Anomalous effects due to the inertial anti-gravitational potential of the sun

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Abstract

It is introduced inertial anti-gravitational potential in to the theory of gravity to stop gravitational collapse at the nuclear density and thus prevent singularities. It is considered effective gravity which includes Newtonian potential and inertial anti-gravitational potential. It is investigated footprints of the effective gravity in the solar system. The inertial anti-gravitational potential of the sun allows to explain the anomalous acceleration of Pioneer 10 and 11, the anomalous increase in the lunar semi-major axis, the residuals of the seasonal variation of the proper angular velocity of the earth, the anomalous increase of the Astronomical Unit, the anomalous shift of the perihelion of mercury.

1 Basic idea

The theory of gravity [1] faces the problem of singularities. In particular, singularities may arise as a result of the gravitational collapse of massive stars [2]. A body contracted to the nucleus density resembles the neutron star. There is a maximum mass for the neutron star of order of the mass of the sun, \( m_{\text{max}} \sim m_\odot \). For the neutron star with the mass less than the maximum mass, the pressure due to the degenerated neutron Fermi gas balances the gravity of the star. For the neutron star with the mass more than the maximum mass, the gravity of the star overcomes the pressure due to the degenerated neutron Fermi gas, and the star goes to the singularity.

As known the nucleus of atom consists of elementary particles, protons and neutrons. Those have the intrinsic angular momentum, spin

\[
\frac{\hbar}{2} = m_p v r_p
\]  

where \( \hbar \) is the Planck constant, \( m_p \) is the mass of proton, \( r_p \) is the radius of proton. The spin of proton (neutron) yields the centrifugal acceleration

\[
w_p = \frac{v^2}{r_p} = \frac{\hbar^2}{4m_p^2 r_p^3}.
\]

This centrifugal acceleration opposes acceleration due to gravity. Consider the neutron star and compare the gravity of the star and the centrifugal acceleration due to the spin of proton (neutron). In view of eq. (2) the centrifugal acceleration due to the spin of proton is of order \( w_p \sim 10^{31} \text{ cm/s}^2 \). Take the maximum possible neutron star of the mass of the visible universe. While adopting the size of the visible universe \( r_U \sim 10^{28} \text{ cm} \), the mass of the visible universe is \( m_U = c^2 r_U / G \sim 10^{56} \text{ g} \). While adopting the density of neutron star \( \rho_{\text{NS}} \sim 10^{14} \text{ g/cm}^3 \), the radius of neutron star for the mass of the visible universe is of order \( r_{\text{UNS}} = (3 m_U / 4 \pi \rho_{\text{NS}})^{1/3} \sim 10^{13} \text{ cm} \). The acceleration due to gravity of the neutron
star of the mass of the visible universe is of order \( w_{\text{UNS}} = \frac{Gm}{r_{\text{UNS}}^2} \sim 10^{21} \text{ cm/s}^2 \). Thus the centrifugal acceleration due to the spin of proton exceeds the gravity of neutron star of any possible mass in the visible universe. The centrifugal acceleration due to the spin of proton prevents collapse of a neutron star. Then one can consider proton incompressible in the gravitational interaction.

The results of experiments show that in electromagnetic, weak, strong interactions elementary particles do not change their size and mass with the momentum conservation law holding true. This means that elementary particles are incompressible in electromagnetic, weak, strong interactions. Let two protons collide with the velocity of light \( c \). Estimate acceleration experienced by the proton under proton colliding. The radius of strong interaction is of order of the size of proton, \( r_p \sim 10^{-13} \text{ cm} \). Then the acceleration experienced by the proton is \( w = c^2/r_p \sim 10^{34} \text{ cm/s}^2 \). The acceleration due to gravity of the neutron star of the mass of the visible universe \( w_{\text{UNS}} \sim 10^{21} \text{ cm/s}^2 \) is much less than that in the strong interaction under proton colliding.

So elementary particles are incompressible hence the neutron star consisting of elementary particles is incompressible. Then the radius of neutron star is a limiting one under contraction of a body with the nuclear density being a limiting one. All the atomic nuclei of a body specifies incompressible volume of the radius of neutron star for a body. Incompressibility of the volume of the radius of neutron star means that the total force at the radius of neutron star must be equal to zero. Then there must be an inertial force to balance the force of gravity at the radius of neutron star. Suppose that a body produces inertial anti-gravitational potential

\[ \Psi = \frac{4\pi}{3} G \rho r_{\text{NS}}^2 \]  

where \( G \) is the Newtonian constant, \( \rho \) is the density of the body, \( r_{\text{NS}} \) is the radius of neutron star for the body. We arrive at the effective gravity which includes Newtonian potential and inertial anti-gravitational potential

\[ \Phi = -\frac{Gm}{r} + \Psi. \]  

For the body contracted to the nuclear density the inertial anti-gravitational potential balances the Newtonian potential thus preventing singularities under the gravitational collapse of the body.

