The impact of vaccination on the modeling of COVID-19 dynamics: a fractional order model

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Abstract The coronavirus disease 2019 (COVID-19) is a recent outbreak of respiratory infections that have affected millions of humans all around the world. Initially, the major intervention strategies used to combat the infection were the basic public health measure, nevertheless, vaccination is an effective strategy and has been used to control the incidence of many infectious diseases. Currently, few safe and effective vaccines have been approved to control the inadvertent transmission of COVID-19. In this paper, the modeling approach is adopted to investigate the impact of currently available anti-COVID vaccines on the dynamics of COVID-19. A new fractional-order epidemic model by incorporating the vaccination class is presented. The fractional derivative is considered in the well-known Caputo sense. Initially, the proposed vaccine model for the dynamics of COVID-19 is developed via integer-order differential equations and then the Caputo-type derivative is applied to extend the model to a fractional case. By applying the least square method, the model is fitted to the reported cases in Pakistan and some of the parameters involved in the models are estimated from the actual data. The threshold quantity ($R_0$) is computed by the Next-generation method. A detailed analysis of the fractional model, such as positivity of model solution, equilibrium points, and stabilities on both disease-free and endemic states are discussed comprehensively. An efficient iterative method is utilized for the numerical solution of the proposed model and the model is then simulated in the light of vaccination. The impact of important influential parameters on the pandemic dynamics is shown graphically. Moreover, the impact of different intervention scenarios on the disease incidence is depicted and it is found that the reduction in the effective contact rate (up to 30%) and enhancement in vaccination rate (up to 50%) to the current baseline values significantly reduced the disease new infected cases.

Keywords Fractional analysis · COVID-19 mathematical model · Vaccination · Data fitting · Stability · Numerical simulations

1 Introduction

Coronavirus infection is a contagious viral disease affecting almost all countries on the globe. This viral infection is caused by Severe Acute Respiratory
Syndrome coronavirus 2 (SARS-CoV-2) and firstly emerged in December of 2019. The COVID-19 was declared a pandemic by WHO in March 2020 due to its spread to millions of people worldwide [1]. COVID-19 infected more people than either of SARS and MERS. The first outbreak of respiratory illness cases due to COVID-19 was identified in Wuhan, Hubei Province, China. Most of the infected people with COVID-19 might be asymptotic, i.e., having no or even mild clinical symptoms. The infected individuals with disease symptoms typically include dry cough, fever, muscle or joint pain and in severe cases, the patient feels shortness of breath. The COVID-19 is life-threatening for those people who are already infected with other diseases like diabetes, kidney disease, obesity, cardiovascular disease. People having chronic respiratory disease are particularly at high risk of dying from the COVID-19. This infection is primarily transferred from one human to another while touching contaminated surfaces or objects. The transmission of viruses also takes place during the inhalation of respiratory droplets from either symptomatically or asymptotically infected people [1, 2].

The COVID-19 is still a serious threat to the human around the globe as the incidence of this infection in some countries is increasing. Therefore, detection, assessment, and interpretation of the immune response to SARS-CoV-2 infection becomes essential. Some diseases can be controlled by vaccinating children under a certain age, like Polio. Also, certain antibiotics and medicines control infection outbreaks around the globe. Infectious disease has been eliminated during the recent past by the use of vaccines. Still, TB, Cholera, Malaria are responsible for mortalities in the world [3]. Although different effective vaccines have been introduced recently, still most countries rely on non-pharmaceutical interventions (NPI’s), such as social distancing, lockdown, and face masks to combat the infection. However, vaccination is an effective strategy in the control of such infectious diseases like the most successful vaccination of smallpox ever recorded [4]. The development of vaccines against COVID-19 has occurred, and different vaccines have been rolled out in countries. As vaccines are a new tool in the battle against COVID-19, so that is why scientists across the world are collaborating in the development of effective vaccines to save human lives [4].

To better understand the dynamics of epidemic illnesses and propose better methods for preventing further transmission, mathematical models might be a useful tool. For forecasting disease dynamics and flattening the infection and mortality curves, new mathematical models are put forward [5–8]. Also, many compartmental mathematical models addressing the impact and importance of vaccination against different infectious diseases have been presented in [9–14]. Recently, the mathematical modeling approach has been used to study the long-term impact of vaccination on the incidence and control of COVID-19 as can be found in the literature and reference therein [15–18]. Fractional Calculus (FC) generalizes the classical integer order calculus and gain more popularity to investigate real-world phenomena due to its many distinguishing features. These operators have been used over time to disclose useful and current studies. In this context, FC and its diverse uses are attracting the interest of many scholars. The concept of fractional order derivative originated with Leibnitz’s question in 1695. Over time, different fractional derivatives are introduced and found their application in various fields. The most common fractional operators are the Riemann–Liouville derivative, Caputo derivative, Caputo–Fabrizio fractional derivative (CF), and Atangana–Baleanu fractional derivative (ABC) [19–22] and many more. The combination of memory and hereditary aspects in modeling COVID-19 infection demonstrates the need of dealing with fractional-order derivatives. Because of the memory effect, the non-integer model includes all previous data from the past, allowing them to determine the epidemic scenarios more precisely. When comparing the fractional-order derivative to the integer-order derivative, it is evident from the literature that fractional operators provide more accurate and deeper results while representing real-life scenarios [23–25]. Recently, Khan and Atangana studied the trends of the pandemic in Wuhan using fractional derivative [26]. A mathematical modeling approach using novel fractional-fractal operators has been adopted for the purpose to study the population level influence of community lockdown over the COVID-19 dynamics [27]. A fractional modeling approach with the Caputo operator has been applied to explore the dynamical patterns of COVID-19 in a better way [28, 29]. The analysis of different scenarios of some interventions on the control of infection is carried out with the help ABC fractional-order epidemic model in [30].

Keeping the above literature in view, the present study is focused on the formulation of a new fractional
model to analyze the dynamics of COVID-19 with the impact of vaccination. The Caputo operator is applied to construct the proposed model. Further, the study’s potential goal is to analyze COVID-19 transmission patterns under some NPIs and particularly vaccination. The parameters involved in the proposed model are estimated using actual pandemic cases in Pakistan. The basic reproduction number and equilibrium points are presented and some of the basic analysis are carried out. The research conclusions of the study may help government and public health authorities to adopt new strategic plans to reduce the spread of outbreaks in the future. Section 2 contains the basic concepts regarding fractional calculus (FC), and model formulation with parameter estimation in Sect. 3 is presented. Fractional model formulation and analysis in Sect. 4. Section 5 includes numerical results and discussion. Finally, the conclusion is accomplished in Sect. 6.

