Fixed               | 80–200  | $C_D$, $C_L$, $St$, $h$                 |

Coupling the heat transfer with VIV shows how one affects the other. It is noted from the above literature that VIV causes an increase in the heat transfer by increasing local and average $h$ and $Nu$, whereas on the other hand, heating the cylinder dampens the VIV response of the cylinder as it lowers $C_{L,f}$, $A/D$, and $f_{ex}$, while increasing average $C_D$. Heating the cylinder has a similar effect on the cylinder response as does $U_e$ at low Reynolds number for unheated cylinder.

13. Tandem, Side-by-Side, and Staggered Cylinders

For studies with two cylinders, the cylinder arrangement strongly affects the interference between the upstream cylinder and the WIV for the downstream cylinder. Possible arrangements for two cylinders can be: tandem, side-by-side, or staggered. Figure 15 shows these arrangements, where $L$ is the streamwise distance between the two cylinders, $T$ is the transverse distance and $P$ is the pitch for staggered cylinders with $a$ as the incidence angle.
Zdravkovich [136] defined the two types of interference, as shown in Figure 16, depending upon the relative distance between the two cylinders: wake interference, when downstream cylinder is in the wake of the upstream cylinder, and proximity interference, when the two cylinders are near to each other, but the downstream cylinder is not in the wake of the upstream cylinder. In proximity interference, the downstream cylinder is not directly in the wake of the upstream cylinder but the presence of one cylinder affects the behavior of the other cylinder and the same is true for side-by-side cylinders.

For cylinders in tandem, Zdravkovich [136] identified three major flow regimes with the first one being the extended body regime ($L^* < 1.2–1.8$). In the extended body regime, also called single bluff body behavior, the separated shear layers from upstream cylinder fully enclose the downstream cylinder without any reattachment and roll up to form vortices behind the downstream cylinder. The gap between the cylinders usually contains stagnant fluid. The presence of downstream cylinder affects the vortex shedding of upstream, however no vortices are shed from the downstream cylinder. Vortex shedding frequency is higher as compared to single cylinder and wake is narrower [137].
The second regime is the reattachment regime \((1.2 - 1.8 < L^* < 3.4 - 3.8)\), also called shear layer reattachment behavior, the separating shear layers from the upstream cylinder reattach onto the surface of the downstream cylinder and vortices are shed from the downstream cylinder. The presence of upstream cylinder affects the behavior of downstream cylinder as compared to single cylinder. This regime can be sub-divided into two regimes, one where the reattachment occurs on the back side of the downstream cylinder and the other where it occurs on the front face of the downstream cylinder, this depends on the \(L^*\) as shown in Figure 17.

![Figure 17. Flow regimes for two tandem cylinders as defined by Zhou and Yiu [138]. Reprinted with permission from Ref. [24]. Copyright 2010, Elsevier.](image)

The third regime is the co-shedding regime \((L^* > 4)\), where vortices are shed from both the cylinders and the vortices shed from upstream cylinder impinge on the surface of downstream cylinder and affects the behavior of the downstream cylinder. More detail on these regimes can be found in Sumner [24].

Igarashi [139] performed experiments and studied the effect of Reynolds number on the Strouhal number and pressure fluctuations for \(Re\) in the range of \(8.7 \times 10^3 - 5.2 \times 10^4\) for tandem cylinders. He introduced a \(Re\) vs. \(L^*\) map for flow behavior for tandem cylinders, shown in Figure 18, which has great engineering importance and is very helpful in understanding flow physics. It can be observed that the boundaries of flow patterns change at low \(Re\) (less than \(2 \times 10^4\)) but remain relatively fixed at higher \(Re\). Alam and Meyer [140] experimentally studied FIV for two circular cylinders with the gap-spacing ratio \((G/D)\) varied from 0.1 to 5 and the attack angle \(\alpha\) from 0° to 180°. In their analysis, galloping did not occur for single cylinder but two cylinders in tandem experienced galloping due to the shear layer-cylinder and boundary layer-cylinder interaction.

Qin et al. [141] experimentally studied FIV for a circular cylinder in the wake of fixed smaller cylinder using PIV. The ratio of upstream to downstream cylinder diameter was varied from 0.2 to 1. They measured the cylinder frequency and vibration response, shedding frequency, surface pressure and flow fields. They identified six flow regimes. They investigated the effect of the ratio of diameters of cylinders on the vibrations, the flow physics for strong vibrations, and the start of the vibrations.

Kim et al. [142,143] experimentally investigated FIV on tandem cylinders. They studied the effect of varying gap spacing ratio and reduced velocity on the vibration response of the system. The experiments were conducted at \(4365 < Re < 74,200\), reduced velocity \(1.5 < U_r < 26\), and spacing ratios \(0.1 < L^* < 3.2\). Five different vibration regimes were identified. Xu et al. [144] conducted multiple tests in a water channel with two tandem cylinders studying flow-induced vibrations. The experiments were conducted at \(28,600 < Re < 114,000\), spacing ratios \(L^* = 1.57, 2.57, 3.57, 4.57\) and low mass damping parameter \(m^*\zeta = 0.0269\). Maximum vibrational response amplitudes of the upstream and downstream cylinders were found to be \(1.41D\) and \(1.71D\), respectively. Sun et al. [145] studied flow-induced vibrations experimentally in a water channel with two tandem cylinders. The experiments were conducted at \(30,000 < Re < 120,000\), spacing ratios \(L^* = 1.57, 2.0,\) and \(2.57,\)
spring stiffness $400 < k < 1200$, and damping ratio $0.02 < \zeta < 0.26$. Huera-Huarte and Jiménez-González [146] experimentally studied FIV on two tandem cylinders with different diameters. The experiments were conducted at $7000 < Re < 28,000$, reduced velocity $4 < U_r < 14$, spacing ratio $L^* = 1.3$, diameter ratio $d/D = 0.12, 0.2, 0.32, 0.4, 0.5, 1$ where $d$ and $D$ represented the downstream and upstream cylinder diameters, respectively. The study focused on the effect of different diameter ratios on the forces and flow. They found that the vibration amplitude is reduced especially for $d/D > 0.2$ compared to the isolated cylinder case. Another case of tandem cylinders with different diameters is conducted by Zhao et al. [67]. They used Petrov–Galerkin finite element technique (PG-FEM), developed by Zhao et al. [147], to solve for URANS equations. They studied both the streamwise and transverse vibrations of an elastically mounted small cylinder located in the wake of larger fixed cylinder. Spacing ratios and reduced velocities were varied and their effect on response amplitude, response frequency, and wake interactions was studied. The study was conducted at $Re = 200$ (based on larger diameter $D$), diameter ratio $d/D = 0.5$, reduced velocity $2 < U_r < 30$, spacing ratio $L^* = 1.5, 2, 3$. They identified four different regimes depending on the spacing ratio and reduced velocity. The vibration amplitudes trends were greatly differed as the reduced velocity was increased.

