Models of $\alpha$ Centauri A and B with and without seismic constraints: time dependence of the mixing-length parameter

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ABSTRACT
The $\alpha$ Cen binary system is a well-known stellar system with very accurate observational constraints to structure of its component stars. In addition to the classical non-seismic constraints, there are also seismic constraints for the interior models of $\alpha$ Cen A and B. These two types of constraint give very different values for the age of the system. While we obtain 8.9 Gyr for the age of the system from the non-seismic constraints, the seismic constraints imply that the age is about 5.6-5.9 Gyr. There may be observational or theoretical reasons for this discrepancy, which can be found by careful consideration of similar stars. The $\alpha$ Cen binary system, with its solar type components, is also suitable for testing the stellar mass dependence of the mixing-length parameter for convection derived from the binaries of Hyades. The values of the mixing-length parameter for $\alpha$ Cen A and B are 2.10 and 1.90 for the non-seismic constraints. If we prioritize to the seismic constraints, we obtain 1.64 and 1.91 for $\alpha$ Cen A and B, respectively. By taking into account of these two contrasting cases for stellar mass dependence of the mixing-length parameter, we derive two expressions for its time dependence, which are also compatible with the mass dependence of the mixing-length parameter derived from the Hyades stars. For assessment, these expressions should be tested in other stellar systems and clusters.

Key words: stars: interior – stars: evolution – stars: individual: $\alpha$ Cen – stars: abundances – binaries: visual – stars: oscillations

1 INTRODUCTION
The observable quantities of a star, such as luminosity, radius and effective temperature, mostly depend on its mass. Chemical composition, rotation and age, however, have a second role in these quantities. The time dependence of the observable quantities are very weak. Therefore, no more or less precise age for stars available, particularly for late-type stars, and this is an essential quantity for our understanding of the evolution of both the far and near Universe. However, it is possible to derive the ages of stars from their seismic properties (Christensen-Dalsgaard, 1988). In this respect, the solar-like components of $\alpha$ Centauri ($\alpha$ Cen) are excellent targets, as their seismic properties are determined by ground-based spectrographic observations. The aim of the present paper is to test the theory of stellar evolution by constructing models for the internal structure of these stars, using all the available constraints.

The internal structures of $\alpha$ Cen A and B have been considered theoretically in many papers. Recent studies by Miglio & Montalban (2005), Eggenberger et al. (2004), Thoul et al. (2003), Morel et al. (2000) and Guenther & DeMarque (2000) have analysed the structure and evolution of these stars in detail. The detection of their seismic properties have increased the scientific interest in these stars. The large and small frequency separations of $\alpha$ Cen A and B have been derived observationally from their p-mode oscillations by Bouchy & Carrier (2002), and Carrier & Bourban (2003), respectively, using spectrographic methods. In addition to the constraints from the fundamental properties of the stars, the interior models of these stars should also satisfy these seismic constraints. Recently, Kjeldsen et al. (2005) also observed $\alpha$ Cen B, and measured the frequencies, which turned out to be quite different from the previous results of Carrier & Bourban (2003). Therefore, the system deserves to be reconsidered by taking this new measurement into account.

The masses (Pourbaix et al. 2002) and radii (Kervella et al. 2003) of $\alpha$ Cen are well known from the cited observations. Although the $\alpha$ Cen system has already been widely
investigated, no agreement has yet been reached about the precise value of the effective temperatures ($T_{\text{eff}}$) and element abundances in the atmospheres of these stars. These quantities are determined by spectroscopic methods. However, using the photometric observation of Bessel (1990) as an alternative, which has unfortunately been neglected in studies on $\alpha$ Cen, we have alternatively derived the effective temperatures and the heavy element abundances of both components.

