Demonstration of Minisuperspace Quantum Cosmology Using Quantum Computational Algorithms on IBM Quantum Computer

Anirban Ganguly,1,∗ Bikash K. Behera,2,† and Prasanta K. Panigrahi2,‡
1Physics Department, Presidency University, Kolkata 700073, West Bengal, India
2Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India

Quantum computers promise to efficiently solve important problems that are intractable on a conventional computer. Quantum computational algorithms have the potential to be an exciting new way of studying quantum cosmology. In quantum cosmology, we learn about the dynamics of the universe without constructing a complete theory of quantum gravity. Since the universal wavefunction exists in an infinite-dimensional superspace over all possible 3D metrics and modes of matter configurations, we take minisuperspaces for our work by constraining the degrees of freedom to particular 3D metrics and uniform scalar field configurations. Here, we consider a wide variety of cosmological models. We begin by analyzing an anisotropic universe with cosmological constant and classical radiation. We then study the results for higher derivatives, Kaluza-Klein theories and string dilaton in quantum cosmology. We use IBM’s Quantum Information Science Kit (QISKit) python library and the Variational Quantum Eigensolver (VQE) algorithm for studying these systems. The VQE algorithm is a hybrid algorithm that uses the variational approach and interlaces quantum and classical computations in order to find the minimum eigenvalue of the Hamiltonian for a given system.

I. INTRODUCTION

Predictions of Superstrings, Supergravity and higher-dimensional Kaluza-Klein theories, which try to unify the four fundamental interactions of nature are for the present, as well as for the near future, well beyond the reach of our particle accelerators1. In quantum cosmology, we attempt to describe a quantum state of the universe, without the actual unification into a “Theory of Everything”. The Einstein-Hilbert action for a particular cosmological model is quantized by the method of canonical quantization. The Wheeler-DeWitt constraint is applied to the obtained Hamiltonian and the equation is solved for the universal wavefunction.

Supercomputers are huge technological breakthroughs of the modern era. However, understanding how atoms, and molecules, interact with each other on a larger scale is beyond the scope of such technologies. Quantum systems are too mathematically complex for today’s limited computing power. For example, it is impossible without approximations to calculate the properties of atoms, which are even slightly more complicated than the simple hydrogen atom.

Here, quantum simulation can provide a solution. Richard Feynman3 introduced the core idea of quantum simulations as long back as in 1982. Intractable quantum problems can be solved by simulating4 them with other controllable quantum systems that resemble them. It is done by running an experiment, called a quantum simulator which yields a description of the system being studied. Thus, quantum simulations are ahead of the most powerful supercomputers and will enable the development of new tools at the forefront of research. For example, its applications in high-temperature superconductivity, would be beneficial for tackling climate change.

Quantum simulation can also have wide-ranging applications in the field of quantum cosmology, the subject of our paper. We use an open-source development kit, “QISKit”, founded by IBM5. QISKit provides tools for various tasks such as creating quantum circuits, performing optimizations, simulations, and computations. The QISKit Aqua library contains an implementation of the VQE algorithm which is the most suited for our purpose. It also provides the classical exact eigensolver algorithm which we use to compare our results. In this paper, we use the VQE algorithm to apply the Wheeler DeWitt equation on four different quantum cosmological models, each with its own significance. We use two qubits for simulating each dimension of a system.

IBM has become a huge platform for researchers in the field of quantum computation and quantum information6, where users are allowed to run quantum computational algorithms on the 5-qubit (ibmqx2, ibmqx4), 16-qubit quantum processors (ibmqx5), and one 32-qubit simulator (IBM qasm simulator). A large number of quantum computing researches such as quantum simulational works7–12, developing quantum error correcting algorithms13–18, testing quantum algorithms19–30, developing quantum applications31–35 have already been performed.

