Three-particle Bell-like inequalities under Lorentz transformations

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Abstract We study the effects of Lorentz transformations on three-particle non-local system states (GHZ and W) of spin $1/2$ particles, using the Pauli spin operator and a three-particle generalization of Bell's inequality, introduced by Svetlichny. In our setup, the moving and laboratory frames used the (same) set of measurement directions that maximally violate Svetlichny’s inequality in the laboratory frame. We also investigate the behavior of Mermin’s and Collins’ inequalities. We find that, regardless of the particles’ type of entanglement, violation of Svetlichny’s inequality in the moving frame is decreased by increasing the boost velocity and the energy of particles in the laboratory frame. In the relativistic regime Svetlichny’s inequality is a good criterion to investigate the non-locality of the GHZ state. We also find that Mermin’s and Collins’ inequalities lead to reasonable predictions, in agreement with the behavior of the spin state, about non-locality of the W state in the relativistic regime. Then, comparing our results with those in which Czachor’s relativistic spin is used instead of the Pauli operator, we find that the results obtained by considering the Pauli spin operator are in better agreement with the behavior of spin state of the system in the relativistic information theory.

Keywords Multi-partite non-locality · Lorentz transformation
1 Introduction

Einstein, Podolsky and Rosen have found the non-local behavior of multiparticle systems in the framework of Quantum Mechanics (EPR) [1]. Moreover, it is shown that the distinguishable particles can behave non-locally [2]. In order to provide a criterion for differing the local and non-local behaviors of systems from each other, Bell introduced his inequality (the Bell’s inequality) which may be violated by the non-local phenomena [3]. The Bell’s inequality is the backbone of subsequent similar works [4,5,6,7]. Aspect and his co-workers observed non-locality in their experiments [8,9,10]. Also, it was proven that one-particle systems can exhibit non-local behavior [11,12].

Previously, it was thought that non-locality leads to entanglement, i.e. the total state of the system of particles can not be written as the product of states of its constitutes, and therefore, the maximum violation of Bell’s inequality is obtainable for maximally entangled states including maximum non-locality [3,13,14,15]. Hence, the amount of entanglement, non-locality and the violation of Bell’s inequality were seen as the same. But, it is shown that there are some mixtures of entangled states which do not violate any two-partite Bell-type inequality [14,15]. It is also shown that two-qubit quantum states can be entangled without violating any Bell-type inequality [16,17]. Moreover, there are states which are not maximally entangled but can maximally violate some types of Bell’s inequality [15,16,20,21,22]. In fact, it is shown that the amount of non-locality is not proportional to the amount of entanglement, this means that it is possible to store a large amount of non-locality into the states which are less entangled [23]. Indeed, unentangled systems may also exhibit the quantum non-locality [24]. Therefore, the relation between entanglement, non-locality and the violation of the Bell’s inequality is not as previously thought [22,17,25].

The entropy associated to non-locality has vast implications in quantum information theory and its related topics [26,27]. However, it is shown that the probability distributions are frame dependant quantities [28] and consequently, the entropy and information are frame dependent. Peres and his colleagues confirmed the frame dependency for the spin entropy of a single free spin-\(\frac{1}{2}\) particle. They have also shown that, due to the fact that the uncertainty in the momentum state of the system will transport to the spin state, the spin entropy is not invariant under the Lorentz transformations (LT) [29]. Therefore, we can conclude that in systems with no uncertainty in their momentum states, the Wigner rotation and thus the spin rotation in the moving frame are unique. Therefore, the spin entropy does not change under LT. The similar results hold for one-particle non-local systems [30]. More debates on the results published by Peres and his colleagues can be found in refs. [31,32]. Additionally, it is useful to mention here that relativistic motions may induce some noises to the quantum cryptographic protocols [33,34].

For high energy particles in the laboratory frame, QM is broken down and we need a more comprehensive theory. In fact, under this condition the particles have a non-negligible antiparticle component [35]. Finally, in order to
study the system, one should either use the Relativistic Quantum Mechanics (RQM) or the Quantum Field Theory (QFT) approaches, depending on the energy of particles [28,35]. Some attempts in which authors take into account the QFT interpretation of phenomena, and study non-locality can be found in [36,37,38,39,40].

Whenever the Quantum Mechanical interpretations of the phenomena are satisfactory in the lab frame, then the system is non-relativistic and the antiparticle component is negligible. Here, we shall not consider any antiparticle production or entanglement effects that could result from scattering. Therefore, in our setup, the lab and moving frames are connected to each other with a LT [29,30,41,42].

The effects of LT on the standard two-particle entanglement have been extensively studied [43,44,45,46,47,48,49,50,51]. In their approaches, there is a moving frame (\(S'\)) which is related to a lab frame (\(S\)) by a LT. The results of the generalizations of the frame dependency, shown by Peres et al. [29] and Vedral et al. [30] in one-particle systems, to the two-particle pure entangled systems [43,44,45] are analogous with the previous studies on one-particle systems. Gravitational effects, the curvature of spacetime [52,53,54,55] and the acceleration [36,37,46,56,57,58,59], do also change the entanglement of the system.

Terashima et al. have considered the Pauli spin operator in order to build the Bell operator, and studied the effects of LT on the standard two-particle entangled systems (one of the Bell states), in which the moving frame takes the same measurement directions for the Bell operator as the lab frame. For their setup, Bell’s inequality is violated to its maximum value in the lab frame, and the value of violation of Bell’s inequality (non-locality), in the moving frame, is a decreasing function of the boost speed and the energy of the particles. Therefore in the moving frame, Bell’s inequality in the \(\beta \to 1\) limit can be violated for different energy levels of the particles in the lab frame [47,48].

