On the linear filters for point source extraction of the Planck mission

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Accepted 2003 ???? ??; Received 2003 ???? ??

Abstract

The manifestation of point sources in the upcoming Planck maps is a direct reflection of the properties of the pixelized antenna beam shape for each frequency, which is related to the scan strategy, pointing accuracy, noise properties and map-making algorithm. In this paper we first compare analytically two filters for the Planck point source extraction, namely, the adaptive top-hat filter (ATHF), the theoretically-optimal filter (TOF). Our analyses are based on the premise that the experiment parameters the TOF assume and require are already known: the CMB and noise power spectrum and a circular Gaussian beam shape and size, whereas the ATHF does not need any a priori knowledge. The analyses show that the TOF is optimal in terms of the gain after the parameter inputs. We simulate the Planck HFI 100 GHz channel with elliptical beam in rotation to test the efficiency of the TOF and the ATHF. We also apply the ATHF on the WMAP Q-band map and the derived map (the foreground-cleaned map by Tegmark, de Oliveira Costa & Hamilton) from the WMAP 1-year data. The uncertainties on the angular power spectrum will harm the efficiency of the TOF. To tackle the real situations for the Planck point source extraction, most importantly, the elliptical beam shape with slow precession and change of the ellipticity ratio due to possible mirror degradation effect, the ATHF is computationally efficient and well suited for the construction of the Planck Early Release Compact Source Catalogue.

Key words: techniques: image processing (cosmic microwave background) methods: analytical

1 Introduction

The observation of cosmic microwave background (CMB) radiation by ESA Planck mission will be able to provide the understanding of our Universe with any key cosmological parameters with unprecedented precision. The signal received from the sky, however, will contain not only the CMB radiation, but also foreground emissions such as synchrotron, dust, free-free emission, Sunyaev-Zel’dovich effect from galaxy clusters and extra-galactic point sources. Removing foreground contaminations is therefore one of the main tasks for the Planck mission. Of these different foreground emissions, the purpose for extraction of extra-galactic point sources has two folds: to separate the foregrounds from the underlying CMB signals to achieve the scientific goal of the Planck mission as this type of foreground contamination is rather non-Gaussian. On the other hand, the whole sky coverage and a wide range of the Planck observing frequencies provide an opportunity for producing the extra-galactic point source catalogue, most notably, the proposed Early Release Compact Source Catalogue (ERCSC), which will be released in roughly 7 months when the full sky is covered once.

Recently, the NASA Wilkinson Microwave Anisotropy Probe (WMAP) experiment (Bennett et al. 2003a) has demonstrated the importance of the component separation tools for extraction of CMB power spectrum from the raw data. One of the sources with localized peculiarities of the CMB sky is from the point source contamination. For the WMAP mission, the catalogue of the radio point sources was obtained (Bennett et al. 2003b; Pierpaoli 2003) and the final CMB map was cleaned from the point source contribution.

The WMAP channels at high frequency correspond to the Planck LFI frequency range, in which the contamination of point sources seems 3 times higher than that from the WMAP. For the Planck HFI channels, the point sources are expected to be more significant regarding the issue of component separation and of point source catalogue.

In this paper, we compare the theoretically-optimal filter (Tegmark & de Oliveira-Costa 1998) (hereafter TOF) and the adaptive top-hat filter (Chiang et al. 2002b) (ATHF) proposed for extra-galactic point source detection. Theoreti-
cally speaking, the IER proposed by Tegmark & de Oliveira-Costa (1998) is optimal in terms of the gain factor. The gain factor is an indicator of the amplitude enhancement of point sources relative to the background. By ‘theoretically’ we mean that the required information input for this latter is known: the CMB and the noise power spectrum, and the circular beam shape and size. This latter is known as a matched filter in the field of signal processing.

A lot of effort has been devoted to the development of other linear IERS (Cayon et al. 2000; Vielva et al. 2001; Sanz, Herranz & Martnez-Gonzlez 2001; Vielva, Tenero & Wambsganss 2002; Barreiro et al. 2002; Vielva et al. 2003), namely the spherical M-exican Hat MHW), and the so-called pseudo-IER. Vielva et al. (2002) and Barreiro et al. (2002) have performed the comparison between the TOF and the pseudo-IER. Vielva et al. (2001, 2003) have applied the MHW on simulation data for the Planck mission.

The Planck in-flight beam shape properties are crucial in connection with the point source extraction. There are many issues regarding the in-flight antenna beam shape properties and its reconstruction (Burgana et al. 2001; Page, Dore & Bouchet 2002; Chiang et al. 2002a N. N. C. 2002). First of all, according to optics calculations the main beam is not circular Gaussian (Burgana et al. 2001), but elliptical, which is normally formulated as bivariate Gaussian functions. Secondly, with the most likely scan strategy, the inclination angle of the beam is not parallel through the sky due to possible precession of its spin axis, but is rather slowly rotating along circular scans. Note that this slow rotation of the antenna beam is different from the balloon-borne experiments such as MAXIMA-1, which has a fast rotating beam (Wu et al. 2001), or WMAP, in which the pixels have histograms of multiple orientations (Page et al. 2003), and therefore a resultant quasi-symmetric pixel beam. Furthermore, there could exist minor degradation effect during the approximately 15-month routine operation of the Planck mission (N. N. C. 2002), which will mean the change of the inclination angle and the shape of the beam. The combined effect from the above-mentioned factors will cause the in-flight beam shape to change with time not only its ellipticity ratio , where , and are the major and minor axis of the ellipse, but also the inclination angle. There are also some minor effects such as the issues on pointing accuracy and the noise properties that could also mean in the beam degradation. The beam which assume a circular Gaussian beam for point source extraction will need corresponding modifications.

