Asymmetric Velocity Distributions from Halo Density Profiles in the Eddington Approach

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ABSTRACT

In the present paper we show how obtain the energy distribution \( f(E) \) in our vicinity starting from WIMP density profiles in a self consistent way by employing the Eddington approach and adding reasonable angular momentum dependent terms in the expression of the energy. We then show how we can obtain the velocity dispersions and the asymmetry parameter \( \beta \) in terms of the parameters describing the angular momentum dependence. From this expression for \( f(E) \) we proceed to construct an axially symmetric WIMP velocity distributions, which for a gravitationally bound system automatically has an velocity upper bound and is characterized by the the same asymmetries. This approach is tested and clarified by constructing analytic expressions in a simple model, with adequate structure. We then show how such velocity distributions can be used in determining the event rates, including modulation, both in the standard as well directional WIMP searches. find that some density profiles lead to approximate Maxwell-Boltzmann distributions, which are automatically defined in a finite domain, i.e. the escape velocity need not be put by hand. The role of such distributions in obtaining the direct WIMP detection rates, including the modulation, is studied in some detail and, in particular, the role of the asymmetry is explored.

Keywords
dark matter, galaxies: halos

1. Introduction

The combined MAXIMA-1 \cite{Hanary:2000}, Wu and et al. \cite{Wu:2001}, Santos and et al. \cite{Santos:2002}, BOMBERG \cite{Mauskopf:2002}, Mosi and et al. \cite{Mosi:2002}, DASI \cite{Halverson:2002}.
(2002) and COBE/DMR Cosmic Microwave Background (CMB) observations Smoot & et al (COBE Collaboration) imply that the Universe is flat Jaffe & et al (2001) and that most of the matter in the Universe is Dark Spergel & et al (2003), i.e. exotic. These results have been confirmed and improved by the recent WMAP Spergel et al. (2007) and Planck Ade et al. (2013) data. Combining the data of these quite precise measurements one finds:

\[ \Omega_b = 0.0456 \pm 0.0015, \quad \Omega_{CDM} = 0.228 \pm 0.013, \quad \Omega_\Lambda = 0.726 \pm 0.015 \]

(the more recent Planck data yield a slightly different combination \( \Omega_{CDM} = 0.274 \pm 0.020, \quad \Omega_\Lambda = 0.686 \pm 0.020 \). It is worth mentioning that both the WMAP and the Plank observations yield essentially the same value of \( \Omega_m h^2 \), but they differ in the value of \( h \), namely \( h = 0.704 \pm 0.013 \) (WMAP) and \( h = 0.673 \pm 0.012 \) (Planck). Since any “invisible” non exotic component cannot possibly exceed 40% of the above \( \Omega_{CDM} \), exotic (non baryonic) matter is required and there is room for cold dark matter candidates or WIMPs (Weakly Interacting Massive Particles).

Even though there exists firm indirect evidence for a halo of dark matter in galaxies from the observed rotational curves, see e.g. the review Ullio & Kamioknowski (2001), it is essential to directly detect such matter in order to unravel the nature of the constituents of dark matter. At present there exist many such candidates: the LSP (Lightest Supersymmetric Particle) Bottino et al. (1997); Arnowitt & Nath (1995, 1996); Ellis & Roszkowski (1992); Gómez & Vergados (2001); Gómez et al. (2000); Ellis & Flores (1991), technibaryon Nussinov (1992); Gudnason et al. (2006), mirror matter Foot et al. (1991); Foot (2011), Kaluza-Klein models with universal extra dimension Servant & Tait (2003); Oikonomou et al. (2007) etc. This makes it imperative that we invest a maximum effort in attempting to detect dark matter whenever it is possible. Furthermore such a direct detection will also unravel the nature of the constituents of dark matter.

The possibility of such detection, however, depends on the nature of the dark matter constituents (WIMPs). Since the WIMP is expected to be very massive, \( m_\chi \geq 30 GeV \), and
extremely non relativistic with average kinetic energy $T \approx 50KeV(m_\chi/100GeV)$, it can be directly detected mainly via the recoiling of a nucleus (A,Z) in elastic scattering. The event rate for such a process can be computed following a number of steps Vergados (2007). In the present work we will focus on the WIMP density in our vicinity and its velocity distribution.

