Sensor fault detection and isolation over wireless sensor network based on hardware redundancy

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Abstract. In order to diagnose sensor faults with small magnitude in wireless sensor networks,
distinguishability measures are defined to indicate the performance for fault detection and
isolation (FDI) at each node. A systematic method is then proposed to determine
the information to be exchanged between nodes to achieve FDI specifications while limiting the
computation complexity and communication cost.

1. Introduction
Wireless sensor networks (WSNs) can provide new methods for information gathering for a
variety of applications, like environment monitoring and human health monitoring. They are
working based on the collaboration of a large number of sensor nodes which integrate the
computation, communication and sensing capabilities. In order to ensure the network quality
of service, the quality of the measurements has to be guaranteed. Distributed fault detection
and isolation schemes are preferred to centralized solutions to diagnose faulty sensors in WSNs.
Indeed the first approach avoids the need for a central node that collects information from every
sensor node, and hence it limits complexity and energy cost while improving safety.

A taxonomy for classification of faults in sensor network is introduced in [1]. Two common
fault classes correspond to dead sensors, namely sensors not reporting any value, on the one
hand and sensor bias, on the other hand. The diagnosis of permanent faults for example caused
by battery depletion in WSNs are considered in [2] and [3]. A spinning tree fault diagnosis
protocol is proposed to identify the crash faults in [2] while considering the time efficiency and
energy consumption. An external operator can be used to replace the faulty nodes based on
the diagnostic information. In [3], a decentralized method is proposed to detect the permanent
faults in the cluster heads which are the leaders of the grouped nodes. According to the energy
and radio range, a cluster manager is chosen to be responsible to detect the fault in the cluster
head by confirming with ‘alive’ message, to notify all the other cluster members and to perform
the recovery by selecting a new cluster head.

Rather than hard faults like complete breakdown of a node, small and slowly developing
malfunctions should also be taken care of. Distributed fault detection algorithms with voting
scheme for sensor faults in wireless sensor networks have been proposed in [4, 5]. In these papers,
all the sensor nodes in the sensor network measure the same quantity. The sensor readings are
compared with their neighbor’s to determine whether they are faulty. Time redundancy is used
in [5] to tolerate transient faults in sensing and communication, which corresponds the errors
caused by some temporary conditions like connectivity issues and will disappear automatically. Distributed fault tolerant methods are used for event region detection in WSNs. The problem is to distinguish the specific events of interest from abnormal measurements. To achieve this goal, spatial correlation between the measurements is exploited by a distributed fault-tolerant Bayesian recognition algorithm in [6] and a clustering approach in [7]. Model-based FDI methods have been developed for distributed fault diagnosis in [8] and in [9], in which the dynamic model of the monitored system is considered.

In order to diagnose an incipient sensor fault, a distributed FDI system in WSNs can be designed with the aid of a measure named distinguishability degree. Distinguishability measures have been used in sensor placement problem for fault diagnosis in [11] [12] [13]. Qualitative detectability and isolability have been defined with a structural method in [11] and with an analytical method in [12]. They are used to decide which sensors to include to achieve fault diagnosis. A quantitative distinguishability measure is defined in a stochastic framework in [14] and it is used as a constraint for a greedy optimization algorithm to find globally optimal sensor sets for diagnosis [13]. The considered cost function is the the number of the selected sensors.

In this paper, we propose a distributed method based on the parity space approach to generate residuals for sensor fault detection and isolation in a WSN. A distinguishability measure is defined in a deterministic framework and a diagnosibility degree is defined for indicating the performance for FDI at each node. Furthermore, a systematic method is presented to determine the information to be exchanged between different nodes in order to achieve, when possible, a given performance level for FDI while the computation complexity and communication cost are kept as small as possible.

The paper is organized as follows. The problem statement is given in section 2. In section 3, the proposed approach for distributed FDI system design in a deterministic framework is described. The distinguishability and diagnosibility degree at each node are defined to characterize the property of the distributed FDI system. The selecting criteria among the neighboring nodes to improve the FDI system property are deduced. Then the proposed approach for distributed FDI system design is described. Next, a simulation example is used to illustrate how to improve the property of fault detection and isolation at a given node with the aid of the neighboring node in section 4.

