Negative Overlap and Lack of the Zero-bias Peak as Majorana Fermions Fingerprints

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We investigate theoretically a setup composed by STM and AFM tips hybridized with a pair of superconducting adatoms hosting Majorana quasiparticles (MQPs). We show that within the adatom beneath the AFM tip an effective topological Kitaev wire emerges, in which entangled MQPs exhibit an overlap emulated by the energy level of such an adatom. For the binding energy $\Delta$ of the Cooper pair delocalized into the adatoms coincident with the tunneling amplitude $t$ between them, namely $\Delta = t$, we find that the adatom coupled to the STM tips hybridizes only with the nearest MQP. This yields the complete suppression of the zero-bias peak typically due to an isolated MQP, in addition to a novel feature: the transmittance profile is revealed as an invariant quantity under the symmetric swap of the energy level position with respect to the zero mode of the isolated MQP. Notably, the overlap concept can be extrapolated to the negative domain for the adatom level below such a mode. These fingerprints thereby are robust novelties within the topological phase that make explicit the rising of bounded MQPs. As for quantum computing is desirable entangled unities for qubits platforms, our results point out that transport measurements can be helpful in the quest for pairs of MQPs.

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I. INTRODUCTION

The physicist Ettore Majorana proposed in the field of high-energy Physics the existence of peculiar fermions that constitute their own antiparticles. In the context of condensed matter Physics, these fermions are Majorana quasiparticles (MQPs) [1, 2]. From the quantum computing perspective, two far apart MQPs can compose a non-local regular fermion acting as a protected qubit, which is indeed decoupled from the host environment and free of the decoherence effect. As a result, the quest for setups supporting MQPs has attracted broad interest from the communities of researchers in the fields of quantum information and transport [3–5], since the qubit made by the entangled MQPs exhibits the aforementioned robustness when the system is driven to a phase known as topological. Noteworthy, the Kitaev wire within such a phase [6] is considered the most promising candidate to this end as the aftermath of the emerging $p$-wave and spinless superconductivity. Indeed, in Kitaev’s setup, MQPs appear as zero-energy modes attached to the edges of the wire.

Experimentally, $p$-wave superconductivity is feasible due to the proximity effect by the employment of an $s$-wave superconductor close to a semiconducting nanowire characterized by a spin-orbit interaction under an external magnetic field [7, 8]. Particularly in the case of transport through a quantum dot (QD) coupled to a MQP [10–24], a zero-bias peak (ZBP) [13, 14] in the conductance is expected to be observed with the amplitude of $G = 0.5G_0$, where $G_0 = e^2/h$ is the quantum of conductance. It is worth mentioning that the ZBP has been detected in conductance measurements through a nanowire of indium antimonide linked to gold and niobium titanium nitride electrodes [21]. Analogously, a ZBP has also been verified in the superconducting system of aluminium next to a nanowire of indium arsenide [22]. However, the ZBP signature may also have another physical origin, for instance, the Kondo effect [25–29] thus turning the experimental ZBP detection inconclusive within a MQP perspective. Moreover, recently an alternative way for the achievement of the topological Kitaev wire has been the employment of magnetic chains on top of superconductors [30–33]. Particularly in Ref. [33], the ZBP observed exhibits a subtle amplitude of the order $10^{-4}G_0$, which is a signal extremely weak due to thermal broadening. Thus in the current context, novel approaches in the pursuit of MQPs become necessary.

Considering the fact that quantum computing requires entangled unities for qubits platforms, in this work we propose a new route for detection of MQPs by looking beyond the ZBP signature, which can be considered as a fingerprint of an isolated MQP and intrinsically hard to be detected. To that end, we consider the setup outlined in Fig. 1 where a conventional superconductor hosts two adatoms coupled to STM (Scanning Tunneling Microscope) and AFM (Atomic Force Microscope) tips. In regard to the possibility of experimental realization of the setup shown in Fig. 1 we stress that multitip STM experiments can be possible, see e.g. Refs. [34–36]. Hence, we trust that in the near future such a setup can be implemented.