The adaptive top-hat IER is also viable for real situations. The ATHF is similar to the so-called Gabor transform of the signal, using a adaptive width of the kernel. The ATHF has a top-hat shape with two cut-off scales working in harmonic domain. The two cut-off scales and in serve to cut down the unwanted power from both sides of the power spectrum. Simultaneously, the two scales retain the essential part of power spectrum where the convolution (by the beam) of point sources is most pronounced.

We also propose a median IERing technique for removing the point source from the maps. The median IER can be used on combination with any methods of point source detection in order to increase the precision of the CMB map reconstruction.

The layout of this paper is as follows. In Section 2 we discuss the antenna beam shape properties by stylily introducing the Planck scan strategy and the definition of the window function. In Section 3 we present the detailed analysis on the theoretically-optimal IER (TOF) (the Matched Filter) and the adaptive top-hat IER (ATHF). In Section 4 we compare by simulations the TOF and the ATHF on gain capability on more realistic situations and apply the ATHF on discussions and conclusion are in Section 5.

2 THE PLANCK ANTENNA BEAM SHAPE PROPERTIES

2.1 The scan strategy

The manifestation of point sources in the upcoming Planck maps is a direct extension of the properties of the pixelized antenna beam shape, which is related to the scan strategy, pointing accuracy, map-making algorithm and the extraction of the systematics from the timelimited data (TOD) set and pixelized maps as well. Before our analysis of point source problem, in particular, for the Planck Early Release Compact Source Catalogue (ERCC), we need to describe the Planck experiment when the TOD contain the information about the signal (and noise) from a large number of circular timelimited scans (Elabrouhi, Patanchon & Audibert 2002; Chiang et al. 2002a). Below we assume for simplicity that the systematics features are already removed and the instrumental noise is Gaussian white noise. In the temporal domain the observed signal is the combined signal of the CMB and foreground signals from the sky (hereafter beam-convolved sky signals), plus random instrument-related noise.

\[ m_1 = d_1 + n_1 \]

where

\[ \mathbf{X} = \mathbf{X}' \]

\[ d_1 = \mathbf{B}_{1/n} a_{n} Y_{n} (\mathbf{r}_i) \]

and \( \mathbf{B}_{1/n} \) is the multipole expansion of the timelimited beam \( \mathbf{B}_{1/n} (\mathbf{r}_i) \). In Eq. (2) \( a_{n} \) are the corresponding multipole coefficients of the CMB and foreground signal expansion on the sphere and \( Y_{n} \) are the spherical harmonics.

Following Tegmark & Efstathiou (1996) we also assume that map-making algorithm is linear. Thus the signal in each pixel \( s_p \) should have the following relation

\[ \mathbf{X} = \mathbf{M}_{1/p} \mathbf{s}_p \]

where \( \mathbf{M}_{1/p} \) is the corresponding pointing matrix and \( \mathbf{s}_p \) is the sky signal convolved by the pixel beam \( \mathbf{B}_{p/n} \).

\[ \mathbf{X}' = \mathbf{B}_{1/n} a_{n} Y_{n} (\mathbf{x}_p) \]

\[ \mathbf{s}_p = \mathbf{B}_{p/n} a_{n} Y_{n} (\mathbf{x}_p) \]

\( x_p \) is the two-dimensional vector with its components \( x_p \) and \( y_p \) denoting the location on the surface of the sphere. The definition of the pointing matrix \( \mathbf{M}_{1/p} \) depends on the scan strategy of an experiment. Below, as a basic model, we will use the model of the scan strategy of the Planck mission which was discussed by Burgana et al. (1998). Let \( s \) be the
Each hour the spin-axis is manoeuvred along the ecliptic same circle 60 times around the spin-axis at \( t = 1 \) r.p.m. Each hour the spin-axis is manoeuvred along the ecliptic plane by 26. Note that for our geometric model, during the spinning of the optical axis around the spin-axis in each step, the inclination of the beam relative to the optical axis are stable. Form modelling of the Planck antenna beam shape we will describe the simplest scan strategy with the spin axis right on the ecliptic plane without any additional (regular) modulation (e.g., it can be above or below the plane due to precession). This model extracts the geometrical properties of the beam asymmetry and their manifestation in the pixelized maps. Under the assumptions mentioned above we can now describe the elliptical beam shape models as follows:

\[
B_t(x, y) = \exp \left[ -\frac{1}{2} (R(U)^T D (R(U)) \right] ,
\]

where

\[
U = \begin{pmatrix} x \\ y \\ \pi \end{pmatrix} ;
\]

\[
R = \begin{pmatrix} \cos \theta & \sin \theta & 0 \\ -\sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{pmatrix} ,
\]

(7)

with \( \theta \) being the inclination angle between the \( x \) axis and the major axis of the ellipse. The matrix describes the beam dispersion along the ellipse principal axes, which can be expressed as

\[
D = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} ,
\]

(8)

The center of the Cartesian coordinate system is denoted by \( x_0 \) and \( y_0 \), at some location with \( z \) along the spin and \( x \) and \( y \) in the plane tangent to the celestial sphere. We will choose the standard position of the local Cartesian coordinate system with \( x \) axis parallel to the scanning direction.