2. Problem statement

A sensor network of $M$ nodes, like the one illustrated in figure 1, can be spatially placed over some region to monitor some target quantities of a system. This network can be described by a graph which is characterized by its adjacency matrix $W$, a $M \times M$ matrix with Boolean entries. The $(i,j)^{th}$ entry is equal to 1 if nodes $i$ and $j$ are connected, namely if they can communicate directly with each other. Otherwise, the entry is equal to 0. The connections between the nodes are called the edges. The neighborhood of a given node, say $i$, is denoted by $N_i$ and it is defined as the set of nodes that are directly connected to node $i$. The number of neighbors of node $i$ is called the degree of node $i$ and is denoted by $m_i$. These nodes can be called one-hop neighbors, for example, in figure 1, node 2 and node 3 are the one-hop neighbors of node 1. The nodes which are not connected with node $i$ but are connected with the neighboring nodes of node $i$ are called proximity neighbors of node $i$, or two-hop neighbors. Node 1’s proximity neighbors are node 4 and node 5. Likewise, the three-hop neighbors, are node 6 and node 7 [10]. The following measurement equation is considered at node $i$:

$$y_i(k) = C_i x(k) + f_i(k)$$

where $x(k) \in R^n$ is a vector that represents the target quantities, $y_i(k) \in R^{p_i}$ is the measurement vector at node $i$, $C_i$ is the measurement matrix at node $i$ that indicates which quantities are measured at node $i$ and $f_i(k) \in R^{p_i}$ is the fault vector. In order to get a variety of measurements
with a limited set of sensors, we assume that there are no redundant sensors at each node. The sensor nodes measure one or several of the target quantities, thus \( p_i \leq n \) and \( C_i \) has full row rank. Only single faults are considered here.

A local sensing model can be defined at each node by considering the measurements available in the neighborhood, namely:

\[
y_i \text{loc}(k) = C_i \text{loc} \bar{x}(k) + f_i \text{loc}(k)
\]

where \( y_i \text{loc}(k) \in R^{p_i \text{loc}} \), \( C_i \text{loc} \in R^{p_i \text{loc} \times n} \) and \( f_i \text{loc}(k) \in R^{p_i \text{loc}} \) are the local measurement vector, local measurement matrix and local fault vector over the network respectively. The following notations will be needed below. \( N_i = \{ i_1, \ldots, i_j, \ldots, i_m \} \) indicates the neighborhood of node \( i \). \( y_i \text{loc}(k) = [y_{i_1}^T(k), \ldots, y_{i_j}^T(k), \ldots, y_{i_m}^T(k)]^T \), \( C_i \text{loc} = [C_{i_1}^T, \ldots, C_{i_j}^T, \ldots, C_{i_m}^T]^T \), \( f_i \text{loc}(k) = [f_{i_1}^T(k), \ldots, f_{i_j}^T(k), \ldots, f_{i_m}^T(k)]^T \) and \( p_{i \text{loc}} = \sum_{j \in N_i} p_i \). Such notations are illustrated in example 1.

**Example 1.** A sensor network made of 7 nodes is considered as an example. The topology of the network is illustrated in figure 1 and the adjacency matrix of this network is thus \( W = \).

![Topology of a sensor network with 7 sensor nodes](image)

The network measures three different quantities which are represented by \( x(k) \in R^3 \). The measurement equation at each node can be described as:

\[
y_i(k) = C_i x(k) + f_i(k)
\]

where \( C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \), \( C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \), \( C_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \), \( C_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \), \( C_5 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \), \( C_6 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \), \( C_7 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \).

The local sensing model at node 1 is generated based on the adjacency matrix:

\[
y_{1 \text{loc}}(k) = C_{1 \text{loc}} x(k) + f_{1 \text{loc}}(k)
\]

where \( y_{1 \text{loc}}(k) = [y_1^T(k), y_2^T(k), y_3^T(k)]^T \), \( C_{1 \text{loc}}(k) = [C_1^T, C_2^T, C_3^T]^T \), \( f_{1 \text{loc}}(k) = [f_1^T(k), f_2^T(k), f_3^T(k)]^T \).
Our objective is to design and implement a distributed monitoring algorithm in order to detect and isolate sensor faults for each sensor node based on the local sensing model. If the FDI system at each node can not accomplish detection and isolation, it can be extended to include information exchange with the other nodes in the network. The process of selecting an extra node is performed with the aid of a distinguishability measure of sensors at each node. The distinguishability is presented and the criteria for selecting a suitable node are analyzed in the next section. Then an on-line algorithm is introduced for the distributed deterministic fault diagnosis system.

3. Distributed sensor fault detection and isolation

The aim of a fault detection and isolation system is to generate fault indicators (also called residuals) and to determine the most likely faulty component through the analysis of the fault signature on these residuals. Since the property of the residual generator can reflect the relation between the residual and faults, detectability and isolability can be defined based on residual generator according to [15] notably. A distinguishability measure which includes detectability and isolability can be used to check whether the residuals can fulfill the FDI tasks. A natural idea for fault detection and isolation which has been investigated for a long time is to design residuals with directional responses namely residuals constrained in a fixed and different direction for each fault vector [16], [17]. Considering the parity space method in the deterministic situation, a fault can be detected by comparing the norm of the residual to a preset threshold, and the fault can be isolated by comparing the orientation of the residual to that of the potential fault directions [18]. These notions are applied to the local model (2), next the performance of the local FDI system is analyzed.

