Aspects of the momentum dependence of the equation of state and of the residual $NN$ cross section, and their effects on nuclear stopping

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With the semiclassical Landau-Vlasov transport model we studied the stopping observable $R_E$, the energy-based isotropy ratio, for the $^{129}$Xe$+^{120}$Sn reaction at beam energies spanning 12A to 100A MeV. We investigated the impacts of the nonlocality of the nuclear mean field, of the in-medium modified nucleon-nucleon ($NN$) cross section and of the reaction centrality. A fixed set of model parameters yields $R_E$ values that favorably compare with the experimental ones, but only for energies below the Fermi energy $E_F$. Above $E_F$ agreement is readily possible, but by a smooth evolution with energy of the parameter that controls the in-medium modification of $NN$ cross section. By comparing the simulation correction factor $F$ applied to the free $NN$ cross section with the one deduced from experimental data [Phys. Rev. C 90, 064602 (2014)], we infer that the zero-range mean field almost entirely reproduces it. Also, in accordance with what has been deduced from experimental data, around $E_F$ a strong reduction of the free $NN$ cross section is found. In order to test the impact of sampling central collisions by multiplicity an event generator (HIPSE) was used. We obtain that high multiplicity events are spread over a broad impact parameter range, but it turns out that this has a small effect on the observable $R_E$ and thus, on $F$ as well.

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I. INTRODUCTION

The ratio between transverse and longitudinal components of kinematical observables is a measure of the conversion of the initial entrance channel motion into intrinsic degrees of freedom in heavy-ion collisions (HICs). Such an observable gives an insight on the rate of a system’s equilibration, of the dissipation of the available energy, as well as of HIC stopping power [1, 2]. Thanks to such an observable, the FOPI Collaboration has evidenced partial nuclear transparency in HIC in the beam-energy range $E_{\text{inc}} \approx 0.1A – 1A$ GeV [3]. More recently, by examining the ratio of transverse to longitudinal energy $R_E$ and linear momentum $R_p$ for the most violent HICs, the INDIRA Collaboration has revealed a substantial reduction of the nuclear stopping power at $E_{\text{inc}} \approx 10A – 100A$ MeV [4]. This stopping observable reaches a minimum around the Fermi energy $E_F$ and stagnates or very weakly increases with the further increase of $E_{\text{inc}}$ at least up to 100A MeV, the upper limit of the energy range available in this study. The above observation is valid for all (mass symmetric) systems studied, with system masses $A_{\text{sys}} = 72 – 394$ a.m.u. It is worth emphasizing that the fusion cross section normalized by the total reaction cross section exhibits an analogously rapid fall-off up to about $E_F$ [4, 5], a behavior especially evident for mass-symmetric systems (cf. Fig. 6 of Ref. [6]).

In a recent publication [6] the above observable $R_E$ was analyzed for the $Z=1$ subset of the same INDIRA data. The $Z=1$ $R_E$ displays a slightly stronger increase with $E_{\text{inc}} > E_F$ for the heavier systems [1] relative to the $R_E$ values obtained in the previous study [4] which included light charged particles and fragments, but also was somewhat more stringent on the selection of the most central events. The authors of Ref. [6] report a minimum of $R_E$ around $E_F$, which is particularly enhanced when $R_E$ is normalized to the Fermi-gas-model prediction of the incoming $R_E$ value at a given $E_{\text{inc}}$. In Ref. [7] it was assumed that protons are predominantly dynamically emitted during the early reaction phase, in accordance with Refs. [9, 10]. Such a hypothesis offers a possibility of extracting information on the in-medium correction for the free nucleon-nucleon ($NN$) cross section $\sigma^{\text{free}}_{NN}$. Following such an argument, starting from the experimental $R_E$ values in Ref. [6], with some basic assumptions about the effects owing to the Pauli-exclusion principle, the nucleon mean free path was extracted and an effective value of the in-medium $NN$ cross section $\sigma_{NN}^{\text{m}}$ was deduced. In the process, a correction factor $F$ was obtained by which $\sigma_{NN}^{\text{m}}$ has to be multiplied at each $E_{\text{inc}}$ to get a proper $\sigma_{NN}^{\text{m}}$ value. The authors found that (i) a significant reduction of $\sigma_{NN}^{\text{m}}$ is present in HICs below 100A MeV and (ii) this change of $\sigma_{NN}^{\text{m}}$ is strongly dependent upon $E_{\text{inc}}$. At the lowest energies the measured $R_E$ is compatible with the full stopping value ($R_E \approx 1$) and the effective $\sigma_{NN}^{\text{m}}$ amounts to about 0.4$\sigma^{\text{free}}_{NN}$. One should keep in mind that the authors claimed a large uncertainty on the factor $F$ below $E_{\text{inc}} \sim 30A$ MeV, a subject for which they have announced a devoted publication [7]. At incident energies around $E_F$ where $R_E$ attains its minimum, $\sigma_{NN}^{\text{m}}$ is reduced to less than one fifth the free $\sigma^{\text{free}}_{NN}$ value ($F \approx 0.17$) and then the effective $\sigma_{NN}^{\text{m}}$ steadily and regularly increases up to half of the free value ($F \approx 0.5$) at

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$E_{\text{inc}} = 100\,\text{A}\,\text{MeV}$ \cite{ref7} (see also Fig. 3 in the present work).