2 Preliminaries on fractional derivative

The basic definitions regarding the FC are as follows:

**Definition 2.1** The Caputo derivative of order \( \alpha \) in \( (n - 1, n) \), [19] is defined as:

\[
C D^\alpha_t (p(t)) = \frac{1}{\Gamma(n - \alpha)} \int_0^t \frac{p^n(h)(t - h)^{n-\alpha}}{(t - h)} dh,
\]

**Definition 2.2** The respective integral in the Caputo sense is defined as:

\[
I^\alpha_t (p(t)) = \frac{1}{\Gamma(\alpha)} \int_0^t \frac{p(h)(t - h)^{\alpha}}{(t - h)} dh, \quad 0 < \alpha < 1, \quad t > 0.
\]

**Definition 2.3** The ABC operator is defined as [22]:

\[
\begin{align*}
ABC D^\alpha_a (p(t)) &= \frac{ABC(\alpha)}{(1 - \alpha)} \int_a^t p(h)E_a[\frac{-\alpha (t - h)^{\alpha}}{1 - \alpha}] dh, \quad 0 < \alpha \leq 1. \\
ABC I^\alpha_a (p(t)) &= \frac{1 - \alpha}{ABC(\alpha)} p(t) + \frac{\alpha}{ABC(\alpha) \Gamma(\alpha)} \int_a^t p(h)(t - h)^{\alpha-1} dh,
\end{align*}
\]

where \( \alpha \in [0, 1) \).

**Definition 2.5** If \( m^* \) is an equilibrium point of the system with Caputo derivative

\[
C D^\alpha_t m(t) = h(t, m(t)), \quad \alpha \in (0, 1),
\]

iff \( h(t, m^*) = 0 \).

3 The COVID-19 vaccine model

The current section deals with the formulation of the proposed compartmental COVID-19 vaccination model to explore the influence of some NPIs, i.e., social distancing coupled with vaccination. However, the vaccine of COVID-19 is available up to some extent and is in the experimental phase but has a vital role to control the pandemic with some other pharmaceutical interventions. To develop the mathematical model the total population is divided into different compartments, the susceptible \( S(t) \), vaccinated \( V(t) \), exposed \( E(t) \), asymptomatic infected \( I_A(t) \), symptomatic infected \( I(t) \) and those who completely recovered from the infection \( R(t) \). Moreover, the cumulative population \( N(t) \) is such that

\[
N(t) = S(t) + E(t) + I(t) + I_A(t) + V(t) + R(t).
\]

The proposed COVID-19 vaccine model results in the form of a nonlinear set of deterministic ordinary differential equations and is mathematically described as follows:

\[
\begin{align*}
\frac{dS}{dt} &= \Delta + \psi_v V - \frac{\beta(I + \beta A)}{N} S - (\mu + \omega_v) S, \\
\frac{dE}{dt} &= \beta(I + \beta A) S - (\kappa + \mu) E, \\
\frac{dI}{dt} &= (1 - r) \kappa E - (\mu + \mu_1 + \phi_1) I, \\
\frac{dI_A}{dt} &= \mu \kappa E - (\mu + \phi_2) I_A, \\
\frac{dV}{dt} &= \omega_v S - (\psi_v + \mu) V, \\
\frac{dR}{dt} &= \phi_1 I + \phi_2 I_A - \mu R,
\end{align*}
\]

along with the initial conditions

\[
S(0) = S_0 \geq 0, \quad E(0) = E_0 \geq 0, \quad I(0) = I_0 \geq 0, \quad I_A(0) = I_{A0} \geq 0, \quad V(0) = V_0, \quad R(0) = R_0 \geq 0.
\]
In the above-considered model (1), $\Delta$ is assumed as birth rate, while $\mu$ be the natural death rate in each class of the model. The vaccination rate of susceptible people is symbolized by $\omega_v$, and the vaccinated population becomes susceptible at rate $\psi_v$. The latent individuals are infected with a rate $\kappa(1 - \gamma)$, where the remaining individuals are shown by $\gamma \kappa$ join the asymptomatic infected. The parameter $\mu_1$ is the COVID-19-induced mortality rate and $\phi_i$ for $i = 1, 2$ shows the rate by which the infected individuals both symptomatically and asymptotically become recovered. The disease transmission is shown by $\beta$, while $\beta_A$ is the transmissibility coefficient of asymptomatic individuals.

3.1 Data fitting and parameter estimation

The nonlinear least squares approach is discussed in this section as it relates to parameter estimation. The COVID-19 vaccination model is used to suit the confirmed infected patients that were reported in Pakistan during a certain time period of the pandemic. Some parameter values, including the rate of recruitment and the rate of natural death, are drawn from published works. We obtain the revised reproduction number $R_0 \approx 1.5839$ by factoring in the parameter values, which are displayed in Table 1. The model prediction (shown by solid blue curve) to the reported cases (shown a red circle) is depicted in Fig. 1, showing a good agreement to the real data curve. Table 1 displays the estimated and fitted model parameter values as a result. The initial condition is taken as $S(0) = 220870336$, $E(0) = 20000$, $I(0) = 4$, $IA(0) = 200$, $V(0) = R(0) = 0$.

