Static three quark potential in the quenched lattice QCD

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We study the static three quark potential using lattice QCD simulation. At zero temperature, the three quark potential is extracted from the baryonic Wilson loop and fitted to the phenomenological form, the sum of the Coulomb term, linearly rising term and the constant. We compare two types of the linear term, “\(Y\)” and “\(\Delta\)” type, and find that the string tension almost coincide with \(\sigma_{\bar{Q}Q}\) in the former case. We also investigate the three quark potential at finite temperature using the Polyakov loop correlation. In the deconfined phase, the screened potential is observed similarly to the case of \(Q\bar{Q}\) system.

1. Introduction

Determination of the static quark-antiquark (\(Q\bar{Q}\)) potential has been one of most vital subjects of the lattice QCD simulation \cite{1}. On the other hand, the status of the three quark (3\(Q\)) potential is much less settled, in spite of its importance in the baryon spectroscopy. In this paper, we present our recent result of lattice calculation of the static three quark potential \cite{2,3}.

It has been widely used the static \(Q\bar{Q}\) potential expressed as the sum of the short range Coulomb term and long range confining linear term. In the case of the 3\(Q\) potential, two distinct treatments of the linear term are frequently applied. In this paper they are denoted as “\(\Delta\)”-type and “\(Y\)”-type forms. The former expresses the linear potential as the sum of two-body potentials, which are proportional to the distance between each pair of quarks. The latter treats the potential with the string linking three quarks at the point with which the total length of the string takes the minimum length, \(L_{\text{min}}\). \(Y\)-type ansatz has been used in the phenomenological studies with the string tension \(\sigma_{3Q} \simeq \sigma_{\bar{Q}Q}\), and successfully describes the gross feature of the baryon spectrum \cite{4}. \(\Delta\)-type ansatz is, however, also used in some model calculations with the string tension \(\sigma_{\Delta}\) effectively around half of \(\sigma_{\bar{Q}Q}\). The lattice simulations have been done mainly in 1980’s \cite{5,6,7}, and there is one recent preliminary work \cite{8}. Among them, several authors interpreted their results as the support of the \(\Delta\)-type picture \cite{5,6,7}. This embarrassing situation is mainly caused by the insufficient extraction of the ground state contribution from the baryonic Wilson loop. In nineties, calculation of \(Q\bar{Q}\) potential became much reliable by making use of the smearing technique which enhances the ground state contribution to the Wilson loop \cite{9}. This procedure is directly applicable to the three quark system, and is expected to enable us reliable extraction of the ground state contribution.

Our first goal of this work is an extraction of reliable static 3\(Q\) potential using the smearing technique. With the lattice result, we discuss preferable description of the potential. These subjects are described in the next section.

Another important subject on the 3\(Q\) potential is the thermal effect on it at finite temperature. Again the lattice computation is much retarded compared with the \(Q\bar{Q}\) potential. We present our preliminary result at \(T > T_c\) in Section 3.

\textsuperscript{*}talk presented by H.Matsufuru at Lattice 2000, Bangalore, India.
The lattice spacing is determined as $a = \beta$ action at a size $12^3$, including translational invariance.

The same constellation is averaged making use of the three static quarks on each three spatial axis. The contracting point as the spatial origin, and locate not depend on this point. We choose the contracting point as the spatial origin, and locate three static quarks on each three spatial axis. The same constellation is averaged making use of the translational invariance.

The numerical simulation is done on the lattice of size $12^3 \times 24$, with the standard Wilson gauge action at $\beta = 5.7$ in the quenched approximation. The lattice spacing is determined as $a \simeq 0.19$ fm by setting the string tension of $Q$-$\bar{Q}$ potential to be $\sigma = 0.89$ GeV/fm. The (baryonic) Wilson loop is measured on 210 configurations with applying the smearing technique to enhance the ground state contribution. The potentials are extracted from the (baryonic) Wilson loop by the single exponential fit in appropriate fit regions, where the Wilson loop is dominated by the ground state contribution.

