Overview of Flow and Hydrodynamic Modeling of Nuclear Collisions

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Abstract. I give a brief overview of flow phenomena in relativistic heavy-ion collisions and the hydrodynamic models used to simulate these systems, aimed at new and young researchers, such as those attending the Hot Quarks 2012 conference.

1. Introduction

In an ultrarelativistic heavy-ion collision, experimenters detect the particles exiting the collision region after the initial collision and any further interactions have long ceased. This consists of anywhere from a few hundred to a few thousand charged hadrons, in addition to neutral hadrons, leptons, and electromagnetic radiation. With this information, we are tasked with inferring as much as we can about the dynamics of the collision and properties of any medium that may have been created during the short duration of the system evolution—hopefully a new, deconfined, phase of matter called the Quark-Gluon Plasma.

2. Measurements

After collecting this information for a large set of collision events, the events are sorted into centrality classes, and analyzed. Centrality selection is usually closely related to the total number of particles produced in a collision, which in a heavy-ion collision is closely related to impact parameter [1] (the latter relationship being much weaker for, e.g., proton-nucleus collisions [2], which were a topic of significant interest when brand new data were presented at this conference [3]). For the set of events in each centrality class, various observables are measured.

In general, the particles detected in a given collision event are governed by some underlying probability distribution, whose azimuthal dependence is often written as a Fourier series:

\[
\frac{dN}{dp_T d\eta d\phi} = \frac{1}{2\pi} \frac{dN}{dp_T d\eta} \left[ 1 + \sum_{n=1}^{\infty} v_n \cos n(\phi - \Psi_n) \right].
\]

(1)

For a general distribution, the coefficients $v_n$ and orientation angles $\Psi_n$ can depend on transverse momentum $p_T$ and pseudorapidity $\eta$, and there can be a different probability distribution for each type of particle and for each collision event (even within a narrow centrality bin).
These coefficients are of significant interest. However, the number of particles detected in a single event is typically insufficient to accurately map out this azimuthal anisotropy of the underlying probability distribution—a single-event measurement of \( v_n \) will typically have at least a 50% statistical uncertainty or worse. Therefore, experimenters look at correlations between detected particles, averaged over an ensemble of events.

The simplest example is a correlation between pairs of particles that are both detected coming from the same event. If each particle in a pair is restricted to be in a narrow bin in phase space, an event-averaged pair correlation can span 5 degrees of freedom: the transverse momentum and pseudorapidity of each particle, and the relative azimuthal angle. (Each collision has a random orientation in azimuth, so one can only measure rotationally invariant event-averaged quantities, and no meaningful information about the average azimuthal angle of the pair can be measured).

The probability distribution of pairs is determined by the underlying one-particle distribution, plus any intrinsic correlation between particle pairs \( \delta \). One can again write the azimuthal dependence as a Fourier series, this time with respect to the relative azimuthal angle between the pair, \( \Delta \phi \) [4],

\[
\frac{dN_{\text{pairs}}}{dp_T^a dp_T^b d\eta^a d\eta^b d\phi^a d\phi^b} = \frac{dN}{dp_T^a d\eta^a d\phi^a} \frac{dN}{dp_T^b d\eta^b d\phi^b} + \delta,
\]

where \((a,b)\) represent bins in transverse momentum and pseudorapidity for the first and second particle, respectively, and any sine term is omitted since it vanishes when averaging over events. The resulting event-averaged Fourier coefficients thus in general measure the quantities:

\[
\langle V_n \Delta \rangle = \langle v_n^a v_n^b \cos n(\Psi_n^a - \Psi_n^b) \rangle + \langle \delta_n \rangle.
\]

If one can choose pairs such that \( \delta_n \) is negligible, and if \( \Psi_n^a \approx \Psi_n^b \), this simplifies to

\[
\langle V_n \Delta \rangle \approx \langle v_n^a v_n^b \rangle.
\]

It is believed that these are reasonably good approximations, and measurements of differential and integrated flow are derived from this observable. E.g., an integrated measurement \( v_n \{2\} \) is the square root of this quantity when neither particle is restricted to a particular bin in phase space [5]:

\[
v_n \{2\} = \sqrt{\langle V_n \Delta \rangle} \approx \sqrt{\langle v_n^2 \rangle}.
\]

It should be noted, however, that it is difficult to determine whether experimental procedures such as imposing a gap in pseudorapidity between the pair really accomplishes the former condition, and the latter is not exactly satisfied even in a purely hydrodynamic calculation [6, 7]. Nevertheless, they are well-defined observables in any case, that can easily be calculated in any theory.

Similarly, one can gain more information about, e.g., the event-by-event distribution of \( v_n \) coefficients and correlations between \( \Psi_n \) angles by looking at correlations between more than two particles [8, 9] or performing more complicated analyses [10, 11].

