MODELING THE RETENTION PROBABILITY OF BLACK HOLES IN GLOBULAR CLUSTERS: KICKS AND RATES

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ABSTRACT

We simulate black hole binary interactions to examine the probability of mergers, black hole growth, and gravitational radiation signals using a specific initial distribution of masses for black holes in globular clusters and a simple semianalytic formalism for dynamical interactions. We include three-body recoil and the latest results in numerical relativity for gravitational radiation recoil. It is found that while 99% of binaries are ejected from low-metallicity, low-mass clusters, metal-rich massive clusters retain 5% of their binaries. An interesting fraction of the ejected binaries, especially those from high-mass, high-metallicity systems, merge on timescales short enough to be gravitational radiation sources during their mergers with rates approaching those expected for galactic field black hole binaries. While the merger rates are comparable, the much larger mass of these binaries and their localization will make them appealing targets for advanced study at the Laser Interferometer Gravitational-Wave Observatory. We single out two possible Milky Way clusters (NGC 6441 and NGC 6388) as having the properties for a good probability of retention.

Key words: binaries: general – black hole physics – globular clusters: general – stellar dynamics

1. INTRODUCTION

Observed black hole masses occupy two regimes, $M_{\text{BH}} \lesssim 100 M_{\odot}$ for black holes formed from core collapse supernova (SN), and supermassive black holes with $M_{\text{BH}} \gtrsim 10^{9} M_{\odot}$, which reside in the centers of galaxies. Observations of some objects, however, have suggested that a middle regime of intermediate-mass black holes (IMBH; review by Miller & Colbert 2004) could exist with masses between stellar and supermassive black holes. Ultraluminous X-ray sources ($L_X \gtrsim 10^{39}$ ergs s$^{-1}$) have been found in intense star-forming regions outside the nuclei of some galaxies (Kaaret et al. 2001; Matsumoto et al. 2001; Fabbiano et al. 2001), and recently even in one globular cluster (Maccarone et al. 2007). The lower limit on the mass for these objects, assuming isotropic emission at the Eddington limit, is a few hundred solar masses. A stellar mass black hole would require special geometry of its accretion disk for sufficient beaming to occur and the accretion to be sub-Eddington (King et al. 2001), or need special conditions on the gas to provide a super-Eddington accretion rate (Beigelman 2001). Conversely, a supermassive black hole ($M \gtrsim 10^9 M_{\odot}$) would experience dynamical friction and sink to the center of its galaxy in too short a time to be plausibly observed at locations within the host galaxies where the IMBH candidates are projected to be seen today (Kaaret et al. 2001). This seems to indicate an intermediate mass for these objects. In the nearby bulgeless galaxy NGC4395, there has been found (Filippenko & Ho 2003) an active galactic nucleus (AGN), the black hole mass for which seems to be $\lesssim 10^7 M_{\odot}$, which is in the upper range of IMBH masses. An object with a similar mass may also exist in the galaxy POX 52 (Barth et al. 2004).

Another possible place to look for IMBHs besides in starburst regions in galaxies is in the centers of globular clusters. Observations have seen an increase in the mass-to-light ratio toward the centers of two globular clusters that might be consistent with a massive object, a $M > 10^4 M_{\odot}$ object in the Andromeda Galaxy cluster G1 (Gebhardt et al. 2002, 2005), a $4 \times 10^4 M_{\odot}$ object found by Noyola et al. (2008) in the cluster $\omega$ Cen, and a few-thousand-solar-mass object in M15 (van der Marel et al. 2002; Gerssen et al. 2002), although in the case of M15, Baumgardt et al. (2003) are able to simulate the observations without an IMBH using smaller compact objects. The velocity dispersion of the central stars in the cores of these globular clusters as compared to the conjectured mass of the IMBH put these clusters on the same $M$–$\sigma$ relation as the bulges of galaxies with supermassive black holes (Gebhardt et al. 2002; van der Marel et al. 2002; Gerssen et al. 2002). While the theory of the origin of the $M$–$\sigma$ relation for supermassive black holes would probably not apply to globular clusters, it is intriguing that, at least in these two cases, the IMBHs in these clusters are consistent with it.

