The injection of phonons into superfluid $^4$He.

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Abstract. The injection of phonons into superfluid helium can give a wide variety of behaviours. The liquid helium is so cold that the ambient phonons can be neglected. Our measurements show that the injected phonons can be strongly interacting and confined to a narrow cone along the normal to the heater, or they can be non interacting and spread out in the half space in front of the heater. These are the extreme behaviours and it is possible to go continuously between them. The actual behaviour is mainly determined by the shape of the phonon dispersion curve in the helium which varies with pressure, and at zero pressure there is anomalous dispersion up to a high phonon energy. This critical energy decreases with pressure until at 19 bar the dispersion is normal. The various behaviours depend on the range of phonon energies that can scatter by the strong three phonon process which is allowed if the dispersion is anomalous. Our phonon-energy dependent detector enables us to analyse the phonon signals which are measured as a function of angle and pressure. The entire range of behaviours can be explained by the complex effects arising from phonon scattering.

1. Introduction

The simple action of a heating pulse in a thin film heater immersed in liquid $^4$He creates a complex behaviour in the helium. There are a train of interactions which determine the spatial distribution of the energy density in the liquid helium. The frequency of the phonons also varies in time and space. This complex behaviour stems from the anomalous dispersion of liquid helium: the phonon energy initially increases slightly faster than linearly with phonon momentum, and then above a certain energy, ($\epsilon_c$), it increases slower than linearly. The initial upward dispersion allows three phonon processes, 3pp, which do not conserve phonon number. This a very strong scattering process and leads to rapid phonon energy decay if the phonon density is low, but to a dynamic equilibrium, with both decay and creation, if the phonon density is high. The 3pp interaction is not allowed as soon as the dispersion becomes normal, so that above the phonon energy $\epsilon_c$, only four phonon processes (4pp) are allowed. In 3pp, the scattering angles are small, but with 4pp, they are large.

Liquid $^4$He is very compressible and not only does sound velocity change rapidly with pressure, but the shape of the dispersion curve also changes. The anomalous dispersion decreases with pressure and it has completely disappeared by 19 bar. The value of the critical energy $\epsilon_c$, decreases with pressure too, so the range of phonon energies, that can scatter by 3pp, decreases. Hence pressure has a profound effect on the frequency and spatial distribution of the phonons injected into liquid $^4$He.

The behaviour of phonons injected into cold liquid $^4$He has been studied in detail in the directions along and close to the normal to the heater. Several unexpected phenomena have been discovered: the creation of high energy phonons from low energy phonons, the creation
of phonon sheets and the formation of hot lines in the helium. These have been analysed theoretically. In this paper we explore the phonon propagation at all angles in the half plane in front of the heater. We present measurements over the full range of pressures and describe the reasons for the different behaviours at different angles and pressures. More details and references can be found in [1].

2. The experiment
A thin film gold heater, 1 mm × 1 mm is rotated opposite to, and 12.3 mm above, a superconducting zinc bolometer. Recent experiments have shown that the sensitive area of the bolometer is ∼ 0.1 mm × ∼ 0.1 mm. It is most likely that the sensitive region is a single superconducting-normal boundary across the track width of ∼ 0.1 mm. The experimental cell was filled with ultra pure liquid ⁴He which could be pressurised up to 24 bar. The temperature of the cell was ∼ 50 mK during measurements.

![Figure 1](image1.png)

**Figure 1.** The angular distribution of the signal, at the time of the l-phonon peak, at various pressures. The heater pulse is 100 ns and the heater power 12.5 mW.

![Figure 2](image2.png)

**Figure 2.** A block diagram of the interactions for θ > 20° and θ < 5°.

3. Results and discussion
The angular distribution of the bolometer signal, shown in figure 1, varies strongly with pressure. We model the signals with the processes described in figure 2. At P = 0 bar, the angular distribution has a mesa shape centred at θ = 0. At larger angles the signal is very much smaller. At P = 6 bar, the mesa is replaced by a cusp-like maximum which persists to P = 12 bar. At P = 15 bar the signal at θ = 0 has decreased but at θ > 20° it has grown so that the relatively small angular peak for θ < 20° is on a broad cosine-like curve for θ > 20°. This trend continues to 17 bar. At P = 20 and 24 bar there is only the cosine-like curve.

The signal at θ > 20° increases slowly with pressure up to P ∼ 15 bar and then rapidly up to P = 19 bar, after which it is nearly constant, as shown in figure 3 for θ = 33°. The signal at θ = 0° is shown in figure 4. The signal shows a strong increase in the range 0 < P < 7.5 bar, a decrease in the range 7.5 < P < 15 bar, an increase in the range 15 < P < 19 bar and, thereafter, remains constant.

