Z-induced FCNCs and their effects on Neutrino Oscillations

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Abstract

Adding singlet neutrinos to the standard model spectrum in general gives rise to Z-induced flavor-changing neutral currents. We study the impact of these currents on matter-induced neutrino oscillations in the sun and in supernovae. While the effects for solar neutrinos are negligible, dramatic effects are possible for supernova neutrinos.

1 Introduction

Neutrino oscillations \cite{1} can provide an appealing solution to the solar neutrino problem, which seems to be increasingly difficult to explain otherwise. All four solar neutrino experiments \cite{2, 3, 4, 5} observe a deficit of $\nu_e$'s compared to the Standard Solar Model (SSM) \cite{6} expectation. In particular, the apparently strongest depletion of intermediate energy neutrinos makes it very difficult if not impossible to account for these observations in any reasonable solar model \cite{7}. Moreover the recent helioseismological data confirms the SSM predictions for the solar density profile \cite{8} strengthening the belief that the solar neutrino problem is due to non-standard neutrino properties.

The features that are required to allow for neutrino oscillations are a non-vanishing mass-squared difference $\Delta_{ij} \equiv m_i^2 - m_j^2$ (implying that at least one neutrino state is massive) and mixing (i.e. the neutrino interaction eigenstates do not coincide with the mass eigenstates). The solar neutrino problem can then be solved by vacuum or matter-enhanced neutrino oscillations \cite{9} with

$$\Delta_{\text{vac}}^\text{sol} \simeq 10^{-10} \text{ eV}^2 \quad \text{or} \quad \Delta_{\text{mat}}^\text{sol} \simeq 10^{-5} \text{ eV}^2.$$  \hspace{1cm} (1)

The upper bounds \cite{10} on the light neutrino masses,
\[ m_{\nu_e} \leq 15 \text{ eV}, \quad m_{\nu_\mu} \leq 0.17 \text{ MeV}, \quad m_{\nu_\tau} \leq 24 \text{ MeV}, \quad (2) \]

are not in conflict with such a solution, although one has to explain why the postulated neutrino masses are so much smaller than the charged fermion masses.

The most popular solution is the Majorana see-saw mechanism \(^{11}\). It requires additional right-handed \(SU(2)_L\) singlet fermions with large Majorana masses which mix with the Standard Model (SM) neutrinos via \(SU(2)_L\) breaking Dirac masses. In its most common version this leads to a \(6 \times 6\) mass matrix of the form

\[
M = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix}. \quad (3)
\]

The entries of the Dirac mass matrix \(M_D\) are typically comparable to the masses of the charged leptons while the entries of the Majorana mass matrix \(M_R\) do not break weak \(SU(2)_L\) and are therefore related to some very high or intermediate New Physics (NP) scale \(\Lambda\). This leads to three ultra-light neutrinos with masses of order \(m_D^2/m_R\) \((m_X \equiv (\det M_X)^{1/3})\), and singlet admixtures in the light mass eigenstates of order \(m_D/m_R\). There are variants of the above see-saw mechanism in which expanded neutrino singlet sectors lead to much larger mixing with the known neutrinos. We return to a discussion of these scenarios later in the paper.

The point we wish to make is the following: In order to explain the solar neutrino problem (and other experimental anomalies like the atmospheric neutrino problem \(^{12}\) and the LSND results \(^{13, 14}\)) one requires small, but non-zero neutrino masses. The most attractive scenarios for obtaining such tiny masses involve see-saw suppression. The important ingredient is the presence of heavy \(SU(2)_L\) singlet neutrinos, which is rather common in NP models. However, mixing of the known neutrinos with singlets may also influence the neutrino oscillations that were invoked to solve the experimental anomalies in the first place, since they modify the neutral current interactions in a way that is flavor dependent. While the existence of this effect is generic in models that allow for neutrino masses, it has not been extensively investigated in the literature \(^{15}\). It is the purpose of this work to give a quantitative analysis of its importance for solar and supernova neutrinos. We find that phenomenological constraints on singlet mixing make its significance for solar neutrinos marginal at best, but allow for dramatic effects in the case of supernova neutrinos.

The paper is structured as follows: In Section 2 we introduce the formalism of \(Z\)-induced Flavor Changing Neutral Currents (FCNCs) and work out the changes to matter-enhanced neutrino oscillations in the presence of non-sequential neutrinos. Phenomenological constraints on neutrino-singlet mixing are discussed in Section 3. The implications for the Mikheyev-Smirnov-Wolfenstein (MSW) solution of the solar neutrino problem are studied in Section 4 and for oscillations of supernova neutrinos in Section 5. We conclude in Section 6.
2 Formalism

2.1 $Z$-mediated FCNCs

Consider a model where the lepton sector of the SM is extended in a non-sequential way (we first consider additional $SU(2)_L$ singlets, then briefly discuss the case of additional triplets in Section 2.3). We group the known $(K)$ and the new $(N)$ interaction eigenstates in the vector $\Psi^I = (\Psi_K, \Psi_N)^T$, which is related to the corresponding vector of light $(L)$ and heavy $(H)$ mass eigenstates $\Psi^M = (\Psi_L, \Psi_H)^T$ by a unitary transformation

$$
\begin{pmatrix}
\Psi_K \\
\Psi_N 
\end{pmatrix} = U
\begin{pmatrix}
\Psi_L \\
\Psi_H 
\end{pmatrix}
$$

where

$$
U = \begin{pmatrix}
U_{KL} & U_{KH} \\
U_{NL} & U_{NH}
\end{pmatrix}.
$$

(4)

The submatrices $U_{KL}$ and $U_{NL}$ describe the overlap of the light eigenstates with the known interaction states and the new interaction states, respectively.

