Effect of phase noise on two-angle DoA estimation using planar arrays

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Abstract. The effect of phase noise on the accuracy of the two-angle direction of arrival (DoA) is investigated. For this purpose, an efficient technique that depends on the correlation between the signals received by the 2D array and its steering matrix is proposed to estimate the DoA. A new model of the phase noise is proposed to get the consequent phase error in the received signals. The effect of the phase noise power and the array size on the accuracy of the estimated two-angle DoA is investigated. The average error in the estimation of the two-angles DoA is found to increase by increasing the power of the phase noise and by decreasing the array size. The numerical investigations are performed through electromagnetic simulation of a multiple plane waves incident on a 2D planar array of printed patch antennas.

1. Introduction

The DoA estimation is one of the key issues in the discipline of array signal processing and finds applications in the fields of radar, sonar, wireless positioning applications, seismology etc. [1]. Many studies in the research work focus on the effect of the additive noise on the accuracy of DoA estimation through the relevant processing algorithms. Among the most popular DoA estimation algorithms, the maximum likelihood (ML) [2], multiple signal classification (MUSIC) [3], and estimation of signal parameters via rotational invariance techniques [4] are commonly used algorithms which accounted for the effect additive white Gaussian noise on the DoA estimation. A DoA estimation algorithm with a model for impulsive noise with an alpha-stable distribution has been reported in [5]. In [6], a modified iterative ML estimation algorithm is proposed to account for non-stationary noise. The present work proposes a new efficient processing algorithm for DoA estimation and, in the same time, investigates the effect of the phase noise on the accuracy of the two-angle DoA considering the arrival of multiple signals. The phase noise is characterized by its single side band power spectral density (PSD) which is defined over an offset frequency band from the carrier frequency [7]. Also, the present work proposes a new mathematical model of the phase noise (of a specific PSD) that enables the calculation of the phase error of the signals received at the array elements. The algorithm proposed for DoA estimation depends on the determination of the locations of the peaks appearing in a function of the correlation matrix elements when the angles of arrival of a received signal match those of the steering matrix. The efficiency of the proposed DoA estimation technique as well as the effect of the phase noise on the accuracy of the retrieved angles of arrival of multiple signals are investigated using planar 2D arrays of point sources as well as printed microstrip patch antennas. The Patch antenna is chosen because of its light weight, small size, good parameters performance and easy array assembling fabrications.
2. Formulation of the problem
This section is divided into two sections. The first section introduces the DoA estimation algorithm and the second section provides the mathematical model of phase noise.

2.1. PCAMA Algorithm for DoA Estimation
A robust processing algorithm is applied to estimate the two-angles DoA of multiple incoming signals using the received signals at the array elements of a planar array. The algorithm depends on the fact that the correlation matrix between the steering matrix of the array and the matrix representing the received signals at the array elements will have its main diagonal elements nearly pure real when the steering matrix is set to get the same direction of maximum radiation of the array as the direction of arrival of one of the received signals. This algorithm is called PCAMA which stands for peaks of the correlation at matched angles.

The steering matrix for $M \times N$ planar array whose elements are isotropic point sources can be written in the form

$$
S(\theta, \phi) = \left[ s_{m,n}(\theta, \phi) \right]_{m=1,2,\ldots,M \atop n=1,2,\ldots,N} = \begin{pmatrix}
  e^{j\psi_{1,1}} & e^{j\psi_{1,2}} & \cdots & e^{j\psi_{1,N}} \\
  e^{j\psi_{2,1}} & e^{j\psi_{2,2}} & \cdots & e^{j\psi_{2,N}} \\
  \vdots & \vdots & \ddots & \vdots \\
  e^{j\psi_{M,1}} & e^{j\psi_{M,2}} & \cdots & e^{j\psi_{M,N}} 
\end{pmatrix},
$$

where $\psi_{m,n}$ is the phase of excitation of the array element on the $m^{th}$ row and $n^{th}$ column; this can be expressed as

$$
\psi_{m,n} = (m-1)\beta_x + (n-1)\beta_y, \quad m = 1,2,\ldots,M, \quad n = 1,2,\ldots,N,
$$

where $\beta_x$, $\beta_y$ are the progressive phase shifts of the array element excitations across the array rows and columns respectively, which can be expressed as

$$
\beta_x(\theta, \phi) = k_o \Delta x \sin \theta \cos \phi, \\
\beta_y(\theta, \phi) = k_o \Delta y \sin \theta \sin \phi,
$$

where $k_o$ is the free space wave number, $\Delta x$ and $\Delta y$ are the spacing between the array elements rows and columns, respectively.

