1. INTRODUCTION

The Møller scattering [1] with polarized electrons has attracted active interest from both experimental and theoretical standpoints for several reasons. It has allowed the high-precision determination of the electron-beam polarization at SLC [2], SLAC [3, 4], JLab [5] and MIT-Bates [6] (and as a future prospect—the ILC [7]). The polarized Møller scattering can be an excellent tool in measuring parity-violating weak interaction asymmetries [8]. The first observation of Parity Violation (PV) in the Møller scattering was made by the E-158 experiment at SLAC [9–11], which studied scattering of 45 to 48-GeV polarized electrons on the unpolarized electrons of a hydrogen target. It results at $Q^2 = -t = 0.026$ GeV$^2$ for the observable parity-violating asymmetry $A_{PV} = (1.31 \pm 0.14 \text{ (stat.)} \pm 0.10 \text{ (syst.)}) \times 10^{-7}$ [12] which allowed one of the most important parameters in the Standard Model (SM)—the sine of the Weinberg angle $\sin \theta_W$—to be determined with accuracy of 0.5%. The MOLLER (Measurement Of a Lepton Lepton Electroweak Reaction) experiment planned at the Jefferson Lab aims to measure the weak charge of the electron and search for new physics. The numerical estimations for the NNLO contribution to the cross section asymmetry are presented.

1 The article is published in the original.

† Deceased.
such a complicated observable as the parity-violating asymmetry to be measured by the MOLLER experiment.

The two-loop EWC to the Born cross section \( \sim \mathcal{M}_0^0 \) can be divided onto two classes: \( Q \)-part induced by quadratic one-loop amplitudes \( \sim \mathcal{M}_1 \), and \( T \)-part—the interference of Born and two-loop amplitudes \( \sim 2 \text{ Re } (\mathcal{M}_0 \mathcal{M}_1^*) \) (here index \( i \) in the amplitude \( \mathcal{M} \) corresponds to the order of perturbation theory). The \( Q \)-part was calculated exactly in [23] (using Feynman-\( \tau \)-Hooft gauge and the on-shell renormalization), where we show that the \( Q \)-part is much higher than the planned experimental uncertainty of MOLLER, i.e. the two-loop EWC are larger than was assumed in the past. The large size of the \( Q \)-part demands detailed and consistent treatment of \( T \)-part, but this formidable task will require several stages. Our first step was to calculate the gauge-invariant double boxes [24]. In this paper we do the next step—we consider the EWC arising from the contribution of a wide class of the gauge-invariant Feynman amplitudes of the box type with one-loop insertions: fermion mass operators [or Fermion Self-Energies in Boxes (FSEB)], vertex functions [or Vertices in Boxes (VB)], and polarization of vacuum for bosons [or Boson Self-Energies in Boxes (BSEB)].

The paper is organized as follows. We define the basic notations in Section 2 and present FSEB, VB, and BSEB in Section 3. In Section 4, we provide the numerical results for asymmetry for the kinematics conditions of the MOLLER experiment and discuss work still to be done in the future. In Appendix A, the mass operators of electron and neutrino are presented. In Appendix B, we show the result for one-loop corrections to vertex functions for the case when only one fermion is on the mass shell. In Appendix C, we consider the polarization of vacuum for the virtual photon, \( Z \)- and \( W \)-boson. The details of calculation of ultraviolet cut-off loop momenta integrals can be found in Appendix D.

2. BASIC NOTATIONS

We consider the process of electron-electron elastic scattering, i.e. Moller process:

\[
ee_-(p_1, \lambda_1) + e_-(p_2, \lambda_2) \rightarrow e_-(p_3, \lambda_3) + e_-(p_4, \lambda_4),
\]

where \( \lambda_i (i = 1, 4) \) are the chiral states of initial and final electrons. The kinematical invariants were defined in the standard way:

\[
s = (p_1 + p_2)^2, \quad t = (p_1 - p_3)^2, \quad u = (p_1 - p_4)^2.
\]

In the MOLLER experiment, the expected beam energy is \( E_{\text{beam}} = 11 \text{ GeV} \), that is \( s = 2mE_{\text{beam}} \approx 0.01124 \text{ GeV}^2 \), where \( m \) is the electron mass \( (p_i^2 = m^2) \).

For the central region of MOLLER (at \( \theta = 90^\circ \) in center-of-mass system of initial electrons), \(-t \approx -u \approx s/2\) thus we can use an approximation that \( s, |t|, |u| \gg m^2 \).

We consider the process (1) in terms of chiral amplitudes \( \mathcal{M}_C \), where \( \lambda = \{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} \) is the chiral state of initial and final electrons. The PV asymmetry to be measured by MOLLER is then defined as

\[
A = \frac{|\mathcal{M}_C^{--} - \mathcal{M}_C^{++}|^2}{\sum_{\lambda} |\mathcal{M}_C^{\lambda}|^2},
\]

\[
\sum_{\lambda} |\mathcal{M}_C^{\lambda}|^2 = 2(8\pi \alpha)^2 s^4 + t^4 + u^4.
\]

In the Born approximation, this asymmetry has a form

\[
A^{(0)} = \frac{s}{2m_w^2 s^4 + t^4 + u^4} \frac{a}{s_W^2},
\]

proportional to

\[
a = 1 - 4s_W^2.
\]

Let us now recall that \( s_W \) (\( c_W \)) is the sine (cosine) of the Weinberg angle expressed in terms of the \( Z \)- and \( W \)-boson masses according to the Standard Model rules:

\[
s_W = \sqrt{1 - c_W^2}, \quad c_W = m_w/m_Z.
\]

Thus, the factor \( a \) is just \( a \approx 0.109 \) and the asymmetry is therefore suppressed by both \( s/m^2 \) and \( a \). Even at \( \theta = 90^\circ \), where the Born asymmetry is maximal, it is extremely small:

\[
A^{(0)} = \frac{s}{9m_w^2 s_W^2} a \approx 9.4968 \times 10^{-8}.
\]

We denote the specific contribution to the asymmetry by the index \( C \), which thus can be BSEB, FSEB, VB or \( \text{IB} = \text{BSEB} + \text{FSEB} + \text{VB} \) for the whole set of diagrams Fig. 1, respectively.

The contribution to the asymmetry \( (\Delta A)_C \) and the relative correction \( D^C_A \) are defined as:

\[
(\Delta A)_C = \frac{|\mathcal{M}_C^{--} - \mathcal{M}_C^{++}|^2}{\sum_{\lambda} |\mathcal{M}_C^{\lambda}|^2},
\]

\[
D^C_A = \frac{(\Delta A)_C}{A^{(0)}},
\]

\[
= \frac{|\mathcal{M}_C^{--} - \mathcal{M}_C^{++}|^2}{|\mathcal{M}_0^{--}|^2 - |\mathcal{M}_0^{++}|^2}.
\]
The relative correction to observable asymmetry from the contribution of type $C$ looks as (see derivation in more details in [19]):

$$\delta_A^C = \frac{A^C - A^{(0)}}{A^{(0)}} = \frac{B_A^C - \delta_A^C}{1 + \delta_A^C},$$

(10)

where the relative correction to unpolarized cross section $\sigma_u$ (we used short notation for differential cross section $\sigma = d\sigma/d(cos\theta)$) is:

$$\delta_A^C = \frac{\sigma_C}{\sigma_u^0}.$$  

(11)

For the two-loop effects where $\delta_A^C$ is small, we can use an approximate equation for relative correction to asymmetry $\delta_A^C \approx D_A^C$.