In the frame of the setup here proposed, a device
II. THE MODEL

A. System Hamiltonian

In order to mimic the system outlined in Fig. 1(a), we employ the Hamiltonian proposed by Leijnse and Flensberg [5], taking two STM tips into account,

\[
\mathcal{H} = \sum_{\alpha k} \tilde{\epsilon}_{\alpha k} c_{\alpha k}^{\dagger} c_{\alpha k} + \sum_{j} \varepsilon_j d_j^{\dagger} d_j + V \sum_{\alpha k} (c_{\alpha k}^{\dagger} d_1 + \text{H.c.}) + (V_{12} \sum_{k \Omega} c_{1k}^{\dagger} c_{2 \Omega} + \Delta d_1^{\dagger} d_2 + \Delta d_1 d_2^{\dagger} + \text{H.c.}),
\]

where the electrons in the STM tip $\alpha = 1, 2$ (STM tip 1 and 2, respectively) are described by the operator $c_{\alpha k}^{\dagger}$ ($c_{\alpha k}$) for the creation (annihilation) of an electron in a quantum state labeled by the wave number $k$ and energy $\tilde{\epsilon}_{\alpha k} = \varepsilon_k - \mu_{\alpha}$, with $\mu_{\alpha}$ as the chemical potential. Notice that the spin degree of freedom within the STM tips is neglected since we consider these tips completely spin polarized. Here we adopt the gauge $\mu_2 = -\Delta \mu$ and $\mu_1 = -\Delta \mu$, with $\mu_2 - \mu_1 = 2\Delta \mu = e\phi$ as the bias between the tips, being $e > 0$ the electron charge and $\phi$ the bias-voltage. Consequently, the transmittance through the setup is a function of the Fermi energy $\varepsilon = \mu_1 = \mu_2$ of the STM tips, where the point $\varepsilon = 0$ corresponds to the MQP zero mode. For the adatoms, $d_j^{\dagger}$ ($d_j$) creates (annihilates) an electron in the state $\varepsilon_j$, with $j = 1, 2$. $V$ is the hybridization between the adatoms and the tips.

Here we show that the adatom under the AFM tip (see Fig. 1) encloses two entangled MQPs as those found in the topological Kitaev wire [6], but connected via an effective overlap emulated by the own energy level of such an adatom. Particularly when the AFM tip tunes this energy level below the MQP zero mode, the overlap concept defined in the negative domain emerges. Thus the AFM tip acts as a metallic gate attached to a QD in order to tune their levels. Our findings are in contrast with those found in the frame of the original Kitaev Hamiltonian, where only positive values are allowed in agreement with the standard overlap concept. This novelty thereby opens the possibility of recognizing MQPs in the absence of the ZBP hallmark, in particular when the AFM tip swaps symmetrically the overlap around the MQP zero mode: it reveals a universality feature of the transmittance. More specifically, we find then distinct situations in which the transmittance shares the same profile as a function of the Fermi level for the STM tips, in particular when the binding energy of the Cooper pair splitted into the adatoms is in resonance with the tunneling term between them. In this case, we show that when the MQP 1 couples solely to the adatom 1, the aforementioned universality is revealed by the entangled MQPs.

This paper is organized as follows: in Sec. III we develop the theoretical model for the system sketched in Fig. 1 by deriving the expression for the transmittance through such a device and the system Green’s functions. The results are present in Sec. III and in Sec. IV we summarize our concluding remarks.
$V_{12}$ is the tip 1-tip 2 coupling, $t$ and $\Delta$ respectively accounts for the hopping term between the adatoms and the binding energy of a delocalized Cooper pair split through the Majorana basis by following the transformation $d_2 = \frac{1}{\sqrt{2}}(\Psi_1 + i\Psi_2)$ and $d_1 = \frac{1}{\sqrt{2}}(\Psi_1 - i\Psi_2)$, in which $\Psi_i^\dagger = \Psi_i$ ($i = 1, 2$) characterizes a MQP operator, thus yielding

$$\mathcal{H}_{\text{MQPs}} = \varepsilon_2 d_1^\dagger d_2 + (t + \Delta)(d_1^\dagger d_2 + d_1 d_2^\dagger) + \text{H.c.}$$

$$= i\varepsilon_2 \Psi_1 \Psi_2 + \frac{(t + \Delta)}{\sqrt{2}}(d_1 - d_2^\dagger)\Psi_1$$

$$+ i\frac{(\Delta - t)}{\sqrt{2}}(d_1 + d_2^\dagger)\Psi_2 + \varepsilon_2 \frac{\Psi_2}{2},$$

