Abstract

We study the implications of the global $U(1)_R$ symmetry present in minimal lepton flavor violating implementations of the seesaw mechanism for neutrino masses. Our discussion is done in the context of explicit minimal type-I seesaw scenarios, where depending on the R-charge assignment different models can be constructed. We study the charged lepton flavor violating phenomenology of a concrete realization paying special attention to $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$.

1 Introduction

The observation of neutrino flavor oscillations constitutes an experimental proof of lepton flavor violation $^1)$. In principle, other manifestations of such effects could be expected to show up in the charged lepton sector as well. However, the lack of a definitive model for neutrino mass generation implies that conclusive
predictions for lepton flavor violating processes can not be made, and even assuming a concrete model realization for neutrino masses, predictions for such effects can only be done if the flavor structure of the corresponding realization is specified.

In this regards the minimal flavor violating hypothesis 2, 3, 4) is a very useful guide for constructing predictive models in which lepton violating signals are entirely determined by the low-energy neutrino data. However minimal lepton flavor violation (MLFV) can not be uniquely implemented and depends on the new physics responsible for neutrino masses. Here considering a type-I seesaw mechanism i.e. taking the new degrees of freedom to be heavy fermionic electroweak singlets (right-handed (RH) neutrinos), we study the implications of the $U(1)_R$ present in MLFV models assuming it is slightly broken. The full analysis is done in the context of a minimal type-I seesaw setup (2 RH neutrinos) where the number of parameters and low energy observables are such that all flavor effects are entirely determined by neutrino observables up to normalizations factors.

## 2 The setups

The kinetic Lagrangian of the standard model extended with two RH neutrinos exhibits the global $G = U(3)_c \times U(3)_\ell \times U(2)_N$ symmetry. This group can be rewritten as $U(1)_Y \times U(1)_L \times U(1)_R \times G_F$ where $U(1)_{Y,L}$ can be identified with global hypercharge and lepton number whereas the $U(1)_R$ is a “new” global symmetry, already mentioned in the introduction 4, 5).

The charges associated with this global transformation (hereafter denoted by $R$) are arbitrary, and thus different $R$-charge assignments define different models. Here we will explicitly consider the seesaw Lagrangian with a slightly broken $U(1)_R$ and discuss a generic model\(^1\) that arises from a particular $R$-charge assignment.

Precisely speaking we refer to the following: $R(N_1, \ell_i, e_i) = +1$, $R(N_2) = -1$, $R(H) = 0$ where $\ell_i, e_i$ and $N_{1,2}$ are respectively the electroweak lepton

\(^1\)A second class of models can be constructed in which the small breaking of $U(1)_R$ allows to decouple the lepton number breaking scale from the RH neutrino mass scale. But this decoupling implies also a suppression of the corresponding Yukawa couplings, thus leading to non-observable charged lepton flavor violating effects. For more details see 6)
define the RH neutrino mass matrix. The diagonalization of the Majorana RH neutrino mass matrix leads to two quasi-degenerate states with masses given by \( M \), and thus are tiny, \( \tilde{\nu} \), are Yukawa vectors in flavor space and \( M \) is the charge conjugation operator, the \( \lambda \)'s are Yukawa vectors in flavor space and \( M_{\alpha} \) are the parameters that define the RH neutrino mass matrix. The diagonalization of the Majorana RH neutrino mass matrix leads to two quasi-degenerate states with masses given by \( M_{N_{1,2}} = M \pm \frac{M_{11} + M_{22}}{2} \epsilon_N \) and in the basis in which the RH neutrino mass matrix is diagonal the Yukawa couplings \( \lambda_{\alpha} \) become \( -\frac{\epsilon}{\sqrt{2}} [\lambda_{k1} + (-1)^a \epsilon \lambda_{k2}] \) \( (k = e, \mu, \tau \) and \( a = 1, 2) \). In terms of these redefined Yukawa couplings, the effective light neutrino mass matrix, up to \( O(\epsilon_N^2) \), is given by

\[
\begin{align*}
\nonumber m_{\nu}^{eff} &= -\frac{\epsilon^2 \lambda}{M} |\lambda_1| |A| \left( \hat{\Lambda}_1^* \otimes \hat{\Lambda}_1^* + \hat{\Lambda}^* \otimes \hat{\Lambda}_1^* \right),
\end{align*}
\]

(2)

with \( \hat{\Lambda}_1^* = \hat{\lambda}_2^* - \frac{M_{11} + M_{22}}{4M} \epsilon_N\hat{\lambda}_1^* \). Note that we have expressed the parameters space vector \( \lambda_1 \) and \( A \) in terms of their unitary vector \( \hat{\lambda}_1, \hat{\Lambda} \) and moduli \( |\lambda_1|, |A| \). Since \( \epsilon_{\lambda} \ll 1 \) small neutrino masses do not require heavy RH neutrinos or small Yukawa couplings, thus potentially implying large lepton flavor violating effects.

