Kaon Condensation and Enforced Charge Neutrality of the Color-Flavor Locked Phase

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Abstract

We investigate the influence of charged kaon condensation in hadronic sector on the formation of the electron-free charge neutral quark matter of equipartition of three light flavors, i.e., a phenomenon called “enforced electrical neutrality” of the color-flavor locked (CFL) phase. We employ the chiral quark model which is assumed to be applicable, “bottom-up,” toward chiral restoration while accessible to kaon condensation in the hadronic sector. Approaching “bottom-up” the high density regime is more appropriate in addressing the properties of compact stars than approaching “top-down” from asymptotic QCD. It turns out that the presence of hadronic kaon condensation makes the electron-free charge-neutral CFL phase energetically more favorable with a relatively small color-superconducting gap $\Delta_0 \sim 20$ MeV compared to $\Delta_0 \sim 70$ MeV required in the absence of such kaon condensation.
1 Introduction

The phenomenon of color superconductivity in high density quark matter at relatively low temperature along with its physical consequences has recently been extensively investigated. For recent reviews, see [1]. It turns out that in high density quark matter with three flavors, the color-flavor-locked (CFL) phase is energetically favored, exhibiting a variety of interesting physics such as enforced electric charge neutrality[2] and kaon condensation [3].

Kaon condensation in the CFL phase (that we shall refer to as “CFL kaon condensation”) was first investigated by Schäfer [3] who showed that negative kaons can be condensed in high-density quark matter. The principal mechanism for this phenomenon is that in the CFL phase, the spectrum of Goldstone bosons is inverted so that the kaons turn out to be the lightest modes. When the electron chemical potential $\mu_e$ is equal to the kaon mass $m_K$, the electrons turn into kaons which then condense. In the Schäfer process, the presence of electrons is essential. Although this resembles what happens in kaon condensation that takes place in hadronic phase [4, 5] (referred to as “hadronic kaon condensation” in what follows) in that it is the kaon mass that plays the key role, there is a crucial difference. In hadronic kaon condensation, it is the conspiracy between matter density and chiral symmetry-breaking (strange quark mass) effect that triggers the phase change, so the larger the symmetry breaking, the earlier the phase change. The CFL kaon condensation, in contrast, exploits the smallness of non-zero symmetry breaking.

The effects of non-zero electron chemical potential and a finite strange quark mass on CFL matter have been studied in [6]. It has been shown there that kaon or pion condensation takes place as a response to an external stress generated by quark masses. In particular, it was argued that if the CFL matter were in contact with a hadronic matter that supports a large electron chemical potential, the surface layer would likely be $K^-$- or $\pi^-$-condensed. The reason why the electron chemical potential induces $K^-$ or $\pi^-$ condensation is that a positive electron chemical potential lowers the energy of negatively charged Goldstone modes in the CFL phase [6],

$$E_{\pi^\pm} = \pm \mu_e + m_{\pi^\pm}, \quad E_{K^\pm} = \pm \mu_e + m_{K^\pm}.$$

A recent study, which does not include the effects of the large electron chemical potential, shows that with nonzero quark masses, the most relevant phases in nature are $CFLK^0$ (3-flavor, color superconducting and $K^0$ condensed phase), $CFLK^-$ and/or $CFLK^+$ depending on densities [7] rather than the symmetric charge neutral CFL phase with equal numbers of $u$, $d$ and $s$ quarks [3]. The existence of meta-stable non-topological domain walls associated with broken $U(1)_Y$ in high density quark matter with $K^0$ condensation has also been investigated by Son [8]. Kaplan and Reddy [9] discussed the existence of stable vortices excited by the spontaneous breaking of $U(1)_Y$ and $U(1)_{EM}$ in $K^0$ and $K^+$ condensations respectively.

The ultimate goal of research in the physics of high density matter is to understand the formation and structure of the interior of compact stars which involve supernova explosion and collapse into dense compact stars. In this process, nature follows the route from low density to high density. The studies cited above all approach the density regime of interest coming down (i.e., “top-down”) from asymptotic density at which QCD provides a controllable tool. Whether or not one can actually access, top-down, with sufficient accuracy the relevant density regime is not yet clear. What we propose to do in this paper is to take the other route, namely, start with the low-density regime which is, though theoretically un-controlled, phenomenologically understood and access bottom-up the relevant density regime. This approach unfortunately
suffers from lack of reliable theoretical tools, with QCD being untractable in the nonperturbative regime, so we are forced to rely on models. This paper is an initial attempt to make a bridge from lower density to what can happen at higher density.