The stopping observables $R_E$ and/or $R_p$ have also been investigated in isospin-dependent quantum molecular dynamics (IQMD) \cite{ref8,ref11,ref14,ref15} and antisymmetrized molecular dynamics (AMD) \cite{ref16} model studies of HICs. All these works were carried out before publication of Ref. \cite{ref3}. Neither of simulation approaches predicts the remarkable in-medium reduction of $\sigma_{NN}^e$ found in Ref. \cite{ref3}. In the AMD study, specific attention has been paid to performing the analysis by meticulously following the experimental procedure for data handling \cite{ref12}. The simulation with in-medium $\sigma_{NN}$ due to Li and Machleidt \cite{ref16} (the free $\sigma_{NN}$) undershoots (overshoots) the data of \cite{ref3}. An agreement with the data could only be reached at $E_{\text{inc}} \geq 80\,\text{A}\,\text{MeV}$ by doubling the theoretically established $\sigma_{NN}^e$ of \cite{ref16}. A systematic investigation of the impact of $\sigma_{NN}$ on $R_E$, however, has not been performed yet. The intention of the present study is twofold:

1. by varying the nuclear equation of state (EOS) and the nuclear equation of state (EOS) and the parametrization of $\sigma_{NN}$, to investigate how the semiclassical Landau-Vlasov (LV) transport model of HIC \cite{ref17,ref18} complies with the experimentally deduced dependence of the stopping observable $R_E$ on $E_{\text{inc}}$ and

2. by varying a simple multiplicative factor $F$ of the free $NN$ cross section, to compare thus obtained values for $F$ with those reported in Ref. \cite{ref7}.

**II. MODEL INGREDIENTS**

Within the semiquantal extension of the Boltzmann transport theory, the highly nonlinear LV equation governs the spatio-temporal evolution of the one-body density distribution function $f(r,p;t)$:

$$\frac{\partial f(r,p;t)}{\partial t} + \{f(r,p;t),H\} = I_{\text{coll}}(f(r,p;t)), \quad (1)$$

which gives the probability of finding at the instant $t$ a particle in the phase-space point $(r,p)$. $\{,\}$ stands for the Poisson bracket, whereas $H$ is the one-body Hamiltonian describing the Coulomb potential and the nuclear mean field. We present the results obtained with a soft nonlocal mean field labeled D1-G1 ($K_\infty = 228\,\text{MeV}$, $m^*/m = 0.67$) due to Gogny \cite{ref12} and those obtained with the standard simplification of the soft zero-range Skyrme interaction due to Zamick ($K_\infty = 200\,\text{MeV}$, $m^*/m = 1.0$) \cite{ref20}. The D1-G1 force is reputed to reproduce fundamental properties of nuclear matter as well as those of finite nuclei \cite{ref14} while the Zamick parametrization of the EOS is, owing to its simplicity, of rather widespread use in a number of microscopic approaches. Details on both the nonlocal and the local parametrizations of the used EOS may be found in Tables I and III of Ref. \cite{ref21}, respectively. We have demonstrated that the LV model is able to correctly describe experimental observations in the intermediate energy regime \cite{ref3,ref10,ref15,ref21,ref22}. The use of only a density dependent EOS is legitimated by the finding \cite{ref11,ref14} that the isospin dependence of the mean field has a weak, if any, influence on isotropy ratios. Experimental $R_E$’s for a number of HICs between various xenon and tin isotopes corroborate this result; cf. Table I of Ref. \cite{ref4}.

The function $f$ is expanded onto a moving basis of coherent states as normalized Gaussians $G_\chi$ ($G_\phi$) with frozen width $\chi$ ($\phi$) in $r$ ($p$) space:

$$f(r,p;t) = A/N \sum_i G_\chi(r-r_i)G_\phi(p-p_i). \quad (2)$$

$A$ is the system mass number and $N$ is the number of coherent states ($N/A$ equals 60 in the present study). The widths $\chi$ and $\phi$ are chosen such as to best reproduce the nuclear ground state characteristics of the two colliding nuclei. The local density reads

$$\rho(r) = \int d^3p \, f(r,p). \quad (3)$$

Gaussians move in the self-consistent mean field and suffer hard scattering between them, controlled by the Uehling-Uhlenbeck collision integral accounting for the fermionic character of interacting particles \cite{ref23}:

$$I_{\text{coll}} = \frac{4a}{m^2} \int d^3p_2 \, d^3p_3 \, d^3p_4 \, \frac{d\sigma_{NN}^e}{d\Omega} \times \delta(p+p_2-p_3-p_4) \times \delta^4\epsilon + \delta^4\epsilon_2 - \delta^4\epsilon_3 - \delta^4\epsilon_4) \times [(1-f)(1-f_2)f_3f_4 - (1-f)(1-f_2)f_3f_4]$$