It has been known for a long time [16] that the presence of new interaction eigenstates in general leads to FCNC neutrino interactions. To make this statement more precise consider the Neutral Current (NC) operator

$$
O_{NC} \equiv |\Psi_K\rangle \langle \Psi_K|,
$$

(5)

where the sum over $K = e, \mu, \tau$ is implicit. Effective four-Fermi couplings for $Z$-mediated neutrino scattering are proportional to $O_{NC}$. Trivially, the matrix elements of this operator in the basis of known interaction eigenstates $\{\Psi_K\}$ are

$$
\langle \Psi_K|O_{NC}|\Psi_K'\rangle = \delta_{KK'},
$$

(6)

while in the basis of light mass eigenstates they are

$$
\langle \Psi_{l}|O_{NC}|\Psi_{l'}\rangle = \langle \Psi_{l}|\Psi_K\rangle \langle \Psi_{l'}|\Psi_{K'}\rangle = U_{Kl}^* U_{K'l'}^{*}\,.
$$

(7)

The latter are in general not equal to $\delta_{ll'}$, since the submatrix $U_{KL}$ is not unitary. Thus there are “flavor”-changing NCs between the light mass eigenstates.

In most cases of interest the heavy mass eigenstates are not kinematically accessible. Hence we need to project the interaction eigenstates onto the subspace of light “propagating” mass eigenstates, if we want to calculate the effective operators at low energies. We denote the projected states by

$$
|\Psi^P_K\rangle \equiv |\Psi_{l}\rangle \langle \Psi_{l}|\Psi_K\rangle = U_{Kl}|\Psi_{l}\rangle.
$$

(8)

Note that these states are not orthonormal to each other, since

$$
\langle \Psi^P_K|\Psi^P_{K'}\rangle = \langle \Psi_{K}|\Psi_{l}\rangle \langle \Psi_{K'}|\Psi_{l}\rangle = U_{Kl}^* U_{K'l'}. 
$$

(9)
In the basis of these “light” interaction eigenstates the operator $O_{NC}$ is represented by the matrix

$$
\langle \Psi_{K'}^P | O_{NC} | \Psi_{K'}^P \rangle = \langle \Psi_{K'}^P | \Psi_{l}^I \rangle \langle \Psi_{l}^I | \Psi_{K''}^P \rangle \langle \Psi_{K''}^P | \Psi_{l}^I \rangle \langle \Psi_{l}^I | \Psi_{K'}^P \rangle = U_{K'l}^{*} U_{K''l} U_{K''l}^{*} U_{K'l},
$$

which is not in general diagonal since the basis $\{ \Psi_{l} \}$ does not span the space on which $O_{NC}$ is defined.

As a simple example consider the case of two known interaction eigenstates $\nu_{\ell}$ and $\nu_{\ell}$ ($\ell = \mu$ or $\tau$) and one new $SU(2)_L$ singlet $\nu_S$. The vector $\Psi^I = (\nu_{e}, \nu_{\ell}, \nu_S)^T$ is connected to the vector of mass eigenstates $\Psi^M = (\nu_1, \nu_2, \nu_h)^T$, where $\nu_1$ and $\nu_2$ are light while $\nu_h$ is heavy, by a unitary transformation

$$
\begin{pmatrix}
\nu_e \\
\nu_{\ell} \\
\nu_S
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} & U_{eh} \\
U_{\ell 1} & U_{\ell 2} & U_{\ell h} \\
U_{S1} & U_{S2} & U_{Sh}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_h
\end{pmatrix}.
$$

The “propagating” neutrinos that are produced in low-energy charged-current interactions together with the charged leptons $e$ and $\ell$ are

$$
\begin{pmatrix}
\nu_{e}^P \\
\nu_{\ell}^P
\end{pmatrix} =
\begin{pmatrix}
U_{e1} & U_{e2} \\
U_{\ell 1} & U_{\ell 2}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2
\end{pmatrix},
$$

i.e. we have projected $\nu_e$ and $\nu_\ell$ onto the $\nu_1 - \nu_2$ plane. Note that since $U_{KL}$ is not unitary, $\nu_{e}^P$ and $\nu_{\ell}^P$ are not orthogonal to each other,

$$
\langle \nu_{e}^P | \nu_{\ell}^P \rangle = U_{e1}^{*} U_{\ell 1} + U_{e2}^{*} U_{\ell 2} = -U_{eh}^{*} U_{\ell h},
$$

and are not properly normalized. Thus we cannot use these states as a proper basis for the description of neutrino oscillations. Instead, we choose an orthonormal basis $\{|\nu_{e}^O\rangle, |\nu_{\ell}^O\rangle\}$ where

$$
|\nu_{e}^O\rangle = \frac{U_{e1}|\nu_1\rangle + U_{e2}|\nu_2\rangle}{\sqrt{|U_{e1}|^2 + |U_{e2}|^2}}
$$

is aligned with $|\nu_{e}^P\rangle$ and

$$
|\nu_{\ell}^O\rangle = \frac{-U_{e2}^{*}|\nu_1\rangle + U_{e1}^{*}|\nu_2\rangle}{\sqrt{|U_{e1}|^2 + |U_{e2}|^2}}
$$

is orthogonal to $|\nu_{e}^O\rangle$. In this basis the off-diagonal matrix elements of $O_{NC}$ have magnitude

$$
|\langle \nu_{e}^O | O_{NC} | \nu_{\ell}^O \rangle| = \frac{|(U_{e1}^{*} U_{\ell 1} + U_{e2}^{*} U_{\ell 2})(-U_{e1}^{*} U_{\ell 2}^{*} + U_{e2}^{*} U_{\ell 1}^{*})|}{|U_{e1}|^2 + |U_{e2}|^2} \simeq |U_{eh}^{*} U_{\ell h}|.
$$
The approximation uses (13) and the fact that experimental constraints, that will be discussed in Section 3, imply $|U_{eh}|, |U_{th}|, |U_{S1}|, |U_{S2}| \ll |U_{sh}| \simeq 1$, from which it follows that

$$1 = |\det U| \simeq |\det \begin{pmatrix} U_{e1} & U_{e2} \\ U_{\ell1} & U_{\ell2} \end{pmatrix}| |U_{sh}|,$$

