Velocity fields in non–Gaussian cold dark matter models

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ABSTRACT
We analyse the large–scale velocity field obtained by N–body simulations of cold dark matter (CDM) models with non–Gaussian primordial density fluctuations, considering models with both positive and negative primordial skewness in the density fluctuation distribution. We study the velocity probability distribution and calculate the dependence of the bulk flow, one–point velocity dispersion and Cosmic Mach Number on the filtering size. We find that the sign of the primordial skewness of the density field provides poor discriminatory power on the evolved velocity field. All non–Gaussian models here considered tend to have lower velocity dispersion and bulk flow than the standard Gaussian CDM model, while the Cosmic Mach Number turns out to be a poor statistic in characterizing the models. Next, we compare the large–scale velocity field of a composite sample of optically selected galaxies as described by the Local Group properties, bulk flow, velocity correlation function and Cosmic Mach Number, with the velocity field of mock catalogues extracted from the N–body simulations. The comparison does not clearly permit to single out a best model; the standard Gaussian model is however marginally preferred by the maximum likelihood analysis.

Key words: Cosmology: theory – dark matter – galaxies: clustering, formation – large–scale structure of Universe – early Universe

1 INTRODUCTION
The cornerstone of current theories of structure formation in the universe is the dominance of a non–baryonic dark matter component. During the last decade the standard cold dark matter (hereafter SCDM) scenario has shown a high predictive power in explaining many observed properties of the large–scale galaxy distribution: the constituents of dark matter in this model are massive particles, which decoupled when non relativistic or have never been in thermal equilibrium; the primordial perturbations are assumed to be Gaussian and adiabatic with a scale–invariant power–spectrum, \( P(k) \propto k^n \), with \( n = 1 \) (the so–called Harrison–Zel’dovich spectral index); the SCDM scenario is also characterized by an Einstein–de Sitter universe with vanishing cosmological constant. The amplitude of the primordial perturbations is usually parametrised by the linear bias factor \( b \), defined as the inverse of the rms mass fluctuation on a sharp–edged sphere of radius 8 \( h^{-1} \) Mpc (in this work we will adopt the value \( h = 0.5 \) for the Hubble constant \( H_0 \) in units of 100 km \( s^{-1} \) Mpc\(^{-1} \)). The COBE DMR detection of large angular scale anisotropies of the cosmic microwave background (Smoot et al 1992; Bennett et al. 1994) can be used to fix the normalization of the model, leading to \( b \approx 1 \). This normalization makes the model completely specified.

However, it is well known that this model presents some serious problems, mostly due to the high ratio of small to large–scale power: in particular, the COBE normalization implies excessive velocity dispersion on Mpc scale (e.g. Gelb & Bertschinger 1994) and is unable to reproduce the slope of the galaxy angular correlation function obtained from the APM survey (Maddox et al. 1990).

To overcome these difficulties many alternatives to this basic model have been proposed: i) “tilted” (i.e. \( n < 1 \)) CDM models (Vittorio, Matarrese & Lucchin 1988; Adams et al. 1992; Cen et al. 1992; Lucchin, Matarrese & Mollerach 1992;

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Tormen et al. 1993, Moscardini et al. 1994), ii) hybrid (i.e. hot plus cold) dark matter models (e.g. Klypin et al. 1993, and references therein), iii) CDM models with a relic cosmological constant (e.g. Efstathiou, Bond & White 1992, and references therein), iv) non-Gaussian CDM models (hereafter NGCDM; Moscardini et al. 1991; Messina et al. 1992).

