Abstract. The light Higgs boson of the Standard Model could arise as the consequence of the weakly broken conformal symmetry in a strongly interacting gauge theory. Here we present a novel idea to study the transition from conformal to confining behavior using an SU(3) gauge theory with four light and eight heavy flavors. This system interpolates between the 12-flavor conformal and the 4 flavor chirally broken theory as the mass of the heavy flavors are varied. We show first results on our determination of the iso-singlet 0^{++} state.

1. Introduction
In 2012 the Higgs boson was discovered at the LHC [1, 2] adding the final missing piece to the electroweak sector of the Standard Model. In the coming years the experiments will improve their precision and resolve further properties of the Higgs particle. It is of particular interest to learn more about the origin of the Higgs boson and the nature of electroweak symmetry breaking. The fundamental, self-interacting scalar as described by the Standard Model poses a theoretical challenge because it is not ultraviolet complete. One viable scenario is to consider the Higgs boson as a composite particle with its own underlying strongly interacting but chirally broken gauge theory [3, 4]. In such composite Higgs models the Higgs boson arises as a fermionic 0^{++} iso-singlet bound state.

QCD-like systems have a plethora of low-energy bound states and we expect to find a similar spectrum in scaled-up QCD models. Viable composite Higgs models have to be very different, predicting that the mass of any non-Goldstone states is much heavier than the iso-singlet scalar. In addition to an unusual spectrum, composite Higgs models have to satisfy other constraints, like predicting the experimentally observed Higgs decay width or the S-parameter correctly. These constraints could be met by a strongly interacting theory with a weakly broken conformal symmetry — a conjecture which can be numerically tested using lattice gauge theory.

Recent lattice results look very promising: there is increasing evidence for an infrared fixed point (IRFP) for 12 fundamental flavors [5–12] and two groups have reported on a low mass scalar in the SU(3) gauge theory: one for 8 fundamental flavors [13], the other for 2 sextet flavors [14]. These investigations are limited by an integer flavor number. \textit{A priori} it is not obvious
that a theory with an integer fermion number will be close enough to the conformal window where the IRFP appears. Also there are many more gauge groups or fermion representations to be considered. To facilitate such a search and overcome the limitation of integer flavor numbers, we proposed to simulate a theory with $N_\ell = 4$ light and $N_h = 8$ heavy flavors [15]. Keeping the four light flavors near the chiral limit, i.e. $m_\ell \approx 0$, we use the eight heavy flavors with mass $m_h \geq m_\ell$ to interpolate between the $N_\ell = 4$ chirally broken theory and the $N_h + N_\ell = 12$ flavor mass-deformed conformal theory. Phenomenologically it may be more interesting to simulate a model with, e.g., 2+1 flavors of sextet fermions since this has exactly three massless Goldstone bosons as needed by electroweak symmetry breaking. Our choice of 4+8 flavors is motivated by our use of the staggered formulation to discretize the fermions which has a multiple of four as natural choice. Furthermore, the above mentioned results indicate that the limiting cases of four and twelve flavors have the desired properties, implying 4+8 flavors should certainly interpolate in the relevant parameter space, although the anomalous dimension of the $SU(3)$ 12-flavor system may be too small ($\gamma^* \approx 0.24$) [9, 11, 16] to satisfy phenomenological walking constraints.

In our first numerical studies presented at Lattice 2014 [17], we demonstrated that the renormalized running coupling in our setup becomes a “walking” coupling in an intermediate energy range. Having a walking regime is necessary to satisfy known phenomenological constraints. We also found that the energy range of walking increases as we lower the mass of the eight heavy flavors, i.e., as we approach the IRFP in the 12-flavor theory, confirming our expectations. Next we study the impact of a walking gauge coupling on the meson spectrum. While the computation of the (connected) meson spectrum of states like the pion, rho, or $a_0$ is standard, the computation of the $0^{++}$ sigma particle brings additional challenges because it requires the evaluation of disconnected contributions. Disconnected contributions arise from quark-anti-quark pairs coupled only via a gluon to the state of interest. The evaluation of those is numerically much more demanding. In this paper we focus on the computation of the mass of the $0^{++}$ meson and show preliminary results for the meson spectrum of our 4+8 flavor theory.

In the following section we introduce the setup of our simulations and provide details on the generated ensembles of gauge field configurations. Next we discuss in Section 3 the details and results of our computation of the connected and disconnected spectrum, before we finally conclude.