(17)
or $| - U_{\ell1}U_{e2} + U_{\ell2}U_{e1}| \simeq 1$. Note that in the oscillation-basis $\{\nu^{O}_{k}\}$ the operator $O_{NC}$ also has non-universal diagonal couplings

$$\langle \nu^{O}_{e}|O_{NC}|\nu^{O}_{e}\rangle = 1 - |U_{eh}|^2 + \frac{|U_{eh}U_{th}|^2}{1 - |U_{eh}|^2},$$

$$\langle \nu^{O}_{\ell}|O_{NC}|\nu^{O}_{\ell}\rangle = \frac{| - U_{\ell2}U_{e1} + U_{e1}U_{\ell2}|^2}{1 - |U_{eh}|^2}. \quad (18)$$

The off-diagonal element in (16) and the deviations from unity of the diagonal matrix elements in (18) are small since they are quadratic in the mixing between the doublet and singlet neutrinos.

### 2.2 NP-coupling of the $W$

The couplings of the $W$-boson are also affected by the presence of a heavy singlet neutrino. The effective four-Fermi couplings for $W$-mediated neutrino scattering in the sun or a supernova are proportional to the Charged Current (CC) operator

$$O_{CC} \equiv |\nu_e\rangle \langle \nu_e|.$$  

(19)

In our preferred basis $\{\nu^{O}_{e}, \nu^{O}_{\ell}\}$ for the description of neutrino oscillations its only non-vanishing matrix element is

$$\langle \nu^{O}_{e}|O_{CC}|\nu^{O}_{e}\rangle = \langle \nu^{O}_{\ell}|\nu^{O}_{\ell}\rangle \langle \nu^{O}_{\ell}|\nu^{O}_{\ell}\rangle = 1 - |U_{sh}|^2.$$  

(20)

The other matrix elements vanish since $\nu^{O}_{\ell}$ is orthogonal to $\nu_{e}$ by definition. Thus also in our orthonormal basis $O_{CC}$ only projects onto the electron neutrino, but with a prefactor that is slightly smaller than the standard one.

### 2.3 Adding Triplets

Additional $SU(2)_{L}$ singlets are not the only relevant extension of the lepton sector. We briefly investigate the effects of neutrino mixing with the neutral component of an $SU(2)_{L}$ triplet. Fermionic triplets arise naturally in the context of supersymmetric extensions of the SM. For example, the $SU(2)_{L}$ gauginos form such a triplet with hypercharge $Y = 0$. However, the neutral component has weak isospin $t_{3L} = 0$ so it does not couple to the $Z$ and the resulting formalism is the same as for singlets.
The neutral component of a triplet with $Y = \pm 1$ does couple to the $Z$. A good example is the superpartner of the neutral component of the Higgs triplet (with $Y = -1$) in supersymmetric left-right symmetric models. It has $t_{3L} = 1$ so we have to replace the NC-operator defined in (3) by

$$O'_{NC} \equiv O_{NC} + 2|\nu_N\rangle\langle\nu_N|.$$  

(21)

In the context of our simple example with two ordinary neutrinos and one exotic, the off-diagonal element of $O'_{NC}$ in the oscillation-basis is given by

$$\langle\nu^O_e | O'_{NC} | \nu^O_\ell \rangle = \langle\nu^O_e | O_{NC} | \nu^O_\ell \rangle = \left( U^*_e U_{N1} + U^*_e U_{N2} \right)(-U^*_{N1} U^*_{e2} + U^*_{N2} U^*_{e1}) \left| U^*_e \right|^2 \left| U^*_e \right|^2,$$

(22)

which is quadratic in the mixing between ordinary and exotic neutrinos, as in (16).

### 2.4 Effective $Z$-induced NP-couplings

In this section we compute the effective four-Fermi couplings, $G^f_N$, for the neutrino flavor-changing reactions $\nu^O_e f \rightarrow \nu^O_\ell f$ ($f = e, u, d$) mediated by the $Z$, in the presence of a neutrino singlet. We note that all three couplings are determined by the single parameter

$$\varepsilon \equiv \langle\nu^O_e | O_{NC} | \nu^O_\ell \rangle,$$

(23)

which essentially gives the ratio between the $Z$-induced flavor-changing NC amplitudes and the usual flavor diagonal NC amplitudes. For simplicity, we will ignore the possibility of CP violation, taking $\varepsilon$ to be real throughout. The flavor-changing four-Fermi couplings follow directly from known results [17] for the flavor-diagonal NCs, which are reviewed below.

The potential $V_{NC}$ due to $Z$-mediated neutrino scattering off the thermal background fermions can be deduced from the relevant (Fierz rearranged) four-Fermi interaction

$$H_f = \frac{G_F}{\sqrt{2}} \bar f \gamma_\mu (g^f_V - \gamma_5 g^f_A) f \nu \gamma^\mu (1 - \gamma_5) \nu.$$  

(24)

The vector and axial couplings are

$$g^f_V = t^f_{3L} - 2q^f \sin^2 \theta_W$$

(25)

$$g^f_A = t^f_{3L},$$

(26)

