Thermodynamics of string black hole with hyperscaling violation

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Abstract

In this paper, we start with black brane and construct specific space-time which violates hyperscaling. In order to obtain the string solution we apply Null-Melvin-Twist and \textit{KK}-reduction. By using the difference action method we study thermodynamics of system to obtain Hawking-Page phase transition. In order to have hyperscaling violation we need to consider $\theta = \frac{d}{2}$. In that case the free energy $F$ is always negative and our solution is thermal radiation without a black hole. Therefore we find that there is not any Hawking-Page transition. Also, we discuss the stability of system and all thermodynamical quantities.

\textbf{Keywords:} String Theory; Black Hole; Hyperscaling Violation; Null-Melvin-Twist; \textit{KK}-Reduction; Thermodynamics.

1 Introduction

As we know the AdS/CFT correspondence provides an analytic approach to study strongly coupled field theory \cite{1, 2, 3, 4}. Recently, we see several paper about development of AdS gravity theories and their conformal field theory dual, in that case the metric background generalized and result is dual to scale-invariant field theories instead of conformal invariant. The scale invariance provided by dynamical critical exponent $z \neq 1$ (the $z = 1$ corresponds to case of the AdS metric) on the following metric, \cite{12},

$$ds^2 = -\frac{1}{r^{2z}} dt^2 + \frac{1}{r^2} \left( dr^2 + dx_i^2 \right),$$

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The corresponding metric will be invariant under following scale transformation,

$$ t \rightarrow \lambda^2 t, \quad x_i \rightarrow \lambda x_i, \quad r \rightarrow \lambda r. $$  

The resulting metric may be a solution of field equations with coupled theories to matter with negative cosmological constant also include an abelian field in the bulk. Space-time metrics that transform covariantly under dilatation have recently been reinterpreted as holography dual to stress tensor of quantum field theories which violates hyperscaling \[10]. Recently, the large class of scaling metrics containing an abelian gauge field and scalar dilaton considered \[7-19], which is presented by the following equation \[11],

$$ ds^2 = r^{-2(d-\theta)/d} \left( r^{-2(z-1)} dt^2 + dr^2 + dx_i^2 \right), $$

where $\theta$ is hyperscaling violation exponent. Note that, this metric is not invariant under scale transformation \[2], but transforms covariantly as,

$$ ds = \lambda^{\theta/d} ds, $$

which defines property of hyperscaling violation in holography language. The corrections of conformal hyperscaling relation in the conformal point of view in large $N_f$ QCD as a concrete dynamical model is given by the Ref. \[20]. Such examples show that QCD can be a candidate for usage hyperscaling. On the other hand we have strong motivation to study a metric with hyperscaling violation. As we know, scale invariance broken under quantum effects in some theories such as QCD. Specially in large scale or low energy, scale invariance broken for massive theories. In that cases, using AdS metric which has scale invariance is not appropriate. For example, we have some problems to calculate form factor or quantum mass spectrum in QCD. Therefore, it is necessary to modify original metric \[21]. Instead modification of AdS metric, it is appropriate to choose suitable metric such as hyperscale metric which has scale violation. In large scale (or $r \rightarrow \infty$) there are good applications of a metric with hyperscaling violation in QCD or string theory. So, in this paper we use metric of the Ref. \[7] to obtain string solution. Then, we discuss physical properties (specially thermodynamics) of mentioned metric to verify our motivation. If our solution will be coincide with known physical rules, then one can use hyperscale metric instead of AdS metric for future works.

