Heavy Vectors in Higgs-less models

Riccardo Barbieri\textsuperscript{a}, Gino Isidori\textsuperscript{a,b}, Vyacheslav S. Rychkov\textsuperscript{a}, Enrico Trincherini\textsuperscript{a}

\textsuperscript{a} Scuola Normale Superiore and INFN, Piazza dei Cavalieri 7, 56126 Pisa, Italy

\textsuperscript{b} INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40 I-00044 Frascati, Italy

Abstract

One or more heavy spin-1 fields may replace the Higgs boson in keeping perturbative unitarity up to a few TeV. By means of two prototype chiral models for the heavy spin-1 bosons, a composite model or a gauge model, we discuss if and how the sole exchange of the same fields can also account for the ElectroWeak Precision Tests. While this proves impossible in the gauge model, the composite model hints to a positive solution, which we exploit to constrain the phenomenological properties of the heavy vectors.
1 Introduction and statement of the problem

The case for the existence of a (relatively light) Higgs boson is strong. In the Standard Model (SM) all its couplings are determined in terms of a single parameter, its mass $m_h$. By taking $m_h$ around 100 GeV, all the ElectroWeak Precision Tests (EWPT) are successfully accounted for. With the inclusion of the Higgs boson the electroweak interactions are consistently described up to energies which are, in principle, indefinitely high. Yet the strong sensitivity of $m_h$ to short distance scales makes it hard to understand its relative lightness and motivates the search for alternative roads. Most of the times one considers an extended Higgs sector, in one direction or another. Relatively less frequently, and generally less successfully, one also tries to do without it at all.

In this last case one or more spin-1 bosons often replace the Higgs boson in trying to keep perturbative unitarity up to a few TeV, one order of magnitude above the Fermi scale, $v = (\sqrt{2}G_F)^{-1/2} \approx 246$ GeV, so that a meaningful comparison with current experiments can be made. While this is possible, the role played by the Higgs boson of the SM in the EWPT proves generally harder to substitute, at least without adding extra ingredients that have little to do with the spin-1 bosons themselves. This is a serious drawback. Not only the competition with the Higgs boson gets weakened but also, unlike in the Higgs boson case, one cannot use the EWPT to determine the very properties of the heavy vector(s) which are phenomenologically crucial. This is the point that we examine in this work by means of two prototype chiral models for the heavy spin-1 bosons: a composite model and a gauge model. As we are going to see, while the gauge model confirms the difficulty, the composite model hints to a possible positive solution, which we exploit to constrain the phenomenological properties of the heavy vectors. The corresponding effective Lagrangians are meant to be valid up a cutoff $\Lambda \approx 4\pi v \approx 3$ TeV.

In connection with the EWPT, a much discussed problem is the effect on the $S$-parameter of the tree level exchange of the heavy spin-1 fields. We shall assume that $S$ at tree level is positive and moderately small, $0 < S^{\text{tree}} \lesssim 0.2$, as compatible with both models in a suitable range of their parameters. At least equally important, however, is the correlation of $S$ with $T$, which, although vanishing at tree level in presence of a suitable custodial symmetry, has a one loop infrared effect

$$\hat{T}_{\text{IR}} = -\frac{3\alpha}{8\pi c_W^2} \ln \left( \frac{\Lambda}{m_W} \right) = -1.2 \cdot 10^{-3} \ln \left( \frac{\Lambda}{m_W} \right), \quad (1.1)$$

which in the SM is cut off by the Higgs boson exchange. By itself, with $\Lambda \approx 3$ TeV, it would be largely incompatible with current data, no matter what happens to $S$. On the other hand the exchange of the heavy vectors in the loop are bound to give a contribution which may mitigate this effect, as they have couplings to pions delaying the loss of unitarity in $WW$-scattering. If one wants to defend the calculability of the EWPT, the full one loop for $T$ must therefore be included. As we will see, the compensating effect of the heavy vectors is more subtle than the one of the Higgs boson; in particular it does not reduce to replacing the logarithmic cutoff $\Lambda$ in (1.1) with the heavy vector mass.

The common starting point of all Higgs-less models in the literature is the $SU(2)_L \times SU(2)_R$ chiral symmetry spontaneously broken down to $SU(2)_{L+R}$. A minimal point of view consists therefore in taking the heavy spin-1 field(s) as (triplet) non-linear representation(s) of $SU(2)_L \times$
The $W$ and $B$ bosons weakly gauge the usual $SU(2)_L \times U(1)_Y$ subgroup, with $Y$ extended to include $B - L$ when acting on fermions, which only couple to $W$ and $B$. While not crucial to our conclusions, for simplicity we shall assume parity conservation as the usual gauge couplings, $g$ and $g'$, are switched off. We shall therefore speak of vector or axial spin-1 fields. This essentially defines the composite model.

Although with some constraints, as we are going to see, a structure of this type may emerge in particular from a gauge theory based on $G = SU(2)_L \times SU(2)_R \times SU(2)^N$ broken to the diagonal subgroup $H = SU(2)_{L+R+...}$ by a generic non-linear $\sigma$-model of the form

$$\mathcal{L}_\chi = \sum_{I,J} v_{IJ}^2 \langle D_\mu \Sigma_{IJ} D^{\mu} \Sigma_{IJ} \rangle, \quad \Sigma_{IJ} \to g_{IJ} \Sigma_{IJ}^+,$$

(1.2)

where $g_{IJ}$ are elements of the various $SU(2)$ and $D_\mu$ are covariant derivatives of $G$. This is the gauge model that we shall consider. It includes as special cases or approximates via deconstruction many of the models in the literature [1, 2, 3, 4].

The paper is organized as follows. In Sect. 2 we give the Lagrangian of the composite model. In Sect. 3 we recall the unitarity constraints. In Sect. 4 we calculate the contributions to $T$ of the heavy vector exchanges in the composite model. In Sect. 5 we do the analogous calculation in the gauge model and discuss the relation between the two models. In Sect. 6 we summarize the properties of the heavy spin-1 field(s) in the composite model as they emerge from the unitarity and the EWPT constraints. The corresponding phenomenology is briefly described in Sect. 7. Conclusions are drawn in Sect. 8. In the Appendices we discuss the sum rules following from the assumption of spin-1 meson dominance, and review the phenomenological parameters of low-lying spin-1 resonances in QCD.

### 2 The Lagrangian of the composite model

The building block is the usual lowest-order chiral Lagrangian for the Goldstone fields

$$\mathcal{L}_\chi^{(2)}(U) = \frac{v^2}{4} \langle D_\mu U (D^\mu U) \rangle,$$

(2.1)

where

$$U = e^{i\vec{\pi}/v}, \quad \vec{\pi} = T^a \pi^a = \frac{1}{\sqrt{2}} \begin{bmatrix} \pi^0 \pi^+ \pi^- \end{bmatrix}, \quad T^a = \frac{1}{2} \sigma^a,$$

$$D_\mu U = \partial_\mu U - i \hat{B}_\mu U + i U \hat{W}_\mu, \quad \hat{W}_\mu = gT^a W_\mu^a, \quad \hat{B}_\mu = g' T^3 B_\mu,$$

(2.2)

and $\langle \rangle$ denotes the trace of a $2 \times 2$ matrix. The invariant kinetic and mass terms for the standard fermions are left understood. Under $SU(2)_L \times SU(2)_R$,

$$U \to g_R U g^+_L.$$

Starting from $O(p^4)$, there are many possible terms which can be added to (2.1). It is well known [5] that in QCD the coefficients of these terms can be near-saturated by the tree-level
exchanges of the low-lying spin-1 resonances (see App. B). Motivated by this fact, we will assume that this spin-1 meson dominance is approximately true in our composite model as well. As we will see, the resulting framework is quite predictive.

