“At Sea”: Reversibility in Teaching and Learning

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Abstract An equal-armed balance at equilibrium—the bar is horizontal—tips into disequilibrium upon displacing a weight. Equilibrium is restored by reversing that move—putting the weight back where it was, or doing the same on the other side. Piaget adopted the idea of equilibration to describe how the intellect, in relating to the world, develops. Equilibrium arises as: our mind adjusts its structures in response to the outer world (accommodation), so our internalized views can take in this outer world (assimilation). That is the process Piaget calls equilibration. Upon undergoing disequilibrium, the intellect employs these equilibrating moves, changing its structures in the process. When the intellect resolves a disturbing problem no matter how it is encountered, the intellect tries to reverse the disturbing feature: how did the familiar situation get to this disturbing one; how might that change be reversed? These equilibrating processes are encouraged as means of teaching and learning in this paper’s math and science examples. The clinical interviewing methodology of Piaget and Inhelder, as adapted by Eleanor Duckworth in the research pedagogy of clinical exploration in the classroom, provides the neutral, safe conditions requisite for these learners and teachers in undergoing disequilibrium, struggling with uncertainty, and constructing new understandings. In beginning to teach through exploration, the author and an undergraduate experimented with free fall motion. Experiencing disequilibrium, the student reconstructed her understanding of time as concurrently continuous and divisible. Seeking to enact methods of Piaget and Duckworth while engaging her, the teacher also experienced disequilibrium.

Keywords Piaget · Science education · Exploratory learning · Equilibrium · Periodicity
Introduction

Long ago, youth left behind the stability of land, founding life and learning on the tumultuous sea. Melville’s Ishmael, like his creator, proclaimed the depth of his education by sea voyage “... a whaling ship was my Yale College and my Harvard.” (Melville 2008, Ch. 24) With pause at calm, then tossed in oncoming waves of space and personal being, in mutual exchange, sailor, boat and sea heaved and swayed, tipped and righted. This back and forth—this responsiveness to whatever arises, along with moment-to-moment refining of balance and resilience amid ever-changing swells—pervades the educational experiences related below.

Like seagoing youth of the past, the learners and teachers, whose work is described in the following accounts, experience an education while “at sea” in classrooms. Seafarers develop “sea legs” that automatically and unconsciously respond to the sea’s shifts in balance. Similarly, when education happens “at sea”, educational practices and structures designed for a land-based scaffolding expose their obstructive character and fall away, supplanted by structures that more adequately respond to the fluid environment.

Being “at sea”—with all its inherent confusions—provides a testing ground where participants face inadequacies in what they suppose, and all the while improvise more seaworthy ways of moving and learning. Learning and teaching develops productively both in smooth travel, as of a boat at equilibrium, and in response to heaving swells, in the efforts of bringing about new equilibria. When in disequilibrium, the mind imagines a similar weight placed on the other side of the boat; or a spatial movement that counterbalances a change in weight, and thus connects back to the starting point. In less concrete puzzles, too, the mind searches for ways that would get the situation back to where it was—ways that could, in the mind, transform the starting point and yet get back to it at will—that could think of it as this way or that way, at will. The sea legs do it dynamically, in the moment.

Weaving between these analogies is equilibrium, an inherently relational process among multiple factors. Twentieth century psychologist Jean Piaget adopted the idea of equilibration to describe how the intellect, in relating to the world, develops. In Piaget’s analysis, equilibrium is the nature and mechanism of the provisional structures of intellect and action, by which beings act and come to ever more inter-relational capacity:

Every structure is to be thought of as a particular form of equilibrium, more or less stable within its restricted field and losing its stability on reaching the limits of the field. But these structures … succeed one another according to a law of development, such that each one brings about a more inclusive and stable equilibrium for the processes that emerge from the preceding level. (Piaget 1981, p. 7)

Equilibrium arises as: our mind adjusts its structures in response to the outer world (accommodation), so our internalized views can take in this outer world (assimilation). That is the process Piaget calls equilibration. Accommodations are made in order that the being’s structures may then assimilate some external matter
that the former version of those structures did not have means to assimilate. Upon undergoing disequilibrium, the intellect employs these equilibrating moves, changing its structures in the process. When the intellect resolves a disturbing problem no matter how it is encountered, the intellect tries to reverse the disturbing feature: how did the familiar situation get to this disturbing one; how might that change be reversed?

…intelligence, capable of all its detours and reversals by action and by thought, tends toward an all-embracing equilibrium by aiming at the assimilation of the whole of reality and the accommodation to it of action, which it thereby frees from its dependence on the initial [here and now]. (Piaget 1981, p. 9)

This equilibrating process has a liberating trajectory, as successive equilibria widen the being’s scope and responsiveness to engage productively with whatever disturbances of equilibrium occur next.

The behavior of a physical balance carries import for portraying these equilibrating processes. An equal-armed balance at equilibrium—the bar is horizontal—tips into disequilibrium upon displacing a weight. Equilibrium is restored by reversing that move—putting the weight back where it was, or doing the same action on the other side. Equilibrium is produced reversibly and by multiple possible moves. Extending the example to learning, suppose a learner has only seen a balance at rest horizontally, when its arms support equal weights. The sight of a balance holding equilibrium with differing weights on either arm might provoke disequilibrium in that learner’s thought. Through working experimentally and thoughtfully with relationships that the balance invariably exhibits, the learner may work toward a new equilibrium in understanding. When the learner can mentally analyze and predict the outcome of any arrangement of weights and distances, the learner will have achieved reversibility in understanding the relationship of balance:

Reversibility is defined as the permanent possibility of returning to the starting point of the operation in question. (Inhelder and Piaget 1958, p. 272)

Reversibility is at play where a being discerns a relationship, even where masked and inaccessible, and acts with that relationship to restore equilibrium in flexible ways. If the relationship being questioned is different from the balance, whether it be mathematical, physical, or psychological, the process of engaging with it will bear out the underlying properties that characterize that relationship and how it is expressed in the world. A reversible means of thinking enables an equilibrium that is both robust and susceptible to further disturbance. For Piaget:

reversibility is the very criterion of equilibrium (as physicists have taught us). To define intelligence in terms of the progressive reversibility of the mobile structures which it forms is therefore to repeat… that intelligence constitutes the state of equilibrium towards which tend all the successive adaptations of a sensori-motor and cognitive nature, as well as all the assimilatory and accommodatory interactions between the organism and the environment. (Piaget 1981, p. 11)
“Progressive reversibility” includes prior and ongoing capacities of mind and action. Strategies and structures formed in the past are reorganized and partake together with those that are emerging. Continuity holds even while a being undergoes disruptive disequilibrium. There are no endpoints of static immobility.

By contrast, conventional education imposes externally defined goals, typically to be attained by following a preset sequence. Traditional science education, based on books, is complementary to reversible processes. After a result is reached by one path, often under explicit direction, learners seldom have opportunities to try out or form other paths. Such educational routes omit a role for reversibility, or lack trust of it as means of developing mind in relation with the world. Recognition that investigation and uncertainty are core in doing science, contributed to establishing inquiry models for science education (NSES 1996). Case studies (Hammer and van Zee 2006; Learning Progressions Project 2013), educational research (Bonawitz et al. 2011; Dean and Kuhn 2006; Zion and Slezak 2005), principles (NSTA 2004a) and standards (NSES 1996) support conducive grounding conditions, such as risk-taking, collaboration, interaction, and investigatory modes of learning and teaching. Such innovative contexts support the science classroom in hosting students in asking questions (NSTA 2004b), teachers in investigating their students’ understanding and their own (Bamberger 2013; Hammer 2000), and even the interdependent creation of knowledge by teacher and students together (Maskiewicz and Winters 2012).

Trust in the educational potential of the reversible, equilibrating processes described by Piaget underlies the works of educators in the examples below, who reject depictions of equilibrium in teaching and learning, as portrayed in conventional education, that are final, compartmentalized, and irreversible. In this paper’s examples, teachers, including myself, research their teaching in classroom and study sessions. Their support of reversibility in teaching and learning moves these teachers to stretch or transgress boundaries that typically frame educational environments. In each example, the immediate learning environment is non-traditional, including sessions that I facilitated with a volunteer, and participants in a nondepartmental seminar that I teach as an emergent process. Teachers and learners are among the many seeming opposites that relate interdependently through equilibrium: in these examples, students are at the same time teachers, and vice versa, across a range of ages including adults.

In this paper, Piaget’s analysis and methodology, as demonstrated in his interviews with children, provides context for exemplary episodes where teachers reshaped problems of science and math in ways that invited students’ thinking into equilibrium and disequilibrium. This paper’s major example relates from my own uncertain efforts as a novice teacher and researcher in beginning to notice, and seek to act upon, equilibrating processes in learning and teaching. My uncertainty gradually deepened, putting me in conditions of realizing that learning experiences such as I sought to support for a learner, are unknown and uncertain to the teacher as well as to learners. Adapted from Galileo’s experimenting, an instructional activity with a weight’s free fall yielded frustrations in an algebra class that I observed when I was a first-term student of educational research. While I was interviewing one student about this activity, she transformed that frustration into productive
disequilibrium. The next term, another of those students volunteered for trial sessions with me, where the two of us, in revisiting that same free fall experiment, were bemused by discrepancies that drew us into making increasingly reversible interpretations of its temporal relationships. This paper’s last example, from 20 years later in my teaching, integrates science and art. In happening upon reflective effects while seated at a window, my recent students and I respond to a wonderment that we shared with Galileo.

Especially in human doings, the shadow of land reaches far beyond that place at sea where its heights are lost to sight. So it is for myself as learner, teacher, facilitator, and for the students in classroom and study settings across the several examples that follow. We are land-lubbers at sea, awkward and easily disconcerted, harkening back to the land’s practices and yet increasingly captivated by the rhythms and constancies of the unknown and unexplained. For us, the vast sea and the frail craft we ply upon it are always at hand, inviting new voyages. As whiffs of sea air and promise of adventure attracted past youths to sea, perhaps the stories here will launch explorations of your own with learning and teaching.

**Piaget’s Clinical Methodology Reveals Equilibrium Processes Opening to Reversibility in the World**

Piaget and colleagues conducted extensive observational and interviewing studies in matters ranging across language (Piaget 1955), spatial reasoning (Piaget and Inhelder 1967; Piaget et al. 1981), and moral judgment (Piaget 1965a). They interviewed hundreds of children individually, where each child responded to provocative materials posing a quandary (Piaget 1965a, b, 1969, 1987a, b; Piaget and Inhelder 1967, 1969, 1975; Piaget et al. 1981). These studies demonstrate that cognitive maturation interrelates the mind with the world increasingly across its growth by means that are not: isolated within the individual; inexorable with the advance of age; or directed by external agents. Instead, the child’s ongoing relation with the outer world generates the destabilizations and equilibrations which draw children into constructing and reorganizing their own thought and action. Equilibrating processes occur with such fluidity and subtlety that it can be difficult to become aware of, and observe, their character while undergoing them. A means of observing and analyzing the dynamic nature of equilibration lies in the clinical methodology developed and practiced by Piaget and colleagues.

