1. Introduction

A traditional global navigation satellite system (GNSS) receiver tracks each satellite independently and each channel does not permit information sharing among other channels. The performance between dynamic and noise suppression is a tradeoff for the tracking loop bandwidth [1, 2]. The integrated navigation system of GNSSs and inertial navigation systems (INSs) was developed to improve the accuracy and robustness of the system [3–6]. The deeply-coupled (also called ultra-tight) navigation system is one of these effective integrated systems. Some recent researches on deeply-coupled navigation systems include the acquisition and loop control algorithms [7, 8], the fusion methods of the GNSS and INS [9, 10] and other related fields [11, 12].

Generally, deeply-coupled navigation systems can be classified as having a centralized filtering architecture or a federated filtering architecture [13–16]. However, the centralized filtering architecture suffers from a high computation burden and complex relationship between in-phase/quadra-phase (I/Q) correlator outputs and INS errors for hardware implementation [15]. Thus, only the federated filtering architecture will be discussed in the following sections of this paper.

In the federated architecture, the I/Q measurements are firstly pre-processed by a series of pre-filters and then the integrated navigation filter is used to process the output of the pre-filters and to restrict the INS errors. The INS navigation solutions and GNSS ephemerides are used to control the numerically controlled oscillators (NCOs) of code and carrier [16].

The pre-filter is a key technology of the deeply-coupled navigation system, which can be mainly divided into two
categories, coherent and non-coherent [17, 18]. The coherent algorithm inputs the GNSS accumulated correlator outputs, i.e. the \( I_s \) and \( Q_s \), directly to the Kalman filter as measurements. The non-coherent algorithm firstly passes the \( I_s \) and \( Q_s \) through code and carrier discriminator functions, similar to those used in conventional GNSS signal tracking. The coherent algorithm bypasses the discriminators, avoiding the introduction of unmodeled nonlinear in the measurement inputs to the Kalman filter, and can reach a less noisy tracking performance than the non-coherent algorithm. However, it is unsuited to applications that require operation under low signal-to-noise environments as both code and carrier-frequency tracking can be maintained at a lower carrier power-to-noise density ratio \( (C/N_0) \) than carrier-phase tracking. On the contrary, the non-coherent algorithm can work well whether there is sufficient \( C/N_0 \) to track carrier phase because the code discriminator function is independent of the carrier phase. This enables the non-coherent algorithm to maintain tracking in weaker signal environments than the coherent algorithm.

In order to improve the accuracy and robustness of the system, an adaptive deeply-coupled GNSS/INS navigation system with hybrid pre-filters processing is proposed in this paper. The existing pre-filter algorithms are analyzed and modified to overcome their shortcomings firstly. Then, a hybrid-based pre-filter processing strategy is introduced. An adaptive hysteresis controller is designed to implement the hybrid pre-filters processing strategy. Finally, the simulation and vehicle tests are conducted to assess the system’s performance.

2. Hybrid-based adaptive pre-filter processing

2.1. Analysis of the existing pre-filter algorithms

There are three main structures of pre-filter, as summarized in [14]. The existing coherent pre-filter algorithm (option \#1 in [14]) follows closely the filter implementation proposed in [19, 20]. The correlator outputs shown in (1) and (2) are used directly as the measurements of the Kalman filter,

\[
I = A \cdot \frac{\sin(\pi \cdot \delta f \cdot T)}{\pi \cdot \delta f \cdot T} \cdot D \cdot R(\delta \tau + \Delta k) \cdot \cos(\delta \Phi) + n_I
\]

\[
Q = A \cdot \frac{\sin(\pi \cdot \delta f \cdot T)}{\pi \cdot \delta f \cdot T} \cdot D \cdot R(\delta \tau + \Delta k) \cdot \sin(\delta \Phi) + n_Q
\]

where \( A \) represents the accumulated amplitude, \( T \) is the integration period, \( D \) is the navigation data bits of the GNSS, \( R(\cdot) \) is the autocorrelation function of the ranging code, \( \delta \tau \) is the code phase bias between the local replica code and the incoming signals and \( \delta f \) is the frequency error (Hz) between the local replica frequency and the incoming signals. \( \Delta k \) is the correlator spacing for early, prompt and delay code, where \( k = -1, 0, 1 \). \( n_I \) and \( n_Q \) are the noise of \( I \) and \( Q \). \( \delta \Phi \) is the average phase error over the integration interval, which can be written as

\[
\delta \Phi = \delta \phi_0 + \frac{1}{2} \delta \phi_0 T + \frac{1}{6} \delta \phi_0 T^2
\]

where \( \delta \phi_0 \) and \( \delta \phi_0 \) indicate the initial phase error and the initial carrier frequency error at the start of the integration interval, respectively, and \( \delta \phi_0 \) is the carrier phase acceleration error.

