Soft mass generation

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We replace the Higgs sector of the electroweak gauge $SU(2)_L \times U(1)_Y$ model of three fermion families with its 'twenty-some' parameters by a horizontal non-vector-like gauge $SU(3)_F$ quantum flavor dynamics with one parameter. With plausible physical assumptions we suggest that the new dynamics generates spontaneously the masses of its eight flavor gluons, of leptons and quarks, and of the intermediate $W$ and $Z$ bosons. Absence of axial anomalies requires neutrino right-handed electroweak singlets and the dynamics then suggests the existence of massive Majorana neutrinos.

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I. The Higgs mechanism [1] of soft (i.e. spontaneous) mass generation in the Standard model [2] is built up on two basic principles: the principle of gauge invariance and the principle of spontaneous breakdown of symmetry. It has its roots in nonrelativistic field theory of quantum fluids where both principles are instrumental for physical understanding of many distinct macroscopic quantum phenomena. Soft mass generation is in fact a necessity: Hard fermion and intermediate boson masses simply ruin the unitary behaviour of scattering amplitudes with longitudinally polarized intermediate vector bosons.

Principles are, however, more general than their particular realizations: "...who has ever heard of a fundamental theory that requires twenty-some parameters?" [3] We think the Higgs mechanism is a phenomenological realization of the principle of spontaneous gauge symmetry breakdown in electroweak interactions in much the same way the Ginzburg-Landau theory is a phenomenological realization of the same principle in superconductivity.

Phenomenological interpretation of the Higgs mechanism means that the massive spinless particle of a scalar field with properties given by the Standard model Lagrangian does not exist.

What is then the 'microscopic' dynamics which generates softly the vastly different masses to the quanta of three electroweakly identical families of massless lepton and quark fields and to the massless $W$ and $Z$ gauge fields? Attempts are numerous [4]. When strongly coupled they are not truly quantitative. Our suggestion belongs to this category. For dynamical mass generation we suggest to gauge properly the flavor index. Resulting is the strong horizontal non-vector-like non-confining $SU(3)_F$ gauge quantum flavor dynamics (QFD). It is defined by its eight "phonons" or eight flavor gluons $C^a_F$ interacting uniquely with each other and with leptons and quarks of both chiralities with one coupling constant $\hbar$.

In perturbation theory the masslessness of fermion fields is protected by chiral symmetry and the masslessness of the gauge fields is protected by gauge symmetry. Massless fields can, however, describe massive particles. This is possible if: (1) The nonlinear Schwinger-Dyson (SD) equations for the chirality-changing fermion proper self-energies $\Sigma$ have energetically favorable non-perturbative symmetry-breaking ultraviolet-finite solutions. (2) In the transverse gauge-field polarization tensor

$$\Pi^{\mu\nu}(q) \equiv (q^2 g^{\mu\nu} - q^\mu q^\nu) \Pi_{ab}(q^2)$$

the scalars $\Pi_{ab}$ develop dynamically the massless poles.

We argue as follows. First, $\Pi_{ab}$ develop dynamically the massless poles. They correspond to the eight composite 'would-be' Nambu-Goldstone (NG) bosons of spontaneously broken global $SU(3)$ underlying QFD due to the flavor gluon self-interactions [5] and the flavor gluon interactions with fermions [6]. This realization of the general Schwinger mechanism [7] is theoretically viable if the underlying interaction is strong at large distances, asymptotically free at small ones [8], and not vector-like [9]. Field-theoretic purity then demands the model to be free of axial anomalies [10]. Phenomenologically, mediating the flavor changing, electric charge conserving processes the flavor gluons have to be rather heavy. For definiteness we consider $M_0 \sim 10^6$ GeV.

