Calculation on seismic resistance of box-shaped structures of large-panel buildings

M K Usarov\(^1\) and G I Mamatisaev\(^2\)

\(^1\)Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, 31 Durmon yuli str., Tashkent 100125, Uzbekistan
\(^2\)Fergana Polytechnical Institute, 86 Fergana str., Fergana 150107, Uzbekistan

E-mail: umakhamatali@mail.ru

Abstract. The article is devoted to the theoretical calculation of the box-shaped structure of large-panel buildings on dynamic effects, taking into account the spatial work of transverse and longitudinal walls under dynamic effects, set by the base displacement according to a sinusoidal law. The problem is solved using the finite difference method.

1. Introduction
Prediction of dynamic behavior and assessment of the stress-strain state of various structures, taking into account physical and geometrical nonlinearity, viscoelastic and anisotropic properties of the material and inhomogeneous structural features under the influence of external effect, are of great importance.

The study in [1] was devoted to the calculation of a two-story reinforced concrete frame structure with crossbars at initial stresses. It is believed that the frame structure in question is susceptible to emergency impact. Using the finite element method, the values of internal forces in structural elements were found.

[2] is devoted to solving the problem of building structures vibrations under the influence of external harmonic load. The displacements of the nodes and the internal forces in the elements of the structure under consideration were determined depending on various parameters of dynamic effect.

The theory of beams, plates and shells has been widely used in the field of the dynamics of structures in calculation of structural elements of buildings and structures.

For buildings and structures, the effects of an earthquake depend on the characteristics of seismic effects (intensity, spectral composition, etc.), the properties of the soils of the base of the construction area, the quality of design and construction [3-7].

In studies [8,9], the problems of parametric resonance were solved for forced vibrations of structural elements made of isotropic material under the action of a uniformly distributed dynamic load.

The model of a box-shaped structure of a building is improved in [10,11] taking into account forces and moments in the contact zones of beam and plate elements interaction. The equations of motion of the box-shaped elements, the boundary conditions in the box base and the contact conditions between the box elements are given; the graphs of plates and beams displacements are constructed. The problem of forced oscillations of a building of spatial box type is considered in the paper; it composes of rectangular panels and interacting beams under dynamic effect set by base displacement according to a...
The finite difference method was used in the problem solution. Numerical results of stresses, displacements in the hazard areas of the box-like building are obtained.

The studies in [12, 13] are devoted to the development of the methods for dynamic spatial calculation of a structure based on the finite difference method in the framework of the bimoment theory, which takes into account the spatial stress-strain state.

Solutions to the problem of transverse and longitudinal vibrations of buildings and structures were obtained using a plate model developed in the framework of the bimoment theory of plates [14,15].

A method has been developed in the paper for dynamic calculation of one box (room) of buildings, called a box-like structure, which consisting of interconnected plate and beam elements.

2. Statement of the problem

The mathematical problem of building vibrations is formulated on the basis of its box-like structure with fixed bottom end(figure 1). The direction of coordinate axes shown in figure 1.

![Figure 1. Box construction of buildings.](image)

Introduce the following notation for the panels of the spatial box-like structure of a building: $E_i, \nu_i, \rho_i$ - are the elastic modulus, Poisson's ratio and density of $i$-wall, respectively.

Assume that panels 1 and 3 work only on bending, and panels 2 and 4 are strained only in their $XOZ$ plane. Based on this assumption, the bending force factors disappear in the area of butt joints of panels 4, 2 and beams. Overlapping (panel 5) is considered as deformable also. The law of it points' motion is determined in accordance with the forms of deformation of its upper edges of the vertical contacted panels.

As a design scheme, a spatial box is considered, making forced vibrations in the horizontal and vertical direction along the small size of the building according to a given law:

$$U_0(t) = A_0 \sin \omega_0 t, \quad V_0(t) = 0.$$  

Where: $A_0$ and $\omega_0$ are the amplitude and frequency of forced oscillations.

In this article, a theoretical calculation of a box structure of large-panel buildings on dynamic effects in the form of (1) is carried out taking into account the spatial work of the internal transverse and external longitudinal walls. Suppose that the external load-bearing walls of a building (panels), perpendicular to the direction of seismic effects, work only on transverse dynamic bending. The interior panels, located in the direction of external influence, are subjected in their plane to tension-compression and shear. An analytical-numerical method is proposed for solving the vibration problem of a building box taking into account spatial strains. In the areas of butt joints of panels, full contact conditions are set to ensure the equality of displacements and stresses.

