Chiral dynamics and pionic 1s states of Pb and Sn isotopes

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Recent accurate data on $1s$ states of $\pi^-$ bound to Pb \cite{1} and Sn \cite{2} isotopes have set new standards and constraints for the detailed analysis of $s$-wave pion-nucleon interactions. This topic has a long history \cite{3} culminating in various attempts to understand the notorious "missing repulsion" in the $\pi$-nucleus interaction: the standard ansatz for the (energy independent) $s$-wave pion-nucleus optical potential, given in terms of the empirical threshold $\pi N$ amplitudes times densities $\rho_{p,n}$ and supplemented by sizable double-scattering corrections, still misses the observed repulsive interaction by a large amount. This problem has traditionally been circumvented on purely phenomenological grounds by introducing an extraordinarily large repulsive real part $(\text{Re}B_0)$ in the $\rho^2$ terms of the $\pi$-nucleus potential. The arbitrariness of this procedure is of course unsatisfactory.

In recent papers \cite{4,5} we have re-investigated this issue from the point of view of the distinct explicit energy dependence of the pion-nuclear polarization operator \cite{4} in a calculation based on systematic in-medium chiral perturbation theory \cite{5,6}. Ref. \cite{4} has also clarified the relationship to a working hypothesis launched previously \cite{7,8}: that the extra repulsion needed in the $s$-wave pion-nucleus optical potential at least partially reflects the tendency toward chiral symmetry restoration in a dense medium. To leading order, this information is encoded in the in-medium reduction of the pion decay constant $f_\pi$, which, by its inverse square, drives the isospin-odd pion-nucleon amplitudes close to threshold. The aim of the this note is to present an updated summary of the situation and to compare with the new Sn data \cite{2}. A detailed assessment of the overall systematics covering the complete pionic atoms data base has recently been given in ref. \cite{9}, using optical potential phenomenology.

The starting point is the energy- and momentum-dependent polarization operator (the pion self-energy) $\Pi(\omega, \vec{q}; \rho_p, \rho_n)$. In the limit of very low proton and neutron densities, $\rho_{p,n}$, the pion self-energy reduces to $\Pi = -(T^+ + T^- \delta \rho)$ with $\rho = \rho_p + \rho_n$ and $\delta \rho = \rho_p - \rho_n$, where $T^\pm$ are the isospin-even and isospin-odd off-shell $\pi N$ amplitudes. In the long-wavelength limit ($\vec{q} \to 0$), chiral symmetry (the Weinberg-Tomozawa theorem) implies $T^- (\omega) = \omega / (2 f_\pi^2) + \mathcal{O}(\omega^3)$. Together with the observed approximate vanishing of the isospin-even threshold amplitude $T^+(\omega = m_\pi)$, it is clear that $1s$ states of pions bound to heavy, neutron rich nuclei are a particularly sensitive source of information for in-medium...
chiral dynamics.

At the same time, it has long been known that term of non-leading order in density (double scattering (Pauli) corrections of order $\rho^{4/3}$, absorption effects of order $\rho^2$ etc.) are important. The aim must, therefore, be to arrive at a consistent expansion of the pion self-energy in powers of the Fermi momentum $k_F$ together with the chiral low-energy expansion in $\omega, |q|$ and $m_\pi$. In-medium chiral effective field theory provides a framework for this approach. We apply it here systematically up to two-loop order, following ref. [4]. Double scattering corrections are fully incorporated at this order. Absorption effects and corresponding dispersive corrections appear at the three-loop level and through short-distance dynamics parameterized by $\pi NN$ contact terms, not explicitly calculable within the effective low-energy theory. The imaginary parts associated with these terms are well constrained by the systematics of observed widths of pionic atom levels throughout the periodic table. (We use $\text{Im} B_0 = -0.063 m_\pi^4$ in the s-wave absorption term, $\Delta \Pi^\text{abs}_S = -8 \frac{\pi}{8} \frac{1}{m_\pi/2M} B_0 \rho_p (\rho_p + \rho_n)$, and the canonical parameterization of p-wave parts, in accordance with refs. [3, 9]). The real part of $B_0$ is still the primary source of theoretical uncertainty. In practice, our strategy is to start from $\text{Re} B_0 = 0$ (as suggested also by the detailed analysis of the pion-deuteron scattering length) and then discuss the possible error band induced by varying $B_0$ within reasonable limits [4].

We proceed by using the local density approximation (with gradient expansion for p-wave interactions, $\vec{q}^2 F(\rho) \rightarrow \vec{\nabla} F(\rho(\vec{r})) \vec{\nabla}$) and solve the Klein-Gordon equation

$$\left(\omega - V_c(\vec{r})\right)^2 + \vec{\nabla}^2 - m_\pi^2 - \Pi(\omega - V_c(\vec{r}); \rho_p(\vec{r}), \rho_n(\vec{r}))\right] \phi(\vec{r}) = 0. \quad (1)$$

Note that the explicit energy dependence of $\Pi$ requires that the Coulomb potential $V_c(\vec{r})$ must be introduced in the canonical gauge invariant way wherever the pion energy $\omega$ appears. This is an important feature that has generally been disregarded in previous analysis.