An important concept in our study is the Galilean holography which is developed in Refs. \[22, 23], where the non-relativistic generalizations of the AdS/CFT correspondence extracted. An expansion of Galilean algebra can be obtained by adding dilation operator and a special conformal transformation to the time and space scale identically. A discussion of non-relativistic conformal symmetry generalization which is known as Schrödinger has been explained in Ref. \[3]. In this discussion, time and space geometry of $d$ dimensions isometry group has Schrödinger symmetry and established over AdS/CFT correspondence. They suggested that the gravity is the holographic dual of the non-relativistic conformal field theories at strong couplings. The next development of Galilean holographic is finite temperature generalization \[24, 25, 26]. In the AdS/CFT correspondence of finite temperature a planar black brane solutions suggested in the Schrödinger space as the holographic dual of
the non-relativistic conformal field theory at finite temperature. The investigation of \( AdS_5 \) geometry near horizon of D3-brane in flat space is investigated [24, 25]. Then, the known Null-Melvin-Twist (NMT) [27, 28, 29] applied to this system. Ref. [26] started with solution of asymptotical black hole metrics which leads to the string solution and characterize the specific non-relativistic conformal field theories to which they are dual. An analysis of these black hole space-time thermodynamics shows that they describe the dual conformal field theory at finite temperature and finite density. It has been shown that, after doing NMT by applying \( KK \)-reduction over \( S^5 \) geometry, the result is extremal black brane also the asymptotic limits is reduced to Schrödinger geometry. The thermodynamic solutions of such black hole discussed by Refs. [24, 25].

The new regularization method has been suggested by the Ref. [30] which is the oldest regularization method [31, 32] with some modification which is subtraction method with an unusual boundary matching.

In the recent work [33], thermodynamics of Schrödinger black holes with hyperscaling violation has been studied. It may be found some overlaps between our work and mentioned paper, however we should note that our system is completely different and application of hyperscaling violation in both systems yields to independent results which are interesting itself. While the primary metrics of both papers are similar but we use NMT method to obtain string solution, also discuss about phase transition and thermodynamics stability.

This paper is organized as the following. In next section we begin with black brane metric and make the corresponding metric which violates hyperscaling. In that case, we apply NMT and \( KK \)-reduction to obtain the string solution of this geometry. In section 3 we use the difference action method and extract the thermodynamics of system in section 4 and discuss Hawking-Page phase transition and thermodynamics stability. In section 5 we summarized our results.

## 2 String Black Brane

Now, we consider the non-extremal D3-brane geometry [30] near horizon, which is obtained by the following action [22, 25],

\[
\begin{align*}
ds^2 &= \left( \frac{r}{R} \right)^2 (-f dt^2 + dy^2 + dx_i^2) + \left( \frac{R}{r} \right)^2 f^{-1} dr^2 + R^2 d\Omega_5^2, \\
\phi &= 0, \quad B = 0, \quad f(r) = 1 - \left( \frac{r_H}{r} \right)^4,
\end{align*}
\]

where \( R \) is the AdS scale, \( x_i = (x_1, x_2) \) and \( r = r_H \) is the location of the horizon, so the metric at \( r_H = 0 \) reduces to the extremal case. \( \phi \) is dilaton and \( B \) is NS-NS two-form. A particularly convenient choice for \( d\phi \) is given by Hopf fibration \( s^1 \to s^5 \to p^2 \) with the following metric,

\[
\begin{align*}
d\Omega_5^2 &= ds_{p^2}^2 + (d\chi + A),
\end{align*}
\]

