**Thermodynamics of a deeply degenerate SU(N)-symmetric Fermi gas**

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Many-body quantum systems can exhibit a striking degree of symmetry unparalleled in their classical counterparts. In real materials SU(N) symmetry is an idealization, but this symmetry is pristinely realized in fully controllable ultracold alkaline-earth atomic gases. Here, we study an SU(N)-symmetric Fermi liquid of $^{87}$Sr atoms, where $N$ can be tuned to be as large as 10. In the deeply degenerate regime, we show through precise measurements of density fluctuations and expansion dynamics that the large $N$ of spin states under SU(N) symmetry leads to pronounced interaction effects in a system with a nominally negligible interaction parameter. Accounting for these effects, we demonstrate thermometry accurate to 1% of the Fermi energy. We also demonstrate record speed for preparing degenerate Fermi seas enabled by the SU(N)-symmetric interactions, reaching $T/T_f = 0.22$ with 10 nuclear spin states in 0.6 s working with a laser-cooled sample. This, along with the introduction of a new spin polarizing method, enables the operation of a three-dimensional optical lattice clock in the band insulating regime.

**Fast preparation**

Our preparation scheme (Fig. 2a) begins with standard laser-cooling techniques developed for alkaline-earth atoms. After two stages of laser cooling, roughly $10^7$ atoms are cooled to 2 nK in a far-off-resonant crossed optical dipole trap (XODT) with a sheet-like geometry. A vertically oriented round optical dipole trap (VODT) forms a dipole in a horizontal optical sheet potential that is provided by an elliptically shaped horizontal optical dipole.
is well described by a three-body loss process with a three-body loss coefficient $k_3 = 4.7(1.2) \times 10^{-21} \text{cm}^3 \text{s}^{-1}$ (see Methods).

The elastic collision rate\(^7\) for a non-degenerate, balanced spin mixture is proportional to $(1 - 1/N) \frac{N \sigma}{T}$, where $\sigma$ is the spatially averaged density for spin state $\sigma$. Assuming a constant total atom number, it is thus advantageous to have all ten spin states populated. We reach an initial collision rate of $1,000 \text{s}^{-1}$ with $N = 10$. Evaporation begins at a trap depth of $20 \mu \text{K}$ with radial trap frequency $\nu_r$, and vertical trap frequency $\nu_z$ in the dimple of $(\nu_r, \nu_z) = (100, 800) \text{Hz}$. The HODT intensity is then reduced in a two-stage ramp down to a final trap depth of a few $100 \text{nK}$ with trap frequencies of $(100, 200) \text{Hz}$.

After 600 ms of evaporation, we reach $T/T_r = 0.22$ with $3 \times 10^6$ atoms per spin state. Slower evaporation leads to lower temperatures, and we achieve $T/T_r = 0.07$ with $5 \times 10^4$ atoms per spin state after evaporating for 2.4 s. This marks a considerable improvement over previous evaporation results, where evaporation stages took around 10 s (ref. \(^{21}\)), limiting the potential of Fermi-degenerate optical atomic clocks\(^8\).

We observe an approximate $1/(N-1)$ scaling of the total evaporation rate with the number of spin states participating in evaporation as shown in Fig. 2d, reflecting the reduction in collisional partners for smaller $N$. Here, each sample is prepared with the same atom number per spin state and $T/T_r$ and is measured after reaching $T/T_r = 0.12$. The final atom number per spin state changes by roughly a factor of two as $N$ varies.