In the present work we consider the last alternative. Physical motivations for this class of models can be given in terms of the effect of relic topological defects (e.g. Scherrer & Bertschinger 1991; Scherrer 1992, and references therein), or in terms of the inflationary dynamics of models containing multiple scalar fields (see e.g. Salopek 1992), or in the frame of the cosmic explosion scenario (e.g. Weinberg, Ostriker & Dekel 1989). NGCDM models have been investigated in a series of papers (Moscardini et al. 1991; Matarrese et al. 1992; Messina et al. 1992; Coles et al. 1993a,b; Moscardini et al. 1993; Borgani et al. 1994), mainly devoted to the analysis of the clustering properties of the matter distribution on large scales, as resulting from N-body simulations. A similar analysis was done by Weinberg & Cole (1992), who, however, performed numerical simulations with scale–free initial power–spectra. Of course, the ultimate probe of the non–Gaussian character of the primordial perturbation field can only be obtained from the analysis of the cosmic microwave background (CMB) anisotropies on large angular scales: a number of statistical tests have been recently proposed in order to detect possible non–Gaussian signatures, such as the skewness of the temperature distribution (Hinshaw et al. 1994) and the genus of iso–temperature contours (Smoot et al. 1994). The analyses performed on the first year COBE data have revealed that non–Gaussian signals are not present on the angular scales probed by the DMR experiment (∼7°), beyond those due to the effects of the cosmic variance (e.g. Scaramella & Vittorio 1991). The implication of this result is that non–Gaussian features cannot be relevant for the large–scale gravitational potential, so, either the primordial fluctuations were indeed Gaussian, or primordial non–random phases were only present on scales below 7°, as it seems the case for anisotropies generated by topological defects (e.g. Coulson et al. 1994, and references therein). Therefore, for the models we consider here, we just have to require that the gravitational potential was significantly non–Gaussian already at redshifts of order of a tenth, when we start to evolve our system, and up to scales as large as ∼10^2 Mpc (much below the COBE scale), as probed by our simulations.

This work is devoted to a detailed study of the velocity field in N–body simulations of NGCDM models, and to a comparison of the large–scale velocity field of a composite sample of optically selected galaxies (1184 galaxies, with known radial peculiar velocities, grouped in 704 objects, from the “Mark II” compilation), with that obtained from the simulations. In particular, we calculate the probability to reproduce the observed properties of the Local Group and the observed values for the bulk flow, the velocity correlation function, and the Cosmic Mach Number of the data in mock catalogues extracted from our simulations. A similar analysis has been performed by Tormen et al. (1993) and Moscardini et al. (1994) in the frame of open and/or tilted CDM models. Contrary to most previous analyses of NGCDM models (Moscardini et al. 1991; Messina et al. 1992), we here fix the ‘present time’ of the simulations in such a way that the linear bias parameter b is one, consistently with the COBE normalization]. The same choice for the normalization has been made by Moscardini et al. (1993), where the effect of primordial non–random phases on the large–scale behaviour of the galaxy angular two–point function has been investigated: it was found that models with initially negatively skewed fluctuations are in principle capable of reconciling the lack of large–scale power of the CDM spectrum with the observed clustering of APM galaxies.

The plan of the paper is as follows. In Section 2 we introduce our skewed CDM models. In Section 3 we discuss the general properties of the velocity field of the simulations, analysing the bulk flow, the one–point velocity dispersion and the Cosmic Mach Number at different smoothing scales. We also analyse the density and velocity probability distributions and power–spectra. Section 4 is instead devoted to the statistical comparison of mock catalogues extracted from the simulations with observational data. Conclusions are drawn in Section 5.

2 SKewed CDM MODELS AND N–Body Simulations

The non–Gaussian statistics considered here are those adopted by Moscardini et al. (1991), namely the Lognormal (hereafter LN) and the Chi–squared with one degree of freedom (hereafter χ^2), chosen as distributions for the peculiar gravitational potential, Φ(x), before the modulation by the CDM transfer function. These distributions actually split in two different types of models: the positive (LN_p and χ^2_p) and negative (LN_n and χ^2_n) models, according to whether the sign of the skewness for linear mass fluctuations, ⟨δ_M^3⟩, is positive or negative respectively.

All our model distributions are set up in such a way that Φ has the “standard” CDM power–spectrum

$$\mathcal{P}_\Phi(k) = \frac{9}{4} \mathcal{P}_0 k^{-3} T^2(k) = \frac{9}{4} \mathcal{P}_0 k^{-3} [1 + 6.8 k + 72.0 k^2/2 + 16.0 k^2]^2,$$

where $\mathcal{P}_0$ is a suitable normalization constant and the CDM transfer function $T(k)$ is taken from Davis et al. (1985) for a flat universe. This choice for the spectrum allows us to make a direct comparison with the Gaussian CDM (hereafter $G$) model.

