Description of dynamics of stock prices by a Langevin approach

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We present a time-dependent Langevin description of dynamics of stock prices. Based on a simple sliding-window algorithm, the fluctuation of stock prices is discussed in the view of a time-dependent linear restoring force which is the linear approximation of the drift parameter in Langevin equation estimated from the financial time series. By choosing suitable weighted factor for the linear approximation, the relation between the dynamical effect of restoring force and the autocorrelation of the financial time series is deduced. We especially analyze the daily log-returns of S&P 500 index from 1950 to 1999. The significance of the restoring force towards the prices evolution are investigated from its two coefficients, slope coefficient and equilibrium position. The new simple form of the restoring force obtained both from statistical and theoretical analyses suggests that the Langevin approach can effectively present the macroscopical and the detail properties of the price evolution.

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I. INTRODUCTION

The analysis of financial data by methods developed for physical systems has a long tradition \cite{1, 2, 3, 4} and has attracted the interest of physicists. One of the most motivated reasons is that it is a great scientific challenge to understand the dynamics of a strongly fluctuating complex system with a large number of interacting elements. In addition, it may be possible that the experience gained in the process of studying complex physical systems might yield new results in economics.

There are many observables generated from financial markets, and one central issue of the research on the dynamics of financial markets is the statistics of price changes which determine losses and gains. The price changes of a time series of quotations \(x(t)\) are commonly measured by returns: \(r := x(t + \tau)/x(t)\), log-returns, or increments: \(\Delta x := x(t + \tau) - x(t)\) at a time scale \(\tau\). In 1900, Bachelier proposed the first model for the stochastic process of returns—an uncorrelated random walk with independent identically distributed Gaussian random variables \cite{1}. However, prices do not follow a signal random walk process \cite{5, 6, 7, 8}. For example, the daily correlation has been known as the daily log-returns correlated with themselves in such a way that positive returns are followed by positive returns as well \cite{9, 10}. Many considerations have been aroused by this effect recently and meanwhile the related research has been reported, not only for daily data \cite{11} but also for high frequency data \cite{12, 13}.

The Langevin equation (LE) which distinguishes the development of sample path into the deterministic and random terms, has been used to deal with the Brownian motion problem. Recently, the Langevin approach was used to analyze the financial time series on scale \cite{14, 15}. Friedlich \textit{et al.} \cite{14} have investigated how price changes \(\Delta x\) on different time scales \(\tau\) are correlated motivated by hierarchical structure of financial time series, which is similar to the energy cascade in hydrodynamic turbulence. They derived a multiplicative Langevin equation from a Fokker-Planck equation (FPE) in the variable scale \(\tau\) and performed the statistical way to distinguish and quantify the deterministic and the random influence on the hierarchical structure of the financial time series in terms of the drift and diffusion parameters, \(D^{(1)}\) and \(D^{(2)}\), respectively \cite{14}.

Different from the former study in which the LE is used to analyze the scales evolutions of finance, yet, in this paper, with the Langevin description, a new insight on the dynamics of the process will be obtained by investigating the time-dependence of log-returns. The time-dependent properties of prices evolution are derived in the way of estimating drift parameter \(A(z)\) of sampled local periods in the sliding window. Then, the relation between \(A(z)\) and autocorrelation \(C\), average return \(\langle z \rangle\), from which the practical significance of \(A(z)\) can be recognized, are resulted both from the statistical time-dependence of \(A(z)\) and some theoretical analyses. Besides, our Langevin description contains, as a particular case with flat-A\((z)\), the effect of daily correlation in log-returns. On the other hand, the form of diffusion parameter \(B(z)\) got in this paper, to some extent, explains the heavy tailed probability densities of price changes.

The research are mainly carried out from the samples of the daily log-returns \(z(t)\) of S&P 500 index from 1950 to 1999, containing 12583 days, thus covering a wide time range with many different economic and political situations.