In the past various velocity distributions have been considered. The one most used is the isothermal Maxwell-Boltzmann velocity distribution with $< v^2 >(= (3/2) v_0^2$ where $v_0$ is the velocity of the sun around the galaxy, i.e. 220 km/s. Extensions of this M-B distribution were also considered, in particular those that were axially symmetric with enhanced dispersion in the galactocentric direction Drukier et al. (1986); Collar & et al. (1992); Vergados (2000). In all such distributions an upper cutoff $v_{esc} = 2.84 v_0$ was introduced by hand, in the range obtained by Kochanek (1996).

In a different approach Tsallis type functions, derived from simulations of dark matter densities were employed, see e.g. recent calculations Vergados et al. (2008) and references there in.

Non isothermal models have also been considered. Among those one should mention the late infall of dark matter into the galaxy, i.e caustic rings Sikivie (1999, 1998); Vergados (2001); Green (2001); Gelmini & Gondolo (2001), dark matter orbiting the Sun Copi et al. (1999), Sagittarius dark matter Green (2002).

The correct approach in our view is to consider the Eddington proposal Eddington (1916), i.e. to obtain both the density and the velocity distribution from a mass distribution, which depends both on the velocity and the gravitational potential. Our motivation in using Eddington (1916) approach to describing the density of dark matter is found, of course, in his success in describing the density of stars in globular clusters. Since this approach adequately describes the distribution of stars in a globular cluster in which the
main interaction is gravitational and because of its generality, we see no reason why such an approach should not be applicable to dark matter that also interacts gravitationally. It seems, therefore, not surprising that this approach has been used by Merritt (1985a) and applied to dark matter by Ullio and Kamionkowski (2001) and by us Owen & Vergados (2003); Vergados & Owen (2007).

It is the purpose of the present paper to extend the previous work obtain a dark matter velocity distribution, which need not be spherically, but they may originate from density profiles that are spherically symmetric. We have constructed a one-parameter family of self-consistent star clusters that are spherically symmetric but anisotropic in velocity space. These were computed modifying the distribution (DF) by including suitable angular momentum factors along the lines suggested by Wojtak et al. (2008) and more recently by Fornasa and Green (2013). Also a one-parameter family of self-consistent star clusters that are spherically symmetric was shown to be anisotropic in velocity space Nguyen & Pedraza (2013) (see also Agsn et al. (2011)). The model was constructed first in the Newtonian limit and then after the first post-Newtonian corrections were computed. To clarify some of the issues involved in this approach, we will concentrate on some cases amenable to analytic solutions like the celebrated Plummer solution (1911). We will show how this method can be used in dark matter searches and leave the case of realistic calculations for a future publication.

2. The Dark Matter Distribution in the Context of the Eddington approach

The introduction the matter distribution can be given as follows

\[ dM = 2\pi f(\Phi(r), v_r, v_t) \, dx \, dy \, dz \, v_t \, dv_t \, dv_r \]  

(1)
where the function $f$ the distribution function, which depends on $\mathbf{r}$ through the potential $\Phi(\mathbf{r})$ and the tangential and radial velocities $v_t$ and $v_r$. We will limit ourselves in spherically symmetric systems. Then the density of matter $\rho(|\mathbf{r}|)$ satisfies the equation:

$$d\rho = 2\pi f(\Phi(|\mathbf{r}|), v_r, v_t) \, v_t \, dv_t \, dv_r$$

(2)

### 2.1. The distribution is a function of the total energy only

The energy is given by $E = \Phi(r) + \frac{v^2}{2}$. Then

$$\rho(r) = 4\pi \int f(\Phi(r) + \frac{v^2}{2}) v^2 dv = 4\pi \int_0^0 f(E) \sqrt{2(E - \Phi)} dE$$

(3)

This is an integral equation of the Abel type. It can be inverted to yield:

$$f(E) = \frac{\sqrt{2}}{4\pi^2} \frac{d}{dE} \int_0^0 \frac{d\Phi}{\sqrt{\Phi - E}} \frac{d\rho}{d\Phi}$$

