Thermodynamics second law and $\omega = -1$ crossing(s) in interacting holographic dark energy model

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October 4, 2018

Abstract

By the assumption that the thermodynamics second law is valid, we study the possibility of $\omega = -1$ crossing in an interacting holographic dark energy model. Depending on the choice of the horizon and the interaction, the transition from quintessence to phantom regime and subsequently from phantom to quintessence phase may be possible. The second transition avoids the big rip singularity. We compute the dark energy density at transition time and show that by choosing appropriate parameters we can alleviate the coincidence problem.

1 Introduction

Recent observations suggest that the universe is undergoing an accelerated expansion [1]. This acceleration may be explained by the assumption that 70% of the universe is filled by a perfect fluid with negative pressure, dubbed dark energy. Some present data seem to favor an evolving dark energy, corresponding to an equation of state (EOS) parameter less than $\omega = -1$ at present epoch from $\omega > -1$ in the near past [2]. Many candidates for dark energy has been proposed such as the cosmological constant [3]: A constant quantum vacuum energy density which fills the space homogeneously, corresponding to a fluid with a constant EOS parameter $\omega = -1$; dynamical fields with a suitably chosen potential to make the vacuum energy vary with time [4], and so on. Recently, using holographic principle, a new candidate for dark energy which is independent of any specific field has been suggested

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Based on the holographic principle (which relates the number of degrees of freedom of a physical system to the area of its boundary), in order to allow the formation of black holes in local quantum field theory, Cohen et al. proposed a relationship between UV and IR cutoff. This yields an upper bound on the zero-point energy density, which by a suitable choice of the infrared cutoff, can be viewed as the holographic dark energy density. In [5], three candidates for the infrared cutoff was proposed: the Hubble radius, the particle horizon and the future event horizon. There was shown that among these options only the future event horizon may be identified with the desired infrared cutoff. To study the coincidence problem and also to have other choices for the infrared cutoff, e.g. the Hubble radius, interaction between dark matter and dark energy may be considered in the holographic dark energy model. As we have mentioned, based on astrophysical data, we may take into account the possibility of \( \omega = -1 \) (phantom divide line) crossing. Therefore dark energy models which can describe phantom divide line crossing, has been also studied vastly in the literature. The phantom like behavior of interacting holographic dark energy was studied in [10], where it was claimed that by selecting appropriate interaction parameters the transition from the dark energy EOS parameter \( \omega_D > -1 \) to \( \omega_D < -1 \) is possible. Despite this, in [11] it was shown that the dark energy effective EOS parameter cannot cross \( \omega_{eff} = -1 \).

In this paper we consider interacting holographic dark energy model and study the ability of the model to describe the transition from quintessence to phantom regime and vice versa. After preliminaries in section two, where we introduce the interacting holographic dark energy model and some of its general properties used in the subsequent sections, in section three we study the possibility of crossing \( \omega = -1 \). In section four we derive necessary conditions for existence of two transitions in our model. The first transition is from quintessence to phantom phase and the second the transition from phantom to quintessence regime. The importance of the second transition lies on the fact that it avoids the big rip singularity. We discuss also the behavior of Hubble parameter and dark energy density at transitions times.

We use \( \hbar = G = c = k_B = 1 \) units throughout the paper.

2 preliminaries

We consider a spatially flat Friedmann–Lemaitre–Robertson–Walker (FRW) universe), with scale factor \( a(t) \)

\[
ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2). \tag{1}
\]
We assume that this universe is filled with dark energy and pressureless dark matter fluids satisfying the following equations of state

\[ \dot{\rho}_D + 3H(1 + \omega_D)\rho_D = -Q \]
\[ \dot{\rho}_m + 3H\rho_m = Q. \]  

(2)

\( H = \dot{a}/a \) is the Hubble parameter, \( \rho_D \) is dark energy density and \( \rho_m \) is the density of cold dark matter. "dot" denotes derivative with respect to the comoving time. \( \omega_D \) is dark energy EOS parameter. \( Q \) denotes the interaction of dark matter with dark energy. In this paper \( Q \) is assumed to be

\[ Q = (\lambda_m\rho_m + \lambda_D\rho_D)H, \]  

(3)

where \( \lambda_m \) and \( \lambda_D \) are two real constants. Different choices such as \( \lambda_m = 0 \), \( \lambda_D = 0 \) and \( \lambda_D = \lambda_m \) has been adopted in literature [8], [10]. The Hubble parameter satisfies

\[ H^2 = \frac{8\pi}{3}(\rho_m + \rho_D) \]
\[ = \frac{8\pi}{3}\rho, \]  

