Abstract—In this paper we propose a method for estimating radial distribution grid admittance matrix using a limited number of measurement devices. Neither synchronized three-phase measurements nor phasor measurements are required. After making several practical assumptions, the method estimates even impedances of lines which have no local measurement devices installed. The computational complexity of the proposed method is low, and this makes it possible to use for online applications. Effectiveness of the proposed method is tested using data from a real-world distribution grid in Vienna, Austria.

I. INTRODUCTION AND MOTIVATION

Many countries worldwide are pursuing the quest for sustainable, economic and livable environment, smart cities in particular. The concepts of smart cities are fundamentally based on harnessing artificial intelligence (AI) technologies and the ongoing electric power grid modernization. Seestadt Aspern, a smart city district in Vienna (Austria) [1], is one such example. Transforming cities into smart cities is based on the integration of distributed energy sources (DERs) and advanced cyber (sensing, communication and control) infrastructures [2]. Also, pro-active participation of customers is also being enabled by embedding smart automation at the end-users level. All these innovations at the grid end-users side require modernization of low-voltage (distribution) grids, as they are emerging as the key enablers of smarter and more sustainable electricity services.

Modernization of distribution power systems required by these profound changes of end users needs offers new opportunities but also raises significant technical and business challenges. To start with, one of the most basic technical challenges is the lack of accurate grid data. This is needed because reliable and efficient operation of a power system requires accurate state estimation and control algorithms which can not be done without good knowledge of electric power grid itself. Recent research highlights that errors in grid admittance parameters could lead to significant loss of efficiency in system operation and even cause instability [3]. This is a difficult problem because the information about the power grid is often outdated due to the limited visibility and observability in large city networks with millions of nodes. The challenge can be tackled by beginning to rely on massive data one can collect using fast sensors, such as the Phasor Measurement Units (PMUs). Recent progress in PMU technologies and their rapidly decreasing cost can provide a means of collecting and utilizing time-series data to improve the accuracy of the grid admittance. Based on these technologies single-line/network impedance estimation methods have been proposed for transmission grids [4], [5]. However, it is realistic to assume that one cannot deploy such sensors and process all that data on all grid lines. To overcome this problem, [7]–[10] propose estimation methods for the case when only a limited number of sensors is available. However, above methods all need PMU measurements from partial or all nodes, since synchronized measurements and phasor measurements are required. Although the price of PMU has dropped, it is still unrealistic and financially demanding to install PMUs (or micro-PMUs) everywhere in distribution grids. In addition, these methods assume that the system is balanced, which is not always the case in distribution grids. The lines in distribution grids are short and are usually a mix of overhead lines and underground cables, and this further introduces numerical problems with grid admittance estimation. It is therefore conjectured in this paper that estimation of admittance in large complex distribution grids without synchronized measurements and phasor measurements remains a challenging task. The main requirement is that any effective estimation method should be robust and should have reasonable computation requirements.

This paper concerns this basic challenge of estimating grid admittance with limited number of conventional (non-PMU) sensors. In this paper we mainly focus on admittance estimation challenges in radial distribution grids. The proposed method is derived assuming that the network topology is known and that the non-synchronized measurements are only installed at the limited number of nodes. Unlike other entirely data-driven approaches, our proposed method uses physical models. A statistical learning process utilizes this physical model. As a result, our method is capable of providing good estimation on both balanced and unbalanced lines. Also an approximate estimate of parameters is computed for lines where no measurement devices are installed. It is emphasized that no phasor measurement is required.

The rest of the paper is organized as follows: Section II provides sensor information and problem formulation. Section III introduces the proposed method. Section IV demonstrates using simulations the effectiveness of the proposed method on
a real-world system. And Section IV concludes the paper.

II. PROBLEM FORMULATION

A. Available sensor type and installation

In Seestadt Aspern inexpensive measurement devices, called Grid Monitoring Devices (GMD) [11], are already installed. Compared with existing PMUs, a single GMD costs less than $200 and can provide measurement within 1% accuracy. As a trade-off, GMDs can only provide non-synchronized three-phase real power, reactive power and voltage magnitude every 2.5 minutes. Voltage phase information is not available.