Recently, two possible formation scenarios for IMBHs within a globular cluster have been proposed. The first involves the process of core collapse, by which the heavier stars in a cluster first sink to the middle through mass segregation. The stellar density goes very high, and it might be sufficient that several stars collide, forming a very large star (a few hundred solar masses) which collapses directly to an IMBH (Beigelman & Rees 1978; Portegies Zwart & McMillan 2002; Freitag et al. 2007). Otherwise, stellar evolution causes the high-mass stars to form black holes, which can become binaries through exchange with existing binaries of lower mass main-sequence or neutron stars (Sigurdsson & Phinney 1993). We assume that the formation of an IMBH by runaway merger does not occur in this case. Three-body interactions, which in the cores of clusters are dominated by interactions with all three objects being black holes, can then begin to work to harden binaries to the point where they merge. It is this scenario that we intend to investigate.

Previous studies of dynamical formation of an IMBH from stellar mass black holes have been performed. Black holes have been shown to be dynamically important in such aspects as the radius–age relation of Magellanic Cloud clusters (Mackey et al. 2007, 2008), and even in galactic nuclei (Lee 1995). Portegies Zwart & McMillan (2000) include a study of the
importance of black hole binaries in clusters to gravitational wave research. Recently, Holley-Bockelmann et al. (2008) did a study of mergers of black holes in a system already containing a black hole of a few hundred solar mass, which represents the next step in IMBH formation after our work. Kulkarni et al. (1993) and Sigurdsson & Hernquist (1993) used only 10 $M_\odot$ black holes, and determined that the formation of $10^3$ objects is possible. On the other hand, Miller & Hamilton (2002) showed that these binaries tend to be ejected before reaching a size at which recoil becomes unimportant, precluding further growth. Since black holes are produced from progenitors with a wide range of masses (20–100 $M_\odot$), and have varied evolution histories (wind losses and mass transfer) just prior to becoming black holes, a distribution in masses may better reflect the actual situation in globular clusters. O’Leary et al. (2006) did a study using the distribution of black hole masses and binary periods as given in Belczynski et al. (2004) and a more complicated method of computing interactions than the semianalytic model that we use. They use the old prescription for gravitational radiation recoil similar to that found in Favata et al. (2004). Numerically simulating black hole mergers through ringdown has now been done (Gonzalez et al. 2006) and definitive recoil velocities have been determined, so that the sole remaining uncertainty in determining the circumstances of black hole mergers is the initial distributions of their masses. Our semianalytic method can more quickly respond to updates in stellar population synthesis models than direct many-body integration.

We describe the conditions under which the simulations were done, including initial conditions of the binary, the analytical form of the three-body interactions, and relevant timescales, and then report results from several simulation runs, with a few parameters (e.g., metallicity) adjusted after each run. Finally, we discuss what the results imply for observed systems and suggest two systems that have a higher probability of harboring an IMBH.

2. SIMULATION CONDITIONS

We simulate the history for a total of 100,000 binaries for each set of initial conditions. The ensemble of initial conditions includes two values each for metallicity and escape velocity. An examination of Belczynski et al. (2004) shows that there exist only two distinct shapes of the period and mass distributions based on metallicity, therefore we only include a qualitative distinction with the changeover at $\log(\text{Fe/H}) = -1.3$ for observed clusters. The escape velocities were chosen as a proxy for several properties to represent moderately sized clusters and heavy clusters; smaller clusters which might have had a lower value for the escape velocity are not expected to retain any interacting binaries due to their extremely shallow potentials. The models’ initial conditions are described in Table 1. Each binary history is run until one of the three fates is determined: ejection as a binary through three-body superelastic

| Table 1 | Initial Conditions of the Models |
|---------|---------------------------------|
| Model   | Metallicity | $v_{esc}$ (km s$^{-1}$) |
| A       | Low         | 30                      |
| B       | High        | 30                      |
| C       | Low         | 50                      |
| D       | High        | 50                      |

Note. All runs used $\sigma_{GC} = 10$ km s$^{-1}$ and $\Delta = 0.4$, as described in the text.

