Anomalous scattering of highly dispersed pulsars

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ABSTRACT

We report multi–frequency measurements of scatter broadening times for nine highly dispersed pulsars over a wide frequency range (0.6 – 4.9 GHz). We find the scatter broadening times to be larger than expected and to scale with frequency with an average power-law index of 3.44 ± 0.13, i.e. significantly less than that expected from standard theories. Such possible discrepancies have been predicted very recently by Cordes & Lazio.

1. Introduction

Fluctuations in the Galactic electron density distribution are responsible for scintillation and scattering of radio signals propagating through the interstellar medium (ISM). The resulting observable phenomena have been studied extensively to investigate the nature of the density irregularities (e.g. Rickett 1977). Pulsars are particularly useful as probes of the medium because of their small angular diameter and spatial distribution with samples on many lines of sight through the Galaxy. Scattering of pulsar signals through an irregular random ISM causes the signal to arrive from different, multiple ray paths with different geometric lengths, so that a pulse which left the source at one instant, arrives at the observer over a typical time interval, \( \tau_{sc} \), commonly the scatter broadening time. Further, along different ray paths the radiation acquires random phases which cause interference in the plane of the observer to produce diffraction patterns. The pattern decorrelates over a characteristic bandwidth \( \Delta \nu_d \).

Both the scatter broadening time and decorrelation bandwidth are strongly dependent on frequency \( \nu \), i.e. \( \tau_{sc} \propto \nu^{-\alpha} \) and \( \Delta \nu_d \propto \nu^\alpha \), and are related to each other by \( 2\pi \tau_{sc} \Delta \nu_d = C \). The “constant” \( C \) is of the order unity but changes for different geometries and different models for the turbulence wavenumber spectrum, \( P_n \) (e.g. Lambert & Rickett 1999). Knowledge of the spectrum would provide valuable insight into the physics of the ISM irregularities. A commonly used description for the wavenumber spectrum is a power-law model with large range between “inner” and “outer” scales, \( \kappa_i^{-1} \) and \( \kappa_o^{-1} \), (e.g. Rickett 1977), i.e.

\[
P_n(q) = \frac{C_n^2}{(q^2 + \kappa_o^2)^{\beta/2}} \exp\left[ -\frac{q^2}{4\kappa_i^2} \right]
\]

where \( q \) is the magnitude of the three-dimensional wavenumber and \( C_n^2 \) is the strength of fluctuation for a given line-of-sight. For \( \kappa_o \ll q \ll \kappa_i \), one obtains a simple power-law model with a spectral index \( \beta \), i.e. \( P_n(q) = C_n^2 q^{-\beta} \) and \( \alpha = 2\beta / (\beta - 2) \) (e.g. Lee & Jokipii 1975). For a pure Kolmogorov spectrum, \( \beta = 11/3 \), we expect \( \alpha = 4.4 \). In turn, by measuring \( \alpha \), one can infer \( \beta \) and hence details about the actual wavenumber spectrum. A value of \( \beta = 4 \) could, for instance, describe a medium with abrupt changes in density caused by randomly placed, discrete clouds along the line-of-sight (Lambert & Rickett 1999).

In this Letter, we report measurements of \( \alpha \) by carefully determining \( \tau_{sc} \) over a wide frequency range for a sample of nine highly dispersed pulsars, selected to be observable at 4.8 GHz and located towards the inner Galaxy. Measurements of \( \tau_{sc} \) can be hindered by possible frequency evolution of the intrinsic pulse shape but we use results from previous studies of pulsars unaffected
Table 1

