Tetraquark molecular states in the $D_s\bar{D}_{s1}$ and $D_s^*\bar{D}_{s0}$ mass spectrum

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Abstract

In the present work, we construct the color-singlet-color-singlet type four-quark currents to investigate the $D_s\bar{D}_{s1}$ and $D_s^*\bar{D}_{s0}$ tetraquark molecular states with the $J^{PC} = 1^{--}$ and $1^{-+}$ via the QCD sum rules, and obtain satisfactory results. We can search for the $D_s\bar{D}_{s1}$ and $D_s^*\bar{D}_{s0}$ tetraquark molecular states with the $J^{PC} = 1^{--}$ and $1^{-+}$ at the BESIII and Belle II in the future.

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1 Introduction

In 2008, the CLEO collaboration measured the cross sections of the processes $e^+e^- \rightarrow D_s^+ D_s^-$, $D_s^{*+} D_s^-$ and $D_s^{*+} D_s^-$ up to the center-of-mass energy 4.26 GeV, and observed no evidence of the $Y(4260)$. In 2010, the BaBar collaboration measured the cross sections of the processes $e^+e^- \rightarrow D_s^+ D_s^-$, $D_s^{*+} D_s^-$ and $D_s^{*+} D_s^-$ up to the center-of-mass energy 6.2 GeV via initial-state radiation (ISR), and observed no evidence of the $Y(4260)$ either. Also in 2010, the Belle collaboration measured the cross sections of the processes $e^+e^- \rightarrow D_s^+ D_s^-$, $D_s^{*+} D_s^-$ and $D_s^{*+} D_s^-$ up to the center-of-mass energy 5.0 GeV via initial-state radiation, and observed that both the $e^+e^- \rightarrow D_s^{*+} D_s^-$ cross section and $R$ ratio exhibit an obvious dip near the mass of the $Y(4260)$. In 2020, the BESIII collaboration measured the cross sections of the processes $e^+e^- \rightarrow D_s^+ D_{s1}(2460)^- +c.c.$ at the center-of-mass energy 4.467 GeV – 4.600 GeV, and $e^+e^- \rightarrow D_s^{*+} D_{s1}(2460)^- +c.c.$ at the center-of-mass energy 4.590 GeV – 4.600 GeV, and observed no obvious charmonium or charmonium-like structure. Recently, the BESIII collaboration measured the cross sections of the processes $e^+e^- \rightarrow D_s^{*+} D_{s0}(2317)^- +c.c.$ and $e^+e^- \rightarrow D_s^{*+} D_{s1}(2460)^- +c.c.$ at the center-of-mass energy 4.600 GeV – 4.700 GeV, and $e^+e^- \rightarrow D_s^{*+} D_{s1}(2536)^- +c.c.$ at the center-of-mass energy 4.660 GeV – 4.700 GeV, and observed no structure in either process.

The assignments of the $Y(4260)$, such as the tetraquark state, hybrid state, conventional charmonium, are still in hot debate. If there exist the color-singlet-color-singlet type tetraquark states $c\bar{s}s\bar{c}$, irrespective of weak bound states or higher resonances, they can decay to their constituents, the $c\bar{s}$ and $s\bar{c}$ color-singlet clusters, through the Okubo-Zweig-Iizuka super-allowed fall-apart mechanism saving feasible in the phase-space. When the experimental data are accumulated, the BESII and Belle II collaborations maybe observe them in the $D_s\bar{D}_s$, $D_s^*\bar{D}_s$, $D_s^*\bar{D}_{s1}$, $D_s\bar{D}_{s1}$, $D_s^*\bar{D}_{s0}$, · · · invariant mass spectrum in the $e^+e^-$ scattering processes, which can shed light on the nature of the $X$, $Y$ and $Z$ states. It is necessary and important to investigate the mass spectrum of the $D_s\bar{D}_s$, $D_s D_{s*}$, $D_s^* D_s$, $D_{s1} D_{s1}$, $D_s^* D_{s0}$, · · · tetraquark molecular states and make reliable predictions.

In Ref. [15], we accomplish the operator product expansion for the correlation functions up to the vacuum condensates of dimension 10 consistently and investigate the ground state hidden-charm tetraquark molecular states without strange, with strange and with hidden-strange, such as the $D\bar{D}_s^*$, $D^*\bar{D}_s^*$, $DD_{s1}$, $D_s D_{s*}$ and $D_s^* D_s^*$ tetraquark molecular states with the $J^{PC} = 0^{++}$, $1^{++}$, $1^{-+}$ and $2^{++}$, via the QCD sum rules comprehensively, and make possible assignments of the existing $X$, $Y$ and $Z$ states, such as the $X_c(3872)$, $Z_c(3900)$, $Z_{cs}(3985/4000)$, $Z_{cs}(4020/4025)$.

