On the coupling constant for $\Lambda(1405)\bar{K}N$

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The value of $\Lambda(1405)\bar{K}N$ coupling constant $g_{\Lambda(1405)\bar{K}N}$ is obtained by fitting it to the experimental data on the total cross sections of the $K^- p \rightarrow \pi^0 \Sigma^0$ reaction. On the basis of an effective Lagrangian approach and isobar model, we show that the value $|g_{\Lambda(1405)\bar{K}N}| = 1.51 \pm 0.10$ could be extracted from the available experimental data by assuming that the $s$–channel $\Lambda(1405)$ resonance plays the dominant role, while the background contributions from the $s$–channel $\Lambda(1115)$, $t$–channel $K^*$ and $u$–channel nucleon pole processes are small and can be neglected. However, the $u$–channel nucleon pole diagram may also give an important contribution in present calculations. After the background contributions are taken into account, the above value of $g_{\Lambda(1405)\bar{K}N}$ is reduced to $|g_{\Lambda(1405)\bar{K}N}| = 0.77 \pm 0.07$, which is not supported by the previous calculations and the recent CLAS measurements.

The theoretical calculations on differential cross sections are also presented, which can be checked by the future experiments.

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I. INTRODUCTION

The reactions induced by $K^-$ meson beam are important tools to gain a deeper understanding of the $\bar{K}N$ interaction and also of the nature of the hyperon resonances. Among those reactions, the $K^- p \rightarrow \pi^0 \Sigma^0$ reaction is of particular interest. Since there are no isospin-1 hyperons contributing here, this reaction gives us a rather clean platform to study the isospin zero $\Lambda$ resonances. Furthermore, it is well known that the inelastic effects are especially important for the low energy $\bar{K}N$ interaction because the $\bar{K}N$ channel strongly couples to the $\pi \Sigma$ channel through $\bar{\Lambda}(1405)$ resonance (spin-parity $J^P = 1/2^-$). Thus, the $K^- p \rightarrow \pi^0 \Sigma^0$ reaction is a good place to study the $\Lambda(1405)$ state, whose structure and properties are still controversial, even it is catalogued as a four-star $\Lambda$ resonance in the Particle Data Group (PDG) review book [1].

In the traditional quark models, the $\Lambda(1405)$ is described as a $p$–wave $q^4$ baryon [2], but it can also be explained as a $\bar{K}N$ molecule [3] or $q^4 q^\dagger$ pentaquark state [3]. Besides, it was also argued, within the unitary chiral theory [4, 5], two overlapping isospin $I = 0$ states are dynamically generated and in this approach the shape of any observed $\Lambda(1405)$ spectrum might depend upon the production process. In a recent experimental study of the $pp \rightarrow pK^+ \Lambda(1405) \rightarrow pK^+ (\pi \Sigma)$ reaction [6], the $\Lambda(1405)$ resonance was clearly identified through its $\pi^0 \Sigma^0$ decay and no obvious mass shift was found, which has been checked in Ref. [9] by using the effective Lagrangian approach with considering only one $\Lambda(1405)$ state. However, the final answer is still absent in the sense that the experimental data could also be well described in the two-resonance scenario [10].

As shown in Refs [11–17], the combination of effective Lagrangian approach and the isobar model is a good method to study the hadron resonances production in the $\pi N$, $N N$, and $\bar{K}N$ scattering. One key issue of this method is the coupling constant of the involved resonance interaction vertex, which can be obtained from the partial decay width. However, if the mass of the resonance is below the threshold of the corresponding channel, such as the $\Lambda(1405)$ ($M = 1405$ MeV) to the $\bar{K}N$ ($m_K + m_N = 1434.6$ MeV) channel, it is impossible to get the coupling constant within the above procedure. For example, the strong coupling constants of $\Lambda(1405)$ resonance were investigated within an extended chiral constituent quark model [18], while in Refs [16, 19], the coupling constant of $g_{N^*(1535)N\rho}$ was obtained from the studies of the $\pi^- p \rightarrow n \rho^0$ reaction. Besides, in Ref. [20], the coupling constant of $g_{N^*(1535)N\rho}$ was studied from the analysis of the $N^*(1535) \rightarrow N \rho^0 \rightarrow N \pi^+ \pi^-$ and the $N^*(1535) \rightarrow N \rho^0 \rightarrow N\gamma$ decays.