Whereas previously (see Sigurdsson & Hernquist 1993) the distribution of black hole masses has been assumed to be single-valued at $10 M_\odot$, we start with a multi-valued initial mass function (IMF) for black holes. Based on Figure 3 of Fryer & Kalogera (2001) for $f = 1$ (fully efficient SN) and a stellar IMF power-law index $\gamma = 2.0$, we propose using a smooth power law for our low-metallicity distribution, which gives the probability of a black hole having mass $M$ proportional to $10^{-0.5 M/M_0}$ for masses in the range $3$–$80 M_\odot$. This is the same analytical form as the low-metallicity ($Z = 0.001$) IMF found by Belczynski et al. (2004) for his standard model parameters. Fryer & Kalogera (2001) calculate a black hole IMF for single progenitor stars, which is used by Belczynski et al. (2004) to predict black hole masses from a range of progenitor masses while also considering binary evolution effects such as common envelope phases. The greatest uncertainty in determining mass functions from binary evolution models comes during the common envelope phase and in calculating mass loss. These are common to any attempt to use a distribution of masses.

The IMFs in Belczynski et al. (2004) are presented as histograms, but we convert these to analytical probability distributions for ease of use with computer-based simulations. Binaries are constructed using the IMF to pick both masses, a distribution in periods taken from Belczynski et al. (2004) as appropriate for the metallicity studied and also converted to an analytical form, and an eccentricity from a thermal distribution ($P(e) = 2e$). We chose the masses independently, as we expect the stars to have, for the most part, developed independently in well-separated binaries. The simulations take place in a regime, being the center of a dense cluster, where all of the stars not heavy enough to have become evolved have been ejected to the outskirts of the cluster, except in the first $100$ Myr when massive stars were evolving rapidly and drastically changing core conditions. We assume that we start after this time, unlike other studies such as Portegies Zwart & McMillan (2002), and have a quasi-static cluster environment. The forms for both the IMF and periods are as follows (Belczynski et al. 2004):

$$P_{\text{low}}(M) = 0.152 \cdot 10^{-0.05 M/M_0}, \quad \text{all } M.$$  

$$P_{\text{high}}(M) = \left\{ \begin{array}{ll} 0.028, & 3 M_\odot \geq M \geq 15 M_\odot, \\ 10^{0.6 M/M_0}, & 15 M_\odot \geq M \geq 55 M_\odot, \\ 0, & 55 M_\odot \geq M \geq 80 M_\odot. \end{array} \right.$$
\[ P(P) = \begin{cases} \sqrt{1 - (P - 1)^2}, & 0 \geq \log P(\text{days}) \geq 2 \text{ low } Z \text{ only}, \\ (P - 2), & 2 \geq \log P(\text{days}) \geq 6 \text{ all } Z. \end{cases} \]

Binaries are subjected to encounters with a third black hole whose mass is randomly drawn from the IMF. The timescale for the encounter,

\[ t_{\text{enc}} = \frac{1.5 \times 10^9}{\mu_1 m_T a_{\text{AU}}^4} \text{ years}, \]

is calculated from Equation (2.9) of Sigurdsson & Phinney (1993) with \( m_1 \) the mass of the third black hole, \( \mu_{12} \) the reduced mass of the binary, \( m_T \) the total mass of all the three objects, the masses being expressed in terms of \( M_\odot, a_{\text{AU}} \) the binary’s semimajor axis, \( n_4 \) the density of stars \( n/10^4 \text{ pc}^{-3} \), and \( v_{10} \) the relative velocity of the third object \( v/10 \text{ km s}^{-1} \), which is taken to be 1 for typical globular clusters which have velocity dispersions on the order of 10 km s\(^{-1}\). For this work we use a value of the dimensionless cross section \( \sigma = 10 \) as defined in Equation (2.7) of Sigurdsson & Phinney (1993), as this value is broadly consistent with interacting systems having mass ratios in the range of those in our simulations, as given in Tables 3A and 3B of Sigurdsson & Phinney (1993). This timescale is compared with the time for merger by emission of gravitational radiation (Peters 1964),

