The size of our causal Universe

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ABSTRACT
A Universe with finite age also has a finite causal scale. Larger scales can not affect our local measurements or modeling, but far away locations could have different cosmological parameters. The size of our causal Universe depends on the details of inflation and is usually assumed to be larger than our observable Universe today. To account for causality, we propose a new boundary condition, that can be fulfill by fixing the cosmological constant (a free geometric parameter of gravity). This forces a cancellation of vacuum energy with the cosmological constant. As a consequence, the measured cosmic acceleration can not be explained by a simple cosmological constant or constant vacuum energy. We need some additional odd properties such as the existence of evolving dark energy (DE) with energy-density fine tuned to be twice that of dark matter today. We show here that we can instead explain cosmic acceleration without DE (or modified gravity) assuming that the causal scale is smaller than the observable Universe today. Such scale corresponds to half the sky at z=1 and 60 degrees at z=1100, which is consistent with the anomalous lack of correlations observed in the CMB.

Key words: Cosmology: dark energy, cosmic background radiation, cosmological parameters, early Universe, inflation

1 INTRODUCTION
One of the most striking changes to Newton’s gravity proposed by Einstein is that energy gravitates. Scientists have since been wondering if vacuum energy $\rho_{\text{vac}}$ (vacuum fluctuations, zero-point fluctuations, quantum vacuum, dark energy or aether) could also gravitate. Measurements of cosmic acceleration (see e.g. Planck Collaboration et al. 2018; Abbott et al. 2019; Tutusaus et al. 2017) point to a model of dark energy (DE) with energy-density fine tuned to be twice that of dark matter today. We show here that we can instead explain cosmic acceleration without DE (or modified gravity) assuming that the causal scale is smaller than the observable Universe today. Such scale corresponds to half the sky at z=1 and 60 degrees at z=1100, which is consistent with the anomalous lack of correlations observed in the CMB.

A Universe with finite age also has a finite causal scale. Larger scales can not affect our local measurements or modeling, but far away locations could have different cosmological parameters. The size of our causal Universe depends on the details of inflation and is usually assumed to be larger than our observable Universe today. To account for causality, we propose a new boundary condition, that can be fulfill by fixing the cosmological constant (a free geometric parameter of gravity). This forces a cancellation of vacuum energy with the cosmological constant. As a consequence, the measured cosmic acceleration can not be explained by a simple cosmological constant or constant vacuum energy. We need some additional odd properties such as the existence of evolving dark energy (DE) with energy-density fine tuned to be twice that of dark matter today. We show here that we can instead explain cosmic acceleration without DE (or modified gravity) assuming that the causal scale is smaller than the observable Universe today. Such scale corresponds to half the sky at z=1 and 60 degrees at z=1100, which is consistent with the anomalous lack of correlations observed in the CMB.

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1 INTRODUCTION
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$$dx^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a(t)^2 (d\chi^2 + \chi^2 d\Omega^2)$$  \hspace{1cm} (1)

is the exact general solution for a mathematically homogeneous and isotropic flat Universe. The scale factor, $a(t)$, describes the expansion of the Universe as a function of time. We can relate $a(t)$ to the energy content of the Universe for a perfect fluid by solving the field equations (see Eq.9-10):

$$R_0^0 = R_{00} = - \left( \frac{3\dot{a}}{a} \right) = 4\pi G (\rho + 3p) - \Lambda$$  \hspace{1cm} (2)

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3}$$  \hspace{1cm} (3)

where $\rho = \rho_m a^{-3} + \rho_r a^{-4} + \rho_{\text{vac}}$ and $\rho_m$ is the pressureless matter density today ($a=1$), $\rho_r$ corresponds to radiation (with pressure $p_r = \rho_r/3$) and $\rho_{\text{vac}}$ represents vacuum energy ($p_{\text{vac}} = -\rho_{\text{vac}}$). \footnote{For easy of notation we focus on the flat case, but our results can easily be extended to the non-flat case or non trivial topology.} One can argue that $\Lambda$ is indistinguishable from $\rho_{\text{vac}}$, because field equations are degenerate to the combination:

$$\rho_\Lambda = \rho_{\text{vac}} + \frac{\Lambda}{8\pi G}$$  \hspace{1cm} (4)

Here we take $\Lambda$ to be a fundamental (geometrical) constant, while $\rho_{\text{vac}}$ depends on the actual energy content of our universe. The measured $\rho_\Lambda$ is very small compared to what we expect for $\rho_{\text{vac}}$. Moreover, $\rho_\Lambda \sim 2.3\rho_m$ today, which seems a remarkable coincidence. Possible solutions to this puzzle are: I) $\Lambda = 0$ and $\rho_\Lambda$ originates only from $\rho_{\text{vac}}$ or some dark

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energy (DE) (Weinberg 1989; Elizalde & Gaztañaga 1990; Carroll et al. 1992; Huterer & Turner 1999; Elizalde 2006), II) ρvac = 0 and we need to fix Λ or Modified Gravity (Gaztañaga & Lobo 2001; Gaztañaga et al. 2002; Lue et al. 2004; Nojiri et al. 2017) or III) there is a cancellation between Λ and ρvac, as we will propose here.

