Offshore wind: Evidence for two-dimensional turbulence and role of sea horizon

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We analyse offshore wind speeds with a high time resolution of one second over a long period of 20 months for different heights above the sea level. Power spectra \( S(f) \) show a scaling behaviour consistent with three-dimensional turbulence at high frequencies, followed by the so-called spectral gap region at lower frequencies, where \( fS(f) \) varies weakly. For frequencies lower than a crossover frequency \( f_{\text{ED}} \), a rapid rise of \( fS(f) \) occurs, which reflects the scaling \( fS(f) \sim f^{-2} \) and \( fS(f) \sim f^{-2/3} \) of the enstrophy and inverse energy cascade of two-dimensional turbulence. Contrary to earlier observations but in agreement with theoretical predictions, the regime of the enstrophy cascade appears at higher frequencies compared to that of the inverse energy cascade. Strikingly, an analysis of the third moment (structure function) \( D_3(\tau) \) of wind speed differences for a given time lag \( \tau \) provides strong further evidence of a transition to two-dimensional turbulence. This is reflected in a rapid change from negative to positive values of \( D_3(\tau) \) at lags close to \( 1/f_{\text{ED}} \). We argue that the physical meaning of \( f_{\text{ED}} \) is connected with the distance of the sea horizon from the measurement point.

Understanding offshore wind properties is a central problem for forecasting wind power and for estimating wind farm power outputs. Due to the turbulent nature of wind flows in the atmosphere, this is a challenging problem. For three-dimensional (3D) homogeneous isotropic turbulence, a description in terms of Kolmogorov’s theory is possible. Its hallmark is a scaling of power spectra (kinetic energy spectra) with the wavenumber \( k \)

Spectra of horizontal wind speeds \( v = (v_x^2 + v_y^2)^{1/2} \), with \( v_x \) and \( v_y \) being the components parallel to the Earth’s surface, show a deviation from K41 scaling. When measurements at a height \( h \) above the surface are performed, isotropic 3D turbulence can no longer be assumed for length scales larger than \( h \). In power spectra of wind speeds, the corresponding crossover frequency \( f_{\text{ED}} \approx \bar{v}_h/h \), with \( \bar{v}_h \) the mean wind speed at height \( h \), marks the high-frequency limit of the so-called spectral gap region [6, 7]. The spectral gap has been discussed controversially regarding its very existence and features. There is evidence now that this gap exists and that its properties are dependent on the measurement height \( h \) (for a review, see Ref. [8]).

Several studies suggest that the spectrum in the spectral gap regime can show a \( f^{-3} \) scaling [9–11] and different models have been developed to explain this scaling [12–17]. Other fitting functions have been proposed also for describing that regime [18, 19]. Recently, it was suggested that the spectral gap results from a superposition of isotropic three-dimensional and isotropic two-dimensional (2D) turbulence [8]. That atmospheric turbulence is characterised by the scaling properties of 2D turbulence has been debated since a long time [20, 21].

In particular, it was suggested that a superposition of an \( f^{-3} \) and \( f^{-5/3} \) scaling can describe wind spectra [8], similar as it was reported earlier for measurements made on board of aircrafts [22]. These findings seem to reflect the signatures of isotropic 2D turbulence derived in the seminal paper by Kraichnan [20]: A regime of \( f^{-3} \) scaling as fingerprint of the enstrophy cascade, and a slightly broader regime of \( f^{-5/3} \) scaling at lower frequencies indicative for the inverse energy cascade.

However, in the description of the wind speed spectra in Refs. [8, 22], the order of the scaling regimes is reversed, i.e. the \( f^{-3} \) scaling is suggested to appear at lower frequencies and the \( f^{-5/3} \) scaling at higher ones. As for the description of the aircraft data, the contradiction to the theoretical predictions was pointed out earlier by Lindborg [21].