(4)

and

\[ \dot{H} = -4\pi((1 + \omega_D)\rho_D + \rho_m) \]
\[ = -4\pi(1 + \omega)\rho. \]  

(5)

\( \rho = \rho_m + \rho_D \) is the total energy density satisfying

\[ \dot{\rho} + 3H(1 + \omega)\rho = 0, \]  

(6)

and \( \omega \) is the parameter of the EOS of the universe. In terms of \( \Omega_D = \rho_D/\rho \), we can write

\[ \omega_D\Omega_D = \omega. \]  

(7)

\( \rho_D \) can be viewed as a holographic energy density

\[ \rho_D = \frac{3}{8\pi}c^2L^2. \]  

(8)

The length scale \( L \) is an infrared cutoff and \( c > 0 \) is a positive numerical constant. We assume

\[ L = \beta R_{FH} + \alpha R_{PH}, \]  

(9)

where \( R_{FH} \) and \( R_{PH} \) are the future and particle event horizons

\[ R_{FH} = a \int t^\infty dt \frac{a}{a} \]
\[ R_{PH} = a \int t^1 dt \frac{a}{a}. \]  

(10)
and $\alpha$ and $\beta$ are two positive numerical constants. By taking $\beta = 0$ and $\alpha = 1$, we arrive at the holographic cosmology horizon adopted in [12]. For $\alpha = 0$ and $\beta = 1$, the infrared cutoff becomes the future event horizon [5]. Generally one can assume that $L$ is a function of $R_{FH}$ and $R_{PH}$ [13]. The time derivative of $L$ is obtained as

$$\dot{L} = HL + \alpha - \beta. \quad (11)$$

Using (4) and (8), $HL$ in the above equation can be written as

$$HL = c\Omega_D^{-\frac{1}{2}}. \quad (12)$$

By relating the entropy of the universe to the infrared cutoff $L$ via

$$S = \pi L^2, \quad (13)$$

the second law of thermodynamics results in $\dot{L} > 0$, leading to $\dot{\rho}_D < 0$. In a phantom dominated universe (identified by $\dot{H} > 0$), using (4) and (8) we get

$$\dot{\Omega}_D = \frac{8\pi}{3H^4} \left( \dot{\rho}_D H^2 - 2H\dot{H}\rho_D \right) < 0. \quad (14)$$

For $\dot{\Omega}_D > 0$ we must have $(LH) < 0$ which leads to $\ddot{L} < 0$. But if one requires $\ddot{L} \geq 0$, then either $\lim_{t \to \infty} \dot{L} = 0$ or $\dot{L}$ becomes positive after a finite time.

In terms of the Hubble parameter, $\omega$ is

$$\omega = -1 - \frac{2\dot{H}}{3H^2}. \quad (15)$$

By substituting $\dot{H} = -H^2 + (\beta - \alpha)H/L + (HL)/L$ (which can be verified using (11)), (15) becomes

$$\omega = -1 - \frac{2}{3} (\beta - \alpha) \sqrt{\Omega_D} + \frac{\dot{\Omega}_D}{3H\Omega_D}. \quad (16)$$

Using (2) one can show that

$$\dot{r} = 3rH \left[ \omega_D + \frac{1}{3} \left( \frac{r + 1}{r} \right) (\lambda_D + r\lambda_m) \right], \quad (17)$$

where we have defined $r = \rho_m/\rho_D$. Therefore, by considering $r = \Omega_D^{-1} - 1$, we obtain

$$\omega_D = -\frac{1}{3H} \frac{\dot{\Omega}_D}{\Omega_D(1 - \Omega_D)} - \frac{\lambda_D}{3(1 - \Omega_D)} - \frac{\lambda_m}{3\Omega_D}. \quad (18)$$

and consequently

$$\omega = -\frac{1}{3H} \frac{\dot{\Omega}_D}{(1 - \Omega_D)} - \frac{\lambda_D\Omega_D}{3(1 - \Omega_D)} - \frac{\lambda_m}{3}. \quad (19)$$
From (16) and (19) we can express $\omega$ and $\dot{\Omega}_D$ in terms of $\Omega_D$:

$$\omega = -\frac{1 + \lambda_D - \lambda_m}{3} \Omega_D - \frac{2(\beta - \alpha)}{3c} \Omega_D^\frac{2}{3} - \frac{\lambda_m}{3},$$

