Modifying the Yamaguchi Four-Component Decomposition Scattering Powers Using a Stochastic Distance

Avik Bhattacharya, Member, IEEE, Arnab Muhuri, Shaunak De, Surendar Manickam, and Alejandro C. Frery, Senior Member, IEEE

Abstract—Model-based decompositions have gained considerable attention after the initial work of Freeman and Durden. This decomposition which assumes the target to be reflection symmetric was later relaxed in the Yamaguchi et al. decomposition with the addition of the helix parameter. Since then many decompositions have been proposed where either the scattering model was modified to fit the data or the coherency matrix representing the second order statistics of the full polarimetric data is rotated to fit the scattering model. In this paper we propose to modify the Yamaguchi four-component decomposition (Y4O) scattering powers using the concept of statistical information theory for matrices. In order to achieve this modification we propose a method to estimate the polarization orientation angle (OA) from full-polarimetric SAR images using the H ellinger distance. In this method, the OA is estimated by maximizing the Hellinger distance between the un-rotated and the rotated $T_{33}$ and the $T_{22}$ components of the coherency matrix $[T]$. Then, the powers of the Yamaguchi four-component model-based decomposition (Y4O) are modified using the maximum relative stochastic distance between the $T_{33}$ and the $T_{22}$ components of the coherency matrix at the estimated OA. The results show that the overall double-bounce powers over rotated urban areas have significantly improved with the reduction of volume powers. The percentage of pixels with negative powers have also decreased from the Y4O decomposition. The proposed method is both qualitatively and quantitatively compared with the results obtained from the Y4O and the Y4R decompositions for a Radarsat-2 C-band San Francisco dataset and an UAS SAR L-band Hayward dataset.

Index Terms—Synthetic aperture radar, radar polarimetry, polarization orientation angle, stochastic distance

I. INTRODUCTION

POLARIMETRIC target decomposition is a technique to characterize scattering mechanisms from polarimetric synthetic aperture radar (SAR) data. Target decomposition techniques can be broadly classified into two categories: (1) coherent decomposition techniques which utilize the information contained in a target scattering matrix $[S]$, and (2) incoherent decomposition techniques which utilize the second-order statistics in terms of covariance matrices ($[C]$ or $[T]$) derived from the scattering matrix $[S]$. Furthermore, incoherent decomposition techniques can be subdivided into eigenvalue/eigenvector based, and model-based. Eigenvalue/eigenvector based decompositions provide a unique solution in terms of the scattering mechanisms $[1]$, $[2]$. However, Ref. $[3]$ raised questions regarding the assignment of each eigenvector to one of the independent scattering mechanisms by the spectral decomposition of the coherency matrix. Nonetheless, the solutions provided by model-based decompositions depend on the assumptions made about the physical scattering model.

Model-based decompositions have gained considerable attention after the initial work of Freeman and Durden $[4]$. Their decomposition assumes the target to be reflection symmetric, i.e., that the co-polarized and the cross-polarized components are always uncorrelated. This assumption was later relaxed in the Yamaguchi et al. decomposition (Y4O) $[5]$, which included the helical scattering as a fourth component. These two decompositions are widely used in both practice and literature because of their simplicity and computational ease. Besides these, the decomposition of Arii et al. $[6]$, $[7]$ and Neumann et al. $[8]$, $[9]$ introduced different scattering models for vegetation volume component. Jagdhuber et al. $[10]$ proposed a full-polarimetric decomposition with multi-angular data acquisition to estimate soil moisture from bare ground and vegetated soils. Cui et al. $[11]$ proposed a complete and an exact decomposition of the coherency matrix in a volume and two single scattering with non-negative scattering powers. Lee et al. $[12]$ investigated the shortcomings of model-based decompositions, and suggested several models to alleviate them. Recently, Jagdhuber et al. $[13]$ developed a hybrid model-based and eigenvalue/eigenvector-based polarimetric decomposition technique with generalized volume model for soil moisture estimation under vegetation cover.

