Affleck-Dine baryogenesis and the Q-ball dark matter in the gauge-mediated SUSY breaking

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We consider the Affleck-Dine baryogenesis comprehensively in the minimal supersymmetric standard model with gauge-mediated supersymmetry breaking. Considering the high temperature effects, we see that the Affleck-Dine field is naturally deformed into the form of the Q ball. In the natural scenario where the initial amplitude of the field and the A-terms are both determined by the nonrenormalizable superpotential, we obtain a narrow allowed region in the parameter space in order to explain the baryon number and the dark matter of the universe simultaneously. Therefore, the Affleck-Dine baryogenesis is successful, although difficult.

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1 Introduction

The Affleck-Dine (AD) mechanism [1] is the most promising scenario for explaining the baryon number of the universe. It is based on the dynamics of a (complex) scalar field $\phi$ carrying baryon number, which is called the AD field. During inflation, the expectation value of the AD field develops at a very large value. After inflation the inflaton field oscillates about the minimum of the effective potential, and dominates the energy density of the universe like matter, while the AD field stays at the large field value. It starts the oscillation, or more precisely, rotation in its effective potential when $H \sim m_{\phi, \text{eff}}$, where $H$ and $m_{\phi, \text{eff}} \equiv |V''(\phi)|$ are the Hubble parameter and the curvature of the potential of the AD field. Once it rotates, the baryon number will be created as $n_B \sim \omega \phi^2$, where $\omega = \dot{\theta}$ is the velocity of the phase of the AD field. When $t \sim \Gamma_\phi$, where $\Gamma_\phi$ is the decay rate of the AD field, it decays into ordinary particles carrying baryon number such as quarks, and the baryogenesis in the universe completes.

However, important effects on the field dynamics were overlooked. It was recently revealed that the AD field feels spatial instabilities [2]. Those instabilities grow very large and the AD field deforms into clumpy objects: Q balls. A Q ball is a kind of nontopological soliton, whose stability is guaranteed by the existence of some charge $Q$ [3]. In the previous work [4], we found that all the charges which are carried by the AD field are absorbed into formed Q balls, and this implies that the baryon number of the universe cannot be explained by the relic AD field remaining outside Q balls after their formation.

In the radiation dominated universe, charges are evaporated from Q balls [5], and they will explain the baryon number of the universe [6]. This is because the minimum of the (free) energy is achieved when the AD particles are freely in the thermal plasma at finite temperature. (Of course, the mixture of the Q-ball configuration and free particles is the minimum of the free energy at finite temperature, when the chemical potential of the Q ball and the plasma are equal to be in the chemical equilibrium. This situation can be achieved for very large charge of Q balls with more than $10^{40}$ or so [3].) Even if the radiation component is not dominant energy in the universe, such as that during the inflaton-oscillation dominant stage just after the inflation, high temperature effects on the dynamics of the AD field and/or Q-ball evaporation are important. Therefore, in this article, we investigate the whole scenario of the AD mechanism for baryogenesis in the minimal supersymmetric standard model (MSSM) with the gauge-mediated supersymmetry (SUSY) breaking in the high temperature universe.

2 Flat directions as the Affleck-Dine field

A flat direction is the direction in which the effective potential vanishes. There are many flat directions in the minimal supersymmetric standard model (MSSM), and they are listed in Refs. [7, 8]. Since they consist of squarks and/or sleptons, they carry baryon
and/or lepton numbers, and we can identify them as the Affleck-Dine (AD) field. Although the flat directions are exactly flat when supersymmetry (SUSY) is unbroken, it will be lifted by SUSY breaking effects. In the gauge-mediated SUSY breaking scenario, SUSY breaking effects appear at low energy scales, so the shape of the effective potential for the flat direction has curvature of order of the electroweak mass at low scales, and almost flat at larger scales. Therefore, the effective potential reads as

$$V(\Phi) = M_F^4 \log \left( 1 + \frac{|\Phi|^2}{M_S^2} \right) + m^2_{3/2} \left[ 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right] |\Phi|^2$$

$$+ \lambda^2 \frac{|\Phi|^{2n-2}}{M^{2n-6}} + \left( \lambda A_\lambda \frac{\Phi^n}{M^{n-3}} + h.c. \right)$$

