Abstract

This study proposed simply design procedure of a single degree of freedom (SDOF) structure equipped with friction dampers. General method is suggested in order to reduce the structural seismic response by using friction dampers. The analysis model was transformed into an equivalent mass-spring-dashpot system by approximating nonlinear friction damping force with equivalent viscous damping force. A closed form solution for dynamic amplification factor (DAF) for steady-state response was derived by the energy balance equation. The equivalent damping ratio was defined by using DAF at natural frequency. The transfer function between input harmonic excitation and output structural response was obtained from the DAF, and the response reduction factor of the root mean square (RMS) for displacements without and with friction dampers was analytically determined. Using the proposed procedure the friction force required for satisfying given target response reduction factor was obtained. Mean response reduction factors matched well with the target values based on the dynamic analysis results. It is concluded that the proposed method is quite simple for the design of friction dampers to reduce seismic response of the structure.

Keywords: Type your keywords here, separated by semicolons;

1. INTRODUCTION

Friction dampers are considered as one of the most efficient energy absorbing devices for building structures against earthquake load. A lot of research have been carried out to investigate their energy-dissipating capacity and to propose a proper design procedure. Energy dissipations of slotted bolted
friction dampers were investigated numerically and experimentally (Grigorian et al. 1993; Li and Reinhom 1995). Fu and Cherry studied the application of a quasi-static design procedure for a friction damped system (Fu and Cherry 1999). They also proposed a code-based seismic design procedure for friction damped frames (Fu and Cherry 2000). A new equivalent linearization technique was proposed for a friction damper-brace system based on the probability distribution of the extreme displacement (Park et al. 2007). Lee et al. proposed design methodology of combined system of bracing and friction dampers for seismic retrofit of structures (Lee et al. 2004). The present study is intended for proposing a simple design process to determine desired control force of a friction damper to satisfy a given target performance of a structure subjected to an earthquake ground excitation. Energy balance of input loading and output building motion is investigated to identify the building-damper system in case of steady-state behavior. A closed form solution for dynamic amplification factor (DAF) is derived by assuming that the friction damped building shows steady-state response, and that Coulomb damping force can be replaced by equivalent viscous damping force. A straightforward methodology is suggested to assess the control efficiency of a friction damped building under an earthquake ground excitation by modifying DAF into transfer function. Then control ratios for displacement responses with and without friction dampers are found analytically. Finally a design procedure is proposed to determine the required damping ratio and friction force to satisfy a given target control ratio. Time history analyses are carried out to check the validity of the proposed procedure.

2. APPROXIMATE EQUIVALENT DAMPING RATIO

Friction dampers are generally installed between stories to reduce inter-story displacements of structures as shown in figure 1. In real situation, the system should consider stiffness of brace \( k_b \), because it requires additional brace in order to install a friction damper. However to increase capacity concerning the energy dissipated of the damper, stiffness of the bracing is good larger and larger. Numerical model assuming that infinite stiffness of the bracing was carried out because of the stiffness of bracing is larger than compared to stiffness of real column.

They generate damping forces characterized by friction damping, the direction of which is opposite to structural motion. The equation of motion of a single-story structure with a friction damper is represented by

\[
\begin{align*}
    m\ddot{u} + c\dot{u} + ku + f_{c}\text{sgn}(\dot{u}) &= F(t) \\
\end{align*}
\]  

(1)
where \( m, c_v, \) and \( k \) are the mass, viscous damping constant, and stiffness of a structure, respectively; \( u, \dot{u}, \) and \( \ddot{u} \) are the inter-story displacement, velocity, and acceleration of the structure, respectively; \( f_c \) and \( F(t) \) are, respectively, the friction force of a damper and external loading; \( \text{sgn}(\dot{u}) \) is the symbolic function defined as -1, 0, and 1, respectively, in case \( \dot{u} < 0, \dot{u} = 0 \) and \( \dot{u} > 0 \). To find an exact solution of equation (1) is dependent on the form of the external load \( F(t) \). It is nearly impossible to obtain an analytical solution for a randomly excited load such as an earthquake, and generally a numerical approach is applied instead. This study first revisited previous approach for identifying a building structure installed with a friction damper under harmonic excitation for reducing steady-state response. By equating the dissipated energy by a friction damper with the energy dissipated by viscous damping for one cycle, friction damping force can be replaced by an equivalent viscous damping force (Chopra 2001). As a result, the equation of motion of a single degree of freedom (SDOF) system with an equivalent viscous damping subjected to a harmonic force can be represented as

\[
m\ddot{u} + (c_v + c_{eq})\dot{u} + ku + f_c \text{sgn}(\dot{u}) = F_0 \sin \omega t
\]