The effective temperature of a model is a function of the mixing-length parameter for convection. Yildiz et al. (2006) have confirmed a very definite relationship between the mixing-length parameter and the stellar mass by fitting the models of late type components of some binaries in Hyades to the observations: $\alpha = 9.19(M/M_\odot - 0.74)^{0.033} - 6.65$. However, controversial results are obtained from studies on binaries by different investigators: while Lastennet et al. (2003), Ludwig & Salaris (1999), and Lebreton et al. (2001) have stated that $\alpha$ is an increasing function of the stellar mass in their studies on some other binaries, Miglio & Montalban (2005), Eggenberger et al. (2004), Morel et al. (2000) have confirmed that $\alpha_B$ is greater than $\alpha_A$ for the $\alpha$ Cen binary system. In these studies and others, at least for the favoured models, however, we note that the model luminosity is less than the observed value for $\alpha$ Cen A, and that the opposite is true for $\alpha$ Cen B. This point needs to be explained. However, the results of the hydrodynamic simulations of convection are in agreement with the decrease $\alpha$. Thus, there may be no single relationship between the mixing-length parameter and the stellar mass and we should therefore search for a more general expression for $\alpha$ than that given above.

According to the above-mentioned studies on the internal structure of $\alpha$ Cen A and B, the age of the system is between 4.85–7.6 Gyr, at least for the favoured or typical models. The lower limit is given by Thevenin et al. (2002) and the upper limit is from Guenther & Demarque (2000). Miglio & Montalban (2005), however, find the age of the system to be 8.9 Gyr from the models for the non-seismic constraints (see Section 4.1). In other words, the seismic and the non-seismic constraints lead to very different ages for $\alpha$ Cen.

The remainder of this paper is organized as follows. In Section 2, we summarize the observed chemical composition, the seismic and the fundamental properties of $\alpha$ Cen A and B. The description of models are given in Section 3, and the results of the model computations are presented and discussed in Section 4. Finally, we give concluding remarks in Section 5.

2 OBSERVED PROPERTIES OF $\alpha$ CEN

2.1 Chemical abundance of $\alpha$ Cen A and B

Many studies have been done on the abundance determination of chemical species of $\alpha$ Cen from the spectra of its components. Most of these studies deal with the abundance of Fe and/or similar heavy elements, which are not abundant in stars. The most complete studies—in the sense that they also deal with the most abundant heavy elements in normal stars such as oxygen, carbon and nitrogen—have been carried out by Neuforge-Verheecke & Magain (1997) and Feltzing & Gilmore (2001, hereafter FG2001). The abundances of 17 elements have been obtained by Neuforge-Verheecke & Magain (1997). They find from the spectra of $\alpha$ Cen A that overabundance relative to solar is 0.25 dex for Fe and 0.21 for the most abundant heavy element oxygen, and a general overabundance of 0.24 dex. They also confirm that there is no significant difference between the chemical compositions of $\alpha$ Cen A and B, but $\alpha$ Cen B is likely more metal rich than $\alpha$ Cen A. Using their equivalent widths, FG2001 have obtained very similar results for the abundance of 13 heavy elements, including oxygen.

By considering Mg, Ca, Si, Ti, Cr and Fe elements, Doyle et al. (2004) have found that the mean overabundance of $\alpha$ Cen A is 0.12±0.06 dex. Contrary to this result, Ecuillon et al. (2006) computed the average value of the oxygen abundance of $\alpha$ Cen A and B as $[O/H]=-0.12$ and -0.06, respectively, from triplet lines by applying non-local thermodynamical equilibrium effects. Such contradictions in the abundance determination $\alpha$ Cen A and B, and the recent revisions on the solar compositions (Asplund et al. 2005) make us cautious about seeing the abundance determination of stars as an issue that has been conclusively studied.

The mixture in FG2001 is taken as the chemical composition of the OPAL opacity tables (Iglesias & Rogers 1996) used in the construction of models for $\alpha$ Cen A and B. For the chemical species that are not observed by FG2001, such as Ne, we adopt the value of 0.25 dex for their abundance relative to the Sun. For comparison, we also construct models with the recent solar mixture given by Asplund et al. (2005, hereafter AGS2005).