(20)

$$\frac{\dot{\Omega}_D}{H} = (\lambda_m - \lambda_D - 1)\Omega_D^2 + (1 - \lambda_m)\Omega_D + \frac{2(\beta - \alpha)}{c} \Omega_D^\frac{2}{3}(1 - \Omega_D).$$

(21)

For an accelerated universe, $\ddot{a}(t) > 0$ (which results in $\omega < -1/3$), we have

$$(1 - \lambda_m) + (\lambda_m - \lambda_D - 1)\Omega_D - \frac{2(\beta - \alpha)}{c} \Omega_D^\frac{2}{3} < 0.$$ 

(22)

In the absence of interaction, acceleration is not possible for $\beta < \alpha$, as was pointed out by [5] for the case $\beta = 0, \alpha = 1$. This was the motivation of [5] to take the future event horizon (instead of the particle horizon) as the infrared cutoff. However the above calculation reveals that in the presence of interaction, inflation may be possible even when the particle horizon is taken as the infrared cutoff. Combining (21) and (22), yields

$$\dot{\Omega}_D < \frac{2(\beta - \alpha)}{c} H \Omega_D^\frac{2}{3}.$$ 

(23)

Hence like the phantom regime, for $\beta < \alpha$ and in an accelerating universe, $\Omega_D$ is decreasing. Applying the assumption $\dot{L} > 0$, gives an upper bound for $\Omega_D$ which depends only on $H$

$$\dot{\Omega}_D < 2H\Omega_D < 2H.$$ 

(24)

3 Crossing $\omega = -1$ in interacting holographic dark energy model

At $\omega = -1$, $u = \Omega_D^\frac{2}{3}$ satisfies the cubic equation

$$\frac{2(\beta - \alpha)}{c} u^3 + (1 + \lambda_D - \lambda_m)u^2 + \lambda_m - 3 = 0.$$ 

(25)

If $\omega = -1$ is allowed, the above equation must have, at least, one positive root which is less than one. Based on Descartes rule, we know that the above equation, at most, has two real positive roots. So, if $\omega = -1$ is crossed, two transitions may be possible, one from quintessence to phantom and the other from phantom to quintessence phase (by quintessence phase (regime) we mean $\omega < -1/3$ and $\dot{H} < 0$). From [15] it is clear that at $\omega = -1$ we have $\dot{H} = 0$. If a transition from quintessence to phantom phase occurs at time $t_1$, we must have $\dot{H}(t_1) = 0$, and $\dot{H}(t < t_1) < 0$ and $\dot{H}(t > t_1) > 0$, therefore $H(t_1)$ must be a local minimum of $H$. In the same way, $H$ must have a local maximum at $t_2$, where the transition
from phantom to quintessence era occurs. In the neighborhood of $t_1$, $\omega$ is a decreasing function while in the neighborhood of $t_2$, $\omega$ is an increasing function of time. So in order to see that if $\omega = -1$ crossing is permissible, we must also consider the behavior of the Hubble parameter near the roots of (25).

In the following we assume that $H > 0$ and $L > 0$. We also assume that the Hubble parameter is a differentiable function of time [14]. Therefore following (11) and (12), $L$ and $u(= \Omega_D^1)$ are also differentiable. Let us consider the Taylor expansion of $H$ at $t = t_i$, where $t_i$ is defined by $\dot{H}(t_i) = 0$ (or $\omega(t_i) = -1$),

$$H = h_0 + h_1 \tau^a + O(\tau^{a+1}), \quad a \geq 2,$$

(26)

where $\tau = t - t_i$, $h_1 = \frac{1}{a!} \frac{d^a H}{dt^a}(t_i)$, and $a$ is the order of the first nonzero derivative of $H$ at $t = t_i$. If $a$ is an even integer and $h_1 > 0(< 0)$, then $H$ has a minimum (maximum) at $t_i$ and the transition occurs at $t_i$. Using (15) we obtain

$$\omega = -1 - \frac{2ah_1}{3h_0} \tau^{a-1} + O(\tau^a).$$

(27)

We consider the following expansion for $u$ at $t = t_i$,

$$u = u_0 + u_1 \tau^b + u_2 \tau^{b+1} + O(\tau^{b+2}), \quad u_1 \neq 0, \quad b \geq 1$$