For some time \( t = 2N_{rot} \), where \( N_{rot} = 60 \) is the number of the sub-scans, the spin axis of the satellite will be stepped along the ecliptic plane with the smallest deviation \( s = s_0 \). For a small part of the sky, we can neglect the rotation of the Cartesian coordinate system and choose the \( x \) and \( y \) axes parallel to \( x \) and \( y \) axis.

The received signals in each pixel depend on the orientation of the pixel beam \( B_t(x) \) and the location of its center \( (x_0, y_0) \). “For the upcoming Planck non-symmetrical spatially-dependent beam, the convolution of the signal with a symmetrical beam in the spherically produces a Gaussian signature coupled with the underlying signal, which acts as the estimation of an angular power spectrum. A thorough analysis on the real beam profile (including the side lobes) and its convolution with the signal is essential to the estimation of the sky power spectrum.”

We begin our analysis from the model of the time-dependent, elliptical beam shape which is applicable up to 30 dB level. At low amplitude part of the beam shape (\(< 30 \) dB) the complexity of the beam estimation increases dramatically. Moreover, due to possible precession of the spin axis (around the standard orientation), which can be described as some additional noise, it is possible to nd some transfor-
The window function can now be expressed as \( W_0^2(k) = \frac{1}{2} \left( \frac{\partial B}{\partial k} \right) B \) (Souradeep & Ratna 2001)

\[
W_0^2(k) = \frac{1}{2} \left( \frac{\partial B}{\partial k} \right) B \quad \text{or} \quad W_0^2(k) = \frac{1}{2} \left( \frac{\partial B}{\partial k} \right) B
\]

where \( k \) is the angle of wave vector \( \mathbf{k} \) in the k space such that \( k_x = k \cos \theta \) and \( k_y = k \sin \theta \). Moreover, for the at sky approximation (and without the influence of the instrument noise and any peculiarities of the scanning such as precension) we can use the assumption of stable beam orientation, \( k, \) const, with the corresponding Fourier image for the elliptical part of the beam shape,

\[
B(k) = \exp \left( \frac{2}{k^2} k_x \sin \theta + k_y \cos \theta \right)^2\frac{1}{2}
\]

where \( k \) is the symmetry parameter. In the at sky approximation, we can use the assumption of the mean beam profile, \( k = const. \) With the corresponding Fourier image for the elliptical part of the beam shape,

\[
B(k) = \exp \left( \frac{2}{k^2} k_x \sin \theta + k_y \cos \theta \right)^2\frac{1}{2}
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(27)

\[ C_s = C_s^{\text{sky}} = \frac{C_s^{\text{sky}}}{W}; \]

where \( C_s^{\text{sky}} \) is the sky signals including CMB and all foregrounds but the point source contribution, \( C_s^{\text{ps}} \) is the noise power spectrum, which is taken as \( (FWHM)^2 \), and \( W \) is the window function. Thus the combined (except point sources) power spectrum in the map is \( C_{\text{tot}} = C_s^{\text{sky}} - C_s^{\text{ps}} \). Note that the integration beam \( B \) and the window function \( W \) do not have any errors or biases caused by possible error degradation effects, the galactic foreground contaminations and the instrumental noise. Thus simple inversion of \( W \) for the power spectrum requires specific renormalization due to the antenna beam shape properties.

The idea to construct the theoretically-optimal filter (TOF) for point source extraction is outlined in TD 98. We can write down the signal as the sum of the point source contribution and the rest (TD 98)

\[ x(r) = g S_i B(r) + a_n Y_n(r); \]

(28)

where \( \delta \) is a Dirac delta function, \( S_i \) is the \( i \)th point source at the direction \( r, a_n \) is the \( n \)th clump of the signal density function with \( \int C \) as in Eq. (27), \( Y_n(r) \) the spherical harmonics, and

\[ g = \frac{1}{2k} \frac{2 \sinh \theta}{kT_{\text{CMB}}}; \]

(29)

The maximum of the signal-to-noise ratio \( (S_i/N_i) \) will give us the TOF shape, where \( S_i \) is the square root of the variance of the point source amplitudes and \( N_i \) now is the rms of the combined signal, both convolved by the TOF \( F \).

2 Similarly, we want to maximize

\[ \frac{P}{P_i} = \frac{g^2 B_n X P \text{S}_i S_i'}{(F \text{B} + F \text{C})} \]

(30)

where \( B_n \) are the clump of the spherical harmonic mode position of the beam pro \( B \), \( F \) is the multiple expansion off \( B_n \), and \( F \) is the beam. We thus can obtain the TOF for each Planck frequency channel.

