Cross-tests of CMB features in the primordial spectra

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Abstract

The recent PLANCK data on the power spectrum of temperature anisotropies of the cosmic microwave background marginally support deviations from the ΛCDM model at several multipoles. With a view towards current and forthcoming observational surveys, we trace these features to other observables like the scalar bispectrum and the tensor power spectrum. A possible detection of such bumps in these channels would increase their statistical significance shedding light on the ultra violet mechanisms responsible for their appearance in the data.

1 Introduction

Inflation [1] is currently the main paradigm of early universe dynamics compatible with the current data which point towards a Gaussian and nearly scale invariant power spectrum of the cosmic microwave background (CMB) temperature anisotropies [2]. Since the proposal of the original idea in the 80’s there has been a vast variety of models realising it, many of which draw inspiration from ultra violet (UV) completions of the QFT and GR frameworks like string theory and supergravity.

An indispensable characteristic of these UV realisations is the presence of new high energy degrees of freedom, whose interaction with inflation may leave imprints on the CMB observables. Like in particle physics, where new particles appear as bumps over the standard model background, such imprints would be manifested in small deviations from the standard cosmological model. Indeed, this is exactly what the PLANCK satellite has shown [3] reconfirming earlier observations from WMAP [4], a fact which has sparked an intense effort to study these features [5].

The difference in the cosmological setup is of course that data are very hard to collect resulting in a low statistical significance of at most 2σ of these bumps [6]. However, as in particle physics, a way to increase the significance of deviations from the known model is to look for them in different channels, or in the cosmological language, in different CMB spectra.

The PLANCK mission is currently constraining the scalar bispectrum, while several ongoing and future surveys are looking for primordial tensor modes. In the meantime, a large amount of data is expected in the next few years from large scale structure observations. In anticipation of the coming data, we workout how these bumps in the power spectrum, if real, would manifest themselves in the scalar bispectrum and the tensor power spectrum.
2 Method

We are interested in computing how the presence of sharp features, which are produced by a brief deviation of the inflaton from the slow-roll dynamics\(^1\), propagate from one spectrum to another. Let us thus define two spectra as \(S_1\) and \(S_2\) and a set of parameters \(\epsilon_i = \{\epsilon, \eta, c_s\}\), where \(\epsilon = H/H^2\), with \(H\) denoting the Hubble scale, \(\eta = \dot{\epsilon}/\epsilon H\) and \(c_s\) the soundspeed of the scalar perturbations. The correlators \(S_i\) could be any among the scalar, tensor power spectra or higher point functions.

The effective field theory of inflation allows us to write down a model independent Lagrangian describing the dynamics of curvature perturbations around the characteristic energy scale of inflation, the Hubble constant \(H\), which may be used to compute the aforementioned correlators. Within this context, the idea is to setup an equation that connects the scalar power spectrum to other spectra.

The relation is set up as follows:

\(\star\) One splits the parameters of the Lagrangian into a slow-roll background and a fast contribution. Then using the \textit{in-in} formalism \cite{7} one computes the corrections on the slow-roll part of \(S_1\) induced by the fast-rolling parts of the theory. At this stage one has equations of the form

\[
\Delta S_1 = \int_0^\infty dt \, D^1_1(\epsilon_i) \, \mathcal{R}^n, \tag{1}
\]

where \(\Delta S_1\) is the correction on the spectrum \(S_1\), \(D^1_1\) is a diferencial operator with respect to time, while \(n \in \mathbb{N}\), is a number counting the order of the correlator \(S_1\).

\(\star\) One then exploits the fact that the de Sitter mode functions \(\mathcal{R} \propto e^{ikt}\) contain the Fourier measure. Using a trick to define parameters in the time interval \([-\infty, \infty]\), one may Fourier invert Eq. (1), ending up with formulas of the following type:

\[
\epsilon_i = \int_{-\infty}^{\infty} dk \, D^k_2 \, \Delta S_1(k), \tag{2}
\]

that is, the parameters \(\epsilon_i\) are written as functions of the spectrum.

