Defining Rough Sets by Extended Logic Programs *

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Abstract. We show how definite extended logic programs can be used for defining and reason with rough sets. Moreover, a rough-set-specific query language is presented and an answering algorithm is outlined. Thus, we not only show a possible application of a paraconsistent logic to the field of rough sets as we also establish a link between rough set theory and logic programming, making possible transfer of expertise between both fields.

1 Introduction

This paper shows how the formalism of rough sets used for processing of uncertain and contradictory data relates to paraconsistent logic programming. This gives a basis for efficient implementation of rough sets in logic programming.

Since mid-eighties rough sets have been a subject of intensive research. The literature on rough sets includes both theoretical studies and reports on applications (for more information and bibliography see the home page of the Rough Set Society http://www.roughsets.org).

A rough set is usually defined by a decision table which can be seen as a finite collection of ground positive and negative datalog facts. The table may include both a positive fact and its negation, thus it may represent inconsistent information. The intuition is that a decision table defines a set, and the inconsistent facts identify elements with uncertain membership. A table may also include multiple occurrences of facts. This makes it possible to introduce a quantitative measure for membership. One of the main concepts is that of the lower approximation of a rough set, corresponding to the elements that the decision table asserts as being only positive examples.

Viewing rows of a decision table as datalog facts gives a basis for extending rough sets to Rough Datalog. In our previous work, we proposed such an extension. Rough datalog makes it possible to define rough sets not only explicitly

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as collections of facts (as the decision tables do) but also implicitly by rules. The fixpoint semantics of rough datalog links the predicates of a program to rough relations. However, having predicates denoting rough relations rather than relations may cause some difficulty in understanding the rules. Furthermore, the intuition of a rule as a definition of a rough set is quite complex, since it has to define both positive facts and negative facts of the defined rough set. Finally, compilation of datalog rules to Prolog, described in [7], may cause explosion of the number of Prolog clauses necessary to deal with negative facts.

In this paper we propose a simplified approach based on the concept of definite extended logic program (DXL programs) [4]. As mentioned above, decision tables for rough sets include explicit negative information. This information can be expressed in DXL programs by using explicit negation. Thus, DXL programs are well suited to represent rough sets. The fixpoint semantics of DXL determines then the rough sets specified by a given program. DXL programs can be easily implemented and queried in pure Prolog. However, DXL is not expressive enough for stating rough-set-specific queries. For example, in DXL it is not possible to query lower approximations of the defined rough sets. To achieve this we propose to extend DXL with a query language tailored for rough sets. We also show how to obtain answers by transforming the queries to usual Prolog queries.

The rough sets denoted by the predicates occurring in a program are similar to the paraconsistent relations used in [1]. The main aim of the work presented in [1] is to introduce an algebraic method to construct the well-founded model [5] for a general deductive database by using paraconsistent relations associated with each predicate symbol of the database. Since the well-founded model is always a consistent interpretation, predicates occurring in a general deductive database denote crisp sets [1]. However, in contrast to [1], the models of the programs proposed in our framework may incorporate contradictions. Consequently, while well-founded models are 3-valued, we use models in a 4-valued logic. Moreover, we deal with explicit negation.

The rest of the paper is organized as follows. Section 2 surveys some basic concepts of rough sets. Section 3 summarizes the semantics of DXL programs and gives an example of how rough sets can be represented via DXL programs. Section 4 discusses a rough-set-specific query language and proposes an algorithm to obtain answers. Section 5 gives some conclusions.

2 Rough sets

This section gives a brief introduction to Rough Sets.

We want to deal with the situation where there are conflicting judgments about classification of a given object. For example, two patients show identical results of clinical tests but one of them has a certain disease and the other does not have it, or the experts looking at the medical record of a patient may disagree on the diagnosis. The concept of rough set makes possible to express such a