The general kinematic law of motion of the box is represented as the sum of the displacement of the base $U_0(t)$ and the relative displacement of the panels $u(x, z, t)$, $\nu(x, z, t)$:

$$u_1 = U_0(t) + u(x, z, t), \quad u_2 = \nu(x, z, t)$$

Here $u, \nu$ - are the displacements of panels working on shear.

When deriving the equation of motion, the panels are considered as thin elastic plates obeying the Kirchhoff-Love hypothesis and each beam is subjected to bending and torsion.
Normal displacements of bending panels points are:
\[ u_z = U_0(t) + W(x, y, t) \]  
(3)

Where \( W(x, y, t) \) is the deflection of bending panels.

The equation of transverse oscillations of panels that work only on bending, taking into account the representation (3), can be written as \([10,11]\):

\[
D(\frac{\partial^4 W}{\partial x^4} + 2 \frac{\partial^4 W}{\partial x^2 \partial y^2} + \frac{\partial^4 W}{\partial y^4}) + \rho h \ddot{W} = -\rho h \ddot{U}_0
\]  
(4)

Where \( D \) - is the cylindrical rigidity of panels under lateral bending.

The equations of motion of panels working in their plane in tension (compression) and shear, taking into account representation (2), are written as follows [10,11]:

\[
B(\frac{\partial^2 u}{\partial z^2} + \frac{1 + \nu}{2} \frac{\partial^2 u}{\partial x \partial z} + \frac{1 - \nu}{2} \frac{\partial^2 u}{\partial y^2}) = \rho h \ddot{u} + \rho h \ddot{U}_0,
\]

\[
B(\frac{\partial^2 v}{\partial x^2} + \frac{1 + \nu}{2} \frac{\partial^2 v}{\partial x \partial z} + \frac{1 - \nu}{2} \frac{\partial^2 v}{\partial z^2}) = \rho h \ddot{v},
\]

(5)

where \( B \) is the cylindrical rigidity of panels under extension and compression.

The equation of the bending oscillations of beams is written in the form [10,11]:

\[
EJ \frac{\partial^4 W^{(b)}}{\partial x^4} + \rho F \ddot{W}^{(b)} = R_y^{(b)} - P_{(b)} - \rho F \ddot{U}_0.
\]

(6)

The equation of torsional oscillations of the beam has the form [5,6]:

\[
\frac{\partial}{\partial x} G I_{kr} \frac{\partial \alpha}{\partial x} = \rho I_{kr} \ddot{\alpha} + M^{(b)}_y + \frac{S}{2} R^{(b)}_y,
\]

(7)

where: \( R_y^{(b)} = -D \frac{\partial}{\partial y} \left[ \frac{\partial^2 W}{\partial y^2} + (2 - \nu) \frac{\partial^2 W}{\partial x \partial y} \right] \) and \( P_{(b)} = B(\frac{\partial U}{\partial z} + \nu \frac{\partial V}{\partial x}) \) are the values of the reactive shear force, bending panel and longitudinal force of the panel working on shear in the area of the butt connection of panels and beam, \( \alpha = \left. \frac{\partial W}{\partial y} \right|_{x=a} \) - is the beam twist angle, \( W^{(b)}(x, t) \) - is the deflection of the beam, \( M_{kr} = EL_{kr} \frac{\partial \alpha}{\partial x} \) - torque, \( EL_{kr} \) - beam torsional rigidity, \( EJ \) - beam flexural rigidity.

The equation of motion of the floor slab is derived based on the following considerations. For simplicity, assume that the floor displacement field is determined by the law:

\[ u_p(y, t) = W(H, y, t), \quad v_p(y, t) = \nu(H, t) \]

Where: \( u_p(y, t) \) and \( v_p(y, t) \) are the horizontal and vertical displacements of floor slab points; \( W(H, y, t) \) and \( \nu(H, t) \) - deflection and displacement of the upper points of panels working on bending and shear.

