Towards the SQL: Status of the direct thermal-noise measurements at the ANU

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Abstract. We present the preliminary results for an experiment that aims to perform direct measurements of suspension thermal noise. The experiment is based on a niobium flexure membrane approximately 200 µm thickness that is operated as a stable inverted pendulum. A 0.25 g mirror suspended by this flexure membrane is used as the end mirror of a Fabry-Perot test cavity. This test cavity has a length of 12 mm and a finesse of about 800. It is mounted at the lowest stage of a quadruple cascaded pendulum suspension, enclosed in a high-vacuum envelope. The length of test cavity is stabilized with 1 Hz bandwidth to a Nd:YAG laser, which itself is stabilized with high bandwidth to the length of a suspended Zerodur reference cavity of finesse 6000.

1. Introduction
For more than a decade numerous research groups have attempted to reach the standard quantum limit (SQL) for an interferometric position measurement [1, 2, 3, 4]. Despite all effort, no such measurement has been reported. In order to achieve the required sensitivity a sensor with very low noise must be employed and all other relevant noise must be well suppressed. Laser noise and seismic noise are among the major noise sources that need to be reduced. The latter can be attenuated by suspending all relevant components of the experimental setup from multiple cascaded pendulums. Thermal noise of these suspensions or of the mirrors and their coatings are further impediments to reaching SQL sensitivity. Thermal noise will also set a limit to the sensitivity of future interferometric gravitational-wave detectors, such as Advanced LIGO, in their most sensitive frequency band. Therefore, great effort is put into understanding and consequently lowering the thermal noise floor for these instruments [5, 6, 7].

Understanding the nature of thermal noise is the inevitable first step to reducing it. The origin of thermal noise is described by the fluctuation-dissipation theorem [8]. The basic statement of the fluctuation-dissipation theorem is that a given oscillator only experiences thermal noise if there is a loss channel present. In other words, every dissipation mechanism causes a fluctuating force on the system, resulting in a displacement fluctuation. The power spectral density \( \hat{x}^2(\omega) \) of this displacement is described by

\[
\hat{x}^2(\omega) = \frac{4 k_B T}{\omega^2} \text{Re} \left( \frac{1}{Z(\omega)} \right)
\]  

(1)
where $\omega$ is the Fourier angular frequency, $k_B$ is the Boltzmann constant, $T$ is the temperature, and $Z(\omega)$ is the mechanical impedance of the oscillator. The common models of energy loss can be divided into categories: viscous or velocity dependent damping such as gas damping, and structural damping such as linear thermo-elastic loss. Other sources of loss are the bulk loss of the material due to internal friction (caused by e.g. interstitials or dislocations), or surface loss (caused by friction due to the presence of micro cracks).

The thermal noise power spectral density, $\hat{x}^2_{\text{struct}}(\omega)$, of a given oscillator, when assuming structural damping, is described by

$$\hat{x}^2_{\text{struct}}(\omega) = \frac{4 k_B T \omega_0^2 \phi}{m \omega [(\omega_0^2 - \omega^2)^2 + \omega_0^4 \phi^2]}$$

where $\omega_0$ is the resonance frequency of the oscillator, $\phi$ is the loss angle of the oscillator, and $m$ is the effective mass of the oscillator. Velocity dependent damping, however, leads to a power spectral density given by

$$\hat{x}^2_{\text{vel}}(\omega) = \frac{4 k_B T \gamma}{m^2 (\omega_0^2 - \omega^2)^2 + \omega_0^4 \gamma^2}$$

where $\gamma$ is the damping coefficient. For a detailed derivation of thermal noise spectral densities, see [9].

While these two different damping mechanisms lead to a similar behavior of the oscillator at its resonance frequency, they differ substantially at both lower and higher frequencies. Below the resonance frequency the velocity damping model leads to a flat spectrum of the thermally driven amplitude, while the structural damping model leads to a roll off proportional to $\omega^{-1/2}$. Above the resonance frequency, a roll off of the thermally driven amplitude proportional to $\omega^{-2}$ is expected for a velocity damped oscillator, while $\omega^{-5/2}$ is expected for a structurally damped oscillator. Hence, there are three frequency regions that need to be investigated in order to get a full understanding of the thermal noise of a given oscillator: below the oscillator’s resonance frequency, at its resonance frequency, and above its resonance frequency. While the resonant peak can be observed in many experiments, only a few direct measurements of thermal noise below resonance have been reported [10, 11]. These measurements were done for internal modes of mirrors of different materials. Other experiments aim to obtain a direct measurement of suspension thermal noise (see e.g. [12, 13]). However, no conclusive results of suspension thermal noise nor any measurements of thermal noise above the resonance frequency have been reported to date.

