Comparison of linear methodologies and not for the evaluation of the vibrational morphodynamics of a pair of gears in different tribological conditions

V Niola*, G Quaremba, C Cosenza, L Russo and S Savino
Department of Industrial Engineering, University of Naples “Federico II”, via Claudio 21, 80125 Napoli (Italy)

vniola@unina.it

Abstract. The gear wheels are the most common mechanism for the transmission of the torque between two organs. The tribological conditions can compromise both their performance and lifetime. Vibration analysis is a valid tool for the assessment of the health of a mechanical system. The following work illustrates a non-linear methodology for evaluating the vibrational dynamics of a single pair of gears, under different tribological conditions. The proposed methodology highlights the presence of damage on the surface of the teeth of the gear pair and evaluates effect on micro-vibratory dynamics due to different lubrication and rotation regimes. Once the vibratory dynamics features of the structure are known, a statistical model has been developed capable of classifying the different tribological regimes.

1. Introduction
The assessment of tribological state is a crucial issue in mechanical systems. Indeed, the tribological condition strongly affects the system function and as a consequence a poor lubrication regime can lead to the system fault and damage. In this context, non-destructive techniques represent a good choice for the system diagnosis [1]. We this regards, we propose the application of non-linear signal processing to the evaluation of the lubrication regime of an unloaded helical gear pair. Signal processing, using discrete wavelet transform (DWT) and artificial neural network, has already been applied for the detection of fault in gear box [2,3]. Moreover, experimental investigations have been carried out to assess surface fatigue wear on spur gear teeth through vibration signal analysis, using continuous wavelet transform, along with specific film thickness studies [4].
Here, a methodology, to determine the lubrication regime of a gear box, is proposed using signals acquired through accelerometer sensors elaborated following a non-linear theory approach [5–7]. The theoretical background, on which the proposed signal processing methodology is based, is summarized in the theory paragraph. A more exhaustive description has been presented in previous works [8–11].
The proposed approach aims not only to the identification of the tribological regime of the system but could be used for the health monitoring of this and other mechanical systems. Furthermore, this methodology could help to test new lubrication oil, especially during the research and development stage and/or to monitor the mechanical system during the working conditions.

2. Experimental setup
In order to detect the gear tribological level, experimental tests have been conducted on a specific gear test rig, reported in Fig. 1.

![Experimental test rig](image)

**Figure 1.** Experimental test rig.

The experimental test rig comprises of an unloaded helical gear pair. The gearwheels are splined to the shafts by locking devices (sit-lock). The helical gear pair is constituted by a pinion, splined on the driveshaft, with 37 teeth and a driven wheel with 33 teeth. Hence, the gear ratio, \( \varepsilon \) is 0.891. The driveshaft is moved by a speed controlled brushless motor. The measurement technique is based on the acquisition of vibrational signals through a DAQ device and an accelerometer sensor. The DAQ device is equipped with a chassis (NI PXIe-1071), a controller (NI PXIe-8840) and a motherboard acquisition module (Sound and Vibration NI PXIe 4497). The acquisition module is designed for high channel count vibration applications. Its main features are a 204.8 kS/s sample frequency range, 16-analog input channels with 24-bit resolution. The sensor is a triaxial accelerometer (Kistler) with a resonant frequency >70 kHz. The accelerometer is placed on test-rig structure through a magnetic mounting base (see Fig1). The accelerometer is mounted so that the z-axis is perpendicular to the driveshaft. The data analysis that we propose needs high sampling frequency. The acquisition module and sensor features allow us to set a sampling frequency of 51200 Hz, which has been chosen according to the Nyquist theorem.

