Destroying a near-extremal Kerr black hole with a charged particle: Can a test magnetic field serve as a cosmic censor?

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We investigate effect of a test magnetic field on the process of destroying near-extremal Kerr black hole with a charged test particle. It has been shown that it would be possible to throw a charged test particle into the near extremal rotating black hole and make it go past the extremality i.e. turn Kerr black hole into the Kerr-Newmann naked singularity. Typically in an astrophysical scenario black holes are believed to be surrounded by a magnetic field. Magnetic field although small, affects motion of charged particles drastically due to the large Lorentz force, as the electromagnetic force is much stronger that the gravity. Thus a test magnetic field can affect the process of destroying black holes and restore the cosmic censorship in the astrophysical context. We show that a test magnetic field would act as a cosmic censor beyond a certain threshold value. We try to gauge the magnitude of the magnetic field by comparing its energy density with that of the change in the curvature induced by the test particle. We find that the magnetic field required in only as strong as or slightly stronger as compared to the value for which its effect of the background geometry is comparable to the tiny backreaction as that of the test particle. In such a case one has to take into account effect of the magnetic field on the background geometry, which is difficult to implement in the absence of any exact near-extremal rotating magnetized black hole solutions. We argue that magnetic field would still act as a cosmic censor.

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I. INTRODUCTION

In this paper we study a Gedanken experiment to destroy a black hole with the infalling test particle. The infalling particle would add to the mass, angular momentum and charge of the black hole and can make it go pass the extremality, thus turning black hole into the naked singularity. Such a process was examined by Wald for the first time, who found that it is impossible to overspin an extremal Kerr black hole by throwing in a neutral test particle \cite{Wald}. If the angular momentum of the infalling particle is large enough for the purpose of overspining the black hole, it turns back before it could enter a black hole. Whereas if the particle enters the black hole it would not add a sufficient amount of angular momentum to overspin the black hole.

Recently it was shown by Jacobsan and Sotiriou (JS) \cite{JS} that it would be possible to destroy a black hole with infalling test particle if we start with a near-extremal configuration, rather than an extremal black hole as in the case of Wald’s analysis. There is a narrow range of the energy and angular momentum of the infalling particles for which it would be possible for it to enter a black hole and also overspin it past the extremality. It was also shown that it would be possible to destroy near-extremal Reissner-Nordström black holes with charged test particle \cite{JS}. The process of destroying rotating black hole with the charged test particle was investigated in \cite{JS}. In all these calculations it was assumed that the test particle follows a geodesic motion and effects of the conservative and dissipative backreaction were ignored. There are investigations which suggest that the radiation reaction and self-force would act as a cosmic censor \cite{Wald, Maeda}. It was also shown that it would be possible to destroy a regular black hole with the test particles even when the backreaction effects are taken into account \cite{Wald, Maeda}.

We approach this issue from a different perspective. Typically in the astrophysical scenarios black holes are surrounded with the magnetic field which would affect the motion of the charged particle and thus influence the process of destroying black hole. With this motivation in mind in this paper we investigate whether or not a magnetic field could possibly serve as a cosmic censor. For this purpose we introduce a test magnetic field on the Kerr spacetime following a procedure that is described in \cite{Maeda}. The Wald solution is recently extended to a black hole that is also moving at constant velocity in \cite{Maeda}. The magnetic field respects the axial symmetry of the Kerr spacetime and it takes a constant value asymptotically at infinity. We show that for large enough value of the magnetic field it serves as a cosmic censor. We try to gauge the magnitude of the magnetic field required by comparing its backreaction on the background geometry as compared to that of a test particle. The trace of the energy-momentum tensor of the magnetic field is a
good measure of its backreaction on the background Kerr spacetime. We compute square root of the difference in the Kretschmann scalar at the horizon between extremal and near-extremal geometry, which is a fair indicator of the backreaction of the test particle on the background spacetime. We show that the backreaction of the magnetic field is as much as or slightly larger than that of the tiny backreaction of the test particle when it starts acting as the cosmic censor. Thus an extremely weak magnetic field is sufficient to restore the cosmic censorship in the process of destroying near extremal Kerr black hole.

This analysis also suggests that we must in principle take into account the backreaction of the magnetic field on the background geometry since change in the metric due to the magnetic field will be comparable to that of the test particle. It is difficult to implement it in the absence of any exact solution representing magnetized near extremal geometry. However we argue that our conclusions won’t change even after taking into account the backreaction of the magnetic field.