In Ref. [16], we construct the color-singlet-color-singlet type four-quark currents to explore tetraquark molecular states $D\bar{D}_s(2420)$ and $D^*\bar{D}_s^*(2400)$ with the $J^{PC} = 1^{--}$ and $1^{-+}$ via the
QCD sum rules by calculating the contributions of the vacuum condensates up to dimension-10. The predictions only support assigning the $Y(4390)$ to be the $D\bar{D}_2$ molecular state with the $J^{PC} = 1^{--}$.

In Ref. [17], we construct the color-singlet-color-singlet type tensor current to explore the neutral $D_s^*\bar{D}_s - D_s\bar{D}_s^*$ tetraquark molecular states with the $J^{PC} = 1^{-+}$ via the QCD sum rules, and obtain the molecule mass $4.67 \pm 0.08$ GeV, which is in very good agreement with the mass of the $X(4630)$ observed later by the LHCb collaboration, $M_{X(4630)} = 4626 \pm 18_{-11}^{+18}$ MeV [18].

All in all, the QCD sum rules is a powerful theoretical approach in exploring the masses and decay widths of the $X$, $Y$ and $Z$ states, and has achieved many successful descriptions in the scenario of tetraquark states, or tetraquark molecular states or tetraquark molecular states [29, 30, 31, 32, 33, 34].

In the present work, we extend our previous works to explore tetraquark molecular states $D_s\bar{D}_s$ and $D_s^*\bar{D}_s^*$ with the $J^{PC} = 1^{--}$ and $1^{-+}$ via the QCD sum rules by accomplishing the operator product expansion up to the vacuum condensates of dimension 10 consistently, and make possible predictions to be confronted to the experimental data at the BESIII and Belle II in the future.

The article is arranged as follows: we obtain the QCD sum rules for the vector tetraquark molecular states in section 2; in section 3, we present the numerical results and discussions; section 4 is reserved for our conclusion.

## 2 QCD sum rules for the vector tetraquark molecular states

Let us write down the correlation functions $\Pi_{\mu\nu}(p)$ in the QCD sum rules,

\[
\Pi_{\mu\nu}(p) = i \int d^4x e^{ipx} \langle 0 | T \{ J_\mu(x) J^\dagger_\nu(0) \} | 0 \rangle ,
\]

where the color-singlet-color-singlet type four-quark currents $J_\mu(x) = J_\mu^1(x), J_\mu^2(x), J_\mu^3(x), J_\mu^4(x)$,

\[
J_\mu^1(x) = \frac{1}{\sqrt{2}} \left\{ \bar{s}(x)i\gamma_5c(x)\gamma_\mu \gamma_5 s(x) - \bar{s}(x)\gamma_\mu\gamma_5 c(x)\bar{c}(x)i\gamma_5 s(x) \right\} ,
\]

\[
J_\mu^2(x) = \frac{1}{\sqrt{2}} \left\{ \bar{s}(x)i\gamma_5 c(x)\bar{c}(x)\gamma_\mu \gamma_5 s(x) + \bar{s}(x)\gamma_\mu\gamma_5 c(x)\bar{c}(x)i\gamma_5 s(x) \right\} ,
\]

\[
J_\mu^3(x) = \frac{1}{\sqrt{2}} \left\{ \bar{s}(x)c(x)\bar{c}(x)\gamma_\mu s(x) + \bar{s}(x)\gamma_\mu c(x)\bar{c}(x)s(x) \right\} ,
\]

\[
J_\mu^4(x) = \frac{1}{\sqrt{2}} \left\{ \bar{s}(x)c(x)\bar{c}(x)\gamma_\mu d(x) - \bar{s}(x)\gamma_\mu c(x)\bar{c}(x)s(x) \right\} .
\]

Under charge conjugation transform $\hat{C}$, the currents $J_\mu(x)$ have the properties,

\[
\hat{C} J_\mu^{1/3}(x) \hat{C}^{-1} = -J_\mu^{1/3}(x) ,
\]

\[
\hat{C} J_\mu^{1/4}(x) \hat{C}^{-1} = +J_\mu^{1/4}(x) ,
\]

the currents $J_\mu(x)$ are eigenstates of the charge conjugation.

In the present work, we choose the local color-singlet-color-singlet type four-quark currents to interpolate the hidden-charm tetraquark states, which have two color-singlet clusters. The color-singlet clusters have the same quantum numbers as the charmed mesons, such as $D_s$, $D_s^*$, $D_s^0$ and $D_s\bar{s}$, except for the masses, as those color-singlet clusters are not necessary to be the physical mesons.

The physical mesons with two valence quarks are spatial extended objects and have mean spatial sizes about the magnitude $\sqrt{\langle r^2 \rangle} \sim 0.5$ fm. In the present work or in the QCD sum rules, though we refer the color-singlet-color-singlet type tetraquark states as the tetraquark molecular states, they are not the usually called molecular states. They have the mean/average spatial sizes
as that of the typical heavy mesons, and are compact objects. The usually called molecular states are loosely bound states consist of the physical mesons, the mean spatial sizes are proportional to the inverse of the binding energies, about 1 fm or larger than 1 fm, which are too large to be interpolated by the local four-quark currents.

