Constraints on anomalous quartic gauge couplings by $W\gamma jj$ production

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ABSTRACT: Though the standard model describes the experiments very well, it is necessary to search for the signal of new physics. One kind of the processes sensitive to the new physics is the vector boson scattering process at the LHC, which can be used to probe the anomalous quartic gauge couplings (aQGC). In this paper, we investigate the aQGC contribution to the $pp \to W\gamma jj$ production at $\sqrt{s} = 13$ TeV. The unitarity bounds imply that the aQGC in this channel is difficult to be observed at current luminosity. After analysing in detail the kinematic and polarization features of the aQGC signal, we found the polarization effects induced by the aQGCs are unique and can discriminate the signal from the SM backgrounds. With the help of the event selection strategies proposed, we calculate the significance of signals of each relevant anomalous quartic gauge-boson operators, and obtain the constraints on the coefficients of these operators at $\sqrt{s} = 13$ TeV LHC with current luminosity. The results indicate that the $pp \to W\gamma jj$ production is powerful for searching for the effect of $O_{M2,3,4,5}$ and $O_{T5,6,7}$ operators.
1 Introduction

In the past few decades, most of the experimental measurements are in good agreement with the standard model (SM) predictions. Searching the new physics beyond standard model (BSM) is the main goal for current and future colliders. The possibility of the existence of new interactions involving the electroweak (EW) symmetry breaking (EWSB) is contemplated in many BSM scenarios, and lead to deviations from the SM predictions. The model-independent approach called the SM effective field theory (SMEFT) [1–3] has been widely used to search for the BSM. In SMEFT, the SM is a low energy effective theory of some unknown BSM theory. When the centre-of-mass (c.m.) energy is not enough to directly produce the new resonance states and the new physics sector is decoupled because the new physics scale is much higher than the EW scale, one can integrate out the new physics particles, then the BSM effects become new interactions of known particles. Formally, the new interactions appear as higher dimensional operators. The operators w.r.t. EWSB up to dimension-8 can contribute to the anomalous trilinear gauge couplings (aTGCs) and anomalous quartic gauge couplings (aQGCs). There are many full models that contain these operators, such as anomalous gauge-Higgs couplings [4, 5], composite Higgs [6, 7], warped extra dimensions [7], 2HDM [8–11], $U(1)_{L_{\mu} - L_{\tau}}$ [12, 13], as well as axion-like particles [14, 15] scenarios.
In this work, we focus on the dimension-8 anomalous quartic gauge-boson operators. The dimension-8 operators can contribute to aTGCs and aQGCs independently, therefore, it is worthwhile to carry out studies of the dimension-8 operators contributing to aQGCs. On the other hand, due to no evidence of dimension-6 operators yet, it is possible that higher dimensional operators contributing to aQGCs exist without dimension-6 operators. This situation arises in the Born-Infeld (BI) theory proposed in 1934 [16]. This theory is a nonlinear extension of Maxwell theory motivated by a “unitarian” standpoint. It could provide an upper limit on the strength of the electromagnetic field. In 1985, the BI theory rebirth in models inspired by M-theory [17, 18]. We note that the constraint on the BI extension of the SM has recently been presented via dimension-8 operators in the SMEFT [19].

To study aQGCs, the vector boson scattering (VBS) process at LHC provides an ideal chance. It is well known that the perturbative unitarity of the longitudinal $W_L Z_L \rightarrow W_L Z_L$ scattering is violated if the Higgs boson is not presented, which sets an upper bound on the mass of the Higgs boson [20]. In other words, with the Higgs boson presented, the Feynman diagrams of the VBS processes cancel each other and the cross-section do not grow with c.m. energy. However, such suppression of cross-section can be relaxed if there were new physics particles, consequently, the cross-section may be significantly increased and a window to detect the BSM is open [21]. Both aTGCs and aQGCs could have effects on VBS process [23–25]. Unlike the aTGCs which also affect the diboson processes and vector boson fusion (VBF) processes etc. [21, 22], the most sensitive process for aQGCs is the VBS process. Historically, VBS has been proposed as a means to test the structure of EWSB since the early stage of planning for the Superconducting Super Collider (SSC) [26]. In the past a few years, the study of the VBS drew a lot of attention. Before the LHC, limits on aQGCs were obtained by $W^+ W^- \gamma \gamma$ and $ZZ \gamma \gamma$ interactions at the LEP [27–31] and the Tevatron[25, 32]. The first report of limits on aQGCs at the LHC is the same-sign $WW$ production [33, 34]. At present, a number of experimental results in VBS have been obtained, including the electroweak-induced production of $Z \gamma jj$ [35, 36], $W \gamma jj$ [37] at $\sqrt{s} = 8$ TeV and $ZZ jj$ [38, 39], $WZ jj$ [40, 41], $W^+ W^+ jj$ [42] at $\sqrt{s} = 13$ TeV. Theoretical studies are also extensively carried out [43–47].

Among those VBS processes, in this paper we consider $W \gamma jj$ production channel via the scattering between $Z/\gamma$ and $W$ bosons. Compared with the $\gamma \gamma \rightarrow W^+ W^-$ process, $W \gamma jj$ channel is more sensitive to the $O_{T_i}$ operators. Compared with the same sign $WW jj$ final state, the $W \gamma jj$ channel can provide constraints on $O_{M_{2,3,4,5}}$ and $O_{T_{5,6,7}}$ operators. On the other hand, the $\gamma \gamma \rightarrow W^+ W^-$ process has smaller energy scale due to the fact that the photons emitting from protons are less energetic than the $Z$ bosons because photons are massless, therefore the signal of aQGCs in $W \gamma jj$ channel is larger. The next-to-leading order (NLO) QCD corrections to the $pp \rightarrow W \gamma jj$ have been computed in Refs. [24, 48], and the K factor is found to be close to one ($K \approx 0.97$ [24]). However, the phenomenology of this channel at $\sqrt{s} = 13$ TeV has been poorly studied previously. In this paper, we present the dimension-8 quartic gauge-boson operators contributing to this channel. Another important issue of SMEFT is its validity. The scattering of vector bosons with aQGCs has amplitude grow as $\mathcal{O}(E^4)$, leading to tree-level unitarity violation at high
enough energy [49–51]. For coefficients of anomalous quartic gauge-boson operators, the unitarity bounds have to be set, which depend on the energy scales of the sub-processes of VBS. We investigate the energy scales of the sub-processes and study the unitarity bounds, and find that the bounds are very strong, indicating that the aQGC effects are difficult to be observed.

Another important phenomenon is the polarization of the $W$ bosons and the resulting angular distribution of the leptons. The polarization of the $W$ and $Z$ bosons plays an important role in testing the SM [52]. The angular distributions are good observables to search for the BSM signals (an excellent example is the $P_5'$ form factor [53, 54]) because the differential cross-sections expose more information than the total cross-section. While the polarization fractions of the $W$ and $Z$ bosons have been extensively studied within the SM [55–60], up to our knowledge, the polarization effects caused by aQGCs have not been studied yet. One of the obstacles to study the polarization is the reconstruction of $W$ boson’s rest-frame. The $W\gamma jj$ channel has the advantage that there is only one neutrino, and thus one can use the observable $L_p$ defined in ref. [57]. With the help of $L_p$, we find that the aQGCs result in unique polarization features. The backgrounds, i.e. the SM contributions to the $pp \to \ell\nu\gamma jj$ ($\ell = e$ or $\mu$) channel are complicated. To reduce the backgrounds, the selection strategy needs to be optimised. The polarization features of the signal, which have not been considered before in the study of aQGCs, can be applied to achieve this goal. By using Monte-Carlo (MC) simulation, we also study the kinematic features. Along with the polarization features we propose several efficient event selection strategies, based on which we study the significance of signals and calculate the expected constraints on the coefficients of the relevant anomalous quartic gauge-boson operators according to current c.m. energy and luminosity of LHC.