Second, interactions of massive flavor gluons with both left-handed and right-handed massless lepton and quark fields can build up the bridges between these two in the form of the fermion symmetry breaking proper matrix self-energies $\Sigma(q^2)$. Basically, the charged lepton and quark masses of three electroweakly identical fermion families differ due to a unique assignment of the the chiral fermion multiplets to triplet and antitriplet representations of QFD. Within the given electric charge the fermion masses differ due to the low-momentum effective sliding coupling depending upon the flavor gluon mass matrix. The prototype mass formula

$$m_f = M \exp[-8\pi^2/h^2]$$

nicely illustrates our task: If we set $M = 10^6$ GeV the "neutrino" mass $m_\nu = 10^{-9}$ GeV is obtained with an effective interaction strength $h^2/4\pi = 2\pi/15 \ln 10$, and the "top quark" mass $m_t = 10^2$ GeV with $h^2/4\pi = 2\pi/4 \ln 10$. We will demonstrate that due to the expected non-analytic dependence of the symmetry-breaking order parameters upon the ef-
effective interaction strengths such an output is conceivable.

Third, there is no mass-generating dynamics in electroweak \( SU(2)_L \times U(1)_Y \) interactions (small coupling constants \( g \) and \( g' \)). The fermion proper self-energies imply, however, also the spontaneous breakdown of the ‘vertical’ electroweak symmetry. Consequently, the Schwinger mechanism applies and the standard analysis of the corresponding Ward identities \([12, 13, 14]\) results in masses of \( W \) and \( Z \) bosons expressed in terms of the fermion proper self-energies by sum rules.

II. Standard model chiral fermions \( q_{fL}^T = (u_{fL}, d_{fL}) \); \( u_{fR}, d_{fR} \); \( I_{fL}^T = (\nu_{fL}, e_{fL}) \); \( e_{fR} \) of three families \((f = 1, 2, 3)\) can transform under \( SU(3)_F \) either as triplets or anti-triplets. Non-vector-like assignments are those in which not all fermion currents coupled to \( C_\mu \) are vectorial.

(i) Assume that \( q_L \) is an \( SU(3)_F \) triplet (i.e. both \( u_L \) and \( d_L \) are triplets). Then \((u_R, d_R)\) can be either \((3, 3)\) or \((3, 3)\), since for the choices \((3, 3)\) and \((3, 3)\) the mass matrices of the \( u- \) and \( d-\)type quarks would come out equal. Without lack of generality choose \((u_R, d_R) = (3, 3)\).

(ii) It follows that \( l_L \) (i.e. both \( \nu_L \) and \( e_L \)) cannot be a triplet. For it were, the charged lepton mass matrix would be equal either to the \( u-\) or the \( d-\)type quark matrix. Hence, \( l_L \) (i.e. both \( \nu_L \) and \( e_L \)) must be an antitriplet.

(iii) Let \( e_R \) be a triplet (case I). At this point we impose the second restriction i.e., the absence of axial anomalies. Anomaly freedom in this case requires introduction of three neutrino right-handed flavor triplets, \( \nu_{NR} \), \( N = 1, 2, 3 \).

(iv) Let \( e_R \) be an antitriplet (case II). Anomaly freedom in this case requires introduction of five neutrino right-handed flavor triplets, \( \nu_{NR} \), \( N = 1, \ldots, 5 \).

Two asymptotically free cases (three or five QFD triplets) of the neutrino right-handed electroweak singlets should not be considered as an ambiguity. Knowledge of the solution of the SD equations for, say, the charged fermion masses would fix the neutrino pattern uniquely.

The QFD generates the full flavor gluon polarization tensor \( \Pi^{\mu\nu}_{ab}(q) \) \([11]\). It must be symmetric in flavor-octet indices by definition and transverse due to the non-Abelian Ward identity. If \( \Pi_{ab}(q^2) \) is proportional to \( \delta_{ab} \), the \( SU(3)_F \) remains unbroken. Terms of the form \( \delta_{ab} \pi^{(1)}_{\mu\nu}(q^2) + d_{abc} \pi^{(2)}_{\mu\nu}(q^2) \) signal the spontaneous breakdown of this symmetry. This is what we assume. \( \Pi^{\mu\nu}_{ab}(q) \) defines the full flavor gluon propagator \( \Delta^{\mu\nu}_{ab}(q) \) (for definiteness written in the transverse Landau gauge):

\[
\Delta^{\mu\nu}_{ab}(q) = -g^{\mu\nu} + g^{\mu\nu}q^2/q^2[(1 + \Pi(q^2))^{-1}]_{ab} \tag{2}
\]

