The problems of unification–mismatch and low $\alpha_3$: A solution with light vector–like matter

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Abstract
The commonly accepted notion of a weak unified coupling $\alpha_X \approx 0.04$, based on the assumption of the MSSM–spectrum, is questioned. It is suggested that the four–dimensional unified string coupling should very likely have an intermediate value ($\sim 0.2 - 0.3$, say) so that it may be large enough to stabilize the dilaton but not so large as to disturb the coupling–unification relations. Bearing this in mind, as well as the smallness of the MSSM unification scale $M_X$ compared to the string scale, the consequences of a previously suggested extension of the MSSM spectrum are explored. The extension contains two vector–like families of quarks and leptons with relatively light masses of order 1 TeV, having the quantum numbers of $16 + \overline{16}$ of $SO(10)$. It is observed that such an extension provides certain unique advantages. These include: (a) removing the stated mismatch between MSSM and string unifications with regard to $\alpha_X$ and to some extent $M_X$ as well, (b) achieving coupling unification with a relatively low value of $\alpha_3(m_Z)$, in accord with its world average value, and (c) following earlier works, providing a simple explanation of the observed inter–family mass–hierarchy. The extension provides scope for exciting new discoveries, beyond those of SUSY and Higgs particles, at future colliders, including the LHC and the NLC.

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1 Introduction

Achieving a complete unity of the fundamental forces together with an understanding of the origin of the three families and their hierarchical masses is among the major challenges still confronting particle physics. Conventional grand unification falls short in this regard in that owing to the arbitrariness in the Higgs sector, it does not unify the Higgs exchange force, not to mention gravity. Superstring theory is the only theory we know that seems capable of removing these shortcomings. It thus seems imperative that the low energy data extrapolated to high energies be compatible with string unification.

It is, however, known [1] that while the three gauge couplings, extrapolated in the context of the minimal supersymmetric standard model (MSSM) meet, at least approximately [2-5], provided $\alpha_3(m_Z)$ is not too low (see later), their scale of meeting, $M_X \approx 2 \times 10^{16} \, GeV$, is nearly 20 times smaller than the expected (one–loop level) string–unification scale $\mathcal{M}_{st}$ of $\mathcal{M}_{st} \approx g_{st} \times (5.2 \times 10^{17} \, GeV) \simeq 3.6 \times 10^{17} \, GeV$.

It seems to us that there is still a second mismatch concerning the value of the unified gauge coupling $\alpha_X$ at $M_X$. Subject to the assumption of the MSSM spectrum, extrapolation of the low energy data yields a rather low value of $\alpha_X \sim 0.04$ [2-5], for which perturbative physics should work well near $M_X$. On the other hand, it is known [4] that non–perturbative physics ought to be important for a string theory near the string scale, in order that it may help choose the true vacuum and fix the moduli and the dilaton VEVs. The need to stabilize the dilaton in particular would suggest that the value of the unified coupling at $M_{st}$ in four dimensions should be considerably larger than 0.04 [8]. At the same time, $\alpha_{st}$ should not be too large, because, if $\alpha_{st} \gg 1$, the corresponding theory should be equivalent by string duality [9] to a certain weakly coupled theory that would still suffer from the dilaton runaway problem [10]. Furthermore, $\alpha_{st}$ at $M_{st}$ should not probably be as large as even unity, or else, the one–loop string unification relations for the gauge couplings [8] would cease to hold near $M_{st}$ (e.g. in this case, the string threshold corrections are expected to
be too large) and the observed (approximate) meeting of the three couplings would have to be viewed as an accident. In balance, therefore, the preceding discussions suggest that an *intermediate value* of the string coupling \( \alpha_{st} \approx 0.2 - 0.3 \) at \( M_{st} \) in four dimensions, which might be large enough to stabilize the dilaton, but not so large as to disturb significantly the coupling unification relations, is perhaps the more desired value. It is thus a challenge to find a suitable variant or alternative to MSSM which removes the mismatch not only with regard to the meeting point \( M_X \), but also with regard to the value of \( \alpha_X \).

A third relevant issue is that the world average value of \( \alpha_3(m_Z) = 0.117 \pm 0.005 \) \cite{11} seems to be low compared to its value that is needed for MSSM unification. Barring possible corrections from GUT threshold and Planck scale effects, the latter is higher than about 0.127, if \( m_{\tilde{q}} < 1 \) TeV and \( m_{1/2} < 500 \) GeV \cite{2-5}.

It is conceivable that the resolution of *all three issues* raised above-i.e. (a) understanding fermion mass–hierarchy, (b) removing incompatibility between MSSM and string unification, and (c) accommodating low \( \alpha_3(m_Z) \)- have a *common denominator*. The purpose of this note is to explore just this possibility, the common denominator in question being a previously suggested extension of the MSSM spectrum \cite{12, 13, 14} that contains two vector–like families and their SUSY partners, having the quantum numbers of \( 16 + \overline{16} \) of \( SO(10) \), all with masses of order 1 TeV.

It has been noted for some time that the existence of two such families enables one to obtain a simple understanding of the observed inter-family mass-hierarchy of the three chiral families \cite{13}. The argument will be presented briefly in Sec. 3. On the experimental front it is interesting to note that although the precision measurements of \( N_\nu \) and of the oblique electroweak parameters (\( S, T \) and \( U \)) disfavor a fourth chiral family, they are rather insensitive to vector-like families \cite{15, 14}.

