The generalized second law of thermodynamics in Hořava-Lifshitz cosmology

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Abstract. We investigate the validity of the generalized second law of thermodynamics in a universe governed by Hořava-Lifshitz gravity. Under the equilibrium assumption, that is in the late-time cosmological regime, we calculate separately the entropy time-variation for the matter fluid and, using the modified entropy relation, that of the apparent horizon itself. We find that under detailed balance the generalized second law is generally valid for flat and closed geometry and it is conditionally valid for an open universe, while beyond detailed balance it is only conditionally valid for all curvatures. Furthermore, we also follow the effective approach showing that it can lead to misleading results. The non-complete validity of the generalized second law could either provide a suggestion for its different application, or act as an additional problematic feature of Hořava-Lifshitz gravity.

Keywords: modified gravity, gravity
1 Introduction

Almost one year ago, Hořava proposed a power-counting renormalizable, ultra-violet (UV) complete theory of gravity [1–4]. Although presenting an infrared (IR) fixed point, namely General Relativity, in the UV the theory possesses a fixed point with an anisotropic, Lifshitz scaling between time and space. These novel features led many authors to examine and extend the properties of the theory itself [5–31], and furthermore to apply it as a cosmological framework, constructing the so-called Hořava-Lifshitz cosmology [32, 33]. Amongst the very interesting physical implications are the novel solution subclasses [34–52], the gravitational wave production [53–58], the perturbation spectrum [59–68], the matter bounce [69–74], the dark energy phenomenology [75–81], the astrophysical phenomenology [82–86], and the observational constraints on the theory [87–89].

A specific direction of the research on the topic is the investigation of the thermodynamic properties of Hořava-Lifshitz gravity, a project that is crucially connected to the black hole properties in such a theory [90–111]. In particular, due to the well known connection between thermodynamics and gravity (see [112] and references therein), one is able to express the field equations as a first law of thermodynamics (however the inverse procedure is not always possible, that is starting from thermodynamics to extract the general field equations, without this implying that the specific gravitational theory is inconsistent\(^1\)). Additionally, the aforementioned thermodynamic interpretation of the field equations can be extended in cosmology, and it is applicable to any horizon provided that the gravitational theory is diffeomorphism invariant [113]. Having in mind that Hořava-Lifshitz gravity is not diffeomorphism invariant, and thus in cosmological frameworks one has to use the apparent horizon [114] instead of an arbitrary one, some authors have investigated the connection between the first law of thermodynamics and the Friedmann equations [115–118].

In order to interpret the Friedmann equations of Hořava-Lifshitz cosmology thermodynamically, one faces two possible approaches. The first and correct one is to consider a universe containing only the matter fluid, and calculate the horizon entropy using the modified relation for the black hole entropy in Hořava-Lifshitz gravity [118]. The second approach

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is to absorb all the extra information of Hořava-Lifshitz cosmology in an effective dark energy component, and thus to consider a universe containing the matter and the effective dark energy fluid in a general-relativity background, that is calculate the horizon entropy using the standard relation for the black hole entropy \[1^{15, 16}\]. Interestingly, while with the second approach one obtains exactly the standard Friedmann equations, with the first one he acquires some additional corrections, and the two results coincide only in the zero-curvature or in the low energy limit \[1^{18}\] (see also \[1^{19}\]). However, we should stress that the first approach is the robust one since it treats the gravitational sector separately and completely, taking into account its radical effects on the geometry, while the second approach defines naively an effective “gravitationally-originated” fluid and treating it as a conventional fluid, not taking into account that gravity is not only an “actor” but a “director”, too. Finally, the better theoretical background of the first approach becomes obvious in the fact that it is followed in all the thermodynamic studies of various alternative gravitational theories \[1^{20}\].

In the present work we are interested in investigating the validity of the generalized second law of thermodynamics in the context of Hořava-Lifshitz cosmology. We follow the exact and robust approach, that is we use the modified entropy relation as it has been calculated in the specific context of Hořava-Lifshitz gravity. Under the equilibrium assumption between the universe interior and the horizon, which is expected to be valid at late cosmological times, we find that the generalized second law is only conditionally valid. For completeness, we also follow the discussed effective approach, showing that it can lead to misleading results. The plan of the work is as follows: in section 2 we present the cosmology of a universe governed by Hořava-Lifshitz gravity and in section 3 we investigate the validity of the generalized second law of thermodynamics. In section 4 we perform the same analysis following the effective approach. Finally, in section 5 we discuss and we summarize the obtained results.

