WZ Sge-type dwarf novae with multiple rebrightenings: MASTER OT J211258.65+242145.4 and MASTER OT J203749.39+552210.3

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(Received 2010; accepted 2010)

Abstract
We report on photometric observations of WZ Sge-type dwarf novae, MASTER OT J211258.65+242145.4 and MASTER OT J203749.39+552210.3 which underwent outbursts in 2012. Early superhumps were recorded in both systems. During superoutburst plateau, ordinary superhumps with a period of 0.060291(4) d (MASTER J211258) and of 0.061307(9) d (MASTER J203749) in average were observed. MASTER J211258 and MASTER J203749 exhibited eight and more than four post-superoutburst rebrightenings, respectively. In the final part of the superoutburst, an increase in the superhump periods was seen in both systems. We have made a survey of WZ Sge-type dwarf novae with multiple rebrightenings, and confirmed that the superhump periods of WZ Sge-type dwarf novae with multiple rebrightenings were longer than those of WZ Sge-type dwarf novae without a rebrightening. Although WZ Sge-type dwarf novae with multiple rebrightenings have been thought to be the good candidates for period bouncers based on their low mass ratio (q) from inferred from the period of fully grown (stage B) superhumps, our new method using the period of growing superhumps (stage A superhumps), however, implies higher q than those expected from stage B superhumps. These q values appear to be consistent with the duration of the stage A superoutbursts, which likely reflects the growth time of the 3:1 resonance. We present a working hypothesis that the small fractional superhump excesses for stage B superhumps in these systems may be explained as a result that a higher gas pressure effect works in these systems than in ordinary SU UMa-type dwarf novae. This result leads to a new picture that WZ Sge-type dwarf novae with multiple rebrightenings...
and SU UMa-type dwarf novae without a rebrightening (they are not period bouncers) are located in the same place on the evolutionary track.

Key words: accretion, accretion disks — stars: novae, cataclysmic variables — stars: dwarf novae — stars: individual (MASTER OT J211258.65+242145.4, MASTER OT J203749.39+552210.3)

1. Introduction

Cataclysmic variables (CVs) are binary star systems that have a white dwarf (primary) and a secondary which fills its Roche lobe and transfers matter to the primary. Dwarf novae (DNe) are one of subtypes of CVs. DNe have outbursts that are well understood as a release of gravitational energy caused by large mass transfer through the disk by the thermal instability. SU UMa-type dwarf novae are a subclass of DNe. They have occasional superoutbursts that are brighter and have longer durations than normal outbursts. During superoutbursts, they show light variations, which have a period few percent longer than the orbital period, called superhumps. It is believed that superhumps are caused by the tidal instability that is triggered when the disk radius expands to the critical radius for the 3:1 resonance [see e.g. Osaki (1996) for a theoretical review]. According to Kato et al. (2009a), SU UMa-type dwarf novae generally show three distinct stages of period variation of superhumps; first stage with a longer superhump period (stage A), middle stage with systematically varying periods (stage B) and final stage with a shorter superhump period (stage C).

WZ Sge-type dwarf novae are a subgroup of dwarf novae [see e.g. Bailey 1979; Downes 1990; Kato et al. 2001]. They are known as systems that have infrequent large-amplitude superoutbursts. Although the general properties of outbursts in WZ Sge-type dwarf novae can be understood with thermal-tidal disk-instability model [see e.g. Osaki 1995, Osaki, Meyer (2003) for WZ Sge], there remain features in WZ Sge-type dwarf novae whose origin is still in dispute.

WZ Sge-type dwarf novae have several characteristic properties. One is the existence of double-wave early superhumps with periods close to the orbital periods in early stage of superoutburst (Kato 2002). Patterson et al. (1981) originally suggested that these humps represent enhanced orbital humps arising from an enhanced mass transfer. Osaki, Meyer (2002) suggested that these humps arise from the 2:1 resonance. Although Patterson et al. (2002) now appears to favor the explanation by Osaki, Meyer (2002) for the origin of early superhumps, Patterson et al. (2002) suggested that an enhanced mass transfer plays a role in WZ Sge-type outbursts — post-superoutburst rebrightenings. Post-superoutburst rebrightenings [also called “echo outburst” by Patterson et al. (2002)] are often seen after superoutbursts of WZ Sge-type dwarf novae. These rebrightenings are classified by their morphology (Imada et al. 2006; Kato et al. 2009a). The mechanism of rebrightenings is still in dispute. There is a suggestion that the enhanced mass-transfer following the superoutburst cause rebrightenings (Hameury 2000; Buat-Ménard, Hameury 2002). On the other hand, it is suggested that persistence of high viscosity in the accretion disk after the termination of the superoutburst produce rebrightenings (Osaki et al. 1997, Osaki et al. 2001). Patterson et al. (2002) wrote “hot-spot eclipses establish that mass transfer is greatly enhanced during superoutburst. This may settle the debate over the origin of echo outbursts in dwarf novae”, while Osaki, Meyer (2003) reported that the conclusion by Patterson et al. (2002) was a result of mis-interpretation of the observation, and there is no evidence of an enhanced mass transfer.

According to the standard evolutionary theory, CVs evolve from a longer to shorter orbital period and finally reach a minimum orbital period when the secondary begins to be degenerate or the thermal and mass-loss time scales become comparable for the secondary. Then the secondary becomes a brown dwarf which cannot remain in hydrogen burning or becomes somewhat oversized for its mass as a result of deviation from thermal equilibrium and the orbital period becomes longer again. The systems whose periods increase after reaching the minimum period are called period bouncers [see e.g. Knigge et al. (2011) for standard evolutionary theory of CVs]. A fraction of known WZ Sge-type dwarf novae are thought to be period bouncers because of their short orbital periods (Uemura et al. 2010; Patterson et al. 2005b; Patterson 2011).

