PHENOMENOLOGICAL STUDY OF EXCITED BARYONS

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Abstract

We study baryon excited states for quark confinement and chiral symmetry breaking. In the first part we discuss spatially deformed baryon excitations. As signals of deformation, we study masses and electromagnetic transitions. Such a study of spatial structure is expected to provide information on quark binding mechanism and hence quark confinement. In the second part, we consider the chiral symmetry for baryons and study positive and negative parity baryons. We show that there are two distinctive representations of chiral symmetry for baryons. We investigate their phenomenological consequences in terms of linear sigma models.

1 Introduction

There are several reasons that hadron physics is one of interesting and hot subjects in current physics. In the last two decades, much progress has been made to reveal rich structure of QCD from asymptotic to non-perturbative regions. Yet essential questions associated especially with non-perturbative properties such as quark confinement and chiral symmetry breaking are not fully understood. Recent developments in experiments at facilities such as TJNAL and COSY are important [1]. Using electromagnetic or hadronic probes, various form factors are measured in detail. Also a new facility SPring8 where highly polarized photon beam is available is expected to contribute to interesting hadron physics [2]. Our objective in the present study is then a phenomenological study of baryon excited states, which are accessible by the new experiments. We investigate properties of baryons and attempt to study non-perturbative aspects of QCD.

In the former part of this report, we discuss spatial structure of excited baryons. Previously, it was pointed out that observed baryon mass spectrum of excitation energies up to around 1 GeV has a structure similar to rotational band, which implies deformed structure for excited baryons [3, 4]. Recently, we have pointed out that such a deformation is a common property for various flavor SU(3) baryons [5, 6]. The study of spatial structure would be useful in understanding quark binding mechanism. In particular, the flavor independent nature would be an indication of gluon dominant dynamics. We study the deformed baryons in terms of a deformed oscillator quark (DOQ) model. Masses and electromagnetic transitions are investigated in detail.

In the second part of this report we discuss the chiral symmetry for baryons. While the role of chiral symmetry has been extensively investigated for mesons, that for baryons is less worked out. For instance, it is not very well understood what the chiral partner of the nucleon should

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be. Chiral partners are those which belong to the same multiplet of chiral symmetry. The fact that is not very well known is that there are two different representations of chiral symmetry when considering both positive and negative parity baryons. They lead to very different results for such as masses and coupling constants toward chiral symmetry restoration \cite{7}. Our purpose is to study the role of chiral symmetry for positive and negative parity baryons, $N$ and $N^*$. We study masses, $\pi NN^*$ couplings and axial charges of $N$ and $N^*$ using linear sigma models \cite{8, 9}.

2 Deformed oscillator quark model

2.1 Masses

Let us begin with the experimental mass spectrum of nucleon excitations as shown in Fig. 1. Data are taken from the particle data group and well observed states with three and four stars are presented \cite{10}. Three negative parity states of masses around 1.7 GeV and one positive parity state $P_{11}(1720)$ are not shown because they are out of systematics which we are interested in here.

From Fig. 1 we observe the followings:

1. There is degeneracy in pairs $(3/2^+, 5/2^+)$, $(1/2^-, 3/2^-)$ and $(5/2^+, 7/2^-)$. This implies that the total spin $j$ of these levels are composed of orbital angular momentum $l$ and spin $S = 1/2$ with negligible spin-orbit forces.

2. The $1/2^+$ state appears as the lowest excited states. This is the Roper resonance whose nature is not well understood.

3. The level spacing between states of different $l$’s exhibits a characteristic feature. In the positive parity sector, the level spacing between higher energy states is larger than that of lower energy states. Furthermore the level spacing of the negative parity states is larger than that of those of the positive parity states.

Here we attempt to describe these properties in terms of an effective model. The third observation concerning the level spacing is particularly important, as it has lead to a model of deformed baryons whose masses appear as rotational band \cite{3, 4}.