where the first term in the second line stands for the Kitaev wire within the topological phase (see Figs. 1(c), 1(d), 1(e) and Ref. 6). Here we emphasize that $\varepsilon_2$ mimics the overlap between the MOPS 1 and 2 found in the nonlocal Kitaev Hamiltonian $i\varepsilon M \Psi_1 \Psi_2$ in which such a quantity is dictated by the coefficient $\varepsilon M \sim e^{-L/\xi}$, with $L$ as the distance between the MOPS at the edges of the wire and $\xi$ as the superconducting coherence length. Consequently, $\varepsilon M \geq 0$ as expected for a finite overlap. As here $\varepsilon M = \varepsilon_2$, the possibility of evaluating such a coefficient within a negative domain is entirely feasible, thus contrasting with the standard Kitaev model where the constraint $\varepsilon M \geq 0$ is fulfilled permanently. In this scenario, we extrapolate the conventional overlap concept by exploring the model regimes $t = \Delta$ and $t \neq \Delta$ combined with $\varepsilon_2 > 0$ and $\varepsilon_2 < 0$ for the situation in which a negative overlap is addressed. We stress that the tuning of the parameters $t$ and $\Delta$ can be performed experimentally just by changing the distance between the adatoms, while the AFM tip controls the level position of the adatom 2. Particularly for $t = \Delta$, we can verify in Eq. 2 that the adatom 1 decouples from the MQP 2 (here sketched by the most right half-sphere in Fig. 1(d)) thus allowing an exclusively connection with the MQP 1 (the most left half-sphere) as the Hamiltonian $H_{\text{MQPs}} = \varepsilon_2 (i\Psi_1 \Psi_2 + \frac{1}{2}) + \frac{1}{2}\Delta(d_1 - d_2^\dagger)\Psi_1$ points out. For $\varepsilon_2 \neq 0$, the transmittance becomes invariant under the symmetric change of $\varepsilon_2$ with respect to the MQP zero mode, in contrast with the case $t \neq \Delta$ where this universality is prevented. Thereafter, it gives rise to distinct transmittance profiles strongly dependent on the sign of $\varepsilon_2$ off the point $t = \Delta$.

It is worth noticing that the standard Majorana ZBP occurs for $t = \Delta$ and $\varepsilon_2 = 0$, thus implying that the conductance $G = 0.5G_0$ arises from the half-fermion given by the MQP 1 completely free of the MQP 2 within this regime (Fig. 1(e)). Thus by fixing $t = \Delta$ and tuning $\varepsilon_2$ symmetrically around the zero mode, we propose that the invariance of the entire lineshape of the transmittance as the novel strategy for detection of MOPS fingerprints, in particular for bounded MOPS in which the ZBP is lacking. As quantum computing requires entangled units for qubits platforms, transport measurements could be helpful in the pursuit for signatures of bounded MOPS instead. These aspects will be minutely covered in Sec. III.

B. Conductance

In what follows we derive the Landauer-Büttiker formula for the zero-bias conductance $G[39]$. Such a quantity is a function of the transmittance $T(\varepsilon)$ as follows:

$$G = \frac{e^2}{h} \int d\varepsilon \left( -\frac{\partial f_F}{\partial \varepsilon} \right) T(\varepsilon)$$

where $f_F$ stands for the Fermi-Dirac distribution.

We begin with the transformations $c_{2k} = \frac{1}{\sqrt{2}}(c_{ck} + c_{ok})$ and $\varepsilon_1 = \frac{1}{\sqrt{2}}(c_{ck} - c_{ok})$ on the Hamiltonian of Eq. 1, which starts to depend on the even and odd conduction operators $c_{ck}$ and $c_{ok}$, respectively. These definitions allow us to express Eq. 1 as $H = H_e + H_o + H_{\text{tun}}$, where

$$H_e = \sum_k \varepsilon_k c_{ck}^\dagger c_{ck} + \varepsilon_1 d_1^\dagger d_1 + \sqrt{2V} \sum_{jk} (c_{ck}^\dagger d_j + \text{H.c.})$$

$$+ V_{12} \sum_{kp} c_{ck}^\dagger c_{ck} + H_{\text{MQPs}}$$

represents the Hamiltonian part of the system coupled to the adatoms via an effective hybridization $\sqrt{2V}$, while

$$H_o = \sum_k \varepsilon_k c_{ck}^\dagger c_{ck} - V_{12} \sum_{kp} c_{ck}^\dagger c_{ck}$$

is the decoupled one. However, they are connected to each other by the tunneling Hamiltonian $H_{\text{tun}} = -\Delta \mu \sum_k (c_{ck}^\dagger c_{ok} + c_{ok}^\dagger c_{ck})$.

