THERMOPHORESIS AND DIFFUSION THERMO EFFECTS ON SHEAR THICKENNING AND SHEAR THINING CASES OF FLUID MOTION PAST A PERMEABLE SURFACE

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Abstract
An effort has been prepared numerically to investigate thermophoresis and diffusion thermo effects on liquid motion past a permeable surface. Motion is managed by the constitutive equation of power law fluid model. External forces appeared in the flow system are Lorenz force due to external magnetic field, buoyancy force. Similarity transformation has been utilized in the methodology part and MATLAB built in bvp4c solver scheme has been adopted to carry out the numerical solutions. Impacts of flow parameters on flow characteristics have been outlined by figures and diagrams.

Keywords: MHD, Power-law fluid, Soret Effect (thermophoresis), Dufoureffect (diffusion thermo), thermal and mass transfer

Nomenclature

\( u, v \) - velocity components along x and y- directions, \( g \) - acceleration due to gravity, \( T, C \) - temperature and concentration of the fluid, \( K_f \) - thermal conductivity, \( n \) - power-law index parameter, \( D_m \) - mass diffusivity, \( M \) - magnetic parameter, \( k \) - permeability paramete \( Gr, Gm \) - Grashof number for heat and mass transfer, \( Du \) - Dufour number, \( Sr \) - Soret number
I. Introduction

It is seen that different mathematical models have been proposed to explain the rheological behavior of fluids used in various science and engineering purposes. Among these, a model which has been most widely used and is frequently encountered in chemical engineering processes is the power-law model. Although this model is merely an empirical relationship between the stress and velocity gradients, it has been successfully applied to non-Newtonian fluids experimentally. Research works on fluid flow using power law constitutive model have been done by Acivos et al. [I], Schowalter [XI], Lee and Ames [IX], Andersson et al. [II], Tai and Char [XIV], Aziz et al. [III] and Sarithaet al. [X].

Simultaneous thermal and mass diffusions phenomenon has two major coupled characteristics: (i) transportation of solutes due to gradient of temperature: thermal-diffusion (Soret) effect or thermophoresis, (ii) transportation of energy due to difference in solute. Further, many researchers (Shateyi et al. [XIII], Cheng [IV], Sharma et al. [XII]) have investigated the boundary layer fluid flow with both Soret and Dufour effects on heat and mass transfers over a vertical plate by taking different methodologies. Dey [V] has investigated the hydro-magnetic boundary flow of dusty non-Newtonian fluid flow over a plate. Jafarimugaddam and Aberoumand [VIII] have derived precise formulations for skin friction and convective heat transfer coefficients for power-law fluids. Zhang et al. [XV] have investigated the heat transfer phenomena of non-Newtonian power-law fluid in pipes with different cross sections. Huang and Yih [VII] have analyzed the heat and mass transfer of non-Newtonian fluids over a vertical plate with the influence of both Soret and Dufour effects. Hirschhom and Madsen [VI] have investigated the MHD boundary layer power-law fluid flow over a flat plate.

Motivated by the works mentioned above, the objective of this work is to study the steady MHD fluid flow using power law fluid model with thermophoresis and diffusion thermo using numerical scheme.

II. Mathematical Formulation

We have taken time independent fluid flow with power law model past a permeable surface with coupled thermo-diffusion and diffusion-thermo effects. Other forces appeared in the flow problem are Lorentz force and buoyancy force. Permeable surface coincides with the axis of x (fig. 1) and component of velocity along the
direction of surface is \( u \) and along \( v \) be velocity component along the normal to the surface. The geometry of the flow model is given in Fig 1.