Scatter Broadening for 9 Pulsars

| PSR   | l   | b   | DM  | \(\tau_{0.6}\) | \(\tau_{0.9}\) | \(\tau_{1.4}\) | \(\tau_{1.6}\) | \(\alpha\) | \(\beta\) | \(k\) |
|-------|-----|-----|-----|--------------|--------------|--------------|--------------|--------|--------|------|
| B1750−24 | 4.3 | 0.5 | 676  | 49(11)       | 25(4)        | 5.2(1.1)     | 3.4^{+0.5}_{-0.4} | 4.9^{+1.3}_{-0.7} | 5.1 |
| B1758−23 | 6.8 | -0.1 | 1074 | 111(19)      | 55(10)       | 8.6(1.7)     | 3.9^{+0.4}_{-0.3} | 4.1^{+0.5}_{-0.4} | 5.7 |
| B1805−20 | 9.5 | -0.4 | 609  | 259(47)      | 45(9)        | 14(7)        | 2.9^{+0.7}_{-0.6} | 6.6^{+4.7}_{-2.7} | 2.7 |
| B1815−14 | 6.4 | 0.6  | 625  | 64(17)       | 15.0(0.3)    | 8.5(0.6)     | 3.5^{+0.5}_{-0.4} | 4.7^{+1.4}_{-0.6} | 2.4 |
| B1817−13 | 17.2 | 0.5  | 782  | 35(3)        | 19.3(1.7)    | 4.0(1.4)     | 3.3^{+0.7}_{-0.6} | 5.0^{+1.9}_{-0.8} | 2.8 |
| B1820−11 | 19.8 | 0.9  | 428  | 43(6)        | 11.9(1.9)    | 3.0^{+0.7}_{-0.6} | 6.0^{+3.1}_{-1.8} | 2.1 |
| B1820−14 | 17.3 | -0.2 | 648  | 52(31)       | 10(6)        | 2.0(0.4)     | 4.1^{+1.2}_{-1.7} | 3.8^{+0.4}_{-0.3} | 0.3 |
| B1821−11 | 19.8 | 0.7  | 582  | 224(180)     | 40(12)       | 8.1(0.8)     | 3(3)         | 0.9(1.6) | 3.5^{+1.0}_{-0.6} | 4.6^{+1.4}_{-0.7} | 2.1 |
| B1849+00 | 33.5 | 0.0  | 680  | 223(24)      | 133(11)      | 36(16)       | 2.8^{+0.6}_{-0.8} | 5.6^{+1.7}_{-0.8} | 54.8 |

Note.—Cols. (2) and (3) give the Galactic longitude and latitude of each pulsar; col. (4) its Dispersion Measure and cols. (5)–(9) the scatter broadening with 3σ-errors in parenthesis at 0.6, 0.9, 1.4, 1.6 and 2.7 GHz. The median of normalized \(\chi^2\) for all fits is 1.37. Col. (10) gives the spectral index of scatter broadening and col. (11) the spectral index of density irregularities for each pulsar with their 1σ-errors. Col. (12) gives the ratio \(k\) of \(\tau_{sc}\) at 1 GHz scaled by \(\alpha\) and \(\tau_{sc}\) at 1 GHz predicted by the Taylor & Cordes (1993) model. Note that, with the exception of PSR B1820−14, our measured scatter broadening times are significantly higher than those predicted by the Taylor & Cordes (1993) model.

Fig. 1.— (Top) Observed profile of PSR B1815−14 at 1.4 GHz and best–fit of the model profile (see section 2). (Bottom) Contours of \(\chi^2\) near its global minimum (indicated by the plus sign) in the \(c\)-\(\tau_{sc}\) plane for the above profile.

by scattering to estimate these pulse shape effects. Although one could attempt to measure \(\Delta \nu_d\) instead, predicted values are well below 50 Hz at 1 GHz and cannot be measured with the frequency resolution of typical filterbanks. This results in a surprisingly small number of published spectral indices (see Cordes, Weisberg & Boriakov 1985, Johnston, Nicastro & Koribalski 1998). Consequently, the data presented here are in fact the first ever measurements of \(\alpha\) for pulsars with high dispersion measure (DM).