Eq.1-3 are a mathematical extrapolation. A physical model requires a mechanism to produce homogeneity that respects causality. Note that the Lorentz invariance or covariance of the field equations in General Relativity (GR) is not enough to warrant that a given solution has a causal structure (see Minguzzi & Sanchez 2006; Howard 2010 and references therein). This is not the case for the FLRW model, because ρ and p are the same everywhere at any fix cosmic time, and this can not have causal explanation for a Universe that has a finite age. If we want a causal explanation for our Universe we can not find the solution directly solving the Cauchy problem in GR, because we are not allowed to setup causal initial conditions for such a problem. The only way to do this is to setup initial conditions that are random with no correlations. This is what happens in the realm of vacuum quantum fluctuations under Heisenberg uncertainty, which do not require a causal mechanism to exist. Inflation could them produce a large and homogeneous universe out of these initial quantum fluctuations. But even for such a case, there is a finite causal scale associated with the duration of inflation.

Particles separated by distances larger than the comoving Hubble radius dh(t) = c/[a(t)H(t)] can’t communicate at time t. Distances larger than the horizon

\[ \eta(a) = c \int_0^t \frac{dt}{a(t)} = \int_0^a d\ln(a) \, dH(a), \]

have never communicated. We know from the cosmic microwave background (CMB) and large scale structure (LSS) that the Universe was very homogeneous on scales that were not causally connected (without inflation). This either means that the initial conditions where acausally smooth to start with or that there is a mechanism like inflation (Dodelson 2003; Liddle 1999; Brandenberger 2017) which inflates regions outside the Hubble radius. During inflation, dh decreases which freezes out communication on comoving scales larger than the horizon \( \chi_3 \) ≥ \( \eta(a_i) = dH(t_i) \) when inflation begins, at \( a_i = a(t_i) \). Inflation also smoothed and stretches out any initial (quantum) inhomogeneities to scales that could be larger than our current horizon. This creates homogeneous and flat patches. When inflation ends, radiation from reheating makes dh grow again. Thus, the scale \( \chi_3 \) is fixed before inflation in comoving coordinates and is the same for all times, while the horizon \( \eta \) and \( dH \) change with time. This is illustrated in Fig.1. Inflation allows the full observable Universe to originate from a very small causally connected homogeneous patch, \( \chi_3 \), which could be as small as the Planck scale. We usually assume that this region \( \chi_3 \) is much larger that our observable universe today: \( \sim 3c/H_0 \). As we approach our epoch (label "now" in the figure), we believe that a mysterious Dark Energy (DE) produces a second inflation that makes dh decrease again. Why a second inflation now? Are both inflations related?

Regardless of the details of inflation, a Universe of finite age will only be causally homogeneous for scales smaller than some cut-off \( \chi < \chi_3 \). We need a boundary condition at

\[ \chi = \chi_3 \text{ to account for the lack of causality (and therefore homogeneity) at larger scales. This results in a cancellation between } \Lambda \text{ and } \rho_{vac} \text{ and could be the cause of the current cosmic acceleration. In §2 we view this problem in Classical Physics, while in §3 we present a relativistic version. In §4 we estimate the size of the causal Universe and discuss the implications for inflation and CMB. We end with some Discussion and Conclusions.} \]
because of lack of causality. This is why boundary terms are usually neglected at infinity. In this spirit, we will require here that test particles should be free ($\vec{g}=0$) or more relevant for a fluid: that boundary terms should be zero (i.e. the flux $\Phi=0$), when outside causal contact, $r>r_3$. For $r_3 \rightarrow \infty$ this condition requires $\Lambda \equiv 0$, as otherwise $\vec{g}$ and $\Phi$ diverge. Observational evidence that $\Lambda \neq 0$ may then indicate that $r_3$ is finite. This agrees with the finite age of the Universe. From Eq.7 the boundary condition $\Phi(r>r_3)=0$, implies:

$$\Lambda = 4\pi G \rho_0(r < r_3)$$

(8)

which is clearly related to the coincidence problem: $\rho_\Lambda \sim 2\rho_0$. Let's next explore this same argument in GR. For this we first need to see what is the relativistic version of Eq.6-7.

