SU(2|1) supersymmetric spinning models of chiral superfields

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Abstract
We construct SU(2|1), d = 1 supersymmetric models based on the coupling of dynamical and semi-dynamical (spin) multiplets, where the interaction term of both multiplets is defined on the generalized chiral superspace. The dynamical multiplet is defined as a chiral multiplet (2, 4, 2), while the semi-dynamical multiplet is associated with a multiplet (4, 4, 0) of the mirror type.

Keywords: supersymmetric mechanics, deformation, spin variables

1. Introduction
A new class of systems of $\mathcal{N} = 4$, $d = 1$ supersymmetric quantum mechanics called ‘Kähler oscillator’ was introduced by Bellucci and Nersessian in [1]. They studied supersymmetric oscillator models on Kähler manifolds with Wess–Zumino (WZ) type terms responsible for the presence of a constant magnetic field. It turned out, the presence of oscillator and WZ terms deforms the standard $\mathcal{N} = 4$ Poincaré supersymmetry to the so called ‘weak supersymmetry’ [2]. In our works [3, 4], we showed that the deformed superalgebra of weak supersymmetry corresponds to SU(2|1) supersymmetry. In these papers we initiated a study of deformed supersymmetric quantum mechanics by employing superfield approach based on the worldline supersymmetry SU(2|1) with a mass dimension deformation parameter $m$. In the limit $m = 0$, models of the standard $\mathcal{N} = 4$ supersymmetric mechanics are restored. Indeed, within this framework we reproduced the models studied earlier at the component level [1, 2, 5] and obtained the new ones, including those constructed via harmonic superspace [6] (see also [7]).

During the study of SU(2|1) supersymmetric mechanics we revealed few peculiar features. One of them is the generalization of a chiral condition given by [4]

$$\left(\cos \lambda \, \bar{D}_i - \sin \lambda \, D_i\right) \varphi = 0,$$

where $D_i$ and $\bar{D}_i$ are SU(2|1) covariant derivatives. It describes a new type of the chiral multiplet (2, 4, 2) defined on the generalized chiral superspace and depending on two deformation
parameters: $\lambda$ and $m$. Exactly this multiplet is a basis for the construction of supersymmetric Kähler oscillator models [1], where the frequency of oscillator and the strength of external magnetic field are identified with

$$\omega = \frac{m \sin 2\lambda}{2}, \quad B = m \cos 2\lambda.$$  \hfill (1.2)

However, both parameters disappear in the limit $m = 0$, because the rotation parameter $\lambda$ becomes just an external automorphism parameter of the standard $\mathcal{N} = 4$ Poincaré supersymmetry. One needs to point out that the corresponding Hamiltonian does not commute with supercharges and is identified with internal $U(1)$ generator of $SU(2|1)$. It can be treated as a central charge only when $\lambda = 0$. Recently, particular superintegrable Kähler oscillator models were considered [8].

With the lapse of time we found out other distinguished features of deformed supersymmetric quantum mechanics. An important one concerns $\mathcal{N} = 4$ ‘mirror multiplets’ [9]. The standard $\mathcal{N} = 4$ multiplets have their mirror counterparts characterized by the interchange of two $SU(2)$ groups which form $SO(4)$ automorphism group of the standard $\mathcal{N} = 4$ Poincaré supersymmetry. Since this interchange has no essential impact on Poincaré supersymmetry, $\mathcal{N} = 4$ multiplets and their mirror counterparts are mutually equivalent when dealing with only one multiplet from such a pair. Deformation to $SU(2|1)$ supersymmetry breaks this equivalence, because the first $SU(2)$ group becomes subgroup of $SU(2|1)$ and the second one is broken. It means that $SU(2|1)$ multiplets differ from their mirror counterparts. We showed this difference in details on the example of $(4, 4, 0)$ multiplets [6].

The main goal of the present paper is to employ $SU(2|1)$ superfield approach to spinning models of chiral superfields instead of harmonic ones. The $SU(2|1)$ supersymmetric spinning models of harmonic superfields [7, 10, 11] followed the construction elaborated in [12, 13], with the dynamical multiplet $(1, 4, 3)$ and the semi-dynamical multiplet $(4, 4, 0)$. The most important property is that both dynamical and semi-dynamical multiplets admit a description in the analytic harmonic superspace [14], like as the $\mathcal{N} = 4, d = 1$ gauge multiplet [15]. The latter one allows to introduce the $U(1)$ gauge symmetry transformations. The gauge invariant coupling term is constructed as an integral over the analytic harmonic superspace and provides the interaction of two multiplets. This term is identified with WZ term and involves semi-dynamical spin variables. Extra $SU(2)$ symmetries are defined in terms of these variables, with respect to which physical states of quantum models carry additional spin quantum numbers. Generalization of the construction to matrix superfields [12] give rise to $\mathcal{N} = 4$ supersymmetric extentions of the widely known Calogero system [17] (see also the review [18]).

In this paper we consider the coupling of the dynamical multiplet $(2, 4, 2)$ and the semi-dynamical multiplet $(4, 4, 0)$ of the mirror type. Both multiplets satisfy the generalized chiral condition (1.1) that allows to introduce the interaction term as a superpotential term. We construct $SU(2|1)$ supersymmetric models based on this coupling, undeformed versions of which were studied in [19]. To compare the results with [19], we consider an example of a model on the pseudo-sphere $SU(1, 1)/U(1)$ (the Lobachevsky space).

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1 Some recent $\mathcal{N} = 4$ deformed and undeformed supersymmetric models with spin variables were studied at the component level in [16].

2 In this paper, superfield description for a semi-dynamical multiplet proceeds from the fermionic multiplet $(0, 4, 4)$. One can make the same comment as in the footnote 7 of [20].
2. SU(2|1) chiral superfields

In [4], we considered a special type of SU(2|1) superspace defined for the standard form of the superalgebra $su(2|1)$:

\[
\{Q^i, \bar{Q}_j\} = 2mI^i + 2\delta^i_j\mathcal{H}, \quad [I^i, I^j] = \delta^i_jI^i - \delta^i_jI^j,
\]

\[
[I^i, \bar{Q}_j] = \frac{1}{2}\delta^i_j\bar{Q}_j - \delta^i_j\bar{Q}_i, \quad [I^i, Q^j] = \delta^i_jQ^j - \frac{1}{2}\delta^i_jQ^i.
\]

(2.1)

The bosonic subgroup includes of SU(2) subgroup composed of $I^i$ ($i = 1, 2$) and U(1) generator $\mathcal{H}$ associated with Hamiltonian. Factorizing the SU(2) subgroup, the superspace is defined as

\[
\frac{SU(2|1)}{SU(2)} \sim \frac{\{\mathcal{H}, Q^i, \bar{Q}_j, I^i\}}{\{I^i\}} = \{t, \theta_i, \bar{\theta}^i\}.
\]

(2.2)

