Surface ultrasonic envelope solitons and wave collapse in solid film–substrate systems

V Grimalsky*, Koshevaya2, E Gutierrez-D1 and O V Kolokoltsev3

1National Institute for Astrophysics, Optics, and Electronics (INAOE), P.O. 51 & 216, ZP 72000, Puebla, Pue., Mexico

2Research Center of Applied Sciences and Engineering (CIICAp), Autonomous University of State Morelos (UAEM), Av. Universidad No. 1001, ZP 62210, Cuernavaca, Mor., Mexico

3National Autonomous University of Mexico (UNAM), Center of Applied Science and Technological Development (CCADET), P.O. 70-186, ZP 04510, Campus UNAM, Mexico, D.F., Mexico

*E-mail: vgrim@inaoep.mx

Abstract. An excitation of ultra-high frequency (100 MHz – 1 GHz) nonlinear envelope solitary acoustic waves, propagating along the interface between a solid film and a solid substrate, is theoretically analyzed. Both the quadratic nonlinearity and the cubic one are important in the case of the envelope waves. When generation of higher harmonics is reduced due to essential waveguide dispersion and the cubic nonlinearity due to the induced zero harmonic is dominating, a possibility of the envelope solitary pulse propagation and the spatial-temporal wave collapse exists, as demonstrated. When the cubic material nonlinearity reduces the associated cubic nonlinear term, there also exists a possibility to observe a wave collapse, if the initial focusing of the input pulse at the first harmonic is applied.

1. Introduction
During the last years, research of nonlinear wave phenomena in bounded solids in ultra-high frequency and microwave ranges has been of great interest [1-14]. Especially, nonlinear wave effects in structures, which include thin solid films, are perspective for experimental observations, due to a possibility to control the linear and nonlinear wave properties.

The surface ultrasonic waves (SAW) possess attracting properties for observing nonlinear phenomena. The surface acoustic waves are relatively slow and are localized at the interfaces. The integral power levels, which are necessary for a manifestation of nonlinearity, are quite low. The combination of materials of the film and the substrate makes possible to manage both the wave dispersion and the nonlinearity.

The nonlinear acoustic wave propagation in the solid layered systems has been considered in many papers [5-14]. The most results of simulations were obtained within an approximation of moderate nonlinearity, where the propagating wave was presented as a superposition of partial harmonics with slowly varying amplitudes, but the transverse profiles of partial harmonics were assumed as weakly changed, in a comparison to the linear case. Those results can be summarized as follows. The
baseband pulses can be excited experimentally [12,13,14] but they are not true solitons, because they
do not preserve their shapes after collisions, as numerical simulations demonstrated [9,14]. In the
experiments, the initial acoustic pulses were quite short: 2 – 20 ns.

The existence of envelope solitons has been pointed out in several papers [3,6,8,12]. The envelope
solitons are governed by the nonlinear Schrödinger equation or its generalizations [12]. But, a
propagation of nonlinear acoustic waves in the case of simultaneous generation of higher harmonics
due to quadratic nonlinearity and the self-action due to cubic nonlinearity was not investigated in
details. Moreover, under different spatial and temporal scales and under different relations between
elastic properties of the film and substrate, the behavior of nonlinear waves may be different. Also an
influence of the transverse inhomogeneity in the plane of the interface is of great interest, especially,
for a realization of spatial-temporal wave collapsing.

In this paper, a possibility of excitation of ultra-high frequency (100 MHz – 1 GHz) moderately
nonlinear solitary surface acoustic waves propagating along the interface between a solid thin film and
a substrate is under investigation. Both quadratic nonlinearity and cubic one are taken into account for
envelope waves. The dynamics of the envelope nonlinear wave is governed by the set of coupled
equations for amplitudes of harmonics, where the self-action due to the induced zero harmonic is
taken into account. Numerical simulations have demonstrated that an existence of envelope solitons
and collapsing pulses essentially depends on the relation of the bulk elastic parameters of contacting
media. An additional possibility to observe spatial-temporal collapse has been demonstrated, if the
initial focusing of the input pulse has been applied.

