An optimal experiment planning in the problem of longevity evaluation at crack propagation stage

I V Gadolina¹, A N Romanov¹, A A Bautin² and L Papic³

¹Mechanical Engineering Research Institute of the Russian Academy of Sciences (IMASH RAN), Moscow, Russia
²TsAGI, Zhukovsky, Russia
³DQM Research Center, Chachak, Serbia

Abstract. A large number of factors influencing the fatigue crack rate are traditionally studied separately. In this case, their interactions are not taken into account, and the experiment does not possess the property of optimality. The article describes the authors' vision of the problem, how it would be possible to approach the solution of the research problem from the point of view of the theory of optimal experiment planning. Initially, it is proposed to consider three basic factors, namely: the influence of loading asymmetry, loading frequency and sequence of overloads. A full factorial experiment with varying factors at extreme levels is considered. The formalization of the factors describing the sequence of overloads is proposed.

The second stage of fatigue - namely, the stage of fatigue crack propagation, traditionally attracts a lot of attention of scientists and engineers. It is primarily due to the increasingly widespread use of the concept of condition-based operation, i.e. when the object continues to operate despite the presence of a macrocrack. The task of assessing the survivability is also relevant when extending the guaranteed resource of the other mechanical engineering products.

Method
The problem of estimating the rate of crack growth is far from being trivial. There are many models for different materials and loading conditions [1-4]. The type of structural element is also important. An additional contribution to the variation of experimental results (as well as calculation results) is due to the random character of loading. It is known that overloads can slow down the rate of fatigue crack propagation. The scientists associate this phenomenon with the formation of a plastic zone, which leads to crack growth time lapse. Many calculation methods are based on the assessment of this influence. In [5], based on numerical simulations using the special method of random sequences imitation [6] and the estimate of the crack growth rate according to Willenborg [4], the effect of the sequence of overload peaks was investigated.

There is a wealth of experimental data on fatigue crack propagation. Perhaps the primary task should be to generalize them. On the other hand, it is interesting to outline further directions of research. For a systematic approach, it is advisable to formulate the problem of planning an optimal experiment to study the phenomenon of crack growth rate in metals. In this case, it is important to...
determine the choice of response--a function that optimally characterizes the stage of crack development.

Following the theory of optimal experiment planning [7], it is recommended to consider the factors possibly influencing the response. If three factors are examined, they are normalized, and the experiment space looks like a unit cube.

![Figure 1. Single cube in factor space.](image)

Having carried out a full factorial experiment (figure 1), one can get an idea of the main trends of influence and outline combinations of factors for further research. Long-term efforts of scientists have led to an understanding of WHICH factors can be chosen as determining factors in the task of crack propagation: the loading asymmetry (X\(_1\)), the loading frequency (X\(_2\)) and sequence of peaks (X\(_3\)). The experimental data for factors X\(_1\) and X\(_2\) and the assignment of the boundaries of their variation would not present serious difficulties. Concerning the factor X\(_3\), there is no formalization so far. Moreover, it would be desirable to characterize the factor as a quantitative one, and not just "yes" - "no". The latter definition is only justified to encode the quality factors.

**Characterization of the sequence of loading peaks**

In [5], in a numerical experiment, the duration of the propagation of a fatigue crack in an aluminium plate was estimated for random processes simulated based on TWIST using the original method of modelling the sequence [6]. The sequence of peaks in the processes varied. Other parameters remained unchanged. Since the evaluation by the Willenborg [4] method takes into account the sequence of peaks (which is essential in reality), the calculated durability turned out to be different. In figure 2, one can see that the difference in durability appeared to be significant (2.5 times), which was ensured by the proper selection of loading processes. On the base of results, presented in figure 2, two marginal processes were chosen: the “Light” and the “Severe” ones, which were further analyzed.

![Figure 2. Calculated longevities (crack lengths) for varied processes.](image)
Figure 3 shows the maxima of the light and severe processes.

![Figure 3](image1.png)

Figure 3. Maximums of two contrast processes.

The cumulative numbers of overloads over 17 kg·cm$^{-2}$ for the shown in figure 3 “Light” and “Severe” processes are shown in figure 4.