At relatively low density, hadrons are the relevant degrees of freedom. Kaon condensations in terms of hadronic variables have been studied extensively since mid 1980’s following the seminal work of Kaplan and Nelson [4]. Kaplan and Nelson showed in tree order with $SU(3)_L \times SU(3)_R$ chiral Lagrangian that kaons could condense at a density around $3\rho_0$. Subsequently a new mechanism for kaon condensations going beyond the tree order which is consistent with kaon-nuclear interactions was proposed by Brown, Kubodera, Rho and Thorsson [5]. This work as well as others that followed it confirmed that the critical density lies in the range $2\rho_0 < \rho_c < 4\rho_0$, more or less in accord with the first prediction of [4]. For reviews see [13]. At densities higher than that of normal nuclear matter, hyperons or more generally strange-quark degrees of freedom – that we shall refer to as strange matter – can become relevant. The effect of hyperons or of strange matter has been studied by several authors. It is found that the presence of strangeness in matter tends to push hadronic kaon condensation to higher densities or depending on parameters – which are hard to pin down – even out of relevant density regime. This aspect is reviewed in [14].

The goal of the present work is to study how $K^-$ condensation aided by electrons that can take place in hadronic sector affects the formation of color superconductivity (CSC) in quark sector. Here we are imagining dialing the density from the regime where hadronic kaon condensation is present to the regime where CSC in the CFL phase is realized. In describing CSC in dense quark matter, one assumes, to start with, that the normal phase of the quark matter is in the form of a Landau Fermi liquid, stable against quark(\textit{q})-anti-quark(\textit{\bar{q}}) pair condensation but unstable against diquark condensation. What we wish to do here is to start from the density regime in which $qq$ pairs are condensed, so that Goldstone bosons of the $\bar{q}q$ type are present and climb up in density to the regime where CSC is presumed to take place. Thus we are approaching Cooper pairing from a sort of non-Fermi-liquid ground state. As a first step to study CSC with such nontrivial vacuum configurations, we consider quark matter in the presence of $K^-$ condensation. To do this, we take the chiral quark model [15] which is believed to be relevant near the chiral phase transition [10, 11, 12]. We first study hadronic $K^-$ condensation with this model and assure that it reproduce more or less what has been obtained in chiral perturbation theory in the hadronic sector. We then extrapolate this model to the density regime of CFL phase and examine the consequence on, among others, the enforced electric charge neutrality nature of the CFL phase. What we are doing can be understood in terms of the NJL (Nambu-Jona-Lasinio) model. In the NJL model, one considers $\langle \bar{q}q \rangle$ and $\langle qq \rangle$ condensations, and finds that the phase with $\langle \bar{q}q \rangle$ condensate is energetically favored in the low density regime while the other one with $\langle qq \rangle$ condensate is at high density. We find that this is true whether a mixed phase exists [16] or not [17]. In fact, the presence of a mixed phase is not crucial.

The structure of this paper is as follows. In section 2, we introduce the chiral quark model and study kaon condensations in the frame work of the model with and without hyperons or strange matter. We investigate the effect of kaon condensation on the enforced charge neutrality of the CFL phase in section 3. We summarize our results in section 4.
2 Kaon Condensation in the Chiral Quark Model

In this section, we describe hadronic kaon condensation in the context of the chiral quark model and compare our results with those obtained in heavy-baryon chiral perturbation theory (HBChPT). To do this, we calculate the in-medium $K^-$ mass using the chiral quark model defined by the Lagrangian

$$\mathcal{L} = \bar{\psi}(iD + V)\psi + g_A\bar{\psi}A\gamma_5\psi - m\bar{\psi}\psi + \frac{1}{4}f^2\pi\text{Tr}(\partial^\mu\Sigma^\dagger\partial_\mu\Sigma) - \frac{1}{2}\text{Tr}(F_{\mu\nu}^\dagger F_{\mu\nu}) + \ldots$$  \hspace{1cm} (1)

where

$$\psi = \begin{pmatrix} u \\ d \\ s \end{pmatrix},$$  \hspace{1cm} (2)

$$D_\mu = \partial_\mu + igG_\mu, \quad G_\mu = G_\mu^aT^a,$$

$$V_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger),$$

$$A_\mu = \frac{i}{2}(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger),$$

$$\xi = e^{i(m/f_\pi)}, \quad f_\pi \simeq 93 \text{ MeV}, \quad \Sigma = \xi\xi$$

and

$$\Pi = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ \\pi^- & K^+ \\pi^0 & K^0 \\bar{K}^- \\frac{1}{\sqrt{6}}\eta \\frac{1}{\sqrt{6}}\eta \\frac{1}{\sqrt{6}}\eta \end{pmatrix}.$$  \hspace{1cm} \hspace{1cm} (3)