![Figure 18](https://example.com/image.png)

**Figure 18.** Vortex shedding regimes for tandem cylinders as a function of non-dimensional spacing ($L^*$) and $Re$. Reprinted with permission from Ref. [15]. Copyright 2016, Elsevier.

Alam et al. [148] investigated the flow over two fixed tandem cylinders at $Re = 6.5 \times 10^4$, and spacing ratio $1 < L^* < 8$. They found that the fluctuating forces on the downstream cylinder strongly depend on the spacing ratio between the cylinders. The values of the fluctuating lift and drag coefficients were found to be 2 and 2.8 times more, respectively, compared to the isolated cylinder at $L^* = 1.4$. Assi et al. [149] studied the FIV interference between two tandem cylinders experimentally. The experiment was conducted at $3000 < Re < 13,000$, and spacing ratios $2 < L^* < 5.6$. The galloping phenomenon was observed for the spacing range of $3 < L^* < 5.6$, which is characterized by increasing vibration response continuously with increasing reduced velocity. Borazjani and Sotiropoulos [150] numerically investigated the FIV for two cylinders in tandem arrangement. The study was conducted at $Re = 200$, and spacing ratios $3 < L^* < 14$. It was established that larger
vibration responses occur for the tandem configuration where the synchronization regime occurs at a wider range. Brika and Laneville [151] experimentally investigated the FIV of a fixed upstream cylinder and elastically mounted downstream cylinder. The study was conducted at $5000 < Re < 27,000$, and spacing ratios $7 < L^* < 25$. This study established that the reduced velocity at which lock-in occurs was higher for the tandem arrangement compared to a single isolated cylinder. In addition, the lock-in happens for a wider range of reduced velocities compared to the isolated cylinder case. Gao et al. [152] experimentally studied the flow structures between an elastically mounted smaller cylinder in the wake of a larger fixed cylinder. The study was conducted at $Re = 7200$, and spacing ratios $L^* = 0.5, 2, 3.5$. It was found that the vibration response and the downstream cylinder’s vortex shedding frequency were both reduced due to the presence of the larger upstream cylinder. However, as the gap spacing ratio was increased, both vibration response and vortex shedding increased for the downstream cylinder. Papaioannou et al. [61] numerically modeled the VIV of two elastically mounted cylinders in tandem. The Arbitrary Lagrangian–Eulerian (ALE) formulation was used to solve for incompressible URANS equations. Their study was conducted at $Re = 160$, and spacing ratios $L^* = 2.5, 3.5, 5$. They found that the range of reduced velocities where synchronization occurs for the upstream cylinder increased as the spacing between the cylinders was decreased. They also found that the upstream cylinder was unaffected by the presence of the downstream cylinder for $L^* = 5$. Kitagawa and Ohta [153] numerically modeled the flow over two fixed tandem cylinders. They used the LES with the Smagorinsky subgrid-scale model. The study was conducted at $Re = 2.2 \times 10^4$, and spacing ratios $2 < L^* < 5$. The critical spacing ratio in this study was found to be $L^* = 3.25$. Below this critical spacing ratio, vortices were shed from the downstream cylinder only. While for ratios greater than the critical, vortices were also shed from the upstream cylinder. This observation on the critical spacing ratio agreed with the results of [154]. The latter numerically modeled the flow over to fixed tandem cylinders and found that the fluid forces on the downstream cylinder changed noticeably at a critical spacing ratio of $L^* > 3.5–4$. They used third-order upwind finite element scheme to solve the Navier–Stokes equations. This change can be attributed to the shed vortices from the upstream cylinder at that spacing ratio. Carmo et al. [155] numerically modeled the flow over two tandem cylinders, where the upstream cylinder was fixed, and the downstream cylinder was free to vibrate. They used spectral/hp element technique to discretize the flow equations, and to solve the dynamics of the cylinder simple Newmark integration method was used. The study was conducted at $100 < Re < 645$, and spacing ratio $L^* = 4$. Their results validated that the existence of wake upstream of the cylinder (WIV) leads to larger oscillation amplitudes compared to the VIV case. Table 5 summarizes some of the work in the literature done on cylinders in tandem, side-by-side, or staggered arrangement to study the flow characteristics and FIV.

Wake induced vibrations are quite different from vortex induced vibrations as presented in the above studies. WIV depends on all the parameters as the VIV and additionally on $L^*$, $T^*$, $P^*$, and $\alpha$. In two cylinders, “wake induced vibration” term can only be used for downstream cylinder as it is affected by the wake of the upstream cylinder, however various studies have shown that the presence of downstream cylinder also affects the behavior of upstream cylinder as compared to the single cylinder.

Table 5. Selected studies for tandem, side-by-side, and staggered cylinders showing the type of methodology, cylinder mounting, Reynolds number ($Re$), and cylinder spacing ($L^*$, $T^*$, or $P^*$). The measurements carried out in each study are also presented, where $A/D = \text{nondimensional vibration amplitude}$, $f = \text{vibration frequency}$, $C_L = \text{lift coefficient}$, $C_D = \text{drag coefficient}$, $C_P = \text{pressure coefficient}$, $St = \text{Strouhal number}$, $FS = \text{flow structure}$, $VF = \text{velocity field}$, and $TF = \text{temperature field}$.

| Researchers         | Methodology  | Mounting          | $Re$          | Cylinder Spacing | Measurements         |
|---------------------|--------------|-------------------|---------------|------------------|----------------------|
| Alam [37]           | Experimental | Fixed             | $1-6.5 \times 10^4$ | $L^* = 1.1-4.5$  | $C_L, C_D, St$       |
| Papaioannou et al. [61] | Numerical    | Elastically supported | 160           | $L^* = 2.5, 3.5, 5$ | $A/D, C_L, C_D, FS$ |