As in the zero-bias regime $\Delta \mu \rightarrow 0$, due to $\varphi \rightarrow 0$, $H_{\text{tun}}$ is a perturbative term and the linear response theory ensures that

$$T(\varepsilon) = \frac{(2\pi V_{12})^2}{\hbar} \rho_e(\varepsilon) \rho_\omega(\varepsilon),$$

where $\rho_\omega(\varepsilon) = -\frac{i}{\hbar} \text{Im}(G_{\Psi_\omega, \Psi_e})$ is the local density of states (LDOS) for the Hamiltonian of Eq. 4 and

$$G_{\Psi_\omega, \Psi_e} = -\frac{i}{\hbar} \theta(\tau) \text{Tr} \{\rho_\omega \{\Psi_e(\tau), \Psi_e^\dagger(0)]_+\}$$

gives the retarded Green’s function in the time domain $\tau$, where $\theta(\tau)$ is the Heaviside step function, $\rho_\omega$ is the density-matrix for Eq. 4, $\Psi_e = f_e + (\pi \Gamma_0)\frac{1}{2} q d_1$ is a field operator, with $f_e = \sum_p c_{cp}$, the Anderson parameter $\Gamma = 2\pi V^2 \rho_0$, with $\rho_0$ as the density of states for the STM tips and $q = (\pi \rho_0 \Gamma)^{-1/2} \left( \frac{\sqrt{2V}}{2V_{12}} \right)$.
To calculate Eq. (7) in the energy domain $\varepsilon$, we should employ the equation-of-motion (EOM) method summarized as follows

\[
(\varepsilon + i0^+) \tilde{G}_{AB} = [A, B^\dagger]_+ + \tilde{G}_{[A, H_e]}B
\]

for the retarded Green’s function $\tilde{G}_{AB}$, with $A$ and $B$ as fermionic operators belonging to the Hamiltonian $H_e$. By considering $A = B = \Psi_e$ and $H_e = H_e$, we find

\[
\tilde{G}_{\Psi_e^\dagger, \Psi_e} = \tilde{G}_{f_e, f_e} + (\pi \rho_0 \Gamma q)^2 \tilde{G}_{d_1d_1} + 2(\pi \rho_0 \Gamma)^{1/2} q \tilde{G}_{d_1f_e}.
\]

From Eqs. (4), (9) with $A = B = f_e$ and (10), we obtain

\[
\tilde{G}_{f_e, f_e} = \frac{\pi \rho_0 (q-i)}{1 - \sqrt{x(q-i)}} \Gamma \tilde{G}_{d_1d_1},
\]

and the mixed Green’s function

\[
\tilde{G}_{d_1f_e} = \frac{\pi \rho_0 (q-i)}{1 - \sqrt{x(q-i)}} \tilde{G}_{d_1d_1},
\]

determined from Eq. (8) by considering $A = d_1$, $B = f_e$ and $H_i = H_e$, with the parameter $x = (\pi \rho_0 V_{12})^2$ and $\tilde{q} = \frac{1}{\sqrt{2}} \sum_k \varepsilon_k \zeta_k$. Here we assume the wide band limit denoted by $\tilde{q} \to 0$.

Additionally, for the Hamiltonian of Eq. (5) we have the LDOS $\tilde{\rho}_0 (\varepsilon) = -\frac{1}{\pi} \text{Im} (\tilde{G}_{f_e, f_e})$, with

\[
\tilde{G}_{f_e, f_e} = -\frac{i}{\hbar} (\tau) \text{Tr} \{ g_o f_o (\tau), f_o^\dagger (0) \} +
\]

and $f_o = \sum_q c_o \tilde{q}$. We notice that $\tilde{G}_{f_e, f_e}$ is decoupled from the adatoms. Thereby, from Eqs. (5) and (12), we take $A = B = f_o$ and $H_i = H_o$ in Eq. (8) and we obtain

\[
\tilde{G}_{f_o, f_o} = \frac{\pi \rho_0 (q-i)}{1 + \sqrt{x(q-i)}}.
\]

