Global Non-ideal Magnetohydrodynamic Simulations of Protoplanetary Disks with Outer Truncation

Haifeng Yang1,3 and Xue-Ning Bai1,2

1 Institute for Advanced Study, Tsinghua University, Beijing, 100084, People’s Republic of China; yanghaifeng@tsinghua.edu.cn
2 Department of Astronomy, Tsinghua University, Beijing, 100084, People’s Republic of China; xbai@tsinghua.edu.cn
3 C.N. Yang Junior Fellow.

Received 2021 August 22; revised 2021 September 7; accepted 2021 September 7; published 2021 November 30

Abstract

It has recently been established that the evolution of protoplanetary disks is primarily driven by magnetized disk winds, requiring a large-scale magnetic flux threading the disks. The size of such disks is expected to shrink with time, as opposed to the conventional scenario of viscous expansion. We present the first global 2D non-ideal magnetohydrodynamic simulations of protoplanetary disks that are truncated in the outer radius, aiming to understand the interaction of the disk with the interstellar environment, as well as the global evolution of the disk and magnetic flux. We find that as the system relaxes, the poloidal magnetic field threading the disk beyond the truncation radius collapses toward the midplane, leading to a rapid reconnection. This process removes a substantial amount of magnetic flux from the system and forms closed poloidal magnetic flux loops encircling the outer disk in quasi-steady state. These magnetic flux loops can drive expansion beyond the truncation radius, corresponding to substantial mass loss through a magnetized disk outflow beyond the truncation radius analogous to a combination of viscous spreading and external photoevaporation. The magnetic flux loops gradually shrink over time, the rates of which depend on the level of disk magnetization and the external environment, which eventually governs the long-term disk evolution.

Unified Astronomy Thesaurus concepts: Protoplanetary disks (1300); Magnetohydrodynamical simulations (1966); Magnetic fields (994)

1. Introduction

The global evolution of protoplanetary disks (PPDs) is largely governed by mechanisms that drive angular momentum transport, where magnetic fields are believed to play a crucial role. Conventionally, the disk is considered to evolve viscously, which leads to outward angular momentum transport, with turbulent viscosity from the magnetorotational instability (MRI, Balbus & Hawley 1991). In PPDs, however, the MRI is found to be largely suppressed or damped due to the weak level of disk ionization (Bai & Stone 2013; Bai 2013; Simon et al. 2013; Gressel et al. 2015), and disk evolution is believed to be primarily driven by a magnetized disk wind, which extracts and carries away disk angular momentum through an outflow mediated by magnetic forces (Blandford & Payne 1982).

Launching magnetized disk winds requires a large-scale poloidal magnetic flux threading the disks, and the disk winds are launched from the disk atmosphere that flows along such poloidal field lines. It is well known that solutions for the magnetized disk wind are intrinsically global, determined by both disk microphysics (that determines the coupling of gas with magnetic fields) and global boundary conditions (that determine the global field configuration). Early studies of disk winds dismissed disk microphysics by treating the disks as razor thin and simplified boundary conditions by imposing the self-similar ansatz (Konigl 1989; Pudritz & Norman 1986; Li 1995; Krasnopolsky et al. 1999). Later studies relaxed the assumption of a razor-thin disk and considered disks of finite thickness, usually inserting a certain level of disk resistivity to mimic turbulent dissipation (e.g., Casse & Keppens 2002; Zanni et al. 2007; Tzeferacos et al. 2009; Sheiknezami et al. 2012). More recent studies on PPDs have increasingly incorporated more realistic disk microphysics by taking into account the disk ionization structure and the resulting non-ideal magnetohydrodynamic (MHD) effects, both in local (Bai & Stone 2013; Bai 2013; Simon et al. 2013) and global (Gressel et al. 2015; Béthune et al. 2017) simulations. Further studies incorporating more realistic thermodynamics reveal that the wind is magnetothermal in nature (Bai et al. 2016), and the mass-loss rate from the PPD disk wind is likely substantial, comparable to wind-driven accretion rates (Bai 2017; Wang et al. 2019; Gressel et al. 2020).

Despite studies that focus on disk microphysics, attention has rarely been paid to global boundary conditions. PPDs have a finite size, as set by initial conditions from star formation. In previous global simulations, disks are usually taken to be infinitely extended, usually leading to a quasi-self-similar wind structure. However, as a PPD is truncated beyond a certain outer radius, the drop in disk density and hence pressure would likely influence the global distribution of magnetic flux and hence the entire wind solution. Consequently, we anticipate that incorporating a disk outer truncation would likely alter our views on two of the most fundamental problems in the theory of disk evolution, stated below.

First, the evolution of the bulk disk mass reservoir. Typically, most of the disk mass is distributed in the outer disk around and beyond the truncation radius, and it is such regions that largely set the disk evolution timescales. The conventional wisdom is that for viscous disk evolution, the outer disk receives angular momentum from the inner disk and expands, known as viscous spreading (e.g., Lynden-Bell & Pringle 1974; Hartmann et al. 1998). By contrast, for wind-driven accretion, as the disk directly loses angular momentum to the wind, it is anticipated that the disk size should decrease over time. This distinction has been considered in
the recent literature to broadly distinguish the two mechanisms by statistical studies of observed disk sizes over age, with tentative evidence in favor of viscous evolution (Tazzari et al. 2017; Najita & Bergin 2018; Trapman et al. 2020). However, as we shall see, incorporating disk truncation complicates the process of wind-driven disk evolution and it is premature to draw conclusions simply based on trends in disk size evolution.

Second, the evolution of the poloidal magnetic flux. Typically, the rate of wind-driven disk accretion and mass-loss rates directly scale with the amount of magnetic flux threading the disks (e.g., Bai & Stone 2013; Bai et al. 2016; Lesur 2021). Therefore, the more fundamental question for global disk evolution is how the magnetic flux evolves. Early works on magnetic flux transport generally belong to the advection-diffusion framework (Lubow et al. 1994), where inward advection of the magnetic flux due to viscously driven accretion competes with outward diffusion from disk (turbulent) resistivity, with later semi-analytical studies that incorporate the disk vertical structure (Rothstein & Lovelace 2008; Guilet & Ogilvie 2012, 2013), and a radial resistivity profile (Okuzumi et al. 2014). Takeuchi & Okuzumi (2014), though these works largely ignored the wind-driven accretion process and additional non-ideal MHD effects (other than resistivity). More recently, magnetic flux transport has been studied in more realistic global disk simulations (Bai & Stone 2017), semi-analytical local calculations (Leung & Ogilvie 2019), and self-similar numerical solutions (Lesur 2021), which generally find outward flux transport whose rate increases with disk magnetization, in addition to other dependencies. It remains to determine, however, how the results are affected by the presence of a disk outer truncation, which will likely yield different field geometries with important consequences for global flux transport.

In this paper, we carry out long-term global non-ideal MHD simulations of PPDs with outer truncation. Our simulations are two-dimensional (2D) and incorporate ambipolar diffusion (AD) as the dominant non-ideal MHD effect for outer PPDs (e.g., Wardle 2007; Bai 2011). For this first study, and due to the high computational demand for long-term evolution, we do not aim to accurately capture the key disk microphysics, but instead parameterize the main physical ingredients behind the microphysical processes. It will offer a better intuition for interpreting simulation outcomes and dependencies, allowing us to systematically address the two outstanding questions mentioned above.

The structure of this paper is as follows. We describe our 2D global MHD simulations of a truncated disk with prescribed ambipolar diffusion and thermodynamics in Section 2. We start with the discussion of the fiducial run and the general evolution picture of the simulations in Section 3, followed by the parameter study focusing on their impacts on the flux transport rate in Section 4. In Section 5, we discuss our major contributions and caveats. We summarize and conclude in Section 6.

2. Method

2.1. Dynamical Equations

We use Athena++, a newly developed finite-volume Godunov’s scheme MHD code that uses constrained transport to enforce divergence free of magnetic fields (Stone et al. 2020). It is a successor of the widely used Athena MHD code with much more flexible coordinate and grid options in addition to a significantly improved performance and scalability. We carry out global simulations of PPDs and solve the MHD equations in conservation form:

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \tag{1}
\]

\[
\frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} - \mathbf{B} \mathbf{B} + \left( P + \frac{B^2}{2} \right) \mathbf{I} \right] = -\nabla \Phi, \tag{2}
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot \left( \left( E + P + \frac{B^2}{2} \right) \mathbf{v} - \mathbf{B} (\mathbf{B} \cdot \mathbf{v}) \right) = -\Lambda, \tag{3}
\]

where \( \rho, \mathbf{v}, \) and \( P \) are gas density, velocity, and pressure, respectively, \( \mathbf{B} \) is the magnetic field, \( E = P/\gamma - 1 + \rho v^2/2 + B^2/2 \) with \( \gamma \) as the adiabatic index, \( \Phi = -GM/r \) is the gravitational potential of the protostar, and \( \Lambda \) is the cooling rate. Note that in code units and the above equations, factors of \( 4\pi \) are absorbed so that magnetic permeability is 1. In addition, we have the induction equation with non-ideal MHD effects

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta_\Omega \mathbf{J} + \eta_\Lambda \mathbf{J}_\perp), \tag{4}
\]

where the \( \eta_\Omega \) and \( \eta_\Lambda \) are ohmic and ambipolar diffusivities, respectively, \( b = \mathbf{B}/B \) is the unit vector for the magnetic field direction, \( \mathbf{J} = \nabla \times \mathbf{B} \) is the current density, and \( \mathbf{J}_\perp = - (\mathbf{J} \times \mathbf{b}) \times \mathbf{b} \) is the component perpendicular to the magnetic field. Note that we mainly consider AD that dominates the outer disk and ignore the Hall effect, which is more important toward higher-density inner disk regions. We retain ohmic resistivity for numerical reasons, to be described in Section 2.2.3.

2.2. Simulation Setup

We perform 2D global simulations in spherical-polar coordinates \((r, \theta)\) assuming axisymmetry. In our data analysis, we also use the cylindrical radius \( R = r \sin \theta \). Our code units are chosen such that \( GM = 1 \) and \( r = r_0 = 1 \) at the inner boundary. This yields a natural unit of time to be \( \Omega_0^{-1} \), where \( \Omega_0 = \sqrt{GM/r_0^3} \) is the Keplerian frequency at the inner boundary. With these, we define temperature as the ratio of the pressure to the density, \( T = P/\rho \), with a code unit of \( GM/r_0 \). We further take the density at the disk midplane at the inner boundary \( \rho_0 = 1 \).