(4)

The above equation can be rewritten as:

$$f(E) = \frac{1}{2\sqrt{2\pi}} \left[ \int_0^0 \frac{d\Phi}{\sqrt{\Phi - E}} \frac{d^2\rho}{d\Phi^2} - \frac{1}{\sqrt{-E}} \frac{d\rho}{d\Phi}|_{\Phi=0} \right]$$

(5)

In order to proceed it is necessary to know the density as a function of the potential. In practice only in few cases this can be done analytically. This, however, is not a problem, since this function can be given parametrically by the set $(\rho(r), \Phi(r))$ with the position $r$ as a parameter. The potential $\Phi(r)$ for a given density $\rho(r)$ is obtained by solving Poisson’s equation.

Once the function $f(E)$ is known we can obtain the needed velocity distribution $f_{rs}(v)$ in our vicinity ($r = r_s$) by writing:

$$f_{rs}(v') = \mathcal{N} f(\Phi(r)|_{r=r_s} + \frac{v'^2}{2})$$

(6)

where $\mathcal{N}$ is a normalization factor.
2.2. Angular momentum dependent terms

The presence of such terms can introduce asymmetries in the velocity dispersions. In such a reasonable model [Wojtak et al. (2008)] we get:

\[
\rho(r) = \int \int \int f(E) \left( 1 + \frac{L^2}{2L_0^2} \right)^{-\beta_{\infty}+\beta_0} L^{-2\beta_0} d^3v.
\]

i.e. by introducing three new parameters. Introducing the new parameters \( L \) and \( E \) in terms of \( v_t \) and \( v_r \) via:

\[
v_t = \frac{L}{r}, \quad v_r = \sqrt{2(E - \Phi)} - \frac{L^2}{r^2} \quad \text{or} \quad v_t = \frac{L_0}{r} \sqrt{2\lambda}, \quad v_r = \sqrt{2\frac{L_0}{r}} \sqrt{x - \lambda}, \quad \lambda = \frac{L^2}{2L_0^2}
\]

we can perform the integration in cylindrical coordinates and get:

\[
\rho(r) = 2^{1/2-\beta_0} L_0^{-2\beta_0} \frac{\pi}{r} \int_0^\Phi f(E) dE \int_0^x \frac{\lambda^{-\beta_0}(\lambda + 1)^{-\beta_{\infty}+\beta_0}}{\sqrt{x - \lambda}} d\lambda
\]

In the above expressions \( x = (r^2/L_0^2)(\Phi - E) \).

Before proceeding further we prefer to write the above formula in terms of dimensionless variables \( \Phi = \Phi_0 \xi, \rho = \rho_0 \eta, E = \Phi_0 \epsilon \) and \( f(E) = \rho_0 \Phi_0^{-3/2} \tilde{f}(\epsilon) \). Thus the last equation becomes:

\[
\eta = 2^{1/2-\beta_0} L_0^{-2\beta_0} \frac{1}{\sqrt{a}} \pi \int_\xi^0 \tilde{f}(\epsilon) d\epsilon \int_0^x \frac{\lambda^{-\beta_0}(\lambda + 1)^{-\beta_{\infty}+\beta_0}}{\sqrt{x - \lambda}} d\lambda
\]

with \( a = \frac{r^2 \Phi_0^2}{L_0^2} \) and \( x = a(\xi - \epsilon) \).

The second integral can be done analytically to yield:

\[
\frac{\sqrt{\pi} x^{1/2-\beta_0} \Gamma(1 - \beta_0)}{\Gamma(\frac{3}{2} - \beta_0)} \ {}_2F_1(1 - \beta_0, -\beta_0 + \beta_{\infty}, 3/2 - \beta_0, -x),
\]

with \( {}_2F_1 \) the usual hypergeometric function. Then Eq. (8) becomes:

\[
\eta = 2^{1/2-\beta_0} L_0^{-2\beta_0} \frac{1}{\sqrt{a}} \pi \frac{\sqrt{\pi} \Gamma(1 - \beta_0)}{\Gamma(\frac{3}{2} - \beta_0)} \int_\xi^0 \tilde{f}(\epsilon) d\epsilon x^{1/2-\beta_0} {}_2F_1(1 - \beta_0, -\beta_0 + \beta_{\infty}, 3/2 - \beta_0, -x)
\]

(10)
In the limit in which $\beta_0 > 0$, $L_0 > \infty$ the last expression is reduced to Eq. (3).

