Four Eccentric Mergers Increase the Evidence that LIGO–Virgo–KAGRA’s Binary Black Holes Form Dynamically

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Abstract

The growing population of compact binary mergers detected with gravitational waves contains multiple events that are challenging to explain through isolated binary evolution. Such events have higher masses than are expected in isolated binaries, component spin tilt angles that are misaligned, and/or nonnegligible orbital eccentricities. We investigate the orbital eccentricities of 62 binary black hole candidates from the third gravitational-wave transient catalog of the LIGO–Virgo–KAGRA Collaboration with an aligned-spin, moderate-eccentricity waveform model. Within this framework, we find that at least four of these events show significant support for eccentricity $e_{10} \geq 0.1$ at a gravitational-wave frequency of 10 Hz (>60% credibility, under a log-uniform eccentricity prior that spans the range $10^{-6} < e_{10} < 0.2$). Two of these events are new additions to the population: GW191109 and GW200208\_22. If the four eccentric candidates are truly eccentric, our results suggest that densely populated star clusters may produce 100% of the observed mergers. However, it remains likely that other formation environments with higher yields of eccentric mergers—for example, active galactic nuclei—also contribute. We estimate that we will be able to confidently distinguish which formation channel dominates the eccentric merger rate after >80 detections of events with $e_{10} \geq 0.05$ at LIGO–Virgo sensitivity, with only $\sim$5 detectably eccentric events required to distinguish formation channels with third-generation gravitational-wave detectors.

Unified Astronomy Thesaurus concepts: Black holes (162); Compact objects (288); High energy astrophysics (739); Astrophysical black holes (98); Gravitational waves (678); Gravitational wave astronomy (675); Gravitational wave detectors (676); Gravitational wave sources (677); Globular star clusters (656); Star clusters (1567); Bayesian statistics (1900); Active galactic nuclei (16)

1. Introduction

The LIGO–Virgo–KAGRA (LVK) Collaboration has so far reported 90 gravitational-wave signals of probable (>50% credible) astrophysical origin (Abbott et al. 2019; Abbott et al. 2021a, 2021b, 2021c), all consistent with coming to mergers of a compact binary: a binary black hole (BBH), binary neutron star (BNS), or neutron star–black hole binary (NSBH). The provenance of these compact-object binary mergers is an open question in gravitational-wave astrophysics. In order for an isolated pair of stars to merge as a compact binary on an observable timescale, it must undergo specific evolutionary scenarios. Typically, isolated binaries must either harden through Roche-lobe overflow mass transfer (van den Heuvel et al. 2017; Neijssel et al. 2019; Bavera et al. 2020; Gallegos-Garcia et al. 2021; Olejak et al. 2021) or common-envelope evolution (Livio & Soker 1988; Bethe & Brown 1998; Ivanova et al. 2013; Kruckow et al. 2016), or be born with a small enough separation that makes chemically homogeneous evolution possible (e.g., de Mink et al. 2010; de Mink & Mandel 2016; Marchant et al. 2016). Compact objects can instead be driven to merge rapidly by dynamical interactions, which can happen in populous environments like star clusters (e.g., Rodriguez et al. 2018a, 2018b; Samsing et al. 2018; Fragione et al. 2020).

The different processes that facilitate the merger of a binary leave their signature on the resulting gravitational-wave signal. Multiple studies have shown how the compact-object masses, spins, and orbital eccentricities inferred from the signal may act as identifiers of different formation scenarios, for both individual events and the contribution of different formation pathways to the entire population (e.g., Stevenson et al. 2015; Gerosa & Berti 2017; Vitale et al. 2017; Bavera et al. 2020; Mapelli 2020; Sedda et al. 2020; Zevin et al. 2021b; Fragione et al. 2022). In addition, the redshift evolution of the merger rate should contain distinct contributions from different formation channels (e.g., Mandic et al. 2016; Franciolini et al. 2022), although this will only be resolvable after $O(100)$ detections (Fishbach et al. 2018).

Nonzero orbital eccentricity is arguably the most robust signature of dynamical formation.\textsuperscript{4} A binary undergoing isolated evolution is expected to circularize before its gravitational-wave frequency reaches the start of the LVK frequency band at 10 Hz (Peters 1964). In contrast, dynamically induced mergers often merge so rapidly that they retain nonnegligible orbital eccentricity at 10 Hz, a quantity that we refer to as $e_{10}$. Robust predictions exist for the distribution of binary eccentricities in dense star clusters (e.g., Samsing & Ramirez-Ruiz 2017; Rodriguez et al. 2018a, 2018b; Samsing 2018; Samsing & D’Orazi 2018; Zevin et al. 2019), with $\sim$5% of mergers in

\textsuperscript{4} Lidov–Kozai resonant oscillations can drive up the eccentricity of a merging binary in an isolated triple system (Kozai 1962; Lidov 1962; Naoz 2016; Antonini et al. 2017), but the relative contribution of this channel to the observable eccentric merger rate is thought to be small even if optimistically low black hole natal kicks and metallicities are assumed (Silsbee & Tremaine 2017; Rodriguez & Antonini 2018).
these environments retaining $\epsilon_{10} \gtrsim 0.1$. Expectations from galactic nucleus and active galactic nucleus (AGN) disks are also becoming clearer, with up to 70% of mergers in AGN disks thought to retain $\epsilon_{10} \gtrsim 0.1$ (e.g., Samsing et al. 2020; Gondán & Kocsis 2021; Tagawa et al. 2021b; Vajpeyi et al. 2022).

Existing detectors are sensitive to eccentricities $\epsilon_{10} \gtrsim 0.05$ for BBH mergers (Lower et al. 2018; Romero-Shaw et al. 2019). Signal detection currently depends on achieving a high signal-to-noise ratio (S/N) when the data are matched-filtered using a quasi-circular signal template, so eccentric signals have reduced detectability compared to quasi-circular signals. Roughly half of the $\sim 7\%$ of mergers in dense star clusters with $\epsilon_{10} \gtrsim 0.05$ are recoverable with such a search (Zevin et al. 2021a). Therefore, accounting for current detector sensitivities and the loss of S/N power when using quasi-circular templates to search for eccentric signals, we expect to be able to measure the eccentricities of $\sim 4\%$ of mergers from dense star clusters. This percentage is an underestimate for the true fraction of detectably eccentric sources recovered, since it is based on the overlap between eccentric and quasi-circular waveforms with otherwise identical parameters.

Although nonzero orbital eccentricity is one calling card of dynamical formation, the component masses of a compact-object merger may also help to distinguish its origins. While pair-instability supernovae prevent black holes from forming between $\sim 60$ and $\sim 130M_\odot$ in isolation (Heger & Woosley 2002; Belczynski et al. 2016; Marchant et al. 2016; Fishbach & Holz 2017; Woosley 2017), dynamical environments can build massive black holes through hierarchical mergers or accretion (e.g., Fishbach et al. 2017; Gerosa & Berti 2019; Rodriguez et al. 2019; Anagnostou et al. 2020; Fragione & Silk 2020; Kimball et al. 2020; Kremer et al. 2020a; Samsing & Hotokezaka 2021; Banerjee 2021; Gerosa & Fishbach 2021; Zevin & Holz 2022). However, the uncertain limits and range of the pair-instability mass gap (Farmer et al. 2019; Belczynski 2020; Sakstein et al. 2020; Woosley & Heger 2021; Ziegler & Freese 2021) reduce the efficacy of compact-object mass as an identifier of formation channel.