2.1. Test variables

The accelerometer signals have been acquired for three lubrication conditions typical of gear tooth contacts: dry (no-oil, N) and two boundary conditions. The two boundary lubrication regimes have been achieved greasing the gearwheel teeth with an initial amount of lubricant, without providing any other oil external feeding. Two different oils have been employed: a high viscosity oil (H) and a low viscosity oil (L). The high viscosity lubricant has a SAE grade equal to 85w140, while the low viscosity lubricant has a SAE grade 75w90. The experimental tests have been performed at two driveshaft speeds: 700 rpm and 900 rpm. Two incremental optical encoders (40000 pulses/rev), fixed on each shaft, allow to control the engine speed. The acquisitions have been carried out after the achievement of the system regime conditions. Data have been acquired for two consecutive runs of 10 seconds each, with a time break between them of 15 seconds.

3. Theoretical background

The nonlinear method adopted in this work starts with the decomposition of the rough accelerometric signal into two mutually orthogonal components (\( \chi_\omega \) and \( \zeta_\omega \), as proposed in previous theoretical works [6,7]. These components can be obtained using the following equations:

\[
\chi_\omega = \sum_{j=1}^{m} x_j \cos(j \omega)
\]
\[ \zeta_\omega = \sum_{j=1}^{m} \chi_j \sin(j\omega) \]  

where \( x_j \) is a time series (in our case, the accelerometric signal), \( \omega \in (0, \pi] \) and \( m=1, 2, ..., N \), where \( N \) is the time series length.

The two orthogonal projectors (1) and (2) allow to determine the vibration morphology. The \( \chi_\omega \) vs \( \zeta_\omega \) plane is able to give qualitative details on the accelerometric signals, such as its regularity, as already discussed in previous works [12]. This plane can help to distinguish regular from chaotic sequences. The regular series are characterized by Lyapunov exponent \( \lambda < 0 \), the chaotic series by a Lyapunov exponent \( \lambda > 0 \) [13].

A better characterization of the numerical sequence can be made computing the mean square displacement \( M_\omega(m) \), of these components \( \chi_\omega \) and \( \zeta_\omega \), according to the following function:

\[
M_\omega(m) = \lim_{N \to +\infty} \frac{1}{N-m} \sum_{j=1}^{N-m} [\chi_\omega(j + m) - \chi_\omega(j)]^2 + [\zeta_\omega(j + m) - \zeta_\omega(j)]^2
\]

where \( m=1, 2, ..., m^* \) and \( m^* << N \) (typically \( m^* = N/10 \)).

Stationary and regular series deal with stable values of the mean square distances. If the time series tends toward a fixed point, the mean square displacement decreases. Conversely, the time series associated to unstable and chaotic systems, are characterized by an increase in the mean square displacement value [6]. The trend of function \( M_\omega(m) \) can be studied subtracting the oscillatory component \( O(\omega, m) \) from \( M_\omega(m) \) obtaining \( \Delta_\omega(m) \):

\[
\Delta_\omega = M_\omega(m) - O(\omega, m)
\]

where

\[
O(\omega, m) = |E[x]|^2 \frac{\sin(m\omega)}{1-\cos(\omega)}
\]

and

\[
|E[x]|^2 = \lim_{N \to +\infty} \frac{1}{N} \sum_{j=1}^{N} x_j
\]

Let us consider \( \rho \) as the correlation coefficient for two vectors, \( x \) and \( y \), with the same length \( n \):

\[
\rho(x, y) = \frac{\text{cov}(x,y)}{\sqrt{\text{var}(x) \text{var}(y)}}
\]

where \text{cov} and \text{var} stand for covariance and variance, respectively.

Now define two vectors:

\[ \xi = [1, 2, \ldots, m^*] \]

\[ \Delta = [\Delta_\omega(1), \Delta_\omega(2), \ldots, \Delta_\omega(m^*)] \]

These two variables \( \Delta \) and \( \xi \) can be used to define the behavior of the time series starting from their correlation:

\[
K_\omega = \rho(\xi, \Delta) = \frac{\text{cov}(\xi, \Delta)}{\sqrt{\text{var}(\xi) \text{var}(\Delta)}}
\]

\( K_\omega \) assumes values close to zero, if the signal is regular (i.e., according to Lyapunov exponent, \( \lambda < 0 \)). Conversely, if the signal exhibits a chaotic trend, \( K_\omega \) tends to 1 (Lyapunov exponent, \( \lambda > 0 \)). It could be also calculated a statistical median of \( K_\omega \), in order to reduce the presence of outliers.

