Sparticle Mass Spectrum in Grand Unified Theories

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We carry out a detailed analysis of sparticle mass spectrum in supersymmetric grand unified theories. We consider the spectroscopy of the squarks and sleptons in SU(5) and SO(10) grand unified theories, and show how the underlying supersymmetry breaking parameters of these theories can be determined from a measurement of different sparticle masses. This analysis is done analytically by integrating the one-loop renormalization group equations with appropriate boundary conditions implied by the underlying grand unified gauge group. We also consider the impact of non-universal gaugino masses on the sparticle spectrum, especially the neutralino and chargino masses which arise in supersymmetric grand unified theories with non-minimal gauge kinetic function. In particular, we study the interrelationships between the squark and slepton masses which arise in grand unified theories at the one-loop level, which can be used to distinguish between the different underlying gauge groups and their breaking pattern to the Standard Model gauge group. We also comment on the corrections that can affect these one-loop results.

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I. INTRODUCTION

Despite its great success, the gauge group $SU(3) \times SU(2) \times U(1)$ remains a completely unexplained feature of the Standard Model (SM) of electroweak and strong interactions. The idea of grand unification [1] is, therefore, one of the most compelling theoretical ideas that goes beyond the Standard Model. In grand unified theories (GUTs), the SM gauge group can be elegantly unified into a simple group. Moreover, the fermion content of the SM can be accomodated in irreducible representations of the unified gauge group. Also, one can understand the smallness of neutrino masses via the seesaw mechanism [2, 3, 4, 5] in some of the grand unified models [6, 7] like SO(10). The renormalization flow of the gauge couplings leads to their unification at a very large scale [8]. However, this picture wherein the SM is embedded into a grand unified theory (GUT) with gauge coupling unification at large scale leads to the well known hierarchy and naturalness problems due to the widely separated scales, the weak scale characterized by the mass of the $Z$-boson ($\sim M_Z$), and the large unification scale characterized by the gauge coupling unification.

Supersymmetry (SUSY) is at present the only known framework [9] in which the hierarchy between the weak scale and the large GUT scale can be made technically natural [10, 11, 12, 13, 14]. Supersymmetry is, however, not an exact symmetry in nature. The precise manner in which SUSY is broken is not known at present. However, the necessary SUSY breaking can be introduced through soft supersymmetry breaking terms that do not reintroduce quadratic divergences in the Higgs mass, and thereby do not disturb the stability of the hierarchy between the weak scale and the GUT scale. Such terms can typically arise in supergravity theories, in which local supersymmetry is spontaneously broken in a hidden sector, and is then transmitted to the visible sector via gravitational interactions. This is what is usually done in the case of the minimal supersymmetric standard model (MSSM), with gravity mediated supersymmetry breaking [15, 16]. However, minimality is only a simplifying assumption, and may not necessarily lead to realistic models.

At present the most direct phenomenological evidence in favour of supersymmetry is obtained from the unification of couplings in GUTs. Precise LEP data on $\alpha_s(m_Z)$ and $\sin^2 \theta_W$ show that standard one-scale GUTs fail in predicting $\sin^2 \theta_W$, given $\alpha_s(m_Z)$ (and $\alpha(m_Z)$), while GUTs based on supersymmetry (SUSY GUTs) [17] are in agreement with the present experimental results. If one starts from the known values of $\sin^2 \theta_W$ and $\alpha(m_Z)$, one finds [18, 19, 20] for $\alpha_s(m_Z)$ the results: $\alpha_s(m_Z) = 0.073 \pm 0.002$ for Standard GUTs and $\alpha_s(m_Z) = 0.129 \pm 0.010$ for SUSY GUTs to be compared with the world average experimental value $\alpha_s(m_Z) = 0.118 \pm 0.002$. Furthermore, one of the most important predictions of grand unification is that, because of the presence of baryon number violating interactions,
proton must decay. In SUSY GUTs proton decay is much slower as compared to the non SUSY case. This is because the unification mass is typically $M_{\text{GUT}} \sim \text{few} \times 10^{16}\text{ GeV}$ in SUSY GUTs, which is about 20-30 times larger than for ordinary GUTs. This makes proton decay via gauge boson exchange negligible and the main decay amplitude arises from dimension-5 operators with higgsino exchange, leading to a rate close but still compatible with existing bounds \cite{21}. Moreover, SUSY provides an excellent dark matter candidate, the neutralino. We finally recall that the range of neutrino masses as indicated by oscillation experiments, when interpreted in the see-saw mechanism, point toward a large scale \cite{22} and give additional support to GUTs.

This naturally leads us to the idea of supersymmetric grand unification. In supersymmetric grand unified theories the soft terms which break supersymmetry are introduced at some large scale (the GUT scale), from where they evolve through renormalization group equations (RGEs) to the electroweak scale \cite{23}. The values of the soft terms so evolved to the weak scale are then used to make predictions for the masses of the superpartners of the SM particles. However, this leaves the question of underlying grand unified gauge group open. In this paper we consider the question whether the underlying grand unified gauge group leaves its imprint on the superpartner masses. This question is of great importance as the Large Hadron Collider (LHC) is expected to start operating within the next couple of years and is likely to discover the supersymmetric partners of the SM particles. Furthermore, there is a possibility of an International Linear Collider (ILC) being constructed, where a precision study of the properties of these states is likely to become a reality \cite{24}. It is, therefore, important to consider what may be learnt from measurement of superpartner masses about the underlying supersymmetric grand unified theory. This question of reconstruction of the underlying parameters \cite{25,26,27,28} of a supersymmetric theory from a measurement of superpartner masses and then using these parameters to distinguish between different underlying grand unified gauge groups is precisely the one which can be addressed in the framework of renormalization group evolution of these parameters. This is because the underlying grand unified gauge group leaves its imprint on the sparticle spectrum through the boundary conditions at the grand unified scale which serve as an input to the RG evolution of these parameters. Thus, by measuring the masses of the sparticles, and hence reconstructing the supersymmetric parameters at the large scale from these masses, we can determine the GUT gauge group of the underlying supersymmetric grand unified theory, and its breaking pattern to the SM gauge group.

In this paper we consider the sparticle spectrum in supersymmetric grand unified theories in detail in order to determine the parameters of the underlying theory at the large GUT scale, which can then be used to determine the underlying grand unified gauge group. We point out at the outset that this analysis is done by analytically integrating the one-loop renormalization group equations with boundary conditions appropriate to the grand unified gauge group. However, we note that two- and three-loop contributions to the renormalization group equations may have significant impact on the results based on the one-loop renormalization group equations. In addition shifts from the DR to the on-shell scheme, as well as theoretical uncertainties may also affect the relations between the high-scale parameters and the physical mass spectrum. Moreover, the uncertainties in the SUSY breaking scale as well as the experimental uncertainties on the sparticle mass spectrum will affect the determination of the underlying parameters. However, results obtained on the basis of analytical solutions of the one-loop renormalization, which are possible only at the one-loop level, are physically transparent, and can serve as a basis for a more precise numerical analysis.

We recall that the SM gauge group can be embedded into a larger gauge group, where an entire SM generation can be fitted into a single (ir)reducible representation of the underlying gauge group. Indeed, there is chain of group embeddings \cite{29} of the SM gauge group into a larger group $SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5) \subset SO(10) \subset E_6 \subset E_7 \subset E_8$. However, in four-dimensional grand unified theories the gauge groups $E_7$ and $E_8$ do not support a chiral structure of the weak interactions, and hence cannot be used as grand unified gauge groups in four dimensions. This leaves out only the three groups, $SU(5)$, $SO(10)$, and $E_6$ as possible unified gauge groups in four dimensions. In this paper we shall consider grand unified supersymmetric theories based on the gauge groups $SU(5)$ and $SO(10)$.

As pointed above, minimalism need not lead to realistic models. In grand unified theories there can be departures from the minimalism that is assumed in the minimal supersymmetric standard model. The nonminimalism can arise because of the boundary conditions at the grand unified scale. This manifests itself in the form of non-universal soft scalar masses at the scale of the breaking of the grand unified gauge group. For example, in the $SU(5)$ supersymmetric grand unified theory, since the fermions (and the sfermions) belong to the $\text{10}$, and $\bar{5}$ representations, respectively, the soft scalar masses are different for fermions belonging to these representations. On the other hand in the $SO(10)$ unification, although all the fermions (and the sfermions) belong to a single $\text{16}$-dimensional representation of the gauge group, the boundary conditions for soft scalar masses are non-universal because of the $D$-term contributions to these masses at the GUT scale. These $D$-term contributions to the soft scalar masses arise because the rank of $SO(10)$ is higher than SM gauge group. In general $D$–term contributions to the SUSY breaking soft scalar masses arise whenever a gauge symmetry is spontaneously broken with a reduction of rank \cite{30}. These $D$–term contributions can have important phenomenological consequences at low energies as they allow one to reach certain regions of parameter space which are not otherwise accessible with universal boundary conditions \cite{31,32,33}. In particular, these $D$-term contributions are likely to help distinguish between different scenarios for breaking of grand unified
gauge group to the Standard Model gauge group $SU(5)$.

Another source of departure from the universality can arise in the gaugino sector. Gaugino masses arise from higher dimensional interaction terms which involve gauginos and auxiliary parts of chiral superfields in a given supersymmetric model. For example, in the $SU(5)$ supersymmetric grand unified theory the auxiliary part of a chiral superfield in these higher dimensional interaction terms can be in the representation $1, 24, 75$, or $200$, or some combination of these, of the $SU(5)$ gauge group. If the auxiliary field of one of the $SU(5)$ nonsinglet chiral superfields obtains a vacuum expectation value (VEV), then the gaugino masses are not universal at the grand unification scale. Since the phenomenology of supersymmetric models depends crucially on the composition of neutralinos and charginos, it is important to investigate the changes in the experimental signals for supersymmetry with the changes in the composition of neutralinos and charginos that may arise because of the changes in the underlying boundary conditions at the grand unification scale. In this paper we shall investigate the implications of the non-universal gaugino masses, as they arise in $SU(5)$ and $SO(10)$ grand unified theories, on the neutralino and chargino mass spectrum.

