Galaxy-cluster associations from gravitational lensing

Xiang-Ping Wu\textsuperscript{1,2}, Francois Hammer\textsuperscript{1,3}

\textsuperscript{1}DAEC, Observatoire de Paris-Meudon, 92195 Meudon Principal Cedex, France
\textsuperscript{2}Beijing Astronomical Observatory, Chinese Academy of Sciences, Beijing 100080, China
\textsuperscript{3}CFHT, PO Box 1597, Kamuela, HI 96743, USA

Received , 1993; accepted 1994

Abstract. We investigate the associations between background galaxies and foreground clusters of galaxies due to the effect of gravitational lensing by clusters of galaxies. Similar to the well-known quasar-galaxy ones, these associations depend sensitively on the shape of galaxy number-magnitude or number-flux relation, and both positive and “negative” associations are found to be possible, depending on the limiting magnitude and/or the flux threshold in the surveys. We calculate the enhancement factors assuming a singular isothermal sphere model for clusters of galaxies and a pointlike model for background sources selected in three different wavelengths, $B$, $K$ and radio. Our results show that $K$ – selected galaxies might constitute the best sample to test the “negative” associations while it is unlikely that one can actually observe any association for blue galaxies. We also point out that bright radio sources ($S > 1$ Jy) can provide strong positive associations, which may have been already detected in 3CR sample.

Key words: gravitational lensing – galaxies: general – galaxies: clustering

Send offprint requests to: Xiang-Ping Wu\textsuperscript{1}
1. Introduction

It has been argued for two decades about the existence of associations between high redshift quasars and low redshift galaxies. If these associations are real but not physical, gravitational lensing has thus far provided the most natural explanations. Indeed, quasar-galaxy associations had been predicted (Gott & Gunn, 1974) even before the discovery of the first lensed quasar pair. It is generally believed that distant quasars can be magnified by gravitational lensing of the matter associated with foreground galaxies, leading to an overdensity of quasars around the galaxies. Nevertheless, the present observational evidences on quasar-galaxy associations appear to be puzzling: positive, null and even negative evidences have been reported in different observations (Narayan, 1992), in contradiction with previous claims based on gravitational lensing. Wu (1993) has very recently noticed that gravitational lensing can not only produce a number increase of background quasars in the vicinities of foreground galaxies, but also can reduce the quasar surface number density, depending sensitively on the “turnover” in the number-magnitude relation and the limiting magnitude of background quasars (Boyle, Shanks and Peterson, 1988). For example, one can expect positive associations for bright quasar samples (limiting magnitude of $B < 19.5$) while “negative” associations for fainter magnitude limits. The latter are the consequence of the flatter slope in quasar counts. Such scenarios have successfully explained the current observations of quasar-galaxy associations.

The motivation of the present paper is then to extrapolate the quasar-galaxy associations to some other larger systems, namely, the associations between background galaxies and foreground clusters (hereafter galaxy-cluster associations), and explore the observational possibilities. Recall that any background sources whose number-magnitude relations do not show the slope of $\log N/dm = 0.4$ is potentially affected by gravitational lensing, resulting in either positive or “negative” associations with foreground objects.

It was noticed a long time ago that there may exist associations between galaxies and galaxy clusters. Roberts, O’Dell & Burbidge (1977) found more nearby 3CR galaxies near the positions of clusters of galaxies although their radio sources might be physically associated with their host clusters. The evidences on these associations have remained unclear until recent years when some of the high redshift 3CR galaxies are found to be gravitationally magnified by foreground clusters of galaxies lying in their lines of sight (Hammer, Nottale & Le Fèvre, 1986; Le Fèvre, Hammer & Jones, 1988; Le Fèvre et al., 1988; Hammer & Le Fèvre, 1990). Indeed, bright radio galaxies are ideal sources to be lensed since they are usually at high redshifts ($\langle z \rangle \approx 1–2$) and the slope of their counts
is extremely steep. At optical wavelengths, the Hubble diagram of the brightest cluster galaxies might be also affected by foreground clusters (Hammer and Nottale (1986)). Moreover, the presence of giant luminous arcs in the cores of rich clusters (Hoag, 1981; Lynds & Petrosian, 1986; Soucail et al, 1987) is a direct evidence that rich clusters might affect the observations of faint background galaxies.