where $t^f_{3L}$ is the weak isospin of the fermion $f$ and $q^f$ is its electro-magnetic charge. We assume unpolarized background fermions and therefore only the $\gamma_0$ component of the fermion
density can contribute. The $\gamma_0 \gamma_5$ mixes “small” and “large” components of the spinor, so the axial coupling does not contribute. For the large component $\gamma_0$ is 1, thus the charged fermion coupling reduces to
\[ \bar{f} \gamma_\mu (g_V^f - \gamma_5 g_A^f) f = \delta_{\mu 0} N_f (t_{3L}^f - 2 q^f \sin^2 \theta_W), \] (27)
where $N_f$ is the fermion number density. Hence the term describing NC interactions in the effective Hamiltonian for neutrino propagation is
\[ H_{NC}^f = \frac{G_F N_f}{\sqrt{2}} (2 t_{3L}^f - 4 q^f \sin^2 \theta_W) \times |\nu_\kappa\rangle \langle \nu_\kappa|, \] (28)
Note that the $\bar{\nu} \gamma^\mu \frac{1}{2} \gamma_\nu$ coupling yields the unit matrix $|\nu_\kappa\rangle \langle \nu_\kappa|$ in the basis of weak neutrino eigenstates, which is just $O_{NC}$. The factor $V_{NC}^f$ multiplying $O_{NC}$ takes the values
\[
V_{NC}^e &= G_F N_e (4 \sin^2 \theta_W - 1) / \sqrt{2}, \quad (29)
V_{NC}^u &= G_F N_u (-8/3 \sin^2 \theta_W + 1) / \sqrt{2}, \quad (30)
V_{NC}^d &= G_F N_d (4/3 \sin^2 \theta_W - 1) / \sqrt{2}, \quad (31)
V_{NC}^p &= G_F N_p (-4 \sin^2 \theta_W + 1) / \sqrt{2}, \quad (32)
V_{NC}^n &= -G_F N_n / \sqrt{2}, \quad (33)
\]
The contributions for the nucleons are obtained by summing the quark potentials, i.e. $V_{NC}^p = 2V_{NC}^u + V_{NC}^d$ and $V_{NC}^n = V_{NC}^u + 2V_{NC}^d$. Note that the electron and the proton contributions cancel in electrically neutral media.

In the presence of neutrino singlets, the effective flavor-changing four-Fermi couplings $G_N^f$ are given by
\[ G_N^f \equiv \frac{\langle \nu_e^O | H_{NC}^f | \nu_e^O \rangle}{\sqrt{2 N_f}} = \frac{V_{NC}^f}{\sqrt{2 N_f}} \varepsilon, \] (34)
Equivalently, the flavor-changing NC potentials are
\[ V_{FCNC}^f = \varepsilon V_{NC}^f. \] (35)
Note that the effective flavor-diagonal NC potentials are decreased slightly by the deviations of the flavor-diagonal matrix elements $\langle \nu_e^O | O_{NC} | \nu_e^O \rangle$ and $\langle \nu_\ell^O | O_{NC} | \nu_\ell^O \rangle$ in eq. (15) from unity.
2.5 Modifications to Matter Oscillations

The MSW mechanism [9] provides an elegant solution to the solar neutrino problem and it might also be important for neutrinos that are produced in a supernova explosion. In Ref. [18] modifications of the MSW solution due to flavor-changing neutrino interactions were considered. Although the new non-diagonal four-Fermi couplings were suggested to stem from the exchange of new heavy particles, the results obtained in [18] can also be used to discuss matter-enhanced neutrino oscillations in the presence of $Z$-induced FCNCs.

We have shown that the projection of the known interaction eigenstates onto the subspace spanned by the light mass eigenstates results in flavor-changing NC neutrino interactions of strength $\varepsilon$ with respect to the flavor-diagonal NCs. Thus, knowing the standard NC-contributions it is straightforward to obtain the correct expressions for the off-diagonal terms in the effective Hamiltonian $H_N$ for neutrino propagation in matter

$$H_N(\nu^O) = \frac{1}{4E} \begin{pmatrix} -\Delta \cos 2\theta + A' & \Delta \sin 2\theta + B \\ \Delta \sin 2\theta + B & \Delta \cos 2\theta - A' \end{pmatrix} (\nu^O_\ell \nu^O_e).$$

(36)

Here $E$ is the neutrino energy, $\Delta \equiv m_2^2 - m_1^2$ is the mass-squared difference of the two vacuum mass eigenstates, $\theta$ is the vacuum mixing angle, and

$$A' \equiv A \left\{ (1 - |U_{eh}|^2) - \frac{N_n}{2N_e} \left[ 1 - |U_{eh}|^2 + \frac{|U_{eh}^* U_{\ell h}|^2}{1 - |U_{eh}|^2} - \frac{|-U_{e2} U_{\ell 1} + U_{e1} U_{\ell 2}|^2}{1 - |U_{eh}|^2} \right] \right\}. \quad (37)$$

$A'$ reduces to the standard induced mass $A \equiv 2E\sqrt{2}G_F N_e$ in the limit where the heavy singlet neutrino decouples ($|U_{S3}| \rightarrow 1$). The parameter

$$B \equiv 4E\sqrt{2}(G_N^e N_e + G_N^u N_u + G_N^d N_d) \quad (38)$$

describes the FCNC contributions from neutrino scattering off electrons and quarks in the sun or supernovae. It is convenient for later purposes to rewrite this as

$$B = 4E\sqrt{2}G_F N_e \varepsilon(N_n/N_e), \quad (39)$$

where

$$\varepsilon(x) \equiv \frac{1}{G_F} \left[ G_N^e + 2G_N^u + G_N^d + (G_N^u + 2G_N^d)x \right]. \quad (40)$$

We have used the fact that the quark densities can be expressed in terms of $N_n$ and $N_e$ (which equals the proton density in neutral matter). To compute $B$ we set $\frac{B}{4E} = \sum_f \varepsilon V_{NC}^f$ and obtain

$$B = -2E\sqrt{2}G_F N_n \varepsilon \simeq \pm 2E\sqrt{2}G_F N_n |U_{eh}^* U_{\ell h}|. \quad (41)$$
Note that due to the cancelation of $V_{NC}^e$ and $V_{NC}^p$ the parameter $B$ is proportional to $N_n$. Interestingly, this implies that the strongest possible effect arises from variation of ratio $x = N_n/N_e$.

