Definition of preferable thickness of metal cylindrical panels with mild camber based on post-buckling behaviour due to compression

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Abstract. Engineer design methodology for metal cylindrical panels with mild camber with post-buckling behavior accepted in case of compression above limit level is suggested. Method is based on geometrically non-linear task analytic solution and is reduced to solution of non-linear equation in panel thickness with limit stress being reached with post-buckling behavior of rigidly supported panel taking into account initial deflection.

1. Introduction
Let us consider load-bearing panel of the aircraft fuselage section with mild camber which is manufactured using welding and permanent strain is possible due to taken design and technology solutions [1]. It should be highlighted that above mentioned permanent strain may be initial deflection (ID) and significantly affect further geometrically non-linear behavior of load-bearing panels in case of loading above limit level. To be definite let us assume that welding along the panel longitudinal axis can lead to ID with transverse waves. According to reference [1] researches results let us assume following parameters of load-bearing panel longitudinal and transverse wave generation: \( m = 1 \) and \( n \geq 1 \).

Further on in this investigation let us consider panel minimal thickness definition analytic solution for an early stage of design using geometrically non-linear correlations taking into account initial deflection (ID) [2] for thin isotropic cylindrical panels with mild camber. Let us take into account peculiarities of load-bearing panels design method based on post-buckling behavior suggested in reference [3].

2. Main correlations and statement of the problem
Let us put down main geometrically non-linear correlations of thin isotropic panels. Equation of strain compatibility taking into account initial deflection is of the following form [2]

\[
L_4(F) - L_2(W) = 0
\]

with

\[
L_4(F) = \frac{1}{E\delta} \left[ \frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} \right],
\]

and

\[
L_2(W) = \frac{1}{E\delta} \left[ \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2} \right].
\]
Let us write non-linear Karman equation of type for isotropic panels taking into account initial deflection of the following form

\[ L_2(W) = \left( \frac{\partial^2 (W + W_{id})}{\partial x \partial y} \right)^2 - \left( \frac{\partial^2 W_{id}}{\partial x \partial y} \right)^2 - \frac{\partial^2 (W + W_{id}) \partial^2 (W + W_{id})}{\partial x^2 \partial y^2} + \frac{\partial^2 W_{id} \partial^2 W_{id}}{\partial x^2 \partial y^2} - \frac{1}{R} \frac{\partial^2 W}{\partial x^2}. \]

Let us note that there are examples of welding along the skin longitudinal edge with ID formation in reference [1].

Let us consider cylindrical panel with mild camber with ID which can be approximated by following function

\[ W_{id} = f_{id} \cdot \sin^2 \frac{2 \pi x}{a} \sin^2 \frac{\pi ny}{b}. \]

Let us present flexure similarly

\[ W = f \cdot \sin^2 \frac{2 \pi x}{a} \sin^2 \frac{\pi ny}{b}. \]

After substitution of equations (4)-(5) into non-linear equation of strain compatibility (1) we derive following equation for stress function:

\[ F = \left( f^2 + 2 f f_{id} \right) E \delta \left( A_1 \cos \frac{2 \pi x}{a} - A_2 \cos \frac{4 \pi x}{a} + A_3 \cos \frac{2 \pi ny}{b} - A_4 \cos \frac{4 \pi ny}{b} - A_5 \cos \frac{2 \pi x}{a} \cos \frac{2 \pi ny}{b} \right) + f E \delta \left( -A_6 \cos \frac{2 \pi x}{a} + A_7 \cos \frac{2 \pi x}{a} \cos \frac{4 \pi ny}{b} \right) \]

\[ \times \cos \frac{4 \pi ny}{b} + T_s y^2 \]

with \( T_s \) being compression force, \( A_1 = \frac{n^2 a^2}{32 b^2}, A_2 = \frac{n^2 a^2}{512 b^2}, A_3 = -\frac{b^2}{32 n^2 a^2}, A_4 = -\frac{b^2}{512 n^2 a^2} \).
\[ A_5 = \frac{n^2a^2b^2}{32(n^2a^2 + b^2)^2}, \quad A_6 = \frac{n^2a^2b^2}{64(n^2a^2 + 4b^2)^2}, \quad A_7 = \frac{n^2a^2b^2}{64(4n^2a^2 + b^2)^2}, \quad A_8 = \frac{a^2}{16\pi^2R}, \]

\[ A_9 = \frac{n^2a^2b^4}{32\pi^2R(n^2a^2 + b^2)^2}. \]

