Photon counting with loop detector

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We propose a design for a photon counting detector capable of resolving multiphoton events. The basic element of the setup is a fiber loop, which traps the input field with the help of a fast electrooptic switch. A single weakly coupled avalanche photodiode is used to detect small portions of the signal field extracted from the loop. We analyze the response of the loop detector to an arbitrary input field, and discuss both the reconstruction of the photon number distribution of an unknown field from the count statistics measured in the setup, and the application of the detector in conditional state preparation.

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Photon counting is an important method of detecting light with applications in many fields including spectroscopy, atmospheric physics, and quantum information processing. Detection of single photons necessarily involves a gain mechanism in order to generate a macroscopic number of photoelectrons from the absorption of a single energy quantum from the incident electromagnetic radiation. Among the most popular of detectors with this characteristic are avalanche photodiodes operated in the Geiger mode.\cite{1} Compared to standard photomultipliers, they have higher quantum efficiency, are more stable and are more robust with respect to external conditions.

Geiger-mode avalanche photodiodes have one drawback: the breakdown current is almost completely independent of the number of absorbed photons. Consequently, it is not possible to determine the number of photons incident on the detector on a time scale short compared to the detector response time. This has serious implications for several applications. For example, it critically affects the performance of practical quantum cryptography systems.\cite{2} It also impairs the fidelity of detectors with this characteristic are avalanche photodiodes operated in the Geiger mode.\cite{1} This has serious implications for several applications. For example, it critically affects the performance of practical quantum cryptography systems.\cite{2} It also impairs the fidelity of practical quantum cryptography systems.\cite{2}

In this paper, we propose a photon counting setup which is capable of resolving multiphoton events. The basic idea is to split the input signal into separate small pieces which are expected to contain less than one photon, and therefore can be detected with an avalanche photodiode without losing information on the photon number. Our method is based on current fiber optic technology and has the important advantage that it requires only a single avalanche photodiode. This contrasts with previous proposals that involve the splitting the input pulse. We will first calculate the response of the detector to a coherent field. This result can be generalized to an arbitrary input field by averaging with an appropriate probability distribution for the field amplitude. In the most general case this probability distribution is given by Glauber’s $P$ representation for the input field. Next, we will use this result to derive a method for reconstructing the photon number distribution from the count statistics for a completely unknown input field.

We begin by calculating the probability distribution $p_I(k)$ of obtaining $k$ counts on the photodiode assuming that the coherent pulse after injection into the loop had intensity $I$. In the derivation, it will be convenient to use the generating function:

$$\tilde{p}_I(z) = \sum_{k=0}^{\infty} z^k p_I(k).$$  \hfill (1)

There are several parameters of the setup relevant to our calculation. Let $t_r$ be the power transmission of the loop for a

FIG. 1: The proposed construction of the loop detector. S, electrooptic switch; C, weak coupler; APD, avalanche photodiode.
single pulse roundtrip, including losses at all the optical elements. Further, let \( t_r \) be the fraction of the light intensity extracted by the coupler \( C \) to the avalanche photodiode. We assume that the photodiode is characterized by the quantum efficiency \( \eta \), and that the probability of registering a dark count is \( p_d \). We will discuss realistic values for all these parameters later, though our analysis is valid for an arbitrary set of parameters satisfying certain approximations.

The assumption of coherent input substantially simplifies the calculations, since the portion of the pulse directed to the APD in each roundtrip is uncorrelated with the field remaining in the loop. Consequently, the counts registered by the avalanche photodiode in each roundtrip are statistically independent. In the \( i \)th circulation of the loop, the intensity of the pulse is \( t_r^{i-1} I \). Of this the light intensity extracted to the photodiode is \( t_r t_c \). Taking the efficiency of the avalanche photodiode to be \( \eta \), the probability of a count in the \( i \)th roundtrip is \( 1 - (1 - p_d) \exp(-\eta t_r t_c^{i-1} I) \), including the possibility of a dark count with probability \( p_d \). For an input pulse in a coherent state the counts on the APD are statistically uncorrelated on each circulation, so that the generating function \( \tilde{\rho}_z(z) \) is simply given by a product of generating functions describing detection events for each round trip:

\[
\tilde{\rho}_z(z) = \prod_{i=1}^{L} \left( z + (1 - z)(1 - p_d) \exp(-\eta t_r t_c^{i-1} I) \right). \tag{2}
\]

Here \( L \) is the total number of circulations of the loop.

The expression derived in Eq. (2) describes the most general situation of arbitrary input intensity. An important limiting case is when the probability of extracting two or more photons from the loop in a single roundtrip via the coupler \( C \) is negligible. Then the logarithm of the generating function \( \tilde{\rho}_z(z) \) can be expanded up to terms linear in \( t_r \) or \( p_d \), assuming that both these parameters are much smaller than one. Further, when the number of roundtrips is large enough to allow all the input signal photons to leak from the loop, we obtain from the logarithmic expansion the following approximate expression for the generating function:

\[
\tilde{\rho}_z(z) \approx \exp \left( (z - 1) \left( \frac{\eta t_c I}{1 - t_r} + L p_d \right) \right). \tag{3}
\]

This formula describes simply the standard Poissonian statistics normally associated with coherent radiation, with the average number of counts equal to \( \eta t_c I / (1 - t_r) + L p_d \). The first term here describes the average number of counts generated by the input light, and the second term corresponds to dark counts. In this regime, the response of the loop detector is described by a single effective efficiency parameter:

\[
\eta_{\text{eff}} = \frac{\eta t_c}{1 - t_r}. \tag{4}
\]

In the ideal limit all the optical elements are lossless, and the only source of attenuation for the pulse trapped inside the loop is extraction of the photons to the avalanche photodiode. Then \( t_r = 1 - t_c \), and the effective efficiency of the loop detector approaches that of the avalanche photodiode, but with the advantage of having photon number resolution. In a realistic case, there are always excess losses in the coupler or the electroptic switch, which decreases the effective efficiency of the loop detector below the level of the APD itself.

