Estimates Long-term Data of Multi-State Model

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Abstract. This study aims to obtain an estimate of transition opportunities from the health status of elderly persons. The phenomenon of changes in health status in the elderly has a chance of transition at each shift in their health status. To obtain an accurate estimate of the transition probability in a multi-status model of health status is indispensable. We estimate the four-state multi-status models from alive/healthy through out-patient sickness, in-patient sickness to death with recovery from either of the two cases of sickness will be used based on medical data elderly. The focus will be on deriving the probabilities of transition from one state to another via the forces of intensities. We then provide a method for estimating unique transition probabilities for both males and females elderly, generally behaved as expected with transition probabilities to sickness states increasing with age. Transition probabilities out of the sick state appear higher for males than females.

1. Introduction

Due to demographic changes, the proportion of elderly patients is constantly increasing in many countries. In the coming decades, Indonesia will face a sharp increase in the number of older people, due to the existence of the large baby-boom until 2045. Simultaneously, the number of older people is growing rapidly as a result of increasing longevity. Although different population situations for the government may project different future populations, all scenarios show significant increases in the numbers of older people [1]. The extent to which this demand will increase depends on the future health status of the elderly population. If the average health status improves, long-term care needs may increase to a lesser extent than the number of older people. On the other hand, if inability increases strongly at very old ages, increasing life expectancies may lead to additional increases in the demand for care.

The health status of the elderly is closely related to nursing innovations. Improvements in health care which part of long-term care may result in better survival. The two main methods used to calculate the future number of disabled elderly and the probability of health transition rates [2]. Many authors have analyzed the estimation problems used by multistate models. Much of this work has drawn on the theory of stochastic processes to obtain new results of interest and to generalize the results of more traditional methods. Such models are most tractable when it is assumed that the process satisfies the Markov property [3]. Under this assumption, the generalizations of many standard results from life contingencies can be done [4]. The expected value and variance of multistate in a Markov model set has been modeled before [5].
The estimation of the intensity of the transition or probability is one of the most important things from the application of the Markov model and also the semi-Markov model for health insurance that requires data where the transition between the status of the population is collected, classification of calculation methods based on the format of statistical data, for example, methods based on healthy probabilities become ill and methods based on the average time spent in sickness[6].

Multi-status model models are built by one-way two-state Markov chain processes from life to death estimated using transitional force and it is a model used for continuous-time stochastic processes in which a person can be moved to a certain limited status that can be developed by adding many statuses that have transition opportunity values. The multistate estimating basis is the estimation transition probabilities [7]. One structure of the simple multistate model is the sick-death model (Health-illness-death model) with 3 statuses, namely "healthy", "sick", and "dead", as below.

![Figure 1. Three state of multi-state sickness model](image)

In general, a multi-status stochastic process uses the intensity of this transition as a major component in the formation of the model and is a bridge that connects the raw data into formulations that exist in the multi-status model such as transition and sedentary opportunities. However, this approach gives rise to a high level of non-linear form if the model is associated with the influence of the covariate, that is, things that might influence the transition opportunity[8].

Aalen proposed an estimator for the integrated hazard under a broad class of counting process models, it is obtained an estimator for the transition probability matrix and subsequently state occupation probabilities [9]. Datta and Satten [10] established that the resulting estimators of state occupation probabilities remained valid even when the process is non-Markovian by proposed an estimator for state occupation probabilities that can handle state-dependent censoring and other flexible models through a weighting function based on the censoring scheme. Estimation of state entry and exit distribution functions are also of interest and can be calculated through normalized sums of state occupation probabilities [11].

2. Method

This research focuses on the estimation of probabilities transition from one state to another using a four-state Markov model which involves the healthy, nursing type one, nursing type two and nursing type three. Then the parametric estimation of the maximum likelihood estimates of the forces of transition \( \mu \), from one state to another, the general case, then the properties of the maximum likelihood estimators, the alternative derivation of the maximum likelihood estimates. The parametric graduation methods of the Nelson-Aalen estimator with their logarithmic modification including to get a finer estimate that using by the Kernel method.

