Applications of relativistic light front dynamics to computing the nucleonic and mesonic components of nuclear wave functions are reviewed. In this approach the fields are quantized at equal values of $\tau = x^0 + x^3$. Our method is to use a Lagrangian, and its associated energy-momentum tensor $T^{\tau \mu}$ to define the total momentum operators $P^\mu$ with $P^+$ as the plus-momentum and $P^-$ the $\tau$-development operator. The motivation for unusual treatment of nuclear physics is the desire to use wave functions, expressed in terms of plus-momentum variables, which are used to analyze high energy experiments such as deep inelastic scattering, Drell-Yan production, $(e,e')$ and $(p,p')$ reactions. This motivation is discussed, and a simple overview of light front dynamics is presented. Some examples of ordinary quantum mechanics are solved to show that the formalism is tractable. The necessary quantization is reviewed and applied to a series of problems: infinite nuclear matter within the mean field approximation; a simple static source theory; finite nuclei using the mean field approximation; low-energy pion-nucleon scattering using a chiral Lagrangian; nucleon-nucleon scattering, within the one boson exchange approximation; and, infinite nuclear matter including the effects of two-nucleon correlations. Standard good results for nuclear saturation properties are obtained, with a possible improvement in the lowered value, 180 MeV, of the computed nuclear compressibility. A complicating feature of our light front dynamics is that manifest rotational invariance is not used as an aid in doing calculations. But for each of the examples reviewed here, manifest rotational invariance emerges in the results of the calculations. Thus nuclear physics can be done in a manner in which modern nuclear dynamics is respected, boost invariance in the $z$-direction is preserved, and in which the rotational invariance so necessary to understanding the basic features of nuclei is maintained. A salient feature is that $\omega$, $\sigma$ and $\pi$ mesons are important constituents of nuclei. It seems possible to find Lagrangians that yield reasonable descriptions of nuclear deep inelastic scattering and Drell-Yan reactions. Furthermore, the presence of the $\sigma$ and $\omega$ mesons could provide a nuclear enhancement of the...
ratio of the cross sections for longitudinally and transversely polarized virtual photons in accord with recent measurements by the HERMES collaboration.
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References
I. INTRODUCTION AND MOTIVATION

The need for a relativistic methodology that is broadly applicable to nuclear physics has never been more apparent. One of our most important sets of problems involves understanding the transition between the hadronic (baryon, meson) and the underlying (quark, gluon) degrees of freedom. Using a relativistic formulation of the hadronic degrees of freedom is necessary to avoid a misinterpretation of a kinematic effect as a signal for the transition. In particular, the goal of understanding future high energy and momentum transfer studies of nuclear targets using exclusive, nearly exclusive or inclusive processes can only be met through using relativistic techniques. The light front approach of Dirac [1] in which the time variable is taken as \( t + z \) and the spatial variables are \( t - z, x, y \) [2]-[5], is one of the promising approaches because the momentum canonically conjugate to \( t - z \), \( p^+ \equiv p^0 + p^3 \), is directly related to the observables.

To be specific, consider the lepton-nucleus deep inelastic scattering observed by the EMC experiment [6,7]. This showed that there is a significant difference between the parton distributions of free nucleons and nucleons in a nucleus. This difference can interpreted as a shift in the momentum distribution of valence quarks towards smaller values of the Bjorken variable \( x \). In the parton model, \( x \) is the ratio of the plus-momentum \( k^+ = k^0 + k^3 \) of a quark to that of the target. If one uses \( k^+ \) as a momentum variable, the corresponding canonical spatial variable is \( x^- = x^0 - x^3 \) and the time variable is \( x^+ = x^0 + x^3 = \tau \) [3]. The \( \tau \)-development operator, which plays the role of the light front Hamiltonian is \( P^- = P^0 - P^3 \), in which \( P^\mu \) is the total momentum operator. In equal time dynamics, the eigenstates of the Hamiltonian \( P^0 \) are complete. In light front dynamics the eigenstates of \( P^- \) are complete:

\[
P^-|n\rangle = p^-_n |n\rangle
\]

\[
I = \sum_n |n\rangle\langle n|.
\]

To do calculations in this framework is to use light front formalism or light front dynamics. We have been attempting to derive the properties of nuclei using this light front formalism.

Light front dynamics applies to nucleons within the nucleus as well as to partons of the nucleons, and this is a useful approach whenever the momentum of initial or final state nucleons is large compared to their mass [4]. For example, this technique can be used for \((e,e'p)\) and \((p,2p)\) reactions at sufficiently high energies. It is important to realize that the mere use of light-front variables for nucleons in a nucleus is not sufficient to guarantee a reasonable result. Shouting relativity at a problem will not make it go away. It is also necessary to include all the relevant features of realistic conventional nuclear dynamics. Combining the two aspects of relativity and realistic physics provides the technical challenge addressed here.

A. An Example

A simple example involving quasi-elastic scattering of high energy electrons from nucleons in nuclei shows how using the light-front approach leads to important simplifications. To be specific, use coordinates in which the four-momentum \( q \) of the exchanged virtual photon
is \( \nu, 0, 0, -\sqrt{Q^2 + \nu^2} \). Here \( Q^2 = -q^2 \), and \( \nu^2 \) are both very large, but \( Q^2/\nu \) is finite (the Bjorken limit). Use the light-cone variables, so that \( q^+ \approx Q^2/2\nu \equiv Mx \) (\( M \) is the mass of a nucleon), \( q^- \approx 2\nu - Q^2/2\nu \), and \( q^- \gg q^+ \).

The scattering cross section for \( e + A \to e' + (A - 1)f + p \), where \( i, f \) represents the initial, final nuclear eigenstate of \( P^- \), and \( p \) the four-momentum of the final proton, takes the form

\[
\frac{d\sigma}{d^3p_f} \sim \sum_f \int d^4p \delta(p^2 - M^2) \delta^{(4)}(q + p_i - p_f - p) |\langle p, f | J(q) | i \rangle|^2 , \tag{1.3}
\]

with the operator \( J(q) \) as a schematic representation of the electromagnetic current. Kinematic factors and terms involving the cross section are ignored here in order to bring out the salient features. Performing the four-dimensional integral over \( p \) leads to the expression

\[
\frac{d\sigma}{d^3p_f} \sim \sum_f \int d^2p_f dp_f^+ \frac{dp_f^+}{p_f^+} \delta \left( (p_i - p_f + q)^2 - M^2 \right) |\langle p = p_i - p_f + q, f | J(q) | i \rangle|^2 . \tag{1.4}
\]

Equation (1.4) can be evaluated most easily by using a variable

\[
k \equiv p_i - p_f , \tag{1.5}
\]

with \( k \equiv (k^+, k_\perp) \), the momentum that the struck nucleon had in the initial state. The simplification emerges from the argument of the delta function, under the Bjorken limit. One finds:

\[
(p_i - p_f + q)^2 - M^2 \approx -Q^2 + q^- (p_i - p_f)^+ = -Q^2 + q^- k^+ . \tag{1.6}
\]

Using this approximation, and changing the variables of integration to \( k \) gives:

\[
\frac{d\sigma}{d^3p_f} \sim \sum_f \int d^2k \, dk^+ \frac{dk^+}{k^+} \delta \left( -Q^2 + q^- k^+ \right) |\langle p = k + q, f | J(q) | i \rangle|^2 . \tag{1.7}
\]

For large values of \( \nu \), one may use the single-nucleon approximation:

\[
|\langle p = k + q, f | J(q)i \rangle| = |\langle f | b_k | i \rangle| , \tag{1.8}
\]

where \( b_k \) is a nucleon destruction operator. Thus we find

\[
\frac{d\sigma}{d^3p_f} \sim \sum_f \int d^2k \, dk^+ \frac{dk^+}{k^+} \delta \left( -Q^2 + q^- k^+ \right) |\langle f | b_k | i \rangle|^2 . \tag{1.9}
\]

The important advantage is that \( p_f^- \) does not appear in the argument of the delta function, or anywhere else in Eq. (1.9). Thus the sum over intermediate states can be replaced by unity, using Eq. (1.2). In the usual equal-time representation, the argument of the delta function is \(-Q^2 + 2\nu(E_i - E_f)\). The energy of the final state appears, and one can not do the sum over states. Using light front dynamics, we re-write Eq. (1.9) as
\[ d\sigma \sim \int \frac{d^2k}{k^+} \frac{dk^+}{k^+} \delta(-Q^2 + q^- k^+) \langle i | b_k^\dagger b_k | i \rangle. \] (1.10)

Only a ground state matrix element is required; this is the simplification we seek. Then integration over \( k^+ \) and noting that the operator \( b_k^\dagger b_k \) is a number operator

\[ d\sigma \sim d^2k_\perp \rho(Mx, k_\perp), \] (1.11)

where \( \rho(Mx, k_\perp) = \langle i | b_{k^+=Mx, k_\perp}^\dagger b_{k^+=Mx, k_\perp} | i \rangle \) is the probability for a nucleon in the ground state to have a momentum \((Mx, k_\perp)\). Integration in Eq. (1.11) leads to

\[ \sigma \sim \int d^2k_\perp \rho(Mx, k_\perp) \equiv f(Mx), \] (1.12)

with \( f(Mx) \) as the probability for a nucleon in the ground state to have a plus momentum of \( Mx \), or the nucleon distribution function. The use of light-front dynamics to compute nuclear wave functions should allow first-principles calculations of \( f(Mx) \). Using light-front dynamics incorporates the experimentally relevant kinematics from the beginning, and therefore is the most efficient way to compute the cross sections for nuclear deep inelastic scattering and nuclear quasi-elastic scattering.

**B. An Apparent Puzzle**

Since much of this work is motivated by the desire to understand nuclear deep inelastic scattering and related experiments, it is worthwhile to review some of the features of the EMC effect \[6–8\]. One key experimental result is the suppression of the structure function for \( x \sim 0.5 \). This means that the valence quarks of bound nucleons carry less plus-momentum than those of free nucleons. This may be understood by postulating that mesons carry a larger fraction of the plus-momentum in the nucleus than in free space \[7,10\]. While such a model explains the shift in the valence distribution, one obtains at the same time a meson (i.e. anti-quark) distribution in the nucleus, which is strongly enhanced compared to free nucleons and which should be observable in Drell-Yan experiments \[11\]. However, no such enhancement has been observed experimentally \[12\], and this has been termed as a severe crisis for nuclear theory in Ref. \[13\].

**C. More Motivation**

The use of light-front dynamics should allow us to compute the necessary nuclear meson distribution functions using variables which are experimentally relevant. The need for a computation of such functions in a manner consistent with generally known properties of nuclei led me to try to compute nuclear properties using light front dynamics. There are other motivations for using the light front formalism that have been emphasized in many reviews \[4\]. One key feature is the belief (or hope) that the vacuum of the theory is trivial because it can not create pairs. Another is that the theory is a Hamiltonian theory and the many-body techniques of equal time theory can be used here too. I also quote the review by Geesaman et al. \[7\] “In light front dynamics LFD, the particles are on mass-shell, and there
are no off-shell ambiguities. However, ... we have little or no experience in calculating the wave function of a realistic nucleus in LFD”. The aim here is to provide such wave functions.

I also like to say: Ask not what the light front can do for nuclear physics; instead ask what nuclear physics can do for the light front. This is to provide a set of non-trivial four dimensional examples with real physics content.

There are have been several serious efforts devoted to solving QCD using light front dynamics [14–16]. This is proving be a formidable task. Much of the technical difficulties are associated with the vanishing gluon mass and the nearly vanishing values of the current quark masses. In equal time dynamics, the vacuum is known to contain both quark and gluon condensates, and recovering the physics of the complicated vacuum using light front dynamics requires a careful treatment of the constraint equations and the related zero-modes. This is discussed nicely in the conference proceeding [17]. See also the most recent review by Brodsky et al. [4].

As an example of the possible difficulties one faces in solving a gauge theory using light front quantization, recall the work of Casher [18]. As explained by Paston et al [20], Casher found that the use of the gauge \( A^+ = 0 \) is necessary. In that gauge the Feynman propagator has an extra power of the plus-momentum in the denominator, which strengthens the infrared singularities and introduces the need for a special regularization. One may ”cut out” the infrared regime momentum space, but this procedure breaks the Lorentz invariance until the regularization is removed. In principle, one can preserve the gauge invariance by using a discrete light cone quantization [4], but one also needs to keep the zero modes. This procedure typically introduces enormous complications. But not all theories are so complicated. In the case of Yukawa theory, which is very similar to our Lagrangians, the difficulty with the infrared regime as already been solved in an convincing manner Ref. [19].

The essential simplification encountered here is in our use of hadronic Lagrangians, which are not gauge theories. For example, consider that, in the theories we consider, all of the particles are massive. No one believes that the vacuum is a condensate of nucleon-anti-nucleon pairs. The vacuum really should be trivial. The implication is that the complications associated with zero modes can legitimately be ignored. Furthermore, the meson masses are much larger (\( \geq 135 \text{ MeV} \)) than typical energy scales (\( \sim 10 \text{ MeV} \)) of nuclear physics. This means that truncation of the Fock space to terms involving only 0,1 or 2 mesons should be a reliable approximation. Thus many of the technical difficulties of using light front dynamics are not present for hadronic Lagrangians. There is no reason to prevent light front dynamics from becoming a standard tool of relativistic nuclear physics.

D. Outline and Scope

The general goal is to provide a series of examples showing that light front dynamics can be used for high energy realistic and relativistic nuclear physics. The goal is to present the physics in as simplified form as possible.

We shall begin with a very brief overview of light front dynamics. Then some simple examples [21], requiring little technology are solved. The formal procedures of light front quantization of a hadronic Lagrangian \( \mathcal{L} \) are discussed next. Our first application of this quantization is a study of infinite nuclear matter within the mean field approximation [22].
The distribution functions $f(y)$ for nucleons and mesons are computed. The results are startling—the vector mesons are found to carry a substantial fraction of the nuclear plus-momentum. This finding is analyzed using a simple static model \[23\] in the following section. Then finite nuclei \[24\] are studied using the mean field approximation. Here one confronts the problem that the use of $x^- = t - z$ as a spatial variable violates manifest rotational invariance. We found \[24\] that rotational invariance re-emerges in the results, after one does the appropriate dynamical calculation. The desire for a realistic approach means that nucleon-nucleon correlations must be included and one must do physics beyond the mean field approximation. Chiral symmetry and a simple example involving pion-nucleon scattering is then discussed \[22\]. This is necessary preparation for the development of a new light-front one boson exchange potential \[25\]. The influence of nucleon-nucleon correlations on the properties of nuclear matter is studied by making the necessary light front calculations. Applications are to compute the nuclear pionic content and to nuclear deep inelastic scattering and Drell-Yan processes. This study \[25\] indicates that the apparent puzzle is not a real puzzle. Then some plans for future research are discussed. The impact of the calculations discussed here is that light front dynamics can be used for high energy realistic and relativistic nuclear physics. Reasonable results for standard properties are obtained. There is a strong desire for an experimental measurement of an observable that has an importance more readily appreciated and that is more easily computable using light front dynamics. As discussed, in Sect. 11, the discovery of the HERMES effect \[26\] that the ratio $R \equiv \sigma_L/\sigma_T$ is significantly enhanced in nuclei may provide such an observable.

The emphasis of this review is on the influence of light front quantization on nuclear physics. Hadronic Lagrangians are used. There are several excellent reviews of light front techniques \[15\], but these are mainly devoted to applications in particle physics \[14, 16\]. I am concerned here mainly with heavy nuclei. Applications of light front techniques to nuclear reactions have been pioneered by Frankfurt and Strikman \[2, 8\]. The present work reviews the first attempts, using realistic inputs, to apply light front techniques to the structure of heavy nuclei. This is the salient feature of the present review. However, it is worth mentioning the differences between the present and previous uses of light front techniques. Apart from Ref. \[27\], almost all of the early work \[2, 8, 28–31\] is devoted to the two-nucleon system, which is of tangential importance here. But this is only one difference. There are also differences in philosophy. Our approach is to choose a Lagrangian, and then determine its consequences for experimental observables. If the consequences are not palatable, we change the Lagrangian. The approach of \[28, 30\] relies on symmetries which are used to write down the allowed form of operators. This is not sufficient to determine uniquely, for example, all of the components of the electromagnetic current operator $J^\mu$. The approaches of \[31, 31\] try to restore manifest rotational invariance by introducing a general direction $\hat{n}$ to replace the three-direction. We expect, and show, that the final calculations of observables do respect this invariance. Our approach is closest to that of Refs. \[2, 8\], which is based on Feynman diagrams. The specification of a Lagrangian, should allow us to go further because all of the relevant diagrams can be defined.
II. WHAT IS LIGHT FRONT DYNAMICS?

This is a relativistic treatment of dynamics in which the fields are quantized at a fixed “time” $\tau = t + z = x^0 + x^3 \equiv x^+$. This means that the independent spatial variable must be $x^- \equiv t - z$ so that the canonical momentum is $p^0 + p^3 \equiv p^+$. The remainder of the spatial variables are given by $\vec{x}_\perp, \vec{p}_\perp$.

The consequence of using $\tau$ as a “time” variable is that the canonical energy is $p^- = p^0 - p^3$. Our notation for four-vectors is to use:

$$A^\pm \equiv A^0 \pm A^3,$$

with

$$A \cdot B = A^\mu B_\mu = \frac{1}{2} \left( A^+ B^- + A^- B^+ \right) - \vec{A}_\perp \cdot \vec{B}_\perp.$$

The key reason for using such unusual coordinates is phenomenological. For a particle with $\vec{v} \approx c \hat{e}_3$, the quantity $p^+$ is BIG. Thus experiments tend to measure quantities associated with $p^+$.

Another important feature is the relativistic dispersion relation $p^\mu p_\mu = m^2$, which in light front dynamics takes the form:

$$p^- = \frac{p^2_\perp + m^2}{p^+}.$$

Thus one has a form of relativistic kinematics that avoids using a square root. Eq. (2.3) is perhaps the most important equation in light front physics. Its consequences appear again and again in this review. If a reader, new to the light front, decides to learn only one of the many equations of this review—this should be the equation.

The main formal consequence of using light front dynamics is that the minus component of the total momentum, $P^-$, is used as a Hamiltonian operator, and the plus component $P^+$ is used as a momentum operator. These are obtained by using the energy-momentum tensor

$$T^{\mu\nu} = -g^{\mu\nu} \mathcal{L} + \frac{\partial \mathcal{L}}{\partial (\partial_\nu \phi_i)} \partial^\nu \phi_i,$$

in which the degrees of freedom are labeled as $\phi_i$. The term $T^{+\mu}$ is the density for the operator $P^\mu$, with

$$P^\mu = \frac{1}{2} \int d^2 x_\perp dx^- T^{+\mu}.$$

The use of a Lagrangian to provide these operators is the salient feature of our approach. The procedures to obtain these operators for the different Lagrangians are discussed below.
III. SIMPLE PROBLEMS

We introduce light front techniques by solving the wave equations for the three dimensional (scalar) harmonic oscillator, and the three dimensional Hulthén potential. We consider a situation in which a single nucleon moves under the influence of an external potential. This external potential arises from interactions with residual $(A-1)$-particle nucleus. The physics of the entire $A$-nucleon system is Lorentz invariant. In the present example the external potential is most simply expressed in the rest frame of the nucleus, for which it independent of time. For simplicity, we neglect the effects of spin and use the Klein-Gordon equation.

A. Three Dimensional Harmonic Oscillator

In ordinary coordinates, the Klein-Gordon equation for a 3 dimensional relativistic oscillator

$$E^2 \phi = \left[ \vec{p}^2 + m^2 + \kappa \vec{x}^2 \right] \phi$$

(3.1)

closely resembles the Schroedinger equation for a non-relativistic harmonic oscillator, and the energy eigenvalues are

$$E_n^2 = m^2 + \omega \left(n + \frac{3}{2}\right),$$

(3.2)

where $\omega^2 = 4\kappa$.

The light front version of the Helmholtz equation is obtained by changing variables to $x^\pm = x^0 \pm x^3$, with

$$\partial^\pm = 2 \frac{\partial}{\partial x^\pm}.$$ 

(3.3)

Then

$$\left( \partial^- \partial^+ - \nabla_\perp^2 + m^2 + \kappa (x_\perp^2 + (x^+ - x^-)^2/4) \right) \phi = 0.$$ 

(3.4)

A potential that is static in the equal time formulation corresponds to a LF-“time”, i.e. $x^+$, dependent potential in light-front coordinates. A static source in a rest-frame corresponds to a uniformly moving source on the light-front because the line $z = z_0$ corresponds to $x^\pm = t \pm z_0$ and $\Delta x^-/\Delta x^+ = 1$. Thus the time dependence is only due to a uniform translation, and it should be easy to transform Eq. (3.4) into a form which contains a potential that does not depend on $x^+$. For this purpose, we consider the equation of motion satisfied by fields obtained by an $x^+$ (LF-time) dependent translation

$$\phi(\vec{x}_\perp, x^-, x^+) \equiv e^{-ix^+P^+/2} \chi(\vec{x}_\perp, x^-, x^+)$$

(3.5)

Using $P^+ = -i2 \frac{\partial}{\partial x^-}$, we find
\[ e^{ix^+p^+/2} f \left( \frac{x^+-x^-}{2} \right) e^{-ix^+p^+/2} = f \left( \frac{-x^-}{2} \right) \]
\[ e^{ix^+p^+/2} \partial^- e^{-ix^+p^+/2} = \partial^- - \partial^+. \] (3.6)

The use of Eqs. (3.5) and (3.6) in Eq. (3.4) then leads to the result

\[ \left( (\partial^- - \partial^+)\partial^+ - \nabla^2_\perp + m^2 + \kappa (x^2_\perp + \left( \frac{x^-}{2} \right)^2) \right) \chi = 0. \] (3.7)

Note that the potential does not depend on \( x^+ \) and is instead a function of \( x^2_\perp + (\frac{x^-}{2})^2 \). The term \( (\frac{x^-}{2})^2 \) can be thought of as playing the role of \( z^2 \), so that the potential is actually spherically symmetric. Indeed, if we simply set \( x^+ = 0 \), then \( x^0 = -x^3 \) and \( x^- = -2x^3 = -2z \).

It is instructive to write the result (3.7) in an operator form:

\[ i\partial^- \chi = \left[ i\partial^+ + \frac{-\nabla^2_\perp + m^2}{i\partial^+} + \frac{1}{\sqrt{i\partial^+}} V \frac{1}{\sqrt{i\partial^+}} \right] \chi, \] (3.8)

where \( V \equiv \kappa (x^2_\perp + (\frac{x^-}{2})^2) \). This looks like an operator version of Eq. (2.3), but including an interaction and an additional \( p^+ \) operator which accounts for the recoil of the \( A - 1 \)-nucleon residual nucleus, see Sect. VII. The factors \( \frac{1}{\sqrt{i\partial^+}} \) are boson normalization factors; see Sect. IV.

