EXEm: Expert Embedding using Dominating Set Theory with Deep Learning Approaches

Abstract

A collaborative network is a social network that is comprised of experts who cooperate with each other to fulfill a special goal. Analyzing the graph of this network yields meaningful information about the expertise of these experts and their subject areas. To perform the analysis, graph embedding techniques have emerged as a promising tool. Graph embedding attempts to represent graph nodes as low-dimensional vectors. In this paper, we propose a graph embedding method, called ExEm, which uses dominating-set theory and deep learning approaches. In the proposed method, the dominating set theory is applied to the collaborative network and dominating nodes of this network are found. After that, a set of random walks is created which starts from dominating nodes (experts). The main condition for constructing these random walks is the existence of another dominating node. After making the walks that satisfy the stated conditions, they are stored as a sequence in a corpus. In the next step, the corpus is fed to the SKIP-GRAM neural network model. Word2vec, fastText and their combination are employed to train the neural network of the SKIP-GRAM model. Finally, the result is the low dimensional vectors of experts, called expert embeddings. Expert embeddings can be used for various purposes including accurately modeling experts’ expertise or computing experts’ scores in expert recommendation systems. Hence, we also introduce a novel strategy to calculate experts’ scores by using the extracted expert embedding vectors. The effectiveness of ExEm is validated through assessing its performance on multi-label classification, link prediction, and recommendation tasks. We conduct extensive experiments on common datasets. The obtained results outperform the baseline algorithms and how effective are our graph embedding approach, especially for higher density networks. Moreover in this study, we present data related to a co-author network formed by crawling the vast author profiles from Scopus.

Keywords Social and collaborative Networks · Graph embedding · Node representation · Dominating set · Expert recommendation system

1 Introduction

Today’s, social networks have emerged as a great platform to generate and share information. Although social networks are rich data sources, a major challenge surrounding the social networks is how to analyze both their structure and...
content. By the way of illustration, Question Answering Community (QAC) is one type of on-line social network that the users’ collaborations are asking or answering questions. On the other hand, in QACs, the network between users are constructed from their answer relations [11]. One of the key problems in these networks is identifying experts to answer the given questions [2]. For this purpose, it is necessary to analyze the social interactions of users and content of the questions asked and the answers replied by them to determine experts [3]. As another example, academic papers are composed of several co-authors. Moreover, the development of collaboration among academic authors constitutes a collaboration network, called the co-author network. In this network, nodes represent authors and an edge between two nodes demonstrates the corresponding authors have published at least one paper together. Hence, the co-author network can be observed as one specific type of social network [4]. Co-author network has been used to recognize the key researchers in a specific area, that can be considered as experts, and analyze the social interactions of these researchers [5].

Graph analytic appears in a wide variety of applications such as node classification, link prediction, clustering, node recommendation, visualization, etc [6][7]. Although many methods have been proposed for the analysis and representation of the graph, they encounter several challenges such as space cost and time complexity. Moreover, graph embedding is an effective solution to represent the graph and perform the analysis. It changes the form related to the representation of graph nodes into a low dimensional space and tries to maintain consistent the structural information and properties of the graph.

Modeling the interactions between users in social networks as a graph and applying graph embedding methods to the extracted graph can yield valuable information. This information can be derived from the structure and content of social networks for different tasks such as classification, link prediction and recommendation [8].

Here we propose a graph embedding method that using dominating-set theory and neural models, called “ExEm” to convert a graph to a low-dimensional vector space. Dominating set theory is one of the graph theory that has been found to have important applications in computer science fields. It has been a classic subject studied in graph theory that considers as a virtual backbone in these areas [9]. A set is dominating if every node in the network is either in the set or a neighbor of a node in this set [10]. ExEm considers a set of random walks that contain at least two dominating nodes. Then, these random walks are stores as a corpus. A SKIP-GRAM neural network model is used to produce the embedding vectors of nodes. In order to train this neural network, Word2vec and fastText and their combination are utilized. Moreover, the effectiveness of graph embedding in different real-world applications motivates us to explore its potential usage in the expert recommendation system and propose a strategy to compute experts’ scores and recommend experts. Furthermore, we present a collaborative network that is constructed based on the gathered information from Scopus. In this network, nodes that are represented authors have multi labels. The labels demonstrate the authors’ subject areas. Edges between authors denote their co-author relationship. On the other hand, in this study, we aim to answer the following research questions:

- **Q1.** Does data gathered from Scopus provide a suitable real dataset for the different tasks such as classification, link prediction, recommendation and so on?
- **Q2.** Does using dominating set theory result in an effective approach for graph embedding?
- **Q3.** How can we extend the obtained node representation in expert recommendation systems?

The remainder of the paper is outlined as follows: Section 2 reviews the related works. Section 3 explains our proposed method for graph embedding in detail. Section 4 presents the descriptions of the gathered dataset from Scopus. Our suggested approach for computing the experts’ is explained in Section 5. In order to verify the proposed framework, extensive experiments are conducted on real-life datasets. The descriptions of these datasets and baseline approaches, the tasks that are used to test our proposed method and parameter setting are presented in Section 6. The experimental results and their analysis are given in Section 7. Finally, Section 8 concludes the paper.

## 2 Related Work

To analyze social network data, previous studies have proven that representation of social network as a graph structure and using graph theories have achieved successful results. Furthermore, deep learning approaches have demonstrated to be a promising technique to analyze information from social networks with complicated structures. Thus, the incorporation of graph-structured data and the deep learning model results in an outstanding feature learning technique, called graph embedding. Graph embedding learns a map of the graph’s nodes to a low-dimensional space features. It provides insight into analyzing users’ activity patterns and their relationships in social networks. In this section, we investigate some of the proposed graph embedding methods by different researches.