**Fig. 1:** Flow model

From conservation principles of mass, momentum, energy and species concentration, the governing equations of fluid motions are:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{k_T}{\rho c_p} \frac{\partial^2 u}{\partial y^2} + \frac{k}{\rho c_p} \left( \frac{\partial u}{\partial y} \right)^n + g \beta(T - T_\infty) + g \beta(C - C_\infty) \tag{2}
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_T}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \frac{k}{\rho c_p} \left( \frac{\partial T}{\partial y} \right)^n + \frac{D_m K_f \partial^2 C}{c_p c_s} \frac{\partial C}{\partial y} \tag{3}
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + \frac{D_m K_f \partial^2 T}{\tau_m} \frac{\partial T}{\partial y} \tag{4}
\]

The boundary conditions for the velocity, temperature and concentration fields are:

\[
u = l \frac{\partial u}{\partial y}, v = v_w, T = T_w + D \frac{\partial T}{\partial y}, C = C_w + E \frac{\partial C}{\partial y} \text{ at } y = 0 \tag{5}
\]

\[u \to U_\infty, T \to T_\infty, C \to C_\infty \text{ as } y \to \infty \tag{6}\]

To obtain the dimensionless forms of (3), (4) and (5), we introduce the following dimensionless similarity variables and stream function \( \psi \) such that \( u = \frac{\partial \psi}{\partial y} \) & \( v = -\frac{\partial \psi}{\partial x} \)

\[
\psi = LU_\infty \left( \frac{x}{Le} \right)^{\frac{1}{n+1}} f(\eta), \eta = \left( \frac{ReL}{x} \right)^{\frac{1}{n+1}} \frac{y}{L}, u = U_\infty f'(\eta), \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{7}
\]
\[ \phi = \frac{C - C_\infty}{C_w - C_\infty}, \quad Pr = \frac{K_T}{\rho C_p U_\infty L^2} \left( \frac{Re L}{x} \right)^{\frac{2}{n+1}}, \quad Ec = \frac{K U_\infty n Re}{\rho C_p L^n (T_w - T_\infty)} \]

\[ M = \frac{\sigma B_r^2 x}{\rho U_\infty}, \quad k = \frac{K x \left( \frac{Re L}{x} \right)^{\frac{n-1}{n+1}} U_\infty^{n-2}}{\rho K_p L^{n-1}}, \quad Gr = \frac{g \beta (T_w - T_\infty) x}{U_\infty^2}, \quad Gm = \frac{g \beta (C_w - C_\infty) x}{U_\infty^2} \]

Making use of the similarity transformations (7), we get the following nonlinear system of differential equations:

\[ -\frac{1}{n+1} f'''' = n(f'')'; \quad \theta'' = -\frac{Pr}{n+1} f'\theta' - Pr Ec (f''') + DuPr f'' \]

\[ \phi'' = -\frac{Sc}{n+1} f'\phi' - SrSc \theta'' \]

The corresponding boundary conditions are as follows:

\[ f = S, f' = df'/d\eta, \quad \theta = 1 + b\theta', \quad \phi = 1 + n\phi' \text{ at } \eta = 0 \]

\[ f' \to 1, \theta \to 0, \phi \to 0 \text{ as } \eta \to \infty \]

Further, various flow characteristics, viscous drag coefficient \( C_f \), local Nusselt number \( Nu_x \) and local Sherwood number \( Sh_x \) are outlines below:

\[ C_f = \frac{2\tau_w}{\rho U_\infty^2}, \quad Nu_x = \frac{q_{w} x}{k_1 (T_w - T_\infty)}, \quad Sh_x = \frac{x J_w}{D_m (C_w - C_\infty)} \]

where, \( \tau_w = \rho_0 \left[ \frac{\partial U}{\partial y} \right]_{y=0}, \quad q_{w} = -k_1 \left[ \frac{\partial T}{\partial y} \right]_{y=0}, \quad J_w = -Dm \left[ \frac{\partial C}{\partial y} \right]_{y=0}. \]

Then we have got the following expressions for these quantities:

\[ f''''(0) = \frac{1}{2} C_f \left( \frac{x Re L}{L} \right)^{\frac{1}{n+1}}, \quad Nu_x = -\theta''(0) \frac{1}{x L} \left( \frac{Re L}{x} \right)^{\frac{1}{n+1}}, \quad Sh_x = -\phi''(0) \frac{1}{x L} \left( \frac{Re L}{x} \right)^{\frac{1}{n+1}} \]

