Analytical Method Used to Calculate Pile Foundations with the Widening Up on a Horizontal Static Impact

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Abstract: This paper presents a methodology for the analytical research solutions for the work pile foundations with surface broadening and inclined side faces in the ground array, based on the properties of Fourier transform of finite functions. The comparative analysis of the calculation results using the suggested method for prismatic piles, piles with surface broadening prismatic with precast piles and end walls with precast wedges on the surface is described.

1. Introduction

In a disjointed waterlogged soils becomes relevant the use of piles with inclined side faces or surface caps, which in practice greatly increases the effectiveness of pile Foundation in solving unusual design problems. However, their use is limited due to the lack of reliable methods for determining the stress-strain state in soils composed of different physical-mechanical characteristics changing along the length with consideration of plastic properties [1-4]. Photographs of the experiments with the pile with surface broadening in the form of prefabricated wedges on a construction site in Astrakhan are presented in Figure 2.

Experience of application of the finished piles with the broadening at the top or corners of the taper helped to reveal that the bearing capacity of such structures is increasing not only due to the increase in the square at the tip, but due to the changes of the work conditions of the soil on the lateral surface of pile and friction forces, which is implemented to a greater extent.

In works [5-9] the authors developed a calculation method for beams and piles with piecewise-constant parameters based on the properties of Fourier transforms of finite functions, which have shown good convergence of calculation results with the results obtained experimentally in the course of conducting field studies and numerical simulations in MIDAS GTS NX and the system solver FEMAP with NE/NASTRAN [10-13].

2. Solution method based on properties of Fourier transforms of finite functions

For solving problems represented by differential equations with piecewise constant parameters it is most convenient to use a solution method based on properties of Fourier transforms of finite functions.
However, in the methods not the task of the joint work of the soil massif at the site of construction of piles with complex geometry widenings located at the top end [14-20].

Consider a pile with surface broadening Figure 1, prismatic piles with precast cap, piles with prefabricated wedges, step piles, immersed in the soil of length $L$ and divide it into four sections with corresponding lengths $l_1$, $l_2$, $l_3$, $l_4$, with piecewise constant bending stiffness $EI_1$, $EI_2$, $EI_3$, $EI_4$ and hardness of the base $k_1$, $k_2$, $k_3$, $k_4$. Take the origin of coordinates in the upper section of each area of the pile, marking them with dots [1-4].

![Diagram of the pile with surface broadening](image)

**Figure 1.** Diagram of the pile with surface broadening, prismatic piles with precast cap pile with modular wedges, speed of the pile.

Write the differential equation of the bending area of the pile with surface broadening in $l_1$ generalized finite functions

$$EI_1 \frac{d^4 u}{dx^4} + k_1 u = EI_1 u(0) \delta'(x - l_1) - EI_1 u'(0) \delta''(x - l_1) + EI_1 u''(l_1) \delta''(x - l_1) +$$

$$+ M(0) \delta'(x + l_1) - M \delta'(x - l_1) + Q(0) \delta(x + l_1) - Q(l_1) \delta(x - l_1) \quad (1)$$

Dividing by the Flexural rigidity and using common designation $4 \beta_1 u = k / EI_1$ get

$$\frac{d^4 u}{dx^4} + 4 \beta_1^4 u = u(0) \delta'(x - l_1) - u'(0) \delta''(x - l_1) + u''(0) \delta''(x - l_1) +$$

$$+ \frac{M(0)}{EI_1} \delta'(x - l_1) - \frac{M(l_1)}{EI_1} \delta'(x - l_1) + \frac{Q(0)}{EI_1} \delta(x) - \frac{Q(l_1)}{EI_1} \delta(x - l_1) \quad (2)$$

$$\tilde{u}_1(v) = \frac{1}{\nu^4 + 4 \beta_1^4} \tilde{Q}(v) \quad (3)$$

In accordance with the theorem of Wiener-Paley-Schwartz function $\tilde{u}_1(v)$ must be an integer, because it is a picture of Fourier, finite functions. Therefore, the numerator $\tilde{Q}(v)$ must be divisible by the denominator $\nu^4 + 4 \beta_1^4$, which corresponds to the condition of divisibility of the function $\tilde{Q}(v)$ in terms $(\nu - \nu_j)$, where $\nu_j, (j = 1, 2, 3, 4)$ are the roots of the denominator:

$$\nu^4 + 4 \beta_1^4 = 0 \quad (4)$$
Get:

$$\tilde{Q}(v_j) = 0; (j = 1, 2, 3, 4)$$

$$\tilde{Q}(v) = \frac{Q(0)}{E_1} + \frac{M(0)}{E_1}(-iv) + u'(0)(-iv)^2 + u(0)(-iv)^3 - \frac{Q(l_j)}{E_1} - \frac{M(l_j)}{E_1}(-iv)e^{\phi_i} - u'(l_j)(-iv)^2 e^{\phi_i} - u(l_j)(-iv)^3 e^{\phi_i};$$

$$Q(0) M(0) u'(0) u(0) Q(l_j) M(l_j) u'(l_j) u(l_j)$$

$$[\begin{array}{cccccccc}
1 / E_1 & -iv / E_1 & -v_1^j & -iv_1^j & -1 / E_1 & iv_1^j e^{\phi_1} / E_1 & -v_1^j e^{\phi_1} / E_1 & -iv_1^j e^{\phi_1} / E_1 \\
1 / E_1 & -iv / E_1 & -v_2^j & -iv_2^j & -1 / E_1 & iv_2^j e^{\phi_2} / E_1 & -v_2^j e^{\phi_2} / E_1 & -iv_2^j e^{\phi_2} / E_1 \\
1 / E_1 & -iv / E_1 & -v_3^j & -iv_3^j & -1 / E_1 & iv_3^j e^{\phi_3} / E_1 & -v_3^j e^{\phi_3} / E_1 & -iv_3^j e^{\phi_3} / E_1 \\
1 / E_1 & -iv / E_1 & -v_4^j & -iv_4^j & -1 / E_1 & iv_4^j e^{\phi_4} / E_1 & -v_4^j e^{\phi_4} / E_1 & -iv_4^j e^{\phi_4} / E_1
\end{array}]$$

$$Q(0) + \frac{M(0)}{E_1}(-iv) - \frac{Q(l_j)}{E_1}(-iv) + v_j^j u'(0) - iv_j^j u(0) - v_j^j u'(l_j) e^{\phi_j} = 0;$$

$$Q(l_j) = \tilde{q}_j(v) + Q(l)e^{\phi_i} + M(l)(-iv)e^{\phi_i} + Elu'(l)(-iv) - Eu(l)(-iv^3) e^{\phi_i};$$

$$Q(0) M(0) u'(0) u(0) Q(l_j) M(l_j) u'(l_j) u(l_j)$$

$$[\begin{array}{cccccccc}
1 / E_1 & -iv(1 + i) / E_1 & -2\beta^2 e^{\phi_1} = 2\beta^2 i; & v_1^j & 2\sqrt{2}\beta e^{\phi_1} = 2\beta^3(1 + i); \\
1 / E_1 & -iv(-1 + i) / E_1 & -2\beta^2 e^{\phi_2} = 2\beta^3 i; & v_2^j & 2\sqrt{2}\beta e^{\phi_2} = 2\beta^3(1 + i); \\
1 / E_1 & -iv(-1 - i) / E_1 & -2\beta^2 e^{\phi_3} = 2\beta^3 i; & v_3^j & 2\sqrt{2}\beta e^{\phi_3} = 2\beta^3(1 - i); \\
1 / E_1 & -iv(1 - i) / E_1 & -2\beta^2 e^{\phi_4} = 2\beta^3 i; & v_4^j & 2\sqrt{2}\beta e^{\phi_4} = 2\beta^3(1 - i);
\end{array}]$$

$$Q(l_j) = \tilde{q}_j(v) + Q(l)e^{\phi_i} + M(l)(-iv)e^{\phi_i} + Elu'(l)(-iv) - Eu(l)(-iv^3) e^{\phi_i};$$