The plan of this paper as follows. In section II we consider the renormalization group evolution of the sfermion masses of the first- and second-generation in the case where the soft masses are universal at the high scale, and then consider the evolution of these masses in $SU(5)$ and $SO(10)$ grand unified theories where these masses are non-universal. We show how the determination of the sfermion masses can be used to determine the soft parameters of the sfermion sector. The determination of these parameters can then be used to distinguish between different underlying supersymmetric grand unified theories. We then consider interrelationships between the squark and slepton masses in supersymmetric grand unified theories, and shown how these can be used to distinguish between different breaking patterns of a grand unified group to the SM gauge group. In section III we consider the non-universality that can arise in the gaugino sector of a supersymmetric grand unified theory, and its implications for the interrelationships between squark and slepton masses. In section IV we consider the effect of gaugino non-universality on the neutralino and chargino mass spectrum. Finally, in section V we consider the interrelationships between sfermion masses of the third generation with universal boundary conditions, as well as in grand unified theories with non-universal soft masses. We conclude with a summary in section VI. In Appendix A and Appendix B we summarize the renormalization group equations and their solutions for the general case which we use in our paper.

## II. SPARTICLE SPECTROSCOPY IN GRAND UNIFIED THEORIES

In this Section we shall consider the sparticle spectrum in grand unified theories. More specifically, we shall consider the case when the underlying grand unified gauge group is either $SU(5)$ or $SO(10)$. With a given gauge group, the sparticle masses are obtained by the renormalization group evolution of the soft supersymmetry breaking mass parameters from the GUT scale to the weak scale, with the boundary conditions on these parameters at the GUT scale determined by the breaking pattern of the grand unified gauge group to the Standard Model gauge group. The underlying GUT group leaves its imprint on the sparticle spectrum through these boundary conditions.

### A. Sfermion masses for first- and second-generation

The renormalization group equations for the soft supersymmetry breaking squark and slepton mass parameters, the Higgs mass parameters, the trilinear couplings, and the Yukawa couplings are well known and are reproduced in the Appendix A. The renormalization group equations for the soft supersymmetry breaking mass parameters from the GUT scale to the weak scale, with the boundary conditions on these parameters at the GUT scale determined by the breaking pattern of the grand unified gauge group to the Standard Model gauge group. The physical masses of the sfermions of the light generations, which we shall denote by $M_{\tilde{g}}, M_{\tilde{q}}$ are easy to obtain. These can be written as

$$ M_{\tilde{u}}^2 = m_{\tilde{u}}^2(t_G) + C_3(M_{\tilde{u}}) + C_2(M_{\tilde{u}}) + \frac{1}{36} C_1(M_{\tilde{u}}) + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) - \frac{1}{5} K, $$

$$ M_{\tilde{d}}^2 = m_{\tilde{d}}^2(t_G) + C_3(M_{\tilde{d}}) + C_2(M_{\tilde{d}}) + \frac{1}{36} C_1(M_{\tilde{d}}) + \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) - \frac{1}{5} K, $$

$$ M_{\tilde{e}}^2 = m_{\tilde{e}}^2(t_G) + C_3(M_{\tilde{e}}) + \frac{4}{9} C_1(M_{\tilde{e}}) + \frac{2}{3} \sin^2 \theta_W M_Z^2 \cos(2\beta) + \frac{4}{5} K, $$

where $\theta_W$ is the weak mixing angle, $K$ is the top mass parameter, and $C_i$ are the coefficients of the Planck scale running.

### B. First- and second-generation sfermions

In grand unified theories the auxiliary part of a

$$ (\tilde{1}, \tilde{2}, \tilde{3}) $$

or the

$$ (\tilde{1}, \tilde{2}, \tilde{3}) $$

The seven third-generation sfermions will be split from the other two generations by the effects of the Yukawa couplings. The physical masses of the sfermions of the light generations, which we shall denote by $M_{\tilde{g}}, M_{\tilde{q}}$ are easy to obtain. These can be written as

$$ M_{\tilde{u}}^2 = m_{\tilde{u}}^2(t_G) + C_3(M_{\tilde{u}}) + C_2(M_{\tilde{u}}) + \frac{1}{36} C_1(M_{\tilde{u}}) + \left(\frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) - \frac{1}{5} K, $$

$$ M_{\tilde{d}}^2 = m_{\tilde{d}}^2(t_G) + C_3(M_{\tilde{d}}) + C_2(M_{\tilde{d}}) + \frac{1}{36} C_1(M_{\tilde{d}}) + \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) - \frac{1}{5} K, $$

$$ M_{\tilde{e}}^2 = m_{\tilde{e}}^2(t_G) + C_3(M_{\tilde{e}}) + \frac{4}{9} C_1(M_{\tilde{e}}) + \frac{2}{3} \sin^2 \theta_W M_Z^2 \cos(2\beta) + \frac{4}{5} K, $$

where $\theta_W$ is the weak mixing angle, $K$ is the top mass parameter, and $C_i$ are the coefficients of the Planck scale running.
\[ M_{d_R}^2 = m_{d_R}^2(t_G) + C_3(M_{d_R}) + \frac{1}{9} C_1(M_{d_R}) - \frac{1}{3} \sin^2 \theta_W M_Z^2 \cos(2\beta) - \frac{2}{5} K, \quad (4) \]

\[ M_{\tilde e_L}^2 = m_{\tilde e_L}^2(t_G) + C_2(M_{\tilde e_L}) + \frac{1}{4} C_1(M_{\tilde e_L}) + \left( - \frac{1}{2} + \sin^2 \theta_W \right) M_Z^2 \cos(2\beta) + \frac{3}{9} K, \quad (5) \]

\[ M_{\tilde \nu_L}^2 = m_{\tilde \nu_L}^2(t_G) + C_2(M_{\tilde \nu_L}) + \frac{1}{4} C_1(M_{\tilde \nu_L}) + \frac{1}{2} M_Z^2 \cos(2\beta) + \frac{3}{9} K, \quad (6) \]

\[ M_{\tilde e_R}^2 = m_{\tilde e_R}^2(t_G) + C_1(M_{\tilde e_R}) - \sin^2 \theta_W M_Z^2 \cos(2\beta) - \frac{6}{5} K, \quad (7) \]

where \( \tan \beta = \frac{v_u}{v_d}, \frac{v_e}{v_d} \) and \( v_d \) being the vacuum expectation values of the two Higgs doublets of the minimal supersymmetric standard model, and where \( C_1, C_2 \) and \( C_3 \) are given by

\[ C_i(t) = \frac{a_i}{2 \pi^2} \int_{\alpha}^{t_G} dt g_i(t)^2 M_i(t)^2, \quad i = 1, 2, 3, \]

\[ a_1 = \frac{3}{5}, \quad a_2 = \frac{3}{4}, \quad a_3 = \frac{4}{3}. \quad (9) \]

and

\[ K = \frac{1}{16 \pi^2} \int_{\alpha}^{t_G} g_i^2(t) S(t) \ dt = \frac{1}{2 b_1} \left[ S(t) - S(t_G) \right], \quad (10) \]

is the contribution of the non-universality parameter \( S \) to the sfermion masses. The non-universality parameter \( S \) is given by Eqs. (A13) – (A15) of Appendix A and \( b_1 = -33/5 \). More explicitly

\[ C_1(t) = \frac{2}{11} \frac{M_I^2(t_G)}{\alpha_I^2} \left[ \alpha_I^2(t_G) - \alpha_1^2(t_G) \right] = M_I^2(t_G) \alpha_1(t), \quad (11) \]

\[ C_2(t) = \frac{3}{2} \frac{M_2^2(t_G)}{\alpha_2^2} \left[ \alpha_2^2(t_G) - \alpha_2^3(t_G) \right] = M_2^2(t_G) \alpha_2(t), \quad (12) \]

\[ C_3(t) = \frac{8}{9} \frac{M_3^2(t_G)}{\alpha_3^2} \left[ \alpha_3^2(t_G) - \alpha_3^3(t_G) \right] = M_3^2(t_G) \alpha_3(t). \quad (13) \]

The sfermion masses (11) - (17) have contributions coming from different sources. First, there is the contribution coming from the mass at the large (GUT) scale denoted by \( m_{\tilde q_{L,R}}, m_{\tilde u_{L,R}} \). Second, there is a contribution from the renormalization group (RG) running of scalar masses down to the experimental scale. Third, the contribution coming from \( D^2 \) term in the scalar potential, which is proportional to \( M_Z^2 \). Finally, there is the contribution from the non-universality of the sfermion masses, which is proportional to \( K \). The contributions coming from corresponding quark and lepton masses is completely negligible for all the sfermions of the light generations, as is the contribution from the sfermion mixings. We note that in arriving at (11) – (17), we have integrated the RGEs to the physical sparticle masses at the initial GUT scale:

\[ m_{Q_R}^2(t_G) = m_{\tilde q_R}^2(t_G) = m_{\tilde q_L}^2(t_G) = m_{\tilde e_R}^2(t_G) = m_{\tilde e_L}^2(t_G) = m_0^2, \quad (15) \]

\[ M_I^2(t_G) = M_R^2(t_G) = M_2^2(t_G) = M_3^2(t_G) = M_{1/2}^2, \quad (16) \]

for which \( S = K = 0 \). In this case all the sfermion masses can be expressed in terms of three parameters only. These are \( m_0^2, M_{1/2}^2 \) and \( \cos(2\beta) \). These three parameters can be determined from a measurement of three of the sfermion masses, say, for example, \( \tilde u_L, \tilde d_L \), and \( \tilde e_R^\ast \). We have from Eqs. (11) - (17)

