Erratum

Erratum: Continuum-wise expansiveness for generic diffeomorphisms (2018 Nonlinearity 31 2982)

Manseob Lee

Department of Mathematics, Mokwon University, Daejeon, 302-729, Republic of Korea
E-mail: lmsds@mokwon.ac.kr

Received 15 January 2019
Accepted for publication 27 February 2019
Published 12 April 2019

Recommended by Dr Eimear O’Callaghan

The opportunity has been taken to update the wording of the original abstract to improve its clarity.

Abstract

Let $M$ be a closed smooth manifold and let $f : M \to M$ be a diffeomorphism. For a $C^1$ generic diffeomorphism $f$, if $f$ is continuum-wise expansive then it is Axiom A without cycles. Let $M = \mathbb{T}^3$ and let $f : \mathbb{T}^3 \to \mathbb{T}^3$ be a diffeomorphism. There is a $C^1$ neighborhood $U(f)$ of $f \in RT(\mathbb{T}^3)$ and a residual set $\mathcal{R} \subset U(f)$ such that for any $g \in \mathcal{R}$, $g$ is not continuum-wise expansive, where $RT(\mathbb{T}^3)$ is the set of all robustly transitive diffeomorphisms on $\mathbb{T}^3$.

In addition to this, in lemma 2.3 of the published article, reference [8] was incorrectly cited. The correct citation should be [15] ([15] Lee M Measure expansiveness for generic diffeomorphisms Dynam. Syst. Appl. accepted).

Lemma 2.3 ([15, lemma 2.4]).

ORCID iDs

Manseob Lee https://orcid.org/0000-0001-9701-6508
Continuum-wise expansiveness for generic diffeomorphisms

Manseob Lee

Department of Mathematics, Mokwon University, Daejeon, 302-729,
Republic of Korea

E-mail: lmsds@mokwon.ac.kr

Received 24 March 2016, revised 23 January 2018
Accepted for publication 28 March 2018
Published 11 May 2018

Abstract
Let $M$ be a closed smooth manifold and let $f: M \to M$ be a diffeomorphism. $C^1$-generically, a continuum-wise expansive satisfies Axiom A without cycles. Let $M = \mathbb{T}^3$ and let $f: \mathbb{T}^3 \to \mathbb{T}^3$. There are a $C^1$ neighborhood $U(f)$ of $f \in RT(\mathbb{T}^3)$ and a residual set $R \subset U(f)$ such that for any $g \in R$, $g$ is not continuum-wise expansive, where $RT(\mathbb{T}^3)$ is the set of all robustly transitive diffeomorphisms on $\mathbb{T}^3$.

Keywords: expansive, continuum-wise expansive, Axiom A, partially hyperbolic, generic
Mathematics Subject Classification numbers: 37D30; 34D10

1. Introduction
Let $M$ be a closed smooth manifold with $\dim M \geq 2$, and let $\text{Diff}(M)$ be the space of diffeomorphisms of $M$ endowed with $C^1$ topology. Denote by $d$ the distance on $M$ induced from a Riemannian metric $\|\cdot\|$ on the tangent bundle $TM$. In differentiable dynamical systems, expansiveness is a very useful notion to investigate for stability theory. For instance, Mañé [16] proved that the $C^1$-interior of the set of expansive diffeomorphisms coincides with the set of quasi-Anosov diffeomorphisms. Here $f$ is quasi-Anosov if for all $v \in TM \setminus \{0\}$, the set $\{\|Df^n(v)\| : n \in \mathbb{Z}\}$ is unbounded. Let $f \in \text{Diff}(M)$. We say that $f$ is expansive if there is $\varepsilon > 0$ such that for any $x, y \in M$ if $d(f^i(x), f^i(y)) < \varepsilon$ for all $i \in \mathbb{Z}$ then $x = y$. Denote by $E$ the set of all expansive diffeomorphisms. From now on, we introduce various expansiveness (N-expansive, countably expansive, measure expansive [18, 19]) which are general notions of original expansiveness. We say that $f$ is N-expansive if there is $\varepsilon > 0$ such that for any $x \in M$, the number of elements of the set $\Gamma_{\varepsilon}(x) = \{y \in M : d(f^i(x), f^i(y)) < \varepsilon \text{ for all } i \in \mathbb{Z}\}$ is less than $N$. Denote by $GE$ the set of all N-expansive diffeomorphisms on $M$. We say that
$f$ is countably expansive if there is $e > 0$ such that for $x \in M$, the number of elements of the set $\Gamma_x(e) = \{y \in M : d(f^i(x), f^i(y)) < e \text{ for all } i \in \mathbb{Z}\}$ is countable, where $e$ is an expansive constant for $f$. Note that if a diffeomorphism $f$ is expansive then $\Gamma_x(e) = \{x\}$ for $x \in M$. Thus if a diffeomorphism $f$ is expansive then $f$ is countably expansive, but the converse is not true (see [19]). For a Borel probability measure $\mu$ on $M$, we say that $f$ is $\mu$-expansive if there is $\delta > 0$ such that $\mu(\Gamma_x(e)) = 0$ for all $x \in M$. In this case, we say that $\mu$ is an expansive measure for $f$. We say that $f$ is measure expansive if it is $\mu$-expansive for every non-atomic Borel probability measure $\mu$. Denote by $\mathcal{ME}$ the set of all measure-expansive diffeomorphisms on $M$. Continuum-wise expansive diffeomorphisms was introduced by Kato [12]. A set $\Lambda$ is nondegenerate if the set $\Lambda$ is not reduced to one point. We say that $\Lambda \subset M$ is a subcontinuum if it is a compact connected nondegenerate subset of $M$.

