A Time-Domain Surface Grinding Model for Dynamic Simulation

Marco Leonesio\textsuperscript{a*}, Paolo Parenti\textsuperscript{b}, Alberto Cassinari\textsuperscript{a}, Giacomo Bianch\textsuperscript{a}, Michele Monno\textsuperscript{b}

\textsuperscript{a}CNR-Institute of Industrial Technology and Automation, Via Bassini 15, 20133, Milan
\textsuperscript{b}Politecnico of Milan, Dept. of Mechanical Engineering, Via La Masa 1, 20156, Milan

* Corresponding author. Tel.: +39-02-23699952; fax: +39-02-23699915. E-mail address: marco.leonesio@itia.cnr.it.

Abstract

The quality of a workpiece resulting from a grinding process is strongly influenced by the static and dynamic behavior of the mechanical system, composed by machine tool, wheel, fixture and workpiece. In particular, the dynamic compliance may cause vibrations leading to poor surface quality. In order to evaluate in advance the process performance in terms of surface quality, a simulation model for surface grinding has been developed, based on workpiece discretization by means of a z-buffer approach. The volume engaged by the wheel is associated to the grinding force by means of a variable specific energy that is a function of the equivalent chip thickness. The model is able to provide static and dynamic grinding force components taking into account the following aspects: nonlinearity of the grinding force with respect to cutting parameters, grinding damping effect, contact stiffness, machine-workpiece dynamics in all the relevant degrees of freedom (radial and tangential both for wheel and workpiece). The implementation in Matlab/Simulink\textsuperscript{™} environment allows an easy connection with any given mechatronic models of the grinding machine. Stable surface grinding tests with force measurements have been performed on a commercial CNC grinding machine for identifying the model parameters; then, the validation was extended to the dynamic case by introducing an artificial wheel unbalance.

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1. Introduction

Several models have been proposed in literature for the dynamic simulation of grinding, whose objective is the representation of the effect of workpiece-wheel relative vibrations.

In [1], a simulation tool was developed for traverse roll grinding, which allows the dynamic simulation of the process in time domain. In this program, the surface of workpiece and grinding wheel are represented by fields of discrete, equal spaced, supporting points. The supporting points enable the calculation conditions of contact in form of the intersection of workpiece and grinding wheel.

Another time-domain dynamic model was presented in [2], which simulates cylindrical plunge grinding processes under general grinding conditions. The model focuses on the prediction of grinding chatter boundaries and growth rates, taking into account several critical issues: the distributed nonlinear force along the contact length, the geometrical interaction between the wheel and workpiece based on their surface profiles, the structure dynamics with multiple degrees of freedom for both the wheel and workpiece, the response delay due to spindle nonlinearities and other effects, and the effect of the motion perpendicular to the normal direction.

A two-dimensional model for the grinding process with two mechanical degrees of freedom is presented in [3]. This model especially deals with the nonlinear grinding contact. It cares for system excitation by stochastically distributed grains as well as for an eccentricity of the wheel due to unbalance. The approach to grinding contact dynamics allows to simulate and study chatter vibrations, workpiece surfaces and wheel wear like they appear in experiment.
Besides the numerical approaches based on a workpiece-wheel discretization, pure analytical representation of process geometry and grinding forces have been used [4][5]. These analytical models are usually subject to strong assumptions that limit their generality: often they are used to get an insight to a particular phenomenon, but they results insufficient when trying to predict surface quality in real practical cases. In [5] the author showed how the onset of chaotic vibrations is possible under specific cutting condition; instead, the effect of torsional vibrations in dynamic instability occurrence is analyzed in [6].

The most of works present in literature usually show complex force models validated on simplified machine dynamics: typically, they are aimed at predicting chatter onset, and not at reproducing the obtained surface finish in time domain.