Thus the substitution of Eqs. (9), (11), and (13) in Eq. (6), leads to

\[
\frac{\tau (\varepsilon)}{\tau_b} = 1 + (1 - q_o^2) \Gamma \text{Im} (\tilde{G}_{d_1d_1}) + 2q_b \Gamma \text{Re} (\tilde{G}_{d_1d_1})
\]

where $\tau_b = \frac{4\pi}{(1+\varepsilon)^2}$ represents the background transmittance, $q_b = \sqrt{\frac{\tau_b}{\tau_e}} = \frac{(1-\varepsilon)}{2\sqrt{\varepsilon}}$ as the Fano parameter [37, 38, 40] and $\Gamma = \frac{\tau_f}{1+\varepsilon}$ is an effective adatom 1-tip coupling.

C. System Green’s functions

By applying the EOM on

\[
\tilde{G}_{d_1d_1} = -\frac{i}{\hbar} (\tau) \text{Tr} \{ g_o [d_1 (\tau), d_1^\dagger (0)] \} +
\]

and changing to the energy domain $\varepsilon$, we obtain the following relation

\[
(\varepsilon - \varepsilon_j - \Sigma) \tilde{G}_{d_1d_1} = 1 - t \tilde{G}_{d_1d_1} - \Delta \tilde{G}_{d_1d_1}
\]

expressed in terms of the self-energy $\Sigma = -(\sqrt{x} + i) \tilde{\Gamma}$ and Green’s functions $\tilde{G}_{d_1d_1}$ and $\tilde{G}_{d_1d_1}$. According to the EOM approach we find

\[
\tilde{G}_{d_1d_1} = -\frac{t \tilde{G}_{d_1d_1}}{(\varepsilon - \varepsilon_2 + i0^+)} + \frac{\Delta \tilde{G}_{d_1d_1}}{(\varepsilon - \varepsilon_2 + i0^+)}
\]

\[
\tilde{G}_{d_1d_1} = -\frac{\Delta \tilde{G}_{d_1d_1}}{(\varepsilon + \varepsilon_2 + i0^+)} + \frac{\tau \tilde{G}_{d_1d_1}}{(\varepsilon + \varepsilon_2 + i0^+)}
\]

and

\[
\tilde{G}_{d_1d_1} = -2t \Delta \tilde{K} \tilde{G}_{d_1d_1},
\]

in which $\tilde{K} = \frac{K}{\varepsilon + \varepsilon_1 + \varepsilon - K}$, with $K = \frac{(\varepsilon + i0^+)}{\sqrt{x^2 + 2\varepsilon_10^+ - (\varepsilon^2 + \varepsilon_2^2 + 2\varepsilon_20^+ - 2\varepsilon_2^2 + \varepsilon_2^2 + 4\varepsilon_20^+)}$, $\Sigma$ as the complex conjugate of $\Sigma$ and $K = \frac{K}{\sqrt{x^2 + 2\varepsilon_10^+ - (\varepsilon^2 + \varepsilon_2^2 + 2\varepsilon_20^+ - 2\varepsilon_2^2 + \varepsilon_2^2 + 4\varepsilon_20^+)}$.

Thus substituting Eqs. (17), (18) and (19) into Eq. (16) the Green’s function of the adatom 1 becomes

\[
\tilde{G}_{d_1d_1} = \frac{1}{\varepsilon - \varepsilon_1 - \Sigma - \Sigma_{\text{MQPs}}},
\]

where

\[
\Sigma_{\text{MQPs}} = K + (2t \Delta)^2 K \tilde{K}
\]

accounts for the self-energy due to the MQPs connected to the adatom 1. Particularly for $t = \Delta = \frac{\sqrt{2}}{2}$, we highlight that the expressions for $K$ and $\Sigma_{\text{MQPs}}$ found in Ref. [13] are recovered.

It is worth mentioning that the Green’s function for the adatom 1 derived here is exact as expected for the quadratic Hamiltonian of Eq. (1). From the experimental perspective, this feature can be reliable by considering the adatoms separated by a certain distance where we can safely disregard the intersites Coulomb repulsion.

