Research on Roll Attitude Control of UAV Based on Active Disturbance Rejection Control

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Abstract. The roll attitude control of small high-speed unmanned aerial vehicle (UAV) is very difficult compared to the conventional low-speed UAV. The dynamic pressure is large and the moment of inertia is too small, which cause the aileron is sensitive to manipulation. The roll attitude may oscillate due to the actuator rate limiting. In response to this feature, the active disturbance rejection control (ADRC) is added to compensate the nonlinear part of the system, so that the system is close to an integral link. The problem of the actuator rate limiting can be solved by tuning the gain of the angular rate control to change the system bandwidth. On the other hand, the rolling motion is, in general, coupled from the yawing and sideslip motions. ADRC can decouple the roll channel and the yaw channel, solving the nonlinear problem of control. Combination with the requirements of frequency and time domain, design the roll attitude controller. The UAV nonlinear simulation results show that the roll attitude control based on ADRC meets the flight control requirements.

1. Introduction
UAV attitude control is the inner loop of the trajectory control and is an important part of the autopilot. Whether the attitude loop control is stable or not directly determines the safety of the aircraft. Currently PID controller still plays a major role in industrial control. The paper adds the extended state observer (ESO), which is the core of the ADRC to estimate and compensate the modeled part of the aircraft model, the unmodeled part and the external disturbance part, so that the system approaches an integral form linearly related to input. Then the PID controller is added to the simple integration to meet the design requirements. The controller has the characteristics of simple structure, easy implementation in the project and less parameter adjustment.

ADRC [1] is a control technique proposed by Jingqing Han researcher to estimate and compensate the uncertainties. It does not need accurate mathematical models and has high robustness and anti-disturbance ability. The active disturbance rejection control technique has been developed for many years, and the theoretical system has been gradually completed and has high engineering application value. [5-7]

In this paper, the lateral motion model of the UAV is firstly established, and the linear state space equation is extracted in a typical steady-state flight. Then the dynamic inverse analysis for the nonlinear motion equation is carried out, and the requirement of state quantity estimation is proposed. The linear extended state observer (LESO) [3] is used for estimation. The attitude control law design is based on the linear model. The effects of the LESO parameters b0, ω are analysed separately. Finally, verification is performed on the nonlinear simulation platform.

2. Building the Aircraft Lateral Model
In this section, the aerodynamic forces and moments models are incorporated into the vector motion
equations to obtain an aircraft model for simulation and analysis. Only four equations related to the lateral model are selected. Reference [2] introduces coordinate system, complete six-degree-of-freedom equation and the aerodynamic forces and moments models.

2.1. The Nonlinear Aircraft Model

There are four equations related to the lateral side of the drone. (1) represents the lateral acceleration equation. (2) and (3) are the angular equations around the x-axis and the z-axis. (4) is the x-axis Euler equation of motion.

\[
\dot{V} = -RU + PW + \frac{F_y}{m}
\]

\[
\dot{P} = \frac{1}{I_x I_z - I_{xz}^2} \left[ I_x \hat{L} + I_{xz}N + \left( (I_y - I_z)I_z - I_{xz}^2 \right) RQ + (I_x - I_y + I_z)I_{xz}PQ \right]
\]

\[
\dot{R} = \frac{1}{I_x I_z - I_{xz}^2} \left[ I_x (I_x - I_y) + I_{xz}^2 PQ - \left( (I_x - I_y + I_z)I_{xz}RQ + I_{xz} \hat{L} + I_z N \right) \right]
\]

\[
\dot{\phi} = P + \tan \theta (Q \sin \phi + R \cos \phi)
\]

\[
[U V W]^T \text{ indicates the forward speed, lateral velocity, and longitudinal velocity of the body axis. [P Q R]^T is the roll rate, the pitch rate and the yaw rate. [L M N]^T is the rolling moment, the pitching moment and the yawing moment. [I_x I_y I_z I_{xz}]^T is moment of inertia. } m \text{ is the aircraft mass. Add a sideslip angle equation.}
\]

\[
\beta = \sin^{-1} \frac{v}{(U^2 + V^2 + W^2)^{1/2}}
\]