3 Constraints

We now discuss experimental constraints on mixing between the ordinary neutrinos and $SU(2)_L$ singlets. A direct bound on $|U_{eh}^* U_{\mu h}|$ comes from the KARMEN [19] experiment searching for “neutrino-oscillations” in the $\bar{\nu}_e - \bar{\nu}_\mu$ channel. The upper bound on the transition probability $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ yields

$$|U_{eh}^* U_{\mu h}|^2 \simeq P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) < 3.1 \times 10^{-3} \ (90\% \ c.l.).$$

This restricts $|U_{eh}^* U_{\mu h}|$ to be less than 0.06, which allows for rather large FCNCs.

Much tighter bounds on mixing with a singlet neutrino are obtained from constraints on lepton universality, CKM unitarity, and the measured $Z$ invisible decay width [20]. Rather than fitting the weak couplings of all known fermions we essentially update the analysis of Ref. [21], which considers the simpler possibility that only the neutrinos mix with exotic fermions. ($|U_{ah}|^2$ is equivalent to $s_{2
u a}^2$ in the notation of [21].) The ratios of the leptonic couplings $g_e, g_\mu, g_\tau$ to the $W$ are given by

$$\left( \frac{g_a}{g_b} \right)^2 = \frac{1 - |U_{ah}|^2}{1 - |U_{bh}|^2}, \quad a, b = e, \mu, \tau. \quad (43)$$

The best test of $e-\mu$ universality of the $W$ couplings comes from comparison of the rates for $\pi \rightarrow \mu\bar{\nu}_\mu$ and $\pi \rightarrow e\bar{\nu}_e$, where the two most precise experiments [22] can be combined [23] to yield $g_\mu/g_e = 1.0012 \pm 0.0016$. The best tests of $\mu-\tau$ universality come from ALEPH measurements of leptonic [24] and hadronic [25] $\tau$ decays. Averaging the two determinations of $g_\tau/g_\mu$ (they are fully consistent with each other) yields $g_\tau/g_\mu = 0.9943 \pm 0.0065$. From the leptonic decays one also obtains [24] $g_\tau/g_e = 0.9946 \pm 0.0064$.

The unitarity constraint for the first row of the CKM matrix is

$$\sum_{i=1,2,3} |V_{ui}|^2 = (1 - |U_{\mu h}|^2)^{-1}. \quad (44)$$

The most recent experimental determination [26] is $\sum_{i=1,2,3} |V_{ui}|^2 = 0.9972 \pm 0.0013$, $2\sigma$ away from the SM value of unity. Finally, the ratio of the $Z$ invisible decay width to the SM prediction is given by [21]

$$\frac{\Gamma_{Z \rightarrow inv}}{\Gamma_{Z \rightarrow inv}^{SM}} = 1 - \frac{|U_{eh}|^2}{6} - \frac{|U_{\mu h}|^2}{6} - \frac{2|U_{\tau h}|^2}{3}. \quad (45)$$
Combining the most recent measurement of $\Gamma_{Z \rightarrow \text{inv}}$ \cite{27}, obtained under the assumption of universal charged lepton couplings to the $Z$, with the SM prediction (for $m_t = 175.6 \pm 5.5$ GeV) yields $\Gamma_{Z \rightarrow \text{inv}}/\Gamma_{Z \rightarrow \text{inv}}^{SM} = 0.995 \pm 0.004$.

We construct a $\chi^2$ function with the experimental measurements discussed above, and derive bounds on the mixing parameters using the MINUIT package. Allowing for singlet mixing with all known neutrinos we obtain the 90% c.l. bounds

$$|U_{eh}|^2 < 0.0049, \quad |U_{\mu h}|^2 < 0.004,$$

and $|U_{\mu h}|^2 = -0.0028 \pm 0.0013$, which is $2\sigma$ away from the standard model but in the “wrong direction” for singlet mixing. This is due to the deviation of the CKM unitarity sum from unity. However, we note that this does not rule out $\nu_\mu$-singlet mixing since a discrepancy in this direction could be accounted for by mixing of the $u$ and $d$ quarks with $SU(2)_L$ singlets. We therefore present a second set of more conservative bounds (90% c.l.) in which all known neutrinos are allowed to mix with a singlet but the unitarity constraint has been eliminated:

$$|U_{eh}|^2 < 0.012, \quad |U_{\mu h}|^2 < 0.0096, \quad |U_{\tau h}|^2 < 0.016.$$

We have not included the possibility of mixing between the charged leptons and exotics, nor have we taken into account correlations between ALEPH’s determinations of $g_\tau/g_\mu$ and $g_\tau/g_\epsilon$ in \cite{24}. Nevertheless we can conclude that our parameter $\varepsilon$ should be less than about 1%, with the largest values possible corresponding to conversion of $\nu_e$ to $\nu_\tau$.

### 4 Solar Neutrino Oscillations

Using the results of Section 2.3 it is straightforward to obtain the survival probability \cite{28, 1, 18} for a $\nu_e$ that was produced in the solar center to be detected as an electron neutrino

$$P_N(\nu_e \rightarrow \nu_e) = \frac{1}{2} + \left( \frac{1}{2} - P_c \right) \cos 2\theta \cos 2\theta_N,$$

where the effective mixing is given by

$$\cos 2\theta_N = \frac{(\Delta \cos 2\theta - A')}{\sqrt{(\Delta \cos 2\theta - A')^2 + (\Delta \sin 2\theta + B)^2}}.$$

The crossing probability $P_c$ is well approximated by \cite{29}

$$P_c = \Theta(A_{prod} - A_{res}) \times \frac{\exp \left[ \pi \gamma N F(\theta)/2 \right] - \exp \left[ \pi \gamma N F(\theta)/2 \sin^2 2\theta \right]}{1 - \exp \left[ \pi \gamma N F(\theta)/2 \sin^2 2\theta \right]}.$$
Here $\gamma_N$ denotes the adiabaticity parameter which is given by [18]

$$
\gamma_N = \frac{\Delta \sin^2 2\theta}{2E \cos 2\theta \left(\frac{dN_e}{dx}\right)/N_e|_{res}} \times |1 + 2\epsilon(x_{res}) \cot 2\theta|^2,
$$

(51)

where $x_{res}$ is the ratio $N_n/N_p$ at the resonance. Using these results we have calculated the suppression rate for the three types of solar neutrino experiments as a function of $\Delta$ and $\sin^2 2\theta$. The calculations have been performed along similar lines to those described in Ref. [18]. We present the individual 95% c.l. contours (dashed for Homestake [2], dotted for the combined gallium experiments [3, 4], and solid for Kamiokande [5]) together with the combined allowed regions (shaded) for various positive (Fig. 1) and negative (Fig. 2) values of $\epsilon$.