Indeed, there are various operators suggested to describe the spin of electron and therefore, the relativistic version of the Stern-Gerlach experiment [60,61,62,63]. Here, since some authors used Czachor’s relativistic spin operator in order to study the relativistic version of EPR [49,50,51,60,63,64,65,66], and because we will point to the results of considering Czachor’s relativistic spin operator, it is useful to introduce this spin operator. Based on Czachor’s proposal [60], the relativistic spin operator of states with zero momentum uncertainty along the unit vector \(\vec{A}\) is

\[
\hat{\vec{A}} = \frac{(\sqrt{1 - \beta^2} \hat{A}_\perp + \hat{A}_\parallel) \sigma}{\sqrt{1 + \beta^2 (\hat{\vec{c}}, \vec{A})^2 - 1}}. \tag{1}
\]

In this equation, \(\sigma\) and \(\hat{\vec{c}}\) are the Pauli spin operator and the unit vector along the particle velocity direction respectively, whiles, \(\beta\) is the particle velocity. In addition, the subscripts \(\perp\) and \(\parallel\) denote the perpendicular and the parallel components of the vector \(\vec{A}\) to the direction of the particle velocity. This operator commutes with the Hamiltonian and covers the Pauli spin operator.
whenever $\vec{A}_\parallel = 0$ meaning that $\hat{A} = \vec{A} \hat{\sigma}$. We should note that the uncertainty principle inhibits such possibility in a realistic experiment [60]. More properties of the Czachor’s relativistic spin operator can be found in [60,61,62,63]. The generalization of this operator to the cases in which we deal with wave-packets instead of a plane wave can be found in [34].

Now consider a setup in which the lab frame uses Czachor’s relativistic spin operator to build the Bell operator, and the special direction for measuring the spin operator leading to $\hat{A} = \vec{A} \hat{\sigma}$, which is similar to the result of considering the Pauli operator [47,60]. Therefore, Bell’s inequality will be violated to its maximum value in the lab frame by choosing proper directions for $\hat{A}$ [49,50,51,60]. Thereinafter, consider another frame moving with respect to the lab frame along an axis which is not parallel to the particles velocity in the lab frame while the spin measurement in the moving frame is also directed along $\vec{A}$. It means that $\vec{A}_\parallel$ is generally non-zero in the moving frame [49,50,51,60]. Therefore, since particles have an extra motion (along the boost direction) in the moving frame compared with the lab frame, $\vec{A}_\perp$ in the moving frame differs from that of the lab frame. Briefly, since the moving frame uses the same direction as the lab frame for measuring the spin operator ($\vec{A}$), $\vec{A}_\parallel$ and $\vec{A}_\perp$ differ from those of the lab frame and $\hat{A} \neq \vec{A} \hat{\sigma}$ in the moving frame. Finally, independent of the energy of the particles, the value of violation of Bell’s inequality decreases as a function of the boost velocity in the moving frame and eventually, Bell’s inequality will be served in the $\beta \to 1$ limit [49,50,51]. We should mention here that the maximum violation of Bell’s inequality is also obtainable in the moving frame by choosing proper directions for $\vec{A}$ in the moving frame [51].

Therefore, it is apparent that the results of studying Bell’s inequality by using Czachor’s relativistic spin operator differ from those in which the Pauli operator is used in order to build the Bell operator. Here, we must note that if there is no uncertainty in the momentum state of each particle in two-particle systems then the maximum violation of Bell’s inequality in the moving frame will be accessible by simultaneous application of the LT on the quantum state and the Bell operator [47,48,51].

There are two well known three-particle pure entangled states including GHZ and W states. The GHZ and W states are written as

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|++-\rangle + |--+\rangle)$$

and

$$|W\rangle = \frac{1}{\sqrt{3}}(|++-\rangle + |+-+\rangle + |-++\rangle),$$

which include the genuine three-partite entanglement [67,68,69,70,71,72,73]. On one hand, both of them are genuine three-partite entangled states while on the other hand, the GHZ state is separable by measuring the spin of one particle. Whereas a spin measurement on the W state leads to a separable
state with probability equal to $\frac{1}{4}$. Indeed, a spin measurement on the W state turns this state into the two-partite entangled state

$$|B\rangle = \frac{1}{\sqrt{2}}(|+\rangle + |\rangle),$$

with probability equal to $\frac{2}{7}$. This remaining part includes the spin state of the other particles. This difference leads to different values for the 3-tangle measure ($\tau$) of entanglement. Indeed, while $\tau$ vanishes for the W state, its value is non-zero for the GHZ state [69,74]. Therefore, one can use the 3-tangle measure to distinguish the GHZ and W states. Moreover, one can always convert nontrivial three-particle entangled states into one of these states by stochastic local operations and classical communication (SLOCC) whiles these two classes of states cannot be converted to each other by a SLOCC [69,71]. These arguments show that although both of the W and GHZ states include genuine three-partite non-locality [67,68,69,70,72,73], but their entanglement type differs from each other [68]. More relations between the W and GHZ states and three-partite non-locality [67,68,69,70,71,72,73] can be found in [67,68,70,72].

Three-particle version of Svetlichny’s inequality ($|S_3|$), as the generalization of Bell’s inequality to the multi-particle systems, is written as [72,73]

$$|S_3| = |E(ABC) + E(ABC') + E(AB'C) + E(A'B'C) - E(A'B'C') - E(A'B'C') - E(AB'C')| \leq 4,$$

where we have

$$E_{GHZ}(ABC) = \langle GHZ|\sigma(n_1) \otimes \sigma(n_2) \otimes \sigma(n_3)|GHZ\rangle = \cos(\phi_1 + \phi_2 + \phi_3),$$

and

$$E_{W}(ABC) = \langle W|\sigma(N_1) \otimes \sigma(N_2) \otimes \sigma(N_3)|W\rangle = -\frac{3}{2} \cos(\theta_1 + \theta_2 + \theta_3) - \frac{1}{2} \cos(\theta_1 \cos \theta_2 \cos \theta_3).$$

In the above equations, $A$ and $A'$ are possible measurements on the first particle and the same relations hold for the second and third particles with possible measurements $B$, $B'$ and $C$, $C'$, respectively. $n_i$ and $N_i$ are the unit vectors in the $x - y$ and $x - z$ planes, respectively. $n_i$ is characterized by the azimuthal angle $\phi_i$, and the maximum possible violation (4$\sqrt{2}$) of $|S_3|$ for the GHZ state will be obtainable if $\Sigma \phi_i = (n + \frac{1}{2})\pi$ and $\Sigma \phi_i = (n + \frac{3}{2})\pi$, where $(n = 0, \pm 1, \pm 2, \ldots)$ [71]. For the W state, $|S_3|$ will be at most violated to the value 4.354 if $\theta_i$ (the polar angle), which specifies the unit vector $N_i$, satisfies the condition $\theta_i = \pi - \theta_i' = 35.264^\circ$ [71]. Loosely speaking, since the maximum violation value of the $|S_3|$ inequality for the GHZ state (4$\sqrt{2}$) is greater than that of the W state (4.354), the same as the 3-tangle measure, one can also use this inequality to distinguish the GHZ and W states [71,68].