If black holes could indeed turn into the naked singularities i.e. if cosmic censorship hypothesis could be violated, it would have serious implications from theoretical as well as observational perspective. Kerr and Kerr-Newmann naked singular geometries are associated with the absence of global hyperbolicity and also with the existence of the closed timelike curves. However it was suggested that string theory could potentially resolve the naked singularities and all these issues would disappear rendering their existence legal. From observational point of view it was shown that the near extremal naked singularities can host ultra-high energy particle collisions and thus can serve as an astrophysical probe of high energy physics. We also note that there are many investigations where it is demonstrated that the naked singularities can form as an end-state of the continual gravitational collapse.

We note that there are many papers analyzing motion of the charged particles in the magnetic field around central objects, analytically as well as numerically. [see, e.g. [25, 32]. The strength of the magnetic field \( B \) is estimated to be the \( B_1 \sim 10^8 G \) for stellar mass black holes of mass \( M \sim 10\, M_\odot \), and \( B_2 \sim 10^4 G \) for the supermassive black holes of mass \( M \sim 10^9 M_\odot \) ([34]). The strength of the magnetic fields near the event horizon of the black hole was also measured recently ([35, 36]).

In Sec. II we describe the process of destroying near-extremal Kerr black hole with the charged test particle. In Sec. III we introduce a test magnetic field on the background of the Kerr black hole and analyze its effect on the motion of the charged particle. In Sec. IV we compare backreaction of the test magnetic field with that of the test particle and analyze whether magnetic field could possibly serve as a cosmic censor. We summarize our concluding remarks of the obtained results in the Sec. V.

In this work we use a system of units in which \( G = c = 1 \).

## II. Particles Motion Around Near-Extremal Kerr Black Hole

In this section we describe the process of destroying near-extremal Kerr black hole with the charged test particle. We calculate the range of the energy and angular momentum of the particle for it to turn Kerr black hole into the Kerr-Newmann naked singularity. We then analyze allowed range of the parameters for which particle can enter the black hole.

We restrict our attention to the particles that follow geodesic motion on the equatorial plane of the Kerr black hole with mass \( M \) and angular momentum \( J \). There are two constants of motion associated with the particle, namely conserved energy \( \delta E \) and conserved angular momentum \( \delta J \). We assume that these quantities are much smaller compared to that of the black hole \( \delta E \ll M, \delta J \ll J \), so that the test particle approximation holds good. Let \( e \) be the charge associated with the test particle which is also assumed to be small. We neglect the radiation reaction and self-force. When particle enters the black hole it adds to the mass, angular momentum and charge of the black hole. The final mass, angular momentum and charge of the black hole are given by \( M + \delta E, J + \delta J \) and \( e \) respectively.

If the particle were to turn a Kerr black hole into the Kerr-Newmann naked singularity, following condition must hold:

\[
(M + \delta E) < \left( \frac{J + \delta J}{M + \delta E} \right)^2 + e^2.
\]

This yields the lower bound on the angular momentum of the particle

\[
\delta J > \delta J_{\text{min}} = (M^2 - J) + 2M\delta E + \delta E^2 - \frac{e^2}{2}.
\]

As it was pointed out in [2], null energy condition on the matter the test particle consists of, puts an upper bound on the angular momentum of the particle

\[
\delta J < \delta J_{\text{max}} = \frac{2Mr_+}{a} \delta E,
\]

where the two parameters, \( a \) and \( M \), denote specific angular momentum and the mass of a Kerr black hole, respectively. In case of the extremal black hole it turns out that \( J_{\text{max}} < J_{\text{min}} \) and thus it is not possible to overspin it.

In this paper we deal with the near-extremal black hole with dimensionless spin parameter close to unity. We take \( J/M^2 = a/M = 1 - 2\epsilon^2 \), with \( \epsilon \ll 1 \) being a small dimensionless parameter. Hereafter we set \( M = 1 \). Maximum and minimum values of \( \delta J \) are given by

\[
\delta J_{\text{min}} = 2\epsilon^2 + 2\delta E + \delta E^2 - \frac{e^2}{2},
\]

\[
\delta J_{\text{max}} = (2 + 4\epsilon)\delta E,
\]
and the allowed range of $\delta E$ is
\[
(2 - \sqrt{2}\sqrt{1 + \left(\frac{\epsilon}{2\epsilon}\right)^2}) \epsilon < \delta E < (2 + \sqrt{2}\sqrt{1 + \left(\frac{\epsilon}{2\epsilon}\right)^2}) \epsilon.
\] (6)

The value of charge $\epsilon$ is taken to be small as compared to $\epsilon$. Thus we have $\delta E$ of the order of $\epsilon$. For the given $\delta E$ we get $\delta J \sim \delta E$. Thus we have $\delta E \ll M$ and $\delta J \ll J$ and the particle under consideration can be thought of as a test particle.