The sensor installation strategy adopted in Seestadt Aspern, GMDs a has been to only install GMDs at one end of a single line, i.e., not to have measurements of both sending and receiving flows and voltages. Also, loads are not measured. Although smart meters have been installed at the end-users level, these data are not available for the grid estimation task due to home privacy concerns.

B. Distribution line impedance approximation in practice

For a variety of single and multi-core cables in distribution grids, their impedances are normally approximated using the formula given in the standard IEC 60909. For example, BICC Electric Cables Handbook gives a formula for inductance as [12]:

\[ L = (K + 0.2 \ln \frac{2S}{d})^{10^{-6}} \]  

where \( L \) is cable inductance \((H/m)\); \( K \) is conductor formation constant; \( S \) is axial spacing between conductors within a cable/in trefoil/flat formulation conductors \((mm)\); \( d \) is conductor diameters \((mm)\).

Notice that \( L \) includes both self and mutual inductance. Consequently, phases of cables can be regarded as decoupled from each other, with \( L \) of each phase.

C. Problem formulation

Instead of using Eqn.(1), a method is derived to estimate line impedances online utilizing GMD measurements. This distribution grid admittance matrix estimation problem is posed as follows:

- Given: historical non-synchronized three-phase measurements \((P, Q\) and \(V)\) of a limited number of buses; a known grid incident matrix \( A \)
- Objective: estimate the impedance \( L \) of each line comprising of the radial distribution grid

III. PROPOSED HYBRID DATA-PHYSICS ESTIMATION METHOD

In this section, we propose a hybrid data-physics estimation method [13] for solving the problem posed in Section II. The method consists of a topology decomposition process and a hybrid data-physics estimation process, shown in Fig.1. The topology decomposition process aims to break a complicated network into a few basic elements. Then, the admittance estimation is achieved by the composition of estimation procedures of basic elements (four cases in Fig.1).

The difference between Case 2 and Case 3 is: we know the real and reactive power flow received at node \( j \) along \( line_{ij} \) in Case 2, while we only know the total power injection at node \( i \) in Case 3. Because it is possible to have multiple branches connected to node \( i \). Therefore, in Case 3 and Case 4, we only provide equivalent line impedance estimation which includes possible local loads. This will be discussed in detail in Section III-B.

2) Network topology decomposition algorithm: Conceptually speaking, the proposed decomposition algorithm first starts with a measured node. Then, the algorithm explores its connecting lines and compares each line with four basic elements. We first define the following notations: \( \Omega_m \) is the set of nodes with GMD installed; \( \Omega_N \) is the set of nodes without GMD; \( \Theta_l \) is the line set; \( line_{ij} \) is the line between node \( i \) and node \( j \); \( \emptyset \) is the empty set.

The algorithm is given in Algorithm 1. After executing Algorithm 1, the system will be decomposed into basic elements listed in Table I. Different estimation processes for each basic element therefore can be invoked, which will be discussed next.

B. Hybrid data-physics estimation algorithms

Observe that distribution grid cables are usually short and GMD measurements are non-synchronized over 2.5 min period. It is reasonable to assume that measured data describes
It can be seen that Ohm’s law provides relations between measured variables and unknown variables. Next, we will show that the Law of Large Number and Assumption 1 can be used to link the measurement data with Eqs. (5) and Eq. (6).

From statistics point of view, all the measured data can be regarded as random variables satisfying certain distributions. In particular, as Assumption 1 implies that $|\theta_j| \approx 0$, it is reasonable to assume that the voltage angle of measured node $\theta_j$ satisfies a distribution with zero mean. To estimate $Z_{ij}$ and $\delta_{ij}$, the method of moments is used [13]. Since $Z_{ij}$ and $\delta_{ij}$ do not change much over time, we thus are interested in the first order approximation from the measured data. Therefore, the physics and data can be linked through Eqs. (5) and Eq. (6) as:

$$Z_{ij}^2 E[I_{ij}^2] = E[(V_i - V_j \cos \theta_j)^2 + (V_j \sin \theta_j)^2]$$

$$E[-V_j \sin \theta_j/(V_i - V_j \cos \theta_j)] = E[\tan(\theta_j - \Phi_j + \delta_{ij})]$$

where $I_{ij}^2 = (P_{ij}^2 + Q_{ij}^2)/V_j^2$ and $E[\ast]$ denotes the expectation of variable ($\ast$).

