Search for New Physics
in the Semileptonic $D_{l4}$ Decays, $D^\pm \rightarrow K\pi l^\pm \nu$

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Abstract

New physics effects through the direct CP violation and the decay rate change are investigated in the semileptonic $D_{l4}$ decays, $D^\pm \rightarrow K\pi l^\pm \nu$, by including a scalar-exchange interaction with a complex coupling. In the decay process, we included various excited states as intermediate states decaying to the final hadrons, $K + \pi$, and found that among the intermediate states only the lowest state ($K^*$) is dominant and the other higher excited states are negligible, contrary to the $B_{l4}$ decays. We also obtained constraints on the new complex coupling within the multi-Higgs doublet model and the scalar leptoquark models.

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1 Introduction

Recently we studied the possibility of probing the direct CP violation in the $B_{l4}$ decays, $B^\pm \to D\pi l^\pm \nu$ [1] and $B^\pm \to \pi\pi l^\pm \nu$ [2], in extensions of the Standard Model (SM) by including a scalar-exchange interaction with a complex coupling in the weak charged current. There we considered, as specific models, the multi-Higgs doublet (MHD) model and the scalar-leptoquark (SLQ) models. And we investigated mainly the direct CP violation effects, i.e., CP-odd asymmetries for maximally-allowed values of the imaginary part of the additional scalar coupling, even though we also found that such a scalar coupling could induce a sizable effect to the CP-even total decay rate. In the present paper, we take a more general approach and investigate new physics effects by considering changes in the (CP-even) total decay rate as well as the (CP-odd) CP violation effects in $D_{l4}$ decays, $D^\pm \to K\pi l^\pm \nu$. Here we find constraints to both real and imaginary parts of the new scalar coupling, by comparing our predictions of the decay rate and the direct CP violation effects with the observable experimental results.

As is well known, in order to observe the direct CP violation effects, there should exist interferences not only through weak CP-violating phases but also with different CP-conserving strong phases. In the decay $D^\pm \to K\pi l^\pm \nu$, we consider it as a two-step process: $D \to (\sum_i \bar{K}_i) l \nu \to K\pi l \nu$, where $\bar{K}_i$ stands for an intermediate state which decays to $K + \pi$. In this picture CP-conserving phases come from the absorptive parts of the intermediate resonances. The relevant resonance states are $K^*$, $K_0^*$ and $K_2^*$ mesons, which decay dominantly to $K\pi$ mode (see Table. 1). As shown in Refs. [1, 2] for the $B_{l4}$ decays, the inclusion of higher excited states could amplify the CP violation effects. However, we now find that this is not true anymore for the $D_{l4}$ decays, because in the $D_{l4}$ decays final state phase spaces are much smaller for those higher excited states than the available phase spaces in the $B_{l4}$ decays. Therefore, we anticipate that the effects of higher excited resonances will be correspondingly reduced and the dominant contributions will come mainly through the lowest state $K^*(892)$. We will discuss more on this later.

In Section 2, we present in detail our formalism for the $D^\pm \to K\pi l^\pm \nu$ decays within the SM and in the models beyond the SM. Section 3 is devoted to the numerical analyses, and concluding remarks are also in Section 3.
Table 1. Properties and branching ratios of $K\pi$ resonances

| Label $i$ | $K_i$               | $J^P$ | $m_i$ (MeV) | $\Gamma_i$ (MeV) | $\mathcal{BR}_i(K_i \to K^+\pi^-)$ |
|-----------|---------------------|-------|-------------|------------------|-----------------------------------|
| 0         | $K_0 = K^*_0(1430)$ | 0$^+$ | 1429        | 287              | 0.62                              |
| 1         | $\tilde{K}_1 = K^*(892)$ | 1$^-$ | 896         | 51               | 0.67                              |
| 2         | $\tilde{K}_2 = K^*_2(1430)$ | 2$^+$ | 1432        | 109              | 0.33                              |

2 Theoretical Details of the $D_{l4}$ Decays

We first describe the formalism within the SM for the decay $D^- \to K\pi l^- \nu$. Its extensions to the models beyond the SM are obtained by rather simple appropriate modifications. The decay amplitudes for the processes of Fig. 1

$$D^-(p_D) \to \tilde{K}_i(p_i, \lambda_i) + W^*(q) \to K^+(p_K) + \pi^-(p_\pi) + l^-(p_l, \lambda_l) + \bar{\nu}(p_\nu)$$

are expressed as

$$A^\lambda_i = -V_{cs} \frac{G_F}{\sqrt{2}} \sum_i \sum_{\lambda_i} \langle l^- (p_l, \lambda_l) | \bar{\nu}(p_\nu)| j^\mu(0) \rangle \langle K_i(p_i, \lambda_i) | J_\mu | D^-(p_D) \rangle$$

$$\times \Pi_i(s_M) \langle K^+(p_K) \pi^-(p_\pi) | \tilde{K}_i(p_i, \lambda_i) \rangle,$$

where $\lambda_i = 0$ for spin 0 states ($K^*_0$), $\lambda_i = \pm 1, 0$ for spin 1 states ($K^*$), $\lambda_i = \pm 2, \pm 1, 0$ for spin 2 states ($K^*_2$), and $\lambda_l$ is the lepton helicity, $\pm \frac{1}{2}$.