which takes into account energy and momentum conservation as well as the Pauli exclusion principle. Here, $m$ denotes the nucleonic mass, $f = (2\pi\hbar)^3 f(r,p;t)/g$ is the occupation number with $g$ the spin-isospin degeneracy \cite{ref18}, $p$ and $p_2$ ($p_3$ and $p_4$) are initial (final) momenta of the scattering particle pair, $\epsilon = \epsilon(p)$ is the single-particle energy, while $\sigma_{NN}^e$ is the in-medium nucleonic cross section. $\sigma_{NN}^e$ is scaled so that a Gaussian-averaged mean-free path is the same as for a nucleon. The cross section dependence on isospin has been reported as crucial for the study of stopping \cite{ref11}. This kind of $\sigma_{NN}^e$ parametrization was proposed by Chen et al. \cite{ref24}, which hereafter we label $\sigma_{NN}^{\text{Chen}}$. This phenomenological $\sigma_{NN}^{\text{Chen}}$ is based on the empirical isospin and energy dependence of the free $NN$ scattering and has been used in both \cite{ref7} and \cite{ref11}, $\sigma_{NN}$ due to Li and Machleidt \cite{ref16}, which accounts for the in-medium effects and is also isospin dependent, is tested too.

**III. RESULTS AND DISCUSSION**

Our stopping observable, the energy-based isotropy ratio, is defined as the ratio between transverse $E_{\text{tran}}$ and longitudinal $E_{\text{long}}$ energy components of reaction ejectiles \cite{ref4,ref6}:

$$R_E = \frac{\sum E_{\text{tran}}}{\sum E_{\text{long}}}. \quad (5)$$
where summation runs over particles of those reaction events that satisfy certain selection criteria. For the LV simulation results, the summation index \( i \) of Eq. (5) runs over the free Gaussians, i.e., those which are not bound in large (residue-like) fragment(s). Among the experimentally studied systems, the \( ^{129}\text{Xe} + ^{120}\text{Sn} \) reaction has been measured at by far the most abundant number of \( E_{\text{inc}} \) values. Consequently, in the present work only the simulation of this system is performed. To acquire stable \( R_{\text{sim}}^{E} \) values, the simulation is carried out up to 600 fm/c at the lowest \( E_{\text{inc}} = 12\text{A} - 32\text{A MeV} \) and up to 240 fm/c at the highest \( E_{\text{inc}} = 80\text{A} - 100\text{A MeV} \). Beyond that time the calculation was continued until 8,000 fm/c, considering only the Coulomb repulsion due to reaction residues. Special care was taken in order to perform our analysis of simulation data as close as possible to experimental conditions.

### A. A density-dependent \( \text{NN} \) cross section

In the experimental analysis \[7\], event selection is based on the charged particle’s multiplicity. The authors selected the most central events that are estimated, in cross-section units, to be equivalent to 50 to 150 mb \[7\]. We adopt the median value of 100 mb for our analysis. This amount corresponds to about 2\% of the total reaction cross section \( \sigma_R \) and, in a geometrical sharp-cut approximation, to \( b \leq 2.0 \text{ fm} \). Consequently, in this sub-section our simulation is limited to \( b \leq 2.0 \text{ fm} \).

Figure \[1\] displays \( R_{\text{sim}}^{E} \) obtained with the momentum-dependent D1-G1 EOS (upper panel) and with the zero-range Zamick EOS (lower panel) for several parametrizations of the in-medium corrected \( \sigma_{\text{NN}} \). For comparison, the experimental \( R_{\text{exp}}^{E} \)’s are shown by filled circles with the corresponding errors \[7\]. As a reference, the \( R_{\text{sim}}^{E} \) results obtained with the in-medium free scattering \( \sigma_{\text{NN}}^{\text{free}} = \sigma_{\text{NN}}^{\text{Chen}} \) \[24\] are displayed by the heavy dotted curves. This empirical \( \sigma_{\text{NN}} \) is used as an input for the in-medium modified \( \sigma_{\text{NN}} \) suggested by Cugnon et al. \[23\]. In their Brueckner \( G \)-matrix in-medium renormalization of the \( \text{NN} \) interaction, they obtained a set of parameters explicitly describing the dependence of \( \sigma_{\text{Chen}}^{\text{Cugnon}} \) on the local density \[23\]. These simulation results are displayed by the red curves and reddish zone in Fig. \[1\]. The zone shows the range of the \( R_{\text{sim}}^{E} \) values limited by the impact parameters \( b = 1 \text{ fm} \) (dashed bordering curve) and \( b = 2 \text{ fm} \) (full bordering curve). \( b = 0 \text{ fm} \) has no weight and \( R_{\text{exp}}^{E} \) at most of the energies is roughly the same for \( b = 0 \text{ and } 1 \text{ fm} \). The heavy curve in each zone represents the \( b \)-weighted \( R_{\text{exp}}^{E} \) value in the range \( b = 0 - 2 \text{ fm} \) and corresponds to \( 2\% \) of \( \sigma_{\text{R}} \).

For both EOS, the \( R_{\text{sim}}^{E} \) values with \( \sigma_{\text{Chen}}^{\text{Cugnon}} \) are very similar to those obtained with \( \sigma_{\text{NN}}^{\text{Chen}} \) (dotted curves). Clearly, in the full energy range investigated here the in-medium effects of \( \sigma_{\text{Chen}}^{\text{Cugnon}} \) have rather weak impact on the \( R_{\text{E}} \) observable. Consequently, as for \( \sigma_{\text{NN}}^{\text{Chen}} \), the compatibility of \( R_{\text{sim}}^{E} \) and \( R_{\text{exp}}^{E} \) for both EOS may be observed at the lowest \( E_{\text{inc}} \) only when the experimental errors are accounted for. In addition, for the Zamick EOS, Fig. \[1\]), the simulation strongly overshoots the data at the highest \( E_{\text{inc}} \)’s.