Galaxy-cluster associations depend on the selection of flux and wavelength of background galaxies. In this paper we consider $B$, $K$ and radio selected galaxies and study their associations with foreground clusters.

2. Number counts and enhancement factors of background galaxies

The ratio of the observed surface number density of background sources to their undisturbed value is referred to as the enhancement factor $q$. $q$ is balanced by two factors: magnification ($A$) and area distortion ($1/A$). The first effect leads to the increase of total number in a flux-limited survey by picking up fainter sources and the second one reduces the surface number density by enlarging the searched area. $q$ can be simply expressed as (Narayan, 1989)

$$q = \frac{N(< m + 2.5 \log A)}{N(< m)} \frac{1}{A} = \frac{N(> S/A)}{N(> S)} \frac{1}{A}.$$ (1)

Where $N$ is the number counts of background sources, $m$ and $S$ denote the apparent magnitude and flux, respectively. Hence, $q$ can be determined by two parameters only: the intrinsic or the undisturbed number counts of background sources and the lensing magnification by foreground objects. Leaving $A$ to be a free parameter, one can calculate the enhancements for different background sources by assuming that the observed number-magnitude or number-flux relations have not been significantly contaminated by gravitational lensing and can, thereby, represent rather well their intrinsic ones. Indeed, it is very unlikely that lensing can actually change the whole behaviours of the total number counts of any kinds of sources [e.g., for quasars see Schneider (1992)].

The cumulative surface number density of galaxies in $B$ and $K$ bands can be well fitted by the following expressions (Tyson, 1988; Cowie & Songaila, 1992)

$$N(< B) = 1.72 \times 10^{4+0.45(B-24)}$$ (2)

and

$$N(< K) = \begin{cases} 
2.1 \times 10^{3+0.63(K-17)}, & K < 17; \\
-5.6 \times 10^{3} + 7.8 \times 10^{3+0.26(K-17)}, & K > 17.
\end{cases}$$ (3)
For radio sources we use the number counts $N(S)$ of Dunlop and Peacock (1990) and Langston et al. (1990) (see also Wu & Hammer, 1993). An important feature of $N(S)$ is that at bright end ($S > 1$ Jy) the slope turns to be very steep, hence one would expect such sources to be significantly affected by lensing, while at faint end $N(S)$ is relatively flat. However, radio sources are not only galaxies but also quasars and the fraction of quasars in radio source surveys varies with flux threshold. The following discussion on
radio galaxy-cluster associations should be then generally considered to include radio quasars although we mainly discuss galaxies as sources.

The enhancement factors against lensing magnification for different galaxies can be calculated by eq.(1), and their results are shown in Fig.1. Because the $B-$ galaxy counts have a constant slope just above 0.4, large enhancement factors of up to 2 are unlikely for realistic lensing objects. Hence no significant associations of blue galaxies with clusters are expected. Associations between clusters and $K-$ selected galaxies are very similar to quasar-galaxy associations (Wu, 1993), and can be either positive and “negative” because of the “knee” of $K \sim 17$ in the counts. Bright radio sources ($S > 1$ Jy) are the most promising objects for searching strong associations with clusters (large $q$ values) while the enhancement of fainter sources with $S < 10$ mJy appears to be negative ($q < 1$). Note, however, that the above calculations require the extrapolations of source number counts into fainter magnitude/flux. The subsequent quantities of $m + 2.5 \log A$ and $S/A$ in the number counts of eq.(1) may exceed observational limiting magnitude or flux threshold when $A$ becomes very large.

3. Galaxy clusters as lenses

Lensing properties may depend on a priori choice of the mass profiles for lensing objects (Wu & Hammer, 1993) whilst the real matter distribution in clusters of galaxies is still poorly known. It is commonly believed that the isothermal model and the King model can both fit fairly well the luminous matter (optical/X-ray) distribution in clusters of galaxies. However, the presence of giant arcs in rich distant clusters implies highly concentrated dark matter in their cores (Hammer, 1991; Wu & Hammer, 1993; Le Fèvre et al., 1993). For the purpose of study of galaxy-cluster associations, only moderate and small magnifications are actually concerned, i.e., the regions to be considered are far from the central cores. It is then convenient to assume a singular isothermal sphere for matter distribution in clusters, which would greatly simplify the calculations.