The currents \( \bar{s}(x)i\gamma_5c(x) \) and \( \bar{s}(x)\gamma_{\mu}c(x) \) have the spin-parity \( J^P = 0^- \) and \( 1^- \), respectively, and couple potentially to the mesons \( D_s \) and \( D_s^* \), respectively. While the currents \( \bar{s}(x)i\gamma_5\gamma_5c(x) \) and \( \bar{s}(x)\gamma_{\mu}\gamma_5c(x) \) have the spin-parity \( J^P = 0^+ \) and \( 1^+ \), respectively, and couple potentially to the mesons \( D_{s0}^* \) and \( D_{s1} \), respectively, as multiplying \( \gamma_5 \) to the currents changes their parity, the net effects of the relative P-wave are embodied implicitly in the \( \gamma_5 \). In the heavy quark limit, the total angular momentum of a heavy-light meson \( \vec{J} = \vec{j}_t + \vec{S}_Q \), the light quark total angular momentum \( \vec{j}_t = \vec{L} + \vec{S}_q \), the \( \vec{L} \) is the light quark angular momentum, the \( \vec{S}_Q \) and \( \vec{S}_q \) are the heavy quark and light quark spins, respectively. There exist two doublets \((0^+, 1^+)\) and \((1^+, 2^+)\) for \( j_t = \frac{1}{2} \) and \( \frac{3}{2} \), respectively. On the other hand, the heavy-light mesons can also be classified in terms of eigenvalues of the light quark angular momentum \( |2S + 1L\rangle \), \( \vec{J} = \vec{S} + \vec{L} \), \( \vec{S} = \vec{S}_q + \vec{S}_Q \) is a sum of the intrinsic quark spins.

In the present case, we choose the currents \( (D_{s0}^*(2317), D_{s1}^*(2460)) \) and \( (D_{s1}(2536), D_{s2}^*(2573)) \). We usually choose the currents \( \bar{s}(x)c(x) \) and \( \bar{s}(x)\gamma_{\mu}\gamma_5c(x) \), which have the total angular momenta, \( \vec{J} = \vec{S} + \vec{L} \), i.e. \( \vec{0} = \vec{1} = \vec{1} + \vec{1} \) and \( \vec{1} = \vec{1} + \vec{1} \) respectively, to interpolate the \( D_{s0}^*(2317) \) and \( D_{s1}^*(2460) \) respectively [35]. In fact, there exists mixing effect between the \( j_t = \frac{1}{2} \) and \( \frac{3}{2} \) states with the spin-parity \( J^P = 1^+ \),

\[
\begin{pmatrix}
1^+ & j_t = \frac{1}{2} \\
1^+ & j_t = \frac{3}{2}
\end{pmatrix}
= \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\frac{3}{2}P_{1} \\
\frac{1}{2}P_{1}
\end{pmatrix},
\]

where the mixing angle \( \tan \theta = \frac{1}{\sqrt{2}} \) in the heavy quark limit [36]. So we cannot exclude the coupling between the current \( \bar{s}(x)\gamma_{\mu}\gamma_5c(x) \) and meson \( D_{s1}(2536) \). In the present work, we will not distinguish the \( D_{s1}^*(2460) \) and \( D_{s1}(2536) \), and use the notation \( D_{s1} \) to represent the color-singlet cluster with the spin-parity \( J^P = 1^+ \).

At the hadron side of the correlation functions \( \Pi_{\mu\nu}(p) \), we isolate the contributions of the ground state vector tetraquark molecular states \( Y \),

\[
\Pi_{\mu\nu}(p) = \frac{\lambda_Y}{M_Y^2 - p^2} \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2} \right) + \cdots,
\]

\[
= \Pi(p^2) \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2} \right) + \cdots,
\]

where the pole residues \( \lambda_Y \) are defined by \( \langle 0|J_\mu(0)|Y(p)\rangle = \lambda_Y \varepsilon_\mu \), the \( \varepsilon_\mu \) are the polarization vectors.

We accomplish the operator product expansion up to the vacuum condensates of dimension-10 consistently and assume vacuum saturation for the higher dimensional vacuum condensates, and write the correlation functions \( \Pi(p^2) \) at the QCD side in the form,

\[
\Pi(p^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds \frac{\text{Im} \Pi(s)}{s - p^2},
\]

through dispersion relation. In calculations, we contract the \( s \) and \( c \) quark fields in the correlation functions with Wick theorem, and substitute the quark lines with the full \( s \) and \( c \) quark propagators \( S_{ij}(x) \) and \( C_{ij}(x) \), respectively,