III. RESULTS AND DISCUSSION

Below we investigate the features of the system Green’s functions by employing the expression for the transmittance (Eq. (14)). According to Eq. (3), this transmittance can be obtained experimentally via the conductance $G$ in units of $G_0 = e^2/h$ for temperatures $T \to 0$. Additionally, we employ values for the Fermi energy $\varepsilon$, $\varepsilon_j$, $t$, and $\Delta$ in units of the Anderson parameter $\Gamma$. 
Particularly in the topological phase $t = \Delta$, the standard zero-bias peak is found. (b) For $t < \Delta$ the transmittance shares the same lineshape of the symmetric situations above ($\varepsilon_2 = 6\Gamma$) and below ($\varepsilon_2 = -6\Gamma$) the MQP zero mode $\varepsilon = 0$, respectively for positive and negative overlaps. The transmittance then becomes an invariant quantity under these conditions. Panels (c) and (d) reveal distinct profiles when the system is driven away ($t \neq \Delta$) from the topological phase.

In Fig. 2 we consider the Fano regime $x = 0$ ($q_b \to \infty$) for the transmittance $T$ of Eq. (14) as a function of the Fermi energy $\varepsilon$. This situation corresponds to the case where the electron tunneling occurs exclusively through such an adatom, due to the strong coupling between it and the STM tips. As predicted by the standard Fano’s theory [37], the transmittance should exhibit a peak around each localized state in the adatom probed by the tips: see the green line shape of panel (a) for the adatom 1 here assumed to be decoupled from the adatom 2 for a sake of simplicity, which leads to the resonance centered at $\varepsilon = \varepsilon_1 = -5$ with maximum amplitude $T = 1$. By keeping this level at such a value and employing $t = \Delta = 4$ combined with $\varepsilon_2 = 0$, it is possible to dissociate the MQPs in the adatom 2 in order to allow the emergence of the ZBP given by $T = 1/2$ expressly due to the leaking of the Majorana zero mode within the MQP 1 into the adatom 1 (see Ref. [14] for a detailed discussion on the leaking effect and Fig. 1(e) with $\varepsilon_2 = 0$). Additionally, the most left resonance in the same curve corresponds to that at $\varepsilon = -5$ found in the green lineshape, in particular with renormalized peak position $\varepsilon \approx -10$ as the aftermath of the connection $\sqrt{2}\Delta$ with the adatom 2, and with higher amplitude ($T > 1/2$) in respect to that for the ZBP ($T = 1/2$). Notice that a third peak in the vicinity of $\varepsilon \approx +10$ is found characterized by $T < 1/2$. Thus in the presence of finite couplings $t$, the original peak at $\varepsilon = \varepsilon_1 = -5$ in the green curve of panel (a) with $T = 1$ is splitted into those at $\varepsilon \approx -10$ and $\varepsilon \approx +10$ both with $T < 1$ as the red lineshape points out. In which concerns the curves for $t < \Delta$ ($t = 2$ and $\Delta = 4$) and $t > \Delta$ ($t = 4$ and $\Delta = 2$), the transmittance is revealed as independent on the strengths $t$ and $\Delta$. Such a behavior attests the situation in which entangled MQPs are absent in the system. Panel (b) of Fig. 2 depicts the situation in which the system is still within the regime $t = \Delta$, but with $\varepsilon_2 \neq 0$. We highlight that such a panel contains the most relevant finding of this work: a universal behavior in the transmittance profile is observed when symmetric values for $\varepsilon_2$ are accounted. The single MQP fingerprint as the ZBP is absent, being replaced by a novel structure of four resonances. This regime is characterized by entangled MQPs (see Fig. 1(d)) which result in the suppression of the ZBP and the splitting of the resonances at $\varepsilon \approx -10$ and $\varepsilon \approx +10$ observed in the red curve of panel (a). For instance, the aforementioned universality is verified providing two identical curves for both values $\varepsilon_2 = 6\Gamma$ (positive overlap) and $\varepsilon_2 = -6\Gamma$ (negative overlap) due to the self-energy $\Sigma_{MQP_2}$ of Eq. (21) for the MQPs, which is dependent on the amplitude $K_\pm$.