2.2. The Linear Aircraft Model

Take a trim state, airspeed 80m / s, height 1500m, level and straight flight. Based on these conditions, the Jacobian matrices are obtained by the numerical linearization.

\[
\dot{\hat{x}} = \begin{bmatrix}
-0.2444 & 0.1219 & 0.0985 & -0.9951 \\
0 & 0 & 1 & 0.9990 \\
-39.7445 & 0 & -2.3996 & 1.3719 \\
25.3239 & 0 & 0.0326 & -0.7316
\end{bmatrix} x + \begin{bmatrix}
0.0000 \\
0 \\
-0.5989 \\
-0.0448
\end{bmatrix} u
\]

\[
\dot{\hat{y}} = \begin{bmatrix}
57.3 & 0 & 0 & 0 \\
0 & 57.3 & 0 & 0 \\
0 & 0 & 57.3 & 0 \\
0 & 0 & 0 & 57.3
\end{bmatrix} x
\]

\[
x = [\beta \ \phi \ P \ R]^T, \ u = [\delta_a \ \delta_r]^T, \ y = [\beta \ \phi \ P \ R]^T, \text{ where 57.3 indicates the radians-to-degree conversion 57.29578.}
\]

2.3. Modal Analysis

| Mode                         | Pole            | Damping | Frequency (rad/s) | Time Constant(s) |
|------------------------------|-----------------|---------|------------------|------------------|
| Dutch roll mode              | -0.592 ± 5.36i  | 0.11    | 5.39             | 1.69             |
| Roll subsidence mode         | -2.21           | 1.0     | 2.21             | 0.452            |
| Spiral mode                  | 0.0219          | -1.0    | 0.0219           | -45.6            |

Table 1 show that there are three modes in the lateral-directional. Dutch roll period is quite short (T = 1.69 s) and the oscillation is very lightly damp (ζ = 0.11). This would make landing in gusty wind conditions difficult. The second mode is simply a stable exponential mode and clearly involves mostly roll rate; it is known as the roll subsidence mode. The time constant of 0.452 indicates a fast roll
response. The third mode has an unstable pole. An unstable spiral mode can cause an aircraft to get into an ever-steeper, coordinated, spiral dive.

3. Control Law Design

The control law design consists of two parts, one is the design of the ADRC controller, and the other is the roll controller. In the ADRC controller, the observer bandwidth is the only parameter need to design. The roll attitude control needs to design the roll rate loop first, and then design the roll angle loop, which are both based on the frequency domain design method. Pay attention to the system bandwidth while ensuring the system time domain response.

3.1. ADRC

3.1.1. Dynamics Inversion

Given \( \dot{x} = f(x) + g(x)u \)

\( y = h(x) \)

Based on the input-output linearization theory for nonlinear systems [4], derive the output \( \dot{y} = \frac{dh}{dx} \dot{x} = \frac{dh}{dx} (f(x) + g(x)u) = F(x) + G(x)u \). If \( G(x) \) is not singular, the relative degree of the system is 1.

Let \( u = G^{-1}(x)(K(y - y_g) - F(x)) \). The result is \( \dot{y} = K(y - y_g) \). It is an integral form.

In dynamic inverse controllers, \( F(x) \) is usually about the parameters of the aircraft model. If the parameters are not accurate, the robustness of the dynamic inverse controller is poor. A new control scheme is proposed, which is to estimate \( F(x) \) with \( \hat{f}(x) \). The controller input is \( u = G^{-1}(x)(K(y - y_g) - \hat{f}(x)) \). When the estimator converges quickly, \( F(x) - \hat{f}(x) \to 0 \), the result is \( \dot{y} = K(y - y_g) \). The transfer function is \( \frac{y(s)}{y_g(s)} = \frac{-bk}{s-bk} \).

Now ESO is used to estimate \( F(x) \). And a method for evaluating the convergence of ESO is given, which compares the actual poles and theoretical poles of the closed-loop system transfer function.