The spin directions of a binary’s components can indicate its formation mechanism (Farr et al. 2017; Stevenson et al. 2017; Talbot & Thrane 2017). Isolated binaries are expected to have spins approximately aligned with the binary angular momentum vector (e.g., Kalogera 2000; Camppanelli et al. 2006; O’Shaughnessy et al. 2017; Gerosa et al. 2018), while dynamically assembled pairs in spherically symmetric environments should have an isotropic distribution of relative spin tilts (e.g., Rodriguez et al. 2016). However, mergers with aligned spins are not necessarily of isolated origin: gas torques in AGN disks can align component spins (Bogdanovic et al. 2007) if the timescale for dynamical interaction is sufficiently long (Liu & Lai 2017; Tagawa et al. 2020), and dynamically assembled binaries in open clusters can have aligned spins because few dynamical encounters can occur before merger (Trani et al. 2021).

Black hole spin magnitudes, too, can distinguish binary formation mechanisms. Merger products should develop high spins as they accumulate angular momentum through repeated mergers, so high spin magnitude can indicate dynamical formation (Fishbach et al. 2017; Kimball et al. 2020; Tagawa et al. 2021a), since the spins of black holes that form via stellar collapse are typically expected to be small. However, chemically homogeneous evolution (Mandel & de Mink 2016; Marchant et al. 2016; Qin et al. 2019), mass transfer and/or tidal locking in tight binaries (Izzard et al. 2003; Valsecchi et al. 2010; Qin et al. 2019; Bavera et al. 2020; Belczynski et al. 2020; Neijssel et al. 2021; Broekgaarden et al. 2022; Zevin & Bavera 2022), and differential rotation between the stellar core and envelope (Hirschi et al. 2005) can also produce rapidly spinning black holes.

In addition to those proceeding via isolated evolution or dynamical formation, mergers can occur between primordial black holes that form through direct collapse of density fluctuations in the early universe. While primordial black holes can have masses and spins that mimic those of isolated or dynamical black holes, with no upper or lower mass limit and preferentially zero spin (unless spun up by accretion over the majority of cosmic history), primordial mergers should have zero eccentricity at detection (Fragiolini et al. 2022; Green & Kavanagh 2021). Nonzero orbital eccentricity is also, therefore, a reliable way to rule out the primordial binary hypothesis.

Within the growing population of LVK observations are events that are challenging to explain through isolated stellar evolution. The first event to breach the pair-instability mass gap at $\sim 90\%$ confidence was GW190521 (Abbott et al. 2020a), which also exhibited signs of spin-induced precession and/or orbital eccentricity greater than 0.1 at 10 Hz (Romero-Shaw et al. 2020b; Calderón Bustillo et al. 2021a; Gamba et al. 2021; Gayathri et al. 2022). A second BBH merger, GW190620, also supports a dynamical formation hypothesis, with $\epsilon_{10} \gtrsim 0.05$ at 74% credibility (Romero-Shaw et al. 2021a). New events that support the dynamical formation hypothesis have emerged in GWTC-2.1 and GWTC-3. These include additional upper-mass-gap events, such as GW190426_19 (Abbott et al. 2021c) and GW200220_06 (Abbott et al. 2021b), and events consistent with having negatively aligned or substantially misaligned component spins, such as GW191109 and GW200129 (Abbott et al. 2021b; Hannam et al. 2022).\footnote{The strength of the support for anti-aligned or misaligned spins in GW200129 and GW191109 is contested by Payne et al. (2022) and H. Tong et al. (in preparation), respectively, who show that this support may be highly dependent on the data cleaning methods used.}

On top of these individual-event clues, population-level hints of dynamical formation come from evidence for hierarchical mergers (Kimball et al. 2021) and the mass-gap-encroaching shape of the inferred mass distribution. Hints of misaligned spins (negative effective spin parameter $\chi_{\text{eff}}$) have been claimed at the population level (Abbott et al. 2021d); some follow-up studies show that the population is consistent with the majority of binaries having nonzero and misaligned component spins, while others argue that the population is consistent with a majority of binaries having $\chi_{\text{eff}} = 0$ and only a small subset having significant positive $\chi_{\text{eff}}$ (Galaudage et al. 2021; Roulet et al. 2021).

Eccentricity is not included in the gravitational-waveform models used by LVK to produce the inferences reported in their catalogs, because incorporating the effects of eccentricity makes physically accurate waveform models too slow for conventional inference methods. In Romero-Shaw et al. (2019, 2020a, 2020b, 2021a), we used an efficient reweighting method to obtain measurements of the orbital eccentricity of gravitational-wave sources up to and including the second LVK gravitational-wave transient catalog, GWTC-2. In this work, we use the same method to analyze additional BBH
candidates from the most recent updates to the catalog of LVK events: GWTC-2.1 (Abbott et al. 2021c) and GWTC-3 (Abbott et al. 2021b). Our results come with important caveats: we cannot distinguish eccentricity from spin-induced precession, our analysis does not include higher-order modes, and we are limited to studying moderate eccentricities ($e_{\text{eff}} \lesssim 0.2$) and restricted spins ($\chi_{\text{eff}} \lesssim 0.6$). The limitations of our method are explained in detail in Section 2. Within our analysis framework, we report an additional two binaries with significant support for $e_{\text{eff}} \geq 0.05$ (>$60\%$ credibility), adding to the building of circumstantial evidence for a dynamically formed subset within the observed mergers.

This paper is structured as follows. In Section 2, we describe our methodology, and note its limitations. In Section 3 we present results from our analysis of new events from GWTC-2.1 and GWTC-3, taking the total number of BBH candidates investigated for signatures of eccentricity to 62. Two events, GW191109 and GW200208_22, have $\geq 70\%$ of their posterior support at $e_{\text{eff}} \geq 0.05$ and have inconclusive but positive natural log (ln) Bayes factors in favor of the eccentric hypothesis, with $\text{ln} B(e_{\text{eff}} \geq 0.05) \gtrsim 1.4$, where we use the convention that $\text{ln} B \gtrsim 8$ constitutes “strong” evidence. We present analyses of eccentricity at the population level in Section 4, and demonstrate that $\geq 80$ detectably eccentric mergers are required to confidently distinguish different dynamical formation scenarios at current detector sensitivity. In Section 5, we conclude with some final thoughts. Results for events with negligible eccentricity are provided in Appendix A.

2. Method

We use a reweighting method (see Payne et al. 2019; Romero-Shaw et al. 2019) to efficiently calculate posterior probability distributions using the aligned-spin eccentric waveform model SEOBNRE (Cao & Han 2017). First, we run an importance-sampling step, performing Bayesian inference using bilby and the bilby_pipe pipeline (Ashton et al. 2019; Romero-Shaw et al. 2020c). We run five parallel analyses with unique seeds for each event with the dynesty sampler (Speagle 2020), utilizing the spin-aligned quasi-circular model IMRPhenomD (Khan et al. 2016) as the “proposal” model. For these initial analyses, we use 1000 live points, 100 walks, and 10 autocorrelation times. For follow-up analyses on eccentric candidates, we use 4000 live points and 200 walks. We use a sampling rate of 4096 Hz and a reference frequency of 10 Hz for every event. We analyze publicly available data from GWTC-2.1 and GWTC-3 (Abbott et al. 2021b, 2021c, 2021e, 2021f) and use detector noise curves generated using BayesWave (Cornish & Littenberg 2015; Littenberg & Cornish 2015).

We reweight the proposal samples obtained in the initial step to our “target” model: SEOBNRE, a spin-aligned eccentric waveform approximant containing the inspiral, merger, and ringdown sections of the signal. Since this is a time-domain model, we use a Fourier transform to obtain frequency-domain waveforms for use in the likelihood, softening the abrupt start of the time-domain inspiral using a half-Tukey window to avoid spectral leakage.