2.2 Fundamental properties of $\alpha$ Cen A and B

For our better understanding of stellar structure and evolution, a very accurate knowledge of stellar masses is required. The determination of masses of $\alpha$ Cen A and B from observations has been the subject of many papers. The most recent determination is that of Pourbaix et al. (2002). According to their findings, the masses of $\alpha$ Cen A and B are as $M_A = 1.105 \pm 0.007 M_\odot$ and $M_B = 0.934 \pm 0.007 M_\odot$, respectively. The radii of the components have been measured by Kervella et al. (2003) as $R_A = 1.224 \pm 0.003 R_\odot$ and $R_B = 0.863 \pm 0.005 R_\odot$. The uncertainties in radii are computed by taking into account only the uncertainty in the parallax. Many investigators have determined the effective temperatures from the spectra of $\alpha$ Cen A and B is done by many investigators (see, for example, Morel et al. 2000 and references therein). According to the results of studies based on spectra, while the effective temperature of $\alpha$ Cen A is between 5830 K and 5720 K, the range of effective temperatures of $\alpha$ Cen B is 5250–5325 K.

In the literature, apparent magnitude (V) and colour (B-V) of $\alpha$ Cen A and B are, in general, taken from The Bright Star Catalogue (Hoffleit & Jaschek 1982): $V_A = -0.01$, $V_B = 1.33$, $(B-V)_A = 0.71$ and $(B-V)_B = 0.88$. Adopting the parallax given in Söderhjelm (1999), $\pi = 747.1 \pm 1.2$ mas, we find the absolute magnitudes of $\alpha$ Cen A and B as 4.358 and 5.697, respectively. Using the tables of Lejeune et al. (1998, hereafter LCB1998) for colours and bolometric correction, prepared for different heavy element abundances, we obtain the required luminosities of $\alpha$ Cen
A and B for selected values of Z. In Table 1 the first three rows for α Cen A and B are for the heavy element abundance of 0.25 dex and 0.20 dex and for the solar metallicity, respectively. For each metal abundance, while we obtain reasonable values for the absolute magnitudes from the tables in comparison with the observed values given above, for the colours, there is no agreement between theoretical and observational results. In the fourth and the eighth rows, we derive the fundamental properties of A and B relative to the Sun are found as 5824 K and 5250 K, respectively, and are in very good agreement with the values found from spectra by Neuforge-Verheecke & Magain (1997).

Indeed, the measurements given in Bessel (1990) and Hoffleit & Jaschek (1982) are very close to each other; the remarkable differences exist between the colours of each star and the measurements of Bessel (1990) are compatible with the theoretical results. Therefore, we adopt the photometric constraints in Bessel (1990) in the calibration process of models for α Cen A and B.

2.3 Seismic properties of α Cen A and B and an age estimation for the system

The most important constraints on the stellar evolution theory from the observations of solar-like oscillations are small (δν13) and large (Δν13) separations between the observed
frequencies. The small separation between the frequencies (Christensen-Dalsgaard 1988) is defined as
\[ \delta \nu_{nl} = \nu_{n,l} - \nu_{n-1,l+2}. \tag{1} \]

In place of this, \( D_0 \) is frequently used as an average effect of oscillations with different values of degree \( l \). We compute \( D_0 \) from \( \delta \nu_{nl} \): \( D_0 = \frac{\delta \nu_{nl}}{6} \) (Kjeldsen et al. 2005). If frequencies for \( l = 3 \) modes are observed, \( D_0 \) is computed from \( \delta \nu_{n0} \) and \( \delta \nu_{n1} \): \( D_0 = (\delta \nu_{n0}/6 + \delta \nu_{n1}/10)/2 \).

The large separation between the frequencies is given below:
\[ \Delta \nu_{nl} = \nu_{n,l} - \nu_{n-1,l}. \tag{2} \]

While the small separation is a sensitive function of physical conditions in central regions where the nuclear evolution proceeds, the large separation is a measure of the mean density.