The Piagetian interviewer—in seeking to reveal and probe the child’s thinking—maintains a neutral stance, striving to be open to whatever each child did and said, without giving clues or holding preconceived expectations based on the adult outlook. Through not being tied to a script and interacting directly with the child, the interviewer improvises questions and the task so as to further elucidate the child’s thoughts and actions (Piaget 1960). Piaget would never be satisfied with the initial answer a child provided to his questions. He would always follow a child’s response, even if correct, with another question that would engage the child in grappling with complexities of the problem.

To illustrate how Piaget’s clinical methodology interacts with a child’s thinking while revealing it, I excerpt from his study of how children come to understand
relations between a part and a whole containing that part (Piaget 1965b, Chapter VII). Child participants in this study ranged in age from 5 to 8; one child, Laur, was interviewed at 5 years 5 months, and again 2 years later. Being presented with a dozen wooden (whole) beads, of which two are white (part) and the others brown (another part), each child was asked whether there were more wooden beads, or more brown ones? Young children affirmed there to be more brown beads, while at the same time stating that all beads are wooden.

Seeking to uncover what underlay this conviction, so incongruous to adults, the interviewer modified the task. For example, the interviewer drew brown and white beads on paper, then asked the child to draw one circle around all the brown ones, and another around all the wooden ones; this child’s second circle only included white beads. Having correctly responded that there would be none left if all wooden beads were placed in a box, another child, Laur, was then asked to consider a more complex problem. If one girl wanted to make a necklace from all the brown beads in that box, and another girl wanted to make a necklace from all the wooden beads, which necklace would be longer? While the interviewer responsively drew out Laur’s interpretation, Laur’s thinking bordered on disequilibrium. Laur questioned what can go into the wooden necklace and voiced alternatives, yet these doubts did not accomplish the accommodation that would be needed for a new equilibrium:

**Laur** Will [the girl] who’s making the wooden necklace only take the white beads?

**Interviewer** No

**Laur** She’ll have the brown ones as well? (N.B. the spontaneous character of these two questions.)

**Interviewer** What do you think?

**Laur** Yes... because they’re made of wood as well.

**Interviewer** Which necklace will be longer...?

**Laur** The one with the brown beads (adapted from Piaget 1965b, Chapter VII, p. 165–166)

Upon further probing by the interviewer, Laur asserted that the wooden bead necklace has just two white beads—“There aren’t any others!!” (Piaget 1965b, p. 166) Piaget inferred that once Laur had imaginatively conceived a necklace of brown ones, he could no longer mentally access those brown beads for imaginatively constructing another necklace. Piaget surmised that Laur did not hold the whole as being in relation with its parts:

... as soon as the whole is divided, even in thought, the parts cease to be included in it, and are merely juxtaposed without synthesis. (Piaget 1965b, p. 171)

On being interviewed about brown and round [instead of wood] beads 2 years later, Laur’s thinking, again disrupted, this time established a new equilibrium:

**Interviewer** Are there more brown beads or more round beads?

**Laur** More brown ones. Oh no! (spontaneously), more round ones, because there are the two white ones as well.
Interviewer Which would be longer, a necklace made with brown ones, or… round ones?

Laur The one with round ones. (Piaget 1965b, p. 176)

For Laur, the parts, consisting of brown beads and white beads, had come into reversible relation with the whole of all round beads, encompassing both brown and white ones. Laur could mentally reuse beads, constructing simultaneously in thought, one necklace of all the brown beads and another of all the round beads. The whole could be divided into parts and reassembled from those parts, reversibly. These operations are not available under irreversible thinking, such as Laur exhibited in the prior interview.

This example, where children engaged with a part’s relation to a whole, characterizes how thought operates through a relationship to discern some way it manifests, and how thought can reverse or invert that operation through maintaining and applying the relationship. While the relationship that engages thought may originate in any specific realm, such as math or physics, the equilibrating process of thinking through relationships is at work in all those diverse contexts. Reversibility, demonstrated here where children’s thinking concurrently decomposed a whole into parts and recomposed its parts, inheres in thought generally, making for its dynamic agency, as Piaget portrayed it:

Every reasoning is a reversible construction, and there are as many different reasonings as there are types of construction...when the child has to think simultaneously of the brown beads and the wooden beads, he has to unite objects and then separate them again in order to construct another union, each element being at the same time in both constructions...mobility [of thought is] necessary for carrying out the operations, for combining and separating them, for constructing and reconstructing simultaneously. It is therefore in terms of reversibility that the difficulties of [intellectual] synthesis must be described... dynamism must be introduced (Piaget 1965b, pp. 180–181)

A driving force for these relational processes is the capacity of generating and projecting multiple possibilities for one’s own thought and actions and for what may occur and inhere in the world beyond. To project possibilities—where no workable way had been apparent before—is freeing, creative, inventive, and observant. Considering possibilities has the mind traveling along both how something might be, and along paths of its possible undoings, developing reversibility in thinking. While explanations or goals tend to circumscribe our thinking along particular lines, Piaget described how release from those limiting factors induces our minds to extend with new flexibility: “if one variation is possible, others are also possible” (Piaget1987a, p. 6).

Possibilities—conceived through awareness deepened by being in disequilibrium—develop our relationships with the world. Throughout her infancy, Piaget played with daughter Lucienne by deftly hiding a prized toy under one of several covers, then watching her efforts to recover it. One day, the nearly year-old child did the hiding! Taking her rattle, “she slips [it] under a rug to bring it out and put it...
under again endlessly” (Piaget 1954, p. 153), thereby creating and testing the alternative possibilities herself!

The hiding game and the infant’s instigation of possibilities through it engendered Piaget’s consideration of analogues with children. Presenting them with an object whose regular or irregular shape was partly hidden from their view, Piaget asked each child “What’s it like…?” “Could it be some other way?” “Do you have an idea?” (Piaget 1987a, p. 30) Mon, a seven-year old, proposed that one object’s exposed triangular tip be completed by a base of either linear or arc-like shape; an eight-year old supposed there could be hundreds of forms that complete it (p. 33); while Arl, a twelve-year old, went further in predicting “One can imagine all possible shapes” (p. 37). Where the younger child extended possible forms from the visible object, the older child projected unlimited options. Each child furthered understanding of the puzzle by means of the range of possibilities that they themselves envisioned.

The world, with the multitude of possibilities it sustains, becomes more evident as the child furthers in the equilibrating processes from which it is inseparable. As Piaget recorded in longitudinal studies with his own babies (Piaget 1952, 1954), while newborns lack boundaries between self and something other, as they go on to encounter disruption and exert themselves responsively, their behavior shows emergent attention to external matters. Where Piaget’s son at 4 months reaches out for a dropped toy as if to bring back what was inseparable from him, 4 months, later, the boy “truly searches for everything that falls from his hands” (Piaget 1954, p. 24). Having watched toys drop, reached for them and often failed, Piaget’s infant engaged the world in evolving equilibrating relations, relations remaking the world as an external interest, available for his actions of mind and body in searching and exploring. Concurrently, the child’s mind is conceiving possibilities, testing these experimentally, and undergoing destabilizing change.

These equilibrating processes, that Piaget identified in the developing child, transpire wherever humans engage the world and come to new understanding, notably in the building of science at personal and even institutional levels. Although no Piagetian interviewer accompanied historical scientists in their researches, through such records as the extensive diaries of nineteenth century experimenter Michael Faraday (1932) historians and scholars (Cavicchi 1997, 2006; Gooding 1990, 2006; Steinle 2002; Tweney 2001, 2006) “glimpse… close interplay of the senses … hands, imagination, systematic and critical reasoning and … institutions and politics of science” (Gooding 2006, p. 42). By analyzing Faraday’s encounters with unexpected disequilibrating phenomena alongside his responses “which restore order” (Gooding 1990, p. 195), cognitive scientist Gooding educed “a theory of the assimilation and the transformative power of novelty” (1990, pp. 25–26).

Widened further to the scale of collective enterprises of science, Piaget and colleague Garcia discerned equilibrating processes in historical science ranging from classical Greek geometry to Newtonian science and beyond (Piaget and Garcia 1989). For the child, and for science, they found that any current theory or outlook deals with change and destabilization by means of:
assimilation of novel elements into preceding structures and the accommodation of [those structures] to new, effective acquisitions. (Piaget and Garcia 1989, p. 25)

All preceding theories, outlooks and understandings become reintegrated into each next equilibrium. Equilibrium held in common across a science community provides both the conditions for its extension through problem-solving, and on rare occasion, the basis for discerning disruptive anomalies that open it to alternatives and “paradigm change” (Kuhn 1970, p. 65), in the models by which philosopher Kuhn (1970, 1977) depicted tradition and change in science. Under these analyses, equilibrium, with its moments of calm, sustains the undertaking of such significant scientific work as theory conceptualization, instrument design and apparatus construction.

In a climate rife with personally generated possibility—such as what Piaget pioneered in observations and clinical interviews with children—the space widens for all kinds of trials of tentative equilibrations. As relations with the world widen, so do the experimental initiatives of child or learner or experimenter like Faraday or investigator like Piaget, in the equilibrating interplay by which action and thought develop:

“discoveries of new means through active experimentation” [constitutes]...a process of learning...which partakes simultaneously of experience and mental construction (Piaget 1954, p. 95)

The experimenting that the child initiates in response to the world- without being directed to do so-involves the child in equilibrating processes of mind and experience that both brings them to ever-more complex predicaments and provides their means of building the cognitive skills by which they will meet those predicaments and form a next equilibrium.

This equilibrating process of learning is the inherent means of education for beings in the world. To instill it into school education depends on the vulnerability of classroom members to partake in disequilibrium, mutually sharing and sustaining each other’s involvement. This relation of mutual vulnerability is suppressed under conventional education, where the stance of possessing knowledge is privileged over the dynamic processes of questioning and not-knowing. Classroom environments “at sea” diverge offshore from the land-based classrooms, where being in disequilibrium can be interpreted as a shortcoming in one’s knowledge and as a perilous vulnerability.

In facilitating a learning environment so antithetical to conventional education, the teacher is subjected to tensions that go beyond the Piagetian interviewer’s role of neutrality and openness. The teacher’s part rests critically on sensitivity and vulnerability, on being responsive to uncertainty wherever it may emerge across her experience and that of learners.

The acceptance of mutuality of vulnerability among all classroom participants is sustained through trust that they develop together interactively. Any classroom has its own capacities for forming trust and letting members’ vulnerabilities surface freely, and for eroding trust and blocking access to vulnerability. Since classroom
life anywhere encompasses a range of trust and vulnerability, there is always some local context by which its members can evolve collective and reflective awareness that engenders trust. For example, through questioning distrust in his classroom and in its surrounding society, a middle-school teacher developed personal realizations and exchanges with students by which there emerged mutual trust and risk-taking in doing experimental science (Sconiers and Rosiek 2000).

Making Space for Disequilibrium: Critical Exploration in the Classroom and its Precedents in ESS

A sense of disequilibrium is only an opportunity for development. We may let that opportunity recede without engaging it; we may raise our guard against acknowledging the vulnerabilities that it could raise. Taking up the opportunity of disequilibrium carries the risk of invalidating what we supposed we knew. In the wake of such foundering, the process of reforming equilibrium is inherently and concurrently personal, collective, and spontaneous: depending on active engagement of minds with outside matter. While instruction may schedule and try to structure change in learners’ thinking, these instructional programs may not lessen the many inhibitions by which learners and their communities miss, avert, or forego opportunities for transforming their thinking.

