On the Performance of DCSK MIMO Relay Cooperative Diversity in Nakagami-$m$ and Generalized Gaussian Noise Scenarios

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Abstract—Chaotic Communications have drawn a great deal of attention to the wireless communication industry and research due to its limitless meritorious features, including excellent anti-fading and anti-intercept capabilities and jamming resistance exempli gratia. Differential Chaos Shift Keying (DCSK) is of particular interest due to its low-complexity and low-power and many attractive properties. However, most of the DCSK studies reported in the literature considered the additive white Gaussian noise environment in non-cooperative scenarios. Moreover, the analytical derivations and evaluation of the error rates and other performance metrics are generally left in an integral form and evaluated using numerical techniques. To circumvent on these issues, this work is dedicated to present a new approximate error rates analysis of multi-access multiple-input multiple-output dual-hop relaying DCSK cooperative diversity (DCSK-CD) in Nakagami-$m$ fading channels (enclosing the Rayleigh fading as a particular case). Based on this approximation, closed-form expressions for the average error rates are derived for multiple relaying protocols, namely the error-free and the decode-and-forward relaying. Testing results validate the accuracy of the derived analytical expressions.

Keywords—DCSK, Multi-Access, Relay, Cooperative Diversity, Chaos Communication, Bit Error Rates, Nakagami-$m$, Rayleigh.

I. INTRODUCTION

Chaotic modulations have drawn a great deal of attention in the wireless communications industry and research community [1]. Chaotic signals are non-periodic random-like signals that are generated from nonlinear dynamic systems. They are naturally well-suited for spread spectrum wireless communications due to their inherently wideband characteristics [2]. In contrast to the classical direct sequence spread spectrum (DSSS) communications that use pseudo-random noise (PN) sequences, chaotic communications utilize chaotic sequences which are directly generated by a discrete-time nonlinear map (e.g., logistic map) [1]. They possess a myriad of merits, in addition to those accomplished by classical DSSS, including mitigating frequency selectivity, jamming-resistance, low probability of interception, easiness of generation, simple transceiver circuits, strong immunity to self-interference, anti-multipath, transmission security, and excellent cross-correlation properties [3]. These features make it an excellent candidate for military scenarios, ultra-dense populated environments, and many other applications [4] [5]. Chaos-based sequences were also proven to reduce the peak-to-average power ratio (PAPR) [5].

Since its introduction in [6] by Parlitz et al., chaotic digital modulation, a great deal of research effort has been devoted to developing new chaotic modulation schemes. Depending on the need for chaotic synchronization, chaotic modulation schemes can be categorized into coherent and non-coherent schemes [4]. Amongst them, the coherent chaos-shift-keying (CSK) and non-coherent differential CSK (DCSK) are the prominent ones. Coherent CSK, akin to coherent modulations, requires perfect knowledge of channel state information and the chaotic synchronization are required to generate a chaotic replica at the receiver side to perform the demodulation [7], and hence are not really easily implementable in fast-fading channels with short coherence time [3]. For example, the proposed chaotic synchronization in [8] is still practically impossible to achieve in noisy environments. Hence, and due to these limitations and challenges, non-coherent chaos-based communications are more promising, and amid them, DCSK is the most attractive one [9].

DCSK, similar to differential phase shift keying (DPSK), is a non-coherent scheme, and so requires no synchronization or channel state information to recover the transmitted messages. However, DCSK is more robust to multipath fading compared than DPSK schemes [7] [10]. DCSK was also extended to the non-binary domain, leading to $M$-ary DCSK [11]. Currently, DCSK was considered for low-power and low-complexity wireless applications such as wireless personal area networks (WPANs) and wireless sensor networks (WSNs). It was also considered recently for short-range ultra-wideband (UWB) communications [1]. An elegant survey that studies recent advances in DCSK and its derivatives (e.g. RM-DCSK [2] a d HE-DCSK [12]) is given in [1] for the reader’s reference.