3 RELATIVISTIC CASE

The symmetries of Einstein’s field equations allow for a cosmological constant $\Lambda$ term (Landau & Lifshitz 1971):

$$R^\nu_\mu + \Lambda g^\nu_\mu = 8\pi G \left( T^\nu_\mu - \frac{1}{2} g^\nu_\mu T \right),$$

(9)

For a perfect fluid with density $\rho$ and pressure $p=\omega \rho$:

$$T^\nu_\mu = (\rho + p) u^\nu u_\mu - \delta^\nu_\mu,$$

(10)

where both $\rho$ and $p$ could change with space-time. For events comoving with the fluid we have $u^0=1$ so that $u^\mu u_\mu = u^0 u_0 = 1$, so that the time-time component of the Ricci curvature is:

$$R^0_0 = 4\pi G (\rho + 3\rho) - \Lambda$$

(11)

3.1 The generalised Gauss’s law

Consider perturbations around Minkowski metric $\eta_{\mu\nu}$ (i.e. around empty space):

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

(12)

where $h_{\mu\nu}$ are small corrections. To linear order in $h_{\mu\nu}$ we have $R_{00} = 1/2 \Box h_{00}$ (Landau & Lifshitz 1971), so that we can defined the gravitational potential as $\phi \equiv h_{00}/2$ to find:

$$R_{00} = R^0_0 = \Box \phi = -\nabla_\mu \nabla^\mu \phi = -\nabla_\mu g^{\mu \nu}$$

(13)

where $g^{\mu \nu} = \nabla^\nu \phi$ is the covariant gravitational acceleration. We can combine this relation with Eq.11 to find:

$$\Box \phi = 4\pi G (\rho + 3\rho) - \Lambda$$

(14)

which is the relativistic generalization of Poisson equation Eq.6. Thus the relativistic version of Gauss law in Eq.7 is then

$$\Phi = \int_{\partial M} d\mu \ g^{\mu \nu} = -\int_M \sqrt{-g} \ d^4x \ [4\pi G (\rho + 3\rho) - \Lambda]$$

(15)

where $M$ is the 4D volume inside the 3D hypersurface $\partial M$. Traditionally, we take such boundary terms to be zero at infinity: $\Phi(\infty) = 0$ (see Eq.4.7.8 in Weinberg 1972).

We can reach a similar expression without the weak field approximation by noticing that the equivalent of Poisson equation is the covariant time-time component of the Field Equations $R^0_0$ in Eq.11. We can then identify the flux directly with:

$$\Phi = -\int_M \sqrt{-g} \ d^4x \ R^0_0$$

(16)

which for a perfect fluid gives the same result as Eq.15.

3.2 Causal Boundary condition

As mentioned in the Introduction, scales larger that $\chi_3$ can have no effect on the metric or the curvature of the universe around us. Mathematically, this appears as retarder Green functions ($\Phi(\chi, t) = \Phi(\chi - ct)$) as solutions to the wave equation Eq.14 (or Eq.11) with appropriate boundary conditions (typically $\chi_3 \rightarrow \infty$). This could result in a non-homogeneous solution for the metric of the Universe on very large scales (see Gaztañaga 2019). An observer situated at the edge of our causal boundary will find a similar solution, but could measure different cosmological parameters, because she sees a different patch of the initial conditions. There should be a smooth background across disconnected regions with an infrared cutoff in the spectrum of inhomogeneities for $\chi > \chi_3$.

Solutions in different regions could be matched as in Sanghái & Clifton (2015).

We usually assume that particles should be free at infinity, because of lack of causality: if there is no cause there should not be any effect. This is why boundary terms are usually set to zero at infinity. For example, to reproduce the field equations Eq.9 in GR, from an action principle we need to neglect these boundary terms (e.g. Landau & Lifshitz 1971; Weinberg 1972). On scales $\chi < \chi_3$ we have a homogeneous expanding Universe with $\rho = \rho_0$. On larger scales we require boundary terms to vanish. In particular we will require $\Phi(\chi > \chi_3) = 0$ in Eq.16, so that there is no flux (i.e. no effects of gravity) beyond the causal scale. This implies:

$$\frac{\Lambda}{8\pi G} = \frac{1}{2M_\delta} \int_{M_1} \sqrt{-g} d^4x \ (\rho + 3\rho) = \frac{\rho >_\delta + \rho >_\delta}{2},$$

(17)

where $M_\delta$ is the volume inside the lightcone to the surface $\partial M_\delta$, where $\chi = \chi_3$. Note how this condition is similar to the one found by Lombriser (2019) and the mechanism for sequestering vacuum energy (Kaloper et al. 2016) from requiring an additional minimization of the Einstein-Hilbert action. Recall how here the scale $\chi_3$ is fixed in comoving coordinates while the horizon $\eta$ and $dH$ change with time (see Fig.1). This is consistent with a constant value for $\Lambda$ in Eq.17.