The potential appearing in Eq. (3.7) does not depend on \( x^+ \) so that there will be solutions of the form:

\[ \chi(x) = e^{-ip^-x^+/2} \chi_n(x^-, \vec{x}_\perp). \] (3.9)

Using this and completing the square, leads to the result

\[ \left( -(\partial^+ + ip^-/2)^2 - \nabla^2_\perp + m^2 + \kappa (x^2_\perp + \left( \frac{x^-}{2} \right)^2) \right) \chi_n = (p^-_n/2)^2 \chi_n. \] (3.10)

One converts the operator \((\partial^+ + ip^-/2)^2\) to \((\partial^+)^2\) using yet another transformation:

\[ \chi(x^-, \vec{x}_\perp) = e^{-ip^-_nx^-/4} F(x^-, \vec{x}_\perp) \] (3.11)

to find

\[ \left( -(\partial^+)^2 - \nabla^2_\perp + m^2 + \kappa \left[ x^2_\perp + \left( \frac{x^-}{2} \right)^2 \right] \right) F = (p^-_n/2)^2 F. \] (3.12)

This is the same form as the equation in the equal time coordinates, and \( p^-_n/2 \) takes on the values of \( E_n \):

\[ (p^-_n/2)^2 = m^2 + \omega(n + 3/2). \] (3.13)

The light front version gives the same results for the eigenvalues as the ET development. So one might wonder why we are doing the light front at all. The point is that we are able to
compute the light front wave functions that depend on $x^-$, or in momentum space depend on $p^+$. The wave function of the ground state of Eq. (3.4) is given by

$$
\chi_0(x^-, \vec{x}_\perp) = e^{ip_0^- x^- / 4} N_0 \exp \left( -\frac{1}{2} \sqrt{\kappa} \left( x^2_\perp + \frac{|x^-|^2}{4} \right) \right).
$$

(3.14)

The number density $n_0(p_\perp, p^+)$ is defined as the square of the momentum space version of $\chi_0$. This quantity is accessible in high energy proton and electron nuclear quasi-elastic reactions. It is useful to define the light front variable

$$
\alpha \equiv \frac{p^+}{(p^-_0 / 2)}.
$$

(3.15)

For a shell-model nuclear target $p^-_0$ is a definite fraction of the total minus-momentum. In the rest frame, the minus-momentum is the same as the plus-momentum. Thus the variable $\alpha$ is a ratio of plus-momenta and $n_0(p_\perp, \alpha)$ is independent of frame.

The Fourier transform of Eq. (3.14) is obtained by multiplying by $e^{ip_\perp \cdot x_\perp} e^{-i\frac{p}{2} x^-}$ and doing the integral over all $x_\perp$ and $x^- / 2$. Then one determines that

$$
n_0(p_\perp, \alpha) = \tilde{N}_0 e^{-p_\perp^2 / 4\kappa} e^{-\frac{(p^-_0 (\alpha - 1))^2}{4\kappa}}.
$$

(3.16)

Note that one finds the same $p_\perp$ distribution for each value of the variable $\alpha$. This is not a general feature of light-front wave functions, as we show in the next sub-section.

In an exact calculation, the wave functions and number density should vanish for values of a plus-momentum fraction variable that are not between 0 and 1. This is referred to as having proper support or satisfying the spectrum condition. This property is not evident here because the system consists of one particle that interacts with an external source which supplies the external potential $V$ and is also responsible for the term $i\partial^+$ on the right hand side of Eq. (3.8). To better understand the support and the origin of the term $i\partial^+$, consider a two-particle problem in which a light particle (nucleon) of mass $m$ interacts with a heavy particle (residual nucleus) of mass $M$. The total four-momentum of the system is denoted as $P^\mu$. For a nucleus with $A$ nucleons, $M \approx (A - 1)m$ and $M \gg m$. This is a weak binding limit (binding energy per particle much less than the mass) appropriate for nuclear physics. The light particle is labeled as 1 and the heavy particle as 2. The plus-momentum fraction carried by the light particle is denoted as $\beta$ with

$$
\beta = \frac{p^+}{P^+},
$$

(3.17)

so that

$$
\frac{\beta}{\alpha} = \frac{p^-_0}{2P^+}.
$$

(3.18)

Using the given masses $m, M$ it will always be true that $\beta \ll 1$, and that $\alpha \sim 1$. The relative momentum is given by

$$
p_\perp = (1 - \beta) p_{1\perp} - \beta p_{2\perp},
$$

(3.19)
with
\[ p_\perp \approx p_{1\perp}. \] (3.20)

The use of these variables allows leads to a simple expression for the kinetic energy

\[ p_1^- + p_2^- = \frac{p_{1\perp}^2}{\beta(1 - \beta)P^+} + \frac{m^2}{\beta P^+} + \frac{M^2}{(1 - \beta)P^+} \] (3.21)
\[ \approx \frac{p_{1\perp}^2 + m^2}{p^+} + p^+ + \frac{M^2}{P^+}. \] (3.22)

in which the weak binding approximation \( \frac{M^2}{P^+} \to 1 \) is used. The right hand side of Eq. (3.22) is the kinetic part of the right hand side of Eq. (3.8). Thus the term \( i\partial^+ = p^+ \) represents the momentum of the heavy particle which supplies the external potential. The use of the kinetic energy operator of Eq. (3.21) instead of that of Eq. (3.8) would lead to obtaining exact support properties.

The approximation does not cause much trouble. For large values of the particles mass \( m, (m \gg \kappa^{1/4}) \) (which corresponds to the situation of nuclear physics in which the product of the nucleon mass and the nuclear radius is a very large number) the value of \( \alpha \) must always be close to unity. In nuclear physics \( \frac{m^2}{\sqrt{\kappa}} \) is of the order of \( (5R_A/Fm)^2 \) which taking \( R_A = 4 \text{ Fm} \), yields \( e^{-400} \) at \( x = 0 \). In that case, there is no problem with the support.

To provide an example, we consider the electromagnetic form factor of the ground state of a heavy nucleus. This has been measured, for many nuclei, in elastic electron scattering, and the sizes of nuclei have been determined as one of the classic achievements of nuclear physics. Interest in this topic has been revised because of a recent proposal to Jefferson Laboratory to use parity-violating electron scattering to measure the neutron radius \[32\]. Determining this to high precision is needed, and can be obtained provided one knows the proton distribution. Therefore it is useful to examine the influence of presumably small effects such as relativistic corrections. One works using a reference frame, the Drell-Yan frame \[33\] in which the plus component of the four vector \( q^\mu \) of the virtual photon vanishes, so that \( Q^2 = -q_\perp^2 = q_{1\perp}^2 \). Now consider the various momenta involved. Suppose initially, \( P_\perp = 0 \). If the light particle absorbs the virtual photon the final momenta are \( p_\perp + q_\perp \) for particle 1 and \( -p_\perp \) for particle 2. The relative momentum is then \( p_\perp + (1 - \beta)q_\perp \). In this case the form factor \( F(Q^2) \) (matrix element of the plus component of the electromagnetic current operator) is given by

\[ F(Q^2) = \int d^2p_\perp d\beta \beta \chi_0(\beta, p_\perp)\chi_0(\beta, p_\perp + (1 - \beta)q_\perp), \] (3.23)

in which the influence of relativity appears in the integral over \( \beta \) and the factor \( \beta \). For the harmonic oscillator ground state we find:

\[ F(Q^2) = N \int d^2p_\perp d\alpha d\epsilon e^{\left(-\frac{p_\perp^2}{\sqrt{\kappa}} + \frac{Q^2(1 - \beta)^2}{4\sqrt{\kappa}}\right)} e^{-\frac{p_\perp^2}{\sqrt{\kappa}}(\alpha - 1)^2}, \] (3.24)

where
\[
\frac{p_0^2}{4} = m^2 + 3\sqrt{\kappa}. 
\] (3.25)

Our purpose here is the study of nuclear physics, so we are interested in the non-relativistic limit and the corrections to it. To this end, we define a variable \( p_z \) using

\[
\alpha = 1 + \frac{p_z}{m}. 
\] (3.26)

The non-relativistic limit of (3.24) is obtained by letting \( m \) approach infinity, but with \( m/M \approx 1/(A - 1) \), and

\[
\beta = \frac{m}{M}\alpha. 
\] (3.27)

Then \( \frac{p_0^2}{4} = m^2 \) and we find

\[
F_{NR}(Q^2) = N_{NR} \int \frac{d^3p}{m} e^{-\left(\frac{p_z^2}{\sqrt{\kappa}} + \frac{Q^2(1 - m/M)^2}{4\sqrt{\kappa}}\right)} 
= e^{-\frac{Q^2(1 - m/M)^2}{4\sqrt{\kappa}}}. 
\] (3.28)

The mean square radius \( -6 \frac{dF(Q^2)}{dQ^2} |_{Q^2=0} \), is given by

\[
R_{NR}^2 = \frac{3}{2} \frac{(1 - m/M)^2}{\sqrt{\kappa}}. 
\] (3.29)

The experimental value of the nuclear radius is given approximately by

\[
R_{exp} = 1.1 A^{1/3} \text{ fm}. 
\] (3.30)

The leading correction to this comes from the product of the factor \( \frac{p_z}{m} \) in the integrand of Eq. (3.24) with a \( \frac{p_z}{M} \) which comes from a term in the exponent. This is correction is of order \( p_z^2/mM \sim \sqrt{\kappa}/mM \). We define a semi-relativistic limit \( SR \) via the use of (3.24) and keeping the leading correction terms. Performing the straightforward evaluations, using \( R_{NR} = R_{exp} \), leads to the result:

\[
\delta \equiv \frac{R_{SR}^2 - R_{NR}^2}{R_{NR}^2} = \frac{\sqrt{\kappa}}{mM}, 
\] (3.31)

or using Eq. (3.30)

\[
\delta \approx \frac{0.055}{A^{2/3}(A - 1)}. 
\] (3.32)

This corresponds to very small \( (8 \times 10^{-6}) \) effects for large nuclei \( A \sim 200 \).
B. Light Front Hulthén Wave Function

Any static potential of the form $V(\vec{x}^2 = x^2 + z^2)$ can be solved on the light front. The transformation (3.3) corresponds to including the $p^+$ term in the $x^+$ development operator and a simple prescription of replacing $z$ by $-x^-/2$ in $V$. We present here the solution for the Hulthén potential. This allows us to demonstrate an interesting contrast between the implications of different forms of potentials. In the equal time formulation we have the wave equation:

$$E^2 \phi = [\vec{p}^2 + m^2 + V^H(\vec{x}^2)] \phi,$$  \hspace{1cm} (3.33)

in which $V^H$ is chosen so that the lowest energy solution is

$$\phi(r) = N(e^{-ar} - e^{-br}),$$  \hspace{1cm} (3.34)

where $b > a$. The eigenenergy is given by

$$E = \sqrt{m^2 - a^2}.$$  \hspace{1cm} (3.35)

The light front version of the wave equation is obtained in the same manner as Eq. (3.12), with the result

$$\frac{p_n^2}{4} \chi(x^-, \vec{x}_-) = \left[\left(-2i \frac{\partial}{\partial x^-}\right)^2 + p_\perp^2 + V^H \left(x_\perp^2 + \frac{x^-^2}{4}\right)\right] \chi(x^-, \vec{x}_-).$$  \hspace{1cm} (3.36)

The lowest value of $p_n^-/2$ is clearly the same as $E$ of Eq. (3.33), and the ground state wave function is given by

$$\chi_0(x^-, \vec{x}_-) = e^{ip_\perp x^- / 4} N_0^H \left[\exp\left(-a\sqrt{x_\perp^2 + \frac{x^-^2}{4}}\right) - \exp\left(-b\sqrt{x_\perp^2 + \frac{x^-^2}{4}}\right)\right].$$  \hspace{1cm} (3.37)

The momentum distribution $n_0^H(p_\perp, p^+)$ obtained here provides an interesting contrast with that of the harmonic oscillator (3.16). We find

$$n_0^H(p_\perp, \alpha) = N_0^H \left[\frac{1}{(a^2 + p_\perp^2 + (p^-_n/2)^2(\alpha - 1)^2)^2} - \frac{1}{(b^2 + p_\perp^2 + (p^-_n/2)^2(\alpha - 1)^2)^2}\right].$$  \hspace{1cm} (3.38)

There is a different $p_\perp$ distribution for each value of $\alpha$, with broader functions of $p_\perp$ obtained for smaller values of $\alpha$; see Fig. 1. The results in the figure are obtained using $m=.94$ GeV, $E=.932$ GeV and $b = 5a$. An experimental hint of such a behavior has been found recently [34].

IV. LIGHT FRONT QUANTIZATION

I discuss the basic aspects in an informal way as possible. For more details, see the reviews and the references. It’s easiest to start by considering one free field at a time. These are the scalar meson $\phi$, the Dirac fermion $\psi$ and the massive vector meson $V^\mu$. 

15
A. Free Scalar field

Consider the Lagrangian

\[ L_\phi = \frac{1}{2}(\partial^+ \phi \partial^- \phi - \nabla_\perp \phi \cdot \nabla_\perp \phi - m_s^2 \phi^2) \]  

(4.1)

The notation is such that \( \partial^\pm = \partial^0 \pm \partial^3 = 2 \frac{\partial}{\partial x^\pm} \). The Euler-Lagrange equation leads to the wave equation

\[ i\partial^- \phi = \frac{-\nabla_\perp^2 + m_s^2}{i\partial^+} \phi. \]  

(4.2)

The most general solution is a superposition of plane waves:

\[ \phi(x) = \int \frac{d^2 k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2} \sqrt{2k^+}} \left[ a(k)e^{-ik^+x} + a^\dagger(k)e^{ik^+x} \right], \]  

(4.3)

where \( k \cdot x = \frac{1}{2}(k^- x^+ + k^+ x^-) - k_\perp \cdot x_\perp \) with \( k^- = \frac{k^2 + m^2}{k^+} \), and \( k \equiv (k^+, k_\perp) \). The \( \theta \) function restricts \( k^+ \) to positive values. Note that

\[ i\partial^+ e^{-ik^+x} = k^+ e^{-ik^+x}. \]  

(4.4)

The value of \( x^+ \) that appears in Eq. (4.3) can be set to zero, but only after taking necessary derivatives.

Deriving the equal \( x^+ \) commutation relations for the fields is a somewhat obscure procedure [35–37], but the result can be stated in terms of familiar commutation relations:

\[ [a(k), a^\dagger(k')] = \delta(k_\perp - k'_\perp) \delta(k^+ - k'^+) \]  

(4.5)
with $[a(k), a(k')] = 0$.

It is interesting [5] to consider the commutation relations between fields at the same value of light front time:

$$[\phi(x), \phi(y)]_{x^+ = y^+} = \frac{-i}{4} \epsilon(x^- - y^-)\delta^{(2)}(x_\perp - y_\perp), \quad (4.6)$$

where $\epsilon(x) = \theta(x) - \theta(-x)$. This commutator, evaluated for $x^0 = y^0$, vanishes because the separation between $x$ and $y$ would necessarily be space-like. However, for $x^+ = y^+$ the separation is $-(x_\perp - y_\perp)^2$ which vanishes for $x_\perp = y_\perp$.

The next step is to compute the Hamiltonian $P^-$ for this system. The conserved energy-momentum tensor is given in terms of the Lagrangian:

$$T^{\mu\nu}_\phi = -g^{\mu\nu}\mathcal{L}_\phi + \frac{\partial\mathcal{L}_\phi}{\partial(\partial_\mu\phi)}\partial^\nu\phi. \quad (4.7)$$

This brings us to the question of what is $g^{\mu\nu}$? This is straightforward, although the results (viewed for the first time) can be surprising:

$$g^{++} = g^{00} + g^{03} + g^{30} + g^{33} = 1 + 0 + 0 - 1 = 0$$

$$g^{ij} = -\delta_{ij}(i = 1, 2, j = 1, 2); \quad g^{+-} = g^{-+} = 2. \quad (4.9)$$

Then one finds that

$$T^{+-}_\phi = \nabla_\perp \phi \cdot \nabla_\perp \phi + m_s^2\phi^2. \quad (4.10)$$

In general, the term $T^{+-}$ is the density for the operator $P^-$:

$$P^- = \frac{1}{2} \int d^2x_\perp dx^- T^{+-}. \quad (4.11)$$

The use of the field expansion (4.3), along with normal ordering and integration leads to the result:

$$P^- = \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(k)a(k) \frac{k^2 + m_s^2}{k^+}. \quad (4.12)$$

One defines a vacuum state $|0\rangle$ such that $a(p) |0\rangle = 0$. Then the creation operators acting on the vacuum give the usual single particle states:

$$P^- a^\dagger(p) |0\rangle = \frac{p^2 + m_s^2}{p^+}a^\dagger(p) |0\rangle. \quad (4.13)$$

The momentum operator $P^+$ is constructed by integrating $T^{++}$:

$$P^+_\phi = \int d^2k_\perp dk^+ \theta(k^+) a^\dagger(k)a(k)k^+. \quad (4.14)$$
It is interesting to consider how the results for $P^-$ are changed if interactions are included. If we add an interaction term of the form $\frac{1}{2}\dot{\phi}V\phi$ one would get an equation for $P^-$ of the form of Eq. (3.8), but without the term $i\partial^+\psi$ on the right hand side.

Another consideration involves the vacuum. Suppose we take the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi \partial^\mu \phi - m^2_\phi \phi^2) + \lambda \phi^4.$$  \hspace{1cm} (4.15)

The operator $\phi$ creates or destroys a particles of plus-momenta $k^+ > 0$. Thus possible terms in which $\lambda \phi^4$ term converts the vacuum $|0\rangle$ into a four particle state vanishes by virtue of the conservation of plus-momentum. The vacuum of $p^+ = 0$ can not be connected to four particles, each having a positive $k^+$. This vanishing simplifies Hamiltonian ($x^+$-ordered perturbation) calculations. However, one must be careful about physics involving $k^+ = 0$. For example, Burkardt [38] constructs non-vanishing perturbative corrections to the self energy of the scalar meson which do involve the spontaneous creation of four scalar mesons from the vacuum. For a detailed discussions, see Ref. [17].

### B. Free Dirac Field

Consider the Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}(\gamma^\mu \frac{i}{2} \partial_\mu - M)\psi,$$  \hspace{1cm} (4.16)

and its equation of motion:

$$(i\gamma^\mu \partial_\mu - M)\psi = 0.$$  \hspace{1cm} (4.17)

A fermion has spin $1/2$, so there can only be two independent degrees of freedom. The standard Dirac spinor has four components, so two of these must represent dependent degrees of freedom. In the light front formalism one separates the independent and dependent degrees of freedom by using projection operators: $\Lambda_\pm \equiv \frac{1}{2}\gamma^0 \gamma^\pm$. Then the independent field is $\psi_+ = \Lambda_+ \psi$ and the dependent one is $\psi_- = \Lambda_- \psi$.

The Dirac equation (4.17) is re-written as

$$\left(\frac{i}{2} \gamma^+ \partial^- + \frac{i}{2} \gamma^- \partial^+ + i\gamma_\perp \cdot \nabla_\perp - M\right) \psi = 0.$$  \hspace{1cm} (4.18)

Equations for $\psi_\pm$ can be obtained by multiplying Eq. (4.18) on the left by $\Lambda_\pm$:

$$i\partial^- \psi_+ = (\alpha_\perp \cdot \frac{\nabla_\perp}{i} + \beta M)\psi_-\hspace{1cm} \text{and} \hspace{1cm} i\partial^+ \psi_- = (\alpha_\perp \cdot \frac{\nabla_\perp}{i} + \beta M)\psi_+,$$  \hspace{1cm} (4.19)

so that the equation of motion of $\psi_+$ becomes

$$i\partial^- \psi_+ = (\alpha_\perp \cdot \frac{\nabla_\perp}{i} + \beta M) \frac{1}{i\partial^+}(\alpha_\perp \cdot \frac{\nabla_\perp}{i} + \beta M)\psi_+.$$  \hspace{1cm} (4.20)
One can make the field expansion and determine the momenta in a manner similar to
the previous section. The key results are

\[ T^+ - \psi^+ = \psi^+ \left( \alpha_\perp \cdot \frac{\nabla_\perp}{i} + \beta M \right) \frac{1}{i\partial^+} \left( \alpha_\perp \cdot \frac{\nabla_\perp}{i} + \beta M \right) \psi^+, \quad (4.21) \]

\[ P^{-} = \sum_\lambda \int d^2 p_\perp dp^+ \theta(p^+) \frac{p_\perp^2 + M^2}{p^+} \left[ b^\dagger(p, \lambda) b(p, \lambda) + d^\dagger(p, \lambda) d(p, \lambda) \right], \quad (4.22) \]

where \( b(p, \lambda), d(p, \lambda) \) are nucleon and anti-nucleon destruction operators.

C. Free Vector Meson

The formalism for massive vector mesons was worked out by Soper \[39\] and later by Yan \[40\] using a different formulation. I generally follow Yan’s approach. The formalism in the references is lengthy and detailed in the references, so I try to state the minimum. There are three independent degrees of freedom, even though the Lagrangian depends on \( V^\mu \) and \( V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu \). These are chosen to be \( V^+ \) and \( V^{+i} \). The other terms \( V^-, V^i, V^{-i} \) and \( V^{ij} \) can be written in terms of \( V^+ \) and \( V^{+i} \).

The expression for the vector meson field operator is

\[ V^\mu(x) = \int \frac{d^2 k_\perp dk^+ \theta(k^+)}{(2\pi)^{3/2} \sqrt{2k^+}} \sum_{\omega=1,3} \epsilon^\mu(k, \omega) \left[ a(k, \omega) e^{-ik^\perp \cdot x} + a^\dagger(k, \omega) e^{ik^\perp \cdot x} \right], \quad (4.23) \]

where the polarization vectors are the usual ones:

\[ k^\mu \epsilon_\mu(k, \omega) = 0, \quad \epsilon^\mu(k, \omega) \epsilon_\mu(k, \omega') = -\delta_{\omega\omega'}, \]

\[ \sum_{\omega=1,3} \epsilon^\mu(k, \omega) \epsilon^\nu(k, \omega) = -(g^{\mu\nu} - \frac{k^\mu k^\nu}{m_v^2}). \quad (4.24) \]

Once again the four momenta are on-shell, with \( k^- = \frac{k^2 + m_v^2}{k^+} \). The commutation relations are

\[ [a(k, \omega), a^\dagger(k', \omega')] = \delta_{\omega\omega'} \delta^{(2+)}(k - k'), \quad (4.25) \]

with \([a(k, \omega), a(k', \omega')] = 0\), and lead to commutation relations amongst the field operators that are the same as in Ref. \[40\].

