A Black Hole Life Preserver

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Abstract

Since no one lives forever, all a life preserver can really do is prolong life for longer than would have otherwise been the case. With this rather limited definition in mind we explore in this paper whether in principle you can take a life preserver with you to protect you (for a while at least) against the tidal forces encountered on a trip inside a black hole.

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It is well advertised that if you fall into a non-rotating Schwarzschild black hole you will be torn apart by tidal forces. As you free-fall in on a radial trajectory you will experience tidal acceleration given by
\[ a_z = (2M/r^3)z, \quad a_x = -(M/r^3)x, \quad a_y = -(M/r^3)y, \]
where, using geometrized units, \( M \) is the mass of the black hole, \( r \) is the circumferential radius appearing in the Schwarzschild metric, and \( x, y, z, t \) is a local freely falling coordinate system. If you fall in feet-first there will be a differential tidal acceleration between your head and feet of
\[ a_{\text{net}} = (2M/r^3)h, \]
where \( h = 1.8 \) meters is your height. It is like being stretched on a rack (by \( a_z \)) and simultaneously crushed in an iron maiden (by \( a_x \) and \( a_y \)). You can stand this up to \( a_{\text{crit}} = 10g = 98 \text{ m s}^{-2} \). Fighter pilots withstand 9g turns for example. But beyond 10g’s, the tidal acceleration will cause pain and dismemberment. Thus, the tidal acceleration becomes torturous at \( r_c = (2Mh/a_{\text{crit}})^{1/3} \). (You are still comfortable as you enter the event horizon of the hole only if \( M > 13,800 \) solar masses, so \( r_c < 2M \).) What is not generally recognized is that the period of torture is relatively brief. Suppose you fall into the black hole from a large distance (i.e., zero kinetic energy at infinity). The proper time for you to fall from radius \( r_c \) to \( r = 0 \) is
\[ t_c = \left( \frac{4M}{3} \right)^{3/2} = \left( \frac{2}{3} \right)(h/a_{\text{crit}})^{1/2} = 0.0904 \text{ sec}, \]
independent of the mass of the black hole. Can you lower \( t_c \), so that you live longer and are tortured for less time?

You can if you take along a life preserver: a ring of mass \( M_R \) and radius \( R \) (see Figure 1). It even looks like a life preserver! The ring produces to first order near the origin \( (x, y, z \ll R) \) a tidal acceleration \( a_z = -(M_R/R^3)z, \quad a_x = (M_R/2R^3)x, \quad a_y = (M_R/2R^3)y, \)
where \( M_R \ll M, \quad R \ll r, \) and \( M_R \ll R \) so that the ring’s gravitational field is approximately Newtonian. If \( M_R/R^3 = 2M/r^3 \), the tidal acceleration by the ring exactly cancels the tidal acceleration by the black hole. If we adjust the forces in the ring so that it is just neutrally stable against collapse due to its own self-gravity, each piece of the ring will fall radially inward keeping \( R \propto r \) during the infall so the tidal force of the black hole is countered all the way in. Of course, this eventually fails when \( R \) becomes smaller than your waist or, more precisely, when \( R \) is no longer much greater than \( h \).

A thin ring of radius \( R \) exerts an acceleration \( a_z \) along the \( z \)-axis: \[ a_z(z) = -M_Rz/(z^2 + R^2)^{3/2}. \]
Consider the ring to be a torus of major radius \( R \) with cross sectional width \( 4\delta \) and height \( 4\epsilon \) where \( \delta, \epsilon \ll R \). It may be approximated as four thin rings, each of mass \( M_R/4 \), located at \( x^2 + y^2 = (R \pm \delta)^2, \quad z = \pm \epsilon \). This torus produces an acceleration in the \( z \) direction
as a function of $z$ along the $z$-axis (for $z \ll R$) of approximately:

$$a_z(z) \approx -\frac{M_R z}{R^3} \left(1 - \frac{3}{2} \frac{z^2}{R^2} + 6 \frac{\delta^2}{R^2} - \frac{9}{2} \frac{\epsilon^2}{R^2}\right).$$

The first term in parentheses exactly cancels the tidal acceleration from the hole leaving a much smaller net acceleration

$$a_{z,\text{net}}(z) \approx -\frac{M_R z}{R^3} \left(-\frac{3}{2} \frac{z^2}{R^2} + 6 \frac{\delta^2}{R^2} - \frac{9}{2} \frac{\epsilon^2}{R^2}\right).$$

How might we make the ring neutrally stable against collapse under its own self-gravity? One way would be to use $Q = M$ material where the electrostatic repulsion exactly cancels the gravitational attraction (Papapetrou [1], Majumdar [2], Bonner and Wickramasuriya [3]). Surrounding yourself with an electrically conducting, spherical-shell Faraday cage of radius $h/2$ would protect you against the large electric fields, while still leaving you subject to the tidal gravitational field of the ring. Since $m_e, m_p \ll e$, and $Q_R = M_R \ll h/2 \ll R$, the gravitational effects of introducing the Faraday cage and excluding the electric field inside are small. And since $h/2 \ll R$ the electric quadrupole fields produced by the Faraday cage at the location of the ring are also fractionally small, of order $(h/2)^5/R^5$, and so do not affect its dynamics significantly. During free-fall collapse $R \propto r$, $\delta \propto r$, $\epsilon \propto r^{-1/2}$ as each part of the ring follows a geodesic trajectory, and we require $-5g \leq a_{z,\text{net}}(z) \leq 5g$ for $-h/2 \leq z \leq h/2$ up until a time $t_c$ from the end. You are neutrally stable at the center to first order because
the tidal force of the black hole cancels that of the ring. (For black holes larger than 7 solar masses, gradients in the strength of the hole’s tidal field across your body are small relative to the maximum values of $a_{z,\text{net}}$ encountered in the situation described above.)