The large and small frequency separations of \( \alpha \) Cen A are determined by Bouchy & Carrier (2002) as 105.5 \( \pm \) 0.1 \( \mu \)Hz and 5.6 \( \pm \) 0.7 \( \mu \)Hz (\( D_0 = 0.93 \pm 0.12 \mu \)Hz), respectively. Carrier & Bourban (2003) find that \( \delta \nu_{n0} = 8.7 \pm 0.8 \mu \)Hz and \( \Delta \nu_{n0} = 161.5 \pm 0.1 \mu \)Hz for \( \alpha \) Cen B. Recently, Kjeldsen et al. (2005) also obtained the seismic properties of \( \alpha \) Cen B using the two-site observations. According to their findings, while the large separation between the frequencies is very similar to that of Carrier & Bourban (2003), the small separation between the frequencies is \( \delta \nu_{n0} = 10.14 \pm 0.62 \mu \)Hz and significantly larger than the previously found separation (8.7 \( \pm \) 0.8 \( \mu \)Hz) by Carrier & Bourban (2003). Kjeldsen et al. (2005) also observed the oscillations with \( l = 3 \) and the small separation between the frequencies of these oscillations and oscillations with \( l = 1 \) is \( \delta \nu_{n1} = 16.73 \pm 0.65 \mu \)Hz. From \( \delta \nu_{n0} \) and \( \delta \nu_{n1} \), \( D_0 = 1.68 \pm 0.08 \mu \)Hz is found.

Fig. 1 shows and observational seismic HR diagram for several stars for which \( D_0 \) and \( \Delta \nu_{nl} \) are available. While the filled circle and star represent \( \alpha \) Cen A and B, respectively, the square shows the position of the Sun. \( D_0 \) and \( \Delta \nu_{nl} \) of the Sun are computed from the data of Bisson group (Chaplin et al. 1999). The seismic data of \( \eta \) Boo (box), \( \beta \) Vir (diamond) and Procyon A (circle) are taken from Carrier, Eggenberger & Bouchy (2005), Carrier et al. (2005), and Martic et al. (2004), respectively. The \( D_0 \) values of \( \alpha \) Cen B and the Sun are very close to each other. This means that the central hydrogen abundance of \( \alpha \) Cen B \( (X_c = 0.34) \) is nearly equal to the solar value (see figure 1 in Christensen-Dalsgaard 1988).

Thus, the age of \( \alpha \) Cen B is about half of its nuclear time scale \( (t_{\text{nuc}}) \). From this confirmation, we can estimate the age of \( \alpha \) Cen using a simple approach. Because the nuclear time-scale is inversely proportional to \( M^{2.5} \) (Böhm-Vitense 1992),
\[ t_{\alpha \text{Cen}} = \left( \frac{M_{\odot}}{M_{\alpha \text{Cen}}} \right)^{2.5} t_{\odot} = 5.5 \text{ Gyr}. \tag{3} \]

The basic observational properties of \( \alpha \) Cen A and B discussed in this section are given in the last two rows of Table 2.

3 DESCRIPTION OF THE MODELS

The characteristics of our code have already been described by Yıldız (2000, 2003, see also references therein), and therefore we shall not go into details. Our equation of state uses the approach of Mihalas et al. (1990) in the computation of the partition functions. The radiative opacity is derived from Iglesias & Rogers (1996), completed by the low temperature tables of Alexander & Ferguson (1994). For the nuclear reaction rates, we use the analytic expressions given by Caughlan & Fowler (1988), and we employ the standard mixing-length theory for convection (Böhm-Vitense 1958). For the diffusion of chemical species, the routines of Thoul, Bahcall & Loeb (1994) are used.

4 RESULTS AND DISCUSSIONS

4.1 Models satisfying the constraints on the fundamental properties of \( \alpha \) Cen A and B

In order to find the five unknowns (i.e. \( X, Z, t, \alpha_A \) and \( \alpha_B \)), we write down four equations for \( L_A, L_B, R_A \) and \( R_B \). The fifth equation for a solution is for the surface value of the \( (Z/X) \) ratio for \( \alpha \) Cen A. For \( (Z/X)_{A,0} = 0.0237 \), the values of five unknowns and the models of \( \alpha \) Cen A and B using these values are given in the first two rows of Table 2. The heavy element mixture of the models is taken from FG2001. According to the solution for non-seismic constraints (NOS model), the age of the system is 8.88 Gyr and \( \alpha \) Cen A is very close to the end of its main-sequence (MS) lifetime. Because the seismic constraints are not used in this solution, the seismic properties of models are not in agreement with the observations; small separations between the frequencies of the models are significantly smaller than the observed separations for both stars. In the last two rows of Table 2, the observational results are listed for comparison. In Fig. 2 the models are plotted in a theoretical HR diagram.
Table 2. Model properties of α Cen A and B. \(\delta \nu_{\alpha 2}, \delta \nu_{\alpha 3}, D_0\) and \(\Delta \nu_0\) are in units \(\mu Hz\). The uncertainty of \(\delta \nu_{\alpha 2}\) and \(\delta \nu_{\alpha 3}\) is about 0.7 \(\mu Hz\). The error in \(\Delta \nu_0\) is 0.1 \(\mu Hz\). The uncertainty of \(D_0\) is 0.12 \(\mu Hz\) for α Cen A and 0.08 \(\mu Hz\) for α Cen B.