D. We need a Lagrangian no matter how bad

It seems to me that one can not do complete dynamical calculations using the light
front formalism without specifying some Lagrangian. One starts \[22\] with \( L \) and derives
field equations. These are used to express the dependent degrees of freedom in terms of
independent ones. One also uses \( L \) to derive \( T^{\mu\nu} \) (as a function of independent degrees of
freedom) which is used to obtain the total momentum operators \( P^\pm \). It is \( P^- \) that acts as
a Hamiltonian operator in the light front \( x^+ \)-ordered perturbation theory.
We start with a Lagrangian containing scalar and vector mesons and nucleons $\psi'$. This is the minimal Lagrangian for obtaining a caricature of nuclear physics because the exchange of scalar mesons provides a medium range attraction which can bind the nucleons and the exchange of vector mesons provides the short-range repulsion which prevents a collapse. It is also useful, for phenomenological purposes, to include scalar meson self-coupling terms. Thus we take

$$
\mathcal{L} = \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2) - \frac{\kappa}{3!} \phi^3 - \frac{\lambda}{4!} \phi^4 - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{m_v^2}{2} \phi^2 
$$

$$
+ \bar{\psi}' \left( \gamma^\mu \left( \frac{i}{2} \partial_\mu - g_v V_\mu \right) - M - g_s \phi \right) \psi',
$$

with the effects of other mesons included elsewhere and below. The equations of motion are

$$
\partial_\mu V^{\mu} + m_v^2 \phi = g_v \bar{\psi}' \gamma^\mu \psi'
$$

$$
\partial_\mu \partial^\mu \phi + m_s^2 \phi = -g_s \bar{\psi}' \phi',
$$

$$
(i \partial^- - g_v V^-) \psi'_+ = (\alpha_\perp \cdot (p_\perp - g_v \bar{V}_\perp) + \beta (M + g_s \phi)) \psi'_-
$$

$$
(i \partial^+ - g_v V^+) \psi'_- = (\alpha_\perp \cdot (p_\perp - g_v \bar{V}_\perp) + \beta (M + g_s \phi)) \psi'_+.
$$

The presence of the interaction term $V^+$ on the left-hand side of Eq. (4.31) presents a problem because one can not easily solve for $\psi_-$ in terms of $\psi_+$. This difficulty is handled by using the Soper-Yan transformation:

$$
\psi' = e^{-i g_s \Lambda(x)} \psi, \quad \partial^+ \Lambda = V^+.
$$

Using this in Eqs. (4.29)-(4.30) leads to the more usable form

$$
(i \partial^- - g_v \bar{V}^-) \psi'_+ = (\alpha_\perp \cdot (p_\perp - g_v \bar{V}_\perp) + \beta (M + g_s \phi)) \psi'_-
$$

$$
i \partial^+ \psi'_- = (\alpha_\perp \cdot (p_\perp - g_v \bar{V}_\perp) + \beta (M + g_s \phi)) \psi'_+.
$$

The cost of the transformation is that one gets new terms resulting from taking derivatives of $\Lambda(x)$. One uses $\bar{V}^\mu$ with

$$
\bar{V}^\mu = V^\mu - \frac{1}{\partial^+} \partial^\mu V^+, \quad \partial^+ \Lambda = V^+.
$$

and $\bar{V}^\mu$ enters in the nucleon field equations, but $V^\mu$ enters in the meson field equations.

We also need the eigenmode expansion for $\bar{V}^\mu$. This is given by

$$
\bar{V}^\mu(x) = \int \frac{dk^+}{(2\pi)^{3/2}} \frac{\theta(k^+)}{\sqrt{2k^+}} \sum_{\omega=1,3} \bar{\epsilon}^\mu(k,\omega) \left[ a(k,\omega)e^{-ik\cdot x} + a^\dagger(k,\omega)e^{ik\cdot x} \right],
$$

where, using Eqs.(4.31) and (4.23), the polarization vectors $\bar{\epsilon}^\mu(k,\omega)$ are

$$
\bar{\epsilon}^\mu(k,\omega) = \epsilon^\mu(k,\omega) - \frac{k^\mu}{k^+} \epsilon^+(k,\omega).
$$

Note that
\[ \sum_{\omega=1,3} \epsilon^\mu(\mathbf{k}, \omega) \epsilon^\nu(\mathbf{k}, \omega) = -(g^{\mu\nu} - g^{+\mu} \frac{k^\nu}{k^+} - g^{+\nu} \frac{k^\mu}{k^+}). \]  

(4.36)

Then we may construct the total four-momentum operator from

\[ P^\mu = \frac{1}{2} \int dx^- d^2x_\perp T^{+\mu}(x^+ = 0, x^-, x_\perp), \]  

(4.37)

with

\[ T^{\mu\nu} = -g^{\mu\nu} L + \sum_r \frac{\partial L}{\partial (\partial_\mu \phi_r)} \partial^\nu \phi_r, \]  

(4.38)

in which the degrees of freedom are labeled by \( \phi_r \). We need \( T^{++} \) and \( T^{+-} \), which are given by

\[ T^{++} = \partial^+ \phi \partial^+ \phi + V^{i\mu} V^{i\mu} + m_v^2 V_\perp V_\perp + 2 \psi_+^\dagger \partial^+ \psi_+, \]  

(4.39)

and

\[ T^{+-} = \nabla_\perp \phi \cdot \nabla_\perp \phi + m^2_\phi \phi^2 + 2 \left( \frac{\kappa}{3!} \phi^3 + \frac{\lambda}{4!} \phi^4 \right) + \frac{1}{4} (V^{+-})^2 + \frac{1}{2} V^{kl} V^{kl} + m^2 \phi \phi \]  

\[ + \bar{\psi} \left( \gamma_\perp \cdot (p_\perp - g_v \bar{V}_\perp) + M + g_s \phi \right) \psi. \]  

(4.40)

This form is still not useful for calculations because the constrained field \( \psi_- \) contains interactions. We follow Refs. [39,41] in expressing \( \psi_- \) as a sum of terms, one \( \xi_- \) whose relation with \( \psi_+ \) is free of interactions, the other \( \eta_- \) containing the interactions. That is, rewrite the second of Eq. (4.32) as

\[ \xi_- = \frac{1}{i \partial^+}(\alpha_\perp \cdot p_\perp + \beta M)\psi_+ \]  

\[ \eta_- = \frac{1}{i \partial^+}(-\alpha_\perp \cdot g_v \bar{V}_\perp + \beta g_s \phi)\psi_+. \]  

(4.41)

Furthermore, define \( \xi_+(x) \equiv \psi_+(x) \), so that

\[ \psi(x) = \xi(x) + \eta_-(x), \]  

(4.42)

where \( \xi(x) \equiv \xi_-(x) + \xi_+(x) \). This separates the dependent and independent parts of \( \psi \).

One needs to make a similar treatment for the vector meson fields. The operator \( V^{+-} \), is determined by

\[ V^{+-} = \frac{2}{\partial^+} \left[ g_v J^+ - m_v^2 V^+ - \partial_i V^{i+} \right]. \]  

(4.43)

Part of this operator is determined by a constraint equation, because the independent variables are \( V^+ \) and \( V^{+i} \). To see this examine Eq (4.43), and make a definition

\[ V^{+-} = v^{+-} + \omega^{+-}, \]  

(4.44)
where
\[ \omega^{+-} = -\frac{2}{\partial^+} J^+. \] (4.45)

The sum of the last term of Eq. (4.40) and the terms involving \( \omega^{+-} \) is the interaction density. Then one may use Eqs. (4.40), (4.42), and (4.44) to rewrite the \( P^- \) as a sum of different terms, with
\[ P_{0N}^- = \frac{1}{2} \int d^2 x_\perp dx^- \bar{\xi} (\gamma_\perp \cdot \mathbf{p}_\perp + M) \xi, \] (4.46)
and the interactions
\[ P_I^- = v_1 + v_2 + v_3, \] (4.47)
with
\[ v_1 = \int d^2 x_\perp dx^- \bar{\xi} (g_v \gamma \cdot \bar{V} + g_s \phi) \xi, \] (4.48)
\[ v_2 = \int d^2 x_\perp dx^- \bar{\xi} \left( -g_v \gamma \cdot \bar{V} + g_s \phi \right) \frac{\gamma^+}{2i\partial^+} \left( -g_v \gamma \cdot \bar{V} + g_s \phi \right) \xi, \] (4.49)
and
\[ v_3 = \frac{g_v^2}{8} \int d^2 x_\perp dx^- \int dy_1^- \epsilon(x^- - y_1^-) \xi_\perp^\dagger (y_1^- , \mathbf{x}_\perp) \gamma^+ \xi_\perp (y_1^-, \mathbf{x}_\perp) \nonumber \times \int dy_2^- \epsilon(x^- - y_2^-) \xi_\perp^\dagger (y_2^- , \mathbf{x}_\perp) \gamma^+ \xi_\perp (y_2^- , \mathbf{x}_\perp), \] (4.50)

where \( \epsilon(x) \equiv \theta(x) - \theta(-x) \). The term \( v_1 \) accounts for the emission or absorption of a single vector or scalar meson. The term \( v_2 \) includes contact terms in which there is propagation of an instantaneous fermion. The term \( v_3 \) accounts for the propagation of an instantaneous vector meson.

Our variational procedure will involve the independent fields \( \psi_\perp \), so we need to express the interactions \( P_{0N}^- \) and \( v_{1,2} \) in terms of \( \xi_\perp \). A bit of Dirac algebra shows that
\[ P^-_\perp \equiv P^-_{0N} + v_1 + v_2 \]
\[ = \int d^2 x_\perp \frac{dx^-}{2} \xi_\perp^\dagger \left[ 2g_v \bar{V}^- \right. \nonumber + \left. \left( \alpha_\perp \cdot (\mathbf{p}_\perp - g_v \bar{V}_\perp) + \beta(M + g_s \phi) \right) \right] \frac{1}{i\partial^+} \left( \alpha_\perp \cdot (\mathbf{p}_\perp - g_v \bar{V}_\perp) + \beta(M + g_s \phi) \right) \xi_\perp. \] (4.51)

It is worthwhile to define the contributions to \( P^\pm \) arising from the mesonic terms as \( P^\pm_s \) and \( P^\pm_v \). Then one may use Eqs. (4.40) and (4.39) along with the field expansions to obtain
\[ P^-_s = \frac{1}{2} \int d^2 x_\perp dx^- \left( \nabla_\perp \phi \cdot \nabla_\perp \phi + m^2_s \phi^2 \right) \]
\[ = \int d^2 k_\perp dk^+ \theta(k^+) a^\dagger(k) a(k) \frac{k^2}{k^+} + m^2_s, \] (4.52)
\[ P_s^+ = \int d^2k_\perp dk^+ \theta(k^+)a^\dagger(k)a(k)k^+, \quad (4.53) \]

\[ P_v^- = \sum_{\omega=1,3} \int d^2k_\perp dk^+ \theta(k^+) \frac{k^2 + m_v^2}{k^+} a^\dagger(k,\omega)a(k,\omega) + v_3, \quad (4.54) \]

and

\[ P_v^+ = \sum_{\omega=1,3} \int d^2k_\perp dk^+ \theta(k^+)k^+a^\dagger(k,\omega)a(k,\omega). \quad (4.55) \]

The term \( v_3 \) is the vector-meson instantaneous term, and we include it together with the purely mesonic contribution to \( P_v^- \) because it is canceled by part of that contribution.

Thus, our result for the total minus-momentum operator is

\[ P^- = P_N^- + P_s^- + P_v^-, \quad (4.56) \]

and for the plus-momentum

\[ P^+ = P_N^+ + P_s^+ + P_v^+, \quad (4.57) \]

where from Eq. (4.39)

\[ P_N^+ \equiv \frac{1}{2} \int d^2x_\perp dx^- 2\xi_+^1 i\partial^+\xi_+. \quad (4.58) \]

**V. INFINITE NUCLEAR MATTER MEAN FIELD THEORY**

The aim of our approach is to do the realistic, relativistic physics of large nuclei using light front dynamics. Since this is a difficult project, it is very worthwhile to consider first the simplest possible example. This is to consider the nucleus of infinite size. One considers a limit in which the radius and nucleon number are each taken to be infinity, but the baryon density, or number of nucleons per unit volume, is taken as finite. Furthermore, a useful first step is to consider the mean field approximation to the dynamics in which nucleons are treated as moving under the influence of self-consistent potentials. Field-theoretic versions of relativistic mean field theory were pioneered by Walecka and exploited heavily by his students and many other workers \[42\]. Until our work, the only light front version of relativistic field theory applied to infinite nuclear matter was that of Glazek and Shakin \[43\] who treated the nucleus as bound under the influence of scalar mesons.

The philosophy \[42\] behind field-theoretic mean field theory is that the nucleonic densities which are mesonic sources are large enough to generate a large number of mesons to enable a classical treatment (replacing the operators which represent the sources by expectation values). In infinite nuclear matter, the volume is taken as infinity so that all positions are equivalent. Then, the nucleon mode functions are plane waves and the nuclear matter ground state is assumed to be a normal Fermi gas, of Fermi momentum \( k_F \) and of large
volume $\Omega$. Under these conditions one finds solutions of Eqs. (4.27) and (4.28) in which the meson fields are constants:

$$\phi = -\frac{g_s}{m_s^2} \langle \bar{\psi} \psi \rangle \equiv -\frac{g_s}{m_s^2} \rho_s \quad (5.1)$$

$$V^\mu = \frac{g_v}{m_v^2} \langle \bar{\psi} \gamma^\mu \psi \rangle = \delta_{0,\mu} g_v \rho_B \quad (5.2)$$

where the expectation values refer to a ground state expectation value, and $\rho_B = \frac{2}{3} \frac{k_3}{\pi^2} F$. This result that $V^\mu$ is a constant. The notion that there is no special direction in space is used. Eq. (4.33), tells us that the only non-vanishing component of $\bar{V}$ is $\bar{V}^\perp = V^0$. Since the potentials entering the light-front Dirac equation (4.32) are constant, the nucleon modes are plane waves $\psi \sim e^{i k \cdot x}$, and the many-body system is a Fermi gas. The solutions of Eq. (4.32) are

$$i \partial^\perp \psi_+ = g_v V^- \psi_+ + \frac{k_\perp^2 + (M + g_s \phi)^2}{k^+} \psi_+ \quad (5.3)$$

Solving the equations (5.1), (5.2) and (5.3) yields a self-consistent solution. The light front eigenenergy ($i \partial^\perp \equiv k^-$) is the sum of a kinetic energy term in which the mass is shifted by the presence of the scalar field, and an energy arising from the vector field. Comparing this equation with the one for free nucleons shows that the nucleons have a mass $M + g_s \phi$ and move in plane wave states. The nucleon field operator is constructed using the solutions of Eq. (5.3) as the plane wave basis states. This means that the nuclear matter ground state, defined by operators that create and destroy baryons in eigenstates of Eq. (5.3), is the correct wave function and that Equations (5.1), (5.2) and (5.3) represent the solution of the approximate field equations, and the diagonalization of the Hamiltonian.

### A. Nuclear Momentum Content

The expectation value of $T^{+\mu}$ is used to obtain the total momentum:

$$P^\mu = \frac{1}{2} \int d^2 x_\perp dx^- \langle T^{+\mu} \rangle \quad (5.4)$$

The expectation value is constant so that the volume $\Omega = \frac{1}{2} \int d^2 x_\perp dx^-$ will enter as a factor. A straightforward evaluation leads to the results

$$\frac{P^-}{\Omega} = m_s^2 \phi^2 + 2 \left( \frac{\kappa}{3!} \phi^3 + \frac{\lambda}{4!} \phi^4 \right) + \frac{4}{(2\pi)^3} \int_F d^2 k_\perp dk^+ \frac{k_\perp^2 + (M + g_s \phi)^2}{k^+} \quad (5.5)$$

$$\frac{P^+}{\Omega} = m_v^2 (V^-)^2 + \frac{4}{(2\pi)^3} \int_F d^2 k_\perp dk^+ k^+ \quad (5.6)$$

To proceed further one needs to define the Fermi surface $F$. The use of a transformation

$$k^+ \equiv \sqrt{(M + g_s \phi)^2 + k_\perp^2 + k^3} \equiv E(k) + k^3 \quad (5.7)$$
to define a new variable \( k^3 \) enables one to simplify the integrals. One replaces the integral over \( k^+ \) by one over \( k^3 \), including the Jacobian factor

\[
\frac{\partial k^+}{\partial k^3} = \frac{k^+}{E}.
\]  

(5.8)

Then one computes the nuclear energy \( E \) as the average of \( P^+ \) and \( P^- \):

\[
E \equiv \frac{1}{2} (P^- + P^+).
\]

The results are the very same expressions as in the original Walecka models. This provides a useful check on the algebra, which is important because this model has been solved in a manifestly covariant fashion.

There is a potential problem: for nuclear matter in its rest frame we need to have \( P^+ = P^- = M_A \). If one looks at the expressions for \( P^\pm \) this result does not seem likely. However, the value of the Fermi momentum has not yet been determined. There is one more condition to be satisfied:

\[
\left( \frac{\partial (E/A)}{\partial k_F} \right)_\Omega = 0.
\]  

(5.9)

Satisfying this equation determines \( k_F \) and for the value so obtained the values of \( P^+ \) and \( P^- \) turn are the same.

Thus we see that our light front procedure reproduces standard results for energy and density. We discuss two sets of results. The first involves the original Walecka model, in which \( \lambda = \kappa = 0 \). Then we may use the parameters of Chin and Walecka \[44\] \( g^2 v^2 M^2/m_v^2 = 195.9 \) and \( g_s^2 M^2/m_s^2 = 267.1 \) to obtain first numerical results. In this case, \( k_F = 1.42 \) \( \text{fm}^{-1} \), the binding energy per nucleon is 15.75 \( \text{MeV} \) and \( M + g_s \phi = 0.56 M \). The last number corresponds to a huge attraction that is nearly canceled by the huge repulsion. Then one may use Eq. (5.6) to obtain the separate contributions of the vector mesons and nucleons, with spectacular results. The use of Eq. (5.6) leads immediately to the unusual result that nucleons carry only 65\% of the plus-momentum. Thus is much less than the 90\% needed to explain the EMC effect for infinite nuclear matter \[43\]. According to Eq. (5.6) the vector mesons must carry the remaining 35\% of the plus-momentum, an amazingly large number.

It is instructive to evaluate the vector \( U_V \equiv g_V V^- \) and scalar \( U_S \) potentials that the nucleon feels with the present parameters of QHD1. These are given by

\[
U_V = \frac{g_V^2}{m_V^2} \rho_B = 330 \text{ MeV}
\]  

(5.10)

\[
U_S = -\frac{g_S^2}{m_S^2} \rho_S = -420 \text{ MeV}.
\]  

(5.11)

These very large potentials, obtained with \( m_V = 783 \text{ MeV} \) and \( m_S = 550 \text{ MeV} \), are the distinctive features of the Walecka model \[42\]. The vector and scalar meson fields are given according to Eqs. (5.1) and (5.2) as

\[
V^0 = U_V/g_V = 28.3 \text{ MeV}
\]

(5.12)

\[
\phi = -43.9 \text{ MeV}.
\]  

(5.13)

The nucleonic momentum distribution is the input to calculations of the nuclear structure functions. This distribution function can be computed from the integrand of Eq.(5.6). The
probability that a nucleon has plus momentum \( k^+ \) is determined from the condition that the plus momentum carried by nucleons, \( P^+_N \), is given by \( P^+_N / A = \int dk^+ k^+ f(k^+) \), where \( A = \rho_B \Omega \). It is convenient to use the dimensionless variable \( y = k^+ / M \) with \( M = M - 15.75 \) MeV. Then we find the result:

\[
f(y) = \frac{3 M^3}{4 k_F^3} \theta(y^+ - y) \theta(y - y^-) \left[ \frac{k_F^2}{M^2} - \left( \frac{E_F}{M} - y \right)^2 \right],
\]

(5.14)

where \( y^+ \equiv \frac{E_F + k_F}{M} \) and \( E_F \equiv \sqrt{k_F^2 + (M + g_s \phi)^2} \). This function is displayed in Fig. 4. The average value of \( y, \langle y \rangle \) can be computed from this distribution:

\[
\langle y \rangle = \int_0^\infty dy \ y f(y) = \frac{E_F}{M}.
\]

(5.15)

The relation to experiments is obtained by recalling that the nuclear structure function \( F_{2A} \) can be obtained from the light front distribution function \( f(y) \) (which gives the probability for a nucleon to have a plus momentum fraction \( y \)) and the nucleon structure function \( F_{2N} \) using the relation:

\[
\frac{F_{2A}(x)}{A} = \int dy f(y) F_{2N}(x/y),
\]

(5.16)

where \( y \) is \( A \) times the fraction of the nuclear plus-momentum carried by the nucleon, and \( x \) is the Bjorken variable computed using the nuclear mass minus the binding energy. This formula is the expression of the usual convolution model, with validity determined by a number of assumptions. Eq. (5.16) is essentially Eq. (5.2) of Frankfurt and Strikman, [8], with a correspondence between our \( A f(y) \) and their \( \int d^2 p_\perp \rho_N^A(\alpha, p_\perp) / \alpha \). The use of the light front formalism enables us to calculate the function \( f(y) \) from the integrand of Eq.(5.6).

The distribution of the vector meson plus-momentum is also an interesting quantity. The mean fields \( \phi, V^\mu \) are constants in space and time. Thus \( V^- \) has support only for \( k^+ = 0 \). The physical interpretation of this is that an infinite number of mesons carry a vanishingly small \( \epsilon \) of the plus-momentum, but the product is 35%. One can also show [23] that

\[
k^+ f_v(k^+) = 0.35 M \delta(k^+).
\]

(5.17)

There is an important phenomenological consequence the value \( k^+ = 0 \) corresponds to \( x_{Bj} = 0 \) which can not be reached in experiments. This means one can’t use the momentum sum rule as a phenomenological tool to analyze deep inelastic scattering data to determine the different contributions to the plus-momentum.

Of course these noteworthy results are caused by solving a simple model for a simple system with a simple mean field approximation. It is necessary to ask if any of the qualitative features of the present results will persist in more detailed treatments. A simple first step is to include the effects of scalar meson self-coupling. We examine the parameters of Ref. [46], and find one set which seems consistent with deep inelastic scattering data. This is \( \kappa = 2500 \) MeV, \( \lambda = 200, m_S = 368.6 \) MeV. Here \( k_F = 1.3 \) fm\(^{-1}\) and the binding energy per nucleon is again 15.75 MeV. The other parameters are \( g_{s^2} = 27.96, g_{v^2} = 52.44 \). The value of \( M + g_s \phi = 0.84 M \). This corresponds to nucleons carrying 90% of the plus momentum, and
FIG. 2. Light front momentum distribution as a function of $y$ for linear and non-linear models.

\[ k^+ f_{\text{non-linear}}(k^+) = 0.10 M \delta(k^+). \] (5.18)

which is roughly consistent with the data [13]. The nucleon momentum distribution is also shown in Fig. 2. The vector and scalar fields are now given by

\[ V^- = \frac{g_V}{m_V} \rho_B = 97 \text{ MeV} \] (5.19)

\[ \phi = -29 \text{ MeV}. \] (5.20)

VI. LIGHT FRONT STATIC SOURCE MODEL FOR NUCLEAR MESONS

The previously discussed calculations of Ref. [22], using a Lagrangian in which Dirac nucleons are coupled to massive scalar and vector mesons [12], treated the example of infinite nuclear matter within the mean field approximation. In this case, the scalar and vector meson fields are constants in both space and time, but only the vector mesons carry a non-zero plus-momentum. The constant nature means that the momentum distribution has support only at $k^+ = 0$. Such a distribution would not be accessible experimentally, so that the suppression of the plus-momentum of valence quarks would not imply the existence of a corresponding testable enhancement of anti-quarks. However, it is necessary to ask if the result is only a artifact of the infinite nuclear size and of the mean field approximation.