(28)

where $b$ is the order of the first nonzero derivative of $u$ at $t_i$. If the solution (26) is permissible, by inserting (27) and (28) in (20) and by comparing the powers of $\tau$ in both sides of (20), we obtain the following results:

$$(i) \quad - \frac{1 + \lambda_D - \lambda_m - 3u_0}{3c} - \frac{2(\beta - \alpha)}{3c} u_0^3 = -1 + \frac{\lambda_m}{3}.$$  

(29)

The roots of this equation specify $u$ at transition time(s). In order that the transition occurs the above equation must have at least one real root in the interval $(0, 1)$. (ii) For

$$\frac{1 + \lambda_D - \lambda_m - \beta - \alpha}{c} u_0 \neq 0,$$

(30)

we obtain

$$- \frac{ah_1}{3h_0} = \left(\frac{1 + \lambda_D - \lambda_m - \beta - \alpha}{c} u_0\right) u_0 u_1, \quad b = a - 1.$$  

(31)

In the case

$$\frac{1 + \lambda_D - \lambda_m - \beta - \alpha}{c} u_0 = 0,$$

(32)

and for $\beta \neq \alpha$, we find

$$\frac{2ah_1}{3h_0} = \frac{(\beta - \alpha)u_0 u_1^2}{c}, \quad a = 2b + 1.$$  

(33)
In this case \( a \) is an odd number and transition does not occur. If \( \beta = \alpha, \)
\( 1 + \lambda D - \lambda m = 0 \), and \( \omega \) in \((20)\), becomes a constant: \( \omega(t) = -1 \), and no transition occurs.

To determine \( a \) and the sign of \( h_1, (21) \) may be used:

\[
2 \dot{u} = H \left[ \frac{2(\alpha - \beta)}{c} u^4 + (\lambda_m - \lambda D - 1)u^3 + \frac{2(\beta - \alpha)}{c} u^2 + (1 - \lambda_m)u \right].
\] (34)

By inserting \((28)\) and \((26)\) into the above equation, we see that the left hand side begins with \( \tau b - 1 \), while if the right hand side does not begin with \( \tau^0 \), it will begin by \( \tau^{(\gamma > b)} \), which is inconsistent with the left hand side. Thereby \( b = 1 \). Hence based on \((31)\) and our previous discussion after \((26)\), if the transition occurs, we must have \( a = 2 \). Note that for the case \((32)\), we obtain \( a = 3 \) which may be corresponding to the inflection point of \( H \). For \( b = 1 \), by equalizing \( \tau^0 \)'s coefficients in both sides of \((34)\), we get

\[
2u_1 = h_0 \left[ \frac{2(\alpha - \beta)}{c} u_0^4 + (\lambda_m - \lambda D - 1)u_0^3 + \frac{2(\beta - \alpha)}{c} u_0^2 + (1 - \lambda_m)u_0 \right],
\] (35)

which using \((29)\) reduces to

\[
u_1 = h_0 u_0 \left[ \frac{\beta - \alpha}{c} u_0 - 1 \right].
\] (36)

Now let us determine the sign of \( h_1 \). Combining \((31)\) and \((36)\) results in

\[
- \frac{ah_1}{3h_0^2} = \left( 1 + \lambda D - \lambda m \right) - \frac{\beta - \alpha}{c} u_0 \left( \frac{\beta - \alpha}{c} u_0 - 1 \right) h_0 u_0^2.
\] (37)

From \((11), (12)\) and \((13)\), by applying the thermodynamics second law we deduce

\[
\frac{\beta - \alpha}{c} u_0 - 1 < 0,
\] (38)

which results in \( u_1 < 0 \). \( \dot{S} = 0 \) is ruled out because \( h_1 \neq 0 \). So we conclude that the sign of \( h_1 \) must be the same as the sign of \((1 + \lambda D - \lambda m)/(-3) - (\beta - \alpha)u_0/c \). For transition from quintessence to phantom phase we must have

\[
1 + \lambda D - \lambda m \left( -3 \right) - \frac{\beta - \alpha}{c} u_0 > 0,
\] (39)

while a transition from phantom to quintessence era requires

\[
1 + \lambda D - \lambda m \left( -3 \right) - \frac{\beta - \alpha}{c} u_0 < 0.
\] (40)