GraRep [11] learns the node representations of weighted graphs. It makes use of the matrix factorization version of skip-gram to obtain high-order proximity [6]. On the other hand, it catches the k-step (k = 1, 2, 3, ...) neighbour relations
and integrates global structural information of the graph into the learning process. The final representation of nodes are provided by concatenating k-step node representations together \[12\].\[13\].

TriDNR \[14\] utilized node structure, node content, and node labels for the graph embedding. It used the DeepWalk approach \[15\] to learn the network representation based on the network structure. Moreover, TriDNR coupled two neural networks to capture the node content and label information. Finally, the obtained representation from network structure, and the node label and attribute were linearly combined together \[16\] \[13\]. \[6\].

Mahmood et al. \[17\] have proposed a geodesic density gradient (GDG) algorithm that is divided a network into a series of relatively small communities \[18\] \[19\]. On the other hand, this study considers a vector for each node with dimensionality equals the number of all nodes. In this vector, every dimension represents the geodesic distance of that node from all other nodes of the network \[6\]. Thus, the network structure can be captured by the geodesic distance vectors. In this way, the nodes with the same region of space belong to the same communities in the original network.

DNGR \[20\] is based on a deep learning approach that comprises three steps. As the first step, it captures information related to the graph structure by proposing a random surfing model and generates a probabilistic co-occurrence matrix. The random surfing model is inspired by the PageRank model. After calculating the PPMI matrix, in the final step, a stacked denoising autoencoder is applied to the matrix to learn low-dimensional vertex representations \[12\].

HOPE \[21\] is considered as a matrix factorization based method. It preserved the asymmetric transitivity property of a directed network in embedding vectors by high-order proximity \[12\]. It is worth-nothing that asymmetric transitivity describes the correlation among directed edges. HOPE derived a general formulation from four high-order proximity measurements \[13\]. Then, it applied generalized Singular Value Decomposition (SVD) to the general formulation to capture the embedding successfully \[7\].

SDNE \[22\] is a deep learning based graph embedding that applied an auto-encoder on the whole graph for social network embedding purpose. It used first and second order proximities and defined two separate functions, \(L_{1st}\) and \(L_{2nd}\), to preserve these proximities, respectively \[13\]. These functions are combined linearly to minimize the joint function as equation \[1\] \[23\].

\[
\begin{align*}
L &= L_{2nd} + \alpha L_{1st} + \nu L_{reg}
\end{align*}
\]

here \(L_{reg}\) represents a regularization term. After minimizing the above function, the \(k\)-th layer representation is captured as embedding of vertex \(v_i\).

Line 23 is another graph embedding technique that minimizes the distance loss function to preserve first-order proximity and second-order proximity as opposed to producing random walks.

Although many network embedding methods are proposed for static networks, recent studies have investigated the embedding methods over the dynamic networks that evolve over time \[24\] \[25\] \[26\]. Goyal et al. \[27\] proposed a deep learning model to capture temporal patterns in dynamic networks for the link prediction task. This study introduced three different architectures using an autoencoder, LSTM, and combination of autoencoder and LSTM. These architectures take as input the adjacent matrix of graph \(A_{t-l} [i], A_{t-l+1} [i], \ldots, A_{t-1} [i]\) and produces a vector \(v_i\) corresponding to the embedding of \(v_i\) at time \(t\). It allows predicting interactions between vertices at each time step. Moreover, in another study \[28\], the authors proposed to compute a dynamic node representation by employing self-attention mechanisms over its neighbors and previous historical representations. Survey \[29\] reviewed the recent representation learning methods for dynamic graphs.

Additionally, some studies have focused on the knowledge graph embedding. A knowledge graph is a directed graph that represents structured information of entities as nodes and their relations as edges \[30\] \[13\]. Research \[31\] embeds the knowledge graph in this manner that entities are closed to each other in the embedding space if they belong to the same semantic category. Authors in \[32\] provide a review of existing approaches used in knowledge graph embedding.

In spite of the fact that graph embedding is a powerful tool for converting data structures into low dimensions, employing all features for this purpose may lead to noise \[33\]. To handle this challenge, one solution is dimensionality reduction. In recent years, many studies have focused on dimensionality reduction for graph embedding. Dimensionality reduction methods are categorized into two groups: feature selection and feature extraction \[34\]. Chen et al. \[33\] proposed a binary feature selector to perform feature selection for graph embedding. It can deal with the feature cardinality in the least-squares formulation. On the other hand, researches in \[33\] proposed a feature extraction based algorithm that reduces the dimensionality of original data by a compatible mapping \[36\].

Moreover, in the last few years, a number of surveys have been presented that attempted to categorize the existing graph embedding methods based on their proposed techniques. Cai et al. \[6\] summarized the researches into five categories: matrix factorization, deep learning, edge reconstruction, graph kernel, and generative model. In this study,
deep learning-based graph embedding is divided into two groups, deep learning graph embedding with and without a random walk. Based on this review, an edge reconstructing based graph embedding technique minimizes the distance loss to preserve first-order proximity and second-order proximity. On the other hand, a matrix factorization based method represents the connections between nodes as a matrix and factorizes this matrix to extract node embedding. Moreover, deep learning based graph embedding techniques with random walks represent a graph as a set of random walks and these random walks are fed into a deep learning method like Skip-Gram to optimize their neighborhood preserving likelihood objectives. In comparison, deep learning based graph embedding methods without random walks apply an autoencoder or deep neural network such as Convolutional Neural Network, on the whole graph. 

Zhang et al. [13] reviewed the state-of-art graph embedding techniques with a different perspective. They classified studies into two classes: unsupervised and semi-supervised network representation learning. Moreover, this survey summarized the existing approaches into five types matrix factorization, random walk, edge modeling, deep learning and hybrid from methodology perspective. Goyal et al. [7] and Cui [12] presented the graph embedding techniques in three categories: factorization based, random walk based and deep learning based.