Here we show that spectra of offshore wind speeds measured at another location in the North Sea show scaling features consistent with the theoretical predictions for 2D turbulence in a frequency interval \( 10^{-6} - 10^{-4} \) Hz. The scaling regimes for the enstrophy and inverse energy in this interval turn out to be quite narrow and it is difficult to identify them clearly. Based on the spectra, we thus can conclude only that our analysis of offshore is indicative for a regime of 2D turbulence.

We provide strong further evidence for such a regime by studying wind speed fluctuations in the time domain, where 2D and 3D turbulence can be distinguished by the sign of the third-order structure function [21, 23, 24]. This function quantifies the third-order moment of velocity differences in space, or, when applying Taylor’s hypothesis [3], in time also. We find that the third-order
FIG. 1. Identification of different scaling regimes and of the spectral gap in power spectra of offshore wind speeds. For this identification, the spectra multiplied by the frequency \( f \) are plotted in a double-logarithmic representation for three different heights \( h = 100 \text{ m}, 60 \text{ m}, \text{ and } 30 \text{ m} \). Open circles refer to \( fS_{\text{ave}}(f) \), where \( S_{\text{ave}}(f) \) is obtained by an averaging over all spectra of sub-sequences without missing values (see “Methods” for details). Full circles refer to the total spectrum \( S_{\text{tot}}(f) \), where linearly interpolated wind speeds were taken in all intervals of missing data. Green crosses mark spectra calculated from ten minutes averaged wind speeds in the period January 2005 to July 2021 (see text). The vertical lines separate the various regimes: the blue line at frequency \( f_{3D} = \bar{v}_h/(\pi h) \) separates the scaling regime of 3D turbulence from the spectral gap interval, the red line at frequency \( f_{2D} = \bar{v}_h/(2\pi d_h) \) separates the spectral gap interval from the regime of 2D turbulence, and the black line at frequency \( f_{x} \) marks the onset of uncorrelated wind speed fluctuations (white noise behaviour). The theoretical scaling laws expected in the regimes of 2D and 3D turbulence are indicated by orange solid lines, as well as the white noise behaviour at very low frequencies.

structure function changes from negative values at small time lags to positive values at large lags, as expected for a transition from 3D to 2D turbulence. The zero-crossing is remarkably sharp and occurs at a certain lag \( 1/f_{2D} \). We argue that \( f_{2D} \) is determined by the distance between the measurement point and the sea horizon.

RESULTS

The wind speeds were measured at the FINO1 platform in the North Sea, which is located about 45 km north from the island Borkum [25]. They were sampled by three-cup anemometers over 20 months, from September 2015 to April 2017, for eight different heights \( h \) between 30 m and 100 m. The time resolution was \( \Delta t = 1 \text{ s} \), yielding time series with \( N \approx 5 \times 10^7 \text{ speed values for each height} \). Further details of the data with information in particular on missing values (NaN entries) are given in the “Methods”.

Diurnal variations of the offshore wind speeds did not show up as significant patterns in spectra or structure functions and we therefore did not apply a corresponding detrending of the data. We furthermore did neither consider seasonal variations nor changes of meteorologic stability [18], because we expect them to have only a weak effect on our principal results. Variations of the atmospheric stratification and boundary layer thickness, which are characterised by the meteorologic stability, can be expected to largely average out. Seasonal variations may affect our findings at very long times and corresponding low frequencies only.

Results of our analysis are presented for the three measurement heights \( h = 30 \text{ m}, 60 \text{ m}, \text{ and } 100 \text{ m} \). The mean \( \bar{v}_h \) and standard deviation \( \sigma_h \) of the wind speeds for these heights are listed in Table I.