Let us start with the $Q$-$\bar{Q}$ potential. It is widely accepted that the lattice result of the static $Q$-$\bar{Q}$ potential is well described by the form

$$V(r) = C - \frac{A}{r} + \sigma r.$$  

In Table 1, we list the results of the fit in three kinds of fit ranges: on-axis fit, fits with off-axis data in the whole $r$ region and at $r \geq 2$. Since our lattice spacing is rather coarse, the fit with the off-axis data may suffer from the rotational symmetry breaking effect. In this sense, the fit with the on-axis data is preferable to determine the parameters. For the $3Q$ potential, however, we inevitably treat the off-axis constellations, and then it would be meaningful to examine the typical size of the rotational symmetry breaking effect in the $Q$-$\bar{Q}$ system. For this purpose, we carry out the fit including the off-axis data with two fit ranges.

The fit using all data (all $r$ in Table 1) results in large $\chi^2$ value, and this is mainly caused by the result at the short distance region where the statistical error is smallest and the cutoff effect is most severe. We alternatively fit at $r \geq 2$, which is more appropriate for comparison with the result of the $3Q$ potential. In this case the fit results in $\chi^2/N_{df} = 3.3$, and we regard this value as a guide of the rotational symmetry breaking effect on this lattice. In both cases, the fit result of the Coulomb coefficient much differs from the value of on-axis fit than the case of the string tension.

Figure 1 shows the lattice result of the $3Q$ potential from the viewpoint of the $Y$-type ansatz. The horizontal axis, $L_{min}$, is the minimum length of the flux lines linking three quarks. Three flux connecting the junction with quarks take the angles $2\pi/3$, which balances three forces with equal strength, in the case that all three angles of the “quark triangle” are less than $2\pi/3$. The data are fitted to the form

$$V(r_1, r_2, r_3) = C_Y - A_Y \left( \frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{31}} \right) + \sigma_Y L_{min}.$$  

The result is listed in Table 1. This form of the potential well describes the observed result, and the string tension is almost same as in the case of $Q$-$\bar{Q}$ potential. The $\chi^2$ of the fit is in the same order as in $Q$-$\bar{Q}$ with fit range $r \geq 2$. Considering the rotational symmetry violation effect at this $\beta$, this seems to be a consistent result.

Alternative ansatz, $\Delta$-type, is represented with the fitting function

$$V(r_1, r_2, r_3) = C_\Delta - A_\Delta \left( \frac{1}{r_{12}} + \frac{1}{r_{23}} + \frac{1}{r_{31}} \right)$$

Figure 2 shows the lattice result of the $3Q$ potential from the viewpoint of the $Y$-type ansatz. The horizontal axis, $L_{min}$, is the minimum length of the flux lines linking three quarks. Three flux connecting the junction with quarks take the angles $2\pi/3$, which balances three forces with equal strength, in the case that all three angles of the “quark triangle” are less than $2\pi/3$. The data are fitted to the form

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Fit results of the $Q$-$ar{Q}$ and the $3Q$ potentials.

Table 1

| $C$          | $A$          | $\sigma$ | $\chi^2/N_{df}$ |
|--------------|--------------|----------|-----------------|
| $3Q(Y)$      | 0.914(20)    | 0.1345(62) | 0.1528(20) | 4.0 |
| $3Q(\Delta)$ | 0.934(20)    | 0.1405(60) | 0.0858(15) | 11. |

on-axis 0.629(16) 0.2793(12) 0.1629(47) 0.59
all r 0.686(7) 0.3351(55) 0.1545(20) 16.
r \geq 2 0.696(24) 0.395(35) 0.1561(38) 3.3

Figure 2. $3Q$ potential from the viewpoint of Y-type ansatz. $L_{min}$ is the minimum length of
the flux linking three quarks.

result at $\beta = 5.7$.

3. Finite temperature result

Now we turn on the temperature and observe how the potential changes with the thermal effect.
In this paper, we focus on the static $3Q$ potential in the deconfinement phase. Above the critical
temperature $T_c$, the quarks and the gluons are expected to be liberated and form a plasma
state, in which the potential is expected to be screened. Then one expects that the screened potential
takes the Yukawa potential like as the $Q$-$\bar{Q}$ potential, and genuine three-body potential
(in Y-type picture) would disappear.

On the finite temperature lattice, the static potential is extracted from the Polyakov loop

\[ P_3(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \langle P(\vec{r}_1) P(\vec{r}_2) P(\vec{r}_3) \rangle = e \cdot \exp(-V_{3Q}(\vec{r}_1, \vec{r}_2, \vec{r}_3) N_T), \]

where the Polyakov loop $P(\vec{x})$ is defined as $P(\vec{x}) = \text{Tr} \prod_{t=0}^{N_t-1} U_4(\vec{x}, t)$. The $Q$-$\bar{Q}$ potential
is also measured in similar way.

