News from the Adriatico Research Conference on “Superconductivity, Andreev Reflection, and Proximity Effect in Mesoscopic Structures”

D. C. Ralph and V. Ambegaokar

Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, NY 14853

The Adriatico Research Conference on “Superconductivity, Andreev Reflection, and Proximity Effect in Mesoscopic Structures” took place at the International Center for Theoretical Physics in Trieste, Italy, July 8-11, 1997. The organizers were Elias Burstein, Leonid Glazman, Teun Klapwijk, and Subodh R. Shenoy. We describe some of the central issues discussed at the conference, along with more personal reflections prompted by new developments.

I. THE PROXIMITY EFFECT AND ANDREEV REFLECTION

Introductory treatments of the superconducting proximity effect – how electrons behave in the vicinity of an interface between a normal metal and a superconductor – typically follow one of two tracks. One is to consider a Ginzburg-Landau picture of a superconductor/normal metal (SN) contact. In this treatment, which is accurate for temperatures close to the superconducting critical temperature $T_c$, one may define a position-dependent electron-pair correlation function $\langle \Psi_\uparrow(x)\Psi_\uparrow(x) \rangle$ which extends from the superconductor into the normal metal, decreasing exponentially (in diffusive materials) on thermal diffusion length scale of $L_T = \sqrt{\hbar D/(k_B T)}$ (here $D$ is the diffusion constant and $T$ is the temperature). This is perhaps a useful pedagogical approach, in that it allows one to think in a simple way that the Cooper pairs in the superconductor may “leak” through a non-zero thickness of normal metal. However, this framework is of limited practical value for temperatures much lower than $T_c$, in that it does not provide a way to calculate experimental quantities such as supercurrents, the tunneling density of states in the normal metal, or the conductance properties for various geometries of superconductors and normal metals made into devices. The problem is that the simple Ginzburg-Landau theory does not properly reflect the energy dependence of electronic properties. In fact, in the low-temperature regime where the electron phase-breaking length is long enough to be ignored, the appropriate length scale for describing the range of pair-correlated electrons diffusing inside a normal metal that is in contact with a superconductor is given by the energy-dependent quantity $L_\varepsilon = \sqrt{\hbar D/\varepsilon}$, where $\varepsilon$ is the electronic energy measured with respect to the Fermi level. This can be much longer than $L_T$. A theory which properly takes this energy dependence into account is the “quasiclassical Green’s function theory”, formulated in general by Eilenberger, and specialized to diffusive systems by Usadel (some of their work being done during post-doctoral stays at our home institution – Cornell!). “Quasiclassical” means that the full, non-equilibrium Gorkov equations of superconductivity are coarse-grained so as to eliminate quantum features on the fine scale of $1/k_F$. An overview on the current status of these methods was given at the conference by Gerd Schön.

A second pedagogical approach to understand the proximity effect is Andreev reflection. In this picture one notes that if an electron is traveling from a normal metal into a superconductor, and it has an energy within a range of the superconducting gap $\Delta$ about the Fermi energy, then it cannot simply be transmitted into the superconductor because of the superconducting gap. Instead, one may take the view that the electron, upon encountering the superconducting interface, produces a Cooper pair which is transmitted into the superconductor, and in the process a hole is retroreflected back into the normal state (in this way conserving charge, energy, and transverse momentum). The physics of Andreev reflection is contained within the quasiclassical Green’s function treatment. However, in addition, more recent formulations have extended a purely scattering-theory approach to include coherent multiple processes in which both Andreev and normal reflections occur at imperfect SN interfaces, and electrons may also be scattered from defects within the metals. The overall transport properties are than calculated in the spirit of the Landauer-Büttiker formula, as a function of the transmission eigenvalues of an overall scattering matrix. Colin Lambert reviewed the development of these methods, and Carlo Beenakker described related results which take into account statistical random-matrix properties of the scattering matrix.

