Entropy generation under the influence of melting heat transfer in stratified Polystyrene-water/kerosene nanofluid flow with velocity slip

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Abstract
The interaction of Polystyrene nanoparticles in the presence of entropy generation optimization along the magnetite Riga plate is the major concern of this study. The analysis deals with entropy generation minimization is well established approach to improve the efficiencies of the thermal characteristics. The performance of nanofluid is studied over a stretchable Riga plate. The water and Kerosene based Polystyrene nanoparticles is exercised to examine the influence of nanoparticles on flow and heat transport features in the region around the stagnation point. The velocity slip mechanism is accounted to observe the modification in flow field. A study is also carried out to analyze the heat transport phenomenon under the the aspects of viscous dissipation and thermal stratification. Additionally, the stimulus of realistic and more general melting surface condition is presented. The problem is modeled in view of Tiwari-Das model. The reduced governing equations are solved analytically using convergent scheme. To predict the behavior of flow velocity, entropy generation and fluid temperature against the significant involved parameters, the curves are plotted and analyzed. Skin friction coefficient and Nusselt number are also shown through graphs. Results depict that entropy generation minimizes by intensifying melting parameter and further in entropy generation minimization, the effect of water-polystyrene nanofluid is dominant over kerosene-polystyrene nanofluid.

1. Introduction
The entropy generation analysis in the field of fluid mechanics is a key component that cannot be overhighlighted because of singular fact that the accomplishment of thermal machines for instant power plants (like water/steam/gas turbines etc.), heat pumps, refrigerators, heat engines and air conditioners, primarily depends upon procurable work. Thus, in order to amplify the efficiency of thermal devices, the causes of entropy generation need to be studied which has been perceive to occur due to heat transfer. Moreover, in the analysis of heat transfer, measure of disorder is important in order to understand the efficiency of the system. In this regard, irreversibility parameter and entropy generation number are helpful. Viscous dissipation and heat transfer are two important factors on which the entropy generation number depends mostly. Also, diverse factors are accountable for irreversibility include Joule dissipation, fluid friction, diffusion and transportation of heat through fixed difference in temperature and so forth. To conserve the energy and maximize the efficiencies, it is significant to reduce or eliminate the irreversibility process. Therefore, the concept of entropy generation is utilized in minimization of heat transfer/viscous dissipation irreversibilities. The entropy generation is studied inside the system by implementing second law of thermodynamics. In this regard, Bejan [1] for the first time used the concept of entropy generation minimization in the flow of convective fluid. Bahiraei et al [2] explored the entropy generation analysis in the flow of water based nanofluid over inclined annulus considering CuO-nanoparticles and mixed convection. Akhbar and Butt [3] disclosed the optimization of entropy generation in peristaltic propagation of water based Copper (Cu) nanoparticles with dissipation effects. Khan et al [4] explained...
the features of entropy generation in Sisko fluid flow deformed by stretching plate. Manay et al [5] discussed the aspects of heat sink in flow of nanomaterial under the theory of entropy generation minimization. Khan et al [6] depicted the salient aspects of mixed convection in nanoliquid flow with entropy generation optimization. Karimzadokhohoei et al [7] scrutinized the entropy generation in water based nanofluids in a microtube with constant heat flux.

The analysis of heat transfer process has become the core topic over the years but very less interest has been show in studying heat transfer through melting phenomenon. Melting heat transfer plays a key role in physics and engineering and in advanced industrial and technological processes. Therefore owing to that the investigators have shown interest to develop the sustainable and effective technologies regarding energy storage. Such implements in which the energy keep in reserve are solar energy, waste heat recovery process, heat and power plants etc. Sensible heat, latent heat and chemical heat are the main three mechanism which are effectively able to store energy. Among these, latent heat method is the most effective and economical for storage of energy. Melting phenomenon directly relates to latent heat. Melting condition implements this source to optimize and store the energy in the material. The stored energy can be easily recovered by freezing the material. The melting of soil, preparation of semiconductor material, thawing of frozen grounds, magma solidification, soil freezing process around the heat exchanger coils in pump (i.e. ground based), melting of permafrost, sewage freezing treatment, the casting of manufacturing process are the key applications of melting phenomenon. Sheikholeslami and Rokni [8] analyzed melting heat phenomenon in nanofluid flow deformed by stretched plate e having magnetic effect. Ibrahim [9] presented the melting heat transfer in the magneto nanofluid flow deformed by stretching sheet saturated in stagnation region. Venkateswarlu et al [10] demonstrated the melting and heat source phenomena in hydromagnetic flow caused by moving sheet with viscous dissipation. Uddin et al [11] presented radiative heat transfer in slip flow of nanoliquid over stretching plate through melting heat condition. Hayat et al [12] examined the melting phenomenon in the hydromagneto flow deformed by variable thicked disc using water based nanofluid. Javed et al [13] illustrated the behavior of chemical species in stratified flow of radiative stagnant fluid past a porous medium considering melting heat condition.

