Boundary-induced wavelength selection in a one-dimensional pattern-forming system

Samuel S. Mao and John R. de Bruyn

Department of Physics
Memorial University of Newfoundland
St. John’s, Newfoundland, Canada A1B 3X7

Stephen W. Morris

Department of Physics and Erindale College
University of Toronto
60 St. George St.,
Toronto, Ontario, Canada M5S 1A7

(October 26, 2018)

Abstract

We have measured the stability boundary for steady electrically-driven convective flow in thin, freely suspended films of Smectic-A liquid crystal. The thinness and layered structure of the films supress two- and three-dimensional instabilities of the convection pattern. As the voltage applied across the film, or the length of the film, is varied, convective vortices are created or destroyed to keep the wave number of the pattern within a stable range. The range of stable wave numbers increases linearly with the dimensionless control parameter $\epsilon$, for small $\epsilon$, and the vortices always appear and disappear at the
ends of the film. These results are consistent with a mechanism for boundary-induced wavelength selection proposed by Cross et al. [Phys. Rev. Lett. 45, 898 (1980)].

47.20-k,47.54+r,47.64+a
The formation and dynamics of patterns in nonequilibrium nonlinear systems has been the subject of much recent study \[1\]. The general behavior of fully three-dimensional systems can be quite complicated, so there is substantial interest in systems which develop simpler, one-dimensional patterns which can be subjected to detailed experimental and theoretical study. Many such systems have been studied, of which Rayleigh-Bénard convection (RBC), in which a pattern of straight convective rolls develops in a thin layer of fluid heated from below, is perhaps the best known \[1,2\]. Although the fluid layer is three-dimensional, the flow pattern just above onset is characterized by a one-dimensional wave vector $k$ and one may speak of a one-dimensional pattern. An important issue in the study of patterns is that of wave number selection \[1\]. Typically, beyond the transition at which a pattern first appears, the base state will be unstable to perturbations within a band of wave numbers. The range of wave numbers over which the pattern is stable is limited to a narrower band of selected wave numbers by various instabilities. In the case of RBC, many of these instabilities involve perturbations in directions other than that of the original wave vector and lead to two- or three-dimensional flow patterns \[3\]. These can be partly supressed by confining the fluid layer to a container which is small in the direction parallel to the roll axis, although the flow velocity field will always be three-dimensional. Some wave number selection mechanisms are purely one dimensional: The Eckhaus instability \[4,5\] is a long-wavelength phase instability which results in the creation or loss of pattern units \(i.e.,\) of convective roll pairs in the case of RBC) in the interior of the experimental cell. Slow spatial variation of the control parameter through the value at which the pattern appears will select a unique wave number \[6,7\]. Another one-dimensional wave number selection mechanism results from the finite size of the experimental system \[8\]. For the case of RBC, the boundary conditions on the flow at end walls perpendicular to $k$ are predicted to allow the creation or destruction of rolls at the end walls, resulting in a change of wave number. Numerical simulations of RBC have shown roll creation and loss at end walls \[9,10\], but this mechanism has not been unambiguously observed in RBC experiments \[11\] due to the three-dimensional nature of the velocity field \[1\].
In this Letter we report measurements of the stability boundary of electrically-driven convective flow in very thin, freely suspended films of smectic-A liquid crystal [12–14]. Smectics-A have a layered structure [15] which makes it possible to make centimeter-size films, fractions of a micron thick, which are completely uniform in thickness. Flow between the layers is very difficult, and a film behaves as a two-dimensional isotropic fluid [13]. When a voltage $V$ is applied across the film, a convection pattern in the form of a one-dimensional array of counter-rotating vortices appears at a well-defined critical voltage $V_c$ [12]. The dimension of the smectic films in the direction of the axis of the convective vortices is extremely small. As a result, instabilities involving a bending or reorientation of the vortex axis are impossible, and we expect only one-dimensional processes to be involved in wave number selection. Indeed, we find that wave number adjustments occur via the creation or loss of vortices at the ends of the film. The shape of the stability boundary we measure is consistent with that expected from the boundary-induced wave number selection mechanism of Cross et al. [8].