The rest of the paper is organized as follows: In Section II, we discuss briefly over the method of canonical quantization of the Einstein-Hilbert action of a universe and the formulation of the Wheeler DeWitt equation for a minisuperspace. In Section III, we provide an introduction to quantum computation. In Section IV, we discuss the discretization method used for the construction of the Hamiltonians corresponding to different models. In Section V, we explain the variational approach to quantum mechanics and the variational quantum eigen-
solver (VQE) algorithm. In Section VI, we verify the Wheeler De Witt equation for different quantum cosmological models using the tools that we have developed. In Section VII, we establish our results and show the optimized quantum circuits representing our solved universal wavefunction. In Section VIII, we conclude by suggesting some future prospects of our research.

II. THE WHEELER DE WITT EQUATION AND MINISUPERSPACES

The Einstein-Hilbert action that results in the Einstein field equations in vacuum, through the principle of least action is given by

$$S_{EH} = \frac{c^4}{16\pi G} \int d^4 x (-g)^{\frac{1}{2}} R$$

(1)

where $R$ is the Ricci scalar, $g$ is the determinant of the metric tensor matrix, $c$ is the speed of light and $G$ is the universal gravitational constant. Total action of the world will be accompanied by Lagrangian terms to account for matter fields\cite{36} as well as the cosmological constant.

$$S = \int \left[ \frac{c^4}{16\pi G} (R - 2\Lambda) + L_M \right] (-g)^{\frac{1}{2}} d^4 x$$

(2)

Other matter or radiation components can also be added to obtain the total action. The topology of our universe is taken as a 3-surface with metric $h_{ij}$ where $i,j = 1,2,3$ with some matter field configuration $\phi$, embedded in a 4-D manifold on which 4-metric is given by

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -(N^2-N_iN^i) dt^2 + 2N_i dx^i dt + h_{ij} dx^i dx^j$$

(3)

where $N=$lapse(arbitrary), $N_i=$shift. The Wheeler De Witt equation\cite{37} $H \psi = 0$ is a constraint on the Hamiltonian $H$ derived from the action in Eq. (1) and $\psi$ represents the universal wavefunction that exists in an infinite-dimensional space of all the possible 3-metrics $h_{ij}(x)$ and matter field configurations $\phi(x)$, called the superspace\cite{38}. To make the problem solvable, we need to restrict the degrees of freedom of the universe to a finite number, giving a finite dimensional space called minisuperspace. Usually, in minisuperspace, the lapse is taken to be homogeneous and the shift is taken as zero. Hence, now, from Eq. (2),

$$ds^2 = -N^2(t) dt^2 + h_{ij} dx^i dx^j$$

(4)

the 3-surface metric $h_{ij}$ is taken as homogeneous so that it can now be described by a finite number of functions of $t$(histories): $q^\nu(t)$ where $\alpha=0,1,2,\ldots,n-1$. The simplest case is the homogeneous and isotropic Friedmann-Robertson-Walker minisuperspace where $h_{ij} dx^i dx^j = a^2(t) d\Omega_3^2$, $d\Omega_3^2$ is the metric on the 3-sphere and $q^\nu(t) = a(t)$. We deal with other more complicated minisuperspaces in our paper as presented in Section V.

III. QUANTUM COMPUTATION

Quantum computation\cite{38} uses the properties of quantum mechanics: superposition and entanglement to encode information in units called the qubits which are fundamentally different from their classical equivalent, bits (0 and 1). Qubits are the 2-state quantum systems which can be physically represented through electron spins and photon polarizations while in the Hilbert space they can be described as,

$$|b\rangle = \alpha_1 |0\rangle + \alpha_2 |1\rangle = \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}$$

where $||\alpha_1|^2 + ||\alpha_2|^2 = 1$ (5)

In a quantum circuit, the qubits are acted on by quantum gates, which can be described in Hilbert space as unitary matrices. Single-qubit gates include rotations of the qubits about the x-, y- and z-axes, the Pauli matrices, and the Hadamard gate which can put a qubit state into an equal superposition state. The most important, two-qubit gate is the controlled-not or “CNOT” gate, which is commonly used to entangle two neighboring qubits. If we begin with n qubits in our quantum circuit, we obtain a state that is a superposition of $2^n$ basis states. So, when we make our measurements, we prepare an ensemble to get a probabilistic distribution of the results. A simple quantum circuit illustrating both superposition and entanglement is shown in Fig. 1.