Following Ref. [5], we describe the heavy spin-1 states by means of antisymmetric tensors. This formalism is particularly convenient since it avoids any mixing of the spin-1 fields with (the derivatives of) the Goldstone fields. Up to field redefinitions, addition of local terms, and appropriate matching conditions for the coupling constants, the results are equivalent to those obtained with the more familiar formalism of vector fields [5].

We consider at most two sets of vector states, $A^{\mu\nu}$ and $V^{\mu\nu}$, with opposite parity, both transforming in the adjoint representation of $SU(2)^L + R$

$$R^{\mu\nu} \rightarrow h R^{\mu\nu} h^\dagger, \quad R^{\mu\nu} = A^{\mu\nu}, V^{\mu\nu}. \quad (2.3)$$

To describe their transformation properties under the full $SU(2)^L \times SU(2)^R$, we introduce the little matrix $u$ [6] via

$$U = u^2.$$

This matrix parametrizes the $SU(2)^L \times SU(2)^R \times SU(2)^L + R$ coset and transforms as

$$u \rightarrow g_R u h^\dagger = h u^g_L,$$

where $h = h(u, g_L, g_R)$ is uniquely determined by this equation. The general transformation of $R^{\mu\nu}$ is then given by the same Eq. (2.3) with $h$ so defined. For $g_L = g_R$ we have $h = g_L = g_R$ independent of $u$, and we recover the linear $SU(2)^L + R$ transformation [6].

The kinetic Lagrangian for heavy spin-1 fields has the form:

$$L_{\text{kin}}(R^{\mu\nu}) = -\frac{1}{2} \langle \nabla_\mu R^{\mu\nu} \nabla_\sigma R_{\sigma\nu} \rangle + \frac{1}{4} M^2_R \langle R^{\mu\nu} R_{\mu\nu} \rangle, \quad (2.4)$$

where the covariant derivative

$$\nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R], \quad \Gamma_\mu = \frac{1}{2} \left[ u^\dagger (\partial_\mu - i \hat{B}_\mu) u + u (\partial_\mu - i \hat{W}_\mu) u^\dagger \right], \quad \Gamma^\dagger_\mu = -\Gamma_\mu, \quad (2.5)$$

ensures that $\nabla_\mu R$ transforms as $R$ under the global $SU(2)^L \times SU(2)^R$ and under the SM gauge group.

The most general $SU(2)^L \times SU(2)^R$ invariant Lagrangian at $O(p^2)$ describing the coupling of these heavy fields to Goldstone bosons and SM gauge fields, invariant under parity, is

$$L_V(R, u) = L_{\text{kin}}(A^{\mu\nu}) + L_{\text{kin}}(V^{\mu\nu}) + \frac{i}{2\sqrt{2}} G_V \langle V^{\mu\nu} [u_\mu, u_\nu] \rangle + \frac{1}{2\sqrt{2}} F_V \langle V^{\mu\nu} (u \hat{W}^{\mu\nu} u^\dagger + u^\dagger \hat{B}^{\mu\nu} u) \rangle + \frac{1}{2\sqrt{2}} F_A \langle A^{\mu\nu} (u \hat{W}^{\mu\nu} u^\dagger - u^\dagger \hat{B}^{\mu\nu} u) \rangle, \quad (2.6)$$

where

$$u_\mu = i u^\dagger D_\mu U u^\dagger = u^\dagger_\mu, \quad u_\mu \rightarrow h u_\mu h^\dagger. \quad (2.7)$$

The phenomenological parameters $G_V, F_{V,A}$ have dimension of mass and, by naive dimensional analysis, we expect them to be of $O(v)$.

The Lagrangian (2.6) does not include any trilinear coupling between the Goldstone fields and a pair of heavy spin-1 fields. We shall have to come back to this point.

3
3 Perturbative unitarity

Up to terms of order \( m_W / \sqrt{s} \), the amplitudes for longitudinal gauge-boson scattering are identical to those of the corresponding Goldstone bosons. In the limit of unbroken \( SU(2)_L + R \) symmetry we can decompose the generic \( W^+_L W^-_L \rightarrow W^+_L W^-_L \) amplitude as \[7\]

\[
\mathcal{A}(W^+_L W^-_L \rightarrow W^+_L W^-_L) = \mathcal{A}(s, t, u) \delta^{ab} \delta^{cd} + \mathcal{A}(t, s, u) \delta^{ac} \delta^{bd} + \mathcal{A}(u, t, s) \delta^{ad} \delta^{bc}. \tag{3.1}
\]

The fixed isospin amplitudes are \[7\]

\[
T(0) = 3\mathcal{A}(s, t, u) + \mathcal{A}(t, s, u) + \mathcal{A}(u, t, s)
\]

\[
T(1) = \mathcal{A}(t, s, u) - \mathcal{A}(u, t, s)
\]

\[
T(2) = \mathcal{A}(t, s, u) + \mathcal{A}(u, t, s)
\]

and the partial wave coefficients:

\[
a^I_l(s) = \frac{1}{64\pi} \int_{-1}^{1} d(cos \theta) P_l(cos \theta) T(I),
\]

\[
t = -\frac{s}{2}(1 - \cos \theta), \quad u = -\frac{s}{2}(1 + \cos \theta).
\]

Evaluating this process at the tree level leads to

\[
\mathcal{A}(s, t, u) = i \mathcal{A}(\pi^+ \pi^- \rightarrow \pi^0 \pi^0) = \frac{s}{v^2} - \frac{G^2_V}{v^4} \left[ 3s + M^2_V \left( 3 - \frac{u}{t-M^2_V} + \frac{s-t}{u-M^2_V} \right) \right], \tag{3.2}
\]

where the first term is the contribution of \( \mathcal{L}_V^{(2)} \). Exchanges of more than one vector spin-1 field are easily included. Axial spin-1 fields do not contribute to \( \mathcal{A}(s, t, u) \).

The cancellation in \(3.2\) of the linear growth with \( s \) occurs for

\[
G_V = v/\sqrt{3}. \tag{3.3}
\]

This result is equivalent to the one obtained by Bagger et al. \[7\] (\( a = 4G^2_V/v^2 \) in their notation). The strongest unitarity constraint is obtained by requiring \( |a^0_0| < 1 \) for any energy up to \( \sqrt{s} = \Lambda \), where

\[
a^0_0 = \frac{M^2_V}{16\pi v^2} \left\{ x \left( 1 - \frac{3G^2_V}{v^2} \right) + \frac{2G^2_V}{v^2} \left[ (2 + x^{-1}) \log(x+1) - 1 \right] \right\}, \quad x = \frac{s}{M^2_V}. \tag{3.4}
\]

Imposing the constraint \( |a^0_0| < 1 \) up to \( \Lambda \simeq 3 \) TeV, we get an allowed region in the \((M_V, G_V)\) plane extending all the way up in \( M_V \), see Fig. 1. Notice that the partial wave grows with energy even for \( G_V = v/\sqrt{3} \), albeit only logarithmically. As a result the unitarity is restored more efficiently for \( G_V \) moderately above \( v/\sqrt{3} \), so that the linear term enters with a small negative coefficient and compensates for the logarithmic growth. This is the situation realized for the \( \rho \) meson in QCD (see App. 1). Of course this mechanism cannot work for arbitrarily high energies, also because at higher energies we have to take inelastic channels into account \[8\].
Figure 1: Summary of unitarity and EWPT constraints (at 95% C.L.) in the $(M_V, G_V)$ plane (see Sect. 6).