Introduction of the inertial anti-gravitational potential means that we consider the gravitating body in the non-inertial frame. Let us start with the Lagrangian of a particle of the unit mass moving in the field of a gravitating body. In the background frame the Lagrangian is given by

\[ L = \frac{v^2}{2} + \frac{Gm}{r} \]  

One can write down inertial anti-gravitational potential as

\[ \Psi = \frac{v_{\Psi}^2}{2}. \]  

This means the gravitating body brings in the frame with an effective velocity \( v_{\Psi} \) directed from the centre of the body along the radial coordinate. While taking the gravitating body
as a rest frame then a particle is in the frame with an effective velocity \( v_\Psi \) directed towards the body. The velocity of a particle with respect to the background frame is

\[
v = v' - v_\Psi.
\]

Substituting eq. (7) into eq. (5) and following the standard procedure [3] we obtain the Lagrangian in the frame with an effective velocity \( v_\Psi \)

\[
L' = \frac{v'^2}{2} + \frac{G m}{r'} + \frac{d v_\Psi}{d t} r'
\]

where \( \frac{d v_\Psi}{d t} \) is the inertial acceleration directed backwards the body. In view of eq. (4) the inertial acceleration is given by

\[
w_{\text{in}} \equiv \frac{d v_\Psi}{d t} = \frac{\Psi}{r'}.
\]

From the Lagrangian eq. (8) one can derive the equation of motion

\[
w = -\frac{G m}{r'^2} + \frac{\Psi}{r'}
\]

where the first term is the Newtonian acceleration due to gravity, the second term is the inertial acceleration due to the inertial anti-gravitational potential. Thus introduction of the inertial anti-gravitational potential does not modify Newtonian gravity.

Let the sun along with the Newtonian potential produce the inertial anti-gravitational potential. Estimate the inertial anti-gravitational potential of the sun. Calculate the density of neutron star as \( \rho_{NS} = \frac{3 m_p}{4 \pi r_0^3} = 3.0 \times 10^{14} \text{ g/cm}^3 \) where \( m_p \) is the mass of proton, \( r_0 = 1.1 \times 10^{-13} \text{ cm} \) is the radius of the nucleus. Then the radius of neutron star for the sun is equal to \( r_{NS} = (3 m_\odot / 4 \pi \rho_{NS})^{1/3} = 1.2 \times 10^{6} \text{ cm} \). In view of eq. (3), the inertial anti-gravitational potential of the sun is equal to \( \Psi_\odot = 5.3 \times 10^{5}\text{ cm}^2/\text{s}^2 \).

## 2 Anomalous acceleration of Pioneer 10 and 11

The inertial acceleration due to the inertial anti-gravitational potential gives contribution into the first order relativistic effect. The inertial acceleration of the earth due to the inertial anti-gravitational potential of the sun is given by \( w_E = \frac{\Psi_\odot}{r_{SE}} \) where \( r_{SE} \) is the distance between the earth and sun. This acceleration can be seen in ranging to distant spacecraft as a blue shift of the frequency of the electromagnetic field

\[
\frac{\Delta \omega}{\omega} \approx \frac{w_E t}{c} = \frac{\Psi_\odot t}{c r_{SE}}.
\]

In ranging to distant spacecraft, the acceleration of the earth outward the sun looks like the acceleration of the spacecraft inward the sun, \( w_{sc} = 2 w_E \). The factor 2 takes into account that the acceleration of the earth gives contribution into the shift of the reference frequency during the time of two-leg light travel while the acceleration of the spacecraft gives contribution into the shift of the observed re-transmitted frequency during the time of one-leg light travel.

Analysis of radio Doppler and ranging data from distant spacecraft [4] indicated that an anomalous inward acceleration is acting on Pioneer 10 and 11, \( w_p = (8.74 \pm 1.25) \times 10^{-8} \text{ cm/s}^2 \). This may be interpreted as the outward acceleration of the earth. In view of eq. (11), determine the inertial anti-gravitational potential of the sun from the anomalous acceleration of Pioneer 10 and 11. Then we obtain \( \Psi_\odot = 2 w_p r_{SE} = 6.6 \times 10^5 \text{ cm}^2/\text{s}^2 \).
3 Polarization of the moon’s orbit