Moreover, the couplings of $\Lambda(1405)$ resonance to the $\bar{K}N$ and $\pi \Sigma$ channels and the ratio of $g_{\Lambda(1405)\bar{K}N}$ and $g_{\Lambda(1405)\pi \Sigma}$, $\mathcal{R} = g_{\Lambda(1405)\bar{K}N}/g_{\Lambda(1405)\pi \Sigma}$, have been intensively studied both experimentally [21] and within various theoretical approaches, for instance, the current algebra [22, 23], potential models [22, 24], dispersion relations [25], asymptotic SU(3) symmetry approach [26], and they are also recently investigated by taking the $\Lambda(1405)$ resonance to be an admixture of traditional three-quark and higher five-quark Fock components [18]. However, the obtained values of $\mathcal{R}$ by different theoreti-
cal methods vary in a large range 3.2 – 7.8, so it is still worth to study the coupling constants \( g_{\Lambda(1405)KK} \) and \( g_{\Lambda(1405)\Sigma \Sigma} \) from other different ways.

The present work we report here is one more in this line, by using an effective Lagrangian approach and the isobar model, we extract the \( \Lambda(1405)KK \) coupling constant \( g_{\Lambda(1405)KK} \) by fitting it to the experimental data on the total cross sections of \( K^- p \to \pi^0 \Sigma^0 \) reaction near threshold. We also calculate the differential cross sections for the \( K^- p \to \pi^0 \Sigma^0 \) reaction with the fitted parameters. These model predictions can be checked by the future experiments. For simplicity we shall here work within the single \( \Lambda(1405) \) state framework with parameters as reported in the PDG [1].

This article is organized as follows. In Sect. II, we show here the relations of these coupling constants in terms of the \( K^- p \to \pi^0 \Sigma^0 \) reaction within the effective Lagrangian method. The combination of isobar model and effective Lagrangian method is a useful theoretical approach in the description of various processes in the resonance production region. In this section, we introduce the theoretical formalism and ingredients for calculating the cross sections of the \( K^- p \to \pi^0 \Sigma^0 \) reaction within the effective Lagrangian method. The basic tree level Feynman diagrams, for the \( K^- p \to \pi^0 \Sigma^0 \) reaction are shown in Fig. I. These include the \( t \)-channel \( K^* \) exchange [Fig. I (a)], the \( u \)-channel proton exchange [Fig. I (b)], and the \( s \)-channel \( \Lambda(1115) \) and \( \Lambda(1405) \) processes [Fig. I (c)]. To compute the contributions of these terms, we use the effective interaction Lagrangian densities as used in Refs. [27–31]:

\[
\mathcal{L}_{K^*KK} = -g_{K^*KK}(\bar{\pi} \cdot \tau \partial \bar{K} - \bar{K} \partial \bar{\pi} \cdot \tau)K^*_\mu, \\
\mathcal{L}_{K^*\Sigma \Sigma} = -i g_{K^{\ast}N\Sigma \Sigma} \bar{N}(\gamma_\mu - \frac{\kappa}{2M_N} \sigma_\mu \partial^\nu)K^{\ast \nu \Sigma \cdot \tau} + \text{H.c.}, \\
\mathcal{L}_{\pi NN} = -i g_{\pi NN} \bar{N} \gamma_5 \pi \cdot \tau N + \text{H.c.}, \\
\mathcal{L}_{K^{\ast}N\Sigma} = -i g_{K^{\ast}N\Sigma} \bar{N} \gamma_5 \Sigma \cdot \tau K + \text{H.c.}, \\
\mathcal{L}_{K^{\ast}N\Lambda} = -i g_{K^{\ast}N\Lambda} \bar{N} \gamma_5 \Lambda K + \text{H.c.},
\]

for the \( t \)-channel \( K^* \) exchange, and

\[
\mathcal{L}_{\pi \Sigma \Lambda} = -i g_{\pi \Sigma \Lambda} \bar{N} \gamma_5 \Lambda \pi \cdot \Sigma + \text{H.c.}, \\
\mathcal{L}_{\Lambda^*\Sigma \Lambda} = -i g_{\Lambda^*\Sigma \Lambda} \bar{N} \gamma_5 \Sigma \cdot \Lambda + \text{H.c.},
\]

for the \( s \)-channel \( \Lambda(1115) \) and \( \Lambda(1405) \) terms.

