Non-resonant Density of States Enhancement at Low Energies for Three or Four Neutrons

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The low energy systems of three or four neutrons are treated within the adiabatic hyperspherical framework, yielding an understanding of the low energy quantum states in terms of an adiabatic potential energy curve. The dominant low energy potential curve for each system, computed here using widely accepted nucleon-nucleon interactions with and without the inclusion of a three-nucleon force, shows no sign of a low energy resonance. However, both systems exhibit a low energy enhancement of the density of states, or of the Wigner-Smith time-delay, which derives from long-range universal physics analogous to the Efimov effect. That enhancement could be relevant to understanding the low energy excess of correlated 4-neutron ejection events observed experimentally in a nuclear reaction by Kisamori et al.\textsuperscript{[1]}

The three- and four-neutron (3n and 4n) systems are intriguing and important problems in few-nucleon fundamental physics that deserve a comprehensive, deep theoretical understanding. While no 4n bound state is generally believed to exist, there have been speculations for decades about the possible existence of a long-lived resonance in the 4-particle scattering continuum. Those early speculations have evolved into renewed interest triggered by the recent experimental observation of an enhanced signal of 4 low energy neutrons emerging together, which they tentatively interpreted as a possible 4n resonance (or bound) state, by Kisamori \textit{et al.}\textsuperscript{[1]}. The present Letter investigates the possible existence of a low energy resonance-like enhancement of the density of states in both the 4n and 3n systems, using well established nucleon-nucleon (NN) interactions, with and without the inclusion of a three-nucleon force (3NF), and also using a simple Gaussian potential adjusted to match the neutron-neutron (nn) scattering length and effective range.

In our study, the low energy regions of the 3n and 4n systems are explored using the adiabatic hyperspherical representation, which has a strong track record of successfully predicting and interpreting resonances for atomic systems.\textsuperscript{[2,3]} Our results with the aforementioned potentials are consistent with strong enhancements of the low-energy density of states (or Wigner-Smith time delay) for both the 3n and 4n systems, although the nature of the potential curves and the eigenphaseshift energy dependences make it clear that the enhanced density of states should not be viewed as a resonance. Moreover, neither the 3n nor the 4n system is close to possessing a bound state. Our analysis also demonstrates how the density of states enhancement can be understood in terms of universal physics considerations that are closely related to the Efimov effect.\textsuperscript{[4–6]}

Remarkably, theoretical treatments to date have not been able to reach a consensus agreement about whether a 3n or 4n resonance exists, consistent with the presently understood NN interaction potentials. The need for more theoretical input into this problem is therefore clear, given the conflicting conclusions reached so far by competing theoretical methods. Specifically, some of the theory published to date is consistent with the claimed experimental observation of a low energy resonance in the 4n system,\textsuperscript{[7,8]} whereas alternative theoretical analyses are incompatible with a resonance or bound state interpretation of the experimental measurement \textsuperscript{[9–16]}. An advantage of the present method based on the adiabatic hyperspherical representation is that the absence of a resonance state is immediately clear visually after inspecting the relevant adiabatic potential energy curve for the system. Moreover, our quantitative calculation shows that a nonresonant density of states enhancement is guaranteed to be present at low energies, owing to the attractive hyperradial potential energy at very long range. Specifically, this connects with the universal behavior of three- and four-fermion systems close to the unitarity limit. We propose that such a density of states enhancement could help to understand the enhanced production of four low energy neutrons in the experiment of Kisamori \textit{et al.}\textsuperscript{[1]}, even in the absence of a tetraneutron resonance state.

The theoretical approach adopted here starts consider-
ing realistic nuclear interaction Hamiltonians. They are constructed by an overall fit of the existing np and pp data and, invoking charge symmetry invariance, they can be applied to describe neutron systems as well. In particular, we have considered the AV18 and AV8’ NN potentials [17] as well as the recent local NN potentials derived within the chiral effective field theory approach [18 [19], in particular the model NV2-1a. With the AV18 potential, we have performed calculations with the inclusion of the Urbana and Illinois 3NFs. [20] [22] It should be noticed that the two-body singlet \( nn \) scattering length is large and negative, believed to be approximately \( a \approx -18 \) fm, consistently reproduced by the NN interactions considered. Motivated by the large value of the \( nn \) scattering length, we have also carried out calculations using a simple single Gaussian potential, adjusted to describe that value and the corresponding effective range, in order to explore connections with universal behavior and the unitary limit of the three- and four-fermion systems.