2 Hořava-Lifshitz cosmology

In this section we briefly review the scenario where the cosmological evolution is governed by Hořava-Lifshitz gravity \[3^{2, 33}\]. The dynamical variables are the lapse and shift functions, \(N\) and \(N_i\) respectively, and the spatial metric \(g_{ij}\) (roman letters indicate spatial indices). In terms of these fields the full metric is written as:

\[
ds^2 = -N^2 dt^2 + g_{ij}(dx^i + N^i dt)(dx^j + N^j dt),
\]

where indices are raised and lowered using \(g_{ij}\). The scaling transformation of the coordinates reads: \(t \rightarrow l^3 t\) and \(x^i \rightarrow lx^i\).

The gravitational action is decomposed into a kinetic and a potential part as

\[
S_g = \int dt d^3x \sqrt{g}N (\mathcal{L}_K + \mathcal{L}_V).
\]

The assumption of detailed balance \[3\] reduces the possible terms in the Lagrangian, and it allows for a quantum inheritance principle \[1\], since the \((D+1)\)-dimensional theory acquires the renormalization properties of the \(D\)-dimensional one. Under the detailed balance condition the full action of Hořava-Lifshitz gravity is given by

\[
S_g = \int dt d^3x \sqrt{g}N \left\{ \frac{2}{\kappa^2} (K_{ij} K^{ij} - \lambda K^2) + \frac{\kappa^2}{2w^4} C_{ij} C^{ij} - \frac{\kappa^2 \mu^2}{2w^2} \sqrt{g} R_{il} \nabla_j R^i_j + \frac{\kappa^2 \mu^2}{8} R_{ij} R^{ij}
\right.

\left. + \frac{\kappa^2 \mu^2}{8(3\lambda - 1)} \left[ 1 - \frac{4\lambda}{4} R^2 + \Lambda R - 3\Lambda^2 \right] \right\},
\]

where

\[
K_{ij} = \frac{1}{2N} (\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i)
\]
is the extrinsic curvature and
\[ C^{ij} = \frac{\epsilon^{ijk}}{\sqrt{g}} \nabla_k \left( R^i_j - \frac{1}{4} R \delta^i_j \right) \] (2.4)

the Cotton tensor, and the covariant derivatives are defined with respect to the spatial metric \( g_{ij} \). \( \epsilon^{ijk} \) is the totally antisymmetric unit tensor, \( \lambda \) is a dimensionless constant and the variables \( \kappa, w \) and \( \mu \) are constants with mass dimensions \(-1, 0 \) and \( 1 \), respectively. Finally, we mention that in action (2.2) we have already performed the usual analytic continuation of the parameters \( \mu \) and \( w \) of the original version of Hořava-Lifshitz gravity, since such a procedure is required in order to obtain a realistic cosmology [34, 38, 96, 115] (although it could fatally affect the gravitational theory itself). Therefore, in the present work \( \Lambda \) is a positive constant, which as usual is related to the cosmological constant in the IR limit.

Lastly, in order to incorporate the (dark plus baryonic) matter component one adds a cosmological stress-energy tensor to the gravitational field equations, by demanding to recover the usual general relativity formulation in the low-energy limit [13, 47, 48]. Thus, this matter-tensor is a hydrodynamical approximation with its energy density \( \rho_M \) and pressure \( p_M \) (or \( \rho_M \) and its equation-of-state parameter \( w_M \equiv p_M/\rho_M \)) as parameters.

Now, in order to focus on cosmological frameworks, we impose the so called projectability condition [11] and use a Friedmann-Robertson-Walker (FRW) metric,
\[ N = 1, \quad g_{ij} = a^2(t) \gamma_{ij}, \quad N^i = 0, \] (2.5)
with
\[ \gamma_{ij} dx^i dx^j = dr^2 + r^2 d\Omega^2_k, \] (2.6)
where \( k = -1, 0, +1 \) corresponding to open, flat, and closed universe respectively. By varying \( N \) and \( g_{ij} \), we obtain the equations of motion:
\[ H^2 = \frac{\kappa^2}{6(3\lambda - 1)} \rho_M + \frac{\kappa^2}{6(3\lambda - 1)} \left[ \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2 a^2}, \] (2.7)
\[ \dot{H} + \frac{3}{2} H^2 = -\frac{\kappa^2}{4(3\lambda - 1)} w_M \rho_M - \frac{\kappa^2}{4(3\lambda - 1)} \left[ \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)} \right] - \frac{\kappa^4 \mu^2 \Lambda}{16(3\lambda - 1)^2 a^2}, \] (2.8)
where we have defined the Hubble parameter as \( H \equiv \frac{\dot{a}}{a} \). The term proportional to \( a^{-4} \) is the usual “dark radiation term”, present in Hořava-Lifshitz cosmology [32, 33], while the constant term is just the explicit cosmological constant. Finally, as usual, \( \rho_M \) follows the standard evolution equation
\[ \dot{\rho}_M + 3H(1 + w_M)\rho_M = 0. \] (2.9)

