Highlighting the Deficiencies in Some Existing Optimal PMU Placement Techniques

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Abstract Synchrophasor technology is receiving a global acceptance for electric power grid Wide Area Measurement System, (WAMS), an important function in a smart power transmission grid. A critical challenge in the synchrophasor technology deployment is the optimal choice of Phasor Measurement Unit (PMU) locations on the power grid. Researchers have proposed several techniques and algorithms in this respect. This work evaluated some of the major techniques and established that the available techniques and the factors they considered are not sufficient for a real-life optimal PMU placement (OPP). It also pointed to a method that could be applied to achieve a practical and robust solution for effective PMU placement for synchrophasor applications in a real-life electric power grid. It, therefore, calls for a shift in paradigm in the studies for the optimal PMU placement locations.

Keywords: optimal PMU placement, placement considerations, synchrophasor system deployment, synchrophasor applications

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1. Introduction

A constant real-time monitoring of the power grid operating conditions is a requirement for the secure operation of a power system. This real-time monitoring of the power grid can be achieved with synchrophasor technology, which uses the PMUs to measure electrical waves at the different locations across the power grid and synchronizes them using a common GPS Satellite time signal. Foundational works in the development and utilization of PMU in electric power grids are found in [1,2]. The major challenging issues confronting the synchrophasor deployment in electric power grids were explored in [3], and they include data quality, cyber security, data transmission/channel characteristics, synchrophasor applications development, technology adaptation, PMU deployment cost, and manpower training.

Optimal PMU placement has been identified as an effective way of reducing the cost of PMU installation, ensuring efficient utilization of available transmission channel bandwidth, and enhancing the performances of the synchrophasor applications deployed into the grid. For the above reasons, researchers have devoted a lot of research energy into solving the problem of optimal PMU placement, and have developed numerous solutions to the OPP, some of which are reviewed in [4,5].

It is important to note that Optimal PMU Placement study in a power grid is supposed to be a major task that is carried out once in preparation for PMU installation on a power grid not a task for mere exercise of mathematical prowess. Hence, the OPP solution should be all encompassing, considering the existing state and possible future state of the power grid, and the characteristics inherent with the grid. The goal of optimal placement is to identify the optimal locations that maximize the benefit across multiple applications as well as offers the least-cost solution by providing for the existing and planned infrastructure upgrades across the power company’s footprint and its neighboring systems [6].

2. OPP Methods

The rules usually considered in minimizing the number of PMUs are spelled out in [7] as follows:
- A PMU placed at a particular bus can measure the voltage phasor of that bus and the current phasors of all the branches radiating from the bus.
- If the voltage and current phasors at one end of a branch are known, the branch current can be calculated using the line impedance.
- If voltage phasors at both ends of a branch are known, the branch current can be calculated using the line impedance.
- Each bus in the network must be observed at least once by PMUs.

Several solution methodologies have been proposed for the OPP problem, some of which are heuristic methods [8,9], while others utilize mathematical algorithms[10-15]. Some of the solutions went further to consider one grid characteristic or another, such as the presence of contingencies, and other occurrences which are capable of
altering the network topology and changing the optimal solution or influencing the performance of the synchrophasor system deployed.

Integer Linear Programming (ILP) approach, which was first proposed in [16], is the most common method used so far in solving the problem of OPP. The popularity of the ILP in OPP studies stems from the fact that it is simple to implement, and it gives room for modeling of some characteristics and conditions inherent in the power grids.