Note that in Eq. (31) the linear assumption is made only in the direction of the Fourier rings and statistically homogeneous. Thus in the spherical harmonic mode position this linear assumption is valid only for the azimuthal number \( m \).

This condition is valid when the sky signal and the pixel noise are Gaussian. If the sky signal (or the pixel noise) is non-Gaussian, however, then Eq. (31) needs corresponding modification. For the following analyses we assume for simplicity that the Gaussian assumption is appropriate for the CMB plus foreground.

The determination of the TOF \( F \) is similar to the definition of functional derivatives of \( \phi \). Suppose that \( \phi \) corresponds to the maximization of the ratio \( R \) from Eq. (31), then a small variation \( \delta \) of the shape from \( \phi \) shall shift the function \( R(\phi + \delta) \) away from the maximum value \( R_{\text{max}} \), which is proportional to \( \delta \) and \( f_\text{max} \), if \( \delta \ll 1 \). So we have

\[ F = \frac{F_\text{max}}{f_\text{max}}; \]

(32)

and obtain the following from use from Eq. (31),

\[ R(\phi) = \frac{g^2}{\phi} \frac{S_i M + Q}{P}; \]

(33)

where

\[ X = \frac{2}{\phi} \frac{1}{\phi} \frac{W}{C} \frac{C}{g}; \]

(34)

\[ X = \frac{2}{\phi} \frac{1}{\phi} \frac{W}{C} \frac{C}{g}; \]

(35)

\[ Q = \frac{X}{\phi} \frac{2}{\phi} \frac{1}{\phi} \frac{W}{C} \frac{C}{g}; \]

(36)

\[ P = \frac{X}{\phi} \frac{2}{\phi} \frac{1}{\phi} \frac{W}{C} \frac{C}{g}; \]

(37)

where

\[ F = \frac{B_n}{W_0}; \]

(38)

As one can see from Eq. (35) and (36) the coefficients are also the functional of the TOF \( F \). Substituting Eq. (39) into Eq. (35) and (36) we obtain the following relationship between and functionals.

\[ \frac{2}{\phi} \frac{1}{\phi} \frac{W}{C} \frac{C}{g}; \]

(39)

Taking into account the definitions of the and functionals and Eq. (40), we reach the conclusion that the TOF \( F \) is the sum

\[ \frac{F_\text{max}}{W_0} \frac{B_n}{W_0} \frac{C}{g}; \]

(40)
If the TOF $\mathcal{F}$ corresponds to the global maximum of the gain (in the class of the analytic functions) then the second functional derivative $\mathcal{R} = \mathcal{F}$ should be negative at $\mathcal{F} = \mathcal{F}$. For all variations around $\mathcal{F}$ this corresponds to the condition $Q < 0$, according to Eq. (33). After substituting $\mathcal{F} = \mathcal{F}$ into Eq. (37) we get

$$Q = \left( \frac{X}{n} \right)^{\frac{1}{2}} \left( \frac{h_n}{h} \right)^{\frac{1}{2}} \left( \frac{h_n}{h} \right)^{\frac{1}{2}}$$

$$= \frac{1}{2} \left( \frac{X}{n} \right)^{\frac{1}{2}} \left( \frac{h_n}{h} \right)^{\frac{1}{2}} \left( \frac{h_n}{h} \right)^{\frac{1}{2}}$$

(43)

where $h = (2^4 + 1) = (4^2) \csc \psi ? \csc \psi$. As one can see from Eq. (43), for any values of the $\mathcal{F}$ functions, the function $Q$ is negative and the $\mathcal{F} \mathcal{R} = \mathcal{F}$ corresponds to the maximum gain in the functional space of the linear $\mathcal{F}$s.

The $\mathcal{F}$ shape of Eq. (41) for point source extraction is a generalization of the TD 98 $\mathcal{F}$, which is obtained under the assumption of circular Gaussian antenna beam shapes. The new element which we get for the complex Gaussian antenna beam shape in Eq. (41) shows that for the TOF construction, instead of the TD 98 m order, we need to know the maximum beam (over all $m$ from Eq. (23) beam principle), the window function $W$, and all the signal properties as well.

One of the most crucial factors acting the efficiency of the TOF is from the systematics of the $\mathcal{F}$ and secondary $\mathcal{F}$ inor surface during the $\mathcal{F}$ (Naselsky et al. 2002). As mentioned in the Introduction, the $\mathcal{F}$ inor degradation could change the ellipticity ratio and the beam size, so the calibration and reconstruction of the $\mathcal{F}$ beam $\mathcal{F}$ antenna beam shape will have direct consequence of Eq. (23) and the window function.

From a practical point of view, when we apply the TOF of Eq. (41) for the $\mathcal{F}$ beam $\mathcal{F}$ source extraction, in particular for the ERCSC, it is necessary to know precisely the power spectrum of the sky signals and the pixel noise properties. It is, however, only possible at the earliest after the analyses of component separation and $\mathcal{F}$ extraction when the $\mathcal{F}$ beam is completed. Conversely, for the realization of the $\mathcal{F}$ extraction it is necessary to know the window function $W$ for all frequency ranges. This means that for the ERCSC construction it is better to use primitive $\mathcal{F}$, which require as little information as possible about the properties of the antenna beam shape and the sky signals.