where \( c_{eq}, F_0, \) and \( \omega \) are the equivalent viscous damping constant, amplitude of harmonic loading, and angular loading frequency, respectively. When all input energy is dissipated, the response is reached at steady-state. Both friction and viscous damping are repeated in stick and slip state every cycle. If the dissipated energy of the structure is equated with the one of an equivalent viscous damping, the equivalent damping constant is easily led (Rao 1995). And then, equivalent viscous damping \( \xi_{eq} \) is also obtained as:

\[
\xi_{eq} = \frac{2}{\pi} \frac{1}{\text{DAF}} R_h
\]

where \( R_h \) and DAF are the force ratio in harmonic excitation \( f_d / F_0 \) and the ratio of amplitude of response \( u_0 \) to static displacement response \( u_{st} = F_0 / k \) as dynamic amplification factor, respectively. Substituting equation (3) into equation (2) leads to

\[
\frac{u_0}{F_0 / k} = \frac{1}{\left(1 - \omega_r^2\right)^2 + \left(2(\xi_v + \xi_{eq})\omega_r\right)^2} \left(\frac{1 - \omega_r^2}{\alpha^2 + (2\omega_r \xi_v)^2}\right)^{\frac{1}{2}}
\]

Since \( u_0 \) exists in \( \xi_{eq} \), as shown in equation (3), solving the quadratic equation in terms of \( u_0 \) obtain the following form of the DAF:

\[
\text{DAF} = \frac{u_0}{u_{st}} = \frac{-\left(\frac{8}{\pi}\right)\omega_r \xi_v R_h + \sqrt{\left(\alpha^2 + (2\omega_r \xi_v)^2\right) - \left(\frac{4}{\pi} \alpha R_h\right)^2}}{\alpha^2 + (2\omega_r \xi_v)^2}\left(\frac{1}{2}\right)
\]
where $\alpha$ is defined as $(1 - \omega^2)$. As the DAF in equation (5) depends on $R_h$, $\omega_r$ and $\xi_v$. Note that as $\omega_r$ approaches 1.0, the magnitude approaches a maximum value for all curves of $R_h$. The magnitude increases as $R_h$ decreases. As the amplitude of the steady-state vibration is affected by changing the damping ratio, it is expected that $R_h$ takes the role of damping ratio. At resonance, i.e. $\omega_r = 1$, DAF in equation (5) becomes

$$\text{DAF} = \frac{1 - 4R_h}{2\pi \xi_v}$$

(6)

As can be observed in equation (15), the steady-state response is guaranteed only when there exists $\xi_v$. If $\xi_v$ is zero, DAF becomes infinite, which means that input energy is greater than the energy dissipated by the friction damper. Through the DAF from equation (5) into the equivalent damping ratio of the equation (3), the invariant equivalent damping ratio, $\xi_{eq,app}$, is derived without DAF as:

$$\xi_{eq,app} = \frac{R_h}{(\pi/4) - R_h \xi_v}$$

(7)

Note that $\xi_{eq,ass}$ is related to both $R_h$ and $\xi_v$. Rewriting equation (7) yields the following equation for the friction force ratio:

$$R_h = \frac{(\pi/4)R_{\xi}}{1 + R_{\xi}}$$

(8)

It can be observed that $R_h$ depends on $R_{eq}$, the ratio of the approximate equivalent damping ratio and the viscous damping ratio, which is $\xi_{eq,app}/\xi_v$.

3. DESIGN PROCEDURE OF A FRICTION DAMPER

To estimate the response of a structure subjected to a random excitation such as an earthquake ground excitation, the frequency contents of the excitation and the transfer function between the excitation and the response need to be known. The mean square response is obtained by integrating the power spectrum of the response over the frequency range of interest, which consists of the multiplication of the transfer function and the power spectrum of the excitation. The behavior of a friction damper is inherently nonlinear and thus its transfer function cannot be obtained. In this study, however, it is assumed that the friction force is small compared with the amplitude of harmonic loading and the steady-state vibration is ensured. Based on this assumption, the approximate equivalent viscous damping ratio is obtained using only the force ratio, $R_h$, and the viscous damping ratio $\xi_v$, as shown in equation (7).

Physical insight into response reduction as a result of damper installation can be provided by observing damping ratio rather than friction force contributed by the friction damper. For design purpose, the damping ratio to be supplied by the damper to achieve a target performance, which is denoted as $\xi_{target}$, can be prescribed regardless of $R_h$ and $\xi_v$. With this in mind, the amplitude of the dynamic displacement obtained in equation (4) can be modified into the following equation using the transfer function $H(\omega)$, obtained as follows:

$$u_0 = H(\omega) F_0$$

(9)
The right-hand side terms in equations (4) and (10) are almost the same, but their interpretations are different. The former includes magnitude of excitation, \( F_{in} \), and therefore cannot be regarded as a transfer function. The latter, however, is considered as a transfer function by prescribing \( \xi_{target} \) regardless of \( F_{0} \). The mean square displacement is obtained by integrating the displacement power spectrum over all frequency range. The power spectrum of the excitation at natural frequency can be considered as constant without introducing significant error in the final results (Crandall and Mark 1973):

\[
\sigma_f^2 = S(\omega_n)\int_{-\infty}^{\infty} |H(\omega)|^2 \, d\omega
\]