For the initial condition, we adopt a two-component model: a truncated disk component and a background surrounding envelope component, to be elaborated below.
2.2.1. Disk Component

For the truncated disk component, we adopt a power-law midplane density distribution with a Gaussian cutoff:

$$\rho_{\text{mid}}(r) = \rho_0 \left( \frac{r}{r_0} \right)^{-\alpha} \exp \left[ -\left( \frac{r}{r_e} \right)^2 \right],$$  \hspace{1cm} (5)

where $r_c$ is the truncation radius, and $\alpha$ is the power-law index of the radial density profile. The form of the density cutoff is not well known observationally, and we choose a Gaussian cutoff that is numerically convenient for its rapid falloff so that a substantial density drop occurs over a reasonable radial range. This profile is also subject to later development and will be modified once disk winds are launched. The truncation radius $r_c$ is associated with a Keplerian rotation period $P_c = 2\pi \sqrt{r_c^3/GM}$, which characterizes the dynamical timescale of the outer disk and which we will normalize time to $P_c$ when we later discuss the long-term evolution.

The temperature profile for the disk component is prescribed with a midplane temperature, $T_{\text{mid}} = T_0 (r/r_0)^{-1}$, and is constant in the cylindrical $z$ direction. In code units, we take $T_0 = 0.01 GM/r_0$, which, together with the power-law index $-1$, grants a constant aspect ratio of $H/R = 0.1$ for simplicity. The corresponding vertical structure of the disk in hydrostatic equilibrium is:

$$\rho_d = \rho_{\text{mid}} \exp \left[ \frac{GM}{T_0} \left( \frac{r}{r_0} \right) \left( \frac{1}{r} \frac{1}{r \sin \theta} \right) \right].$$  \hspace{1cm} (6)

where $\rho_d$ denote the disk component density. The initial velocity has zero $r$ and $\theta$ components. In order to balance the radial pressure gradient and the gravity without magnetic fields in this truncated disk setup, the $\phi$ component of velocity takes the following form:

$$v_\phi^2 = \frac{GM}{r} - (\alpha + 1) T_0 \frac{r_0}{r \sin \theta} \frac{r_0 r \sin \theta}{r_c^2}.$$  \hspace{1cm} (7)

Note that due to the existence of the third term, $v_\phi^2$ can be negative in some atmosphere regions. We set $v_\phi = 0$ in such cases.

For our fiducial run, we choose $r_c = 30 r_0$, corresponding to $P_c \approx 103200 G_0^{-1}$, and $\alpha = 9/4$, corresponding to a surface density profile of $\Sigma(r) = \Sigma_0 (r/r_0)^{-1.25} \exp[-(r/r_c)^2]$.

The magnetic field in the disk component is initialized to be purely poloidal with a constant plasma $\beta_0$ parameter, defined as $\beta_0 = P/P_B$, $P_B = B^2/2$, in the midplane. To do so, we first calculate the midplane magnetic flux through a numerical integral:

$$\Phi(r) = \int_{r_{\text{min}}}^r 2 \pi r' dr' \sqrt{2 \rho_{\text{mid}}(r) T_{\text{mid}}(r)/\beta_0},$$  \hspace{1cm} (8)

where $r_{\text{min}} = 0.1 r_0$ is the smallest radius in the midplane with penetration of magnetic field lines. With this definition, the midplane vector potential is given by $A_{\phi,\text{mid}}(r) = \Phi(r)/(2 \pi r)$, and it can be easily verified that the poloidal field given by $B = \nabla \times (A_{\phi,\text{mid}})$ equals $\sqrt{2 \rho B/\beta_0}$ at the midplane.

The dependence of the vector potential on $\theta$ is taken to be (Zanni et al. 2007):

$$A_\phi(r, \theta) = A_{\phi,\text{mid}}(r) [1 + (m \tan \theta)^2]^{-5/8},$$  \hspace{1cm} (9)

where $m$ is a parameter that determines how much the fields bend and we set $m = 1$ throughout this work. For pure poloidal magnetic fields, $A_\phi$ is sufficient to reconstruct the magnetic fields.

Given Equation (8) and the density and temperature profiles, the total magnetic flux in the disk can be integrated as:

$$\Phi_d = 2 \pi r_c^2 \sqrt{\rho_0 T_0} \frac{\sqrt{2 r_c}}{r_0} \left( \frac{r_0}{r} \right)^{(1+\alpha)/2} \Gamma \left( \frac{3 - \alpha}{4} \right),$$  \hspace{1cm} (10)

where $\Gamma(x)$ is the special Gamma function. Note that here we have set $r_{\text{min}} = 0$ to make the integration analytic. As most magnetic flux resides in the outer disk, the choice of $r_{\text{min}}$ is unimportant for this purpose. This flux serves as a normalization factor for the magnetic flux in our diagnostics later.

2.2.2. Combination withEnvelope Component

For the envelope component, we adopt a simple power-law model. The density has the same power-law profile as the disk component, except for the Gaussian cutoff: $\rho_{\text{en}}(r/r_0)^{-\alpha}$, while it is constant in the $\theta$ direction. The envelope density at the inner boundary $\rho_{\text{en}}$ is taken as $10^{-9}$ in our fiducial run. The envelope component is completely at rest, with all three components of velocity being zero throughout the simulation domain. The density is summed up to obtain the total density of our model, while the velocities are set as the density-weighted values. The initial density profile can be viewed in the upper left panel of Figure 1.

The envelope temperature has the same power-law index as the disk component at small radii, but with a larger leading figure, $T_{\text{en}} = T_0 (r/r_0)^{-1}$. We call this the “corona temperature,” whose meaning will become obvious soon. At large radii, the temperature transitions to a constant plateau value $T_{\text{cor}}$:

$$T_e = \begin{cases} T_0 & r \ll r_k, \\ T_{\text{cor}} & r \gg r_k, \end{cases}$$  \hspace{1cm} (11)

where $r_k = r_0 (T_{\text{cor}}/T_e)$ is where these two profiles meet. These two temperature profiles are connected smoothly through a circular arc in the $(\log(r), T)$ plane. We take $T_{\text{cor}} = 0.001$ in the fiducial run.

The final temperature of our model is the density-weighted temperature of these two components. At large radii, the temperature is determined by the envelope component. At small radii, the system transitions from the midplane temperature $T_{\text{mid}}$ to the corona temperature $T_{\text{cor}}$ ($> T_{\text{mid}}$) in the polar regions. In reality, the atmosphere/corona of the disk is heated to higher temperatures by absorbing the stellar far-ultraviolet radiation (Glassgold et al. 2004; Walsh et al. 2012). Since we

---

Footnote 5: In the midplane, this is inside the disk inner boundary, and field lines originating from this location will enter the simulation domain through the upper/lower parts of the inner boundary.

Footnote 6: Note that both the power-law temperature profile at the inner radius and the constant temperature at a large radius are straight lines when plotted in the $(\log(r), T)$ plane. These two lines can be connected through an arc of a circle that is tangent to both of these two lines. The exact form of the transition will be set once the center of the circle is given. The center of the circle is chosen to be located at 1.1 $\log(r_k)$ in the $(\log(r), T)$ plane.
aim to set up a disk with a constant aspect ratio, we enforce the temperature of the disk component to transition from the midplane value $T_{\text{mid}}$ to the corona value $T_{\text{cor}}$. This transition occurs around $\theta = \pi/2 \pm \theta_{\text{trans}}(H/r)$, with a transition width of 0.1. The far-ultraviolet photons heating the corona will also ionize the atmosphere region, leading to weaker non-ideal MHD effects. Because of this, we also use the same transition height for the AD, which will be discussed in more detail in Section 2.2.3. The transition height is $\theta_{\text{trans}} = 4$ in the fiducial run. The full initial temperature profile can be viewed in the upper right panel of Figure 1.

Note that this combination of disk and envelope components makes the initial disk component not in hydrostatic equilibrium. Without rotation, the envelope component will also tend to fall toward the central object. However, it does not matter much, since no equilibrium exists once we introduce magnetic fields, driving accretion and outflows. The prescriptions above simply serve to provide an educated guess for the initial density and temperature structure, and the system will relax toward a new steady state. Also, the envelope component will soon be pushed outside of the simulation domain, as we will see in Section 3.

The magnetic field in the envelope is in the $z$ direction if converted to a cylindrical coordinate with a uniform magnitude of $B_z = 3 \times 10^{-10}$ in the fiducial run. This value is chosen such that, at around $r = 100 R_0$, the envelope has a plasma $\beta \approx 10^5$. The envelope magnetic field strength is also modified when we change the plasma $\beta_0$ for the disk component to ensure a similar plasma $\beta$ in the outer region. The vector potential for this uniform magnetic field is simply $A_{\phi} = (1/2) r \sin(\theta) B_{r,e}$. We have experimented with different choices of the envelope component and the results are largely independent of any specific choice because, as we shall see, the envelope component is quickly blown away after the simulation starts. The vector potential $A_{\phi}$ of the disk and envelope components are added up to set up the total initial magnetic field in our simulation. The initial poloidal field lines are overlaid in the upper left panel of Figure 1.

As the system evolves, we relax the temperature at the rate of the local Keplerian frequency at the midplane. This corresponds to a cooling rate of:

$$\Lambda = \frac{\rho}{\gamma - 1} \Omega(T_{\text{int}} - T),$$

where $T_{\text{int}}$ is the initial temperature, and $\Omega = \sqrt{GM/r^3}$ is the local Keplerian angular speed. We also require the ratio with the initial temperature to be no larger than 5.

### 2.2.3. Non-ideal MHD Coefficients

For the non-ideal MHD effects, we mainly consider ambipolar diffusion (AD), which is the dominant effect in the outer regions of PPDs. In this work the AD diffusivity is prescribed through the dimensionless AD Elsässer number $A_{m} = v_{\text{A}}^2/\eta_{\text{m}} \Omega$, where $\Omega$ is the disk Keplerian frequency in terms of spherical $r$ and $v_{\text{A}} = B_{r,e}^{1/2}$ is the Alfvén velocity. We note that unless charged grains become the dominant charge carrier, $\eta_{\text{m}} \propto B^2$ and hence $A_{m}$ is independent of field strength, and its value is typically on the order of unity in the outer disk (Bai 2011). We set $A_{m} = 0.3$ in the bulk disk, which gradually transitions to $A_{m} = A_{m_{\text{out}}}$ beyond $r = r_c$, and to $A_{m} = 100$ in the disk atmosphere. The latter ($A_{m} = 100$) mimics a strong boost in the level of ionization due to far-ultraviolet (FUV) radiation (Perez-Becker & Chiang 2011), which has become the standard practice in recent disk simulations. This transition occurs around $\theta = \pi/2 \pm \theta_{\text{trans}}(H/r)$ with a transition width of $1(H/r)$. This transition of $A_{m}$ resembles that in the temperature profile given their similar physical origins. The value of $A_{m_{\text{out}}}$ is more uncertain and likely depends on the interstellar environment, and we choose $A_{m_{\text{out}}} = 1$ as the fiducial value. In practice, the $A_{m}$ transitions from its inner value to $A_{m_{\text{out}}}$ around the transition radius $r_c$, and the transition width is 20$r_0$. The $A_{m}$ profile can be viewed in the lower left panel of Figure 1.