II. FORMALISM AND INGREDIENTS

For the coupling constants in the above Lagrangian densities, we take \( g_{\pi NN} = 13.45 \) (obtained with \( g_{\pi NN}^2/4\pi = 14.4 \)), \( g_{K^{\ast}N\Lambda} = -13.98 \), \( g_{\pi \Sigma \Lambda} = 9.32 \), and \( g_{K^{\ast}N\Sigma} = 2.60 \) as that determined within SU(3) flavor symmetry [32]. For the \( K^*N\Sigma \) couplings, we take \( g_{K^{\ast}N\Sigma} = -2.36 \) and \( \kappa = -0.47 \) as used in Ref. [32] for the calculation of \( K\Lambda \) photoproduction. While the coupling constants \( g_{K^*K^*} \) and \( g_{K^*\Sigma \Sigma} \) are determined from the experimentally observed partial decay widths of the \( K^* \to K\pi \) and \( \Lambda(1405) \to \pi \Sigma \), respectively,

\[
\Gamma_{K^* \to K\pi} = \frac{g_{K^*K^*}^2 [\bar{p}_\pi^c.m.]^3}{2\pi m_{K^*}}, \\
\Gamma_{\Lambda^* \to \pi \Sigma} = \frac{3 g_{\Lambda^*\Sigma \Sigma}^2}{4\pi} (E_{\Sigma} + m_{\Sigma}) |\bar{p}_\Sigma| / M_{\Lambda^*},
\]

where

\[
|\bar{p}_\pi^c.m.| = \sqrt{[m_{K^*}^2 - (m_K + m_\pi)^2]/2m_{K^*}}, \\
E_{\Sigma} = \frac{M_{\Sigma^*}^2 + m_{\Sigma}^2 - m_\Sigma^2}{2M_{\Lambda^*}}, \\
|\bar{p}_\Sigma| = \sqrt{E_{\Sigma}^2 - m_\Sigma^2}.
\]

With masses \( m_{K^*} = 893.1 \text{ MeV} \), \( m_K = 495.6 \text{ MeV} \), \( m_\pi = 138.04 \text{ MeV} \), and \( M_{\Lambda^*} = 1405.1^{+1.6}_{-1.0} \text{ MeV} \), total decay widths \( (\Gamma_{K^*} = 49.3 \text{ MeV} \) and \( \Gamma_{\Lambda^*} = 50 \pm 2 \text{ MeV} \), and decay branching ratio of \( K^* \to K\pi \) and \( \Lambda(1405) \to \pi \Sigma \) \( \text{Br}(K^* \to K\pi) \sim 1 \) and \( \text{Br}(\Lambda(1405) \to \pi \Sigma) \sim 1 \),

![FIG. 1: Feynman diagrams for the reaction $K^- p \to \pi^0 \Sigma^0$. In these diagrams, we show the definitions of the kinematical $(p_1, p_2, p_3, p_4)$ and polarization variables $r_1, r_2$ those we use in our calculation.](image-url)
we obtain $g_{K^*K\pi} = 3.25$, and $|g_{\Lambda^\ast\Sigma}| = 0.90 \pm 0.02$ by considering the uncertainties of the total decay width and the mass of $\Lambda(1405)$ resonance.

With the effective Lagrangian densities given above, we can easily construct the invariant scattering amplitude,

$$M_i = \bar{u}_{r_2}(p_2) A_i u_{r_1}(p_1),$$

(11)

where $i$ denotes the $s-$, $t-$ or $u-$channel process, and $\bar{u}_{r_2}(p_2)$ and $u_{r_1}(p_1)$ are the spinors of the outgoing $\Sigma^0$ baryon and the initial proton, respectively. The reduced $A_i$ read,

$$A_{s(\Lambda(1115))}^{(1115)} = -ig_{\bar{K}KN\pi} s \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \Sigma \S
with $\Lambda = 0.6$ GeV for all the channels, and the fitted parameter $g_{\Lambda^*K_N}g_{\Lambda^*\Sigma}$ is $-0.70 \pm 0.06$. From the value of $g_{\Lambda^*\Sigma}$, we can get $|g_{\Lambda^*K_N}| = 0.77 \pm 0.07$.