**Definition 1.1.** A diffeomorphism $f$ on $M$ is said to be continuum-wise expansive if there is a constant $e > 0$ such that for any nondegenerate subcontinuum $\Lambda$ there is an integer $n = n(\Lambda)$ such that $\text{diam}^n(\Lambda) \geq e$, where $\text{diam} = \sup\{d(x, y) : x, y \in \Lambda\}$ for any subset $\Lambda \subset M$. Such a constant $e$ is called a continuum-wise expansive constant for $f$.

Note that every expansive diffeomorphism is continuum-wise expansive diffeomorphism, but its converse is not true (see [12, example 3.5]). Denote by $\mathcal{CE}$ the set of all continuum-wise expansive diffeomorphisms of $M$. In [4], Artigue showed that

$$\mathcal{E} \supset \mathcal{G}\mathcal{E} \supset \mathcal{C}\mathcal{E} = \mathcal{ME} \supset \mathcal{CW}\mathcal{E},$$

where $\mathcal{C}\mathcal{E}$ is the set of all countably expansive diffeomorphisms on $M$. For a $C^1$ perturbation expansive diffeomorphism, we can find the following result (see [5, 13, 21, 22]). Denote by $\text{int} A$ the $C^1$-interior of a set $A$ of $C^1$-diffeomorphisms of $M$.

**Theorem 1.2.** Let $f \in \text{Diff}(M)$. Then we have the following

$$\text{int} \mathcal{E} = \text{int} \mathcal{G}\mathcal{E} = \text{int} \mathcal{ME} = \text{int} \mathcal{CW}\mathcal{E}.$$

Let $\Lambda$ be a closed $f$-invariant set. We say that $\Lambda$ is hyperbolic if the tangent bundle $T_\Lambda M$ has a $Df$-invariant splitting $E^s \oplus E^u$ and there exists constants $C > 0$ and $0 < \lambda < 1$ such that

$$\|Df^n|E^s|\| \leq C\lambda^n \quad \text{and} \quad \|Df^{-n}|E^u|\| \leq C\lambda^n$$

for all $x \in \Lambda$ and $n \geq 0$. If $\Lambda = M$ then $f$ is said to be Anosov. It is well known that if a diffeomorphism $f$ is Anosov then it is quasi-Anosov, but the converse is not true (see [10]). Thus if a diffeomorphism $f$ is Anosov then $f$ is expansive, $N$-expansive, measure expansive, countably expansive and continuum-wise expansive.

We say that $f$ satisfies Axiom A if the non-wandering set $\Omega(f)$ is hyperbolic and it is the closure of $P(f)$. A point $x \in M$ is said to be non-wandering for $f$ if for any non-empty open set $U$ of $x$ there is $n > 0$ such that $f^n(U) \cap U \neq \emptyset$. Denote by $\Omega(f)$ the set of all non-wandering points of $f$. It is clear that $P(f) \subset \Omega(f)$.