\[\text{Using rough set terminology, a crisp set has an empty boundary region.}\]
situation. More precisely, the situation can be described as follows. We have a universe of objects, each of them characterized by a tuple of attribute values and by a decision attribute classifying the object. For simplicity, we assume two-valued (say “yes” or “no”) classification. This can be seen as a definition of a set consisting of all objects with the decision attribute “yes”. However, objects with identical attribute values may have different values of the decision attribute. Since we only access objects by their attribute values, the double classification describes the boundary region, where we cannot be sure whether the object belongs to the defined set or not. Thus intuitively, a rough set $S$ is defined by indicating the elements of a universe which belong to $S$ and elements which do not belong $S$, while these two categories need not be disjoint. Usually, it is also assumed that the union of these categories covers the universe. In practice this may be achieved by the assumption that the elements which do not appear in the decision table are implicitly classified as not belonging to the defined set. In this paper we do not make this assumption. This makes it possible to distinguish between the tuples whose membership in $S$ is explicitly negated and those for which we have no membership evidence. This distinction is well known in the field of logic programming, while it seems not to be discussed in the context of rough sets. Notice that our assumption does not exclude the possibility that the union of both categories covers the universe, and thus generalizes the usual approach.

**Example 1.** The following table contains patient records with the symptom attributes *temperature*, *cough*, *headache*, *muscle-pain* and the diagnosis done by a doctor which says whether or not the patient has flu. The table defines a rough set, since it includes different diagnoses for some cases with identical symptoms.

| temp   | cough | headache | muscle-pain | flu  |
|--------|-------|----------|-------------|------|
| normal | no    | no       | no          | no   |
| subfev | no    | yes      | yes         | no   |
| subfev | no    | yes      | yes         | yes  |
| subfev | yes   | no       | no          | no   |
| subfev | yes   | no       | no          | yes  |
| high   | no    | no       | no          | no   |
| high   | yes   | no       | no          | no   |
| high   | yes   | no       | no          | yes  |
| high   | yes   | yes      | yes         | yes  |

The intuitions discussed above can be formalized by the following definitions.

An attribute $a$ is a function $a : U \rightarrow V_a$, where $U$ is a universe of objects. The set $V_a$ is called the *value domain* of $a$.

We assume that tuples of values provide the only way of referring to objects. Two objects are *indiscernible* with respect to a selected set of attributes, if both have the same values for these attributes. Clearly, the indiscernibility relation is an equivalence on objects and its equivalence classes are sets of objects which are characterized by identical tuples of attribute values. We assume that the tuples
provide the only access to objects. Hence, the technical definitions are expressed in terms of tuples. As illustrated by the table above, we specify a rough set \( S \) by classifying tuples of attribute values as positive or negative examples.

**Definition 1.** A rough set \( S \) is a pair \( (S^+, S^-) \) such that \( S^+, S^- \subseteq V_{a_1} \times \cdots \times V_{a_n} \), for some non-empty set of attributes \( \{a_1, \ldots, a_n\} \).

The components \( S^+ \) and \( S^- \) will be called the positive region (or the positive information) and the negative region (or the negative information) of \( S \), respectively.

We will also use the following notion.

**Definition 2.** A rough complement of a rough set \( S = (S^+, S^-) \) is the rough set \( \overline{S} = (S^-, S^+) \).

Using rough set terminology, given a rough set \( S = (S^+, S^-) \), the sets \( S^+ \) and \( (S^+ - S^-) \) correspond to the upper approximation and the lower approximation of \( S \), respectively. Thus, the approximations of \( \overline{S} \) are: \( S^- \) (the upper approximation) and \( (S^- - S^+) \) (the lower approximation). The set \( S^+ \cap S^- \) is called the boundary (region) of \( S \). Intuitively, the lower approximation of \( S \) (or \( \overline{S} \)) refers to the elements that can certainly be classified as (not) members of \( S \). The elements in the boundary may belong to \( S \) (or \( \overline{S} \)), but we cannot be sure. It is easy to see that both the lower approximation and boundary of \( S \) are subsets of the upper approximation of \( S \).

The following definition, adopted from [1], formalizes the idea of the decision system (also called decision table) used to define rough sets.

**Definition 3.** A (binary) decision system is a pair \( D = (U, A \cup \{d\}) \), where \( U \) is a universe of objects, and \( A \cup \{d\} \) is a non-empty finite set of attributes, such that \( d : U \rightarrow \{\text{true, false}\} \). We allow that for some \( u \in U \) all attribute values, including the value of \( d \), are undefined.

For a given \( u \in U \) and set of attributes \( A = \{a_1, \ldots, a_n\} \), we denote by \( A(u) \) the tuple \( \langle a_1(u), \ldots, a_n(u) \rangle \). Recall that \( A \) may be undefined for some \( u \). Thus, \( A \) is a partial function on objects.