where \( \chi \) is the local coordinate on Hopf fibre and \( A \) is the one-form on \( P^2 \), and \( ds_{p^2}^2 \) is metric on \( P^2 \) [26]. We need to consider two isometry directions as \( dy \) and \( d\phi \) for Melvinization.
process, where \(dy\) is along the world-volume, \(d\phi\) is along the \(S^5\) and \(y\) is one of three spatial coordinates. Now, we begin with the metric (5) includes hyperscaling violation in the black hole solution according to the Ref. [7],
\[
ds^2_{d+2} = \left( \frac{r}{R} \right)^2 \left( \frac{d}{r} \right)^{2\theta/d} \left( -\left( \frac{R^2}{r} \right)^{-2(z-1)} f \, dt^2 + dy^2 + dx_i^2 \right) + \left( \frac{R}{r} \right)^2 \left( \frac{d}{r} \right)^{2\theta/d} \left( \frac{dr^2}{f} + R^2 d\Omega_5^2 \right), \tag{7}
\]
where \(d = 3\), and \(r_F\) is scale which is obtained from dimensional analysis [7]. Finite temperature effects in theories with hyperscaling violation studied, in that case, in the gravity side, we have \(r_F < r_h\). From null energy condition (NEC) as \(T_{\mu\nu} n^\mu n^\nu \geq 0\) [7, 34] and null vectors satisfy the \(n^\mu n^\nu = 0\) condition. The above conditions lead us to obtain the following relations,
\[
(d - \theta)(d(z - 1) - \theta) \geq 0,
\]
\[
(z - 1)(d + z - \theta) \geq 0. \tag{8}
\]
In order to satisfy our following results with equation (11) we need to consider \(z = 1\) (because \(z = 1\) in \(\theta \to 0\) limit gives the AdS metric). From the first relation of (8), one can obtain,
\[
(\theta \leq 0, \quad d \geq \theta), \quad \text{or} \quad (\theta \geq 0, \quad d \leq \theta). \tag{9}
\]
Now, we apply NMT to the metric (7) with \(z = 1\), and obtain,
\[
ds^2_{d+2} = K^{-1} \left( \frac{r}{R} \right)^2 M \left[ -(1 + b^2 r^2 M^2) f dt^2 - 2b^2 r^2 f M^2 dt dy + (1 - b^2 r^2 f M^2) dy^2 + K dx_i^2 \right],
\]
\[
+ M \left( \frac{R}{r} \right)^2 f^{-1} dr^2 + M K^{-1} R^2 \eta^2 + M R^2 ds_{y^2}^2, \tag{10}
\]
and,
\[
\phi = -\frac{1}{2} \ln K,
\]
\[
B = \frac{M^2}{K} \left( \frac{r}{R} \right)^2 b(f dt + dy) \wedge \eta,
\]
\[
K = 1 - (f - 1)b r^2 M^2, \tag{11}
\]
where \(\eta = (d\chi + \mathcal{A})\), \(M = \left( \frac{r}{R} \right)^{2\theta/d}\), and also \(b\) has \([L^{-1}]\) dimension. If we perform the KK-reduction on \(S^5\) for the non-extremal solution (10), we obtain,
\[
ds^2_{d+2} = K^{-2/3} \left( \frac{r}{R} \right)^2 M \left[ -(1 + b^2 r^2 M^2) f dt^2 - 2b^2 r^2 f M^2 dt dy + (1 - b^2 r^2 f M^2) dy^2 + K dx_i^2 \right]
\]
\[
+ K^{1/3} M \left( \frac{R}{r} \right)^2 f^{-1} dr^2, \tag{12}
\]
and,
\[ \phi = -\frac{1}{2} \ln K, \]
\[ A = \frac{M^2}{K} \left( \frac{r}{R} \right)^2 b(\text{d}t + \text{d}y), \]  
(13)
where \( A \) is one-form field in Einstein frame. It is useful to work in the following light-cone coordinates,
\[ x^+ = bR(t + y), \quad \text{and}, \quad x^- = \frac{1}{2bR}(t - y). \]  
(14)
So, the solution is,
\[ ds^2_{d+2} = K^{-2/3} \left( \frac{r}{R} \right)^2 M \left[ -\left( \frac{f - 1}{(2bR)^2} \right) fM^2 \right] \text{d}x^+ \text{d}x^- - (1 + f) \text{d}x^+ \text{d}x^- \]
\[ + \ (bR)^2 (1 - f) \text{d}x^- + K \text{d}x_i^2 \]
\[ + K^{1/3} M \left( \frac{R}{r} \right)^2 f^{-1} \text{d}r^2, \]  
(15)
and,
\[ \phi = -\frac{1}{2} \ln K, \]
\[ A = \frac{M^2}{K} \left( \frac{r}{R} \right)^2 b \left[ \frac{f + 1}{2bR} \text{d}x^+ + bR(1 - f) \text{d}x^- \right]. \]  
(16)
The equation (15) is the same as the equation (5) in Ref. [30] with additional \( M = \left( \frac{r}{R} \right)^{2\theta/d} \).
By consideration \( x^+ \) coordinate as the time, the recent metric under scale transformation \( x^+ \to \lambda x^+, \ x_i \to \lambda x_i, \ r \to \lambda^{-1} r, \ x_- \to \lambda^2 x_- \) and \( d = 2\theta \) transforms covariantly as the equation (11), and it is violates hyperscaling.
The extremal case coming from \( f = 1 \), and non-extremal case approaches this at asymptotically large \( r \). The last metric on the light-cone coordinates in the equation (14) gives extremal case which is independent of the parameter \( b \). So, \( b \) is unphysical and thus cannot give any physical quantity. One can interpret this result in the zero-temperature limit [26].
The metric background (15) is a solution of the effective action. In non-extremal case, for the \( \theta = 0 \) we have the following action [30],
\[ S_5 = \frac{1}{16\pi G_5} \int dx^5 \sqrt{-g} \left[ R - \frac{4}{3} (\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{4} R^2 e^{-2\phi/3} F_{\mu \nu} F^{\mu \nu} - 4A_\mu A^\mu - \frac{V}{R^2} \right], \]  
(17)
where \( G_5, g \) and \( R \) are the 5 dimensional Newton constant, the determinant of 5 dimensional metric and the scalar curvature respectively. \( F = dA \) is two-form field and the potential \( V \) is defined by the following expression,
\[ V = 4e^{2\phi/3}(e^{2\phi} - 4). \]  
(18)
By setting $\phi = 0$, the above action reduces to the extremal action \[22\]. As we know, in case of $\theta \neq 0$, the shape of action \[17\] will conserve, but in this process the potentials $V$ and corresponding field $\phi$ will be changed. Because the $K$ will be changed by parameter $\theta$.