The central theme of the conference was a clear consensus that both flavors of theory, the quasiclassical Green’s function method and the scattering matrix approach, are equivalent in the regimes in which they are both applicable, and that their main results are well-supported by recent experiments. In the words of Michel Devoret, the proximity effect and Andreev reflection are “two sides of the same coin.” Nathan Argaman went so far as to show that the Usadel equations of the Green’s function theory for diffusive metals may be derived within a purely scattering-matrix approach based on multiple Andreev scattering. The two types of theories do have slightly different ranges of applicability. Green’s function methods begin to break down in applications to nm-scale devices
with just a few conducting channels, a regime in which the scattering matrix methods are particularly suited. Scattering matrix methods can also be used more easily to model sample-to-sample variations in the mesoscopic size regime. However the Green’s functions methods are otherwise more general, as they may include effects of a wide range of interactions, and also possibilities for the modification of superconducting regions due to their contact with normal metals, not included in present scattering-matrix treatments. The Green’s functions are the sole means to calculate quantities such as magnetization, that cannot be related directly to transmission coefficients.

The driving force for renewed interest in the proximity effect in the last 5 years is that new technologies for the fabrication of small electronic devices have allowed SN devices to be studied in size regimes that have never before been accessible – smaller than both the low-temperature phase-breaking length for electrons ($L_\phi$), and also $L_T$. These samples have been used to test the proximity-effect theories through measurements of densities of states, transport properties, and magnetization.

Density of states: Michel Devoret reported tunneling measurements on small copper wires connected at one end to superconducting aluminum pads. The density of states at different distances from the interface was probed with tunnel junctions a few 10’s of nm wide. The results displayed excellent agreement with calculations performed by Gerd Schön’s group within the quasiclassical theory. As predicted, the density of states in the normal metal was depressed for energies below an scale corresponding to $hD/x^2$, where $x$ was the distance from the interface. This is the energy range over which electrons diffusing a distance $x$ will all remain in phase. For the best fit to theory, an effective spin-flip scattering time of approximately 65 ps was required. The explanation of this somewhat unexpectedly short time is perhaps at this moment unclear.

Reentrant resistance: Measurements of the transport properties of well-characterized diffusive SN devices having a variety of different geometries were described by representatives of several groups, including M. H. Devoret, H. Courtois, and B. J. van Wees. In accord with theory, the resistance of diffusive SN wires shows a “reentrance effect” as a function of temperature, meaning that as the temperature is lowered below $T_c$, the resistance first decreases and then rises to begin to approach the normal state value as the temperature goes to zero. Both quasiclassical theory and matrix scattering approaches predict that at $T = 0$ the resistance of a disordered SN wire is precisely the normal state resistance, and the temperature scale for the minimum in the resistance is the Thouless energy $hD/L^2$, divided by $k_B$ ($L$ is the length of the normal region of the wire).

Interferometer devices: A clever trick employed by many groups (including V. Petrashov, M. H. Devoret, H. Courtois, and B. J. van Wees) is to make “interferometer” devices which consist of an open loop of superconductor whose ends are attached to different points of a normal metal conductor. An applied magnetic field then acts to change the relative superconducting phase of the two ends, and allows phase-dependent measurements of Andreev-scattering effects. The results of these experiments are all apparently in good qualitative accord with predictions. In particular, for interferometer devices in which there is good metallic contact between the superconductors and the normal metals, the conductance oscillates periodically with the superconducting phase difference, with a relative amplitude which scales roughly as $hD/(L^2 k_B T)$, so that as a function of increasing temperature the oscillation amplitude falls slowly as $1/T$ rather than exponentially. The interpretation of this result is straightforward, in that while electrons at the Fermi energy within an energy window of $k_B T$ contribute to the total conductance, only those within an energy window of $hD/L^2$ remain in phase over the sample.