Here, $m$ denotes the constituent quark mass generated by spontaneous chiral symmetry breaking and is approximately $350\text{MeV}$. To investigate kaon condensation, we need the symmetry-breaking term

$$\mathcal{L}_M = -\frac{1}{2}c_1\bar{\psi}(\xi^\dagger M\xi^\dagger + \xi M\xi)\psi$$  \hspace{1cm} (4)

where $c_1 \approx 1$ [18] and

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}.$$

In the chiral quark model, one has the advantage of a systematic chiral power counting (as summarized in Appendix) to account for the interactions of constituent quarks with Goldstone bosons and of weak-coupling expansion for interactions with gluons (with $\alpha_s \approx 0.23$) [13]. Within the scheme, there are no free parameters to the order we are considering in contrast to the plethora of uncertainties in coupling constants present in the models involving hyperons [14].
We begin with the relevant part of the Lagrangian for the problem at hand which is of the form

\[ \mathcal{L}_K = \frac{i}{4f_\pi^2} [\bar{u}(K^- \partial K^- - \partial K^+ K^-)u + \bar{s}(K^- \partial K^+ - \partial K^- K^+)s] + \frac{1}{2f_\pi^2} (m_u + m_s) [\bar{u}K^+ K^- u + \bar{s}K^- K^+ s]. \] (5) (6)

The lowest order self-energy correction with this Lagrangian comes from the graph shown in Fig. 1.

Before going into detailed calculations, one can see qualitatively what can happen with and without strange matter. By “strange matter,” we mean, within the framework of chiral quark model, non-zero ground state expectation value of the strange quark pair, \( \langle \bar{s}s \rangle_F \). This will be zero if the strange quark Fermi sea is empty. Hyperon matter in terms of hadronic degrees of freedom is replaced in this scheme by a strange-quark Fermi sea. One can see from (5) that \( u \) quarks decrease (increase) the in-medium mass of \( K^- \) (\( K^+ \)), while \( s \) quarks decrease (increase) the in-medium mass of \( K^+ \) (\( K^- \)). Therefore we expect that at some high densities, the in-medium \( K^+ \) and \( K^- \) masses can become degenerate, i.e \( m_{K^+}^* = m_{K^-}^* \). This interesting result seems, however, at odds with, for example, the results in Ref. [5] in which \( m_{K^+}^* (m_{K^-}^*) \) is found to increase (decrease) with increasing density. This discrepancy can be understood by noting that in Ref. [5], hyperons (or strange matter) are absent. One can understand simply why in the absence of strange-quark Fermi sea, a \( K^- \) gets lighter while a \( K^+ \) gets heavier. In the chiral quark picture, \( K^- \approx \bar{u}s \) and \( K^+ \approx u\bar{s} \), so we expect that putting a \( K^+ \) on top of the Fermi seas of \( u \) and \( d \) quarks is suppressed by the \( u \) quark in the Fermi sea. Now let us increase the baryon density and see what happens there. As density increases, strange quarks start making a strange Fermi sea and eventually at very high density, we can have a Fermi sea composed of equal numbers of \( u \), \( d \) and \( s \) quarks. In terms of the quark picture given above, it is reasonable to expect that the masses of \( K^- \) and \( K^+ \) become equal at very high density. Thus the CFL picture arises naturally in this picture.