2. Basic equations and linear dispersion equation
The surface acoustic wave propagation in the system thin solid film – solid substrate is under
investigation. Below, we consider isotropic contacting media without slipping. Usually, the Lagrange
coordinate frame is used within the solid media [1,2,5,15,16,17]. The dynamic equation for the
mechanical displacement vector $u$ in solids has the next form [1,16,17]:

$$
\rho_0 \frac{\partial^2 u_{ij}}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j}; \sigma_{ij} = \sigma_{ij}^L + \sigma_{ij}^{NL}
$$

(1)

where $\rho_0$ is the unperturbed density of solid and $\sigma_{ij}$ is the Piola – Kirchhoff tensor.

For an isotropic solid, the expression of $\sigma_{ij}$ is [1,3]:

$$
\sigma_{ij} = \rho_0(s_l^2 - 2s_t^2)\text{div}u\delta_{ij} + 2\rho_0s_t^2u_{ij} +
+ C(div\bar{u})^2\delta_{ij} + (\rho_0s_t^2 + A/4) \left( \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} \right) +
+ (\rho_0(s_t^2 - 2s_l^2) + B) \frac{\partial u_i}{\partial x_j} \text{div}u + (A/4) \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} + B \frac{\partial u_j}{\partial x_i} \text{div}u +
+ (\rho_0(s_l^2 - 2s_t^2)/2 + B/2)(\frac{\partial u_i}{\partial x_m})^2\delta_{ij} + (B/2) \frac{\partial u_i}{\partial x_m} \frac{\partial u_m}{\partial x_i} \delta_{ij}
$$

(2)

In such an expansion, the quadratic nonlinear terms are given. The coefficients in nonlinear terms
are combinations of both material nonlinear modules ($A,B,C$) and linear ones, due to a presence of
geometric nonlinearity [3]. Here $s_\nu$ are longitudinal and transverse acoustic velocities; $u_i$ are the
components of a mechanic displacement. For a sake of simplicity, we have assumed in the figures
presented below that the following condition is satisfied: $|C| >> |A|, |B|$. Thus, the nonlinear term
$C(div\bar{u})^2\delta_{ij}$ in the expansion (2) is assumed as dominating. For some glass materials, this inequality is
valid [1]. In a general case, the coefficients of nonlinear equations are expressed through the combination of all material nonlinear and linear modules, but the governing equations for slowly varying amplitudes preserve the same structure. The results of the simulation presented below are of qualitative character and do not depend essentially on the values of nonlinear modules. The wave dissipation due to viscosity, proportional to the square of frequency, is also taken into account, which is not presented in equation (1).

The ordinary boundary conditions at the interface film - substrate are used; namely, the components of displacement vector $u_i$ and the normal components of the tension force $\sigma_{ijn}n_j$ are the same, where $n$ is vector normal to the unperturbed interface (vector $n$ is directed along $OX$-axis).

The geometry of the system under consideration is described here. The axis $OZ$ is directed horizontally along the wave propagation in the interface plane, $OX$ one is directed vertically upwards, see Figure 1a. The thickness of the film ($-h < x < 0$) is $h = \text{const}$. The solid substrate occupies the space $x<-h$. The surface $x=0$ is assumed as free.

In this system, linear surface acoustic waves can propagate with the displacement components $u = (u_x, 0, u_z)$. The dispersion equation for the sound waves has been obtained from subjecting the partial solutions of the linearized dynamic equation (1) in each region to the boundary conditions, and it can be expressed in the following matrix form:

$$\det(E_1F_1 + E_2F_2) = 0,$$

$$E_1 = \begin{pmatrix}
(1 - q_t^2)\cos(q_t h); & 2\cos(q_t h) \\
-2q_t^2\sin(q_t h); & (1 - q_t^2)\sin(q_t h) \\
2 - \gamma(1 + \tau_t^2); & 2\tau_t^2(1 - \gamma) \\
\gamma(1 + \tau_t^2) - 1 + \frac{q_t^2}{k^2}; & \frac{\tau_t}{k}(2\gamma - 1 + \frac{q_t^2}{k^2})
\end{pmatrix};$$

$$F_1 = \begin{pmatrix}
\gamma(1 + \tau_t^2) - 1 + \frac{q_t^2}{k^2}; & \frac{\tau_t}{k}(2\gamma - 1 + \frac{q_t^2}{k^2})
\end{pmatrix};$$

$$\begin{pmatrix}
-2\frac{q_t^1}{k}\sin(q_t h); & \frac{1}{k}\sin(q_t h) \\
(1 - \frac{q_t^2}{k^2})\cos(q_t h); & 2\cos(q_t h)
\end{pmatrix};$$