A full \textit{ab initio} microscopic study of \( \sigma_{\text{NN}}^{m} \) based upon the Dirac-Brueckner approach to nuclear matter was performed by Li and Machleidt \[16\]. Besides dependence on energy, isospin, and density of \( \sigma_{\text{NN}}^{\text{Chen}} \) for this \( \sigma_{\text{NN}}^{\text{Li-Machleidt}} \), we have added an explicit dependence on angle. In contrast to the scattering of neutrons, which is taken as isotropic, those between neutron and proton \( \sigma_{np} \) and between protons \( \sigma_{pp} \) are anisotropic in accordance with the fit of Ref. \[21\], which is given in detail in the Appendix. Similarly to above, the corresponding values of \( R_{\text{sim}}^{E} \) are displayed by the blue curves and bluish zone in Fig. \[1\]. Again, the compatibility of \( R_{\text{sim}}^{E} \) with \( R_{\text{exp}}^{E} \) is unsatisfactory. Nevertheless, for the D1-G1 EOS and \( E_{\text{inc}} \lesssim E_{\text{F}} \), the slope of the isotropy ratio excitation function is correct but the simulation somewhat undershoots the experimental points: \( R_{\text{E}}^{\text{sim}} \) may be taken as compatible with the lower edges of experimental errors on \( R_{\text{E}}^{\exp} \). For the Zamick EOS, Fig. \[1\]b), the compatibility with \( R_{\text{E}}^{\exp} \) exists at low \( E_{\text{inc}} \) and around \( E_{\text{inc}} \approx 60\text{A MeV} \), but the general features of the data are poorly reproduced. Manifestly, none of the above parametrizations of \( \sigma_{\text{NN}}^{m} \) and EOS can account for the observed behavior of the \( R_{\text{E}} \) stopping observable in the full energy range.

The parameters in the above \( \sigma_{\text{NN}}^{m} \) are of a fixed value. By an expansion around the saturation value \( \rho_0 \), Klakow et al. have suggested a simple parametrization for the dependence of \( \sigma_{\text{NN}}^{m} \) on the evolving nuclear density \[27\],

\[
\sigma_{\text{NN}}^{m} = \sigma_{\text{NN}}^{\text{free}}(1 + \alpha \frac{\rho}{\rho_0}), \tag{6}
\]

where \( \rho \) is evaluated locally according to Eq. (4), and \( \alpha \) is a free parameter assumed to reduce the cross section, thus it is strictly negative. As before, for \( \sigma_{\text{NN}}^{m} \) the value taken is the empirical \( \sigma_{\text{NN}}^{\text{Chen}} \). The authors have recommended for \( \alpha \) the domain \([-0.5, -0.1] \) \[27\]. In our simulation \( \alpha \) is varied between \(-0.1 \) and \(-0.6 \). These \( R_{\text{sim}}^{E} \) are presented in Fig. \[1\] by the thinner dashed curves with variable dash size. They display a more or less regular dependence on both \( E_{\text{inc}} \) and \( \alpha \). For the nonlocal EOS and \(-0.6 \leq \alpha \leq -0.5 \) the \( R_{\text{exp}}^{E} \) values at \( E_{\text{inc}} \lesssim E_{\text{F}} \) are well reproduced in both slope and absolute value; cf. Fig. \[1\]b). At energies higher than \( E_{\text{F}} \), however, for each \( E_{\text{inc}} \) another and regularly increasing value of the parameter \( \alpha \) is required such that, at the highest \( E_{\text{inc}} \) here considered, it should become positive, implying an in-medium enhancement rather than a reduction of \( \sigma_{\text{NN}}^{\text{Chen}} \) at \( E_{\text{inc}} \gtrsim 80\text{A MeV} \). Let us mention that \( R_{\text{sim}}^{E} \) with \( \sigma_{\text{NN}}^{\text{Chen}} \) corresponds to that of \( \sigma_{\text{NN}}^{\text{Klakow}} \) with \( \alpha = -0.1 \) in the full range of \( E_{\text{inc}} \) considered and for both EOS. Simulation results with \( \sigma_{\text{NN}}^{\text{Chen}} \) and D1-G1 EOS are comparable to \( \sigma_{\text{NN}}^{\text{Chen}} \) with \( \alpha = -0.6 \) and \( E_{\text{inc}} \gtrsim 50\text{A MeV} \). For the Zamick EOS of Fig. \[1\]b) one does not find a range of \( E_{\text{inc}} \) of stable value of the parameter \( \alpha \) that gives
In conclusion, neither choice of $\sigma_{NN}^m$ allows for a unique description of experimental observation. One faces the fact that every model study, ours and previous [8, 11–14], fails to reproduce with a single set of parameters the INDRA experimental results in the full energy range studied [4, 7]. In particular, all models but [15] predict steadily decreasing values of $R_{E}^{\text{sim}}$ when $E_{\text{inc}}$ increases, while experimental $R_{E}^{\exp}$ results display a break in the slope around the Fermi energy $E_F$.

