PHENOMENOLOGICAL QUANTUM GRAVITY: THE BIRTH OF A NEW FRONTIER?

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Abstract

In the last years a general consensus has emerged that, contrary to intuition, quantum-gravity effects may have relevant consequences for the propagation and interaction of high energy particles. This has given birth to the field of “Phenomenological Quantum Gravity” We review some of the aspects of this new, very exciting frontier of Physics.

1 Introduction

In the late 50’s John Weeler [1] made clear that when gravity (described by general relativity) is coupled to quantum mechanics, the concept of space-time itself changes: in fact space-time becomes dynamical and when examined at very small distances, near \( l_P = \sqrt{\frac{\hbar c}{G_N^3}} \approx 10^{-33} \text{ cm} \) (\( G_N \) being the Newton constant and \( l_P \) is called the “Planck distance”) has to show
violent fluctuations making for instance impossible to define a distance. The
emergence of these phenomena has been generically named space-time foam,
but their effects were comfortably thought to be visible only in processes not
testable in laboratory physics.

However the Universe has more surprises than we might expect.

One of the consequences of general relativity (and in fact historically
preceeding it) is that in flat space-time, as is approximately ours at least
for distances much smaller than the scale of the Universe, Lorentz Invari-
ance should hold. This has been tested with high precision. It is however
important to remind that relativistic invariance stems from experimental
facts and as such has to be put under scrutiny. It is in fact expected that
in space-time foam regime violations of relativistic invariance might appear.
So (minuscule) departures from relativistic invariance will in general signal
the onset of the QG regime.

And in fact Nature provides us with very sensitive tools to test Lorentz
Invariance: the Cosmic Microwave Background Radiation (CMBR) and
other universal radiations. On this radiation UHE cosmic ray protons can
interact and loose energy quite efficiently and it is expected that their spec-
trum bends at the highest energies. It is the famous Greisen, Zatsepin and
Kuzmin (GZK) cut-off [2], with a threshold at $E \approx 5 \times 10^{19}$ eV. The energy
lost by protons in the CMBR is such that they can only travel for about
100 $M_p$, before being brought below the threshold. So the spectrum should
show a sharp decrease above the threshold, the amount being related to the
distribution and evolution of possible sources.

Notwithstanding the extreme energy of the threshold, the process involved
is a low energy one, and in fact one of the best known experimentally:
the pion photoproduction $\gamma p \rightarrow N\pi$ whose cross-section is extremely well
known, in the frame in which the target proton is at rest. This absorption
appears at extreme energies just because it is the process above, but ex-
amined in a reference frame where the photon has an extremely low energy
($\approx 10^{-3}$ eV). It is therefore clear that a verification of the presence of the
GZK cut-off would imply a verification of Lorentz boosts between frames
with a Lorentz factor $\gamma_L \approx 5 \cdot 10^{10}$.

This fact was discovered already few years after the theoretical deter-
mination of the GZK cut-off, in 1971 in a paper by Kirshinitz and Chechin
[3], which went largely unnoticed. At the time there were already CR ex-
periments (Haverah Park, Yakutsk) with the possibility of detecting CR
primaries at these energies, but the situation was not clear. In their pio-
neping work they wrote “Primary protons with energy above $5 \cdot 10^{19}$ eV are
expected to be strongly slowed down by the interaction with the background thermal radiation. However, no break is observed in the CR spectrum in this region. It is of course premature in this circumstances....” and the key observation was “The point is that the primary protons have a uniquely large Lorentz factor $\gamma > 5 \times 10^{10}$ larger by many order of magnitudes than in any other experiment.”

With these premises they proposed a modified relativity theory to introduce small violations in the dispersion relation of particles at sufficiently high energies in such a way to account for the absence of the so-called GZK feature in the spectrum. This may be taken as the official date of birth of “phenomenological quantum gravity” although it might not have been named in such way those days.

The point, as we will see in a moment, is that there are processes in which possible tiny violations of Lorentz invariance, normally absolutely negligible, can be in some way amplified by peculiar physical situations (here, the extreme Lorentz factor between the frames in which the GZK effect is expected and the laboratory in which cross sections are measured), generally typical of astrophysical contexts, in such a way to induce in principle measurable effects. Theories in which relativistic invariance is modified are then in general falsifiable, and in fact many classes of them have already been.