Power spectra

Our results for the power spectra are shown in Fig. 1a-c for the three heights \( h = 30 \text{ m}, 60 \text{ m} \text{ and } 100 \text{ m} \), respectively. In this figure, \( fS \) vs. \( f \) is plotted in a double logarithmic representation, which allows us to identify different frequency regimes in a clear way. When calculating the spectra we have used two methods to cope with longer periods of missing values: In the first method, we determined power spectra \( S_\alpha(f) \) separately for all time intervals \( \alpha \) with existing successive data and averaged these in bins equally spaced on the logarithmic frequency axis. The averaged values \( S_{\text{ave}}(f) \) are shown by the open circles in Fig. 1. Details of the calculations of \( S_{\text{ave}}(f) \) are given in the “Methods”.

In the second method, each interval of successive missing values was linearly interpolated between the two wind speed values terminating the interval. The resulting series covers the total time span of 20 months and we calculated its power spectrum \( S_{\text{tot}}(f) \). This spectrum can not
be valid for frequencies \( f \gtrsim 1/\bar{v}_h \) and perhaps lower frequencies. However, as shown in Fig. 1, the spectra \( S_{\text{tot}}(f) \) (full circles) agree with \( S_{\text{ave}}(f) \) in the intermediate frequency range \( 10^{-4} \text{Hz} \lesssim f \lesssim 10^{-2} \text{Hz} \). This demonstrates that \( S_{\text{tot}}(f) \) is reliable for low frequencies \( f < 1/\bar{T}_{\text{saw}} \). The combination of \( S_{\text{tot}}(f) \) and \( S_{\text{ave}}(f) \) allows us to discuss the wind speed spectrum over the entire frequency range.

**Transition to 2D turbulence**

Above a frequency \( \sim \bar{v}_h/h \) (with \( \bar{v}_h \) the mean wind speed, see Table I) we see in Fig. 1 the signature of 3D turbulence, i.e. a behaviour consistent with the K41 scaling. We define

\[
\frac{\bar{v}_h}{\pi h}
\]

as the border of this frequency regime, which is marked by the vertical blue lines in the figure. The K41 scaling behaviour is indicated by the solid lines with slope \((-2/3)\). When \( f \) becomes smaller than \( f_{3D} \), it enters the spectral gap, where the K41 scaling is absent.

We argue below that the spectral gap terminates at a low frequency

\[
f_{2D} = \frac{\bar{v}_h}{2\pi d_h}
\]

which is determined by the distance between the sea horizon and the measurement point. This distance is given by

\[
d_h = \sqrt{2r_h h + h^2} \approx \sqrt{2r_h h},
\]
where \( r_e \) is the Earth’s radius. The factor two in (4) is motivated by our results for the third-order structure function \( D_3(\tau) \) (see below). The frequency \( f_{sd} \), defined in (4) is indicated by the red vertical lines in Fig. 1. For \( f \ll f_{sd} \), \( fS \) displays a sudden rapid increase towards lower \( f \). This indicates that \( f_{sd} \) is indeed a characteristic frequency for describing offshore wind turbulence.

For frequencies below \( f_{sd} \) the results in Fig. 1 indicate a scaling \( fS \sim f^{-2} \) characteristic for the enstrophy cascade followed by a scaling \( fS \sim f^{-2/3} \) at lower frequencies, corresponding to the inverse energy cascade. These scaling laws are marked by the two solid lines in the frequency intervals around \( 3 \times 10^{-5} \) Hz and \( 6 \times 10^{-6} \) Hz.

The crossover frequency between the enstrophy and inverse energy cascade regimes is close to a frequency 1/day. At this crossover frequency one could have expected a peak to occur due to diurnal variations. Such a peak has indeed been observed in the early analysis of onshore wind data by Van der Hoven [6]. A diurnal peak does not occur in Figs. 1a-c for the three different heights. We believe this is because of weaker diurnal temperature variations of oceans compared to land masses. That signatures of scaling features for 2D turbulence can be identified in the power spectra in Figs. 1a-c could in fact be enabled by the absence of a diurnal peak.