$$- c_H H^2 |\Phi|^2 + \left( \lambda a_H \frac{\Phi^n}{M^{n-3}} + h.c. \right)$$

$$+ c_{T}^{(1)} T^2 |\Phi|^2 + c_{T}^{(2)} T^4 \log \frac{|\Phi|^2}{T^2},$$

where $\Phi$ is the complex scalar field representing the flat direction, $M_S \sim M_F^2/m_\phi$ is a messenger scale. The second term comes from the gravity mediation effects, since the gravity effects always exist. The $K$ is the coefficient of the one-loop corrections $[10]$, and is usually negative. However it may be positive in some cases, and Q balls are not formed until the amplitude of the field becomes as small as it stays in the logarithmic potential. We call it delayed case. $M = 2.4 \times 10^{18}$ GeV is the reduced Planck mass. The second line represents for the nonrenormalizable terms. The third line shows terms which depends on the Hubble parameter $H$, during inflation and inflaton oscillation stage which starts just after inflation. $c_H$ is a positive constant and $a_H$ is a complex constant with a different phase from $A_\lambda$ in order for the AD mechanism to work. The last line is the finite temperature potential. The first and second term denote direct and indirect coupling of the flat direction to thermal bath $[11, 12]$, respectively.

### 3 Affleck-Dine mechanism

It was believed that the Affleck-Dine (AD) mechanism works as follows $[1, 7]$. During inflation, the AD field are trapped at the value determined by the following conditions: $V'_H(\Phi) \sim V'_{NR}(\Phi)$ and $V'_{AH}(\Phi) \sim 0$. Therefore, we get

$$\phi_{min} \sim (HM^{n-3})^{1/(n-2)},$$

$$\sin[n\theta_{min} + \arg(a_H)] \sim 0,$$

where $c_H, \lambda \sim 1$ are assumed, and $\Phi = (\phi e^{i\theta})/\sqrt{2}$. After inflation, the inflaton oscillates around one of the minimum of its effective potential, and the energy of its oscillation dominates the universe. During that time, the minimum of the potential of the AD field
are adiabatically changing until it rolls down rapidly when \(H \approx \omega \equiv |V'(\phi)/\phi|^{1/2}\). Then the AD field rotates in its potential, producing the baryon number \(n_B = \dot{\theta \phi^2}\). After the AD field decays into quarks, baryogenesis completes.

In order to estimate the amount of the produced baryon number, let us assume that the phases of \(A_\lambda\) and \(a_H\) differ of order unity. Then the baryon number can be estimated as

\[
n_B \sim H^{-1} \frac{\partial V_A}{\partial \phi} \phi \sim H^{-1} V_A \sim \omega^{-1} m_{3/2}^n \frac{\phi^n}{M_{n-3}} \sim \left(\frac{m_{3/2}}{\omega}\right) \omega \phi^2 = \varepsilon n_B^{(\text{max})},
\]

where \(\varepsilon \equiv (m_{3/2}/\omega)\), \(n_B^{(\text{max})} \equiv \omega \phi^2\), and \(H \sim \omega\) is used in the second line. Notice that the contribution from the Hubble A-term is at most comparable to this. When \(\varepsilon = 1\), the trace of the motion of the AD field in the potential is circular orbit. If \(\varepsilon\) becomes smaller, the orbit becomes elliptic, and finally the field is just oscillating along radial direction when \(\varepsilon = 0\). We call \(\varepsilon\) as the ellipticity parameter below.

When the logarithmic potential is dominant, \(\omega \sim m^2/\phi\), so the ellipticity parameter varies, and it may be very small. On the other hand, when the potential is dominated by the gravity effect, \(\omega \sim m_{3/2}\). Then \(\varepsilon \simeq 1\) always holds.