In all our calculations it has been found that the use of a particular form of NN potential, with or without the inclusion of the 3NF, has comparatively little influence on the results; in particular the inclusion of 3NFs only slightly modified the potential curves around 1 - 2 fm, making them more repulsive. The 3n and 4n Schrödinger equations are then solved in the adiabatic hyperspherical representation [2, 23–25], which has a proven track record in correctly predicting resonances, especially in atomic and molecular physics contexts. After one diagonalizes the fixed-hyperradius Hamiltonian, \( H_{\rho=\text{const.}} \), the \( \rho \)-dependent eigenvalues \( U_\nu(\rho) \) act as adiabatic potential energy curves (and couplings \( W_{\nu,\nu'} \)) that often make it immediately and visibly clear whether or not there is a resonance, and they yield an immediate interpretation if a resonance does exist [14]. Note for reference that two successful predictions and interpretations of atomic shape resonances, carried out within the adiabatic hyperspherical framework in Refs. [2, 3], were eventually confirmed by both experiment [20, 27] and by other theory for the singlet electronic \( L^+ = 1^+ \) states of the negative ions \( \text{H}^- \) and \( \text{Ps}^- \).

The greatest numerical challenge in the present study is the calculation of the \( 4n \) and \( 3n \) potential energy curves \( U_\nu(\rho) \) and the elements of the coupling matrix operator \( W_{\nu,\nu'}(\rho) = \frac{\hbar^2}{2\mu} \langle \Phi_\nu | \frac{\partial^2}{\partial \rho^2} | \Phi_{\nu'} \rangle + \langle \Phi_{\nu'} | \frac{\partial^2}{\partial \rho^2} | \Phi_\nu \rangle \), where \( \Phi_\nu \) are the adiabatic eigenfunctions. Our approach tackles this variationally at each value of \( \rho \), by expanding the unknown adiabatic eigenfunctions (\( \Phi_\nu \)) into a basis set. Two different choices of the basis set have been implemented in our study. The first is a set of coupled hyperspherical harmonics and spinors adapted to the symmetry of interest, e.g., \( J^m = 0^+ \) for the tetraneutron. The second type of basis set implemented to solve the fixed-\( \rho \) Schrödinger equation is a linear combination of correlated Gaussian functions. [28] [31] Following diagonalization of \( H_{\rho=\text{const.}} \) at each \( \rho \), a Rayleigh-Ritz upper bound on the exact potential \( U_\nu(\rho) \) is obtained. The following theorem is important for our subsequent analysis below: When the hyperradial Schrödinger equation is solved in the lowest potential curve, including also just the diagonal nonadiabatic coupling terms \( W_{\nu,\nu'}(\rho) \), the lowest computed energy of the system will be a rigorous upper bound to the exact ground state energy. Much of our detailed analysis of the resonance physics has been performed at the level of the adiabatic approximation, which neglects off-diagonal coupling terms. We have conducted tests of this approximation as well, and they confirm its general validity for the \( 3n \)- and \( 4n \)-systems considered here.

To understand the basic idea of the formulation, consider first the one-dimensional hyperradial Schrödinger equation. The single adiabatic term variational ansatz for the wavefunction is written for \( N \) particles in their relative frame as: \( \Psi(\rho, \Omega) = \rho^{-(3N-4)/2} \Phi_0(\rho; \Omega) F_0(\rho) \), where \( \Phi_0(\rho; \Omega) \) is the lowest adiabatic eigenfunction of \( H_{\rho=\text{const.}} \) with eigenvalue \( U_0(\rho) \) and repulsive diagonal correction term \( W_{00}(\rho) \). The radial equation then takes the form:

\[
-\frac{\hbar^2}{2\mu} \frac{d^2}{d\rho^2} F_0(\rho) + (U_0(\rho) - E) F_0(\rho) = 0,
\]

where the full, effective adiabatic potential in the lowest channel, including the diagonal correction term, is:

\[
u_0(\rho) = U_0(\rho) + W_{00}(\rho).
\]

Note that \( u_0(\rho) \) includes the effective centrifugal term \( \frac{\hbar^2}{2\mu} (3N-6)(3N-4) \) associated with the elimination of first order hyperradial derivatives from the effective radial Schrödinger equation. Here \( \mu \) is a reference mass (we use \( \mu = m/2 \) with \( m \) the neutron mass), and the hyperradius \( \rho \), for a system of equal mass particles, is defined by the relation \( \rho^2 = \frac{2}{\pi} \Sigma_{i<j} r_{ij}^2 \), where \( r_{ij} \) is the distance between neutrons \( i \) and \( j \). Alternative representations for the hyperradius, including generalizations to unequal masses using Jacobi coordinates, can be found in review articles, e.g. [4] [5].