Luckily, it is possible to support the transition to 2D turbulence by switching the analysis to the time domain and analysing the behaviour of \( D_3(\tau) \). In the recent past, theoretical studies based on the Navier-Stokes equations have shown that \( D_3(\tau) \) should in general be positive for 2D turbulence [21, 23, 24], while it is negative for 3D turbulent flows [1]. Accordingly, \( D_3(\tau) \) is expected to change sign at a time lag close to \( 1/f_{sd} \).

Figures 2a-c show that this is indeed the case. To make the changes in \( D_3(\tau) \) clearly visible, we have plotted \( D_3(\tau)^{1/3} \) in this figure. The time \( \tau_{sd} = 1/f_{sd} \) is marked by the vertical red solid lines, which intersect the curves \( D_4(\tau) \) almost at their minima. For \( \tau < \tau_{sd} \), \( D_4(\tau) \) is negative. When \( \tau \) becomes larger then \( \tau_{sd} \), \( D_4(\tau) \) increases rapidly and crosses zero slightly above \( \tau_{sd} \). This finding strongly supports the crossover to 2D turbulence for frequency scales \( f < f_{sd} \). The close matching of \( \tau_{sd} \) with the lag of minimal \( D_3(\tau) \) is the reason, why we introduced the factor two in our definition of \( f_{sd} \) in (4).

As for the time \( \tau_{sd} = 1/f_{sd} \), it is not very significant in the third-order structure functions in Figs. 2a-c. This is different for the kurtosis \( \kappa(\tau) \) shown in Figs. 2d-f in a double-logarithmic representation. In all respective graphs, we have marked \( \tau_{sd} \) by a vertical blue line. The kurtosis rapidly decreases with \( \tau \) for \( \tau < \tau_{sd} \). This decrease is consistent with the downward energy cascade in 3D turbulence and the intermittency corrections to K41 scaling [26], which predicts \( \kappa(\tau) \sim \tau^{-4\mu/9} \). The parameter \( \mu \) is the intermittency factor and quantifies the amplitude of the logarithmic correction in the scaling of the energy dissipation rate with scale \( r \). Values of \( \mu \) lie in the range 0.2-0.5 [27–29]. The orange lines drawn in Figs. 2d-f have slopes corresponding to \( \mu = 0.45 \). The kurtosis values for \( \tau = 1 \text{s} \) and \( \tau = 2 \text{s} \) deviate from the respective lines, but one should keep in mind that for time lags approaching one second, differences of wind speeds measured by cup anemometers lose precision. That \( \kappa(\tau) \) is much larger than three for small \( \tau \) reflects fat non-Gaussian tails in the distribution of wind speed fluctuations for short times [30]. For \( \tau \approx \tau_{sd} \), the kurtosis crosses over to a plateau-like regime at larger lags \( \tau \). When \( \tau \) approaches \( \tau_{sd} \), \( \kappa(\tau) \) again decreases until reaching a value \( \kappa(\tau) \approx 3 \) for \( \tau \gtrsim \tau_{sd} \), reflecting Gaussian distributed wind speed fluctuations.

The time \( \tau_x \) has a value of about 4 days and corresponds to a frequency \( f_x = 1/\tau_x \), where \( fS(f) \) in Figs. 1a-c runs through a peak maximum. This peak is known as the synoptic peak. If we assume Taylor’s hypothesis to hold even at large time scales of order \( \tau_x \), the corresponding spatial scale \( r_x = \sqrt{\nu_x \tau_x} \approx 3 \times 10^3 \text{km} \) is the typical linear dimension of low and high pressure areas.

For \( r \gtrsim r_x \), one should not expect correlations between wind speed fluctuations to be present. Accordingly, a constant white noise spectral behaviour should occur for \( f < f_x \). To test this expectation, one needs very long time series to suppress numerical noise in the spectra. The FINO1 project [25] also provides ten minutes averaged wind speeds in the long period January 2005 until July 2021. Taking these data, we calculated power spectra \( S_{10\text{min}}(f) \) with the same method as used for obtaining \( S_{tot} \). The results are represented by the green crosses in Figs. 1a-c and agree with \( S_{tot} \) and \( S_{ave} \) for frequencies below \( f_{sd} \). In the low-frequency regime \( f < f_x \), they indeed show a behaviour \( fS_{10\text{min}} \sim f \) of a white noise spectrum. The particular high value of \( S_{10\text{min}} \) at the frequency of 1/year reflects annual variations.