† Even though the statistical analysis of CMB anisotropies on large angular scales for non–Gaussian models cannot be reduced to calculating the rms fluctuation we assume here that the effect of non–Gaussian statistics on the COBE DMR scale is small, so that we can safely use the standard normalization, leading to $b \approx 1$.  

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We use a particle–mesh code with \( N_p = 128^3 \) particles on \( N_g = 128^3 \) grid–points. The box–size is \( L = 260 \, h^{-1} \) Mpc. We run two independent simulations for each of the five considered models.

The primordial gravitational potential is obtained by the convolution of a real function \( \tau(x) \) with a random field \( \varphi(x) \),

\[
\Phi(x) = \int d^3 y \, \tau(y-x) \varphi(y).
\] (2.2)

The field \( \varphi \) is obtained by a non–linear transformation on a zero–mean Gaussian process \( w \), with unit variance and power–spectrum \( \propto k^{-3} \); the function \( \tau \) is fixed by its Fourier transform,

\[
\tilde{\tau}(k) \equiv \int d^3 x e^{-ik \cdot x} \tau(x) = T(k) F(k),
\] (2.3)

where \( F(k) \) is a positive correction factor which we applied to get the exact CDM initial power–spectrum of Eq.(2.1). The non–linear transformations from \( w \) to \( \varphi \) is \( \varphi(x) \propto e^{w(x)} \) and \( \varphi(x) \propto w^2(x) \) for LN and \( \chi^2 \) respectively (Coles & Barrow 1987; Coles & Jones 1991; Moscardini et al. 1991).

The clustering dynamics and the present large–scale structure have been shown (Moscardini et al. 1991) to strongly depend upon the sign of the primordial skewness: positive models rapidly cluster to a lumpy structure with small coherence length, while negative models build up a cellular structure by the slow merging of shells around primordial voids.

3 GENERAL PROPERTIES OF VELOCITY FIELDS

The velocity field of NGCDM models has not received as detailed a study as its Gaussian counterpart; a preliminary analysis was done by Moscardini et al. (1991). For this reason, before comparing the observed cosmic velocity field with the prediction of our models, we would like to briefly discuss the role of primordial density skewness on velocity field statistics. In this section we will compute some general statistics in the idealized situation of a perfect knowledge of the three–dimensional velocity field. We will carry out our analysis on the four skewed models (positive and negative LN and \( \chi^2 \)), plus the Gaussian model. For each model we will analyse the velocity field defined on a \( 128^3 \) mesh. The field is obtained using the same procedure as in Kofman et al. (1994). First we interpolated mass and momentum from the particle distribution on a regular cubic grid by means of a Triangular Shaped Cloud (TSC) scheme (see Hockney & Eastwood 1981). We then smoothed the result using a further Gaussian filter, to define the velocity field also in underdense or empty regions of the simulations. The Gaussian window had a radius of 275 km s\(^{-1}\), close to the dynamical resolution of the simulations. Finally, the velocity at a grid–point was defined as the local ratio of momentum and mass density.

Since the normalization criteria of the simulations here analysed differ from those used in the previous papers, we will also present the density distribution function and mass–spectrum as auxiliary tools for our analysis.

Due to the fact that the velocity field is quite sensitive to long wavelength fluctuations, following Strauss, Cen & Ostriker (1993), we included in our simulations the effect of waves larger than the box size. To do so we added to the velocity of each particle the linear \( rms \) bulk velocity of a cube of \( 260 \, h^{-1} \) Mpc, the size of our simulations. The direction of the bulk velocity was chosen at random for each simulation. The value of such a bulk flow, approximately \( 220 \) km s\(^{-1}\), was calculated for the Gaussian CDM model; however, being a linear correction, we could safely apply it also to the non–Gaussian simulations.