It is interesting to note that models of electroweak symmetry breaking in 5D and their $N$-site deconstructions, to be discussed in Sect. 5, apparently cannot access the region $G_V \geq v/\sqrt{3}$. For instance in the 3-site model [9] one has $G_V = v/2$, so that the vector exchange cancels only $3/4$ of the linear $s$ growth of the amplitude. In those models the linear growth can be canceled only by the exchange of the full tower of resonances, i.e. in 5D or in the limit of infinitely many sites, and even then the amplitude continues to grow logarithmically.

The analogue of the amplitude (3.2) in the SM with the Higgs boson exchange is

$$A(s, t, u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2},$$

$$a_0^0 = \frac{1}{16\pi} \frac{M_H^2}{v^2} \left( \frac{\log(x + 1)}{x} - \frac{3/2}{x - 1} - \frac{5/2}{x} \right), \quad x = \frac{s}{M_H^2}.$$  

This partial wave has a fixed limit at large $s$, which however grows with $M_H$. Because of this, unlike in the vector-boson case, $M_H > 1.2$ TeV is not compatible with the unitarity bound [10].

Finally, we remind that the loss of unitarity associated with the chiral Lagrangian description of fermion masses intervenes at energies well above 3 TeV.
4  $T$ at one loop in the composite model

The tree level exchange of the heavy vectors leads to well known contributions to the parameters $\hat{S}, \hat{W}, \hat{Y}$ of Ref. [11]:

$$
\Delta \hat{S} = g^2 \left( \frac{F_V^2}{4M_V^2} - \frac{F_A^2}{4M_A^2} \right), \quad \Delta \hat{W} = \left( \frac{g}{g'} \right)^2 \Delta Y = g^2m_W^2 \left( \frac{F_V^2}{4M_V^4} + \frac{F_A^2}{4M_A^4} \right),
$$

(4.1)

whereas it leaves $T$ untouched because of the protecting $SU(2)_{L+R}$ symmetry. As already pointed out, however, it is crucial to include the loop effects in $T$, which arise from the diagrams of Fig. 2. The corresponding calculation is in fact significantly simplified by noticing that, up to corrections of relative order $m_W^2/M_R^2$,

$$
\hat{T} = \frac{Z^{(+)}(0)}{Z^{(0)}} - 1
$$

(4.2)

where $Z^{(+)}$, $Z^{(0)}$ are the wave-function renormalization constants of the charged and neutral Goldstone bosons computed in the Landau gauge for the light vectors. This follows from the Ward identities of the global $SU(2)_L$ symmetry [12] and from the fact that it is only in the Landau gauge that the global $SU(2)_L \times U(1)_Y$ is preserved and the Goldstone bosons are kept massless. A further simplification occurs by setting to zero the $SU(2)_L$ gauge coupling, which does not break the custodial symmetry, so that only the B-boson exchange produces an effect in $T$.

From the diagram of Fig. 2b, which is there in the SM, one obtains the infrared effect in (1.1). The contribution from the remaining diagrams is also readily obtained by means of the propagator for the spin-1 fields of mass $M$, vector or axial, in the antisymmetric tensor paramatrization that we are using [5]

$$
D_{\mu\nu,\rho\sigma}(k) = i \frac{g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho}}{k^2 - M^2} - i \frac{P_{\mu\nu,\rho\sigma}(k)}{k^2 - M^2},
$$

(4.3)

$$
P_{\mu\nu,\rho\sigma}(k) = -g_{\mu\rho}g_{\nu\sigma}k^2 - k_{\mu}k_{\rho}g_{\nu\sigma} + k_{\mu}k_{\rho}g_{\nu\sigma} - (\mu \leftrightarrow \nu).
$$

(4.4)

Except for special values of the couplings, the dominant contribution to $T$ from the heavy spin-1 exchanges comes from the diagram of Fig. 2b, which is the only one to diverge quadratically. Adding this contribution to the infrared term we have

$$
\Delta \hat{T}\bigg|_{\text{generic}} = -\frac{3\alpha}{8\pi c^2_W} \ln \left( \frac{\Lambda}{m_W} \right) + \frac{3\pi\alpha}{c^2_W} \left[ \frac{F_A^2}{4M_A^2} + \left( \frac{F_V - 2G_V}{2M_V} \right)^2 \right] \frac{\Lambda^2}{16\pi^2 v^2} + \ldots
$$

(4.5)

To make contact with the gauge model as defined in the Introduction and with the existing literature on the subject, it is useful to consider the same calculation of $T$ for the special case $F_A = 0$, $F_V = 2G_V$, which makes the $\Lambda^2$-term in (4.5) to vanish. In this case all the four diagrams in Fig. 2 contribute with a logarithmic term or, explicitly,

$$
\Delta \hat{T}\bigg|_{F_V=2G_V,F_A=0} = -\frac{3\alpha}{8\pi c^2_W} \left\{ \ln \left( \frac{M_V}{m_W} \right) + \left[ \left( 1 - \frac{2G_V^2}{v^2} \right)^2 + \frac{G_V^2}{v^2} \right] \ln \left( \frac{\Lambda}{M_V} \right) + \mathcal{O}(1) \right\}
$$

(4.6)

The remaining two diagrams do not contain a quadratic divergence because the second term in (4.3), which dominates at high momenta, vanishes when contracted with $k_{\mu}$ in any of the four indices.
The result of Ref. [9] is recovered from (4.6) for $G_V = v/2$. This is a particular 3-site gauge model with two links connecting $SU(2)_L$ and $SU(2)_R$ to one extra $SU(2)$ respectively. One can obtain $G_V < v/2$ in this model, still preserving parity, by having one extra link connecting $SU(2)_L$ and $SU(2)_R$ with each other. However, for any choice of $G_V/v$ the vector contribution does not compensate the dominant infrared term.

Two interesting questions arise at this point. In the QCD case, where the Lagrangian (2.6) is known to describe well the properties of the spin-1 resonances and, especially, their contribution to the low energy pion-interactions, is there any evidence for $F_A \neq 0$ and/or $F_V \neq 2G_V$? Furthermore, if indeed $F_A \neq 0$ and/or $F_V \neq 2G_V$, what cuts off the quadratic divergence in (4.5)?