The optimal placement of PMUs is formulated as follows:

\[
\min \sum_{i=1}^{N} c_i x_i
\]  

subject to

\[ s.t. A_{PMU} X \geq b_{PMU} \]  

where \( c_i \) is the cost of PMU installation at bus \( i \), \( x_i \) is the PMU placement variable, and \( A_{PMU} \) is an N-by-N matrix defined for and N-bus power grid with elements defined as

\[
a_{i,j} =\begin{cases} 
1 & \text{if } i = j \\
1 & \text{if } i \text{ and } j \text{ are connected} \\
0 & \text{otherwise}
\end{cases}
\]  

And \( b_{PMU} \) is an N-element column vector defined as

\[
b_{PMU} = \begin{bmatrix} 1 \\ \vdots \\ \vdots \\ 1 \end{bmatrix}_{NX1}
\]  

### 3. Placement Considerations

Some of the necessary considerations in searching for the optimal locations of PMUs are vividly described in work in [6]. These considerations are categorized into applications, infrastructural, and operational considerations as shown in Figure 1. An OPP study should first analyze the power grid and find out which of the placement considerations apply to the grid and the intended synchrophasor applications.

### 4. Deficiencies in the Existing Models

Many of the current optimal placement models yielded solutions, which will cease to be optimal when the power grid is subjected to its practical operating conditions. Such a solution is not desirable in a predominantly data-driven electricity grid because a compromise in data quality or data availability can be of a tremendously adverse effect. The various yardsticks with which some of the models are considered insufficient are:

Factors considered

Most of the OPP formulations focused on determining the number of PMUs required for full coverage of the grid. In those formulations, the cost of PMU placement and full grid coverage were the only constraints considered. Some later models considered the fact that certain conditions and occurrences could render an observable system unobservable. They went on to consider one factor or another including contingencies, controlled islanding, etc. Figure 2 demonstrates some of the factors considered and how they are considered in some of the present OPP solution models. A close observation of the power grid still reveals that including one factor or another in the formulation is not sufficient because a regular power grid could be affected simultaneously by a combination of these factors at varying degrees. A good model should consider all these factors in their various degrees of effects they have on the synchrophasor system deployment.

Variations and distributions of characteristics

Each power grid is unique in structure and operational characteristics. For instance, some grids may have predominantly radial structures while others have loop structures. Also, some buses in the grid may be located in areas where they are susceptible to climate-induced contingencies while others are located where they play vital roles in renewable resources integrations. Some buses in a power grid may never be a candidate for locating a PMU, either due to their locations or their natures. These characteristics affect the behaviors of the grids and dictate the numbers and optimal locations of PMUs required for the synchrophasor applications on the system. An optimal placement study for a power grid can only be effective if it considers all the essential factors that affect the performances of synchrophasor applications, and the way the factors are distributed on the power grid. Figure 3 [17] shows a transmission network with some of the various occurrences that can affect the behavior of the network. These occurrences are also the factors that should be considered in PMU placement. Figure 4 gives an illustration of a distribution of power grid characteristics. For a network that is characterized as depicted in Figure 4, an OPP formulation should be able to model the three broad considerations plus the cost of placement as applies to the individual buses in the grid to yield a truly optimal solution.

Non-uniformity in the cost of placement at the buses

Most of the placement models assumed a uniform PMU installation cost for all of the buses in the system. The report in [18] identified the different factors that contribute to the effective cost of PMU placement per bus on the power grid. Analysis of those factors shows that the cost of PMU placement varies from one bus to another. The differences in placement cost per bus are often so significant that neglecting them poses significant effects on the accuracy of the resulting solutions. It follows; therefore, that optimal placement should not assume a uniform placement cost per bus but should consider the placement costs as they occur. The Per Unit (P.U.) placement cost for each of the buses in the grid can be established by taking the lowest placement cost estimated in a bus in the grid as the base cost and use it to divide the placement costs estimated for all the buses in the grid.
Figure 1. Some necessary OPP solution considerations

Figure 2. A demonstration of factors considered in available OPP models

Figure 3. An example of the distribution of a network’s monitoring needs [17]
5. Demonstration with IEEE Test Systems