It is known from universality studies that for \( N \)-particle systems dominated by a large magnitude two-body scattering length \( a \), their lowest long range hyperradial potential energy curve in the continuum has the following asymptotic form, at \( \rho \to \infty \):

\[
u_0(\rho) \to \frac{\hbar^2}{2\mu} \left( \frac{l_{\text{eff}} (l_{\text{eff}} + 1)}{\rho^2} + C \frac{a}{\rho^3} \right),
\]

where \( C \) and \( l_{\text{eff}} \) depend on the number of particles and their statistics; their values are given in Table I below for the symmetries considered in the present study. The adiabatic correction term \( W_{00}(\rho) \) decays asymptotically at least as fast as \( \rho^{-4} \) for the \( 3n \) and \( 4n \) systems and therefore has no role in the above decomposition.

For the present problem, where the \( nn \) scattering length is large and negative, the attractive long range term proportional to \( a/\rho^3 \) has key implications for the low energy Wigner-Smith time-delay [52] [53]. \( Q = 2\hbar d\delta/dE \), which also measures the density of states of
In particular the density of states diverges like \( E^{-1/2} \) as \( E \to 0 \) since the scattering phaseshift \( \delta(E) \) at low energy can be seen perturbatively to equal \( \delta \to -Cak/(2l_{\text{eff}} + 2l_{\text{eff}}^2) \) as the wavenumber \( k \to 0 \).

### Table I. Unitarity (subscript \( u \)) and non–unitarity (no subscript) long–range \((\rho \to \infty)\) coefficients of the lowest adiabatic potential

| \( N \) | \((LS)J^π\) | \( l_{\text{eff}} \) | \( C \) | \( l_{\text{eff},u}^{(a)} \) | \( l_{\text{eff},u}^{(b)} \) |
|---|---|---|---|---|---|
| 3 | \((\frac{1}{2})\frac{3}{2}\) | 5/2 | 15.22 | 1.275 | 1.2727(1) |
| 4 | \((00)0^+\) | 5 | 86.68 | 2.027 | 2.0091(4) |

Next consider the numerical computation of the adiabatic hyperspherical potential energy curves for the 3n and 4n systems. The most technically demanding aspect of the present study is the diagonalization of the fixed-\( \rho \) Hamiltonian to determine the eigenvalues, interpreted as potential energy curves \( U_\rho(\rho) \) and the diagonal adiabatic corrections \( W_{\nu,\nu}(\rho) \). We use two different variational basis sets, an expansion into hyperspherical harmonics (extremely accurate at small and intermediate values of \( \rho \)) \([28, 30]\) and an expansion into correlated Gaussian basis functions (more accurate at large \( \rho \)) \([28–30]\).

The lowest adiabatic hyperspherical potential energy curves in the most attractive symmetries of the 4n and 3n systems, namely \( 0^+ \) and \( \frac{3}{2}^- \) respectively, are plotted in Fig. 1. At a glance it is immediately apparent that the lowest potential curve for both systems is totally repulsive, and moreover positive at all hyperradii, which guarantees both that there is no bound state and that there can be no resonance state in the low energy range below 10 MeV. Nevertheless there is extensive attraction in the system, which is apparent from the fact that the potential curve lies everywhere well below the upper dashed curve which would apply if there were zero interaction between the neutrons. Over much of the range of \( \rho \), in fact, both systems are slightly closer to the unitary-limiting potentials that would emerge if the two-body potential was made even more attractive to give an infinite singlet n-n scattering length (i.e., closer to the lower dashed curves in Fig. 1), than to the noninteracting limit.