The existence of two vector–like families together with three chiral families and the associated form of the \( 5 \times 5 \) fermion mass matrix was in fact derived in the context of a SUSY preon model \cite{12, 16}. Such a spectrum could well emerge, however, even
if quarks, leptons and Higgs bosons are elementary, e.g. from a superstring theory. In view of its prospects for providing exciting discoveries at the LHC and NLC, we propose to explore here whether such a spectrum might have some additional advantages, *in the context of an elementary quark–lepton–Higgs theory*, in bridging the gap between MSSM and string unifications mentioned above, and simultaneously accommodating low $\alpha_3$. Before proceeding, we note a few alternative suggestions which have been proposed to address some of these issues.

First, a very intriguing suggestion in this regard has recently been put forth by Witten [17]. Using the equivalence of the strongly coupled heterotic $SO(32)$ and the $E_8 \times E_8$ superstring theories in $D = 10$, respectively to the weakly coupled $D = 10$ Type I and an $M$–theory, he observed that the 4-dimensional gauge coupling and $M_{st}$ can both be small, as suggested by MSSM extrapolation of the low energy data, without making the Newton’s constant unacceptably large. While this observation opens up a new perspective on string unification, its precise use to make $\alpha_{st} \approx 0.04$ at $M_{st}$ would seem to run into the dilaton runaway problem as in fact noted in Ref. [17]. Furthermore, lowering $M_{st}$ to $2 \times 10^{16}$ GeV would mean that the heavy string states, very likely including color triplets, would have masses $\sim 2 \times 10^{16}$ GeV. Generically, this might lead to the problem of rapid proton decay through dimension 5 operators. The case of larger $\alpha_X$ and $M_X$ proposed here (see later) would seem to fare better in overcoming these potential difficulties.

A second way in which the mismatch between $M_X$ and $M_{st}$ could be resolved is if superstrings yield an intact grand unification symmetry like $SU(5)$ or $SO(10)$ with the right spectrum – i.e., three chiral families and a suitable Higgs system including an adjoint Higgs at $M_{st}$, and if this symmetry would break spontaneously at $M_X \approx (1/20$ to $1/50) M_{st}$ to the standard model symmetry. However, as yet, there is no realistic string–derived GUT model [18]. Furthermore, for such solutions, there is the likely problem of doublet-triplet splitting and rapid proton decay.

A third alternative is based on string–derived standard model–like gauge groups
and attributes the mismatch between $M_X$ and $M_{st}$ to the existence of new matter with intermediate scale masses ($\sim 10^9 - 10^{13}$ GeV), which may emerge from strings [19]. Such a resolution is in principle possible, but it would rely on the delicate balance between the shifts in the three couplings and on the existence of very heavy new matter which in practice cannot be directly tested by experiments. Also, within such alternatives, as well as those based on non-standard hypercharge normalization [20] and/or large string–scale threshold effects [21], $\alpha_X$ typically remains small ($\sim 0.04$), which is not compatible with the need for a larger $\alpha_X$, as suggested here.

2 The Extended Supersymmetric Standard Model (ESSM)

Bearing in mind the discussions above, we study the running of the coupling constants within the variant spectrum of quarks and leptons proposed some time ago [12, 13, 14] that assumes the standard model gauge symmetry but extends the MSSM spectrum by adding to it two light vector-like families $Q_{L,R} = (U, D, N, E)_{L,R}$ and $Q'_{L,R} = (U', D', N', E')_{L,R}$, two Higgs singlets ($H_S$ and $H_\lambda$) and their SUSY partners, all at about 1 TeV. We will refer to this variant as the Extended Supersymmetric Standard Model (ESSM). The combined sets ($Q_L|Q'_{R}$) and ($Q'_L|Q_R$) transform as 16 and 16 of $SO(10)$ respectively. It is interesting to note that the allowed extensions of MSSM in the low energy region are rather limited. Barring addition of singlets, ESSM is in fact the only extension of the MSSM, containing complete families of quarks and leptons, that is permitted by measurements of the oblique electroweak parameters and $N_\nu$ on the one hand, and renormalization group analysis on the other hand. The former restricts one to add only vector–like (rather than chiral) families [15], i.e. only pairs of 16 + $\overline{16}$ of $SO(10)$, whereas the latter states that no more than one such pair can be added, or else the gauge couplings would grow too rapidly and would become nonperturbative far below the unification scale [22]. While in this note, we do not address the derivation of such a spectrum in string theories, it is worth noting that
the emergence of pairs of $27 + 27$ of $E_6$ or $16 + 16$ of $SO(10)$ in addition to chiral multiplets is rather generic in string theories \[23\].

Now if the three couplings meet (at least approximately) for MSSM having 3 families, i.e. three $16$'s, at a position $M_X$, they are guaranteed to meet at the same position in the one-loop approximation for ESSM, having an extra pair of $16 + 16$. But the extra pair having masses $\sim 1\, TeV$ will inevitably raise the value of $\alpha_X$ at the meeting point, as desired. However, they will not raise $M_X$, in one loop. But once $\alpha_X$ is raised to 0.2 to 0.3 (see discussions later), two-loop effects are expected to be important especially near $M_X$. Our main task thus is to examine whether these two-loop effects for the ESSM spectrum, including contributions from gauge as well as Yukawa interactions, would still retain the meeting of the three couplings while raising $\alpha_X$ as well as $M_X$.