To the first question there is a neat answer. By integrating out the heavy spin-1 fields at tree level, one shows (See Sect. 5) that a trilinear coupling $\pi - v_\mu(q) - a_\nu(k)$ arises between the pion field and the external vector and axial currents carrying respectively momentum $q$ and $k$. Furthermore this coupling at $q^2 = k^2 = 0$ is non vanishing if and only if $F_A \neq 0$ and/or $F_V \neq 2G_V$ (see App. B). Precise measurements of the radiative pion decay, $\pi^+ \to e^+\nu\gamma$, show that this same coupling is phenomenologically required [13]. A non-vanishing $F_A$ is also required by the significant partial width $\Gamma(a_1 \to \pi\gamma)$ (see App. B).

We will therefore assume that, in analogy to QCD, $F_A \neq 0$ and/or $F_V \neq 2G_V$, and try to take advantage of the positive second term in (4.5). In order to convince ourselves that this quadratically divergent term corresponds to a true physical effect, it is important to understand a mechanism which cuts off the quadratic divergence. This would simultaneously provide us with an idea for the appropriate $\Lambda$. The mechanism that we propose involves the trilinear interactions between the Goldstone pions and a pair of heavy spin-1 fields

$$L_{2V}(R, u) = ig^A_j \langle A^{\mu\nu}[\nabla_\rho V_{j\rho}, u_\mu] \rangle + ig^V_j \langle V^{\mu\nu}[\nabla_\rho A_{j\rho}, u_\mu] \rangle.$$  \hspace{1cm} (4.7)

These couplings are required if one imposes the vanishing at high momentum of the form factors

$$\langle A|v_\mu(q)|\pi\rangle, \quad \langle V|a_\mu(q)|\pi\rangle,$$  \hspace{1cm} (4.8)

for each vector or axial field. As explicitly indicated with the index $j$, this may involve, in general, more than one heavy state. In App. A we discuss explicitly the constraints of this type, as well as those arising from the high momentum behaviour of the pion form factor and of the left-right two-point function ($\langle v_\mu v_\nu \rangle - \langle a_\mu a_\nu \rangle$) (the analogue of the Weinberg sum rules in QCD). From the couplings in (4.8) more diagrams contribute to $\hat{T}$ with two heavy spin-1 particles in the

![Figure 2: One-loop contributions to $\hat{T}$ generated by $L^{(2)}$ and by couplings of the heavy vectors at most linear in the heavy fields.](image)
intermediate states (see Fig. 3). With the form factors in (4.8) softened by the couplings (4.7), the quadratic divergence in (4.5) disappears and Λ gets replaced by a heavy particle mass. We will assume that this cancellation involves heavy states of mass close to the cutoff, so that (4.5) continues to be a good estimate of $T$ with $\Lambda \approx 3 \text{ TeV}$.

5 The $SU(2)^N$ gauge model

As anticipated, we consider a generic model based on $G = SU(2)_L \times SU(2)_R \times SU(2)^N$, with the $SU(2)^N$ subgroup fully gauged and with $G$ spontaneously broken into the diagonal subgroup $H$. The Lagrangian of the model is

$$L = L_{\text{gauge}} + L_\chi, \quad L_{\text{gauge}} = -\sum_I \frac{1}{4g_I^2} (\omega_I^{\mu \nu})^2, \quad \omega_I^{\mu \nu} = \partial^\mu \omega_I^\nu - \partial^\nu \omega_I^\mu - i[\omega_I^\mu, \omega_I^\nu], \quad (5.1)$$

with $L_\chi$ given in (1.2).

This model has been analyzed in [4] and argued to give $\Delta S_{\text{tree}} \gtrsim N/6\pi$, in accordance with a previous result in 5D [14]. Here we are mainly interested in the relation of this model with the composite model of Section 2. To this end we analyze its low energy limit after the tree level integration of the heavy vectors and we compute $T$ at one loop. Unlike the case of the composite model, we show that the gauge model does not have an on shell $\pi - v_\mu - a_\nu$ coupling of $O(p^4)$ and that it cannot account for the EWPT by heavy vector exchanges only. The reader not interested in the gauge model or in its connection with the composite model can skip this Section.

The symmetry breaking term of the model can be put in the form

$$L_\chi = \sum_{I,J} v_I^2 \langle (\Omega_I^\mu - \Omega_J^\mu)^2 \rangle, \quad (5.2)$$

where

$$\Omega_I^\mu = \sigma_I^\dagger \omega_I^\mu \sigma_I + i\sigma_I^\dagger \partial^\mu \sigma_I$$

are gauge transformations of the original fields, and $\sigma_I$ are the elements of $SU(2)_I/H$ (the generalizations of little $u$: $\sigma_I \xrightarrow{G} g_I \sigma_I h^\dagger$). The link fields can be expressed in terms of $\sigma_I$ as:

$$\Sigma_{AB} = \phi_{AB} \sigma_A \sigma_B^\dagger,$$

where $\phi_{AB}$ is singlet under $G$ describing extra Higgs-like degrees of freedom. We will assume In what follows that these extra scalars are decoupled, $\phi_{AB} \equiv 1$. This can be achieved by adding to
the Lagrangian (5.1) placket-like mass terms $\left(\Sigma_{Ai} \ldots \Sigma_{jB}\Sigma_{AB}\right)$ for each non-sequential link $\Sigma_{AB}$ and sending these masses to infinity.

We look for a description in terms of the SM gauge fields and the $SU(2)_L \times SU(2)_R$ Goldstone $u$ defined via

$$\sigma_R = \sigma^\dagger_L =: u.$$  

We hide the Goldstone boson fields of the non-SM groups in the heavy $\Omega_{I \neq L,R}^\mu$, which corresponds to a partial gauge-fixing. These fields transform under $SU(2)_L \times SU(2)_R$ via

$$\Omega^\mu_I \rightarrow h\Omega^\mu_I h^\dagger + ih\partial^\mu h^\dagger .$$

The $\Omega^\mu_I$ can be decomposed with respect to parity as:

$$\Omega^\mu_I = V_I^\mu + A_I^\mu , \quad \Omega^\mu_{P(I)} = V_I^\mu - A_I^\mu$$  \hspace{1cm} (5.3)

$$V_I^\mu \rightarrow hV_I^\mu h^\dagger + ih\partial^\mu h^\dagger , \quad A_I^\mu \rightarrow hA_I^\mu h^\dagger$$  \hspace{1cm} (5.4)

Defining

$$V_K^{\mu\nu} = \partial^\mu V_K^\nu - \partial^\nu V_K^\mu - i[V_K^\mu, V_K^\nu] , \quad D_V^\mu A_K^\nu = \partial^\mu A_K^\nu - i[V_K^\mu, A_K^\nu] ,$$  \hspace{1cm} (5.5)

we can write the gauge Lagrangian in the form:

$$\mathcal{L}_{\text{gauge}} = \mathcal{L}_{\text{gauge,SM}} - \sum_{I \neq L,R} \frac{1}{4g_I^2} \left[ \langle (V_I^{\mu\nu} - i[A_I^\mu, A_I^\nu])^2 \rangle + \langle (D_V^\mu A_I^\nu - D_V^\nu A_I^\mu)^2 \rangle \right].$$  \hspace{1cm} (5.6)