Although the mechanism of rebrightenings are unknown, the rebrightenings have been thought to be related with evolutionary stages. Imada et al. (2006) and Kato et al. (2009a) classified the rebrightenings of WZ Sge-type dwarf novae into four types by their light curve shapes: long duration rebrightenings (type-A), multiple rebrightenings (type-B), single rebrightening (type-C), and no rebrightening (type-D). Kato et al. (2009a) indicated there is a relation between these rebrightening types and $p_{\text{rot}}$ versus $P_{\text{SH}}$ [see figure 37 of Kato et al. (2009a)]. This result implies that rebrightening type generally reflects the system parameters (the nature or the evolutionary state) of the system, although different superoutbursts in the same system sometimes show different rebrightening types [AL Com: Uemura et al. (2008); WZ Sge: Patterson et al. (1981)]. In EZ Lyn, however, two superoutbursts that have been observed so far were with type-B rebrightenings (Pavlenko et al. 2007; Kato et al. 2009b; Kato et al. 2012), suggesting that the same object tends to show the same type of rebrightenings. The cases were also true for UZ Boo (Kuulkers et al. 1996; Kato et al. 2009a) and WZ Sge (1978 and 2000; Patterson et al. 1981; Patterson et al. 2002; Ishioka et al. 2002). We here assume as the starting point that WZ Sge-type dwarf novae can be generally categorized by the rebrightening type.

EG Cnc, one of WZ Sge-type dwarf novae, is known to have had a superoutburst with multiple rebrightenings
from the fractional superhump excess, defined as $M_{\text{P}}$. Patterson et al. (1998) calculated its mass ratio $q = 0.027$ from the fractional superhump excess, defined as $\varepsilon \equiv (P_{\text{sh}} - P_{\text{orb}})/P_{\text{orb}}$. Using an average white dwarf mass of 0.7 $M_\odot$, this suggests very low secondary mass $M_2 \approx 0.02 M_\odot$. After then, it has been generally considered that objects with multiple rebrightenings are good candidates of period bouncers.

In this paper, we present two WZ Sge-type dwarf novae which exhibited multiple rebrightenings. MASTER OT J211258.65+242145.4 was discovered by the Mobile Astronomical System of the Telescope-Robots (MASTER; Gorbovskoy et al. 2013) network (Denisenko et al. 2012, hereafter MASTER J211258). The quiescent counterpart was identified with a 20-th magnitude star. The large outburst amplitude (~7 mag) was suggestive of a WZ Sge-type dwarf nova (vsnet-alert 14697). MASTER OT J203749.39+552210.3 was discovered by MASTER network (Balanutsa et al. 2012, hereafter MASTER J203749). No previous outburst is known. The quiescent counterpart was identified with a 20-th magnitude star. The large outburst amplitude (more than 7 mag) was again suggestive of a WZ Sge-type dwarf nova. These objects turned out to be WZ Sge-type dwarf novae with multiple rebrightenings. We report in this paper on the development of their outbursts and superhumps and discuss the implications from the recorded variation of superhump periods. In Section 2, we briefly show a log of observations and the analysis method. In Section 3 and Section 4, we present the results of the observations of MASTER J211258 and MASTER J203749, respectively. In Section 5, we make a discussion on the results.

### Table 1. Log of observations of MASTER J211258.

| Start* | End* | mag† | error‡ | $N^\dagger$ | obs§ | sys# |
|--------|------|------|--------|------------|------|------|
| 4.4570 | 4.5690 | 13.855 | 0.004 | 203 | deM | C |
| 4.5147 | 4.6119 | 13.815 | 0.002 | 308 | Mas | C |
| 4.6663 | 4.8883 | 13.384 | 0.002 | 235 | UJH | C |
| 4.6923 | 4.8649 | 13.698 | 0.003 | 96 | BJA | V |
| 4.6932 | 4.8818 | 13.907 | 0.001 | 664 | LCO | C |
| 4.7362 | 4.8962 | 13.891 | 0.001 | 287 | SRI | C |
| 4.7637 | 4.9541 | 13.902 | 0.001 | 458 | SWI | V |
| 5.3734 | 5.5402 | 13.544 | 0.001 | 419 | DPV | C |
| 5.4514 | 5.5950 | 13.998 | 0.001 | 483 | Mas | C |
| 5.6468 | 5.8412 | 14.061 | 0.001 | 425 | LCO | C |

*B1JD−2456100.
†Mean magnitude.
‡$1-\sigma$ of the mean magnitude.
§Number of observations.
#Observer’s code.

### Table 2. Log of observations of MASTER J203749.

| Start* | End* | mag† | error‡ | $N^\dagger$ | obs§ | sys# |
|--------|------|------|--------|------------|------|------|
| 25.1165 | 25.2574 | 15.166 | 0.001 | 166 | Mas | C |
| 25.9345 | 26.1221 | 15.171 | 0.002 | 445 | KU | C |
| 25.9755 | 26.1298 | 15.236 | 0.003 | 486 | KU | C |
| 26.1294 | 26.3079 | 15.302 | 0.002 | 207 | Mas | C |
| 26.9299 | 27.1284 | 15.318 | 0.002 | 486 | KU | C |
| 28.2880 | 28.4762 | 15.308 | 0.002 | 240 | deM | C |
| 28.6062 | 28.7205 | 15.184 | 0.003 | 141 | Kra | C |
| 29.2833 | 29.4669 | 15.338 | 0.004 | 230 | deM | C |
| 29.5441 | 29.7184 | 15.247 | 0.004 | 167 | Kra | C |
| 29.9108 | 30.0583 | 15.270 | 0.005 | 304 | KU | C |
| 30.5411 | 30.7152 | 15.259 | 0.005 | 167 | Kra | C |
| 31.5406 | 31.7128 | 15.326 | 0.004 | 165 | Kra | C |
| 32.5403 | 32.7101 | 15.420 | 0.003 | 163 | Kra | C |
| 33.5390 | 33.7068 | 15.536 | 0.003 | 161 | Kra | C |
| 36.4837 | 36.6453 | 15.853 | 0.004 | 122 | UJH | C |
| 36.5406 | 36.6977 | 15.912 | 0.002 | 137 | Kra | C |
| 36.6880 | 36.8481 | 15.945 | 0.005 | 100 | GFB | C |
| 37.2884 | 37.3456 | 16.198 | 0.005 | 76 | PXR | V |
| 37.5412 | 37.6065 | 16.031 | 0.002 | 134 | Kra | C |
| 37.6903 | 37.8487 | 16.088 | 0.005 | 100 | GFB | C |
| 37.9127 | 38.1294 | 16.418 | 0.004 | 524 | KU | C |
| 38.5368 | 38.6939 | 16.851 | 0.007 | 136 | Kra | C |
| 38.9180 | 39.1222 | 18.341 | 0.021 | 500 | KU | C |
| 39.5366 | 39.6909 | 18.923 | 0.017 | 103 | Kra | C |
| 43.2612 | 43.2671 | 16.972 | 0.034 | 5 | deM | C |
| 43.5344 | 43.5405 | 16.414 | 0.013 | 5 | Kra | C |
| 43.7419 | 43.8332 | 16.395 | 0.005 | 58 | GFB | C |
| 44.2618 | 44.3051 | 16.866 | 0.007 | 30 | deM | C |
| 44.7044 | 44.8490 | 17.551 | 0.013 | 91 | GFB | C |
| 45.2640 | 45.2699 | 18.184 | 0.045 | 5 | deM | C |
| 45.6895 | 45.8433 | 18.768 | 0.033 | 86 | GFB | C |
| 46.6888 | 46.8426 | 16.502 | 0.006 | 96 | GFB | C |
| 50.2631 | 50.2720 | 17.189 | 0.016 | 7 | deM | C |
| 51.4569 | 51.4569 | 17.100 | 0.016 | 1 | MUY | C |
| 54.6967 | 54.8100 | 18.469 | 0.098 | 48 | GFB | C |
| 55.3672 | 55.3738 | 17.007 | 0.031 | 10 | PXR | V |
| 55.6799 | 55.8201 | 16.600 | 0.013 | 85 | GFB | C |
| 56.7061 | 56.8165 | 18.106 | 0.049 | 66 | GFB | C |
| 57.6870 | 57.8128 | 19.348 | 0.033 | 78 | GFB | C |