Disordered-enhanced Andreev reflection at tunnel barriers: Devices in which a superconductor and a normal metal are not in metallic contact, but are separated by a tunnel barrier, can also exhibit Andreev reflection processes. For ballistic metal samples joined by tunnel barriers, this effect is predicted to be very weak. However, disorder in the normal metal can enhance Andreev processes by orders of magnitude (a factor of 1000 for the Saclay group), due to an effect dubbed “reflectionless tunneling”. The mechanism may be understood in analogy to the Fabry-Perot interferometer in optics. In a disordered sample, an electron of energy $\varepsilon$ may be viewed as taking a path in which it undergoes many ordinary reflections from the SN tunnel junction and disorder, thereby returning to the tunnel junction many times. At each reflection from the SN interface, there will be a small amplitude for Andreev reflection. However, because Andreev reflection is a retro-reflection process, the reflected hole state (corresponding at $V = 0$ to the electron energy $-\varepsilon$) will have almost precisely the same trajectory as the electron path (but in reverse), and the quantum-mechanical phase accumulated by the hole between reflections at the SN interface will match that of the electron. The end result is that the amplitudes for Andreev reflection at each scattering event at the SN tunnel junction will add constructively, producing a much larger tunneling signal than if the processes were added incoherently. With a voltage applied across the tunnel junction, the differences in energy (and hence wavelength) of the electron and reflected hole states will grow and the constructive interference will gradually be degraded, leaving a zero-bias peak in the conductance. Another related effect, “giant Andreev reflection”, was predicted by Beenakker and observed by van Wees for the geometry of a ballistic constriction in series with a disordered conductor.

Resistance increases due to superconductivity: Frank Wilhelm proposed a theory to describe the counterintuitive experimental result (Petrashov) that the resistance of a wide, diffusive normal metal wire in contact with a superconductor can increase as the sample is cooled.
through the superconductor’s $T_c$. The explanation appears to be a geometrical effect – a consequence of the quasi-2-dimensional nature of a wide wire and the fact that current and voltage probes were positioned on opposite sides of the wire.

Supercurrents: Experiments on diffusive SNS devices exhibiting supercurrents were not discussed at the conference in as much depth as conductance measurements. However, from the work of Courtois, it seems clear that supercurrents (at least in “long” devices where the N region is longer than the coherence length) are governed by the same energy scale, the Thouless energy $E_c = \hbar D/L^2$, which plays the central role in conductance measurements. The typical magnitude of the critical current at $T = 0$ is $I_c = E_c/R_n$, where $R_n$ is the normal state resistance of the device and the length scale in $E_c$ is the extent of the normal region.

Ballistic samples: Conductance measurements for 2-dimensional electron gas (2DEG) samples in which electron motion is ballistic, or quasi-ballistic (as opposed to diffusive) were presented by H. Takayanagi and A. F. Morpurgo. Morpurgo described a breakdown of the idea of simple retroreflection of the hole in Andreev scattering, when the interface contains disorder on the scale of the electron wavelength. Nevertheless, he argued that his experiments could still be described well by a semiclassical ray-tracing procedure in which the possible paths for electrons and holes were added coherently. It appears that additional work is still required to test the behavior of even smaller and cleaner devices (such as those containing ballistic point contacts) where semiclassical ideas break down and a fully quantum picture is required. It also seems that the characterization of ballistic 2DEG samples can be quite difficult, particularly concerning the quality of the coupling between the superconductor and the 2DEG. In present-generation devices, electron scattering is likely much stronger at this interface than in the 2DEG away from the superconductor.

Magnetization: Magnetization measurements appear to be an area which may provide a challenge for existing theory. Joe Imry described measurements made by A. C. Mota (Zurich) on the susceptibility of fine superconducting wires (Nb) with a thin coating of normal metal (Cu or Ag). As a function of decreasing temperature, the susceptibility becomes increasingly diamagnetic as the sample is cooled below the superconducting $T_c$, as is expected due to the proximity effect in the normal metal. However, when the samples are cooled even farther, to the low mK range and below, the susceptibility in some wires reaches an extremum and then turns around, so that in the $T = 0$ limit some wires display a susceptibility that is even more paramagnetic than at the superconductor’s $T_c$. Imry speculated that this behavior may be related to specific “whispering gallery” modes in the normal metal which may decouple from the superconductor well below $T_c$.