The scaling behaviours \( f_{3d} \sim 1/h \) and \( f_{2d} \sim 1/\sqrt{h} \) in Eqs. (3) and (4) mean that both \( f_{3d} \) and \( f_{2d} \) should shift to lower frequencies with increasing measurement height. This is in agreement with the shifts of the blue and red vertical lines in Figs. 1a-c. The frequency \( f_{sd} \) should shift less strongly than \( f_{3d} \), which implies that the spectral gap interval is expected to shrink with increasing height \( h \), as well as the scaling regime of 2D turbulence when assuming \( f_x \) to be independent of \( h \) [black vertical lines in Figs. 1a-c, indicating the frequency of the synoptic peak]. Also the shrinkage of the respective frequency regimes is seen in Figs. 1a-c. A conclusive quantitative check of the functional dependence of \( f_{sd} \) and \( f_{2d} \) on \( h \) is, however, not possible here for the limited range of measurement heights between 30 m and 100 m.

Role of the sea horizon

Let us now reason, why we conjecture that the frequency \( f_{sd} \) in (4) is related to the distance \( d_h \) in (5). This interpretation arises from the idea that self-similar features in turbulent wind dynamics, as reflected in power-law scaling of spectra and structure functions, should
be expected if scale-dependent external influences on the wind field are absent.

For an observation point at a height $h$ above the sea surface, scale-dependent influences are absent for length scales $r < h$. In that regime, the K41 scaling features of isotropic turbulence are observed. For distances $r > h$, this scaling is no longer observed because the wind field is affected by the Earth’s surface on the respective length scales. The breakdown of K41 scaling at $r \simeq h$ has been found also in experiments [31] and in direct numerical simulations [32].

The question emerges, whether power law scaling could occur for some length scales $r > h$, as conjectured in the transition scenario from 3D to 2D turbulence. For answering this question, let us consider a domain of influence $S_h(r)$ of the Earth’s surface on the wind field as it is seen from the observation point at a height $h$. How $S_h(r)$ varies with $r$ depends on the distance $d_h$, see Fig. 3a: For $h < r < d_h$, $S_h(r)$ is dependent on $r$ and given by all points on the Earth’s surface with distances $r' \leq r$ to the observation point. For $r > d_h$, by contrast, $S_h(r) = S_h(d_h)$ is independent of the scale $r$.

We characterise the corresponding regimes of spatial scales $r$ by assigning to $S_h(r)$ the solid angle $\Omega_h(r)$ indicated in Fig. 3a. This cone is formed by all lines connecting the observation point with boundary points of $S_h(r)$. Setting $\Omega_h(r) = 0$ for $r \leq h$, it holds

$$
\Omega_h(r) = \begin{cases} 
0, & r \leq h, \\
2\pi \left(1 - \frac{r^2 + d_h^2}{2r(r_E + h)}\right), & h \leq r \leq d_h, \\
2\pi \left(1 - \frac{d_h}{r_E + h}\right), & r \geq d_h.
\end{cases}
$$

(6)

The three different spatial regimes according to (6) are shown in Fig. 3b. In the regimes of small and large $r$, $\Omega_h(r)$ is constant and we can expect scaling regimes of 3D and 2D turbulence to occur. Because both $h$ and $d_h$ are much smaller than $r_E$, $\Omega_h(r) \simeq 2\pi$ for $r \geq d_h$. In the intermediate regime $h \leq r \leq d_h$, $\Omega_h(r)$ increases with $r$, corresponding to the increasing impact of the Earth’s surface on the wind field. We interpret this regime, where we do not expect power law scaling due to self-similarity, as the spectral gap in the spatial domain.