We must ensure that the particle in the allowed range of the energy and angular momentum starting from a distant location indeed enters the black hole if it were to turn it into a naked singularity. Thus we need to understand the geodesic motion of the particle. As stated earlier we focus on the particle that is restricted to move on the equatorial plane. The motion of the particle in the radial direction can be described in terms of the effective potential as stated in the equation below. Thus we must analyze the behavior of the effective potential to understand whether or not particle enters the black hole:
\[
\ddot{r}^2 + V_{\text{eff}}(r, \tilde{E}, \tilde{J}) = 0,
\] (7)

where $\tilde{E} = \delta E/m$ and $\tilde{J} = \delta J/m$, and $m$ is the rest mass of the particle.

For the chosen value of the energy $\delta E$ one can write the allowed range of the angular momentum of the body falling into the black hole as
\[
(2 + 3\epsilon)\delta E - \frac{\epsilon^2}{2m} < \delta J < (2 + 4\epsilon)\tilde{E}.
\] (8)

As we have already mentioned, the initial black hole is nearly extremal, but now we can be somewhat more quantitative. For example, we take $\epsilon = 10^{-2}$. The spin parameter of the Kerr black hole in this case is given by $a = 0.9998$. We can imagine even smaller values of $\epsilon$ in principle.

We parametrize the range of allowed specific angular momentum in the following way
\[
\delta J = (2 + b \epsilon) \tilde{E} - (4 - b) \tilde{c},
\] (9)

where $b \in [3, 4]$ and $\tilde{c} = \epsilon^2/2m$ is charge parameter.

We now investigate the effective potential for the radial motion of the particle. The charged particle is assumed to start from a distant location falling in towards the black hole. We must make sure that $V_{\text{eff}} < 0$ everywhere outside the horizon so that the particle enters the black hole. The effective potential that appears in (7), is given by
\[
V_{\text{eff}} = -\frac{\tilde{E}^2}{2} \left[ 1 - \frac{3 + b\tilde{c}(b - 4) + 4b\epsilon + (4 + b^2)\epsilon^2}{\tilde{r}^2} + \frac{2 - 2b\tilde{c}(b - 4) + 4b\epsilon - 4\tilde{c}(4 - b)\epsilon + 2(4 + b^2)\epsilon^2}{\tilde{r}^3} \right] - \gamma \left( \frac{8b + (4 - b)^2\gamma - 32}{4\tilde{r}^2} - \frac{4b + (4 - b)^2\gamma - 16}{2\tilde{r}^3} \right),
\] (10)

where $\gamma = \tilde{c}/\epsilon$. Here we assume that the specific energy of the infalling particle is large $\delta E >> 1$ as in [2].

We compute the maximum value of the effective potential attained at a location outside the horizon and write it as $V_{\text{eff}} = -\delta E^2 V_{\text{eff}} b/2$. For particle to enter the event horizon $V_{\text{eff}}$, must be positive at the radial location stated above where effective potential attains maximum. For given value of $\tilde{c}$ the parameter $b$ must take a value in the range $(3, b_{cr})$ as it can be seen in Fig 1. The upper critical values of the parameter $b_{cr}$ for the different values of charge parameter $\tilde{c}$ are tabulated in Table 1.

| $\epsilon$   | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
|-------------|------|------|------|------|------|------|
| $\tilde{c}$  | $10^{-3}$ | $10^{-4}$ | $5 \times 10^{-5}$ | $10^{-5}$ | $5 \times 10^{-6}$ | 0 |
| $b_{cr}$    | 3.9863 | 3.8846 | 3.8034 | 3.6093 | 3.5558 | 3.4641 |

The range of parameter $b$ is given in $[3, 4]$. As shown in [2] the allowed range for the uncharged particle to enter the black hole is given by $[3, \sqrt{12}]$. As it can be seen in Fig 1 the effective potential $V_{\text{eff}} b$ at $r_{\text{max}}$ is positive in this range. Here we analyze the process of destroying Kerr black hole with charged particle. The allowed range of parameter $b$ for which test particle enters the near-extremal black hole and turns it into the Kerr-Newmann naked singularity increases as we increase the charge parameter. As it can be seen from Fig 1 the intersection point of effective potential curve moves towards $b = 4$. Consequently it becomes easier to destroy the black hole if incoming particle is charged.

