Accuracy Analysis for Manipulator of Satellite Antenna
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Abstract. To solve the problem of tolerance design for manipulator of satellite antenna, DH method is used to build the error field model. Based on this model, the error field, the max error range, the position and attitude error of manipulator end in each direction can be calculated, and the results can be the reference for tolerance design of manipulator. The simulation example of a three-joints manipulator illustrate the method we used is feasible. This research focus on the model of the error field which can be the reference for the accuracy design and precision assemble of satellite antenna.

Introduction
In order to avoid the impact on satellite antenna from inner electromagnetic components, deployable antenna is commonly used to keep a certain distance away from the satellite, considering the payload layout. The deployable antenna can be positioned by the deployment mechanism and the biaxial drive mechanism [1]. The former is mostly a 3-degree-of-freedom extending arm which is extended to the predetermined position and locked after the satellite is on orbit, while the latter is in charge of the real-time tracking of the target. The errors of the two both have a significant impact on the pointing accuracy of the antenna. Scholars generally focus on the latter as the technology has been widely used in military communication satellites, data relay satellites and other satellites [2-4]. However, the impact of the error on the pointing accuracy of the antenna extension arm is rarely studied.

The static error of the extending arm has a great impact on satellite pointing as it no longer adjusts after being fixed on the right location and unfolded. The engineering practice has proved that the accuracy design of the extending arm is of much importance in the development of the satellite antenna. At present, the design of the tolerances is based on experience and the error range can not be accurately determined, resulting in the majority of the prototype production can not meet the accuracy requirements, should be corrected and reconstructed, causing a lot of waste in manpower, material and financial resources. This has becoming a practical problem to be solved urgently.

Combined with the static error analysis method of robot [5], a calculation model of the error region for the end of the extending arm is proposed based on DH method. This can serve as the reference for tolerance design and then realize the accuracy design. The simulation example in the end illustrates the engineering feasibility of the method above and the engineering guiding significance of the research results.

Calculation Model of the Error Region for the End of the Extending Arm
The initial design of the extending arm can be modeled and calculated through the error region, therefore the error range of the end position and attitude can be found to determine whether the tolerance design is reasonable.

The Calculation Model of the Error Range based on DH Method
A theoretical value of the end position and attitude of the extending arm can be calculated from the designed value of each serial link between the base and the end. As the serial links each has a designed tolerance, allowing the actual result of its manufacture or assembly within a deviation range
enclose the designed value. So the end position and attitude will end up in a range of deviation near the theoretical value, which is called the error range.

Take an articulated single arm as an example, as is shown in Figure 1. The designed value of the arm length is \( L \), the tolerance is \( \Delta L \), the angle of the articulation is \( \theta \), and the tolerance is \( \Delta \theta \), so the theoretical value of the end position can be recursively calculated as \( O_1 \). Due to the tolerances of the two serial links, the envelope scope formed by \( B, B', C', C \) is the error range of the end position, which means the end position of the arm made by this size and tolerance will fall within this range.

The spatial shape of the error range is special due to the complex structure of the extension arm and the large number of types and huge amount of intermediate serial links. Yet in the engineering practice we generally focus on the maximum error range of the end position and attitude of the extending arm, that is, the boundary value of the error range. Therefore it is possible to obtain discrete terminal error region by traversing and calculating several typical positions of each error link within the tolerance range.

![Figure 1. The error range diagram.](image)

Take the structure in Figure 1 as an example, the number of steps is 2, which means the tolerance range of each error link is divided into two parts and the two sides of boundary together with the middle value are calculated respectively. The number of the total traversal steps of the two error links is \( 2^2 = 9 \), therefore the end of the single arm pod should probably be in the nine positions as \( O_1, O_2, O_3, A, A', B, B', C, C' \), which make up the error range of the end position. And the two positions \( B' \) and \( C' \) become the maximum error region, as they are the farthest from the theoretical position \( O_1 \).

DH method is a reasonable way to establish the fixed coordinate system for each linkage of the robot. Supposing there are \( m \) links of the extending arm of which the errors need to be analyzed, find out these links and make them equivalent to translational or rotational articulation joints. First, they are numbered from the base to the end, and the fixed coordinate system for each link is established. Four parameters \( \{d_i, \theta_i, a_i, \alpha_i\} \) are used to show the relative position and direction between coordinate system \( i-1 \) and system \( i \) [6], as shown in Figure 2.