In section, I review a previous investigation [23] of the dependence on nuclear size. The technique was to construct the scalar and vector meson states for a model in which the nucleus is represented as a static source of radius $R$ and mass $M_A$. In this case, $M_A$ is very large for large $A$, and the nucleus acts as an extended static source. In the mean field approximation, the nucleus acts as a static source if it is infinitely heavy. Then a coherent state is the ground state of the Hamiltonian.
A. Scalar Meson Distribution

We take the scalar mesons, as coupled to a large static nucleus represented by a scalar source \( J(\vec{r}) \). Such a system is described by the Lagrangian density

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{m_{S}^{2}}{2} \phi^{2} + J \phi. \tag{6.1}
\]

To be specific, consider a spherical nucleus of radius \( R \) with constant density, so that

\[
J(\vec{r}) = J_{0} \Theta(R - |\vec{r}|). \tag{6.2}
\]

The quantity \( J_{0} \) can be thought of as arising from the product of a scalar meson coupling constant \( g_{S} \) and an appropriate scalar density \( \rho_{S} \) with

\[
J_{0} = g_{S} \rho_{S}. \tag{6.3}
\]

This is a static source in the rest frame, expressed in the usual \((t, \vec{r})\) coordinates.

The essential observation for the LF formulation of systems coupled to static sources is that static sources in a rest-frame correspond to uniformly moving sources in a LF framework \[47\]. In particular, a static charge distribution in a rest-frame \( J_{RF}(\vec{x}_{\perp}, x^{3}) \) corresponds to a uniformly moving charge distribution on the LF, where at \( x^{+} = 0 \)

\[
J_{LF}(\vec{x}_{\perp}, x^{-}) = J_{RF}(\vec{x}_{\perp}, z = -x^{-}/2). \tag{6.4}
\]

Using the usual technique to construct \( T^{\mu\nu} \), one finds

\[
P^{-} = \sum_{k^{+}, \vec{k}_{\perp}} \left[ \left( \frac{m_{S}^{2} + \vec{k}_{\perp}^{2}}{k^{+}} + k^{+} \right) a_{k^{+}, \vec{k}_{\perp}}^{\dagger} a_{k^{+}, \vec{k}_{\perp}} - 2 \frac{1}{\sqrt{2k^{+}}} \left( \tilde{J}_{RF}(\vec{k}_{\perp}, k^{+}) a_{k^{+}, \vec{k}_{\perp}}^{\dagger} + h.c. \right) \right], \tag{6.5}
\]

where

\[
\tilde{J}_{RF}(\vec{k}_{\perp}, k^{+}) = \frac{1}{(2\pi)^{3/2}} \int d^{2}x_{\perp} \frac{dx^{-}}{2} e^{-i k_{-} \cdot x_{\perp}} e^{ik^{+}x^{-}} g_{S} \rho_{S} \Theta(R - \sqrt{x_{\perp}^{2} + (x^{-}/2)^{2}}), \tag{6.6}
\]

\[
= \frac{1}{(2\pi)^{3/2}} \frac{4\pi R^{3}}{3} \frac{3j_{1}(kR)}{kR}, \tag{6.7}
\]

with

\[
k \equiv \sqrt{k_{\perp}^{2} + k^{+2}}. \tag{6.8}
\]

The interaction depends only on a combination of momenta that corresponds to the magnitude of a vector \( k \) with a positive \( z \) component, \( k^{+} \). The form factor \( \frac{3j_{1}(kR)}{kR} \) can be well approximated \[48\] by the expression \( e^{-k^{2}R^{2}/10} \) so that the typical momenta are bounded by \( \sqrt{10}/R \). The term proportional to \( k^{+} \) arises from the plus momentum of the heavy source and its inclusion is necessary to maintain the rotational invariance of the light-front approach, see Sect. III.
The LF Hamiltonian $P^-$ [Eq. (6.5)] is quadratic in the scalar fields and its ground state is a coherent state

$$|\psi_0\rangle_{LF} \propto \prod_{k^+, \vec{k}_\perp} \exp \left( \frac{2J^*_RF(\vec{k}_\perp, k^+)a^\dagger_{k^+\vec{k}_\perp}}{\sqrt{2k^+\left(m_k^2 + k^2\right)}} \right) |0\rangle. \quad (6.9)$$

The action of Eq. (6.5) on $|\psi_0\rangle_{LF}$ yields again the same state with eigenvalue

$$P^- = -2 \int_0^\infty \frac{dk^+}{k^+} \int d^2k_\perp \left| J_{RF}(\vec{k}_\perp, k^+) \right|^2 \frac{1}{m_k^2 + k^2}. \quad (6.10)$$

We replace the integral $\int_0^\infty dk^+$ by $\frac{1}{2} \int_{-\infty}^{\infty} dk^+$ so that

$$P^- = -(gS\rho_S)^2 \int \frac{d^3k}{(2\pi)^3} \left( \frac{4\pi R^3}{3} \right) \left( \frac{3j_1(kR)}{kR} \right)^2 \frac{1}{m_S^2 + k^2}. \quad (6.11)$$

Integration of the above yields the result that

$$\lim_{R \to \infty} P^-= -(gS\rho_S)^2 \frac{4\pi R^3}{3} \frac{1}{m_S^2} . \quad (6.12)$$

The LF-momentum distribution for the scalar mesons

$$\rho_S(\vec{k}_\perp, k^+) = \langle \psi_0 | a^\dagger_{k^+\vec{k}_\perp} a_{k^+\vec{k}_\perp} | \psi_0 \rangle \quad (6.13)$$

can be calculated using Eq. (6.3), with the result

$$\rho_S(\vec{k}_\perp, k^+) = \frac{2k^+ \left| J_{RF}(\vec{k}_\perp, k^+) \right|^2}{\left[ m_S^2 + k^2 \right]^2} . \quad (6.14)$$

Since $\tilde{J}_{RF}(\vec{k}_\perp, k^+)$ is strongly peaked for $k \sim 1/R$, the momentum distribution is also trivially peaked near small momenta (for $R \gg 1/m_S$). However, $\rho_S$ vanishes at $k^+ = 0$, so that it necessarily is very small. Note that the light frame momentum distribution function and the related plus-momentum distribution can only be obtained using the light front formulation.

The plus-momentum carried by the meson field is of great interest here. We compute this using Eq. (6.14), to obtain

$$\langle k^+ \rangle = \frac{1}{3} \int d^3k \frac{\vec{k}^2}{m_S^2 + k^2} \frac{\left| \tilde{J}_{RF}(\vec{k}) \right|^2}{\left[ m_S^2 + k^2 \right]^2} . \quad (6.15)$$

where we used rotational invariance of the source in the rest-frame. It is interesting to express the quantity $\langle k^+ \rangle$ in terms of a coordinate space integral. We find:

$$\langle k^+ \rangle = \int d^3r d^3r' J(\vec{r}) \left( 1 - \frac{m_S}{2} |\vec{r} - \vec{r}'| \right) \frac{e^{-m_S|\vec{r} - \vec{r}'|}}{12\pi |\vec{r} - \vec{r}'|} \right] J(\vec{r}). \quad (6.16)$$
The quantity in brackets has a volume integral of 0 (with the integration variable as $|\vec{r} - \vec{r}'|$). Thus the integral receives non-vanishing contributions only from regions near the nuclear surface, as is expected from the notion that the scalar meson field would be constant for a nucleus of infinite size and so receives nonzero Fourier components only from the regions near the surface of the nucleus. This means that $\langle k^+ \rangle \propto R^2$ which gives a far smaller magnitude than the $R^3$ behavior of the binding energy. In particular,

$$\langle k^+ \rangle \approx \frac{1}{R^2}$$

which vanishes in the limit $R \to \infty$, in accord with Ref. [22] which shows that $\langle k^+ \rangle$ vanishes for the case of infinite nuclear matter.

A detailed investigation of the scalar meson momentum distribution, $f_S(x)$, probability that a scalar meson carries a momentum fraction

$$x \equiv \frac{k^+}{M_N}$$

is given in Ref. [23].

### B. Vector Meson Distribution

The calculation of vector meson distributions is based on the formalism in Ref. [22], which used the light front quantization procedure of Ref. [39,40]. The model we consider is defined by taking the vector mesons $V^\mu$ to be coupled to a large nuclear source of baryon current $J^\mu$. Thus the relevant Lagrangian density $L_V$ is given by

$$L_V = -\frac{1}{4} V^{\mu\nu} V_{\mu\nu} + \frac{m^2}{2} V^{\mu} V_{\mu} - J^{\mu} \bar{V}_{\mu}$$

where $\bar{V}$ of Eq. (4.33) enters in the interaction term, with $\bar{V}^+ = 0$.

Note that

$$\bar{J}^{\mu}_{LF}(\vec{k}_\perp, k^+) = \delta(\mu, 0) \bar{J}^V_{RF}(\vec{k}_\perp, k^+),$$

where

$$J^V_{RF} = g_V \rho_B \Theta(R - r),$$

because there is no special direction in space. The nucleus is in its rest frame.

The main difference between vector mesons and scalar mesons is the appearance of the polarization vector $\epsilon^\mu$ in the coupling of vector mesons to the nucleon current (4.34). The transverse components of the nucleon current vanish in our model, and $\bar{\epsilon}^+ = 0$, so that the only non-zero component of $\bar{\epsilon}^\mu$ is $\mu = -$.

The coherent state takes the form

$$|\psi^V_0 \rangle \propto \left[ \prod_{k^+, \vec{k}_\perp} \exp \left( \frac{2 \bar{J}^{\mu}_{RF}(\vec{k}_\perp, k^+) \epsilon^\mu a(k, \omega)}{\sqrt{2k^+ (m^2 + k^2_{\perp} + k^+)}} \right) \right] |0\rangle,$$
where

\[ \tilde{J}^V_{RF}(\vec{k}_\perp, k^+) = g_V \rho_B \sqrt{\frac{2}{\pi R^3}} J_1(kR) \left( \frac{1}{k} \right), \]  

(6.23)

with \( k \equiv \sqrt{\vec{k}_\perp^2 + k^+}. \) The expressions for the energy and momentum distribution are obtained by taking expectation values using this wave function. The expressions obtained are similar to those for the scalar mesons except that a factor of \((\bar{\epsilon}^-)^2\) summed over all polarization states is present. The polarization sum is given by Eq. (4.35), with \( \mu = -\), and \( k^- = \frac{k^2 + m_V^2}{k^+} \), so that

\[ \sum_{\omega=1,3} \bar{\epsilon}^-(\vec{k}, \omega) \bar{\epsilon}^-(\vec{k}, \omega) = 4 \frac{m_V^2 + \vec{k}^2}{k^+}. \]  

(6.24)

We shall see that the denominator factor \( k^+ \) plays a large role in causing the vector meson momentum distribution function to be large for small values of \( k^+ \).

The light front Hamiltonian \( P^- \) consists of three terms: the kinetic energy (Eq. (2.24) of Ref. [22]); the linear coupling and, the effects of the instantaneous vector meson exchange. Thus the light-front energy of the nucleus due to coupling to the vector meson field consists of a contribution which arises from physical vector meson intermediate states, and an instantaneous interaction which exactly cancels the most infrared singular terms of the term due to dynamical mesons. This term is obtained by canonical light front quantization and e.g. is included in Eq. (2.48) of Ref. [22]. One takes the matrix element of the light front Hamiltonian to obtain the ground state energy of the vector meson field coupled to the fixed source:

\[ P^-_0(V) = 2 \int_0^\infty \frac{dk^+}{k^+} \int d^2k_\perp \frac{\tilde{J}^V_{RF}(\vec{k}_\perp, k^+)}{m_V^2 + k^2 + k^+}, \]  

(6.25)

which is related to the vector meson contribution to the binding energy \( P^-_0 \), which except for the sign (reflecting the fact that scalar meson give rise to attraction, while vector mesons give rise to repulsion between nucleons) is of the same form as the scalar result Eq. (6.10).

The result of taking matrix elements of the plus-momentum operator in the coherent state leads to the result:

\[ \rho_V(\vec{k}_\perp, k^+) = \left| \tilde{J}^V_{RF} \right|^2 \left\{ \frac{2}{k^+ m_V^2 + k^2} - \frac{2k^+}{[m_V^2 + k^2]^2} \right\}. \]  

(6.26)

The second term on the right hand side of Eq. (6.26) is (except for the sign and a differing mass) identical to the momentum distribution of scalar mesons. The contribution of this second term to the momentum (per nucleon) carried by the vector mesons vanishes in the nuclear matter limit and explicit numerical evaluation shows that for finite nuclei it is negligible compared with the the first term. This singular term

\[ \rho_V^{\text{sing}}(\vec{k}_\perp, k^+) \equiv \frac{2 \left| \tilde{J}^V_{RF} \right|^2}{k^+ m_V^2 + k^2} \]  

(6.27)
is more interesting since it diverges as \( k^+ \rightarrow 0 \). By direct comparison one can verify that the contribution from this term to the total momentum carried by the vector mesons is identical to the contribution of the vector mesons to the rest-frame energy of the nucleus

\[
\int_0^\infty dk^+ \int d^2k_\perp \rho^\text{sing}_V k^+ = \int_0^\infty dk^+ \int d^2k_\perp \frac{2 |J_{RF}|^2}{m_V^2 + k^2} = P_0^-(V). \tag{6.28}
\]

This means that in the infinite nuclear matter limit the momentum carried by the vector mesons is given by

\[
\langle k^+_V \rangle = P_0^-(V). \tag{6.29}
\]

This result that the momentum carried by vector mesons equals the \( P^- \) due to vector mesons holds regardless whether or not there is also a scalar interaction present. Both scalar and vector interaction are rather large in nuclei (of opposite sign, such that their net effect is small), so vector mesons may carry a substantial fraction of the nucleus’ momentum. In the previous section, we have shown that scalar mesons carry only a small fraction so the net plus-momentum carried by the mesons is essentially the potentially very large plus-momentum carried by the vector mesons.

The result that vector mesons carry a large part of the nuclear plus-momentum, which was obtained in Ref. [22] using more general arguments, is at first surprising since the vector meson field (very much like the scalar meson field) is constant in space and time for nuclear matter in the mean field approximation. Therefore, one would expect that the vector meson field for nuclear matter contains only quanta with vanishing + momentum. It thus seems paradoxical that vector mesons nevertheless carry a finite fraction of the nucleus’ + momentum in this limit. To resolve this apparent paradox, we study the momentum distribution arising from the crucial singular piece \( \rho^\text{sing}_V \) of Eq. (6.27).

A +–momentum distribution function per nucleon \( (A = 4\pi R^3 \rho_B) \)is defined using the variable of Eq. (6.18). Then, the use of Eq. (6.27) leads to the result

\[
f_V(x) \equiv \frac{3M_N}{4\pi R^3 \rho_B} \int d^2k_\perp \rho^\text{sing}_V(k_\perp, k^+), \tag{6.30}
\]
or

\[
x f_V(x) = R^3 \frac{6}{\pi} g_V^2 \rho_B \int_{xM_NR}^{\infty} dy \frac{1}{y^2 + m_V^2} |j_1^2(y)|. \tag{6.31}
\]

A qualitative understanding may be gained by noting that the quantity \( j_1^2(y)/y \) peaks at \( y \approx 1.6 \), and that \( m_V R \) and \( M_N R \) are very large numbers. Thus, in the limit \( R \rightarrow \infty \) the integral vanishes unless \( x = 0 \). In the limit of infinite \( R \) there is a sharp distinction between the results for \( x = 0 \) and for non-zero values, no matter how small. If \( x = 0 \), the integral can be done. The net result is that

\[
\lim_{R \rightarrow \infty} x f_V(x) = \left( \frac{g_V}{m_V} \right)^2 \frac{\rho_B}{M_N} \delta(x), \tag{6.32}
\]
in accord with the result expected from earlier work. The present meaning of the function \( \delta(x) \) is that integrals over \( x \) including this delta function are non-vanishing provided the
lower limit is infinitesimally close to zero. We follow Ref. [22] and again use the parameters of of Chin and Walecka [44]. Then the use of Eq. (6.32) leads to $\langle x_V \rangle = 0.348$, which agrees with the result of Ref. [22].

The distribution function $f_V(x)$ of Eq. (6.31) for finite values of $R$ is shown in Fig. (3). These results show the typical behavior of distributions that approach delta functions. The approach to the delta function limit is slow. All of the results show a significant spread and there is support for values of $x$ such that $x > 0.1$ for $0 < R < 6$ fm. The key physics question to be addressed is whether or not the distribution is non-zero for values of $x$ that are too small to be observable. This is discussed in Sect. 11.

The scalar and vector meson states for a mean field model of large nuclei have been constructed using coherent states, and the meson distribution functions have been obtained. The light-front momentum distribution for scalar mesons in the mean field approximation is localized near $k^+ \rightarrow 0$, and the total momentum carried by the scalar mesons in the mean field approximation is vanishingly small for nuclear matter. This calculation thus confirms the results of Ref. [22]: even though scalar mesons contribute to the nuclear binding in nuclear matter, they carry only a vanishing fraction of the momentum in the mean field calculation.

Also in accord with Ref. [22] is the present result that vector mesons do contribute to the plus-momentum of the nucleus in the same limit of infinite nuclear matter. The momentum fraction carried by vector mesons in mean field approximation and in the nuclear matter limit is given by the ratio between the the vector meson contribution to the potential energy and the nuclear mass. This result is obtained even though the vector meson distribution functions are zero for non-zero $x$ for infinitely large nuclei. This is because $xf_V(x) \propto \delta(x)$.

For nuclear radii $R$ corresponding to realistic large nuclei, the vector meson distributions, while strongly peaked at low values of $x$, are wide enough so that some fraction of the vector mesons could be observable. Thus the limitation of the support to $k^+ = 0$ found in Ref. [22], does not occur. However, the predictive power of the momentum sum rule is still vitiated because a significant fraction of the mesons are hidden at small values of $x_{Bj}$. 

FIG. 3. The quantity $xf_V(x)$ vs. $x$ for nuclei of radii 3,4,5,6, 10 fm.
VII. MEAN FIELD THEORY FOR FINITE-SIZED NUCLEI

It is important to make calculations for finite nuclei because all laboratory experiments are done for such targets or projectiles. The most remarkable feature of all of nuclear physics is that the shell model is able to explain the magic numbers. Rotational invariance causes the $2j + 1$ degeneracy of the single particle orbitals, and full occupation leads to increased binding. But light front dynamics does not make rotational invariance manifest because the different components $x^-, x_\perp$ of the spatial variable are treated differently. There is only an explicit invariance for rotations of the $xy$ plane about the $z$ axis. However, any set of correct final results must respect rotational invariance. The challenge of making successful calculations of the properties of finite nuclei is important to us.

In the following we review the work of Ref. [24] Let’s start with a schematic discussion of how it is that we will be able to find spectra which have the correct number of degenerate states. The concern here is with systems with a large number of nucleons, so that applying the usual light front Fock state expansion is too difficult. Instead, a variational calculation, a light front version of Hartree-Fock theory, is needed. The light front Hamiltonian, is $P^-$, but minimizing the expectation value of that operator would lead to nonsensical results because $P^- = M_A^2/P^+$. One can reach zero energy by letting $P^+$ be infinite. This difficulty is avoided by performing a constrained variation, in which the total light front momentum is fixed by including a Lagrange multiplier term proportional to the total momentum in the light front Hamiltonian. We minimize the expectation value of $P^+$ subject to the condition that the expectation values of $P^-$ and $P^+$ are equal. This is the same as minimizing the expectation value of the average of $P^-$ and $P^+$.

We use a simple example to demonstrate the need to include the plus-momentum along with the minus momentum. Consider a nucleus of $A$ nucleons of momentum $P_A^+ = M_A$, $P_A^\perp = 0$, which consists of a nucleon of momentum $(p^+, p^\perp)$, and a residual $(A-1)$ nucleon system which must have momentum $(P_A^+ - p^+, -p^\perp)$. The kinetic energy $K$ is given by the expression

$$K = \frac{p^2_\perp + M^2}{p^+} + \frac{p^2_\perp + M^2_{A-1}}{P_A^+ - p^+}. \quad (7.1)$$

In the second expression, one is tempted to neglect the term $p^+$ in comparison with $P_A^+ \approx M_A$. This would be a mistake. Instead expand

$$K \approx \frac{p^2_\perp + M^2}{p^+} + \frac{M^2_{A-1}}{P_A^+} \left(1 + \frac{p^+}{P_A^+}\right)$$

$$\approx \frac{p^2_\perp + M^2}{p^+} + p^+ + M_{A-1}, \quad (7.2)$$

using $M^2_{A-1}/P_A^2 \approx 1$ for large $A$. The second term is the plus-momentum operator mentioned above. For free particles, of ordinary three momentum $p$ one has $E^2(p) = p^2 + m^2$ and $p^+ = E(p) + p^3$, so that

$$K \approx \frac{(E^2(p) - (p^3)^2)}{E(p) + p^3} + E(p) + p^3 + M_{A-1} = 2E(p) + M_{A-1}, \quad (7.3)$$

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We see that $K$ depends only on the magnitude of a three-momentum and rotational invariance is restored. The physical mechanism of this restoration is the inclusion of the recoil kinetic energy of the residual nucleus.

A. Nucleon Mode Equation

The key feature of Ref. [24] was the derivation of the equation that defines the nucleon single-particle modes. This was done by minimizing the value of $P^-$ subject to the constraint that the expectation values of $P^+$ and $P^-$ were equal. These equations are given in Eq. (4) of the short paper, and (3.28) of the long paper. Here we present a more intuitive derivation [21] of the nucleon single particle wave equation. The first step is to develop the light front formalism for potentials that are static in the nuclear rest frame. Let us start from the Dirac equation for a static potential in the usual $(t, r)$ (ET) coordinates

$$i\gamma^\mu \partial_\mu - m - g_S V_S(\vec{r}) - g_V \gamma^0 V_0(\vec{r}) \psi^{ET} = 0. \tag{7.4}$$

Since $\gamma^0 = (\gamma^+ + \gamma^-)/2$ and since couplings using the “bad” component $\gamma^-$ are difficult to handle in the light front framework, we perform the transformation

$$\psi^{ET} = e^{ig_V \Gamma} \psi', \tag{7.5}$$

where

$$\partial_3 \Gamma = V_0, \tag{7.6}$$

and $\Gamma$ does not depend on time. Using the transformation (7.5), one finds

$$0 = \left[ i\gamma^\mu \partial_\mu + i\vec{\gamma}_\perp \cdot (\vec{\partial}_\perp \Gamma) - m - g_S V_S(\vec{r}) - g_V \left( \gamma^0 + \gamma^3 \right) V_0(\vec{r}) \right] \psi', \tag{7.7}$$

which may be expressed in terms of light front coordinates ($\vec{x}_\perp = \vec{r}_\perp$) as

$$\left[ \frac{1}{2} \left( i\gamma^+ \partial^- + i\gamma^- \partial^+ \right) + i\vec{\gamma}_\perp \cdot \left( \vec{\partial}_\perp + g_v \vec{\partial}_\perp \Gamma(\vec{x}_\perp, \frac{x^+ - x^-}{2}) \right) - m \right. \left. - V_S(\vec{x}_\perp, \frac{x^+ - x^-}{2}) - V^+ V_0(\vec{x}_\perp, \frac{x^+ - x^-}{2}) \right] \psi' = 0. \tag{7.8}$$

Even though the potential is static in the equal time formulation, the Dirac equation for the same potential in light-front coordinates is light front-“time”, i.e. $x^+$, dependent. This is because a static source in a rest-frame corresponds to a uniformly moving source on the light-front. Given that the time dependence of the external fields is only due to a uniform translation, we transform Eq. (7.8) into a form which contains only static (with respect to $x^+$) potentials. For this purpose, we consider the equation of motion satisfied by Dirac fields which are obtained by an $x^+$ (light front -time) dependent translation

$$\psi'(\vec{x}_\perp, x^-, x^+) \equiv e^{-ix^+ P^+/2} \psi(\vec{x}_\perp, x^-, x^+) \tag{7.9}$$
The techniques of Sect. III, in particular Eq. (3.6), are used to show that the equation of motion for $\psi$ takes the form:

$$\begin{align*}
\left[ \frac{1}{2} i \gamma^+ (\partial^- - \partial^+) + \frac{1}{2} i \gamma^- \partial^+ + i \bar{\gamma}_\perp \cdot (\bar{\partial}_\perp + i g_v \Gamma(\bar{x}_\perp, -\frac{x^-}{2})) - m \\
- g_s V_s(x_\perp, -\frac{x^-}{2}) - g_v \gamma^+ V^-(x_\perp, -\frac{x^-}{2}) \right] \psi = 0,
\end{align*}$$

with $V^- = V_0$. The translated fields satisfy an equation of motion with potentials that do not depend on $x^+$. Moreover, the static potentials evaluated at $\vec{r}$ correspond to light front potentials evaluated at $(x_\perp, -\frac{x^-}{2})$. An even simpler derivation of this can be obtained from evaluating $z = (x^+ - x^-)/2$ at $x^+ = 0$ which says $z = -x^-/2$.