Our experimental apparatus is similar to that used previously [12–14]. The long sides of the smectic-A film were supported by two 23 µm-diameter tungsten wire electrodes. The ends of the film were supported by thin plastic wipers which rested on the wires. The separation of the electrodes $d$ was adjustable; for the work reported here $0.66 \text{ mm} < d < 2 \text{ mm}$. One of the end wipers could be driven with a motorized micrometer, allowing variation of the film length $l$ in the range $0 < l < 30 \text{ mm}$. Films were made by bringing the two wipers together, placing a small amount of liquid crystal [16] on the place where they joined, and then slowly drawing them apart. Films consist of an integer number of smectic layers (one layer = 3.16 nm) and can be made uniformly thick. They can maintain their uniform thickness even in the presence of strong convection, and even when the length of the film is changed, as it is easier for the film to exchange material with the electrodes or the wipers than to form a partial smectic layer. A voltage is applied between the two electrodes, and steady convection starts at $V = V_c$ and persists up to a certain voltage, beyond which the flow becomes unsteady. The film holder was temperature controlled to $\pm0.1 ^\circ \text{C}$ over a
given run. All runs were performed at temperatures in the range 25 ± 1 °C, well below the smectic-A–nematic transition at 33 °C.

The smectic films were viewed through a microscope with a color ccd video camera. A small amount of incense smoke was admitted into the experimental housing and some smoke particles settled on the film. These particles were advected by flow in the film. Their motion was followed by shining a collimated beam of white light onto the film from below, and observing the scattered light.

The drawing procedure produces films of various uniform thicknesses $s$. Since $V_c$ and thus the dimensionless control parameter $\epsilon = (V/V_c)^2 - 1$ depend on $s$, it must be accurately measured. We determined the film thickness in two ways. Since $s$ is on the order of a wavelength of visible light, the films show bright interference colors when viewed in reflected white light. The color of the film can be calculated in terms of the CIE chromaticity diagram, and with practice the thickness of films up to about 100 layers thick can be determined to an accuracy of ±2 layers from their color. We also determined the thickness with an accuracy of ±1 layer from reflectivity measurements. These two techniques together permitted unambiguous determination of the film thickness to an accuracy of ±1 layer.

Our measurements of the stability boundary of the steady convective state were made using two techniques. In the first, a stable pattern was prepared by increasing the applied voltage to a chosen value above the onset of convection. Patterns with wave numbers $k$ close to $k_c$, the wave number at onset, were easily obtained by increasing $V$ slowly through the pattern onset, while a sudden jump from below to substantially above $V_c$ resulted in a pattern with $k$ different from $k_c$. Roughly speaking, jumping to values of $\epsilon$ in the range $1 \lesssim \epsilon \lesssim 4$ gave $k < k_c$, while jumping to $\epsilon \gtrsim 4$ resulted in a pattern with $k > k_c$. The voltage was then increased or decreased in small steps and the flow pattern monitored. The pattern required about 50 vortex turnover times, corresponding to about 1 minute, to equilibrate after a change in $V$. Typically, varying the voltage eventually led to a wave number adjustment through either the creation or the loss of a vortex, which always occurred at the ends of the film, not in its interior.
In the second type of measurement, a stable pattern was prepared as above. The film length was then changed at constant thickness by slowly (≈ 20 µm/s) moving the motorized wiper. This resulted in a stretching (for increasing \( l \)) or a compression (for decreasing \( l \)) of the vortex pattern, and periodically led to the creation or loss, respectively, of one or more vortices when \( k \) reached the stability boundary. As above, the creation and destruction of vortices occurred at the ends of the film [19,20].

Figure 1 shows the stability range of the steady vortex pattern measured by varying the voltage across the film at fixed film length. Data from thirty-six runs, using films with thickness between 2 and 45 layers, and with aspect ratios in the range \( 3 < l/d < 15 \), were combined to produce the stability boundary in Fig. 1. The data shown represent the maximum and minimum values of \( \epsilon \) at which a given wave number state was observed. For some films, wave number changes occurred inside the plotted boundary. This may be due to variations in the end conditions in different runs, as discussed below. No systematic variation in the position of the boundary with either film thickness or aspect ratio was detected.

Figure 2 shows the wavelength \( \lambda \) of the pattern in two experiments in which the film length was changed by moving the end wiper. As \( l \) is increased (solid circles), \( \lambda \) increases, to accommodate the change in length at fixed number of vortices. Eventually \( \lambda \) reaches a value above which the pattern is unstable. At this point one or more new vortices form at the end of the film, and the mean wavelength of the pattern decreases back to a stable value. When the film length is decreased (open circles) the opposite process occurs, with a vortex disappearing at the end of the film when \( \lambda \) decreases below a stability boundary. As shown in Fig. 2, the appearance and disappearance of vortices is always hysteretic.