The ADM form of metric is,

$$ds^2_{d+2} = K^{1/3} \left( \frac{r_F}{r} \right)^{2(\theta/d)} \left( \frac{R}{r} \right)^2 f^{-1} dr^2$$

$$+ K^{-2/3} \left( \frac{r}{R} \right)^2 \left( \frac{r_F}{r} \right)^{2(\theta/d)} \left[ K dx_i - \frac{1}{(bR)^2(1-f)} \left( \frac{r}{R} \right)^2 \left( \frac{r_F}{r} \right)^{4(\theta/d)} \right] f dx^+ dx^-$$

$$+ K^{-2/3} \left( \frac{r}{R} \right)^2 \left( \frac{r_F}{r} \right)^{2(\theta/d)} (bR)^2 (1-f) \left( dx^+ - \frac{1+f}{2(bR)^2(1-f)} dx^- \right)^2.$$  \[19\]

By using the corresponding metric, we obtain the angular velocity of the horizon $\Omega_H$, which interpreted as chemical potential associated with the conserved quantities along the $x^-$ direction,

$$\Omega_H = \frac{1}{2(bR)^2}. \quad \[20\]$$

Note that we have mentioned two kinds of hypersurfaces; the time-like boundary at a large fixed $r$ and the space-like surface at a fixed time $x^+$ whose time is described by the ADM form. In the extremal case there is a problem with $g_{--}$ component in calculation of difference action ($g_{--} = 0$).

### 3 The Difference Action

The metric \[15\] gives the extremal solution near the boundary (the large $r$) and interpreted as the finite temperature generalization of the Galilean holography \[24, 25, 26\]. We want to consider the thermodynamics of this system in the finite temperature. In order to calculate the thermodynamics, we use difference action method \[30, 31, 32\].

According to the Ref. \[30\], first we continue analytically $x^+$ to $ix^+$ and put the system into a box by cutoff $r = r_B$. The cutoff $r_B$ is larger than the scale $R$ but it is finite. We subtract the action of the extremal solution from the non-extremal one. We note here each action include two terms such as bulk and Gibbons-Hawking surface term. To do such process, we have to match the geometries of metrics in $r = r_B$ wall. As mentioned earlier, the $g_{--}$ component of the extremal case has been degenerated in the metric \[15\], so we cannot match metrics in the wall. In order to remove this problem, we match the boundary metric of the extremal geometry to the non-extremal one only for the $x^-$ constant. So, we rescale appropriately
three dimensional slices \((x^+, x^i)\). We obtain scaled extremal metric as a following,

