Influence of multi-valued diagnostic signals on optimal sensor placement

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Abstract. In this paper the comparison of results of optimal sensor placement with the use of binary and multi-valued diagnostic signals is given. The exoneration assumption was introduced and its effects were discussed. The influence of multi-valued diagnostic signals on several fault isolability metrics were discussed. The optimal sensor placement problem under budgetary constraints is formulated. A branch-and-bound algorithm solving this problem is described. It is used in context of three tanks system with 25 possible diagnostic signals.

1. Introduction
The fault isolability is one of the elementary features characterizing the quality of a diagnosis. Therefore, it is used for a comparison of effectiveness of different methods of Fault Detection and Isolation (FDI).

The performance of a FDI system for a given industrial process is strongly dependent on available measurements. A FDI system performance figure can often be improved by installing additional sensors providing additional information about a process. However, it is vital to achieve the best possible FDI system performance without additional costs. The problem of optimal sensor selection can be understood as a combinatorial problem of selecting the optimal set of measurements.

In recent years, numerous papers discussed different problems of the optimal sensor placement. They usually consider the required minimum fault isolability of the diagnostic system [1, 2]. Some of proposed methods also maximize designed fault isolability using heuristic methods, e.g. genetic algorithms [3].

The model-based FDI considers faults as deviations from nominal values of process parameters or as unknown process inputs. Faults are detected when system model and measurements behave differently. In [4], a method for searching for the optimal sensor set based on Analytical Redundancy Relations (ARRs) is proposed. First, all ARRs are found under the assumption that all sensor candidates are installed. Then, a sensor set is selected that minimizes the cost while satisfying detectability and isolability requirements. However, this solution is computationally expensive. A modified, incremental approach, using Minimal Structurally Overdetermined (MSO) sets, was proposed in [5]. In [6] the Binary Integer Programming is used to find the optimal sensor set using the set of all possible MSO sets. FDI requirements were ensured by means of non-linear constraints. The resulting problem is computationally difficult to solve. This method was further improved in [7] and [8]. There, FDI requirements were specified as linear constraints. As the cost function is linear, the problem falls into Binary Integer...
Linear Programming (BILP). It can be efficiently solved with branch-and-bound algorithm with standard Linear Programming (LP) solver. Those methods were thoroughly compared in [9]. Budgetary constraints were analyzed in [10]. Branch-and-bound algorithm is used to obtain the optimal solution. Regardless of chosen method, simple, qualitative methods of analysis of fault isolability are insufficient. Generalized, quantitative method of fault isolability analysis is required.

This paper presents the analysis of different approaches to the optimal sensor placement problem with multi-valued diagnostic signals. The main contribution of this paper is assessment of the influence of multi-valued diagnostic signals in optimal sensor placement problem. Both resulting sensor sets and computational complexity of optimization procedure were taken into account. Application of multi-valued signals in fault diagnosis often allows to obtain more accurate diagnosis. Nevertheless, designing such FDI system is usually more time consuming and therefore expensive. An efficient methodology of designing multi-valued FDI systems is required. Different metrics of isolability were used as a performance indices in order to analyze their effect on the optimization results. The model of three tank system was used as a benchmark.

The paper is organized as follows. In Section 2, the preliminary definitions used in this work are given. Section 3 defines different metrics of fault isolability. Section 4 presents the proposed optimization procedure. Section 5 describes the example of a three tank system. In Section 6 the achieved results are presented and discussed. Conclusions and final remarks section finalizes this paper.

2. Preliminaries
A signal sensitive to faults is considered as a diagnostic signal in FDI. A symptom is a value of a diagnostic signal which indicate fault or faults. In the case of multi-valued diagnostic signals, one fault type may generate different values of each diagnostic signal. A fault signature is a vector of diagnostic signal values associated with a particular fault [11]. The specific vector of values of diagnostic signals is called an alternative signature [11]. In case of binary diagnostic signals each fault has exclusively one alternative signature, while in case of multi-valued diagnostic signals there might be multiple alternative signatures.

