Can one ever prove that neutrinos are Dirac particles?

Martin Hirsch, Rahul Srivastava, and José W. F. Valle

AHEP Group, Institut de Física Corpuscular – C.S.I.C./Universitat de València, Parc Científic de Paterna. C/ Catedrático José Beltrán, 2 E-46980 Paterna (Valencia) - SPAIN

According to the “Black Box” theorem the experimental confirmation of neutrinoless double beta decay \(0\nu2\beta\) would imply that at least one of the neutrinos is a Majorana particle. However, a null \(0\nu2\beta\) signal cannot decide the nature of neutrinos, as it can be suppressed even for Majorana neutrinos. In this letter we argue that if the null \(0\nu2\beta\) decay signal is accompanied by a \(0\nu4\beta\) quadruple beta decay signal, then at least one neutrino should be a Dirac particle. This argument holds irrespective of the underlying processes leading to such decays.

PACS numbers: 24.80.+y, 14.60.Lm, 14.60.Pq, 12.60.-i

Ever since the early days of neutrino physics, there has been a debate about the nature of neutrinos i.e. whether they are Dirac or Majorana fermions. The debate has origins in the fact that, although most of the known fermions (except neutrinos, whose nature is yet to be ascertained) are Dirac particles and hence four-component spinors, the fundamental irreducible spinorial representations of the Poincaré group are actually two-component. However, the Poincaré group describes just the kinematics, and does not represent the full unbroken symmetry of nature.

Apart from spacetime symmetry, particle theories also have “internal symmetries” for example “gauge symmetries”, such as the \(SU(3)_C \otimes SU(2)_L \otimes U(1)_EM\) of our cherished Standard Model (SM). According to the gauge paradigm, these symmetries dictate the dynamics of all fundamental processes amongst elementary particles. The SM gauge group is spontaneously broken by the celebrated Brout-Englert-Higgs mechanism, but not completely. As far as we know from experiments, an \(SU(3)_C \otimes U(1)_EM\) gauge symmetry remains unbroken. This symmetry then dictates the dynamics of fundamental processes at energies below the electroweak symmetry breaking scale. Thus at energy or temperature scales well below the electroweak breaking scale, one must not only take into account the invariance under the Poincaré group, but also under the unbroken \(SU(3)_C \otimes U(1)_EM\) gauge group. Thus, any fermion carrying a non-zero color or electric charge cannot have a Majorana mass term, since such term would necessarily break the \(SU(3)_C \otimes U(1)_EM\) gauge symmetry. This implies that, although two-component spinors are indeed fundamental, the requirement that color and electromagnetic charges remain conserved, forces all the quarks and charged leptons to be Dirac particles. On the basis of this argument it has been argued in [5] that, thanks to their complete charge neutrality, only neutrinos can be – and should be – Majorana fermions. However, nature need not follow our theoretical prejudices, so that only experiments can settle whether neutrinos are Dirac or Majorana particles.

Thanks to the small neutrino mass \(m_\nu\) and the V-A nature of the weak interaction, discerning the nature of neutrinos from experiments is a formidable task. A basic difference between Dirac and Majorana fermions resides in the CP phases present in their mixing matrices [5]. Indeed, the sensitivity to the physical Majorana phases present in neutrino to anti-neutrino oscillations [6] is well below any conceivable test. Likewise, electromagnetic properties of neutrinos [7–9] have a hidden dependence on \(m_\nu\). Indeed, all observables sensitive to the Majorana nature of neutrinos end up being suppressed by a power of \(m_\nu\). The small scale of the active neutrino masses makes such differences very tiny.

However, there is a potentially feasible process which may settle the issue, namely the neutrinoless double beta decay, which has long been hailed as the ultimate test concerning the nature of neutrinos. Indeed, if \(0\nu2\beta\) decay is ever observed, its amplitude can always

[1] In SM extensions there may be feasible complementary probes of lepton number violation at collider energies [10,11].
be “dressed” so as to induce a Majorana mass, ensuring that at least one of the neutrinos is of Majorana type \[15\], as illustrated in Fig. 1. See Ref. \[16\] for recent discussions.

![Diagram](image_url)

Figure 1. The “Black Box” theorem states that a 0ν2β signal ensures that at least one neutrino is Majorana in nature \[15\].

However, the non-observation of 0ν2β decay so far \[18\]–\[21\] has raised the intriguing possibility that neutrinos might well be Dirac particles. Several well motivated high-energy completions of the SM do lead to naturally light Dirac-type neutrinos \[22\]–\[25\]. Alternatively, the absence of a 0ν2β signal is not inconsistent with the Majorana nature of neutrinos, since the decay amplitude may be suppressed as a result of a destructive interference amongst the three active neutrinos, even if they are Majorana type \[26\], \[27\]. Thus, although the observation of 0ν2β decay would necessarily imply that at least one neutrino species is Majorana in nature, the converse is not true: a negative 0ν2β decay signal does not tell us anything about the nature of neutrinos.