**Definition 4.** A rough set \( D \) specified by a decision system \( D = (U, A \cup \{d\}) \) is a pair \( (D^+, D^-) \), where

\[
D^+ = \{ A(u) | u \in U \text{ and } d(u) = \text{true} \}, \\
D^- = \{ A(u) | u \in U \text{ and } d(u) = \text{false} \}.
\]

**Example 2.** Consider the rough set Flu specified by the decision system of Example 1.

It is easy to check that the lower approximation of Flu is the singleton \( \{\langle \text{high, yes, yes, yes} \rangle\} \). The set \( \{\langle \text{normal, no, no, no} \rangle, \langle \text{high, no, no, no} \rangle\} \) is the lower approximation of the rough set \( \overline{\text{Flu}} \). The boundary region of \( \text{Flu} \) consists of all other remaining tuples in the decision table.
A binary decision system can be equivalently represented by a set of literals. We illustrate the idea on the decision table of Example 1. We assume $\text{flu}$ to be a 4-ary predicate letter. Each row of the table is then represented by a literal with the argument values stated in the row. The literal is positive if the decision attribute's value is “yes” and negative otherwise. Thus, we obtain the set of literals:

$$\{-\text{flu}(\text{normal}, \text{no}, \text{no}, \text{no}), -\text{flu}(\text{subfev}, \text{no}, \text{yes}, \text{yes}), \text{flu}(\text{subfev}, \text{no}, \text{yes}, \text{yes}), \cdots\}.$$ 

### 3 Definite Extended Logic Programs

This section recalls the concept of Definite Extended Logic Programs and relates them to rough sets. Definite extended logic programs extend classical definite logic programs with explicit negation. Similar ideas were discussed by many authors, see e.g. [3,10,11,12]. We follow here the presentation of the survey paper [4].

As discussed above, a rough set $S$ can be defined by providing explicitly a set of literals with the same predicate letter. The positive literals (e.g. $s(t_1, \cdots, t_n)$) identify the tuples in the positive region of $S$, while the negative literals (e.g. $\neg s(t_1, \cdots, t_n)$) determine its negative region. This can be seen as an alternative representation of a decision system.

Definite Extended Logic Programs provide a more general way of defining sets of literals.

**Definition 5.** A definite extended logic program (DXL program) is a set of rules of the form

$$H : -B_1, \cdots, B_n. \quad (n \geq 0)$$

where $H, B_1, \cdots, B_n$ are literals.

Notice that rules extend definite clauses by allowing negative literals, both in the head and in the body. In the sequel, the rules with empty bodies (facts) will be written in the form $H$.  

The semantics of DXL programs is defined by viewing each negated literal $\neg p(t_1, \cdots, t_n)$ as a positive literal $p^-(t_1, \cdots, t_n)$, with a new predicate symbol $p^-$. In this way, a DXL program $\mathcal{P}$ is transformed into a definite program $\mathcal{P}'$. The standard least Herbrand model semantics $\mathcal{M}_{\mathcal{P}'}$ of $\mathcal{P}'$ is a set of ground atoms, over the original and the new predicate symbols. The semantics of the DXL program $\mathcal{P}$, $\mathcal{M}_{\mathcal{P}}$, is defined by replacing each atom of the form $p^-(t_1, \cdots, t_n) \in \mathcal{M}_{\mathcal{P}'}$ by the corresponding negative literal $\neg p(t_1, \cdots, t_n)$.

Clearly, in general $\mathcal{M}_{\mathcal{P}}$ may include an atom together with its negation. Thus, a DXL program $\mathcal{P}$ may introduce inconsistencies. This is what is needed to be able to define rough sets. Each predicate symbol $p$, with arity $n \geq 0$, occurring in $\mathcal{P}$ denotes the rough relation (set)

$$\mathcal{P} = \{(t_1, \cdots, t_n) \mid p(t_1, \cdots, t_n) \in \mathcal{M}_{\mathcal{P}}\}, \{(t_1, \cdots, t_n) \mid \neg p(t_1, \cdots, t_n) \in \mathcal{M}_{\mathcal{P}}\}.$$
For a model theoretic semantics for DXL programs based on the four-valued Belnap’s logic the reader is referred to [2].

We now show an example of a definition of rough sets by a DXL program.