The angular distribution at P > 19 bar is most easily understood. The injected spectrum of phonons propagates unchanged to the bolometer so the angular distribution is that emitted by
Figure 3. The signal at the time of the l-phonon peak, at an angle of $\theta = 33^\circ$, is shown as a function of pressure, for a heater power 12.5 mW. The injected phonons decay to low energies below $P \sim 15$ bar.

Figure 4. The signal at the time of the l-phonon peak, at the centre of the angular distribution, i.e. $\theta = 0$, is shown as a function of pressure, for a heater power 12.5 mW. The signal has some contribution from the h-phonons at intermediate pressures.

The measured cosine-like behaviour is consistent with the transmission of phonons from the heater to the liquid $^4$He via the background channel. This is due to scattering at the surface of the heater, so that a phonon in the heater creates two or more phonons in the liquid $^4$He. We see that there is no sign of an acoustic transmission channel which would give a narrow peak centred on $\theta = 0$. This is as expected because the gold film heater is rough.

The behaviour at $0 < P < 19$ bar is due to two effects; the interaction of phonons and the change in the sensitivity of the bolometer with incident phonon energy. These two effects can be seen to have a profound influence on the size of the signal, when one remembers that nearly the same energy is injected into liquid $^4$He at all pressures. At large angles, the small signal at low pressures is due to the decay of the phonons into many low energy phonons that have a very small transmission probability into the detector. As pressure increases, fewer phonons decay and a greater fraction of the phonon energy density is in higher energy phonons which have a higher transmission probability into the detector. At small angles, the signal is large due to high phonon energies; at low pressure this is due to a high density of phonons which strongly interact which keeps the phonon temperature relatively high, and at high pressures it is due to fewer phonons decaying, as for large angles We model the signals at $\theta > 20^\circ$ and $\theta = 0$ with the processes described in figure 2.

The temperature of the injected phonons is $\sim 0.6$ K, and is largely independent of heater power. However the intensity of the spectrum scales with heater power. At zero pressure, phonons emitted at small angles to the heater normal, $\theta < \sim 5^\circ$ strongly interact and form a dynamic equilibrium. The temperature of these l-phonons, at the bolometer, is (coincidentally) found to be 0.6 K, which is a little lower than 0.7 K which was expected from the theoretical h-phonon creation rate. However there appears to be more higher-energy phonons than in a Bose-Einstein spectrum at a temperature of 0.6 K.

At $\theta = 0$ the l-phonons are thermalised for pressures 0 to 7.5 bar but are less than completely thermalised at higher pressures. The thermalisation decreases to near zero by 15 bar, see the dotted line in figure 6. The interaction rate diminishes with angle and is zero at $\theta \sim 20^\circ$.
For $\theta > 20^\circ$ the injected phonons with $\epsilon < \epsilon_c$, only spontaneously decay to low energies and give little signal. This is presumably due to the low phonon density at large $\theta$. The phonons with $\epsilon > \epsilon_c$ are not scattered and contribute nearly all the signal. This signal increases with pressure as $\epsilon_c(P)$ decreases with pressure and fewer phonons decay, see figure 5.

At angles near to the heater normal and for pressures 0 to 7.5 bar, about half the energy in phonons with $\epsilon < \epsilon_c$, in the injected spectrum, go to h-phonons. From 7.5 bar to 15 bar the creation of h-phonons decreases approximately linearly to zero, see the dash-dot line in figure 6. The narrow cusp-like peaks in the angular distribution at 6 and 12 bar are due to h-phonons being created in a narrow cone in momentum space, in agreement with theory.

4. Conclusions
We have measured the phonon signal, at the time of the l-phonons, as a function of angle, for phonons emitted into cold liquid $^4$He by a small plane heater. The signal varies greatly with pressure even though the injected energy is the same. This is due to the energy spectrum of the phonons changing from the injected spectrum, through 3pp and 4pp interactions. As the sensitivity of the zinc bolometer increases linearly with phonon energy, the signal is greatly decreased if the phonons decay to low energies. At low pressures, 3pp scattering is allowed up to high phonon energies and this leads to a strongly interacting group of l-phonons at a relatively high temperature. At large angles $\theta > 20^\circ$, the injected energy density is low and the injected phonons only decay to low energies. As pressure increases, the range of phonon energies that can interact, gradually decreases to zero, and for $P > 19$ bar the angular distribution is that injected by the heater. The creation of h-phonons, through 4pp interactions, leads to l-phonon sheets being formed at $P \sim 0$ bar, and cusp-like angular distributions at 6 to 12 bar.

We acknowledge R.V. Vovk for making the heaters and bolometers and C.D.H. Williams for use of his program for some data analysis.

[1] D. H. S. Smith and A. F. G. Wyatt, Phys. Rev. B 76, 224519 (2007).