![Quantum Circuit](image)

FIG. 1. Quantum circuit generating the state, $|\Psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$.

IV. DISCRETIZATION OF THE HAMILTONIANS

For the computation of any Hamiltonian to be possible, we need to work with a finite number of elements so instead of having a continuous eigen spectrum of position we need to discretize the space\cite{39}. If we discretize a 2-dimensional space with $x,y\in[-L,L]$ such that we have $N$ eigenvalues for each of $x$ and $y$, then we obtain a mesh of $N^2$ spatial elements. Each element corresponds to a specific eigenvalue for $x$ and $y$. If our mesh is centered at $[0,0]$, our position operator will be an $N^2N$ matrix with
the position eigenvalues as the diagonal elements. For 2 qubits and N=4,

\[
x^d = \begin{bmatrix}
-2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (6)

For higher dimensional models, we operate our Pauli gates on different spaces corresponding to different dimensions. The decomposed Hamiltonian for a 2-dimensional quantum harmonic oscillator will then be \((H^d \otimes I) + (I \otimes H^d)\) which from Eq. (10) can be written as:

\[
\begin{align*}
1.178572(I \otimes I \otimes I \otimes I) \\
-0.196429(I \otimes Y \otimes I \otimes I) \\
+0.0982143(I \otimes Z \otimes I \otimes I) \\
-0.0982143(X \otimes I \otimes I \otimes I) \\
+0.196429(X \otimes Y \otimes I \otimes I) \\
+0.196429(Z \otimes Z \otimes I \otimes I) \\
-0.196429(I \otimes I \otimes I \otimes Y) \\
-0.0982143(I \otimes I \otimes I \otimes Z) \\
+0.196429(I \otimes I \otimes X \otimes I) \\
+0.196429(I \otimes I \otimes X \otimes Y) \\
+0.196429(I \otimes I \otimes Z \otimes I) \\
+0.196429(I \otimes I \otimes Z \otimes Z)
\end{align*}
\] (11)

In this way, we can simulate any N-dimensional Hamiltonian into a quantum circuit. In our work, we will utilize this method to deal with 2-dimensional cosmological models.

V. THE VARIATIONAL QUANTUM EIGENSOLVER ALGORITHM

Any normalized wavefunction can be written as a superposition of its eigenstates:

\[
\psi = \sum c_n \psi_n
\] (12)

such that \(H \psi_n = E_n \psi_n\) and \(1 = \langle \psi | \psi \rangle = \sum |c_n|^2\). Now, since the ground state energy \(E_g \leq E_n\), the expectation value can be written as,

\[
\langle H \rangle = \langle \psi | H | \psi \rangle = \sum E_n |c_n|^2 \geq E_g \sum |c_n|^2 = E_g
\] (13)

Thus, we obtain the variational principle of quantum mechanics\(^41\): \(\langle H \rangle \geq E_g\). In the VQE algorithm, we can take any trial wavefunction \(\psi\) in the form of a quantum circuit. For this, we use the Ry variational form provided in the QISKit library. Then for obtaining the tightest bound on our ground state, we have to minimize the expectation \(\langle H \rangle\) with respect to the parameters of our circuit. This minimization is carried out through a classical optimization method called the simultaneous perturbation stochastic approximation (SPSA)\(^42\). Once the quantum-classical hybrid algorithm (VQE)\(^43\) is carried out, we get the optimized variational form and the upper bound for the ground state energy.
VI. QUANTUM COSMOLOGICAL MODELS

A. Bianchi-Type IX universe with $\Lambda$, $\gamma$ and small anisotropies

Most of the works in quantum cosmology have been around homogeneous and isotropic Friedmann-Robertson-Walker minisuperspace models with topology $S^3$. However, considering anisotropic universes may allow us to obtain a more realistic insight into the fate of our universe. A Kantowski-Sachs minisuperspace, for example, is an anisotropic universe with an extra anisotropic degree of freedom and topology $S^1 \times S^2$. The Bianchi-Type IX minisuperspace, on the other hand, has two anisotropic degrees of freedom. It is given by the metric:

$$ds^2 = -\sigma^2 N^2(t)dt^2 + \frac{\sigma^2}{4} a^2(t)(e^{2\beta(t)})_{ij}\sigma^i\sigma^j$$  \hspace{0.5cm} (14)

where

$$\sigma^1 = \cos\psi d\theta + \sin\psi \sin\theta d\phi$$
$$\sigma^2 = \sin\psi d\theta - \cos\psi \sin\phi d\phi$$
$$\sigma^3 = d\psi + \cos\theta d\phi$$  \hspace{0.5cm} (15)