3.1. Residual Generation

A residual can be generated based on the local sensing model (2) with the parity space approach. The condition of existence for physical redundancy requires the existence of a non zero left null space of the measurement matrix, i.e. \( \text{rank}(C_{i,loc}) < p_{i,loc} \). Here we assume it is true that a non zero left null space of the local measurement matrix exists. A basis of the left null space of the measurement matrix at each node can be derived according to the definition: \( \Omega_{i,loc} C_{i,loc} = 0 \). Multiplying the model equation (2) on the left by \( \Omega_{i,loc} \) yields the residual \( r_{i,loc}(k) \).

\[
\begin{align*}
r_{i,loc}(k) &= \Omega_{i,loc} y_{i,loc}(k) \\
&= \Omega_{i,loc} f_{i,loc}(k)
\end{align*}
\]

The computational form (4) corresponds to the implementation step and it is used to generate the residual on line. The internal form (5) expresses the residual as a function of the fault vector.

\[
\begin{align*}
r_{i,loc}(k) &= \Omega_{i,loc} y_{i,loc}(k) \\
&= \Omega_{i,loc} f_{i,loc}(k)
\end{align*}
\]

3.2. Distinguishability measure in deterministic context based on parity space

The directions of the residual vector upon occurrence of fault \( l \) can be defined with the \( l^{th} \) column of \( \Omega_{i,loc} = [\omega_{l,loc}^1, \omega_{l,loc}^2, ..., \omega_{l,loc}^{m_i}] \), where \( \omega_{l,loc}^j \) is the \( l^{th} \) column of matrix \( \Omega_{i,loc} \). Define the set \( F_{i,loc} = \{ f_{i_1}, f_{i_2}, ..., f_{i_{m_i}} \} \) which includes all the sensor faults in the neighborhood of node \( i \). The sensor faults are detectable if

\[
\forall f_k \in F_{i,loc}, \| \omega_{l,loc}^j \| > 0
\]

The \( l^{th} \) sensor fault \( f_l \in F_{i,loc} \) is isolable if:

\[
\forall f_q \in F_{i,loc} \text{ with } f_q \neq f_l, \{ \theta_{lq} | \theta_{lq} \text{ is the angle between } \omega_{l,loc}^j \text{ and } \omega_{l,loc}^q \} \cap \{0, \pi\} = \emptyset
\]
Table 1. Deterministic distinguishability at node $i$

| $D$ | $NF$ | $f_{i1}$ | $f_{i2}$ | $\cdots$ | $f_{im}$ |
|-----|------|---------|---------|----------|---------|
| $f_{i1}$ | 1, if $\|\omega_{i,loc}^l\| > 0$ | 0 | $\arccos \theta_{12}$ | $\arccos \theta_{1m}$ |
| $f_{i2}$ | 1, if $\|\omega_{i,loc}^l\| > 0$ | $\arccos \theta_{21}$ | 0 |
| $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ | $\vdots$ |
| $f_{im}$ | 1, if $\|\omega_{i,loc}^m\| > 0$ | $\arccos \theta_{m1}$ | $\arccos \theta_{m2}$ | 0 |

Table 2. Distinguishability matrix at node 1

| $D$ | $NF$ | $f_{i1}$ | $f_{i2}$ | $f_{i3}$ | $f_{i4}$ | $f_{i5}$ | $f_{i6}$ |
|-----|------|---------|---------|---------|---------|---------|---------|
| $f_{i1}$ | 1 | 0 | 1.5708 | 1.5708 | 3.1416 | 1.5708 | 1.5708 |
| $f_{i2}$ | 1 | 1.5708 | 0 | 1.5708 | 1.5708 | 3.1416 | 1.5708 |
| $f_{i3}$ | 1 | 1.5708 | 1.5708 | 0 | 1.5708 | 2.0944 | 1.5708 |
| $f_{i4}$ | 1 | 3.1416 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 |
| $f_{i5}$ | 1 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 0 | 1.5708 |
| $f_{i6}$ | 1 | 1.5708 | 3.1416 | 1.5708 | 1.5708 | 1.5708 | 0 |

So the detectability property is characterized by a binary value. The isolability can be calculated through the above defined angles: the isolability between fault $f_l$ and fault $f_q$ is related to the angles between their corresponding column vectors in $\Omega_{i,loc}$. It can be determined from $\cos \theta_{lq} = \frac{\omega_{i,loc}^l \cdot \omega_{i,loc}^q}{\| \omega_{i,loc}^l \| \| \omega_{i,loc}^q \|}$ [19]. Moreover, the closer the angles between the two fault directions is to $90^\circ$, the more easily the two faults can be isolated.