While, the GHZ and W states can violate the $|S_3|$ inequality [37,67,68,70,71,72], three-particle systems with bi-partite non-locality cannot violate the
Svetlichny's inequality can also be used to distinguish the bi-partite non-locality and the non-locality stored in the GHZ and W states [67,68,70,71].

There are also two other Bell-like inequalities for n-particle systems derived by Mermin (M) and Collins et al. (M') which in the three-particle case are written as [75,76]:

\[ |M| = |E(ABC') + E(AB'C) + E(A'BC) - E(A'B'C')| \leq 2, \]
\[ |M'| = |E(ABC) - E(A'B'C) - E(A'BC') - E(AB'C')| \leq 2. \] (8)

One gets the values 4 and 3.046 as the maximum amount of violation of these inequalities for the GHZ and W states respectively [71]. Therefore, these inequalities can also be used to distinguish the GHZ and W states. Gisin et al. have shown that only the GHZ state can violate \(|M|\) and \(|M'|\) to the values greater than \(2\sqrt{2}\) [77]. Here, it is useful to mention that there is no set of measurements violating \(|M|\) and \(|M'|\) to the values greater than \(2\sqrt{2}\) simultaneously [71]. Finally, we should note that since the maximum violation amounts of the three-particle Bell-like inequalities for the GHZ state are greater than that of the W state, the three particles are more entangled in the GHZ state than the W state [71]. The latter may be supported by this fact that the value of 3-tangle measure for these states differs from each other [69,74].

When \(|S_v|\) is violated to its maximum value, for the \(|M|\) and \(|M'|\) inequalities, one will find

\[ |M| = |M'| = \frac{|S_v|}{2}. \] (9)

Therefore, if we use the set of measurements that violate \(|S_v|\) to its maximum value, then \(|M|\) and \(|M'|\) will also be violated [71]. But, since there is no set of measurements that simultaneously violate the \(|M|\) and \(|M'|\) inequalities to the amounts greater than \(\frac{|S_v|}{2}\), the violations of \(|M|\) and \(|M'|\) to their maximum violation amounts (4 and 3.046 for the GHZ and W states, respectively) cannot happen simultaneously [71]. In addition, if we consider the GHZ state and the special set of measurement angles violating \(|S_v|\) to the value \(4\sqrt{2}\), we will find \(|M| = |M'| = 2\sqrt{2}\) which according to Gisin et al. [71], we can conclude that there is no genuine three-partite entanglement in the GHZ state. This result is fully inconsistent with some attempts which claim that the GHZ state includes genuine three-partite entanglement [67,68,69,70,71,72,73]. This example shows that the \(|M|\) and \(|M'|\) inequalities are weaker criteria than the \(|S_v|\) inequality in order to study the GHZ state [71]. We also see that the dependency of the three-particle generalizations of Bell's inequality to the set of measurements is extremely more than that of Bell's inequality itself.

It seems that, in non-relativistic regimes, one can use \(|S_v|\) as the proper three-particle Bell-like inequality in order to study the GHZ and W states [71,72,87]. This is of course due to the following points: (i) \(|M|\) and \(|M'|\) are violated by a hybrid local-nonlocal hidden variables model, and the same
situation holds for those inequalities which include four correlation functions $(E(ABC))$. (ii) It is shown that the upper bound of Mermin’s inequality is not correct and should be revised. (iii) It has been shown that if $|M|$ is violated to its maximum value, then $|M'|$ will be satisfied and vice versa. In addition, for the GHZ state, there is no set of the measurement angles that violate $|M|$ and $|M'|$ to the values bigger than $2\sqrt{2}$ simultaneously. Therefore, it seems that we will confront a contradiction if we compare the results of $|M|$ and $|M'|$ either with each other or that of $|S_v|$. (iv) The $|M|$ and $|M'|$ inequalities are weaker than what is needed to study the multi-partite non-locality. (v) The violation amounts of $|S_v|$ for the GHZ and W states are different and therefore $|S_v|$ can distinguish these states.

We must note that the correlation functions in $|S_v|$ are stronger than those needed for detecting the non-locality stored in the GHZ and W states, but it seems that $|S_v|$ can detect these non-localities, and one can use $|S_v|$ in the study of the non-relativistic systems. In fact, the $|S_v|$ inequality is offended from the violation of the no-signalling constraint, but this shortcoming of $|S_v|$ can be eliminated by either considering bi-partite correlations which satisfy the no-signalling constraint or using the time-ordered bi-partite correlation functions. In fact, such corrections eliminate the possibility of forming the grandfather-type paradoxes. For a comprehensive review on this subject, see ref. and references therein.

Authors in ref. have studied the effects of LT on the GHZ state. They have used the $|M|$ inequality, considering the Pauli spin operator, and the special set of measurement angles that violate $|M|$ to its maximum value in the lab frame. They have also used the same measurement angles for the moving frame in order to evaluate $|M|$. In addition, they have shown that the violation amount of the $|M|$ inequality in the moving frame is a decreasing function of the boost velocity and the energy of the particles. Finally, they found that, in the $\beta \to 1$ limit, the violation amount of $|M|$ depends on the energy of particles in the lab frame. These results are in line with previous studies on the two-particle non-local systems. Therefore, we may conclude that the behavior of non-locality under LT is independent of the number of particles.