Although $Z$-induced FCNCs modify the individual contours at very small $\sin^2 2\theta$, the combined “small-angle solution” is (almost) unchanged for $\epsilon = \pm 0.02$ and the large-angle solution is not affected at all. In fact, even for larger $\epsilon$ there are no dramatic changes to the standard small-angle MSW-solution. In light of the experimental constraints on $\epsilon$ presented in Section 3 we conclude that $Z$-induced FCNCs must have negligible impact on solar neutrino oscillations. The cancelation of the NC potentials from electron and proton scattering leads to effects proportional to the neutron density $N_n$, which in our sun is at most half of the charged particle densities $N_e = N_p$. This is unlike scenarios with flavor-changing neutrino interactions induced by new heavy particle exchange [30, 31, 32, 33, 34], e.g., supersymmetric models without $R$-parity, where the different contributions from scattering off quarks and electrons can add up constructively.

5 Supernova Neutrino Oscillations

Supernova explosions are intense neutrino sources. Due to the huge supernova densities these neutrinos can be resonantly converted for a large range of the parameters $\Delta$ and $\sin^2 2\theta$. The impact of neutrino oscillations has been discussed extensively in [35, 36, 37, 38]. Also it has been noted that neutrino oscillations might help to solve the shock reheating problem [39]. In Refs. [40, 41] resonant neutrino conversions were studied in the presence of supersymmetric $R$-parity violating interactions and $Z$-induced FCNCs, but for massless neutrinos. In this Section we investigate how FCNCs due to mixing of the known neutrinos with singlets alter the MSW resonant conversion of massive neutrinos emerging from the neutrino-sphere of a supernova. The object we investigate is the survival probability $P(\nu_e \rightarrow \nu_e)$. It has been shown in Ref. [36] that $P(\nu_e \rightarrow \nu_e)$ and $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ are the only quantities that must be specified to determine how neutrino oscillations mix the fluxes. We assume that the neutrino parameters are such that matter-enhanced neutrino oscillations can occur for the neutrinos but not the anti-neutrinos. Moreover, we neglect neutrino-neutrino scattering [38] which is justified for neutrino propagation outside the neutrino-sphere. For the point we wish to make it is sufficient to discuss the impact of FCNCs on the survival probability itself, so we do not convolute $P(\nu_e \rightarrow \nu_e)$ with the
predicted neutrino fluxes and cross sections in order to compute the experimental rates. (Note that from the neutrino data of SN1987a [12] one cannot obtain a reasonable energy spectrum. Moreover, it might well be that NP processes, like Z-induced FCNCs, have an effect on the flux of produced neutrinos, which has not been considered in current supernova simulations [13].)

As shown in Ref. [36], the following modifications of the solar MSW formalism are required in the case of supernova neutrinos:

(a) The electron-density at the neutrino-sphere \((N_e)_{\text{prod}} \sim 10^{35}\) cm\(^{-3}\) is larger than the solar core density \((N_e)_{\text{core}} \sim 10^{25}\) cm\(^{-3}\) by about ten orders of magnitude. Since the supernova neutrinos are not much more energetic than solar neutrinos, an adiabatic threshold energy

\[
E_A \equiv \frac{\Delta \cos 2\theta}{2\sqrt{2}G_F (N_e)_{\text{prod}}} \quad (52)
\]

of the order of a few MeV will scale the adiabatic band by a factor \(\sim 10^{10}\) with respect to the solar case, to \(\Delta_{\text{max}} \sim 10^6\) eV\(^2\). Above this value neutrinos cannot be resonantly converted, since they are produced below the resonance. Note that \(E_A\) is not changed by flavor-changing neutrino interactions [18].

(b) Unlike the solar density profile, which decays (roughly) exponentially, the supernova density \(\rho(r)\) is predicted to decrease like \(1/r^3\) (\(r\) being the distance to the core outside the neutrino-sphere), but for the sake of generality we will just assume a power law (with \(\alpha > 0\)):

\[
\rho(r) = \rho(R) \left(\frac{r}{R}\right)^{-\alpha}. \quad (53)
\]

For a density described by (53) the scaling factor \(N_e'/N_e\) that appears in the adiabaticity factor \(\gamma_N\) [defined in Eq. (51)] is not constant (like for an exponentially decaying density profile), but:

\[
\frac{N_e'}{N_e} = \frac{-\alpha}{r} = \left(\frac{-\alpha}{R}\right) \left[\frac{N_e(r)}{N_e(R)}\right]^{1/\alpha}, \quad (54)
\]

where we assume that the electron number density \(N_e = \rho Y_e/m_N\) \((m_N\) is the nucleon mass) is proportional to the mass density \(\rho\) (in fact \(Y_e \simeq 0.4\) is constant to a good approximation outside the neutrino-sphere). To obtain \(\gamma_N\) we have to take \(N_e\) in (54) at the resonance

\[
N_e^{\text{res}} = \frac{\Delta \cos 2\theta}{2\sqrt{2}G_F E}. \quad (55)
\]

resulting in
\[
\gamma_N = \frac{\sin^2 2\theta |1 + 2\epsilon(x_{res})\cot 2\theta|^2 R}{2E\Delta \cos 2\theta} \left( \frac{2\sqrt{2}G_FN_e(R)}{\Delta \cos 2\theta} \right)^{1/\alpha} \\
= E^{\frac{1}{\alpha}} \Delta^{\frac{\alpha-1}{\alpha}} (\cos 2\theta)^{-\frac{\alpha+1}{\alpha}} \left| \sin 2\theta + 2\epsilon(x_{res})\cos 2\theta \right|^2 \\
\times \frac{R}{2\alpha} \left[ 2\sqrt{2}G_FN_e(R) \right]^{1/\alpha}.
\] 