The HH expansion includes the eigenfunctions of the grand angular momentum operator \( K^2 \), with eigenval-
Figure 2. Elastic scattering phaseshift versus the square root of the energy for the CGHS calculation using the AV8’ potential, for the 4n 0+ symmetry as the upper magenta curve, and for the 3n 3/2− symmetry as the lower magenta curve. Both cases show the proportionality to \( \sqrt{E} \) dependence that holds in the zero-energy limit, a consequence of the \( \rho^{-3} \) long range potential energy term. The solid blue points that lie almost exactly on top of these curves are computed using the lowest 3n and 4n hyperspherical potential curves based on a simple 2-body Gaussian potential interaction (see text).

Key evidence for our conclusions derives from the energy dependent scattering phaseshift \( \delta(E) \) in the lowest adiabatic channel representing the 3n to 3n continuum and for the 4n to 4n continuum, shown in Fig.2. Note that, while the results shown here have been obtained in the single-channel adiabatic hyperspherical approximation, numerical tests have also been carried out with full coupled-channel calculations of the multichannel scattering matrix and time delay eigenvalues; there we include all diagonal and off-diagonal nonadiabatic couplings \( W_{\nu,\nu'} \), and the results agree quantitatively with the adiabatic results presented here.

Again, 3NFs have only a minor effect on these systems at short distances, without modifying the long range part. This relative unimportance appears to be a consequence of the greater Pauli repulsion on a system of 3 or more neutrons, which suppresses the probability for more than two neutrons to come close to each other. This suppression does not occur for a mixed system of up to four protons and neutrons which can all penetrate to much closer inter-particle or hyperradial distances simultaneously. For this reason, our simple adiabatic potential curve analysis is adequate to explain the absence of both bound and resonant states of the 3n and 4n systems.

The Wigner-Smith time delay, defined in general as \( Q(E) = i\hbar dS^1/dE \), which reduces for a single potential curve to \( 2\hbar d\delta(E)/dE \), also can be viewed (after division by \( 2\pi\hbar \)) as the density of states enhancement associated with particle interactions. \( Q(E) \) is reported in Fig.3 for the 3n and 4n systems, in each case for both the AV8’ and the simple gaussian interaction; it has been rescaled by \( \sqrt{E} \) since the product remains finite at \( E \to 0 \). But most critically for our conclusions, the density of states shows no local maximum that would be expected for a low energy resonance in either system. Both curves do make clear the \( E^{-1/2} \) dependence of \( Q(E) \) in the zero energy limit, a consequence of the \( \rho^{-3} \) term in the long-range potentials for both the 3n and 4n systems.

Consider now the relationship between our present conclusions and some of the alternative theoretical investigations that have been carried out previously for the 3n and 4n systems. The studies closest to the present spirit, as true scattering theory treatments, are Refs. [2, 10, 11]. There is a strong attraction in the 3n and 4n systems, evidently, but this attraction competes with strong Pauli repulsion. While the attraction does create a negative \( \rho^{-3} \) term in the long range hyperradial potential, it cannot overcome the \( \rho^{-2} \) repulsion that is far larger for three or four neutrons than would be the case if two or even one of the particles would be replaced by a proton.

One fundamental question is the extent to which the 3n and 4n systems fit the pattern of universality that has been well-established for cold fermionic atom systems [3, 12, 13], especially in the context of the BEC-BEC crossover problem [14]. We tackle this question by introducing a very simple attractive potential with a single Gaussian for the singlet \( nn \) interaction, with a strength and range adjusted to give the correct singlet
n-n scattering length and effective range. Two different choices for the triplet nn interaction have been tested, either neglecting it altogether or setting a gaussian that reproduces the AV8’ p-wave scattering volume and effective range; those two models are indistinguishable on the scale of Figs.1-3. Results from this simple gaussian Hamiltonian for the 3n system are shown in the inset of Fig.1(a) as blue points on top of the AV8’s results shown as the solid magenta potential curve; remarkably, the results are nearly indistinguishable.

Finally, we can speculate about the experimental observation of enhanced 4n coincident events in the observation of Kisamori et al.\[1\]. Even though, in the analysis of that experiment, those enhanced low energy events seemed to indicate existence of a low energy tetraneutron, we speculate that the dramatically enhanced low energy density of states that is evident in our calculations (increasing as $1/\sqrt{E}$) could be the origin of the strong low energy 4n signal. This enhancement of the 4n density of states is predicted to exist even though no resonance and no bound state exists for the tetraneutron system.