Boundary conditions play a vital role in material processing technologies and significantly modify the characteristics of manufactured products. A type of boundary condition is generally studied in literature namely no-slip condition at boundary. There is another type of boundary condition is called slip condition in which the velocity of fluid at solid boundary is not zero relative to the boundary. However no-slip condition does not always applicable in real life applications, it is inadequate in various natural, industrial and bioengineering processes. Such processes involve different velocities of the object and fluid particles which is demonstrated as boundary slip. Fluid slip occurs in different applications such as inexpensive lubricating, the polishing of inner cavities and artificial heart valves, optical coatings and refrigeration equipment. Therefore the slip boundary condition has received immense attraction of researchers. Slip phenomenon in radiative flow of magneto water based nanomaterial deformed in inclined channel is examined by Abbas et al [14]. Features of partial slip and magnetic field strength in nanofluid deformed due to rotating disk are demonstrated by Mustafa [15]. Ramya et al [16] presented slip phenomenon in magnetite nanofluid flow over a sheet stretched nonlinearly. Impact of slip conditions and thermal stratification towards variable thicked Riga plates of stagnation point flow is explored by Anjum et al [17]. Sobamowo et al [18] analyzed the slip effects in squeezed flow of nanofluid under the magnetic field. Hayat et al [19] reported the mixed convection in nanofluid flow near the stagnation region past through non-Darcy porous medium with slip mechanism.

In the recent aforementioned works, the researchers have furnished tremendous work of entropy generation in nanofluid [20] flow incorporating numerous nanoparticles like Cu, Al2O3, Ag, carbon nanotubes etc. However no attempt has been made where the entropy generation aspect is used in nanofluid analysis considering polystyrene nano particles. Hence our main objective is to fill such void. These particles have significant applications in biomedical such as plastic surgery treament, enzyme activities and soil microorganisms. Further these nanoparticles are friendly suitable to soil microbes and in their involved processes. The study of slip mechanism in nanofluid flow near the region of stagnation point along the stretchable Riga plate is performed. A polystyrene nanoparticles is incorporated with base fluids as water and kerosene. Entropy generation analysis is carried out with viscous dissipation. Further thermal stratification and melting boundary condition are executed for the heat transportation. The formulated equations are simulated using a homotopic approach [20–29]. The performance of different physical parameters on characteristics of flow, temperature and entropy generation are graphed and reported. Skin friction and Nusselt number against pertinent parameters are discussed and shown graphically. We compared our results of skin friction in the limiting case and found excellent agreement between the present and already published work.
2. Mathematical formulation

We consider steady flow of an incompressible nano fluid through the Riga plate. The flow in stagnation point region is generated by linear stretching with stretching rate \( a > 0 \). Two types of nanofluids namely water-polystyrene and kerosene-polystyrene are implemented. A plate is placed horizontally along \( x \)-axis and \( y \)-axis is taken in normal direction. The flow and heat transport mechanisms are addressed by considering velocity slip and melting condition. Entropy generation is debated during the flow for impacts of viscous dissipation and thermal stratification. The governing equations subject to boundary layer approximations are expressed as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, 
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = U_e \frac{dU_e}{dx} + v_{nf} \frac{\partial^2 u}{\partial y^2} + \frac{\pi dM}{8\rho_{nf}} \exp \left( -\frac{\pi}{a1^2} \right), 
\]

\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha_{nf} \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{nf}}{(\rho c_p)_{nf}} \left( \frac{\partial u}{\partial y} \right)^2, 
\]

The implemented boundary conditions are

\[
\begin{align*}
&u = U_w(x) = ax + \beta \mu_{nf} \frac{\partial u}{\partial y}, \quad k_{nf} \frac{\partial T}{\partial y} = \rho_{nf} (\lambda + c_s(T_m - T_0)) v(x, y), \quad T = T_m = T_0 + cx \quad \text{at} \quad y = 0, \\
&u \to U_s(x) = bx, \quad T \to T_{\infty} = T_0 + \epsilon_1 x \quad \text{as} \quad y \to \infty,
\end{align*}
\]