It is worth noting that there have been past attempts \[24\] to study the question of the meeting of the coupling constants by adding new families (chiral or vector) to the MSSM spectrum. Our approach and results will differ, however, from those of the past attempts because (i) We use a specific (yet most economical) pattern of the Yukawa coupling matrix (see below) which is tied to our desire to understand the inter-family mass hierarchy \[12,13\]. (ii) We include the contributions of these Yukawa couplings on the running of the gauge couplings in two–loop, which turns out to be quite important, but which have been neglected in past attempts. (iii) We use smoothed out threshold effects near the TeV scale \[3-5\]. (iv) And finally, owing to the beneficial effects of the Yukawa couplings (see later), we stay within semi-perturbative limits with $\alpha_X \sim (0.2 - 0.3)$, in contrast to $\alpha_X \sim \mathcal{O}(1)$ in Ref. \[24\] so that our results using the two loop $\beta$-functions are expected to be more reliable.

3 The Yukawa coupling matrix in ESSM

Following Ref. \[13,12\], it is known that the inter–family mass–hierarchy is reproduced simply if the three chiral famililies $q_{L,R}^i, i = 1 - 3$ derive their masses primarily
through their off–diagonal mixings with the two vector–like families $Q_{L,R}$ and $Q'_{L,R}$. Short of deriving such a mass–matrix from a string theory, we will assume suitable discrete symmetries (see later) which ensure this feature. To a good approximation the corresponding $5 \times 5$ Yukawa coupling matrix, near the presumed unification scale, is thus assumed to have the simple form:

$$h_{f,c}^{(o)} = \begin{pmatrix} q_i^L & Q_L & Q'_L \\ \bar{q}_R \Omega \begin{pmatrix} O & X_f H_f & Y_c H_S \\ Y_c^\dagger H_S & z_c H_\lambda & 0 \\ X_f^\dagger H_f & 0 & z'_f H_\lambda \end{pmatrix} \end{pmatrix}. \quad (1)$$

Here the symbol $q, Q, \text{ and } Q'$ stand for quarks as well as leptons, and $i = 1, 2, 3$. The subscript $f$ denotes $u, d, l$ or $\nu$, while $c = q$ or $l$ denotes quark or lepton color. $H_f$ with $f = u, d$ denotes the familiar two Higgs doublets, while $H_S$ and $H_\lambda$ are Higgs singlet fields. If the Yukawa couplings satisfy left–right, up–down as well as quark–lepton symmetries at the string scale, we would have $X_f = X'_f, Y_c = Y'_c$ and $z = z'$, and these couplings would be independent of flavor and color indices $f$ and $c$ at that scale. The zeros appearing in Eq. (1) are expected to be corrected by terms of order 1 MeV through VEVs inserted into higher dimensional operators.

The Higgs fields $H_\lambda, H_S$ and $H_f$ are assumed to acquire VEVs so that $\langle H_\lambda \rangle \sim 1 \text{ TeV}, \langle H_S \rangle \sim \langle H_u \rangle \sim 250 \text{ GeV}$ and $\langle H_d \rangle \ll \langle H_u \rangle$. To see the reason for family mass hierarchy, though not essential assume for simplicity $X_f = X'_f$ and $Y_c = Y'_c$ for a moment and denote $X_f^T = (x_1, x_2, x_3)f$ and $Y_c^T = (y_1, y_2, y_3)c$. Regardless of the values of these Yukawa couplings, one can always rotate the basis vectors so that $Y_c^T$ is transformed to the form $\hat{Y}_c^T = (0, 0, 1)y_c$, $X_f^T$ simultaneously to the form $\hat{X}_f^T = (0, p, 1)x_f$, and similarly $X'_f$ and $Y'_c$. It is thus apparent why one family is massless (barring corrections of order 1 MeV), despite lack of any hierarchy in the Yukawa couplings $(x_1, x_2, x_3)f$ and $(y_1, y_2, y_3)c$: this one is naturally identified with the electron family. At the unification scale one obtains $m_{t,b,\tau}^{(0)} \approx (2x_f y_c)(\langle H_S \rangle \langle H_f \rangle / (z \langle H_\lambda \rangle))$ and $m_{c,s,\mu}^{(0)} \approx m_{t,b,\tau}^{(0)}(p^2/4)$. A value of $p \approx (1/4 \text{ to } 1/5)$, which is in the realm of naturalness, thus provides a big hierarchy of about $(1/64 \text{ to } 1/100)$ between the masses
of the \((c, s, \mu)\) and \((t, b, \tau)\) at the string scale. Thus the presence of two vector–like families helps to provide a simple explanation of the inter–family mass–hierarchy: 
\[ m_{u,d,e} \ll m_{c,s,\mu} \ll m_{t,b,\tau} \] 