| Researchers                  | Methodology | Mounting  | Re    | Cylinder Spacing | Measurements |
|-----------------------------|-------------|-----------|-------|------------------|--------------|
| Zhao et al. [67]            | Numerical   | Elastically supported | 200   | $L^* = 1.5, 2, 3$ | A/D, f, FS    |
| Alam [73]                   | Numerical   | Elastically supported | 150   | $L^* = 1.5, 3.0$ | A/D, FS      |
| Zhao et al. [112]           | Numerical   | Elastically supported | 4267  | $L^* = 1.2-2.8$  | A/D, FS      |
| Lin et al. [137]            | Experimental | Fixed       | $1 \times 10^4$ | $L^* = 1.15-5.1$ | VF           |
| Zhou and Yiu [138]          | Experimental | Fixed       | 7000  | $L^* = 2$        | VF           |
| Kim et al. [142,143]        | Experimental | Elastically supported | 4365–74,200 | $L^* = 0.1-3.2$  | A/D, FS      |
| Xu et al. [144]             | Experimental | Elastically supported | 28,600–114,000 | $L^* = 1.57, 2.57, 3.57, 4.57$ | A/D, f, C_f |
| Sun et al. [145]            | Experimental | Elastically supported | 30,000–120,000 | $L^* = 1.57, 2, 2.57$ | A/D, f, FS |
| Huarte and Gonzalez [146]   | Experimental | Elastically supported | 7000–28,000 | $L^* = 1.3$ | A/D, f, C_f, C_D, C_f, FS |
| Alam et al. [148]           | Experimental | Fixed       | $6.5 \times 10^4$ | $L^* = 1-8$ | C_f, C_D, C_f, St, FS |
| Assi et al. [149]           | Experimental | Elastically supported | 3000–13,000 | $L^* = 2-5.6$ | A/D, FS      |
| Borazjani and Sotiropoulos [150] | Numerical   | Elastically supported | 200   | $L^* = 3-14$ | A/D, FS      |
| Brika and Laneville [151]   | Experimental | Elastically supported | 5000–27,000 | $L^* = 7-25$ | A/D, FS      |
| Gao et al. [152]            | Experimental | Elastically supported | 7200  | $L^* = 0.5, 2, 3, 5$ | Vorticity, FS |
| Kitagawa and Obta [153]     | Numerical   | Fixed       | $2.2 \times 10^4$ | $L^* = 2-5$ | C_f, C_D, C_f, St, FS |
| Kondo and Matsukuma [154]    | Numerical   | Fixed       | 1000  | $L^* = 2-5$ | C_f, C_D, FS |
| Carmo et al. [155]          | Numerical   | Elastically supported | 100–645 | $L^* = 4$ | A/D, C_f, FS |
| Brika and Laneville [156]   | Experimental | Elastically supported | 5100–27,500 | $L^* = 7-25$ | A/D          |
| Sumner et al. [157]         | Experimental | Fixed       | 1200–3000 | $T^* = 1-3$ | St, VF       |
| Sumner et al. [158]         | Experimental | Fixed       | 850, 1350 | $P^* = 1-5$ | St, VF       |
| Zhou et al. [159]           | Experimental | Fixed       | 1800  | $T^* = 1.5-3$ | VF           |
| Farrant et al. [160]        | Numerical   | Fixed       | 100, 200 | $L^* = 5$, $T^* = 4$ | C_f, C_D, St |
| Meneghini et al. [161]      | Numerical   | Fixed       | 100–200 | $L^* = 1.5-4$ | C_f, C_D, St |
| Ozono et al. [162]          | Experimental | Fixed       | 2500–7500, $3 \times 10^4$ | $L^* = 1-4$ | C_f, St       |
| Zhou et al. [163]           | Experimental | Fixed       | 5800  | $T^* = 1.5-3$ | VF           |
| Wang et al. [164]           | Experimental | Fixed       | 120–1.65 $\times 10^5$ | $T^* = 1.13, 1.7, 3$ | Vorticity |
| Xu et al. [165]             | Experimental | Fixed       | 1000–1.4 $\times 10^4$ | $T^* = 1.2-1.6$ | St, VF       |
| Chen et al. [166]           | Numerical   | Fixed       | 750   | $T^* = 2.7, 4$ | C_f, VF      |
| Jester and Kallinderis [167] | Numerical   | Fixed       | 80, 1000 | $L^* = 0-10$ | C_f, C_D, St |
| Kang [168]                  | Numerical   | Fixed       | 40–160 | $T^* = 1-6$ | C_f, C_D, St |
| Alam et al. [169,170]       | Experimental | Fixed       | $5.5 \times 10^4$ | $T^* = 1.1-6$ | C_f, C_D, C_f, St, FS |
| Akosile and Sumner [171]    | Experimental | Fixed       | 50,000 | $P^* = 1.125-1.25$ | C_f, C_D, C_f, St |
| Sumner and Richards [172]   | Experimental | Fixed       | $3.2 \times 10^4$ | $P^* = 2-2.5$ | C_f, C_D, St |
| Sumner [173]                | Experimental | Fixed       | $5 \times 10^4$ | $P^* = 1.125-1.25$ | C_f, C_D, C_f, St |
| Xu and Zhou [174]           | Experimental | Fixed       | 800   | $L^* = 1-15$ | St           |
| Mittal and Kumar [175]      | Numerical   | Elastically supported | 1000 | $L^* = 5.5$ | A/D, C_f, C_D, FS |
| Brun et al. [176]           | Experimental | Fixed       | 1000–14,300 | $T^* = 1.583$ | St, VF       |
Table 5. Cont.