\[ t_{GW} = 3.151 \times 10^{17} \text{ yr} (\frac{a}{\text{AU}})^4 \times \left( \frac{M_\odot}{m_1} \right) \left( \frac{M_\odot}{m_2} \right) \left( \frac{M_\odot}{m_1 + m_2} \right) g(e) = (1 - e^2)^{-1/2} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right). \]

For \( t_{\text{enc}} < t_{GW} \), the encounter takes place. This entails choosing a new eccentricity from the thermal distribution and a change in the semimajor axis \( a \) such that the binding energy of the binary is changed by

\[ \Delta = 1 - \frac{a_{\text{in}} m_a m_b}{a_{\text{out}} m_1 m_2}. \]

where \( a_{\text{in}} \) and \( a_{\text{out}} \) are the starting and ending semimajor axes of the binary for the encounter, \( m_1 \) and \( m_2 \) are the masses of the two original objects in the binary, and \( m_a \) and \( m_b \) are the masses of the two new objects. Based on the results in Sigurdsson & Phinney (1993), we assume that the two most massive of the three interacting objects form the new binary, leading to the possibility of membership change. If there is not a change, \( a \) is simply reduced by a factor of \( (1 - \Delta) \). With an exchange in membership, it is possible for \( a \) to increase dramatically. For simplicity, we choose a fixed value of \( \Delta = 0.4 \), characteristic of the mean energy transferred in the same encounters from Sigurdsson & Phinney (1993) that gave us our value for \( \sigma \). This is warranted if there are multiple encounters per system before ejection. For a check on the adequacy of a single value of \( \Delta \), we ran one set of simulations allowing \( \Delta \) for each interaction to vary in a normal distribution around 0.4. Even with a variance of 0.2, the effect was negligible. We do not track stellar interactions; implicitly we assume that we are in a regime where there are multiple black holes that have formed a dense subcore in the cluster, and that the interactions of these black holes dominate the fate of any binary. In general, interactions of the subcore with stars are not important. They become very important in late stages, particularly for the “last” black hole binary. The discussion of stellar interactions with the binary is beyond the scope of this paper.

Besides changes in the internal dynamics of the binary, the conservation of momentum among the two systems requires the binary to recoil. The magnitude of the recoil is

\[ v_{\text{rec}} = \frac{m_e}{m_T} \sqrt{\frac{m_3(m_1 + m_2)}{m_2(m_a + m_b)} a_{\text{GC}} + \frac{2 \Delta m_T G m_1 m_2}{m_a(m_a + m_b)a_{\text{in}}}. \]

If the recoil is smaller than the assumed escape velocity of the globular cluster, then the time is incremented by \( t_{\text{enc}} \), and the run continues by choosing a black hole mass independently from the mass distribution. The dynamical friction timescale for the binary is approximately \( \langle m \rangle / M_\text{BH} t_r \) (O’Leary et al. 2006), where \( \langle m \rangle \) is the average stellar mass and \( t_r \) is the relaxation timescale. Very few binaries recoil with a velocity within the narrow range of velocity required to have a turning point of several half-mass radii, and these are highly radial orbits. Therefore, the core relaxation time applies. The core relaxation times for most clusters are \( 10^7-10^8 \text{ years} \) (Harris 1996), so most binaries will return to the core in less than a million years. If the binary is ejected, the run is stopped and \( t_{GW} \) is calculated for the binary. Those for which \( t_{GW} \) is less than \( 10^7 \text{ years} \) may be field gravitational radiation sources. The run will also be stopped once \( t_{\text{enc}} > t_{GW} \), at which time the recoil velocity from asymmetric emission of gravitational radiation is calculated. We used the zero spin expression for the gravitational radiation recoil from Gonzalez et al. (2006),

