Extended Cluster Model for Light, and Medium Nuclei

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The structures, the electromagnetic transitions, and the beta decay strengths of exotic nuclei are investigated within an extended cluster model. We start by deriving an effective nuclear Hamiltonian within the $S_2$ correlation operator. Tensor forces are introduced in a perturbative expansion which includes up to the second order terms. Within this Hamiltonian we calculate the distributions and the radii of $A=3, 4$ nuclei. For exotic nuclei characterized by $n$ valence protons/neutrons we excite the structure of the closed shell nuclei via mixed modes formed by considering correlations operators of higher order. Good results have been obtained for the calculated transitions and for the beta decay transition probabilities.

Keywords: Cluster model, exotic nuclei, electromagnetic transitions, beta decay.

1. Introduction

The data obtained at RIA, Riken and GSI have given new life to the old Nuclear Physics. Nuclear structures of proton and neutron rich nuclei are and will be investigated giving a new insight to the fundamental observable such as the nuclear forces in the proton-proton and neutron-neutron components, shell closure far from stability, magnetic properties of weakly excited nuclear states, and many others. The theoretical analysis of these data requires a reliable nuclear model which can reproduce the data of stable nuclei and be extrapolated to predict or at least reproduce the experimental results. The Cluster Correlation Model offers effective tools in this direction. We have to depart, however, from a perturbative scheme which is generally used to treat the two body correlation. Generalization to the perturbative method to include the many body correlation is in this paper realized within an extended, non-perturbative Cluster Correlation Model. One of the central challenges of theoretical nuclear physics is the
attempt to describe unknown properties of the exotic systems in terms of a realistic nucleon-nucleon (NN) interaction. In order to calculate matrix elements with the singular interaction (hard core) we have to define effective correlated Hamiltonians.

Correlation effects in nuclei have been first introduced in nuclei by Villars,\(^1\) who proposed the unitary-model operator (UMO) to construct effective operators. The method was implemented by Shakin\(^2\) for the calculation of the G-matrix from hard-core interactions. Non perturbative approximations of the UMO have been recently applied to odd nuclei in Ref.\([3]\) and to even nuclei in Ref.\([4]\). The basic formulas of the Dynamic Correlation Model and of the Boson Dynamic Correlation Model (BDCM) presented in the above quoted papers have been obtained by separating the n-body correlation operator in short- and long-range components. The short-range component is considered up to the two body correlation while for the long range component the three and four body correlation operators have been studied. The extension of the correlation operator to high order diagrams is especially important in the description of exotic nuclei (open shell). In the short range approximation the model space of two interacting particles is separated in two subspaces: one which includes the shell model states and the other (high momentum) which is used to compute the G-matrix of the model. The long range component of the correlation operator has the effect of generating a new correlated model space (effective space) which departs from the originally adopted one (shell model). The amplitudes of the model wave functions are calculated in terms of non linear equation of motions (EoM). The derived systems of commutator equations, which characterize the EoM, are finally linearized. Within these generalized linearization approximations (GLA) we include in the calculation presented in the paper up to the \(((n+1)p1h)\) effective diagrams. The linearized terms provide, as explained later in the text, the additional matrix elements that convert the perturbative UMO expansion in an eigenvalue equation. The n-body matrix elements needed to diagonalize the resulting eigenvalue equations are calculated exactly via the Cluster Factorization Theory (CFT).\(^5\)

2. The \(S_2\) correlated Hamiltonian

In order to describe the structures and the distributions of nuclei we start from the following Hamiltonian:

\[
H = \sum_{\alpha\beta} \langle \alpha|t|\beta \rangle a_\alpha^\dagger a_\beta + \sum_{\alpha\beta\gamma\delta} \langle \Phi_{\alpha\beta}|v_{12}|\Phi_{\gamma\delta} \rangle a_\alpha^\dagger a_\beta^\dagger a_\delta a_\gamma
\]  

(1)
where \( v_{12} \) is the singular nucleon-nucleon two body potential. Since the two body states \(|\alpha\beta\rangle\) are uncorrelated the matrix elements of \( v_{12} \) are infinite. This problem can be avoided by taking matrix elements of the Hamiltonian between correlated states. In this paper the effect of correlation is introduced via the \( e^{iS} \) method. In dealing with very short range correlations only the \( S_2 \) part of the correlation operator needs to be considered.