The Binary Diagnostic Matrix (BDM) or Incidence Matrix is a widely used form of notation of relation between binary diagnostic signals and faults. It has a form of a column matrix. Each fault signature constitutes one column. Each row displays sensitivity of a given diagnostic signal to each individual fault. The analogous form of notation in the case of multi-valued diagnostic signals is Fault Information System (FIS) defined in [12].

There are different definitions of fault isolability. Generally, faults are considered isolable when at least some of their signatures are different [13]. In the case of multi-valued diagnostic signals, it is possible that a pair of faults is conditionally isolable i.e. isolable for some alternative signatures and unisolable for others.

Commonly, in FDI an exoneration assumption is accepted. It states that a lack of a symptoms exonerates a fault. It means that all symptoms must appear for a fault isolation. This assumption is not always valid. Due to dynamics of symptoms and different sensitivity to faults, they may not appear simultaneously or may even not appear at all.

3. Metrics of fault isolability
There are known quantitative methods of determining fault isolability for pairs of faults. In the simplest case, Hamming distance between binary fault signatures can be used. Examples of other, more advanced metrics, using an internal form of residuals, can be found in [14] and [15]. Unfortunately, those measures can be only used when analyzing pairs of faults. To analyze a whole diagnostic system other methods are required.
Let us consider a simple example of a BDM given in the Tab. 1. It consists of three binary diagnosis signals detecting four possible faults. This example will be further used in this paper.

3.1. Diagnosability degree
One of the most commonly used measures of fault isolability is diagnosability degree [1, 2, 3, 16, 17]. It was defined in [17]. The value of this measure is calculated in two steps.

(1) The set of all considered faults is divided into disjoint subsets of unisolable faults. Those sets are called D-classes.

(2) The number of D-classes denoted as $D_c$, is divided by the number of all considered faults. The acquired ratio is called diagnosability degree:

$$
D_c \frac{\text{card}(F)}{\text{card}(D)}.
$$

In the special case when all fault are isolable, the number of D-classes is equal to the number of all considered faults. Diagnosability degree is then equal to 1.

If all faults are unisolable then diagnosability degree is equal to $\frac{1}{\text{card}(F)}$.

Unfortunately, it has some disadvantages. Fault diagnosability for each pair of faults is defined only for binary diagnostic matrices and does not apply to multi-valued diagnostic matrices, such as Fault Information System FIS [18].

Considering the example given in the Tab. 1, there are three D-classes of unisolable faults: $\{f_1\}$, $\{f_2, f_3\}$, $\{f_4\}$. Therefore diagnosability degree is equal to $3/4$.

3.2. Diagnosis accuracy
In [19], a simple fault isolability measure was defined. It is called a diagnosis accuracy. It is calculated as the reciprocal of an average number of faults in a diagnosis.

$$
\left[\frac{\sum_{d_i \in D} \text{card}(d_i)}{\text{card}(D)}\right]^{-1},
$$

where $d_i$ denotes the $i^{th}$ diagnosis from the set of all diagnoses $D$. Only single faults will be analyzed in this work.

When all diagnoses point out only single faults, then value of diagnosis accuracy is equal to 1. In general case value of diagnosis accuracy is in $(0, 1)$.

The value of this measure depends on the chosen diagnosis approach. For example, in the case of binary diagnostic matrix, diagnosis accuracy delivers different results depending whether the exoneration assumption is used or not. This results from ignoring the part of information regarding lack of some symptoms. Nevertheless, in some cases such approach is justified, because

| $s_1$ | $s_2$ | $s_3$ |
|------|------|------|
| 1    | 1    | 1    |
| 1    | 1    | 1    |
| 1    |      |      |

**Table 1.** An example of Binary Diagnostic Matrix.
after occurrence of a fault, symptoms do not appear simultaneously. Different interim diagnoses are considered with appearance of successive symptoms.

Average number of faults in a diagnosis can be difficult to calculate in case of complex diagnostic systems with multi-valued or continuous diagnostic signals or taking into account sequences of appearance of symptoms. In this case the number of possible diagnoses, grows exponentially with a number of faults.

