Applying the mathematical optimization model in water distribution management

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Abstract. This paper aims to analyse the concept of water distribution system management and the modalities of including the optimization tools for increasing the process efficiency. Water distribution systems are often unable to respond quickly and efficiently to consumers demands. Water is a non-renewable resource and should therefore be treated with special interest. Sustainable development and optimization are the steps to take. A mathematical optimization model is presented in the second part of the paper with explanations regarding the steps followed. The result demonstrates the efficiency, speed and accessibility of using graphical-analytical optimization methods in the field of water distribution system.

1. Introduction
Water is an essential source for life. Human society has developed a number of systems in an attempt to facilitate access to water, improve water quality and use it responsibly. There is a wide range of water that is used in urban agglomeration of the large cities: rainwater, bodies of water, drinking water, wastewater. The urban water system is a complex consisting of several subsystems: the water treatment station, the water storage, the water distribution system, the consumers (industrial and household), the waste water collection system [1]. These systems are generally equipped with intelligent SCADA elements. The first attempts of automation were made in the 1970s on water treatment systems in order to reduce operating costs. The systems were oversized and often their operation was not correlated with the load fluctuations. Nowadays, more emphasis is placed on efficiency in order to take over the peaks of loads that have increased with the population and the standard of living [2]. The same problem is encountered in water distribution systems. In this scientific work, it is desired to applying the mathematical optimization model in a water distribution system to determine the best parameters for optimal management. The mathematical model is based on the graphical-analytical optimization methods.

2. Water distribution management
Water distribution management is a challenging issue that is closely linked to global warming, quality of life, the water circuit in nature. Managing water distribution in big cities means managing all the functions of the water distribution system.

The water distribution system consists of pipes, valves, meters, pumps, storage tanks, etc. and has the role of transporting water from the source to the consumer.

At the consumer level the water must have the microbiological quality, the required flow and pressure. Water distribution is the “urban metabolism” [3]. All over the world, efforts are being made to reduce losses, adapt to the demands of people and ensure the reliability of water infrastructure. The
concept of water sensitive urban design aims to incorporate all the water systems that serve a city, focusing on stormwater management and domestic wastewater [3].

Water losses can have different causes: system leaks, system damage, unauthorized consumers (consumers that are not counted) or apparent losses. Leakage volumetric losses represent over 70% of total water losses [4]. Some of the main causes that can lead to the appearance of volumetric losses are: excessive pressure in certain points of the network, mechanical factors that may occur (road traffic loads, accidental hit), the irresponsible operation of the network or the existence of an old network.

The most efficient way to control the losses in the system is with the help of pressure monitoring indifferent points of the network [5]. In such a system the pressure anomalies are analysed. According to the severity of the problems losses in the water system are classified as follows [6]:

- background losses (low flow, they are very hard to detect using classic methods like acoustic detection),
- unreported losses (moderate flow and long duration, their identification requires time),
- reported losses (high flow losses and short duration, these are visible problems).

Long-term constructing transitions of water distribution management require a correlation between society, urban development of the region and the structure of the water system. These three factors are interdependent, the modification of one leading to implicit modification of the others.

In [7] an overall approach is proposed based on three modules:

- Social Transitions Module is related to the stormwater management and its role is to explore the possible outcomes and to generate from a large portfolio a set of typical pathways,
- the Urban Development Module is an agent-based modelling solution that aims to simulate the population density evolution on certain urban areas,
- biophysical module is related to design and performance of the urban water system.

This approach based on three basic modules has a high degree of accuracy in optimizing long-term urban water systems. The algorithm underlying these models is a mathematical optimization algorithm. Optimization is the act of obtaining the best result under given circumstances. In practice, any system or installation must be controlled and management decisions are required in several stages. The ultimate goal of all these decisions is to minimize the effort required and maximize the desired benefit. Since the required effort or desired benefit in any practical situation can be expressed as a function of certain decision variables, optimization can be defined as the process of finding the conditions that give the maximum or minimum value of a function. Optimization can also be defined as choosing, from the multitude of possible solutions to a problem, the one that is best in relation to a predefined criterion.