\(\star\) One then computes another correlator, say \(S_2\), to obtain another equation similar to (1):

\[
\Delta S_2 = \int_{-\infty}^{\infty} dt \, D^2_2(\epsilon_i) \, \mathcal{R}^m. \tag{3}
\]

Finally, one substitutes the \(\epsilon_i\)'s from Eq. (2) obtaining formulas of the form

\[
\Delta S_1 = D^k_4[\Delta S_2]. \tag{4}
\]

With such a formula at hand, one may compute how features in the spectrum \(S_2\) propagate to the correlator \(S_1\).

\(^1\)Note that inflation is not interrupted by this mechanism. One can easily realise a setup where \(\epsilon\) remains small but its derivative, encoded in \(\eta\), gets large – \(\mathcal{O}(0.5)\) – during a short time interval. This is enough to produce features in the power spectrum.
3 Results

3.1 Scalar Power/Bispectrum Correlation

Here, based on Ref. [8], we apply the aforementioned algorithm to the case where $S_2 = P_R$ is the power spectrum and $S_1 = B_R$, the scalar bispectrum. We assume that features are present only in the Hubble parameters, while $c_s$ is kept constant. The result is:

$$B_R(k_1, k_2, k_3) \propto \left[ (1 + x^2 + y^2) \frac{x + y + xy}{16} + \frac{x^2 + y^2 + (xy)^2}{8} - \frac{xy}{8} \right] (1 - n_R) + \frac{xy}{8} \alpha_R, \quad (5)$$

where $k_1 = k$, $k_2 = xk$, $k_3 = yk$ and

$$1 - n_R = d \log k \log P_R, \quad \alpha_R = d^2 \log k \log P_R, \quad (6)$$

are the spectral index and its running, respectively. Before applying it to real data, one may first test the above formula. We did that using a typical model with a step in the inflationary potential known to produce features. We computed numerically the left hand side and the right hand side (RHS) of Eq. (5) and found excellent agreement. Next, we may input the PLANCK power spectrum data into the RHS of our formula (5) and get a prediction of the scalar bispectrum in the case of features in the Hubble flow parameters. We present the results in Fig. 1.

![Figure 1](https://via.placeholder.com/150)

Figure 1: Left panel: bispectrum amplitude at the squeezed limit, $k_3 \sim 0$. The feature at $k = 0.06 \text{ Mpc}^{-1}$ corresponds to the $\ell = 800$, $2\sigma$ feature of the angular power spectrum. Right panel: difference between featureless and featureful bispectrum amplitudes for the scale $k_1 = 0.06 \text{ Mpc}^{-1}$. Red and blue contours indicate the $k$-regions where this difference is nonzero by more than $2\sigma$.

\[\text{2}^\text{In this work, we applied a different method in order to correlate the two spectra based on the generalised slow-roll formalism [9]. However, the exact same formula has been obtained in [10] using the Fourier inversion. Due to lack of spacetime, we have chosen to comment only on the latter, which we have also used to obtain the rest of the results presented here.}

\[\text{3}^\text{If features are present in both parameters, one cannot invert Eq. (1) to get the relation (2). However, this is possible if one assumes a relation between the two quantities [11].}\]
3.2 Scalar/Tensor Power Spectrum Correlation

Next, based on Ref. [12], we apply the procedure to the case where $S_2 = P_S$ is the scalar power spectrum and $S_1 = P_T$ is the tensor one. The result is:

$$\Delta P_T = -6 \int d\ln k \, \epsilon \Delta P_S,$$  \hspace{1cm} (7)

We see that features in the tensor spectrum are suppressed due to slow-roll, which was known before [13], but also because of the integral structure of Eq. (7), which smooths out any feature of $P_S$. This indicates that, the tensor power spectrum remains practically scale invariant even if the scalar spectrum admits scale dependent features.

Note that this statement does not categorically exclude local enhancements – i.e. features – in the tensor spectrum. It just says that under fairly general assumptions the features appearing in the PLANCK data, e.g. the $\ell = 20$ dip in the angular power spectrum, will not be observable in the tensor spectrum. Reversing the statement, the very interesting case of the detection of a feature in $P_T$ at multipole $\ell = 20$, would either point to an astrophysical origin of the bump, or would be an indicator of a very exotic inflationary model.