$$E_2 = \begin{pmatrix}
2\frac{\tau_t}{k}(1 - \gamma); & 2 - \gamma(1 + \tau_t^2) \\
\gamma(1 + \tau_t^2) - 1 + \frac{q_t^2}{k^2}; & \frac{\tau_t}{k}(2\gamma - 1 + \frac{q_t^2}{k^2})
\end{pmatrix};$$

$$F_2 = \begin{pmatrix}
\frac{\tau_t}{k}(2\gamma - 1 + \frac{q_t^2}{k^2}); & \gamma(1 + \tau_t^2) - 1 + \frac{q_t^2}{k^2}
\end{pmatrix};$$

where $c$, $s$, are longitudinal and transverse bulk velocities in the film and the substrate, respectively; $\rho_1, \rho_2$ are densities; $\gamma = \rho_2/\rho_1$, $q_t = ((\alpha c, \gamma) - k^2)^{1/2}$, $\tau_t = (k^2 - (\alpha s, \gamma))^{1/2}$.

The dispersion curve for the lowest acoustic mode is presented in figure 1b. The contact of the Si(Ge)O$_2$ film with Si substrate is considered. The following parameters are used: $h = 2.5 \, \mu m$, $c_s = 2.5 \cdot 10^5 \, cm/s$, $c_t = 4.0 \cdot 10^5 \, cm/s$, $\rho_1 = 3.0 \, g/cm^3$ (SiGeO$_2$); $s_t = 5.0 \cdot 10^5 \, cm/s$, $s_s = 8.4 \cdot 10^5 \, cm/s$, $\rho_s = 2.33 \, g/cm^3$ (Si). A crystalline anisotropy in Si is neglected here.
Figure 1. The geometry of the problem (a). The film occupies the region \(-h < x < 0\), the substrate is \(x < -h\). The free surface is \(x = 0\). Linear wave dispersion for surface acoustic wave in the layered solid system film – substrate where \(\omega_{\text{crit}} = 2.72 \cdot 10^9 \text{s}^{-1}\) for used parameters of Si(Ge)O₂ film – Si substrate (b). Corresponding dependencies of phase (curve 1) and group (curve 2) velocities on frequency (c). The thickness of the film is \(h = 2.5 \mu\text{m}\).

These waves possess an essential dispersion [3], see figure 1c. For the following consideration, we note that \(\partial^2 k / \partial \omega^2\) has different signs in the frequency regions \(\omega < \omega_{\text{crit}}\) and \(\omega > \omega_{\text{crit}}\), where \(\omega_{\text{crit}}\) is the value of the frequency of the wave where the inflection of dispersion curve takes place: \(\partial^2 k / \partial \omega^2 = 0\).

3. Envelope solitons and wave collapse

In the case of envelope waves, or wave packets, the spectrum of the wave is localized near the certain carrier frequency and its higher harmonics. Below, a dependence of the wave amplitudes on the transverse coordinate \(y\) in the plane of the interface is also taken into account. The travelling envelope wave is a set of wave packets interacting with each other:

\[
\frac{\partial u_z}{\partial t} = \frac{c_I}{2} \sum_j A_j(z, \eta) f_{ij}(x) e^{i(\omega_j t - j k(z) z)} + \text{c.c.}
\]
The dynamics of nonlinear envelope waves is described by a set of partial differential equations for complex slowly varying amplitudes \( A_j(z,y,\eta) \) for the first (\( j=1 \)) and several (\( 1<j\leq5 \)) higher harmonics:

\[
i\omega_j \left( \sum_{m<j} P_{m,j} A_mA_{m-j} + 2 \sum_{m=0} P_{m+j} A_{m+j} \right) + \frac{\partial A_j}{\partial z} + \frac{1}{v_j} \frac{1}{\eta} \frac{\partial A_j}{\partial \eta} + ig_j \frac{\partial^2 A_j}{\partial \eta^2} + i\eta \frac{\partial^2 A_j}{\partial y^2} + i(k_j - jk_1)A_j + \Gamma_j A_j + i\sum_i N_{j,m} |A_m|^2 A_j
\]