3. INSERTION OF MASS OPERATOR, VERTEX AND VACUUM POLARIZATION FUNCTIONS TO THE BOX TYPE AMPLITUDE

The numerical value of loop momentum squared $|k|^2$ in the box-type amplitudes with the heavy boson exchange is large compared with the square of electron mass $|k|^2 \gg m_e^2$, since if $|k|^2$ is far from $m_{W,Z}^2$ the contribution is suppressed with the mass of heavy boson squared in denominator. So we can use the asymptotic expressions for the one-loop vertex functions as well as the mass and vacuum polarization operators. Using the well-known approach [25, 26] which successfully employed for the box-type chiral amplitudes in [24] (see also [27]), we can write for the direct $ZZ$-box chiral amplitude of “+++” type:

$$\mathcal{M}_{+++}^Z = \frac{i\alpha^2 (1 + a)^4 6s^2t}{4c_s^4m_z^4} \int d\tau \int d\omega \int d\omega' I_{ZZ}^z(\tau),$$

(13)

where “+$” sign corresponds to the chiral amplitudes $\mathcal{M}_{+++}^z$. The expression for the box amplitude with $Z\gamma$-exchange is similar:

$$\mathcal{M}_{++-}^Z = \frac{i2\alpha^2 (1 + a)^4 6s^2t}{4c_s^4m_z^4} \int d\tau \int d\omega \int d\omega' I_{Z\gamma}^z(\tau).$$

(14)

At last, for $\gamma\gamma$-exchange amplitude we have:

$$\mathcal{M}_{++-}^Z = \frac{i\alpha^2 6s^2t}{4c_s^4m_z^4} \int d\tau \int d\omega \int d\omega' I_{\gamma\gamma}^z(\tau).$$

(15)

In all the above cases, the integration variable is related to the loop momentum as $\tau = -k^2/m_z^2$. The lower limit of integration $z = -t/m_z^2$ for $\mathcal{M}_{++-}^z$ is introduced to avoid the double counting for the region of small loop momenta $-k^2 < s$, where we use the Yennie–Frautchi–Suura approach [28].

Finally, the contribution to $\mathcal{M}_{++-}^Z$ arises from the box-type Feynman diagram with two $W$-boson exchange:

$$\mathcal{M}_{--\cdot}^Z = \frac{-i\alpha^2 6s^2t}{2s_w^4g/m_w^4} \int \frac{d\tau_w}{(1 + \tau_w)^2} I_{WW}^w(\tau_w),$$

(16)
The structure of the quantities $I_{\nu}$ in (13)—(16) corresponds to three types of radiative corrections, FSEB, VB and BSEB, respectively:

$$I_{ZZ} = 2M_e + 4V_{ee}z + 2\Pi_{ZZ},$$
$$I_{Z\gamma} = 2M_e + 2V_{ee}z + 2V_{e\gamma} + \Pi_{Z\gamma} + \Pi_{\gamma},$$
$$I_{\gamma\gamma} = 2M_e + 4V_{e\gamma} + 2\Pi_{\gamma\gamma},$$
$$I_{WW} = 2M_{\nu} + 4V_{e\nu} + 2\Pi_{WW}. \tag{17}$$

Here, we use the dimensionless quantities for the product of fermion Green function and the truncated mass operators of electron $M_e$ and neutrino $M_\nu$ (see Appendix A):

$$M_{e, \nu} = \frac{i k}{k^2} M_{e, \nu}. \tag{18}$$

The vertex function $V_{ee}^\mu(k^2)$ with one electron on the mass shell and another electron off the mass shell is normalized as

$$V_{ee}^\mu(k^2) = -ie\gamma^\mu V_{ee}(k^2), \quad V_{ee}(0) = 0. \tag{19}$$

The vertex function $V_{ee\gamma}(k^2)$ is normalized at the point $k^2 = m_Z^2$:

$$V_{ee\gamma}(k^2) = \frac{ie}{4c_w s_w} \gamma^\mu V_{eez}(k^2), \quad V_{eez}(m_Z^2) = 0, \tag{20}$$

and similarly for $evW$-vertex function we have:

$$V_{ev\gamma}(k^2) = \frac{ie}{\sqrt{2s_w}} \omega_{\gamma} V_{ev\gamma}(k^2), \tag{21}$$

$$V_{ev\gamma}(m_Z^2) = 0.$$

The explicit expressions for the vertices $V_{ee\gamma}$, $V_{eez}$ and $V_{ev\gamma}$ are given in Appendix B.

The dimensionless products of boson Green function with the relevant regularized polarization operator $\Pi_{\nu}(q) = \Pi(q^2)g_{\nu\nu} + B(q^2)q_{\nu}q_{\nu}$, are defined as:

$$\Pi_{\gamma} = -\frac{i}{q^2} \Pi_{\gamma\gamma}(q^2), \quad \Pi_{\gamma\gamma}(0) = 0;$$
$$\Pi_{Z} = \frac{-i}{q^2 - m_Z^2} \Pi_{ZZ}(q^2),$$
$$\Pi_{ZZ}(m_Z^2) = \frac{\partial}{\partial q^2} \Pi_{ZZ}(m_Z^2) = 0;$$
$$\Pi_{Z\gamma} = -\frac{i}{q^2} \Pi_{Z\gamma}(q^2), \quad \Pi_{Z\gamma}(0) = 0;$$
$$\Pi_{W} = \frac{-i}{q^2 - m_W^2} \Pi_{WW}(q^2),$$
$$\Pi_{WW}(m_W^2) = \frac{\partial}{\partial q^2} \Pi_{WW}(m_W^2) = 0. \tag{22}$$

The structure $\mathcal{B}(q^2)q_{\nu}q_{\nu}$ does not contribute due the gauge invariance. The explicit expression for the “truncated” quantities are given in Appendix C.

4. NUMERICAL RESULTS AND CONCLUSION

For the numerical calculations, we use the central kinematical point of the MOLLER experiment and $\alpha$, $m_W$ and $m_Z$ in accordance with the Particle Data Group [29]. The effective quark masses used for the vector boson self-energy loop contributions are extracted from the shifts in the fine structure constant due to hadronic vacuum polarization $\Delta a_{\text{had}}^{(5)}(m_Z^2) = 0.02757$ [30]. For the mass of Higgs boson, we take $m_H = 125$ GeV.