Besides AD, we also artificially introduce ohmic resistivity near the inner boundary as well as the midplane region, which solely serves to better stabilize the simulations (e.g., Cui & Bai 2020). In the midplane region, adding such an artificial resistivity helps suppress the MRI turbulence and stabilize the current sheet that inevitably develops where $B_{\phi}$ changes sign. The midplane resistivity is prescribed as $\eta_{\text{m}} = f_{\text{m}} c_{\text{s}} H$, with $c_{\text{s}}$ being the sound speed. We take $f_{\text{m}} = 0.02$ as the fiducial value. It gradually transitions to 0 around $\theta = \pi/2 \pm 0.1$. We have tested that the exact strength of the artificial resistivity has virtually no effect on our results (see Figure 4). We also introduced an artificial resistivity near the inner boundary

---

$^7$ The transition in this work is prescribed as following. Let $f(x)$ be the function in question, which transitions between two constants $f_1$ and $f_2$ around $x = x_d$ over width $\delta x$. We then adopt:

$$f(x) = f_1 + \frac{1}{2}(f_2 - f_1) \left( \tanh \left( \frac{x - x_d}{\delta x/2} \right) + 1 \right)$$

In what follows, we will adopt this formula for the transition unless noted otherwise.

$^8$ It is not needed in 3D, which can properly capture the MRI turbulence (e.g., see Cui & Bai 2021), which we will consider in the future.
within $1.5r_0$, which is on order of $0.1c_s H$. The strength of the artificial resistivity, in units of $c_{sH}$, can be viewed in the lower right panel of Figure 1.

### 2.2.4. Grid Setting and Boundary Conditions

Because we run simulations over a very long time, it generally leads to substantial changes to the overall surface density profile, which may destabilize regions near the disk inner boundary. We consider our simulations as numerical experiments under well-controlled conditions, and such changes would undermine the nature of our simulations. To remedy its negative influence, we relax the density within the truncation radius $r < r_c$ to the initial value over a timescale of 200 times the local Keplerian period. This approach has little influence on the overall gas dynamics, but allows us to smoothly run the simulations for dozens of orbits at $r = r_c$ to properly diagnose the results.

We use reflecting boundary conditions in the $\theta$ direction and there is a $2^\circ$ cone near both poles. For the inner $r$ boundary condition, we set the hydrodynamic variables in the ghost zones to our initial condition. In addition, the $v_\phi$ is capped by a rigid body rotation that has the same velocity at $r_0$ with a Keplerian rotation. At the outer boundary in the $r$ direction, the density is extrapolated assuming a power-law index $-\alpha$, the same as for the initial density profile. The temperature is extrapolated as $T \sim r^{-\alpha}$. The $v_r$ and $v_\phi$ are copied to the ghost zones, but only allow outflow ($v_r > 0$). The $v_\phi$ is extrapolated according to the Keplerian rotation profile. The magnetic fields are extrapolated with $B_r \sim r^{-2}$, $B_\theta \sim r^0$, and $B_z \sim r^{-1}$, for both the inner and outer boundary conditions.

The fiducial run has 448 cells along the $r$ direction with a logarithmic spacing, and 160 cells along the $\theta$ direction. The $\theta$ grid is designed such that the ratio between adjacent cells is constant, with a decreasing cell size toward the midplane, where the cell size is matched with the radial grid. The ratio between the sizes of adjacent cells in the $\theta$ direction is 1.01 in the fiducial run. This grid gives about 7.8 cells per scale height in the fiducial run and most other runs.

### 2.3. Simulation Runs

We list all our simulation runs in Table 1. Our fiducial run is labeled Fid. Because we have introduced artificial resistivity, we test whether the results are sensitive to it by conducting two more runs etaM1 and etaM4, which have half and twice the fiducial midplane resistivity, respectively. Run hi_res doubles the resolution for a convergence study. In runs Rc10 and Rc20, we change $r_c$ to 10$r_0$ and 20$r_0$ (and $r_{\text{max}}$ to 100 and 200, accordingly) to examine how our simulation results scale with the outer truncation radius. To test the impact of the envelope component, we set up runs dblRhoe/hlfRhoe with double/half the fiducial envelope density, and run Te1e2/Te2e2 with a higher envelope temperature ($T_{\text{en}} = 0.01$ and 0.02) to make it gravitationally unbound within our simulation.

---

**Table 1**

| Run   | Resolution | $\beta_0$ | $r_c$, $r_{\text{max}}$ | $z_{\text{trans}}$ | $\alpha$ | $\rho_e$ | $T_{\text{en}}$ | $H/R$ | $d\Psi_{\text{max}}/dt$ |
|-------|------------|----------|------------------------|-------------------|------|-------|---------------|-------|------------------|
| Fid   | 448 × 160 | 0.02     | 10$^3$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001            |
| hi_res| 896 × 320 | 0.02     | 10$^3$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001            |
| etaM1 | 448 × 160 | 0.01     | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001            |
| etaM4 | 448 × 160 | 0.04     | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001            |
| Te1e2 | 448 × 160 | 0.02     | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.01             |
| Te2e2 | 448 × 160 | 0.02     | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.02             |
| dblRhoe | 448 × 160 | 0.02     | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 2 × 10$^{-9}$ | 0.001 |
| hlfRhoe | 448 × 160 | 0.02     | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 5 × 10$^{-10}$ | 0.001 |
| Rc10  | 448 × 160 | 0.02     | 10$^3$                 | 10,100            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001            |
| Rc20  | 448 × 160 | 0.02     | 10$^3$                 | 20,200            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001            |
| alpha1.5 | 448 × 160 | 0.02   | 10$^4$                 | 30,300            | 4.0  | 1.5   | 1.0           | 10$^{-9}$ | 0.001            |
| alpha2.0 | 448 × 160 | 0.02   | 10$^4$                 | 30,300            | 4.0  | 2.0   | 1.0           | 10$^{-9}$ | 0.001            |
| alpha2.5 | 448 × 160 | 0.02   | 10$^4$                 | 30,300            | 4.0  | 2.5   | 1.0           | 10$^{-9}$ | 0.001            |
| AMout0.3 | 448 × 160 | 0.02 | 10$^4$                 | 30,300            | 4.0  | 2.25  | 0.3           | 10$^{-9}$ | 0.001 |
| AMout10 | 448 × 160 | 0.02  | 10$^4$                 | 30,300            | 4.0  | 2.25  | 10            | 10$^{-9}$ | 0.001 |
| AMout100 | 448 × 160 | 0.02 | 10$^4$                 | 30,300            | 4.0  | 2.25  | 100           | 10$^{-9}$ | 0.001 |
| zt3.5  | 448 × 160 | 0.02  | 10$^4$                 | 30,300            | 3.5  | 2.25  | 1.0           | 10$^{-9}$ | 0.001 |
| zt4.5  | 448 × 160 | 0.02  | 10$^4$                 | 30,300            | 4.5  | 2.25  | 1.0           | 10$^{-9}$ | 0.001 |
| beta3  | 448 × 160 | 0.02 | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001 |
| beta5  | 448 × 160 | 0.02 | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001 |
| HoR05  | 448 × 160 | 0.02 | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001 |
| HoR15  | 448 × 160 | 0.02 | 10$^4$                 | 30,300            | 4.0  | 2.25  | 1.0           | 10$^{-9}$ | 0.001 |

Note. In the above table, $f_{\beta M}$ is the parameter to control the strength of the artificial resistivity in the midplane; $\beta_0$ is the initial plasma $\beta$ of the poloidal field; $r_c$ is the truncation radius; $r_{\text{max}}$ is the maximum radius in the simulation domain; $z_{\text{trans}}$ defines the transition height for both Ambipolar diffusion and the temperature; $\alpha$ is the power-law index for the density profile; $\text{Amout}$ is the AD Elsässer number at large radius; $\rho_e$ is the density at $r_0$ for the envelope component; $T_{\text{en}}$ is the temperature plateau at large radius for the envelope component. In the last column, we also list the measured rate of flux transport $d\Psi_{\text{max}}/dt$, in units of $\Phi_s/\rho_c$. See Sections 3.2 and 4 for more details.
domain. Runs alpha1.5, alpha2.0, and alpha2.5 have a different power-law index \( \alpha \) for the density profile of the disk component, while runs HoR05 and HoR15 modify the \( H/R \) of the disk component by changing its midplane temperature. To examine the impact of the ionization level in the external environment, we set up runs AMout0.3, AMout10, and AMout100 with \( \alpha_{\text{out}} \) = 0.3, 10, and 100, respectively. Through runs zt3.5 and zt4.5, we study the role of transition heights in the disk for temperature and AD, with \( z_{\text{trans}} \) = 3.5 and 4.5, respectively. Finally, we modify the disk magnetization in runs beta3 and beta5 by changing the initial poloidal plasma \( B_0 \) to \( 10^3 \) and \( 10^2 \), respectively. The envelope magnetic field \( B_{\text{envelope}} \) is also changed in proportion.

For all simulations, we start with a field free of a magnetic field to \( 10 \Omega_0^{-1} \) before imposing the magnetic field. All simulations are run to a time of at least 20 times the Keplerian period at \( r_c \), (roughly \( 2 \times 10^4 \Omega_0^{-1} \) in the fiducial run), allowing us to achieve a quasi-steady state.