3 Proposed Method

As it was previously mentioned, the aim of this paper is to map a graph to a low-dimensional vector space. Thus, for this goal, a dominating-set-based method is proposed, called ExEm, which constructs random walks that include at least two dominating nodes. ExEm solves the graph embedding problem by considering multi-label classification, link prediction, and node recommendation tasks. ExEm is implemented in four steps. The following subsections describe each step in detail. The required notations are defined in Table 1.

| Notations | Comment |
|-----------|---------|
| \( N \) | Number of graph nodes |
| \(|E|\) | Number of edges |
| \( D \) | Number of dominating nodes |
| \( R \) | Number of random walks |
| \( L_{R} \) | Length of random walks |
| \( E_{d} \) | Dimensional node embedding |
| \( C \) | Number of classes in classification task |
| \( d_{max} \) | Maximum degree |
| \(|T|\) | Number of triangles |
| \(|C_{cof|f}\) | Average clustering coefficient |
| \( V(u)_{i} \) | ith feature of vector representation related to node \( u \) |

3.1 Step1 : Preprocessing

Data preprocessing plays an important role in social network analysis. The goal of preprocessing is to convert the original dataset to an acceptable format for discovering beneficial information or recognizing patterns from the social network dataset [37]. In this way, the first step of ExEm is preprocessing the dataset. Firstly, the relations between the nodes of the dataset are considered and the graph of the dataset is constructed. Then, if nodes contain assigned elements such as attributes, labels, and tags, they are analyzed. Finally, the output of this step is a graph \( G = \langle V, L, E \rangle \) that \( V \), \( L \) and \( E \) demonstrate nodes of the graph, the corresponding element values of nodes, and edges of the graph, receptively. Figure 1 shows the preprocessing task and as it can be observed the input is a dataset and the output is a graph representation of the dataset.

3.2 Step2 : finding a dominating set

In this step of ExEm, a dominating set of the corresponding graph \( G \) is generated. A subnet of nodes, \( D \), is called a dominating set if every node is either in \( D \) or adjacent to a node in \( D \) [9]. The dominating set has been a critical capability in network science and its applications. It has been found to be an effective approach to detect, monitor and control the behavior of nodes in graphs. Dominating sets are able to perform various critical tasks in network systems, such as network controllability, social influence propagation, and finding high-impact optimized subsets in protein interaction networks [38]. In our study, the dominating set \( D \) is produced with an algorithm provided by authors in [39]. Figure 4 presents the procedure of finding a dominating set for a given graph. First, one of the nodes is randomly
selected and added to the dominating set $D$. After that this node and its neighbours are removed from a set of graph vertexes. Then, another random node is chosen from reminded vertexes and inserted into $D$. The mentioned steps are continued until there is no node in the graph vertex set. In our study, the dominating set $D$ is produced by Algorithm provided by authors in [39]. The flow chart of the process related to finding dominating nodes is illustrated in figure 3.

Consider the graph in figure 4 as an example. After applying the dominating set algorithm on the graph, one possible selection for dominating nodes is $A_3$ and $A_5$ that shown in figure 5. It is obvious that all nodes in the graph are accessible by $A_3$ and $A_5$.

### 3.3 Step3 : Random walks and making corpus

After estimation of the dominating set related to the graph, it’s time to apply random walks on the graph. Each random walk of ExEm represents a sentence that its’ words are nodes instead of vocabulary. After that, a corpus is created by concatenating these random walks. For this purpose, a starting node is selected from the dominating set, then a neighbor of it is randomly chosen, and is moved to this neighbor; the procedure is continued until the maximum length, $T$, of random walk, is achieved. The main condition for constricting these random walks is the existence of at least another dominating node. Figure 6 depicts the mechanism of making random walks in the proposed method.
Figure 3: Step2 : finding a dominating set

1. Select randomly a vertex $v \in V$
2. Let $D = \{v\}$
3. If $V - (D \cup N(D)) = \emptyset$ then stop.
4. Select randomly a vertex $w \in V - (D \cup N(D))$
5. $D \leftarrow D \cup w$
6. Repeat steps 3-5 until $V - (D \cup N(D)) = \emptyset$

Dominating nodes of graph
3.4 Step 4: learning part

In random walk models, a node and a random walk are regarded as a word and sentence, respectively. In this environment, the neighborhood of the node can be observed as the co-occurrence of words in the text. Furthermore, the skip-gram model is the most efficient way of counting the co-occurrence \[40\]. The aim of SKIP-GRAM in node representations is predicting the nodes surrounding a target node. On the other hand, the SKIP-GRAM would count the number of times node \( j \) appears within a certain window of \( w \). Mathematically, this model tries to maximize the average log probability:

\[
\frac{1}{T} \sum_{t=1}^{N} \sum_{w \leq j \leq w, j \neq 0} \log p(n_{t+j} \mid n_{t})
\]  

(2)

where \( n_1, n_2, ..., n_T \) are nodes in the corpus. For instance, in the random walks "\( n_1 n_2 n_3 n_4 n_5 \)", the input would be "\( n_3 \)" whereas the output is "\( n_1 \), "\( n_2 \), "\( n_4 \), and "\( n_5 \), assuming \( w \) is 5. All the input and output have the same dimension and are one-hot encoded. The neural network includes one hidden layer that its dimension is equal to the embedding size of node representation. The graph in figure 7 visualizes the network structure.