It turns out that due to the structure of (2) the vectors \( \lambda_1 \) and \( A \) can be entirely determined by means of the solar and atmospheric mass scales and mixing angles, up to the factors \( |\lambda_1| \) and \( |A| \). The expression depend on the light neutrino mass spectrum, in the normal case they read \( 7 \)

\[
\begin{align*}
\lambda_1 &= |\lambda_1| \hat{\lambda}_1 = \frac{|\lambda_1|}{\sqrt{2}} \left( \sqrt{1 + \rho} U_3^* + \sqrt{1 - \rho} U_2^* \right), \\
A &= |A| \hat{A} = \frac{|A|}{\sqrt{2}} \left( \sqrt{1 + \rho} U_3^* - \sqrt{1 - \rho} U_2^* \right),
\end{align*}
\]

(3)

where \( U_i \) denote the columns of the leptonic mixing matrix and

\[
\rho = \frac{\sqrt{1 + r} - \sqrt{r}}{\sqrt{1 + r} + \sqrt{r}}, \quad r = \frac{m_{\nu_2}^2 - m_{\nu_3}^2}{m_{\nu_3}^2 + m_{\nu_2}^2}.
\]

(5)

3 Lepton flavor violating processes

With potentially large Yukawa couplings and RH neutrinos in the TeV ballpark, charged lepton flavor violating processes could be expected to have large rates.
Figure 1: Radiative LFV decay branching ratio $BR(\mu \rightarrow e\gamma)$ for normal light neutrino mass spectra as a function of the common RH neutrino mass. The upper horizontal dashed line indicates the current limit on $BR(\mu \rightarrow e\gamma)$ from the MEG experiment \(^8\), whereas the lower one the future experimental sensitivities \(^9\).

In what follows we analyze $l_i \rightarrow l_j \gamma$ and $l_i^- \rightarrow l_j^- l_j^+ l_j^+$.  

3.1 $l_i \rightarrow l_j \gamma$ processes

The decay branching ratios can be approximated by \(^6\),

$$BR(l_i \rightarrow l_j \gamma) \simeq \frac{\alpha}{1024\pi^2} \frac{m_i^5}{M^4} \frac{|\lambda_1|^4}{\Gamma_{Tot}^l} |\hat{\lambda}_i \hat{\lambda}_j^*|^2,$$  \hspace{1cm} (6)

where $\Gamma_{Tot}^l$ stands for the total decay width of the corresponding decaying charged lepton $l_i$.

Among these lepton flavor violating processes, presently the $\mu \rightarrow e\gamma$ transition is the most severely constrained. The MEG collaboration recently established an upper bound of $2.4 \times 10^{-12}$ at the 90\% C.L. \(^8\). So from now on we focus on that process. To quantify this effect, we randomly generate low energy observables in their $2\sigma$ ranges and the parameters $|\lambda_1|$ and $M$ in the ranges $[10^{-5}, 1]$ and $[10^2, 10^6]$ GeV allowing RH neutrino mass splittings in the range $[10^{-8}, 10^{-6}]$ GeV.

The results for the normal mass spectrum case are displayed in figure 1. It can be seen that $BR(\mu \rightarrow e\gamma)$ can reach the current experimental limit reported by the MEG experiment \(^8\) for RH neutrino masses $M_N < 0.1$ TeV, 1 TeV, 10 TeV provided $|\lambda_1| \gtrsim 2 \times 10^{-2}$, $10^{-1}$, 1, respectively.
Figure 2: Decay branching ratio $BR(\mu^- \rightarrow e^- e^+ e^-)$ for normal light neutrino mass spectra as a function of common RH neutrino mass. The upper horizontal dashed line indicates the current bound for $\mu^- \rightarrow e^+ e^- e^-$ placed by the SINDRUM experiment \(^{11}\), whereas the lower one future experimental sensitivities \(^{12}\).

3.2 $l_i^- \rightarrow l_j^- l_j^- l_j^+$ processes

The decay branching ratios for these processes have been calculated in \(^{10, 6}\). The most constrained process in this case is $\mu^- \rightarrow e^+ e^- e^-$ for which the SINDRUM experiment has placed a bound on the decay branching ratio of $10^{-12}$ at 90\% C.L. \(^{11}\). Following the same numerical procedure used in the previous section we calculate the decay branching ratio for $\mu^- \rightarrow e^+ e^- e^-$. The results are shown in figure 2. Again, as in the $\mu \rightarrow e\gamma$ case, it can be seen that the decay branching ratio can saturate the current experimental bound for RH neutrino masses $M_N < 0.1 \text{ TeV}, 1 \text{ TeV}, 10 \text{ TeV}$ provided $|\lambda_1| \gtrsim 2 \times 10^{-2}, 10^{-1}, 1$, respectively.

4 Conclusion

The presence of an extra global $U(1)_R$ in the seesaw Lagrangian allows the construction of different types of models, all of them determined by the R-charge assignments of the lepton sector. We have considered a concrete realization and analyzed its consequences for the most promising charged lepton flavor violating decays, namely $\mu \rightarrow e\gamma$ and $\mu^- \rightarrow e^- e^+ e^-$. Our analysis shows that for a large mass range of the lepton flavor violating mediators these processes might be observed in near future experiments.
5 Acknowledgements

I would like to thank D. Aristizabal Sierra and J. F. Kamenik for the fruitful collaboration. D. Aristizabal Sierra also for the help during the preparation of this work.

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