\[
\begin{align*}
 ds_{d+2}^2 &= \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{\frac{2(\theta/d)}{2}} \left[ \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{\frac{4(\theta/d)}{4}} H_B^2 dx^+ dx^- - 2i H_B dx^+ dx^i + G_B^2 dx_i^2 \right] \\
 &\quad + \left(\frac{R}{r}\right)^2 \left(\frac{r_F}{r}\right)^{\frac{2(\theta/d)}{2}} dr^2, \\
 \phi &= 0, \\
 A &= i \left(\frac{r}{R}\right)^2 \left(\frac{r_F}{r}\right)^{\frac{\theta}{d}} \frac{H_B}{R} dx^+, \\
 (21)
\end{align*}
\]

where,

\[
\begin{align*}
 H_B &= \left[ K(r_B)^{-2/3} \left( f(r_B) - \frac{1}{(2bR)^2} \right) + \left(\frac{r_B}{R}\right)^2 \left(\frac{r_F}{r_B}\right)^{\frac{4(\theta/d)}{4}} f(r_B) \right]^{1/2} \left(\frac{r_B}{R}\right)^{-1} \left(\frac{r_F}{r_B}\right)^{\frac{(-2\theta/d)}{-2}} , \\
 G_B &= K(r_B)^{1/6} . \quad (22)
\end{align*}
\]

The difference action \((S - S_0)\) will be as,

\[
S_0 = S_{0bulk} + S_{0GH}, \quad \text{and}, \quad S = S_{bulk} + S_{GH}, \quad (23)
\]

where both \(S_{0bulk}\) and \(S_{bulk}\) are action \((17)\), but the \(S_{0bulk}\) evaluate on the extremal solution \((21)\) and the \(S_{bulk}\) calculate on the non-extremal solution \((15)\). Also \(S_{0GH}\) and \(S_{GH}\) are the Gibbons-Hawking surface term,

\[
S_{0GH} = -\frac{1}{8\pi G_5} \int dx^4 \sqrt{g_B} (Tr K_0), \quad (24)
\]

where \(g_B\) is the determinant of the boundary first fundamental form, and \((Tr K_0)\) is the trace of the boundary second fundamental form. We calculate the difference action in the limit of \(r_B \to \infty\), which is not divergent,

\[
\lim_{r_B \to \infty} (S - S_0) = \frac{V_4}{16\pi G_5} \frac{r_H^4}{R^5} \left( 1 - \frac{\theta}{d} \right) \left(\frac{r_F}{r_H}\right)^{3\theta/d}, \quad (25)
\]

where \(V_4\) is volume of four dimensions space-time. It shown that this result agree with Ref. [30] without hyperscaling violation.

### 4 Thermodynamics

Now, we use results of the previous section to study the thermodynamics of system. In that case the Hawking temperature can be obtained from surface gravity as \(\beta = \frac{2\pi}{\kappa}\) where \(\kappa\) is surface gravity,

\[
\kappa^2 = -\frac{1}{2} \left(\nabla^a \xi^b\right) \left(\nabla_a \xi_b\right), \quad (26)
\]
where $\xi$ is the killing vector field which is obtained by following expression,

$$\xi = \frac{1}{bR} \frac{\partial}{\partial t} = \partial_+ + \Omega_H \partial_-,$$

and corresponding $\beta$ is obtained by,

$$\beta = \frac{4}{d + 1 - \theta} \frac{\pi b R^3}{r_H}.$$  

(28)

The killing generator of the event horizon not only has components along the boundary time translation direction $x^+$, but also along light-like direction $x^-$. From the gravitational point of view it is therefore a system with chemical potential for $x^-$ directions,

$$\mu = \frac{1}{2(bR)^2}. $$  

(29)