In this study to train the SKIP-GRAM model, we use two state-of-the-art word embedding methods, "Word2Vec" \[40\] and “fastText” \[41\]. ExEm(fastText), ExEm(Word2vec) and ExEm(fastText+Word2vec) respectively demonstrate the implementation of ExEm with “fastText”, “Word2Vec”, and combination of “fastText” and “Word2Vec”. Furthermore, we solve the optimization problem by performing Stochastic gradient descent (SGD) parameters similar to the method proposed by \[42\]. Figure 8 shows the summary form of learning part.

4 Data Description

There are some motivations for creating the labeled collaborative network dataset that we enumerate in the following. Recently, there has been an increasing interest in graph embedding. The feature representations obtained through graph embedding techniques are evaluated on learning tasks such as multi-label classification. In the multi-label classification task, one or more labels are assigned to each node. A certain fraction of nodes and all their labels are fed into the input of the graph embedding in the training phase. The task is to predict the labels for the remaining nodes \[43\]. Karate, BlogCatalog, Wikipedia, and Protein–Protein Interactions (PPI) are the most used labeled datasets to estimate the efficiency of a proposed graph embedding method. These labeled datasets are types of social networks and biology networks. To the best of our knowledge, the labeled collaborative network does not exist in order to evaluate a graph embedding approach. Moreover, it is necessary for the existence of a labeled collaboration network dataset for the usage in supervised machine learning methods in expert finding or detecting communities of experts in collaboration networks. Although, it is possible to discover datasets for the collaboration network of authors such as Arxiv AstroPhysics or Arxiv High Energy Physics Theory, but these is a demand for a labeled collaborative network dataset for the mentioned goals.
Figure 6: Step 3: random walks and making corpus

1. Define \( j = 1 \)
2. If \( j < R \) continue:
   - Initialize \( walk = \{ \} \)
   - Select a random vertex \( d \in D \)
   - Let \( walk = \{ d \} \)
   - Select a random vertex \( u \in N(d) \)
   - Add \( u \) to \( walk \)
   - If \( \text{len}(walk) \leq T_w - 2 \) continue:
     - Select a random vertex \( v \in N(u) \)
     - Add \( v \) to \( walk \)
   - If \( walk \cap D \geq 2 \) continue:
     - Take \( walk \) as the corpus
3. Increment \( j = j + 1 \)

This process continues until the desired corpus size is reached.
Figure 7: SKIP-GRAM architecture

Figure 8: Step4 : learning part
In this way, we figure out Scopus is an adequate source that consists of a wide number of authors and articles from scientific areas. Scopus is Elsevier’s abstract and citation database and covers thousands of titles, millions of author profiles and billions of cited references. It provides the required labeled collaborative dataset. Hence, we try to build a co-author network from authors’ co-authorship in the publications and assign authors’ subject areas as their labels. In other words, usage of the collected dataset can be enumerated as:

- Evaluation of graph embedding techniques performance
- Multi-label classification task
- Link prediction task
- Community detection task
- Expert finding task

What are the differences between this dataset and previous ones? There are several datasets for collaborative networks that are listed in Table 2. These datasets are investigated based on the number of nodes, the number of edges and other parameters. It is worth noting that this information is collected from the source [44]. To the best of our knowledge, there is no dataset that is gathered from Scopus. Our paper provides a vast labeled co-authorship network from Scopus by crawling authors whose labels are their subject areas.

4.1 Data gathering

Scopus allows users to do accurate searches and provides integration into other platforms using APIs. Scopus APIs exhibit curated abstracts and citation data from all scholarly journals. Further, Scopus has Python library, called Scopus, to extract data from the Scopus database. There are some limitations in order to retrieve data from Scopus using python code. One limitation is to register on Scopus and obtain API Key. The second is to be in an institution’s network. Scopus offers classes to interact with different APIs. Additionally, authors with publications indexed in Scopus have their profiles and are automatically assigned as a unique Scopus author identifier. Figure 9 shows an example of an author’s profile in Scopus. To locate an author’s profile, AuthorRetrieval(AuthorID) class is used that extracts the different types of information data about an author, including references, citations of work, h-index, subject areas, and co-authors.

1 Scopus: [https://www.scopus.com/](https://www.scopus.com/)
To form our desired collaborative network, we have chosen 20 experts from the Arnetminer expert list related to “Information Extraction” topic. After that, we have obtained these experts’ identifiers from Scopus by the employment of AuthorSearch class. Then, these authors’ information is retrieved through the AuthorRetrieval class. Our network graph is extended with a 2-hop expansion. That means we have gathered the information of co-authors of experts and the co-authors of these co-authors in the next steps. Finally, we have gathered 27473 authors’ information. It is worth noting that because the name of subject areas are long, we have used the shortened forms of them, called “Abbreviation”. The abbreviations and their full names that are obtained from our gathered dataset are listed in Table 3.
4.2 Data analysis

To analyze the network, we examine a number of classic network metrics to provide a clear understanding of the network. The definitions of these metrics are:

**Degree**: The degree of an author in a co-author network demonstrates the number of one’s co-authors [45].

**Clustering coefficient**: The clustering coefficient of a node is the fraction of its pairs of neighbors that are directly connected with each other [45, 46].

**Triangle**: A triangle is a three-node subgraph that are connected by either two or three undirected nodes [47].

**Modularity**: The quality of the detected communities is determined by its modularity. The modularity is a scalar value between -1 and 1 that measures the strength of the division of a network into communities. On the other hand, it measures the density of links inside communities as compared to links between communities [48].

The visualization can help understand and debug the graph. Hence, we have visualized our graph by Gephi, which is an open-source network analysis and visualization software [49]. The visualization of our graph dataset is presented as figure 10. In this figure each node represents an author and edges show the authors’ co-author relationship.