One possibility for the new class of the linear $\mathcal{F}$ comes from Eq. (34) at $M = 0$ but for the non-analytical shape of the $\mathcal{F}$, which is not in the scope of this paper. In the next section we will demonstrate that the maximization of the gain factor can be obtained using a simple top-hat $\mathcal{F}$.

### 3.2 The Adaptive Top-Hat Iter (ATHF)

Chiang et al. (2002b) introduce another class of iterative methods to the Gabor transform (Gabor 1946; Hansen, Gorski & Hivon 2003): the adaptive top-hat $\mathcal{F}$ (ATHF), which is implemented in the harmonic space with two adaptive cut-off parameters $\mathcal{F}^{(2)} = \left( \begin{array}{cc} \cdot & \cdot \\ \cdot & \cdot \end{array} \right)$, where $\mathcal{F}$ is a Heaviside function, $\mathcal{F}_{in}$, and $\mathcal{F}_{ax}$ are two adaptive parameters. The ATHF allows us to investigate the best class of non-analytical functions for the ERCSC. Point source extraction by the ATHF is based on the idea that each point source in the map (and the TOD as well) manifests itself as a non-Gaussian feature which is convolved with the beam shape. The goal, therefore, is to distinguish these non-Gaussian features from the CM B signal and pixel noise by using some general properties of the CM B map only. The ATHF is generalized from the amplitudes and phases of the signal and pixel noise by using some general properties of the CM B map only. The ATHF is based on the idea that each point source in the map (and the TOD as well) manifests itself as a non-Gaussian feature which is convolved with the beam shape. The goal, therefore, is to distinguish these non-Gaussian features from the CM B signal and pixel noise by using some general properties of the CM B map only. The ATHF is based on the idea that each point source in the map (and the TOD as well) manifests itself as a non-Gaussian feature which is convolved with the beam shape. The goal, therefore, is to distinguish these non-Gaussian features from the CM B signal and pixel noise by using some general properties of the CM B map only.
in pushing the gain factor higher. The steps of increasing $\gamma_{\text{ax}}$ are carried on up to the step where the gain factor dips after reaching a maximum value. We can then tune the $\gamma_{\text{ax}}$ parameter in the similar way by lowering it from $\gamma_{\text{ax}}^{(0)} = \gamma_{\text{ax}}^{(0)}$.

Tuning of the cut-o scale of the ATHF will produce the so-called Brownian motion of the CMB peaks, as compared to motions within the beam size for beam-convolved point sources (Chiang & Naozsky 2003), which is useful in lowering the threshold of the criterion for point source detection.

For practical implementation of the iteration scheme, however, it is not necessary to start from $(\gamma_{\text{in}}, \gamma_{\text{ax}}) = (\gamma_{\text{in}}^{(0)}, \gamma_{\text{ax}}^{(0)})$. In order to minimize the time needed for the realization of the iteration scheme, we can take as the first step from the suggested sets of $\gamma_{\text{in}}$ and $\gamma_{\text{ax}}$ for all the Planck channels (Chiang et al. 2002b). Below to obtain the axial gain from the ATHF, we can use the set of treatment at that for the TD 98 filter, but treating $k_{\text{in}}$ and $k_{\text{ax}}$ as variables. Again, the same assumptions are made on the properties of the signal and noise similar to the TD 98 filter and its generalization TOF Eq. (39), namely the mean beam, the window function, and the power spectra of the signal and noise are known. For analysis we will use the at sky approximation for a star all path of the sky. The generalization of the analysis for the whole sphere can be done without special oddication.

Let us start with Eq. (31) for the at path of the sky, the cut-o shape of the ATHF transforms the $k_{\text{in}}$ and $k_{\text{ax}}$ parameters on to the sums at both input in tovar and the aniator,

$$
R(\gamma_{\text{in}}, \gamma_{\text{ax}}) = g^2(\gamma_{\text{in}}) \frac{\mathcal{C}(\gamma_{\text{ax}})}{(\gamma_{\text{ax}})^2} \frac{d}{dk} \frac{1}{B(k)} \left[ 1 + \frac{k_{\text{ax}}^2}{k_{\text{in}}^2} \right],
$$

(44)

where the shape of the $\mathcal{C}(\gamma_{\text{ax}}) = (k_{\text{in}} k_{\text{ax}}) (k_{\text{in}} k_{\text{ax}} k)$ now is quite peculiar. By the definition, the perturbations of the $k_{\text{in}}$ and $k_{\text{ax}}$ parameters, $k_{\text{in}}$ and $k_{\text{ax}}$, produce the perturbations of the $\mathcal{C}$ shape.

$$
f(k) = \left( k_{\text{in}} k_{\text{in}} k_{\text{ax}} + k_{\text{ax}} k_{\text{ax}} k_{\text{in}} \right) f\left( k_{\text{in}}, k_{\text{ax}} \right) + \left( k_{\text{in}} k_{\text{ax}} + k_{\text{ax}} k_{\text{in}} \right) f\left( k_{\text{ax}}, k_{\text{in}} \right),
$$

