Bag Formation from Gauge Condensate

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As it is well known, one can lower the energy of the trivial perturbation QCD vacuum by introducing a non-vanishing chromomagnetic field strength. This happens because radiative corrections produce an effective action of the form $\int (F_{\mu\nu}F^{\mu\nu})$ with $f(y_0) = 0$ for some $y_0 \neq 0$. However, a vacuum with a non zero field strength is not consistent with Poincare Invariance (PI). Generalizing this type of effective action by introducing, in the simplest way, a four index field strength $\partial_{[\mu} A_{\nu\alpha\beta]}$, which can have an expectation value without violating PI, we are lead to an effective action that can describe both a confinement phase and a perturbative phase of the theory. In the unconfined phase, the 4-index field strength does not introduce new degrees of freedom, while in the confined phase both 4-index field strength and ordinary gauge fields are not true degrees of freedom. The matching of these phases through membranes that couple minimally to the 3-index potentials from which the 4-index field strength derive, leads automatically to the MIT bag boundary conditions for the gauge fields living inside the bubble containing the perturbative phase.

I. INTRODUCTION

The fact that some condensates of gauge field want be present in the vacuum of QCD in $3 + 1$ dimensions, has been argued from the well known instability of the perturbative QCD vacuum studied in [1]. There, at the one loop level, it was found that a condensate of the form

$$\langle A_{ia} \rangle = \frac{B}{2} \epsilon_{ijk} x_j \delta_{\alpha 3},$$

for example in the case of an $SU(2)$ gauge theory, can have an energy lower than that of the perturbative vacuum. In fact the one loop contribution to the vacuum energy of a state with a chromomagnetic field is

$$E_{\text{vac}} = \frac{1}{2} B^2 \left[ 1 + bg^2 \ln \left( \frac{B}{\mu} \right) \right] \quad (1)$$

where $b = \frac{11 C_A}{48 \pi^2}$, $C_A$ being the adjoint Casimir operator ($= N$ for $SU(N)$). $E_{\text{vac}}$ given by (1) has a minimum below zero at a non zero value of $B$, leading to the existence of a condensate. Once the existence of a condensate of gauge field is accepted, some problems are, of course, evident, since an explicit expectation value of $\langle A_{ia} \rangle$ breaks Lorentz invariance for example. Attempts to restore translation and rotation invariance were made in [2–10]. In [11], the rotation and homogeneity has been addressed but still a full Poincare invariance formulation seems hard to obtain this way.

Clearly, thinking about the non-stability of a gauge field condensate with the reasonable requirement of Poincare symmetry of the physical vacuum will be a non-trivial task, to say the least. One can think of different pictures where this is achieved. For example, in [12] in the case of QCD in $2 + 1$ dimensions, Poincare invariance is restored, while condensate are allowed in the vacuum but these all possible vector potentials, with all possible directions in color and real space are integrated with equal strengths. Such random background of coloured field is argued to lead to confinement of static charges [13], for other types of background gauge fields, color confinement effects have also been found [14].

The above mentioned pictures where the conflict between the existence of a gauge condensate and Lorentz invariance use in a fundamental way at least one quantum mechanical concept, that is that of averaging in order to achieve an effectively Poincare invariant situation. Those effects can not be represented by a classical effective action. There is, however, a known classical gauge field action which can acquire an expectation value without violating Poincare invariance. This is the case of a four index field strength $F_{\mu\rho\alpha\beta}$, which derives from a 3-index potential according to

$$F_{\mu\rho\alpha\beta} = \partial_{[\mu} A_{\nu\alpha\beta]} = \partial_{\nu_1} V_{\alpha\beta}$$

where $V_{\alpha\beta} \rightarrow A_{\gamma\alpha\beta} + \partial_{\gamma} V_{\alpha\beta}$

In 4-dimensional flat spacetime, it is an identity that

$$\varepsilon_{\mu\rho\alpha\beta} = 0$$

and

$$\varepsilon_{\mu\rho\alpha\beta} \neq 0$$

if any index is repeated and $\varepsilon_{\mu\rho\alpha\beta}$ is totally anti-symmetric. $(\varepsilon)$ can be non zero and this is not in conflict with Poincare invariance of the vacuum.