Dataset statistics are demonstrated in Table 4. We have gathered 27473 unique authors and 285231 connections between them. The author with id 34769751400 is the one with the highest degree, 2147. We list the top-10 authors that have the highest degrees in Table 5. Figure 11 is the visualization of authors that the larger numbers in size of the author’ identifier denotes the higher degree of the author. Additionally, Figure 12 shows the degree distribution of the network.
The degree distribution is defined as counting how many nodes have each degree \[50\]. Equation \[3\] demonstrates the definition of degree distribution.

\[
P_{\text{deg}}(K) = \text{fraction of nodes in the graph with degree } k
\]  

Additionally, figures 12b and 12c indicate the distributions of in-degree, the number of head ends adjacent to a node and out-degree, the number of the tail ends adjacent to a node, if the graph is considered as directed one. Furthermore, we calculate the eigenvector centrality distribution of our network as shown in figure 12d. Eigenvector centrality is one method for computing the influence of a node in a network. Each node's centrality is the sum of the centralities of the nodes that it is connected to. So, a high eigenvector score means that a node is connected to many nodes who themselves have high scores \[51\].

Some insight into the network structure can be obtained by throwing out information about the network except for the degrees of its nodes \[50\]. For example, the clustering coefficient that shows the tendency of authors to cluster together. The average clustering coefficient is 0.889 in our prepared dataset.

Moreover, the obtained modularity, 0.912, exposes that our network is constructed in a professional way. That means our network has dense connections between the authors within communities and sparse connections between authors in different communities. The number of detected communities is 49. Detection of communities is useful to insight the structure of complex networks, and extract useful information from them. In figure 13 nodes are colored according to the community detection result of applying the proposed method in the study \[48\]. Also the distribution of authors in

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**Table 5: Top-10 authors with the highest degree**

| Identifier       | Degree |
|------------------|--------|
| 34769751400      | 2147   |
| 57203666468      | 949    |
| 7102307605       | 896    |
| 7004756014       | 714    |
| 26642959500      | 683    |
| 7201745855       | 662    |
| 8905931300       | 659    |
| 1604391700       | 615    |
| 7005614807       | 602    |
| 7101702977       | 600    |

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(a) Degree distribution

(b) In-degree distribution

(c) Out-degree distribution

(d) Eigenvector centrality distribution

Figure 12: Computing various degree distributions on the graph.

Each community is denoted in figure 14. From this figure, it can be concluded that authors in different subject areas are connected to each other and work together as co-authors. So, it is evidence for the goodness of the collected dataset.

Finally, figure 15 shows how many authors belong to each label. It can be observed from the figure that the most number of authors have label "COMP". Authors with label "ENGI" stand in second place. Moreover, from this figure it can be concluded that the constructed network covers authors from different scientific areas. It should be noted that labels with a higher percentage of 1% are displayed in this figure.

5 Case Study

In this section, we have investigated the procedures of the proposed method on the gathered data from Scopus for recommending experts. As the first step, we have considered the experts and their abbreviations (subject areas) as nodes and their labels, respectively. Moreover, the relations between experts are defined as co-authors. Figure 10 shows the graph that is derived from Scopus after applying the preprocessing step. In the second step, the process depicted in figure 3 is applied to the graph. Figure 16 presents the dominating nodes of Scopus that are shown with green color. After finding dominating nodes, the random walks with starting from nodes of dominating sets are created with attention to the existence of another dominant node. Figure 17 is the example of the random walks with length 5. Then, the corpus is fed as input into the SKIP-GRAM model of FastText and Word2Vec. The result is the low dimensional vectors of experts, called expert embedding. By these vectors, it is possible to predict the label sets of experts through analyzing training experts with known label sets. Therefore, all experts are with labels. As we know an expert recommendation system takes a user’s query in the term of input and then provides a list of experts sorted by the degree of their relevant expertise with the given query. Figure 18 introduces our proposed method for computing experts’ scores by using
Figure 13: Visualization of the detected communities based on the algorithm proposed in [48]

Figure 14: Distribution of authors in each community

Figure 15: Percentage of authors related to each label
the expert embedding vectors in order to make recommendation experts. The user’s query, that is a topic, is injected into the input of the system. Then, experts whose labels include this topic are extracted and make a cluster. Subsequently, we compute the center of this cluster by taking the mean of all the expert embedding vectors in the group. Finally, similarity measure functions such as Euclidean, Cosine, and Manhattan can be employed to calculate the distance between each expert and the centroid. This similarity is considered as an expert’s score.
Figure 18: The process of proposed method for computing experts’ scores

\[ c_j = \frac{\sum l = 1^* u_i}{n} \]

\[ S(u_i) = sim(u_i, c_j) \]
Table 6: Summary descriptions of datasets

| Name            | $|V|$  | $|E|$  | Labels | Tasks          |
|-----------------|------|------|-------|--------|---------------|
| BlogCatalog     | 10,312 | 333,983 | 39    | ✓✓     | -             |
| PPI             | 3890 | 76,584 | 50    | ✓✓     | -             |
| Wikipedia       | 4777 | 184,812 | 40    | ✓       | -             |
| arXiv(Astro-PH) | 18772 | 396,160 | -     | -✓✓     | -             |
| Scopus          | 27,473 | 285,231 | 27    | -       | ✓             |

6 Experimental Evaluation

In the present section, we provide an overview of the datasets which we used for different tasks. Next, we specify tasks to evaluate our proposed algorithm. We also introduce a number of baseline algorithms in order to compare our method against them. Finally, we explain the used parameter settings.

6.1 Dataset

In the following, we list and describe the datasets that we conduct experiments on them.