Under Assumption 1 ($E[\theta_j] = 0$) above two equations can be further simplified as:

$$Z_{ij}^2 E[I_{ij}^2] = E[(V_i - V_j \cos \theta_j)^2]$$

$$E[\tan(-\Phi_j + \delta_{ij})] = 0$$

According to the LLN, the expectation in above equations can be approximated by the mean of measured data. Before introducing specific algorithms for each case, we first define the feasible region of voltage magnitude $V_{\text{min}}$ and $V_{\text{max}}$. $V_{\text{min}} = 0.95 V_{\text{nominal}}, V_{\text{max}} = 1.05 V_{\text{nominal}}$, where $V_{\text{nominal}}$ is the rated nominal voltage magnitude.

1) Case 1: The available measurements are three-phase real/reactive power received at node $j$:

$$P = (P_{ji,a}[1], ..., P_{ji,c}[T]) \quad Q = (Q_{ji,a}[1], ..., Q_{ji,c}[T])$$

three-phase voltage magnitude of node $i$ and node $j$:

$$V_i = (V_{i,a}[1], ..., V_{i,c}[T]) \quad V_j = (V_{j,a}[1], ..., V_{j,c}[T])$$

See Algorithm 2 for detail.

2) Case 2: GMD is only installed at node $j$. Thus, the available measurements are three-phase real/reactive power received at node $j$ ($P/Q$) and three-phase voltage magnitude of node $j$ ($V_j$). In normal operation, voltage should satisfy the feasibility requirement. This indicates that voltage magnitude of node $i$ ($V_i$) can vary between $V_{\text{min}}$ and $V_{\text{max}}$. Estimation method is listed in Algorithm 3.

3) Case 3: GMD is installed at node $i$. Besides voltage magnitude at node $i$ ($V_i$), only three-phase total real/reactive power injection are available: $P_i = (P_{i,a}[1], ..., P_{i,c}[T])$ and $Q_i = (Q_{i,a}[1], ..., Q_{i,c}[T])$. Note that power flow along line $ij$ is unknown. We therefore assume that real and reactive power are equally shared by all case 3 type lines connected to node $i$. The proposed estimation method is given in Algorithm 4.
Algorithm 2 Estimation algorithm for case 1

Input: Three-phase measurements: P, Q, V, \(V_j\)
1: Calculate: the expectation of \(V_j\) and \(I_{ij}\)
2: \(\Phi_j = \text{acos} \left( \frac{|P|}{\sqrt{P^2+Q^2}} \right)\) \(\Rightarrow E[\Phi_j] = \text{mean}(\Phi_j)\)
3: \(I_{ij} = \text{sign}(P) \sqrt{P^2+Q^2} \Rightarrow E[I_{ij}] = \text{mean}(I_{ij})\)
4: \(E[V_i] = \text{mean}(V_i), E[V_j] = \text{mean}(V_j)\)
5: Estimate impedance magnitude using Eqn.(7):
\[
Z_{ij} = E[|V_i - I_{ij}|^2]
\]
6: Estimate impedance angle using Eqn.(8):
\[
\delta_{ij} = E[\Phi_j]
\]
Output: \(Z_{ij}, \delta_{ij}\)

Algorithm 3 Estimation algorithm for case 2

Input: Three-phase measurements: P, Q, V, \(V_j\)
1: Calculate: \(E[V_i], E[I_{ij}]\) and \(E[\Phi_j]\) (same as in case 1)
2: Estimate the upper bound of impedance magnitude:
if \(E[I_{ij}] > 0\) then
4: Upper bound: \(Z_{ij} = \frac{V_{\max} - E[V_i]}{E[I_{ij}]}\)
else if \(E[I_{ij}] < 0\) then
6: Upper bound: \(Z_{ij} = \frac{V_{\min} - E[V_i]}{E[I_{ij}]}\)
else
8: Upper bound: \(Z_{ij} = +\infty\)
Estimate impedance angle using Eqn.(8):
10: \(\delta_{ij} = E[\Phi_j]\)
Output: \(Z_{ij}, \delta_{ij} = 0\) and \(\delta_{ij}\)

4) Case 4: We do not have GMD and thus no measurements are available related to the line. However, we can still approximate the impedance by making the following assumption. The proposed method for case 4 is shown in Algorithm 5.