### B. Global modification of the free $NN$ cross section

Being clearly unable to reproduce the experimental data with different parametrizations of the residual $NN$ cross section, with or without momentum dependence of the force, let us concentrate on our second task that is, by following Ref [7], to infer the multiplicative factor $F$ between the in-medium $NN$ cross section $\sigma_{NN}^m$ and the free $\sigma_{NN}^{\text{free}}$ one:

$$\sigma_{NN}^m = F \sigma_{NN}^{\text{free}}.$$  

As previously done and as in [6], we take $\sigma_{NN}^{\text{free}} = \sigma_{NN}^{\text{Chen}}$ [24]. Of course, this simple cross-section normalization factor $F$ cannot completely describe the rather complex modification of the free $NN$ interaction occurring in the nuclear medium. In particular, such a $\sigma_{NN}^m$ is frozen during a reaction course and depends only indirectly on $E_{\text{inc}}$. Nevertheless, the prescription of Eq. (7) allows one to get an insight into the global in-medium effects on nuclear medium stopping properties and enables a comparison of the factor $F_{\text{sim}}$ obtained in our simulation with $F_{\text{exp}}$ of Ref. [3].

Figure 2 displays $R_{E}^{\text{sim}}$ as a function of $E_{\text{inc}}$ and the $NN$ cross-section factor $F$ for the two effective interactions. In the D1-G1 EOS case, Fig. 2(a), the parameter $F$ takes values 0.2, 0.5, 0.8, 1.0, 1.2, and 1.5. For the Zamick EOS, Fig. 2(b), it is varied between 0.1 and 0.8.
in steps of 0.1 fm. As in Fig. 1, $R_{E}^{\text{sim}}$ are for central HIC with $b \leq 2$ fm, where $b = 1$ fm (2 fm) results are represented by the thin dashed (full) curves that border the (colored) zone of each of the $F$ values. As before, the heavy curve in each zone shows the $b$-weighted $R_{E}^{\text{sim}}$ that corresponds to 2% of $\sigma_R$. $R_{E}^{\text{sim}}$ displays a regular dependence on $E_{\text{inc}}$ and $F$. In accordance with expectation and corroborating the results of Fig. 1, higher $\sigma_{NN}^{m}$ (larger $F$) implies higher stopping power of HICs. Unlike experimental $R_{E}^{\exp}$ and like our results of Fig. 1, as well as of a number of previous theoretical works [8, 11–13], the LV-simulation $R_{E}^{\text{sim}}$ steadily decreases with $E_{\text{inc}}$ for all $F$ without a minimum around $E_{F}$. At the lowest $E_{\text{inc}}$ the mean field completely dominates the course of the collision, and for each $F$ value and both EOS $R_{E}^{\text{sim}}$ is compatible with $R_{E}^{\exp}$. For $E_{\text{inc}} \leq 45$ A MeV, $R_{E}^{\exp}$ is well reproduced by the $F = 0.5$ curve [Fig. 2(a)] and by the $F = 0.1$ one [Fig. 2(b)], respectively. Again, a single value of $F$ cannot reproduce experimental results. However, similarly to the case of the parameter $\alpha$ of Eq. (6), by allowing $F$ to change with $E_{\text{inc}}$ one may find a set of $F$ values to achieve an agreement between $R_{E}^{\text{sim}}$ and $R_{E}^{\exp}$. The behavior of both the parameter $\alpha$ and the factor $F$ with $E_{\text{inc}}$ corroborates the experimental finding [7] that the effective in-medium cross section $\sigma_{NN}^{m}$ drastically changes with $E_{\text{inc}}$ and that around $E_{F}$ there is a break in this dependence.

We take the $b$-weighted $R_{E}^{\text{sim}}$ as the starting point to infer information about the correction factor $F$ by which one would have to multiply $\sigma_{NN}^{m}$ to comply with $R_{E}^{\exp}$. The procedure is evidenced in the inset of Fig. 3 in which the D1-G1 EOS at 50 A MeV is shown as an example. The horizontal red line and reddish background zone display the $R_{E}^{\exp}$ value and its uncertainty, respectively, at 50 A MeV. Blue circles joined by a broken line are the LV simulation $R_{E}^{\text{sim}}$ as a function of $F$ at the same energy. The crossing of this broken line with the red line and the edges of the reddish zone give the most appropriate value for the factor $F$ of Eq. (7) and its uncertainty, respectively.