**Example 3.** We consider the rough relation Flu of Example 1 and a rough relation Patient with the same attributes as Flu extended with the new ones: identification, age and sex. Intuitively, the universe of relation Patient is a set of people who visited a doctor. Its decision attribute shows whether a person has to be treated for some disease and, therefore, has to be considered a patient. The decision may be made independently by more than one expert. All decisions are recorded, what might make the relation rough. The example relation is defined by the following decision table.

| id | age | sex | temp  | cough | headache | muscle pain | patient |
|----|-----|-----|-------|-------|----------|-------------|---------|
| 1  | 21  | m   | normal| no    | no       | no          | no      |
| 2  | 51  | m   | subfev| no    | yes      | yes         | yes     |
| 3  | 18  | f   | subfev| no    | yes      | yes         | no      |
| 3  | 18  | f   | subfev| no    | yes      | yes         | yes     |
| 4  | 18  | m   | high  | yes   | yes      | yes         | yes     |

In order to know who are the people to be treated for flu, we define a new rough set $F_t$. Intuitively, these are people possibly qualified as patients, who may have flu according to the decision table of Example 1. We may also state that the people not treated for flu are those not qualified as patients; or those qualified as patients who may not have flu.

This can be expressed as the following DXL program $P$.

\[
ft(Id) :\neg\text{patient}(Id, Age, Sex, Fev, C, Ha, Mp), \\
\text{flu}(Fev, C, Ha, Mp).
\]

\[
\neg ft(Id) : \neg \text{patient}(Id, Age, Sex, Fev, C, Ha, Mp).
\]

\[
\neg ft(Id) : \text{patient}(Id, Age, Sex, Fev, C, Ha, Mp), \\
\neg \text{flu}(Fev, C, Ha, Mp).
\]

As explained above, the semantics of this program determines the rough relation $F_t$. Thus, we can conclude that person 4 is definitely qualified for flu treatment (i.e. belongs to the lower approximation of $F_t$). Persons 2 and 3 may or may not be treated for flu (i.e. belong to the boundary of $F_t$) and person 1 is not certainly qualified for flu treatment (i.e. belongs to the lower approximation of $\neg F_t$).

**4 Rough Set Queries**

The transformed version $P'$ of a DXL program $P$, defined above, may be used by a Prolog system for answering queries about rough sets. Notwithstanding
the incompleteness of Prolog we conclude, that whenever the query evaluation terminates and succeeds, we obtain an answer showing an instance of the query consisting of the elements of the least model.

Other systems exist that can answer queries w.r.t to a normal program, for instance, XSB-Prolog (for more details see http://xsb.sourceforge.net/). Hence, also those systems could be used to implement our query answering algorithm.

4.1 A Query Language for Rough Sets

Since the proposed query answering technique refers to the least model of the transformed program, in the terminology of rough sets the answer concerns the upper approximations of the defined rough sets. For example, consider program $P$ of Example 3, the answer yes to the query $\texttt{?}(P, \texttt{ft}(4))$ means that person 4 belongs to the upper approximation of the rough set $\text{Ft}$. However, it may also be important to check whether a given element is in the lower approximation of a rough set, or what are the elements in the boundary region of a given set. Thus, we propose to extend DXL with the following rough set specific queries.

**Definition 6.** A rough query $Q$ is a pair $\texttt{?}(P, q)$, where $P$ is a DXL program and $q$ is defined by the following abstract syntax rules

$$q \rightarrow q' \mid a?$$

$$q' \rightarrow l \mid \exists \mid q_1, q_2,$$

where $l$ is a literal and $a$ is an atom.

Let $Q = \texttt{?}(P, q)$ be a query, given a DXL program $P$. Then, $Q$ is a simple query if $q$ is a literal $l$, or of the form $\exists$ or $\forall$, where $a$ is an atom. A composite query is a sequence of simple queries, separated by commas. A composite query is interpreted as a conjunction of simple queries.

Let $P$ be a DXL program and $R$ be the rough relation denoted by predicate $r$ of $P$. First, we explain intuitively how the answer to a ground simple query can be obtained. The answer to a ground simple query may only be yes or no.