| Researchers            | Methodology | Mounting     | Re          | Cylinder Spacing | Measurements |
|------------------------|-------------|--------------|-------------|------------------|--------------|
| Alam and Sakamoto [177]| Experimental| Fixed        | $5.5 \times 10^4$ | $P^* = 1.1-6$, $\alpha = 10-75\degree$ | $St$         |
| Alam et al. [178]      | Experimental| Fixed        | $350, 5.5 \times 10^4$ | $P^* = 1.1-6$, $\alpha = 10-75\degree$ | $C_L, C_D, C_P$, FS |
| Sumner et al. [179]    | Experimental| Fixed        | $3.2 \times 10^4$ | $P^* = 1.125-4$, $\alpha = 0.90\degree$ | $C_L, C_D, St$ |
| Sharman et al. [180]   | Numerical   | Fixed        | 100         | $L^* = 2-10$     | $C_L, C_D, C_P, St$ |
| Akbari and Price [181] | Numerical   | Fixed        | 800         | $P^* = 1.1-3.5$, $\alpha = 0-70\degree$ | $St$         |
| Agrawal et al. [182]   | Numerical   | Fixed        | 73          | $T^* = 1.7, 3.5$ | $C_L, C_D, C_P, VF$ |
| Sumner and Schenstead [183]| Experimental| Fixed | $(3.2-7.4) \times 10^4$ | $P^* = 1.125-4$, $\alpha = 0-9\degree$ | $C_L, C_D, C_P$, St |
| Carmo and Meneghini [184]| Numerical | Fixed | 160–320     | $L^* = 1.2-8$   | $C_L, C_D, St$ |
| Deng et al. [185]      | Numerical   | Fixed        | 220–270     | $L^* = 1.5-8$   | $C_D, St$, VF |
| Huang et al. [186]     | Numerical   | Fixed        | 150         | $L^* = 1-1.9$   |VF |
| Papaioannou et al. [187]| Numerical | Fixed        | 100–1000    | $L^* = 1.1-5$   | $C_L, C_D, St$ |
| Kondo and Matsukuma [188]| Numerical | Fixed        | 1000        | $L^* = 3-4$, $T^* = 0-1.1$ | $C_L, C_D, C_P$ |
| Liu et al. [189]       | Numerical   | Fixed        | 40–100      | $T^* = 1.1-3$   | $C_L, C_D, St$ |
| Wang et al. [190]      | Numerical   | Fixed        | 60–200      | $L^* = 1.7$     | $C_L, C_D, St$ |
| Alam and Zhou [191]    | Experimental| Fixed        | 4.7 $\times 10^4$ | $T^* = 1.1-1.2$ | $C_L, C_D$ |
| Alam and Zhou [192]    | Experimental| Fixed        | $5.5 \times 10^4-6.5 \times 10^4$ | $L^* = 1.1-9$ | $C_L$, phase angle |
| Mizushima and Ino [193]| Numerical   | Fixed        | 20–80       | $T^* = 1.3-2$   | $St$, FS     |
| Hu and Zhou [194,195]  | Experimental| Fixed        | 300, 7000   | $P^* = 1.2-4$, $\alpha = 0-90\degree$ | $St$, VF     |
| Carmo et al. [196]     | Numerical   | Fixed        | 200–350     | $T^* = 5-5.8$, $\alpha = 0-31\degree$ | $C_D, St$, VF |
| Lee et al. [197]       | Numerical   | Fixed        | 100         | $L^* = 1-5$, $T^* = 1-5$ | $C_L, C_D, C_P, St$ |
| Zhou et al. [198]      | Experimental| Fixed        | 1500, $2 \times 10^4$ | $P^* = 1.2-4$, $\alpha = 0-90\degree$ | $St$         |
| Kumar et al. [199]     | Experimental| Fixed        | 250, 350, 450 | $T^* = 1.1, 1.7, 3$ | $St$, VE, vorticity |
| Singha and Sinhamahapatra [200]| Numerical | Fixed | 40-150      | $L^* = 1.2-5$   | $C_L, C_D, St$ |
| Assi et al. [201]      | Experimental| Elastically supported | $0.25 \times 10^4-2.5 \times 10^4$ | $L^* = 4, 5, 6, 8, 10, 20$ | $A/D, f$, FS |
| Wang et al. [202]      | Numerical   | Fixed        | 60, 80, 100 | $L^* = 1-12$    | $C_L, C_D$, VF |
| Carmo et al. [203,204]| Numerical   | Fixed        | 160–320, 80–500 | $L^* = 1.2-10$ | $C_L, C_D$ |
| Dehkordi et al. [205]  | Numerical   | Fixed        | 100, 200    | $L^* = 1.5-5.5$ | $C_L, C_D, St$, VF |
| Vakil and Green [206]  | Numerical   | Fixed        | 1-20        | $L^* = 1.1-31$  | $C_L, C_D$ |
| Saryghad et al. [207]  | Numerical   | Fixed        | 100, 200    | $T^* = 1.5-4$   | $C_L, C_D, C_P, VF$ |
| Alam and Meyer [140,208]| Experimental| Fixed        | 55,000      | $P^* = 1.1-6$, $\alpha = 0-90\degree$ | $C_L, C_D, C_P$, St, FS |
| Tsutsui [209]          | Experimental| Fixed        | $3.8 \times 10^4-1.3 \times 10^5$ | $L^* = 1.2-1.3$ | $C_L, C_D, C_P$, FS |
| Koda and Lien [210]    | Numerical   | Fixed        | 160–220     | $L^* = 1.5-8$   | $C_L, C_D, St$ |
| Wong et al. [211]      | Experimental| Fixed        | 1500–2 $\times 10^4$ | $P^* = 1.2-6$, $\alpha = 0-90\degree$ | $St$         |
There exist numerous studies of flow over multiple heated cylinders. Investigating such configurations can lead to a better understanding of the physics for more complicated structural interactions. Mahir and Altaç [215] numerically analyzed the flow over two fixed tandem cylinders with convective heat transfer. Finite volume method was used to solve the unsteady Navier–Stokes and energy equations. The study was conducted at $Re = 100, 200$, and spacing ratios $L^* = 2, 3, 4, 5, 7, \text{ and } 10$. Vorticity, local and mean Nusselt numbers and isotherms are observed to analyze the flow. The mean $Nu$ of the upstream cylinder was found to be close to that of an isolated single cylinder for $L^* \geq 4$. As this happened, the heat transfer rate from the downstream cylinder was found to be 80% of the upstream cylinder. Figure 19 shows how the local Nusselt number varies for upstream and downstream cylinder at $Re$ of 100 and $L^*$ of 3. For upstream cylinder, the maxima of local Nusselt number occurs at front stagnation point where flow impinges on the cylinder surface and for downstream cylinder the maxima occur on the top and bottom surface of the cylinder due to the reattachment of the separated shear layers from the upstream cylinder onto the downstream cylinder.

Dhiman et al. [216] numerically simulated the fluid flow over two fixed tandem cylinders with convective heat transfer. Finite volume method and implicit techniques were used to solve the URANS and energy equations. To account for turbulence, k-kl-$\omega$ model was used. Pressure coefficients, drag forces, lift forces, Strouhal number and Nusselt number were studied in relation to gap spacing ratio. The study was conducted at $11,000 < Re < 41,000$, and spacing ratios $1.2 < L^* < 4$. By observing the flow field and heat transfer they found $L^* = 2.2$ as the critical spacing ratio where the maximum heat transfer rate occurred. They developed a correlation to calculate average $Nu$ as:

$$Nu = 0.182Re^{0.6568}Pr^{0.33}, \text{ for } 1.1 \times 10^4 \leq Re \leq 4.1 \times 10^4$$

(23)
Harimi and Saghafian [217] numerically simulated the flow over tandem cylinders with convective heat transfer. The numerical simulations were performed using finite volume scheme and the overset grid method. The study was conducted at \( Re = 100 \) and 200, and spacing ratios \( L/D = 2, 3, 4, 5, 7, 10 \). They found that the flow periodicity was shattered for \( L^* > 5 \). They presented two correlations to calculate the Nusselt number from upstream and downstream cylinder:

\[
Nu_{\text{upstream}} = 0.1248 Re^{0.48} Pr^{0.363} \left[-0.1055 L^* + \frac{0.0542 + 70L^*}{15.7855L^* 0.8612}\right]
\]

\[
Nu_{\text{downstream}} = 0.073 Re^{0.595} Pr^{0.407} \left[-3.886 L^* + -1.621 + \frac{70L^*}{13.256L^* 0.0953}\right]
\]

Both these relations are valid for \( L^* \leq 10, 100 \leq Re \leq 200, \) and \( 0.7 \leq Pr \leq 7 \).