The potential $\Psi_\odot$ gives contribution into the polarization of the moon’s orbit in the direction of the sun. While adopting the sum, $G m_\odot/r_{SE} + \Psi_\odot$, as an effective gravitational potential of the sun at the radius $r_{SE}$, we reveal an additional inertial potential along the moon’s orbit

$$V \approx \frac{\Psi_\odot r_{EM}}{r_{SE}} \sin \Omega_{MET}$$

where $r_{EM}$ is the distance between the earth and moon, $\Omega_{ME}$ is the angular velocity of the moon. An observer at the earth treats this as an additional outward acceleration of the moon

$$w_M \approx \frac{\Psi_\odot}{r_{SE}} | \sin \Omega_{MET}|$$

where $| \sin |$ is the modulus of the function. The additional outward acceleration of the moon average for the period of revolution of the moon is given by

$$w_{av}^M \approx \frac{\Psi_\odot r_{EM}}{\sqrt{2} r_{SE}}.$$

This can be seen in lunar laser ranging as a velocity

$$v_{av}^M = \frac{w_{av}^M r_{EM}}{c} = \frac{\Psi_\odot r_{EM}}{\sqrt{2} c r_{SE}}.$$ 

There is a difference in the rate of the lunar semi-major axis increases obtained from telescopic observations and from lunar laser ranging, $\dot{a}_{LLR} - \dot{a}_{tel} = 1.29 \text{ cm/yr} = 4.1 \times 10^{-8} \text{ cm/s}$ [5]. It is used telescopic observations of the secular deceleration of the proper angular velocity of the earth from which the rate of the lunar semi-major axis increase is determined. In view of eq. (15), determine the inertial anti-gravitational potential of the sun from this value. Then we obtain $\Psi_\odot = 6.9 \times 10^5 \text{ cm}^2/\text{s}^2$.

4 Seasonal variation of the angular velocity of the earth

Under the variation of the earth-sun distance the inertial anti-gravitational potential of the sun yields the variation of the angular velocity of the earth around the sun

$$\frac{\Delta \Omega_{ES}}{\Omega_{ES}} = -\frac{\Psi_\odot r_{SE}}{G m_\odot} e \cos \varphi$$

where $e$ is the eccentricity, $\varphi$ is the angle (longitude) of the earth’s orbit around the sun. This looks like the variation of the proper angular velocity of the earth with respect to remote stars

$$\frac{\Delta \Omega_E}{\Omega_E} \approx -\frac{\Psi_\odot r_{SE}}{G m_\odot} e \cos 23.5^0 \cos \varphi$$

where $\cos 23.5^0$ accounts for the tilt of the earth’s axis of rotation to the perpendicular to the earth-sun plain. Observation [6] gives the residuals of the seasonal variation of the proper angular velocity of the earth, $\Delta \Omega_E/\Omega_E = -(9 \pm 2) \times 10^{-10} \cos \varphi$. In view of eq. (17), determine the inertial anti-gravitational potential of the sun from this value. Then we obtain $\Psi_\odot = 5.2 \times 10^6 \text{ cm}^2/\text{s}^2$. 

4
5 Secular increase of Astronomical Unit

Under the variation of the earth-sun distance the inertial anti-gravitational potential of the sun is seen as an anomalous potential of the earth

$$V_E = \Psi_\odot e |\cos \varphi|.$$  \hfill (18)

This gives contribution into a second order relativistic shift of the reference frequency in the planet ranging

$$\frac{\Delta \omega}{\omega} = \frac{\Psi_\odot e}{\sqrt{2} c^2}$$  \hfill (19)

where the factor $1/\sqrt{2}$ accounts for averaging for the period. While interpreting the shift as a first order effect this mimics the velocity of the earth outward the sun

$$v_E = \frac{\Psi_\odot e}{\sqrt{2} c}.$$  \hfill (20)

Krasinsky and Brumberg reported [7] the anomalous positive secular trend in the Astronomical Unit $(d/dt)AU = 15 \pm 4$ m/cy obtained from analysis of the data on ranging to the major planets (mars, venus, mercury). Pitjeva gets the smaller value about 5 metres per century, see discussion to the talk by Standish [8]. In view of eq. (20), determine the velocity of the earth adopting the inertial anti-gravitational potential of the sun $\Psi_\odot = 6.6 \times 10^5$ cm$^2$/s$^2$. Then we obtain $v_E = 2.6 \times 10^{-7}$ cm/s = 8.3 m/cy that may explain the anomalous secular trend in the Astronomical Unit.