The contribution relevant to the observed asymmetry is the interference of the two-loop box-type amplitudes with the Born amplitudes $M_{\gamma\gamma}$. The contribution to the matrix element squared (i.e. cross section) has the form:

$$\left|M_{\gamma\gamma} - M_{\gamma\gamma}^{IB}\right|^2 = 2(1 + P_{\nu})$$
$$\times \left[(M_{ZZ} + M_{Z\gamma} + M_{WW} + M_{\gamma\gamma}) \mathcal{M}_{\nu}^\mu + M_{\gamma\gamma} \mathcal{M}_{\nu}^\mu \right]. \tag{23}$$

In the right-hand side of this equation, we assume that the amplitudes are taken in the same chiral state corresponding to the state of left-hand side. Note that the intermediate states with $W^\pm$ bosons and Faddeev–Popov ghosts $G_W$ contribute to the mass and vertex operators in the $\mathcal{M}_{\gamma\gamma}$ chiral amplitude. Since the parameter $a$ is very small, we can present the final result as:

$$\left|M_{\gamma\gamma} - M_{\gamma\gamma}^{IB}\right|^2 = -H(a)$$
$$+ (H(-a) + Y) = -2a \frac{\partial H(a)}{\partial a}\bigg|_{a \to 0} + Y, \tag{24}$$

d and thus the relative correction $D_{\lambda}^{IB}$ has the form:

$$D_{\lambda}^{IB} = \frac{t^2 u^2}{128(\pi a)^2 (s^2 + t^2 + u^2)}$$
$$\times \left(-2a \frac{\partial H(a)}{\partial a}\bigg|_{a \to 0} + Y \right) \frac{1}{A_0(a)} \tag{25}$$

We define $H$ and $Y$ as:

$$H = H_{ZZ} + H_{Z\gamma} + H_{WW} + H_{\text{mix}},$$
$$Y = Y_{ZZ} + Y_{Z\gamma} + Y_{WW} + Y_{\text{mix}}, \tag{26}$$

where the first four terms in both $H$ and $Y$ correspond to the box-type amplitudes with $ZZ$, $Z\gamma$, $WW$ bosons exchanged between electrons, and the last term corresponds to the cases with $Z$ or $\gamma$ and the mixed boson Green function with polarization operator $\Pi_{\gamma\gamma}$.
Using the following relations (see, for example, [24] and [27])

\[
\frac{1}{gl'} \left( \frac{1}{gl'} - \frac{1}{ed'} \right) = -\frac{1}{st^2} \quad \frac{1}{gl'} \left( \frac{1}{gl'} - \frac{u}{ed'} \right) = \frac{2}{st} \tag{27}
\]

we obtain the following numerical results:

\[
H_{ZZ} = -\frac{3\alpha^3 \pi (1 + a)^4}{8(cws)^4 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau}{(1 + \tau)^2} \times \left[ 2(M_c^2 + (1 + a)^2 M_Z^2) + 2\Pi_{ZZ} + 4(V^\gamma + (1 + a)^2 V_{\gamma\gamma}) \right] = -1.653 \times 10^{-13} (1 + a)^4 (81.36 - 1.1293 (1 + a)^2);
\]

\[
H_{YY} = -\frac{3\alpha^3 \pi}{8(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau}{(1 + \tau)^2} \times \left[ 2M_e^2 W^\gamma + 2(V^\gamma + V_{\gamma\gamma}^\gamma + (1 + a)^2 (V_{\gamma\gamma}^\gamma + V_{\gamma\gamma}^\gamma)) \right] = -9.155 \times 10^{-11} (1 + a)^3 \times (4.30744 - 0.04567 (1 + a)^3);
\]

\[
Y_{ZZ} = -\frac{12\alpha^3 \pi}{8(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau}{\tau (1 + \tau)} \times \left[ 2(M_c^2 + (1 + a)^2 M_Z^2) + 2\Pi_{ZZ} + 2(V^\gamma + V_{\gamma\gamma}) \right] = 6.974 \times 10^{-11};
\]

\[
H_{YY} = \frac{12\alpha^3 \pi (1 + a)^3}{(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau}{\tau (1 + \tau)} \times 2(M_c^2 + (1 + a)^2 M_Z^2) + 2\Pi_{ZZ} + 4(V^\gamma + (1 + a)^2 V_{\gamma\gamma}) \right] = -3.094 \times 10^{-12} (1 + a)^2 \times (-2.5238 - 5.04456 \times 10^{-6} (1 + a)^2);
\]

\[
H_{ZZ} = \frac{12\alpha^3 \pi (1 + a)^3}{(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau}{\tau (1 + \tau)} \times 2(M_c^2 + 2V^\gamma + (1 + a)^2 V_{\gamma\gamma}) \right] = -4.4261 \times 10^{-17}, \quad Y_{WW} = 0;
\]

\[
Y_{WW} = \frac{8\alpha^3 \pi}{(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau_{WW}}{(1 + \tau_{WW})^2} \times [2M_c^2 + 2\Pi_{WW} + 4V_{\epsilon\epsilon}] = -3.36 \times 10^{-10}.
\]

The "mixed"-type amplitude in two-loop approximation has two different contributions \( (H, Y)_{mix} = (H, Y)_{mix}^{(1)} + (H, Y)_{mix}^{(2)} \). The first contribution is associated with the two-loop box-type amplitude:

\[
H_{mix}^{(1)} = \frac{6\alpha^3 \pi (1 + a)}{(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau_{WW}}{(1 + \tau_{WW})^2} \times R_{WW}(\tau_{WW}) \left( \frac{1}{2} (1 + a)^2 \right) = -1.10029 \times 10^{-9} (1 + a)^2 \times (0.007746 - 0.00340 (1 + a)^2), \quad Y_{mix}^{(1)} = 0.
\]

\[
R_{WW}(\tau_{WW}) = \frac{\alpha c_w}{8\pi s_w} \left( \frac{3}{4} - 2(3 - 2 c_w^2) \frac{1}{\tau_{WW}} \right) L_{WW}(\tau_{WW}),
\]

\[
L_{WW}(\tau_{WW}) = \int d\log (1 + x(1 - x)\tau_{WW}).
\]

The second contribution arises from the interference of the Born-type amplitude with the mixed Green function and the box type one-loop amplitude with the \( W\gamma\) exchange:

\[
H_{mix}^{(2)} = \frac{48\alpha^3 \pi (1 + a)}{(cws)^2 m_{Zu}^2} \int_{0}^{\infty} \frac{d\tau_{WW}}{(1 + \tau_{WW})^2} \times R_{WW}(\tau_{WW}) \left( \frac{1}{2} (1 + a)^2 \right) = -3.982 \times 10^{-13} (1 + a), \quad Y_{mix}^{(2)} = 0.
\]

The contributions to the asymmetry from the transition polarization operator \( \Pi_{\gamma\gamma} \) with leptons in the fermion loop are proportional to higher powers of \( a \), which is small. The same reasoning is valid for the quark–antiquark state contribution. Specifically, it enters with the factor

\[
(2/3)(1 - (8/3) s_w^2) - (1/3)(1 - (4/3) s_w^2) = a^3/3.
\]

The contributions from \( (W^\gamma W^\gamma), (W^\gamma G_{\gamma\gamma}), (c_w^2 G_{\gamma\gamma}) \) intermediate states are considered in Appendix C.