In this section we described the process of destroying near-extremal Kerr black hole with a charged test particle. In the next section we introduce a test magnetic field around the Kerr black hole and analyze its effect on the process of destroying black hole.
III. PARTICLE MOTION AROUND BLACK HOLE IN A MAGNETIC FIELD

We analyze the effect of magnetic field in the process of destroying Kerr black hole with a charged particle. We describe a process to set up a test magnetic field on the spacetime containing Kerr black hole. The magnetic field takes a constant value at infinity and is oriented along the axis of symmetry of the Kerr geometry. The motion of the charged particle could be get influenced by the test magnetic field and it can also affect the process of destroying black hole. We try to understand whether or not magnetic field can stop particles with the appropriate values of geodesic parameters from entering the black hole and thus it would serve as a cosmic censor.

The metric of the Kerr geometry in the Boyer-Lindquist coordinates is given by

$$ds^2 = -\left(\frac{\Delta - a^2 \sin^2 \theta}{\Sigma}\right)dt^2 - \frac{2a \sin^2 \theta (r^2 + a^2 - \Delta)}{\Sigma}dtd\phi + \frac{(r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta}{\Sigma} \sin^2 \theta d\phi^2 + \frac{\Sigma}{\Delta} dr^2 + \Sigma d\theta^2,$$

where $\Sigma = r^2 + a^2 \cos^2 \theta$ and $\Delta = r^2 + a^2 - 2Mr$.

Kerr space-time admits two Killing vectors, $\xi^a_i = (\partial/\partial t)^a$ and $\xi^a_{(\phi)} = (\partial/\partial \phi)^a$. They satisfy two Killing equations

$$\xi_{a;\beta} + \xi_{\beta;\alpha} = 0,$$

which can be written in the form $\Box \xi^a = 0$. The vacuum Maxwell equations for vector potential $A^a$ have the same form $\Box A^a = 0$ in the Lorentz gauge. Thus the vector potential can be described as combination of the Killing vectors

$$A^a = C_1 \xi^a_{(t)} + C_2 \xi^a_{(\phi)}.$$

We select the integration constants $C_1 = 0$ and $C_2 = B/2$ and the vector potential can be written as

$$A^a = \frac{B}{2} \xi^a_{(\phi)}.$$

It can be shown from the asymptotic properties that $B$ turns out to be the magnetic field at infinity which is uniform and oriented along the axis of symmetry.

The covariant components of the 4-vector potential of the electromagnetic field will take the form

$$A_t = -\frac{1}{2\Sigma} \left\{ aB \left[ \Delta (1 + \cos^2 \theta) + (r^2 - a^2) \sin^2 \theta \right] - 2aB \left( \Sigma - 2Mr \right) \right\},$$

$$A_t = A_\theta = 0,$$

$$A_\phi = \frac{1}{\Sigma} \left\{ \frac{B}{2} \left[ \Delta a^2 (1 + \cos^2 \theta) + r^2 - a^4 \right] - 2QMBo^3 \right\} \sin^2 \theta.$$

We write down the conserved quantities for particle motion in the equatorial plane

$$\pi_t = -g_{\mu\nu}(\xi^a_{(t)})^\mu \pi^\nu = g_{tt} \pi^t + g_{t\phi} \pi^\phi + eA_t,$$

$$\pi_\phi = -g_{\mu\nu}(\xi^a_{(\phi)})^\mu \pi^\nu = g_{\phi t} \pi^t + g_{\phi\phi} \pi^\phi + eA_\phi,$$

where $\pi^\nu$ is the four velocity defined by $\pi^\nu = \frac{dx^\nu}{d\tau}$, $\tau$ is the proper time for timelike geodesics.

Solving equations (10) and (17) we write down the equation of motion for the charged particle motion in the Maxwell field around black holes

$$\pi^t = \frac{1}{r^2} \left[ a(\pi_\phi + a\pi_t) + \frac{r^2 + a^2}{\Delta} P \right],$$

$$\pi^\phi = \frac{1}{r^2} \left[ (\pi_\phi + a\pi_t) + \frac{a}{\Delta} P \right],$$

$$(\pi^r)^2 = \frac{P^2 - \Delta [r^2 + (\pi_\phi + a\pi_t)^2]}{r^4},$$

where $P = (r^2 + a^2)(-\pi_t) - a\pi_\phi$. 

