Sources and Distributions of Dark Matter*

Pierre Sikivie

I. Newton Institute, University of Cambridge, Cambridge, CB3 0EH, UK
and
Physics Department, University of Florida, Gainesville, FL 32611, USA

In the first section, I will try to convey a sense of the variety of observational inputs that tell us about the existence and the spatial distribution of dark matter in the universe. In the second section, I will briefly review the four main dark matter candidates, taking note of each candidate’s status in the world of particle physics, its production in the early universe, its effect upon large scale structure formation and the means by which it may be detected. Section 3 concerns the energy spectrum of (cold) dark matter particles on earth as may be observed some day in a direct detection experiment. It is a brief account of work done in collaboration with J. Ipser and, more recently, with I. Tkachev and Y. Wang.

*To appear in the Proceedings of the Conference “Trends in Astroparticle Physics”, Stockholm, Sweden, Sept. 22–25, 1994, Nucl. Phys. B. Proc. Supplements, edited by L. Bergstrom, P. Carlson, P.O. Hulth and H. Snellman.
Ref. [1] is a list of works which I have consulted in preparing this rather cursory overview, and which the reader should turn to for more complete and in-depth information.

I. DARK MATTER OBSERVATIONS

In 1932, Oort[2] studied the motion of galactic disk stars in the vertical direction, i.e., perpendicular to the disk. By applying a version of the virial theorem to the distribution of vertical star velocities, he obtained an estimate of the density of the galactic disk in the solar neighborhood:

\[ \rho_{\text{disk}} \simeq 1.2 \times 10^{-23} \text{gr/cm}^3. \]  

(1.1)

On the other hand, if one adds up the densities of all the matter “seen” in stars and interstellar gas, plus what is expected from stellar remnants, mainly white dwarfs, one finds considerably less than the dynamical estimate of Eq. (1.1), of order half thereof. So there is dark matter in the galactic disk. Because this dark matter is in the disk rather than the halo, we expect it to be dissipative, which means in all probability that it is baryonic dark matter.

In 1933, Zwicky[3] used measurements of the line-of-sight velocities of galaxies in the Coma cluster to estimate the mass of that cluster using the virial theorem. The result he obtained in this way is approximately 400 times the mass inferred by counting the number of galaxies in the cluster and assigning to each a mass \( \sim 10^{11} M_\odot \) typical for the luminous part of a spiral galaxy. The masses of the luminous parts of some spiral galaxies had already been determined by measuring their rotation curves up to distances from their centers of order their disk radii. Smith[4] obtained a similar result for the Virgo cluster.

In 1973, Ostriker and Peebles[5] pointed out that the tendency of galactic disks to be unstable towards a large-scale bar mode can be cured by assuming the existence of a spherical halo of dark matter with mass within the disk radius of order the disk mass \( (6 \times 10^{10} M_\odot \text{ for our galaxy}) \). Thus the galactic mass within a sphere with radius equal to the disk radius would be roughly half in the disk and half in an unseen spherical halo. Of course, by Birkhoff’s theorem, the argument does not say anything about halo matter outside the disk radius.

During the seventies, the rotation curves of spiral galaxies were measured[6] over much larger distances than before, in many cases extending the rotation curve to distances several times the disk radius. In all cases, the rotation velocity was found to be constant (i.e., independent of radius \( r \)) or slightly rising, up to the last measured point. Balancing centrifugal and gravitational forces, one has

\[ \frac{GM(r)}{r^2} = \frac{v_{\text{rot}}(r)^2}{r} \]  

(1.2)

where \( M(r) \) is the mass interior to \( r \) and \( v_{\text{rot}}(r) \) is the rotation velocity at \( r \). If only luminous matter were contributing to the galactic mass, we would have \( M(r) \sim \text{constant} \) and hence \( v_{\text{rot}}(r) \sim r^{-\frac{1}{2}} \), for \( r > \text{disk radius} \). Instead, the data show \( v_{\text{rot}}(r) \sim \text{constant} \).
there and hence $M(r) \sim r$. The implication is that there is a halo of dark matter whose density $\rho_{dm}(r) \sim \frac{1}{r^2}$ at large $r$. The halo distribution is usually modeled by the function

$$\rho_{dm}(r) = \frac{\rho_{dm}(0)}{1 + \left(\frac{r}{a}\right)^2}$$

where $a$ is called the core radius. For our own galaxy, $v_{rot} \simeq 220\text{km/s}$, $a \simeq \text{few kpc}$, and :

$$\rho_{dm}(r_\odot) \simeq \frac{1}{2} \cdot 10^{-24}\text{gr/cm}^3$$

where $r_\odot \simeq 8.5\text{kpc}$ is our distance to the galactic center. The estimate (1.4) of the local dark halo density is based upon models\cite{7} of the galactic mass distribution developed in the early 80’s by Bahcall and Soneira, and Caldwell and Ostriker. However, the discovery of an abundance of microlensing events in the direction of the galactic bulge has stimulated a lot of recent work on the galactic mass distribution and this will likely result in a more precise determination of the galactic halo parameters.