3. Result and discussion

Assume that multistate evolution can be represented as a sample path of a time-continuous, inhomogeneous Markov chain with a finite state space [12]. We denote transition probabilities and transition intensities, respectively, by

\[
P_q(t) = \Pr(X_{t+1} = s_{t+1} = j | X_t = s_t = i) \text{ ................. (1)}
\]
\[ \mu_k(t) = \sum_{j \neq i} \lim_{u \to t} \frac{p_{ij}(t,u)}{u-t} \]
\[ = \lim_{u \to t} \sum_{j \neq i} \frac{p_{ij}(t,u)}{u-t} \] .................................(2)

From the development of the three state Markov sickness model a multistate model will be considered consisting of the following statuses:

State 1: Healthy elderly condition
State 2: Mild pain elderly condition
State 3: Severe elderly condition
State 4: Death

By Chapman –Kolmogorov equation,

\[ p_{ij}(s,t) = \sum_k p_{ik}(s,t) p_{kj}(t,t+h) \] .................................(3)

In this case we shall use:

\[ p_{ik}(s,t+h) = \sum_{j=1}^{j=3} p_{ik}(s,t) p_{kj}(t,t+1) \text{ for } j = 1,2,3,.... \]

\[ p_{i1}(s,t+h) = p_{i1}(s,t) p_{11}(t,t+h) + p_{i2}(s,t) p_{21}(t,t+h) + p_{i3}(s,t) p_{31}(t,t+h) \]

\[ = p_{i1}(s,t) \left[ 1 - (\mu_{12} + \mu_{13} + \mu_{14}) h + O(h) \right] + p_{i2}(s,t) \{0\} + p_{i3}(s,t) \{\mu_{i3} h + O(h)\} \]

\[ + p_{i2}(s,t) \{\mu_{i2} h + O(h)\} \]

\[ \therefore \frac{d}{dt} p_{i1}(s,t) = p_{i1}(s,t) \left[ -(\mu_{12} + \mu_{13} + \mu_{14}) \right] + p_{i2}(s,t) 0 + p_{i3}(s,t) \{\mu_{i3}\} + p_{i4}(s,t) \mu_{i1} \] .................................(4)

\[ p_{i2}(s,t+h) = p_{i1}(s,t) p_{21}(t,t+h) + p_{i2}(s,t) p_{22}(t,t+h) + p_{i3}(s,t) p_{32}(t,t+h) \]

\[ p_{i4}(s,t) p_{42}(t,t+h) \] .................................(5)
\[ p_{14}(s,t+h) = p_{14}(s,t) p_{14}(t,t+h) + p_{12}(s,t) p_{23}(t,t+h) + p_{13}(s,t) p_{34}(t,t+h) \]

\[ = p_{14}(s,t) \left[ \mu_{14}h + O(h) \right] + p_{12}(s,t) \mu_0 + p_{13}(s,t) \left[ \mu_{34}h + O(h) \right] \]

\[ + p_{14}(s,t) \left[ 1 - \left\{ \mu_{41} + \mu_{42} + \mu_{43} \right\} h + O(h) \right] \]

\[ \therefore p'(s,t) = p(s,t)Q \]...............(6)

Where

\[ Q = \begin{bmatrix}
- (\mu_{12} + \mu_{13} + \mu_{14}) & \mu_{12} & \mu_{13} & \mu_{14} \\
0 & 0 & 0 & 0 \\
\mu_{31} & \mu_{32} & - (\mu_{31} + \mu_{32} + \mu_{34}) & \mu_{34} \\
\mu_{41} & \mu_{42} & \mu_{43} & - (\mu_{41} + \mu_{42} + \mu_{43})
\end{bmatrix} \]

Time \( a_i \) and leaves at time \( b_i \) if he has not died by this time[13]. Let us make the simplifying assumption that \( \mu_i = \mu, a \leq t \leq b \), where \( a = \inf_{i \leq s \leq a} a_i, b := \sup_{i \leq s \leq a} b_i \). We wish to estimate the one and only unknown parameter of the model, namely \( \mu \).

First of all, we know that \( T_i > a_i \). So it is best to consider each of the variables \( T_i \) which in distribution, equals \( T_i - a_i \) given that \( T_i > a_i \). We observe that random variables \( X_i = T_i \mid \delta_i \) and \( \delta_i = I(T_i \leq l_i) \) where \( l_i := b_i - a_i \).