Yet, teaching can soften the air, loosen the ground, sculpt valleys bubbling with intriguing sounds, shape melodic rises punctuated with reverberating chords, and provide gathering places stocked with handy and intriguing tools. Teaching can encourage the emergences of curiosity, befuddlement and community that give free play to swings between balance and off-balance, and the unsteady constructions that make for ever-evolving equilibrium in understanding.

The process of making a classroom “at sea” is a liberating act that acknowledges explicitly what “land-based” curricula leaves implicit: that the world is complex and that many mysteries and doubts remain despite our best efforts to explain it. One pedagogy that seeks to safeguard experiences of equilibrium and disequilibrium on the part of all participants, both learners and teachers, is critical exploration in the classroom, a practice developed by Eleanor Duckworth (2001b, 2006a, d, e) and her students (Cavicchi 1999, 2009; Cavicchi et al. 2009; Chiu 2009; Cifone 2001, 2013; Cirino 2001; Critical Explorers 2010; Delaney 2001; Gill 2004; Harouni 2013, 2015; Hughes-McDonnell 2000, 2009; Julyan 1988; Knox 2001; Lowry 2006; Magau 2001; McKinney 2004; Harouni 2015; Quintero 2001; Rauchwerk 2005; Rowe 1987; Schneier 2001, 2018 this issue; Shorr 2006; Shorr et al. 2013; Tierney 1988; Yang 2014, 2018 this issue). Work with mentors Jean Piaget (1960) and Bärbel Inhelder (Inhelder et al. 1974) sensitized Duckworth to attain neutral openness in following and eliciting anyone’s thinking, whether that of a child or adult.

The above authors and others practice critical exploration in the classroom across the full range of teaching settings from primary school to medical school, from small to large classes, from private to public schools, with subject areas ranging from science to art to foreign languages, and in schools around the world. The teacher of critical exploration is a researcher seeking, like Piaget, to understand and
probe learners’ actions and thinking while learners investigate provocative materials. Having investigation as the engine for classroom activities entails interdependency of investigation on the parts of learners and teacher: where learners are intellectually engaged and questioning, the teacher concurrently finds grounds to investigate, respond, question, and seek to extend. Any dropping away of investigative spirit by learners and/or teacher impairs and halts the cycling of investigative learning and teaching.

The onset of such exploratory involvement is “the first act of teaching” (Hawkins 2002, p. 60) for Duckworth’s colleague at ESS (Elementary Science Study), the philosopher David Hawkins. He characterized the relation through which that act happens as necessarily triangular: among learner[s], teacher, and subject matter of the world. In critical exploration classrooms, the third partner in that triangle, the subject matter, meets the learners in its full and vibrant complexity. It is accessible to them to investigate directly by hand, mind, senses, and tools.

A poem standing on its own without literary commentary (Schneier 2001, 2018 this issue); a falling object and instrumentation that records its fall (see below) and the moon in the sky as we observe it without explanatory guides (Yang 2014, 2018 this issue) are among the complex subject matter that figure in triangular investigative relations with learners and teachers in this issue’s accounts. In supporting the triangular relation, the teacher faces the demanding task of designing problems and materials that bring learners into contact with the world, and of valuing and keeping open the dilemmas that arise in learners’ thinking and contact with it. By means of the complexity and integrity of the problems and materials, learners’ experiences, questions, and observations expose and draw out the properties and relationships inhering in that matter. The matter of the world itself is the source and testbed of learner’s investigation.

The exemplary problems described above, Piaget’s question about brown and wooden beads and dropping his infant’s favored toy, sustain an amazing potential for ever-deepened involvement of the learner. Such problems and complex materials require a sensitive partnership with the interviewer (who will be a teacher in the cases below) who seeks to welcome the learners’ deepening involvement as it goes into disequilibrium, conceives new possibilities, and eventually—but by no pre-constrained or predictable route—finds its own reversible resolutions which are genuine and germane to the material at hand. In response to the learner’s emerging quandaries, the teacher’s imagination is constantly stretching in considering and seeking out yet further materials and problems that might put the learner’s quandaries into a new light, or open a window onto some area of the issue that the learner has not yet noticed or engaged. Such preparations and work pose challenges and conflicts for the teacher, as indicated in examples below.

These principles and practices came to be articulated and piloted in the 1960s science curriculum project initially directed by David Hawkins, the Elementary Science Study (ESS 1970; Romney and Neuendorffer 1973). The mandate of integrating science understanding and technical expertise into American culture, along with apprehensions stoked by Sputnik’s success and the Cold War, gave rise to a host of US federally funded science curriculum projects. Top professional scientists and educators developed and classroom-tested new curriculum programs
addressing all school levels and science areas (Rudolph 2002; DeBoer 1991). Spearheading these programs was the high school physics course (PSSC 1960) authored by prominent MIT physicists whose ground-breaking researches in physics included contributions to the Manhattan Project. Upon classroom-testing PSSC, the physicist-developers realized that only through having been nurtured in curiosity and exploration during elementary school, would high school students sustain the interest requisite for it (Griffith and Morrison 1971; Goldstein 1992). Acting on this insight, the physicists established ESS to foster the “natural curiosity of children and their freedom from preconceptions of [science as difficult]” (ESS 1970, p. 7). During its decade of duration, teachers together with scientists made up the hundred-member ESS development staff, producing philosophy, pedagogy and curricular materials that trialed with children, prepared teachers in summer institutes, and publicly reached 10–15% of US school districts (Griffith and Morrison 1971; DeBoer 1991).

Diverse materials of the world were put into children’s hands: clay (ESS 1966a); ice cubes (ESS 1966b); earthworms (ESS 1969a); balance and weights (ESS 1969b). Tools such as microscopes (ESS 1965) and printing materials (ESS 1969d) extended children’s means of observing and recording their experience. Open-ended questions, such as “what do you think will happen when the clay is placed in water?” (ESS 1966a, p. 5) encouraged children’s discussions and investigative actions. While ESS teachers’ guides for 60 investigatory areas abounded with photos, quotes from engaged children, descriptions of materials, and encouragements for follow-up activities and questions (ESS 1966a, 1968, 1969c), no conventional textbooks for children were produced.

Inseparable from opportunities for children to become engrossed in science experiences went the ESS curriculum’s emphasis on children’s agency in generating theory and questions. The physicists’ firsthand involvement with uncertainty, as productively fostering disequilibrium and equilibrium in doing science, grounded their commitment and trust for engaging children in the classroom with “deeper questions” of:

how we know what we claim to know, of evidence…of the effects of scale…of the nature of uncertain and partial knowledge (Griffith and Morrison 1971, p. 30).

Concurrent with ESS, federal funding supported scientists and educators in producing and trialing two other elementary science curricula (Bredermen 1982) of differing pedagogical stances. One, SAPA (Science: a Process Approach), was adopted in 9% of US school districts (DeBoer 1991). Under SAPA, students were to explicitly apply hierarchically organized science process skills according to a prescribed procedure, including observe, compare, classify, measure, experiment, and evaluate (SAPA 1965). Where ESS teachers were trained to facilitate open-ended explorations of children and catalyze their thinking (Rogers and Voekler 1970), such opportunities were rare under SAPA. The SAPA teachers’ role was explanatory and directive, including “seeking specific student response[s]” through teacher questions (Stefanich 1976, p. 384) based on textbook materials. A contemporary classroom observation study reported that ESS teachers fostered
enhanced student involvement in science, as compared to textbook-driven SAPA teachers, whose narrow convergent practices restricted student participation (Baker 1970). Having trialed both curricula, one Indiana school district adopted ESS, citing its “abundance of open-ended experiments emphasizing student involvement in the processes of science” (Baptiste and Turner 1972, p. 8). Advocating for science education where learners are actively constructing meaning, a subsequent critique refuted SAPA and ensuing analogue curricula by arguing that: science consists of no codifiable process such as SAPA presents; science skills are not transferrable, from teacher to learner or by learner to new contexts as SAPA premises; children enter the classroom with science ideas of their own, which SAPA did not acknowledge yet which matter to their learning (Millar and Driver 1987).

The ESS openness for children to explore—without being directed—made space for personal and collective experiences of disequilibrium within an evolving equilibrium. Duckworth relates such a development during a fifth-grade unit on pendulums (ESS 1969c). Children were shown a film depicting a swinging container having a hole that leaked sand as it swung back and forth. On seeing more sand piled under the outer termini of its swing, than at the midpoint, one child said “I don’t get it. Why isn’t it [the sand] the same all along…?” (Duckworth 2006b, p. 66) This observation carried the germ of destabilizing another child’s prior claim that a pendulum always swings at the same rate. While adults in the room kept silent, the children continued to watch. Exclaiming over the dearth of sand in the mid-swing area, conjecturing that the pendulum went faster there, and inferring that as it switches directions at the swing’s endpoints, the pendulum actually stops, each child delved further into the phenomena while breaking ranks with the first claimant on the issue. Through having formed critical observations and projected ideas of their own, the children came to consensus, co-sharing a new equilibrium in their understanding to which each contributed. Equal among these was the claimant child who resisted succumbing to the disequilibrating evidence posed by the others, and yet who was supported in eventually coming to realize that things were otherwise than what he had argued.

Such equilibrating experiences, arising through the children’s exploring and interactions, are transformative for the teacher and observer who values and encourages their development. On behalf of ESS, David Hawkins embraced this awareness of mutuality and vulnerability among learners and teachers:

> We who have been involved in this study of science and children have ourselves been changed in the process… we have been liberated. Those who knew children before science have now seen the former (and ourselves as well) … as inventors, as analysts and synthesizers… lovers of nature. Those of us who knew science first and children later have an altered and more childlike view of science… full of unexpected delights (ESS 1970, Forward)

**Teachers Partake in Disequilibrium and Open Possibilities for All**

In parallel with opening and sustaining a classroom environment where learners are safe in allowing the vulnerabilities of disequilibrium and in mustering creative
responses to it, it is also necessary to secure space where the teacher may undertake comparable risks and thus initiate novelty. By emphasizing a static form of equilibrium that teachers are expected to embody and students to work toward, school education often closes off their access to engage dynamically with equilibrium and disequilibrium. For unbalancing and rebalancing of understanding to recur out of the fabric of ongoing educational activities, the teacher needs to be fluidly involved in going into the unknown. Core in such a teacher’s work is the opening of multiple, diverse, concurrent complex possibilities for engagement, activities, discussion and reflection—by both students and teacher.

Like children in Piaget’s study described in a preceding section, such work has the teacher poised at the verge of a sea of potential shapes for learning. The teacher aspires to sense those possibilities as fully as Arl, the twelve-year old who said of a partially obscured object “One can imagine all possible shapes” (Piaget 1987a, p. 37). However in practice the teacher may correlate more with the view of younger children who imposed a particular or limited form onto it.