A distinguishability matrix can be derived to analyze the detectability and isolability property of the sensor faults in the neighborhood of a given node. This matrix is illustrated in table 1 for node $i$. The column $NF$ represents the detectability of each sensor fault and any $(l, q)^{th}$ element from the other columns characterizes the isolability between fault $l$ and $q$. Unlike in [14], the distinguishability matrix here is symmetric because of the considered definition in the deterministic framework. A notion of diagnosibility degree can be introduced on the basis of the distinguishability matrix. To this end, the indices of the sensors whose faults are detectable and isolable are stored in a set denoted $DI_i$ (the so-called diagnosable set), the indices of the sensors whose faults can not be detected are stored in a set denoted $ND_i$ and the indices of the ones whose faults can not be isolated are stored in $NI_i$.

Definition 1. (Diagnosibility degree) The diagnosibility degree $d_i$ at node $i$ can be defined as $d_i = p_{di}/p_{j,loc}$ where $p_{di}$ is the cardinality of the set $DI_i$.

If $d_i = 1$, all the sensor faults in the neighborhood can be detected and isolated. Otherwise, if $d_i < 1$, node $i$ needs to exchange more information with other nodes to improve the detectability and isolability.

The following example illustrates the distinguishability matrix and the diagnosibility degree.

Example 2. The distinguishability matrix at node 1 in the previous example can be generated
based on the local sensing model (1) and the matrix $\Omega_{\text{loc}}$. It is shown in Table 2. $f_{11}$, $f_{12}$ and $f_{13}$ represent the three sensor faults at node 1, $f_{21}$ and $f_{22}$, $f_{31}$ and $f_{32}$ represent the sensor faults at node 2 and at node 3 respectively.

From the distinguishability matrix, we can deduce the set $ND_1 = \emptyset$ which means all the sensor faults are detectable at node 1. The non isolable set is $NI_1 = \{f_{11}, f_{21}, f_{12}, f_{31}\}$. Therefore, the diagnosibility degree is $d_1 = \frac{2}{3}$ at node 1.

### 3.3. Selection process based on detectability and isolability

If the diagnosibility degree is less than 1, like for node 1 in the above example, one can determine whether there is suitable information from the other nodes in the network that can improve diagnosibility. Since the nodes can communicate with their neighbors directly, the selection process starts with the neighboring nodes. The scheme is illustrated below and it is proven that the distinguishability can be improved by exchanging information with a suitable node.

Consider two parity space residuals at two neighboring nodes, say node $i$ and node $j$:

$$
\begin{align*}
    r_{i,\text{loc}}(k) &= \Omega_{i,\text{loc}} y_{i,\text{loc}}(k) = \Omega_{i,\text{loc}} C_{i,\text{loc}} x(k) + \Omega_{i,\text{loc}} f_{i,\text{loc}}(k) \\
    r_{j,\text{loc}}(k) &= \Omega_{j,\text{loc}} y_{j,\text{loc}}(k) = \Omega_{j,\text{loc}} C_{j,\text{loc}} x(k) + \Omega_{j,\text{loc}} f_{j,\text{loc}}(k)
\end{align*}
$$

As defined in the previous section, the sets $F_{i,\text{loc}}$ and $F_{j,\text{loc}}$ represent all the sensor faults in the neighborhood of node $i$ and node $j$. Since the local sensing model at each node is formed based on the measurements from their neighborhood, the neighboring nodes always have a set of common faults. Let $F_{ij}$ denote the set of faults that are common to nodes $i$ and $j$. The set of fault specific to node $i$ (node $j$) are respectively denoted as $F_{i,j} = F_{i,\text{loc}} \setminus F_{ij}$ and $F_{j,i} = F_{j,\text{loc}} \setminus F_{ij}$. The respective fault vectors associated to these sets are defined as: $f_{ij} \in F_{ij}, \bar{f}_{ij} \in F_{i,j}, \tilde{f}_{ij} \in F_{j,i}$. The corresponding measurement vectors are denoted $y_{ij}, \bar{y}_{ij}$ and $\tilde{y}_{ij}$. The measurement matrix $C_{i,\text{loc}}$ can also be divided into two parts: the part associated to the common faults $C_{ij}$, and the part associated to the non common faults $C_{ij}$.