The leptonic and hadronic currents are defined, respectively, as

$$j^\mu = \bar{\psi}_l \gamma^\mu (1 - \gamma_5) \psi_l,$$

$$J^\mu = \bar{\psi}_c \gamma^\mu (1 - \gamma_5) \psi_s.$$ (3)

We assume that the resonance contributions of the intermediate states can be treated by the Breit-Wigner form, which is written in the narrow width approximation as

$$\Pi_i(s_M) = \frac{\sqrt{m_i \Gamma_i / \pi}}{s_M - m_i^2 + i m_i \Gamma_i},$$

where $s_M = (p_K + p_\pi)^2$ and the $m_i$’s and $\Gamma_i$’s are the masses and widths of the resonances, respectively (see Table 1). For the decay parts of the resonances we use

$$\langle K^+(p_K) \pi^-(p_\pi) | \tilde{K}_i(p_i, \lambda_i) \rangle = \sqrt{\mathcal{BR}_i Y^\lambda_i_{A_{max}}(\theta^*, \phi^*)},$$

(5)
Figure 1: Diagrams for $D^- \to \tilde{K}_i W^* \to K^+ \pi^- l^- \bar{\nu}_l$ decays within the SM.

where $Y^m_l(\theta, \phi)$ are the $J = l$ spherical harmonics, and the angles $\theta^*$ and $\phi^*$ are those of the final state $\pi^-$ specified in the $\tilde{K}_i$ rest frame, as defined in Fig. 2c. The couplings of $\tilde{K}_i$ to $K\pi$ are effectively taken into account by the branching fractions, $BR_i(\tilde{K}_i \to K^+ \pi^-)$.

In order to obtain the full helicity amplitude of the $D^- \to K\pi l^- \bar{\nu}_l$ decay, we first consider the amplitude of $D^- \to \tilde{K}_i l^- \bar{\nu}_l$, denoted as $M^\lambda_{\lambda_i}$:

$$M^\lambda_{\lambda_i} = -V_{cs} \frac{G_F}{\sqrt{2}} \langle \tilde{K}_i(p_i, \lambda_i) | J^\mu | 0 \rangle \langle K(p, \lambda) | J^\mu | D^-(p_D) \rangle.$$  \hfill (6)

We express the matrix elements $M^\lambda_{\lambda_i}$ into the following form:

$$M^\lambda_{\lambda_i} = V_{cs} \frac{G_F}{\sqrt{2}} \sum_{\lambda_W} \eta_{\lambda_W} L^\lambda_{\lambda_W} H^\lambda_{\lambda_W},$$  \hfill (7)

where for the decays $D \to \tilde{K}_i W^*$ and $W^* \to l\bar{\nu}$, respectively,

$$H^\lambda_{\lambda_W} = \epsilon^*_{W\mu}(\tilde{K}_i(p_i, \lambda_i) | J^\mu | D^-(p_D) \rangle,$$

$$L^\lambda_{\lambda_W} = \epsilon_{W\mu}(l^\mu(p_l, \lambda_l) \bar{\nu}(p_{\bar{\nu}}) | j^\mu | 0 \rangle,$$  \hfill (8)

in terms of the polarization vectors $\epsilon_w \equiv \epsilon(q, \lambda_w)$ of the virtual $W$. These $\epsilon_w$'s satisfy the relation

$$-g^{\mu\nu} = \sum_{\lambda_W} \eta_{\lambda_W} \epsilon^\mu_w \epsilon^{*\nu}_w,$$  \hfill (9)

where for the decays $D \to \tilde{K}_i W^*$ and $W^* \to l\bar{\nu}$, respectively,
where the summation is over the helicities $\lambda_W = \pm 1, 0, s$ of the virtual $W$, with the metric $\eta_\pm = \eta_0 = -\eta_s = 1$.

We evaluate the leptonic amplitude $L_{\lambda_W}^\lambda_{l\bar{\nu}}^W$ in the rest frame of the virtual $W^*$ (see Fig. 2b). Using the 2-component spinor formalism [4] with explicit polarization vectors [2], we find

$$
L_{-\pm}^\lambda_0 = 2 \sqrt{q^2} vd_\pm, \quad L_{-0}^\lambda = -2 \sqrt{q^2} vd_0, \quad L_{-s}^\lambda = 0, \\
L_{+\pm}^\lambda_0 = \pm 2m_lvd_0, \quad L_{+0}^\lambda = \sqrt{2}m_lv(d_+ - d_-), \quad L_{+s}^\lambda = -2m_lv,
$$

(10)

where

$$
v = \sqrt{\frac{1 - m_l^2}{q^2}}, \quad d_\pm = \frac{1 \pm \cos \theta_l}{\sqrt{2}}, \quad \text{and} \quad d_0 = \sin \theta_l.
$$

(11)

Here we show only the sign of $\lambda_l$ as a superscript on $L$. Note that the $L_+^\lambda$ amplitudes are proportional to the lepton mass $m_l$, and the scalar amplitude $L_{-s}^\lambda$ vanishes due to angular momentum conservation.