We also assume that the QFD generates the fermion symmetry breaking proper self energies \( \Sigma \) which give rise to the fermion masses. Later we find the symmetry breaking parts the validity of the Ward identities properly.

\[
S(p) = (p + \Sigma^+ + \Sigma^+) - (p + \Sigma)^2 / (p + \Sigma)^2 \tag{3}
\]

\[
S(p) = (p + \Sigma^+)(p^2 - \Sigma^+)^{-1} F_L + (p + \Sigma)^2 (p^2 - \Sigma^+)^{-1} P_R \tag{4}
\]

\[
\text{Massiveness of particles is a strong coupling low momentum phenomenon:} \quad \text{The fermion and flavor gluon symmetry breaking self-energies become important at low } q^2. \quad \text{At high } q^2 \text{ both } \Sigma(p^2) \text{ and } \Pi(q^2) \text{ simply acquire their known symmetric perturbative form.}
\]

III. Flavor gluon mass generation. Here we follow the analysis of the Ward identities with flavor gluons in accordance with \([3]\): Divergences of the full vertices \( \Gamma^{\mu\nu\lambda}_{abc}(p + q, p) \) (three-flavor-gluon vertex) and \( \Gamma^{ij\kappa}_{ijkl}(p + q, p) \) (fermion-flavor-gluon vertex) at vanishing momenta are expressed in terms of the full inverse flavor gluon and fermion propagators i.e., in terms of \( \Pi \) and \( S \), respectively. We assume that the ghost propagators do not play any dynamical role in the generically nonperturbative reasoning. This assumption is manifest in the ‘pinch technique’ \([10]\). If the symmetry is unbroken the Ward identities are fulfilled trivially. If \( \Pi_{ab} \) and \( S \) develop the symmetry breaking parts the validity of the Ward identities requires the massless poles in the vertices themselves. They correspond to the ‘would-be’ NG bosons composed by construction from both flavor gluons and from all fermion species in the world:

\[
\Gamma^{\mu\nu\lambda}_{abc}(p + q, p)|_{pole} = P^{\mu\nu\lambda}_{b\ast c}(p + q, p) \frac{i}{q^2} \hbar (-iq^\mu)\mathcal{A}_{da}(q^2) \tag{3}
\]

\[
\Gamma^{ij\kappa}_{ijkl}(p + q, p)|_{pole} = P^{ij\kappa}_{j\ast i}(p + q, p) \frac{i}{q^2} \hbar (-iq^\mu)\mathcal{A}_{da}(q^2) \tag{4}
\]

\[
- i q^\mu \mathcal{A}_{da}(q^2) \equiv [\mathcal{P}_{C,da}(q) + \sum_{f} \mathcal{P}_{f,da}(q)] \tag{5}
\]

Physical interpretation of this decomposition should be clear: (1) There are eight ‘would-be’ NG bosons composed both of the flavor gluons and of all fermions in the model. (2) \( \mathcal{P}_{b\ast c} \) is the effective coupling of the NG boson with flavor gluons. (3) \( \mathcal{P}_{j\ast i} \) is the effective coupling of the NG boson with the fermion \( f \). (4) \( \mathcal{P}_{C,da}(q) \) and \( \mathcal{P}_{f,da}(q) \) are the vectorial tadpole UV finite loop integrals. They convert in terms of the effective vertices \( P \), the elementary vertices and the full flavor gluon and fermion.
propagators both the flavor gluon components and the fermion components of the `would-be' NG bosons to the flavor gluons. (5) The crucial effective bilinear derivative vertex between the flavor gluon octet and the `would-be' NG boson octet is given by \[ \tilde{h}_2 \].

The vertex \( \tilde{h}_2 \) gives rise to the massless pole in the longitudinal part of the flavor gluon polarization tensor \( \Pi \). Although its transversality is saved by contributions which we cannot compute explicitly, it follows from it that the flavor gluon mass matrix \( M_{ab}^2(q^2) \) is given by the formula \[ -q^2 \Pi_{ab}(q^2) = M_{ab}^2(q^2) = \sum_d \Lambda_{ad}(q^2) \Lambda_{bd}(q^2) \] (6)

Practical applications will demand diagonalization of the mass matrix \( M_{ab}^2(0) \) and introduction of the flavor gluon mass eigenstates.