Note that if, like \([10]\), we take \( \alpha = 0, \beta = 1 \) and \( \lambda_m = \lambda D \), transition from quintessence to phantom is not allowed.
It is also instructive to study the behavior of $L$ at $t_i$. Assuming that $\dot{L} > 0$, by inserting the Taylor expansion of $L$ at $t = t_i$ up to the order $\tau^2$ in (11), we obtain $\dot{L} = [L_0 + L_1 \tau + O(\tau^2)] + \alpha - \beta$, where $L_0 = L(t_i)$ and $L_1 = \dot{L}(t_i)$. Hence $\dot{L} = h_0 L_0 + \alpha - \beta$. (12) results in $u = c/(LH)$, therefore

$$u_0 = \frac{c}{L_0 h_0}, \quad u_1 = -\frac{c}{L_0} \left(1 + \frac{\alpha - \beta}{h_0 L_0}\right),$$

(41)

which is consistent with (36).

As a summary we have shown that in order that a transition phase occurs: (i) (29) must have at least a positive real root in the interval $(0,1)$ (ii) At these roots the Hubble parameter (if it is differentiable) must have the Taylor expansion (26), with an even integer $a$. In interacting holographic dark energy model we obtained $a = 2$ and verified that quintessence to phantom phase transition and vice versa occur provided (39) and (40) hold respectively.

4 Two transitions in interacting holographic dark energy model

In this part we try to investigate the ability of the system to return to the quintessence regime from the phantom phase. This may be interesting because it avoids the big rip singularity which may be encountered in phantom models.

Let us write (29) as

$$pu_0^3 + qu_0^2 + 1 = 0,$$

(42)

where

$$p = \frac{2(\beta - \alpha)}{c(\lambda_m - 3)}, \quad q = 1 + \frac{\lambda_D - \lambda_m}{\lambda_m - 3}. $$

(43)

In order to have two transitions, (12), must possess two real positive roots, which we denote by $u_{01}, u_{02}$ in the interval $(0,1)$. $p$ and $q$ are real numbers, hence the third root must be also real. Therefore the discriminant of (42), i.e. $-27p^2 - 4q^3$, must be positive

$$\left(\frac{p}{2}\right)^2 + \left(\frac{q}{3}\right)^3 < 0.$$  

(44)

From $\sum_{i \neq j} u_{0i} u_{0j} = 0$, we find that the third root, $u_{03}$, is negative and $|u_{03}| < 1$. So using $0 < -u_{03} u_{02} u_{01} = 1/p < 1$, we deduce $p > 1$. Also following Descartes rule of sign, having two positive roots is only possible when $p > 0$ and $q < 0$. 

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The Sturm sequence constructed from (42) is \( S(x) = \{ \mathcal{P}(x), \mathcal{P}_1(x), \mathcal{P}_2(x), \mathcal{P}_3(x) \} \), where
\[
\begin{align*}
\mathcal{P}(x) &= px^3 + qx^2 + 1 \\
\mathcal{P}_1(x) &= 3px^2 + 2qx \\
\mathcal{P}_2(x) &= \frac{2q^2x}{3p} - 1 \\
\mathcal{P}_3(x) &= -\frac{9p}{4q^3}(27p^2 + 4q^3).
\end{align*}
\] (45)

We have \( S(0) = \{ 1, 0, -1, -9p(4q^3 + 27p^2)/(4q^4) \} \), \( S(1) = \{ p + q + 1, 3p + 2q, -1 + 2q^2/(9p), -9p(4q^3 + 27p^2)/(4q^4) \} \). If we expect to have two real roots in the interval \((0, 1)\), using (44) and by applying the Sturm theorem, we find
\[
\begin{align*}
p + q + 1 &> 0 \\
\frac{p}{2} + \frac{q}{3} &> 0 \\
\left( \frac{q}{3} \right)^2 - \frac{p}{2} &> 0.
\end{align*}
\] (46)

These conditions and (44), may be resumed as
\[
- q - 1 < p < 2 \left( -\frac{q}{3} \right)^2, \quad q < -3.
\] (47)

Now let us consider (39) and (40). A transition from quintessence to phantom phase may be occurred at \( t_1 \) \( (u(t = t_1) = u_{01}) \), if \( h_1 > 0 \)
\[
\frac{1 + \lambda_D - \lambda_m}{-3} - \frac{\beta - \alpha}{c} u_{01} > 0.
\] (48)