Constant $\sigma^2 = \frac{l^2}{2\pi^2}$ and $N(t)$ is a lapse function. $\beta$ is a time independent traceless diagonal matrix. We use the parametrization:

$$\beta_{11} = \beta_+ + \sqrt{3}\beta_-$$
$$\beta_{22} = \beta_+ - \sqrt{3}\beta_-$$
$$\beta_{33} = -2\beta_+$$  \hspace{0.5cm} (16)

As explained in Section II, we can obtain the Einstein-Hilbert action for the minisuperspace. On adding components for the cosmological constant $\Lambda$ and $\gamma$ in the action, the Wheeler De Witt equation $H\psi = 0$ takes the form:

$$\left[\frac{\partial^2}{\partial a^2} - \frac{1}{a^2}\left(\frac{\partial^2}{\partial \beta_+^2} + \frac{\partial^2}{\partial \beta_-^2}\right) + (\Lambda a^4 - a^2 + \gamma) + 8a^2(\beta_+^2 + \beta_-^2)\right] \psi = 0$$  \hspace{0.5cm} (17)

For small anisotropies $\beta_+$ and $\beta_-$, the equation depends only on $\beta = (\beta_+^2 + \beta_-^2)$ so we can restrict it to a single anisotropic degree of freedom and we get the following equation:

$$\left[\frac{\partial^2}{\partial a^2} - \frac{1}{a^2}\frac{\partial^2}{\partial \beta^2} + (\Lambda a^4 - a^2 + \gamma) + 8a^2\beta^2\right] \psi = 0$$  \hspace{0.5cm} (18)

The 2-dimensional Hamiltonian for our system is:

$$H_1 = -\frac{\beta_+^2}{a^2} + \frac{\beta_-^2}{a^2} + (\Lambda a^4 - a^2 + \gamma) + 8a^2\beta^2$$  \hspace{0.5cm} (19)

We can now use the VQE algorithm to verify the Wheeler De Witt constraint on this Hamiltonian. Our resulting ground state eigenvalue is $0.4829572576229935 \pm 0.1203210787358038$ for classical radiation density constant $\gamma = 0.99$ and cosmological constant $\Lambda = 2.4800000000000011$. The classical exact eigenvalue algorithm gives the result as $2.0707928270104423 \times 10^{-5}$. Fig. 3 shows the optimized quantum circuit representing our resulting solution for the universal wavefunction.

B. Higher Derivative Cosmology

For isotropic and anisotropic universes alike, it has been found in general that it is necessary to include a massive scalar field in order to yield wavefunctions that on classical approximation, gives Lorentzian solutions of the field equations, which have a period of exponential expansion and approach isotropy at the present epoch, thus agreeing with observations. However, it is possible to avoid the inclusion of this massive scalar field term by resorting to quantum corrections to the gravitational action. These perturbations will lead to an effective gravitational action containing higher order curvature terms. We take the action to be of the form:

$$S = -\frac{1}{16\pi} \int d^4x\sqrt{g}(R - \alpha C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + \beta R^2)$$
$$- \frac{1}{8\pi} \int d^3x\sqrt{h}(K(1+2\beta R) - 4\alpha K_{ij}C^{\mu\nuij}\eta_\mu \eta_\nu)$$  \hspace{0.5cm} (20)

where $C_{\mu\nu\rho\sigma}$ is the Weyl Tensor and $\eta^\alpha$ is the unit normal to the boundary. The simplest minisuperspace model i.e. a homogeneous and isotropic universe is taken so the C-squared term is zero and can be neglected. The $R^2$ term on the other hand, behaves like a massive scalar field. The metric taken is of the form:

$$ds^2 = \frac{4a^2}{3\pi}(d\eta^2 + d\omega_3^2)$$  \hspace{0.5cm} (21)

where

$$d\eta = \frac{dt}{a}$$  \hspace{0.5cm} (22)