The system model for this implementation is written as follows:

\[
\begin{bmatrix}
\delta \phi_0 \\
\delta \phi_0 \\
\delta I_m \\
\delta I_m
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\delta \phi_0 \\
\delta \phi_0 \\
\delta I_m \\
\delta I_m
\end{bmatrix} \\
+ \begin{bmatrix}
w_A \\
w_{mp} \\
w_{clock} \\
w_{drift} \\
w_{accel}
\end{bmatrix}
\]

where \( \beta \) converts units of rad/s into units of chips per second. \( w_A \) is the process noise for the amplitude; \( w_{mp} \) is the process noise for the code phase error to account for code multipath effects; \( w_{clock} \) is the process noise for the clock bias; \( w_{drift} \) is the process noise for the clock drift; and \( w_{accel} \) is the process noise for the phase acceleration (which is related to the receiver dynamics).

For the first existing non-coherent pre-filter algorithm (option \#2 in [14]), the system model is the same as option \#1, whereas the measurements are changed to the output of the carrier discriminator function and combination of \( I \) and \( Q \), which are shown as follows:

\[
Z_1 = \delta \phi = \arctan(Q_f/I_f)
\]

\[
Z_2 = \sqrt{I_f^2 + Q_f^2} = A \cdot \frac{\sin(\pi \cdot \delta f \cdot T)}{\pi \cdot \delta f \cdot T} \cdot D \cdot R(\delta \tau + \Delta k)
\]

The relationship between states and measurements in option \#1 would be nonlinear, and option \#2 has the problem of the measurement noise correlation originating from the nonlinear combination of \( I_s \) and \( Q_s \) data. Besides, the signal amplitude item in option \#1 may suffer the initial value setting problem and cause a convergence speed of the filter in some cases like a sudden change of the signal amplitude. Conversely, we can exclude the signal amplitude \( A \) from the system model and estimate it separately and quickly using the \( C/N_0 \) estimator. This change can avoid the initial signal amplitude value setting of the filter, speed up the convergence speed of the filter, and also reduce the dimensions of the Kalman filter. The signal amplitude item in option \#2 is not needed and can also be excluded from the state vector.

The system model of the second existing non-coherent pre-filter algorithm (Option \#3 in [14]) is shown as follows:

\[
\begin{bmatrix}
\delta \phi_0 \\
\delta \phi_0 \\
\delta I_m \\
\delta I_m
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
\delta \phi_0 \\
\delta \phi_0 \\
\delta I_m \\
\delta I_m
\end{bmatrix} \\
+ \begin{bmatrix}
w_A \\
w_{accel} \\
w_{Lm}
\end{bmatrix}
\]
where \( A \) is the normalized signal amplitude, \( \delta p_0 \) is the pseudorange error, \( \delta \rho_0 \) is the pseudorange error rate, \( \delta f_0 \) is the pseudo-range error accelerator, and \( \delta \rho_{\text{iono}} \) is the ionospheric error.

The accuracy of the pseudorange information and ionospheric correction error information (especially in single-point operation) are usually not accurate enough for carrier phase tracking in vector-tracking and deeply-coupled mode. Therefore, option \( \#3 \) is not appropriate for carrier tracking, as concluded by \([14]\).

2.2. Modification and improvement of the pre-filter algorithms

2.2.1. Coherent pre-filter algorithm. Firstly, we exclude the signal amplitude \( A \) from the system model and estimate it separately using the C/N0 estimator. The detail of \( A \) in (1) and (2) can be further described as

\[
A = \sqrt{2 \cdot (c/n_0) \cdot T \cdot \sigma_{\text{IQ}}} \tag{8}
\]

where \( c/n_0 \) represents the carrier power-to-noise density (unit of Hz), \( T \) is the integration interval, and \( \sigma_{\text{IQ}} \) is the standard deviation of noise.

Then, the state vector and the system model of the coherent pre-filter algorithm can be modified as

\[
X_{\text{coh}} = \begin{bmatrix} \delta \tau \cr \delta \phi_0 \cr \delta f_0 \cr \delta \rho_0 \cr \delta \rho_{\text{iono}} \cr 1 \cr \alpha \end{bmatrix} T
\]

\[
\begin{align*}
R_{\text{mp}} &= \begin{bmatrix} 0 & 0 & \beta & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \end{bmatrix} \\
R_{\text{coh}} &= \begin{bmatrix} W_{\text{mp}} \\ W_{\text{clock}} \\ W_{\text{drift}} \\ W_{\text{accel}} \\ \end{bmatrix}
\end{align*}
\]

(9)

It is known that in the coherent pre-filter algorithm, the GNSS accumulated correlator outputs \( I_p \) and \( Q_p \), directly to the Kalman filter as measurements in the coherent algorithm. The six-measurement model with measurements in-phase and quadra-phase prompt, early and late (\( I_p, Q_p, I_L, Q_L, I_F, Q_F \)) are used as the pre-filter algorithm, shown as

\[
Z_{\text{coh}} = A \cdot D \cdot h(X_{\text{coh}}) + n_{IQ} \tag{11}
\]

\[
\begin{align*}
I_p &= A \cdot \text{sinc}(\delta f \cdot T) \cdot D \cdot R(\delta \tau + \Delta_{-1}) \cdot \cos(\delta) \\
Q_p &= A \cdot \text{sinc}(\delta f \cdot T) \cdot D \cdot R(\delta \tau + \Delta_0) \cdot \sin(\delta) \\
I_F &= A \cdot \text{sinc}(\delta f \cdot T) \cdot D \cdot R(\delta \tau + \Delta_{+1}) \cdot \cos(\delta) \\
Q_F &= A \cdot \text{sinc}(\delta f \cdot T) \cdot D \cdot R(\delta \tau + \Delta_{+1}) \cdot \sin(\delta)
\end{align*}
\]