The claim for insufficiencies of some of the existing OPP solution models is demonstrated using IEEE Test Systems (IEEE 14-Bus System and IEEE 30-Bus System). The IEEE 14-Bus system is shown in Figure 5. The popular solutions achieved for these test systems are given in Table 1. The test systems were subjected to different contingency conditions to observe what happened to the system observability. The Test Systems were also subjected to a condition of non-uniformity in the PMU placement cost per bus. The results are shown in Table 2 to Table 6. Table 7 shows that modeling one consideration or another into the OPP model does not guarantee an adequate optimal solution because the sets of solutions achieved with the different considerations differ from one another and do not satisfy the constraints of one another.

| System          | General Solution | Few References |
|------------------|------------------|----------------|
| IEEE 14-Bus      | 2, 7, 10, 12 or 2, 6, 7, 9 | Ref[8], Ref[9], Ref[10], Ref[11], Ref[12], Ref[13], Ref[14], Ref[15] |
| IEEE 30-Bus      | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | Ref[14], Ref[15] |

Table 2. Effects of contingency locations on OPP solutions for ieee-14 bus system

| Grid State                   | Placement Solution | Unobserved Bus | Number of PMUs | Solution | Comment |
|------------------------------|--------------------|----------------|----------------|----------|---------|
| Normal                       | 2, 7, 10, 13       | None           | 4              | 2, 7, 11, 13 | All buses were observed |
| Contingency (Line 6-13)      | 6                  | 4              | 2, 6, 7, 9     | Bus 6 became unobserved, and solution has to change |
| Contingency (Line 2-5)       | 5                  | 4              | 2, 7, 11, 13   | Bus 5 became unobserved, and solution has to change |
| Contingency (Line 9-10)      | None               | 4              | 2, 7, 11, 13   | All buses were still observed but a new optimal solution still emerged |
| Contingency (Line 1-2)       | 1                  | 5              | 4, 5, 8, 11, 13| Bus 1 became unobserved, and the solution has to change to 5 buses |
| Contingency (Line 2-3)       | 3                  | 5              | 4, 5, 8, 11, 13| Bus 3 became unobserved, and the solution has to change to 5 buses |
| Contingency (Line 7-8)       | 8                  | 4              | 2, 6, 8, 9     | Bus 8 became unobserved, and solution has to change |
| Shut down Bus 2              | 5                  | 4              | 4, 5, 8, 11, 13| The solution changed to 5 buses |

Table 3. Effects of differences in placement cost per bus on OPP solutions for ieee-14 bus system

| Cost Cases                   | Number of PMUs | Optimal Solution | Comment |
|------------------------------|----------------|------------------|---------|
| Case: Uniform Placement cost | 4              | 2, 7, 10, 13     | A set of 4 buses optimal solution |
| Case: Bus 2 = 2P.U. cost     | 5              | 4, 5, 8, 11, 13  | A new set of 4 buses optimal solution |
| Case: Bus 7 = 2P.U. cost     | 4              | 2, 6, 8, 9       | A new set of 4 buses optimal solution |
| Case: Bus 10 = 2P.U. cost    | 5              | 2, 7, 9, 11, 13  | A new set of 5 buses optimal solution |
| Case: Bus 13 = 2P.U. cost    | 5              | 2, 7, 11, 12, 14 | A new set of 5 buses optimal solution |
| Case: Bus 2, 13 = 2P.U. cost | 6              | 3, 5, 7, 11, 12, 14 | A new set of 6 buses optimal solution |
| Case: Bus 2, 7, 10, 13 = 2P.U. cost each | 7 | 3, 5, 8, 9, 11, 12, 14 | A new set of 7 buses optimal solution |
Table 4. Combined effects of both placement contingency locations and placement cost per bus on OPP solutions for ieee-14 bus system