4 Renormalization Group Analysis for ESSM

We have performed a full two–loop analysis of the relevant renormalization group equations of the gauge couplings including the contributions of the Yukawa couplings as given in Eq. (1). To two–loop order, the RGE for the gauge coupling evolution are given by

\[
\frac{d\alpha_i}{dt} = \frac{b_i}{2\pi} \alpha_i^2 + \sum_{j=1}^3 \frac{b_{ij}}{8\pi^2} \alpha_i^2 \alpha_j - \frac{\alpha_i^2}{2\pi} \left( \frac{1}{16\pi^2} \right) b_{iYuk} \tag{2}
\]

where the coefficients \(b_i\) and \(b_{ij}\) are:

\[
b_i = \left( \begin{array}{c} 2n_g + \frac{3}{5}n_H \\ -6 + 2n_g \\ -9 + 2n_g \\ \end{array} \right) ; \quad b_{ij} = \left( \begin{array}{ccc} \frac{38}{15}n_g + \frac{9}{5}n_H & -6n_g + \frac{9}{5}n_H & \frac{88}{15}n_g \\ \frac{6n_g + \frac{9}{5}n_H}{11} & -24 + 14n_g + 7n_H & \frac{8n_g}{3} \\ \frac{11}{15}n_g & 3n_g & -54 + \frac{68}{5}n_g \end{array} \right). \tag{3}
\]

Here \(n_g\) is the total number of generations plus anti–generations and \(n_H\) is the number of pairs of Higgs doublets. For the case of ESSM, \(n_g = 5, n_H = 1\), corresponding to 3 chiral and two vector–like families, and one pair of Higgs doublets \(H_u\) and \(H_d\). The coefficients \(b_{iYuk}\) appearing in Eq. (2) are given by

\[
b_{1Yuk} = \frac{26}{5}(x_u^2 + x_u^2) + \frac{14}{5}(x_d^2 + x_d^2) + \frac{18}{5}(x_l^2 + x_l^2) + \frac{2}{5}(y_q^2 + z_q^2)
\]

\[
+ \frac{6}{5}(y_l^2 + z_l^2 + k_1^2) + \frac{16}{5}(x_l^2 + y_l^2) + \frac{4}{5}(z_l^2 + y_d^2) + \frac{12}{5}(x_l^2 + y_l^2)
\]

\[
b_{2Yuk} = 6(x_u^2 + x_u^2) + 6(x_d^2 + x_d^2) + 2(x_l^2 + x_l^2) + 6(y_q^2 + z_q^2) + 2(y_l^2 + z_l^2 + k_1^2)
\]

\[
b_{3Yuk} = 4(x_u^2 + x_u^2) + 4(x_d^2 + x_d^2) + 4(y_q^2 + z_q^2) + 2(z_l^2 + y_u^2) + 2(z_l^2 + y_d^2). \tag{4}
\]

In addition to the Yukawa couplings given in Eq. (1), we have assumed the following terms in the superpotential:

\[
W \sim k_1 H_u H_d H_\lambda + \frac{k_2}{6} H_\lambda^3 + \frac{k_3}{6} H_S^3, \tag{5}
\]
which gives masses to all the Higgses and Higgsinos and which consists of the most general set of superpotential terms consistent with a $Z_3 \times Z_3$ symmetry. Under this symmetry, which ensures the Yukawa coupling matrix, Eq. (1), the three chiral families $16_i$ transform as $(\omega, 1)$, the vector families transform as $16 \sim (1, \omega)$, $\overline{16} \sim (\omega, 1)$ and Higgs doublets as $10_H \sim (\omega^2, \omega^2)$ and the Higgs singlets as $H_\lambda \sim (\omega^2, \omega^2)$, $H_c \sim (\omega, 1)$, where $\omega^3 = 1$.

We study the evolution of the gauge couplings using two–loop RGE (i.e., Eq. (2)) from $m_Z$ upwards by dividing the momentum-range to two regions: Region I: $(m_Z \leq \mu \leq \mu_0 \sim 10^M)$: Here $M$ denotes the mass of the heaviest particle ($\approx 1 - 2$ TeV) in the ESSM spectrum and $\mu_0$ denotes the momentum scale upto which inclusion of threshold effects is important [3-5]. Region II: $(\mu_0 \leq \mu \leq 10^{18}$ GeV): In this region, we treat all particles as massless. Taking the couplings at $\mu_0$ as boundary values we use Eqs. (2)-(4) to extrapolate them upwards.

For region I, since the masses are spread from $m_Z$ to $M \approx 1.5 - 2$ TeV, we integrate Eq. (2) piecewise from one threshold ($m_1$) to the next ($m_2$) by first using the $\theta$–function approximation for each threshold and using appropriate two–loop $\beta$–function coefficients ($\tilde{b}_i$ and $\tilde{b}_{ij}$) for each subregion, which are not exhibited here. These include contributions from all particles with masses $\leq m_1$ to the evolution of the couplings in the range $m_1 \leq \mu \leq m_2$. Thus ignoring the contributions from the Yukawa couplings for a moment, and replacing $b_i$ and $b_{ij}$ in Eq. (2) by $\tilde{b}_i$ and $\tilde{b}_{ij}$ for the sub–region $m_1 \rightarrow m_2$, Eq. (2) can be integrated analytically to yield:

$$\tilde{\alpha}_i^{-1}(\mu) = \tilde{\alpha}_i^{-1}(m_1) - \frac{\tilde{b}_i}{2\pi} \ln \left( \frac{\mu}{m_1} \right) - \frac{1}{4\pi} \sum_j \frac{\tilde{b}_{ij}}{\tilde{b}_j} \ln \left[ \frac{\tilde{\alpha}_j(\mu)}{\tilde{\alpha}_j(m_1)} \right].$$

(6)