For the $D \rightarrow \tilde{K}_i$ transition through the weak charged current

$$
J^\mu = V^\mu - A^\mu,
$$

(12)

the most general forms of matrix elements are,

for $K_0^*(0^+)$ states:
\[ \langle K_0^*(p_i)|V_\mu|D(p_D)\rangle = 0, \]
\[ \langle K_0^*(p_i)|A_\mu|D(p_D)\rangle = u_+(q^2)(p_D + p_i)_\mu + u_-(q^2)(p_D - p_i)_\mu; \]

for \( K^*(1^-) \) states:
\[ \langle K^*(p_i, \epsilon_1)|V_\mu|D(p_D)\rangle = ig(q^2)\epsilon_{\mu\nu\rho\sigma}\epsilon_1^\nu(p_D + p_i)^\rho(p_D - p_i)^\sigma, \]
\[ \langle K^*(p_i, \epsilon_1)|A_\mu|D(p_D)\rangle = f(q^2)\epsilon_1^\mu + a_+(q^2)(\epsilon_1^\mu \cdot p_D)(p_D + p_i)_\mu + a_-(q^2)(\epsilon_1^\mu \cdot p_D)(p_D - p_i)_\mu; \]

for \( K_2^*(2^+) \) states:
\[ \langle K_2^*(p_i, \epsilon_2)|V_\mu|D(p_D)\rangle = ih(q^2)\epsilon_{\mu\lambda\beta\delta}\epsilon_2^{\lambda\beta\delta}p_D\lambda(p_D + p_i)\lambda(p_D - p_i)^\mu, \]
\[ \langle D_2^*(p_i, \epsilon_2)|A_\mu|D(p_D)\rangle = k(q^2)\epsilon_2^{\mu\nu}\lambda(p_D + p_i)_\mu + b_+(q^2)(\epsilon_2^{\mu\nu}\lambda(p_D + p_i)_\mu + b_-(q^2)(\epsilon_2^{\mu\nu}\lambda(p_D - p_i)_\mu, \]

where \( \epsilon_1 \) and \( \epsilon_2 \) are the polarization vectors of the spin 1 and spin 2 states, respectively.

Using the above expressions and the polarization vectors \( \epsilon \), we find the non-zero \( D \to \bar{K}_iW^* \) amplitudes,

for \( i = 0, \quad H^0_{\lambda_W} \equiv S^0_{\lambda_W}, \)
\[ S^0_0 = -u_+(q^2)\sqrt{Q_+Q_-}, \]
\[ S^0_s = -\left( u_+(q^2)\frac{(m_D^2 - s_M^2)}{q^2} + u_-(q^2)\sqrt{q^2} \right), \]

for \( i = 1^{(c)}, \quad H^{\lambda_1}_{\lambda_W} \equiv V^{\lambda_1}_{\lambda_W}, \)
\[ V^0_0 = \frac{1}{2\sqrt{s_Mq^2}}\left[ f(q^2)(m_D^2 - s_M^2 - q^2) + a_+(q^2)Q_+Q_- \right], \]
\[ V^0_1 = \frac{\sqrt{Q_+Q_-}}{2\sqrt{s_Mq^2}}\left[ f(q^2) + g(q^2)\sqrt{Q_+Q_-} \right], \]
\[ V^0_s = -\frac{\sqrt{Q_+Q_-}}{2\sqrt{s_Mq^2}}\left[ f(q^2) + a_+(q^2)(m_D^2 - s_M^2) + a_-(q^2)^2 \right], \]

for \( i = 2, \quad H^{\lambda_2}_{\lambda_W} \equiv T^{\lambda_2}_{\lambda_W}, \)
\[ T^0_0 = -\frac{1}{2\sqrt{6}}\sqrt{Q_+Q_-}q^2\left[ k(q^2)(m_D^2 - s_M^2 - q^2) + b_+(q^2)Q_+Q_- \right], \]
\[ T^0_1 = \frac{1}{2\sqrt{2}}\sqrt{\frac{Q_+Q_-}{s_M}}[k(q^2) + h(q^2)\sqrt{Q_+Q_-}], \]
\[ T^0_s = -\frac{1}{2\sqrt{6}}\frac{Q_+Q_-}{s_M\sqrt{q^2}}[k(q^2) + b_+(q^2)(m_D^2 - s_M^2) + b_-(q^2)^2], \]
where

\[ Q_\pm = (m_D \pm \sqrt{s_M})^2 - q^2. \]  

(17)

Combining all the formulae, we can write the SM helicity amplitudes of \( D^- \rightarrow K^+\pi^-l^-\bar{\nu}_l \) decays as

\[
\mathcal{A}_\lambda = V_{cs} \frac{G_F}{\sqrt{2}} \left[ \sum_{\lambda=0,s} \eta_\lambda L_\lambda^\lambda (\Pi_{K_0^*} S_\lambda^0 Y_0^0 + \Pi_{K_1} V_\lambda^0 Y_1^0 + \Pi_{K_2^*} T_\lambda^0 Y_2^0) + \sum_{\lambda=\pm 1} L_\lambda^\lambda (\Pi_{K_1} V_\lambda^\lambda Y_1^\lambda + \Pi_{K_2^*} T_\lambda^\lambda Y_2^\lambda) \right],
\]