**Spin manipulation**

To manipulate the spin composition of the atom sample and prepare a spin-polarized gas, we apply a spin-selective optical potential to the atoms after evaporation. Past procedures have used optical Stern–Gerlach techniques to separate out spin states during the time of flight\(^{-16,17}\), but our method, the tensor Stark shift spin selector (TenS4), creates a spin-selective force on the atoms from the tensor Stark shift of a laser while the atoms remain trapped in the XODT. Atoms with the same magnitude of the nuclear spin state $|m_f\rangle$, where $m_f$ represents the projection of the total angular momentum $F$ along the quantization axis, experience a small differential force due to an applied magnetic field of 5 G. This magnetic field is too small to fully remove $m_f = -9/2$ and $-7/2$. We thus conventionally remove these spins via optical pumping before evaporative cooling. The TenS4 beam is offset from the atoms such that the a.c. Stark shift varies across the atomic sample by hundreds of nanokelvin, which causes a spin-dependent modification in the combined optical and gravitational potential of the atoms (Fig. 2e). The TenS4 laser is blue-detuned from the \(^1P_0, F = 11/2\) transition by 266 MHz (Fig. 2b), where the polarizability from \(^3P_0, F = 11/2\) cancels the polarizability from \(^3P_0, F = 9/2\) for nuclear spin state $m_n = +9/2$. As a result, atoms with $m_n = +9/2$ are unaffected by the TenS4 laser, whereas all other spin states are subject to a repelling force. A detailed experimental protocol is provided in the Methods.

The spin purity after applying the TenS4 laser for 10 ms is measured by loading the atoms into a deep 3D optical lattice. The spin population for each nuclear spin state is then read out through selected $\pi$-pulse excitations on the clock transition. We measure 92% of the atoms in the target $m_n = +9/2$ state, as shown in Fig. 2f, and an atom number of $3.3 \times 10^4$ after the TenS4 beam is applied, in rough agreement with one-eighth of the initial atom population of $2.5 \times 10^4$. The temperature of the sample increases by only ~10%. Our technique provides spin-state selectivity without optical excitation and, as a result, does not cause light-induced heating, overcoming issues typically associated with optical pumping schemes. The spin distillation technique enables us to load a single spin with $T/T_r = 0.2$ into a 3D optical lattice with a total preparation time under 3 s (Fig. 2a).

**Characterization of SU(N)-enhanced interactions**

Having prepared a high-density, deeply degenerate SU(10) gas, we demonstrate in the following that a nominally very weakly...
interacting quantum system with interaction parameter $k_a \ll 1$, where $k_a$ is the magnitude of the Fermi wave vector, can develop striking interaction effects due to SU($N$) enhancement. The nuclear spin degree of freedom substantially modifies the character of the gas towards an interacting multicomponent Fermi liquid with subtle consequences for correlation analysis and thermometry.

To investigate this intriguing quantum system and illuminate the role of SU($N$) symmetry, we perform measurements that characterize the system's thermodynamics. A key quantity in this context is the isothermal compressibility $\kappa = \frac{1}{\rho} \frac{\partial \rho}{\partial P}$, where $\rho$ denotes the particle density and $P$ the chemical potential. For $^{87}$Sr with $a_{Bohr} = 97a_{Bohr}$, where $a_{Bohr}$ is the atomic Bohr radius, the interaction contacts are repulsive and a decreased compressibility would be expected compared with an ideal Fermi gas. These density fluctuation measurements are performed on average $\overline{\rho}$ atoms, the corresponding generalized force is the local chemical potential $\mu$. The relative number fluctuations $\eta = \Delta \overline{\rho} / \overline{\rho}$ are related to the susceptibility $\Delta \overline{\rho} / \Delta \mu$ via $\eta = \sqrt{k_B T \kappa}$, where $k_B$ is the Boltzmann constant. Although the equation of state of a classical ideal gas dictates that $\eta = 1$, a value of $\eta = 3/2T/T_F$ would be expected for a deeply degenerate ideal Fermi gas. These excitations fluctuate the degeneracy pressure in the gas. Combined with the compressibility reduction due to repulsive interactions, it would be expected, to first order in temperature and interactions (see Methods), that:

$$\eta = \frac{3}{2} \left( \frac{T}{T_F} \right)^\frac{2}{3} \frac{k_B a}{(N-1)}$$

which suggests that even in $k_a \ll 1$ limit the interaction effects become non-negligible due to the $(N-1)$-fold SU($N$) enhancement.

Our density fluctuation measurements are performed on expanded gas clouds. After abruptly turning off the harmonic confinement ($\nu_i = 130$ Hz, $\nu_s = 240$ Hz) the quantum degenerate sample...
that contains in total $10 \times 59,000$ atoms such that $k_a a = 0.07$ freely expands over 11.5 ms. We then obtain line-of-sight integrated density profiles via absorption imaging. Following the protocol described in refs. 44,45, we run this experiment in a repeated fashion so that for each projected subregion of the cloud containing $N$ atoms we can measure the statistical variance $\Delta N^2$. Figure 3 shows the results obtained from 400 individual images together with a calibration line derived from noise measurements on a thermal gas. Pronounced noise suppression down to about 25% of thermal noise in the centre of the sample indicates that the gas is deeply in the quantum regime.

