Screening of heavy quark free energies at finite temperature and non-zero baryon chemical potential

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(Dated: November 6, 2018)

We analyze the dependence of heavy quark free energies on the baryon chemical potential \( \mu_b \) in 2-flavour QCD using improved (p4) staggered fermions with a bare quark mass of \( m/T = 0.4 \). By performing a 6\(^{th}\) order Taylor expansion in the chemical potential which circumvents the sign problem. The Taylor expansion coefficients of colour singlet and colour averaged free energies are calculated and from this the expansion coefficients for the corresponding screening masses are determined. We find that for small \( \mu_b \) the free energies of a static quark anti-quark pair decrease in a medium with a net excess of quarks and that screening is well described by a screening mass which increases with increasing \( \mu_b \). The \( \mu_b \)-dependent corrections to the screening masses are well described by perturbation theory for \( T \gtrsim 2T_c \). In particular, we find for all temperatures above \( T_c \) that the expansion coefficients for singlet and colour averaged screening masses differ by a factor 2.

PACS numbers: 11.15.Ha, 11.10.Wx, 12.38Gc, 12.38.Mh

I. INTRODUCTION

Numerical studies of QCD provided quite detailed information about the properties of matter at high temperature and vanishing net baryon density \cite{1}. In particular, the screening of static quark anti-quark sources at large distances and their renormalization has been analyzed in quite some detail \cite{2–4}. Compared to this our knowledge on the dependence of the equation of state and screening at non-zero baryon number density, or equivalently, non-zero baryon chemical potential (\( \mu_b \)) is rather limited. The \( \mu_b \)-dependence of the QCD partition function \cite{5} and the HTL-resummation of the pressure \cite{6} have been evaluated only recently. Although in leading order of high temperature perturbation theory the dependence of the Debye screening mass on \( \mu_b \) is well-known \cite{7} neither the temperature range for the validity of this perturbative result nor the generic features of screening of heavy quark free energies at non-zero \( \mu_b \) have been analyzed so far with non-perturbative methods in the vicinity of the QCD transition \textsuperscript{1}.

Recently studies of the equation of state have successfully been extended to non-vanishing baryon chemical potential using Taylor expansions \cite{9} around \( \mu_b = 0 \) as well as reweighting techniques \cite{10} and imaginary chemical potentials \cite{11}. We will use here the former approach to analyze the screening of static quark anti-quark sources at non-zero \( \mu_b \), \textit{i.e.} in a medium with a non-vanishing net quark density. We evaluate the Taylor expansion coefficients for correlation functions of heavy quark anti-quark pairs and deduce from this expansion coefficients for the screening mass.

For this work we analyzed gauge field configurations using a p4-improved staggered fermion action with \( N_f = 2 \) degenerate quark flavours. We used the same data sample that recently has been generated by the Bielefeld-Swansea collaboration for the analysis of the equation of state \cite{9}. This sample consists of 1000 up to 4000 gauge field configurations available for several bare gauge couplings below and above the transition temperature. The lattice size is \( 16^3 \times 4 \) and the bare quark mass, \( m/T = 0.4 \), corresponds to a pion mass of about 770 MeV. In addition we generated 1000 configurations at \( T = 3T_c \) and 1600 at \( T = 4T_c \) to check for the approach to the high temperature perturbative regime. In addition to gauge invariant colour averaged free energies we also have analysed colour singlet free energies. To do so all gauge configurations have been transformed to Coulomb gauge before evaluating the Polyakov loop correlation functions.

This paper is organized as follows. In the next section we discuss the general setup for calculating Taylor expansions for heavy quark free energies. Our numerical results for singlet and colour averaged free energies are presented in section III. In section IV we discuss the determination of \( \mu_b \)-dependent corrections to screening masses from the free energies. Our conclusions are given in section V. In an appendix we give detailed expressions for the Taylor expansion coefficients of purely gluonic observables.

\textsuperscript{1} A first attempt to calculate heavy quark free energies at non-zero quark chemical potential has been discussed in \cite{8}.
II. TAYLOR EXPANSION OF HEAVY QUARK FREE ENERGIES

A heavy (static) quark $Q$ at site $x$ is represented by the Polyakov loop,

$$L(x) = \prod_{x_4=1}^{N_r} U_4(x,x_4),$$

which is an SU(3) matrix. A heavy anti-quark $\bar{Q}$ is described by the corresponding hermitian conjugate matrix. The free energy of a $QQ$-pair separated by a distance $r$ is then calculated from the expectation value of the correlation function of $L(0)$ and $L^\dagger(r)$ where $r$ points to a site with distance $r$ to 0. The dependence on the baryon chemical potential $\mu_b$ or quark chemical potential $\mu \equiv \mu_b/3$ is established solely through the fermion determinant, $\det M(\{U_\rho(x)\}, \mu, \hat{m})$, with $\hat{m}$ denoting the bare quark mass. In order to avoid the sign problem, which arises from $\Re(\det M)$ not being positive definite for $\mu \neq 0$, Taylor expansions in the quark chemical potential are used. This allows us to perform our simulations at zero chemical potential thereby restricting us to small chemical potentials.