Here \( u \) and \( v \) represents the velocity components corresponding to horizontal and vertical directions respectively, wall stretching and free stream velocities are symbolized as \( U_w \) and \( U_e \) respectively, \( \mu_{nf}, v_{nf} \) represent the nanofluid dynamic and kinematic viscosities respectively, \( j_0 \) represents the applied current density, \( M (=M_0 x) \) represents the magnetization, distance between magnets and electrodes is represented by \( a \), \( \alpha_{nf} \) describes thermal diffusivity of nanofluid, \( T \) and \( T_m \) reflect the fluid temperatures inside and outside the boundary layer, \( T_m (=T_0 + cx) \) demonstrates the melting surface temperature, \( \rho_{nf} \) represents the fluid’s density, \( c_s \) depicts the surface heat capacity, \( k_{nf} \) reflects the nanofluid’s thermal conductivity, \( \beta \) depicts velocity slip factor, \( \lambda \) represents latent heat, \( T_0 \) depicts reference temperature, \( (c_p)_{nf} \) represents nanofluid’s heat capacity whereas \( a, b, \epsilon_1, \epsilon_1 \) are the positive dimensional constants.

The thermophysical properties of nanofluids are mathematically expressed as follows:

\[
\begin{align*}
\mu_{nf} &= \frac{\mu_f}{(1 - \phi_f)^{2.5}}, & v_{nf} &= \frac{v_{nf}}{\rho_{nf}}, & \rho_{nf} &= (1 - \phi) \rho_f + \phi \rho_s, \\
\alpha_{nf} &= \frac{k_{nf}}{(\rho c_p)_{nf}}, & k_{nf} &= k_f + 2k_f - 2\phi(k_f - k_s) \quad \text{and} \quad k_f = k_f + 2\phi(k_f - k_s),
\end{align*}
\]

where \( \mu_{nf}, \phi, \rho_s, \rho_f \) represent viscosity, nanoparticle volume fraction, density of solid particles and density of base fluid respectively. However \( k_f, k_s \) and \( k_{nf} \) demonstrate the thermal conductivities of fluid, solid particles and nanofluid respectively (see table 1).

| Table 1. Thermophysical features of nanoparticle and base fluids. |
|---------------------------------------------------------------|
|                      | Nanomaterial | Base fluids          |
|----------------------|--------------|----------------------|
| Physical Properties  | Polystyrene  | Water                |
| \( \rho (\text{kg m}^{-3}) \) | 1053         | 997                  |
| \( c_p (\text{J/kgK}) \) | 1210         | 4179                 |
| \( k (\text{W/mK}) \)  | 0.16         | 0.613                |

Transformations of the form

\[
\eta = y \sqrt{\frac{a}{\nu_f}}, \quad u = axf' (\eta), \quad v = -\sqrt{\nu_f} f (\eta), \\
\theta (\eta) = \frac{T - T_m}{T_{\infty} - T_0},
\]