The corresponding best fitting results of the Fit I for the total cross sections are shown in Fig. 2 comparing with the experimental data. We also show, in Fig. 2, the experimental data with larger error from Ref. [37] and one data point from Ref. [37] for comparison. The solid line represents the full results, while the contributions from $s-$channel $\Lambda(1405)$ resonance, $s-$channel $\Lambda(1115)$, $t-$, and $u-$channel diagrams are shown by dashed, dotted, and dot-dashed lines, respectively. From Fig. 2 one can see that we could describe the near threshold data of $K^-p \to \pi^0\Sigma^0$ reaction quite well, and the $s-$channel $\Lambda(1405)$ resonance and $u-$channel proton pole give the dominant contributions, while the $s-$channel $\Lambda(1115)$ process and $t-$channel $K^*$ exchange give minor contribution. Theoretically, it is important to find some observables to distinguish the relative roles of individual mechanisms. In Fig. 3 the corresponding theoretical calculation results for the differential cross sections at $p_{lab} = 0.15$ GeV [Fig. 3 (a)], $p_{lab} = 0.25$ GeV [Fig. 3 (b)], and $p_{lab} = 0.35$ GeV [Fig. 3 (c)] are shown, which can be tested by future experiments.

For the role of $u-$channel proton pole diagram, since there could be large SU(3) flavor symmetry violation, as summarized in Ref. [38] (see Table II of this reference), the value of $g_{\Sigma N}$ lies in a very wide range. Besides, the contributions from $s-$channel $\Lambda(1115)$ and $t-$channel diagram are very small, so, next, we try the fit with considering the contribution from only $s-$channel $\Lambda(1405)$ resonance (Fit II). In this case, we have two free parameters which are $g_{\Lambda^*K_N}g_{\Lambda^*\Sigma}$ and the cut off parameter $\Lambda_s^{(1405)}$. The fitted results are $g_{\Lambda^*K_N}g_{\Lambda^*\Sigma} = 1.36 \pm 0.08$, which gives $|g_{\Lambda^*K_N}| = 1.51 \pm 0.10$, and $\Lambda_s^{(1405)} = 3.00 \pm 2.62$ GeV, with a large $\chi^2/dof = 1.7$. The corresponding results for the total cross sections are shown in Fig. 4 with the solid line. We also show the 90% confidence-level band obtained from the statistical uncertainties of the fitted parameters. The results show that we can also give a reasonable description for the experimental data by only including the $s-$channel $\Lambda(1405)$ resonance. However, even by considering the errors of the theoretical calculation, we still cannot give a reasonable description for the data points from Ref. [37].

The value of $|g_{\Lambda^*K_N}| = 1.51 \pm 0.10$ is comparable with the value, 1.84 [39], which was obtained from the separable potential model [39]. In contrast to the unitary chiral potential model [39], in which the mean values and standard deviations obtained from the best $\chi^2$ fit for each $(g_{\Lambda^*K_N}g_{\Lambda^*\Sigma}$ and $\Lambda_s^{(1405)})$ pair, we calculate the total cross sections. We plot all these results and throw away the upper 5% and the lower 5%, then the band, shown in Fig. 4 is obtained.

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3 We generate pairs of parameters $(g_{\Lambda^*K_N}g_{\Lambda^*\Sigma}$ and $\Lambda_s^{(1405)})$ from a two-dimensional correlated Gaussian distribution with the mean values and standard deviations obtained from the best $\chi^2$ fit. For each $(g_{\Lambda^*K_N}g_{\Lambda^*\Sigma}$ and $\Lambda_s^{(1405)})$ pair, we calculate the total cross sections. We plot all these results and throw away the upper 5% and the lower 5%, then the band, shown in Fig. 4 is obtained.

FIG. 3: $K^-p \to \pi^0\Sigma^0$ differential cross sections at different energies. Results have been obtained from the Fit I. The solid line represents the full results, while the contribution from $s-$channel $\Lambda(1405)$ resonance, $s-$channel $\Lambda(1115)$, $t-$channel $K^*$, and $u-$channel proton processes are shown by the dashed, dotted, dot-dashed, and dot-dot-dashed lines, respectively.
theory \cite{6}, the separable model produces only a single \Lambda(1405) pole and this is consistent with our assumption that we only include one \Lambda(1405) state in the present calculation for simplicity.

