Evolution of the interacting viscous dark energy model in Einstein cosmology

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Abstract

In this paper we investigate the evolution of the viscous dark energy (DE) interacting with the dark matter (DM) in the Einstein cosmology model. Using the linearizing theory of the dynamical system, we find, in our model, there exists a stable late time scaling solution which corresponds to the accelerating universe, and we also find the unstable solution under some appropriate parameters. In order to alleviate the coincidence problem, some authors considered the effect of quantum correction due to the conform anomaly and the interacting dark energy model. But if we take into account the bulk viscosity of the cosmic fluid, the coincidence problem will be softened just like the interacting dark energy cosmology model. That’s to say, both the non-perfect fluid model and the interacting models of the dark energy can alleviate or soften the singularity of the universe.

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I. INTRODUCTION

It’s well known that recent data from Ia supernova (SN Ia) \cite{1} and microwave background (CMB) radiation \cite{2} have provided strong evidences for a spatially flat and accelerating universe in the present time. The origin of accelerating expansion is regarded that the universe is dominated by an exotic component with the negative pressure called ”dark energy” which constitutes 70 percent of the energy density of the universe, and dark matter about 26 percent. There are several candidates for dark energy: The first is the cosmological constant \cite{3}, and the second is the so-called dynamic candidates such as: Phantom \cite{4}, quintessence \cite{5}, K-essence \cite{6} and quintom \cite{7}. The difference of these candidates for dark energy is the size of the parameter $\omega_E$, namely the ration of the pressure and energy density of the dark energy. For quintessence, the state equation is given by the relation between the pressure $p_E$ and the energy density $\rho_E$, i.e. $p_E = \omega_E \rho_E$, where $-1 < \omega_E < -1/3$. The borderline case of $\omega_E = -1$ of the extraordinary quintessence covers the cosmological constant term.

The negative pressure of the dark energy may be the cause of the acceleration of the present Universe. However, the nature of the dark energy still remains a complete mystery. No more than five years ago, some physicists \cite{8} found that, if we only assumed the cosmic fluid to be ideal, i.e. nonviscous, it must bring out the occurrence of a singularity of the universe in the far future. There are two methods to modify or soften the singularity. The first is the effect of quantum corrections due to the conformal anomaly \cite{9}. The other is to consider the bulk viscosity of the cosmic fluid \cite{10}. The viscosity theory of relativistic fluids was first suggested by Eckart, Landau and Lifshitz \cite{11}. In recent years some physicists \cite{12} also took into account the bulk viscous cosmology. In this paper we consider that the bulk viscous dark energy is characterized by energy density $\rho_E$ and pressure $p_E$ as $p_E = \omega_E \rho_E + \beta \rho_E^\text{d}$.  

Up to now the model, which the dark energy is considered as the perfect fluid state, suffers the coincidence problem \cite{13}. That’s to say, why the dark energy (DE) and the dark matter (DM) are comparable in size exactly right now. In recent years some interacting DE models have been brought out to overcome the problem. In these models they all are assumed that there exists a nonzero interaction between DE and DM in the Universe and gauges DE transfers to DM which allows us to create an equilibrium balance in the evolution of the Universe, so that the density of DE keeps the same order as that of DM at late times.
Some authors [14] investigated dynamical behaviors of the dark energy models with only the dark energy linear equation of state interacting with dark matter in different cosmology models. They all found that the universe will enter an era dominated by dark energy and dark matter with interaction between them, and accelerate in late time under some proper parameters of the dynamical system. In this paper we focus on extending the equation of state of dark energy to nonlinear term, i.e. bulk viscous fluid, to investigate in what way the nonlinear term affects the evolution of the cosmology.

II. DYNAMICS OF THE INTERACTING VISCOUS DARK ENERGY MODEL IN THE FLAT FRW UNIVERSE

Base on the Einstein General Relativity theory, the standard Friedman equation acts as follows in the flat FRW universe

\[ H^2 = \frac{8\pi G}{3} (\rho_M + \rho_E), \]  

(1)

and the conservation relation of the cosmological total energy is

\[ \dot{\rho} + 3H (\rho + p) = 0, \]  

(2)

where \( G \) is the gravitational constant, and the total energy density is \( \rho = \rho_M + \rho_E \).