- **BlogCatalog** [52]: This is a labeled network of social relationships that nodes demonstrate the bloggers and edges show the friendship connection among the bloggers. The labels of nodes represent blogger interests.
- **Protein–Protein interactions(PPI)** [53]: This is a subgraph of the biological network. In this graph, nodes are proteins and edges indicate the pairwise physical interactions between proteins. The labels of nodes are obtained from the gene sets.
- **Wikipedia** [54]: This is a network of co-occurrence words related to Wikipedia’s articles. The labels are shown the Part-of-Speech (POS) tags.
- **arXiv(Astro-PH)** [55]: This is a collaboration network that is constructed from the collaborations between authors’ papers submitted to the e-print arXiv and Astro Physics category. On the other hand, nodes are authors and edges contain the co-authored relationships between nodes.

We also evaluate the performance of our algorithm on Scopus as described in section 4. The descriptions of the datasets are summarized in Table 6. The task shows the usage of datasets. The details of tasks are represented in 6.2 subsection.

6.2 Tasks

- **Multi-label classification**: One of the tasks that increasingly used by modern applications is multi-label classification. In this task, nodes in a graph are associated with a set of target labels [56].
- **Link prediction**: The low-dimensional vectors of nodes encode rich information about the network structure and have application in various information processing tasks such as link prediction [6]. The task of link prediction is to propose missing links between two nodes in the incomplete graph.
- **Recommendation**: Node recommendation is the task of recommending top nodes of interest to a given query according to certain specifications [6]. The types of recommended nodes are miscellaneous; in this paper, recommended nodes are experts who their research interests and expertise are most similar to a given query.

6.3 Baseline algorithms

To approve the performance of Expert2ve we compare it against the following baselines algorithms:

- **DeepWalk** [15]: DeepWalk represents a graph as a set of simple unbiased random walks and predicts the local neighborhood of nodes via these random walks. Hierarchical softmax with a binary-tree structure operates over the full vertex set to approximate the normalizing factor [57].
- **Node2vec** [58]: Node2vec is the extended version of DeepWalk with a more elaborate random walk. It insinuates breadth-first sampling (BFS) and depth-first sampling (DFS) for biased random walks. It is noted that negative sampling is applied to a set of random samples to compute the normalizing factor.
6.4 Parameter settings

The proposed algorithm is implemented in Python 3.7 and the experiments are conducted on a Corei7 system with Ubuntu OS. The parameters of our experiments have been set as follows: the dimensional node embedding, \( E_d = 128 \), length of random walks \( L_R = 80 \) and number of random walks \( R = 10 \). It should be noted that the dimension of embedding related to ExEm(fastText+Word2Vec) is 256. We select the best values for \( p \) and \( q \) in Node2vec from \( 0/25, 0/5, 1, 2, 4 \) as proposed in [58].

7 Evaluation Results

In this section, the experimental results of different tasks under different dataset are reported and analyzed.

7.1 Number of dominating nodes

As mentioned before, we applied the dominating set algorithm, described in figure 3, to the preprocessed networks in order to find their dominating nodes. Figure 19 graphically illustrates the number of dominating nodes for the different datasets. Moreover, it can be concluded that the number of dominant nodes approximately increases with the number of the network nodes and it nearly obeys a linear equation.

7.2 Probability of one more dominating node

We start each sequence with a dominating node, as there should be one more dominating node to fulfill the condition, and the probability of equation 4 indicates the presence of less than two dominating nodes. It is equivalent to tossing a coin 79 times and seeing no heads, or in other words, we toss a coin 79 times and all of them are tails. That means we do not ingestive on existing another dominating node in each random walk in order to achieve the least time complexity. Table 5 gives the probability of no more than two dominating nodes in random walks related to the corpus of different datasets. These values are obtained according to equation 5.

\[
P(\text{No More Than Two Dominating Nodes}) = \prod_{i=1}^{79} P_i(\text{Not Dominating At Trial}) = \\
\prod_{i=1}^{79} \frac{\text{Dominating Set Size}}{\text{All dataset nodes}} \approx \frac{1}{2^{79}}
\]

\[
P(\text{No More Than Two Dominating Nodes}) = \frac{1}{2^{79}} \times \text{number of random walks}
\]
Table 7: The probability of no more than two dominating nodes in random walks related to the corpus of different datasets

| Dataset            | Probability             |
|--------------------|-------------------------|
| BlogCatalog        | $8.18 \times e^{(-19)}$ |
| PPI                | $2.81 \times e^{(-19)}$ |
| Wikipedia          | $2.64 \times e^{(-19)}$ |
| arXiv(Astro-PH)    | $9.37 \times e^{(-19)}$ |
| Scopus             | $1.24 \times e^{(-19)}$ |
| Instagram          | $1.79 \times e^{(-19)}$ |

7.3 Multi-label classification

A good node embedding can give the network structure as input and predicts the node labels. That means a multi-label classification task is one of the ways for evaluating the performance of a graph embedding approach. Hence, we compare the effectiveness of ExEm method with the two other relevant approaches under the multi-label classification task. For this purpose, we use a one-vs-rest logistic regression for classification. Moreover, for the train part, we randomly select 10% to 90% of nodes as training data and evaluate the achievements on the remaining nodes.

Figure 20 presents the results of Micro-F1 and Macro-F1 scores for different approaches under BlogCatalog, Scopus, PPI and Wikipedia. From the results, we can see how the using dominating set theory allows ExEm to outperform the other benchmark algorithms.

For the BlogCatalog dataset, ExEm presents significant improvements over Micro-F1. DeepWalk and Node2vec do not provide good performance because they only need local information means the adjacent neighbors. On the other hand, ExEm uses intermediate information consisting of the adjacent neighbors and their neighbors [59]. It can also be concluded that ExEm(Word2vec) achieves the lowest results than ExEm(fastText) and ExEm(fastText+Word2vec). The reason is that ExEm(fastText+Word2vec) represents more features of nodes because it combines the embedding vectors of two other models and this provides more features. Though ExEm(fastText) can construct the vector for a node if the node does not appear in the training corpus.