\[ m_0^2 = \frac{1}{X_U} \left[ 3(c_{\tilde d_L} + c_{\tilde u_L}) M_{\tilde e_R}^2 - 3 c_{\tilde e_R} (M_{\tilde d_L}^2 + M_{\tilde u_L}^2) \right. \]

\[ + 2(-2 c_{\tilde d_L} M_{\tilde e_R}^2 - c_{\tilde u_L} (3 M_{\tilde d_L}^2 + M_{\tilde e_R}^2) + 3 c_{\tilde e_R} M_{\tilde u_L}^2 + c_{\tilde e_R} (2 M_{\tilde d_L}^2 + M_{\tilde u_L}^2)) \sin^2 \theta_W \right], \quad (17) \]

\[ M_{1/2}^2 = \frac{1}{X_U} \left[ 3(M_{\tilde d_L}^2 - 2 M_{\tilde e_R}^2 + M_{\tilde u_L}^2) + 2(M_{\tilde d_L}^2 + 3 M_{\tilde e_R}^2 - 4 M_{\tilde u_L}^2) \sin^2 \theta_W \right], \quad (18) \]

\[ \cos(2\beta) = -\frac{1}{X_U M_Z^2} \left[ 6 c_{\tilde u_L} (M_{\tilde d_L}^2 - M_{\tilde e_R}^2) + c_{\tilde e_R} (M_{\tilde d_L}^2 - M_{\tilde u_L}^2) + c_{\tilde d_L} (M_{\tilde e_R}^2 - M_{\tilde u_L}^2) \right]. \quad (19) \]
where

\begin{align*}
X_U &= 3(c_{dL} - 2c_{eR} + c_{uL}) + 2(c_{dL} + 3c_{eR} - 4c_{uL}) \sin^2 \theta_W, \\
c_{dL} &= \tau_3(M_{dL}) + \tau_2(M_{dL}) + \frac{1}{36} \tau_1(M_{dL}), \\
c_{uL} &= \tau_3(M_{uL}) + \tau_2(M_{uL}) + \frac{1}{36} \tau_1(M_{uL}), \\
c_{eR} &= \tau_1(M_{eR}). \\
\end{align*}

Measurement of the remaining four doubly degenerate sparticle masses overdetermines the unknown parameters. This allows for a check of the consistency of underlying assumption of universal soft mass parameters. If the seven doubly degenerate sparticle masses cannot be fit with three parameters, then new physics beyond that of universal soft masses at the large scale must be invoked.

In Fig. 1 we plot the parameters \(m_0, M_{1/2} \) and \(\cos(2\beta)\) for the case of universal boundary conditions (15) and (16) as a function of sparticle masses. In this Fig. we have taken \(M_{dL} \) fixed at 0.5 TeV, and varied \(M_{\tilde{u}_R}\) in a range dictated by the requirement that \(\cos(2\beta)\) lies in the range of \(-1 \leq \cos(2\beta) \leq 0\) (we take \(\tan \beta \geq 1\)). The mass of the selectron \(M_{\tilde{e}_R}\) is varied over a reasonable range for purposes of illustration. We have chosen what we consider to be a typical set of values. It may be observed that the effects due to \(\cos(2\beta)\) leads to a very small splitting between the \(u_L\) and \(d_L\) squarks and there is no known mechanism to lift the near degeneracy any further. The degeneracy would be exact here, and in the case of non-universal boundary conditions which are to be discussed later, but for electroweak symmetry breaking effects. Clearly larger values of \(M_{\tilde{e}_R}\) lead to larger values of \(m_0\), while they lead to smaller values of \(M_{1/2}\) for the other two squark masses held fixed, which is dictated by the renormalization group flow. Once these three parameters are determined from the measurements of these three sparticle masses, they may be inserted back into the expressions for the other sparticle masses and compared with the experimental values of these other masses. The predictions of universal boundary conditions may be readily studied in this manner. We will see, however, in the next section that the discussion becomes significantly more complicated for the case of non-universal boundary conditions which occur in grand unified theories.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The parameters of the supersymmetric standard model with universal boundary conditions as a function of the sparticle masses. The first frame shows the universal scalar mass parameter \(m_0\) as a function of \(M_{\tilde{u}_L}\) and \(M_{\tilde{e}_R}\), with \(M_{dL} = 0.5\). All masses are in TeV. The second frame shows \(M_{1/2}\) as a function of sparticle masses. All parameters are as in the first frame. The third frame shows \(\cos(2\beta)\) as a function of sparticle masses, with parameters as in the first plot.}
\end{figure}

1. The SU(5) supersymmetric grand unified theory

Supersymmetric models with universal squark and slepton mass parameters at the large scale are economic in terms of new parameters, but could to some extent be unrealistic. One of the realistic models which goes beyond the assumption of universality of soft scalar masses is the SU(5) supersymmetric GUT. Since \(Q_L, u_R, e_R\) all lie in a 10-dimensional representation, and \(L, d_R\) lie in a 5 representation of SU(5), the boundary conditions for the soft masses for squarks and sleptons can be written as

\begin{align*}
m_{Q_L}^2(t_G) &= m_{u_R}^2(t_G) = m_{e_R}^2(t_G) = m_{10}^2, \\
m_{LL}^2(t_G) &= m_{d_R}^2(t_G) = m_{5}^2.
\end{align*}
Here \( m^2_{10} \) and \( m^2_{1} \) are the common squared soft scalar masses corresponding to the \( 10 \)- and \( \overline{5} \)-dimensional representations, respectively of \( SU(5) \), at the unification scale. For the \( SU(5) \) supersymmetric grand unified theory, the universal gaugino mass condition \( \text{10} \) continues to hold. We note that the two Higgs doublets of the minimal supersymmetric standard model belong to two different representations of \( SU(5) \), and as such their soft parameters are unrelated to each other. Since, we don’t need the Higgs mass parameters in our calculations, we do not write these explicitly.

The boundary conditions \( \text{24} \) and \( \text{25} \) are valid for each generation. Since there is no boundary condition relating masses of particles in different generations, we have four input parameters, namely, \( m^2_{\overline{5}}, m^2_{10}, M_{1/2}, \) and \( \cos(2\beta) \), in terms of which the squark and slepton masses of each light generation can be computed. These input parameters, together with the non-universality parameter \( K \), can be determined by a measurement of five sfermion masses. Taking these to be \( \tilde{u}_L, \tilde{d}_L, \tilde{d}_R, \tilde{u}_R, \tilde{e}_R \), we have from Eqs. \( \text{11} \) - \( \text{17} \)

\[
m^2_{\overline{5}} = \frac{1}{5X_5} \left[ -c_{\alpha R}(M^2_{d_L} + 5M^2_{\tilde{d}_R} + 2M^2_{\tilde{d}_R} + M^2_{\tilde{u}_L}) + c_{\alpha R}(M^2_{d_L} + 5M^2_{\tilde{d}_R} + M^2_{\tilde{u}_L} - 2M^2_{\tilde{u}_R}) \\
-5c_{\alpha R}(M^2_{d_L} - M^2_{\tilde{d}_R} + M^2_{d_L} - M^2_{\tilde{u}_R}) + (c_{d_L} + c_{\alpha L})(5M^2_{d_R} - M^2_{\tilde{e}_R} + M^2_{\tilde{u}_R}) \right],
\]

(26)

\[
m^2_{10} = \frac{1}{3X_5} \left[ c_{\alpha R}(-3M^2_{d_L} + 2M^2_{d_L} - 3M^2_{\tilde{u}_L}) - c_{\alpha R}(2(M^2_{d_L} + M^2_{\tilde{u}_L} + M^2_{\tilde{u}_R}) + (c_{d_L} + c_{\alpha L})(2M^2_{\tilde{e}_R} + 3M^2_{\tilde{u}_R}) \right],
\]

(27)

\[
M^2_{1/2} = \frac{1}{X_5} \left[ M^2_{d_L} - M^2_{\tilde{d}_R} + M^2_{d_L} - M^2_{\tilde{u}_R} \right],
\]

(28)

\[
\cos(2\beta) = \frac{1}{X_5M^2_{\tilde{u}_R}(\sin^2\theta_W - 1)} \left[ c_{\alpha R}(M^2_{\tilde{u}_L} - M^2_{\tilde{d}_L}) + c_{\alpha R}(M^2_{\tilde{u}_L} - M^2_{\tilde{d}_L}) + c_{d_R}(M^2_{\tilde{e}_R} - 2M^2_{\tilde{u}_L} + M^2_{\tilde{u}_R}) \right],
\]

(29)

where

\[
X_5 = c_{d_L} - c_{\tilde{e}_R} + c_{\tilde{u}_L} - c_{\tilde{u}_R},
\]

(30)

\[
c_{\alpha R} = \tau_3(M_{\tilde{u}_R}) + \frac{4}{9}\tau_3(M_{\tilde{u}_R}),
\]

(31)

\[
c_{d_R} = \tau_3(M_{\tilde{d}_L}) + \frac{1}{9}\tau_3(M_{\tilde{d}_R}),
\]

(32)

Since the two Higgs doublet superfields \( H_d \) and \( H_u \) of the MSSM lie in different representations of \( SU(5) \), \( M^2_{H_u}(t_G) \) and \( M^2_{H_d}(t_G) \) are not equal. Thus, the parameter

\[
S(t_g) = m^2_{H_u}(t_g) - m^2_{H_d}(t_g),
\]