### 2.4. Diagnostics

In this section, we present several diagnostics that will aid our interpretation. The first diagnostic is the mass flow/accretion rate, which is very important in understanding the dynamics in the disk. Its dependence can be derived from the momentum conservation equation (Equation (2)). In cylindrical coordinates, the \( \phi \) component of this equation reads:

\[
\partial_t (\rho v_\phi) + \frac{1}{R^2} \partial_R (R^2 M_{R\phi}) + \frac{1}{R} \partial_\phi M_{\phi\phi} + \partial_z M_{\phi z} = -\frac{1}{R} \partial_\phi \Phi,
\]

where the total stress tensor is:

\[
M = \rho v v - B B + \left( P + \frac{B^2}{2} \right) I.
\]

Multiplying both sides of Equation (13) with \( R^2 \), taking the azimuthal average, and ignoring the time derivative because we focus on quasi-steady states, we get:

\[
\frac{\partial}{\partial R} \left( R^2 M_{R\phi} \right) + R^2 \frac{\partial}{\partial z} M_{\phi z} \approx 0,
\]

where \( \bar{X} \) denotes the azimuthal average of the quantity \( X \). Plugging in the total stress tensor, we get:

\[
\frac{\partial}{\partial R} \left( R^2 \rho \bar{v}_\phi \bar{v}_\phi - R^2 \bar{B}_R \bar{B}_\phi \right) + R^2 \frac{\partial}{\partial z} \left( \rho \bar{v}_z \bar{v}_\phi - \bar{B}_z \bar{B}_\phi \right) \approx 0.
\]

Note that in our non-ideal MHD simulations in 2D, the gas is largely laminar and we can pull \( \bar{v}_\phi \) out of the azimuthal average so that, e.g., \( \rho \bar{v}_R \bar{v}_\phi \approx \rho \bar{v}_R \cdot \bar{v}_\phi \). In addition, we have \( \partial_t (\rho \bar{v}_R \bar{v}_\phi) + R \partial_\phi (\rho \bar{v}_R \bar{v}_\phi) = 0 \) in the steady stage due to mass conservation (Equation (1)). With these, we obtain:

\[
\rho \bar{v}_R \approx \frac{\partial}{\partial R} \left( R^2 \bar{B}_R \bar{B}_\phi \right) + R^2 \frac{\partial}{\partial z} \left( \bar{B}_R \bar{B}_\phi \right) - R^2 \rho \bar{v}_z \frac{\partial \bar{v}_\phi}{\partial z}.
\]

In general, the first two terms in the numerator dominate, resulting from magnetic stresses, whereas the third term provides a correction at higher altitude where the outflow mass flux becomes significant. This result will help us interpret the flow structure seen in our simulations.

Our simulations are scale-free, with mass-accretion/mass-loss rates measured in code units. For unit conversion, consider a typical T Tauri disk with mass \( \sim 0.01 M_\odot \) and truncation radius \( r_c \approx 90 \) au: the mass-accretion rate in physical units is:

\[
\dot{M}_{\text{phys}} \approx 5.1 \times 10^{-8} M_\odot/\text{yr} \left( \frac{\dot{M}_{\text{code}}}{10^{-3}} \right) \left( \frac{M_{\text{disk,phys}}}{0.01 M_\odot} \right) \times \left( \frac{M_{\text{star,phys}}}{M_\odot} \right)^{1/2} \left( \frac{R_{\text{phys,phys}}}{90 \text{au}} \right)^{-3/2}.
\]

In the above, subscripts “phys” and “code” denote quantities in physical and code units, respectively. Note that this unit conversion formula applies mainly to our fiducial run and other simulations with the same resolution and disk aspect ratio. Under the same assumption, the temperature conversion is simply \( GM/r_0 \sim 300 \text{/(km/s)}^2 \). The sound speed in the envelope with a temperature of \( T_\text{envelope} = 0.001 \) is roughly 0.5 km s\(^{-1}\), which corresponds to \( \sim 30 \) K. Unit conversion for other runs should be straightforward, but are not shown here.

Finally, we present the diagnostics for magnetic flux transport. The rate of flux transport is set by the \( \phi \) component of the electric field as (Bai & Stone 2017):

\[
\frac{d\Phi}{dt} = -2\pi R e \Phi
\]

This can be further decomposed as:

\[
\frac{d\Phi}{dt} = -2\pi R [v_{BR} B_z - v_{Bz} B_R] + \eta_0 J_0\]

where \( E_{0,1} = v_{BR} B_z - v_{Bz} B_R \) and \( E_{0,NI} = (v_{d,BR} B_z - v_{d,Bz} B_R) + \eta_0 J_0 \). The ion-neutral drift velocity due to the ambipolar diffusion (note that we have ignored the Hall effect, so that ions and electrons have the same velocity), given by:

\[
v_{AD} = \frac{\eta_0}{B} J \times B = \frac{\nabla \times B}{\rho \Omega \cdot Am}.
\]

This decomposition will help us interpret the microphysical mechanism of the magnetic flux transport.

### 3. The Fiducial Run

In this section, we focus on the fiducial simulation run and its high-resolution counterpart. The results are diagnosed in great detail, as they are representative and contain all major aspects of the conclusions of this paper.

#### 3.1. General Evolutionary Picture

Following the initial condition shown in Figure 1, snapshots of the subsequent evolution of the fiducial run are shown in Figure 2. We chose to show only the simulation with a high resolution (hi_res) because the results show no differences in snapshots, aside from a higher resolution. Different columns represent snapshots of the simulation at different time. The three rows, from top to bottom, show the density, mass flow in the \( r \) direction, and the magnetic field structure, respectively.
The magnetic field lines are shown as contours of constant magnetic flux surface in the third row. We emphasize that the initial condition is unrealistic and we are primarily interested in the quasi-steady state after evolving the simulations for a sufficiently long time after which the system largely forgets its initial conditions. Accordingly, the progress of the simulations can be roughly divided into three stages.

At the first stage, magnetic disk winds are launched initially upwards, then radially outwards. At the same time, the toroidal component of the magnetic field $B_{\phi}$ starts to build up due to the differential rotation in the system. Gradually, the disk wind pushes all of the envelope away and dominates the simulation domain. After about one orbital period at the truncation radius $r_c$, $(t \gtrsim 1000)$, represented by the third column, the envelope material is pushed away and all of the initial magnetic fluxes in the envelope are lost through the outer boundary. During this process, the material lifted by the disk wind falls down to the midplane carrying magnetic fluxes with it. This results in magnetic reconnection at the outer region of the simulation domain, which breaks the open field lines and forms loops of poloidal magnetic fields enclosing the outer disk (see the first two panels of Figure 2).

After the rapid relaxation in the first stage, the second stage is a slower relaxation process in which we observe the
adjustments and fluctuations of density, gas flow, and magnetic field structures beyond the truncation radius (not shown in Figure 2). These variations are still a reflection of initial conditions, which are again not the focus of this study. Toward the end of this stage, corresponding to about 10 orbits at $r_c$, all field loops encircling the outer disk region are smooth and round, with a stable flow structure (e.g., see Figures 4, 11), marking that we have reached a quasi-steady state.

We choose the third, quasi-steady, stage to start from $t = 14,000 \alpha_0^{-1}$. During this stage, the magnetic loops gradually shrink and eventually reconnect and disappear from the loop center, which is around $r = 50$. It is this stage that likely reflects the reality and will be the focus of our study. While we have run the simulation to a much longer time (e.g., see Figure 4), we note that the loss of magnetic flux (through dissipation of poloidal field loops) also changes the dynamics. We thus only consider time intervals after reaching this third stage, while the system still keeps most of its magnetic flux. For most of the discussions, we average all dynamical quantities from $t = 14,000 \alpha_0^{-1}$ to $t = 20,000 \alpha_0^{-1}$ for diagnostics, unless noted explicitly otherwise.

There are several important characteristic radii that are important in the diagnostics of the disk dynamics. In addition to the truncation radius $r_c$, there exists a transition radius $R_{t,v}$ in the midplane where the radial flow changes direction. The net motion of disk material at $R > R_{t,v}$ is outward, while for $R < R_{t,v}$ matter is accreting, as can be seen in Figures 3 and 5. The hydrodynamics is closely related to the poloidal magnetic field structure, which forms loops beyond the truncation radius. The center of the loop is defined as $R_{t,\phi}$, which is the location where the 2D magnetic flux function $\Phi(r, \theta)$ is maximized, defined as:

$$\Phi(r, \theta) = \int_0^\theta B_r(r, \theta') \times 2\pi r^2 \sin \theta' d\theta'.$$ (19)

We will see that $R_{t,v}$ and $R_{t,\phi}$ are closely related later in Section 3.4.

Across all three stages in our simulations, we observe a steady transport of magnetic fluxes, angular momentum, and mass in the disk region within the truncation radius $r_c$. If we zoom in to just this part of the simulation, we find results similar to those discussed in Bai & Stone (2017), at a qualitative level. This disk region mainly serves as a sanity check and will not be the focus of our discussion in this paper, but measurements of some fundamental quantities will be presented later in this section for a more quantitative comparison.

### 3.2. Accretion Rate and Mass Loss

We start by analyzing accretion and mass-loss rates in our fiducial simulation. The presence of a disk outer truncation presents some ambiguity regarding where the disk boundary is when calculating the disk mass-loss rates. In Figure 3, we show the mean density and velocity vectors in stage three in the upper left panel, where the velocity vectors are in unit lengths to enhance the visibility. In this figure, we define a contour to calculate the accretion rate and mass loss quantitatively. This contour consists of three parts: two constant $\theta$ lines with $\theta = \pi/2 \pm \delta_\theta$, where $\delta_\theta = 3.5(H/r)$, and one arc at $r = 100$. Here, we define the contour with $3.5(H/r)$ lines instead of $z_{\text{trans}}(H/r)$, with $z_{\text{trans}} = 4$. This is partly because our diagnostics, written in cylindrical coordinates, are more accurate for geometrically thin disks. Also, our transition for

![Figure 3](image-url)
The mass loss through the disk region, defined as the mass outflow across the contour, is calculated as a function of cylindrical radius $R$ as follows:

$$\frac{dM}{d \log R} = \frac{2\pi R^2 \rho \Omega_p}{\cos \delta_b},$$

(20)

where $\cos \delta_b$ accounts for a geometric correction in surface area, as we measure the mass-loss rate per logarithmic cylindrical radius. This differential mass-loss rate is measured up to the truncation radius $r_c$. We can see from the bottom panel of Figure 3 that the $dM/d \log R$ through the disk wind is smaller than the accretion rate by a factor of a few, but the integrated mass loss over radius is usually larger in the wind (by a factor of 2–3 in the fiducial run). We comment that the strong mass loss is a typical outcome when conducting simulations with simplified thermodynamics around the wind launching region (e.g., Bai & Stone 2017; Cui & Bai 2020; Rodenkirch et al. 2020). Simulations with a more realistic treatment of thermal physics tend to yield milder mass-loss rates (Wang et al. 2019; Gressel et al. 2020). We thus do not pursue further discussions on disk mass loss in this work.

