Optical elements with extended depth of focus and arbitrary distribution of intensity along the focal segment obtained by angular modulation of the optical power

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Abstract. Light Sword Lens (LSL), i.e., an optical element with extended depth of focus (EDOF) characterized by angular modulation of the optical power in its conventional form is characterized by a linear relationship between the optical power and the angular coordinate of the corresponding angular lens sector. This dependence may be manipulated in function of the required design needs. In the present communicate this additional degree of freedom of design is used for elimination of the LSL shape discontinuity.

1. Introduction
One of possible ways to classify the family of EDOF optical elements with explicitly given transmittance can be made according to their mapping function, i.e., the relationship between the part of the element’s area, from which light is focused into the corresponding point of the focal segment and this point. The most famous example are axicons, where the mapping function ties an infinitesimal annulus within the aperture of the element with the corresponding focal point [1-5]. In this communicate we will focus our attention on the LSL, where the mapping function joins the infinitesimal sector of the element’s surface and the given focal point [6-8].

LSL has been shown recently to have promising properties for presbyopia compensation [9-12], and therefore in the present communicate we would like to address a possibility of the compensation of the shape discontinuity, which is characteristic for the LSL in its hitherto design.

2. Elimination of the LSL’s discontinuity of shape
The distribution of the optical path difference due to the LSL is given as follows [9], [10]:

\[ \Delta l(r, \theta) = \frac{D_0 r^2}{2} - \frac{\Delta D \theta^2}{4\pi} \]  

where \((D_0, D_0 + \Delta D)\) stands for the range of the optical powers created by the element. The phase distribution of the kinoform counterpart of the LSL, the shape of the LSL itself, as well as the geometry of the focal segment formation process is shown in Fig.1.
As one can see, a characteristic feature of the LSL is a discontinuity of its shape (phase discontinuity in the case of the LSL’s diffractive counterpart) due to the direct neighborhood of sectors given by $\theta=0$ ($\Delta l=D_0$) and by $\theta=2\pi$ ($\Delta l=D_0+\Delta D$). This discontinuity can be a source of an additional boundary wave forming an undesired interference pattern with the main wave forming the focal pattern or in the incoherent case may be a cause of the noise increase. Additionally, from purely mechanical point of view such sharp step may be reason of inconvenience and may cause additional problems during the manufacture of the LSL.

Fig.1. LSL: a) phase distribution of the diffractive kinoform version; b) refractive version; c) geometrical scheme of the design – the infinitesimal angular sector of the LSL with a parameter $\theta$ focuses light into a point $P(\theta)$ of the focal segment.
Here we would like to draw attention for another feature of LSL’s, namely the linear dependence of
the optical power on the angular coordinate $\theta$ given by Eq.(1) can be changed and consequently the
constant value of the axial intensity within the whole focal segment in principle can be substituted by
an arbitrary one. The reasons for doing so can be various. One of motives could be a requirement to
make some range or ranges of objects location privileged. In our case we are choosing a possibility to
remove the phase (or shape in the refractive version) discontinuity present on the radius corresponding
to transition from 0 to $2\pi$ radians. Let us see that there could be found at least two motives for such
proceeding. First one is of technological nature: refractive LSLs are easier to fabricate when such
discontinuities of shape do not take place. The second one, in the case of possible ophthalmologic
application is of physiological nature: it turns out that a typical value of the jump on the $2\pi$ line, e.g.,
in the case of a presbyopia correcting lens would be of order of several tens of micrometers.
Indeed, assuming the radius of the lens $R=4 \text{ mm}$, the focal length $f_0 = 18.5 \text{ mm}$ (which results from
typical value of optical power $D_0 = 54D$ and formula $f_0 = 1/D_0$) and $\Delta f = 1.5 \text{ mm}$, what in turn results
from the assumed range of optical power equal to $4D$ we obtain that

$$h = \Delta DR^2/2 \text{ or } \Delta f R^2/2f_0$$

i.e., $35 \mu\text{m}$. (2)

In order to eliminate this discontinuity we introduce into the LSL surface an angular sector of width
equal to $\Theta$ radians, where the dioptric power increases gradually from the value $D_0$ to the value
$D_0 + \Delta D$ oppositely to the remaining sector occupied by common LSL in such a way that both junctions
remain continuous (Fig.2). The optical features of such element for satisfactorily great value of $\Theta$
remain almost unchanged, what allows substituting the old structure with the new one.