The contact conditions at the joints of the floor and the bending wall have the form

\[
-R^b_x + \eta_0 \rho_p h_b h_p \ddot{W}_{p,k} = h_b h_p \frac{\partial^2 \varepsilon^b_y}{\partial y^2} - \eta_0 \rho_p h_b h_p \ddot{U}_0.
\]

(8)
Where: \( x^b_{\gamma} = \frac{G \partial W}{\partial y} \), \( \eta_0 = \frac{bc}{2(bh_b + ch_c)} \) - is the ratio of the floor slab area in plan to the cross-sectional areas of the walls working on bending and shear, \( \rho_p \) - is the floor slab density, \( h_p, h_b, h_c \) - thickness of the floor slab and bending and shear panels, \( b \) и \( c \) - box length and width.

In the areas of butt joints of panels and beam elements, full contact conditions regarding kinematic and force factors are accepted.

Now consider the contact conditions between the elements of the box and the boundary conditions at the base and in the upper part of the building. At the junction of panels and beams, we have contact kinematic conditions

\[
W(x,0,t) = U(x,c,t) = W(b,x,t), \quad V(x,c,t) = \pm H_1 \frac{\partial W(b)}{\partial x}
\]  

(9)

The minus sign is taken when the beam is connected to the panels with concave sides.

Note that the equations of motion (5) and (6) are taken as the contact force conditions in the butt joints of panels and beams.

The mass of the floor slab and the portion of the mass of floor slab on the walls are written as follows:

\[
M_0 = \rho_p bh_b, \quad m_{pb} = \eta_0 \rho_p bh_b h_p, \quad m_{pc} = \eta_0 \rho_p bh_b h_p
\]

where: \( M_0 \) - mass of the floor slab, \( m_{pb} \) и \( m_{pc} \) - the portion of the mass of floor slab on the walls, working on bending and shear.

Based on accepted assumption that the floor is not deformable, the boundary conditions on the upper part of the box are written in the form of kinematic and force conditions

\[
W = u_t(y,t), \quad U = W(b) = u_t(b,t), \quad V = 0, \quad M_{xx} = 0, \quad M_{x}^0 = 0,
\]

(10)

where: \( M_0 \) - is the mass of the floor slab, \( H \) - is the thickness of shear panels, \( \tau_{xy} \) и \( R_x \) - are the shear stress and reactive force of panels working on shear and bend, \( u_t(t) \) - is the horizontal displacement of rigid floor.

The boundary conditions at the base of the building are similar to the rigid fixing. The lower part of the building moves with the base;

\[
W = U = W(b) = u_0(t), \quad V = 0,
\]

(11,a)

and the rotation angle is absent

\[
\frac{\partial W}{\partial x} = 0, \quad \frac{\partial W(b)}{\partial x} = 0.
\]

(11,b)

The contact conditions at the joints of the floor and wall working on shear relative to contact shear stress are written as

\[
h_n \tau^b_{zy} + h_c \tau^c_{zx} - \rho h_c \rho h_p \ddot{u}_{p,k} = \rho h_c \dot{h}_p \ddot{V}_0.
\]

(12)

The contact conditions at the joints of the floor and the shear wall relative to the contact normal stress are written as

\[
\sigma_{xx} - \rho h_p \ddot{u}_{p,k} = \rho h_p \dot{V}_0(t).
\]

(13)

Where \( \tau^c_{zx} = G_c \left[ \frac{\partial \vartheta}{\partial z} \right]_{x=H} \), \( \sigma^c_{xx} = E_p \left[ \frac{\partial \vartheta}{\partial z} \right]_{x=H} \).

The modes of oscillations (2) and (3) must satisfy the equations of motion (4)-(7), boundary and contact conditions (8)-(13). The problem is solved by the method of finite-difference schemes.
3. Method of the solution

The general solution to the problem of forced oscillations of a bending box panel is described by a function represented as the sum of the solution to the problem of forced and natural oscillations:

\[ W(x, y, t) = A_0 W_v(x, y) \sin \omega_0 t + C_1 W_1(x, y) \sin p_1 t, \]  

where \( p_1 \) - the first natural frequency, \( C_1 \) - is a constant, to be determined, \( W_1(x, y) \) form of forced oscillations, \( W_v(x, y) \) – the first fundamental mode of oscillations.

