UV completion of an axial, leptophobic, $Z'$

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Abstract

The $Z'$-portal is one of most popular and well-explored scenarios of dark matter (DM). To avoid the strong constraints coming from dilepton resonance searches at the LHC and direct detection of DM, it is usually required that in addition to being leptophobic, the $Z'$ is axially coupled to either the (fermionic) DM or the standard model (SM) quarks. The first possibility has been extensively studied both in the context of simplified model and UV complete scenarios. However, the studies on the second possibility are largely confined to simplified models only. Here, we construct the minimal UV completion of these models satisfying both the criteria of leptophobia and purely axial $Z'$–quark coupling. The anomaly cancellation conditions demand highly non-trivial structures, not only in the dark sector, but also in the Higgs sector.
1 Introduction

One of the most popular scenarios for dark matter (DM) consists of a Standard Model (SM) fermionic singlet, \( \chi \) (the DM particle) coupled to SM fields via a massive \( Z' \) gauge boson \([1-24]\). Typically, the most severe constraints on these kinds of models come from di-lepton production at the LHC \([25, 26]\) and from DM direct-detection experiments \([27]\). Concerning the first ones, a usual strategy is to consider leptophobic models, so that the \( Z' \) just couples to quarks in the SM sector. Similarly, spin-independent direct-detection cross-section is drastically suppressed if the \( Z' \) has axial couplings either to the DM particle or to the quarks (or both) \([9, 13, 19, 28-30]\).

Usually, the phenomenological analyses have been done in the context of simplified DM models, where the DM particle and the \( Z' \) mediator are the only extra fields (see e.g. \([31]\)). The corresponding parameter-space is then spanned by the \( Z' \)–mass, its coupling to the DM particle and the coupling(s) to the SM fields.

However, the above view becomes over-simplified when one takes into account theoretical constraints, in particular those coming from the requirement of anomaly cancellation. In this sense, there have been a number of studies exploring possible ultraviolet (UV) completions of the leptophobic \( Z' \) scenario when the \( Z' \) boson has vectorial coupling to quarks and (preferably) axial coupling to the DM particle \([29, 32, 33]\). In that case, the most important conclusion that emerges is that the dark sector (DS) has to be enlarged beyond the most simplified picture. More precisely, the minimal DS consists of the DM particle, \( \chi_{L,R} \), a \( SU(2) \) doublet, \( \psi_{L,R} \), and a \( SU(2) \) singlet, \( \eta_{L,R} \). On the other hand, the charges of these fields under the extra \( U(1) \) are fixed by anomaly cancellation, thus reducing the effective parameter-space; although there appear new parameters related to the extra stuff in the DS.

The complementary scenario, when the leptophobic \( Z' \) boson has purely axial coupling to the SM quarks and vectorial/axial vector coupling to the DM particle has been often considered in phenomenological analyses (usually in the context of simplified models) \([9, 19, 28, 30]\); but its possible UV completions remain mostly unexplored, except for Ref.\([19]\). The main goal of this paper is to determine the form of the minimal DS for this scenario, consistent with anomaly cancellation, and the complete set of consistent assignments of ordinary and extra hypercharges to the various fields. As for the vectorial case, the DS must be extended with respect to the usual assumptions in simplified models; actually the extension is larger than for the vectorial case.

In addition, we show that for any, minimal or not, UV completion, the consistency of the scenario requires the Higgs sector to contain at least three Higgs doublets.

In Sec.2 we outline the structure of the Higgs sector as required by the conditions of leptophobia and axial couplings to quarks. In Sec.3 we derive the constraints on the particle content of the models coming from the anomaly cancellation conditions. In Sec.4, we present the minimal scenario consistent with all the requirements, giving a complete account of the possible assignments of charges to the various fields. Finally, we conclude in Sec.5.

2 Constraints in the Higgs sector

Let us start by showing that a leptophobic \( Z' \) axially coupled to quarks requires a Higgs sector consisting of, at least, three Higgs doublets. If the Higgs sector contains just one Higgs (as in the
then the invariance under the extra gauge factor, $U(1)_{Y'}$, of the fermionic Yukawa couplings

$$y_i^u \bar{L}_i H e_i, \quad y_i^d \bar{Q}_i H u_i, \quad y_i^d \bar{Q}_i H d_i$$  \hspace{1cm} (1)$$

($y_i$ are the Yukawa coupling constants, with $i$ a family index), forces the $Y'$-charge of the Higgs to vanish, $Y'_H = 0$, in order to satisfy the leptophobia assumption ($Y'_L = Y'_e = 0$). On the other hand, since $Y'_Q = -Y'_u = -Y'_d$ (axial-coupling assumption), the invariance of the above hadronic Yukawa couplings implies $Y'_{Q_i} = Y'_{u_i} = Y'_{d_i} = 0$, so there is no coupling at all to quarks.