This definition implies the existence of the following components [8-9]:

1. a technical problem consisting of the mathematical calculation of a solution;
2. existence of several solutions for the same problem;
3. a criterion for selecting the optimal solution.
4. establishing the optimal solution.

There is no single method available for efficiently solving all optimization problems. Therefore, a series of optimization methods have been developed to solve different types of optimization problems. Optimal search methods are also known as mathematical programming techniques and are generally studied as part of operational research. Linear programming takes data from real-world problems and transposes them into a series of mathematical formulas. Such a problem is characterized by a linear objective function of the form:

\[ f(x_1, x_2, ..., x_n) = c_1x_1 + c_2x_2 + ... + c_nx_n \]  
(1)

and a number of constraints represented by a system of linear equations of equality type. The process of transposing constraints from the real world into linear functions is an essential step in solving linear programming problems [10-11]. A constraint is restrictive if the right and the left member are equal when the decision variables are replaced by the optimal values. If the two members of the constraint are not equal after the decision variable is replaced, constraint is not restrictive.

In order to solve such a problem, the mathematical function must have a certain form, known as the standard form.
This form has the positive variables and constraints are equal which have the positive terms on the right side. The general mathematical form of a linear programming problem must comply with the restrictions:

\[ \sum_{j=1}^{n} a_{ij}x_j = d_i, \, i = 1, \ldots, m \]  

(2)

\[ x_j \geq 0, \, j = 1, \ldots, n \]  

(3)

If the constraint (2) is not an equality, a variable will be added or subtracted (it is known as a programmable element) so that the mathematical relation is brought in the form of an equality. The importance of the optimization problem lies in the fact that it corresponds to the purpose general optimization of the used resources in order to meet certain objectives [12]. The numerical solution of linear programming problems is based on the fact that the solution of the problem is on the border of the admitted solutions.

The simplest method of solving is the geometric or graphical-analytical method. This method is limited to solve problems that have less than or equal to three decision variables. On the other hand, the simplex method is considered one of the oldest solving methods and it is based on George Dantzig's algorithm [13], whose complexity increases directly in proportion to the number of variables associated with the problem.

### 3. Case study

An application of the linear programming is proposed as case study considering the problem of optimizing the benefit in the case of an urban water system.

**Problem description:** drinking water in the urban system considered is provided from two sources: raw water and used water. The quality assurance of the drinking water is obtained through by a complex treatment process consisting of a series of treatment steps:

- **mechanical treatment step,** with the role of removing the floating solids in water, here is the disintegration, separation of fats and decanting of water;
- **chemical treatment step,** with the role of removing some of the impurities found in water, most often using the coagulation-flocculation process, in which the basic reactant is the chlorinated ferrous sulphate FeSO4;
- **biological treatment step,** which uses cultures of microorganisms to transform hazardous substances into degradation products and sludge that does not affect the environment;
- **a tertiary treatment** has the role of removing excess substances and disinfecting the water.

In this case study, the chemical step of water treatment for an urban area is considered. The treatment consists of introducing of the chemical substances as follows:

- **Raw water (C1) treatment:**
  
  S1-150 g/m³; S2-250 g/m³; S3-125 g/m³; S4-150 g/m³.

- **Used water (C2) treatment:**
  
  S1-300 g/m³; S2-500 g/m³; S3-400 g/m³; S4-150 g/m³.

From the infrastructure point of view, the water treatment capacities for each substance are:

S_{m1} = 90 kg, S_{m2} = 180 kg, S_{m3} = 150 kg, S_{m4} = 60 kg. The cost of producing the drinking water is: 4 lei / m³ for water providing from used water, and 2 lei / m³ for water obtained from raw water. The optimization problem is defined: how to get the maximum benefit for obtaining drinking water from the two categories of water sources, at a charge in the given circumstances.