A purely gluonic observable $O$ like the Polyakov loop $L(x)$ or a corresponding correlation function does not explicitly depend on the quark chemical potential; it is calculated in terms of link variables $U_\rho(x)$ of the gauge field configuration which do not explicitly depend on $\mu$. Any $\mu$-dependence of the expectation value $\langle O \rangle_\mu$ thus arises from the $\mu$-dependence of the Boltzmann weights in the QCD partition function, i.e. the $\mu$-dependence of the fermion determinant. Therefore we can apply the same method that was used for the power series expansion of the equation of state; expanding the fermion determinant in powers of $\mu$ leads to a power series of our purely gluonic observable,

$$\langle O \rangle_\mu = \langle O \rangle \cdot (1 + o_1 \mu + o_2 \mu^2 + \cdots),$$

where $\langle O \rangle$ denotes the expectation value of $O$ evaluated for vanishing chemical potential. We consider observables like the colour averaged and singlet $QQ$-correlation functions,

$$O^{\text{av}}(r) = \frac{1}{N} \frac{1}{N_c} \sum_{x,y} \Tr L(x) \Tr L^\dagger(y),$$

$$O^1(r) = \frac{1}{N} \frac{1}{N_c} \sum_{x,y} \Tr L(x) L^\dagger(y),$$

where the sum refers to all sites $x, y$ with $\|x - y\| = r$ and $N$ is the number of these $x, y$-pairs. As $O^{\text{av,1}}$ and the corresponding expectation values are strictly real for every single gauge field configuration the odd orders in the expansion vanish as was argued in [12]. For observables like the Polyakov loop itself or static quark-quark correlations...
like \( \text{Tr} L(0) \text{Tr} L(r) \) we also have to take into account the odd orders which are in general non-vanishing. In the appendix we give explicit formulas for calculating the expansion coefficients of an arbitrary gluonic observable up to sixth order in \( \mu/T \). We have used this to evaluate the first three, non-vanishing expansion coefficients of the purely real observables considered here.

We extract the colour averaged free energy of a static quark anti-quark pair from the Polyakov loop correlation function

\[
F_{Q\bar{Q}}^{\text{av}}(r, T, \mu) = -T \ln \langle O_{\text{av}}(r) \rangle_{\mu} 
\]

and the colour singlet free energies from

\[
F_{Q\bar{Q}}^{1}(r, T, \mu) = -T \ln \langle O^{1}(r) \rangle_{\mu} .
\]

We renormalize the Polyakov loop as described in [13] such that at short distances and vanishing chemical potential the singlet free energy \( F_{Q\bar{Q}}^{1}(r, T, 0) \) matches the zero temperature heavy quark potential. This also fixes the renormalization of the Polyakov loop and all its correlation functions. In particular, this renormalizes also the colour averaged free energies. As all our calculations have been performed on lattices with temporal extent \( N_{\tau} = 4 \) the smallest available distance at which this matching could be performed is \( r_{0} = 1/4T \).

In order to determine the expansion coefficients of the colour averaged (av) and singlet (1) free energies,

\[
F_{Q\bar{Q}}^{x}(r, T, \mu) = f_{Q\bar{Q},0}^{x}(r, T) + f_{Q\bar{Q},2}^{x}(r, T) \left( \frac{\mu}{T} \right)^2 + f_{Q\bar{Q},4}^{x}(r, T) \left( \frac{\mu}{T} \right)^4 + f_{Q\bar{Q},6}^{x}(r, T) \left( \frac{\mu}{T} \right)^6 + O \left( \left( \frac{\mu}{T} \right)^8 \right) ,
\]

FIG. 2: The 2\textsuperscript{nd} order coefficients of the singlet (a,c) and colour averaged (b,d) free energies for some selected temperatures below (a,b) and above (c,d) \( T_{c} \).
with $x = \text{av}$ and 1, we apply (A18) to the corresponding Polyakov loop correlation functions. We again note that these are strictly real on every gauge field configuration and thus have an expansion in even powers of $\mu/T$. Explicit formulas used for the calculation of the expansion coefficients $f_{\text{av}}^{Q\bar{Q},n}(r,T)$ and $f_{1}^{Q\bar{Q},n}(r,T)$ are given in the appendix.