So the residual at node $i$, assuming that $r_{j,\text{loc}}$ is transmitted to node $i$, can be written as:

$$
\begin{align*}
    r_{i,\text{new}} &= \begin{bmatrix} r_{i,\text{loc}}(k) \\ r_{j,\text{loc}}(k) \end{bmatrix} \\
    &= \begin{bmatrix} \Omega_{ij} & \Omega_{ij} \\ \Omega_{ji} & 0 \end{bmatrix} \begin{bmatrix} y_{ij} \\ \bar{y}_{ij} \end{bmatrix} + \begin{bmatrix} \Omega_{ij} & \Omega_{ij} \\ 0 & \Omega_{ji} \end{bmatrix} \begin{bmatrix} C_{ij} \\ \bar{C}_{ij} \end{bmatrix} x(k) + \begin{bmatrix} \Omega_{ij} & \Omega_{ij} \\ 0 & \Omega_{ji} \end{bmatrix} \begin{bmatrix} f_{ij} \\ \tilde{f}_{ij} \end{bmatrix}
\end{align*}
$$

where $
\Omega_{ij} = [\omega_{ij}^1, \omega_{ij}^2, \ldots, \omega_{ij}^n]$ represents the part of $\Omega_{i,\text{loc}}$ corresponding to the common faults from $F_{ij}$ and $\Omega_{ij} = [\bar{\omega}_{ij}^1, \bar{\omega}_{ij}^2, \ldots, \bar{\omega}_{ij}^{n_{i,\text{loc}}}]$ is the part of $\Omega_{i,\text{loc}}$ associated to the faults from $F_{ij}$. The following result can be stated on the basis of (6).

**Proposition 1.** A non detectable fault at node $i$, say fault $l$, which is common to nodes $i$ and $j$ ($f_{ij}^l \in F_{ij}$), can be made detectable by transmitting $r_{j,\text{loc}}$ to node $i$ if and only if the fault $f_{ij}^l$ can be detected at node $j$.

**Proof.** According to the definition of detectability, fault $f_{ij}^l$ is detectable if $\| \omega_{ij}^l \| > 0$ where $\omega_{ij}^l$ is the $l$th column of $\Omega$. Hence, if a sensor fault $f_{ij}^l$ that belongs to the common set at node $i$ can not be detected, it means $f_{ij}^l \in F_{ij}$, $f_{ij}^l \neq 0$ and $\| \omega_{ij}^l \| = 0$. Then the detectability can be improved if and only if $\| \omega_{ij}^l \| \neq 0$ because $\| \omega_{ij}^l \| \neq 0$ after combining $r_{i,\text{loc}}$ and $r_{j,\text{loc}}$. \qed
Proposition 2. Consider a detectable fault $f_{ij}^l \in F_{ij}$ that cannot be isolated from $f_{ij}^q \in F_{ij}$. Then isolability of $f_{ij}^l$ from $f_{ij}^q$ can be achieved by transmitting $r_{j,loc}$ to node $i$ if and only if the two faults can be isolated at node $j$.

Proof. According to the definition of isolability, fault $f_l$ is isolable from fault $f_q$ if $-1 < \cos \theta_{lq} < 1$, where $\theta_{lq}$ is the angle between column vector $\omega_l$ and any other column $\omega_q$ of matrix $\Omega$. If a sensor fault $f_{ij}^l$ can be detected but can not be isolated from $f_{ij}^q$ at node $i$, $\omega_{ij}^l$ and $\omega_{ij}^q$ in matrix $\Omega_{ij,loc}$ are linearly dependent, namely $\omega_{ij}^l = \alpha \omega_{ij}^q$. The isolability between fault $f_{ij}^l$ and $f_{ij}^q$ can be improved if and only if the two faults can be isolated at node $j$, that is, $\exists \gamma$ s.t. $\omega_{ji}^l = \gamma \omega_{ji}^q$. Thus the angle between the two columns is changed such that $-1 < \cos \left( \begin{bmatrix} \omega_{ij}^l \\ \omega_{ji}^q \\ \omega_{ji}^q \end{bmatrix} , \begin{bmatrix} \omega_{ij}^q \\ \omega_{ji}^q \\ 0 \end{bmatrix} \right) < 1$. It indicates the isolability can be ensured by adding the residual information from node $j$. □

Proposition 3. Consider a detectable fault $f_{ij}^l \in F_{ij}$ that cannot be isolated from $f_{ij}^q \in F_{ij}$ at node $i$, then fault isolation can be achieved by transmitting $r_{j,loc}$ to node $i$ if and only if $f_{ij}^l$ is detectable at node $j$.