(5)

where \( z \) is the coordinate along the wave propagation, \( v_j = (\partial k/\partial \omega)|_{\omega_j} \) are group velocities of the harmonics, \( g_j = -0.5 \cdot (\partial^2 k/\partial \omega^2)|_{\omega_j} \) are corresponding dispersion coefficients, \( \eta = t - z/v \) is the “running time”; \( \omega_j, k_j \) are carrier frequencies and wave numbers for harmonics; the parameter \( \Gamma \) describes the dissipation. The coefficients \( P_{m,j} \) are expressed through the quadratic nonlinear modules of the film and the substrate. The term with \( \partial^2 A/\partial y^2 \) presents an influence of diffraction in the interface plane; \( p_j = 1/(2kj) \). The cubic terms, leading to self-action \( |A_m|^2 A_j \) in equation (5), are due to the induced zero harmonic [3]. But also an influence of the cubic material and geometric nonlinearity on these terms can be essential, as discussed below. The equation describing the induced zero harmonic is obtained from the equation (1) \((\sigma_1)|_{\omega_0} = 0\) and it has the following form

\[
\rho_1 c_1^2 \frac{\partial u_1}{\partial x} |_{\omega=0} + C(div u)^2 |_{\omega=0} = 0, -h < x < 0;
\]

(6)

\[
\rho_2 s_1^2 \frac{\partial u_1}{\partial x} |_{\omega=0} + C(div u)^2 |_{\omega=0} = 0, 0 < x < -h
\]

The main part of the energy of envelope wave (wave packet) lies within the frequency interval \( \omega_0 - \Delta \omega < \omega < \omega_0 + \Delta \omega \) where \( \Delta \omega \sim 5 \cdot 10^{-7} \). The carrier frequency of the wave is chosen within the region of the essential wave dispersion, and namely the effective cubic nonlinear terms in equation (5) are dominating here.

Direct numerical simulations of equation (5) have been done for input envelope pulses at different carrier frequencies of the first harmonic and for different material combinations of the film and the substrate. The parameters of the system have been chosen as in the section 2 and the value of the nonlinear coefficient in both media is assumed as \( |C| = 2 \times 10^{12} \text{ g-cm}^{-2} \text{ s}^{-1} \). The results of simulations are tolerant to some changes of parameters of the system; they are qualitatively the same, when such a change is within 30-50%. The nonlinear dynamics of envelopes does not depend on the total number of higher harmonics, if such a number has been chosen as 4 - 5.

The results demonstrate a possibility of forming the temporal bright envelope soliton in such a system, when the difference of bulk longitudinal and shear velocities in the film and the substrate is essential: \( s/c_s \geq 1.5, s/c_t \geq 1.5 \).

The higher harmonics are generated due to the quadratic nonlinearity. When the wave dissipation parameter \( \Gamma \) is proportional to \( \omega \), the self-action dominates over the influence of higher harmonics, which leads to the soliton propagation in the region of envelope frequency \( \omega > \omega_m \) (figure 2). The simulation has been done here for a transversely wide pulse \((\partial \eta/\partial y = 0)\). The spatial-temporal dependencies of the square of the amplitude of the vertical component of the velocity at the free surface of the film \( x=0 \) for the first and second harmonics \( |V_{x,1,2}|^2 \) are presented; the carrier frequency of the first harmonics is \( \omega_1 = 2.9 \times 10^9 \text{ s}^{-1} \). The dissipation coefficient of the first harmonic is \( \Gamma_1 = 1 \text{ cm}^4 \).
Figure 2. Evolution of a transversely wide envelope pulse. Dynamics of $|V_x|^2$ at the free surface of the film: (a) for the input first harmonics and (b) for the first harmonics at $z = 0.6$ cm (where the temporal soliton is formed). The carrier frequency is $\omega_1 = 2.9 \times 10^9$ s$^{-1} > \omega_{\text{crit}}$ in the case of generation of higher harmonics in Si(Ge)O$_2$ glass – Si structure. The thickness of the film is $h = 2.5$ µm.