\[ v_{GW} = 1.20 \times 10^4 \eta \sqrt{1 - 4n(1 - 0.93\eta)} \text{ km s}^{-1}, \]

where \( \eta \) is the symmetric mass ratio defined using \( q = a/(1 + q)^2 \), with \( a \) being the mass ratio of the two objects in the binary \((0 \geq q \geq 1, \ 0 \geq \eta \geq 0.25) \). Stellar mass black holes, unlike supermassive black holes in galaxy centers that have accreted most of their mass from a thin disk where the spin goes to 0.98, are not likely to have a large spin parameter. O’Shaughnessy et al. (2005b) find, by analogy with neutron star birth spins (Lorimer et al. 2005; Kramer et al. 2003; Migliazzo et al. 2002), that expected spins should be less than 0.1, unless otherwise spun up by fallback from the SN explosion. Burrows et al. (2007) find that only rapidly spinning cores may produce the phenomena called hypernovae. Since these types of objects are rare, we may infer that most SNe that produce black holes make slowly spinning ones. Any accretion that does occur while the black hole is in a binary with a mass donor is expected by Belczynski et al. (2008) to increase the spin parameter \( a = J/M^2 \) beyond 0.5. From the fully spin-dependent form of the recoil velocity (Campanelli et al. 2007), and using the approximation of \( 1/\sqrt{2} \) for the values of the sine and cosine of the angles, we find that \( v_{GW}(a) / v_{GW}(a = 0) \) goes above 2 for values of \( a \gtrsim 0.4 \) except for extreme mass ratios which are more sensitive to spin. The merged object is then ejected or retained in the globular cluster depending on the magnitude of \( v_{GW} \).

3. RESULTS

As seen in Table 2, the most likely conditions for a black hole to be retained are in massive, metal-rich clusters. The change in mass distribution with metallicity is the main driver of whether or not a binary may be retained. What changes most between the two metallicities is the distribution of mass ratios, seen in
there is a peak at distribution of initial mass ratios for high-metallicity binaries, binaries is nearly constant above 0.2, which is the condition Figure 1. The initial distribution of mass ratios for metal-poor $M_\odot$ the black hole IMF having two peaks near 10 and 45 $M_\odot$ from the indicated distribution, as is done initially in each simulation. The dotted line shows the distribution of mass ratios for ejected systems show one peak of 20–30 $M_\odot$ see at 120 $M_\odot$, with a slight break downward at 60 $M_\odot$. For the high-metallicity systems, there is a peak around 40–60 $M_\odot$, and another between 80 and 100 $M_\odot$, which reflects the underlying initial mass distribution. For ejected objects, the low-metallicity systems show a monotonic decline from 15 $M_\odot$ to the maximum mass seen at 120 $M_\odot$, with a slight break downward at 60 $M_\odot$. For the high-metallicity systems, there is a peak around 40–60 $M_\odot$ where the distribution of masses for retained objects has a deficit. The currently used recoil velocity function has its peak at a mass ratio of about a third; if a binary in the high-metallicity model consists of one member from each of the two regions, it will allow the binary to become harder before ejecting it. The binaries for the most part stay within their host galaxy. Figure 2 shows the distribution of velocities of the ejected binaries. For the two low-metallicity models, which have binaries that start with tight orbits having log $P$(days) $< 2$, there are a few systems (1%) that are ejected with a velocity higher than 300 km s$^{-1}$, but most binaries (and all of the high metallicity ones) have $v_{\text{esc}} < 200$ km s$^{-1}$. This means that, while they leave their parent cluster, they are still confined to their parent galaxy unless it is a dwarf galaxy.