In Figures 1a and 1b we plot the projected particle positions and the smoothed peculiar velocity taken from a slice of depth \( 4 \, h^{-1} \) Mpc, for the Lognormal and Chi–squared models respectively. A comparison with a similar plot for the Gaussian model is possible looking to Figure 1b in Moscardini et al. (1994), where the same scale and normalization for the velocity field was used. Even if the times here considered are different, a first glance confirms previous results. The longer time evolution amplifies the properties of skew–positive models: a lumpy structure, with isolated knots surrounded by large regions of quasi–uniform density. On the contrary, the shorter evolution does not completely prevent the skew–negative models from displaying their cellular structure with long filaments, extended sheets and large voids.

3.1 Density and Velocity Probability Distributions

Figure 2 shows for our models the density fluctuation probability distribution (left panel), its power–spectrum (central panel) and the velocity modulus \( v \equiv |\vec{v}| \) distribution function (right panel). In Table 1 we report also the first and second moment of the velocity distributions and the second moment of the density fluctuation distributions. One can see that time evolution has preserved the memory of the primordial skewness of the density field in all simulations. In fact, negatively skewed models show a larger number of voids and underdense regions, and develop fewer non–linear overdensities than the positively skewed models do. The bump shown in the density histogram at \( 0 \lesssim \delta \lesssim 1 \) points out the action of gravity on the initial underdensities. The Gaussian model exhibits an intermediate behaviour, with a larger number of moderate overdensities but a shorter tail on the right. This is also confirmed by the values listed in Table 1.

The density power–spectra are given in the second panel of Figure 2. The fast decay of the curves at high \( k \) is due to

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Figure 1. a) Slices with thickness $4h^{-1}$ Mpc for the skew-negative (top row) and skew-positive Lognormal models (bottom row). Left column: projected particle positions. Right column: projected peculiar velocity field after smoothing by a Gaussian filter with width $275$ km s$^{-1}$.

Table 1. Moments of Probability Distribution Functions.

| Model | $\sigma_\delta$ | $\langle |\vec{v}| \rangle$ km s$^{-1}$ | $\sigma_v$ km s$^{-1}$ |
|-------|----------------|---------------------------------|-----------------|
| $G$   | 1.15           | 637                             | 260             |
| $LN_n$| 0.76           | 536                             | 310             |
| $LN_p$| 1.26           | 461                             | 279             |
| $\chi^2_n$| 0.79          | 562                             | 274             |
| $\chi^2_p$| 1.34          | 474                             | 275             |

The smoothing introduced by the TSC interpolation used to define the density field on a regular cubic grid. The primordial phase correlation present in the skewed models has caused non-linear effects also on the scale of the box. In fact, in the skew-positive models some power has been transferred from long to short wavelengths; this is another way to say that these models have more non-linear structure at small scales. The skew-negative ones show instead a general lower amplitude for the power-spectrum: the “cross talk” between modes has slowed down the growth rate for $P(k)$ at all scales. Recalling that velocities are sensitive to large scale power and looking at $P(k)$ we may expect to find a higher mean velocity for the Gaussian model, longer tails of very high velocities (due to small scale non-linear structures) for the positively skewed models and in general lower velocities for the negatively skewed models.

The velocity distributions are in fact different from one another. The Gaussian model has the largest rms velocity of all, whereas the skew-positive ones have a larger number of very high velocities. The difference is statistically significant: after our smoothing, in the skew-positive models there are, for example, nearly 2000 grid-points with a velocity greater than 2000 km s$^{-1}$, whereas the Gaussian distribution has no grid point with such a high velocity. As for the negative models, while the $\chi^2_n$ indeed has low velocities (it is the model with the least non-linear structure of all), the $LN_n$ exhibits a singular behaviour, with a tail of high velocities as the positive models have: a possible explanation of this tail is the gravitational “push" from voids and underdense regions, which are larger and more extreme in this model than in the $\chi^2_p$.  