For simplicity we consider a pointlike source for the background galaxy and leave the discussion about extended sources to the next section. The magnification of a point source by a singular isothermal sphere is

$$A = \frac{\theta}{\theta - \theta_c}.$$  \hspace{1cm} (4)

Where $\theta$ measures the distance to the center of a galaxy cluster with a critical radius (Einstein radius) of

$$\theta_c = 4\pi \left(\frac{\sigma}{c}\right)^2 \frac{D_{ls}}{D_s}.$$  \hspace{1cm} (5)
Here $D_{ds}$ and $D_s$ are the angular diameter distances from the lensing cluster and from the observer to the distant sources, respectively.

Without the detailed knowledge of velocity dispersion in clusters of galaxies, the typical features of $q$ with the limiting magnitude/flux threshold can be found by using the critical radius ($\theta_c$) as distance unit. Fig. 2 illustrates the enhancement factors for $B$-

![Fig. 2. The dependence of enhancement factor $q$ on limiting magnitude/flux threshold. The search regions are chosen in the unit of the critical radius ($\theta_c$). (a) $B$-selected galaxies; (b) $K$-selected galaxies; (c) Radio sources.](image)
and radio selected galaxies within the regions of 1.5, 2, 4 and 8 critical radii. As it is expected, blue galaxy-cluster associations are independent of limiting magnitude. It is very likely that a null evidence on such associations may be reached because of the corresponding low value of \( q < 1.1 \). Conversely to this, infrared galaxies turn to present noticeably both positive and “negative” associations. For \( K < 17 \) galaxies, a high value of \( q \) is expected, especially when one looks at the regions near the critical radius. At the fainter end \( (K > 18) \), a considerably smaller number of infrared galaxies would be found, leading to “negative” associations. This “negative” effect is even much stronger than in the quasar-galaxy associations (Wu, 1993). The turnover point appears between \( K = 17 \) and \( K = 18 \) because of the break of number counts at \( K \approx 17 \). It is then suited to choose a survey limiting magnitude well outside this range (below or above) in order to study \( K \) galaxy-cluster associations. For radio sources, it is very hopeful to observe large enhancements at high flux limits. Choosing a flux threshold of \( S_{5\text{GHz}} = 3-4 \) Jy would provide relatively larger values of \( q \). Although the “negative” associations exist for \( S_{5\text{GHz}} < 30 \) mJy, it is unlikely that one can easily see them nowadays without considerably large radio telescope time.

For a further discussion of clusters of galaxies as lensing objects, one needs to consider two parameters appearing in the critical radius \( \theta_c \): the distance factor \( D_{ds}/D_s \) and the velocity dispersion \( \sigma \). If one can eliminate the factor of \( D_{ds}/D_s \) in eq.(5), the uncertainties in distances to foreground clusters and background galaxies would vanish in the calculation of enhancement factor \( q \). This can be indeed reached by properly choosing nearby clusters of galaxies and distant sources so that \( D_{ds}/D_s \approx 1 \). Nevertheless, the distance parameter \( D_{ds}/D_s \) only affects the critical radius. Therefore, taking into account of the actual distances of the clusters and the galaxies is equivalent to having a lens with smaller critical radius, which does not change the \( q \) in shape shown in Fig.2. The following calculations will then be made for the “maximum” Einstein radius assuming \( D_{ds}/D_s = 1 \).

An exact treatment of finding \( q \) for background galaxies would be made by considering the luminosity \( (L) \) distribution of clusters of galaxies, then to estimate their corresponding velocity dispersions \( (\sigma) \) throughout a quasi- Faber-Jackson relation and finally to integrate over the redshift space. This procedure of statistical lensing by clusters of galaxies has been extensively investigated by Wu and Hammer (1993). Nevertheless, statistical lensing by clusters of galaxies may still have some uncertainties today, mainly resulting from the fact that the \( L - \sigma \) relation for clusters of galaxies has not been very well determined. Here we simply consider two individual clusters, one with large \( \sigma \) (rich
Fig. 3. The enhancement factors $q$ of $B$-selected galaxies versus search ranges ($\theta$) in nearby poor ($\sigma = 600$ km/s) and very rich ($\sigma = 1200$ km/s) galaxy clusters.