2. Numerical setup

We use a lattice action based on nHYP [18] smeared staggered fermions and the plaquette gauge action containing fundamental and adjoint terms [9, 19]. Gauge field ensembles are generated using the hybrid Monte Carlo (HMC) update algorithm [20] as implemented in the FUEL software package [21]. The combined choice of the gauge and fermion actions has been extensively tested in projects using various number of fermion flavors (see e.g. [9, 19]) and proven to provide numerically stable simulations for our choice of parameters. Moreover, taste breaking effects of staggered fermions are largely suppressed when nHYP smeared. So far we generated ensembles with a range of different light masses $m_\ell$ and heavy masses $m_h$, analyzing two different volumes $24^3 \times 48$ and $32^3 \times 64$ at one value of the gauge coupling ($\beta = 4.0, \beta_a = -\beta/4$ [22]). In this paper we focus on the 4+8 flavor ensembles summarized in the left panel of Fig. 1, a total of 21 different ensembles at five different values of the light quark mass $m_\ell$ and three different values of the heavy quark mass $m_h$. Work is still in progress to extend some of the existing runs and add new ensembles at different light or heavy quark masses. The color coding in the above mentioned plot highlights in green, orange and red those ensembles which are most useful for our determinations of the $0^{++}$ scalar with the colors indicating finite size effects (negligible for green and severe for red). As expected we observe finite size effects becoming significant when reducing the masses. Simulations with $m_\ell = 0.035$ turned out to have a too large light mass to
reliably extract the $0^{++}$ and are not considered in our analysis.

The right plot in Fig. 1 shows the lattice scale of our ensembles and how it depends on the light and heavy quark mass. We determine the lattice scale using the concept of gradient or Wilson flow [23–25] which defines the quantity $t_0$. In order to reduce $O(a^2)$ lattice artifacts we consider the $t$-shift improved gradient flow by shifting the Wilson flow parameter $t$ by a small constant $\tau_0$ [10]. Empirically we find that $\tau_0 = 0.1$ is close to optimal and removes nearly all cut-off effects of $t_0$ [17]. We denote our $t$-shifted scale by $\tilde{t}_0$ and use the quantity $\sqrt{8\tilde{t}_0}$ to compare the lattice scales of our different ensembles as shown in the right plot of Fig. 1. Reducing cut-off effects is important since currently we only have data at one $\beta$ value which does not allow us to take a proper continuum limit. As is shown in the plot, our scale has a strong, non-linear dependence on $m_\ell$ and changes significantly with $m_h$. Simulating at lower values of $m_\ell$, as needed to approach the chiral limit, will therefore force us to simulate on larger volumes. As a rule of thumb, values of $\sqrt{8\tilde{t}_0} \lesssim L/5$ are typically considered to be safe from finite size effects as verified by comparing the full ($24^3 \times 48$) and open ($32^3 \times 64$) symbols. We observe a significant difference between the two volumes only for the simulation with the lightest input masses, $m_\ell = 0.005$, $m_h = 0.060$.

### 3. Calculating the Light flavor spectrum

As mentioned above, the iso-singlet $0^{++}$ meson receives special attention in models beyond the Standard Model as it could be a Higgs candidate. Its mass is considerably more difficult to measure numerically than other, non-singlet mesons because both quark-line connected and disconnected diagrams contribute to the correlator

$$C_{0^{++}}(t) \equiv \frac{N_f}{4} C_{\text{disc}}(t) - C_{\text{conn}}(t).$$

(1)
The disconnected contribution is given by
\[ C_{\text{disc}}(t) = \sum_{t_0} \left( \langle \bar{\psi}(t_0) \psi(t) \rangle - \langle \bar{\psi}(t) \psi(t_0) \rangle \right) \left( \langle \bar{\psi}(t_0 + t) \psi(t) \rangle - \langle \bar{\psi}(t) \psi(t_0 + t) \rangle \right) \]  
\[ \text{(2)} \]

where \( \langle \bar{\psi}(t) \psi(t) \rangle \) denotes the ensemble average of the fermion condensate. Accurately measuring the ensemble average of the condensate is essential for these calculations as large errors in \( \langle \bar{\psi}(t) \psi(t) \rangle \) can destroy the correlator of Eq. (2) at large \( t \).