To simulate the finite temperature system, we adopt the anisotropic lattice. An advantage
of the anisotropic lattice for our study is that it enables us to work out several values of temperature
without varying the lattice cutoff. As the gauge field action, we use the Symanzik improved action at the tree level.

The numerical calculation is done on the lat-
tices of sizes $16^2 \times 24 \times N_t$ at $\beta = 4.56$ and $\gamma_G = 3.45$, in the quenched approximation. At $N_t = 96$, which is realized as the zero temperature lattice, we determine the renormalized anisotropy as $\xi = 3.95(2)$ from the Wilson loops in the spatial-spatial and temporal-spatial planes. The static $Q$-$Q$ potential at $T = 0$ gives the cutoffs $a_{v-1} = 1.61(1)$ GeV and $a_{v-1} = 6.36(5)$ GeV. The critical temperature is estimated by the rapid change of the Polyakov loop and its susceptibility, and is found between $N_t = 24$ and 25, and slightly near to $N_t = 24$. For the investigation of the static potentials in the deconfined phase, we generate 60 configurations at $N_t = 20$, which roughly corresponds to $T \simeq 1.22T_c$.

Figure 3 shows the $Q$-$Q$ potential and the $3Q$ potential in the deconfined phase. At the long distance, The Polyakov loop correlators approach to the multiples of the expectation value of single Polyakov loop: $P_2 \rightarrow \langle P \rangle (P)^*$ and $P_3 \rightarrow \langle P \rangle^3$. These contributions are subtracted from the potentials. The $Q$-$Q$ potential above $T_c$ is known to behave as the Yukawa-type potential [10]. In present calculation, the Debye screening mass of the $Q$-$Q$ potential strongly depends on the fit range and is estimated around 0.5 – 1 GeV. At this $N_t$, we have measured the 3Q potential with only equilateral constellations. In Figure 3, we show the 3Q potential divided by 3, and find good agreement with the $Q$-$Q$ potential,

$$V_{3Q}(r_{12} = r_{23} = r_{31} = r) \simeq 3V_{QQ}(r) \quad (T > T_c). \quad (7)$$

Above $T_c$, since the quarks are deconfined, there is no reason to assume that the combination of quarks are in the color-singlet states. The question is whether the 3Q potential can be represented as the sum of two body potential. Our present result shows that the $V_{3Q}/3$ behaves as almost same as the $Q$-$Q$ potential at least for the equilateral $3Q$ constellations. For more definite conclusion, we need precise results including other $3Q$ constellations and a systematic comparison between $V_{3Q}$ and $V_{QQ}$ at various $T$.

The calculations have been done on NEC SX-4 at Research Center for Nuclear Physics, Osaka University and NEC HSP at INSAM (Institute for Nonlinear Science and Applied Mathematics), Hiroshima University.

REFERENCES

1. K. Schilling, Nucl. Phys. B (Proc. Suppl.) 83-84 (2000) 140, and references therein.
2. T.T. Takahashi, H. Matsufuru, Y. Nemoto and H. Suganuma, hep-lat/0006007.
3. H. Matsufuru, Y. Nemoto, H. Suganuma, T.T. Takahashi and T.Umeda, Proc. of "Quantum Chromodynamics and Color Confinement", in press.
4. S. Capstick and N. Isgur, Phys. Rev. D 34 (1986) 2809.
5. R. Sommer and J. Wosiek, Phys. Lett. B 149 (1984) 497; Nucl. Phys. B 267 (1986) 531.
6. J. Kamesberger, G. Eder, M.E. Faber, H. Leeb and H. Markum, Proc. of “Few-Body Problems in Particle, Nuclear, Atomic and Molecular Physics”(1987) 529.
7. H.B. Thacker, E. Eichten and J.C. Sexton, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 234.
8. G.S. Bali, hep-lat/0001312.
9. APE Collaboration (M.Albanese et. al.), Phys. Lett. B 192 (1987) 163.
10. M. Gao, Phys. Rev. D 41 (1990) 626.
11. F. Karsch, Nucl. Phys. B 205 (1982) 285.
12. K. Symanzik, Nucl. Phys. B226 (1983) 187.