For a two-Higgs doublet (THDM) model things are similar. Suppose that in the THDM under consideration $u$– and $d$–quarks couple to the same Higgs, say $H_1$. This is the case of Type I and lepton-specific THDMs [34]. Then, the invariance of the hadronic Yukawa couplings,

$$y_i^u \bar{Q}_i H_{1u_i}, \quad y_i^d \bar{Q}_i H_{1d_i}$$  \hspace{1cm} (2)$$

plus the axial requirement ($Y'_Q = -Y'_u = -Y'_d$) imply $Y'_{H_1} = Y'_{Q_i} = Y'_{u_i} = Y'_{d_i} = 0$.

Suppose now that $d$–quarks couple to a Higgs doublet, say $H_2$, different to that of $u$–quarks, say $H_2$. This is the case of Type II and flipped THDMs [34]. Since one of the two Higgses must couple to leptons, either $Y'_{H_1} = 0$ or $Y'_{H_2} = 0$. Then the axial condition plus the invariance of the hadronic Yukawa couplings,

$$y_i^u \bar{Q}_i H_{2u_i}, \quad y_i^d \bar{Q}_i H_{2d_i}$$  \hspace{1cm} (3)$$

imply $Y'_{H_1} - Y'_{H_2} = 0$, and thus finally all the $Y'$ hypercharges are vanishing in the SM sector. Consequently, the minimal number of Higgses to implement a leptophobic $Z'$ with axial couplings to quarks is three, say $H_u, H_d, H_i$, each one of them coupled specifically to $u$–quarks, $d$–quarks and leptons respectively. This conclusion is completely general, independently of the UV completion of the model.

3 Constraints from anomaly cancellation

Let us now obtain the conditions that anomaly cancellation imposes on the dark sector. From now on we will assume that the $U(1)_{Y'}$ group is flavour-blind. This is a sensible assumption since, otherwise, a not-too-heavy $Z'$ would naturally lead to dangerous FCNC. On top of that, if the $U(1)_{Y'}$ charges of $u$– and $d$–quarks are family-dependent, the off-diagonal terms of the corresponding Yukawa matrix (necessary to reproduce the observed CKM matrix) would be forbidden unless they arise from the coupling of the quarks to extra Higgs-doublets. This would lead to further extensions of the Higgs sector. Besides, the mass-eigenstates of the quarks would not have well-defined $U(1)_{Y'}$ charges, thus spoiling their axial coupling to the $Z'$.

Therefore, the three generations of the SM fermions transform under the gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$, as

$$Q \ (3, \ 2, \ \frac{1}{6}, \ Y'_Q ),$$

$$u_R \ (3, \ 1, \ \frac{2}{3}, \ -Y'_Q ),$$

$$d_R \ (3, \ 1, \ -\frac{1}{3}, \ -Y'_Q ),$$

$$L \ (1, \ 2, \ -\frac{1}{2}, \ 0 ),$$

$$e_R \ (1, \ 1, \ -1, \ 0 ).$$  \hspace{1cm} (4)$$

In addition, we will often take $Y_Q' = 1$ with no loss of generality (it is a normalization factor for the extra hypercharge).

The first consequence of these axial $U(1)_{Y'}$ charges of quarks is that there are six new anomalies to be considered:

$$
\begin{align*}
SU(3)_C^2 & \times U(1)_{Y'}, \\
SU(2)_L^2 & \times U(1)_{Y'}, \\
U(1)_Y^3 & \times U(1)_{Y'}, \\
U(1)_Y & \times U(1)_Y^2, \\
U(1)_{Y'} & , \\
U(1)_{Y'}^3. 
\end{align*}
$$

(5)

Out of them, only the fourth one is cancelled inside the SM sector. Hence, the existence of a dark sector (DS) to implement anomaly cancellation is compulsory. Since we are interested in the minimal DS able to do that job, all the DS fermions, say $f$, must be vectorial under the ordinary hypercharge, $U(1)_Y$, i.e. $Y_{f_L} = Y_{f_R} \equiv Y_f$, so that the four SM anomalies, $SU(3)_C^2 \times U(1)_Y$, $SU(2)_L^2 \times U(1)_{Y'}$, $U(1)_Y^3$ and $U(1)_Y$, are kept vanishing. Otherwise, the DS has to be further increased (this holds for all the scenarios analyzed in the paper).