\[
S_{ij}(x) \equiv i \frac{\delta \bar{s}_i \hat{L}}{2\pi^2 x^3} - i \frac{\delta \bar{s}_i m_s}{4\pi^2 x^2} \left( \frac{12}{192} \right) - i \frac{\delta \bar{s}_i \hat{L}}{32\pi^2 x^2} \left( \frac{1152}{1152} \right) - i \frac{\delta \bar{s}_i \hat{L}}{32\pi^2 x^2} \left( \frac{1}{8} \right),
\]

\[
C_{ij}(x) \equiv i \frac{\delta \bar{s}_i \hat{L}}{2\pi^2 x^3} - i \frac{\delta \bar{s}_i m_s}{4\pi^2 x^2} \left( \frac{12}{192} \right) - i \frac{\delta \bar{s}_i \hat{L}}{32\pi^2 x^2} \left( \frac{1152}{1152} \right) - i \frac{\delta \bar{s}_i \hat{L}}{32\pi^2 x^2} \left( \frac{1}{8} \right),
\]

\[
\text{Im} \Pi(s) = \frac{\text{Im} \Pi(s)}{s - p^2},
\]

\[
\Pi(p^2) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds \frac{\text{Im} \Pi(s)}{s - p^2},
\]
\[ C_{ij}(x) = \frac{i}{(2\pi)^4} \int d^4ke^{-ik\cdot x} \left\{ \frac{\delta_{ij}}{k-m_c} - \frac{g_sG^\alpha_{\alpha,\beta}t^{\alpha}_{ij}}{4} \frac{(k+m_c)\sigma^{\alpha\beta} + (k+m_c)\sigma^{\alpha\beta}}{(k-m_c)^2} \right. \]
\[ \left. - \frac{g_s^2(t^\alpha t^\beta)_{ij} G^\alpha_{\alpha,\beta} G^\beta_{\mu\nu} (f^{\alpha\beta\mu\nu} + f^{\alpha\beta\mu\nu} + f^{\alpha\beta\mu\nu})}{4(k^2-m_c)^6} + \ldots \right\}, \]
\[ f^{\alpha\beta\mu\nu} = (k+m_c)\gamma^\alpha(k+m_c)\gamma^\beta(k+m_c)\gamma^\mu(k+m_c)\gamma^\nu(k+m_c), \]

(8)
to facilitate the cumbersome tasks, where \( t^n = \frac{\lambda^n}{3} \), the \( \lambda^n \) is the Gell-Mann matrix. In the full s-quark propagator, see Eq.(7), the s-quark mass \( m_s \) is taken as a mass quantity and is treated perturbatively, direct calculations indicate that such a perturbative treatment of the s-quark mass does not modify the dispersion relation comparing to the massless light quarks. For more technical details, one can consult Refs.\[26, 27, 28, 33, 34.\]

Then we implement the quark-hadron duality below the continuum thresholds \( s_0 \) and accomplish Borel transform in regard to the variable \( P^2 = -p^2 \) to acquire the QCD sum rules,
\[ \lambda^2 Y \exp \left( -\frac{M^2}{T^2} \right) = \int_{4m^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{T^2} \right), \]
the explicit expressions of the QCD spectral densities \( \rho_{QCD} \) are available via contacting the corresponding author by E-mail.

In the present work, we take account of the vacuum condensates \( \langle \bar{s}s \rangle, \langle \bar{q}G^2q \rangle, \langle \bar{q}G^2q \rangle, \langle \bar{s}s \rangle^2, \langle \bar{s}s \rangle \langle \alpha/G^2 \rangle, \langle \bar{s}s \rangle \langle \bar{q}G^2q \rangle, \langle \bar{s}s \rangle \langle g_sG_5S \rangle \) and \( \langle \bar{s}s \rangle^2 \langle \alpha/G^2 \rangle \). The four-quark condensate \( g_s^2 \langle \bar{s}s \rangle^2 \) originates from the matrix elements \( \langle \bar{s}\gamma^\mu t^aD_\mu G_{\lambda\tau}^a \rangle, \langle \bar{s}_D\bar{s}_D D_\mu D_\nu D_\mu D_\nu \rangle \) and \( \langle \bar{s}_D\bar{s}_D D_\mu D_\nu D_\mu D_\nu \rangle \), rather than originates from the radiative corrections of the \( \langle \bar{s}s \rangle^2 \), the strong fine structure constant \( \alpha_s = \frac{g^2}{4\pi} \) appears at the tree level. We adopt the truncations \( n \leq 10 \) and \( k \leq 1 \) consistently, the operators of the orders \( O(\alpha_s^n) \) with \( k > 1 \) are discarded \[26, 27, 28, 33, 34.\] The condensates \( \langle g_s^2G^2 \rangle, \langle \bar{q}G^2q \rangle^2, \langle \bar{q}G^2q \rangle \langle \bar{q}G^2q \rangle \) have the dimensions of mass 6, 8, 9 respectively, but they are vacuum expectations of the operators of the order \( O(\alpha_s^3/2), O(\alpha_s^2), O(\alpha_s^3/2) \) respectively, and are discarded for a consistent treatment.