(11)

where \( \sigma_f \) and \( S(\omega_n) \) are, respectively, the mean displacement and the power spectrum of the excitation at natural frequency, \( \omega_n \). Substituting equation (10) into equation (11) and performing integration result in

\[
\sigma_f^2 = \frac{\pi}{2} \frac{S(\omega_n)}{2(\xi_v + \xi_{target})m^2\omega_n^3}
\]

(12)

The vibration control effect of the damper is defined by equation (12) with the mean square displacement obtained without a friction damper (i.e. \( \xi_{eq} = 0 \)) and taking a square root, which is

\[
J_f = \sqrt{1 + \frac{\xi_{target}}{\xi_v}}
\]

(13)

Since equation (13) is obtained by the process that mean square displacement with friction damper is normalized by the mean square displacement obtained without friction damper, it is governed by the ratio \( R_{\xi,\text{target}} = \xi_{target} / \xi_v \), not by the intrinsic damping \( \xi_v \). The response reduction factor, \( J_f \), is composed of the ratio of the target damping and the inherent viscous damping ratios. It can be deduced that the displacement response reduction factor presented in equation (13) is the same as the response reduction factors for the velocity and the acceleration, since the friction damper affects damping ratio only. It is appropriate to reorganize equation (13) in terms of the response reduction factor to obtain \( \xi_{target} \):

\[
\xi_{target} = \frac{1 - J_f^2}{J_f^2} \xi_v
\]

(14)

For design of a friction damper the response reduction factor is prescribed first and then the corresponding target damping ratio is determined using the above equation.

4. VERIFICATION FOR SEISMIC EXCITATION

The previous section dealt with the process for designing a friction damper to satisfy a given target response reduction factor. The process first began with prescribing a target response reduction factor considering a trade-off between damper cost and control effectiveness. And then the corresponding target equivalent damping ratio was chosen, and finally the required friction force to meet the target
performance was obtained. In order to verify the proposed process, numerical time history analysis of a single-story structure installed with a designed friction damper was carried out for 6 seismic records, which are El Centro (EC), Helena (HE), Mexico (ME), Northridge (NO), Hachinohe (HA) and state building (ST) earthquakes. The structural properties for this study are that the natural frequency, $f_n = 0.2, 0.5$ and the inherent viscous damping, $\xi_v$ is 0.02. The force ratio $R_h$ and the response reduction factor, $J_f$, are governed by $R_\xi$ and $R_\xi^{\text{target}}$, respectively, not by $\xi_v$ as shown in equations (8) and (14), respectively. To design a friction damper with the proposed procedure, firstly select a desired response reduction factor, $J_f$. Second, obtain target damping ratio $\xi_{\text{target}}$ using equation (14). Third, obtain $R_h$ using equation (8) with $\xi_{\text{target}}$ in place of $\xi_{\text{eq.app}}$. Finally, determine $f_d$ using $R_h$, and then the numerical analysis is performed using it. The following figure 2 is illustrated by numerical analysis in seismic excitations according to the described procedure.

![Figure 2: Verification of $J_f$ according to variable seismic excitation.](image)

The continuous lines and dot lines in figure 2 are a target response reduction factor and a numerical response reduction factor through seismic excitation analysis. Figures 2 (a)~(c) are performed when structural natural frequency is 0.2 and figures 2 (d)~(f) are expressed about $f_n = 0.5$. The proposed procedure generally indicates the valid values.

5. CONCLUSIONS

This study presented a simple design process of a friction damper for controlling seismic responses by a numerical analysis approach. A closed form solution for a dynamic amplification factor (DAF) was derived by assuming that the friction damped structure showed steady-state response with small friction damping force, and that Coulomb damping force could be replaced by an equivalent viscous damping force. DAF turned out to be narrow banded and dependent on equivalent viscous damping ratio at natural
frequency. Based on these observations the equivalent viscous damping ratio was derived from friction force ratio and inherent viscous damping ratio. The equation for DAF was transformed into a transfer function by adopting the value of DAF at natural frequency and rearranging the equation by prescribing target damping ratio. Then response reduction factor of displacement responses with and without friction dampers was found analytically, and a design procedure was proposed to determine the required damper friction force to satisfy a given target response reduction factor. Time history analysis of a SDOF system with a friction damper was carried out to check if the given target response reduction factor was satisfied using ten earthquake records. The analysis results showed that the mean response reduction factors obtained by numerical time history analyses matched well with the target response reduction factors. The proposed design procedure has the advantage that the friction damper consistent with target response reduction factor is simply designed without the complex non-linear analysis. Based on the analysis results it was concluded that the proposed procedure could be used for designing a friction damper to control structural responses to satisfy a given target performance.

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