Coming back to the symmetry breaking term, the quadratic form (5.2) can be brought by a field rotation to a diagonal form in terms of mass eigenstates of different parity:

$$\mathcal{L}_\chi = \sum_{n=1}^{N_V} v_{V_n}^2 \langle \hat{V}_n^\mu \rangle^2 + \sum_{n=1}^{N_A} v_{A_n}^2 \langle \hat{A}_n^\mu \rangle^2 + \frac{v^2}{4} \langle \langle u^\mu \rangle^2 \rangle$$  \hspace{1cm} (5.7)

$$\hat{V}_n^\mu = \hat{V}_n^\mu - i\beta_n \Gamma^\mu , \quad \hat{A}_n^\mu = \hat{A}_n^\mu - \alpha_n u^\mu$$  \hspace{1cm} (5.8)

$$\hat{V}_n^\mu = \sum_{I=1}^{N_V} b_I^n V_I^\mu , \quad \hat{A}_n^\mu = \sum_{I=1}^{N_A} a_I^n A_I^\mu$$  \hspace{1cm} (5.9)

There are $N_V = [(N + 1)/2]$ heavy spin-1 vectors, and $N_A = [N/2]$ heavy axials. In (5.8) we separated explicitly their $\Omega_{L,R}$ components

$$\Gamma^\mu = \frac{1}{2i} (\Omega_R^\mu + \Omega_L^\mu) , \quad u^\mu = \Omega_R^\mu - \Omega_L^\mu ,$$

which coincide with the fields already encountered in Sect. 2. The heavy mass eigenstates transform homogeneously:

$$\hat{V}_n^\mu \rightarrow h\hat{V}_n^\mu h^\dagger , \quad \hat{A}_n^\mu \rightarrow h\hat{A}_n^\mu h^\dagger$$

\footnote{Note that $A_I^\mu = 0$ if $I = P(I)$.}

\footnote{This transformation can be taken orthogonal in the vector sector. In the axial sector the orthogonal transformation has to be followed by a non-orthogonal linear transformation needed to separate the pure $\langle \langle u^\mu \rangle^2 \rangle$ term.}

\footnote{[$a$] $\equiv$ maximal integer $\leq a$.}
which follows from the fact that the quadratic form (5.2) has a zero eigenvector $\hat{V}_0^\mu = \sum_i \Omega_i^\mu$, and from orthogonality of eigenvectors.

The complete Lagrangian $\mathcal{L}_\chi + \mathcal{L}_{\text{gauge}}$ in the form of (5.6), (5.7) describes in general terms an $(N + 2)$-site model with arbitrary number of links, including non-sequential ones. A particularly simple example of this general setup is the 3-site model of Ref. [9]. Here there is only one heavy vector, $\hat{V}_1 = V_1$ ($N_A = 0$, $N_V = 1$), with $\beta_V = 1$ and $v_V = v$, and the only free parameter of the model is $M_V = \sqrt{2} g_1 v$.

It is crucial that the gauge model predicts the appearance of the commutator term $[A^\mu_I, A^\nu_I]$ in the pure gauge Lagrangian (5.6). The coefficient of this term is not determined by the symmetry (5.4) and is usually set to zero in phenomenological Lagrangians based on the massive Yang-Mills formulation [5] [15].

### 5.1 Low-energy limit and comparison with the composite model

To analyze the structure of the theory at low energies in the $SU(2)_{L,R}$ sector, we can express the full Lagrangian in terms of the heavy fields $\hat{V}_n, \hat{A}_n$. The fields $V_I^\mu, A_I^\mu$ have expression of the form:

$$V_I^\mu = \Gamma^\mu + b_I^\mu \hat{V}_n, \quad A_I^\mu = \varepsilon_I u^\mu + a_I^\mu \hat{A}_n, \quad \varepsilon_I = a_I^\mu \alpha_n,$$

which are found by inverting (5.9). The field $\Gamma^\mu$ enters with unit coefficient to ensure the correct transformation properties. This leads to:

$$\mathcal{L}_{\text{gauge}} = - \sum_{I \neq L,R} \frac{1}{4 g_I^2} \left[ \left( \frac{1}{2} f_{\perp}^{\mu\nu} + i \frac{\varepsilon_I}{4} [u^\mu, u^\nu] - i \varepsilon_I^2 [u^\mu, u^\nu] \right)^2 + \langle (\varepsilon_I f_{\perp}^{\mu\nu})^2 \rangle \right] + \Delta \mathcal{L}(\hat{V}_n^\mu, \hat{A}_n^\mu, u),$$

where $f_{\perp}^{\mu\nu} = u \hat{W}^{\mu\nu} u^\dagger \pm u^\dagger \hat{B}^{\mu\nu} u$.

Integrating out the heavy $\hat{V}_n^\mu$ and $\hat{A}_n^\mu$, it follows that the interaction terms in $\Delta \mathcal{L}(\hat{V}_n^\mu, \hat{A}_n^\mu, u)$ contribute to Green’s functions with external $SU(2)_{L,R}$ fields only at $O(p^6)$ (see Ref. [4]). The only $O(p^4)$ contributions to light-field amplitudes are the contact terms in (5.11). In particular, those contributing to bilinear and trilinear couplings have the following general form

$$\Delta \mathcal{L}_{\text{gauge}}^{(4)} = -\beta \left[ \langle (f_{\perp}^{\mu\nu})^2 \rangle + i \langle f_{\perp}^{\mu\nu} [u^\mu, u^\nu] \rangle \right] - \alpha \left[ \langle (f_{\perp}^{\mu\nu})^2 \rangle - i \langle f_{\perp}^{\mu\nu} [u^\mu, u^\nu] \rangle \right],$$

with only two (positive) free parameters: $\beta$, arising by the vector-meson sector, and $\alpha$ arising by the axial-vector sector. For any choice of these two couplings $\Delta \mathcal{L}_{\text{gauge}}^{(4)}$ gives a vanishing on-shell $\pi - a_\mu - v_\mu$ coupling, which contradicts the QCD phenomenology of $\pi \to l^+ \nu_l \gamma$ decays (see App. [13]).

For comparison, the $O(p^4)$ bilinear and trilinear couplings of light fields obtained integrating out the heavy vectors in the composite models are

$$\Delta \mathcal{L}_{\text{composite}}^{(4)} = -\frac{F_Y^2}{8 M_V^2} \langle (f_{\perp}^{\mu\nu})^2 \rangle - i \frac{F_Y G_V}{4 M_V^2} \langle f_{\perp}^{\mu\nu} [u^\mu, u^\nu] \rangle - \frac{F_A^2}{8 M_A^2} \langle (f_{\perp}^{\mu\nu})^2 \rangle.$$
Comparing this result with $\Delta L^{(4)}_{\text{gauge}}$, we find that the vector-meson sectors of the two models have the same structure for $F_V = 2G_V$. In the specific case of the 3-site model, requiring the same overall normalization of the vector terms we also find the relation $F_V = v$.

Contrary to the vector sector, the $O(p^4)$ structures in the axial-vector sectors are always different in the gauge and composite scenarios: the two models coincide only in the (trivial) case $F_A = \alpha = 0$.

5.2 One-loop contributions to $T$

Without performing an explicit calculation, we can generalize the 3-site result in (4.6) and show that also in the general $N$-site case there are no large contributions to $T$ beside the infrared term.