(33)

is nonzero in this case. This parameter, or equivalently the parameter \( K \), can be expressed in terms of the sfermion masses as

\[
K = \frac{1}{6X_5(\sin^2\theta_W - 1)} \left[ 3c_{\alpha R}(M^2_{d_L} - 2M^2_{\tilde{d}_R} + M^2_{\tilde{u}_L}) + 3(c_{d_L} + c_{\alpha L})(M^2_{d_R} - M^2_{\tilde{u}_R}) \\
+ 2(c_{\alpha R}(-3M^2_{d_L} + 2M^2_{d_L} - 3M^2_{\tilde{u}_L}) - c_{d_R}(4M^2_{d_L} - M^2_{\tilde{e}_L} + M^2_{\tilde{u}_L}) + c_{\alpha L}(-5M^2_{d_L} + M^2_{\tilde{e}_L} + 4M^2_{d_R}) \sin^2\theta_W \\
+ c_{e_R}(-3M^2_{d_L} + 2M^2_{\tilde{d}_L} - 2M^2_{\tilde{u}_L} - 2(-4M^2_{d_L} + M^2_{\tilde{u}_L} + 3M^2_{\tilde{u}_R}) \sin^2\theta_W) \right].
\]

(34)

In Fig. \( \text{2} \) we plot the parameters \( m_{\overline{5}}, m_{10}, M^2_{1/2}, K \) and \( \cos(2\beta) \). We have taken \( M_{\tilde{d}_L} = 0.55 \) TeV and varied \( M_{\tilde{u}_R} \) in a range dictated by the requirement that \( \cos(2\beta) \) lies in the range of \(-1 \leq \cos(2\beta) \leq 0 \) as before. The mass of the selectron \( M_{\tilde{e}_R} \) is varied over a reasonable range for purposes of illustration. Note that in the case of universal boundary conditions we would expect to see the right handed squarks to be somewhat lighter than the left handed squarks, and also that in that case exact degeneracy would not be possible. Here it would be possible to have exact degeneracy of the states due to the interplay between the non-universality parameter contributions balancing the electroweak contribution. Such a possibility, although highly simplified makes our discussion simple and is useful for purposes of illustration. We have chosen what we consider to be a typical set of values. It may be observed that the effects due to \( \cos(2\beta) \) leads to a very small splitting between the \( u_L \) and \( d_L \) squarks and there is no mechanism to lift the near degeneracy any further. It may be from our expressions that a spectrum of the sort depicted here requires a large splitting between \( m_{\overline{5}} \) and \( m_{10} \), whereas \( M_{1/2} \) remains relatively less sensitive.
As discussed above, the simplest grand unified theory which results in non-universal masses for the sfermions is $SU(5)$ GUT. The rank of $SU(5)$ is the same as that of SM gauge group. On the other hand SM can be embedded into a larger gauge group like $SO(10)$. However, since the rank of $SO(10)$ is higher than that of SM gauge group, the breaking of $SO(10)$ to the SM gauge group involves the reduction of rank by one unit. There is an additional $U(1)$ factor beyond that of the Standard Model, which must be broken. Thus, in the case of $SO(10)$ unified theory there will be $D$-term contributions to the soft supersymmetry breaking scalar masses. These $D$-term contributions have important phenomenological consequences at low energies as these allow one to reach regions of parameter space which are not otherwise accessible with universal boundary conditions. These contributions can help distinguish between different scenarios for breaking of grand unified symmetry at high energies.

The $D$-term contributions to the soft scalar masses will depend on the manner in which $SO(10)$ gauge group is broken to the SM gauge group. When $SO(10)$ breaks via its maximal subgroup $SU(5) \times U(1)_Z$, with $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_X$, there are two possibilities for the hypercharge generator of the SM gauge group. In the “conventional” embedding via $SU(5)$, the hypercharge generator $Y$ of the SM is identified with the generator $X$ of $U(1)_X$. On the other hand, in the “flipped” embedding the hypercharge generator is identified with a linear combination of the generators $X$ and $Z$.

Apart from the “natural” subgroup $SU(5) \times U(1)$, the group $SO(10)$ also has “natural” subgroup $SO(6) \times SO(4)$. Since $SO(6)$ is isomorphic to $SU(4)$, and $SO(4)$ is isomorphic to $SU(2) \times SU(2)$, $SO(10)$ contains $SU(4) \times SU(2) \times SU(2)$. We shall focus on the signatures of the $SO(10)$ breaking to the SM via its two natural subgroups, and try to find distinguishing features of the sparticle spectrum in the two cases.

As in the case of $SU(5)$ GUT, the solutions of the RG equations for the soft scalar masses involve the values of these masses at the initial GUT scale. These initial values will be determined by the pattern of the breaking of the grand unified group to the SM gauge group. For the case of breaking of $SO(10)$ to the Standard Model gauge group via its maximal subgroup $SU(5) \times U(1)$, these initial values are given by \cite{32, 33, 49, 50, 51}:

\begin{align}
    m^2_{QL} (t_G) &= m^2_{dR} (t_G) = m^2_{tR} (t_G) = m^2_{L_1} + g^2_{10} D,
    \\
    m^2_{LL} (t_G) &= m^2_{dR} (t_G) = m^2_{L_2} - 3g^2_{10} D,
\end{align}

FIG. 2: The parameters of $SU(5)$ supersymmetric grand unified theory as a function of sparticle masses. The first frame shows the plot of $m_\tilde{G}$ as a function of $M_{\tilde{G}_L}$ and $M_{\tilde{G}_R}$, with $M_{\tilde{G}_L} = 0.5, M_{\tilde{G}_R} = M_{\tilde{G}_R} = 0.55$, with all masses in TeV. The second frame shows the plot of $m_{10}$, whereas the third, fourth, and fifth frames show the variations of $M_{1/2}$, $K$, and $\cos(2\beta)$ as a function of sparticle masses. All parameters are as in the first frame.
\[ m_{H_u}(t_G) = m_{16}^2 - 2g_{10}^2D, \]
\[ m_{H_d}(t_G) = m_{10}^2 + 2g_{10}^2D. \]

at the SO(10) breaking scale \( M_G \), where the normalization and sign of \( D \) is arbitrary. Here \( m_{16} \) and \( m_{10} \) are the common soft scalar masses, corresponding to the 16– and 10– dimensional representations, respectively of SO(10), at the unification scale, and \( g_{10} \) is the SO(10) gauge coupling. We note here that in the breaking of SO(10) the rank is reduced by one, and hence the \( D \)-term contribution to the soft masses is expressed by a single parameter \( D \). We also note that \( g_{10} \) and \( D \) enter the boundary conditions (35) – (38) in the combination \( g_{10}^2D \), and, therefore, constitute only one parameter. Thus, in the case of SO(10) grand unified theory, there are four input parameters, the same as in the case of SU(5) grand unified theory, which determine the light sfermion sector. We can take these to be \( m_{16}^2 \), \( g_{10}^2D \), \( M_{1/2} \), and \( \cos(2\beta) \). In addition there is the non-universality parameter \( K \). Using the solutions (1) – (7) of the RG equations, these parameters can be determined in terms of squark and slepton masses as

\[
m_{16}^2 = \frac{1}{4X_5} \left[ -c_{a_R}(2M_{d_L}^2 + M_{d_R}^2 - M_{e_R}^2 + 2M_{\tilde{e}_L}^2) + c_{\tilde{e}_R}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2) - c_{d_R}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2) + (c_{d_L} - c_{\tilde{e}_R})(M_{d_L}^2 + M_{e_R}^2 + 2M_{\tilde{e}_L}^2) \right] - \frac{c_{d_R}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2) - c_{d_L}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2)}{4X_5},
\]
\[
g_{10}^2D = \frac{1}{20X_5} \left[ -c_{a_R}(2M_{d_L}^2 - 5M_{d_R}^2 + M_{e_R}^2 + 2M_{\tilde{e}_L}^2) + c_{d_R}(5M_{d_R}^2 - 3M_{e_R}^2 - 2M_{\tilde{e}_L}^2) - c_{d_L}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2) + 5c_{\tilde{e}_R}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2) + 5c_{d_L}(M_{d_L}^2 + M_{e_R}^2 - M_{d_R}^2 - M_{\tilde{e}_L}^2) \right].
\]

All other quantities are same as in the SU(5) case.

In Fig. 3 we show the variation of the SO(10) parameters \( m_{16} \) and \( g_{10}^2D \) as a function of sfermion masses. The rest of the parameters \( M_G \), \( \cos(2\beta) \) and \( K \) for SO(10), and are not shown here.

It is important to note that once \( m_{16}^2 \) and \( g_{10}^2D \) are obtained, the boundary conditions for all the squark and slepton soft masses are fixed through (35) – (38). The masses of all the first and second generation squarks can then be obtained from the solutions (1) – (7) of the RG equations.

Another important aspect of (35) – (38) is that the boundary condition for the difference of the Higgs mass parameters is given by \( S(t_G) = m_{H_u}(t_G) - m_{H_d}(t_G) = -4g_{10}^2D \). This quantity can, thus, be determined from the measurement of sfermion masses of the light generations through the relation (40). This implies that the value of the difference of the Higgs mass parameters at the electroweak scale, which is crucial for the electroweak symmetry breaking, can be obtained from the difference of the RG equations (A7) and (A8) with the boundary condition determined by the observed masses of the sfermions through the relation (40). This is different from what happens in SU(5), where the difference of the Higgs mass parameters at the electroweak scale cannot be obtained in terms of the sfermion masses alone.

