Quantum Interference Effects in Hořava-Lifshitz Gravity

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The relativistic quantum interference effects in the spacetime of slowly rotating object in the Hořava-Lifshitz gravity as the Sagnac effect and phase shift of interfering particle in neutron interferometer are derived. We consider the extension of Kehagias-Sfetsos (KS) solution in the Hořava-Lifshitz gravity for the slowly rotating gravitating object. Using the covariant Klein-Gordon equation in the nonrelativistic approximation, it is shown that the phase shift in the interference of particles includes the gravitational potential term with the KS parameter $\omega$. It is found that in the case of the Sagnac effect, the influence of the KS parameter $\omega$ is becoming important due to the fact that the angular velocity of the locally non rotating observer is increased in Hořava gravity. From the results of the recent experiments we have obtained lower limit for the coupling KS constant as $\omega \simeq 1.25 \cdot 10^{-25} \text{cm}^2$. Finally, as an example, we apply the obtained results to the calculation of the UCN (ultra-cold neutrons) energy level modification in the gravitational field of slowly rotating gravitating object in the Hořava-Lifshitz gravity.

Keywords: Hořava gravity; Neutron interferometer; Sagnac effect.

PACS Nos.: 04.50.-h, 04.40.Dg, 97.60.Gb.
1. Introduction

One of the biggest difficulties in attempts toward the theory of quantum gravity is the fact that general relativity is non-renormalizable. This would imply loss of theoretical control and predictability at high energies. In January 2009, Petr Hořava proposed a new theory of quantum gravity with dynamical critical exponent equal to $z = 3$ in the UV (Ultra-Violet) in order to evade this difficulty by invoking a Lifshitz-type anisotropic scaling at high energy. This theory, often called Hořava-Lifshitz gravity, is power counting renormalizable and is expected to be renormalizable and unitary.\(^\text{11}\)

Having a new candidate theory for quantum gravity, it is important to investigate its astrophysical and cosmological implications. Thus the Hořava theory has received a great deal of attention and since its formulation various properties and characteristics have been extensively analyzed, ranging from formal developments, cosmology, dark energy, dark matter, and spherically symmetric or axial symmetric solutions.

In the paper Ref. 27 the possibility of observationally testing Hořava gravity at the scale of the Solar System, by considering the classical tests of general relativity (perihelion precession of the planet Mercury, deflection of light by the Sun and the radar echo delay) for the Kehagias-Sfetsos asymptotically flat black hole solution of Horava-Lifshitz gravity has been considered. The stability of the Einstein static universe by considering linear homogeneous perturbations in the context of an Infra-Red (IR) modification of Hořava gravity has been studied in the paper 28. In the paper Ref. 29 author considered potentially observable properties of black holes in the deformed Hořava-Lifshitz gravity with Minkowski vacuum: the gravitational lensing and quasinormal modes.

The role of the tidal charge in the orbital resonance model of quasiperiodic oscillations in black hole systems and in neutron star binary systems have been studied intensively. The motion of test particles around black hole immersed in uniform magnetic field in Hořava gravity and influence of $\omega$ parameter on radii of innermost stable circular orbit have been studied in papers Ref. 32, 33.

The experiment to test the effect of the gravitational field of the Earth on the phase shift in a neutron interferometer were first proposed by Overhauser and Colella. Then this experiment was successfully performed by Collela, Overhauser and Werner. After that, there were found other effects, related with the phase shift of interfering particles. Among them the effect due to the rotation of the Earth, which is the quantum mechanical analog of the Sagnac effect, and the Lense-Thirring effect which is a general relativistic effect due to the dragging of the reference frames. So we do not consider the neutron spin in this paper.

In the paper Ref. 39 a unified way of study of the effects of phase shift in neutron interferometer was proposed. Here we extend this formalism to the case of slowly rotating stationary gravitational fields in the framework of Hořava-Lifshitz gravity in order to derive such phase shift due to either existence or nonexistence of the KS
The Sagnac effect is well known and thoroughly studied in the literature, see e.g. paper Ref. 40. It presents the fact that between light or matter beam counter-propagating along a closed path in a rotating interferometer a fringe shift $\Delta \phi$ arises. This phase shift can be interpreted as a time delay $\Delta T$ between two beams, as it can be seen below, does not include the mass or energy of particles. That is why we may consider the Sagnac effect as the "universal" effect of the geometry of space-time, independent of the physical nature of the interfering beams. Here we extend the recent results obtained in the papers Ref. 41, 42 where it has been shown a way of calculation of this effect in analogy with the Aharonov-Bohm effect, to the case of slowly rotating compact object in Horava-Lifshitz gravity.