We differentiate Eq.(9) in regard to \( \tau = \frac{1}{m^2} \), and eliminate the pole residues \( \lambda_Y \) to acquire the QCD sum rules for the molecule masses,
\[ M^2_Y = \frac{\frac{4}{3} \int_{4m^2}^{s_0} ds \rho_{QCD}(s)e^{-\tau s} \left( \frac{4}{3} \int_{4m^2}^{s_0} ds \rho_{QCD}(s)e^{-\tau s} \right)}{\frac{4}{3} \int_{4m^2}^{s_0} ds \rho_{QCD}(s)e^{-\tau s}}. \]

(10)

3 Numerical results and discussions

At the QCD side, we adopt the standard values of the vacuum condensates \( \langle \bar{q}q \rangle = -(0.24 \pm 0.01\, \text{GeV}^4), \langle \bar{s}s \rangle = (0.8 \pm 0.1)(\bar{q}q), \langle g_sG_5S \rangle = m_0^2(\bar{s}s), m_0^2 = (0.8 \pm 0.1)\, \text{GeV}^2, \langle \frac{\bar{q}G^2q}{4\pi} \rangle = (0.33\, \text{GeV})^4 \) at the energy scale \( \mu = 1\, \text{GeV} \) \[37, 38, 39, 40.\] and take the \( \overline{\mathcal{M}} \) masses \( m_c(m_c) = (1.275 \pm 0.025)\, \text{GeV} \) and \( m_s(\mu = 2\, \text{GeV}) = (0.95 \pm 0.005)\, \text{GeV} \) from the Particle Data Group \[41.\] In addition, we take account of the energy-scale dependence of the quark condensates, mixed
quark condensates and $\overline{\text{MS}}$ masses in regard to the renormalization group equation [42].

\[
\langle \bar{s}s \rangle (\mu) = \langle \bar{s}s \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{12}{33 - 2n_f}},
\]

\[
\langle \bar{s}g_s\sigma Gs \rangle (\mu) = \langle \bar{s}g_s\sigma Gs \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{4}{33 - 2n_f}},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{12}{33 - 2n_f}},
\]

\[
m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{12}{33 - 2n_f}},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0t} \left[ 1 - b_1 \log t + \frac{b_2^2 (\log^2 t - \log t - 1) + b_0b_2}{b_0^2} \right],
\]

as the strong fine-structure constant $\alpha_s$ already appears at the tree level, where $t = \log \frac{\mu}{M}$, $b_0 = \frac{33 - 2n_f}{12\pi^2}$, $b_1 = \frac{153 - 19n_f}{24\pi^2}$, $b_2 = \frac{2857 - 303n_f + 25n_f^2}{126\pi^2}$, $\Lambda = 213 \text{ MeV}$, 296 MeV and 339 MeV for the quark flavors $n_f = 5$, 4 and 3, respectively [41]. As we investigate the tetraquark molecular states with hidden-charm and hidden-strange, it is natural to choose the quark flavors $n_f = 4$, and evolve the QCD spectral densities $\rho_{\text{QCD}}(s)$ to the suitable energy scales $\mu$ to extract the molecule masses.

In the present work, we take the energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2m_c)^2}$ with the updated value of the effective $c$-quark mass $M_c = 1.85 \text{ GeV}$ to acquire the suitable energy scales of the QCD spectral densities [16, 33, 34]. We introduce the effective heavy quark mass $M_c$ and divide the tetraquark molecular states into both the heavy degrees of freedom $2M_c$ and light degrees of freedom $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2M_c)^2}$ by setting $m_u = m_d = 0$. We can also consider the light flavor $SU(3)$ breaking effects, and acquire the light degrees of freedom $\mu = \sqrt{M_{X/Y/Z/T}^2 - (2M_c)^2 - 2m_s(\mu)}$, in other words, $\mu + 2m_s(\mu) = \sqrt{M_{X/Y/Z/T}^2 - (2M_c)^2}$.

We can rewrite the energy scale formula in the form,

\[
M_{X/Y/Z}^2 = \mu^2 + \text{Constants},
\]

where the Constants have the value $4M_c^2$ and fitted by the QCD sum rules [24, 25, 33, 34], the predicted tetraquark (molecule) masses and the pertinent/suitable energy scales of the QCD spectral densities have a Regge-trajectory-like relation [33]. In calculations, we take account of the light-flavor $SU(3)$ mass-breaking effects by subtracting a small $s$-quark mass to obtain the modified energy scale formula $\mu = \sqrt{M_{X/Y/Z}^2 - (2M_c)^2 - 2m_s(\mu)}$. Analysis of the $J/\psi$ and $\Upsilon$ with the famous Cornell potential, i.e. the Coulomb-potential-plus-linear-potential, leads to the constituent quark masses $m_c = 1.84 \text{ GeV}$ and $m_b = 5.17 \text{ GeV}$ [14]. We can set the effective $c$-quark mass $M_c = m_c = 1.84 \text{ GeV}$, which is consistent with the updated value $M_c = 1.85 \text{ GeV}$. In numerical calculations, we add an uncertainty $\delta \mu = \pm 0.1 \text{ GeV}$ considering the uncertainty of the $M_c$.