The action from Eq. (20) is then:

$$S = -\int d\eta\left(\frac{1}{a^2} - \frac{a''}{a^3} + \frac{9}{2\pi\beta}\left(\frac{1}{a^2} - \frac{a''}{a^3}\right)\right)a^4$$  \hspace{0.5cm} (23)
Taking $Q = a(1 + 2\beta R)$ and $x = Q-a, y = Q+a$ for convenience, our Hamiltonian

$$H_2 = -\frac{p_x^2}{4} + \frac{p_y^2}{4} - \frac{(y^2 - x^2)}{4} + \frac{x^2}{8\beta}(x - y)^2$$  \hspace{1cm} (24)$$

The VQE algorithm for $\tilde{\beta} = 0.42808219$ yields ground state energy eigenvalue $-0.07876069717108591 \pm 1.3971389629283328$. On applying the classical exact eigensolver algorithm, we get the eigenvalue as $-1.6494040350599992$. Fig. 4 shows the optimized quantum circuit generating the state vector.

\section*{C. String Dilaton Cosmology}

String theory is one of the foremost proposals of unification into a theory of everything. It is relevant to cosmology as it can be investigated at Planck scale. We take the tree level dilaton-graviton string-effective action\(^{48}\) for a 4-dimensional spacetime as

$$S = \int d^4x \sqrt{-g} e^{-2\phi} [R + \Lambda + 4(\partial\phi)^2 + ..]$$  \hspace{1cm} (25)$$

where $\phi$ stands for the dilaton field. The dots indicate other antisymmetric tensor terms and scalar matter fields that we ignore. The minisuperpace is taken as a homogeneous and isotropic Friedman-Robertson-Walker metric. Using the transformation $2\Phi = 2\phi - 3\ln a$ and taking $z = \ln a$, we get the Hamiltonian for the system as

$$H_3 = -\frac{p_x^2}{12} + \frac{p_y^2}{16} + 2\Lambda e^{-4\Phi}$$  \hspace{1cm} (26)$$

The result of using the VQE algorithm to verify the Wheeler-DeWitt equation gives ground state energy $E_0 = 0.10969043206316431 \pm 0.02731991298884283$ and the classical eigensolver algorithm yields $6.15903864812147 \times 10^{-5}$ taking cosmological constant $\Lambda = 0.581$. The optimized variational circuit is shown in Fig. 5.

\section*{D. Kaluza-Klein Theory}

Kaluza-Klein theory\(^{49}\) attempts to unify gravity with electromagnetism by allowing for a 5th dimension beyond the usual four dimensions of spacetime. It is considered as a precursor to string theory. The 4-dimensional part of the model can be taken as a Friedmann closed space. The extra dimension $y$ is supposed to be disconnected from the rest so that the $y$ dependence can be factorized out of the metric components. So,

$$ds^2 = \lambda^2(y)(-N^2 dt^2 + a^2(t)dy^2 + \phi^2(t)dy^2)$$  \hspace{1cm} (27)$$

where $d\Omega^2$ is the metric on a unit 3-sphere, and $\phi$ is a scalar field. The 5-dimensional Einstein field equations yield the 4-dimensional Einstein field equations, Maxwell’s electromagnetic field equations and an equation for the scalar field. Taking $q_1 = a(a + \phi)/2$ and $q_2 = a(a - \phi)/2$, we get the Hamiltonian:

$$H_4 = \frac{N}{24\pi^2}[k(q_1 - q_2) + (p_1^2 - p_2^2)]$$  \hspace{1cm} (28)$$

On applying the VQE algorithm for this Hamiltonian taking $k = 10^{-6}$, we get the ground state eigenvalue $E_0 = -0.03330932911207404 \pm 0.19977149262935648$. The classical exact eigensolver algorithm gives the eigenvalue as $-1.571428000014587$. The optimized quantum circuit representing the universal wavefunction is shown in Fig. 6.

