First estimates of the $B_c$ wave function from the data on the $B_c$ production cross section

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In the framework of perturbative QCD and nonrelativistic bound state formalism, we calculate the production of $B_c$ and $B_c^*$ mesons at the conditions of the CDF and LHCb experiments. We derive first estimations for the $B_c$ wave function from a comparison with the available data.

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I. INTRODUCTION

The family of $B_c^{(*)}$ mesons is an interesting though poorly explored part of the quarkonium world. Although some properties of these mesons may look apparently different from the ones of the hidden-flavor onium states, their inner structure must be similar and driven by the same physics. Studying the $B_c^{(*)}$ properties is important on its own and can provide an additional cross check of the exploited theoretical models.

The flavor composition of $B_c^{(*)}$ mesons excludes the convenient strong and electromagnetic decays channels that could be used as a prompt measure of the nonrelativistic wave function. Instead, we will try to obtain an estimate of this essential parameter via considering the production process. We will rely on the data collected by the CDF Collaboration at the Fermilab Tevatron at 1.8 TeV [1] and 1.96 TeV [2] and by the LHCb Collaboration at CERN LHC at 7 TeV [3] and 8 TeV [4].

II. THEORETICAL FRAMEWORK

In the theory, the production of $B_c^{(*)}$ mesons at the LHCb conditions is dominated by the $O(\alpha_s)^4$ partonic subprocess

$$g + g \rightarrow B_c^{(*)} + b + \bar{c},$$

where $B_c^{(*)}$ may denote either pseudoscalar $B_c$ (spin=0) or vector $B_c^*$ (spin=1) bound state of the charm and beauty quarks. The evaluation of the relevant 36 Feynman diagrams is straightforward and is described in every detail in Ref. [5]. The only innovation made in the present calculation is in using the $k_t$-factorization approach. The advantage of the latter comes from the ease of including the initial state radiation corrections that are efficiently taken into account in the form of the evolution of gluon densities. Then, in accordance with the $k_t$-factorization prescription [5], the initial gluon spin density matrix is taken in the form of the evolution of gluon densities. Then, in accordance with the $k_t$-factorization prescription [6], the initial gluon spin density matrix is taken in the form $\epsilon_g^{\mu
u} = k_T^{\mu} k_T^{\nu}/|k_T|^2$, where $k_T$ is the component of the gluon momentum perpendicular to the beam axis. In the limit when $k_T \rightarrow 0$, this expression converges to the ordinary $\epsilon_g^{\mu
u} = -g^{\mu\nu}/2$, and we recover the results of collinear approach [7–9]. This work is the first calculation of the $B_c^{(*)}$ hadronic production with $k_t$-factorization.

The perturbative part of our calculation is performed according to the formula

$$d\sigma(pp \rightarrow B_c b\bar{c}X) =$$

$$\frac{\alpha_s^4}{12 \pi^2} |\mathcal{R}(0)|^2 \sum_{\text{spins}} \sum_{\text{colors}} |\mathcal{M}(gg \rightarrow B_c^* b\bar{c})|^2$$

$$\times \mathcal{F}_g(x_1, k_{T1}^2, \mu^2) \mathcal{F}_g(x_2, k_{T2}^2, \mu^2) \, dk_{T1}^2 \, dk_{T2}^2$$

$$\times dp_{B_c}^2 \, dp_{B_c}^2 \, dy_{B_c} \, dy_{b} \, dy_{c} \, d\phi_1 \, d\phi_2 \, d\phi_{B_c} \, d\phi_c \, 2\pi \, 2\pi \, 2\pi \, 2\pi,$$

and we use the JH’2013 (set 2) [10] parametrization for the transverse momentum dependent (TMD, or unintegrated) gluon distribution $\mathcal{F}_g(x, k_{T1}^2, \mu^2)$.

The absolute normalization of the cross section depends on a non-perturbative parameter: the radial (color singlet) wave function at the origin of the coordinate space $|\mathcal{R}(0)|^2$ [11–13]. The so-called color octet contributions are not important as they are suppressed by the relative velocity counting rules. Note by the way that the gluon fragmentation mechanism (known to dominate the production of $J/\psi$ mesons at high transverse momenta) is not applicable to our case.

