Mono-pulse radar angle estimation algorithm under low signal-to-noise ratio

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Abstract. In the case of low signal-to-noise ratio (SNR), in order to improve the angle measurement accuracy of the mono-pulse radar seeker, a target angle estimation algorithm is proposed. The algorithm takes the target angle as a state variable under the Bayesian framework, expands it to the state vector, performs state modeling and observation modeling on it, and builds a particle-based eye angle estimator, which improves the target angle estimation performance. The effectiveness of the algorithm is verified by the simulation data of the medium pulse repetition frequency radar seeker.

1. Introduction

Detection and state estimation are the basic tasks of radars, and detection and estimation accuracy under low signal-to-noise ratios is an important indicator for evaluating radar performance. Target detection and state estimation under low SNR are important challenges in the development of radar. By increasing information input, the target can be effectively detected under low SNR, but the angle measurement accuracy cannot meet the requirements of closed-loop control. The problem of target angle estimation under low SNR needs to be solved urgently.

For mono-pulse radar, the traditional angle estimation algorithm is mono-pulse angle measurement. The general idea is to use the sum channel information to detect the target, and then compare the amplitude or phase of the difference channel to obtain the angle information [1]. However, the single pulse angle measurement method has higher requirements for the amplitude and phase consistency of the sum and difference channels. To solve this problem, literature [2] and literature [3] give corresponding amplitude and phase compensation methods, which effectively improve the angle measurement performance. Literature [4] derives the formulas of weighted fusion mono-pulse angle estimation based on the four-channel mono-pulse theory. The algorithm is used in phased array radar, which improves the accuracy of angle measurement compared with the traditional single pulse angle measurement method. In the case of low SNR, there is a contradiction between instantaneous angle measurement and coherent accumulation, literature [5] uses the total variation method to solve this contradiction.

The above angle measurement methods only use the current data of sum and difference, and the angle measurement performance cannot meet the accuracy requirements under the condition of low SNR. In order to improve the accuracy of angle measurement, the length of information input can be increased, and the target angle is used as a state variable to filter it recursively. Aiming at the problem of target angle estimation under low SNR, this paper establishes the state description based on the TBD detector.
output and the antenna line-of-sight mixed coordinate system. The modeling process of the angle estimator is given in the Bayesian framework. The angle estimator is highly non-linear and uses particle implementation. It is verified by simulation experiments that the algorithm can effectively estimate the target angle under low SNR.

2. Bayesian recursion

The overall framework is shown in the figure 1 below:

\[
X_k = [r_k, \dot{r}_k, \alpha, \dot{\alpha}, \bar{O}_k] \rightarrow \text{state modeling} \rightarrow f_{k+1|k}(X_{k+1} | X_k) \rightarrow \text{observation} \rightarrow f_{k+1|k+1}(X_{k+1} | Z_{k+1}) \rightarrow \text{Bayesian paradigm} \rightarrow \text{Particle realization}
\]

**Figure 1.** Block diagram of filter structure

The prediction steps and update steps are as follows:

\[
f_{k+1|k}(x | Z^{(k)}) = \int f_{k+1|k}(x | x') f_k(x') f_k(x | Z^{(k)}) dx'
\]

**Equation 1**

\[
f_{k+1|k+1}(x | Z^{(k+1)}) = \frac{f_{k+1}(Z_{k+1} | x) f_{k+1|x}(x | Z^{(k)})}{\alpha}
\]

**Equation 2**

\[
\alpha = \int f_{k+1}(Z_{k+1} | x) f_{k+1|x}(x | Z^{(k)})
\]

**Equation 3**

where \( f_{k+1|k}(x | x') \) is Markov transition density, \( f_{k+1|x}(x | Z^{(k)}) \) is the likelihood function, and \( \alpha \) is the normalization factor.