**DISCUSSION**

Our analysis shows that the correlation behaviour of offshore wind speed fluctuations at times between a few hours and several days is in agreement with the theory of 2D isotropic turbulence. We believe that this is the first time that an agreement with the theoretical predictions has been demonstrated in atmospheric turbulence. Signatures of 2D turbulence are reflected in power spectra $S(f)$, but it is difficult to identify them clearly in such spectra. This is due to the limited extent of the frequency regimes, where the scaling according to the enstrophy and inverse energy cascade show up. Our analysis of the third-order structure function gives more clarity and yields strong support for the conjecture of a transition to 2D turbulence: close to the transition, the third-order structure function changes rapidly from negative to positive values.

We suggest to consider the spectral gap as the regime separating the scaling regimes of 3D and 2D turbulence. The upper frequency $f_{3D}$ of this gap is given by $f_{3D} \sim \bar{v}_h/h$, where $h$ is the height of the measurement point above the sea surface and $\bar{v}_h$ is the mean wind speed at that height. The lower frequency of the gap is argued to be determined by the distance $d_h$ of the sea horizon seen from the measurement point. This yields $f_{2D} \sim \bar{v}_h/d_h \sim \bar{v}_h/\sqrt{2r_Eh}$, i.e. a scaling of the crossover frequency $f_{2D}$ with the inverse square root of the measurement height ($r_E$ is the Earth’s radius). The values $f_{2D}$ obtained from our analysis are in agreement with this conjecture.

A point of concern regarding these findings is that the
data sampled at the FINO1 platform could have been affected by near wind farms that were put into operation since May 2010. Accordingly, wake effects could have influenced the turbulent velocity field around the measurement mast. To check this, we have additionally analysed wind speed data between May 2008 and December 2009. These data contain 25% NaN values, which are about 250 times more than in the data set used for our analysis discussed above. Nevertheless, the results for the 2008-2009 period show the same scaling behaviour of 2D and 3D turbulence. Within the gap separating the scaling regimes, it is possible that the third-order structure function becomes positive already for $\tau < 1/f_{2D}$. This goes along with a steeper (negative) logarithmic slope of $f S(f)$ at corresponding frequencies. Following the idea that the spectral gap is reflecting a mixture of 3D and 2D turbulent behaviour [8], we can interpret these findings as follows: For the data sampled between April 2015 and September 2017, features of 3D turbulence are prevailing throughout the gap regime, while for the data sampled between 2008 and 2009, features of 2D turbulence become dominant already before reaching time scales of order $1/f_{2D}$. This suggests that $1/f_{2D}$ is in fact the time above which features of 3D turbulence are no longer visible.

As for the question whether the proposed scaling of $f_{2D}$ with $h$ is valid, we could not obtain a conclusive answer. This is due to the fact that the measurement heights of wind speed data analysed here span a rather limited range from 30 to 100 m. We expect that in the near future local data of wind speeds over long time periods are available for larger heights above the sea level. These can be measured, for example, by the Light Detection and Ranging (LIDAR) technique [33, 34]. An analysis of corresponding data can be used to test our conjecture.

With the ever increasing computational power, it may by furthermore possible to analyse the proposed scenario also by large-scale simulations based on the Navier-Stokes equations. While length scales comparable to the distance of the sea horizon ($d_h \approx 36$ km for $h = 100$ m) can not be reached by direct numerical simulation [35, 36], this may be different for large eddy simulation in the future [37–39].

Our findings shed new light onto the characterisation of wind speed fluctuations from micro- to mesoscales and beyond, including an interpretation of the spectral gap regime. We hope that they contribute to a better understanding of the cascade of scales involved in wind energy science [5].