The contribution of each individual particle denoted by $\tilde{b}_i$ to the regional one–loop coefficients $\tilde{b}_i$ is listed in Table 1. The corresponding $\tilde{b}_{ij}$ are not exhibited here. Following this procedure in successive steps (i.e., $m_Z \rightarrow m_1 \rightarrow m_2 \rightarrow m_3, ..., M \rightarrow \mu_0$) we obtain $\tilde{\alpha}_i^{-1}(\mu_0)$. Since the leading log contributions have already been included in Eq. (6), we finally add to $\tilde{\alpha}_i^{-1}(\mu_0)$ obtained as above, the sum of the non-logarithmic threshold corrections for each new particle – ie. $\tilde{\Delta}_i(\mu_0) \equiv \sum (\Delta_i - \text{leading log term})$ –
Table 1: Threshold function coefficients appearing in Eq. (8) for various particles in ESSM. 
(Q, U', D', L, E') are the vector family fermions and a tilde denotes SUSY particle.

as well as contributions from the top and the Yukawa couplings of vector–like quarks to obtain

\[ \alpha^{-1}_i(\mu_0) = \tilde{\alpha}_i^{-1}(\mu_0) + \tilde{\Delta}_i(\mu_0) + \Delta^\text{top}_i + \Delta^\text{Yuk}_i \]  
(7)

where \( \Delta^\text{top}_i = (0.138, 0.158, 0.090) \) for \( m_t = 180 \text{ GeV} \) [2,3] while \( \Delta^\text{Yuk}_i = (0.026, 0.032, 0.023) \).

To evaluate \( \Delta_i \) and thus \( \tilde{\Delta}_i \) we use exact one-loop threshold functions given by [3,25]

\[ \Delta^{F,S}_i(m, \mu_0) = \frac{b_i}{2\pi} \left[ K^{F,S}(m/m_Z) - K^{F,S}(\mu_0/m) \right] . \]  
(8)

\[ K_F(q/m) = \frac{w^2}{2} \left[ 1 - \frac{(w^2 - 3)}{2w} \ln \left( \frac{w + 1}{w - 1} \right) \right] \]

\[ K_S(q/m) = 1 - w^2 + \frac{1}{2} w^3 \ln \left( \frac{w + 1}{w - 1} \right) \]  
(9)

Here \( (F, S) \) denote (fermion, scalar) and \( w(q/m) \equiv \sqrt{1 + 4m^2/q^2} \).

The values of \( \Delta_i \)'s would depend somewhat, as in MSSM, on the assumed masses of the new particles. Considerations based on (a) QCD renormalization effects which enhance the masses of \( (Q, \tilde{Q}, \tilde{q}, \tilde{g}) \) relative to \( (L, \tilde{L}, \tilde{l}, \tilde{W}) \), (b) the need to avoid unnatural fine-tuning, (so that \( m_{\tilde{q}} \leq 1 \text{ TeV}, |m_Q - m_{\tilde{Q}}| \leq 300 \text{ GeV} \) and (c) simplicity of
analysis, we assume the pattern: $m_Q \approx 1 - 2 \, TeV \geq m_{\tilde{Q}} \geq m_L \sim m_{\tilde{L}} \sim m_{\tilde{q}} \sim m_H \approx m_{\tilde{H}} \geq m_{\tilde{t}} \geq m_{\tilde{\bar{q}}} \geq m_{\tilde{\bar{W}}} \approx 80 - 200 \, GeV$. The QCD renormalization effects are taken from our preliminary analysis as a guide, which will be presented elsewhere. Owing to the added importance of the two-loop effects in ESSM, even if gaugino masses were universal at $M_X$, we obtain (ignoring Yukawa effects for this purpose) $m_{\tilde{g}}/m_{\tilde{W}} \approx 2$. This is in contrast to the one-loop value of $m_{\tilde{g}}/m_{\tilde{W}} \approx \alpha_3/\alpha_2 \approx 3.5$, for MSSM. Using this as a rough guide and also allowing for the possible lack of universality at $M_X$, we will vary $m_{\tilde{g}}/m_{\tilde{W}}$ in the range of 1.5 to about 3 for ESSM.

To study the evolution of the $\alpha_i$’s in region II ($\mu_0 \leq \mu \leq 10^{18} \, GeV$), we will assume here that all the relevant Yukawa couplings involving the third family are large at $M_X$—i.e., $x_i \sim x_i' \sim y_i \sim y_i' \sim z \sim z' \approx 1 - 2$, so that they approach their fixed point values near the electroweak scale [26]. We have derived the full set of one-loop RGE for the evolution of the Yukawa couplings of the ESSM. For brevity, these equations are not presented here [27]. Solving these coupled RGE Eqs. (2)-(4), and using typical values of $M_X \approx 10^{17} \, GeV$ and $\alpha_X \sim 0.25$ (see later), we find that the Yukawa couplings acquire their near-fixed point values at 1 TeV, given by:

\[
\begin{align*}
x_u' &= 0.896, \ y_q' = 0.746, \ x_u = 0.896, \ z_q = 0.740, \ z_u' = 0.554, \ y_u = 0.559, \\
x_d' &= 0.871, \ x_d = 0.872, \ z_d' = 0.533, \ y_d = 0.538, \ x_d' = 0.368, \ y_d' = 0.251 \\
x_l &= 0.396, \ z_l = 0.273, \ z_l' = 0.185, \ y_l = 0.184, \ x_l' = 0.332, \ y_l' = 0.152 \\
k_1 &= 0.010, \ k_2 = 0.214, \ k_3 = 0.217
\end{align*}
\]

These will be taken as their input values at 1 TeV [26].