We use standard priors on R.A. $\alpha$, decl. $\delta$, source inclination $\theta_{\text{ins}}$, polarization $\phi$, coalescence phase $\psi$, and geocent time $t_0$. Our prior on mass ratio $q$ is uniform between 0.125 and 1, and our priors on the aligned component spin magnitudes $\chi_1$, $\chi_2$ are capped at the SEOBNRE maximum of $\pm 0.6$. We use uniform priors on chirp mass $M$ and priors on luminosity distance $d_L$ that are uniform in the source frame. When reconstructing the eccentricity distribution with SEOBNRE, we employ a log-uniform prior on eccentricity, which covers the range $10^{-4} \leq e_{\text{eff}} \leq 0.2$. We marginalize over phase and coalescence time to mitigate definitional differences between the two models.

Including all events in GWTC-3, we report results for 62 BBH candidates in total. In this work, we present analyses of events added to the catalog in GWTC-2.1 and GWTC-3; for analyses of events added to the catalog in earlier LVK publications using the same method as used here, see Romero-Shaw et al. (2019, 2021a). We reserve detailed analyses of other events, including binaries that contain neutron stars, for future work. The events that we do not discuss in this work are as follows:

1. Events for which the reweighting process fails to achieve an adequate effective sample size. The number of effective samples in a posterior distribution after reweighting is

$$n_{\text{eff}} = \frac{\left(\sum_{i=1}^{n} w_i\right)^2}{\sum_{i=1}^{n} w_i^2}.$$  \hspace{1cm}(1)

We deem any events with $n_{\text{eff}} < 100$ undersampled, and do not include them in this work. Potential causes of undersampling are explored in Appendix B.

2. Binaries likely to contain at least one neutron star. These include those confidently designated as BNSs (GW170817 and GW190425), NSBH binaries (GW191219, GW200105, and GW200115), and events with secondaries of ambiguous mass that may be black holes or neutron stars (GW190814, GW190917, and GW200210). We neglect BNS and likely NSBH events from the analysis presented in the main body of this paper because their formation mechanisms may be drastically different from those of BBH mergers, and we wish to make statements about the formation channels that produce the BBH mergers in our population. When studied with the reweighting method, we find that GW190814A fails to achieve a high reweighting efficiency; this is expected, since higher-order modes are known to be present in this signal, and both models used in this work incorporate only the $\ell = 2$ mode (Abbott et al. 2020b). GW190412, which is thought to be an unequal-mass black hole binary with mass ratio $q \sim 0.27$, also has higher-order mode content, and also ends up undersampled. We do not provide posterior probability distributions for GW190917 and GW200210, despite their adequate sample size after reweighting; their mass ratio posteriors rival against the lower-mass-ratio prior limits, implying that the true probability distribution extends below the lowest value of mass ratio contained within the prior, and the eccentricity distribution inferred for both events is uninformative. We leave eccentric analyses of systems containing low-mass black holes or neutron stars for future work.

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6. BNS mergers have so far been found to be consistent with quasi-circularity (Lenon et al. 2020; Romero-Shaw et al. 2020a).
The Astrophysical Journal, 940:171 (18pp), 2022 December 1
Romero-Shaw, Lasky, & Thrane

2.1. Caveats

Our analysis method leads to our results having the following caveats:

1. Since the waveform model that we employ, SEOBNRE, does not support misaligned spins, we are not able to disentangle the effects of orbital eccentricity and spin-induced precession on the signal. This may lead us to infer nonzero eccentricity for a quasi-circular system undergoing spin precession; see Romero-Shaw et al. (2020b).

2. SEOBNRE enforces a dimensionless aligned-spin magnitude upper limit of 0.6. Any binary that is truly highly spinning will produce a signal that is poorly specified by our choice of waveform model, and will therefore bias our results.

3. Similarly, the upper limit of our eccentricity prior is 0.2 at a detector-frame gravitational-wave frequency of 10 Hz. Any binary with an eccentricity higher than this will not be correctly specified by our choice of waveform model. However, we have seen in the case of GW190521 that systems consistent with larger eccentricities (Calderón Bustillo et al. 2021b; Gamba et al. 2021; Gayathri et al. 2022) can still show signs of high eccentricity in our analyses, railing against the model-enforced upper limit of the prior (see, e.g., Romero-Shaw et al. 2020b). Such railing, in which the peak of the posterior distribution is seen to "pile up" at the prior boundary, implies that the truly most probable region of the parameter space likely exists outside the range covered by the prior.

4. Waveforms produced with IMRPhenomD and SEOBNRE do not contain higher harmonics, which may mislead inference when higher harmonics are present in the data. Neglecting higher-order modes can bias recovery of parameters like spins and mass ratios (Shaik et al. 2020), so we expect that inferences of eccentricity would also be biased. Although there are no tailored studies to assess the eccentricity bias when higher-order modes are neglected, quasi-circular and spin-precessing waveform templates with higher-order modes may be more likely mistaken for eccentric waveforms than those without higher-order modes (Romero-Shaw et al. 2020b). Recent analyses including both eccentricity and higher-order modes (Iglesias et al. 2022) have obtained qualitatively consistent results with respect to previous analyses neglecting higher-order modes.

5. SEOBNRE sets the initial argument of periapsis based on the fixed starting frequency of waveform generation. This parameter is therefore not adjustable and cannot be sampled over. We anticipate that being able to sample over this parameter could lead to a shift in the locations of the peaks of the recovered eccentricity distributions in events with high S/N (≈ 30), but the consequences of neglecting this parameter for the events studied here are likely to be small (Clarke et al. 2022).

6. Different waveform models and simulations use different definitions of eccentricity, and use different prescriptions to set initial conditions. This means that an eccentricity inferred with SEOBNRE does not exactly equate to the eccentricity that would be inferred with another model, and that comparisons to predictions made by simulations of dynamical environments should be taken as indicative rather than absolute. Work is ongoing to establish a translation guide between the eccentricities defined by various simulations (Knee et al. 2022).

7. We restrict our analyses to events that have been flagged as likely compact binary merger signals by LVK searches. In order to be flagged as such, a signal must bear significant resemblance to a quasi-circular inspiral track so that it achieves a high match with the waveforms used in those templated searches. As a result, the events we analyze are highly likely to have small or negligible eccentricities.

3. Eccentricity Measurements

In Figure 1, we provide marginal one-dimensional eccentricity posterior distributions for 62 BBH candidates. We note that we have removed two events that are found to have below-threshold significance in GWTC-2.1 (GW190924A and GW190909A;
Table 1
Summary of the Detector-frame Eccentricity Measurements for Newly Analyzed Events in GWTC-3 with a ln Bayes Factor Greater than 0 for the Hypothesis that $e_{10} \geq 0.05$ and Number of Effective Samples $\geq 100$

| Event Name | $e_{10} \geq 0.1$ (%) | $e_{10} \geq 0.05$ (%) | ln $B(e_{10} \geq 0.1)$ | ln $B(e_{10} \geq 0.05)$ | $n_{\text{eff}}$ |
|------------|------------------------|------------------------|------------------------|------------------------|----------------|
| GW190403   | 16.64                  | 27.53                  | 0.83                   | 0.56                   | 6358          |
| GW190805   | 14.33                  | 23.78                  | 0.19                   | 0.03                   | 719           |
| GW191105   | 10.58                  | 18.52                  | 0.29                   | 0.12                   | 718           |
| **GW191109** | **62.52**              | **71.53**              | **1.89**               | **1.49**               | **7125**      |
| GW191126   | 26.28                  | 33.79                  | 1.27                   | 0.92                   | 293           |
| GW191127   | 23.68                  | 33.33                  | 1.20                   | 0.88                   | 436           |
| **GW200208 22** | **70.78**             | **73.06**              | **1.94**               | **1.39**               | **219**       |
| GW200209   | 16.58                  | 26.63                  | 0.64                   | 0.44                   | 43,848        |
| GW200216   | 10.62                  | 22.43                  | 0.26                   | 0.25                   | 3108          |
| GW200322   | 12.57                  | 21.37                  | 0.12                   | 0.07                   | 10,203        |

