Quantum Control of the States of Light in a Mach-Zehnder Interferometer

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Abstract. The experiment proposed by Elitzur and Vaidman is modified by replacing the fully absorbing obstacle (the ‘bomb’) with a non-linear optical medium. In this form the involved Mach-Zehnder interferometer works as a quantum beam splitter that produces NOON states which can be controlled if the input is the appropriate linear combination of two-mode Fock states.

1. Introduction

In a previous work [1] we have shown that the probabilities of detection in the well known Elitzur-Vaidman experiment [2] can be manipulated if the fully absorbing obstacle (the ‘bomb’) is substituted by a semitransparent object. Moreover, we found that the transparency of the obstacle determines either the particle-like or the wave-like behavior of the test photon as this occurs in the ‘delayed choice’ experiment proposed by Wheeler [3].

In the present contribution we follow the idea of a quantum beam splitter (QBS) discussed in [4] and include this in the Elitzur-Vaidman experiment to analyze the case when the obstacle is substituted by a non-linear optical medium. Our point is that the QBS works as a refinement of the Elitzur-Vaidman experiment and permits the manipulation of the photon-states that are injected into a Mach-Zehnder interferometer. Indeed, injecting $N$ photons into one of the channels of a conventional beam splitter one obtains a superposition of two-mode Fock states at the output [5]. In contrast, a QBS produces the NOON states [6]

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left( |N, 0\rangle + e^{i\xi} |0, N\rangle \right),$$

where $\xi$ is a phase. Remarkably, the entangled states (1) are very useful to obtain high-precision phase measurements; they have been generated up to $N = 5$ by multi-photon interference of down-converted light with a classical coherent state [7].

We are interested in the case when the following state is used as the input in a QBS

$$|\psi_{in}\rangle = |\psi_b, 0\rangle = \sum_{n=0}^{r} c_n |n, 0\rangle,$$

where $r$ is a non-negative integer defining the number $(r + 1)$ of Fock states $|n\rangle$ involved in the superposition. Hereafter the two-mode Fock states $|n, m\rangle := |n\rangle \otimes |m\rangle$ correspond to the
biphoton states formed by \( n \) photons in the horizontal arm, and \( m \) photons in the vertical arm of the Mach-Zehnder interferometer shown in Fig. 1. For details and properties of the tensor product ‘\( \otimes \)’ see, e.g. [8]. As we are going to see, the non-linear medium used instead the fully absorbing obstacle of the Elitzur-Vaidman experiment would be useful to steering the state (2) to a destination.

2. The modified Elitzur-Vaidman experiment

The setup shown in Fig. 1 represents a QBS, this is a Mach-Zehnder interferometer with a non-linear medium \( O_{\chi} \) in its upper horizontal arm which plays the role of a refined ‘bomb’ in the Elitzur-Vaidman experiment. The operators representing the different devices in the optical-bench are as follows [4,9]:

\[
\begin{align*}
BS &= \exp[i\theta(b \otimes a^\dagger + b^\dagger \otimes a)], \\
M &= \exp\left[i\frac{\pi}{2}(b \otimes a^\dagger + b^\dagger \otimes a)\right], \\
\Phi &= \exp[i\phi(N_H \otimes I)], \\
O_{\chi} &= e^{-i\pi(b^\dagger b)^2/2}.
\end{align*}
\]  (3)

The straightforward calculation shows that the output state is given by

\[
|\psi_{\text{out}}\rangle = \frac{e^{i\pi/4}}{\sqrt{2}} \left( |0, \psi_b^\prime\rangle + |\psi_{b^\prime\prime}, 0\rangle \right)
\]  (4)

where

\[
|\psi_b^\prime\rangle = \sum_{n=0}^{r} c_n t^n |n\rangle, \\
|\psi_{b^\prime\prime}\rangle = \sum_{n=0}^{r} c_n t^{2n-1} |n\rangle.
\]  (5)

The above expressions depend explicitly on the coefficients \( c_n \) that define the initial state (2). The probability of finding the output (4) in the initial state acquires the form

\[
P_{\psi_{b^0},0} = \frac{1}{2} \left( |c_0|^2 + \sum_{n=0}^{r} \sum_{m=0}^{r} |c_n|^2 |c_m|^2 (-1)^{n+m} \right).
\]  (6)

