Stacking non-BPS D-Branes

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Abstract

We present a candidate supergravity solution for a stacked configuration of stable non-BPS D-branes in Type II string theory compactified on $T^4/Z_2$. This gives a supergravity description of nonabelian tachyon condensation on the brane worldvolume.
I. INTRODUCTION

The seminal work of Sen [1–3] underlined the importance of understanding stable non-BPS branes in string theory. Non-BPS branes provide us with new ways to construct stable non-supersymmetric states in string theory, which ultimately may lead to realistic brane world models as exact solutions. It seems likely they will lead to new insights into the outstanding phenomenological difficulties of string theory. Review articles on non-BPS branes may be found in [4].

In this paper we consider a class of non-BPS branes with an integer valued conserved charge [5,6] that arise from non-BPS branes on a $T^4/Z_2$ orbifold of Type II string theory. The $Z_2$ symmetry acts by reflection in the four toroidal directions combined with $(-1)^{F_L}$ ($F_L$ is the left-moving fermion number). The orbifolding removes the open string tachyons, making the configuration stable. The brane is charged with respect to a twisted sector Ramond-Ramond gauge field of the orbifolded Type II string theory. States that only carry charge under this gauge field will not be supersymmetric, as there is no associated central charge.

If the radii of the torus are fixed to a particular value the branes do not interact at first order in perturbation theory and there is a no-force condition similar to that of the BPS case [7]. One expects there to exist a classical supergravity solution when a large number of such branes are stacked on top of each other. Studies of this supergravity solution appear in [8–13]. Related supergravity solutions for coinciding branes and anti-branes have appeared in [12–13]. In particular, [8–14] construct supergravity solutions with the appropriate gauge charge, but with naked singularities. These solutions violate the scalar no-hair theorems of general relativity. One therefore expects such solutions will be unstable. This was confirmed in [8], where a repulsive force was found to act on a probe non-BPS brane in their classical background solution.

Starting with the six-dimensional supergravity action arising from Type II string theory on $K3$, we find solutions compatible with the no-hair theorems with the same conserved charge as the configuration of non-BPS branes. These are argued to be the classical solution corresponding to the stable ground state of the coinciding non-BPS branes. The solutions are six-dimensional analogues of the well known black p-branes (see for example [14–17]). The solutions we construct are free of naked singularities.

In [18] the worldvolume theory of coinciding stable non-BPS branes has been studied. Although individual branes are tachyon-free, non-abelian tachyons appear when branes are
coincident. It was argued there should be a stable ground state for such a configuration of branes, where these tachyons condense at a nontrivial minimum. The supergravity solution we construct will presumably correspond to this stable minimum. The tension to charge ratio we find is a factor of $\sqrt{2}$ larger than one would expect starting from the perturbative string theory result $[5,6]$. This yields a nontrivial prediction for the strongly coupled groundstate of the worldvolume theory of the non-BPS branes.

II. GENERAL BLACK BRANE SOLUTIONS

In this section we review the construction of general black brane solutions in arbitrary dimensions (see, for example, [12]). The starting point for the description of a brane charged with respect to one gauge field in a D-dimensional spacetime is the action

$$S = \int d^Dx \sqrt{-g} \left[ R - \frac{1}{2} \nabla^M \phi \nabla_M \phi - \frac{1}{2(p+2)} e^{a\phi} F_{[p+2]}^2 \right], \quad (2.1)$$

where $x^M, M = 0, \ldots, D - 1$ are the spacetime coordinates, $g$ is the determinant of the metric, $R$ the Ricci scalar, $\phi$ is the dilaton field, $F_{[p+2]}$ is the $(p+2)$-form field strength of the $(p+1)$-form gauge potential $A_{[p+1]}$, that is $F_{[p+2]} = dA_{[p+1]}$ and $a$ is a constant governing the coupling between the dilaton and the gauge potential. In general the effective action contains the metric tensor, various ranks of antisymmetric tensor fields and many scalars. However it can be shown that (2.1) is a consistent truncation of the whole action, so that the solutions of the equations of motion obtained by varying this action are particular solutions of the full theory. The equations of motion obtained by varying the above action can be expressed as

$$R_{MN} = \frac{1}{2} \nabla_M \phi \nabla_N \phi + S_{MN}$$

$$\nabla_M \left( e^{a\phi} F^{MM_1\ldots M_{p+1}} \right) = 0$$

$$\Box \phi = \frac{a}{2(p+2)!} e^{a\phi} F^2 , \quad (2.2)$$

with

$$S_{MN} = \frac{1}{2(p+1)!} e^{a\phi} \left( F_{MM_1\ldots M_{p+1}} F^{MM_1\ldots M_{p+1}} - \frac{p+1}{(p+2)(D-2)} F^2 g_{MN} \right). \quad (2.3)$$