The paper is organized as follows. In Sec. II, taking the daily log-returns as an example, we generally discuss the application of Langevin approach to log-returns series. In Sec. III, we show the results and discussions. Finally, the summary and the outlook of this paper are given in Sec. IV.

II. THE LANGEVIN APPROACH TO LOG-RETURNS SERIES

For a time series of prices or market index values \(x(t)\), the log-return \(z(t) \equiv z(t)\), over a time scale \(\tau\) is defined as the
forward change in the logarithm of \( x(t) \),
\[
z(t)_\tau \equiv \ln x(t + \tau) - \ln x(t).
\] (1)

The behavior of daily log-return \( z(t)_{1d} \) as a stochastic variable is described by the following LE:
\[
dz = A(z)dt + B(z)dw,
\] (2)
where the drift parameter \( A(z) \) and diffusion parameter \( B(z) \) respectively describe the deterministic and the random influences on the time process of log-returns, and \( dw \) denotes the increment of a standard Wiener process. It is assumed that, within each sampling window the parameters may depend on the log-returns, but not explicitly on time (stationary). Thus, the drift and diffusion parameters of the sampled period can be extracted from the sampled data by simply using the definition
\[
A(z) = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \langle Z(t + \Delta t) - z \rangle |_{Z(t) = z}
\] (3)
\[
B(z)^2 = \lim_{\Delta t \to 0} \frac{1}{\Delta t} \langle (Z(t + \Delta t) - z)^2 \rangle |_{Z(t) = z}.
\] (4)

Here \( \langle \cdots \rangle \) denotes the averaging operator and \( Z(t + \Delta t) \) is a realization of the LE (2). From Eq. (3), it is obvious that the drift parameter \( A(z) \) is the average increment of unit time under the condition \( Z(t) = z \), which represents the deterministic influences. \( B(z) \) is the deviation of \( A(z) \) which pictures the random influences. It has been known that the autocorrelation of the log-returns decays very fast which is usually characterized by a correlation time much shorter than a trading day [5]. When the time increment \( \Delta t \) is larger than 1 day, the daily log-returns can be considered as the result of many uncorrelated ‘shocks’. Thus, in this paper, \( \Delta t \) is mainly set as 1 day. Compared to the length of time window \( T \), \( \Delta t = 1d \) approximately accords with the limit in Eq. (3) and Eq. (4).

Based on the samplings all over the long series, the statistical results of \( A(z) \) and \( B(z)^2 \) which are respectively estimated by Eq. (4) and Eq. (4) have their simple and general forms. The results of \( A(z) \) are close to a linear form, and that of \( B(z)^2 \) are close to a parabolic form,
\[
A(z) = az + b
\] (5)
\[
B(z)^2 = a'z^2 + b'z + c'.
\] (6)

Fig. 1 presents the statistical results of \( A(z) \) and \( B(z)^2 \) (circles) which are estimated from the daily log-returns time series of S&P 500 index from 28 May, 1962 to 10 Jun. 1970 (window length \( T = 2000d \)). The statistical results for large \( z \) are more noisy and uncertain than the points near the origin, because these border points are visited rarely by the trajectory. On the contrary, as viewed from statistics, the more one given \( z_i \) is visited, the more times the averaging operator \( \langle \cdots \rangle |_{Z(t) = z} \) in Eq. (3) works, which would produce more accurate and reasonable \( A(z_i) \). Thus, while approximating \( A(z_i) \) \((i = 1, 2, ..., n)\) with linear form and \( B(z_i)^2 \) \((i = 1, 2, ..., n)\) with parabolic form by Least-squares fit, the effect of the visited probability for each \( z_i \) should be considered, with each \( z_i \) corresponding to its own weight.