Finally, we are ready to present final numerical value for the relative corrections considered in this paper to the observable cross section asymmetry. The one-loop (NLO) corrections [18, 19] give the biggest contribution,

\[
\delta_A^{NLO} = -0.6953.
\]

Several categories of the NNLO contributions (\( Q \)-part and double boxes) are calculated in [23] and [24] and give the following values:

\[
\delta_A^{NLO + Q} \approx -0.6553,
\]

\[
\delta_A^{double box} \approx D_A^{double box} = -0.0101.
\]

Summing up all the contributions in (25), the numerical result of the class of the gauge-invariant Feynman amplitudes considered in this paper (boxes with one-loop insertions of fermion mass operators, vertex functions and polarization of vacuum for bosons) is:

\[
\delta_A^{IB} \approx D_A^{IB} = -0.0039.
\]

As one can see, the relative correction we obtained is much less than the expected MOLLER experimental
error, but it still a non-negligible contribution to the MOLLER error budget. Most likely, the entire set of two-loop corrections will be smaller than the experimental statistical error, but, in the light of the MOLLER success depending so crucially on its precision, the two-loop corrections still need to be controlled.

As the low-energy precision experiment, MOLLER is complementary to the LHC efforts and may discover new physics signal that could escape LHC detection. However, for the MOLLER experiment to produce meaningful physics, the uncertainties in the NNLO EWC must be much smaller than the MOLLER statistical error. Clearly, there is a need for a complete study of the two-loop electroweak radiative corrections in order to meet the MOLLER precision goals.

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APPENDICIES

A. MASS OPERATORS

Here, we will define the explicit form of the quantities $M_e$, $M_\nu$ which enter to $I_y$ from (17). The quantity $M_e$ has the following form:

$$M_e = M_e^I + (a \pm \gamma_5)^2 M_e^Z + \omega_\nu M_e^\nu. \quad (34)$$

The explicit expression for the truncated mass operator in QED was found by E. Karplus and M. N. Kroll in 1950 [31, 32]:

$$M_e^I = \frac{ie}{2\pi m} (p - m)^I \left[ \frac{1}{2(1 - \rho)} \left( 1 - \frac{1 - 3 \mu \log \rho}{1 - \rho} \right) \right. $$

$$+ \left. \frac{\hat{\rho} + m}{m^2} \left( \frac{1}{2(1 - \rho)} \left( 2 - \rho + \frac{\rho^2 + 4 \rho - 4 \log \rho}{1 - \rho} \right) \right) + 1 + 2 \log \frac{\lambda}{m^2}, \right] \quad (35)$$

$$\rho = 1 - \frac{p^2}{m^2}.$$ 

It is useful to note that the expression in the square brackets is finite at $\rho \rightarrow 1$. In the limit of large $\tau = -p^2/m^2$ with logarithmical accuracy we have

$$M_e^I = M_e^I(\tau) \cdot \hat{\rho} \approx \frac{\alpha}{4\pi} \log (\tau) \cdot \hat{\rho}, \quad \tau \gg 1. \quad (36)$$

This mass operator contribution to the integral in (15) with logarithmical accuracy gives:

$$-\frac{1}{m^2} \int_\infty^{\frac{1}{m^2}} d\tau \frac{d\tau}{\tau} M_e^I(\tau) = -\frac{\alpha}{4\pi} \log \frac{1}{m^2}. \quad (37)$$

The mass operators induced by additional $Z$ and $W$ bosons have the following form:

$$M_e^Z = \frac{\alpha}{2\pi(4\pi s^W)} \int_0^1 (1 - x) \log (1 + \tau x) dx, \quad (38)$$

$$M_e^W = \frac{\alpha}{\pi s^W} \int_0^1 (1 - x) \log (1 + \tau x) dx. \quad (39)$$

B. VERTICES

The general form of the vertex function is $V^\nu(k) = A_\lambda^\nu + Bk^\nu$; the term $Bk^\nu$ inserted in the box-type amplitude gives no contribution due to the gauge invariance. The vertex function with one electron on the mass shell and other electron off the mass shell $V_{ee}(p, p - k, k) = -ie\epsilon^\mu V_{e\nu}(k^2)$, normalized as $V_{e\nu}(0) = 0$, has three contributions:

$$V_{ee} = V_{ee}^\nu + (a \pm \gamma_5)^2 V_{ee}^Z + \omega_\nu V_{ee}^\nu. \quad (40)$$

First, let us consider the QED-type contribution with the virtual photon intermediate state $V_{ee}^\nu$. The standard procedure of joining denominators and performing the loop momenta integration leads to

$$V_{ee}^\nu(k^2) = \frac{\alpha}{4\pi} \int_0^1 \int_0^1 d\tau_z d\tau_y \log \left( \frac{\Lambda^2}{D} + \frac{k^2 b\bar{b}}{2D} \right),$$

$$b = xy, \quad \bar{b} = 1 - b, \quad (40)$$

$$D = (m^2 - k^2 x(1 - x)) y^2$$

$$+ (1 - y) \lambda - y - (1 - y)(k^2 - 2p^2k),$$

where $\Lambda$ is cut-off regularization parameter. Since the sub-set of the diagrams considered here is gauge invariant on its own, it was not essential for us to use the dimensional regularization scheme providing gauge invariance, so we simply applied the cut-off technique. There is no significant numerical difference between two schemes in this situation.

The renormalization procedure consists in subtraction at $k = 0$ and leads to:

$$V_{ee}^\nu(\tau_e) = -\frac{\alpha}{4\pi} \int_0^1 \log (1 + x(1 - x)\tau_e) dx,$$

$$\tau_e = \frac{k^2}{m^2}. \quad (41)$$
The contribution of this vertex function to the integral in (15) has the form:

\[
-\frac{i}{m_Z^2} \int_{-\frac{1}{m_Z^2}}^{1} \frac{d\tau}{\tau^2} V'_{ee\tau}(\tau) \approx \frac{\alpha}{4\pi} \left(1 - \log \frac{-\tau}{m_Z^2}\right). \tag{42}
\]

The other contributions are:

\[
V^Z_{ee\tau} = \frac{\alpha}{2\pi(4\epsilon_{W}\epsilon_{W})^2} \times \int_{0}^{1} dy \int_{0}^{1} dy' \left(\log \frac{1-y-b\bar{b}}{1-y+b\bar{b}} - \frac{b\bar{b}\tau}{2(1-y+b\bar{b}\tau)} - \frac{b\bar{b}}{2(1-y+b\bar{b}\tau)}\right),
\]

\[
V^W_{ee\tau} = \frac{\alpha}{4\pi}\frac{1}{s_W^2} \times \int_{0}^{1} dy \int_{0}^{1} dy' \left(3\log \frac{y_{cW}^2 + \tau b\bar{b}}{y_{cW}^2 - b\bar{b}} - \frac{\tau b(b\bar{b}+b)}{2(y_{cW}^2 + \tau b\bar{b})} + \frac{\tau b}{2(y_{cW}^2 + \tau b\bar{b})}\right), \tag{43}
\]