The paper is organized as follows: we first discuss the corresponding dimension-8 anomalous quartic gauge-boson operators relevant to the $W\gamma$ production in VBS processes. Then we analyse the partial waves unitarity bounds for the $\gamma W \to \gamma W$ process and the $ZW \to \gamma W$ process. Finally, we discuss the feature of the signals of aQGCs and sensitivities to the aQGCs in the $\ell\nu\gamma jj$ channel at the LHC.

2 Operator basis

If we assume that the new physics is at a scale much higher than the weak scale, an effective Lagrangian can be written in terms of an expansion in higher dimensional operators,

$$\mathcal{L}_{SMEFT} = \mathcal{L}_{SM} + \sum_i \frac{C_{6i}}{\Lambda^2} O_{6i} + \sum_j \frac{C_{8j}}{\Lambda^4} O_{8j} + \ldots, \tag{2.1}$$

where $O_{6i}$ and $O_{8j}$ are dimension-6 and dimension-8 operators, $C_{6i}/\Lambda^2$ and $C_{8j}/\Lambda^4$ are corresponding Wilson coefficients. The effects of BSM are described by higher dimensional operators which are suppressed by an energy scale $\Lambda$. Considering one fermion generation, 86 independent operators out of 895 baryon number conserving dimension-8 operators can contribute to QGCs and TGCs. [25].
We follow Refs. [61] and [62] to list all dimension-8 operators affecting aQGCs, they are

$$\mathcal{L}_{aQGC} = \frac{2}{\Lambda^4} O_{S_0} + \sum_{j=0}^{7} \frac{f_M}{\Lambda^4} O_{M_j} + \sum_{k=0}^{9} \frac{f_T}{\Lambda^4} O_{T_k}$$  \hspace{1cm} (2.2)

with

$$O_{S_0} = (D_\mu \Phi)^\dagger D_\mu \Phi \times [(D^\nu \Phi)^\dagger D^\nu \Phi], \hspace{1cm} O_{S_1} = (D_\mu \Phi)^\dagger D_\mu \Phi \times [(D^\nu \Phi)^\dagger D^\nu \Phi],$$

$$O_{S_2} = (D_\mu \Phi)^\dagger D_\mu \Phi \times [(D^\nu \Phi)^\dagger D^\nu \Phi]$$  \hspace{1cm} (2.3)

$$O_{M_0} = \text{Tr} \left[ \widehat{W}_{\mu \nu} \widehat{W}^{\mu \nu} \right] \times [(D^\beta \Phi)^\dagger D^\beta \Phi], \hspace{1cm} O_{M_1} = \text{Tr} \left[ \widehat{W}_{\mu \nu} \widehat{W}^{\nu \beta} \right] \times [(D^\beta \Phi)^\dagger D^\mu \Phi],$$

$$O_{M_2} = [B_{\mu \nu} B^{\mu \nu}] \times [(D^\beta \Phi)^\dagger D^\beta \Phi] \hspace{1cm} O_{M_3} = [B_{\mu \nu} B^{\mu \beta}] \times [(D^\beta \Phi)^\dagger D^\mu \Phi]$$

$$O_{M_4} = \left[ (D_\mu \Phi)^\dagger \widehat{W}_{\beta \nu} D^\mu \Phi \right] \times B^{\beta \nu}, \hspace{1cm} O_{M_5} = \left[ (D_\mu \Phi)^\dagger \widehat{W}_{\beta \nu} D_\nu \Phi \right] \times B^{\beta \mu} + h.c.,$$

$$O_{M_6} = (D_\mu \Phi)^\dagger \widehat{W}_{\beta \nu} D_\nu \Phi, \hspace{1cm} O_{M_7} = (D_\mu \Phi)^\dagger \widehat{W}_{\beta \nu} D_\nu \Phi,$$

$$O_{T_0} = \text{Tr} \left[ \widehat{W}_{\mu \nu} \widehat{W}^{\mu \nu} \right] \times \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\alpha \beta} \right], \hspace{1cm} O_{T_1} = \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\mu \beta} \right] \times \text{Tr} \left[ \widehat{W}_{\mu \beta} \widehat{W}^{\alpha \nu} \right],$$

$$O_{T_2} = \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\mu \beta} \right] \times \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\mu \beta} \right], \hspace{1cm} O_{T_3} = \text{Tr} \left[ \widehat{W}_{\mu \nu} \widehat{W}^{\mu \nu} \right] \times B_{\alpha \beta} B^{\alpha \beta},$$

$$O_{T_5} = \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\nu \beta} \right] \times B_{\mu \beta} B^{\nu \alpha}, \hspace{1cm} O_{T_7} = \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\mu \beta} \right] \times B_{\beta \nu} B^{\nu \alpha},$$

$$O_{T_6} = \text{Tr} \left[ \widehat{W}_{\mu \nu} \widehat{W}^{\mu \nu} \right] \times \text{Tr} \left[ \widehat{W}_{\alpha \beta} \widehat{W}^{\alpha \beta} \right], \hspace{1cm} O_{T_9} = \text{Tr} \left[ \widehat{W}_{\mu \nu} \widehat{W}^{\mu \nu} \right] \times B_{\beta \nu} B^{\nu \alpha},$$

where $\widehat{W} = \sigma \cdot \vec{W}$ with $\sigma$ the Pauli matrix and $\vec{W} \equiv \{W^1, W^2, W^3\}$. Different operators are related to different channels. The operators $O_{M_1,2,3,4,5,7}$ and $O_{T_{0,1,2,5,6,7}}$ operators will contribute to the $\gamma W jj$ channel, while $O_{S_i}$ will not though they contribute to the same sign $WW jj$ channel.

The aQGC vertices relevant to $\gamma W jj$ channel are $\gamma \gamma W^+ W^-$ and $\gamma Z W^+ W^-$ vertices. Extracting from the operators, the vertices are

$$V_{AZWW,0} = F^{\mu \alpha} Z_{\mu \beta} (W_\alpha^+ W^- - W_\alpha^- W^+), \hspace{1cm} V_{AZWW,1} = F^{\mu \alpha} Z_{\alpha} (W^+ W^- + W^+ W^-),$$

$$V_{AZWW,2} = F^{\mu \alpha} Z_{\mu \alpha} W^0, \hspace{1cm} V_{AZWW,3} = F^{\mu \alpha} Z^3 (W^+ W^- + W^+ W^-),$$

$$V_{AZWW,4} = F^{\mu \alpha} Z^3 (W^+ W^- + W^+ W^-), \hspace{1cm} V_{AZWW,5} = F^{\mu \nu} Z_{\mu \alpha} W^0,$$

$$V_{AZWW,6} = F^{\mu \alpha} Z_{\mu \beta} (W^+ W^- + W^+ W^-), \hspace{1cm} V_{AZWW,7} = F^{\mu \alpha} Z_{\mu \beta} (W^+ W^- + W^+ W^-).$$  \hspace{1cm} (2.6)

$$V_{2AZWW,0} = F^{\mu \nu} F^{\mu \nu} W^+ W^-, \hspace{1cm} V_{2AZWW,1} = F^{\mu \nu} F^{\mu \nu} W^+ W^-$$

$$V_{2AZWW,2} = F^{\mu \nu} F^{\mu \nu} W^+ W^-, \hspace{1cm} V_{2AZWW,3} = F^{\mu \alpha} F^{\mu \alpha} W^+ W^-.$$

$$V_{2AZWW,4} = F^{\mu \alpha} F^{\mu \alpha} W^+ W^-, \hspace{1cm} V_{2AZWW,5} = F^{\mu \alpha} F^{\mu \alpha} W^+ W^-.$$