Table 6. Selected studies for heated cylinders in tandem, side-by-side, and staggered arrangement showing the type of methodology, cylinder mounting, Reynolds number (\( Re \)), and cylinder spacing (\( L^*, T^*, \) or \( P^* \)). The measurements carried out in each study are also presented, where \( f = \) vibration frequency, \( C_{L} = \) lift coefficient, \( C_{D} = \) drag coefficient, \( C_{P} = \) pressure coefficient, \( St = \) Strouhal number, \( Nu = \) Nusselt number, \( h = \) convective heat transfer coefficient, \( FS = \) flow structure, \( VF = \) velocity field, and \( TF = \) temperature field.

| Researchers          | Methodology           | Mounting   | \( Re \)     | Cylinder Spacing | Measurements            |
|----------------------|-----------------------|------------|--------------|------------------|-------------------------|
| Kumar and Jayavel [131] | Numerical simulation | Fixed      | 50–200       | L* = 1.3–6       | \( C_{L}, C_{D}, C_{P}, St, Nu, h \) |
| Zhou and Yiu [138]   | Experimental          | Fixed      | 7000         | L* = 1.5–3       | \( VF, TF \)            |
| Zhou et al. [159]    | Experimental          | Fixed      | 1800         | L* = 1.5–3       | \( VF, TF \)            |
| Zhou et al. [163]    | Experimental          | Fixed      | 5800         | T* = 1.5–3       | \( VF, TF \)            |
| Mahir and Altac [215] | Numerical simulation | Fixed      | 100, 200     | L* = 2, 3, 4, 5, 7, 10 | \( C_{L}, C_{D}, C_{P}, St, Nu, vorticity, isotherms \) |
| Dhiman et al. [216]  | Numerical simulation | Fixed      | (2.1–3.5) \( \times 10^4 \) | L* = 1.2–4       | \( C_{L}, C_{D}, C_{P}, St, Nu, vorticity, pathlines \) |
| Harimi and Saghafian [217] | Numerical simulation | Fixed      | 100, 200     | L* = 2, 3, 4, 5, 7, 10 | \( C_{L}, C_{D}, St, Nu, vorticity, isotherms \) |
| Kostic and Oka [218] | Experimental          | Fixed      | 1.3 \( \times 10^4–4 \times 10^4 \) | L* = 1.6–9       | \( C_{L}, C_{D}, C_{P}, Nu \) |
| Sisodia [219]        | Numerical simulation | Fixed      | 10–45        | L* = 3a = 0–180° | \( C_{L}, C_{D}, St, Nu, vorticity \) |
| Zafar and Alam [220] | Numerical             | fixed      | 200          | L* = 6          | \( C_{L}, C_{D}, St, Nu, vorticity \) |
| Zhang et al. [221]   | Numerical simulation | Fixed      | 100          | G = 1–8         | \( FS, TF, vorticity \) |

To the best of our knowledge very little work has been done for multiple heated cylinders undergoing FIV. Most of the studies for heated cylinders are done on fixed cylinders whereas the FIV analysis is done on unheated cylinders, or single heated cylinder. There is a huge research gap in this area as FIV of multiple cylinders is usually accompanied by the heat transfer as can be seen in the applications of nuclear fuel rods, power plants tubes, and heat exchangers.

### 15. 2-DOF Vibration

All the discussion prior to this section was related to 1 degree of freedom (DOF) motion of the cylinder, i.e., cylinder was free to oscillate only in one direction (transverse or streamwise), however, FIV is not a 1-DOF phenomena and is more complex than initially thought. Therefore, to fully understand it we have to consider it in 2-DOFs, i.e., the cylinder can freely oscillate in transverse (\( y \)) and streamwise (\( x \)) direction simultaneously and this mixed motion has effect on the vortex formation and the pattern of vortices. In a recent
trend, researchers are considering the 2-DOF motion of the cylinder to study FIV. Figure 20 shows the typical arrangement for cylinders under 2DOF motion.

Figure 20. Cylinders free to oscillate in 2DOF. Reprinted with permission from Ref. [63]. Copyright 2017, Elsevier.

For circular cylinder free to vibrate in two axes, the cylinder traces a characteristic pattern of figure “8” [222–224]. Zdravkovich [225] stated that “the two-degree-of-freedom response is the interaction of the streamwise synchronization at $U_r = 2.5$ with the transverse synchronization at $U_r = 5$”. Sarpkaya [226] identified the range of reduced velocity $1.7 < U_r < 2.3$ as the initial instability zone where vibrations occur in the streamwise direction along with symmetric vortex shedding, and the range of $2.8 < U_r < 3.2$ as the second zone of instability with alternating vortex shedding. In the experimental work of Chen and Jendrzejczyk [227], beating mode at $U_r = 4.53$ was observed. It must be noted that these findings only occur in the case of 2-DOF cases and not in 1-DOF work.

Chen et al. [59] carried out numerical simulations and immersed boundary method (IBM) was used to solve for fluid–solid interaction. They studied the 2DOF FIV for two side-by-side cylinders and classified the response of both the cylinders into four branches depending upon $T^*$ as shown in Figure 21. They observed that soft lock-in dominated the initial branch ($2.8 < U_r < 3.7$) whereas lock-in dominated the lower branch ($3.7 < U_r < 7.3$), whereas lock-in was not observed in stationary branch ($0 < U_r < 2.8$) and desynchronized branch ($7.3 < U_r < 10$). They defined soft lock-in as “when the vibration frequency lies between the vortex shedding frequency of the stationary case and the natural frequency of the cylinder”. They observed that the amplitude of vibration abruptly increased in initial branch and had maximum value in the lower branch due to the lock-in. They also observed that with the increase in reduced velocity, the frequency of vibration decreased. The vortex shedding mode in the stationary branch was found to be same as that of the single cylinder. They found that due to the 2DOF motion of the cylinders, at certain conditions the cylinders attained staggered arrangement.
Figure 21. (a) Nondimensional oscillation amplitudes, (b) vibration frequencies, and (c) shifts in the equilibrium position due to VIV of two side-by-side cylinders at $L^* = 2$. SB: stationary branch, IB: initial branch, LB: lower branch, DB: desynchronization branch. Reprinted with permission from Ref. [59]. Copyright 2019, Elsevier.