In turn, the probability of the transition \( |\psi_{\text{in}}\rangle \rightarrow |k, 0\rangle \) is given by

\[
P_{k,0} = \begin{cases} 
|c_0|^2 & \text{if } k = 0, \\
\frac{|c_k|^2}{2} & \text{if } k \neq 0.
\end{cases}
\]  (7)
3. Applications

Let us take the coefficients $c_n$ such that

$$|c_n|^2 = \binom{r}{n} p^n (1-p)^{r-n}, \quad 0 \leq p \leq 1, \quad n \leq r. \quad (8)$$

Then, the superposition (2) is an optimized binomial state [10] which is parameterized by $p$ and $r$. In Fig. 2 we show the behavior of the probability (6) for this case. At $p = 0$ the probability is maximum since only the coefficient $c_0$ is different from zero and the initial state is the vacuum. At $p = 1$ the superposition (2) is equal to the Fock state $|r\rangle$ because only the coefficient $c_r$ is different from zero. The output (4) is the entangled NOON state with $N = r$.

![Figure 2](image.png)

**Figure 2.** The probability $P_{\psi,b,0}$ defined in (6) as a function of $p$ for the coefficients (8). The superposition (2) is an optimized binomial state.

On the other hand, the probability (7) is shown in Fig. 3 for three different values of the parameter $p$. As we can see, the maximum of the probability is shifted to the right as $p$ increases and reaches its highest value in the vicinities of either $p = 0$ or $p = 1$.

![Figure 3](image.png)

**Figure 3.** The probability $P_{k,0}$ of finding $k$ photons in the horizontal arm at the output of the Mach-Zehnder interferometer for the coefficients defined in (8) with $p = 0.1$ (a), $p = 0.5$ (b) and $p = 0.9$ (c).

Another useful linear combination (2) is obtained by taking

$$c_n = \exp(-|\alpha|^2/2) \frac{\alpha^n}{\sqrt{n!}}, \quad \alpha \in \mathbb{Z}, \quad (9)$$

at the limit $r \to \infty$. Then the distribution of the Fock states in the horizontal channel is ruled by the Poisson distribution so that the input is $|\alpha,0\rangle$, with $|\alpha\rangle$ a coherent state [10]. The output (4) is reduced to the two-mode coherent state

$$|\psi_{\text{out}}\rangle = \frac{\cos^{\pi/4}}{\sqrt{2}} |0, -\alpha\rangle + \frac{\sin^{\pi/4}}{\sqrt{2}} |0, i\alpha\rangle, \quad (10)$$
Figure 4. The probability (11) of finding the output (10) in the initial state $|\alpha,0\rangle$ as a function of the mean number of photons $\bar{n}$.

which has been already reported in, e.g [11–13]. The probability (6) of finding the output in the initial state is now given by

$$P_{\alpha,0} = \frac{1}{2} e^{-2|\alpha|} \left(e^{-2|\alpha|} + 1\right),$$

and depends on the mean number of photons involved in the initial superposition $|\alpha|^2 = \bar{n}$. Indeed, $P_{\alpha,0}$ decays exponentially as $\bar{n} \to \infty$, see Fig. 4.

In turn, the probability (7) of finding $k$ photons in the horizontal arm of the interferometer is depicted in Fig. 5 for $\bar{n} = 10$. As expected, the probability is maximum in the vicinity of $k = 10$

Figure 5. The probability $P_{k,0}$ of finding $k$ photons in the horizontal arm at the output of the Mach-Zehnder interferometer for the coefficients defined in (9) with $\bar{n} = 10$.

4. Conclusions

We have shown that using different linear combinations of two-mode Fock states as the input in the Elitzur-Vaidman experiment give rise to output states which can be controlled if the fully absorbing obstacle is changed by a non-linear optical medium. The NOON states as well the two-mode coherent states already reported in the literature on the matter have been recovered as particular cases. Further insights are in progress and will be reported elsewhere.

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