\hspace{1cm} (2.7)
\[
\begin{array}{|c|c|c|c|}
\hline
\text{coefficient} & \text{constraint} & \text{coefficient} & \text{constraint} \\
\hline
f_{M_0}/A^4 (\text{TeV}^{-4}) & [-6.0, 5.9]^{[42]} & f_{T_0}/A^4 (\text{TeV}^{-4}) & [-0.62, 0.65]^{[42]} \\
f_{M_1}/A^4 (\text{TeV}^{-4}) & [-8.7, 9.1]^{[42]} & f_{T_1}/A^4 (\text{TeV}^{-4}) & [-0.28, 0.31]^{[42]} \\
f_{M_2}/A^4 (\text{TeV}^{-4}) & [-26, 26]^{[37]} & f_{T_2}/A^4 (\text{TeV}^{-4}) & [-0.89, 1.02]^{[42]} \\
f_{M_3}/A^4 (\text{TeV}^{-4}) & [-43, 44]^{[37]} & f_{T_3}/A^4 (\text{TeV}^{-4}) & [-3.8, 3.8]^{[37]} \\
f_{M_4}/A^4 (\text{TeV}^{-4}) & [-40, 40]^{[37]} & f_{T_4}/A^4 (\text{TeV}^{-4}) & [-2.8, 3.0]^{[37]} \\
f_{M_5}/A^4 (\text{TeV}^{-4}) & [-65, 65]^{[37]} & f_{T_5}/A^4 (\text{TeV}^{-4}) & [-7.3, 7.7]^{[37]} \\
f_{M_6}/A^4 (\text{TeV}^{-4}) & [-13.3, 12.9]^{[42]} & & \\
\hline
\end{array}
\]

Table 1: The constraints on the coefficients obtained by experiments.

and the coefficients are

\[
\begin{align*}
\alpha_{Z\nu W,0} &= \frac{\alpha^2}{s_W^2} \left( \frac{c_W^2}{s_W} f_{M_5} - f_{M_5} - \frac{c_W}{s_W} f_{M_1} + 2 \frac{c_W}{s_W} f_{M_3} + \frac{c_W}{s_W} f_{M_7} \right), \\
\alpha_{Z\nu W,1} &= \frac{\alpha^2}{s_W^2} \left( -\frac{1}{2} \left( \frac{c_W}{s_W} + \frac{s_W}{c_W} \right) f_{M_5} - f_{M_5} - \frac{c_W}{s_W} f_{M_3} \right), \\
\alpha_{Z\nu W,2} &= \frac{\alpha^2}{s_W^2} \left( \frac{c_W^2}{s_W} f_{M_4} - f_{M_4} + 2 \frac{c_W}{s_W} f_{M_6} - 4 \frac{c_W}{s_W} f_{M_2} \right), \\
\alpha_{Z\nu W,3} &= \frac{\alpha^2}{s_W^2} \left( -\frac{c_W}{s_W} f_{M_4} - f_{M_4} \right), \\
\alpha_{Z\nu W,4} &= \frac{\alpha^2}{s_W^2} \left( \frac{1}{2} \left( \frac{c_W}{s_W} + \frac{s_W}{c_W} \right) f_{M_5} - f_{M_5} - \frac{c_W}{s_W} f_{M_3} \right), \\
\alpha_{Z\nu W,5} &= \frac{2c_W s_W}{\Lambda^4} (f_{T_0} - f_{T_0}), \\
\alpha_{Z\nu W,6} &= \frac{c_s^2 s_W}{\Lambda^4} (f_{T_2} - f_{T_2}), \\
\alpha_{Z\nu W,7} &= \frac{c_s^2 s_W}{\Lambda^4} (f_{T_1} - f_{T_1}),
\end{align*}
\]

(2.8)

\[
\begin{align*}
\alpha_{2\nu W,0} &= \frac{\alpha^2}{s_W^2} \left( f_{M_0} + \frac{c_W}{s_W} f_{M_4} + 2 \frac{c_W}{s_W} f_{M_2} \right), \\
\alpha_{2\nu W,1} &= \frac{\alpha^2}{s_W^2} \left( \frac{1}{2} f_{M_7} + 2 \frac{c_W}{s_W} f_{M_5} - f_{M_5} - 2 \frac{c_W}{s_W} f_{M_3} \right), \\
\alpha_{2\nu W,2} &= \frac{1}{\Lambda^2} \left( s_W^2 f_{T_0} + c_W^2 f_{T_5} \right), \\
\alpha_{2\nu W,3} &= \frac{1}{\Lambda^2} \left( s_W^2 f_{T_2} + c_W^2 f_{T_5} \right), \\
\alpha_{2\nu W,4} &= \frac{1}{\Lambda^2} \left( s_W^2 f_{T_1} + c_W^2 f_{T_5} \right).
\end{align*}
\]

(2.9)

Note that the \( V_{Z\nu W,0,1,2,3,4} \) vertices and \( V_{\nu AW,0,1} \) vertices are from \( O_{M_i} \) operators and are dimension-6 vertices, and the other vertices are from \( O_{T_i} \) operators and are dimension-8 vertices.

The constraints on the coefficients of the corresponding operators from the experiments are listed in table 1.

3 Unitarity bounds

Unlike in the SM, the cross-section of the VBS process with aQGCs can grow with c.m. energy. Such feature opens a window to detect the aQGC couplings at higher energies. However, the cross-section with aQGCs will violate unitarity at certain energy. The violation of unitarity indicates that SMEFT is no longer valid to describe the phenomenon at such high energies and that new physics particles will emerge.

Consider the process \( V_1 \lambda_1 V_2 \lambda_2 \rightarrow V_3 \lambda_3 V_4 \lambda_4 \), where \( V_i \) are vector bosons, \( \lambda_i \) correspond to the helicities of \( V_i \), and therefore \( \lambda_i = \pm 1 \) for photons, and \( \lambda_i = \pm 1, 0 \) for \( W^\pm, Z \) bosons.
The amplitudes of the process can be expanded as [63, 64]

\[
\mathcal{M}(V_1, \lambda_1, W_{\lambda_2}^+ \rightarrow \gamma\lambda_3, W_{\lambda_4}^+) = 8\pi \sum_J (2J + 1) \sqrt{1 + \delta_{\lambda_1\lambda_2}} \sqrt{1 + \delta_{\lambda_3\lambda_4}} e^{i(\lambda - \lambda')\phi} d^J_{\lambda\lambda'}(\theta) T^J
\]

(3.1)

where \( V_1 \) is \( \gamma \) or \( Z \) boson, \( \lambda = \lambda_1 - \lambda_2, \lambda' = \lambda_3 - \lambda_4 \), \( \theta \) and \( \phi \) the zenith and azimuth angles of the \( \gamma \) in the final state, and \( d^J_{\lambda\lambda'}(\theta) \) are the Wigner \( d \)-functions [63]. The partial wave unitarity bound is \(|T^J| \leq 2\) [64] which is widely used in previous works [65–68].

In the following discussions, we denote \( \hat{s} = (p_{V_1} + p_{V_2})^2 \). Note that \( \sqrt{\hat{s}} \) is not the c.m. energy of protons. The strongest bounds are set by the amplitudes that grow fastest with \( \sqrt{\hat{s}} \), so it is sufficient to only keep those leading terms.