Chung [63] in his numerical study noted that at all the conditions, the downstream cylinder experienced larger streamwise vibration as compared to the upstream cylinder and stated that “the assumption of 1-DOF (transverse only) motion for 2-DOF VIV is either oversimplified or even incorrect”. They observed that the cylinders repel and attract each other for all non-tandem and for most of the tandem configuration. Assi [228] found that for tandem arrangement, upstream cylinder sheds vortices as a single cylinder, at $L^* > 4$. For downstream cylinder, both streamwise and crossflow amplitudes increased with increasing $U_r$. Prasanth and Mittal [229,230] studied VIV of two circular cylinders in tandem and staggered arrangement at low $Re$. Their numerical simulations involved solution of incompressible URANS equations. They observed that at higher Reynolds number, in tandem arrangement, the downstream cylinder experienced much larger vibration amplitude as compared to the single cylinder case, and in staggered configuration, large streamwise amplitude occurred for downstream cylinder. They found that the downstream
cylinder experienced an orbital motion for most $U_r$ and a figure-eight motion at some $U_r$, in staggered configuration, whereas for tandem arrangement only figure-eight motion was observed. Gao et al. [75] numerically simulated the flow over three tandem cylinders at $Re$ of 150, with varying spacing ratio $L^*$ from 2–5 and reduced velocity $U_r$ from 3–14. The solution of URANS equations was numerically sought using fourth order Runge–Kutta method, and $k-\omega$ turbulence model was used. In their study of FIV, they observed that the middle cylinder and downstream cylinder both had large response amplitudes at all values of $L^*$. They identified the mechanism behind the large amplitudes as the gap flow in proximity regime, and WIV in the co-shedding regime. They found that in comparison with the single cylinder, for the upstream cylinder in tandem arrangement both the streamwise and transverse response amplitudes increased greatly in the lock-in regime.

Wang et al. [65] used ALE scheme to numerically study WIV of downstream cylinder in tandem configuration with stationary upstream cylinder and observed that the dominant wake pattern was 2S for Reynolds number of 50 and 100, while increasing it to 150 and 200, P+S vortex shedding pattern was seen at lower $U_r$. They developed a map for shedding pattern comparing 1DOF with 2DOF as shown in Figure 22. They also highlighted the need for 3D simulation for 2DOF cases.

Tu et al. [58] investigated 2DOF WIV of a circular cylinder downstream of the square cylinder, numerically using semi-implicit characteristics-based split (CBS) finite element algorithm. They observed 2S, 2P and 2T modes as shown in Figure 23. Vortex shedding patterns are shown for different values of the ratio length of square cylinder to diameter of circular cylinder ($d/D$) and reduced velocity ($U_r$). Figure 24 shows the map for vortex shedding modes from their results.

![Figure 22](image-url)
Yang et al. [11] numerically studied 2DOF FIV of a single heated cylinder at \( Re \) of 100. They used coupled implicit solver for pressure-velocity coupling, second order scheme for momentum and energy equations, and fourth-order Runge–Kutta scheme for cylinder motion equations. They observed that the maximum heat transfer occurred at \( U_r = 6 \), and found that the average \( Nu \) increased by 5.73\% when compared to that of stationary cylinder, and by 2.46\% as compared to cylinder moving in transverse direction only (1DOF). They highlighted the importance of studying FIV in 2DOF. Table 7 shows some of the work done on FIV of cylinder(s) considering 2DOF.

**Figure 23.** Different vortex shedding modes. Reprinted with permission from Ref. [58]. Copyright 2017, JVE International.

**Figure 24.** Map of vortex shedding as developed by Tu et al. [58]. □ = 2S, ◇ = 2S-8, Δ = 2P, ○ = 2T, solid and hollow figures denote steady and wake in the gap between cylinders, respectively. Reprinted with permission from Ref. [58]. Copyright 2017, JVE International.
highlighted the importance of studying FIV in 2DOF. Table 7 shows some of the work done on FIV of cylinder(s) considering 2DOF.

Table 7. Flow over circular cylinder with 2DOF motion showing the type of methodology, number of cylinders, cylinder arrangement, Reynolds number (Re), reduced velocity (Ur), cylinder spacing (L*, T*, or P*), angle of incidence (α), mass ratio (m*), and damping ratio (ζ).

| Researchers          | Methodology      | Number of Cylinders | Cylinders Arrangement | Re  | Geometric Parameters | Ul r |
|----------------------|------------------|---------------------|-----------------------|-----|----------------------|------|
| Yang et al. [11]     | Numerical        | 1                   | -                     | 100 | m* = 1.38            | 3–10 |
| Tu et al. [58]       | Numerical        | 2                   | Tandem                | 120 | L* = 10              | m* = 6 | 3–26 |
| Chen et al. [59]     | Numerical        | 2                   | Side-by-side          | 100 | T* = 2.0–5.0, m* = 2 | 0–10 |
| Papaioannou et al. [61]| Numerical    | 2                   | Tandem                | 160 | L* = 2.5, 3.5, 5     | 3–10 |
| Han et al. [62]      | Numerical        | 4                   | Square arrangement    | 80, 160 | L* = 5              | m* = 6 | 3–14 |
| Chung [63]           | Numerical        | 2                   | Side-by-side, tandem, and staggered | 100 | L* = 1.1, 1.3, 1.5, 1.7, 1.9  30°, 60°  α = 0, 30, 60°  m* = 2  T* = 0  ζ = 0 | 2–16 |
| Wang et al. [64]     | Numerical        | 3                   | Equilateral-triangle arrangement | 150 | L* = 4             | m* = 2  | 3–12 |
| Wang et al. [65]     | Numerical        | 2                   | Tandem                | 50–200 | L* = 5.5            | 3–30 |
| Assi [228]           | Experimental (Water channel) | 2     | Tandem and staggered | 2000–25,000 | L* = 4 | T* = 0–3, 1.6  m* = 0.7  0.1  ζ = 0.003 | 1–11 |
| Prasanth et al. [229,230] | Numerical     | 2                   | Tandem and staggered | 100 | L* = 5.5            | m* = 10  | 2–15 |

The above studies have shown that FIV is a very complex phenomena and cannot be fully studied by considering 1DOF only, therefore the need for allowing the cylinder(s) to move in 2DOF. The added degree of freedom affects the behavior of cylinders and also the heat transfer characteristics as presented by the above literature.

16. Rough Cylinders

Cylinders/pipes/tubes when exposed to operational conditions may experience fouling on their surface. Fouling is the buildup of unwanted particles on solid surfaces. It can be external or internal, for the sake of this paper, only external fouling will be discussed. External fouling can be of different types depending upon what types of condition the cylinder is subject to. Corrosion/oxidation of metallic pipes, dust accumulation, plant plankton or algae growth on the riser pipes submerged in the sea are all types of fouling. The resultant effect of fouling is that it alters the external geometry of the cylinder which in turn affects the flow behavior, vortex shedding, and heat transfer as the fouling material is usually non-conductive.

