Comparison of optical spectral devices in the framework of system approach

V Kazakov, O Moskaletz, M Vaganov
St. Petersburg State University of Aerospace Instrumentation, St. Petersburg 190000, Russia

Abstract. In the framework of solving the problem of comparison and evaluation problem of the spectral devices performance and quality two methods of optical spectra measurement are considered: by using diffraction grating spectral device and multichannel optical spectrometer. Comparison has been performed based on the matrix representation of the measuring results of the optical radiation energy spectrum by each device. Complex and power spectrum spread functions are obtained. Parameters of spectral devices which are defined its spectral resolution are established.

1. Introduction
The problem of comparing and evaluating the performance and quality of spectral devices, formulated in [1], has a multicriterial character. Development of existing traditional methods and the emerging new methods and technical devices of spectroscopy, for example [2], stimulates its further development.

Two types of spectral devices are compared, which has different principles of working: a diffraction spectral device based on a transmission grating and a multichannel optical spectrometer [2], which carries out spectral decomposition based on the resonance phenomenon, i.e. the spectral decomposition is implemented by the principle of the narrow-band optical filtration in n parallel channels.

Comparison of devices of different classes is possible if there is a common description of the spectra and the unity approach to the estimation of its main characteristics. This problem is solved on the basis of the developed system approach [3] to the description of the working of the above mentioned spectral devices, which makes it possible to establish the input-output connection of the spectral device, which is the most important problem in the theory of spectral measurements [4].

The system approach is based on the methods of the signals theory and the linear systems theory where the connection between an input and output of a device is given as a linear integral operator, the kernel of which is spread function, which is exhaustive characteristic of linear system. In the case of an optical spectral device the spread function is as a device's reaction to monochromatic radiation. It allows determining its most important characteristic – its spectral resolution, as well as the errors in spectral measurements. The result of this approach is the matrix representation of the spread functions of spectral devices, which is the basis for their comparison and evaluation of their effectiveness and quality.

2. Diffraction grating spectral device
The optical scheme of the diffraction grating spectral device is shown in Figure 1.
Forming optics

Lens
Grating

φ1
φ2
CCD

Processing unit
Optical coherent Fourier processor

Figure 1. The optical scheme of the diffraction grating spectral device.

In contrast to the well-known methodology for describing the action of spectral devices, for example [5], in this work the obtaining of the energy spectrum spread function of a diffraction grating spectral device is based on the principles of wave optics, the fundamental positions of the theory of linear systems [6] and signal theory [7,8] and methods of radiooptics [9] and bases on a consistent consideration of the passage of optical radiation through the whole spectral device from its input aperture to the result of photodetection.

The diffraction grating performs spatial modulation of the incident wave in accordance with its transparency function \( T(\xi) \), represented in the form of an expansion in the Fourier series at its aperture \( L \) [10]:

\[
T(\xi) = \sum_{n=-\infty}^{\infty} \frac{\sin(n\Omega_g \frac{\tau_g}{2})}{n\Omega_g} \cdot \sin[(N + \frac{1}{2})n\Omega_g T_g] \frac{\exp(in\Omega_g \xi)}{L \cdot \sin\left(\frac{n\Omega_g T_g}{2}\right)},
\]

where \( L \) – grating aperture, \( \Omega_g = 2\pi / T_g \), \( T_g \) – diffraction grating period, \( N \) – the number of transparency element in the diffraction grating.

Further transformation of the field by the optical system of the device can be described by using optical coherent Fourier processor and thus obtain an input-output connection of the device for the complex apparatus spectrum in +1 diffraction order:

\[
S_{\omega}(\omega, t) = M \int_{\Delta\Omega} S(\omega') \exp(i\omega't) \frac{\sin[(\omega(x) - \omega') \frac{T_x}{2}]}{(\omega(x) - \omega') \frac{T_x}{2}} d\omega',
\]

where \( \Delta\Omega \) – analyzed frequency band, \( S(\omega') \) – complex spectrum of analyzed radiation, \( M \) – proportional coefficient, \( T_x = Lx / 2c_0F \), \( c_0 \) – velocity of light.

