Hydrodynamic Modeling of Swirling Binary Mixture Gas–Particle Flows Using a Second-Order-Moment Turbulence Model

Yang Liu,* Ziyun Chen, Yongju Zhang, and Lixing Zhou

ABSTRACT: The polydisperse behaviors of a binary ultralight–heavy mixture particle flow in a swirling axisymmetric chamber were investigated based on a developed second-order-moment gas–particle turbulent model. A binary particle Reynolds stress transport equation to depict the anisotropic interactions between gas-mixture particles and binary ultralight–heavy particles was established to close the governing equations. Hydrodynamic parameters, including particle number density, particle and gas velocities, and fluctuation velocities, Reynolds stress tensors, and their invariants, turbulent kinetic energy, and vortex structure, are numerically simulated. The detailed effects of the density, the diameter of the particle, the Stokes number, and the ultralight particle mass loading ratios on the flow status were studied. It is shown that normal and shear Reynolds stresses and kinetic turbulent energies of mixture particles have been redistributed, particularly, they are very sensitive to the mass loading ratios. Higher particle mass loading ratios enhanced the anisotropic characteristics. The particle number density at central regions of the farthest downstream is approximately three times larger than those of smaller mass loading ratios. Larger Stokes number particles reinforced the axial fluctuations up to 1.2 times that of the light particles, whereas ultralight particles increased tangential fluctuation to 2.5 times for axial ones.

INTRODUCTION

Fullerene and derivatives are the kinds of fundamental and irreplaceable materials that have been widely applied in the field of chemical engineering, sustainable energy material, and biotechnological engineering.1 Synthesis technique using the graphite combustion approach in an axisymmetric swirling chamber is a promising strategy due to large-scale industrial advantages. As a new functional carbon granular material, the expanded graphite particle has a loose, porous internal structure resulting in a relatively smaller density on the order of 102, which is only a few tenths of that of conventional materials. Because of the extremely low density, they are easily dispersed in the gas phase and exhibit unique behaviors. The polydisperse mixture swirling gas–particle flow with different sizes or densities and effects on hydrodynamic behaviors needs to be further studied due to the complex transport processes as a result of centrifugal and Coriolis forces, and the unclear interaction mechanism between the particles and gas phase turbulence. Recirculation and swirling flow affect the intensity of momentum, heat, and mass transfer and strongly influence the multiphase turbulent flow structure.2–5 Therefore, the current understanding of the processes of multiphase turbulent mixing, diffusion, and transport remains insufficient.

Numerous experiments and computational investigations on gas–particle swirling flows have been carried out. Traditional experimental measurements using the particle image velocity (PIV), phase Doppler particle analyzers (PDPAs), high-speed cameras (HSCs), etc. have successfully solved the hydro-
dynamics of swirling gas—particle turbulent flows, such as mean and fluctuation velocities, mass flux, particle residence time, correlation between particle size and residence time, etc.6–14 In recent years, the computational fluid dynamics (CFD) method has been effectively developed due to doable and applicable readily, especially for diverse operation conditions. Broadly speaking, the physical and mathematical models used for numerical simulation are classified into the Euler–Euler two-fluid continuum model15–22 and the Euler–Lagrange discrete particle model.22–28 Regarding the Euler–Euler model, both gas and particle phases are considered as continuous and as full interpenetration phases to each other. First, a radial function distribution to consider particle–particle interactions and the renormalization group (RNG) turbulent model have been employed to describe the extra recirculation zones of gas turbulence in vertical swirling gas particles.15,16 The kinetic theory of a granular flow model to predict the wide-size distributions of a circulated fluid bed was applicable.17,18 The turbulence modulations with strong anisotropy and closely determined by the flow structure in the sudden expansion of two-phase flows were clarified.19 An improved two-phase turbulent Reynolds stress model to predict the swirling particle dispersions behind the sudden tube expansion was also performed.20,21 Applications of the unified second-order-moment (USM) model for a single-particle phase in gas–particle turbulent flows were proposed.22 Compared to the discrete particle model, the two-fluid model has been dominantly popularized for simulation of a large-scale facility due to a lower CPU time. Both hard and soft sphere models to model the dispersed particles in a bubble fluidized bed and a circulated fluidized bed were developed to predict the dense particle flows.23,24 A semiempirical algebraic subgrid-scale (SGS) eddy viscosity model to explain the effect of particles on gas stress distributions was established for the first time.25 Similarly, a gas SGS kinetic energy equation model to consider the effect of particles on the gas SGS stress was used.26,27 However, additional closure correlations for Reynolds stress transport equations that describe the interactions between gas and particle phases by means of two- or four-way coupling strategies are required urgently.

