Development and advancement in the wheel-rail rolling contact mechanics

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Abstract. Realistic modelling of wheel-rail contact is necessary for true analysis of railway vehicle system. This study reviews the rail-wheel contact models with their related computer software programs in detail. The phenomenon of creepage and creep-force is studied in detail in this paper. An insight is presented to linear and non-linear model developed by Carter, Kalker, Johnson & Vermeulen, Halling, Haines and Ollerton and related programs based on their theories. Advanced wheel-rail contact theory is also reviewed in this paper.

Keywords: Longitudinal creepage, Lateral creepage, Contact patch, Spin, Adhesion, Wheel-rail interaction.

1. Introduction
A fundamental element of rail vehicle curving and support system is its wheel axle. The behaviour of a railway vehicle system is highly influenced by its wheel-rail interactive forces. The wheel and rail profile influence the creep-adhesion-wear characteristics and determines the vehicle performance. The atmospheric conditions, moisture, humidity, oil, dirt, snow and such other factors also, influence adversely, the wheel-rail interaction and affect the performance indices of the vehicle. Wheel and rail physical and mechanical properties are also very crucial in this context. The ever increasing normal axle loads of rail vehicles cause problems with the track and vehicle components and affect the vehicle performance. The wheel-rail interaction is indeed a complex problem. The problem of rolling contact for two bodies was first solved by Hertz in 1895. Both bodies were considered elastic half spaces. Continuum rolling contact theory was stated by Carter in 1926 [1]. Carter considered the wheel as a cylinder along with the rail as an infinite half-space. This theory was two dimensional and accurate solution was obtained. Considering wheel-rail interaction solution of three-dimensional rolling contact was required. For solving this problem, numerical solutions of rolling contact was essential. This was solved first by Johnson [2, 3] and Haines and Ollerton [4]. Later on Johnson and Vermeulen [5] presented the approximate solution of rolling contact considering longitudinal as well as lateral creepages. However in order to deal with most general solution the contact patch needed to be discretized. The integral equations then needed to be solved by boundary element method. The solutions of these problems were achieved by Kalker [6-10]. The Kalker’s program CONTACT deals the arbitrary surfaces of two bodies in contact that result the non-elliptic contact patches. Longitudinal,
lateral and spin creep are considered in CONTACT. Linear and non-linear, steady-state as well as non-steady state solutions can be achieved by CONTACT. However due to half space assumption and higher computational time there are limitations associated with CONTACT.

Kalker’s program FASTSIM was able to overcome these limitations of CONTACT where the contacting bodies are not modelled as elastic half-spaces but an elastic foundation model is considered. For wheel-rail contact at wheel flange and at rail gauge corners, ratio of contact ellipse semi axis along and across the track (a/b) may be as high as 16 and modifications in Kalker’s ‘numerical theory’ which is valid up to contact ellipse ratio of 2 is required. This issue to an extent was resolved in Kalker’s ‘exact theory’ and modified program DUVOROL which is valid up to contact ellipse ratio of 10. However few analyses e.g. wheels moving on corrugated rails requires a non-steady state contact mechanics and further modification in CONTACT was essential. Grob-Thebing [11] considered the solutions of non-steady state rolling contact. Grob-Thebing [11] modified Kalker’s program CONTACT.

The situation of high fluctuations of creepages i.e. curve squealing was analysed by Fingberg [12], who combined linear, steady state contact mechanics and a decreasing creep curve and Periard [13], who modified FASTSIM. However these attempts were inadequate for curve squealing analysis. Researchers [14] have analysed different aspects of wheel-rail contact i.e. kinetic coefficient of friction, effect of temperature on creep, influence of micro roughness, wheel-rail contact with FEM. Wheel-rail interaction is an endless field of research.

2. Wheel-rail interaction and creep

Creep appears if two rigid bodies are under axial compressive load and permitted to roll over one another. Contact patch is obtained around contact point. Creep forces are generated as the wheel-rail surfaces have different strain rates. Wheel and rail are subjected to heavy loads and rigid body assumption is not possible hence pure rolling is not achieved. Normal load at wheel-rail surface develops local elastic deformation at contact region so called contact patch. If wheel-rail surface are perfectly smooth and invariable curvature is achieved contact region, Hertz theory is valid and contact patch found to be an ellipse, and normal pressure at contact region considered to be distributed semi-ellipsoidal.