Note. The two events with greater than 50% of their posterior support at $e_{10} \geq 0.05$ are emboldened. The second and third columns provide the percentage of posterior support for $e_{10} > 0.1$ and $e > 0.05$, two values typically used as thresholds for currently detectable binary eccentricity at 10 Hz. Further, these values are somewhat arbitrary, as each individual signal has a different detectable eccentricity threshold. Furthermore, simulations that quote eccentricity resulting from dynamical binary formation (e.g., Wen 2003; Gondán et al. 2018; Rodriguez et al. 2018a, 2018b; Samsing et al. 2018; Gondán & Kocsis 2021). We restrict the eccentricity prior to astrophysically motivated ranges for these Bayes-factor calculations to avoid contamination from quasi-circular samples, which do not represent the eccentric hypothesis. We present in Figure 2 marginal posteriors on log($e_{10}$) for the 10 newly analyzed events in GWTC-3 that have positive ln Bayes factors in favor of the eccentric hypothesis when compared to the quasi-circular hypothesis. We adopt the convention that a “detection” of eccentricity is not made unless ln $B \geq 8$, and so do not claim that any of these events definitively prefer the eccentric model over the quasi-circular model. Posterior probability distributions on all parameters of all analyzed events are provided online.⁷

3.1. New Events in GWTC-3 with Majority Posterior Support for $e_{10} \geq 0.05$

There are two new events that show significant evidence for eccentricities above $e_{10} = 0.05$ within their posterior probability distributions, with ≥50% of their posterior probability support above $e_{10} = 0.05$. For these events, and for the two eccentric candidates GW190521 and GW190620 (Romero-Shaw et al. 2020b, 2021a), we conduct our analysis with more aggressive sampler settings (4000 live points and 200 walks) to obtain a higher number of effective samples. These results are shown in Figure 1 and all other figures, and in Table 1. The waveforms corresponding to the median posterior parameters for GW190521, GW190620, GW191109, and GW200208 are shown in Figure 3.

GW191109 was found by the LVK analysis to have the highest support for negative spin of all new GWTC-3 events. Our initial quasi-circular analysis recovers this preference for negatively aligned spins, but when we reweight to the eccentric posterior, higher spin magnitudes have lower weights. This is similar to the reweighting behavior observed for eccentric candidate GW190620 (Romero-Shaw et al. 2021a), although we retain some appreciable deviation from the spin prior for $\chi_1$ after reweighting for GW191109. Other parameters are consistent with those recovered in the LVK analysis. Proposal (quasi-circular) posteriors for GW191109 are shown in Figure 4 in teal, and target (eccentric) posteriors are overplotted in gray.

We find that GW191109 has 72.19% of its posterior support above $e_{10} = 0.05$, 62.63% of its posterior above $e_{10} = 0.1$, and a ln Bayes factor of ln $B = 1.49$ (1.89) in favor of the $e_{10} > 0.05$ ($e_{10} \geq 0.1$) hypothesis relative to the quasi-circular hypothesis. For this event, we obtain a reweighting efficiency of 4.52%, and $n_{\text{eff}} = 7125$. Visible in Figure 1 is the double-peaked structure of the eccentricity posterior for GW191109: the main peak is at the upper limit of the eccentricity prior, $e_{10} = 0.2$, but there is a subdominant mode at $e_{10} \approx 0.1$. The eccentric (target) posterior distribution on the phase of coalescence, $\phi$, for GW191109 has periodic peaks that are not present in the quasi-circular (proposal) posterior distribution. Since GW191109 is relatively high-mass, its $e_{10}$ measurement gives the shape of its orbit just a few cycles before the coalescence itself. Having distinctly nonzero orbital eccentricity at this point means that the orbit close to merger is elongated, so the gravitational-wave emission varies more strongly with $\phi$. Additionally, $\phi$ is likely to closely correlate with the argument of periapsis, $\omega$; while $\phi$ describes the shape of the orbit, $\omega$ describes the angle of rotation of the orbit itself. Since $\omega$ is set indirectly through the reference frequency $f_{\text{ref}}$ and eccentricity $e_{10}$ in our analyses, the narrow range of $e_{10}$ and fixed $f_{\text{ref}}$ may restrict $\omega$ enough that only certain values of $\phi$ can produce waveforms consistent with the data.

³ github.com/IsobelMarguarethe/eccentric-GWTC-3
We find that, when analyzed with our default sampling settings, the mass ratio posterior for GW200208_22 rails against the lower limit of the prior, indicating that the lower limit is too high to capture the full extent of the posterior. Therefore, for this event, we reduce the lower-mass-ratio prior limit from 0.125 to 0.025. We notice also that the posterior rails against the upper limits of $\chi_1$ and $\chi_2$ at 0.6, implying that the true distribution extends above the upper limits allowed by the prior. However, since this is a limit enforced by SEOBNRE, we do not relax this limit. In Figure 5, we show proposal (quasi-circular) posteriors for GW200208_22 in teal and target (eccentric) posteriors in gray.

We find that GW200208_22 has 76.71% of its posterior support above $e_{10} = 0.05$, with 73.52% above $e_{10} = 0.1$, and a ln Bayes factor for $e_{10} \geq 0.05$ ($e_{10} \geq 0.1$) relative to the quasi-circular hypothesis of 1.45 (1.92). In our analysis, GW200208_22 has a a source-frame chirp mass of $17.38^{+3.10}_{-4.54} M_\odot$. The reweighting efficiency for GW200208_22 is relatively low at 0.18%, consistent with our expectations for low chirp mass (long-duration) eccentric signals. Additionally, the waveform model choice has been shown to be important for this event (Abbott et al. 2021b), so posterior differences that are not correlated with eccentricity may be a result of waveform systematics. We obtain 219 effective samples for this event.

Our posteriors peak at a lower total mass ($\sim 41 M_\odot$) than that found in the LVK analysis ($\sim 63 M_\odot$), although the posteriors do overlap: while the LVK posteriors and our IMRPhenomD posteriors are multimodal, the eccentric posterior favors the lower-mass peak. The median primary mass recovered with the eccentric model ($\sim 25 M_\odot$) is significantly lighter than the median of the LVK analysis ($\sim 51 M_\odot$), for the same reason. As a result, the median luminosity distance recovered is roughly 1 Gpc smaller than the median LVK result. It is possible for eccentric systems to masquerade as higher-mass quasi-circular systems when their gravitational-wave signals are analyzed assuming quasi-circularity, since eccentricity can drive a binary
to merge on a faster timescale and at a lower frequency (see, e.g., Calderón Bustillo et al. 2021b; Favata et al. 2022). In
addition to the inclusion of eccentricity, a reason for the discrepancy in median posterior parameters may be our
enforced limit on the spin prior magnitude due to the
limitations of SEOBNRE: while the LVK analysis finds that
\( \chi_1 \geq 0.29 \) (90\% credibility) with 51\% of its posterior support
above \( \chi_1 = 0.8 \), we infer a posterior that rails against the upper
limit of the prior at \( \chi_1 = 0.6 \).

As in the LVK analysis, the mass ratio posterior recovered with
IMRPhenomD is quite flat, with a slight preference for
unequal masses \( (m_2/m_1 \approx 0.3) \). When we reweight to
SEOBNRE, the long tail out to high masses is downweighted
and only the peak at chirp mass \( \mathcal{M} \approx 16 M_\odot \) remains. More
equal mass ratios are favored by the eccentric model over
unequal masses. Higher values of \( \chi_1 \) and \( \chi_2 \) are also
downweighted, with the eccentric model preferring samples
with high values of eccentricity and low values of spin. Again,
the reason for the difference may be the enforced spin prior
limitation. It is possible that the data is best fit by a waveform
with \( \chi_1 \approx 0.9 \) and \( e_{10} = 0 \), but prefers a waveform with \( \chi_1 = 0 \) and \( e_{10} = 0.2 \) to one with \( \chi_1 \approx 0.5 \). In this case, because of our
restricted prior, the vast majority of the eccentric posterior is at
high eccentricities with low spins. Previous analyses have
shown that when spin amplitude is restricted, higher eccentricities
may be favored (see O’Shea & Kumar 2021).