3.3 Vacuum Energy does not gravitate

Inside $\chi < \chi_3$, we can use Eq.1-3 with $\rho = \rho_M + \rho_r + \rho_{vac}$ and $\rho = \rho_r / 3 - \rho_{vac}$, so that we can write Eq.17 as:

$$\frac{\Lambda}{8\pi G} \leq \frac{\rho_m >_\delta}{2} + \frac{\rho_r >_\delta - \rho_{vac}}{\rho_{vac}} = \frac{\rho_{vac}}{\rho_{vac}}.$$

(18)
where $\rho_3$ is the mean matter and radiation contribution in the integral of Eq.17. The values of $\rho_m$ and $\rho_r$ evolve with space-time, so that $\rho_3$ is the average contribution inside the volume $M_3$, while the vacuum density contribution is constant (by definition). We can combine Eq.4 with Eq.18:

$$\rho_\Lambda = \frac{\Lambda}{8\pi G} + \rho_{\text{vac}} = \rho_3 - \rho_{\text{vac}} + \rho_{\text{vac}} = \rho_3,$$

(19)

which shows that vacuum energy cancels out and can not affect the observed value of $\rho_\Lambda$. In this respect we can conclude that vacuum does not gravitate. This result is independent of the value of $\chi_3$ or the value $\rho_{\text{vac}}$, which could both be infinite (as it follows from Quantum Field Theory in the case of $\rho_{\text{vac}}$).

4 THE SIZE OF OUR CAUSAL UNIVERSE

Sometime in our past, at time $t_i$, inflation (or a similar mechanism) blow the initial quantum fluctuations and create a large homogeneous patch for our universe. In terms of our comoving coordinates, its size is $\chi_3 = [c/(a(t_i)H(t_i))]$, where $H(t_i)^2 = 8\pi/3G\rho(t_i)$ is given by the (potential) energy of inflation $\rho(t_i) - V(\phi)$. This comoving scale has remained constant and outside causal contact through out the evolution of the Universe. As we don’t know the values of $a_i$ or $\rho(t_i)$ it seems impossible to estimate how large $\chi_3$ is from current observations or first principles.

But imagine that DE does not exist. Then, the horizon of our expanding universe (that emerged after inflation ended) will eventually reach $\chi_3$ at some time $a_3$. This is illustrated in Fig.2. If we assume that vacuum energy does not evolve after inflation (i.e. $\omega_{\text{vac}} \approx -1$), we can use Eq.17-19 to estimate:

$$\rho_\Lambda = \rho_3 = \frac{\int_{M_3} \sqrt{-\eta} d^3x (\rho_m + 2\rho_r)}{\int_{M} \sqrt{-\eta} d^3x}.$$  

(20)

We can actually only do this calculation at time $a_3$ because (from current observations) we only know the full content of the Universe at that time. At earlier times, part of the causal region is outside our horizon. Thus, we can use our measurements of $\rho_\Lambda$, $\rho_m$ and $\rho_r$ to estimate $a_3$ and therefore $\chi_3$.

The horizon after inflation (see Eq.5) is:

$$\chi(a) = \eta(a) - \eta(a_c)$$  

(21)

where $a_c$ represents the end of inflation. We then have $\chi_3 = \chi(a_3) = \eta(a_3)$ where $a_3$ is the time when the causal boundary enters the horizon after inflation and $a_i$ the beginning of inflation. Fig.2 illustrate this. We calculate $\rho_3$ in Eq.20 as the integral to $\chi_3$ in the light-cone:

$$\rho_3 = \frac{\int_{H_i} \int_{\chi_3} \frac{d^3x}{a^3} a^3 (\rho_m a^3 + 2\rho_r a^3)}{2 \int_{H_i} \int_{\chi_3} \frac{d^3x}{a^3} a^3}.$$  

(22)

where $a = a(\chi)$ in Eq.21 and Eq.5. For $H(a)$ we use Eq.3 with $\Omega = \rho_{H}/\rho_c$, $\rho_c = 3H_0^2/8\pi G$ and $\Omega_m = 4.2 \times 10^{-5}$ (Planck Collaboration et al. 2018) for a flat Universe $\Omega_m = 1 - \Omega_\Lambda - \Omega_\gamma$. We find $\chi_3$ from Eq.22 numerically using $\Omega_\Lambda = \Omega_\Lambda \equiv \rho_3/\rho_c \simeq 0.69 \pm 0.01$.