| Star | \(L/L_\odot\) | \(R/R_\odot\) | \(T_{eff}\) | \(X_0\) | \(Z_0\) | \((Z/X)_S\) | \(\alpha\) | \(t(10^6y)\) | \(\delta \nu_{\alpha 2}\) | \(\delta \nu_{\alpha 3}\) | \(D_0\) | \(\Delta \nu_0\) | \(cc\) | MODEL |
|------|-------|-------|-------|-------|-------|---------|-------|-------|-------|-------|-------|-------|-------|-------|
| A    | 1.545 | 1.224 | 5822  | 0.703 | 0.0328| 0.0237 | 0.0316| 2.10  | 8.88  | 2.7   | ...... | 0.45  | 106.5 | FG2001 | NOS  |
| B    | 0.507 | 0.864 | 5246  | 0.703 | 0.0328| 0.0267 | 0.0364| 1.90  | 8.88  | 6.6   | 15.8  | 1.30  | 164.6 | FG2001 | NOS  |
| A    | 1.545 | 1.224 | 5822  | 0.669 | 0.0322| 0.0237 | 0.0316| 1.64  | 5.70  | 2.7   | 12.4  | 0.30  | 107.4 | FG2001 | SIS  |
| B    | 0.584 | 0.863 | 5436  | 0.669 | 0.0322| 0.0271 | 0.0388| 1.91  | 5.70  | 19.6  | 16.8  | 1.68  | 164.6 | FG2001 | SIS  |
| B    | 0.506 | 0.862 | 5248  | 0.669 | 0.0322| 0.0271 | 0.0389| 1.58  | 5.70  | 9.50  | 19.0  | 1.74  | 163.5 | FG2001 | SIS914|
| A    | 1.543 | 1.223 | 5822  | 0.676 | 0.03 | 0.0198 | 0.0316| 1.65  | 5.57  | 6.4   | ...... | 0.94  | 107.6 | FG2001 | SIS3 |
| B    | 0.584 | 0.863 | 5441  | 0.676 | 0.03 | 0.0271 | 0.0388| 1.93  | 5.57  | 29.9  | 18.4  | 1.70  | 164.6 | FG2001 | SIS3 |
| A    | 1.549 | 1.223 | 5828  | 0.713 | 0.023 | 0.0161 | 0.0211| 1.71  | 5.90  | 5.49  | ...... | 0.91  | 106.7 | FG2001 | SIS23|
| B    | 0.569 | 0.863 | 5400  | 0.713 | 0.023 | 0.0192 | 0.0259| 1.98  | 5.90  | 9.31  | 18.5  | 1.70  | 164.4 | FG2001 | SIS23|
| A    | 1.544 | 1.223 | 5828  | 0.697 | 0.0233| 0.0159 | 0.0213| 1.67  | 5.70  | 5.6   | ...... | 0.93  | 106.8 | AGS2005| SISAGS|
| B    | 0.572 | 0.863 | 5409  | 0.697 | 0.0233| 0.0192 | 0.0265| 1.96  | 5.70  | 9.38  | 18.6  | 1.71  | 165.0 | AGS2005| SISAGS|
| A    | 1.544 | 1.224 | 5824  | ...... | 0.0237 | 0.0364 | 0.0264 | 0.0391 | 0.0267 | 0.0399 | ...... | 10.14 | 167.3 | 1.68 | 161.5 | obs  |
| B    | 0.507 | 0.863 | 5250  | ...... | 0.0267 | 0.0391 | 0.0267 | 0.0399 | 0.0267 | 0.0399 | ...... | 10.14 | 167.3 | 1.68 | 161.5 | obs  |

\*\*Derived from the photometric data of Bessel (1990) using the tables for colours and bolometric corrections of LCB1998.