With more advanced experimental and numerical techniques, study can be done in a more realistic way rather than simplifying the problem. Recently, more work is being done to study the effect of surface roughness on the flow behavior. In laboratory,
roughness can be created on the surface of the cylinder to mimic the effect of fouling [231]. Adding roughness on the surface of the cylinder affects the FIV. PTC (passive turbulence control)-to-FIV Map [92,232–235] was used to enhance and suppress FIV of two cylinders in tandem [236,237]. Figure 25 shows how the surface roughness can be added in experimental and numerical studies. For studies related to rough surfaces, $K_s$ is the roughness parameter and technically it defined as the height of the irregular surface.

Figure 25. Creating roughness on the surface of cylinder: (a) netting, (b) dimples. Reprinted with permission from Ref. [238]. Copyright 2015, Elsevier. (c) numerically creating small beads on the surface. Reprinted with permission from Ref. [75]. Copyright 2020, Elsevier.

Chang et al. [239] investigated the influence of $Re$ and surface roughness on VIV of a circular cylinder. They observed that the roughness strip altered the dynamic behavior of the cylinder by suppressing VIV for $U_r < 10$, while induced galloping for $U_r > 10$. Gao et al. [75,240] highlighted the need for 3D study as fouling on the surface of the cylinder is not uniform and varies spanwise across the length of the cylinder. Figure 26 shows the dimensionless amplitudes in in-line ($A_x/D$) and transverse ($A_y/D$) direction along with the normalized vibration frequency ($f_s/f_n$) for smooth and rough cylinder, where $f_s$ is the vibration frequency and $f_n$ is the natural frequency. They observed that the lock-in phenomenon occurred for all surface roughnesses, but the lock-in region was not very sensitive to the variation of the surface roughness. The respective vibration modes which show the cylinder trajectory at each value of reduce velocity is also drawn in Figure 26. This shows that how the vibration behavior of the cylinder varies depending upon the flow conditions. Some of the observed modes are the “figure-eight”, “distorted figure-eight”, “elongated loop”, and the “vertical line”. The characteristic figure-eight pattern appears when the streamwise vibration frequency is twice of the transverse vibration frequency. Whereas the “vertical line” shows that the cylinder vibrates only in transverse direction. They also observed that with increasing surface roughness Strouhal number increased, and average drag coefficient decreased (Figures 26 and 27).

Dierich and Nikrityuk [241] numerically studied the effect of surface roughness on the Nusselt number and found that the Nusselt number decreases rapidly as the degree of roughness increases. Finite volume method was used to discretize the Navier–Stokes and energy equations. IBM technique was used to simulate heat and fluid flow over the rough cylinder. In their study for $Re$ in the range of 10–200, they found that in the dimples the air became stagnant and acted as an insulator which increased the boundary layer thickness thereby decreasing the convective heat transfer and therefore $Nu$. They called the stagnation of air as the “isolation effect”. Table 8 shows selected studies for flow over circular cylinder with specific surface roughness.
Dierich and Nikrityuk [241] numerically studied the effect of surface roughness on the Nusselt number and found that the Nusselt number decreases rapidly as the degree of roughness increases. Finite volume method was used to discretize the Navier–Stokes and energy equations. IBM technique was used to simulate heat and fluid flow over the rough cylinder. In their study for $Re$ in the range of $10–200$, they found that in the dimples

Figure 26. Nondimensional transverse and streamwise amplitude and frequency response for (a) smooth and (b) rough cylinder. Reprinted with permission from Ref. [75]. Copyright 2020, Elsevier.

Figure 27. Strouhal and mean drag coefficient for smooth and rough cylinders at various reduced velocities. Reprinted with permission from Ref. [75]. Copyright 2020, Elsevier.
Table 8. Various studies considering flow over rough cylinder(s) showing the type of methodology, number of cylinders, Reynolds number ($Re$), reduced velocity ($Ur$), cylinder spacing ($L^*$, $T^*$, or $P^*$), spring constant ($k$), mass ratio ($m^*$), damping ratio ($\zeta$), nondimensional gap from the wall ($e/D$), and the nondimensional surface roughness ($K_s/D$). The measurements carried out in each study are also presented, where $A/D = $ nondimensional vibration amplitude, $f_{ex} = $ vibration frequency, $C_L = $ lift coefficient, $C_D = $ drag coefficient, $St = $ Strouhal number, $Nu = $ Nusselt number, $FS = $ flow structure.

| Researchers          | Methodology | Number of Cylinders | $Re$  | Geometric Parameters | $Ur$ | Surface Roughness $K_s/D$ | Measurements                      |
|----------------------|-------------|---------------------|-------|----------------------|------|---------------------------|----------------------------------|
| Gao et al. [75]      | Numerical   | 1                   | 5000  | $m^* = 2.6,$ $\xi = 0.0036$ Gap from wall $(e/D) = 2$ | 1–14 | 0, 0.005, 0.01, 0.02     | $St, A/D, f_{ex}, FS, vorticity$ |
| Sun et al. [145]     | Experimental| 2 (Tandem)          | 30,000–120,000 | $L^* = 1.57, 2, 2.57$ $m^* = 1.34$ $k = 400, 1200$ $\xi = 0.02–0.26$ | 2.92–15.33 | 0.0095                     | $A/D, f_{ex}, vorticity$         |
| Zhou et al. [238]    | Experimental| 1                   | 6000–80,000 | - Fixed cylinder     | 0.0028–0.025 | $C_D, C_L$                   |                                  |
| Gao et al. [240]     | Numerical   | 1                   | 5000  | $m^* = 2.6,$ $\xi = 0.0036$ Gap from wall $(e/D) = 0.8, 2$ | 1–14 | 0.01, 0.02                | $St, A/D, f_{ex}, C_D, C_L, FS, vorticity$ |
| Dierich and Nikrityuk [241] | Numerical | 1                   | 10–200 | - Fixed cylinder     | 0.01–0.5 | $C_D, Nu$                   |                                  |
| Skeide et al. [242]  | Experimental| 1                   | 20,000–160,000 | - Fixed cylinder | 0.1, 0.2, 0.42 | $C_D, vorticity$              |                                  |
| Han et al. [243]     | Numerical   | 1                   | 1500–10,500 | $m^* = 2.6,$ $\xi = 0.0036$ | 2–14 | 0.005, 0.01, 0.02         | $A/D, vorticity$                 |

Fouling affects the flow behavior, FIV response and heat transfer characteristics for cylinder(s). Fouling can be replicated in laboratory or modeled numerically by applying roughness on the surface of the cylinder. In order to study the effect of fouling, more accurate roughness models are needed to precisely mimic it. Using 2D models also oversimplifies the problem as fouling is not constant and can vary spanwise across the length of the cylinder. Usually roughness suppresses FIV, however in certain conditions it might aggravate FIV. More research is required in this field to fully understand the effect of fouling on the heat transfer and FIV response of the cylinder.