A possibility of soliton propagation can be explained qualitatively as follows. In the case of a great phase mismatch $(k_l - jk_1)$, equation (5) can be approximately reduced to the nonlinear Schrödinger equation for the single amplitude $A_1$:

$$\frac{\partial A_1}{\partial z} + ig_1 \frac{\partial^2 A_1}{\partial \eta^2} + i\eta \frac{\partial^2 A_1}{\partial \eta^2} + iR |A_1|^2 A_1 + \Gamma_1 A_1 = 0;$$

where

$$R = N_{1,1} - \frac{(2\omega P_{1,2})^2}{k_2 - 2k_1}; \quad A_j = \frac{\omega_j}{(k_j - jk_1) - i\Gamma_j} P_{k_j} A_j A_{j-1}.$$

If the inequality $g_j R > 0$ is satisfied, then the nonlinear Schrödinger equation allows the pulse (bright) envelope soliton propagation and a modulation instability also takes place. This fact is valid in the region $\omega > \omega_{\text{crit}}$, if the difference of elastic properties of materials of the film and the substrate is essential: $g_j > 0$, $R > 0$, where $N_{1,j} > 0$. The distance of forming envelope solitons is of about 0.5 cm. Note that the amplitude of the velocity of the free surface of the film at the second harmonic may be smaller or greater than at the first one, but the energy flux is two orders larger for the first harmonic. The last statement can be explained by a difference of transverse profiles of the first and second harmonics. Our simulations have demonstrated that it is impossible to exclude the amplitudes of higher harmonics from equation (5) in a general case, even when the wave dispersion is essential. This fact differs from the results of the paper [12], where only the equation for the first harmonic was considered. To get correct results, it is necessary in a general case to consider the equations for higher harmonics, too, because the difference of phase velocities of the first and the second harmonics is not large, as well seen from figure 1c.

The influence of transverse inhomogeneity of the input pulse can lead to essential changes of the nonlinear dynamics. In this system, an inequality $p_j R > 0$ takes place; thus, nonlinear spatial focusing also takes place. The simulation of two-dimensional dynamics shows that it is possible to observe the formation of the temporal soliton, when the initial width of the pulse is $\geq 0.3$ cm. Otherwise, the joint action of wave dispersion and diffraction leads to the collapse phenomenon. In figure 3, the results of numerical simulations are presented for the transversely localized input pulse. The profiles of the input pulse and of the pulse under a maximal compression are given in figures 3a and 3b. Analogous dependencies for the second harmonics are presented in figures 3c and 3d.
Figure 3. A distribution of $|V_2|^2$ at the free surface of the film of the first harmonics in the case of transversely inhomogeneous pulse in Si(Ge)O$_2$ glass – Si structure: a) input pulse ($z=0$), b) the pulse under strong compression ($z = 0.6$ cm); c) the second harmonic at $z = 0.1$ cm; d) the second harmonic at $z = 0.6$ cm. The thickness of the film is $h = 2.5$ µm.

If the carrier frequency is $\omega_1 > \omega_{cr}$, the bright solitons can propagate here only in the case when the generation of higher harmonics is reduced, i.e., when the linear wave dispersion is essential. Numerical simulations have been done within a framework of equation (5). It has been confirmed in the case of a SiO$_2$ film – Si substrate system. In the last case, the parameters are: $c_t = 3.7 \times 10^5$ cm/s, $c_l = 5.8 \times 10^5$ cm/s, $\rho_1 = 2.2$ g/cm$^3$ (SiO$_2$); $s_t = 5.0 \times 10^5$ cm/s, $s_l = 8.4 \times 10^5$ cm/s, $\rho_2 = 2.33$ g/cm$^3$ (Si). The thickness of the film is $h = 2.5$ µm. Note that the spatial solitons and the spatial-temporal collapse are possible there only under additional suppression of the generation of higher harmonics.

In the simulations pointed above, only the effective cubic nonlinearity due to the induced second harmonic has been taken into account. But in a general case the nonlinear coefficients $N_{ij}$ in equation (5) change due to essential nonlinear cubic terms, added to the expansion (2). When such an influence increases the nonlinear module $N_{11}$, the results are qualitatively the same. But, there exists a problem for emerging wave collapse, when the value $N_{11}$ decreases. Numerical simulations with two times decreased value of $N_{11}$ have demonstrated that the generation of higher harmonics prevents the collapsing of the input pulse.

To obtain the collapsing of the pulse at the first harmonic, it is possible to use some initial focusing of the input pulse. Such an initial focusing is equivalent to the phase modulation of the input envelope.
pulse with respect to the coordinate $y$. In the simulations, it has been assumed that the initial pulse is excited by the circular antenna, see Fig. 4. Therefore, at $z = 0$, the slowly varying amplitude of the pulse at the first harmonic gets the multiplier $\exp(-ik_1|CC'|)$. The distance $|CC'|$ can be expressed approximately as $|CC'| = (l/2)(y_m/2)^2 - (y - y_m/2)^2)/R$. Here $R$ is the distance from the antenna to the focus point, $y_m$ is the transverse width of the film ($y_m << R$).