The odd $\epsilon^i$ transformations are given by

\[
\delta t = i(\epsilon_1\bar{\theta}^2 + \epsilon_2\theta_1), \quad \delta \theta_i = \epsilon_i + 2m\theta_k\theta_i, \quad \delta \bar{\theta}^i = \epsilon^i - 2m\epsilon \bar{\theta}^2 \bar{\theta}^i,
\]

(2.3)

\[
(\theta_i) = \bar{\theta}^i, \quad (\epsilon_i) = \epsilon^i.
\]

In the limit $m = 0$, the standard ‘flat’ supersymmetry transformations are restored. One can easily pick out the chiral subspace $\{t_\chi, \theta_\chi\}$, where

\[
t_\chi = t + i\bar{\theta}^2\theta_k - im(\bar{\theta}^2\theta_k)^2, \quad \delta t_\chi = 2i\epsilon \theta_k, \quad \delta \theta_\chi = \epsilon_i + 2m\bar{\theta}^2 \theta_i.
\]

(2.4)

We also define the generalized chiral coordinates

\[
i_\chi = t + i\bar{\theta}^2\theta_k, \quad \hat{\theta}_i = \left(\cos \lambda \theta_k e^{\frac{1}{2}m\theta_k} + \sin \lambda \bar{\theta}_i e^{-\frac{1}{2}m\theta_k}\right) \left(1 - \frac{m}{2} \bar{\theta}^2 \theta_k\right).
\]

(2.5)

They are closed under the following supersymmetry transformations:

\[
\delta \theta_i = \cos \lambda \left(\epsilon_i e^{\frac{1}{2}m\theta_k} + m e^\theta \bar{\theta}_i \theta_k e^{-\frac{1}{2}m\theta_k}\right) + \sin \lambda \left(\epsilon_i e^{-\frac{1}{2}m\theta_k} + m e^\theta \bar{\theta}_i \theta_k e^{\frac{1}{2}m\theta_k}\right),
\]

(2.6)

\[
\delta \theta_k = 2i \left(\cos \lambda e^{\frac{1}{2}m\theta_k} e^{\frac{1}{2}m\theta_k} - \sin \lambda e^{\frac{1}{2}m\theta_k} e^{\frac{1}{2}m\theta_k}\right).
\]

Chiral superfields, defined on the left chiral subspace $\{t_\chi, \theta_\chi\}$, satisfy the chiral condition

\[
\bar{\mathcal{D}}_{\chi}^i \left(\hat{t}_\chi, \hat{\theta}_i\right) = 0,
\]

(2.7)

where the covariant derivatives $\bar{\mathcal{D}}_\chi$ and $\bar{\mathcal{D}}^i$ are modified as

\[
\bar{\mathcal{D}}_\chi = \cos \lambda \bar{\mathcal{D}}_\chi - \sin \lambda \mathcal{D}_\chi, \quad \bar{\mathcal{D}}^i = \cos \lambda \mathcal{D}^i + \sin \lambda \bar{\mathcal{D}}^i, \quad \lambda \in [0, \pi/2).
\]

(2.8)

In the basis $\{\hat{t}_\chi, \hat{\theta}_i, \hat{\theta}^i\}$, the covariant derivative $\bar{\mathcal{D}}_\chi$ is written as
\[ \bar{D}_j = - \left[ 1 + \frac{m \cos \lambda}{2} \tilde{\theta}_k \tilde{\theta}_k - \frac{m \sin \lambda}{4} \left( \tilde{\theta}_k \tilde{\theta}_k + \tilde{\theta}_k \tilde{\theta}_k \right) + \frac{m^2}{16} \left( \tilde{\theta}_k \tilde{\theta}_k + \tilde{\theta}_k \tilde{\theta}_k \right) \right] \frac{\partial}{\partial \tilde{\theta}_j} \] (2.9)

Here, we ignore the matrix SU(2) generators \( \tilde{I}_i \), because the generalized chiral superfields \( \bar{D}_i \) cannot carry any external SU(2) indices.\(^3\)

The SU(2) invariant measure is written as
\[ d\zeta = dt d^2 \tilde{\theta} d^2 \tilde{\theta} \left( 1 + 2m \tilde{\theta}_k \tilde{\theta}_k \right). \] (2.10)

One can write this measure in the basis \( \{ t, \tilde{\theta}, \tilde{\theta} \} \) as
\[ d\zeta = dt d^2 \tilde{\theta} d^2 \tilde{\theta} \left[ 1 + m \cos \lambda \left( \tilde{\theta}_k \tilde{\theta}_k - \frac{m}{2} \left( \tilde{\theta}_k \tilde{\theta}_k + \tilde{\theta}_k \tilde{\theta}_k \right) \right) \right]. \] (2.11)

The chiral measure \( d\zeta_L d^2 \tilde{\theta} \) is also invariant under the transformations (2.6):
\[ d\zeta_L = d\zeta d^2 \tilde{\theta}, \quad \delta (d\zeta_L) = 0. \] (2.12)

2.1. Dynamical multiplet (2, 4, 2)

The multiplet (2, 4, 2) is described by a complex superfield \( \Phi \) subjected to the chirality condition
\[ \bar{D}_j \Phi = 0. \] (2.13)

Its general solution reads
\[ \Phi (\tilde{t}_L, \tilde{\theta}) = z + \sqrt{2} \tilde{\theta}_k \xi^k + \tilde{\theta}_k \tilde{\theta}_k B. \] (2.14)

The superfield is not deformed, but transformation properties of its components are still deformed:
- \( \delta z = -\sqrt{2} \cos \lambda \xi^k e^{\frac{1}{2}mt} - \sqrt{2} \sin \lambda \xi^k e^{-\frac{1}{2}mt} \)
- \( \delta \xi^k = \sqrt{2} e^k (i \cos \lambda \tilde{z} - \sin \lambda B) e^{\frac{1}{2}mt} - \sqrt{2} e^k (i \sin \lambda \tilde{z} + \cos \lambda B) e^{\frac{1}{2}mt} \),
- \( \delta B = -\sqrt{2} \cos \lambda \xi^k \left( i e^k + \frac{m}{2} \xi^k \right) e^{-\frac{1}{2}mt} + \sqrt{2} \sin \lambda \xi^k \left( i e^k - \frac{m}{2} \xi^k \right) e^{\frac{1}{2}mt} \). (2.15)

The deformation parameters \( m \) and \( \lambda \) also appear in (2.11).

Superfield kinetic action for the chiral superfield \( \Phi \) is given by
\[ S_{\text{kin}} = \frac{1}{4} \int d\zeta K (\Phi, \bar{\Phi}), \] (2.16)

where \( K (\Phi, \bar{\Phi}) \) is a Kähler potential. The component Lagrangian is written as

\(^3\)The explanation is given in appendix B of [21].
The component Lagrangian contains the standard kinetic term

$$\mathcal{L}_{\text{kin.}} = g \dot{\bar{z}} \bar{z} + i \frac{1}{2} g \left( \zeta \dot{\xi} - \xi \dot{\zeta} \right) + g B \theta - \frac{1}{2} B \partial_\xi \xi \dot{\xi} - \frac{1}{2} B \partial_\xi \xi \dot{\xi}$$

with the metric $g(z, \bar{z}) := \partial_z \partial_{\bar{z}} \bar{z} (z, \bar{z})$.