### 3.1 Partial wave expansion of \( \gamma W \rightarrow \gamma W \) process

We calculate the partial wave expansion of the \( \gamma W^+ \rightarrow \gamma W^+ \) amplitudes with one dimension-8 operators at a time. Table 2 shows the results of the leading terms. There are also leading terms which can be obtained with the relation \( \mathcal{M}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}(\theta) = (-1)^{\lambda_1-\lambda_2-\lambda_3+\lambda_4} \mathcal{M}_{-\lambda_1,-\lambda_2,-\lambda_3,-\lambda_4}(\theta) \), therefore they are not presented. Denoting \( \mathcal{M}^{fX} \) as the amplitude with only \( O_X \) operator, for \( O_{M2,3,4,5,7} \) and \( O_{T5,6,7} \), the amplitudes can be derived by using Eq. (2.9) and written as

\[
\begin{align*}
\mathcal{M}^{fM4}(\gamma W^+ \rightarrow \gamma W^+) &= \frac{c_W f_{M4}}{s_W f_{M0}} \mathcal{M}^{fM0}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fM2}(\gamma W^+ \rightarrow \gamma W^+) &= \frac{2 c_W f_{M2}}{s_W f_{M0}} \mathcal{M}^{fM0}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fM3}(\gamma W^+ \rightarrow \gamma W^+) &= \frac{2 c_W f_{M3}}{s_W f_{M1}} \mathcal{M}^{fM1}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fM5}(\gamma W^+ \rightarrow \gamma W^+) &= -\frac{2 c_W f_{M5}}{s_W f_{M1}} \mathcal{M}^{fM1}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fM7}(\gamma W^+ \rightarrow \gamma W^+) &= -\frac{1}{2} \frac{f_{M7}}{f_{M1}} \mathcal{M}^{fM1}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fT5}(\gamma W^+ \rightarrow \gamma W^+) &= \frac{c_W f_{T5}}{s_W f_{T0}} \mathcal{M}^{fT0}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fT6}(\gamma W^+ \rightarrow \gamma W^+) &= \frac{c_W f_{T6}}{s_W f_{T1}} \mathcal{M}^{fT1}(\gamma W^+ \rightarrow \gamma W^+), \\
\mathcal{M}^{fT7}(\gamma W^+ \rightarrow \gamma W^+) &= \frac{c_W f_{T7}}{s_W f_{T2}} \mathcal{M}^{fT2}(\gamma W^+ \rightarrow \gamma W^+).
\end{align*}
\]

(3.2)

The partial wave expansion of the amplitudes in the l.h.s. of Eq. (3.2) can be easily obtained from the r.h.s., therefore these are also not shown in table 2.

In table 2, the largest channels are marked with stars, which will lead to the strongest
Table 2: The partial wave expansion of the $\gamma W \to \gamma W$ scattering. We present only the leading order. Note that the leading order processes which can be directly related to the processes in this table are not presented for simplicity. The processes set the strongest bounds are marked by a ‘*’. $\theta$ and $\phi$ are zenith and azimuth angles of the $\gamma$ in the final state.

| amplitude | leading order | expansion |
|-----------|--------------|-----------|
| $\mathcal{M}(\gamma p W^+ \to \gamma^{-} W^0)$ | $\left| f_M \right|^2 \frac{e^{i2\nu^2\sin^4(\frac{\theta}{2})}}{A^2} s^2$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2 \left( \frac{3}{4} d_{1,-1} - \frac{1}{4} d_{1,-1} \right)^* $ |
| $\mathcal{M}(\gamma p W^+ \to \gamma^{-} W^0)$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2 \left( \frac{3}{4} d_{1,-1} - \frac{1}{4} d_{1,-1} \right) $ |
| $\mathcal{M}(\gamma p W^+ \to \gamma^{-} W^0)$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2 \left( \frac{3}{4} d_{1,-1} - \frac{1}{4} d_{1,-1} \right) $ |
| $\mathcal{M}(\gamma p W^+ \to \gamma^{-} W^0)$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2$ | $\frac{f_M}{A^2} e^{i2\nu^2\sin^4(\frac{\theta}{2})} s^2 \left( \frac{3}{4} d_{1,-1} - \frac{1}{4} d_{1,-1} \right) $ |

3.2 Partial wave expansion of $Z W \to \gamma W$ process

Similarly, we calculate the partial wave expansion of the $Z W^+ \to \gamma W^+$ amplitudes which are shown in Table 3.
| amplitude | leading order | expansion |
|-----------|---------------|-----------|
| \(\mathcal{M}(Z_+W_0^+ \to \gamma_-W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_+W_0^+ \to \gamma_+W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_+^+ \to \gamma_-W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_0^+ \to \gamma_+W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_0^+ \to \gamma_+W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_0^+ \to \gamma_-W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_0^+ \to \gamma_+W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_0^+ \to \gamma_-W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |
| \(\mathcal{M}(Z_0W_0^+ \to \gamma_+W_0^+)\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2\) | \(-\frac{f_{M_0} c \epsilon \epsilon e^{+} \epsilon^{-} \epsilon^{+} \epsilon^{-} \sin^4 \left(\frac{\theta}{2}\right)}{\Lambda^4} s_2^2 \left(\frac{3}{4} d_1^3 - \frac{1}{4} d_2^3\right)\) \* |

**Table 3:** The partial wave expansion of the \(ZW \rightarrow \gamma W\) scattering. We present only the leading order. Note that the leading order processes which can be directly related to the processes in this table are not presented for simplicity. The processes set the strongest bounds are marked by a \(*\). \(\theta\) and \(\phi\) are zenith and azimuth angles of the \(\gamma\) in the final state.
From Eq. (2.8), one have
\[
\mathcal{M}^{f_{m_2}}(ZW^+ \rightarrow \gamma W^+) = -\frac{f_{m_2}}{f_{m_0}} \mathcal{M}^{f_{m_0}}(ZW^+ \rightarrow \gamma W^+),
\]
\[
\mathcal{M}^{f_{m_3}}(ZW^+ \rightarrow \gamma W^+) = -\frac{f_{m_3}}{f_{m_1}} \mathcal{M}^{f_{m_1}}(ZW^+ \rightarrow \gamma W^+),
\]
\[
\mathcal{M}^{f_{r_5}}(ZW^+ \rightarrow \gamma W^+) = \frac{f_{r_5}}{f_{r_0}} \mathcal{M}^{f_{r_0}}(ZW^+ \rightarrow \gamma W^+),
\]
\[
\mathcal{M}^{f_{r_6}}(ZW^+ \rightarrow \gamma W^+) = \frac{f_{r_6}}{f_{r_1}} \mathcal{M}^{f_{r_1}}(ZW^+ \rightarrow \gamma W^+),
\]
\[
\mathcal{M}^{f_{r_7}}(ZW^+ \rightarrow \gamma W^+) = \frac{f_{r_7}}{f_{r_2}} \mathcal{M}^{f_{r_2}}(ZW^+ \rightarrow \gamma W^+),
\]
(3.4)

which are not shown in table 3. From table 3 and Eq. (3.4), the strongest bounds are
\[
\left| \frac{f_{m_0}}{\Lambda^4} \right| \leq \frac{512\pi M_W^2 s_W}{c_W e^2 v^2 s^2}, \quad \left| \frac{f_{m_1}}{\Lambda^4} \right| \leq \frac{768\pi M_W^2 s_W}{c_W e^2 v^2 s^2}, \quad \left| \frac{f_{m_2}}{\Lambda^4} \right| \leq \frac{256\pi M_W^2 s_W}{c_W e^2 v^2 s^2}, \quad \left| \frac{f_{m_3}}{\Lambda^4} \right| \leq \frac{384\pi M_W^2 s_W}{c_W e^2 v^2 s^2},
\]
\[
\left| \frac{f_{r_0}}{\Lambda^4} \right| \leq \frac{512\pi M_W M_Z s_W^2}{e^2 v^2 s^2}, \quad \left| \frac{f_{r_1}}{\Lambda^4} \right| \leq \frac{1024\pi M_W M_Z s_W^2}{e^2 v^2 s^2}, \quad \left| \frac{f_{r_2}}{\Lambda^4} \right| \leq \frac{1536\pi M_W^2 s_W}{e^2 v^2 c_W s^2},
\]
\[
\left| \frac{f_{r_3}}{\Lambda^4} \right| \leq \frac{40\pi}{c_W s_W s^2}, \quad \left| \frac{f_{r_4}}{\Lambda^4} \right| \leq \frac{24\pi}{c_W s_W s^2}, \quad \left| \frac{f_{r_5}}{\Lambda^4} \right| \leq \frac{64\pi}{c_W s_W s^2},
\]
\[
\left| \frac{f_{r_6}}{\Lambda^4} \right| \leq \frac{40\pi}{c_W s_W s^2}, \quad \left| \frac{f_{r_7}}{\Lambda^4} \right| \leq \frac{24\pi}{c_W s_W s^2}, \quad \left| \frac{f_{r_8}}{\Lambda^4} \right| \leq \frac{64\pi}{c_W s_W s^2},
\]
(3.5)