4 Charge evaporation at high temperature

4.1 Gauge-mediation type Q balls

Since the Q-ball formation takes place nonadiabatically, (almost) all the charges are absorbed into produced Q balls \([4, 13]\). Thus, the baryon number of the universe cannot be explained by the remaining charges outside Q balls in the form of the relic AD field. However, at finite temperature, the minimum of the free energy of the AD field is achieved for the situation that all the charges exist in the form of free particles in thermal plasma \([5]\). This situation is realized through charge evaporation from Q-ball surface \([5, 14]\). In spite of the fact that the complete evaporation is the minimum of the free energy, the actual universe is filled with the mixture of Q balls and surrounding free particles, since the evaporation rate becomes smaller than the cosmic expansion rate at low temperatures. (As we mentioned in the Introduction, the free energy is minimized at the situation that some charges exist in the form of free particles (in thermal plasma) and the rest stays inside Q balls, if the Q-ball charge are large enough for the chemical equilibrium \([\text{IV}]\). In this case, the charge of the Q ball should be \(Q \sim \eta_B^{-4} \sim 10^{40}\).) The nonadiabatic creation of Q balls and the later charge evaporation takes place, since the time scale of evaporation is much longer than that of the Q-ball formation. Notice that the charge of the Q ball is conserved for the situation that the evaporation of charges is not effective. However, the energy of the Q ball decreases as the temperature of the universe decreases. This happens because the Q ball and surrounding plasma are in thermal equilibrium: i.e., the Q balls and thermal plasma have the same temperatures.
The rate of charge transfer from inside the Q ball to its outside is determined by the diffusion rate at high temperature and the evaporation rate at low temperature. When the difference between the chemical potentials of the plasma and the Q ball is small, chemical equilibrium is achieved and charges inside the Q ball cannot come out. Therefore, the charges in the ‘atmosphere’ of the Q ball should be taken away in order for further charge evaporation. This process is determined by the diffusion. The diffusion rate is given by

$$\Gamma_{\text{diff}} \equiv \frac{dQ}{dt} \sim -4\pi D R_Q \mu_Q T^2 \sim -4\pi A T,$$

where $D = A/T$ is the diffusion coefficient, and $A = 4 - 6$, $\mu_Q \sim \omega$ is the chemical potential of the Q ball. On the other hand, the evaporation rate is

$$\Gamma_{\text{evap}} \equiv \frac{dQ}{dt} \sim -\zeta (\mu_Q - \mu_{\text{plasma}}) T^2 4\pi R_Q^2 \sim -4\pi \frac{\zeta T^2 Q^{1/4}}{M_F},$$

where $\mu_{\text{plasma}} \ll \mu_Q$ is the chemical potential of thermal plasma, and $\mu_Q \sim \omega \sim M_F Q^{-1/4}$ is used in the second line. $\zeta$ change at $T = m_\phi$ as

$$\zeta = \begin{cases} \left(\frac{T}{m_\phi}\right)^2 & (T < m_\phi), \\ 1 & (T > m_\phi). \end{cases}$$

Therefore, we get

$$\Gamma_{\text{evap}} = \frac{dQ}{dt} = \begin{cases} -4\pi \frac{T^2 Q^{1/4}}{M_F} & (T > m_\phi), \\ -4\pi \frac{\zeta T^2 Q^{1/4}}{m_\phi^2 M_F} & (T < m_\phi). \end{cases}$$

Integrating the above expressions, we finally obtain the total evaporated charge as

$$\Delta Q \sim 4.6 \times 10^{17} \left(\frac{m_\phi}{\text{TeV}}\right)^{-2/3} \left(\frac{M_F}{10^6\text{GeV}}\right)^{-1/3} \left(\frac{Q}{10^{24}}\right)^{1/12}.$$  

### 4.2 Gravity-mediation type Q balls

Now we will show the evaporated charges for the ‘new’ type of stable Q ball. The evaporation and diffusion rate have the same forms in terms of Q-ball parameters $R_Q$ and $\omega$. The only differences are that we have to use the features for the ‘gravity-mediation’ type Q ball, such as

$$R_Q \sim |K|^{-1/2} m_{3/2}, \quad \omega \sim m_{3/2},$$
and the transition temperature when $\Gamma_{\text{evap}} = \Gamma_{\text{diff}}$ becomes $T_\ast \equiv A^{1/3} |K|^{1/6} (m_{3/2} m_\phi^2)^{1/3}$. As in the ‘usual’ type of Q balls, where the potential is dominated by the logarithmic term, the charge evaporation near $T_\ast$ is dominant, and the total evaporated charges are found to be

$$\Delta Q \sim 10^{20} \left( \frac{m_{3/2}}{\text{MeV}} \right)^{-1/3} \left( \frac{m_\phi}{\text{TeV}} \right)^{-2/3}. \quad (10)$$

