Constraints on $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ from White Dwarf Seismology

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Abstract. For many years, astronomers have promised that the study of pulsating white dwarfs would ultimately lead to useful information about the physics of matter under extreme conditions of temperature and pressure. We can now make good on that promise. Using observational data from the Whole Earth Telescope and a new analysis method employing a genetic algorithm, we empirically determine the central oxygen abundance of the helium-atmosphere variable white dwarf GD 358. We use this value, combined with detailed evolutionary calculations of the internal chemical profiles to place constraints on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ nuclear reaction cross-section.

1. Introduction

During helium burning in the core of a red giant star, only two nuclear reactions are effectively competing for the available helium nuclei: [1] the triple-$\alpha$ process, which fuses three helium nuclei into carbon, and [2] the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction, which combines the carbon with another helium nucleus to produce oxygen. The triple-$\alpha$ reaction has been measured very precisely in the laboratory, but the same is not true of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. Our current knowledge of its rate at stellar energies is the result of a very uncertain extrapolation from high energy measurements. This translates into similarly large uncertainties in our understanding of every astrophysical process that depends on this reaction, including supernovae explosions and galactic chemical evolution.

The relative success of the two reactions inside a red giant determines the final mixture of carbon and oxygen in the resulting white dwarf star. Since the triple-$\alpha$ process is already well constrained, a measurement of the internal $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction will provide additional useful information about the internal structure of the white dwarf.

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composition of a white dwarf can improve our understanding of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction. Although we cannot see directly inside a white dwarf star, a fortunate circumstance allows us to learn about the interior. After shedding its red giant cocoon to form a planetary nebula, the white dwarf begins to cool. As it fades, it will pass through one of several narrow ranges in temperature that may induce subtle periodic vibrations, sending seismic waves deep through the interior and bringing information to the surface in the form of brightness variations. Careful observations of these periodic changes reveal patterns that can be reproduced with a fair degree of accuracy using relatively simple computer models of white dwarf stars. By adjusting the characteristics of the model to provide the closest possible match to the observations, we can infer the internal composition and structure of the actual white dwarf.

2. Method

In the past decade, the observational requirements of white dwarf seismology have been satisfied by the development of the Whole Earth Telescope—a group of astronomers distributed around the globe who cooperate to observe these stars continuously for up to two weeks at a time (Nather et al. 1990). This instrument is now mature, and has provided a wealth of seismological data on the different varieties of pulsating white dwarf stars.

In an effort to bring the analysis of these data to the level of sophistication demanded by the observations, we have recently developed a new model-fitting method based on a genetic algorithm (Metcalfe 2001). The underlying ideas for genetic algorithms were inspired by Charles Darwin’s notion of biological evolution through natural selection. The basic idea is to solve a problem by evolving the best solution from an initial set of random guesses. In practice, this approach allows us to find the global solution in a parameter-space with $10^{10}$ grid points by performing only a few $\times 10^6$ model evaluations. To complete the calculations on a reasonable timescale, we designed and built a specialized metacomputer to run the models in parallel (Metcalfe & Nather 2000).

The initial application of this new method to the well-observed pulsating white dwarf GD 358 demonstrated that the models are very sensitive to the internal composition and structure (Metcalfe, Nather, & Winget 2000). In a recent follow-up study, an extension to the method finally yielded a preliminary constraint on the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction (Metcalfe, Winget, & Charbonneau 2001). More precise constraints require additional detailed simulations of the internal chemical profiles of white dwarfs (cf. Salaris et al. 1997), and a thorough investigation of the systematic uncertainties.

3. Results

Metcalfe, Winget, & Charbonneau (2001) derived a central oxygen abundance for GD 358 of $X_O = 84 \pm 3$ percent, with the transition from constant oxygen beginning at $q = 0.49 \pm 0.01 \, m/M_*$. To evaluate this result, we calculated new internal oxygen profiles for a 0.65 $M_\odot$ white dwarf model using the same method and code described in Salaris et al. (1997), but updated to use the nuclear
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Figure 1. The internal oxygen profiles for a 0.65 $M_\odot$ white dwarf model using the NACRE rates (solid line), and the $\pm 1\sigma$ limits for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate (dashed lines). Also shown are profiles using the rate that matches the central oxygen mass fraction derived for GD 358 (dark solid line) with the $\pm 1\sigma$ limits (dark dashed lines), and when overshooting is included (dotted line).

reaction rates from the NACRE collaboration (Angulo et al. 1999) rather than from Caughlin et al. (1985).

The total rate of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction at stellar energies in the NACRE compilation corresponds to an astrophysical S factor at 300 keV of $S_{300} = 200 \pm 80$ keV-b, yielding a central oxygen abundance between 0.53 and 0.78. To generate profiles with a higher central oxygen abundance, we simply scaled the value of $S_{300}$ in the simulations. Matching the central oxygen mass fraction inferred for GD 358 required a value of $S_{300} = 370 \pm 40$ keV-b, or $360 \pm 40$ keV-b when convective overshooting was included (see Figure 1).

The oxygen profiles from the simulations performed without overshooting consistently show the transition from constant oxygen beginning near a fractional mass of $q \sim 0.5$, regardless of the assumed value for the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate. This is in good agreement with the value for $q$ found by Metcalfe, Winget, & Charbonneau (2001) even though they did not use evolutionary profiles. With convective overshooting included ($\alpha = 0.20 \, H_p$), this transition moves out to $q \sim 0.6$ and the shape of the profile is otherwise similar.
4. Future Work

We are now working to quantify the systematic errors of this method by repeating both the model-fitting procedure and the chemical profile simulations with different assumptions. Early results suggest that these uncertainties are not much larger than the internal errors. We also have plans to apply this method to additional DBV stars as new data becomes available.

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Discussion

J. Christensen-Dalsgaard : How sensitive are your results to uncertainties in the physics, e.g., the equation of state?

T. Metcalfe : We are in the process of quantifying the model-dependent uncertainties due to the physical assumptions built into both the white dwarf pulsation code and the models of the internal chemical profiles. At this time I can tell you that the systematic errors are comparable to the internal errors.

G. Handler : Isn’t there some chance that the “best” solution gives unrealistic results? I was somewhat surprised by the high He-layer mass you found.

T. Metcalfe : We were also initially surprised by the He-layer mass, but it provides a much better fit to the data. We have applied the fitting method to simulated data, and the probability that we have not found the optimal solution is extremely small.

H. Shibahashi : Could you briefly tell us the advantages and the disadvantages of the genetic algorithm?

T. Metcalfe : This optimization method is global, objective, and very efficient compared to a full grid search. However, it is still fairly computationally intensive. If you want the global solution, you have to pay for it—the genetic algorithm simply gives you a substantial discount.