The relationship between the spatial coordinate \( x \) and the spectral frequency \( \omega \) is given by the expression:

\[
\omega(x) = 2\pi c_0 F / T_g x,
\]

where \( F \) – focal length of the lens.
The processing of the complex apparatus spectrum is carried out using a linear CCD, where each element performs narrow-band filtering, determined by the length of the element along the axis of spatial frequencies $n$, equal to:

$$
\Delta n = \frac{(2\pi c) F T_e x_n^2}{2 \Delta x_n},
$$

or

$$
\Delta x_n = \frac{T_e x_n^2 (\Delta n_x (x_n))}{4 \pi c F}.
$$

According to the general theory of photodetection, the equivalent photodetection scheme by one element of the linear CCD can be described by the structural scheme, which is shown in Figure 2.

![Figure 2. The equivalent scheme of the photodetector](Image)

The photocurrent $i_f$ is given by:

$$
i_f = \gamma q_e \frac{p}{h \omega'},
$$

here $\gamma$ denotes the quantum efficiency; $q_e$ is the charge of the electron; $h$ is Planck's constant; $p$ corresponds to the power of an optical radiation; $\omega'$ is the angular frequency of an optical radiation fallen on the photodetector.

The power of an optical radiation $p$ fallen on the photo-detector is defined as

$$
p = \int \int p \Delta s,
$$

where $A$ is area of the sensitive surface of the photodetector; $\Delta s = n \Delta s$, $n$ is a single normal to the sensitive surface of the photodetector; $P$ is Poynting vector which is given by,

$$
P = E \times H = |E \times H| s = \sqrt{\varepsilon / \mu} |E| s = \sqrt{\mu / \varepsilon} |H| s,
$$

here $E$ denotes the electric field vector; $H$ is magnetic field vector, $s$ means a unit vector; $\varepsilon$ is the permittivity of the material; $\mu$ is the permeability of the material.

Substitution instead of the values of the vector $E$ of the complex apparatus spectrum, as well as the spatial and temporal integration operations, according to the algorithm shown in Fig. 2, allows us to represent the result of the photodetection in the form:

$$
\tilde{I}_b = B \int_{\omega_b - \Delta \omega_b}^{\omega_b + \Delta \omega_b} \sin^2 \left[ \frac{(\omega(x) - \omega') T_n}{2} \right] G(\omega') d\omega' = B \int_{\omega_b - \Delta \omega_b}^{\omega_b + \Delta \omega_b} \sin^2 (\cdot) \cdot G(\omega') d\omega'.
$$
The obtained expression for the photocurrent in each element of the linear CCD allows us to write a mathematical expression that describes the input-output connection of the spectral device in the matrix form for the energy spectrum when the spectrum is detected on all elements of the linear CCD:

\[
\| G(\omega') \| = B \int_{\omega_0 - \Delta \omega}^{\omega_0 + \Delta \omega} \text{diag} \{ A_n(\omega,\omega') \} \cdot \| G(\omega') \| d\omega',
\]

where \( G(\omega') \) – the energy spectrum of analyzed signal; \( \text{diag} \{ A_n(\omega,\omega') \} \) – energy spectrum spread function of a spectral device of the optical range in a matrix form; \( A_n(\omega,\omega') = \text{sinc}^2(\cdot) \).

Thus, the results of the reading of the spectrometric information are given in the form of the reference values of the energy spectrum averaged over the size of the sensitive element of the linear CCD and the integration time \( T_R \).

3. Multichannel optical spectrometer

The optical scheme of the multichannel optical spectrometer is shown in Figure 3.