3.1. Relation between omega and spectral frequency
Starting from the mean square displacement, $M_\omega(m)$, we can obtain the relationship between $\omega$ (rad) and the spectral frequency in Hz. Comparing the mean square displacement, $M_\omega(m)$ with the power spectrum $P(f)$:

$$M_\omega(m) = \lim_{N \to +\infty} \frac{1}{N-m} \sum_{j=1}^{N-m} \left\{ e^{i\omega j \Delta x} \right\}^2$$

$$P(f) = \lim_{N \to +\infty} \frac{1}{N} \left| \sum_{j=1}^{N} e^{2\pi i f \Delta x} x_j \right|^2 \tau_s^2$$

where $\tau_s$ is the sampling period, we achieve:

$$\omega = 2\pi f \frac{\tau_s}{\tau_s}$$

The Eq. 13 allows to convert $\omega$ in radians into frequency $f$ in Hertz and vice versa.

### 4. Results

For each lubrication regime and for two rotation speeds of the primary shaft, accelerometer signals have been acquired. The rough signals are not able to discriminate between the different dynamical behaviour of the system. Thus, the spectrum $K_\omega vs \omega$ (defined in equation 11) can be used to highlight the different behavior due to the several lubrication regimes and to identify them. It is necessary to choose an appropriate $\omega$, namely $\omega^*$, that allows to best discriminate between the acquired accelerometer signals.

As showed in the following $K_\omega vs \omega$ spectrums, reported in Fig. 2, an $\omega^*$ value of 1.472 rad is a good choice. The $K_\omega vs \omega$ spectrum for the signals associated to the three lubrication regimes have been reported for both the speed regimes of 700 rpm (Fig.2A) and 900 rpm (Fig.2B), respectively. $K_\omega vs \omega$ spectrum assumes values from 0 to 1. Values around 1 represent chaotic signals, conversely, values around 0 are associated to regular numerical series, as discussed in the previous Theory paragraph.

During the second acquisition run, the high-viscosity regime values overlap the first run data of the low-viscosity regime. The increase in the driveshaft speed reduces the difference between the spectrum values, therefore, in Fig. 2B, it is more difficult to discriminate between the signals with respect to Fig. 2A. The high-speed regime (900 rpm) leads to an earlier decay of the gear lubrication film.

![Figure 2. $K_\omega vs \omega$ spectrum for the two driveshaft speeds: A) 700 rpm and B) 900 rpm.](image)

For each speed, three lubrication regimes are reported: low-viscosity oil, L, red colored data; high-viscosity oil, H, blue colored data and no-oil regime, N, in black.

Moreover, we have computed the $M_\omega(m)$ functions (equation 3) and the $M_\omega^*(m)$ for the different cases. Afterwards, we have interpolated the $M_\omega^*(m)$ with a linear function, denominated $M'_\omega^*(m)$, and we have calculated two parameters that are the slope (Fig. 3) and the intercept (Fig. 4) of this function. The slope
parameter is not able to identify the different tribological regimes for both the driveshaft speeds (Fig. 3A-B). Indeed, all the lines, corresponding to three lubrication regimes, overlap each other.

**Figure 3.** Weak identification index: the slope of the $M'_\omega(m)$ function for A) 700 rpm and B) 900 rpm.

We can consider it as a *weak* identification index. On the contrary, the $M'_\omega(m)$ intercept, given in Fig. 4, allows us to identify the different system behaviors dealing with the corresponding lubrication regimes influenced by the wear levels. Thus, the intercept parameter can be considered as a *strong* identification index. The $M'_\omega(m)$ intercept index is also influenced by the driveshaft speed. Let us consider a single tribological regime, for example the non-oil (N). Comparing Fig. 4A and B, it is possible to observe different mean values of the index for both the driveshaft speeds: around 0.75 (Fig. 4A) and 0.92 (Fig. 4B), for 700 rpm and 900 rpm, respectively. In addition, it is possible to observe that the higher is the speed, the lower is the time occurring, for the boundary tribological regimes (H and L), to reach to the dry condition (N).