That the result (7.10) is the same as the equations for $\psi_\pm$ in Ref. [24], can be seen by making a decomposition into a dynamical and a constraint equation. Multiplication of Eq. (7.10) by $\gamma^+$ from the left yields a constraint equation

$$i \partial^+ \psi_\pm = \left[ i \bar{\alpha}_\perp \cdot (\bar{\partial}_\perp + i g_v \Gamma) + \beta m + V_s \right] \psi_\pm$$

(7.11)

where $\psi_\pm \equiv \frac{1}{2} \gamma^0 \gamma^\pm \psi$. Multiplication of Eq. (7.10) by $\gamma^-$ from the left yields an equation for $\psi^+$:

$$i (\partial^- - \partial^+ - i g_v V^-) \psi_+ = \left[ i \bar{\alpha}_\perp \cdot (\bar{\partial}_\perp + i g_v \Gamma) + \beta m + V_s \right] \psi_-.$$  

(7.12)

One may use the constraint equation (7.11) to eliminate $\psi_-$ in Eq. (7.12) to obtain the equation of motion for the dynamical degrees of freedom. The results (7.11) and (7.12) are the desired equations. The difference between these and the original fermion field equations (4.32) is the appearance of the term $-i \partial^+$ which appears on the left side of the equation.

**B. Dynamical Meson Fields**

The light-front Schrödinger equation for the complete nuclear ground-state wave function $| \Psi \rangle$ is

$$P^- | \Psi \rangle = M_A | \Psi \rangle.$$  

(7.13)

We choose to work in the nuclear rest frame so that we also need

$$P^+ | \Psi \rangle = M_A | \Psi \rangle.$$  

(7.14)

As explained above, one must minimize the expectation value of $P^-$ subject to the condition that the expectation value of $P^+$ is equal to the expectation value of $P^-$. This is the same as minimizing the average of $P^-$ and $P^+$, which is the rest-frame energy of the entire system. To this end we define a light-front Hamiltonian

$$H_{LF} \equiv \frac{1}{2} (P^+ + P^-).$$  

(7.15)
We stress that $H_{LF}$ is not usual the Hamiltonian, because the light-front quantization is used to define all of the operators that enter.

The wave function $| \Psi \rangle$ consists of a Slater determinant of nucleon fields $| \Phi \rangle$ times a mesonic portion

$$| \Psi \rangle = | \Phi \rangle \otimes | \text{mesons} \rangle,$$  \hspace{1cm} (7.16)

and the mean field approximation is characterized by the replacements

$$\phi \rightarrow \langle \Psi | \phi | \Psi \rangle$$

$$V^\mu \rightarrow \langle \Psi | V^\mu | \Psi \rangle.$$  \hspace{1cm} (7.17)

The essential difference of our approach and the procedure to handle the original Walecka model is that the fields are treated as dynamical objects. The equations for the meson fields are derived in terms of expectation values of creation and destruction operators. This means that other expectation values involving meson fields, such as the plus-momentum density, also have non-zero values.

We detail the treatment of the expectation value of $\phi(x)$. Meson self-coupling terms are ignored here. Consider the quantity $H_{LF}a(k) | \Psi \rangle$, and use commutators to obtain

$$H_{LF}a(k) | \Psi \rangle = [H_{LF}, a(k)] | \Psi \rangle + M_A a(k) | \Psi \rangle.$$  \hspace{1cm} (7.18)

The operators $P_s^{\pm}$ of Eqs. (4.52) and (4.53) and the standard commutation relations allow one to obtain

$$[H_{LF}, a(k)] = -\frac{k_{\perp}^2 + k^+ + m_s^2}{2k^+}a(k) + \frac{J(k)}{(2\pi)^{3/2}\sqrt{2k^+}},$$  \hspace{1cm} (7.19)

where

$$\frac{J(k)}{(2\pi)^{3/2}\sqrt{2k^+}} = \frac{1}{2}[P^-_s, a(k)],$$  \hspace{1cm} (7.20)

and $P^-_s$ is given in Eq. (4.47).

We use Eqs. (4.48)-(4.50), and take the commutator of the interactions $v_i$ with $a(k)$. Then re-express the results in terms of $\xi$ to obtain

$$J(k) = -\frac{1}{2}g_s \int d^2x_\perp dx^- e^{ik\cdot x} \bar{\xi}(\mathbf{x}) \xi(\mathbf{x}).$$  \hspace{1cm} (7.21)

Take the overlap of Eq. (7.18) with $| \Psi \rangle$ to find

$$\langle \Psi | a(k) | \Psi \rangle \frac{k_{\perp}^2 + k^+ + m_s^2}{2k^+} = \langle \Psi | J(k) | \Psi \rangle \frac{(2\pi)^{3/2}\sqrt{2k^+}}{(2\pi)^{3/2}\sqrt{2k^+}}.$$  \hspace{1cm} (7.22)

Multiply Eq. (7.22) by a factor $\sqrt{2k^+/(2\pi)^{3/2}} e^{-ik\cdot x}$ and add the result to its complex conjugate. The integral of the resulting equation over all $k_{\perp}$ and $k^+ > 0$ leads to the result
\[
\left( -\nabla_\perp^2 - \left( 2 \frac{\partial}{\partial x^\perp} \right)^2 + m_s^2 \right) \langle \Psi \mid \phi(x) \mid \Psi \rangle = \
\langle \Psi \mid \int \frac{d^2 k_\perp dk^+ \theta(k^+)}{(2\pi)^3} \left( J(k) e^{-i k \cdot x} + J^\dagger(k) e^{+i k \cdot x} \right) \mid \Psi \rangle.
\]

(7.23)

The evaluation of the right-hand-side of Eq. (7.23) proceeds by using Eq. (7.21) and its complex conjugate. The combination of those two terms allows one to remove the factor \( \theta(k^+) \) and obtain a delta function from the momentum integral. That \( \frac{1}{2} k^+ x^- \) appears in the exponential leads to the removal of the factor \( \frac{1}{2} \) of Eq. (7.21). One can also change variables using

\[ z \equiv -\frac{x^-}{2}, \quad x \equiv (z, x_\perp). \]  

(7.24)

The minus sign enters to remove the minus sign between the two terms of the factor \( k \cdot x \). Then one may use a simple definition

\[ -\nabla_\perp^2 - \left( 2 \frac{\partial}{\partial x^\perp} \right)^2 \equiv -\nabla^2. \]  

(7.25)

Note Ref. [23] obtained the relation (7.24) by examining the space-time diagram for a static (independent of \( x^0 \)) source. The net result is that

\[
\left( -\nabla^2 + m_s^2 \right) \langle \Psi \mid \phi(x) \mid \Psi \rangle = -g_s \langle \Psi \mid \bar{\xi}(x) \xi(x) \mid \Psi \rangle,
\]

(7.26)

which has the same form as the equation in the usual equal-time formulation. Note that the right hand side of Eq. (7.26) should be a function of \(|x|\) for the spherical nuclei of our present concern. Our formalism for the nucleon fields uses \( x_\perp \) and \( x^- \) as independent variables, so that obtaining numerically scalar and vector nucleon densities that depend only \( x_\perp^2 + (x^-/2)^2 \) will provide a central, vital test of our procedures and mean field theory. This does occur [24], so the scalar field \( \langle \Psi \mid \phi(x) \mid \Psi \rangle \) depends only \(|x|\) according to (7.26).

We stress that the use of Eq. (7.24) is merely a convenient way to simplify the calculation — using it allows us to treat the \( \perp \) and minus spatial variables on the same footing, and to maintain explicit rotational invariance. We will obtain the mesonic plus-momentum distributions from the ground state expectation value of operators expressed in terms of light front coordinates.

The procedure of Eqs. (7.18) to (7.26) can also be applied to the vector fields, [24]. After some manipulations mandated by the appearance of the barred vector potential, one finds

\[
\left( -\nabla^2 + m_v^2 \right) \langle \Psi \mid \bar{V}^\mu(x) \mid \Psi \rangle = g_v \langle \Psi \mid \bar{\xi}(x) \gamma^\mu \xi(x) \mid \Psi \rangle.
\]

(7.27)

C. Finite Nucleus Solutions, Results and interpretation of the eigenvalues

The main content of the nucleon mode equation is that it is the usual Dirac equation expressed in terms of light front variables. It should not be a surprise that the mode equation
of the ET theory turns out to be the same as that of the light front theory. In the former, one minimizes the energy of the chosen single determinant wave function. In the light front one minimizes $P^-$ subject to the constraint that the expectation value of $P^+$ is the same as that of $P^-$. For the correct wave function, each of $P^\pm$ is one half the mass or energy of the nucleus.

The nuclear wave function $|\Psi\rangle$ consists of a Slater determinant of nucleon fields $|\Phi\rangle$ times a mesonic portion, and the mean field approximation is characterized by the replacements: $\phi \rightarrow \langle \Psi | \phi | \Psi \rangle, V^\mu \rightarrow \langle \Psi | V^\mu | \Psi \rangle$. We quantize the nucleon fields using

$$\psi(x) = \sum_n \langle x^-, x^\perp | n \rangle e^{-i p_n x^+/2} b_n$$

(7.28)

in which the variable $x$ represents both the spatial variables $x^\perp, x^-$ and the spin, isospin indices, and $b_n$ obeys the usual anti-commutation relations. The meson fields are also treated as functions of these variables. The Slater determinant $|\Phi\rangle$ is defined by allowing $A$ nucleon states to be occupied.

The use of Eq. (7.28) in Eqs. (7.11) and (7.12) leads to

$$p^-_n | n \rangle_+ = \left( i \partial^- + 2 g_v \vec{V}^- \right) | n \rangle_+ + (\alpha_\perp \cdot (p_\perp - g_v \vec{V}_\perp) + \beta (M + g_s \phi)) | n \rangle_- ,$$

(7.29)

$$i \partial^+ | n \rangle_- = (\alpha_\perp \cdot (p_\perp - g_v \vec{V}_\perp) + \beta (M + g_s \phi)) | n \rangle_+ ,$$

(7.30)

$$| n \rangle_\pm = \Lambda_\pm | n \rangle ,$$

(7.31)

which together with Eqs. (7.26) and (7.27) forms the self-consistent set of equations to be solved. The first step of the solution procedure is to use a representation [49] in which $| n \rangle_\pm$ are each represented as Pauli spinors. The light front quantization respects manifest rotational invariance for rotations about the $z$-axis. Thus each single-particle state has a good $J_z$. We use a momentum representation for the longitudinal variable in which the values of $p^+$ take on those of a discrete set: $p^+_m = (2m + 1)\pi/(2L)$ where $m \geq 0$ and $L$ is a quantization length. This means that the nucleon wave functions have support only for $p^+ \geq 0$. This spectrum condition is a requirement for exact solutions for any theory. The coordinate space representation is used for the $\perp$ variables. Then we have

$$\langle p^+_m, x_\perp | n \rangle_+ = \left[ U^{(n)}_m(x_\perp) e^{iJ_z-1/2} \phi \over L^{(n)}_m(x_\perp) e^{iJ_z+1/2} \phi \right] ,$$

(7.32)

in which the upper (lower) entrees of the Pauli spinor correspond to $m_s = 1/2(-1/2)$, and $x_\perp \equiv (x_\perp, \phi)$. When Eq. (7.32) is used in Eq. (7.12), one finds that the equations do not depend on the magnitude of $J_z$; solutions for $\pm$ a given magnitude of $J_z$ are degenerate. The functions $U_m(x_\perp)$ and $L_m(x_\perp)$ are expanded in a basis of B-splines of degree five in $x_\perp$ [50].

The technical aspects of the solution procedure are detailed in Refs. [24], so I concentrate on summarizing the results. If our solutions are to have any relevance, they should respect rotational invariance. The success in achieving this is examined in Table I which gives our results for the spectra of $^{16}$O and $^{40}$Ca. Scalar and vector meson parameters are taken from Horowitz and Serot [51], and we have ignored electromagnetic effects. Note that the $J_z = \pm 1/2$ spectrum contains the eigenvalues of all states, since all states must have a
The relation (7.34) tells us that \( \langle p^+, x_\perp \mid n \rangle \) peaks at \( p^+ \approx M - g_V V^0 \), with a width of the order of the inverse of the radius of the entire nucleus. Therefore, neglecting \( p^+ < 0 \) is only a
minor approximation, which is the reason why our above LF calculation, with $p^+ < 0$ gives $P_n^-/2$ that are approximately equal to the ET results.

Eq. (7.34) shows that the influence of the vector potential is to remove plus-momentum from the nucleons. Furthermore, the large value of the nuclear radius causes the region of support to be very narrow, so that $\langle p^+, x_\perp | n \rangle$ is very small for negative values of $p^+$.

D. Nuclear momentum content and lepton-nucleus deep inelastic scattering

Table 7.2 \cite{24} gives the contributions to the total $P^+$ momentum from the nucleons, scalar mesons, and vector mesons for $^{16}$O, $^{40}$Ca, and $^{80}$Zr, as well as the nuclear matter limit. The vector mesons carry approximately 30\% of the nuclear plus-momentum. The technical reason for the difference with the scalar mesons (which have negligible effect) is that the evaluation of $a^\dagger(k, \omega) a(k, \omega)$ counts vector mesons “in the air” and the resulting expression contains polarization vectors that give a factor of $\frac{1}{k^+}$ which enhances the distribution of vector mesons of low $k^+$. The results for the nucleon and vector meson plus-momentum distributions are shown in Figs. 7.1 and 7.2 \cite{24}. As the size of the nucleus increases the enhancement of the distribution at lower values of $k^+$ becomes more evident.

It is worthwhile to see how the present results are related to lepton-nucleus deep inelastic scattering experiments. We find that the nucleons carry only about 70\% of the plus-momentum. The use of our $f_N$ in standard convolution formulae lead to a reduction in the nuclear structure function that is far too large ($\sim 95\%$ is needed \cite{7}) to account for the reduction observed \cite{7} in the vicinity of $x \sim 0.5$. The reason for this is that the quantity $M + g_s \phi$ acts as a very small nucleon effective mass of about 670 MeV. While such a low value is needed to reproduce the nuclear spin orbit force, it causes difficulties, not only in
FIG. 5. Vector meson plus-momentum distribution $y f_v(y)$, for $^{16}$O, $^{40}$Ca, and $^{80}$Zr.

| Nucleus | $P_N^+/A$  | $P_s^+/A$  | $P_v^+/A$  | $P^+/A$  |
|---------|-------------|-------------|-------------|-----------|
| $^{16}$O | 704.7       | 6.4         | 221.8       | 932.9     |
| $^{40}$Ca | 672.6       | 4.7         | 253.3       | 930.6     |
| $^{80}$Zr | 655.2       | 3.6         | 270.2       | 929.0     |
| NM      | 569.0       | 0.0         | 354.2       | 923.2     |
deep inelastic scattering, but also in understanding the quasi-elastic \((e,e')\) reaction \[32\]. The use of other Lagrangians \[53–55\] will lead to improved results. Including effects beyond the mean field lead to a significant effective tensor coupling of the iso-scalar vector meson \[56\], and to an increased value of the effective mass. Such effects are incorporated in Brueckner theory, and a light-front version \[25\] could be applied to finite nuclei with better success in reproducing the data.

A simple way to improve the phenomenology is to modify \(\mathcal{L}\), by for example including scalar meson self coupling terms: \(\phi^3, \phi^4\). A wide variety of parameter sets reproduce the binding energy and density of nuclear nuclear matter \[40\]. For one set, nucleons carry 90% of \(P^+\), so that vector mesons carry 10%, see Sect. 5. This could be acceptable. There is a problem with this parameter set, the related nuclear spin-orbit splitting is found to be too small \[51\]. This is not so bad, since there are a variety of non-mean field mechanisms which can supply a spin orbit force. Thus one finds a need to go beyond mean field theory. This involves the introduction of light front Brueckner theory.

VIII. CHIRAL SYMMETRY AND PION-NUCLEON SCATTERING

The essence of Brueckner theory is the use of an in-medium nucleon-nucleon scattering matrix to derive the self-consistent fields. This scattering matrix is derived from a realistic nucleon-nucleon potential. This means that the influence of pions and approximate chiral symmetry must be incorporated. In this section, we discuss the light-front quantization of a chiral Lagrangian, and review its application to low energy pion-nucleon scattering.

The starting point is the definition of a non-linear chiral model in which the nuclear constituents are nucleons \(\psi\) (or \(\psi'\)), pions \(\pi\) scalar mesons \(\phi\) and vector mesons \(V^\mu\). The Lagrangian without the influence of chiral symmetry is given above in Eq. (4.26). The additional pieces which include pions and implement chiral symmetry is \(\mathcal{L}_\chi\), with

\[
\mathcal{L}_\chi = + \frac{1}{4} f^2 \text{Tr}(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{4} m_\pi^2 f^2 \text{Tr}(U + U^\dagger - 2) - \bar{\psi}' ((M + g_\pi \phi)(U - 1)) \psi'.
\] (8.1)

The unitary matrix \(U\) can be chosen from amongst three forms \(U_i\):

\[
U_1 = e^{i \gamma_5 \tau \cdot \pi / f}, \quad U_2 = \frac{1 + i \gamma_5 \tau \cdot \pi / 2f}{1 - i \gamma_5 \tau \cdot \pi / 2f}, \quad U_3 = \sqrt{1 - \pi^2 / f^2 + i \gamma_5 \tau \cdot \pi / f},
\] (8.2)

which correspond to different definitions of the fields.

The pion-nucleon coupling here is chosen as that of linear representations of chiral symmetry used by Gursey \[58\], with the the Lagrangian approximately \((m_\pi \neq 0)\) invariant under the chiral transformation

\[
\psi' \rightarrow e^{i \gamma_5 \tau \cdot a} \psi', \quad U \rightarrow e^{-i \gamma_5 \tau \cdot a} U e^{-i \gamma_5 \tau \cdot a}.
\] (8.3)

One may transform the fermion fields, by taking \(U^{1/2} \psi'\) as the nucleon field. One then gets Lagrangians of the non-linear representation \[59\]. In this case the early soft pion theorems are manifest in the Lagrangian, and the linear pion-fermion coupling is of the pseudovector
type. However, the use of light front theory, requires that one find an easy way to solve the constraint equation that governs the fermion field. The constraint is handled in a usual fashion if Eq. (8.1) is used, and we’ll show that this contains the early soft pion theorems.

The constant \( M_f \) plays the role of the bare pion-nucleon coupling constant. If \( f \) is chosen to be the pion decay constant, the Goldberger-Trieman relation yields the result that the axial vector coupling constant \( g_A = 1 \), which would be a problem for the Lagrangian, unless loop effects can make up the needed 25% effect. Corrections of that size are typical of order \( (M_f)^3 \) effects found in the cloudy bag model \[18\] for many observables, including \( g_A \).

There are no explicit \( \Delta' \)’s in the above Lagrangian. We note that treating the higher order effects of the pion-nucleon inherent in this Lagrangian is likely to lead to a resonance in the (3,3) channel of pion-nucleon scattering. Such resonance effects can be included in the two-pion exchange contribution to nucleon-nucleon scattering. However, the \( \Delta \) really is dominated by its 3-quark component \[48\] and carrying out such computations seems cumbersome. Thus it would be useful to incorporate the \( \Delta \) in the Lagrangian.

The scalar meson of Eq. (4.26) is kept, even though the effects of \( \pi - \pi \) interactions, which could lead to similar effects, are now included in the Lagrangian. We follow many authors and include a scalar meson to simplify calculations and neglect the \( \pi - \pi \) interactions of Eq. (8.1). Note that, in the present Lagrangian, the scalar meson \( \phi \) is not a chiral partner of the pion- the chiral transformation is that of Eq. (8.3).

It is now worthwhile to mention a subtle feature regarding chiral symmetry in light front formalisms. Chiral invariance is defined as invariance under the transformation defined by Eq. (8.3) if the equal time formalism is used. Now the independent fermion variable is \( \psi_+ \) and \( \psi_- \) is a functional of this. Thus chiral invariance is the invariance under the transformation

\[
\psi_+ \rightarrow e^{i\gamma_5 \tau \cdot a} \psi_+ ,
\]

which is not the same as Eq.(8.3) \[60,16\]. However, the \( T^{+-} \) (or light front Hamiltonian density) derived from \( \mathcal{L} + \mathcal{L}_\chi \) is invariant under the transformation (8.4) if the pion mass is neglected \[22\], so the usual chiral properties are obtained in these light front dynamics.

We begin by showing that, if one starts with a non-linear representation of chiral symmetry, the requirement of solving the constraint equation for the minus component of the fermion field leads one to a Lagrangian of the Gursey-type linear representation.

The focus is on chiral properties and pion-nucleon scattering, so we momentarily dispense with the vector and non-chiral \( \phi \) meson fields, and examine only the following fermion-pion term of a non-linear representation:

\[
\mathcal{L}_{N\pi} = \bar{N} \left[ \gamma_\mu i \partial^\mu - M + \frac{1}{1 + (\pi/2f)^2} \left( \frac{1}{2f} \gamma_5 \tau \cdot \partial^\mu \pi - \left( \frac{1}{2f} \right)^2 \gamma^\mu \tau \cdot \pi \times \partial^\mu \pi \right) \right] N. \quad (8.5)
\]

The most general non-linear realization does not specify the ratio of the two pion-nucleon couplings. Next obtain the fermion field equation and make the usual decomposition:

\[
\bar{N}_+ \equiv \Lambda_+ N \text{ with } \quad \left( i\partial^+ - O^+ \right) N_+ = [\alpha_+ \cdot (p_+ - O_+)] N_+ + \beta M N_+ ,
\]

\[
\left( i\partial^- - O^- \right) N_- = [\alpha_- \cdot (p_- - O_-)] N_- + \beta M N_- , \quad (8.6)
\]
in which the operator $O^\mu$ is defined as

$$O^\mu \equiv \frac{-1}{1 + (\pi/2f)^2} \left( \frac{1}{2f} \gamma_5 \tau \cdot \partial^\mu \pi - \left( \frac{1}{2f} \right)^2 \tau \cdot \pi \times \partial^\mu \pi \right). \quad (8.7)$$

We need to remove the $O^+$ term from the left hand side of the equation for $N_-$. This can be done by defining, in analogy with the Soper-Yan transformation, a unitary operator $F$ and fermion field $\chi$ such that

$$N = F \chi \quad (8.8)$$

with

$$i \partial^+ F = O^+ F. \quad (8.9)$$

The identity \[58\]

$$U_2^\dagger \partial^\mu U_2^{-1} = iO^\mu, \quad (8.10)$$

where $U_2$ is given in Eq. (8.2), helps a good deal. Its use in Eq. (8.9), combined with the condition $\partial^\mu (U_2 U_2^{-1}) = 0$, leads to the result

$$F = U_2^\dagger, \quad (8.11)$$

so that using Eqs. (8.11) and (8.8) in (8.6) yields

$$i\partial^- \chi_+ = [\alpha_\perp \cdot p_\perp + \beta MU_2] \chi_-, \quad i\partial^+ \chi_- = [\alpha_\perp \cdot p_\perp + \beta MU_2] \chi_+. \quad (8.12)$$

This is of the desired form in which no interactions appear on the left-hand-side of the equation for $\chi_-$. Thus the use of light front quantization mandates that the pion-nucleon interactions be of the form of Eq. (8.1), in which the possibility of using $U_1$ or $U_3$ arises from a field re-definition of that Lagrangian.