4 Formulation of Caputo model

The fractional-order derivative-based epidemic models are more prevalent and offer greater insights into the dynamical patterns of illness. The Caputo derivative with non-integer order is the most commonly used in the literature. The present section reformulates the classical integer order COVID-19 vaccine model described in (1) in the light of Caputo fractional order. Utilizing the Caputo derivative instead of classical derivative in the model (1), the resulting epidemic model leads to the fractional differential equations system, given by:

\[
\begin{align*}
C D^\alpha_0 S(t) &= \Delta + \psi_v V - \frac{\beta(I + \beta_A IA)}{N} S - (\mu + \omega_v) S, \\
C D^\alpha_0 E(t) &= \frac{\beta(I + \beta_A IA)}{N} S - (\kappa + \mu) E, \\
C D^\alpha_0 I(t) &= (1 - r) \kappa E - (\mu + \mu_1 + \phi_1) I, \\
C D^\alpha_0 IA(t) &= r \kappa E - (\mu + \phi_2) IA, \\
C D^\alpha_0 V(t) &= \omega_v S - (\psi_v + \mu) V, \\
C D^\alpha_0 R(t) &= \phi_1 I + \phi_2 IA - \mu R.
\end{align*}
\]

Let

\[
\lambda = \frac{\beta(I + \beta_A IA)}{N},
\]

and

\[
k_0 = (\mu + \omega_v), k_1 = (\kappa + \mu), k_2 = (\mu + \mu_1 + \phi_1), k_3 = (\mu + \phi_2), k_4 = (\psi_v + \mu).
\]

The model in (2) can be summarized as

\[
\begin{align*}
C D^\alpha_0 S(t) &= \Delta + \psi_v V - \lambda S - k_0 S, \\
C D^\alpha_0 E(t) &= \lambda S - k_1 E, \\
C D^\alpha_0 I(t) &= (1 - r) \kappa E - k_2 I, \\
C D^\alpha_0 IA(t) &= r \kappa E - k_3 IA, \\
C D^\alpha_0 V(t) &= \omega_v S - k_4 V, \\
C D^\alpha_0 R(t) &= \phi_1 I + \phi_2 IA - \mu R.
\end{align*}
\]
Table 1  Biological description of parameters with corresponding estimated values

| Notation | Details | Time unit per/day | Source |
|----------|---------|-------------------|--------|
| Δ        | Recruitment rate | μ * N(0) | Estimated |
| μ        | Natural death rate | \( \frac{1}{0.7 \times 365} \) | [31] |
| μ₁      | Infection-induced mortality rate | 0.022 | |
| φ₁      | Recovery rate of individuals in \(I(t)\) class | 0.4958 | Fitted |
| φ₂      | Recovery rate of individuals in \(I_A(t)\) class | 0.1110 | Fitted |
| β       | Disease transmission rate | 0.6022 | Fitted |
| β₁      | Transmissibility relative to \(I_A(t)\) class | 0.7459 | Fitted |
| κ       | Transmission rate from \(E\) to \(I\) | 0.5171 | Fitted |
| r       | Proportion of exposed people join \(I_A\) class | 0.8833 | Fitted |
| \(ω_v\) | Vaccination rate of susceptible class | 0.0313 | Fitted |
| \(ψ_v\) | loss of immunity | 0.0233 | Fitted |

with

\[
\mathbb{R}^6_+ = \{ y \in \mathbb{R}_+^6 | y \geq 0 \} \quad \text{and} \quad y(t) = \left( S(t), E(t), I(t), I_A(t), V(t), R(t) \right)^T .
\]

(4)

In order to proceed further, we first present the basic analysis of the vaccine model in Caputo case (3).

4.1 Invariant region

The invariant region to study the COVID-19 vaccine model in fractional case (3) is constructed as follows:

\( y \subset \mathbb{R}^6_+ \),

such that

\[
\left( S(t), E(t), I(t), I_A(t), V(t), R(t) \right) \in \mathbb{R}^6_+ ; S(t) + E(t) + I(t) + I_A(t) + V(t) + R(t) \leq \frac{\Delta}{\mu}.
\]

Further, we present the following lemma.

**Lemma 4.1**  The region described above \( y \in \mathbb{R}^6_+ \) is positively invariant for COVID-19 vaccine model (3) with the corresponding initial conditions in \( \mathbb{R}^6_+ \).

**Proof**  For the model (3) we have,

\[
C D_{0,t}^\alpha N(t) = C D_{0,t}^\alpha S(t) + C D_{0,t}^\alpha E(t) + C D_{0,t}^\alpha I(t) + C D_{0,t}^\alpha I_A(t) + C D_{0,t}^\alpha V(t) + C D_{0,t}^\alpha R(t).
\]

Hence,

\[
C D_{0,t}^\alpha N(t) = \Delta - \mu N(t) - \mu_1(t) \leq \Delta - \mu N(t),
\]

we have

\[
C D_{0,t}^\alpha N(t) + \mu N(t) = \Delta.
\]

The utilization of Laplace transform leads to

\[
L[C D_{0,t}^\alpha N(t) + \mu N(t)] = L[\Delta],
\]

\[
s^\alpha N(s) - s^{\alpha-1} N(0) + \mu N(s) \leq \frac{\Delta}{s},
\]

\[
N(s) \leq \frac{\Delta}{s(s^\alpha + \mu)} + N(0) \frac{s^{\alpha-1}}{s^\alpha + \mu}.
\]

The utilization of inverse Laplace leads to

\[
N(t) = N(0) E_{\alpha,1}(-\mu t^\alpha) + \Delta t^\alpha E_{\alpha,\alpha+1}(-\mu t^\alpha),
\]

(5)

with the generalized Mittag-Leffler function is given by the following infinite series

\[
E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{Z^k}{\Gamma(\beta + \alpha k)}.
\]

Finally, the equation (5) leads to the following result

\[
\lim_{t \to \infty} N(t) \leq \frac{\Delta}{\mu}.
\]

Thus, the biologically feasible region is constructed as:

\[
\mathbb{R}^6_+ = \{ y \in \mathbb{R}_+^6 | y \geq 0 \}, \quad \text{where} \quad y(t) = \left( S(t), E(t), I_A(t), I(t), V(t), R(t) \right)^T.
\]

To proceed further, first we recall the generalized mean values theorem [33].

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Lemma 4.2 [33] Let suppose that $\mathcal{H}(y) \in C[c_1, c_2]$ and $C D^p_t \mathcal{H}(y) \in (c_1, c_2]$, then

$$\mathcal{H}(y) = \mathcal{H}(c_1) + \frac{1}{\Gamma(p)} (C D^p_t \mathcal{H})(\chi)(t - c_1)^p,$$

where $c_1 \leq \chi \leq t$, for all $t \in (c_1, c_2]$.

Corollary 1 [33] Suppose that $\mathcal{H}(y) \in C[c_1, c_2]$ and $C D^p_t \mathcal{H}(y) \in (c_1, c_2]$, where $p \in (0, 1]$. Then if

(i) $C D^p_t \mathcal{H}(y) \geq 0, \forall y \in (c_1, c_2)$,

(ii) $C D^p_t \mathcal{H}(y) \leq 0, \forall y \in (c_1, c_2)$,

then $\mathcal{H}(y)$ is non-decreasing.