(12)

Besides, the ideal autocorrelation function \( R(\cdot) \) is not differentiable and also not realistic when the input RF signal has passed through a band-pass filter\([21-23]\). A more realistic autocorrelation function, which is a 6th order polynomial approximation of \( R(\cdot) \), is used in the local filter as follows:

\[
R(\tau) \approx x_0 \tau^6 + x_5 \tau^5 + x_4 \tau^4 + x_3 \tau^3 + x_2 \tau^2 + x_1 \tau + x_0. \tag{13}
\]

As shown in figure 1, the blue curve denotes the actual measured curve coming from the GNSS IF signal collector, the black curve is the nominal autocorrelation curve and the red curve denotes the fitting curve of the 6th order polynomial approximation (prompt branch). The other two curves are fitting curves of the 6th order polynomial approximation for early and late branches, respectively. It can be seen that the fitting curves of the 6th order polynomial approximation are more realistic than the nominal autocorrelation curve, especially when the code phase error is small. Although the approximation needs more computation than the nominal autocorrelation function, it can get a better accuracy of the autocorrelation function.

To solve the nonlinear problem and get a higher filter accuracy, a five-degree cubature Kalman filter (5th-CKF) for the coherent pre-filter is proposed. The 5th-CKF uses a series of cubature points to propagate the a priori and a posteriori statistical characteristics. The core of the CKF is a cubature transformation based on the spherical-radial rule\([24, 25]\).

The five-degree spherical-radial cubature rule’s points and weights can be calculated as follows:

\[
\begin{align*}
\xi_0 &= 0 \\
\xi_1 &= \sqrt{n + 2e_i} \\
\xi_2 &= -\sqrt{n + 2e_i} \\
\xi_3 &= \sqrt{n + 2s_i^+} \\
\xi_4 &= -\sqrt{n + 2s_i^+} \\
\xi_5 &= \sqrt{n + 2s_i^-} \\
\xi_6 &= -\sqrt{n + 2s_i^-}
\end{align*}
\]

(14)

where \( e_i \) denotes a unit vector in the direction of coordinate axis i. \( s_i^+ \) and \( s_i^- \) are described as

\[
\begin{align*}
\begin{cases}
s_i^+ = \sqrt{1/2}(e_i + e_l) \\
s_i^- = \sqrt{1/2}(e_l - e_i)
\end{cases}
\end{align*}
\]

(15)

\( (i = 1, 2, \ldots, n(n-1)/2; \ j < l, \ l = 1, 2, \ldots, n). \)

The coherent pre-filter based on 5th-CKF works as follows.
2.2.1. Time update. The posterior probability density of $x_{k-1}$ is known in previous update $p(x_{k-1}) = N(x_{k-1|k-1}, P_{x_{k-1|k-1}})$. The Cholesky decomposition of $P_{x_{k-1|k-1}}$ is calculated as follows:

$$P_{x_{k-1|k-1}} = S_{x_{k-1|k-1}}S_{x_{k-1|k-1}}^T.$$  \hfill (16)

The cubature points are calculated as

$$\begin{aligned}
X_{0,k-1|k-1} &= S_{x_{k-1|k-1}}\hat{S}_0 + \hat{x}_{k-1|k-1} \\
X_{r,k-1|k-1} &= S_{x_{k-1|k-1}}\xi_r + \hat{x}_{k-1|k-1},
\end{aligned}$$  \hfill (17)

(\(r = 1, 2, \ldots, 6; \quad i = 1, 2, \ldots, n\)).

Then, the sample points are obtained by propagating the above cubature points through the system model in (10), as follows:

$$\begin{aligned}
X_{s,0,k-1} &= F(X_{0,k-1|k-1}) \\
X_{s,r,k-1} &= F(X_{r,k-1|k-1}),
\end{aligned}$$  \hfill (18)

(\(r = 1, 2, \ldots, 6; \quad i = 1, 2, \ldots, n\))

where $F$ is the system matrix of the coherent pre-filter.