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**References**

[1] K. Kisamori, S. Shimoura, H. Miya, S. Michimasa, S. Ota, M. Assie, H. Baba, T. Baba, D. Beaumel, M. Dozono, T. Fujii, N. Fukuda, S. Go, F. Hammache, E. Ideguchi, N. Inabe, M. Itoh, D. Kameda, S. Kawase, T. Kawabata, M. Kobayashi, Y. Kondo, T. Kubo, Y. Kubota, M. Kurata-Nishimura, C. S. Lee, Y. Maeda, H. Matsubara, K. Miki, T. Nishi, S. Noji, S. Sakaguchi, H. Sakai, Y. Sasamoto, M. Sasano, H. Sato, Y. Shimizu, A. Stolz, H. Suzuki, M. Takaki, H. Takeda, S. Takeuchi, A. Tamii, L. Tang, H. Tokieda, M. Tsumura, T. Uesaka, K. Yako, Y. Yanagisawa, R. Yokoyama, and K. Yoshida, Candidate Resonant Tetraneutron State Populated by the He-4 (He-8, Be-8) Reaction, Phys. Rev. Lett. 116, 052501 (2016).

[2] C. D. Lin, Feshbach and shape resonances in the e-H $^1p^o$ system, Phys. Rev. Lett. 35, 1150 (1975).

[3] J. Boteroaund C. H. Greene, Resonant photodetachment of the positronium negative ion, Phys. Rev. Lett. 56, 1366 (1986).

[4] J. P. D’Incao, Few-body physics in resonantly interacting ultracold quantum gases, J. Phys. B 51, 043001 (2018).

[5] S. T. Rittenhouse, J. von Stecher, J. P. D’Incao, N. P. Mehta, and C. H. Greene, J. Phys. B 44, 172001 (2011).

[6] C. H. Greene, P. Giannakeas, and J. Pérez-Ríos, Universal few-body physics and cluster formation, Rev. Mod. Phys. 89, 035006 (2017).

[7] A. M. Shirokov, G. Papadimitriou, A. I. Mazur, I. A. Mazur, R. Roth, and J. P. Vary, Prediction for a Four-Neutron Resonance, Phys. Rev. Lett. 117, 182502 (2016).

[8] S. Gandolfi, H.-W. Hammer, P. Klos, J. E. Lynn, and A. Schwenk, Is a trineutron resonance lower in energy than a tetraneutron resonance?, Phys. Rev. Lett. 118.
A. Deltuva and R. Lazauskas, Tetraneutron resonance in K. Fossez, J. Rotureau, N. Michel, and M. Ploszajczak, Can tetraneutron be a narrow resonance?, Phys. Rev. Lett. 119, 032501 (2017)

A. Deltuva, Tetraneutron: Rigorous continuum calculations. Physics Letters B 782, 238 (2018)

J. G. Li, N. Michel, B. S. Hu, W. Zuo, and F. R. Xu, Ab initio no-core Gamow shell-model calculations of multineutron systems, Phys. Rev. C 100, 054313 (2019)

A. Deltuva and R. Lazauskas, Tetraneutron resonance in the presence of a dineutron, Phys. Rev. C 100, 044002 (2019)

A. Deltuva and R. Lazauskas, Comment on “Is a Trineutron Resonance Lower in Energy than a Tetraneutron Resonance?” Reply, Phys. Rev. Lett. 123, 069202 (2019)

E. Hiyama and M. Kamimura, Study of various few-body systems using Gaussian expansion method (GEM), Frontiers in Physics 13, 132106 (2018)

R. B. Wiringa, V. Stoks, and R. Schiavilla, An Accurate nucleon-nucleon potential with charge independence breaking, Phys. Rev. C 51, 38 (1995) arXiv:nucl-th/9408016

M. Piarulli, L. Girlanda, R. Schiavilla, A. Kievsyky, A. Lovato, L. E. Marcucci, S. C. Pieper, M. Viviani, and R. B. Wiringa, Local chiral potentials with ∆-intermediate states and the structure of light nuclei, Phys. Rev. C 94, 054007 (2016) arXiv:1606.06335 [nucl-th]

M. Piarulli, L. Girlanda, R. Schiavilla, A. Kievsky, A. Lovato, L. E. Marcucci, S. C. Pieper, M. Viviani, and R. B. Wiringa, Local chiral interactions with ∆-intermediate states and the structure of light nuclei, Phys. Rev. C 94, 054007 (2016) arXiv:1606.06335 [nucl-th]

B. S. Pudliner, V. R. Pandharipande, J. Carlson, S. C. Pieper, and R. B. Wiringa, Quantum Monte Carlo calculations of nucleii with A < 7, Phys. Rev. C 56, 1720 (1997)

S. Pieper, V. Pandharipande, R. Wiringa, and J. Carlson, Realistic models of pion-exchange three-nucleon interactions, Phys. Rev. C 64, 014001 (2001)

J. Carlson, V. Pandharipande, and R. Wiringa, 3-Nucleon Interaction in 3-Body, 4-Body and Infinity-Body Systems, Nuclear Physics A 401, 59 (1983).