The calculation proceeds as in Section 4. Since the effective couplings of $B_\mu$ to pions and to the heavy gauge fields are only in $L_\chi$, it is more convenient to avoid the field redefinitions which move $B_\mu$ into $L_{\text{gauge}}$. The only unavoidable redefinition is the shift $A_\mu^I \rightarrow A_\mu^I + (2\varepsilon_I/v)\partial_\mu \pi$, necessary to obtain a canonical kinetic term for the pions. This shift produces an effective trilinear coupling $\sim \bar{A}_n \bar{V}_m \pi$ from the gauge Lagrangian in (5.6).

At the one loop level we have at most logarithmic UV divergences. This can be easily understood by analogy with the SM in the $m_H \rightarrow \infty$ limit. We are interested in the parametric dependence on the masses and coupling constants of the coefficient of the log and of finite terms, in order to isolate possible enhancement factors. As far as the diagrams in Fig. 2 are concerned, the parametric dependence is always of the type

$$\Delta \hat{T} \sim \frac{(g')^2}{16\pi^2} \sum g_{Vn}^2 \left( \frac{\beta_n v_{Vn}^2}{v} \right)^2 \times F_{\text{loop}} \sim \frac{(g')^2}{16\pi^2} \sum \beta_n^2 v_{Vn}^2 v^2 \sim \frac{(g')^2}{16\pi^2}, \quad F_{\text{loop}} \sim \frac{1}{M_{Vn}^2}, \quad (5.14)$$

and similarly for the axial vectors. Here $M_{Vn} \sim g_{Vn} v_{Vn}$ and $g_{Vn}$ are the effective coupling of the heavy fields obtained from $L_{\text{gauge}}$. We took into account that there cannot be any enhancement due to $\beta_n^2$ or $v_{Vn}^2/v^2$ factors: being the matrix elements of an orthogonal rotation matrix the coefficients $|\beta_n|$ are bounded by unity; also, by the perturbative diagonalization of the mass matrix one gets $\beta_n \sim v/v_{Vn}$ in the limit $v_{Vn} \gg v$.

The case of the diagrams in Fig. 3 generated by the $\bar{A}_n \bar{V}_m \pi$ trilinear coupling, is slightly more complicated. However, after identifying the parametric dependence of the trilinear terms in (5.6) from the effective coupling of the heavy fields, one finds also in this case $\Delta \hat{T} \sim (g')^2/(16\pi^2)$.

This general conclusion can also be reached by noting that, since in the gauge model $M_{Vn} \sim g_{Vn} v_{Vn}$, we cannot enhance $\Delta \hat{T}$ by a ratio $M_{Vn}/M_{Vm}$ (as in the composite case): this would correspond to a singularity in the limit $g_{Vn} \rightarrow \infty$ or $g_{Vm} \rightarrow 0$, while the one-loop amplitude must remain regular in this limit.

6 Unitarity and EWPT constraints put together

We can discuss now if the sole exchange of the spin-1 fields can account for the EWPT and, in the positive case, how their properties are constrained. From Eq. (4.6) and the discussion in Sect. 5 we conclude, as anticipated, that the EWPT cannot be satisfied in the gauge model without extra,
Figure 4: Areas of $S$ and $T$ covered in the composite model for different ranges of the free parameters. The two ellipses denote the experimentally allowed region at 68% and 95% CL. The small cross at negative $T$ values is the $F_V = F_A = G_V = 0$ point (infrared logs only).

not calculable, ingredients. We thus restrict our attention to the composite model of Sect. 2. For our purposes the significant constraints come solely from $S$ and $T$. The constraints from LEP2 on $W$, $Y$ and on the cubic gauge-boson vertexes are easily satisfied. The key equations are therefore (4.5) for $T$ and

$$\Delta \hat{S} = \frac{g^2}{96\pi^2} \ln \left( \frac{\Lambda}{m_W} \right) + g^2 \left( \frac{F_V^2}{4M_V^2} - \frac{F_A^2}{4M_A^2} \right) + \ldots ,$$

for $S$. We assume that the dots in both of these equations give negligible contributions for $\Lambda \approx 3$ TeV.

We also assume that below $\Lambda$ there are at most one vector and one axial spin-1 particles. The relevant parameters are therefore $M_V, G_V$ and two dimensionless ratios $F_V/M_V, F_A/M_A$.

The expectation for $S$ and $T$ in the composite model is represented in Fig. 4 for three different values of $M_V$. The other parameters are constrained by requiring that: i) the tree-level contribution to $S$ (the second term in (6.1)) is positive; ii) $0 < F_V/G_V < 2$ or 2.5; iii) for every $M_V$ the unitarity constraint on $G_V$, $|a_0| < 1$, is satisfied. As manifest in the Figure, the saturation of the unitarity bound is not critical. The boundary of the small $F_V$ region, $F_V/G_V < 1$, is also shown.

The cumulative constraints from unitarity and the EWPT are illustrated in Fig. 1, where we consider in particular two special cases, physically distinct by the spectrum of the spin-1 particles:

\footnote{Residual finite terms are fixed by matching $\Delta \hat{S}$ and $\Delta \hat{T}$ at $\Lambda = 1$ TeV and $F_A = F_G = G_V = 0$, to the SM values of $S$ and $T$ computed for $m_H = 1$ TeV with the central value of $m_{top} = 171.4$ GeV [17].}
I. a single vector spin-1 field below \( \Lambda \), denoted as \( F_V \ll 2 G_V \), \( F_A = 0 \);

II. one gauge-like vector and one axial state below \( \Lambda \), denoted as \( F_V = 2 G_V \), \( F_A \neq 0 \).

In Case I we therefore imagine that it is the quadratically divergent vector contribution that provides the positive \( \Delta T \) necessary to cancel the negative IR term, while the tree level contribution to \( S \), saturated by the same vector resonance, is within experimental bounds. This may happen if \( F_V \ll 2 G_V \). In Case II, on the other hand, the crucial role is played by the axial spin-1, which provides the positive one-loop \( \Delta T \) and partially cancels the tree level \( \Delta S \) of the vector (which by itself would be too large). We also consider the more general case, when both heavy vectors contribute to both \( S \) and \( T \):

III. with one vector and one axial state below \( \Lambda \), without special constraints on \( F_V/G_V \) except for the upper bound \( F_V < 2.5 \, G_V \), and with \( F_A \neq 0 \).

7 Phenomenology

From a phenomenological point of view the main consequence of the previous Section, as manifest from Fig. 1, is that, unlike the pure unitarity constraint, the EWPT require a relatively light spin-1 vector. In turn, together with unitarity, this bounds \( G_V \) between about 120 and 180 GeV. Less is known about the axial spin-1, since it does not influence the \( \pi \pi \), or \( W_L W_L \), scattering amplitude and it enters the EWPT only through the combination \( F_A/M_A \).

In the allowed region of the \((M_V, G_V)\) plane the cross section for \( \sigma(pp \to V \to WZ + 2\text{jets}) \) at the LHC via vector boson fusion, taking into account the leptonic branching ratios of \( W \) and \( Z \), and applying the cuts suggested in Ref. \[18\] on the jet variables, is between 0.3 and 3 fb. The coupling of the heavy vector to the fermions proceeds only through its mixing with the \( W \) and the \( Z \) and is therefore highly suppressed. As a consequence the production by quark-antiquark annihilation is irrelevant, as is probably irrelevant also the search of the neutral vector decaying into a pair of charged leptons.