The stability boundary determined from data such as that in Fig. 2 is plotted in Fig. 3, which shows the range of wave numbers observed at different values of \( \epsilon \). Data from 20 runs with films having \( s \) between 3 and 65 layers and \( 2 < l/d < 15 \) are shown. Again there were no systematic variations in the boundary with either film thickness or aspect ratio.

The stability boundaries for \( \epsilon < 2.5 \), measured with these two techniques, are shown superimposed in Fig. 4. They are consistent in their general shape over the whole range.
of measurements, and for \(k - k_c < 0\) and at low \(\epsilon\), the two boundaries agree very well. The quantitative differences in other parts of the diagram may be due to variations in the conditions at the ends of the films, as the small amount of liquid crystal remaining on the wipers after the film has been drawn is different in every run.

Cross et al. have studied the effects of endwalls on wave number selection in RBC [8]. The convective flow velocity must go to zero near an end wall, and as a result the amplitude of the convection pattern is weaker near the wall than in the bulk. In the theory of Cross et al., this implies that it is easier to create or destroy convection rolls at the ends of the experimental cell than in the bulk. They calculated the range of stable wave numbers for this situation and found it to be linear, i.e., \(\epsilon_B \propto |(k - k_c)/k_c|\), in contrast to the quadratic boundary found for the Eckhaus instability [1,4,5]. Thus at small enough \(\epsilon\), this boundary-induced wave number selection mechanism will result in a narrower stable band than would the Eckhaus instability. The slope of the lower-\(k\) boundary of the stable region must be negative, but that of the higher-\(k\) boundary can have either sign, depending, for RBC, on the fluid properties and on the exact nature of the boundary conditions at the end walls [8].

For small \(\epsilon\), the measured stability boundary is linear with negative slope for \(k < k_c\). The dotted curve plotted in Fig. 4 is a fit to both sets of data for \(\epsilon < 1\), \(k < k_c\) of the function \(\epsilon = a(k - k_c)/k_c + b((k - k_c)/k_c)^2\) with \(a\) and \(b\) parameters; it describes the data in this range well. Furthermore, in our experiments vortices form or vanish at the ends of the system. These results are consistent with what would be expected from the boundary-induced wave number selection mechanism [8], and inconsistent with what would be expected from the Eckhaus instability [1,4,5,7]. For \(k > k_c\) the measured stability boundary is less well-determined, but within our uncertainties it appears more linear than parabolic, and so is also consistent with the mechanism of Cross et al. [8].

Measurements of the velocity field in a convecting film indicate that the amplitude of the convection decreases near the ends, as expected from a no-slip boundary condition on the flow at the rigid end walls [22]. However, the actual end conditions in our experiments involve more than just the flow condition, as the electric field which drives the flow will be
modified by the presence of the ends of the film, and possibly also by the conductivity of the excess liquid crystal which remains on the wipers. In the context of the theory of Ref. [8], such details might be expected to affect the slopes of the branches of the stability boundary, but not their linear form.

In summary, we have observed wave number selection due to end conditions in electrically-driven convection in freely suspended smectic films. Because of the extreme two-dimensionality of the films, effects due to three-dimensional flow, or to bending of the convection roll axis, do not exist in this system. Wave number adjustments occur via the creation or loss of vortices at the ends of the film, and the band of stable wave numbers broadens linearly for small $\epsilon$. These results are consistent with those expected from the theory of Cross et al. [8], and not with the bulk Eckhaus mechanism.

J. de B. is grateful to M. Cross for a helpful discussion. This research was supported by the Natural Sciences and Engineering Research Council of Canada.
REFERENCES

[1] M.C. Cross and P.C. Hohenberg, Rev. Mod. Phys. 65, 851 (1993).

[2] G. Ahlers, in Lectures in the Sciences of Complexity, edited by D. Stein (Addison, Reading, MA, 1989), p. 175.

[3] F. Busse, in Hydrodynamic Instabilities and the Transition to Turbulence, edited by H.L. Swinney and J.P. Gollub (Springer, Berlin, 1984), p. 91.