One-step state prediction $\hat{x}_{k|k-1}$ is then obtained by the weighted linear combination of sample points, as follows:

$$\begin{aligned}
\hat{x}_{k|k-1} &= w_0X_{s,0,k-1|k-1} + w_1 \frac{n(n-1)}{2} \sum_{i=1}^{n} X_{s,r,k-1|k-1} \\
&+ w_2 \sum_{i=1}^{n} X_{s,0,k-1|k-1},
\end{aligned}$$  \hfill (19)

One-step prediction error covariance $P_{x|k-1}$ is updated as follows:

$$\begin{aligned}
P_{x|k-1} &= w_0X_{s,0,k-1|k-1}X_{s,0,k-1|k-1}^T \\
&+ w_1 \frac{n(n-1)}{2} \sum_{i=1}^{n} X_{s,r,k-1|k-1}X_{s,r,k-1|k-1}^T \\
&+ w_2 \sum_{i=1}^{n} X_{s,0,k-1|k-1}X_{s,0,k-1|k-1}^T - \hat{x}_{k|k-1}^2
\end{aligned}$$  \hfill (20)

2.2.2. Measurement update. The Cholesky decomposition of $P_{a|k-1}$ is calculated as follows:

$$P_{a|k-1} = S_{a|k-1}S_{a|k-1}^T.$$  \hfill (21)

The cubature points are calculated as

$$\begin{aligned}
X_{0,a|k-1} &= S_{a|k-1}\hat{S}_0 + \hat{x}_{a|k-1} \\
X_{r,a|k-1} &= S_{a|k-1}\xi_r + \hat{x}_{a|k-1},
\end{aligned}$$  \hfill (22)

(\(r = 1, 2, \ldots, 6; \quad i = 1, 2, \ldots, n\)).

Then, the sample points are obtained by propagating the above cubature points through the measurement equation in (11), as follows:

$$\begin{aligned}
Z_{n|0,k-1} &= A \cdot D \cdot h(X_{0,k-1|k-1}) \\
Z_{n|r,k-1} &= A \cdot D \cdot h(X_{r,k-1|k-1}),
\end{aligned}$$  \hfill (23)

(\(r = 1, 2, \ldots, 6; \quad i = 1, 2, \ldots, n\))

One-step measurement prediction $\hat{z}_{k|k-1}$ is then obtained by the weighted linear combination of sample points as follows:

$$\begin{aligned}
\hat{z}_{k|k-1} &= w_0Z_{n,0,k-1|k-1} + w_1 \frac{n}{2} \sum_{i=1}^{n} Z_{n,i,k-1|k-1} \\
&+ w_2 \sum_{i=1}^{n} Z_{n,0,k-1|k-1},
\end{aligned}$$  \hfill (24)

The auto-correlation covariance matrix $P_{zz,k|k-1}$ is obtained as

$$\begin{aligned}
P_{zz,k|k-1} &= w_0Z_{n,0,k-1|k-1}Z_{n,0,k-1|k-1}^T \\
&+ w_1 \frac{n}{2} \sum_{i=1}^{n} Z_{n,i,k-1|k-1}Z_{n,i,k-1|k-1}^T \\
&+ w_2 \sum_{i=1}^{n} Z_{n,0,k-1|k-1}Z_{n,0,k-1|k-1}^T
\end{aligned}$$  \hfill (25)

The cross-correlation covariance matrix $P_{za,k|k-1}$ is calculated as follows:

$$\begin{aligned}
P_{za,k|k-1} &= w_0Z_{n,0,k-1|k-1}Z_{n,a,k-1|k-1} \\
&+ w_1 \frac{n}{2} \sum_{i=1}^{n} Z_{n,i,k-1|k-1}Z_{n,a,k-1|k-1} \\
&+ w_2 \sum_{i=1}^{n} Z_{n,0,k-1|k-1}Z_{n,a,k-1|k-1} \\
&- \hat{x}_{k|k-1}Z_{n,0,k-1|k-1}^T + R_k.
\end{aligned}$$  \hfill (26)

The Kalman filter gain is calculated as follows:

$$W_k = P_{za,k|k-1}P_{a|k-1}^{-1}.$$  \hfill (27)

The state estimation $\hat{x}_{k|k}$ is calculated as follows:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + W_k(\hat{z}_{k} - \hat{x}_{k|k-1}).$$  \hfill (28)

The state estimation error covariance $P_{k|k}$ is calculated as follows:

$$P_{k|k} = P_{k|k-1} - W_kP_{za,k|k-1}W_k^T.$$  \hfill (29)

where $\hat{x}_{k|k}$ and $P_{k|k}$ are used in the next iteration.

2.2.2. Non-coherent pre-filter algorithm. For the non-coherent pre-filter algorithm, we simplify the state vector and the system model as

$$X_{\text{nonCoh}} = [\delta \tau, \delta f, \delta \alpha]^T.$$  \hfill (30)

$$\begin{aligned}
\delta f &= \begin{bmatrix} \delta \tau \\ \delta f \end{bmatrix} \\
&= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \delta \tau \\ \delta f \end{bmatrix}
\end{aligned}$$  \hfill (31)