As a last step, requiring these expressions to coincide with the standard Friedmann equations, in units where \( c = 1 \) we set [32, 33]:
\[ G_{\text{cosmo}} = \frac{\kappa^2}{16\pi(3\lambda - 1)} \]
\[ \frac{\kappa^4 \mu^2 \Lambda}{8(3\lambda - 1)^2} = 1, \] (2.10)
where \( G_{\text{cosmo}} \) is the “cosmological” Newton’s constant. We mention that in theories with Lorentz invariance breaking (such is Hořava-Lifshitz one) the “gravitational” Newton’s constant \( G_{\text{grav}} \), that is the one that is present in the gravitational action, does not coincide with
the “cosmological” Newton’s constant $G_{\text{cosmo}}$, that is the one that is present in Friedmann equations, unless Lorentz invariance is restored [121]. For completeness we mention that in our case

$$G_{\text{grav}} = \frac{\kappa^2}{32\pi},$$  \hspace{1cm} (2.11)

as it can be straightforwardly read from the action (2.2). Thus, it becomes obvious that in the IR ($\lambda = 1$), where Lorentz invariance is restored, $G_{\text{cosmo}}$ and $G_{\text{grav}}$ coincide.

Using the above identifications, we can re-write the Friedmann equations (2.7), (2.8) as

$$H^2 + \frac{k}{a^2} = \frac{8\pi G_{\text{cosmo}}}{3} \rho_M + \frac{k^2}{2\Lambda a^4} + \frac{\Lambda}{2},$$  \hspace{1cm} (2.12)

$$\dot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_{\text{cosmo}}w_M \rho_M - \frac{k^2}{4\Lambda a^4} + \frac{3\Lambda}{4}.  \hspace{1cm} (2.13)$$

3 Generalized second law of thermodynamics

Having presented the cosmological scenario of a universe governed by Hořava-Lifshitz gravity, we proceed to an investigation of its thermodynamic properties, and in particular of the generalized second thermodynamic law (although there is still missing a robust proof for its general validity) [122–132]. As it is usual in the literature, one considers the universe as a thermodynamical system. However, a priori it is not trivial what should be the “radius” of the system in order to acquire a consistent description. This subject becomes more important under the light of use of black-hole physics [133] in a cosmological framework [134, 135], that is connecting the ‘radius’ and ‘area’ of the universe with its temperature and entropy respectively. As we mentioned in the Introduction, the thermodynamic interpretation of field equations can be applicable for any horizon, provided that the gravitational theory is diffeomorphism invariant [112, 113], however the apparent horizon is widely used in the literature either in flat [136–138] or in non-flat FRW geometry [139, 140]. In the case of Hořava-Lifshitz gravity, the breaking of diffeomorphism invariance leaves us the apparent horizon as a reasonable choice.

The dynamical apparent horizon, a marginally trapped surface with vanishing expansion, is in general determined by the relation $h^{ij}\partial_i\tilde{r}\partial_j\tilde{r} = 0$, which implies that the vector $\nabla\tilde{r}$ is null (or degenerate) on the apparent horizon surface [114]. In a metric of the form $ds^2 = h_{ij}dx^idx^j + \tilde{r}^2d\Omega_2^2$, with $h_{ij} = \text{diag}(-1, a^2/(1 - kr^2))$, $i, j = 0, 1$, it writes [114]:

$$\tilde{r}_A = \frac{1}{\sqrt{H^2 + \frac{k}{a^2}}}.  \hspace{1cm} (3.1)$$

In summary, we consider the universe as a thermodynamical system with the apparent horizon surface being its boundary.

Let us now proceed to the investigation of the generalized second law of thermodynamics. We are going to examine whether the sum of the entropy enclosed by the apparent horizon and the entropy of the apparent horizon itself, is not a decreasing function of time. Simple arguments suggest that after equilibrium establishes and the universe background geometry becomes FRW, all the fluids in the universe acquire the same temperature $T$ [141], which is moreover equal to the temperature of the horizon $T_h$ [136–140], otherwise the energy flow would deform this geometry [142]. However, although this will certainly be the situation at late times, that is when the universe fluids and the horizon will have interacted for a long time, it is ambiguous if it will be the case at early or intermediate times (for instance
the present CMB temperature is of the order of 1 Kelvin, while the horizon temperature is many orders of magnitude below this figure). However, in order to avoid non-equilibrium thermodynamical calculations, which would lead to lack of mathematical simplicity and generality, the assumption of equilibrium, although restricting, is widely disseminated in the generalized-second-law literature \[112, 113, 136–142\]. Thus, we will follow this assumption and we will have in mind that our results hold only at late times of the universe evolution.