Motivated by the advantages of DCSK, significant amount of research work was carried to study and analyze the theoretical performance and fundamental limits of the DCSK systems under different channel conditions such as the additive white Gaussian noise (AWGN) and Rayleigh fading channels with single-input single-output (SISO) links. However, in realistic and practical communication scenarios, the transmitters may not be able to use multiple antennas due to size, complexity, power, cost and other constrains. To tackle this design issue, a novel MIMO relay DCSK cooperative-diversity system that used a single-transmit antenna, multi-antenna relay and multi-antenna receiver, was analyzed in [13]. The different links in the design were assumed to be subjected to Nakagami-$m$ fading and AWGN channels. The authors derived a closed-form analytical expression that was given in an integral form, which required numerical evaluation due to its complexity. Moreover, the AWGN noise assumption may sometimes be inaccurate and unsuitable to model the noise conditions [2].

To circumvent on these issues, and inspired by [13], this paper is dedicated to present a novel performance analysis of the same MIMO relay DCSK cooperative-diversity proposed by the authors of [13], with the assumption of an additive white generalized Gaussian noise (AWGGN) instead. Moreover, to
simplify the analytical derivations in the error rate analysis, we propose an approach to approximate the analysis and use it in our mathematical derivations. The numerical evaluations show an agreement with the approximated expressions.

The remaining part of this paper is structured as follows. In section II, an overview of the DCSK and the deployed cooperative scheme are presented. The analytical derivations and the new DCSK bit error rate approximation are given in section III. Simulation results in the different fading and noise scenarios and different configurations of the deployed cooperative scheme are presented and compared with Monte Carlo simulation showing excellent agreements. Finally, the paper findings are summarized in section IV.

II. SYSTEM MODEL

A. Principle of DCSK Modulation

The transmitter and the receiver structures of the DCSK system are shown in Fig. 1 and Fig. 2, respectively. In the DCSK, the \( t \)th transmitted symbol is assumed to be either +1 or −1, with equal probabilities. During the \( t \)th transmission, the transmitted signal, \( s_k \), is given by:

\[
s_k = \begin{cases} 
    x_{k} & k = 2(l-1)M + 1, ..., (2l-1)M \\
    b_i x_{k-M}, & k = (2l-1)M + 1, ..., 2lM 
\end{cases},
\]

where \( 2M \) is the spreading factor. At the receiver, the received signal \( r_k \), that is the input signal to the correlator, is given by:

\[
r_k = \varphi s_k + N.  \tag{2}
\]

where \( \varphi \) is the fading parameter and \( N \) is the additive noise to the signal. The \( l \)th decoded symbol, \( c_l \), is compared to threshold 0 to decide \( \tilde{b}_l \). The decision and hence the recovered symbol \( \tilde{b}_l \) is given by:

\[
\tilde{b}_l = \begin{cases} 
    +1, & c_l \geq 0 \\
    -1, & c_l < 0
\end{cases}.
\]

Reverting to [14], and making use of the relation between the error function the \( Q \)-function [15], the BER of the decoding data bits is given by:

\[
\text{BER}_{\text{DCSK}} = \frac{1}{2} \text{erfc} \left( \frac{\sqrt{2} \gamma}{2} \right) = Q \left( \frac{\sqrt{2} \gamma}{2} \right), \tag{4}
\]

where \( \gamma \) is the signal-to-noise ratio, which is exactly the same BER performance expression of the DDPSK-Walsh Coding reported in [16], and hence, the results derived in the later sections apply for both DCSK and DDPSK-Walsh Coding.

B. Multi-Access MIMO Relay DCSK-CD

Consider a two-hop network system model with \( n \) users (sources), one multi-antenna destination and a multi-antenna relay. We assume that each transmitter has a single antenna, and that there is no cooperation between users to transmit messages. As such, only the multi-antenna relay helps in transmitting messages to the final receiver. Furthermore, assume that a transmission period is divided into two phases (or time slots), the broadcast phase and cooperate phase, respectively. In the broadcast phase, users broadcast their messages to other terminals (the relay and the receiver). In the cooperate phase, the relay cooperates with the users to send their messages to the destination. The system model, assuming two users for illustration only, is presented in Fig. 3, where each user can support only one transmit antenna, the relay has \( M_R \) antennas used under ideal condition (error free (EF) at relay antennas) or decode-and-forward (DF) protocol, and the destination has \( M_D \) receive antennas [2] [13].