*B1JD−2456200.
†Mean magnitude.
‡$1-\sigma$ of the mean magnitude.
§Number of observations.
| Observer’s code.
#Filter. “C” means no filter (clear).
Table 4. Comparison stars for MASTER J211258.

| Observers        | Comparison star  |
|------------------|------------------|
| deM HMB          | GSC 2186.232     |
| Mhh              | GSC 2190.396     |
| OKU SWI DPV UJH  | GSC 2190.309     |
| OKU              | GSC 2190.546     |

Table 5. Comparison stars for MASTER J203749.

| Observers | Comparison star |
|-----------|-----------------|
| deM Kra   | GSC 3954.1468   |
| KU        | GSC 3954.649    |
| Mhh       | GSC 3954.2015   |

2. Observation and Analysis

The photometry log is given in tables 1 and 2. The continuation of table 1 can be seen on the electronic version. The observations are composed of those obtained by observers listed in table 3 and AAVSO observers. Comparison stars are shown in tables 4 and 5. The unfiltered CCD magnitudes are close to V for outbursting CVs.

All the observed times were corrected to barycentric Julian days (BJDs). The log of the observation is listed in table 2. Before making the analysis, we corrected zero-point differences between different observers by adding a constant to each observer.

In making period analysis, we used phase dispersion minimization (PDM) method (Stellingwerf 1978). We subtracted the global trend of the outburst light curve by subtracting smoothed light curve obtained by locally-weighted polynomial regression (LOWESS, Cleveland 1979) before making the PDM analysis. The 1-σ error in periods of the PDM analysis was determined by the methods in Fernie (1989), Kato et al. (2010).

We used a variety of bootstrapping in estimating the robustness of the result of PDM. We typically analyzed 100 samples which randomly contain 50% of observations, and performed PDM analysis for these samples. The bootstrap result is shown as a form of 90% confidence intervals in the resultant \( \theta \) statistics.

3. MASTER OT J211258.65+242145.4

3.1. Overall Light Curve

Figure 1 shows the overall light curve of MASTER J211258. The object was first detected in superoutburst on June 24 (BJD 2456104). Although the early rise was hardly observed, the recorded possible maximum brightness of \( V = 13.72 \) on about BJD 2456104.57 and the long duration of existence of early superhumps (subsection 3.2) suggest that the detection was made within a few days of the start of the superoutburst. The main superoutburst lasted until BJD 2456125, followed by a rapid decline. The object then faded below \( V = 18 \). On BJD 2456130, the first rebrightening (\( V = 16.32 \) at maximum) was recorded. Similar rebrightenings repetitively occurred eight times in total.

The light curve is composed of different segments: BJD 2456104–2456116, during which early superhumps were present, BJD 2456116–2456126, during which superhumps were present, and the following eight rebrightenings starting on BJD 2456130.

3.2. Early Superhumps

Since the orbital period is considered to be very close to the early superhump period (Ishioka et al. 2002b), we used the period of the early superhump as the orbital period.

The early superhumps were fortunately well recorded. The doubly humped early superhumps with a period of 0.059732(3) d were recorded (figure 2) during the earliest 12 nights of observations. The mean amplitude of early superhumps was \( \sim 0.05 \) mag. Figure 3 shows the nightly variation of the profile of early superhumps.

3.3. Ordinary Superhumps

The fractional superhump excess, an observational measure of the precession rate of the accretion disk, is defined by \( \epsilon \), as mentioned in the introduction.

A period analysis indicated the presence of a very stable period of 0.060291(4) d during BJD 2456116–2456126 (figure 4). Figure 5 shows the nightly variation of the profile of ordinary superhumps. The amplitude of superhumps was 0.1–0.2 mag. They became larger until BJD 2456119, and then became smaller. The fractional superhump excess was 0.009376(7).

3.4. \( O-C \) Analysis of Ordinary Superhumps

We determined the times of maxima of ordinary superhumps in the way given in Kato et al. (2009a). The resultant times are listed in table 6.

The \( O-C \) curve of MASTER J2112 is shown in figure 6. The stage A (\( E \leq 38 \)) and stage B (\( E \geq 50 \)) are clearly seen. In stage A, superhumps have a long period and the period is decreasing. In stage B, \( P_{\text{dot}} (=\dot{P}/P) \) is almost zero. The mean periods for these stages were 0.06158(5) d (stage A) and 0.060221(9) d (stage B), respectively.
Table 3. Observers and equipment.

| Code | Observer/Site          | Equipment                                      |
|------|------------------------|------------------------------------------------|
| Mas  | G. Masi                | 43.2 cm telescope and STL6303E                 |
| deM  | E. de Miguel           | 28cm telescope and QSI-518 wsg                |
| UJH  | J. Ulowetz             | 23.5cm telescope and QSI-583 wsg              |
| LCO  | C. Littlefield         | -                                              |
| KU   | Kyoto University       | 40cm telescope and ST-9 CCD                   |
| GFB  | W. Goff                | -                                              |
| Kra  | T. Krajci              | 28cm telescope                                 |
| Mhh  | H. Maehara             | 25cm telescope and ST-7XME                     |
| SWI  | W. Stein               | 35.56 cm telescope and ST10-XME               |
| SRI  | R. Sabo                | -                                              |
| OKU  | Osaka Kyoiku U.        | 51cm telescope and ST-10 CCD                  |
| DPV  | P. Dubovsky            | 28 cm telescope and Meade DSI ProII CCD       |
| DKS  | S. Dvorak              | -                                              |
| HMB  | F.-J. Hambsch          | 40cm telescope and FLI ML16803 CCD            |
| PXR  | R. Pickard             | -                                              |
| MEV  | E. Morelle             | -                                              |
| PSD  | S. Padovan             | -                                              |
| MUY  | E. Muyllaert           | -                                              |

*Observer’s code used in table 1 and table 2.