Get the coefficient matrix of the boundary element (pile i)

$$Q(0) M(0) u'(0) u(0) Q(l_j) M(l_j) u'(l_j) u(l_j)$$

$$[\begin{array}{cccccccc}
1 / E_1 & -i\beta(1 + i) / E_1 & -2\beta^2 e^{\phi_1} = 2\beta^3 i; & v_1^j & 2\sqrt{2}\beta e^{\phi_1} = 2\beta^3(1 + i); \\
1 / E_1 & -i\beta(-1 + i) / E_1 & -2\beta^2 e^{\phi_2} = 2\beta^3 i; & v_2^j & 2\sqrt{2}\beta e^{\phi_2} = 2\beta^3(1 + i); \\
1 / E_1 & -i\beta(-1 - i) / E_1 & -2\beta^2 e^{\phi_3} = 2\beta^3 i; & v_3^j & 2\sqrt{2}\beta e^{\phi_3} = 2\beta^3(1 - i); \\
1 / E_1 & -i\beta(1 - i) / E_1 & -2\beta^2 e^{\phi_4} = 2\beta^3 i; & v_4^j & 2\sqrt{2}\beta e^{\phi_4} = 2\beta^3(1 - i);
\end{array}]$$

$$Q(l_j) = \tilde{q}_j(v) + Q(l)e^{\phi_i} + M(l)(-iv)e^{\phi_i} + Elu'(l)(-iv) - Eu(l)(-iv^3) e^{\phi_i};$$

However, the Flexural rigidity at the area of the pile l_i (of surface widening) value is not piecewise constant, and piecewise variable and depends on the geometry of the broadening. The stiffness of the piles in the area of broadening depends on the angle of inclination of the side faces α and width broadening b_i Figure 2. Therefore, for the pile with surface broadening Figure 1 and piles of modular wedges Figure 1. Flexural rigidity on the i-th section of the broadening can be determined from the design scheme presented in figure 2 according to the formulas:

$$EI_{li} = E \left[ \frac{b_i l_i^3}{12} - \frac{b_i A_i}{36} \right]; E I_{li} = E \left[ \frac{b_i l_i^3}{12} - \frac{b_i A_i}{36} \right].$$
Figure 2. Calculation scheme for determining the Flexural rigidity of the piecewise-variable plots of the upper end of the piles with broadening and sloping of the side piles.

Therefore, the matrix (10) coefficients of the boundary element (piles $l_1$) due to changes in bending stiffness along the height of the pile's cross-section will look like:

$$
\begin{bmatrix}
Q(0) & M(0) & u'(0) & u(0) & Q(l_i) & M(l_i) & u'(l_i) & u(l_i) \\
1/El & -i\beta(1+ i)/El & -2\beta i & -i2\beta(-1+ i) & -1/El & i\beta(1+ i)e^{\pi/2} / El & -2\beta ie^{\pi/2} & -i2\beta(-1+ i)e^{\pi/2} \\
1/El & -i\beta(-1+ i)/El & 2\beta i & -i2\beta(1+ i) & -1/El & i\beta(-1+ i)e^{\pi/2} / El & 2\beta ie^{\pi/2} & -i2\beta(1+ i)e^{\pi/2} \\
1/El & -i\beta(-1- i)/El & -2\beta i & -i2\beta(-1- i) & -1/El & i\beta(-1- i)e^{\pi/2} / El & -2\beta ie^{\pi/2} & -i2\beta(-1- i)e^{\pi/2} \\
1/El & -i\beta(1- i)/El & 2\beta i & -i2\beta(-1- i) & -1/El & i\beta(1- i)e^{\pi/2} / El & 2\beta ie^{\pi/2} & -i2\beta(1- i)e^{\pi/2}
\end{bmatrix}
$$

(12)

By analogy, calculate the differential equation of bending sections of the pile $l_3$ of the generalized finite functions.