Eq. (7) allows the calculation of moments of the velocity. In particular following the procedure of Wojtak et al. (2008) one finds:

\[ \langle \upsilon_t^2 \rangle = 2 \left( \frac{L_0}{r} \right)^2 (2 - \beta_0) \frac{\int_{\xi}^{0} \tilde{f}(\epsilon)d\epsilon x^{3/2-\beta_0} \, _2F_1(2 - \beta_0, -\beta_0 + \beta_\infty, 5/2 - \beta_0, -x)}{\int_{\xi}^{0} \tilde{f}(\epsilon)d\epsilon x^{1/2-\beta_0} \, _2F_1(1 - \beta_0, -\beta_0 + \beta_\infty, 3/2 - \beta_0, -x)} \tag{11} \]

\[ \langle \upsilon_r^2 \rangle = \left( \frac{L_0}{r} \right)^2 (1 - \beta_0) \frac{\int_{\xi}^{0} \tilde{f}(\epsilon)d\epsilon x^{3/2-\beta_0} \, _2F_1(1 - \beta_0, -\beta_0 + \beta_\infty, 5/2 - \beta_0, -x)}{\int_{\xi}^{0} \tilde{f}(\epsilon)d\epsilon x^{1/2-\beta_0} \, _2F_1(1 - \beta_0, -\beta_0 + \beta_\infty, 3/2 - \beta_0, -x)} \tag{12} \]

The extra factor of 2 in the case of the tangential velocity can be understood, since there exist two such components. The moments of the velocity are, of course, functions of the three parameters of the model. The model clearly can accommodate asymmetries in the velocity dispersion, even if the density is spherically symmetric.

Eq. (8) can be inverted to yield the distribution function $\tilde{f}(\epsilon)$, even though this is technically more complicated than in the standard Eddington approach without the angular momentum factors. Given the function $\tilde{f}(\epsilon)$ we define the quantities:

\[ \Lambda_t = (2 - \beta_0) \int_{\xi}^{0} \tilde{f}(\epsilon)d\epsilon \, _2F_1(2 - \beta_0, -\beta_0 + \beta_\infty, 5/2 - \beta_0, -x), \tag{13} \]

\[ \Lambda_r = (1 - \beta_0) \int_{\xi}^{0} \tilde{f}(\epsilon)d\epsilon \, _2F_1(1 - \beta_0, -\beta_0 + \beta_\infty, 5/2 - \beta_0, -x) \tag{14} \]

Then the asymmetry parameter $\beta$ defined by:

\[ \beta = 1 - \frac{\langle \upsilon_t^2 \rangle}{2 \langle \upsilon_r^2 \rangle}, \tag{15} \]

is given by:

\[ \beta = 1 - \frac{\Lambda_t}{\Lambda_2}. \tag{16} \]
The axially symmetric velocity distribution, with respect to the center of the galaxy, is thus obtained from \( f(E) \) as described in the Appendix below. Clearly for a given matter density profile, both the distribution function \( \tilde{f}(\epsilon) \) as well as the integrals \( \Lambda_t \) and \( \Lambda_r \) are functions of the parameters \( r_s \beta_0 \beta_\infty \) and \( L_0 \). So is the asymmetry parameter \( \beta \). The above equations get simplified in the following cases:

1. In the limit in which \( \beta_0 = 0 \) and \( \beta_\infty = -1 \). Then

\[
\eta = 4\pi \int_\xi^0 \tilde{f}(\epsilon) d\epsilon \sqrt{2(\epsilon - \xi)} \left( 1 + \frac{2}{3} a(\epsilon - \xi) \right), \quad a = \frac{r^2 \Phi_0}{L_0^2}
\]  

\[
< v_t^2 > = \frac{2 L_0^2}{15 r^2} \int_\xi^0 \tilde{f}(\epsilon) d\epsilon \sqrt{\epsilon - \xi} \frac{(5 + 4a(\epsilon - \xi))}{(1 + (2/3)a(\epsilon - \xi))}
\]  

\[
< v_r^2 > = \frac{1}{15 r^2} \int_\xi^0 \tilde{f}(\epsilon) d\epsilon \sqrt{\epsilon - \xi} \frac{(5 + 2a(\epsilon - \xi))}{(1 + (2/3)a(\epsilon - \xi))}
\]  

\[
\beta = 1 - \frac{\int_\xi^0 \tilde{f}(\epsilon) d\epsilon (5 + 4a(\epsilon - \xi))}{\int_\xi^0 \tilde{f}(\epsilon) d\epsilon (5 + 2a(\epsilon - \xi))}
\]  

2. \( \beta_\infty = 1, \beta_0 = 0 \).

In this case:

\[
\frac{1}{\sqrt{a}} \frac{\sqrt{\epsilon - \xi}}{2F_1(1 - \beta_0, -\beta_0 + \beta_\infty, 3/2 - \beta_0, -x)} \rightarrow \frac{1}{\sqrt{a}} \frac{\sinh^{-1}(\sqrt{x})}{\sqrt{1 + x}}
\]

This function is very complicated to handle. Note however that for sufficiently small values of \( a \) one finds that the above expression for \( x = a(\epsilon - \xi) \) is reduced to:

\[
2\sqrt{\epsilon - \xi} \left( 1 - \frac{2}{3} a(\epsilon - \xi) \right)
\]

We thus recover the previous formula with just a change of sign in \( a \). The corresponding expressions for the velocity dispersions become:

\[
\Lambda_t \leftrightarrow 2 \left( \sqrt{x} - \frac{\sinh^{-1}(\sqrt{x})}{\sqrt{1 + x}} \right), \quad \Lambda_r \leftrightarrow 4 \left( -\sqrt{x} + \sqrt{1 + x} \sinh^{-1}(\sqrt{x}) \right)
\]

In the limit of small \( a \) we again recover the previous expressions with \( a \rightarrow -a \).
3. The case of $L >> L_0$.

In this case the integral equation:

$$\eta = \pi \sqrt{2\pi a^{-\beta_\infty}} \frac{\Gamma(1 - \beta_\infty)}{\Gamma(3/2 - \beta_\infty)} \int_0^{\infty} (\epsilon - \xi)^{1/2 - \beta_\infty} \tilde{f}(\epsilon) d\epsilon$$

(21)

can be solved exactly (see Appendix below) to yield:

$$\tilde{f}(\epsilon) = \frac{a^{\beta_\infty}}{\pi^{2\sqrt{2\pi}}} \frac{\Gamma(3/2 - \beta_\infty) \sin(\pi(1/2 - \beta_\infty))}{\Gamma(1 - \beta_\infty)(1/2 - \beta_\infty)}$$

$$\frac{d}{d\epsilon} \int_\epsilon^0 (\xi - \epsilon)^{-1/2 + \beta_\infty} d\eta/\xi d\xi, \quad (22)$$

provided $\eta(0) = 0$. In this case, however, we find that

$$\beta = 1 - \frac{\Lambda_t}{\Lambda_r} = 1 - \frac{\Gamma(2 - \beta_\infty)}{\Gamma(1 - \beta_\infty)} = 1 - \beta_\infty, \quad \beta_\infty < 1$$

regardless of the velocity distribution.