In order to model the above picture, we consider a simple non-relativistic quark model with a deformed oscillator potential. We call this the deformed oscillator quark (DOQ) model. The hamiltonian of the DOQ model is then given by

$$H = \sum_{i=1}^{3} \left[ \frac{p_i^2}{2M_i} + \frac{1}{2}M_i(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2) \right]. \tag{1}$$

We may think that this hamiltonian is an effective hamiltonian with the mean field single particle potential which could be derived from the fundamental quark-quark interaction. In the DOQ model, variation of the oscillator parameter is allowed under the condition of volume conservation, $\omega_x\omega_y\omega_z = \omega_0^3$. We do not include more terms as meson degrees of freedom for an exclusive study of deformation effects.
Table 1: Physical parameters of the DOQ model.

| N  | $E_{\text{int}} [\hbar \omega]$ | $\omega^{-1}_x : \omega^{-1}_y : \omega^{-1}_z$ | shape   | $\hbar^2/2I [\hbar \omega_0]$ | $\langle l^2 \rangle$ |
|----|-------------------------------|---------------------------------|--------|---------------------------|-------------------|
| 0  | 3                             | 1 : 1 : 1                       | spherical | 0                         | 0                 |
| 1  | 3.780                         | 1 : 1 : 2                       | prolate  | 0.126                     | 3                 |
| 2  | 4.327                         | 1 : 1 : 3                       | prolate  | 0.072                     | 8                 |

The system of (1) has been worked out [4, 11]. After removing the center-of-mass motion, the intrinsic energy is given by

$$E_{\text{int}}(N_x, N_y, N_z) = \omega_x(N_x + 1) + \omega_y(N_y + 1) + \omega_z(N_z + 1),$$

where $N_i$ are the sum of oscillator quanta for two internal degrees of freedom, $\lambda$ and $\rho$. Variation of the intrinsic energy (2) under $(\delta \omega_x, \delta \omega_y, \delta \omega_z)$ for each principal quantum number, $N = N_x + N_y + N_z$, leads naturally to deformed intrinsic states for $N \neq 0$. Energies and the corresponding shapes for several low $N$’s are summarized in Table 1.

After performing the angular momentum projection [12], we obtain the rotational energy

$$E(N, l) = E_{\text{int}}(N) + \frac{\hbar^2}{2I} \left[(l + 1) - \langle l^2 \rangle \right]$$

for states of orbital angular momentum $l$. Here $I$ is the moment of inertia, and $\langle l^2 \rangle$ is the average angular momentum of the deformed intrinsic state [11]. Numbers for these quantities are tabulated also in Table 1.

We fix the constituent quark mass $M = 300$ MeV and adjust the absolute masses so that the nucleon $N(939)$ is reproduced. Taking the oscillator parameter $\omega_0 = 607$ MeV, we obtain theoretical mass spectrum as shown in Fig. 1. The total spin $j$ is given by the coupling of the intrinsic spin $S = 1/2$ of three quarks with the orbital angular momentum $l$, with the spin-orbit coupling being ignored. We find that experimental masses are nicely reproduced by the simple mass formula (3). In particular, it is remarkable that the lowest $1/2^+$ (Roper) state is reproduced in the DOQ model. We have also applied this idea to flavor SU(3) baryons [5, 6] and find a good agreement between the theory and experiment. The fact that the deformed picture is commonly applied to various SU(3) baryons suggests that the underlying dynamics would be dominated by gluons or flavor independent interactions.

As a prediction of the DOQ model, we consider masses of high spin states, e.g., $j^P = 13/2^+$ state. In the DOQ model, this state has the orbital angular momentum $l = 6$ whose energy is expected to appear around 3.5 GeV. This may be well compared with predictions of other models. For instance, the string model a la the Regge theory predicts the mass of 13/2$^+$ state to be around 2.5 GeV. The difference is substantial and should be studied in future experiments.

### 2.2 Electromagnetic transitions

If spatial deformation is sufficiently developed, we expect evidences in various transition amplitudes. Here we study electromagnetic decays of excited baryons. Data are available in the form of helicity amplitudes [10].

Theoretically, we study matrix elements of the electromagnetic interaction

$$H_{EM} = e J_\mu A^\mu \rightarrow -e \vec{J} \cdot \vec{A}. \quad (4)$$
For simplicity we adopt the non-relativistic current:

\[ \vec{J} = \frac{1}{2m} \left( u_f^\dagger (i \vec{\nabla} - i \vec{\nabla}) u_i + \vec{\nabla} \times (u_f^\dagger \vec{\sigma} u_i) \right) \tau_\mu, \]  

with standard notations. The computation of matrix elements of Eq. (4) is, though complicated, straightforward. Details will be reported elsewhere \[13\].