3.3 Partial wave unitarity bounds

In VBS process, the initial states are protons, therefor the $\sqrt{s}$ is a distribution. The unitarity bounds indicate that those events with large enough $\sqrt{s}$ could not be described by SMEFT correctly. However, using the operator basis we cannot distinguish the $\gamma W \rightarrow \gamma W$ process and $ZW \rightarrow \gamma W$ process, therefor we set the unitarity bounds by requiring all events satisfy the strongest bounds. From Eqs. (3.3) and (3.5), the strongest bounds are
\[
\left| \frac{f_{m_0}}{\Lambda^4} \right| \leq \frac{512\pi M_W^2 s_W}{c_W e^2 v^2 s^2}, \quad \left| \frac{f_{m_1}}{\Lambda^4} \right| \leq \frac{768\pi M_W^2 s_W}{c_W e^2 v^2 s^2}, \quad \left| \frac{f_{m_2}}{\Lambda^4} \right| \leq \frac{256\pi M_W^2 s_W}{c_W e^2 v^2 s^2}, \quad \left| \frac{f_{m_3}}{\Lambda^4} \right| \leq \frac{384\pi M_W^2 s_W}{c_W e^2 v^2 s^2},
\]
\[
\left| \frac{f_{r_0}}{\Lambda^4} \right| \leq \frac{512\pi M_W M_Z s_W^2}{e^2 v^2 s^2}, \quad \left| \frac{f_{r_1}}{\Lambda^4} \right| \leq \frac{1024\pi M_W M_Z s_W^2}{e^2 v^2 s^2}, \quad \left| \frac{f_{r_2}}{\Lambda^4} \right| \leq \frac{1536\pi M_W^2 s_W}{e^2 v^2 c_W s^2},
\]
\[
\left| \frac{f_{r_3}}{\Lambda^4} \right| \leq \frac{40\pi}{s_W c_W s^2}, \quad \left| \frac{f_{r_4}}{\Lambda^4} \right| \leq \frac{24\pi}{s_W c_W s^2}, \quad \left| \frac{f_{r_5}}{\Lambda^4} \right| \leq \frac{64\pi}{s_W c_W s^2},
\]
\[
\left| \frac{f_{r_6}}{\Lambda^4} \right| \leq \frac{40\pi}{s_W^2 s^2}, \quad \left| \frac{f_{r_7}}{\Lambda^4} \right| \leq \frac{32\pi}{c_W^2 s^2}, \quad \left| \frac{f_{r_8}}{\Lambda^4} \right| \leq \frac{64\pi}{c_W^2 s^2}.
\]
(3.6)

We use the Monte Carlo simulations to investigate the distribution of $\sqrt{s}$ with the help of MadGraph5_aMC@NLO toolkit \[69–72\]. The $\sqrt{s}$ can be obtained as $\sqrt{(p_\gamma + p_{W^+})^2}$ where $p_{\gamma,W^+}$ are momenta of the final states. We simulate the $pp \rightarrow jj\gamma W^+$ process.
The distribution of $\sqrt{s}$ at different c.m. energy of protons obtained by MC simulation of $pp \to jjW^{+}\gamma$ process with only aQGCs. The distributions of all $O_{M_i}$ and $O_{T_i}$ operators are similar, therefore they are combined.

induced by only one aQGC at a time at $\sqrt{s} = 13$, 14 [73], 27 [74], 50 [75] and 100 TeV [76]. Note that the coefficients of the operators can be factorized out and should not affect the kinematic features such as $\sqrt{s}$. We run the simulation with the coefficients at the upper bounds listed in table 1. The distributions of all $O_{M_i}$ operators are similar, so are the $O_{T_i}$ operators. But there are differences between $O_{M_i}$ operators and $O_{T_i}$ operators. The results are shown in figure 1. By requiring 95% events are within the $\sqrt{s}$, the corresponding $\sqrt{s}$ are listed in table 4.

Using the results in table 4 and Eq. (3.6), we calculate the corresponding unitarity bound of each dimension-8 operators, which is listed in table 5. From table 5, we find the following points.

- By comparing table 5 and table 1, we see that the unitarity bounds are stronger than the bounds set by experimental data. This indicates that the signals of aQGCs are
Table 4: The $\sqrt{s}$ such that 95% events are included. This result is obtained by MC simulation.

| $\sqrt{s}$ (TeV) | $\sqrt{s}_{O_{M_{t}}}$ (TeV) | $\sqrt{s}_{O_{\gamma}}$ (TeV) |
|------------------|-----------------------------|-----------------------------|
| 13               | 5.6                         | 6.1                         |
| 14               | 6.0                         | 6.6                         |
| 27               | 11.6                        | 12.6                        |
| 50               | 21.3                        | 23.4                        |
| 100              | 40.8                        | 46.2                        |

difficult to be observed by current luminosity.

- The violation of unitarity can be avoided by unitarization methods such as K-matrix unitarization \cite{77} or by putting form factors into the coefficients \cite{23, 24}, as well as via dispersion relations \cite{43, 44}. Therefore if there were aQGC signals outside of the unitarity bound, one needs to study the unitization schemes. To compare with experimental data, we present our results without unitization in this paper.

- One can compare the $\gamma\gamma \rightarrow W^+W^-$ process with our result. Because the photons are massless, the $\sqrt{s}$ of photons are much smaller and grow slowly with $\sqrt{s}$ \cite{47, 78}. Therefore, the unitarity bounds of $\gamma\gamma \rightarrow W^+W^-$ process are much looser. However, also due to smaller $\sqrt{s}$, the signal of this process is more difficult to be observed.

- Since different initial states have different energy scales, the $Z\gamma W^+W^-$ vertices and the $\gamma\gamma W^+W^-$ vertices can have different unitarity bounds if they are considered separately such as in ref. \cite{47}. Considered individually, the bounds on $\gamma\gamma W^+W^-$ vertices should be looser because the $ZW \rightarrow \gamma W$ process is more energetic than $\gamma W \rightarrow \gamma W$. In this paper, we use the operators same as those used in the analysis of experiments, therefore we cannot set different unitarity bounds on $Z\gamma W^+W^-$ and $\gamma\gamma W^+W^-$ vertices. However, it is more convenient to compare our results with the limits obtained in experiments.

4 The signal and the backgrounds of aQGCs

In this analysis, the SM process $pp \rightarrow jj\ell^+\nu\gamma$ is treated as the irreducible background, while signal is defined as the events generated with only aQGC and with one dimension-8 operator at a time and with the coefficients at the upper bounds in table 1. To investigate the change of VBS process due to the aQGCs, we generate the signal without the $s$-channel diagrams (figure 2. (b)), since those contributions are not VBS contribution therefore are meant to be cut off. The typical Feynman diagrams for the SM backgrounds can be seen in figure 3. The Feynman diagrams contributing to the SM backgrounds are often categorized as EW VBS, EW non-VBS and QCD contributions as shown in figure 3. (a), (b) and (c) respectively.
Figure 2: The typical anomalous aQGC diagrams contribute to $jj\ell^+\nu\gamma$ final states. Similar as in the SM, there are also VBS contributions as depicted in (a) and non-VBS contributions as shown in (b).

Figure 3: The backgrounds are the processes contribute to $jj\ell^+\nu\gamma$ final states in the SM. The typical EW-VBS diagrams are shown in (a), EW-non-VBS diagrams are shown in (b), and the typical QCD diagrams are shown in (c).
After fast detector simulation, the final states are not exactly $jj\ell^+\nu\gamma$. To ensure a high quality track of the $W\gamma jj$ candidate, a minimum number of composition is required. We denote the number of jets, photons and charged leptons as $N_j$, $N_\gamma$ and $N_{\ell^+}$, respectively. Events are selected by requiring the $N_j \geq 2$, $N_\gamma \geq 1$ and $N_{\ell^+} = 1$. We analyse the kinematic features and polarization features of the events after these particle number cuts.