At the beginning, we tentatively set the masses of the molecular states to be the sum of the physical masses of the two charmed mesons, which correspond to the two color-singlet clusters inside the molecular states, and obtain the energy scales $\mu$ through the modified energy scale formula. Then we calculate the molecule masses with the QCD sum rules by searching for the best Borel parameters $T^2$ and continuum threshold parameters $s_0$ via trial and error, and examine whether or not the modified energy scale formula is satisfied. We vary the molecule masses therefore the energy scales $\mu$ slowly and steadily until reach the satisfactory results, and acquire the Borel parameters, continuum threshold parameters, pole contributions and optimal energy scales, which are shown plainly in Table[1].
From Table 1, we can see that the central values of the pole contributions are larger than 50%, the pole dominance criterion can be satisfied very well. In calculations, we observe that in the Borel windows, the dominant contributions come from the perturbative terms, the contributions come from the vacuum condensates of dimension 10, the highest dimensional vacuum condensates, are much less than 1%, the convergent behaviors of the operator product expansion are very good.

Now we take account of all uncertainties of the input parameters, and obtain the values of the masses and pole residues of the vector tetraquark molecular states with hidden-charm and hidden-strange, which are shown plainly in Fig.1 or Table 1. From Table 1 we can infer that the threshold parameters and the predicted masses satisfy the relation \( \sqrt{s_0} = M_Y + (0.4 \sim 0.6) \text{GeV} \), which is consistent with our naive expectation, in addition, we can also infer that the modified energy scale formula is well satisfied. In Fig.1 we plot the tetraquark molecule masses with variations of the Borel parameters at much larger intervals than the Borel windows, where the regions between the two short vertical lines are the Borel windows. From the figure, we can see clearly that there appear platforms really in the Borel windows, and we expect to make reasonable predictions, which can be confronted to the experimental data in the future.

From Table 1 we can see that the mass-splitting for the two molecular states with the \( J^{PC} = 1^{-} \) is rather large, while the mass-splitting for the two molecular states with the \( J^{PC} = 1^{+} \) is rather small. We simplify the analysis to explore the origination of the mass-splitting by taking the limit \( m_s \to 0 \). For the \( D_s \bar{D}_{s1} \) and \( D_s^* \bar{D}_{s0} \) molecular states with the \( J^{PC} = 1^{-} \), the contributions of the quark condensate \( \langle \bar{s}s \rangle \) are zero, the contributions of the mixed condensate \( \langle \bar{s}g_s \sigma Gs \rangle \) are of the same magnitude but opposite sign. For the \( D_s \bar{D}_{s1} \) and \( D_s^* \bar{D}_{s0} \) molecular states with the \( J^{PC} = 1^{+} \), the contributions of the vacuum condensates \( \langle \bar{s}s \rangle \) and \( \langle \bar{s}g_s \sigma Gs \rangle \) are canceled out with each other severely, the net contributions are very small. The different contributions of the vacuum condensates \( \langle \bar{s}s \rangle \) and \( \langle \bar{s}g_s \sigma Gs \rangle \) lead to the different behaviors of the mass-splittings of the molecular states with the \( J^{PC} = 1^{-} \) and \( 1^{+} \).

In Table 2 we also present the predictions for the masses of the hidden-charm tetraquark molecular states with one P-wave constituent (or color-singlet cluster) in our previous works [16, 17, 45]. The predictions \( M_{D_{s1}^{(*)}}(1^{-}) \) is greatly suppressed by the Belle collaboration and has the quantum numbers \( J^{PC} = 1^{+} \) \([18]\). The \( Y(4630) \) was observed in the \( J/\psi \phi \) invariant mass spectrum in the \( B^+ \to J/\psi \phi K^+ \) decays by the LHCb collaboration and has the number quantum numbers \( J^{PC} = 1^{-} \). The \( Y(4630) \) was observed in the \( \Lambda_c^+ \Lambda_c^- \) invariant mass spectrum in the exclusive process \( e^+e^- \to \gamma \Lambda_c^+ \Lambda_c^- \) by the Belle collaboration, and has the mass \( M_Y = 4634^{+5+8}_{-8-7} \text{MeV} \) and width \( \Gamma_Y = 92^{+21+10}_{-24-21} \text{MeV} \), respectively \([47]\), which are consistent with that of the charmonium-like state \( Y(4660) \) within errors \([41]\). However, precise measurement of the \( e^+e^- \to \Lambda_c^+ \Lambda_c^- \) cross section near the threshold by the BESIII collaboration indicates that there might exist a bound state below the \( \Lambda_c^+ \Lambda_c^- \) threshold, which differs from the \( Y(4630/4660) \) remarkably \([48, 49]\). In Ref. [60], we take the \( Y(4660) \) as the tetraquark state with the \( J^{PC} = 1^{--} \) and study the strong decays \( Y(4660) \to J/\psi f_0(980), \eta_c \phi, \chi_{c0} \phi, D_s \bar{D}_s, D_s^* \bar{D}_s^*, D_s \bar{D}_s^*, D_s^* \bar{D}_s, \psi \pi^+ \pi^-, J/\psi \phi \) with the QCD sum rules based on rigorous quark-hadron quality, and observe that the decay to \( J/\psi \phi \) is greatly suppressed or forbidden. The predicted width \( \Gamma(Y(4660)) = 74.2^{+29.2}_{-19.2} \text{MeV} \) supports assigning the \( Y(4660) \) to be the \( [\bar{s}c]_P [\bar{s}c]_A - [s\bar{c}]_A [\bar{s}c]_P \) type tetraquark state with the \( J^{PC} = 1^{--} \). The \( X(4630) \), \( \Lambda_c^+ \Lambda_c^- \) resonance and \( Y(4660) \) are different particles.