A diffeomorphism $f$ is $\Omega$-stable if there is a $C^1$-neighborhood $U(f)$ of $f$ such that for any $g \in U(f)$ there is a homeomorphism $h : \Omega(f) \rightarrow \Omega(g)$ such that $h \circ f = g \circ h$, where $\Omega(g)$ is the non-wandering set of $g$.

A subset $\mathcal{G} \subset \text{Diff}(M)$ is called residual if it contains a countable intersection of open and dense subsets of $\text{Diff}(M)$. A dynamic property is called $C^1$ generic if it holds in a residual subset of $\text{Diff}(M)$. Arbieto [3] proved that if a $C^1$ generic diffeomorphism $f$ is expansive then it is Axiom A without cycles. Lee [13] proved that if a $C^1$ generic diffeomorphism $f$ is $N$-expansive then it is Axiom A without cycles. Very recently, Lee [15] proved that if a $C^1$
generic diffeomorphism $f$ is measure expansive then it is Axiom A without cycles. From that, we consider $C^1$ generic continuum-wise expansive diffeomorphisms. The following is a main result.

**Theorem A.** For $C^1$ generic $f \in \text{Diff}(M)$, if $f$ is continuum-wise expansive then it is Axiom A without cycles.

We say that a $f$-invariant closed set $\Lambda$ admits a *dominated splitting* if the tangent bundle $T\Lambda M$ has a continuous $Df$-invariant splitting $E \oplus F$ and there exist constants $C > 0$ and $0 < \lambda < 1$ such that
\[
\|Df^n|_{E(x)}\| \cdot \|Df^{-n}|_{F(f^n(x))}\| \leq C \lambda^n
\]
for all $x \in \Lambda$ and $n \geq 0$. If the dominated splitting can be written as a sum
\[
T\Lambda M = E_1 \oplus E_2 \oplus \cdots \oplus E_i \oplus E_{i+1} \oplus \cdots \oplus E_k,
\]
then we say that the sum is *dominated* if for all $i$ the sum
\[
(E_1 \oplus E_2 \oplus \cdots \oplus E_i) \oplus (E_{i+1} \oplus E_{i+2} \oplus \cdots \oplus E_k)
\]
is dominated. Note that the decomposition is called the *finest* dominated splitting if we cannot decompose in a non-trivial way subbundle $E_i$ appearing in the splitting.

The set $\Lambda$ is *partially hyperbolic* if there is a dominated splitting $E \oplus F$ of $T\Lambda M$ such that either $E$ is contracting or $F$ is expanding.

**Definition 1.3.** We say that a compact $f$-invariant set $\Lambda \subset M$ is strongly partially hyperbolic if the tangent bundle $T\Lambda M$ has a dominated splitting $E^s \oplus E^c \oplus E^u$ and there exist $C > 0$ and $0 < \lambda < 1$ such that for all $v \in E^s$, we have $\|Df^n(v)\| \leq C \lambda^n \|v\|$ for all $n \geq 0$, and for all $v \in E^u$, we have $\|Df^{-n}(v)\| \leq C \lambda^n \|v\|$ for all $n \geq 0$, where $E^c$ is the central direction of the splitting.

Note that if $\Lambda$ is hyperbolic for $f$ then it is strongly partially hyperbolic and $E^c$ is not empty, that is, $E^c = \{0\}$. For a partially hyperbolic diffeomorphism, Burns and Wilkinson [6] showed the following lemma.

**Lemma 1.4.** Let $\Lambda$ be a compact $f$-invariant set with a partially hyperbolic splitting,
\[
T\Lambda M = E^s \oplus E^c \oplus E^u.
\]
Let $E^{\epsilon,ij} = E^s \oplus E^c_i \oplus \cdots \oplus E^c_k \oplus E^u$ and $E^{\epsilon,ui} = E^s_1 \oplus \cdots \oplus E^c_i \oplus E^u$ and consider their extensions $\tilde{E}^{\epsilon,ij}$ and $\tilde{E}^{\epsilon,ui}$ to a small neighborhood of $\Lambda$. Then for any $\epsilon > 0$ there exist constants $R > r > r_1 > 0$ such that for any $x \in \Lambda$, the neighborhood $B(x,r)$ is foliated by foliations $\tilde{\omega}^u(x), \tilde{\omega}^s(x), \tilde{\omega}^{\epsilon,ij}(x)$ and $\tilde{\omega}^{\epsilon,ui}(x)(i = 1, \ldots, k)$ such that for each $\sigma \in \{u,s,(cs,i),(cu,i)\}$ the following properties hold.