![Figure 2. The DH parameter diagram.](image)
There are $k$ out of $4m$ parameters of these links which can be adjusted and the value of which can be obtained by traverse. For one particular link (either length or angle), the designed value of the length of angle is $x$, of the tolerance is $y$, if the number of the traversing steps is $n$, which means the tolerance range is divided into $n$ parts and each of them will be calculated once, then the value of this link is set respectively as follow:

$$e = x - y + \frac{2yi}{n}, i = 0, 1, \ldots, n$$  \hspace{1cm} (1)

From Eq. 1, when the number of the traversing steps is taken as $n$, there are $n+1$ values of each link being calculated, so the total number of the traversing steps calculating the $k$ variable parameters will be $(n+1)^k$.

Due to the complex structure of the extending arm, taking boundary tolerance values for all the links does not always make the end be in the maximum error region, so the number of the steps should be a little bit larger to make the calculation more precise. But this may lead to exponential increase in total traversal steps, thus the number of the steps must be appropriately determined.

**The End Position, Attitude and Error Matrices Represented by DH Parameters**

Using the DH parameters for translation and rotation, the homogeneous transformation matrix between two adjacent coordinate systems can be obtained as follow:

$$A_{i-1} = \text{Trans}_{i}(d_i)\text{Rot}_{i}(\theta_i)\text{Trans}_{i}(a_i)\text{Rot}_{i}(\alpha_i)$$

$$= \begin{bmatrix}
    c_i & -c_\alpha c_i & s_i & a_i c_i \\
    s_i & c_\alpha & c_i & a_i s_i \\
    0 & -s_\alpha c_i & s_\alpha & a_i \\
    0 & 0 & 0 & 1
\end{bmatrix}$$  \hspace{1cm} (2)

In Eq. 2, $s_i = \sin \theta_i$, $c_i = \cos \theta_i$, $s_\alpha_i = \sin \alpha_i$, $c_\alpha_i = \cos \alpha_i$.

The end position and attitude matrix of the end relative to the base can be obtained by the multiplication of the homogeneous transformation matrices between coordinate systems:

$$T_0^m = A_0^1 A_1^2 \cdots A_{n-1}^n A_n$$  \hspace{1cm} (3)

Mark $V_{ij}$ as the DH parameter $\{d_i, \theta_i, a_i, \alpha_i\}$ of linkage $i$, so the end position and attitude matrix $T_0^m$ is a function of $V_{ij}$, recognized as $T_0^m = f(V_{ij})$. The partial derivative of the end position and attitude function to a certain DH parameter $j$ of a certain link $i$ is:

$$\frac{\partial f}{\partial V_{ij}} = A_{ij} A_{i+1}^2 \cdots \frac{\partial A_{n-1}^i}{\partial V_{ij}} \cdots A_n$$  \hspace{1cm} (4)

In which $i=1, 2, \ldots, m$; $j=1, 2, 3, 4$;

Mark the error of the DH parameter of linkage $i$ as $\{\Delta d_i, \Delta \theta_i, \Delta a_i, \Delta \alpha_i\}$. According to the function relationship between the end position and attitude and the adjustable variables and structural parameters of each linkage, the static error of the end position and attitude is:

$$\Delta f \approx \sum_{i=1}^{n} \left( \frac{\partial f}{\partial \theta_i} \right) \Delta \theta_i + \sum_{i=1}^{n} \left( \frac{\partial f}{\partial d_i} \right) \Delta d_i$$

$$+ \sum_{i=1}^{n} \left( \frac{\partial f}{\partial a_i} \right) \Delta a_i + \sum_{i=1}^{n} \left( \frac{\partial f}{\partial \alpha_i} \right) \Delta \alpha_i$$  \hspace{1cm} (5)

So the envelope scope formed by the $(n+1)^k$ values of $\Delta f$ will be the static error range of the end position and attitude of the extending arm.
Simulation Example

Accuracy Analysis Model and Tolerance Distribution Hypothesis of the Extending Arm

As shown in Figure 3, the extending arm model of satellite antenna has 3 rotating joints, respectively, the root expansion joint, the root rotary joint and the expansion joint between arms. The rotation angle of each joint is denoted respectively by $\theta_1$, $\theta_2$, $\theta_3$, with their rotation angle range $\theta_1 \in [0, 135]^\circ$, $\theta_2 \in [0, 89]^\circ$ and $\theta_3 \in [0, 176]^\circ$. The DH parameter is utilized to establish the transformation relationship from the base coordinate system $O_XO_YO_Z$ to the end coordinate system $O_XO_YO_Z$, and the amount of translation of each coordinate system in the figure is respectively set as $l_1=50\text{mm}$, $l_2=60\text{mm}$, $l_3=180\text{mm}$, $l_4=3400\text{mm}$, $l_5=80\text{mm}$, $l_6=105\text{mm}$, the DH parameters of the extending arm model are shown in table 1.