Segregation or mixing polydisperse characteristics of binary mixture particles composed of different particle sizes or densities are entirely different from those of a single-particle system. With respect to dense particles, for the first time, a binary mixture model with the same granular temperature correlation was established for two kinds of particle mixtures. The question is that the dissipation energy originating from inelastic particle–particle collisions cannot be considered.27 Then, an unequal granular temperature model based on the kinetic theory of granular flows to model binary mixture particles was developed.28 A new two-granular temperature model that involved a kinetic theory of granular flow in terms of unequal granular temperatures between particle phases was proposed, which is a function of particle pressure, binary radial distribution function, viscosity, particle collision dissipation, and conductivity. It can be used to close the transport equations of each particle phase.29–31 As for the dilute particle flow in a swirling chamber, the two-equation k–ε models are generally used to predict the hydrodynamics. The disadvantage is that the anisotropic characteristics for two-phase turbulence stresses cannot be described, resulting in considerable errors in modeling strongly nonequilibrium flows with high velocity gradients and flow curvatures.32–34 To solve it, a second-moment closure (SMC) has to consider the dispersed phase for Reynolds stress transport. Thus, anisotropy has been partially considered for both mean and fluctuating parameters in a two-phase separated flow without a sudden expansion pipe.35,36 Furthermore, a series of second-order-moment two-phase turbulent models, such as the k–ε–Ap model, the unified second-order-moment (USM) model, the subgrid-scale USM (SGS-USM) model, the USM-θ particle temperature model, etc., are proposed to completely determine the anisotropic behaviors. They have successfully predicted the hydrodynamics of higher swirling intensity gas–particle turbulent flows.38–43 As mentioned above, they are all based on the Reynolds averaged Navier–Stokes equations (RANS) method. The advantageous large eddy simulation (LES) approach has an effective algorithm for modeling a swirling gas–particle flow because it can predict accurately the coherent structure and instantaneous flow characteristics. However, the unbeatable restrictions are the huge CPU time for large-size domains, the unreasonable two-phase subgrid-scale model in high shear stress flow regions, and unclear mechanism on subgrid-scale stresses affected by the disperse phase.44–46

As for the expanded graphite (EP) flow in a cyclone separator, the interaction force between gas and ultralight particles is much larger than that of the particle gravity, and secondary particle breakage is mainly caused by particle collisions with different diameters. Even a lower inlet velocity is also able to separate this ultralight particle; however, they are very different from those of heavy particles.27 It should be noted that the detailed polydisperse behaviors of binary ultralight/heavy mixture swirling flows in a chamber have not been revealed so far. The aim of the present study is to explore the polydisperse characteristics in binary mixture particle-laden swirling flows as well as the effects of particle density, diameter, and ultralight particle mass loading ratios. An improved second-order-moment binary mixture particle flow model was proposed, in which anisotropic particle dispersions were fully considered by means of the established Reynolds stresses transport equations. The mixing hydrodynamic characteristics in a swirling turbulent flow are discussed in detail (Figure 1).

Figure 1. Sketch of the axisymmetric swirling flow chamber.
RESULTS AND DISCUSSION

In this simulation, the density and the diameters of ultralight particles (expanded graphite) are set to 21.5 kg/m³, 15 μm, and 60 μm, respectively. Ultralight particle mass loading ratios are defined as \( \chi = \frac{m_{\text{light}}}{m_{\text{light}} + m_{\text{heavy}}} \) and are set to 0.1 and 0.5; the heavy particle is the glass bead employed in the experiment. Figure 2 shows the grid resolution test on the time-averaged heavy particle velocity. Three kinds of grid sizes, coarse size of 60 × 68, medium size of 240 × 272, and fine size of 480 × 544, are compared to test the independence of grid systems. We can see that the present data using the coarse grid system have larger errors, especially for the near-wall zone. The acceptable maximum error is approximately 2.5% using medium and fine grid systems. Thus, simulation results are independent of grid resolution accordingly. Validations of the axial and tangential particle velocities corresponding to the experiment are shown in Figure 3. The W-shaped profiles with the annular reversed flow region along the axial direction and the typical Rankine-vortex structures along the tangential direction are in line with the experimental measurements. Certainly, due to the limitation of the RANS algorithm in capturing the instantaneous hydrodynamics and coherent flow structures, errors still exist within approximately 3%. Although the LES algorithm is advantageous, the current investigation is limited to a single-particle phase, rather than a particle mixture.

In Figure 4, the distributions of polydisperse particle streamlines on a single heavy particle with a diameter of 60 μm, \( \chi = 0.1 \), \( \chi = 0.5 \), with mixture diameters of 60 and 15 μm are indicated. The flow structure shows an extensive change after sudden expansion and recirculation zone. The secondary vortex is produced, clearly located in the corner region, and the single gas-phase flow displays a similar feature. In the sections of \( x/R = 1.25 \) and \( r/R = 0.4 \), the highest negative velocities within the recirculation regime can be observed, which are in good agreement with the experimental data of \( x/R = 1.22 \) and \( r/R = 0.38 \) and those of LES simulations in reference. Meanwhile, the largest width and the diameter of recirculation flow are continuously reduced with downstream flow. With an increase in \( \chi \), the maximum radial width of central-reversed flow is decreased; the strength is damped with the streamwise direction as well. The reasons could be that the smaller lighter particles arise from the primary nozzle and it is easier to follow the carrier gas phase because of the smaller Stokes number of St.

In Figure 3, validation of the axial and tangential particle velocities by measurement: (a) axial particle velocity and (b) tangential particle velocity.