It is natural for a physical scientist to define the weighted factor of \( z_i \) as its probability,
\[
P(z_i) = N(z_i)/T,
\] (7)
where the \( N(z_i) \) is the frequency of \( z_i \) within the time window of length \( T \). Fig. 1 shows the approximations of the statistical results of \( A(z) \) and \( B(z)^2 \) with equal-weight (dashed lines) and weighted factor \( P(z_i) \) (solid lines). The values of the fitting error \( \sigma_A \) and \( \sigma_B \) (the mean standard deviations from approximations of \( A(z) \) and \( B(z)^2 \)) with weighted factor \( P(z_i) \) are visibly lower than those with equal-weight. Therefore, while approximating the statistical results, the point at \( z_i \) is ended the weight \( P(z_i) \) which is correlated to the frequency of \( z_i \) within the window.

For the given return value \( z_i \), the frequency \( N(z_i) \) also depends on the length of time windows. Thus, it is worth mentioning the influences from the length \( T \) on the approximations. The relations between \( T \) and the fitting errors of
\(A(z)\) and \(B(z)^2\) are investigated. To estimate the error purely made by window length \(T\), not by information the time series embodied, the daily log-returns series is randomly rearranged and the fitting errors \(\sigma_A\) and \(\sigma_B\) vs. \(T = 100d \sim 13000d\) are calculated (Fig. 2). It was found that the weighted fitting errors \(\sigma_A\) and \(\sigma_B\) of the approximations with weighted factor \(P(z_i)\) decline quickly with a perfect power-law behavior, \(\sigma_A = 8.178 \times 10^{-3} \cdot T^{-0.49022}\) and \(\sigma_B = 1.249 \times 10^{-4} \cdot T^{-0.31122}\). This behavior suggests that the statistical results of \(A(z)\) (or \(B(z)^2\)) with larger \(T\) is more feasible to be approximated with linear (or parabolic) form. Note incidentally that, only the results with the same window length \(T\) could be compared since different \(T\) corresponds to different fitting errors.

The algorithm for the detection of the time-dependence of drift term \(A(z)\) in this paper can be described as follows: sample the long log-returns series with a sliding window of short length \(T\), and compute \(A(z)\) for each location. The results estimated from the time window which samples a given local period, present the corresponding local characters of financial markets. The algorithm is more sensitive than merely studying transient behavior. The comment for the selection of window length \(T\) is to choose \(T\) long enough so that the averages in Eq. 4 and Eq. 5 are statistically meaningful but not so long as to lose the temporal resolution. In the results presented below, 250\(d\) and 20000\(d\) window lengths and 5\(d\) overlapping (window shift by 5 days per time) are used. The corresponding fitting errors are, \(\sigma_{A,250d} = 5.46 \times 10^{-4}\), \(\sigma_{B,250d} = 2.24 \times 10^{-5}\), and \(\sigma_{A,20000d} = 1.97 \times 10^{-4}\), \(\sigma_{B,20000d} = 1.17 \times 10^{-5}\). It is expected that the variation of \(A(z)\) as a function of time can accurately indicate interesting dynamical changes in financial process.

III. RESULTS AND DISCUSSIONS

In the LE 2, the drift parameter \(A(z)\) could be seen as an action of potential, with \(A(z) = -\nabla V(z)\). Noted that \(A(z)\) has a linear form with negative slope, it could be interpreted as the effect of linear state-dependent restoring force with symmetrical potential well,

\[
V(z) = -\frac{a}{2} z^2 - b z - \frac{b^2}{2a},
\]

where \(z\) presents the position of one given particle enslaved to it. The sketch maps of \(A(z)\) and \(V(z)\) are showed in Fig. 3 in which the equilibrium position \((A(z_0) = 0)\) is \(z_0 = -b/a\).

Fig. 3 shows the time series of log-returns \(z(t)\), restoring force \(A(z)\) and potential \(V(z)\) from 26 May. 1983 to 1 Jan. 1993. It is easy to find that the vibrancy of log-returns \(z\) presented seems to be similar to force \(A(z)\) and potential \(V(z)\), which can be clearly seen from several large events marked in this figure, and the restoring force \(A(z)\) always presents converse effect to log-returns \(z\). In some large events, the potential \(V(z)\) is exceedingly large.