In general, the apparent horizon $\tilde{r}_A$ is a function of time. Thus, a change $d\tilde{r}_A$ in time $dt$ will lead to a volume-change $dV$, while the energy and entropy will change by $dE$ and $dS$ respectively. However, since in the two states there is a common source $T_{\mu\nu}$, we can consider that the pressure $P$ and the temperature $T$ remain the same \[136–140\]. Such a consideration is standard in thermodynamics, where one considers two equilibrium states differing infinitesimally in the extensive variables like entropy, energy and volume, while having the same values for the intensive variables like temperature and pressure. In this case the first law of thermodynamics writes $T dS = dE + P dV$, and therefore the dark-matter entropy reads (the universe contains only the dark-matter fluid and we neglect the radiation sector.):

$$dS_M = \frac{1}{T} \left( P_M dV + dE_M \right),$$

(3.2)

where $V = 4\pi \tilde{r}_A^3/3$ is the volume of the system bounded by the apparent horizon and thus $dV = 4\pi \tilde{r}_A^2 d\tilde{r}_A$. We mention here that a thermodynamic identity of the form of (3.2) has a universal validity, and the information about a given system is only encoded in the form of the entropy functional $S(E, V)$ \[112\]. In particular, in the case of normal materials, this entropy arises because of our coarse graining over microscopic degrees of freedom which are not tracked in the dynamical evolution, however, in the case of spacetime the existence of horizons for a particular class of observers makes it mandatory that these observers integrate out degrees of freedom hidden by the horizon.

Dividing (3.2) by $dt$ we obtain

$$\dot{S}_M = \frac{1}{T} \left( P_M 4\pi \tilde{r}_A^2 \dot{\tilde{r}}_A + \dot{E}_M \right).$$

(3.3)

In this relation the time derivative of the apparent horizon writes

$$\dot{\tilde{r}}_A = H \tilde{r}_A \left[ 4\pi G_{\cosmo} (1 + w_M) \rho_M + \frac{k^2}{\Lambda \sigma^4} \right],$$

(3.4)

as it easily arises differentiating the Friedmann equation (2.12) and using (2.9).

In order to connect the thermodynamically relevant quantities, namely the energy $E_M$ and pressure $P_M$, with the cosmologically relevant ones, namely the energy density $\rho_M$ and the pressure $p_M$, we can straightforwardly use:

$$E_M = \frac{4\pi}{3} \tilde{r}_A^3 \rho_M$$

(3.5)

$$P_M = w_M \rho_M.$$ 

(3.6)

Inserting the time-derivative of (3.5), along with (3.6), into (3.3), and using (2.9), we obtain:

$$\dot{S}_M = \frac{1}{T} \left( 1 + w_M \right) \rho_M 4\pi \tilde{r}_A^2 \left( \dot{\tilde{r}}_A - H \tilde{r}_A \right).$$

(3.7)
At this stage, we have to connect the temperature of the matter fluid $T$ to that of the horizon $T_h$. As we have said, although at early and intermediate times these two temperatures do not coincide in general, at late times, after the establishment of equilibrium, they become equal, that is $T = T_h$. Now, $T_h$ has to be related to the geometry of the universe. Note that although the association of a temperature to a horizon was historically related to black holes, it was soon realized that the study of quantum field theory in any spacetime with a horizon shows that all horizons possess temperatures $[143–145]$. In particular, an observer who is accelerating through the vacuum state in flat spacetime perceives a horizon and will attribute to it $[146]$ a temperature $T = \kappa_a/2\pi$ proportional to her acceleration $\kappa_a$ (for a review, see $[147–153]$). We stress that the relation connecting the horizon temperature with the geometry of the universe depends only on this geometry and not on the gravitational sector of the scenario. Thus, for spherical (FRW) geometry and according to the generalization of black hole thermodynamics $[133]$ to a cosmological framework, the temperature of the horizon is related to its radius through $[112]$

$$T_h = \frac{1}{2\pi \bar{r}_A},$$

(3.8)

either in general relativity $[134, 135, 139, 140]$, in modified gravitational theories $[120]$, or in Hořava-Lifshitz gravity $[109]$.

As a last step, we have to connect the entropy of the horizon to its radius $\bar{r}_A$ (or equivalently to each area). As usual this relation will be the corresponding one for black holes, but with the apparent horizon instead of the black-hole horizon, and thus it obviously depends on the particular gravitational sector of the scenario. In the case of black holes in Hořava-Lifshitz gravity, and under the detailed balance condition, this expression is known $[91, 108, 109]$ and thus its cosmological application straightforwardly leads to:

$$S_h = \frac{\kappa^2}{32\Lambda G^2_{\text{cosmo}}} \left[ \Lambda \bar{r}_A^2 + 2k \ln \left( \sqrt{\Lambda \bar{r}_A} \right) \right],$$

(3.9)

where we have also made use of the identifications (2.10). Note that in the extraction of this relation the authors have neglected quantum buoyancy effects near the black hole horizon, however such effects are expected to bring about lower-order corrections $[154]$ and thus relation (3.9) captures the main behavior.