Looking for the general black p-brane solutions of these equations we do not require Poincaré
invariance on the $d \equiv p + 1$ world-volume of the brane. We look instead for the most general solutions that preserve the symmetry
\[ S = SO(p) \times SO(D - p) , \tag{2.4} \]
that is a rotational symmetry in the spatial directions transverse to the brane and on the brane. We can split the spacetime coordinates as $X^M = (x^\mu, y^m)$ where $x^\mu$ ($\mu = 0, ..., d - 1$) are the spatial coordinates on the brane worldvolume and $y^m$ ($m = d, ..., D - 1$) are the coordinates on the transverse spacetime directions. The ansatz for the metric, given the symmetry (2.4), is
\[ ds^2 = e^{2A(\rho)} \left( -f(\rho)dt^2 + \eta_{\alpha\beta} dx^\alpha dx^\beta \right) + e^{2B(\rho)} \left( \delta_{mn} dy^m dy^n \right) , \tag{2.5} \]
where $\alpha = 1, ..., d - 1$ and $\rho \equiv \sqrt{y^m y^m}$ is the isotropic radial coordinate in the transverse space. The ansatz for the dilaton is simply $\phi(x^M) = \phi(\rho)$ and that for the gauge potential, for the case of an electric brane, is
\[ A_{\mu_1\mu_2...\mu_d} = e^{A(\rho)} \epsilon_{\mu_1\mu_2...\mu_d} , \text{ other components zero} , \tag{2.6} \]
where $\epsilon_{\mu_1\mu_2...\mu_d}$ is the usual antisymmetric tensor ($\epsilon_{01...d} = 1$). The equivalent ansatz in terms of the field strength is
\[ F_{m\mu_1\mu_2...\mu_d} = \epsilon_{m\mu_1\mu_2...\mu_d} \partial_m e^{A(\rho)} , \text{ other components zero} . \tag{2.7} \]
The general solution with this ansatz is constructed in [12]. If we require an asymptotically flat metric and a regular horizon, we find the solutions:
\[ ds^2 = -\left[ 1 - \left( \frac{r_+}{r} \right)^d \right] \left[ 1 - \left( \frac{r_-}{r} \right)^d \right] \frac{d^4}{\Delta(d-2)} \frac{1}{d!} dt^2 + \left[ 1 - \left( \frac{r_-}{r} \right)^d \right] \frac{d^4}{\Delta(d-2)} \frac{1}{d!} dx^\alpha dx_\alpha \]
\[ + \left[ 1 - \left( \frac{r_+}{r} \right)^d \right] \frac{1}{d!} dr^2 + \left[ 1 - \left( \frac{r_-}{r} \right)^d \right] \frac{2a^2}{\Delta} d\Omega_{d+1}^2 \]
\[ e^\phi = \left[ 1 - \left( \frac{r_-}{r} \right)^d \right] \frac{2a}{d} , \]
\[ F = \lambda \ast \epsilon_{D-p-2} , \tag{2.8} \]
where $r_\pm$ are the positions of the inner and outer horizons, $\epsilon_n$ is the volume form on the unit $n$ sphere, $\tilde{d} = D - p - 3$,
\[ \Delta = \frac{2dd}{D-2} + a^2 , \tag{2.9} \]
and the charge $\lambda$ is

$$\lambda = \frac{2\tilde{d}}{\sqrt{\Delta}}(r_+-r_-)^\frac{\tilde{d}}{2}. \quad (2.10)$$

In writing this solution we have changed to Schwarzschild coordinates via $\rho^d = r^d - r_-^d$. In components the field strength is

$$F_{\mu_1...\mu_{n-1}} = -\frac{2}{\sqrt{\Delta}}\epsilon_{\mu_1...\mu_{n-1}}\partial_m \left(\frac{r_-}{r}\right)^\tilde{d}. \quad (2.11)$$

In the extremal limit $r_+ = r_-$. These are the expressions for the metric tensor, the dilaton and the field strength for a black p-brane in a D-dimensional spacetime. We will show that these are the relevant solutions for the problem we are discussing.