2. Observations and Data analysis

Dual circular polarization observations were made both with the 100-m radio telescope at Effelsberg (1.4, 2.7 and 4.9 GHz) and the 76-m Lovell telescope at Jodrell Bank (0.6, 0.9, 1.4 and 1.6 GHz). Most of the pulse profiles recorded at Effelsberg were obtained with a coherently de-dispersing backend, the Effelsberg Berkeley Pulsar Processor (Backer et al. 1997). For pulsars listed in Table 1, the total available bandwidth ranges from 28 to 45 MHz at 1.4 and 2.7 GHz and up to 128 MHz at 4.85 GHz. At lower frequencies, we often obtain higher sensitivity by employing an incoherent hardware de-disperser consisting of 4 × 60 channels of 0.667 MHz, providing a bandwidth of
40 MHz per polarization. Resulting dispersion broadening times \( t_{DM} \), for the pulsars observed lie in a range of \( 0.8 \text{ ms} \leq t_{DM} \leq 1.3 \text{ ms} \). Detailed descriptions of the hardware set-up can be found in Seiradakis et al. (1995) and Kramer et al. (1999).

At Jodrell Bank, the pulse profiles were obtained for each polarization with an incoherent de-disperser using filterbanks with varying number of channels and bandwidths. At 0.6 GHz we used 32 channels of 0.125 MHz bandwidth (2.0 ms \( \leq t_{DM} \leq 5.4 \text{ ms} \)), at 0.9 GHz 32 channels of 0.25 MHz each (1.1 ms \( \leq t_{DM} \leq 2.0 \text{ ms} \)), at 1.4 GHz we employed 32 channels of 1 MHz each (1.7 ms \( \leq t_{DM} \leq 3.2 \text{ ms} \)), while we employed 8 channels of 5 MHz bandwidth at 1.6 GHz (0.8 ms \( \leq t_{DM} \leq 10.0 \text{ ms} \)). Details of the system can be found in Gould & Lyne (1998).

Typical observation times were between 30 – 60 min, depending on observing frequency, bandwidth and flux density of the individual source. Incoming signals were folded with the topocentric pulse period and two circular polarizations were added to produce total power profiles. The signal-to-noise ratios of our profiles range from 13 to 67.

In order to measure the scatter broadening, \( \tau_{sc} \), we performed least-squares fits of an artificial model profile, \( P^M(t) \), representing the scattered intrinsic pulse shape, to the observed profile. For each pulsar, a template \( P^T(t) \) is constructed as a sum of Gaussian components fitted to an unscattered, high–frequency profile using a procedure developed by Kramer et al. (1994, 1999). Usually, the high–frequency profile used is observed at 4.9 GHz, except for PSR B1820–14 and PSR B1815–14 where we use data taken at 1.6 GHz and 2.7 GHz, respectively, because of the low signal-to-noise of their 4.9 GHz–profiles. For the given DMs, we can expect the scatter broadening to be negligible at these frequencies (Bhattacharya et al. 1992), but we discuss the validity of this assumption in more detail later. For each lower frequency, \( P^M(t) \) is the convolution of the template \( P^T(t) \) with the impulse response function characterizing the scatter broadening, \( s(t) \), the dispersion smearing across the filterbank channel, \( d(t) \), and the instrumental impulse response, \( i(t) \),

\[
P^M(t) = P^T(t) \otimes s(t) \otimes d(t) \otimes i(t)
\]

where \( \otimes \) denotes convolution (Ramachandran, Mitra & Deshpande 1997). The rise times of the receivers and backends are small enough to consider the effect of \( i(t) \) to be negligible, while \( d(t) \) is a rectangular function of width \( t_{DM} \) for incoherent de-dispersion and of zero width for coherent de-dispersion. Williamson (1972) pointed out that multi–path scattering due to both thick and thin screens leads to an exponential decay of the pulse giving \( s(t) = \exp(-t/\tau_{sc}) \). In general, \( s(t) \) can assume different functional forms for different geometries reflecting also different rise times of scattered pulses (Lambert & Rickett 1999) which could lead to uncertainties in the estimation of \( \tau_{sc} \). However, the frequency dependence of the scatter broadening time, i.e. \( \alpha \), can be expected to remain unaffected as we discuss later.

The best fit of the model pulse shape to that observed is obtained by minimizing the normalized \( \chi^2 \) value,

\[
\chi^2 = \frac{1}{(N - 4)\sigma_{off}^2} \sum_{i=1}^{N} \left[ P^O_i - P^M_i(a, b, c, \tau_{sc}) \right]^2
\]

where \( P^O \) is the observed profile, \( \sigma_{off} \) the off-pulse rms, and \( N \) the number of bins in the profile. During the fit, we adjust four free parameters of the model pulse profile \( P^M \), i.e. an amplitude scale-factor, \( a \), constant offsets in baseline \( 4 \), \( b \), and phase, \( c \), and the scatter broadening time \( \tau_{sc} \).