![Figure 1: The dependence of value of the effective potential at given maximum radius $r_{max}$ on the parameter $b$ is plotted here for the different values of charge parameter $c$. The allowed range of $b$ increases as we increase the charge $c$.](image)
The effective potential for the radial motion of charged particle at the equatorial plane $\theta = \pi/2$ of the Kerr black hole placed in an external magnetic field is given by

$$V_{eff}(r) = \frac{\Delta [r^2 + (\pi \varphi + a \pi t)^2]}{2r^4} - P^2.$$  \hfill (21)

Now we discuss and analyze charged particle motion around a Kerr black hole immersed in a uniform magnetic field. Again we analyze the effective potential. Using the Eq. (21), the effective potential for radial motion can be given as

$$V_{eff} = -\frac{1}{2r^2} \left[ \left( r^2 + a^2 + \frac{2M a^2}{r} \right) \left( \delta E^2 - \frac{\beta^2}{4M^2} \Delta \right) - \left( 1 - \frac{2M}{r} \right) \delta J^2 - \frac{4Ma \delta E \delta J}{r} - \Delta \left( 1 - \frac{\beta \delta J}{M} \right) \right],$$  \hfill (22)

where the magnetic parameter $\beta = eBM/m$ measures the influence of the magnetic field on charged particle motion.

We study the effective potential in order to understand the effect of magnetic field on the process of destroying black hole. We would like to be sure that the particle with the energy and angular momentum in the appropriate range as described in the earlier section will start from infinity and fall towards the black hole without encountering any turning point.

The effective potential can be written in the following form

$$V_{eff} = -\frac{\delta E^2}{2} \left( V^b_{eff} + V^\beta_{eff} \right),$$  \hfill (23)

where $V^\beta_{eff}$ is defined as follows

$$V^\beta_{eff} = \gamma \left[ \frac{2 + be - 8\delta - 2b \delta + \beta(1 - 4e^2)}{\delta E} - \frac{(4 + 2be) \delta E - (8\delta - 2b \delta) \delta E^2}{r} + \frac{(16b \delta - 64\delta + 3\delta)}{4r^2} \right].$$  \hfill (24)

We have expanded the potential (23) out to second order in $\epsilon$.

We now try to understand the effect of the test magnetic field on the motion of the test particle and see whether or not it can serve as a cosmic censor in a process of destroying black hole.

For a particle with a given mass and charge, the magnetic parameter $\beta$ increases with the increasing magnetic field. For low values of the magnetic fields and parameter $\beta$ it will not be possible for magnetic field to prevent particles from entering the black hole and turning it into the naked singularity. However with increase of the magnetic field and parameter $\beta$ the motion of the charged particles is significantly affected. We plot effective potential at the maximum $V^b_{eff} + V^\beta_{eff}$ as a function of parameter $b$. The allowed range of the angular momenta for which it is possible to destroy black hole is given by $(3, b_{cr})$ where $V^b_{eff} + V^\beta_{eff}$ is positive. As we can see from the Fig 2, that $b_{cr}$ tends to decrease as we increase the magnitude of parameter $\beta$. A at a certain critical value of $\beta$ we have $b_{cr} = 3$. Beyond this value it is not possible for the charged particle to enter the black hole, and the test magnetic field serves as a cosmic censor.

The effective potential for the radial potential is plotted in Fig 3. When the magnetic field is zero, the maximum of effective potential is negative, thus allowing infalling particle to enter the black hole. As we increase the magnetic field the height of maximum tends to increase. Beyond certain value of the magnetic field maximum value crosses zero and is positive. Thus the infalling particle will turn back and will not be able to enter the black hole.

We have shown that if the magnetic field is sufficiently large it can prevent infalling particle from entering the black hole and thus could in principle serve as a cosmic censor. In the next section we try to gauge how large is the critical magnetic field by comparing its backreaction with that of the test particle.

**IV. ANALYZING THE BACKREACTION OF MAGNETIC FIELD ON BACKGROUND SPACETIME**

We have shown that the test magnetic field can potentially serve as the cosmic censor preventing a particle that can turn near-extremal black hole into a naked singularity in the absence of magnetic field, from entering the black hole. In this section we try to understand how large is the threshold magnetic field. As we describe below we do that by comparing the strength of the perturbation of the magnetic field on the background spacetime with that of the test particle.

