Scaling Behavior for the Susceptibility of the Vacuum in a Polarization Model for the Cosmos

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ABSTRACT:

Based on a model of Winterberg, where the vacuum is made up of a two component, positive and negative mass superfluid/supersolid, we derive scaling laws for the polarization of space, i.e., the vacuum. Upon expansion of the universe, this vast assembly (sea) of positive, and negative mass planckions form a rigid, ether-like, medium, which at sufficiently low temperatures, can be polarized through gravitational alignment/ordering of planckion mass dipoles. Two models for susceptibility of the vacuum as a function of the cosmic scale parameter, \( \alpha \), are presented. We also consider the possibility that Newton’s constant can scale, i.e., \( G^{-1} = G^{-1}(\alpha) \), to form the most general scaling laws for polarization of the vacuum. The positive and negative mass of the planckion, is inextricably related to the value of, \( G \), and as such, both are intrinsic properties of the vacuum. Scaling laws for the non-local, cosmic susceptibility, \( \bar{\chi}(\alpha) \), the cosmic polarization, \( \bar{P}(\alpha) \), the cosmic macroscopic gravitational field, \( \bar{g}(\alpha) \), and the cosmic gravitational field mass density, \( \bar{\rho}_{gg}(\alpha) \), are worked out, with specific examples. At the end of recombination, i.e., the era of last scattering, using the polarization to explain dark matter, and the gravitational field mass density to explain dark energy, we find that, \( (\Omega_{rad,1}, \Omega_{\text{b,1}}, \Omega_{c,1}, \Omega_{\Lambda,1}) = (0.37, 0.19, 0, 0.44) \). While this is an unconventional assignment, differing from the \( \Lambda \text{CDM} \) model, we believe this is correct, and we give our reasons why. Among them is the fact that localized dark matter (LDM) contributions can be much higher in this epoch than smeared values for susceptibility. The above assignments are cosmic averages, and will not apply locally. We also evaluate the transition from ordinary matter dominance, to dark matter dominance, for the cosmos as a whole. We obtain for the transition points, \( z = 1.66 \), for susceptibility model I, and, \( z = 2.53 \), for susceptibility model II.
I Introduction

Recently [1], we formulated a model to explain the density parameters in the $\Lambda CDM$ model, i.e., why, $(\Omega_{b,0}, \Omega_{c,0}, \Omega_{A,0}) = (.0486, .2589, .6911)$, in the present epoch. We were able to make identifications for dark matter, and dark energy, based on a polarization model for the vacuum, which is thought to be epoch specific. We argued that, within this model, dark matter and dark energy are related. In our scheme, dark matter is bound mass formed in the surrounding vacuum due to the ordering of inherent positive and negative mass dipoles, induced by ordinary matter. Dark energy, on the other hand, is an energy density associated with a macroscopic cosmic gravitational field formed by taking into account both source mass, and bound mass. We can calculate the total gravitational field that free and bound mass produces by appealing to Gauss’s law. Even though the gravitational fields are formally calculated at the surface of the observable Hubble bubble, they hold point for point throughout space, as non-local, smeared, spherically symmetric quantities. When distance scales in excess of 100 $Mpc$ are considered, the universe is of uniform density being homogeneous, and isotropic.

In the Friedmann equation, bound, or polarized, mass has its own mass density, $\bar{\rho}_B$. This is a smeared, non-local quantity. Locally we can also have, $\bar{\rho}_B(\vec{x})$, and this we can also call dark matter, localized dark matter ($LDM$), but cosmically this gets replaced by, $\bar{\rho}_B$. Dark energy, as mentioned, is the mass density associated with gravitational fields due to both, ordinary, source matter, and polarized, bound matter. This permeates the universe and leads to a smeared mass density in Friedmann’s equation, $\bar{\rho}_{gg}$. There are actually two terms relating to $\bar{\rho}_{gg}$ because $\bar{\rho}_{gg}$ can be further decomposed as, $\bar{\rho}_{gg} = \bar{\rho}_{AA} + \bar{\rho}_{AB}$, where, $\bar{\rho}_{AA}$, refers to a pure source mass gravitational field contribution, and, $\bar{\rho}_{AB}$, is a coupling of the source gravitational field, the applied field, with the polarized, or induced, gravitational field. It turns out that, $\bar{\rho}_{AB} = (\chi/K) \bar{\rho}_{AA}$, where, $\chi$, is the cosmic gravitational susceptibility, and, $K = 1 - \chi$, equals the relative gravitational permittivity. This is a non-local equation. Cosmically, both $\chi$ and $K$, are thought to be epoch dependent functions, which depend on, $\alpha$, the cosmic scale parameter. When there is no susceptibility, both, $\bar{\rho}_B$, and, $\bar{\rho}_{AB}$, vanish. All three mass density terms, $\bar{\rho}_B$, $\bar{\rho}_{AA}$, and, $\bar{\rho}_{AB}$, are necessitated by analogy to electrostatics, and therefore, should not be ignored in Friedmann’s equation.

Locally, due to the anti-screening feature of gravistatics, the total macroscopic gravitational field, $\vec{g}(\vec{x})$, which is the vector sum of source gravitational field, $\vec{g}^{(0)}(\vec{x})$, and induced gravitational field, $\vec{g}^{(1)}(\vec{x})$, adds up to a sum greater than the applied source field, $\vec{g}^{(0)}(\vec{x})$. The local susceptibility, $\chi(\vec{x})$, in gravistatics, is related to the relative gravitational permittivity, $K(\vec{x})$, by the equation, $K = 1 - \chi$, versus the customary, $K = 1 + \chi$, which holds in electrostatics. The, $K = 1 + \chi$, in electrostatics, leads to screening where, $\vec{E} = \vec{E}^{(0)}/K < \vec{E}^{(0)}$. In gravistatics, however, we have anti-screening, since, $\vec{g} = g^{(0)}(\vec{x}) < g^{(0)}$. In the present epoch, we calculated the cosmic smeared susceptibility for the universe as a whole [1]. Its value was found to equal, $\chi = .842$, which allowed us to explain the present values for the density parameters in Friedmann’s equation. This susceptibility gave us a smeared, or cosmic
average value for the macroscopic gravitational field in the amount of, \( \bar{g} = 2.387 \, E - 9 \, m/s^2 \). This value holds point for point within the cosmos, as any observer, no matter the location, would deduce this same value. This value could also be justified using other calculations.

Our model borrowed heavily from the works of Hajdukovíc [2-5], and Winterberg [6-12]. Hajdukovíc first suggested that polarization of the vacuum, may, in some fashion, lead to an explanation of dark matter. The anti-screening feature intrigued him and seemed a natural fit for the halo effect, rotation curves, virial movement of galaxies within superclusters, gravitational lensing, etc. He later looked at dark energy in the hopes of finding a suitable polarizable medium. He focused on the creation and annihilation of virtual particles within the vacuum, specifically \( \pm \pi \) pairs, where the mass equals, \( (\pi^\pm) = 2.488 \, E - 28 \, kg = 139.57 \, MeV/c^2 \). He applied this to the cosmological constant problem as an explanation for dark energy. This sea of virtual charged particles was his answer to polarization of the vacuum, and dark energy.

Winterberg was another author intrigued by the cosmological constant problem, and offered an explanation for the zero point energy associated with the vacuum. Due to a seemingly infinite number of harmonic oscillators, he argued correctly, we believe, that the vacuum cannot be empty. He introduced the bold idea that the vacuum is, in reality, comprised of fundamental positive, and negative mass particles, which he called planckions. They have positive and negative the Planck mass, \( \pm M_p = \pm 2.178 \, E - 8 \, kg \), are real particles, versus the virtual particles of Hajdukovíc, and together form a stiff, and very dense two component superfluid. We prefer supersolid. This vast assembly (sea) of very densely packed, positive, and negative mass particles, permeate all of space as they constitute the vacuum. Due to a mass compensating effect, already at the submicroscopic level, \( (< 10^{-18} \, meters) \), they form an ether-like medium, the vacuum, which is seemingly empty. The total mass density, the total gravitational pressure, and the net entropy of the vacuum all add up to zero in the undisturbed state.

Moreover, through thermally induced CBR photon bombardment, energy/ momentum can be exchanged/ transferred between planckions, and the blackbody photons, as they both occupy the same space. These collisions with photons make for inherent fluctuations within the vacuum. The position and momentum of other material particles, when placed within, or upon, such a space, cannot be determined precisely. Thus, the Heisenberg uncertainty relation has an explanation. Because the underlying space is oscillating and vibrating, a material particle is much like a ship when placed upon the ocean, which is rocking to and fro. This Zitterbewegung, or erratic, chaotic, and random motion makes it impossible to fix both its momentum and position simultaneously. In the Winterberg model, both quantum mechanics, and the general theory of relativity, are treated as two asymptotic limits. His theory is also very interesting in that it is very deterministic/ mechanistic in its approach, using Boltzmann type arguments to derive quantum mechanical equations from first principles, such as the Schroedinger equation, an equation, which is supposedly based on probability and thus non-deterministic.

In the Winterberg model, two like planckions, whether they have positive or negative mass, will simultaneously repel and attract each other. When brought too close together they repel, and
when separated by too large a distance, they attract. In short, they maintain a fixed distance between nearest neighbors. Unlike mass planckions also simultaneously attract, and repel, but indirectly. When unlike mass planckions undergo elastic collisions with each other, they pass through each other as if the other particle from the other species didn’t exist [12]. However, the two species occupy the same space, and thus, indirectly, they are forced close to particles of the opposite species. These unlike mass planckions also strive to maintain a fixed distance of separation from one another.

This is in contrast to the model of Hajdukovic, where, as mentioned, the charged $\pi^\pm$ particles are virtual, and two like charges repel, and two unlike charges attract. For the specifics of these two respective models, we refer the reader to the original references. In paper [1], both theories were discussed briefly. Our model is really an extension of these two separate models, with some major modifications built in.