2.1 Kaon condensation without strange matter

Consider kaons in a medium that consists only of up and down quark Fermi seas without strange Fermi sea. We believe this system to be analogous to hadronic matter without hyperons. To do
this, we treat the quarks as “heavy” and use the heavy-fermion approximation as in HBChPT. The kaon self-energy computed with Fig. 1 in this approximation is

\[-i\Sigma_K(q_0) = i\left[\frac{3}{4} \frac{(m_u + m_s)}{f_\pi^2} + \frac{3}{4} \frac{q_0}{f_\pi^2}\right] \rho_B\]

(7)

where $\rho_B$ is the baryon density $\rho_B = \frac{2}{3} \rho_Q$ with $\rho_Q$ being quark density. To obtain the in-medium kaon mass, we need to solve the dispersion equation

\[m_K^* = m_K^2 + \Sigma_K(q_0 = m_K^*)\]

(8)

Defining $x = \frac{m_K^*}{m_K}$ and taking $m_u \approx 6$ MeV, $m_s \approx 240$ MeV as in [13] and $m_K \approx 500$ MeV, we rewrite the dispersion equation

\[x^2 + 0.24cx + 0.12c - 1 = 0\]

(9)

where $c = \rho_B/\rho_0$, i.e., the ratio of baryon density to normal nuclear matter density. For instance taking $c=1$, $\rho_B = \rho_0$, we get

\[m_{K^-}^* \approx 410 \text{ MeV}, \quad m_{K^+}^* \approx 533 \text{ MeV}.\]

(10)

In HBChPT, one obtains $m_{K^-}^* \approx 360$ MeV [13]. At higher densities, we get

\[
\begin{align*}
    c = 2 : & \quad m_{K^-}^* \approx 330 \text{ MeV} \\
    c = 3 : & \quad m_{K^-}^* \approx 260 \text{ MeV} \\
    c = 4 : & \quad m_{K^-}^* \approx 193 \text{ MeV}.
\end{align*}
\]

(11)

Noting that for densities of $(2 \sim 4)\rho_0$, $200$ MeV $< \mu_e < 300$ MeV, we expect kaon condensation to take place around $\rho \approx 3\rho_0$.

We consider these results to be unreliable for two reasons. First we are ignoring strange quark matter which is unjustified within the model. Second the parameters in the Lagrangian must be density-dependent as recently shown in a different context [19]. We shall consider the former in the next subsection. Here we take into account the density dependence of the parameters, we adopt the BR scaling approach [22] and use $f_\pi^* = (1 - 0.15\rho/\rho_0)f_\pi$ to compare with the results in Ref. [21]. The results we get are

\[
\begin{align*}
    c = 1 : & \quad m_{K^-}^* \approx 380 \text{ MeV} \\
    c = 2 : & \quad m_{K^-}^* \approx 190 \text{ MeV}
\end{align*}
\]

The effective chiral Lagrangian with BR scaling used in Ref. [21] gives $m_{K^-}^* \approx 330$ MeV in symmetric nuclear matter at $\rho = \rho_0$.

In summary, we find that without strange quark matter, kaon condensation occurs roughly at about $\rho = 3\rho_0$ independently of whether BR scaling is incorporated or not.

### 2.2 Kaon condensation with strange matter

It has been argued that hyperons appear around $(2 \sim 4)\rho_0$ [14], the range of density close to the critical density for kaon condensation. We study in this subsection the effect of strangeness...
presence. Here for simplicity, we shall take hyperon or strange quark population as an external parameter to be varied.

To describe kaon condensation in the presence of strangeness which is relevant in higher densities than without, we have to treat the quarks relativistically since the constituent quark mass must drop [19]. The relativistic self-energy of kaon of Fig. 1 reads

$$\Sigma_{Kq0} = -\frac{(m_u + m_s)}{2f^2} \left[ 3M_u I(q^u_F, M_u) + 3M_s I(q^s_F, M_s) \right]$$

where

$$I(q_F, M) = \frac{1}{2\pi^2} \left[ q_F \sqrt{q^2_F + M^2} - M^2 \ln \left( \frac{q_F + \sqrt{q^2_F + M^2}}{M} \right) \right],$$

$$\rho_q = \frac{1}{\pi^2 q^3_F}.$$  \hspace{1cm} (12)

Here $m_q$ is the current quark mass and $M_q$ the constituent quark mass. It is immediately clear from the second line of (12) that the presence of strange matter will increase $K^-$ mass. However, the negatively charged kaon mass will drop with density as long as $\rho_u > \rho_s$. It is convenient to