\( \chi = 0.0006 \). These particles responded quickly to the reversed gas flow and then were entrained substantially toward the secondary recirculation region. Therefore, the size in the maximum core is decreased. Furthermore, fluid kinetic energy was attenuated by the rushing small-lighter particles along with downstream flow. The large-heavy particles, due to the large inertia and lagging velocity somewhat with the gas inlet velocity, were prone to penetrate the central-reversed flow region rather than in the direction to the wall-normal region.

Figure 5 shows the distributions of normalized particle number density for mixture particles with diameters of 60, 15 μm and \( \chi = 0.1 \), \( \chi = 0.5 \), respectively. Starting from the cross section (\( x = 112 \) mm), the particle began to accumulate at near-wall regions as a result of an increase in the tangential velocity, turbulent diffusion, and centrifugal force action. This trend is enhanced with downstream flow and reached the maximum values in the section of \( x = 315 \) mm. Under these conditions, all particles are moved from the central region and accumulated gradually near the wall. The formation of a single peak value at the near inlet region and the two peaks at the downstream region indicated that large-heavier particles are likely to penetrate the central-reversed flow and not followed with carrier gas; small-lighter particles are primarily carried by gas and can be easily shifted to the outer edge of the recirculation zones. As \( \chi = 0.1 \)
reaches up to $\chi = 0.5$, much more small-lighter particles arise from the inlet region. During the motion experiences toward downstream zones, some particles were retrapped into the recirculation zone after getting reflected from the wall. Thus, particle number densities at central regions farthest downstream are approximately 3.0 times larger than those of smaller mass loading ratios.

Figure 6 shows the distributions of the axial and tangential particle velocities of different densities, diameters of 60 and 15 $\mu$m, and $\chi = 0.1$, $\chi = 0.5$ at different downstream sections, respectively. As shown in Figure 6a, for axial velocities, two peaks with W-shaped profiles found at the inlet region are similar to those of a single heavy particle, and they gradually evolved into flat profiles in the streamwise direction. Heavy particle velocities at the central core region are greater than those of ultralight particles under the action of inertia and complicated turbulent diffusion. As for tangential velocities, the typical Rankine-vortex rotation plus free vortex structures are captured in Figure 6b. At the near inlet region, lighter particles were subjected to be thrown out from central-reversed flow zone toward out-edge regions by centrifugal force. After that, they were accelerated back toward the inlet direction and moved radially outward as depicted by negative velocities. It is noted that the discrepancies in the velocities under different Stokes numbers less than unity are not very evident when the mass loading ratio is light. However, as it increased, the effects of particle diameters became evident as indicated at the downstream sections of $x = 112$ mm and $x = 195$ mm, especially for those of heavy particles. Under the same mass loading ratios, the axial velocities with a larger Stokes number of 0.055 are 2.8 times larger than the smallest ones. Close to the inlet regions, the effects on the mean velocity are negligible regardless of particle diameter and these effects are strengthened by the larger diameter at the far streamwise region.

Figure 7 displays the distributions of root-mean-square (rms) axial and tangential particle fluctuation velocities of different densities, diameters, and $\chi$ values at the downstream different locations. As shown in Figure 7a, ultralight particles with small Stokes numbers of 0.0006 and 0.004 are smaller than those of heavy particles because their absorption for kinetic energy is rapid. With increasing mass loading ratios, the heavy fluctuation velocities are intensified along the central downstream region. It is found that the tangential fluctuation velocity decreases for the high mass loading ratio near the inlet flow and those heavy particles are obvious, as represented in Figure 7b. Moreover, compared to larger particles with the same mass loading ratios, smaller particles are very less. The reasons are that heavy and light particles have a completely different history throughout the central-reversed flow region and swirling flow toward the wall-normal direction as mentioned above. The effects of mass loading ratios are most sensitive to those at near inlet regions due to the vigorous swirling motion of light particles. It seems that with increasing mass loading ratio, smaller particles attenuated the turbulence fluctuations at the near inlet flow regions and expanded in the central downstream region. In contrast, with the same mass loading ratios, larger Stokes number particles generated strong fluctuations due to less increases in the number flow rate. As a whole, heavy particles reinforce the fluctuation along the axial direction up to 1.2 times those of light particles, as well as light particles intensify those tangential direction up to 2.5 times. The effects of particle density, diameter, and mass loading ratio appear to be rather complicated. In spite of the fact that particle number flow is the most important influencing factor in comparison with the effects of particle diameter, mass loading on turbulent modulation in single particle flow, however, reports regarding mixture particle flows have not been published.46
To date, satisfactory closure transports have not been obtained due to unclear understanding so far, i.e., a simple closure correlation based on a nondimensional analysis, in which the kinetic energy term is always greater than zero. A binary particle Reynolds stress equation for mixture particles to fully consider anisotropic behaviors was proposed for the first time in this study.

Figure 8 depicts the distributions of root-mean-squared axial−axial and axial−tangential fluctuation velocities between gas and particles under different densities, diameters, and ultralight particle mass loading ratios. It is observed that they exhibited distinctively anisotropic characteristics as shown in Figure 8a,b. Fluctuation amplitudes of light particles near the inlet region are 3.0 times larger than those downstream due to more retaining particles and radical motions entrained by annular gas, as well as higher than those of heavy particles up to 2.0 times at near region. Fluctuations of heavy particles are larger than those of light particles in downstream regions because they are prone to penetrate the reversed flow region and the majority of light particles preferentially accumulated along the shear layer. An increase in mass loading ratio intensifies the axial−axial interactions for heavy particles and strengthens the axial−tangential ones for light particles.