2.2. Semi-dynamical mirror multiplet (4, 4, 0)

In [6], we studied the deformed mirror multiplet (4, 4, 0) in the framework of harmonic superspace. Here we consider its generalization to the chiral superspace (2.5). The generalized mirror multiplet (4, 4, 0) is described by a pair of chiral superfields $Y^A$ ($A = 1, 2$) satisfying the constraints

$$\tilde{\mathcal{D}}_i Y^A = 0, \quad \bar{\mathcal{D}}^i \bar{Y}^A = 0, \quad \bar{\mathcal{D}}_i Y^A = \tilde{\mathcal{D}}_i \bar{Y}^A, \quad \left( Y^A \right) = \bar{Y}_A. \quad (2.18)$$

Their solution reads

$$Y^A \left( \eta, \theta \right) = y^A + \sqrt{2} \theta_j \psi^{jk} + \bar{\theta}_k \bar{\psi}^{jk} \bar{A}.$$  \hspace{1cm} (2.19)

where

$$\overline{\left( y^A \right)} = \bar{Y}_A, \quad \left( \psi^{jk} \right) = \psi_{jk}. \quad (2.20)$$

The corresponding component field transformations are

$$\delta y^A = -\sqrt{2} \cos \lambda \xi \psi^{jk} e^{2\eta^A} - \sqrt{2} \sin \lambda \xi \psi^{jk} e^{-2\eta^A},$$

$$\delta \bar{y}_A = \sqrt{2} \cos \lambda \xi \psi^{jk} e^{2\eta^A} - \sqrt{2} \sin \lambda \xi \psi^{jk} e^{-2\eta^A}, \quad (2.21)$$

$$\delta \psi^{jk} = \sqrt{2} \xi \left( \cos \lambda \psi^A - \sin \lambda \bar{\psi}^A \right) e^{2\eta^A} - \sqrt{2} \xi \left( \cos \lambda \bar{\psi}^A + \sin \lambda \psi^A \right) e^{-2\eta^A}. \quad (2.22)$$

The invariant superfield action is written as

$$S_{(4, 4, 0)} = \frac{1}{2} \int d\zeta L \left( Y, \bar{Y} \right). \quad (2.23)$$

The component Lagrangian contains the standard kinetic term $G \xi^A \bar{\psi}_A$, where the metric $G$ of four-dimensional manifold is defined as

$$G(y, \bar{y}) := \Delta_3 L(y, \bar{y}), \quad \Delta_3 = -2 \xi^{AB} \partial_A \bar{\psi}_B. \quad \partial_A = \frac{\partial}{\partial y^A}. \quad (2.23)$$

$$\partial_B = \frac{\partial}{\partial \bar{y}^B}. \quad (2.23)$$
In order to describe the semi-dynamical spin multiplet we must drop such kinetic terms. So, we take the limit $G = 0$ that gives the residual Lagrangian of the first order in time derivatives (WZ type Lagrangians) where the function $L(y, \bar{y})$ satisfies the Laplace equation

$$\Delta y L = 0. \quad (2.24)$$

Another option is given by the superpotential term

$$S_{\text{pot}} = \frac{\mu}{2} \int d\zeta h (Y^A) + \frac{\mu}{2} \int d\bar{\zeta} h (\bar{Y}_A). \quad (2.25)$$

One can consider the function $h (y^A)$ in (2.26) as a holomorphic subset of the solutions found in [6], where the general construction of WZ type Lagrangians was given.

Here, we are interested in coupling of this semi-dynamical multiplet with the chiral super-field $\Phi$ that can be described only by a superpotential term. So, we skip the construction of WZ type Lagrangians from (2.22) and present the superpotential term written as

$$S_{\text{pot}} = \int dt \mathcal{L}_{WZ},$$

$$\mathcal{L}_{WZ} = \mu \left[ i \dot{Y}^A \partial_\lambda h (Y^A) + \frac{1}{2} \psi^i A \psi^B \partial_A \partial_B h (Y^A) + \text{c.c.} \right]. \quad (2.26)$$

This Lagrangian contains no deformation parameters, so it is invariant as well under the standard ($m = 0$) supersymmetry transformations. Indeed, the superpotential term must be invariant under the transformations closed on the centerless $N = 4$ super Virasoro algebra [22], which is an infinite dimensional superalgebra possessing both deformed and undeformed superalgebras.

2.3. Gauged mirror multiplet $(4, 4, 0)$

By analogy with [7], we can also expect that chiral superfields are subjected to the local U(1) transformations

$$\left( Y^A \right)' = e^{(\Lambda - \bar{\Lambda}) Y^A}, \quad \mathcal{I} Y^A = \frac{1}{2} (\sigma^3)_{\tilde{A}}^A Y^\tilde{A},$$

$$\left( \bar{Y}^A \right)' = e^{(\Lambda - \bar{\Lambda}) \bar{Y}^A}, \quad \mathcal{I} \bar{Y}^A = \frac{1}{2} (\sigma^3)^A_{\bar{A}} \bar{Y}^\bar{A}, \quad (2.27)$$

where $\mathcal{I}$ is a U(1) generator and $\Lambda = \Lambda \left( \hat{t}_L, \hat{\theta} \right), \bar{\Lambda} = \bar{\Lambda} \left( \hat{t}_R, \bar{\theta} \right)$. Gauged superfields satisfy the new gauge invariant constraints

$$\left( \tilde{D}^I + i \left[ \tilde{D}^I, \mathcal{I} \right] \right) Y^A = 0, \quad \left( \mathcal{D}^I - i \left[ \mathcal{D}^I, \mathcal{I} \right] \right) Y^A = 0, \quad (2.28)$$

$$\left( D^I - i \left[ D^I, \mathcal{I} \right] \right) Y^A = \left( \mathcal{D}^I + i \left[ \mathcal{D}^I, \mathcal{I} \right] \right) \bar{Y}^A, \quad (2.29)$$

where the real superfield $X$ is a gauge superfield transforming as

$$X' = X + \Lambda + \bar{\Lambda}. \quad (2.30)$$
Superspace expansion of the gauge superfield $X$ displays 8 bosonic and 8 fermionic components:

$$X(t, \bar{\theta}, \theta) = x + \hat{\theta}_k \chi^k - \bar{\theta}^k \bar{\chi}_k + 2\bar{\theta}^k \hat{\theta}_k A + \bar{\theta}_k \bar{\theta}^j D + \bar{\theta}^k \bar{\theta} D + \bar{\theta}_i \bar{\theta}_j B^{ij}$$

$$+ \bar{\theta}^k \hat{\theta}_k \left( \bar{\theta}^j \zeta^j - \bar{\theta} \zeta \right) + \hat{\theta}_i \bar{\theta}_j \bar{\theta} C,$$