Longitudinal force creates divergence with a pure rolling motion. Ratio of difference relative speed for wheel and rail with wheel forward velocity is termed as longitudinal creepage. Lateral creepage is ratio of change in lateral velocity and wheel forward velocity. Ratio of difference in wheel and rail circular speed perpendicular to contact patch with wheel forward velocity is defined as spin creepage. Provided the longitudinal creepage becomes negligible, it has to be accounted with elastic strains in contact region. When wheel rolls over rail, unstrained material approaches contact patch towards its leading edge. As material approaches towards contact patch, wheel-rail relative speed equates strain difference rate and surfaces are interlocked. Longitudinal tangential stress rises linearly with leading edge distance. In same way lateral creepage increases shear stresses. Longitudinal creepage along with lateral creepage develops forces and are function of respective creepage. In presence of spin, elastic strain pattern is relatively complex. For this condition, when material approach towards contact region wheel-rail relative velocity is linear function of separation from core of contact region thus strain field is curvilinear. This result to raise lateral force and subsequently spin moment. Increase in creepages together with spin increases shear stresses, and as shear stresses increase beyond normal force × coefficient of friction, slipping occurs. If region of adhesion near contact patch where surfaces are interlocked gradually reduces as creepage increases. For very large creepage, slipping occurs on entire contact patch and creep force is normal force × coefficient of friction. When longitudinal together with lateral creepage occurs at same time then with moderate creepage creep force may be superposed, however with high creepages in region of sliding, direction of tangential stresses is opposite to net relative speed. Outcome is therefore that creep forces are function of lateral-longitudinal and lateral creepages-spin together. Although lateral forces are function of spin with moderate spin, for higher spin sliding occurs on a significant portion of contact patch and lateral force becomes negligible. Figure 1 describes the creep force and creepage relation. Linear creep theory
describes that creep forces and moments varies linearly with relative linear and angular speeds of wheel-rail at contact point. These non-dimensional ratios of linear speeds are defined as lateral or longitudinal creep on the basis of direction of relative linear speed. Directions of longitudinal lateral and spin creep on wheel are shown in figure 2.

3. Wheel-rail contact theories

3.1 Kalker’s Linear Theory

With extremely low magnitude of longitudinal, lateral creepage and spin (i.e. $\xi^l$, $\xi^l$ and $\xi^w$) slip region is negligible so that its influence may be ignored. Adhesion zone, then seal the whole contact region.
Kalker’s linear longitudinal, lateral creep force & spin creep moment-creepage [8] relationship are expressed as

\[ F_x = -f_{33} \xi_3 \]  
(1)

\[ F_y = -f_{11} \xi^y - f_{12} \xi^\psi \]  
(2)

\[ M_z = f_{12} \xi^y - f_{22} \xi^\psi \]  
(3)

Note that in Equations 1, 2, 2 and 3 are the creep coefficients, which are defined by Kalker as

\[ f_{11} = (ab)GC_{22}, \quad f_{12} = (ab)^{3/2}GC_{23} \]

\[ f_{22} = (a/b)^2GC_{33}, \quad f_{33} = (ab)^{3/2}GC_{11} \]  
(4)

where

a, b = semi-axis of ellipse  
G = shear modulus of rigidity  
\( C_{ij} \) = creepage and spin coefficients

In linear analysis of rail vehicle on tangent track, it is usual to assume that the normal force is constant. However, in curving and when the flange comes into contact, the dependence of the normal force on the wheel axle and other component motions must be included.

Although laboratory results show good agreement with the linear theory, experiments with contaminated surfaces and full-scale railway wheel axles indicate that the creep coefficients may be as much as 50% less than those given by the theory. These discrepancies are because of surface deterioration of the surfaces. Although there is not yet any analytical treatment that accounts for these effects, empirical evidence indicates that Kalker’s coefficients should be multiplied by the factor \( f/0.6 \), where \( f \) = actual coefficient of friction, and 0.6 = friction value factor used by Kalker in his calculations.

Kalker’s linear theory is extensively utilized to evaluate rail vehicle performance indices i.e. eigenvalues, vehicle ride [15-36].