In this paper we study quantum interference effects in particular the Sagnac effect and phase shift effect in a neutron interferometer in the Horava model which is organized as follows. In section 2, we start from the covariant Klein-Gordon equation in the Horava model and consider terms of the phase difference of the wave function. Recently Granit experiment 43 verified the quantization of the energy level of ultra-cold neutrons (UCN) in the Earth’s gravity field and new, more precise experiments are planned to be performed. Experiments with UCN have high accuracy and that is the reason to look for verification of the gravitational effects in such experiments. In this section as an example we investigate modification of UCN energy levels caused by the existence of KS (Kehagias and Sfetsos) parameter $\omega$. In section 3 we consider interference in Mach-Zender interferometer and in Section 4 we study the Sagnac effect in the background spacetime of slowly rotating object in Hořava gravity.

Throughout, we use space-like signature $(-, +, +, +)$, geometrical units system (However, for those expressions with an astrophysical application we have written the speed of light explicitly.). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

2. The Phase shift

The four-dimensional metric of the spherical-symmetric spacetime written in the ADM formalism [20,27,28,29] has the following form:

$$ds^2 = -N^2c^2dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$  

where $N, N^i$ are the metric functions to be defined.

The IR (Infrared) - modified Horava action is given by (see for more details Ref. 1, 2, 20, 27, 28, 29)

$$S = \int dt dx^3 \sqrt{-g} N \left[ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda g K^2) - \frac{\kappa^2}{2\rho_g} C_{ij} C^{ij} + \frac{\kappa^2}{2\rho_g^2} \epsilon^{ijk} R_{il} \nabla_j R^k_l + \frac{\kappa^2}{8} R_{ij} R^{ij} + \frac{\kappa^2}{8} \frac{\rho_2}{(3\lambda_g - 1)} \left( \frac{4\lambda_g - 1}{4} R^2 - \Lambda W R + 3\Lambda^2 \right) + \frac{\kappa^2}{8} \frac{\rho_2}{(3\lambda_g - 1)} R \right],$$  

(2)
where $\kappa, \lambda, \nu, \mu, \omega$ and $\Lambda_W$ are constant parameters, the Cotton tensor is defined as

$$C^{ij} = \epsilon^{ikl} \nabla_k (R^l_j - \frac{1}{4} R^l_l),$$  (3)

$R_{ijkl}$ is the three-dimensional curvature tensor, and the extrinsic curvature $K_{ij}$ is defined as

$$K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i),$$  (4)

where dot denotes a derivative with respect to $t$.

Imposing the case $\lambda g = 1$, which reduces to the action in IR limit, one can obtain the Kehagias and Sfetsos (KS) asymptotically flat solution \cite{44} for the metric outside the gravitating spherical symmetric object in Horava gravity:

$$ds^2 = -N^2 c^2 dt^2 + N^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2,$$

$$N^2 = 1 + \omega r^2 - \sqrt{r(\omega^2 r^3 + 4\omega M)}$$

where $M$ is the total mass, $\omega$ is the KS parameter and the constant $\Lambda_W = 0$ is chosen.

Up to the second derivative terms in the action, one can easily find the known topological rotating solutions given in Ref. \cite{21, 45}. This metric in the slow rotation limit has the form:

$$ds^2 = -N^2 c^2 dt^2 + N^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\varphi^2 - 2(1 - N^2)ac \sin^2 \theta dt d\varphi,$$  (6)

here $a$ is the specific angular momentum of the gravitating object.

Using the Klein-Gordon equation

$$\nabla^\mu \nabla_\mu \Phi - (mc/\hbar)^2 \Phi = 0,$$  (7)

for particles with mass $m$ one can define the wave function $\Phi$ of interfering particles as \cite{39}

$$\Phi = \Psi \exp \left( -imc^2/\hbar t \right),$$  (8)

where $\Psi$ is the nonrelativistic wave function.