The distributions of masses for the retained merged objects are shown in Figure 3. For low-metallicity systems, the distribution is flat up to 60 $M_\odot$, after which it drops. High-metallicity systems show one peak of 20–30 $M_\odot$, and another between 80 and 100 $M_\odot$, which reflects the underlying initial mass distribution. For ejected objects, the low-metallicity systems show a monotonic decline from 15 $M_\odot$ to the maximum mass seen at 120 $M_\odot$, with a slight break downward at 60 $M_\odot$. For the high-metallicity systems, there is a peak around 40–60 $M_\odot$ where the distribution of masses for retained objects has a deficit. The currently used recoil velocity function has its peak at a mass ratio of about a third; if a binary in the high-metallicity model consists of one member from each of the two regions, it will have a total mass of about 40–60 $M_\odot$, and a mass ratio of 0.3–0.4, and will most likely be ejected, whereas a binary with both members from the same region will have a mass of either 20 or 80–100 $M_\odot$ and $q$ close to 1 and be retained (if it survives three-body interactions).

While the binaries are in the globular cluster, they may go through short-lived phases with large semimajor axes due to exchanges of membership. These stages may be important in transferring angular momentum from the binary to the cluster.
Figure 2. Plot of the recoil velocities of ejected binaries. The solid line is for model A (low mass, low metallicity). The dotted line shows model B (low mass, high metallicity). The short dashed line is for model C (high mass, low metallicity). The long dashed line is for model D (high mass, high metallicity). The plots start at the escape velocity for the cluster. The high velocity tail for models A and C is due to binaries from the low-metallicity period distribution which have small initial separations since \( v_{\text{rec}} \) is inversely proportional to \( a \). This also is the reason why the high-mass cluster models are shifted to higher velocities (at least for \( v_{\text{rec}} \) between 50 and 200 km s\(^{-1}\)), as the binaries are able to become more tightly bound before being ejected. The fraction of binaries at each point covers a 5 km s\(^{-1}\) bin.

Figure 3. Top histogram showing distribution of masses for the black holes retained upon merger. The bins are 5 \( M_\odot \) wide and show the log of the fraction in each bin. The solid line is for metal-poor systems, while the dashed line is for metal-rich systems. The model dependence is most visible in the second of these with the lack of merged black holes at 40–60 \( M_\odot \) and the sharp dropoff above 110 \( M_\odot \). We see that there are a substantial fraction (1%–5%) of black holes that remain which have masses above 100 \( M_\odot \), which is a common definition for the lower boundary IMBH masses. The bottom plot shows the mass distribution for those objects ejected upon merger by gravitational radiation, with the same convention for the lines. Both of these plots are normalized to the number of black holes that undergo fate 2 (ejection) or fate 3 (retention), the number of which for each model is given. These objects show a “complimentary” distribution to the retained objects, especially for the metal-rich clusters where the peak mass of ejected objects fits nicely into the deficit of retained objects.

4. DISCUSSION

The event rate from merging black hole binaries can be calculated from the fraction of systems that merge within a Hubble time and the relative contributions from low- and high-metallicity systems and light or massive clusters. To conservatively estimate the event rate, we assume 100 globular clusters per galaxy (e.g., the Milky Way is currently thought to have about 150) and 100 BH per globular cluster \( (N_{\text{BH}} \sim 10^{-4} N_\star) \). We assume that the break between high versus low metallicity is at an [Fe/H] of \(-1.3\) and that light globulars have \( v_{\text{esc}} \) of less than 30 km s\(^{-1}\) and heavy globulars have \( v_{\text{esc}} \) above this value. The escape velocity for a globular cluster is given by \( v_{\text{esc}} = \sqrt{2\Phi_0} \), where \( \Phi_0 \) is the central potential of the cluster; \( W = \Phi_0/\sigma^2 \) is the King parameter and is correlated with the cluster concentration, and \( \sigma \) is approximately equal to the velocity dispersion except in the case for shallow globulars. We determine the concentration and thus \( W \) from the catalog of Harris (1996), while one-dimensional velocity dispersion data were obtained from Pryor & Meylan (1993). For the 56 Milky Way clusters for which we could determine the escape velocity, we find that the percentage of clusters in each of our models is as follows: A 45% (25), B 21% (12), C 20% (11), and D 14% (8). Including data from Table 2 on the number of mergers within a Hubble time, we find that ejected binaries account for \( \sim 640 \) mergers per galaxy in a Hubble time, with another 335
coming from those binaries that merge while still in the cluster. Over a Hubble time, this gives a rate of $10^{-7}$ per year per galaxy. These are very conservative estimates for the rate, as the Milky Way is assumed to have about 150 globular clusters, and giant ellipticals can have on the order of $10^3$. Assuming a value of 300 globular clusters per galaxy and 300 black hole binaries per cluster, there would be an order of magnitude jump in the rate to $10^{-6}$ per year per galaxy. We also expect a further increase in rate from the additional mergers produced by black holes that are retained after their first merger. Galactic binary BH merger rates are estimated at $10^{-6}$ per year (O’Shaughnessy et al. 2005a).