An interesting comment is in order regarding the value of $m_b/m_\tau$. Naively, without the assistance of the Yukawa couplings, owing to the large ratio $\alpha_3(M_X)/\alpha_3(m_Z)$, $m_b$ would be much too big compared to experiments at the low scale, if it were equal to $m_\tau$ at $M_X$. However, with the effects of the Yukawa couplings included, we obtain $m_b/m_\tau \approx 2.53$ at 1 TeV, which is compatible with observation.
To determine the gauge couplings at $m_Z$ we follow the mass dependent subtraction procedure (MDSP) [3], which is suited to include the non–logarithmic threshold effects. We denote the initial values of the couplings at $m_Z$ in the MDSP scheme by $\hat{\alpha}_i(m_Z)$ [28]. Following Ref. [4, 3], we choose $G_F = 1.6639 \times 10^{-5} GeV^{-2}, m_Z = 91.187 GeV$ and $\alpha^{-1}(0) = 137.036$ as input values (rather than $\alpha_{em}(m_Z)$ and $\sin^2\theta_W(m_Z)$ of the $\overline{MS}$ scheme), together with a value for $m_t \approx 180 GeV$ and a chosen ESSM–spectrum to determine $\hat{\alpha}_1$ and $\hat{\alpha}_2$ at $m_Z$. We next choose a varying input value for $\hat{\alpha}_3(m_Z) \approx 0.12-0.127$ in the MDSP scheme [28] and extrapolate the three gauge couplings upward, for a given spectrum, to test unification. Following preceding discussions, we consider a few cases for the spectrum as noted below.

**Case 1:** $m_{\tilde{W}} = 75 GeV, m_{\tilde{g}} = 250 GeV, m_{\tilde{t}} = m_{\tilde{H}} = m_{\tilde{H}} = 400 GeV, m_{\tilde{q}} = 600 GeV, m_L = m_{\tilde{L}} = 900 GeV, m_Q = m_{\tilde{Q}} = 2.2 TeV$. Using Eq. (9) and the input values of $G_F, m_Z$ and $\alpha(0)$, this choice yields $\Delta_i(\mu_0 = 20 TeV) = (1.26, 1.40, 1.24)$ and $\hat{\alpha}_{1,2}(m_Z) = (1/59.56, 1/29.90)$ in the MDSP scheme [28]. Using these and an input $\hat{\alpha}_3(m_Z) = 0.127$, we determine $\alpha_i^{-1}(\mu_0)$ by means of Eqs. (6)-(8), which we use in turn to extrapolate to higher values of $\mu$ with the help of Eqs. (2)-(4). As can be seen from Fig. 1, the three gauge couplings meet at a scale $M_X \approx 10^{17} GeV$ (to within 2% difference from each other), with a unified value of the gauge couplings $\alpha_X \approx 0.24$.

**Case 2:** $m_{\tilde{W}} = 75 GeV, m_{\tilde{g}} = 215 GeV, m_{\tilde{t}} = m_{\tilde{H}} = m_{\tilde{H}} = 300 GeV, m_{\tilde{q}} = 500, m_L = m_{\tilde{L}} = 500 GeV, m_Q = m_{\tilde{Q}} = 1.5 TeV$: For this case [29], the couplings meet almost perfectly at $M_X \approx .8 \times 10^{17} GeV$ with $\alpha_X \approx 0.25$ and $\hat{\alpha}_3(m_Z) = 0.125$ (see Fig. 2).

**Case 3:** $m_{\tilde{W}} = 90 GeV, m_{\tilde{g}} = 170 GeV, m_{\tilde{t}} = m_{\tilde{H}} = m_{\tilde{H}} = 400 GeV, m_{\tilde{q}} = 600 GeV, m_L = m_{\tilde{L}} = 900 GeV, m_Q = m_{\tilde{Q}} = 2.2 TeV$. Here we get perfect meeting with $M_X \approx .7 \times 10^{17} GeV, \alpha_X \approx .22$ and $\hat{\alpha}_3(m_Z) = .123$ (see Fig. 3).

While we have not explored the parameter space pertaining to the spectrum of the new particles and variation in $\alpha_3(m_Z)$ in any detail, we find it indeed remarkable that
the three couplings meet, even perfectly for many cases, for a fairly wide variation in the ESSM spectrum beyond what we have exhibited here [30]. The corresponding values of $\alpha_X$, $M_X$ and $\tilde{\alpha}_3(m_Z)$ in ESSM are found to lie in the ranges of [31]:

\begin{equation}
\alpha_X \approx (.2 - .3); M_X = (.7 - 1.2) \times 10^{17} \text{ GeV}, \tilde{\alpha}_3(m_Z) = .122 - .128.
\end{equation}

Thus we see that ESSM leads to coupling–unification, with an intermediate value of $\alpha_X$, and a lower value of $\alpha_3(m_Z)$ than that needed for MSSM unification, just as desired. The resulting $M_X \sim 10^{17}$ GeV is higher than the MSSM value, but it is still lower than the one–loop string–unification scale of Ref. [6], which, for $\alpha_X \approx 0.25$, yields $M_{st} \approx 7 \times 10^{17}$ GeV. This remaining gap between $M_X$ and $M_{st}$ may have its resolution in part due to the increased importance of two–loop string threshold effects, corresponding to an intermediate value of $\alpha_X$, which could lead to significant corrections to the one–loop formula for $M_{st}$ [6], and in part due to the relative importance of three and higher loop effects, which may shift $M_X$ (see remarks below). In other words, considering the proximity of $M_X \sim 10^{17}$ GeV to the expected string scale of $(5-8) \times 10^{17}$ GeV, contributions from the infinite tower of heavy string–states, which have been neglected in the running of $\alpha_i$’s, and quantum gravity may play an important role in bridging the relatively small gap between $M_X$ and $M_{st}$ [32].