Meanwhile, the measurements are designed as

$$Z_{\text{nonCoh}} = [\delta \tau, \delta f]^T.$$  \hfill (32)

$$\begin{aligned}
\begin{bmatrix} \delta \tau \\ \delta f \end{bmatrix} &= \begin{bmatrix} \sqrt{\delta \tau^2 + \delta f^2} \\ \sqrt{\delta \tau^2 + \delta f^2} \\ \arctan(\frac{\delta \tau}{\delta f}) \end{bmatrix} \\
&= \begin{bmatrix} n_\tau \\ n_f \end{bmatrix}
\end{aligned}$$  \hfill (33)
The narrow-to-wideband power ratio, $P_{nw}$, is simply the ratio of the two power measurements. However, to reduce the noise, the measurement is averaged over $K$ iterations. Thus,

$$\mu_P = \frac{1}{K} \sum_{i=1}^{K} P_{nw}. \quad (36)$$

Finally, the measured carrier carrier power-to-noise density (Hz) is derived as a function of the power ratio measurement, shown as

$$\frac{c}{n_0} = \frac{M \mu_P - 1}{\tau (M - \mu_P)}. \quad (37)$$

Besides, the standard deviation of the can be calculated as follows:

$$\sigma \left( \frac{c}{n_0} \right) = \frac{M}{\tau} \left( \frac{M - 1}{M - \mu_P} \right)^{3/2} \sigma(P_{nw}). \quad (38)$$

Just as the $C/N_0$ value contains errors, a single threshold used to decide whether to switch the system mode or not may cause an abnormal frequency switch between the system's modes. To avoid this case, the adaptive hysteresis controller is designed based on the thought of a hysteresis-comparator circuit, which is shown in figure 3.
are the position of the user and 10th and s non-coherent pre-filter th satellite respectively. The measurement be written as up (ENU) frame.

The deep fusion of the combined GNSS and INS is accom-
plished by an integrated navigation filter. The states vector matrix \( X_{\text{nav}} \) of the navigation filter is shown as

\[
X_{\text{nav}} = \begin{bmatrix} \delta \rho_x, \delta \phi_x, \delta v_x, \delta v_n, \delta \theta_x, \delta \theta_n, \\
\delta \rho_y, \delta \phi_y, \delta v_y, \delta v_w, \delta \theta_y, \delta \theta_w \end{bmatrix}^T.
\]

(39)

The navigation filter estimates the errors of the user’s three position, three velocity, three attitude, three gyroscope bias, three acceleration bias, clock bias and clock drift, respectively. The position error states are shown in a geodetic coordinate system. The velocity error states are shown in an east-north-up (ENU) frame.

The outputs of the hybrid pre-filters are taken as measurement for the integrated filter based on the errors of the replica code and carrier signals having relationships with the residual errors of the INS. The measurement of the integrated filter can be written as

\[
Z_{\text{nav}} = \begin{bmatrix} \delta \rho_1, \delta \rho_2, \cdots, \delta \rho_n \\
\delta \rho_1, \delta \rho_2, \cdots, \delta \rho_n \end{bmatrix}^T
\]

where \( \delta \rho_1 \) and \( \delta \rho_n \) represent the pseudorange and pseudorange rate residual of jth satellite, respectively. The measurement states are derived from the following equation:

\[
\begin{bmatrix} \delta \rho \\ \delta \dot{\rho} \end{bmatrix} = \begin{bmatrix} \delta \tau \cdot \frac{c}{f_{\text{carrier}}} \\ - \delta f \cdot \frac{c}{f_{\text{carrier}}} \end{bmatrix}
\]

(41)

where \( \delta \tau \) and \( \delta f \) are the code phase error and carrier frequency error coming from the pre-filter, respectively. \( f_{\text{carrier}} \) and \( f_{\text{coded}} \) denote the normalized carrier frequency and the ranging code chipping rate of GNSS signals. \( c \) is the speed of light.

The observation matrix given below in (42) is linearized at each measurement epoch to accommodate the error measurements from each channel:

\[
H_{\text{nav}} = \begin{bmatrix} s_1^n \ T_s \ T_s \ 0_{1 \times 12} \ 1 \ 0 \\
0_{1 \times 3} \ t_1^n \ t_1^n \ t_1^n & 0_{1 \times 10} & 1 \\
\vdots & \vdots & \vdots & \vdots
\end{bmatrix}
\]

(42)

where

\[
\begin{bmatrix} s_1^n \ T_s \ T_s \ 0_{1 \times 12} \ 1 \ 0 \\
0_{1 \times 3} \ t_1^n \ t_1^n \ t_1^n & 0_{1 \times 10} & 1 \\
\vdots & \vdots & \vdots & \vdots
\end{bmatrix}
\]

\[
\begin{bmatrix} e_1^n \ T_s \ T_s \ 0_{1 \times 12} \ 1 \ 0 \\
0_{1 \times 3} \ t_1^n \ t_1^n \ t_1^n & 0_{1 \times 10} & 1 \\
\vdots & \vdots & \vdots & \vdots
\end{bmatrix}
\]

(43)

\[
\begin{bmatrix} \delta \rho_1, \delta \phi_1, \delta v_1, \delta v_n, \delta \theta_1, \delta \theta_n \\
\delta \rho_1, \delta \phi_1, \delta v_1, \delta v_n, \delta \theta_1, \delta \theta_n \end{bmatrix} = \begin{bmatrix} -\lambda \cos \phi \sin \lambda \cos \phi \cos \lambda \\
-\lambda \cos \phi \sin \lambda \cos \phi \cos \lambda \end{bmatrix}
\]

(44)

\[
\begin{bmatrix} \delta \tau \cdot \frac{c}{f_{\text{carrier}}} \\ - \delta f \cdot \frac{c}{f_{\text{carrier}}} \end{bmatrix}
\]

(45)

where \( X_u \) and \( X_j \) are the position of the user and jth GNSS satellite, respectively.