Following Ref. [2] we therefore calculate an “effective Hamiltonian” by using only the \( S_2 \) correlation operator obtaining:

\[
H_{\text{eff}} = e^{-iS_2} H e^{iS_2} = \sum_{\alpha\beta} \langle \alpha|t|\beta\rangle \hat{a}_\alpha^\dagger \hat{a}_\beta + \sum_{\alpha\beta\gamma\delta} \langle \Psi_{\alpha\beta}|v_{12}|\Psi_{\gamma\delta}\rangle \hat{a}_\alpha^\dagger \hat{a}_\beta \hat{a}_\gamma \hat{a}_\delta
\]

(2)

where \( v_{12} \) refers to the long-range part of the nucleon-nucleon force diagonal in the relative orbital angular momentum, after the separation.\(^6\)

\( v_{12} = v_{12}^l + v_{12}^s \)

(3)

The separation is made in such a way that the short range part produces no energy shift in the pair state.\(^6\) In doing shell model calculation with the Hamiltonian Eq. (2), we remark: a) only the long tail potential plays an essential role in the calculations of the nuclear structure i.e.: the separation method and the new proposed \( v_{\text{low-}k} \) method show a strong analogy and b) the \( v_{\text{od}}^T \) must be included as an additional re-normalization of the effective Hamiltonian Eq. (2).

In Eq. (2) the \( \Psi_{\alpha\beta} \) is the two particle correlated wave function:

\[
\Psi_{\alpha\beta} = e^{iS_2} \Phi_{\alpha\beta}
\]

(4)

In order to evaluate the effect of the tensor force on the \( \Psi_{\alpha\beta} \) we calculate:

\[
w(r) = V_{T}^{\text{od}} \frac{Q}{\Delta E} u(r) = V_{T}^{\text{od}} \frac{Q}{\Delta E} |(\tilde{n}\ell S), J' : NL, J\rangle
\]

(5)

where \( Q \) is a momentum dependent projection operator, \( \Delta E(k_1, k_2) \) the energy denominator and \( \tilde{n}\ell \) the correlated two particle state in the relative coordinates. In Eq. (5) \( u(r) \) is generated as in Ref.\(^2\) by a separation distance calculation for the central part of the force in the \( ^3S_1 \) state. The wave function obtained in this way (full line) heals to the harmonic-oscillator wave function (dashed line) as shown in Fig. 1. The result obtained for Eq. (5) calculated with the tensor force of the Yale potential\(^8\) is given also in Fig. 1 left where we plot for the harmonic oscillator size parameter \( b=1.41 \) fm:

\[
\Psi(\vec{r}) = [u(r)Y_0^0(\Omega_\vec{r}) + w(r)Y_2^1(\Omega_\vec{r})]
\]

(6)
Fig. 1. Left: The $u(r)$ and $w(r)$ wave functions of the deuteron, with quantum numbers $^3S_1$ and $^3D_1$, plotted as function of $r$; Right: Distributions of $^3H$ and $^3He$.