In a similar setup, Moradi has considered Czachor’s relativistic spin operator instead of the Pauli operator in order to evaluate the $|M|$ inequality. He found that the violation amount of the $|M|$ inequality in the moving frame, in the $\beta \to 1$ limit, depends on the energy of the particles in the lab frame, which is against the two-particle entangled system studies. Therefore, it seems that the behavior of non-locality under LT depends on the number of particles, and it is independent of the nature of non-locality. This result is fully inconsistent with the previous result by You et al. Due to the fact that the authors in references and have used the $|M|$ inequality in order to study the GHZ state, their results are doubted. Bearing the differences between the GHZ and W states in mind, since authors in did not consider the W state, one can not generalize their results to the W state.
It has been shown that, in the moving frame, by considering the GHZ and W states and the Czachor's relativistic spin operator to evaluate three-particle Bell-like inequalities (|M|, |M'| and |Sv|), and also, using the same special set of measurements that violate |Sv| to its maximum value in the lab frame, these inequalities will be satisfied in the $\beta \to 1$ limit, independent of the energy of particles in the lab frame \[65\] which is in line with two-particle studies \[49, 50, 51\]. Therefore, in the $\beta \to 1$ limit, Bell’s inequality and multi-particle Bell-like inequalities will be satisfied under LT independent of the number of the entangled particles, their energies in the lab frame and type of their entanglement, if and only if the moving observer uses Czachor’s relativistic spin operator as well as the special set of measurements that violate either |Sv| or Bell’s inequality, depending on the considered system, to its maximum value in the lab frame. Finally, it seems that the |Sv| inequality is a good witness for detecting non-locality in relativistic multi-particle systems \[65\].

It is useful to note that non-local systems, it was shown that the maximum violation of Bell’s inequality and its generalization to the three-particle systems in the non-accelerated moving frame is accessible if one applies LT on the spin state and the spin operator simultaneously \[47, 48, 51, 64, 66, 83\]. Moreover, the acceleration effects on two and three-particle entangled states have also been studied \[37, 38, 84, 85\]. It is shown that the |Sv| inequality can be violated for any finite value of acceleration whiles Bell’s inequality cannot be violated for sufficiently large but finite acceleration. Therefore, it seems that the effects of acceleration on two-particle entanglement differs from those of three-particle entangled states such as the GHZ state \[37\]. It is useful to note that the |Sv| inequality is also satisfied for infinite value of acceleration \[37, 38\].

Here, we study the behavior of three-particle non-local systems, which either include the GHZ state or the W state, under LT. In order to investigate the behavior of multi-particle Bell-like inequalities under LT we use the Pauli spin operator and the same set of measurements for the moving and lab frames that violate the |Sv| inequality to its maximum value in the lab frame. We also, compared the results obtained from using |M| and |M'| with those obtained from using the |Sv| inequality. We show that, in the relativistic regime, the inequalities using the Pauli spin operator instead of Czachor’s operator to study the entanglement are in more agreement with the behavior of the quantum mechanical spin states transformed by the LT. Throughout this paper we set the light velocity equal to one (c = 1) for simplicity.

The paper is organized as follows. In section (II), we introduce the Wigner rotation, consider the Pauli spin operator, and show that the |Sv| inequality is a good witness for studying the GHZ state. Thereinafter, we consider the W state, and point to the weakness of |Sv| compared with the |M| and |M'| inequalities in the relativistic regime. Finally, we find that the |M| and |M'| inequalities can lead to reasonable predictions about the behavior of the GHZ and W states under LT. Throughout the article, the results of considering Czachor’s operator are also addressed. Section (III) is devoted to a summary and concluding remarks.
2 The three particle non-local system under LT

Here, we consider a situation in which Quantum Mechanics is enough for describing the system in the lab and thus moving frames. This situation is considered by many authors for both of the low and high energy particles, and some of these efforts can be found in refs. [29,30,31,32,33,34,43,44,45,46,47,48,49,50,51,64,65,66]. In the lab frame (S) for a system, including three spin-$\frac{1}{2}$ particles with the spin state $|\psi\rangle$ and the momentum state $|\vec{p}_1\vec{p}_2\vec{p}_3\rangle$, the state of the system is written as

$$|\xi\rangle = |\vec{p}_1\vec{p}_2\vec{p}_3\rangle|\psi\rangle,$$

where $\vec{p}_i = p_0\hat{z}$, $\forall i$. Now, consider a moving frame ($S'$) which moves along the x axis ($\beta = \beta\hat{x}$). In the $S'$ frame, the state of the system is

$$|\xi\rangle' = |\vec{p}_1\vec{p}_2\vec{p}_3\rangle'|D(W(\Lambda, p_i))|\psi\rangle.$$  

$|\vec{p}_1\vec{p}_2\vec{p}_3\rangle'$ denotes the momentum state of the system in the moving frame, and $D(W(\Lambda, p_i))$ is the spin-$\frac{1}{2}$ Wigner representation of the Lorentz group for the $i$th particle [41,42]:

$$D(W(\Lambda, p_i)) = \cos \frac{\Omega_{p_i}}{2} + i\sigma_y \sin \frac{\Omega_{p_i}}{2}.$$  

In this equation, $\sigma_y$ is the Pauli matrix where $\Omega_{p_i}$ is called the Wigner angle and will be evaluated as

$$\tan \Omega_{p_i} = \frac{\sinh \alpha \sinh \delta}{\cosh \alpha + \cosh \delta}. $$  

Here, $\cosh \delta = \frac{m}{\beta}$ and $\cosh \alpha = \sqrt{1 - \beta^2}$.