Fig. 2. Early superhumps in MASTER J211258 (BJD 2456104–2456116). (Upper): PDM analysis. (Lower): Phase-averaged profile.

Fig. 3. Nightly variation of the profile of early superhumps in MASTER J211258.
The $P_{\text{dot}}$ for stage B ($E \geq 50$) was $+0.8(1.0) \times 10^{-5}$. The peak of the superhump amplitudes is close to the time of stage A-B transition. A major increase in the period took place during the final part of stage B ($E \geq 50$).

During stage A, superhumps with a period of 0.06158(5) d were observed and the fractional superhump excess was 0.0310(1).

### 4. MASTER OT J203749.39+552210.3

#### 4.1. Overall Light Curve

Figure 7 shows the overall light curve of MASTER J203749. The object was first detected in superoutburst ($V = 15.1$ at maximum) on October 22 (BJD 2456225). The short duration of the recorded early superhump stage suggests that the earlier part of the outburst was missed. The main superoutburst lasted until BJD 2456238, followed by a rapid decline. The object then faded below $V = 19$. On BJD 2456243, the first rebrightening ($V = 16.37$ at maximum) was recorded. Similar rebrightenings repetitively occurred at least four times in total, although the number of observations was not sufficient to detect other potential rebrightenings.

The object developed early superhumps during BJD...
Table 6. Times of superhump maxima in MASTER J211258 (continued).

| E  | maximum time$^*$ | error  | $O - C$$^†$ | $N$$^‡$ |
|----|------------------|--------|-------------|--------|
| 113| 56122.3643       | 0.0008 | 0.0010      | 49     |
| 114| 56122.4239       | 0.0005 | 0.0003      | 64     |
| 115| 56122.4819       | 0.0004 | −0.0022     | 63     |
| 116| 56122.5432       | 0.0007 | −0.0012     | 44     |
| 117| 56122.7844       | 0.0007 | −0.0016     | 146    |
| 118| 56122.8440       | 0.0007 | −0.0023     | 148    |
| 119| 56122.9054       | 0.0006 | −0.0013     | 136    |
| 120| 56123.6886       | 0.0010 | −0.0029     | 83     |
| 121| 56123.7474       | 0.0009 | −0.0022     | 140    |
| 122| 56123.8089       | 0.0009 | −0.0026     | 169    |
| 123| 56123.8698       | 0.0011 | −0.0029     | 175    |
| 124| 56123.9302       | 0.0016 | −0.0029     | 66     |
| 125| 56124.2904       | 0.0010 | −0.0049     | 92     |
| 126| 56124.3592       | 0.0037 | 0.0036      | 43     |
| 127| 56124.4106       | 0.0009 | −0.0055     | 52     |
| 128| 56124.4721       | 0.0011 | −0.0043     | 117    |
| 129| 56124.5310       | 0.0007 | −0.0058     | 102    |
| 130| 56124.5881       | 0.0013 | −0.0091     | 58     |
| 131| 56124.6566       | 0.0058 | −0.0009     | 58     |
| 132| 56124.7135       | 0.0033 | −0.0044     | 64     |
| 133| 56124.7704       | 0.0026 | −0.0079     | 64     |
| 134| 56124.8337       | 0.0029 | −0.0049     | 65     |
| 135| 56124.8888       | 0.0026 | −0.0102     | 72     |
| 136| 56125.2584       | 0.0007 | −0.0028     | 66     |
| 137| 56125.3804       | 0.0010 | −0.0016     | 43     |
| 138| 56125.4347       | 0.0010 | −0.0077     | 59     |
| 139| 56125.4960       | 0.0014 | −0.0068     | 88     |
| 140| 56125.5565       | 0.0012 | −0.0067     | 83     |
| 141| 56125.6205       | 0.0021 | −0.0030     | 65     |
| 142| 56125.7410       | 0.0044 | −0.0033     | 20     |
| 143| 56125.7947       | 0.0033 | −0.0100     | 29     |
| 144| 56125.8617       | 0.0061 | −0.0034     | 23     |
| 145| 56126.2210       | 0.0012 | −0.0063     | 66     |
| 146| 56126.2866       | 0.0018 | −0.0010     | 49     |
| 147| 56127.2114       | 0.0016 | 0.0181      | 66     |
| 148| 56127.2615       | 0.0025 | 0.0078      | 66     |

$^*$BJD−2400000.
$^†C = 2456115.5410 + 0.060375E$.
$^‡$Number of points used to determine the maximum.

Fig. 5. Nightly variation of the profile of ordinary superhumps in MASTER J211258.

2456225–2456227. During BJD 2456227–2456238, superhumps appeared.

4.2. Early Superhumps

It was likely that the final stage of early superhumps was observed. The mean period of 0.06051(18) d was recorded during BJD 2456225–2456227 (figure 8). We identified this period to be the orbital period. Figure 9 shows the nightly variation of the profile of early superhumps. The data give a larger error than in MASTER J211258 because a period of the observations was short than in MASTER J211258.

4.3. Ordinary Superhumps

Ordinary superhumps with a period of 0.061307(9) d developed during BJD 2456227–2456238 (figure 10). Figure 11 shows the nightly variation of the profile of ordinary superhumps. The growth of ordinary superhumps was clearly seen. The amplitude increased until around BJD 2456231, and then became smaller. The fractional superhump excess was 0.01334(15).
Fig. 6. The $O-C$ curve of MASTER J211258 during the superoutburst. (Upper:) $O-C$ diagram of superhumps in MASTER J211258. An ephemeris of BJD 2456115.541+0.060375$E$ was used to draw this figure. (Lower:) Light curve. The data were binned to 0.01 d.

Fig. 7. Overall light curve of MASTER J203749. The data were binned to 0.01 d.