We mention that since we desire to remain general we have not imposed the IR limit of Hořava-Lifshitz gravity, and thus in the above relation we have not set $\lambda = 1$. Although such a choice would be very reasonable concerning the late-time epochs of the universe, at very early times, where the universe is very small and the UV features of the theory are revealed, the divergence of $\lambda$ from 1 may be significant. However, since the equilibrium assumption, which allowed us to equalize the horizon temperature with that of the universe interior and perform the above calculations, is justified only at late times, later on we will impose the IR limit of the obtained relations. In other words, investigating the intrinsic running character of $\lambda$ is very interesting, but it is not need to be performed in detail for the present work, since in the end of the day one has to imply the late time limit. Such an investigation would indeed be very interesting as an independent work, without focusing on thermodynamics, where one could impose Renormalization Group methods to model its running.

As expected, in relation (3.9) the first term corresponds to the standard (general relativity) result, while the second term is the novel one, arising from Hořava-Lifshitz gravity.
Now, differentiating (3.9) we finally acquire
\[
\dot{S}_h = \frac{\kappa^2}{16G^2_{\text{cosmo}}} \dot{r}_A \dot{r}_A + \frac{\kappa^2 k}{16\Lambda G^2_{\text{cosmo}}} \dot{r}_A. \tag{3.10}
\]

Let us now proceed to the calculation of the total entropy variation. Adding relations (3.7) and (3.10), with \( T \) given by (3.8), we find:
\[
\dot{S}_{\text{tot}} = \dot{S}_M + \dot{S}_h = 8\pi^2 \dot{r}_A^3 \left( \dot{r}_A - H \dot{r}_A \right) \left( 1 + w_M \right) \rho_M + \frac{\kappa^2}{16G^2_{\text{cosmo}}} \left( \dot{r}_A + \frac{k}{\Lambda A^4} \right) \dot{r}_A. \tag{3.11}
\]

Thus, substituting also \( \dot{r}_A \) by (3.4) we result to:
\[
\dot{S}_{\text{tot}} = \dot{r}_A^3 H \left[ 8\pi^2 \dot{r}_A^3 \left( 1 + w_M \right) \rho_M + \frac{\kappa^2 k}{16G^2_{\text{cosmo}} \Lambda A^4} \right] \left[ 4\pi G_{\text{cosmo}} \left( 1 + w_M \right) \rho_M + \frac{k^2}{\Lambda A^4} \right] \\
+ \frac{\pi(3\lambda - 1)\dot{r}_A^3 H k^2}{G_{\text{cosmo}} \Lambda A^4} + \left[ \left( \frac{3\lambda - 1}{2} \right) - 1 \right] 8\pi^2 \dot{r}_A^3 H \left( 1 + w_M \right) \rho_M. \tag{3.12}
\]

Finally, at late times, where equilibration between the horizon and the universe interior had been established, \( \lambda \) has taken its IR limit (\( \lambda = 1 \)) and thus the above relation becomes:
\[
\dot{S}_{\text{tot}} = \dot{r}_A^3 H \left[ 8\pi^2 \dot{r}_A^3 \left( 1 + w_M \right) \rho_M + \frac{2\pi k}{G \Lambda A^4} \right] \left[ 4\pi G \left( 1 + w_M \right) \rho_M + \frac{k^2}{\Lambda A^4} \right] + \frac{2\pi \dot{r}_A^3 H k^2}{G \Lambda A^4}, \tag{3.13}
\]

where we have simplified the notation using \( G \) for the Newton’s constant, since in the IR limit \( G_{\text{cosmo}} \) and \( G_{\text{grav}} \) coincide.

Relations (3.12) and (3.13) provide the expression for the total entropy variation rate in a universe governed by Hořava-Lifshitz gravity. Let us examine its sign. Obviously, for a flat or closed universe \( \dot{S}_{\text{tot}} \geq 0 \) (we remind that due to analytic continuation in the present work \( \lambda \) is always positive and \( \lambda \geq 1 \)) and thus the generalized second law of thermodynamics is valid. We mention that this result holds independently of the matter equation-of-state parameters and of the background geometry, provided it is FRW.

However, for an open universe (\( k = -1 \)), in order to acquire \( \dot{S}_{\text{tot}} \geq 0 \) one has to have a non-zero matter component. In the limiting case where matter is absent one can easily see that \( \dot{S}_{\text{tot}} \geq 0 \) if \( \Lambda A^4 \geq 1 \), or using the explicit form for \( \dot{r}_A \) (from relations (3.1) and (2.12)) if \( \Lambda A^2 \geq 1 \). Thus, the generalized second law is always violated for sufficiently small scale factors (provided that we are still in the late-time cosmological regime). The reason for this arises from the modified horizon entropy relation (3.9). Clearly, the presence of the curvature as a coefficient of the correction term means that \( \dot{S}_h \) can be negative for \( k = -1 \) unless \( \Lambda A^2 \geq 1 \). Thus, since in the absence of matter \( \dot{S}_{\text{tot}} \) and \( \dot{S}_h \) coincide, the aforementioned violation of the generalized second law is implied.