(18)

where

\[
\Pi_{K_0^*} \equiv \sqrt{BR_0} \Pi_0 \quad \Pi_{K_1} \equiv \sqrt{BR_1} \Pi_1 \quad \Pi_{K_2^*} \equiv \sqrt{BR_2} \Pi_2.
\]

(19)

The differential partial width can be expressed as

\[
d\Gamma(D^- \rightarrow K^+\pi^-l^-\bar{\nu}_l) = \frac{1}{2m_D} \sum_{\lambda_l} |\mathcal{A}_\lambda|^2 \frac{(q^2 - m_l^2)\sqrt{Q_+ Q_-}}{256\pi^3 m_D^2 q^2} d\Phi_4,
\]

(20)

where the 4 body phase space \( d\Phi_4 \) is

\[
d\Phi_4 \equiv ds_M \cdot dq^2 \cdot d\cos \theta^* \cdot d\cos \theta_l \cdot d\phi^*.
\]

(21)

So far we have established the SM formalism for the \( D_{l4} \) decays. Now we extend the virtual \( W \)-exchange part in Fig. 1 by including an additional scalar interaction with the complex coupling. Then, the decay amplitudes for \( D^- \rightarrow K^+\pi^-l^-\bar{\nu}_l \) can be expressed as

\[
\mathcal{A}_\lambda = -V_{cs} \frac{G_F}{\sqrt{2}} \sum_{\lambda_l} \sum_{\lambda_l} \left[ \langle l^- (p_l, \lambda_l) | \bar{\nu} (p_\nu) | j^\mu | 0 \rangle \langle \tilde{K}_i (p_i, \lambda_i) | J_\mu | D^- (p_D) \rangle \right.

\[ + \zeta \langle l^- (p_l, \lambda_l) | \bar{\nu} (p_\nu) | j_\mu^\dagger | 0 \rangle \langle \tilde{K}_i (p_i, \lambda_i) | J_\mu | D^- (p_D) \rangle \right]

\times \Pi_s(s_M) \langle K^+ (p_K) \pi^- (p_\pi) | \tilde{K}_i (p_i, \lambda_i) \rangle,
\]

(22)

where the scalar currents are

\[
j_s = \bar{\psi}_\nu (1 - \gamma_5) \psi_l, \quad J_s = \bar{\psi}_c (1 - \gamma_5) \psi_s.
\]

(23)
Here the parameter $\zeta$, which parameterizes contributions from physics beyond the SM, is in general a complex number. By using the Dirac equation for the leptonic current, $q_\mu j^\mu = m_\ell j_\ell$, the amplitude can be written as

$$A^{\lambda_\ell} = -V_{cs} \frac{G_F}{\sqrt{2}} \sum_i \sum_{\lambda_\ell} \langle l^- (p_\ell, \lambda_\ell) \bar{\nu} (p_\nu) | j^{\mu\dagger} | 0 \rangle \langle \bar{K}_i (p_i, \lambda_i) | \Omega^\dagger | D^- (p_D) \rangle \times \Pi_i (s_M) \langle K^+ (p_K) \pi^- (p_\pi) | \bar{K}_i (p_i, \lambda_i) \rangle,$$

(24)

where the effective hadronic current $\Omega^\dagger$ is defined as

$$\Omega^\dagger \equiv J^\mu + \zeta q_\mu J_\ell.$$

(25)

In this case the amplitudes $M^{\lambda_\ell}_{\lambda_i}$ of $D \to \bar{K}_i l \bar{\nu}$ have the same form as the previous SM case (6) except for the modification in the hadronic current part due to the additional scalar current:

$$M^{\lambda_\ell}_{\lambda_i} = \frac{G_F}{\sqrt{2}} V_{cs} \sum_{\lambda_\ell W} \eta_{\lambda_\ell W} L_{\lambda W}^{\lambda_\ell} \mathcal{H}_{\lambda W}^{\lambda_\ell},$$

(26)

where $\mathcal{H}_{\lambda W}^{\lambda_\ell}$ stands for the hadronic amplitudes modified by the scalar current $J_\ell$. Using the equations of motion for $c$ and $s$ quarks, we get within the on-shell approximation

$$J_\ell = (p_c^\mu - p_s^\mu) \left[ \frac{V_\mu}{m_c - m_s} - \frac{A_\mu}{m_c + m_s} \right].$$

(27)

Later we use the approximation, $(p_c^\mu - p_s^\mu) \approx (p_D^\mu - p_{\bar{K}_i}^\mu) \equiv q^\mu$, which has been generally assumed in quark model calculations of the form factors. After explicit calculation, we find that the additional scalar current modifies only the scalar component of $\mathcal{H}_{\lambda W}^{\lambda_\ell}$:

$$\mathcal{H}_s^0 = (1 + \zeta') H_s^0,$$

and

$$\mathcal{H}_{\lambda W}^{\lambda_\ell} = H_{\lambda W}^{\lambda_\ell} \text{ for } \lambda_W = 0, \pm 1,$$

(28)

where

$$\zeta' = \frac{q^2}{m_t (m_c + m_s)} \zeta.$$

(29)

