Binary Star Population with Common Proper Motion in Gaia DR2

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Abstract—We describe a homogeneous catalog compilation of common proper motion stars based on Gaia DR2. A preliminary list of all pairs of stars within the radius of 100 pc around the Sun with a separation less than a parsec was compiled. Also, a subset of comoving pairs, wide binary stars, was selected. The clusters and systems with multiplicity larger than 2 were excluded from consideration. The resulting catalog contains 10358 pairs of stars. The catalog selectivity function was estimated by comparison with a set of randomly selected field stars and with a model sample obtained by population synthesis. The estimates of the star masses in the catalogued objects, both components of which belong to the main-sequence, show an excess of “twins”, composed by stars with similar masses. This excess decreases with increasing separation between components. It is shown that such an effect cannot be a consequence of the selectivity function only and does not appear in the model where star formation of similar masses is not artificially preferred. The article is based on the talk presented at the conference “Astrometry yesterday, today, tomorrow” (Sternberg Astronomical Institute of the Moscow State University, October 14–16, 2019).

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1. INTRODUCTION

A significant proportion of the stellar population is concentrated in the binary stars. According to some estimates [1], a half of all main-sequence stars are components of binary and multiple systems. Binaries are an important component of the Galaxy’s stellar population, which affects the stellar system evolution as a whole. In addition, the parameters of stellar pairs can provide us important information about star formation.

It is generally accepted that the stars are born in large groups, which eventually decay into separate systems. The vast majority of binary and multiple stars are the remainders of such groups. Thus, examining a multiple system, its components likely formed simultaneously, under the same conditions [2]. The study of stellar systems with various characteristics allows one to advance in solving many astrophysical problems (see, i.e., the discussion in [3]). The dynamic connection between the components of pairs itself makes it possible to evaluate some of the physical parameters of them directly. Interaction between components of close pairs leads to the formation of various astronomical objects attractive for the study, e.g., Novae, type Ia Supernovae, pulsars, and symbiotic binary stars.

In the present paper, we consider the population of wide binary stars. Thus, the pairs with separation of components are so large, their evolution proceeds in the same way as in single stars. Among such binaries, there are both pairs with observed orbital motion and pairs with a common proper motion (comoving ones) with orbital periods from several thousand to millions of years and with the distance between components up to many thousands of astronomical units [4]. Wide binaries are loosely bound and can be easily destroyed by the heterogeneities of the Galaxy’s potential, for instance, due to the close passage of giant molecular clouds. This makes such systems valuable indicators of the Galaxy’s dynamic environment. The distribution of wide binary stars over masses of components and the distance between them makes it possible to elucidate the features of star formation processes.

The present study is aimed to the population properties of wide binaries and common proper motion stars at the distance of up to 100 pc from the Sun, identified in the study of the catalog of candidates for pairs with a common proper motion [5], created on the basis of the Gaia DR2 database.

Section 2 briefly outlines the principles of creating the catalog and refining it. In Section 3, the selected population parameters of binary stars with common proper motion are investigated and discussed. Conclusions are drawn in Section 4.

2. THE CATALOG OF BINARY AND COMOVING STARS

Candidate binary stars were selected among the objects of the Gaia DR2 catalog that have parallaxes 10 mas ≤ π ≤ 100 mas, which correspond, when esti-
mating the distance to the star by the parallax \( \frac{1000}{\varpi} \) (pc), to the stars located inside a spherical layer with an inner radius of 10 pc and an outer radius of 100 pc around the Sun. The lower limit of the distance to the Sun is due to computational constraints related to the procedure of candidate selection (see Appendix). The upper limit of distance (100 pc) is selected for a number of the following reasons. Within this volume, the characteristic relative parallax error does not exceed 10%, which, in fact, allows us to use the parallax value for estimating distances as described above (see, for example, [6, 7]). In addition, Gaia DR2 completeness limit for faint stars is \( G \approx 17^m \) [8], which means that it includes all stars of the lower part of the main-sequence within 100 pc, at least up to the spectral subtype M4.5 (such stars, according to Mamajek’s data,1 have absolute magnitude \( G \approx 12^m \)). Within the Gaia horizon (up to the apparent magnitude \( G \approx 21^m \)) are all dwarfs, at least down to the spectral subtype M10.