### 3.2.1. Mass Loss Beyond Truncation Radius

Beyond the truncation radius, the “wind base” as well as the bulk disk are not well defined. The magnetic field also starts to form loops, and the flow direction becomes more horizontal. We will discuss the dynamics and mass flow in this region beyond the truncation radius more carefully in Section 3.4. For the purpose of measuring mass-loss rates beyond the truncation radius, we simply quote a single value, by integrating the mass flux through the remaining parts of the contour (from $r_c$ to radius 100 and the arc). The result is shown in the green horizontal line in the bottom panel of Figure 3. Note that the value that we quote is the net mass-loss rate, where only mass loss from the region is counted, whereas mass flux through the region cancels out.

Again, we see the appreciable mass-loss rate beyond $r_c$ is comparable to the accretion rates in the bulk disk. Note that such mass loss is usually attributed to external photoevaporation, as a result of (primarily) UV heating from nearby massive stars (e.g., Hollenbach et al. 1994; Clarke 2007). Despite the fact that our simulations treat the thermodynamics external to the disk in a very rough manner by prescribing the temperature, we emphasize that the temperature that we prescribe is insufficient for the gas beyond the disk truncation radius to drive an outflow on its own (at least up to a radius of $\sim 1000$ for $T_{d0} = 0.001$). Therefore, the outflow seen in our simulations is likely of magnetic origin.

We have conducted some further analysis to probe the origin of the mass loss beyond $r_c$ (but typically within $r_c$), discussed in the Appendix, and conclude that about 70% of this mass-loss rate can be attributed to the continuation of a magnetized disk wind launched beyond $r_c$, which is unbound, and about 20% can be attributed to a decretion flow to be discussed in Section 3.4. These flows can be further seen in the streamline plot in Figure 5 and are smoothly joined at large disk radii.

### 3.3. Magnetic Flux Evolution

In the scenario of wind-driven accretion, long-term disk evolution is largely governed by how the magnetic flux evolves in disks. We have already seen from simulation snapshots the evolution of the magnetic fluxes. More quantitatively, in Figure 4, we show the “flux evolution curve,” which is the maximum value of the magnetic flux $\Phi$ defined as Equation (19) across the simulation domain plotted against time. Note that the time unit here is the orbital period at $r_c$ ($P_c$), and the $\Phi_{\text{max}}$ is normalized with the total flux in the disk $\Phi_d$ defined in Equation (10).

Where the maximum magnetic flux $\Phi_{\text{max}}$ is achieved depends on the evolutionary stage. At the beginning (the first stage), $\Phi_{\text{max}}$ is always achieved at the disk outer boundary $r_{\text{max}}$. As the disk wind pushes the envelope away, magnetic fluxes originally threading through the envelope component are quickly lost through the outer boundary. This process is exhibited as a sudden drop in Figure 4. After all of the envelope fluxes are pushed away forming magnetic field loops (second and third stages), $\Phi_{\text{max}}$ decays over time, and upon entering stage 3, the rate at which flux is lost is largely steady. To quantify this flux-loss rate in the quasi-steady stage, we fit the curve between 10$P_c$ and 20$P_c$ with a straight line, shown as the dotted line in Figure 4. We do not use data beyond about 20$P_c$ because the dynamics will be affected by the loss of flux toward later times. The slope of this line for the fiducial run is $-0.003436\Phi_d/P_c$, corresponding to a flux-loss timescale of $\sim 300P_c$.

Also plotted are the results for runs with different artificial resistivity ($\eta_{\text{M1}}$ and $\eta_{\text{M4}}$), higher resolution ($\text{h}_{\text{res}}$), higher envelope temperature ($\text{T}_{\text{e2}}$ and $\text{T}_{\text{e24}}$), and different initial envelope density ($\text{dRhoeh}$ and $\text{hRhoeh}$). For these simulations, the flux evolution curves are generally similar to each other. The rate of flux loss ($d\Phi_{\text{max}}/dt$) is also tabulated in Table 1. We verify that the introduction of artificial resistivity does not affect our results much other than to stabilize the simulations. We also find that the initial parameters in the envelope component, either density or temperature, have little impact on the end result. This is reasonable, since the entire envelope is quickly pushed
away during the first stage. Parameters that do affect the loss rate of magnetic flux will be discussed in Section 4.

3.4. Flow Structure Beyond Truncation Radius

In this subsection, we focus on the flow structure beyond the disk truncation radius. In Figure 5, we show a streamline plot averaged between 14, 000Ω\(_{\text{K}}^{-1}\) and 20, 000Ω\(_{\text{K}}^{-1}\). We can see that in the midplane region, there exists a transition point (marked with a cyan star at \(R_{\text{t},\psi} = 49.7\)). Inside this radius, gas flows inward, which is essentially the wind-driven accretion flow. Outside of this radius, however, the gas flows outward, indicative of a decretion flow. In other words, the outer disk expands. This is contrary to the conventional wisdom that the entire disk shrinks in the wind-driven disk evolution scenario, as we study in further detail below.

This transition point closely follows the location of the loop center (at \(R_{\text{t},\psi} = 49.4\)), as we shall see more clearly in Figure 11. The reason can be qualitative understood. We see that the toroidal field configuration is largely unchanged within and beyond the loop center. By contrast, the vertical field changes sign across the loop center. As a result, the Maxwell stress exerted on the disk, \(B_B\psi\) (more specifically, the sign of the Lorentz force), changes sign when moving beyond the loop center. This sign change leads to a reversal of the magnetic torque, and hence a transition from accretion to decretion flow.

In Figure 6, we show the vertical profiles of the radial mass flux \(\nu\Phi\) averaged from 14, 000Ω\(_{\text{K}}^{-1}\) to 20, 000Ω\(_{\text{K}}^{-1}\) as solid lines, as well as the prediction based on the right-hand side of Equation (14) as dotted lines. We choose \(R = 30\) as a representative location inside the transition point and \(R = 65\) as a representative location outside. We see that Equation (14) reasonably well reproduces the observed accretion/decretion flow structure within/beyond the loop center. While there are deviations near the midplane, the agreement is reasonable given the approximate nature of the equation.

As discussed in Section 3.2 and the Appendix, the outflow mass flux beyond ±2\(H\) is connected to the wind launched from beyond \(r_c\). As seen in Figure 6, the radial velocity increases rapidly beyond ±2\(H\) and can reach up to \(\sim 0.65v_K\) at ±3\(H\) (not shown) and continues to be accelerated toward larger radii to exceed \(v_K\). This flow is essentially unbound, similar to a standard disk wind. This flow is analogous to but physically different from external photoevaporation, where recent 2D simulations of Haworth & Clarke (2019) illustrate that a substantial fraction of the mass loss arises from the surface of the outer disk. We will further discuss this analogy in Section 5.2.

The bump in Figure 6 near \(z = 0\), however, best exhibits the magnetically driven decretion flow. The flow speed is about 0.014\(v_K\) and remains bounded, which can give an appreciable mass flux given a higher gas density at the midplane. With a thickness of \(\sim 0.5H\), we find the outward mass flux it carries is about \(\sim 20\% (\sim 2 \times 10^{-5})\) of the total mass-loss rate beyond \(r_c\). This is exactly analogous to the conventional scenario of viscous spreading, implying that the wind-driven evolution of PPDs can expand over time.

3.4.1. Density Dispersion

With mass outflowing beyond the loop center analogous to viscous spreading, here we analyze the dispersion of the gas surface density. In Figure 7, we show a time sequence of the disk surface density profile (the integral of the gas density along the \(\theta\) direction). We choose to plot only the part beyond the truncation radius \((r > r_c = 30)\), which is the focus of this work, and this region is not affected by the density replenishment scheme inside \(r_c\).

The upper four snapshots corresponds to stage 1 where the envelope is pushed away, establishing the density profile beyond the truncation radius. The last of them at time \(t = 1300\) (about one Keplerian period at \(r_c = 30\)) sets a baseline for our further analysis. After \(t = 1300Ω_{\text{K}}^{-1}\), the column density profile evolves very slowly. To better visualize the evolution, we plot the ratio of the surface density profile to that at \(t = 1300Ω_{\text{K}}^{-1}\) on
a linear scale. We can see that the surface density inside of \( r = 50 \) is gradually depleted due to the steady accretion flow. At the same time, an excess appears around \( t = 60\text{–}70 \). This excess in surface density is comparable to the local surface density (on the order of several tens of percent). The development of this excess is directly related to the decration flow discussed earlier. Over time, the excess gradually flattens, as mass is further transported toward larger radii. Overall, the density dispersion we observe is indeed qualitatively analogous to expectations for viscous expansion. We with conduct a more quantitative comparison in Section 5.1.

3.5. Detailed Analysis of Flux Evolution

To better understand the magnetic flux evolution in our simulations, in Figure 8, we show both the total \( E_{\phi,\text{tot}} \) (left panel) and its non-ideal contribution \( E_{\phi,\text{NI}} \), normalized by the product of the local Keplerian velocity \( v_K \) and the poloidal magnetic field strength \( B_{\text{pol}} = \sqrt{B_r^2 + B_z^2} \). Positive and negative \( E_{\phi} \) are colored red and blue, correspondingly, and the color value thus marks the rate at which the field line is transported relative to \( v_K \).

We first see from the left panel of Figure 8 that the total \( E_{\phi} \) is largely positive throughout the system. In the disk region, up to disk radii within the loop center where the poloidal magnetic field has a positive \( z \)-component, a positive \( E_{\phi} \) directly translates to the loss of magnetic flux with outward flux transport. Similarly, beyond the loop center where \( B_z \) is negative, a positive \( E_{\phi} \) leads to inward transport of magnetic flux. Combining the above, a positive \( E_{\phi} \) essentially describes the shrinking of the field loop, and hence the dissipation and loss of magnetic flux. More generally, a positive (negative) \( E_{\phi} \) means the shrinking and dissipation of field loops if magnetic fields go clockwise (anti-clockwise) in the \((R, z)\) plane. We see from the right panel of Figure 8 that in the bulk disk, as well as in regions beyond the loop center, \( E_{\phi,\text{tot}} \approx E_{\phi,\text{NI}} \). Therefore, the shrinking of magnetic flux loops is mainly due to the non-ideal MHD effects, i.e., ambipolar diffusion. In the following, we analyze flux transport more quantitatively.

In Figure 9, we show the \( \theta \) profiles of mean gas velocity, ambipolar drift velocity (left panels), and the \( E_{\phi} \) normalized by \( v_K B_{\text{pol}} \) (right panels). The upper panels show the results at \( r = 20 \), i.e., the disk region. The poloidal field is dominated by a vertical component in the bulk disk, and the results are qualitatively similar to those reported by Bai & Stone (2017) in
their Figure 5. We can see that the $E_{\phi}$ curve is vertically flat in the disk region, meaning a steady transport of magnetic fluxes across different heights. The exact value of the flux transport rate, however, is smaller than that in Bai & Stone (2017) by roughly a factor of 3. This difference likely results from the different treatment in the wind launching region, where we use $z_{\text{trans}} = 4$ for our fiducial run as opposed to $z_{\text{trans}} = 3$ in their work; more discussions follow in Section 4.