For \( t > t_1 \) the system becomes phantom dominated until \( t = t_2 \) \( (u(t = t_2) = u_{02}) \), i.e. when the second transition occurs, provided \( h_1 < 0 \) at \( t_2 \)
\[
\frac{1 + \lambda_D - \lambda_m}{-3} - \frac{\beta - \alpha}{c} u_{02} < 0.
\] (49)

From (14) we find that \( u \) is a decreasing function of time in phantom dominated era, hence \( u_{01} > u_{02} \). So in order that (48) and (49) become consistent we must have
\[
\beta - \alpha < 0, \quad 1 + \lambda_D - \lambda_m > 0.
\] (50)

The existence of \( u_{01} \) and \( u_{02} \) which satisfy (48) and (49), may be verified as follows: Using the condition posed on \( p \) and \( q \), we obtain
\[
0 < \frac{2q}{3p} = -\frac{(1 + \lambda_D - \lambda_m)c}{3(\beta - \alpha)} < 1.
\] (51)
The Sturm sequence at $-2q/(3p)$ is

$$S(-2q/(3p)) = \{4q^3/(27p^2) + 1, 0, -4q^3/(27p^2) - 1, -9p(4q^3 + 27p^2)/(4q)^4\}.$$  \hspace{1cm} (52)

So by invoking the Sturm theorem, one can verify that one of the positive roots, which based on our discussion in previous paragraph we take $u_{02}$, is located in $(0, -2q/(3p))$ while the other, i.e. $u_{01}$ belongs to $(-2q/(3p), 1)$. As we have previously mentioned (see discussion after (25)), $\omega$ is a decreasing function of time in the neighborhood of $u_{01}$ and an increasing function of time in the vicinity of $u_{02}$, whence for a differentiable $\omega$, we must have $\dot{\omega} = 0$ at a point in the phantom regime. To verify this claim, we note that in an accelerating universe $\dot{H} + H^2 > 0$, and for $\alpha > \beta$,

$$\langle HL\rangle = (\dot{H} + H^2)L + (\alpha - \beta)H > 0.$$

Therefore $HL$ is an analytic differentiable increasing function of time. Thereby $u$, is a decreasing function of time, as we showed before (see (23)) via another method. If $u = 0$ occurs at a finite time $t$, then for $t > t$, $u(t > t) < 0$ which conflicts with definition of $u$. In addition $u(t) = 0$ leads to $H(t)L(t) = \infty$ which conflicts with continuity of $H$ and $L$.

As a test of our results, we have plotted $\omega + 1$, (20), in terms of $u$ for an interacting holographic dark energy model with parameters $\{\beta = 0, \alpha = 1, c = 1, \lambda_D = 3.9, \lambda_m = 2.5\}$ (see fig. (1)). In this example $\omega + 1$ has two zero in the interval $(0, 1)$, $u_{01} = 0.86$ corresponding to $\Omega_D = 0.75$ and $u_{02} = 0.72$ corresponding to $\Omega_D = 0.52$. For $u_{02} < u < u_{01}$, the system is in phantom phase, i.e. $\omega < -1$, and for $u > u_{01}$ and $u < u_{02}$ the system is in quintessence phase. Note that for an accelerating universe and when $\alpha > \beta$, $u$ is a decreasing function of time (see (23) and our discussion in the previous paragraph) and the directions of $t$ and $u$-axis are opposite. $\dot{\omega} = 0$ occurs in the phantom regime: $u = 0.8$, and at $u = 0$.

5 Summary

In this paper we considered the holographic dark energy model with a general interaction between dark matter and dark energy, see (3). We took the infrared cutoff as a linear combination of the future an particle horizon see (9). We derived an expression for EOS parameter of the universe, $\omega$, in terms of the ratio of dark energy density and total energy density of the flat FRW
Figure 1: $\omega + 1$ as a function of $u$, for $\{\beta = 0, \alpha = 1, c = 1, \lambda_D = 3.9, \lambda_m = 2.5\}$

space-time, $\Omega_D$, see (20). Using (20) and the expression obtained for time derivative of $\Omega_D$ (21), and by assumption that the thermodynamics second law is still valid, we studied the possibility (and necessary conditions) for quintessence to phantom phase transition and vice versa with a differentiable Hubble parameter. Using some theorems about solutions of cubic equation satisfied by $\Omega_D^{1/2}$ see (25), we showed that such transitions occur provided we appropriately choose the parameters of the system see (47) and (50).

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