II. SUPERCONDUCTING BREAK JUNCTIONS AND MULTIPLE ANDREEV REFLECTION

One of the most beautiful and interesting experiments discussed at the conference was the work of Elke Scheer and collaborators at Saclay, reported by Christian Urbina. They were able to make detailed measurements of the current traveling via discrete quantum mechanical modes in atomic-scale superconducting (Al) break junctions. This work may be viewed as a continuing development of the point-contact spectroscopy technique pioneered by Igor Yanson, and reviewed by him at the conference. The beauty of the new Saclay work is that an analysis of the conductance for applied voltages less than the superconducting gap of the electrodes (i.e., the subharmonic gap structure) was used to characterize the transmission coefficient for each of the active transport channels in the atomic-scale contact. The theory of the subharmonic gap structure, developed by J. C. Cuevas and collaborators (Madrid) using a Hamiltonian approach, and described by V. Shumeiko and D. V. Averin in a scattering-theory picture, is remarkably in such good shape that the transmission coefficients of 6 or more different quantum channels may be determined simultaneously. For aluminum break junctions, the Saclay group found, using fits to the theory of the Madrid group, that at least three partially-transmitting channels were required to describe their data, no matter how small they made their contact. Atomic-scale aluminum contacts are therefore different than 2DEG point contacts where transport may occur via a single, fully transmitting conductance channel. Urbina (referencing Cuevas et al.) speculated that the behavior of the aluminum contacts was rooted in chemistry – each aluminum atom can contribute 3 combinations of hybridized s and p atomic orbitals which lead to transport channels in a wire. Aluminum electrodes joined together by (purely s-like) gold atoms, on the other hand, can be reduced in size to the point that only a single quantum mode contributes to the conductance.

In view of the fact that all the channels observed so far in the aluminum break junctions are only partially transmitting, it seems very puzzling that histograms of the values of the total conductance in these samples still show peaks near quantized values of conductance (integer multiples of $2e^2/h$), corresponding to individual, fully-transmitting quantum channels.

In a poster presentation, P. Dieleman et al. from T. M. Klapwijk’s group described shot noise measurements of superconductor-insulator-superconductor tunnel junctions which display subharmonic gap structure due to multiple Andreev reflection. At values of voltage corresponding to subharmonic peaks, they found an increase in the magnitude of the shot noise above the classical value, consistent with the view that in multiple Andreev reflection several electrons are transmitted simultaneously.
III. SUPERCONDUCTIVITY IN NM-SCALE PARTICLES

One afternoon of the conference was given over to consideration of the nature of superconductivity in nm-scale metal particles, small enough that the discrete electrons-in-a-box energy-level spacing is comparable to the superconducting gap. One of us (D.C.R) described experiments in which these discrete levels were measured in single aluminum nanoparticles by a tunneling technique, and the effects on the spectra of a variety of different forces and interactions, including superconducting pairing, were analyzed. Andrei Zaikin reviewed theoretical results that as the level spacing in a metal sample is increased to approach the superconducting gap, the pairing parameter within BCS theory should be different for even and odd numbers of electrons. It was not clear how this effect might be measured experimentally, for two reasons. The first is that the even and odd pairing parameters may not be observables – what is measured in a tunneling experiment are energy differences between states with even and odd numbers of electrons, so that it is not a trivial matter to separate the different pairing parameters. Also, for nanoparticles in the size range where even and odd differences become interesting, the level spacing due to independent-electron quantum confinement becomes comparable to gaps due to superconductivity, and it is not clear how to separate these two effects.

Jan von Delft described a model for how spin pair-breaking due to an applied magnetic field will affect the eigenstates in a small superconducting particle. Von Delft found that the discreteness of the electronic spectrum may change the nature of the superconducting transition, compared to the theory of Clogston and Chandrasekhar (C&C) which describes well the transitions observed in thin film samples where the electronic spectrum is effectively a continuum. In the C&C theory, the tunneling threshold changes discontinuously at the transition field, but for a small particle this may be continuous, if the transition from the superconducting state involves the flipping of just a single electron spin.