3.3 Features in the Bispectrum

Finally, based on Ref. [15], we want to see how features appear in the bispectrum if a correlation between spectra is not possible because e.g. all parameters admit a brief period of fast changes. Here, $S_1$ and $S_2$ are the bispectrum shape function evaluated at two mode configurations with momenta $k_1 = k$, $k_2 = x_1 k$, $k_3 = y_1 k$ and $p_1 = k$, $p_2 = x_2 k$, $p_3 = y_2 k$. This leads us to two main results:

3.3.1 Bispectrum Consistency Relation for Features

If the background parameters experienced brief deviations from slow-roll during inflation then the bispectrum should obey the following relation:

$$S_R(k, x, y) = \left[ \alpha_1 S_1(x, y) - \alpha_2 S_2(x, y) \right] S_R \left( \frac{1 + x + y}{1 + x_1 + y_1} k, x_1, y_1 \right)$$

$$- \left[ \beta_1 S_1(x, y) - \beta_2 S_2(x, y) \right] S_R \left( \frac{1 + x + y}{1 + x_2 + y_2} k, x_2, y_2 \right),$$  \hspace{1cm} (8)

where $\alpha_{1,2}$ and $\beta_{1,2}$ are parameters depending on $x_{1,2}$ and $y_{1,2}$ and the partial shapes $S_{1,2}$ can be found in [15]. For example, fixing equilateral $(x_1, y_1) = (1, 1)$ and flat $(x_2, y_2) = (1/2, 1/2)$ shapes, we obtain

$$S_R(k, x, y) = \frac{18(x + y + xy) - 15(1 + x^2 + y^2)}{(1 + x + y)^2} S_R \left( \frac{1 + x + y}{3} k, 1, 1 \right)$$

$$- 16 \frac{x + y + xy - (1 + x^2 + y^2)}{(1 + x + y)^2} S_R \left( \frac{1 + x + y}{2} k, \frac{1}{2}, \frac{1}{2} \right).$$ \hspace{1cm} (9)

\[4\text{For example, in models with non Bunch-Davies vacuum, one can obtain observable features in } P_T \text{ [14].}\]
implying that the amplitude of the bispectrum at any point \((k, x, y)\) should be related to the corresponding amplitudes evaluated at \(\left(\frac{1+x+y}{3}k, 1, 1\right)\) and \(\left(\frac{1+x+y}{2}k, 1/2, 1/2\right)\). This can be viewed as a consistency relation since it holds only in the case of features. We have tested the above formula numerically using a simple step model as in Sec. 3.1.

3.3.2 Featured Bispectrum Templates

Motivated by the form of the consistency relation of Eq. (8), we may construct templates for the featured bispectrum. We thus replace the amplitudes on the RHS of Eq. (8) with a sine function which is a typical choice to model oscillating features in the spectra [16]. We have:

\[
S_{\alpha\beta}(k, x, y) = S_{\alpha_1\alpha_2}(x, y) \sin [(1+x+y)\omega_1 k + \phi] + S_{\beta_1\beta_2}(x, y) \sin [(1+x+y)\omega_2 k + \phi],
\]

(10)

where the partial modulating shapes, including the known – equilateral, orthogonal, flattened – templates, can be found in [15].

Interestingly, the template contains two frequencies \(\omega_1\) and \(\omega_2\) stemming from the factors \(\frac{1}{1+x_1+y_1}\) and \(\frac{1}{1+x_2+y_2}\) in the amplitudes involved in the consistency relation (8). In our construction it is evident that oscillating features may involve as many frequencies as couplings in the cubic Lagrangian. This is in agreement with the PLANCK results which favour such a multifrequency distribution for features [16].

4 Concluding Remarks

If the deviations from the ΛCDM line observed by PLANCK and WMAP are real indicators of new physics then they should also show up in other spectra. In these works we have traced how features appear in different observables. We found that i) the bispectrum should be enhanced around the scale \(k = 0.06\ \text{Mpc}^{-1}\) if during inflation the Hubble parameters experience short deviations from slow-roll without interrupting inflation; ii) the tensor power spectrum remains scale invariance unless some exotic mechanism produces the features; iii) in the case where all background parameters admit nontrivial time dependence, the bispectrum obeys certain consistency relations. In the case of oscillating spectra, there can be as much frequencies involved as cubic couplings in the Lagrangian, while all the known templates may appear as modulating shapes.