The positive and negative mass planckion particles which form the vacuum can be gravitationally polarized, if conditions are right. Gravitational dipoles of positive and negative the Planck mass will form, which can be ordered or aligned creating a net susceptibility, and net polarization, which can be measured macroscopically. The dipole moments can be of two kinds. The first type of dipole moment is transitory and random due to collisions with blackbody photons. When bombarded with these photons, they oscillate about their center of mass, which for a positive and negative mass planckion dipole pair, is at spatial infinity. These thermally induced oscillations produce non-vanishing root mean square amplitudes, leading to dipole moments, which are non-vanishing. Here, however, there can be no net polarization in space due to the random and chaotic nature of blackbody bombardment. The vector sum, of these individual dipole moments, we argue, add up to zero because no single direction in three-dimensional space can be singled out.

The second type of dipole moment can set in at a much reduced temperature. These are due to gravitational fields produced by source matter, and in the surrounding regions of space we will have a net polarization, if the gravitational fields are strong enough, the dipole moment large enough, and if the ambient temperatures are not too excessive given the gravitational field strength. It is really a tug of war situation, where the gravitational field promotes order, and the temperature frustrates all such attempts. Locally, a polarization cloud will form, about the free or source mass distribution, $\rho_F(\vec{x})$, built up from quarks and leptons. The source mass distribution, $\rho_F(\vec{x})$, produces an applied field in the surrounding space, $\overline{g}^{(0)}(\vec{x})$, which takes on the same symmetry as $\rho_F(\vec{x})$. That could be spherical symmetry, cylindrical symmetry, rectangular symmetry, etc. The, $\overline{g}^{(0)}(\vec{x})$, in turn, induces a polarized gravitational field, $\overline{g}^{(1)}(\vec{x})$, within the vacuum, the gravitic, which is our gravitational version of a dielectric. The total macroscopic gravitational field, $\overline{g}(\vec{x}) = \overline{g}^{(0)}(\vec{x}) + \overline{g}^{(1)}(\vec{x})$, is greater than the original field, $\overline{g}^{(0)}(\vec{x})$, and we have anti-screening. All gravitational fields reflect the symmetry of the original source distribution, $\rho_F(\vec{x})$.

This second type of dipole moment will allow for a net polarization of the vacuum. Cosmically, when averaged over the entire universe, we obtained [1], $\overline{P}_0 = \epsilon_0 \overline{\chi_0} \overline{g_0} = 2.396 \text{ kg/m}^2$, in the
present era. In this equation, $\chi_0$, is the cosmic susceptibility, a smeared quantity. The, $\overline{G}_0 = 2.387 E - 9 m/s^2$, is the cosmic net macroscopic gravitational field, another smeared quantity, obtained from Gauss’s law, which holds point for point in the universe, but only if huge distance scales are considered. This value takes into account both source and bound matter within the universe. The gravitational permittivity, $\varepsilon_0$, is defined, by analogy to electrostatics, as, $\varepsilon_0 \equiv 1/(4\pi G_0) = 1.192 E9 (MKS) $, where, $G_0$, is Newton’s constant. The values for, $\overline{\chi}_0$, and, $\overline{G}_0$, above, were imposed upon us in order to make sense of the present-day density parameters, $\Omega_i$, in Friedmann’s equation, within the $\Lambda CDM$ model. It is important to realize that for polarization to exist a-priori, either locally or cosmically, the positive and negative mass planckions must be spatially anchored or locked in position, somehow. The specific mechanism of anchoring was discussed in some detail in section, II, of reference, [1].

The models presented by, Hajdukovic, Winterberg, and ultimately ours, which is a modified version of the Hajdukovic/ Winterberg models (see the previous paper for the differences) are unconventional. In this work, we wish to build upon our model, as presented in reference, [1]. We want to look into the specifics of gravitational dipole formation, and present two scaling laws for the cosmic susceptibility, $\overline{\chi} = \overline{\chi}(a)$, where, $a$, is the cosmic scale parameter dependent on CBR temperature. In fact, we wish to take this further and develop scaling laws for the macroscopic variables of interest, which were introduced in reference, [1]. This is our primary objective.

A second goal, and just as important, is to consider a cosmologically varying gravitational constant. We wish to build in this additional feature, as it ties in directly with the mass value for the planckions, as well as other considerations presented elsewhere [13-15]. The Planck mass is only a constant, if Newton’s constant, $G_0$, is a true constant of nature. If $G$ varies cosmologically, very slowly in the current era so as not to upset, too much, the accepted and very successful $\Lambda CDM$ model, we have planckion masses which change with cosmological time. For the broadest possible scaling laws, and to provide an intimate connection in later work between electrostatics and planckion theory, we will include this possibility. The limit where, $G$, is a constant is easily taken into account, in all our formulas, and we retrieve all the standard $\Lambda CDM$ results in this particular limit. We have specific reasons for including this option, as we shall see in our third paper.

The outline of this paper is as follows. In section II, we include the possibility that, $G = G(a)$. There are many reasons for assuming this, which were discussed elsewhere. Some of those reasons include alleviating the cosmological constant problem, explaining the observed value of the quintessence parameter, which is, $\omega = -.98$, versus the, $\omega = -1$, which the $\Lambda CDM$ model assumes. Another reason is the renormalization of gravity. Gravity will not exist at extremely high energy/ momentum exchanges, and there is no theory to renormalize. As mentioned, these reasons were discussed at length in previous work, cited above, and will not be repeated here.

We will however, make use of two specific models for, $G = G(a)$, which were called models, $A$, and, $B$, in references [13-15]. In our view, $\overline{\chi}(a)$, and, $G(a)$, are both intrinsic properties of the vacuum, although the latter does not require any source mass. Elementary particles, i.e.,
quarks and leptons, only came into existence, at temperatures below, 10 E16 Kelvin (about 1 TeV). If Newton’s constant varies at all, it was calculated that, $G^{-1}(a)$, surfaces, or forms, at an inception CBR temperature of about, 6.20 E21 Kelvin, in model A, and, 7.01 E21 Kelvin, in model B. In fact, both models, $A$, and, $B$, lead to very similar inception temperatures even though they are modeled quite differently, by two very distinct and separate functions. The polarization of space, on the other hand, requires much reduced CBR temperatures, as source mass is needed, and that source mass, made up of quarks and leptons, didn’t even begin to freeze out until well below, 1 TeV, or, about 10 E16 Kelvin [16-19]. So, the polarization of space seems to have little to do with the cosmic development of, $G^{-1}$.

There is, however, an intimate connection. The mass of the constituent positive and negative mass planckions, is directly determined by the value of, $G$. The positive and negative mass planckions are the building blocks of the vacuum, and the principle players in determining polarization, i.e., macroscopic ordering, or alignment, of gravitational dipole moments.

If we wish to see how the vacuum evolves, it would be a mistake not to include the possibility that the masses of the planckions can vary, as they comprise and populate the vacuum. Should $G$ not vary cosmologically, all equations in this paper, and in other work, can easily be modified to include this contingency, as well.

In section III, we give two models for cosmic susceptibility, $\tilde{\chi} = \tilde{\chi}(a)$. We first differentiate between ionic and orientation polarization, and show that both lead to essentially the same results, both qualitatively, and quantitatively. Model, $I$, is based on ionic polarization, whereas model, $II$, has orientation polarizability as its basis. Ionization polarization is conceptually simpler, has non-permanent gravitational dipoles, except those which form when an external gravitational field is applied. This will lead to what we call a charging capacitor model, where bound mass, $M_B$, builds up via the relation, $M_B = (\chi/K) M_F = [\chi/(1 - \chi)] M_F$. The, $M_F$, is the source mass, enclosed within a specific Gaussian surface, and, $M_B$, is the bound mass within that same surface. This can be treated as a cosmic equation, where the entire cosmos is considered, and then, $\tilde{\chi} = \tilde{\chi}(a)$ is a smeared quantity. As the universe expands and cools, bound or polarized mass will slowly build up within the universe as a function of the cosmological scale parameter, $a$. In the present epoch, we have found that, $M_{B,0} = 5.327 M_{F,0}$.

Orientation polarization, model, $II$, is somewhat more complicated, and assumes pre-existing dipoles before any applied field is introduced. When a source field is applied, these dipole moments will orient, or align, themselves in the sense of the applied field, if conditions are right, in order to minimize their gravitational potential energy. Those pre-existing dipoles could be due to blackbody photon bombardment, as these photons would cause non-vanishing root mean square amplitudes for the oscillating positive and negative mass planckion pairs, which make up the vacuum. Those gravitational dipoles would then be thermally induced by constant and random blackbody photon collisions. What remains is for this dipole axis to align itself in the sense of the applied field against the disruptive effects of temperature, to cause a net macroscopic polarization. This model is treated much like magnetization in a paramagnet, and a Langevin function will be introduced to model, $\tilde{\chi} = \tilde{\chi}(a)$. In this model the universe can be
broken up into competing domains, much like in magnetization, where some regions polarize (magnetize) and others do not.

Both models, $I$ and $II$, are non-linear, one parameter functions, which mimic order parameter behavior. They start from a zero value, at a specific inception temperature, rise dramatically initially, and as the temperature cools, start to level off and flatten. Eventually they reach a saturation value, which for cosmic gravitational susceptibility, is equal to one. At very high temperatures, both functions are inversely proportional to $CBR$ temperature.

In section IV, macroscopic quantities, important in a discussion of polarization for the cosmos will be considered. We will derive the scaling laws for these quantities as the universe expands. We will also focus on one or two epochs of special interest, such as the era of last photon scattering, 380,000 years after the big bang. With our two models we will see that when the $CBR$ temperature was about, 3000 Kelvin, the cosmic $\bar{\chi}(a)$ values are rather small. However the localized values for, $\chi(\vec{x})$, can still be very large. The coolest regions in the universe will have the greatest amount of local susceptibility, and thus those coolest pockets will have the greatest amount of dark matter. This can be important in interpreting the acoustic peaks in the power spectrum correctly. We also consider the cosmological point where dark matter starts to dominate over ordinary matter. Cosmically, this happened rather recently, when the universe as a whole is considered. Local deviations seem to follow their own rules, when it comes down to scaling. Finally, in section V, we present our summary and conclusions.