In the present situation, both parameters $GM/rc^2$ and $a/r$ are sufficiently small and their higher order terms can be neglected. Therefore, to the first order in $M$, $\omega$ and neglecting the terms of $O((\nu/c)^2)$, the Klein-Gordon equation in Horava-Lifshitz gravity becomes

$$i\hbar \frac{\partial \Psi}{\partial t} = \frac{\hbar^2}{2m} \left[ \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial}{\partial r}) - \frac{L_z^2}{r^2 \hbar^2} \right] \Psi + \frac{mc^2 \omega r^2}{2} \left( 1 - \sqrt{1 + \frac{4GM}{c^2 \omega r^3}} \right) \Psi$$

$$-ac\omega \left( 1 - \sqrt{1 + \frac{4GM}{c^2 \omega r^3}} \right) L_z \Psi,$$  (9)
where we have used the following notations:

\[ L^2 = -\hbar^2 \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right), \]

\[ L_z = -i \hbar \frac{\partial}{\partial \varphi}, \]

which correspond to the square of the total orbital angular momentum and \( z \) component of the orbital angular momentum operators of the particle with respect to the center of the Earth, respectively.

After the coordinate transformation \( \varphi \to \varphi + \Omega t \), where \( \Omega = \Omega_{\oplus} \) is the angular velocity of the Earth, we obtain the Schrödinger equation for an observer fixed on the Earth in the following form:

\[ i\hbar \Psi_t = H_0 \Psi + H_1 \Psi + H_2 \Psi + H_3 \Psi, \]

where

\[ H_0 = -\hbar^2 \frac{1}{2m} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{L^2}{2mr^2}, \quad H_1 = \frac{mc^2 \omega r^2}{2} \left( 1 - \sqrt{1 + \frac{4GM}{c^2 \omega r^3}} \right), \]

\[ H_2 = -\Omega L_z, \quad H_3 = -a \omega c \left( 1 - \sqrt{1 + \frac{4GM}{c^2 \omega r^3}} \right) L_z. \]

\( H_0 \) is the Hamiltonian for a freely propagating particle, \( H_1 \) is the Horava-Lifshitz gravitational potential energy, \( H_2 \) is concerned to the rotation, \( H_3 \) is related to the effect of dragging of the inertial frames. The phase shift terms due to \( H_1, H_2 \) and \( H_3 \) are

\[ \beta_{\text{Sag}} \simeq \frac{2m\Omega \cdot S}{\hbar}, \]

\[ \beta_{\text{drag}} \simeq \frac{2Gm}{hc^2 R^3} \mathbf{J} \left[ S - 3 \left( 1 - \frac{2GM}{c^2 \omega R^3} \right) \left( \frac{\mathbf{R} \cdot \mathbf{S}}{R} \right) \frac{\mathbf{R}}{R} \right], \]

where \( \mathbf{R} \) represents the position vector of the instrument from the center of the Earth, \( S = S \mathbf{n}, \) \( S \) is the area of the interferometer, and \( \mathbf{n} \) is the unit normal vector.

If we assume that the Earth is a sphere of radius \( R \) with uniform density then

\[ \mathbf{J} = \frac{2}{5} MR^2 \Omega, \]

and

\[ \beta_{\omega} = \frac{\omega R^2}{5} \left( 1 - \sqrt{1 + \frac{4GM}{c^2 \omega R^3}} \right) \beta_{\text{Sag}}, \quad \beta_{\omega} = \frac{\omega R^2}{5} \left( 1 - \frac{1 - \frac{2GM}{c^2 \omega R^3}}{\sqrt{1 + \frac{4GM}{c^2 \omega R^3}}} \right) \beta_{\text{Sag}}, \]

if \( \mathbf{R} \) is perpendicular and parallel to \( S \), respectively. Here \( g = GM/R^2 \) is the free fall acceleration of Earth.
Now one can easily calculate the phase shift due to the gravitational potential. For the purpose of the present discussion, the quasi-classical approximation is valid and the phase shift

$$\beta_{H_{grav}} = \beta_{ABD} - \beta_{ACD} \simeq -\frac{1}{\hbar} \int H_1 dt = \frac{m^2 c^2 S \lambda \omega R}{2\pi \hbar^2} \left(1 - \frac{1 + \frac{g}{c^2 \omega R}}{\sqrt{1 + \frac{4g}{c^2 \omega R}}} \right) \sin \phi,$$

is given by the integration along a classical trajectory. Here $S = d_1 d_2$ is the area of interferometer, $\lambda$ is de Broglie wavelength (see the Fig. 1).

Recently published paper Ref. [46] describes the precise measurement of the gravitational redshift by the interference of matter waves in the gravitational field of the Earth. Comparing their experimental results with our theoretical predictions one can easily obtain the lower limit on the value of KS parameter $\omega \simeq 2.4 \times 10^{-27}$ cm$^{-2}$.