We are now able to give the following result.

4.2 Positivity and boundedness

Theorem 4.1 The solution of model (3) is non-negative and bounded remain in $\mathbb{R}^6_+$ for all time $t > 0$.

Proof For the positivity of the solution of the COVID-19 vaccine model in Caputo it is necessary fact is to prove that on each hyperplane bounding the positive orthant, the vector field points to $\mathbb{R}^6_>$. The system (2), leads to

$$C D^\alpha_0, S(t) = \Delta + \psi_y V \geq 0,$$

$$C D^\alpha_0, E(t) = \lambda S \geq 0,$$

$$C D^\alpha_0, I(t) = (1 - r) \kappa E \geq 0,$$

$$C D^\alpha_0, I_A(t) = r \kappa E \geq 0,$$

$$C D^\alpha_0, R(t) = \phi I + \phi_2 I_A \geq 0.$$

Thus, by utilizing the fact in corollary 1, it is deduced that all the solutions will be in $\mathbb{R}^6_+$.

4.3 Model equilibria

To evaluate the equilibria of the proposed model (3), we need to solve the following linearized system:

$$C D^\alpha_0, S(t) = C D^\alpha_0, E(t) = C D^\alpha_0, I(t) = C D^\alpha_0, I_A(t)$$

$$= C D^\alpha_0, V(t) = C D^\alpha_0, R(t) = 0.$$

Theorem 4.2 The model (3) has at most two equilibrium points.

The disease-free equilibrium (DFE) is given by

$$D_0 = (S^0, 0, 0, 0, V^0, 0) = \left(\frac{\Delta}{\mu}, 0, 0, 0, \frac{\Delta \omega_1}{\psi_y \omega_y - k_0 k_4}, 0\right).$$

The endemic equilibrium (EE) of the COVID-19 vaccine Caputo model (3) denoted by $D_1(S^{**}, E^{**}, I^{**}, I_A^{**}, V^{**}, R^{**})$ with

$$\begin{align*}
S^{**} &= \frac{\Delta k_4}{\lambda^{**} k_4 + k_0 k_4 - \psi_y \omega_y}, \\
E^{**} &= \frac{\Delta \lambda^{**} k_4}{k_1 (\lambda^{**} k_4 + k_0 k_4 - \psi_y \omega_y)}, \\
I^{**} &= \frac{\Delta k \lambda^{**} k_4}{k_2 k_1 (\lambda^{**} k_4 + k_0 k_4 - \psi_y \omega_y)}, \\
I_A^{**} &= \frac{\Delta \lambda^{**} k_4}{k_3 (\lambda^{**} k_4 + k_0 k_4 - \psi_y \omega_y)}, \\
V^{**} &= \frac{\Delta \omega_0}{\lambda^{**} k_4 + k_0 k_4 - \psi_y \omega_y}, \\
R^{**} &= \frac{\Delta k \lambda^{**} k_4}{k_2 k_3 k_4 (\lambda^{**} k_4 + k_0 k_4 - \psi_y \omega_y)}. 
\end{align*}$$

The force of infection yields,

$$\lambda^{**} = \frac{\beta (I^{**} + \beta A I_A^{**})}{N^{**}}. $$

Substituting (7) in to (8), the non-zero equilibria of the model satisfies the equation given by

$$d_0 \lambda^{**} + d_1 = 0,$$

where

$$d_0 = k_4 \{k k_3 (1 - r) (\mu + \phi_1) + k_2 (k_3 \mu + \kappa r (\mu + \phi_2))\},$$

$$d_1 = \mu k_1 k_2 k_3 (k_4 + \omega_y) (1 - R_0).$$

Here, it should be noted that from (9) that $d_0 = 0$ is positive while $d_1$ can be positive or negative depending on the value of $R_0$. To have a unique positive endemic equilibrium for our proposed model then $R_0 > 1$ and hence the existence of positive equilibrium exists.

4.4 The basic reproduction number

To derive the most crucial quantity called the basic reproduction number $R_0$, with the help of well-known Next-generation steps we proceed as:

Let $x = (E, I, I_A, V)^T$, then we have

$$F = \begin{pmatrix}
0 & \frac{\beta k_4}{k_4 + \omega_y} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},

\quad V = \begin{pmatrix}
k_1 & 0 & 0 & 0 & 0 & 0 \\
0 & k_2 & 0 & 0 & 0 & 0 \\
-(1 - r) \kappa & 0 & k_3 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix},$$

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Table 2  The sensitivity indices of model parameters

| Parameter | sensitivity |
|-----------|-------------|
| \( \mu_1 \) | -0.0015545 |
| \( \phi_1 \) | -0.0350339 |
| \( \phi_2 \) | -0.063058 |
| \( \beta \) | +1.00 |
| \( \beta_A \) | +0.963409 |
| \( \kappa \) | +0.000078 |
| \( r \) | +0.68645 |
| \( \omega_v \) | -0.572835 |
| \( \psi_v \) | +0.571842 |

Thus the resulting expression of \( R_0 \) is obtained as:

\[
R_0 = \frac{\beta \kappa k_4 (k_2 \beta_A + k_3 (1 - r))}{k_1 k_2 k_3 (k_4 + \omega_v)}.
\]

4.5 Sensitivity indices of \( R_0 \)

This part of the paper presents the importance of the model parameters on the basic reproductive number \( R_0 \) in order to indicate the most influential parameters. We assess the chosen parameters’ individual normalized sensitivity indices, which are shown in Table 2. The differentiability of the parameter \( \Upsilon \) determines the normalized sensitivity index of \( R_0 \). The following formula is used for this purpose [34].

\[
S_{\Upsilon}R_0 = \frac{\Upsilon}{R_0} \frac{\partial R_0}{\partial \Upsilon}.
\]

The parameters having a positive index value in Table 2 show that \( R_0 \) will increase with the increase in these parameters. More specifically, the increase (or decrease) in the values of \( \beta, \beta_A, r, \psi_v \) and \( \kappa \) up to 10% will increase (or decrease) \( R_0 \) by 10%, 9.634%, 6.864%, 5.718% and 0.015%, respectively. On the other hand, the parameters with negative index value have an inverse relation with \( R_0 \). That is the increase in the values of parameters \( \phi_2, \phi_1 \) and \( \omega_v \) up to 10% will decrease \( R_0 \) by 9.630%, 0.350% and 5.78%.