The operation of the multichannel optical spectrometer with transmitting analyzed optical signals by the optical fiber is described as follows: A forming optics transmits optical radiation from surroundings to the common input of the fiber-optical bundle which is situated in its focal length. The fiber-optical bundle is used for transmitting radiation on at the given distance from source of an optical radiation. The optical radiation passed through the fiber-optical bundle is transmitted to each channel of spectrometer for the further spectral decomposition. Each channel contains the narrow-band optical filter, which has been set on the certain wavelength. Receiving of spectrometric information in a multichannel optical spectrometer is carried out by a photodetector set in each channel. After spectral decomposition and photodetecting analyzed signal is processed by the signal processing unit. The received spectroscopic information about analyzed optical radiation is displayed on the recorder. The results of receiving the spectrometric information in a multichannel optical spectrometer are presented in matrix form. These are the sampling values of the energy spectrum averaged in the frequency band of each narrowband interference filter.
Taking into account the specifics of the considered device the process of receiving of the energy spectrum estimation of the optical signal in one channel can be represented by a functional scheme shown in figure 4 [11].

The complex spectrum of analyzed signal is proportional to the intensity of the electrical components of the optical radiation, so the mathematical form of the sequence of operations presented in the figure 4 has the form:

\[
G_k(\omega) = \frac{T_k}{2} \int \frac{t_k(t)}{2} \left| \mathbf{S}_{\omega k}(\omega,t) \right|^2 dt,
\]

where \( t_k(t) \) is current of the photodetector for the channel \( k \); \( P_k(\omega_k) \) is coefficient of the spectral sensitivity of the photodetector; \( T_k \) is integration time; \( t_0 = -\frac{T_k}{2} \).

**Figure 4.** Functional scheme of receiving of the energy spectrum estimation

In paper [11] it is shown that the expression describing receiving of an energy spectrum of an optical signal in the \( k \) channel of the spectrometer is defined as:

\[
G_k(\omega) = \frac{2}{\Delta \omega_k} \int W_{\omega k}(\omega_k,\omega') \cdot G(\omega') d\omega'.
\]

Here \( G(\omega') \) represents the energy spectrum of analyzed signal, \( G(\omega') = |S(\omega')|^2 \); \( 2\Delta \omega_k \) designates the bandwidth of optical filter, \( W_{\omega k}(\omega_k,\omega') \) denotes a "partial" the energy spread function of channel \( k \) of the multichannel optical spectrometer.

Then the receiving of an energy spectrum of an optical signal by a multichannel spectrometer can be represented in matrix form:

\[
\left\| G_{\omega k}(\omega) \right\| = \int_{-\infty}^{\infty} \text{diag} \left\{ W_{\omega k}(\omega_k,\omega') \right\} \cdot \left\| G(\omega') \right\| d\omega',
\]

where \( \text{diag} \left\{ W_{\omega k}(\omega_k,\omega') \right\} \) is energy spectrum spread function of a multichannel optical spectrometer in a matrix form.

Expression (13) describes the energy spectrum in the form of its reference values and establishes a connection between the real energy distribution by the frequencies (energy spectrum) and the spectral energy distribution by the frequencies obtained experimentally using a multichannel spectrometer.

4. Conclusion

The comparison of the spectral instruments of a different class: the diffraction grating spectral device and a multichannel optical spectrometer based on their presentation in the form of multichannel linear systems has been considered in this paper.
Despite the different principle of operation, the result of spectrum measurement by each device was obtained in a unified matrix form. Thus, the requirement to generality of the spectra description and the unity of approach to the estimation of the main characteristics of the spectral devices was accomplished. Received the energy spread functions of spectral devices in matrix form allow to compare spectral devices, in particular, on criterion of their resolution capability.

The frequency filtration in the diffraction grating spectral device is performed by each element of the CCD line. Thus, the resolution is determined by the parameters of the device: the period of the grating, the focal length of the lens and the geometric dimensions of the sensitive element of the CCD line.

The resolution of a multichannel optical spectrometer is determined only by the bandwidth of the optical filter set in each channel.

It should also be noted that the matrix representation of the measurement results in the form sampling values of the energy spectrum by each device is the basis for the application of the sampling theorem in the frequency domain and allows establishing a continuous function describing the energy spectrum.

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