One way that a teacher prepares for exploratory class sessions is by imagining how the various entries, materials and areas of students’ exploration may interact, thinking more broadly than the confines of any textbook or conventional syllabus. During class, the teacher stays largely neutral during students’ selection of, and negotiation among, those possibilities—or of yet other possibilities that students raise that the teacher had not envisioned. Anywhere along these multiple ways, students and teacher are apt to find their thinking and expectations cast into disarray. Those are the disequilibrating passages that merit pause to look closer and consider possibilities for investigation anew, through debate and reflection. Teaching involves being observant to identify such passages and to support the accompanying deepening and vulnerability, including uncertainties that arise for the teacher. Doing so engages the teacher intellectually in ways that are not supported by the traditional classroom setting and by the packaged curricula that direct classroom lessons.

For a teacher used to being a source of knowledge, this can be an uncomfortable skill to learn. Yet—once learned—the skill is easy to master: it involves letting the phenomena become the grounded truths under study, rather than and instead of words and writings describing the phenomena. Teaching through this method requires less expertise in remembering words and explanations, balanced with more expertise in observing and reflecting: skills that are applicable to teaching a variety of subjects.

The intermingling of spontaneity and extended puzzlement among students and teacher are manifest in Duckworth’s account (Duckworth 2001a) of a seminar where she involved teacher education students in exploring floating and sinking. In a follow-up to a class activity where colored rubber bands were found to either float or sink depending on their colors, one student brought from home a collection of colored rubber bands that baffled all, including the teacher who borrowed them to examine further. White ones always sank while those of other colors fluctuated statistically among floating and sinking. Meanwhile, balloons (that the teacher had provided, supposing students might want to release air from them to produce bubbles underwater) were put to an entirely different use by learners. Filled with varying amounts of water, not air, and placed in water, the balloons floated, even when greatly distended with water and heavy.
Unforeseen by the teacher, this student-initiated experiment astounded everyone and its disequilibrating effects persisted. Revisiting it in a later session, Duckworth floated a water balloon in the tub before them; then asked for students’ thoughts. One raised a concern about whether the amount of water in the tub mattered to the balloon’s floating. Considering this student’s doubt as oblique to the discussion at hand and following through, not with it but with what was building in that other discussion, Duckworth replaced the water balloon with a rigid-sided container. Of her own uncertainty in doing so, she reflected:

To make a connection with their thoughts, I was groping as much as they were, and I moved toward the helpful questions stumbling as much as they did in moving toward the helpful ideas.” (Duckworth 2001a, p. 23)

The “helpful ideas” for the water balloon emerged, not that day but in the seminar’s final session, prodded by a school child in one teacher education student’s class who had said of the puzzle: “Water goes with water, and the balloon floats” (p. 32). Those words of a child were as enigmatic as the phenomenon, until the group investigated water balloons yet again.

By filling balloons to differing extents, testing, observing and discussing what they saw, the class came to equilibrium through a collective process of their own. That process had strayed along paths and experiments that seemed unlikely to yield productive thinking even to this perceptive teacher. For example, the floating of water balloons in water was a quandary discovered by students among the many possibilities of what can come of balloons. While this possibility was not envisioned by their teacher in advance, she welcomed it, together with the uncertainties that it raised in her teaching and their learning. Being evoked through a context where the students freely admitted disequilibrium, that possibility sustained their engagement to the point where they performed the intellectually reversible act of supposing the water out of its balloon, with the balloon floating beside its water, as equivalent to the floating, yet heavy, water balloon.

Sara Egan, a museum educator, experienced a yet more trenchant impact of “what can happen when teachers invite their students to observe” (Harouni 2013, p. 130). It “caught her off-guard” (p. 138) when a high school student observed that those artisans who made the crafts on display in the art gallery where the class met, would not have been allowed to use the finished crafted objects, such as ornate silver pitchers. Being a fan of Isabella Gardener, that gallery’s founder, the teacher had never entertained such a critique herself. Yet she summoned the openness to respond by asking students to describe emotions they felt in that gallery. Her students offered that they felt “belittled”. While someone like President Obama would be allowed to touch things in there, they were not allowed to touch them (p. 138). These remarks challenged Egan’s world-view about museums. She honored this disruption by giving room for it in discussions. Critically examining the heritage that museums project, Egan returned to her next class with materials attuned to concerns that students perceived, yet she had not. Coming to such a response—a new equilibrium in her role as a museum educator—required (and developed) courage to hear struggles with the complex values and heritages of culture as experienced by her students, the artisans, and even herself.
In opening class time to questioning initiated by students, which Duckworth and Egan had not expected, these teachers risked disequilibrium in how they understood the material and their role. Upon taking on those questions emergent from their students, the teachers encouraged possibilities for investigation and discussion that raised issues new for them, such as what students might do with balloons, or how a cultural institution might diminish those it seeks to celebrate and teach. Not only did learners move beyond the matters that stood before them to enact reversible thinking, such as by imaging water being both in and out of the balloon, so did the teachers. By considering perspectives other than her original view, then reversibly assuming a new sense of her own, Egan found herself changed as an educator.

**Reversible Thought**

Over 30 years ago, a young teacher noticed how the vibrancy of urban teens being themselves in their high school’s halls had utterly sublimated once those same students were seated in classroom rows (Schneier 2018, this issue). In meetings with small groups around lessons in math, English and science, teacher Lisa Schneier sought to invite their inner being into open play with those school exercises. Trust eked gradually into those relationships, to where—as the teens applied their minds to the school work—Schneier came into a momentarily leveled balance in the teacher and learner relationships; she glimpsed sense in their personal constructive processes.

Reacting to the fractional notation used in math problems and written out on paper by Schneier, their teacher, students concurred with Seth who argued that 1/4 is larger than 1/2, just as 4 is larger than 2. Schneier invited further engagement with the problem by asking her students first to imagine something they wanted to divide among five participants (including the teacher) and then to draw it on paper. During the drawing and division activity, students’ surety started to fray in different ways. To Tony’s tentative ruminating among multiple possibilities “five … no … one-fifth … no … one”, Seth rejoined with the sole definitive outcome of “one-fourth” (Schneier 2018, this issue). He argued that it came out that way since four pieces were left-over when that one was removed.

Weeks later, Annie, whose participation in the above discussion was limited to a shrugged assent with Seth, came to engage openly with these matters. Of three floor tiles before her and Schneier, Annie proficiently applied Seth’s protocol in saying that if one tile were taken away, that tile would be one-half. Standing on one side of all three tiles while Annie stood on the other side of them, Schneier asked Annie how she would divide the tiles “in half between us” (Schneier 2018, this issue).

She paused and started to point to a line between two of the tiles

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but then slowly moved her finger to the middle tile and drew it downward in a line

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Looking at me, she said, “It’s confusing. There’s two halves.” (Schneier 2018, this issue)

With her teacher silently beside her, Annie continued to ponder the floor, at a cusp of uncertainty before possibilities that her mind provided. A complication in those possibilities arises through instructional use of abstract representations which appear arbitrary and meaningless to many students. In Schneier’s analysis, the fractional notation \( \frac{1}{2} \) was barren of its functional meaning for Seth and Annie. Under the pseudo-meaning these students supplied to fractional notation, when one piece is removed from a group, that piece’s portion of the whole group is represented by a fraction whose bottom number is the number of remaining pieces. Annie pondered two methods of decomposing a whole into parts, but one—the one that seemed to be the “preferred” version at the school—was corrupted, not a genuine possibility.

Tony and eventually Annie commence the unsteady mental acts to compose, decompose, and recompose the part and its whole, the drawn shape with its divisions, the tiles with the floor pattern. In doing so, they were hampered by the symbols through which educators designate reversibility in the concept of a fractional part of a whole. These students’ prior education had failed to connect them to the power of symbolic notation to assist in reversible thinking: to propose a relation, work it through in the mind, and then undo or revise it again. Instead of setting students’ thinking afloat on a substantial medium that supports endless reversible ways to paddle or travel, those same symbols recede into what Schneier evokes poignantly by “empty shadows” (Schneier 2018, this issue). Stranded in a sea of emptiness, these learners are bereft of medium that could sustain their minds in motion; they are left reaching after rote answers dropped from above on an irreversible fishing line.

The relationship provides the mind of learner or teacher with means to act reversibly. If the relationship cannot be accessed, thought cannot make a reversible and integrative understanding of the material in which a relationship inheres. In such cases, as thought engages with the material, its various observations may isolate from each other, or link by distortion [such as Seth’s proposal]. When a structure of thought whose links are irreversible or distorted encounters something discrepant to that structure, the mind’s resources for acting on the discrepancy are limited. It lacks the underlying relationship by which equilibrium and disequilibrium retain, lose, and restore balance.

Schneier’s story carries a further destabilization, reverberating across schooling, both then and now. That destabilization pivots on the incommensurability between everyday school classrooms, and what goes into making a space safe for experiences of disequilibrium among children, teens, learners, and teachers. Tools of schooling, curricula and instructional roles are sealed against opportunities for pausing in unexpected puzzlement (Yang 2018, this issue); for brooding back and forth in thought, action and materials; for observing something outside ourselves recurrently enough to construe its habits even as these perhaps undo one’s own reasoning about it; or for listening deeply enough to impart space and connection among what went unsaid and unacknowledged. By stepping without the weight of our minds attached,
we do not break through the brittle crust that accretes between school participants and the stuff of the world; by aiming for pre-set goals, we do not see the divergent and destabilizing passages in our own learning journeys; by staying between ruled lines, we restrict thought to rely only upon irreversible pathways.

**Provoking Reversibility between Algebraic Symbols and the World**

Something like that brittle crust of ice—impenetrable and unacknowledged—separated the instructional physics I encountered as a physics teacher and student from everyday physical phenomena. That barrier was inessential. From my own curiosity and that of others in and out of classrooms and in history, I knew that the world engenders inquisitive responses of endless vibrancy. On putting this conviction to my first educational test with a group of schoolteachers as my students (Cavicchi 2009), I was moved and compelled by their joy in deepening understanding through experiments we devised together. Inspired by this example to develop genuine investigation as a practice of learning and teaching physics, I commenced doctoral studies in education.

A course research project in my first doctoral term had me observing a college algebra class and interviewing its students about their experiences with math in everyday life and in that course. This algebra class (Kime and Clark 1996) was an experiment in practicing algebraic analyses through examples from social, physical, and life sciences and in having students collaborate in discussion, open-ended problems, and computer graphics assignments. At the invitation of the course’s development team, I proposed that the parabola, a form of relationship that the course studied with graphs and algebraic expressions, be illustrated by a physical behavior that exhibits this relationship. As the physical example, we chose a standard introductory physics exercise: the relation between distance traversed by a weight falling freely and the time elapsed in its fall. During the term that I observed, course instructors arranged for their math students to visit a physics lab as guests to collect free fall data, and plot that data to produce a curve demonstrating how the distance fallen by a weight grows as the square of the time since its release. As context for this activity, I was writing a supplementary reading on the experimenting by Galileo through which this relation emerged (Cavicchi 1996).

I observed the session when the algebra students conducted the free fall lab and analyzed it. I noticed that the only person to manipulate the Behr free fall apparatus was the physics lab instructor. For each pair of students, he produced a paper tape marked with the falling weight’s position at successive equal time intervals (Fig. 1). Instructed to use a meter-stick to measure the distance between the first mark on the tape and each next mark, students attended to how the tape’s marks matched with the ruler’s divisions and negotiated which numerical value appeared closest to each mark.