The mergers are expected to be delayed from the formation of the clusters, which, in the case of globulars, is close to the beginning of the universe. The last interaction before the binary is ejected typically happens when the semimajor axis is 0.1–1 AU, giving a $t_{\text{enc}}$ of $10^8$–$10^9$ years. The timescales for the gravitational merger of the ejected binaries span a wide range of values ($5 < \log t_{\text{GW}} < 20$). Figure 5 shows the distribution of merger timescales for ejected binaries for each of the models. We find that the percentage of binaries which merge between 1 and 10 Gyr is 2.6% for model A, 2.3% for model B, 4.3% for model C, and 8.2% for model D. While we have used a thermal distribution ($P(e) = 2e, \langle e \rangle = 0.67$) for the eccentricity after an exchange, the three-body study by Sigurdsson & Phinney (1993) found that this works for equal-mass exchanges, but for nonequal masses, the eccentricities may be higher ($\langle e \rangle \approx 1 - 1.3(m_3/m_2)$). This does not affect recoil velocities, but the $t_{\text{GW}}$ would be shortened, and the expected rates of black hole mergers would increase by a factor of a few. If we choose black hole binaries as we do for the simulation runs and determine their $t_{\text{GW}}$ without any interactions, we find that for metal-rich systems, only 0.1% merge in less that a Hubble time, whereas 7% of low-metallicity binaries do so. We explain this as a model-dependent result—the low-metallicity period distribution includes systems which have periods shorter than 100 days while the metal-rich distribution does not. Interactions
We find that, while galactic mergers have chirp masses of 3–6 
\( M_\odot \) due to exchanges undergone while the binary was in the cluster.

In conclusion, we find that within our simplified model assumptions, most black hole binaries are ejected through

1 Recent spectroscopy by Zepf et al. 2008 favors the lower mass interpretation of this object (Zepf et al. 2008).
gravitational three-body interaction from the cluster into the general potential of the galaxy. Of those binaries that survive to merge by gravitational radiation, about 2/3 to 1/2 are ejected through gravitational radiation, recoil. Between 0.5% and 3.5%, depending on metallicity and cluster escape velocity, of all black hole binaries in clusters are predicted to be retained upon merger of the binary, with typical final masses of 20–50 M☉, but in some instances over 100 M☉. Of course if other formation channels dominate, or there is significant gas accretion after the dynamical interaction phase, then the final black hole masses may be very different (higher if there is significant accretion). We find that the rate per galaxy of black hole binary mergers from the globular cluster population, through gravitational radiation, is competitive with the total merger rate from the parent galaxy, but biased toward higher masses. While most globular clusters in massive galaxies probably form at high redshift, this suggests that black hole binary coalescence from clusters in low mass, nearby star-forming galaxies may be a significant contributor to the total high-frequency gravitational radiation, signal in the local universe. The current results are dependent on the exact form of the initial conditions of mass distributions and period distributions obtained from population synthesis. As the formation mechanisms for black holes become more well understood, it would be appropriate and easy to refine the results in this paper.

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