Neutrino condensates at center of galaxies as background for the MSW mechanism

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The possibility is explored that neutrino condensates, possible candidates for the explanation of very massive objects in galactic centers, could act as background for the Mikheyev-Smirnov-Wolfenstein mechanism responsible of neutrino oscillations. Assuming a simple neutrino star model with constant density, the lower limit of the mass squared difference of neutrino oscillations is inferred. Consequences on neutrino asymmetry are discussed.

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I. INTRODUCTION

The nature of matter condensates at galaxy centers \cite{1} is nowadays widely discussed since no current proposed model seems fully able to fit the data. For example, recent observations toward the Center of our Galaxy indicate that it hosts a super-massive dark compact object, Sagittarius A$^*$ (Sgr A$^*$) \cite{2}, which is an extremely loud radio source of estimated mass $M \sim 2.61 \times 10^6 M_\odot$ and radius $R \sim 0.016 \text{pc} \sim 30.6\text{lds}$ \cite{3,4} and several evidence for the presence of similar objects have been found in other galaxies, in quasars, and active galactic nuclei in a mass-range $10^6 \div 10^9 M_\odot$.

The most natural candidates to explain such peculiar objects could be either a single super-massive black hole or a very compact cluster of stellar size black holes \cite{4}. This last case has been ruled out by stability criteria \cite{5} which give maximal lifetimes of the order of $10^8$ years which are much shorter than the estimated age of standard galaxies \cite{6}. The first hypothesis is much more supported since similar super-massive black holes allow to explain the central dynamics of several galaxies as M87 \cite{7,8}, or NGC4258 \cite{9}. However, if Sgr A$^*$ were a super-massive black hole, its luminosity should be more than $10^{40}\text{erg/s}$, in contrast to observations which give a bolometric luminosity of $10^{37}\text{erg/s}$ (this is the so called blackness problem or the black hole on starvation). Besides, the most recent observations probe the gravitational potential at a radius larger than $4 \times 10^4$ Schwarzschild radii of a black hole of mass $2.6 \times 10^6 M_\odot$ \cite{3}, so that unambiguous proof that the super-massive object at the center of Galaxies is a black hole is still lacking.

Several alternative models have recently appeared in literature: for example, in \cite{10}, the hypothesis that the Galactic Center could consist of a super-massive boson star is investigated and some proposals have been done in order to seek a signature like, for example, the Cerenkov effect \cite{11}.

Viollier et al. \cite{12} proposed that the dark matter at the center of galaxies could be made by non-baryonic matter (massive neutrinos) which interacts gravitationally forming super-massive balls in which the degeneracy pressure of fermions balances their self–gravity. Such neutrino balls could be formed in the early epochs during a first–order gravitational phase transition and their dynamics could be reconciled.
with some adjustments to the Standard Model of Cosmology [12]. The neutrino mass required for fitting observational data by mean of super-massive degenerate neutrino stars is $10\text{keV} \leq m_\nu \tau \leq 25\text{keV}$, $m_\nu$ being the mass of $\tau$ neutrinos making up the stars. The gravitational effects of such a structure on the stars orbiting around it can be investigated by gravitational lensing [13], while astrophysical constraints and signatures can be investigated by next generation of X-ray satellites as XEUS or Constellation-X [14]. In [14], a more physical model is discussed taking into account a neutrino ball in gravitational equilibrium of a semi-degenerate fermion gas. Density and pressure within the ball are defined by adopting a formalism based on a distribution function in phase space, which allows to consider neutrinos with a degeneracy degree varying from the center to the border of the system. Limiting cases are fully degenerate fermion systems and the classical isothermal spheres well-known in literature.

In this paper, we take into account the fact that a condensate of massive neutrino could naturally act as the background for the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism which gives rise to neutrino oscillations. In Sect.II, we outline the simplest neutrino star model in order to define the environment where such oscillations should happen. The MSW effect and the active-sterile oscillations are discussed in Sect.III. Conclusions are drawn in Sect.IV.