In order to play the role of the DM particle, the DS must contain a neutral particle, singlet under $SU(3)_C \times U(1)_{em}$. The simplest possibility is a fermion, $\chi_{L,R}$, singlet under the whole SM gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$. Then, additional fields in the DS are needed in order to cancel the anomalies of Eq.(5); in particular those associated to $SU(3)_C^2 \times U(1)_{Y'}$ and $SU(2)_L^2 \times U(1)_{Y'}$, which require non-trivial representations under $SU(3)_C \times SU(2)_L$. Thus the cheapest option (if viable) would be to use one extra particle, say $\Gamma_{L,R}$, transforming as $(3,2)$. However, the corresponding equations for anomaly-cancellation read

$$
\begin{align*}
12Y_Q' + 2(Y_{\Gamma_L}' - Y_{\Gamma_R}') &= 0, \\
9Y_Q' + 3(Y_{\Gamma_L}' - Y_{\Gamma_R}') &= 0,
\end{align*}
$$

(6)

which are compatible only if $Y_Q' = 0$.

Consequently, we have to incorporate additional fields to the DS. The most economical alternative is to consider, beside the DM particle $\chi_{L,R}$, one $SU(3)_C$ triplet, $\Phi_{L,R}$, and one $SU(2)_L$ doublet, $\psi_{L,R}$. Hence, the DS spectrum reads

$$
\begin{align*}
\chi_L & \ ( \ 1, \ 1, \ 0, \ Y_{\chi_L}' \ ), \\
\chi_R & \ ( \ 1, \ 1, \ 0, \ Y_{\chi_R}' \ ), \\
\Phi_L & \ ( \ 3, \ 1, \ Y_{\Phi}, \ Y_{\Phi_L}' \ ), \\
\Phi_R & \ ( \ 3, \ 1, \ Y_{\Phi}, \ Y_{\Phi_R}' \ ), \\
\psi_L & \ ( \ 1, \ 2, \ Y_{\psi}, \ Y_{\psi_L}' \ ), \\
\psi_R & \ ( \ 1, \ 2, \ Y_{\psi}, \ Y_{\psi_R}' \ ).
\end{align*}
$$

(7)
Now, the cancellation conditions for the six anomalies of Eq.(5) read

\[ 12Y_Q' + (Y_{\Phi_L} - Y_{\Phi_R}) = 0 \]
\[ 9Y_Q' + (Y_{\psi_L} - Y_{\psi_R}) = 0 \]
\[ \frac{11}{2}Y_Q' + 3Y_{\Phi}^2(Y_{\Phi_L} - Y_{\Phi_R}) + 2Y_{\psi}^2(Y_{\psi_L} - Y_{\psi_R}) = 0 \]
\[ 3Y_{\Phi}(Y_{\Phi_L}^2 - Y_{\Phi_R}^2) + 2Y_{\psi}(Y_{\psi_L}^2 - Y_{\psi_R}^2) = 0 \]
\[ 36Y_Q'^3 + 3(Y_{\Phi_L}^3 - Y_{\Phi_R}^3) + 2(Y_{\psi_L}^3 - Y_{\psi_R}^3) + (Y_{\chi_L}^3 - Y_{\chi_R}^3) = 0 \]
\[ 36Y_Q' + 3(Y_{\Phi_L} - Y_{\Phi_R}) + 2(Y_{\psi_L} - Y_{\psi_R}) + (Y_{\chi_L} - Y_{\chi_R}) = 0 \]

It is worth-noticing that the first three equations imply

\[ Y_Q'(72Y_{\Phi}^2 + 36Y_{\psi}^2 - 11) = 0 \]

which only has non-trivial solution \( Y_Q' \neq 0 \) if \( Y_{\psi} = \pm 1/2, \ Y_{\Phi} = \pm 1/6 \). Solving the complete set of equations (8) we find the 8 possible assignments of charges for the DS, which are presented in Table 1. Note that all charges are given in terms of two parameters, \( Y_Q' \) and \( Y_{\psi} \) which are arbitrary. Furthermore, as mentioned above, there is a trivial factor of proportionality for all \( Y' \)-charges, so one can take e.g. \( Y_Q' = 1 \) with no loss of generality.