We obtain best–fit values and uncertainties for \( \tau_{sc} \) from the \( \chi^2 \)-contours in the \( c-\tau_{sc} \) plane. An example for an observed profile, the best–fit model profile and \( \chi^2 \)-contours for the fit is shown in Figure 1.

3. Factors affecting scatter-broadening-measurements

Measuring \( \tau_{sc} \) can be difficult since the intrinsic pulse shape often changes slowly with frequency in two distinct ways. First, outer profile components often have flatter flux density spectra than central components (Rankin 1993, Lyne & Manchester

\[\footnote{usually close to zero}\]
Fig. 2.— Scatter broadening times $\tau_{sc}$ with their $3\sigma$-errors as a function of observing frequency $\nu$ for nine pulsars. The lines correspond to the linear fit of the form $y = \alpha x + K$ where $y = \log(\tau_{sc})$ in ms and $x = \log(\nu)$ in GHz. The dashed lines are examples of the expected dependence due to a Kolmogorov spectrum, i.e. $\alpha = 4.4$. The bottom right most panel shows a linear function with an average spectral index $\langle \alpha \rangle = 3.44$.

Fig. 3.— Spectral index of scatter broadening $\alpha$ as a function of DM for our sample of pulsars as well as for earlier measurements from Cordes et al. (1985) and Johnston et al. (1998). The dotted line $\alpha = 4.4$ indicates the spectral index predicted by a pure Kolmogorov spectrum. The dashed line $\alpha = 4.0$ represents the lowest possible index predicted within standard theories of the turbulent medium.

Second, pulse widths decrease with increasing frequency (Sieber, Reinecke & Wielebinski 1975). While changes in widths are significant below $\sim 1$ GHz, at higher frequencies the width usually saturates to a constant value (Thorsett 1991, Xilouris et al. 1996, Mitra & Rankin 2001). Both effects, if not carefully accounted for, can give rise to inaccurate estimation of $\tau_{sc}$. As we use high frequency profiles to create our templates, stronger outer components could make the template too wide for the lower frequency profiles. Due to this effect, we may tend to underestimate $\tau_{sc}$ at lower frequencies. Alternatively, at frequencies around 1 GHz, it is also possible to mistake an evolving trailing profile component for parts of a scattering tail. This aspect is however not always present and hard to predict. In contrast, virtually every pulsar’s width is increasing towards lower frequencies. The model profile would then be too small, overestimating $\tau_{sc}$ for frequencies well below 1 GHz.

In order to quantify the impact of these possibly competing effects, we have simulated “typical” pulse profiles and their frequency evolution, made
them subject to scatter broadening and applied our analysis procedure under the additional presence of noise. While confirming the above effects, the combined profile evolution hardly affects our results, i.e. the derived scatter broadening times agree very well with the true values. Width evolution does not change the actual values of the scatter broadening times significantly. The largest impact is when unnoticed outer components slowly evolve at high frequencies, which in our sample is only seen for PSRs B1820−11 and B1849+00. Our simulations show that increased error bars are sufficient to account for these effects, so that we conservatively quote 3σ error bars for all scatter broadening times in what follows. We additionally confirm by determinations of α using scatter broadening times measured at the two lowest frequencies only that our results are unaffected by possible changes in pulse profiles.

4. Results and Discussion

Table 1 summarizes our measurements of τ_{sc}. For PSRs B1750−24, B1758−23, B1805−20, B1815−14 and B1820−11, these can be compared to those obtained by Clifton et al. (1992) at a single frequency. We find our more accurate scatter times to be in very good agreement with their results. Note that all but one of the scatter broadening times are factors of several larger than predicted by the Taylor & Cordes (1993) model. However, this model is poorly constrained for distant, heavily scattered pulsars.