(2.31)

where

$$\langle \bar{\chi} \rangle = x, \quad \langle \bar{A} \rangle = A, \quad \langle D \rangle = \bar{D}, \quad \langle B^{ij} \rangle = -B_{ij}, \quad B^{ij} = B^{ji},$$

$$\langle \chi' \rangle = \bar{\chi}_i, \quad \langle \zeta' \rangle = \bar{\zeta}_i.$$

(2.32)

However, from the constraints (2.28) and (2.29) we obtain the additional constraint\(^4\)

$$\bar{D}_i \bar{D}_j X = 0.$$

(2.33)

It kills half of the components of (2.31) as

$$X(t, \bar{\theta}, \theta) = x + \hat{\theta}_k \chi^k - \bar{\theta}^k \bar{\chi}_k + 2\bar{\theta}^k \hat{\theta}_k A + \bar{\theta}_k \bar{\theta}^j D + \bar{\theta}^k \bar{\theta} D$$

$$+ i\bar{\theta}^k \hat{\theta}_k \left( \bar{\theta} \chi^j + \bar{\chi}_j \right) - \frac{1}{4} \bar{\theta}_i \bar{\theta}_j \bar{\theta} \bar{\theta} \bar{\chi}.$$

(2.34)

This superfield describes the mirror multiplet $(1, 4, 3)$ that differs from the ordinary multiplet $(1, 4, 3)$ [3] because of the deformation. Using the $U(1)$ gauge freedom (2.30), we can choose the WZ gauge

$$X_{WZ} = 2\bar{\theta}^k \hat{\theta}_k A, \quad A' = A - \hat{\alpha}.$$

(2.35)

Thus, it can be interpreted as a mirror counterpart of the ‘topological’ gauge multiplet [15] described by the harmonic superfield $V^{++}$ in the WZ gauge. One can introduce accompanying chiral superfields

$$V_{WZ} \left( \hat{t}_l, \bar{\theta} \right) = \bar{\theta}_l \bar{\theta}^j A, \quad \bar{V}_{WZ} \left( \hat{t}_R, \theta \right) = \bar{\theta}^j \hat{\theta}_k A,$$

(2.36)

satisfying

$$\bar{D}_i X_{WZ} = \bar{D}_j V_{WZ}, \quad \bar{D}_i X_{WZ} = -\bar{D}_j \bar{V}_{WZ}.$$

(2.37)

The triplet of superfields, given by (2.35) and (2.36), has a harmonic superfield description in the flat superspace $m = 0$ with respect to the second SU(2) subgroup [9]. One could ascribe SU(2) indices to this triplet as $V^{(s')}$ and harmonize them. In the deformed super-

\(^4\) Actually, the gauge superfield $X$ can be considered as a prepotential for the vector multiplet $(3, 4, 1)$ given by the superfield $V_j = \bar{D}_j \bar{D}_j X$ [23] satisfying the constraints

$$\bar{D}_j V_{j_1} = \bar{D}_j V_{j_2} = 0.$$

One can check that it is invariant under the gauge transformations (2.30). Hence, the following constraint (2.33) implies that all physical degrees of freedom of (2.31) can be gauged away, i.e. the gauge multiplet becomes ‘topological’ [15].
symmetric mechanics we have no such description, because the second SU′(2) symmetry is broken.

According to the chiral condition (2.28), the superfield solution (2.19) is modified as
\[
Y^A \left( t, \tilde{\theta}, \bar{\theta} \right) = e^{-\Lambda t} Y^A \left( \tilde{t}, \theta \right), \quad \left( Y^A \right)' = e^{2\Lambda t} Y^A,
\]
\[
Y^A \left( t, \tilde{\theta}, \bar{\theta} \right) = e^{\Lambda t} Y^A \left( \tilde{t}, \theta \right), \quad \left( Y^A \right)' = e^{-2\Lambda t} Y^A.
\]
(2.38)

Solving (2.29), the left chiral superfield \( Y^A_L \) has the \( \theta \)-expansion
\[
Y^A_L \left( \tilde{t}, \theta \right) = y^A + \sqrt{2} \theta \psi^A + i \bar{\theta} \partial^A \nabla_i y^A,
\]
(2.39)
where
\[
\nabla_i = \partial_i + 2\Lambda \mathcal{I}.
\]
(2.40)

It is necessary to replace time derivatives by \( \nabla_i \) in the transformations (2.21). The residual local \( \alpha = \alpha(t) \) transformations of component fields must be written as
\[
\left( Y^A \right)' = e^{2\alpha t} Y^A, \quad \left( \psi^A \right)' = e^{2\alpha t} \psi^A,
\]
\[
\left( \nabla_i \psi^A \right)' = e^{2\alpha t} \nabla_i \psi^A, \quad \mathcal{A}' = \mathcal{A} - \alpha.
\]
(2.41)

Taking into account these transformations and the supersymmetry transformations (2.6), one can check that the real superfield (2.35) transforms as (2.30) with the parameter
\[
\Lambda \left( \tilde{t}, \theta \right) = i \alpha \left( \tilde{t} \right) + 2\mathcal{A} \left( \tilde{t} \right) \left( \cos \lambda \ c^A e^{i k \tilde{t}} - \sin \lambda \ c^A e^{-i k \tilde{t}} \right)
\]
(2.42)

The superfield \( Y^A \) transforms with the same parameter according to (2.27).

Finally, the superpotential (2.25) must be written in terms of \( Y^A Y^B \equiv Y^A_L Y^B_L \), since it is the only gauge invariant object defined on the left chiral subspace. One can define also the invariant F-term
\[
S_{FI} = -\frac{c}{4} \left[ \int d\appa \mathcal{V}_{WZ} + \int d\bar{\appa} \bar{\mathcal{V}}_{WZ} \right] \Rightarrow \mathcal{L}_{FI} = -c\mathcal{A}, \quad c = \text{const},
\]
(2.43)

which is a counterpart of Fayet–Iliopoulos term defined on the analytic harmonic superspace [15].

### 3. Coupling of dynamical and semi-dynamical multiplets

In this section we consider the coupling of dynamical and semi-dynamical multiplets identified with the chiral multiplet \((2, 4, 2)\) and the gauged mirror multiplet \((4, 4, 0)\), respectively.

#### 3.1. Interacting superpotential term

According to (2.27), the simplest interacting superpotential term reads
\[
S_{int.} = \frac{\mu}{2} \int d\appa \ Y^A Y^B f \left( \Phi \right) + \frac{\mu}{2} \int d\bar{\appa} \bar{Y}^A \bar{Y}^B \bar{f} \left( \bar{\Phi} \right),
\]
(3.1)
where \( f \) is an arbitrary holomorphic function of \( \Phi \). The Lagrangian is then given by

\[
\mathcal{L}_{\text{int.}} = \mu \left[ i (\gamma^1 \nabla_i \tilde{\gamma}_1 - \gamma^2 \nabla_i \tilde{\gamma}_2) f + \psi^i_1 \psi^2_i f + B^1 \gamma^2 \partial_i f \\
+ \xi^i (\psi^i_1 \gamma^1 - \psi^2_i \gamma^2) \partial_i f - \frac{\xi^2}{2} \gamma^2 \gamma^2 \partial_i \partial_i f + \text{c.c.} \right].
\]

(3.2)

It is straightforward to check that the Lagrangian is invariant under the local U(1) transformations (2.41).