**III. NON-BPS BRANE SOLUTIONS OF TYPE II SUPERGRAVITY ON $T^4/Z_2$**

The starting point for the analysis of the particular case we are addressing, is the action coming from a truncation of the ten-dimensional type II supergravity theory compactified on an orbifolded torus $T^4/Z_2$. We will be interested in black p-brane solutions related to non-BPS branes. We therefore take $p$ even for Type IIB, and $p$ odd for Type IIA. Actually we will examine a truncation of the full six-dimensional gravity theory describing the metric, the gauge potential and five scalars as discussed in [8]. The truncated action in Einstein frame is

$$S = \int d^6x \sqrt{-g} \left[ R - \frac{1}{2} (\nabla \phi)^2 - \frac{1}{2} (\nabla \eta_b)^2 - \frac{1}{2(p+2)!} \epsilon^{a\phi} F_{[p+2]}^2 \right], \quad (3.1)$$

where $g$ is the determinant of the six dimensional metric, $\phi$ is the dilaton field, $\eta_b$ with $b = 1, ..., 4$ are scalar fields associated with the size of the $T^4$ and $F_{[p+2]}$ is the $(p+2)$-form field strength for a $(p+1)$-form gauge potential $A_{[p+1]}$. Here $a = (1-p)/\sqrt{2}$.

Varying this action one obtains equations of motion with the structure of (2.2) with additional terms involving derivatives of the scalars $\eta_b$, and the additional equations of motion

$$\Box \eta_b = 0. \quad (3.2)$$

\footnote{In order to recover the conventions of appendix B of [8] we must renormalize $\varphi = \phi/\sqrt{2}$, along with a compensating rescaling of $a$.}
First, we note that the scalars $\eta_b$ have an equation of motion that can be integrated once immediately. No hair theorems show that the only scalar field which approaches a constant at infinity and is regular at the event horizon of a black brane solution is everywhere constant [19]. Otherwise the solution will have a naked singularity, and in line with the cosmic censorship conjecture, it is expected such a solution will generically be unstable. As we are looking for a non-singular solution, or at least a solution with singularities hidden by a horizon, we can set the $\eta_b$ to constants. They then decouple from the other equations of motion. This is an important step toward the solution of the problem, since we are now left with the equations of motion examined in the first section, that leads to the black brane solutions for a six-dimensional spacetime. Note that the $\eta_b$ are non-constant for the elementary non-BPS brane [9]. We therefore propose that once the nonabelian open-string tachyons on the worldvolume of the branes have condensed to a stable minimum, the $\eta_b$’s decouple from the solution. The critical radius of the elementary non-BPS brane, below which it becomes unstable, corresponds to $\eta_b = 0$.

We can carry over the results of the previous section, identifying $D = 6$ and $\Delta = 2$, which leads to $\tilde{d} = 3 - p$. We will focus on the cases $p < 3$. Using the results of the previous section, we can straightforwardly write down the solutions

$$ ds^2 = -\left[1 - \left(\frac{r_+}{r}\right)^{3-p}\right]\left[1 - \left(\frac{r_-}{r}\right)^{3-p}\right]^{(1-p)/2} dt^2 + \left[1 - \left(\frac{r_+}{r}\right)^{3-p}\right]^{3-p/2} dx^\alpha dx_\alpha $$

$$ + \left[1 - \left(\frac{r_-}{r}\right)^{3-p}\right]^{-1}\left[1 - \left(\frac{r_-}{r}\right)^{3-p}\right]^{2(2-p)/3} \frac{r^2 - \tilde{r}_+^2}{2(3-p)} dr^2 + r^2 \left[1 - \left(\frac{r_-}{r}\right)^{3-p}\right]^{(1-p)^2/2} d\Omega_{5-d}^2 $$

$$ e^\phi = \left[1 - \left(\frac{r_-}{r}\right)^{3-p}\right]^{\frac{(p-1)}{\sqrt{2}}} $$

$$ F = \lambda \epsilon_{5-d}, \quad (3.3) $$

and $\lambda = \sqrt{2}(3 - p)(r_+r_-)^{(3-p)/2}$.

This solution depends on only two parameters $r_+$ and $r_-$ which are related to the mass and gauge charge, consistent with the no hair theorems. We can explicitly see that the singularity located at $r_-$ is hidden by an horizon at $r_+$. At this stage this is a black brane solution of the six-dimensional theory. The Hawking temperature of these solutions is

$$ T = \frac{3 - p}{4\pi r_+} \left[1 - \left(\frac{r_-}{r_+}\right)^{3-p}\right]^{\frac{p-2}{p-3}}. \quad (3.4) $$
This leads to a positive specific heat in the extremal limit for $p = 0, 1$, and a negative specific heat for $p = 2$. We therefore expect the black membrane will suffer from a Gregory-Laflamme instability \[20\], but the black D-particle ($p = 0$) solution and the black D-string should be stable.