In the following, the time dependence of restoring force \(A(z)\) will be discussed including the equilibrium position \(z_0\) and the slope coefficient \(a\). In addition, discussions of the diffusion parameter \(B(z)\) and the error analysis will be given.

A. Equilibrium Position \(z_0\)

From Langevin equation 17, the so called equilibrium position \(z_0\), which is the zero value of negative-sloped linear drift term, corresponds to the minimum of the potential well. From a physical point of view, the average displacement \(\langle z \rangle\) of an oscillating particle in the potential well defined by Eq. 5 should also be the minimum of the potential well, \((-b/a, 0)\). Thus, we get,

\[
z_0 \simeq \langle z \rangle.
\]

The average displacement \(\langle z \rangle\) was directly obtained from the log-returns series, and the equilibrium position \(z_0\) was calculated from Eq. 6. The time dependence of \(z_0\) and \(\langle z \rangle\) coincide with each other very well all over the ranges [see Fig.
the falling trend could be estimated if $z_0 = -a/b < 0$. Thus, the equilibrium position $z_0$ of restoring force $A(z)$ is comprehended as the ‘trend index’ of stock prices. Furthermore, the stock prices and their log-returns are macroeconomic indicators which are widely used because of the strong correlation between financial markets and economic development. In this case, the equilibrium position $z_0$, which is derived from the LFs description of financial time series, would be another important indicator of macroeconomics.

B. Slope Coefficient $\alpha$

In Fig. 6(c), the time dependence of the slope coefficient $\alpha$ calculated from the daily log-returns of S&P 500 with $T = 250d$ and $2000d$ are plotted. The ranges of the slope coefficient $\alpha$ with $T = 250d$ and $2000d$ are $[-1.127, -0.574]$ and $[-1.009, -0.699]$ respectively, both of which are close to the value $-1$. As shown in Fig. 3 we know that in a certain given position $z_i$, steeper (flatter) slope of $A(z)$ corresponds to larger (smaller) restoring force $A(z_i)$. Thus, one can imagine the mechanism of our model: it would take few times for larger forces (slope: $a < -1$) to draw particles from one side of the equilibrium position $z_0$ to another side, which we called ‘$z_0$-crossing’ action for the moment, and more times for smaller forces (slope: $-1 < a < 0$). To discuss the aforementioned mechanism more accurately, normalized daily log-returns are used,

$$g = \frac{z - \langle z \rangle_T}{\nu}, \nu = \sqrt{\langle z^2 \rangle_T - \langle z \rangle_T^2},$$

which has zero mean value, $\langle g \rangle_T = 0$. Here the standard deviation $\nu$ of log-returns is defined as the time averaged volatility $\nu$ and the $\langle \cdots \rangle_T$ denotes an average over the entire length of the series within time window $T$. From Eq. 9, $g$ has the equilibrium position $z_0$ equaling zero, so that the $z_0$-crossing action could be reduced to the sign convert of $g$. Thus the mechanism can be described as follows: The sign of $g$ changes frequently while the slope is quite steep; on the contrary, same signs congregate together and sequences of consecutive ‘+’ or ‘−’ appear when the slope is flat.

The sign series of daily log-returns has been considered to study the daily correlation in log-returns. Those researchers got the conditional dynamics from the sequence of consecutive ‘+’ and ‘−’. In this paper, however, we will compare the sequence of the same sign with the converting sign of two neighboring days to check the relationship between $a$ and $z_0$-crossing.