**Problem solving** includes the following solving steps are applied:

#### 3.1. Initial data centralization

The problem data are centralized in table 1.
### Table 1. Quantities and capacities of chemical treatment substances needed for obtaining drinking water from raw and used water for a cycle of treatment

| Treatment restrictions | Raw water (C1) (g/m³) | Used water (C2) (g/m³) | Capacity (kg) |
|------------------------|-----------------------|------------------------|---------------|
| S1                     | 150                   | 300                    | 90            |
| S2                     | 250                   | 500                    | 180           |
| S3                     | 125                   | 400                    | 150           |
| S4                     | 150                   | 150                    | 60            |
| Water selling price[lei/m³] | 2                    | 4                      |               |

3.2. Setting variables
Variable are: $x$ for raw water treatment and $y$ for used water treatment.

3.3. Establishing the mathematical model of the restrictions and the objective function
The mathematical model which expresses all restrictions described in table 1 consists of the following system:

\[
\begin{align*}
150x + 300y & \leq 90,000 \\
250x + 500y & \leq 180,000 \\
125x + 400y & \leq 150,000 \\
150x + 150y & \leq 60,000 \\
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

Based on the given data, the objective function is written as:

\[
f = 2x + 4y
\]

3.4. Solving by graphical method
The graphical representation was done using software from the Autodesk package. The package contains software services for architecture, engineering, manufacturing and education industries. In figure 1 the graphical representation of lines D1, D2, D3, D4, corresponding to the relations (4)-(7) is shown.

3.5. Establishing the admissible solution area
The set of admissible solutions is determined taking into account the conditions of inequalities in (4)-(9). Shaded area of figure 2 represents the admissible solution.

3.6. Obtaining the graphical solution
The mathematical function

\[
(d_0) = -\frac{2}{4}x = -0.5x
\]

is represented graphically with the help of the extreme points Also, the values of the intersection points A, B, C, in figure 3, are determined using the graphical method.

3.7. Establishing the values of objective function for ABCD points
The following values for the objective function (4) are obtained, corresponding to the intersection points A, B, C:
For point A:
\[ f(0,300) = 2 \times 0 + 4 \times 300 = 1200 \] (12)

For point B:
\[ f(200,200) = 2 \times 200 + 4 \times 200 = 1200 \] (13)

For point C:
\[ f(400,0) = 2 \times 400 + 4 \times 0 = 800 \] (14)

3.8. Solving the system:
\begin{align*}
150x + 300y &= 90000 \\
150x + 150y &= 60000
\end{align*}
(15) (16)

it is possible to determinate the solution: \( x=200 \) and \( y=200 \) which means the best optimization plan is \( C1=200 \text{ m}^3 \) and for \( C2=200 \text{ m}^3 \).

3.9. Establishing the benefit and the optimum planning
The benefit is determined:
\[ f = 2x + 4y = 2 \times 200 + 4 \times 200 = 1200 \] (17)
and the optimum planning is obtained:

\[ S1: 150 \times 200 + 300 \times 200 = 90\,000 \text{ g/m}^3 \]
\[ S2: 250 \times 200 + 500 \times 200 = 150\,000 \text{ g/m}^3 \]
\[ S3: 125 \times 200 + 400 \times 200 = 105\,000 \text{ g/m}^3 \]
\[ S4: 150 \times 200 + 150 \times 200 = 60\,000 \text{ g/m}^3 \]

4. Conclusions

For an efficient management of the water treatment system presented in the studied case, the parameters obtained from the calculations must be respected. This example demonstrates the simple and efficient way by which short and very short-term optimizations can be made without the use of complex computing systems. The graphical methods of solving optimization problems can be adapted to a number of problems. A disadvantage of this method is that the mathematical formulation of the problem must be defined as a system of linear functions.

Although this method has limitations, it can be used in operation without investing a considerable amount of money in different software and hardware solutions. The graphical method for solving linear problems is efficient and simple to be used but it is only a first step that forms the basis of an efficient management system of water resources in urban agglomerations.

To be able to maintain a sustainable evolution of water usage we must integrate modules like Societal Transitions, Urban Development, and Biophysical which will help to predict the future challenges.

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