The experiment described in the next section is set up to investigate the thermal-noise driven displacement of a flexure suspension below, at, and above its resonance frequency. The aim of this experiment is to improve our understanding of thermal noise in order to allow us to create a setup that can reach the SQL. The SQL setup will use a very low frequency oscillator to convert quantum fluctuations of the light field into displacement fluctuations. These measurements will require the thermal noise floor of the oscillator to be very low.

2. Experimental setup

The layout of our thermal noise experiment is schematically illustrated in Figure 2: A stabilized 500 mW Nd:YAG NPRO is used as the interrogating laser source. The laser amplitude is stabilized by employing a standard noise eater. The laser frequency is stabilized to the length of a Zerodur reference cavity. This linear reference cavity has a round-trip length of 40 cm and a finesse of $F \simeq 6000$. It is housed inside a high-vacuum envelope at a pressure of about $5 \cdot 10^{-5}$ Pa ($5 \cdot 10^{-7}$ mbar). Thereby, it is decoupled from acoustic noise and the influence of fluctuations of the refractive index of the air is minimized. To decouple the reference cavity from seismic noise, it is suspended via two steel wire loops from a pair of Marval18 cantilever springs. Pairs
of aluminium plates and NdFeB magnets provide eddy-current damping of the suspension in all six degrees of freedom. Figure 1 shows a photograph of the suspended reference cavity.

The control signals for the laser frequency stabilization are derived using the Pound-Drever-Hall RF-sideband scheme. The required RF modulation is provided by a phase modulator, resonant at 76 MHz. The actuators for the frequency stabilization are the laser temperature, which is controlled for the slow actuation up to approximately one Hertz, and the piezo mounted on the laser crystal, which is used for the faster actuation. The overall bandwidth of the stabilization loop is about 60 kHz (see Figure 1).

This stabilized laser is used to interrogate a test cavity, which contains the mirror suspension under investigation. In order to isolate the test cavity from acoustic noise, it is mounted inside a high-vacuum tank. The tank contains a volume of about $5.5 \, \text{m}^3$ and is held below $10^{-4} \, \text{Pa}$ ($10^{-6} \, \text{mbar}$). To isolate the test cavity from seismic noise it is suspended from a four stage cascaded pendulum. The uppermost pendulum stage is formed by a 1.2 m long steel wire that supports a cross shaped aluminium mass of about 3 kg. This first stage supports a pair of Marval18 Euler-buckles providing vertical isolation for the second pendulum stage. The second stage is suspended from the Euler-buckles by a 0.34 m long steel wire. This stage is made of two individual masses with an overall weight of 50 kg. It is designed as a self-damped rocking stage: A strong cross coupling between the pendulum and pitch modes leads to a differential motion of the two individual parts of the stage which are damped using eddy-currents. NdFeB magnets are attached to one part while copper plates are attached to the other part. This arrangement allows eddy-current damping from a seismically isolated platform without the need to suspend a second pendulum. Such isolation systems are described in detail in e.g. [14, 15]. The second stage supports another set of Euler-buckles from which the third pendulum stage is suspended. This penultimate stage is formed by a 0.34 m steel wire and cross shaped aluminium mass of about 9 kg. The test cavity itself is mounted on a vacuum compatible breadboard that forms the last pendulum stage. The breadboard has the dimensions 60 cm by 60 cm by 5 cm and weighs about 35 kg. It is suspended via four 0.25 m long steel wires from the penultimate stage. Two steering mirrors, a quarter-wave plate, and the last lens of the mode-matching telescope are also

![Figure 1. Left: The suspended Zerodur reference cavity. Right: Total gain of the frequency stabilization loop.](image)
Figure 2. The experimental setup. The frequency of the 500 mW Nd:YAG laser is stabilized to a Zerodur reference cavity of finesse $\mathcal{F} = 6000$ with a bandwidth of about 60 kHz. The reference cavity is suspended in its own high-vacuum system. The flexure suspension under investigation is implemented in the test cavity, which is suspended from a quadruple cascaded pendulum system inside the main high-vacuum envelope.

mounted on this breadboard.