The effect of the test particle on the background spacetime can be understood in terms of the change in the
Kretschmann scalar for the Kerr metric is given by the following expression

\[
K = R^\alpha{}_{\dot{\beta}{}_{\dot{\mu}}{}_{\dot{\nu}}} R_{\alpha{}_{\dot{\beta}{}_{\dot{\mu}}{}_{\dot{\nu}}}{}_{\dot{\sigma}{}_{\dot{\rho}}}} = \frac{48M^2}{(r^2 + a^2x^2)^6} \times \left( (r^2 + a^2x^2)^2 - 16a^2r^2x^2 \right), \tag{25}
\]

where \(x = \cos \theta\).

We calculate the square root of the difference of Kretschmann scalar at the horizon for extremal and near-
extremal geometries, subtract and take a square root:

\[ K = (K_1 - K_2)^{1/2} \]

\[
= 6 \sqrt{2} \left[ 2\sqrt{2}\left(\frac{1 - 7x^6 + 35x^4 - 21x^2}{(1 + x^2)^7}\right)^{1/2}(\epsilon)^{1/2} - \frac{7 - 2x^{10} + 47x^8 - 224x^6 + 434x^4 - 182x^2}{(1 + x^2)^8}(\epsilon)^{3/2}\right],
\]

which measures the change in the curvature in the background spacetime due to the test particle.

We now calculate the trace of the energy-momentum tensor of the test magnetic field. The energy-momentum tensor of the electromagnetic field is given by

\[ T^{\mu\nu} = \frac{1}{4\pi} (F_{\mu\sigma} F^{\nu\sigma} - \frac{1}{4} g^{\mu\nu} F_{\sigma\lambda} F^{\sigma\lambda}). \]  

(27)

The trace of the energy-momentum tensor of the magnetic field is given by

\[
T = \frac{B^2}{\Sigma(2Mr + \Sigma)} \left[ \frac{(2Mr + \Sigma) (\Xi - 4M^2r^2 + \Delta(2Mr + \Sigma))}{2\Sigma^2} \right] \left\{ -\frac{4\Sigma^4x^2 ((\Xi - 4a^2Mr) (a^2 + r^2) - a^2\Delta\Sigma (1 - x^2))^2}{\Upsilon^2} \right. \\
\left. -a^2M^2 \left( \Xi\Sigma (1 - x^2) + r (1 - x^2) (\Xi - 4a^2Mr - 2r^3\Sigma + a^2(M - r)\Sigma (1 + x^2)) \right)^2 \right. \\
\left. +2 \left( (\Xi - 4M^2r^2 + 2\Sigma^2 - \Delta(2Mr + \Sigma)) x (a^6M^2r^2 (x^2 - 1) (1 + x^2)^2 \\
+ \Sigma^2 (\Xi - 4a^2Mr - 2r^3\Sigma + a^2(M - r)\Sigma (1 + x^2))^2 \right) \right. \\
\left. - \frac{1}{\Upsilon^2} (\Xi + 4M^2r^2 - 2\Sigma^2 - \Delta(2Mr + \Sigma)) (1 - x^2)^2 \right. \\
\left. \times \left( M^2r^2 (\Xi - 4a^2Mr - 2r^3\Sigma + a^2(M - r)\Sigma (1 + x^2))^2 \\
- a^2x(\Xi^2 + a^2(4Mr + \Sigma) - 1) - a^2(2Mr + \Sigma) (4a^2Mr + 4Mr^3 + \Delta\Sigma (1 - x^2))^2 \right) \right\}.
\]

(28)

Here we have \( \Upsilon = \left[ a^2 + 2r^2 + a^2(2x^2 - 1) \right] \). At the horizon trace is given by

\[
T = g_{\mu\nu} T^{\mu\nu} = \frac{4B^2 \left( (1 + x^2)^4 (3 - 2x - 5x^2 - 3x^3 + x^4 + x^6 + x^7) + x (x^2 - 1)^2 (3 + x^2) \right)}{(1 + x^2)^8 (3 + x^2)},
\]

(29)

We now find the critical value of the magnetic field for which the square root of change in the Kretschmann scalar between the extremal and near-extremal configurations at horizon is equal to the trace of the energy momentum tensor of the magnetic field at the horizon.

\[ K = (K_1 - K_2)^{1/2} \sim T = g_{\mu\nu} T^{\mu\nu}, \]

We set \( x = \cos \theta = 0 \) as we restrict ourselves to the equatorial plane.