In order to study the thermodynamics of system, we use the following free energy $^{30,35,36}$,

$$F = -(16\pi G_5) V_3^{-1} \lim_{r_H \rightarrow \infty} (S - S_0)$$

$$= -\beta \left( \frac{r_H^4}{R^5} \right) \left( 1 - \frac{\theta}{d} \right) \left( \frac{r_F}{r_H} \right)^{\frac{3\theta}{d}}$$

$$= -\frac{\pi^4 R^3_t}{4\mu^2 \beta^3} \left( 1 - \frac{\theta}{d} \right) \left( \frac{\beta r_F}{\pi R^2} \right)^{\frac{3\theta}{d}} \left( \frac{4}{d + 1 - \theta} \right)^{4 - \frac{3\theta}{d}} (2\mu)^{\frac{3\theta}{2d}},$$

(30)

where $V_3$ is the integration over $x^{-i}$, and equal to $V_4 \beta^{-1}$. So we obtain entropy as,

$$S = \beta \left( \frac{\partial F}{\partial \beta} \right)_\mu - F$$

$$= \frac{4\pi b r_H^3}{R^2} \left( 1 - \frac{\theta}{d} \right) \left( 4 - \frac{3\theta}{d} \right) \left( \frac{r_F}{r_H} \right)^{\frac{3\theta}{d}}.$$

(31)

These equations, in the case of $\theta = 0$, agree with the Ref. $^{30}$. Also we can obtain,

$$E = \left( \frac{\partial F}{\partial \beta} \right)_\mu - \mu \beta^{-1} \left( \frac{\partial F}{\partial \mu} \right)_\beta$$

$$= \frac{r_H^4}{R^5} \left( 1 - \frac{\theta}{d} \right) \left( 1 - \frac{3\theta}{2d} \right) \left( \frac{r_F}{r_H} \right)^{\frac{3\theta}{d}},$$

$$Q = -\beta^{-1} \left( \frac{\partial F}{\partial \mu} \right)_\beta$$

$$= -\frac{4b^2 r_H^4}{R^3} \left( 1 - \frac{\theta}{d} \right) \left( 1 - \frac{3\theta}{4d} \right) \left( \frac{r_F}{r_H} \right)^{\frac{3\theta}{d}}.$$  

(32)

In equation (30) we have two conditions for $F$ such $F > 0$ and $F < 0$. In the case of $F < 0$ we have two conditions as $\theta < d$ or $\theta > d + 1$, and in the case of $F > 0$ we have
\(d < \theta < d+1\). So, in the case of \(\theta = d\) and \(\theta = d+1\) we have Hawking-Page phase transition. As mentioned before we take \(\theta = d/2\), so we have always negative \(F\). So, our solution is thermal radiation without a black hole and we have not any Hawking-Page phase transition. As we know, in order to calculate the stability of system we need to obtain the Hessian of \(\beta(E - \mu Q) - S\) with respect to the thermodynamic variables \((r_H, b)\) and evaluate it at the on-shell values of \((\beta, \mu)\). In the case of \(\theta = 0\) it recovers the results of the Ref. \([30]\). In the case of hyperscaling violation with condition of \(\theta > \frac{1}{2}\), the results will be positive and the system is thermodynamically stable. Here, also we check the first law as \(dE = TdS + \Omega_H dQ\) and satisfy by the above quantities.

## Conclusion

In this paper, we considered the black brane metric and made corresponding metric which is violate hyperscaling. By using the difference action method we obtained the thermodynamical quantities such as \(\beta, Q, S, E,\) and \(F\). In the case of \(F > 0\) we achieved two conditions as \(\theta < d\) or \(\theta > d+1\). And also for \(F > 0\) we arrived at \(d < \theta < d + 1\). Two above conditions lead to Hawking-Page phase transition \((\theta = d, \theta = d + 1)\). But in this paper we have always negative \(F\) because our condition was \(\theta = \frac{d}{2}\) and we have not such phase transition. Also we discussed the stability of system which agree with the Ref. \([30]\) in \(\theta = 0\). We have shown that in the case of hyperscaling violation the \(\theta\) must be \(\theta > \frac{1}{2}\) which is covered by our condition. In general we can say that the system has thermodynamical stability. Therefore one can use hyperscale metric instead of AdS metric to avoid technical problems in boundary because of scale symmetry breaking. For future work we focus on this subject and use a hyperscale metric instead of AdS metric to calculate form factor in QCD. Finally we verified that the first law of thermodynamics is valid.