In Table 2, we summarize various helicity amplitudes \( A_{1/2} \) and \( A_{3/2} \) with tentative identification of quark model states with physical baryon states. We make the following observations.

1. In many channels, theoretical predictions are roughly consistent with data with small dependence on deformation. The reason that the effect of the deformation is not very significant is that the number of relevant degrees of freedom (the valence quarks) is much less than, for example, that of deformed nuclei which show clearly evidences of deformation.

2. The decay of the Roper \( N(1440) \) is not reproduced; the sign of the amplitude is wrong.

As a final remark of the DOQ model, the present treatment involves a non-orthogonality problem. We have worked out the re-diagonalization of the non-orthogonal hamiltonian and examined its effect on masses and transition amplitudes \[13\]. It turns out that the effect is not very large and the present results we have shown here are qualitatively unchanged. Therefore, we consider that the problem of the decay of the 1/2+ state is still an unsolved problem which we have to work out in future.

3  Chiral symmetry of baryons

In the latter part of this report, we discuss chiral symmetry of baryons \[8, 9\]. Although chiral symmetry and its spontaneous breakdown play important role for low energy hadron physics, its actual role for baryon dynamics is not well understood. For example, when chiral symmetry is restored, we expect the appearance of degenerate particles which belong to the same multiplet of the chiral group (chiral partners). Since these particles have opposite parities, one might expect that the \( N(939) \equiv N \) and \( N(1535) \equiv N^* \) might be candidates of such chiral partners. If this would be the case, it is an interesting question what properties of \( N \) and \( N^* \) are governed by chiral symmetry. In the following we consider the chiral \( SU(2)_L \times SU(2)_R \) group.

The fact that is not very well known is that when we consider two kinds of baryons, there are two distinctive representations of the chiral group. One is called the naive and the other mirror representation \[7\]. It turns out that they lead to very different results for properties such as the mass, meson-baryon couplings and axial charges.

Let us define the naive nucleons \( (N_1, N_2) \) and the mirror nucleons \( (\psi_1, \psi_2) \) such that they transform under \( SU(2)_L \times SU(2)_R \) chiral transformations as

\[ N_{1R} \rightarrow RN_{1R}, \quad N_{1L} \rightarrow LN_{1L}, \]

\[ N_{2R} \rightarrow RN_{2R}, \quad N_{2L} \rightarrow LN_{2L}. \]  

(6)
Table 2: Helicity amplitudes in units of $GeV^{-1/2} \times 10^{-3}$. The column indicated as DOQ is for results of the DOQ model with appropriate deformation; $d = 3$ for $N = 2$ positive parity states and $d = 2$ for $N = 1$ negative parity states. The column indicated as IK is for the spherical limit ($d = 1$) which correspond to the results of Koniuk-Isgur [14].

| Proton | $A_{1/2}^{p}$ | DOQ | KI | Exp | $A_{3/2}^{p}$ | DOQ | KI | Exp |
|--------|---------------|-----|----|-----|---------------|-----|----|-----|
| $1/2^+$ | $^2S_S$ | 109 | 22.6 | -68±5 | - | - | - | $P_{11}(1440)$ |
|        | $^2S_{MS}$ | -14.6 | -15.9 | +5±16 | - | - | - | $P_{11}(1710)$ |
| $3/2^+$ | $^2D_S$ | 70.9 | 111 | 52±39 | -23.5 | -36.7 | -35±24 | $P_{13}(1720)$ |
|        | $^2D_{MS}$ | -3.8 | -5.9 | -17±10 | 47.0 | 73.5 | 127±12 | $F_{15}(1680)$ |
| $1/2^-$ | $^4P_{MS}$ | 151 | 156 | 74±11 | - | - | - | $S_{11}(1535)$ |
|        | $^4P_{MS}$ | 0 | 0 | 48±16 | - | - | - | $S_{11}(1650)$ |
| $3/2^-$ | $^2P_{MS}$ | 24.8 | 25.6 | -23±9 | 138 | 143 | 163±8 | $D_{13}(1520)$ |
|        | $^4P_{MS}$ | 0 | 0 | -22±13 | 0 | 0 | 0±19 | $D_{13}(1700)$ |
| $5/2^-$ | $^4P_{MS}$ | 0 | 0 | 19±12 | 0 | - | 19±12 | $D_{15}(1675)$ |