| \( Y_{\psi} \) | 1/2 | 1/2 | -1/2 | -1/2 |
|----------------|-----|-----|------|------|
| \( Y_{\Phi} \) | 1/6 | -1/6 | 1/6  | -1/6 |
| \( Y_{\Phi_R} \) | \( \frac{2}{3}(17 - 2Y_{\psi}') \) | \( \frac{2}{3}(17 - 2Y_{\psi}') \) | \( \frac{2}{3}(17 - 2Y_{\psi}') \) | \( \frac{2}{3}(17 - 2Y_{\psi}') \) |
| \( Y_{\Phi_L} \) | \( \frac{2}{3}(1 - 2Y_{\psi}') \) | \( \frac{2}{3}(1 - 2Y_{\psi}') \) | \( \frac{2}{3}(1 - 2Y_{\psi}') \) | \( \frac{2}{3}(1 - 2Y_{\psi}') \) |
| \( Y_{\psi} \) | \( Y_{\psi} - 9 \) | \( Y_{\psi} - 9 \) | \( Y_{\psi} - 9 \) | \( Y_{\psi} - 9 \) |
| \( Y_{\chi_R} \) | \( -9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) | \( -9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) | \( -9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) | \( -9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) |
| \( Y_{\chi_L} \) | \( 9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) | \( 9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) | \( 9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) | \( 9 \pm \frac{1}{2} \sqrt{22Y_{\Phi}^2 - 198Y_{\psi}^2 + 2747/6} \) |

Table 1: Charge assignments for the DS of Eq.7 satisfying anomaly cancellation conditions of Eq.8. \( Y_{\psi} \) is a free parameter. The expressions correspond to the normalization \( Y_Q' = 1 \). In general, one should understand \( Y_{\psi}' \) above as \( Y_{\psi}'/Y_Q' \) for all fermions \( f \) (including \( Y_{\psi}' \) inside the expressions). The two \( \pm \) signs are correlated, so for each value of \( Y_{\psi}' \) there are 8 solutions.

The DS of Eq.(7) with the charges of Table 1 represents the most economical UV completion of a leptophobic \( Z' \) with axial couplings to quarks. Nevertheless, the fact that the dark quarks (\( \Phi \)) have electric charge \( Q_{\Phi} = \pm 1/6 \) strongly suggests the existence of stable baryons with fractional electric charge, e.g. \( \pm 1/2 \), which would be cosmologically disastrous [35, 36]. Hence, we consider this possibility unrealistic.

There is another, in principle equally economical, alternative for the DS when the DM particle is the neutral component of the doublet, \( \psi \). This requires \( Y_{\psi} = \pm 1/2 \) from the beginning. Then, one could try to satisfy the anomaly-cancellation conditions just with the addition of a \( SU(3)_C \) triplet,
Φ_{L,R} (to cancel the color anomaly) plus a singlet field, η_{L,R}. The corresponding spectrum of the DS is similar to the previous case:

\[ \begin{align*}
\psi_L & (1, 2, Y_\psi, Y'_{\psi_L}), \\
\psi_R & (1, 2, Y_\psi, Y'_{\psi_R}), \\
\Phi_L & (3, 1, Y_\Phi, Y'_{\Phi_L}), \\
\Phi_R & (3, 1, Y_\Phi, Y'_{\Phi_R}), \\
\eta_L & (1, 1, Y_\eta, Y'_{\eta_L}), \\
\eta_R & (1, 1, Y_\eta, Y'_{\eta_R}),
\end{align*} \]

and the cancellation conditions for the six anomalies of Eq. (10) read

\[ \begin{align*}
12Y'_Q + (Y'_{\Phi_L} - Y'_{\Phi_R}) &= 0 \\
9Y'_Q + (Y'_{\psi_L} - Y'_{\psi_R}) &= 0 \\
\frac{11}{2}Y'_Q + 3Y^2_\Phi(Y'_{\Phi_L} - Y'_{\Phi_R}) + 2Y^2_\psi(Y'_{\psi_L} - Y'_{\psi_R}) + Y^2_\eta(Y'_{\eta_L} - Y'_{\eta_R}) &= 0 \\
3Y_\Phi(Y'_{\Phi_L}^2 - Y'_{\Phi_R}^2) + 2Y_\psi(Y'_{\psi_L}^2 - Y'_{\psi_R}^2) + Y_\eta(Y'_{\eta_L}^2 - Y'_{\eta_R}^2) &= 0 \\
36Y^3_Q + 3(Y^3_{\Phi_L} - Y^3_{\Phi_R}) + 2(Y'_{\psi_L}^3 - Y'_{\psi_R}^3) + (Y'_{\eta_L}^3 - Y'_{\eta_R}^3) &= 0 \\
36Y'_Q + 3(Y'_{\Phi_L} - Y'_{\Phi_R}) + 2(Y'_{\psi_L} - Y'_{\psi_R}) + (Y'_{\eta_L} - Y'_{\eta_R}) &= 0.
\end{align*} \]