The lower panels show the results at $r = 50$, which is about the center of the magnetic flux loops. We can see that the gas poloidal velocity is close to zero near the midplane; thus transport is mainly mediated by ambipolar drift. The ambipolar drift velocities show similar structures with those at $r = 20$, largely due to the fact that the vertical profile of the toroidal field remains similar. However, the poloidal field near this region becomes largely horizontal, rather than vertical. As a result, flux transport is mainly mediated by $v_{\text{dr}}$, that brings the radial field toward the loop center. This process is rapid, as exhibited in the bottom right panel, with a central spike in $E_{\phi}/B_{\text{pol}}$. We note that if we simply plot $E_{\phi}$, the corresponding vertical profile becomes largely flat (i.e., a steady dissipation of magnetic flux toward loop center). Therefore, rapid transport is compensated for by the weak poloidal field near the loop center. In other words, the poloidal field is weakened or “diluted,” which can also be seen from the “emptiness” of the constant-$\Phi$ contours near the loop center from Figures 2 and 8.

Finally, we note that the flux transport rate at the innermost region is initially very fast, leading to a rapid depletion of magnetic fluxes near the inner boundary, as can be seen in Figure 2. This may be physical because of the shorter dynamical timescale, but it is also affected by the inner boundary condition. This phenomenon is interesting in its own right, but also demands more realistic simulations (with a better treatment of ionization and thermodynamics) for further study. In this paper, we restrict ourselves mainly to flux transport and dissipation beyond $r_c$.

### 4. Parameter Study

In this section, we conduct a parameter study by varying one parameter at a time while fixing other parameters. The sets of simulations include:

1. The transition height for the AD Elsässer number $A_m$, $z_{\text{trans}} = 3.5, 4.5 \ (zt3.5 \text{ and zt4.5 in Table 1})$;
2. The value of $A_m$ beyond the disk truncation radius, $A_{\text{Mout}} = 0.3, 10, 100 \ (\text{AMout0.3, AMout10, and AMout100 in Table 1})$;
3. The truncation radius $r_c = 10, 20 \ (\text{Rc10 and Rc20 in Table 1})$; note that the maximum radius $r_{\text{max}}$ is also modified accordingly to ensure $r_{\text{max}} = 10r_c$;
4. The power-law index for the disk density profile, $\alpha = 1.5, 2.0, 2.5 \ (\text{alpha1.5, alpha2.0, and alpha2.5 in Table 1})$;
5. The initial disk magnetization $\beta_0 = 10^3, 10^5 \ (\text{beta3 and beta5 in Table 1})$;
6. The disk aspect ratio $H/R = 0.05, 0.15 \ (\text{HoR05 and HoR15 in Table 1})$.

Overall, we find that all of the simulations show an evolutionary trend similar to that depicted in Figure 2, and we discuss quantitative differences below.

The long-term system evolution is better characterized by the evolution of the magnetic flux. We show in Figure 10 the evolution of $\Phi_{\text{max}}$ for these simulations (normalized by $\Phi_{\text{d}, \text{max}}$ from Equation (10)), grouped by different sets of parameters mentioned above. We can see that almost all of the runs, upon entering stage 3, lose magnetic flux linearly over time. We measure the slope of this linear relation, corresponding to the rate of flux losses, and show results in the insets, from which we can identify important trends in the parameter dependence. Note that the scale for the y-axis is the same for all five insets except for that in the lower left panel comparing runs with different initial plasma $\beta_0$.

First, we verify from simulations with different $r_c$ that the rate of flux transport, when measured in $P_c$ (Keplerian period at $r_c$), is independent of the specific choice of $r_c$. This result indicates that the overall rate of flux transport in disks is largely set by the disk truncation radius, or simply the disk size. Together with our other simulations with varying $\eta_0$, $T_e$, and $\rho_0$, they establish the robustness of our results against numerical parameters.

Among all parameters, the rate of flux transport depends most sensitively on $z_{\text{trans}}$ and the initial plasma $\beta_0$. A stronger disk magnetization (smaller $\beta_0$) leads to a faster rate of transport. This trend was seen in recent wind simulations of full disks (Bai & Stone 2017; Lesur 2021), and it holds in truncated disks. In particular, increasing the magnetization from $\beta_0 = 10^3$ to $10^5$ leads to a dramatic increase in the rate by a factor of $\sim 10$, whereas a lower magnetization with $\beta_0 = 10^3$ leads to a modest reduction by $\sim 40\%$. A lower/higher disk wind base (smaller $z_{\text{trans}}$) enhances/reduces the loss rate of the disk magnetic flux. A slight reduction of $z_{\text{trans}}$ from our fiducial value of $4.0$–$3.5$ increases the rate of flux loss by more than a factor of $2.5$, whereas increasing $z_{\text{trans}}$ to $4.5$ reduces the transport rate by a factor of $\sim 2$. This new trend that we identify here strengthens the importance of properly capturing the physics of disk wind launching. It also helps explain the discrepancies in the absolute flux-loss rate measured in the recent literature. For instance, the rate of flux loss in Bai & Stone (2017) is about a factor 3 faster than that obtained in our simulations, and they adopted $z_{\text{trans}} = 3$.\footnote{By contrast, Lesur (2021) found a much smaller transport rate, though it is not straightforward to define an equivalent $z_{\text{trans}} = 3$ in his calculations: his $A_m \text{ profile varies in a different manner as } \exp(c/M)^\lambda \text{ with } \lambda = 3$, and he assumes the gas to be isothermal, hence there is no temperature transition.}

The Elsässer number at large radius ($A_{\text{Mout}}$) has moderate effects. In general, except for the last data point with $A_{\text{Mout}} = 100$, a larger $A_m$ number results in slower loss rate of magnetic flux. This result also exemplifies the influence of outer boundary conditions on the rate of flux transport.

By varying the density profile in the disk, parameterized by $\alpha$, we also vary the radial distribution of the magnetic flux (since the initial radial profile of the et poloidal field has a constant plasma $\beta_0$). Our parameter study shows that it has a minor impact on the rate at which fluxes are lost. The main trend is that a steeper outward gradient of the radial flux profile (larger $\alpha$) leads to a faster rate of flux loss, which is in line with the flux transport mediated by diffusion. The rate of flux transport appears to be insensitive to the disk aspect ratio for $H/R \lesssim 0.1$. This rate is modestly enhanced when the disk becomes thicker ($H/R = 0.15$), although for $z_{\text{trans}} = 4$, the location of the wind base can no longer be considered to be in a geometrically thin system.

By contrast, Lesur (2021) found a much smaller transport rate, though it is not straightforward to define an equivalent $z_{\text{trans}} = 3$ in his calculations: his $A_m$ profile varies in a different manner as $\exp(c/M)^\lambda$ with $\lambda = 3$, and he assumes the gas to be isothermal, hence there is no temperature transition.
Figure 10. The same as Figure 4, but for runs with varying parameters. The upper left panel compares runs with different transition heights $z_{\text{trans}}$. The upper right panel compares runs with different levels of AD coefficients at large radius. The middle left panel compares runs with different truncation radii $r_c$. The middle right panel compares runs with different power-law indices for the density profile. The lower left panel compares runs with different levels of magnetization (initial plasma $\beta_0$). The lower right panel compares runs with different aspect ratios of the disk. In all panels, the fiducial run is plotted as solid black curves. The insets in all panels show the rate of magnetic flux loss $d\Phi_{\text{max}}/dr$ vs. the parameter of interest in that panel. The scale in all insets is the same, except in the lower left panel comparing different runs with varying plasma $\beta_0$. 

The Astrophysical Journal, 922:201 (18pp), 2021 December 1 Yang & Bai
5. Discussion

There are three major aspects of our simulation results that merit further discussion, as we detail in Sections 5.1–5.3. As this is a pilot study, our simulations are also subject to a number of caveats that will be discussed in Section 5.4.

5.1. Analogy with Viscous Spreading

Canonically, models of long-term disk evolution assume the outward transport of disk angular momentum mediated by viscosity. As a result, most materials accrete, transferring their angular momentum to the outer disk, which expands. There is a transition radius separating the accreting and expanding regions, and this transition radius itself increases with time. Here, we compare the outcome of our simulation results to that of the viscous evolution model to show the analogy and potential differences.

5.1.1. Expectations from Viscous Spreading

For a disk with a viscosity profile of $\nu \propto R^{3}$, the viscous evolution equation of surface density has a similarity solution, with the transition radius evolving as (Lynden-Bell & Pringle 1974; Hartmann et al. 1998):

$$R_t = R_1 \left[ \frac{T}{2(2 - \gamma)} \right]^{1/\gamma}, \quad (21)$$

where $R_1$ is a radial scale factor, $T = t/t_\nu + 1$ is a nondimensional time with $t_\nu = (R_1^2/\nu_1)/3/(2 - \gamma)^2$ being the viscous scaling time, and $\nu_1$ is the viscosity at $R_1$. Taking the derivative of Equation (21), one gets the expansion rate of the transition radius:

$$\frac{dR_t}{dt} = \frac{R_1}{t_\nu} \frac{1}{2 - \gamma} \left[ \frac{T}{2(2 - \gamma)} \right]^{-1/\gamma}. \quad (22)$$

For simplicity, we will assume $\gamma = 1$. Results assuming other $\gamma$ are generally not very different. In this case, we have $dR_t/dt = R_1/t_\nu$, with $t_\nu = R_1^2/(3\nu_1)$. For a typical $\alpha$ disk with $\nu_1 = \alpha_c H$, we find:

$$\frac{dR_t}{dt} = 3\alpha \left( \frac{H}{R} \right)^2 v_K. \quad (23)$$

This rate of spreading is directly linked to the radial flow velocity. Using Equations (17) and (21) from Hartmann et al. (1998), one can derive the radial profile of the radial velocity, for the $\gamma = 1$ case, as:

$$v_K = \frac{3}{2} \frac{\nu_1}{R_1} \left( \frac{R}{R_t} - 1 \right) = \frac{3}{2} \alpha \left( \frac{H}{R} \right)^2 v_K \left( \frac{R}{R_t} - 1 \right). \quad (24)$$

5.1.2. Expansion of Transition Radius $R_t$

A magnetized disk wind is expected, conventionally, to extract angular momentum from the disk, driving the entire disk to accrete. Our simulations show that this is not the case due to the formation of magnetic field loops beyond the truncation radius. Instead, there exists a similar transition radius, which separates accretion and decretion flows analogous to viscous spreading. We have also seen that the mass flux beyond the transition radius is a sizable fraction of the accretion flow. It remains to examine how the transition radius itself evolves.