Proof. In the considered case $\omega_{ij}^l = \alpha \omega_{ij}^q$, the isolability can be ensured if sensor fault $f_{ij}^l$ can be detected at node $j$ namely $\omega_{ji}^l \neq 0$. Indeed $-1 < \cos \left( \begin{bmatrix} \omega_{ij}^l \\ \omega_{ij}^q \\ 0 \end{bmatrix} , \begin{bmatrix} \omega_{ij}^q \\ \omega_{ji}^q \\ 0 \end{bmatrix} \right) < 1$, hence the angle between these two fault directions is neither $0^\circ$ nor $180^\circ$ any more. □

We can explain the propositions from the physical point of view in the following. Here a special case is used in which each sensor only measures a single component of $x$.

Detectability If a sensor fault $f_{ij}^l \in F_{ij}$ can not be detected, it means there is only one sensor measuring the quantity. Then the detectability can be improved by adding the residual information from node $j$ which includes the information from the sensors measuring the same quantity.

Isolability If a sensor fault $f_{ij}^l$ can be detected but can not be isolated from $f_{ij}^q$ at node $i$, we can infer that there are only two sensors measuring the same quantity and they can not be distinguished from each other. Then the isolability of the $l$th fault can be improved if there is another sensor in the neighborhood measuring the same quantity.

Criteria to improve certain diagnosibility of faults According to propositions 1 to 3 and the above analysis, the residual information from node $j$ is helpful only when it can fulfill some conditions. The criteria for selecting a suitable node to improve the diagnosibility of the fault $f_{ij}^l$ at node $i$ can be deduced:

Criterion 1 The sensor fault $f_{ij}^l$ is detectable at node $j$. Adding the residual information from node $j$ ensures the detectability of $f_{ij}^l \in F_{ij} \cap NI_i$. Moreover, it also ensures the isolability of $f_{ij}^l \in F_{ij} \cap NI_i$ from $f_{ij}^q \in F_{ij} \cap NI_i$.

Criterion 2 The sensor $f_{ij}^l$ is isolable at node $j$. Adding the residual information from node $j$ ensures the isolability of $f_{ij}^l \in F_{ij} \cap NI_i$ from $f_{ij}^q \in F_{ij} \cap NI_i$.

Criterion 3 In order to avoid bringing other undiagnosable faults, we have to set a constraint on node $j$ that the fault $f_{ji}^l \in F_{ji}$ should be detectable and isolable at node $j$.
Now we can summarize the steps of computing the distinguishability, checking the diagnosibility and selecting a suitable node if necessary:

1. generate the parity space $\Omega_{i,loc}$ based on the local sensing model;
2. compute the distinguishability matrix according to Table 1 on the basis of the columns of $\Omega_{i,loc}$;
3. classify the faults into the detectable and isolable set $DI_i$, the non detectable set $ND_i$ and the non isolable set $NI_i$, then compute the diagnosibility degree $d_i$;
4. if $d_i < 1$, one has to associate a neighboring node to node $i$ to ensure fault detection and isolation at node $i$ according to the above mentioned criteria;

There are two different schemes to accomplish the selection, either off-line or on-line. Moreover, an iterative algorithm can be exploited for choosing a suitable neighboring node as described in the following section.

**Off-line** If the topology of the network is fixed and static, based on the global information on the network, the above 4-step procedure can be realized for each node, in order to determine the residual(s) to be exchanged with the neighboring node, as well as the corresponding decision logic.

**On-line** If the topology of the network is mobile, each node can know its identifier and the identifiers of its neighbors [2]. Assuming that the global measurement matrix $C_G = [C_{i_1}^T, \cdots, C_{j}^T, \cdots, C_{M}^T]^T$ is known by each node, the nodes can form the local measurement matrix from the global measurement matrix and the identifiers of its neighbors, like firstly forming the circles $N_1$, $N_2$, $N_3$ in figure 2. Then the diagnosable sets $DI_i$ can be exchanged between node and its neighboring nodes to determine a suitable node which can improve the diagnosibility according to the criteria.

### 3.4. Selection optimization

If $d_i < 1$, there could be more than one suitable node able to increase the diagnosibility degree at node $i$. Besides more than one node might be required to improve the diagnosibility. Therefore a criterion for node selection is needed, as well as a methodology for searching among the neighboring nodes.
To describe such a methodology, let us introduce some notations. Let $O_i^l$ denote the set of $l$ hop-neighbours of node $i$ which fulfills the criteria to improve the distinguishability. Let $\bar{d}_i^l$ denote the highest achievable value of the diagnosibility degree for a diagnosis system based on the available measurements in the set of nodes $S_i^l = O_i^1 \cup O_i^2 \cup \cdots \cup O_i^l$.

Furthermore, to characterize the communication and computation costs associated to the introduction of a given node, say node $j$, in the diagnosis system at node $i$, let us introduce the following cost function:

$$C_{ij} = E_{g_{ij}} + Dim_{ij} \tag{7}$$

where $E_{g_{ij}}$ represents the number of edges between node $i$ and node $j$, and $Dim_{ij}$ denotes the dimension of the residual $r_{j,loc}$. The first term can be seen as a communication cost, while the second one characterizes a computation cost.