Figure 2 displays the measured frequency dependence of τ_{sc} from which we determine the spectral indices α listed in Table 1. Earlier multi-frequency observations of pulsars with low DMs (see Cordes, Weisberg & Boriakoff 1985, Johnston, Nicastro & Koribalski 1998) derived spectral indices that are consistent with a pure Kolmogorov spectrum, i.e. α = 4.4 (Figure 3). In contrast, for our highly dispersed pulsars we find all individual values of α to be significantly lower than α = 4.4 and usually even lower than α = 4.0 of a “β = 4−model” (Lambert & Rickett 1999). The average spectral index is ⟨α⟩ = 3.44±0.13 (see also Figure 2).

Deriving the spectral index of the wavenumber spectrum from equation (1), β = 2α/(α − 2), we obtain values that are significantly larger than those expected from standard theories such as β = 11/3 or β = 4 (Table 1). Interestingly, Cordes et al. (1985) predicted departures from the Kolmogorov spectrum such as wavenumber enhancements to be detected in multi-frequency observations of high-DM pulsars. Other observational inconsistencies with a simple Kolmogorov spectrum have been explained by steeper spectra (β > 4, e.g. Goodman & Narayan 1985), the influence of an inner scale (Coles et al. 1987), or physically slightly different models (Lambert & Rickett 1999, 2000). It is, however, interesting to note that recent results suggest that observations along lines of sight of enhanced scattering at low Galactic latitudes are described by a Kolmogorov spectrum in an extended medium (e.g. Stinebring et al. 2000, Lambert & Rickett 2000). While the DMs of the pulsars in these studies are typically lower, we observe an apparently different behavior.

Lambert & Rickett (1999) point out that the scatter broadening function, s(t), depends to some extent on the actual geometry and spectral model, although our χ² values show that our profiles can be described accurately by the model. In the worst case, according to their results, our individual scatter broadening times could be consistently under- or over-estimated by an amount which is typically well covered by our 3σ error-bands. While τ_{sc} computed for a given frequency may change slightly, its spectral dependence, α, shows only minor variations well with the quoted uncertainties when testing it by increasing (or decreasing) each τ_{sc} by a fixed percentage. We conclude that our weak frequency dependence, α < 4, is real and cannot be explained by simple application of standard theories.

Anisotropic irregularities may lead to variations in the broadening functions for different frequencies. This causes essentially the phenomena of anomalous scattering in the ISM as discussed recently by Cordes & Lazio (2001). In particular, one usually assumes that the transverse extent of the scattering screen is arbitrarily large and that the strength of the scattering is uniform across the screen. However, if this assumption is relaxed and/or pulsar distances are large, several anomalous effects can emerge that can in principle flatten the observed frequency dependence of τ_{sc} by reducing the amount of scattering apparent at lower frequencies. In our case, the observed anom-
lous behaviour could be caused by scattering at multiple screens with finite extension transverse to our line-of-sight. As a consequence, less radiation reaches the observer at lower frequencies since some of the radiation that would be scattered by an infinite screen is now lost. For such scattering geometries, one can expect to detect several breaks in the $\tau_{sc}$ power-law spectra (see e.g. Figure 3. of Cordes & Lazio 2001). A finer frequency sampling and higher signal-to-noise ratios may be able to reveal these spectral features in future observations.

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Note added in proof. – The equation $\beta = 2\alpha/(\alpha - 2)$ is only valid for $\beta < 4$. As shown in Table 1 this results, however, in values of $\beta$ larger than 4. For turbulence spectra with $\beta > 4$, a relationship $\beta = 6 - 8/\alpha$ is valid (Cordes et al., 1986, ApJ, 310, 737; Romani et al., 1986, MNRAS, 220, 19). Applying this to $\alpha$ listed in Table 1 results however in values $\beta < 4$. We conclude again that our results are not consistent with standard theory. We also note that a scatter broadening function $s(t) = 1/\sqrt{t} \exp(-t/\tau_{sc})$, which is a conceivable alternative shape in the limit of one-dimensional scatterers that might pertain to situations with large scattering (Cordes 2001, private communication), does not appropriately describe our scattered profiles, in contrast to $s(t)$ used in the Letter.

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