\[6\]
Mass conservation law vanishes identically and equations (1) to (4) are reduced to
\[
\left( \frac{1}{(1-\phi)^2(1-\phi + \phi_0^2b)} \right) f''' + ff'' - (f')^2 + Sf' + \frac{Q}{(1-\phi + \phi_0^2b)} \exp(-Bf) = 0, \quad (7)
\]
\[
\left( \frac{k_{nf}/k_f}{1 - \phi + \phi_0^2b} \right) \theta'' + Pr(f\theta' - f''\theta) - PrSi f' + \frac{PrEc}{(1-\phi)^{2.5}} \left(1 - \phi + \phi_0^2b\right) f''\theta = 0, \quad (8)
\]
The boundary conditions take the form
\[
\frac{k_{nf}/k_f}{1 - \phi + \phi_0^2b} M f'(0) + Pr f'(0) = 0, \quad f'(0) = 1 + \frac{L}{(1-\phi)^{2.5}} f''(0), \quad (9)
\]
\[\theta(0) = 0, \quad f'(\infty) \to S, \quad \theta(\infty) \to 1 - S, \quad (10)\]
where \(S\) represents ratio of rates, \(Q\) represents modified Hartman number, \(B\) dimensionless length parameter, \(Pr\) Prandtl number, \(Si\) thermal stratification parameter, \(Ec\) demonstrates Eckert number, \(L\) represents length parameter and \(M\) represents melting parameter which are mathematically represented as follows:
\[
Pr = \frac{\mu_{nf} f}{k_f}, \quad S = \frac{b}{a}, \quad Q = \frac{\pi_0^2 M_0}{8 \rho f a^2}, \quad B = \frac{\sqrt{\gamma}}{a}, \quad S_1 = \frac{c}{a}, \quad Ec = \frac{U_w^2}{\epsilon_{nf}(T_m - T_0)},
\]
\[
L = \beta \sqrt{\frac{\alpha k}{\gamma_f}}, \quad M = \frac{\epsilon_{nf}(T_m - T_0)}{\lambda + c(T_m - T_0)}. \quad (11)
\]
Mathematical expressions for surface drag and heat transfer rate or Nusselt number for the considered flow are as follows:
\[
C_f = \frac{\tau_w}{\rho_f U_w}, \quad Nu_x = \frac{x_{fl} \tau_w}{k_f(T_m - T_0)}, \quad (12)
\]
\[
\tau_w = \mu_{nf} \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -\nu_{nf} \left( \frac{\partial T}{\partial y} \right)_{y=0}. \quad (13)
\]
Dimensionless forms of these quantities are
\[
C_f \text{Re}^{1/2} = \frac{1}{(1-\phi)^{2.5}} f''(0), \quad Nu_x \text{Re}^{-1/2} = \frac{k_{nf}}{k_f} \frac{\theta'(0)}{1 - S}, \quad (14)
\]
where \(\text{Re} = U_w \text{Re/\gamma_f}\) depicts Reynolds number.

### 3. Analysis of entropy generation

Here our main motto is to compute the irreversibility of the system via entropy generation. Thus entropy generation is defined as follows:
\[
E_G = \frac{k_{nf}}{(T_m - T_0)^2} \left( \frac{\partial T}{\partial y} \right)^2 + \frac{\theta_{nf}}{(T_m - T_0)} \left( \frac{\partial u}{\partial y} \right)^2. \quad (15)
\]
The first term is responsible for the system irreversibility due to heat transfer while second term is responsible for heat generation via viscous dissipation. The dimensionless relation for entropy generation is expressed as
\[
Ns = \frac{(T_m - T_0)^2}{k_{nf}(T_m - T_0)^2} E_G. \quad (16)
\]
by implementing transformations, we have
\[
Ns = \frac{\theta'^2}{(1 - S_1)^2} + \frac{EcPr}{(1 - S_1)(1-\phi)^{2.5}} \frac{k_f}{k_{nf}} \theta'^2. \quad (17)
\]
The Bejan number for each point location of boundary layer is expressed as \[1\]

\[
Be = \frac{\text{Entropy generation due to thermal irreversibility}}{\text{Total entropy generation}},
\]

\[
Be = \frac{\theta^2}{(1 - S)^2} + \frac{\theta^2}{(1 - S)^2 + \frac{q}{(1 - S)(1 - \phi)^2} \frac{k_{af}}{Prk_f}}.
\] (18)

4. Homotopic solutions

Homotopic method is adopted to solve the governing flow equations. For this method, initial approximations as well as linear operators have been essentially taken into account which are mentioned below

\[
f_0(\eta) = S \eta + \frac{(1 - \phi)^{2.5}(1 - S)}{(1 - \phi)^{2.5} + L}(1 - \exp(-\eta)) - \frac{k_{af}/k_f M}{(1 - \phi + \phi \frac{q}{\eta})} Pr,
\]

\[
\theta_0(\eta) = (1 - S)(1 - \exp(-\eta)),
\] (19)

\[
L_f(\eta) = \frac{d^2 \eta}{d \eta^2} - \frac{df}{d \eta}, \quad L_\theta(\eta) = \frac{d^2 \theta}{d \eta^2} - \theta,
\] (20)

\[
L_f [A_i^* + A_i^* \exp(\eta) + A_i^* \exp(-\eta)] = 0,
\] (21)

\[
L_\theta [A_i^* \exp(\eta) + A_i^* \exp(-\eta)] = 0,
\] (22)

where \(A_i^* (i = 1, 2, \ldots, 5)\) represent the arbitrary constants.