To perform a comparison with the data (see below) we also need to calculate the production of the ordinary $B^+$ mesons. Again, we do that in the $k_t$-factorization approach with the same gluon density [10] as above, and with Peterson [14] fragmentation function with $\epsilon = 0.0126$ for the formation of $B^+$ mesons from $b$-quarks. The consistency of this setting was shown in a previous publication [15].
III. NUMERICAL RESULTS AND DISCUSSION

The data we wish to compare with are presented in the form of the ratio of the $B_{c}^{(+)}$ to $B^{+}$ production cross sections times the relevant branching fractions. All these results accumulate the statistics from both $B_{c}$ and $B_{c}^{*}$ mesons and include also their charge conjugate states.

Ref. [11] reports for the fiducial phase space defined as $p_{T}^{B_{c}} > 6$ GeV, $p_{T}^{B^{+}} > 6$ GeV, $|y_{B_{c}}| < 1$, $|y_{B^{+}}| < 1$:

$$\frac{\sigma(B_{c}) Br(B_{c} \rightarrow J/\psi \ell \nu)}{\sigma(B^{+}) Br(B^{+} \rightarrow J/\psi K)} = 0.132 \pm 0.063. \quad (3)$$

Hereafter, in the experimental references $B_{c}$ will denote a combined sample of $B_{c}$ and $B_{c}^{*}$ mesons. Within the specified kinematic cuts, we obtain, from Eq.(2):

$$\sigma^{\text{theor}}(B_{c}) = |\mathcal{R}(0)|^2 \cdot 0.247 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 0.516 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c} + B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 0.763 \text{ nb/GeV}^3. \quad (4)$$

We also have for the production of $B^{+}$ mesons

$$\sigma^{\text{theor}}(B^{+}) Br(B^{+} \rightarrow J/\psi K^{+}) = 7.13 \text{ nb}, \quad (5)$$

where we have used the decay branching fraction value

$$Br(B^{+} \rightarrow J/\psi K^{+}) = 1.026 \cdot 10^{-3} \quad (6)$$

taken from the Particle Data book [14].

Ref. [2] reports for $p_{T}^{B_{c}} > 6$ GeV, $p_{T}^{B^{+}} > 6$ GeV, $|y_{B_{c}}| < 0.6$, and $|y_{B^{+}}| < 0.6$:

$$\frac{\sigma(B_{c}) Br(B_{c} \rightarrow J/\psi \ell \nu)}{\sigma(B^{+}) Br(B^{+} \rightarrow J/\psi K)} = 0.211 \pm 0.024 \pm 0.023 \quad (7)$$

Within the above cuts, we obtain

$$\sigma^{\text{theor}}(B_{c}) = |\mathcal{R}(0)|^2 \cdot 0.177 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 0.3646 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c} + B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 0.541 \text{ nb/GeV}^3, \quad (8)$$

and

$$\sigma^{\text{theor}}(B^{+}) Br(B^{+} \rightarrow J/\psi K^{+}) = 4.99 \text{ nb}. \quad (9)$$

Ref. [3] reports for $p_{T}^{B_{c}} > 4$ GeV, $p_{T}^{B^{+}} > 4$ GeV, $2.0 < |y_{B_{c}}| < 4.5$, and $2.0 < |y_{B^{+}}| < 4.5$:

$$\frac{\sigma(B_{c}) Br(B_{c} \rightarrow J/\psi \pi^{+})}{\sigma(B^{+}) Br(B^{+} \rightarrow J/\psi K)} = 0.0061 \pm 0.0012. \quad (10)$$

Under these conditions, we have:

$$\sigma^{\text{theor}}(B_{c}) = |\mathcal{R}(0)|^2 \cdot 2.37 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 3.03 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c} + B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 5.40 \text{ nb/GeV}^3, \quad (11)$$

and

$$\sigma^{\text{theor}}(B^{+}) Br(B^{+} \rightarrow J/\psi K^{+}) = 27.3 \text{ nb}. \quad (12)$$