Suppose that \( N_{ij}(t) \) is the total number of transitions from status \( i \) to \( j \), at intervals [0, \( t \)] and for example also states the number of individuals in the current status \( i \), then the estimated intensity of the transition from status \( i \) to \( j \), based on Nelson-Aalen, is:

\[ \hat{A}_{ij}(t) = \int_{0}^{t} \frac{I(Y_i(s) \neq 0)}{Y_i(s)} dN_{ij}(s) \].........................(7)

\[ I(Y_i(s) \neq 0) = \begin{cases} i & Y_i(s) \neq 0 \\
0 & Y_i(s) = 0 \end{cases} \].........................(8)

Based on the classification of the age group of observations from 135 data, counted the number of nursing home residents who make the transition from state 1 to state 2, state 1 to death, from state 2 care to death, as well as from state 1 to state 1 and from state 2 care to state 3, obtained:

| Table 1. The number of transitions by age interval and type of transition. |
|---|---|---|---|---|---|---|---|
| Ages | state 1 to state 2 | state 1 to state 3 | state 2 to state 1 | state 2 to state 4 | state 3 to state 2 | state 3 to state 4 |
| 65-70 | 12 | 3 | 2 | 2 | 1 | 1 | 2 |
| 71-75 | 21 | 4 | 1 | 1 | 0 | 1 | 2 |
| 76-80 | 19 | 6 | 3 | 2 | 1 | 2 | 1 |
| 81-85 | 13 | 5 | 3 | 0 | 2 | 1 | 2 |
| 86-90 | 10 | 3 | 2 | 1 | 2 | 1 | 3 |
The parameter estimates for graduating transition intensities for each age interval for both males and females are presented in Table 2.

| Age    | State | Male        | Female       |
|--------|-------|-------------|--------------|
|        | 1     | 2           | 3            | 4             | 1   | 2   | 3   | 4 |
| 65-70  | 1     | 0.7182      | 0.0243       | 0.5170        | 0.0163 | 0.0556 | 0.0014 | 0.1031 | 0.0662 |
|        | 2     | 0.1000      | 0.1122       | 0.5212        | 0.0050 | 0.8093 | 0.8384 | 0.0294 | 0.1606 |
|        | 3     | 0.0365      | 0.4935       | 0.3112        | 0.6561 | 0.0054 | 0.1481 | 0.0605 | 0.5876 |
| 71-75  | 1     | 0.8433      | 0.1231       | 0.0303        | 0.0219 | 0.7434 | 0.2367 | 0.0084 | 0.0217 |
|        | 2     | 0.2811      | 0.7186       | 0.0808        | 0.0521 | 0.2374 | 0.6638 | 0.0794 | 0.0198 |
|        | 3     | 0.0388      | 0.2985       | 0.5178        | 0.1231 | 0.0279 | 0.2985 | 0.6039 | 0.5300 |
| 76-80  | 1     | 0.6785      | 0.1067       | 0.0243        | 0.0416 | 0.7182 | 0.1067 | 0.0720 | 0.0178 |
|        | 2     | 0.2666      | 0.5229       | 0.1122        | 0.0300 | 0.1000 | 0.5229 | 0.0556 | 0.0306 |
|        | 3     | 0.0234      | 0.2921       | 0.4935        | 0.1963 | 0.0365 | 0.2921 | 0.8093 | 0.1031 |
| 81-85  | 1     | 0.6086      | 0.5170       | 0.0010        | 0.0728 | 0.5877 | 0.5170 | 0.0054 | 0.0294 |
|        | 2     | 0.1639      | 0.5212       | 0.1820        | 0.0458 | 0.1473 | 0.5739 | 0.2809 | 0.0605 |
|        | 3     | 0.0050      | 0.3112       | 0.5044        | 0.2228 | 0.0014 | 0.2048 | 0.6969 | 0.1137 |
| 86-90  | 1     | 0.6561      | 0.9823       | 0.0662        | 0.0634 | 0.8384 | 0.4017 | 0.0133 | 0.0398 |
|        | 2     | 0.0812      | 0.7682       | 0.1606        | 0.1103 | 0.1481 | 0.8870 | 0.5930 | 0.0620 |
|        | 3     | 0.0199      | 0.2222       | 0.5876        | 0.1763 | 0.0192 | 0.1088 | 0.5383 | 0.1518 |

Table 2. shows the one-year health state transition probability matrix of different ages and genders. The samples are first grouped by age and gender, and the individuals of each group are classified in accordance with the state definition standard. Then, the states of individuals at the end of the period are tracked in each category so as to calculate the ratio of the number of people in each state at the end of the period to that at the beginning of the period.

4. Conclusion
Based on the studies, the research concludes that the transition intensities or forces of transition and probabilities for both males and females generally behaved as expected with transition probabilities to sickness states increasing with age. Transition probabilities out of the sick state appear higher for males than females. Transition out of the healthy/able state to outpatient and inpatient sickness states appears higher for females than males. Furthermore, mortality in the in-patient sickness state is higher for males than females.

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