![Figure 3. The extending arm model of satellite antenna.](image)

Table 1. DH parameters.

| $i$ | $a_i$ | $a_j$ | $d_i$ | $\theta_i$ |
|-----|------|------|------|-----------|
| 1   | 0    | 0    | $l_1$ | 0         |
| 2   | 0    | -90  | $l_2$ | $\theta_1$|
| 3   | 0    | 90   | $l_3$ | $\theta_2$|
| 4   | 0    | 0    | $l_4$ | -90       |
| 5   | 0    | -90  | 0    | $\theta_3$|
| 6   | $l_5$| 90   | $-l_6$| 90       |

Table 2. Tolerance distribution of the extending arm model.

| Parameters | Designed value | Tolerance value | Variable types |
|------------|----------------|-----------------|----------------|
| 1 $l_1$    | 50mm           | 0.05mm          | Translation    |
| 2 $\theta_1$| $0/135^\circ$  | 0.02°           | Rotation       |
| 3 $l_2$    | 60mm           | 0.05mm          | Translation    |
| 4 $l_3$    | 180mm          | 0.1mm           | Translation    |
| 5 $\theta_2$| $0/89^\circ$  | 0.02°           | Rotation       |
| 6 $l_4$    | 3400mm         | 0.1mm           | Translation    |
| 7 $\theta_3$| $0/176^\circ$ | 0.02°           | Rotation       |
| 8 $l_5$    | 80mm           | 0.05mm          | Translation    |
| 9 $l_6$    | 105mm          | 0.05mm          | Translation    |
The tolerances are set and defined types for all the structures of the extending arm of the satellite antenna, as is shown in table 2.

**Analysis of End Error Range of the Extending Arm**

The nominal value of end position and attitude in expansion state can be obtained by recursive calculation using DH parameters in Table 1.

The nominal position vector of the end is: \([2663.162, 61.521, -2489.961]\) mm

The nominal attitude angle of the end is: \([49.23, -0.7071, -90.7071]\)^°

Taking all the dimensional tolerances into consideration, setting the traversal step as 1, the total steps \(2^9 = 512\), the maximum error range of end position is 1.446 mm and the maximum angular deviation of end attitude is 0.0447° by calculation, the error range of the end is shown in Figure 4.

![Figure 4. The end error range of extending arm.](image)

When the end position reaches the maximum error range, the end position vector of the arm is: \([2662.42, 61.599, -2491.2]\) mm

Compared with nominal position vector, the position error in each of \(xyz\) direction is: \([-0.743885, 0.0789668, -1.23783]\) mm

When the end attitude reaches the maximum angle deviation, the end attitude angle of the arm is: \([49.216, -0.747, -90.693]\)^°

Compared with nominal attitude angle, the angle deviation in each of \(xyz\) direction is: \([-0.0139, -0.0399, 0.0144]\)^°

According to the analysis method above, the engineering designer can find out in the early stage of design, the maximum error range of the end position and attitude of the extending arm under current tolerance design circumstance, and how much error does each component of the end position and attitude direction have when the maximum error range is reached, and then determine whether the tolerance of each link is designed properly or needs modification.

**Summary**

In this paper, modeling and analysis are carried out to solve the problem of the design of the tolerance of the extending arm. And the calculation of an extending arm model of the common 3-joint-type satellite antenna is used as an example to simulate and explain. The DH method is used to carry out the error range model of the end position and attitude of the extending arm, which can calculate the maximum error range and the posture deviation on each direction under the current designed tolerance, therefore can serve as the tolerance design reference of each link.
In summary, the DH method is used to analyze the accuracy of the end position and attitude of the extending arm, which can be utilized to optimize the tolerance, so that the extending arm can quickly reach the limited range of the tolerance, which has engineering guide significance to the accuracy design and precision assembly of the satellite antenna.

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