3. Asymmetries in the velocity distribution

Proceeding as above we get the function $f_{(\beta_\infty, \beta_0, L_0)}(E)$. We then proceed to construct a velocity distribution, which is characterized by the same asymmetry in velocity dispersion along lines similar to those previously adopted by Binney & Tremaine (2008), i.e. by considering models of the Osipkov-Merritt type Osipkov (1979); Merritt (1985a,b). Thus the velocity distribution in our vicinity ($r = r_s$) is written as:

$$f_{r_s}(\mathbf{v}) = \mathcal{N}(1 + \alpha_s) f_{0,0,\infty} \left( \Phi(r_s, s) + \frac{v_r'^2}{2} + (1 + \alpha_s) \frac{v_t'^2}{2} \right)$$

(23)

where $v_r'$ and $v_t'$ are the radial, i.e. outwards from the center of the galaxy, and the tangential components of the velocity, with respect to the center of the galaxy. The parameter $\alpha_s = \beta/(1 - \beta)$ can be determined by calculating the moments of the velocity as above, i.e. it is a function of the parameters $L_0, \beta_0$ and $\beta_\infty$. since these parameters are
usually treated as phenomenological parameters, we will treat $\beta$ phenomenologically. We note that this function is only axially symmetric and the normalization constant $N$ is the same as in the case of $\alpha_s = 0$. The isotropic case follows as a special case in the limit $\alpha_s \to 0$.

The characteristic feature of this approach is that the velocity distribution automatically vanishes outside a given region specified by a cut off velocity $v_m$, given by $v_m = \sqrt{2|\Phi(r_s)|}$.

4. A simple test density profile

Before proceeding further we will examine a simple model, amenable to analytic solution, i.e. the famous Plummer solution and leave the case of realistic density profiles, like, e.g., those often employed for a future publication. It is well known that a spherical density distribution of the type

$$\eta = \frac{\rho(x)}{\rho_0} = \frac{1}{(1 + x^2/3)^{5/2}}, \quad x = \frac{r}{a},$$

which is sometimes used as an ordinary matter profile, leads to a potential of the form

$$\xi = \frac{\Phi(x)}{\Phi_0} = -\frac{1}{(1 + x^2/3)^{1/2}}, \quad \Phi_0 = 4\pi G N a^2 \rho_0$$

It is interesting to remark that the Plummer solution naturally arises in a model involving self-consistent star clusters studied the Newtonian limit as well as after the first post-Newtonian corrections were computed.

From these we obtain the desired relation:

$$\eta(\xi) = -\xi^5, \text{ with } \eta'(\xi) = -20\xi^3, \eta(\xi)|_{\xi=0} = 0, \frac{d\eta}{d\xi}|_{\xi=0} = 0$$

Then the solution to Eq. 17 is given by

$$\tilde{f}(x) = \frac{16e^{-ax}}{a^{5/2}\pi x}.$$
\[ e^{ax} \left( \sqrt{a} \sqrt{x} (2ax(2ax - 5) + 15) - 15 \right) + 15 \sqrt{ax} \]

\[ - 15a \sqrt{\pi} \text{erfi} \left( \sqrt{ax} \right), \quad x = -\epsilon \]  

(27)

This leads to a velocity distribution

\[ f_{\xi(xs)}(y) = \tilde{f}(\xi(xs) - y^2/2) \]  

(28)

where \( \xi(xs) \) is the value of the potential in our vicinity. In our simple model \( \xi(xs) \approx \sqrt{3}/2 \). We also used a larger value \( \xi(xs) = 10 \).

1. The choice \( a > 0 \)

The obtained velocity distribution properly normalized is exhibited in Fig. 1. We notice that the dependence on \( a \) is very mild.

We next compute the asymmetry parameter \( \beta = 1 - \Lambda_t/\Lambda_r \) as a function the potential

\[ 4\pi y^2 f_{\xi(xs)}(y) \rightarrow \]  

(a)  

\[ 4\pi y^2 f_{\xi(xs)}(y) \rightarrow \]  

(b)  

\[ y \rightarrow \]  

Fig. 1.—: We show the properly normalized velocity distribution obtained in our simple model for various values of \( a \) for the value \( \xi(xs) = \sqrt{3}/2 \) (a) and a larger, perhaps more realistic, value \( \xi(xs) = 10 \) (b). The obtained velocity distribution depends mildly on \( a \).

\[ \xi \] for various values of \( a \). This is exhibited in Fig. 2. The asymmetry is negative, opposite to what is commonly believed, see e.g. Drukier et al. (1986); Collar & et al.
(1992); Vergados (2000); Evans et al. (2000), Hansen et al. (2006), Vergados et al. (2008), i.e. it does not lead to enhanced dispersion in the galactocentric direction, regardless of the values of $\xi$. Thus the positive values of $a$ are not acceptable, i.e. the choice $\beta_{\infty} = -1$, $\beta_0 = 0$ is not physically acceptable.

\[ \xi \rightarrow \]

Fig. 2.—: The asymmetry parameter $\beta = \Lambda_t/\Lambda_r$ as a function of $\xi$ for values of $a = 0, 0.25, 0.50, 0.75, 1.0, 1.25, 1.50$ increasing downwards.