3.2. Other New Events in GWTC-3 with Nonnegligible Support for \( e_{10} \geq 0.05 \)

Another two candidates from O3b, GW191126 and
GW191127, have \( >30\% \) of their posterior support at
\( e_{10} \geq 0.05 \) and \( \ln B(e_{10} \geq 0.1) > 1.0 \). Their marginal eccentricity posterior distributions show peaks at our prior upper
limit of \( e_{10} = 0.2 \) and nonnegligible tails down to lower eccentricities. While we also analyze these events with more
aggressive sampler settings, we do not discuss these events in
detail here; we reserve a detailed analysis of these events, in
conjunction with the marginal eccentric candidates presented in
Romero-Shaw et al. (2021a), for future work.

3.3. Notable New Events in GWTC-3 with \( e_{10} \leq 0.05 \)

3.3.1. Mass-gap Events

GW190426_19 has the highest mass of all binary mergers
reported by LVK, with both components more massive than
predicted by isolated evolution: \( m_1 = 106.9^{+41.6}_{-25.2} M_\odot \),
\( m_2 = 76.6^{+26.2}_{-33.6} M_\odot \) (Abbott et al. 2021c). Our analysis of
GW190426_19 recovers parameters consistent with those
recovered in the LVK analysis, including the slight deviation
from the prior at higher values of \( \chi_{eff} \). Another high-mass event, GW200220_06, is found by LVK to have mass-gap
components: \( m_1 = 87^{+40}_{-25} M_\odot \), \( m_2 = 61^{+26}_{-25} M_\odot \) (Abbott et al.
2021b), consistent with our findings. We find that
GW190426_19 and GW200220_06 do not contain hints of orbital eccentricity.

GW190426_19 and GW200220_06 have high reweighting
efficiencies of 72\% and 83\%, respectively, with 11\% and 13\%
of their posterior support above \( e_{10} = 0.05 \). Reweighting to
SEOBNRE pushes the preferred mass ratio and source-frame
chirp mass to slightly lower values for GW190426B, but this
shift does not appear correlated with eccentricity; we put the
difference down to waveform systematics. There is virtually no
difference between the quasi-circular and eccentric posteriors
for GW200220_06.

Being consistent with quasi-circular at 10 Hz does not mean
that these binaries are not dynamically formed: we expect only
\( \sim 4\% \) of our detected mergers from globular clusters (GCs)
to retain detectable eccentricity at this frequency (Zevin et al.
2021a), and more massive mergers circularize at lower
frequencies than their lower-mass counterparts. It is none-
theless worth noting that the eccentricity, spin magnitude, and
spin tilt measurements for these systems are inconclusive. If
they contained merger remnants, which became bound through
dynamical interactions, their dimensionless spin magnitudes
should be \( \chi_{eff} \sim 0.7 \), and their spin tilt angles would likely be
misaligned (e.g., Pretorius 2005; González et al. 2007; Buonanno et al. 2008). Alternatively, such massive binaries may
form in isolation if the pair-instability mass gap is
narrower than predicted (as suggested in the wake of
GW190521 by, e.g., Costa et al. 2021), or if our standard
priors on mass ratio are misleading inference (also suggested to
explain GW190521 by Fishbach & Holz 2020).
3.3.2. Spinning Events

A number of the events that strongly support nonzero spins are undersampled after the reweighting process: GW200129, which exhibits support for signs of spin-induced precession; GW191204, which has a $\chi_{\text{eff}}$ posterior tightly constrained away from zero; and GW191216, which has negligible support for $\chi_{\text{eff}} = 0$ (Abbott et al. 2021b). However, some are adequately sampled—GW191103, for example, which has 175 samples after reweighting. The marginal eccentricity posterior for this event is uninformative, and correlated with spin: lower magnitudes of $\chi_1$ are favored for samples with $e_{10} \geq 0.05$. The eccentric reweighting process disfavors larger values of $\chi_1$, while the marginal $\chi_2$ posterior is relatively unchanged. GW191103A has only 17% of its posterior support above $e_{10} = 0.05$ after reweighting.
The Astrophysical Journal, 940:171 (18pp), 2022 December 1

### 3.4. Spin-induced Precession or Eccentricity?

There are currently no waveform models that incorporate the simultaneous effects of eccentricity and spin-induced precession on the signal. Since the two effects can cause similar phase and amplitude modulations in gravitational-wave signals (e.g., Calderón Bustillo et al. 2021b), they can cause spin-aligned analyses to recover eccentricity, or quasi-circular analyses to recover misaligned spins (Romero-Shaw et al. 2020b; Calderón Bustillo et al. 2021a). Therefore, any nonzero eccentricity measurements that we infer in our analysis may, in actuality, be caused by the binary having misaligned spins.

While we cannot simultaneously infer the presence of spin-induced precession and eccentricity, we can attempt to deduce which effect is more likely to be present in the signal. We perform analyses on GW190521, GW190620, GW191109, and GW200208_22 using a precessing waveform approximant; see Romero-Shaw et al. (2020b) for an extensive comparison between the eccentric and spin-precessing hypotheses for GW190521. For the study presented here, we employ one of the preferred waveforms used in the transient catalog of LVK (Abbott et al. 2021a, 2021b, 2021c), the quasi-circular spin-precessing model IMRPhenomXPHM (Pratten et al. 2021), allowing the full range of available spin orientations in our priors and component spin magnitudes up to 0.89, and using the same aggressive sampler settings as employed for our follow-up analyses using SEOBNRE.

Table 2 contains the relative ln Bayes factors of the spin-aligned, eccentric hypothesis (calculated from our SEOBNRE posteriors above thresholds of \(e_{10} = 0.05\) and 0.1) against the quasi-circular, spin-precessing hypothesis (calculated from our IMRPhenomXPHM posteriors) for GW190521, GW190620, GW191109, and GW200208_22. There is a marginal preference for the eccentric hypothesis for GW190521, GW190620, and GW200208_22, and a marginal preference for the spin-precessing hypothesis for GW191109; however, in no case is the evidence for either hypothesis overwhelming. Thus, each of our potentially eccentric candidates could be eccentric, spin-precessing, or both.

### 4. Eccentricity in the Population

As the catalog of mergers grows, it becomes increasingly likely that random noise fluctuations emulate the effects of eccentricity in a subset of BBH merger signals. Additionally, while we highlight four events that have clear peaks above \(e_{10} = 0.05\) in their eccentricity posterior probability distributions, there are multiple other events that show significant support for \(e_{10} \geq 0.05\). We wish to quantify the fraction of observed mergers that truly support the eccentric merger hypothesis, without assuming any specific formation channel. We perform population analyses under the hypothesis that some binaries have some support for eccentricity \(e_{10}\) above some threshold eccentricity, \(e_{\text{thresh}}\). We consider two possibilities: that \(e_{\text{thresh}} = 0.05\) and that any eccentricity lower than this is not detectable, and a more conservative hypothesis with \(e_{\text{thresh}} = 0.1\). We calculate a likelihood for \(f\), the fraction of the population support for \(e_{10} \geq e_{\text{thresh}}\):

\[
\mathcal{L}(d|f) = \prod_k \left( \int_{e_{\text{min}}}^{e_{\text{max}}} \, de \, \pi(e) \mathcal{L}(d_k|e) 
+ (1-f) \int_{e_{\text{thresh}}}^{e_{\text{max}}} \, de \, \pi(e) \mathcal{L}(d_k|e) \right). \tag{2}
\]

Here, \(k\) represents each event in our population, \(e_{\text{min}}\) and \(e_{\text{max}}\) are our eccentricity prior bounds, and \(\pi(e)\mathcal{L}(d_k|e)\) is the marginal posterior probability distribution for the eccentricity of event \(k\). Drawing proposals for the value of \(f\) from a uniform prior and computing \(\mathcal{L}(d|f)\) over this range produces a posterior probability distribution for \(f\). This posterior is plotted in Figure 6 for both \(e_{\text{thresh}}\) conditions.