A major advancement was made with the orientation compensation application in model-based decompositions. This was necessary because of the fact that a target with different orientations in the plane orthogonal to the radar line of sight (LOS) will have different polarimetric responses. A number of decomposition methods with orientation compensation have been proposed $[7]$, $[14]$, $[15]$. The fundamental idea behind such compensation is to minimize the cross-polarization component. The orientation compensation can, to a certain extent, reduce the overestimation of the volume power in a model-based decomposition and increase the double-bounce power. Most notably among these methods are Lee et al. $[14]$, the three-component model-based decomposition by An et al. $[15]$, the orientation compensated four-component decomposition by Yamaguchi et al. (Y4R) $[16]$, and the generalized four-component decomposition by real and complex rotation of the coherency matrix by Singh et al. $[17]$. The decomposition...
methods of Arii et al. [7] and Chen et al. [18] also use orientation angle in their scattering models.

In general, orientation angle (OA) estimation methods can be broadly categorized into two groups according to the input data: (1) OA derived from a Digital Elevation Model (DEM), and (2) OA derived from PolSAR data. The slope and the azimuth obtained from the DEM is used to estimate the OA. Apart from the DEM derived orientation angle there are few other methods available in the literature which directly use PolSAR data to compute the orientation angle. The phase difference between the RR-LL (Right-Right and Left-Left) circular polarizations has been used in [19] to estimate the orientation angle. The polarization orientation shift is used to infer terrain slopes [20]. In a later study, Kimura et al. [21] computed the shifts in the polarization orientation angle in built-up areas. The beta (β) angle obtained from the eigen-decomposition of the coherency matrix in Cloude-Pottier decomposition [1] is also used as a measure of the orientation angle. For deterministic (coherent) scatterers, the orientation angle (diagonalization angle) is obtained from the Cameron et al. decomposition [22]. Xu et al. [23] estimate the target deorientation angle by minimizing the cross-polarization.

In this paper, we propose to modify the Yamaguchi four-component decomposition (Y4O) scattering powers using the concept of statistical information theory for matrices. In order to achieve this modification, we first introduce the concept of stochastic divergence in Section II. In Section III, we propose a method to estimate the polarization OA from full-polarimetric SAR images using a stochastic distance. In this method, the OA is estimated by maximizing the stochastic distance between the un-rotated and the rotated T33 and the T22 elements of the coherency matrix [T]. In Section IV, the powers of the Yamaguchi four-component model-based decomposition (Y4O) are then modified using the relative stochastic distance between the T33 and the T22 elements of the coherency matrix at the estimated OA. The results show that the overall double-bounce powers over rotated urban areas improve significantly with the reduction of volume powers. The percentage of pixels with negative powers is also reduced from the Y4O decomposition. In this work, it is obtained with h(x) = x/2 and \( \beta(x) = (x^{1/2} - 1)^2 \), then (1) becomes

\[
d^h_\phi(D_1, D_2) = 1 - \int \sqrt{f_1 f_2},
\]
as it is also a distance.

In the following we will instantiate this distance under the most important model for full PolSAR data: the Wishart law. Denote \( z = [S_1, S_2, \ldots, S_p]^T \) a complex random vector with \( p \) polarizations (\( p = 3 \) for reciprocal medium or monostatic radar) where \( S_1, S_2 \text{ and } S_3 \) are either \( S_{HH}, S_{HV}, S_{VH}, S_{VV} \) or \( S_{HH} + S_{VV}, S_{HH} - S_{VV}, 2S_{HV} \) are the elements of a scattering vector in the Lexicographic and Pauli bases, respectively. In PolSAR data analysis, \( z \) is often assumed to obey a zero mean multivariate complex Gaussian distribution characterized by the following density:

\[
f(z; \Sigma) = \frac{1}{\pi^p | \Sigma |} \exp(-z^* \Sigma^{-1} z),
\]