In [10,11], taking into account zero initial conditions, solutions (14) can be rewritten in the form:

\[ W(x, y, t) = A_0 \left( \sin \omega_0 t - \frac{\omega_0}{p_1} \sin p_1 t \right) W_v(x, y). \]  

The expressions for displacement of the panel working on shear have the form

\[ u(x, z, t) = A_0 \left( \sin \omega_0 t - \frac{\omega_0}{p_1} \sin p_1 t \right) u_v(x, z, t), \]

\[ v(x, z, t) = A_0 \left( \sin \omega_0 t - \frac{\omega_0}{p_1} \sin p_1 t \right) v_v(x, z, t). \]

The kinematic functions of the beams can be written as:

\[ W^{(l)}(x, t) = A_0 \left( \sin \omega_0 t - \frac{\omega_0}{p_1} \sin p_1 t \right) W^{(l)}_v(x), \]

\[ \alpha^{(l)}(x, t) = A_0 \left( \sin \omega_0 t - \frac{\omega_0}{p_1} \sin p_1 t \right) \alpha^{(l)}_v(x). \]

Thus, a general solution to the problem is constructed in the form of (15)-(17), satisfying the boundary conditions (11) and the contact conditions between the floors (8) and (12), (13) panels and beams (6), (7) and zero initial conditions.

The problem of determining the unknown coordinate functions in expressions (15)-(17) is solved by the finite difference method.

The proposed methodology for calculating the box-like structure of a building [10,11] allows us to estimate the maximum stresses in the internal transverse and external longitudinal bearing panels, as well as in the butt joints of these elements. This design diagram of the box structure complements the design scheme developed in the framework of the continuous plate model [12,13], which describes the spatial deformation of the building as a whole and takes into account all types of strains and stresses. It should be noted that the continuum plate model of the building was developed on the basis of the bimoment theory of plates [14,15], which was created without simplifying hypotheses in the framework of the three-dimensional theory of elasticity.

4. Analysis of numerical results.

The mechanical and geometric characteristics of the box materials are as follows: Bendable panels have the same elastic characteristics: elastic modulus \( E = 20000 \text{ MPa} \); density \( \rho = 2.5 \text{ t/m}^3 \); Poisson's ratio \( \nu = 0.3 \). For a shear panel: elastic modulus \( E = 7500 \text{ MPa} \), density \( \rho = 1.2 \text{ t/m}^3 \), Poisson's ratio \( \nu = 0.3 \).

Consider the fluctuations of the box without taking into account and window openings. The geometric dimensions are as follows: for bending panels, thickness \( h_y = 0.5 \text{ m} \), height \( H = 3.25 \text{ m} \) and length \( b = 6 \text{ m} \), and for shear panels, thickness \( h_v = 0.25 \text{ m} \); height \( H = 3.25 \text{ m} \) length \( c = 6 \text{ m} \).

Proceed to discuss the obtained numerical results of stress calculations.

Figure 2 shows the graphs that characterize the changes in the maximum normal contact stress \( \sigma_{zz} \) in the average vertical section of the panels and in the areas of butt joints of panels and beams.
The stresses in the cross section of the edge of the panels located on the side of load impact take negative values, therefore, are in compression conditions. At the points of the opposite edge of the panel, the stress takes positive values, i.e. this zone is a tension zone.

In Figure 2 shown that in panels working on shear, the edge of the panels on the side of the dynamic load is compressed, and on the opposite side it is tensile. The stress $\sigma_{zz}$ on the top of the panel is greater than the stress on the bottom.

Figure 3 shows the diagrams of the maximum compressive and tensile stresses $\sigma_{yy}$ of bending panels. The maximum stress $\sigma_{yy}$ values are found in the upper part of the bending panel. In the contact zones of the panels, the stress $\sigma_{yy}$ are tensile.

The maximum value of the normal stress $\sigma_{yy}$ in the panels working on bending without account for the window opening was obtained equal to: $-\sigma_{yy} = -0.814 MPa$.

The maximum value of the normal stress $\sigma_{yy}$ in the panels working on bending with account for the window opening is obtained approximately 5 times less than the value obtained without account for the window opening $\sigma_{yy} = -4.528 MPa$.

Figure 2. Changes in stress $\sigma_{zz}$ of shear panels:

a) at the edges of the panels on the side load of loaf impact; b) the vertical section of the panels; c) the opposite edges of the panels.

a) 

b) 

c)
Figure 3. Change in stress $\sigma_{yy}$ of panels working for bending in height: a) middle; b) the edge of the window opening; c) the edge of the panel.

In Figure 4 the laws of shear stress variation along the height of shear panels are shown.

