The second-class current decays $\tau \rightarrow \pi \eta(\eta') \nu_\tau$ in the NJL model including the interaction of mesons in the final state

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Abstract

The effect of the interaction of mesons in the final state is additionally considered within the description of $\tau \rightarrow \pi \eta(\eta') \nu_\tau$ decays. This interaction is taken into account at the level of production of intermediate pions. One of them, in turn, might be transited into $\eta$ or $\eta'$ mesons. Our results do not exceed the experimentally established branching fractions, and they are in agreement with the results of other theoretical studies.

1 Introduction

The hadronic $\tau$ lepton decays play an important role in the study of the intrinsic properties of the fundamental QCD theory. The decays $\tau \rightarrow \pi \eta(\eta') \nu_\tau$ are interesting examples of the second class current decays. For the first time, these types of currents were considered by Weinberg [1]. These decays are suppressed by G-parity violation and can occur only due to the mass difference between light $u$ and $d$ quarks. This difference also ensures the transition of pions into $\eta$ and $\eta'$ mesons.

Early experiments to study the $\tau \rightarrow \pi \nu_\tau$ decay were carried out with the CLEO [2,3] and ALEPH [4] detectors. However, stricter limits for branching fractions with a large number of events are set in the experiments of the BaBar [5,7] $Br(\tau \rightarrow \pi \eta \nu_\tau) < 9.9 \times 10^{-5}$, $Br(\tau \rightarrow \pi \eta' \nu_\tau) < 4.0 \times 10^{-6}$ and the Belle [8,9] $Br(\tau \rightarrow \pi \eta \nu_\tau) < 7.3 \times 10^{-5}$ collaborations at $e^+e^-$ colliders. Also, in the recent work [10], on the base of analysing Belle data the authors presented the branching fractions $Br(\tau \rightarrow \pi \eta \nu_\tau) = 4.4 \times 10^{-5}$.

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From a theoretical point of view, these processes were investigated within the framework of various phenomenological models \([11–20]\).

At the same time, in the paper \([21]\), the processes \(\tau \to \pi \eta(\eta')\nu_\tau\) are described in the framework of the standard and extended Nambu – Jona-Lasinio (NJL) model. The standard NJL model describes 4 meson nonets in the ground state and their interactions within the \(U(3) \times U(3)\) chiral symmetry \([22,26]\). The extended model includes intermediate mesons in both the ground and first radially excited states without breaking of the \(U(3) \times U(3)\) chiral symmetry \([27,30]\).

In recent years, papers have been published where the effect of interaction of mesons in the final state (FSI) considered by taking into account the meson loops describing the exchange of a vector meson between outgoing pseudoscalar mesons. These are, for example, such decays as \(\tau \to [\pi \pi, K \pi, K \eta]\nu_\tau\) \([31–33]\). After that, it turned out to be correct implementation of the generated mesons interaction in the previously considered processes \(\tau \to \pi \eta(\eta')\nu_\tau\) \([21]\). For this, we can use the recently obtained results in the description of \(\tau \to \pi \pi \nu_\tau\) \([31]\) decay with considering FSI. Herewith, one should take into account the transitions \(\pi^0 - \eta(\eta')\) in the final state. Thus, FSI will be taken into account here at the level of intermediate pions. Our results are in satisfactory agreement with the experimental limitations, as well as in qualitative agreement with the estimates obtained by previous authors in other theoretical models.

It should be noted that when describing FSI using meson loops, we had to go beyond the limitations in which the NJL model was formulated. Namely, we consider a higher order in \(1/N_C\) in perturbation theory, where \(N_C\) is the number of colors in QCD.

## 2 The decay \(\tau \to \pi \pi \eta \nu_\tau\) including FSI

A description of the \(\tau \to \pi \pi \nu_\tau\) decay with consider of the FSI effect was obtained in the paper \([31]\) with satisfactory agreement with the current experimental data. For the description of the \(\tau \to \pi \pi \eta \nu_\tau\) decay, we use the amplitude from \([31]\) with an additional \(\pi^0 - \eta\) transition in the final state. The corresponding Feynman diagrams are presented in Figures 1-2. As a result, for the amplitude of the vector channel of the \(\tau \to \pi \eta \nu_\tau\) decay we obtain

\[
\mathcal{M}(\tau \to \pi \eta \nu_\tau) = G_F V_{ud} T_{\pi \eta} l_\mu M_\rho^2 \left(1 - \frac{i\sqrt{2} \Gamma_\rho}{M_\rho^2}\right) \frac{1}{M_\rho^2} \left[(p_\eta - p_\pi)^\mu + g_\rho^2 \left(a(s)p_\eta^\mu - b(s)p_\rho^\mu\right)\right]. \tag{1}
\]