### II $G = G(a)$ Models

We are interested in the scaling behavior of macroscopic quantities relevant to our polarization model. One of these quantities is Newton’s constant, $G$. There are many reasons why $G$ could vary with cosmological time [13-15], and we include this possibility here. We keep in mind, however, that our formulas are easily modified, should $G$ turn out to be a true constant of nature. All results in reference, [13], revert to the standard $\Lambda CDM$ model, in the limit where the quintessence parameter, $w$, equals negative one. We assumed, namely, in reference [13], that, $w = -0.98$, a slight deviation from the $\Lambda CDM$ assumed value of negative unity. The value, $w = -0.98$, is what is actually observed, although, in fairness, $w = -1$, is easily accommodated within observational error. Choosing, $w = -0.98$, allowed us to derive two specific functions for $G(a)$, which we called models, $A$, and, $B$. Except in the very early universe, the deviation from the predictions of the $\Lambda CDM$ model, were slight.

A cosmologically varying $G$ has a long and interesting history, starting with the work of Dirac and his large number hypothesis [20-22], already formulated in 1936. He was among the first to recognize that $G$ is unusual because of its very weak value when compared to the other coupling constants, and its inherent canonical dimension. Soon afterwards, Jordan [23-26] related a cosmologically time varying $G$ to Hubble’s constant. Since then, there have been many attempts to observe such a variation, with limited success. Some of that history is presented in reference, [13], and will not be repeated here. It is extensive. We mention it here only to give some context.
Model, in reference [13], assumes a $G^{-1}$ scaling behavior as follows,

$$G^{-1}(T) = G_\infty^{-1}(1 - e^{b/T}) \quad (model\ A) \quad (2 - 1)$$

In equation, $(2 - 1), T$ stands for the CBR temperature, and $G_\infty^{-1}$ is a saturation value, achieved in the limit where, $T \to 0$. The constant, "$b$", was determined to equal, $b = 11.663$ Kelvin, by fixing the quintessence parameter to equal, $w = - .98$. In model, $A$, the, $G_\infty^{-1} = 1.014 \; G_0^{-1}$, where $G_0$ is Newton’s constant.

Another way to write equation, $(2 - 1)$, is to make use of the cosmic scale parameter, $a$, defined by, $a \equiv T_0/T = R/R_0 = (1 + z)^{-1}$. All subscripts, “$0$”, denote the current era, and we are using the convention where, $a_0 = 1$. The, $R$, stands for the Hubble radius, the, $T$, denotes CBR temperature, and the, $z$, equals the redshift. In the present epoch, $T_0 = 2.725$ Kelvin. When re-expressed in terms of the cosmic scale parameter, equation, $(2 - 1)$, reads,

$$G^{-1}(T) = G_\infty^{-1}(1 - e^{-4.28a})$$

$$= 1.014 \; G_0^{-1}(1 - e^{-4.28a}) \quad (model\ A) \quad (2 - 2)$$

This equation came into being at a temperature estimated to be, 6.20 E21 Kelvin. We are close to full saturation in the present epoch since, $G_0 = .986 \; G_\infty$. Saturation will occur at roughly, $a \cong 10$, i.e., when the universe is roughly ten times its current radius in this model. Equation, $(2 - 2)$, was modeled as a charging capacitor. What is charging up as a function of cosmological time, is the mass squared of the planckions, as will be seen shortly.

Model $B$ assumes an entirely different scaling law. Here,

$$G^{-1}(T) = G_\infty^{-1}[\coth(b/T) - T/b] \quad (model\ B) \quad (2 - 3)$$

Again, $G_\infty^{-1}$ is the saturated value, applicable in the limit where the CBR temperature, $T \to 0$. The constant, “$b$”, was determined to equal, $b = 48.15$ Kelvin, in order to guarantee that the quintessence parameter, $w = - .98$ . Here, in model, $B$, $G_\infty^{-1} = 1.054 \; G_0^{-1}$.

A second way to rewrite equation, $(2 - 3)$, is to make use of the identity, $a = T_0/T = 2.725/ T$. Substituting this into equation, $(2 - 3)$, and making use of the numerical value for “$b$”, we find,

$$G^{-1}(a) = G_\infty^{-1}[\coth(17.67a) - 1/(17.67a)]$$

$$= 1.054 \; G_0^{-1}[\coth(17.67a) - 1/(17.67a)] \quad (model\ B) \quad (2 - 4)$$

This order parameter surfaced at a Curie temperature of roughly, 7.01 E21 Kelvin, which is very close to the value above, in model A. This is remarkable because both functions, indicated by equations, $(2 - 1)$, and, $(2 - 3)$, are entirely different and quite distinct from one another. In model, $B$, Newton’s constant, $G_0$, is also close to the final saturation value as, $G_0 = .949 \; G_\infty$. Effective saturation in model, $B$, is achieved when the cosmic CBR temperature drops to one-half current value, or when the Hubble radius is twice the current radius. Model, $B$, is modeled much like magnetization, and we call this model the magnetization model for $G$. Both, $G^{-1}$,
and magnetization, $M$, have the same inherent canonical dimension. It should be noted that both equations, $(2 - 3)$, and, $(2 - 4)$, involve the Langevin function, $L(x) = \coth x - 1/x$, where in this instance, the variable, $x = b/T = 17.67a$. The Langevin function is often used to model paramagnetism. We can think of space as somehow consisting of polarized (magnetic) domains, which can be ordered.

The inverse Newtonian “constant”, $G^{-1}$, in both models, $A$, and, $B$, are one-parameter, non-linear functions, which have specific inception temperatures, and rise dramatically at very high temperatures. In fact, both models give a $G^{-1}(a)$ value, which is inversely proportional to temperature at very high temperatures. More correctly, if $T_C$ equals the inception temperature, then, $G^{-1}$, is proportional to, $1/(T - T_C)$, which is typical order parameter behavior. As the universe expands, and the CBR temperature cools, the $G^{-1}(a)$ functions will start to level off and flatten. Close to saturation, the $G^{-1}$ approaches a constant value, $G_0^{-1}$. In the current era, we are close to full saturation as, $G_0 \equiv G_\infty$. When plotted as a function of cosmic scale parameter, $a$, both equations, $(2 - 2)$, and, $(2 - 4)$, look very similar.

The inverse Newtonian gravitational constant, $G^{-1}(a)$, is directly related to planckian mass. To see this, we start with the formal definition of the Planck mass,

$$M_{Pl} \equiv (hc/G)^{1/2} \tag{2 - 5}$$

We square this result, and rewrite the mass as a field,

$$M_{Pl}^2 = hc G^{-1} = < 0|\varphi^2|0 > \tag{2 - 6}$$

Here, the $M_{Pl}^2$ is no longer a constant, but the vacuum expectation value (VEV) of a scalar field, $\varphi$, squared. As the scalar field squared, $\varphi^2$, freezes out of the vacuum, the $G^{-1}$ will change its value, a process lasting eons. In our scenario, $G^{-1}$ is no longer a constant, and neither is the Planck mass. We identify the scalar field in equation, $(2 - 6)$, with the scalar field of Jordan, first introduced in 1937 [23].

It should be noted that $M_{Pl}^2$ has the same canonical dimensions as magnetization in condensed matter physics, or $M_{W_{\pm}}^2$ in particle physics. Thus, it could very well be an order parameter based on inherent dimension alone. In the theory of weak interactions, it is well known that $M_{W_{\pm}}^2$ is essentially the inverse Fermi constant, $G_F^{-1}$, which effectively fades at high energies, and is only constant below 100 GeV. The, $M_{W_{\pm}}$, is the mass of the $W_{\pm}$ boson. Newton’s constant, and the Fermi constant, are the only two known coupling constants in physics, which have an inherent dimension, and that canonical dimension is the same for both. It can be expressed as inverse mass or inverse momentum, squared. We are modeling the gravitational constant much like the Fermi constant in the electro-weak interaction.

The current value for, $G$, is, of course, $G_0 = 6.674 E - 11$ (MKS). If we insert this into equation, $(2 - 5)$, then we obtain the familiar Planck mass, $M_{Pl} = 2.178 E - 8 kg$. Using this value, we can write in place of equation, $(2 - 6)$, the following expression.

$$M_{Pl}^2 = hc G^{-1} = (G_0/G) (2.178 E - 8 kg)^2 \tag{2 - 7}$$
Since, \( G^{-1} \), will increase, with an increase in cosmological time, so too will, \( M_{Pl} \). The Planck mass will start out from a zero value in our models, and increase in accordance with equation, (2 - 7).

As a specific example for our formulas, equations, (2 - 2), and, (2 - 4), let us consider the era of last scattering, where the CBR temperature was, \( T_1 = 3000 \) Kelvin. This specifies a particular epoch, where, \( a_1 = T_0/T_1 = 2.725/3000 = 1100^{-1} \). We substitute this value into both equations, (2 - 2), and, (2 - 4), and find,

\[
\frac{G_1}{G_0} = 254 \quad \text{(model A)} \quad (2 - 8a)
\]

\[
\frac{G_1}{G_0} = 177 \quad \text{(model B)} \quad (2 - 8b)
\]

Both functions give a larger \( G \) value for this cosmological time, when the universe was \( 1/1100 \) its present radius. By equation, (2 - 7), both the positive and negative planckion mass, are reduced in magnitude, by a factor of, \( 1/\sqrt{254} = .063 \), and \( 1/\sqrt{177} = .075 \), respectively, in this epoch versus today.

III Two Models for Cosmic Susceptibility, \( \chi(a) \)

Two types of polarization will be considered, ionization polarization, and orientation polarization. For each, we will present a specific function, \( \chi(a) \). Ionic polarization, model, \( I \), involves induced gravitational dipole moments. Consider a source gravitational field, \( \vec{g}^{(0)} \), pointing from right to left. The positive mass planckion will get displaced from its equilibrium position, and move slightly to the left, being attracted to the source mass. Call that displacement, \( \vec{d}_+ \). The negative mass planckion will also get shifted, but to the right, being repelled by the source field, \( \vec{g}^{(0)} \). Seeing that the source field is uniform, we can expect the displacement of the negative mass to equal in magnitude the positive mass displacement. However, the sense of direction is opposite, i.e., \( \vec{d}_- = -\vec{d}_+ \). The induced dipole moment is thus, \( \vec{p}_d = M_{Pl} \vec{d} = M_{Pl} (2d_+) (-\hat{i}) \), where, \( \hat{i} \), is a unit vector pointing from left to right. This is the simplest kind of polarization possible, where, \( d_+ \), will depend on the amount of the applied field, \( \vec{g}^{(0)} \). The full macroscopic field, \( \vec{g} \), is the vector sum of the source field, \( \vec{g}^{(0)} \), and the induced field, \( \vec{g}^{(1)} \), both pointing in the same direction, \( -\hat{i} \). By definition, \( \vec{p}_d \), will always point from the negative mass to the positive mass, just like for charges in electrostatics.