Assumption 2: Given a node \(i\), we assume that all case 4 type lines connected to node \(i\) have same impedance \(Z\) and \(\delta\).

C. Computational complexity and online implementation

It can be seen that proposed algorithms only require basic algebraic operations and corresponding computational complexity is \(O(n)\). Therefore, it is possible to implement the method online which has been validated on a subnet in Asperrn smart city. Results and required data size analysis are discussed in Section IV.

IV. EVALUATION WITH REAL-WORLD SYSTEM

A. System description

Fig. 2: An exemplary distribution subnet in Asperrn Smart City

The Seestadt Asperrn distribution system has 11 radial subnets connecting to their substations in the normal condition.

Algorithm 4 Estimation algorithm for case 3

Input: \(P_i, Q_i, V_i\) and \(N_{branch} = 1\)
1: Calculate: \(E[P_j], E[Q_j] \leftarrow \text{LowerBound 1}(node j)\);
2: \(E[P_i], E[Q_i] \leftarrow \text{LowerBound 2}(node i, node j, N_{branch})\);
3: Calculate: the approximated real/reactive power injection to the line \(E_{ij}\) using:
4: \(P_{ij} = \frac{E[P_j] - E[P_i]}{N_{branch}}, Q_{ij} = \frac{E[Q_j] - E[Q_i]}{N_{branch}}\)
5: Estimate: Resistance and reactance of equivalent impedance as:
\[
R_{ij} = E[V_i]^{2} \Rightarrow X_{ij} = \frac{E[Q_i]^{2}}{E[P_i]^{2}}
\]
Output: \(Z_{ij} = \sqrt{R_{ij}^{2} + X_{ij}^{2}}, \delta_{ij} = \text{atan}(\frac{X_{ij}}{R_{ij}})\)

function LOWERBOUND 1(node j)
7: for \(m \in \text{child nodes of node } i\) do
8: if \(line_{jm}\) has been estimated then
9: \(P_{jm} = E[I_{jm}]^{2}Z_{jm}\cos(\delta_{jm}) - E[P_{jm}]\)
10: \(Q_{jm} = E[I_{jm}]^{2}Z_{jm}\sin(\delta_{jm}) - E[Q_{jm}]\)
else
12: \(P_{jm} = 0, Q_{jm} = 0\)
13: Return: \(P_{0} = \sum P_{jm}, Q_{0} = \sum Q_{jm}\)
14: function LOWERBOUND 2(node i, node j, \(N_{branch}\))
15: for \(n \in \text{child nodes of node } i, \& n \neq j\) do
16: if \(line_{in}\) has been estimated then
17: \(P_{in} = E[I_{in}]^{2}Z_{in}\cos(\delta_{in}) - E[P_{in}]\)
18: \(Q_{in} = E[I_{in}]^{2}Z_{in}\sin(\delta_{in}) - E[Q_{in}]\)
else
20: \(N_{branch} = N_{branch} + 1\)
21: \(E[P_{in}, Q_{in}] \leftarrow \text{LowerBound 1}(node n)\)
22: Return: \(P_{0} = \sum P_{in}, Q_{0} = \sum Q_{in}\) and \(N_{branch}\)

Algorithm 5 Estimation algorithm for case 4

1: let \(N_{branch} = 1\)
2: \((Z, \delta, N_{branch}) \leftarrow \text{Case4}(node i, N_{branch})\)
3: Calculate: \(Z_{ij} = Z/N_{branch}, \delta_{ij} = \delta\)
4: Assign \(Z_{ij}, \delta_{ij}\) to all case 4 type lines in Function Case4.
Output: \(Z_{ij} = \sqrt{R_{ij}^{2} + X_{ij}^{2}}, \delta_{ij} = \text{atan}(\frac{X_{ij}}{R_{ij}})\)

function CASE4(node i, \(N_{branch}\))
6: Find: parent node of node \(i \rightarrow \text{node } k\)
7: if node \(k\) has GMD then
8: \((Z_{ki}, \delta_{ki}) \leftarrow \text{call: Algorithm 4}(line_{ki})\)
9: else
10: \((Z, \delta, N_{branch}) \leftarrow \text{Case4}(node k, N_{branch})\)
11: \(N_{branch} = N_{branch} + 1\)
12: Return: \(Z_{ki}\), \(\delta_{ki}\), \(N_{branch}\)
Output: \(Z_{ij} = N_{branch}, \delta_{ij} = \delta\)