### III. Results and Discussion

Solving the above differential equations, analysis is done graphically. Here, we have taken Prandtl number, \( Pr = 0.71 \) which physically characterizes air at \( 20^\circ \text{C} \) at 1 atmospheric pressure and Schmidt number, \( Sc = 0.6 \) corresponds to water vapor with air during diffusing medium. The Power-law index parameter is taken to be
$n = 0.4, 1, 1.4$ such that $n < 1$ resembles shear thinning fluid (pseudo-plastic fluid) and $n > 1$ represents the shear thickening fluid (dilatants fluid) and the case corresponding to $n = 1$ represents Newtonian fluid. Also, the Eckert number ($Ec$) is kept fixed as 0.01 throughout this discussion.

**Velocity Profiles for Various Values of Flow Parameters**

The figures (2)-(5) illustrate the velocity distributions for different values of flow parameters. From the figure (2), it is observed that the motion is accelerated with Power-law index parameter. Further, it can be interpreted that the Power-law index parameter helps to increase the boundary layer thickness of the fluid motion. The boundary layer thickness of the shear thinning (pseudo plastics, $n < 1$) fluids is more thicker as compared to the shear thickening (dilatants, $n > 1$) fluids. The shear thinning fluid experiences maximum speed in the neighborhood of the plate than the other fluids (Newtonian and shear thickening fluid). Acceleration in fluid motion is also observed the permeability parameter $k$ which is seen in both the cases of Newtonian[Fig. 3(a)] and shear thinning [Fig. 3(b)] fluids (see Fig. 3). Due to applications of the magnetic field, a resistance type of force termed as ‘Lorentz force’ is developed which drops the speed of the fluid. This theoretical result is in perfect match with the result obtained in our result, i.e., the magnetic parameter $M$ decelerates both Newtonian and pseudo plastic (shear thinning) fluid motions (Fig. 4). From the figure (5), it is seen that the speed of the Newtonian fluid rises due to Soret number ($Sr$). It is physically interpreted that greater Soret number leads to a larger temperature difference which helps to amplify the velocity of the fluid.

![Fig.2: Velocity Profile $f'(\eta)$ against $\eta$ for various values of power-law index $n$.](image)
Fig. 3: Velocity distribution \( f(\eta) \) against \( \eta \) for various values of \( k \).

(a) Newtonian fluid (\( n = 1 \))

(b) Shear thinning fluid (\( n < 1 \))
(b) Shear thinning fluid (n < 1)

**Fig. 4:** Velocity distribution $f'(\eta)$ against $\eta$ for (Newtonian and Shear thinning fluids) various value magnetic parameter $M$.

**Fig. 5:** Velocity profile $f'(\eta)$ for various values of Soret number (Sr).

**Temperature Profiles for Various Values of Flow Parameters**

The temperature distributions for different values of flow parameters are graphically discussed in figures (6)-(8). From the Fig. 6, it is observed that the temperature of the fluid rises with increasing values of Power-law index parameter. In other words, the thickness of the thermal boundary layer of shear thickening (n>1) fluid is more than other fluids. From figure (7), it is noticed that the Dufour number leads to reduce the temperature of Newtonian and pseudo plastic fluids in the neighborhood of the plate. It is physically interpreted that larger concentration difference helps to drop the temperature of the fluid in the vicinity of the surface. But, Soret number leads to rise the temperature of the Newtonian and shear thinning fluids in the neighborhood of the surface of the plate [see Fig. 8(a) and 8(b)].
Fig. 6: Temperature Profile $\theta(\eta)$ against $\eta$ for various values of power-law index $n$.

(a) Newtonian fluid ($n=1$).

(b) Shear thinning fluid ($n<1$).

Fig. 7: Temperature Profile $\theta(\eta)$ against $\eta$ for various values (Newtonian fluid and shear thinning fluid) of Dufour number $Du$.
(a) Newtonian fluid ($n=1$).

(b) Shear thinning fluid ($n<1$).

Fig. 8: Temperature profile (Newtonian and Shear thinning fluids) for various values of Soret number (Sr).