Figure 4. Changes in stresses $\tau_{zx}$ along the height of shear panels: a) left edges; b) the middle; c) the right edges.

From Figure 4 it is seen that the maximum value of the shear stress $\tau_{zx}$ is found in the middle of the lower part of the panel. In addition, the maximum value of shear stress $\tau_{zx}$ was found at the edge of the panels, on the side of the dynamic load impact.

Note that the values calculated on each vertical section $\tau_{zx}$ are the contact stresses between the panels and the beams.
5. Conclusions
1. The equations of motion of the panels and beams points of the box-type buildings, the boundary, contact and initial conditions of the problem of forced vibrations are given in the paper;
2. Within the framework of the finite difference methods, a method for dynamic calculation of stress in beam and panel elements of box-shaped structures of buildings has been developed;
3. The laws of changes in the maximum stress values in the characteristic sections of panels working in bending and shear are presented in graphs.

References
[1] Le T Q T, Lalin V V and Bratashov A A 2019 Static accounting of highest modes in problems of structural dynamics Magazine of Civil Engineering (St. Petersburg: Peter the Great St. Petersburg Polytechnic University) 88 3-13 DOI: 10.18720/mce.88.1
[2] Vatin N I, Kuznetsov V D and Nedviga E S 2011 Installation errors in calculating large-panel buildings Magazine of Civil Engineering (St. Petersburg: Peter the Great St. Petersburg Polytechnic University) 6 35-40 DOI:10.5862/mce.24.3
[3] Pshenichkina V.A., Zolina T.V., Drozdov V.V., Harlanov V.L. Methodology for assessing the seismic reliability of buildings with high floors // Bulletin of the Volgograd State University of Architecture and Civil Engineering. Series: Construction and Architecture. 2011. No. 25. S. 50-56.
[4] Khachatryan S.O. Spectral-wave theory of earthquake resistance // Earthquake engineering. Safety of facilities. 2004. No. 3. P. 58–61.
[5] Radin V.P., Trifonov O.V., Chirkov V.P. A model of a multi-story frame building for calculations of intense seismic effects // Earthquake-resistant construction. Safety of facilities. 2001. No. 1. C. 23-26.
[6] Tyapin A.G. Calculation of structures for seismic effects, taking into account interaction with a soil base. M.: DIA Publishing House, 2013.399 p.
[7] Chopra Anil K. Elastic response spectrum: a historical note // Earthquake Engineering and Structural Dynamics. 2007. Vol. 36. No. 1. Pp. 3—12.
[8] M. M. Mirsaidov, R. A. Abdikarimov, N. I. Vatin, V. M. Zhgutov, D. A. Khodzhaev, and B. A. Normuminov, Mag. Civ. Eng. (2018)
[9] M. M. Mirsaidov, R. A. Abdikarimov, and D. A. Khodzhaev, PNRPU Mech. Bull. (2019)
[10] Usarov M, Mamatissaev G, Yarashov J and Toshmatov E 2020 Non-stationary oscillations of a box-like structure of a building Journal of Physics: Conference Series 1425 012003 DOI:10.1088/1742-6596/1425/1/012003
[11] Usarov M, Mamatissaev G, Toshmatov E and Yarashov J 2020 Forced vibrations of a box-like structure of a multi-storey building under dynamic effect Journal of Physics: Conference Series 1425 012004 DOI:10.1088/1742-6596/1425/1/012004
[12] Yarashov J, Usarov M and Ayubov G 2019 Study of longitudinal oscillations of a five-storey building on the basis of plate continuum model E3S Web of Conferences 97 04065 (Form 2019 04065) DOI:org/10.1051/e3sconf/20199704065
[13] Toshmatov E, Usarov M, Ayubov G and Usarov D 2019 Dynamic methods of spatial calculation of structures based on a plate model E3S Web of Conferences 97 04072 (Form 2019 04072) DOI:org/10.1051/e3sconf/20199704072
[14] Usarov D, Turajonov K and Khaidov S 2020 Simulation of free vibrations of a thick plate without simplifying hypotheses Journal of Physics: Conference Series 1425 012115 DOI:10.1088/1742-6596/1425/1/012115
[15] Usarov M K 2015 Buckling of orthotropic plates with bimoments Magazine of Civil Engineering (St. Petersburg: Peter the Great St. Petersburg Polytechnic University) 53(1) 80–90 DOI: 10.5862/MCE.53.8