## References

[1] J. M. Maldacena, ”The large \(N\) limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 1113 (1999)] [arxiv:hep-th/9711200].

[2] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, ”Gauge theory correlators from non-critical string theory,” Phys. Lett. B 428, 105 (1998) [arxiv:hep-th/9802109].

[3] E. Witten, ”Anti-de Sitter space and holography,” Adv. Theor. Math. Phys. 2, 253 (1998) [arxiv:hep-th/9802150].

[4] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, ”large \(N\) field theories, string theory and gravity,” Phys. Rept. 323, 183 (2000) [arxiv:hep-th/9905111].

[5] S. Kachru, X. Liu and M. Mulligan, ”Gravity dual of Lifshitz-like Fixed Points,” Phys. Rev. D78 (2008) 106005, [arXiv:0808.1725].
[6] M. Tylor, ”Non-relativistic holography,” [arXiv:0812.0530]. M. Cadoni, S. Mignemi”, Phase transition and hyperscaling violation for scalar black branes”, JHEP 1206 (2012) 056. [arXiv:1205.0412 [hep-th]]

[7] X. Dong, S. Harrison, S. kachru, G. Torroba, and H. Wang, ”Aspects of holography for theories with hyperscaling violation,” JHEP 1206, 041 (2012) [arXiv:1201.1905v4 [hep-th]].

[8] S. S. gubser and F. D. Rocho, ”Peculiar properties of a charged dilatonic black hole in Ads5,” Phys. Rev. D 81, 046001 (2010) [arXiv:0911.2898 [hep-th]].

[9] K. Goldstein, S. Kachru, S. Prakash and S. P. Trivedi, ”Holography of Charged Dilaton Black Hole,” JHEP 1008, 078 (2010) [arXiv:0911.3586 [hep-th]].

[10] M. Cadoni, G. D’Appollonio and P. Pani, ”Phase transitions between Ressner-Nordstrom and dilatonic black hole in 4D Ads spacetime,” JHEP 1003, 100 (2010) [arXiv:0912.3520 [hep-th]]; M. Cadoni, P. Pani, ”Holography of charged dilatonic black branes at finite temperature,” JHEP 1104, 049 (2011) [arXiv:1102.3820 [hep-th]].

[11] C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, ”Effective Holographic Theories for low-temperature condensed matter system,” JHEP 1011, 151 (2010) [arXiv:1005.4690 [hep-th]].

[12] E. Perlmutter, ”Domain Wall Holography for Finite Temperature Scaling Solutions,” JHEP 1102, 013 (2011) [arXiv:1006.2124 [hep-th]].

[13] G. Bertoldi, B. A. Burrington and A. W. Peet, ”Thermal behavior of charged dilatonic black branes in Ads and UV completions of Lifshitz-like geometries,” Phys. Rev. D 82, 106013 (2010) [arXiv:1007.1464 [hep-th]]; G. Bertoldi, B. A. Burrington, A. W. Peet and I. G. Zadeh, ”Lifshitz-like black brane thermodynamics in higher dimensions,” Phys. Rev. D 83, 126006 (2011) [arXiv:1101.1980 [hep-th]].

[14] K. Goldstein, N. Iizuka, S. Kachru, S. Prakash, S. P. Trivedi and A. Westphal, ”Holography of Dyonic Dilaton Black Branes,” JHEP 1010, 027 (2010) [arXiv:1007.2490 [hep-th]].

[15] N. Iizuka, N. Kundu, P. Narayan and S. P. Trivedi, ”Holographic Fermi and Non-Fermi Liquids with Transitions in Dilaton Gravity,” [arXiv:1105.1162 [hep-th]].

[16] P. Berglund, J. Bhattacharyya and D. Mattingly, ”Charged Dilatonic Ads Black Brane in Arbitrary Dimensions,” [arXiv:1107.3090 [hep-th]].

[17] N. Ogawa, T. Takayanagi and T. Ugajin, ”Holographic Fermi Surface and Entanglement Entropy,” [arXiv:1111.1023 [hep-th]].