2.1 the GHZ state

In the laboratory frame, the entangled particles are in the GHZ state. Thus

$$|\psi\rangle = |\text{GHZ}\rangle,$$

and the maximum violation of the $|S_\nu|$ inequality ($4\sqrt{2}$) in the $S$ frame is obtainable by using the special set of angles including $\phi_i = \frac{\pi}{4}$ and $\phi_i' = \frac{3\pi}{4}$. This set of measurements yield $|M| = |M'| = 2\sqrt{2}$, as we have pointed in the introduction, which indicates that the system does not contain the genuine three-partite entanglement [77]. Also, we know that this is not true for the GHZ state because the GHZ state includes the genuine three-partite entanglement [67,68,69,70,71,72,73]. Therefore, we see again that this example can clarify our assertion about the more sensitivity of the three-particle inequalities to measurements with respect to Bell’s inequality [74]. It is useful to remind that
since there is no set of measurements that simultaneously violate $|M|$ and $|M'|$ to the values bigger than $2\sqrt{2}$, the maximum violation amounts of $|M|$ and $|M'|$ cannot be obtained simultaneously [71].

The spin state in the moving frame, $(|GHZ\rangle^A = \prod_{i=1}^A D(W(A, p_i))|GHZ\rangle)$, is written as

$$|GHZ\rangle^A = (\cos(\frac{\Omega_p}{2}))^3|GHZ\rangle + (\sin(\frac{\Omega_p}{2}))^3|GHZ\rangle$$

$$+ \sqrt{\frac{3}{2}} \sin(\frac{\Omega_p}{2}) \cos(\frac{\Omega_p}{2}) [(\sin(\frac{\Omega_p}{2}) + \cos(\frac{\Omega_p}{2}))|W\rangle$$

$$+ (\sin(\frac{\Omega_p}{2}) - \cos(\frac{\Omega_p}{2}))|\overline{W}\rangle],$$

where $(|GHZ\rangle = \frac{1}{\sqrt{2}}(|++--\rangle - |---+\rangle)$ and $(|\overline{W}\rangle = \frac{1}{\sqrt{3}}(|+--\rangle + |---+\rangle + |++--\rangle)$. Simple calculations yield

$$E_{\overline{W}}(ABC) = (\overline{W}\rangle \sigma(\tilde{N}_1) \otimes \sigma(\tilde{N}_2) \otimes \sigma(\tilde{N}_3)|\overline{W}\rangle$$

$$= -\frac{2}{3} \cos(\theta_1 + \theta_2 + \theta_3) - \frac{1}{3} \cos \theta_1 \cos \theta_2 \cos \theta_3$$

and

$$E_{GHZ}(ABC) = (|GHZ\rangle \sigma(\tilde{n}_1) \otimes \sigma(\tilde{n}_2) \otimes \sigma(\tilde{n}_3)|GHZ\rangle)$$

$$= -\cos(\phi_1 + \phi_2 + \phi_3).$$

In these equations, $\tilde{n}_i$ and $\tilde{N}_i$ have the same definitions as those of the GHZ and W states respectively. In addition, these equations tell that every set of measurements used to detect the non-locality in either the W state or the GHZ state can also be used for $|\overline{W}\rangle$ and $|GHZ\rangle$ respectively. Using Eq. (6) for correlation functions $E(ABC)$, we get

$$E(ABC) = |\langle GHZ| \sigma(\tilde{n}_1) \otimes \sigma(\tilde{n}_2) \otimes \sigma(\tilde{n}_3)|GHZ\rangle^A = [(\cos(\frac{\Omega_p}{2}))^6$$

$$- (\sin(\frac{\Omega_p}{2}))^6] \cos(\varphi_1 + \varphi_2 + \varphi_3) - \frac{3}{4} (\sin \Omega_p)^2 \cos \Omega_p \cos(\varphi_1 + \varphi_2 - \varphi_3)$$

$$+ \cos(\varphi_1 - \varphi_2 + \varphi_3) + \cos(-\varphi_1 + \varphi_2 + \varphi_3),$$

(18)

which is compatible with Eq. (8) in the $\Omega_p = 0$ limit. Inserting $\phi_i = \frac{\pi}{4}$ and $\phi'_i = \frac{3\pi}{4}$ into the above equation, we find

$$E(ABC) = (-\frac{\sqrt{2}}{2})[(\cos(\frac{\Omega_p}{2}))^6 - (\sin(\frac{\Omega_p}{2}))^6] - \frac{9\sqrt{2}}{8} (\sin \Omega_p)^2 \cos \Omega_p$$

$$= -E(A'B'C'),$$

$$E(A'BC') = E(AB'C') = E(A'B'C) = (\frac{\sqrt{2}}{2})[(\cos(\frac{\Omega_p}{2}))^6 - (\sin(\frac{\Omega_p}{2}))^6]$$

$$- \frac{3\sqrt{2}}{8} (\sin \Omega_p)^2 \cos \Omega_p,$$

(19)

$$E(A'BC) = E(AB'C) = E(ABC) = -E(A'B'C).$$
For the $|M|$, $|M'|$ and $|S_v|$ inequalities, we get

$$|M| = |M'| = \frac{|S_v|}{2} = | - 2\sqrt{2}((\cos(\frac{\Omega_p}{2}))^6 - (\sin(\frac{\Omega_p}{2}))^6)|. \quad (20)$$

It is straightforward that, in the $\Omega_p \rightarrow 0$ limit, the results of the $S$ frame are accessible, as a desired result. In addition and for the ultra relativistic regimes ($\beta \rightarrow 1$), calculations lead to

$$|M| = |M'| = \frac{|S_v|}{2} \sim \frac{1 + 3\Gamma^2}{\sqrt{2}\Gamma^3}, \quad (21)$$

where $\Gamma = \frac{1}{\sqrt{1 - v_0^2}}$ and $v_0$ is the energy factor and the velocity of particles in the $S$ frame respectively. We have also used the approximation, $\sin \Omega_p \sim \sqrt{1 - \Gamma^{-2}}$ to derive Eq. (21).