At the present time, there are no experimental candidates for the \( D_s \bar{D}_{s1} \) and \( D_s^* \bar{D}_{s0} \) tetraquark molecular states with the \( J^{PC} = 1^{-} \) and \( 1^{+} \). In the scenario of tetraquark molecular states, we can assign the \( X(3872) \), \( Z_c(3900/3885) \), \( Z_{cs}(3985/4000) \), \( Z_c(4020/4025) \), \( Y_c(4390) \), \( X_c(4630) \) and \( Z_b(10610/10650) \) to be the tetraquark molecular states tentatively based on the predicted masses...
Table 1: The Borel parameters, continuum threshold parameters, pole contributions, energy scales, masses and pole residues of the vector tetraquark molecular states.

| Molecule         | $T^2$(GeV$^2$) | $\sqrt{s_0}$(GeV) | pole   | $\mu$(GeV) | $M_Y$(GeV) | $\lambda_Y$(10$^{-2}$GeV$^2$) |
|------------------|----------------|-------------------|--------|------------|------------|-------------------------------|
| $D_sD_{s1}$ (1−) | 3.3 − 3.7      | 5.0 ± 0.1         | (45 − 64)% | 2.3        | 4.48 ± 0.08 | 4.47 ± 0.61                   |
| $D_sD_{s1}$ (1+) | 3.6 − 4.0      | 5.2 ± 0.1         | (44 − 63)% | 2.7        | 4.71 ± 0.10 | 5.89 ± 0.74                   |
| $D_s^*D_{s0}^*$ (1−) | 3.9 − 4.3 | 5.3 ± 0.1         | (44 − 62)% | 2.9        | 4.80 ± 0.08 | 7.46 ± 0.87                   |
| $D_s^*D_{s0}^*$ (1+) | 4.0 − 4.4  | 5.3 ± 0.1         | (43 − 61)% | 2.9        | 4.79 ± 0.08 | 7.49 ± 0.86                   |

Table 2: Assignments of the tetraquark molecular states with one P-wave cluster.

from our previous QCD sum rules calculations [15,16,17,33,34],

\[
X(3872) = \frac{1}{\sqrt{2}} (D\bar{D}^* - D^*\bar{D}) \quad \text{(with 1^{++})},
\]

\[
Z_c(3900/3885) = \frac{1}{\sqrt{2}} (D\bar{D}^* + D^*\bar{D}) \quad \text{(with 1^{+-})},
\]

\[
Z_c(4020/4025) = D^*\bar{D} \quad \text{(with 1^{--})},
\]

\[
Z_{cs}(3985/4000) = \frac{1}{\sqrt{2}} (D\bar{D}^*_s ± D^*\bar{D}_s) \quad \text{(with 1^{±±})},
\]

\[
Y_c(4390) = \frac{1}{\sqrt{2}} (D\bar{D}_1 - D_1\bar{D}) \quad \text{(with 1^{--})},
\]

\[
X_c(4630) = \frac{1}{\sqrt{2}} (D^*_s\bar{D}_{s1} - D_{s1}\bar{D}^*_s) \quad \text{(with 1^{−+})},
\]

\[
Z_b(10610) = \frac{1}{\sqrt{2}} (B\bar{B}^* + B^*\bar{B}) \quad \text{(with 1^{+-})},
\]

\[
Z_b(10650) = B^*\bar{B} \quad \text{(with 1^{++}),}
\]

(13)

the $D_s\bar{D}_{s1}$ molecular states were also discussed in Ref. [51] recently.