More than 30 years after the experimental situation concerning UHECRs is still unclear: previous largest experiments, AGASA [4] and HiRes [5] do not provide strong evidence either in favor or against the detection of the GZK feature [6]. A substantial increase in the statistics of events, as expected with the Auger project [7] will clarify the scenario in one or two more years.

2 Relativistic invariance modifications from Quantum Gravity

As anticipated above, there is a priori no guarantee that, when space-time becomes dynamical, relativistic invariance is preserved down to the smallest distances (or highest energies).

Several attempts to construct a model for QG have been done. They basically share a new interpretation for space-time: it is no more a given background for physical objects These attempts include Loop QG, some string-based model and the space-time foam approach.

However the status of QG theories, although much more evolved than
even a few years ago, does not still allow to describe a realistic low (compared to the Planck) energy limit where the effects of modification of relativistic invariance should be experimentally verifiable/falsifiable.

Therefore several models have been proposed as effective theories that should try to catch some of the possible new QG physics at large but still sub-Planckian energy scales.

All these approaches predict some modification of basic physical principles. The following is a non-exhaustive cumulative list of the different possibilities. The first is the possibility of modification of Poincaré and Lorentz symmetries. Depending on the specific model they can still be exactly realized as well as explicitly broken (introducing a preferred reference frame) or kept but in a deformed way. The energy-momentum (dispersion) relation is generally modified including extra terms that can be of fixed or stochastic nature. Generally a new invariant physical scale \( l_p \) or the Planck energy \( E_p \) is introduced and this scale can (possibly) coexist with the standard invariant: the (low energy limit of) light speed \( c \), which may in fact acquire an energy dependence. Other possible effects are indetermination in position and/or momentum measurements due to the fluctuating nature of the space-time structure and the appearance of new non-linear composition laws for energy and momentum of multiparticle states \( i.e. P_{\text{tot}} \neq \sum_i P_i \).

Many of these possibilities have been investigated trying to find possible experimental signatures for new physics even at energy scales much smaller the \( 10^{28} \) eV that correspond to the Plank energy.

Astroparticle physics is a privileged arena for such studies both for the availability of very energetic particles and for the possibility to consider their motion along large (cosmological) distances. Among the others the large distance propagation of photons with energy dependent velocity [8, 9, 10] and modifications induced in the standard synchrotron radiation emission process have been considered to put limits on possible Lorentz Invariance (LI) breaking [11, 12, 13].

Another interesting possibility to test such models is to consider physical processes with a kinematic energy threshold, which is in turn very sensitive to the smallest violations of LI. This is the case for UHECRs and TeV gamma rays. UHECRs are expected to suffer severe energy losses due to photopion production off the photons of the cosmic microwave background (CMB), and this should suppress the flux of particles at the Earth at energies above \( \sim 10^{20} \) eV, the so called GZK feature. Super-TeV energy photons from sources at cosmological distances are expected to undergo electron-positron production in interactions with low energy photons of the far infra
In both cases a very large $\gamma$ factor is involved in moving from the laboratory to the center of mass reference frame. The sharply defined thresholds can be substantially shifted (or even disappear) if a small LI breaking term is introduced giving potential for investigation in this field. The new phenomena, if present, should show up in modification of expected UHECRs spectrum.

Some authors \cite{14, 15, 16, 17} have invoked possible violations of LI as a plausible explanation to some puzzling observations related to the detection of ultra high energy cosmic rays (UHECRs) with energy above the GZK feature, and to the unexpected shape of the spectrum of photons with super-TeV energy from sources at cosmological distances.

Both types of observations have in fact many uncertainties, either coming from limited statistics of very rare events, or from accuracy issues in the energy determination of the detected particles, and most likely the solution to the alleged puzzles will come from more accurate observations rather than by a violation of fundamental symmetries.

For this reason, from the very beginning we proposed \cite{18} that cosmic ray observations should be used as an ideal tool to constrain the minuscule violations of LI, rather than as evidence for the need to violate LI.

We adopt some reasonable choice to parametrize the LI violations predicted by QG models, consider the theoretical consequences and compare with experimental data. If the features in the spectrum related to the processes thresholds are indeed found this will provide limits on LI violation scale. If such features are absent this will allow us to reject some models but, for the moment, not to prove the existence of LI breaking new phenomena.

\section{More on Lorentz invariance modifications}

The recipes for the modifications of LI generally consist of requiring modification (either explicit or stochastic) of the dispersion relation of high energy particles. This modification is an effective way to describe their propagation in the “vacuum”, now affected by quantum gravity (QG) phenomena. This effect is generally parametrized by introducing a mass scale $M$, expected to be of the order of the Planck mass, that sets the scale for QG to become effective.