As mentioned earlier, we will assume that the fading model of the separate links to be independent Nakagami-\( m \) channels that are subjected to additive generalized Gaussian noise [17] [18], and the channel state remains constant during each transmission period. The probability density function (PDF) for a gamma random variable (RV) is given by [19] [20]:

\[
f(y) = \frac{y^{a-1} e^{-y/b}}{b^a \Gamma(a)}, \tag{5}
\]

where \( a \) and \( b \) are positive real numbers. Henceforth, the gamma RV of (5) is represented as \( G(a, b) \). To carry the rest of the analysis, we shall recall the following two theorems that are related to gamma RVs [2] [13] [21].

**Theorem 1**: Given \( N \) independent gamma RVs \( X_1, X_2, ..., X_N \), where \( X_k \sim G(a_k, b) \) (\( k = 1, ..., N \)), the sum of these variables \( X = \sum_{k=1}^{N} X_k \) is also a gamma RV, given as \( X \sim G \left( \sum_{k=1}^{N} a_k, b \right) \).
Theorem 2: Given $N$ independent gamma RVs $X_1, X_2, \ldots, X_N$, where $X_k \sim G(\alpha_k, \beta_k)$ $(k = 1, \ldots, N)$ where $\beta_1 \neq \beta_2 \neq \cdots \neq \beta_N$, then the sum of these variables $X = \sum_{k=1}^{N} X_k$ has a PDF that is expressed as:

$$f(x) = \sum_{i=0}^{\infty} \frac{\eta \Gamma(i+\alpha)}{\Gamma(\alpha)} \frac{x^{i+\alpha-1} e^{-\frac{x}{\beta}}}{\beta^{i+\alpha}},$$

with $\eta = 1$, $c = \prod_{k=1}^{N} \left( \frac{\beta_k}{\alpha_k} \right)^{\alpha_k} \frac{1}{\Gamma(\alpha)} \frac{1}{\Gamma(\alpha)}$, $i = 0, 1, \ldots, j = \sum_{k=1}^{N} a_k + i - 1$, $j = 1, 2, \ldots, p = \sum_{k=1}^{N} a_k > 0$, and $b_0 = \min(b_k)$.

Furthermore, to simplify the ABER analysis, we propose to approximate (4), for a fixed value of $M$, as a sum of scaled decaying exponential functions, given as:

$$Q_{\alpha} \left( \sqrt{\frac{y^2}{2y+M}} \right) \approx \sum_{r=1}^{\alpha} \delta_r e^{-\beta_r y},$$

where the fitting parameters, $\delta_r$ and $\beta_r$, are obtained using nonlinear curve fitting (using the MATLAB® curve-fitting Marquardt-Levenberg algorithm), with sample fitting values for different cases of $\alpha$ being presented in Table II, assuming $M = 32$, a value commonly used in the literature.

**Table II: Fitting Parameters of $Q_{\alpha}(\sqrt{y})$ Approximation**

| $\alpha$ | $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ | $\delta_6$ | $\delta_7$ | $\delta_8$ |
|----------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1        | 0.1078     | 0.4294     | -0.009     | 0.1788     | 0.5424     | 0.2477     | 0.7834     | 0.1044     |
| 1.5      | 0.4410     | 0.9055     | -0.0928    | -0.1217    | 0.2113     | 0.6214     | 0.3212     | 0.6197     |
| 2        | 0.2520     | 0.3976     | -0.611     | -0.4621    | 0.5162     | 0.2283     | 0.4096     | 0.2243     |
| 2.5      | 0.6083     | -1.0650    | 0.2360     | 0.8107     | 0.2341     | 0.3982     | 0.6922     | 0.2534     |

The relative absolute error plots of this approximation are given in Fig. 4. With simple and direct variable transform to the approx. in (9), $Q_{\alpha}(\cdot)$ approx. is straightforward.