II. 4-INDEX FIELD STRENGTH AND PHASES OF THE THEORY: CONFINING AND PERTURBATIVE

We consider an effective action that depends on the contributions

$$y = F_{\mu\nu} F^{\mu\nu} + \varepsilon_{\mu\rho\alpha\beta} \partial_{\mu} A_{\nu\alpha\beta}$$

where $B = \frac{11 C_A}{48 \pi^2}$, $C_A$ being the adjoint Casimir operator ($= N$ for $SU(N)$). $E_{\text{vac}}$ given by (1) has a minimum below zero at a non zero value of $B$, leading to the existence of a condensate. Once the existence of a condensate of gauge field is accepted, some problems are, of course, evident, since an explicit expectation value of $\langle A_{ia} \rangle$ breaks Lorentz invariance for example. Attempts to restore translation and rotation invariance were made in [2–10]. In [11], the rotation and homogeneity has been addressed but still a full Poincare invariance formulation seems hard to obtain this way.

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and where the contribution of the gauge fields to the Lagrangian density is of the form

$$S = \int f(y) d^4x \quad F_{\mu\nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu - g f^{abc} A_\mu^b A_\nu^c \quad (6)$$

We can also include a the coupling of gauge fields to fermions (quarks), this will introduce another term in the action (6), which will be added to \( f(y) \), of the form

$$\sum_{i=1}^3 \bar{\Psi}_i (\partial - g A_\mu \tau_\alpha - m_i) \Psi_i \quad (7)$$

It is straightforward to see that a variation of this action with respect to three form \( A_{\alpha\beta} \) will bring about the following equation

$$\varepsilon^{\mu\nu\alpha\beta} \partial_\mu \left( \frac{\partial f(y)}{\partial y} \right) = 0 \quad (8)$$

That gives \( \frac{\partial f(y)}{\partial y} = \omega = \text{Constant} \), this means that for a truly non-linear function \( f(y) \) (we exclude here therefore the case \( f(y) = ay + b \), \( a \) and \( b \) being constants, for which \( \frac{\partial f(y)}{\partial y} = a \) without imposing conditions on \( y \))

$$y = F_{\mu\nu}^a F^{\mu\nu a} + \varepsilon^{\mu\nu\alpha\beta} \partial_\mu A_{\alpha\beta} = \text{Constant} \quad (9)$$

Therefore \( A_{\alpha\beta} \) does not represent new degrees of freedom, since the field strength \( F_{\mu\nu\alpha\beta} = \partial_\mu A_{\nu\alpha\beta} = \varepsilon_{\mu\nu\alpha\beta} \), which represent the only physical gauge invariant object under the gauge transformations (3) can be solved from (9) in terms of \( F_{\mu\nu}^a \).

The variation (9) with respect to the gauge field \( A_{\alpha\beta} \) gives the following equation of motion

$$\partial_\mu \left( \frac{\partial f(y)}{\partial y} F^{\mu\nu a} \right) - g \frac{\partial f(y)}{\partial y} f^{abc} A_\mu^b F^{\mu\nu c} = 0 \quad (10)$$

If \( \frac{\partial f(y)}{\partial y} = \omega \neq 0 \), i.e. we are not in the lower energy phase of the theory (the true vacuum). In this phase, the equations of motion are indistinguishable from those of classical QCD and they define a perturbative phase. If we turn our attention to the true vacuum state, we see that a drastically different situation arises. In the true vacuum state \( y = y_0 \) and \( \frac{\partial f(y)}{\partial y} |_{y=y_0} = 0 \), the gauge field equations of motion disappear and there is no meaningful equation for the gauge fields in this phase\(^1\), except for (9), which can be solved for any gauge field configurations by appropriately choosing \( A_{\alpha\beta} \). That means that in this phase any gauge field configuration is allowed and that all gauge field configurations have constant energy density \( f(y_0) \). The canonical momenta associated to \( A_\mu^a \)

all vanishes in the phase \( \frac{\partial f(y)}{\partial y} |_{y=y_0} = 0 \)

$$\pi_\mu^a = \frac{\partial L}{\partial \dot{A}_\mu^a} = 0 \quad (11)$$

The vanishing of all the canonical conjugate momenta in the true vacuum phase means that, in this phase, the gauge fields are not true dynamical variables. In fact according to the Dirac constraint Hamiltonian theory, the first order constraints (11) generate the gauge invariance, generated by

$$Q = \int A_\mu^a \pi_{\mu a} d^3x \quad , \quad A_\mu^a = [A_\mu^a, Q] = \Lambda_\mu^a \quad (12)$$