For $p$ incoming signals, let the received voltage due to the $p^{th}$ incoming signal under the effect of phase noise be,

$$
X_p = \alpha_p S_p e^{j\phi_e}, \quad p = 1,2,\ldots,P,
$$

where $\alpha_p$ is constant proportional to the strength of the $p^{th}$ arriving signal, $S_p$ is the steering matrix, and $\phi_e$ is the error in phase of the received signal due to the phase noise.

Consider the following correlation matrix

$$
R_p(\theta, \phi) = E\{S(\theta, \phi)X_p^H\}, \quad p = 1,2,\ldots,P,
$$

where the operator $E\{\cdot\}$ means the expected or mean value of the operand, $S(\theta, \phi)$ is the steering matrix at $(\theta, \phi)$ direction, $H$ is the hermitian operator.

It is clear that in the absence of noise ($\phi_e = 0$), the main diagonal elements of the matrix $R_p(\theta, \phi)$ are approximately pure real at $\theta = \theta_p$ and $\phi = \phi_p$. For an antenna array of size $N \times M$, the $R_p$ matrix can be written as,
\[ \mathbf{R}_p(\theta, \phi) = \left[ r_{p_{m,n}}(\theta, \phi) \right]_{m=1,2,\ldots,M, n=1,2,\ldots,N} \quad p = 1, 2, \ldots, P \]  

(6)

At low noise to carrier power ratio (NCR) or high signal to noise ratio (SNR), each of the complex quantities \( r_{p_{m,n}}(\theta, \phi) \) has its imaginary part tending to zero as the pair \( (\theta, \phi) \) approaches \( (\theta_p, \phi_p) \). Hence, the following quantity is important and can be used to detect the directions of arrival of multiple signals.

\[ d_p(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} \frac{|\text{imag}(r_{p_{m,n}})|}{|r_{p_{m,n}}|^4}, \quad m = n \]  

(7)

Consequently, the peaks of the following function, when plotted against \( \theta \) and \( \phi \) in the half-space domain, \( 0 < \theta < \frac{\pi}{2}, \quad 0 < \phi < 2\pi \), give the two angles determining the DoA for all the incoming signals.

\[ P_{\text{CAMA}}(\theta, \phi) = \sum_{p=1}^{P} \frac{1}{d_p(\theta, \phi)}, \quad p = 1, 2, \ldots, P, \]  

(8)

### 2.2. Modelling the phase noise

Investigating the effect of the phase error encountered due to the phase noise is of particular importance as the techniques of DoA estimation depend on the correctness and accuracy of the phases of the signals received at the array elements. The phase noise in the frequency domain is commonly characterized by the single side band PSD, \( L(f) \), which is defined as the noise power in 1 Hz bandwidth at an offset frequency, \( f \), from the carrier frequency relative to the carrier power [7, 8].

\[ L(f) = \frac{\text{Noise power in 1Hz bandwidth}}{P_r}, \]  

(9)

where \( P_r \) is the carrier power.

Let the signal \( s(t) \) received at the port of an antenna element of the array be expressed as

\[ s(t) = V_o e^{i(2\pi f_r t + \psi_{m,n} + \varphi_e(t))}, \]  

(10)

where \( V_o \) is the received signal amplitude, \( f_r \) is the received signal frequency, \( \psi_{m,n} \) is the phase due to the path travelled by the signal received at the element \( (m,n) \) of the 2D array, and \( \varphi_e(t) \) is the unknown instantaneous value of the phase error due to the phase noise. The discrete frequency domain samples \( S_k \) of the received signal are constructed as,

\[ S_k = S(f_k) = A_k e^{i\psi_k}, \quad k = 1, 2, \ldots, K \]  