It is straightforward to check that the first five equations lead to

\[ Y'_Q(72Y^2_\Phi + 36Y^2_\psi - 36Y^2_\eta - 11) = 0. \]

Keeping in mind that \( Y_\psi = \pm 1/2 \), it is easy to see that this equation does not have non-trivial solutions \( Y'_Q \neq 0 \) for which \( Y_\Phi = n/3 \), with \( n \) integer. Again, this suggests the existence of stable baryons with fractional electric charge, which is cosmologically unacceptable. So, we consider this possibility unrealistic as well.

Anyway, we have worked out the complete set of equations (11), finding again 8 possible assignments of charges for the DS of Eq.(10), which are presented in Table 2.

To summarize, the two minimalistic UV completions, Eqs.(7, 10), examined in this section are not phenomenologically viable, so we have to go a step forward by adding, at least, one extra \( SU(3)_C \times SU(2)_L \) singlet. This leads to our final minimal scenario, which is discussed in the next section.

4 The minimal scenario

From the above discussion, the minimal (viable) DS for a leptophobic mediator, \( Z' \), axially coupled to quarks, consists of four particles: \( \chi_{L,R}, \Phi_{L,R}, \psi_{L,R}, \eta_{L,R} \), with \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{Y'} \) representations:
Table 2: Charge assignments for the DS of Eq.10 satisfying anomaly cancellation conditions of Eq.11, with $\Sigma = \frac{1}{2\sqrt{6}}\sqrt{(18Y^2 - 1)(132Y'^2_{\psi_R} - 1188Y'_{\psi_R} + 2747)}$. The free parameters are $Y_{\eta}, Y_{\psi'_{\eta}}$. For each choice of them, the remaining charges are obtained recursively following the order of the table. In each column the ± signs are not correlated, thus leading to 8 solutions in total. As for Table 1 the normalization $Y_\Omega = 1$ has been assumed.

\[
\begin{array}{|c|c|c|}
\hline
Y_\psi & 1/2 & -1/2 \\
\hline
Y_\Phi & \pm\frac{1}{\sqrt{6}}\sqrt{18Y^2 + 1} & \pm\frac{1}{\sqrt{6}}\sqrt{18Y^2 + 1} \\
Y_{\psi'_{\eta}} & Y_{\psi'_{\eta}} - 9 & Y_{\psi'_{\eta}} - 9 \\
Y_{\eta'} & \pm\Sigma - 9Y_{\eta'_{\psi_R}} + (81Y_{\eta}/2 - 9) & \pm\Sigma - 9Y_{\eta'_{\psi_R}} + (81Y_{\eta}/2 - 9) \\
Y_{\eta'}^r_{\eta} & \frac{1}{2\sqrt{3}}(Y_{\eta'}^2_{\eta} + 72(Y_{\phi}^2 - 2)) & \frac{1}{2\sqrt{3}}(Y_{\eta'}^2_{\eta} + 72(Y_{\phi}^2 - 2)) \\
Y_{\psi'_{\eta}} & \frac{1}{72Y_{\phi}}(Y_{\eta}(Y_{\eta}^2 - Y_{\eta'}^2_{\psi_R}) + 9((48Y_{\phi} + 9) - 2Y_{\psi'}^r)) & \frac{1}{72Y_{\phi}}(Y_{\eta}(Y_{\eta}^2 - Y_{\eta'}^2_{\psi_R}) + 9((48Y_{\phi} + 9) - 2Y_{\psi'}^r)) \\
Y_{\psi'} & Y_{\psi'}^r - 12 & Y_{\psi'}^r - 12 \\
\hline
\end{array}
\]

We have assumed here that the $\chi$ particle has vanishing hypercharge, in order to play the role of DM, but the latter could also be played by the neutral component of $\psi$ (if $Y_{\psi} = \pm1/2$).