So far, we constructed a formalism for $D_{-14}^-$ decays. Since the initial $D^-$ system is not CP self-conjugate, any genuine CP-odd observable can be constructed by considering both
the $D^-$ decay and its charge-conjugated $D^+$ decay, and by identifying the CP relations of their kinematic distributions. Before constructing possible CP-odd asymmetries explicitly, we calculate the decay amplitudes for the charge-conjugated process $D^+ \to K^−\pi^+l^+\nu_l$. For the charge-conjugated $D^+$ decays, the amplitudes can be written as

$$\bar{A}^\lambda_i = -V^*_{cs}G_F\sqrt{2}\sum_i\sum_\lambda_i\langle l^+(p_l, \lambda_l)\nu(p_\nu)|j^\mu|0\rangle\langle \tilde{K}_i(p_i, \lambda_i)|\Omega^\dagger_i|D^+(p_D)\rangle \times \Pi_i(s_\lambda)(K^-(p_K)\pi^+(p_\pi)|\tilde{K}_i(p_i, \lambda_i)).$$

(30)

Similarly to the $D^-$ decay, the leptonic amplitudes $\bar{L}^\lambda_i$ for $D^+$ decay are

$$L^\pm_\mp = -2\sqrt{q^2}vd_\mp, \quad L^0_\pm = -2\sqrt{q^2}vd_0, \quad L^+_s = 0,$$

$$\bar{L}^\pm_\mp = \pm 2m Lv_0, \quad \bar{L}^0_\mp = \sqrt{2}m Lv(d_+ - d_-), \quad \bar{L}^0_s = -2m Lv.$$

(31)

And the transition amplitudes $\bar{H}^\lambda_i_{K_i}$ for $D^+ \to \tilde{K}_iW^*$ are given by a simple modification of the amplitudes $H^\lambda_i_{K_i}$ of the $D^-$ decays:

$$\bar{H}^\lambda_i_{K_i} = H^\lambda_i_{K_i}\{g \to -g, \ h \to -h, \ f_\pm \to -f_\pm; \ \zeta \to \zeta^*\},$$

(32)

which is expected from the property that vector currents change sign under the charge conjugation.

It is easy to see that if $\zeta$ is real, the amplitude (24) of the $D^-$ decay and (30) of the $D^+$ decay satisfy the CP transition relation:

$$A^\pm(\theta^*, \phi^*, \theta_l) = \eta_{CP}A^\mp(\theta^*, -\phi^*, \theta_l),$$

(33)

where $\theta^*$ and $\phi^*$ in $\bar{A}^\lambda_i$ are the angles of the final state $\pi^+$, while those in $A^\lambda_i$ are for $\pi^-$. Then, with a complex phase $\zeta$, $d\Gamma/d\Phi_4$ can be decomposed into a CP-even part $S$ and a CP-odd part $D$:

$$\frac{d\Gamma}{d\Phi_4} = \frac{1}{2}(S + D).$$

(34)

The CP-even part $S$ and the CP-odd part $D$ can be easily identified by making use of the CP relation (33) between $D^-$ and $D^+$ decay amplitudes, and they are expressed as

$$S = \frac{d(\Gamma + \Gamma)}{d\Phi_4}, \quad D = \frac{d(\Gamma - \Gamma)}{d\Phi_4},$$

(35)
where $\Gamma$ and $\mathcal{T}$ are the decay rates for $D^-$ and $D^+$, respectively. And we use the same kinematic variables $\{s_m, q^2, \theta^*, \theta_1\}$ for the $d\mathcal{T}/d\Phi_4$ except for the replacement of $\phi^* \rightarrow -\phi^*$, as shown in Eq. (33). The CP-even $S$ term and the CP-odd $D$ term can be obtained from $D^\pm$ decay probabilities. The CP-even quantity $S$ is

$$S = 2C(q^2, s_m) \Sigma,$$

with

$$\Sigma = \left( L_0^s S_0^0 Y_0^0 \right)^2 |\Pi_{K_0}^*|^2 + |\langle V^- \rangle \Pi_{K^*} |^2 + |\langle T^- \rangle \Pi_{K_2} |^2$$

$$+ 2(L_0^s S_0^0 Y_0^0) \text{Re}(\Pi_{K_0}^* \Pi_{K^*}^* \langle V^- \rangle^* + \Pi_{K^*} \Pi_{K_2}^* \langle T^- \rangle^*) + 2\text{Re}(\Pi_{K^*} \Pi_{K_2}^* \langle V^- \rangle \langle T^- \rangle^*)$$

$$\pm |\Pi_{K_0}^*|^2 |L_0^s S_0^0 Y_0^0 - (1 + \zeta') L_2^s S_2^0 Y_0^0|^2$$

$$\pm |\Pi_{K^*}|^2 |\langle V^+ \rangle|^2 + (L_2^s S_2^0 Y_1^0)^2 |1 + \zeta' |^2 - 2(L_2^s T_2^0 Y_2^0) \text{Re}(\langle V^+ \rangle) \text{Re}(1 + \zeta')$$

$$\pm |\Pi_{K_2}^*|^2 |\langle T^+ \rangle|^2 + (L_2^s T_2^0 Y_2^0)^2 |1 + \zeta' |^2 - 2(L_2^s T_2^0 Y_2^0) \text{Re}(\langle T^+ \rangle) \text{Re}(1 + \zeta')$$