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3.2 Bulk flow, one–point velocity dispersion and Cosmic Mach Number

In order to further characterize the velocity field of each model, we computed the bulk flow, the mean residual velocity and their ratio (the so called Cosmic Mach Number) in top–hat spheres of different radii. We defined these statistics as follows. The bulk flow is the amplitude of the centre of mass velocity of the sphere: \( v_{\text{bulk}}(R) = \left| \frac{1}{n} \sum_{i=1}^{n} \vec{v}_i \right| / n \), where the sum extends to the \( n \) grid–points falling within a distance \( R \) from the centre of the sphere; the one–point velocity dispersion \( \sigma_v \) of the sphere is the \( \text{rms} \) velocity measured in the reference system comoving with the bulk flow: \( \sigma_v^2(R) = \frac{1}{n} \sum_{i=1}^{n} (\vec{v}_i - \vec{v}_{\text{bulk}})^2 \), where the sum is extended to the same grid–points; the Cosmic Mach Number is the ratio between the two: \( M(R) = v_{\text{bulk}}(R)/\sigma_v(R) \).

In particular, for each model we randomly selected 100 different grid–points from the simulations, and calculated the mentioned statistics in top–hat spheres centred on them, for a radius of the sphere ranging from \( 5 \ h^{-1} \ \text{Mpc} \) to \( 130 \ h^{-1} \ \text{Mpc} \). Figure 3 shows the mean values obtained by averaging the results from the 100 estimates. For clarity we prefer not to plot the error bars: they are always less than 5% for all statistics and different models.

The bulk flow is plotted in the left panel and the velocity dispersion in the central one. Note that the saturation of the bulk flow at large scales is due to the random velocity we added to each particle in order to include the effect of wavelength larger than the box–size, as discussed above. In both cases the different models show a behaviour consistent with the velocity distribution function and density power–spectra. The Gaussian model, with more power on large scales and a higher \( \text{rms} \) total velocity (shown in Table 1), has the highest figures for both the bulk flow and the residual velocities at all scales. Skew–positive models, which have larger velocity tails but less power on the largest scales and lower \( \text{rms} \) velocities, show the lowest values for both the bulk flow and the velocity dispersion. Finally, the skew–negative models, with an intermediate \( \text{rms} \) velocity, also have intermediate values for the bulk flows and residual velocity.

The similar trends for the bulk flow and velocity dispersion in the different models implies a wash up of most differences when looking at the Cosmic Mach Number (right panel of Figure 3), except perhaps at the smallest scales. The five curves in the panel are in fact much closer to each other than any observational estimate with the data available today could possibly distinguish. We do not expect that adding the observational uncertainties to the simulated data (see the next section) can in

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Figure 3. Left panel: the density fluctuation probability distribution. Central panel: the density power–spectrum. Right panel: the probability distribution for the velocity modulus $|v|$. The $G$, $LN_\alpha$, $LN_\beta$, $\chi^2_\alpha$ and $\chi^2_\beta$ models are represented by solid, dotted, short–dashed, long–dashed and dot–dashed lines respectively. In the central panel only, the heavy solid line refers to the linear prediction.

Figure 4. Bulk flow $v_{bulk}$ (left panel), one–point velocity dispersion $\sigma_v$ (central panel) and Cosmic Mach Number $\mathcal{M}$ (right panel) versus the radius $R$. The models are represented as in Figure 2.

any way disentangle this situation. This last result shows how, at least in the present framework, the Cosmic Mach Number is pretty insensitive to the underlying statistics (Gaussian or skewed) of models with otherwise identical cosmological parameters.

4 COMPARISON WITH OBSERVATIONAL DATA
4.1 Sample and catalog construction

In this section we use the same observational data considered in Tormen et al. (1993), where more details can be found. Our catalogue is a compilation of peculiar velocities from the “Mark II” data, and contains 1184 galaxies, including spirals, ellipticals and S0. In this way we assume that different types of galaxies are tracers of the same velocity field; this assumption was supported by Kolatt & Dekel (1994), who recently analysed the velocity fields traced separately by ellipticals and spirals. We reduced distance uncertainties by grouping the galaxies following the rules in the original papers (Lynden-Bell et al. 1988; Faber et al. 1989); at the end our sample consists of 704 objects.