Vertex function \(V^\nu_{ee\tau} = -iG^\nu(a \pm \gamma_5)V_{ee\tau}\), \(G = e/(4\epsilon_{W}\epsilon_{W})\) has four different contributions:

\[
V_{ee\tau} = \omega_\nu V^\nu + (a \pm \gamma_5) V^Z + \omega_\nu V^W + V^{2\nu}, \tag{44}
\]

and is normalized as \(V_{ee\tau}(k^2 = m_Z^2) = 0\). These contributions are

\[
V^\nu = -\frac{\alpha}{4\pi} \log \tau,
\]

\[
V^Z = \frac{1}{4\pi(4\epsilon_{W}\epsilon_{W})^2} \int_{0}^{1} dy \int_{0}^{1} dy' \left(\log \frac{1-y-b\bar{b}}{1-y+b\bar{b}} - \frac{b\bar{b}\tau}{2(1-y+b\bar{b}\tau)} - \frac{b\bar{b}}{2(1-y+b\bar{b}\tau)}\right),
\]

\[
V^W = \frac{\alpha}{4\pi}\frac{1}{s_W^2} \times \int_{0}^{1} dy \int_{0}^{1} dy' \left(3\log \frac{y_{cW}^2 + \tau b\bar{b}}{y_{cW}^2 - b\bar{b}} - \frac{\tau b(b\bar{b}+b)}{2(y_{cW}^2 + \tau b\bar{b})} + \frac{\tau b}{2(y_{cW}^2 + \tau b\bar{b})}\right),
\]

\[
V^{2\nu} = \frac{\alpha}{2\pi}\frac{1}{s_W^2} \times \int_{0}^{1} dy \int_{0}^{1} dy' \left(3\log \frac{y_{cW}^2 + \tau b\bar{b}}{y_{cW}^2 - b\bar{b}} - \frac{\tau b(b\bar{b}+b)}{2(y_{cW}^2 + \tau b\bar{b})} + \frac{\tau b}{2(y_{cW}^2 + \tau b\bar{b})}\right).
\]

And finally, the vertex function \(V^\mu_{ee\tau} = \frac{i\gamma_{\mu}\alpha}{\sqrt{2}} V_{ee\tau}\) as well contains three contributions:

\[
V_{ee\tau} = V^{ZW} + V^{WZ} + V^{\nu\tau}, \tag{46}
\]

and is normalized as \(V_{ee\tau}(\tau = -c_{W}^2) = 0\) and \(V^{ZW} = V^{WZ}\). So the contributions are:

\[
V^{ZW} = \frac{\alpha}{4\pi}\frac{1}{s_W^2} \int_{0}^{1} dy \int_{0}^{1} dy' \left(-3\log \frac{y_{cW} + \tau b\bar{b}}{y_{cW}^2 - c_{W}^2 b\bar{b}} + \frac{\tau b(b\bar{b}+b)}{2(y_{cW}^2 + \tau b\bar{b})} + \frac{c_{W}^2 b(b\bar{b}+b)}{2(y_{cW}^2 + \tau b\bar{b})}\right),
\]

\[
V^{WZ} = \frac{\alpha}{4\pi}\frac{1}{s_W^2} \int_{0}^{1} dy \int_{0}^{1} dy' \left(3\log \frac{b_{cW} + \tau + c_{W}^2}{b_{cW}^2} - \frac{\tau b}{2(c_{W}^2 + \tau b)} - 1 + \frac{1}{4}\log \frac{m_Z^2}{m^2} + \frac{1}{4}\log \frac{m^2}{\lambda^2}\right).
\]

C. POLARIZATION OPERATORS

While considering the vacuum polarization operators of photon, \(Z\)- and \(W\)-boson at one loop, one should recall that the regularization implies the double subtraction procedure. The “truncated” operators imply including only the vertices of interaction of bosons with the fermion loop. From now on, we will omit index “tr”. The general form of the polarization operator is:

\[
\Pi_{\mu\nu}(q) = g_{\mu\nu}\Pi(q^2) + q_{\mu}q_{\nu}B(q^2). \tag{47}
\]

We only need to consider a part of polarization tensor proportional to \(g_{\mu\nu}\). The reason is the gauge invariance of the whole set of the double-box amplitudes, which leads to a zero contribution for terms proportional to \(q_{\mu}q_{\nu}\) tensor.

Let’s define \(\Pi^\nu\) as:

\[
\Pi^\nu = -\frac{i}{q^2}\Pi^\nu(q^2). \tag{48}
\]

It has five types of contributions, corresponding to the intermediate state of lepton-antilepton pairs, quark-antiquark pairs, \(W^+W^-\) and the charged ghost state \(W^\mu G_w\):
The contribution of leptons and quarks are associated with the quadratic divergent integral over the loop momentum:

\[
\frac{1}{4} \int \frac{dk}{(k^2 - m^2)((k - q)^2 - m^2)} \times \text{Sp}[\hat{q} + m\gamma, (k - \hat{q} + m)\gamma].
\]

Using the set of divergent integrals (see Appendix D) and performing the regularization procedure, we include the contribution of leptons and quarks as

\[
\Pi^l + \Pi^q = \frac{\alpha}{\pi \tau} \sum_{i=e,\mu,\tau} G(\tau, \sigma^i) + 3 \sum_{q = u, d, s, \ldots} Q^q G(\tau, \sigma^q),
\]

where

\[
G(\tau, \sigma) = \frac{1}{3}(\tau - 2\sigma)L(\frac{\tau}{\sigma}) + \frac{1}{9}\tau, \\
L(z) = \int_0^1 dx \log(1 + x(1 - x)z), \\
\sigma^f = \frac{m_i^2}{m_z^2}, \quad \tau = -\frac{q^2}{m_z^2}.
\]

The factor 3 takes into account the number of quark colours. The last three contributions in (49) are

\[
\Pi^{WW} + \Pi^{G_1 G_2} + \Pi^{G_2 G_2}
\]

\[
= -\frac{\alpha}{12\pi \tau} \left(1 + (5\tau - c_w^2)\frac{L(\tau)}{c_w^2}\right),
\]

the known result for Feynman-t’Hooft gauge used in [34, 35].

The polarization operator for Z-boson has seven types of contributions:

\[
\Pi^Z = \Pi^l + \Pi^q + \Pi^l + \Pi^{Z W^- W^+} + \Pi^{G_1 G_2} + \Pi^{G_2 G_2},
\]

where we used the definition

\[
\Pi^Z = -\frac{i}{q^2 - m_z^2} \Pi^Z(q^2).
\]

The contribution of lepton \(\Pi^l\), quark \(\Pi^q\) and the neutrino \(\Pi^\nu\) loops can be calculated in the non-renormalized approach:

\[
\Pi^l = \frac{\alpha}{12\pi} \left(q^2 \log \frac{\Lambda^2}{q^2} + O(q^4)\right) g_{\mu\nu}.
\]

The renormalization of \(R(\tau)\) for any contribution to the polarization operator of Z-boson consist of the following replacement:

\[
R(\tau) \rightarrow R(\tau) - R(-1) - (\tau + 1)R'(-1).
\]

In particular, for example:

\[
-\frac{q^2 \log \frac{q^2}{m^2}}{2} \rightarrow m_z^2 F(\tau), \quad F(\tau) = \tau \log \tau - 1 - \tau.
\]

Keeping in mind that there are three generations of charged leptons, neutrinos, and quarks, we obtain:

\[
\Pi^{l+q+v} = \frac{\alpha}{12\pi \tau} \left[3 + \frac{3}{4(s_w c_w)^2}\right] + \frac{1}{2(s_w c_w)^2}(1 - 2s_w^2 + 20\frac{4}{9} s_w).
\]