(11)

where the discrete phases \( \psi_k \) are generated as a sequence of uniformly distributed random numbers in the closed interval \([ -\pi, \pi] \) and the discrete magnitudes \( A_k \) are calculated from the PSD of the phase noise as follows,

\[ A_k = \sqrt{P_r L(f_k)}, \]  

(12)

where \( P_r \) is the power of the received signal. The Discrete Inverse Fourier Transform (DIFT) is then applied to get the time samples of the noisy received signal,

\[ s(t) = \text{DIFT}(S_k) \]  

(13)

The corresponding instantaneous phase error, \( \varphi_e(t) \) can be calculated as,
\[
\varphi_e(t) = \tan^{-1}\left(\frac{\text{imag}(s(t))}{\text{real}(s(t))}\right) - (2\pi f_c t + \psi_{m,n})
\] (14)

3. Results and discussions

In this section, the PCAMA technique is applied to estimate the DoA of one incoming wave using the signals received at the elements of a planar array composed of 5 × 5 isotropic point sources assuming the carrier frequency is 1 GHz. The separation between the elements is in both x and y directions is 15 mm. For the same array, the PCAMA technique is applied to estimate the two-angles of arrival (\(\theta, \phi\)) under the effect of phase noise with a specific PSD. The effect of the power of the phase noise on the resulting errors of the estimated two angles of arrival (\(\theta, \phi\)) is then investigated for different array sizes. Since the isotropic point source is a hypothetical source that doesn’t exist in real life and in order to simulate a realistic array, a printed array of 8 × 8 microstrip patches is used to estimate the two-angle DoA under the effect of phase noise for multiple signals coming from different directions. For this purpose, electromagnetic simulation is performed using the CST® commercially available software package. It should be noted that, in the following discussions the actual angles of arrival are designated as (\(\theta, \phi\)) whereas their estimated values are designated as (\(\hat{\theta}, \hat{\phi}\)); and the estimation error is absolute the difference between the actual and estimated values.

3.1. Estimation of DoA using PCAMA Technique

A planar array of 5 × 5 point sources with an interelement spacing of \(\lambda/2\) operating at 1 GHz is used to detect the DoA of an incoming signal from the direction defined by \(\theta = 25^\circ, \phi = 210^\circ\). In the absence of the phase noise, the function \(P_{\text{CAM}}(\theta, \phi)\), is calculated using (5) and is plotted as shown in Figure 1. It is clear in the figure that the two angles of arrival are exactly retrieved at \(\hat{\theta} = 25^\circ, \hat{\phi} = 210^\circ\). It should be noted that the function \(P_{\text{CAM}}(\theta, \phi)\) is normalized to its maximum value for convenience of presentation.

![Figure 1](image1.png)

**Figure 1.** Normalized \(P_{\text{CAM}}(\theta, \phi)\) for an incoming signal on an array of 5 × 5 elements of point sources from the direction \(\theta = 25^\circ, \phi = 210^\circ\).

3.2. Estimation of DoA under the Effect of Phase Noise

The effect of phase noise on the error in the estimation of DoA is examined in the presence of phase noise with PSD as that presented in Figure 2a. The calculated phase noise samples \(\varphi_e(t)\) are plotted in Figure 2b. Phase noise is added to the received signals of the elements of an array of 5 × 5 point sources described in section 3.1. The incoming signal direction is \(\theta = 25^\circ, \phi = 210^\circ\). The ensemble average of the error in \(\theta\) and \(\phi\) is calculated due to an ensemble of 100 phase noise samples separated in time by interval of 10 ms and the results are shown in Figure 3. Under the effect of a phase noise whose single side band PSD is that shown in Figure 2, the plot of the function \(P_{\text{CAM}}(\theta, \phi)\) is obtained.
and presented in Figure 4, where the function $P_{CAMA}(\theta, \phi)$ is normalized to its maximum value. The peak of the $P_{CAMA}(\theta, \phi)$ function occurs at $\hat{\theta} = 28^\circ, \hat{\phi} = 215^\circ$.

Figure 2. Phase Noise characteristics (a) $L(f)$ (b) Corresponding phase error time samples.