II. THE NEUTRINO STARS MODEL

In this section, we shortly recall the simplest model of heavy neutrino condensates, bound by gravity (for details, see [12,15]). In the Thomas–Fermi model for fermions, the Fermi energy $E_F$ and the gravitational potential binding the system are related by (in natural units)

$$\frac{k_F^2(r)}{2m_\nu} - m_\nu \Phi(r) = E_F = -m_\nu \Phi(r_0),$$

(1)

where $\Phi(r)$ is the gravitational potential, $k_F$ is the Fermi wave number and $\Phi(r_0)$ is a constant chosen to cancel the gravitational potential for vanishing neutrino density. The constant $r_0$ is the estimated size of the halo of the ball. If one takes into account a degenerate Fermi gas, one gets $k_F(r) = (6\pi^2 n_\nu(r)/g_\nu)^{1/3}$, where $n_\nu(r)$ is the neutrino number density, assumed being the same for neutrinos and antineutrinos within the halo. The number $g_\nu$ is the spin degeneracy factor. If in the center of the neutrino condensate there is a baryonic star (which is approximated as a point source), the gravitational potential will obey a Poisson equation where neutrinos (and antineutrinos) are the source terms,

$$\nabla^2 \Phi = -4\pi G m_\nu n_B,$$

(2)

where $n_B = N/V$ is the neutrino background number density with $N$, the number of neutrinos making up the condensate, $N = M/m_\nu$, and $V = 4\pi R^3/3$ its volume. We have

$$n_B = \frac{N}{V} = \frac{3Mm_\nu}{4\pi R^3}.$$

(3)

Eq.(2) is valid everywhere except at the origin. In the case of spherical symmetry, the Poisson equation reduces to a radial Lané–Emden differential equation [12].

The general solution of (2), has scaling properties and it is able to reproduce the observations [12]. In fact, a degenerate neutrino star of mass $M = 2.6 \times 10^6 M_\odot$, consisting of neutrinos with mass $m \geq 12.0\text{keV}$ for $g_\nu = 4$, or $m \geq 14.3\text{keV}$ for $g_\nu = 2$, does not contradict the observations. Considering a standard accretion disk, the data are in agreement with the model if Sgr A* is a neutrino star with radius $R = 30.3$ ld ($\sim 10^5$ Schwarzschild radii) and mass $M = 2.6 \times 10^6 M_\odot$ with a luminosity $L \sim 10^{37}\text{erg/s}$. Similar results hold also for the dark object ($M \sim 3 \times 10^6 M_\odot$) inside the center of M87.

For our purposes, we assume a simple model where the neutrino density is constant around a point-like baryonic mass (e.g. a baryonic star). Such a model has been also considered in [13] where lensing experiments have been proposed for probing the existence of neutrino balls, without invoking the presence
of super-massive black holes at the center of our Galaxy. If \( n_B = \text{constant} \), it follows, from Eq. (2), that \( \Phi \sim r^2 \), in agreement with the Newton theorem applied to a spherical mass distribution. Such a situation is achieved considering a Fermi gas at temperature \( T = 0 \) [13].

III. MSW EFFECT AND \( \nu_\tau - \nu_S \) OSCILLATION

In the framework of Violler et al. model, it is natural to investigate the possibility that neutrinos crossing the neutrino background making up the supermassive condensate, could oscillate owing to the mechanism equivalent to MSW effect [16]. To be more specific, assuming that \( \tau \) neutrinos are the constituent of neutrino balls, we explore the possibility that a beam of \( \tau \) neutrinos (\( \nu_\tau \)) propagating in the \( \nu_\tau \)-background, could oscillate in sterile neutrinos \( \nu_s (\nu_\tau \to \nu_s) \) through a neutral current interaction. The neutrino \( \nu_s \) has the property that does not undergo electroweak interactions.