I followed students to a separate computer lab. As one partner read the data aloud, the other typed it in, pairing each distance measurement with the sequence number of the corresponding mark. While students were not expected to work out graphing protocols on their own, the instructions provided on a handout did not provide for seamless data entry, plotting and printing. Panic struck one group that
needlessly feared their laboriously entered data was lost; another group lost, then retyped, their data three times. Others struggled with incomplete or mangled printouts. Unacquainted with the software (Hausknecht and Kowalczyk 2000–2011), I could not assist when students asked me if they were doing something “right”. To one student’s query “What’s the experiment objective of this thing?”, I heard another respond “Who knows?” (Cavicchi 1993).

To my view, this trial of a lab lesson in a math class proved ineffective and lacking in meaning. Anita, a 40-year-old interviewee, confided, “To be very honest, I did not find [the free fall experiment] useful. I hope I didn’t hurt your feelings”. Saying “I need to know”, I encouraged her to share more. At the fore were tedium and frustrations with measurement, data entry and plotting, echoing all that I observed.

Yet our further conversation yielded something that I had not witnessed: a potent confusion that came to be exposed through struggling with the free fall homework.
Anita It was entirely confusing to me...All along we were dealing with x and y...suddenly I was thinking in terms of time and seconds and centimeters and it took me for hours to realize that – wait a minute- that’s the same thing...The whole free fall experiment suddenly combined two things (Cavicchi 1993, Nov. 30)

“For hours” Anita hung in disequilibrium. The assignment sheet (Kime and Clark 1993) freely interchanged the x–y formalism with the measured physical entities of time and space. Yet for this learner, no such relation existed until she constructed it, eventually construing how graphs, spreadsheets and math could work with something other than algebraic symbols x and y: “that’s the same thing”. An underlying premise of the algebra course, that math is expressed in the world around us, emerged unexpectedly as the means of her personal synthesis. She realized the import of this synthesis through the opportunity for reflection afforded by my interview. A basic assumption of the course assignment—that algebraic symbols designate relations of the world—became its unplanned lesson for Anita.

The reorganization that Anita accomplished in her understanding was profound. While she was already adept at solving and graphing equations with x and y, for her those algebraic symbols existed in a math space with no referent. Like Julia in Schneier’s article (2018, this issue) who applied a distorted translation of the meaning of fraction symbols, Anita had manipulated x and y reversibly as symbols in solving and composing equations. Yet the symbols’ reversibility was limited for her; Anita could not bring it to bear on problems of the world. By contrast, through her new understanding, Anita’s reversible thinking extended to the world. Those x and y symbols became an aid in describing and analyzing relationships of things in the world. As with 12-year old Arl (Piaget 1987a) above, Anita envisioned a multitude of applications for algebra, saying “I want to see: how does it connect to everything else” (Cavicchi 1993).

Although her struggle with the free fall assignment provoked this productive disequilibrium, Anita identified its origins in specific course assignments in social science statistics and readings (Kline 1967). Now, Anita felt empowered to initiate her own mathematical analyses within everyday experience. For example, after talking with a software company (before internet-based software distribution) about how the cost of a diskette dissuaded them from distributing upgrades, Anita had the idea to calculate the impact of that extra cost, thereby reaching a new appreciation for business concerns: “these marketing decisions started to make sense to me” (Cavicchi 1993, Nov. 30).

When I elicited and reported Anita’s story, I did not know how to see it as a learning process. Weeks of observations and interviews left me with students’ words and classroom descriptions, but in all that data I could not yet discern the passages, interactions and journeys of learning. As with the algebra students’ sense that x and y symbols operated disjoint from the world, for me the words and records of a classroom stayed separate from any learning happening within.

For me and for Anita, what might produce reversible interconnections between symbolic representations and processes of math, or of learning, was missing. Feeling myself without means to see and make interconnections between my data
and experiences of learning, I regarded with doubt my inference that students’ “new understanding of [algebraic] symbols derived from the building of a link to something…in the familiar world” (Cavicchi 1994a, p. 23). “Without an opportunity to question the process” of learning, I felt I had not seen “the process of [students’] confusions”. I needed to observe and know something I didn’t know I needed to know—the unfolding engagement of the learner with all its uncertainties and syntheses.

I doubted that “the questions about teaching and learning that remain for me” could be adequately addressed through the methods I employed in studying the algebra class. Inside my questioning of the observation and interview methodology was my sense of being without reversible means of operating with data I collected during my student project and in a previous educational study where I interviewed students (Di Stefano 1996). I heard students tell of frustration and resolution, but those were only stories to me and did not bring my thinking into the acts, possibilities and development of theirs. I had yet to learn to see, value and evolve relationships bearing equilibrium and disequilibrium among experiences of learners and myself. Further reversibility would emerge through learning and teaching lived as ever-interchanging and dynamic relationships, as the following examples suggest.

An alternative opportunity to interact with experiences of learning emerged in my final interview with a student after she completed the algebra course’s final exam. Like Anita, this student, Halle, also “felt confused” by the free fall homework; a subsequent class activity moved in her a “realization of what it was all about” (Cavicchi 1993). When I asked for specifics, Halle could not summon any, saying “it’s really hard to pinpoint …after” an experience has passed. Instead, she proposed the idea of researching that process by following one student closely throughout a semester. Resonating with this proposal, I later asked if she would be such a learner through participating with me outside of a course context. Halle agreed.

**Reversible Thought Develops Through Disequilibrium**

We were both first-year students; Halle in college, I in doctoral studies. Meeting in a physics lab or outdoors for regular sessions of an hour or more that spring and summer, together we explored physical motions, periodicity, teaching and learning in experiences that evolved in response to what we noticed and wondered. I was a beginner in the practice of engaging, following and extending a learner’s thinking that Eleanor Duckworth later called critical exploration in the classroom.

Sessions with Halle were my first research efforts in supporting a learner as an explorer. The work that I did in conducting and reporting on these sessions constituted my research project for the graduate level course that Eleanor Duckworth taught on practicing critical exploration in the classroom and understanding Piaget. Immediately after each session, I transcribed its audiotape and wrote a detailed field report. My reports integrated: lists of experimental materials; dialogue excerpts; diagrams, illustrations and data; narrative descriptions; my reflections and struggles in following Halle’s understandings; ideas for future
activities. Duckworth’s written comments on each report contributed to my thoughts for the next session, as did discussion with my classmates in Duckworth’s course, teaching fellow Susan Rauchwerk, and physicist Philip Morrison (a developer of the PSSC and ESS curricula). As preparation before sessions, I sought to widely envision what we might notice and do, what other materials or tools might facilitate those investigations, patterns that might emerge in the phenomena or our thinking about it. Being a novice in envisioning widely, I began by deriving these preparatory thoughts from the vicinity of physics I had encountered. Yet as our experimenting together brought us to ever-more provocative questions and observations, the depictions already traced within physics receded for me and I felt “drawn to retain the complexity” in whatever engaged us (Cavicchi 1994b).

Our undertaking nearly collapsed at its start. In my proposal for doing a research project with Halle, I wrote that we would investigate time. I imagined that this investigation could traverse many possible areas of science, ranging from measurement to relativity. I collected a stash of tools and materials that could be provocative for considering time. I brought these materials to our first session, but I did not unpack them. Wanting the student to own and develop her own investigation, I refrained from providing any specific entry or material. My doing so reveals that I did not yet grasp how a teacher of critical exploration engages learners with materials which, by their complex nature, evoke endless possibilities for investigation. Instead, I tried to start the session with endless possibilities, yet lacking a medium for their encounter.

Desirous to encourage every possible response, my first question was so expansive as to give Halle nothing to do or examine. I asked: how might we understand “when and where, time and space”. Halle was without grounding:

Halle I don’t know… Where to go with that. (Cavicchi 1994b, Feb. 17)

As the interview sagged and Halle might have abandoned it, a passerby inquired the time.

This interruption was about time, the very theme which underlay my thoughts for our sessions and my broad question. Now Halle found a place for herself within that theme; she was an observer. Her exchange with the passerby posed to her both the role of the observer, and the challenge of fulfilling that role. Events and emotions interfere. As she portrayed it, such an observer is there to “live the moment… Just let it happen…each moment is similar.” Halle’s conception of an observer who lives fully self-aware while not imposing any of that self on the observations being made, had me bemused yet sensing its relation to observing in science. She described the flow of time as a river, a continuum, where one moment is no different than the next. Seeking to form my questions from her ideas and drawing on her words “flowing time” or “moment”, I asked how we might become such an observer and what means that observer might have for noticing time. Halle supposed that even a rock is not always the same. Listening to her, I pictured a rock in my mind, examining it for change.

With my thoughts seeking experimental extensions of hers, it came as a surprise to me when Halle reverted to my original question about “when and where”. She now discovered a way into it, by the very experiment that had brought us together:
Halle The free fall! Yeah! Oh yeah! I think that’s what free fall is about: when and where.

I shared with Halle my delight for that connection, and that I had not foreseen it. With gathering enthusiasm for “fun” of doing the free fall experiment in the light of when and where, Halle envisioned its educational contrast with conventional physics instruction:

Halle Instead of [the physics lab where]… free fall is: you drop a ball and this is what happens. …It might be more interesting to look at … the movement of time through space, and how the moment.. changes (1994b, Feb. 17)

Unlike my broad initial question, the free fall experiment proposed by Halle carried the potential for multiple investigative experiences whose interrelating of thought and action are means of coming to reversible understanding.

After our session, I arranged with the physics lab to use their Behr free fall apparatus (Fig. 1). A switch on this apparatus releases a metal weight so its fall traverses a vertical space between two parallel wires. A waxed paper tape, which is the length of the fall, is positioned between the weight and the wires. At periodic time intervals, a spark jumps from one wire (at high potential) wire to the other wire (grounded) transiting through the metal weight. A spark marks the tape at the position where the weight was during each pulse. The pulse interval, variable with a dial, is usually set at 1/60 s. The fall takes about ½ second, producing about 30 marks on the tape. The marked tape is torn off for analysis; fresh tape is extended before a next release.

The opportunity to explore with the formidable apparatus imbued us with playfulness. The contrast between the single use to which it was usually put, and the possibilities before us, was giddying to me. Exclaiming over having a “choice”—unlike usual instructional science labs where students have no choice—Halle proceeded to invent experiments! She released the weight once without switching on the sparks, then released it again, with the sparks on, and again.

Next, Halle did something that had not occurred when the algebra class did the lab, when each team analyzed only one tape from one free fall event performed by the lab instructor. She placed the curling tapes from successive falls side-by-side on the table. With this simple act, Halle took ownership of data she generated through releasing the weight and expanding her experience with its fall. As this side-by-side comparison of the fragile tapes with their subtle dots held her gaze, her thinking came into a disequilibrium that drew ever-deepening involvement.

Recalling from algebra class that each mark is made 1/60 s after the previous one, Halle discerned how the lengthening spacing of successive dots belies the equality of their time intervals:

Halle Each 1/60th second it [space between marks] gets longer and longer as it [the weight] goes down… Not the moment in time, but the weight is traveling faster. (Cavicchi 1994b, Feb. 25)

Yet it was not the tapes’ indication of accelerated motion, but something I regarded as ancillary, that perturbed her:
Halle Why is it?...Why wouldn’t it [marks on the tape] be exactly the same every time you do the experiment? (Cavicchi 1994b, Feb. 25)

Placing two tapes side-by-side provided her means to see that the dots on one tape were not spaced identically with those on the other (Fig. 2). Two different releases of the identical object from the identical height resulted in different impressions on the tapes. Welcoming Halle’s curiosity, I affirmed her interest in that difference and asked what we might do to look into it. Halle conjectured that tapes might receive differing dots if the sparking, or the weight’s fall speed, was not the same during one fall, as during the other.