(a) Almost tangency of invariant distributions. For each $y \in B(x,r)$, the leaf $\tilde{\omega}^\sigma(x)$ lies in a cone of radius $\epsilon$ about $\tilde{E}^\sigma(y)$.
(b) Coherence. $\tilde{\omega}^{\epsilon,ij}_x$ subfoliates $\tilde{\omega}^{\epsilon,ui}_x$ and $\tilde{\omega}^{\epsilon,ui}_x$ subfoliates $\tilde{\omega}^{\epsilon,ij}_x$ for each $i \in \{1, \ldots, k\}$.
(c) Local invariance. For each $y \in B(x,r)$ we have $f(\tilde{\omega}^\sigma(x,r_1)) \subset \tilde{\omega}^\sigma_{f^n(x)}(f(y))$ and $f^{-1}(\tilde{\omega}^\sigma(x,r_1)) \subset \tilde{\omega}^\sigma_{f^{-1}(x)}(f^{-1}(y))$, where $\tilde{\omega}^\sigma(x,r_1)$ is the connected components of $\tilde{\omega}^\sigma(x) \cap B(y,r_1)$ containing $y$.
(d) Uniqueness. $\tilde{\omega}^u(x) = \omega^u(x,r)$ and $\tilde{\omega}^u(x) = \omega^u(x,r)$. 

2984
We say that a diffeomorphism \( f \) has a **homoclinic tangency** if there is a hyperbolic periodic point \( p \) whose invariant manifolds \( W^s(p) \) and \( W^u(p) \) have a non-transverse intersection. The set of \( C^1 \) diffeomorphisms that have some homoclinic tangencies will be denoted \( \mathcal{H}^T \). For a homoclinic tangency, Pacifico and Vieitez [20] proved that surface diffeomorphisms presenting homoclinic tangencies can be \( C^1 \)-approximated by non-measure expansive diffeomorphisms. From the result, Lee [14] proved that if \( f \) has a homoclinic tangency associated to a hyperbolic periodic point \( p \), then there is a \( g \) \( C^1 \)-close to \( f \) such that \( g \) is not continuum-wise expansive.

**Proposition 1.5** ([7, theorem 1.1]). The diffeomorphism \( f \) in a dense \( G_\delta \) subset \( \mathcal{G} \subset \text{Diff}(M) \setminus \mathcal{H}^T \) has the following properties.

(a) Any aperiodic class \( \mathcal{C} \) is partially hyperbolic with a one-dimensional central bundle. Moreover, the Lyapunov exponent along \( E^c \) of any invariant measure supported on \( \mathcal{C} \) is zero.

(b) Any homoclinic class \( H_f(p) \) has a partially hyperbolic structure

\[
T_{H_f(p)}M = E^s \oplus E^c_1 \oplus \cdots \oplus E^c_k \oplus E^u.
\]

Moreover, the minimal stable dimension of the periodic orbits of \( H_f(p) \) is \( \dim E^c \) or \( \dim E^u + 1 \). Similarly, the maximal stable dimension of the periodic orbits of \( H_f(p) \) is \( \dim E^c + k \) or \( \dim E^u + k - 1 \). For every \( i = 1, \ldots, k \), there exist periodic points in \( H_f(p) \) whose Lyapunov exponent along \( E^c_i \) is arbitrarily close to 0. In particular, if \( f \in \mathcal{G} \), then \( f \) is partially hyperbolic.

Recently, Pacifico and Vieitez [20] proved that there is a residual subset \( \mathcal{G} \) of \( \text{Diff}(M) \setminus \mathcal{H}^T \) such that for any Borel probability measure \( \mu \) absolutely continuous with respect to Lebesgue, \( f \) is \( \mu \)-expansive. Lee [15] showed that there is a partially hyperbolic diffeomorphism such that it is not measure expansive. From the facts, we consider continuum-wise expansiveness for partially hyperbolic diffeomorphisms. The set \( \Lambda \) is transitive if there is a point \( x \in \Lambda \) such that \( \omega(x) = \Lambda \), where \( \omega(x) \) is the omega limit set of \( f \). We say that the set \( \Lambda \) is robustly transitive if there are a \( C^1 \)-neighborhood \( \mathcal{U}(f) \) of \( f \) and a neighborhood \( U \) of \( \Lambda \) such that for any \( g \in \mathcal{U}(f) \), \( \Lambda_g(U) = \bigcap_{n \in \mathbb{Z}} g^n(U) \) is transitive, where \( \Lambda_g(U) \) is the continuation of \( \Lambda \). Let \( M = \mathbb{T}^3 \) and let \( f : \mathbb{T}^3 \to \mathbb{T}^3 \) be a diffeomorphism.