Figure 3. Ensemble average of the average error in (a) $\theta$ and (b) $\phi$, for 100 phase error samples of the phase noise PSD shown in Fig. 2.

Figure 4. Normalized PCAMA for an incoming signal from direction $\theta = 25^\circ, \phi = 210^\circ$.

3.3. Dependence of DoA Estimation Accuracy on the Array Size under the Effect of Phase Noise

The average errors of the estimated angles of arrival ($\hat{\theta}, \hat{\phi}$) due to the effect of phase noise are investigated with increasing the NCR for different sizes of the receiving antenna array. It is assumed that one signal is arriving at the receiving array from the direction defined by $\theta = 30^\circ, \phi = 50^\circ$. In Figure 5a and 5b the average error in $\theta$ and $\phi$ angles, respectively, is plotted versus the NCR for array sizes $3 \times 3$, $5 \times 5$, $7 \times 7$, and $10 \times 10$. It is clear in the two figures that the average error in
estimating the incoming signal direction increases with increasing the NCR. On the other hand, increasing the array size has the effect of decreasing the average errors of the estimated angles of arrival.

![Graph showing average error in (a) θ and (b) φ, with increasing the NCR for different sizes of the receiving array.](image)

**Figure 5.** Average error in (a) θ and (b) φ, with increasing the NCR for different sizes of the receiving array.

### 3.4. Patch Antenna Array for the DoA detection of Multiple Signals

The array of isotropic point sources is hypothetical and cannot be implemented. In this section, a printed planar array of square patch antennas placed above a ground plane is used to achieve more practical investigation of the efficiency of the PCAMA technique to estimate the two-angle DoA of multiple signals under the effect of phase noise. The commercially available CST® electromagnetic simulation package is used for this purpose. An 8 × 8 planar array of probe feed square patches printed on FR4 substrate is placed in the x-y plane. The substrate thickness is 1.5 mm. The patch side length is 20 mm and the spacing between elements in both x- and y- directions is 40 mm. The operating frequency is 3.5 GHz, the array configuration is shown in Figure 6a. The steering matrix for the array is calculated in the range 0° < θ < 45° and 0° < φ < 360° such that the side lobe level is ≤ −13 dB, and the gain ≥ 18 dB. The patch antenna array beam steering is shown in Figure 6b.

![Patch antenna array configuration and beam steering.](image)

**Figure 6.** Patch antenna (a) array configuration, (b) beam steering.

The steering matrix is saved for post processing. For electromagnetic simulation, four plane waves are assumed incident on the microstrip patch array and coming from different directions as listed in table 1. In real case, the incoming signals will be affected by different phase noise power. Thus, the estimated angles of arrival of the incoming waves are obtained for different values of the NCR as
listed in table 1. Figure 7 shows the normalized PCAMA for four incoming signals, each has a different noise to carrier power ratio.

| Incoming Signal Direction | NCR (dB) | Estimated DoA |
|---------------------------|----------|---------------|
| $\theta = 20^\circ, \phi = 50^\circ$ | -30      | $\hat{\theta} = 20^\circ, \hat{\phi} = 50^\circ$ |
| $\theta = 45^\circ, \phi = 100^\circ$ | -16      | $\hat{\theta} = 44^\circ, \hat{\phi} = 99^\circ$ |
| $\theta = 5^\circ, \phi = 300^\circ$ | -10      | $\hat{\theta} = 4^\circ, \hat{\phi} = 298^\circ$ |
| $\theta = 25^\circ, \phi = 200^\circ$ | -7       | $\hat{\theta} = 25^\circ, \hat{\phi} = 204^\circ$ |

**Figure 7.** Normalized PCAMA of four incoming signals on a planar array of patch antennas.

4. Conclusions
An efficient processing algorithm for estimating the DoA using planar arrays is investigated. A mathematical model for phase noise is introduced to study the effect of phase noise on the error in DoA estimation. The error in detecting the DoA is found to decrease by increasing the array size and by decreasing the phase noise power. A realistic array of 8x8 patch antennas is considered to accurately detect the two-angle direction of multiple incoming signals from different direction in the presence of phase noise.

5. References
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