Without referring to any specific model, we write a modified dispersion relation obeying the following postulates:
1) modifications are universal, \textit{i.e.} do not depend on particle type;
2) modifications preserve rotational invariance;
3) modifications are an high energy phenomenon, vanishing at low momenta.

With these conditions we write the following expression:

\[ E^2 - p^2 = \mu(m, p/M) = m^2 + p^2 f(p/M) \]  

(1)

This deformed dispersion relation has been proposed by several authors [19, 20, 21, 22] and is the most popular in the literature.

Just for completeness we note that another possibility compatible with the dimensional analysis exists: it refers to the so called conformal models of LI modifications and was considered by Kirzhnits and Chechin in their paper. It accounts to introduce the extra (respect to the standard case) term proportional to the particle mass squared instead that to \( p^2 \). When considering thresholds modifications this last possibility gives no detectable effects for UHECRs propagation if \( M \) is the Planck mass.

Already at this stage we can intuitively understand why modification of the dispersion relations can sensitively affect the threshold values: the right-hand side of the modified dispersion relation can be thought as a (momentum dependent) effective mass, and the thresholds do depend explicitly on rest masses: we therefore expect strong modifications.

The standard way to proceed is to expand the last term in rhs of (1) and this, at lowest order, gives a term of the form

\[ E^2 - p^2 = m^2 + \eta (p/M)^\alpha \]  

(2)

where \( \alpha \) is model dependent and \( \eta \) a real parameter of order one. To get a quick result and some physical insight we can argue that, for massive particles, the above extra term in dispersion relation becomes relevant for the kinematics of particle interactions when its modulus is comparable with the particle squared mass. For the protons (\textit{i.e.} for the GZK case) we get immediately the following numbers for the critical momentum \( p_c \), where we may expect changes (in the following formula we fix \( M \) to the Planck mass value):

\[ \alpha = 1 \quad \rightarrow \quad p_c = (m_p^2 M^2)^{\frac{1}{4}} \simeq 10^{15} \text{eV} << M \]

\[ \alpha = 2 \quad \rightarrow \quad p_c = (m_p^2 M^2)^{\frac{1}{4}} \simeq 10^{18} \text{eV} << M \]
In both cases we see that the value of $p_c$ is much smaller than the Planck mass scale, justifying *a posteriori* the Taylor expansion. This gives another indication that if we modify the dispersion relation with terms related to some scale (the Planck mass in our case), the resulting particle kinematics can indeed be sensitive to such changes already at much lower energy scales. In other words we do not need Planck scale experiments to detect effects related to new physics at Planck scale.

A detailed calculation of photopion and $e^+e^-$ threshold production for high energy protons and photons interacting with low energy background photons has been carried out [18]. In this calculation the conservation of total energy and momentum of incoming and outgoing particles is assumed.

If the total energy and momentum of multiparticle states are calculated as usual (just the sum of the contribution of each particle) and we assume that the scale parameter $M$ is the Planck mass we find that the GZK feature could be absent (the threshold goes to infinity) when we consider $\eta$ negative, or, for positive sign, shifted downward by five ($\alpha = 1$) or one ($\alpha = 2$) order of magnitude respect to the standard case.

Notice that (as an aside) the same equations that do describe the modifications of the thresholds also imply that for positive sign some decay processes like $\gamma \rightarrow e^+e^-$ can happen at high energies and this severely restricts the range of allowed modification parameters. From a theoretical point of view notice this means that physics might be different in different reference frames, as expected when Lorentz symmetry is violated.

The same calculations can be done in the framework of Doubly Special Relativity. In this case the theory is constructed in such a way that the relativity principle is still valid: no privileged reference system exists. The (non linear) deformed boost in momentum space require a change in the dispersion relation as the one previously considered but also a different definition of total energy and momentum in multiparticle states. For the DSR1 [21] and DSR2 [22] models we have (at lowest order in $p/M$) [23]:

$$E_{\text{tot}} = E_1 + E_2 - \frac{1}{2M} (p_1p_2 + p_2p_1) + O\left(\frac{1}{M^2}\right)$$

$$E_{\text{tot}} = E_1 + E_2 + \frac{1}{M} (E_1E_2 + E_2E_1) + O\left(\frac{1}{M^2}\right)$$

In this case basically no new particle processes (like photon decay) are kinematically allowed and, for the GZK case, the momentum threshold is basically the same as in standard case [24].