Now the conditions for the cancellation of the six anomalies of Eq.(5) read

\[
\begin{align*}
12Y'_Q + (Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) &= 0 \\
9Y'_Q + (Y_{\psi_{LL}}^r - Y_{\psi_{RR}}^r) &= 0 \\
\frac{11}{2}Y_{Q_L}^r + 3Y_{\phi_{LL}}^r(Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) + 2Y_{\psi_{LL}}^r(Y_{\psi_{LL}}^r - Y_{\psi_{RR}}^r) + Y_{\eta}^r(Y_{\eta_{LL}}^r - Y_{\eta_{RR}}^r) &= 0 \\
3Y_{\phi}^r(Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) + 2Y_{\psi}(Y_{\psi_{LL}}^r - Y_{\psi_{RR}}^r) + Y_{\eta}(Y_{\eta_{LL}} - Y_{\eta_{RR}}^r) &= 0 \\
36Y_{Q_L}^r + 3(Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) + 2(Y_{\psi_{LL}}^r - Y_{\psi_{RR}}^r) + (Y_{\eta_{LL}}^r - Y_{\eta_{RR}}^r) + (Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) &= 0 \\
36Y_{Q_L}^r + 3(Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) + 2(Y_{\psi_{LL}}^r - Y_{\psi_{RR}}^r) + (Y_{\eta_{LL}}^r - Y_{\eta_{RR}}^r) + (Y_{\phi_{LL}}^r - Y_{\phi_{RR}}^r) &= 0.
\end{align*}
\]

This set of equations is difficult to handle. However, it becomes much more tractable by going into a Gröbner basis for them [37]. This provides a set of equations, equivalent to (14), in which the unknowns can be trivially solved in sequential order, much as in Gaussian elimination for a system of linear equations. Normalizing the extra hypercharges as $Y_{\Omega} = 1$, the equivalent set of equations reads

\[
\begin{align*}
\chi_L & (1, 1, 0, Y_{\chi_{LL}}^r), \\
\chi_R & (1, 1, 0, Y_{\chi_{RR}}^r), \\
\Phi_L & (3, 1, Y_{\phi}, Y_{\phi_{LL}}^r), \\
\Phi_R & (3, 1, Y_{\phi}, Y_{\phi_{RR}}^r), \\
\psi_L & (1, 2, Y_{\psi}, Y_{\psi_{LL}}^r), \\
\psi_R & (1, 2, Y_{\psi}, Y_{\psi_{RR}}^r), \\
\eta_L & (1, 1, Y_{\eta}, Y_{\eta_{LL}}^r), \\
\eta_R & (1, 1, Y_{\eta}, Y_{\eta_{RR}}^r).
\end{align*}
\]
\[2Y^2_Y (Y'_{\psi_L} - Y'_{\psi_R} - 18) + 72Y^2_\Phi + 36Y^2_\psi - 11 = 0 \quad (15)\]
\[Y'_{\psi_L} - Y'_{\psi_R} + Y'_{\eta_L} - Y'_{\eta_R} - 18 = 0 \quad (16)\]
\[Y'_{\chi_L} - 3(-A - 72Y^2_\Phi) + Y'_{\chi_R} - 3(A + 72Y^2_\Phi) + 4Y'_{\chi_R}( -81(-3B + 8C) - 36Y'_{\eta_R}(C - A) + Y'_{\eta_R}^2A)\]
\[+ 2Y'_{\chi_R} - 2(-9(-3B + 4C) - Y'_{\eta_R}D) - Y'_{\chi_L} - 2(-3Y'_{\eta_R}B + 2(9(4C - 3B) + Y'_{\eta_R}D))\]
\[+ Y'_{\chi_L} - 3Y^2_{\chi_R} - B + 4Y'_{\chi_R}(9(4C - 3B) + Y'_{\eta_R}D) + 4(81(8C - 3B) - Y'_{\eta_R}^2A + 361Y'_{\eta_R}(C - A))\]
\[+ 72(-18Y'_{\eta_R}(2C - A) + Y'_{\eta_R}^2A + 3(8Y_2^2(70 - 27Y'_{\psi_R} + 3Y'_{\psi_R}^2)\]
\[-3(99 + 36C + (-405 + 36Y'_{\psi_R} - 4Y'_{\psi_R}^2) = 0 \quad (17)\]
\[Y'_{\psi_L} - Y'_{\psi_R} + 9 = 0 \quad (18)\]
\[(Y'_{\eta_R} + Y'_{\eta_R} + 18)(Y'_{\eta_R} - Y'_{\eta_R})(Y'_{\psi_R} - Y'_{\psi_R} + 18)\]
\[+ 18(2Y'_{\psi_R} - 12) + Y'_{\psi_R}(Y'_{\psi_R} - 9) - Y'_{\eta_R}(Y'_{\eta_R} + 18)) + 258 = 0 \quad (19)\]
\[Y'_{\eta_R} - Y'_{\eta_R} + 12 = 0 , \quad (20)\]