From the fitted parameters of Fit II, we find an unrealistic central value of 3.00 GeV for the cutoff \lambda^{[1405]} parameter, with a large error (2.62 GeV), which indicates that the \chi^2 is rather insensitive to this parameter, so, next we fix the cut-off at some values, then fit the only one parameter \gamma_{\Lambda+} to the total cross sections data, from which we can get the fitted coupling constant \gamma_{\Lambda+} as a function of the cut-off parameter \lambda^{[1405]}.

The results are shown in Fig. 5. We can see from the figure that the value of \gamma_{\Lambda+} is stable at 1.5 within a very wide range of the cut-off parameter \lambda^{[1405]}.

From the values \gamma_{\Lambda+} = 1.51 \pm 0.10 and \gamma_{\Lambda+} = 0.90 \pm 0.02, we can easily obtain the ration \gamma = \gamma_{\Lambda+}/\gamma_{\Lambda+} = 1.68 \pm 0.12 \footnote{If we take the values 0.77 from Fit I and 1.51 from Fit II as the lower and upper limit for the \gamma_{\Lambda+}, respectively, then we can get \gamma_{\Lambda+} = 1.14 \pm 0.37, which gives \gamma = 1.27 \pm 0.41.}, which is smaller than those obtained from different models: 2.19 obtained by using an algebra-of-currents approach \cite{22}, and 2.61 \pm 1.34 extracted from the coupled-channel analysis of the \bar{K}p scattering \cite{41}. We show these values and also the couplings of \Lambda^{[1405]}KN and \Lambda^{[1405]}\Sigma in the Table I for comparison. From Table I we find that even the values of \gamma are different, but the values for \gamma_{\Lambda+} are similar within the errors.

\begin{table}[h]
\begin{center}
\caption{Couplings of \Lambda(1405)\bar{K}N and \Lambda(1405)\Sigma.}
\begin{tabular}{|c|c|c|}
\hline
\g_{\Lambda+} & \g_{\Lambda+} & \g_{\Lambda+} \\
KN & \Sigma & \Sigma \\
\hline
1.64 & 0.75 & 2.19 \footnote{[22]} \\
2.10 \pm 0.71 & 0.77 \pm 0.30 & 2.61 \pm 1.34 \footnote{[40]} \\
1.51 \pm 0.10 & 0.90 \pm 0.02 & 1.68 \pm 0.12 \footnote{This work} \\
\hline
\end{tabular}
\end{center}
\end{table}

On the other hand, in analysis of the line shapes of the \Sigma\pi final states in the production reaction \gamma p \rightarrow \bar{K}^{+}+(\Sigma\pi), it is convenient to parameterize the scattering amplitude, in the isospin zero sector, as the Breit-Wigner function BW(W), which has been used in Ref. \cite{41},

\begin{equation}
BW(W) = \frac{1}{W^2 - M_{\Lambda}^2 + iM_{\Lambda}\Gamma_{\Lambda}(W)},
\end{equation}

where \textit{W} is the invariant mass of the \Sigma\pi system, and \Gamma_{\Lambda}(W) is the energy dependent width that accounts for all the decay channels, this is because of that in the \gamma p \rightarrow \bar{K}^{+}+(\Sigma\pi) reaction, the \NK channel opens within the range of the mass distribution of the \Sigma\pi system.

Considering the \Lambda^{[1405]}\bar{K}N coupling, by using the Flatté prescription \cite{42}, the energy dependent total decay width for the \Lambda^{[1405]} resonance, is \footnote{We mention that in the present calculation, since the \NK channel is opened, so we just use a constant total decay width for \Lambda(1405) resonance, in such a way we can also reduce the number of free parameters. Furthermore, we do not continue the decay momentum to imaginary values, which means below the threshold of decay channel, we take the decay momentum as zero.}
\[ \Gamma_{\Lambda^*}(W) = \frac{3g_{\Lambda^*}^2}{4\pi} [E_{\Xi} + m_{\Sigma}] |p_{\Sigma}^*| W + \frac{g_{\Lambda^*}^2 \Lambda KN}{2\pi} [E_N + m_N] |p_{\Sigma}^*| \theta(W - m_\Lambda - m_N), \]  
\[ (20) \]
with,
\[ E_{\Xi/N} = \frac{W^2 + m_{\Xi/N}^2 - m_{\pi/K}^2}{2W}, \]  
\[ (21) \]
\[ |p_{\Sigma}^*/N| = \sqrt{E_{\Xi/N}^2 - m_{\Sigma/N}^2}. \]  
\[ (22) \]