We now come to the case of SO(10) breaking via its other maximal subgroup SO(10) \( \supset SU(4)_{PS} \times SU(2)_L \times SU(2)_R \). As in the case of breaking via the \( SU(5) \times U(1) \) subgroup, there appear \( D \)-term contributions to the soft scalar masses.
when the rank of the gauge group reduces from 5 to 4 at the intermediate Pati-Salam symmetry breaking scale $M_{PS}$. For this case of breaking, the initial values of soft masses are given by \[49, 50\]

\[
\begin{align*}
m^2_{Q_L}(M_{PS}) &= m^2_L + g_L^2 D, \\
m^2_{d_R}(M_{PS}) &= m^2_R - (g_L^2 - 2g_{2R}) D, \\
m^2_{\tilde{e}_R}(M_{PS}) &= m^2_R + (3g_L^2 - 2g_{2R}) D, \\
m^2_{L_L}(M_{PS}) &= m^2_L - 3g_L^2 D, \\
m^2_{\tilde{d}_R}(M_{PS}) &= m^2_R - (g_L^2 + 2g_{2R}) D,
\end{align*}
\]

at the Pati-Salam breaking scale. Here $D$ represents the $D$-term contributions whose normalization is arbitrary. We note that the boundary conditions \([41]-[45]\) do not depend on a particular choice of the Higgs representation which breaks the Pati-Salam group, but is fixed only by the symmetry breaking pattern. Here, $m_L, m_R$ are the soft masses corresponding to the $SU(2)_L, SU(2)_R$ gauge groups, and $g_4, g_{2R}$ are the $SU(4)_P, SU(2)_R$ gauge couplings, respectively. Furthermore, the initial soft sfermion masses are parametrized by four parameters, which we take to be $m^2_L, m^2_R, g_4^2 D$, and $g_{2R}^2 D$ respectively. Also the gauge coupling $g_4^2, g_{2R}^2$ can be determined from the low-energy gauge coupling $\alpha_i(m_Z)$ $(i = 1, 2, 3)$ as a function of $M_{PS}$ alone.

The four parameters $m^2_L, m^2_R, g_4^2 D$, and $g_{2R}^2 D$ together with $M_{1/2}$, $K$ and $\cos(2\beta)$ constitute a set of seven parameters in terms of which the light sfermion mass spectrum can be determined. However, the system of equations which determine these seven parameters in terms of seven sparticles masses is degenerate and hence no solution exists. Thus, in this case we cannot determine the underlying parameters in terms of the sfermion masses alone.

### B. Sum Rules

The fact that the renormalization group equations for the soft mass parameters for the first two generations can be solved analytically allows us to determine the parameters of the underlying supersymmetric theory. This can also allow us to distinguish between different grand unified gauge groups. In this subsection we show that in grand unified theories, the squark and slepton masses obey certain relations, which follow from the renormalization group evolution and the boundary conditions at the GUT scale.

For the case of non-universal boundary conditions for the scalar masses which obtain in both $SU(5)$ and $SO(10)$ models, we eliminate $m_{\tilde{q}}, m_t$ from the solutions of the renormalization group equations of the soft scalar masses \([11]-[7]\) to obtain the following two sum rules for the sfermion masses:

\[
\begin{align*}
2M^2_Q - M^2_{d_R} - M^2_{\tilde{e}_R} &= (C_3 + 2C_2 - \frac{25}{18} C_1), \\
M^2_Q + M^2_{\tilde{d}_R} - M^2_{\tilde{e}_R} - m^2_{L} &= (2C_3 - \frac{10}{9} C_1),
\end{align*}
\]

where we have used the notation

\[
M^2_Q = \frac{1}{2}(M^2_{3L} + M^2_{3L}), \quad M^2_L = \frac{1}{2}(M^2_{\tilde{e}_L} + M^2_{\nu_L}).
\]

These sum rules are valid for $SU(5)$ supersymmetric grand unified theory. Although, the rank of $SO(10)$ gauge group is larger than the SM (and $SU(5)$), these sum rules are also valid in a supersymmetric $SO(10)$ grand unified theory when $SO(10)$ breaks to the SM gauge group via its maximal $SU(5) \times U(1)$ subgroup, irrespective of whether the embedding of the SM in $SU(5) \times U(1)$ is conventional or flipped one.

On the other hand in the case of $SO(10)$ breaking via its other maximal subgroup $SO(10) \supset SU(4)_P \times SU(2)_L \times SU(2)_R$, using the boundary conditions \([11]-[15]\), we obtain the sum rule

\[
m^2_{Q_L} + M^2_{\tilde{d}_R} - M^2_{\tilde{e}_R} - m^2_{L} = (2C_3 - \frac{10}{9} C_1),
\]

which is the only sum rule valid in this case. Thus, this sum rule serves as a crucial distinguishing feature of $SO(10)$ breaking via the Pati-Salam subgroup. If both the sum rules \([16]\) and \([17]\) are seen to hold experimentally, then in the context of grand unification, the underlying gauge group is either $SU(5)$ or $SO(10)$, with the breaking of $SO(10)$ taking place via its $SU(5)$ subgroup. On the other hand, if only the sum rule \([18]\) is seen to hold experimentally, then the breaking of $SO(10)$ must take place via the Pati-Salam subgroup.
We now turn to the the right hand side of the sum rules (46) and (47) which involve the functions \(C_i\). These functions can be written in terms of quantities whose values can be inferred from experiment. In terms of the gluino mass \(M_\tilde{g} = M_3(t_\tilde{g})\), we can write \(C_3(t)\) from (14) as

\[
C_3(t) = \frac{8}{9} \frac{M_\tilde{g}^2}{\alpha_3^2(t_\tilde{g})} [\alpha_3^2(t) - \alpha_3^2(t_G)],
\]

(49)

where we have used the fact that gaugino masses run as (51) irrespective of the breaking pattern [34, 35, 49] to the Standard Model gauge group so long as the underlying gauge group is unified into a simple gauge group at a high mass scale \(M_G\). We note that (51) is a result of one-loop renormalization group equations, and does not hold at the two loop level [49]. It follows from (50) that

\[
\frac{M_i(t)}{\alpha_i(t)} = \frac{M_i(t_G)}{\alpha_i(t_G)},
\]

(50)

We then have

\[
\frac{M_1(t)}{\alpha_1(t)} = \frac{M_2(t)}{\alpha_2(t)} = \frac{M_3(t)}{\alpha_3(t)} = \frac{M_{1/2}}{\alpha_G},
\]

(51)

where \(\alpha_1(t_G) = \alpha_2(t_G) = \alpha_3(t_G) \equiv \alpha_G\) is the grand unified gauge coupling. We note that the gaugino masses always satisfy the relation (51) irrespective of the breaking pattern [34, 35, 49] to the Standard Model gauge group so long as the underlying gauge group is unified into a simple gauge group at a high mass scale \(M_G\). We note that (51) is a result of one-loop renormalization group equations, and does not hold at the two loop level [49]. It follows from (50) that

\[
M_i(t) = \alpha_i(t) \frac{M_\tilde{g}}{\alpha_3(\tilde{g})}.
\]

(52)

Using the above, we can now express the functions \(C_1\) and \(C_2\) in terms of the gluino mass and the corresponding gauge couplings. We have [51]

\[
C_1(t) = \frac{2}{11} \frac{M_\tilde{g}^2}{\alpha_3^2(t_\tilde{g})} [\alpha_3^2(t) - \alpha_3^2(t_G)],
\]

(53)

\[
C_2(t) = \frac{3}{2} \frac{M_\tilde{g}^2}{\alpha_3^2(t_\tilde{g})} [\alpha_3^2(t) - \alpha_3^2(t_G)],
\]

(54)

We note that the gluino mass in (49), (53) and (54) is the one-loop gluino mass and not the pole mass, although these are related. Using these results for \(C_i\), we can write the sum rules (46) and (47) as follows:

\[
2M_Q^2 - M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2 = \frac{M_\tilde{g}^2}{9\alpha_3^2(t_\tilde{g})} \left[ \frac{8}{9} \alpha_3^2(t) - 3\alpha_3^2(t_G) + \frac{25}{99} \alpha_3^2(t) + \frac{184}{99} \alpha_G^2 \right],
\]

(55)

\[
M_Q^2 + M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2 - M_L^2 = \frac{M_\tilde{g}^2}{9\alpha_3^2(t_\tilde{g})} \left[ \frac{16}{9} \alpha_3^2(t) + \frac{20}{99} \alpha_3^2(t) - \frac{196}{99} \alpha_G^2 \right].
\]

(56)

As before, using a supersymmetric threshold of 1 TeV, and the values \(M_G = 1.9 \times 10^{16}\) GeV, \(\alpha_G = 0.04\), \(\alpha_1(1\, TeV) = 0.0173\), \(\alpha_2(1\, TeV) = 0.0328\), \(\alpha_3(1\, TeV) = 0.091\), we can finally write our sum rules in terms of experimentally measurable masses as (at a scale of 1 TeV)

\[
2M_Q^2 - M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2 \approx 0.9M_\tilde{g}^2,
\]

(57)

\[
M_Q^2 + M_{\tilde{d}_R}^2 - M_{\tilde{e}_R}^2 - M_L^2 \approx 1.4M_\tilde{g}^2.
\]

(58)

These relations, which relate the sfermion masses to the gluon mass, are valid in \(SU(5)\) as well as \(SO(10)\) supersymmetric theories.
III. SPARTE SPECTROSCOPY WITH NONUNIVERSAL GAUGINO MASSES

In the previous section we have considered the sparticle spectrum in supersymmetric grand unified theories when the gaugino masses are universal at the GUT scale. We have shown how the parameters of the underlying supersymmetric grand unified theory can be determined from a measurement of sparticle masses. However, universal gaugino masses are a very special case that can arise in grand unified theories. In this section we shall explore the changes in the sparticle spectrum when the gaugino masses are non-universal at the GUT scale in the context of $SU(5)$ and $SO(10)$ supersymmetric grand unified theories.