![Figure 4](image)

**Figure 4.** The initial focusing of the envelope pulse. $OABCD$ is the circular antenna; $OF = R >> y_m$ is the radius. $F$ is the focus point. $OD = y_m$ is the transverse width of the film.

Typical results of simulations are shown in figure 5. The distance to focus can be chosen as 0.5 - 2 cm. A possibility to obtain simultaneous spatial and temporal compression can be explained by a different evolution of transverse widths of the envelope pulses at the first and higher harmonics, because of the difference of longitudinal wave numbers at the carrier frequencies. The growth of the maximal intensity of the first harmonic is twice bigger here, in a comparison with the purely linear case of focusing the pulse.
Figure 5. A distribution of the square of the velocity $|V_x|^2$ at the free surface of the film of the input pulse (a); the pulse at the first harmonic under the strongest compression, $z = 0.4$ cm (b); the pulse in second harmonics ($z = 0.4$ cm), in the case of transverse inhomogeneous pulse in Si(Ge)O$_2$ glass – Si structure. Initial focusing of input pulse is used in the case of the reduced nonlinear coefficient $N_{1,1}$. The distance to the focus is 0.6 cm. The thickness of the film is $h = 2.5$ µm.

4. Conclusions
Simulations of the envelope nonlinear wave dynamics in the layered system, which includes a film of a thickness of 1 – 5 µm and half-infinite substrate, have demonstrated a possibility of the formation of ultra-high frequency envelope solitons and collapsing pulses. The frequency range, where envelope soliton propagation is possible, lies higher than a certain critical frequency, determined by the thickness of the film and the acoustic velocities in the film and the substrate. The generation of higher harmonics affects the soliton dynamics. The propagation of envelope solitons and the wave collapse are possible only when the difference between bulk acoustic velocities in the film and the substrate is essential. In the case when the cubic nonlinear elastic modules in the equations of motion decrease the value of effective cubic nonlinear coefficient, there exists a possibility to use the initial focusing of the input pulse to observe simultaneously both the temporal and spatial compression.

5. Acknowledgement
The authors acknowledge the support of CONACyT, Mexico.
References
[1] Naugolnykh K and Ostrovsky L 1998 *Nonlinear Wave Processes in Acoustics* (Cambridge, New York: Cambridge University Press)
[2] Nelson D F 1979 *Electric, Optic, and Acoustic Interactions in Solids* (New York: Wiley)
[3] Mayer A P 1995 *Phys. Reports* 256 237
[4] Parker F and Talbot F M 1985 *J. Elasticity* 15 389
[5] Hamilton MF, Il’inskii Yu A, and Zabolotskaya E A 1999 *J Acoust Soc Am* 105 639
[6] Porubov A V and Samsonov A M 1995 *Int J Non-Linear Mechanics* 30 861
[7] Fu Y B and Hill S L B 2001 *Wave Motion* 34 109
[8] Eckl C, Schloemann J, Mayer A P, Kovalev A S, and Maugin G A 2001 *Wave Motion* 34 35
[9] Eckl C, Kovalev A S, and Mayer A P 1998 *Phys Rev Lett* 81 983
[10] Lomonosov A M and Hess P 2002 *Phys Rev Lett* 89 095501
[11] Kovalev A S, A.P. Mayer A P, Eckl C, and Maugin G A 2002 *Phys Rev E* 66 036615
[12] Mayer A P and Kovalev A S 2003 *Phys Rev E* 67, 066603
[13] Lomonosov A M, Hess P, and Mayer A P 2002 *Phys Rev Lett*. 88 076104
[14] Eckl C, Kovalev A S, Mayer A P, Lomonosov A M, and Hess P 2004 *Phys Rev E* 70 044604
[15] Mingxi Deng 2004 *J Phys D: Appl Phys* 37 1385
[16] Maugin G A 1999 *Nonlinear Waves in Elastic Crystals* (New York: Oxford Univ. Press)
[17] Auld B A 1973 *Acoustic Fields and Waves in Solids* (New Uork: Wiley) vol 2