Here \(G_F\) is the Fermi constant, \(V_{ud}\) is the element of the Cabibbo-Kobayashi-Maskawa matrix; \(l_\mu\) is the lepton current; \(g_\rho\) is the \(\rho \to \pi^+ \pi^-\) decay constant \([22,31]\); \(s = (p_\eta + p_\pi)^2\); \(M_\rho = 775.26 \pm 0.23\) MeV, \(\Gamma_\rho = 149.1 \pm 0.8\) MeV are \(\rho\) meson mass and width \([34]\). The intermediate \(\rho\) meson is described by the Breit-Wigner propagator \([31]\). The multipliers \(a(s)\) and \(b(s)\) in the momenta of outgoing particles appear as the functions of \(s\) due to FSI. These functions read

\[
a(s) = I_{\rho \pi} \frac{M_\pi^2(M_\eta^2 - M_\pi^2)}{M_\rho^2} - I_{\rho 3\pi} \frac{M_\rho^4(-M_\eta^2 + 7M_\rho^2 + 6M_\pi^2 + s)}{6M_\rho^2} - I_{\rho 4\pi} \frac{M_\rho^8(23M_\eta^2 + 2M_\pi^2 - 5s)}{6M_\rho^2} + 4I_{\rho 5\pi} \frac{M_\rho^8(4M_\eta^2 + 2M_\rho^2 - s)}{6M_\rho^2}, \tag{2}
\]

\[
b(s) = I_{\rho \pi} \frac{M_\pi^2(M_\eta^2 - M_\pi^2)}{M_\rho^2} - I_{\rho 3\pi} \frac{M_\rho^4(-M_\eta^2 + 7M_\rho^2 + 6M_\pi^2 + s)}{6M_\rho^2} - I_{\rho 4\pi} \frac{M_\rho^8(23M_\eta^2 + 2M_\pi^2 - 5s)}{6M_\rho^2} + 4I_{\rho 5\pi} \frac{M_\rho^8(4M_\eta^2 + 2M_\rho^2 - s)}{6M_\rho^2}, \tag{2}
\]
the substitution obtained for the decay constant \cite{22}. The comparison of the amplitudes where \( m \) is the value of the cutoff parameter is given in the Table 1.

The amplitudes of these transitions in the NJL quark model take the form \cite{21}

\[
T_{\pi\eta} = 2g_{\pi\eta}^2 \left[ \left( 2I_1^{m_d} + M_{\pi(\eta')}^2 I_2^{m_d} \right) - \left( 2I_1^{m_u} + M_{\eta(\eta')}^2 I_2^{m_u} \right) \right] \sin(\theta) \cos(\bar{\theta}) \frac{1}{M_{\pi}^2 - M_{\eta(\eta')}^2},
\]

where \( m_u = 280 \text{ MeV} \) and \( m_d = 283.7 \text{ MeV} \) are the constituent quark masses \cite{22}; \( \theta = -54^\circ \) is the mixing angle of \( \eta \) and \( \eta' \) mesons \cite{35}; \( g_{\pi} = m_u / M_{\pi} \) is the \( \pi \rightarrow \mu \nu_{\mu} \) decay constant \cite{22}. The comparison of the amplitude \( T_{\pi\eta(\eta')} \) and coupling constant \( g_{\rho\pi\eta(\eta')} \) is given in the Table 1.

|            | NJL model | \cite{15,16} | \cite{18} |
|------------|-----------|-------------|-----------|
| \( T_{\pi\eta} \) | 1.55 \times 10^{-2} | 1.34 \times 10^{-2} | (0.98 \pm 0.03) \times 10^{-2} |
| \( T_{\pi\eta'} \) | 6.80 \times 10^{-3} | (3 \pm 1) \times 10^{-3} | (0.25 \pm 0.14) \times 10^{-3} |

|            | NJL model | \cite{15,16} | \cite{13,14} |
|------------|-----------|-------------|-----------|
| \( g_{\rho\pi} \) | 9.51 \times 10^{-2} | 8.04 \times 10^{-2} | 8.50 \times 10^{-2} |
| \( g_{\rho\pi'} \) | 4.17 \times 10^{-2} | (1.8 \pm 0.6) \times 10^{-2} | < 2.5 \times 10^{-2} |

Table 1: The comparison of \( T_{\pi\eta(\eta')} \) and coupling constant \( g_{\rho\pi\eta(\eta')} \).