The contribution of \(W^+ W^-\) pair in the intermediate state to the Z-boson polarization operator looks like:

\[
\Pi^{W^+ W^-} = \frac{\alpha c_w^2}{8\pi s_w} \left[\frac{19\tau - 16}{6} \right] \left[\frac{1}{c_w^2} \right] - \left[\frac{19\tau - 16}{6} \right] \left[\frac{1}{c_w^2} \right] \approx -0.248,
\]

\[
c_2 = \int_0^1 \frac{x(1-x)}{1-x(1-x)/c_w^2} dx \approx 0.226.
\]

The contribution of the charged ghosts \(G_w^+\) is:

\[
\Pi^{G_w^+ G_w^-} = \frac{\alpha(1 - 2s_w^2)^2}{4\pi(c_w s_w)^2} \frac{1}{1 + \tau} \left[\frac{1}{12} + \frac{1}{3} c_w^2\right] \left[\frac{1}{c_w^2} \right] - \left[\frac{1}{c_w^2} \right] c_1 \left[\frac{1}{12} - \frac{1}{3} c_w^2\right] c_2,
\]

\[
\Pi^{G_w^+ G_w^-} = \frac{\alpha s_w^4}{2\pi} \left[\frac{1}{1 + \tau} \left[\frac{1}{c_w^2} \right] - c_1 \left[\frac{1}{12} - \frac{1}{3} c_w^2\right] c_2\right]
\]

And, finally, the contribution from the state with ghosts \(G_{1,2}\) is:

\[
\Pi^{G_{1,2}} = \frac{\alpha}{12\pi} \frac{1}{(c_w s_w)^2} \frac{1}{1 + \tau} \left[\frac{1}{12} - \frac{1}{3} c_w^2\right] c_2 \left[\tau(A(\tau) - A(-1)) + (\tau + 1)A'(\tau)\right],
\]

with explicit form of \(A(\tau)\) given in Appendix D.
where factor 4 corresponds to the number of pairs \(d + \bar{s}\)(\(u + c\)). Defining the dimensionless combination:

\[
\Pi^W = -\frac{i}{q^2 - m_W^2} \Pi_w^Z(q^2) = \Pi^W(\tau_w),
\]

we write

\[
\Pi^W = \Pi^W_{\gamma^3} + \sum_q \Pi^q_w + \Pi^W_{\gamma^2} + \Pi^{WG_2}_w + \Pi^{WG_2\gamma}_w + \Pi^{GW_\gamma}_w + \Pi^{GW\gamma}_w.
\]

From now on, when considering the definite contributions to \(\Pi^W\), we imply that \(\tau \to \tau_w\). Let us first consider the contributions from fermions. For the state with a charged lepton and the corresponding antineutrino we obtain:

\[
\sum^\gamma_q \Pi^q_w = 3\frac{\alpha}{24\pi s^3_W} \frac{1}{1 + \tau} F(\tau),
\]

with function \(F\) given in (57). Factor 3 corresponds to the number of lepton generations. The contribution of quark states is:

\[
\sum_{q = u, d, s, c} \Pi^q_w = 4\frac{\alpha}{24\pi s^3_W} \frac{1}{1 + \tau} F(\tau),
\]

where factor 4 corresponds to the number of pairs \((d + \bar{s})(u + c)\). The for the \(WZ\) state we have:

\[
\Pi^W_{WZ} = -\frac{\alpha c^2_w}{4\pi s^3_W(1 + \tau_w)} \times [\Psi(\tau_w) - \Psi(-1) - (1 + \tau_w)\Psi'(-1)],
\]

\[
\Psi(z) = \left(4z - 1 - \frac{1}{c^2_w} \right) \int_0^1 \log \left( x + \frac{1 - x}{c^2_w} + x(1 - x)z \right) dx
\]

\[
- \left( \frac{1}{12} z + \frac{1}{c^2_w} \right) \int_0^1 \log (1 + x(1 - x)z) dx + \frac{1}{2} s^2_w
\]

\[
\times \int_0^1 \int_0^1 \log \left( y + (1 - y) \frac{1}{c^2_w} + y(1 - y)z \right) dy,
\]

\[
\Psi(-1) = 0.226, \quad \Psi'(-1) = -1.26.
\]

Now we consider the intermediate states \((W, G_2)\) and \((G_\mu, G_2)\). For the insertion to the box amplitude we have:

\[
\Pi^{WG_2} + \Pi^{GW_\gamma} = -\frac{\alpha}{2\pi} \frac{1}{1 + \tau}
\]

\[
\times \int_0^1 d\tau \log \left( x + (1 - x)(1 + \tau x)c^2_w \right)
\]

\[
- \tau \left( A(\tau) - A\left( -\frac{\tau}{c^2_w} \right) \right) - (\tau + c^2_w) A\left( -\frac{1}{c^2_w} \right),
\]

with \(A(\tau)\) taken with \(\gamma = 1/c^2_w\).

For the last two terms we have:

\[
\Pi^{WG_2\gamma} + \Pi^{GW_\gamma} = \frac{\alpha}{4\pi(\tau + 1)} \left( -4Q(\tau) + \frac{5}{3\pi} R(\tau) \right),
\]

\[
Q(\tau) = \tau \int_0^1 d\tau ' (1 + \tau x) + 3 + (1 + \tau) \left( 1 + \frac{1}{2} \log \frac{m^2}{\lambda^2} \right),
\]

\[
R(\tau) = \frac{6}{\tau^2} - \frac{15}{2} + 11 + 6\left( 1 + \frac{\tau}{\tau} \right)^3
\]

\[
\times \log (1 + \tau) - 20 - 27(1 + \tau),
\]

Note that the term \(\log (m^2/\lambda^2)\) in the expression for \(Q(\tau)\) is compensated by the corresponding contributions from the two-box amplitudes.

Let us now consider the contributions to the transition polarization \(\Pi^{Z\gamma}_w = \Pi^{Z\gamma}_w\) and define the dimensionless function

\[
\Pi^{Z\gamma}_w = -\frac{i}{q^2} \Pi^{Z\gamma}_w(q^2).
\]

As shown above, the fermions contribution is proportional to \(a^2\) and can be omitted. The contributions of \((W^+W^-), (W^\pm G_\mu^\pm), (G_\mu^\mp G_\mu^\pm)\) to \(\Pi^{Z\gamma}_w\) are, respectively:

\[
-i\frac{\alpha c_w}{8\pi s_w} \left( \frac{19}{6} - \frac{16}{3\tau_w} \right) L(\tau_w),
\]

\[
-i\frac{\alpha c_w}{8\pi s_w} \left( \frac{1}{6} + \frac{2}{3\tau_w} \right) L(\tau_w),
\]

\[
+i\frac{\alpha c_w}{2\pi s_w^2} L(\tau_w).
\]

Thus, the total is:

\[
\Pi^{Z\gamma}_w = -\frac{i}{q^2} \Pi^{Z\gamma}_w \left( -3 + \frac{1}{\tau_w} (6 - 4c_w^2) \right) L(\tau_w).
\]
D. LOOP INTEGRALS
AND REGULARIZATION