(45)

As one can see from Eq. (45) these perturbations correspond to $\mathcal{F}^2(\gamma_{\text{in}} \gamma_{\text{ax}})$ at the ranges $k_{\text{ax}} k_{\text{ax}} k_{\text{ax}} k_{\text{in}}$ and $k_{\text{in}} k_{\text{ax}} k_{\text{in}} k_{\text{ax}}$. Thus the values of the $k_{\text{in}}$ and $k_{\text{ax}}$ parameters through the same treatment in the last section is not appropriate. From a theoretical point of view that means that for a given top-hat shape of the $\mathcal{C}$ the point of an $\gamma_{\text{ax}}$ is a of the gain parameter does not correspond to any solution for Eq. (34).

In order to find the optimal $k_{\text{in}}$ and $k_{\text{ax}}$ values we use instead the following conditions $R(\gamma_{\text{in}}, \gamma_{\text{ax}}) = 0$ and $R(\gamma_{\text{in}}, \gamma_{\text{ax}}) = 0$. For Eq. (44) these conditions lead to

$$
2B(\gamma_{\text{ax}}) \frac{d}{dk} \frac{1}{B(k)} \left[ 1 + \frac{k_{\text{ax}}^2}{k_{\text{in}}^2} \right] = 0,
$$

(46)

and

$$
2B(\gamma_{\text{in}}) \frac{d}{dk} \frac{1}{B(k)} \left[ 1 + \frac{k_{\text{in}}^2}{k_{\text{ax}}^2} \right] = 0.
$$

(47)

We get from Eq. (46) and Eq. (47)

$$
W^2(\gamma_{\text{in}}) C(\gamma_{\text{in}}) = B(\gamma_{\text{in}}) \frac{d}{dk} \frac{1}{B(k)} \left[ 1 + \frac{k_{\text{in}}^2}{k_{\text{ax}}^2} \right] C(\gamma_{\text{in}}) C(\gamma_{\text{in}}),
$$

(48)

Apart from a trivial solution in Eq. (48), with undetected value of the $k_{\text{in}} = k_{\text{ax}} = k_{\text{in}}$ there are nontrivial solutions. For comparison with other terms we use the same circular Gaussian model for the antenna beam and the window function, $W^2(k) = B(k)$, for $k = k_{\text{in}}$, we have $B(k_{\text{in}}) C(\gamma_{\text{in}}) C(\gamma_{\text{in}}) = B(k_{\text{in}}) C(\gamma_{\text{in}}) C(\gamma_{\text{in}})$. We can assume that $k_{\text{in}} = k_{\text{in}} < 1$.

$$
B(\gamma_{\text{in}}) = \exp\left( \frac{k_{\text{in}}^2}{2} \right),
$$

(49)

We further have the following two assumptions. At the range of $k < k_{\text{in}}$, the power spectrum of the combined...
signal. C (k, n) is mainly determined by the sky signals, and the pixel noise contribution is not significant, so

\[ C(k, n) = \frac{C^{\mathrm{sky}}}{k_{\mathrm{sky}}^2} \quad (50) \]

On the other hand, for the range of \( k < k_{\alpha x} \), the power spectrum is mainly determined by the pixel noise power spectrum, which leads to

\[ C(k, n) = \frac{C^{\mathrm{sky}}}{k_{\mathrm{sky}}^2} \quad (51) \]

The assumptions of Eq. (49), (50), and (51) are illustrated in Fig. 1, in which we show the simulated power spectrum for the Planck High Frequency Instrument (HFI) 100 GHz frequency channel. Eq. (50) and Eq. (51) lead to the following relationship between \( k_{\alpha x} \) and \( k_{\mathrm{sky}} \) parameter,

\[ \frac{C^{\mathrm{sky}}}{k_{\mathrm{sky}}^2} = \frac{C^{\mathrm{sky}}}{k_{\alpha x}^2} \quad (52) \]

which leads to

\[ k_{\alpha x} = \left( \frac{2}{5} \ln \frac{C^{\mathrm{sky}}}{C^{\mathrm{pix}}} \right)^{1/2} \quad (53) \]

The ratio \( R \) can then be obtained from Eq. (44)

\[ R_{A\mathrm{THF}} = \frac{g^2 X_i^2}{S^2_{1/2} C^{\mathrm{sky}}(k_{\alpha x})} \quad (54) \]

We can further solve Eq. (47) by inserting Eq. (7), which then becomes (with the integral part of \( C^{\mathrm{sky}} \) and \( C^{\mathrm{pix}} \) deformed)

\[ \frac{1}{\frac{2}{5} C^{\mathrm{sky}}(k_{\alpha x})} = \frac{1}{k_{\alpha x}^2} + \frac{1}{k_{\mathrm{sky}}^2} \quad (55) \]

If the \( C^{\mathrm{sky}} \) is \( A^3 \), Eq. (55) can be approximated and we reach

\[ k_{\alpha x} = k_{\mathrm{sky}} + \frac{1}{k_{\alpha x}^2} \quad (56) \]

Note that we use the simple trapezoidal rule for the integration, which works better when the index \( n \) is smaller.