That is \( A_\mu^a \) can be transformed into anything we want. This is in agreement with ideas concerning the confinement phase of QCD in [19], where it was argued that the gauge invariance in the confinement phase is bigger then the non-confinement phase. In Polyakov’s language, we must have “non-Abelian third kind gauge invariance” (chapter 5 in [19]).

### III. DYNAMICAL COUPLING IN THE PERTURBATIVE VACUUM

We would like to show that equation (10) in either phases with constant \( \omega \), including the current \( j^{\mu a} \), can be written in the original Yang-Mills form by rescaling the gauge fields \( A_\mu^a \) and defining a new coupling constant in the following way

$$A_\mu^a \to \tilde{A}_\mu^a = \omega \frac{\dot{A}_\mu^a}{\omega} \quad , \quad g \to \tilde{g} = g \frac{\sqrt{\omega}}{\omega} \quad (13)$$

with those definitions the field strength tensor rescales to

$$\tilde{F}_{\mu\nu}^a = \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu - \tilde{g} f^{abc} \tilde{A}_\mu^b \tilde{A}_\nu^c \quad (14)$$

and equation (10), including the fermionic fields, reads

$$\partial_\mu \tilde{F}_{\mu\nu}^a - \tilde{g} f^{abc} \tilde{A}_\mu^b \tilde{F}_{\mu\nu c} = -i \frac{1}{4} \tilde{g} \sum_{i=1}^3 \bar{\Psi}_i \gamma^\nu \tau_\alpha \Psi_i \quad (15)$$

the effective coupling constant that results depends on the vacuum though \( \omega \). As one approaches the true vacuum phase, where \( \omega \) is equal to zero, we see that the effective coupling constant approaches infinity this is consistent with the idea that such a vacuum represent a confining phase of the theory.

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\(^1\) If one also considers fermionic fields, one finds that the fermionic current, \( j^{\mu a} = -\frac{i}{2} \bar{\Psi} \sum_{i=1}^3 \gamma^\nu \tau_\alpha \Psi_i \), identically vanishes since the term representing the gauge fields contribution to the equation of motion disappears in this phase
IV. BAG FORMATION AND DERIVATION OF THE MIT BOUNDARY CONDITIONS

We now address the question of the possible coexistence of these two phases, that is whether it is possible for example to form bags of perturbative phase in the middle of a true vacuum state, the confinement phase. As we will see, the answer to this question is positive. Such an effect can be produced if the minimal coupling of the 3-index gauge potential $A_{\alpha\beta}$ to 2 + 1 membranes is considered \[15\]. That is we add to the action \[15\] a term of the form

$$S_{int} = \lambda \int A_{\alpha\beta} \frac{\partial z^\gamma}{\partial x^\alpha} \frac{\partial z^a}{\partial x^\beta} \varepsilon_{abc} d^3\xi$$ \hspace{1cm} (16)

Where $z(\xi)$ represents the location of the $2 + 1$ dimensional membrane in the four dimensional spacetime and where $\xi^a$ and $\varepsilon_{abc}$ are, respectively, the coordinates defined on the membrane and the totally anti-symmetric 3-dimensional Levi-Civita tensor.

Here we present the study of the theory \[15\] neglecting gravitational effects, having in mind a model for the strong interaction alone and in this context effects not considered in \[15\]. We can include a coupling of gauge fields to fermions (quarks). In such a case one must consider the following action

$$S = \int \left[ f(y) + \sum_{i=1}^{3} \Psi_i (i\gamma^\mu \partial_\mu - m_i) \Psi_i \right] d^4x + \int \delta^4(x-x(\xi))A_{\alpha\beta} \frac{\partial x^\gamma}{\partial x^\alpha} \frac{\partial x^a}{\partial x^\beta} \varepsilon_{abc} d^3\xi d^4x \hspace{1cm} (17)$$