Four-way coupling methods to describe the interactions between light and heavy particle collisions are adopted. Figure 9 shows the distributions of root-mean-squared axial−axial and tangential−tangential correlations between ultralight and heavy particles of different densities, diameters, and mass loading ratios. It seems that the distributions of normal and shear components of Reynolds stresses are very complicated. The normal stresses are far greater than those of shear stresses. Large Stokes number particles and smaller mass loading ratios are favorable for spreading anisotropic behaviors, in which shear stresses were suppressed by increasing the mass loading ratios. As the heavy particle readily penetrates the central-reversed flow rather than toward the wall region, the light particle is able to respond to the reversed flow with negative velocity and preferential accumulation at the shear layer. It follows that normal and shear stresses have different appearances under the combined effects of centrifugal force and turbulent diffusion.

Figure 5. Distributions of the normalized particle number density for mixture particles with different diameters, densities, and ultralight particle mass loading ratios: (a) $\chi = 0.1$ and (b) $\chi = 0.5$.

Figure 6. Axial and tangential particle velocities of different densities, diameters, and ultralight particle mass loading ratios: (a) axial and (b) tangential.
The invariants of Reynolds stress tensor may indicate the important geometrical and physical characteristics of mixture particle swirling flows, which are defined as follows

\[ I_1 = u'_1 u'_1 + u'_2 u'_2 + u'_3 u'_3 \]  
\[ I_2 = u'_1 u'_2 + u'_1 u'_3 + u'_2 u'_3 + \begin{vmatrix} u'_1 u'_1 & u'_2 u'_3 \\ u'_1 u'_3 & u'_2 u'_2 \end{vmatrix} \]  
\[ I_3 = u'_1 u'_1 u'_1 u'_3 + u'_2 u'_2 u'_2 u'_3 + u'_3 u'_3 u'_3 u'_3 \]

Figure 7 shows the distributions of the stress tensor invariants \( I_1, I_2, \) and \( I_3 \) of mixture particle flow. The values of invariants \( I_1 \) under different Stokes numbers are similar. When \( \chi \) is small, its values at near inlet regions \((x = 3 \text{ mm} \text{ and } x = 52 \text{ mm})\) are lower than those of larger mass loading ratios and are larger than those in the far downstream region. This trend is also true for \( I_2 \) and \( I_3 \) invariants. When the diameter is increased, invariants \( I_1, I_2, \) and \( I_3 \) are decreased compared to those of small particles. In addition, the values far downstream are far less than those near the inlet. The profiles of turbulent kinetic energy are given in Figure 11. Larger \( \chi \) and Stokes number values contributed to the augment effects because smaller particles can more easily and rapidly absorb the kinetic energy. Figure 12 shows the vorticity maps of light and heavy particle swirling flows under different densities, diameters, and mass loading ratios \((1/s, r−w \text{ plane})\). The shedding vortices from the inlet region rolled up by the boundary layer with the vortices from the shear layer underwent a process of pairing, merging, and breakup. Smaller Stokes number particles do not change the particle preferential
accumulation in the vortices. Particle inertia increases with particle size and density, which weakens the accumulation at the vortex edge. The motion of relatively small particles is mainly governed by large-scale structures, and large eddy structures were destroyed by large-size particles. Even if large particles changed more dramatically than smaller ones, vortices scale in the flow field prevents their motion from being affected by the vortices. Distinct vortex structures have not been formed because axial particle movements are more rigid and have larger inertia than the gas phase. For a single heavy particle, the maximum values are found at the border of central-reversed flow and shear layer regions, as shown in Figure 12a. Compared to heavy particles, the lengths of central and recirculation flow regions of ultralight particles are larger. With an increase in particle mass loading ratios, more ultralight particles were released from the primary jetting and peaks began to migrate toward annular recirculation regions and incurred the intensive dispersions along the radial direction. The swirling flow structures of mixture particles are considerably complicated, and heavy and ultralight particles have very different vortices due to complex multiphase turbulent diffusion, interactions between gas and particle, particle and particle, and particle inertia with Stokes numbers.

**CONCLUSIONS**

In this work, a second-order-moment mixture binary particle turbulent model was developed to numerically simulate the expanded graphite swirling turbulent flows. Polydisperse behaviors of particle number density, the mean and fluctuation velocity, Reynolds stress tensor and invariants, and turbulent kinetic energy were obtained to analyze the effects of Stokes numbers. The anisotropic characteristics of interphase interactions and hydrodynamics of mixture particles under strongly swirling flow conditions were investigated. Ultralight and heavy particles have experienced different histories throughout due to particle inertia, interactions of gas and particle phases, and complicated turbulent diffusions. Normal and shear components in Reynolds stress tensors of gas and particles were redistributed. Small sizes of ultralight particles with lower Stokes numbers enhanced the tangential fluctuations in the streamwise...
The momentum equations of the gas phase are given as follows

$$\frac{\partial}{\partial t} (\alpha_{fl} \rho_{fl}) + \frac{\partial}{\partial x_j} (\alpha_{fl} \rho_{fl} \mathbf{u}_{fl}) = 0 \quad (m = 1, \ldots, M + 1)$$