*In general, we have $\rho_B = (\rho_u + \rho_d + \rho_s)/3$. In case of the symmetric nuclear matter characterized by $\rho_u = \rho_d \equiv \rho_Q$ and $\rho_s = 0$, we get $\rho_Q = (3/2)\rho_B$. 
define \( y \) as
\[
\rho_B = \frac{1}{3} (\rho_u + \rho_d + \rho_s) = \frac{1}{3} (1 + 1 + y) \rho_Q
\]  
(14)
where \( \rho_u = \rho_d = \rho_Q \) and \( \rho_s = y \rho_Q \). The in-medium kaon mass is obtained from the solution of the dispersion relation
\[
D_K^{-1}(\omega) = \omega^2 - m_K^2 - \Sigma_\rho(\omega) = 0, \quad \text{with } \omega = m_K^*.
\]  
(15)
where \( m_K^* \) denotes the in-medium kaon mass. Taking \( m_u = 6 \text{ MeV}, m_d = 12 \text{ MeV}, \) and \( m_s = 240 \text{ MeV} \) as in [4], we obtain the results given in Fig. 2, Fig. 3 and Table 1. As expected, the mass of \( K^- \) (thin solid line) decreases with density. However, it decreases too slowly, thereby shifting the onset for kaon condensation to a much higher density, \( \rho > 8 \rho_0 \) or even excluding it. This is essentially the result found when hyperons are introduced in the conventional way in a hadronic field theory [14]. In medium, however, this treatment is incomplete since it ignores the “intrinsic” density dependence of the parameters of the Lagrangian required by matching to QCD as described in [19, 20]. In specific terms, this means that we have to take into account that masses and coupling constants must scale with density. We do not know how to compute the density dependence from first principles, so to proceed, we resort to BR scaling [22] and take
\[
\frac{f_{\pi}^*}{f_{\pi}} = \frac{M_q^*}{M_q} = \frac{1}{1 + 0.28 \rho_B/\rho_0}.
\]  
(16)
Inserting this into the dispersion equation (15), we obtain the results summarized in Fig. 2, Fig. 3 and Table 1.

Table 1 : In-medium kaon mass without BR scaling (second column) and with BR scaling (third column) when \( y = 1 \).

| \( c \) | \( m_{K^+}^* = m_{K^-}^* \) (MeV) | \( m_{K^+}^* = m_{K^-}^* \) (MeV) |
|---|---|---|
| 1 | 463 | 440 |
| 2 | 426 | 320 |
| 2.84 | 394 | 26 |
| 5 | 300 | . |
| 8.3 | 30 | . |

In Table 1, we consider a quark matter composed of equal numbers of \( u, d \) and \( s \) quarks and find that \( K^+ \) and \( K^- \) masses become equal as expected. The electron chemical potential is known to be lowered substantially when hyperons or strange quarks are present in matter. For instance, it can be as low as \( \sim 150 \text{ MeV} \) for \( \rho_0 \leq \rho_B \leq 6 \rho_0 \) [23]. For a given baryon density and a lepton number per baryon, the Fermi momenta of \( u, d, s, \nu \) and electron are determined by the constraints of charge neutrality and \( \beta \) equilibrium [24] from which the electron Fermi momentum or the electron chemical potential is found to be \( 108 \text{ MeV} < \mu_e < 223 \text{ MeV} \) depending on the value of the lepton number per baryon at \( \rho = \rho_0 \).

As one can see in Fig. 2 and Fig. 3, with BR scaling, the in-medium kaon masses are \( \sim 100 \text{ MeV} \) around \( \rho \sim 3 \rho_0 \). Thus we arrive at the qualitative conclusion that kaon condensation can take place around \( \rho \sim 3 \rho_0 \) regardless of the presence or absence of strange matter.
3 Kaon Condensation and Color-Flavor Locking

In this section, we turn to the question of whether and how the presence of hadronic kaon condensation discussed above influences the enforced charge neutrality of the CFL phase. Note that here we are not talking about kaon condensation in the CFL phase. As discussed above, we expect $K^{-}$ condensation in the vicinity of $\rho \sim 3\rho_0$ even with strange matter when BR scaling is incorporated. Therefore we start by assuming that $K^{-}$ condenses in dense neutron-star medium and write

$$\langle K^- \rangle = v \exp^{-i\mu K t}.$$  \hfill (17)

Given this condensation, the interaction terms in (5) will change the chemical potential of $u$ and $s$ quarks while (6) will reduce quark masses. Focusing on the change in the chemical potentials, we write

$$L = \bar{\psi}(i \not\partial + \bar{\mu} \gamma_0 - \mu_e Q \gamma_0)\psi + \delta \mu_K (\bar{u} \gamma_0 u - \bar{s} \gamma_0 s)$$  \hfill (18)

where $\bar{\mu}$ is the baryon chemical potential, $\mu_e$ the chemical potential for electric charge and $\delta \mu_K$ denotes a correction from kaon condensation,

$$\delta \mu_K = \frac{v^2}{2 f_\pi^2} \mu_K.$$  

From (18), we observe

$$\mu_u = \bar{\mu} - \frac{2}{3} \mu_e + \delta \mu_K$$
\[
\mu_s = \bar{\mu} + \frac{1}{3}\mu_e - \delta \mu_K. \tag{19}
\]

We note here that kaon condensation decreases the strange-quark chemical potential \(\mu_s\) since it provides the system with an economic way of piling up *strange* quarks without increasing their Fermi momenta.