Astrophysically it is interesting to apply the obtained result for the Hamiltonian of the particle, moving around rotating gravitating object in Hořava gravity, to the calculation of energy level of ultra-cold neutrons (UCN) (as it was done for slowly rotating space-times in the papers Ref. [47, 48]. The effect of the angular momentum perturbation of the Hamiltonian $H_2 = \Omega L_z$ on the energy levels of UCN was studied in [43] and subsequent papers. Our purpose is to generalize this correction to the case
of the gravitating object (the Earth in particular case) in Hořava model. Denote as $\psi$ the unperturbed non-relativistic stationary state of the 2-spinor (describing UCN) in the field of the rotating gravitating object in Hořava gravity. Then we have

$$H_3\psi = i\hbar ac\omega \left( 1 - \sqrt{1 + \frac{4GM}{\omega c^2 r^3}} \right) \frac{\partial \psi}{\partial \varphi} = i\hbar ac\omega r \sin \theta \left( 1 - \sqrt{1 + \frac{4GM}{\omega c^2 r^3}} \right) \nabla \psi \cdot e_\varphi,$$

(19)

Here

$$\nabla \psi = \frac{\partial \psi}{\partial r} e_r + \frac{1}{r} \frac{\partial \psi}{\partial \theta} e_\theta + \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial \varphi} e_\varphi,$$

(20)

is the Laplacian in the spherical coordinates. By adopting new Cartesian coordinates $x, y, z$ within $e_x \equiv e_\varphi$ and axis $z$ being local vertical, when the stationary state is assumed to have the form

$$\psi(x) = \psi_n(z)e^{i(k_1 x + k_2 y)},$$

(21)

one can easily derive from (19) that

$$H_3\psi = i\hbar k_1 ac\omega r \sin \theta \left( 1 - \sqrt{1 + \frac{4GM}{\omega c^2 r^3}} \right) \frac{\partial \psi}{\partial x} =$$

$$\hbar k_1 ac\omega r \sin \theta \left( 1 - \sqrt{1 + \frac{4GM}{\omega c^2 r^3}} \right) \psi = -mu_1 ac\omega r \sin \theta \left( 1 - \sqrt{1 + \frac{4GM}{\omega c^2 r^3}} \right) \psi,$$

(22)

where the following notation

$$u_1 \equiv u \cdot e_\varphi, \quad u \equiv \frac{\hbar}{m}(k_1 e_x + k_2 e_y)$$

(23)

has been used.

Following to the papers Ref. 47, 48 one can compute "KS parameter $\omega$" modification of the energy level as the first-order perturbation:

$$(\delta E)_\omega \simeq \langle \psi | H_3 \psi \rangle = -mu_1 \int ac\omega r \sin \theta \left( 1 - \sqrt{1 + \frac{4GM}{\omega c^2 r^3}} \right) |\psi|^2 dV.$$

(24)

Assume $r = (R + z) \cos \chi$ (where $\chi$ is the latitude angle) and $\sin \theta$ to be equal to 1, that is $\theta = \pi/2$. Assuming now $z \ll R$ one can extend (24) as

$$
(\delta E)_\omega \simeq -mu_1 acR \cos \chi \left[ 1 - \sqrt{1 + \frac{4GM}{c^2 \omega R^3 \cos^3 \chi}} \right] + \frac{6GM}{c^2 \omega R^4 \cos^3 \chi} \left( 1 + \frac{4GM}{c^2 \omega R^3 \cos^3 \chi} \right)^{-1/2} \int z|\psi|^2 dV \right],
$$

(25)

We remember that $\int z|\psi|^2 dV$ is the average value $\langle z \rangle_n$ of $z$ for the stationary state $\psi = \psi_n$. For the further calculation we use formula for $\langle z \rangle_n$ from 47

$$\langle z \rangle_n = \frac{2}{3} \frac{E_n}{mg}.$$

(26)
Now one can easily estimate the relative "KS parameter $\omega$" modification of the energy level $E_n$ of the neutrons as

$$\frac{(\delta E)_\omega}{E_n} \approx -\frac{4u_1a}{cR\cos^2\chi} \left(1 + \frac{4g}{c^2\omega R \cos^3\chi}\right)^{-1/2}$$  \hspace{1cm} (27)$$