where \( \Sigma \) is the Hermitian positive definite covariance matrix, \( v^* \) denotes the conjugate of \( v \), and \( v^T \) its transpose. In order to
increase the signal-to-noise ratio, $L$ independent and identically distributed samples are averaged to form the $L$-looks covariance matrix

$$Z = \frac{1}{L} \sum_{\ell=1}^{L} z(\ell) z(\ell)^T. \quad (3)$$

The $p \times p$ Hermitian positive definite matrix $Z$ follows a scaled complex Wishart distribution whose density is

$$f(Z) = \frac{L^{pl}}{\Gamma_p(L)} |Z|^{L-p} \exp\left[-L \text{Tr}(\Sigma^{-1} Z)\right], \quad (4)$$

where $\text{Tr}$ is the trace and $\Gamma_p(L) = \pi^{p(p-1)/4} \prod_{\nu=1}^{p} \Gamma(L - \nu + 1)$.

The Hellinger distance between two scaled complex Wishart distributions with the same number of looks $L_i = L_j = L$ is given by

$$d_H(\Sigma_i, \Sigma_j) = 1 - \left[\frac{\left|\Sigma_i^{-1} + \Sigma_j^{-1}\right|^{-1}}{2 \sqrt{|\Sigma_i||\Sigma_j|}}\right]^L, \quad (5)$$

where we indexed the distributions by their parameters. Consequently the Hellinger distance for the 1-dimensional ($p = 1$) multilook intensity distribution is given by

$$d_H(\sigma_i^2, \sigma_j^2) = 1 - \left[\frac{\sigma_i \sigma_j}{2(\sigma_i^2 + \sigma_j^2)}\right]^L. \quad (6)$$

Expression (6) is the Hellinger distance between two Gamma distributions with same shape parameter $L$ and expected values $\sigma_i^2$ and $\sigma_j^2$. In the context of PolSAR data we can associate $\sigma_i^2$ and $\sigma_j^2$ of the two probability density functions with the diagonal elements $T_{\ell\ell}$, $T_{4\ell}(\theta)$ for $\ell = 1, 2, 3$ of the un-rotated and the rotated Hermitian positive definite coherency matrices respectively; cf. (7).

### III. Polarization Orientation Angle Estimation

In this section we estimate the polarization OA from full-polarimetric SAR data by using a stochastic distance measure between the elements of the coherency matrix, which is assumed to follow a complex Wishart distribution.

The OA is zero for reflection symmetric media, but OA shifts are induced for surfaces with azimuthal tilts and for buildings oriented perpendicular to the radar LOS. Apart from these, the OA is appreciable for low frequency (L and P band) data in forested areas due to surface topography. In this context, a recent adaptive-model based decomposition with topographic polarization orientation compensation (TPOC) has been proposed in [39]. In this, the volume orientation is removed from the generalized volume component, and the conventional adaptive model-based decomposition is modified by introducing the TPOC concept.

In general, the primary effect of compensating the OA is in the reduction of the cross-polarization ($T_{33}$) component and the increase in the co-polarized ($T_{22}$) component. Following the proposal of Lee and Ainsworth [14], the Hermitian positive definite coherency matrix $[T]$ is unitarily rotated by $[U_3]$:

$$[T(\theta)] = [U_3][T][U_3]^{-1},$$

$$[T] = \begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
T_{12} & T_{22} & T_{23} \\
T_{13} & T_{23} & T_{33}
\end{bmatrix}, \quad [U_3] = \begin{bmatrix}
\sigma_1^2 & \rho_{12} & \rho_{13} \\
\rho_{12} & \sigma_2^2 & \rho_{23} \\
\rho_{13} & \rho_{23} & \sigma_3^2
\end{bmatrix},$$

$$[T] = \begin{bmatrix}
T_{11} & T_{12} & T_{13} \\
T_{12} & T_{22} & T_{23} \\
T_{13} & T_{23} & T_{33}
\end{bmatrix}, \quad [U_3] = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(2\theta) & \sin(2\theta) \\
0 & -\sin(2\theta) & \cos(2\theta)
\end{bmatrix}. \quad (7)$$