To calculate loop integrals, we perform the Wick rotation of the loop momentum \( k (k_0 \to ik_0, k^2 = -k^2_E < 0). \) In order to regularize ultra-violet divergence, we introduce the cut-off parameter \( \Lambda \) so \( k^2_E < \Lambda^2 \), and all of the kinematical invariants much less (i.e. \( \Lambda^2 \gg |p,p| \)). The final result will be independent of \( \Lambda \) after the renormalization procedure. Let us now list all the integrals we need:

\[
\int \frac{k^2 \, dk}{(k^2 - \Lambda^2)^3} = \log \frac{\Lambda^2}{2} - \frac{3}{2}, \quad \int \frac{dk}{(k^2 - \Lambda^2)^2} = \log \frac{\Lambda^2}{2} - 1,
\]

\[
\int \frac{dk}{(k^2 - \Lambda^2)^3} = -\frac{1}{2 \Lambda^2}, \quad \int \frac{dk}{(k^2 - \Lambda^2)^4} = \frac{1}{6 \Lambda^4}, \quad (68)
\]

\[
\int \frac{(k^2)^2 \, dk}{(k^2 - \Lambda^2)^4} = \log \frac{\Lambda^2}{2} - \frac{11}{6}, \quad \int \frac{k^2 \, dk}{(k^2 - \Lambda^2)^3} = -\frac{1}{3 \Lambda^2}.
\]

Here, we use the notation \( dk \equiv d^4k/(i\pi^2) = k^2 \, dk_E \), where \( k_E \) is the Euclidean 4-vector (i.e. \( k^2_E = k^2_1 + k^2_2 + k^2_3 + k^2_4 > 0 \)) and omit the terms of order \( O(D/\Lambda^2) \). We also use the consequence of the integrand symmetry:

\[
\int (k^2) k_\mu \, dk = 0 \quad (69)
\]

for any function \( f(k^2) \). The standard procedure of shifting variable in loop integrals [32] leads to:

\[
\int \frac{dk}{(k^2 - b^2)^2} = \log \frac{\Lambda^2}{d} - 1, \quad \int \frac{k_\mu \, dk}{(k^2 - b^2)^2} = b_\mu \left( \log \frac{\Lambda^2}{d} - \frac{3}{2} \right).
\]

Let us consider the divergent integrals with \( A \equiv k^2 - m^2 \) and \( B \equiv (q - k^2) - m^2 \):

\[
\int \frac{dk}{AB} = L_\Lambda - 1 - L, \quad \int \frac{k_\mu \, dk}{AB} = \frac{1}{2} q_\mu \left( L_\Lambda - \frac{3}{2} - L \right),
\]

\[
\int \frac{k_\mu k_\nu \, dk}{AB} = g_\mu \nu \left\{ -\frac{\Lambda^2}{4} + \frac{q^2}{72} - \frac{m^2}{4} + \frac{1}{2} \left( \frac{m^2 - q^2}{6} \right) L_\Lambda \right\}, \quad (70)
\]

\[
+ \frac{1}{3} \left( \frac{q^2}{4} - m^2 \right) L + q_\mu q_\nu \left\{ \frac{1}{3} L_\Lambda - \frac{5}{9} + \frac{1}{3} \left( m^2 - q^2 \right) L \right\},
\]

where

\[
L_\Lambda = \log \frac{\Lambda^2}{m^2},
\]

\[
L = L(\tau) = \int_0^1 dx \log (1 + x(1 - x)\tau), \quad \tau = \frac{q^2}{m^2}.
\]

By contracting indices in the tensor integral (70), we obtain:

\[
\int \frac{k^2 \, dk}{AB} = -\frac{\Lambda^2}{2} - \frac{q^2}{2} + 2m^2 L_\Lambda - m^2 L. \quad (71)
\]

According the renormalization procedure, we can omit terms having the form \( aq^2 + bm^2 \) and \( (aq^2 + dm^2) L_\Lambda \).

Let us consider now the general integral of the form

\[
I_{\mu \nu} = \int \frac{k_\mu k_\nu \, dk}{(k^2 - m^2)(k^2 - q^2 - m^2)}. \quad (72)
\]

Now, let us use the following algebraic identity:

\[
\frac{1}{(q-k)^2 - m^2} = \frac{1}{k^2 - m^2} + \frac{2qk-q^2}{(k^2 - m^2)^2} + \frac{2qk-q^2}{(k^2 - m^2)^2} \left( \frac{q-k}{q^2 - m^2} \right). \quad (73)
\]

Due to our renormalization convention, we can omit the first and the second terms in the right-hand side of this equation so the integral reads as:

\[
I_{\mu \nu} = \int \frac{k_\mu k_\nu (2qk-q^2)^2 \, dk}{(k^2 - m^2)(k^2 - m^2) ((q-k)^2 - m^2)}. \quad (74)
\]

First, we combine the factors \( (k^2 - m^2)^2 \) and \( (k^2 - m^2)^2 \) in the denominator using the Feynman trick:

\[
\frac{1}{a^2 b} = 2 \int \frac{(1-x) \, dx}{a(1-x) + bx^4}, \quad (75)
\]

and obtain

\[
\frac{1}{(k^2 - m^2)(k^2 - m^2)} = 2 \int_0^1 \frac{(1-x) \, dx}{(k^2 - M_x^2)^3}, \quad M_x^2 = (1-x)m^2 + xM^2.
\]

Next, we join the resulting expression with the factor \( ((k^2 - m^2)) \) with the similar Feynman identity:

\[
\frac{1}{c^3 d} = 3 \int_0^1 \frac{(1-y)^3 \, dy}{(c(1-y) + dy)^3}. \quad (76)
\]
and, finally, get:

\[
\frac{1}{(k^2 - M^2)^3} \int_0^1 \frac{(1 - y)^2 dy}{\delta(k - yq^2 - m_i^2)}
\]

where

\[
d = \tau_1 y(1 - y) + \mu^2,
\]

\[
\mu^2 = x(1 - y) + \gamma[y + (1 - x)(1 - y)],
\]

\[
\tau_1 = -\frac{q^2}{m_i^2}, \quad \gamma = \frac{m_2^2}{m_i^2}.
\]

Thus, we have the logarithmically-divergent loop momentum integral, which allows the operation of the loop momentum shifting \( k = \bar{k} + qy \). After that, we can use the loop integrals from the beginning of this Appendix. Now, we have:

\[
I_{\mu\nu} = A(\tau_1, \gamma) q^2 g_{\mu\nu} + O(q^4),
\]

\[
A(\tau_1, \gamma) = -\int_0^1 dx \int_0^1 dy (1 - x)(1 - y)^2
\]

\[
\times \left( \log d - \frac{\tau_1 (1 - 2y)^2}{2d} \right),
\]

therefore the renormalization procedure for this integral has the form:

\[
\tau_1 A(\tau_1, \gamma) \rightarrow \tau_1 A'(\tau_1, \gamma) - A(-1, \gamma) + (1 + \tau_1) A'(-1, \gamma),
\]

where \( A(-1, \gamma) \approx -0.0896 \) and \( A'(-1, \gamma) \approx 0.00654 \) for \( \gamma = m_h^2/m_Z^2 = 1.879 \).