In this work, a model for surface grinding has been developed, based on workpiece discretization by means of a z-buffer approach. The material removal is computed numerically and it takes into account the workpiece waviness due to vibrations occurrence: vibrations may be due to wheel unbalance, external disturbances, workpiece regenerative chatter (if there are pass overlaps), or other dynamic instabilities related to all the relevant degrees of freedom (radial and tangential both for wheel and workpiece). The following aspects are taken into account: nonlinearity of the grinding force with respect to cutting parameters, wheel contact stiffness, and the measured machine-workpiece relative dynamics.

The force model, based on the specific energy, is relatively simple and is compliant with the most of literature [7]. Nevertheless, it suffices to predict grinding force evolution in time domain and surface geometry (including waviness) in presence of vibrations, after being calibrated with reduced set of stable grinding tests.

In section 2 the modeling approach is properly described, both in regard to the material removal computation and the force model. In section 3, the model identification procedure is outlined. The experimental campaign aimed to parameters identification and model validation is presented in section 4. Discussion and conclusions are exposed in section 5.

2. Model description

The grinding model receives as input the relative position of the wheel and workpiece along tangential, radial and traverse component, and it returns the tangential and radial force components, the wheel torque and the actual material removal rate (MRR).

The workpiece is discretized by means of a z-buffer representation (see Fig. 1): basically, it is regarded as a squared block of segments along infeed direction (Z).

The wheel is regarded as a perfect disc that engages the z-buffer determining an interference volume. The volume engaged at each time step yields the instantaneous material removal rate that is associated to the grinding force by means of a specific energy.

2.1. Force model

The overall grinding force is decomposed in its tangent and normal components with respect to a reference frame located at the ideal contact point between the wheel and the workpiece, where the tangent direction corresponds to the wheel cutting velocity and the normal direction corresponds to the radius connecting the point under consideration with the wheel center.

When considering the engagement zone on Fig. 2, the normal and tangent component is referred to each point on the arc of contact: tangential component is directed as the cutting velocity, while normal component is orthogonal to the cutting velocity itself.

Based on this latter definition, the local force model for tangential and normal cutting forces, needed to remove each volume element of the z-buffer, is based on an actual specific energy. Namely,

\[ f_{\text{t}} = \frac{E_c \cdot \text{MRR}}{V_r} \quad f_{\text{n}} = \mu f_{\text{t}} \quad (1) \]

where

- \( E_c \): nominal specific energy
- \( \text{MRR} \): material removal rate associated to the considered \( i^{th} \) volume element
- \( V_r \): wheel peripheral velocity
- \( \mu \): ratio between the normal and tangential force components

The “local” material removal rate is computed dividing the removed volume element by the simulation time step corresponding to the removal occurrence:
In order to take into account even rubbing and ploughing components, according to the most of literature, the actual specific energy varies with respect to the average chip thickness associated to each grit. Let $h_i$ be the average chip thickness cut by each grit in $\Delta V'$: the corrected specific energy can be estimated starting from the following relationship ([8] p. 12):

$$E_c \propto h_i^n \Rightarrow E_c = Ch_i^n$$  \hspace{1cm} (3)$$

where $C$ is a constant taking into account grit shape factor and density, and $n$ defines the exponential dependence of the actual specific energy on the chip thickness.

Usually, the chip thickness is estimated starting from the actual infeed value considering a nominal cutting geometry, that is the contact length, and the ratio between wheel and workpiece diameter. As the developed model is claimed to handle any engagement geometry, the chip thickness must be computed for each volume element.

A general expression yielding the average chip thickness for each volume element is derived here below: the infeed component of the volume element $a_i$ is divided by the number of grits that “sweep” the volume in the corresponding time step. Introducing the linear density of active grits $\lambda$, the volume element width $b$ and the discretization length $\Delta l$, it yields:

$$h_i = \frac{a_i}{\Delta V_{\text{in}}} = \frac{\Delta l}{b\Delta l\lambda V_{\text{in}}} = \frac{\Delta V'}{b\Delta l\lambda V'} = \frac{MRR}{b\Delta l\lambda V'}$$ \hspace{1cm} (4)$$

Substituting eq.(4) into eq.(3), the specific energy expression becomes:

$$E_c = K \left( \frac{MRR}{V'} \right)^n$$ \hspace{1cm} (5)$$

with

$$K = C \left( \frac{1}{b\Delta l\lambda} \right)^n$$

2.2. Engagement computation

The engagement condition of the wheel into the workpiece is traced back to the shortening of the $z$-buffer segments that interfere with the wheel. The segment reduction $\Delta z$ is computed for each time step knowing the actual and the previous $z$ coordinate of the intersection between the wheel and the $z$-buffer segments, see Fig. 2.