The new chiral terms of $\tilde{L}_\chi$ lead to additions to the potentials $v_1, v_2$ given above in Eqs. (4.48,4.49). These additions are:

$$v_1^\chi = \int d^2x_\perp dx^- \bar{\xi} (U - 1)(M + g_s \phi) \xi, \quad (8.13)$$

and

$$v_2^\chi = \int d^2x_\perp dx^- \bar{\xi} \left[ (U - 1)(M + g_s \phi) \frac{\gamma^+}{2p^+} \left( -g_v \gamma \cdot \bar{V} + (U - 1)(M + g_s \phi) \right) \right] \xi$$

$$- \int d^2x_\perp dx^- \bar{\xi} \left[ g_v \gamma \cdot \bar{V} \frac{\gamma^+}{2p^+} (U - 1)(M + g_s \phi) \right] \xi. \quad (8.14)$$

The term $v_1^\chi$ accounts for the emission or absorption of any number of pions through the operator $U - 1$. The term $v_2^\chi$ includes contact terms in which there is propagation of an instantaneous fermion.
The first test for any chiral formalism is to reproduce the early soft pion theorems \[61\]. Here we concentrate on low energy pion-nucleon scattering because of its relation to the nucleon-nucleon force. We work to second order in \(1/f\) in this first application. In this case, each of the \(U_i\) takes the same form:

\[
U = 1 + i\gamma_5 \frac{\tau \cdot \pi}{f} - \frac{1}{2f^2} \pi^2.
\] (8.15)

This expression is to be used in the potentials \(v_1^\chi\) and \(v_2^\chi\) of Eqs. \((8.13)\) and \((8.14)\). Substitution leads to the approximations

\[
v_1^\chi \approx \frac{M}{f} \int d^2x_\bot dx^- \xi \left[ i\gamma_5 \frac{\tau \cdot \pi}{2p^+} i\gamma_5 \tau \cdot \pi \right] \xi,
\] (8.16)

and

\[
v_2^\chi \approx \frac{M^2}{f^2} \int d^2x_\bot dx^- \xi \left[ i\gamma_5 \tau \cdot \pi \frac{\gamma^+}{2p^+} i\gamma_5 \tau \cdot \pi \right] \xi
\] (8.17)

The second-order scattering graphs are of three types and are shown as time \(x^+\) ordered perturbation theory diagrams in Fig. 8.1. The kinematics are such that \(\pi(q)N(k) \rightarrow \pi(q')N(k')\), with \(P_i = q + k\) and \(P_f = q' + k'\). The iteration of \(v_1\) to second order yields the direct and crossed graphs of Fig. 8.1a. In this formalism \(v_1\) is proportional to the matrix element of \(\gamma_5\) between spinors, so it is proportional to the momentum of the absorbed or emitted pion. Thus the terms of Fig. 8.1a vanish near threshold. The terms of Fig. 8.1b are generated by the \(\bar{u}\gamma_5v\) terms of \(v_1^\chi\). Using the field expansions:

\[
\pi(x) = \int \frac{d^2k_\bot dk^+ \theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}} \left[ a(k)e^{-ik\cdot x} + a^\dagger(k)e^{ik\cdot x} \right],
\] (8.18)

and

\[
\xi(x) = \int \frac{d^2k_\bot dk^+ \theta(k^+)}{(2\pi)^{3/2}\sqrt{2k^+}} \sum_{\lambda=\pm} \left[ u(k,\lambda)e^{-ik\cdot x}b(k,\lambda) + v(k,\lambda)e^{+ik\cdot x}d^\dagger(k,\lambda) \right],
\] (8.19)

in the expression \((8.16)\) for \(v_1^\chi\) leads to the result that plus-momentum is conserved and the plus momentum of every particle is greater than zero. This means that the first of Fig. 8.1b vanishes identically and the second vanishes for values of the initial pion plus momentum that are less than twice the nucleon mass. The net result is that only the instantaneous term of \(v_2^\chi\) and the \(\pi^2\) term of \(v_1^\chi\) (shown in Fig. 8.1c) remain to be evaluated.

Proceeding more formally, we evaluate the S-matrix given by

\[
S = T_+ e^{-\frac{i}{2} \int_{-\infty}^{\infty} dx^+ \hat{P}_i^-(x^+)},
\] (8.20)

where \(T_+\) is the \(x^+\) (light-front time) ordering operator and \(\hat{P}_i^-\) is the interaction representation light front Hamiltonian. Then

\[
(S - 1)_{fi} = -2\pi i\delta(P_i^- - P_f^-) \langle f | T(P_i^-) | i \rangle,
\] (8.21)
with
\[
T(P_i^-) = P_i^- + \frac{1}{P_i^- - P_0^-}T(P_i^-)
\] (8.22)

The evaluation proceeds by using the field expansions in the expressions for \( \psi_1^\uparrow \) and \( \psi_2^\uparrow \). Integrating over \( d^2x_\bot dx^+ \) and evaluating the result between the relevant initial and final pion-nucleon states leads to the result that each contribution to the S-matrix is proportional to a common factor,
\[
\frac{\delta^{(2,\bot)}(P_i - P_f)}{2(2\pi)^3\sqrt{k' + k + q' + q}}
\]
which combines with the result of the required integration over the light cone time \( (x^+) \) to provide the necessary momentum conservation and flux factors. The remaining factor of each term is its contribution to the invariant amplitude \( M \). The result is
\[
M = \tau_i\tau_f \frac{M^2}{f^2} \frac{\bar{u}(k')\gamma^+u(k)}{2(k^+ + q^+)} + \tau_f\tau_i \frac{M^2}{f^2} \frac{\bar{u}(k')\gamma^+u(k)}{2(k^+ - q^+)} - \delta_{ij} \frac{M}{f^2} \bar{u}(k')u(k)
\] (8.23)
where the three terms here correspond to the three terms of Fig. 8.1c At threshold, \( k' = k = M \) and \( q' = q = m_\pi \), so the role of cancellations in the reduction of the term proportional to \( \delta_{ij} \) is immediately apparent. In more detail, one finds
\[
M = \delta_{ij} \frac{2m_\pi^2}{f^2} + 2i\epsilon_{f\uparrow\tau_n} \frac{m_\pi M}{f^2}
\] (8.24)
to leading order in \( m_\pi/M \). The weak nature of the \( \delta_{ij} \) term and the presence of the second Weinberg-Tomazow term is the hallmark of chiral symmetry [61].

The same results could be obtained using the linear sigma model, with \( \sigma \) exchange playing the role of the \( \pi^2 \) term of Eq.(8.13).

IX. NUCLEON-NUCLEON SCATTERING ON THE LIGHT FRONT

The correlations between nucleons are caused by the nucleon-nucleon interaction. Thus a necessary first step towards a light-front theory of nuclear correlations is the derivation of a light-front theory of the nucleon-nucleon interaction. Previous work [22] showed that the light-front version of the Lippmann-Schwinger equation, the Weinberg equation, can be transformed (except that the retardation effect is kept) into the Blankenbecler-Sugar equation [78]. Kinematic invariance under boosts in the three-direction is maintained, and a one-boson exchange potential, is in reasonably good agreement with the NN phase shifts, is obtained [25].

It is worthwhile to begin by reviewing [22,23] how using the light-front Hamiltonian of Eqs. (4.46-4.50) and (8.13), (8.14) leads to the one-boson exchange potential. This derivation is useful in understanding the full nuclear wave function discussed in Sect. X. Consider the scattering process \( 1 + 2 \rightarrow 3 + 4 \), of Fig. 7.

The use of second-order perturbation theory shows that the lowest-order contribution to the nucleon-nucleon scattering amplitude is given by
FIG. 6. Low energy pion-nucleon scattering, $x^+$-ordered graphs. (a) Second-order effects of the $\bar{u}\gamma_5u$ term $v_1^\chi$. (b) Second-order effects of the $\bar{u}\gamma_5v$ and $\bar{v}\gamma_5u$ terms of $v_1^\chi$. (c) Effects of the instantaneous fermion propagation terms of $v_2^\chi$, and of the $\pi^2$ term of $v_1^\chi$. The terms $v_i^\chi$ are defined in Eqs. (8.16,8.17).

\begin{equation}
\langle 3, 4 | K | 1, 2 \rangle = \langle 3, 4 | (v_1 + v_1^\chi)g(P_{ij}^-)(v_1 + v_1^\chi) + v_3 | 1, 2 \rangle,
\end{equation}

with

\begin{equation}
g_0(P_{ij}^-) \equiv \frac{1}{P_{ij}^- - P_0^-},
\end{equation}

where $P_{ij}^-$ is the negative component of the total initial or final momentum which are the same. In constructing the NN potential one uses conservation of four-momentum between the initial and final NN states in constructing the NN potential. The expression (9.1) yields a one-boson exchange approximation to the nucleon-nucleon potential.

It is worthwhile to discuss the energy denominator $P_{ij}^- - P_0^-$ in more detail. To be specific, suppose that $k_1^+ > k_3^+$. Then the emitted meson of mass $\mu$ has momentum $k$ with $k^+ = k_1^+ - k_3^+, k_\perp = k_{1\perp} - k_{3\perp}$ and $k^- = \frac{k_1^+ + \mu^2}{k^+}$. Then

\begin{equation}
P_{ij}^- - P_0^- = P_{12}^- - P_0^-- P_{34}^- - P_0^- = (k_1^- - k_3^-) - \frac{k_\perp^2 + \mu^2}{k_1^+ - k_3^+}.
\end{equation}
FIG. 7. $x^+$-ordered graphs One boson exchange contributions to nucleon-nucleon scattering. $x^+$-ordered graphs. The numbers 1-4 represent the momentum, spin and charge states of the nucleons. Here $k_1^+ > k_3^+$. (a) meson propagation terms (b) instantaneous vector meson exchange of $v_3$, Eq. (2.48)

The interaction $K$ also contains a factor of $k^+$ in the denominator, so that the relevant denominator is

$$D \equiv k^+(P_{ij}^- - P_0^-) = (k_1^+ - k_3^+)(k_1^- - k_3^-) - k_1^2 - \mu^2 = q^2 - \mu^2, \quad (9.4)$$

with

$$q \equiv k_1 - k_3. \quad (9.5)$$

This last familiar form involves the four-momentum transfer between nucleons 1 and 3 leads to the Yukawa-type potentials that include the effects of retardation. It is also useful to explore the form of the energy denominator using light-front variables by first defining the plus-component, $P^+$, of the initial and final total momentum. We may also define $k_1^+ = xP^+$ and $k_3^+ = x'P^+$ in which $x$ and $x'$ ($x > x'$), as ratios of plus-momenta, are invariant under Lorentz transformations in the three-direction. Then using (9.3), we find
This quantity is also invariant under Lorentz transformations in the three-direction. This expression is to be used only if \( k_1^+ > k_3^+ \). If \( k_3^+ > k_1^+ \), then use a version of expression (9.6) in which \( x \) and \( x' \) are interchanged.

A straightforward evaluation of Eq. (9.6) using Eqs. (4.48–4.50) leads to the result

\[
\langle 3,4|K|1,2\rangle = 2 \langle 3,4|V|1,2\rangle \frac{M^2\delta^{(2,+)}(P_i - P_f)}{\sqrt{k_1^+ k_2^+ k_3^+ k_4^+}},
\]

(9.7)

where \( \delta^{(2,+)}(P_i - P_f) \equiv \delta^{(2)}(P_i - P_f)\delta(P_i^+ - P_f^+) \) and \( V \) is the standard expression for the sum of the \( \pi, \phi \) and vector meson exchange potentials:

\[
\langle 3, 4|V|1,2\rangle = \langle 3, 4|V(\phi) + V(\pi) + V(V)|1,2\rangle. \tag{9.8}
\]

The operator \( K \) is twice the usual two-nucleon potential times a factor which includes the light front phase space factor and a momentum-conserving delta function.

For the exchange of scalar and pseudoscalar mesons, only the term \( (v_1 + v_1^\gamma)g_0(P_i^-)(v_1 + v_1^\gamma) \) enters, and one finds

\[
\langle 3, 4|V(\phi, \pi)|1,2\rangle = \frac{\bar{u}(4)\Gamma u(2) \bar{u}(3)\Gamma u(1)}{4M^2(2\pi)^3(q^2 - \mu^2)}. \tag{9.9}
\]

The notation is that \( u(i) \) is the Dirac-spinor for a free nucleon of quantum numbers \( i \), and \( \Gamma \) is either of the form \( g_s \) or \( i g_\pi \gamma_5 \). The derivation of the contribution of vector meson exchange proceeds by including the meson exchange \( v_1g_0(P_i^-)v_1 \) plus the meson instantaneous term \( v_3 \), and the result takes the familiar form:

\[
\langle 3,4|V(V)|1,2\rangle = -g_\mu v_\mu v_1 \frac{\bar{u}(4)\gamma_\mu u(2)\bar{u}(3)\gamma_\mu u(1)}{4M^2(2\pi)^3(q^2 - m_\pi^2)}. \tag{9.10}
\]

The expressions (9.9) and (9.10) represent the usual \[2\] expressions for the chosen one-boson exchange potentials, if no form factor effects are included. The sum of the amplitudes arising from each of the individual one-boson exchange terms gives the invariant amplitude to second order in each of the coupling constants. The factors \( \frac{1}{4M^2} \) in Eqs. (9.9) and (9.10) can be thought of as re-normalizing the spinors so that \( \bar{u}u = 1 \), and the factors \( \frac{\sqrt{M}}{\pi} \) of Eq. (9.7) serve to further change the normalization to \( u^\dagger u = 1 \).

These amplitudes are strong, so computing the nucleon-nucleon scattering amplitude and phase shifts requires including higher order terms. One may include a sum which gives unitarity by including all iterations of the two particle irreducible scattering operator \( K \) through intermediate two-nucleon states. One first removes kinematic factors by defining a \( T \)-Matrix \( T \) using

\[
\mathcal{M} \equiv 2T \frac{M^2\delta^{(2,+)}(P_i - P_f)}{\sqrt{k_1^+ k_2^+ k_3^+ k_4^+}}, \tag{9.11}
\]

50
to find that
\[
\langle 3, 4|T|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \sum_{\lambda_5, \lambda_6} \int \langle 3, 4|V|5, 6 \rangle \frac{2M^2}{k_5^+k_6^- P_i^- - (k_5^- + k_6^-)} d^2k_5^+dk_6^- (5, 6|T|1, 2).
\]
(9.12)

One realizes that Eq. (9.12) is of the form an equation, similar that of Weinberg \cite{77}, derived for the scattering of nucleons by Frankfurt and Strikman \cite{2}, by expressing the plus-momentum variable in terms of a light-front momentum fraction \(\alpha\) such that
\[
p_5^+ = \alpha P_i^+,
\]
and using the relative and total momentum variables:
\[
k_\perp \equiv (1 - \alpha)k_5^\perp - \alpha k_6^\perp,
\]
\[
P_{i\perp} = k_5^\perp + k_6^\perp.
\]
(9.14)

Then,
\[
\langle 3, 4|T|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle \frac{2M^2}{\alpha(1 - \alpha) P_i^2 - \frac{k_5^2 + M^2}{\alpha(1 - \alpha)} + i\epsilon} d^2k_{i\perp}d\alpha (5, 6|T|1, 2),
\]
(9.15)

where \(P_i^2\) is the square of the total initial four-momentum, otherwise known as the invariant energy \(s\) and \(\frac{k_i^2 + M^2}{\alpha(1 - \alpha)}\) is the corresponding quantity for the intermediate state. Because the kernel \(V\) is itself invariant under Lorentz transformations in the three-direction and the integral involves \(k_\perp\) and \(\alpha\) the procedure of solving this equation gives \(T\) with the same invariance. Note that we use the labels \(k_i\) to designate momenta in the intermediate state, and \(k_i\) for the initial and final states.

Equation (9.15) can in turn be re-expressed (in the center of mass frame) as the Blankenbecler-Sugar (BbS) equation \cite{78} by using the variable transformation \cite{79}:
\[
\alpha = \frac{E(k) + k^3}{2E(k)},
\]
(9.16)
with \(E(k) \equiv \sqrt{k \cdot k + M^2}\), and \(P_i^2 = 4(k \cdot k + M^2)\). The result is:
\[
\langle 3, 4|T|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle \frac{M^2}{E(p) k_{i\perp}^2 - k^2 + i\epsilon} \frac{d^3p}{k_i^2 - k_5^2 + i\epsilon} (5, 6|T|1, 2),
\]
(9.17)

which is the desired equation. Rotational invariance is manifestly obeyed. The three-dimensional propagator is exactly that of the BbS equation. There is, one difference between Eq. (9.17) and the standard BbS equation. Our one-boson exchange potentials depend on the square of the four momentum \(q^2\) transferred when a meson is absorbed or emitted by a nucleon. Thus the energy difference between the initial and final on-shell nucleons is included and \(q^0 \neq 0\). This non-zero value is a consequence of the invariance of \(D\) of Eq. (9.6) under
Lorentz transformations in the three-direction. The usual derivation of the BbS equation from the Bethe-Salpeter equation specifies that \( q^0 = 0 \) is used in the meson propagator. Including \( q^0 \neq 0 \) instead of \( q^0 = 0 \) increases the range of the potential relative to the usual treatment, and its consequences are explored below. One can convert Eq. (9.17) into the Lippmann-Schwinger equation of non-relativistic scattering theory by removing the factor \( M/E(p) \) with a simple transformation [80].

A. Realistic One-Boson Exchange Potential

So far we have reviewed how the light front technique is used to derive nucleon-nucleon potentials in the one-boson exchange (OBE) approximation and use these in an appropriate wave equation (9.17). It is also true that this procedure yields potentials essentially identical to the Bonn OBEP potentials [73,74] and these potentials lead to a good description of the NN data [25].

The Bonn one-boson exchange potentials employ six different mesons, namely, \( \pi, \eta, \omega, \rho, \sigma \) and the (isovector scalar) \( \delta/a_0 \) meson. The present formalism can account for the \( \pi, \eta, \omega \) and \( \sigma \) in an approximately chiral invariant manner. We wish to add in couplings \( \bar{\psi} \tau \cdot \delta \psi \) and \( \bar{\psi} \tau \cdot \rho \gamma_\mu \psi \) in a chiral invariant manner. Simply adding such terms to the Lagrangian of Eq. (6.19) would lead to a violation of the approximate symmetry of Eq. (8.3). However, one can redefine the operator \( \bar{\psi} U \psi \) so that the symmetry remains. We replace the operator \( \bar{\psi} U \psi \) in the Lagrangian (6.19) by \( \bar{\psi} \tilde{U} \psi \)

\[
\tilde{U} \equiv e^{i \frac{1}{2} \tau \cdot \rho \gamma_\mu} e^{\frac{1}{2} \tau \cdot \delta} U e^{\frac{1}{2} \tau \cdot \rho \gamma_\mu} e^{\frac{1}{2} \tau \cdot \delta} e^{i \frac{1}{2} \tau \cdot \rho \gamma_\mu}. \tag{9.18}
\]

Then the new Lagrangian is invariant under the transformation

\[
\psi' \rightarrow e^{i \gamma_5 \tau a} \psi', \quad \tilde{U} \rightarrow e^{-i \gamma_5 \tau a} \tilde{U} e^{-i \gamma_5 \tau a}. \tag{9.19}
\]

In the applications the exponential is expanded to first order in the meson fields.

The final term to be included is the tensor \( \sigma_{\mu \nu} q^\nu \) part of the \( \rho \)-nucleon interaction. The presence of such a tensor interaction makes it difficult to write the equation for \( \psi_- \) as \( \psi_- = 1/p^+ \cdots \psi_+ \). This is a possible problem because the standard value of the ratio of the tensor to vector \( \rho \)-nucleon coupling \( f_\rho/g_\rho \) is 6.1, based upon Ref. [81]. Reproducing the observed values of \( \varepsilon_1 \) and P-wave wave phase shifts requires a large value \( f_\rho/g_\rho \); see Ref. [82]. However our Lagrangian generates such a term via vertex correction diagrams (which are the origin of the anomalous magnetic moment of the electron in QED). Thus the procedure of Ref. [25] was to simply add the necessary tensor terms to the one boson exchange potential. Recently Cooke [71] has provided the necessary light front quantization of a Lagrangian containing the tensor \( \sigma_{\mu \nu} q^\nu \) part of the \( \rho \)-nucleon interaction.

This brings us to the treatment of divergent terms used in Ref. [25]. The definition of any effective Lagrangian requires the specification of such a procedure. Meson \( (m) \) nucleon form factors \( F_m(q^2) \) were introduced which reduce the strength of the coupling for large values of \( -q^2 \). This is also the procedure of Refs. [73,75]. In principle, calculating the higher order terms using the correct Lagrangian can lead to consistent calculations of these form factors. We use a more phenomenological approach here.
The net result is that the one-boson exchange treatment of the nucleon-nucleon potential and the T-matrix resulting from its use in the BbS equation is essentially the same as the one-boson exchange procedure of Refs. [72–75]. The only difference is the keeping of the retardation effects—the square of the four-vector momentum transfer enters in our potentials.

B. Results for the two-nucleon system

Following established procedures [73,74], the coupling constants and cutoff masses of the six OBE amplitudes are varied within reasonable limits such as to reproduce the two-nucleon bound state (deuteron) and the two-nucleon scattering data below the inelastic threshold (about 300 MeV laboratory kinetic energy).

In Table III, we show the meson parameters for the Light-Front (LF) OBEP of Ref. [25] together with the predictions for the deuteron as well as low-energy neutron-proton (np) scattering. For comparison, we also give the parameters from an OBEP that was previously constructed and applied in the Dirac-Brueckner approach to nuclear matter [73,75]. The latter uses the Thompson formalism [83] which is very similar to the BbS formalism. Note that the Thompson OBEP uses \( n_\alpha = 1 \) also for vector meson form factors, which explains the differences in the vector meson cutoff masses between the two OBEP.