In this limit $\sin \frac{\Omega}{2} \sim \sqrt{\Gamma^{-1}}$ and from Eq. (15), we get

$$|GHZ\rangle^A \sim |GHZ\rangle, \quad (22)$$

for low energy particles ($\Gamma \rightarrow 1$), and

$$|GHZ\rangle^A \sim \frac{1}{2} (| - - \rangle + \sqrt{3} |W\rangle), \quad (23)$$

for high energy particles ($\Gamma \rightarrow \infty$). In addition, for the high energy particles in the $\beta \rightarrow 1$ limit from Eq. (21) we get

$$E(ABC) \sim 0. \quad (24)$$

Eq. (21) shows that in the $\beta \rightarrow 1$ limit: (i) the high energy particles ($\Gamma \rightarrow \infty$) satisfy all of the inequalities which is compatible with Eq. (23). (ii) in accordance with Eq. (22), the inequalities will be violated to their violation value in the $S$ frame for the low energy particles ($\Gamma \rightarrow 1$). Our result is in line with the previous studies on the standard two-particle entanglement (Bell states) \cite{47,48}. Therefore, we see that the violation of the inequalities in the $\beta \rightarrow 1$ limit depends on the energy of the particles in the $S$ frame. This result is compatible with the previous studies by You et al. \cite{83} and in line with the Moradi’s calculations, in which Czachor’s relativistic spin operator are used instead of the Pauli operator, \cite{64,66}. It is useful to note that since we assumed that we work in the Quantum Mechanic framework, the high energy limit ($\Gamma \rightarrow \infty$) is problematic. Indeed, Quantum Mechanics has no desired efficiency in this limit, and we need to consider more comprehensive theories such as RQM and QFT \cite{35}. Finally, we think that the survey of this limit may lead to useful outcomes about the quality of consistency between Quantum Mechanics and LT (Special Relativity) which may lead to the some desired predictions about the effects of relative motion on the Quantum Mechanical phenomena such as the relativistic version of the Stern-Gerlach experiment \cite{41,47,60}.
In addition, by using Czachor’s relativistic spin operator instead of the Pauli operator one gets

$$|M| = |M'| = \frac{|S_v|}{2} = \frac{2|\cos \Omega_p|}{\sqrt{2 - \beta^2}} (\cos^2 \Omega_p + 3(1 - \beta^2)), \quad (25)$$

which leads to

$$|M| = |M'| = \frac{|S_v|}{2} \sim \frac{2}{\Gamma^3}, \quad (26)$$

in the ultra relativistic limit ($\beta \to 1$) [65]. Eqs. (25) and (26) indicate that, independent of the energy of the particles in the lab frame, the inequalities are satisfied in the $\beta \to 1$ limit which is inconsistent with the results of Eqs. (20) and (21). Although, Eqs. (25) and (26) are in line with previous studies on the Bell states [49,50,51] but, they have full contradiction with the asymptotic behavior of the spin state of system (Eq. (22) and (23)).

As we have mentioned in the introduction, since authors in [64,66,83] have used the $|M|$ inequality in order to study the behavior of the GHZ state under LT, their results are doubted. Here, we used the $|S_v|$ inequality and get the similar results as obtained in the references [17,18,61,66,63]. Therefore, based on Eqs. (20) and (25) and the asymptotic behavior of the spin state in the moving frame (Eqs. (22) and (29)), we think that the results of studying the effects of LT on the non-locality of GHZ state will be compatible with the LT of this Quantum Mechanical state if the Pauli spin operator, the $|S_v|$ inequality and the special set of measurements, violating $|S_v|$ to its maximum violation amount ($4\sqrt{2}$), are considered. In addition, bearing the difficulties of the $|S_v|$, $|M|$ and $|M'|$ inequalities in mind, our investigation shows that the inequalities with correlation functions stronger than those of $|M|$ and $|M'|$, and weaker than those of the $|S_v|$ inequality may be used to study the effects of LT on the GHZ state which is in line with non-relativistic study [79].

We should note that since there is no uncertainty in the momentums of the particles [29,30,43,44,45], if the moving observer applies LT on both of the spin operators and the quantum state of the system simultaneously, the maximum violation of the inequalities will be obtainable in the $S'$ frame [17,45,51,61,66,60,83].

2.2 The W state

In the lab frame, consider a situation in which the W state is the spin state of particles which move along the $z$ direction with the same momentum. In the moving frame, which moves along the $x$ direction with $\beta = \beta \hat{x}$, the Wigner rotation satisfies Eq. (12) and by following the procedure of the previous section for the W state we get

$$|W\rangle^A = \sqrt{3} \sin\left(\frac{\Omega_p}{2}\right) \cos\left(\frac{\Omega_p}{2}\right) [-\cos\left(\frac{\Omega_p}{2}\right) |++\rangle + \sin\left(\frac{\Omega_p}{2}\right) |--\rangle]$$
\[ + |(\cos(\frac{\Omega_p}{2}))^3 - 2 \cos(\frac{\Omega_p}{2})(\sin(\frac{\Omega_p}{2}))^2|W\rangle \\
+ |2 \sin(\frac{\Omega_p}{2})(\cos(\frac{\Omega_p}{2}))^2 - (\sin(\frac{\Omega_p}{2}))^3|\bar{W}\rangle, \] 

(27)

where \(|W\rangle\) is the spin state in the moving frame. In addition, calculations for the correlation function lead to

\[ E_W(\theta_1\theta_2\theta_3) = A \cos \theta_1 \cos \theta_2 \cos \theta_3 + B \sin \theta_1 \sin \theta_2 \sin \theta_3 + C \cos(\theta_1 + \theta_2 + \theta_3) + D(\cos \theta_1 \sin \theta_2 \cos \theta_3 + \sin \theta_1 \cos \theta_2 \cos \theta_3), \] 

(28)

where we have used these abbreviations

\[ A = -\frac{1}{3}(\cos \Omega_p)^3 + \frac{7}{6}(\sin \Omega_p)^2 \cos \Omega_p, \]
\[ B = \sin \Omega_p[2(\cos \Omega_p)^2 - (\sin \Omega_p)^2], \]
\[ C = \frac{7}{3}(\sin \Omega_p)^2 \cos \Omega_p - \frac{2}{3}(\cos \Omega_p)^3, \]
\[ D = \sin \Omega_p[-\frac{7}{3}(\cos \Omega_p)^2 + \frac{2}{3}(\sin \Omega_p)^3]. \] 

(29)