The gravitational potential energy here for dipole ordering is, \( U = -\vec{p}_d \cdot \vec{g}^{(2)} = -M_{Pl} (2d_+) \cdot \vec{g}^{(2)} \), where, \( \vec{g} \), gets replaced by the localized field, \( \vec{g}^{(2)} \). The localized field is sometimes called the local Lorentz field, or the “molecular field” in electrostatics, and it takes into account the other neighboring dipoles in the vicinity. This is the field that a particular dipole directly experiences within the lattice. If there is no displacement of positive and negative mass, no dipole is formed. In this situation, the symmetry between the positive and negative masses within the undisturbed vacuum prevents a particular direction in space being singled out. The
A second type of polarization is orientation polarization, model, II. Here we have permanent or inherent dipoles within the medium (vacuum). These will try to self-organize and align in a particular direction in an applied $\overline{g^{(0)}}$ field against the disruptive effects of temperature. The gravitational potential energy here is, $U = -\overline{p_d} \cdot \overline{g^{(2)}} = -|\overline{p_d}| \left| \overline{g^{(2)}} \right| \cos(\theta)$, where, $\overline{g^{(2)}}$, is, again, the local Lorentz field, or molecular field, described above, which also takes into account the gravitational field produced by the neighboring dipoles. The permanent dipoles will orient themselves three-dimensionally in a $\overline{g^{(2)}}$ field, in order to achieve the lowest possible potential energy, against a backdrop of CBR temperature, which will attempt to disrupt/ frustrate any such attempts. The permanent dipoles can be due to inherent and constant collisions with CBR blackbody photons. This will cause oscillations about the center of mass for dipole planckion pairs, and a root mean square amplitude for simple harmonic motion results. In a $\overline{g^{(2)}}$ field, the axis of vibration/ oscillation would want to align itself with the gravitational field, with the positive mass facing the source.

If the vector sum of the individual gravitational dipoles can overcome temperature, we can have partial, or even full, alignment. In either case, we then have polarization in the amount

$$\overline{p} = n_{MAX} \overline{< p_d >}$$

(3-1)

where, $n_{MAX}$, is the maximum gravitational dipole density, $n_{MAX} = n_{MAX}(\overline{x})$, and, $\overline{< p_d >}$, is some average taking into account thermal disturbances. It turns out that, in the case of orientation polarization,

$$\overline{< p_d >} = \overline{p_d} \overline{< \cos(\theta) >} = \overline{p_d} L(x) = \overline{p_d} \left[ \coth(x) - 1/x \right]$$

(3-2)

In equation, (3-2), $L(x)$, is the Langevin function, defined as $L(x) \equiv [\coth(x) - 1/x]$. This Langevin function can be viewed as a probability or percentage of total dipole alignment. The Langevin function depends on ambient temperature, $T$, dipole moment, $\overline{p_d}$, and, $\overline{g^{(2)}}$ field. The argument of the Langevin function, $x$, in equation, (3-2), is namely defined as,

$$x \equiv -U/(k_B T) = \left( \frac{|\overline{p_d}|}{|\overline{g^{(2)}}|} \right) / (k_B T)$$

(3-3)

The, $k_B$, is Boltzmann’s constant.

If dealing with an expanding universe, and space on a grand scale, all quantities in the definition of, $\overline{x}$, above, are smeared quantities. Then, we would write in place of equation, (3-3),

$$\overline{x} = a \overline{x_0} \equiv a (\overline{p_d} \overline{g^{(2)}})/(k_B T_0)$$

(3-4)
In this equation, $T_0$, represents the present CBR temperature, $T_0 = 2.725$ Kelvin, and, $a$, is the cosmic scale parameter, $a = T_0/T = (1 + z)^{-1}$. The redshift is specified by the variable, $z$. Equations, (3–3) and (3–4), look similar, but one is local, equation (3–3), and the other, equation (3–4), is cosmic, where all variables are smeared cosmic averages, which hold only when huge distance scales are considered. A local equation is one where all variables depend on position, $\mathbf{x}$. For $\chi(a)$, we choose $\chi(\mathbf{x}) = L(\mathbf{x})$, where, $\mathbf{x}$, is specified by equation, (3–4). This is model, I, for cosmic susceptibility. One will note that the maximum value for cosmic susceptibility, $\chi(\mathbf{x}) = L(\mathbf{x})$, is unity.

In the case of ionic polarization, model, I, we will use a different function for, $\chi = \chi(a)$, not the Langevin function. Instead of choosing, $\chi(\mathbf{x}) = L(\mathbf{x})$, as specified in equation, (3–2), we will use, instead,

$$\chi(x) = [1 - e^{-x}] \quad \text{(model I)} \quad (3–5)$$

The variable, $x$, is defined as in equation, (3–4). We are looking at cosmic susceptibility, $\chi(x) = \chi(a)$, which holds only when the universe is taken as a whole. The maximum value for equation, (3–5), is also one. It is achieved in the limit where, $x \rightarrow \infty$, or equivalently, when, $T \rightarrow 0$. The Langevin function has those same limits.

The physical motivation for equation, (3–5), is somewhat different than that of equation, (3–2). Equation, (3–2), treats the cosmic susceptibility as a kind of magnetization. Localized domains in space, create an average or smeared cosmic value, and, $L(x) = L(a) = \chi(a)$, is the result. Equation, (3–5), on the other hand, looks more like a charging capacitor model where bound mass for the universe is being “charged” up within the gravitic, which is what we call the vacuum. From previous work [1], remember that bound mass, or polarized mass, identified as dark matter, $M_B$, is related to source mass, $M_F$, by means of the equation,

$$M_B = (\chi/K) M_F = [\chi/(1 - \chi)] M_F \quad (3–6)$$

This is a non-local equation. As the universe expands, and the CBR temperature decreases, cosmic susceptibility, $\chi(a)$, will increase. And so, the bound mass will build up as a function of cosmological time, but not linearly. As $\chi$ gets larger, non-linearly according to equation, (3–5), the bound mass will increase even more dramatically because of the denominator decreasing at the same time. The $M_F$ value stays the same. In the current epoch, it was determined that the cosmic value for $\chi$ equals, $\chi_0 = \chi(a_0 = 1) = .842$.

The counterpart to equation, (3–5), will hold for orientation polarization. This we call model, II, our magnetization model for cosmic susceptibility. Once more, this can be written in terms of a Langevin function as,

$$L(x) = \chi(x) = [\coth(x) - 1/x] \quad \text{(model II)} \quad (3–7)$$

Equations, (3–5) and (3–7), are two quite distinct functions, and yet, when plotted, look remarkably similar. They are both one parameter, nonlinear functions, and both mimic order parameter behavior. The one parameter that has to be fixed in both models is, $x_0 \equiv$
See equation, (3 − 4). What we are really determining is dipole gravitational potential energy, \( U \), for both models, \( I \), and \( II \), since \( T_0 = 2.725 \) Kelvin. See equation, (3 − 3). This dipole energy, \( U \), determines dipole ordering, or alignment, in a \( g^{(2)} \) field. Once \( x_0 \) is determined, we are in a position to find the cosmic susceptibility for both our models, \( I \), and, \( II \), using equations, (3 − 5), and (3 − 7), respectively. We keep in mind that, \( = a \ x_0 \), where, \( a \), is cosmic scale parameter.

To find the parameter, \( x_0 \), in either model, we use the present epoch value for cosmic susceptibility. This has been found \([1]\) to equal, \( \chi_0 = \bar{\chi}(a_0 = 1) = .842 \). Inserting this value in equation, (3 − 5), and solving gives,

\[
x_0 = 1.845 \quad x = a \ x_0 \quad (model \ I) \quad (3 − 8)
\]

For equation, (3 − 7), we proceed likewise. Set the right hand side equal to \( .842 \), keeping in mind that this holds for, \( a = 1 \), and solve for, \( x_0 \). The result is,

\[
x_0 = 6.338 \quad x = a \ x_0 \quad (model \ II) \quad (3 − 9)
\]

With these values for, \( x_0 \), we can easily find, \( x = a \ x_0 \), for any given cosmological epoch. We just have to specify the cosmic scale parameter, \( a \), or the redshift. Substituting the \( x \) value in the appropriate equation, (3 − 5), or (3 − 7), will give us our cosmic susceptibility.

One may have noticed that equations, (3 − 5), and (3 − 7), bear a striking resemblance to equations, (2 − 2), and (2 − 4), in section \( II \). This is no accident. A charging capacitor model, or a magnetization model, seem to us very good models, for both, \( \bar{\chi} = \bar{\chi}(a) \), and, \( G^{-1} = G^{-1}(a) \). We emphasize however that they both model entirely physical processes. The inverse Newton “constant”, \( G^{-1}(a) \), has an inception temperature of about, \( 10 \ E21 \) Kelvin, and effectively models the development of Planckian mass squared. See equations, (2 − 6), or (2 − 7). The, \( \bar{\chi}(a) \), on the other hand, models cosmic susceptibility, or polarization of space, when the cosmos is treated as a whole, and as a smeared value. The inception temperature for \( \bar{\chi}(a) \) is much less, probably after BBM (Big Bang Nucleosynthesis), or, \( T \leq 10 \ E9 \) Kelvin. At CBR temperature of, \( 3000 \) Kelvin, the \( \bar{\chi}(a) \) is already about a thousand times smaller than what it is today, as we shall see next.