The value of diagnosis accuracy depends on the exoneration assumption. Returning to the example given in the Tab. 1, with exoneration assumption there are three possible diagnoses: \( \{f_1\}, \{f_2, f_3\}, \{f_4\} \). Average number of faults in a diagnosis equals 4/3 and diagnosis accuracy is equal to 3/4. In case of binary diagnostic signals, under exoneration assumption, the value of diagnosis accuracy is always equal to diagnosability degree, because possible diagnoses are identical with D-classes.

Without exoneration assumption there are also possible three diagnoses: \( \{f_1, f_2, f_3\}, \{f_2, f_3\}, \{f_4\} \). The diagnosis accuracy is then equal to \( [6/3]^{-1} = 1/2 \).

3.3. Isolability index

Isolability index is often used, as the fault isolability measure [20, 10]. It is defined as a number of ordered pairs of isolable faults. Maximum number of isolability index depends on a number of considered faults. If isolability relation is asymmetric, then maximal isolability index is equal to \( n(n-1) \), where \( n \) is the number of considered faults. Similarly to diagnosability degree, the isolability index is defined for binary diagnostic signal.

Following the example given in the Tab. 1, with exoneration assumption there are ten ordered pairs of faults if the first fault is isolable from the second: \( (f_1, f_2), (f_1, f_3), (f_1, f_4), (f_2, f_1), (f_2, f_4), (f_3, f_1), (f_3, f_4), (f_4, f_1), (f_4, f_2), (f_4, f_3) \). It is worth noticing that with exoneration assumption this relation is symmetric.

Without exoneration assumption there are eight such pairs: \( (f_1, f_2), (f_1, f_3), (f_1, f_4), (f_2, f_4), (f_3, f_1), (f_4, f_1), (f_4, f_2), (f_4, f_3) \).

3.4. Isolability measure

In [21] a new measure of fault isolability for multi-valued diagnostic systems was proposed. It is calculated in two steps.

1. Calculate for each ordered pair of faults:

\[
D(f_i, f_j) = \frac{\text{card}(S_{i,j})}{\text{card}(S_i)}
\]  \( (3) \)

where \( S_i \) is the set of all possible alternative signatures of \( f_i \), and \( S_{i,j} \) is the set of alternative signatures of \( f_i \) excluding \( f_j \). If faults are unconditionally isolable [18] then \( D(f_i, f_j) = 1 \), if they are unconditionally unisolable then \( D(f_i, f_j) = 0 \), otherwise \( D(f_i, f_j) \in (0, 1) \).

2. Calculate the measure as:

\[
\frac{1}{(n-1)n} \sum_{i=1}^{n} \sum_{j=1 \atop j \neq i}^{n} D(f_i, f_j).
\]  \( (4) \)

Value of this measure has a practical interpretation. It is an average fraction of all single faults that are excluded during each diagnosis. There are \( n+1 \) possible diagnoses: a faultless state and \( n \) single fault states.

This approach for calculation of isolability measure can be used without modifications for any bi-valued form of notation of diagnostic relation, such as directions in residual space or a sequences of symptoms.
Table 2. Values of $D(f_i, f_j)$.

|   | $f_1$ | $f_2$ | $f_3$ | $f_4$ |
|---|------|------|------|------|
| $f_1$ | 0    | 0    | 0    | 1    |
| $f_2$ | 1    | 0    | 0    | 1    |
| $f_3$ | 1    | 0    | 0    | 1    |
| $f_4$ | 1    | 1    | 1    | 0    |

The isolability measure can be calculated for the example given in the Tab. 1. The values of $D(f_i, f_j)$ are presented in the Tab. 2. Using those values we can calculate isolability measure as

$$\frac{1}{(n-1)n} \sum_{i=1}^{n} \sum_{j=1}^{n} D(f_i, f_j) = 2/3.$$  

4. Optimization Problem Formulation

The optimal sensor set is determined using following conditions.

(1) A solution that allow all faults to be detectable is preferred i.e. for each fault there is at least one diagnostic signal sensitive to this fault.