6 Advance of the perihelion of mercury

The inertial anti-gravitational potential of the sun should lead to the precession of the Keplerian orbit of a planet under its motion in the gravitational field of the sun. A small addition $\Psi_\odot$ to the potential of the sun causes the shift of the perihelion of planet's orbit per revolution by the value [3]

$$\delta \varphi = \frac{\partial}{\partial M} \frac{2m^2}{M} \int_0^\pi r^2 \Psi_\odot d\varphi.$$  \hfill (21)

where $m$ is the mass of the planet, $M$ is the angular momentum. The non-perturbed orbit is given by

$$r = \frac{p}{1 + e \cos \varphi}$$  \hfill (22)

with

$$p = \frac{M^2}{G m^2 m_\odot} \quad p = a(1 - e^2)$$  \hfill (23)

where $p$ is the orbit's latus rectum, $a$ is the semi-major axis. Integration of eq. (21) with the use of eqs. (22),(23) gives the shift (advance) of the perihelion of a planet due to the potential $\Psi_\odot$ per revolution

$$\delta \varphi \approx \frac{6\pi a(1 - e^2) \Psi_\odot}{G m_\odot}.$$  \hfill (24)

Determine the inertial anti-gravitational potential of the sun from the data on the anomalous shift of the perihelion of mercury, 43 arcseconds [1]. Then we obtain $\Psi_\odot = 6.4 \times 10^5$ cm$^2$/s$^2$. 


7 Discussion

We have considered effective gravity which includes Newtonian potential and inertial anti-gravitational potential. We have investigated footprints of the inertial anti-gravitational potential of the sun. We have shown that the inertial anti-gravitational potential of the sun allows to explain the anomalous acceleration of Pioneer 10 and 11, the anomalous increase in the lunar semi-major axis, the residuals of the seasonal variation of the proper angular velocity of the earth, the anomalous increase of the Astronomical Unit, the anomalous shift of the perihelion of mercury. The theoretical estimate of the inertial anti-gravitational potential of the sun is $\Psi_{\odot} = 5.3 \times 10^5 \text{cm}^2/\text{s}^2$. The inertial anti-gravitational potential of the sun determined from the anomalous acceleration of Pioneer 10 and 11 is $\Psi_{\odot} = 6.6 \times 10^5 \text{cm}^2/\text{s}^2$, from the anomalous increase in the lunar semi-major axis $\Psi_{\odot} = 6.9 \times 10^5 \text{cm}^2/\text{s}^2$, from the residuals of the seasonal variation of the proper angular velocity of the earth $\Psi_{\odot} = 5.2 \times 10^5 \text{cm}^2/\text{s}^2$, from the anomalous shift of the perihelion of mercury $\Psi_{\odot} = 6.4 \times 10^5 \text{cm}^2/\text{s}^2$. The inertial anti-gravitational potential of the sun is seen as an anomalous potential of the earth in the planet ranging. This mimics the velocity of the earth outward the sun and may explain the anomalous increase of the Astronomical Unit. Thus the data from the above observations may be considered as a support for the effective gravity with the inertial anti-gravitational potential.

The Shapiro effect [1] contributing to ranging is a powerful tool for exploring gravity. The Shapiro effect being dependent of the gravitational potential is not sensible to the inertial anti-gravitational potential because it is fixed with radius. Thus the inertial anti-gravitational potential cannot be seen in ranging data. It is worth noting that the authors of [4] consider the phenomenological model, p.11.5 quadratic in time model, which fits well the Pioneer effect and ranging data. This model adds a quadratic in time term to the light time as seen by the DSN station. It mimics a line of sight acceleration of the spacecraft and could be thought of as an expanding space model. Phenomenologically this model is similar to the effective gravity with the inertial anti-gravitational potential. The expansion of space may be interpreted as an effective velocity due to the inertial anti-gravitational potential.

The inertial anti-gravitational potential of the sun provides an explanation of the anomalous shift (advance) of the perihelion of mercury alternative to the general relativity [1]. The theory of relativity may be reinterpreted in a way that relativistic effects pertain only to electromagnetism while gravitation remains non-relativistic. In this case effects of general relativity such as gravitational redshift, time delay due to gravity arise due to extension of special relativity with the use of the principle of equivalence. The motion of bodies in the gravitational potential is described within the Newtonian mechanics.

References

[1] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).

[2] S.L. Shapiro and S.A. Teucolsky, *Black Holes, White Dwarfs, and Neutron Stars* (Jhon Wiley and Sons, New York, 1983).

[3] L. Landau and Ye. Lifshitz, *Mechanics* (Pergamon Press, Oxford, 1960).
[4] J.D. Anderson et al., Phys. Rev. D65 (2002) 082004.

[5] Yu.V. Dumin, Adv. Space Res. 31 (2003) 2461.

[6] C.E. Dutton and F.W. Fallon, in: Int. Conf. Earth Rotat. and Terr. Ref. Frame, Columbus, Ohio (1985) V. 2, p. 450.

[7] G.A. Krasinsky, V.A. Brumberg, Celes. Mech. Dyn. Astron. 90 (2004) 267.

[8] E.M. Standish, in: IAU Coll. N196, ed. D.W. Kurtz, Cambridge University Press (2005) p. 163, see discussion on p. 176.