The sign-cases of a given day and its previous day are: ‘++’, ‘−−’, ‘−+’ and ‘+−’. One can define ‘++’ and ‘−−’ as the sign-sustained cases, and ‘+−’ and ‘−+’ as the sign-convert cases. Incidentally, the contribution of ‘++’ to the sign-sustained cases was shown to be a little more than ‘−−’ on average over the whole series. Then, the time series of signs of $g$ is investigated by counting the frequencies of sign-sustained cases, $m$, and sign-convert cases, $n$. The time dependence of the proportion $\beta = m/n$ and the slope coefficient $\alpha$ calculated from the daily log-returns of S&P 500 with
Thus, the mechanism of the daily correlation in log-returns is that the sign of the day’s log-return is more likely to be different from its previous day. In detail, in one sampled window, the slope coefficient \( a \) with \( \Delta t = 1 \) day. The lengths of sliding windows are \( T = 250d \) (left) and \( T = 2000d \) (right), and windows shift by 5 days per time. (d) The correlation function between \( a \) and \( C \). Their linear approximations (solid lines) respectively are: \( C = 0.9942 + 0.9958a \) (left) with standard deviation \( SD = 3.44 \times 10^{-3} \), and \( C = 0.9932 + 0.9938a \) (right) with standard deviation \( SD = 6.44 \times 10^{-4} \). The lengths of sliding windows are \( T = 250d \) and \( 2000d \) are plotted in Figs. 6(a) and (c). The good similarity of \( \beta \) and \( a \) proves that the slope coefficient is related to the \( z \)-crossing action and reflects the correlation of neighboring daily log-returns. In detail, in one sampled period, while the slope of the restoring force is flat, the given day’s sign of log-return is more likely to be the same as its previous day; on the other hand, the slope is steep, the given day’s sign is more likely to be different from its previous day. Thus, the mechanism of the daily correlation in log-returns is qualitatively explained by the restoring force.

It’s easy to notice from the time dependence of \( a \) and \( \beta \) [Fig. 6(a)(c)] over the whole series, most of the periods have their slope \( a \) larger than \(-1 \), and \( \beta \) larger than \(1\), with mean values \( \bar{a}_{T=250d} = -0.883 \), \( \bar{a}_{T=2000d} = -0.874 \), and \( \bar{\beta}_{T=250d} = 1.242 \), \( \bar{\beta}_{T=2000d} = 1.223 \). These imply that, in practice, the flat-slope restoring force is more prevalent than the steep-slope one, and the case of sign-sustained is more than that of the sign-converter. Consequently, from the general appearance of sign-sustained cases, the same conclusion was reached as that of [11]: the return of the price during a given day can be correlated with the previous day, in particular with the sign of the previous day. However, it is worth noticing that, the analysis with Langevin approach is more general because it contains the positive daily correlation as the particular case with flat-slope restoring force, and non-correlation with steep-slope restoring force.

On the other hand, the autocorrelation function \( C(\Delta t') \), a typically important statistic of stochastic processes, is always used to investigate pairwise correlation of the log-returns of a financial asset. In the following, we compared it with the slope coefficient \( a \) from mathematical relations and statistical results. It is known that,

\[
C(\Delta t') = \frac{\langle z(t) \cdot z(t + \Delta t') \rangle_T - \langle z(t) \rangle_T^2}{\langle z(t)^2 \rangle_T - \langle z(t) \rangle_T^2},
\]

where \( \langle \ldots \rangle_T \) denotes time averaging over all the trading days within the sampled local period with length \( T \), \( \Delta t' \) is time increment.

For the weighted linear Least-squares fit, which has been used to approximate the statistical results of \( A(z) \), the weighted objective function [18][19] can be written as,

\[
q = \sum_{i=1}^{n} W_i [y_i - (a x_i + b)]^2.
\]

where \( W_i \) presents the weight of the point \( (x_i, y_i) \). The values of \( a \) and \( b \) corresponding to the minimal values of function \( q \) are something to be sought for. Thus from Eq. (12), the solution of slope coefficient \( a \) in \( y(x) = ax + b \) is achieved,

\[
a = \frac{\sum_i W_i \sum_i x_i y_i W_i - \sum_i x_i W_i \sum_i y_i W_i}{\sum_i W_i \sum_i x_i^2 W_i - (\sum_i x_i W_i)^2}.
\]

In the previous discussion, the weight of \( z_i \) was defined as its probability \( P(z_i) \) (Eq. (7)). Thus one get,

\[
a = \frac{\sum_i P(z_i) \sum_i z_i A(z_i) P(z_i) - \sum_i z_i P(z_i) \sum_i A(z_i) P(z_i)}{\sum_i P(z_i) \sum_i z_i^2 P(z_i) - (\sum_i z_i P(z_i))^2}.
\]