(2) If multiple solutions offer the same detectability, the solution with higher isolability metric is chosen.

(3) If there are multiple solutions with the same detectability and isolability metric, the cheapest one is selected.

The optimization problem described in this section is non-linear and computationally difficult to solve. There are generalized techniques that will allow to find a solution. The optimization problem was solved with the branch-and-bound algorithm [10]. For each sensor there was defined a discrete decision variable with three possible values: positive, negative and unknown. In the beginning, all decision variables are set to undecided.

If a current solution is infeasible, i.e. the cost of chosen sensors exceeds available budget, the solution is rejected. Current best fully decided solution is retained by algorithm. The upper bound of currently analyzed solution is compared to current best solution according to criteria given above. If the achieved result is worse, then analyzed solution can be rejected, because it will not lead to solution better than current best. This operation is called bounding.

During each step of the algorithm, after bounding, one undecided variable is chosen and two new intermediate solutions are created, one with positive value of this variable and the other with negative. This operation is called branching. Those solutions are then added to solution queue.

5. Three Tank System example

The proposed sensor placement optimization approach was tested on the example of a FIS for a system of three tanks arranged in series (TTS) as in Fig. 1. In total 16 faults were considered (Tab. 3). Seven proposed new sensor locations with cost estimation are presented in (Tab. 4). The cost of $CV_v$ is 0, because this signal is already available in the diagnosed system.

By using method presented in [23], the 25 possible diagnostic signals were generated, using sensors depicted in the Tab. 4. Tri-valued diagnostic signals were considered. The method proposed in this paper was tested for all budget values lower or equal to the total cost of all proposed sensors.
Table 3. Considered faults.

| Fault | Description               |
|-------|---------------------------|
| $f_1$ | measurement chain $FI102$ fault |
| $f_2$ | measurement chain $LI103$ fault |
| $f_3$ | measurement chain $LI104$ fault |
| $f_4$ | measurement chain $LIC105$ fault |
| $f_5$ | measurement chain $X1101$ fault |
| $f_6$ | control signal fault      |
| $f_7$ | valve actuator fault       |
| $f_8$ | valve fault                |
| $f_9$ | pump fault                 |
| $f_{10}$ | low water level fault     |
| $f_{11}$ | clogging between tanks $T1$ and $T2$ |
| $f_{12}$ | clogging between tanks $T2$ and $T3$ |
| $f_{13}$ | clogged output flow from tank $T3$ |
| $f_{14}$ | leak from tank $T1$        |
| $f_{15}$ | leak from tank $T2$        |
| $f_{16}$ | leak from tank $T3$        |

Table 4. Sensor locations and costs.

| Symbol | Measurement | Cost |
|--------|-------------|------|
| $F_1$  | Input flow  | 5    |
| $P_v$  | Valve position | 2  |
| $L_1$  | Level in $T1$  | 1    |
| $L_2$  | Level in $T2$  | 1    |
| $L_3$  | Level in $T3$  | 1    |
| $CV_v$ | Control signal | 0  |
| $p_{zp}$ | Pressure on pump inlet | 1 |
| $n$    | Pump rotational speed | 2  |

Figure 1. Three tank system [22].

6. Results
Two main cases were analyzed. In the first one, the BDM obtained from Tab. 5 was used. In the second case, three-valued diagnostic signals were used as presented in Tab. 5. In each case, the optimization process was repeated for each appropriate isolability metric and for each reasonable budget value. The optimization results from the first case are presented in the Tab. 7. Similarly, the results from the second case are presented in the Tab. 6. For each combination of metrics and available budgets the optimal sensor set is presented with the corresponding metric value.