A. Three-form field variation

We start by performing a variation of the action \[17\] with respect to the three-form $A_{\alpha\beta}$, this variation leads to the following expressions which we will denote by $I_1$, $I_2$

$$I_1 = \int \frac{\partial d^4x}{\partial y} A_{\alpha\beta} \frac{\partial f(y)}{dy} \hspace{1cm} (18)$$

and

$$I_2 = -\varepsilon^{\mu\nu\alpha\beta} \frac{\partial d^4x}{\partial y} \frac{\partial f(y)}{dy} + \lambda \int \delta^4(x-x(\xi)) \frac{\partial x^\gamma}{\partial x^\alpha} \frac{\partial x^a}{\partial x^\beta} \varepsilon_{abc} d^3\xi d^4x \hspace{1cm} (19)$$

Turning our attention to the second part of this variation, $I_2$, one has the following equation

$$\varepsilon^{\mu\nu\alpha\beta} \frac{\partial d^4x}{\partial y} \frac{\partial f(y)}{dy} = \lambda \int \delta^4(x-x(\xi)) \frac{\partial x^\gamma}{\partial x^\alpha} \frac{\partial x^a}{\partial x^\beta} \varepsilon_{abc} d^3\xi d^4x \hspace{1cm} (20)$$

We are interested in obtaining spherically symmetric solutions, in which the membrane forms a closed surface, dividing space into inside and outside regions. Then the normal to the membrane is given by \[17\]–\[18\]:

$$\frac{1}{\sqrt{\gamma}} \frac{\partial x^\mu}{\partial y^a} \frac{\partial x^a}{\partial \xi^\beta} \varepsilon_{\mu\nu\alpha\beta} = -\epsilon n^\mu \hspace{1cm} (21)$$

Multiplying \[20\] by $n^\sigma$ and $\varepsilon_{\sigma\mu\nu\alpha\beta}$ and using \[21\], we see that equation \[20\] shows that on either side of the membrane $\frac{\partial f(y)}{dy}$ is constant, and that these two values of $\frac{\partial f(y)}{dy}$ differ in magnitude $|\lambda|$. We will choose $\frac{\partial f(y)}{dy}$ to be zero outside the membrane (the outer region is the lower energy phase of the theory, the true vacuum) and have a non-vanishing value inside, in this manner we bring about the two different phases of the theory, the confining and perturbative phases.

For the first part of this variation, $I_1$, we notice that since one can not use \[20\], this integral will vanish from pure variational principles arguments, that is, the "end" points are held fixed, so $\delta A_{\alpha\beta}$ is zero at the ends.

B. Gauge Field Variation

We now turn to the variation of the action \[17\] with respect to $A_{\alpha}^a$, we again denote the different parts by $I_3$ and $I_4$ given by

$$I_3 = \int \frac{\partial d^4x}{\partial y} F^\mu d^4x \hspace{1cm} (22)$$

and

$$I_4 = \int \left( -\frac{\partial d^4x}{\partial y} F^{\mu\alpha} - \frac{\partial f(y)}{dy} f^{abc} A_{\alpha}^b F^{\nu\mu} \right) d^4x + \frac{-i}{4} g \sum_{i=1}^{3} \Psi_i \gamma^\nu \tau_3 \Psi_i A_{\alpha}^a d^4x \hspace{1cm} (23)$$

The second part of this variation, $I_4$, will give us the gauge field equations of motion

$$\frac{\partial d^4x}{\partial y} F^{\mu\alpha} + g f^{abc} A_{\alpha}^b \frac{\partial d^4x}{\partial y} F^{\nu\mu} = \gamma^\mu$$

$$= -\frac{i}{4} g \sum_{i=1}^{3} \Psi_i \gamma^\nu \tau_3 \Psi_i$$ \hspace{1cm} (24)

2 the normal is defined as $\sqrt{\gamma} \frac{\partial x^\mu}{\partial y^a} \frac{\partial x^a}{\partial \xi^\beta} \varepsilon_{\mu\nu\alpha\beta}$, where $d = D – 1$, in flat space $\sqrt{\gamma} = 1 \hspace{1cm} (13)–(14)$, this normal points from the inside to the outside and $\epsilon = \pm 1$.\[
This equation, when expanded with the use of (20) and the discussion that followed it, contains elements multiplied by a delta-function and a step-function.