**Theorem B.** There is a \( C^1 \) neighborhood \( \mathcal{U}(f) \) of \( f \in \mathcal{RT}(\mathbb{T}^3) \) and a residual set \( \mathcal{R} \subset \mathcal{U}(f) \) such that for any \( g \in \mathcal{R} \), \( g \) is not continuum-wise expansive, where \( \mathcal{RT}(\mathbb{T}^3) \) is the set of all robustly transitive diffeomorphisms on \( \mathbb{T}^3 \).

2. Proof of theorems

2.1. Proof of theorem A

Let \( M \) be as before, and let \( f \in \text{Diff}(M) \). The following Franks’ lemma [9] will play an essential role in our proofs.

**Lemma 2.1.** Let \( \mathcal{U}(f) \) be any given \( C^1 \) neighborhood of \( f \). Then there exist \( \epsilon > 0 \) and a \( C^1 \) neighborhood \( \mathcal{U}_0(f) \subset \mathcal{U}(f) \) of \( f \) such that for given \( g \in \mathcal{U}_0(f) \), a finite set \( \{x_1, x_2, \ldots, x_N\} \), a neighborhood \( U \) of \( \{x_1, x_2, \ldots, x_N\} \) and linear maps \( L_i : T_{x_i}M \to T_{g(x_i)}M \) satisfying \( \|L_i - D_{x_i}g\| \leq \epsilon \) for all \( 1 \leq i \leq N \), there exists \( \tilde{g} \in \mathcal{U}(f) \) such that \( \tilde{g}(x) = g(x) \) if \( x \in \{x_1, x_2, \ldots, x_N\} \cup (M \setminus U) \) and \( D_{x_i}\tilde{g} = L_i \) for all \( 1 \leq i \leq N \).
Lemma 2.2 ([15, lemma 2.2]). If \( f \in \text{Diff}(M) \) has a non-hyperbolic periodic point, then for any neighborhood \( U(f) \) of \( f \) and any \( \eta > 0 \), there are \( g \in U(f) \) and a curve \( \gamma \) with the following property:

1. \( \gamma \) is \( g \) periodic, that is, there is \( n \in \mathbb{Z} \) such that \( g^n(\gamma) = \gamma \);
2. the length of \( g^i(\gamma) \) is less than \( \eta \), for all \( i \in \mathbb{Z} \);
3. \( \gamma \) is normally hyperbolic with respect to \( g \).

By the persistency of normally hyperbolic manifolds, we know that there is a neighborhood \( U(g) \) of \( g \) such that for any \( \tilde{g} \in U(g) \) there is a curve \( \tilde{\gamma} \) close to \( \gamma \) such that all properties of \( \gamma \) listed in lemma 2.2 is also satisfied for \( \tilde{\gamma} \).

For \( f \in \text{Diff}(M) \), we say that \( f \) is the \textit{star diffeomorphism} (or \( f \) satisfies the \textit{star condition}) if there is a \( C^1 \)-neighborhood \( U(f) \) of \( f \) such that all periodic points of \( g \in U(f) \) are hyperbolic. Denote by \( \mathcal{F}(M) \) the set of all star diffeomorphisms. Aoki [2] and Hayashi [11] showed that for any dimension case, if \( f \in \mathcal{F}(M) \) then \( f \) is Axiom A without cycles. The following notion was introduced by Yang and Gan [24]. For any \( \epsilon > 0 \), a \( C^4 \) curve \( \eta \) is called a \( \epsilon \)-\textit{simply periodic curve} of \( f \) if (i) \( \eta \) is diffeomorphic to \([0,1]\) and its two endpoints are hyperbolic periodic points of \( f \), (ii) \( \eta \) is periodic with period \( \pi(\eta) \) and \( L(f^i(\eta)) < \epsilon \) for any \( i \in \{1,2,\cdots,\pi(\eta)\} \), where \( L(\eta) \) denotes the length of \( \eta \), and (iii) \( \eta \) is normally hyperbolic.