**Fig. 4: Relative Absolute Error for the approx. in (9).**

B. ABER with Error Free Protocol for the Relay

The average bit error rates (ABER) due to a fading channel can be evaluated by averaging the conditional bit error rate probability of the noisy channel using the PDF of the fading envelope. With the DCSK generalized error expression in (11) with the AWGGN, the averaging process can be, in general, expressed as [2]:

$$P_e = \int_{0}^{\infty} f(y) Q_{\alpha} \left( \sqrt{\frac{y^2}{2y+M}} \right) dy,$$

where $f(y) = f(y_D)$ for the error free (EF) protocol is the fading PDF which, using theorem 1 and 2, is given at the destination as:

$$f(y_D) = \psi y^{m-1} e^{-\beta y_D},$$

with $\psi = \frac{\alpha_n}{\Gamma(\gamma)}, m = a + \sigma_{RD}$, and $\beta = \sqrt{2 \beta_{SD}^2} = \sqrt{2 \beta_{RD}^2}$, or

$$f_D(y) = \sum_{i=0}^{\alpha} \psi_i y^{m-1} e^{-\beta y_D},$$

with $\psi_i = \frac{\eta_i e^{-\frac{x_i}{\beta}}}{\Gamma(\alpha + 1) \beta^{\alpha_i}}$, $m = \rho + i$, $\beta = \frac{1}{\beta_0}$, and $\beta_{SD} \neq \beta_{RD}$.

### III. AVERAGE ERROR RATES ANALYSIS

In this section, the analytical derivation of the average error rates of the proposed system model will be presented.

A. Novel Approximation of the Generalized $Q$-Function

It was shown in [14] that the BER of the DCSK is given as in (4). This expression is based on the Gaussian approximation and is very difficult to manipulate analytically. Furthermore, the assumption of the AWGN environment therein might not be the case in some realistic and practical communication scenarios. Hence, in this work, we extend further the Gaussian approximation, given in terms of the Gaussian $Q$-function, to the generalized Gaussian approximation, given in terms of the generalized $Q$-function, which is given in [17] as:

$$Q_{\alpha}(x) = \int_{0}^{\infty} e^{-\alpha u} \left( 1 + \frac{u^2}{2} \right)^{-\frac{1}{2}} du = \frac{\alpha^{-1/2}}{\sqrt{\pi}} \frac{1}{\Gamma(\alpha)} \Gamma(1/\alpha, \alpha x).$$

where $\alpha_0 = (\sqrt{3}/\alpha)(\Gamma(1/\alpha))$ and $\Gamma(\cdot)$ and $\Gamma(\cdot, \cdot)$ are the gamma and the incomplete gamma functions [23]. Table I illustrates how the noise special models can be achieved from (8).

**Table I: Relation Between $Q_{\alpha}(x)$ and Special Noise Models.**

| Noise Dist. | Impulsive | Laplacian | Gaussian | Uniform |
|-------------|-----------|-----------|----------|---------|
| $x$         | 0.0       | 1.0       | 2.0      | $\infty$ |
Note that (12) is very difficult to be integrated and solved in a closed-form. Hence, the approx. in (11) is used to simplify the integrand. Using the approx. in (11) and the expression in (13.a), (12) can be evaluated as:

$$P_e \approx \sum_{t=1}^T \psi \delta_t \int_0^{\infty} y \hat{m}_{t} - 1 e^{-[\beta + \mu_y] y} dy = \sum_{t=1}^T \psi \Gamma(\hat{m}_{t}), \quad (14.a)$$

with $$\psi = \frac{\phi}{(\beta + \mu_y)}. $$ Similarly, using (9) and (11.b), (10) can be solved as:

$$P_e = \sum_{t=0}^{\infty} \sum_{\tau=1}^{4} \psi \delta_t \int_0^{\infty} y \hat{m}_{t} - 1 e^{-[\beta + \mu_y] y} dy = \sum_{t=0}^{\infty} \sum_{\tau=1}^{4} \psi \Gamma(\hat{m}_{t}), \quad (14.b)$$

with $$\psi = \frac{\phi}{(\beta + \mu_y)}. $$ These expressions are new, simple and very efficient to evaluate (take few seconds to process) using standard software packages, e.g. MATLAB.