4.6 Local stability of DFE

**Theorem 4.3** The COVID-19 vaccine Caputo model (3) is LAS at the DFE if all eigenvalues \( \sigma \) of the linearized matrix of (3) fulfill \( |\arg (\sigma)| > \frac{\alpha \pi}{2} \).

**Proof** For local stability, let first we evaluate the following Jacobian matrix at DFE

\[
J(D_0) = \begin{pmatrix}
-k_0 & -\frac{\beta k_4}{k_4 + \omega_v} & -\frac{\beta k_4 \beta_A}{k_4 + \omega_v} & 0 \\
0 & -k_1 & -\frac{\beta k_4 \beta_A}{k_4 + \omega_v} & 0 \\
0 & 0 & -k_2 & 0 \\
0 & \omega_v & 0 & 0 & -k_4 \\
0 & 0 & \phi_1 & \phi_2 & 0 & -\mu
\end{pmatrix}.
\]

The characteristic equation obtained as,

\[
(\sigma + \mu)(\sigma^5 + a_1 \sigma^4 + a_2 \sigma^3 + a_3 \sigma^2 + a_4 \sigma + a_5) = 0,
\]

the eigenvalue \( -\mu \) has a negative real part while for the interpretation of remaining eigenvalues the following coefficient list involve in (13) obtained as

\[
a_1 = k_0 + k_1 + k_2 + k_3 + k_4,
\]

\[
a_2 = k_2 (k_3 + k_4) + k_4 (k_1 + k_3) + k_0 (k_1 + k_2 + k_3) + k_1 k_3 (1 - R_1) + k_1 k_2 (1 - R_2) + \mu (k_4 + \omega_v),
\]

\[
a_3 = k_2 \mu (k_1 + k_2 + k_3)(k_4 + \omega_v) + k_1 k_2 (1 - R_2)(k_0 + k_4) + k_1 k_2 k_3 (1 - (R_1 + R_2)) + k_2 k_3 (k_0 + k_4) + k_1 k_3 (1 - R_1)(k_0 + k_4),
\]

\[
a_4 = \mu k_1 k_3 (1 - R_1)(k_4 + \omega_v) + \mu k_1 k_2 (1 - R_2)(k_4 + \omega_v) + k_1 k_2 k_3 (1 - (R_1 + R_2))(k_0 + k_4) + \mu k_2 k_3 (k_4 + \omega_v),
\]

\[
a_5 = \mu k_1 k_2 k_3 (k_4 + \omega_v)(1 - R_0),
\]

where,

\[
R_1 = \frac{\beta \kappa k_4 r \beta_A}{k_1 k_3 (k_4 + \omega_v)}, \quad \text{and} \quad R_2 = \frac{\beta \kappa k_4 (1 - r)}{k_1 k_2 (k_4 + \omega_v)}.
\]

Clearly, \( a_i > 0 \) for \( i = 1, 2, ..., 5 \) under the condition \( R_0 < 1 \). Moreover, we have

\[
arg (\omega_k) = \frac{\pi}{m} + k \frac{2 \pi}{m} > \frac{\pi}{m} > \frac{\pi}{2m},
\]

for \( k = 0, \cdots, (m - 1) \).

Similarly, it can be shown that the arguments of the of equation \((\sigma^5 + a_1 \sigma^4 + a_2 \sigma^3 + a_3 \sigma^2 + a_4 \sigma + a_5) = 0\) are all greater than \( \frac{\pi}{2m} \) if \( R_0 < 1 \), having an argument less than \( \frac{\pi}{2m} \) for \( R_0 > 1 \). Thus we deduced that the DFE is LAS for \( R_0 < 1 \),

**Lemma 4.3** The DFE of the Caputo COVID-19 model with vaccination (3) is unstable if \( R_0 > 1 \).
4.7 Global stability of DFE

The global asymptotic stability (GAS) of the model (3) at DFE $D_0$ is demonstrated through the Lyapunov function approach.

**Theorem 4.4** If $R_0 \leq 1$ then DFE of the model (3) is GAS.

**Proof** The Lyapunov function for the proof of stated result is given as follows:

$$
\mathcal{V}(E, I, A) = \left( \frac{R_0k_3}{\beta \beta_A} \right) E + \left( \frac{k_3}{k_2 \beta_A} \right) I + I_A.
$$

(14)

By taking the Caputo derivative of (14) leads to

$$
C D_t^\alpha \mathcal{V}(E, I, A) = \left( \frac{R_0k_3}{\beta \beta_A} \right) C D_t^\alpha E

+ \left( \frac{k_3}{k_2 \beta_A} \right) C D_t^\alpha I + I_A,

= \frac{R_0k_3}{N \beta \beta_A} \left[ \beta S(I + \beta_A I_A) - k_1 E \right]

+ \left( \frac{k_3}{k_2 \beta_A} \right) \left( (1 - r) E - k_2 I \right)

+ (\gamma k E - k_3 I_A).

\leq \frac{R_0k_3}{\beta \beta_A} \left[ \beta (I + \beta_A I_A) - k_1 E \right]

+ \left( \frac{k_3}{k_2 \beta_A} \right) \left( (1 - r) E - k_2 I \right)

+ (\gamma k E - k_3 I_A), \quad S \leq N,

= \left( \frac{R_0k_3}{\beta A} - \frac{k_3}{\beta A} I \right) + \left( R_0k_3 I_A - k_3 I_A \right)

+ \gamma k E - \frac{R_0k_3}{\beta \beta_A} k_1 E

+ \left( \frac{k_3}{k_2 \beta_A} \right) (1 - r) E,

= \frac{k_3}{\beta A} \left[ R_0 - 1 \right] I + k_3 \left( R_0 - 1 \right) I_A,

\leq \frac{k_3}{\beta A} \left[ R_0 - 1 \right] (I + \beta_A I_A),

\leq 0.