### 4.1 Kinematic features of the signal

As introduced, in the SM the VBS process does not grow with $\sqrt{s}$, which opens a window to detect aQGC signals. To focus on the VBS process, we use the standard VBS/VBF cut [24]. We only impose $|\Delta y_{jj}|$ which is defined as the difference pseudo rapidity of the hardest two jets. The differential cross-sections as functions of $|\Delta y_{jj}|$ are shown in figure 4. (a). The distributions are similar for each class of operators (i.e. $O_{M_0}$ or $O_{T_0}$), but are different for $O_{M_i}$ and $O_{T_i}$. Therefor we only present $O_{M_0}$ and $O_{T_0}$ in figure 4. (a) as examples.

The cross-sections of the $\gamma W^+ \rightarrow \gamma W^+$ and $ZW^+ \rightarrow \gamma W^+$ grow with $\sqrt{s}$, therefore one can expect that the event numbers of the signals are larger when the $\hat{s}$ is larger. On the other hand, with an energetic $W^+$ boson, the leptons produced by the $W^+$ boson approximately fly along the direction of the $W^+$ boson in the c.m. frame, as a result, one can expect a large missing transverse momentum $|p_T|$.

| $f_{M_0}/\Lambda^4$ | $f_{M_1}/\Lambda^4$ | $f_{M_2}/\Lambda^4$ | $f_{M_3}/\Lambda^4$ | $f_{M_4}/\Lambda^4$ | $f_{M_5}/\Lambda^4$ | $f_{M_6}/\Lambda^4$ | $f_{T_0}/\Lambda^4$ | $f_{T_1}/\Lambda^4$ | $f_{T_2}/\Lambda^4$ | $f_{T_3}/\Lambda^4$ | $f_{T_4}/\Lambda^4$ |
|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| $[13 \text{ TeV}]$ | $[14 \text{ TeV}]$ | $[27 \text{ TeV}]$ | $[50 \text{ TeV}]$ | $[100 \text{ TeV}]$ |
| $\leq 1.02$         | $\leq 0.77$         | $\leq 0.055$        | $\leq 0.0049$       | $\leq 0.00036$      |
| $\leq 1.53$         | $\leq 1.16$         | $\leq 0.083$        | $\leq 0.0073$       | $\leq 0.00054$      |
| $\leq 0.28$         | $\leq 0.21$         | $\leq 0.015$        | $\leq 0.0013$       | $\leq 0.000099$     |
| $\leq 0.42$         | $\leq 0.32$         | $\leq 0.023$        | $\leq 0.0020$       | $\leq 0.00015$      |
| $\leq 0.49$         | $\leq 0.37$         | $\leq 0.027$        | $\leq 0.0023$       | $\leq 0.00017$      |
| $\leq 0.87$         | $\leq 0.66$         | $\leq 0.047$        | $\leq 0.0041$       | $\leq 0.00031$      |
| $\leq 3.47$         | $\leq 2.64$         | $\leq 0.19$         | $\leq 0.017$        | $\leq 0.0012$       |
| $\leq 0.22$         | $\leq 0.16$         | $\leq 0.012$        | $\leq 0.00099$      | $\leq 0.000065$     |
| $\leq 0.13$         | $\leq 0.094$        | $\leq 0.0071$       | $\leq 0.0000$       | $\leq 0.000039$     |
| $\leq 0.34$         | $\leq 0.25$         | $\leq 0.019$        | $\leq 0.0016$       | $\leq 0.00010$      |
| $\leq 0.12$         | $\leq 0.086$        | $\leq 0.0065$       | $\leq 0.00055$      | $\leq 0.000036$     |
| $\leq 0.071$        | $\leq 0.052$        | $\leq 0.0039$       | $\leq 0.00033$      | $\leq 0.000022$     |
| $\leq 0.19$         | $\leq 0.14$         | $\leq 0.010$        | $\leq 0.00087$      | $\leq 0.000057$     |

**Table 5:** The constraints on the coefficients by requiring the satisfaction of partial wave unitarity.
In view of the difference between the kinematic features of $O_{M_i}$ and $O_{T_i}$ operators, we therefore adopt different cuts for searching for the signals of different operators which are summarized in section 4.3.

### 4.2 Polarization features of the signal

The signals of $W\gamma jj$ induced by aQGCs have unique polarization features. One can see from tables 2 and 3, for $O_{M_i}$, the leading contributions of the signals are those with longitudinal $W^+$ bosons in the final states, while for $O_{T_i}$, both left-handed and right-handed $W^+$ bosons dominate. The polarization of the $W^+$ bosons can be inferred by the momentum of the charged leptons in the $W^+$ boson rest-frame, the so called helicity frame, as

$$
\frac{d\sigma}{d\cos\theta^*} \propto f_L \frac{(1 - \cos(\theta^*))^2}{4} + f_R \frac{(1 + \cos(\theta^*))^2}{4} + f_0 \sin^2(\theta^*),
$$

where $\theta^*$ is the angle between flight directions of $\ell^+$ and $W^+$ boson (between $p_{\ell^+}$ and the $z$-axis) in the helicity frame, and $f_L$, $f_R$ and $f_0 = 1 - f_L - f_R$ are the fraction of the left-handed, right-handed and longitudinal polarization, respectively. Since the momenta...
of the neutrinos are invisible, it is difficult to reconstruct the momenta of the $W^+$ bosons and boost the leptons to the rest frame of $W^+$ bosons. However, when the transverse momentum of the $W^+$ boson is large, $\cos(\theta^*)$ can be obtained approximately as $\cos(\theta^*) \approx 2(L_p - 1)$ with $L_p$ defined as

$$L_p = \frac{\mathbf{p}_T^f \cdot \mathbf{p}_T^W}{|\mathbf{p}_T^W|^2},$$

(4.2)

where $\mathbf{p}_T^W = \mathbf{p}_T^f + \mathbf{p}_T$. In the signal events of $O_{M_i}$ or $O_{T_i}$ operators, the polarization fractions of $W^+$ bosons in the final states are different from the SM. The polarization fractions can be categorized as four patterns, the SM pattern, the $O_{M_i}$ pattern, the $O_{T_{0,5}}$ pattern and $O_{T_{1,2,6,7}}$ pattern. Neglecting the events with $L_p \not\in [0, 1]$, the differential cross-section as functions of $L_p$ is shown in figure 5. The $O_{M_0}$, $O_{T_0}$ and $O_{T_2}$ are chosen to represent the $O_{M_i}$ pattern, the $O_{T_{0,5}}$ pattern and $O_{T_{1,2,6,7}}$ pattern correspondingly.

As presented in tables 2 and 3, the polarizations of $W^+$ bosons is related to $\theta$ which is the angle between the outgoing photon and the $z$-axis of c.m. frame of the sub-process. Since the protons are energetic, we can assume that the flight directions of initial states of the sub-processes $\gamma W^+$ and $ZW^+$ are the same as $pp$ initial states. By doing so $\theta$ could be approximately estimated by the angle between outgoing photons and $z$-axis of c.m. frame of protons, which is denoted as $\theta'$. The correlation features between $\theta'$ and $L_p$ can be used to extract the aQGC signals from the SM backgrounds. Therefore, we investigate $\theta'$.
Figure 6: The differential cross-section as functions of $L_p$ and $\cos \theta'$. Each bin corresponds to $\Delta L_p \times \Delta (\cos \theta) = 0.04 \times 0.08$ ($25 \times 25$ bins).