Being the admixture of the two components, circa 4%, the wave function Eq. (6) can be associated to the deuteron wave function. Let us use then the Hamiltonian Eq. (2) to calculate the structure of the $A=3$ nuclei. Here we propose to calculate the ground state of $^3H$, $^3He$, and $^4He$ within the EoM method which derive the eigenvalue equations by working with the $e^{iS_2}$ operator on the wave functions of the $A=3, 4$ nuclei Ref. From the diagonalization of the eigenvalue equation of the three particles, we obtain an energy difference $\Delta E(^3H-^3He)=0.78$ MeV and the distributions and radii given in Fig. 1 Right. By extending the commutator to a four particle state we obtain for the ground state of $^4He$ the binding energy of $E=28.39$ Mev and the rms radius of 1.709 fm. In dealing with complex nuclei however the $(S_i, i=3 \cdots n)$ correlations should also be considered. The evaluation of these diagrams is, due to the exponentially increasing number of terms, difficult in a perturbation theory. We note however that one way to overcome this problem is to work with $e^{i(S_1+S_2+S_3+\cdots+S_i)}$ operator on the Slater’s determinant by keeping the n-body Hamiltonian uncorrelated. Via the long tail of the nuclear potential the Slater determinant of the “n” particle systems are interacting with the excited Slater’s determinants formed by the (“n” particles+(mp-mh) mixed-mode excitations). The amplitudes of the different determinants are calculated via the EoM method. After having performed the diagonalization of the n-body Hamilton’s operator we can calculate the form of the effective Hamiltonian which, by now, includes the complete set of the commutator equations. The method is here applied to $^6He$, $^{11}Be$, $^{14}C$, $^{15}O$, and $^{17}O$. A detailed formulation of the model my be found in Ref.9
3. Results

In order to perform structure calculations for complex nuclei, we have to define the CMWFs base, the “single-particle energies” and to choose the nuclear two-body interactions. The CMWFs are defined as in Ref.9 by including mixed valence modes and core-excited states. The base is then orthonormalized and, since the single particle wave functions are harmonic oscillators, the center-of-mass (CM) is removed. The single-particle energies of these levels are taken from the known experimental level spectra of the neighboring nuclei. For the particle-particle interaction, we use the G-matrix obtained from Yale potential.10 These matrix elements are evaluated by applying the $e^S$ correlation operator, truncated at the second order term of the expansion, to the harmonic oscillator base with size parameter $b=1.76$ fm. As elucidate in Refs. [3] and [4] the effective two-body potential used by the DCM and the BDCM models is separated in low and high momentum components. Therefore, the effective model matrix elements calculated within the present separation method and those calculated by Kuo7 in the $v_{\text{low-k}}$ approximation are pretty similar. The adopted separation method and the $v_{\text{low-k}}$ generate two-body matrix elements which are almost independent from the radial shape of the different potentials generally used in structure calculations.

The particle-hole matrix elements could be calculated from the particle-particle matrix elements via a re-coupling transformation. In this contribution we present application of the $S_3$ correlated model to the charge distributions of $^6\text{He}$, $^{11}\text{Be}$, and to the electromagnetic transitions of neutron rich Carbon and Oxygen isotopes. The beta decay strengths from the ground state $^{14}\text{N}$ to the excited states of $^{14}\text{C}$ are also calculated. In Fig. 2) Left three distributions are given for $^6\text{He}$: 1) the correlated charge distribution calculated with the full $S_3$ operator, 2) the correlated charge distribution calculated with the partial $S_3$ operator obtained by neglecting the folded diagrams, 3) the charge distribution calculated for two correlated protons in the $1s_\uparrow^2$ shell. The full $S_3$ correlation operator therefore increases the calculated radii. In Fig. 2) Right the charge distribution for $^{11}\text{Be}$ is given. A charge radius of 3.12 fm has been obtained. Calculations are performed in a mixed $S_3$ and $S_5$ system. The results obtained for the Carbon and Oxygen isotopes are in the following presented as function of the increasing valence neutrons. It is worthwhile to remark that the high order correlation operators generate the interaction of the valence particles with the closed shell nucleus. The correlation model treats therefore consistently the “A” particles of the isotopes.
By using generalized linearization approximations and cluster factorization coefficients we can perform exact calculations. In following Tables an over all $b=1.76$ fm has been used.

In Table 1, 3) we give the calculated magnetic moments and rms radii for one-hole and for one-particle in $^{16}$O. The energy splitting between the ground- and the second (first) excited states and the electromagnetic transitions for the two isotopes are given in Tables 2, 4).