In Fig. 6 we show the results for the module square of BW(W) as a function of W. The solid line stands for the results obtained with a constant total decay width (50 MeV), while the dashed and dotted line are obtained with \( g_{\Lambda^*} KN = 1.51 \) and \( g_{\Lambda^*} KN = 0.77 \) by using the energy dependent width with the form in Eq. \( (20) \), respectively. From the results, we find that the Breit-Wigner mass will be pushed down if we use the energy dependent width, and there is a clear drop in the module squared distribution at the \( \Lambda KN \) threshold with a larger value \( g_{\Lambda(1405) KN} = 1.51 \). However, the smaller value \( g_{\Lambda(1405) KN} = 0.77 \) could not give a clear drop for the module square distribution at the \( \Lambda KN \) threshold. The very recent experimental results measured by the CLAS Collaboration in Ref. [41] show that there is a sharp drop of the \( \Sigma \pi \) mass distributions, and this could be reproduced by using the above BW(W) formalism with a large coupling constant \( g_{\Lambda(1405) KN} \).

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6}
\caption{The module square of the Breit-Wigner function for \( \Lambda^*(1405) \) vs W with a constant total decay width (solid line) and an energy dependent width with the form in Eq. \( (20) \) with \( g_{\Lambda^*} KN = 1.51 \) (dashed line) and \( g_{\Lambda^*} KN = 0.77 \) (dotted line).}
\end{figure}

\section{Summary}

In this work, the value of \( \Lambda(1405) \Lambda KN \) coupling constant \( g_{\Lambda(1405) \Lambda KN} \) is obtained by fitting it to the low energy experimental data of the \( K^- p \rightarrow \pi^0 \Sigma^0 \) reaction. On the basis of an effective Lagrangian approach, we show that the value of \( \Lambda(1405) \Lambda KN \) coupling constant \( g_{\Lambda(1405) \Lambda KN} = 0.77 \pm 0.07 \) can be extracted from the available low energy experimental data of the total cross section of the \( K^- p \rightarrow \pi^0 \Sigma^0 \) reaction by including the \( s \)-channel \( \Lambda(1405) \) resonance, \( s \)-channel \( \Lambda(1115) \) process, \( t \)-channel \( \Lambda \) and \( u \)-channel proton pole diagrams. On the other hand, by including only the contribution from the \( s \)-channel \( \Lambda(1405) \) resonance, we get \( g_{\Lambda(1405) \Lambda KN} = 1.51 \pm 0.10 \), which is supported by the previous calculations [22, 40] as shown in Table 1.

Due to the violation of SU(3) flavor symmetry, the contribution from \( u \)-channel could be small, and we show that the differential cross sections shown in Fig. 6 could help us clarifying whether the \( u \)-channel contribution is important or not.

We also calculate the module square of Breit-Wigner function for the \( \Lambda(1405) \) resonance with an energy dependent total decay width with the form as the Flatté prescription [42]. The results show that the Breit-Wigner mass of \( \Lambda(1405) \) resonance will be pushed down, and there is a clear drop in the module square distribution at the \( \Lambda KN \) threshold if we take a larger value \( g_{\Lambda(1405) \Lambda KN} = 1.51 \). The clear drop could explain the sharp drop that was found in the \( \Sigma \pi \) mass distribution in the recent experimental measurements [41] by the CLAS Collaboration.

Finally, we would like to stress that the coupling constant \( g_{\Lambda(1405) \Lambda KN} \) is important for studying the \( \Lambda(1405) \) resonance in the \( \gamma p \rightarrow K^+ \Lambda(1405) \rightarrow K^+ (\pi \Sigma) \) reaction and also in the \( pp \rightarrow pK^+ \Lambda(1405) \rightarrow pK^+ (\pi \Sigma) \) reaction. More and accurate data for these reactions can be used to improve our knowledge on the structure and properties of \( \Lambda(1405) \) state, which are, at present, still controversial.

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