Usefulness of $Q = M$ material is limited by electron-positron pair creation, whose rate is given by the relevant limit of Schwinger’s formula (Schwinger [4]), valid when the electric field $E$ varies slowly over an electron Compton wavelength:

$$\frac{dN}{dV dt} = \frac{\alpha E^2}{\pi^2 l_p^2} \exp \left( -\frac{\pi m_e^2}{|eE| l_p^2} \right),$$

(3)

where, using geometrized units, $\alpha$ is the fine structure constant, $l_p$ is the Planck length, $e$ and $m_e$ are the electron charge and mass, and we integrate $dV$ over all space. The ring has a total charge $Q_R = eN_0 = MR$ and a discharge timescale $T_c = N_0/(dN/dt)$ since positrons will be repelled by the positively charged ring and escape while electrons will be trapped and discharge the ring. We estimate $T_c$ by noting that, near $t_c$, if $a = 2\epsilon = 2\delta \ll R$, the discharge timescale for the torus should be approximately equal to the discharge timescale for an infinite cylinder of radius $a$ and charge per unit length of $Q_R/2\pi R$. Integrating Schwinger’s equation over all space then gives:

$$T_c = \left( \frac{\pi l_p^2 R}{4\alpha e MR} \right) \left[ x^4 \Gamma(-4, x) + \Gamma(0, x) \right]^{-1},$$

(4)

where $\Gamma$ are incomplete gamma functions and $x = \frac{\pi^2 m_e^2 R a}{e l_p^2 M_R}$. As the ring shrinks, the discharge timescale becomes exponentially shorter. Discharge is unimportant as long as $T_c \gg t_c$. Using the above equations we establish an optimal solution where at the endpoint of usefulness, $t_c = 3.46 \times 10^{-3}$ sec, $R = 28.47$ m, $a = 2\epsilon = 2\delta = 1.8$ m, $M_R = 2.145 \times 10^{-6} M_{\text{earth}}$, and $T_c = 1.3$ sec $\gg t_c$, with 50% of the pairs being created near the surface of the torus (within $\pm 3\%$ of the radius $a = 1.8$ m, the Faraday cage making a negligible correction to the calculation).

Thus, the ring effectively counters black hole tides of 6760 g’s across your body, allowing you to live longer and be tortured for only 1/26th as long as before. If you were killed in 0.0904 seconds, pain signals starting at your waist (where you were being pulled apart) traveling along the fast pathways (6 - 30 m/sec) – those telling you something is happening and where – would not have time to arrive, so it would not “hurt” much. By contrast, being killed in $3.46 \times 10^{-3}$ sec, you really wouldn’t know what hit you. In this example the number of pairs created prior to $t_c$ is large, of order $1.5 \times 10^{25}$, and the Faraday cage would have to
protect you. However, if \( M_R = 9.451 \times 10^{-7} M_{\text{earth}} \), \( a = 2 \epsilon = 2 \delta = 1.8 \text{ m} \), \( R = 24.16 \text{ m} \), at \( t_c = 4.08 \times 10^{-3} \text{ sec} \), the pair creation rate would be lowered so that, on average, no pairs would be created prior to \( t_c \).

Finally, we would note that instead of falling in feet first, a better posture would be to lie in fetal position with the line connecting your shoulders pointing toward the black hole, thus squeezing your body into a cylinder 18 inches high and 36 inches in diameter, taking advantage of the fact that the tidal acceleration is smaller by a factor of 2 in the \( x, y \) directions than in the \( z \) direction, giving \( t_c = 0.0455 \text{ sec} \). A massive ring as described above (at \( t_c \) with \( R = 7.242 \text{ m} \), \( a = 2 \epsilon = 2 \delta = 0.4572 \text{ m} \), \( M_R = 1.393 \times 10^{-7} M_{\text{earth}} \)) allows you to survive unscathed down to \( t_c = 1.75 \times 10^{-3} \text{ sec} \).

It is interesting that, using only normal matter, we may in principle counteract tidal forces encountered in extreme situations. This might also find application in trips near neutron stars or small black holes (without falling in) where an adjustable-radius, actively-oriented life preserver might enable you to venture closer than would otherwise have been the case and still return safely home from the adventure. When entering a region of unknown tidal forces one might want to take along a large spherical shell of \( Q = M \) material. Then, as needed, rings or small spherical masses could be drawn in from the shell to smaller radii to counteract the vacuum tidal fields encountered. A general tidal field produced by an arbitrary distribution of distant matter in the Newtonian limit could be countered by a series of rings, each ring counteracting each point mass in the distant mass distribution. Strong tidal forces due to a gravitational wave (moving along the \( z \) axis) could be countered by pulling in six small spherical masses on the \( x \) and \( y \) axes out of phase with the gravitational wave. For the case where you fall into an unperturbed Kerr black hole metric along the rotation axis to take advantage of the axisymmetry, you would need to pull in a ring at first as in the Schwarzschild case, but would need to manually adjust its radius as a function of time, finally enlarging it to large radius and replacing it with two small equidistant spherical masses drawn in along the previous ring axis of symmetry upon entering the region where \( r < (3)^{1/2} a \), where the tide from the black hole changes sign. As you approach the inner Cauchy horizon at \( r_- \), however, other effects would have to be considered, such as infalling photons and gravitons and black hole evaporation. Quantum gravity effects would also have to be considered, as well as limits on the magnitude of the tidal forces you can counter as we have discussed Schwarzschild case. Still, the results presented here prompt one to ask
whether masses carried along might allow you to survive longer than would otherwise have been possible.

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