\(\beta\) Derived from abundances found by FG2001. For the elements not observed by them, an average overabundance of 0.25 dex relative to solar is assumed.

\(\gamma\) Derived from \(\beta\), assuming an overabundance of 0.04 dex relative to α Cen B (observed for Al and Si, for example).

4.2 Calibration of models with seismic constraints

As discussed in the previous subsection, the small separations of NOS models of α Cen A and B are significantly smaller than the observed values. Therefore, models satisfying the seismic constraints must have a smaller age. However, models of α Cen A and B require a high value for age, such as 8.88 Gyr, in order to fit the observed difference between the luminosities of α Cen A and B. Because there is no simultaneous solution for \(L_A, L_B\) and the seismic constraints, we therefore take into account the luminosity of one component only. If we leave aside the luminosity of α Cen B and use all the remaining constraints including seismic measurements, we find a significantly smaller age for the system (SIS model) than predicted by NOS models. The SIS models of α Cen A and B are given in the third and fourth rows of Table 2, respectively. In contrast to the NOS models, the value of the mixing-length parameter found for α Cen A is smaller than that of α Cen B. However, the reason of this result could be that the luminosity of the SIS model for α Cen B is 15 per cent greater than the observed luminosity.

If this excess is a result of observational uncertainty about the mass of α Cen B, then its required mass is 0.914 \(M_\odot\). The properties of the model with this mass are given in the fifth row of Table 2. The mixing-length parameter of this model is less than the corresponding model of α Cen A.

Because the abundance determination of stars is not an issue that has been pursued exhaustively, we also construct models for α Cen A and B with two different \(Z\). For \(Z = 0.03\) (sixth and seventh rows in Table 2) and \(Z = 0.023\) (eighth
and ninth rows), putting aside the differences between the initial hydrogen abundances, we find very similar models. In order to see the effect of the heavy element mixture, we also obtain models with the recent solar heavy element mixture (AGS2005) in place of that of FG2001.

In Fig. 3, the time run of the models of α Cen A and B (SIS models) are plotted in a seismic HR diagram. The filled circle and diamond show α Cen A and B, respectively, and the Sun is represented by ○. While the dotted lines mark the evolutionary tracks, the solid lines show the isochrones. The isochrone line at the top is for 0.1 Gyr and the next line is for 1 Gyr. The increment between the other isochrone lines is 1 Gyr, except the last one. The isochrone line at the bottom is for 5.7 Gyr.

4.3 On the alternative seismic HR diagrams

Seismic and non-seismic constraints impose very different ages on the models of α Cen A and B. For the small separation between the frequencies, however, oxburgh & Vorontsov (2003) and Mazumdar (2005) propose alternative expressions for a better representation of physical conditions in central regions of the late-type stars. Roxburgh & Vorontsov (2003), for example, propose to use

\[ d_{01}(n) = \frac{1}{8} \left( \nu_{n-1,0} - 4\nu_{n-1,1} + 6\nu_{n,0} - 4\nu_{n,1} + \nu_{n+1,0} \right) \]

in place of \( D_0 \) (or \( \delta\nu_{22} \)). They also confirm that the ratio of small to large separations \( (r_{01} = d_{01}(n)/\Delta\nu_{21}) \) for acoustic oscillations is more sensitive to physical conditions in the stellar core. From the observed seismic data of α Cen A and B, we find that \( r_{01} \) is 0.018 and 0.025, respectively. The uncertainty in \( d_{01}(n) \) is, in principle, the same as for the small separations. Then, the uncertainty in \( r_{01} \) is 0.007 for α Cen A and 0.004 for α Cen B. For the SIS model, we compute the ratio \( r_{01} = d_{01}(n)/\Delta\nu_{21} \): \( r_{01} = 0.045 \) for α Cen A and \( r_{01} = 0.012 \) for α Cen B. Very similar results are also obtained for the other models listed in Table 2. While the value of \( r_{01} \) for the model of α Cen A is significantly larger than the observed \( r_{01} \) and does not show an explicit time dependence, the observed \( r_{01} \) of α Cen B is close to the zero-age main-sequence (ZAMS) value of the SIS model rather than the value at 5.7 Gyr.