Figure 7: The differential cross-sections as functions of $r_{1,2}$. 
and $L_p$ for $O_{M_0}$, $O_{T_0}$ and $O_{T_2}$ as representations of the $O_{M_i}$, $O_{T_{0,5}}$ and $O_{T_{1,2,6,7}}$ operators. According to the results shown in figure 6, we define

$$r_1 = \left(1 - |\cos(\theta')|\right)^2 + \lambda_p^2, \quad r_2 = \left(1 - |\cos(\theta')|\right)^2 + (1 - L_p)^2.$$  \hspace{1cm} (4.3)

We propose to use $r_1$ and $r_2$ to discriminate the signals of $O_{T_{0,5}}$ and $O_{T_{1,2,6,7}}$ operators from the corresponding the SM backgrounds, respectively. The differential cross-sections as functions of $r_{1,2}$ of $O_{T_{0,2}}$ compared with differential cross-section of the SM are shown in figure 7. The cuts due to the polarization features are summarized in the next sub-section.

**4.3 Summary of the cuts**

For different operators, the kinematic features and polarization features are different. Therefore we propose to use the different cuts $cut^X$ to search for the certain signal of corresponding operator $O_X$. These cuts can be categorized into the following three types: $cut^{M_i}$, $cut^{T_{0,5}}$ and $cut^{T_{1,2,6,7}}$, which are summarised in table 6. We use $M_{\ell\gamma} > 800$ GeV, therefore the cut $|M_{\ell\gamma} - M_Z| > 10$ GeV is also satisfied, the latter is used to reduce the backgrounds from $Z \rightarrow \ell\ell$ with one $\ell$ mis-tagged as a photon [37], while the cut on $M_{\ell\gamma}$ we are using has similar effects.

The results of the cuts are shown in tables 7, 8 and 9. The large SM backgrounds can be effectively reduced by our selection strategy.

| $cut^{M_i}$ | $cut^{T_{0,5}}$ | $cut^{T_{1,2,6,7}}$ |
|------------|----------------|----------------|
| $|\Delta y_{jj}| > 2.0$ | $0 \leq L_p \leq 1, \ r_2 > 0.15$ | $0 \leq L_p \leq 1, \ r_2 > 0.15$ |
| $|p_T| > 120$ GeV | $|p_T| > 75$ GeV | $|p_T| > 75$ GeV |
| $M_{\ell\gamma} > 800$ GeV | $M_{\ell\gamma} > 800$ GeV | $M_{\ell\gamma} > 800$ GeV |
| $\cos(\theta_{\ell\gamma}) < 0$ | $\cos(\theta_{\ell\gamma}) < 0$ | $\cos(\theta_{\ell\gamma}) < 0$ |
| $\cos(\Delta\phi_{\gamma,m}) < -0.75$ | | |
| $\cos(\Delta\phi_{\ell,m}) > 0$ | | |

**Table 6:** The three classes of cuts we proposed.

The cross-sections induced by $O_{M_{0,1,2}}$ and $O_{T_{0,1,2}}$ operators are small because we use the coefficients constrained by the same sign $W^+W^+jj$ channel [42]. For now, we neglect the effects of the s-channel diagrams in figure 2. (b) and possible interferences. With the same luminosity and 95\% CL upper limits on $f_X/\Lambda^4$ as in ref. [42], the significance of signal $\mathcal{S}_{stat} = N_S/\sqrt{N_S + N_B}$ can be calculated, where $N_S$ and $N_B$ denote the number of the signal and background events, respectively. Take $\mathcal{S}_{stat}$ for $O_{M_0}$ as an example, we find that $\mathcal{S}_{stat}$ can only be 1.82. Such a rough estimate shows that the significance of signal of $O_{M_{0,1,7}}$ operators is smaller than those in the same sign $WWjj$ channel. Therefore, same sign $WWjj$ is a better channel to detect the signals of $O_{M_{0,1,7}}$ operators and $O_{T_{0,1,2}}$ operators than $W^\gammajj$. However, the constraints on $O_{M_{2,3,4,5}}$ operators and the $O_{T_{5,6,7}}$ operators are not given by the same sign $WWjj$ channel in ref. [42]. For this reason, we concentrate on the $O_{M_{2,3,4,5}}$ and $O_{T_{5,6,7}}$ operators in the following.
Table 7: The cut flow on $O_{M_i}$.

| Channel | no cut | $N_{j,\gamma,\ell^+}$ | $|\Delta y_{jj}|$ | $|p_T|$ | $M_{\ell\gamma}$ | $\theta_{\ell\gamma}$ | $\Delta \phi_{\nu,m}$ | $\Delta \phi_{\ell,m}$ |
|---------|--------|----------------------|-----------------|--------|-----------------|-----------------|----------------|----------------|
| SM      | 9497   | 3008                | 1166            | 93.12  | 0.301           | 0.253           | 0.222          | 0.190          |
| $O_{M_0}$ | 0.413  | 0.273               | 0.237           | 0.218  | 0.203           | 0.199           | 0.186          | 0.186          |
| $O_{M_1}$ | 0.469  | 0.313               | 0.273           | 0.255  | 0.238           | 0.233           | 0.221          | 0.221          |
| $O_{M_2}$ | 67.18  | 43.12               | 37.98           | 34.97  | 32.59           | 31.99           | 30.33          | 30.30          |
| $O_{M_3}$ | 92.42  | 60.43               | 53.46           | 49.76  | 46.64           | 45.62           | 43.83          | 43.81          |
| $O_{M_4}$ | 46.14  | 30.21               | 26.73           | 24.08  | 21.83           | 21.39           | 20.25          | 20.22          |
| $O_{M_5}$ | 184.9  | 122.1               | 107.1           | 99.41  | 92.21           | 89.89           | 85.70          | 85.66          |
| $O_{M_7}$ | 0.306  | 0.205               | 0.178           | 0.166  | 0.155           | 0.152           | 0.144          | 0.144          |

Table 8: The cut flow on $O_{T_0,5}$.

| Channel | no cut | $N_{j,\gamma,\ell^+}$ | $r_1$ | $|p_T|$ | $M_{\ell\gamma}$ | $\theta_{\ell\gamma}$ |
|---------|--------|----------------------|------|--------|-----------------|----------------|
| SM      | 9497   | 3008                | 1423 | 247.7  | 0.760           | 0.696          |
| $O_{T_0}$ | 1.01   | 0.664               | 0.562| 0.499  | 0.477           | 0.465          |
| $O_{T_5}$ | 71.24  | 46.82               | 39.85| 35.25  | 33.74           | 32.91          |

Table 9: The cut flow on $O_{T_{1,2,6,7}}$.

5 Cross-sections and significance of signals

To investigate the signals of the aQGCs, one should investigate how the cross-section is modified by adding dimension-8 operators to the SM Lagrangian so that the effects of interference are included. In this section, we investigate the $pp \rightarrow jj\ell^+\nu\gamma$ process with all Feynman diagrams including non-VBS aQGC diagrams, such as figure 2. (b), and with all possible interference effects. In this case, the total cross-sections are bilinear functions of the coefficient $f_X/\Lambda^4$. We assume that the cuts will not affect the shape of the functions which is true as will be shown later. The cross-section after cut, which is denoted as $\sigma_X$, can be approximately expressed as

$$\sigma_X = \sigma_{SM}^{cut} \left( 1 + b_X \frac{f_X}{\Lambda^4} + c_X \left( \frac{f_X}{\Lambda^4} \right)^2 \right) ,$$ (5.1)
Figure 8: The cross sections as a function of $f_{Mi}/\Lambda^4$ (a) and $f_{Tj}/\Lambda^4$ (b) by fitting the MC results to Eq. (5.1).

### Table 10: The $b_X, c_X$ obtained by fitting, where $b_X, c_X$ are undetermined coefficients in Eq. (5.1). The results of fittings are also shown in figure 8.

| Operator | $b_X$ | $c_X$ | Operator | $b_X$ | $c_X$ |
|----------|-------|-------|----------|-------|-------|
| $O_{M2}$ | 0.141 | 0.158 | $O_{T_5}$ | 0.283 | 1.13 |
| $O_{M3}$ | -0.0165 | 0.0892 | $O_{T_6}$ | -0.104 | 2.16 |
| $O_{M4}$ | 0.100 | 0.0440 | $O_{T_7}$ | 0.0400 | 0.311 |
| $O_{M5}$ | 0.0792 | 0.0730 |

where $b_X$ and $c_X$ are dimensionless coefficients. Note that for different class of cuts, $\sigma_{SM}^{cut_{M_i}} = 0.190$ fb, $\sigma_{SM}^{cut_{T_{0.5}}}$ = 0.696 fb and $\sigma_{SM}^{cut_{T_{1.2.6.7}}}$ = 0.538 fb.