As potential candidates for binary components, not all Gaia sources in this field are considered, but only those satisfying the quality limits of the astrometric and photometric solutions (see [9], as well as Gaia DR2 Known Issues,2 and the recommendations on Gaia DR2 astrometry use).3

The following restrictions on the astrometric solution quality were applied: first, the RUWE (Renormalized Unit Weight Error) parameter should not exceed 1.4 and, second, the nominal relative parallax error should not exceed 10%. The quality of the photometric solution was checked by the limit on the Flux Excess Factor [9]. The introduction of these filters results in rejection of many weak sources concentrated mainly in the direction of the Galactic center with a density that is much higher than expected from the notion on the star distribution in the Galaxy. We assume that these sources may be more distant stars. Due to the sky background and the high density of sources in the Galaxy plane, the determination reliability of parallaxes is low.

These criteria are met by 40% of Gaia sources within a radius of 100 pc. These 242122 sources are treated as the stars among which we search for binary and common proper motion stars.

To decide whether each particular pair of stars is a binary system, one needs to compare the parameters of both objects. In order to do this for different parameters of all possible pairs in the ensemble is extremely impractical. Therefore, a preliminary list of possible pairs of stars located closer than 1 pc to each other was compiled. The determination of the star position in the three-dimensional space involves the distance to the Sun defined as \( d (\text{pc}) = \frac{1000}{\varpi} (\text{mas}) \). For compilation optimization of such a list, which is too time-consuming for a simple solution of the problem, we implement the algorithm described in detail in the Appendix.

We completely remove the objects located in the regions of the sky from our list corresponding to the known open cluster of Hyades, as well as to the moving group (according to other sources, the cluster) Mamajek 1, since it is not possible to distinguish binary stars in these areas. For the pairs from the resulting list (39445 pairs), relative motion parameters were calculated—the difference of proper motions \( \Delta \mu \), projection of the relative motion (in the units of linear velocity \( dV = |\mathbf{r}|\Delta \mu \)). Figure 1 shows how the distance between which \( d \leq 1 \) pc stands out among a subset of stellar pairs with similar proper motion of the components in the ensemble of star pairs. Moreover, if the distance between components is considered, it becomes noticeable how the ensemble separates into two subsets also according to its physical separation: pairs with a large difference of their proper motions are predominantly wider than those with a small difference of \( \Delta \mu \) (see Fig. 2).

We treat this as a separation of “random coincidences” and comoving/binary stellar pairs, and introduce an empirical criterion that relates the physical separation and the difference in proper motions: \( \log(\Delta \mu) < 2 - 1.4\sqrt{S} \). Here, \( S \) is the physical separation in parsecs. We include pairs that satisfy this criterion in our catalog and consider them as candidate binary systems that are bound (either gravitationally bound wide binary stars or the members of moving groups). Independently, the applied criterion can be evaluated by comparing radial velocities for the star pairs in which they are known for both components. There are 3593 pairs of such pairs in total (9%); of these, 1636 satisfy the suggested empirical criterion.

For radial velocities, we construct a diagram similar to Fig. 1 for proper motions (Fig. 3). It is seen that the radial velocities of the pair components that satisfy the accepted empirical selection criterion agree well with each other. Among 46 pairs that satisfy the accepted criterion, in which the radial velocity difference between the components exceeds 10 km/s, the nominal determination errors of the radial velocity in 32 cases are large enough to explain this discrepancy. A data comparison on the remaining 14 pairs with the SIMBAD and BBD databases [10] shows that they include rotating variables (according to the type of variability in the General Catalog of Variable Stars [11]) and spectroscopic binary stars, which is why listed Gaia DR2 radial velocity can be associated with

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1 https://www.pas.rochester.edu/~emamajek/EEM_dwarf_UBVJHK_colors_Teff.txt
2 https://www.cosmos.esa.int/web/gaia/dr2-known-issues
3 https://www.cosmos.esa.int/documents/29201/1770596/Lindegren_GaiaDR2_Astrometry_extended.pdf/1ebddb25-f010-6437-cb14-0e360e2d9f09
Fig. 1. Distribution of the pair components according to their proper motion according to declination (left) and right ascension (right). Position of the individual points along two axes shows the proper motion of two components. A much densely populated diagonal is visible, corresponding to the pairs with a common proper motion.