We numerically estimate the obtained modification using the typical parameters for the Earth: $u_1 \simeq +10^3$ cm/s, $\omega \sim 10^{-24}$ cm$^2$, $\cos\chi \simeq 0.71$, $a \simeq 3.97 \times 10^6$ cm, $g \simeq 10^3$ cm/s$^2$ and $R \simeq 6.4 \times 10^8$ cm

$$\frac{(\delta E)_\omega}{E_n} \simeq 2 \times 10^{-13}.$$  \hspace{1cm} (28)$$

From the obtained result (28) one can see, that the influence of $\omega$ parameter will be stronger in the vicinity of compact gravitating objects with small $R$. Recent experiments on measuring energy levels of UCN has an error $\sim 10^{-10}$, which does not allow to obtain the influence of $\omega$ parameter on energy levels of UCN. Further improvements of the experiments would give either exact value or lower limit for the above mentioned parameter.

3. The interference in a Mach-Zehnder-type interferometer

The components of the tetrad frame for the proper observer for metric (39) are

$$e^\mu_i = N^{-1} (1, 0, 0, 0) , \quad e^i_\mu = -N \left(1, 0, 0, \frac{(1 - N^2)a \sin^2 \theta}{N^2}\right) ,$$

$$e_\rho^\mu = N(0, 1, 0, 0) , \quad e^\rho_\mu = N^{-1}(0, 1, 0, 0) ,$$

$$e_\theta^\mu = r^{-1} (0, 0, 1, 0) , \quad e^\theta_\mu = r(0, 0, 1, 0) ,$$

$$e_\varphi^\mu = \frac{1}{r \sin \theta} \left(-\frac{(1 - N^2)a \sin^2 \theta}{N^2}, 0, 0, 1\right) , \quad e^\varphi_\mu = r \sin \theta(0, 0, 0, 1) ,$$

and the acceleration of the Killing trajectories is

$$a_\mu = \frac{1}{2} \partial_\mu \ln(-g_{00}) ,$$

and we obtain for the nonvanishing component of the acceleration:

$$a_\varphi = \omega r \left(1 - \frac{M}{\omega r^2} \sqrt{1 + \frac{4M}{\omega r^2}}\right) \left(1 + \omega^2 r^2 - \omega^2 r^2 \sqrt{1 + \frac{4M}{\omega r^2}}\right)^{-1/2} .$$

(32)

The nonvanishing orthonormal ("hatted") components of rotation tensor of the stationary congruence $\chi_{\mu \nu}$ in the slowly rotating Hořava-Lifshitz gravity are given by
\[
\chi_{\hat{r}\hat{\phi}} = \frac{a\omega \sin \theta}{1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}} \left(1 - \frac{1 + \frac{M}{\omega r^3}}{\sqrt{1 + \frac{4M}{\omega r^3}}} \right), \quad (33)
\]

\[
\chi_{\hat{\theta}\hat{\phi}} = \frac{a\omega \cos \theta}{\sqrt{1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}}} \left(1 - \sqrt{1 + \frac{4M}{\omega r^3}} \right)} \quad (34)
\]

The simple form of the vector potential of the electromagnetic field \(A_\mu\) in the Lorentz gauge in the spacetime (6) is

\[
A_\alpha = C_1 \xi_\alpha^t + C_2 \xi_\alpha^\phi \quad (35)
\]

One can write the total energy of the particle in the weak field approximation in the following form:

\[
E = p(\xi) + E_{pot} = p(\xi) + e_p A_t \quad (36)
\]

where \(e_p\) is electric charge of the particle. This is interpreted as total conserved energy consisting of gravitationally modified kinetic and rest energy \(p(\xi)\), a modified electrostatic energy \(e_p A_t\).

For the further use note the measured components of the electromagnetic field, which are the electric \(E_\alpha = F_{\alpha\beta} u^\beta_{obs}\) and magnetic \(B_\alpha = (1/2)\eta_{\alpha\beta\mu\nu} F^{\beta\mu} u^\nu_{obs}\) fields:

\[
B_\rho = -\frac{B \cos \theta}{2}, \quad B_{\theta} = -\frac{B \sin \theta}{2} \left(1 + \omega r^2 - \omega r^2 \sqrt{1 + \frac{4M}{\omega r^3}} \right)^{1/2}, \quad (37)
\]