2. The choice $\beta_{\infty} = 1$, $\beta_0 = 0$.

In this case we will explore the regime of negative absolutely small values of $a$ The velocity distribution obtained is exhibited in Fig. 3.

5. The velocity distribution in WIMP searches

The asymmetric velocity distribution in the galactic frame can be written as:

\[ g(\beta, y') = \frac{1}{1 - \beta} f_{0,0,\infty} \left( \Phi(r_s) + \frac{1}{2} \left( \frac{1}{1 - \beta} (y'^2 - \beta y_r'^2) \right) \right) \] (29)
Fig. 3.—: We show the properly normalized velocity distribution obtained in our simple model for negative values of $a$, i.e. $a = 0, -0.1, -0.2, -0.3, -0.4, -0.5$ for the value $\xi(xs) = \sqrt{3}/2$ (a) and a larger, perhaps more realistic, value $\xi(xs) = 10$ (b). The obtained velocity distribution depends mildly on $a$ in (a) and it is noticeable in (b). In the plots $a$ is increasing from left to right.

This function depends on two variables. In order to compare with the previous results we exhibit in Fig. 6(b) the dependence on the asymmetry of its angular average. The values of $\beta$ employed were related to $a$ as above. We intend, however, to treat $\beta$ as a free parameter.

The results shown here exhibit the same trends as those obtained by using, e.g., Tsallis functions (see Vergados et al. (2008)).

Our next task is to transform the velocity distribution from the galactic to the local frame. The needed equation, see e.g. Vergados (2012), is:

$$y \rightarrow y + \hat{v}_s + \delta \left( \sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{v_s} \right), \quad y = \frac{v}{v_0} \quad (30)$$

with $\gamma \approx \pi/6$, $\hat{v}_s$ a unit vector in the Sun’s direction of motion, $\hat{x}$ a unit vector radially out of the galaxy in our position and $\hat{y} = \hat{v}_s \times \hat{x}$. The last term in the first expression of
Fig. 4.—: The asymmetry parameter $\beta = \Lambda_t/\Lambda_r$ as a function of $\xi$ for values of $a$ the same as in Fig. 3. In the plots $a$ is increasing upwards.

Fig. 5.—: The asymmetry parameter $\beta = \Lambda_t/\Lambda_r$ as a function of $a$ for values of $\xi(r_s) = (1, 2, 4, 6, 8, 10)/2$. In the plots $\xi(r_s)$ is increasing upwards. Note that on the x-axes the opposite of $a$ is indicated.
Eq. (30) corresponds to the motion of the Earth around the Sun with $\delta$ being the ratio of the modulus of the Earth’s velocity around the Sun divided by the Sun’s velocity around the center of the Galaxy, i.e. $v_0 \approx 220\text{km/s}$ and $\delta \approx 0.135$. The above formula assumes that the motion of both the Sun around the Galaxy and of the Earth around the Sun are uniformly circular. The exact orbits are, of course, more complicated [Green (2003)], but such deviations are not expected to significantly modify our results. In Eq. (30) $\alpha$ is the phase of the Earth ($\alpha = 0$ around June 3rd).²

5.1. Standard non directional experiments

We have seen that in the galactic frame in the presence of asymmetry $\beta$ the relevant quantity is:

$$y_x^2 + \frac{1}{1 - \beta} (y_y^2 + y_z^2) = \frac{1}{1 - \beta} (y'^2 - \beta y_x'^2)$$