According to the standard model, the \( \nu - \nu \) scattering is described by the effective Lagrangian density

\[
L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} [\bar{\nu}_\tau \gamma^\mu (1 - \gamma^5) \nu_\tau] [\bar{\nu}_B \gamma^\mu (1 - \gamma^5) \nu_B] \tag{4}
\]

where \( \nu_B \) indicates background neutrino, i.e. \( \nu_B \equiv \nu_\tau \). For simplicity in what follows we write \( \nu \) instead of \( \nu_L \equiv (1 - \gamma^5) \nu/2 \). Neutrinos forming the condensate are non relativistic so that the axial current contribution is negligible, as well as the spatial components of the 4-current since it is related to the average velocity of \( \nu_B \). Then, by considering the average over the background, the 0-component of the 4-current gives, in agreement with standard procedure [16],

\[
< \bar{\nu}_B \gamma^0 \nu_B > = < \nu_B^\dagger \nu_B > \equiv n_B . \tag{5}
\]

Eq. (5) adds an amounts of \( \sqrt{2} G_F n_B \) to the energy of neutrinos propagating within the neutrino star. Equation of evolution of neutrinos is (for simplicity we consider only the radial motion)

\[
\frac{d}{dr} \begin{pmatrix} \nu_\tau \\ \nu_s \end{pmatrix} = \Lambda \begin{pmatrix} \nu_\tau \\ \nu_s \end{pmatrix}, \tag{6}
\]

where the mixing matrix is defined as [16]

\[
\Lambda \equiv \begin{pmatrix} -\frac{\Delta m^2}{2E} \cos 2\theta + a & \frac{\Delta m^2}{2E} \sin 2\theta \\ \frac{\Delta m^2}{2E} \sin 2\theta & \frac{\Delta m^2}{2E} \cos 2\theta \end{pmatrix}, \tag{7}
\]

where \( a = \sqrt{2} G_F n_B \). The mass eigenstates \( |\nu_1 > \) and \( |\nu_2 > \) are determined by diagonalizing the mixing matrix in (7): One writes the mass eigenstates as a superposition of flavor eigenstates

\[
|\nu_1 > = \cos \tilde{\theta} |\nu_\tau > - \sin \tilde{\theta} |\nu_s > , \tag{8}
\]

\[
|\nu_2 > = \sin \tilde{\theta} |\nu_\tau > + \cos \tilde{\theta} |\nu_s > ,
\]

where the mixing angle \( \tilde{\theta} \) is defined as

\[
\tan 2\tilde{\theta} = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2aE}. \tag{9}
\]

The corresponding eigenvalues are

\[
\lambda_{1,2} = \pm \frac{\Delta m^2}{4E} \sqrt{(\cos 2\tilde{\theta} - b) \cos 2\tilde{\theta} + \sin^2 2\tilde{\theta}}, \tag{10}
\]

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being \( b = 2aE/\Delta m^2 \). The resonance condition occurs when

\[
\cos 2\theta = b.
\]

(11)

We remember that the neutrino density \( n_B \) is assumed constant. From Eq. (11), and using Eq.(3) one infers the relation

\[
\Delta m^2 \geq 7 \times 10^{-14} \left( \frac{30.6 \text{ld}}{R} \right)^3 \frac{E}{\text{GeV}} \frac{M}{M_\odot} \frac{eV}{m_{\nu_e}} eV^2.
\]

(12)

Neutrino stars are characterized by the parameters \( R \sim 30 \text{ld} \) and \( M \sim 2.6 \times 10^6 M_\odot \), thus Eq. (12) becomes

\[
\Delta m^2 \geq 1.4 \times 10^{-7} \frac{E}{\text{GeV}} \frac{eV}{m_{\nu_e}} eV^2.
\]

(13)