Fig. 2. Distribution of stellar pairs in the “physical separation—difference of proper motions” diagram. Stellar pairs are divided into two groups, the criterion for the formal separation between them is indicated in the figure by a dashed line.

effects other than spatial motion. For two pairs identified with binary stars HD 53229 and HD 95123, the radial velocities of the weak components are determined to be +568 and −715 km/s, respectively, while the radial velocities of the main components are 14 and 28 km/s. This may be a manifestation of an incorrect determination effect of radial velocities in dense stellar fields (see Gaia DR2 Known Issues, as well as...
3. RESULTS

3.1. Catalog Completeness

Completeness of the created catalog of wide binary systems at the distance of 10 to 100 pc from the Sun is determined by several factors. First, it is limited by completeness of Gaia DR2. It is known that the Gaia DR2 catalog is substantially incomplete for stars brighter than approximately $G \approx 6^m$, while it is almost complete only for the magnitude range $12 \leq G \leq 17$ [8]. In addition, the catalog may be incomplete for the stars with large proper motion (there are more such stars in the immediate vicinity of the Sun) and in the dense stellar fields.

Inhomogeneous coverage of the celestial sphere by Gaia DR2 sources in the microscale (reflecting the scanning law), as well as the complete removal of the sources associated with the clusters from the created catalog, should not critically affect the properties of the obtained sample.

In addition, the completeness of the resulting binary star catalog is affected by the method of selecting sources from the Gaia DR2: (i) we selected only stars with parallaxes; (ii) we applied the filters to select "astrometrically clean" solutions. In this case, the filter by the relative parallax error eliminates, predominantly, more distant stars. The probability of passing these filters (i.e., having a sufficiently high quality of astrometric and photometric solutions) particularly is lower than the average one for unresolved binary stars and binary components with pronounced orbital motion, and is equal to zero for pairs with angular distance between components less than or equal to 2 mas. For angular distances between components larger than 2 mas, the relation between limiting angular resolution and the brightness difference between components is expressed by the relation $\Delta G_{\text{lim}} \propto \log \rho$ for $\rho \geq 2$ mas (see also the discussion in [13]). Obviously, this selection effect predominantly distinguishes pairs with a small brightness difference at small angular distances (see Fig. 4).

Even after excluding the regions of space in which the stars of the Hyades and Mamajek 1 clusters are located, the catalog contains a noticeable number of pairs, with one or both components as members of...
more than one pair. As the ratio of the distances between components in the groups of stars consist more than of one pair shows, they may be hierarchical multiple star systems; however, these are mainly stars of moving groups. About 400 pairs that have a common component with another pair were found, which were removed from the catalog in order to avoid distortions in the characteristic analysis of the ensemble of binary stars. After elimination of multiple stars, 9977 pairs remained in the sample.

Generally, one would expect that some of the components of the catalogued pairs (10–13%, based on the statistics of multiplicity [3, 14]) can be represented by unresolved binary stars (see also the discussion of pairs with different radial velocities of the components in Section 2). On the other hand, the catalog compilation procedure for filtering sources according to the quality of the astrometric and photometric solutions should reduce this fraction. A study of solution quality indicators (RUWE, Flux Excess Factor) performed by the DR2 authors for known close binary stars (\(\rho \leq 2\) mas) taken from the identifier catalog of binary and multiple stars ILB [15] showed the following: 3537 close pairs were identified as an unresolved binary star with Gaia DR2 sources for which the parallax values \(\varpi \geq 10\). Out of these sources, Gaia DR2 quality of the solution filter rejected 2180 or 62%. For comparison, only 8% of objects do not pass the same filter when identifying Gaia DR2 sources with Hipparcos stars. This suggests that the use of recommended filters has particularly led to a significant reduction of the number of components that are unresolved binary stars.

### 3.2. Analysis of Sample Features

Let us consider the distribution of sample pairs over the angular separation of components (Fig. 5a), over the distance between components in the projection onto the celestial sphere (Fig. 5b), and in the linear space (Fig. 5c). The distribution of the angular separations between components (Fig. 5a) in the logarithmic scale turns out to be flat in the 3 mas range and decreases at larger values. Although the parallax value is involved in the calculation of both linear distances, the accuracy of its determination is more significant for the distance in the three-dimensional space. This is reflected in the distributions: the projection distribution of the distances between components (Fig. 5b) is similar to the distribution in Fig. 5a, linearly decreasing in the logarithmic scale beyond 300–400 AU (consistently with the estimate of 10\(^{2.5}\) AU [13]). The distribution over the distance between components in the three-dimensional space in the logarithmic scale increases to \(\approx 0.5\) pc, i.e., to 10\(^{7}\) AU.