\section*{VII. QUANTUM CIRCUITS}

We present the optimized quantum circuits that generate the universal wavefunction for each of the models taken. The initial state for the VQE algorithm was taken as the Ry variational form provided by the IBM Qiskit library. The circuit in Fig. 3 generates the universal wavefunction for Hamiltonian $H_1$ representing a Bianchi-IX type universe with classical radiation density constant $\gamma$, cosmological constant $\Lambda$ and small anisotropies. Fig. 4 shows the circuit for generating the statevector corresponding to the Hamiltonian $H_2$ obtained for our higher derivative cosmological model. Fig. 5 shows the circuit generating the universal wavefunction for Hamiltonian $H_3$ of the string dilaton cosmological model taken. The circuit in Fig. 6 is the representation of $H_4$ corresponding to a Kaluza-Klein model of quantum cosmology.
FIG. 3. Optimized quantum circuit generating the state $\psi$ for Hamiltonian $H_1$

FIG. 4. Optimized quantum circuit generating the state $\psi$ for Hamiltonian $H_2$
FIG. 5. Optimized quantum circuit generating the state $\psi$ for Hamiltonian $H_3$

FIG. 6. Optimized quantum circuit generating the state $\psi$ for Hamiltonian $H_4$
VIII. CONCLUSION

It has been previously shown how quantum computing is an efficient tool for studying the homogeneous isotropic FRW minisuperspace with conformally coupled scalar fields. Here, we have successfully utilized the VQE algorithm for finding the universal wavefunction solutions consistent with the Wheeler De Witt constraint applied on anisotropic minisuperspaces, Kaluza-Klein models, higher derivative quantum cosmological models, and string dilaton cosmology. It yielded the upper bounds on the ground state energy and the optimized quantum circuits that generate the required state vectors. The variational method has thus proved beneficial in circumventing the problematic necessity of boundary conditions to study the universal wavefunction and can be used in the future for studying other cosmological models or even extending minisuperspace quantum cosmology to the actual superspace, in which the wavefunction exists. The simulation was done using 2 qubits for each dimension of all the 2-dimensional systems considered. The backend used was the “state vector simulator” provided by QISKit.