To quantitatively interpret the noise data beyond first order and decouple Pauli suppression and SU(N)-enhanced interaction contributions, we calculate the expected line-of-sight integrated number fluctuations based on a kinetic approach46,47, using the collisional Boltzmann–Vlasov equation with a mean-field interaction term. The Boltzmann–Vlasov equation describes the evolution of the semi-classical phase-space distribution $f(r, p)$ with position $r$ and momentum $p$:

$$\left( \partial_t + \frac{p}{m} \cdot \nabla_r - \nabla_r [U(r) + V_{MF}(r)] \cdot \nabla_p \right) f = I_c(f).$$

The phase-space distribution evolves due to ballistic motion (second term), forces from the harmonic trapping potential $U = m/2 \sum_i (2\pi \nu_i)^2 r_i^2$, mean-field interactions with strength $V_{MF} = g(N-1)n$ where $g = 4\pi a^2 a/m$, and collisions that are described by the integral $I_c(f)$ (see Methods)48,49. Solving the Boltzmann–Vlasov equation in equilibrium and for finite temperature allows us to obtain the real-space density $n$, from which we can compute the compressibility and thus the number fluctuations in trap and after time of flight. By fitting this model to the observed fluctuations, we extract $T/T_i = 0.16 \pm 0.01$. At these low temperatures, $I_c(f)$ plays no role.

![Fig. 3 | Local density fluctuations of an SU(N) degenerate gas.](image)

Density fluctuations after 11.5 ms time of flight for a degenerate cloud with $N = 10$ nuclear spin states. The data are fitted using an SU(N) interacting model to extract $T/T_i = 0.16$, with shading representing a 2σ uncertainty of ±0.02 $T/T_i$. Fitting the data instead to a non-interacting ideal Fermi gas gives $T/T_i = 0.13$, showing an interaction-induced suppression of ~20%. The difference between the interacting and non-interacting fits is much less than the scatter in the data, highlighting the indistinguishability between interacting and non-interacting systems when measuring density fluctuations alone at the given signal-to-noise ratio. The total density fluctuations are 25% of that of the thermal gas. A thermal cloud density fluctuations alone at the given signal-to-noise ratio. The total contributions, we calculate the expected line-of-sight integrated number fluctuations based on a kinetic approach46,47, using the collisional Boltzmann–Vlasov equation with a mean-field interaction term. The Boltzmann–Vlasov equation describes the evolution of the semi-classical phase-space distribution $f(r, p)$ with position $r$ and momentum $p$:

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![Fig. 4 | Cloud anisotropy.](image)