Here, $\alpha_{fl}$, $\rho_{fl}$, $\mathbf{u}_{fl}$, and $g$ refer to the volume fraction of gas and particles, the density and the velocity, the fluid pressure, and the gravity, respectively. $\tau_{fl}$ represents the relaxation time of the particle and $\tau_{rs}$ is the gas viscous stress tensor. The right-hand-side terms denote the gravitational force without buoyancy effects, the pressure gradient, the molecular viscosity, and the momentum exchange between gas and particle phase $s$ due to the drag force. $\tau_{fl}$ and $\tau_{rs}$ are calculated as

$$\tau_{fl} = \frac{2}{3} \frac{\mu_{s}}{\tau_{rs}}$$

The momentum equations for each particle phase $s$ ($s = 1,2,\ldots,M$) are very important to indicate the interphase interactions of gas–particle and particle–particle phases and are as follows.
Here, the right-hand-side terms represent the gravitational force without considering the buoyancy, the pressure gradient, the molecular viscosity, the momentum exchange between gas and particle phase, and the exchange terms between particles, phase \( m \) and phase \( s \). The mixture particle parameters of the restitution coefficient \( q_{ms} \), is defined as a function of particle fluctuation velocity.

\[
q_{ms} = c_m u_m^{s_m}
\]

The mixture particle parameters of the restitution coefficient, diameter, and mass are defined by

\[
d_m = \frac{d_s + d_m}{2} \quad m_m = m_s + m_m \quad \epsilon_m = \frac{\epsilon_s + \epsilon_m}{2}
\]

The radial function distribution \( g_{ms} \) is

\[
g_{ms} = \frac{1}{1 - \alpha_s/\alpha_{ms,max}} + \frac{6d_d}{d_s + d_m} \chi + 8\left(\frac{d_d}{d_s + d_m}\right)^2 \left(1 - \alpha_s/\alpha_{ms,max}\right)^2
\]

\[
\chi = 2\pi(n_s d_s^2 + n_d d_d^2)/3
\]

\[
\alpha_s = \alpha_s + \alpha_i
\]

\[
\alpha_{ms,max} = 0.64, \text{ which is the maximum total volume fraction of particles.}
\]

**Reynolds Stress Equations of Gas and Particle Phases.**

The gas Reynolds stress equation is

\[
\frac{\partial \langle \rho \overline{u_i u_j} \rangle}{\partial t} + \frac{\partial \langle \rho \overline{u_i u_j} \overline{u_k u_l} \rangle}{\partial x_k} = D_{sij} + P_{sij} + \Pi_{sij} - \epsilon_{sij} + \sum_{\gamma=1}^{M} G_{sij,\gamma}
\]

where the right-hand-side terms represent the diffusion term, the shear production term, the pressure—strain term, the dissipation term, and the interaction term between gas and particle phase, respectively.

The Reynolds stress equations of particle phases \((s = s, m)\) are

\[
\frac{\partial \langle \rho \overline{u_i u_j} \rangle}{\partial t} + \frac{\partial \langle \rho \overline{u_i u_j} \overline{u_k u_l} \rangle}{\partial x_k} = D_{sij} + P_{sij} + \Pi_{sij} - \epsilon_{sij} + \sum_{\gamma=1}^{M} G_{sij,\gamma}
\]

where the right-hand-side terms stand for the diffusion term, the shear production term, the pressure—strain term, the dissipation term, and the interaction between gas and particle, respectively. The last term is used to describe the interaction between particle phase \( s \) and \( m \), which are as follows

\[
D_{sij} = \frac{\partial}{\partial x_i} \left( C_{fl,sij} \frac{k_s}{\epsilon_i} \frac{\partial \overline{u_i u_j}}{\partial x_i} \right)
\]

\[
P_{sij} = -\frac{\partial}{\partial x_j} \left( \frac{\partial \overline{u_i u_j}}{\partial x_j} \right)
\]

\[
\Pi_{sij} = \Pi_{sij,1} + \Pi_{sij,2}
\]

\[
\Pi_{sij,1} = C_{r,ij} \frac{\epsilon_s}{k_s} (u_{ij}^s - 3k_s^s) - C_{H} \frac{\epsilon_s}{k_s} (u_{ij}^s - 3k_s^s)
\]

\[
\Pi_{sij,2} = \frac{2}{3} \delta_s \alpha_p \alpha_s
\]

\[
G_{sij,\gamma} = \sum_{\gamma=1}^{M} \left( \frac{\rho_s}{\tau_{s,\gamma}} (u_{ij}^{s,\gamma} - 2u_{ij}^{s,\gamma}) + \frac{\rho_m}{\tau_{m,\gamma}} (u_{ij}^{m,\gamma} - 2u_{ij}^{m,\gamma}) \right)
\]