4.1 Inflation and the coincidence problem

Do the results in previous section, e.g. Eq.23, depend on the observer? An astronomer in a galaxy at $z=9$, when $a = 0.1$, will measure $\Omega_m$ to be $\Omega_m(\theta) = \Omega_m(\theta) = 0.20 \pm 0.3$ today. So we can see that the scale of our causal universe is slightly smaller than our observable universe today. Because $\chi_3$ is smaller than 2$r$ times our observable horizon, we should be able to see this horizon in our past lighcone at $\theta(z) = \chi_3/z(z)$. At $z \sim 1$ about half of the sky ($\theta(z) \sim 180$ deg) is causally disconnected. At larger redshifts this boundary tends to a fix value $\theta(z) \sim 60$ deg, depending on $\chi_3$ (and therefore $\Lambda$). This has implications for CMB observations (see section 4.2). In Appendix B we discuss how this results change in the presence of DE. But our proposal is that DE is not needed to explain cosmic acceleration. If we set $\Omega_m = 0$ we find $\chi_3 = 0.86$ and $\chi_3 = 3.081$, so our results are not very sensitive to the details of the Early Universe after inflation.
a \chi_{i} = c \chi_{i}^{-1} \sim (0.3176 \pm 0.0006) H_{0} \quad (25)

where \( H_{i} = H(t_{i}) \) or \( a_{i} = a(t_{i}) \). This shows how \( a_{i} \) determines \( \chi_{i} \), while \( a_{0} \) determines when it re-enters the horizon (see Fig.2), and therefore how large \( \rho_{g} \) is. The Hubble rate during inflation \( H_{I} \) is proportional to the energy of inflation. During reheating this energy is converted into radiation: \( H_{I}^{2} = \Omega_{k} H_{I}^{2} v_{e}^{-3} \), with \( a_{e} \equiv e^{N} a_{i} \). We can combine with Eq.25 to find:

\[
a_{i} \chi_{i} = \frac{H_{I}}{H_{I}} e^{-2N} \Omega_{I}^{1/2} \left( \chi_{i}^{2} H_{0}/c \right) \sim 4 \times 10^{8} \, \text{i} \text{planck}
\]

where for the second equality we have used the canonical value of \( N \approx 60 \) and \( H_{i} \approx H_{I} \), which also yields \( a_{i} \approx 1.56 \times 10^{-33} \) and \( H_{I} \approx 10^{10} \, \text{GeV} \). The condition \( a_{i} \chi_{i} > \text{i} \text{planck} \) requires \( N < 70 \), close to the value found in Dodelson & Hui (2003). Thus, the whole causal size of our Universe \( \chi_{i} \) could result from a quantum fluctuation at the Planck scale \( \text{i} \text{planck} \). Such vacuum fluctuation could generate an inflationary expansion. After \( N \approx 70 \) e-folds inflation ends with reheating, which results into matter and radiation today. Thus, this model links the cosmological constant scale to the Planck scale via inflation.

### 4.2 Implications for CMB

The (look-back) comoving distance to the surface of last scattering \( a_{s} \approx 9.2 \times 10^{-4} \) (Planck Collaboration et al. 2018) is \( \chi_{\text{CMB}} = \eta(1) - \eta(a_{s}) \approx 3.145 \, \text{deg} \). This is similar to our estimate for \( \chi_{3} \) in Eq.23. Thus, we would expect to see no correlations in the CMB on angular scales \( \theta > \theta_{l} \approx \chi_{\text{CMB}} / 60 \approx 0.7 \). The lack of structure seen in the CMB on these large scales is one of the well known anomalies in the CMB data, see Schwarz et al. (2016) and references therein. This lack of correlations above 60 deg. has been interpreted as a universe with non trivial topology (Luminet et al. 2003). Fig.3 shows a comparison of the measured CMB temperature correlations (points with error-bars) with the ΛCDM prediction for an infinite Universe (continuous line). There is a very clear discrepancy which Copi et al. (2009) estimates to happen in only 0.025 per cent of the realizations of the infinite ΛCDM model. The significance of this discrepancy is model dependent. If errors are estimated from the data (and not from the model) ΛCDM is strongly ruled out by this measurement (Gaztañaga et al. 2003). Even assuming a ΛCDM model, the lack of large scale correlations in w_{2}(\theta) represent an odd alignment of lower order multipoles of the angular power spectrum \( c_{l} \) (Schwarz et al. 2016).