In the left panel of Figure 11, we plot the evolution of the transition radius $R_t$, defined by the location where the radial velocity changes sign in the midplane, and the evolution of the location where the magnetic flux $\Phi$ is maximized ($R_{t,\Phi}$, i.e., the loop center) from run hi_res. We see that $R_{t,\Phi}$ closely tracks the center of the magnetic flux loops. Following some relaxation processes in stage 2 where the field loop center oscillates, the two closely match each other in stage 3 (which is one main reason we choose $t \approx 14P_0$ as the starting time of stage 3). Interestingly, in stage 3, we observe that the transition radius also moves outward. This is again analogous to the scenario of viscous spreading.

Fitting the linear part in the left panel of the Figure 11, we get $dR_t/dt \sim 1.4 \times 10^{-3} v_K$, where the Keplerian velocity was taken at $r = 50$. If this expansion rate were due to viscous spreading, we find an effective $\alpha \approx 0.047$.

By examining our other simulation runs, we find that the evolution trend of $R_{t,\Phi}$ is mainly sensitive to the value of $\dot{M}_{out}$.
reflecting the ionization fraction in the envelope. The rate of \(dR_{e,\Phi}/dt\) can be modified by other parameters at some modest level but without a sign change. Here, we will focus on the role of \(Am_{\text{out}}\), and the results are shown in the right panel of Figure 11. We can see that runs with a relatively small \(Am_{\text{out}} \lesssim 10\) and hence a lower ionization level tend to have \(R_{e,\Phi}\) expand over time, whereas runs with a large \(Am_{\text{out}} = 100\) (and hence a higher ionization level) tend to have \(R_{e,\Phi}\) decrease with time.

Given the importance of \(Am_{\text{out}}\), here we give a rough estimate of the correspondence between the value of Am and the ionization fraction. At the transition radius of \(r = 50\), we find \(\rho = 8.5 \times 10^{-6}\) in code units, averaged over 10000–20,000 resolutions for the hi_res run. Using the same unit conversion method discussed in Section 2.4, we find \(\rho_0 = 9.2 \times 10^{-12}\) g cm\(^{-3}\). Assuming HCO+ as the dominant ion with \(\langle \sigma v \rangle \approx 2 \times 10^{-3}\) cm\(^3\) s\(^{-1}\), we obtain the ionization fraction \(x_i \approx 3 \times 10^{-5} Am\). The real ionization fraction in the outer regions of PPDs is largely uncertain. Calculations from Cleaves et al. (2015) for the IM Lup disk suggests that \(x_i\) is between \(10^{-11}\) to \(10^{-9.5}\), translating to \(Am = 0.004 \pm 0.1\); thus the disk evolution likely follows an overall expanding path. Note that they have adopted a substantially reduced cosmic-ray ionization rate with \(\zeta_{\text{CR}} < 10^{-19}\) s\(^{-1}\), whereas the value of \(Am\) could be brought to order unity for more standard ionization rates. Nevertheless, the exact ionization rate, and hence the ionization level, is still highly uncertain and likely depends on the external environment, and a higher \(Am\) value is likely when the disk is in the vicinity of some massive stars (see the next subsection). In such cases, despite having a deceleration flow beyond \(R_{e,\Phi}\), the disk itself still likely shrinks over time due to the contraction of \(R_{e,\Phi}\).

5.1.3. Radial Flow Speed Beyond \(R_{e,\Phi}\)

To compare our radial velocity with that of the 1D viscous model, we calculate the density-weighted average of the radial velocity vertically between \(-3H\) to \(3H\) at \(R = 65r_0\). We get \(v_R(R = 65r_0) = 8.7 \times 10^{-3} \Phi\). Taking \(R_{e,\Phi} \sim 49r_0\), we find that \(\alpha = 1.88\) if this radial flow velocity were due to viscous spreading. This value is apparently too large for any reasonable viscous disk models, partly because the mass outflow is dominated by high-velocity wind flows (which is analogous to external photoevaporation). If we limit ourselves to only the mass flux in the midplane region between \(\theta = \pi/2 \pm 0.05\) but still average over the entire disk column density, we find \(v_{R,\text{mid}} = 2.7 \times 10^{-3}\), which translates to \(\alpha = 0.58\). This is still more than one order of magnitude larger than the value inferred from \(dR_{e,\Phi}/dt\).

The large apparent equivalent \(\alpha\) value here reflects the highly efficient nature of magnetic torques exerted on the outer disk. As it is analogous to (but reversed relative to) the wind torque that transports angular momentum vertically, it is more efficient by a factor of \(\sim R/H\) than the equivalent viscous torque assuming similar field strengths (e.g., Wardle 2007; Bai & Goodman 2009).

In Figure 12, we show the three different radial velocity profiles, \(v_R\) at the midplane, \(\bar{v}_R\), and \(v_{R,\text{mid}}\) defined above. Also plotted are the radial velocity profiles for a viscous spreading disk with \(\alpha = 0.047, 1.9,\) and 0.58. The difference in radial velocity profiles between the disk wind-driven flow and viscous spreading is clearly shown. We see that the mean radial velocity not only requires a large effective \(\alpha\), it also increases more rapidly over radius than that of a typical viscous model. This may offer clues to help disentangle the deceleration flow found in the wind-driven disk evolution from that of standard viscous evolution, although more systematic modeling is beyond the scope of this work.

Based on the discussions above, we can tentatively conclude that the overall gas disk size, under typical conditions, likely expands over time, and this is accompanied by a deceleration flow. The overall scenario qualitatively matches all aspects of viscous spreading in canonical models of disk evolution, except that the mass flux of the deceleration flow well exceeds the viscous counterpart. Given that our simulation settings are highly simplified, we are not yet in a position to develop a long-term disk evolutionary model for a quantitative comparison. That said, our results suggest that it is premature to distinguish the mechanisms that drive disk angular momentum transport by searching for a signature of disk expansion. By comparing gas disk sizes at different disk evolution stages, there is tentative evidence of a trend of larger disk size (Najita & Bergin 2018; Trapman et al. 2020). We therefore argue that such signatures are equally likely to be the outcome of wind-driven disk evolution, based on our findings.

5.2. Relation to External Photoevaporation

External photoevaporation has been considered as another important mechanism that drives disk dispersal. This occurs when stars are strongly irradiated by the UV photons from nearby massive stars, which can be a common situation as stars form in clusters. This leads to strong mass loss beyond the gravitational radius \(r_g\), where gas is heated such that its sound speed approaches the escape velocity (Hollenbach et al. 1994; Johnstone et al. 1998), although later studies found that it can be relaxed to small radii of \(\gtrsim 0.1-0.2r_g\) (Adams et al. 2004; Facchini et al. 2016). It is anticipated that external photoevaporation can be important in disk dispersal even for a very mild environment with \(G_{\text{FUV}} = 10-30G_0\) (Facchini et al. 2016; Haworth et al. 2018), where \(G_0\) is the Habing unit of UV radiation (Habing 1968) representing the interstellar radiation field, though most previous studies were based on simple 1D calculations. More recent 2D hydrodynamic simulations with
extensive photodissociation physics have found a modest increase of the mass-loss rate (Haworth & Clarke 2019).

In our fiducial run, the temperature of the ambient medium is set to be constant at $T_{\infty} = 0.001(\text{GM}/r_0)$. The gravitational radius is roughly $1000r_0$, which is well beyond our simulation domain and about 20 times our transition radius. Therefore, external photoevaporation is not expected to operate. We have seen that we still have a strong mass outflow emanating from beyond the truncation radius. These flows can be interpreted as an extension of the disk winds, but launched beyond the truncation radius. They reside above a couple disk scale heights, typically having much higher speeds than the midplane decretion flow, but their flow properties are strongly affected by non-ideal MHD effects (e.g., gas does not flow along field lines). Such flows are exactly analogous to external photoevaporation yet they are mediated by magnetic fields. It is conceivable that in reality, both magnetic and thermal effects matter, similar to the picture of magnetothermal disk winds (Bai et al. 2016), but the non-ideal MHD nature of this outer region may preclude developing a semi-analytical theory.

It is interesting to note that strong external photoevaporation usually implies a higher ionization fraction in the disk surroundings, or a larger $A_{\text{out}}$. From our results, we see that the transition radius $R_t$ will shrink over time, suggestive of a shrinking disk size. This adds to another layer of analogy with the picture of external photoevaporation, which also reduces the disk size despite viscous spreading occurring (Clarke 2007; Anderson et al. 2013). Nevertheless, our rather rough treatment of thermodynamics restrains us from drawing more quantitative conclusions, and we will defer more detailed comparisons to future studies.

\subsection*{5.3. Global Transport of Magnetic Fluxes}

The transport of magnetic flux governs long-term disk evolution, and it is intrinsically a global problem. Our work fills an important gap in the study of flux transport by providing more realistic outer boundary conditions. We have shown that the formation of magnetic flux loops is inevitable beyond the truncation radius, and flux transport at a global scale is governed by the shrinking of such field loops. We have identified several interesting trends as discussed earlier. These trends indicate that conditions both in the disk (e.g., $z_{\text{trans}}$) and beyond the truncation radius (e.g., $A_{\text{out}}$) matter for the global rate of flux transport, and thus exemplify the notion that flux transport is intrinsically a global problem.

This situation is very different from conventional theoretical models of flux transport, where the disk is typically assumed to be infinitely extended radially to permit radially self-similar-type solutions. Under such assumptions, recent works have started to incorporate a more detailed disk vertical structure and also to consider the role of disk winds (Leung & Ogilvie 2019; Lesur 2021). We anticipate that such approaches may still be applicable well within the truncation radius $r_t$, but new theories are needed for regions beyond the disk truncation to determine the global rate of flux transport.