5 Q-ball formation

Q-balls are produced while the AD field is rotating. We obtained the relation between the typical charge of the Q balls and the initial amplitude of the AD field \[16\]. Here we only show the results. For the gauge-mediation type of Q balls, we have

$$Q = \beta \left( \frac{\phi_0}{m(T)} \right)^4, \quad (11)$$

where $\beta \simeq 6 \times 10^{-4}$, and $m(T) = M_F$ for $T < M_F$, while $m(T) = T$ for $T > M_F$. For the gravity-mediation type,

$$Q = \beta' \left( \frac{\phi_0}{m_{3/2}} \right)^2, \quad (12)$$

where $\beta' \simeq 6 \times 10^{-3}$. These are consistent with the analytical estimation: $Q \sim k_{\text{res}}^{-3} n_b$, where $k_{\text{res}}^{-1}$ is the size of the resonance mode, and $n_b \sim \omega \phi_0^3$ is the baryon density.

6 General constraints for Q-ball scenarios

We would like to see whether there is any consistent cosmological scenario for the baryogenesis and the dark matter of the universe, provided by large Q balls. In the Q-ball scenario, the baryon number of the universe should be explained by the amount of the charge evaporated from Q balls, $\Delta Q$, and the survived Q balls become the dark matter. If we assume that Q balls do not exceed the critical density of the universe, i.e., $\Omega_Q \lesssim 1$, and the baryon-to-photon ratio as $\eta_B \sim 10^{-10}$, the condition can be written as

$$\eta_B = \frac{n_B}{n_\gamma} \simeq \frac{\varepsilon n_Q \Delta Q}{n_\gamma} \simeq \frac{\varepsilon n_0 \Omega_Q \Delta Q}{n_\gamma M_Q} \simeq \frac{\varepsilon \rho_e \Omega_Q \Delta Q}{n_\gamma,0 M_Q}, \quad (13)$$

where $\Omega_Q$ is the density parameter for the Q ball. Since the evaporated charge explains the baryon number in the universe, while survived Q-balls becomes the dark matter, we can have the Q-ball charge to explain the right amount of both baryons and dark matter in the universe, using $M_Q \simeq m_\phi Q^{3/4}$, $\rho_{e,0} \sim 8h_0^2 \times 10^{-47}\text{GeV}^4$, and $n_{\gamma,0} \sim 3.3 \times 10^{-39}\text{GeV}^3$, where $h_0(\sim 0.7)$ is the Hubble parameter normalized with $100\text{km/sec/Mpc}$. It reads as

$$Q \sim 3.2 \times 10^{17} \varepsilon^{3/2} \Omega_Q^{3/2} \left( \frac{\eta_B}{10^{-10}} \right)^{-3/2} \left( \frac{m_\phi}{\text{TeV}} \right)^{-1} \left( \frac{M_F}{10^6\text{GeV}} \right)^{-2}. \quad (14)$$
We call this baryon-dark matter (B-DM) condition. We can also put another condition, namely, survival condition, which implies that the Q ball should survive from the evaporation: \( Q \gtrsim \Delta Q \). It is expressed as

\[
Q \gtrsim 1.2 \times 10^{17} \left( \frac{m_\phi}{\text{TeV}} \right)^{-8/11} \left( \frac{M_F}{10^6 \text{GeV}} \right)^{-4/11}.
\] (15)

Finally, Q balls must be stable against the decay into nucleons to become the dark matter of the universe, which we call it the stability condition:

\[
Q \gtrsim 10^{24} \left( \frac{M_F}{10^6 \text{GeV}} \right)^4.
\] (16)

\section{Consistent cosmological Q-ball scenarios}

Now we can see whether there is any consistent scenario to explain the amounts of baryons and dark matter of the universe naturally, which means that the initial conditions and the A-terms are provided by the non-renormalizable superpotential. The details can be seen in Ref.\cite{16}, so we will summarize the results here. There are only three successful cases. The first one is when the potential is dominated by the thermal logarithmic term, which is shown in Fig. 1. Here, the successful scenario is achieved if the parameters are as follows: \( M_F \sim 5 \times 10^2 \text{ GeV}, T_{RH} \sim 5 \times 10^4 \text{ GeV}, \) and \( Q \sim 2 \times 10^{24} \).