### 3.2. Total Lagrangian

The total Lagrangian of the interacting model is given by the sum of the kinetic Lagrangian (2.17), the interacting superpotential term (3.2) and Fayet–Iliopoulos term (2.43):

\[
\mathcal{L} = \mathcal{L}_{\text{kin.}} + \mathcal{L}_{\text{int.}} + \mathcal{L}_{\text{FI}}.
\]

(3.3)

Eliminating the auxiliary fields \( B \) and \( \psi^A \) by their equations of motion and performing the following redefinition

\[
y^1 = v(f + \bar{f})^\frac{1}{2}, \quad \tilde{y}_1 = \bar{v}(f + \bar{f})^{-\frac{1}{2}},
\]

\[
y^2 = \bar{w}(f + \bar{f})^{-\frac{1}{2}}, \quad \tilde{y}_2 = w(f + \bar{f})^{-\frac{1}{2}},
\]

\[
\xi^i = g^{-\frac{1}{2}} \eta^i, \quad \xi_j = g^{-\frac{1}{2}} \bar{\eta}_j,
\]

we obtain the total on-shell Lagrangian (up to full time derivatives) as

\[
\mathcal{L} = g^2 \ddot{z} + \frac{i}{2} \left( \gamma^i \dot{\eta}_i - \gamma^j \bar{\eta}_j \right) + \frac{i}{2} \mu \left( \dot{v} \bar{w} + w \bar{v} - \bar{v} \bar{w} - \bar{v} \bar{w} \right)
\]

\[
- i \frac{m}{2} \cos 2\lambda \left( \dot{\gamma}_j \partial_j K - \dot{\gamma}_k \partial_k K \right) + \frac{i \mu}{2} \left( \dot{\gamma}_j f - \dot{\gamma}_k f \right) \frac{1}{(f + \bar{f})} (\dot{v} \bar{w} - \dot{w} \bar{v})
\]

\[
+ i \frac{2}{(f + \bar{f})} \left( \dot{\gamma}_j g - \dot{\gamma}_k g \right) g^{-1} \eta_j \bar{\eta}_k - m \cos 2\lambda \eta_j \bar{\eta}_k - \frac{\mu \gamma_j \partial_j f}{(f + \bar{f})^2} (\dot{v} \bar{w} - \dot{w} \bar{v}) g^{-1} \eta_j \bar{\eta}_k
\]

\[
- \frac{\mu \gamma \dot{v} \bar{w}}{(f + \bar{f})} \frac{(\partial_j \partial_j f - \partial_k \partial_k f)}{2} - \frac{\partial_j \partial_k f \partial_j f}{(f + \bar{f})} - \frac{\partial_j \partial_k f}{(f + \bar{f})} g^{-1} \dot{\gamma}_j \eta_j \bar{\eta}_j
\]

\[
+ \frac{m}{4} \sin 2\lambda \left[ \left( \partial_j \partial_j K - g^{-1} \partial_j K \partial_j g \right) g^{-1} \eta_j \bar{\eta}_j + \left( \partial_j \partial_j K - g^{-1} \partial_j K \partial_j g \right) g^{-1} \eta_j \bar{\eta}_j \right]
\]

\[
- g^{-1} \left( \frac{\mu \gamma \dot{v} \bar{w} \dot{\gamma}_j f}{(f + \bar{f})} - \frac{m \sin 2\lambda}{2} \partial_j K \right) \left( \frac{\mu \gamma \dot{v} \bar{w} \dot{\gamma}_j f}{(f + \bar{f})} - \frac{m \sin 2\lambda}{2} \partial_j K \right)
\]

\[
+ \frac{1}{4} \left( \partial_j \partial_j K - g^{-1} \partial_j K \partial_j g \right) g^{-2} \eta_j \bar{\eta}_j \dot{\gamma}_j \bar{\eta}_j + [i \mu (v \bar{w} + w \bar{v}) - c] A.
\]

(3.5)

Looking at the last term, one concludes that the U(1) gauge field \( A \) plays the role of a Lagrange multiplier enforcing the constraint

\[
\mu (v \bar{w} + w \bar{v}) - c = 0.
\]

(3.6)
3.3. Hamiltonian and Noether charges

Our next step is a formulation of a classical Hamiltonian mechanics. We perform Legendre transformation with the exclusion of the Lagrange multiplier $A$. As a result, we obtain the classical Hamiltonian

$$
\mathcal{H} = g^{-1} \left[ p_z - \frac{i}{2} m \cos 2\lambda \partial_z K + \frac{i \mu \partial_z f \sin 2\lambda}{2 (f + f)} + \frac{i}{2} g^{-1} \partial_z g \eta^k \right]
$$

$$
	imes \left[ p_{\bar{v}} + \frac{i}{2} m \cos 2\lambda \partial_{\bar{v}} K - \frac{i \mu \partial_{\bar{v}} f \sin 2\lambda}{2 (f + f)} - \frac{i}{2} g^{-1} \partial_{\bar{v}} g \eta^k \right]
$$

$$
+ \frac{m \cos 2\lambda}{2} \eta^k \bar{\eta}_k + \frac{\mu \partial_z f \partial_{\bar{v}} f}{(f + f)} (v \bar{v} - w \bar{w}) g^{-1} \eta^k \bar{\eta}_k
$$

$$
+ \frac{\mu \bar{w} \bar{v}}{(f + f)} \left[ \frac{\partial_z \partial_{\bar{v}} f}{2} - \frac{\partial_{\bar{v}} \partial_z f}{2} - \frac{\partial_z f}{2} g^{-1} \partial_z g \right] g^{-1} \eta^k \bar{\eta}_k
$$

$$
+ \frac{\mu \bar{w} \bar{v}}{(f + f)} \left[ \frac{\partial_z \partial_{\bar{v}} \bar{f}}{2} - \frac{\partial_{\bar{v}} \partial_z \bar{f}}{2} - \frac{\partial_z \bar{f}}{2} g^{-1} \partial_z g \right] g^{-1} \eta^k \bar{\eta}_k
$$

$$
- \frac{m \sin 2\lambda}{4} \left[ (\partial_z \partial_{\bar{v}} K - g^{-1} \partial_z K \partial_z g) g^{-1} \eta^k \bar{\eta}_k + (\partial_z \partial_{\bar{v}} K - g^{-1} \partial_z K \partial_z g) g^{-1} \bar{\eta}_k \eta^k \right]
$$

$$
+ g^{-1} \left( \frac{\mu \bar{w} \bar{v}}{(f + f)} - \frac{m \sin 2\lambda}{2} \partial_z K \right) \left( \frac{\mu \bar{w} \bar{v}}{(f + f)} - \frac{m \sin 2\lambda}{2} \partial_z K \right)
$$

$$
- \frac{1}{4} (\partial_z \partial_{\bar{v}} g - g^{-1} \partial_z g \partial_{\bar{v}} g) g^{-2} \eta^k \bar{\eta}_k \eta^j \bar{\eta}_j.
$$