Sensing that some consistency held between any two falls, she wondered if the overall pattern of dots on one tape might be spaced in proportion with that on another. On examining those patterns closely, Halle’s perplexity grew. As she looked further into the weight’s fall along a tape, the more its dots became out-of-sync with the position of those on the comparison tape (Fig. 3). Having her attention drawn to the fall’s end, she raised a new question, about where that endpoint occurred on the tape.

In trying to grasp what perplexed Halle, I sensed how ambiguous our evidence actually was. Her observation was astute; our tapes lacked any indication about where that endpoint occurred. A vague awareness of inadequacy in the physics rendition of the lab was emerging for me. I said I was confused. As Halle rebounded in proposing a reference mark to be made at the tape’s bottom, she articulated possibilities for how fall and spark might interrelate that she had not considered before:

Halle Is the last spark where it [weight] actually stopped? Is it?
Author Now I am completely confused about when and where!
Halle I think we need to mark the tape at the very bottom, where it [weight] stops! So that we know whether the spark sparked where it stops, or before it stops! (Cavicchi 1994b, Feb. 25)

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**Fig. 2** A scan of the first segment of one tape is placed above a scan of another. The first dot on each tape is at the left (number labels have been added to the tape’s scan). The first dot on one tape is not at the same place as on the other tape. The spacing between corresponding dots, on the two tapes, are different. The pulse interval, for each tape, was 1/60 s. In each illustrated tape, the left side is earlier in the fall than the right side of the tape.
She did not pursue this proposal immediately. When Halle acted on it later, so much intervened that I had lost touch with her rationale for marking the tape’s bottom. However, for Halle, her uncertainty under-girded her experimenting, gaining palpable form in two new tapes whose bottoms she marked. Voicing my confusion, I asked her to explain what she saw. On one tape the final dot was further above her hand-drawn bottom mark, than on the other. She offered two sources for this discrepancy: that the weight fell faster in the run whose final dot recorded closer to the bottom, and/or that the spark mechanism was involved. Halle was developing as an experimenter, both by assimilating the dots’ positions to her expectations about speed and by watching the apparatus as it sparked as a means of modifying [accommodating] her understanding to its function.

As Halle’s queries took forms beyond what I envisaged, my mind was too unsteady to surmise what interlinked her thought. I experienced her statements and questions as isolated. Through lacking a grasp of the relationships underlying her thinking, I could not integrate reversibly among the possibilities and ideas that she raised.

We explored the spark mechanism. Over and over, we watched and listened to the noisy sparks, with and without releasing the weight. Adjusting the variable frequency dial, we released falls with sparks triggered at intervals of 1/30 and 1/5 s. The 1/30 s tape accorded with Halle’s expectation in presenting half as many dots as our usual tapes (Fig. 4). We faced the 1/5 s fall with great suspense, Halle with stopwatch in hand, seeking to check its timing. I described that moment:

The fall happened so quickly, and was so surprising, that we did not know what to make of it. I was used to watching the glow of sparks along the whole apparatus, but this time did not even know where to look for them. There were only 2 or 3 dots on the tape. (Cavicchi 1994c, Feb. 28)

Too nonplussed to stop the watch, Halle exclaimed “I don’t know!…I have to find another way.” The inadequacy of the stopwatch for measuring the spark interval’s brevity both thrust upon Halle what she did not know and yet had her contemplating other means of checking its measure. Concurrently, this trial revealed what I did not know in the domain of teaching. It humbled me as a teacher to witness how the learner did not respond to what was most salient for me about that trial: that the

Fig. 3 The last dot recorded on the top tape is at a different position from the last dot recorded on the bottom tape (see Fig. 2 for the early dots on these same tapes). The difference in spacing between corresponding dots, is greater than it was near the falls’ beginning.
cycle, by which sparks came on and off, occurred independent of the weight’s position in its fall. While the sparsity of sparks in the 1/5 s interval clarified for me what I already understood, that observation did not of itself resolve the picture for Halle.

Releases of the weight with the spark frequency set at 1/5 s occupied us for an entire session. The recording of two 1/5 s releases on the same tape introduced new discordance (Fig. 5). The first dot of one fall was separated from the other fall’s first dot. Halle exclaimed:

**Halle** That’s really weird! Why is that like that?! (Cavicchi 1994b, Mar. 4)

To investigate, we recorded five 1/5 s trials on the same tape, labeled each dot with its trial number, then examined the relative positions and spacing of these dots. The tapes showed that if a fall’s first dot was lower than its counterpart dot in another fall, then the spacings between all successive dots in that fall were longer (Fig. 6). To Halle, a longer spacing indicated a greater speed, thus she conjectured that some falls recorded greater speeds than others. This finding appeared to her to conflict with the weight’s release always from the same height. Restating our first session’s

**Fig. 4** Intervals of 1/30 s intervene between successive sparks during the fall of the weight recorded by the top tape; intervals of 1/60 s intervene between sparks for the fall recorded on the bottom tape. About half as many dots are recorded on the top tape.

**Fig. 5** Two successive falls of the weight are recorded on this same tape. The first dot of the first fall appeared above (more to the left here) the first dot of the second fall. The pulse interval was 1/5 s for each fall.
question of how “when” might relate to “where”, I asked if that time and space relation could be part of what made the tape “confusing”.

Reexamining the tape, Halle construed that the weight, always “gathering speed” as it falls, would be going faster at the point of the lower first spark. With this analysis, Halle had begun to form means of a new equilibrium. She associated longer spacings between successive dots in a fall having a lower first spark dot, with greater speed of the weight (at that point).

This understanding was tentative and not yet reversible. For Halle, the longer spacings between dots belonged to falls with greater overall falling speed; she did not reversibly apply the relation between dot spacing and motion to simultaneously apply to different moments in the same fall, and to different moments in other falls. As indicated in the passages to come, she expressed this relation between dot spacing and the weight’s speed in ways that show she comes to see more in it.

What made those first spark dots appear at different positions remained nondescript for Halle. However, Halle did not view this non-coincidence as a matter of physics; instead she attributed it to my lack of “coordination”. She viewed the differing positions of first dots for different falls as due to my failure to perform simultaneously and consistently the separate acts of releasing the weight and switching on sparks. To invite her to consider other possibilities for the sparks’ non-coincidence, I expressed uncertainty about what goes on at the moment of switching on sparks. Halle averred that voltage would then be present, but “there’s no spark”.

Halle’s undiminished commitment to my role as the source of the non-coincidence figures in her next proposal. To investigate non-simultaneity in my acts, she asked me to deliberately delay switching on the sparks until well after I released the weight. We laughed loudly at the prospect of amplifying my uncoordination! The inverse option—delaying my release of the weight until well after the sparks had started—occurred to me after the session. I regretted then that I had not proposed it.

The first dot was recorded far down the tape. I asked if the weight was at that very spot when I had switched on the sparks. Halle agreed, then wavered:

**Halle:** Yeah. No. No. Oh. So. The options are… (Cavicchi 1994b, Mar. 4)
Halle’s repetitive voicing of the alternatives repetitively is similar to what Schneier observed with students Mary and Jenny during moments when their thinking swayed in disequilibrium (Schneier 2018, this issue). Halle now envisioned various possibilities for the weight’s position: at the dot or “somewhere else” above it. After a long pause where I was silent too, Halle pictured the sparks as going on and off. Now she voiced the sparks’ repetition: “spark, spark, spark, spark”.

Then, saying that a spark might not occur at the very moment of switching the sparks on, she imagined the already falling weight as “halfway between two sparks” at that switching moment. Since the weight had already dropped and gathered speed, when the next spark came on it, she said it recorded longer spacings for the weight, than in some previous fall. Halle said “so that’s why it’s confusing.” As I heard her overlay the spark’s periodicity with the weight’s falling motion and acknowledge that the weight might be anywhere, I also ascribed to her an understanding that she had not stated [or made yet]: that the varying extent, by which the weight might have fallen before the first sparking event occurred, accounted for the differing positions of first dots on the tapes.

Uncertain, she pleaded for my validation: “just agree with me”. Although I heard her reasoning as viable and did not yet see its limits, I did not provide agreement. Searching for ways for understanding to arise from the work, not the teacher, I asked if this argument helped her with the other tapes. Reviewing the tape with multiple recorded falls, she now extended her tentative understanding of the relation between spacing between dots and falling speed in saying that the longer successive spacings of one fall, compared to another, were due to the weight having “fallen and gathered speed” before reaching its first spark. (Cavicchi 1994b, Mar. 4)

Halle reaffirmed that my “coordination” was responsible for the offset of one fall’s first dot, from that of another fall. This surprised me. A source of my surprise lay in my obliviousness to having flowed my reasoning into hers so as to override and obscure hers. As a novice researcher in the Piagetian tradition, I regarded the supplanting of my understanding over the learner’s thinking, as a failing on my part. This concern would not arise in conventional education where teachers are assigned the objective of bringing what students do and think into convergence with correct science terms and concepts (Abrahams and Reiss 2012).

My sense that my grasp of a learners’ thinking was inhibited by my own, was destabilizing for my perception of teaching. Previously, I had aspired to thoroughly work out whatever I taught. Now I recognized a downside to having a thorough mental picture. The relationships carried in my mind blocked me from seeing how a learner might draw on the same evidences that I saw, to weave relationships that were functional, yet differently founded. Our explorations brought each of us to disequilibrium in what we supposed we understood: for her, about time and motion; for me, about teaching and learning.

Engrossing us across several sessions, the free fall apparatus took us to questions and thoughts that might not find a hearing under typical physics instruction. Acquainted to its use for one ½ second fall, when the lab instructor stopped by, he asked “Anything wrong? You are staring at it!” (Cavicchi 1994b, Mar. 4) For us, the more we observed with it, the more there was to ponder. In preparing for sessions, I imagined explorations involving other tools which I brought along:
stopwatch, metronome, egg timer, alarm clock, tape measure, carbon paper… Yet we scarcely touched these. During our sessions, I was so intent in following what emerged for Halle that I could not marshal means of transitioning to other options. My seeking to hear her thinking by asking questions, and restating what I heard, intensified my awareness of my uncertainty and limits. On sharing my uncertainty with Halle, she acknowledged how we both stepped in unknown ground.

My teacher for this study, Eleanor Duckworth, encouraged our process saying

> If you’re in the area of what you’re interested in, then don’t short-circuit the struggles that might be at the very heart of your own questions. (Cavicchi 1994c, Mar. 13)

Entangled in that heart were multiple modes of bringing the singular event of free fall together with recurrent periodicity of the spark mechanism. Reexamining the tapes at our next session, Halle expressed a deepened relational understanding of the differing dot patterns of successive falls: “the spark is at a different place at a different stage in its fall.” Mindful of having forged a new understanding through her own efforts, she celebrated “So Glad I figured that out!” (Cavicchi 1994b, Mar. 9) I forestalled that synthesis from short-circuiting by asking again about the offset among the first dots of different falls.

