Restoration of Entanglement by Spectral Filters

K. Usami1,2, Y. Nambu2,3, S. Ishizaka2,3, T. Hiroshima2,3, Y. Tsuda4,5, K. Matsumoto4, and K. Nakamura1,2,3

1 Department of Material Science and Engineering, Tokyo Institute of Technology, 4259 Nagatsuta-chou, Midori-ku, Yokohama, Kanagawa, 226-0026, Japan
2 CREST, Japan Science and Technology Corporation (JST), 3-13-11 Shibuya, Shibuya-ku, Tokyo, 150-0002, Japan
3 NEC Fundamental Research Laboratories, 34 Miyaquigaoka, Tsukuba, Ibaraki, 305-8501, Japan
4 ERATO, Japan Science and Technology Corporation (JST), 5-28-3, Hongo, Bunkyo-ku, Tokyo, 113-0033, Japan
5 Institute of Mathematics, University of Tsukuba, Tsukuba, Ibaraki, 305-8571, Japan

(Received November 20, 2018)

We experimentally demonstrate that entanglement of bi-photon polarization state can be restored by spectral filters. This restoration procedure can be viewed as a new class of quantum eraser, which retrieves entanglement rather than just interference by manipulating an ancillary degree of freedom.

The quantum state which has entanglement is defined that it is composed of more than two quantum systems and cannot be represented as a mixture of direct product state of its subsystems [1]. Its non-local properties have been studied by testing the so-called Bell-CHSH inequality [2-3]. Entanglement plays the most important role in quantum information technologies [4], such as quantum cryptography [4], quantum teleportation [5,6], quantum dense coding [7], and quantum computation [8], which have recently attracted a great deal of attention. It is known that the more the states are entangled, the better the state works as a resource for quantum information processing. It is therefore important to answer the following questions: how to create the entangled states so as to maximize their entanglement, how to distill poorly entangled states into highly entangled states, and how to restore the entanglement of a state that potentially possesses more entanglement. The third question is the focus of this letter.

Hereafter, we will restrict our consideration to the polarization states of photons. Several methods to obtain polarization-entangled bi-photon have been reported [9]. Recently, Kwiat et al. devised an easy and efficient method for producing an arbitrary polarization-entangled state [10]. They used cw-pumped spontaneous parametric down-conversion (SPDC), by which one parent photon in the cw pump beam is split into two polarization-entangled daughter photons via two nonlinear optical crystals, conserving energy and momentum and satisfying the type-1 phase matching condition [10]. Under cw-pumped SPDC, the converted photon pair has high purity (low entropy) and high degree of entanglement, and it can be considered the following Bell state [10],

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B),$$

(1)

where $|H\rangle$ ($|V\rangle$) stand for a horizontal (vertical) polarized photon state and subscripts A and B represent two separate observers, Alice and Bob. On the other hand, it has been reported that under femtosecond pulse-pumped SPDC, the converted photon pair has poor purity and entanglement, due to the complicated dispersion and phase matching associated with an ultra-short pump pulse [9]. However, femtosecond pulse-pumped SPDC is indispensable for manipulating entangled photon pairs, because of its capability to generate two or possibly more bi-photons simultaneously [10-12].

In this letter, we report on an experiment of a disentanglement eraser [10], which restores entanglement in a bi-photon polarization state generated by two-crystal, femtosecond-pulse-pumped SPDC. In this experiment, spectral filters play the role of an effective disentanglement eraser, which stretch the coherent time of two bi-photon wave packets and thus erase the information concerning which crystal generates a bi-photon. By experimentally reconstructing the density matrices, we evaluate how much entanglement (concurrence [17]) can be restored by a disentanglement erasing. In order to obtain the reliable density matrices, we employ two techniques: quantum tomography [11,18] and a maximum likelihood method [19].