a-c, Line-of-sight integrated atomic density for $t_{tof} = 0.5$ ms (a), 3.5 ms (b) and 8.5 ms (c) in terms of the number of atoms per $1.37 \mu m^2$. d, Aspect ratio of a cloud of cold atoms with $N = 10$ nuclear spin states released from an XODT for variable expansion times (red circles). Here, $\sigma_v (\sigma_r)$ represents the vertical (horizontal) $1/e^2$ Gaussian width of the cloud. An identical aspect ratio is extracted using a Fermi-Dirac fit. After ~3 ms, the aspect ratio passes through unity (black dashed line), a clear signature of interactions in the gas. At long timescales, the sample approaches an aspect ratio of 1.12 (grey line). The sample has an initial trap asymmetry of $\nu_r/\nu_v = 6.4$. e, Aspect ratio versus initial trap asymmetry of a degenerate gas for $N = 10$ (red circles) and $N = 1$ (blue squares) after time-of-flight expansion for 15.5 ms. f, Aspect ratio versus temperature. Data are shown with roughly the same atom number per shot. The data are fitted using an interesting model that includes both a mean-field interaction and an additional collisional term. Neglecting the collisional term fails to explain the results (grey dashed line). All error bars represent the s.e.m. and are smaller than the data points.
To illustrate the interaction-induced compressibility change, we also fit a non-interacting model to the noise data, which gives an apparent $T/T_r = 0.13 \pm 0.01$, indicating a compressibility reduction of $\sim 20\%$ due to interactions. More precisely, we find that the compressibility in the centre of the trap is reduced by 18% compared with a non-interacting gas at the same density and temperature. This percentage is comparable to the ratio of 21% between the interaction energy in a small volume $V$ at the centre of the cloud, $g/2(N-1)n_F^2 V$, and the total energy of a non-interacting Fermi gas at the same density, $3/5E_F n_F V$. Clearly, without prior knowledge of the interaction parameter, one cannot distinguish between a colder or more repulsively interacting system. Having a full thermodynamic description at hand, as discussed in more detail later, we also perform global profile fits of the acquired images (see Fig. 5) to numerically calculated density distributions and find $T/T_r = 0.17 \pm 0.01$, which is in good agreement with the temperature derived from density noise measurements. We want to emphasize again that even though the two underlying physical mechanisms that lead to the observed suppression of density fluctuations are fundamentally different, it is impossible to distinguish these two contributions by performing density noise measurements alone at the given signal-to-noise ratio.

To unambiguously distinguish between temperature and interaction effects, we study the expansion dynamics of the cloud after being released from the trap. In addition to the kinetic energy of the gas, interactions provide a release energy that is mapped to momentum after a long time of flight. As the mean-field energy preferentially pushes atoms along the direction of the largest density gradient, this conversion produces an anisotropic distribution after a long time of flight (see Fig. 4a–c). The expansion can be described via scaling solutions of the time-dependent Boltzmann–Vlasov equation (see Methods)\textsuperscript{16–22}. Unlike in trap, the effect of interactions on the density after expansion cannot be partly captured by a lower temperature, and result in a non-unitary aspect ratio at long times. This is in stark contrast to the expansion behaviour of an ideal Fermi gas where expansion is purely ballistic and after a long time of flight reflects the isotropic momentum distribution even if the confining potential is anisotropic.

Figure 4d displays the aspect ratio of a SU(10) atom cloud measured after variable expansion times $t_{\text{off}}$ out of a harmonic trap with $\nu_x = 125$ Hz and $\nu_z = 800$ Hz. The sample contains 50,000 atoms per spin component at $T/T_r = 0.16$. Initially, the atom cloud reproduces the trap’s asymmetry. As $t_{\text{off}}$ becomes larger than 1, the spatial density distribution is determined more and more by the momentum distribution in the gas. Observing an inversion of the aspect ratio beyond 1 is an unambiguous signature of the interactions modifying the isotropic momentum distribution during time of flight\textsuperscript{43}.

To further explore this behaviour, we present in Fig. 4e measurements of the expanded cloud aspect ratio ($t_{\text{off}} = 15.5$ ms) as a function of the confinement asymmetry for a ten-component gas and a spin-polarized gas, both at $T/T_r = 0.16$. In the non-interacting case $N = 1$ (blue data points), the aspect ratio is always 1 as expected in the long time-of-flight limit. Finally, we show in Fig. 4f the dependence of the observed cloud aspect ratio on $T/T_r$ for a fixed initial confinement with $\nu_x = 130$ Hz and $\nu_z = 725$ Hz and unchanged $t_{\text{off}} = 15.5$ ms. Atomic interactions include a mean-field term $\propto a$ and a collision integral $I(f)/a^2$; see equation (1). The latter, however, is only relevant in the presence of high collision rates and its pronounced effect is observed when the gas is relatively hot (Fig. 4f). In comparison, the interaction energy and kinetic energy become comparable at low temperatures, and the collisional rate is suppressed by Fermi statistics.

All measurements are well reproduced by our quantitative model (see Methods). To emphasize the role of $N$ in modifying the dynamics, we also plot in Fig. 4d–f the behaviour expected for $N = 1$, 4, 7, 10 spin states using the validated theoretical model. As aspect ratio measurements via Gaussian fits are fairly immune to most imaging artifacts, they can be exploited to perform precise thermometry of the interacting Fermi gas.