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\[5\]The same template can be written with log arguments in the sine functions, which is another typical profile modeling features [16].
References

[1] A. H. Guth, Phys. Rev. D 23, 347 (1981) ; A. D. Linde, Phys. Lett. B 108, 389 (1982)

[2] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A13 (2016) [arXiv:1502.01589 [astro-ph.CO]].

[3] N. Aghanim et al. [Planck Collaboration], Astron. Astrophys. 594, A11 (2016) [arXiv:1507.02704 [astro-ph.CO]] ; P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A20 (2016) [arXiv:1502.02114 [astro-ph.CO]].

[4] G. Hinshaw et al. [WMAP Collaboration], Astrophys. J. Suppl. 208, 19 (2013) [arXiv:1212.5226 [astro-ph.CO]].

[5] A. Achúcarro, J. O. Gong, G. A. Palma and S. P. Patil, Phys. Rev. D 87, no. 12, 121301 (2013) [arXiv:1211.5619 [astro-ph.CO]] ; A. Achúcarro, V. Atal, P. Ortiz and J. Torrado, Phys. Rev. D 89, no. 10, 103006 (2014) [arXiv:1311.2552 [astro-ph.CO]] ; A. Achúcarro, V. Atal, B. Hu, P. Ortiz and J. Torrado, Phys. Rev. D 90, no. 2, 023511 (2014) [arXiv:1404.7522 [astro-ph.CO]] ; J. Torrado, B. Hu and A. Achúcarro, arXiv:1611.10350 [astro-ph.CO] ; A. G. Cadavid, A. E. Romano and S. Gariazzo, Eur. Phys. J. C 76, no. 7, 385 (2016) [arXiv:1508.05687 [astro-ph.CO]] ; A. Gallego Cadavid, A. E. Romano and M. Sasaki, arXiv:1703.04621 [astro-ph.CO] ; A. Gallego Cadavid, A. E. Romano and S. Gariazzo, Eur. Phys. J. C 77, no. 4, 242 (2017) [arXiv:1612.03490 [astro-ph.CO]] ; A. Achúcarro, J. O. Gong, S. Hardeman, G. A. Palma and S. P. Patil, Phys. Rev. D 84, 043502 (2011) [arXiv:1005.3848 [hep-th]] ; A. Achúcarro, J. O. Gong, S. Hardeman, G. A. Palma and S. P. Patil, JCAP 1101, 030 (2011) [arXiv:1010.3693 [hep-ph]] ; J. R. Ferguson, H. F. Gruetjen, E. P. S. Shellard and M. Liguori, Phys. Rev. D 91, no. 2, 023502 [arXiv:1410.5114 [astro-ph.CO]] ; J. R. Ferguson, H. F. Gruetjen, E. P. S. Shellard and B. Wallisch, Phys. Rev. D 91, no. 12, 123506 (2015) [arXiv:1412.6152 [astro-ph.CO]] ; P. D. Meerbeg, M. Münchmeyer and B. Wandelt, Phys. Rev. D 93, no. 4, 043536 (2016) [arXiv:1510.01756 [astro-ph.CO]] ; S. Mooij, G. A. Palma, G. Panotopoulos and A. Soto, JCAP 1609, no. 09, 004 (2016) [arXiv:1604.03533 [astro-ph.CO]] ; J. Chluba, J. Hamann and S. P. Patil, Int. J. Mod. Phys. D 24, no. 10, 1530023 (2015) [arXiv:1505.01834 [astro-ph.CO]] ; X. Gao, D. Langlois and S. Mizuno, JCAP 1310, 023 (2013) [arXiv:1306.5680 [hep-th]] ; R. Saito, M. Nakashima, Y. i. Takamizu and J. Yokoyama, JCAP 1211, 036 (2012) [arXiv:1206.2164 [astro-ph.CO]] ; X. Gao, D. Langlois and S. Mizuno, JCAP 1210, 040 (2012) [arXiv:1205.5275 [hep-th]] ; S. Mizuno, R. Saito and D. Langlois, JCAP 1411, no. 11, 032 (2014) [arXiv:1405.4257 [hep-th]] ; T. Noumi and M. Yamaguchi, JCAP 1312, 038 (2013) [arXiv:1307.7110 [hep-th]] ; J. O. Gong, K. Schalm and G. Shiu, Phys. Rev. D 89, no. 6, 063540 (2014) [arXiv:1401.4402 [astro-ph.CO]] ; X. Gao and J. O. Gong, JHEP 1508, 115 (2015) [arXiv:1506.08894 [astro-ph.CO]] ; J. O. Gong and M. Yamaguchi,
Phys. Rev. D 95, no. 8, 083510 (2017) [arXiv:1701.05875 [astro-ph.CO]] ; S. Gariazzo, O. Mena, H. Ramirez and L. Boubekeur, Phys. Dark Univ. 17, 38 (2017) [arXiv:1606.00842 [astro-ph.CO]] ; S. Gariazzo, L. Lopez-Honorez and O. Mena, Phys. Rev. D 92, no. 6, 063510 (2015) [arXiv:1506.05251 [astro-ph.CO]] ; X. Chen, JCAP 1201, 038 (2012) [arXiv:1104.1323 [hep-th]] ; D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1408, 048 (2014) [arXiv:1405.2012 [astro-ph.CO]] ; D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, Phys. Rev. Lett. 113, no. 7, 071301 (2014) [arXiv:1404.0360 [astro-ph.CO]] ; D. K. Hazra, A. Shafieloo, G. F. Smoot and A. A. Starobinsky, JCAP 1609, no. 09, 009 (2016) [arXiv:1605.02106 [astro-ph.CO]].