Thus, $C D_t^\alpha \mathcal{V}(t)$ is negative for $R_0 \leq 1$ and zero when $I = I_A = 0$. So, the largest compact invariant set in $\Omega$ is the singleton set $D_0$. Thus, the model (2) is globally asymptotically stable at $D_0$ when $R_0 < 1$. □

4.8 Global stability of endemic equilibrium

$$
\begin{align*}
\Lambda &= (\lambda^*_A + k_0)S^* - \psi_v V^*, \\
k_1 E^* &= \lambda^*_1 S^*, \\
k_2 I^*_A &= (1 - r) \kappa E^*, \\
k_3 I^*_A &= \kappa \kappa E^*.
\end{align*}
$$

(15)

$$
\lambda^*_1 = \beta (I^* + \beta_A I^*_A).
$$

**Theorem 4.5** The system (3) at unique (EE), i.e. $D_1$ is GAS in the region $\Omega$, if $R_0 > 1$.

**Proof** To prove the desired result, consider the nonlinear Lyapunov function as defined below and by taking the time-fractional derivative and corresponding unique endemic equilibrium $D_1$.

$$
\mathcal{L}(t) = \int_{S^*}^S \left( 1 - \frac{S^*}{x} \right) dx + \int_{E^*}^E \left( 1 - \frac{E^*}{x} \right) dx

+ \left( \frac{k_1}{(1 - r) \kappa} \right) \int_{I^*_A}^I \left( 1 - \frac{I^*_A}{x} \right) dx

+ \left( \frac{k_1}{\kappa \kappa} \right) \int_{I^*_A}^V \left( 1 - \frac{V^*}{x} \right) dx

+ \int_{V^*}^V \left( 1 - \frac{V^*}{x} \right) dx + \int_{R^*}^R \left( 1 - \frac{R^*}{x} \right) dx.
$$

By taking the time fractional derivative of above equation and then from (15), we have

$$
\left( 1 - \frac{S^*}{S} \right) C D_t^\alpha S = \left( 1 - \frac{S^*}{S} \right) \left[ \psi_v S^* + k_0 S^* \right]

- \psi_v V^* + \psi_v V - \lambda_1 S - k_0 S

= \beta S^* I^* \left( 1 - \frac{SI^*}{S^*} - \frac{S^*}{S} \right)

+ \frac{I^*_A}{I^*_A}

+ \psi_v V^* - \frac{S^* V^*}{SV^*} + \frac{S^*}{S} + \frac{V^*}{V^*}.
$$

(16)

$$
\left( 1 - \frac{E^*}{E} \right) C D_t^\alpha E(t) = \left( 1 - \frac{E^*}{E} \right) \left( \lambda S^* - \lambda S \right)

- \frac{E^*}{E} + \frac{SI^*}{S^* E^*}

+ \beta S^* I^* \left( 1 + \frac{IS}{S^* S^*} \right)

+ \psi_v V^* - \frac{E^*}{E} + \frac{SI^*}{S^* E^*}.
$$

(17)

$$
\left( \frac{k_1}{(1 - r)} \right) \left( 1 - \frac{I^*_A}{I^*} \right) C D_t^\alpha I^*_A(t)
$$
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\[ = k_1 \left( 1 - \frac{I^{**}}{I} \right) \left( E - E^{**} \frac{I}{I^{**}} \right) \]
\[ = \beta S^{**} I^{**} \left( 1 + \frac{E}{E^{**}} - \frac{I}{I^{**}} - \frac{I^{**} E}{I} \right) \]
\[ + \beta A S^{**} I^{**} \left( 1 - \frac{I}{I^{**}} - \frac{I^{**} E}{I} + \frac{E}{E^{**}} \right) \]
(18)

\[ = \left( \frac{k_1}{k^r} \right) \left( 1 - \frac{I^{**}}{I} \right) C D^\rho(t) \]
\[ = \left( \frac{k_1}{k^r} \right) \left( 1 - \frac{I^{**}}{I} \right) \left( \kappa R \frac{E}{E^{**}} - \kappa E^{**} \frac{I}{I^{**}} \right) \]
\[ = \lambda^{**} S^{**} \left( 1 + \frac{E}{E^{**}} - \frac{I}{I^{**}} - \frac{E^{**} I}{E^{**} I} \right) \]
\[ + \beta S^{**} I^{**} \left( 1 + \frac{E}{E^{**}} - \frac{I}{I^{**}} - \frac{E^{**} I}{E^{**} I} \right) \]
(19)

Thus, the solution of the 3 tends to its \( (D_1) \) for associated reproduction number as \( t \to \infty \). Thus, at \( EEP \) \( (D_1) \) the system (3) is globally asymptotically stable when \( R_0 > 1 \).

5 Numerical treatment of the Caputo model

5.1 Iterative scheme

In this section of the study, a detailed numerical approach for the iterative solution of the COVID-19 transmission model in Caputo sense (3) is described. The fractional Adams–Bashforth–Moulton is used for this. The vaccination COVID-19 model shown in (3) may be thoroughly described as in the following form in order to achieve the required scheme:

\[ \left\{ \begin{array}{l}
C D^\rho(t) u(t) = G(t, u(t)), \quad 0 < t < T \\
u^{(p)}(0) = u_0^{(p)}, \quad p = 0, 1, \ldots, v, \quad v = [\eta],
\end{array} \right. \]
(23)

where \( u = (S, V, E, I, I_A, R) \in \mathbb{R}^6 \), and the continuous real valued vector function is expressed by the function \( G(t, u(t)) \). The aforementioned problem (23) was transformed into the following structure using the integral in the Caputo case:

\[ u(t) = \sum_{p=0}^{v} u_0^{(p)} I^p + \frac{1}{\Gamma(\alpha)} \int_0^t (t-x)^{\alpha-1} G(x, u(x)) dx. \]
(24)