Despite the accepted limitation of the relative parallax error by 10%, for the distances between components typical for binary stars, small relative parallax
errors can lead to large relative errors in the determined distances between the stars. To separately consider the parameter distribution of a more refined, albeit less complete sample, a subsample of binary stars is introduced in which the nominal error estimate in the parallax distance does not exceed 0.1 pc: \( \delta_d < 0.1 \) (we name it sample I). In Fig. 5, distributions for sample I are represented by a gray-filled histogram. It can be seen that the introduction of a limit on the error of the distance significantly reduces the binary fraction with extremely large sep-

Fig. 5. Normalized distributions (top to bottom) over angular separation between components (a), over projection of the physical separation between components (b), and over the physical separation between components in three-dimensional space (c). The black line shows all binaries, by gray filling are indicated systems that satisfy requirement of the error in determination of the distance being not larger than 0.1 pc for both components.
rations (more than 50,000 AU) and leads to a shift of the distribution maximum in the logarithmic scale to the lower values.

We use absolute magnitudes in the photometric band $G$ and the color obtained from the difference in magnitudes in the BP and RP Gaia bands to construct the Hertzsprung–Russell diagram (Fig. 6). According to the position in the diagram, it is possible to distinguish the components populating the main-sequence for which we estimate the masses from absolute magnitudes in the band $G$, using Mamajek’s tables. Moreover, we neglect the probability that the catalog contains unresolved binary stars and stars evolving to the main-sequence, although such objects may be present in the sample.

This allows us to proceed to the mass ratio distribution for pairs in which both components are supposedly located on the main-sequence (see Fig. 7). In the region $0.3 \leq q \leq 0.9$, the distribution looks close to a planar one. In the region $q \geq 0.95$, an excess of binary stars is detected (the so-called twin stars, with components of close mass values). It is important to find out whether such a feature reflects the nature of wide binary stars or if it is related to the effects of sample selection.

3.3. Catalog Selectivity Function

As discussed above, selection effects that affect the sample are a combination of the Gaia DR2 selectivity function which has predetermined quality decision filters for single stars and the pair selectivity function. We create a sample of random star pairs from Gaia DR2 with $\sigma < 15$ mas (to limit the sample size), selecting them with the same solution quality requirements that were imposed on the pair components in Section 2. Using Hertzsprung–Russell diagram, we select the pairs with both components belonging to the main sequence, estimate their component masses, and construct the ratio distribution of the masses of the weak (“secondary”) and bright (“main”) components. Figure 7 shows the mass distribution for the sample of wide binary stars in comparison with a similar distribution for the synthesized random sample. It is observed that the selectivity function of Gaia DR2 in combination with the applied filters for the solution quality of the components cannot be the reason for the appearance of the “peak of twins”.

In order to investigate how the pair selectivity function associated with the different visibility of stars with different magnitude contrasts at the same angular distance can distort the observed distribution of mass ratios, we will model this effect using binary population synthesis code [16] for various assumptions about the mechanisms (“scenarios”) of binary system formation [17]. Kroupa [18] initial mass function (IMF) was used and various scenarios of combining the com-

![Fig. 6. “Color–absolute magnitude” diagram for all components of the binary stars of the catalog.](image-url)
component masses were assumed: independent distribution of them and/or distribution of the component mass sum according to the IMF. For scenarios assuming that the distribution of the component mass ratio \( f(q) \) is a free parameter, various options were considered, including \( f(q) \propto C \). For the distribution over semi-major axes of the orbits, a power function \( f(a) \propto a^{\beta} \) was chosen. Since it was found that the value of the power does not affect the distribution shape over mass ratio of components, the value \( \beta = -1 \) was adopted. The distribution over eccentricity of the orbits was taken to be flat. For each of such combinations of initial conditions, the vicinity of the Sun within 100 pc was simulated. Star formation rate over 14 Gyr was assumed to be \( 15e^{-t/15} \), where \( \tau = 7 \) Gyr [19].