\[
E_\varphi = \frac{aB \omega r}{\sqrt{1 + \omega r^2}} \left[2 \left(1 + \frac{M}{\omega r^3} \right) - \sqrt{1 + \frac{4M}{\omega r^3}} + \frac{3M}{\omega r^3} \sin^2 \theta \right], \quad E_{\theta} = 0 \quad (38)
\]

where \(F_{\alpha\beta} = A_{\beta,\alpha} - A_{\alpha,\beta}\) is the field tensor, \(\eta_{\alpha\beta\mu\nu} = \sqrt{-g} \epsilon_{\alpha\beta\mu\nu}\) is the pseudotensorial expression for the Levi-Civita symbol \(\epsilon_{\alpha\beta\mu\nu}\), \(g \equiv det|g_{\alpha\beta}|\).
Now one can obtain the total phase shift as
\[
\Delta \phi = \mathcal{E} S \left[ -\frac{\mathcal{E}}{p_0} \left( \cos \beta a_r - \cos \gamma \sin \beta a_\theta - \sin \gamma \sin \beta a_\phi \right) \\
- \frac{1}{p_0} \left( \cos \beta \partial_\mu \mathcal{E}_{pot} - \cos \gamma \sin \beta \partial_\mu \mathcal{E}_{pot} - \sin \gamma \sin \beta \partial_\mu \mathcal{E}_{pot} \right) \\
+ \sin \beta \chi_{\partial_\mu} + \cos \gamma \cos \beta \chi_{\partial_\mu} + \sin \gamma \cos \beta \chi_{\partial_\mu} \right] \\
+ e_p S \left( \sin \beta B_r + \cos \gamma \cos \beta B_\theta + \sin \gamma \cos \beta B_\phi \right),
\]
(39)
where \( \partial_\mu = e_\nu \partial_\nu \), \( \gamma \) is the angle of the baseline with respect to \( e_\phi \) and \( \beta \) is the tilt angle. Therefore one can independently vary the angles \( \beta \) and \( \gamma \), and extract from the phase shift measurements the following combinations of terms:
\[
\Delta \phi (\beta = 0, \gamma = 0) = \\
\mathcal{E} S \mathcal{W} \left[ \frac{a \sin \theta}{A^2} + \frac{a B r e_p A}{p_0} (2 - \sin^2 \theta) - \frac{\mathcal{E}_r}{p_0 A} \right] - \frac{1}{2} e_p BS \sin \theta A,
\]
(40)
\[
\Delta \phi \left( \beta = \frac{\pi}{2}, \gamma = \frac{\pi}{2} \right) = S \cos \theta \left( \frac{a \omega E K}{A} - \frac{e_p B}{2} \right),
\]
(41)
\[
\Delta \phi \left( \beta = \frac{\pi}{2}, \gamma = 0 \right) = S \cos \theta \left[ \frac{a \omega E K}{A} \left( 1 + \frac{B r \sin \theta A}{p_0} \right) - \frac{e_p B}{2} \right],
\]
(42)
\[
\Delta \phi \left( \beta = 0, \gamma = \frac{\pi}{2} \right) = -\frac{\omega r S \mathcal{E}^2 \mathcal{W}}{A p_0} \left[ 1 - \frac{a e_p B A^2}{\mathcal{E}} (2 - \sin^2 \theta) \right],
\]
(43)
where
\[
\left( 1 - \frac{1 + \frac{M}{\omega r^2}}{\sqrt{1 + \frac{4M}{\omega r^2}}} \right) = \mathcal{W}, \quad \left( 1 - \sqrt{1 + \frac{4M}{\omega r^2}} \right) = \mathcal{K}.
\]
(44)

Using above obtained results one can estimate lower limit for KS parameter \( \omega \). Using the results of the Earth based atom interferometry experiments \( ^{56} \) would give us an estimate \( \omega \approx 1.25 \times 10^{-25} \text{cm}^{-2} \).