REFERENCES

1. C. Møller, “Zur Theorie des Durchgangs schneller Elektronen durch Materie,” Ann. Phys. 406, 531 (1932).

2. M. Swartz et al., “Observation of target electron momentum effects in single arm Moller polarimetry,” Nucl. Instrum. Methods Phys. Res. A 363, 526 (1995).

3. P. Steiner, A. Feltham, I. Sick, M. Zeier, and B. Zihlmann, “A high-rate coincidence Moller polarimeter,” Nucl. Instrum. Methods Phys. Res. A 419, 105 (1998).

4. H. Band, G. Mitchell, R. Prepost, and T. Wright, “A Moller polarimeter for high-energy electron beams,” Nucl. Instrum. Methods Phys. Res. A 400, 24 (1997).

5. M. Hauger, A. Honegger, J. Jourdan, G. Kubon, T. Petitjean, D. Rohe, I. Sick, G. Warren, H. Woehrle, J. Zhao, R. Ent, J. Mitchell, D. Crabb, A. Tobias, M. Zeier, and B. Zihlmann, “A high precision polarimeter,” Nucl. Instrum. Methods Phys. Res. A 462, 382 (2001).

6. J. Arrington, E. J. Beise, B. W. Filippone, T. G. O’Neill, W. R. Dodge, G. W. Dodson, K. A. Dow, and J. D. Zumber, “A variable energy Moller polarimeter at the MIT bunches linear accelerator center,” Nucl. Instrum. Methods Phys. Res. A 311, 39 (1992).

7. G. Alexander and I. Cohen, “Moller scattering polarimetry for high-energy e^+e^- linear colliders,” Nucl. Instrum. Methods Phys. Res. A 486, 552 (2002).

8. E. Derman and W. J. Marciano, “Parity violating asymmetries in polarized electron scattering,” Ann. Phys. (N.Y.) 121, 147 (1979).

9. K. S. Kumar, E. Hughes, R. Holmes, and P. Souder, “Precision low-energy weak neutral current experiments,” Mod. Phys. Lett. A 10, 2979 (1995).

10. K. Kumar, “The E158 experiment,” Eur. Phys. J. A 32, 531 (2007).

11. P. Anthony et al. (SLAC E158 Collab.), “Observation of parity nonconservation in Moller scattering,” Phys. Rev. Lett. 92, 181602 (2004).

12. P. Anthony et al. (SLAC E158 Collab.), “Precision measurement of the weak mixing angle in Moller scattering,” Phys. Rev. Lett. 95, 081601 (2005).

13. D. Androic et al. (Qweak Collab.), “First determination of the weak charge of the proton,” Phys. Rev. Lett. 111, 141803 (2013).

14. W. T. H. van Oers (MOLLER Collab.), “The MOLLER experiment at Jefferson Lab: search for physics beyond the standard model,” AIP Conf. Proc. 1261, 179 (2010).

15. J. Benesch et al., http://hallaweb.jlab.org/12GeV/Moller/downloads/DOE_Proposal/DOE_Moller.pdf (2011).

16. K. S. Kumar, “Parity-violating Moeller scattering,” AIP Conf. Proc. 1182, 660 (2009).

17. J. Benesch et al. (MOLLER Collab.), “The MOLLER experiment: an ultra-precise measurement of the weak mixing angle using Moller scattering: arXiv:1411.4088.

18. A. G. Alekseev, S. G. Barkanova, A. N. Ilyichev, and V. A. Zykunov, “Electroweak radiative corrections for polarized Moller scattering at future 11 GeV JLab experiment,” Phys. Rev. D: Part. Fields 82, 093013 (2010).

19. A. G. Alekseev, S. G. Barkanova, and V. A. Zykunov, “Precise calculations of observables of polarized Moller scattering: from JLAB to ILC energies,” Phys. At. Nucl. 75, 209 (2012).

20. A. G. Alekseev, S. G. Barkanova, A. N. Ilyichev, Yu. G. Kolomensky, and V. A. Zykunov, “One-loop electroweak corrections for polarized Moller scattering at different renormalization schemes and conditions,” arXiv:1010.4185.

21. A. Czarnecki and W. J. Marciano, “Electroweak radiative corrections to polarized Moller scattering asymmetries,” Phys. Rev. D: Part. Fields 53, 1066 (1996).

22. F. J. Petriello, “Radiative corrections to fixed target Moller scattering including hard bremsstrahlung effects,” Phys. Rev. D: Part. Fields 67, 033006 (2003).

23. A. G. Alekseev, S. G. Barkanova, Yu. G. Kolomensky, E. A. Kuraev, and V. A. Zykunov, “Quadratic electroweak corrections for polarized Moller scattering,” Phys. Rev. D: Part. Fields 85, 013007 (2012).

24. A. G. Alekseev, S. G. Barkanova, Yu. M. Bystritskiy, E. A. Kuraev, A. N. Ilyichev, and V. A. Zykunov, “Parity violating Moller scattering asymmetry up to the two-loop level,” arXiv:1202.0378.
25. F. A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans, W. Troost, and T. T. Wu, “Multiple bremsstrahlung in Gauge theories at high-energies. 2. Single bremsstrahlung,” Nucl. Phys. B 206, 61 (1982).

26. F. A. Berends, R. Kleiss, P. de Causmaecker, R. Gastmans, and T. T. Wu, “Single bremsstrahlung processes in Gauge theories,” Phys. Lett. B 103, 124 (1981).

27. A. I. Ahmadov, Yu. M. Bystritskiy, E. A. Kuraev, A. N. Ilyichev, and V. A. Zykunov, “One-loop chiral amplitudes of Moller scattering process,” Eur. Phys. J. C 72, 1977 (2012).

28. D. R. Yennie, S. C. Frautschi, and H. Suura, “The infrared divergence phenomena and high-energy processes,” Ann. Phys. (N.Y.) 13, 379 (1961).

29. C. Amsler et al. (Particle Data Group Collab.), Phys. Lett. B 667, 1 (2008).

30. F. Jegerlehner, “Hadronic contributions to the photon vacuum polarization and their role in precision physics,” J. Phys. G 29, 101 (2003).

31. R. Karplus and N. M. Kroll, “Fourth-order corrections in quantum electrodynamics and the magnetic moment of the electron,” Phys. Rev. 77, 536 (1950).

32. A. I. Akhiezer and V. B. Berestetskij, Quantum Electrodynamics (Nauka, Moscow, 1981; Wiley, New York, 1965).

33. A. G. Aleksejevs, S. G. Barkanova, Yu. M. Bystritskiy, A. N. Ilyichev, E. A. Kuraev, and V. A. Zykunov, “Double-box contributions to Moeller scattering in the standard model,” Eur. Phys. J. C 72, 2249 (2012).

34. T.-P. Cheng and L.-F. Li, Gauge Theory of Elementary Particle Physics (Oxford Univ. Press, USA, 1988).

35. A. Denner, “Techniques for calculation of electroweak radiative corrections at the one loop level and results for W physics at LEP-200,” Fortschr. Phys. 41, 307 (1993).