IV. MEAN FIELD THEORIES AND BEYOND

Igor Aleiner described recent work with Altshuler on the theory of a tunneling anomaly in a superconductor in a magnetic field above the paramagnetic limit. The interesting result is that even though the mean field order parameter is zero in this regime, there is a singularity in the density of states due to fluctuations in the order parameter.

A. F. Andreev reported on his published work in which broken gauge symmetry is taken as the central defining characteristic of superconductivity, requiring a modified form of statistical mechanics. Some in the audience (including one of the authors, V.A., in closing remarks) expressed the view that “broken symmetry” is merely a mean-field treatment of the interacting system, and that residual effects such as those discussed by Aleiner (above) also contain important physics.

In his closing remarks, V.A. also said that since the conference was in some ways in honor of Andreev, it might be worth noting that a superconducting quasiparticle is built up from repeated virtual Andreev scattering against the mean-field order parameter. This can be seen by writing the electron propagator as

\[ G(\varepsilon, z) = \frac{z + \varepsilon}{z^2 - \varepsilon^2 - |\Delta|^2} = \frac{1}{z - \varepsilon - \Delta^*(z + \varepsilon)^{-1}\Delta}. \]

The last term in the last denominator (the electron self-energy) shows a normal electron being converted into a normal hole via the Andreev process, which here cannot be energy conserving because there is no analog of the voltage across an interface.

V. LOOKING TO THE FUTURE

Also presented at the conference were topics that look more to the future, in that mysteries remain which suggest avenues for future work.

Andreev processes involving localized states: Z. Ovadyahu described data from superconductor-insulator-normal metal tunnel junctions in which the barrier material was the Anderson insulator indium oxide, which contains a high density of localized electronic states. Conductance measurements showed unusual zero-bias signals, and also features at voltages well above the superconducting gap. Igor Yanson noted that the above-gap features are similar to signals seen in metallic SN point contacts; however it seems to us that the details of the zero-bias signals are at least suggestive that something more interesting than pinholes may be at work. Andreev reflection involving localized states could conceivably serve as an excellent model system for exploring the interplay of Coulomb charging effects, Kondo physics, and superconducting pairing. Similar themes were touched upon by A. Golub in his talk, and R. Fazio and R. Raimondi in a poster.

Nonequilibrium effects: The study of nonequilibrium processes such as charge imbalance and phase slip centers has a proud history that was reviewed in a talk by M. Tinkham. However, new non-equilibrium experiments in the mesoscopic regime continue to show unanticipated behavior, particularly when the samples are exposed to AC signals, as described by V. Chandrasekhar.

d-wave superconductors: Thus far all the proximity-effect devices that we have described utilized conventional s-wave superconductors. Yu. S. Barach and Y. Tanaka provided a theoretical discussion of a variety of ways in which tunnel junctions made using high-\(T_c\) d-wave superconductors would produce qualitatively different results. These include an anomalous temperature...
dependence for the Josephson current, the existence of quasiparticle states bound to the tunnel barrier, and surface pair breaking. Actually fabricating well-controlled high-\( T_c \) tunnel junctions will be a daunting task because of their difficult chemistry, but O. Fischer showed that low-temperature STM studies of the superconducting cuprates are already providing interesting results. He demonstrated striking differences in tunneling spectra for the electronic states in the cores of magnetic vortices in high-\( T_c \) materials, as compared to s-wave NbSe\(_2\).

**Electron-electron interactions:** The subject of electron-electron interactions in metals was not the focus of any scheduled presentations, but a recent paper by Mohanty, Jariwala, and Webb \[7\] was the object of much informal debate. This work proposes that zero-point fluctuations of the electromagnetic environment can produce electron dephasing in mesoscopic devices. Michel Devoret also mentioned puzzling results out of Saclay, where direct measurements of electron energy relaxation processes have suggested a scaling form that is not compatible with a present understanding of interaction processes.