In the case of the Scopus network, as the results present, both evaluation metrics have more value than the BlogCatalog network because the Scopus network has the highest density in comparison to three other datasets. ExEm has significant improvements in the Macro-F1 score over Node2vec and DeepWalk. For Micro-F1 value, ExEm(fastText+Word2vec) and ExEm(fastText) display better results, respectively. When training data is 80% and 90%, Node2vec obtains higher consequences than ExEm(Word2vec). Moreover, DeepWalk gives weak outputs in the dataset.

For Wikipedia dataset, the results indicate that ExEm outperformed baseline algorithms, considering both Micro and Macro-F1. The reason is that a virtual backbone is formed by the induced hierarchy of dominating set can significantly control the structure of networks. While DeepWalk and Node2vec suffer from local structure information. It is important to note that the Wikipedia dataset is denser than BlogCatalog and PPI [8].

PPI has the lowest density among the other datasets. In this case, the achievements of ExEm are improved for both Micro-F1 and Macro-F1 scores.

Finally, it can be concluded that our proposed method has significant improvement for networks with high density. That means the results of multi-label classification indicate how effective are our graph embedding approach, especially for higher density networks. In these networks, one of the efficient ways of retrieving information is finding the dominating set and using it as a backbone [60].

7.4 Link prediction

For this task, we consider different popular binary operators to provide an edge representation for node pair $(u, v)$. These operators are defined by the following equations [8, 58, 61]:

$$
Average = \frac{V(u)_i + V(v)_j}{2} \tag{6}
$$

$$
Hadamard = V(u)_i \times V(v)_j \tag{7}
$$
Figure 20: Performance comparison of 5 methods (DeepWalk, Node2Vec, ExEm(Word2vec), ExEm(fastText), and ExEm(fastText+Word2vec)) by considering their achieved values of Micro-F1 and Macro-F1 for various datasets under different train-test split ratio for multi-label classification task (dimension of embedding is 128)
Table 8: Area Under Curve scores of different algorithm for link prediction. (a)Average, (b)Hadamard, (c)Weighted–L1, and (d)Weighted–L2

| Op     | Algorithm                      | Dataset       | PPI     | arXiv(Astro-PH) |
|--------|--------------------------------|---------------|---------|-----------------|
|        | Common Neighbors                | PPI           | 0.7142  | 0.8153          |
|        | Jaccard’s Coefficient           | arXiv         | 0.7018  | 0.8067          |
|        | Adamic-Adar                     | PPI           | 0.7126  | 0.8315          |
|        | Pref. Attachment                | arXiv         | 0.6670  | 0.6996          |
|        | DeepWalk                        | PPI           | 0.6923  | 0.7066          |
|        | LINE                            | PPI           | 0.6330  | 0.6516          |
|        | Node2vec                        | arXiv         | 0.7944  | 0.9340          |
| (a)    | ExEm(fastText)                  | PPI           | 0.9116  | 0.9803          |
|        | ExEm(Word2vec)                  | arXiv         | 0.9104  | 0.9788          |
|        | ExEm(fastText+Word2vec)         | arXiv         | 0.9038  | 0.9798          |
|        | DeepWalk                        | arXiv         | 0.7249  | 0.8902          |
|        | LINE                            | arXiv         | 0.7192  | 0.9366          |
|        | Node2vec                        | arXiv         | 0.9614  | 0.9862          |
| (b)    | ExEm(fastText)                  | arXiv         | 0.9590  | 0.9874          |
|        | ExEm(Word2vec)                  | arXiv         | 0.9609  | 0.9872          |
|        | ExEm(fastText+Word2vec)         | arXiv         | 0.9689  | 0.9848          |
|        | DeepWalk                        | arXiv         | 0.6292  | 0.8468          |
|        | LINE                            | arXiv         | 0.6292  | 0.8468          |
|        | Node2vec                        | arXiv         | 0.9642  | 0.9877          |
| (c)    | ExEm(fastText)                  | arXiv         | 0.9656  | 0.9873          |
|        | ExEm(Word2vec)                  | arXiv         | 0.9689  | 0.9848          |
|        | ExEm(fastText+Word2vec)         | arXiv         | 0.9730  | 0.9875          |
|        | DeepWalk                        | arXiv         | 0.6236  | 0.8477          |
|        | LINE                            | arXiv         | 0.7106  | 0.8862          |
|        | Node2vec                        | arXiv         | 0.6236  | 0.8477          |
| (d)    | ExEm(fastText)                  | arXiv         | 0.9715  | 0.9876          |
|        | ExEm(Word2vec)                  | arXiv         | 0.9691  | 0.9805          |
|        | ExEm(fastText+Word2vec)         | arXiv         | 0.9730  | 0.9875          |

\[
\text{Weighted} \quad L1 = |V(u)_i - V(v)_j|
\]  

\[
\text{Weighted} \quad L2 = |V(u)_i - V(v)_j|^2
\]

As investigated in the study \[62\], link prediction can be addressed as a binary classification problem. In this case, the class label of each node pair is positive if a directed link exists between the node pair. On the other hand, if there is no link between the node pair, then the label of the paired node is negative. Hence, the link prediction problem is treated as an optimization problem that its objective function is the \(\text{AUC}\) score. \(\text{AUC}\) score is the most common evaluation metric to evaluate the accuracy of the prediction in the link prediction task. The larger \(\text{AUC}\) score is, the higher the probability that there is a connection between node \(u\) and node \(v\) for the pair of nodes \((u, v)\) \[62\].