Using Bubnov-Galerkin method procedure following non-linear equation can be derived:

\[
D\frac{\pi^4f}{4a^3b^3}(3n^4a^4 + 2n^2a^2b^2 + 3b^4) + E\delta \frac{\pi^4n^2}{2ab}\left(f + f_{id}\right)(f^2 + 2ff_{id}) \left(A_1 + A_2 + A_3 + A_4 + A_5 + \frac{A_6}{2}\right) + \frac{A_7}{2} + E\delta \frac{\pi^2b}{2Ra}\left[A_1 + \frac{A_2}{2}\right](f^2 + 2ff_{id}) + \left(A_8 + \frac{A_9}{2}\right)f = \frac{3\pi^2bT_x(f + f_{id})}{16a}, \quad (6)
\]

which associates acting compression force, panel thickness \(\delta\), flexure amplitude \(f\) and ID. So, with given compression force \(T_x\) and known panel thickness flexure amplitude \(f\) is defined by equation (6) and membrane stress is calculated using Airy stress function (3). Let us put down equation for longitudinal membrane stress

\[
\sigma_x = \frac{1}{\delta} \frac{\partial^2 F}{\partial y^2} = -\frac{4\pi^2n^2E}{b^2}\left(f^2 + 2ff_{id}\right)A_1 - \frac{4\pi^2n^2Ef}{Rb^2}A_2 - \frac{T_x}{\delta} \quad (7)
\]

with

\[
A_1 = \left[A_3\cos \frac{2\pi ny}{b} + 4A_4\cos \frac{4\pi ny}{b} + A_5\cos \frac{2\pi x}{a}\cos \frac{2\pi ny}{b} + A_6\cos \frac{2\pi x}{a}\cos \frac{2\pi ny}{b} + 4A_7\cos \frac{2\pi x}{a}\cos \frac{4\pi ny}{b}\right]
\times \cos \frac{4\pi ny}{b}, \quad (8)
\]

\[
A_2 = \cos \frac{2\pi x}{a}\cos \frac{2\pi ny}{b}.
\]

It should be highlighted that extremum values of functions \(A_1\) and \(A_2\) do not depend on thickness and load and are defined by correlations of panel geometrical parameters.

### 3. Engineer design methods

To begin with let us consider design task for thin cylindrical panels with mild camber based on post-buckling state assuming that applied materials design characteristics are known and there is no ID (\(\text{fid} \approx 0\)). Let us assume that wave generation parameters meet above mentioned conditions that is the most probable panel strain due to taken technology solutions is transverse one. Let us state provisions of engineer method for fuselage section load-bearing skin thickness definition.

At first let us assume that buckling is impossible at limit loading level. In this case thin skins thicknesses \(\delta_{\text{stab.}}\) can be defined based on stability conditions.

Secondly with loads above limit level buckling is possible and skin thickness \(\delta_{\text{post-buckl.}}\) can be defined with limit stress being reached with post-buckling behavior which is described by geometrically non-linear correlations. General methodology of load-bearing panels design based on post-buckling state is shown in reference [4].
Thirdly panel maximal thickness \( \max(\delta_{\text{stab}}, \delta_{\text{post-buckl}}) \) is defined since normally indicated values are calculated for different loading levels using different criteria.

Let us consider in further details application of design method. Let us transfer equation (6) for linear case for stability task with small flexures \((f \to 0)\):

\[
\frac{E\delta^3}{12(1-\mu^2)} \frac{\pi^4}{4a^3b^3} \left(3n^4a^4 + 2a^2b^2n^2 + 3b^4\right) + E\delta \frac{\pi^2n^2b}{2Ra} \left(A_8 + \frac{A_9}{2}\right) = \frac{3\pi^2bT_{\text{xlim}}^2}{16a},
\]

from which we derive analytic equation in panel thickness \( \delta \) based on stability conditions with limit loading level. In this case it is also necessary to use equation \( \hat{\sigma}_{T_x} \lim / \hat{\sigma}_{n^2} = 0 \) for definition of critical value \( n_{\text{cr}} \) corresponding to panel buckling.