The orientation angle $\theta \in [-\pi/8, \pi/8]$ is estimated by minimizing the $T_{33}(\theta)$ component, i.e., $d_{H_{T_{33}}}(\theta)/d\theta = 0$, with this:

$$2\theta = \frac{1}{2} \tan^{-1}\frac{-2\Re(T_{23})}{T_{33} - T_{22}}. \quad (8)$$

The effect of orientation on the three diagonal elements of the coherency matrix is as follows:

1. $T_{11} = |HH + VV|^2/2$ is roll invariant for any $\theta$.
2. $T_{22} = |HH - VV|^2/2$ always increases or remains the same after the OA compensation, and
3. $T_{33} = 2|HV|^2$ always decreases or remains the same after OA compensation.

In this work, we have used a statistical information theoretic measure to estimate the OA from full-polarimetric SAR imagery: the OA is estimated by maximizing the Hellinger distance between the $T_{33}$ and the $T_{22}$ elements of $[T]$.

The proposed method estimates the OA by first maximizing the Hellinger distance between the un-rotated ($\sigma_3^2$, $\sigma_2^2$) and the rotated ($\sigma_3^2(\theta)$, $\sigma_2^2(\theta)$) elements, over the $[-\pi/4, \pi/4]$ range, leading to two candidate angles:

$$\phi_3 = \arg\max_{-\pi/4 \leq \theta \leq \pi/4} \left\{1 - \frac{2 \sqrt{\sigma_3^2 \sigma_2^2(\theta)}}{\sigma_3^2 + \sigma_2^2(\theta)}\right\}^L, \quad \text{and}$$

$$\phi_2 = \arg\max_{-\pi/4 \leq \theta \leq \pi/4} \left\{1 - \frac{2 \sqrt{\sigma_2^2 \sigma_2^2(\theta)}}{\sigma_2^2 + \sigma_2^2(\theta)}\right\}^L. \quad (9)$$

Two maxima are found at $\phi = \phi_{3,2}$ and $\phi = \phi_{3,2} \pm \pi/4$, and the OA is chosen such that the Hellinger distance ($d_{H_{\phi}}$) corresponding to $\sigma_2^2$ is greater than the Hellinger distance ($d_{H_{\phi}}$) corresponding to $\sigma_2^2$ either at $\phi = \phi_{3,2}$ or at $\phi = \phi_{3,2} \pm \pi/4$. This condition corresponds exactly to the case mentioned earlier, where the cross-polarized component is minimized, whereas the other peak corresponds to the situation where the cross-polarized component is incorrectly maximized. Finally, the OA ($\theta_0$) is obtained by wrapping $\phi$ in the range $[-\pi/8, \pi/8]$ using (10), which is then compared with the OA obtained by the method stated in [14]:

$$\theta_0 = \begin{cases}
\phi + \pi/4 & \text{if } \phi < -\pi/8, \\
\phi - \pi/4 & \text{if } \phi > \pi/8, \\
\phi & \text{otherwise.}
\end{cases} \quad (10)$$

The implementation of the proposed method is given in Algorithm 1. The code is freely available at [https://github.com/avikcsre/SD_Y4O/](https://github.com/avikcsre/SD_Y4O/).
The maxima estimated by the proposed methodology are,

\[ \text{algorithm} \]