Fig. 4. The same as Fig. 3 but for $K$-selected galaxies.
cluster) and another with low $\sigma$ (poor cluster). Fig.3 - 5 give the results of $q$ derived from two nearby clusters with $\sigma = 1200$ km/s and $\sigma = 600$ km/s, respectively, for the three wavelength selected galaxies, assuming that redshift of the sources is much larger than redshift of the lenses. Compared to Fig.2, the similar features except the amplitudes are shown. It strengthens that nearby and very rich clusters of galaxies are the best targets to test the galaxy-cluster associations.

4. Discussion

4.1. Extended sources

The above calculations have been made under the assumption that background galaxies are pointlike. We now discuss if the extended sources would contribute any significant effects on the $q$ derived from the pointlike sources. Indeed at small virtual impact parameters, magnification factor reaches a maximum value for an uniform luminous disk (formation of an Einstein ring) instead of a linear increase of magnification for a point source. This type of magnification distribution is valid for any spherical lenses, and such an example for a point lens can be found in Bontz (1979). Hence, lensing magnification for an extended source by a spherical lens can be approximated by using the magnification
for a pointlike source till the point where magnification exceeds its maximum value for a uniform disk source.

For a singular isothermal sphere as deflector, the maximum magnification of a uniform disk with radius of \( R \) is

\[
A_{max} = 1 + 8\pi \left( \frac{\sigma}{c} \right)^2 \frac{D_{ds}}{R}. \tag{6}
\]

Taking a standard cosmological scenario of \( \Omega = 1 \) and \( H_0 = 50 \text{ km/s/Mpc} \), eq.(6) reads

\[
A_{max} = 1 + 116.35 \left( \frac{\theta_c}{10''} \right) \left( \frac{R}{10 \text{kpc}} \right)^{-1} h_{50}^{-1}. \tag{7}
\]

A simple numerical calculation shows that for galaxies having redshifts ranging from 0.2 to 3, \( A_{max} \) is not smaller than \( \sim 10 \) even if \( \theta_c \) is taken to be 10" (Note that this critical radius corresponds to a poor galaxy cluster with \( \sigma = 600 \text{ km/s} \), see Fig.3). Indeed, this maximum magnification is large enough for the consideration of associations. Recall that only giant arc events can reach such a high magnification of \( \sim 10 \). Therefore, if one does not choose the regions very close to the critical line, the pointlike model for background galaxy would be a good approximation. For instance, using \( \theta = 1.5\theta_c \) in Fig.2 gives \( A = 3 \) and using \( \theta = 50'' \) in Fig.3 with \( \theta_c = 41'' \) gives \( A \approx 6 \), both of which are smaller than the maximum magnification of \( \sim 10 \) provided that the source was an extended galaxy with a luminous disk of 10 kpc. Indeed, disk radii of galaxies are usually smaller than 10 kpc, implying that for moderate magnification the point source hypothesis is reasonable.

4.2. Optical galaxies

As were shown in Fig.1–3, blue galaxy-cluster associations are unlikely detectable because the slope of the blue galaxy counts is very close to 0.4. Moreover, the relative excess of faint blue galaxies, provided by the fact that there is no apparent turn-over in their number-magnitude relation, is probably due to an excess of dwarves at low redshifts (Cowie, Songaila & Hu, 1992; Tresse et al, 1993), the latter being not the ideal sources for studying galaxy-cluster associations.

\( K \)-selected galaxies span rather a larger range of redshifts. The median redshift of galaxies is 0.6 for \( K = 18–19 \) (Cowie & Songaila, 1992), appearing to be the good targets for studying galaxy-cluster associations. Taking \( K = 19.5 \) to be the limiting magnitude in the survey would result in a significant effect of \( q < 1 \) (see Fig.4), which might provide a better example than in the quasar-galaxy surveys to test the “negative” associations. Certainly, brighter galaxies in \( K \) can be also used to see the strong positive associations.
However, contamination by cluster galaxies (especially ellipticals which are concentrated towards cluster centers) would render a difficulty for a simple analysis based on images, implying that a dedicated (and telescope time consuming) spectroscopic work would be required.

4.3. Radio sources

Bright radio sources provide the largest enhancement due to their steep slope of number counts at $S > 1$ Jy, constituting the ideal sources to find enhancement factor $q$ for galaxy-cluster associations by gravitational lensing. Indeed, five out the 23 3CR radio galaxies at $z \geq 1$ have been found to having foreground clusters of galaxies within about 1 arcminute near the lines of sight (Hammer & Le Fèvre, 1990), while the probability of observing these pairs randomly in the sky is only $0.05 \sim 0.06$. It then remains very promising to soon have a more statistically significant sample of radio galaxy-cluster associations which can be selected from the known bright radio source surveys. Thus, there might be no need for further radio observations for such a purpose and the work of extracting $q$ in the catalogues of bright radio source and clusters of galaxies is underway.