Our source of entangled bi-photon is almost the same as the source in Ref. [10]. Two thin nonlinear crystals, each of which is a 0.15-mm-thick BBO crystal cut for the type-1 phase matching, are attached so that their optical axes lie in planes perpendicular to each other (Fig. 1(a)). The plane including the optical axes of the first (second) crystal and the propagating direction of the pump beam defines vertical (horizontal). If the polarization of the pump beam is set to 45° to the horizontal, the pump beam state is given by the direct product of the horizontally polarized and the vertically polarized coherent states with the same amplitude. Therefore, the following two probability amplitudes can be “coherently” superposed: (1) two horizontally polarized photons generated in the first crystal or (2) two vertically polarized photons generated in the second one. Thus, the converted photons can be considered as a polarization-entangled state as given by Eq. (1). Under femtosecond pulse-pumped SPDC, however, two bi-photon wave packets, i.e., two probability amplitudes, can be distinguished because of

arXiv:quant-ph/0107121v1 24 Jul 2001
the considerably short coherent time of the pump pulse [20]. We will refer to this distinguishability as which-crystal information. This which-crystal information usually degrade entanglement of the converted photon pair. Figure 2 shows a space-time diagram, which depicts the space-time difference between two wave packets. Which-crystal information, in other words, bi-photon wave packets appears in the density matrix as an ancillary degree of freedom, though we are only concerned with the polarization degree of freedom. We define two bi-photon (temporal) wave packets, one of which is generated by the first crystal and the another by the second one, as $|\psi_1(t)\rangle_T$ and $|\psi_2(t)\rangle_T$, respectively. They can be expressed as the inverse Fourier transform, as follows:

$$
|\psi_1(t)\rangle_T = \frac{1}{\sqrt{2\pi}} \int d\omega_1 g(\omega_1) e^{-i\omega_1 t} |\omega_1\rangle
$$

$$
|\psi_2(t)\rangle_T = |\psi_1(t + \tau)\rangle_T = \frac{1}{\sqrt{2\pi}} \int d\omega_2 g(\omega_2) e^{-i\omega_2(t+\tau)} |\omega_2\rangle.
$$

(2)

Here we assume that $|\psi_2(t)\rangle_T = |\psi_1(t + \tau)\rangle_T$, which means two bi-photon wave packets are the same, apart from the time difference $\tau$. This time difference is mainly due to group velocity dispersion (Fig. 3). In Eq. 3 $g(\omega)$ and $|\omega_i\rangle$ represent the spectral amplitude and eigenstate of the bi-photon state with energy $\hbar \omega$ for each photon, respectively, and $|\omega_i\rangle$ constitute a complete orthonormal system (i.e., $\int d\omega |\omega_i\rangle \langle \omega_i| = 1$, $\langle \omega_i | \omega_j | = \delta(\omega_i - \omega_j)$). Then the state of whole system obtained by pulse-pumped SPDC can be expressed as

$$
|\Omega_{ABT}(t)\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |H\rangle_B |\psi_1(t)\rangle_T + |V\rangle_A |V\rangle_B |\psi_2(t)\rangle_T).
$$

(3)

What is intriguing about the state of Eq. 3 is that its entanglement can be restored by manipulating the ancillary degree of freedom, i.e., the bi-photon wave packets, like quantum erasing. Traditional quantum erasing, e.g., in Young’s double-slit configuration [21], involves only two subsystems, principal system A and a ancilla (tag) system T. In contrast to such a traditional quantum erasing, the state given by Eq. 3 involves three subsystems: two principal systems A and B, and a tag system T. This framework corresponds to that of a new type of quantum eraser, i.e., a disentanglement eraser proposed by Garisto and Hardy [10], which can be restored entanglement between the principal systems A and B, rather than just the interference by a suitable manipulation of the tag system T. The question now arises: how can the tag system be manipulated in order to erase which-crystal information? For erasing which-crystal information marked via a wave packet, we employed narrow bandwidth spectral filters in front of two photon counting detectors at the expense of the available flux of entangled photon pairs. Although there are some reports that the quantum interference visibility in femtosecond-pulse-pumped SPDC can be restored by erasing the distinguishability with spectral filters [22], they treat two principal systems, A and B, as a single subsystem as a whole. Thus, their eraser can be viewed as a conventional one. Recently, Teklemariam et al. reported the three-spin disentangled eraser on a liquid-state NMR [23]. But their effective pure state is separable at any moments of the processing as mentioned by Braunstein et al. [24].