3. NCO feedback control

The NCO feedback control is based on the integrated navigation solution to the GNSS tracking loops forms another important part of the deep-coupling strategy. The correct position and velocity states of the INS and the estimated clock states are converted into pseudoranges and range rates (Doppler...
frequency), and subsequently used to update the code and carrier NCOs.

The Doppler frequency for the \( j \)th tracking channel is predicted using (46) or can be a filtered version of its measurement:

\[
\hat{f}^j = \frac{-\left((\hat{\mathbf{v}}_u - \mathbf{v}_j^\parallel) \cdot \mathbf{e}^j + \hat{t}_j - t_j^\parallel\right)}{c} f_{\text{carrier}0}
\]  

(46)

where \( \mathbf{v}_j^\parallel \) and \( t_j^\parallel \) are the velocity vector and clock drift of the \( j \)th satellite. \( \mathbf{v}_u \) and \( t_u \) are the user’s velocity vector and clock drift. \( c \) is the speed of light.

Then, the carrier frequency is generated as

\[
\hat{f}^j_{\text{carrier}} = f_{\text{IF}} + \hat{f}^j + f_{\text{nco}}
\]

\[
= f_{\text{IF}} - \left((\hat{\mathbf{v}}_u - \mathbf{v}_j^\parallel) \cdot \mathbf{e}^j + \hat{t}_j - t_j^\parallel\right) \frac{f_{\text{carrier}0}}{c} + f_{\text{nco}}
\]  

(47)

where \( f_{\text{nco}} \) is the carrier NCO correction item which is generated by the estimated carrier phase errors after passing the carrier loop filter.

The pseudorange for the \( j \)th tracking channel is predicted in (48):

\[
\hat{\rho}^j = \left\| \mathbf{X}_u - \mathbf{X}_s^j \right\| + \hat{r}^j
\]

\[
= \sqrt{\left(\hat{x}_u - x_s^j\right)^2 + \left(\hat{y}_u - y_s^j\right)^2 + \left(\hat{z}_u - z_s^j\right)^2} + \hat{r}^j
\]  

(48)

where \( x_u^j, y_u^j, z_u \) and \( r_u \) are the user’s predicted position and clock bias. \( x_s^j, y_s^j, z_s^j \) are the \( j \)th satellite’s position. The code frequency is generated as

\[
\hat{f}^j_{\text{code},k+1} = f_{\text{code}0} \cdot \left[1 - \frac{\hat{\rho}^j_{k+1} - \hat{\rho}^j_k}{c \cdot \tau_N}\right]
\]  

(49)

where \( \tau_N \) is the code NCO update period, \( k \) denotes the \( k \)th update of the NCO.

Another way of generating the code frequency is by using the carrier-aided code structure, which can be expressed as follows:

\[
\hat{f}^j_{\text{code}} = f_{\text{code}0} \cdot \left(1 + \hat{f}^j \cdot \frac{1}{f_{\text{carrier}0}}\right).
\]  

(50)

4. Results and discussions

4.1. Test description

Two sets of kinematic data were used to compare the performance of the adaptive deeply-coupled system with hybrid pre-filters processing. The first kinematic data were collected using a hardware GNSS simulator to assess the performance of the modified pre-filter algorithms. The second data were collected using vehicle tests to compare the performance of the adaptive deeply-coupled system with hybrid pre-filters processing (Hybrid-DC), the single coherent deeply-coupled method (Coherent-DC) and the single non-coherent deeply-coupled method (nonCoherent-DC) under a GNSS-challenged environment.

4.1.1. Simulation test description. The HWA-RNSS 7300 hardware simulator is a multiple constellation and frequency GNSS simulator. The GNSS IF signal collector is a digital down converter that can receive GNSS signals through the GNSS antenna and then convert the high-frequency GNSS signals down to lower frequency signals. The data collection process is shown in figure 4.

In this simulation, the GPS L1 CA signals are simulated. The parameters of the simulated errors are set by the simulator and the known broadcast ephemeris which is stored in the simulator. The simulated errors include ionospheric error (using the Klobuchar model), tropospheric error (using the Hopfield model), the errors of broadcast orbits, the satellites’ clock errors and relativistic effect. All these errors are preset by the simulator.

Besides, the true trajectory files from the hardware simulator are used to simulate the IMU information. Table 1 shows the detail parameters defined in the simulation test system.

| Parameter                             | Values        |
|---------------------------------------|---------------|
| Gyro bias                             | 1 deg h\(^{-1}\) |
| Gyro noise density                    | 0.004 deg/s/√Hz |
| Accelerometer bias                    | 0.005 mg/√Hz |
| Accelerometer noise density           | 0.015 mg/√Hz |
| GNSS sampling frequency               | 16.369 MHz   |
| GNSS IF frequency                     | 3.996 MHz    |
| Coherent integration time             | 10 ms        |
| Pre-filter period                     | 100 Hz       |
| Integrated filter period              | 10 Hz        |

Figure 4. Data collection process with GNSS hardware simulator.