\[
\lambda \left( \int \delta^4(x-x(\xi)) \left( \frac{\partial \delta^4}{\partial \xi^a} - \frac{\partial \delta^4}{\partial \xi^b} \right) \right) F^{\mu \nu a}
+ \frac{\partial f(y)}{\partial y} \left( \partial_\mu F^{\mu \nu a} + g f^{abc} A^b_\mu F^{\mu \nu c} \right) = j^{\nu a}
\]

(25)

Where the first part is actually a delta-function (on the surface) times the normal (by using (21)) multiplied by the field strength tensor, and the function \( \frac{\partial f(y)}{\partial y} \) is a generalized step function. Following this naive approach, one could conclude that this contribution, containing a delta-function, must vanish by itself. This will lead to the conclusion that \( n_\mu F^{\mu \alpha} = 0 \) on the surface, thus, deriving the well known MIT boundary conditions.

We will now follow a more rigorous formalism, i.e., the variational principle, to derive this result. To do this we first notice that the \( I_3 \) part of the variation, given in (22), contains the generalized step-function \( \frac{\partial f(y)}{\partial y} \), where now the use of (20) is allowed. The appearance of \( \frac{\partial f(y)}{\partial y} = 0 \) (outside the membrane) "cuts-off" the outer region of the integral, i.e., the "outer end point". Furthermore, since there is a discontinuity in \( \frac{\partial f(y)}{\partial y} \), and its "on-surface value" is not well defined, it can take any value from zero to \( \lambda \) so we can not say anything about \( \frac{\partial f(y)}{\partial y} \) and \( \delta A^\alpha_\nu \) at the surface.

Following these arguments, we conclude that the vanishing of this integral, (22), which can be written as

\[
\int_S \frac{\partial f(y)}{\partial y} F^{\mu \nu a} n_\mu \delta A^a_\nu ds
\]

must be due to the boundary condition \( n_\mu F^{\mu \alpha} = 0 \) on the surface, and not because the potential \( A^\alpha_\nu \) is held fixed on the membrane. We then have a derivation of the famous and well known MIT boundary conditions from an action and variational principles. One should also note that there is no current in the confining phase, meaning the fermion fields \( \Psi_j(x) \) are associated with the inner part, the perturbative phase, and \( n_\mu j^{\mu \alpha} = 0 \) on the surface as well.

V. APPLICATION TO TMT

Two Measure Theory (TMT) is a new class of gravity theories based on the idea that the action integral may contain a new metric-independent measure of integration. For example, in \( D = 4 \) space-time dimensions the new measure density can be built out of four auxiliary scalar fields \( \varphi^i \) (\( i = 1, 2, 3, 4 \)):

\[
\Phi(\varphi) = \frac{1}{4!} \varepsilon^{\mu \nu \alpha \beta} \varepsilon_{ijkl} \partial_\mu \varphi^i \partial_\nu \varphi^j \partial_\alpha \varphi^k \partial_\beta \varphi^l
\]

(27)

An action which incorporate this new measure, and the previous idea will take the following form

\[
S = \int_M \Phi(\varphi) d^4 x + \lambda \int_\Sigma A_{\nu \alpha \beta} d x^\nu \wedge d x^\alpha \wedge d x^\beta
\]

(28)

\[
y = L_g (g_{\mu \nu}, R_{\mu \nu}(\Gamma)) + L_m + \frac{\varepsilon^{\mu \nu \alpha \beta} \partial_\mu A_{\nu \alpha \beta}}{\sqrt{-g}}
\]

(29)

We will relax the Levi-Civita condition on the metric and obtain the field equations following the Palatini approach, in which metric and connection are regarded as two physically independent entities [21]. This implies that both metric and connection must be determined by solving the respective equations obtained through the application of the variational principle.