### III. Numerical Results on $Q\bar{Q}$ Free Energies

In Fig. 1-4 we show the leading and higher order expansion coefficients up to sixth order in $\mu/T$ expressed in units of the string tension$^2$. We do not include data for all temperature values analyzed by us because for $T \gg T_c$ they have very small absolute values and for $T < T_c$ they suffer from large statistical errors and are still consistent with

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$^2$ We use $\beta_c = 3.649$ [9, 12] as the bare pseudo-critical coupling. Results on the string tension [14], $a\sqrt{\sigma}$, are then used to set the temperature scale in units of $T$.  

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FIG. 5: The coefficients for the singlet and colour averaged free energies at infinite distance $rT$ versus temperature. They have been obtained from a weighted average of $f_{Q\bar{Q},n}^\text{av}(r,T)$ and $f_{Q\bar{Q},n}^1(r,T)$ at the five largest distances.

The leading order results, $f_{Q\bar{Q},0}^\text{av}(r,T)$ and $f_{Q\bar{Q},0}^1(r,T)$ are consistent with previous analyses of static quark anti-quark free energies performed in 2-flavour QCD at $\mu = 0$ on the same data set [3]. For the second order expansion coefficients we display separately results below (Fig. 2(a), (b)) and above (Fig. 2(c), (d)) the $\mu = 0$ transition temperature, $T_c$. As can be seen the second order expansion coefficients are always negative and increase in magnitude in the vicinity of $T_c$.

The corresponding results for the 4th and 6th order expansion coefficients are shown in Fig. 3 and Fig. 4, respectively. Here we only show results above $T_c$; below $T_c$ the expansion coefficients are consistent with being zero within errors even at rather short distances and errors grow large for $rT \geq 1$. We note that all expansion coefficients shown in Figs. 2 to 4 vanish at small distances. This shows that a quark anti-quark pair is not affected by the surrounding medium if its size becomes small. This observation also justifies our procedure to renormalize the Polyakov loop by matching the $\mu = 0$ singlet free energy to the $T = 0$ heavy quark potential. The renormalization constant is independent of $\mu$.

Also close to $T_c$, where the $\mu$-dependence of the free energies is strongest, the absolute values of the fourth and sixth order expansion coefficients are of the same order as or smaller than the second order expansion coefficient. Therefore the 4th and 6th order contributions rapidly become negligible for $\mu/T < 1$.

Although the errors are large for the higher order expansion coefficients they show that at high temperature the 2nd and 4th order expansion coefficients are opposite in sign, $f_{Q\bar{Q},2}^\text{av}(r,T) < 0$ and $f_{Q\bar{Q},4}^\text{av}(r,T) > 0$. This is consistent with the expectation that at high temperature the asymptotic large distance value of the heavy quark free energy is proportional to the value of the Debye mass [15]. In this limit one obtains alternating signs of the expansion.
coefficients of the heavy quark free energies when one expands the perturbative Debye mass [7],

\[
\frac{m_D(T, \mu)}{g(T)T} = \frac{m_{D,0}(T)}{g(T)T} \sqrt{1 + \frac{3N_f}{(2N_c + N_f)\pi^2} \left(\frac{\mu}{T}\right)^2},
\]  

(7)
with $m_{D,0}(T) = g(T)T \sqrt{\frac{2\pi}{3} + \frac{N_f}{6}}$ denoting the Debye mass for vanishing baryon chemical potential. Although the statistical significance of our results for $f_{Q\bar{Q},6}^{av}(T)$ rapidly drops with increasing temperature this pattern of alternating signs seems to be valid also at sixth order at least for temperatures $T \geq 1.05 T_c$.

Except for temperatures close to the transition temperature the asymptotic behaviour of the free energies is reached at distances $r T \geq 1.5$. We determined their large distance value by taking the weighted average of the values at the five largest distances. The results are shown in Fig. 5. We note that $\left| f_{Q\bar{Q},2}^{av}(\infty, T) \right|$ have a pronounced peak at $T_c$. This also holds for $\left| f_{Q\bar{Q},2}(r, T) \right|$ evaluated at any fixed distance $r$. In fact, $f_{Q\bar{Q},2}^{av}(r, T)$ is proportional to the second derivative of a partition function including a pair of static sources, $Q, Q\bar{Q}$. It thus shows the characteristic properties of a susceptibility in the vicinity of a (phase) transition point.