We used the results of our N–body simulations to construct mock catalogues. Our analysis relies on the assumption that the galaxy peculiar velocities give an unbiased signal of the actual velocity field. Because of this, our mock catalogues should not be interpreted as “galaxy catalogues”, but just as an appropriate mask applied to the full three dimensional velocity field in order to mimic the observations. In this sense we are allowed to neglect velocity bias effects (Carlberg, Couchman & Thomas 1990; Couchman & Carlberg 1992), which are in any case believed to mostly affect small scale peculiar velocities.

We located in each simulation 500 “observers” in grid–points with features similar to those of the Local Group (LG) (e.g. Gorski et al. 1989; Davis, Strauss & Yahil 1991; Strauss, Cen & Ostriker 1993; Moscardini et al. 1994). Three different requirements were imposed to each of these observers:

i) its peculiar velocity $v$ is in the range of the observed LG motion, $v_{LG,obs} = 627 \pm 22 \text{ km s}^{-1}$ (Kogut et al. 1993);

ii) the local flow around the mock LG is quiet with a small local ‘shear’, $S \equiv |v - \langle v \rangle|/|v| < 0.2$, where $\langle v \rangle$ is the average velocity of a sphere of radius $R = 750 \text{ km s}^{-1}$ centred on the LG;

iii) the density contrast in the same sphere is in the range $-0.2 \leq \delta \leq 1.0$.

The reference frame was built from each LG position imposing that the velocity vector of the central point had the CMB dipole direction ($l = 276 \pm 3^\circ$, $b = 30 \pm 3^\circ$; Kogut et al. 1993), with the direction of the remaining axis randomly selected. Next, we constructed our catalogues by collecting, for each of the 704 positions of the observed objects, the closest particle in the simulation and considering the radial component of its velocity with respect to the LG position.

We then introduced in the simulated catalogues the galaxy distance errors present in the real data, by perturbing each distance and radial peculiar velocity with a Gaussian noise (e.g. Dekel, Bertschinger & Faber 1990), $r_{i,p} = r_i + \xi_i \Delta r_i$ and $u_{i,p} = u_i - \xi_i \Delta r_i + \eta_i \sigma_f$, where $\xi_i$ and $\eta_i$ are independent standard Gaussian variables; $\Delta r_i$ is the estimated galaxy distance error and $\sigma_f = 200 \text{ km s}^{-1}$ is an estimate of the Hubble flow noise.

Finally, let us comment that the low resolution of the numerical code is not a problem in the comparison with real data. In fact, the exclusion from the catalogue of objects in virialized regions, the grouping technique discussed above and the Hubble flow noise adopted in the model to calculate the peculiar velocities correspond to an intrinsic smoothing in the observed dataset on a scale comparable to the numerical resolution.

4.2 Results of statistical tests

In our comparison between the real sample and the mock catalogues, we will use four different observables: the Local Group constraints, the bulk flow, the Cosmic Mach Number and the velocity correlation function. For definitions and assumptions adopted here we refer to Moscardini et al. (1994). The results for the skewed models are shown in Figure 4 and in Table 2: they can be compared with analogous results for the Gaussian model presented in Moscardini et al. (1994), where that model was analysed in the frame of non–scale invariant CDM models.

First, we estimated the capability of our models to reproduce the characteristics of the observers (Local Groups), i.e. their velocity, local shear and local density contrast, as previously described. In Table 2 we report the percentage of grid–points that separately fulfil each constraint [$P(v)$, $P(\delta)$ and $P(S)$ for the velocity, density and shear constraints respectively] and altogether $P(LG)$, both for the skewed models and the Gaussian one. As in similar previous analyses, we are not able to discriminate between the models using the constraint on the flow quietness: the differences are very low and the constraint is poorly stringent. When we consider the velocity distribution, the Gaussian model is preferred to all skewed ones, even if the skew–negative are better than the skew–positive ones. The behaviour is opposite when the models are compared by the density contrast constraint: skew–negative models have a larger number of grid–points in the observed range. The balancing of these results gives similar total probability of reproducing all the LG characteristics at the same time for Gaussian and skew–negative models, while the skew–positive ones have smaller percentages. In any case, the differences among the considered models are not so large as naively expected by considering different primordial distributions of the gravitational potential. This is probably due to the fact that we are considering models with the same normalisation, i.e. $b = 1$. In Tormen et al. (1993) and Moscardini et al. (1994) it was found that the total probability $P(LG)$ turns out to be strongly dependent on the bias parameter but almost independent of other parameters, such as the spectral index and the density parameter $\Omega$.