The above analysis has been performed under the detailed-balance condition, since in this case the relation for black-hole (and thus for the apparent horizon) entropy, is well known. However, detail balance is not at all a requirement and one can straightforward go beyond it and repeat the aforementioned steps, but with the new entropy relation that will arise from the corresponding black hole solutions (and of course with the new \( \dot{r}_A \) that will arise from the new Friedmann equations). Extracting the black hole entropy in Hořava-Lifshitz gravity without detailed balance is an independent task of its own, and one can have a variety of results according to the specific form of detail-balance breaking [101]. Thus, one should in principle
examine the validity of the generalized second law in each case separately. However, in all cases our result will be qualitatively maintained. The reason is the following: As it has been extensively stated in the literature \[32, 33\], Hořava-Lifshitz cosmology coincides completely with ΛCDM if one ignores the curvature and imposes the IR limit. Thus, all modifications in the black hole entropy of various versions of non-detailed-balance Hořava-Lifshitz gravity will depend on the curvature, while the entropy relations will always include the standard (general relativity) basic term. Note however that the sign of the correction term will also depend on the sign of the coefficients of the higher curvature terms needed to break the detail balance, and thus the overall sign will be arbitrary.\footnote{We thank S. Mukohyama for this comment.} Therefore, for all the specific versions of Hořava-Lifshitz gravity, the resulting $S_{\text{tot}}$ will be different, but always of the structure of (3.13), that is with a conditionally negative contribution for all possible curvatures.

4 Generalized second law of thermodynamics: an effective but inconsistent approach

In the previous section we examined the validity of the generalized second law of thermodynamics in Hořava-Lifshitz cosmology, following the robust approach of considering a universe containing only matter and incorporating the extra information of Hořava-Lifshitz gravity through the modified entropy relation. However, as we discussed in the Introduction, in the case of the first law of thermodynamics some authors have chosen a different approach, that is to absorb all the extra information of Hořava-Lifshitz gravity into an effective dark energy fluid, and thus considering a universe containing matter plus this effective fluid, in a general relativity background \[115, 116\]. However, this approach does not possess the transparent theoretical justification of the previous one, and can be misleading in the case where gravitational phenomena are important (for example one could find wrong black-hole properties in such an “effective” universe comparing to those of the real one). In order to reveal this problematic behavior and in order to be complete and comparable with part of the literature, in this section we investigate the generalized second law following this effective description of Hořava-Lifshitz cosmology.

4.1 Hořava-Lifshitz cosmology with effective dark energy fluid

Observing the Friedmann equations (2.7), (2.8) one can introduce an effective dark-energy sector defining the energy density and pressure as

$$\rho_{\text{DE}} \equiv \frac{3\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} + \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)}$$ \hspace{1cm} (4.1)

$$p_{\text{DE}} \equiv \frac{\kappa^2 \mu^2 k^2}{8(3\lambda - 1)a^4} - \frac{3\kappa^2 \mu^2 \Lambda^2}{8(3\lambda - 1)},$$ \hspace{1cm} (4.2)

which after the identifications (2.10) become

$$\rho_{\text{DE}} \equiv \frac{1}{16\pi G_{\text{cosmo}}} \left( \frac{3k^2}{\Lambda a^4} + 3\Lambda \right)$$ \hspace{1cm} (4.3)

$$p_{\text{DE}} \equiv \frac{1}{16\pi G_{\text{cosmo}}} \left( \frac{k^2}{\Lambda a^4} - 3\Lambda \right).$$ \hspace{1cm} (4.4)
Therefore, the effective dark energy incorporates the contributions from the dark radiation term (proportional to $a^{-4}$) and from the cosmological constant. It is straightforward to show that $\rho_{\text{DE}}$ satisfies the standard evolution equation:

$$
\dot{\rho}_{\text{DE}} + 3H(1 + w_{\text{DE}})\rho_{\text{DE}} = 0,
$$

(4.5)

where $w_{\text{DE}} \equiv p_{\text{DE}}/\rho_{\text{DE}}$ is the effective dark energy equation-of-state parameter. Finally, using the above definitions, we can re-write the Friedmann equations (2.12), (2.13) in the standard form:

$$
H^2 + \frac{k}{a^2} = \frac{8\pi G_{\text{cosmo}}}{3}(\rho_M + \rho_{\text{DE}}) 
$$