**Closure Correlations of Interphase Reynolds Stresses.**

The closure correlation for gas—particle interactions in eq 23 is

\[
\frac{\partial \overline{u_i u_j}^s}{\partial t} + (\overline{u_i u_j}) \frac{\partial \overline{u_i u_j}^s}{\partial x_k} = D_{sij} + P_{sij} + \Pi_{sij} - \epsilon_{sij} + G_{sij}
\]

where the right-hand-side terms stand for the diffusion term, the shear production term, the pressure—strain term, the dissipation term, and the gas—particle interaction term, respectively. To reveal the fully anisotropic characteristics for interactions between particle \( s \) and \( m \) (see eq 24), we developed a new correlation as given in eq 26. The right-hand-side terms are the particle diffusion term, the particle shear production term, the particle pressure—strain term, the particle dissipation term, and the particle—particle interaction term, which are as follows
The expansion section. The central inlet pipe contains the primary coaxial inlet pipes and a cylinder chamber with a sudden produced a swirling air jetting. The swirling number is de proposed mathematical model, algorithm, and solver.6 The with experimental data was performed for validation of the sources is set for the gas and the particle phases. A comparison of the parameters for modeling and simulation in this research. In this simulation, the swirling number $s$ is set to 0.47. Larger or smaller $s$ means variation of the annular velocity, that is, larger $s$ corresponds to a strong annular velocity and vice versa. Under the larger annular velocity condition, the ultralight particles respond quickly to the swirling flow and much more prone to be entrained toward secondary recirculation regions, as well as for those of particles with smaller Stokes number. Meanwhile, it was disturbed and enhanced the heavier particle dispersions at both axial central and radially swirling flow directions.

### Table 1. Parameters and Boundary Conditions of Simulations

| parameter | unit | value |
|-----------|------|-------|
| diameter of EP and glass particles, $d_i$ | $\mu$m | 15/60, 15/60 |
| density of the expanded graphite and glass particles, $\rho_g$ | kg/m$^3$ | 21.9, 2500 |
| density of the gas, $\rho_e$ | kg/m$^3$ | 1.225 |
| viscosity of the gas, $\mu_e$ | Pa·s | 1.8e−05 |
| restitution coefficient of particle–particle, $e_i$ | 0.95 |
| diameter of the primary inject, $d_i$ | mm | 32 |
| diameter of the annular jet, $d_a$ | mm | 64 |
| total length, $l$ | mm | 960 |
| ultralight particle mass loading ratios, $\eta$ | 0.1/0.5 |
| swirling number, $s$ | 0.47 |
| flow rate in the primary jet, $q_{1p}$ | g/s | 9.9 |
| flow rate in the annular jet, $q_{1a}$ | g/s | 38.3 |
| inlet Reynolds number, $Re_i$ | 26 200 |
| Stokes numbers of glass particles $St_i$, 15 $\mu$m/60 $\mu$m | 03/5.5e−02 |
| Stokes numbers of EP particles $St_i$, 15 $\mu$m/60 $\mu$m | 02/4.1e−03 |
| boundary conditions | | | |
| inlet conditions of particles | uniform |
| inlet conditions of the gas | parabolic |
| normal Reynolds stress | isotropic |
| shear Reynolds stress | Eddy viscosity |
| wall | non-slip |
| outlet | Neumann condition |

In summary, the governing equations and their closure transport equations were performed completely as mentioned above. As for the single-particle phase, we can deduce that the particle phase $s$ is equal to $m$. Thus, these will be reduced to a single-particle phase.

### Numerical Procedures and Boundary Conditions

Numerical solutions were carried out using the finite volume method in which the mean transport equations for both gas–particle mixtures, as well as their Reynolds stress models are solved on a staggered grid system. The quadratic upstream interpolation for convective kinematics (QUICK) procedure and the central difference scheme for the diffusion terms are utilized. The computational domain is first divided into a finite number of control volumes and then the differential equations are integrated over this certain control volume. The velocity correction approach to satisfy the continuity criteria is used through the semi-implicit method for pressure linked equations corrected (SIMPLEC) algorithm coupling velocity and pressure. In the meantime, the tridiagonal marching algorithm, line-by-line iteration, and under-relaxation algorithm are served. The convergence criterion of $4.0 \times 10^{-5}$ for residual mass sources is set for the gas and the particle phases. A comparison with experimental data was performed for validation of the proposed mathematical model, algorithm, and solver. The experimental setup is shown in Figure 1, which consists of two coaxial inlet pipes and a cylinder chamber with a sudden expansion section. The central inlet pipe contains the primary air–particle mixtures, and the surrounding coaxial annulus produced a swirling air jetting. The swirling number is defined by

$$ s = \frac{2 \int_{0}^{l} \rho m u_w r^2 \, dr}{D_w \int_{0}^{l} \rho u_w^2 \, dr} $$

The detailed properties, boundary conditions, and inlet conditions mentioned above are shown in Table 1. It lists all of the authors declare no competing financial interest.
ACKNOWLEDGMENTS

The authors are grateful for the financial support of the National Key Research and Development Program of China (No. 2017YFC0702600).