In the main panel of Fig. 3 we show, by the open circles and squares joined by dashed curves, the thus obtained $F$ values plotted against $E_{\text{inc}}$ for the D1-G1 and Zamick EOS, respectively. Within experimental errors, the $R_{E}^{\text{sim}}$ values for $E_{\text{inc}} \leq 20$ A MeV are roughly compatible with any $F^{\exp}$ value and are not reported. The LV model with the highly recommended nuclear interaction D1-G1 for the range of energies of the present study and with the empirical $NN$ cross section $\sigma_{NN}^{m}$ predicts, for all energies studied, about twice higher $F$ values compared to those suggested by Fig. 10 of Ref. [7]; these are presented in Fig. 3 as black filled circles, with the gray area showing their uncertainties. In contrast to this, when experimental and simulation uncertainties are accounted for, the zero-range (local) Skyrme interaction in the Zamick implementation $F_{\text{sim}}$ is compatible with $F^{\exp}$ above $E_{\text{inc}} \approx 35$ A MeV. Let us underline that $F_{\text{sim}}$ for both EOS display a minimum around $E_{F}$. The minimum is relatively more pronounced than the one suggested by $F^{\exp}$ and it is somewhat shifted in energy. The Zamick EOS gives a $F_{\text{sim}}$ that reduces the free $\sigma_{NN}$ at all $E_{inc}$ while the D1-G1 EOS gives $F_{\text{sim}} > 1$ at $E_{inc} \gtrsim 80$ A MeV.

### C. Centrality versus multiplicity

The most evident difference between a simulation and an experimental data analysis is in the reliability of the assessments of reaction impact parameter $b$. Experimental selection of the most central collisions is made by assuming that there is a biunivocal correspondence between the reaction violence, i.e., the multiplicity of particles in

![Fig. 2.](image-url) FIG. 2. (Color online.) $R_{E}^{\text{sim}}$ as a function of incident energy for the central Xe + Sn reaction and several values of the $\sigma_{NN}^{m}$ scaling factor $F$ of Eq. (7). Upper (lower) panel shows results obtained with the D1-G1 (Zamick) EOS. The colored zones and curves have the same meaning as in Fig. 1 but here for the scaled $\sigma_{NN}$ of Chen et al. [24]. For more details see the caption of Fig. 1 and the text.
a reaction event, and the reaction centrality. In a simulation the centrality is an input variable, thus it is under full control. In comparing simulation results and the earlier INGRID study of $R_E$ and $R_p$ \ref{0} it has been underlined that selecting events via multiplicity strongly mixes events of different impact parameters over a rather broad span in $b$ \cite{12, 28}. Thus, let us examine the $b$ vs multiplicity relationship and its influence on the isotropy ratio. For that purpose we use the semidynamical general-purpose event-generating code HIPSE (Heavy-Ion Phase-Space Exploration) intended to describe HICs at intermediate energies \cite{29}. At each energy 100 000 events are generated in the range $b = 0 – 7$ fm. Let us note that, according to the expression of $\sigma_R$ in Ref. \cite{30}, the above range in $b$ is equivalent to $0.27 \sigma_R – 0.30 \sigma_R$, depending on $E_{inc}$. At $E_{inc} = 50$ A MeV the simulation was performed in the full impact parameter range of the $^{129}$Xe + $^{120}$Sn reaction, i.e., $b = 0 – 13$ fm, in order to verify that in the non-covered range ($b = 7 – 13$ fm) the high multiplicity events, in which we are interested, are not present. By passing the generated events through a sophisticated INGRID-device geometry and detection-acceptance filter \cite{31} we found that it has no appreciable effect on the $Z = 1$ $R_E$ values. Mostly, the change in $R_E$ due to this filter is below 0.5%.

Selecting the range $b = 0 – 2$ fm, in a geometrical sharp-cut approximation, corresponds to 1.82% to 1.92% of $\sigma_R$ in the studied $E_{inc}$ range, i.e., between 104 and 116 mb. These values fall in the middle of the cross section values of the selected subset of the most violent INGRID data events analyzed in Ref. \cite{7}. Ideally, the reactions with $b \leq 2$ fm should correspond to 8163 out of the total 100 000 generated events. In reality, there were on the average 8109 such events with a fluctuation up to 3% from energy to energy. We denote this precise number of events $N_{0–2}$ to search for, in the full ensemble of 100 000 events, the subset of events with the highest multiplicity that is by number of events closest to $N_{0–2}$. By $M_{0–2}$ we label both the lowest multiplicity of the thus selected subset as well as the subset of events itself at each $E_{inc}$.

Let us check the behavior of the most violent $M_{0–2}$ HIPSE events. As a kind of ”background”, in Fig. 4 we show by the thin black line the $b$-distribution histogram of the full 100 000 event data set for each second studied $E_{inc}$. The $b$ distribution of the $M_{0–2}$ events is shown by the red-line yellow-filled histogram. These high-multiplicity events are generated in a large domain of $b$ values which extends up to 5 fm. To make the $M_{0–2}$ $b$ distribution better visible, it is enlarged to the full frame size by the yellow red-line histogram. A Gaussian fit to it clearly demonstrates that the normal-law of data statistics correctly reproduces the distribution of $M_{0–2}$ subset over $b$’s. These events are in minor part (3% to 29%) belonging to the $b \leq 2$ fm subset of the full data set (hatched part of black histogram). From the Gaussian fit one infers that the maximum of these high-multiplicity events is about $b \approx 3$ fm and that it slightly decreases with the increasing $E_{inc}$.
Finally, let us apply the HIPSE $M_{0,-2}$ $b$ distribution to the LV simulation results. Taking the Gaussian fit values of Fig. 4 as the weights for the integer values of $b$, the $b$-averaged $R_{E}^{\text{sim}}$ are obtained for each studied value of the factor $F$ of Eq. (7). By this method, for the $F=1$ case these $R_{E}^{\text{sim}}$ are, in millibarn units, also equivalent to 0.02 $\sigma_{R}$. $R_{E}^{\text{sim}}$ extracted from thus averaged $R_{E}^{\text{sim}}$ is in Fig. 9 shown by dot-dashed curves and open triangles, upright and reversed, for the D1-G1 EOS and Zamick EOS, respectively. For the nonlocal D1-G1 EOS the two $b$-averaging intervals give strictly the same $R_{E}^{\text{sim}}$ for $E_{\text{inc}} \leq 50$ A MeV. At $E_{\text{inc}} = 80$ A and 100 A MeV the respective $R_{E}^{\text{sim}}$ values differ by about 20% but are mutually compatible when errors are accounted for. For the zero-range Zamick EOS in the full $E_{\text{inc}}$ interval, two $b$-averaging intervals give compatible predictions for the $F_{\text{sim}}$ values although for $E_{\text{inc}} \geq E_{F}$ the more stringent centrality results are in somewhat better agreement with the $F_{\text{exp}}$ values.