A uniform grid on \([0, T]\) with step size \( h = \frac{T}{N}, N \in \mathbb{N} \), where \( t_n = nh, n = 0, 1 \ldots, N \) is taken into consideration in order to execute the integration required in (24). As a result, the system mentioned in (3) may be expressed as follows:

\[ S_{n+1}(t) = S_0 + \frac{\alpha^{h}}{\Gamma(\alpha + 2)} \left\{ \Delta + \psi_v V^p \right\} + \beta (I^p + \beta A I A^p) \frac{S^p}{N^p} - k_0 S^p \]
\[ - \beta (I^p + \beta A I A^p) \frac{S^p}{N^p} - k_0 S^p \]
\[ + \frac{h^a}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{l,n+1} \left\{ \Delta + \psi_v V_j \right\} + \beta (I_j + \beta A I_A) \frac{S_j}{N_j} - k_0 S_j \]
\[ E_{n+1}(t) = E_0 + \frac{h^a}{\Gamma(\alpha + 2)} \left\{ (\beta (I^p + \beta A I A^p) \frac{S^p}{N^p} - k_1 E^p) + \beta (I_j + \beta A I_A) \frac{S_j}{N_j} - k_1 E_j \right\}
\]
Further, we have in the above expressions

\[ I_{n+1}(t) = I_0 + \frac{h^\alpha}{\Gamma(\alpha + 2)} \left\{ (1 - r)\kappa E^p - k_2 I^p \right\} \]

\[ + \frac{h^\alpha}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{j,n+1} \left\{ (1 - r)\kappa E_j - k_2 I_j \right\}, \]

\[ I_{A,n+1}(t) = I_{A0} + \frac{h^\alpha}{\Gamma(\alpha + 2)} \left\{ r\kappa E^p - k_3 I_A^p \right\} \]

\[ + \frac{h^\alpha}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{j,n+1} \left\{ r\kappa E_j - k_3 I_{Aj} \right\}, \]

\[ V_{n+1}(t) = V_0 + \frac{h^\alpha}{\Gamma(\alpha + 2)} \left\{ \omega_n S^p - k_4 V^p \right\} \]

\[ + \frac{h^\alpha}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{j,n+1} \left\{ \omega_n S_j - k_4 V_j \right\}, \]

\[ R_{n+1}(t) = R_0 + \frac{h^\alpha}{\Gamma(\alpha + 2)} \left\{ \phi_1 I^p + \phi_2 I_A^p - \mu R^p \right\} \]

\[ + \frac{h^\alpha}{\Gamma(\alpha + 2)} \sum_{j=0}^{n} b_{j,n+1} \left\{ \phi_1 I_j + \phi_2 I_{Aj} - \mu R_j \right\}, \]

where,

\[ S_{n+1}^p(t) = S_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \theta_{j,n+1} \left\{ \Delta + \psi V_j \right\} \]

\[-\beta(I_j + \beta A_{Aj}) \frac{S_j}{N_j} - k_0 S_j \}], \]

\[ E_{n+1}^p(t) = E_0 \]

\[ + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \theta_{j,n+1} \left\{ (\beta(I_j + \beta A_{Aj}) \frac{S_j}{N_j} - k_1 E_j \right\}], \]

\[ I_{n+1}^p(t) = I_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \theta_{j,n+1} \left\{ (1 - r)\kappa E_j - k_2 I_j \right\}], \]

\[ I_{A,n+1}(t) = I_{A0} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \theta_{j,n+1} \left\{ r\kappa E_j - k_3 I_{Aj} \right\}, \]

\[ V_{n+1}(t) = V_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \theta_{j,n+1} \left\{ \omega_n S_j - k_4 V_j \right\}, \]

\[ R_{n+1}(t) = R_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} \theta_{j,n+1} \left\{ \phi_1 I_j + \phi_2 I_{Aj} - \mu R_j \right\}. \]

Further, we have in the above expressions

\[ b_{j,n+1} = \begin{cases} \frac{n^{\alpha+1} - (n - \alpha)(n + 1)^\alpha}{\alpha}, & j = 0 \\ \frac{(n - j + 2)^{\alpha+1} + (n - j)^{\alpha+1} - 2(n - j + 1)^{\alpha+1}}{\alpha}, & 1 \leq j \leq n, \end{cases} \]

\[ \theta_{j,n+1} = \frac{\alpha}{\alpha} \left[ (n - j + 1)^\alpha + (n - j)^\alpha \right], \quad 0 \leq j \leq n, \]

with \( i = 1, 2, 3 \).

5.2 Simulation results

In this section, the Caputo fractional COVID-19 vaccine model (3) is simulated using the iterative procedure discussed in the previous section. The purpose of simulation results is to explore the population-level influence of various levels (scenarios) in NPIs such as reducing contacts via social-distancing, self-isolation, avoiding travels and vaccination on the transmission of COVID-19 infection. The time level is considered up to 500 days and the estimated baseline values of parameters used for simulation purpose are presented in Table 1. Initially, in graph 2, we assess the dynamics of various population groups for different five values of fractional order \( \alpha \) demonstrating the convergence of the model. The set of simulation in Fig. 1(a-d) is carried out to assess the impact of variation in memory index \( \alpha \) and the effective community disease transmission rate \( \beta \) on the symptomatically infected individuals. Figure 1a shows the impact of baseline value and for various levels of decrease in the baseline values, i.e., 10%, 20%, and 30% reduction in estimated value of \( \beta \) on the symptomatically infected individuals for integer case \( \alpha = 1 \). It is found that the reduction in community transmission due to the implementation of other NPIs significantly rescued the pandemic peaks. A similar interpretation is carried out for three fractional values of \( \alpha \) and can be found in Fig. 1(b-d). Further, the effect of reduction in effective contact rate \( \beta \) at different levels on the dynamics of asymptomatically COVID-19 cases is demonstrated in Fig. 4. A noticeable decrease in the cumulative asymptomatic cases is found when \( \beta \) is reduced up to 30% to its estimated baseline value. This interpretation for fractional values of \( \alpha \) is depicted in 4(b-d) showing slighter high peaks in comparison with integer case. In this section,
the Caputo fractional COVID-19 vaccine model (3) is simulated using the iterative procedure discussed in the previous section. The purpose of simulation results is to explore the population-level influence of various levels (scenarios) in NPIs such as reducing contacts via social distancing, self-isolation, avoiding travels and vaccination on the transmission of COVID-19 infection. The time level is considered up to 500 days and the estimated baseline values of parameters used for simulation purpose is presented in Table 1. Initially, in the first graph 2, we assess the dynamics of various population groups for different five values of fractional order $\alpha$ demonstrating the convergence of the model. The set of simulations in Fig. 1(a-d) is carried out to assess the impact of variation in memory index ($\alpha$) and the effective community disease transmission rate ($\beta$) on the symptomatically infected individuals. Figure 1a, show the impact of baseline value and for various levels of decrease in the baseline values, i.e., 10%, 20%, and 30% reduction in an estimated value of $\beta$ on the symptomatically infected individuals for integer case ($\alpha = 1$). It is found that the reduction in community transmission due to the implementation of other NPIs significantly rescued the pandemic peaks. A similar interpretation is carried out for three fractional values of $\alpha$ and can be found in Fig. 1(b-d). Further, the effect of reduction in effective contact rate ($\beta$) at different levels on the dynamics of asymptotically COVID-19 cases is demonstrated in Fig. 4. A noticeable decrease in the cumulative asymptomatic cases is found when $\beta$ is reduced up to 30% to its estimated baseline value. This interpretation for fractional values of $\alpha$ is depicted in 4(b-d) showing slighter high peaks in comparison with integer case.