Phase shifts for np scattering are shown in Fig. 1 of the long paper of Ref. [25] for all partial waves with \( J \leq 2 \). A typical result is shown here in Fig. 8. Over-all, the reproduction of the NN data [84,85] by our LF OBEP is quite satisfactory and certainly as good as by OBEP constructed within alternative relativistic frameworks. Based upon these results, we applied this OBEP to the relativistic nuclear many-body problem.
TABLE III. Potential parameters and predictions for the deuteron and low-energy \( np \) scattering. For the deuteron, the binding energy \( B_d \), the \( D \)-state probability \( P_D \), the quadrupole moment \( Q_d \), and the asymptotic \( D \)-state over \( S \)-state ratio \( D/S \) are given. Low-energy \( np \) scattering is parameterized in terms of \( a_{np} \) and \( r_{np} \) in \( ^1S_0 \) and \( a_t \) and \( r_t \) in \( ^3S_1 \), where \( a \) denotes the scattering length and \( r \) the effective range. The nucleon mass is \( M = 938.919 \) MeV.

|                  | Light-Front OBEP | Thompson OBEP\(^a\) | Empirical\(^b\) |
|------------------|------------------|---------------------|-----------------|
|                  | \( m_\alpha \) (MeV)  | \( g^2_\alpha/4\pi \) \([f/g]\) | \( \Lambda_\alpha \)(GeV) | \( g^2_\alpha/4\pi \) \([f/g]\) | \( \Lambda_\alpha \)(GeV) | \( g^2_\alpha/4\pi \) \([f/g]\) |
| \( \pi \)       | 138.04           | 14.0                | 1.2             | 14.6            | 1.2             | 13.5 – 14.6       |
| \( \eta \)      | 547.5            | 3                   | 1.5             | 5               | 1.5             | \( \leq 5 \)       |
| \( \rho \)      | 769              | 0.9 [6.1]           | 1.85            | 0.95 [6.1]      | 1.3             | 0.6(1) [6.6 \( \pm \) 1.0] |
| \( \omega \)    | 782              | 24.5 [0.0]          | 1.85            | 20.0 [0.0]      | 1.5             | 24 \( \pm \) 5 \( \pm \) 7 |
| \( a_0 \)       | 983              | 2.0723              | 2.0             | 3.1155          | 1.5             |                     |
| \( \sigma \)    | 550              | 8.9602              | 2.0             | 8.0769          | 2.0             |                     |

Deuteron

|                  |                 |                    |                 |
|------------------|-----------------|-------------------|-----------------|
| \( B_d \) (MeV)  | 2.2245           | 2.2247             | 2.224575(9)     |
| \( P_D \) (%)    | 4.53            | 5.10              | —               |
| \( Q_d \) (fm\(^2\)) | 0.270\(^c\) | 0.278\(^c\)      | 0.2860(15)      |
| \( D/S \)        | 0.0250           | 0.0257             | 0.0256(4)       |

Low-Energy \( np \) Scattering

|                  |                 |                    |                 |
|------------------|-----------------|-------------------|-----------------|
| \( a_{np} \) (fm) | –23.745         | –23.747            | –23.748(10)     |
| \( r_{np} \) (fm) | 2.671            | 2.664              | 2.75(5)         |
| \( a_t \) (fm)   | 5.494            | 5.475              | 5.424(4)        |
| \( r_t \) (fm)   | 1.856            | 1.828              | 1.759(5)        |

\(^a\)Potential B of Brockmann and Machleidt [\(^75\)].

\(^b\)For more comprehensive information on the empirical data and references, see Table 4.1 and 4.2 of Ref. [\(^73\)].

\(^c\)Meson-exchange current contributions not included.
X. CORRELATED NUCLEONS IN INFINITE NUCLEAR MATTER

I begin with an outline of the procedure and then review the formalism of Ref. [25]. The first step is to derive a light front version of the nucleon-nucleon interaction. This is most easily done within the framework of the one boson exchange approximation. The resulting nucleon-nucleon potential \( V(NN) \) describes phase shifts reasonably well, as discussed in Sect.IX. The corresponding interaction density is \( V(NN) \). The basic Lagrangian density contains a free nucleon term \( L_0(N) \), a free meson term \( L_0(\text{mesons}) \) and an interaction term \( L_I(N, \text{mesons}) \) but does not contain \( V(NN) \). Thus one adds this term and subtracts it:

\[
\mathcal{L} = L_0(N) - V(NN) + \mathcal{L}_m
\]

\[
\mathcal{L}_m = L_I(N, \text{mesons}) + L_0(\text{mesons}) + V(NN).
\]

We use the term \( L_0(N) - V(NN) \) to obtain a first solution \( |\Phi\rangle \) to the many-body problem. The term \( \mathcal{L}_m \) accounts for mesonic content of Fock space, and we present [25] a scheme to incorporate the effects of \( \mathcal{L}_m \) and calculate the full wave function \( |\Psi\rangle \). Our procedure allows us to assess whether or not \( V(NN) \) has been chosen well. If it has, the effects of \( \mathcal{L}_m \) can be treated perturbatively.

Solving for \( |\Phi\rangle \) is no easy task—it demands a special non-perturbative treatment. One introduces a mean field \( U_{MF} \) which acts on single nucleons.

\[
L_0(N) - V(NN) = L_0(N) - U_{MF} + (U_{MF} - V(NN)).
\]

The operator \( U_{MF} \) is chosen to minimize the effects of \( \langle \Psi | U_{MF} - V(NN) |\Psi\rangle \). There is a well-known procedure, called Brueckner theory, which is used to determine \( U_{MF} \). In schematic terms:

\[
U_{MF} \sim G \times \rho,
\]

in which \( G \) is a nucleon-nucleon scattering matrix, as modified by the Pauli principle, \( \rho \) is the nuclear density, and the \( \times \) represents a convolution.

The result [25] is a rather complete theory in which the full wave function \( |\Psi\rangle \) includes the effects of both NN correlations and explicit mesons. This theory is reviewed next.

A. Nucleonic Truncation For The Many-Body Problem

One starts with a Lagrangian and can derive a light front Brueckner theory from first principles [25]. The nuclear wave function for the ground state of infinite nuclear matter at rest is defined as \( |\Psi\rangle \), and we wish to solve the equation

\[
P^-|\Psi\rangle = M_A|\Psi\rangle,
\]

in which \( P^- \) is the light-front Hamiltonian discussed above. For a nuclear system at rest we must have also

\[
P^+|\Psi\rangle = M_A|\Psi\rangle.
\]
The operator $P^-$ can be separated as follows:

$$ P^- = P_0^- (N) + J $$ \hspace{1cm} (10.7)

in which $P_0^- (N)$ is the kinetic contribution to the $P^-$ operator, giving $p^2 + m^2$ for the minus-momentum of free fermions. The operator $J$ is the sum of the terms of Eqs. (10.48)-(10.50), (5.13) and (5.14):

$$ J \equiv v_1 + v_2 + v_3 + v_1^\chi + v_2^\chi $$ \hspace{1cm} (10.8)

An approximate solution of Eq. (10.5) is obtained in two stages. First the nucleons-only part of the Hilbert space is constructed. This involves the assumption that using a nucleon-nucleon interaction $K$ accounts for the meson-nucleon dynamics. This assumption in then relaxed, and the formalism necessary to construct the best possible potential and to include meson degrees of freedom in the wave function is reviewed. Our Hamiltonian ($P^-$) contains no important terms in which the vacuum can spontaneously emit particles. (Some of the meson self interaction terms do involve non-zero creation from the vacuum \[38\], but their influence on the physics would enter only through a medium modifications of the nucleon mass which are not part of standard relativistic Brueckner theory.) This simplifying feature of the dominant terms causes the derivations to look very similar to those of non-relativistic theory, even though the treatment is relativistic.

All of the interactions in the Lagrangian are expressed in terms of the meson-nucleon vertex functions and contact terms represented by the operator $J$. The traditional approach is to introduce a two-nucleon potential and temporarily eliminate the meson degrees of freedom. This accomplished by subtracting and adding the two-nucleon potential to the Lagrangian. It is worthwhile to define the P-minus operator so obtained as $P_0^-$ with:

$$ P_0^- \equiv P_0^- (N) + K. $$ \hspace{1cm} (10.9)

The operator $K$ is given in Eq. (9.1). The complete P-minus operator is given by

$$ P^- = P_0^- + H_1, $$ \hspace{1cm} (10.10)

with

$$ H_1 \equiv J - K + P_0^- (m) $$ \hspace{1cm} (10.11)

where $P_0^- (m)$ accounts for the contributions to $P^-$ of mesons which do not involve interactions with the nucleons. The formal problem of choosing the best $K$ by minimizing the effects of $H_1 = J - K$ is discussed below in sub-section X.E.

The purely nucleonic part of the full wave function is defined as $| \Phi \rangle$, and is the solution of the light front Schroedinger equation

$$ P_0^- | \Phi \rangle = \left( P_0^- (N) + K \right) | \Phi \rangle = M_0 | \Phi \rangle. $$ \hspace{1cm} (10.12)

The eigenvalue problem stated above is considerably simpler than the initial one, but does contain the full complications of the nuclear many-body problem. We shall next review the derivation of the light front Brueckner Hartree-Fock \[86\] approximations.
B. Light Front Brueckner Hartree-Fock Approximation

The interaction $K$ that appears in Eq. (10.12) is strong and the scattering amplitude is obtained, as discussed in Section 9, by solving the light front version of the Lippmann-Schwinger equation which treats the interaction between two nucleons to all orders in $K$. One needs to find an all-orders treatment for the ground state. This can be done, and one can also find the Slater determinant $|\phi\rangle$, recall Eq. (10.13), that leads to the best approximation for the energy $M_0$ of the full nucleonic wave function $|\Phi\rangle$:

$$P_{MF}^{-} |\phi\rangle = P_0^{-}(N) |\phi\rangle = m_0 |\phi\rangle. \quad (10.13)$$

The operator $P_0^{-}(N)$ is expressed in terms of nucleon field operators defined by the single particle basis:

$$k^-|i\rangle = k^-_i|\phi\rangle \quad (10.14)$$

$$k^-_i = \frac{k_{1i}^2 + (M + U_S)^2}{k_i^+} + U_V. \quad (10.15)$$

This is a generalization of Eq. (5.3) in which self-consistent potentials, $U_S$ and $U_V$, to be defined below, replace the meson field terms, $g_S\phi$ and $g_V\bar{V}$, in the equation defining the nucleon mode functions. The details of the related spinors are given in Ref. [25].

Both of the states $|\phi\rangle$ and $|\Phi\rangle$ are eigenstates of a $P$-minus operator, and both are eigenstates of the operator $P_0^+(N)$. We shall use standard techniques to derive a perturbation theory which employs the appropriate transition matrix generated by $K$ to obtain an expression for the state $|\Phi\rangle$ in terms of $|\phi\rangle$. Thus we write

$$|\Phi\rangle = |\phi\rangle + \Lambda |\Phi\rangle \quad (10.16)$$

with

$$\Lambda \equiv 1 - |\phi\rangle\langle\phi|. \quad (10.17)$$

The use of algebra leads to the result

$$|\Phi\rangle = |\phi\rangle + \frac{1}{M_0 - \Lambda(P_0^{-}(N) + K)\Lambda} \Lambda K |\phi\rangle. \quad (10.18)$$

We can obtain a useful expression for $M_0$ by acting with the operator $\langle\phi | (P_0^{-}(N) + K)$ on the left of Eq. (10.18) and using $\langle\phi | \Phi\rangle = 1$. Then we find

$$M_0 = \langle\phi | P_0^{-}(N) + K |\phi\rangle + \langle\phi | K\Lambda \frac{1}{M_0 - \Lambda(P_0^{-}(N) + K)\Lambda} \Lambda K |\phi\rangle, \quad (10.19)$$

and with Eq. (10.13)

$$M_0 - m_0 = \langle\phi | K |\phi\rangle + \langle\phi | K\Lambda \frac{1}{M_0 - \Lambda(P_0^{-}(N) + K)\Lambda} \Lambda K |\phi\rangle. \quad (10.20)$$
Thus

\[ M_0 - m_0 = \langle \phi \mid X \mid \phi \rangle, \]  

with

\[ X = K + K \Lambda \frac{1}{M_0 - \Lambda P_0^-(N)} \Lambda X. \]  

(10.22)

The operator \( X \) is a many-body operator acting on all nucleons via the iterations of the two-nucleon interaction \( K \). We shall make the independent pair approximation of including only pair-wise interactions. Thus we approximate

\[ \langle \phi \mid X \mid \phi \rangle \approx \langle \phi \mid \sum_{i<j} \Gamma_{i,j}(P_{ij}^-) \mid \phi \rangle \equiv \langle \phi \mid \Gamma \mid \phi \rangle, \]  

(10.23)

where \( \Gamma_{i,j} \) is a two-nucleon operator which is a solution of the integral equation

\[ \Gamma_{i,j}(P_{ij}^-) = K_{ij} + K_{ij} \frac{\Lambda}{P_{ij}^- - \Lambda P_0^-(N)} \Lambda \Gamma_{i,j}(P_{ij}^-). \]  

(10.24)

The notation \( i,j \) refers to a pair of particles. The relevant matrix element is expressed using the eigenstates of Eq. (5.3) as

\[ \langle 3, 4 \mid \Gamma(P_{1,2}^-) \mid 1, 2 \rangle = \langle 3, 4 \mid K \mid 1, 2 \rangle + \sum_{\lambda_5,\lambda_6} \int \langle 3, 4 \mid K \mid 5, 6 \rangle \frac{2M^*}{k_5^+k_6^+} \frac{d^2k_{5\perp}dk_5^+ Q}{P_{1,2}^- - (k_5^- + k_6^-) + i\epsilon} \langle 5, 6 \mid \Gamma \mid 1, 2 \rangle, \]  

(10.25)

in which we define

\[ M^* \equiv M + U_S. \]  

(10.26)

The operator \( Q \) is the two-body version of \( \Lambda \) and projects the momenta \( k_5 \) and \( k_6 \) above the Fermi sea. The factor \( \frac{2M^*}{\sqrt{k_1^+k_2^+k_3^+k_4^+}} \) appears in each of the terms of Eq. (10.25), so it is worthwhile to define a Bruckner \( G \)-Matrix \( G \) using

\[ \Gamma \equiv 2G \frac{M^*2\delta^{(2,+)}(P_i - P_f)}{\sqrt{k_1^+k_2^+k_3^+k_4^+}}. \]  

(10.27)

To follow the steps of Sect. IX in converting Eq. (10.23) into one of a more familiar form, in which rotational invariance is manifest, one needs to know the values of

\[ P_{i,2}^- = k_1^- + k_2^-, \]  

(10.28)

which for the case of relevance here in computing the nuclear expectation value in the independent pair approximation, is the same as \( k_3^- + k_4^- \). The single-particle minus-momentum eigenvalues are given according to the light front Eq. (10.15). Our approximation is that
$U_V$ is independent of orbital $i$. Thus this potential cancels out in computing the difference $P_i^- - (k_5^- + k_6^-)$ and the energy denominator is as in the free space considerations of Sect. III, except that the mass of the nucleon is replaced by $M + U_S$. Thus the previous derivation of an equivalent three-dimensional integral equation that is manifestly covariant and rotationally invariant proceeds as before.

One expresses the plus-momentum variable in terms of a light-front momentum fraction $\alpha$ of Eqs. (9.16) and (9.14) so that one obtains

$$
\langle 3, 4|G|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle 2M^* \frac{d^2k_+ d\alpha}{\alpha(1-\alpha)} \frac{d^2k_{\perp} Q}{P_i} \left( \frac{k_i^2 + M^*}{\alpha(1-\alpha)} \langle 5, 6|G|1, 2 \rangle \right),
$$

(10.29)

where $P_i^2$ is square of the total initial four-momentum, computed using $M + U_S$ for the nucleon mass.

Equation (10.29) can in turn be re-expressed as a medium-modified Blankenbecler-Sugar (BbS) equation [78] by using the medium-modified version of the variable transformation [79]:

$$
\alpha = \frac{E_k^* + k^3}{2E_k^*},
$$

(10.30)

with $E_k^* = \sqrt{k^2 + M^*}$. The result is:

$$
\langle 3, 4|G|1, 2 \rangle = \langle 3, 4|V|1, 2 \rangle + \int \sum_{\lambda_5, \lambda_6} \langle 3, 4|V|5, 6 \rangle \frac{M^*}{E_k^*} \frac{d^3k Q}{k_i^2 - k_{\perp}^2} \langle 5, 6|G|1, 2 \rangle,
$$

(10.31)

which is the desired equation.

The Brueckner light front Hartree-Fock (BHF) approximation is defined by taking the mean field to be calculated from the average G-matrix according to

$$
U_l = \langle \bar{l} | (U_S + \gamma^0 U_V^0)^{BHF} | l \rangle = \sum_{m<F} \langle \bar{l} \ m | G | l \ m \rangle_a.
$$

(10.32)

The sum over occupied orbitals $l, m$ gives

$$
\sum_{l<F} U_l = 2 \langle \phi | G | \phi \rangle.
$$

(10.33)

We use this BHF mean-field to determine the value of $m_0$ via Eq. (10.13). Then the use of Eq. (10.23) in Eq. (10.21) determines the value of $M_0$, as the eigenvalue of $P_0^-$. But $M_0$ is also the eigenvalue (or in this case the expectation value) of $P_0^+$. The minimization of $P_0^-$ subject to the constraint that the expectation value of $P_0^+$ is the value of $P_0^-$ leads to (5.3):

$$
\frac{P_0^-}{\Omega} = \frac{4}{(2\pi)^3} \int_F d^2k_+ dk^+ \left \{ \frac{k_+^2 + (M + U_S)^2}{k^+} - 2 \frac{1}{2} \sum_{\lambda} \frac{\tilde{u}(k, \lambda)}{\sqrt{2k^+}} U_S \frac{u(k, \lambda)}{\sqrt{2k^+}} \right \} + \langle \phi | \Gamma | \phi \rangle
$$

(10.34)

$$
\frac{P_0^+}{\Omega} = \frac{4}{(2\pi)^3} \int_F d^2k_+ dk^+ k^+.
$$

(10.35)
Note that the quantity $k^+$ is defined as

$$k^+ \equiv E_k^* + k^3. \quad (10.36)$$

Taking the average of equations (10.34) and (10.35), and using the basis of Eq. (10.15) leads to our result for the BHF version of the nuclear mass:

$$M_0 = \sum_{l<F} \epsilon_l - \frac{1}{2} \sum_{l,m<F} \langle \bar{l}m | G | l m \rangle_a, \quad (10.37)$$

in which $\epsilon_l$ is the eigenvalue of the equal time self-consistent Dirac equation for the nuclear modes. This is equivalent to the usual expression of Ref. [73,75].

C. Results for Energy versus Density

The formalism of the previous section is used to calculate the energy per nucleon in nuclear matter as a function of density, Eq. (10.37). Our result is plotted in Fig. 9 by the solid line. The curve has a minimum value of $E/A = -14.71$ MeV at $k_F = 1.37$ fm$^{-1}$, and gives an incompressibility of $K = 180$ MeV at the minimum. These predictions agree well with the empirical values $E/A = -16 \pm 1$ MeV, $k_F = 1.35 \pm 0.05$ fm$^{-1}$, and $K = 210 \pm 30$ MeV [87].

To better understand our predictions, it is useful to compare with the results from alternative relativistic approaches. Brockmann and Machleidt [73] predict $E/A = -13.6$ MeV, $k_F = 1.37$ fm$^{-1}$, and $K = 250$ MeV at saturation, using the equal-time formalism and their ‘Potential B’. The greatest difference occurs for the incompressibility which is predicted
smaller by the light front Brueckner theory implying a softer equation of state. This can be partially attributed to the medium effect that comes from the meson propagators in the light front approach and that is absent in the equal time (ET) approach. In the ET approach the meson propagators are

\[ \frac{i}{-q^2 - m_\alpha^2} = \frac{i}{-(k' - k)^2 - m_\alpha^2}. \]  

(10.38)

In nuclear matter, the free-space LF meson propagators are replaced by

\[ \frac{i}{(E'^* - E^*)^2 - (k' - k)^2 - m_\alpha^2}, \]  

(10.39)

while the ET propagators undergo no changes. The medium effect on the LF meson propagators enhances them off-shell which leads to more binding energy. This is demonstrated in Fig. 9 where the difference between the dotted and solid curve is generated by the medium effect on the meson propagators.

There is another difference that arises from a technical issue in the solution of the transcendental equation for the G-matrix. We obtain new values of the mean fields

\[ M^*(k_F) = M_N + U_S = 718 \text{ MeV} \]  

and

\[ U_V = 165 \text{ MeV}. \]

The mean field potentials obtained here from the G-matrix are considerably smaller than those of mean field theory in which the potential is used. The implications for nuclear deep inelastic scattering are discussed in Sec. 10.G.

The most important medium effect in relativistic approaches to nuclear matter comes from the use of in-medium Dirac spinors representing the nucleons in nuclear matter (‘Dirac effect’). This effect (and the medium effect on meson propagators) is absent in the conventional (non-relativistic) Brueckner calculation which yields the dashed curve in Fig. 9. A characteristic for all non-relativistic Brueckner theory calculations are the results that the saturation density is too high, or the binding energy is too low.

**D. Meson degrees of Freedom**

The nucleonic wave function \(| \Phi \rangle\) of Eq. (10.18) gives us the purely nucleonic part of the Fock space, in which the effects of the mesons have been replaced by the two-nucleon interaction \(K\). However, the full wave function is \(| \Psi \rangle\) of Eq. (10.5). We need to assess whether \(| \Phi \rangle\) is a good approximation to \(| \Psi \rangle\) and to determine the mesonic plus-momentum distributions.

To obtain \(| \Psi \rangle\) recall the relation between the full \(P^-\) operator and the one of Eq. (10.12) \((P_0^- = P_0^-(N) + K)\) corresponding to the nucleonic wave function \(| \Phi \rangle\):

\[ P^- = P_0^- + J - K + P_0^-(m). \]

(10.40)

Using this in Eqs. (10.3) and (10.12) yields

\[ | \Psi \rangle = | \Phi \rangle + \frac{1}{M_A - \Lambda_\Phi P^- \Lambda_\Phi} \Lambda_\Phi (J - K) | \Phi \rangle \]  

(10.41)
where $\Lambda_\Phi = 1 - |\Phi\rangle\langle\Phi|$. 