With input specified in details in ref. [4], we have solved eq. (1) with the explicitly energy dependent pion self-energy, obtained in two-loop in-medium chiral perturbation theory for the s-wave part, adding the time-honored phenomenological p-wave piece. The results for the binding energies and widths of 1s and 2p states in pionic $^{205}$Pb are shown in Fig. 1 (triangles). Also shown for comparison is the outcome of a calculations using a "standard" phenomenological (energy independent) s-wave optical potential,

$$\Pi_S = -T_{\text{eff}}^+ \rho - T_{\text{eff}}^- \delta \rho + \Delta \Pi^\text{abs}_S, \quad (2)$$

with $T_{\text{eff}}^+ = T_0^+ - \frac{3k_F(\omega)}{8\pi^2} \left[(T_0^+)^2 + 2 (T_0^-)^2\right]$ and the amplitudes $T_0^\pm \equiv T^\pm(\omega = m_\pi)$ taken fixed at their threshold values. This approach fails and shows the "missing repulsion" syndrome, leading to a substantial overestimate of the widths. Evidently, a mechanism is needed to reduce the overlap of the bound pion wave functions with the nuclear density distributions. The explicit energy dependence in $T^\pm$ provides such a mechanism: the replacement $\omega \rightarrow \omega - V_c(\vec{r}) > m_\pi$ increases the repulsion in $T^-$ and disbalances the "accidental" cancellation between the $\pi N$ sigma term $\sigma_N$ and the range term proportional to $\omega^2$ in $T^+$, such that $T^+(\omega - V_c) < 0$ (repulsive).

Uncertainties in $\text{Re} B_0$, in the radius and shape of the neutron density distribution, and in the input for the sigma term $\sigma_N$ have been analysed in ref. [4]. Their combined effect falls within the experimental errors in Fig. 1.
Figure 1. Binding energies and widths of pionic 1s and 2p states in $^{205}\text{Pb}$. Experimental data from [1]. Full triangles: results of two-loop in-medium chiral perturbation theory, keeping the explicit energy dependence in the s-wave polarization operator. Open circles: energy independent potential as described in text (see ref. [4] for details). Note that Re$B_0 = 0$ in both cases.

Figure 2. Binding energies, $E_{\text{bind}}$, and widths, $\Gamma$, of pionic 1s states in Sn isotopes. The curves show predictions [10] based on the explicitly energy dependent pionic s-wave polarization operator calculated in two-loop in-medium chiral perturbation theory [4]. The sensitivity to the $\pi N$ sigma term (input) is also shown. Data from ref. [2].

Using the same (explicitly energy dependent) scheme we have predicted binding energies and widths for pionic 1s states bound to a chain of Sn isotopes. These calculations [10] include a careful assessment of uncertainties in neutron distributions. Results are shown in Fig. 2 in comparison with experimental data [2] reported at PANIC’02 after the calculations. This figure also gives an impression of the sensitivity with respect to variations of the (input) $\pi N$ sigma term.

We now come to an important question of interpretation: do we actually ”observe” fingerprints of (partial) chiral symmetry restoration in the high-precision data of deeply bound pionic atoms with heavy nuclei, as anticipated in refs. [7, 8]? Is this observation related to the ”missing s-wave repulsion” that has been recognized (but not resolved in a consistent way) by scanning the large amount of already existing pionic atom data?

To approach this question, recall that pionic atom calculations are traditionally done with energy-independent phenomenological optical potentials instead of explicitly energy dependent pionic polarization functions. Let us examine the connection between these two seemingly different approaches by illustrating the leading-order driving mechanisms.

Consider a zero momentum pion in low density matter. Its energy dependent leading-order polarization operator is $\Pi(\omega) = -\left[T^+(\omega)\rho + T^-(\omega)\delta\rho\right]$, and the in-medium dispersion equation at $\vec{q} = 0$ is $\omega^2 - m_\pi^2 - \Pi(\omega) = 0$. The chiral low-energy expansion of the
off-shell amplitudes $T^\pm(\omega)$ at $\vec{q} = 0$ implies leading terms of the form:

$$T^+(\omega) = \frac{\sigma_N - \beta \omega^2}{f_\pi^2}, \quad T^-(\omega) = \frac{\omega}{2 f_\pi^2},$$

(3)

where $f_\pi = 92.4$ MeV is the pion decay constant in vacuum and $\sigma_N \simeq 0.05$ GeV. The empirical $T^+(\omega = m_\pi) = (-0.04 \pm 0.09) \text{ fm} \simeq 0$ sets the constraint $\beta \simeq \sigma_N/m_\pi^2$.

Expanding $\Pi(\omega)$ around the threshold, $\omega = m_\pi$, we identify the commonly used effective (energy-independent) s-wave optical potential $U_S$ as:

$$2 m_\pi U_S = \frac{\Pi(\omega = m_\pi, \vec{q} = 0)}{1 - \partial\Pi/\partial\omega^2},$$

(4)

where $\partial\Pi/\partial\omega^2$ is taken at $\omega = m_\pi$. Inserting (3) and assuming $\delta \rho \ll \rho$ one finds:

$$U_S \simeq -\frac{\delta \rho}{4 f_\pi^2} \left(1 - \frac{\sigma_N \rho}{m_\pi^2 f_\pi^2}\right)^{-1} = -\frac{\delta \rho}{4 f_\pi^2(\rho)},$$

(5)

with the replacement $f_\pi \to f_\pi^*(\rho)$ of the pion decay constant representing the in-medium wave function renormalization. The expression (3) is just the one proposed previously in ref. [7] on the basis of the relationship between the in-medium changes of the chiral condensate $<\vec{q}q>$ and the pion decay constant associated with the time component of the axial current. The explicitly energy dependent chiral dynamics encoded in $\Pi(\omega)$ ”knows” about these renormalization effects. Their translation into an equivalent, energy-independent potential implies $f_\pi \to f_\pi^*(\rho)$ as given in eq. (5). This statement holds to leading order. Whether (important) higher order corrections permit a similar interpretation needs still to be further explored.

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