In this paper, \( a \) is calculated by this statistic formula. Substituting Eq. (3) and Eq. (7) into Eq. (14), we get,

\[
a = \frac{\langle z \cdot A(z) \rangle_T - \langle z \rangle_T \langle A(z) \rangle_T}{\langle z^2 \rangle_T - \langle z \rangle_T^2} = \frac{1}{\Delta t} \left[ \frac{\langle z(t) \cdot z(t + \Delta t) \rangle_T - \langle z(t) \rangle_T \langle z(t + \Delta t) \rangle_T}{\langle z(t)^2 \rangle_T - \langle z(t) \rangle_T^2} - 1 \right].
\]

FIG. 6: (Color online) Sliding window analysis of the time-dependence of (a) the proportion of sign-convert cases and sign-sustained cases \( \beta \), (b) the autocorrelation function \( C_{\Delta t=1} \), and (c) the slope coefficient \( a \) with \( \Delta t = 1d \). The lengths of sliding windows are \( T = 250d \) (left) and \( T = 2000d \) (right), and windows shift by 5 days per time. (d) The correlation function between \( a \) and \( C \).
When $T$, compared to $\Delta t$, is sufficiently long, the stationary assumption of financial time series is: $\langle z(t) \rangle_T \approx \langle z(t + \Delta t) \rangle_T$. Then, compared Eq. (15) with Eq. (16), a simple relation between $a$ and $C$ will be found,

$$a(\Delta t) \approx \frac{1}{\Delta t} [C(\Delta t) - 1]$$  \hspace{1cm} (16)

which is valid for any value of $\Delta t$ because of no limit to $\Delta t$ during the derivation. However, since the lack of correlation for $\Delta t > 1d$, only the case with $\Delta t = 1d$ is analyzed. The statistical result $C_{\Delta t=1d}$ is compared with the slope coefficient $a$, and the correlation function between $a$ and $C$ exhibited the good effectiveness of Eq. (16) showed in Fig. 6. The analytical results indicate that the sign-cases, slope coefficient $a$, and autocorrelation function $C_{\Delta t=1d}$ reflect the similar properties of time series. Known that correlations observed in financial time series show the incompleteness of the efficient market hypothesis [11], the three coefficients $C$, $a$, and $\beta$ may probably indicate the degree of market efficiency. From the same tendency of the three coefficients showed in Fig. 6 with $T = 2000d$ (right), two conclusions will be arrived at: (i) the market lost efficiency from 1961 to 1976 relatively, since the values is much larger than the remaining 25 years; (ii) the market tended to be more and more efficient from 1968 to 1999 because of the decreasing trend of the value.

From the preceding analysis of the equilibrium position $z_0$ and the slope coefficient $a$, the final form of restoring force $A(z)$ will be got by substituting Eq. (6) and Eq. (16) into Eq. (6).

$$A(z) = \frac{1}{\Delta t} (C - 1)(z - \langle z \rangle).$$  \hspace{1cm} (17)

This new form as a function of the traditional qualities, $C$ and $\langle z \rangle$, is a more direct way to understand the dynamical behavior of time series. To the financial data, the information given by $A(z)$ mixes features of the macroscopical properties together with the detail of the prices evolution: the macroscopical trend of prices is presented by the equilibrium position $z_0$, and the detail correlation between two neighboring days is exhibited by the slope coefficient $a$.

Eq. (17) can be informatively rewritten as $A(z) = -(z - z_0)/t_0$. $t_0$ is the characteristic relaxation time, $t_0 = -\Delta t/a$, and reflects the same properties of the log-returns as the slope coefficient $a$ does, but $t_0$ is more visualized. The maximum values of $t_0$, which were calculated from the sliding windows of various length, are all less than 2 days, and the average values of $t_0$ with $T = 250d$ and $T = 2000d$ are $t_0|_{250d} = 1.153d$ and $t_0|_{2000d} = 1.157d$. Thus, the effect of the restoring force $A(z)$ of one given day decays with the characteristic relaxation time less than 2 days.