Beyond their directly visible manifestation through cloud ellipticity, interaction-modified expansion dynamics can also be identified by carefully inspecting high-signal-to-noise-ratio absorption images. Figure 5 illustrates that a presumably round line-of-sight integrated density profile $n(x, z)$ still contains a systematic interaction signature. After $t_{\text{off}} = 11.5$ ms, we observe density profiles that at first sight appear circularly symmetric (first row) for the experimental data (first column), the non-interacting (second column) and the interacting models (third column). Interactions are revealed in the transpose anisotropy of the density distribution, defined as $n(x, z) - n(z, x)$, shown in the second row. Both the experimental data and the interacting model exhibit pronounced lobes that are not visible in the non-interacting case. Finally, a one-dimensional measure of this anisotropy can be defined by integrating over one of the axes ($\int dx (n(x, z) - n(z, x))$) as shown in the third row. In the integrated transpose anisotropy, we observe peaks symmetric to the centre of the cloud, the heights of which are sensitive to $T/T_r$ for the experimental data and the interacting model, whereas in the non-interacting case this anisotropy is considerably reduced, inverted and insensitive to temperature. Thus, the anisotropy reveals the interacting nature of the Fermi gas in a single absorption image and can serve as a precise temperature probe.
Conclusions
We have demonstrated that SU(N) symmetry substantially enhances interaction dynamics in a quantum degenerate Fermi gas. This allows us to reach ultralow temperatures in short timescales. Creating a spin-polarized degenerate sample with a total preparation time under $3 s$ is important for the realization of atomic clocks probing engineered quantum states of matter. The many-body problem for the dilute repulsively interacting Fermi gas can be solved exactly, and we have shown with high precision how the additional spin degree of freedom systematically modifies thermodynamic properties in the bulk gas. This opens a path to future quantum simulators capable of systematically exploring SU(N)-symmetric Fermi systems in periodic potentials.

Online content
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Density profiles.

Solving the Boltzmann–Vlasov equation in equilibrium, we obtain

\[ f(\mathbf{r}, \mathbf{p}) = \frac{1}{e^{\frac{E(\mathbf{r}, \mathbf{p})}{k_B T}} - 1} \]

which has to be solved self-consistently, as it depends on \( n(\mathbf{r}) \). In practice, we solve \( f(\mathbf{r}, \mathbf{p}) \) iteratively in steps \( i \). We start by ignoring the interaction term, and determine \( n_0 \) from \( f^{(0)}(\mathbf{r}, \mathbf{p}) \), where \( n_0 \) is the number of atoms per spin species that defines \( f^{(0)} \). Having \( f^{(i)} \), we compute the density \( n \) and the corresponding interaction energy \( V_{\text{int}} \). This defines \( f^{(i+1)} \), from which we determine \( n^{(i+1)} \) via normalization. We then set \( f = n^{(i)}f^{(i+1)} \), \( V_{\text{int}} \), and time \( t = 0.9 \). Iterating this procedure leads to convergence in 5–10 iterations for the parameters we consider.

Number fluctuations.

We compute the expected number fluctuations directly from the obtained density profiles. The fluctuation–dissipation theorem shows how the number fluctuations of the gas are related to the isothermal compressibility: \( \Delta n^2/N = nk_B T \), where the isothermal compressibility \( \kappa = \frac{1}{P V} \frac{\partial V}{\partial P} \).

In the harmonic trap the chemical potential varies as \( \mu(\mathbf{r}) = \mu - U(\mathbf{r}) \). Thus, the derivative with respect to \( \mu \) can be replaced by a derivative with respect to one of the spatial directions as \( \frac{\partial}{\partial \mu} = \frac{\partial}{\partial z} \), which we can directly evaluate based on the computed density profiles.

Dynamics.

To obtain the dynamics we use the scaling factor method\(^{(1)}\)–\(^{(4)}\) with the ansatz

\[ f(\mathbf{r}, \mathbf{p}, t) = \frac{1}{\Pi(\Delta s^2)} f^{(i)}_{\mathbf{r}} \left( \frac{\mathbf{p}}{\xi_i \lambda_i} \right) \]

for the out-of-equilibrium distribution function of the gas, where \( i = x, y, z \) labels the spatial directions.