Algorithm 1 can be considered for the selection of a relevant set of nodes to build a diagnosis system at node $i$. This allows one to retain the solution with the lowest computational and communication costs.

Algorithm 1 Selection algorithm

- Compute $\bar{d}_i^0 = d_i$,(where $d_i$ is the distinguishability at node $i$ for a diagnosis system based on the local model at node $i$)
- Set $n = 0$, ($n$ stands for $n$ hop neighbors)
- while $n < n_{\text{max}}$ do
  - $n = n + 1$
  - solve $\max d_i^n_{s_j,\in S_i^n}$
- end while
- Let $S_{i,\text{max}}^{n_{\text{max}}} = \{s_{i_1}, \cdots, s_{i_q}\}$ denote the set of all solutions that reach the highest diagnosibility degree $d_{i,\text{max}}^n$.
- To determine the set of nodes to be retained for the diagnosis system at node $i$, solve $\min_{s_j \in S_{i,\text{max}}} \sum_{s_i \in S_i} C_{ij}$, (where $C_{ij}$ is defined in (7))

Remark 1. Depending on different design requirements, constraints can be added to specify the faults with respect to which detectability and/or isolability should be ensured. For example, increasing one of the elements in the distinguishability matrix could be an objective.

3.5. Implementation of the distributed fault diagnosis system

Algorithm 2 summarizes the implementation steps in the simple situation where one single additional residual is necessary for achieving a diagnosibility degree equal to 1 at node $i$.

The decision rules include thresholds $h_d$ and $h_{\text{isol}}$ that are introduced to account for uncertainties in the measurement model (1).

4. Simulation

Here we use the same example as in section 2 to illustrate the process of selecting the suitable neighboring node and the improved performance of the fault detection and isolation system at node 1.

According to the topology, node 1 has two neighbors node 2 and 3 and their distinguishability matrices are illustrated in table (3) and (4) respectively. The analysis of these tables yields: $ND_2 = \emptyset$, $NI_2 = \emptyset$, $ND_3 = \emptyset$ and $NI_3 = \emptyset$ which means all the sensor faults can be detected and isolated at node 2 and node 3.
Algorithm 2 on-line diagnosis algorithm in a deterministic framework

1. residual generation: \( r_{i,loc}(k) = \Omega_{i,loc}y_{i,loc}(k) \);

2. residual exchanging: \( r_{i,new} = \begin{bmatrix} r_{i,loc}(k) \\ r_{j,loc}(k) \end{bmatrix} \);

3. fault detection: if \( \| r_{i,new} \| > h_d \) indicate that there is a fault, where \( h_d \) is the detection threshold;

4. if a fault is detected, fault isolation can be implemented: if \( \arccos(r_{i,new},\omega_{i,new}^l) < h_{isol} \) decide that fault \( l \) has occurred, where \( \omega_{i,new}^l \) is the \( l \)th column of matrix \( \Omega_{i,new} = [\omega_{i,new}^1,\omega_{i,new}^2,...,\omega_{i,new}^n] \) and \( h_{isol} \) is the isolation threshold;

### Table 3. Distinguishability matrix at node 2

| \( D \) | \( NF \) | \( f_{11} \) | \( f_{12} \) | \( f_{13} \) | \( f_{21} \) | \( f_{22} \) | \( f_{31} \) | \( f_{32} \) | \( f_{41} \) | \( f_{42} \) |
|------|------|------|------|------|------|------|------|------|------|------|
| \( f_{11} \) | 1    | 0    | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 |
| \( f_{12} \) | 1.5708 | 0    | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 |
| \( f_{13} \) | 1    | 1.5708 | 0    | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 |
| \( f_{21} \) | 2.0944 | 1.5708 | 1.5708 | 0    | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| \( f_{22} \) | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 0    | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| \( f_{31} \) | 1    | 2.0944 | 1.5708 | 1.5708 | 1.5708 | 0    | 1.5708 | 2.0944 | 1.5708 |
| \( f_{32} \) | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 1.5708 | 0    | 1.5708 | 2.0944 |
| \( f_{41} \) | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 1.5708 | 0    | 1.5708 |
| \( f_{42} \) | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 1.5708 | 0    |

Given the non isolable set \( NI_1 = \{ f_{11}, f_{21}, f_{12}, f_{31} \} \), node 2 and node 3 can both be selected based on the criteria stated in section 3.3. Node 1 node 2 and node 3 have the common faults set \( \{ f_{11}, f_{12}, f_{13}, f_{21}, f_{22}, f_{31}, f_{32} \} \). Faults of fault 1 can be isolated from fault 21 and fault 12 can be isolated from fault 31 at node 2 and node 3. Besides the uncommon faults \( \{ f_{41}, f_{42} \} \) and \( \{ f_{51}, f_{52}, f_{53} \} \) can be detected and isolated at node 2 and node 3 respectively.