| Grid State | Optimal Solution | Number of PMUs | Optimal Solution | Comment |
|------------|------------------|----------------|------------------|---------|
| Normal     | 2, 7, 10, 13     | 4              |                  | A set of 4 buses optimal solution |
| Contingency (Line 1-2); Bus 4 = 2P.U. cost | 5 | 1, 2, 7, 10, 13 | A new set of 5 buses optimal solution |
| Contingency (Line 1-2); Buses 4,13 = 2p.U. cost | 6 | 1, 3, 7, 11, 12, 14 | A new set of 6 buses optimal solution |
| Contingency (Line 1-2); Bus 13 = 2P.U. cost | 6 | 4, 5, 7, 10, 12, 14 | A new set of 6 buses optimal solution |
| Contingency (Line 2-3); Bus 4 = 2P.U. cost | 5 | 1, 3, 7, 11, 13 | A new set of 5 buses optimal solution |
| Contingency (Line 2-3); Bus 11 = 2P.U. cost | 6 | 4, 5, 6, 7, 10, 13 | A new set of 6 buses optimal solution |
| Contingency (Line 6-13); Bus 2 = 2P.U. cost | 5 | 4, 5, 8, 11, 13 | A new set of 5 buses optimal solution |
| Contingency (Line 6-13); Buses 2,11 = 2P.U. cost | 6 | 3, 5, 6, 7, 10, 13 | A new set of 6 buses optimal solution |

Table 5. Effects of contingency locations on OPP solutions for ieee-30 bus system

| Grid State | Placement Solution | Unobserved Bus | Number of PMUs | Solution | Comment |
|------------|--------------------|----------------|----------------|----------|---------|
| Normal     | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | None | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 30 | All buses were observed |
| Contingency (Line 27-29) | 30 | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 30 | Bus 30 became unobserved, and solution set has to change |
| Contingency (Line 19-20) | None | 10 | 1, 2, 6, 9, 10, 12, 18, 24, 25, 27 | A new set of 10 buses optimal solution |
| Contingency (Line 2-5) | 5 | 10 | 3, 6, 7, 9, 10, 12, 15, 20, 25, 27 | Bus 5 became unobserved, and solution set has to change |
| Contingency (Line 12-14) | None | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | A new set of 10 buses optimal solution |
| Contingency (Line 29-30) | None | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | A new set of 10 buses optimal solution |
| Contingency (Line 27-28) | None | 10 | 2, 3, 6, 9, 10, 12, 15, 19, 25, 27 | A new set of 10 buses optimal solution |
| Shut down Bus 6 | 7, 8 | 10 | 3, 5, 9, 10, 12, 15, 19, 25, 27, 28 | Buses 7 and 8 became unobserved, and solution set has to change |

Table 6. Effects of differences in placement cost per bus on OPP solutions for ieee-30 bus system

| Cost Cases | Number of PMUs | Optimal Solution | Comment |
|------------|----------------|------------------|---------|
| Case: Uniform Placement cost | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | A set of 10 buses optimal solution |
| Case: Bus 6 = 2P.U. cost | 10 | 3, 5, 8, 9, 11, 14, 18, 24, 26, 30 | A new set of 10 buses optimal solution |
| Case: Bus 27 = 2P.U. cost | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 30 | A new set of 10 buses optimal solution |
| Case: Bus 12 = 2P.U. cost | 11 | 2, 3, 6, 9, 10, 13, 15, 17, 19, 25, 27 | A new set of 11 buses optimal solution |
| Case: Bus 10 = 2P.U. cost | 10 | 1, 2, 6, 9, 10, 12, 15, 19, 25, 27 | A new set of 10 buses optimal solution |
| Case: Bus 10 = 3P.U. cost | 11 | 2, 3, 6, 9, 12, 15, 17, 20, 22, 25, 27 | A new set of 11 buses optimal solution |
| Case: Bus 2 = 1.5P.U. cost | 10 | 3, 5, 6, 9, 10, 12, 15, 19, 25, 27 | A new set of 10 buses optimal solution |

Figure 6. Schematic of an adequate OPP model
A necessary step forward is to develop an optimal PMU placement model that incorporates all the necessary factors, which are considered in PMU placement, and in the way they occur in the given power grid.

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