with

\[A = -11 + 36Y^2_\psi\]
\[B = A - 24Y^2_\Phi\]
\[C = Y_\eta(-9 + 2Y'_{\psi_R})Y_\psi\]
\[D = (22 - 72Y^2_\psi) . \quad (21)\]

The free (arbitrary) parameters in the previous equations (15-20) are

\[\{Y_\eta, Y_\psi, Y_\Phi, Y'_{\chi_R}, Y'_{\eta_R}\} . \quad (22)\]

This means, in particular, that we can choose all the ordinary hypercharges of the DS, so that there are no cosmological problems related to factional electric charges. Now, each equation in (15-20) solves one parameter in terms of the precedent ones, so it is trivial, once the initial parameters (22) have been chosen, to obtain the others in terms of them. The sequence of reduction goes as

\[\{Y_\eta, Y_\psi, Y_\Phi, Y'_{\chi_R}, Y'_{\eta_R}\} \rightarrow Y'_{\chi_L} \rightarrow Y'_{\eta_L} \rightarrow Y'_{\psi_R} \rightarrow Y'_{\psi_L} \rightarrow Y'_{\Phi_R} \rightarrow Y'_{\phi_L} . \quad (23)\]

Note that for all the equations the eliminations are linear, and thus completely trivial and unambiguous, except for Eq.(17), which is a second-order equation and therefore implies a double solution for \( Y'_{\psi_R} \) (and thus for the subsequent variables in the sequence (23)).

Eqs.(15-20) represent the general solution for the possible hypercharges and extra-hypercharges of the minimal DS (13). In order to gain some intuition on the scenario we can particularize the general solution for concrete values of the hypercharges. E.g. for \( Y_\eta = 1, Y_\psi = 1/2, Y_\Phi = 1/3, \) we get\(^1\)

\(^1\)The set of equations (24-29) is of course equivalent to the set (15-20) for the above values of \( Y_\eta, Y_\psi, Y_\Phi \). However we have obtained them by replacing those values in the initial equations (14) and then going into a Gröbner basis.
\[
Y'_{\chi_L} - Y'_{\chi_R} - 15 = 0 \tag{24}
\]
\[
-3 + Y'_{\eta_L} - Y'_{\eta_R} = 0 \tag{25}
\]
\[
371 - 300Y'_{\chi_R} - 20Y'_{\chi_R}^2 + 78Y'_{\eta_R}^2 - Y'_{\eta_R}^2 - 18Y'_{\psi_R}Y'_{\psi_R} + 51Y'_{\psi_R}^2 = 0 \tag{26}
\]
\[
9 + Y'_{\psi_L} - Y'_{\psi_R} = 0 \tag{27}
\]
\[
9 - Y'_{\eta_R} + 4Y'_{\Phi_R} + 3Y'_{\psi_R} = 0 \tag{28}
\]
\[
-9 - Y'_{\eta_R} + 4Y'_{\Phi_L} + 3Y'_{\psi_R} = 0 , \tag{29}
\]

with the same sequence of reduction as (23). Some particular solutions to Eqs.(24-29) are shown in Table 3. Clearly, the extra hypercharges get rather bizarre (non-rational) values.

| \( Y'_{\chi_R} \) | 1 | 1 | 1 | 1 | 2 | 2 |
| \( Y'_{\eta_R} \) | 1 | 1 | 2 | 2 | 1 | 1 |
| \( Y'_{\psi_R} \) | 16 | 16 | 16 | 16 | 17 | 17 |
| \( Y'_{\psi_L} \) | 4 | 4 | 5 | 5 | 4 | 4 |
| \( Y'_{\phi_R} \) | 0.260 | 9.621 | 0.404 | 9.830 | -0.440 | 10.323 |
| \( Y'_{\phi_L} \) | -8.739 | 0.621 | -8.595 | 0.830 | -9.440 | 1.323 |
| \( Y'_{\phi_R} \) | 9.804 | 2.783 | 9.946 | 2.877 | 10.330 | 2.257 |
| \( Y'_{\phi_L} \) | 13.045 | 10.705 | 13.9821 | 11.625 | 13.221 | 10.530 |