| Neutron | $A_{1/2}^{n}$ | DOQ | KI | Exp | $A_{3/2}^{n}$ | DOQ | KI | Exp |
|--------|---------------|-----|----|-----|---------------|-----|----|-----|
| $1/2^+$ | $^2S_S$ | -73.0 | -15.1 | +39±15 | - | - | - | $P_{11}(1440)$ |
|        | $^2S_{MS}$ | 4.9 | 5.3 | -5±23 | - | - | - | $P_{11}(1710)$ |
| $3/2^+$ | $^2D_S$ | -20.1 | -31.5 | -2±26 | 0 | 0 | -43±94 | $P_{13}(1720)$ |
|        | $^2D_{MS}$ | 24.7 | 38.5 | 31±13 | 0 | 0 | -30±14 | $F_{15}(1680)$ |
| $1/2^-$ | $^4P_{MS}$ | -125 | -130 | -72±25 | - | - | - | $S_{11}(1535)$ |
|        | $^4P_{MS}$ | 12.9 | 13.4 | -17±37 | - | - | - | $S_{11}(1650)$ |
| $3/2^-$ | $^2P_{MS}$ | -61.3 | -63.4 | -64±8 | -138 | -143 | -141±11 | $D_{13}(1520)$ |
|        | $^4P_{MS}$ | 5.8 | 6.0 | 0±56 | 30.0 | 31.0 | -2±44 | $D_{13}(1700)$ |
| $5/2^-$ | $^4P_{MS}$ | -26.0 | -30.0 | -47±23 | -36.8 | -42.4 | -69±19 | $D_{15}(1675)$ |

and

$$
\psi_{1R} \rightarrow R \psi_{1R} , \quad \psi_{1L} \rightarrow L \psi_{1L} , \\
\psi_{2R} \rightarrow L \psi_{2R} , \quad \psi_{2L} \rightarrow R \psi_{2L} .
$$

In the naive representation the left and right components transform in the same way, while in the mirror representation they transform in the opposite way. The reason that the latter is possible is that while the chirality of the fermion is associated with the Lorentz group, that of chiral symmetry is associated with internal symmetry. Therefore, the left and right handed fermions can take both representations of the internal chiral group.

Now we construct linear sigma models which respect chiral symmetry. Knowing the transformation (8) and (7) together with that of the meson field, the chiral invariant lagrangians for the nucleon sector can be written up to fourth order in mass dimension:

$$
\mathcal{L}_{\text{naive}} = \bar{N} \partial \gamma N + \bar{N} \partial N + a \bar{N} (\sigma + i \gamma_5 \cdot \pi) N + b \bar{N} (\sigma + i \gamma_5 \cdot \pi) N + c \bar{N} (\gamma_5 \sigma + i \gamma_5 \cdot \pi) N - N (\gamma_5 \sigma + i \gamma_5 \cdot \pi) N + \mathcal{L}_M ,
$$

$$
\mathcal{L}_{\text{mirror}} = \bar{\psi} \gamma \bar{\psi} + \bar{\psi} i \gamma_5 \bar{\psi} + m_0 (\bar{\psi} \gamma_5 \psi - \bar{\psi} \gamma_5 \psi) + \bar{\psi} (\sigma + i \gamma_5 \cdot \pi) \psi + b \bar{\psi} (\sigma - i \gamma_5 \cdot \pi) \psi .
$$
Here $a$, $b$ and $c$ are coupling constants. What should be emphasized here is that in the mirror Lagrangian, the chiral invariant mass term is allowed with the mass parameter $m_0$. Thus both $\psi_1$ and $\psi_2$ remain massive with a degenerate mass when chiral symmetry is restored (parity doubling of baryons) \[.\]