4 APPLICATION OF THE AHTF ON SIMULATIONS AND WMAP DATA

4.1 Simulations of the Planck HFI 100 GHz channel

We carry out the simulations of the Planck HFI 100 GHz channel for point source extraction by the TOF and the AHTF. The simulation area is 25.6 deg², and the pixel size is 3.0 arc min. The \( C^{\mathrm{MB}} \) and the \( C^{\mathrm{PIX}} \) are 403 \( 10^5 \) and 627 \( 10^6 \), respectively. Point sources with amplitudes above 2 \( C^{\mathrm{MB}} \) at this channel would be washed out by both (symmetric) beams with the 5 cm beam (see the 5th column of Table 3 in Chiang et al. 2002b)), so we add randomly 50 point sources whose amplitudes are 1.5 \( C^{\mathrm{MB}} \) to check the efficiency. Obviously the number of point sources is too large for this frequency channel, but it serves as the demonstration of the efficiency of extraction. We specifically use an elliptical beam shape in the simulation with ellipticity ratio \( \alpha = 1.3 \) (Burigana et al. 1998). We designate the elliptical beam size such that \( \frac{a}{b} = 2/3 \), where \( a = \text{FWHM} = 8.1 \text{arcmin} \). For the HFI 100 GHz channel the
Figure 4. The adaptive top-hat iterated maps from the foreground-cleaned map (top) by Tegmark, de Oliveira-Costa & Hamilton (2003) and the WMAP Q-band map (bottom). The filtering ranges for both maps are shown in Fig. 3. In the top panel, the circled are peaks with amplitudes above 5 $S_p$ after filtering, which are the point source residues from the component separation. One interesting feature is that not all the residues we recover are extragalactic point sources. The dash circles are not peaks, but dips. In the bottom panel we retain the positions of the circles for comparison, from which those peaks are point sources. This is to prove that most of the peaks we retrieve from the FCM are point source residues.
FWHM is 10.7 arc min. In order to simulate more realistic situations, we put this beam in rotation while scanning across the simulation area in order to investigate how the elliptical beam could act on the efficiency of point source extraction for the TOF and the ATHF.

The period of rotation of the beam is 2 along both sides of the simulation square. Although the rotation period is too large for such size of beam, it nevertheless can elucidate the effect of elliptical beam shape regarding point source extraction.

Without exact information of the input beam size and orientation, the optimum form of the TOF is by inserting a circular beam function into Eq. (4.1), where the window function \( W = \exp(-R^2) \). In Fig. 2 we plot the point source extraction rate from the TOF for the 100 point sources. The plot is the extraction percentage of point sources against the inserted to the TOF (in term of 100 G Hz channel). We would like to examine, for in-ight elliptical beam shapes, how the supplied beam function for the TOF can act the extraction rate. In the dotted curve, the power spectrum is precisely known. The short-dash curve is the case when the slope of the CM power spectrum has 5 per cent core tail, whereas the long-dash line 5 per cent less tail. The horizontal line marks the ATHF extraction percentage. With the exact power spectrum, the TOF is able to detect 85 per cent of the point sources with an amplitude 15 CM when the supplied is chosen in between , and . For the case of 5 per cent core tail in power spectrum, the supplied beam function will need adjustment to a smaller value to ensure the TOF have a maximal resonance, and adjustment to a larger for the case of 5 per cent less tail. On the other hand, the ATHF can reach 74 per cent of extraction rate without any information. From Fig. 2 it is not surprising that the simple beam of the beam function is 0.1 for the uncertainties in the power spectrum. However, as the cleaning of foreground contain inaction precedes the detection in the angular power spectrum, and during the whole sky scan of the Planck mission, the possible degradations of the internal sources could change the beam size and shape, the e ciency of the TOF will be hampered.

4.2 The ATHF on the WMAP derived map

In this subsection we apply the ATHF to the derived map from the WMAP 1-year data. The derived maps are produced by Tegmark, de Oliveira-Costa & Hamill (2003) performing an independent component separation analysis from the WMAP beam. They are the foreground-cleaned map (FCM) and the WMAP cleaned map (WMF). The FCM was tested to be non-Gaussian (Chiang, Næsæs, Verhodanov & Way 2003), partly due to galactic emission. Here we investigate the possible point source residues from their component separation.

We use the ATHF on the FCM as it is the map with scientific significance. In Fig. 3 we show the power spectra of the FCM and the WMAP 1-year map with the corresponding beam functions (the shaded area) of the ATHF: \((h_{50}, h_{100}) = (200, 400)\) for the FCM and \((400, 900)\) for the WMAP Q-band map. It is easy to see that the CMB (plus foreground) power spectra (convolved with the beam) and noise level note that the presentation of the angular power spectrum in Fig. 3 is the standard form at, i.e. C with the factor \((1+1)^4\), which is different from Fig. 1 used for analysis in the previous Section. We therefore apply the ATHF with beam functions covering the con joinction of the CMB and noise power curves. The choice of the beam function follows the rule of the thumb described in Chiang et al. (2002b): the \(h_{50}\) to exclude most of the CMB power, the \(h_{100}\) the noise power, and both to keep the part being convolved by the beam.