(56)

Thus, the effective non-adiabatic threshold energy is

\[
E_{NA} = \Delta (\cos 2\theta)^{-\frac{\alpha+1}{\alpha}} \left| \sin 2\theta + 2\epsilon(x_{res})\cos 2\theta \right|^2 \times \left[ 2\sqrt{2}G_FN_e(R) \right]^{\frac{1}{2\alpha}} \times \left( \frac{\pi R}{4\alpha} \right)^{\frac{\alpha}{2\alpha}} \\
= 3.7 \times 10^9 \text{ MeV} \left( \frac{\Delta}{\text{eV}^2} \right) \cos^{-2} 2\theta \left| \sin 2\theta + 2\epsilon(x_{res})\cos 2\theta \right|^3,
\] 

(57)

where for the last line we have taken typical supernova values, i.e., \( R = 10^7 \text{ cm} \) for the radius of the neutrinosphere, \( \rho(R) = 10^{12} \text{ g/cm}^3 \) for the density at \( R \) and \( \alpha = 3 \). From (57) it is clear that the non-adiabatic band starts off at similar values of \( \Delta_{min} \sim 10^{-9} \text{ eV}^2 \) as in the sun, but has a different slope of \(-3/2\) (for \( \epsilon(x_{res}) = 0 \)) in the (logarithmic) \( \Delta - \sin^2 2\theta \) plane.

(c) Since the central supernova density is so huge, the “higher” \( e - \tau \) resonance almost always precedes the “lower” \( e - \mu \) resonance. Thus the authors of Ref. [36] have pointed out that a proper treatment of supernova neutrino oscillations should be done within a 3-flavor formalism. Moreover they have noted in [44] that for small mixing angles the survival probability factorizes into

\[
P(\nu_e \rightarrow \nu_e) = P_l(\nu_e \rightarrow \nu_e) \times P_h(\nu_e \rightarrow \nu_e),
\] 

(58)

where \( P_{Lh} \) are the standard two-level survival probabilities for the lower and higher resonances, respectively. Since flavor-changing neutrino interactions become important when the mixing is small (i.e. \( \tan 2\theta \lesssim |\epsilon| \)), eq. (58) is sufficient for our analysis. We will assume that \( \Delta_l \) and \( \sin^2 2\theta_l \) of the lower resonance are fixed by the standard MSW-solution to the solar neutrino problem (rather than fixing the ratios of \( \Delta_l/\Delta_h \) and \( \sin^2 2\theta_l/\sin^2 2\theta_h \) at some arbitrary value) in order to obtain a two-dimensional plot. Then, at fixed energy \( E_\nu \), the survival probability \( P(\nu_e \rightarrow \nu_e) \) is just a constant \( P_l \) multiplying \( P_h(E_\nu, \sin 2\theta_h, \Delta_h, \epsilon) \) which we show in Fig. 3. The plot exhibits the features of the supernova “MSW-triangle” that we discussed in (a) and (b): The adiabatic band appears at very large \( \Delta_{max} \sim 10^6 \text{ eV}^2 \) and the non-adiabatic band starts off at \( \Delta_{min} \sim 10^{-9} \text{ eV}^2 \) for \( \sin^2 2\theta = 1 \) extending to very small mixing \( \sin^2 2\theta \sim 10^{-10} \) where it connects to the adiabatic band.

The important consequence of this is that the supernova triangle is sensitive to even tiny FCNCs. One can see this clearly in Fig. 4 and Fig. 5 where we show the survival probability.
for various positive and negative $\varepsilon$. The effect can be easily understood in terms of the adiabaticity parameter $\gamma_N$. Without FCNCs $\gamma \propto \Delta^{(2/3)} \sin^2 2\theta$ for small vacuum mixing. Thus for smaller $\Delta$ (and fixed $\sin^2 2\theta$) the propagation is less adiabatic and there will be a minimal value $\Delta_{\text{min}}$ for each value of $\sin^2 2\theta$ where most of the neutrinos “cross-over”, resulting in a large survival probability. The non-adiabatic band is the contour defined by $\Delta_{\text{min}}(\sin^2 2\theta)$ which separates the regions where the adiabatic conversion is efficient (above) and where it is not (below). As we have mentioned, this band is a straight line (with slope $-3/2$) if there are no FCNCs. However, if $\varepsilon \neq 0$ then the adiabaticity parameter behaves like $\gamma_N \propto \Delta^{(2/3)} | \sin 2\theta + 2\varepsilon |^2$ for small vacuum mixing. Thus for $\sin 2\theta \ll |\varepsilon|$, $\Delta_{\text{min}}$ is determined by $\varepsilon$, the strength of the FCNCs, and not by the vacuum mixing $\sin 2\theta$ as can be seen in Fig. 4 and Fig. 5. Simply, in this regime the off-diagonal elements of the effective Hamiltonian in (33) are dominated by the FCNC term rather than the mixing term. Note that for positive $\varepsilon$ (corresponding to negative $\varepsilon$) the two competing contributions from FCNCs and mixing can cancel each other resulting in a vanishing $\gamma_N$ which implies a large survival probability around $\sin 2\theta_{\text{div}} = -2\varepsilon$ (for a more detailed discussion see Ref. [18]).

The important result is that neutrino FCNC effects can be very significant for supernova neutrinos.