Starting from $z$-buffer segments reduction, a 1st order approximation is adopted for the computation of the elements volume. The engaged volume is approximated with a trapezium, that can be computed starting from segments reduction as it follows:

$$\Delta V' = \frac{\Delta z_i + \Delta z_{i+1}}{2} \cdot \Delta y \cdot \Delta x$$ \hspace{1cm} (6)$$

where:

- $\Delta z_i$ and $\Delta z_{i+1}$ are the reductions of two subsequent segments of the $z$-buffer;
- $\Delta y$ is the $z$-buffer discretization along feed direction;
- $\Delta x$ is the $z$-buffer axial discretization along traverse direction.

2.3. Local forces integration

The forces $f_i$ associated to the removed volume $\Delta V'$ should be applied at the center of force, whose position is determined solving the equilibrium of the volume element at its nodes. In order to simplify the computations, an approximation is introduced: $f_i$ is considered to be applied at the $i+1$ radius extreme (a null force is associated to the 1st radius).

Then, each is projected on the main reference frame (see Fig. 2):

$$f_{\text{TOT}} = \sum f_i R(\beta_{i+1})$$ \hspace{1cm} (7)$$

where $R(\beta_{i+1})$ is the rotation matrix between the local reference frame located at the extreme of radius $i+1$ and the global reference frame.

2.4. Contact stiffness

Due to normal force, the grinding grits are subject to a displacement that can be modelled by means of a so-called contact stiffness $K_c$, see [9][10]. In this model it is
assumed that each grain of the surface of the wheel is supported by a single linear spring [11]; however, a dependence on wheel-workpiece velocity ratio can be easily introduced, as suggested in [10]. The contact stiffness changes the actual depth of cut and, consequently, the volume element and the corresponding force. The compliance is due to a contact pressure that should be computed solving the equilibrium of all the volume elements, given the boundary conditions and the congruence equations for each node (i.e. segments extremes). Herein an approximation is introduced consisting in considering each segment as a linear spring compliant in the normal direction. Assuming that the infeed variation due to the local compliance is negligible enough to not affect the actual specific energy Ec, the grinding force becomes:

\[
f'_n = \frac{E_c}{V \Delta t} \mu \frac{\Delta z_0 + \Delta z_{11}}{2} \cdot \Delta y/\Delta x \tag{8}
\]

with:

\[
\Delta z_0 = \Delta z = \frac{f'_n}{K_c} \tag{9}
\]

Substituting eq.(9) into eq.(8), it yields:

\[
f'_n = \frac{f'_n^*}{1 + \frac{f'_n^*}{K_c \left( \frac{\Delta z_0 + \Delta z_{11}}{2} \right)}} \tag{10}
\]

where the ideal normal force \( f'_n^* \) is introduced representing the normal force obtained with an infinite contact stiffness.

### 2.5. Output filtering

The model includes only the machine dynamics relevant for the computation of the surface defects associated to vibration (i.e. waviness), that are restricted to a given bandwidth due to geometrical filtering.

On the other side, the effect of high frequency dynamics on the compliance at lower frequency is taken into account by a residual compliance term. In order to avoid numerical instability problem due to the residual compliance term, renouncing to represent the system response at high frequency, a discrete 1st order low pass filter has been introduced on all the model outputs. The time constant of the filter must be chosen to filter the numerical excitation without making any relevant machine-process dynamic: it implies that the workpiece discretization must be fine enough, i.e. it must satisfy the following relationship:

\[
\frac{v_w}{\Delta y} \gg \bar{\omega}
\]

where \( \bar{\omega} \) is the maximum machine resonance worth to be considered, \( v_w \) is the workpiece velocity and \( \Delta y \) is the z-buffer resolution along \( v_w \) direction.