[4] W. Eckhaus, Studies in Nonlinear Stability Theory (Springer, New York, 1965).

[5] P. Manneville, Dissipative Structures and Weak Turbulence (Academic, Boston, 1990).

[6] L. Kramer, E. Ben-Jacob, H. Brand, and M.C. Cross, Phys. Rev. Lett. 49, 1891 (1982).

[7] D.S. Cannell, M.A. Dominguez-Lerma, and G. Ahlers, Phys. Rev. Lett. 50, 1365 (1983); M.A. Dominguez-Lerma, D.S. Cannell, and G. Ahlers, Phys. Rev. A 34, 4956 (1986).

[8] M.C. Cross, P.G. Daniels, P.C. Hohenberg, and E.D. Siggia, Phys. Rev. Lett. 45, 898 (1980); J. Fluid Mech. 127, 155 (1983).

[9] J.C. Mitais, P. Haldenwang, and G. Labrosse, in Proc. VIII International Heat Transfer Conference, edited by C.L. Tien, V.P. Carey, and J.K. Ferell (Harper and Row, New York, 1986), p. 1545.

[10] W. Arter, A. Bernoff, and A.C. Newell, Phys. Fluids 30, 3840 (1987).

[11] B. Martinet, P. Haldenwang, G. Labrosse, J.C. Payan, and R. Payan, in Cellular Structures in Instabilities, edited by J.E. Wesfreid anad S. Zaleski (Springer-Verlag, Berlin, 1984), p. 33.

[12] S.W. Morris, J.R. de Bruyn, and A.D. May, Phys. Rev. Lett. 65, 2378 (1990); J. Stat. Phys. 64, 1025 (1991).

[13] S.W. Morris, J.R. de Bruyn, and A.D. May, Phys. Rev. A 44, 8146 (1991).
[14] S.W. Morris, Ph.D. thesis, University of Toronto, 1991 (unpublished).

[15] P.G. de Gennes, *The Physics of Liquid Crystals* (Clarendon, Oxford, 1979).

[16] The material used was 4,4′-n-octylcyanobiphenyl (8CB) doped with 7.5 ± 0.2 mM/l tetracyanoquinodimethan (TCNQ) to control the nature of the ionic species in the liquid crystal.

[17] E.B. Sirota, P.S. Pershan, L.B. Sorensen, and J. Collett, Phys. Rev. A 36, 2890 (1987).

[18] C. Rosenblatt and N. Amer, Appl. Phys. Lett. 36, 432 (1980).

[19] At higher voltages, close to the onset of the unsteady state, we occasionally observed vortices created or destroyed in the interior of the film. Away from that transition, the wavelength adjustments always took place at the ends.

[20] Techniques analogous to ours were used by Dominguez-Lerma *et al.* [7] to measure the stability boundary of Taylor-Couette flow; they observed the creation or loss of Taylor vortex pairs in the interior of the system, and their measured stability boundary was in quantitative agreement with the calculated Eckhaus stability boundary [21].

[21] H. Riecke and H.G. Paap, Phys. Rev. A 33, 547 (1986).

[22] S. Mao, unpublished.
FIGURES

FIG. 1. The stable wave number range for steady electrically-driven convection, measured by increasing (circles) or decreasing (triangles) the applied voltage.

FIG. 2. The pattern wavelength as function of length when the film length was varied. The wavelength plotted is the mean over the pattern, excluding the vortices at the ends of the film. The solid and open circles were obtained by increasing and decreasing \( l \), respectively. Arrows indicate wavelength changes caused by the creation or loss of vortices. a) \( s = 50 \) layers, \( \epsilon = 1.0 \); Single vortices are gained or lost at the arrows. b) \( s = 25 \) layers, \( \epsilon = 3.0 \); two vortices are gained simultaneously at the downward arrows; they are lost one-at-a-time at the upward arrows.

FIG. 3. The stability boundary determined by varying the film length at constant \( \epsilon \), as in Fig. 2.

FIG. 4. The stability boundaries from Figs. 1 and 3 plotted together for low \( \epsilon \). The symbols correspond to those in Figs. 1 and 3. The dotted line is a fit to the data for \( \epsilon < 1, k < k_c \) to the form \( \epsilon = a(k - k_c)/k_c + b((k - k_c)/k_c)^2 \) which gives \( a = -2.2 \pm 0.5 \) and \( b = 3.2 \pm 2.3 \).