The highest-probability \(f\) representing the fraction of observed BBHs with \(e_{10} \geq 0.05\) is 0.40, corresponding to 25 mergers, while the maximum posterior \(f\) for BBHs observed with \(e_{10} \geq 0.1\) is 0.19, corresponding to 12 mergers. We can exclude \(f = 0\) with greater than 2\(\sigma\) credibility in both cases. We therefore conclude that the population support for eccentricity in GWTC-3 is consistent with a nonnegligible fraction of mergers exhibiting detectable eccentricity at 10 Hz.

The fact that we include only the adequately sampled events in the calculation of \(f\) may bias our results. Reweighting can fail
4.1. Implications for Population Formation Channels

Simulations suggest that 5%–10% of BBH mergers in dense star clusters should enter the LVK sensitivity band with $e_{10} \geq 0.05$ (see, e.g., Rodriguez et al. 2018a, 2018b, 2019; Samsing 2018; Samsing et al. 2019; Zevin et al. 2019; Kremer et al. 2020b). As noted above, Zevin et al. (2021a) showed that, from a simulated population of mergers from the CMC Cluster Catalog (Kremer et al. 2020b), only 3.9% of GC mergers could be detected with $e_{10} \geq 0.05$, assuming a templated search using quasi-circular waveforms with otherwise identical parameters to the injected eccentric signals. Under this assumption, we expect to detect only 56% of sources with $e_{10} \geq 0.05$, since gravitational-wave signals from compact binary mergers are detected using search methods that assume quasi-circular inspirals. This fraction may be higher, if eccentric signals achieve maximum S/N against quasi-circular templates with different source parameters (as suggested by the results of, e.g., O’Shea & Kumar 2021). However, if the parameters preferred by the quasi-circular model are totally removed from the true quasi-circular parameters, then the eccentricity posterior obtained through the reweighting method is unlikely to both recover a high eccentricity and be well sampled; therefore, we use 3.9% as our expected eccentric fraction. The predictions in Zevin et al. (2021a) were obtained using a different eccentric waveform model, TEOBResumS (Nagar et al. 2018); for an overview of key differences between TEOBResumS and SEOBNRE, see Knee et al. (2022). Since the overlap between SEOBNRE and TEOBResumS is $\mathcal{O} \geq 90\%$ in LVK noise over the eccentricity range studied here (Knee et al. 2022), we assume in this paper that the results of Zevin et al. (2021a) are robust to waveform choice. However, we caution that this may not be the case; a repeated study using SEOBNRE is required to obtain waveform-model-specific predictions. We reserve this for future work, and note that our distributions for $B_s$ based on predictions using the alternative waveform model are therefore approximate.

4.1.1. Comparing Detector-frame Eccentricity Measurements to Simulation Predictions

Eccentricity distributions obtained from simulations of dense star clusters are naturally quoted at a reference frequency of 10 Hz in the source frame, while we measure eccentricity at a reference frequency of 10 Hz in the detector frame. Since redshifting pushes detector-frame frequencies lower than their source-frame origins, this means that the lower limits that we report for possibly eccentric events are overly conservative for binaries in the source frame. On the flip side, upper limits reported for noneccentric events are less conservative for source-frame binaries. We convert measurements of eccentricity into the source frame by establishing the source-frame frequency corresponding to a detector-frame frequency of 10 Hz: $f_{\text{source}} = 10(1 + z)$ Hz. We then back-evolve $e_{10}$ from $f_{\text{source}} = 10(1 + z)$ Hz to $f_{\text{source}} = 10$ Hz using Peters’s equations (Peters 1964).

An additional complication comes from conflicting definitions of the reference frequency at which eccentricity is quoted. The reference frequency of the SEOBNRE model, $f_{\text{SEOBNRE}}$, is defined relative to a closed Keplerian orbit, with a semimajor axis that changes as the binary inspirals. Meanwhile, simulations of cluster mergers report eccentricities defined at the peak frequency of gravitational-wave emission, $f_{\text{peak}}$. Within SEOBNRE, an eccentric correction is applied such that the minimum frequency of gravitational-wave content is $f_{\text{SEOBNRE}}/\text{min} = f_{\text{SEOBNRE}} / (1 - e_{\text{SEOBNRE}}^{\text{peak}})^{1.5}$. This means that at $f_{\text{SEOBNRE}} = 10$ Hz, the maximum difference between $f_{\text{SEOBNRE}}$ and $f_{\text{SEOBNRE}}/\text{min}$ is 0.63 Hz (for $e_{\text{SEOBNRE}} = 0.2$). We start analysis for most events from 20 Hz, the exception being GW190521, which we start from 11 Hz. Therefore, this internal eccentric correction does not cause the waveform to start within our analysis band. However, our reported $e_{10}$ measurements are not directly comparable to the predictions of GC simulations, since (Wein 2003)

$$f_{\text{peak}} = f_{\text{SEOBNRE}} (1 - e_{\text{SEOBNRE}}^{2})^{-1.5}(1 + e_{\text{SEOBNRE}}^{2})^{1.954}.$$  \hspace{1cm} (3)

The maximal difference between $f_{\text{SEOBNRE}}$ and $f_{\text{peak}}$ is therefore 3.2 Hz (for $e_{\text{SEOBNRE}} = 0.2$).

We convert from an eccentricity distribution defined at $f_{\text{SEOBNRE}} = 10$ Hz in the detector frame to its equivalent at $f_{\text{peak}} = 10$ Hz in the source frame using Equation (3) and Peters’s equations (Peters 1964). We caution that these results be taken as indicative rather than exact measurements: evolving back to $f_{\text{peak}} = 10$ Hz pushes $f_{\text{SEOBNRE}}$ to lower frequencies. Therefore, we are implicitly assuming that the inspiral of the system is unperturbed at lower frequencies, corresponding to earlier times.

In the left-hand panel of Figure 7, we plot measured median and upper 90% credible intervals on eccentricity at 10 Hz in the detector frame, $e_{10}$, against the source-frame chirp mass $M$ of the 62 BBHs analyzed in this work. On the right-hand side, we plot median and upper 90% credible intervals for $e_{10,\text{peak-source}}$ the eccentricity at a peak gravitational-wave frequency of 10 Hz in the source frame, following the conversions described above. The number of binaries with $e_{10,\text{peak-source}} \geq 0.05$ is the same as that with detector-frame eccentricity at 10 Hz Keplerian frequency $e_{10} \geq 0.05$: we find four binaries above this threshold in both cases.
While all other events are shown in gray. Undersampled events are not plotted. Negligible-eccentricity sources; high-eccentricity (size after reweighting, we are biased toward detecting low- or probability distribution in order to retain a sufficient distribution overlaps enough with the quasi-circular posterior. Since we require that the eccentric posterior probability is zero, we use the posterior, i.e., we ignore parts of the likelihood. Therefore, we may be underreporting the number of eccentric mergers in the observed population as a Poisson counting problem. The likelihood of detecting four eccentric binaries within 62 mergers is represented by the teal curve, with the gray-shaded rectangle indicating the region in which the prior probability goes to zero ($B_1 > 1$).