4.1. Zeroth-order problem

\[
(1 - q) L_f[\bar{f}(\eta; q) - f_0(\eta)] = q h_f N_f[\bar{f}(\eta; q)],
\]

\[
(1 - q) L_\theta[\bar{\theta}(\eta; q) - \theta_0(\eta)] = q h_\theta N_\theta[\bar{\theta}(\eta; q)],
\] (23)

\[
\frac{k_{af}/k_f}{(1 - \phi)^{2.5}(1 - \phi + \frac{q}{\eta})} M \frac{\partial^2 \bar{f}(0; q)}{\partial \eta^2} + Pr f^0(0; q) = 0, \quad \bar{f}^\prime(0; q) = 1 + \frac{L}{(1 - \phi)^{2.5}(1 - \phi + \frac{q}{\eta})} \bar{f}^\prime(0; q),
\]

\[
\bar{f}^\prime(\infty; q) = S, \quad \bar{\theta}(0; q) = 0, \quad \bar{\eta}(\infty; q) = 0,
\] (25)

\[
N_f[\bar{f}(\eta; q), \bar{\theta}(\eta; q)] = \left(1 - \phi + \phi \frac{q}{\eta}\right) \frac{\partial \bar{f}(\eta; q)}{\partial \eta} + \bar{f}(\eta; q) \frac{\partial^2 \bar{f}(\eta; q)}{\partial \eta^2}
\]

\[
- \left(\frac{\partial \bar{f}(\eta; q)}{\partial \eta}\right)^2 + \frac{Q}{(1 - \phi + \phi \frac{q}{\eta})} \exp(-B\eta),
\] (26)

\[
N_\theta[\bar{\theta}(\eta; q), \bar{f}(\eta; q)] = \left(1 - \phi + \phi \frac{q}{\eta}\right) \frac{\partial^2 \bar{\theta}(\eta; q)}{\partial \eta^2} + Pr \left(\frac{\partial \bar{f}(\eta; q)}{\partial \eta}, \frac{\partial \bar{f}(\eta; q)}{\partial \eta} - \theta(\eta; q) - S1 \frac{\partial \bar{g}(\eta; q)}{\partial \eta}
\]

\[
+ \frac{Ec}{(1 - \phi)^{2.5}(1 - \phi + \phi \frac{q}{\eta})} \left(\frac{\partial \bar{f}(\eta; q)}{\partial \eta}\right)^2,
\] (27)

where \(q\) represents embedding parameter and having value \([0, 1]\) while non-zero auxiliary parameters are represented by \(h_f\) and \(h_\theta\).
4.2. mth-order deformation problems

\[ \mathcal{L}_f[f_m(\eta) - \chi_m f_{m-1}(\eta)] = h_f \mathcal{R}_m^f(\eta), \]

\[ \mathcal{L}_\theta[\theta_m(\eta) - \chi_m \theta_{m-1}(\eta)] = h_\theta \mathcal{R}_m^\theta(\eta), \]

\[ \frac{k_{nf}}{k_f} (1 - \phi) \left[1 - \phi + \phi \frac{k_{s}}{k_f}\right] + \Pr f_m'(0) = 0, \quad f_m'(0) - \frac{L}{(1 - \phi)^2 \left(1 - \phi + \phi \frac{k_{s}}{k_f}\right)^2} f_m''(0) = 0, \]

\[ f_m'(\infty) = 0, \quad \theta_m(0) = 0, \quad \theta_m(\infty) = 0, \]

\[ \mathcal{R}_m^f(\eta) = \left(1 - \phi + \phi \frac{k_{s}}{k_f}\right) f_m^{m-1} + \sum_{k=0}^{m-1} \left(f_{m-1-k} f_k' - f_{m-1-k} f_k''\right) \]

\[ + S \left(1 - \chi_m\right) + \left(1 - \phi + \phi \frac{k_{s}}{k_f}\right) \exp(-B\eta), \]

\[ \mathcal{R}_m^\theta(\eta) = \left(1 - \phi + \phi \frac{k_{s}}{k_f}\right) \theta_m^{m-1} + \Pr \sum_{k=0}^{m-1} \left(1 - \phi + \phi \frac{k_{s}}{k_f}\right) \left[\frac{f_{m-1-k} \theta_k' - f_{m-1-k} \theta_k''}{(1 - \phi)^2} \right] f_m^{m-1-k} f_k'' \]

\[ - \Pr S \theta_m', \]

\[ \chi_m = \begin{cases} 0, & m \leq 1 \\ 1, & m > 1 \end{cases}. \]