In the lagrangians (8) and (9), there is a mixing term between the two nucleons, when chiral symmetry is spontaneously broken ($\sigma \rightarrow \sigma_0$). The non-diagonal mass terms can be diagonalized by the physical fields, and the resulting eigenvalues are given by

\[
m_{\pm} = \frac{\sigma_0}{2} \left( \sqrt{(a + b)^2 + 4c^2} \mp (a - b) \right), \quad \text{Naive} \quad (10)
\]

\[
m_{\pm} = \frac{1}{2} \left( \sqrt{(a + b)^2 \sigma_0^2 + 4m_0^2} \mp (a - b)\sigma_0 \right) \quad \text{Mirror} \quad (11)
\]

A schematic plot of the masses are shown in Fig 2. In the naive case, positive and negative parity nucleons degenerate with the vanishing mass when chiral symmetry is restored, while they have the finite mass $m_0$ in the mirror case.

We can also study other physical quantities. Here we just quote a few results of interest.

1. The $\pi N N^*$ coupling constant vanishes in the naive case, while it can be finite in the mirror case. The discrepancies in the previous QCD sum rule analyses \[18, 19\] can be explained by the different choices of the interpolating field corresponding to either naive or mirror representation.

2. Chiral multiplets are different for the naive and mirror models. In the naive case, positive and negative parity nucleons belong to separate multiplets, while they do in the same multiplet in the mirror case \[8\].

3. The relative sign of the axial charges of the positive and negative parity nucleons is different.

These properties are summarized in Table 3. They can be used as signals to distinguish the chiral representations of baryons. In particular the behaviors of coupling constants \[20\] and axial charges are interesting. As pointed out in recent report \[21\], density dependence of the $\pi N N^*$ coupling is also a useful signal. Experimentally, we are able to study these quantities associated with $N(1535)$ (assuming that $N(1535)$ is the chiral partner of the nucleon) through productions of an $\eta$ meson. Such processes have been studied, but not much emphasis has been put on chiral symmetry of baryons. Certainly chiral symmetry of baryons is an interesting topic in both theory and experiment.

4 Summary

We have overviewed baryon excited states based on two different points of view. Low lying mass spectrum up to about 1 GeV suggests an interesting possibility of deformed baryons. A simple DOQ model has been quite useful in reproducing the mass spectrum including the Roper state which has been difficult to be reproduced in conventional quark models. Such properties should be further studied through various form factors. Not only electromagnetic decays, a systematic study of strong decays should also be useful.
Table 3: Comparison of the naive and mirror constructions

|                | naive                  | mirror                |
|----------------|------------------------|-----------------------|
| definition     | $N_{2R} \rightarrow RN_{2R}$ | $\psi_{2R} \rightarrow L\psi_{2R}$ |
|                | $N_{2L} \rightarrow LN_{2L}$ | $\psi_{2L} \rightarrow R\psi_{2L}$ |
| mass in the    |                         |                       |
| symmetric phase| 0                      | $m_0$ (finite)        |
| $\pi NN^*$ coupling | 0              | $(a + b)/\cosh \delta$ |
| chiral partner | $N_+ \leftrightarrow \gamma_5 N_+$, $N_- \leftrightarrow \gamma_5 N_-$ | $\psi_+ \leftrightarrow \psi_-$ |
| $g_A^{NN}, g_A^{N^*N^*}$ | positive       | negative              |

We have also emphasized the role of chiral symmetry for baryons and pointed out that there are two distinctive realizations of chiral symmetry. They predict very different results for baryon properties, and we need more works in understanding the role of chiral symmetry for baryons.

So far we have discussed the two aspects of baryons, spatial structure which is perhaps related to quark confinement, and chiral symmetry, in an independent way. We do not yet know their link from a more fundamental point of view. We need more microscopic theories which can describe both quark confinement and chiral symmetry breaking. For this, we should mention that there is a promising theory such as the DGL theory [22]. In any event, baryon physics is an exciting subject from which we expect to learn more on non-perturbative nature of QCD.

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Figure Captions

Fig. 1  Observed masses of nucleon excitations as compared with the results of the DOQ model.

Fig. 2  A schematic plot of masses of $N$ and $N^*$ as functions of $\sigma_0$. 