![Figure 1: The quark diagram of the decays \( \tau \rightarrow \pi\eta(\eta')\nu_\tau \) in the NJL model](image)

We can calculate the decay width of the studied decay \( \tau \rightarrow \pi\eta\nu_\tau \) by the formula

\[
\Gamma(\tau \rightarrow \pi\eta\nu_\tau) = \frac{1}{2} \cdot \frac{1}{256\pi^3 M_{\pi}^3} \int_{s_-}^{s_+} ds \int_{t_-}^{t_+} dt |M|^2,
\]

where \( M \) is the decay amplitude.
where the variables are defined as 
\[ s = (p_\tau - p_\nu)^2 = (p_\eta + p_\pi)^2, \quad t = (p_\tau - p_\eta)^2 = (p_\pi + p_\nu)^2. \]

The limits of integration have the form
\[ s_+ = M_\tau^2, \quad s_- = (M_\eta + M_\pi)^2, \]
\[ t_\pm(s) = \frac{1}{2} \left[ M_\tau^2 + M_\eta^2 + M_\pi^2 - s \pm \frac{M_\tau^2}{s} (M_\eta^2 - M_\pi^2) \right] \pm \sqrt{D(s)} \],

where
\[ D(s) = s^{-2} \left( s - M_\tau^2 \right) \left( s - (M_\eta + M_\pi)^2 \right) \left( s^2 - s \left( M_\tau^2 + (M_\pi - M_\eta)^2 \right) + M_\tau^2 (M_\pi - M_\eta)^2 \right). \]

The obtained results and the experimentally measured values for the branching fractions of the \( \tau \to \pi\eta\nu_\tau \) process are given in Table 2. In the table comparisons with the results of other theoretical studies are also given. In our calculations, we did not take into account the FSI effect in the scalar channel since contribution is an order of magnitude less than in the vector channel. Moreover, the Lagrangian of the interaction of the \( a_0 \) meson with the pion and the \( \eta \) meson in the minimum order does not contain derivatives [31].

3 The decay \( \tau \to \pi\eta'\nu_\tau \) in the extended NJL model including FSI

To describe the \( \tau \to \pi\eta'\nu_\tau \) decay due to a higher threshold for the final meson products, it is necessary to take into account the first radially excited states as intermediate ones. Therefore, we will use the extended NJL model [27–30]. A fragment of the chiral quark-meson Lagrangian of the extended NJL model containing the mesons involved in the process under consideration has the form

\[ \Delta L_{int} = \bar{q} \left[ \frac{1}{2} \gamma^\mu \sum_{j=\pm} \lambda^\rho_j (A_\rho p_\mu^j + B_\rho p_\mu^j) + i \gamma^5 \sum_{j=\pm,0} \lambda^\pi_j (A_\pi \pi^j + B_\pi \pi^j) \right. \]
\[ \left. + i \gamma^5 \sum_{j=u,s} \lambda^\eta_j A_\eta \eta^j \right] q, \]
where $q$ and $\bar{q}$ are $u$, $d$ and $s$ quark fields with constituent quark masses $m_u \approx m_d = 280$ MeV, $m_s = 420$ MeV, excited mesonic states are marked with a hat, $\lambda$ are linear combinations of the Gell-Mann matrices \[30\],

$$A_M = \frac{1}{\sin(2\theta_M^0)} \left[ g_M \sin(\theta_M + \theta_M^0) + g'_M f_M k_\perp \sin(\theta_M - \theta_M^0) \right],$$

$$B_M = \frac{-1}{\sin(2\theta_M^0)} \left[ g_M \cos(\theta_M + \theta_M^0) + g'_M f_M k_\perp \cos(\theta_M - \theta_M^0) \right]. \tag{11}$$

The subscript $M$ indicates the corresponding meson; $\theta_\pi = 59.48^\circ$, $\theta_\rho = 81.8^\circ$, $\theta_\pi^0 = 59.12^\circ$, $\theta_\rho^0 = 61.5^\circ$ are the mixing angles \[30\].