A. Non-singlet chiral superfields

In grand unified supersymmetric models non-universal gaugino masses are generated by a non-singlet chiral superfield $\Phi^\alpha$ that appears linearly in the gauge kinetic function $f(\Phi)$, which is an analytic function of the chiral superfields $\Phi$ in the theory. The chiral superfields $\Phi$ are classified into a set of gauge singlet superfields $\Phi^s$, and gauge non-singlet superfields $\Phi^\alpha$, respectively under the grand unified group. If the auxiliary part $F_{\Phi}$ of a chiral superfield $\Phi$ in $f(\Phi)$ gets a VEV, then gaugino masses arise from the coupling of $f(\Phi)$ with the field strength superfield $W^\alpha$. The Lagrangian for the coupling of gauge kinetic function to the gauge field strength is written as

$$L_{g.k.} = \int d^2\theta f_{ab}(\Phi) W^a W^b + h.c.,$$

where $a$ and $b$ are gauge group indices, and repeated indices are summed over. The gauge kinetic function $f_{ab}(\Phi)$ is

$$f_{ab}(\Phi) = f_0(\Phi^s) \delta_{ab} + \sum_n f_n(\Phi^s) \frac{\Phi^\alpha_{ab}}{M_P} + \cdots,$$

where as indicated above the $\Phi^s$ and the $\Phi^\alpha$ are the singlet and the non-singlet chiral superfields, respectively. Here $f_0(\Phi^s)$ and $f_n(\Phi^s)$ are functions of gauge singlet superfields $\Phi^s$, and $M_P$ is some large scale. When $F_{\Phi}$ gets a VEV $\langle F_{\Phi} \rangle$, the interaction (59) gives rise to gaugino masses:

$$L_{g.k.} \supset \langle F_{\Phi} \rangle_{ab} \frac{\lambda^a \lambda^b}{M_P} + h.c.,$$

where $\lambda^a, b$ are gaugino fields. Note that we denote by $\lambda_1$, $\lambda_2$ and $\lambda_3$ the $U(1)$, $SU(2)$ and $SU(3)$ gauginos, respectively.

Since the gauginos belong to the adjoint representation of $SU(5)$, $\Phi$ and $F_{\Phi}$ can belong to any of the following representations appearing in the symmetric product of the two 24 dimensional representations of $SU(5)$:

$$(24 \otimes 24)_{Symm} = 1 \oplus 24 \oplus 75 \oplus 200.$$  (62)

On the other hand the corresponding symmetric product of the two 45 dimensional representations of $SO(10)$ grand unified theory has the following decomposition:

$$(45 \otimes 45)_{Symm} = 1 \oplus 54 \oplus 210 \oplus 770.$$  (63)

In the minimal, and the simplest, case $\Phi$ and $F_{\Phi}$ are assumed to be in the singlet representation of $SU(5)$ (or $SO(10)$), which implies equal gaugino masses at the GUT scale. However, as is clear from the decomposition (62), $\Phi$ can belong to any of the non-singlet representations $24$, $75$, and $200$ of $SU(5)$, in which case these gaugino masses are unequal but related to one another via the representation invariants $[36]$. In Table I we show the ratios of resulting gaugino masses at tree-level as they arise when $F_{\Phi}$ belongs to various representations of $SU(5)$, and tabulate $a$ and $b$ defined so that at the unification scale we have the ratio

$$M_1 : M_2 : M_3 = a : b : 1.$$  (64)

Similarly, in Table II we show the corresponding $a$ and $b$ for the $SO(10)$ grand unified theory. For definiteness, we shall study the case of each representation separately, although an arbitrary combination of these is also allowed.  

1 It has been pointed out [52] that non-universal scalar masses can also arise from non-singlet chiral superfields, which are subdominant.
B. Sum Rules

We are now in a position to write down interrelationships between the squarks and sfermions for the case of non-universal gaugino masses. Recalling from the case of universal gaugino masses that

$$\frac{M_i(t)}{\alpha_i(t)} = \frac{M_i(t_G)}{\alpha_i(t_G)},$$

we have for the non-universal case

$$\frac{1}{a} \frac{M_1(t)}{\alpha_1(t)} = \frac{1}{b} \frac{M_2(t)}{\alpha_2(t)} = \frac{M_3(t)}{\alpha_3(t)} = \frac{M_{1/2}}{\alpha_G},$$

where $\alpha_1(t_G) = \alpha_2(t_G) = \alpha_3(t_G) \equiv \alpha_G$ is the grand unified gauge coupling, and $a$, $b$ are as in (53). Using this, the sum rules (55) and (56) are now generalized for the case of non-universal gaugino masses to:

$$2M_Q^2 - M_{\tilde{g}_R}^2 - M_{\tilde{g}_L}^2 = \frac{M_9^2}{\alpha_3^2(t_3)} \left[ \frac{8}{9} \alpha_2^2(t_3) - 3b^2 \alpha_2^2(t) + \frac{25a^2}{99} \alpha_1^2(t) + \frac{(297b^2 - 25a^2 - 88)}{99} \alpha_2^2 \right],$$

$$M_Q^2 + M_{\tilde{g}_R}^2 - M_{\tilde{g}_L}^2 = \frac{M_9^2}{\alpha_3^2(t_3)} \left[ 16 \alpha_2^2(t) + \frac{20a^2}{99} \alpha_1^2(t) - \frac{4(5a^2 + 44)}{99} \alpha_2^2 \right].$$

Using the values for various couplings, as in the universal gaugino mass case, these sum rules can finally be written as

$$2M_Q^2 - M_{\tilde{g}_R}^2 - M_{\tilde{g}_L}^2 \simeq K_1M_9^2,$$

$$M_Q^2 + M_{\tilde{g}_R}^2 - M_{\tilde{g}_L}^2 \simeq K_2M_9^2,$$

where $K_1$ and $K_2$ are given in Table III and Table IV for $SU(5)$ and $SO(10)$, respectively.

We should point out here that the determination of the parameters of the scalar sector as described in the previous Section can be carried out in the case of non-universal gaugino masses with the replacements $\tau_1 \rightarrow a^2 \bar{c}_1$, $\tau_2 \rightarrow b^2 \bar{c}_2$, with $a$ and $b$ given in Table II and Table III.

IV. NEUTRALINOS AND CHARGINOS

As seen in the previous section, in supersymmetric theories with an underlying grand unified gauge group, the gaugino masses need not be equal at the GUT scale. This may have important consequences for the neutralino

| $F_\Phi$ | $a$ | $b$ |
|---|---|---|
| 1 | 1 | 1 |
| 24 | $-1/2$ | $-3/2$ |
| 75 | $-5$ | 3 |
| 200 | 10 | 2 |

**TABLE I:** The constants $a$ and $b$ for the various representations of SU(5).

due to inverse powers of $M_G$ and read:

$$\mathcal{L} \propto \frac{\langle F_\Phi F_\Phi \rangle_{ij}}{M_G^2} \phi_i^\dagger \phi_j,$$

where the $\phi_{i,j}$ are the scalar fields. Recalling that for SU(5), the left-handed SM fermions and their superpartners lie in $\overline{5}$ and 10-dimensional representations, while the light Higgs bosons lie in the 5 and $\overline{5}$, we need to consider the tensor products $10 \otimes \overline{10} = 1 \oplus 24 \oplus 75$ and $5 \otimes \overline{5} = 1 \oplus 24$, and deduce that for $\phi$ lying in 1 and 24 non-universality can be generated by this mechanism. For SO(10) with SM fermions lying in the 16- and the light Higgs bosons in the 10-dimensional representation, and the tensor decompositions $16 \otimes 16 = 1 \oplus 45 \oplus 210$, and $10 \otimes 10 = 1, 45, 54$, we conclude that the only instance when non-universality can be generated through this mechanism is when $\phi$ lies in the singlet representation. We do not consider this type of non-universality any further in this paper.
and chargino mass spectrum. We recall that neutralino is a much favored candidate for the lightest supersymmetric particle (LSP). Thus, its composition is of crucial importance for supersymmetric phenomenology. In this section we shall consider some aspects of neutralino and chargino phenomenology when the gaugino masses are non-universal at the grand unified scale.

We start by recalling the neutralino mass matrix in supersymmetric models in the basis

\[ \psi_j^0 = (-i\lambda', -i\lambda_3, \psi_{H_1}^1, \psi_{H_2}^2), \quad j = 1, 2, 3, 4, \]  

which can be written as \[15\]

\[ M = \begin{pmatrix}
M_1 & 0 & -M_Z \cos \beta \sin \theta_W & M_Z \sin \beta \sin \theta_W \\
0 & M_2 & M_Z \cos \beta \cos \theta_W & -M_Z \sin \beta \cos \theta_W \\
-M_Z \cos \beta \sin \theta_W & M_Z \cos \beta \cos \theta_W & 0 & -\mu \\
M_Z \sin \beta \sin \theta_W & -M_Z \sin \beta \cos \theta_W & -\mu & 0
\end{pmatrix}, \]  

(72)

where \(\lambda'\) and \(\lambda_3\) are the two-component gaugino states corresponding to the \(U(1)_Y\) and the third component of the \(SU(2)_L\) gauge groups, respectively, and \(\psi_{H_1}^1, \psi_{H_2}^2\) are the two-component Higgsino states corresponding to the two Higgs superfields \(H_1\) and \(H_2\) of the MSSM. We shall denote the eigenstates of the neutralino mass matrix by \(\lambda_{10}, \lambda_{02}, \lambda_{03}, \lambda_{04}\) labeled in order of increasing mass. Since some of the neutralino masses resulting from diagonalization of the mass matrix can be negative, we shall consider the squared mass matrix \(M^\dagger M\). Note that the masses and the compositions of neutralinos are determined by the soft supersymmetry breaking gaugino masses \(M_1\) and \(M_2\), the supersymmetric Higgs mixing parameter \(\mu\), and the ratio of the vacuum expectation values of the two neutral Higgs bosons \(H_1^0\) and \(H_2^0\), \((H_1^0)/(H_2^0) = \tan \beta\).

We now briefly consider the question of the composition of the lightest neutralino with non-universal gaugino masses at the GUT scale which follows from the neutralino mass matrix. Using the fact that the ratio \(\alpha_2/\alpha_1\) is \(\sim 2\) at the scale of 1 TeV, we conclude that for the case of \(SU(5)\), with \(F_\Phi\) lying in \(1\) and \(24\), the ratio \(M_2 \gg M_1\), and hence the lightest neutralino is mostly bino-like \[13\]. On the other hand, with \(F_\Phi\) lying in the \(75\)-dimensional representations, there are several possibilities depending on the values of the parameters. The lightest neutralino can be a bino for small values of \(M_2\), a wino for slightly larger values of \(M_2\), and a higgsino for \(M_2 \gtrsim 300\) GeV for values of \(\tan \beta \gtrsim 10\). Finally, for the \(200\)-dimensional representation of \(SU(5)\), the lightest neutralino can be either a wino of a higgsino, depending on the values of \(M_2\) and \(\tan \beta\).