Looking perhaps even farther into the future, F. Hekking reported calculations of the properties of superconductors coupled to the interacting electrons in one-dimensional Luttinger liquids, and H. Mooij speculated as to the use of Josephson-junction devices for quantum computations. There is no way to know at this point the prospects for whether the quantum coherence of Josephson junctions can be controlled sufficiently to allow for real quantum computations, but we expect that the macroscopic-quantum physics to be learned in this effort will be fascinating. An important intermediate goal on the way to computation will be to attain sufficiently long coherence times to produce a quantum clock using Josephson junctions, something long sought but without success to date.

**VI. PERSONAL REFLECTIONS**

Perhaps one way to summarize the present status of the theory of the superconducting proximity effect might be to paraphrase a remark made by Gerd Schön concerning the quasiclassical theory, “You give the theory to students, they solve some differential equations, and before long they come back with results!” Good agreement between recent experiments and theory give considerable confidence that a reasonably comprehensive understanding of (s-wave) superconducting/normal metal interfaces is close at hand. Before embarking upon triumphalism, however, we note that while the Green’s function theory needed to explain most of the recent generation of proximity-effect experiments was complete long before the experiments were begun, there was still a delay of some years after the first experiments before their explanation was generally appreciated. Part of the difficulty undoubtedly lay in uncertainty about experimental parameters (especially the quality of the interfaces between the superconductors and normal metals), but we also believe that there continue to be important issues of accessibility in the theory. We suggest that one of our goals, as a field of study, should be the further development of reliable tools for working intuition, so that those who live happy lives without benefit of Green’s functions may have good pictures with which to begin to understand superconducting/normal metal devices, and a clear prescription for how to proceed in reliable modeling. We need popularizers, not prophets.

Important steps along these lines were reported at the conference. Yuli Nazarov’s “circuit theory” formulation for the quasiclassical Green’s functions is a valuable contribution, though it still cannot be said to be optimally “user friendly”. We find the scattering-matrix theories of Andreev reflection in SN devices to be very important as a more intuitive approach than the Green’s function theory in many situations. Nathan Argaman’s poster was particularly interesting in this regard, as it showed explicitly that the main formulas describing the proximity effect in the quasiclassical Green’s function theory can in fact be derived from a simple picture involving nothing more than multiple Andreev reflection. In addition, we appreciated the strategy that Bart van Wees took in his talk to help build intuition. In the spirit of Nazarov’s circuit theory, he considered the nature of Andreev scattering in the important cases of a tunnel barrier, a disordered wire, and a ballistic constriction, and then he considered what happens when these elements are combined.

**VII. ACKNOWLEDGMENTS**

In addition to the sponsorship of the International Center for Theoretical Physics (ICTP), this Adriatico Research Conference was aided by support funds from the United States Air Force European Office of Aerospace Research and Development. The authors’ work is supported by the US NSF (Grants DMR-9407245, DMR-9632275, DMR-9705059), ONR (Grant N00014-97-1-0745), and the A. P. Sloan Foundation.

[1] G. Deutscher and P. G. de Gennes, in *Superconductivity*, edited by R. D. Parks (Marcel Dekker, New York, 1965), Vol. II p. 1005.
[2] G. Eilenberger, *Z. Phys.* 214, 195 (1968).
[3] K. D. Usadel, *Phys. Rev. Lett.* 25, 507 (1970).
[4] A. F. Andreev, *Zh. Eksp. Teor. Fiz.* 46, 1823 (1964) [*Sov. Phys. JETP* 9, 1228 (1964)].
[5] G. E. Blonder, M. Tinkham, and T. M. Klapwijk, *Phys. Rev. B* 25, 4515 (1982).
[6] A. M. Clogston, *Phys. Rev. Lett.* 9, 266 (1962); B. S. Chandrasekhar, *Appl. Phys. Lett.* 1, 7 (1962).
[7] P. Mohanty, E. M. Q. Jariwala, and R. A. Webb, *Phys. Rev. Lett.* 78, 3366 (97).