The secondary constraint (3.6). Poisson (Dirac) brackets are imposed as

$$
\{ \partial_z z \}_{PB} = -1, \quad \{ \partial_{\bar{v}} z \}_{PB} = -1, \quad \{ \eta^k, \bar{\eta}_j \}_{PB} = -i \delta^k_j, \quad \{ \eta^k, \bar{v} \}_{PB} = i \mu^{-1}, \quad \{ \bar{w}, \bar{v} \}_{PB} = i \mu^{-1}.
$$

SU(2|1) supercharges are

$$
Q' = \sqrt{2} e^{\frac{i m}{g} \frac{1}{4}} \left\{ \cos \lambda \eta \left[ p_z - \frac{i}{2} m \partial_z K + \frac{i \mu \partial_z f \sin 2\lambda}{2 (f + f)} + \frac{i}{2} g^{-1} \partial_z g \eta^k \bar{\eta}_k \right] \right. 
$$

$$
- \sin \lambda \eta \left[ p_{\bar{v}} - \frac{i}{2} m \partial_{\bar{v}} K - \frac{i \mu \partial_{\bar{v}} f \sin 2\lambda}{2 (f + f)} - \frac{i}{2} g^{-1} \partial_{\bar{v}} g \eta^k \bar{\eta}_k \right] \right\}
$$

$$
- \frac{\sqrt{2} i \mu}{(f + f)} \left( \cos \lambda \bar{w} \bar{v} \partial_z \bar{f} \eta - \sin \lambda \bar{w} \bar{v} \partial_z f \eta \right),
$$

$$
Q_j = \sqrt{2} e^{\frac{i m}{g} \frac{1}{4}} \left\{ \cos \lambda \eta \left[ p_z + \frac{i}{2} m \partial_z K - \frac{i \mu \partial_z f \sin 2\lambda}{2 (f + f)} - \frac{i}{2} g^{-1} \partial_z g \eta^k \bar{\eta}_k \right] \right. 
$$

$$
+ \sin \lambda \eta \left[ p_{\bar{v}} + \frac{i}{2} m \partial_{\bar{v}} K + \frac{i \mu \partial_{\bar{v}} f \sin 2\lambda}{2 (f + f)} + \frac{i}{2} g^{-1} \partial_{\bar{v}} g \eta^k \bar{\eta}_k \right] \right\}
$$

$$
- \frac{\sqrt{2} i \mu}{(f + f)} \left( \cos \lambda \bar{w} \bar{v} \partial_z f \eta \bar{\eta}_j + \sin \lambda \bar{w} \bar{v} \partial_z \bar{f} \eta \right). \quad (3.9)
$$
With respect to the brackets (3.8), they close on the superalgebra \( su(2|1) \):

\[
\{ Q^i, Q_j \}_{PB} = -i (2 m Q^i + 2 \delta^i_j \mathcal{H} ), \quad \{ I^j, I^k \}_{PB} = -i ( \delta^j_i I^k - \delta^k_i I^j ),
\]

\[
\{ I^j, Q_i \}_{PB} = -i \left( \frac{1}{2} \delta^j_i Q_i - \delta^j_i Q_i \right), \quad \{ I^j, Q^k \}_{PB} = -i \left( \delta^j_i Q^k - \frac{1}{2} \delta^j_i Q^k \right), \tag{3.10}
\]

where the \( SU(2) \) generators are

\[
I^j = \eta^j \bar{\eta}_j - \frac{\delta^j_i}{2} \eta^i \bar{\eta}_j. \tag{3.11}
\]

It should be noted that the Hamiltonian (3.7) can always be redefined as a central charge when \( \lambda = 0 \) (see the example in the next Section).

There is another \( SU(2) \) subgroup that is generated by spin variables as

\[
S_3 = \mu (v \bar{v} - w \bar{w}), \quad S_+ = \mu v \bar{w}, \quad S_- = \mu w \bar{v}. \tag{3.12}
\]

Indeed, one can check these generators form the \( su(2) \) algebra:

\[
\{ S_3, S_\pm \}_{PB} = \mp i S_\pm, \quad \{ S_+, S_- \}_{PB} = -2 i S_3. \tag{3.13}
\]

Since the Hamiltonian (3.7) contains these \( SU(2) \) generators, it commutes only with the quadratic Casimir operator

\[
C_{SU(2)} = S_+ S_- + (S_3)^2. \tag{3.14}
\]

Taking into account (3.6), the Casimir operator is determined by the constant

\[
C_{SU(2)} = \frac{c}{4}. \tag{3.15}
\]

Its quantum counterpart (up to the ordering ambiguity) is given by \( (c \approx 2s) \)

\[
C_{SU(2)} = s (s + 1), \tag{3.16}
\]

where \( s \) is a spin of the quantum system.

Quantization of supersymmetric spinning models can be performed under the prescription of [13] (see also [10, 11]). Unlike the latter, the problem considered here has a more complex interaction with spin variables. There is also a peculiar fact that the Hamiltonian of supersymmetric ‘Kähler oscillator’ models is associated with the \( U(1) \)-generator of (2.1). It has an impact on the energy spectrum, such that bosonic and fermionic states of a given \( SU(2|1) \) representation have different energy values. Another feature of \( SU(2|1) \) supersymmetry is that it can lead to spontaneously broken supersymmetry [6, 10].