Since node 2 and node 3 both fulfill the distinguishability specification, the optimization step in algorithm 2 is considered, node 2 and node 3 both satisfy \( d^x_2 > d_1 \) and \( d^x_1 = d^x_2 = 1 \). Besides,
in (7) $E g_1^2 = E g_2^2 = 1$. The residual generated at node 2 is $r_{2,loc} \in R^6$ and the residual at node 3 is $r_{3,loc} \in R^7$, so $Dim_1^2 = 6$ and $Dim_3^2 = 7$. The cost function for node 1 associated to node 2 and node 3 are $C_1^2 = 7$ and $C_3^2 = 8$ respectively. Therefore, node 2 corresponds to the optimal selection.

After adding the information from node 2, the distinguishability matrix at node 1 is improved. The angles between $f_{11}$ and $f_{21}$, $f_{12}$ and $f_{31}$ which indicate the isolability are closer to 90°(1.5708) than before as can be seen from the table (5). All the sensor faults can be diagnosed with the aid of node 2. The diagnosability at node 1 is $d_1 = 1$.

### Table 4. Distinguishability matrix at node 3.

| $D$ | $NF$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{21}$ | $f_{22}$ | $f_{31}$ | $f_{32}$ | $f_{51}$ | $f_{52}$ | $f_{53}$ |
|-----|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $f_{11}$ | 1 | 0 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| $f_{12}$ | 1 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 |
| $f_{13}$ | 1 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 | 1.5708 | 1.5708 | 1.5708 | 1.5708 |
| $f_{21}$ | 1 | 2.0944 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| $f_{22}$ | 1 | 1.5708 | 1.5708 | 1.9106 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.9106 | 1.5708 | 1.5708 |
| $f_{31}$ | 1 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| $f_{32}$ | 1 | 1.5708 | 1.5708 | 1.9106 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.9106 | 1.5708 |
| $f_{51}$ | 1 | 2.0944 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 |
| $f_{52}$ | 1 | 1.5708 | 2.0944 | 1.5708 | 2.0944 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 |
| $f_{53}$ | 1 | 1.5708 | 1.5708 | 1.9106 | 1.5708 | 1.5708 | 1.5708 | 1.9106 | 1.5708 | 1.5708 | 0 |

### Table 5. Distinguishability matrix at node 1 with extra information from node 2

| $D$ | $NF$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{21}$ | $f_{22}$ | $f_{31}$ | $f_{32}$ | $f_{41}$ | $f_{42}$ |
|-----|------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| $f_{11}$ | 1 | 0 | 1.5708 | 1.5708 | 2.3664 | 1.5708 | 1.5708 | 1.5708 | 1.9584 | 1.5708 |
| $f_{12}$ | 1 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 | 2.3664 | 1.5708 | 1.5708 | 1.9584 |
| $f_{13}$ | 1 | 1.5708 | 1.5708 | 0 | 1.5708 | 2.0944 | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| $f_{21}$ | 1 | 2.3664 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.5708 | 1.9584 | 1.5708 |
| $f_{22}$ | 1 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 0 | 1.5708 | 2.0944 | 1.5708 | 1.5708 |
| $f_{31}$ | 1 | 1.5708 | 2.3664 | 1.5708 | 1.5708 | 1.5708 | 0 | 1.5708 | 1.5708 | 1.9584 |
| $f_{32}$ | 1 | 1.5708 | 1.5708 | 2.0944 | 1.5708 | 2.0944 | 1.5708 | 0 | 1.5708 | 1.5708 |
| $f_{41}$ | 1 | 1.9584 | 1.5708 | 1.5708 | 1.9584 | 1.5708 | 1.5708 | 1.5708 | 0 | 1.5708 |
| $f_{42}$ | 1 | 1.5708 | 1.9584 | 1.5708 | 1.5708 | 1.5708 | 1.5708 | 1.9584 | 1.5708 | 0 |

5. Conclusion

A systematic way to design and implement a local FDI system at each node of a sensor network has been presented. It resorts to a distinguishability measure to indicate the performance of the FDI system and to select a suitable node in the network. The specified fault detection and isolation performance can be achieved while the communication and computation cost is taken into account.

In the presented work, measurement noise is not accounted for. Should this noise be significant, the scheme proposed in this paper can be developed in a stochastic framework, by resorting to a distance measure based on the Kullback-Leibler divergence.
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