### METHODS

#### Data set

The time series contain sequences of wind speed data with a resolution $\Delta t = 1$ s and also sequences of missing values (NaN entries) of different lengths. The NaN entries require special care for the data analysis, in particular when calculating power spectra. Single missing values occur typically once a day, i.e. at about every $10^5$th entry in the time series. We have replaced a corresponding “not a number” (NaN) entry at a time $t_{NaN}$ by the interpolated value between the two wind speeds at the times $t_{NaN} \pm \Delta t$. The fraction $F_{NaN}$ of remaining NaN entries in the resulting time series $v_t$ of wind speeds is given in Table I. Time intervals with successive NaN entries are typically much longer than one second, indicating a temporary failure of the measurement device. The mean duration $T_{NaN}$ of the respective intervals is 12 minutes for the measurement heights $h = 60$ m and 100 m, and almost one hour for $h = 30$ m, see Table I.

#### Calculation of averaged spectra

The power spectra $S_{ave}(f)$ are an average over power spectra $S_\alpha(f)$ calculated from sequences of existing successive wind speeds. Let $\{v_n\}_\alpha = \{v^{(\alpha)}_n\mid n = 0, \ldots, N_\alpha - 1\}$ be the $\alpha$th sequence of wind speeds without NaN values, $\alpha = 1, \ldots, N_{seq}$, where $N_{seq}$ is the number of these sequences. The discrete Fourier transform of $\{v_n\}_\alpha$ is

$$
\hat{v}_m^{(\alpha)} = \sum_{n=0}^{N_\alpha - 1} v_n^{(\alpha)} e^{-2\pi i m n / N_\alpha}, \quad m = m_{\min}^{(\alpha)}, m_{\min}^{(\alpha)} + 1, \ldots, m_{\max}^{(\alpha)},
$$

(7)

where $m_{\min}^{(\alpha)} = \lfloor((N_\alpha - 1)/2)\rfloor$ and $m_{\max}^{(\alpha)} = \lfloor(N_\alpha/2)\rfloor$. The power spectrum of $\{v_t\}_\alpha$ at the frequency

$$
f_m^{(\alpha)} = \frac{m}{T_\alpha},
$$

(8)

with $T_\alpha = N_\alpha \Delta t$, is

$$
S_m^{(\alpha)} = S_{-m}^{(\alpha)} = \frac{2}{T_\alpha} |\hat{v}_m^{(\alpha)}|^2, \quad m = 1, \ldots, m_{\max}^{(\alpha)}.
$$

(9)

We averaged these values $S_m^{(\alpha)}$ for frequencies $f_m^{(\alpha)}$ in bins with equidistant spacing on a logarithmic frequency axis.

| $h$   | 100 m | 60 m | 30 m |
|-------|-------|------|------|
| $\bar{v}_h$ [ms$^{-1}$] | 9.2   | 8.6  | 8.2  |
| $\sigma_h$ [ms$^{-1}$]   | 4.8   | 4.6  | 4.3  |
| $F_{NaN}$                | 0.09% | 0.09%| 0.35%|
| $T_{NaN}$                | 12 min| 13 min| 55 min|
The bin size was \( b = 10^{0.1} \approx 1.26 \). The left and right border of the \( j \)th bin are denoted as \( f_j^- \) and \( f_j^+ \), respectively. The averaged power spectrum in the \( j \)th bin is

\[
\bar{S}_j = \sum_{\alpha=1}^{N_{\text{max}}} \sum_{m=1}^{m_{\text{max}}} S_m(\alpha) I_j(f_m(\alpha)) \frac{1}{\sum_{\alpha=1}^{N_{\text{max}}} \sum_{m=1}^{m_{\text{max}}} I_j(f_m(\alpha))}, \tag{10}
\]

where \( I_j(.) \) is the indicator function of the \( j \)th bin interval \( [f_j^-, f_j^+] \), i.e. \( I_j(f) = 1 \) for \( f \in [f_j^-, f_j^+] \) and zero otherwise. The \( \bar{S}_j \) value gives \( S_{\text{ave}}(f) \) at the frequency \( f = (f_j^- f_j^+)/2 \).

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