As a numerical example of equation, (3 − 5), and (3 − 7), let us evaluate both \( \bar{\chi}(a) \) values at a CBR temperature of, \( 3000 \) Kelvin, the era of last photon scattering. The appropriate scale parameter value here is, \( a_1 = T_0 / T_1 = 2.725 / 3000 = 1100^{-1} \). In model, \( I \), we substitute this \( a_1 \) value, together with the \( x_0 \) value specified in equation, (3 − 8), into equation, (3 − 5). We find that

\[
\chi_1 = \chi(a_1 = 1100^{-1}) = 1.675 \ E \ − \ 3 \quad (model \ I) \quad (3 − 10)
\]

This is much less than the current cosmic value of, \( \chi_0 = .842 \). In fact, it is about \( .002 \) as large. For model, \( II \), we proceed likewise. We substitute the \( a_1 \) value above, and the \( x_0 \) value as indicated by equation, (3 − 9), into equation, (3 − 7). Doing this, and evaluating the result renders,
\[ \chi_1 = \chi(a_1 = 1100^{-1}) = 1.920 E - 3 \quad (\text{model II}) \quad (3 - 11) \]

This result is also much less than the current value for cosmic susceptibility. It is about .0023 times as large. We notice that at this CBR temperature, both models give much reduced values for cosmic susceptibility.

For what is needed later, let us also evaluate, the corresponding cosmic, \( K_1 = 1 - \chi_1 \), values in the above two models. We find that,

\[ K_1 = K(a_1 = 1100^{-1}) = .9983 \quad (\text{model I}) \quad (3 - 12a) \]
\[ K_1 = K(a_1 = 1100^{-1}) = .9981 \quad (\text{model II}) \quad (3 - 12b) \]

There is virtually no cosmic polarization, and hence, the relative permittivity is close to unity. Finally we evaluate the ratio, \( \chi_1/K_1 \), at the end of recombination. Using the results of equations, \((3 - 10)\), \((3 - 11)\), and, \((3 - 12a,b)\), we obtain,

\[ \chi_1/K_1 = 1.678 E - 3 \quad (\text{model I}) \quad (3 - 13a) \]
\[ \chi_1/K_1 = 1.924 E - 3 \quad (\text{model II}) \quad (3 - 13b) \]

These ratios are very small. In the present epoch, by contrast, we have, \( \chi_0/K_0 = .842/.158 = 5.327 \). The values indicated above for, \( \chi_1, K_1 \), and, \( \chi_1/K_1 \), are cosmic averages, or smeared quantities. They do not hold locally.

Locally, \( \chi = \chi(\vec{x}) \), and we cannot use the, \( x_0 \), values listed above, in equations, \((3 - 8)\), and \((3 - 9)\). The gravitational field is totally different locally, and not a smeared value. Also, we have different values for the gravitational dipole moments, and ambient temperature. To make a long story short, the potential energy is different, and we can no longer use the cosmic values in equations, \((3 - 3)\), and \((3 - 4)\), which have been determined. The local values for, \( \chi = \chi(\vec{x}) \), can be quite large in the era of last photon scattering, even at much, much higher temperatures than 3000 Kelvin. Remember that the CBR temperature, \( T_1 = 3000 \) Kelvin, is a thermal average holding for the universe as a whole, in that era. What counts for local susceptibility, is the local dipole moment, the local gravitational field, and the local ambient temperature, all of which have to be specified before we can use a variation of our models, I, and, II. In principle, however, it is possible to model these local situations, as well.

We have seen that equation, \((3 - 1)\), is one way to specify polarization, Another way is to use a macroscopic formulation [1], \( \vec{P} = \varepsilon \chi \vec{g} \), a result familiar from electrostatics, but now applied to gravistatics. Equating both equations gives,

\[ \vec{P} = \varepsilon \chi \vec{g} = n_{MAX} \langle \vec{p_d} \rangle \]
\[ = n_{MAX} \chi \vec{p_d} \]
\[ = n \vec{p_d} \quad (3 - 14) \]
In this equation, $\tilde{g}$, is the macroscopic gravitational field, the, $n_{\text{MAX}}$, stands for the maximum gravitational dipole density, and, $n$, equals the effective dipole density, taking susceptibility into account. Only a proportion of dipole moments will self-organize, be ordered, or align macroscopically. For gravitational polarization, the gravitational permittivity, $\varepsilon$, is defined by $\varepsilon \equiv 1/(4\pi G)$, where, $G$, is Newton’s constant. In the present epoch, $\varepsilon = \varepsilon_0 = 1.192 \times 10^9 \ (\text{MKS})$. As always, we leave open the possibility that, $G$, can vary. See section, II.

Equation, $(3 - 14)$, can be thought of as a cause and effect relation. A gravitational field will produce a net polarization, but only if there is a net susceptibility. In other words, $\tilde{g}^{(1)} = \chi \tilde{g}$, must exist. If the susceptibility is unequal to zero, then we have a net effective macroscopic dipole alignment, or ordering, in the amount, $n = \chi n_{\text{MAX}}$. The equation can be interpreted both, locally, or cosmically, like so many of our equations. If a cosmic equation, then the, $\tilde{g} (a)$, the, $\tilde{\chi} (a)$, and the, $\tilde{p}_d (a)$, are all smeared values, holding for the universe as a unit whole. We would also have smeared effective dipole number densities, $\tilde{n}(a)$, and smeared maximum dipole number densities, $\tilde{n}_{\text{MAX}} (a)$.

IV The Scaling Behavior of Cosmic Gravitational Fields, Dark Matter, and Dark Energy

We next consider the scaling laws for the macroscopic quantities introduced in reference [1]. Upon expansion of the universe, we wish to determine how the cosmic gravitational fields, the cosmic polarization, and the net bound mass density, change as a function of cosmic scale parameter. We also have gravitational field mass densities, which scale, and we wish to see how. First a quick review.

Dark matter was identified [1] as the mass produced within the vacuum, due to dipole alignment, or ordering within the gravitic, the vacuum. This is what we referred to as bound mass. We had four mass density terms in Friedmann’s equation,

$$H^2 = \frac{(8\pi G / 3) (\rho_{\text{Rad}} + \rho_F + \rho_B + \rho_{gg})}{(4 - 1)}$$

The first, $\rho_{\text{Rad}}$, is the mass/energy density associated with radiation. Although this is a negligible contribution in the current epoch, it becomes the dominant term in the early universe. It is well known that blackbody radiation due to photons and neutrinos scale as,

$$\frac{\rho_{\text{Rad}}}{\rho_{\text{Rad},0}} = a^{-4}$$

All subscripts, “0”, on variables refer to the present epoch. Variables without a subscript refer to other cosmological epochs. The, $a$, is the cosmic scale parameter. In the present epoch, the radiative component has the value, $\rho_{\text{Rad},0} = 0.0 \times 10^{-5} \times (8.624 \times 10^{-27} \text{ kg} / \text{m}^3)$, relatively small when compared to the other contributions on the right hand side of equation, $(4 - 1)$. All values for mass densities are taken from the latest WMAP/Planck cosmological data collaboration [27-29].
The second contribution to total mass density on the right hand side of equation, \((4 - 1)\), is, \(\rho_F\). This is due to ordinary mass found in the universe, made up of quarks and leptons. We sum up the individual masses of all the gases, molecules, atoms, stars, planets, galaxies, etc. to come up with a total mass, and then divide by the Hubble volume, to arrive at a, \(\rho_F\), value. Its current value is close to, \(\rho_{F,0} = \Omega_{F,0} \rho_0 \cong (0.0486)(8.624 \ E - 27 \ kg/m^3)\). This is also well known to scale as,

\[
\rho_F/\rho_{F,0} = a^{-3} \quad (4 - 3)
\]

All mass densities, in Friedmann’s equation, and in this section, are smeared values holding for distance scales in excess of, 100 \(Mpc\). Only then is the universe fairly homogeneous and isotropic. Technically we should have bars over all such quantities indicating a cosmic average. We often dispense with placing the bars over smeared quantities for ease of writing.

The third term on the right hand side of equation, \((4 - 1)\), is, \(\rho_B\), which we identify as dark matter. As mentioned, this is bound mass, which is produced within the vacuum, and surrounds ordinary matter. This contribution is due to the positive and negative mass planckions forming dipoles within the vacuum gravitic, and net macroscopic ordering, or alignment, of such dipoles, within that space. In the present epoch, the current estimate for dark matter amounts to, \(\rho_{B,0} = \Omega_{B,0} \rho_0 = (0.2589)(8.624 \ E - 27 \ kg/m^3)\). This will not scale like ordinary matter in our model. Counter to the \(\Lambda CDM\) standard model, we propose a different scaling law. Our scaling law for, \(\rho_B\), is,

\[
\rho_B/\rho_{B,0} = [(\chi/K)/(\chi_0/K_0)] (\rho_F/\rho_{F,0}) = [(\chi/K)/(\chi_0/K_0)] a^{-3} \quad (4 - 4)
\]

This follows since, \(\bar{\rho}_B\), is related to, \(\bar{\rho}_F\), via the relation \([1]\),

\[
\bar{\rho}_B = (\chi/K)\bar{\rho}_F \quad (4 - 5)
\]

Equation, \((4 - 5)\), also follows from equation, \((3 - 6)\). We know the ratio, \(\chi_0/K_0\), in the present epoch. This equals, \(\chi_0/K_0 = (.842/.158) = 5.329\). For the, \(\chi/K\), value in another epoch, we need to specify the scale parameter, \(a\), and then use either equations, \((3 - 5)\), \((3 - 8)\), for model, \(I\), or, equations, \((3 - 7)\), \((3 - 9)\), for model, \(II\). In both models, \(a = ax_0\).

As a particular example, we can consider the era of last photon scattering. There, the \(CBR\) temperature was, \(T_1 = 3000\ Kelvin\). The, \(\chi_1/K_1\), values have already been worked out for models, \(I\), and \(II\), and are indicated in equations, \((3 - 13a, b)\). Substituting these values into equation, \((4 - 4)\), we find that

\[
\rho_{B,1}/\rho_{B,0} = 4.191 \ E5 \quad (4 - 6a) \quad (model I)
\]
\[
\rho_{B,1}/\rho_{B,0} = 4.805 \ E5 \quad (4 - 6a) \quad (model II)
\]

Both of these ratios are far less than the \(\Lambda CDM\) standard model value. In the standard model, we would expect the dark matter mass density to scale as source matter density, and in place of the right hand sides of equations, \((4 - 6a, b)\), we would have instead, \(a^{-3} = 1100^3 = \)
Clearly equations, \((4 - 6a, b)\), indicate far lessor values. Dark matter, in our models, is virtually non-existent, as a cosmic average, at the end of recombination. There is little net cosmic polarization of space in this epoch. We keep in mind, however, that localized values for dark matter can still be quite large. We postpone further discussion on this point until later.