\begin{algorithm}
\begin{algorithmic}[1]
\State \textbf{Data:} \( [T] \) matrix
\State \textbf{Result:} Orientation compensating angle \((\theta_0)\)
\Statex \begin{align*}
\text{begin} & \quad [T]' \leftarrow [U_3(\theta)][T][U_3(\theta)]^{-1} \quad \forall \theta \rightarrow [-\pi/4, +\pi/4] ; \\
\text{Find common peaks in } d_H & \phi_3 \leftarrow d_H(\sigma_3^2, \sigma_3^2(\theta)) \text{ and } d_H & \phi_2 \leftarrow d_H(\sigma_2^2, \sigma_2^2(\theta)) ; \\
\text{At } \phi(3,2): & \quad \text{if } d_H & \phi_3 > d_H & \phi_2 \quad \text{then} \\
\text{Select } & \phi \rightarrow \phi_3 ; \\
\text{else} & \quad \text{Select } & \phi \rightarrow \phi_2 ; \\
\text{if } \phi > \pi/8 & \text{then} \\
\theta_0 & \rightarrow \phi - \pi/4 ; \\
\text{else if } \phi < -\pi/8 & \text{then} \\
\theta_0 & \rightarrow \phi + \pi/4 ; \\
\text{else} & \quad \theta_0 & \rightarrow \phi ;
\end{align*}
\end{algorithmic}
\end{algorithm}

In the following we use a coherency matrix \( [T] \) extracted from an urban area, to illustrate the proposed methodology:

\[
[T] = \begin{bmatrix}
4.56 & 2.28 + 0.72i & 0.02 + 0.67i \\
2.28 - 0.72i & 6.06 & 1.90 + 0.27i \\
0.02 - 0.67i & 1.90 - 0.27i & 3.50
\end{bmatrix}.
\]

The maxima estimated by the proposed methodology are, approximately, at \( \phi = -31^\circ \) and at \( \phi = 14^\circ \), as shown in Fig. 2. The orientation compensating angle at \( \theta_0 = 14^\circ \) is chosen because it satisfies the aforementioned criterion of \( d_H(\sigma_3^2, \sigma_3^2(\phi)) \geq d_H(\sigma_2^2, \sigma_2^2(\phi)) \).

Apart from the Hellinger distance, any other distances mentioned in Section III can be used to estimate the OA. An example is shown with the Kullback-Leibler (KL) distance in Fig. 2 to estimate the OA. It can be seen that the OA is also correctly estimated by using the KL distance for the coherency matrix given in (11). Our choice was based on computational cost, since the Hellinger distance is the least expensive among the available ones.

IV. SD-Y4O

Presently, all the decomposition techniques that account for the overestimation of the volume power due to target orientation [7], [14]--[18] do so either by rotating the coherency matrix or by considering different volume scattering models. In this work, a modification of the scattered powers for the Yamaguchi four component decomposition (Y4O) is obtained using the relative Hellinger distance. Fig. 2 shows the relative distance \( \delta_H \) between the two Hellinger distances \( d_H(\sigma_3^2, \sigma_3^2(\phi)) \) and \( d_H(\sigma_2^2, \sigma_2^2(\phi)) \) computed at \( \phi \) (the unwrapped orientation angle in the \([-\pi/4, \pi/4]\) range). This relative distance is used to modify the Y4O scattering powers.

According to Lee and Ainsworth [14], the amount of increase in double-bounce power is not equal to the amount of decrease.
in the volume power. It has been shown that the amount of decrease in the volume power component is usually greater than the amount of increase in the double-bounce power component. Moreover, the increase in surface power is smaller compared to the increase in double-bounce power. Due to the roll-invariant property, the helix power remains unaltered after rotation.

In the following we quantify the effect of the proposed rotation on the decomposition powers of a particular target:

$$0 \leq \delta_H = d_H(\sigma_2^2, \sigma_3^2(\phi)) - d_H(\sigma_2^2, \sigma_2^2(\phi)) \leq 1.$$  \hfill (12)

From the previous section, it can be observed that the OA estimation is independent of $L$, but the relative distance $\delta_H$ increases with increasing $L$ until a maximum is attained, and then it rapidly decreases to zero as shown in Fig. 3. The Y4O decomposition powers are then modified with the estimated $\delta_H = \delta_H^m$ for $L = L_m$, where $\delta_H^m$ is the maximum attainable divergence between $\sigma_3$ and $\sigma_2$.