Fig. 8. Early superhumps in MASTER J203749 (BJD 2456225–2456227). (Upper): PDM analysis. (Lower): Phase-averaged profile.

4.4. $O-C$ Analysis of Ordinary Superhumps

The times of superhump maxima are listed in table 7. The $O-C$ curve of MASTER OT J2037 is shown in figure 12, clearly composed of stages A ($E \leq 30$) and B ($E \geq 36$). The mean periods for these stages were 0.06271(11) d (stage A) and 0.061307(9) d (stage B), respectively.

Disregarding stage A superhumps ($E \leq 30$), a marginally positive $P_{\text{dot}}$ of $+2.9(1.0) \times 10^{-5}$ was recorded. A major increase in the period also took place during the final part of stage B ($E \geq 36$).

During stage A, superhumps with a period of 0.06271(11) d were recorded and the fractional superhump excess was 0.0365(2).

The analysis showed $P_{\text{dot}}$ during stage B of both MASTER J211258 and MASTER J203749 were positive. The positive period derivatives were reported in some dwarf novae including WZ Sge-type dwarf novae. The mechanism that causes positive period derivatives of superhump periods is unknown.
Table 7. Times of superhump maxima in MASTER J203749.

| E | maximum time | error  | O – C | N   |
|---|--------------|--------|-------|-----|
| 0 | 56228.33     | 0.0012 | −0.0186 | 62 |
| 1 | 56228.39     | 0.0013 | −0.0168 | 62 |
| 2 | 56228.46     | 0.0014 | −0.0134 | 54 |
| 3 | 56228.64     | 0.0006 | −0.0146 | 60 |
| 4 | 56228.70     | 0.0007 | −0.0122 | 51 |
| 5 | 56229.28     | 0.0050 | 0.0095  | 23 |
| 6 | 56229.33     | 0.0006 | 0.0030  | 59 |
| 7 | 56229.39     | 0.0005 | 0.0037  | 62 |
| 8 | 56229.46     | 0.0006 | 0.0054  | 44 |
| 9 | 56229.58     | 0.0003 | 0.0063  | 47 |
| 10| 56229.64     | 0.0003 | 0.0062  | 47 |
| 11| 56229.70     | 0.0006 | 0.0049  | 37 |
| 12| 56229.95     | 0.0006 | 0.0079  | 125 |
| 13| 56230.01     | 0.0025 | 0.0005  | 75 |
| 14| 56230.57     | 0.0003 | 0.0080  | 46 |
| 15| 56230.63     | 0.0003 | 0.0082  | 47 |
| 16| 56230.69     | 0.0003 | 0.0074  | 47 |
| 17| 56231.55     | 0.0005 | 0.0030  | 15 |
| 18| 56231.61     | 0.0004 | 0.0037  | 70 |
| 19| 56231.67     | 0.0006 | 0.0077  | 47 |
| 20| 56232.53     | 0.0032 | 0.0076  | 15 |
| 21| 56232.59     | 0.0005 | 0.0011  | 47 |
| 22| 56232.65     | 0.0006 | 0.0014  | 47 |
| 23| 56232.71     | 0.0010 | 0.0019  | 26 |
| 24| 56233.57     | 0.0004 | 0.0010  | 47 |
| 25| 56233.63     | 0.0005 | 0.0004  | 47 |
| 26| 56233.69     | 0.0007 | 0.0018  | 41 |
| 27| 56236.51     | 0.0017 | 0.0043  | 35 |
| 28| 56236.57     | 0.0009 | 0.0016  | 80 |
| 29| 56236.64     | 0.0012 | 0.0015  | 70 |
| 30| 56236.69     | 0.0014 | 0.0063  | 46 |
| 31| 56236.76     | 0.0008 | 0.0051  | 30 |
| 32| 56236.81     | 0.0019 | 0.0076  | 32 |
| 33| 56237.55     | 0.0014 | 0.0048  | 32 |
| 34| 56237.62     | 0.0011 | 0.0034  | 42 |
| 35| 56237.68     | 0.0010 | 0.0034  | 46 |
| 36| 56237.74     | 0.0015 | 0.0050  | 29 |
| 37| 56237.80     | 0.0015 | 0.0031  | 32 |
| 38| 56237.92     | 0.0023 | 0.0049  | 82 |
| 39| 56237.99     | 0.0019 | 0.0033  | 104 |
| 40| 56238.06     | 0.0038 | 0.0062  | 118 |
| 41| 56238.12     | 0.0035 | 0.0048  | 95 |
| 42| 56238.55     | 0.0009 | 0.0116  | 34 |
| 43| 56238.61     | 0.0016 | 0.0022  | 43 |
| 44| 56238.67     | 0.0023 | 0.0019  | 42 |
| 45| 56238.99     | 0.0028 | 0.0141  | 121 |
| 46| 56239.09     | 0.0039 | −0.0079 | 121 |
| 47| 56239.58     | 0.0034 | −0.0078 | 32 |
| 48| 56239.65     | 0.0059 | 0.0023  | 32 |

*BJD−2400000.
†C = 2456228.358 + 0.060135E.
‡Number of points used to determine the maximum.
Table 8. The systems with the increase of brightening during evolutionary stage of ordinary superhumps.

| Object      | Year | Rebrightening | Reference                                      |
|-------------|------|---------------|------------------------------------------------|
| V466 And    | 2008 | A             | Kato et al. (2009a)                            |
| NN Cam      | 2009 | -             | Kato et al. (2010)                             |
| EG Cnc      | 1996 | B             | Patterson et al. (1998)                        |
| AL Com      | 1995 | A             | Nogami et al. (1997), Patterson et al. (1996), Kato et al. (2009a) |
| V592 Her    | 2010 | -             | Kato et al. (2009a)                            |
| EZ Lyn      | 2010 | B             | Kato et al. (2012)                             |
| WZ Sge      | 2001 | A             | Ishioka et al. (2002a), Kato et al. (2009a)    |

*Classification of the rebrightenings of WZ Sge-type stars (subsection 5.6).
†Possible increase.

Fig. 10. Ordinary superhumps in MASTER J203749 (BJD 2456227–2456238). (Upper): PDM analysis. (Lower): Phase-averaged profile.

Fig. 11. Nightly variation of the profile of ordinary superhumps in MASTER J203749.
significantly too small for its short duration of stage A superhumps (stage A superhumps were not identified in this object; we estimated the duration of stage A to be less than 3 d from the epochs of early superhumps and fully grown superhumps). We propose that $q$ of EG Cnc should be re-examined.