3. Hydrodynamic Models
If there is a sufficient separation between macroscopic and microscopic scales; that is, if a medium is created whose constituents interact strongly enough, the system is described by relativistic
hydrodynamics. Models that include hydrodynamic evolution have been quite successful at describing a large number of correlation observables. Along with various other observables involving high momentum particles (“jet quenching”) and heavy flavor bound states (“J/Ψ” suppression), this has lead to the standard interpretation of a strongly coupled quark gluon plasma having been produced in these collisions.

The equations of ideal relativistic fluid dynamics are given simply by conservation equations (energy, momentum, and any relevant conserved charges),

$$\partial_\mu T^{\mu\nu} = 0,$$

under the assumption of isotropy in the zero-momentum frame:

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu},$$

$$j^\mu = j^\mu_{\text{ideal}} = n_i u^\mu,$$

where $\epsilon$ is the energy density in the zero momentum frame (“local rest frame”, $T^{0i} = 0$), $u^\mu$ is the velocity relating this frame to the lab frame, $p$ is the isotropic pressure, and $n_i$ the conserved charge density in the local rest frame. For example, a system in local thermal equilibrium will have this isotropic form, and is therefore described by these equations. Small deviations from local thermal equilibrium can be described by adding terms to these ideal forms, with each term having a corresponding transport coefficient (shear viscosity $\eta$, bulk viscosity $\zeta$, etc.).

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal}} + \Pi^{\mu\nu},$$

$$\Pi^{\mu\nu} \equiv \eta \nabla^\mu u^\nu + \zeta \partial_\mu u^\nu (g^{\mu\nu} - u^\mu u^\nu) + ...$$

Here $\nabla^\mu u^\nu$ is the gradient of the fluid velocity, $\partial^\mu u^\nu$, made traceless and orthogonal to the fluid velocity, while the second term is proportional to the trace. The transport coefficients are determined by the underlying dynamics of the medium in question, with the rule of thumb that stronger interactions result in smaller viscosity. The formalism is valid if the length scale set by these transport coefficients is much smaller than the macroscopic length scale set by the gradients.

Finally, the equations are closed by an equation of state $p = p(\epsilon)$.

Thus, a system in this regime has universal behavior, with the only connection to the microscopic physics of the medium in question coming through the equation of state and transport coefficients. These can be calculated from an underlying theory (e.g., the equation of state can be reliably calculated with lattice QCD) or taken as free parameters and constrained by experimental data.

To model a heavy-ion collision, one must also have a description of the earliest stages of the collision, in order to provide initial conditions that can be evolved in time with hydrodynamics. This early stage is not well understood, and a number of models have been used to generate initial conditions, using various physical pictures and assumptions. These include MC-Glauber [12, 13], NEXUS [14, 15], EPOS [16, 17], UrQMD [18, 19], AMPT [20, 21], DIPSY [22, 23, 24], MC-KLN [25, 26], MC-rcBK [27, 24], IP-Glasma [28, 29], etc., and there is much ongoing study [30].

Finally, at the end of the hydrodynamic evolution, one must convert from a fluid to a particle description. The formalism for this is the Cooper-Frye formalism [31], which is well-understood in the ideal case, but has significant uncertainties in the viscous case, since it depends on the microscopic dynamics of the system [32]. Essentially, particles are emitted independently from a fluid cell with a distribution in the fluid rest frame

$$f(p^\mu) = f_0(p \cdot u) + \delta f(p^\mu).$$
In ideal hydrodynamics, one has a thermal distribution $f_0$, while in general there is a viscous correction $\delta f$. These particles are often simply treated as free particles (which can decay if they are unstable), or one can allow rescattering in a kinetic description (e.g., using models such as UrQMD [18, 33] or JAM [34, 26]).

4. Results

Most models for the initial conditions are relatively smooth and uniform in the longitudinal direction—QCD matter tends to form flux tubes or string-like structures between the leading edge of colliding nuclei. This causes the distribution of particles (Eq. (1)) to vary slowly with (pseudo-)rapidity, and therefore causes the distribution of pairs to depend little on relative pseudorapidity $\Delta \eta$. On the other hand, the asymmetry $v_n$ causes correlation of particles in relative azimuth $\Delta \phi$ according to Eq. (4). This agrees with the observed long-range structure of measured two-particle correlations, which in particular see a large number of particle pairs with the same azimuthal angle, but very different pseudorapidity (the “ridge”), which is difficult to explain in the absence of collective behavior in the collision system.

Various other generic features of correlations are also consistent with a hydrodynamic interpretation [35, 36], and actual hydrodynamic calculations have been very successful at describing a large number of 2-, 3-, 4-, 5-particle correlations and beyond [15, 37, 38]. Nevertheless, work is ongoing in understanding as much as we can about the correlations seen in these experiments, and what information we can extract from these data.

5. Summary

This was a brief overview of flow phenomena and hydrodynamic calculations in relativistic heavy-ion collisions, giving enough context and terminology for a young physicist to begin to read current literature of the field.

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