$$\pm 2\text{Re}(\Pi_{K_0}^* \Pi_{K^*}^*) [(L_0^s S_0^0 - L_2^s S_2^0) Y_0^0 \text{Re}(\langle V^+ \rangle) - (L_0^s S_0^0 Y_0^0) (L_2^s T_2^0 Y_2^0) \text{Re}(1 + \zeta')$$

$$- (L_2^s S_2^0 Y_0^0) \text{Re}(\langle V^+ \rangle) \text{Re}(\zeta') + (L_2^s S_2^0 Y_0^0) (L_2^s T_2^0 Y_2^0) |1 + \zeta' |^2$$

$$\pm 2\text{Im}(\Pi_{K_0}^* \Pi_{K^*}^*) \text{Im}(\langle V^+ \rangle) [(L_0^s S_0^0 - L_2^s S_2^0) Y_0^0 - (L_2^s S_2^0 Y_0^0) \text{Re}(\zeta')]$$

$$\pm 2\text{Re}(\Pi_{K_0}^* \Pi_{K_2}^*) [(L_0^s S_0^0 - L_2^s S_2^0) Y_0^0 \text{Re}(\langle T^+ \rangle) - (L_0^s S_0^0 Y_0^0) (L_2^s T_2^0 Y_2^0) \text{Re}(1 + \zeta')$$

$$- (L_2^s S_2^0 Y_0^0) \text{Re}(\langle T^+ \rangle) \text{Re}(\zeta') + (L_2^s S_2^0 Y_0^0) (L_2^s T_2^0 Y_2^0) |1 + \zeta' |^2$$

$$\pm 2\text{Im}(\Pi_{K_0}^* \Pi_{K_2}^*) \text{Im}(\langle T^+ \rangle) [(L_0^s S_0^0 - L_2^s S_2^0) Y_0^0 - (L_2^s S_2^0 Y_0^0) \text{Re}(\zeta')]$$

$$\pm 2\text{Re}(\Pi_{K_2}^* \Pi_{K^*}^*) \text{Re}(\langle V^+ \rangle \langle T^+ \rangle^*) - (L_2^s T_2^0 Y_2^0) \text{Re}(\langle V^+ \rangle) - (L_2^s T_2^0 Y_2^0) \text{Re}(\langle V^+ \rangle) \text{Re}(\zeta')$$

$$- (L_2^s V_2^0 Y_2^0) \text{Re}(\langle T^+ \rangle) \text{Re}(1 + \zeta') + (L_2^s V_2^0 Y_1^0) (L_2^s T_2^0 Y_2^0) |1 + \zeta' |^2$$

$$\pm 2\text{Im}(\Pi_{K_2}^* \Pi_{K^*}^*) \text{Im}(\langle V^+ \rangle) \text{Im}(\langle T^+ \rangle) \text{Im}(\langle V^+ \rangle) - (L_2^s T_2^0 Y_2^0) \text{Im}(\langle V^+ \rangle) - (L_2^s T_2^0 Y_2^0) \text{Im}(\langle V^+ \rangle) \text{Re}(\zeta')$$

$$+ (L_2^s V_2^0 Y_0^0) \text{Im}(\langle T^+ \rangle) \text{Re}(1 + \zeta'),$$

and the CP-odd quantity $D$ is

$$D = -2\text{Im}(\zeta') C(q^2, s_m) \Delta,$$

with

$$\Delta = 2 \left[ \text{Im}(\langle V^+ \rangle) \{ (L_2^s V_2^0 Y_0^0) \Pi_{K^*} |^2 + (L_2^s S_2^0 Y_0^0) \text{Re}(\Pi_{K_0}^* \Pi_{K^*}^*) \right]$$
\[ + (L_+ T_0 Y_2^0) \text{Re}(\Pi K \cdot \Pi K^*_2) \}
\]
\[ + \text{Im}(\langle T^+ \rangle) \{ (L_+ T_0 Y_2^0) |\Pi K^*_2 |^2 + (L_+ S_0 Y_0^0) \text{Re}(\Pi K_0^* \Pi K^*_2) \} + (L_+ V_0 Y_1^0) \text{Re}(\Pi K_0^* \Pi K^*_2) \}
\]
\[ + \text{Re}(\langle V^+ \rangle) \{ (L_+ T_0 Y_2^0) \text{Im}(\Pi K \cdot \Pi K^*_2) - (L_+ S_0 Y_0^0) \text{Im}(\Pi K_0^* \Pi K^*_2) \}
\]
\[ - \text{Re}(\langle T^+ \rangle) \{ (L_+ V_0 Y_1^0) \text{Im}(\Pi K \cdot \Pi K^*_2) + (L_+ S_0 Y_0^0) \text{Im}(\Pi K_0^* \Pi K^*_2) \}
\]
\[ + (L_+ S_0 Y_0^0) (L_+ V_0 Y_1^0) \text{Im}(\Pi K_0^* \Pi K^*_2) + (L_+ S_0 Y_0^0) (L_+ T_0 Y_2^0) \text{Im}(\Pi K_0^* \Pi K^*_2) \}, \quad (39) \]

where

\[ \langle V^\pm \rangle \equiv \sum_{\lambda=0,\pm 1} L_+ V_0^\lambda Y_1^\lambda, \quad \langle T^\pm \rangle \equiv \sum_{\lambda=0,\pm 1} L_+ T_0^\lambda Y_2^\lambda, \quad (40) \]

and the overall function \( C(q^2, s_M) \) is given by

\[ C(q^2, s_M) = |V_{ub}|^2 \frac{G_F^2}{2} \frac{1}{2m_D} \frac{(q^2 - m_2)}{256\pi^3 m_D q^2}. \quad (41) \]