For the $\eta'$ meson, the factor $A$ takes a slightly different form. This is due to the fact that in the case of the $\eta'$ meson four states are mixed \[30\]:

$$A_{\eta'} = -0.32 g_{\eta' u} - 0.48 g_{\eta' u} f_u k_\perp,
A_{\eta'} = 0.56 g_{\eta' s} + 0.3 g_{\eta' s} f_s k_\perp. \tag{12}$$

Here $f(k_\perp^2) = (1 + dk_\perp^4) \Theta(\Lambda^2 - k_\perp^2)$ is the form-factor describing the first radially excited meson states. The slope parameters, $d_{uu} = -1.784 \times 10^{-6}\text{MeV}^{-2}$ and $d_{ss} = -1.737 \times 10^{-6}\text{MeV}^{-2}$, are unambiguously fixed from the condition of constancy of the quark condensate after the inclusion of radially excited states and depends only on the quark composition of the corresponding meson.

The quark-meson coupling constants have the form

$$g_\rho = \left(\frac{2}{3} I_{20}\right)^{-\frac{1}{2}},
\quad g'_\rho = \left(\frac{2}{3} I_{20}\right)^{-\frac{1}{2}},
\quad g_\pi = \left(\frac{A_3^2 - k_\perp^2}{m_u^2 - k_\perp^2}\right)^{-\frac{1}{2}},
\quad g'_\pi = \left(4 I_{20}\right)^{-1},$$

$$g_{\eta' u} = \left(\frac{4}{Z_{\eta' u}} I_{20}\right)^{-\frac{1}{2}},
\quad g_{\eta' s} = \left(4 I_{20}\right)^{-\frac{1}{2}},
\quad g_{\eta' s} = \left(4 I_{20}\right)^{-\frac{1}{2}}, \tag{13}$$

here $Z_\pi \approx Z_{\eta' u}$ and $Z_{\eta' s}$ are additional renormalization constants appearing in the $\pi - a_1$ and $\eta - f_1(1420)$ transitions \[30\].

Integrals appearing in the quark loops are

$$I_{m_1 m_2} = -\frac{i N_c}{(2\pi)^4} \int \frac{f_m(k_\perp^2)}{(m_u^2 - k_\perp^2)^{m_1}(m_s^2 - k_\perp^2)^{m_2}} \Theta(\Lambda_3^2 - k_\perp^2) d^4 k, \tag{14}$$

where $\Lambda_3 = 1030$ MeV is the cutoff parameter \[30\].

Using the extended NJL model for the $\tau \rightarrow \pi\eta'\nu_\tau$ process, after taking into account FSI, we obtain the following amplitude:

$$\mathcal{M}(\tau \rightarrow \pi\eta'\nu_\tau) = G_F V_{ud} Z_\tau T_{\pi\eta'\nu_\tau} \left\{ [\mathcal{M}_c + \mathcal{M}_\rho + \mathcal{M}_{\rho\bar{\rho}}]^{\mu\nu}(p_\eta - p_\pi)_\nu + [\mathcal{M}_c(\text{loop}) + \mathcal{M}_{\rho(\text{loop})} + \mathcal{M}_{\rho\bar{\rho}(\text{loop})}]^{\mu\nu}(a(s)p_\eta - b(s)p_\pi)_\nu \right\}, \tag{15}$$

here the functions $a(s)$ and $b(s)$ are obtained by replacing $M_{\eta'}^2 \rightarrow M_{\eta'}^2$ in accordance with the definitions \[30\] and \[31\]. The terms in the square brackets \[15\] describe the contributions from the contact diagram and diagrams with intermediate $\rho$, $\bar{\rho}$ mesons in the ground and first radially excited states:

$$\mathcal{M}_c^{\mu\nu} = \left[ 1 - C_\rho^2 \frac{m_u^2}{m_a^2} \right] g^{\mu\nu},$$
\[ \mathcal{M}_\rho^{\mu\nu} = C_\rho^2 \left[ 1 - 4I_{20}^{\rho a_1} m_u^2 M_{a_1}^2 \right] \frac{g^{\mu\nu} p^2 - p^{\mu} p^{\nu}}{M_\rho^2 - p^2 - i \sqrt{p^2} \Gamma_\rho}, \]

\[ \mathcal{M}_{\rho}^{\mu\nu} = e^{i\pi} C_\rho^2 \left[ 1 - 4I_{20}^{\rho a_1} \frac{C_\rho m_u^2}{M_{a_1}^2} \right] \frac{g^{\mu\nu} p^2 - p^{\mu} p^{\nu}}{M_\rho^2 - p^2 - i \sqrt{p^2} \Gamma_\rho}, \]