Fig. 5 also shows that at large distances, within the statistical errors of our analysis, the expansion coefficients for the colour averaged and singlet free energies approach identical values,

$$f_{Q\bar{Q},n}^{av}(\infty, T) = f_{Q\bar{Q},n}^{1}(\infty, T) ,$$

where

$$f_{Q\bar{Q},n}^{x}(\infty, T) = \lim_{r \to \infty} f_{Q\bar{Q},n}^{x}(r, T) , \quad \text{with } x = \text{av, 1} .$$

This has been noted before at $\mu = 0$ and suggests that at large distances, e.g. for $r T \geq 1.5$, the quark anti-quark sources are screened independently from each other; their relative colour orientation thus becomes irrelevant.

Including all terms up to sixth order we calculated the singlet and colour averaged free energies at a few values of temperature are shown in Fig. 6. Similar results hold for the colour averaged free energies. The free energies decrease relative to their values at $\mu/T = 0$ for all temperatures above and below $T_c$. At small distances the curves always agree within errors. With increasing distance a gap opens up which reflects the decrease in free energy at non zero $\mu$. As indicated by the asymptotic values $f_{Q\bar{Q},2}^{av}(\infty, T)$, which give the dominant $\mu$-dependent contribution at large distances, the medium effects are largest close to the transition temperature and become smaller with increasing temperature.

### IV. SCREENING MASSES

For temperatures above $T_c$ and large distances $r$ the heavy quark free energies are expected to be screened,

$$\Delta f_{Q\bar{Q}}^{av}(r, T, \mu) = F_{Q\bar{Q}}^{av}(\infty, T, \mu) - F_{Q\bar{Q}}^{av}(r, T, \mu) ,$$

$$\sim \frac{1}{r^n} e^{-m^{av,1}(T, \mu)} r$$

with $n = 1, 2$ for the singlet and colour averaged free energies respectively. In the infinite distance limit we thus can extract the screening masses,

$$m^{av,1}(T, \mu) = - \lim_{r \to \infty} \frac{1}{r} \ln \left( \Delta F_{Q\bar{Q}}^{av}(r, T, \mu) \right) .$$

We use this as our starting point to derive a Taylor expansion for the screening masses. Expanding the logarithm in eq. 11 in powers of $\mu/T$ it is obvious that also the screening masses are even functions in $\mu/T$,

$$m^{av,1}(T, \mu) = m^{av,1}_0(T) + m^{av,1}_2(T) \left( \frac{\mu}{T} \right)^2 + m^{av,1}_4(T) \left( \frac{\mu}{T} \right)^4 + m^{av,1}_6(T) \left( \frac{\mu}{T} \right)^6 + O \left( \left( \frac{\mu}{T} \right)^8 \right) .$$

To analyze the approach of the various expansion coefficients to the large distance limits we introduce effective masses, $m_{eff,n}(r, T)$, with $x = \text{av, 1},$

$$m_{eff,2}(r, T) = - \frac{1}{r} \frac{\Delta f_{Q\bar{Q},2}^{x}(r, T)}{\Delta f_{Q\bar{Q},0}^{x}(r, T)} ,$$

$$m_{eff,4}(r, T) = - \frac{1}{r} \left[ \frac{\Delta f_{Q\bar{Q},4}^{x}(r, T)}{\Delta f_{Q\bar{Q},0}^{x}(r, T)} - \frac{1}{2} \left( \frac{\Delta f_{Q\bar{Q},2}^{x}(r, T)}{\Delta f_{Q\bar{Q},0}^{x}(r, T)} \right)^2 \right] ,$$

$$m_{eff,6}(r, T) = - \frac{1}{r} \left[ \frac{\Delta f_{Q\bar{Q},6}^{x}(r, T)}{\Delta f_{Q\bar{Q},0}^{x}(r, T)} - \frac{1}{2} \frac{\Delta f_{Q\bar{Q},4}^{x}(r, T) \Delta f_{Q\bar{Q},2}^{x}(r, T)}{\Delta f_{Q\bar{Q},0}^{x}(r, T)^2} + \frac{1}{3} \left( \frac{\Delta f_{Q\bar{Q},2}^{x}(r, T)}{\Delta f_{Q\bar{Q},0}^{x}(r, T)} \right)^3 \right] .$$
FIG. 7: Example for the distance dependent effective masses converging at large distance to the expansion coefficients for the screening mass which are represented by the horizontal lines.