4. Model on \( SU(1, 1)/U(1) \)

Let us consider the model on the coset space \( SU(1, 1)/U(1) \), that is in fact a deformation of the model studied in [19]. It is given by the Kähler potential
that leads to the metric
\[ g = \frac{1}{(1 - \gamma z\bar{z})^2}. \] (4.2)
To eliminate the terms \( \sim \mu v\eta\bar{\eta}^k \) in (3.7), we choose the holomorphic function \( f(z) \) as\(^5\)
\[ f = \frac{1 + \sqrt{z\bar{z}}}{1 - \sqrt{z\bar{z}}} \] (4.3)
To simplify the notations, we also perform a redefinition in the phase space that preserves the brackets (3.8):
\[ v \to (1 - \sqrt{z\bar{z}})^{1/2}(1 - \sqrt{z\bar{z}})^{-1/2} v, \quad \bar{v} \to (1 - \sqrt{z\bar{z}})^{-1/2}(1 - \sqrt{z\bar{z}})^{1/2} \bar{v}, \]
\[ w \to (1 - \sqrt{z\bar{z}})^{1/2}(1 - \sqrt{z\bar{z}})^{-1/2} w, \quad \bar{w} \to (1 - \sqrt{z\bar{z}})^{-1/2}(1 - \sqrt{z\bar{z}})^{1/2} \bar{w}, \] (4.4)
\[ p_c \to p_c - \frac{i\sqrt{\mu} (v\bar{w} - w\bar{v})}{2(1 - \sqrt{z\bar{z}})^{1/2}}, \quad p_c \to p_c + \frac{i\sqrt{\mu} (v\bar{w} - w\bar{v})}{2(1 - \sqrt{z\bar{z}})^{1/2}}. \]
Thereby, the Hamiltonian reads
\[
\mathcal{H} = (1 - \gamma z\bar{z})^2 \left[ \pi - \frac{i(1 - \cos 2\lambda) m\bar{z}}{2(1 - \gamma z\bar{z})} \right] \left[ \bar{\pi} + \frac{i(1 - \cos 2\lambda) m\bar{z}}{2(1 - \gamma z\bar{z})} \right] + \frac{m \cos 2\lambda}{2} \eta^i \bar{\eta}_k + \gamma \mu (v\bar{w} - w\bar{v}) \eta^i \bar{\eta}_k + \frac{\gamma m \sin 2\lambda}{2} (\bar{z}^2 \eta^i \bar{\eta}_k + \bar{z}^2 \bar{\eta}^i \eta_k) - \frac{\gamma}{2} \bar{\eta}_i \eta^j \eta^j + \left( \sqrt{\gamma} \mu v\bar{w} - \frac{m \sin 2\lambda}{2} \right) \left( \sqrt{\gamma} \mu w\bar{v} - \frac{m \sin 2\lambda}{2} \right), \] (4.5)
where
\[
\pi = p_c - \frac{i\bar{z}}{1 - \gamma z\bar{z}} \left[ \frac{m}{2} + \frac{\gamma \mu}{2} (v\bar{w} - w\bar{v}) - \gamma \eta^j \bar{\eta}_j \right], \\
\bar{\pi} = p_c + \frac{i\bar{z}}{1 - \gamma z\bar{z}} \left[ \frac{m}{2} + \frac{\gamma \mu}{2} (v\bar{w} - w\bar{v}) - \gamma \eta^j \bar{\eta}_j \right]. \] (4.6)
Supercharges are written in the form
\[
Q^i = \sqrt{2} e^{\frac{i\mu}{2} m} (1 - \gamma z\bar{z}) \left[ \cos \lambda \eta^j \pi - \sin \lambda \bar{\eta}^j \left( \bar{\pi} - \frac{im\bar{z}}{1 - \gamma z\bar{z}} \right) \right] - \sqrt{2\gamma} e^{\frac{i\mu}{2} m} i\mu \left( \cos \lambda w\bar{v}\eta^j - \sin \lambda \eta \bar{w}v\eta^j \right), \]
\[
\bar{Q}_j = \sqrt{2} e^{-\frac{i\mu}{2} m} (1 - \gamma z\bar{z}) \left[ \cos \lambda \bar{\eta}^i \bar{\pi} + \sin \lambda \eta^i \left( \pi + \frac{im\bar{z}}{1 - \gamma z\bar{z}} \right) \right] - \sqrt{2\gamma} e^{-\frac{i\mu}{2} m} i\mu \left( \cos \lambda v\bar{w}\eta^i + \sin \lambda \bar{v}w\eta^i \right). \] (4.7)
\(^5\)In the limit \( \gamma = 0 \) to the flat space, the coupling with the spin multiplet disappears since the holomorphic function (4.3) becomes trivial: \( f = 1 \).
SU(2) generators are given by the same expression (3.11).

**Limit** $\lambda = 0$. In distinct from [19], the Hamiltonian (4.5) contains terms ($\sim m \sin^2 2\lambda$) breaking the target space symmetry SU(1, 1). For instance, the corresponding bosonic Hamiltonian is characterized by a term such as the oscillator term $m^2 \sin^2 2\lambda \bar{z}z/4$. We can define only the remaining U(1) generator $J_3$,

$$J_3 = i(z p_x - z p_z) + \frac{\mu}{2}(v\bar{v} - w\bar{w}) - \eta^k \bar{\eta}_k,$$  

(4.8)

that commutes with all SU(2|1) generators. Thus, to restore the symmetry SU(1, 1) we take the limit $\lambda = 0$. In this limit, we can introduce an external U(1) generator $F$ given by the expression

$$F = \frac{1}{2} \left[ \eta^k \bar{\eta}_k - \mu (v\bar{v} - w\bar{w}) \right],$$  

(4.9)

that rotates the relevant supercharges for $\lambda = 0$ as

$$\{F, \Omega^{(\lambda=0)}\}_{\text{PB}} = -\frac{i}{2} \Omega^{(\lambda=0)}, \quad \{F, \bar{\Omega}^{(\lambda=0)}\}_{\text{PB}} = \frac{i}{2} \bar{\Omega}^{(\lambda=0)}.$$  

(4.10)

Hence, we can define a central charge generator of the extended superalgebra $su(2|1)$ as

$$H = H|_{\lambda=0} + mF.$$  

(4.11)

and take it as a new Hamiltonian of the system, while the generator $F$ becomes an internal U(1) generator of the extended superalgebra.

Deformed SU(1, 1) symmetry generators are written as

$$J_3 = i(z p_x - z p_z) + \frac{\mu}{2}(v\bar{v} - w\bar{w}) - \eta^k \bar{\eta}_k,$$

$$J_+ = p_z - \gamma \bar{z}^2 p_z - iz \left[ \frac{m}{2} + \frac{\gamma \mu}{2}(v\bar{v} - w\bar{w}) - \gamma \eta^k \bar{\eta}_k \right],$$

$$J_- = p_z - \gamma \bar{z}^2 p_z + iz \left[ \frac{m}{2} + \frac{\gamma \mu}{2}(v\bar{v} - w\bar{w}) - \gamma \eta^k \bar{\eta}_k \right].$$  

(4.12)

Indeed, they commute with all SU(2|1) generators including $F$, and form the central extension of the $su(1, 1)$ algebra:

$$\{J_3, J_{\pm}\}_{\text{PB}} = \pm iJ_{\pm}, \quad \{J_+, J_-\}_{\text{PB}} = -2i\gamma J_3 - im,$$  

(4.13)

where the deformation parameter $m$ is a central charge. Defining its Casimir operator

$$C_{SU(1,1)} = J_+ J_- - \gamma \left( J_3 + \frac{m}{2\gamma} \right)^2,$$  

(4.14)

we obtain the following expression for the Hamiltonian (4.11):

$$H = \gamma C_{SU(2)} + C_{SU(1,1)} + \frac{m^2}{4\gamma}.$$  

(4.15)

In the limit $m = 0$, we reproduce the model studied in [19].

In the limit $\gamma = 0$, the generators $J_{\pm}$ become magnetic translation generators forming the Heisenberg algebra with the external U(1) automorphism group generator $J_3$:
\[ \{J, J_{\pm}\}_{PB} = \pm iJ_{\pm}, \quad \{J_{\pm}, J_{\mp}\}_{PB} = -im, \quad \{J_{3}, J_{\pm}\}_{PB} = \pm iJ_{\pm}. \tag{4.16} \]

Spin variables drop out of the model leaving only the dynamical multiplet \((2, 4, 2)\) [3].