### C. The diffusion parameter $B(z)$

The diffusion parameter $B(z)$, which usually has the form shown in Eq. (6), corresponds to a state-dependent linear multiplicative noise term $B(z)dw$ in Eq. (6). That is to say, the

![FIG. 7: (Color online) (a) Daily log-returns of S&P 500 index from 1950 to 1999. (b) The time-dependence of fitting errors $\sigma_A$ and $\sigma_B$ with $T = 250d$. (c) The time averaged volatility of the same series with $T = 250d$.](image-url)
TABLE I: The results from the log-return series of the S&P 500 index on the time scale τ from 1d to 5d. The average slope \( \bar{a} \), the average value of autocorrelation \( \bar{C} \), the average proportion of the sign-sustained and sign-convert cases \( \bar{\beta} \), the characteristic relaxation time \( \tau_0 \) with \( T = 250d \) and 2000d are listed in this table.

| \( \tau(d) \) | \( \bar{a} \) 250d | \( \bar{a} \) 2000d | \( \bar{C} \) 250d | \( \bar{C} \) 2000d | \( \bar{\beta} \) 250d | \( \bar{\beta} \) 2000d | \( \tau_0(d) \) |
|---|---|---|---|---|---|---|---|
| 1 | -0.883 | -0.874 | 0.115 | 0.125 | 1.242 | 1.223 | 1.153 | 1.137 |
| 2 | -0.467 | -0.462 | 0.533 | 0.539 | 2.723 | 2.252 | 2.178 | 2.187 |
| 3 | -0.303 | -0.299 | 0.697 | 0.702 | 3.205 | 3.137 | 3.395 | 3.414 |
| 4 | -0.224 | -0.222 | 0.776 | 0.779 | 4.136 | 4.059 | 4.416 | 4.633 |
| 5 | -0.182 | -0.177 | 0.818 | 0.825 | 4.986 | 4.886 | 5.759 | 5.822 |

E. The results of the log-return series \( z(t)|_{\tau \neq 1d} \)

In the way of our Langevin approach to the log-return series \( z(t) \), with \( \tau = 2d, 3d, 4d, 5d \) are also studied. It can be concluded that, the forms of \( A(z) \) and \( B(z) \) [Eq. (5) and Eq. (6)], the correlation function between the equilibrium position \( z_0 \) and the average log-return \( \langle z \rangle \) [Eq. (9)], the correlation function between slope \( \bar{a} \) and autocorrelation \( \bar{C} \) [Eq. (16)], and the new form of restoring force \( A(z) \) [Eq. (17)] are all effective for these series. Table 1 lists the results from the log-return series of the S&P 500 index on the time scale \( \tau \) from 1d to 5d.

IV. SUMMARY AND OUTLOOK

In this paper, we present a coarse-grain time-dependent Langevin description of the dynamics of stock prices, which is proved to be effective by the results obtained from analyzing the S&P 500 index. The time dependence of drift parameter \( A(z) \), which was considered as the restoring force, was investigated by the simple sliding windows algorithm. Significantly, while choosing the right weighted factor (Eq. (7)) to approximate the statistical results of \( A(z) \), the linear approximation of \( A(z) \) can reflect both the macroscopical and the detail properties of the price evolution, and the final form of the restoring force Eq. (17) can be achieved from analytical methods. The macroscopical trend of price could be investigated from the equilibrium position \( z_0 \), and the daily correlation in log-return was exhibited by the flat slope coefficient \( \bar{a} \). The mechanism of our model is discussed by analyzing the sign series of log-retuns. Therefore, from the restoring force \( A(z) \) in Langevin approach, one can get the properties of experimental data or the properties of financial markets. Furthermore, it must be pointed out that the random force \( B(z) \) also plays an important role in the dynamics of financial markets, which will be addressed further in the future study.

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