3. Bayesian modelling

3.1. State model

Considering that the detection output signal of mono-pulse radar has a simple description form under the antenna system, and the target misalignment angle and line-of-sight angular velocity are estimated in the antenna system and line-of-sight system respectively, this paper chooses the antenna and line-of-sight hybrid coordinate system to describe the target state [6].

The state vector at the selected moment \( k \) is:

\[
X_k = [r_k, \dot{r}_k, \alpha, \dot{\alpha}, \bar{O}_k] \rightarrow \text{state modeling} \rightarrow f_{k+1|k}(X_{k+1} | X_k) \rightarrow \text{observation} \rightarrow f_{k+1|k+1}(X_{k+1} | Z_{k+1}) \rightarrow \text{Bayesian paradigm} \rightarrow \text{Particle realization}
\]

\[
X_k = [r_k, \dot{r}_k, \alpha, \dot{\alpha}, \bar{O}_k] \rightarrow \text{state modeling} \rightarrow f_{k+1|k}(X_{k+1} | X_k) \rightarrow \text{observation} \rightarrow f_{k+1|k+1}(X_{k+1} | Z_{k+1}) \rightarrow \text{Bayesian paradigm} \rightarrow \text{Particle realization}
\]

\[
\begin{align*}
X_k & = [r_k, \dot{r}_k, \alpha, \dot{\alpha}, \bar{O}_k] \\
& \rightarrow \text{state modeling} \rightarrow f_{k+1|k}(X_{k+1} | X_k) \\
& \rightarrow \text{observation} \rightarrow f_{k+1|k+1}(X_{k+1} | Z_{k+1}) \\
& \rightarrow \text{Bayesian paradigm} \rightarrow \text{Particle realization}
\end{align*}
\]

**Equation 4**

where, \( r_k \) and \( \dot{r}_k \) are the distance between the projectile and the target under the line of sight system and the rate of change of the distance; \( \alpha \) and \( \dot{\alpha} \) are the Y-axis and Z-axis misalignment angles of the target under the antenna system; \( \alpha \) and \( \dot{\alpha} \) are the line-of-sight angular velocity in the line of sight coordinate system, respectively around the Y axis and Z axis rotation; \( \bar{O}_k \) is the target average RCS.

**Figure 2.** Antenna line-of-sight hybrid coordinate system

As shown in Figure 2, in the antenna line-of-sight hybrid coordinate system, the radar seeker is tracking the target stably, where \( O X_H Y_H Z_H \) represents the antenna coordinate system and \( O X_P \) represents the X-
axis direction of the line-of-sight coordinate system. Due to the limitation of detection accuracy and the relative movement of the missile and target, the seeker's electrical axis cannot accurately point to the target, and always has a certain angular deviation from the target, that is, the misalignment angles $\varepsilon^x_k$ and $\varepsilon^y_k$ in the state vector. The misalignment angle is also the Euler angle $(\varepsilon_k^x, -\varepsilon_k^y, 0)'_{xyk}$ from the antenna system to the line of sight system.

Motion modeling is mainly a statistical description of the time-varying law of the target state. After deduction, this article establishes the following equation:

$$x_{k+1} = F_k x_k + G_k^w w_k$$  \hspace{1cm} (5)

where: $F_k$, $G_k^w$ are the state transition matrix and the noise input matrix, respectively, expressed as:

$$F_k = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -T & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad G_k^w = \begin{bmatrix} T^2/2 & 0 & 0 & 0 & 0 \\ T & 0 & -1 & 0 & 0 \\ 0 & 0 & -T/r_k & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & T/r_k \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$w_k = [w^1_k, w^2_k, w^3_k, w^4_k, w^5_k, w^6_k]^T$ is the process noise vector, $w_i \sim \mathcal{N}(0, \sigma^2_{w_i})$, $i = 1, \cdots, 6$ obeys the zero mean Gaussian distribution with different variances.