3.1.2. LESO

\[ \begin{bmatrix} z_1(s) \\ z_2(s) \end{bmatrix} = \begin{bmatrix} \frac{\beta_{01}s + \beta_{02}}{s^2 + \beta_{02}s + \beta_{02}} & \frac{b_0s}{s^2 + \beta_{02}s + \beta_{02}} \\ \frac{-\beta_{01}s + \beta_{02}}{s^2 + \beta_{02}s + \beta_{02}} & \frac{b_0s}{s^2 + \beta_{02}s + \beta_{02}} \end{bmatrix} \begin{bmatrix} y(s) \\ u(s) \end{bmatrix} \]  \hspace{1cm} (8)

If \( \beta_{01} = 2\omega \), \( \beta_{02} = \omega^2 \), \( K = \frac{-\omega^2}{b_0} \), \( \omega = 10 \), \( b_0 = -0.5989 \),
\[ z_2(s) = L_1(s)y(s) + L_2(s)u(s) = \frac{Ks}{(s+\omega)^2}y(s) + \frac{\omega^2}{(s+\omega)^2}u(s) \]  

(9)

3.2. Attitude Control

![Figure 2. Roll Control Closed-loop with PD and ADRC Controller](image)

According to the Figure 2, the control law is

\[ \delta_a = K_d(P - P_g) + z_2 \]  

(10)

\[ P_g = K_p(\varphi_g - \varphi) \]  

(11)

3.2.1. Roll Rate Control

According the Figure 2, the inner loop transfer function is

\[ \frac{P(s)}{P_g(s)} = \frac{1}{-K_dG_d(s) - L_1(s)G_d(s) - L_2(s) + 1} \]

Draw the system \( \Phi(s) = \frac{-G_d(s)}{-L_1(s)G_d(s) - L_2(s) + 1} \) root locus map in Figure 3. Choose the gain \( K_d = 0.2 \).

![Figure 3. Root Locus Map (K_d)](image)  

![Figure 4. Bode Diagram](image)

Table 2 shows the closed-loop system zero and pole. The dominant pole is \(-3.5954 + 0.0000i\). Amplitude margin and phase margin of the roll rate control are shown in Figure 4. The phase margin is 104°. The amplitude margin is \(\infty\). The inner loop bandwidth \( \omega_b = 4.05 \) rad/s. The bandwidth of the actuator is 28 rad/s. The control system bandwidth meets design requirements.
Table 2. Closed-loop System Zero and Pole Distribution

| Pole Zero       | Pole      | Zero       |
|----------------|----------|------------|
| -12.6594 ± 5.1621i | -10.0000 ± 0.0000i |
| -1.9375 ± 5.0107i   | -1.8677 ± 4.9575i   |
| -3.5954 + 0.0000i   | -0.2023 + 0.0000i   |
| -0.2009 + 0.0000i   | 0.0120 + 0.0000i    |
| 0.0120 + 0.0000i    |           |

3.2.2. Roll Control

According to equation (11), the transfer function is

\[ \frac{\phi(s)}{\theta_p(s)} = \frac{K_p}{s + K_p} \]

It is a typical first-order integral link. The bandwidth frequency is \( \omega_b = K_p \). And the roll angle controller is the next loop of the roll angular rate controller. According to the bandwidth design requirements, \( K_p = 1.0 \) is selected.

3.3. \( b_0 \) Analysis

Table 3. Closed-loop System Zero and Pole Distribution

| Dominant pole | Phase margin | Amplitude margin | Bandwidth |
|---------------|--------------|------------------|-----------|
| \( b_0 \)     | -3.5954 + 0.0000i | 104              | ∞         | 4.05     |
| 10 * \( b_0 \)| -3.5236 ± 4.1548i | 72.7             | ∞         | 8.28     |
| 0.1 * \( b_0 \)| -0.5925      | 97               | ∞         | 0.58     |

\( b_0 \) is the only model-related parameter in the LESO. It is highly robust. It represents the magnification of the input. If \( b_0 \) is the 10 times of the true value, the angular rate control system can be simplified to a second-order system with a pair of conjugate complex roots as the dominant pole. The phase margin of the system is reduced, and bandwidth frequency is increased. If \( b_0 \) is 1/10 of the true value, the system degenerates into a first-order system with a small real root as the dominant pole. The phase margin of the system is reduced, and bandwidth frequency is reduced.