### 6 Conclusions

We have argued that neutrino singlets are almost unavoidable in any framework that attempts to solve neutrino anomalies by neutrino oscillations. The additional heavy neutrinos give rise to $Z$-induced FCNC interactions in the effective matter propagation matrix, and the question of whether or not they can be neglected is rather a quantitative than qualitative one. We have worked out the resulting modifications to the MSW mechanism in order to study FCNC effects on matter-enhanced neutrino oscillations in the sun and in supernovae. We have found that while phenomenological constraints rule out significant changes in the solar neutrino MSW-solutions, the impact of FCNCs on the survival probability of supernova neutrinos can be large, even for very small singlet components in the standard neutrino mass eigenstates.

We conclude by asking whether values of $\varepsilon$ which are large enough to be of relevance to supernova neutrinos naturally occur in scenarios employing a neutrino see-saw mechanism. For example, this is easily seen not to be the case for the original see-saw matrix in eq. (3). The singlet-doublet mixing is of order $m_D/m_R$ so that $\varepsilon \sim m_D^2/m_R^2$. This should be $\gtrsim 10^{-5}$ for light-heavy mixing effects to be relevant for supernova neutrinos. But in this case the light neutrino masses would only be suppressed by a factor $m_D/m_R \gtrsim 10^{-3}$ with respect to the Dirac masses. If the latter are reasonably large, e.g., of the order of the charged lepton masses, this would be inconsistent with the ultra-light neutrinos usually required to explain the solar neutrino problem (although strictly speaking only $\Delta$ has to be tiny).

However, as already noted in the introduction, there are variants of the above scenario which can lead to much larger singlet admixtures. In a popular alternative, the Majorana
“double see-saw” \[43\], two singlets are added (per generation) leading to a $9 \times 9$ mass matrix of the form

$$M' = \begin{pmatrix}
0 & M_D & 0 \\
M_D^T & 0 & M_R \\
0 & M_R^T & M_S
\end{pmatrix}.$$ \[59\]

For $m_D, m_S \ll m_R$ ($m_X \equiv (\det M_X)^{1/3}$) singlet-doublet mixing is still of order $m_D/m_R$ but the light neutrino masses are of order $m_S(m_D/m_R)^2$. Hence in this framework one can obtain ultra-light neutrinos while having a ratio $m_D/m_R \gtrsim 10^{-3}$ that induces FCNCs that are significant for the MSW-effect in supernovae. Since the Dirac masses of the neutrinos have to be smaller than the weak scale, $m_{weak}$, it follows that as long as the mixing is $\sim m_D/m_R$ the $Z$-induced FCNCs are potentially relevant to our discussion if $m_R \lesssim 100$ TeV.

Finally, vectorlike pairs of $SU(2)_L$ singlet quarks and leptons with large masses $m_V$ are often introduced in order to suppress known quark and charged lepton masses relative to the weak scale via a generalized “Dirac see-saw” \[13\], leading to left-handed singlet components in the ordinary charged fermion mass eigenstates of order $m_{weak}/m_V$. If vectorlike pairs of neutral singlets are also present in such a scenario the known neutrinos could be expected to mix with the left-handed singlets at same order as the charged fermions, in addition to mixing with right-handed singlets responsible for an ultra-light Majorana mass see-saw. (In the absence of a Majorana see-saw the vectorlike singlets would typically lead to neutrino Dirac masses which are of same order as the charged lepton masses.) As in the above example, singlet mixing at the level of interest for supernova oscillations, i.e., $m_{weak}/m_V \gtrsim 10^{-3}$, would correspond to a New Physics mass scale $m_V \lesssim 100$ TeV. The right-handed neutrino Majorana mass scale $m_R$ could be arbitrarily large \[14\], thus allowing for ultra-light neutrino masses which are consistent with the MSW solution for solar neutrinos.

**Note added:** When this work was near completion we learned of another paper \[48\] that analyzed the effects of FCNCs on supernova oscillations in the context of supersymmetric models with broken $R$-parity, arriving at effects of similar magnitude to those presented in our analysis.

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Figure 1: The combined allowed regions for the solar neutrino experiments with $\varepsilon \geq 0$ as indicated above each plot. The dotted contours correspond to the combined gallium experiments, the dashed contours to the Homestake experiment and the solid contours to the Kamiokande experiment. The shaded areas indicate the 95% c.l. combined allowed regions in the $\sin^2 2\theta - \Delta$ plane.
Figure 2: The combined allowed regions for the solar neutrino experiments with $\varepsilon \leq 0$ as indicated above each plot. The dotted contours correspond to the combined gallium experiments, the dashed contours to the Homestake experiment and the solid contours to the Kamiokande experiment. The shaded areas indicate the 95% c.l. combined allowed regions in the $\sin^2 2\theta - \Delta$ plane.
Figure 3: The MSW-contours for supernova neutrinos with $\varepsilon = 0$ at one discrete energy ($E_\nu = 10$ MeV). The shading indicates the value of the survival probability $P_h(\nu_e \rightarrow \nu_e)$ in the $\sin^2 2\theta - \Delta$ plane: White corresponds to $0.9 \leq P_h \leq 1.0$ and the darkest area corresponds to $0.0 \leq P_h \leq 0.1$.
$\Delta \text{[eV}^2\text{]}$

Figure 4: The MSW-contours for supernova neutrinos with $\varepsilon > 0$ (indicated above each plot) at one discrete energy ($E_\nu = 10$ MeV). The shading indicates the value of the survival probability $P_h(\nu_e \rightarrow \nu_e)$ in the $\sin^2 2\theta - \Delta$ plane: White corresponds to $0.9 \leq P_h \leq 1.0$ and the darkest area corresponds to $0.0 \leq P_h \leq 0.1$. 
Figure 5: The MSW-contours for supernova neutrinos with \( \varepsilon < 0 \) (indicated above each plot) at one discrete energy (\( E_\nu = 10 \text{ MeV} \)). The shading indicates the value of the survival probability \( P_h(\nu_e \rightarrow \nu_e) \) in the \( \sin^2 2\theta - \Delta \) plane: White corresponds to \( 0.9 \leq P_h \leq 1.0 \) and the darkest area corresponds to \( 0.0 \leq P_h \leq 0.1 \).