Fig. 4 shows the extracted maps of the FCM (top panel) and the WMAP Q-band map (bottom). The circled in the top panel are circled points with no amplitudes above 5 . One interesting feature in Fig. 4 is that not all the residues we recover are extragalactic point sources. The dash circles are not peaks, but dips. In the bottom panel we retain the positions of the circles for comparison, from which those peaks are point sources. This is to prove that most of the peaks we retrieve from the FCM are point source residues. We do not intend to recover all residues from one single iteration of the ATHF but simply demonstrate the usefulness and quickness of the ATHF on the real data. Application of the ATHF on the WMAP 5-year channel maps will be on a separate paper.

One important issue related to elliptical beam − convolved point source extraction is the removal of the point source. Namely, circular Gaussian profile is used for cleaning the elliptical beam − convolved point sources from the beam after detection of the point source position. Due to the ellipticity of the beam and pixel noise, there are considerable residues after the cleaning by circular Gaussian profile. To eliminate the residual peaks, we apply a simple algorithm that is modified from the so-called hybrid median used in signal processing. The hybrid median replaces the targeting pixel with average of the neighboring pixels from the same row, column and two diagonals. In the modified cascading version, the targeted region (3 × 3 pixels) centered at the residual peak is replaced with the following algorithm (see Fig. 5). The central pixel is replaced by the median of the 8 pixels from the same row, column, and two diagonals right outside the targeted region. The four pixels at the four corners in the region can then be decided by the median of their corners, which exclude the already-replaced central pixel. The rest 4 pixels can be replaced by the median of their neighbouring pixels from the same row and column.

5 CONCLUSIONS

We have discussed analytically the three main linear -filters for point source extraction of the Planck mission. The TOF is optimal in terms of the gain factor. The so-called optimal pseudo- filter is at its best asymptotic at the far ends to the TOF under certain circumstance. Both of these linear filters require the experiment parameters such as the beam shape and size.

Note that the calculation is based on the simple geometric model of the beam. As we mentioned in the beginning,

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\[ \text{http://lamda.gsfc.nasa.gov/product/wmap} / \]
the replaced (a residual peak located at \((i;j)\)) from the cleaning by circular Gaussian pro le, we can apply the cascading hybrid median lter to replace the inner 3 x 3 pixels. First of all, we replace the central pixel \((i;j)\) by the median of the 8 pixels from the same row, column \(n\), and the diagonals outside the targeted region, i.e., by the median of pixels \((i;1)\), \((1;i)\), \((i+1;j)\), \((i;j+1)\), \((i+1;j+1)\), \((i+2;j)\), \((i+2;j+1)\), \((i+1;j+2)\), \((1;i+1)\), \((1;i+2)\), and \((i+2;i+2)\). Once the central pixel is decided, we can in turn replace the four corners by the median of its diagonals, e.g., the pixel \((i+1;j+1)\) can be replaced by the median of \((i+1;j+2)\), \((i;1;j+2)\), \((i+2;j+2)\), and \((i+2;j)\) and the replaced \((i;j)\). Then the last 4 pixels can be replaced by their neighbouring pixels of the same row and column \(n\), e.g., \((i+1;j)\) by the median of \((i+1;j+1)\), \((i+2;j)\), \((i+2;j+2)\) and the replaced \((i;j)\).

Figure 5. The cascading hybrid median ltering scheme. For a residual peak located at \((i;j)\) from the cleaning by circular Gaussian pro le, we can apply the cascading hybrid median lter to replace the inner 3 x 3 pixels. First of all, we replace the central pixel \((i;j)\) by the median of the 8 pixels from the same row, column \(n\), and the diagonals outside the targeted region, i.e., by the median of pixels \((i;1)\), \((1;i)\), \((i+1;j)\), \((i;j+1)\), \((i+1;j+1)\), \((i+2;j)\), \((i+2;j+1)\), \((i+1;j+2)\), \((1;i+1)\), \((1;i+2)\), and \((i+2;i+2)\). Once the central pixel is decided, we can in turn replace the four corners by the median of its diagonals, e.g., the pixel \((i+1;j+1)\) can be replaced by the median of \((i+1;j+2)\), \((i;1;j+2)\), \((i+2;j+2)\), and \((i+2;j)\) and the replaced \((i;j)\). Then the last 4 pixels can be replaced by their neighbouring pixels of the same row and column \(n\), e.g., \((i+1;j)\) by the median of \((i+1;j+1)\), \((i+2;j)\), \((i+2;j+2)\) and the replaced \((i;j)\).

ACKNOWLEDGMENTS

This paper was supported by D ann arks G rundbokningsfund through its support for the establishment of the Theoretical Astrophysics Center. The authors are grateful for Tegm ark et al. for their processed maps. We acknowledge the use of healpix (Gorski, H iron & W ander 1999) package and the gles code (O orshkevich et al. 2003) for whole-sky data processing. W e thank O. Verkhodanov for the help with the gles package.

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