In summary, ESSM predicts (a) an intermediate value of $\alpha_X$ which may help stabilize the dilaton, (b) a value of $M_X \sim M_{st} \geq 10^{17}$ GeV, would fare better than the case of $M_{st} \sim 2 \times 10^{16}$ GeV [17] in avoiding the potential problem of rapid proton decay induced through $d = 5$ operators and (d) a lower $\alpha_3(m_Z)$ than the case of MSSM.

These appear to be distinct advantages of ESSM over MSSM.

Before concluding, the following points are worth noting.

(i) Even if ESSM–unification might be closer to the truth, it provides a simple reason why the couplings appear to meet, at least approximately, even for MSSM. As alluded to before, the reason is that in one loop, unification of couplings in one scheme implies that for the other, though with a vastly different $\alpha_X$. The two models differ only in two loop and thereby in the resulting values of $M_X$, $\alpha_X$ as well as $\alpha_3(m_Z)$.
(ii) The two loop gauge coupling contribution (i.e., $b_{ij}$ terms in Eq. (2)) which raise the slopes of $\alpha_i$, together with the softening effects of the Yukawa contributions which do the opposite, turned out to play an important role [33] in achieving unification for ESSM. It is the interplay of these two contributions which leads to a good meeting of the three gauge couplings (Fig. 1-3) with a low $\alpha_3(m_Z)$.

(iii) Although 3–loop effects could be important especially in fixing $M_X$, we expect our calculation based on 2–loop contributions presented here to be still fairly reliable, at least for the range $m_Z \leq \mu \leq 10^{15}$ GeV for which the couplings are small (i.e., $\alpha_{1,2} \leq 0.12, \alpha_3 \leq 0.18$, see Fig. 1-3). By the time $\mu$ rises to $10^{15}$ GeV, the three couplings, especially $\alpha_1$ and $\alpha_2$, begin turning sharply upward together in a manner that the tendency of the three curves to converge to a common meeting point is already apparent (see Fig. 1-3). Owing to the coupled RGE for the three $\alpha_i$, we suspect that this tendency would persist in three and higher loops [32].

(iv) A related remark: if $\alpha_X$ has an intermediate value, so that it may help stabilize the dilaton, the relative importance of two and possibly higher loops near $M_X$, compared to the case of MSSM, cannot be avoided. Yet as shown here, unification can already be seen quite visibly in two loops in the sense commented above.

(v) This preliminary work of ESSM motivates further study of the evolution of the gauge couplings, fermion and scalar masses as well as of radiative symmetry breaking in ESSM with the inclusion of two–loop evolution of Yukawa couplings and three–loop effects in Eq. (2).

(vi) Last but not least, ESSM predicts [12-16] two complete vector–like families with leptonic and quark members having masses in the ranges of (200 GeV–1 TeV) and (500 GeV–2.5 TeV) respectively. Their mass–pattern, mixing and decay modes as well as characteristic signals have been considered in detail in Ref. [14]. To mention just a few such signals, pair production of vector–like quarks at LHC and/or future version of SSC would lead to systems such as $(b\bar{b} + 4Z + W^{+}W^{-})$ and $(b\bar{b} + 2Z + W^{+}W^{-})$, while an $e^+e^-$ collider (NLC) could produce $E^-E^+$ and even flavor–violating $N\bar{\nu}_\tau$ pair...
appreciably, followed by the decay $N \rightarrow Z + \nu_{\tau} \rightarrow (e^+ e^-) + \nu_{\tau}$. Furthermore, once the relevant momentum transfer for sub-processes exceeds about $m_Q$ in hadronic colliders, the corresponding $\alpha_3$ would grow significantly due to contributions from virtual (or real) heavy quark pairs and their SUSY partners. This would manifest for example in enhancement of JET cross sections, even below threshold for production of real heavy quarks of a nature recently reported by the CDF group [34]. Even though the CDF findings may or may not reflect truly new physics, the phenomenon should reappear in high $p_T$–processes of future colliders including the LHC if the vector–like quarks with masses as above exist.

To conclude, ESSM, possessing two extra vector–like families with masses of order 1 TeV [35], provides (a) a simple explanation of the inter–family mass–hierarchy [12,13] as well as (b) unification with a higher $\alpha_X \sim .2 - .3$, a higher $M_X \sim 10^{17}\text{ GeV}$ and a lower $\alpha_3(m_Z)$ compared to MSSM [31], just as desired. The emergence of an extra pair of $16 + \bar{16}$ is rather generic in string theories. But the derivation of the ESSM spectrum together with a standard model–like gauge symmetry and Yukawa coupling matrix, as assumed here, from a string theory remains an important task. Owing to the advantages mentioned above, ESSM appears to be an attractive, yet falsifiable, alternative to MSSM.