A small causal universe also predicts similar lack of correlation for CMB polarization measurements. We also expect variations of \( c_{l} \) on smaller scales (e.g. around the first acoustic peak \( l \approx 200 \)) for \( c_{l} \) estimated over different regions of the sky (separated by \( \theta > 60 \) deg.), which is another known CMB anomaly (Planck Collaboration et al. 2014). Early evidence for these variations (Gaztañaga et al. 1998) were interpreted as non-Gaussian initial conditions.

We can also predict \( \Omega_{\Lambda} \) from the lack of CMB correlations. From Fig.3 we roughly estimate \( \theta_{l} \approx 60 \pm 3 \) deg. to find (using Eq.22) \( \Omega_{\Lambda} = 0.7 \pm 0.1 \). (the larger the angle the smaller \( \Omega_{\Lambda} \)). But note that this rough estimate does not take into account the foreground (late) ISW and lensing effects (Fosalba et al. 2003; Das & Souradeep 2014), which add non primordial correlations to the largest scales. This requires further investigation. Also note that this estimate for \( \Omega_{\Lambda} \) corresponds to the size of disconnected regions at the location of the CMB, which might be slightly different to the value near us, as we see a different patch of the primordial Universe (see below). Note also that there are temperature differences on scales larger \( \theta_{l} \), but they are not correlated, as expected in causality disconnected regions. Nearby regions are connected which creates a smooth transition across disconnected regions.

### 5 DISCUSSION AND CONCLUSIONS

ΛCDM in Eq.1-3 assumes that \( \rho \) is constant everywhere at a fixed comoving time. This requires acausal initial conditions (Brandenberger 2017) unless there is inflation, where a
tiny homogeneous and causally connected patch, the Causal Universe $\chi_3$, was inflated to be very large today. Regions larger than $\chi_3$ are out of causal contact. Here we require that test particles become free (or the relativistic flux is zero) as we approach $\chi_3$. No cause should produce no effect. This leads to Eq.17, which is the main result in this paper. If we ignore the vacuum, this condition requires: $Λ = 8\pi G \rho_0$, where $\rho_0$ is the matter and radiation inside $\chi_3$ (Eq.20). This leads to Eq.17, which is the main result in this paper. For constant vacuum ($\omega = p/\rho = -1$), we find $\chi_3 \approx 3c/H_0$ for $Ω_Λ = Ω_M = 0.7$. We can also estimate $\chi_3$ as $c/(aH)$ when inflation begins, see Eq.25. After inflation $\chi_3$ freezes out until it re-enters causality at $a_0 = 0.93$, close to now ($a = 1$). This starts a new inflation (as $ρ_Λ = ρ_0 > ρ_m$) which keeps the causal boundary frozen. Thus a finite $\chi_3$ explains why $ρ_0 \approx 2ρ_m$. It also predicts that CMB temperature should not be correlated above $θ > θ_0 \approx 60$ deg. A prediction that matches observations (see Fig.3). This is one of the well known anomalies measured in the CMB. One would also expect the CMB spectrum to be anisotropic on the largest scales, which is another well known measured anomaly (see Planck Collaboration et al.2014). One can reverse this argument to use the lack of CMB correlations above $θ_0 \approx 60$ deg, to estimate $\chi_3 \approx θ_0 / CMB$. Together with condition $ρ_0 = ρ_Λ$, this provides a prediction of $Ω_Λ \approx 0.7 \pm 0.1$, which is independent of other measurements for $Ω_Λ$. More work is needed to account for the late ISW and lensing and to interpret the CMB measurements with a metric that is not homogeneous (Gaztañaga 2019).

Note that because the Universe is not strictly homogeneous outside a causal region, the causal boundary for observers far away from us could be slightly different from ours, because they see a different patch of the Universe which could have slightly different energy content. Continuity across nearby disconnected regions forces these differences to be small, but it is impossible to quantify this without a model for the initial conditions and a better understanding of the process that generates the primordial homogeneity. In general such differences could affect structure formation, galaxy evolution and CMB observations. The fact that we can measure a concordance picture from different observations with the $Λ$CDM model indicates that these differences must be small. But tensions between measurements of cosmological parameters (or fundamental constants of nature) from very different redshifts (eg between CMB and local measurements) or different parts of the sky at high redshifts (such as dipolar variation of fine structure constant in Webb et al. 2011) could be related to such in-homogeneities, rather than to evolution of the Dark Energy (DE) equation of state or other more exotic explanations.