3.2. Johnson and Vermeulen’s Nonlinear Theory

Kalker’s linear creep theory serves well in most analyses. However, a nonlinear creep force theory that consider the limitations provided by coefficient of friction for limitations provided by coefficient of friction for rolling without relative spin among two bodies has been developed by Johnson & Vermeulen [5]. Johnson & Vermeulen [5] expanded the concept of arbitrary smooth half spaces to pure creep excluding spin creep. Contact patch transferring a tangential force was unequally broken in two separate regions i.e. slip region and adhesion region. Adhesion area was considered as an ellipse and it was in contact with leading edge of contact ellipse. Each ellipse is acted through a semi elliptical tangential traction, and total tangential traction is obtained by subtraction. The resulting tangential force is formulated as follows.

\[ B = \int_0^{\pi/2} \cos^2 \theta (\sqrt{1-k^2 \sin^2 \theta})^{-1} d\theta \]

\[ D = \int_0^{\pi/2} \sin^2 \theta (\sqrt{1-k^2 \sin^2 \theta})^{-1} d\theta \]  
(Complete elliptical integral, \( k \leq 1 \))

\[ C = \int_0^{\pi/2} \cos^2 \theta \sin^2 \theta (\sqrt{1-k^2 \sin^2 \theta})^{-1} d\theta \]  
(5)

and
\[ \phi = B - \sigma (D - C) \]
\[ \psi_i = B - \sigma (a^2 / b^2)C \quad (\text{when} \ b \geq a, \ k = \sqrt{1 - a^2 / b^2}) \]
\[ \phi = [D - \sigma (D - C)](b / a) \]
\[ \psi_i = [D - \sigma C](b / a) \quad (\text{when} \ b \leq a, \ k = \sqrt{1 - b^2 / a^2}) \]

where
\[ \xi = \text{normalised longitudinal creepage:} \ (\pi abG \xi) / fN \phi, \]
\[ \eta = \text{normalised lateral creepage:} \ (\pi abG \xi) / fN \psi, \]
\[ \tau = \sqrt{\xi^2 + \eta^2} \]
\[ N = \text{normal force} \]
\[ G = \text{shear modulus of rigidity} \]
\[ \xi, = \text{longitudinal creep} \]
\[ \xi, = \text{lateral creep} \]
\[ \sigma = \text{Poisson’s ratio} \]
\[ f = \text{coefficient of friction} \]

If \( F(F_x, F_y) \), is total tangential force, then
\[ \frac{F}{fN} = \begin{cases} 
\frac{1}{2}[(1 \tau)(\frac{1}{3} \tau^3 - 1)(\xi + \eta)] \quad \text{for} \quad |\tau| \leq 3 \\
-(1 \tau)(\xi + \eta) \quad \text{for} \quad |\tau| \geq 3 
\end{cases} \quad (7) \]

Since \( \sigma \neq 0 \) and usually \( \phi \neq \psi \), the total force is not totally obtained in creep direction. Johnson and Vermeulen’s theory, a modification of Carter’s theory, is therefore limited to state for pure longitudinal & lateral creep, i.e., spin = 0.

3.3. Strip Theory of Halling and of Haines & Ollerton
Halling [37] and Haines and Ollerton [4] evaluated the rough estimation of elliptical contact with pure longitudinal creepage. The contact area has been broken with several strips parallel to the direction of rolling, and the strip has been evaluated on the basis of extension of Carter’s two-dimensional theory. Interface among adjacent strip has been neglected totally. Based upon the theory [4, 37], with every slice that has constant \( y \), Carter’s solution is true and is independent at different values of \( y \). Theory [4, 37] was validated through a field test using a photo elastic stress technique and known as strip theory which is valid for pure longitudinal creepage.

Kalker [7] modified the strip theory for normal state and incorporated longitudinal-lateral creepages with a small spin. Halling [37] and Haines and Ollerton [4] solutions are restricted for long-contact ellipses along lateral direction for moderate spin. Thus, this theory is not so popular for solution of problem of rail wheel interaction. The strip theory evaluated the exact shape of slip and adhesion area.