IV. SUMMARY AND CONCLUSIONS

The semiclassical Landau-Vlasov (LV) transport model was used to study the energy-based isotropy ratio $R_{E}$ of Eq. 5 for the $^{129}$Xe + $^{120}$Sn reaction in the wide incident energy range $12 A \leq E_{\text{inc}} \leq 100$ A MeV. The focus of the present work is twofold:

1) an appreciable reduction of $R_{E}$ of Ref. 7. The model predicts the appearance of a break in the slope of the multiplicative factor $F$ after a minimum located near $E_{F}$. However, the agreement or disagreement between the absolute values of $F_{\text{sim}}$ and $F_{\text{exp}}$ should be considered with some caution due to two possible causes. On one hand, the value of factor $F$ may be altered by reaction centrality. Accordingly, an investigation of $R_{E}$ with a quasidynamical event generator HIPSE 29 was carried out. It reveals that the event selection based on multiplicity and the geometrical sharp-cut approximation is not a correct centrality selector. Indeed, corroborating earlier findings 12,28, we show that this selection approach strongly mixes events of different impact parameters over a rather broad span of $b$ values; cf. Fig. 4.

When a properly weighted contribution of $b$’s involved in the high-multiplicity events is accounted for, the isotropy ratios calculated for the thus relaxed centrality requirement and those strictly central do not differ much. The thus extracted $F_{\text{sim}}$ does not change much as well. On the other hand, the derived $F_{\text{exp}}$ values are based on a number of strong assumptions that allowed the link between the stopping ratio $R_{E}$ and the in-medium $NN$ cross section 8. Hence, besides further experimental and theoretical considerations of the stopping observable $R_{E}$ intended to disentangle the remaining ambiguities a study of other related observables may shed some fresh light on the subject. In addition, the experimentally observed strong and rapid change of the effective in-medium residual $NN$ cross section beyond the Fermi energy urges for an ab initio theoretical analysis of this problem, the solution of which might lie in the way the exclusion principle is accounted for 8 and/or by incorporating the recent observation of short-range correlations in nuclei 22,33. Their consequences for transport descriptions of heavy-ion reactions are of high interest and need to be investigated.

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The angular dependence of the nucleon-nucleon (NN) cross section $\sigma_{\text{NN}}$ is expressed as

$$
\sigma(\theta)_{\text{c.m.}}(\text{mb}/\text{sr}) = f(E, \theta) \times \frac{\sigma_{\text{tot}}(\text{mb})}{4\pi}, \quad (A.1)
$$

where $\sigma_{\text{tot}} = \sigma_{\text{NN}}^{\text{Li-Machleidt}}$ is the total elastic cross section due to Li and Machleidt [10]. The dimensionless weighting factor $f(E, \theta)$ is equal to unity for the scattering between neutrons ($f_{\text{nn}} = 1$) and is increasingly anisotropic as energy increases for neutron-proton scattering (the $f_{\text{np}}$ case), and especially becomes strongly forward-backward peaked for the scattering between protons ($f_{\text{pp}}$). The parametrization of the angular dependence of $\sigma_{\text{np}}$ is defined as [26]

$$
f(E, \theta)_{\text{np}} = \frac{A_1 \cos^3 \theta - A_2 \cos^3 \theta + A_3}{A_1/5 + A_3}, \quad (A.2)
$$

where, for the purpose of the fitting, the coefficients $A_k$ at each energy are expressed by the following functional dependence:

$$
A_k(E) = a_{i,k} + \frac{b_{i,k}(E - c_i)}{c_i}, \quad k = 1, 2, 3. \quad (A.3)
$$

Parameters $a_{i,k}$, $b_{i,k}$, and $c_i$ are fixed by fitting the experimental $\sigma_{\text{np}}$ data at nine beam energies $e_i$ between 26 and 319 MeV, index $i$ running over energies. $E$ and $e_i$ are expressed in MeV units. Between these energies the parameters are assumed to change linearly with $E$. The values of these parameters are given in Table I and $f(E, \theta)_{\text{np}}$ is shown in Fig. 5(a).