5.3. Mass-Ratio from Stage A Superhumps

Quite recently, Osaki, Kato (2013), proposed an important interpretation: the dynamical precession rate at the 3:1 resonance is represented in the growing stage of superhumps (stage A) when the eccentric wave is still confined to the location of the resonance. Kato, Osaki (2013) systematically studied stage A superhumps in SU UMa-type dwarf novae and confirmed that the mass ratios determined from fractional superhump excesses of stage A superhumps are in good agreement with those determined from quiescent eclipse observations or radial-velocity study. Kato, Osaki (2013) also confirmed that the evolutionary sequence using these mass ratios is in good agreement with the most recent evolutionary sequence determined from modern precise eclipse observations [such as Littlefair et al. (2008)]. Having the theoretical background (Osaki, Kato 2013), we used this relation to estimate the mass ratio.

This relation gives mass ratios of MASTER J211258 and MASTER J203749 $q = 0.081(2)$ and $q = 0.097(8)$ respectively.

The traditional method to estimate mass ratios from fractional superhump excesses of stage B superhumps, using

$$\epsilon = 0.16(2)q + 0.25(7)q^2$$

(Kato et al. 2009a) or

$$\epsilon = 0.18q + 0.29q^2$$

(Patterson et al. 2005a), gives mass ratios of MASTER J211258 and MASTER J203749 $q = 0.054$ and $q = 0.080$, respectively.

The mass ratios from stage A superhumps are higher than the mass ratios from stage B superhumps. This discrepancy will be discussed in subsection 5.7.

5.4. Duration of the Early Superhump Stage

The early superhumps were observed for 12 d in MASTER J211258 and for 3 d in MASTER J203749 (we must note that MASTER J203749 was not detected sufficiently early, and this duration is a lower limit of the actual duration). Table 9 shows the duration of the early superhump stage of well-observed WZ Sge-type dwarf novae.

Early superhumps are thought to be a manifestation of the 2:1 resonance (Osaki, Meyer 2002). During the stage of early superhumps, the 2:1 resonance is dominant since the development of the 2:1 resonance is expected to suppress the 3:1 resonance (Lubow 1991, Osaki, Meyer 2003). Therefore the duration of the early superhump stage is expected to be dependent on the disk mass at beginning of the outburst and the disk radius of 2:1 resonance relative to the Roche lobe. It is expected that small $q$ and large disk mass lead to long duration of the early superhump stage because the disk radius of 2:1 resonance is related to $q$.

In EG Cnc, this picture suggests that the comparatively short duration (less than 10 d, even if we consider the maximum duration of the observational gap) of the early superhump stage appears to be incompatible with the very small mass ratio $q = 0.027$ estimated by Patterson et al. (1998). Either actual $q$ may be larger or the disk mass of EG Cnc at the start of the superoutburst may have been exceptionally small.

5.5. Period in Post-Superoutburst State

During final part of the superoutburst, an increase in the superhump period was seen both in MASTER J211258 and MASTER J203749. The tendency is similar to that of EZ Lyn, the eclipsing WZ Sge-type dwarf nova with multiple rebrightenings (Kato et al. 2012). The $O-C$ diagram of EZ Lyn is shown in figure 13.

Lubow (1992) suggested that disk precession is due to a combination of effects of direct axisymmetric tidal forces from secondary, which is the most important, gas pressure in the eccentric mode and resonant wave stresses. Pressure forces act to decrease the precession rate. The tidal effect dominates for superhump binaries and produces a net prograde precession. The gas pressure effect produces a retrograde contribution and decrease the precession rate. The inward propagation of the eccentricity
Table 9. Duration of the early superhump stage.

| Object     | Year | Duration* | References                  |
|------------|------|-----------|-----------------------------|
| EG Cnc     | 1996 | 2–10      | Matsumoto, Schmeer (1996)   |
| ASAS J0233 | 2006 | 8         | Kato et al. (2009a)         |
| V455 And   | 2007 | 7         | Kato et al. (2009a)         |
| V466 And   | 2008 | 11        | Kato et al. (2009a)         |
| AL Com     | 1995 | 5         | Kato et al. (1996)          |
| DV Dra     | 2005 | 5†        | Kato et al. (2009a)         |
| BW Scl     | 2011 | 10        | Kato et al. (2013a)         |
| WZ Sge     | 2001 | 11        | Ishioka et al. (2002c)      |
| OT J0120   | 2010 | 11        | Kato et al. (2010)          |

*Unit d.
†”[]” represents the lower limit.

wave is accompanied by an increase of the pressure effect. Therefore the pressure effect is larger in stage B than in stage A. Furthermore, it is expected that the decrease in the pressure effect due to the transition to a cool state leads to the increase in the precession rate during final part of the superoutburst, which results in the increase in the superhump period. The increase in the period in MASTER J211258 and MASTER J203749 during the final part of superoutburst is exactly what is expected for this interpretation.

From Kato, Osaki (2013), Kato et al. (2013b), the fractional superhump excesses (in frequency unit) of the stage A and post-superoutburst superhumps are expressed as follows:

\[ \epsilon^*(\text{post}) = Q(q)R(r_{\text{post}}) \] (3)

and

\[ \epsilon^*(\text{stage A}) = Q(q)R(r_{3:1}) \] (4)

where \( r_{3:1} \) is the radius of 3:1 resonance

\[ r_{3:1} = 3(-2/3)(1 + q)^{-1/3} \] (5)

\[ \epsilon^* = 1 - P_{\text{orb}}/P_{\text{SH}} \]

\( r_{\text{post}} \) is the dimensionless disk radius immediately after the outburst measured in units of binary separation \( A \).

\[ Q(q) = \frac{1}{2} \frac{q}{\sqrt{1 + q}} \] (6)

and

\[ R(r) = \frac{1}{2} \sqrt{r} b^{(1)}_{3/2}(r) \] (7)

where \( b^{(j)}_{s/2} \) is the Laplace coefficient

\[ \frac{1}{2} b^{(j)}_{s/2}(r) = \frac{1}{2\pi} \int_0^{2\pi} \frac{\cos(j\phi)\,d\phi}{(1 + r^2 - 2r \cos\phi)^{s/2}} \] (8)

Using the relation as follows:

\[ \frac{\epsilon^*(\text{stage A})}{\epsilon^*(\text{post})} = \frac{R(r_{3:1})}{R(r_{\text{post}})} \] (9)

we can obtain \( R(r_{\text{post}}) \). In MASTER J211258, post-superoutburst superhumps with a period of 0.06050(6) d were recorded in the final part of the superoutburst (BJD 2456124.5–2456126.5). The equation (3) gives \( r = 0.33 \) using \( q \) from stage A superhump. Kato et al. (2013) showed 0.30 ≤ \( r \) ≤ 0.38 for the post-superoutburst state of WZ Sge-type dwarf novae. The agreement appears to be good.