While developing a new theory of flux transport is beyond the scope of this work, we here make a simple attempt by comparing the rate of flux loss and the characteristic rate of magnetic dissipation by ambipolar diffusion. When considering a characteristic length scale of $R_p$ at the loop center, the latter is given by

$$ \frac{1}{\Phi_i} \frac{d\Phi_{\text{max}}}{dt} \bigg|_{\text{AD}} \sim \frac{\eta_{\text{d}}}{R_p^2} = \frac{4\pi (H/R)^2}{A_m \beta} P_c^{-1}. \quad (25) $$

When considering $H/R \sim 0.1$, $\beta \sim 100$ in the midplane, $A_m \sim 1$, we obtain a rate of $\sim 1.3 \times 10^{-3} P_c^{-1}$, which is smaller than but comparable to the rate obtained in our simulations of $\sim 3.5 \times 10^{-3} P_c^{-1}$. However, if we replace $R_p^2$ by $H^2$ at radius $R_p$, relevant to dissipation near the loop center, the rate becomes much higher and is inconsistent with our measured flux dissipation rate. This disparity is already hinted at in Section 3.5, where we find a rapid dragging of the poloidal field toward the loop center from above and below thanks to ambipolar drift, leading to a dilution of field lines. If flux transport is mainly mediated by diffusion, we see that the characteristic length scale is better chosen to be close to $R_p$.

Nevertheless, while the scenario of diffusive transport is consistent with the trend that enhancing AD in the outer disk (smaller $A_{\text{out}}$) leads to a faster transport, this trend does not scale linearly with $A_{\text{out}}$ based on our simulations. It is likely that a combination of diffusive and advective transport is needed to account for the observed results, which contribute differently at different radii and heights, as seen in Figure 9.

Using the same conversion method as discussed in Section 2.4, we have $r_0 = 3$ au, which corresponds to a transition radius of 150 au and a truncation radius of 90 au. The Keplerian period at $r_t$ is thus $P_c = 850$ yr. The rate of flux dissipation measured above with $d\Phi_{\text{max}}/dt \sim 3.5 \times 10^{-3} \Phi_i/P_c$ can be directly translated to a flux dissipation timescale of $286P_c = 0.24$ Myr. This is a lot shorter than the typical disk lifetime of 2–3 Myr (Haisch et al. 2001; Mamajek 2009). On the other hand, if the real rate of flux loss in disks is not far from what we have measured, the rapid flux loss might imply an early transition from a magnetically driven evolution to a primarily hydrodynamically driven evolution, and would have major implications for long-term disk evolution theory and planet formation.

\subsection*{5.4. Caveats}

As this is a first study, we made several simplifications to the problem to ease the computational effort. While we expect our simulations to capture the most important physics, some caution should be exercised as we discuss below.

First, our treatment of disk chemistry and thermodynamics is highly simplified, leading to several free parameters. Some of the parameters that we vary systematically (such as $z_{\text{trans}}$ and $A_{\text{out}}$), can be considered as reasonable proxies for physical conditions around the disk, while this is less certain for a few other fixed parameters, especially with respect to the temperature prescription and the $A_m$ profile. In reality, the temperature in the outer disk is mainly determined by irradiation both from the central object and from external sources. Coupled with it are photochemistry and ionization chemistry, which determine cooling in the disk atmosphere as well as the ionization level in the system (e.g., Haworth et al. 2016; Wang et al. 2019; Gressel et al. 2020). We thus anticipate that our
The Astrophysical Journal, 922:201 (18pp), 2021 December 1

Yang & Bai

calculations are subject to such systematics that demand more realistic calculations that incorporate such microphysical processes, but the trends identified in our work are likely robust.

Another important caveat is that our simulations are two-dimensional assuming axisymmetry. In particular, given the level of the Am value in the disk region and beyond, the disk is expected to be MRI turbulent, as has recently been demonstrated in full 3D global simulations (Cui & Bai 2021). Besides being turbulent, another main difference is that, unlike our 2D case where the toroidal field changes sign exactly at the midplane, the toroidal field changes sign at random heights that dynamically evolve, which affects the global flow structures. Moreover, the magnetic flux is found to concentrate along quasi-axisymmetric sheets, which may reduce the global rate of flux transport. It is yet to be seen how the MRI turbulence beyond the truncation radius affect the dissipation of the large-scale poloidal magnetic flux loops encircling the outer disk, which we leave for future work.

6. Conclusions

In this work, we performed the first global non-ideal MHD simulations of PPDs with outer truncation. Our simulations follow the recently established paradigm of disk evolution driven by magnetized disk winds, yet they complement previous studies by incorporating more realistic boundary conditions to mimic disk outer truncation. As the disk outer regions typically contain most of the disk mass and largely govern the disk evolutionary timescale, we aim to clarify issues related to the long-term evolution of PPDs and their interplay with the interstellar environment. Given that wind-driven accretion is largely controlled by the amount of the poloidal magnetic flux threading disks, we pay special attention to the global transport of magnetic flux.

Starting from an hour-glass shaped field threading the disk, we find that as the disk launches a magnetized disk wind, which quickly pushes away the surrounding material, there is a loss of disk pressure support and, as a result, poloidal field lines collapse beyond the truncation radius, reconnect, and form field loops. The loops gradually relax toward a quasi-steady state that is largely independent of initial conditions, and it is this state that best represents realistic PPDs. We have conducted a large parameter survey on top of a detailed study with a fiducial set of parameters, and our main findings are as follows.

1. The center of the poloidal field loop encircling the outer disk separates the disk into accretion (inside loop center) and decretion (outside loop center) flow regions, as a natural consequence of the magnetic field geometry around the loop.
2. Unless the disk outer region is well ionized, the loop center migrates outward over time, and the overall outer disk evolution is directly analogous to viscous spreading even in the absence of viscosity.
3. Launching of disk winds extends to beyond the truncation radius despite the non-ideal coupling of gas with the magnetic field, leading to significant mass loss analogous to external photoevaporation but without thermal driving.
4. Global evolution of the poloidal magnetic flux is largely governed by dissipation within the poloidal field loops.

The rate of dissipation is sensitive to both disk conditions and the outer boundary conditions from disk truncation.

Our results imply that in the wind-driven accretion scenario, disk sizes will likely grow, rather than shrink as is conventionally assumed, with time, similar to the scenario of viscously driven disk accretion. The results are qualitatively consistent with the tentative observational inference of larger disk sizes for older disks (Najita & Bergin 2018; Trapman 2020), and such observational evidence is insufficient to distinguish between the two driving mechanisms of disk angular momentum transport. We do note, however, that the gas velocity in the decretion flow is much faster than that of the viscous counterpart, which may lead to different surface density profiles. Recent observations already start to reveal gas surface density profiles at outer truncation regions (Dullemond et al. 2020; Zhang et al. 2021), and further modelings are needed to distinguish between the two scenarios.

Our results also imply that mass loss through the disk outer boundary is not necessarily be thermally driven, but can equally well be magnetically driven. In reality, it is likely that both thermal and magnetic effects matter, which jointly drive the disk outflows in a manner analogous to magnetothermal disk winds.

Our study highlights the importance of outer boundary conditions, besides disk microphysics, in determining the global magnetic flux transport in disks, calling for an improved, more global theory of flux transport. As flux evolution and disk evolution are coupled, a global picture of disk evolution would be incomplete without a solid understanding of magnetic flux transport.

Finally, our simulations are in 2D, with a simplified treatment of thermodynamics and chemistry. While we anticipate the trends identified in our study are robust, extensions to 3D as well as the incorporation of more realistic radiative physics and chemistry are needed to yield more quantitative and realistic results.

We thank the anonymous referee for a prompt report with useful suggestions, and Can Cui for kindly sharing her problem setup with us and for fruitful discussions. This work is supported by the National Key R&D Program of China (No.2019YFA0405100). Numerical simulations are conducted on TianHe-1 (A) at the National Supercomputer Center in Tianjin, China, and on the Orion cluster at the Department of Astronomy, Tsinghua University.

Appendix

Composition of Mass Outflow Beyond Truncation Radius: Further Analysis

To further analyze the origin of the mass flow beyond $r_c$ more quantitatively, we integrate the mass flux through the constant $\delta = \theta - \pi/2$ part of the contour shown in Figure 3. The cumulative flux as a function of $r$ is shown in the left panel of Figure 13. Note that the mass flow from both the upper and lower sides of the disk are added. We can see that the cumulative mass flux first increases, then decreases, starting from $r \sim 45$, indicating gas is flowing into the region enclosed by the contour. At the end of the constant $\delta$ lines, we continue the integration to calculate the cumulative mass flux along the constant-$r$ arc, from $\delta = \pm 0.35$ down to $\delta = 0$. Again, the mass flux from the upper and the lower side are added together. We also do the same integration changing the constant-$r$ arc to...
\[ r = 65 \text{ and } r = 85. \] Note that the starting points of the curves are changed depending on the amount of mass flux across the constant \( \delta \) contour inside of the radius of interest. The results for all three radii are plotted in the right panel of Figure 13. The intersection of the three vertical dotted lines with the black line in the left panel correspond to the starting point of the three curves on the right with matching colors.

The overall cumulative mass flux along the constant-\( \delta \) line and constant-\( r \) arc indicates that, first, there is wind launching beyond \( r_c \). The orientation of the wind is such that it first penetrates out of the constant-\( \delta \) lines, but then bends downward (due to disk truncation) to penetrate into these contour lines. The wind then exits from the outer constant-\( r \) arc, leading to cancellation (\( \delta \) reaches \( \pm 0.3 \)). A further increase of the mass flux in the arc section indicates a net mass loss beyond \( r_c \). From the arc at \( r = 65 - 100 \), we see a steady increase up to \( \delta = \pm 0.2 \). We interpret this steady increase as a continuation of the disk wind from beyond \( r_c \). From the streamline plot in Figure 5, we see these flows are launched typically within transition radius \( r_c \). They are also heavily modified by non-ideal MHD processes (as opposed to the standard wind scenario), as the wind and magnetic fields are not well coupled. From Figure 13, we can roughly infer that this part of the outflow accounts for about \( \sim 70\% \) of the quoted total mass-loss rate beyond \( r_c \). To some extent, this flow is somewhat analogous to the mass outflow resulting from external photoevaporation.

The cumulative mass flux enters a plateau roughly between \( \delta = \pm 0.2 \) and \( \pm 0.05 \), where it increases very slowly. Then, very close to the midplane within \( \delta = \pm 0.05 \), the outward mass flux experiences another rapid increase, and by reaching \( \delta = 0 \), the total cumulative mass flux reaches the value quoted in Section 3.2.1. This outward mass flux near \( \delta = 0 \) corresponds to the decretion discussed in detail in Section 3.4, and is analogous to viscous spreading. More quantitatively, we find that this outward mass flux corresponds to about 20% of the quoted total mass loss rate beyond \( r_c \).

**Figure 13.** Left: the cumulative mass flux through the upper and lower contour shown in Figure 3 between \( r_c \) and \( r \). Right: the cumulative mass flux through the outer arc at \( r = 65, 85 \), and 100, correspondingly. See the text for more details. The data are taken from run hi_res after averaging between \( r = 14,000 \text{ to } 20,000 \text{ km s}^{-1} \).