Ultra High Energy Cosmic Rays Diffusion in an Expanding Universe

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Abstract: We study the solution of the diffusion equation for Ultra-High Energy Cosmic Rays in the general case of an expanding universe, comparing it with the well known Syrovatsky solution obtained in the more restrictive case of a static universe. The formal comparison of the two solutions with all parameters being fixed identically reveals an appreciable discrepancy. This discrepancy is less important if in both models a different set of best-fit parameters is used.

Introduction

Diffusive propagation of Ultra High Energy Cosmic Rays (UHECR) in extragalactic space has been recently studied by [1, 2, 8, 3] using the Syrovatsky solution (see [9]) of the diffusion equation. The Syrovatsky solution is obtained under the restrictive assumptions of time-independent diffusion coefficient \(D = D(E)\) and energy losses of particles \(dE/dt = b(E)\). Recently two papers appeared [5, 6] solving the problem of the generalization of the diffusion equation (and its solution) in the case of an expanding universe, i.e. in the case of time dependent diffusion coefficient and energy losses. In these works an analytic solution of the diffusion equation in an expanding universe was found, valid in the general case of time-dependent diffusion coefficient and energy losses, we will refer to this solution as the Berezinsky-Gazizov (BG) solution [5]. In the present paper we will compare, following the approach of [6], the spectra computed in the generalized case (BG solution) and the spectra obtained with the Syrovatsky solution as in the above cited papers. The diffusion equation for ultra-relativistic particles propagating in an expanding universe from a single source, as obtained in [5], reads

\[
\frac{\partial n}{\partial t} - b(E,t) \frac{\partial n}{\partial E} + 3H(t)n - n \frac{\partial b(E,t)}{\partial E} - \frac{D(E,t)}{a^2(t)} \nabla_x^2 n = \frac{Q_s(E,t)}{a^3(t)} \delta^3(\vec{x} - \vec{x}_g),
\]

where the coordinate \(\vec{x}\) corresponds to the comoving distance and \(a(t)\) is the scaling factor of the expanding universe, \(n = n(t,\vec{x},E)\) is the particle number density per unit energy in an expanding volume, \(dE/dt = -b(E,t)\) describes the total energy losses, which include adiabatic \(H(t)E\) as well as interaction \(b_{int}(E,t)\) energy losses. \(Q_s(E,t)\) is the generation function, that gives the number of particles generated by a single source at coordinate \(\vec{x}_g\) per unit energy and unit time.

According to [5], the spherically-symmetric solution of Eq. (1) is

\[
n(x_g,E) = \int_0^{\infty} dz \frac{dt}{dz}(z) Q_s[E_g(E,z),z] \exp[-x_g^2/4\lambda(E,z)] \frac{dE_g}{dE}(E,z),
\]

where

\[
\frac{dt}{dz}(z) = -\frac{1}{H_0 (1 + z)\sqrt{\Omega_m (1 + z)^3 + \Lambda}}
\]

with cosmological parameters \(\Omega_m = 0.27\) and \(\Lambda = 0.73\),

\[
\lambda(E,z) = \int_0^{z} dz' \left| \frac{dt'}{dz'} \right| \frac{D(E_g,z')}{a^2(z')},
\]
In the present paper we will not perform a detailed discussion of the proton diffusion in the general case of an expanding universe, we will address this issue in a forthcoming paper [4], our main goal here is to perform a detailed comparison of the BG solution with the Syrovatsky solution. As already discussed in [6], the difference between these two solutions is substantial at energies $E \lesssim 3 \times 10^{18}$ eV, where the effect of the universe expansion (in particular, of the CMB temperature growth with red-shift) is not negligible. The high energy tail of the UHECR spectrum is less affected by the expansion of the universe, nevertheless it is interesting to test the compatibility of the BG and Syrovatsky spectra at these energies where a substantial agreement of the two is expected.