Fig. 9: Concentration Profile $\phi(\eta)$ against $\eta$ for various values of power-law index $n$.

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Concentration Profiles for Various Values of Flow Parameters

The figures (9)-(11) describe concentration distribution of the fluids for different values of flow parameters. The Power-law index parameter \( n \) leads to increase the concentration of the fluid [see Fig. 9]. The concentration of the shear thickening fluid is more than the shear thinning and Newtonian fluids. From figures 10(a) and 10(b), it is seen that there is a reduction of concentrations with increasing values of Soret number \( (Sr) \). Thus, larger values of Soret number indicate the smaller concentration difference which reduces the concentration of the fluids. But there is an acceleration of mass concentration with Dufour number \( (Du) \) [see figures 11(a) and 11(b)]. It is physically interpreted that higher values of Dufour number denotes greater concentration gradient which helps to grow the concentration of the fluids.

\[\text{Fig.10: Concentration Profile } \phi(\eta) \text{ against } \eta \text{ (Newtonian and Shear thinning fluids) for various values of Soret number } Sr.\]
The figure (12) illustrates the heat transfer rate of the fluid for various values of Power-law index parameter $n$. In the vicinity of the surface, the heat transfer rate is decelerated with power law index parameter. Maximum and minimum heat transfer rates at the surface of the plate are experienced by shear thinning fluid and shear thickening fluid respectively.

**Fig.11:** Concentration Profile $\phi(\eta)$ against $\eta$ for (Newtonian and Shear thinning fluids) various values of Dufour number $Du$.

**Fig.12:** Sketch of Heat Transfer rate for various values of power-law index $n$. 

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The numerical values of skin friction coefficient and Sherwood number are shown by tables (1) and (2). From table (1), it is seen that the numerical values of skin friction coefficient reduces with increasing values of Soret number (Sr) and the magnitude of skin friction coefficients of pseudo plastic (shear thinning) fluid are higher than that of Newtonian fluid. But a reverse effect is noticed with the Dufour effects on skin friction coefficient of the fluids. From table (2), it is observed that both Soret and Dufour effect enhance the mass accumulation rate (Sherwood number) of the fluids.

Table. 1: Numerical values of skin friction coefficient

| Values of skin friction coefficient \( f'(0) \) | Du | n = 0.4 | n = 1.0 | Sr  | n = 0.4 | n = 1.0 |
|---------------------------------------------|----|---------|---------|-----|---------|---------|
| 0.5                                         |    | 2.4681  | 1.8496  | 0.5 | 2.5958  | 1.7888  |
| 1.0                                         |    | 2.4765  | 1.8582  | 1.5 | 2.4390  | 1.5401  |
| 1.5                                         |    | 2.4825  | 1.8593  | 2.0 | 2.3572  | 1.4125  |

Table. 2: Numerical values of Sherwood number \(-\phi'(0)\) for

\[ M = 0.2, k = 0.3, Gr = 1.5 \text{ and } Gm = 1.2. \]

| Values of Sherwood number \(-\phi'(0)\) | Du | n = 0.4 | n = 1.0 | Sr  | n = 0.4 | n = 1.0 |
|----------------------------------------|----|---------|---------|-----|---------|---------|
| 0.5                                    |    | 0.5192  | 0.3664  | 0.5 | 0.5380  | 0.3746  |
| 1.0                                    |    | 0.5382  | 0.3782  | 1.5 | 0.6895  | 0.4506  |
| 1.5                                    |    | 0.5604  | 0.3902  | 2.0 | 0.7676  | 0.4993  |

IV. Conclusion

From the above discussion, we have concluded that:

I. The velocity, temperature and concentration of the shear thickening (dilatants \( n > 1 \) ) fluid are higher than Newtonian and shear thinning fluids.

II. Due to influence of Soret number (Sr), the velocity and temperature of the fluids rise, but concentration of the fluids reduces.

III. Due to influence of Dufour number (Du), the temperature of the fluids reduces and concentration of the fluids enhances.