![Fig.2. Modified LSL without the phase discontinuity in a kinoform version with an angular sector of oppositely growing dioptric power of width $\Theta = \pi/4, \pi/2, 3\pi/4, \text{ and } \pi$, respectively.](image)

We conducted numerical simulations of imaging for all elements shown in Fig.2 as well as for
previous version of LSL, marked in the upper row of Fig.3 as LSL, yet with phase discontinuity and
without any angular sector by means of software based on the modified convolution method [13, 14].
We assumed monochromatic, spatially incoherent light with a wavelength 555 nm corresponding to
the highest sensitivity of the human eye in photopic vision [15, 16].
It can be seen that the quality of imaging, though continuously deteriorating with the increasing angle of the discontinuity removing sector, still offers satisfactorily broad range of angles, where remains comparable with the conventional LSL.

3. Conclusions

Summing up it can be said that the elements with angular modulation of optical power turn out to be an alternative for the purposes of imaging with extended depth of focus. Such conclusion results from independence of their optical power on aperture [12] and better quality of obtained images in comparison with EDOF optical elements characterized by symmetry of revolution. Moreover, these elements, as shown in the present communicate, allow additionally for flexible manipulation of their distribution of the optical power and thereby for easier adaptation of the element for the particular needs.

On the other hand it should be bear in mind that the price for these possibilities is their imperfect imaging as well as tiny waving of produced images for different points along the focal segment [7-11].
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5. References
[1] J.H. McLeod J H 1954 J. Opt. Soc. Am. 44 592
[2] Dyson J 1958 Proc. R. Soc. London Ser. A 248 93
[3] Vasara A, Turunen J, and Friberg A.T. 1989 J. Opt. Soc. Am. A 6 1748
[4] Jaroszewicz Z 1997 Axicons. Design and propagation properties, in Research and Development Treatises vol.5, ed. M Pluta (Warsaw: Polish Chapter of SPIE)
[5] Sochacki J, Kołodziejczyk A, Jaroszewicz Z, and Bará S 1992 Appl. Opt. 31, 5326
[6] Kołodziejczyk A, Bará S, Jaroszewicz Z, and Sypek M 1990 J. Mod. Opt. 37, 1283
[7] Mikula G, Jaroszewicz Z, Kołodziejczyk A, Petelczyc K, and Sypek M 2007 Opt. Express 15, 9184
[8] Ares García J, Bará S, Gomez García M, Jaroszewicz Z, Kołodziejczyk A, and Petelczyc K 2008 Opt. Express 16, 18371
[9] Petelczyc K, Ares García J, Bará S, Jaroszewicz Z, Kakarenko K, Kołodziejczyk A, and Sypek M 2011 Opt. Express 19, 8693
[10] Petelczyc K, Bará S, Ciro López A, Jaroszewicz Z, Kakarenko K, Kołodziejczyk A, and Sypek M 2011 Opt. Express 19, 25602
[11] Arias Galego A, Bará S, Jaroszewicz Z, and Kołodziejczyk A 2012 Optom. Vis. Sci. 89, 1702
[12] Petelczyc K, Ares Garcia J, Bará S, Jaroszewicz Z, Kołodziejczyk A, Sypek M 2009 Phot. Lett. Poland 1, 55
[13] Sypek M 1995 Opt. Commun. 116, 43
[14] Sypek M, Prokopowicz C, and Gorecki M 2003 Opt. Eng. 42, 3158
[15] Valberg A, 2005 Light Vision Color (New York: John Wiley & Sons).
[16] Gross H, Blechinger F, and Achtner B, 2008 Survey of Optical Instruments vol.4 Handbook of Optical Systems, (Weinheim: Wiley-VCH).