4. The Sagnac effect in the Horava gravity

It is well known that the Sagnac effect for counter-propagating beams of particles on a round trip in an interferometer rotating in a flat space-time may be obtained by a formal analogy with the Aharonov-Bohm effect. Here we study the interference process of matter or light beams in the spacetime of slowly rotating compact gravitating object in braneworld in terms of the Aharonov-Bohm effect \(^{57} \). The phase shift
\[
\Delta \phi = \frac{2m u_0}{c \hbar} \oint_C A_G \cdot dx
\]
(45)
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is detected at uniformly rotating interferometer and the time difference between the propagation times of the co-rotating and counter-rotating beams is equal to

\[ \Delta T = \frac{2u_t}{c^3} \oint_C \mathbf{A}_G \cdot d\mathbf{x}. \]  
(46)

In the expressions (45) – (46) \( m \) indicates the mass (or the energy) of the particle of the interfering beams, \( \mathbf{A}_G \) is the gravito-magnetic vector potential which is obtained from the expression

\[ (\mathbf{A}_G)_i \equiv c^2 u_t a \frac{u_i}{u_0}, \]  
(47)

and \( u^\alpha(x) \) is the unit four-velocity of particles:

\[ u^\alpha \equiv \left\{ \frac{1}{\sqrt{-g_{tt}}} 0, 0, 0 \right\}, \quad u_\alpha \equiv \{-\sqrt{-g_{tt}}, g_{it}/\sqrt{-g_{tt}} \}. \]  
(48)

From (4) and coordinate transformation \( \varphi \to \varphi + \Omega t \), where \( \Omega = \Omega_0 \) one can see that the unit vector field \( u^\alpha \) along the trajectories \( r = R = \text{const} \) will be

\[ u_t = -(u^t)^{-1}, \quad u_\phi = [\Omega r^2 - a(1 - N^2)]u^t, \]  
(49)

where we have used the following notation

\[ u^t = [N^2 - \Omega^2 r^2 + 2a\Omega(1 - N^2)]^{-1/2}. \]  
(50)

Now inserting the components of \( u^\alpha \) into the equation (47) one can obtain

\[ \mathbf{A}_G^\phi = -[\Omega r^2 - a(1 - N^2)](u^t)^2. \]  
(51)

Integrating vector potential as it is shown in equations (45) and (46) one can get the following expressions for \( \Delta \phi \) and \( \Delta T \) (here we returned to the physical units):

\[ \Delta \phi = \frac{4\pi m}{\hbar} r^2 (\Omega - \bar{\omega})u^t, \quad \Delta T = \frac{4\pi}{c^2} r^2 (\Omega - \bar{\omega})u^t. \]  
(52)

where \( \bar{\omega} \) is the angular velocity of Lense-Thirring.

Following to the paper Ref. 57 one can find a critical angular velocity \( \bar{\Omega} = -ac \left( 1 - \sqrt{1 + \frac{4M}{\omega r^3}} \right) \),

(53)

which corresponds to zero time delay \( \Delta T = 0 \). \( \bar{\Omega} \) is the angular velocity of zero angular momentum observers (ZAMO).

5. Conclusion

We have studied quantum interference effects including e.g. the phase shift and time delay in Sagnac effect in the spacetime of rotating gravitational objects in Hořava gravity and found that they can be affected by the KS parameter \( \omega \). Then, we have derived an additional term for the phase shift in a neutron interferometer due to the presence of KS parameter and studied the feasibility of its detection with the help
of "figure-eight" interferometer. We have also investigated the application of the obtained results to the calculation of energy levels of UCN and found modifications to be rather small for the Earth but more relevant for the compact astrophysical objects. The result shows that the phase shift for a Mach-Zehnder interferometer in spacetime of gravitational object in Hořava gravity is influenced by the KS parameter $\omega$. Obtained results can be further used in laboratory experiments to detect the interference effects related to the phenomena of Hořava gravity. Recently authors of the paper Ref. 20 from Solar system tests obtained values for parameter $\omega$ as follow: $\omega \simeq 3.1 \cdot 10^{-26}$ cm$^2$ (from perihelion precession of the Mercury), $\omega \simeq 4.4 \cdot 10^{-26}$ cm$^2$ (light deflection by the Sun), $\omega \simeq 1.8 \cdot 10^{-25}$ cm$^2$ (radar echo delay). Here we have estimated lower limit for parameter $\omega$ as $\omega \simeq 2.4 \times 10^{-27}$ cm$^{-2}$ using the experimental results of the recent paper Ref. 40 on the precise measurement of the gravitational redshift by the interference of matter waves.

Acknowledgments

The work was supported by the UzFFR (projects 5-08 and 29-08) and projects FA-F2-F079 and FA-F2-F061 of the UzAS. This work is partially supported by the ICTP through the OEA-PRJ-29 project. Authors gratefully thank Viktoria Morozova for useful discussions. AB acknowledges the TWAS for associateship grant. AA and AB thank the IUCAA for the hospitality where the research has been completed.

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