This prompts us to search for processes beyond the simplest 0ν2β decay which can also shed light upon the nature of neutrinos.\footnote{Observation of a non-zero mass in KATRIN, together with non-observation of 0ν2β decay would also favour Dirac neutrinos.} We will specifically focus on the two lowest 0ν2nβ processes characterized by \(n = 1, 2\), namely, the neutrinoless double beta decay 0ν2β and the neutrinoless quadruple beta decay.

An experimental search for the 0ν4β process has been recently performed by the NEMO-3 collaboration, using \[^{150}\text{Nd}\] \[28\]. The possible existence of 0ν4β decays has been first suggested in \[29\], and it is expected to arise in a number of models with family symmetries leading to Dirac neutrinos \[30\]–\[32\]. Here we argue that the combination of the 0ν2β and 0ν4β processes may be enough to settle the nature of neutrinos within a very broad class of models.

In order to proceed let us first look at the 0ν2β process and the neutrino mass generation from the symmetry point of view. In the Standard Model the neutrinos are massless and there is an accidental global “classically conserved” \(U(1)_L\) symmetry in the lepton sector associated to Lepton number for all the leptons in SM \[3\]. By just adding right handed neutrinos \(\nu_R\) sequentially to the SM particle content one can give mass to neutrinos without breaking the lepton number symmetry. In such a case neutrinos will necessarily be Dirac particles and the 0ν2nβ; \(n \geq 1\) decays will all be absent.

We now turn to the cases when this lepton number is broken down to a discrete \(Z_m\) subgroup \((m \geq 2\) which remains conserved. Notice that a \(U(1)\) symmetry only admits \(Z_m\) subgroups, where \(Z_m\) is a cyclic group of \(m\) elements, characterized by the property that if \(x\) is a non-identity group element, then \(x^m \equiv x\). The \(Z_m\) groups only admit one-dimensional irreducible representations, conveniently represented by using the \(n\)-th roots of unity, \(\omega = e^{\frac{\pi i}{m}}\), where \(\omega^m = 1\). If lepton number is broken to a \(Z_m\) subgroup (with neutrinos transforming non-trivially under \(Z_m\) by the new physics responsible for neutrino mass generation, then we have two possible cases:

\[
U(1)_L \rightarrow Z_m \equiv Z_{2n+1} \text{ where } n \geq 1 \text{ is a positive integer} \Rightarrow \text{Neutrinos are Dirac particles} \\
U(1)_L \rightarrow Z_m \equiv Z_{2n} \text{ where } n \geq 1 \text{ is a positive integer} \Rightarrow \text{Neutrinos can be Dirac or Majorana} \tag{1}
\]

If the \(U(1)_L\) is broken to a \(Z_{2n}\) subgroup, then one can make a further broad classification

\[
\nu \sim \omega^n \text{ under } Z_{2n} \Rightarrow \text{Majorana neutrinos} \tag{2} \\
\nu \sim \omega^n \text{ under } Z_{2n} \Rightarrow \text{Dirac neutrinos} \tag{3}
\]

depending on the charges of neutrinos under the unbroken \(Z_{2n}\) symmetry. For neutrinos transforming non-trivially under any unbroken \(Z_{2n+1}\) symmetry, they must be Dirac particles. For neutrinos transforming non-trivially under the \(Z_{2n}\) symmetry, they can be Majorana if and only if \(\nu \sim \omega^n\). For any other transformation neutrinos will be Dirac particles. Thus, from a symmetry point of view, in contrast to popular belief, the Majorana neutrinos are the special ones, emerging only

\[3\] There is an additional accidental global \(U(1)_B\) symmetry associated to conserved Baryon number. While B and L are separately anomalous at the quantum level, there are anomaly free combinations, such as \(U(1)_{B-L}\). For simplicity here we discuss only \(U(1)_L\), though our argument remains valid for \(U(1)_{B-L}\).
for certain transformation properties under the unbroken residual $Z_{2n}$ symmetry.