The relevant widths for \( g' = 0 \) and up to corrections of order \( m_{2V}/M_V^2 \) are

\[
\Gamma(V^+ \to W_L^+ Z_L) \approx \Gamma(V^0 \to W_L^+ W_L^-) \approx \frac{G_V^2 M_V^3}{48 \pi v^4} \approx 9 \text{ GeV} \left( \frac{G_V}{150 \text{ GeV}} \right)^2 \left( \frac{M_V}{600 \text{ GeV}} \right)^3, \tag{7.1}
\]

and

\[
\Gamma(V^0 \to \ell^+ \ell^-) = \frac{g^4 F_V^2}{384 \pi M_V}, \tag{7.2}
\]

so that

\[
\frac{\Gamma(V^0 \to \ell^+ \ell^-)}{\Gamma(V^0 \to W_L^+ W_L^-)} \approx \frac{g^4 F_V^2}{8 G_V^2 M_V} v^4 \approx 6 \cdot 10^{-4} \frac{F_V^2}{G_V^2} \left( \frac{600 \text{ GeV}}{M_V} \right)^4. \tag{7.3}
\]

Notice the rather narrow total width of \( V \), compared to the width of the SM Higgs boson of the same mass. This fact can be partially explained by the fact that: i) three vectors rather than one scalar boson unitarize the \( W_L W_L \) scattering, so that the unitarization is achieved for relatively lower values of the relevant cubic coupling; ii) the vectors decay in to \( W_L W_L \) pairs in
$P$ wave (the unitarization of $S$-wave $W_L W_L$ scattering occurs via $t$-channel exchange) contrary to $S$-wave decay of the Higgs boson.

The $V^+$ production at LHC is supposedly easier to search for, compared to the $V^0$, because of the relatively easier reconstruction of the $W^+ Z$ versus the $W^+ W^-$ final state. Generalizing the results of Ref. [18] to our framework, with 100 fb$^{-1}$ one should expect a clear signal (more than 5$\sigma$) for the $V^+$ in the whole allowed region in Fig. 1 (for related phenomenological studies see also [19]). As already mentioned, the searches at LEP2 both in the $l^+ l^-$ and in the $W^+ W^-$ channels do not add any significant constraint on Fig. 1 even in the low mass region.

8 Summary and conclusions

As said at the beginning, the double role played by a relatively light Higgs boson in allowing a consistent description of the EWPT in the SM and in unitarizing the $W_L W_L$ scattering speaks strongly in favour of its discovery at the LHC. Any alternative point of view has a hard time in replacing this role in a convincingly calculable (rather than ad hoc) way. In this work we have analyzed up to which point one or more spin-1 particles can be such an alternative.

While the presence of and the motivation for spin-1 particles emerges in different contexts, we have focussed on their phenomenological description. To this end, building on the much suggestive $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$ symmetry, we have considered two different models:

- A composite model, inspired by (but also departing from) QCD, where the heavy bosons are triplet representations of $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$;
- A gauge model where $SU(2)_L \times SU(2)_R / SU(2)_{L+R}$ is extended to $SU(2)^N / SU(2)_{diag}$ and the heavy vectors emerge from the breaking of the extended gauge group.

Both model are described by effective Lagrangians with a cutoff $\Lambda \approx 4\pi v \approx 3$ TeV. Each Lagrangian would support counterterms associated with the EWPT parameters $S$ and $T$. To defend the calculability of the EWPT we have to take them negligible.

We have argued that the gauge model is a special case of the composite model, while the composite model cannot be reduced to the gauge model except for special values of the parameters. As known for example from work in QCD, only the composite model can account for some phenomenological properties of pion dynamics. This difference proves in fact essential to offer a potentially consistent description of the EWPT in terms of the sole exchange of the heavy vectors in the composite model.

The problem of the $S$-parameter in Higgs-less models, positive and potentially too large, is well known. Its compatibility with data depends crucially, however, on its correlation with $T$, which receives a large negative infrared effect. $S$ and $T$ must therefore be discussed together, which requires the inclusion of the full $T$ at one loop. In the composite model, the crucial effect in the $T$-parameter comes from the loop diagram in Fig. 2b with derivative couplings to the Goldstone field of the heavy spin-1 particles. This is the only quadratically divergent diagram. As such, for light enough spin-1 fields, it can compensate the logarithmically divergent infrared effect which does not involve the exchange of any heavy particle, as illustrated in Fig. 4. If we impose that suitable correlation functions among composite states have good ultraviolet behavior, the cutoff
A in the quadratically divergent diagram gets replaced by a heavy vector mass. We assume this mass to be close to the cutoff, so that the naive estimate of $T$ with the cutoff itself continues to be reasonable.

On this basis, the EWPT constraints can be crossed with the ones coming from unitarity in longitudinal gauge-boson scattering at high energy. The corresponding significant restriction in the parameter space of the lightest vector spin-1 state is summarized in Fig. 1. Note how the unitarity constraint by itself would not give a strong upper bound on $M_V$. This is a crucial feature in connection with the searches at the LHC, as we comment in Sect. 7.

It goes without saying that we do not know if a phenomenological model like this can emerge from any fundamental theory of electroweak symmetry breaking. In particular we are sharply aware of the fact that this has left us with many free parameters to play with. As repeatedly said, it is hard to compete with the Higgs boson picture of electroweak symmetry breaking. Yet we find it interesting that there be a plausible mechanism that can account both for the EWPT and the unitarity in terms of heavy spin-1 exchanges and that, under suitable hypotheses, this can constrain in a significant way the properties of the lightest of them, most likely accessible to LHC searches.

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A High momentum behavior of two and three point functions

The consistency of the theory requires a smoothing in the ultraviolet of the correlation functions involving external SM gauge fields or, more generally, external vector or axial sources of $SU(2)_L \times SU(2)_R$, with respect to the potential bad behaviour inferred from the lowest-order chiral Lagrangians. This implies a series of sum rules for the free parameters of the model.

Starting from the two-point functions,

$$i \int d^4x e^{iqx} [\langle 0|\pi^\mu_i(x)v^\nu_j(0) - a^\mu_i(x)a^\nu_j(0)|0\rangle] = \delta_{ij}\eta^{\mu\nu} q^2 \Pi_{LR}(q^2) + O(q^4q^\nu), \quad (A.1)$$

the cancellation of tree-level contributions to $q^4\Pi_{LR}(q^2)$ at high $q^2$ leads to the well-known Weinberg sum rules,

$$\sum_{i=1}^{N_V} F^2_{V_i} - \sum_{i=1}^{N_A} F^2_{A_i} = v^2, \quad \sum_{i=1}^{N_V} F^2_{V_i} M^2_{V_i} - \sum_{i=1}^{N_A} F^2_{A_i} M^2_{A_i} = 0, \quad (A.2)$$
for a generic model with \( N_V \) vector and \( N_A \) axial fields (with \( F_V \) and \( G_V \) defined as in Sect. 2).