Finally, Ref. [4] reports for $p_{T}^{B_{c}} < 20$ GeV, $p_{T}^{B^{+}} < 20$ GeV, $2.0 < |y_{B_{c}}| < 4.5$, and $2.0 < |y_{B^{+}}| < 4.5$:

$$\frac{\sigma(B_{c}) Br(B_{c} \rightarrow J/\psi \pi^{+})}{\sigma(B^{+}) Br(B^{+} \rightarrow J/\psi K)} = 0.0068 \pm 0.0002; \quad (13)$$

and our predictions read:

$$\sigma^{\text{theor}}(B_{c}) = |\mathcal{R}(0)|^2 \cdot 4.92 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 5.63 \text{ nb/GeV}^3,$$
$$\sigma^{\text{theor}}(B_{c} + B_{c}^{*}) = |\mathcal{R}(0)|^2 \cdot 10.55 \text{ nb/GeV}^3, \quad (14)$$

and

$$\sigma^{\text{theor}}(B^{+}) Br(B^{+} \rightarrow J/\psi K^{+}) = 65.33 \text{ nb}. \quad (15)$$

The above data have to be combined with the experimentally measured [17] ratio of the branching fractions

$$Br(B_{c} \rightarrow J/\psi \pi^{+})/Br(B_{c} \rightarrow J/\psi \mu\nu) = 0.047 \quad (16)$$

and with the theoretically calculated [18] decay branching fraction

$$Br(B_{c} \rightarrow J/\psi \pi^{+}) = 0.0033. \quad (17)$$

FIG. 1: The ratio of the $B_{c}^{(+)}$ to $B^{+}$ production cross sections (13) as a function of the transverse momentum for different rapidity intervals. Grey band indicates the uncertainty in the determination of the $B_{c}^{(*)}$ radial wave function; yellow band represents uncertainties coming from the renormalization scale. Experimental points are from LHCb [4].
The original [18] prediction of 0.0029 was corrected [4] to 0.0033 for the latest measurement of the $B_c$ lifetime.

Making the necessary substitutions and comparing Eqs. (3), (7), (10), and (13) with theoretical predictions we deduce the following estimations for the radial wave function:

$$|R(0)|^2 = 4.40 \pm 2.00 \text{ GeV}^3 \quad \text{Ref. [1]}$$
$$|R(0)|^2 = 6.91 \pm 0.08 \text{ GeV}^3 \quad \text{Ref. [2]}$$
$$|R(0)|^2 = 5.15 \pm 0.10 \text{ GeV}^3 \quad \text{Ref. [3]}$$
$$|R(0)|^2 = 7.05 \pm 0.20 \text{ GeV}^3 \quad \text{Ref. [4]} \quad (18)$$

These can be summarised in a mean-square average value

$$|R(0)|^2 = 5.88 \text{ GeV}^3 \quad (19)$$

with an error of ±0.64 GeV$^3$ and ±1.07 GeV$^3$ at the 60% and 80% confidence level, respectively.

We conclude our analysis with showing the ratio (13) in the differential form, as a function of the transverse momentum for several rapidity intervals (see Fig. 1). The sensitivity to the renormalization scale is high, as a reflection of the high power of $\alpha_S(\mu_R^2)$ in the key subprocess (1). The central values of the cross sections correspond to the conventional choice $\mu_R^2 = \mu_{B,T}^2 + m_{B_c}^2$; the theoretical uncertainty band (yellow area in Fig. 1) is obtained by varying $\mu_R$ around its default value by a factor of 2.

Our extracted values of $|R(0)|^2$ are higher than the predictions [19] of potential models which range from 1.508 GeV$^3$ for the logarithmic potential [20], through 1.642 GeV$^3$ and 1.710 GeV$^3$ for the Buchmüller-Tye [21] and power low [22] potentials up to 3.102 GeV$^3$ for the Cornell potential [23]. This may be taken as an evidence of the importance of radiative corrections (the latter are known to be large for $J/\psi$ mesons). Another possible interpretation may guess that the conventional choice of $\mu_R$ somehow overestimates the momentum transfer in the hard process. Any way, the agreement between the theory and the data is rather satisfactory and shows no fundamental problems in describing the data.

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