Now the simplest $Z_m$ group to which the $U(1)_L$ can break is $Z_2$. This case is special, as it only offers two possibilities for neutrino transformation i.e. $\nu \sim +1$ or $-1$, both of which satisfy Eq. (2) and only allows for Majorana neutrinos. Breaking $U(1)_L$ to $Z_2$ is quite simple, through a Majorana mass term $\nu \nu$ arising effectively from new physics, as is the case of Weinberg’s dimension 5 operator $L^c \Phi L$ [33]. Most popular in the literature, this case covers a big chunk of model setups, which typically involve breaking of lepton number to a residual $Z_2$ symmetry. This also induces a nonzero $0\nu2\beta$ decay amplitude, as this decay is now allowed by the symmetry. The converse is also true, namely, if the $0\nu2\beta$ decay process is allowed, it always implies that lepton number is broken and the associated new physics is bound to generate Majorana mass terms [3]. Notice that, since the higher $0\nu2n\beta$ beta process are also allowed by the residual $Z_2$ symmetry, they all will also occur through “multiples” of the basic $0\nu2\beta$ process, $0\nu2n\beta \equiv n(0\nu2\beta)$ and thus we have $\Gamma_{0\nu2n\beta} \ll \Gamma_{0\nu2\beta}$.

![Figure 2](image2.png)

**Figure 2.** $0\nu4\beta$ arising as a double $0\nu2\beta$ process

We now turn to the case of $U(1)_L$ broken to higher symmetries, with neutrinos transforming non-trivially under the residual $Z_m$ symmetry. Clearly if $U(1)_L$ breaks to an $Z_{2n+1}$ symmetry, the lowest possible allowed neutrinoless beta decay process will be $0\nu(2n + 1)\beta$, where $n$ is a positive integer. But such processes are forbidden, as can be easily seen. Consider, for simplicity $0\nu3\beta$. This process would require us to write down a 9-fermion operator, which is of course not possible. Hence in such cases no neutrinoless beta decay of any order below $0\nu(2n+1)\beta$ are possible and neutrinos can only be Dirac particles [34–35].

The more interesting case is when $U(1)_L$ breaks to even residual $Z_{2n}$ symmetries, with $n > 1$. As already mentioned, in such cases both Dirac and Majorana neutrinos are possible, depending on how they transform under the $Z_{2n}$ symmetry. Also, irrespective of the Dirac or Majorana nature of neutrinos, if $U(1)_L$ breaks to an even residual $Z_{2n}$ symmetry, there is an associated $0\nu2n\beta$ processes allowed by the residual symmetry. However, an important distinction comes for the case of Dirac or Majorana neutrinos. As mentioned above, if neutrinos transform as $\omega^n$ under the $Z_{2n}$ symmetry, they must be Majorana particles. Moreover, in this case not only the $0\nu2n\beta$ process is allowed, but all other lower dimensional $0\nu2n_1\beta$ processes, where $n_1 < n$ is a positive integer, are also allowed. However, if neutrinos are Dirac particles, then for the case of $Z_{2n}$ symmetry, it follows that $\nu \sim \omega^n$. This implies that the lowest process allowed by $Z_{2n}$ symmetry is $0\nu2n\beta$ decays, all other lower dimensional processes being forbidden by the unbroken residual $Z_{2n}$ symmetry.

![Figure 3](image3.png)

**Figure 3.** The quadruple beta decay process is allowed by a residual $Z_4$ symmetry irrespective of the nature of neutrinos.

This is better illustrated by the simple example of $U(1)_L$ breaking to a $Z_4$ residual symmetry, called quarticity. Such a breaking has been accomplished within concrete realistic gauge models [29–32]. As already mentioned, in this case both Dirac as well as Majorana neutrinos can be Dirac particles.

---

4 Notice that the Majorana mass term might be generated at the loop level, and need not be the dominant source of $0\nu2\beta$ decay.

5 Notice that although the lowest $0\nu(2n + 1)\beta$ is forbidden, the higher dimensional $0\nu(2n + 1)\beta$ processes ($n$ is a even integer and $\alpha \geq 2$) are still allowed by $Z_{2n+1}$ symmetry.
neutrinos are possible. Neutrinos will be Dirac if they transform as $\omega$ or $\omega^2$; with $\omega^4 = 1$. They will be Majorana otherwise, if they transform trivially or transform as $\omega^2$ under $Z_4$. However, the quadruple beta decay $0\nu4\beta$ illustrated in Fig. 3 is always allowed, irrespective of the nature of neutrinos.