### 3. Model parameters identification

The model parameters identification has been performed on “static” cutting tests i.e. characterized by a negligible level of vibrations, in order to simplify the experimental identification procedure. In such a condition the material removal rate is assumed to be well approximated by its nominal value:

\[
MRR = V_c a
\]

where \( a \) is the overall actual infeed.

Then, substituting eq.(5) in eq.(1) and posing \( \varepsilon = 1-n \), the model of the normal grinding force agrees to the following expression ([8], p. 172):

\[
F_n = K b \left( \frac{V_c}{V_a} \right)^\varepsilon \tag{13}
\]

In static condition, the model parameters \( K, \varepsilon \) and \( \mu \) can be identified on the basis of the average value of \( F_n \) and \( F_t \). Moreover, a logarithmic transformation is introduced to make the model linear and allow the exploitation of a simple linear regression; by applying the logarithm to the both terms of eq.(1) and eq.(2), it yields:

\[
\ln F_n = \ln K b + \varepsilon (\ln V_c - \ln V_a + \ln a) \tag{14}
\]

\[
\ln F_t = \ln K b - \ln \mu + \varepsilon (\ln V_c - \ln V_a + \ln a) \tag{15}
\]

The over-determined linear system to be solved for the parameters identification is written here below:

\[
\begin{bmatrix}
\ln F_n \\
\ln F_t
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & \ln V_c - \ln V_a + \ln a \\
1 & 1 & \ln V_c - \ln V_a + \ln a
\end{bmatrix}
\begin{bmatrix}
\ln K b \\
\ln \mu
\end{bmatrix}
\]

It has to be noted that the \( a_i \) values to feed the identification system must be actual, namely the nominal infeed must be depurated by the total machine and wheel compliance. In order to do that, the actual infeed was measured by a dial gauge after each pass (details are given in the following section).

Finally, a least squares solution can be found for eq.(16); then, \( K \) and \( \mu \) are obtained by an exponential anti-transformation.
4. Experimental campaign

The experimental campaign for model parameters identification and validation has been conducted on an Ermando Rosa Iron™ (see Fig. 3), with an aluminum oxide wheel, resin bonded, by Rappold™ (dimension: 400x57x127, code: 7A36IBJ15), characterized by a quite large grit size and a soft grade.

Machine dynamics in feed and infeed directions (Y and Z respectively, including the cross terms) have been characterized by hammer test at wheel hub. Then, a modal parameters identification has been performed using SDTools™ software package, obtaining a MIMO state-space model to import in Simulink™ environment.

Fig. 3. Machine used for the experimental campaign

The same identification provides the machine static stiffness value, that resulted in 30N/μm. At the same time, a finite element analysis of the contact between wheel and workpiece has been conducted to determine the wheel contact stiffness: the Young module of the wheel was matched with the frequencies resulting from a modal analysis in free-free. The contact stiffness was estimated in 200 N/μm, namely, much greater than the machine.

Then, an experimental plan has been conducted, adopting a grinding parameters set coherent with the machine specification. The parameters levels are reported in Table 1.

| Parameter Name | Level 1 | Level 2 | Level 3 | Level 4 |
|----------------|---------|---------|---------|---------|
| Infeed (mm)    | 0.005   | 0.01    | 0.015   | 0.02    |
| Table vel. (m/min) | 5    | 10     | 15     | 20     |
| Wheel vel. (rpm) | 1277  | 1322   | 1490   | -      |

As the velocity of the hydraulic table is regulated in open loop by means of a potentiometer, a wire tachometer was used to measure the actual velocity.

The grinding forces were measured by means of a Kistler™ piezo dynamometer 9255B connected to a National Instrument™ DAQ (NI9215). The workpiece was constituted by a block of low-carbon steel (Fe510 - EN10027) with more or less the same dimensions of the dynamometer (260x210x40mm). The lubricant fluid was a Castrol SYNTILO 81E (5%).