Notice that $K_\pm = \frac{2\Gamma(\varepsilon - \varepsilon_2)}{\varepsilon_2^2 + (2\Delta - 6\Gamma)^2}$ within this situation thus implying in $K_\pm(\varepsilon_2) = K_\pm(-\varepsilon_2)$ as well as $\Sigma_{MQP_2}(\varepsilon_2) = \Sigma_{MQP_2}(-\varepsilon_2)$, which then ensure the invariance of the transmittance profile at the point $t = \Delta$. For regions where the constraint $t \neq \Delta$ is fulfilled, we have $K_\pm \propto \pm \varepsilon_2(\varepsilon^2 - \Delta^2)$ thus allowing a strong dependence on the sign of $\varepsilon_2$. In panels (c) and (d) of the same figure, the transmittance respectively for $t < \Delta$ ($t = 2$ and $\Delta = 4$) and $t > \Delta$ ($t = 4$ and $\Delta = 2$) exhibit distinct behaviors as expected when we adopt symmetric values $\varepsilon_2 = 6\Gamma$ and $\varepsilon_2 = -6\Gamma$. Therefore in both limits $t > \Delta$ and $t < \Delta$, the influence of the negative overlap $\varepsilon_2 < 0$ on the transmittance is made explicit once $\Sigma_{MQP_2}(\varepsilon_2) \neq \Sigma_{MQP_2}(-\varepsilon_2)$.

Fig. 3 holds within the Fano limit $x = 1$ ($q_b = 0$) where the electron tunneling between the STM tips prevails, thus resulting in Fano antiresonances instead of peaks in the transmittance profiles as a function of the Fermi energy. Panel (a) first displays the case in which the adatom 1 is free of the adatom 2 represented by the green curve characterized by a dip at $\varepsilon = \varepsilon_1 = -5$. By using finite values for $t$ and $\Delta$ combined with $\varepsilon_2 = 0$, we can ob-

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**Figure 2.** (Color online) Transmittance as a function of the Fermi energy of the STM tips within the Fano regime $x = 0$ ($q_b \to \infty$): (a) for several cases in the parameters $t$ and $\Delta$ with $\varepsilon_2 = 0$. Particularly in the topological phase $t = \Delta$, the standard zero-bias peak is found. (b) For $t < \Delta$ the transmittance shares the same lineshape of the symmetric situations above ($\varepsilon_2 = 6\Gamma$) and below ($\varepsilon_2 = -6\Gamma$) the MQP zero mode $\varepsilon = 0$, respectively for positive and negative overlaps. The transmittance then becomes an invariant quantity under these conditions. Panels (c) and (d) reveal distinct profiles when the system is driven away ($t \neq \Delta$) from the topological phase.

**Figure 3.** (Color online) Transmittance as a function of the Fermi energy of the STM tips within the Fano regime $x = 1$ ($q_b = 0$): panels (a)-(d) display the same features of Fig. 2 in the opposite regime of interference.
Figure 4. (Color online) Transmittance as a function of the Fermi energy of the STM tips within the Fano regime $x = 0.5$ ($q_b \approx 0.35$): panels (a)-(d) display the same features of Figs. 2 and 3, thus attesting that the robustness of the topological phase is protected against the Fano effect as well as the symmetric change of the level in the adatom 2.

Figure 5. (Color online) Density plots for the transmittance as a function of the Fermi energy of the STM tips and the overlap $\varepsilon_2$ tuned by the AFM tip in distinct Fano limits: (a) $x = 0$ ($q_b \to \infty$), (b) $x = 1$ ($q_b \approx 0$), (c) $x = 0.5$ ($q_b \approx 0.35$) for $t = \Delta$ and (d) $x = 0.5$ ($q_b \approx 0.35$) in the situation $t \neq \Delta$. The symmetrical panels (a), (b) and (c) suggest that a pair of MQPs is formed in which only the MQP 1 couples to the adatom 1 (see Fig. 1(d)). In panel (d), the absence of the mirror symmetry under analysis arises from the simultaneous coupling of MQPs 1 and 2 with the adatom 1. It occurs via the distinct amplitudes $(t + \Delta)$ and $(\Delta - t)$ as Eq. (2) ensures for $t \neq \Delta$.

