LOCALIZATION ON FAT BRANES AS THE SOURCE OF NEUTRINO MIXING

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The localization of fermions in extra dimensions, proposed by Arkani-Hamed and Schmaltz, is discussed as the source of the phenomenon of particle mixing. We work out the example of neutrinos in detail.

The possibility of existence of additional dimensions is an old idea and has been extensively studied in the literature (see e.g.\(^1\)). Only recently this topic received a serious back up from the string theory, which requires at least six new spatial dimensions. The usual way is to assume that the additional dimensions are very small, therefore till now undetectable. During last few years Arkani-Hamed, Dimopoulos and Dvali (ADD) suggested that this does not need to be true.\(^2\) The so-called “large extra dimensions” have been shown to explain the gauge hierarchy problem and the mass hierarchies between families to some extent. In these models we assume that our world is confined to a 3-brane, which means that all standard model (SM) particles are constrained to propagate within a hypersurface, embedded in higher-dimensional space (the bulk). Only gravity is permitted to occupy the bulk as well, which resolves the gauge hierarchy problem.

One of the possible extensions of ADD model has been proposed in Ref.\(^3\), where the brane has a non-zero width in the extra dimension. This allows to localize the SM particles at different points in the extra dimension, therefore separating them spatially. The main motivation for such a setup is that by careful adjusting the distances between the particles one obtains suppressions in interactions and may solve the mass hierarchy problem. The next step is to accommodate the mixing between particles (like quarks and neutrinos) in this picture. This problem has been partially addressed in, e.g.,\(^4\) for the case of linear extra dimension.

In the present paper we show, using the newest neutrino data, that the neutrino mixing may be explained by the ADD model with fat brane in the shape of a hyper-tube. We leave the issue of the mass hierarchy as well as more detailed discussion of the shape of extra dimension to a forthcoming paper.

Let us denote the coordinates by \(\{x^\mu, y\}\), \(\mu = 1 \ldots 4\), where \(y\) corresponds to
the 5th spatial dimension. We assume the shape of the fermion wave function to be
\[ \Psi(x^\mu, y) = \psi(x^\mu)\chi(y), \] (1)
with \( \psi \) being the usual Dirac spinor. For the extra-dimension function \( \chi \) we postulate a Gaussian shape:
\[ \chi(y) = \frac{1}{\sigma\sqrt{2\pi}} \exp \left( -\frac{(y - y_0)^2}{2\sigma^2} \right), \] (2)
localized around the point \( y_0 \) in the extra dimension; \( \sigma \) corresponds to the width of the function. The Gaussian seems to be the most natural candidate for \( \chi \) but, in principle, other functions are possible.

It is a common practice to compactify the extra dimensions, although there were many attempts to discuss particle masses and mixings assuming the extra dimension to be an interval.\(^4\) In such a case, however, there may be additional effects occurring at the ends of the interval, like for example tunneling or leaking of the fields from the brane to the bulk. To avoid such situations we assume that the extra dimension forms a circle, i.e. we identify \( y + L \equiv y + 2\pi R \sim y \).

The mixing between two particles comes in our model from non-zero overlaps of Gaussians in the extra dimension. The probability of transition from a particle \( a \) to particle \( b \) (or from \( b \) to \( a \)), is proportional to the overlap squared and is given by
\[ \text{Prob}(a \leftrightarrow b) = \frac{1}{2\pi(\sigma_a^2 + \sigma_b^2)} \exp \left( -\frac{(y_a - y_b)^2}{2(\sigma_a^2 + \sigma_b^2)} \right). \] (3)

In the case of neutrinos, their mixing is usually described by \( |\nu_i\rangle = U_{ij} |\nu_j\rangle \), where \( i = e, \mu, \tau \) labels the flavor states and \( j = 1, 2, 3 \) labels the mass eigenstates. The standard parametrization of the unitary matrix \( U \) in terms of three mixing angles is, neglecting possible phases,
\[ U = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13}
\end{pmatrix} \] (4)
where \( s_{ij} = \sin\theta_{ij}, \ c_{ij} = \cos\theta_{ij}, \) and \( \theta_{ij} \) are the mixing angles between the eigenstates labeled by indices \( i \) and \( j \). The recent global analysis of neutrino oscillations\(^5\) yields the best fit values: \( 0.23 < \sin^2\theta_{12} < 0.39, 0.31 < \sin^2\theta_{23} < 0.72, \sin^2\theta_{13} < 0.054, \) which correspond to the following mixing matrix squared
\[ |U|^2 = \begin{pmatrix}
    (0.58 - 0.77) & (0.23 - 0.36) & (0.00 - 0.05) \\
    (0.16 - 0.23) & (0.08 - 0.53) & (0.31 - 0.67) \\
    (0.07 - 0.18) & (0.24 - 0.55) & (0.26 - 0.69)
\end{pmatrix}. \] (5)
Each entry of \(|U|^2\) represents the probability of finding neutrino of one state in another state, thus we have
\[ (|U|^2)_{ij} = \text{Prob}(\nu_i \leftrightarrow \nu_j), \] (6)
which should be understood as a set of nine equations, with the LHS taken from Eq. (5) and the RHS having the form of Eq. (3). We have analyzed numerically this set of equations, obtaining values of possible $\sigma$'s and $y$'s.

The results for the widths are presented on Fig. 1. The disallowed regions come from the form of Eq. (3), for which a logarithm will appear in the solutions. The widths of the allowed regions depend on the uncertainty in entries of $|U|^2$. Despite being in some cases difficult to recognize, the same pattern emerges on all nine panels of Fig. 1. The corresponding localizations of the Gaussians are shown on Fig. 2. Here, we have fixed the localization of $\nu_1$ at $y = 0$ on the circle. The length of the compactified extra dimension was set to 2 in some arbitrary units of length. The symmetric pattern is present because of the mirror symmetry in localizations, which will show up in the cases of all periodic shapes (it means simply that we can count
the distance going clockwise or anticlockwise along the circle). This figure does not show the detailed alignment of the fields. In order to give a full presentation one should find all the patterns which show up in the results and classify them. This is, however, beyond the scope of the present paper. One general feature is immediately visible, namely that there are preferred locations for each neutrino type. We have checked, that it is independent of the length of the circle.

In summary, we have shown that the observed neutrino mixing pattern may have its source in the geometry of our world. The not mentioned here mass hierarchy among neutrinos finds also an explanation in our picture.

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![Graphs showing possible localizations of neutrinos for \( L = 2 \).](image)
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