The state equation is in linear Gaussian form, so the Markov transition density is:

$$f_{k+1|k}(x^p_{k+1|k}) = f(x^p_{k+1|k}|F_k x_{k|k})$$  \hspace{1cm} (6)

where $x^p_{k+1|k}$ represents the predicted value of the angle state at time $k+1$; $P_{k+1|k}$ represents the predicted value of the noise covariance of the angle state at time $k+1$; the calculation formula is as follows:

$$x^p_{k+1|k} = F_k x_{k|k}$$  \hspace{1cm} (7)

$$P_{k+1|k} = G_k^w \text{Diag}([\sigma^2_{w_1}, \cdots, \sigma^2_{w_6}]) G_k^w^T$$  \hspace{1cm} (8)

3.2. Observation model

Suppose the detector outputs a single observation point $z_k$, which may come from the target or noise. For medium repetition frequency monopulse radar, the observation point $z_k$ is composed of range-Doppler serial number and three-channel complex amplitude value, the specific form is:

$$z_k = [n_r, n_f, s^1_{x}, s^1_{y}, s^2_{x}, s^2_{y}, s^3_{x}, s^3_{y}, s^4_{x}, s^4_{y}, s^5_{x}, s^5_{y}, s^6_{x}, s^6_{y}]^T$$

where, $n_r, n_f \in \mathbb{Z}$ is the sequence number of the distance-Doppler unit corresponding to the observation point; $s^i_{x}, s^i_{y} (i = x, y, z)$ is the real part (I) and imaginary part (Q) of the channel $i$ complex observation of the observation point.

3.2.1. Observation of target area. For a Rayleigh target, given that the target amplitude $a$ obeys the Rayleigh distribution, and the phase $\phi_t$ obeys the uniform distribution on $[0, 2\pi]$, the real and imaginary parts of a obey the Gaussian distribution and are independent of each other \cite{[1]}, which is expressed as:

$$f (a') = \mathcal{N} (a'; 0, \frac{\rho}{2})$$

$$f (a^Q) = \mathcal{N} (a^Q; 0, \frac{\rho}{2})$$  \hspace{1cm} (10)

where, $a' = a \cos (\phi_t)$, $a^Q = a \sin (\phi_t)$, $\rho$ represents the average power, which is obtained from the radar equation.

The observation equation of the target \cite{[7,8]} is expressed as:

$$z_3 = s^4_{x} = a' + v^x_3$$

$$z_4 = s^4_{y} = a' + v^y_4$$
\[ z_5 = s_x^i = \alpha_x a^i + v_x^i \]
\[ z_6 = s_x^q = a^q + v_x^q \]
\[ z_7 = s_y^q = \alpha_y a^q + v_y^q \]
\[ z_8 = s_z^q = \alpha_z a^q + v_z^q \]  
where:

- \( v_x^i, v_x^q (i = x, y, z) \) are the real and imaginary parts of the noise signal of channel \( i \), satisfying \( v_k^i \sim \mathcal{N}(0, \mathbf{R}_{k+1}) \), \( j = I, Q \); due to the isolation effect of each channel, \( \mathbf{R}_{k+1} \) is a diagonal matrix, and \( \mathbf{R}_{k+1} = \text{Diag}([\sigma_x^2, \sigma_y^2, \sigma_z^2]) \);
- \( \alpha_y, \alpha_z \) is the target DOA in the \( y, z \) axis, which is related to the offset angle, half-power beam width \( \theta_B \), and the single pulse angle coefficient \( \kappa_m \), specifically \( \alpha_y = \frac{k_m v_y}{\theta_B}, \alpha_z = \frac{k_m v_z}{\theta_B} \).