![Fig. 3. The variation of $\delta_H$ with $L$ for $\phi = 14^\circ$. A maximum of $\delta_H = \delta_H^m$ is obtained for a certain value of $L = L_m$.](image)

In the proposed methodology (SD-Y4O), we modify the Yamaguchi four component decomposition powers based on the maximum relative Hellinger distance ($\delta_H^m$) by:

$$P_v^n = P_v(1 - \delta_H^m),$$ \hfill (13a)

$$P_d^n = P_d + \alpha P_v \delta_H^m,$$ \hfill (13b)

$$P_s^n = P_s + \beta P_v \delta_H^m,$$ \hfill (13c)

$$P_c^n = P_c,$$ \hfill (13d)

where $P_v$, $P_d$, $P_s$ and $P_c$ are the volume, double-bounce, surface and helix power components from the Yamaguchi four-component decomposition (Y4O), respectively. The new four-component powers: $P_v^n$, $P_d^n$ and $P_s^n$ are obtained by either deducting or adding the volume power component ($P_v$) of the Y4O decomposition, adjusted by the relative distance $\delta_H^m$ respectively.

The modulating factor $\delta_H^m$ is adjusted by the positive parameters $\alpha$ and $\beta$ with $\alpha + \beta = 1$ for $P_d^n$ and $P_s^n$ respectively in order to conserve the total power ($TP = P_s + P_d + P_v$). Moreover, adhering to the criteria in [14], i.e., the increase in the double-bounce scattering power is more than the increase in the surface scattering power, the ranges of the two parameters are set to $0.5 \leq \alpha \leq 1$ and $0 \leq \beta \leq 0.5$. The $\alpha$ parameter for each pixel is obtained by linearly mapping the estimated orientation angle ($\phi \in [0, \pi/4]$) to $[0.5, 1.0]$. The maps of the $\alpha$ and $\beta$ parameters for the UAVSAR L-band image are shown in Fig. 4.

![Fig. 4. The $\alpha$ and the $\beta$ parameters maps for the UAVSAR L-band Hayward image.](image)

It should be noticed that the linear mapping of the estimated OA to the $\alpha$ parameter does not guarantee maximum $P_d$ (and $P_s$) powers to be observed at/near $45^\circ$. The $P_d$ and $P_s$ powers are modified by using the relative Hellinger distance $\delta_H^m$, which is a non-linear function of the OA angle, multiplied by the $\alpha$ parameter. Figs. 5a and 5b show the 2d scatter plots of $\alpha \delta_H^m$ and $\beta \delta_H^m$ versus $\theta$, respectively, with the contours representing the density for oriented urban areas as shown in Fig. 5a. It can be seen that the maximum density of points lies between $\theta = 20^\circ$ to $\theta = 35^\circ$.

![Fig. 5. (a) The Pauli RGB of a oriented urban area, (b) and (c) the scatterer plot along with density showing the variation of $\alpha \delta_H^m$ and $\beta \delta_H^m$ with $\theta$](image)

In the following section the SD-Y4O method is quantitatively compared with the Y4O and Y4R decomposition scattering.
powers for a Radarsat-2 C-band and a UAVSAR L-band.

V. RESULTS AND DISCUSSION

A fine-beam quad polarization (FQ9) Radarsat-2 C-band image over San Francisco with a spatial resolution of $8 \text{ m} \times 8 \text{ m}$, and a UAVSAR L-band data with a spatial resolution of $12 \text{ m} \times 12 \text{ m}$ (multilooked 3 times in both range and azimuth) are used in this study. The $P_s$, $P_d$ and $P_v$ powers are obtained from the Y4O and Y4R decomposition algorithms given in [5] and [16], respectively. The $P_s$, $P_d$ and the $P_v$ power components corresponding to the two decompositions (Y4O and Y4R) and the proposed modification (SD-Y4O) are shown in Fig. 7 for the Radarsat-2 C-band San Francisco image. The Pauli RGB compositions of the Radarsat-2 and the UAVSAR images are shown in Fig. 6 along with high resolution optical images of patches of urban areas (both aligned and rotated with respect to the LOS), and of a forested area.