An expression for the nuclear mass $M_A$ can be obtained by multiplying Eq. (10.41) by $\langle \Phi | P^- \rangle$ using $\langle \Phi | \Psi \rangle = 1$ to obtain the result:

$$M_A = \langle \Phi | P^- | \Phi \rangle + \langle \Phi | (J - K)\Lambda_\Phi \frac{1}{M_A - \Lambda_\Phi P^- \Lambda_\Phi} \Lambda_\Phi (J - K) | \Phi \rangle. \quad (10.42)$$

For the purely nucleonic wave function $|\Phi\rangle$ we have

$$\langle \Phi | P^- | \Phi \rangle = M_0 + \langle \Phi | (J - K) | \Phi \rangle, \quad (10.43)$$

so that

$$M_A = M_0 + \langle \Phi | J - K | \Phi \rangle + \langle \Phi | (J - K)\Lambda_\Phi \frac{1}{M_A - \Lambda_\Phi P^- \Lambda_\Phi} \Lambda_\Phi (J - K) | \Phi \rangle. \quad (10.44)$$

The difference between $M_A$ and $M_0$ is the expectation value of the operator $O$, which satisfies the integral equation

$$O = J - K + (J - K)\Lambda_\Phi G_0(M_A)\Lambda_\Phi O, \quad (10.45)$$

with

$$G_0(M_A) \equiv \Lambda_\Phi \frac{1}{M_A - \Lambda_\Phi P^- \Lambda_\Phi} \Lambda_\Phi. \quad (10.46)$$

The lowest relevant order of Eq. (10.45) is given by

$$O \approx J - K + (J - K)\Lambda_\Phi G_0(M_A)(J - K).$$

The one-boson exchange interaction $K$ is given by Eq. (9.1) (which now includes also the effects of Sect. 9.B) and we may determine if the expectation value $\langle \Phi | O | \Phi \rangle$ is reasonably small. If this is true, the quantity $M_0$ would be a good approximation to the true eigenvalue of the $P^-$ operator $M_A$. In the one-boson exchange approximation

$$\langle \Phi | O | \Phi \rangle \approx \langle \Phi | v_3 - K | \Phi \rangle + \langle \Phi | J G_0(M_A)J | \Phi \rangle \quad (10.47)$$

$$= \langle \Phi | v_3 - K | \Phi \rangle + \langle \Phi | (v_1 + v_1^\chi)G_0(M_A)(v_1 + v_1^\chi) | \Phi \rangle. \quad (10.48)$$

The use of Eq. (9.1) yields

$$\langle \Phi | O | \Phi \rangle \approx \langle \Phi | (v_1 + v_1^\chi) \left( G_0(M_A) - g_0(P^-_{ij}) \right) (v_1 + v_1^\chi) | \Phi \rangle, \quad (10.49)$$

where the term $P^-_{ij}$ is specified in Eqs. (10.25)-(10.13). But within the independent pair approximation (in which one includes only the energy (minus-momentum) differences for a chosen pair of nucleons)

$$G_0(M_A) \equiv g_0(P^-_{ij}), \quad (10.50)$$

and the expectation value of $O$ vanishes.

Thus within our approximations, it is consistent to say that the exact nuclear mass $M_A$ is well approximated by $M_0$. This means that we have shown that it is acceptable to remove the explicit mesons for calculations of the nuclear mass. The simplicity of the derivation of this result is made possible by the dynamical simplicity of the vacuum, which is one of the defining features of light front field theory.
E. Momentum distributions

We now indicate how to compute the + components of momentum. Look at $T^{++}$ as given by Eq. (4.39). The plus momentum carried by the scalar meson is given by

$$P^+(\phi) = \int d^2k \, k^+ a^\dagger(k)a(k), \quad (10.51)$$

while that of the pion is given by

$$P^+(\pi) = \int d^2k \, a^\dagger(k) \cdot a(k), \quad (10.52)$$

and that of the vector meson is given by

$$P^+(\omega) = \sum_{\omega=1,3} \int d^2k \, a^\dagger(k,\omega)a(k,\omega). \quad (10.53)$$

The procedure [25]. is to evaluate the expectation value of the number operators in the wave function $|\Psi\rangle$ of Eq. (10.41). The meson destruction operator annihilates the first term of that equation but finds a non-zero result when acting on the second term. Here only the scalar term is considered. The evaluation of (10.51) using (10.41) leads to

$$P^+(\phi) = \int d^2k \, k^+ \langle \Phi | J^\dagger(k) \frac{1}{(M_A - \Lambda P - \Lambda)^2} J(k) | \Phi \rangle$$

$$\approx \int d^2k \, k^+ \langle \Phi | J^\dagger(k) \frac{1}{(M_A - \Lambda P_0 - \Lambda)^2} J(k) | \Phi \rangle, \quad (10.54)$$

in which the approximation is motivated by the near equality of $M_A$ and $M_0$ and the expectation that the impulse approximation evaluation of the meson exchange potential is valid. The term $J(k)$ is defined in Eq. (7.20).

The operator $J^\dagger \cdots J$ to be evaluated has one and two body pieces. The one-body terms are related to a shift in the self energy of the nucleon caused by the medium. In infinite nuclear matter the ratio of pairs to single nucleons is infinite so that the number density is well approximated by the two nucleon terms of Eq. (10.51). The evaluation is simplified by the use of Eq. (10.50) and noting that the relevant matrix element is the same as occurring in the one-boson exchange operator $K$ except that the denominator is squared. Thus the momentum density $n_\phi(k)$, defined by

$$P^+(\phi) \equiv \int d^2k \, k^+ n_\phi(k), \quad (10.55)$$

is given as a derivative of the scalar-meson exchange contribution to the nucleon-nucleon potential:

$$n_\phi(k) \approx -2 \langle \Phi | \frac{\partial V_\phi(P\gamma_{ij},k)}{\partial P_{ij}} | \Phi \rangle \quad (10.56)$$

with
\[
\frac{\partial V_\phi(P_{ij}, k)}{\partial P_{ij}} \equiv \left[ J^I_i(k) \frac{1}{(M_A - \Lambda P^-_A)^2} J^I_j(k) \right] \tag{10.57}
\]

in which the notation \(i, j\) specifies that only two-nucleon contributions are included. Note that \(|\Phi>\) is the correlated ground state. Note that the expectation value is taken using the single particle basis specified by Eq. (10.15). Note that [24]

\[
-\frac{\partial}{\partial P^-_{ij}} \left( P^-_{ij} - P^-_0 \right) = \frac{k^+}{(q^2 - m^2_\phi)} \tag{10.58}
\]

This means that evaluating the plus-momentum distribution for scalar mesons is the same as evaluating the expression for the scalar meson contribution to the nuclear potential energy, except that the potential \(V_\phi(k)\) (the dependence on \(P^-_{ij}\) is suppressed) is multiplied by the factor \(-k^+/(q^2 - m^2_\phi)\). The net result is that

\[
n_\phi(k) = \sum_{l,m} \langle l m | \Omega^I_{lm} V_\phi(k) \frac{(-k^+)}{(q^2 - m^2_\phi)} \Omega_{lm} | l m \rangle_A, \tag{10.59}
\]

where \(\Omega_{lm}\) is the Moeller scattering operator for the two-nucleon state \(l m\). One finds a similar expression for the pionic terms.

We can get the total number of each kind of meson (except the \(\omega\)) using a sum rule. Consider the schematic form of the equation for the G-matrix

\[
G(P^-_{ij}) = V(P^-_{ij}) + V(P^-_{ij}) \frac{Q}{\Delta E} G(P^-_{ij}), \tag{10.60}
\]

where \(\frac{Q}{\Delta E}\) is a schematic representation of the propagator of Eq. (10.25). Differentiating with respect to the denominator \(P^-_{ij}\) appearing in a given meson \(m\) exchange potential yields the result:

\[
\frac{\partial G^m(P^-_{ij})}{\partial P^-_{ij}} \equiv (1 + G \frac{Q}{\Delta E}) \frac{\partial V^m(P^-_{ij})}{\partial P^-_{ij}} (1 + \frac{Q}{\Delta E} G), \tag{10.61}
\]

in which the label \(m\) refers to the type of meson. The potential \(V^m\) appearing in Eq. (10.60) is simply the Fourier transform of the potential \(V^m(q)\). Thus an examination of Eq. (10.59) and its pionic generalization that, considering the pion for example,

\[
N_\pi \equiv \int d^3q n_\pi(q) = \sum_{\alpha,\beta < F} \langle l m | \frac{\partial G^\pi(P^-_{ij})}{\partial P^-_{ij}} | l m \rangle_A. \tag{10.62}
\]

Numerical evaluation of Eq. (10.62) leads to the result that \(N_\pi/A = 0.05\). This is smaller than the 18% of Friman et al [88] because we use scalar mesons instead of intermediate \(\Delta\) states to provide the bulk of the attractive force.
F. Apparent Puzzle Resolved

The values of $U_S$, $U_V$ and $N_\pi$ allow us to assess the deep inelastic scattering of leptons from our version of the ground state of nuclear matter. Using $M^*(k_F) = 718$ MeV and neglecting the influence of two-particle-two-hole states to approximate $f(k^\pm)$ using Eq. (5.15) shows that nucleons carry 83% (as opposed to the 65% of mean field theory [22]) of the nuclear plus momentum. This represents a vast improvement in the description of nuclear deep inelastic scattering. The minimum value of the ratio $F_{2A}/F_{2N}$, obtained from the convolution formula (5.16) is increased by a factor of twenty towards the data as extrapolated in Ref. [45]. But this calculation provides only a lower limit of the nucleon contribution because of the neglect of effects of the two-particle-two hole states. We estimate that nucleons with momentum greater than $k_F$ would substantially increase the computed ratio $F_{2A}/F_{2N}$ because $F_{2N}(x)$ decreases very rapidly with increasing values of $x$ and because $M^*$ would increase at high momenta.

Turn now to the experimental information about the nuclear pionic content. The Drell-Yan experiment on nuclear targets [12] showed no enhancement of nuclear pions within an error of about 5%-10% for their heaviest target. No substantial pionic enhancement is found in (p,n) reactions. Understanding this result is an important challenge to the understanding of nuclear dynamics [13]. Here we have a good description of nuclear dynamics, and our 5% enhancement is roughly consistent [89], within errors, with the Drell-Yan data.

XI. COMING ATTRACTIONS

The preceding sections have been concerned with reviewing published work devoted to applications of light front dynamics with hadronic Lagrangians to heavy nuclear systems. Only the first studies of mean field theory and Brueckner theory have been completed. Many more detailed elaborations of these studies are possible and will be needed. Much work in applying light front dynamics to nuclear physics remains to be done. The purpose of the present section is to briefly discuss a few new directions of our research.

A. Quark-composite nucleons in the nucleus

The first new direction is to consider the nucleons as composite systems of quarks. So far nuclear effects enter in deep inelastic scattering only through the influence of the nucleon distribution function $f(y)$, recall Eq. (5.16). However the internal wave functions of the nucleons and therefore structure functions $F_{1,2}$ could be modified by the presence of the nuclear medium. One model which includes such effects is the quark meson coupling model (QMC) [54], initiated by Guichon and developed by Thomas, Saito and collaborators. Here the nucleons are treated as three quarks confined in a bag or under the influence of a confining potential [55]. The nucleus is bound by the exchange of mesons between quarks in different nucleons. This model has the nice property that the scalar and vector potentials are much weaker than in the usual QHD theory. This means that nucleons carry a large (but not unity) fraction of the nuclear momentum, so that one can describe the nuclear deep inelastic scattering data. We [90] have been developing a light front version of the theory. In
this model, the light quarks move relativistically within nucleons which move relativistically within the nucleus. The second part can be handled by using the techniques discussed in the present review. To proceed further, one needs to obtain a light front description of the three-quark system. The binding of two particles is well-described within light front dynamics, see the reviews and our Sect. 3. Therefore, as a first step, we consider the nucleon as a bound state of a quark and a di-quark. In a nucleus the quark moves under the influence of the interactions with other nucleons as well as under the confining effects of the di-quark. The preliminary results are that the saturation properties of the system as about the same as in the usual QMC model. This means that we will be able to evaluate nuclear deep inelastic scattering with nuclear modified versions of the nucleon structure function.

B. Few Body Problems

Applications of light front dynamics to nuclear physics began with studies of the deuteron [2, 28, 30]. The existence of many previous extensive applications, was one reason behind my decision to concentrate on heavy nuclei. However, there are many planned experiments on the deuteron. The Lagrangian approach of this review could be applied to the deuteron and also to bound states of three particles.

We have begun by doing some toy model calculations using a version of the Wick-Cutkosky model in which two heavy scalar particles are bound by the exchange of a light scalar particle [91]. This model had already been applied [92] in light front calculations, with considerable success in understanding rotational invariance in scattering problems. The first step of Ref. [91] was to study the breaking of manifest rotational invariance, present in light front dynamics. There can be many bound states in this model, so a test of rotational invariance is to reproduce the degeneracies of the eigenvalues. We found that the use of an effective Hamiltonian that takes into account two meson exchanges, leads to significant improvement in the degree to which the computed spectra have the correct degeneracies. This is much improved compared to results obtained when only one meson exchange is included in the effective Hamiltonian. The largest improvement occurs when the states are weakly bound.

A more detailed study of the ground state using carefully computed one and two boson exchange potentials [93] shows that the light front techniques reproduce the results of exact model calculations [94], provided the binding energy of the two particle system is small compared to the mass of the heavy particles.

The next step is to use our Lagrangian to describe deuteron form factors measured in electron and neutrino scattering. Much of the previous work has been devoted to using the impulse approximation to compute form factors measurable in electron- and neutrino-interactions with the deuteron. In the light front this means that one evaluates the matrix elements of the so-called good or \( J^+ \), \( A^+ \) components of the electromagnetic and axial-vector currents. We studied this some time ago [95] and found that the computed axial form factor is very sensitive to the choice of matrix element of \( A^+ \) used to extract the form factor \( F_A(Q^2) \). This is because of an inherent ambiguity arising from the spin-one nature of the deuteron: there are two independent matrix elements, but only one operator \( A^+ \) is used in the impulse approximation. The D-state of the deuteron was responsible for most of the numerical effect.
Keister [90] has studied relativistic effects in the deuteron axial form factor and confirms the dominant role of the D-state in observable effects beyond the non-relativistic limit. The present formalism, based on the use of a Lagrangian should allow us to compute the matrix elements of operators other than $J^+, A^+$. In practice, this means that including two-body exchange currents effects of the $|\pi NN\rangle$ Fock-state components will be necessary.

A needed important effort would be to apply light front dynamics to the three-nucleon and three-quark systems. Studies of this problem are in their infancy [97–99]. One can use the light front version of relative momentum variables and write done the light front version of the Fadeev equations. These should be solvable using modern high-speed computers.

C. Possible experimental signature-HERMES Effect

So far we have shown that light front dynamics can be applied to the physics of heavy nuclei. The standard good results for binding energy and density have been reproduced (with some improvement in the computed value of the compressibility of nuclear matter [25]). The salient new finding is that vector mesons carry a significant fraction of the nuclear plus-momentum, and that the meson fields can be treated in a quantum fashion that enables other matrix elements to be computed. The use of the simplest field theories leads to very strong meson fields, which account for an experimentally unacceptably large plus-momenta. This problem is ameliorated by the use of more complicated Lagrangians or by the use of Brueckner theory.

Thus it seems that nuclei can be well-described using the light front approach. But it would be better to go further. In particular, it would be highly desirable to find specific experimental evidence for a previously unappreciated feature of nuclei that is most conveniently computed within the framework of the light front approach. The HERMES collaboration discovery in high energy, low momentum transfer ($0.5 \text{ GeV}^2 < Q^2 < 2 \text{ GeV}^2$) positron-nucleus scattering that the value of $R \equiv \sigma_L/\sigma_T$ is enhanced (by a factor as big as 5) in nuclei [26] may provide the experimental evidence we desire.

The large nuclear value of $R$ occurs as the result of two significant effects: $\sigma_L$ is enhanced by a factor of two or more and $\sigma_T$ is depleted by as much as 50%. Since a small value of $R$ is the signature of spin 1/2 partons [100] it is natural to attempt an explanation of this unusual finding in terms of the mesonic field of nuclei. We found [101] we find that it is possible to reproduce the qualitative features of the data using the nuclear $\omega$ and $\sigma$ fields which are responsible for binding nuclei. A mechanism in which a virtual photon strikes a nuclear $\omega$ meson turning it into a $\sigma$ meson gives a large enhancement to $\sigma_L$. To reproduce the small value of $\sigma_T$ it is necessary to find a mechanism which interferes destructively with the dominant process of $\gamma^*$ conversion to a vector meson which is scattered onto its mass shell by the target. The process of a virtual photon striking a nuclear $\sigma$ meson and converting it into a vector meson can provide the necessary destructive interference. Both of these mechanisms depend on previously unmeasured photon-meson coupling constants and form factors, so that reproducing the HERMES data is not a necessary consequence of our approach. However, we are able to argue that the data do not violate laws of physics or or disagree with other experiments. The model of Ref. [101] contains many specific predictions which should be easy to test.
XII. SUMMARY

This paper begins with a discussion of the motivation for employing light front dynamics to compute the wave functions of nuclei, Sect. 1. The main reason is that high energy experiments are best analyzed in terms of the plus-momenta of the particles involved. Sect. 2 contains a brief description of light front dynamics which stresses the importance of Eq. (2.3) and the use of a Lagrangian to define the total momenta $P^+$ and $P^-$ which are the light front plus-momentum and $\tau$ development operators.

Since light front technology can appear cumbersome and recondite to a new reader, Sect. 3 is devoted to explaining how ordinary potentials [21] are handled using light front technology. The influence of relativistic corrections to means square radii of heavy nuclei is examined and found to vary as $A^{-5/3}$, Eq. (3.32). Light front nuclear momentum distributions are examined using a toy Hulthén model and the separate $p_+$ and $p_\perp$ dependences are interesting, Fig. 3.1.

Light front quantization of hadronic Lagrangians (without pions and chiral symmetry) is presented in Sect. 4. An important element is the transformation (4.31) of Soper [39] and Yan [40] of the Fermion fields used to separate the dependent and independent fermion degrees of freedom.

The formalism is applied to infinite nuclear matter under the mean field approximation, Sect. 5, [22]. Standard results for saturation properties are reproduced. The special feature of the light front formalism the use of the plus momentum as one of the canonical variables. This enables a close contact with the experimental variables used to analyse deep inelastic scattering and any experiment in which there is one large momentum. This feature is exploited here in the derivation (within the mean field approximation) of the nucleonic and mesonic distribution functions for infinite nuclear matter, Eqs. (5.14)-(5.18). The vector mesons are shown to carry a significant fraction of the nuclear plus momentum, but only a zero plus-momentum, and therefore do not participate in nuclear deep inelastic scattering or Drell-Yan experiments.

This restriction to zero plus momentum is investigated in Sect. 6, using a simple static model [23]. The feature that a static source in the usual coordinates corresponds to a source moving with a constant velocity in light front coordinates is exploited. The dependence of the vector meson distribution function on the nuclear radius is studied, with the key result reproduced in Fig. 6.1. The plus momenta are no longer restricted to zero, except in the limiting case of infinite nuclear matter. For finite nuclei, of radius $R_A$ the plus-momentum distributions are spread, with $k^+ \sim 1/R_A$. Thus experimental verification or disproof is possible.

Sect. 7 is devoted to reviewing the application of light front dynamics to finite nuclei [24]. The necessary technique is to minimize expectation value of the sum $P^- + P^+$. This leads to a new set of coupled equations (7.30) and (7.29) for the single nucleon modes. These depend on the meson fields of Eqs. (7.26) and (7.27). The most qualitatively startling feature emerging from the derivation is that the meson field equations (7.26) and (7.27) are the same as that of the usual theory, except that $z$ of the equal-time theory translates to $-x^-/2$ of the light-front version. This can be understood in a simple manner by noting that light-front quantization The general argument is that this result emerges from the feature that a static source in the usual coordinates corresponds to a source moving with a constant
velocity in light front coordinates, as in Sect. 6.

Even though the meson field equations of the light-front and equal-time theories are the same, there are substantial and significant differences between the two theories. In our treatment, the mesonic fields are treated as quantum field operators. The mean field approximation is developed by replacing these operators by their expectation values in the complete ground state nuclear wave function. This means that the ground state wave function contains Fock terms with mesonic degrees of freedom. We can therefore compute expectation values other than that of the field and obtain the mesonic momentum distributions, Table 7.2 and Fig. 7.2. This feature has been absent in standard approaches.

We obtain an approximate solution (7.34) of our nucleon mode equation. Our nucleon mode functions are approximately a phase factor times the usual equal-time mode functions (evaluated at \( x^- = -2z \)). This shows that the energy eigenvalues of the two theories should have very similar values. But the wave functions are different—the presence of the phase factor explicitly shows that the nucleons give up substantial amounts of plus momentum to the vector mesons.

A new numerical technique, is introduced in Ref. [24] to solve the coupled nucleon and meson field equations. Our results, Table 7.1, display the expected \( 2j_\alpha + 1 \) degeneracy of the single nucleons levels, and the resulting binding energies are essentially the same as for the usual equal-time formulation. This indicates that the approximation (7.34) is valid.

The present results for finite nuclei, which use the original Walecka model, are not consistent with experiments on lepton-nucleus deep inelastic scattering and \((e, e')\) reactions. This is because, in \(^{40}\text{Ca}\) for example, the nucleons carry only 72% of the plus momentum. This is a result of the quantity \( M + g_s \phi \), which acts as a nucleon effective mass, is very small, about 670 MeV. The use of a small effective mass and a large vector potential enables a simple reproduction of the nuclear spin orbit force [12,51]. However, the use of a different Lagrangian, including non-linear couplings between scalar mesons should provide significant improvement as indicated by the light front results for infinite nuclear matter, Eq. (5.18).

Another interesting possibility would be to obtain a light-front version of the quark-meson coupling model [54,55,90], in which confined quarks interact by exchanging mesons with quarks in other nucleons. This model, also has smaller magnitudes of the scalar and vector potentials.

In any case, these kinds of nuclear physics calculations can be done in a manner in which modern nuclear dynamics is respected, boost invariance in the \( z \)-direction is preserved, and in which the rotational invariance so necessary to understanding the basic features of nuclei is maintained.

Sect. 8 reviews how the light front quantization of a chiral Lagrangian can be accomplished. The resulting formalism can be applied to many problems of interest to nuclear physics. Soft pion theorems for pion-nucleon scattering, Eq.(8.24) are reproduced.

Sect. 9 reviews the our light front studies [24,25] of nucleon-nucleon scattering. The \( T \)-matrix is shown to be manifestly covariant, using the one boson exchange approximation to the nucleon-nucleon potential. An essential feature is the the meson propagators of Eqs. (9.4) and (9.4) contain the influence of retardation. Light front quantization is used to obtain a nucleon-nucleon potential which yields phase shifts in good agreement with data. See, for example, Fig. 9.2.

Sect. 10 reviews our derivation [25] of the Brueckner Hartree-Fock equations. Applying
our light front OBEP, the nuclear matter saturation properties are reasonably well reproduced. The binding energy per nucleon is 14.7 MeV and $k_F = 1.37\text{fm}^{-1}$. A value of the compressibility, 180 MeV, that is smaller than that of alternative relativistic approaches is obtained. This is largely due to the light front requirement that retardation terms be kept. The derivation starts with a meson-baryon Lagrangian, so we are able to show that replacing the meson degrees of freedom by a NN interaction is a consistent approximation, and the formalism allows one to calculate corrections to this approximation in a well-organized manner. The simplicity of the vacuum in our light front approach is an important feature in allowing the derivations to proceed. The mesonic Fock space components of the nuclear wave function are obtained also, and aspects of the meson and nucleon plus-momentum distribution functions are computed. We find that there are about 0.05 excess pions per nucleon.

There are promising implications of the Brueckner theory results for studies of nuclear deep inelastic scattering and Drell-Yan reactions. The nucleons probably carry about 90% of the nuclear plus momentum, enough for rough accord with data. Furthermore having 0.05 excess pions is consistent within experimental errors, with the Drell-Yan data.

A few future directions are discussed in Sect. 11. Despite the extreme length of this review, much work in applying light front dynamics to nuclear physics remains to be done. For example, only the first studies of mean field theory and Brueckner theory have been completed. Many more detailed elaborations for heavy nuclei, few-body systems and composite nucleons are possible and will be needed. I hope that this review will stimulate the reader to future research using light front dynamics.

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