Further on let us transfer non-linear equation (6) in panel thickness of the following form:

\[
\frac{E\delta^3}{12(1-\mu^2)} \frac{\pi^4}{4a^3b^3} \left(3n^4a^4 + 2a^2b^2n^2 + 3b^4\right) + E\delta \frac{\pi^2n^2b}{2ab} f^2 \left(A_1 + A_2 + A_3 + A_4 + A_5 + \frac{A_6}{2} + \frac{A_7}{2}\right) + E\delta \\
\times \frac{\pi^2b}{2Ra} \left[A_1 + \frac{A_2}{2}\right] + \left[A_8 + \frac{A_9}{2}\right] = \frac{3\pi^2bT_{\text{ult}}}{16a}.
\]

With action of ultimate loads which differ from limit loads for safety factor. In this case according to design methodology based on post-buckling state \([3-4]\) let us assume that for example with post-buckling behavior longitudinal stress reach limit values in terms of static strength \(\sigma_x = \sigma_B\). Let us transfer membrane stress defined by correlations (7) into second degree equation in flexure amplitude \(f\) assuming compression stress is positive:

\[
\sigma_x = \frac{4\pi^2n^2E}{b^2} f^2 A_1 + \frac{4\pi^2n^2Ef}{Rb^2} A_2 + \frac{T_{\text{ult}}}{\delta} = \sigma_B.
\]

Deriving amplitude \(f\) from the last equation and substituting it into equation (9) required non-linear equation for panel best possible thickness with compression flow \(T_{\text{xult}}\) action can be obtained. Maximum stress in cylindrical panels with mild camber reaches limit values in points \(x_{\text{cr]}, y_{\text{cr}}\) defined by magnitude of function \(A_i(x,y)\) (8) which depends only on correlation of panel geometrical parameters \(a\) and \(b\).

Now let us consider peculiarities of task of panels design based on post-buckling state taking into account formation of ID \((\text{fid} \neq 0)\) which depends on taken technological solutions. Let us assume that ID amplitude \(f_{\text{id}}\) can be defined by expertise as dependence on panel thickness

\[
f_{\text{id}}(\delta) = \Omega(\delta) = A\delta^2 + B\delta + C,
\]

with \(A, B, C\) being coefficients obtained by tests.

Then substituting ID as function \(f_{\text{id}}(\delta)\) into above mentioned method correlations non-linear equation in panel thickness with post-buckling state acceptability in case of limit stress being reached with ID can be obtained. For above mentioned case let us transfer equation (6) of the following form:

\[
\frac{E\delta^3}{12(1-\mu^2)} \frac{\pi^4 f}{4a^3b^3} \left(3n^4a^4 + 2a^2b^2n^2 + 3b^4\right) + E\delta \frac{\pi^2n^2b}{2ab} \left(f + A\delta^2 + B\delta + C\right) \left(f^2 + 2Af\delta^2 + B\delta + C\right)
\]
\[ \left( A_1 + A_2 + A_3 + A_4 + A_5 + \frac{A_6}{2} + \frac{A_7}{2} \right) + E \delta \pi^2 b^2 \left( A_1 + \frac{A_2}{2} \right) \left( f^2 + 2f \left( A_0 \delta^2 + B \delta \right) + C \right) + \left( A_8 + \frac{A_9}{2} \right)f = \frac{3\pi^2 b T_x^\text{ult} \left( f + A_0 \delta^2 + B \delta + C \right)}{16a}, \]

and equation (7) for longitudinal membrane stress of the following form:

\[ \frac{4\pi^2 n^2 E}{b^2} \left( f^2 + 2f \left( A_0 \delta^2 + B \delta + C \right) \right) A_1 + \frac{4\pi^2 n^2 E f}{R b^2} A_0 A_2 + \frac{T_x^\text{ult}}{\delta} = \sigma_n. \]  

In this case method can be framed similarly to the above-mentioned procedure. It is necessary to derive flexure amplitude \( f \) from the second-degree equation (11) and substitute it into equation (10). As a result, non-linear equation in panel best possible thickness with limit values being reached in points \( \left( x_{\text{cr}}^{(i)}, y_{\text{cr}}^{(i)} \right) \).

4. Conclusion

Engineer method defining best possible thickness of cylindrical panel with mild camber based on post-buckling behavior with limit values being reached. Method can be used at an early design stage for choosing structural and technological solutions in order to support developed aircraft structures minimal weight.

References
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