17. Conclusions and Future Prospects

Flow-induced vibrations (FIV), which is a sub-field of fluid–solid interaction (FSI), comprises of two types of vibrations: VIV and WIV. Bluff body subjected to flow at certain conditions experiences FIV; however, in this review only circular cylinders are considered as this is the most commonly found shape in practice. Due to its immense industrial and practical importance, FIV has been extensively studied. In this review, the authors attempted to summarize the basics of FIV, the major findings of the past and present research. Generally, the parameters affecting FIV are the Reynolds number, spacing ratio, damping coefficient, mass ratio and reduced velocity. Changing each of these parameters affects the behavior of the cylinder in terms of its Strouhal number, vortex shedding, vibration frequency and response amplitude. This work differs from other review articles available in the literature on FIV in a way that previous works did not consider the FIV of heated or rough cylinders, while most of the reviews considered only one degree of freedom motion of the cylinder. In this work all these aspects are considered along with the flow over single and multiple heated and unheated cylinders. Additionally, the effect of surface roughness and the two-degree of freedom motion is considered.

After extensively reviewing the work on FIV of circular cylinders, it is noted that despite a huge literature available, still FIV phenomena is not fully understood. No work has been done with multiple cylinders heated at different levels of temperature. As this occurs physically in heat exchangers, this study needs to be carried out for more accurate
analysis of the heat exchangers. Only a few researchers have coupled heat transfer with the fluid flow. VIV causes an increase in the heat transfer characteristics by increasing local and average heat transfer coefficient and Nusselt number, whereas heating the cylinder dampens the VIV response of the cylinder as it lowers fluctuating lift coefficient, vibration frequency and amplitude, and increases the drag. Heating the cylinder is analogous to running the flow at low Reynolds numbers for unheated cylinder. There is a huge research gap in studying the FIV coupled with heat transfer although it is commonly experienced in practice, e.g., nuclear fuel rods, power plants tubes, and heat exchangers. The need is to carry out the experiments that mimic the actual cylinder surface- and the flow-conditions. Recent developments in the experimental techniques using wind tunnel with PIV and vibrometer has proved to be quite useful. Care must be given to study the flow over the cylinder(s) as 3D and not simplifying it as 2D. As for the numerical methods, with the emergence of supercomputers, attempt must be made to carryout DNS for the 3D cylinder(s) to fully resolve the flow without modeling it.

For multiple cylinders, WIV depends on the distance between the cylinders, and the inclination angle. WIV response for both the downstream and upstream cylinders is different from the VIV response of a single isolated cylinder even if all the conditions are the identical. Moreover, FIV cannot be accurately studied by restricting the cylinder motion only in one direction. It is a complex phenomenon, and the two-degree of freedom response is different from the one degree of freedom response of the cylinder. Therefore, more work needs to be done considering FIV in transverse and streamwise directions simultaneously. Lastly, as the cylinders are exposed to the external environment, their surface is modified due to the fouling which affects the heat transfer and flow characteristics. The effect of fouling is studied using roughness models and simplified in two-dimensions; however, fouling is a 3D phenomenon. In order to study FIV in a more practical way, the effect of heat and roughness along with two-degree of freedom motion needs to be investigated.

Despite the extensive literature on the FIV and the flow over the cylinder(s), no attempt has been made to provide the general engineering guidelines that can be applied to each application. Attempt should be made to first provide the guidelines for a specific application, such as marine risers or heat exchanger tubes, and then generalizing them so that they can be applied to any engineering application with slight modifications. This will have a great impact on the field as it will ease the work of engineers. Furthermore, more research needs to be done on different shapes of cylinders as circular cross-section is not the only shape for cylinders in practice.

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Nomenclature

| Symbol | Unit | Definition |
|--------|------|------------|
| Re     | -    | Reynolds number |
| St     | -    | Strouhal number |
| Nu     | -    | Nusselt number |
| Ur     | -    | Reduced velocity |
\( f_{ex} \) Hz frequency of vibration of a body
\( f_{st} \) Hz vortex shedding frequency of a body at rest
\( f_{vs} \) Hz vortex shedding frequency of a body in motion
\( D \) m diameter of the cylinder
\( l \) m length of the cylinder
\( U \) m/s freestream velocity
\( \rho \) kg/m\(^3\) density of the fluid
\( \mu \) N s/m\(^2\) dynamic viscosity of the fluid
\( C_{lf} \) - fluctuating lift coefficient
2D - two-dimensional
3D - three-dimensional
\( AR \) - aspect ratio
\( C_L \) - lift coefficient
\( C_D \) - drag coefficient
\( C_p \) - pressure coefficient
\( m \) kg mass of cylinder
\( k \) N/m spring constant
\( k \) m\(^2\)/s\(^2\) turbulent kinetic energy
\( \epsilon \) m\(^2\)/s\(^3\) rate of dissipation of turbulent kinetic energy
\( \kappa \) m\(^2\)/s\(^2\) laminar kinetic energy
\( \omega \) m\(^2\)/s\(^3\) specific dissipation rate
\( c \) N-s/m damping coefficient
\( x \) m streamwise displacement
\( \dot{x} \) m/s streamwise velocity
\( \ddot{x} \) m/s\(^2\) streamwise acceleration
\( F \) N force
\( y \) m transverse displacement
\( \dot{y} \) m/s transverse velocity
\( \ddot{y} \) m/s\(^2\) transverse acceleration
\( F_l \) N lift force
\( \zeta \) - damping ratio
\( f_n \) Hz structural natural frequency
\( w_n \) rad/s circular natural frequency
\( F_n \) - dimensionless structural natural frequency
\( X \) - dimensionless streamwise displacement
\( \dot{X} \) - dimensionless streamwise velocity
\( \ddot{X} \) - dimensionless streamwise acceleration
\( Y \) - dimensionless transverse displacement
\( \dot{Y} \) - dimensionless transverse velocity
\( \ddot{Y} \) - dimensionless transverse acceleration
\( m^* \) - mass ratio
\( A \) m amplitude of oscillation
\( A/D \) - amplitude to diameter ratio
\( t \) s time
\( G \) - gap ratio
\( FP \) - flow pattern
\( FS \) - flow structures
\( VF \) - velocity field
\( TF \) - temperature field
\( H/D \) - dimensionless transverse pitch
\( \text{rms} \) - root mean square
\( h \) W/m\(^2\)K convective heat transfer coefficient
\( L^* \) - dimensionless streamwise center-to-center distance between cylinders
\( T^* \) - dimensionless transverse center-to-center distance between cylinders
\( P \) - pitch for staggered cylinders
\( P^* \) - dimensionless pitch
\( \alpha \) deg incidence angle for staggered cylinders
\( K_s \) m height of irregular surface
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