In the limit of large distances these relations define the expansion coefficients of the colour averaged and singlet screening masses,

$$m_{n}^{\text{av,1}}(T) = \lim_{r \to \infty} m_{\text{eff},n}^{\text{av,1}}(r, T).$$

As will become obvious in the following the effective masses defined above show only little $r$-dependence. They are thus suitable for a determination of the $\mu$-dependent corrections to the screening masses. This is not the case for the leading order, $\mu$-independent, contribution. In order to determine $m_{1}^{0}(T)$ we use an ansatz for the large distance behaviour of the singlet free energy motivated by leading order high temperature perturbation theory,

$$f_{Q\bar{Q},0}^{1}(r, T) = f_{Q\bar{Q},0}^{1}(\infty, T) - \frac{4}{3} \frac{\alpha_{0}(T)}{r} e^{-m_{1}^{0}(T)r}.$$  

We fit our data to this equation using $\alpha_{0}(T)$ and $m_{1}^{0}(T)$ as fit parameters where $f_{Q\bar{Q},0}^{1}(\infty, T)$ is determined as described in the previous section. We choose the same fitting procedure as in [3] namely averaging results received from five fit windows with left borders between $rT = 0.8$ and $rT = 1.0$ and right border at $rT = 1.73$. While the above ansatz is known to describe rather well the large distance behaviour of the color singlet free energy, it also is known that the sub-leading power-like corrections are much more difficult to control in the case of the colour averaged free energy. For this reason we will analyze here only the leading order contribution to the singlet screening mass.

Results for effective masses in the singlet channel are shown in Fig. 7 as function of $rT$ for one value of the temperature. As can be seen the asymptotic value is indeed reached quickly before the errors grow large at distances...
FIG. 8: Expansion coefficients of screening masses in the colour singlet \( (m^1_n) \) and colour averaged \( (m^\text{av}_n) \) channels. Except for \( n = 0 \) \( m^{\text{av}}_n(T) \) is determined from the \( r \to \infty \) limit of (13). The lines are the first order perturbative predictions according to eq. 20. The dotted lines in (a) show the 1\( \sigma \)-range of the \( \chi^2 \)-fit with parameter \( A \).

\( rT \gtrsim 1 \). The expansion coefficients \( m^{\text{av},1}_n(T) \), \( m^{\text{av},1}_4(T) \) and \( m^{\text{av},1}_6(T) \) are thus well determined from the plateau values of these ratios. Similar results hold in the colour averaged channel. We found the left border of the plateau to lie between \( rT = 0.48 \) close to \( T_c \) and \( rT = 0.23 \) for \( T > 1.15T_c \). Results for the various expansion coefficients are shown in Fig. 8. This figure shows that at high temperatures the \( \mu \)-dependent corrections to the screening mass of the colour averaged free energies \( m^{\text{av}}_n(T, \mu) \) are twice as large as those of the (Debye) screening mass in the singlet channel, \( m^1_n(T, \mu) \). This is expected from perturbation theory, which suggests that the leading order contribution to the colour singlet free energy is given by one gluon exchange while the colour averaged free energy is dominated by two gluon exchange. Using resummed gluon propagators then leads to screening masses that differ by a factor of 2,

\[
m^1_n(T) = \frac{1}{2} m^{\text{av}}_n(T) , \quad n = 2, 4, 6 ,
\]

Our results suggest that this relation holds already close to \( T_c \) (Fig. 8). We thus have no evidence for large contributions from the magnetic sector, which is expected to dominate the screening in the colour averaged channel at asymptotically large temperatures [16] and which would violate the simple relation given in eq. 16.

In order to compare the expansion coefficients with perturbation theory we need to specify the running coupling \( g(T) \). Following [3] we use the next-to-leading order perturbative result for the running of the coupling with temperature but allow for a free overall scale factor. We thus fit our data on the \( T \)-dependence of the leading order (\( \mu = 0 \))
screening mass by the ansatz,

\[ m_1^T(T) = A \cdot \frac{2}{\sqrt{3}} g(T) T, \]  

(17)

with the 2nd order perturbative running coupling,

\[ g(T)^2 = \left[ \frac{29}{24\pi^2} \ln \left( \frac{\bar{\mu} T}{\Lambda_{\text{MS}}} \right) + \frac{115}{232\pi^2} \ln \left( \frac{\bar{\mu} T}{\Lambda_{\text{MS}}} \right) \right]^{-1}, \]  