3.4. Kalker’s Non Linear Theories
Most exact estimation of nonlinear interaction of wheel-rail problem is that of Kalker [8]. Kalker’s linear theory found limited to moderate creepage where creep coefficients are function of contact ellipse dimension, normal load along with elastic constants. Moreover Kalker determined the high creepage with numerical theory. Kalker obtained the solutions with non-dimensional mode. For high
creep, the force elements depend on respective creepage components. At extremely high creepage, the whole contact assumes condition of pure sliding. Moreover wheel-rail surface deterioration cannot be ignored which develops very fine boundary layer among contact region, which reduces available friction. Several researchers analysed with uncontaminated, fresh surfaces $\mu$ is nearly 0.6 therefore Kalker’s analytical theory is validated. The results obtained by Kalker’s non-linear theory is not valid for $a/b$ (contact patch ratio) greater than 2. Several researchers evaluated that if contact lies among wheel-rail on wheel flange together with rail gauge corner, $a/b$ is nearly 16. DUVOROL program is based on Kalker’s “Exact Theory”, which is able to analyse the rail-wheel interaction for larger values of $a/b$. This program is able to evaluate non dimensional creep force-creep value up to $a/b$ of 10.

A fast method of handling the creep force-creepage data was provided by computer code “NUCARS”. NUCARS together with VAMPIRE Program was extensively used by the Association of American Railroads and by British Rail. Along with “Exact Theory”, Kalker has also formulated “Simplified Nonlinear Theory”. These two theories are different for their assumptions related to the shear displacement-stress relationship together with normal stress pattern over contact patch. Abovementioned assumptions minimises simulation time however limited for problem of identical materials and shapes. FASTSIM program is based on the “simplified theory” [9]. This computer code was used by several researchers to evaluate creep forces for all wheel-rail contact. Same approach was also adopted by the program MEDYNA. FASTSIM takes less computational time than DUVOROL.

Kalker’s simplified theory for Non-Hertzian contact was embodied in program “CONTACT” [8]. In CONTACT longitudinal, lateral and spin creep can be considered. CONTACT can provide linear and non-linear, steady-state as well as non-steady state solutions of the problems. It is therefore CONTACT is used for many practical problems. However CONTACT has certain limitations that the computational time associated with CONTACT is so high and is difficult to use online. Secondly in CONTACT both wheel together with rail are consider ed an elastic half spaces, assumption is justified as long as contact dimensions are small as compared with the curving radii. One another limitation that CONTACT is restricted to elastic problems only and therefore is not able to deal with plastic deformation in the rail head.

4. Development of wheel-rail contact problem with boundary element and finite element method

4.1. Wheel-rail contact with boundary element method

Transient state theory is used when $L$ (figure 3) approaches the value $a$ (i.e. contact patch semi minor axis length). This is true for corrugation analysis where $L$ can be as small as 2 cm whereas the radius $a$ is between 0.5 and 1 cm. An even more critical situation is curve squealing. This wavelength $L$ becomes smaller than the contact radius $a$ (figure 3) [11].
Figure 3. Characterization of transient state theory [11].

For rolling noise analysis or when a wheel is moving over moderately corrugated rail, it is possible to use linear transient state theory. All fluctuations of state variables with respect to a reference state are assumed as negligible. This case has been considered by Grob-Thebing [11], who modified Kalker’s program CONTACT. Linear, steady state contact mechanics can be described by creep coefficients $C_{ij}$ which have been calculated by Kalker [8].

$$
\begin{align*}
\begin{bmatrix}
F_x \\
F_y \\
M_{sp}
\end{bmatrix} &=
\begin{bmatrix}
C_{11} & 0 & 0 \\
0 & C_{22} & C_{23} \\
0 & -C_{23} & C_{33}
\end{bmatrix}
\begin{bmatrix}
\xi^x \\
\xi^y \\
\xi^{sp}
\end{bmatrix}
\end{align*}
$$

(8)

with $c^2 = a_o b_0$

$a_o$ = semi major axis of ellipse, $b_o$ = semi minor axis of ellipse

For linear, transient state theory along with harmonically varying creepages a similar law can be formulated. Instead of creep forces and creepages now the amplitudes $\Delta F_x$, etc of harmonically varying creep forces can be calculated from the amplitudes of harmonically varying creepages $\Delta \xi^x$, etc.

The creep coefficients are complex values and depend on $a / L = a \Omega / 2 \pi \xi^x$. The creep coefficient therefore is a frequency response function which can be represented in two ways. The first way is complex plane, the alternative way is amplitude together with phase.