**Diffusive energy spectra of UHECR**

In the present calculations we used a simplified description of the IMF evolution with redshift, namely we parametrize the evolution of magnetic configuration $(l_c, B_c)$ as

$$l_c(z) = l_c/(1 + z), \quad B_c(z) = B_c (1 + z)^{2 - m},$$

where the term $(1 + z)^2$ describes the depletion of the magnetic field with time due to the magnetic flux conservation and $(1 + z)^{-m}$ due to MHD amplification of the field. The critical energy $E_c(z)$ found from $r_L(E) = l_c(z)$ is given by

$$E_c(z) = 0.93 \times 10^{18} (1 + z)^{1 - m} \frac{B_c}{1 \text{ nG}}$$

for $l_c = 1$ Mpc. The maximum redshift used in the calculations is $z_{\text{max}} = 4$.

Following [2], we have computed the diffuse flux assuming a distribution of sources on a lattice with spacing $d$ and an injection spectrum, equal for all sources, given by

$$Q_s(E) = \frac{q_0(\gamma_g - 2)}{E_0^2} \left( \frac{E}{E_0} \right)^{-\gamma_g},$$

where $E_0$ is a normalizing energy (we used $E_0 = 1 \times 10^{18}$ eV) and $q_0$ represents the source luminosity in protons with energies $E \geq E_0$. The corresponding emissivity $\mathcal{L}_0 = q_0/d^3$, i.e. the energy production rate in particles with $E \geq E_0$.
Figure 1: Convergence of the diffusive solution to the universal spectrum when the distance between sources diminishes from 50 to 10 Mpc shown by numbers on the curves. The number densities of particles calculated in rectilinear and diffusive approximations, respectively, are equal at $r \sim l_d$, where $Q$ is the rate of particle production. We calculated the number densities of protons $n(E, r)$ numerically for both modes of propagations with energy losses of protons taken into account, and the transition is taken from the equality of the two spectra. We know that this recipe is somewhat rough and an interpolation between the two regimes is required [2]. However, this interpolation is somewhat difficult because the diffusive regime sets up at distances not less than six diffusion lengths $l_d$. At distances $l_d \leq r \leq 6 l_d$ some intermediate regime of propagation is valid. When studied in numerical simulations (e.g., [10]), the calculated number density $n(E, r)$ satisfies the particle number conservation $4 \pi r^2 n u = Q$, where $u$ is the streaming velocity, while with a simple interpolated spectrum this condition is not fulfilled a priori. In the present paper we will not address this problem, that will be studied in a forthcoming paper [4], assuming the rough recipe for the transition between diffusive and rectilinear regimes depicted above. This computation scheme can produce artificial features in the spectra, that are useful as a mark of the transition between the two regimes.

The direct comparison of the BG and Syrovatsky solutions of the diffusion equations is not possible because they are embedded in different cosmological environments. While the BG solution is valid for an expanding universe, the Syrovatsky solution is valid only for a static universe. Using two
Figure 3: Best fit comparison of the BG (expanding universe) and Syrovatsky (static universe) solutions, for $\gamma_g = 2.7$, $L_0 = 2.4 \times 10^{45}$ erg/Mpc$^3$yr and $d = 30$ Mpc. The magnetic field configuration assumed is $B_c = 0.1$ nG and $l_c = 1$ Mpc with different diffusion regimes as indicated on the plot.

The comparison of the two solutions is given in Figures 2 and 3 in the case of $B_c = 0.1$ nG and $l_c = 1$ Mpc with a source spacing $d = 30$ Mpc. From these figures one can see a reasonably good agreement between the Syrovatsky solution, embedded in a static universe model, with the BG solution for an expanding universe at energies $E > 1 \times 10^{18}$ eV, at smaller energies appears a noticeable discrepancy between the two solutions that is natural and understandable as discussed in the introduction. We conclude stating that, from a physical point of view, the second method of comparison is more meaningful and it gives a substantial agreement of the spectra obtained in the two cases.

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