Stellar evolution was described by approximate formulas of Hurley et al. [20]. Then, the simulated sample was compared with the part that would be “visible” under given conditions, including the requirements that both components are main-sequence stars, the restrictions on the apparent magnitude correspond to the Gaia DR2 restrictions \( 6^m \leq G \leq 17^m \) and the pair selectivity function (Section 3.1). In the studied scenarios, the catalog selectivity function for binary systems did not lead to a pronounced peak formation of close mass components. Moreover, if (in the scenarios of pair formation allowing an independent distribution of component mass ratios) a similar peak is introduced artificially, the adopted “observational” filters do not significantly affect the presence and shape of this peak. Figure 8 shows the normalized distributions of the model sample over mass ratios of components for the pairs “existing” in the model under given constraints for some combinations of initial conditions, compared to the “observable” ones. Presented are the following scenarios: RP scenario—“random pairing” (both components are formed independently with masses that are selected from the IMF, top panel, left); PCRP scenario—“primary constrained random pairing” (the main component is formed like in RP scenario, the secondary one—with the mass from the IMF, provided that it is not more massive than the main component, top panel, right); and PCP scenario—“primary constrained pairing” (the main component is formed like in RP scenario, the mass of the secondary is determined by an independently specified distribution over mass ratios of components, bottom panel for the case of a flat distribution by mass of components (left), and a flat distribution with a peak (step) at \( q \geq 0.9 \) for (right)). In these scenarios, the use of the catalog selectivity function that limits the pair visibility with close components with a large brightness difference does not lead to the formation of a “twins peak” in the distribution over component mass ratio if it was absent in the initial sample. The peak in the initial distribution according to the PCRP scenario disappears when the observations are simulated, due to the fact that it is formed by the least massive stars, rejected by the limit of the apparent magnitude. The peak of the twins in the initial distribution according to the PCP scenario with a flat distribution.
and superimposed peak remains in the observed distribution.

3.4. Twin Stars

The excess of twin stars among wide binaries should be recognized as really existing. This result was obtained independently of a detailed study [21] and supports the conclusions drawn there.

The excess of twins becomes less pronounced with increasing separation of components: Fig. 9 shows how the distribution over mass ratios becomes flatter at large physical separations. To the right in the same figure, the distribution for sample I with a restriction on the distance error is shown. In the sample with a more stringent restriction on the error of the distance between components, the peak of twins is more narrow and more pronounced. There can be several reasons for this, and it is possible that the resulting effect is achieved by their combined effect. First, the average distance between the components in the sample I is smaller (and at closer distances, the proportion of systems with twin components is higher). This may be due to the fact that the predominant formation mechanism of twin stars is not effective for the widest pairs. On the other hand, it can be expected that the introduction of a restriction on the error in determined distance for the catalogued stars leads to the decrease in the contamination of the sample by optical pairs, the distance between which is underestimated due to the parallax errors. The admixture of optical pairs in which the distribution of mass ratio falls in the range \(0.4 \leq q \leq 1\) (Fig. 7) should lead to the decrease in the star fraction with large \(q\). Therefore, the smaller it is, the more pronounced the “twin peak” should be.

A detailed review and discussion of possible channels for the formation of an excess of wide binary stars with close masses of components are presented in [21] and references therein. It is important to note that the most probable mechanism is the competitive accretion onto the forming pair components in a common protostellar disk, in which the component with an initially

![Fig. 8. Normalized distribution over mass ratio of components \(q\) for a binary star ensemble modeled under various assumptions about formation of star pairs within 100 pc from the Sun (gray filled histogram) and its observed part with an imitation of the catalog selectivity function (black contour histogram), see the text for details.](image-url)
smaller mass, moving in the orbit with a large radius, can come up in mass with the initially more massive star. Currently, the problem of this mechanism is that the largest observable protostellar disks around forming binaries have a radius of the order of several hundred AU (e.g., [22]), while the excess of twin stars, though decreasing with increasing separation of components, continues to be significant up to the distances exceeding \(3 \times 10^4\) AU.

With the further increase of component separation, the star excess with close masses continues to decrease and completely disappears for pairs with a calculated separation larger than \(8 \times 10^4\) AU, for which the distribution over component mass ratio becomes flat over the entire range \(0.3 \leq q \leq 1\). Such a mass ratio distribution can be the result of a combination of a number of factors (the presence of initially binary stars whose orbit underwent dynamic broadening [23]; presence of a share of pairs formed during cluster decay [24]; and admixture of optical pairs). Finally, in the analysis of the distribution dependence over component mass ratios on their separation, it should be considered that the accuracy of the separation estimates itself strongly depends on the determination accuracy of parallaxes.