Our first step will be to take the variation with respect to \( \varphi^3 \) of the new measure field \( \Phi \), this result of this variation is given by

\[
\partial_\mu f(y) = 0 \rightarrow \frac{\partial f(y)}{\partial y} \frac{\partial y}{\partial x^\mu} = 0
\]

(30)

The non-trivial solution of this equation is when \( \frac{\partial y}{\partial x^\mu} = 0 \), which implies \( y = \text{Const} \), this also implies \( \frac{\partial f(y)}{\partial y} = \text{Const} \), therefore, there is no jump in \( \frac{\partial f(y)}{\partial y} \) across the boundary, in contrast to the previous section. Here the discontinuity will manifest in a new parameter \( \chi \), which we will present next by taking the variation with respect to the three-form \( A_{\alpha \beta} \), this part of the variation is very much like the previous one, only this time the jump across the boundary is in the ratio of the two measures\(^3\)

\[
\chi \equiv \frac{\Phi}{\sqrt{-g}} \frac{\partial f(y)}{\partial y}
\]

(31)

which means that \( \chi \) is a generalized step function.

For the gravitational part of the action, the \( g^{\mu \nu} \) variation will contribute the following equation

\[
\chi \left( \frac{\partial (L_g + L_m)}{\partial g^{\mu \nu}} + g_{\mu \nu} \varepsilon^{\rho \sigma \alpha \beta} \partial_\rho A_{\sigma \alpha \beta} \right) = 0
\]

(32)

We have seen that \( y \) is constant, call it \( y_0 \), we can then use this to eliminate the 3-form field contribution

\[
\frac{1}{2\sqrt{-g}} g_{\mu \nu} \varepsilon^{\rho \sigma \alpha \beta} \partial_\rho A_{\sigma \alpha \beta}
\]

by substituting it with \( y_0 - \)

\(^3\) the jump is in the ratio of the two measures \( \frac{\Phi}{\sqrt{-g}} \) since we have already concluded that \( \frac{\partial f(y)}{\partial y} \) is constant
\[ \mathcal{L}(g_{\mu\nu}, R_{\mu\nu}(\Gamma)) - \mathcal{L}_{\text{matter}} \text{ in (32), this give rise to} \]

\[ \chi \left( \frac{\partial(L_g + L_m)}{\partial g_{\mu\nu}} - \frac{L_g + L_m}{2} g_{\mu\nu} + \frac{\gamma_0}{2} g_{\mu\nu} \right) = 0 \quad (33) \]

Since \( \chi \) is a generalized step function we conclude that equation (33), assuming \( \chi \neq 0 \) on either side of the membrane, actually describes two space-time regions, separated by a thin shell, and these two regions contain a dynamically generated cosmological constant.

VI. SUMMARY AND FUTURE WORK

We have seen that an effective action with a four index field strength \( \partial\mu A_{\nu\alpha\beta} \) can describe both a confinement and perturbative phases. In the unconfined phase, the 4-index field strength does not introduce new degrees of freedom, while in the confined phase both 4-index field strength and ordinary gauge fields are not true degrees of freedom. Matching the two phases through membranes that couple minimally to the 3-index potentials, from which the 4-index field strength derive, leads automatically to the MIT-bag boundary conditions, from a purely dynamical transformation, the total derivative term coming from the transformation

\[ \delta \left( e \left[ R(\omega, e) - \bar{\psi}_\mu \gamma^{\mu\nu\lambda} D_\nu \psi_\lambda \right] \right) = \partial_\mu \left[ e \left( \delta \zeta^\mu \right) \right] \quad (35) \]

In (23) we have considered a theory describing confinement, in flat spacetime, through a non-linear Maxwell term in the form of

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{f}{2} \sqrt{-g} F_{\mu\nu} F^{\mu\nu} \quad , \quad f > 0 \quad (36) \]

When we coupled this Lagrangian to gravity we find that a genuinely charged matter source of gravity and electromagnetism may appear electrically neutral to an external observer. The “charge-hiding” effect occurs in a self-consistent wormhole solution which connects a non-compact “universe”, comprising the exterior region of Schwarzschild (anti-)de-Sitter (or purely Schwarzschild) black hole beyond the internal (Schwarzschild) horizon, to a Levi-Civita-Bertotti-Robinson-type (“tube-like”) “universe”, with two compactified dimensions, via a wormhole “throat” occupied by the charged timelike brane. In this solution the whole electric flux produced by the charged timelike brane is expelled into the cosmological constant.

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