For supersymetric \(SO(10)\) grand unified theory, at a scale of 1 TeV, \(M_2 \gg M_1\) for models \(A, B, C\), and hence the lightest neutralino is bino-like. For model \(D\), the magnitude of \(|a/b| \simeq 4\) and hence the lightest neutralino is no longer a bino-like state.

| Case | \(F_\Phi\) | \(a\) | \(b\) |
|------|-------------|------|------|
| A    | 1           | 1    | 1    |
| B    | 54          | \(G_{422}\) | -1   | -3/2 |
| C    | 54          | \(SU(2) \times SO(7)\) | 1    | -7/3 |
| D    | 210         | \(H_{51}\) | -96/25 | 1   |

TABLE II: The constants \(a\) and \(b\) for \(SO(10)\). Different cases are labelled as A, B, C and D, in the notation of ref. \[53\].

| \(F_\Phi\) | \(K_1\) | \(K_2\) |
|-----------|------|------|
| 1         | 0.9  | 1.4  |
| 24        | 1.1  | 1.4  |
| 75        | 1.4  | 0.6  |
| 200       | -2.5 | -1.7 |

TABLE III: The quantities \(K_1\) and \(K_2\) for different representations of \(SU(5)\).
It is useful to evaluate the upper bound on the mass of the lightest neutralino and the mass of the next to lightest neutralino in the grand unified models. We recall that a bound on the mass of the lightest neutralino can be obtained by using the fact that the smallest eigenvalue of $M^\dagger M$ is smaller than the smallest eigenvalue of its upper left $2 \times 2$ submatrix, thereby resulting in the upper bound \[54, 55\]

$$M_{\chi_1^0}^2 \leq \frac{1}{2}[M_1^2 + M_2^2 + M_Z^2 - \sqrt{(M_1^2 - M_2^2)}] + M_Z^2 - 2(M_1^2 - M_2^2)M_Z^2 \cos 2\theta_W].$$ (73)

Similarly, one can write down the upper bound on the mass of the second lightest neutralino from the structure of the neutralino mass matrix [46].

In Fig. 4 and Fig. 5, we plot the upper bounds on the mass of the lightest and the second lightest neutralino for the $SU(5)$ and $SO(10)$ supersymmetric grand unified theories, respectively. For the case of $SO(10)$ model, we see that the bounds on the mass of the lightest neutralino for models A, B, C referred to in Table II are numerically very similar. The upper bound in the case of model D is significantly larger due to the numerical factor $-96/25$ that is present in the gaugino mass ratio at the unification scale. The bounds on the mass of the next to lightest neutralino are significantly different for the different models as can be seen from the second panel in Fig. 5.

![FIG. 4: Upper bound on the mass of the lightest neutralino and that of the next to lightest neutralino as a function of the gluino mass for the four different cases that arise in $SU(5)$ supersymmetric grand unified theory. The solid line corresponds to 1, the dashed to 24, the dotted to 75 and the dashed-dotted to 200 representations of $SU(5)$, respectively. All masses are in TeV.](image)

![FIG. 5: Upper bound on the mass of the lightest neutralino and that of the second lightest neutralino as a function of the gluino mass for the SO(10) model. The solid line corresponds to A, the dashed to B, the dotted to C and the dashed-dotted to D, respectively. The notation is as in Table II All masses are in TeV.](image)

The chargino mass matrix in the basis $(-i\lambda^+, \psi^2_{H_1})$ can be written as

$$
\begin{pmatrix}
M_2 & \sqrt{2}M_W \sin \beta \\
\sqrt{2}M_W \cos \beta & \mu
\end{pmatrix},
$$

(74)

where $\lambda^+ = (1/\sqrt{2})(\lambda^1 - \lambda^2)$, with $\lambda^1$ and $\lambda^2$ being the first and second component of $SU(2)_L$ gaugino, and $\psi^2_{H_1}$ is the charged fermionic component of the $H_1$ superfield. Taking the trace of the neutralino and chargino squared mass matrices, we find the sum rule

$$
M_{\chi}^2 = 2(M_{\chi_1^0}^2 + M_{\chi_2^0}^2) - (M_{\chi_1^0}^2 + M_{\chi_2^0}^2 + M_{\chi_3^0}^2 + M_{\chi_4^0}^2) = (b^2a_2^2 - a^2a_1^2)\frac{M_0^2}{\alpha_3} + 4m_W^2 - 2m_Z^2.
$$

(75)
Using the values of parameters $a$ and $b$ in Tables I and III the coefficient of $M_3^2$ in (73) can be calculated for various representations of $SU(5)$ and $SO(10)$ grand unified theories. For $SU(5)$ the coefficient is 0.01, 0.28, 0.26 and $-3.09$ for 1, 24, 75 and 200 representations, respectively, whereas for $SO(10)$ this coefficient is evaluated to be 0.01, 0.26, 0.67 and $-0.40$, respectively for the models $A$, $B$, $C$, $D$ in Table III. In Fig. 6 we plot $M_{sum}^2$ as a function of the gluino mass for $SU(5)$ and $SO(10)$ models. Thus, once the neutralino, chargino and gluino masses are measured experimentally, this sum rule can be used to distinguish between the gaugino non-universality that arises in different supersymmetric grand unified theories.

V. SUM RULES FOR THE THIRD GENERATION

In section II we have shown how a measurement of sparticle masses of the light generations can be used to determine the parameters of the underlying supersymmetric theory. We also obtained relationships between squarks and sleptons of the light generations for different supersymmetric grand unified models. We made use of the explicit solutions of the RG equations to arrive at these results. It is worthwhile to ask whether similar relations can be obtained for the squark and slepton masses of the third generation. In this section we discuss this issue.

The renormalization group equations for the soft mass parameters of squarks and sleptons of third generation involve the Yukawa couplings of the third generation. These are catalogued in Appendix A. Because of the dependence on the squark and slepton masses of the third generation, these equations cannot be solved analytically, unlike the corresponding equations for the light generations. However, the solutions for the third generation RG equations can be written down in a closed form [56]. These closed form expressions for the solutions of the third generation RG equations are written down in the Appendix B. Note that we must add the $D$-term contributions on the right hand side of the eqs. (B1) - (B5) to get the masses of the sfermions at the weak scale. We first consider the case when the soft masses of the third generation are universal at the GUT scale:

$$m_{L,L}^2(t_G) = m_{R,L}^2(t_G) = m_{R,R}^2(t_G) = m_{\tilde{t}_R}^2(t_G) = m_{\tilde{H}_d}^2,$$  

$$M_{\tilde{t}}^2(t_G) = M_{\tilde{Q}}^2(t_G) = M_{\tilde{b}}^2(t_G) = M_{\tilde{\tau}}^2 = M_{3/2}^2,$$  

where now the values of the soft masses of sfermions refer to the third generation. As in the case of light generations, we define the average values of the squark and slepton masses for the $SU(2)_L$ doublets of the third generation:

$$M_{\tilde{Q},3}^2 = \frac{1}{2}(M_{\tilde{Q}_L}^2 + M_{\tilde{Q}_R}^2), \quad M_{\tilde{L},3}^2 = \frac{1}{2}(M_{\tilde{L}_L}^2 + M_{\tilde{L}_R}^2).$$

Using the solutions of the RG equations for the third generation [B1] - [B5], as well as the solutions for the Higgs mass parameters [B6] - [B7], we obtain the mass relation:

$$[M_{\tilde{Q},3}^2 - 2M_{\tilde{t}_R}^2 + M_{\tilde{b}_R}^2 + M_{\tilde{\tau}_R}^2 - M_{\tilde{\tau}_L}^2] + (m_{H_d}^2 - m_{H_u}^2) = \frac{10}{3} \sin^2 \theta_W M_Z^2 \cos 2\beta.$$  

However, if the boundary conditions for the soft scalar masses are non-universal, as happens in $SU(5)$ and $SO(10)$ grand unified theories, then the mass relation (78) is modified to

$$[M_{\tilde{Q},3}^2 - 2M_{\tilde{R}}^2 + M_{\tilde{R}}^2 + M_{\tilde{\tau}_R}^2 - M_{\tilde{\tau}_L}^2] + (m_{H_d}^2 - m_{H_u}^2) = \frac{10}{3} \sin^2 \theta_W M_Z^2 \cos 2\beta.$$
We note that the Higgs mass parameters enter the relations (78) and (79) by virtue of the fact that the evolution of soft masses for the third generation is coupled to evolution of the Higgs mass parameters through the large Yukawa couplings of the third generation.

VI. SUMMARY AND DISCUSSION

In this paper we have considered the spectrum of sparticles as it arises in $SU(5)$ and $SO(10)$ supersymmetric grand unified theories. We have shown through an analytical solution of the one-loop renormalization group equation how the measurement of the sparticle masses can be used to obtain the values of parameters of an underlying supersymmetric theory. It may, thus, be possible to distinguish, from the reconstruction of these parameters, between different supersymmetric grand unified theories. We have emphasized that non-universality of the soft scalar masses can arise because of an underlying grand unified theory. We have also discussed the non-universality that can arise in the gaugino sector, and its implications for the sparticle spectrum. In particular, we have considered different interrelationships between sfermion masses and the gluino mass that can arise in supersymmetric grand unified theories. We have also considered the implications of an underlying grand unified theory for the neutralino and chargino spectrum, which may be of direct relevance for sparticle searches at the LHC.