Table 1: Magnetic moment (nm) and rms (fm) of the ground state of $^{15}$O with $J = \frac{1}{2}^{-}; \; T = \frac{1}{2}$

|       | DCM   | Exp. $^{11}$ |
|-------|-------|--------------|
| Magnetic Moment (nm) | .70   | 7.19          |
| rms (fm)      | 2.74  | 2.73(3)       |

Table 2: Energy splitting between the ground and the second excited states and the corresponding electromagnetic transitions for $^{15}$O.

|       | DCM   | Exp. $^{11}$ |
|-------|-------|--------------|
| $\Delta E_{\frac{1}{2}^{-} - \frac{3}{2}^{-}}$ | 5.41  | 5.24         |
| Ratio $\frac{BE(E2, \frac{1}{2}^{-} - \frac{3}{2}^{-})}{BE(M2, \frac{1}{2}^{-} - \frac{3}{2}^{-})}$ | .15   | .10          |

Table 3: Magnetic moment (nm) and rms (fm) of the ground state of $^{17}$O with $J = \frac{5}{2}^{+}; \; T = \frac{1}{2}$

|       | DCM   | Exp. $^{11}$ |
|-------|-------|--------------|
| Magnetic Moment (nm) | -1.88 | -1.89        |
| rms (fm)      | 2.73  | 2.72(3)      |

Table 4: Energy splitting between the ground- and the first excited states and the $E2$ transition for $^{17}$O.
In Table 5) we give the calculated results for the energy splitting between the ground- and the $2^+$ excited state and the corresponding electromagnetic transition for the $^{14}$C. The commutator equations involve $S_2$ and $S_3$ diagrams.

| Energy (MeV) | DCM | Exp. |
|--------------|-----|------|
| $\Delta E_{1^+ 2^+}$ | 0.87 | 0.89 |
| Transition ($e^+m^- \rightarrow 2^+$) | DCM | Exp. |
| $BE(E2; 1^+ \rightarrow 2^+)$ | 2.10 | 2.18 \pm 0.16 |

Table 5: Calculated energy splitting and $BE(E2; 2^+ \rightarrow 0^+)$ transition for $^{14}$C

In Table 6) preliminary results for the calculated reduced transition probabilities from the ground state of $^{14}$N to the $0^+, 1^+, 2^+$ states of $^{14}$C are given. The calculated strengths reproduce reasonably well the experimental values.$^{15}$

| $^{14}$N | $J^+ T$ | $^{14}$C | $J^+ T$ | Energy (MeV) | $B(GT)$ |
|---------|--------|---------|--------|-------------|---------|
| $^{14}$N | $1^+ 0$ | $0^+ 1$ | 0.0 | 0.06 |
| $^{14}$N | $0^+ 1$ | $0^+ 1$ | 7.81 | 0.15 |
| $^{14}$N | $1^+ 1$ | $1^+ 1$ | 12.17 | 0.12 |
| $^{14}$N | $2^+ 1$ | $2^+ 1$ | 7.38 | 0.42 |
| $^{14}$N | $2^+ 1$ | $2^+ 1$ | 8.38 | 0.50 |
| $^{14}$N | $2^+ 1$ | $2^+ 1$ | 10.91 | 0.35 |

Good results have been overall obtained for the transitions with a neutron effective charge varying between 0.1- to 0.12-$e_n$.

4. Conclusion and Outlook
In this contribution we have investigated the effect of the microscopic correlation operators on the exotic structure of the Carbon and Oxygen isotopes. The microscopic correlation has been separated in short- and long-range correlations according to the definition of Shakin. The short-range correlation has been used to define the effective Hamiltonian of the model while the long-range correlation is used to calculate the structures and the distributions of exotic nuclei. As given in the work of Shakin, only the two-body short-range correlation need to be considered in order to derive the effective Hamiltonian especially if the correlation is of very short range. For the
long range correlation operator the three body component is important and should not be neglected. Within the three body correlation operator, one introduces in the theory a three body interaction which compensates for the use of the genuine three body interaction of the no-core shell model. Within the $S_2$ effective Hamiltonian, good results have been obtained for the ground state energies and the distributions of $^3\text{H}$, $^3\text{He}$, and $^4\text{He}$. The higher order correlation operators $S = 3 \cdots n$ have been used to calculate the structure and the electromagnetic transitions of ground and first excited states for the isotopes of Carbon and Oxygen. By using generalized linearization approximations and cluster factorization coefficients we can perform expedite and exact calculations.

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