In summary, if \( b_0 \) is too small, the response time of the angular rate control system becomes longer. From the perspective of the design of the angle loop, a larger gain \( K_p \) is needed to match the bandwidth. Obviously, it affects the roll angle gain design. Therefore, the value of \( b_0 \) can be slightly larger and should not be too small.

3.4. \( \omega \) Analysis

The LESO controller is similar with the integration. Both can get rid of the steady state error in closed-loop system. From the perspective of the frequency domain, the controller improves the system type, which reduce the static error of the step response. The proof is given below.

Let \( G_0(s) = \frac{K}{\prod_{i=1}^{m}(s-z_i)} \), the open-loop system transfer function is

\[
G(s) = \frac{K_dG_0(s)}{1-L_2(s) - L_1(s)G_0(s)} = \frac{K_dK^*(s+\omega)^2[\prod_{j=1}^{m}(s-z_j)]}{s(s+2\omega)[\prod_{i=1}^{m}(s-p_j)+\omega^2[\prod_{j=1}^{m}(s-z_j)]} \]

It shows that the LESO controller adds two zeros \( z_1 = z_2 = -\omega \) to the original zero points. Two poles are added to the original poles, one of which is \( s_1 = 0 \), but completely changes the other poles. Simplified the open-loop transfer function

\[
G(s) = \frac{K_dK^*(s+2\omega)[\prod_{j=1}^{m}(s-z_j)]}{s(s+2\omega)[\prod_{i=1}^{m}(s-p_j)+\omega^2[\prod_{j=1}^{m}(s-z_j)]} \]

Consider closed-loop systems, the closed-loop system transfer function is

\[
\phi(s) = \frac{-G(s)}{1-G(s)} = \frac{-K_dK^*(s+\omega)^2[\prod_{j=1}^{m}(s-z_j)]}{s(s+2\omega)[\prod_{i=1}^{m}(s-p_j)+\omega^2[\prod_{j=1}^{m}(s-z_j)]-K_dK^*(s+\omega)^2[\prod_{j=1}^{m}(s-z_j)]} \]

If \( \omega \to \infty \), then \( \phi(s) = \frac{-K_dK^*}{s-K_dK^*} \).

The theoretical pole is \( s = K_dK^* = -6.8634 \). Table 4 is the actual pole with different \( \omega \).
Table 4. The Actual Pole with Different $\omega$

| $\omega$ | Actual pole |
|----------|-------------|
| 100      | -6.52       |
| 30       | -5.76       |
| 10       | -3.58       |

4. Simulation
The initial conditions of the UAV simulation: airspeed 80m/s, height 1500m, angle of attack 5.7°. The control law design is evaluated by the step response. For the roll rate loop, the roll rate command is 1°/s. The Fig 5 is the step response of roll rate. It shows that the settling time is 1s without overshoot. For the roll angle loop, the roll angle command is 1°, the Fig 6 is the roll angle response and the roll angular rate response. The settling time is 3s without overshoot. The roll angle and roll rate can track commands well and meet time domain response requirements.

Figure 5. Step Response of Roll Rate

Figure 6. Step response of roll

5. Conclusions
Aiming at the problem that small high-speed UAV have high rolling frequency and is easy to diverge, a new control method based on ADRC is proposed. The control method has strong anti-disturbance capability and high robustness. At the same time, the decoupling of roll and yaw is realized, and the nonlinear problem in the control system is solved. And analyse the effects of the parameters $b_0$ and $\omega$ in LESO controller. Verification on the nonlinear simulation platform, the control system has good tracking performance and meets the design requirements.
6. References

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