5.6. Post-Superoutburst Rebrightenings

The rebrightenings of MASTER J211258 are considered to be type-B rebrightenings based on the classification by Imada et al. (2006). Figure 14 shows \( P_{\text{dot}} \) versus \( P_{\text{SH}} \), including the data reported in Kato et al. (2009a) and Nakagawa et al. (2013). The filled circles, filled squares, filled triangles, and open circles represent type-A, type-
Fig. 14. $P_{\text{SH}}$ versus $P_{\dot{P}}$ for each type of rebrightenings. The star marks represent the two MASTER objects studied in this work. The other data are from Kato et al. (2009a). The filled circles, filled squares, filled triangles, and open circles represent type-A, type-B, type-C and type-D, respectively.

Table 10. WZ Sge-type dwarf novae with multiple rebrightenings. The data are from Kato et al. (2009a), Kato et al. (2012), Kato et al. (2013a) and Mroz et al. (2013).

| Object      | Year | $N_{\text{reb}}$ | $P_{\text{SH}}$ |
|-------------|------|------------------|-----------------|
| UZ Boo      | 1994 | 2                | 0.061743(38)    |
| UZ Boo      | 2003 | 4                | 0.061922(33)    |
| DY CMi      | 2008 | 6                | 0.060736(9)     |
| EG Cnc      | 1996 | 6                | 0.060337(6)     |
| AL Com      | 2008 | 4†               | 0.057174(6)     |
| VX For      | 2009 | 5                | 0.061327(12)    |
| EZ Lyn      | 2006 | 11               | 0.059537(31)    |
| EZ Lyn      | 2010 | 6                | 0.059630(16)    |
| EL UMa      | 2010 | 4†               | 0.06045(6)†     |
| 1RXs J0232  | 2007 | 4                | 0.066166(11)    |
| OGLE-GD-DN-001 | 2007 | 4               | 0.06072(2)      |
| MASTER J2037 | 2012 | 3†              | 0.061307(9)     |
| MASTER J2112 | 2012 | 8               | 0.060291(4)     |

*Number of rebrightenings.
†“]” represents the lower limit.
‡During the rebrightening phase.

Table 11. WZ Sge-type dwarf novae without a rebrightening.

| Object      | Year | $P_{\text{SH}}$ |
|-------------|------|-----------------|
| SV Ari      | 2011 | 0.055524(14)    |
| V455 And    | 2007 | 0.057133(10)    |
| V466 And    | 2008 | 0.057203(15)    |
| V592 Her    | 1998 | 0.056498(13)    |
| V592 Her    | 2010 | 0.056607(16)    |
| V1108 Her   | 2004 | 0.057480(34)    |
| GW Lib      | 2007 | 0.054095(10)    |
| BW Sco      | 2011 | 0.055000(8)     |
| HV Vir      | 2002 | 0.058260(17)    |
| SDSS J1339  | 2011 | 0.058094(7)     |
| OT J0238    | 2008 | 0.053658(7)     |
| OT J2138    | 2010 | 0.055019(12)    |
| OT J2109    | 2011 | 0.060045(26)    |

Table 13. Fractional superhump excesses (in frequency unit) from stage A superhumps for SU UMa-type dwarf novae without a rebrightening with the period range of 0.05875–0.06458 d.

| Object      | $P_{\text{orb}}$ | $P_{\text{SH}}$ | $\epsilon$ |
|-------------|------------------|-----------------|-------------|
| V1040 Cen   | 0.06030          | 0.06243(6)      | 0.0341(10)  |
| WX Cet      | 0.05826          | 0.06029(8)      | 0.0336(14)  |
| PU CMa      | 0.05669          | 0.05901(21)     | 0.0392(37)  |
| MM Hya      | 0.05759          | 0.06163(24)     | 0.0656(42)  |
| SW UMa      | 0.05681          | 0.05893(8)      | 0.0360(14)  |
| SDSS J1610  | 0.05687          | 0.05881(9)      | 0.0330(16)  |
| OT J1044    | 0.05909          | 0.06084(2)      | 0.0287(3)   |
| OT J2109    | 0.05865          | 0.06087(6)      | 0.0364(10)  |

*Unit d.
†Fractional superhump excess (in frequency unit).
Fig. 16. $P_{\text{orb}}$ versus $\epsilon^*$ for stage A superhumps. The data are from our result, Kato et al. (2009a), Kato et al. (2012), and Kato et al. (2013a). Filled circles and filled stars represent SU UMa-type dwarf novae without a rebrightening and WZ Sge-type dwarf novae with multiple rebrightenings, respectively.

Fig. 17. $P_{\text{orb}}$ versus $\epsilon^*$ for stage B superhumps. The data are from our result, Kato et al. (2009a), Kato et al. (2012), and Kato et al. (2013a). Filled circles and filled stars represent SU UMa-type dwarf novae without a rebrightening and WZ Sge-type dwarf novae with multiple rebrightenings, respectively.

Table 12. Fractional superhump excesses (in frequency unit) of WZ Sge-type dwarf novae with multiple rebrightenings.

| Object   | $P_{\text{orb}}$ | $P_{\text{SH}}^\dagger$(stage B) | $\epsilon^*$(stage B) | $P_{\text{SH}}^\dagger$(stage A) | $\epsilon^*$(stage A) |
|----------|------------------|----------------------------------|------------------------|----------------------------------|------------------------|
| UZ Boo   | -                | 0.061831(36)                    | -                      | 0.06354(24)                      | -                      |
| DY CMi   | -                | 0.060736(9)                     | -                      | -                                | -                      |
| EG Cnc   | 0.05997          | 0.060337(6)                     | 0.00608(10)            | -                                | -                      |
| AL Com   | 0.056668         | 0.057174(6)                     | 0.00885(11)            | 0.05795(18)                      | 0.0221(32)             |
| VX For   | -                | 0.061327(12)                    | -                      | -                                | -                      |
| EZ Lyn   | 0.059005         | 0.059584(24)                    | 0.00970(41)            | 0.06077(7)                       | 0.0290(12)             |
| 1RXS J0232| -                | 0.066166(11)                    | -                      | -                                | -                      |
| MASTER J2037 | 0.06051    | 0.061307(9)                     | 0.01316(15)            | 0.06271(11)                      | 0.0352(18)             |
| MASTER J2112 | 0.05973    | 0.06029(4)                      | 0.00929(67)            | 0.06158(5)                       | 0.0301(8)              |

|$^\dagger$Unit d.
|$^\dagger$Fractional superhump excess (in frequency unit).
5.7. Evolutionary State