### 3.2.2. Observation of non-target area

For the observation of the non-target area, the range doppler observation \( z_c \) obeys a uniform distribution within the range doppler gate, namely
\[ c_{k+1}(z_c) = \frac{1}{N_r N_f} \]  

The magnitude observation \( z_a \) vector satisfies:
\[ z_3 = s_x^i = c^i + v_x^i \]
\[ z_4 = s_y^i = \gamma_y c^i + v_y^i \]
\[ z_5 = s_y^q = c^q + v_y^q \]
\[ z_6 = s_z^q = \gamma_y c^q + v_z^q \]  
where:

- \( c^i, c^q \) is the real and imaginary part of the \( x \) channel, which is a priori unknown deterministic parameter, which can be obtained by maximum likelihood estimation.
- \( \gamma_y^i, \gamma_y^q \) is the DOA angle of the background noise, assuming that they are independent of each other and have \( v^i \sim \mathcal{N}(0, \sigma_y^2) \), \( i = y, z \).

### 3.2.3. Likelihood function

The range doppler position and the three-channel observation complex amplitude value are independent of each other. This article is divided into two parts, let \( z_a = [z_3, z_4, z_5, z_6, z_7, z_8]^T \) and \( z_c = [z_1, z_2]^T \), if the observation comes from the target, the target likelihood function can be expressed as:
\[ g_{k+1}(z_c|\mathbf{x}_{k+1}) = s_{k+1}(z_c|\mathbf{x}_{k+1}) \cdot s_{k+1}(z_a|\mathbf{x}_{k+1}) \]  
where \( s_{k+1}(z_c|\mathbf{x}_{k+1}) \) represents the spatial distribution of the target coordinate position; \( s_{k+1}(z_a|\mathbf{x}_{k+1}) \) represents the spatial distribution of the target amplitude observation.

Due to the exponential attenuation characteristic of the formula, the target observation is local. It can be considered that each strong scattering point only affects the limited rectangular area of \( N_r^{k+1} \cdot N_f^{k+1} \) around it. Therefore, it is assumed that the observation \( z_c \) obeys a uniform distribution in the area \( N_r^{k+1} \cdot N_f^{k+1} \), which is expressed as:
\[ s_{k+1}(z_c|\mathbf{x}_{k+1}) = \frac{1}{N_r^{k+1} \cdot N_f^{k+1}} \]  

According to the radar observation equation, \( s_{k+1}(z_a|\mathbf{x}_{k+1}) \) can be expressed as:
\[ s_{k+1}(z_a|\mathbf{x}_{k+1}) = \mathcal{N}(s^i, 0, Q_{k+1}) \cdot \mathcal{N}(s^q, 0, Q_{k+1}) \]
\( s^j = \begin{bmatrix} s^j_x, s^j_y, s^j_z \end{bmatrix}^T (j = 1,Q) \) is the real and imaginary parts of the observation vector of the complex amplitude;

- \( Q_{k+1} \) is the covariance matrix, which can be expressed as

\[
Q_{k+1} = R_{k+1} + \frac{\rho}{2} [1, \alpha_y, \alpha_z]^T [1, \alpha_y, \alpha_z],
\]

and \( \rho \) is the average power at the target observation.

If the observation comes from a noisy background, its likelihood function can be expressed as:

\[ g_{k+1}(C_{k+1|x}^p) = c_{k+1}(z_c) \cdot c_{k+1}(z_a) \] (17)

where \( c_{k+1}(z_c) \) is shown in the formula, and the amplitude distribution is:

\[
c_{k+1}(z_a) = \mathcal{N}(s^j; \hat{c}^j y, R_{k+1} + (\hat{c}^j)^2 R') \cdot \mathcal{N}(s^0; \hat{c}^0 y, R_{k+1} + (\hat{c}^0)^2 R')
\] (18)

\( \hat{c}^j (j = I, Q) \) is the maximum likelihood estimate of the magnitude of the non-target observation.

\( R' = \text{Diag}([1, \sigma_\theta^2, \sigma_\phi^2]) \) is the covariance matrix of \( y \).