(4.6)

$$
\ddot{H} + \frac{3}{2}H^2 + \frac{k}{2a^2} = -4\pi G_{\text{cosmo}}(\rho_M + p_M + \rho_{\text{DE}} + p_{\text{DE}}).
$$

(4.7)

### 4.2 Generalized second law of thermodynamics

Following the effective approach to Hořava-Lifshitz cosmology we assume that the universe contains the matter and the dark energy fluid, in a general relativity background. Thus, all the extra information is included inside $\rho_{\text{DE}}$ and $p_{\text{DE}}$, and the problem is equivalent with the one in Einstein gravity but with two fluids. Repeating the steps of section 3 for two fluids we easily find

$$
\dot{S}_{\text{DE}} = \frac{1}{T}(1 + w_{\text{DE}})\rho_{\text{DE}}4\pi \tilde{r}_A^2 (\dot{\tilde{r}}_A - H\tilde{r}_A),
$$

(4.8)

$$
\dot{S}_M = \frac{1}{T}(1 + w_M)\rho_M4\pi \tilde{r}_A^2 (\dot{\tilde{r}}_A - H\tilde{r}_A),
$$

(4.9)

while in this case differentiation of the Friedmann equation (4.6), using also (2.9) and (4.5), gives

$$
\dot{\tilde{r}}_A = 4\pi G_{\text{cosmo}}H \tilde{r}_A^3 [(1 + w_M)\rho_M + (1 + w_{\text{DE}})\rho_{\text{DE}}].
$$

(4.10)

Note that as we discussed above, at late times, due to equilibrium the temperature of the two fluids will be the same, and moreover equal to the temperature of the horizon $T_h$. This horizon temperature is still given by (3.8), since the corresponding relation depends only on the cosmological geometry. Finally, since the (effective) gravitational sector in now the standard general relativity, the horizon entropy will be given by the corresponding standard relation

$$
S = \frac{4\pi \tilde{r}_A^2}{4G_{\text{grav}}}
$$

(4.11)

Differentiating (4.11) we acquire

$$
\dot{S}_h = \frac{4\pi}{(3\lambda - 1)G_{\text{cosmo}}} \tilde{r}_A \dot{\tilde{r}}_A,
$$

(4.12)

where we have also made use of the identifications (2.10) and (2.11).

Adding relations (4.8), (4.9) and (4.12), we find:

$$
\dot{S}_{\text{tot}} \equiv \dot{S}_{\text{DE}} + \dot{S}_M + \dot{S}_h = 8\pi^2 \tilde{r}_A^2 (\dot{\tilde{r}}_A - H\tilde{r}_A) \left[(1 + w_{\text{DE}})\rho_{\text{DE}} + (1 + w_M)\rho_M\right] + \frac{4\pi}{(3\lambda - 1)G_{\text{cosmo}}} \tilde{r}_A \dot{\tilde{r}}_A
$$

(4.13)
and thus, substituting also $\dot{r}_A$ by (4.10) we result to:

$$
\dot{S}_{\text{tot}} = 32\pi^3 G_{\text{cosmo}} \tilde{r}_A^6 H \left[ (1 + w_{\text{DE}}) \rho_{\text{DE}} + (1 + w_M) \rho_M \right]^2 \\
+ \left[ \left( \frac{2}{3 \lambda - 1} \right) - 1 \right] 8\pi^2 H \tilde{r}_A^4 \left[ (1 + w_{\text{DE}}) \rho_{\text{DE}} + (1 + w_M) \rho_M \right].
$$

(4.14)

Finally, since the above results, based on the equilibrium assumption, hold only for late times, we have to impose the IR limit ($\lambda = 1$) and therefore relation (4.14) becomes

$$
\dot{S}_{\text{tot}} = 32\pi^3 G \tilde{r}_A^6 H \left[ (1 + w_{\text{DE}}) \rho_{\text{DE}} + (1 + w_M) \rho_M \right]^2 \geq 0,
$$

(4.15)

where we have simplified the notation using $G$ for the Newton’s constant, since in the IR limit $G_{\text{cosmo}}$ and $G_{\text{grav}}$ coincide.

We mention here that the effective approach can be straightforwardly extended beyond the detail balance condition, in a much more easier way than the approach of the previous section. In particular, since one uses the standard (general relativity) relation for horizon entropy, he does not need to examine the black hole properties at all, but only to extract the Friedmann equations and absorb all the new information in a modified effective dark energy fluid. Thus, result (4.15) will be still valid, but $\rho_{\text{DE}}$ can now have many contributions according to the specific extension beyond the detail balance.