Notice that all subnets can be decomposed into four basic line elements. In order to evaluate the proposed methods, a subnet in Asperrn smart city is considered, whose topology with sensor installation map is shown in Fig. 2.

GMD IDs are directly marked next to their installed nodes in Fig. 2. One month (May 2018) measurements (around 15,000 data points) are used as input data. It is worthwhile mention-
ing that measurement noise and errors are unavoidable. For example, data points are missing for certain dates. Also, we observe from measurements that not all lines are balanced.

B. Estimation results discussion

In the test, the benchmark impedance value for each line is calculated by multiplying line length and corresponding equivalent resistance and inductance value found in the data sheet. The estimation results for case 1 and case 2 type lines are listed in the Table II.

| Line Nodes   | L02-L03 | L03-L04 | L02-L09 | L02-L08 | L01-L08 | L01-L07 | L01-L21 | L07-L06 | L09-L05 |
|--------------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Error (%)    | 21.90%  | 22.66%  | 20.19%  | 17.01%  | 39.81%  | 34.73%  | 126.0%  | 153.6%  |
| Absolute Error (Ω) | 0.0036   | 0.0012  | 0.0009  | 0.0005  | 0.0045  | 0.011   | 0.0255  | 0.001   |
| Absolute Error (°) | 7.73     | 9.41    | 1.11    | 7.55    | 3.18    | 2.35    | 1.614   | 1.68    |

It can be seen that the proposed algorithm managed to estimate the line impedance based on GMD data. As can be seen from Table II, the estimation results are accurate. Although the average percentage error is around 23%, the absolute error of both magnitude and angle are very small. In particular, the error of the Line between L09 and L05 is the largest, at around 153%. However, the absolute error of the angle is less than 1.8 degree, which is even smaller than the other lines.

Given that real line impedance is unknown, percentage error may not be a good measure on estimation accuracy. However, a higher percentage error may indicate that the actual GMDs installation on the line L09-L05 is different from the information found in the data sheet. Or some additional devices have been installed on the line in practice which has changed the line impedance but such situation has not been reflected in the manual. This conjecture is supported by the result that the estimated impedance magnitude is higher than the benchmark value. It is worthwhile to have another GMD installed at L05 side. Therefore, more accurate line impedance could be obtained and the corresponding result can be used to validate the proposed estimation method.

For the rest case 3 and case 4 type lines, as discussed in Section III, we can only provide equivalent impedances which includes line impedances and connecting local loads. Thus, our estimation results for these two type lines are close to their actual value only when local loads are small. Because of this observation, we omit case 3 and 4 estimation results for brevity. However, it should be mentioned that such a situation can be improved if additional information from local smart devices are provided, such as data from E-meters, etc.

C. Sensitivity analysis

We conduct sensitivity analysis on selected lines with respect to the size of data, since computation time depends on the size of utilized data points. The result is shown in Fig. 3. The estimation error tends to decrease as more data points are utilized. However, it does not improve much when more than 5000 data points are used. Notice that one day measurements is around 500 data points. Thus, in practice, there is no need to use all available data points. 5-10 days measurements can provide a reasonably good result.

V. CONCLUSIONS

In this paper we propose a novel algorithm which is able to estimate the radial distribution admittance by only using non-synchronized measurements from limited locations. More importantly, phasor measurement is not required. It should be also emphasized that the computational complexity of the proposed hybrid data-physics method is remarkably low, resulting in an easy online implementation with a small board. Furthermore, our preliminary tests, conducted on a real-world system, show that the proposed method performs well when noise and unbalanced network are considered. In the future, we plan to combine the proposed real-time estimation with the state estimation to improve the operation efficiency.

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