C. ABER with Decode-and-Forward Protocol for the Relay

With the DF protocol, and following a similar approach to that of the EF protocol, the ABER can be obtained in a closed-form using the relation [2] [13] [24]:

$$BER_{DF} = BER_{SR} \cdot BER_{SD} + (1 - BER_{SR}) \cdot BER_{D}, \quad (15)$$

where each of the BER_{SR} and BER_{SD} are given exactly as in (14.a), while replacing the parameters $\{\hat{m}_{t}, \beta\}$ with $\{\alpha_{SR}, \beta_{SR}\}$ and $\{\alpha_{SD}, \beta_{SD}\}$, respectively, and BER_{D} is exactly the same as the ABER of the EF protocol, of either form of (14.a) or (14.b) depending on (13.a) and (13.b), respectively. Hence, (15) is finally given by:

$$BER_{DF} = \sum_{\tau=1}^{4} \sum_{\sigma=1}^{4} \psi_{\tau} \Gamma(\hat{m}_{SR}) \Gamma(\hat{m}_{SD}) + \sum_{\tau=1}^{4} \psi_{\tau} \Gamma(\hat{m}_{D}) - \sum_{\tau=1}^{4} \sum_{\sigma=1}^{4} \psi_{\tau} \Gamma(\hat{m}_{SR}) \Gamma(\hat{m}_{SD}), \quad (16)$$

with $\Psi_{1} = \bar{\psi}_{SR} \bar{\psi}_{SD}$ and $\Psi_{2} = \bar{\psi}_{D} \bar{\psi}_{SR}$. This expression is novel and is the first to address multi-access MIMO DCSK-CD, which is also applicable for multi-access MIMO DDCSK-Walsh Coding-CD.

IV. SIMULATION RESULTS

In this section, illustrative testing scenarios for the derived analytcal expressions are plotted over a range of average SNR using different system’s setups, Nakagami-\textit{m} fading and noise conditions. The results are also compared with the numerically evaluated expressions of the studied cases. In the next plots, the solid lines are the results obtained from numerical evaluations, while the patterns are generated from the derived analytical expressions. Moreover, we assume that $d_{SD} = d_{SR} = 1$, $M = 32$ and $M_{D}=1$, unless stated otherwise, in the following four test scenarios.

In the first test, assuming AWGN environment (i.e. $\alpha=2$) and $L=\text{m}=2$, and $M_{D}=3$, two cases are shown for the ABER for each of the EF and DF. In case 1, a Rayleigh fading (i.e. $m = 1$) is assumed for all links, while in case 2, Nakagami-m is assumed with $m = 4$. The results are shown in Fig. 5, where one can see the close match between numerical and the derived EF and DF analytical ABER expressions. In the second test scenario, we assume the exact configurations as of the first scenario but with the assumption of Laplacian noise instead, and the results are shown in Fig. 6, where an excellent agreement can be seen between the approximate analytical and numerical results. The last two scenarios repeats the first two, while replacing the values of $L$, $n$ and $M_{D}$ with 3, 3 and 4, respectively. The results are given in Fig. 7 and Fig. 8 and prove the accuracy and the validity of our derived results.
V. CONCLUSION

In this paper, novel performance analysis of ABER in multi-access MIMO relay DCSK-CD system is presented. The analysis is based on a new approximation for the error rates in DCSK systems, which also applies to DDCSK-Walsh Coding. The analysis assumed Nakagami-m fading, with an AWGGN environment that includes the Gaussian (AWGN), the Laplacian, and other noise models as special cases. Testing results showed that the derived analytical expressions are very accurate. The proposed evaluation approach can be also extended in a very similar way to approximate the error rates as well as other performance metrics to many chaos wireless communications systems. As a future work, we will study the usability of chaos communications in high mobility wireless networks such as VANETs [25].