IV. Fermion mass generation. Structure of the SD equations for the chiral symmetry changing fermion proper self energies \( \Sigma(p^2) \) (NJL type self-consistency condition [17]) of electrically charged fermions is easily read off the interaction Lagrangian of QFD using the general form of the massive fermion propagator. It is shown in Fig.1. The (bare) flavor gluon propagator is taken in the Feynman gauge as suggested by the pinch technique [16].

![Figure 1: Structure of the SD equation for chirality changing Sigma of a fermion (charged lepton or quark) psi. T_{L,R} are the triplet \( \lambda \) or antitriplet \( -\frac{1}{2} \lambda^* \) generators of a given chiral fermion psi_{L,R}.](image)

The neutrino SD equation is more subtle due to possible Majorana mass terms. Here we merely point out that for three generations the hard Majorana mass terms are prohibited by symmetry and the complete neutrino self energy (Dirac plus Majorana) must be generated dynamically. Such a work is in progress.

The integration in Fig.1. extends over all momenta and the SD equations must be improved by taking into account properly the momentum-dependent sliding coupling \( \tilde{h}_2 \). We know no way of knowing \( \tilde{h}_2 \) at low momenta other than solving the theory [18]. It is likely that it is dominated by the exchanges of the composite `would-be' NG bosons with the effective vertices \( P_{bc,d}^{\lambda} \) and \( P_{ij,d}^{f} \) to both gluons and fermions, respectively.

To proceed we write

\[ \frac{1}{k^2} = \frac{1}{k^2} \left\{ [1+\Pi(k^2)]^{-1} + \Pi(k^2)[1+\Pi(k^2)]^{-1} \right\} \] (7)

and argue as follows:

1. At high momenta \( \Pi \) is given by perturbation theory. The first term in (7) when used in Fig.1. gives rise to flavor insensitive \( \Sigma \) due to massless flavor gluon exchange. With asymptotically free flavor insensitive interaction strength \( \tilde{h}_2^2 \) the first term in (7) when used in Fig.1. gives rise to \( \Sigma \)s due to massless flavor gluon exchange with the bare charge should be ignored.

2. At low momenta the non-perturbative \( \Pi \) is given by [16]. The first term in (7) corresponds to the massive gluon exchanges with a bare charge \( \frac{2}{k^2}(1+\Pi)^{-1} = (k^2 - M^2)^{-1} \) and in Fig.1. it should be ignored. The second term in (7) when used in Fig.1. gives rise to \( \Sigma \)s due to massless flavor gluon exchange with the low-momentum \( \tilde{h}_2^2 \) running to a non-perturbative IR stable fixed point:

\[ \tilde{h}_2^2 \approx h_2^2 \{ 1 + \Pi(k^2) \}^{-1} \] (8)

The corresponding matrix SD equations, still rather schematic, hopefully illustrate the main point: The symmetry-breaking form of \( \Pi \) implies that at low momenta the fermion self energies \( \Sigma \) differ in different flavor channels by the low-momentum flavor sensitive interaction strengths [8], basically due to the low momentum symmetry breaking flavor gluon self energy.

At this exploratory stage we are merely able to illustrate that the low momentum Ansatz [8] for the sliding coupling is bona fide responsible for strong suppression of the fermion mass with respect to the huge flavor gluon mass. We replace both \( M_{ab}^2(k^2) \) and the fermion self energies \( \Sigma_{ij}(p^2) = \Sigma_{ij}(0) = m_{ij} \) by real numbers \( M^2 \) and \( m \), respectively. The SD equation in Fig.1. turns into an algebraic equation

\[ m = \frac{h_2^2}{16\pi^2} \int_0^\infty dk^2 \frac{M^2}{k^2 + M^2} \frac{m}{k^2 + m^2} \] (9)

with solution \( m = M \exp[-8\pi^2/h^2] \) announced earlier in the paper.