Now let us perform Fierz re-arrangement for the four-quark currents $J_\mu$ both in the color space
Figure 1: The masses with variations of the Borel parameters $T^2$, where the $A$, $B$, $C$ and $D$ correspond to the tetraquark molecular states $D_s D_{s1}^{*}(1^{+})$, $D_s D_{s1}^{*}(1^{−})$, $D_s^* D_{s0}^{*}(1^{−})$ and $D_s^* D_{s0}^{*}(1^{+})$, respectively. The regions between the two short vertical lines are the Borel windows.
and Dirac-spinor space,

\[
2\sqrt{2} J_{\mu}^1 = \frac{1}{3} i \bar{\gamma}_\mu s \bar{c} c - \frac{1}{3} i \bar{s} s \bar{c} \gamma_\mu c - \frac{1}{3} i \bar{s} \gamma_\mu \gamma_5 s \bar{c} \sigma_{\mu\beta}\gamma_5 c + \frac{1}{3} \bar{s} \sigma_{\mu\beta}\gamma_5 \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c + \cdots,
\]

\[
2\sqrt{2} J_{\mu}^2 = \frac{1}{3} \bar{s} \sigma_{\mu\beta}s \bar{c} \gamma_\mu c + \frac{1}{3} \bar{s} \gamma_\mu \gamma_5 s \bar{c} \sigma_{\mu\beta}c - \frac{1}{3} i \bar{s} \gamma_\mu \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c - \frac{1}{3} \bar{s} \sigma_{\mu\beta}\gamma_5 \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c + \cdots,
\]

\[
2\sqrt{2} J_{\mu}^3 = \frac{1}{3} i \bar{s} \gamma_\mu \gamma_5 s \bar{c} \gamma_\mu c - \frac{1}{3} i \bar{s} \gamma_\mu \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c - \frac{1}{3} \bar{s} \sigma_{\mu\beta}\gamma_5 \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c + \cdots,
\]

\[
2\sqrt{2} J_{\mu}^4 = -\frac{1}{3} i \bar{s} \sigma_{\mu\beta}s \bar{c} \gamma_\mu c + \frac{1}{3} i \bar{s} \gamma_\mu \gamma_5 s \bar{c} \sigma_{\mu\beta}c - \frac{1}{3} i \bar{s} \gamma_\mu \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c - \frac{1}{3} i \bar{s} \sigma_{\mu\beta}\gamma_5 \gamma_5 s \bar{c} \gamma_\mu \gamma_5 c + \cdots, \tag{14}
\]

to illustrate the two-body strong decays. The components $\bar{s}\Gamma c$ with $\Gamma, \Gamma' = 1, \gamma_\mu, \gamma_\mu \gamma_5, \cdots$ couple potentially to a series of $c\bar{c}-s\bar{s}$-type meson-pairs, or $c\bar{c}s\bar{s}$-type tetraquark molecular states, which decay to their components via the Okubo-Zweig-Iizuka super-allowed fall-apart mechanism. We can investigate the $D_s \bar{D}_s$ and $D_s^* \bar{D}_s^0$ tetraquark molecular states with the $J^{PC} = 1^{--}$ and $1^{+-}$ through the two-body strong decays,

\[
D_s \bar{D}_s (1^{--}) \rightarrow \chi_{c0}, J/\psi f_0(980), J/\psi f_1(1285), J/\psi \eta, h_c \eta, \eta_c \phi, \eta_c h_1(1415),
\]

\[
D_s \bar{D}_s (1^{+-}) \rightarrow J/\psi \phi, J/\psi h_1(1415), h_c \phi, \chi_{c1} \phi, \eta_c h_1(1415),
\]

\[
D_s^* \bar{D}_s^0 (1^{--}) \rightarrow \chi_{c0}, J/\psi f_0(980), J/\psi f_1(1285), J/\psi \eta, h_c \eta, \eta_c \phi, \eta_c h_1(1415),
\]

\[
D_s^* \bar{D}_s^0 (1^{+-}) \rightarrow J/\psi \phi, J/\psi h_1(1415), h_c \phi, \chi_{c1} \eta, \eta_c \phi, \eta_c h_1(1415), \tag{15}
\]

besides the final states $D_s \bar{D}_s$, $D_s \bar{D}_s$, $D_s^* \bar{D}_s^0$ and $D_s^* \bar{D}_s^0$.

### 4 Conclusion

In this article, we construct the color-singlet-color-singlet type four-quark currents to investigate the $D_s D_{s1}$ and $D_s^* \bar{D}_s^0$ tetraquark molecular states with the $J^{PC} = 1^{--}$ and $1^{+-}$ via the QCD sum rules. We accomplish the operator product expansion up to the vacuum condensates of dimension-10 consistently, take account of the $SU(3)$ mass-breaking effects and adopt the modified energy scale formula to choose the best energy scales of the QCD spectral densities, then extract the masses and pole residues of the tetraquark molecular states in the suitable Borel windows, which satisfy the two fundamental criteria of the QCD sum rules. We can search for the $D_s D_{s1}$ and $D_s^* \bar{D}_s^0$ tetraquark molecular states with the $J^{PC} = 1^{--}$ and $1^{+-}$ at the BESIII and Belle II in the future, and confront the present predictions to the experimental data.

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