For the value chosen in this paper \( \epsilon = 0.01 \), the critical value of the magnetic field is given by

\[ B_{cr} \sim 0.8591. \]

(31)
We may take the backreaction of magnetic field with that of test particle we have to calculate the value of the magnetic field and compare it with the critical value. Eliminating the charge $\epsilon$ in the expression for the parameter $\beta$ one can get

$$
\beta = \sqrt{\frac{2\tilde{c}}{m} MB}.
$$

(32)

This equation allows us to relate the magnetic field $B$ with $\beta$ and $\tilde{c}$ for the given mass of the test particle $m$. We may take $M = 1$. Since we assume that $\delta E \sim 2\delta E/m$ is large and $\delta E \sim \epsilon$, we take $m \sim \epsilon^2$.

We now analyze the effective potential for the same values of parameters we have chosen in the previous section and see whether magnetic field can act as a cosmic censor by calculating the critical value $b_{cr}$. We compute and compare the magnetic field with the critical value.

We have tabulated the values of $b_{cr}$ for different values of $\beta > 0$ in Table II and for $\beta < 0$ in Table III for the values of $\beta$ where magnetic field is sufficiently small as compared to the critical value, we find that $b_{cr}$ initially increases and then it tends to decrease as we increase parameter $\beta$. As long as magnetic field is smaller as compared to the critical value, we get $b_{cr} > 3$ and thus there is an allowed range of values of $b$ for which it is possible for charged particle to enter the black hole and turn it into a naked singularity. For the values of $\beta$ for which there is no allowed range of $b$, we find that the magnetic field is slightly larger than the critical value.

In Table IV we have tabulated the values of $\tilde{c}$ for the given mass of the test particle $m = 1$. Since we assume that $\tilde{c} \sim \epsilon$, we take $m \sim \epsilon^2$.

We now analyze the effective potential for the same values of parameters we have chosen in the previous section and see whether magnetic field can act as a cosmic censor by calculating the critical value $b_{cr}$. We compute and compare the magnetic field with the critical value. For the values of $\beta$ where magnetic field is sufficiently small as compared to the critical value, we find that $b_{cr}$ initially increases and then it tends to decrease as we increase parameter $\beta$. As long as magnetic field is smaller as compared to the critical value, we get $b_{cr} > 3$ and thus there is an allowed range of values of $b$ for which it is possible for charged particle to enter the black hole and turn it into a naked singularity. For the values of $\beta$ for which there is no allowed range of $b$, we find that the magnetic field is slightly larger than the critical value.
Table III: $b_{cr}$ is tabulated for the different values of $\beta < 0$. The ratio of magnetic field to the critical value is also specified. Magnetic field starts acting as a cosmic censor i.e. $b_{cr} < 3$ when the magnetic field is around 10 times larger than the critical value.

| $\epsilon$ | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
|------------|------|------|------|------|------|------|
| $\tilde{c}$ | $10^{-3}$ | $10^{-4}$ | $5 \times 10^{-5}$ | $10^{-5}$ | $5 \times 10^{-6}$ | $10^{-6}$ |
| $b_{cr}$   | 3.86380697 | 3.88469117 | 3.80341795 | 3.609301958 | 3.55579171 | 3.514164012 |
| $B = 0$    | $B = 10^3 B_{cr}$ |
| $\beta$    | $3.84 \times 10^1$ | $1.21 \times 10^1$ | 8.59 | 3.84 | 2.72 | 1.21 |
| $b_{cr}$   | 2.86150423 | 3.00864202 | 3.07347623 | 3.336533166 | 3.340286437 | 3.343298255 |
| $B = B_{cr}$ | $B = 10^{-1} B_{cr}$ |
| $\beta$    | 3.84 | 1.21 | $8.59 \times 10^{-1}$ | $3.84 \times 10^{-1}$ | $2.72 \times 10^{-1}$ | $1.21 \times 10^{-1}$ |
| $b_{cr}$   | 3.977197683 | 3.875208802 | 3.794570716 | 3.605006776 | 3.550946383 | 3.511239776 |
| $B = B_{cr}$ | $B = 10^{-3} B_{cr}$ |
| $\beta$    | $3.84 \times 10^{-2}$ | $1.21 \times 10^{-2}$ | $8.59 \times 10^{-3}$ | $3.84 \times 10^{-2}$ | $2.72 \times 10^{-2}$ | $1.21 \times 10^{-2}$ |
| $b_{cr}$   | 3.986196724 | 3.884373267 | 3.803082454 | 3.609096180 | 3.555556457 | 3.514012326 |
| $B = B_{cr}$ | $B = 10^{-5} B_{cr}$ |
| $\beta$    | $3.84 \times 10^{-4}$ | $1.21 \times 10^{-4}$ | $8.59 \times 10^{-5}$ | $3.84 \times 10^{-4}$ | $2.72 \times 10^{-4}$ | $1.21 \times 10^{-4}$ |
| $b_{cr}$   | 3.9863562364 | 3.8847636533 | 3.8038893024 | 3.609283622 | 3.555790463 | 3.51450243 |
| $B = B_{cr}$ | $B = 10^{-6} B_{cr}$ |
| $\beta$    | $3.84 \times 10^{-6}$ | $1.21 \times 10^{-6}$ | $8.59 \times 10^{-7}$ | $3.84 \times 10^{-6}$ | $2.72 \times 10^{-6}$ | $1.21 \times 10^{-6}$ |
| $b_{cr}$   | 3.986380696 | 3.884691115 | 3.803417948 | 3.609301957 | 3.55579169 | 3.514163998 |