The general solutions \((f_m, \theta_m)\) of equations (28) and (29) are represented by

\[ f_m(\eta) = f_m^*(\eta) + A_1^* e^{\eta} + A_2^* e^{-\eta}, \]

\[ \theta_m(\eta) = \theta_m^*(\eta) + A_1^* e^{\eta} + A_2^* e^{-\eta}, \]

here \(f_m^*(\eta)\) and \(\theta_m^*(\eta)\) demonstrates the special solutions where \(A_1^*\) depicts arbitrary constants.

4.3. Convergence analysis

The convergence of the computed analytic solutions is necessary and important part of the analysis. To ensure the convergence region, we have demonstrated graphically the \(h\)-curves in the figures 1 and 2. Auxiliary parameters have ranges for water-polystyrene nanofluid \(-1.3 \leq h_f \leq -0.3, -1.4 \leq h_\theta \leq -0.3\) while for kerosene-polystyrene nanofluid are \(-1.6 \leq h_f \leq -0.8, -1.4 \leq h_\theta \leq -0.7\).

5. Discussion

Here, we demonstrate how the representative parameters influence the velocity and temperature distributions. Hence figures are produced. The velocity field for different values of slip parameter \(L\) is plotted in figure 3. For both water-polystyrene and kerosene-polystyrene nanofluids, this figure indicates a decrease in velocity by increasing \(L\). Due to greater slip parameter \(L\), weak adhesive forces tend to decrease the deformation in fluid and consequently decays the nanofluid velocity. It is also estimated here that the velocity for water-polystyrene nanofluid is stronger than the velocity for kerosene-polystyrene nanofluid. The variation of nanofluid velocity
for diverse values of melting parameter \( M \) is highlighted through figure 4. Here, the velocity field follows a growing trend for both water-polystyrene and kerosene-polystyrene nanofluids by increasing \( M \). In fact, greater values of melting parameter \( M \) helps in developing the melting rate which is responsible for transfer extra heat from hotter fluid towards the surface. In this way, greater convective flow is observed and resultantly velocity field enhances. Furthermore, comparison reveals that the velocity for the water-polystyrene nanofluid is dominant over kerosene-polystyrene nanofluid. In figure 5, the results for velocity field are sketched by considering the nano particle volume fraction \( \phi \) for two type of nanofluids namely water-polystyrene and kerosene-polystyrene. It is seen that velocity field is escalating function of \( \phi \). However greater velocity exist at the surface. It is also perceived that magnitude of velocity reduces in case of kerosene-polystyrene nanofluid as compared to water-polystyrene nanofluid. Figure 6 displays how the modified Hartmann number \( Q \) influences the velocity field for the two nanofluids i.e. water-polystyrene and kerosene-polystyrene. Greater modified Hartmann number \( Q \) makes an effective growth in magnitude of velocity. It is seen that electric field intensifies by incrementing the value of modified Hartmann number \( Q \) against the whole stretching surface and consequently grows the velocity distribution. Also water-polystyrene nanofluid velocity makes a dominant
impact over the velocity of kerosene-polystyrene nanofluid. Figure 7 illustrates the velocity field for some values of ratio parameter $S$ for water-polystyrene and kerosene-polystyrene nanofluids. The velocity field dominates for higher ratio parameter $S$. Further $S = 1$ corresponds to no boundary layer thickness. It means that the fluid and sheet both have the same velocity and also no deformation or disturbance occurs in the fluid. The cases $S < 1$ and $S > 1$ corresponding to maximum velocity exists at the surface and away from the wall. Further velocity is dominant for water-polystyrene in comparison to kerosene-polystyrene nanofluid. Figure 8 discloses the performance of Eckert number $Ec$ on temperature field. It is seen that temperature increases with dominating values of $Ec$ for both types of nanofluids. As Eckert number $Ec$ increases, the drag force between fluid particles increase which causes more heat transport and alternatively the temperature field grows. The temperature field for diverse values of melting parameter $M$ is presented in figure 9. For both water-polystyrene and kerosene-polystyrene nanofluids, the temperature field weakens with an increment of $M$. In fact, when melting parameter $M$ grows, extra heat is transported from the heated fluid towards the surface. Therefore, temperature field shows decreasing behavior. Moreover, water-polystyrene nanofluid velocity shows higher...
magnitude than the velocity of kerosene-polystyrene nanofluid. Figure 10 shows how the nano particle volume fraction \( \phi \) affects the temperature distribution. One sees that temperature curve becomes elevated for higher nano particle volume fraction \( \phi \) for both considered nanofluids. Figure 11 reports the field of temperature for different values of stratification parameter \( S_1 \). This figure illustrates decline in temperature of the nanofluid under boundary layer by increasing \( S_1 \). Higher stratification parameter \( S_1 \) relates to greater melting surface temperature which reduces heat transfer and it shows quantitative reducing behavior of temperature field by changing the involved parameter. However rate of decay in water-polystyrene is faster than kerosene-polystyrene nanofluid. In figure 12, the skin friction coefficient is plotted in the presence of ratio parameter \( S \) and modified Hartmann number \( Q \) for both water/kerosene based polystyrene nanoparticle. By increasing ratio parameter \( S \) and modified Hartmann number \( Q \), the values of skin friction coefficient become smaller for both cases of present of water and kerosene based polystyrene nanofluids. It is interesting to mention that the magnitude of skin friction in case of polystyrene-water nanofluid effectively become minimum as compare to polystyrene-kerosene nanofluid. In figure 13, the Nusselt number is plotted for both polystyrene-water and
polystyrene-kerosene nanofluids by incorporating impacts of different stratification and melting parameters. The minimum magnitude of Nusselt number is observed for polystyrene-water/kerosene nanofluids in the presence of growing stratification parameter $S_1$ and melting parameter $M$. However heat transfer rate is dominant for water-polystyrene nanofluid in comparison to kerosene-polystyrene.