From a hydromechanical standpoint a generalization of cosmic theory so as to encompass viscosity is most nature. In recent years Misner et.al [10] interestingly investigated the viscous cosmology. In this paper we consider the general equation of state equation of the viscous dark energy as

\[ p_E = \omega_E \rho E + \beta \rho_E^d. \]  

(3)

Under considering the interaction term \( Q \) between the dark energy and dark matter, the evolution equations are

\[ \dot{\rho}_E + 3H[(1 + \omega_E) \rho_E + \beta \rho_E^d] = -Q, \]  

(4)

\[ \dot{\rho}_M + 3H \rho_M = Q. \]  

(5)

Differentiating Eq.(1), then putting Eq.(2) into it, we can get

\[ \dot{H} = -\frac{8\pi G}{2} (\rho_M + \rho_E + p_E). \]  

(6)
If we introduce the follow dimensionless variables
\[ x = \frac{8\pi G \rho_E}{3H^2}, \quad y = \frac{8\pi G \rho_M}{3H^2}, \quad \frac{d}{dN} = \frac{1}{H} \frac{d}{dt}, \]
(7)
where \( N \equiv \ln a \) is the number of e-folding to present the cosmological time. The interacting term \( Q = 3bH \rho_E \) is between the dark energy and dark matter. When the the coupling constant \( b \) is positive, it means that the dark energy converts into dark matter. Under these conditions we can construct the follow autonomous dynamics system by Eqs.(1), (3), (4) and (5) for the interacting viscous dark energy model in the flat FRW universe.

\[ x' = f(x, y) = -3(1 + b + \omega_E)x - \beta x^d + x[3y + 3(1 + \omega_E)x + \beta x^d] \]
\[ y' = g(x, y) = 3bx - 3y + y[3y + 3(1 + \omega_E)x + \beta x^d]. \]
(8) (9)

The dynamics is the general form \( X' = F(X) \), where \( X \) is the column matrix constituted by the auxiliary variables and the prime denotes derivative with respect to \( N = \ln a \). In order to analyze the stability of the dynamics system, we must linearly expand the dynamical system near the critical points due to the linearizing theory of dynamical system. So we can acquire the stability properties of the dynamical system from the eigenvalues of linearizing matrix.

In the following section we will focus on evolution of the dynamical system in the phase space and analyze its stability.

III. EVOLUTION AND STABILITY ANALYSIS OF THE AUTONOMOUS DYNAMICAL SYSTEM

In order to investigate the evolution and its stability of the given dynamical system in the phase space, we take the parameter \( d = 2 \) with lowest order nonlinear term of the viscous dark energy \( p_E \) to its density \( \rho_E \) for simplicity. We firstly solve the equations \( x' = f(x, y) = 0 \) and \( y' = g(x, y) = 0 \) to get the critical points:

Point I: \((x_C^I, y_C^I) = (0, 1), \)
and Point II: \((x_C^{II}, y_C^{II}) = \left( \frac{\beta - 3\omega_E - \gamma}{2\beta}, \frac{\beta + 3\omega_E + \gamma}{2\beta} \right), \)
where $\gamma = \sqrt{\beta^2 + 12\beta b + 6\omega_E + 9\omega_E^2}$.

For the Point I, it means that the universe is dominated by the dark matter. The eigenvalues of the linearizing matrix for Point I:

$$\lambda_1^I = 3, \lambda_2^I = -3(b + \omega_E).$$

(10)

We can see that when $b < -\omega_E$, the two eigenvalues are all positive, so the critical point I is saddle. When $b > -\omega_E$, so the critical point I is node. So we find Point I is an unstable critical, that’s to say, the universe only dominated by the dark matter is unstable.

For the Point II, the existence of the critical point II denotes to the era dominated both the dark energy and dark matter in the late-times of our universe, which is the same as classical Einstein cosmology [15]. In Fig.1 we showed the phase diagram of the autonomous dynamical system in the $(x, y)$ phase space with the parameters $\beta = 0.5, b = 0.8, \omega_E = -1.2, d = 2$.

In order to investigate the stability of the critical point II, we linearize the dynamical system near the critical point II as follows:

$$\delta x' = [-3(b + 1 + \omega_E) + 3y_C + 6(1 + \omega_E)x_C - \beta dx_C^{d-1} + \beta(d + 1)x_C^d] \delta x + 3x_C \delta y,$$

$$\delta y' = [3b + 3(1 + \omega_E)y_C + \beta dy_C x_C^{d-1}] \delta x + [-3 + 6y_C + 3(1 + \omega_E)x_C + \beta x_C^d] \delta y.$$ (11) (12)

From the above linearized equations, we can define the following linearizing matrix

$$M = \begin{pmatrix}
\frac{\partial f}{\partial x} |_{x_C, y_C} & \frac{\partial f}{\partial y} |_{x_C, y_C} \\
\frac{\partial g}{\partial x} |_{x_C, y_C} & \frac{\partial g}{\partial y} |_{x_C, y_C}
\end{pmatrix},$$ (13)

where the four elements of the matrix are

$$\frac{\partial f}{\partial x} |_{x_C, y_C} = -3(b + 1 + \omega_E) + 3y_C + 6(1 + \omega_E)x_C - \beta dx_C^{d-1} + \beta(d + 1)x_C^d,$$

(14)

$$\frac{\partial f}{\partial y} |_{x_C, y_C} = 3x_C,$$

(15)

$$\frac{\partial g}{\partial x} |_{x_C, y_C} = 3b + 3(1 + \omega_E)y_C + \beta dy_C x_C^{d-1},$$

(16)

$$\frac{\partial g}{\partial y} |_{x_C, y_C} = -3 + 6y_C + 3(1 + \omega_E)x_C + \beta x_C^d.$$ (17)
FIG. 1: The phase diagram of the interacting viscous dark energy model in the flat FRW universe with the parameters $\beta = 0.5, b = 0.8, \omega_E = -1.2, d = 2$. The point $C_{II}$ is the critical point of the dynamical system.