A first observation is that $\dot{S}_{\text{tot}}$ is slightly different that the one calculated in the previous section, and the two results coincide only in the IR and zero-curvature limit. This difference has been discussed in the case of the first thermodynamical law \cite{118}, where coincidence is achieved in the same limit too. However, the qualitative difference between the two approaches, that is the exact and the effective one, is that now we obtain that $\dot{S}_{\text{tot}}$ is always non-negative in the IR limit and thus the generalized second law would be always valid in the late-time universe. This partially misleading result reveals that in subjects where gravity is involved, one cannot follow a completely effective approach and absorb all the gravitational phenomena inside conventional components. Doing so one does not take into account the richness of the effects of gravity. The fact that the results of the two approaches coincide for zero curvature in the IR is straightforwardly explained, since in this case the correction term in the modified entropy relation (3.9) is zero.

5 Discussion and conclusions

In this work we investigated the validity of the generalized second law of thermodynamics in a universe governed by Hořava-Lifshitz gravity. Considering the universe as a thermodynamical system bounded by the apparent horizon, we calculated the entropy time-variation of the universe-content as well as that of the horizon, under the assumption of thermal equilibrium which is expected to hold at late times. We stress that we followed the theoretically robust approach, that is to consider that the universe contains only matter, while the effect of the novel gravitational sector of Hořava-Lifshitz gravity was incorporated through the modified black-hole (and consequently horizon) entropy. Additionally, although we performed our calculations for an arbitrary $\lambda$, that is not only in the IR limit of the theory, in the end, and in order to be consistent with the equilibrium assumption, we have to focus on late times and thus impose the IR limit. We found that at late times, under detailed balance, for flat and closed universe the generalized second law is always valid, while for open geometry it is...
only conditionally valid. Going beyond detailed balance the generalized second law is only conditionally valid for all curvatures.

The possible violation of the generalized second thermodynamical law in Hořava-Lifshitz cosmology could lead to various conclusions (apart from putting the generalized second law itself or the equilibrium assumption into question). Although conditional validity has been found for some horizon choices in other cosmological scenarios too [155, 156], one could examine if using an alternative horizon would lead to a complete validation. Usual alternative choices such is the Hubble radius $H^{-1}$, or the future event horizon $R_h = \int_0^\infty da/(Ha^2)$, lead to the same result, that is to conditional validity. Nevertheless, one could still search for a suitable horizon in order to acquire full validity. However, one should have in mind that in Hořava-Lifshitz gravity the physical meaning of a horizon might differ from that in general relativity. In particular, in the later the static black-hole horizon and the de Sitter horizon are Killing horizons, allowing for an application of Euclidean [133] or thermofield [157] methods in order for the horizon temperature to acquire a physical meaning. On the other hand, in Hořava-Lifshitz gravity under the projectability condition, there exists a preferred time slicing which is not orthogonal to the horizon, which could furthermore be an emergent concept in the IR limit [85], or particle-dependent (each particle has its own “light speed” and thus it sees its own “horizon”) [101]. Thus, the interpretation of the horizon temperature may be different in the two theories. Finally, as was discussed in [101], even without the projectability condition, Hořava-Lifshitz gravity still shares the uncertainties of non-relativistic theories, which can make the notions of entropy and temperature not well-defined.

A second conclusion could be that the conditional violation of the generalized second law is an additional problem along with the discussed (not few) conceptual and theoretical problems of Hořava-Lifshitz gravity [11, 13, 15, 24, 25, 28]. However, the logarithmic correction in the black-hole entropy, which is the cause of the conditional violation, is not a so exotic term, and indeed it has the same form with the quantum corrections on the standard result calculated in loop quantum gravity (much earlier than the appearance of Hořava-Lifshitz gravity) [158–173]. Thus, one could deduce that at least this point could not be a problematic feature of Hořava-Lifshitz gravity. Finally, one can still put into question the fact that the black hole production inside the apparent horizon and its effect on entropy, as well as possible entropy bounds on matter, have been neglected.

In order to be comparable with part of the literature and for completeness, we also performed the whole analysis following the effective approach, that is to absorb all the extra information of Hořava-Lifshitz gravity in an effective dark energy sector and consider the resulting universe in a general relativity background. Clearly this approach is not theoretically robust, and the fact that our final result is partially different than the one of the exact approach, offers an additional argument against the naive effective incorporation of gravity in applications where gravitational phenomena are important (such are the thermodynamic properties of the universe). This was already known, and that is why in all thermodynamic investigations in other modified gravitational theories the authors followed the exact approach instead of the effective one (see [120] and references therein).

In conclusion, we see that the thermodynamic properties of Hořava-Lifshitz gravity, either concerning the first law of thermodynamic and its relation to the Friedmann equations, or concerning the generalized second law, is a very interesting subject that deserves further investigation. However, one has to be careful and incorporate consistently the novel features of the theory. In the end of the day, the knowledge acquired from these studies will contribute to the acceptance or rejection of Hořava-Lifshitz gravity as a description of nature.
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