In the non-relativistic regime \((\Omega_p = 0)\) we get

\[ E_W(\theta_1\theta_2\theta_3) = -\cos \theta_1 \cos \theta_2 \cos \theta_3 + \frac{2}{3}(\cos \theta_1 \sin \theta_2 \sin \theta_3 + \cos \theta_2 \sin \theta_1 \sin \theta_3 + \cos \theta_3 \sin \theta_2 \sin \theta_1), \] 

(30)

which is the same as Eq. (31). Now, by inserting \(\theta_i = \theta\) and \(\theta_i' = \Pi - \theta = \theta'\) we find

\[ E_W(\theta_1\theta_2\theta_3) = A(\cos \theta)^3 + B(\sin \theta)^3 + C \cos(3\theta) + D(3(\cos \theta)^2 \sin \theta), \]
\[ E_W(\theta_1'\theta_2'\theta_3') = -A(\cos \theta)^3 + B(\sin \theta)^3 - C \cos(3\theta) + D(3(\cos \theta)^2 \sin \theta), \]
\[ E_W(\theta_1'\theta_2\theta_3') = -A(\cos \theta)^3 + B(\sin \theta)^3 - C \cos \theta - D(\cos \theta)^3 \sin \theta) \]
\[ = E_W(\theta_1\theta_2\theta_3') = E_W(\theta_1\theta_3\theta_2'), \]
\[ E_W(\theta_1'\theta_2'\theta_3') = A(\cos \theta)^3 + B(\sin \theta)^3 + C \cos \theta - D(\cos \theta)^3 \sin \theta) \]
\[ = E_W(\theta_1\theta_2\theta_3) = E_W(\theta_1'\theta_2'\theta_3'). \] 

(31)

Thus for the inequalities, we get

\[ |M| = | -2A(\cos \theta)^3 + 2B(\sin \theta)^2 - 6D(\cos \theta)^2 \sin \theta + C(\cos 3\theta - 3 \cos \theta)|, \]
\[ |M'| = | -2A(\cos \theta)^3 - 2B(\sin \theta)^2 + 6D(\cos \theta)^2 \sin \theta + C(\cos 3\theta - 3 \cos \theta)|, \]
\[ |S_o| = |M + M'| = | -4A(\cos \theta)^3 + 2C \cos 3\theta - 6C \cos \theta|. \] 

(32)

In the non-relativistic limit \((\Omega_p \to 0)\) and by using \(\theta = 35.264^\circ\), Eq. (32) is consistent with the measurements in the lab frame. In the high velocity limit
($\beta \to 1$), we reach at

\[
\begin{align*}
A & \rightarrow \frac{3}{2T^3} + \frac{7}{6T} \\
B & \rightarrow \sqrt{1 - \frac{1}{T^2}} \left( \frac{3}{T^2} - 1 \right) \\
C & \rightarrow \frac{3}{T^3} + \frac{7}{3T} \\
D & \rightarrow \sqrt{1 - \frac{1}{T^2}} \left( -\frac{3}{T^2} + \frac{2}{3} \right),
\end{align*}
\]

and finally we get

\[
\begin{align*}
|M| & \sim \left| \frac{9.797}{T^3} - \frac{7.620}{T} + 1.14 \sqrt{1 - \frac{1}{T^2} \left( \frac{9}{T^2} - 2.19 \right)} \right|, \\
|M'| & \sim \left| \frac{9.797}{T^3} - \frac{7.620}{T} - 1.14 \sqrt{1 - \frac{1}{T^2} \left( \frac{9}{T^2} - 2.19 \right)} \right|, \\
|S_v| & \sim \left| \frac{19.594}{T^3} - \frac{15.236}{T} \right|,
\end{align*}
\]

(33)

for the inequalities. In addition, for the spin state in the moving frame ($|W\rangle^A$), in the $\beta \to 1$ limit and the low energy regime ($I \to 1$), we reach

\[|W\rangle^A \sim |W\rangle,\]

(35)

where for the high energy particles ($I \to \infty$) we get

\[|W\rangle^A \sim \frac{1}{2} \left( \sqrt{3} |GHZ\rangle + \frac{1}{\sqrt{2}} (|W\rangle - |\bar{W}\rangle) \right).\]

(36)

Eq. (34) shows that, in the $\beta \to 1$ limit, the inequalities in the moving frame are violated to their violation value in the $S$ frame for the low energy particles ($I \to 1$). This result is supported by the asymptotic behavior of the spin state of system in this limit (35). In addition, if we use Czachor's relativistic spin operator instead of the Pauli spin operator then the inequalities will be satisfied in this limit (65) leading to a full contradiction with Eq. (35). Therefore, it seems that the predictions of the $|M|$ and $|M'|$ inequalities considering the Pauli spin operator are in more agreement with the behavior of the entangled spin state transformed by the LT compared with those in which Czachor’s spin operator are considered.

Let us note that the $|S_v|$ inequality for the high energy particles ($I \to \infty$) is satisfied in the $\beta \to 1$ limit (64). In addition, in the $\beta \to 1$ limit, the violation amount of the $|M|$ and $|M'|$ inequalities for the high energy particles are more than those of the low energy particles (64). Therefore, there is a contradiction between the result of $|S_v|$ and those of $|M|$ and $|M'|$. 
In order to solve this contradiction, we evaluate
\[
E(ABC)_{W\overline{W}} = \langle W | \sigma(\widetilde{N}_1) \otimes \sigma(\widetilde{N}_2) \otimes \sigma(\widetilde{N}_3) | \overline{W} \rangle = -\frac{2}{3} \sin(\theta_1 + \theta_2 + \theta_3)
\]
\[
+ \frac{1}{3} \sin \theta_1 \sin \theta_2 \sin \theta_3 = \langle \overline{W} | \sigma(\widetilde{N}_1) \otimes \sigma(\widetilde{N}_2) \otimes \sigma(\widetilde{N}_3) | W \rangle,
\]
where \( \widetilde{N}_i \) are the same vectors used in order to evaluate the correlation functions \( E_W(ABC) \) of the W state. If we use \( |\tau\rangle = \frac{1}{\sqrt{2}} (|W\rangle - |\overline{W}\rangle) \) state instead of the W state in Eq. (7), simple calculations lead to
\[
E(ABC)_{\tau} = E_W(ABC) - E(ABC)_{W\overline{W}}
\]
and
\[
\Delta E(ABC) = E(ABC)_{\tau} - E_W(ABC)
\]
\[
= -\frac{1}{3} \sin \theta_1 \sin \theta_2 \sin \theta_3 + \frac{2}{3} \sin(\theta_1 + \theta_2 + \theta_3).
\]
Bearing \( \theta_i = \pi - \theta_i' = 35.264^\circ \) in mind, we get \( \Delta E(ABC) > 0 \) for the measurements \( (A,A',B,...) \). This analysis shows that the particles in the \( |\tau\rangle \) state are more entangled than the particles in the W state and therefore, the violation amount of the inequalities for the \( |\tau\rangle \) state should be more than those of the W state. Once again, we note that although the high energy limit is problematic in the Quantum Mechanics framework, it is at least mathematically useful to study this limit.