We simulate the fractional COVID-19 model (3) in order to explore the effect of variation in transmission rate relative to asymptomatic COVID-19 cases ($\beta_A$) on the dynamics cumulative infected case as shown in Figs. 5 and 6. The results obtained in 5 show that reduction in $\beta_A$ with different levels (increase in baseline social-distancing compliance) significantly reduces the cumulative symptomatic infected cases. Moreover, it observed that the decrease in $\beta_A$ (with the same levels as in $\beta$) reduces the infected curve peaks slightly faster. This interpretation for a fractional case can be found in Fig. 5b and c. The impact of variation in $\beta_A$ and fractional order $\alpha$ is presented in Fig. 6(a-c).

Assessment of the impact of vaccination

To study the community-wide impact of vaccination coverage on disease incidence, we simulate the COVID-19 vaccine model (3). The simulation results are carried out demonstrating the dynamics of cumulative symptomatic and asymptomatic COVID-19 cases by enhancing the vaccination coverage at 10%, 30%, and 50%, to the estimated baseline value. The resulting graphical interpretation is depicted in Figs. 7 and 8. While the rest of the parameters and initial conditions related to the other intervention (social-distancing) are kept at their baseline values given in Table 1. From the graphical results in 7 and 8, a significant reduction in the peaks of infected curves is observed with an increase in vaccination coverage ($\omega_v$). Particularly, a 50% enhancement in the vaccination coverage ($\omega_v$) dramatically reduced the infection peaks and leads to disease elimination. In conclusion, these simulations show that the proper implementation of available vaccines will lead to the elimination of pandemics not only in the selected region but in the whole world if the vaccination rate is moderately high enough.

Finally, we simulate the model to assess the potential impact of the reduction in the baseline value of parameter $\psi_v$ (the loss of immunity). The reduction in $\psi_v$ is made with 10%, 20%, and 30% rates to baseline value while the remaining parameters are considered as given in Table 1. The simulation results for the symptomatic and asymptomatic COVID-19 cases are shown in Figs. 9 and 10, respectively. From these simulation results, it is noticed that reduction in $\psi_v$ notably flatten the pandemic peaks.

6 Conclusion

The mathematical modeling approach with real data analysis has proven an effective tool to provide a deeper insight about the transmission dynamics. This work presents a formulation of a new fractional epidemic model coupled with rigorous mathematical analysis to study the transmission dynamics novel COVID-19 pandemic under the effect of vaccination. The proposed model is based on the well-known classical Caputo fractional derivative. Moreover, the population-level impact of other non-pharmaceutical interventions, i.e., social-distancing to reduce the effective contacts (to control the pandemic) is also assessed in the simula-
The model was first developed in the classical integer order case and then extended to fractional order by incorporating the Caputo fractional order derivative. The purpose of using the Caputo derivative in the study is to provide a deeper analysis of the transmission dynamics of COVID-19. Initially, the basic qualitative analysis of the Caputo COVID-19 vaccine model was carried out. The expression of the most crucial quantity known as the basic reproduction number is computed by using the concept of the next-generation method. The disease-free equilibrium is locally and globally asymptotically stable, when $R_0 < 1$. The disease dynamics have been described with the help of a vaccination strategy. After the rigorous analysis, the model in the Caputo case is solved numerically via an efficient iterative approach. Finally, using the estimated and fitted values of the parameters, the model is simulated to depict the influence of some non-pharmaceutical interventions and the vaccination efficacy on the transmission dynamics and control of the ongoing novel pandemic. In simulation results, we also depict the importance of fractional order ($\alpha$) describing
Fig. 3 The impact of reduction in community contact rate $\beta$ (through social-distancing) on cumulative symptomatic COVID-19 cases for $\alpha = 1$, $\alpha = 0.95$, $\alpha = 0.85$ and $\alpha = 0.80$.
Fig. 4  The impact of reduction in community contact rate $\beta$ (through social-distancing) on cumulative asymptomatic COVID-19 cases for $\alpha = 1$, $\alpha = 0.95$, $\alpha = 0.85$ and $\alpha = 0.80$
Fig. 5 Influence of reduction in contact rate relative to asymptomatically-infected individuals $\beta_A$ versus the cumulative symptomatic COVID-19 cases for $\alpha = 1$, $\alpha = 0.90$ and $\alpha = 0.80$.
Fig. 6 Influence of reduction in contact rate $\beta_A$ versus the cumulative asymptomatic COVID-19 cases for $\alpha = 1$, $\alpha = 0.90$ and $\alpha = 0.80$
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Fig. 7 Graphical assessment of impact of vaccination coverage over cumulative symptomatic COVID-19 cases where, a $\alpha = 1$, b $\alpha = 0.90$.

Fig. 8 Graphical assessment of impact of vaccination coverage over cumulative asymptomatic COVID-19 cases where, a $\alpha = 1$, b $\alpha = 0.90$. 
Fig. 9 Assessment of variation in parameter $\psi_v$ on cumulative over cumulative symptomatic COVID-19 cases where, a $\alpha = 1$, b $\alpha = 0.90$

Fig. 10 Assessment of variation in parameter $\psi_v$ on cumulative over cumulative asymptomatic COVID-19 cases where, a $\alpha = 1$, b $\alpha = 0.90$
the impact of memory index on the disease dynamics. Simulations of our model showed, that the reduction in the effective contact rate (up to 30%) and enhancement in vaccination rate (up to 50%) to the current baseline values significantly reduced the disease incidence. In conclusion, by accelerating the low transmission rate followed by preventive measures (such as social distancing, following strict SOPs) and achieve herd immunity by using a vaccine with high efficacy, the results are even better for curtailing the pandemic.

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Declarations

Conflict of Interest The authors declare that they have no conflict of interest.

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