Figure 4: The value of the effective potential at maximum as a function of $b$ is plotted for positive as well as negative values of $\beta$ and for charge parameter $\tilde{c} = 10^{-3}$. The magnetic field is sufficiently smaller than the critical value. $b_{cr}$ initially increases and then it decreases. We have $b_{cr} > 3$. Thus when backreaction of the magnetic field can be ignored it does not serve as a cosmic censor.

test particle one must take into account the effect of magnetic field on the background spacetime while analyzing whether or not it can act as a cosmic censor. Various exact magnetized black hole solutions have been obtained \[11, 12\]. However there is no exact solution that represents near-extremal black hole. Thus it is a daunting task to take into account the effect of magnetic field on the background metric. However as it was shown in \[12\] in the context of the magnetized Reissner-Nordström black hole that the small magnetic field does not change either the horizon radius or the extremality condition. Similar results are also expected to hold good in the context of the near extremal rotating black hole. It seems that the effect of the magnetic field on the background metric does not assist the process of destroying black holes. Thus the conclusions obtained by ignoring the backreaction are expected to hold good even when the backreaction is taken into account.
Table IV: The value of parameter $\beta_{cr}$ at which we have $b_{cr} = 3$ and magnetic field starts acting as a cosmic censor is tabulated here. The ratio of magnetic field to the critical value is also specified. We find that the magnetic field is around 10 times larger than the critical magnetic field $B_{cr}$ when it starts acting as a cosmic censor.

| $b_{cr} = 3$ | $\epsilon$ | $m$ | $\hat{c}$ |
|-------------|-----------|-----|--------|
|             | 0.01      | 0.0001 | $10^{-3}$ |
|             | 0.01      | 0.0001 | $10^{-4}$ |
|             | 0.01      | 0.0001 | $5 \times 10^{-5}$ |
|             | 0.01      | 0.0001 | $5 \times 10^{-6}$ |

V. CONCLUSIONS

In this paper we studied the process of turning black hole into a naked singularity by throwing in the test particle with appropriate values of the geodesic parameters. It is possible to turn a near extremal Kerr black hole into a Kerr-Newmann naked singularity using a charged test particle. Typically in the astrophysical context black holes are surrounded with a magnetic field which would exert a Lorentz force on the charged particle affecting its motion. Thus we study the effect of the test magnetic field on the process of destroying black hole. We invoke a weak magnetic field which takes a constant value at infinity and is aligned with the axis of symmetry of Kerr geometry. We show that for a sufficiently large values of magnetic field it is not possible for a particle with the appropriate values of geodesic parameters to enter the black hole and turn it into the naked singularity. Thus it appears that test magnetic field could serve as a cosmic censor. To gauge the strength of the requisite magnetic field we compute trace of its energy momentum tensor and compare it with the square root of the change in the Kretschmann scalar at the horizon between extremal and near-extremal configurations which is a measure of the effect of the test particle on the background spacetime. We find that when the magnetic field acts as a cosmic censor its backreaction is slightly larger than that of the test particle. Therefore we need an extremely small magnetic field to restore the cosmic censorship in the process of destroying near extremal Kerr black hole with a charged test particle. However since the backreaction of the magnetic field is stronger than that of the test particle one must take account its effect on the metric which is difficult to implement in the absence of any near extremal rotating magnetized black hole solution. But since, neither the horizon radius nor the extremality condition for the magnetized Reissner-Nordström black hole is affected due to the small magnetic field, it seems that the magnetic field does not assist the process of destroying black hole. Therefore the results obtained without considering the backreaction of the magnetic field are expected to hold good even in the presence of the backreaction and magnetic field would act as a cosmic censor.

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