NOMENCLATURE

cl central line
d diameter
D diffusion term
e restitution coefficient of particle
g gas, gravity acceleration, radial distribution function
I invariants of stress tensor
G source term, interaction term
k kinetic energy
n number density
p production term
P pressure
R correlation term
Re particle Reynolds number
sw swirling number
St Stokes number
t time
V,ν velocity
ω,ζ vortice

Greek Alphabets

α volume fraction
δ Kronic—Delta unit tensor
ρ density
e dissipation rate
μ dynamic viscosity
ν kinematic viscosity
Π pressure—strain term
P shear production
ϕ moment transfer coefficient
χ ultralight particle mass loading ratio
τ stress, response time

Superscripts

′ fluctuation
̅ averaged

Subscripts

i,j,k,l coordinate directions
gs gas and particle
h heavy
l laminar
sm particle index
r relaxation
ul ultralight particle

REFERENCES

(1) Margadonna, S.; Prassides, K. Encyclopedia of Materials: Science and Technology; Pergamon Press: Oxford, 2001; pp 34—67.
(2) Gupta, A.; Lilley, D. G.; Syred, N. Swirl Flows; Abacus Press: Tunbridge Wells, 1984; pp 32—67.
(3) Zhou, L. X. Theory and Numerical Simulation Modeling of Turbulent Gas-Particle Flows and Combustion; CRC Press: Beijing, 1993; pp 56—90.
(4) Gidaspow, D. Multiphase Flow and Fluidization: Continuum and Kinetic Theory Description; Academic Press: New York, 1994; pp 45—60.
(5) Crowe, C.; Sommerfeld, M. S.; Tsuji, Y. Multiphase Flows with Droplets and Particles; CRC Press: Boca Raton, 1998; pp 67—88.
(6) Sommerfeld, M.; Qiu, H. H. Detailed measurements in a swirling particulate two-phase flow by a phase Doppler anemometer. Int. J. Heat Fluid Flow 1991, 12, 20—28.
(7) Sommerfeld, M.; Qiu, H. H. Characterization of particle-laden, confined swirling flows by a phase-doppler anemometer. Int. J. Heat Fluid Flow 1993, 19, 1093—1127.
(8) Sommerfeld, M.; Ando, A.; Qiu, H. H. Swirling, particle-laden flows through a pipe expansion. J. Fluids Eng. 1992, 114, 648—656.
(9) Fessler, J. R.; Eaton, J. K. Turbulence modification by particles in a backward-facing step flow. J. Fluid Mech. 1999, 394, 97—117.
(10) Li, H.; Tomita, Y. Particle velocity and concentration characteristics in a horizontal dilute swirling flow pneumatic conveying. Powder Technol. 2000, 107, 144—152.
(11) Gilliland, I.; Fritsching, U.; Bauckhage, K. Measurement of phase interaction in dispersed gas particle two-phase flow. Int. J. Multiphase Flow 2001, 27, 1313—1332.
(12) Kosivczech, W.; Cessous, A.; Trinite, M.; Lecordier, B. Simultaneous velocity field measurements in two-phase flows for turbulent mixing of sprays by means of two-phase PIV. Exp. Fluids 2005, 39, 895—908.
(13) Sciaccitano, A.; Wienes, B.; Scarano, F. PIV uncertainty quantification by image matching. Meas. Sci. Technol. 2013, 24, No. 045302.
(14) Rajamanickam, K.; Basu, S. On the dynamics of vortex-droplet interactions, dispersion and breakup in a coaxial swirling flow. J. Fluid Mech. 2017, 827, 572—581.
(15) Mansoori, G. A.; Carnahan, N.; Starling, F.; Leland, T. W. Equilibrium thermodynamic prosperity of the mixture of hard spheres. J. Chem. Phys. 1971, 54, 1523—1525.
(16) Lu, H. L.; He, Y. R.; Liu, Y. Numerical simulations of multi-component dense-solid flow. J. Eng. Thermophys. 2002, 23, 369—371.
(17) Liu, Y.; Lu, H. L.; Liu, W. T. Model and simulation of gas-solid flow with wide size distributions in circulating fluidized bed. J. Chem. Eng. Ind. 2003, 54, 106—1071.
(18) Marzouk, O. A.; Huckaby, D. Simulation of a swirling gas-particle flow using different k-epsilon models and particle-parcel relationships. Eng. Lett. 2010, 18, 1—11.
(19) Wang, B.; Zhang, H. Q.; Wang, X. L. Large eddy simulation of particle-laden turbulent flows over a backward-facing step considering two-phase two-way coupling. Adv. Mech. Eng. 2013, 325, 1—11.
(20) Pakhomov, M. A.; Terekhov, V. I. Second moment closure modelling of flow, turbulence and heat transfer in droplet-laden mist flow in a vertical pipe with sudden expansion. Int. J. Heat Mass Transfer 2013, 66, 210—222.
(21) Pakhomov, M. A.; Terekhov, V. I. Numerical simulation of turbulent swirling gas-dispersed flow behind a sudden tube expansion. Thermophys. Aeromech. 2015, 22, 597—608.
(22) Zhou, L. X. Comparison of studies on flow and flame structures in different swirl combustors. Aerosp. Sci. Technol. 2018, 80, 29—37.