We can take the extremal limit of the solution (3.3), by setting $r_+ = r_- \equiv r_0$

$$ds^2 = -\left[1 - \left(\frac{r_0}{r}\right)^{3-p}\right]^{\frac{3-p}{2}}\left(dt^2 + dx^\alpha dx_\alpha\right)$$

$$+ \left[1 - \left(\frac{r_0}{r}\right)^{3-p}\right]^{\frac{3-p+2p-11}{2(3-p)}}\left(dr^2 + \left[1 \left(\frac{r_0}{r}\right)^{3-p}\right]^2 r^2 d\Omega^2_{5-d}\right)$$

$$e^\phi = \left[1 - \left(\frac{r_0}{r}\right)^{3-p}\right]^{\frac{3-p}{2}}\frac{2}{r^2}$$

$$F = \lambda \epsilon_{5-d}. \quad (3.5)$$

The extremal solution is Poincaré invariant along the D-brane worldvolume as expected. The unstable modes of \[20\] become degenerate in the extremal limit, and entropy arguments suggest the extremal state should be stable \[21\]. This solution will be our candidate for the stable end-point of tachyon condensation on the worldvolume of coincident non-BPS D-branes. The solution has a singular horizon in the extremal limit (rather than a naked singularity), which is typical of dilatonic BPS solutions. Taking the extremal limit in the expression (3.4) for the cases $p = 0, 1$ one can see that the temperature goes to zero, whereas in the case $p = 2$ it tends to a the finite value $1/4\pi r_0$.

We can compute the mass and charge of the solution by examining the asymptotic behavior of the fields at infinity:

$$g_{00} \sim -\left(1 - \frac{3-p}{2} \left(\frac{r_0}{r}\right)^{3-p}\right)$$

$$A_{0\ldots p} \sim \sqrt{2} \left(1 - \left(\frac{r_0}{r}\right)^{3-p}\right) . \quad (3.6)$$

If we demand the charge matches that of $N$ coincident branes, then comparing to \[8\], we find a tension larger by a factor $\sqrt{2}$ than that of an elementary non-BPS brane. Recall the elementary non-BPS brane has the same tension as a BPS D-brane, but two units of charge \[9\]. We interpret the change in the tension of the stacked branes as a result of tachyon condensation on the worldvolume. Because the solution is non-supersymmetric, we also expect renormalization of the tension versus the perturbative result.

The Born-Infeld action for a static D $p$-brane probe in this background takes the form
\[ S = -M \int d^{p+1} \xi \ e^{-a_0/2 - \sum_b \eta_b/2} \sqrt{-\det g_{\alpha\beta}} + Q \int A_{p+1}. \] (3.7)

Where \( M \) is the probe tension, and \( Q \) is proportional to the probe charge. Inserting the extremal background solution, and setting the \( \eta_b = 0 \) corresponding to the critical radius of the \( T^4 \) directions for convenience, one obtains

\[ S = -\int d^{p+1} \xi \left( 1 - \left( \frac{r_0}{r} \right)^{3-p} \right) \left( M - \sqrt{2}Q \right). \] (3.8)

If we take the probe to be a set of stacked branes, \( M/Q = \sqrt{2} \), the static potential vanishes, as one might expect for an extremal black hole solution. However, if we take \( M/Q = 1 \) as appropriate for an elementary non-BPS probe brane, we find a repulsive force. We can therefore expect the stacked configuration to slowly discharge due to pair production of elementary non-BPS branes via the Schwinger mechanism. As is apparent from (3.8) the force on the pair will be very small near the horizon of a large black hole. The rate of production will go like \( \exp(-\text{const.} \ M^2 r_0) \). The black hole will be absolutely stable only in the limit of a large number of branes.

**IV. CONCLUSIONS**

Starting from an action describing a consistent truncation of ten dimensional type II superstring theory compactified on an orbifolded four dimensional torus \( T^4/Z_2 \) we have considered stable classical solutions describing a stack of non-BPS D-branes. We have argued the stable solution should correspond to the simplest extremal black hole solution with the same gauge charge and spacetime symmetry as the set of branes. This yields a nontrivial prediction for the stable groundstate of the nonabelian tachyon condensation on the worldvolume of the stacked branes. It seems likely these ideas may be generalized straightforwardly to other brane systems where one expects a supergravity description of tachyon condensation to a stable ground state [13].

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