Table 12 lists fractional superhump excesses (in frequency unit) of WZ Sge-type dwarf novae with multiple rebrightenings (type-B). We find the 95% quantile of superhump periods of WZ Sge-type dwarf novae with multiple rebrightenings between 0.05875 and 0.06458 d (figure 15). In this period region, there are many SU UMa-type stars without a rebrightening. We therefore make a comparison between them and WZ Sge-type dwarf novae with multiple rebrightenings. The fractional superhump excesses (in frequency unit) from stage A superhumps and stage B superhumps are also listed in table 13 and table 14, respectively.

Figure 16 and figure 17 show the relation between $P_{\text{orb}}$ versus $\epsilon^*$ for WZ Sge-type dwarf novae with multiple rebrightenings (listed in table 12) and SU UMa-type dwarf novae (listed in tables 13 and 14).

Figure 16 shows $\epsilon^*$ from stage A superhumps of SU UMa-type dwarf novae without a rebrightening (filled circles) and WZ Sge-type dwarf novae with multiple rebrightenings (filled stars). Figure 17 shows $\epsilon^*$ from stage B superhumps of SU UMa-type dwarf novae without a rebrightening (filled squares) and WZ Sge-type dwarf novae with multiple rebrightenings (filled triangles).

$\epsilon^*$ from stage A superhumps of WZ Sge-type dwarf novae with multiple rebrightenings are located in a region with a little smaller $\epsilon^*$ than those of SU UMa-type dwarf novae without a rebrightening. Using the relation between $\epsilon^*$ of stage A superhumps and $q$ (subsection 5.3), $q$ of WZ Sge-type dwarf novae with multiple rebrightenings are estimated close to (only slightly smaller than) those of SU UMa-type dwarf novae without a rebrightening. However, they are two different populations using $\epsilon^*$ from stage B superhumps. $\epsilon^*$ from stage B superhumps of WZ Sge-type dwarf novae with multiple rebrightenings are clearly smaller than that of SU UMa-type dwarf novae without a rebrightening.

Murray (2000) formulated the hydrodynamical precession $\omega$ in the form:

$$\omega = \omega_{\text{dyn}} + \omega_{\text{pres}}$$

where $\omega_{\text{dyn}}$ and $\omega_{\text{pres}}$ are the dynamic precession and the pressure contribution to the precession. $\omega_{\text{pres}}/\omega_{\text{orb}}$ corresponds to $\epsilon^*_A - \epsilon^*_B$, where $\epsilon^*_A$ and $\epsilon^*_B$ are $\epsilon^*$ from stage A and that of stage B, respectively. $\epsilon^*_A - \epsilon^*_B$ is generally similar between those two types of systems. It is shown that $\epsilon^*_A - \epsilon^*_B \approx 0.010$ in SU UMa-type dwarf novae without a rebrightening and $\approx 0.015$ in WZ Sge-type dwarf novae with multiple rebrightenings.

We consider two possible interpretations of this result of great interest. The first interpretation, $q$ from stage B superhumps is indeed the case and that $q$ from stage A does not reflect the true $q$. The second interpretation, $q$ from stage B superhumps does not reflect the true $q$. If the former interpretation is correct, the small $q$ of WZ Sge-type dwarf novae with multiple rebrightenings suggest that the systems are good candidates for period bouncers. If the latter interpretation is the case, there is insignificant difference in $q$ between WZ Sge-type dwarf novae.
with multiple rebrightenings and SU UMa-type dwarf novae without a rebrightening. In the latter interpretation, small \( q \) from stage B superhumps may be explained as a result of a stronger pressure effect during stage B. These two possibilities should be tested by future studies. We, however, have no idea why growing superhumps arise from a radius other than the 3:1 resonance, and we are in favor of the latter interpretation.

Our result suggests a new picture that both classes of objects are located in the same place in the evolutionary trace (figure 18). The difference between these classes may be a result of different mass transfer rates. This possibility should be explored by future studies.

6. Summary

We obtained photometric observations of WZ Sge-type dwarf novae, MASTER OT J211258.65+242145.4 and MASTER OT J203749.39+552210.3 in 2012. The early superhumps with a period of 0.059732(3) d (MASTER J211258) and of 0.06050 d (MASTER J203749) were recorded. During superoutburst, ordinary superhumps with a period of 0.060291(4) d (MASTER J211258) and of 0.061307(9) d (MASTER J203749) were observed. The \( O - C \) curve clearly showed the presence of stage A and stage B. Both dwarf novae exhibited multiple rebrightenings after the main superoutburst.

In both dwarf novae, the increase of brightness were seen during the evolutionary stage of ordinary superhumps. Compared with other systems showing this phenomenon, we can see a possible tendency that those systems exhibit long rebrightening or multiple rebrightenings.

From the distribution of superhump periods, those superhump periods of type-B stars are systematically longer than those of type-D stars.

We estimated binary mass ratios by a new method using the period of superhumps in SU UMa-type dwarf novae during stage A superhumps. The method gave higher mass ratios for these objects than those from stage B superhumps by the traditional method. This suggests that the stage B superhump period is shorter due to effects of gas pressure. This result leads to a different picture of the evolutionary state in WZ Sge-type dwarf novae with multiple rebrightenings although they have been expected to be good candidates for period bouncers due to the small mass ratios expected from the period of stage B superhumps.

In the final part of the superoutburst, the increase in the superhump periods was seen in both systems. We presented an interpretation that the increase in the superhump periods is a result of the decrease of pressure effect during the final stage of the superoutburst and showed that the derived disk radius is in fair agreement with the previous work.

We acknowledge with thanks the variable star observations from the AAVSO International Database contributed by observers worldwide and used in this research. This work is deeply indebted to outburst detections and quick announcement by the MASTER network. We are grateful to many amateur observers.

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