The tests consisted in single passes at the nominal grinding parameters established in the experiments plan. A reference surface is created, before each pass, by a complete grinding until spark-out. Before and after each pass the Z coordinate of the reference surface is measured in order to deduce the actual infeed necessary for the identification procedure.

4.1. Identification Results

The methodology outlined in Section 3 was applied to identify the model parameters, obtaining the values reported in Table 2:

Table 2. Identified parameters

| Parameter Name | Value 1 | Value 2 | Value 3 | Value 4 |
|----------------|---------|---------|---------|---------|
| K (J/mm³)      | 25.99   | 0.11569 | 1.8572  |         |

A comparison between simulated and experimental results is depicted in Fig. 4. The relative error on the mean forces value reaches the 25% in some experiments. It has to be observed that the force behavior is sometime characterized by an anomalous curved shape: a possible explanation of the phenomenon is concerned with the workpiece compliance, that may varies along the workpiece length; further investigations will be conducted to support this claim.

Fig. 4. Comparison between measured and simulated mean forces

4.2. Validation on the dynamic case

The model has been validated in the case of dynamic grinding, i.e. during vibrations onset. An artificial wheel unbalance has been introduced during the cutting tests in order to induce cutting vibrations: three values of
unbalance have been tested: 80/160/240 μm. The same disturbance has been simulated with the model obtaining the related force output in tangential and radial direction.

A result example is depicted in Fig. 5. The overall dynamic component of the measured force is higher than the simulated one due to unavoidable measure noise and other sources of excitation that are neglected in the model, like the interaction between grits and workpiece material.

Future experiments will be performed aiming at improving the capability of the model to predict instability occurrence due to wheel regeneration (wheel wear modeling) and other types of instabilities due to state-dependent, nonlinear force field (e.g. modes coupling instability, etc.).

Further developments will regard the integration of stochastic features in the force model, aiming at taking into account grits-workpiece interaction.

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References

[1] Weck M., Hennes N., Schulz A., Dynamic Behaviour of Cylindrical Traverse Grinding Processes, CIRP Annals - Manufacturing Technology 2001, 50 (1), 213-216.
[2] Hongqi Li., Shin Y.C., A Time-Domain Dynamic Model for Chatter Prediction of Cylindrical Plunge Grinding, Journal of Manufacturing Science and Engineering, 2006, 128(-),404-415.
[3] Biera J. et al., Time-domain dynamic modeling of the external plunge grinding process, International Journal of Machine Tools and Manufacture, 1997, 37(11),1555-1572.
[4] Hashimoto F., Kanai A., Growing Mechanism of Chatter Vibrations in Grinding Processes and Chatter, Annals of the CIRP, 1984,33(1).
[5] Stanescu N.D., Chaos in Grinding Process, transactions on applied and theoretical mechanics, 2009, 4(4).
[6] Mannan M.A. et al., Torsional vibration effects in grinding, CIRP Annals - Manufacturing Technology, 2000. 49(1), 249-252.
[7] Inasaki, I. et al., Grinding – Chatter Origin and suppression, CIRP Annals - Manufacturing Technology 2001,50(2), 515-534.
[8] Marinescu, I.D., Hitchiner M., Uhlmann E., Rowe B. W., Inasaki I, Handbook of Machining with grinding wheels, CRC Press, Taylor & Francis Book.
[9] Thompson, R., On the doubly regenerative stability of a grinder: the effect of contact stiffness and wave filtering. Journal of engineering for industry, 1992, 114.
[10] Ramos J.C., et al., A simplified methodology to determine the cutting stiffness and the contact stiffness in the plunge grinding process. International Journal of Machine Tools and Manufacture, 2001, 41(1), 33-49.
[11] Snoeys R., Wang I.C., Analysis of the static and dynamic stiffnesses of the grinding wheel surface, in: Proc. Of the 9th Int. Mach. Tool Des. and Res. Conf., Birmingham, 1968, pp. 1133–1148.