REFERENCES

[1] Y. Fang, G. Han, P. Chen, F. C. M. Lau, G. Chen and L. Wang, “A Survey on DCSK-based Communication Systems and Their Application to UWB Scenarios,” IEEE Communications Surveys & Tutorials, pp. 1-34, March, 2016.

[2] E. Salahat, D. Shehada and C. Y. Yeun, “Novel Performance Analysis of Multi-Access MIMO Relay Cooperative RM-DCSK over Nakagami-m Fading Subject to AWGGN,” in IEEE 82nd Vehicular Technology Conference (VTC Fall), Boston, MA, September, 2015.

[3] G. Kaddoum, “Design and Performance Analysis of a Multislot OFDM Based Differential Chaos Shift Keying Communication System,” IEEE Trans. on Communications, vol. 64, no. 1, pp. 249 - 260, Nov., 2015.

[4] H. Yang, W. K. S. Tang, G. Chen and G.-P. Jiang, “System Design and Performance Analysis of Orthogonal Multi-Level Differential Chaos Shift Keying Modulation Scheme,” IEEE Trans. on Circuits and Systems I: Regular Papers, vol. 63, no. 1, pp. 146-156, Jan., 2016.

[5] F. J. Escribano, G. Kaddoum, A. Wagemakers and P. Giard, “Design of a New Differential Chaos-Shift Keying System for Continuous Mobility,” IEEE Trans. on Communications, vol. 64, no. 5, pp. 2066-2078, Mar., 2016.

[6] U. Parlitz, L. O. Chua, L. Kocarev, K. S. Halle and A. Shang, “Transmission of Digital Signals by Chaotic Synchronization,” Int. J. Bifurcation and Chaos, vol. 2, no. 4, pp. 973-977, April, 1992.

[7] G. Kaddoum, E. Soujiri and Y. Nijasure, “Design of a Short Reference Noncoherent Chaos-Based Communication Systems,” IEEE Trans. on Communications, vol. 64, no. 2, pp. 680 - 689, Jan., 2016.

[8] L. M. Pecora, T. L. Carroll and G. A. Johnson, “Fundamentals of Synchronization in Chaotic Systems, Concepts, and Applications,” Int. J. Bifurcation Chaos, vol. 74, p. 520-543, 1997.

[9] G. Kaddoum and E. Soujiri, “NR-DCSK: A Noise Reduction Differential Chaos Shift Keying System,” IEEE Trans. on Circuits and Systems II: Express Briefs, vol. 63, no. 7, pp. 648 - 652, Feb., 2016.

[10] Y. Xia, C. K. Tse and F. C. M. Lau, “Performance of Differential Chaos-Shift-Keying Digital Communication Systems over a Multipath Fading Channel with Delay Spread,” IEEE Trans. on Circuits and Systems II: Express Briefs, vol. 51, no. 12, pp. 680–684, Dec. 2004.

[11] G. Kis, “Performance Analysis of Chaotic Communication Systems,” Ph.D. Dissertation, Budapest University of Technology and Economics, Budapest, Hungary, Sept., 2005.

[12] H. Yang and G. P. Jiang, “High-Efficiency Differential-Chaos-Shift-Keying Scheme for Chaos-based Non-Coherent Communication,” IEEE Trans. on Circuits and Systems II: Express Briefs, vol. 59, no. 5, p. 312–316, May, 2012.

[13] Y. Fang, J. Xu, L. Wang and G. Chen, “Performance of MIMO Relay DCSK-CD Systems over Nakagami Fading Channels,” IEEE Trans. on Circuits and Systems I: Regular Papers, vol. 60, no. 3, p. 757–767, Mar., 2013.