Table 1. Parameters defined in the system.

GNSS simulator. The GNSS IF signal collector is a digital down converter that can receive GNSS signals through the GNSS antenna and then convert the high-frequency GNSS signals down to lower frequency signals. The data collection process is shown in figure 4.

The experiment equipment is carried on a car, shown in figure 5(a). Figure 5(b) shows the data collection system. The GNSS IF signal collector is the same as above in figure 4. The IMU is manufactured by Inertial Labs. The reference system is shown in figures 5(c) and (d). Two ProPak6 receivers are used as rover receiver (configured as a GNSS/INS version) and base station receiver, respectively. The detail parameters defined in the vehicle test system are the same as the simulation test, which is shown in table 1.
The proposed system works in single-point mode, while the reference system works in RTK mode and provides the precise position and velocity results as references to evaluate the performance of the proposed system.

4.2. Test results and discussions

4.2.1. Simulation test. During the test, the parameters of visible satellites and received signal power were set and shown in table 2.

The existing coherent pre-filter (E-Coherent), modified coherent pre-filter (M-Coherent), existing non-coherent pre-filter (E-nCoherent) and modified non-coherent pre-filter (M-nCoherent) methods are compared firstly for their tracking and navigation performance.

The carrier phase lock indicator (PLI) is used as the data analysis criterion. The PLI can be expressed as

$$\text{PLI} \approx \cos(2\delta\varphi).$$

The PLI is equal to 1 when the phase is perfectly locked and it is equal to –1 when the phase has no lock. We chose four satellites – SV2, SV10, SV17 and SV28 – to analyse the performance of the different algorithms.

Figure 6 shows the variation in carrier PLI of SV17 and SV28 using different pre-filter algorithms. Table 3 shows the PLI value statistics.

The Doppler frequency reference value from the hardware simulator is used to assess the tracking accuracy of Doppler frequency for the different algorithms. Figures 7–10 shows the Doppler frequency errors of SV17, SV28, SV2 and SV10 using different algorithms, respectively. Table 4 shows the Doppler frequency error statistics.

As shown in figures 7–10 and table 4, the Doppler frequency errors of the M-Coherent algorithm are reduced around 42% of the errors of E-Coherent algorithm for high signal-to-noise signals. Meanwhile, the errors of M-nCoherent algorithm are reduced around 41% and 29% of the errors of the E-nCoherent algorithm for high signal-to-noise signals and low signal-to-noise signals, respectively. In addition, both the coherent algorithms reach a better Doppler frequency tracking accuracy than the non-coherent algorithms for high signal-to-noise signals. However, the non-coherent algorithms can track lower C/N0 signals (i.e. SV2 and SV10) which means that the non-coherent algorithms have stronger robustness than coherent algorithms.

The position and velocity errors using different pre-filter algorithms are shown in figures 11 and 12, respectively. Tables 5 and 6 show the position and velocity error statistics, respectively. It is noted that the start epoch of figures 11 and 12...
The position error results shown in figure 11 and table 5 indicate that the position errors of the M-Coherent algorithm are reduced around 30%, 26% and 48% of the errors of the E-Coherent algorithm in east, north and up direction, respectively. The position errors of the M-nonCoherent algorithm are reduced around 10%, 2% and 22% of the errors of the E-nonCoherent algorithm in east, north and up direction, respectively. It can be seen that both of the coherent algorithms reach a better position accuracy than the non-coherent algorithms. Besides, all the coherent and non-coherent algorithms have a better position accuracy than the GNSS-only solution.

The velocity error results shown in figure 12 and table 6 indicate that the velocity errors of the M-Coherent algorithm are reduced around 45%, 46% and 39% of the errors of the E-Coherent algorithm in east, north and up direction, respectively. The velocity errors of the M-nonCoherent algorithm are reduced around 42%, 36% and 37% of the errors of the E-nonCoherent algorithm in east, north and up direction, respectively. It can be seen that both of the coherent algorithms reach a better velocity accuracy than the non-coherent algorithms.
a better velocity accuracy than the non-coherent algorithms. Besides, all the coherent and non-coherent algorithms have a better velocity accuracy than the GNSS-only solution.

The simulation test results indicate that the coherent algorithms can reach a better tracking and position accuracy while the non-coherent algorithms have a stronger robustness to track lower signal-to-noise signals. The proposed modified coherent and non-coherent algorithms (i.e. $M$-Coherent and $M$-nonCoherent) are proven improvements of both tracking and position performance compared to the existing

| Method          | $E$  | $N$  | $U$  |
|-----------------|------|------|------|
| GNSS-only       | 2.39 | 1.73 | 2.79 |
| $E$-Coherent    | 1.11 | 0.76 | 1.58 |
| $M$-Coherent    | 0.78 | 0.57 | 0.82 |
| $E$-nonCoherent | 1.69 | 1.07 | 2.20 |
| $M$-nonCoherent | 1.52 | 1.04 | 1.71 |
coherent and non-coherent algorithms (i.e. E-Coherent and E-nonCoherent).