Substitute the roots, squares and cubes of the roots, which will need for further algebraic transformations:

$$
v_1 = \sqrt{2}\beta e^{\pi/2} = \beta(1+ i); \quad v_2 = 2\beta^2 e^{\pi/2} = 2\beta^2 i; \quad v_3 = 2\sqrt{2}\beta e^{\pi/2} = 2\beta^3(1+ i);$$

$$
v_1 = \sqrt{2}\beta e^{3\pi/2} = \beta(-1+ i); \quad v_2 = 2\beta^2 e^{3\pi/2} = -2\beta^2 i; \quad v_3 = 2\sqrt{2}\beta e^{3\pi/2} = 2\beta^3(1- i);$$

$$
v_1 = \sqrt{2}\beta e^{5\pi/2} = \beta(-1- i); \quad v_2 = 2\beta^2 e^{5\pi/2} = 2\beta^2 i; \quad v_3 = 2\sqrt{2}\beta e^{5\pi/2} = 2\beta^3(-1- i);$$

(13)

Get the coefficient matrix of the boundary element (pile $l_3$), but also piecewise constant Flexural rigidity:
\[
\begin{bmatrix}
Q(0) & M(0) & u'(0) & u(0) & u'(l_z) & u(l_z)
\end{bmatrix}
\begin{align*}
1 / EI_3 & -i \beta (1 + i) / EI_3 & -2 \beta^2 i & -i2 \beta^3 (1 + i) & -2 \beta^2 i & -i2 \beta^3 (1 + i) e^{i \omega t / \eta z}, \\
1 / EI_3 & -i \beta (-1 + i) / EI_3 & 2 \beta^2 i & -i2 \beta^3 (1 + i) & 2 \beta^2 i & -i2 \beta^3 (1 + i) e^{i \omega t / \eta z}, \\
1 / EI_3 & -i \beta (-1 - i) / EI_3 & -2 \beta^2 i & -i2 \beta^3 (-1 - i) & -2 \beta^2 i & -i2 \beta^3 (-1 - i) e^{i \omega t / \eta z}, \\
1 / EI_3 & -i \beta (1 - i) / EI_3 & 2 \beta^2 i & -i2 \beta^3 (-1 - i) & 2 \beta^2 i & -i2 \beta^3 (-1 - i) e^{i \omega t / \eta z},
\end{align*}
\] (14)

Table 1. Initial data for calculation.

| Design of piles | The size and shape of the broadening (mm) | Soil conditions |
|-----------------|------------------------------------------|-----------------|
| 1 2 3 4         |                                          |                |
| 1 Prismatic pile \( (L_{ca}=8000, \text{cross section } 300x300) \) | L_{sw} = | the modulus of elasticity \( E = 3.24 \times 10^7 \) kN/m², |
| 2 Pile with surface broadening \( (L_{ca}=8000, \text{cross section } 300x300) \) | B= | the ratios of base: \( k_1 = 1 \times 10^5 \) kN/m², \( k_2 = 2 \times 10^6 \) kN/m², \( k_3 = 4.5 \times 10^7 \) kN/m², \( k_4 = 8 \times 10^8 \) kN/m² |
| 3 Prismatic piles with precast cap \( (L_{ca}=8000, \text{cross section } 300x300) \) | L_{sw} = |                |
| 4 Pile with modular wedges on the surface. \( (L_{ca}=8000, \text{cross section } 300x300) \) | L_{sw} = |                |

3. Conclusion
The calculation results present on the chart Figure 3.

Figure 3. the displacement Values of the upper end of the pile with surface broadening and normal prismatic piles horizontal static loading is obtained analytically by the method of Fourier transforms of finite functions: 1 - pile with precast wedges on the surface; 2 - prismatic piles with precast cap; 3 - pile with surface broadening.
Photographs of the experiments with the pile with surface broadening in the form of prefabricated wedges on a construction site in Astrakhan are presented in Figure 4-6.

![Figure 4. Photographs of the experiments with the pile with surface broadening.](image)

![Figure 5. Photographs of the experiments with the pile with surface broadening dip the wedges around the piles.](image)

![Figure 6. The finished pile with wedges.](image)

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