Lemma 2.3 ([8, lemma 2.4]). There is a residual set \( \mathcal{G} \subset \text{Diff}(M) \) such that for any \( f \in \mathcal{G} \),

- either \( f \) is a star,
- or for any \( \epsilon > 0 \) there is a periodic curve \( \gamma \) such that the length of \( f^n(\gamma) \) is less than \( \epsilon \), for any \( n \in \mathbb{Z} \).

Lemma 2.4 ([5, lemma 2.2]). Let \( C \subset M \) be a continuum. \( f \) is continuum-wise expansive if and only if there is \( \delta > 0 \) such that for all \( x \in M \), if a continuum \( C \subset \Gamma_\delta(x) \) then \( C \) is a singleton.

Proof of theorem A. Let \( f \in \mathcal{G} \) be continuum-wise expansive. Suppose by contradiction that \( f \notin \mathcal{F}(M) \). From lemma 2.3, for any \( \epsilon > 0 \) there is a periodic curve \( \gamma \) such that the length of \( f^i(\gamma) \) is less than \( \epsilon \), for any \( i \in \mathbb{Z} \). Let \( \Gamma_\epsilon(x) = \{ x \in M : d(f^i(x), f^i(y)) \leq \epsilon \} \) for all \( i \in \mathbb{Z} \). Since \( f^n(\gamma) = \gamma \) for some \( n \in \mathbb{Z} \), we know \( \gamma \in \Gamma_\epsilon(x) \). By lemma 2.4, \( \gamma \) should be a singleton which is a contradiction since \( \gamma \) is a nontrivial continuum. Thus if \( f \in \mathcal{G} \) is continuum-wise expansive then it is Axiom A without cycles. \( \square \)

2.2. Proof of theorem B

In this section, let \( M = \mathbb{T}^3 \) and let \( f : \mathbb{T}^3 \to \mathbb{T}^3 \) be a diffeomorphism. In [17, theorem B], Mañé constructed a robustly nonhyperbolic transitive diffeomorphism \( f \in \text{Diff}(\mathbb{T}^3) \). By [8, theorem B], every robustly transitive diffeomorphism \( f \) on \( \mathbb{T}^3 \) is partially hyperbolic. Thus we can find a partially hyperbolic diffeomorphism \( f \) on \( \mathbb{T}^3 \) such that \( f \) is robustly nonhyperbolic transitive.

Remark 2.5. There is a partially hyperbolic diffeomorphism \( f \) on \( \mathbb{T}^3 \) such that \( f \) is robustly nonhyperbolic transitive.

Lemma 2.6 ([8, corollary D] and [1, theorem 4.10]). There is a residual set \( \mathcal{G}_1 \subset \mathcal{RT}(\mathbb{T}^3) \) such that for any \( f \in \mathcal{G}_1 \), \( f \) is strongly partially hyperbolic, and \( \mathbb{T}^3 \) is the homoclinic class \( H(f, p) \), for some hyperbolic periodic point \( p \).
Lemma 2.7 ([24, lemma 2.1]). There is a residual set $G_2 \subset \text{Diff}(M)$ such that for any $f \in G_2$ and any hyperbolic periodic point $p$ of $f$, we have the following:

for any $\epsilon > 0$, if for any $C^1$ neighborhood $U(f)$ of some $g \in U(f)$ has a $\epsilon$-simply periodic curve $\eta$ such that two endpoints of $\eta$ are homoclinically related with $p_\eta$ then $f$ has a $2\epsilon$-simply periodic curve $\zeta$ such that the two endpoints of $\zeta$ are homoclinically related to $p$.