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[22] Similar results as regards the RGE would follow if one introduces, instead of $16 + \overline{16}$, four pairs of $5 + \overline{5}$ of $SU(5)$ (or equivalently four $10$’s of $SO(10)$) in the light spectrum, although there will be differences in details between these alternatives in two loop. Such new matter ($5 + \overline{5}$ pair), would not, however, provide the beneficial effects of $16 + \overline{16}$ as regards an understanding of the inter–family mass–hierarchy in the down as well as in the up sector [12, 13].

[23] One may also envisage models in which the mass of a single surviving vector–like pair ($16 + \overline{16}$) as well as mixing of $\overline{16}$ with chiral $16$’s are protected by SUSY and/or a $U(1)$ symmetry that remains unbroken upto low energies.

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[25] It is instructive to note that for $m_Z \leq m \leq \mu$, $\Delta_i^F = (\hat{b}_i/2\pi)\left[-\ln(\mu/m) + 5/6 + 1/10(m_Z/m)^2 - 3(m^2/\mu^2) + O\{(m^4/\mu^4)\ln(\mu/m), (m_Z^4/m^4)\}\right]$. This shows that for $m \sim m_Z$ or $\mu$, the leading log–term gives a poor approximation.

[26] This also has the virtue that our results will be essentially free from the arbitrariness in the detailed choice of the Yukawa couplings.

[27] These have also been derived independently by J.B. Kim, Ph. D. Thesis, submitted to University of Maryland (Dec. 1995).

[28] There are differences between the $\overline{MS}$ and the MDSP couplings $\hat{a}_i$, which for typical SUSY spectrum are given by $(\alpha_i(m_Z)|_{\overline{MS}} - \hat{a}_i(m_Z)) \approx (0.8, 0.9)\%$, for
These differences arise mainly due to light quark thresholds \[3, 4\]. We have been informed by M. Bastero-Gil (private communications and paper to appear) that there is a larger negative difference for \((\alpha_3(m_Z)|_{\overline{MS}} - \hat{\alpha}_3(m_Z)) \approx -8\%\) for typical SUSY spectrum which is mainly due to the constant term arising from the finite part of the contribution of massless gluons to the relevant combination of the vacuum polarization and the vertex function. We thank M. Bastero-Gil for many discussions clarifying these issues. For a discussion of the scheme independence of the observable quantities such as the \(R\)-parameter or the \(Z\)-hadronic width, in spite of the scheme–dependence in \(\alpha_3\), see e.g. S.G. Gorishni, A.L. Kataev and S.A. Larin, Nuovo Cimento 92 A, 119 (1986).

[29] The small variations \((\leq 0.3\%)\) in \(\dot{\alpha}_{1,2}(m_Z)\) due to change in spectra in going from Case 1 to Case 2 will be ignored in this note.

[30] Since string-threshold effects are expected to introduce corrections to the three \(\alpha_i\)’s, differing by a few percent near \(M_X\) [19], we regard cases of almost perfect meeting (Fig. 2-3) as being on par with those for which the meeting occurs to within (2-3)\% (as in Fig. 1).

[31] Including the \(-8\%\) difference between \(\alpha_3(m_Z)|_{\overline{MS}}\) and \(\hat{\alpha}_3(m_Z)\) [28], the values of \(\hat{\alpha}_3(m_Z) \approx .122 - .128\) in the MDSP scheme needed for unification in ESSM would correspond to \(\alpha_3(m_Z)|_{\overline{MS}} \approx .112 - .118\). This should be compared with the value \(\alpha_3(m_Z)|_{\overline{MS}} \geq .127\) needed for MSSM unification, barring GUT and string threshold effects, on the one hand, and the world average value of \(.117 \pm .005\) [11], on the other hand. Both ESSM and MSSM values are still subject to string and possibly unification–scale threshold effects of a few percent [19].

[32] We are tempted to conjecture that with \(\alpha_X \approx .2 - .3\), the increased importance of the contribution from the heavy string tower of states might in fact lead to a sharp slowing down in the growth of the \(\alpha_i\)’s as \(\mu\) exceeds \(10^{17}\) \(GeV\) (compare with Figs. 1-3), so that they may approach their unified fixed point value (with \(\beta_i \rightarrow 0\)) as \(\mu\)
reaches or exceeds $M_{st}$. Such a behavior, although hard to demonstrate with the present state of the art, would seem to be in accord with the intrinsic finiteness of the string theory. The true $M_X$ and $\alpha_{st}$ in this case would be somewhat higher than that exhibited by the meeting point in Fig. 1-3.

[33] The importance of two-loop contributions in ESSM arises especially because the one-loop $\beta$-function for $\alpha_3$ almost vanishes, i.e, $b_3 = -9 + 2n_g = +1$ for ESSM, but $-3$ for MSSM, see Eq. (2).

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[35] Although in this paper we have assumed the standard model gauge symmetry below the string scale, the ESSM spectrum would still help raise $\alpha_X$ even if a higher symmetry like $G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)_C$ or one of its subgroups emerges from strings, which breaks to the standard model symmetry at a scale $\sim 10^{16}$ GeV. These cases need to be studied separately.

**Figure Caption**

Figs. 1-3: Plots of $\alpha_i^{-1}$ (Fig 1) and $\alpha_i$ (Figs. 2-3) as a function of $\mu$. 
Fig. 1 (Case 1)
Fig. 2 (Case 2)

Fig. 3 (Case 3)