Table 3: Explicit examples of extra-hypercharge assignments in the minimal DS, Eq.(13), that lead to anomaly cancellation. (For the non-rational charges, only the first decimals are shown.) The extra-hypercharges of the SM quarks, Eq.(4), are normalized as \( Y'_{Q} = 1 \). The ordinary hypercharges of the DS fermions are \( Y_{\eta} = 1 \), \( Y_{\psi} = 1/2 \) and \( Y_{\Phi} = 1/3 \).

One could try to get additional examples of UV completion by going beyond the minimal DS studied in this paper. One possibility, examined in Ref.[19], is to consider a DS consisting of a whole SM-like vectorial family. In this way, a model (named Model 4) was obtained in [19] with rational (though still weird) extra-hypercharges.

Another (even less economical, but somehow trivial) solution is to assign to every SM fermion a DS fermion with the same representation and charges, but opposite chirality. In this way all fermions form vectorial pairs and anomaly cancellation is automatic. This obvious possibility was also noticed in Ref.[19]. Then the DM fermion, \( \chi_{L,R} \), must correspond to a couple of right-handed neutrinos with non-vanishing extra-hypercharge. Besides, the Higgs sector must be further extended to incorporate Yukawa couplings for both charged and neutral leptons. Notice that in a scenario of this kind, the remarkable anomaly cancellation inside the ordinary SM would be a (weird) accident.

5 Conclusions

The \( Z' \)-portal is one of most popular and well-founded scenarios of dark matter (DM). However, it is subject to severe experimental and observational constraints, in particular those coming from
di-lepton production at the LHC and from DM direct-detection experiments. Consequently, it is often required that the $Z'$ boson has couplings which are (i) leptophobic, (ii) axial either with the DM particle or with the quarks (or both). Condition (ii) leads to spin-dependent direct-detection cross-section, maybe with velocity suppression.

Most of the analyses so far have been performed in the context of simplified models. However, it is convenient to consider their possible UV completions, not only for the sake of theoretical consistency but also from phenomenological point of view. It turns out that, e.g. the requirement of anomaly cancellation implies the existence of an extended dark sector (beyond a lone DM particle) and strong correlations between the $U(1)'$ charges of the SM and the dark sector (DS) fermions. This is of great importance for phenomenological analyses, as well as for evaluations of the relic density.

Concerning UV completions, the case in which the leptophobic $Z'$ has axial couplings to the DM has been well studied in the current literature. However, the complementary case, when the $Z'$ presents axial couplings to the quarks is still essentially unexplored (except for Ref. [19]).

In this paper we have considered the latter scenario, building up the minimal DS (from the point of view of the spectrum) that is anomaly-free and contains a candidate for DM particle. It turns out that the most economical possibilities are not phenomenologically viable since they contain fractional electrically-charged particles. Then, the minimal DS consists of four particles: a SM singlet (the DM particle), $\chi_{L,R}$, a $SU(3)_C$ triplet, $\Phi_{L,R}$, a $SU(2)_L$ doublet, $\psi_{L,R}$ and a $SU(3)_C \times SU(2)_L$ singlet, $\eta_{L,R}$, see Eq. (13). This means, in particular, that the minimal DS is larger than the analogous one when the $Z'$ has vectorial coupling to quarks.

Regarding the possible assignments of (ordinary and extra) charges to the various fields, the complete set of solutions to the anomaly-cancellation conditions can be expressed in a convenient form using a Gröbner basis, as explained in Eqs. (15-23). It turns out, in particular, that it is possible to choose the hypercharges of the DS fields, so that no fractional electric-charge states are present. Then the consistency equations become simpler. Still, the set of solutions contains two free parameters, which we have chosen as $Y'_{\chi R}$, $Y'_{\eta R}$, as indicated in Eq. (23). However, the solutions imply the existence of non-rational $U(1)'$ charges. Some examples are given in table 4. In any case, the existence of the extra states in the DS should not be ignored in phenomenological analyses. Going beyond the minimal DS it is possible to get rational (but still weird) charges.

Concerning the Higgs sector, we have shown that the consistency of Yukawa couplings requires at least three Higgs states giving mass to $u$-quarks, $d$-quarks and charged leptons, respectively. This result holds for any consistent (minimal or not) DS.

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