The result does depend on neutrino energy and the mass of the background neutrinos \( (m_{\nu_e} \sim 10 \text{keV}) \). From (13) we derive the allowed solutions for \( \Delta m^2 \):

\[
E \sim \text{MeV}, \quad \Delta m^2 \geq 10^{-14} \text{eV}^2,
\]

\[
E \sim \text{GeV}, \quad \Delta m^2 \geq 10^{-11} \text{eV}^2,
\]

\[
E \sim \text{TeV}, \quad \Delta m^2 \geq 10^{-8} \text{eV}^2,
\]

which agree with the best fit for active-sterile neutrinos of \( \tau \) type, \( \Delta m^2 \leq 10^{-6} \) obtained from the big bang nucleosynthesis predictions and in the absence of lepton asymmetry. The case of neutrinos with very high energy

\[
E \sim \text{PeV}, \quad \Delta m^2 \geq 10^{-5} \text{eV}^2,
\]

is excluded.

In presence of lepton symmetry, the above bound \( \Delta m^2 \leq 10^{-6} \) becomes \( \Delta m^2 \leq 1 \), and the range of energy \( 1 \div 10^6 \text{GeV} \) is permissible in order that neutrino stars might induce an appreciable MSW effect.

These results have non trivial consequences in relation to neutrino asymmetries in the early Universe, widely discussed in Refs. [17,19].

IV. DISCUSSION AND CONCLUSIONS

In this paper we have investigated the active-sterile neutrino oscillations for neutrinos propagating in a neutrino star which acts as background matter. Taking into account the neutral current weak interaction, we have studied the resonance condition, in analogy to MSW mechanism for solar neutrinos.

The basic results here obtained are two-fold. From one side, our results show that the present data on neutrino oscillations do not contradict a possible existence of such exotic super-massive objects. Actually much more investigation on this results is necessary. First of all, we should take into account more realistic models where the density of the condensate changes with the radius [14]. Furthermore, one should investigate the \( \nu_e : \nu_\mu : \nu_\tau \) ratio which becomes \( \nu_e : \nu_\mu : \nu'_\tau \), where \( \nu'_\tau \) are the \( \tau \)-neutrinos number reduced (with respect to the expected ones) as a consequence of the \( \nu_\tau - \nu_\mu \) oscillation induced by the neutrino star environment (for this topic, see also [18]). On the other side, our results show that oscillations affected by \( \text{matter} \), i.e. induced by neutrino stars, could have effects on the generation of neutrino asymmetries in the Universe [19]. The latter occur as a consequence of active-sterile neutrino
oscillations, which generate a discrepancy of neutrino and antineutrino number density (such a subject is widely discussed in Refs. [19]). The lepton number of a neutrino flavor $f$ is defined as

$$L_f = \frac{n_{\nu_f} - n_{\bar{\nu}_f}}{n_{\gamma}(T)},$$

(14)

where $n_{\nu_f}$ ($n_{\bar{\nu}_f}$) is the number density of neutrinos (antineutrinos), and $n_{\gamma}$ one of photons at temperature $T$. Constraints on $L_f$ are derived from the cosmic microwaves background and from big bang nucleosynthesis. In the case of neutrinos $\nu_\tau$, which are of interest in our case, bounds on the lepton number are $|L_{\nu_\tau}| \leq 6.0$. The creation of the lepton number determines a strong modification on the upper limit of the mass squared difference value. In fact, it goes from $\Delta m^2 \sim 10^{-6}\text{eV}^2$, coming from the big bang nucleosynthesis predictions, to $\Delta m^2 \geq 1\text{eV}^2$ [19,20].

As final comment, it is worth to point out that if at the center of galaxies a black hole is present, then no oscillation phenomena could occur, unless a violation of the equivalence principle is invoked [21,22]. This is opposite to results obtained in this paper, where $\nu_\tau$-background matter affects oscillations giving a new scenario and a strong signature for future investigations aimed to probe the (possible) existence of such exotic objects, the neutrino balls, at the center of galaxies.

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