4.4 Variability of the mixing-length parameter with time

As stated above, Yıldız et al. (2006) have confirmed a relationship between the mixing-length parameter for convection and the stellar mass from the late-type stars of the Hyades open cluster. According to this relationship, for α Cen A and B, \( \alpha_A = 2.06 \) and \( \alpha_B = 1.78 \). A similar relationship is found for the models of α Cen A and B fitted into the non-seismic constraints of the system. The mixing-length parameters for these models are \( \alpha_A = 2.10 \) and \( \alpha_B = 1.90 \). These parameters are qualitatively in very good agreement with the results of Yıldız et al. (2006) from the binaries of the Hyades open cluster: the higher the mass, the greater the mixing-length parameter (case 1).

However, this relationship may have different forms as the stars evolve away from the ZAMS (see below). For all

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Figure 4. The solid line represents the stellar mass dependence of the mixing-length parameter given by Yıldız et al. (2006) for the Hyades stars. We try to derive more general expressions for \( \alpha \) in terms of the structure of the convective zones rather than the stellar mass. For two typical cases, we plot \( f_1 \) and \( f_2 \) as functions of some physical quantities of the convective zone (such as density and temperature at the base of convective zone and at the surface). See the text for the definitions of \( f_1 \) and \( f_2 \).

Figure 5. The time dependence of the mixing-length parameter for a Cen A predicted by \( f_1 \) (circles) and \( f_2 \) (stars).
models satisfying the seismic constraints, for example, contrary to case 1, $\alpha_A$ is less than $\alpha_B$ (case 2). In case 2, at least for $\alpha$ Cen A, $\alpha$ may be a decreasing function of time. Neither case 1 nor case 2 gives the same values for $\alpha_A$ and $\alpha_B$. This means that $\alpha$ is not a simple function of stellar mass and the mass dependence found by Yıldız et al. (2006) (see their figure 4) is a special case of a general situation about the ZAMS. So, we describe $\alpha$ as a function of some quantities pertaining to the convective zone of each model rather than stellar mass. In Fig. 4, we plot two such functions, $f_1$ (circles) and $f_2$ (stars), with respect to stellar mass:

\begin{align}
    f_1 &= 2.5 - \rho_{\text{bcz}} \left( \frac{2.7}{T_{\text{bcz}}} \right)^4 - \rho_{\text{ph}} \left( \frac{1.4}{T_{\text{ph}}} \right)^4, \\
    f_2 &= \left( \frac{M_{\text{bcz}}}{\rho_{\text{bcz}}} \right)^{0.5} \left( \frac{T_{\text{bcz}}}{3.8} \right)^4,
\end{align}

Here, the subscripts bcz and ph mean that the corresponding quantity pertain to the base of convective zone and the photosphere, respectively. $r_{\text{bcz}}$ is the radius of the base of the convective zone and $M_{\text{bcz}}$ is the mass inside the sphere with this radius. $r_{\text{bcz}}$ and $M_{\text{bcz}}$ are in solar units. The solid line in Fig. 4 represents the fitting curve $\alpha(M) = 9.19(M/M_{\odot} - 0.74)^{0.053} - 6.65$ derived by Yıldız et al. (2006). The three functions predict very similar values of $\alpha$ for a given mass. However, there are very important differences between these three functions. While $\alpha(M)$ is constant in time, $f_1$ and $f_2$ are implicitly functions of time because the structure of the convective zone changes in time.

In Fig. 5, $f_1$ (circles) and $f_2$ (stars) are plotted as a function of time for $\alpha$ Cen A (SISZ23 model). While $f_1$ is an increasing function of time, $f_2$ is a decreasing function. We also see how $f_1$ and $f_2$ vary in time for $\alpha$ Cen B. In Fig. 6, we see that both $f_1$ (circles) and $f_2$ (stars) are increasing functions of time and predict almost the same value for $\alpha$ Cen B for a given time.