The fourth term on the right hand side of equation, \((4 - 1)\), is, \(\rho_{gg}\). This we interpret as dark energy \([1]\). It is really made up of two separate components, a part which does not depend on polarization, and another part, which does. Dark energy is interpreted as the energy density associated with gravitational fields, due to both source matter, and bound matter. Following electrostatics, we claimed that,

\[
\rho_{gg} = \frac{1}{(2c^2)} K \varepsilon \, g^2 \\
= \Omega_{gg} \, \rho \\
= \frac{1}{(2c^2)} \, \varepsilon \, g^{(0)} \\
= \frac{1}{(2c^2)} \, \varepsilon \, g^{(0)}(g^{(0)} + g^{(1)}) \\
= \frac{1}{(2c^2)} \, \varepsilon \, g^{(0)}g^{(0)} + \frac{1}{(2c^2)} \, \varepsilon \, g^{(0)}g^{(1)} \\
= \rho_{AA} + \rho_{BB} \tag{4-7}
\]

where,

\[
\rho_{AA} \equiv \frac{1}{(2c^2)} \, \varepsilon \, g^{(0)}g^{(0)} \tag{4-8a}
\]

\[
\rho_{AB} \equiv \frac{1}{(2c^2)} \, \varepsilon \, g^{(0)}g^{(1)} \\
= (\chi/K) \, \rho_{AA} \tag{4-8b}
\]

Equations, \((4 - 8a, b)\), are formal definitions. The gravitational field mass density associated with just ordinary, or source matter, is, \(\rho_{AA}\). This is proportional to, \(g^{(0)}\) squared, as indicated by equation, \((4 - 8a)\). The gravitational field mass density associated with, \(g^{(0)}\), coupled to the gravitational field associated with bound matter, \(g^{(1)}\), is, \(\rho_{AB}\). By equation, \((4 - 8b)\), this involves both the source gravitational field, \(g^{(0)}\), and the polarized gravitational field, \(g^{(1)}\). In the limit where the cosmic susceptibility vanishes, the contribution, \(\rho_{AB} \to 0\).

From equation, \((4 - 5)\), which is a non-local equation, we saw how bound mass density, or dark matter, is related to free, or source, mass density. Dark matter is formed in the space surrounding ordinary matter, and for dark matter a non-vanishing susceptibility is needed. It should come as no surprise then, that in the second line of equation, \((4 - 8b)\), we have a similar relation, but now relating the gravitational field mass densities. For, \(\rho_{AB}\), susceptibility is also needed. If susceptibility vanishes, then we only have the following contributions to mass density, \(\rho_{Rad}, \rho_{F}\), and \(\rho_{AA}\), in Friedmann’s equation.

We next consider the scaling behavior for dark energy, \(\rho_{gg}\). This is made up of two components, \(\rho_{AA}\), and, \(\rho_{AB}\). We first focus on the \(\rho_{AA}\) component, defined by equation,
(4 – 8\(a\)). This equation is really a smeared equation, and should properly have bars over the quantities, \(\rho_{AA}\), and, \(g^{(0)}\), as these are averages for the universe as a whole, and epoch dependent. Because they are cosmic averages, it would be a mistake to used smeared relations to determine scaling. Instead we will argue as follows.

Let us imagine the universe as a three dimensional sphere, within which we place dots representing significant mass sources such as galaxies. Around each dot, draw dashed concentric bubbles, some smaller in radius, some larger depending on how much source mass is present. These dashed bubbles represent the localized susceptibility field, i.e., the extent to which, \(\bar{g}^{(0)}(\bar{x})\), reaches, and polarizes the surrounding vacuum. In some instances, there will be no dashed bubble, because there is no localized susceptibility. In those instances where susceptibility prevails, the dashed bubbles are gravitationally bound to the source mass, or source mass distribution. As such, the \(\rho_{AA}\) must scale like ordinary matter. We obtain,

\[
\rho_{AA}/\rho_{AA,0} = \rho_{F}/\rho_{F,0} = a^{-3} \tag{4–9}
\]

What does the expanding is the space between concentric bubbles, and not the bubbles themselves.

Equation, (4 – 9), also makes sense from a conservation of energy point of view. The ratio of, \(\rho_{AA}\), to, \(\rho_{F}\), must stay more or less constant as the universe expands. Therefore, \(\rho_{AA}/\rho_{F} = \rho_{AA,0}/\rho_{F,0}\), and equation, (4 – 9), follows using equation, (4 – 3). The, \(\rho_{AB}\), on the other hand, involves an interaction (coupling) with the surrounding space, i.e., vacuum. This can and will involve a different scaling law than that for pure source matter.

Let us use the second line in equation, (4 – 8b), to determine this scaling law. From this equation, it should be apparent that,

\[
\rho_{AB}/\rho_{AB,0} = [(\chi/K)/(\chi_0/K_0)] (\rho_{AA}/\rho_{AA,0}) = [(\chi/K)/(\chi_0/K_0)] a^{-3} \tag{4–10}
\]

The, \(\chi_0/K_0 = (0.842/.158) = 5.329\), as before. See the discussion following equation, (4 – 5). We also can make use of the models from the previous section to determine the ratio, \(\chi/K\). Use either equations, (3 – 5), with (3 – 8), for model, I , or, equations, (3 – 7), with (3 – 9), for model, II. In both models, \(x = a x_0\). All we need to do is specify the cosmic parameter, \(a\), or redshift, \(z\), and we can evaluate the cosmic ratio, \(\chi/K\).

We’ll work out one numerical example. Let us consider the end of recombination, where the CBR temperature is, \(T_1 = 3000\) Kelvin. The, \(\chi_1/K_1\), values have been evaluated. See equations, (3 – 13a, b). We substitute these values into equation, (4 – 10), and find,

\[
\rho_{AB,1}/\rho_{AB,0} = (3.149 E - 4) (1100)^3 \text{ (model I) } \tag{4–11a}
\]

\[
\rho_{AB,1}/\rho_{AB,0} = (5.734 E - 4) (1100)^3 \text{ (model II) } \tag{4–11b}
\]

These are small values when compared to the, \(\rho_{AA,1}/\rho_{AA,0} = \rho_{F,1}/\rho_{F,0} = (1100)^3\), specified by equation, (4 – 9).
The total dark energy mass density, \( \rho_{gg} \), is the sum of \( \rho_{AA} \), and \( \rho_{AB} \). This can be written as,

\[
\rho_{gg} = (1 + \chi / K) \rho_{AA} = \rho_{AA} / K
\]

We have utilized the second line in equation, \((4 - 8b)\), and also recalled that, in gravistatics, the identity, \((K + \chi) = 1\), holds. From equation, \((4 - 12)\), we obtain,

\[
\rho_{gg} / \rho_{gg,0} = (K_0 / K) (\rho_{AA} / \rho_{AA,0}) = (K_0 / K) a^{-3}
\]

The relative gravitational permittivity in the present epoch is, \( K_0 = .158 \). At the end of recombination where, \( T_1 = 3000 \text{ Kelvin} \), we find using equations, \((3 - 12a, b)\),

\[
\rho_{gg,1} / \rho_{gg,0} = (.1583) (1100)^3 \quad \text{(model I)} \quad (4 - 14a)
\]

\[
\rho_{gg,1} / \rho_{gg,0} = (.1583) (1100)^3 \quad \text{(model II)} \quad (4 - 14b)
\]

There is no difference between the two models. Dark energy scales according to equations, \((4 - 14a, b)\). It will be noticed that none of the mass density scaling laws involve, \( G \), Newton’s constant. This will be different when we look at the smeared, or cosmic, gravitational field strengths.

Before we consider the individual cosmic gravitational field scaling laws, let us evaluate the various contributions to mass density in the era of last scattering. We have all the relations needed. We start with radiation mass density, \( \rho_{Rad} \). From equation, \((4 - 2)\), we find that,

\[
\rho_{Rad} = a_1^{-4} \rho_{Rad,0} = (1100)^4 (8.3 E - 5) \rho_0
\]

\[
= 1.2152 E8 \rho_0 \quad (4 - 15)
\]

The total mass density in the present epoch, \( \rho_0 \), equals, \( \rho_0 = 8.624 E - 27 \text{ kg/m}^3 \). This \( \rho_0 \) value corresponds to a present rate of expansion of, \( H_0 = 67.74 \text{ km/(s \cdot Mpc)} \). For ordinary matter, we use equation, \((4 - 3)\). At CBR temperature, \( T_1 = 3000 \text{ Kelvin} \), we obtain,

\[
\rho_F = a_1^{-3} \rho_{F,0} = (1100)^3 (.0486) \rho_0
\]

\[
= 0.6469 E8 \rho_0 \quad (4 - 16)
\]

Dark matter comes next. For this we use either equation, \((4 - 6a)\), or equation, \((4 - 6b)\), as our scaling law. We find,

\[
\rho_{B,1} = (4.191 E5)(.2589) \rho_0 = 1.085 E5 \rho_0 \quad \text{(model I)} \quad (4 - 17a)
\]

\[
\rho_{B,1} = (4.805 E5)(.2589) \rho_0 = 1.244 E5 \rho_0 \quad \text{(model I)} \quad (4 - 17b)
\]

And, finally we have dark energy. This scaling law is determined by either one of equations, \((4 - 14a, b)\). Using these equations, we can claim that, in the era of last photon scattering,

\[
\rho_{gg,1} = (.1583)(1100)^3 (.6911) \rho_0 = 1.456 E8 \quad \text{(model I)} \quad (4 - 18a)
\]
\[ \rho_{gg,1} = (\cdot 1583)(1100)^3(\cdot 6911) \rho_0 = 1.456 \, E8 \quad (\text{model II}) \quad (4-18b) \]

The sum of all the various contributions in Friedmann’s equation, equation, (4 – 1), at the end of recombination, is, \( \rho_1 = \rho_{Rad,1} + \rho_{F,1} + \rho_{B,1} + \rho_{gg,1} = 3.3192 \, E8 \, \rho_0 \). This holds for susceptibility model, \( I \). For model, \( II \), the sum is, \( \rho_1 = 3.3193 \, E8 \, \rho_0 \), virtually no difference from model, \( I \). We define the density parameters at the end of recombination by the equation, \( \Omega_{l,1} = \rho_{l,1}/\rho_1 \). Thus, at, \( T_1 = 3000 \, \text{Kelvin} \), we find that,

\[
(\Omega_{Rad,1}, \Omega_{F,1}, \Omega_{B,1}, \Omega_{gg,1}) = (\cdot366, \cdot195, 0, \cdot439) \quad (\text{models I & II}) \quad (4-19)
\]

This result holds for both susceptibility models, \( I \), and, \( II \). There is no difference in values between the two susceptibility models, when taken to three significant figures.