The invariant profiles for the transmittance found in panels (b) of the Figs. 2, 3 and 4 as well as those (a), (b) and (c) for Fig. 5 are due to the symmetric swap of the adatom 2 level around the MQQ zero mode which reveals the formation of a pair of MQPs within such an adatom when $t = \Delta$. It couples the MQQ 1 to the adatom 1 with amplitude $\sqrt{2}\Delta$ as Eq. (2) ensures. However in the regime $t \neq \Delta$, the pair of MQQs still exists within the adatom 2, but with the MQQs 1 and 2 hybridized distinctly with the adatom 1 via the strengths $(t + \Delta)$ and $(\Delta - t)$, respectively. This feature is then probed by the symmetric tuning of the overlap $\varepsilon_2$ around the MQQ zero mode, which yields the panels (c)-(d) of the Figs. 2, 3 and 4 in addition to the density plot in panel (d) of Fig. 5. Therefore, the aforementioned mechanism ruling the transmittance profiles via the couplings of the MQQs with the adatom 1 is encoded by the self-energy $\Sigma_{MQP}$ of Eq. (21).

IV. CONCLUSIONS

In summary, we have explored theoretically a setup composed by STM and AFM tips over superconducting adatoms in which that under the latter tip encloses a pair of MQPs. Particularly for the situation where only

serve the crossover from the regime $t \neq \Delta$ ($t = 2$ with $\Delta = 4$ for the blue lineshape and $t = 4$ with $\Delta = 2$ in the case of the curve for the orange color) towards the point $t = \Delta = 4$, where we can clearly realize in the red curve the emergence of a dip with amplitude $T = 1/2$, which is due to the MQQ 1 decoupled from the MQQ 2 analogously to the opposite Fano regime of interference ($x = 0$ and $q_b \to \infty$) found in Fig. 2(a). In presence of the overlap $\varepsilon_2 \neq 0$, the zero-bias dip disappears according to the curves with $\varepsilon_2 = 6\Gamma$ (positive overlap) and $\varepsilon_2 = -6\Gamma$ (negative overlap) as found in panel (b) of the same figure. As in Fig. 2(b) we also report a universality feature in the transmittance profile, which still arises from the condition $\Sigma_{MQP} (\varepsilon_2) = \Sigma_{MQP} (-\varepsilon_2)$ apart from the Fano parameter as we can notice in Eq. (21). For $t \neq \Delta$ coincident curves no longer exist and the universal behavior is not verified as pointed out by panels (c) and (d), which have the same set of parameters as in Fig. 2. Thereby the transmittance is protected against the Fano effect as well as the symmetric change in the level of the adatom 2.

To make explicit that the robustness of the topological phase obtained with $t = \Delta = 4$ is achievable for any Fano ratio $q_b$, we present in Fig. 4 the case $x = 0.5$ ($q_b \approx 0.35$) in which both paths $V$ and $V_{12}$ of Eq. (1) compete on an equal footing. For this situation, we find intermediate Fano profiles where the underlying physics of Figs. 2 and 3 is still the same. Moreover, the invariance with the overlap $\varepsilon_2$ in the transmittance becomes clearer if we look to its density plot spanned by the axes $\varepsilon$ (Fermi level) and $\varepsilon_2$. Fig. 5(a) is for $\varepsilon_1 = -\Delta$ and $t = \Delta = 4$; it exhibits the case $x = 0$ ($q_b \to \infty$) for the regime of Fano interference, which shows the mirror symmetry under consideration with respect to the vertical axis placed at $\varepsilon_2 = 0$ (see the vertical dashed lines in the same figure). Notice that such a feature also manifests itself in panels (b) and (c), respectively in the limits $x = 1$ and $x = 0.5$. Here the orange color designates perfect insulating regions and those conducting are represented by red color. In panel (d) of the current figure, this mirror symmetry signature is broken just by using $t \neq \Delta$ as expected ($\Delta = 2$ and $t = 4$).
the nearest MQP hybridizes with the other adatom, a
universal behavior within the transmittance is revealed
when the AFM tip tunes symmetrically the energy level
of its adatom around the MQP zero mode. Remarkably,
this level plays the role of the MQP overlap found in the
original topological Kitaev wire, in which only positive
values are allowed. In this scenario, a negative overlap
therefore can be emulated by the adatom level found be-
values are allowed. In this scenario, a negative overlap

tuality is constituted
of its adatom around the MQP zero mode. Remarkably,
when the AFM tip tunes symmetrically the energy level
of MQPs, which is relevant as a qubit platform in quan-
tum computing. Our results show that we should look
forward to the ZBP signature in the quest for entangled
Majorana fermions.

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