![Figure 4](image2.png)

Figure 4. Cumulative numbers of overloads for two contrast processes.

It can be seen that the less damaging process (light) has more overloads, and they are distributed with an offset to the beginning of the realization.

When planning an experiment to study the effect of overloads, it is supposed to describe the distribution of overloads along the realization length using a family of beta distributions of a random variable $X \sim B (\alpha, \beta)$ [8], for which it is true:
\[ f(x) = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1 - x)^{\beta-1} \]

Where: \( \alpha, \beta > 0 \) are arbitrary and fixed parameters; \( B(\alpha, \beta) = \int_0^1 x^{\alpha-1} (1 - x)^{\beta-1} \, dx \) — beta-function.

For the problem of the distribution of overloads along the length of the realization, the parameters were taken as \( \alpha = 1 \) and \( \beta \sim \text{var.} \).

Figure 5 shows two distributions from this family, which can be used to quantitatively describe the distribution of overloads along the length of the realization in the design of an experiment. The distribution with parameters \( \alpha = 1 \) and \( \beta = 5 \) describes the situation when the overloads are shifted closer to the beginning of the realization, thus providing a smaller damaging effect (closer to the “light” family, figure 3.). The distribution with parameters \( \alpha = 1 \) and \( \beta = 0.2 \) describes the opposite situation— the overloads are moved to end of the realization, and the damage is bigger (closer to the “severe” family, figure 3). To describe the processes, the realization length \( x \) and the number of overloads were normalized to unit.

Figure 5. Two distribution from of beta family.

As one can see from the figure 5, the chosen from beta-family distributions, namely, with the parameters \( \alpha = 1 \) and \( \beta \sim 0.2 \ldots 5 \) represent well the possible shapes of overloads distribution during the testing.

Conclusions

In the problem of studying the patterns of fatigue crack development, it is desirable to use the theory of optimal experiment planning. Factors selected: asymmetry, frequency, the sequence of peaks. Based on a numerical experiment carried out earlier, proposals were made to formalize the responsible for the sequence of peaks factor.

In the problem of sequence impact on longevity, only one model (namely Willenborg’s) was considered. Many other ones exist, and some of them are more advanced and up-to-date (see, for example, [1]). They were not mentioned in this paper.

The question of choosing the testing protocol also matters. Apart from the factors mentioned, one can name the climatic conditions of the testing, the shape of the cycle, the crack length registration.
procedure, etc. To reasonably decrease the result scatter, the number of tests on one position specimens (in each from 8 cube tops, figure 1) should be taken at least three.

The question remains, what exactly should be taken as a response in planning an experiment: the Paris exponent, the speed on a stable section, or something else. The authors hope to discuss this issue with the conference participants.

Mostly the calculations and modelling, as well as the graphic, were performed in [9].

References

[1] Sunder R 2012 J. of ASTM Int. 9(1) pp 1-32
[2] Romanov A N, Nesterenko G I and Filimonova N I 2018 Damage accumulation under variable loading of cyclically hardening material at the stages of formation and development of cracks J. of Machinery Manufacture and Reliability 47(5) pp 414-19
[3] Lebedinskii S G 2018 Design modeling of propagation of the fatigue cracks in the steel of molded parts of the railway structures J. of Machinery Manufacture and Reliability 47(1) pp 62–66
[4] Gadolina I V et al 2020 Studying the crack growth rate variability by applying the Willenborg’s model to the Markov’s simulated trials Advances in Intelligent Systems, Computer Science and Digital Economics CSDEIS 2019 1127
[5] Makhutov N A et al 2020 Modeling a random sequence of extremes of loads for fatigue tests under irregular loading Vestnik mashinostroenija 5 pp 13-19
[6] Willenborg J, Engle R H and Wood H A 1971 AFFDL-TM-71-1 FBR WPAFB OH
[7] Adler Ju P 2014 Introduction into the experiment planning Tutorial (Moscow: MISIS)
[8] Mosteller F and J Tukey 1977 Data Analysis and Regression: A Second Course in Statistics (Addison-Wesley Pub Co)
[9] R Core Team 2020 A language and environment for statistical computing R Foundation for Statistical Computing Vienna Austria