Proof of theorem B. Let $U(f)$ be a $C^1$ neighborhood of $f \in \mathcal{RT}(T^3)$ and let $f \in G = G_1 \cap G_2$. Let $p$ be a hyperbolic periodic point of $f$. Then for any $g \in G \cap U(f)$, we have $T^3 = H_g(p_\eta)$, where $p_\eta$ is the continuation of $p$. Since $H_g(p_\eta)$ is not hyperbolic, from [23, section 4], for any $\epsilon > 0$, there is $g_\eta \in G \cap U(f)$ such that $g_\eta$ has an $\epsilon$-simply periodic curve $\eta$, whose endpoints are homoclinically related to $p_\eta$. Note that $\eta$ is a closed and connected set, so it is a nontrivial continuum. For $x \in T^3$, we set $\Gamma_\epsilon(x) = \{y \in T^3 : d(g_1^i(x), g_1^i(y)) \leq \epsilon \text{ for all } i \in \mathbb{Z}\}$. Since $g_1^{\pi(p_\eta)}(\eta) = \eta$, we have

$\Lambda(g_1^{\pi(p_\eta)}(\eta)) = \Lambda(\eta) < \epsilon$.

Clearly $\eta \subset \Gamma_\epsilon(x)$. Since $\eta$ is compact and connected, so $\eta$ is not a singleton, which is a contradiction. □

Acknowledgment

The author would like to deeply thank all the reviewers for their insightful and constructive comments. This work is supported by a Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Science, ICT & Future Planning (No-2014R1A1A105002124) and (No. 2017R1A2B4001892).

References

[1] Abdenur F, Bonatti C and Crovisier S 2011 Nonuniform hyperbolicity for $C^1$-generic diffeomorphisms Isr. J. Math. 183 1–60
[2] Aoki N 1992 The set of Axiom A diffeomorphisms with no cycles Bol. Soc. Bras. Mat. 23 21–65
[3] Arbieto A 2011 Periodic orbits and expansiveness Math. Z. 269 801–7
[4] Artigue A 2016 Robustly $N$-expansive surface diffeomorphisms Discrete Continuous Dynam. Syst. 36 2367–76
[5] Artigue A and Carrasco-Olivera D 2015 A note on measure expansive diffeomorphisms J. Math. Anal. Appl. 428 713–6
[6] Burns K and Wilkinson A 2010 On the ergodicity of partilly hyperbolic systems Ann. Math. 171 451–89
[7] Crovisier S, Sambarino M and Yang D 2015 Partial hyperbolicity and homoclinic tangencies J. Eur. Math. Soc. 17 1–49
[8] Díaz L J, Pujals E R and Ures R 1999 Partial hyperbolicity and robust transitivity Acta Math. 183 1–43
[9] Franks J 1971 Necessary conditions for stability of diffeomorphisms Trans. Am. Math. Soc. 158 301–8
[10] Franks J and Robinson C 1976 A quasi-Anosov diffeomorphism that is not Anosov Trans. Am. Math. Soc. 223 267–78
[11] Hayashi S 1992 Diffeomorphisms in $\mathcal{F}^1(M)$ satisfy Axiom A Ergod. Thoer. Dyn. Syst. 12 233–53
[12] Kato H 1993 Continuum-wise expansive homeomorphisms Can. J. Math. 45 576–98
[13] Lee M 2016 General expansiveness for diffeomorphisms from the robust and generic properties
J. Dynam. Cont. Syst. 22 459–64
[14] Lee M 2016 Continuum-wise expansive homoclinic classes for generic diffeomorphisms Publ.
Math. Debrecen 88 193–200
[15] Lee M Measure expansiveness for generic diffeomorphisms Dynam. Syst. Appl. accepted
[16] Mañé R 1975 Expansive Diffeomorphisms (Lecture Notes in Mathematics vol 468) (Berlin:
Springer)
[17] Mañé R 1978 Contributions to the stability conjecture Topology 17 383–96
[18] Morales C A 2012 A generalization of expansivity Discrete Continuous Dynam. Syst. 32 293–301
[19] Morales C A and Sirvent V F 2013 Expansive Measures (Colóquio Brasileiro de Matemática vol
29) (Rio de Janeiro: IMPA)
[20] Pacifico M J and Vieites J L 2015 On measure expansive diffeomorphisms Proc. Am. Math. Soc.
143 811–9
[21] Sakai K 1997 Continuum-wise expansive diffeomorphisms Publica. Mate. 41 375–82
[22] Sakai K, Sumi N and Yamamoto K 2014 Measure-expansive diffeomorphisms J. Math. Anal. Appl.
414 546–52
[23] Sambarino M and Vieitez J 2006 On C1-persistently expansive homoclinic classes Discrete
Continuous Dynam. Syst. 14 465–81
[24] Yang D and Gan S 2009 Expansive homoclinic classes Nonlinearity 22 729–33