Our analysis has been based on the solutions of the one-loop renormalization group equations supplemented by the boundary conditions flowing from the underlying grand unified gauge group. Thus, the grand unified symmetry leaves its imprint on the superparticle spectra through these boundary conditions. However, there may be significant corrections to our results. These can arise from higher loop contributions to the renormalization group equations, use of a particular renormalization scheme as well as theoretical uncertainties. Since we have considered boundary conditions arising from different grand unified theories, GUT scale threshold can also affect our results. In testing our results, there will also uncertainties arising from sparticle masses, which will propagate to the underlying parameters. We have not considered these corrections in our paper. Our aim has been to show through transparent analytic calculations how different grand unified theories can be tested through a measurement of sparticle masses. Corrections to our results can be calculated only through a detailed analysis. We hope to calculate these corrections in a future publication.

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APPENDIX A: RENORMALIZATION GROUP EQUATIONS

The renormalization group equations for the soft supersymmetry breaking mass parameters and the Higgs mass parameters can be written as

$$16\pi^2 \frac{d}{dt} m_{1,2}^2 = 2y_t^2 X_t^2 + 2y_b^2 X_b^2 - \frac{2}{15} g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{1}{3} g_1^2 S,$$

$$16\pi^2 \frac{d}{dt} m_{1,2}^2 = 4y_t^2 X_t^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 - \frac{4}{5} g_1^2 S,$$

$$16\pi^2 \frac{d}{dt} m_{1,2}^2 = 4y_b^2 X_b^2 - \frac{8}{15} g_1^2 M_1^2 - \frac{32}{3} g_3^2 M_3^2 + \frac{2}{5} g_1^2 S,$$

$$16\pi^2 \frac{d}{dt} m_{1,2}^2 = 2y_t^2 X_t^2 - \frac{6}{5} g_1^2 M_1^2 - 6g_2^2 M_2^2 - \frac{3}{5} g_1^2 S,$$

$$16\pi^2 \frac{d}{dt} m_{1,2}^2 = 4y_t^2 X_t^2 - \frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S,$$

$$16\pi^2 \frac{d}{dt} m_{1,2}^2 = 4y_b^2 X_b^2,$$
Here \( A \) is a measure of the nonuniversality of the soft scalar masses. Note that we have included in the above the renormalization group equation for the soft mass parameter for the right handed sneutrino, and

\[
\begin{align*}
16\pi^2 \frac{d}{dt} m^2_{H_u} &= 6g_1^2 X^2_b + 2g_2^2 X^2_\tau - \frac{6}{5} g_1^2 M^2_1 - 6g_2^2 M^2_2 - \frac{3}{5} g_1^2 S, \\
16\pi^2 \frac{d}{dt} m^2_{H_u} &= 6g_1^2 X^2_t + 2g_2^2 X^2_\tau - \frac{6}{5} g_1^2 M^2_1 - 6g_2^2 M^2_2 + \frac{3}{5} g_1^2 S,
\end{align*}
\] (A7)

where \( t \equiv \ln(Q/M_G) \), with \( M_G \) being the initial GUT scale, \( y_i (i = t, b, \tau, \nu) \) are the Yukawa couplings, \( M_{3,2,1} \) are the running gaugino masses and \( g_{3,2,1} \) are the corresponding gauge couplings associated with the SM gauge group (\( \alpha_i \equiv g_i^2/4\pi, g_1 \) is in GUT normalization), and

\[
\begin{align*}
X_i^2 &= (m^2_{H_u} + m^2_{L_L} + m^2_{L_R} + A_i^2), \\
X_b^2 &= (m^2_{H_u} + m^2_{L_L} + m^2_{L_R} + A_b^2), \\
X_\tau^2 &= (m^2_{H_u} + m^2_{L_L} + m^2_{L_R} + A_\tau^2), \\
X_{\nu}^2 &= (m^2_{H_u} + m^2_{L_L} + m^2_{L_R} + A_{\nu}^2), \\
S &= \text{Tr}(Y m^2) = m^2_{H_u} - m^2_{H_d} + \sum_{\text{families}} (M^2_{Q_L} - 2M^2_{u_R} + M^2_{d_R} - M^2_{L_L} + M^2_{e_R}).
\end{align*}
\] (A8)

The quantity \( S \) evolves according to

\[
\frac{dS(t)}{dt} = \frac{66}{5} \tilde{\alpha}_1 S(t),
\] (A14)

which has the solution

\[
S(t) = S(t_G) \left( \frac{\tilde{\alpha}_1(t)}{\tilde{\alpha}_1(t_G)} \right).
\] (A15)

We note that if \( S = 0 \) at some initial scale, which would be the case if all the soft sfermion and Higgs masses are equal at that scale,

\[
S = \text{Tr}(Y m^2) = m^2_0 \text{Tr}Y = 0,
\] (A16)

then the RG evolution will maintain it to be zero at all scales. However, in a typical GUT, like \( SU(5) \) or \( SO(10) \), this is not the case, and \( S \) is nonzero at the scale where the GUT gauge group is broken. In this paper, unless otherwise stated, we shall take \( S \) to be nonzero.

The renormalization group equations for the gauge couplings, the gaugino masses, the Yukawa couplings, and the \( A \) parameters can be written as

\[
\begin{align*}
16\pi^2 \frac{d}{dt} g_i &= -b_i g_i^3, \\
16\pi^2 \frac{d}{dt} M_i &= -2 b_i M_i g_i^2, \\
\frac{d}{dt} Y_k &= Y_k \left( \sum_l a_{kl} Y_l - \sum_i c_{ki} \alpha_i \right), \\
\frac{d}{dt} A_k &= \sum_l a_{kl} A_l - \sum_i c_{ki} \alpha_i M_i,
\end{align*}
\] (A17 - A20)

where

\[
\begin{align*}
b_i &= \{-33/5, -1, 3\}, \\
c_{ti} &= \{13/15, 3, 16/3\}, \\
c_{bi} &= \{7/15, 3, 16/3\}, \\
c_{ri} &= \{9/5, 3, 0\}, \\
a_{di} &= \{6, 1, 0, 1\}, \\
a_{bi} &= \{1, 6, 1, 0\}, \\
a_{ri} &= \{0, 3, 4, 1\}, \\
a_{vi} &= \{3, 0, 1, 4\},
\end{align*}
\] (A21 - A25)
APPENDIX B: SOLUTION OF RENORMALIZATION GROUP EQUATIONS

Here we present the solutions of the renormalization group equations for the third generation given in Appendix A in a closed form.

\[
m^2_{t_L, b_L} = \tilde{m}^2_{t_L, b_L}(t_G) + \frac{48C_3 + 58C_2 - 55/6C_1}{122}, \quad \frac{17(S_t - \Sigma^0_t) + 20(S_b - \Sigma^0_b) - 5(S_\tau - \Sigma^0_\tau) - 1}{5} K, \quad (B1)
\]

\[
m^2_{t_R} = \tilde{m}^2_{t_R}(t_G) + \frac{54C_3 - 72C_2 + 24C_1}{122}, \quad \frac{-8(S_t - \Sigma^0_t) + 48(S_b - \Sigma^0_b) - 2(S_\tau - \Sigma^0_\tau) + 4}{5} K, \quad (B2)
\]

\[
m^2_{b_R} = \tilde{m}^2_{b_R}(t_G) + \frac{42C_3 - 56C_2 + 112/6C_1}{122}, \quad \frac{-8(S_b - \Sigma^0_b) + 48(S_b - \Sigma^0_b) - 12(S_\tau - \Sigma^0_\tau) - 2}{5} K, \quad (B3)
\]

\[
m^2_{\tilde{c}_L, \tilde{b}_L} = \tilde{m}^2_{\tilde{c}_L, \tilde{b}_L}(t_G) + \frac{30C_3 + 82C_2 - 103/6C_1}{122}, \quad \frac{3(S_t - \Sigma^0_t) - 18(S_b - \Sigma^0_b) + 35(S_\tau - \Sigma^0_\tau) + 3}{5} K, \quad (B4)
\]

\[
m^2_{\tilde{c}_R, \tilde{b}_R} = \tilde{m}^2_{\tilde{c}_R, \tilde{b}_R}(t_G) + \frac{60C_3 - 80C_2 + 80/3C_1}{122}, \quad \frac{6(S_t - \Sigma^0_t) - 36(S_b - \Sigma^0_b) + 70(S_\tau - \Sigma^0_\tau) - 6}{5} K, \quad (B5)
\]

\[
m^2_{\tilde{t}_R} = \tilde{m}^2_{\tilde{t}_R}(t_G) + \frac{-90C_3 - 2C_2 - 57/6C_1}{122}, \quad \frac{-9(S_t - \Sigma^0_t) + 54(S_b - \Sigma^0_b) + 17(S_\tau - \Sigma^0_\tau) + 3}{5} K, \quad (B6)
\]

\[
m^2_{\tilde{H}_u} = \tilde{m}^2_{\tilde{H}_u}(t_G) + \frac{-102C_3 + 14C_2 - 89/6C_1}{122}, \quad \frac{63(S_t - \Sigma^0_t) - 12(S_b - \Sigma^0_b) + 3(S_\tau - \Sigma^0_\tau) - 3}{5} K, \quad (B7)
\]

where \( C_i \) are defined in \[8\], and

\[
\Sigma_t = (m^2_{\tilde{H}_u} + m^2_{t_L} + m^2_{t_R}), \quad (B8)
\]

\[
\Sigma^2_b = (m^2_{\tilde{H}_d} + m^2_{b_L} + m^2_{b_R}), \quad (B9)
\]

\[
\Sigma^2_\tau = (m^2_{\tilde{H}_d} + m^2_{\tilde{b}_L} + m^2_{\tilde{b}_R}), \quad (B10)
\]

\[
\Sigma^0_k = \Sigma_k(t_G), \quad k = t, b, \tau. \quad (B11)
\]

The values of \( \Sigma_k \) completely define those of the soft masses for squarks, sleptons and Higgs bosons due to linear relations which follow from the RG equations \[56\]. Note that we have included here the contribution coming from the nonuniversality of the soft masses through the parameter \( K \) which was neglected in \[56\].

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