For $χ_3 > 3c/H_0$ we can not explain cosmic acceleration with $ω = -1$, because the resulting $ρ_Λ$ in Eq.20 would be very small. We need evolving DE with equation of state $ω_{DE} > -1$ and $ρ_0 = ρ_{DE}$ today (see Appendix B). But DE gives no clue as to why $ρ_{DE} \approx 2ρ_0$ today and can not explain the anomalous lack of CMB correlations at large scales. We can apply Occam’s razor to argue that there is no need for DE or Modify Gravity. Comparing Fig.1 with Fig.2, we can see that the measured cosmic acceleration today can be explained by a finite causal scale $χ_3 \approx 3c/H_0$, which is expected for Universe with a finite age and a mechanism like inflation.

We have speculated that the finite causal size of our universe could result from a quantum fluctuation at the Planck scale $\ell_{Planck}$ which produces an inflated expansion by a factor $\approx e^{70}$, leading to a reheating into matter and radiation today. This eventually results into late time cosmic acceleration. The existence of such finite causal scale can be tested further with observations of the variation of cosmological or fundamental parameters over cosmological scales.

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APPENDIX A: HOEKKE’S LAW

In Newton’s gravity, a point mass $m$ generates a radial acceleration $\vec{g} = -\vec{V}\phi = -Gm/r^2$, where $\phi$ is the Newtonian potential. The $\Lambda$ term in GR’s field equations, Eq.9, corresponds here to an additional term which is linear in $r$, as in Hooke’s law:

$$\vec{g} = -\vec{V}\phi = \vec{g}_{\text{Newton}} + \vec{g}_{\text{Hooke}} = -\frac{G}{r^2} + G_2 m \vec{r} \tag{A1}$$

Hooke’s constant, $G_2$, can be related to $\Lambda$, as we will see below. This Hooke’s term is unique in that it is the only distance dependence (other than the inverse square law) that has a key property for gravity: that a spherical mass shell of arbitrary density $\rho(R)$ produces a gravitational field which is identical to a point source of equal mass in its center:

$$\vec{g}_{\text{Hooke}} = -G_2 \int_{\text{shell}} d^3 R \, (\vec{\tilde{r}} - \vec{R}) \rho(\vec{R}) = -G_2 m \vec{r} \tag{A2}$$

where $\vec{R}$ covers the shell and $\vec{\tilde{r}}$ is some position outside. This is a key property of gravity, needed to treat the Universe as a whole and sustain Gauss’s and Birkhoff’s theorems. Thus in Classical Mechanics the above law of gravity Eq.(A1) is consistent with the symmetries and observations, as long as $G_2$ is small enough. In fact, Newton, and other scientists, had already noticed this, but did not consider Hooke’s term for lack of observational evidence (Calder & Lahav 2008, and references therein).

Notice how Eq.(A1) diverges for $r \to \infty$. We expect instead that $\vec{g} \to 0$, as particles should be free at infinity (as there could be no causal connection). This explains why on theoretical grounds we would expect $G_2 = 0$ even if this relation is allowed by the symmetries of the problem. On the other hand, if causality ends at some finite distance $r_\Lambda$, as the Universe has a finite age, then we see that Newton’s law alone can not describe a causal gravitational interaction. We need to also have Hooke’s law. Requiring that gravitational forces $\vec{g}$ in Eq.(A1) vanished at the causal boundary $r_\Lambda$:

$$\vec{g}(r = r_\Lambda) = 0 \quad \Rightarrow \quad G_2 = \frac{4\pi G}{3\sqrt{3}} \tag{A3}$$

so that Hooke’s gravitational force is repulsive and only becomes comparable to Newton’s gravity for separations comparable to $r_\Lambda$.

We can estimate the flux using Eq.(A1) to find:

$$\Phi = \int_{\partial V} d\vec{A} \cdot \vec{\tilde{g}} = -4\pi Gm_{\Lambda} - 3VG_2m_\Lambda \tag{A4}$$

where $m_{\Lambda}$ is the mass inside $V$ and $m_\Lambda$ is all the mass in all the (causally connected) universe. As expected, $\Phi = 0$ in Eq.(A4) reproduces Eq.(A3) for $V = V_\Lambda$. For the Newtonian term, masses outside $V$ do not contribute to $\Phi$ because for a small cone center outside and crossing $V$ the in going flux exactly cancels with the outgoing flux (as the inverse square law in $\vec{g}$ compensates the increase in the area crossed then the cone goes inside to when is going outside). This is not the case for the Hooke’s law, where the difference in flux is just proportional to the volume inside $V$. As all the masses contribute equally, we have:

$$\Phi_{\text{Hooke}}(V) = -G_2 \sum_m (\vec{r}) \int_{\partial V} (\vec{\tilde{r}} - \vec{\tilde{r}}) d\vec{A} = -3VG_2m_\Lambda \tag{A5}$$

where the total mass is $m_\Lambda = \sum_m m$. This result is also true in arbitrary number of dimensions (see Wilkins (1986)). This explains why Hooke’s law behaves like vacuum energy (i.e. like negative pressure: $\omega = -1$) or $\Lambda$ in GR: the density remains constant as we increase the volume! Comparing Eq.(A4) with Eq.(7) we have:

$$\Lambda = -3G_2m_\Lambda = 4\pi Gp_\Lambda \tag{A6}$$

where in the last step we have also used Eq.(A3), or equivalently $\Phi = 0$ in Eq.(A4). This of course agrees with Eq.(8), which shows that Eq.(A1) and Eq.(6) are equivalent.

This result provides a Classical Physic’s interpretation of the cosmological constant: it is just related to Hooke’s constant $G_2$ and the total mass $m_\Lambda$ in the (causal) universe. Note how $\Lambda$ can be zero if either $G_2 = 0$ or if $m_\Lambda = 0$. Note also how $G_2$ is well defined for infinite volume and in this case $\Lambda = 0$ because $m_\Lambda/V_\Lambda$ also goes to zero. This relation establishes a clear connection between the value of $\Lambda$ and the matter-energy content in the Universe, as required by causality. It also gives new light into Mach’s Principle.

APPENDIX B: EFFECTIVE DARK ENERGY (DE)

Here we generalize the results of section 4 to the case with DE. If vacuum energy suffers a phase transition or changes with time, as could have happened during inflation, the cancellation presented in section 3.3 will not happen (because
\( \omega \neq -1 \) and an evolving \( \rho_{\text{vac}} \) (which we usually call Dark Energy) could contribute to the effective value of \( \rho_\Lambda \). Consider the general case of DE after inflation:

\[
\begin{align*}
\rho_{\text{DE}}(a) &= \rho_{\text{vac}} + \rho_{\text{DE}} a^{-3(1+\omega)} \\
\rho_{\text{DE}}(a) &= -\rho_{\text{vac}} + \omega \rho_{\text{DE}} a^{-3(1+\omega)},
\end{align*}
\]

where (by definition) only one component of DE is evolving. We then have from Eq.17 and Eq.4:

\[
\rho_\Lambda = \rho_\Lambda + \rho_{\text{DE}} \left[ 1 + \frac{1+3\omega}{2} a^{-3(1+\omega)} \right].
\]

(B2)

where \( \hat{a}_\Lambda \) is some mean value of \( a \) in the past light-cone of \( a_\Lambda \) in Eq.22. This reduces to \( \rho_\Lambda = \rho_\Lambda \) for \( \omega = -1 \). For \( a_\Lambda \rightarrow \infty \) we have \( \rho_\Lambda \rightarrow 0 \) because \( \rho_{m}(a) \) and \( \rho_{r}(a) \) tend to zero as we increase \( a_\Lambda \). The same happens with \( \hat{a}_\Lambda^{-3(1+\omega)} \) for \( \omega > -1 \), so that:

\[
\rho_\Lambda = \rho_{\text{DE}} \quad \text{for} \quad a_\Lambda \rightarrow \infty \quad \& \quad \omega > -1.
\]

(B3)

So evolving DE could produce the observed cosmic acceleration in an infinitely large Universe. This solution does not explain why \( \rho_\Lambda = \rho_{\text{DE}} \approx 2\rho_m \). The original motivation to introduce DE was to explain how vacuum energy \( \rho_{\text{vac}} \) could be as small as the measured \( \rho_\Lambda \) (Weinberg 1989; Huterer & Turner 1999). But we have shown in Section 3.3 that the causal boundary condition explains why \( \rho_{\text{vac}} \) does not contribute to \( \rho_\Lambda \) and also results in \( \rho_\Lambda \approx 2\rho_m \). This removes the motivation to have DE, as it represents an unnecessary complication of the model (Occam’s razor).

If observations find \( \omega \) to be significantly larger than \( \omega = -1 \) this will indicate that something like DE exist. The actual measured value of \( \omega \) will not directly tell us the size of \( \chi_3 \) unless we also have some model for \( \rho_{\text{DE}} \). But very accurate measurements for the evolution of \( \rho_\Lambda \) might actually be able to separate a component that is constant (i.e. \( \rho_\Lambda \)) from a component that is evolving (i.e. DE).

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