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* anirbangangulynostalgic@gmail.com
† bkb18rs029@iiserkol.ac.in
‡ prprasanta@iiserkol.ac.in
1 D. Solomons, The wave function of the universe, University of Cape Town, (1994).
2 J. J. Halliwell, Introductory lectures on quantum cosmology, in Quantum cosmology and baby universes, 159, World Scientific, (1991).
3 R. P. Feynman, Simulating Physics with Computers, Int. J. Theor. Phys. 21, 467 (1982).
4 R. D. Somma, Quantum simulations of one dimensional quantum systems, Quantum Inf. Comput. 16, 1125 (2016).
5 A. C. Santos, The IBM Quantum Computer and the IBM Quantum Experience, Rev. Bras. Ens. Fis. 39, e1301 (2017).
6 IBM Quantum Experience. http://www.research.ibm.com/ibm-q/.
7 A. A. Zhukov, S. V. Remizov, W. V. Pogosov, and Y. E. Lozovik, Algorithmic simulation of far-from-equilibrium dynamics using quantum computer. Quantum Inf. Process. 17, 223 (2018).
8 R. Malik, R. P. Singh, B. K. Behera, and P. K. Panigrahi, First Experimental Demonstration of Multiparticle Quantum Tunneling in IBM Quantum Computer, DOI:10.13140/RG.2.2.18542.77129
9 G. R. Malik, S. S. Behera, S. Kumar, B. K. Behera, and P. K. Panigrahi, Observation of Geometric Phase in a Molecular Aharonov-Bohm System Using IBM Quantum Computer, arXiv:1909.00298
10 M. Schuld, M. Fingerhuth, and F. Petruccione, Implementing a distance-based classifier with a quantum interference circuit, Europhys. Lett. 119, 60002 (2017).
11 Manabputra, B. K. Behera, and P. K. Panigrahi, A Simulational Model for Witnessing Quantum Effects of Gravity Using IBM Quantum Computer, arXiv:1806.10229.
12 O. Viyuela et al., Observation of topological Uhlmann phases with superconducting qubits, npj Quantum Inf. 4, 10 (2018).
13 D. Ghosh, P. Agarwal, P. Pandey, B. K. Behera, and P. K. Panigrahi, Automated error correction in IBM quantum computer and explicit generalization, Quantum Inf. Process. 17, 153 (2018).
14 J. Rolfe, D. Headley, N. Chancellor, D. Horsman, and V. Kendon, Protecting quantum memories using coherent parity check codes, Quantum Sci. Technol. 3, 035010 (2018).
15 S. Satyajit, K. Srinivasan, B. K. Behera, and P. K. Panigrahi, Nondestructive discrimination of a new family of highly entangled states in IBM quantum computer, Quantum Inf. Process. 17, 212 (2018).
16 R. Harper and S. Flammia, Fault-Tolerant Logical Gates in the IBM Quantum Experience, arXiv:1806.02359.
17 R. K. Singh, B. Panda, B. K. Behera, and P. K. Panigrahi, Demonstration of a general fault-tolerant quantum error detection code for (2n+ 1)-qubit entangled state on IBM 16-qubit quantum computer, arXiv:1807.02883.
18 A. Warke, B. K. Behera, and P. K. Panigrahi, The first three-qubit and six-qubit full quantum multiple error-correcting codes with low quantum costs, DOI: 10.13140/RG.2.2.18542.77129
19 D. V. Babukhin, A. A. Zhukov, and W. V. Pogosov, Non-destructive classification of quantum states using an algorithmic quantum computer, arXiv:1909.05596
20 D. García-Martín, and G. Sierra, Five Experimental Tests on the 5-Qubit IBM Quantum Computer, J. App. Math. Phys. 6, 1460 (2018).
21 S. Gangopadhyay, Manabputra, B. K. Behera, and P. K. Panigrahi, Generalization and demonstration of an entanglement-based Deutsch-Jozsa-like algorithm using a 5-qubit quantum computer. Quantum Inf. Process. 17, 160 (2018).
22 I. Yalçınkaya, and Z. Gedik, Optimization and experimental realization of the quantum permutation algorithm, Phys. Rev. A 96, 062339 (2017).
23 P. Balasubramanian, B. K. Behera, and P. K. Panigrahi, Circuit implementation for rational quantum secure com-
munication using IBM Q Experience beta platform, DOI: 10.13140/RG.2.2.28733.31207.

24 B. K. Behera, A. Banerjee, and P. K. Panigrahi, Experimental realization of quantum cheque using a five-qubit quantum computer, Quantum Inf. Process. 16, 312 (2017).

25 U. Alvarez-Rodriguez, M. Sanz, L. Lamata, and E. Solano, Quantum Artificial Life in an IBM Quantum Computer, Sci. Rep. 8, 14793 (2018).

26 B. K. Behera, T. Reza, A. Gupta, and P. K. Panigrahi, Designing quantum router in IBM quantum computer, Quantum Inf. Process. 18, 328 (2019).

27 S. K. Rajiuddin, A. Baishya, B. K. Behera, P. K. Panigrahi, Experimental realization of quantum teleportation of an arbitrary two-qubit state using a four-qubit cluster state, DOI: 10.13140/RG.2.2.23102.13121

28 Y. Chatterjee, V. Devrari, B. K. Behera, and P. K. Panigrahi, Experimental realization of quantum teleportation using coined quantum walks, arXiv:1908.01348

29 A. P. Dash, S. K. Sahu, S. Kar, B. K. Behera, and P. K. Panigrahi, Explicit demonstration of initial state construction in artificial neural networks using NetKet and IBM Q experience platform, DOI: 10.1117/12.23102.13121

30 S. Dutta, A. Susa, S. Dutta, S. Roy, B. K. Behera, and P. K. Panigrahi, Demonstration of Quantum Circuit Design Methodology for Multiple Regression, arXiv:1811.01726

31 B. K. Behera, S. Seth, A. Das, and P. K. Panigrahi, Demonstration of entanglement purification and swapping protocol to design quantum repeater in IBM quantum computer, Quantum Inf. Process. 18, 108 (2019).