[18] L. Huijse, S. Sachdev and B. Swingle, ”Hidden Fermi surface in compressible state of gauge-gravity duality,” Phys. Rev B 85, 035121 (2012) [arXiv:1112.0573 [cond-mat.str-el]].
[19] E. Shaghoulian, ”Holographic Entanglement Entropy and Fermi Surfaces,” [arXiv:1112.2702 [hep-th]].

[20] Y. Aoki, T. Aoyama, M. Kurachi, T. Mashawa, K. Nagai, H. Ohki, A. Shibata, K. Yamawaki, and T. Yamazaki, ”Study of the conformal hyperscaling relation through the Schwinger-Dyson equation,” [arXiv:1201.4157 [hep-lat]].

[21] S. J. Brodsky, G. F. de Teramond ”AdS/CFT and Light-Front QCD,” [arXiv:0802.0514 [hep-ph]].

[22] D. T. Son, ”Toward an AdS/cold atoms correspondence: a geometric realization of the Schrodinger symmetry,” Phys. Rev. D 78, 046003 (2003) [arXiv:0804.3972 [hep-lat]].

[23] K. Balasubramanian and J. McGreevy, ”Gravity duals for non-relativistic CFTs,” Phys. Rev. Lett. 101, 061601 (2008) [arXiv:0804.4053 [hep-lat]].

[24] J. Maldacena, D. Martelli and Y. Tachikawa, ”Comments on String theory backgrounds with non-relativistic conformal symmetry,” JHEP 0810, 072 (2008) [arXiv:0807.1100 [hep-th]].

[25] C. P. Herzog, M. Rangamani and S. F. Ross, ”Heating up Galilean holography,” JHEP 0811, 080 (2008) [arXiv:0807.1099 [hep-th]].

[26] A. Adams, K. Balasubramanian and J. McGreevy, ”Hot Spacetime for Cold Atoms,” JHEP 0811, 059 (2008) [arXiv:0807.1111 [hep-th]].

[27] M. Alishahiha and O. J. Ganor, ”Twisted backgrounds, pp-waves and nonlocal field theories,” JHEP 0303, 006 (2003) [arXiv:hep-th/0301080].

[28] E. G. Gimon, A. Hashimoto, V. E. Hubeny, O. Lunin and M. Rangamani, ”Black string in asymptotically plane wave geometries,” JHEP 0308, 035 (2003) [arXiv:hep-th/0306131].

[29] J. Sadeghi, B. Pourhassan ”Energy loss and jet quenching parameter in a thermal non-relativistic, non-commutative Yang-Mills plasma,” Acta Physica Polonica B, 43 (2012) 1825 [arXiv:1002.1596 [hep-th]].

[30] D. Yamada, ”Thermodynamics of Black Holes in Schrödinger Spaces,” Class. Quant. Grav. 26, 075006 (2009) [arXiv:0809.4928 [hep-th]].

[31] S. W. Hawking and D. N. Page, ”Thermodynamics of Black Hole in Anti-De Sitter Space,” Commun. Math. Phys. 87, 577 (1983).

[32] G. W. Gibbons and S. W. Hawking, ”Action Integrals And Partition Function In Quantum Gravity,” Phys. Rev. D 15, 2752 (1977).
[33] J. Sadeghi, B. Pourhassan, F. Pourasadollah, "Thermodynamics of Schrödinger black holes with hyperscaling violation," Physics Letters B 720 (2013) 244249 [arXiv:1209.1874[hep-th]]

[34] K. Narayan, "On Lifshitz scaling and hyperscaling violation in string theory," [arXiv:1202.5935[hep-th]]

[35] J. Sadeghi, B. Pourhassan, M. Rostami, Z. Sadeghi, "Thermodynamics of Near-Extremal Solutions of Einstein-Maxwell-Scalar Theory," Int. J. Theor. Phys. 52 (2013) 2564

[36] A. Pourdarvish, J. Sadeghi, H. Farahani, B. Pourhassan, "Thermodynamics and Statistics of Gdel Black Hole with Logarithmic Correction," Int. J. Theor. Phys. 52 (2013) 3560