- The answer to a query $\texttt{?}(P, r(t_1, \ldots, t_n))$ $\texttt{?}(P, \neg r(t_1, \ldots, t_n))$ is yes iff the tuple $(t_1, \ldots, t_n)$ belongs to the positive region (negative region) of the rough relation $R$, defined by $P$. Otherwise, the answer is no.
- The answer to a query $\texttt{?}(P, \exists(t_1, \ldots, t_n))$ $\texttt{?}(P, \forall(t_1, \ldots, t_n))$ is yes iff the tuple $(t_1, \ldots, t_n)$ belongs to the lower approximation of $R$ ($\neg R$). Otherwise, the answer is no.
- The answer to a query $\texttt{?}(P, \texttt{r}(t_1, \ldots, t_n))$ is yes iff the tuple $(t_1, \ldots, t_n)$ belongs to the boundary region of $R$. Otherwise, the answer is no.

The ground query $\texttt{?}(P, r(t_1, \ldots, t_n))$ questions what is known about atom $r(t_1, \ldots, t_n)$ in the least model of $P$. Four cases are possible. Tuple $(t_1, \ldots, t_n)$
may belong to the boundary region of the denoted rough set $R$, to its lower approximation, to the lower approximation of $\neg R$, or to none of these. The respective answers will be: $\top$, yes, no, and $\bot$. Notice that $\top$ represents the existence of contradictory information and $\bot$ represents absence of information. Although this kind of queries are not strictly needed because the same information can be obtained with several simple queries, they might be useful in practice. For instance, for a given $n$-ary predicate $r$, the query $\? (P, r(X_1, \cdots, X_n)\?)$ classifies all possible $n$-ary tuples with respect to the membership of the rough relation denoted by $r$.

A natural extension to non-ground simple queries $\? (P, q)$ case (i.e. $q$ contains some variables) gives as answer the set of all valuations $\theta$ for which the query instance $\? (P, \theta(q))$ satisfies the above mentioned conditions. The answer no represents the empty set of valuations, and the answer yes corresponds to the set of all ground valuations of the variables of the query.

The answer to a query of the form $\? (P, r(t_1, \cdots, t_n)\?)$, where $r(t_1, \cdots, t_n)$ is a non ground atom, is a triple of sets ($A_1, A_2, A_3$): set $A_1$ corresponds to the instances of the query that belong to the boundary of $R$; set $A_2$ corresponds to the instances of the query that belong to the lower approximation of $R$; set $A_3$ corresponds to the instances of the query that belong to the lower approximation of $\neg R$. Obviously, answers to this type of queries can be obtained by issuing the simple queries $\? (P, \overline{r}(t_1, \cdots, t_n))$, $\? (P, \overline{\overline{r}}(t_1, \cdots, t_n))$ and $\? (P, \overline{\neg r}(t_1, \cdots, t_n))$.

The above ideas can be easily extended to the case of composite queries. Note that a query of the form $\? (P, q?)$ cannot be involved in a composite query.

Example 4. Consider the program $P$ of Example 3, defining the rough relation $\mathcal{R}t$. We may pose queries like $\? (P, ft(3))$, $\? (P, \overline{ft}(X))$ or $\? (P, \overline{\overline{ft}}(4))$. The obtained answer would then be: yes for the first query; $\{x=2, x=3\}$ for the second one; and no for the last one.

4.2 Implementing Rough Queries in Prolog

As already discussed the simple literal queries for a DXL program $P$ can be directly answered in Prolog by using the transformed version $P'$ of $P$.

We now show how the remaining queries can also be answered by transforming them to Prolog queries for $P'$. We define the following transformation $\tau$ of simple queries to Prolog queries, where not denotes Prolog negation as failure ($\land+$).

$$\tau(Q) = \begin{cases} 
q(t_1, \cdots, t_n) & \text{if } Q \equiv q(t_1, \cdots, t_n) \\
\neg q(t_1, \cdots, t_n) & \text{if } Q \equiv \neg q(t_1, \cdots, t_n) \\
q(t_1, \cdots, t_n), \neg q(t_1, \cdots, t_n) & \text{if } Q \equiv q(t_1, \cdots, t_n) \\
\neg q(t_1, \cdots, t_n), \neg q(t_1, \cdots, t_n) & \text{if } Q \equiv \neg q(t_1, \cdots, t_n) \\
q(t_1, \cdots, t_n), q(t_1, \cdots, t_n) & \text{if } Q \equiv \overline{q}(t_1, \cdots, t_n) \\
\neg q(t_1, \cdots, t_n), \neg q(t_1, \cdots, t_n) & \text{if } Q \equiv \overline{\overline{q}}(t_1, \cdots, t_n) 
\end{cases}$$