From equation, (4 – 19), we notice that at the end of recombination, our models predict negligible dark matter. Radiation amounts to roughly, \( 37\% \), of the total amount. Ordinary matter accounts for a further, \( 20\% \). The rest is dark energy, which is, \( 44\% \), of the total. These values go totally counter to what is commonly assumed. In the \( \Lambda CDM \) model, dark matter scales like ordinary matter, and there is no dark energy in this epoch. In the standard cosmological model, the expected result is, \( (\Omega_{Rad,1}, \Omega_{F,1}, \Omega_{B,1}, \Omega_{gg,1}) = (.229, .122, .649, 0) \). In the \( \Lambda CDM \) model, dark energy does not scale, and even in the extended quintessence models, dark energy barely scales. In both situations dark energy is quite negligible at the end of recombination. Here it is not.

This brings us to a dilemma. Dark matter is thought to be needed at recombination in order to aggregate ordinary matter in gravitational potential wells, without which, the present structure of the universe would be difficult to explain. Also, when looking at the CBR power spectrum obtained from \( WMAP/Planck \) satellite data, the height of the third acoustic peak stands in a certain proportion/ relation to the height of the first peak. The third peak is identified with dark matter, whereas the first peak denotes ordinary matter. At the end of recombination, one could expect that, \( \Omega_{B,1}/\Omega_{F,1} = (.2589/.0486) \), just as is the case in the present epoch.

We will still maintain, however, that equation, (4 – 19), is correct. There are several caveats which must be considered. The first is that the localized dark matter contributions are much different than the smeared or cosmic average, contributions. Even though dark matter effectively disappears at this CBR temperature cosmically, localized dark matter does not. In fact, localized dark matter (LDM) must be much higher in value near the somewhat cooler source matter, since in the cosmic voids, where there is little to no source matter, there can also be little to no dark matter. If the average cosmic value for dark matter is weak, and if, in the voids, there is negligible dark matter, then near the source masses we must have localized values for dark matter which are particularly strong to compensate for the close to zero values in the voids. Second, as indicated in equations, \( (2 - 8a, b) \), Newton’s gravitational constant has a much higher value. This would help aggregate ordinary matter into gravitational potential energy wells, perhaps even without the need for localized dark matter. Third, as we shall see shortly, the gravitational fields have enhanced values due to an increase in \( G \) value. Those stronger gravitational fields would also enhance clumping of ordinary matter. Finally, dark
energy, itself, may even play a role in the aggregation of ordinary matter. Dark energy is a mass density which involves gravitational fields in the surrounding vacuum. This mass density can exert a pressure on ordinary matter. This will be made clearer in a follow-up paper. In summary there are many reasons, and permutations, to explain ordinary matter aggregation. As to the height of the third acoustic peak in relation to the first, we leave that to the power spectrum experts. This is clearly an area for further research and study.

As another example of the cosmic susceptibility scaling law behavior as it relates to mass density, we consider the cosmic era where dark matter dominates over ordinary matter for the universe as a whole. This must have happened fairly recently, because at the end of recombination, we saw that cosmic dark matter was insignificant. To find this point in cosmological time, we set,

\[ \rho_B = \rho_F \]
\[ (\chi_2/K_2) \rho_F = \rho_F \]
\[ \chi_2 = 1 - \chi_2 \]
\[ \chi_2 = 1/2 \]

(4 - 20)

For cosmic susceptibility model, , we use equations, (3 - 5) , with (3 - 8) , in order to fix the value for the cosmic scale parameter, \( a_2 = (1 + z_2)^{-1} \). Setting equation, (3 - 5), equal to, \( 1/2 \), as indicated by equation, (4 - 20), we find that,

\[ [1 - e^{-1.845 a_2}] = .5 \]

(4 - 21)

If we consider cosmic susceptibility model, , we would have to use, instead, equations, (3 - 7) , with (3 - 9) , in order to fix this parameter, \( a_2 \). Demanding that equation, (3 - 8) equal the right hand side of equation, (4 - 20), we have the condition that,

\[ [\coth(6.338 a_2) - 1/(6.338 a_2)] = .5 \]

(4 - 22)

Both equations, (4 - 21), and (4 - 22), are easily solved. The solutions are,

\[ a_2 = .376 \quad z_2 = 1.66 \quad (model \ I) \]
\[ a_2 = .2835 \quad z_2 = 2.53 \quad (model \ II) \]

(4 - 23a)\( (4 - 23b) \)

The two models give different predictions, with model, II, indicating an earlier epoch for dark matter dominance.

It is now time to look at the gravitational field scaling laws. These gravitational fields are all cosmic average quantities. As smeared values, they do not apply locally. We start with equation, (4 - 9), and use our definition, (4 - 8a). This allows us to re-express equation, (4 - 9) , as,

\[ \varepsilon \frac{g^{(0)}_2}{\varepsilon_0 \frac{g^{(0)}_2}{0}} = a^{-3} \]

(4 - 24)
We next bring the gravitational permittivity terms over to the right hand side, and keep in mind that, $\varepsilon \equiv 1/(4\pi G)$. This renders,

$$\frac{g^{(0)}_1}{(g^{(0)}_0)^2} = (G/G_0) a^{-3} \quad (4 - 25)$$

Taking the square root of both sides of this equation gives us the cosmic gravitational field scaling law, due to source mass in the universe.

$$\frac{g^{(0)}}{(g^{(0)}_0)} = (G/G_0)^{1/2} a^{-3/2} \quad (4 - 26)$$

This scaling law does involve the Newtonian constant. If $G$ does not scale, then, obviously, $= G_0$, and the right hand side above simplifies to, $a^{-3/2}$.

As an example, we consider the era of last scattering, when, $a_1 = 1100^{-1}$. For this epoch, the $G$ values have been calculated, and they are given by equations, $(2 - 8a, b)$. Substituting these values into equation, $(4 - 46)$, results in,

$$\frac{g^{(0)}_1}{(g^{(0)}_0)} = 5.77 E5 \quad (model A) \quad (4 - 27a)$$

$$\frac{g^{(0)}_1}{(g^{(0)}_0)} = 4.85 E5 \quad (model B) \quad (4 - 27b)$$

The cosmic gravitational field due to source mass is enhanced by the factor, $\sqrt{G/G_0}$.

Another cosmic gravitational field is that due to both source mass, and bound or polarized mass. This gravitational field was designated as, $\bar{g}$. We know, however, that, $g^{(0)} = K \bar{g}$. Using this relation, we can claim for, $\bar{g}/\bar{g}_0$, the following scaling behavior.

$$\frac{\bar{g}}{\bar{g}_0} = (K_0/K) (G/G_0)^{1/2} a^{-3/2} \quad (4 - 28)$$

For this result, we have made use of equation, $(4 - 26)$. This scaling law also involves
Newton’s constant, but in addition, the susceptibility scaling laws, because of the factor, $(K_0/K)$, on the right hand side.

As a numerical example, we focus on, $a_1 = 1100^{-1}$, the end of recombination. The $G$ values are again given by equations, $(2 - 8a, b)$. We also have the appropriate $K$ values, for our two susceptibility models, $I$, and, $II$. These are found in equations, $(3 - 12a, b)$. Substituting all these values into equation, $(4 - 28)$, renders,

$$\frac{\bar{g}_1}{\bar{g}_0} = 9.13 E4 \quad (model A, I) \quad (4 - 29a)$$

$$\frac{\bar{g}_1}{\bar{g}_0} = 9.13 E4 \quad (model A, II) \quad (4 - 29b)$$

$$\frac{\bar{g}_1}{\bar{g}_0} = 7.68 E4 \quad (model B, I) \quad (4 - 29c)$$

$$\frac{\bar{g}_1}{\bar{g}_0} = 7.68 E4 \quad (model B, II) \quad (4 - 29d)$$
There is virtually no difference between cosmic susceptibility models. The variation is due to the \( G \) model chosen, \( A \) or \( B \).

Finally, as far as cosmic gravitational fields go, we still have, \( \overline{g}^{(1)} \), which is the contribution due to just bound, or polarized, mass in the universe. Here we will make use of the fundamental relation, \( \overline{g}^{(1)} = \chi \bar{g} \). We start with equation, \( (4 - 28) \), and multiply this equation through by the factor, \( \chi / \chi_0 \). This allows us to write,

\[
\frac{\overline{g}^{(1)}}{g_0^{(1)}} = \left( \frac{\chi}{\chi_0} \right) \left( \frac{K_0}{K} \right) \left( \frac{G}{G_0} \right)^{1/2} a^{-3/2}
\]

(4 - 30)

Again, both the gravitational constant, and the susceptibility model come into play. The scaling behavior is complicated, even in the limit where Newton’s constant stays the same when switching epochs.

We will work out one numerical example here. Let, \( a_1 = 1100^{-1} \), a familiar example. The scaling laws for \( G \) are indicated by equations, \((2 - 8a, b)\). For the ratio, \( (\chi_1 / K_1) \), use equations \((3 - 13a, b)\). We also keep in mind that, \( \chi_0 / K_0 = .842 / .158 \). Inserting all of this into equation, \((4 - 30)\), gives us the following scaling behavior,

\[
\begin{align*}
\frac{\overline{g}_1^{(1)}}{g_0^{(1)}} &= 1.82 \ E2 & \text{(model } A, I) & & (4 - 31a) \\
\frac{\overline{g}_1^{(1)}}{g_0^{(1)}} &= 2.09 \ E2 & \text{(model } A, II) & & (4 - 31b) \\
\frac{\overline{g}_1^{(1)}}{g_0^{(1)}} &= 1.53 \ E2 & \text{(model } B, I) & & (4 - 31c) \\
\frac{\overline{g}_1^{(1)}}{g_0^{(1)}} &= 1.76 \ E2 & \text{(model } B, II) & & (4 - 31d)
\end{align*}
\]

These cosmic (smeared) gravitational fields do not increase by nearly as much as the other cosmic (smeared) gravitational fields. But then, this is field associated with dark matter, which, in and of itself, fades very rapidly cosmically.