(18)

where we use \( T_c/\Lambda_{\text{MS}} = 0.77(21) \) and the scale \( \bar{\mu} = 2\pi \) as in [3]. Fitting our data to eq. 17 with fit parameter \( A \), yields

\[ A = 1.397(18), \]  

(19a)

which is almost identical to the result in [3] where the data for \( T = 3T_c \) were still missing. Our fit result is included in Fig. 8. We also compare the temperature dependence of \( m_2^T(T) \), \( m_4^T(T) \), and \( m_6^T(T) \) with corresponding expansion coefficients of the perturbative Debye mass which result from an expansion of (7) using (17) as the 0th order. These expansion coefficients are alternating in sign,

\[ m_{D,2}(T) = \frac{\sqrt{3}}{4\pi^2} \cdot Ag(T), \]  

\[ (20a) \]

\[ m_{D,4}(T) = -\frac{3\sqrt{3}}{64\pi^4} \cdot Ag(T), \]  

\[ (20b) \]

\[ m_{D,6}(T) = \frac{9\sqrt{3}}{512\pi^6} \cdot Ag(T). \]  

\[ (20c) \]

At least for the second order coefficient \( m_2^T(T) \) we find that this yields a satisfactory description of the numerical results for \( T \geq 2T_c \). Eq. 20 shows that subsequent terms differ by about an order of magnitude, which explains why our signal for a non-zero contribution \( m_1^T(T) \) is rather poor for \( n > 2 \).

From (20) we find \( m_{D,2}(T)/m_{D,0}(T) = 3/8\pi^2 \) which is independent of \( A \) and \( g(T) \) and is compared with our numerical results in Fig. 9(a). We note that the perturbative value for this ratio is already reached for \( T/T_c \geq 2 \).

In Fig. 9(b) we show the \( \mu \)-dependence of the singlet screening mass for a small values of \( \mu/T \). Here we included only contributions from the 0th and 2nd order expansion in the calculation of \( m^T(\mu, T)/T \).
V. CONCLUSIONS

We have analyzed the response of colour singlet and colour averaged heavy quark free energies to a non-vanishing baryon chemical potential and have calculated the resulting dependence of screening masses on the chemical potential. Using a Taylor expansion in $\mu/T$ we get stable results for the leading, non-vanishing correction, $m_2^2(T)$, which is $\mathcal{O}(\mu/T^2)$. We find that this correction in absolute units as well as its ratio with the leading order screening mass, $m_0^2(T)$, is large in the vicinity of the transition temperature. The ratio $m_2^2(T)/m_0^2(T)$ is in agreement with perturbation theory for $T \gtrsim 2T_c$ indicating that the expansion coefficients $m_1^a(T)$ receive the same multiplicative rescaling as the leading order screening mass.

A calculation of the $\mu$-dependent corrections to the screening mass in the colour averaged channel shows that these corrections are twice as large as those in the color singlet channel for all temperatures $T > T_c$. This agreement with leading order perturbation theory indeed is quite remarkable as it suggests that the leading contribution to the $\mu$-dependent corrections of the colour averaged screening mass is due to two-gluon exchange.

The higher order expansion coefficients of the screening mass vanish within statistical errors at temperatures larger than $1.2T_c$. The analysis of the asymptotic behaviour of the free energies themselves, however, suggests that these corrections are non-zero but small at high temperature and have alternating signs. This is consistent with the leading order perturbative result for the Debye mass subsequent expansion coefficients of which drop by more than an order of magnitude and alternate in sign as they arise from an expansion of a square root.

Our results thus suggest that at least for small values of the chemical potential and fixed temperature the screening length in a baryon rich quark gluon plasma decreases with increasing value of the chemical potential. This is consistent with the expectation that the transition to the high temperature phase shifts to lower temperatures at non-zero baryon chemical potential.

VI. ACKNOWLEDGMENTS

This work has been supported partly through the DFG under grant KA 1198/6-4, the GSI collaboration grant BI-KAR and a grant of the BMBF under contract no. 06BI106. MD is supported through a fellowship of the DFG funded graduate school GRK 881. The work of FK has been partly supported by a contract DE-AC02-98CH1-886 with the U.S. Department of Energy.