Note that the CP-odd term is proportional to the imaginary part of the parameter \( \zeta \) and the lepton mass. Therefore, in order to investigate CP violation in the \( D_{\mu 4} \) decays, we have to consider massive leptonic \( D_{\mu 4} (D^\pm \to K \pi \mu^\pm \nu) \) decays.

### 3 Numerical Analyses and Conclusions

First, we calculate \( D_{\mu 4} \) decay rates through the most dominant resonance, \( K^*(892) \), by varying values of the scalar coupling and then compare those results with the present experimental branching ratio \([3]\),

\[ \text{BR}[D^+ \to (K^*(892)^0 \to K^- \pi^+) \mu^+ \nu_\mu] = (2.9 \pm 0.4)\%. \quad (42) \]

In Fig. 3 we show the allowed parameter space of the complex scalar coupling. For example, using the present experimental result of Eq. \([12]\), one can constrain the value of the scalar coupling down to

\[ |\zeta| \sim 5.0 \]

at 2–σ level. In our numerical analyses, we use the so-called ISGW2 form factors \([1]\) in \( D \to \tilde{K}_i \) transition amplitudes in Eq. \([13]\).

As mentioned earlier, study of CP violation effects can give a further constraint to the imaginary part of the scalar coupling. We consider the so-called optimal observable. An
Figure 3: The allowed parameter space of the complex scalar coupling $\zeta$ at each confidence level, by comparing the theoretical decay rate with the experimental result, Eq. (42), for $D^+ \rightarrow (K^*(892))^0 \rightarrow K^-\pi^+)\mu^+\nu_\mu$ decay. The horizontal shaded region represents the attainable 2–σ limits on the imaginary part through the optimal CP-odd asymmetry (see Table 2).

An appropriate real weight function $w(s_M, q^2, \theta^*, \theta_l, \phi^*)$ is usually employed to separate the CP-odd $D$ contribution and enhances its analysis power through the CP-odd quantity,

$$\langle wD \rangle \equiv \int [wD] d\Phi_4 .$$  \hspace{1cm} (43)

And the analysis power is determined by the parameter,

$$\varepsilon = \frac{\langle wD \rangle}{\sqrt{\langle S \rangle \langle w^2 S \rangle}} .$$  \hspace{1cm} (44)

For the analysis power $\varepsilon$, the number $N_D$ of the $D$-mesons needed to observe CP violation at 1–σ level is

$$N_D = \frac{1}{B_r \cdot \varepsilon^2} .$$  \hspace{1cm} (45)

From the above relation, we can also deduce the bound on the CP-odd parameter for given $N_D$ at an arbitrary confidence level. Certainly, it is desirable to find the optimal weight function with the largest analysis power. It is known [7] that when the CP-odd contribution to the total rate is relatively small, the optimal weight function is approximately given as

$$w_{opt}(s_M, q^2; \theta^*, \theta_l, \phi^*) = \frac{D}{S} \Rightarrow \varepsilon_{opt} = \sqrt{\frac{\langle D^2 \rangle}{\langle S \rangle}} .$$  \hspace{1cm} (46)
Table 2. Attainable 2-σ limits on the imaginary part of the scalar coupling, $\zeta$, through the optimal observable, with the number of $D$-mesons, $N_D = 10^8$

| Intermediate states          | Attainable 2-σ limits |
|------------------------------|-----------------------|
| $K^*$, $K_0^*$ and $K_1^*$   | $|\text{Im}(\zeta)| = 0.121$ |
| $K^*$ only                   | $|\text{Im}(\zeta)| = 0.126$ |

We adopt this optimal weight function in our numerical analyses.

In Table 2, we show the attainable 2-σ limits on the imaginary part of the scalar coupling $\zeta$ through the optimal observable. In order to estimate effects from higher excited resonances, we separately present the result obtained by including only the lowest state $K^*$ as an intermediate state. We can easily see that in $D_{14}$ decays the effect of higher excited resonances is very small. We note the authors of Ref. [8] analyzed the possibility of probing CP-violation by extracting T-odd angular correlations in $D \rightarrow K^*(\rightarrow K\pi)l\bar{\nu}$ decay and found that the effects can be detected in some cases.