5. Conclusions

We have extrapolated the quasar-galaxy associations into larger systems, namely galaxy-cluster associations, which result from gravitational lensing by foreground clusters of galaxies. We found that these associations are indeed observable, especially for $K-$ and radio selected galaxies. Similar to quasar-galaxy associations, enhancement factor describing galaxy-cluster associations depends sensitively on number-magnitude or number-flux relations of background galaxies. We have pointed out the importance of the choice of the limiting magnitude or of the flux threshold in the surveys, which relate closely to the enhancement factors whether they are larger than unity (positive associations) or smaller than unity (“negative” associations). $K-$ selected galaxies turn to be the good targets to test “negative” associations” while the radio sources would provide the strongest positive ones, already detectable in the bright radio samples like 3CR.

To easily confirm galaxy-cluster associations, one needs to choose nearby rich clusters of galaxies in the sense that the distance $D_{ds}$ should be very close to $D_s$. This may help to provide large Einstein radius, leading to large areas for searching associated background galaxies enough far away from the central core of galaxy cluster. Rich clusters have large Einstein radius and provide significant enhancement factors. Therefore, nearby Abell clusters with $\sigma > 1000$ km/s would be strongly recommended for the statistical study
of galaxy-cluster associations. A direct knowledge of velocity dispersion in the cluster of galaxies would be also very useful to estimate in advance the Einstein radius and then the theoretical enhancement factor.

Yet, the present paper deals with only the simple models for matter distribution in and spatial distribution for clusters of galaxies, and the evolution of clusters of galaxies have not been included. Furthermore, the pointlike model for luminous disks of background galaxies needs to be improved. Both the detailed theoretical consideration and observational tests for galaxy-cluster associations will be made soon.

Acknowledgements. WXP wishes to thank CNRS and K.C. Wong Foundation for financial support.

References

Bontz R.J., 1979, ApJ, 233, 402
Boyle R.J., Shanks T., Peterson B.A., 1988, MNRAS, 235, 935
Cowie L.L., Songaila A., 1992, in First Light in the Universe: Stars or QSO’s ?, eds. B.Rocca-Volmerange et al. (Editions Frontieres), 147
Cowie L.L., Songaila A., Hu, E., 1992, Nature, 354, 460
Dunlop J.S., Peacock J.A., 1990, MNRAS, 247, 19
Gott III J.R., Gunn J.E., 1974, ApJ, 190, L105
Hammer F., 1991, ApJ, 383, 66
Hammer F., Le Fèvre O., 1990, ApJ, 357, 38
Hammer F., Nottale L., 1986, A&A, 167, 1
Hammer F., Nottale L., Le Fèvre O., 1986, A&A, 169, L1
Hoag A., 1981, BAAS, 13, 799
Langston G.I., Conner S.R., Heilin M.B., Lehar J., Burke B.F., 1990, ApJ, 353, 34
Le Fèvre O., Hammer F., Angolin M.C., Gioia I.M., Luppino, G.A, 1993, ApJ, in press
Le Fèvre O., Hammer F., Jones, J., 1988, ApJ, 331, L73
Le Fèvre O., Hammer F., Nottale, L., Mazure, A., Christian, C., 1988, ApJ, 324, L1
Lynds R., Petrosian V., 1986, BAAS, 18, 1014
Narayan R., 1989, ApJ, 339, L53
Narayan R., 1992, in Gravitational Lenses, eds. R.Kayser, T.Schramm and L.Nieser (Springer-Verlag), 88
Roberts D.H., O’Dell S.L., Burbidge G.R., 1977, ApJ, 216, 227
Schneider P., 1992, A&A,
Soucail G., Mellier Y., Fort B., Picat J.P., 1987, A&A, 172, L14
Tresse L., Hammer F., Le Fèvre O., Proust D., 1993, A&A, 277, 53
Tyson J.A., 1988, AJ, 96, 1
Wu X.P., 1993, A&A, submitted
Wu X.P., Hammer F., 1993, MNRAS, 262, 187

This article was processed by the author using Springer-Verlag \LaTeX A&A style file 1990.