J. Macek, Properties of autoionizing states of He, J. Phys. B 1, 831 (1968).

U. Fano, Correlations of two excited electrons, Rep. Prog. Phys. 46, 97 (1983).

E. Garrido, A. Kievsky, and M. Viviani, Breakup of three particles within the adiabatic expansion method, Phys. Rev. C 90, 014607 (2014)

H. C. Bryant, B. D. Dieterle, J. Donahue, H. Sharifian, H. Tootoochi, D. M. Wolfe, P. A. M. Gram, and M. A. Yates-Williams, Observation of Resonances near 11 eV in the Photodetachment Cross Section of the H⁻ Ion, Phys. Rev. Lett. 38, 228 (1977).

K. Michishio, T. Kanai, S. Kuma, T. Azuma, K. Wada, I. Mochizuki, T. Hyodo, A. Yagishita, and Y. Nagashima, Observation of a shape resonance of the positronium negative ion, Nature Communications 7, 11060 (2016)

J. von Stecher and C. H. Greene, Correlated gaussian hyperspherical method for few-body systems, Phys. Rev. A 80, 022504 (2009)

D. Rakshitand D. Blume, Hyperspherical explicitly correlated gaussian approach for few-body systems with finite angular momentum, Phys. Rev. A 86, 062513 (2012)

K. M. Dailyand C. H. Greene, Extension of the correlated gaussian hyperspherical method to more particles and dimensions, Phys. Rev. A 89, 012503 (2014)

Y. Suzuki, Adiabatic hyperspherical potentials with localized correlated gaussians, Phys. Rev. C 101, 014002 (2020)

E. P. Wigner, Lower limit for the energy derivative of the scattering phase shift, Phys. Rev. 98, 145 (1955)

F. T. Smith, Lifetime matrix in collision theory, Phys. Rev. 118, 349 (1960)

M. Aymar, C. H. Greene, and E. Luc-Koenig, Multichannel Rydberg spectroscopy of complex atoms, Rev. Mod. Phys. 68, 1015 (1996).

X. Y. Yimand D. Blume, Trapped unitary two-component fermi gases with up to ten particles, Phys. Rev. A 92, 013608 (2015)

A. Kievsky, S. Rosati, M. Viviani, L. E. Marcucci, and L. Girlanda, A high-precision variational approach to three- and four-nucleon bound and zero-energy scattering states, J. Phys. G - Nuclear and Particle Physics 35, 063101 (2008)

L. E. Marcucci, J. Dohet-Eraly, L. Girlanda, A. Gnech, A. Kievsky, and M. Viviani, The hyperspherical harmonics method: A tool for testing and improving nuclear interaction models, Frontiers in Physics 8, 69 (2020)

K. M. Daily, A. Kievsky, and C. H. Greene, Adiabatic hyperspherical analysis of realistic nuclear potentials, Few-Body Syst. 56, 753 (2015).

E. Hiyama, private communication (2017).

D. Blume, J. Von Stecher, and C. H. Greene, Universal properties of a trapped two-component Fermi gas at unitarity, Phys. Rev. Lett. 99, 233201 (2007)

P. Naidon and S. Endo, Efimov physics: a review, Reports on Progress in Physics 80, 056001 (2017)

D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, Scattering properties of weakly bound dimers of Fermionic atoms, Phys. Rev. A 71, 012708 (2005).

D. S. Petrov, C. Salomon, and G. V. Shlyapnikov, Diatomic molecules in ultracold Fermi gases - novel composite bosons, J. Phys. B 38, S645 (2005).

C. Regaland D. Jin, Experimental realization of bcs-bec crossover physics with a fermi gas of atoms, Adv. At. Mol. Opt. Phys. 54, 1 (2007).