Her relational understanding had not disturbed her conviction that this offset derived from my “coordination”. In articulating this argument, she conceived a means of removing my “coordination” from the experiment. I was to switch the spark mechanism on first, then release the weight. As the inverse to the previous session’s release of the weight well before activating the sparks, I had felt remiss in not proposing this test previously. Now Halle developed it through her own investigatory thinking. Her reasoning in producing this discriminatory experimental test became an opening for me to trust the equilibrating process and potential of the learner’s investigation, as source and resource for working through their quandaries.

Heightened suspense attended our release of multiple falls recording on the same tape, while the spark mechanism remained on throughout. Under her interpretation that was not yet fully reversible to all possible releases and falls, Halle expected the dots from these multiple falls would identically overlap producing “the exact same pattern”. Instead each fall’s first dot appeared somewhere different (Fig. 7). Halle exclaimed over the dots’ non-coincidence; we laughed at the demise of her theory about my coordination.

![Fig. 7](image)

> Location of first recorded dots in four separate falls of the weight. The weight was released AFTER the sparks were activated.

**Fig. 7** Four successive falls of the weight are recorded on this tape. The top of the fall is at the left side. The first dots in each fall are labeled with the trial number. The weight was released after the spark mechanism was turned on. The pulse interval was 1/5 s for each fall. In trial 2, the spark was triggered much earlier in the fall (more to the left) than in trial 1
This further destabilization was generative. Drawing on her trust in the consistency of 1/5 s between sparks, Halle integrated their repetitive rhythm together with the make-up of intervals. She portrayed each spark interval as containing yet smaller divisions of time. The weight’s release might occur within any of these internal divisions; it was already falling when its first spark recorded:

**Halle** It’s not … at the same point in the 1/5 s that you let it [weight] drop [each time]; it’s … at the beginning, or halfway through… or at the end of 1/5 s (Cavicchi 1994b, Mar. 9)

Halle now had full grasp of the understanding of the offset among the tape’s first dots that I had preemptively ascribed to her the week before. She had built a theoretical model by means of intellect, interacting with experiments that she initiated and changing in its structures. Her thinking about the 1/5 s intervals developed a flexibility and reversibility not accessible to her previously:

**Halle** 1/5 s can start anywhere… It isn’t one kind of fixed idea…The end [of one 1/5 s] is the same as the beginning of the next one… (Cavicchi 1994b, Mar. 9)

This perspective propelled her invention of a thought experiment. She proposed that we record “a zillion” falls on the same tape. The dots on that tape would compose a continuous line, holding and melding:

**Halle** Every moment, every split second of time, every whatever of time… (Cavicchi 1994b, Mar. 9)

This eloquent synthesis was wended through all the disequilibria and equilibrating possibilities that came about as Halle questioned, experimented and proposed ideas of her own.

Moments of time flow both split and continuous. Halle asked for my agreement without depending on it; she knew. The moment of time that Halle raised in our first conversation bore a heart of her investigation, opening understandings not originally accessible. What Halle accomplished intellectually, with understanding time was equivalent to integrating the part with the whole, like the association of brown with wood beads by older children in Piaget’s study. In Halle’s case, the part was an interval of any duration; she had developed this interval as reversible with the whole stream of time carrying all intervals concurrently. That reversibility was not truly available to her earlier, although her description of time as a river during our first session might seem to suggest it.

Often I felt anxiety when Halle did not do, notice, or appreciate what I wanted, saw, or considered crucial. Somehow—without heeding many clues that I rely on—she built a resilient, reversible interpretation: one that we tested and extended in later sessions. Amid my questioning of what I was doing there, my failures to observe the relationships underlying Halle’s thinking and my doubts about ever responding with means by which she might take her own thinking further, she had moved on with her mind. Unlike my study of the algebra course, where Anita reported on reorganizing her thinking, here I was somehow participating with Halle in all the experiments, interactions, reversals and changes in path by which...
understanding grew—for both of us. That process was involuted in a way that instructional practices never prepared me to anticipate, yet a way that I was learning to value and encourage.

Experiments that evolved as Halle manipulated the free fall apparatus yielded discordant output. The persistence of those discordances perturbed thinking for both of us. Bit by bit, those perturbations exposed ways our thinking and work entrained incongruities with the phenomena of time and exploration that engaged us. In staying with these phenomena, as troubling as our unsteadiness was, there was always something more to wonder about, inviting our lingering with the apparatus and admitting the disequilibrium it sustained. We might have concluded that something constituted an answer or a lesson, and stopped investigating. Yet we went on throughout these free fall experiments, and many beyond. The “lessons” we uncovered through abiding the discomforts of disequilibrium were unsettling to our specific constraints and at the heart of understanding time and exploration.

One lesson for me lay in the emergence of Halle’s experimenting and thinking through her moment-to-moment effort to understand. I saw Halle’s agency in seeking understanding as harmonious with how foundational understandings in science came to be. I suspected that the actual processes of development in science might be as involuted as were our explorations. Although our story seemed “rare” to me, perhaps it was not. I felt that the inherently involuted processes of doing science were short-circuited out of science publications and science instruction (Cavicchi, Mar. 13). My wonderment about these interrelations of science, history, learning and development continues, keeling into further disequilibrium, having unknown puzzlements lodged at the core.

**Shadows in Reversible Dialogue with Galileo**

Long winter afternoons—in the lab and in hallways, under the dark sky and among early printed books—provide a “raft at sea” for university students and myself in observing, exploring and wondering. We are engaged together in the seminar I teach, which continues to evolve through my lifelong struggle with the problems of teaching. The result of my learning in the experiences related above with Halle, this seminar has us taking up the risks and challenges of exploratory teaching and learning without many constraints that typically impose upon education. With no prerequisite, open to undergraduate and graduate students alike, it is offered as a non-departmental elective through MIT’s Edgerton Center under the title “Recreate Historical Experiments: Inform the Future with the Past”.

With this seminar, I strive for the curriculum to emerge through and by means of students’ experiences and discussions. There is no pre-set syllabus; my syllabus advises incoming students to “welcome the unexpected” and “stay with experiences of uncertainty, doubt and wonder” (Cavicchi 2018). Winter sessions are 4 h in duration, allowing diverse activities to flow across each shared meeting. My knowledge as the teacher is not used to chart a route toward specified outcomes. What evolves as a result is unknown in advance. I research what happens by listening, recording, documenting and responding to what is ongoing among students during sessions and in their reflective writing. I seek to open and render
their work as investigation. My documentary and responsive activities provide
students with ways of seeing their experience as investigation and curriculum, and
at the same time provide me with grounds for imagining and supporting future
developments of their investigations. Art and science, learning and teaching,
thought and action, are in continuity, not opposition, in the experiences and
discussions that students and I create together in classroom labs, the outdoors,
museums and other spaces where our inquiries take us.

In striving to be receptive to spontaneity and disequilibrium, I stock our meeting-
spaces with materials having the potential to evoke immediate fascination and
sustained engagement: weights and string (Cavicchi 2008a, 2011, 2012); balances;
planks, lights and balls (Cavicchi 2013b); paper and drawing compasses
(Cavicchi 2014); canvas stretchers and tripods; lenses and mirrors (Cavic-
chi 2007, 2009, 2011, 2013b); telescopes; historical volumes, artifacts
(Cavicchi 2008b), architectural environment and photographic equipment (Cavic-
chi 2017) … The potential of these materials is so diverse and extensive that what
draws students’ fuller involvement is usually not something I previously envisioned.
Coming upon a generative artifact or observation, we may stay with it—or resume
later. Class sessions fluidly weave activities with discussions of observations and
readings by Galileo and others, affording many opportunities to meet Galileo.

Through his conjoined capacity to observe and to wonder, Galileo Galilei opened
possibilities never glimpsed before in matters as mundane as a falling weight and as
austere as the content and workings of the heavens. Irrepressible curiosity for those
observations and possibilities led him to venture so far outside the authoritative
explanations of his time as to see inadequacy in both the explanations and their
making. Welcoming the condition of disequilibrium as genuine, honest and
productive, Galileo developed uncertainty as a means of learning in the world.
Integrating observation, experiment, geometry and literary dialogue as equivalent
means of inferring and expressing relationships, he achieved profound reversibility
in understanding the world and our investigative experience.

Among many ways we meet Galileo, the dialogue becomes particularly fluent
when my students and I find ourselves ambushed by some everyday occurrence.
One frigid afternoon, having observed the sun with solar filter eyeshades, telescopes
fitted with solar filters, and other instruments from within a floor-to-ceiling window,
five of us sat beside that window. Taking a break from the solar observing, students
resumed their ongoing discussion of examples where something can be inverted into
its reciprocal. In a previous session, using a homemade ruler balance and penny
weights, they had come upon reciprocal behaviors while recreating the balance
activities of children interviewed by Piaget and Inhelder (1958), and in Galileo’s
works (Galilei 1978).

This afternoon’s discussion was opened by Yang Yan, then a doctoral student in
education who had taken my seminar in several previous terms following her
participation in Eleanor Duckworth’s course Teaching and Learning, the source for
her moon study (2018, this issue). Yang posed the question to her classmates “What
is a reciprocal?” Responding, Lydia, a college freshman, described the fraction 5/1
as the inverse of the fraction 1/5. Creating a new example with the gesture of
upending her hand, Lydia asked if a visual image inverts by a similar flip.
After a passage of intently pondering Lydia’s question, Yang’s face suddenly transformed to excitement. She exclaimed that right outside the window, she saw an analogy of image inversion! She pointed to a shadow on a fluted pillar, evidently cast by a horizontal bar in the window frame (Fig. 8). From Yang’s present position looking up at it, that shadow curved like a ‘U’. If she moved her perspective, would that shadow’s curve invert?

Yang Suppose I am looking [down] from very high up, what would that shadow look like?

Gathering around the window, everyone sighted through it, observing an upward arc in the bar’s shadow. Lydia asked if Yang could get “tall enough” to look down on it. After standing on a bench of insufficient height, Yang placed a large trash can on it. With classmates’ supporting her standing on the can, she gained the perspective of looking down (Fig. 9):

Yeah! It [shadow] is slightly curved [upside-down U] Come and see.  
(Cavicchi 2013a)

Oblivious to Yang’s invitation that her classmates look for themselves, I passed over my camera for her to photograph the effect from both high and low vantages. Her digital images absorbed the class in collective fascination, not only for the up and down arcing shadows, but also for fluctuating sunlight intensities and their own reflections (Fig. 10, left). Saying “so that’s an example” of inversion in an image, Yang brought that spontaneously initiated activity to a close.

I encouraged the class to take advantage of the fleeting sunlight by revisiting geometrical explorations conducted with their own bodies’ sun-cast shadows (Fig. 10, middle). Yet in doing so, I was also torn about turning the class aside from that window phenomenon. Did the students suppose they had settled it? I was aware that it involved something beyond what the class noticed, something that might bring a latent assumption into question. I need not have worried; the class roamed into its perplexity through walks of their curiosity.

Resuming their seats at the window, discussion ensued for another hour. Before returning to the classroom, I invited the class to try “one last thing with the sun”.