Finding reliable low momentum dominated matrix symmetry breaking self-consistent fermion and flavor gluon self-energies which define the fermion and the flavor gluon mass spectrum is an exceedingly difficult task for future work. Life with nonperturbative QCD taught us, however, to be meek.

IV. Masses of W and Z bosons are the necessary consequence of the dynamically generated fermion masses: The fermion proper self-energies \( \Sigma(p^2) \) generated by strong QFD break spontaneously also the `vertical'
$SU(2)_L \times U(1)_Y$ symmetry down to $U(1)_{em}$. Consequently, as before, the properties of three composite `would-be' NG bosons can be extracted from the $SU(2)_L \times U(1)_Y$ Ward identities [12] [13] [14]. For simplicity we consider the fermion proper self energies diagonal.

\[
\Gamma_W(p+q,p) = \frac{g}{2v_2} \gamma^\alpha(1 - \gamma_5) - \frac{g^2}{q^2} \left[(1 - \gamma_5) \Sigma_I(p+q) - (1 + \gamma_5) \Sigma_D(p)\right],
\]

\[
\Gamma_Z(p+q,p) = \frac{g}{2 \cos \theta_W} \left[t_3 \gamma^\alpha(1 - \gamma_5) - 2Q \gamma^\alpha \sin^2 \theta_W - \frac{g}{q^2} [\Sigma(p+q) + \Sigma(p)] \gamma_5\right].
\]

When the electroweak gauge interactions are switched on as weak external perturbations, the $W$ and $Z$ bosons dynamically acquire masses. Their squares are defined as the residues at single massless poles of the $W$ and $Z$ polarization tensors:

\[
m_W^2 = \frac{1}{4} g^2 \left[ m_I^2 \Sigma_I;D(0) + m_D^2 \Sigma_D;I(0) \right] \quad (10)
\]

\[
m_Z^2 = \frac{1}{4} (g^2 + g'^2) \left[ m_I^2 \Sigma_I;I(0) + m_D^2 \Sigma_D;D(0) \right] \quad (11)
\]

In the formulas above $U$ and $D$ abbreviate upper and nether fermions in electroweak doublets, respectively. The neutrinos are considered as massive Dirac fermions for simplicity. The quantities $I$ in (10) [11] defined in [13] are the UV finite loop integrals depending upon $\Sigma$s. If the proper self-energies $\Sigma_I$ and $\Sigma_D$ were degenerate the Weinberg relation $m_W^2 / m_Z^2 \cos^2 \theta_W = 1$ would be fulfilled. Quantitative analysis of departure from this relation demands quantitative knowledge of the functional form of proper self-energies. At present we can only refer to an illustrative model analysis of [13].

V. Present model of soft generation of masses of the Standard model particles by a strong-coupling dynamics, having just one unknown parameter $\hbar$, is either right or plainly wrong. Reliable computation of the fermion mass spectrum is, however, far away. Ultimately, masses should be related. (1) One elaborated example of mass relations is the sum rules for the intermediate boson masses $m_W$ [10] and $m_Z$ [11]. The implication is interesting: There is no generic Fermi scale in the model. The intermediate boson masses are merely a manifestation of the large top quark mass [13] [14]. (2) Detailed analysis of the uniquely defined neutrino sector is a challenge. The very existence of sterile neutrinos introduced for anomaly freedom should have experimental consequences in neutrino oscillations and in astrophysics [21]. (3) The fermion SD equations can fix also the fermion mixing parameters. (4) It is natural to expect that the unitarization of the scattering amplitudes with longitudinal polarization states of massive spin one particles proceeds in the present model via the massive composite `cousins' of the composite `would-be' NG bosons. Its practical implementation is obscured, however, by our ignorance of the detailed properties of the spectrum of strongly coupled $SU(3)_F$.

In conclusion we may perhaps defend ourselves by paraphrasing the godfather of the Higgs mechanism, Fritz London [21]: `the model at which we have arrived is distinguished by its uniqueness in such a way that we could hardly avoid writing it down'.

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