In this study, we use the Logistic Regression classifier to predict the existence or non-existence of edges related to the network. We randomly hide 50% of the network edges for each dataset in order to test the performance of different embedding methods on this task. Moreover, the dimension of embedding for all approaches except ExEm(fastText+Word2vec) is 128. We have compared ExEm against well-known baselines that have the best results on link prediction. Table 8 shows the summarized result of the \(\text{AUC}\) score for different methods on link prediction. According to these results, ExEm reveals a better performance in comparison with the other approaches on both datasets. The structure of random walks in ExEm, which is explained in Section 3, allows it effectively monitor the network by help of dominating nodes. On the other hand, efficient information can be achieved through dominating set construction from social networks. Though DeepWalk and Node2vec are random walk based methods their walks do not provide enough information about nodes. It should be considered that ExEm(fastText) and ExEm(fastText+Word2vec) have better results than ExEm(Word2vec) in most cases.

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Table 9: Area Under Curve scores of Scopus dataset for different operations

| Algorithm                | Average | Hadamard | Weighted-L1 | Weighted-L2 |
|--------------------------|---------|----------|-------------|-------------|
| DeepWalk                 | 0.7551  | 0.9398   | 0.8818      | 0.8904      |
| Node2vec0                | 0.8034  | 0.9424   | 0.9005      | 0.9079      |
| ExEm(Word2vec)           | 0.9907  | 0.9936   | 0.9917      | 0.9923      |
| ExEm(fastText)           | 0.9909  | 0.9974   | 0.9923      | 0.9929      |
| ExEm(fastText+Word2Vec)  | 0.9915  | 0.9933   | 0.9925      | 0.9933      |

Figure 21: The value of F1-score which is achieved by ExEm along with the other two related approaches on Scopus dataset for recommendation task for three topics IE, NLP and ML.

Additionally, the detailed $AUC$ score achieved by DeepWalk, Node2vec and ExEm for the Scopus dataset is exhibited in table 9. According to this table, considering all methods, the $AUC$ score has reached higher values in the Hadamard operation. Moreover,

7.5 Recommendation

As described earlier in this paper, a novel strategy is introduced for computing experts’ scores using the expert embedding vectors and recommending experts whose scores are maximum. In this case study, first, we select three topics: information extraction (IE), natural language processing (NLP), and machine learning (ML) from Arnetminer as query input. Moreover, the related people lists of these topics are used to construct the ground truth for the evaluation of the recommendation task on the Scopus dataset. Also, cosine similarity measures the similarity between the embedding vectors of expert and centroid and finds the nearest experts to the centroid. The score of each expert is determined by the value of this similarity.

As it is clear, the result of the $F1-score$ shown in figure 21 demonstrates the better performance of the proposed method. Moreover, ExEm(fastText+Word2vec) has gained the highest values among all of the competitors. ExEm(Word2vec) takes the second-ranking position and provides better performance in comparison to ExEm(fastText) in the recommendation task. DeepWalk and Node2vec represent similar achievements but the lowest F1-score compared with ExEm.

In order to evaluate the performance of our proposed method for determining experts’ scores, we compare it against PageRank, one of the algorithm that is used by many researches to calculate experts’ score in the graph. PageRank counts the number and quality of links to a node to determine a rough estimate of how important the node is. The result of their comparisons are reported in figure 22. It is clear that in three topics, ExEm outperformed the others. The reason is that Pagerank only considers the local information that contains the degree of nodes. On the other hand, ExEm regards immediate information including the degree of nodes and their neighbors.

Now, it is time to answer the questions that asked in section 1:

- **A1.** The analyzes in section 4.2 prove the advantage of our collected dataset for different usages. The value of the clustering coefficient shows the efficiency of Scopus data for clustering task. The modularity value demonstrates that this dataset can be used for community detection. Moreover, the results of experiments in
Figure 22: The value of F1-score which is achieved by ExEm along with the other two related approaches on Scopus dataset for recommendation task for three topics IE, NLP and M.

section of [7] is an important evidence for usefulness of the prepared data for multi-label classification, link prediction and expert recommendation tasks.

• A2. The results related to the experiments of different tasks demonstrate that using dominating set theory brings about embedding vectors which show the best performance in diverse tasks. The reason is that dominating nodes are the backbone of the graph and know more information about their neighbors. On the other hand, they have intermediate knowledge concerning the graph. Thus, this causes to create intelligent random walks and better learn features of nodes.

• A3. As explained in section [5], we proposed a novel strategy for computing experts’ scores using the expert embedding vectors and recommending experts with the highest scores. From another point of view, the designed structure can calculate the scores of experts with the vectors extracted from various graph embedding techniques. Also, it can be applied to any type of graph with a special example of the graph related to the relationship between questioner and answerer in QAC’s such as StackOverflow and Quara.

8 Conclusion

In this paper, a graph theory-based method has been proposed for graph embedding with a special focus on recommending experts in collaborative networks. This method has been called ExEm. For implementing the ExEm, first, a dataset is preprocessed. In this process, nodes of the dataset are recognized and their topics are retrieved and are assigned to them as their labels. In the next step, the dominating set theory is applied to the graph and dominating nodes of the graph are found. Then, random walks are created with considering this point that these walks start with dominating nodes and comprise of at least another dominating node. After constructing the slightly walks, they are considered as input of the neural network which is the final part of the presented ExEm method. Word2Vec, fastText and their combination are employed to train the neural network of the SKIP-GRAM model.

To evaluate the performance of the ExEm method experiments were conducted on 4 real-life datasets. Furthermore, the DeepWalk and Node2Vec approaches were used in order to compare them with the ExEm method. Considering the efficiency of the embedding vectors in the experiments of multi-label classification, link prediction and recommendation tasks, ExEm performs better than the other 2 relevant methods. The reason is that dominating nodes are considered in the process of ExEm method. Dominating nodes are regarded as the backbone of the networks and can control the hole networks. They provide intermediate information about the network including the nodes and their neighbors. Also, this research represented a new strategy for calculating experts’ scores using expert embedding vectors.

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