4. CONCLUSIONS

Using Gaia DR2, a catalog of common motion pairs in the 100 pc vicinity from the Sun was compiled. Candidate pairs, pre-selected on the base of component separation in the three-dimensional space, were filtered by a selected empirical criterion that takes into account proper motions and physical separation between components of the pairs, as well as a catalog including about 10,000 binary and common proper motion stars. The incompleteness extent of the resulting catalog is determined mainly by the combination of the Gaia DR2 catalog selectivity function for single stars, the Gaia DR2 selectivity function for star pairs, and the filter system of astrometric and photometric quality adopted for the selection of Gaia DR2 sources. In particular, it was shown that there is a relationship between the limiting brightness difference for a pair of stars and the angular distance at which they can be detected, up to 10 mas. The form of this dependence is suggested and shown that the minimum angular distance between components in the catalog is 2 mas as the result of the use of solution quality filters, and the share of unresolved binaries among the catalog pairs is significantly (possibly up to 60%) lower than the average for the field stars.

Common motion groups of stars with the number of members exceeding 2 were removed from the catalog.

It is shown that the distribution of catalog pairs by separation in the three-dimensional space demonstrates a distribution that differs from the distribution over the projection of separations onto celestial sphere due to parallax determination errors. To reduce this effect, a subsample of binaries with strong restrictions on the error of determination of the distance is selected.

Using Hertzsprung–Russell diagram, the pairs of stars are selected, both components of which are presumably on the main-sequence. For such pairs, the masses of components are estimated. Independently from [21], a confirmation of existence of an excess of binary systems—“twins”—with close masses of components is obtained. It is shown that the detection of the peak in the catalog sample is not a consequence of

Fig. 9. Normalized distributions over mass ratio of components \(q = M_2/M_1\) for binary stars with the separation in the three-dimensional space lower than 0.15 pc (solid line) and exceeding 0.15 pc (filled histogram). Left panel shows distribution for the entire catalog, right panel—for the subsample with a restriction on the determination error of the distance of 0.1 pc.
its selectivity function. It is shown that this “twins peak” becomes less pronounced with increasing separation of components. However, it disappears completely only at the calculated separations greater than $8 \times 10^4$ AU. Like the estimate made in [21], this value significantly exceeds the size of the observed protostellar disks, where the binaries are formed. At the same time, the presence of an excess of twin stars suggests that they are formed during a process in which the masses of the components are dependent on each other. This requires a study of possible mechanisms for a significant increase of separation of components of pairs, occurring also during decay of stellar clusters.

**APPENDIX**

**THE ALGORITHM FOR COMPILING THE LIST OF STELLAR PAIRS**

In this section, we describe the algorithm developed to create a list of Gaia DR2 source pairs that are close enough to each other in three-dimensional space $\alpha, \delta, \omega$. Creation of a list of binaries implies pairwise operations on the stars, which will take $CN^2$ of computer time the multitude of $N$ stars. This will make such calculations too long for any significantly large star sample. Of course, it makes sense to compare the parameters of those stars only that have at least some chance of being in a bound pair. There is no need to spend time to be sure that two stars located at a distance of 50 pc from the Sun in opposite parts of the celestial sphere are not a physical couple. It is possible to introduce a criterion that will consider whether it makes sense to perform detailed calculations for the given pair of stars (for example, a criterion can be a large projected angular separation), and to determine whether this criterion is fulfilled, skipping further calculations if it is not satisfied. However, this criterion calculation carried out for all pairs of stars in the sample is still a challenge of complexity $CN^2$: backtracking from further calculations if this criterion is not met, then $C$ is reduced.

It seems more efficient to create an algorithm that would make a list of potential pairs in the first approximation. This algorithm can be optimized as much as possible to reduce the sample in which complexity $CN^2$ calculations would be performed. With a list of candidates for pairs instead of a star list, calculations can be performed in a row rather than pairwise over all stars, according to the existing list, which is much faster. The creation of such algorithm was the first (and significant) part of this study.

Thus, at the first stage of the study, the goal was to transform the task $CN^2$ into a task $CN$. All stars were selected as candidates for pairs located in three-dimensional space at a distance less than 1 pc to each other. The distance was calculated based on the celestial coordinates of the components $\alpha_1, \delta_1$ and the estimates of the distances to them, calculated as the reciprocal of the parallax $\omega$. Using such estimates is permissible due to the fact that, in the ensemble under study, there are sufficiently close stars with characteristic determination error of the parallax of Gaia DR2 $\frac{\delta \omega}{\omega} < 10\%$.