We now turn to the problem of reconstructing the photon number distribution of an unknown input field from the statistics of counts measured using the loop detector. Let \( g(n) \) be the photon number distribution of the pulse immediately after injection into the fiber loop. The probability \( p(k) \) of observing \( k \) photocounts is given by the linear combination:

\[
p(k) = \sum_n w(k|n) g(n) \tag{5}
\]

where \( w(k|n) \) are conditional probabilities of registering \( k \) counts on the photodiode given exactly \( n \) photons injected into the loop. These conditional probabilities are defined exclusively by the parameters of the setup. The explicit form of these conditional probabilities can be found from our previous calculation of the generating function for a coherent input. For a coherent input pulse, the photon number distribution is Poissonian, \( g(n) = e^{-I} I^n / n! \). Inserting this into Eq. (5), multiplying both its sides by \( z^k \) and performing the summation over \( k \) yields:

\[
e^{-I} \tilde{\rho}_z(z) = \sum_{k=0}^{\infty} \sum_{n=0}^{\infty} w(k|n) \frac{z^k I^n}{n!} \tag{6}
\]

where \( \tilde{\rho}_z(z) \) is defined in Eq. (2). Thus the conditional probabilities \( w(k|n) \) can be identified from the generating function \( e^{-I} \tilde{\rho}_z(z) \) for a coherent input by expanding it into a double power series in the parameters \( I \) and \( z \). This procedure can be easily performed using any of the standard computer programs for symbolic algebraic calculations. Given the explicit form of the conditional probabilities \( w(k|n) \) and the measured count statistics \( p(k) \), one can then use one of the standard methods for solving linear systems to reconstruct the photon number distribution \( g(n) \). In Fig. 2 we show the results of a Monte Carlo simulation of the loop detector operation, using the singular value decomposition method to reconstruct the photon number distribution. The detector is able to provide very good estimates of the photon number distribution of both classical and nonclassical inputs. The accuracy of the reconstruction can be estimated using standard statistical tools.

In a number of quantum information processing protocols, determination of the photon number has to be made on the single-shot basis in order to infer nondestructively the state of another quantum system that is entangled with the detected radiation. In such an application, the performance of the detector can be characterized by the confidence:

\[
C_k = \frac{w(k|n) g(n)}{\sum_n w(k|n) g(n)} \tag{7}
\]

describing a posteriori probability that a \( k \)-click event has been triggered by the \( k \)-photon component of the input field state. Here \( g(n) \) is the a priori photon number distribution of the input field, and \( w(k|n) \) are the conditional probabilities calculated before. In practice, the confidence can be optimized by tuning the splitting ratio of the coupler \( C \) used in the construction of the loop detector.
An important question is whether realistic values for the parameters of the loop detector enable it to function in the manner shown in Fig. 2. In the visible region, silicon avalanche photodiodes can exhibit efficiencies greater than 75% for wavelengths near 700 nm.\[10, 11, 12\] The dark count rates for these devices can be as low as 25 Hz, which is negligible. This rate may be further reduced by operating the photodiode in the gated mode. The APD parameters become less favorable beyond the 1 μm wavelength region, where germanium or InGaAs materials are used.\[13, 11, 12\] As with single photon detection, the quantum efficiency of the photodiodes imposes the upper limit on the performance of the loop detector. A second important factor is the excess losses inside the fiber loop, which attenuate the pulse in each roundtrip in addition to the fraction extracted by the coupler C. These excess losses lead to two opposing constraints on the extraction of the light from the loop to the photodiode, parameterized by the coupling losses $t_c$. On one hand, we have seen that in order to recover the complete photon number distribution, $t_c$ should be as small as possible. On the other hand, if too small a power is extracted from the loop, then a large number of roundtrips are required to measure the count statistics. Consequently, substantial excess losses in the loop would mean that most of the photons would disappear inside the loop before ever reaching the photodiode, and would give no signal at all. The optimal value of $t_c$ is some intermediate value which allows one to detect a substantial fraction of photons in the input signal and at the same time provides a sufficiently large number of multiphoton events. A good test of whether both these conditions are satisfied is to check the singularity of the matrix composed of the conditional probabilities $p(k|n)$ for the range of photon numbers which are expected in the input signal. If the matrix is not singular, then the count statistics provides sufficient information to reconstruct the complete input photon number distribution from the experimental data.

In conclusion, we have proposed and analyzed a design for a photon counting detector capable of resolving multiphoton detection events using standard laboratory technology. The setup uses commonplace fiber-optic components and does not require extreme operating conditions.

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