Further complicating matters, only a stochastic estimate of the matrix element \( \langle \bar{\psi}(t) \psi(t) \rangle \) is feasible. Care must be and is taken to ensure no bias is introduced by incorrectly combining stochastically measured quantities. We compute it on each considered configuration using \( N_r \) \( U(1) \) noise sources \( \eta_i \) with the property
\[ \lim_{N_r \to \infty} \frac{1}{N_r} \sum_i \eta_i^t (\vec{x}, t) \eta_i (\vec{y}, t') = \delta_{\vec{x}, \vec{y}} \delta_{t, t'}. \]  
\[ \text{(3)} \]

Inverting the staggered Dirac operator \( D((\vec{x}, t), (\vec{y}, t')) \) on the noise source \( \eta_i \) gives
\[ \phi_i (\vec{x}, t) = D((\vec{x}, t), (\vec{y}, t'))^{-1} \eta_i (\vec{y}, t'), \]  
\[ \text{(4)} \]

and we can compute the desired matrix element as
\[ \langle \bar{\psi}(t) \psi(t) \rangle = \lim_{N_r \to \infty} \frac{1}{N_r} \sum_i \sum_{\vec{x}} \eta_i^t (\vec{x}, t) \phi_i (\vec{x}, t). \]  
\[ \text{(5)} \]

In order to enhance the signal, we use \( N_r = 6 \) noise sources spreading over the full 4-d volume of the lattice diluted in time, color, as well as even/odd in space [26]. This gives a reasonable balance between numerical costs and the signal-to-noise ratio. Our dilution pattern is consistent with the similar calculation performed in Ref. [14]. Moreover, we use an improved operator for the chiral condensate, unique to naive and staggered fermions, which further reduces variance in our measurement [27].

The above parameter choices require 1728 inversions of a single color fermion matrix for each \( 24^3 \times 48 \) configuration, and 2304 inversions for each \( 32^3 \times 64 \) configuration. Using our full ensembles we are able to determine the disconnected correlators to a few percent precision this way and extract the mass of the corresponding \( 0^{++} \) states. For comparison, for a \( 24^3 \times 48 \) (\( 32^3 \times 64 \)) lattice volume, it suffices to perform 18 (24) inversions of a single color fermion matrix to evaluate the quantities of the connected spectrum, e.g., the pion and rho meson, using wall sources to determine their mass with a precision to better than a percent on our ensembles. This illustrates the cost and difficulties of disconnected correlator calculations. As an example we show the resulting connected and disconnected scalar correlators on the \( 32^3 \times 64 \), \( m_\ell = 0.010 \), \( m_h = 0.060 \) ensemble in the left panel of Fig. 2. Due to the much higher costs we have presently measured the disconnected correlator on only 89 configurations, with further measurements in progress.

General features of the \( C_{0^{++}} \) correlators are nonetheless present: For small \( t \) the connected correlator dominates the combination and the effect of the disconnected contribution is insignificant until \( t \gtrsim 8 \). There the signal of \( C_{\text{disc}} \) is very noisy and we are not able to fit the mass reliably. As an alternative Refs. [12–14] advocate to consider the disconnected correlator only. As long as the \( 0^{++} \) state is lighter than the non-singlet \( a_0 \) state, the iso-singlet scalar mass can be extracted directly from \( C_{\text{disc}} \). From either correlator, values are determined by a fully correlated non-linear fit to a single \( 0^{++} \) state, with statistical errors estimated by a single-elimination jackknife. In practice, fitting to just \( C_{\text{disc}} \) works surprisingly well and one heuristic
The explanation may be that excited states in the connected and the $0^{++}$ correlators cancel. Using $C_{\text{disc}}$ only we obtain values in agreement with $C_{0^{++}}$ but with smaller statistical uncertainty.

We close this section by showing in the right panel of Fig. 2 preliminary results for the light flavor pion, the rho, and the $0^{++}$ scalar, obtained on our 4+8 ensembles with $m_h = 0.060$ using $32^3 \times 64$ volumes for $m_\ell = 0.005$ and 0.010 and $24^3 \times 48$ for $m_\ell = 0.015$ and 0.025. The data shown are rescaled by $\sqrt{8t_0}$ to account for differences in the lattice scale. We observe the $0^{++}$ mass is lighter than the pion at large $m_\ell$, crosses the pion mass, and becomes heavier toward the chiral limit. While the pion and $0^{++}$ may still be degenerate within two sigmas of our statistical uncertainty even at our lightest mass, our data reveal a clear trend which requires further data to be confirmed. Our preliminary results have the potential to be of great phenomenological interest and have thus far not been observed in other studies of conformal or near-conformal systems [12–14]. We have further measurements in progress, both increasing our statistics on existing ensembles and extending our analysis of the $0^{++}$ to additional ensembles.