Fig. 8 Left: Classmates look out the window at the “shadow” of the window bar, as it strikes a fluted column. Middle: The dark line of the “shadow” appears as a U shaped curve in each fluting. Right: Diagram
Again using their bodies to cast shadows, Yang and freshman Sarah found their shadows stretched further down the hall than the hour before, even reaching onto its far wall (Fig. 10, right).

Back in our classroom, Madhu, our guest teaching-researcher (Duckworth 2006c, 185) during three sessions, shared prints by M.C. Escher (n.d.) from her laptop (Fig. 11, left). She described these prints as examples of inversion and compensation, themes the class had encountered in reading Inhelder and Piaget (1958). Viewing these prints was an activity that Madhu had prepared to contribute for that day’s session.

Yet those works of contemporary art brought about an unexpected realization about the science of what we had observed with the shadows out the window—a serendipitous experience. Madhu asked the class for other reciprocal relations they had encountered. Lydia and Sarah described inverse behaviors of instruments they used: a telescope’s far end shrinks the image; a slide rule divides and multiplies; a balance flips when a weight is moved across its pivot. Madhu reflected on the recurring theme of balance; Yang rejoined that the balance’s reciprocity provides an analogy for our process. In expressing this analogy, she recalled her method of viewing the shadow cast by the window bar across the pillar. She had moved her body so as to see it from below, and then climbed high to view it from above.
Yang [my bodily transposition]...is like balance: one point of view, another point of view... that is beyond any particular view of the shadow... (Cavicchi 2013a)

Perhaps the shared activity of making and watching shadows of her body moved Lydia toward a new disequilibrium about the window bar’s shadow that the class had investigated, yet not fully probed. It came to surprise us how the art of Escher reopened the shadow analogy, with questions that deepened into a new understanding. In initiating that process, pensively, tentatively, Lydia asked:

Lydia I was wondering: why you could see that shadow [of the window bar on the pillar]!?
Yang What a great question!

Yang erupted with delight as Lydia struggled to articulate what was so strange about that “shadow”.

Lydia Because — the sun wasn’t hitting from behind us!!
Yang You know, I didn’t think about it!…
Author You didn’t think about it then?
Yang Now I feel like: the window glass reflects the light but the frame stuff doesn’t … (Cavicchi 2013a)

As Yang now described it, sunlight reflected off the windowpanes to fall on the pillar, producing bright areas there, whereas it did not reflect from the window’s bar (Fig. 11, Middle). The “shadow” was not a shadow; it was a gap in the window’s reflected light.
Sarah recalled how sunlight glinting off a watch face makes a bright spot elsewhere. Captivated by Sarah’s story yet sustaining her own uncertainty, Lydia held out her hand so as to block the ceiling light:

**Lydia** If I put my hand here, there is a shadow. It [window bar effect] is different…

With escalating, breath-taking, excitement, Lydia gestured to MC Escher’s prints (n.d.) where figure and ground, dark and light, interchange reversibly.

**Lydia** Actually it[window bar effect] is the opposite [of a shadow], it is everything but !

Everyone exclaimed with the collectively created transformation in their understanding of that “shadow”:

**Madhu** That’s amazing! We should call it something else then.

**Yang** Anti shadow! It makes me think of Anti-Galileo! (Cavicchi 2013a)

Yang alluded to Lodovico delle Colombe’s (1612) retort to Galileo’s Copernicanism:

**Colombe** I should like to become an Anti-Galileo out of respect to [Aristotle]… (Galilei 1957, p. 149; Colombe 1612, p. 317)

In the historical case, Colombe’s appeal to authority roused Galileo and his “Galileist” colleagues into openly promulgating their observationally grounded arguments.

Lydia’s further pondering drew us to a core of that observational practice:

**Lydia** There must be so many things that you walk by and never notice. That blow your mind! … If I hadn’t actually sat down and thought about that, I would have never known. I would have thought: that’s a shadow …(Cavicchi 2013a)

To have the capacity for the everyday world to “blow your mind” involves the intellectual courage of being vulnerable, susceptible to undergoing disequilibrium over seemingly well-established matters. Through acting on the opportunity to reflect with her classmates, Lydia developed a community in which their own courage was discovered and expressed.

That community, with its reversibility in relationships, was deeply understood by others. Addressing Galileo directly as a companion explorer in her final reflective paper, Yang wrote:

The trusting relationships developed as our exploration evolved. It is hard to say which causes which; they are more intertwined and mutually influential. The journey of exploration has been filled with compassion and respect without which the soil for seeds of curiosity became dry and barren; without which the room and light that cultivated growth became squeezed and blocked. Through this collective journey, our intellectual curiosity and
autonomy as well as our capacity of following and serving others developed hand in hand. (Yang 2013, p. 3)

Through their community, the class rediscovered, for themselves, an effect and its underlying principle, that Galileo too had observed. With his 1610 telescopic observations, Galileo had discerned the moon’s rugged surface and estimated the heights of its mountain peaks when illuminated within its shadow (Galilei 1989, pp. 51–52). Galileo corroborated this finding in his famous 1632 Dialogue that challenged prevailing assumptions about earth’s immobility and heavens’ perfection. In explaining how a mirror-smooth moon would be invisible along most viewing directions, whereas light reflected from an unevenly-surfaced moon would be visible from anywhere, he appealed to the reflective effect encountered by my students. Salviati, Galileo’s spokesperson, contrasted reflection from a mirror (analogous to our window) with that from a rough wall (analogous to our window’s bar; Fig. 11 right):

Salviati …you see that the reflection of that little flat mirror, where it is thrown there under the balcony, shines strongly; and the rest of the [balcony] wall, which received a reflection from the wall to which the mirror is attached, is not lighted up… (Galilei 1967, p. 77)

Salviati discerned relationships of balance: sunlit mirror with its reflection; sunlit wall with opposing wall. Similarly, the class found their own ways to relationships of balance through noticing shadows, including their own; exploring from multiple viewpoints; admiring MC Escher’s prints; giving weight to their wonderment and uncertainty. Like Halle, they had integrated part and whole in yet another context, that of reversibility between figure and ground, reflection and shadow.

Unexpected and welcomed, the disequilibrium of Lydia’s question (“why you could see that shadow”) emerged from, and deepened, the whole range of observations and relationships that engaged these students—and Galileo. Any genuine question carries analogous potential for opening processes of disequilibrium and equilibrium. Interrelatedness of the underlying physical behaviors across everyday observations provided the integrity of our explorations, by way of material that only seems to be sporadic and arbitrarily convened. Resilience and balance in the world and in our investigative community provide coherence that accommodates us in rendering new understandings, even as our lookout vessel continually heaves and dips. Reversibility inhering in relational character of the natural phenomena sustains us in moving reversibly with our minds among its evidences and the relational understandings that we develop.

Closing

Being at sea with classmates and the yarns of past shipmates has us taking on the “sea legs” of discovering balance amid endemic disequilibrium that we come to acknowledge as intrinsic to teaching and learning. Walks made with these “sea legs”, as narrated across this paper, differ from the fixed routes that characterize education in many contexts. From a landed vantage, all that travel, bobbing,
displacement across immense seas, appears extraneous; a straight line more economically links the two points of a voyage’s start and end. Those same perspectives of land and sea counterpose in the example of an artist’s pen drawing throughout a voyage that Galileo developed in analogy to Earth’s motion. Understanding the reversibility in thought on which such concurrence of perspectives is contingent, Galileo astutely chose the guise of his beloved friend, the curious Sagredo, to voice this observation:

Sagredo …from the pen’s motion a whole narrative … sketched in thousands of directions …Yet the actual, real, essential movement marked by the pen point would have been only a line … (Galilei 1967, p. 172).

For the mates on board, every action is made as interdependent dance where, in the process of each move evolving the next, many possible moves are exercised and become tightly enfolded into whatever comes next. Those enfolded moves include:

- each pendulum swing watched by children in the elementary science classroom;
- two divisions that Annie made of three tiles;
- distinctions and concurrences that Anita perceived among x and y and seconds and centimeters;
- dots recorded on tape by sparks triggered at equal time intervals in the fall of a weight and their non-coincidence when compared across separate falls;
- the window bar’s “shadow” as viewed from below and above, and as delineated by light’s obstruction or reflection;
- my uncertainties in construing Halle’s accounting of the non-coincident dot patterns or in diverting my class among several sunshine observations.

Each of these enfoldings encompasses a multiplicity of possibility identified by those directly engaged in disequilibrating activity as learners and teachers. When these participants encounter surging seas, those compacted possibilities are on hand to unfurl, brace, rebalance, reverse or invert in provoking yet more flexible thinking. Anita later analyzed the software company’s marketing constraints; on observing a strobe light, Halle grasped its analogy with the spark mechanism and re-envisioned the free fall weight’s release as occurring “halfway through the darkness” (Cavicchi 1994e); through study of Galileo’s geometry using sunlight and diagrams, Lydia and Sarah realized its relation to a problem in their recent physics course, opening up something previously closed to them.

When these possibilities and uncertainties open out in thoughtful observation and exchange, as by participants in the narratives above, something is emerging through their minds, bodies, life, and community. Teaching and learning become reversible acts in continual interchange. Seeking to facilitate learners in constructing relationships that characterize the material they study, the teacher hears evidences of the relationships that are at play in learners’ thought. Learners’ own curiosity and sense of true balance urges their acknowledgement of inadequacy in some way their thinking held itself in precarious balance; in response, they improvise and test the robustness of alternative means of balancing. Uncertainties and inferences of teaching reverse into uncertainties and inventions of learning. All these reversible
processes are sustained by the relational nature of: teaching and learning, investigative thought, and the study material or phenomena.

Yet a relational perspective and reversible practices are too often peripheral to science education. Participants in my studies, including Anita, Halle and Lydia, had previously passed book-based math and physics courses at high school and university, but they had not learned to internalize the concepts relationally. In a turn-around of Melville’s phrase, their “Yale College and Harvard” had not provided a counterpart to the deeply affecting experience of a “whaling ship”. Momentum for everyday instruction is routinely imposed externally on classroom participants. For example, the organization of science curricula under cumulative, hierarchical structures of knowledge, where “core ideas” are predefined and mapped (NGSS 2013), leaves little room where learners might freely develop relationships of their own, encounter disequilibria in their understanding of those relationships, and negotiate more resilient equilibria. As Schneier (2018, this issue) expressed in her still-provocative essay on teaching and research, prescriptive, narrowing assumptions about the nature of learning are taken for granted so pervasively across education that there are seldom cracks by which learning and teaching might escape the confinement of those assumptions.

Equilibrating processes engender the reversible understandings that characterize historical and professional science, from the experimenting and theorizing of Galileo and Michael Faraday to such major conceptual and institutional shifts as those attending the acceptance of a heliocentric model for the universe amid a geocentric culture. In parallel, through welcoming and taking part in these equilibrating processes, teachers and learners conceive, test, refine and develop reversible understandings of math, time, shadow, poetry and other subject areas presented in this issue’s examples. Reversibility empowers further dynamic engagement with the world for learners and teachers, in contrast to the dead-end conferred by the irreversibility of many instructional steps. This study offers intertwined vision, theory and practice for teachers and learners anywhere to step into their own unsteady bark, and move relationally with it and each other into unknown and ever-fascinating seas.

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Conflict of interest There is no conflict of interest in this study.
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