6. Production of entropy

In this portion, we display graphical results depicting the performance of eminent parameters on entropy generation phenomenon. Figure 14 illustrates the entropy generation for some values of Eckert number $Ec$. It reveals that variation of $Ec$ enhances the entropy generation. Physically higher Eckert number $Ec$ is responsible for the production of extra heat and therefore entropy generation enhances. In this figure, the polystyrene-water nanofluid minimizes the magnitude of the entropy generation as compared to the polystyrene-kerosene nanofluid. Hence polystyrene-water nanofluid behaves as the best coolant which causes entropy generation decline in the flow. To capture the aspect of melting parameter $M$ on entropy generation, figure 15 is portrayed.
Here, entropy generation curve determines decreasing behavior for enlarge melting parameter $M$. The values of entropy generation are lower for polystyrene-water nano fluid and higher for polystyrene-kerosene nano fluid. However minimum entropy is generated for the case water-polystyrene nano fluid. Figure 16 determines the impact of stratification parameter $S_1$ on entropy generation. Here, entropy generation shows intensifying performance for stratification parameter $S_1$. Polystyrene-kerosene nano fluid secures the highest magnitude of entropy generation and polystyrene-water nano fluid attains the lowest magnitude of the entropy generation. Therefore, minimization in entropy is achieved for water-polystyrene nano fluid in comparison to kerosene-polystyrene nano fluid. Figure 17 demonstrates the streamlines behavior of flow for both polystyrene-water and polystyrene-kerosene nano fluids respectively.

Table 2 reflects the comparison of skin friction in limiting sense. It is revealed that our results are in good agreement with the previous published work.
The above table reflects that all the results are matched in excellent order.

7. Closing remarks

Here we have investigated the slip flow of water and kerosene based nanofluid considering polystyrene as nanoparticles in the neighborhood of stagnation point. The melting heat transfer is carried out in the presence of stratification and viscous dissipation effects. A theoretical analysis of entropy generation is also presented. The key points of this work is listed as follows:

- Velocity distribution become higher corresponding to greater values of $Q$ and $M$. Also velocity is higher in case of polystyrene-water nanofluid.
- Velocity field has declined behavior for higher $L$.
- Temperature field reflects decreasing behavior for water-polystyrene and kerosene-polystyrene nanofluids by increment in $M$ and $S_1$.
- Drag force decays by increasing ratio parameter $S$ and modified Hartman number $Q$.
- Minimization of entropy generation occurs for higher melting parameter $M$ especially for water-polystyrene nanofluid.

It is hoped that the current research work will be used as stimulus and motivation for biomedical treatment especially, in nano-medicine and plastic surgery treatments. These polystyrene particles are more appropriate for soil microbes and involved processes therein.

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