Following\(^{(8)}\), we take the moments of \( \mathbf{r}, \mathbf{p} \), and \( \mathbf{p}^2 \) to obtain a closed set of differential equations for the scaling parameters \( \lambda_i \) and \( \theta_i \):

\[ \dot{\lambda}_i = (2n_{\mathbf{s}})^\frac{3}{2} \dot{\lambda}_i - (2n_{\mathbf{s}})^\frac{3}{2} \frac{\theta_i}{\xi_i} \]

and

\[ \dot{\theta}_i + 2 \frac{\lambda_i}{\xi_i} \dot{\theta}_i = \left( \theta_i - \frac{\mu}{\xi_i} \right)/\tau \]

where \( \xi_i = \frac{\sqrt{2n_{\mathbf{s}}}}{2^{3/2}(N-1)^{1/2} \pi^{1/2}} \) accounts for the mean-field interaction with \( \cdots \) being phase-space averages with respect to the equilibrium distribution, dotted variables refer to their time derivatives and \( \tau = 1/33 \sqrt{a} \).

Time of flight.

To study the expansion after switching off the trap, the second term in equation (10) is set to 0, and the nonlinear coupled differential equations are solved for the scaling parameters, which, when plugged into the scaling ansatz, yield the phase-space distribution after time of flight.

For the non-interacting case, the equations for the expansion can be solved explicitly. In the collisionless regime, \( \dot{\theta}_i = \dot{\lambda}_i \). Using the non-interacting value of the initial root mean square radius, \( < x_i^2 >_0 = (l = 0) = \frac{1}{8} \frac{\lambda_i}{a} \frac{1}{2^{3/2} N_{\mathbf{s}}^{1/2}} \), we obtain after time of flight \( < x_i^2 > = \frac{5}{2} < x_i^2 > (l = 0) = \frac{1}{8} \frac{\lambda_i}{a} \frac{1}{2^{3/2} N_{\mathbf{s}}^{1/2}} \), such that the ratio of different directions approaches 1, and the cloud becomes isotropic.

Viral expansion for the number fluctuations.

An alternative approach to derive the thermodynamic quantities is provided by viral expansions of the partition function of the interacting many-body system\(^{(1-4)}\).

Methods

Transparency laser. The transparency light is derived from an extended cavity diode laser that is filtered by a volume Bragg grating. Owing to amplified spontaneous emission, we see a lifetime in the dimple of 5 s. The beam is linearly polarized along y, and a small magnetic field is applied along z. To separately extract the number of atoms in the dimple and reservoir, the HODT is extinguished. Atoms in the dimple are guided by the VODT which has a small angle with respect to gravity, whereas atoms in the reservoir undergo free expansion. This spatially separates the atoms in the dimple and the reservoir, and atom numbers can be respectively counted through absorption imaging. To recycle, the VODT is then extracted for atoms in the dimple by calculating \( T_1 \), and measuring the temperature, which is determined by fitting the reservoir and dimple atoms to a Gaussian fit after extinguishing the XODT after the long time of flight.

Three-body loss. To determine \( k_3 \), we load a thermal gas with \( T = 1.45 \) \( \mu \text{K} \) into the dimple part of the recompressed dipole trap. Starting from an initial central density of \( n = 3.9 \times 10^{10} \text{cm}^{-3} \), we measure over the next 10 s a decay of the total atom number \( \Delta \) as a function of the holding time \( \tau \). The observed atom loss is modelled as

\[ \Delta = k_3 \int_0^{\tau} n(\mathbf{r}, \tau) dV \, , \]

where \( n(\mathbf{r}, \tau) \) is the density at position \( \mathbf{r} \) and time \( \tau \). We find \( k_3 = 4.7(2) \times 10^{-3} \text{cm}^{-3} \text{s}^{-1} \). This is a factor of 2 larger than the recent lattice-based three-body measurement that showed agreement with a universal van der Waals model\(^{(3)}\). Discrepancies in three-body loss between bulk gas measurements and predictions have been seen before\(^{(1-3)}\), demonstrating the challenge in accounting for inhomogeneous density profiles. Under our experimental conditions, single- and two-body contributions are expected to be negligible over a time interval of 10 s and, using a corresponding model, we find them to be statistically insignificant.