In summary, the cosmic gravitational fields scale similarly irrespective of the model combination chosen. All these scaling laws involve a variation in \( G \) value, if Newton’s constant does, in fact, scale. Otherwise we set, \( G = G_0 \), in all of the above equations. The factor, \( (G / G_0)^{1/2} \), is the same in all scaling laws for cosmic gravitational fields. This factor, at the end of recombination, equals, \( \sqrt{254} = 15.9 \), for model, \( A \), and, \( \sqrt{177} = 13.3 \), for model, \( B \). The increased gravitational fields can contribute to the aggregation of ordinary matter in this epoch.

To close this section, we give one final scaling law, and that is for cosmic polarization, \( \bar{P} \). It is known that the cosmic polarization is given by the equation, \( \bar{P} = \varepsilon \chi \bar{g} = \varepsilon \chi \bar{g} = \varepsilon \overline{g}^{(1)} \). See equation, \((3 - 14)\). From this equation, it should be apparent that,

\[
\overline{P} / P_0 = (\varepsilon / \varepsilon_0) \left( \frac{\overline{g}^{(1)}}{g_0^{(1)}} \right) \\
= (G_0 / G) \left( \chi / \chi_0 \right) \left( K_0 / K \right) \left( G / G_0 \right)^{1/2} a^{-3/2}
\]

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\[ \frac{P_1}{P_0} = (\chi / \chi_0) \left( \frac{K_0}{K} \right) \left( \frac{G_0}{G} \right)^{1/2} a^{-3/2} \quad (4 - 32) \]

Use of equation, \((4 - 30)\), has been made. This scaling law depends on both the \(G\) scaling and \(\chi\) scaling. We have two models for each quantity which can scale, and thus four combinations.

To understand the numbers, let us look at our familiar example, \(a_1 = 1100^{-1}\). For the \(G\) variation, equations, \((2 - 8a, b)\), can be used. For the ratio, \(\chi_1/K_1\), use equations, \((3 - 13a, b)\). If we insert all these values into our cosmic polarization scaling law, equation, \((4 - 32)\), we find,

\[
\begin{align*}
\frac{P_1}{P_0} &= .72 \quad \text{(model A, I)} \quad (4 - 33a) \\
\frac{P_1}{P_0} &= .83 \quad \text{(model A, II)} \quad (4 - 33b) \\
\frac{P_1}{P_0} &= .87 \quad \text{(model B, I)} \quad (4 - 33c) \\
\frac{P_1}{P_0} &= .99 \quad \text{(model B, II)} \quad (4 - 33d)
\end{align*}
\]

Surprisingly, the net polarization, in the era of last photon scattering, is about the same as the net polarization in the current epoch. In the present epoch, \(P_0 = 2.396 \text{ kg/m}^2\). We are within, \(72 \%-99\%\), of this current value, depending on the model combination.

### V Summary and Conclusions

We have considered the gravitational susceptibility of space assuming that space is made up of a vast assembly (sea) of positive and negative mass particles, called planckions. These particles, first put forward by Winterberg, form a very stiff, two component superfluid/ supersolid, interact with particles within their species, and offer possible explanations for the vacuum energy, quantum mechanical indeterminacy (the Heisenberg relation), the Schroedinger equation, and, now, dark matter/ dark energy. It is specifically the polarization of space and bound mass, which leads to dark matter. For dark energy, we are led to gravitational field mass density, due to both source, as well as bound, mass, within the universe. For the polarization of space, gravitational dipoles are needed, which can be ordered or aligned in some sense. These are formed from the positive and negative mass planckions, which are assumed to be real, versus virtual, sub-nuclear particles. We presented two specific models for cosmic susceptibility, equations, \((3 - 5)\), with \((3 - 8)\), which we call model, \(I\), and equation, \((3 - 7)\), with \((3 - 9)\), which is referred to as model, \(II\). These susceptibilities do not hold locally, but cosmically as smeared quantities, which hold for the universe as a whole. Cosmic susceptibility is thought to be epoch dependent, and can be expressed in terms of the cosmic scale parameter, \(a\), as, \(\chi = \tilde{\chi} = \tilde{\chi}(a)\).

With the help of our two models for, \(\tilde{\chi}(a)\), we can predict how space, i.e., the vacuum, will polarize as a function of cosmological time. We worked out several numerical examples. We can also have a localized version of susceptibility, \(\chi = \chi(\tilde{x})\), where no specific models are given. For that we need a comprehensive microscopic theory, which is being worked on. The
ratio of applied gravitational field, $g^{(0)}$, which promotes order, to ambient temperature, which promotes disorder, is crucial. The applied field will lead through a series of steps to the molecular field, $g^{(2)}$, which is what the individual dipole experiences within the superliquid/supersolid. This takes into account the gravitational field set up by the neighboring dipoles. What is also important in determining, $\chi (\vec{X})$, are the gravitational dipole moments themselves. These dipole moments, $\vec{p}_d = M_{Pl} \vec{d}$, are formed from the positive and negative mass planckions comprising the vacuum. The theory is involved, and will be left for another paper. We keep in mind, however, that even if the cosmic susceptibility is quite low in certain epochs, the localized values for susceptibility within the same epoch can be quite high.

This result is significant because we have worked out the cosmic susceptibility at the end of recombination, the era of last photon scattering, 380,000 years after the big bang. The results are given in equations, (3 – 10), and (3 – 11). For comparison, the present epoch value for cosmic susceptibility is, $\chi_0 = .842$. In the era of last scattering, the cosmic susceptibility is very small, leading to virtually no cosmic dark matter in this epoch. Localized pockets of dark matter, however, can exist at this CBR temperature of 3000 Kelvin (a cosmic average), and even, at much higher temperatures. The bullet cluster has considerable dark matter, and it is known that the temperature in the surrounding space is very, very high. This tells us that the gravitational fields, and dipole moments are substantial enough to overcome the disruptive effects of ambient temperature. Localized dark matter (LDM) is probably needed for aggregation of ordinary matter at the end of recombination. Other mechanisms, however, can also contribute to the clumping of ordinary matter into gravitational wells in this epoch.

The polarization of the vacuum will also depend on, $\gamma$, Newton’s constant. Newton’s constant determines the mass of the positive and negative mass planckions. See equation, (2 – 6), where this is made explicit. We believe that Newton’s constant may vary cosmologically with time, and we include that possibility in this paper. The reasons for this are presented elsewhere, and are glanced over here. In a follow up paper, there is also compelling evidence for this conjecture. For the most general scaling laws for the polarization of space, we include such a term. Two models for, $G^{-1} = G^{-1}(a)$, were included. Model, A, has equation, (2 – 2), as its basis. Model, B, uses a different function to model, $G^{-1}(a)$, namely equation, (2 – 4). All the equations for scaling in this paper, can accommodate both scenarios, a varying $G$, or a non-varying $G$. For a constant $G$ value, simply set, $G = G_0$, in all equations. With the help of two sets of scaling laws, one set for, $G(a)$, and another set for, $G_0$, we can predict how the polarization of space on a cosmic level will evolve.

Dark matter, and to some extent, dark energy, are thought to depend on the susceptibility of the vacuum. We also have cosmic polarization, $\vec{P} = \varepsilon \chi \vec{g} = \varepsilon \bar{\chi} \vec{g} = \varepsilon \overline{g^{(1)}}$, where the cosmic susceptibility and cosmic gravitational field, are smeared quantities holding for the universe as a whole. The gravitational permittivity is defined by, $\varepsilon \equiv 1/(4\pi G)$. Dark matter is given by equation, (4 – 5). The scaling law is relation, (4 – 4). This scaling law involves, $\chi = \bar{\chi} = \overline{\chi}(a)$. Dark energy is identified as equations, (4 – 7), with (4 – 8a, b). The scaling laws, here, are equations, (4 – 9), and (4 – 10). At the end of recombination, dark matter scales numerically by the amount indicated in equations, (4 – 6a, b). For dark energy, in the era of
last scattering, we have the specific increases over present value, specified by equations, 
\((4 − 14a, b)\). Also, the density parameters at the end of recombination have been worked out. Their relative weightings are indicated by equation, \((4 − 19)\). Although this is a somewhat unconventional assignment/prediction, we believe it is correct. We gave several reasons for how this could be reconciled with power spectrum data, not the least being that localized pockets of dark energy \((LDM)\) can survive at this temperature, and, in fact, at much, much higher temperatures. There is a fundamental difference between cosmic dark matter \((CDM)\), and, localized dark matter \((LDM)\). Localized dark matter follows its own rules.

The ordinary matter to dark matter transition in the cosmos can be determined using our cosmic susceptibility models. We obtained either equation, \((4 − 23a)\), or \((4 − 23b)\), depending on the model. These values are for the universe as a whole. We also determined the gravitational field scaling laws as one changes epochs. These will depend on any variation in Newton’s constant, if a cosmological variation exists. We have equations, \((4 − 26)\), \((4 − 28)\), and \((4 − 30)\). Particular numerical values have been worked out at the end of recombination. Those results are presented in equations, \((4 − 27a, b)\), \((4 − 29a, b, c, d)\), and \((4 − 31a, b, c, d)\), assuming \(G\) varies according to either equation, \((2 − 8a)\), or, \((2 − 8b)\). If \(G\) does not vary, minor modifications have to be made in those equations. Finally the cosmic polarization scaling has been ascertained. We believe equation, \((4 − 32)\), is valid. At, \(a_1 = 1100^{-1}\), the era of last photon scattering, we obtain equations, \((4 − 33a, b, c, d)\). The results are surprising because there is virtually little change in cosmic polarization from the current era.

We are currently working on a detailed microscopic theory of space as it relates to positive and negative mass planckions. Other work is in progress. Results should soon be forthcoming.

**Acknowledgment:**

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