In drawing conclusions from this kind of studies we have to keep in mind
that there are two main problems. The first is related to the up to now relatively poor and conflicting experimental data on UHECR spectrum. This will be discussed in more detail in the next section. The second is related to the limitation of approaches based uniquely on kinematic analysis: the present impossibility to include the dynamical effects of the full theory makes quantitative conclusions questionable (even if it seems reasonable to expect modifications to dynamics to be proportional to the energy scale divided by $M$ and hence highly suppressed for physics below GZK scale).

We conclude this section by remarking that it is also possible to assume that modifications are of stochastic nature, i.e. $\eta$ in Eq. 2 becomes a random variable. In fact we can think that this is more natural since we do expect that the space-time itself shows fluctuations near the Planck scale, and that modifications as the ones discussed above emerge as the result of some averaging over quantum fluctuations. Many aspects of this approach are discussed in the contribution of J.Y. Ng to this workshop [31]. Here we want only remark that, when propagation becomes stochastic, in general both signs of $\eta$ are possible, and in fact unavoidable. This has many striking consequences, as discussed in detail in [32], the most unexpected being that all charged particles do emit photons in the vacuum at all energies, in principle loosing catastrophically energy.

In this case the threshold can be written as

$$p_{th} \simeq \left( \frac{m^2 M \omega}{\delta} \right)^{1/4}$$

where $m$ is the particle mass, $\omega$ is the photon energy and $\delta$ is some combination of fluctuating coefficients. Clearly $p_{th} \to 0$ if $\omega \to 0$ and this will eventually result in a stability crisis for all charged particles [32, 33]. It is not clear at present how to avoid this problem, but certainly this will set conditions on allowable theories.

4 The Ultra High Energy Cosmic Ray spectrum: present state and perspectives for Phenomenological Quantum Gravity

The Cosmic Ray spectrum at and above the GZK cut-off is presently known with large statistical and systematic errors; moreover the two largest experiments till now have presented conflicting evidence (although at less than
3 π level): AGASA does not find evidence of the cut-off while the spectrum presented by HiRes shows the expected decrease.

A new generation of experiments is developing and already the Pierre Auger Observatory (P.A.O, see [34] in these proceedings), still in its building phase, is the largest experiment in the world. When the flux will be delivered at the 2005 summer conferences, a statistic analogous to the total AGASA statistic will be employed, although systematic errors are likely to be still relatively large.

Then in a year or two the question of the existence of a sharp decrease of the spectrum will be settled and we will be left with only two possibilities:

- The bend in the spectrum will be found at the expected position, as in the HiRes data: then some kind of relativistic invariance must hold at least up to $10^{20}\ eV$, like in normal Lorentz Invariant theory, as well as in most DSR approaches\(^1\). It is in general difficult to distinguish between the two approaches. In some DSR flavours, however, real photons can acquire an energy-dependent speed, and this can be tested in future satellite experiments like GLAST. Finally, since we do not know the sources of Cosmic Rays at these energies, but we know that acceleration becomes less and less efficient at high energies, we cannot exclude that the budget of UHECRs sources is simply vanishing. In principle this possibility can be tested if the spectrum at energies sub-GZK is known with large precision, a measurement that P.A.O. can accomplish.

- No bending will be found. It is tempting to conclude that this will signal the onset of new physics connected to quantum gravity and violations of relativistic invariance. However this is at present unjustified, the reason still being that we do not know the sources at these energies: several alternative possibilities exist. However the sources should be relatively nearby ($D < 100\ Mpc$). If (with the statistic allowed by P.A.O.) a statistically significant correlation of super-GZK events with very distant sources will be found, than propagation in the Universe (and therefore relativistic invariance) will be at a question.

As a final speculation, we remark that if it were possible to detect the spectrum of a single source, an exponential decrease (rather than a bend) is expected. This can be done with photons. Some controversial claim

\(^1\)At least one example exists of a ad hoc DSR theory that produces large shift of thresholds, without violating frame independence [35].
has already been reported at tens of TeV energies, where absorption is on
the FIRB, whose spectrum is however affected by systematics. The same
measurement on the CMBR would imply detection of PeV photons, with
the bonus that the interaction length becomes smaller than the galactic
radius. So the detection of even a single extragalactic (say, in the Magel-
lanic Clouds) PeV source, although terribly demanding from an experimen-
tal point of view, would unambiguously signal departure from relativistic
invariant propagation [36].

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