Notice that, if neutrinos are Majorana particles transforming as $1, \omega^2$ under the $Z_4$ symmetry, the lower dimensional $0\nu2\beta$ diagram of Fig. 1 is also allowed by the $Z_4$ symmetry. By dimensional power counting one sees that the $0\nu2\beta$ decay is induced by a dim-9 operator, whereas $0\nu4\beta$ decay is induced by dim-18 operator. Barrer extremely fine tuned cancellations, one naively expects that $\Gamma_{0\nu2\beta} \gg \Gamma_{0\nu4\beta}$. We estimate $R = \Gamma_{0\nu2\beta}/\Gamma_{0\nu4\beta}$ for two simple cases: (a) $0\nu2\beta$ and $0\nu4\beta$ induced by effective $d = 9$ and $d = 18$ operators, respectively. And, (b) $0\nu2\beta$ induced by a Majorana neutrino mass, while $0\nu4\beta$ is induced by the “lepton quarticity” $d = 6$ operator. We find for case (a):

$$R = \frac{Q_{3\beta}(\frac{q}{\Lambda})^2 q^6}{Q_{4\beta}(\frac{q}{\Lambda^2})^2 q^{18}} \sim 10^{82},$$

while case (b) gives:

$$R = \frac{Q_{3\beta}(\frac{G_{F}(\nu_{L})}{\Lambda})^2 q^2}{Q_{4\beta}(\frac{G_{F}}{\Lambda^2})^2 q^{10}} \sim 10^{30}.$$  

Here $Q_{3\beta}$ and $Q_{4\beta}$ are the Q-values of the decays, both of order MeV. $G_F$ is the Fermi constant, $q \simeq 0.1$ GeV is the typical momentum transfer in the nucleus and $\Lambda$ is the scale characterizing the new physics. The numbers correspond to $\Lambda \sim 1$ TeV and for the mass mechanism we have included that the resulting neutrino mass should be at the level of the current experimental bound.

Thus for Majorana neutrinos one naively expects to first see neutrinoless double beta decay, if at all. In contrast, for Dirac neutrinos, the dim-18 neutrinoless quadruple beta decay process is still allowed by $Z_4$ symmetry, while “conventional” neutrinoless double beta decay process is forbidden. Therefore, barring exceptional cases, if future experiments were to observe neutrinoless quadruple beta decay \cite{28} without a positive neutrinoless double beta decay signal, then neutrinos should be Dirac particles. This conclusion can be easily generalized to higher $Z_m$ symmetries and higher $0\nu2n\beta$ decays.

Another important conclusion is that, for neutrinos to be Majorana particles, lepton number $U(1)_L$ must be broken to an even $Z_{2n}$ subgroup under which neutrinos must transform in a very special way. Such possible “special nature” of Majorana neutrinos following from symmetry considerations is at odds with popular prejudices. In such a case all $0\nu2n\beta$ neutrinoless beta decay processes can be potentially induced, as they are all allowed by the $Z_{2n}$ symmetry. If neutrinos do not transform appropriately then they must be Dirac particles. In such a case the lowest possible neutrinoless beta decay process allowed by symmetry is $0\nu2n\beta$ instead of the “conventional” $0\nu2\beta$ decay. All lower dimensional neutrinoless beta decay process are forbidden by $Z_{2n}$ symmetry. If, by contrast, $U(1)_L$ is broken to an odd $Z_{2n+1}$ symmetry with neutrinos transforming non-trivially under it, then neutrinos are necessarily Dirac particles and all neutrinoless beta decay process of any dimension lower than $0\nu2(2n + 1)\beta$ decay are forbidden. It may also happen that $U(1)_L$ is either completely broken with no residual subgroup or is broken in such a way that the neutrinos transform trivially under the residual discrete lepton number $Z_m$ symmetry. In either of these cases, neutrinos can again be Majorana and all $0\nu2n\beta$ processes, including the $0\nu2\beta$ decay are allowed by symmetry. In such a scenario also, on dimensional grounds, one should expect to first observe $0\nu2\beta$ decay before observing any higher $0\nu2n\beta$ process.

The whole discussion above leads to an important conclusion concerning the nature of neutrinos, namely, if we first observe a higher $0\nu2n\beta$ decay, unaccompanied by a lower dimensional neutrinoless beta decay signal, such as $0\nu2\beta$ decay, that would be a strong indication in favour of the Dirac nature of at least one neutrino. This statement holds in general, with the possible exception of very special cancellations as present say, in the quasi-Dirac neutrino situation\cite{26,27}, which would require fine tuning at the $\sim 10^{-30}$ level. In contrast, if neutrinos are Majorana particles, then we should first observe $0\nu2\beta$ decay, if at all, well before observing any higher dimensional $0\nu2n\beta$ decays. In short, we have argued that, should a $0\nu4\beta$ decay signal ever be established, unaccompanied by $0\nu2\beta$ decays, then one would rule out Majorana neutrinos.

ACKNOWLEDGMENTS

We wish to thank S.C. Chulil for helpful discussions. Work supported by the Spanish grants FPA2017-85216-P and SEV-2014-0398 (MINECO), and PROMETEOII/2014/084 (Generalitat Valenciana). The Feynman diagrams are drawn using Jaxodraw\cite{56}.