IV. The Dufour number (Du) amplifies the magnitude of skin friction coefficient and Soret number (Sr) acts an opposite manner on skin friction coefficients.
References

I. Acrious, A., Shah, M.J., Peterson E.E., “Momentum and heat transfer in laminar boundary layer flow on non-newtonian fluids past external surfaces”, AIChE Journal. vol. 6, pp: 312–316, 1960.

II. Andersson, H.I., Bech, K.H. and Dandapat, B.S., “Magnetohydrodynamic flow of a power law fluid over a stretching sheet”, International Journal of Non-Linear Mechanics. vol. 72, pp: 929–936, 1992

III. Aziz, A., Ali, Y., Aziz, T. and Siddique, J., “Heat Transfer analysis for stationary boundary layer slip flow of a power low fluid in a Darcy porous medium with plate suction/injection”, PLoS ONE. Vol.10 (9), doi:10.1371/journal.pone.0138855, 2015

IV. Cheng, C.Y., “Soret and Dufour effects on mixed convection heat and mass transfer from a vertical wedge in a porous medium with constant wall temperature and concentration”, Transport in Porous Media. vol. 94, pp: 123–32, 2012.

V. Dey, D., “Non-Newtonian effects on hydromagnetic dusty stratified fluid flow through a porous medium with volume fraction”, Proceedings of the National Academy of Sciences, India Section A: Physical Sciences, 86(1), pp: 47-56, 2016.

VI. Hirschhorn J., Madsen M., Mastroberardino A. and Siddique J.I., “Magnetohydrodynamic boundary layer slip flow and heat transfer of power-law fluid over a flat plate”, Journal of Applied Fluid Mechanics, 9(1), pp: 11-17, 2016.

VII. Huang, C.J. and Yih, K.A., “Heat and Mass Transfer on the Mixed Convection of non-Newtonian fluids over a vertical wedge with Soret/Dufour effects and Internal Heat Generation: Variable wall Temperature/Concentration”, Transport in Porous Media. vol.130, pp: 559-576, 2019.

VIII. Jafarimughaddam, A. and Aberoumand, S., “Exact approximations for skin friction coefficient and convective heat transfer coefficient for a class of power-law fluids flow over a semi-infinite plate: Results from similarity solution”, Engineering Science and Technology: an International Journal. vol. 20(3), pp: 1115-1121, 2016.

IX. Lee, S.Y., Ames, W.F., “Similar solutions for non-Newtonian fluids”, AIChE Journal. vol. 12, pp: 700–708, 1960.

X. Saritha, K., Rajasekhar, M.N. and Reddy, B.S. “Combined effects of soret and dufour on mhd flow of a Power-law fluid over flat plate in slip flow regime”, International Journal of Applied Mechanics and Engineering. vol. 23(3), pp: 689-705, 2018
XI. Schowalter, W.R., “The application of boundary layer theory to power law pseudo plastic fluids: similar solutions”, AIChE Journal. vol. 6(1), pp: 24-28, 1960.

XII. Sharma, B.K., Gupta, S., Vamsikrishna, V. and Bhargavi, R.J., “Soret and Dufour effects on an unsteady MHD mixed convective flow past an infinite vertical plate with Ohmic dissipation and heat source”, AfrikaMathematika, vol. 25, pp. 799–821, 2014.

XIII. Shateyi, S., Motsa, S.S. and Sibanda, P., “The effects of thermal radiation, Hall currents, Soret and Dufour on MHD flow by mixed convection over a vertical surface in porous media”, Mathematical Problems in Engineering, Article ID 627475, 2010

XIV. Tai, B.C. and Char, I.M. “Soret and Dufour effects on free convection flow of non-Newtonian fluids along a vertical plate embedded in a porous medium with thermal radiation”, International Communications in Heat and Mass Transfer. vol. 37, pp. 480-483, 2010

XV. Zhang, H., Kang, Y., and Xu, T., “Study on Heat Transfer of non-Newtonian Power-law fluid in pipes with different cross sections”, Procedia Engineering. vol. 205, pp. 3381-3388, 2017.