Since we still have a small population and many uncertainties from both our method and our theoretical models, we cannot constrain $B_1$ very tightly or confidently at this stage. Since we require that the eccentric posterior probability distribution overlaps enough with the quasi-circular posterior probability distribution in order to retain a sufficient sample size after reweighting, we are biased toward detecting low- or negligible-eccentricity sources; high-eccentricity ($e_{10} > 0.2$) sources may be mistaken for quasi-circular mergers if they are not undersampled. Therefore, we may be underreporting the true number of eccentric mergers in the set of 62 adequately sampled events we consider. Conversely, spin effects can mislead inferences of eccentricity (e.g., Romero-Shaw et al. 2020b; Calderón Bustillo et al. 2021b; O’Shea & Kumar 2021); the signals that we infer to show the fingerprints of eccentricity may in fact be due to spin-induced precession. If four of our eccentric candidates are truly eccentric, and if all eccentric mergers are formed within GCs, then GCs must contribute at least 35% of observed mergers, at 95% credibility (to calculate this we use the posterior, i.e., we ignore parts of the likelihood greater than 1).

4.1.2. Constraining the Fractional Contribution of Mergers Formed in Dense Star Clusters

We can now use the fraction of binaries detected with $e_{10} \geq 0.05$ to constrain the fractional presence of mergers produced in dense star clusters within the population. Consistent with Zevin et al. (2021a), we call this the branching fraction, $B_1$. In Figure 8, we plot in teal the likelihood on $B_1$ calculated following Zevin et al. (2021a), which treats the number of eccentric mergers in the observed population as a Poisson counting problem. The likelihood of detecting four eccentric binaries within 62 mergers is represented by the teal curve, with the gray-shaded rectangle indicating the region in which the prior probability goes to zero ($B_1 > 1$).

Since we still have a small population and many uncertainties from both our method and our theoretical models, we cannot constrain $B_1$ very tightly or confidently at this stage. Since we require that the eccentric posterior probability distribution overlaps enough with the quasi-circular posterior probability distribution in order to retain a sufficient sample size after reweighting, we are biased toward detecting low- or negligible-eccentricity sources; high-eccentricity ($e_{10} > 0.2$) sources may be mistaken for quasi-circular mergers if they are not undersampled. Therefore, we may be underreporting the true number of eccentric mergers in the set of 62 adequately sampled events we consider. Conversely, spin effects can mislead inferences of eccentricity (e.g., Romero-Shaw et al. 2020b; Calderón Bustillo et al. 2021b; O’Shea & Kumar 2021); the signals that we infer to show the fingerprints of eccentricity may in fact be due to spin-induced precession. If four of our eccentric candidates are truly eccentric, and if all eccentric mergers are formed within GCs, then GCs must contribute at least 35% of observed mergers, at 95% credibility (to calculate this we use the posterior, i.e., we ignore parts of the likelihood greater than 1).

4.2. A Population Model for the Eccentricity Distribution

There are multiple pathways that can lead to BBH mergers with detectable eccentricity in the LIGO–Virgo sensitivity band. As the population grows, it may be possible to distinguish mergers from these different channels by studying the shape of the population eccentricity distribution. In Figure 9, we illustrate a simplified population model containing three mechanisms that may produce eccentric mergers with distinct eccentricity distributions: mergers facilitated by interactions within GCs, binary mergers occurring within field triples, and mergers inside AGN. These populations are represented as follows:

1. GCs: We represent the distribution shown in Figure 1 of Zevin et al. (2021a) using a Gaussian mixture model containing four Gaussians in $\log_{10}(e_{10})$ at $\mu = \{-6.9, -4.9, -1.5, 0\}$ with widths $\sigma = \{0.5, 1.0, 1.1, 0.1\}$. These peaks, respectively, correspond to mergers that form in clusters and are ejected before they merge; mergers that occur after dynamical interactions within clusters; gravitational-wave capture mergers; and gravitational-wave capture mergers that become bound within the LVK band, merging with eccentricities close to unity.

2. Triples: Binaries can be driven to merge rapidly in isolation if gravitational energy is removed from their orbit by a third object with which they are bound. In some cases, the eccentricity of these binaries can be amplified through Lidov–Kozai oscillations (e.g., Kozai 1962; Lidov 1962). Following Lower et al. (2018) and references therein, we model the primary component expected eccentricity distribution from field triples as a single Gaussian in $\log_{10}(e_{10})$ centered at $\mu = -3$ with width $\sigma = 0.7$ for simplicity. We ignore the small higher-eccentricity peak expected for $\sim5\%$ of field triples since the contribution from this channel is expected to be small (e.g., Silsbee & Tremaine 2017; Rodriguez & Antonini 2018), and we include it in our model for illustrative purposes only (we do not include it in our toy population model analyses).

3. AGN: In the dense center and accretion disk of an AGN, binaries can be driven to merge through dynamical interactions. The eccentricity distribution of dynamical mergers inside AGN differs from that expected in GCs.
strength of the GC hypothesis relative to the AGN hypothesis, taking \( \ln B_{\text{GCs}} \geq 8 \) as a confident preference for the GC hypothesis. We study only the range of eccentricities that is accessible to each era of the detector:

1. **LVK sensitivity limit:** Although sensitivity to eccentricity varies slightly with mass (see Appendix C of Romero-Shaw et al. 2021a), we take \( \epsilon_{10} \geq 0.05 \) as the range within which existing detectors can measure eccentricity. This is consistent with our own measurements, as well as with the predictions of Lower et al. (2018).

2. **ET/CE sensitivity limit:** We use samples from the range of eccentricities to which the ET and CE are expected to be sensitive; this is approximately \( \epsilon_{10} \geq 10^{-4} \) (Lower et al. 2018).

For each \( N \) and for each sensitivity threshold, we perform parameter estimation on 50 mock populations. We plot our results in Figure 10, where the average \( \ln B_{\text{GCs}} \) for each \( N \) obtained is marked with a diamond. We find that we will be able to confidently distinguish a BBH merger population dominated by those formed in GCs from one in which mergers form in AGN with \( \geq 80 \) detectably eccentric events. This is a lower limit, since real detections have nonnegligible uncertainties. Additionally, it will be trickier to disentangle the contributions of multiple channels that contribute comparable numbers of eccentric binaries to the population. We leave a detailed simulation study that accounts for these complications for future work.

5. Conclusion

In this work, we analyze 26 BBH signal candidates newly added to the LVK catalog of gravitational-wave transients in GWTC-2.1 and GWTC-3 for signs of orbital eccentricity using an aligned-spin, eccentric waveform model. We find that two of these events have significant support for detectable eccentricity at 10 Hz: GW191109 and GW200208_22. Together with two events from GWTC-2, GW190521 and...
GW190620 (Romero-Shaw et al. 2021a), four of the 62 BBHs we have studied show significant support for measurable eccentricity. At just over 6%, this is slightly more than the ∼4% of mergers that should be expected with detectable eccentricity from GCs (Zevin et al. 2021a), and may indicate that there are more eccentric mergers in the population than can be explained by dense star clusters alone. However, as we show in Section 4, the difference can be explained by Poisson noise. Additionally, some of these mergers may be spin-precessing binaries masquerading as eccentric BBHs in our analyses due to our enforced assumption of spin alignment.

While we cannot say conclusively that these four binaries are eccentric and spin-aligned, our results do show that these signals are consistent with those that contain hints of orbital eccentricity, and are better fit by eccentric waveforms than they are by quasi-circular waveforms. If they are eccentric, and/or have large spin magnitudes and/or spin tilts, then these binaries are difficult to explain through isolated evolution scenarios and add weight to the hypothesis that the BBHs detected by LVK are dynamically formed.