In order to import the above observation into the CFL phase, we briefly summarize the results of Ref. [2]. It was shown in Ref. [2] that quark matter in the color-flavor locked phase is automatically charge neutral and no electrons are required. To demonstrate this, they perform a simple calculation with two flavors called 1 and 2 and assume \(m_1 = 0\) and \(m_2 = m_s \neq 0\). The chemical potentials of the quarks are

\[
\mu_1 = \bar{\mu} - \delta \mu \\
\mu_2 = \bar{\mu} + \delta \mu \tag{20}
\]

where \(\mu_e = 2\delta \mu\). Here we can identify the quark 1 with an *up* quark and 2 with a *strange* quark.

Comparing the free energy of the BCS state and that of the unpaired state, they obtain

\[
\Omega_{BCS} - \Omega_{normal} = \frac{\bar{\mu}^2}{\pi^2} \left[ \frac{m_s^2}{4\bar{\mu}} - \delta \mu \right]^2 - \frac{\Delta_0^2}{2}. \tag{21}
\]

Then it is easy to see that the BCS state is the global minimum of the system if

\[
\frac{m_s^2}{4\bar{\mu}} - \delta \mu \mid < \frac{\Delta_0}{\sqrt{2}} \tag{22}
\]

Table 2: The chemical potential difference \(\delta \mu_t\) (MeV) as a function of baryon density with different sets of parameters with \(\mu_e = 150\) MeV. Here I, II and III denote the values taken from Tables 3, 4 and 5 of Ref. [26] respectively. The rotation angle (in degrees) is defined by \(\theta \equiv (\sqrt{2}\nu)/f_{\pi}\).

| I | \(\rho_B\) | \(\theta_{min}\) | \(\delta \mu_t\) | II | \(\rho_B\) | \(\theta_{min}\) | \(\delta \mu_t\) | III | \(\rho_B\) | \(\theta_{min}\) | \(\delta \mu_t\) |
|---|---|---|---|---|---|---|---|---|---|---|---|
|   |   |   |   |   |   |   |   |   |   |   |   |
| 4.18 | 0 | 75 | 3.08 | 0 | 75 | 2.42 | 0 | 75 |
| 4.68 | 28.9 | 66 | 3.58 | 39.3 | 57 | 3.42 | 80.5 | 0.13 |
| 5.18 | 40.6 | 56 | 4.58 | 67.3 | 23 | 4.42 | 98.8 | -37 |
| 5.68 | 48.9 | 48 | 5.58 | 80 | 2 | 5.42 | 105.7 | -53 |

In Table 2, we display the values of \(\delta \mu_t\) for different sets of parameters given in Ref. [24]. The results in Table 2 indicate that kaon condensations could facilitate electric charge neutrality of CFL phase within a wide range of parameter choice.

From the development made so far, it is clear what we should do to see the effect of hadronic kaon condensation [1]. The key point is that with kaon condensation, the condition [23] is modified to

\[
\frac{m_s^2}{4\bar{\mu}} - \delta \mu_t \mid < \frac{\Delta_0}{\sqrt{2}} \tag{23}
\]

\[^{1}\text{More properly, we should compare the free energy of the BCS state with that of the state with kaon condensation, but in this work we are looking at the effects of kaon condensation only to the change of quark chemical potentials. Comparing the ground state energy of the CFL phase with that of the phase with kaon condensation is being investigated and will be reported elsewhere.}\]
where $\delta \mu_t = \delta \mu - \delta \mu_K$. Since $\mu_e = \mu_K$, we can rewrite

$$\delta \mu_t = \frac{1}{2} \mu_e (1 - \frac{v^2}{f_\pi}).$$

(24)