Three types of land cover classes are considered in the Radarsat-2 image: (1) Area “A” is a highly oriented urban region, (2) Area “B” is a region with vegetation cover, and (3) Area “C” is the Golden Gate bridge. Over area “A”, the double-bounce power has increased from 2% for Y4O to 17% for SD-Y4O, and the surface power has increased from 3% for Y4O to 9% for SD-Y4O, as shown in Fig. 7(d)-(f). However, as expected, the powers over area “B” remain unchanged. The double-bounce power over the area “C” has increased from 74% for Y4O to 82% for SD-Y4O. A small change of 3% for the surface power from Y4O to SD-Y4O can also be observed over this area. This could be attributed to the water body partly present in the area marked as “C”.

Also three samples from the L-band UAVSAR image of the Hayward region of San Francisco are analyzed. In this region, there are certain urban areas which are approximately $10^\circ$ to $20^\circ$ rotated away from the LOS, as can be seen in Fig. 1(a)-(c). A small sample, identified as “A”, is used from this oriented urban area to compare the powers obtained from Y4O, Y4R and SD-Y4O. A major improvement can be seen in “A” with the reduction of the volume power from 60% for Y4O to 27% for SD-Y4O. The double-bounce power has increased from 30% to 52% compared to Y4R, which has only increased to 36%, as shown in Fig. 8(d)-(f) and in Table I. These improvements can be correctly attributed to the fact that the area under consideration is a dense urban area rotated about the LOS.

The surface scattering power has also improved from 9.1% for Y4O to 19.8% for SD-Y4O. However, it can be observed that for the urban area “B”, which is facing towards the LOS, the scattering powers obtained from the three methods (Y4O, Y4R and SD-Y4O) are comparable, as shown in Fig 8(g)-(i) and in Table II. This suggests that the proposed method is useful for extracting proper scattering powers in oriented urban areas. In the case of non-oriented urban areas, the three methodologies provide similar results. Another patch, marked as “C” in this image, is a forested area. The powers obtained by the three decompositions are similar with the volume scattering power being the dominant type.

| Methods | $P_s$ | $P_d$ | $P_v$ |
|---------|-------|-------|-------|
| Y4O     | 0.05  | 0.17  | 0.33  |
| Y4R     | 0.09  | 0.20  | 0.26  |
| SD-Y4O  | 0.11  | 0.30  | 0.14  |

| Methods | $P_s$ | $P_d$ | $P_v$ |
|---------|-------|-------|-------|
| Y4O     | 0.37  | 1.67  | 0.16  |
| Y4R     | 0.35  | 1.69  | 0.14  |
| SD-Y4O  | 0.38  | 1.70  | 0.10  |

A major concern associated with model-based decompositions is the occurrence of negative powers in double-bounce and surface scatterings. Many ad-hoc techniques have been proposed which force the negative powers to be positive. The negative powers issue was discussed in [15], and in [40] the constraint of nonnegative eigenvalue was proposed to mitigate the problem. However, in this work, the double-bounce and...
Fig. 7. Radarsat-2 C-band San-Francisco image (a)-(d)-(g)-(j): Yamaguchi four component decomposition without rotation of the coherency matrix (Y4O) for area 'A', 'B' and 'C' respectively, (b)-(e)-(h)-(k): Yamaguchi four component decomposition with rotation of the coherency matrix (Y4R) for area 'A', 'B' and 'C' respectively, (c)-(f)-(i)-(l): Y4O decomposition scattering powers modified with the Hellinger distance (SD-Y4O) for area 'A', 'B' and 'C' respectively.