We have focused on the particular dynamical formation environment of globular star clusters in this paper. However, we emphasize that this is due to the robustness of predictions from this channel, as opposed to any sign that this particular dynamical formation environment is preferred over other dynamical formation environments. In fact, this formation environment may be particularly unlikely when other parameters of these systems are considered. Three of the four potentially eccentric events have notably high primary masses, all with median source-frame measurements above the tentative pair-instability supernova mass-gap lower limit of ∼55 M⊙ (e.g., Heger & Woosley 2002; Belczynski et al. 2016; Woosley 2017; Marchant & Moriya 2020; the exception, GW200208_22, has a median source-frame primary mass of ∼25 M⊙ after reweighting to the eccentric model). The relatively low escape velocities of GCs (typically ≤120 km s⁻¹; Gnedin et al. 2002; Antonini & Rasio 2016; Baumgardt & Hilker 2018) mean that merger remnants are more often kicked out of GCs than out of environments with deeper central potential wells, such as AGN or nuclear star clusters (e.g., Antonini & Rasio 2016; Fragione et al. 2020, 2022; Ford & McKernan 2021; Mahapatra et al. 2021). Additionally, because GCs undergo mass segregation soon after their formation at high redshift, it is likely that the heaviest binaries merge outside of the observable range of current detectors (Antonini & Rasio 2016; Romero-Shaw et al. 2021b). To assess the probability of formation in GCs relative to the probability of formation in AGN, we would need a multi-dimensional version of the toy model analysis demonstrated in Section 4.2 incorporating robust predictions for eccentricity, mass, spin, and redshift from these different environments; see, e.g., Yang et al. (2019), McKernan et al. (2020), Samsing et al. (2020), Tagawa et al. (2020, 2021b, 2021c), Gayathri et al. (2021), and Vajpeyi et al. (2022) for a range of recent predictions for BBH mergers occurring in AGN.

There are many roads to forming merging BBHs dynamically, and we show in this paper that in the future when eccentricity can be tightly constrained, it will be possible to disentangle contributing dynamical channels with ≥80 detectably eccentric mergers at current detector sensitivity limits. As detectors improve, the threshold for detectable eccentricity decreases, so fewer detectably eccentric events are required to distinguish contributions from different dynamical channels. Even so, without a method to confidently measure eccentricity and the full range of spin effects simultaneously, it will not be possible to use the method we have demonstrated to identify the dominant dynamical formation channel. We will therefore turn our focus toward extracting measurements of eccentricity from the constraints of spin-aligned, moderately spinning waveform models in the future.

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**Appendix A**

**Likely Nongeocentric New Events in GWTC-3**

In Table 3, we provide summary statistics for the newly studied events in GWTC-3 that are both adequately sampled and have $\ln B(e_{10} \geq 0.05) < 0$. Marginal posterior distributions on $\log_{10}(e_{10})$ are displayed in Figure 11. Full posterior distributions on all parameters of these events are provided online.8

Table 3

Summary of the Detector-frame Eccentricity Measurements for Newly Studied Events in GWTC-3 with Negligible Support for the Hypothesis that $e_{10} \geq 0.05$

| Event Name | $e_{10} \geq 0.1$ (%) | $e_{10} \geq 0.05$ (%) | $\ln B(e_{10} \geq 0.1)$ | $\ln B(e_{10} \geq 0.05)$ | $n_{\text{eff}}$ |
|------------|------------------------|------------------------|--------------------------|--------------------------|---------------|
| GW190426_16| 3.25                   | 10.69                  | -0.86                    | -0.54                    | 1170          |
| GW190916   | 4.55                   | 11.71                  | -0.59                    | -0.35                    | 573           |
| GW190926   | 6.75                   | 15.61                  | -0.15                    | -0.10                    | 42,171        |
| GW191103   | 9.20                   | 17.82                  | -0.14                    | -0.09                    | 175           |
| GW191222   | 0.00                   | 9.40                   | -1.68                    | -0.96                    | 118           |
| GW191230   | 6.75                   | 14.35                  | -0.46                    | -0.34                    | 10,661        |
| GW200112   | 0.77                   | 7.94                   | -0.88                    | -0.50                    | 39,271        |
| GW200128   | 2.66                   | 11.60                  | -0.83                    | -0.36                    | 11,755        |
| GW200208   | 3.57                   | 11.86                  | -0.70                    | -0.42                    | 53,832        |
| GW200219   | 3.50                   | 11.53                  | -0.74                    | -0.43                    | 36,630        |
| GW200220_06| 4.19                   | 13.16                  | -0.57                    | -0.31                    | 20,910        |
| GW200220_12| 7.47                   | 16.20                  | -0.26                    | -0.14                    | 13,586        |
| GW200224   | 4.34                   | 16.17                  | -0.52                    | -0.03                    | 38,653        |
| GW200302   | 2.44                   | 9.15                   | -1.08                    | -0.38                    | 164           |
| GW200306   | 5.73                   | 14.18                  | -0.44                    | -0.25                    | 4995          |
| GW200308   | 7.60                   | 16.24                  | -0.03                    | -0.03                    | 672           |
| GW200311   | 0.78                   | 6.70                   | -1.61                    | -0.83                    | 51,442        |
| GW200316   | 0.77                   | 18.46                  | -0.67                    | -0.02                    | 131           |

*Note.* Columns are as described in the notes for Table 1.

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8 github.com/IsobelMarguarethe/eccentric-GWTC-3
Figure 11. Marginal posterior distributions on $\log_{10}(e_{10})$ for newly studied events in GWTC-3 that have negligible support for $e_{10} \geq 0.05$. We label each panel with the name of the event.
Appendix B
Undersampled Events in GWTC-3

We consider any event with $n_{\text{eff}} < 100$ to be undersampled. With such few samples, measurements can be misleading, so we do not include in this work analyses of events in GWTC-3 for which the reweighting process is unsuccessful. However, even when an event is undersampled, it can be informative to study a scatter plot of weights versus eccentricities to gain an understanding of the reason for the undersampling. Three examples are shown in Figure 12, in which the $n_{\text{eff}}$ (see Equation (1)) dominating samples are highlighted. First, the locations of the scatter points across the eccentricity range are informative: if the event has no support at all for high eccentricities, it will not be possible for highly eccentric samples to be drawn, so there will be no scatter points at high eccentricities.

Second, the distribution of weights across the eccentricity range is informative. If highly weighted samples are spread evenly across the range of eccentricities—as is the case for our first two examples—waveform systematics are a probable suspect for underweighting: these samples are likely to reside in an area of the wider parameter space (for example, a particular combination of masses and spins) for which the IMRPhenomD waveform does not well represent the $e_{10} = 0 \ SEOBnre$ waveform. If instead the dominating samples are localized to a particular part of the eccentricity range—as is the case for our final example, GW200129—the situation is both more interesting and more frustrating. In this case, it is likely that there is strong support for eccentricity in the data, but that the eccentric posterior does not overlap the quasi-circular posterior to an adequate extent for the reweighting method to be efficient.

![Figure 12](image-url)

*Figure 12.* Scatter plots of ln weight against $\log_{10}(e_{10})$ for three example undersampled events. We label each panel with the name of the event. In each plot, the $n_{\text{eff}}$ most highly weighted samples are highlighted as teal dots. These dots are the points that dominate the reweighted posterior. By plotting the weights against eccentricity in this way, we can see trends that hint at the reason for the undersampling. GW190412 (three effective samples; top left panel) and GW191129 (49 effective samples; top right panel) have highly weighted samples spread across the eccentricity range, and have relatively low weights for highly eccentric samples. This hints at waveform systematics being the root of undersampling; if we removed the $n_{\text{eff}}$ most highly weighted samples from these distributions, the resulting posterior on eccentricity would be mostly uninformative, with decreasing support above $e_{10} \approx 0.1$. Meanwhile, the plot for GW200129 (one effective sample; bottom panel) shows an overall trend toward higher weights for higher eccentricities. If we removed the most highly weighted sample from this distribution, the posterior would still be undersampled, with the most highly weighted sample remaining above $e_{10} = 0.1$. To obtain a well-sampled posterior, we would need to remove most of the samples at high eccentricities, hinting that there is strong support for eccentricity in the data and that the eccentric posterior does not have enough overlap with the quasi-circular posterior for the former to be well sampled using the reweighting method.