[6] D. K. Hazra, A. Shafieloo and G. F. Smoot, JCAP 1312, 035 (2013) [arXiv:1310.3038 [astro-ph.CO]] ; D. K. Hazra, A. Shafieloo and T. Souradeep, JCAP 1411, no. 11, 011 (2014) [arXiv:1406.4827 [astro-ph.CO]].

[7] J. M. Maldacena, JHEP 0305, 013 (2003) [astro-ph/0210603].

[8] S. Appleby, J. O. Gong, D. K. Hazra, A. Shafieloo and S. Sypsas, Phys. Lett. B 760, 297 (2016) [arXiv:1512.08977 [astro-ph.CO]].

[9] E. D. Stewart, Phys. Rev. D 65, 103508 (2002) ; [astro-ph/0110322]. J. Choe, J. O. Gong and E. D. Stewart, JCAP 0407, 012 (2004) [hep-ph/0405155].

[10] G. A. Palma, JCAP 1504, no. 04, 035 (2015) [arXiv:1412.5615 [hep-th]].

[11] S. Mooij, G. A. Palma, G. Panotopoulos and A. Soto, JCAP 1510, no. 10, 062 (2015) Erratum: [JCAP 1602, no. 02, E01 (2016)] [arXiv:1507.08481 [astro-ph.CO]].

[12] G. A. Palma, B. Pradenas, W. Riquelme and S. Sypsas, Phys. Rev. D 95, no. 8, 083519 (2017) [arXiv:1612.09253 [astro-ph.CO]].

[13] W. Hu, Phys. Rev. D 89, no. 12, 123503 (2014) [arXiv:1405.2020 [astro-ph.CO]].

[14] B. J. Broy, Phys. Rev. D 94, no. 10, 103508 (2016) Addendum: [Phys. Rev. D 94, no. 10, 109901 (2016)] [arXiv:1609.03370 [hep-th]].

[15] J. O. Gong, G. A. Palma and S. Sypsas, JCAP 1705, no. 05, 016 (2017) [arXiv:1702.08756 [astro-ph.CO]].

[16] P. A. R. Ade et al. [Planck Collaboration], Astron. Astrophys. 594, A17 (2016) [arXiv:1502.01592 [astro-ph.CO]].