The surface powers obtained from the original Yamaguchi four component decomposition which show negative powers are modified by the two positive quantities, $\alpha P_v^m$ and $\beta P_v^m$. Hence, it can be expected that the number of pixels with small negative powers in double-bounce and surface scattering can be made positive with the addition of the two positive quantities, but the number of pixels with large negative powers may still remain negative. A comparison of the negative powers for the two datasets is shown in Table IV. It can be seen that the percentage of pixels with negative powers have decreased from Y4O to SD-Y4O for both the datasets and is comparable to Y4R.
Fig. 8. UAVSAR L-band Hayward area image (a)-(d)-(g)-(j): Yamaguchi four component decomposition without rotation of the coherency matrix (Y4O) for area ‘A’, ‘B’ and ‘C’ respectively, (b)-(e)-(h)-(k): Yamaguchi four component decomposition with rotation of the coherency matrix (Y4R) for area ‘A’, ‘B’ and ‘C’ respectively, (c)-(f)-(i)-(l): Y4O decomposition scattering powers modified with the Hellinger distance (SD-Y4O) for area ‘A’, ‘B’ and ‘C’ respectively.
TABLE III  
AVERAGE DECOMPOSITION POWERS OVER A FORESTED AREA (AREA “C”) IN FIG. 13(A)-(C))

| Methods  | $P_d$ | $P_v$ | $P_s$ |
|----------|--------|--------|--------|
| Y4O      | 0.01   | 0.02   | 0.38   |
| Y4R      | 0.02   | 0.02   | 0.37   |
| SD-Y4O   | 0.02   | 0.02   | 0.37   |

TABLE IV  
PERCENTAGE OF PIXELS WITH NEGATIVE POWERS

| Dataset     | Method | % of Pixels with Negative Powers |
|-------------|--------|---------------------------------|
| UAVSAR      | Y4O    | 17                              |
|             | Y4R    | 14                              |
|             | SD-Y4O | 13                              |
| Radarsat-2  | Y4O    | 8                               |
|             | Y4R    | 6                               |
|             | SD-Y4O | 6                               |

VI. CONCLUSIONS  
The over-estimation of the volume power and, consequently, the underestimation of the double bounce and the surface powers in the Y4O decomposition in rotated urban areas is of major concern. In order to alleviate this issue, the Y4R decomposition was proposed. Several other decomposition techniques have been recently proposed which address this issue by using different scattering models. In this work we propose a stochastic distance based measure to modify the powers estimated from the Y4O decomposition for oriented targets.

We estimate the orientation angle of the target using a criteria based on the Hellinger distance between the $T_{33}$ and $T_{22}$ elements of the coherence matrix [T]. Using this stochastic distance, we have proposed a method that systematically modifies the $P_s$, $P_d$ and the $P_v$ powers to obtain better estimates. Thus, the physical property of the target, i.e., the orientation angle, is used to make modifications to the powers. The results obtained by the proposed method are encouraging.

An L-band UAVSAR dataset over Hayward is used in this work due to the presence of rotated urban areas. The dataset is decomposed with the Y4R, Y4O and the proposed method. It can be seen that, on oriented urban areas, there is an increase in the $P_s$ power by 11% and an increase in the $P_d$ power by 22% from Y4O to SD-Y4O with a corresponding reduction of the volume power. A similar analysis is also performed on a Radarsat 2 C-band San Francisco dataset. An increase of 15% in the $P_d$ power is observed for the proposed method. However, for the area with forest cover, the three powers from all the decompositions are almost identical. The authors believe that due to its simplicity, the proposed method can be easily used to modify the Y4O decomposition output, which is among the most popular model-based decomposition techniques in SAR polarimetry.

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Alejandro C. Frery (S’92–SM’03) received the B.Sc. degree in Electronic and Electrical Engineering from the Universidad de Mendoza, Argentina. His M.Sc. degree was in Applied Mathematics (Statistics) from the Instituto de Matemática Pura e Aplicada (IMPA, Rio de Janeiro) and his Ph.D. degree was in Applied Computing from the Instituto Nacional de Pesquisas Espaciais (INPE, São José dos Campos, Brazil). He is currently the leader of LaCCAN – Laboratório de Computação Científica e Análise Numérica, Universidade Federal de Alagoas, Maceió, Brazil. His research interests are statistical computing and stochastic modelling.