Now to see the effect of kaon condensation, we take the example of Ref.\[2\]. With $m_s = 200$ MeV, $\bar{\mu} = 400$ MeV and $\mu_e = 2\delta \mu = 150$ MeV, the authors of \[2\] obtained $\Delta_0 > 50$ MeV from (22). With kaon condensation, for instance with $\theta_{\min} = \sqrt{2} \frac{\omega}{f_\pi} \approx 75$ (in degrees) from Ref. \[26\], we obtain $\Delta_0 / \sqrt{2} > |25 - 11|$ and therefore $\Delta_0 > 20$ MeV. We take this to imply that with kaon condensation, the enforced charge neutrality of the CFL phase is energetically more favored with relatively smaller color superconducting gap compared to the case without kaon condensations.

4 Summary

In this work, we studied, bottom-up, charged kaon condensation aided by electrons within the framework of chiral quark model and their possible effects on the charge neutral CFL phase. When a quark matter is in contact with a hadronic matter, a large electron chemical potential can be supported by the hadronic matter. We have shown that kaon masses drop generically to $\sim 100$ MeV in the vicinity of $\rho = 3\rho_0$ when BR scaling is incorporated, and hence an electron chemical potential around $\mu_e \sim 100$ MeV will be able to trigger electron-aided kaon condensation. In the presence of hyperons or strange quarks, the electron chemical potential comes out to be $\sim 150MeV$ for $\rho_0 \leq \rho_B \leq 6\rho_0$ \[23, 24\]. Consequently kaon condensations can take place at around $\rho = 3\rho_0$ whether strange matter is present or not. Extrapolating to higher density, it is argued that kaon condensation as formulated in the model can render the enforced charge neutrality of the CFL phase energetically more favorable with relatively small color superconducting gap than without kaon condensation. This argument is not without caveats, however, and in order to make it firmer, it will be necessary to compare the ground state energy of the CFL phase with that of the phase with kaon condensation. This issue is under investigation.

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Appendix A

In this appendix, we state the power counting rules for the chiral quark model. Since we can treat the gluons perturbatively with $\alpha_s \approx 0.28$, it suffices to focus on Goldstone bosons and quarks.

The most general vertex in the chiral quark model in cutoff regularization takes the form \[18\],

$$(2\pi)^4 \delta^4 \left( \sum p_i \left( \frac{\pi f_\pi}{f_\pi \Lambda} \right)^A \left( \frac{\gamma^\mu \gamma^\nu}{\Lambda} \right)^B \left( \frac{p_i^\mu p_i^\nu}{\Lambda} \right)^D \Lambda^2 \left( \frac{m_i}{\Lambda} \right)^{\Delta \chi} \right) / 2$$

where $\Delta \chi$ is the chirality violation at the vertex, e.g., $\Delta \chi = 2$ for each $m$. For notational simplicity, we write $\Lambda$ for $\Lambda_{\chi SB} = 4\pi f_\pi$.

Now as far as the Goldstone boson sector is concerned, the counting rule is the same with the one given in standard chiral perturbation theory (ChPT). Including quarks is straightforward since the constituent quark mass can be considered small compared to $\Lambda_{\chi SB} \sim 1$ GeV and hence $m_Q \sim p$ where $p$ is a typical momentum scale. Each quark propagator contributes $-1$ power of $p$, each Goldstone boson propagator contributes $-2$ power of $p$, each derivative and quark mass in the interaction terms contribute $+1$ power of $p$ respectively and each four momentum integration contributes $+4$ power of $p$.

Putting all the powers together, the chiral dimension $D$ of a given amplitude with $L$ loops, $I_{GB}$ internal meson lines, $I_Q$ quark lines, $N_{GB}^n$ mesonic vertices and $N_{GBQ}^d$ meson-quark vertices comes out to be

$$D = 4L - 2I_{GB} - I_Q + \sum_n n N_{GB}^n + \sum_d d N_{GBQ}^d.$$  \hspace{2cm} (A.2)

For connected diagrams, we can use the topological relation

$$L = I_{GB} + I_Q - \sum_n (N_{GB}^n + N_{GBQ}^n) + 1$$  \hspace{2cm} (A.3)

to get

$$D = 2L + 2 + I_Q + \sum_n (n - 2) N_{GB}^n + \sum_d (d - 2) N_{GBQ}^d.$$  \hspace{2cm} (A.4)

\[†\] Note that in the mass-independent subtraction scheme, the only dimensional parameter in the amplitude is the momentum $p$.  

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