In the case of three point functions, from the pion vector form factor,

\[
\langle \pi(p')|v^\mu(q)|\pi(p) \rangle = F_V^\pi(q^2)(p + p')^\mu, \tag{A.2}
\]

the condition \( F_V^\pi(q^2) \to 0 \) implies

\[
\sum_i F_{Vi} G_{Vi} = v^2. \tag{A.3}
\]

Similarly, defining

\[
\langle A(p', \epsilon)|v^\mu(q)|\pi(p) \rangle = F_{A\pi}^\mu(q^2)((pq)\epsilon_\mu - q^\mu(p\epsilon)) + O((p')^2), \tag{A.4}
\]

\[
\langle V(p', \epsilon)|a^\mu(q)|\pi(p) \rangle = F_{V\pi}^\mu(q^2)((pq)\epsilon_\mu - q^\mu(p\epsilon)) + O((p')^2), \tag{A.5}
\]

the conditions \( F_{A\pi}^\mu(q^2) \to 0 \) and \( F_{V\pi}^\mu(q^2) \to 0 \) lead to

\[
2 \sum_{j=1}^{N_V} g_j^A F_{Vi} = F_{Ai}, \quad 2 \sum_{j=1}^{N_A} g_j^V F_{Ai} = F_{Vi} - 2G_{Vi}, \tag{A.6}
\]

with \( g_j^A \) and \( g_j^V \) defined as in Eq. (4.7). These last two conditions ensure the cancellation of the quadratic divergence in \( T \) of the composite model.

In the gauge models some of these sum rules are satisfied in a trivial way. For instance, in the 3-site model all terms in Eq. (A.6) are identically zero. In the more general case of composite models we expect the sum rules to be satisfied summing over the first few sets of resonances (not necessarily a single set). With two, non-degenerate sets of vector and axial-vector states, one light and the other close to the cut-off, all the sum rules can be satisfied choosing arbitrary values for the light parameters \( M_V, F_V, G_V, \) and \( F_A/M_A \), within the ranges discussed in Sect. 6.

B Heavy spin-1 resonances in QCD

Low-energy QCD is a theory with spontaneously broken approximate \( SU(3)_L \times SU(3)_R \) global symmetry. For comparison with the EWSB we will ignore the heavier \( s \) quark and consider the \( SU(2)_L \times SU(2)_R \) subgroup. In this Appendix we collect experimentally measured parameters of spin-1 resonances in QCD. If the EWSB were described by a QCD-like theory, all the parameters with dimension of mass would have to be scaled up by the ratio \( v/F_\pi \), where the pion-decay constant

\[
F_\pi \simeq 93.3 \text{ MeV}
\]

is the QCD scale of \( SU(2)_L \times SU(2)_R \) breaking.

The lightest vector and axial spin-1 QCD resonances are the \( \rho \equiv V \) and the \( a_1 \equiv A \):

\[
M_\rho \simeq 775 \text{ MeV}, \quad \Gamma_\rho \simeq 150 \text{ MeV},
\]

\[
M_{a_1} \simeq 1230 \text{ MeV}, \quad \Gamma_{a_1} = 250 \div 600 \text{ MeV}.
\]
The $\rho$ width is totally dominated by $\rho \to \pi\pi$, which allows the determination of $G_V$. From the tree-level amplitude we get

$$G_V \simeq 60 \text{ MeV} \ (M_\pi = 0) \quad \text{or} \quad G_V \simeq 67 \text{ MeV} \ (M_\pi = 135 \text{ MeV}) \ . \quad (B.1)$$

Note that as long as we neglect light quark masses (or explicit chiral symmetry breaking terms), we have no reasons to prefer the value of $G_V$ evaluated in the chiral limit ($M_\pi = 0$), versus the one obtained with the physical pion mass.

If rescaled to the electroweak scale, the $\rho$ meson would correspond to a 2 TeV spin-1 boson. The rescaled $G_V$ is at the lower limit of the unitarity band in Fig. 1, in agreement with the hypothesis that the $\rho$-meson plays a key role in unitarizing $\pi\pi$ scattering in QCD.

The $F_V$ and $F_A$ are determined from $\Gamma(\rho \to e^+e^-)$ and $\Gamma(a_1 \to \pi\gamma)$, respectively \[5\]

$$F_V = 157 \text{ MeV} \ ,$$
$$F_A = (120 \pm 25) \text{ MeV} \ . \quad (B.2)$$

Finally, the coupling $g^A$ can be determined from $\Gamma(a_1 \to \rho\pi) \simeq 60\%\Gamma_{a_1}$ \[20\]. Since the gauge model relation $F_V = 2G_V$ is satisfied at a 20\% level, it is reasonable to assume that $g^V \ll 1$ (see Eq. (4.7)), so that the first operator in (4.7) dominates this width. Under this assumption, we extract:

$$g^A = 0.50 \pm 0.15 \ .$$

It is now interesting to see if the above resonance parameters, extracted from experiment, satisfy the remaining sum rules of Appendix [A]. Both Weinberg sum rules are satisfied within the experimental errors, although the favored values of $F_V$ are different, which indicates potential importance of higher-mass resonances. The Vector Meson Dominance relation for the pion form factor \[A.3\] is well satisfied by the sole $\rho$ contribution if we choose $G_V$ in the lower range of (B.1), indicating a small but non negligible breaking of the $F_V = 2G_V$ relation. The first of the two sum rules \[A.6\] is also in good agreement for the above $g^A$. This means that in QCD the quadratic divergence in the charged-neutral pion wavefunction renormalization difference, present due to $F_A \neq 0$ (see Eq. (4.5)), is likely cut off at a scale of $M_\rho$ ($\sim 2$ TeV in electroweak units). Similarly, we can expect the contribution of the quadratic divergence proportional to $F_V - 2G_V$ to be cut off by the $M_{a_1}$ mass ($\sim 3$ TeV in electroweak units).

In principle, another interesting source of information is provided by the axial pion form factor, or the $\pi - \nu_\mu(q) - \bar{d}_\nu(k)$ coupling at $q^2 = k^2 = 0$, measured with high precision in $\pi^+ \to e^+\nu\gamma$ \[13\]:

$$\langle \gamma(\epsilon, q) | \bar{u}_\gamma \gamma_5 \gamma d | \pi(p) \rangle = \frac{i e}{M_\pi} \mathcal{F}_A [(pq)\epsilon - (pe)q_\mu] + \text{pion pole term} \quad (B.3)$$

$$\mathcal{F}_A^{\exp} = 0.0115 \pm 0.0004 \ . \quad (B.4)$$

As stressed in Sect. 4 and 5, the non-vanishing of $\mathcal{F}_A$ is a clear evidence for $F_A \neq 0$ and/or $F_V \neq 2G_V$. Indeed the $O(p^4)$ value of $\mathcal{F}_A$ obtained by spin-1 resonance exchange is \[5\]

$$\mathcal{F}_A^{\text{vectors}} = \sqrt{2} M_\pi \left( \frac{F_A^2}{M_A^2} - \frac{F_V(F_V - 2G_V)}{M_V^2} \right) \ . \quad (B.5)$$

---

7Notice that \[5\] used a different value of $M_{a_1}$. 

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Since axial and vector contributions in (B.5) turn out to have opposite sign, $F_A^{\exp}$ does not provide, by itself, a clear independent determination of $F_A$ and $F_V - 2G_V$. However, it provides a useful cross-check of the whole picture: using $F_V$ and $F_A$ from (B.2) as input values, one extracts $G_V = (70 \pm 10)$ MeV, in good agreement with (B.1).

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