5. Conclusions

In this paper, we proposed new models of single-particle SU(2|1) supersymmetric mechanics with the use of dynamical, semi-dynamical and gauge multiplets. As an alternative of harmonic superspace, we exploited the generalized chiral superspace \((2, 5)\). The generalized chiral multiplet \((2, 4, 2)\) was introduced in [4]. For the mirror multiplet \((4, 4, 0)\), we defined the gauge invariant constraints \((2.28)\) and \((2.29)\). As turned out, the gauge superfield \(X\), bonded with these constraints, satisfies \((2.33)\) and describes the mirror multiplet \((1, 4, 3)\). As an example, we considered in details the deformed model on the pseudo-sphere \(SU(1, 1)/U(1)\). It would be interesting to reproduce and generalize the quantum model on sphere \(S^2 \sim SU(2)/U(1)\) [24] studied with an arbitrary number of supersymmetries corresponding to the supergroup \(SU(n|1)\).

The most natural question is whether it is possible to consider superconformal models. During our study of superconformal models of the trigonometric type [21], we suggested a simple criteria for construction of superconformal models. With this experience we tried to obtain superconformal symmetry for considered models, but it is impossible at least for a single-particle model. For example, if we take two chiral dynamical multiplets then suitable superconformal models can possibly be obtained. This may be a good challenge for future research.

It is tempting to study the mirror counterpart of the multiplet \((1, 4, 3)\) [3]. Another interesting problem is the application of the gauging procedure [7] to the deformed multiplets \((4, 4, 0)\) [6]. It can give rise to general actions of the ordinary and mirror multiplets \((3, 4, 1)\) [6].

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References

[1] Bellucci S and Nersessian A 2003 (Super)oscillator on CP(N) and constant magnetic field Phys. Rev. D 67 065013
Bellucci S and Nersessian A 2003 (Super)oscillator on CP(N) and constant magnetic field Phys. Rev. D 71 089901 (erratum)
Bellucci S and Nersessian A 2003 Supersymmetric Kähler oscillator in a constant magnetic field Proc. of the Workshop Supersymmetries and Quantum Symmetries (SQS’03) (Dubna, Russia July 24–29, 2003) pp 379–84

\[^6\] SU(2|1) constraints for the ordinary one were solved in appendix C of [25].
[2] Smilga A V 2004 Weak supersymmetry Phys. Lett. B 585 173–9
[3] Ivanov E and Sidorov S 2014 Deformed supersymmetric mechanics Class. Quantum Grav. 31 075013
[4] Ivanov E and Sidorov S 2014 Super Kähler oscillator from SU(2|1) superspace J. Phys. A: Math. Theor. 47 292002
[5] Römelsberger C 2006 Counting chiral primaries in $N = 1, d = 4$ superconformal field theories Nucl. Phys. B 747 329–53
Römelsberger C 2007 Calculating the superconformal index and Seiberg duality (arXiv:0707.3702 [hep-th])
[6] Ivanov E and Sidorov S 2016 SU(2|1) mechanics and harmonic superspace Class. Quantum Grav. 33 055001
[7] Fedoruk S and Ivanov E 2016 Gauged spinning models with deformed supersymmetry J. High Energy Phys. JHEP11(2016)103
[8] Shmavonyan H 2019 $\mathbb{C}^N$-Smorodinsky–Winternitz system in a constant magnetic field Phys. Lett. A 383 1223–8
Ivanov E, Nersessian A and Shmavonyan H 2020 Symmetries of deformed supersymmetric mechanics on Kähler manifolds Phys. Rev. D 101 025003
[9] Ivanov E and Niederle J 2009 Bi-harmonic superspace for $N = 4$ mechanics Phys. Rev. D 80 065027
[10] Fedoruk S, Ivanov E and Sidorov S 2018 Deformed supersymmetric quantum mechanics with spin variables J. High Energy Phys. JHEP01(2018)132
[11] Fedoruk S, Ivanov E, Lechtenfeld O and Sidorov S 2018 Quantum SU(2|1) supersymmetric Calogero–Moser spinning systems J. High Energy Phys. JHEP04(2018)043
[12] Fedoruk S, Ivanov E and Lechtenfeld O 2009 Supersymmetric Calogero models by gauging Phys. Rev. D 79 105015
[13] Fedoruk S, Ivanov E and Lechtenfeld O 2009 $OSp(4|2)$ superconformal mechanics J. High Energy Phys. JHEP08(2009)081
Fedoruk S, Ivanov E and Lechtenfeld O 2010 New $D(2,1,\alpha)$ mechanics with spin variables J. High Energy Phys. JHEP04(2010)129
[14] Ivanov E and Lechtenfeld O 2003 $N = 4$ supersymmetric mechanics in harmonic superspace J. High Energy Phys. JHEP09(2003)073
[15] Delécluse F and Ivanov E 2006 Gauging $N = 4$ supersymmetric mechanics Nucl. Phys. B 753 211–41
Delécluse F and Ivanov E 2007 Gauging $N = 4$ supersymmetric mechanics II: (1, 4, 3) models from the (4, 4, 0) ones Nucl. Phys. B 770 179–205
[16] Kozyrev N, Krivonos S, Lechtenfeld O and Sutulin A 2018 SU(2|1) supersymmetric mechanics on curved spaces J. High Energy Phys. JHEP05(2018)175
Krivonos S, Lechtenfeld O and Sutulin A 2018 N-extended supersymmetric Calogero models Phys. Lett. B 784 137–41
Chernyavsky D 2019 On $OSp(4|2)$ superconformal mechanics J. High Energy Phys. JHEP02(2019)170
Galajinsky A and Lechtenfeld O 2019 Spinning extensions of $D(2,1,\alpha)$ superconformal mechanics J. High Energy Phys. JHEP03(2019)069
[17] Polychronakos A P 1991 Integrable systems from gauged matrix models Phys. Lett. B 266 29–34
[18] Polychronakos A P 2006 The physics and mathematics of Calogero particles J. Phys. A: Math. Gen. 39 12793–845
[19] Bellucci S, Kozyrev N, Krivonos S and Sutulin A 2012 $N = 4$ chiral supermultiplet interacting with a magnetic field Phys. Rev. D 85 065024
[20] Ivanov E 2011 Harmonic superfields in $N = 4$ supersymmetric quantum mechanics Symmetry, Integrability Geometry Methods Appl. 7 015
[21] Ivanov E, Sidorov S and Toppan F 2015 Superconformal mechanics in SU(2|1) superspace Phys. Rev. D 91 085032
[22] Holanda N L and Toppan F 2014 Four types of (super)conformal mechanics: D-module reps and invariant actions J. Math. Phys. 55 061703
[23] Ivanov E A and Smilga A V 1991 Supersymmetric gauge quantum mechanics: superfield description Phys. Lett. B 257 79–82
[24] Hong S-T, Lee J, Lee T H and Oh P 2007 Isospin particle on $S^2$ with arbitrary number of supersymmetries Mod. Phys. Lett. A 22 1481–92
[25] Ivanov E, Lechtenfeld O and Sidorov S 2016 SU(2|2) supersymmetric mechanics J. High Energy Phys. JHEP11(2016)031