In the local frame the components $y_x, y_y, y_z$ of the velocity vector $\mathbf{y}$ are thus given by:

$$y_r = y_x = \frac{1}{s_c} (y \cos \phi \sin \theta + \delta \sin \alpha), \quad y_t = \sqrt{y_y^2 + y_z^2},$$
$$y_y = \frac{1}{s_c} (y \sin \theta \sin \phi - \delta \cos \alpha \cos \gamma), \quad y_z = \frac{1}{s_c} (y \cos \theta + \delta \cos \alpha \sin \gamma + 1),$$
$$y = \frac{v}{v_0}$$

where $s_c$ is a suitable scale factor to bring the WIMP velocity into units of the sun’s velocity, $y = v/v_0$, i.e. $s_c = \sqrt{|\Phi_0|/v_0}$. One finds

$$\frac{1}{1 - \beta} (y'^2 - \beta y_x'^2) \rightarrow Y^2 = \frac{1}{s_c^2} \frac{1}{1 - \beta} \left(-\beta (\delta \sin(\alpha) + y \cos(\phi) \sin(\theta))\right)^2 +$$

²One could, of course, make the time dependence of the rates due to the motion of the Earth more explicit by writing $\alpha \approx (6/5)\pi (2(t/T) - 1)$, where $t/T$ is the fraction of the year.
Thus the velocity distribution for the standard (non directional) case becomes:

\[
g_{\text{nodir}}(Y) = \frac{1}{1 - \beta f_{0,0,\infty}} \left( \Phi(r_s) + \frac{1}{2} Y^2 \right)
\]

(33)

5.2. directional experiments

In the Eddington theory the asymmetric velocity distribution is given by:

\[
g_{\text{dir}}(X) = \frac{1}{1 - \beta f_{0,0,\infty}} \left( \Phi(r_s) + \frac{1}{2} X^2 \right)
\]

(34)

where \( f \) is the symmetric normalized velocity distribution with respect to the center of the galaxy, \( \beta \) is the asymmetry parameter and \( X \) is given, Vergados & Moustakidis (2011), by:

\[
X^2 = \frac{1}{(1 - \beta)s_c^2}
\left( \sqrt{3}\delta\cos\alpha\cos\Phi - 2\sqrt{1 - \xi^2}\cos\phi + 2\delta\sin\alpha\sin\Phi \right)^2
-
\beta \left( 2\sqrt{1 - \xi^2}\cos\phi - (\delta\cos\alpha + 2)\sin\Theta \right) +
\delta\cos\Theta \left( 2\cos\Phi\sin\alpha - \sqrt{3}\cos\alpha\sin\Phi \right)^2 +
\left( 2\xi y + (\delta\cos\alpha + 2)\cos\Theta + \delta\sin\Theta \left( 2\cos\Phi\sin\alpha - \sqrt{3}\cos\alpha\sin\Phi \right) \right)^2 +
\left( -2\sqrt{1 - \xi^2}\cos\phi + (\delta\cos\alpha + 2)\sin\Theta \right) +
\delta\cos\Theta \left( \sqrt{3}\cos\alpha\sin\Phi - 2\cos\Phi\sin\alpha \right)^2.
\]

(35)

The direction of the WIMP velocity is specified by \( \xi = \cos\theta \) and \( \phi \). The direction of observation is specified by the angles \( \Theta \) and \( \Phi \).
6. Discussion

In the present work we studied how one can construct the velocity distribution in the Eddington approach starting from dark matter density profiles. By modifying the distribution function by suitable angular momentum functions one can obtain asymmetric velocity distributions as well. We clarified some of the issues involved in this approach by considering a simple model which can yield analytic solutions. Results of realistic calculations for dark matter searches, employing the present technique and using realistic density profiles \cite{Navarro:1996, Ullio:2001, Vergados:2007}, will appear elsewhere \cite{Moustakidis:2014}.

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Fig. 6.—: We show the angular average of the properly normalized velocity distribution for values of the asymmetry parameter $\beta = (0.0, 0.1, 0.2, 0.3, 0.4, 0.5)$. In the plots $\beta$ is increasing from right to left). The results depend on the value of the potential in our vicinity. Here the value of $\xi(xs) = 10$ was adopted.
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