The test cavity is composed of two half-inch diameter mirrors. It has a length of 12 mm and a finesse of $\mathcal{F} \simeq 800$. The incoupling mirror is glued to a piezo-electric transducer that allows us to adjust the cavity length in order to hold the cavity on resonance with the incident laser light. According to demands, a bandwidth between 1 Hz and a few kHz can be chosen for the test cavity stabilization loop. The required control signals are derived via the Pound-Drever-Hall RF-sideband scheme. The end mirror of the test cavity is only 1 mm thick and hence weighs only about 0.25 g. It is suspended by means of an inverted pendulum, which is created by gluing the mirror to a niobium flexure membrane of about 200 $\mu$m thickness. Niobium is a low loss material that was also used to build the resonant bar gravitational-wave detector NIOBÉ [16]. The bulk Q of niobium has been inferred by Baker et al. to be $Q \simeq 1.8 \cdot 10^5$ [17]. The inverted pendulum has an effective mass of about 0.5 g leading to a resonance frequency of 302 Hz. Hence, this

Figure 3. The small mirror mounted on the niobium flexure membrane. The actual membrane is located inside the bigger structure, which provides end stops for the mirror motion to prevent it from reaching amplitudes exceeding 0.5 mm.
setup allows us to study thermal effects of a pendulum suspension at relatively high frequencies. At these frequencies the measurements benefit strongly from the seismic isolation and the roll off of other noise sources, such as electronic noise. Since the incoupling mirror of the test cavity is attached to a sturdy mount of about 600 g, its displacement fluctuations can safely be neglected when compared with the fluctuations of the flexure mounted mirror. Figure 3 shows a photograph of the mirror mounted to flexure suspension.

3. Measurements and Preliminary Results

The test cavity was locked to the laser frequency with a bandwidth of less than 1 Hz for the measurement of the flexure-suspension displacement spectrum presented here. Hence, the calibrated Pound-Drever-Hall error signal provides a direct measurement of the displacement for the frequencies shown in Figure 4. The calibration of the error signal was carried out using the modulation sidebands at 76 MHz as a frequency reference to measure the cavity linewidth.

During the test cavity noise measurement, the laser frequency was stabilized to the reference cavity length. The residual frequency noise, due to the finite loop gain and bandwidth, was derived using the error signal of the frequency stabilization loop. This error signal was then corrected for the loop gain to obtain the displacement equivalent noise for the test cavity. With the test cavity held off resonance, the beam reflected off the test cavity was used to derive an indication of the noise of the test cavity readout system, including photodiode and related electronics. Hence, this trace includes the residual laser amplitude noise as well.

Figure 4, trace (a), shows the amplitude spectral density of the length equivalent noise of the test cavity. Features of the measured curve include the resonant peak of the flexure suspension at 302 Hz and the internal resonances of the suspension support structure at 62 Hz. Internal resonances of the suspended breadboard and internal modes of the Euler buckles cause peaks

![Figure 4](image-url)

**Figure 4.** Displacement equivalent amplitude spectral densities: (a) the noise of the test cavity, (b) detection noise, and (c) inferred residual laser frequency noise.
between 430 Hz and 650 Hz. The origin of the two broad peaks between one and two kilohertz is not yet identified. Below 60 Hz the measurement is limited by readout noise, trace (b), while it is dominated by laser frequency noise above 7 kHz, trace (c). The peaks at 3 kHz and 8.2 kHz are caused by higher order modes of the flexure suspension, the twist mode and first higher order flexing mode, respectively.

Optical non-linearities, such as the optical spring [18], broaden the resonance peak even when operating the test cavity with moderate input light power of a few milliwatts. These effects prevent us from determining the FWHM of the peak.

4. Conclusion and Future work
We have set up an experiment to measure thermal displacement noise of a small suspension structure. Preliminary results using a niobium flexure membrane show that we may be measuring thermal noise in a broad band (approximately one and a half decades) about resonance. Without an independent measurement of the flexure suspension’s quality factor, however, we are unable to provide a comparison with the models, nor determine which loss mechanism dominates.

Future work will include ring-down time measurements of the flexure suspension to determine its $Q$. In order to improve the measurement noise floor we will stiffen the suspension support structure to shift its resonance to higher frequencies such that the experiment benefits more strongly from the existing isolation. Furthermore, we will implement an extra suspension stage that will be mounted on the breadboard. This extra suspension stage will include a pair of cantilever springs for additional vertical isolation. This setup will decouple the test cavity from breadboard resonances and internal modes of the Euler springs. In a further experiment, a niobium flexure suspension of different frequency and $Q$ will be used to verify whether the measured displacement spectrum of the test cavity is thermal-noise limited.

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