In order to get explicit meaning for our analyses, we now consider specific models beyond the SM. As specific extensions of the SM, we consider four types of scalar-exchange models which preserve the symmetries of the SM [9]: one of them is the multi-Higgs-doublet (MHD) model [10] and the other three are the scalar-leptoquark (SLQ) models [11, 12]:

• (i) Assuming that all but the lightest of the charged scalars effectively decouple from fermions, the effective Lagrangian of the MHD model contributing to the decay $D \rightarrow K\pi l\bar{\nu}_l$ is given at energies considerably low compared to $M_H$ by

$$\mathcal{L}_{\text{MHD}} = 2\sqrt{2}G_F V_{cs} \frac{m_l}{M_H^2} \left[m_s X Z^* (\bar{c} L_s R) + m_c Y Z^* (\bar{c} R s L)\right] (\bar{l} R \nu_L),$$  \hspace{1cm} (47)

where $X, Y$ and $Z$ are complex coupling constants which can be expressed in terms of the charged Higgs mixing matrix elements. From the effective Lagrangian for the MHD model, we obtain scalar coupling $\zeta_{\text{MHD}}$

$$\zeta_{\text{MHD}} = \frac{m_l m_c}{M_H^2} \left\{ \left( \frac{m_s}{m_c} \right) X Z^* - Y Z^* \right\}.$$  \hspace{1cm} (48)

The present bound [10] on $\zeta_{\text{MHD}}$ is

$$|\zeta_{\text{MHD}}| < 0.0029 \quad \text{for } \mu \text{ family.}$$  \hspace{1cm} (49)
Therefore, the MHD bound is already too stringent to use this \( D_{\mu 4} \) decay mode for further constraining the coupling constant.

- (ii) The effective Lagrangians for the three SLQ models [3, 11] contributing to the decay \( D \to K \pi l \nu \) are written, after a few Fierz rearrangements, in the form

\[
\mathcal{L}^I_{\text{SLQ}} = -\frac{x_{2j}y_{2j}^*}{2M_{\phi_1}^2} \left[ (\bar{s}_L c_R)(\bar{\nu}_{1L} l_R) + \frac{1}{4}(\bar{s}_L \sigma^{\mu \nu} c_R)(\bar{\nu}_{1L} \sigma_{\mu \nu} l_R) \right] + \text{h.c.},
\]

\[
\mathcal{L}^{II}_{\text{SLQ}} = -\frac{y_{2j}y_{2j}^*}{2M_{\phi_2}^2} \left[ (\bar{s}_L c_R)(\bar{\nu}_{2L} \gamma_\mu c_L) + \frac{1}{4}(\bar{s}_L \sigma^{\mu \nu} c_R)(\bar{\nu}_{2L} \gamma_\mu \sigma_{\mu \nu} c_L) \right] + \text{h.c.},
\]

\[
\mathcal{L}^{III}_{\text{SLQ}} = -\frac{z_{2j}z_{2j}^*}{2M_{\phi_3}^2} (\bar{s}_L \gamma_\mu c_L)(\bar{c}_l \gamma_\mu \nu_{1L}) + \text{h.c.},
\]

where \( j = 1, 2 \) for \( l = e, \mu \), respectively, and the coupling constants \( x_{ij}^{(l)} \), \( y_{ij}^{(l)} \) and \( z_{ij} \) are in general complex numbers so that the CP symmetry is violated in the scalar-fermion Yukawa interaction terms. Then we find that the derived SLQ model couplings are

\[
\zeta^{I}_{\text{SLQ}} = -\frac{x_{2j}x_{2j}^*}{4\sqrt{2}G_F V_{ub} M_{\phi_1}^2},
\]

\[
\zeta^{II}_{\text{SLQ}} = -\frac{y_{2j}y_{2j}^*}{4\sqrt{2}G_F V_{ub} M_{\phi_2}^2},
\]

\[
\zeta^{III}_{\text{SLQ}} = 0.
\]

Although there are at present no direct constraints on the SLQ model CP-odd parameters in (51), rough constraints on the parameters can be obtained by the assumption [13] that \( |x_{2j}^{(l)}| \sim |x_{2j}| \) and \( |y_{2j}^{(l)}| \sim |y_{2j}| \). That is to say, the leptoquark couplings to quarks and leptons belonging to the same generation are of a similar size. Then the experimental upper bounds yield [13]

\[
|\zeta^{I}_{\text{SLQ}}| < 6.23, \quad |\zeta^{II}_{\text{SLQ}}| < 15.56 \quad \text{for } \mu \text{ family}.
\]

Therefore using these \( D_{\mu 4} \) decays, one could extract much more stringent constraints on \( \zeta^{I,II}_{\text{SLQ}} \), as shown in Table 2.
To summarize, we have investigated new physics effects through the semileptonic $D_{\mu 4}$ decays, $D^\pm \rightarrow K\pi\mu^\pm\nu_\mu$, where we extended the weak charged current by including a scalar-exchange interaction with a complex coupling. We found that by comparing the theoretical decay rate with the observed experimental value, one can constrain both real and imaginary parts of the complex scalar coupling. We also investigated the direct CP violation effects, and found that one can constrain further the imaginary part through the CP-odd optimal asymmetry. We considered as specific models the multi-Higgs doublet model and the scalar leptoquark models, and found that one can extract much more stringent constraints on the scalar-leptoquark couplings, $\zeta_{SLQ}^{I,H}$, through the decay mode $D^\pm \rightarrow K\pi\mu^\pm\nu$.

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