5 CONCLUSIONS

The masses and radii of the components of $\alpha$ Cen, which are essential quantities for stellar modelling, are accurately determined from the observations. Using the photometric measurements of Bessel (1990) and the tables for colours and bolometric corrections of LCB1998, we find the absolute magnitudes of $\alpha$ Cen A and B and then their luminosities and surface heavy element abundances: $L_A = 1.544L_{\odot}$, $L_B = 0.507L_{\odot}$, $0.074$ dex and $0.125$ dex for the metallicities of $\alpha$ Cen A and B, respectively ( $Z_A = 0.0237$ and $Z_B = 0.0267$, taking the metallicity of the tables of colours and bolometric correction as $Z=0.02$). This difference between the metal abundances of $\alpha$ Cen A and B can be interpreted as the result of the diffusion process (settling), which is faster at the bottom of convective zone of $\alpha$ Cen A than $\alpha$ Cen B.

The calibration of models of $\alpha$ Cen A and B into these non-seismic constraints yields the age of the system as 8.88 Gyr. The mixing-length parameters for these models are $\alpha_A = 2.10$ and $\alpha_B = 1.90$. These values of $\alpha$ are qualitatively in very good agreement with the results of Yıldız et al. (2006) from the binaries of the Hyades open cluster: The higher the mass, the greater the mixing-length parameter. However, this relationship may have different forms as the stars evolve away from the ZAMS (see below). The seismic properties of these models are very different from the observations. By comparing small separations between the oscillations, we confirm that both models are much older than the observed stars.

The reason for such a great age for the system is that the observed luminosity of $\alpha$ Cen A is much greater than that of $\alpha$ Cen B according to their masses. Because $\alpha$ Cen A evolves faster than $\alpha$ Cen B, an old age is required for a simultaneous agreement between the models and observations. However, we should also question the accuracy of the observed values (e.g. the radii; see below).

Recently, del Peloso et al. (2006) determined the age of the galactic thin disc using Th/Eu cosmochemistry from the analysis of selected stars, including the components of $\alpha$ Cen, as 8.8 Gyr. This age is very close to the age we found from the models of $\alpha$ Cen A and B for non-seismic constraints.

According to seismic properties of its components, the $\alpha$ Cen system has a much smaller age than 8.8 Gyr; in fact, it is about 5.6-5.9 Gyr. With such an age, however, the luminosities of $\alpha$ Cen A and B can not simultaneously be fitted into the observed luminosities. Therefore, there is no constraint on luminosity of $\alpha$ Cen B in the calibration of these models. Because of this strategy, the model luminosities of $\alpha$ Cen B based on the seismic properties are 15 percent larger than the observed luminosity. To remove this discrepancy, either the mass of $\alpha$ Cen B can be reduced to 0.914 $M_{\odot}$ or its radius can be 7 per cent higher than the observed radius in Kervella et al. (2003). However, the discrepancy may be a result of some processes, such as rotational mixing, which are not involved in the model computations.

For all the models that also satisfy the seismic proper-
ties, in contrast to the models for non-seismic constraints, \( \alpha_B \) is greater than \( \alpha_A \). If this is the case, then the mixing-length parameter is not a simple function of stellar mass (Yıldız et al. 2006) at all evolutionary phases, but also a function of time. Therefore, we derive two expressions \( (f_1 \) and \( f_2; \) see Section 3.4) based on the structure of the convective zone of a given model and also compatible with the results in Yıldız et al. (2006) near the ZAMS. While one of the expressions gives \( \alpha_A \) as an increasing function of time, the other gives it as a decreasing function. Meanwhile, according to both expressions, \( \alpha_B \) increases. Such expressions may also explain why the mixing-length parameter varies for stars from phase to phase (see, for example, Ferraro et al. 2006).

We also compute small separations between the oscillation frequencies from the alternative expressions given by Roxburgh & Vorontsov (2003) for models and the observed seismic data. In this case, we obtain more complicated situation than in the classical seismic HR diagram. The small separation of models for \( \alpha \) Cen A computed from the expression given by Roxburgh & Vorontsov (2003) shows no explicit time dependence.

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