It has been tried several times in the past to validate the creep-force/creep relations which were calculated within the frame of Carter’s and Kalker’s theory [8]. The most comprehensive summery can be found in a BR report [38]. In recent years new experimental results have been obtained due to the development of modern drive systems. The experimental results show strong deviations from the theoretical curves with high creepages. In figure 4 three main deviations can be observed:

A. The measured curves show a large dispersion in contrast to the smooth curve of the theoretical relationship.

B. A higher creepages is obtained for the maximum value of the measured traction force than predicted in theory.

C. Beyond the maximum the measured traction forces decrease whereas a constant value is obtained in the theory.

It should be mentioned that the curves cannot be compared quantitatively:

The theoretical curves are valid for contact point quantities whereas the experimental results have been measured at the axle. However, qualitatively the deviations remain the same, even if the same
quantities are used for theory and measurement. There is a broad interest in the locomotive industry in predicting the creep force/creep behaviour for high creepages.

Figure 4. Comparison between (a) the experimental measurements of traction forces (b) the tangential contact force of Kalker’s theory [8].

The idea, that the creep force decreasing for high creepages can be modelled by evaluating coefficient of static together with kinetic friction that depend on sliding velocity [39]. Later on Nielsen and Theilor [40] stated that Ohyama’s analytical considerations were incorrect and it is insufficient to introduce a static and a kinetic coefficient of friction at high creepages. In both cases Carter’s solution is obtained.

A decrease in creep force/creep curve is obtained even when coefficient of friction alone depends on the sliding speed. Reason of reduction in friction with sliding speed remains the issue of interest for researchers. Rick [41] was first one who assumed that friction depends on surface temperature. He stated that coefficient of friction is proportional to the shear yield stress as friction follows theory of galling which is progressive welding and destroying of junctions; the yield shear stress itself depends on the local temperature. The temperature can be determined from frictional power in contact patch.

Ertz and Knothe [42] analysed that traction forces are function of creepage together with wheel velocity. This was due to the fact that frictional power increases with sliding speed and sliding speed is product of creepage and vehicle speed.

Even new wheel-rail surfaces have roughness. Greenwood [43] investigated this issue using a stochastic approach. Several researchers in the progress [44-46] analysed the influence of surface roughness on normal and tangential contact problems using deterministic approach.

Knothe and Theiler [47] investigated that it is impossible to measure the wheel profile with three-dimensional accuracy. Therefore the wheel profile is Fourier analysed and Fourier terms with wavelengths larger than a filtering wavelength were considered. It was analysed [47] that the maximum contact stresses for rough surfaces are several time higher than the Hertzian normal pressure, though a run in surface has been for analysis.

The non-steady state normal and tangential contact problem has to consider two rough surfaces that are sliding over each other. This problem has not been solved even for two-dimensional case. From these analysis it is found that slope of creep force/creep curve i.e. creep coefficient decreases for rough surfaces. As with normal contact, a relevant part of the deformation energy is concentrated in the boundary layer. The rough surface behaves like an elastic layer attached to half space.

4.2. Wheel-rail interaction with FEM

Problem of rolling contact is nonlinear and researchers developed large number of computer codes using finite element method considering all nonlinearities [48-60].

Disintegration of rigid body kinetics with translational kinetics of rail is shown in figure 5.
Figure 5. Disintegration of rigid body kinetics with translational kinetics of rail [48].

ALE approach along with different FE codes e.g. ABAQUS [55] has been followed by several researchers. The ALE formulation follows a movable net over the material body or spatial discretization of wheel-rail surface through the finite element discretization. If surfaces during motion i.e. straight lines in the translational case and circles for rotational motions the ALE formulation is extremely difficult, that is why not been applied in roadway formulations. Wheel-rail discretization with a dense net in perspective contact area is described with figure 6.

Figure 6. Discretization of wheel-rail in contact area [58].

Solution of wheel-rail contact problem using finite element is a broad area of investigation for researchers.

5. Conclusions
The solution of the problem of wheel-rail interaction has been an area of endless research. Because of development in capacity for simulation time, attention is made on multi-dimensional models using numerical simulation covering even more aspects like thermal behaviour together with its effects on creep. However simplified less dimensional models provide efficient simulations and computational
time is also less. Despite of development in wheel-rail contact problems few important aspects like the prediction of wear are still to be answered accurately.

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