The next problem was to determine which stars are subject to pairwise comparison in order to verify that the proximity criterion is met. Let us look for star pairs that have a distance from the Sun at least not less than some fixed value $d$. Obviously, starting with some angular separation between two stars $\theta$, their physical separation cannot be less than the assigned allowed maximum. By solving a simple geometric problem, this angular separation is equal to $\theta = 2 \arcsin(p / 2d)$, where $p$ is the maximum physical separation, in our case—1 pc. Using this, in the sky, we can select rectangular in coordinates ($\alpha, \delta$) regions ($\alpha_1, \delta_1, \alpha_2, \delta_2$) (hereinafter, we will call them “sections”), the width along both axes will be equal to the above-mentioned maximum separation. Thus, we would know that a star at one side of such a section (for example, $\alpha > \alpha_1$) cannot be a pair to a star on the opposite side ($\alpha < \alpha_1$). It is important to note that later in the Section on compiling the binary list, the expression “cannot be a pair” is used for convenience in the sense “cannot be closer than 1 pc from each other”. Partitioning into similar sections gives us an opportunity to not consider pairs located at the distance exceeding the width of a section. It is worth noting that the size of such a rectangular section over $\alpha$ will be larger than over $\delta$: for large declinations, the same angular separation corresponds to a greater distance over right ascension: $|\alpha_2 - \alpha_1| = \theta / \cos(\delta)$.

Partitioning into sections is done within large areas equal to 1/4 of a complete circle in right ascension and about twenty degrees in declination. Within one large area, the correction $|\alpha_2 - \alpha_1| = \theta / \cos(\delta)$ is accepted as the largest possible (i.e., it is selected for the largest declination value within the given area). Thus, each large area has its own rectangular grid of sections.

Upon the creation of the described partition, we check all possible pairs within one large area and obtain a gain in the computational time. Thus, for any star, a pair can be found either in the same region or in the neighboring one. Thus, two actions are accomplished: checking the distances between all stars in pairs within the same section, and checking the distances between all stars in the neighboring sections (that is, only the connections between the sections). The algorithm for performing these two actions is implemented in such a way that repetitions do not occur. The execution sequence is graphically depicted in a Fig. 10 example by one of the steps in the middle of the passage over a large area. The passage is made in...
the direction of increasing right ascension, and is shifted to the next section of the declinations array, when the right ascension passage for specific declination is completed. At each step, a section is selected, and the following are checked: the presence of star pairs within the site, the presence of star pairs of this site, and four out of the eight adjacent sites. The remaining four “neighborhoods” will be checked in the next steps (see Fig. 10).

Within one large region, the algorithm ensures that all stars separated by less than 1 pc are added to the list of potential pairs. After this, the potential pairs formed by stars located on the opposite sides of the boundaries between large regions are checked separately. Again, only those stars that have separation not exceeding the maximum separation from the boundary of the regions pass the test. After combining all these regions for different declinations, only polar regions remain unchecked, in which the above-described algorithm is not suitable due to the tendency to infinity of the factor $1/\cos(\delta)$. Verification of the pairs in the circumpolar regions is implemented for the entire section with a declination radius selected in such a way that it exceeds $\theta$. The final result is a list of pairs for all parts of the celestial sphere.

The entire above-described procedure is performed for a “layer”, limited by two values of the distance from the tested stars to the Sun (calculated by Gaia parallax): $d_i < d < d_f$. Respectively, the maximum angular separation $\theta$ is determined by the smallest possible distance $d_f$. For this, the 10 pc $\leq d \leq$ 100 pc sample was divided into four layers: from 10 to 25 pc, from 25 to 50 pc, from 50 to 75 pc, and from 75 to 100 pc. For each layer, the values of $\theta$ and partition into sections are determined separately. Thus, despite the larger number of stars in the outer layers, we partially compensate for the increase in computational time by reducing the size of the sections. The pairs formed by stars located on the opposite sides of the boundaries of these spherical layers are checked separately: this is achieved by taking these layers with an overlap equal to the maximum selected physical separation (in our case, 1 pc) and removal of the pairs, both of which are in the overlapping region of one of these sets. Thus, we avoid including them in the list of pairs more than once. Due to the overlap, those pairs that are located on the opposite sides of the layer boundary are taken into account.

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\[6\] http://www.starlink.ac.uk/topcat/ [27].