The second case is the delayed case with the Q-balls formed when the thermal logarithmic term is dominated, shown in Fig. 2. In this case, the allowed parameter sets are \( m_{3/2} \sim 0.1 \text{ GeV}, M_F \sim 5 \times 10^4 \text{ GeV}, T_{RH} \sim 5 \text{ GeV}, \) and \( Q \sim 2 \times 10^{24} \).

The last case is the delayed case with the Q-balls formed when the zero-temperature logarithmic term is dominated, shown in Fig. 3. In this case, the allowed parameter sets are \( m_{3/2} \sim 0.1 \text{ GeV}, M_F \sim 5 \times 10^4 \text{ GeV}, T_{RH} \sim 5 \text{ GeV}, \) and \( Q \sim 10^{20} \).

As can be seen, we need very low reheating temperature and very low \( M_F \), which represents for the expectation value of the F-term of the messenger sector, for all the successful scenarios. Therefore, the model is constrained rather severely in order to explain the baryons and the dark matter of the universe in these Q-ball scenarios.

\section{Conclusion}

We have investigated thoroughly the Q-ball cosmology (the baryogenesis and the dark matter) in the gauge-mediated SUSY breaking model. Taking into account thermal effects, the shape of the effective potential has to be altered somewhat, but most of the features of the Q-ball formation derived at zero temperature can be applied to the finite temperature case with appropriate rescalings. We have thus found that Q balls are actually formed through Affleck-Dine mechanism in the early universe.
Figure 1: Summary of constraints on the parameter space $(Q, M_F)$ with $n = 6$ and $m_\phi = 100$ GeV in the generic logarithmic potential where the thermal terms are dominated when the Q-ball formation occurs. Lines (a), (b), and (c) denote the B-DM, survival, and stability conditions, respectively. Line (d) shows the conditions $T \gtrsim M_F$.

We have sought for the consistent scenario for the dark matter Q ball, which also provides the baryon number of the universe simultaneously. For the consistent scenario, we adopt the nonrenormalizable superpotential in order to naturally give the initial amplitude of the AD field and the source for the field rotation due to the A-term. As opposed to our expectation, very narrow parameter region could be useful for the scenario in the situations that the Q balls are produced just after the baryon number creation. In addition, we have seen that current experiments have already excluded most of the successful parameter regions.

Of course, if the A-terms and/or the initial amplitude of the AD field are determined by other mechanism, the cosmological Q-ball scenario may work. Then, the stable dark matter Q balls supplying the baryons play a crucial role in the universe.

We have also found the new situations that the Q-ball formation takes place when the amplitude of the fields becomes small enough to be in the logarithmic terms in the potential, while the fields starts its rotation at larger amplitudes where the effective potential is dominated by the gravity-mediation term with positive K-term. This allows to produce Q balls with smaller charges while creating larger baryon numbers. In this
Figure 2: Summary of constraints on the parameter space \((Q, M_F)\) for the delayed-formed \(Q\) balls with the thermal logarithmic term domination for \(m_\phi = 100\) GeV and \(n = 6\). Lines (a), (b), and (c) denote the B-DM, survival, and stability conditions, respectively. Lines (d) and (e) show the conditions \(T_{eq} \gtrsim M_F\) and \(m_{3/2} \lesssim 1\) GeV, respectively.

situation, there is wider allowed regions for naturally consistent scenario, although the current experiments exclude most of the parameter space. Notice also that rather too low reheating temperature is necessary for larger \(n\) scenario to work naturally. This aspect is good for evading the cosmological gravitino problem, while it is difficult to construct the actual inflation mechanism to get such low reheating temperatures, in spite of the fact that the nucleosynthesis can take place successfully for the reheating temperature higher than at least 10 MeV.

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Figure 3: Summary of constraints on the parameter space \((Q, M_F)\) for the delayed-formed Q balls with the generic logarithmic term domination for \(n = 6\) and \(m_\phi = 100\) GeV. Lines (a), (b), and (c) denote the B-DM, survival, and stability conditions, respectively. Line (d) represents the condition \(M_F \gtrsim T_{eq}\), and line (e) is just the upper limit for \(M_F\).

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