For evaluating entanglement of a bi-photon generated by pulse-pumped SPDC with two nonlinear optical crystals, we employed linear tomographic measurement [11, 18] and a maximum likelihood method [19]. Figure 2(b) shows our experimental setup to generate polarization-entangled photon pairs and to evaluate its entanglement. A parent photon in the frequency-tripled laser pulse of a mode-locked Ti: Sapphire laser (central wavelength: 266 nm, pulse duration: ~100 fs and average power: ~150 mW) splits into a polarization-entangled photon pair (central wavelength: 532 nm) via

\[\text{FIG. 2. Space-time diagram of two wave packets generated by femtosecond-pulse-pumped SPDC. In our experiment, the time difference between two wave packets are about 100fs. This difference is mainly due to the group-velocity dispertion.}\]
two BBO crystals, as mentioned before. The entangled photons are emitted into a cone with a half-opening angle of 3.0° and detected by photomultipliers (Hama-
matsu, H7421-40) with efficiencies of ˜40%. Quarter wave plates (QWP(A) and QWP(B)), half wave plates (HWP(A) and QWP(B)) and polarizing beam splitters in front of the detectors, can be used to project the polarization state of down-converted photons onto any kind of product states of polarization by coincidence counting measurements (in our experiments, a coincidence window of 6 ns). We chose four projection states for Alice, \( \langle H|,|V|,|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) \), \(|R\rangle = \frac{1}{\sqrt{2}}(|H\rangle + i|V\rangle) \) and for Bob, \( \langle H|,|V|,|D\rangle = \frac{1}{\sqrt{2}}(|H\rangle + |V\rangle) \), \(|L\rangle = \frac{1}{\sqrt{2}}(|H\rangle - i|V\rangle) \) respectively, and these 16 kinds of the joint projective measurements were made for the linear tomography. In order to reduce background noise and select the frequency-degenerate bi-photons at 532 nm, we employed interference filters centered at 532 nm (bandwidth (FWHM) of 8.0 nm, peak transmissivity of 52%) in front of the detectors.

Figure 3(a) shows the reconstructed density matrix numerically reconstructed from the experiment. As expected, the off-diagonal parts of the density matrix were retrieved, and we obtained its concurrence of about 0.74 and von Neumann entropy of about 0.39. Furthermore, from the polarization-correlation measurement (Fig.3(b)), Bell-CHSH’s value, S in Ref. 3, is about 2.4 which indicates the violation of Bell-CHSH inequality. Therefore the results show that we have succeeded in restoration of entanglement from the so-called hidden non-local state (C ≈ 0.21) to the explicitly non-local state (C ≈ 0.74) by a disentanglement eraser.

For theoretically estimating how much entanglement should be restored by our disentanglement erasing model, let us go back to Eq. 4. To obtain the reduced density matrix of the principal system alone, we perform a partial trace over the tag system, then we get \( \rho_{AB} = \text{Tr}_T [\Omega_{ABT}(t)] \) (4). It can be rewritten as

\[
\rho_{AB} = \frac{1}{2} \sum_{\{A,B\}} \langle H|_A |H|_B \langle H|_A |H|_B + |V|_A |V|_B \langle V|_A |V|_B + C \langle H|_A |H|_B \langle V|_A |V|_B + C^* \langle V|_A |V|_B \langle H|_A |H|_B, \tag{4}
\]

where \( C = \frac{1}{2\pi} \int \langle \omega' | \psi_1(t) \rangle \langle \psi_2(t) | \omega' \rangle d\omega' \) and according to the Wiener-Khintchin theorem, we get

\[
C = \frac{1}{2\pi} \int \psi_1^* (t + \tau) \psi_1(t) dt. \tag{5}
\]
Here we define the complex function in Eq. 5 as \( \psi_1(t) = \frac{1}{\sqrt{2\pi}} \int g(\omega) e^{-i\omega t} d\omega = A(t)e^{i\theta(t)} \), where \( A(t) \) and \( \theta(t) \) respectively represent the envelope of the amplitude and the phase (including the chirp term) of the bi-photon state. Since the concurrence of the spectral filter of 1.2 nm-bandwidth with the following three assumptions: (1) the pulse shape of wave packet \( |\psi_1(t)\rangle \) is Gaussian centered at \( t = 0 \), whose pulse duration determined by the bandwidth of the spectral filters; (2) this Gaussian pulse is transform-limited; (3) the time difference \( \tau \approx 100 \) fs. Thus this calculated results are consistent with our experiment. The difference between the value derived by our theoretical model and that obtained by experiment might be due to other distinguishabilities of the entangled states in the experiment and the assumptions in the theoretical estimation.