Applying a very crude estimate, the polar angle dependence of $\sigma_{\text{np}}$ is defined as [26]

$$
f(E, \theta)_{\text{pp}} = \begin{cases} 
B_1 \exp(B_2 \theta), & \theta < \theta_0, \\
B_3, & \theta_0 < \theta < \pi - \theta_0, \\
B_1 \exp(B_2(\pi - \theta)), & \theta > \pi - \theta_0, 
\end{cases} \quad (A.4)
$$

where coefficients $B_k$ are expressed by the following functional dependence:

$$
B_k(E) = a_{i,k} + \frac{(\alpha_{i+1,k} - \alpha_{i,k})(E - c_i)}{\epsilon_{i+1} - \epsilon_i}, \quad k = 1, 2, 3. \quad (A.5)
$$

Due to indistinguishability of particles, coefficients $B_1$ and $B_3$ are divided by 2. At each energy $E$ the limiting angle reads $\theta_0 = \ln(B_3/B_1)/B_2$. The overall angular distribution normalization is given by the value of $\sigma_{\text{NN}}^{\text{Li-Machleidt}}$, and Eq. (A.3) is used to define, on Monte Carlo grounds, the angle into which a couple of charged Gaussians is scattered in a $p - p$ collision. The parameters $\alpha_{i,k}$ are fixed by fitting experimental differential cross sections $\sigma_{\text{pp}}$ at six energies $\epsilon_i$ ranging from 5 to 144 MeV denoted by the index $i$. As above, $E$ and $\epsilon_i$ are expressed in MeV units. Between these energies, parameters are assumed to change linearly with $E$. The values of these parameters are given in Table II and $f(E, \theta)_{\text{pp}}$ is shown in Fig. 5(b).

### Table I. Coefficients of neutron-proton scattering angular distribution function $f_{\text{np}}$ of Eq. (A.2) as parametrized by Eqs. (A.3).

| $E$ (MeV) | $e_i$ (MeV) | Ref. | $a_{i,1}$ | $a_{i,2}$ | $a_{i,3}$ | $b_{i,1}$ | $b_{i,2}$ | $b_{i,3}$ | $c_i$ |
|----------|-------------|------|------------|------------|------------|----------|----------|----------|-----|
| $E < 26$ | 0           |      |            |            |            |          |          |          |     |
| $26 \leq E < 35$ | 26 | [34] | 0 | 0 | 1 | -0.426 | 2.372 | 0.32 | 0.35 | -0.40 | 10 |
| $35 \leq E < 45$ | 35 | [35] | 0.97 | -0.426 | 2.372 | 0.32 | -0.127 | -0.18 | 8 |
| $45 \leq E < 53$ | 45 | [35] | 1.29 | -0.073 | 1.97 | 0.32 | -0.127 | -0.18 | 8 |
| $53 \leq E < 63$ | 53 | [35] | 1.61 | -0.2 | 1.79 | -0.04 | 0.51 | 0.16 | 10 |
| $63 \leq E < 73$ | 63 | [35, 37] | 1.57 | 0.31 | 1.95 | 0.33 | -0.59 | -0.30 | 10 |
| $73 \leq E < 90$ | 73 | [35] | 1.9 | -0.28 | 1.65 | 0.9 | 0.205 | -0.16 | 17 |
| $90 \leq E < 130$ | 90 | [36, 38] | 2.8 | -0.075 | 1.49 | 1.2 | -0.465 | -0.094 | 40 |
| $130 \leq E < 319$ | 129 | [39] | 3.0 | -0.54 | 1.396 | -1.3 | 0.665 | -0.81 | 189 |
| $E \geq 319$ | 319 | [40] | 2.69 | 0.125 | 0.588 | 0 | 0 | 0 | 1 |

### Table II. Coefficients of proton-proton scattering angular distribution function $f_{\text{pp}}$ of Eq. (A.4) as parametrized by Eqs. (A.5).

| $E$ (MeV) | $e_i$ (MeV) | Ref. | $a_{i,1}$ | $a_{i,2}$ | $a_{i,3}$ |
|----------|-------------|------|------------|------------|------------|
| $E < 5$ | 0           |      |            |            |            |
| $5 \leq E < 9.9$ | 5 | [41] | 5176.1 | -8.91 | 100.0 |
| $9.9 \leq E < 19.7$ | 9.9 | [42] | 1795.6 | -9.29 | 52.62 |
| $19.7 \leq E < 39.4$ | 19.7 | [43] | 1071.0 | -12.0 | 24.95 |
| $39.4 \leq E < 68$ | 39.4 | [44] | 1382.2 | -19.26 | 11.16 |
| $68 \leq E < 144$ | 68 | [45] | 1880.5 | -26.77 | 6.16 |
| $E \geq 144$ | 144 | [46] | 4008.8 | -45.92 | 3.99 |

$A_k(E) = a_{i,k} + \frac{b_{i,k}(E - c_i)}{c_i}, \quad k = 1, 2, 3.$
FIG. 5. (Color online.) Dimensionless weighting factor $f(E, \theta)$ which modulates total elastic cross section as a function of polar angle $\theta_{\text{cm}}$ and nucleon incident energy $E$ for the scattering of neutron and proton (upper panel) and between protons (lower panel), where the applicate axis is in logarithmic scale.

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