APPENDIX A: CALCULATION OF EXPANSION COEFFICIENTS

The $\mu$-dependent expectation value of a complex quantity $\mathcal{O}$ is

$$\langle \mathcal{O} \rangle_\mu = \frac{1}{Z_\mu} \int D U \mathcal{O} \Delta e^{-S} = \frac{\int D U \mathcal{O} \Delta e^{-S}}{\int D U e^{-S}},$$

(A1)

where $Z_\mu$ is the partition function for finite $\mu$ and where $\Delta = (\det M(\mu))^{N_f/4}$ is the determinant of the fermion matrix. In the following we denote expectation values for vanishing $\mu$ as $\langle \cdots \rangle_0 = \langle \cdots \rangle_0$. We define

$$L_n = \left. \frac{d^n \ln \Delta}{d \mu^n} \right|_{\mu=0} = \frac{N_f}{4} \left. \frac{d^n \ln \det M}{d \mu^n} \right|_{\mu=0},$$

(A2)
The $L_n$ can be written as traces over the inverse of the fermion matrix and its derivatives

$$L_1 = \frac{N_f}{4} \text{Tr} \left( M^{-1} \frac{\partial M}{\partial \mu} \right) \bigg|_{\mu=0}$$

(A3)

$$L_2 = \frac{N_f}{4} \text{Tr} \left( M^{-1} \frac{\partial^2 M}{\partial \mu^2} - M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right) \bigg|_{\mu=0}$$

(A4)

$$L_3 = \frac{N_f}{4} \text{Tr} \left( M^{-1} \frac{\partial^3 M}{\partial \mu^3} - 3M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} + 2M^{-1} \frac{\partial^2 M}{\partial \mu M^{-1} \frac{\partial M}{\partial \mu}} \right) \bigg|_{\mu=0}$$

(A5)

$$L_4 = \frac{N_f}{4} \text{Tr} \left( M^{-1} \frac{\partial^4 M}{\partial \mu^4} - 4M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^3 M}{\partial \mu^3} - 3M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^2 M}{\partial \mu^2} + 12M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^3 M}{\partial \mu^3} \right)$$

$$-6M^{-1} \frac{\partial^2 M}{\partial \mu M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right) \bigg|_{\mu=0}.$$  

(A6)

$$L_5 = \frac{N_f}{4} \text{Tr} \left( M^{-1} \frac{\partial^5 M}{\partial \mu^5} - 5M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^4 M}{\partial \mu^4} - 10M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^3 M}{\partial \mu^3} + 20M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^3 M}{\partial \mu^3} \right)$$

$$+30M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^2 M}{\partial \mu^2} - 60M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2}$$

$$+24M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \bigg|_{\mu=0}.$$  

(A7)

$$L_6 = \frac{N_f}{4} \text{Tr} \left( M^{-1} \frac{\partial^6 M}{\partial \mu^6} - 6M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^5 M}{\partial \mu^5} - 15M^{-1} \frac{\partial^3 M}{\partial \mu^3} M^{-1} \frac{\partial^4 M}{\partial \mu^4} - 10M^{-1} \frac{\partial^3 M}{\partial \mu^3} M^{-1} \frac{\partial^3 M}{\partial \mu^3} \right)$$

$$+30M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^4 M}{\partial \mu^4} + 60M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^3 M}{\partial \mu^3} + 60M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^3 M}{\partial \mu^3}$$

$$M^{-1} \frac{\partial^3 M}{\partial \mu^3} + 30M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^2 M}{\partial \mu^2} - 120M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2}$$

$$-180M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2} M^{-1} \frac{\partial^2 M}{\partial \mu^2} - 90M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial^2 M}{\partial \mu^2}$$

$$+360M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} M^{-1} \frac{\partial M}{\partial \mu} \right) \bigg|_{\mu=0}.$$  

(A8)

From

$$M^\dagger(\mu) = \gamma_5 M(-\mu) \gamma_5$$

(A9)

it follows that $L_n$ is real for even and imaginary for odd $n$. Using $\Delta = e^{in \Delta}$ we find

$$\Delta(\mu) = \Delta(0) \left( 1 + D_1 \mu + D_2 \mu^2 + \cdots + D_6 \mu^6 + O(\mu^7) \right),$$

(A10)

where

$$D_1 = L_1,$$

(A11a)

$$D_2 = \frac{1}{2} (L_1^2 + L_2),$$

(A11b)

$$D_3 = \frac{1}{6} (L_1^3 + 3L_1 L_2 + L_3),$$

(A11c)

$$D_4 = \frac{1}{24} (L_1^4 + 6L_1^2 L_2 + 3L_2^2 + 4L_1 L_3 + L_4),$$

(A11d)

$$D_5 = \frac{1}{120} (L_1^5 + 10L_1^3 L_2 + 15L_1^2 L_3 + 10L_2 L_3 + 5L_1 L_4 + L_5),$$