FIG. 2: Critical point coordinates $(x_C(\text{solid}), y_C(\text{dash}))$ for fixed $\beta = 0.5, b = 0.8$ and $d = 2$.

FIG. 3: Critical point coordinates $(x_C(\text{solid}), y_C(\text{dash}))$ for fixed $b = 0.8, \omega_E = -1.2$ and $d = 2$.

FIG. 4: Critical point coordinates $(x_C(\text{solid}), y_C(\text{dash}))$ for fixed $\beta = 0.5, \omega_E = -1.2$ and $d = 2$. 
The eigenvalues for the linearizing matrix of the dynamical system near the critical point II are:

\[
\lambda_{1}^{II} = \frac{1}{4\beta} (6\beta + 2\beta^2 + 18\beta b + 9\beta \omega_E + 9\omega_E^2 - 2\beta \gamma + 3\gamma \omega_E \\
- (36\beta^2 - 72\beta^2 b + 36\beta^2 b^2 - 36\beta^2 \omega_E + 36\beta^2 b \omega_E \\
- 108\beta \omega_E^2 + 18\beta^2 \omega_E^2 + 216\beta b \omega_E^2 + 108\beta \omega_E^3 + 162\omega_E^4 \\
- 36\beta \gamma \omega_E + 36\beta b \gamma \omega_E + 18\beta \gamma \omega_E^2 + 54\gamma \omega_E^3)^{\frac{1}{2}},
\]  
(18)

\[
\lambda_{2}^{II} = \frac{1}{4\beta} (6\beta + 2\beta^2 + 18\beta b + 9\beta \omega_E + 9\omega_E^2 - 2\beta \gamma + 3\gamma \omega_E \\
+ (36\beta^2 - 72\beta^2 b + 36\beta^2 b^2 - 36\beta^2 \omega_E + 36\beta^2 b \omega_E \\
- 108\beta \omega_E^2 + 18\beta^2 \omega_E^2 + 216\beta b \omega_E^2 + 108\beta \omega_E^3 + 162\omega_E^4 \\
- 36\beta \gamma \omega_E + 36\beta b \gamma \omega_E + 18\beta \gamma \omega_E^2 + 54\gamma \omega_E^3)^{\frac{1}{2}},
\]  
(19)

FIG. 5: Eigenvalues \(\lambda_1^{II}\) (left column) and \(\lambda_2^{II}\) (right column) of linearizing matrix of the dynamical system near critical point II for fixed \(\omega_E = -1.2\) and \(d = 2\). The second row is viewed through the \(b\) axial direction on the first row picture.

Because of the complexity of the above two eigenvalue expresses, we can not simply determine them positive or negative depending on the parameters. In this paper we mainly perform the numerical simulations of Eqs.(18) and (19) in Fig.5. From Fig.5 we can see that there are same region of parameters \(\beta, b\) where the eigenvalues \(\lambda_1\) and \(\lambda_2\) are both negative.
This means that critical point II is stable point as showed in Fig.1. Moreover if we investigate the stable point position \((x_C, y_C)\) with the parameters which showed in Figs.2-4, we can find that the universe will be dominated by dark matter both the interaction coefficient \(b\) and the viscous fluid coefficient \(\beta\) become stronger. At the same time, for the critical point II, we can see that the coordinates \((x_C, y_C)\) in the phase space is not vanished in its stable region, which tell us that the coincidence problem will be alleviated in the universe, and we also can see that the interaction term and the viscous dark energy will do the same effect on the alleviating the cosmological coincidence problem.

IV. CONCLUSIONS

In this paper we have investigated the evolution of the viscous cosmology model which dark energy interacts with dark matter. Using the linearizing theory of dynamical system, we found, in our model, there exists a stable late time scaling solution which corresponds to the accelerating universe, and we also found the unstable solution under some appropriate parameters.

We all know that, in order to alleviate the coincidence problem, some authors considered the effect of quantum correction due to the conform anomaly: such as dynamical Casimir effect with conformal anomaly, or dark fluid with conformal anomaly \([9]\). And some authors accounted some interacting dark energy models which can also soften the coincidence problem \([14]\). In this paper that we found that, if we take into account the bulk viscosity of the cosmic fluid, the viscosity softens the coincidence problem as the interacting dark energy cosmology models. That’s to say, both the non-perfect fluid model and the interacting models of the dark energy can alleviate or soften the singularity of the universe.

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