In the upper panels of Figure 4 we plot the distributions of bulk flow amplitudes calculated from our mock catalogues. The continuous vertical line refers to the observed value: $v_{bulk} = 306 \pm 72 \text{ km s}^{-1}$, with a misalignment angle $\alpha = 54^\circ \pm 13^\circ$ with respect to the direction of the CMB dipole. The distributions appear similar for different models, even if skew–positive
models show a slightly longer tail toward high amplitude. We checked that a Maxwellian distribution provides a good fit of the data, as shown by a Kolmogorov–Smirnov test for all the considered models.

In Table 2, we report the probability $P(v)$ that the simulated bulk flows have amplitude in the interval $[v_{\text{obs}} - \sigma_{\text{obs}}, v_{\text{obs}} + \sigma_{\text{obs}}]$ and misalignment angle $\alpha$ in the analogous interval. These results show that the Gaussian model reproduces the real data more frequently than the skewed ones.

The central panels of Figure 4 show the distributions for the Cosmic Mach Number as calculated from our simulated catalogues, and compares them with the value observed for our galaxy sample: $M = 0.24 \pm 0.06$ (shown by the vertical line). In Table 2, we report the probability $P(M)$ that the simulated Cosmic Mach Number is inside the observed interval. This statistic turns out to be once more the less stringent one, but on its basis we can conclude that the Gaussian model is preferred. The skew–positive models seem to have a larger probability for higher values of $M$. Once again, using a Kolmogorov–Smirnov test we find that the distributions for $M$ are well fitted by a Maxwellian function.

As a last statistic, we considered the velocity correlation function and in particular its linear integral $J_v$ from the origin to the maximum considered pair separation, $R_{\text{max}} = 5,000\,\text{km}\,\text{s}^{-1}$ (see Gorski et al. 1989; Tormen et al. 1993). For our real catalogue we find $J_v/(100\,\text{km}\,\text{s}^{-1})^3 = 237.9 \pm 61.5$. The lower panels of Figure 4 show the distributions of $J_v$ obtained from our simulated catalogues while the percentages $P(J_v)$ of the simulated catalogues whose value of $J_v$ is less than one standard deviation different from the observed one are reported in Table 2. The behaviour of different non–Gaussian models is similar, but worse than the Gaussian model in reproducing the observational data.

Finally, we performed a Maximum Likelihood analysis to compare all the models. Calling $\vec{C}$ the random vector of the statistics used to constrain the simulated Local Groups, $\vec{C} = (v_{\text{LG}}, S, J)$, and $\vec{S}$ the vector of all the other statistics, $\vec{S} = (v_{\text{bulk}}, \alpha, M, J_v)$, the joint distribution of $\vec{C}$ and $\vec{S}$, under the condition $\vec{C} = \vec{C}_{\text{obs}}$, is $P(\vec{C}_{\text{obs}}, \vec{S}) = P(\vec{C}_{\text{obs}}|\vec{C}_{\text{obs}})P(\vec{S}|\vec{C}_{\text{obs}})$. For a given model $H$, the likelihood function reads $L(H) = P(\vec{C}_{\text{obs}}|H)P(\vec{S}_{\text{obs}}|\vec{C}_{\text{obs}}, H)$. The joint conditional likelihood $P(\vec{S}_{\text{obs}}|\vec{C}_{\text{obs}}, H)$ of $v_{\text{bulk}}$, misalignment angle $\alpha$, Cosmic Mach Number $M$ and correlation integral $J_v$ has been computed by counting the number of simulated catalogues that have, at the same time, $v_{\text{bulk}}$, $\alpha$, $M$ and $J_v$ consistent with the observed values, within the quoted error bars. Table 2 reports, for all the considered models, the resulting values for the joint likelihood $L(H)$. Unlike our previous works (Tormen et al. 1993; Moscardini et al. 1994), here we are not allowed to use the relative likelihood in order to give confidence intervals to our results, since the differences between our models are not of parametric kind.