Lastly, due to the different viscosity of the lubricants, L-regime intercept values are always greater than the H-regime intercept ones.

**Figure 4.** Strong identification index: the intercept of the $M'_\omega(m)$ function for A) 700 rpm and B) 900 rpm.
5. Conclusions
In this study, a methodology to evaluate the lubrication regime of a mechanical system is proposed. The methodology deals with signal processing analysis starting from the acquisition of accelerometer data. The experimental setup mainly comprises of an unloaded gear pair, an acquisition unit and accelerometer sensors. The signal processing uses the approach of non-linear theory for numerical series.

The proposed methods have been able to identify the different lubrication regimes of the unloaded gear pair based on the evaluation of some indexes. The strong index (\(M'\omega^*(m)\) intercept) has led to the identification of the three lubrication regimes as much as to the evaluation of the driveshaft speed effects. This advanced signal processing approach could be further applied to monitor the health state of this and other mechanical systems.

Acknowledgments
The authors thank Gennaro Stingo and Giuseppe Iovino (Department of Industrial Engineering), Mario Minocchi and Davide Marcone (IT Laboratory) at the University of Naples Federico II, for their fundamental technical support during the tuning stages of the test rig.

References
[1] Tan C K, Irving P and Mba D 2007 A comparative experimental study on the diagnostic and prognostic capabilities of acoustics emission, vibration and spectrometric oil analysis for spur gears Mech. Syst. Signal Process. 21 208–33
[2] Saravanan N and Ramachandran K I 2009 Fault diagnosis of spur bevel gear box using discrete wavelet features and Decision Tree classification Expert Syst. Appl. 36 9564–73
[3] Saravanan N and Ramachandran K I 2010 Incipient gear box fault diagnosis using discrete wavelet transform (DWT) for feature extraction and classification using artificial neural network (ANN) Expert Syst. Appl. 37 4168–81
[4] Amarnath M and Sujatha C 2015 Surface Contact Fatigue Failure Assessment in Spur Gears Using Lubricant Film Thickness and Vibration Signal Analysis Tribol. Trans. 58 327–36
[5] Takens F 1981 Detecting strange attractors in turbulence pp 366–81
[6] Gottwald G A and Melbourne I 2005 Testing for chaos in deterministic systems with noise Phys. D Nonlinear Phenom. 212 100–10
[7] Gottwald G A and Melbourne I 2004 A new test for chaos in deterministic systems Proc. R. Soc. London. Ser. A Math. Phys. Eng. Sci. 460 603–11
[8] Vincenzo N, Giuseppe Q and Aniello F 2008 The detection of gear noise computed by integrating the Fourier and Wavelet methods WSEAS Trans. Signal Process. 4
[9] Amoresano A, Langella G, Niola V and Quaremba G 2014 Advanced image analysis of two-phase flow inside a centrifugal pump Adv. Mech. Eng. 2014
[10] Niola V, Quaremba G and Avaglano V 2007 Vibration monitoring of gear transmission Proc. 9th WSEAS Int. Conf. SIMULATION, Model. Optim. 74–9
[11] Niola V and Quaremba G 2011 The Gear Whine Noise : the influence of manufacturing process on vibro-acoustic emission of gear-box Proceeding of 10th WSEAS international conference on electronics, hardware, wireless and optical communications, and 10th WSEAS international conference (Cambridge, UK: World Scientific and Engineering Academy and Society (WSEAS)) pp 175–9
[12] Allouis C, Amoresano A, Capasso R, Langella G, Niola V and Quaremba G 2018 The impact of biofuel properties on emissions and performances of a micro gas turbine using combustion vibrations detection Fuel Process. Technol. 179 10–6
[13] Rosenstein M T, Collins J J and De Luca C J 1993 A practical method for calculating largest Lyapunov exponents from small data sets Phys. D Nonlinear Phenom. 65 117–34