Therefore, the total observation likelihood function can be expressed as:

\[ f_{k+1}(Z_{k+1} | x^p_{k+1}) = g_{k+1}(Z_{k+1} | x^p_{k+1}) \cdot g_{k+1}(C_{k+1} | x^p_{k+1}) \] (19)

Because equation (19) is highly nonlinear, it is difficult to obtain a recursive analytical solution for the Bayesian filter equation. In this paper, the particle filter method is used to estimate the target angle, that is, the particle system \{\( w^i_{k|k}, x^i_{k|k} \}_{i=1}^{N} \} is used to replace the posterior distribution into the Bayesian filter equations of equations (1) and (2).

### 4. Simulation

This paper takes the last-stage mid-frequency radar seeker as an example to verify the feasibility of the algorithm through simulation experiments. The relevant parameters are set as follows.

The head-on attack in the horizontal plane satisfies the collision triangle constraint. The relevant parameter settings are shown in Table 1.

| Distance | Observation period | Beamwidth | Resolution unit | Internal noise characteristics | Variance setting |
|----------|--------------------|-----------|-----------------|------------------------------|-----------------|
| 9km      | 4ms                | 6°        | \( N_f = 250 \) | \( \sigma_i = 1, i = x, y, z \) | \( \sigma_{w_x} = \sigma_{w_y} = \sigma_{w_z} = 0.603 \) |
| 4km      |                     |           | \( N_f = 256 \) |                              | \( \sigma_{w_x} = \sigma_{w_y} = 1 \) mrad |
|          |                     |           |                 |                              | \( \sigma_e = 0.05 \) |
|          |                     |           |                 |                              | \( \sigma_\theta = 1/4 \) |

Through the scene setting, the simulation produces the true value of the target state, and the result is shown in Figure 3 and Figure 4. It can be seen from Figure 3 that as the distance between the radar and the target decreases, the average RCS of the target remains at 0.5m². Figure 4 shows the true value of the azimuth and pitch misalignment angle \( \varepsilon_y, \varepsilon_z \), and as the distance decreases, the misalignment angle \( \varepsilon_y, \varepsilon_z \) keeps increasing.
According to the system parameters and the true value of the target state, the SNR at different distances can be calculated. According to the true value of the target state, the SNR at different moments can be calculated, which is expressed as:

$$\text{SNR} = \frac{k_p \bar{\sigma}}{r^4 (2A_0^2)} \cdot G_a$$

(20)

where, $\bar{\sigma}$ is the average RCS of the target, $r$ is the true distance of the projectile and target, $G_a$ is the antenna gain coefficient, $A_0 = 10$ is the noise amplitude, it obeys the Rayleigh distribution, and $k_p$ is the gain of the radar system. The reference target parameters are the average RCS $\bar{\sigma}_0 = 1 m^2$, distance $r_0 = 10 km$, and echo power $P_0 = 20$dB.

The change of signal-to-noise ratio is shown in Figure 5. It can be seen from the figure that the SNR continues to increase with the continuous decrease of the distance between the projectile and the eye. This is because the echo power is inversely proportional to the fourth power of the distance.

The $\varepsilon_{y_{est}}, \varepsilon_{z_{est}}$ represents the estimation results of the azimuth and pitch misalignment angles, and the $\varepsilon_{y_{true}}, \varepsilon_{z_{true}}$ represents the true values of the azimuth and pitch misalignment angles. Through the analysis of Figure 6, it can be seen that the algorithm in this paper can estimate the target misalignment angle well. As the distance continues to decrease, the SNR continues to increase, and the misalignment angle estimation error continues to decrease.

5. Conclusion

This paper proposes a target misalignment angle estimation algorithm under low SNR. The modeling process of the target angle estimator is deduced in detail, and the corresponding time update equation and observation update equation are given. Simulations verify the effectiveness of the algorithm. The algorithm can effectively track the azimuth and pitch angle of the target, and as the SNR increases, the accuracy of the angle measurement continues to improve. Subsequent work can extend the algorithm to multi-target situations, using multi-target filters to expand the scope of application.
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