The achieved results are very close despite the type of the isolability metrics. In each case, if a budget is greater than 7, the results are identical for all considered metrics. This is due to the fact that the sensor set $\{CV_v, L_1, L_2, L_3, P_v, n, p_{zp}\}$ is the cheapest set providing detectability of all considered faults and detectability had highest priority in the optimization procedure. With
Table 5. Diagnostic signals and the expected directions of its change caused by faults.

| Signals (sensor sets) | $f_1$ | $f_2$ | $f_3$ | $f_4$ | $f_5$ | $f_6$ | $f_7$ | $f_8$ | $f_9$ | $f_{10}$ | $f_{11}$ | $f_{12}$ | $f_{13}$ | $f_{14}$ | $f_{15}$ | $f_{16}$ |
|-----------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|---------|----------|----------|----------|----------|----------|----------|
| $s_1(F_1, CV_v, p_{zp}, n)$ | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     |         |          |          |          |          |          |          |
| $s_2(F_1, P_v, p_{zp}, n)$ | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_3(P_v, CV_v)$           | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_4(L_1, L_3)$            | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_5(L_1, L_3)$            | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_6(L_3, CV_v, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_7(L_2, L_3)$            | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_8(L_1, L_3)$            | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_9(F_1, L_2)$            | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{10}(P_v, L_2, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{11}(L_2, CV_v, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{12}(F_1, L_2, L_3)$    | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{13}(L_1, L_2, L_3)$    | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{14}(L_2, L_3, CV_v, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{15}(P_v, L_2, L_3, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{16}(L_1, L_2)$         | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{17}(F_1, L_1)$         | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{18}(F_1, L_1, L_2)$    | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{19}(P_v, L_1, L_2, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{20}(L_1, CV_v, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{21}(P_v, L_1, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{22}(F_1, L_1, L_3)$    | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{23}(L_1, L_3, CV_v, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{24}(P_v, L_1, L_3, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |
| $s_{25}(L_1, L_2, CV_v, p_{zp}, n)$| ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±     | ±       | ±        | ±        | ±        | ±        | ±        | ±        |

the available budget equal 13 all proposed sensors are selected in each case.

There are 4 differences in optimization results obtained form multi-valued and binary
diagnostic signals. They were marked bold in the Tab. 6. The sensor sets from the Tab.
7 that are different than those from the Tab. 6 were analyzed. Their corresponding values
of isolability measures obtained with FIS were calculated. Diagnosis accuracy with exoneration
for the set \(\{CV_v, P_v\}\) equals 0.13. Diagnosis accuracy without exoneration for the same set is
0.11. Both values are identical as in the case of BDM because the signal \(s_3\) does not provide new
isolability options in multi-valued case. The set \(\{CV_v, L_2, L_3, P_v, n, p_{zp}\}\) with diagnosis
accuracy without exoneration gives value 0.32. Finally, isolability measure of the set \(\{CV_v, L_1, L_2, L_3, P_v\}\)
is 0.69.

The results obtained with BDM are close to optimal results obtained with FIS. This
shows that result of binary optimization can be chosen as reasonable initial guess for
optimization algorithm used for multi-valued diagnostic signals. It is especially important
because computational complexity of analyzed metrics of isolability is directly proportional to
an average number of alternative fault signatures. In analyzed TTS example, the optimization
procedure for binary diagnostic signals is on average 2.1 times faster than for multi-valued ones.

It is also worth noticing that in all the cases the value of isolability measure obtained for FIS
is greater than for BDM. Similarly, greater value of isolability is obtained where exoneration
Table 6. Results of optimization with various measures of isolability for the set of multi-valued diagnostic signals.

| Available budget | 2: from 3 to 4: | 5: | 6: | 7: | from 8 to 12: | 13: |
|------------------|----------------|-----|-----|-----|---------------|-----|
| Diag. acc. exoner. | 0.17: L₁ L₂ L₃ | 0.56: CV₁ L₁ L₂ L₃ | 0.63: CV₁ L₁ L₂ L₃ | 0.67: CV₁ L₁ L₂ L₃ | 0.67: CV₁ L₁ L₂ L₃ | 0.75: CV₁ L₁ L₂ L₃ | 0.81: CV₁ L₁ L₂ L₃ |
| Diag. acc. no exoner. | 0.17: L₁ L₂ L₃ | 0.31: CV₁ L₁ L₂ L₃ | 0.31: CV₁ L₁ L₂ L₃ | 0.33: CV₁ L₁ L₂ L₃ | 0.34: CV₁ L₁ L₂ L₃ | 0.54: CV₁ L₁ L₂ L₃ | 0.75: CV₁ L₁ L₂ L₃ |
| Isol. measure | 0.33: L₁ L₂ L₃ | 0.53: CV₁ L₁ L₂ | 0.73: CV₁ L₁ L₂ | 0.83: CV₁ L₁ L₂ | 0.85: CV₁ L₁ L₂ | 0.93: CV₁ L₁ L₂ | 0.97: CV₁ L₁ L₂ |