4. Conclusions and outlook
Exploring the phenomena of gauge-fermion systems near the conformal window is interesting in its own and may in addition reveal new “physics” which may find its application in beyond the Standard Model theories describing e.g. the Higgs boson as composite particle. We study a novel model of four light and eight heavy flavors which allows us to tune arbitrarily close to the conformal fixed point of the 12-flavor system while still being chirally broken in the infrared limit. Of special interest is determining the mass of the iso-singlet scalar which could be a Higgs candidate. This computation is in particular challenging because its measurement contains both a connected and a disconnected contribution. Using stochastic estimates for the disconnected contribution, our preliminary results reveal the very interesting case of an iso-singlet scalar which is lighter than the pion at large fermion mass, but heavier for smaller masses. Our current findings will have to be confirmed by further simulations.

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References

[1] Aad G et al. (ATLAS Collaboration) 2012 Phys.Lett. B716 1–29 (Preprint 1207.7214)
[2] Chatrchyan S et al. (CMS Collaboration) 2012 Phys.Lett. B716 30–61 (Preprint 1207.7235)
[3] Yamawaki K, Bando M and Matumoto K i 1986 Phys. Rev. Lett. 56(13) 1335–1338
[4] Appelquist T, Terning J and Wijewardhana L C R 1991 Phys. Rev. D 44(3) 871–877
[5] Appelquist T, Fleming G, Lin M, Neil E and Schaich D 2011 Phys.Rev. D84 054501 (Preprint 1106.2148)
[6] Hasenfratz A 2012 Phys.Rev.Lett. 108 061601 (Preprint 1106.5293)
[7] Aoki Y, Aoyama T, Kurachi M, Maskawa T, Nagai K i et al. 2012 Phys.Rev. D86 054506 (Preprint 1207.3060)
[8] DeGrand T 2011 Phys.Rev. D84 116901 (Preprint 1109.1237)
[9] Cheng A, Hasenfratz A, Liu Y, Petropoulos G and Schaich D 2014 Phys.Rev. D90 014509 (Preprint 1401.0195)
[10] Cheng A, Hasenfratz A, Liu Y, Petropoulos G and Schaich D 2014 JHEP 1405 137 (Preprint 1404.0984)
[11] Lombardo M, Miura K, da Silva T J N and Pallante E 2014 (Preprint 1410.0298)
[12] Aoki Y, Aoyama T, Kurachi M, Maskawa T, Nagai K i et al. 2013 Phys.Rev.Lett. 111 162001 (Preprint 1305.6006)
[13] Aoki Y et al. (the LatKMI Collaboration) 2014 Phys.Rev. D89 111502 (Preprint 1403.5000)
[14] Fodor Z, Holland K, Kuti J, Nogradi D and Wong C H 2014 PoS LATTICE2013 062 (Preprint 1401.2176)
[15] Brower R, Hasenfratz A, Rebbi C, Weinberg E and Witzel O 2014 (Preprint 1410.4091)
[16] Cheng A, Hasenfratz A, Petropoulos G and Schaich D 2013 JHEP 1307 061 (Preprint 1301.1355)
[17] Brower R, Hasenfratz A, Rebbi C, Weinberg E and Witzel O 2014 (Preprint 1411.3243)
[18] Hasenfratz A, Hoffmann R and Schaefer S 2007 JHEP 0705 029 (Preprint hep-lat/0702082)
[19] Cheng A, Hasenfratz A, Petropoulos G and Schaich D 2013 PoS LATTICE2013 088 (Preprint 1311.1287)
[20] Duane S, Kennedy A, Pendleton B and Roweth D 1987 Phys.Lett. B195 216–222
[21] Osborn J et al. Framework for unified evolution of lattices (FUEL) URL http://usqcd-software. github.io/FUEL.html
[22] Hasenbusch M and Necco S 2004 JHEP 0408 005 (Preprint hep-lat/0405012)
[23] Narayanan R and Neuberger H 2006 JHEP 0603 064 (Preprint hep-th/0601210)
[24] Lüscher M 2010 Commun.Math.Phys. 293 899–919 (Preprint 0907.5491)
[25] Lüscher M 2010 JHEP 1008 071 (Preprint 1006.4518)
[26] Foley J, Jimmy Juge K, O’Cais A, Peardon M, Ryan S M et al. 2005 Comput.Phys.Commun. 172 145–162 (Preprint hep-lat/0505023)
[27] Kilcup G and Sharpe S R 1987 Nucl.Phys. B283 493