### 5.1 Fitting of the cross-section

After scanning over the parameter space of $f_X/\Lambda^4$ from table 1, we can obtain the $\sigma_X$ by fitting. The results are shown in figure 8. One can find that $\sigma_X$ are indeed approximately bilinear functions. The $b_X$ and $c_X$ are listed in table 10.

Another interesting result is the effects of the s-channel aQGC diagrams and the possible interferences. In table 11, we compare the cross-section of the full signal (including s-channel aQGC and with interference) with that of only aQGC VBS processes. The results in the ‘only aQGC’ column come from tables 7, 8 and 9, and the results in the ‘full’ column are $\sigma_X - \sigma_{SM}^{cut_X}$ obtained by using MC with the coefficients at the upper limits of table 1. In all cases, one can find that $\sigma_X < \sigma_{SM}^{cut_X} + \sigma_{aQGC}$, where $\sigma_{aQGC}$ represents the cross-section of only aQGC VBS process after cuts. The results indicates that the s-channel aQGC diagrams and the possible interferences have important effects and will reduce the cross-section significantly. In figure 8, the cross-sections are approximately mirror symmet-
Table 11: The cross-sections of $\sigma_X - \sigma_{SM}^{cutX}$ (in the full column) compared with the cross-section of only VBS aQGCs (as same as the last column of tables 7, 8 and 9, and denoted as $\sigma_{aQGC}$). In all cases, one can find that $\sigma_X < \sigma_{SM}^{cutX} + \sigma_{aQGC}$.

| Operator | only aQGC | full   | Operator | only aQGC | full   |
|----------|-----------|--------|----------|-----------|--------|
| $O_{M2}$ | 30.30     | 21.22  | $O_{T_5}$| 32.91     | 14.24  |
| $O_{M3}$ | 43.81     | 34.42  | $O_{T_6}$| 24.16     | 10.06  |
| $O_{M4}$ | 20.22     | 15.11  | $O_{T_7}$| 25.31     | 10.44  |
| $O_{M5}$ | 85.66     | 62.67  |          |           |        |

Table 12: The constraints on the operators at LHC with $\mathcal{L} = 137.1$ fb$^{-1}$.

| Coefficients | $\mathcal{S}_{stat} \leq 2$ | $\mathcal{S}_{stat} \leq 3$ | $\mathcal{S}_{stat} \leq 5$ |
|--------------|-----------------------------|-----------------------------|-----------------------------|
| $f_{M_2}/\Lambda^4$ | $[-2.24, 1.35]$ | $[-2.72, 1.83]$ | $[-3.63, 2.74]$ |
| $f_{M_3}/\Lambda^4$ | $[-2.22, 2.41]$ | $[-2.88, 3.07]$ | $[-4.11, 4.30]$ |
| $f_{M_4}/\Lambda^4$ | $[-4.62, 2.34]$ | $[-5.51, 3.24]$ | $[-7.22, 4.95]$ |
| $f_{M_5}/\Lambda^4$ | $[-3.15, 2.07]$ | $[-3.86, 2.78]$ | $[-5.21, 4.12]$ |
| $f_{T_5}/\Lambda^4$ | $[-0.552, 0.327]$ | $[-0.658, 0.433]$ | $[-0.846, 0.620]$ |
| $f_{T_6}/\Lambda^4$ | $[-0.325, 0.373]$ | $[-0.416, 0.464]$ | $[-0.576, 0.624]$ |
| $f_{T_7}/\Lambda^4$ | $[-0.981, 0.854]$ | $[-1.22, 1.09]$ | $[-1.64, 1.51]$ |

Table 12: The constraints on the operators at LHC with $\mathcal{L} = 137.1$ fb$^{-1}$.

5.2 Significance of signals and constraints

By using Eq. (5.1) and table 10, the significance of signals $\mathcal{S}_{stat}$ can be easily obtained for any given luminosity. From $\mathcal{S}_{stat}$, we can calculate the constraints of the coefficients. If the significance of signal is smaller than $\mathcal{S}_{stat}$, the coefficient can be constraint as [47]

\[
f_X \Lambda^4 > -b_X - \frac{2c_X \mathcal{S}_{stat} \left( \sqrt{4 \sigma_{SM}^{cutX} \mathcal{L} + \mathcal{S}_{stat}^2} + \mathcal{S}_{stat} \right)}{\sigma_{SM}^{cutX} \mathcal{L}} + \frac{b_X^2}{2c_X},
\]

\[
f_X \Lambda^4 < -b_X + \frac{2c_X \mathcal{S}_{stat} \left( \sqrt{4 \sigma_{SM}^{cutX} \mathcal{L} + \mathcal{S}_{stat}^2} + \mathcal{S}_{stat} \right)}{\sigma_{SM}^{cutX} \mathcal{L}} + \frac{b_X^2}{2c_X},
\]

where $\mathcal{L}$ is the luminosity. The constraints on the coefficients $f_{M_2,3,4,5}$ and $f_{T_5,6,7}$ at different luminosity and different $\mathcal{S}_{stat}$ are shown in figures 9 and 10.
Figure 9: The constraints on $f_{M2,3,4,5}$ at different $\mathcal{L}$ and different $S_{\text{stat}}$.

Figure 10: The constraints on $f_{T5,6,7}$ at different $\mathcal{L}$ and different $S_{\text{stat}}$. 
The total luminosity $\mathcal{L}$ for the years 2016, 2017 and 2018 sum up to about $\mathcal{L} \approx 137.1\text{fb}^{-1}$ [79]. The constraints of coefficients at such luminosity are shown in table 12. By comparing the constraints from 8 TeV CMS experiments in table 1, our constraints reduce the allowed parameter space by almost one order of magnitude.

6 Summary

The accurate measurement of VBS processes at the LHC is very important to understand the SM and to search for BSM. In recent years, the VBS processes draw a lot of attention, and have been studied extensively. To investigate the signal of BSM, a model independent approach known as SMEFT is frequently used, and the effects of BSM show up as higher dimensional operators. The VBS processes can be used to probe dimension-8 anomalous quartic gauge-boson operators. In this paper, we focus on the effect of aQGCs in the $pp \rightarrow W\gamma jj$ process. The operators concerned are summarized, and the corresponding vertices are obtained.

We study the constraints of the coefficients of the operators by using the partial wave unitarity bounds at different energies. Due to the fact that there are massive $W^+$ or/and $Z$ bosons in the initial state of the subprocess, and that the massive particles emitting from protons can carry a large fraction of the momentum of the proton, the c.m. energy of the subprocess is found to be at the same order as the c.m. energy of protons. As a consequence, at large c.m. energy of protons, the unitarity bounds are very strict, and the constraints set by the experiments have not yet entered the unitarity bounds. The results indicate that the aQGCs are difficult to be observed by current luminosity.

To study the discovery potential of the aQGCs, we investigate the kinematic and polarization features of the signal induced by aQGCs. With the help of MC, we are able to identify several kinematic features and provide a set of efficient cuts. Based on the analysis of the polarization caused by aQGCs, we propose novel cuts to select the signal of $O_T$ operators. For $O_{M_0,1,7}$ and $O_{T_0,1,2}$ operators, we conclude that the same sign $WW$ scattering at the LHC has a better sensitivity. For the rest of the operators, i.e. the $O_{M_2,3,4,5}$ and $O_{T_5,6,7}$ operators, we study the statistical significance of signals. The expected constraints on these operators at current luminosity are presented. We find that, compared with the case at $\sqrt{s} = 8$ TeV, the constraints at 13 TeV would become less by one order of magnitude.

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