is taken into consideration. The optimization results for budget value 7 with exoneration assumption are in all cases identical to those with available budget equal 6, whereas without that assumption they are different. This implies that smaller differences between solution are taken into consideration without adoption of exoneration assumption.

7. Conclusion
In this work the influence of multi-valued diagnostic signals on optimal sensor placement problem was discussed on an example. The results for both types of diagnostic signal were similar in most cases. Therefore, using results from optimal sensor placement is a good initial guess for optimal sensor placement with multi-valued diagnostic signals. It would allow to early ignore groups of possible solutions using branch-and-bound algorithm and result in significant decrease in time needed to solve optimization problem. Following algorithm can be used to efficiently solve optimal sensor placement problem for FIS:

1. Transform FIS into BDM by replacing nonzero values with 1.
2. Solve the optimal sensor placement problem for BDM.
3. Solve the original problem using the solution from step (2) as a initial guess.

The exoneration assumption affects results of optimal sensor placement. When designing diagnostic system it is important to analyze if this assumption can be met. If the expected time between occurrence of the first and last symptoms is much greater than the time needed to formulate diagnosis, then assumption of exoneration may generate false diagnoses.

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Table 7. Results of optimization with various measures of isolability for the set of binary diagnostic signals.

| 2: | 5: | 6: | 7: | 13: |
|---|---|---|---|---|
| Diagnosis accuracy with exoneration: | | | | |
| 0.13: | 0.50: | 0.56: | 0.63: | 0.63: | 0.69: | 0.75: |
| CV_e P_e L_1 L_2 L_3 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 |
| L_3 P_e L_3 n_p | L_3 n_p | L_3 n_p | L_3 n_p | L_3 P_e n_p | L_3 P_e n_p |
| Diagnosis accuracy without exoneration: | | | | |
| 0.11: | 0.205: | 0.214: | 0.23: | 0.24: | 0.39: | 0.55: |
| CV_e P_e L_1 L_2 L_3 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 |
| L_3 P_e L_3 n_p | L_3 n_p | L_3 n_p | L_3 n_p | L_3 P_e n_p | L_3 P_e n_p |
| Isolability Measure: | | | | |
| 0.26: | 0.48: | 0.64: | 0.73: | 0.75: | 0.87: | 0.93: |
| L_1 L_2 L_1 L_2 L_3 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 |
| L_3 P_e L_3 n_p | L_3 n_p | L_3 n_p | L_3 n_p | L_3 P_e n_p | L_3 P_e n_p |
| Diagnosability degree: | | | | |
| 0.13: | 0.50: | 0.56: | 0.63: | 0.63: | 0.69: | 0.75: |
| CV_e P_e L_1 L_2 L_3 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 |
| L_3 P_e L_3 n_p | L_3 n_p | L_3 n_p | L_3 n_p | L_3 P_e n_p | L_3 P_e n_p |
| Isolability Index with exoneration: | | | | |
| 126: | 194: | 218: | 218: | 224: | 230: |
| L_1 L_2 L_1 L_2 L_3 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 |
| L_3 P_e L_3 n_p | L_3 n_p | L_3 n_p | L_3 n_p | L_3 P_e n_p | L_3 P_e n_p |
| Isolability Index without exoneration: | | | | |
| 63: | 115: | 154: | 175: | 179: | 208: | 223: |
| L_1 L_2 L_1 L_2 L_3 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 | CV_e L_1 L_2 |
| L_3 P_e L_3 n_p | L_3 n_p | L_3 n_p | L_3 n_p | L_3 P_e n_p | L_3 P_e n_p |

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