### 4.2.2. Vehicle test

The vehicle test is conducted to compare the performance of the adaptive deeply-coupled system with hybrid pre-filters processing (Hybrid-DC), the single coherent deeply-coupled method (Coherent-DC) and the single non-coherent deeply-coupled method (nonCoherent-DC) under GNSS-challenged environment. The Hybrid-DC, Coherent-DC and nonCoherent-DC methods consist of the proposed modified coherent and non-coherent algorithms.

Figure 13 shows the trajectory of the vehicle test. The car passes through the building, seen as a sheltered environment as shown in figure 13.

It can be seen in figure 14 that all C/N₀ values of the satellites decrease rapidly when the car passes through the building. During the pass time, almost all the satellites are blocked. After that, the signals recover gradually.

Figure 15 shows the tracking modes for different satellites using the Hybrid-DC method. The tracking mode of each channel switches automatically according to the signal quality to search for an optimal performance.

Figure 13 shows the trajectory of the vehicle test. The car passes through the building, seen as a sheltered environment as shown in figure 13.

It can be seen in figure 14 that all C/N₀ values of the satellites decrease rapidly when the car passes through the building. During the pass time, almost all the satellites are blocked. After that, the signals recover gradually.

Table 6. RMS velocity error statistics.

| Method           | $V_E$  | $V_N$  | $V_U$  |
|------------------|--------|--------|--------|
| GNSS-only        | 0.181  | 0.125  | 0.196  |
| E-Coherent       | 0.026  | 0.021  | 0.031  |
| M-Coherent       | 0.015  | 0.011  | 0.019  |
| E-nonCoherent    | 0.110  | 0.077  | 0.117  |
| M-nonCoherent    | 0.064  | 0.049  | 0.073  |

Figure 13. The trajectory of the vehicle test.

Figure 14. C/N₀ values of different satellites.

Figure 15. (a) and (b) The tracking modes for different satellites using the Hybrid-DC method.

The tracking information of $I_P$ and $Q_P$ for SV7 and SV27 are shown in figures 16 and 17. It can be seen that the signals are blocked at about 26s for both SV7 and SV27 when the car begins passing through the building. The channels are maintained mainly by the navigation feedback information (state 1). The channels using the Coherent-DC method lose lock at about 38s (state 2) and return to re-acquisition and re-tracking (state 3) at about 46s for SV7 and about 40s for SV27. However, the channels using the nonCoherent-DC and Hybrid-DC methods remain in state 1 until the car
passes out of the building and recover tracking quickly (state 4). These results indicate that both of the nonCoherent-DC and Hybrid-DC methods have a stronger robustness for maintaining and recovering the blocked signal quickly.

Figure 18 shows the number of available satellites during the operating time. It can be seen that the proposed Hybrid-DC method can track the most number of satellites all the time. The nonCoherent-DC method also has a better tracking performance than the Coherent-DC method due to its better robustness.
The position and velocity errors using different methods are shown in figures 19 and 20, respectively. Tables 7 and 8 show the position and velocity error statistics, respectively.

The position error results shown in figure 19 and table 7 indicate that the position errors of the Hybrid-DC method are reduced around 15%, 20% and 26% of the errors of the Coherent-DC method and around 32%, 34% and 59% of the errors of the nonCoherent-DC method in east, north and up direction, respectively. Besides, all the coherent, non-coherent and hybrid algorithms have a better position accuracy than the GNSS-only solution.

The velocity error results shown in figure 20 and table 8 indicate that the velocity errors of Hybrid-DC method are reduced around 10%, 13% and 29% of the errors of the Coherent-DC method and around 41%, 40% and 37% of the errors of the nonCoherent-DC method in east, north and up direction, respectively. Besides, all the coherent, non-coherent and hybrid algorithms have a better velocity accuracy than the GNSS-only solution.

The vehicle results indicate that the proposed Hybrid-DC method reaches an optimal tracking and navigation performance by combining the advantages of the coherent and non-coherent algorithms. In the test, the robustness of the non-coherent mode enables the Hybrid-DC method to track more satellites especially in a GNSS-challenged environment. Meanwhile, the coherent mode helps the Hybrid-DC method to reach a better accuracy.

5. Conclusions and future work

The existing pre-filters of the deeply-coupled structures are modified and the improvement of the tracking performance are proved by the simulation test. An adaptive deeply-coupled GNSS/INS navigation system with hybrid pre-filters processing is proposed to combine the advantages of the coherent and non-coherent algorithms. The vehicle test results show that the proposed system can achieve accuracy and robustness performance preferably under a GNSS-challenged environment, compared to the single coherent and non-coherent deeply-coupled method.

Note that only the GPS constellation and L1 CA signal are designed and simulated in the presented system, other GNSS constellations (such as Beidou, GLONASS and Galileo) can be included in the future work to realize joint tracking and navigation so as to reach a better performance.

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