It is worthy to mention that the disentanglement eraser decreased the von Neumann entropy of the principal systems. On the other hand, in the recent experiment of the entanglement distillation using Procrustean method [8], von Neumann entropy of the systems increased.

In summary, we have shown that entanglement of the bi-photon polarization state generated by pulse-pumped SPDC can be restored by the narrow-bandwidth spectral filtering. We found that spectral filtering effectively erased the possibilities of distinguishing the two bi-photon wave packets; thus, entanglement of the principal system (polarization state) can be restored. This erasing procedure will open up the alternative possibility of entanglement manipulation and quantum information processing.

We gratefully acknowledge useful discussions with S. Kono, B. -S. Shi, and A. Tomita.

[1] M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, (Cambridge University Press, 2000)
[2] J. F. Clauser, et al., Phys. Rev. Lett. 23, 880 (1969)
[3] A. Aspect, P. Grangier, and G. Roger, Phys. Rev. Lett. 49, 91 (1982); G. Weihs, et al., Phys. Rev. Lett. 81, 5039 (1998)
[4] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991); T. Jennewein, et al., Phys. Rev. Lett. 84, 4729 (2000); D. S. Naik, et al., Phys. Rev. Lett. 84, 4733 (2000); W. Tittel, et al., Phys. Rev. Lett. 84, 4737 (2000)
[5] C. H. Bennett, et al., Phys. Rev. Lett. 70, 1895 (1993)
[6] D. Bouwmeester, et al., Nature (London) 390, 575 (1997)
[7] D. Boschi, et al., Phys. Rev. Lett. 80, 1121 (1998); A. Furusawa, et al., Science 282, 706 (1998); Y. -H. Kim, S. P. Kulik, and Y. Shih, Phys. Rev. Lett. 86, 1370 (2001)
[8] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992); K. Mattle, et al., Phys. Rev. Lett. 76, 4656 (1996)
[9] Y. -H. Kim, et al., Phys. Rev. A 63, 062301 (2001) and reference therein
[10] P. G. Kwiat, et al., Phys. Rev. A 60, R773 (1999)
[11] A. G. White, et al., Phys. Rev. Lett. 83, 3103 (1999)
[12] P. G. Kwiat, et al., Science 290, 498 (2000)
[13] P. G. Kwiat et al., Nature (London) 409, 1041 (2001)
[14] M. Zukowski, et al., Phys. Rev. Lett. 71, 4287 (1993); M. Zukowski, A. Zeilinger, and H. Weinfurter, Annals of the N.Y. Acad. of Sciences 755, 91 (1995)
[15] J. -W. Pan, et al., Phys. Rev. Lett. 80, 3891 (1998); J. -W. Pan, et al., quant-ph/0104047; D. Bouwmeester, et al., Phys. Rev. Lett. 82, 1345 (1999); J. -W. Pan et al., Nature (London) 403, 515 (2000)
[16] R. Garisto and L. Hardy, Phys. Rev. A 60, 827 (1999)
[17] W. K. Wootters, Phys. Rev. Lett. 80, 2245 (1998)
[18] K. Usami, Ms. of Engineering thesis, Tokyo Institute of Technology, Japan (2001)
[19] D. F. James, et al., quant-ph/0103122
[20] Y. -H. Kim, S. P. Kulik, and Y. Shih, Phys. Rev. A 63, 060301(R) (2001)
[21] M. O. Scully, B. -G. Englert, and H. Walther, Nature (London) 351, 111 (1991)
[22] G. Di Giuseppe, et al., Phys. Rev. A 56, R21 (1997); W. P. Grice, et al., Phys. Rev. A 57, R2289 (1998)
[23] G. Teklemariam, et al., Phys. Rev. Lett. 86, 5845 (2001)
[24] S. L. Braunstein, et al., Phys. Rev. Lett. 83, 1054 (1999)
[25] R. F. Werner, Phys. Rev. A 40, 4277 (1989); S. Popescu, Phys. Rev. Lett. 72, 797 (1994)