(A11e)

$$D_6 = \frac{1}{720} (L_1^6 + 15L_1^4 L_2 + 45L_1^2 L_3^2 + 15L_2^2 + 20L_1^3 L_3 + 60L_1 L_2 L_3$$

$$+ 10L_3^2 + 15L_1^5 L_4 + 15L_2 L_4 + 6L_1 L_5 + L_6).$$

(A11f)
We immediately see that $D_n$ is real for even and imaginary for odd $n$. Because

$$Z_\mu = \langle 1 + D_1 \mu + \cdots D_6 \mu^6 \rangle + O(\mu^7) \tag{A12}$$

is real one has $\langle D_n \rangle = 0$ for odd $n$. We consider the case where the observable $O$ is independent of $\mu$. The expectation value (A1) then becomes

$$\langle O \rangle_\mu = \frac{\langle O \rangle + \langle OD_1 \rangle \mu + \cdots + \langle OD_6 \rangle \mu^6}{1 + \langle D_2 \rangle \mu^2 + \cdots + \langle D_6 \rangle \mu^6} + O(\mu^7). \tag{A13}$$

Expanding in powers of $\mu$ we get

$$\langle O \rangle_\mu = \langle O \rangle (1 + O_1 \mu + (-D_2 + O_2) \mu^2 + (-D_2 O_1 + O_3) \mu^3 + (D_2^2 - D_4 - D_2 O_2 + O_4) \mu^4 + (D_2^2 O_1 - D_4 O_1 - D_2 O_3 + O_5) \mu^5 + (-D_2^3 + 2D_2 D_4 - D_6 + D_2^2 O_2 - D_4 O_2 - D_2 O_4 + O_6) \mu^6) + O(\mu^7) \tag{A14}$$

where we use the notation

$$O_i = \frac{\langle OD_i \rangle}{\langle O \rangle}, \tag{A15a}$$

$$D_i = \langle D_i \rangle. \tag{A15b}$$

In the case that $O$ is strictly real on every configuration $OD_n$ is imaginary for odd $n$. In order to keep $\langle O \rangle_\mu$ real $\langle OD_n \rangle$ has to vanish for odd $n$ and the preceding expansion simplifies to

$$\langle O \rangle_\mu = \langle O \rangle (1 + (-D_2 + O_2) \mu^2 + (D_2^2 - D_4 - D_2 O_2 + O_4) \mu^4 + (-D_2^3 + 2D_2 D_4 - D_6 + D_2^2 O_2 - D_4 O_2 - D_2 O_4 + O_6) \mu^6) + O(\mu^8), \tag{A16}$$

i.e. this formula is applicable to the correlation function in eq. 3.

Because free energies are calculated from logarithms of correlation functions we give here the expansion coefficients of the logarithm of an observable $O$ which can be obtained by inserting the above expansion into the expansion of the logarithm. For a generic, not necessarily real observable the expansion is

$$\ln \langle O \rangle_\mu = \ln \langle O \rangle + O_1 \mu + \left( -D_2 - \frac{1}{2} O_2^2 + O_2 \right) \mu^2 + \left( \frac{1}{3} O_1^3 - O_1 O_2 + O_3 \right) \mu^3 + \left( \frac{1}{2} D_2^2 - D_4 - \frac{1}{4} O_1^3 + O_1 O_2 - O_2 O_3 - O_3 \right) \mu^4 + \left( \frac{1}{5} O_1^5 - O_1^3 O_2 + O_1 O_2^2 + O_2^3 O_3 - O_2 O_3 \right) \mu^5 + \left( -\frac{1}{3} D_2^3 + D_2 D_4 - D_6 - \frac{1}{6} O_1^6 + O_1^4 O_2 - \frac{3}{2} O_1^2 O_2^2 + \frac{1}{3} O_3^3 - O_3 O_3 \right) \mu^6 + O(\mu^7). \tag{A17}$$

For real observables this reduces to

$$\ln \langle O \rangle_\mu = \ln \langle O \rangle + (-D_2 + O_2) \mu^2 + \left( \frac{1}{2} D_2^2 - D_4 - \frac{1}{2} O_2^2 + O_4 \right) \mu^4 + \left( -\frac{1}{3} D_2^2 + D_2 D_4 - D_6 + \frac{1}{3} O_2^3 - O_2 O_4 + O_6 \right) \mu^6 + O(\mu^8). \tag{A18}$$

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