TenS4 laser. The TenS4 laser overlaps with the VODT and the transparency beam, and has a 30 \( \mu \text{m} \) waist with a peak intensity of 0.15 kW cm\(^{-2} \). The beam is linearly polarized along y, and along this axis a small magnetic field of 5 G is also applied, producing a small differential force between \( m_1 \) states of opposite signs. Owing to our relatively small magnetic field and optical power, the TenS4 beam does not provide enough force to completely remove all other nuclear spin states. Consequently, we conventionally remove atoms with \( m_1 = -9/2 \) and \( m_1 = -7/2 \) before evaporation using optical pumping to aid with spin selectivity. To ensure that we are not addressing a molecular resonance, we employ the relaxation time approximation\(^{(48)}\), which yields the phase-space distribution after time of flight.

Methods

Equilibrium solution. Solving the Boltzmann–Vlasov equation in equilibrium, we obtain

\[ f(\mathbf{r}, \mathbf{p}) = \frac{1}{e^{\frac{E(\mathbf{r}, \mathbf{p})}{k_B T}} - 1} \]

Relaxation time approximation. Instead of a detailed treatment of the collisions between particles, we employ the relaxation time approximation\(^{(1)}\), which approximates

\[ I_1[f] = \delta f - \frac{f}{r} \, , \]

where \( f \) approaches the local equilibrium state \( f_0 \), over a characteristic relaxation time \( \tau \).

We follow refs.\(^{(42, 43)}\) and approximate the relaxation time as

\[ (2n_{\mathbf{s}})^{-1} = (N - 1) \left( \frac{4}{5} \frac{31}{3} \frac{\pi}{\mu_0} \right)^2 \frac{a_{\text{nm}}^2}{\mu_0} F_0(T/T_0) \, , \]

where \( F_0(T/T_0) \) is a universal function of \( T/T_0 \) as defined in ref.\(^{(1)}\), and \( a_{\text{nm}} \) is the harmonic oscillator length. \( F_0 \) vanishes at \( (T/T_0) \) at low temperatures, a signature of Fermi statistics, is of order 1 in the intermediate temperature regime and vanishes as \( (T/T_0)^{-1} \) at higher temperatures.

\[ f(\mathbf{r}, \mathbf{p}) = \frac{1}{e^{\frac{E(\mathbf{r}, \mathbf{p})}{k_B T}} - 1} + 1 \]

where \( n = n(\mathbf{r}) \) and \( \tau \) is time. Using the non-interacting value of the initial root mean square radius, \( < x_i^2 >_0 = (l = 0) = \frac{1}{8} \frac{\lambda_i}{a} \frac{1}{2^{3/2} N_{\mathbf{s}}^{1/2}} \), such that the ratio of different directions approaches 1, and the cloud becomes isotropic.
As we are interested in the number fluctuations, and thus $\kappa$, we start from the expression for the chemical potential of a homogeneous Fermi gas at low temperature and weak interactions, adapted to the SU($N$) case
\[
\mu(n, T, a) = E_F \left[ 1 - \frac{1}{\pi} \left( \frac{T}{T_F} \right)^2 + \frac{12}{\pi^2} (N - 1) a \right] + \frac{411 - 2(11 - 2 \ln 2)}{15 \pi^2} (a)^2 (N - 1) + C T^2 a^2,
\]
(12)
where $C$ is a constant independent of $n$. We can then evaluate the compressibility from the dependence of $n$ on the Fermi parameters.

For simplicity we only keep terms up to first order, to obtain for $\Delta \tilde{N}^2 / \tilde{N}$ the equation presented in the main text.

Data availability
The datasets generated and analysed during the current study are available from the corresponding author L.S. on reasonable request.

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Author contributions
L.S., C.S., R.B.H., A.G., L.Y., W.R.M. and J.Y. contributed to the experimental measurements. T.B. and A.M.R. developed the theoretical model. All authors discussed the results, contributed to the data analysis and worked together on the manuscript.

Competing interests
The authors declare no competing interests.

Additional information
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