Even if the presently available observational data appear to be not so stringent in discriminating between the scenarios here considered, on the basis of our analysis we find that the best model is the Gaussian one; however, the difference with respect to the $\chi^2_n$ is not so large. On the contrary, the skew–positive models have a lower likelihood (approximately one third of that of the Gaussian model) and seem not to provide a good alternative to the standard Gaussian scenario. These results are coherent with our previous analyses of the same models, mostly based on the clustering properties: a positive primordial skewness does not help in overcoming the difficulties of the standard CDM scenario, while the skew–negative models cannot be excluded by present data.

5 CONCLUSIONS

In this paper we have analysed the large–scale velocity field in the context of non–Gaussian CDM models. Weinberg & Cole (1992) partially analysed the properties of peculiar velocities in their non–Gaussian models, assuming scale–free initial conditions. Our work presents the first detailed study of large–scale motions as resulting from primordial non–random phases in a CDM scenario.

Unlike previous analyses on the same models, mostly devoted to the study of the matter distribution and its clustering properties, in this work we found that the sign of the primordial skewness of the density field provides a poor discriminatory power. All our non–Gaussian models tend to have lower velocity dispersion and bulk flow than the standard Gaussian CDM model. We interpret this as due to the effect of primordial phase correlation which, in skew–positive models, causes power to be transferred from large scales (important for velocities) to small scales, whereas in skew–negative models it causes a general slowing down of the growth of the power–spectrum at all scales. This result is different to our earlier findings due to the different normalization here applied, which is dictated by the COBE data, i.e. $b \approx 1$. 

Table 2. Local Group constraints and Likelihood functions.

| Model | $P(v)$ | $P(\delta)$ | $P(S)$ | $P(LG)$ | $P(v_b)$ | $P(M)$ | $P(J_v)$ | $L$ |
|-------|--------|-------------|--------|---------|---------|--------|---------|-----|
| $G$   | 6.41   | 37.26       | 73.90  | 1.73    | 13.2    | 46.6   | 24.4    | 0.076|
| $LN_n$| 3.70   | 59.32       | 84.60  | 1.64    | 10.6    | 37.4   | 17.0    | 0.059|
| $LN_p$| 2.48   | 51.18       | 86.41  | 0.99    | 5.8     | 30.4   | 16.2    | 0.022|
| $\chi^2_n$| 4.84   | 55.15       | 79.38  | 2.08    | 7.6     | 32.4   | 13.6    | 0.037|
| $\chi^2_p$| 2.98   | 40.37       | 80.48  | 1.95    | 7.8     | 33.8   | 19.0    | 0.025|
In particular, this choice does not allow our skew–negative models to fully develop their dynamical properties, discovered in previous studies, where it was found that only after a lengthy evolution these models could achieve the right slope for the correlation function, assumed to indicate the “present time”. These non–Gaussian models, therefore, only experience moderate non–linear evolution (as shown by the low value of $\sigma_3$ in Table 1), which makes their large–scale velocity field still sensitive to the initial conditions (i.e. to the CDM power–spectrum) and only marginally dependent, through mildly non–linear effects, on its primordial kurtosis and on the skewness of the density fluctuations. This very fact implies that the comparison of our non–Gaussian models with observational data does not clearly permit to single out a best model: the standard Gaussian model is marginally preferred by the maximum likelihood analysis.

Moreover, these results suggest that, contrary to naive expectations, primordial non–random phases do not help in producing large–scale bulk motions such as those indicated by the Lauer & Postman (1994) analysis, which is one of the most challenging observational results for the present structure formation scenarios.
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