(23) Hoomans, B. P. B.; Kuipers, J. A. M.; et al. Discrete particle simulation of bubble and cluster formation in a two-dimensional gas-fluidized bed. Chem. Eng. Sci. 1996, 51, 99—108.
(24) Xu, B. H.; Yu, A. B. Numerical simulation of the solid-fluid flow in a fluidized bed by combining discrete particle method with computational fluid dynamics. Chem. Eng. Sci. 1997, 52, 2785—2809.
(25) Yoo, S.; Nishikawa, H.; Umekage, T. Numerical simulation of air and particle motion in group-B particle turbulent fluidized bed. Powder Technol. 2001, 118, 32—44.
(26) Zhou, H. S.; Flamant, G. D.; Gauthier, J. D.; et al. Lagrangian approach for simulating the gas-particle flow structure in a circulating fluidized bed riser. Int. J. Multiphase Flow 2002, 28, 1801—1821.
(27) Helland, E.; Occelli, R.; Tadrist, L. Computational study of fluctuating motions and cluster structures in gas-particle flows. Int. J. Multiphase Flow 2002, 28, 199—223.
(28) Fang, C. C.; Xu, J. L.; Liu, H. F. Influences of the cross angle on the dispersion characteristics in a dense gas-particle coaxial jet. Ind. Eng. Chem. Res. 2017, 56, 1739—1749.
(29) Jenkins, J.; Mancini, F. Kinetic theory for binary mixtures of smooth, nearly elastic spheres. Phys. Fluids A: Fluid Dyn. 1989, 1, 2050–2059.
(30) Mathiesen, V.; Solberg, T.; Hjertager, T. Predictions of gas-particle flow with an Eulerian model including a realistic particle size distribution. Powder Technol. 2000, 112, 34–43.
(31) Lu, H.; Gidaspow, D. Hydrodynamics of binary fluidization in a riser: CFD simulation using two granular temperatures. Chem. Eng. Sci. 2003, 58, 3777–3792.
(32) Lu, H.; Gidaspow, D.; He, Y. R. Hydrodynamics modeling of binary mixture in gas bubbling fluidized bed using the kinetic theory of granular flow. Chem. Eng. Sci. 2003, 58, 1197–1205.
(33) Lu, H.; Zhao, Y. H.; Ding, J. M.; Gidaspow, D.; et al. Investigation of mix-segregation of mixture particles in gas-solid fluidized bed. Chem. Eng. Sci. 2007, 62, 301–317.
(34) Liu, Y.; Zhou, L. X.; Xu, C. X. Numerical simulation of instantaneous flow structure of swirling, and non-swirling coaxial jet particle laden turbulence flows. Physica A 2010, 389, 5380–5389.
(35) Liu, Y.; Lu, H. L.; Liu, W. T. Model and simulation of gas-solid flow with wide size distributions in circulating fluidized bed. J. Chem. Ind. Eng. 2003, 54, 106–1071.
(36) Manceau, R.; Hanjalic, K. Elliptic blending model: a new near-wall Reynolds stress turbulence closure. Phys. Fluids 2002, 14, 744–754.
(37) Fadai-Ghotbi, A.; Manceau, R.; Boree, J. Revisiting URANS computations of the backward facing step flow using second moment closures. Flow, Turbul. Combust. 2008, 81, 395–410.
(38) Zhou, L. X.; Chen, T. Simulation of strongly swirling gas-particle flows using USM and k-ε-kp two-phase turbulence models. Powder Technol. 2001, 114, 1–11.
(39) Zhou, L. X.; Xu, Y.; Fan, L. S.; et al. Simulation of swirling gas-particle flows using an improved second-order moment two-phase turbulence model. Powder Technol. 2001, 116, 178–189.
(40) Liu, Y.; Zhou, L. X.; Xu, C. X. Large-eddy simulation of swirling gas-particle flows using a USM two-phase SGS stress model. Powder Technol. 2009, 19, 183–188.
(41) Liu, Y.; Liu, X.; Li, G. H.; Jiang, L. X. Numerical prediction effects of particle-particle collisions on gas-particle flows in swirl chamber. Energy Convers. Manage. 2011, 52, 1748–1754.
(42) Liu, Y.; Zhang, Y. J.; Zhou, L. X. Numerical study on bubble-liquid two-phase turbulent hydrodynamics in extremely narrow shape bioreactor. Int. Commun. Heat Mass Transfer 2019, 108, No. 104286.
(43) Liu, Y.; Zhang, L.; Zhou, L. X. Development of modeling and simulation of bubble liquid hydrodynamics in bubble column. Energy Sci. Eng. 2020, 8, 327–339.
(44) Apte, S. V.; Mahesh, K.; Moin, P.; Oefelein, J. C. Large-eddy simulation of swirling particle-laden flows in a coaxial-jet combustor. Int. J. Multiphase Flow 2003, 29, 1311.
(45) Yan, J.; Gui, N.; Xie, G. N.; Gao, J. S. Direct numerical simulation of particle-laden swirling flows on turbulence modulation. Math. Probl. Eng. 2014, 3, 1–12.
(46) Oefelein, J. C.; Oefelein, V.; Drozda, G. Large eddy simulation of swirling particle laden flow in model axisymmetric combustor. Proc. Combust. Inst. 2007, 31, 2291–2299.
(47) Zhou, H. L.; Hu, Z. Q.; Zhang, Q. L.; Lv, X.; et al. Numerical study on gas-solid flow characteristics of ultra-light particles in a cyclone separator. Powder Technol. 2019, 344, 784–796.