32 A. Pal, S. Chandra, V. Mongia, B. K. Behera, and P. K. Panigrahi, Solving Sudoku Using Quantum Computation, DOI: 10.13140/RG.2.2.19777.86885

33 V. Singh, B. K. Behera, and P. K. Panigrahi, Design of Quantum Circuits to Play Bingo Game in a Quantum Computer, DOI: 10.13140/RG.2.2.22727.34720.

34 S. Mahanti, S. Das, B. K. Behera, and P. K. Panigrahi, Quantum robots can fly; play games: an IBM quantum experience, Quantum Inf. Process. 18, 219 (2019).

35 N. Mishra, S. C. Rayala, B. K. Behera, and P. K. Panigrahi, Automation of Quantum Braitenberg Vehicles Using Finite Automata: Moore Machines, Quantum Inf. Process., DOI: 10.1007/s11128-019-2512-2

36 A. Zee, QFT in a Nutshell, Princeton University Press (2010).

37 T. P. Shestakova, Is the Wheeler - DeWitt equation more fundamental than the Schrodinger equation?, Int. J. Mod. Phys. D 27, 1841004 (2018).

38 M. A. Nielsen and I. L. Chuang, Quantum Computation and Quantum Information, Cambridge University Press, (2010).

39 V. K. Jain, B. K. Behera, and P. K. Panigrahi, Quantum Simulation of Discretized Harmonic Oscillator on IBM Quantum Computer, DOI: 10.13140/RG.2.2.26280.93448

40 Quantum Fourier Transform, https://en.wikipedia.org/wiki/Quantum$\_\$Fourier$\_\$transform

41 D. J. Griffiths, Introduction to Quantum Mechanics, Pearson Prentice Hall (2004).

42 J. C. Spall, Multivariate stochastic approximation using a simultaneous perturbation gradient approximation, IEEE Transactions on Automatic Control 37, 332-341 (1992).

43 A. Peruzzo, J. McClean, P. Shadbolt, M.-H. Yung, X.-Q. Zhou, P. J. Love, A. Aspuru-Guzik, and J. L. O’Brien, A variational eigenvalue solver on a photonic quantum processor, Nat. Comm. 5, 2413 (2014).

44 R. Laflamme and E. P. S. Shellard, Quantum Cosmology and Recollapse, Phys. Rev. D 35, 2315 (1987).

45 P. Amsterdam, Wavefunction of an anisotropic universe, Phys. Rev. D 31, 1.2 (1985).

46 P. Pedram, On the conformally coupled scalar field quantum cosmology, Phys. Lett. B 671, 1-6 (2009).

47 S. W. Hawking and J. C. Luttrel, Higher derivatives in quantum cosmology, Nuclear Phys. B 247, 250-260 (1984).

48 S. Capozziello and R. De Ritis, Minisuperspace and Wheeler-DeWitt Equation for String Dilaton Cosmology, Int. J. Mod. Phys. D 2, 3 (1993).

49 M. I. Beciu, Canonical Quantum Kaluza-Klein Theory, Il Nuovo Cimento 90B, (1985).

50 C. D. Kocher, M. McGuigan, Simulating 0+1 Dimensional Quantum Gravity on Quantum Computers: Minisuperspace Quantum Cosmology and the World Line Approach in Quantum Field Theory, 2018 New York Scientific Data Summit (NYSDS), IEEE (2018)

51 A. Vilenkin, M. Yamada, Tunneling wave function of the universe, Phys. Rev. D 98, 066003 (2018).

52 J. B. Hartle and S. W. Hawking, Wave function of the universe, Phys. Rev. D 28, 2960 (1983); Adv. Ser. Astrophys. Cosmol. 3, 174 (1987).

53 Department of Physics, IIT Guwahati, Surjyamukhi Road, North, Amingaon, Guwahati, Assam 781039, India