Let $P$ be a DXL program. We now claim that the answers obtained by Prolog evaluation of the query $\tau(Q)$ w.r.t to the program $P'$ coincide with the answers
Let $p$ be a predicate letter occurring in $\mathcal{P}$. By the construction of $\mathcal{P}'$, it follows that an atom $p(t_1, \cdots, t_n)$ belongs to $\mathcal{M}_\mathcal{P}$ iff it also belongs to $\mathcal{M}_{\mathcal{P}'}$. Moreover, a negative literal $\neg p(t_1, \cdots, t_n) \in \mathcal{M}_\mathcal{P}$ iff the atom $p^-(t_1, \cdots, t_n) \in \mathcal{M}_{\mathcal{P}'}$. Recall that $\mathcal{P}'$ is a definite program. If a simple query $q(t_1, \cdots, t_n)$ or $q^-(t_1, \cdots, t_n)$ w.r.t $\mathcal{P}'$ fails in Prolog, then it has no (ground) instances in $\mathcal{M}_{\mathcal{P}'}$. In view of that, it can be easily checked that each of the five cases of the definition of $\tau$ satisfies our claim. Take for example a lower approximation rough query $\tau(\mathcal{P}, q(t_1, \cdots, t_n))$. Assume that the Prolog answer to $\tau(q(t_1, \cdots, t_n))$ w.r.t $\mathcal{P}'$ returns a valuation $\theta$. Thus, $\theta(q(t_1, \cdots, t_n))$ is in $\mathcal{M}_{\mathcal{P}'}$, hence in $\mathcal{M}_\mathcal{P}$. On the other hand, if $\theta(q^-(t_1, \cdots, t_n))$ fails then $\theta(q^-(t_1, \cdots, t_n)) \notin \mathcal{M}_{\mathcal{P}'}$. Thus, $\theta(\neg q(t_1, \cdots, t_n))$ is not in $\mathcal{M}_\mathcal{P}$. Consequently, $\theta(q(t_1, \cdots, t_n))$ belongs to the lower approximation of the rough set $\mathcal{Q}$ denoted by predicate $q$, as required. One should also consider the case when the Prolog query $\tau(\mathcal{Q})$ fails w.r.t $\mathcal{P}'$. This means that $q(t_1, \cdots, t_n)$ fails (i.e. there is no instance of $q(t_1, \cdots, t_n)$ that belongs to the upper approximation of $\mathcal{Q}$) or that whenever $q(t_1, \cdots, t_n)$ succeeds with a valuation $\theta$ then not $\theta(q^-(t_1, \cdots, t_n))$ fails (i.e. $\theta(\neg q(t_1, \cdots, t_n))$ is in the negative region of $\mathcal{Q}$). Thus, in both cases there is no instance of the query which belongs to the lower approximation of the rough relation $\mathcal{Q}$.

5 Discussion and Conclusions

The contribution of the paper is twofold. First, it establishes a link between logic programming and rough set theory that makes possible to combine techniques originating from both fields. Second, we show an application of the techniques developed in the area of paracconsistent logic.

We relate DXL programs to rough sets: we have shown that the least model of any DXL program can be seen as a family of rough relations. Although this observation is technically very straightforward, it opens for use of Prolog for defining and manipulation of rough sets. To our knowledge this approach is novel as concerns rough sets. It improves and simplifies our recent work on rough datalog [8], by providing more flexible technique for defining negative regions of rough sets, which results in simplification of the semantics.

The language of rough queries brings the specificity of rough sets to paraconsistent logic programming. It should be clear that with this language, mainly due to the use of lower approximations, we implicitly introduce a very restricted form of default negation into DXL. A natural question is whether the lower approximations should be introduced into bodies. There may be example applications such that the reference to lower approximations in the rules may be desirable. However, so far the interest of rough sets community for nonmonotonic reasoning seems to be rather limited.

Extension of the language with lower approximations in the body would require a more sophisticated semantics. However, such an extension would still not allow a free use of default negation. An interesting question is then whether these restrictions make it possible to provide a simple and intuitive semantics.
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