\[ \mathcal{M}_{c(\text{loop})}^{\mu\nu} = g_\rho^2 Z^2 C_\rho^2 \left[ 1 - C_\rho^2 \frac{6m_u^2}{M_{a_1}^2} \right] \left[ 1 - 4I_{20}^{\rho a_1} m_u^2 M_{a_1}^2 \right] \frac{g^{\mu\nu}}{M_\rho^2 - p^2 - i \sqrt{p^2} \Gamma_\rho}, \]

\[ \mathcal{M}_{\rho(\text{loop})}^{\mu\nu} = e^{i\pi} g_\rho^2 Z^2 C_\rho^2 \left[ 1 - 4I_{20}^{\rho a_1} \frac{C_\rho m_u^2}{M_{a_1}^2} \right] \left[ 1 - 4I_{20}^{\rho a_1} m_u^2 M_{a_1}^2 \right] \frac{g^{\mu\nu} p^2 - p^{\mu} p^{\nu}}{M_\rho^2 - p^2 - i \sqrt{p^2} \Gamma_\rho}, \]

where \( M_{a_1} = 1230 \pm 40 \text{ MeV}, \) \( M_{\rho'} = 1465 \pm 25 \text{ MeV}, \) \( \Gamma_{\rho'} = 400 \pm 60 \text{ MeV} \) are masses and width of \( a_1 \) and \( \rho' \) mesons, respectively, given in PDG [34]. Following the paper [21], for the excited states we use the phase factor \( e^{i\pi} \). The constants \( C_\rho \approx 0.95 \) and \( C_\rho \approx 0.31 \) are taken from [30].

Integrals with the vertices from the Lagrangian [10] in the numerator, which were also used in the amplitude, take the form:

\[ I_{M,\ldots}^{\mu_1 \ldots \mu_n} = -i \frac{N_c}{(2\pi)^4} \int \frac{A_M \ldots B_M}{(k^2 - m_2^2)^2} \Theta(\Lambda_2^2 - k^2) d^4 k, \quad (16) \]

where \( A_M, B_M \) are defined in [11].

The results obtained using the amplitude (15) are presented in Table 2.

| \( \text{NJL model} \) | \( \text{Br}(\tau \to \pi\eta\nu_\tau) \times 10^5 \) | \( \text{Br}(\tau \to \pi\eta'\nu_\tau) \times 10^7 \) |
|-----------------------|---------------------------------|----------------------------------|
| 12                    | 1.69                            | 1.17                             |
| 13 14                 | 1.36                            | 2 – 14                           |
| 15 16                 | 0.4 – 2.9                       | 0.6 – 2.1                        |
| 17                    | 0.33                            |                                  |
| BaBar [6, 7]          | < 9.9                           | < 40                             |
| Belle [9]             | < 7.3                           |                                  |

Table 2: The comparison of the branching fractions for the decays \( \tau \to \pi\eta(\eta')\nu_\tau \).

It worth to note, that the similar results for the branching fraction were obtained in the work [12] using the chiral perturbation theory (ChPT) with resonances, and in [13, 14] using the non-standard \( V - A \) scalar weak interaction. The ChPT model is close to our
model since it is also based on chiral symmetries of strong interactions. Also, the results obtained in the papers [15,16] using the vector dominance do not contradict our results. The decay widths calculated in this work overestimates the calculated results in the paper [17]. The reason might be in quite different parameterization of the form factors.

The theoretical uncertainty of the NJL model can be estimated at $\sim 5\%$. The source of this uncertainty is the effects of chiral symmetry breaking [30].

4 Conclusions

In recent works [31,32], it was shown that taking FSI into account plays an important role in describing the $\tau$ decays with pseudoscalar mesons in the final state. In the present paper, we have confirmed this by taking into account the corresponding corrections to the previously described $\tau \rightarrow \pi\eta(\eta')\nu_\tau$ decays. Furthermore, our results do not contradict the experimental data, and are in agreement with the number of authors [12,14]. In [18], these processes are also studied within the framework of the chiral perturbation theory with resonances. In this case, the parametrizations of vector and scalar form factors were used. Also, in the work [20], possible contributions from New Physics for the studied decays were estimated.

In the above theoretical works, the FSI effect was taken into account by parametrizing the corresponding form factors. We took FSI into account due to the exchange of pions by the $\rho$ meson in the P wave. Herewith, we had to go beyond the lower order $1/N_C$ expansion, in which the NJL model was formulated.

The Belle II experimental collaboration presented the research program in recent works [36,37]. The upcoming experiment will allow to study the second class current decays more accurately. We hope that our results will receive experimental confirmation.

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7
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