Comparing Eqs. (35) and (36), we conclude that the violation amount of the inequalities should be increased as a function of the energy of particles. Therefore, Eq. (35) is in agreement with the predictions of the \(|M|\) and \(|M'|\) inequalities for high energy particles while the \(|S_v|\) inequality fails to predict this behavior [53]. This result indicates that the \(|S_v|\) inequality is not a good witness for studying the non-locality stored in the W state in the relativistic regimes. From Eqs. (36) and (17), it is apparent that if the moving frame observer applies the measurements used to detect the non-locality of the GHZ state, the \(|S_v|\) inequality will be violated to its maximum violation value, this is due to the fact that the GHZ state is produced in this limit. In addition, this result shows that the multi-particle Bell-like inequalities are more sensitive to the directions of measurements in the relativistic regimes compared with the Bell’s inequality which is in agreement with the relativistic [65] and non-relativistic studies [71]. Bearing the differences between the entanglement of the W state and that of the GHZ state together with the results of previous subsection in mind, it seems that the behavior of the W state differs from that of the Bell states [17,18] and the GHZ state [83], this shows that the behavior of non-locality depends on both the number of the entangled particles and their type of entanglement. If one uses Czachor’s relativistic spin operator instead of the Pauli spin operator then, in the \( \beta \rightarrow 1 \) limit, the inequalities will be satisfied independent of the particles energy [65] which is again indicating that the inequalities considering the Pauli spin operator have better agreement.
with the behavior of the entangled spin state affected by the LT compared with those in which Czachor’s spin operator are considered.

3 Summary and Conclusion

The behavior of the GHZ and W states under the LT is investigated. In addition, considering the Pauli spin operator, we studied the behavior of three well-known classes of the multi-partite Bell-like inequalities including the $|S_v|$, $|M|$ and $|M'|$ inequalities under the LT. We compared our results with those in which Czachor’s relativistic spin operator is used instead of the Pauli operator. We used the behavior of the spin state under LT as criterion in order to choose the appropriate inequalities and the proper spin operator. In our setup, the moving and lab frames use the same set of measurements violating the $|S_v|$ inequality to its maximum violation amount ($|S_v|m$) in the lab frame. By these measurements, the $|M|$ and $|M'|$ inequalities are violated to the same value ($\frac{|S_v|m}{2}$) in the lab frame simultaneously. Our results indicate that: (i) Since the predictions of the $|S_v|$, $|M|$ and $|M'|$ inequalities, when the Pauli operator is considered as the spin operator, are compatible with the asymptotic behavior of the spin states in the moving frame, the Pauli operator is more compatible with LT of the quantum mechanical spin state compared with Czachor’s spin operator. This result is in full agreement with the two and one-particle studies [89]. (ii) The general behavior of the GHZ and W states under the LT, in the $\beta \to 1$ limit, is independent of the type of entanglement of the entangled particles if one considers the $|S_v|$ inequality and the special set of measurements violating $|S_v|$ to its maximum violation amount in the lab frame. It is due to the fact that, in the $\beta \to 1$ limit, the violation amount of the $|S_v|$ inequality in the moving frame is equal to that of the lab frame for the low energy particles whereas it is not violated for the high energy particles. Moreover, we found that the $|S_v|$ inequality can be considered as a reasonable witness for studying the non-locality stored in the GHZ state in the relativistic regime. Also, our study shows that the inequalities with correlation functions stronger than those of the $|M|$ and $|M'|$ inequalities, and weaker than those of $|S_v|$ can be used in order to study the effects of LT on the GHZ and W states supported by the non-relativistic study [70]. (iii) The predictions of the $|M|$ and $|M'|$ inequalities are always consistent with the asymptotic behavior of spin state of the system if the lab and moving frames use the special set of measurements violating $|S_v|$ to its maximum violation amount in the lab frame. These behaviors show that the $|M|$ and $|M'|$ inequalities can be considered as reasonable inequalities for studying the effects of LT on the W state. (iv) Since the behavior of the W state under the LT differs from those of the GHZ and Bell states, it seems that the behavior of non-locality depends on the number of the entangled particles and their type of the entanglement. This result is supported by the behavior of the inequalities under the LT. (v) The violation amount of the $|S_v|$, $|M|$ and $|M'|$ inequalities for the low energy particles measured in the moving frame, in the $\beta \to 1$ limit, is equal to that of the
The same result is reported in the case of the two-particle systems \[47,48\]. Therefore, we note that this result is independent of the number of the entangled particles and their type of entanglement. Finally, we should note that a Stern-Gerlach type experiment is needed for investigating our results.

By comparing our results with the results reported in \[64,65,66,83\], we can conclude that the sensitivity of the three-particle Bell-like inequalities to the set of measurements in the relativistic regimes is more than that of the Bell’s inequality \[47,48,49,50,51\] which is in line with the non-relativistic \[71\] and the relativistic studies \[65\].

Acknowledgements

We are grateful to the anonymous reviewers for their worthy hints and constructive comments which help us increase our understanding of the subject. This work has been supported financially by Research Institute for Astronomy and Astrophysics of Maragha (RIAAM) under research No.1/3720 – 76.

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