In 1972, J. Einasto, A. Kaasik and E. Saar\cite{8} studied 105 pairs of galaxies, the members of each pair being close on the sky and assumed to be gravitationally bound to each other. They compared the distance between each pair to its relative velocity to obtain an estimate of its reduced inner mass. Of course, only the line-of-sight velocities and the angular projections of distance onto the sky are measured and therefore an average over many pairs must be performed to try and eliminate the effects of projection and ignorance of orbit eccentricities. At any rate, these authors find that the galactic mass increases with distance, approximately linearly, up to masses of order $10^{13}M_\odot$. This study and others\cite{1} imply that galactic halos extend very far out. I do not know of anything that contradicts the assumption that galactic halos extend all the way to radii of order 1–2 Mpc, where the halo density, falling off as $\frac{1}{r^2}$, becomes equal to the average intergalactic dark matter density.

There are a number of methods to estimate the average dark matter density on scales larger than the typical intergalactic or intercluster distance ($\gtrsim 10\text{ Mpc}$). Density perturbations on such large scales are still in the linear regime of their growth by gravitational instability. Let us describe a particular method. If a region has an overdensity $\delta \rho$ in excess of the average density $\rho$, neighboring galaxies will have an excess gravitational attraction towards that region and consequently deviate from perfect Hubble flow. One writes:

$$\vec{v} = H_0\vec{r} + \vec{v}_p$$

where $H_0$ is the Hubble expansion rate, $\vec{r}$ is the position relative to the center of an overdensity and $\vec{v}_p$ is called the peculiar velocity. It is found\cite{9} that in the linear regime around a single, spherically symmetric overdensity

$$\vec{v}_p = -H_0\frac{1}{3}\Omega^{0.6}\frac{\delta \rho}{\rho},$$

where

$$\Omega = \frac{\rho}{\rho_{crit}} = \frac{8\pi G \rho}{3H_0^2}.$$
\( \rho_{\text{crit}} \) is the critical density for closing the universe. The \( \Omega \) dependence on the RHS of Eq. (1.6) is a close fit to the actual \( \Omega \) dependence for vanishing cosmological constant. Eq. (1.6) affords a way to determine \( \Omega \) by measuring peculiar velocities \( \vec{v}_p \) and comparing them with observed overdensities \( \frac{\delta \rho}{\rho} \). However, \( \frac{\delta \rho}{\rho} \) cannot be measured directly. What can be done is count galaxies, measuring their average density \( n_G \) and local overdensities \( \delta n_G \). Unfortunately, the relationship between \( \frac{\delta \rho}{\rho} \) and \( \frac{\delta n_G}{n_G} \) is not known. It is parametrized by a fudge factor \( b \), called the “bias parameter”:

\[
\frac{\delta n_G}{n_G} = b \frac{\delta \rho}{\rho}.
\]

(1.8)

So the method of peculiar velocities actually measures \( \frac{\Omega^{0.6}}{b} \). A number of authors\(^{[10]} \) have analyzed galaxy distributions in this way with the result:

\[
\frac{\Omega^{0.6}}{b} = 1 \pm 0.3.
\]

(1.9)

Most attempts to determine the bias parameter from first principles yield \( b > 1 \). Eq. (1.9) suggests then that, when measured on the largest scales, the value of the density parameter \( \Omega \) is consistent with a critically closed universe (\( \Omega = 1 \)). On the other hand, the measurements on these large scales are very imprecise.

Of course, \( \Omega = 1 \) is strongly favored on ‘theoretical’ grounds. A \( \Omega \neq 1 \) universe will deviate from \( \Omega = 1 \) more and more as time goes on, in pretty much the same way as a pencil standing nearly vertically on its point will fall over. For a universe to stay near \( \Omega = 1 \) for a long time, it has to be extraordinarily close to \( \Omega = 1 \) to start with. This problem of initial conditions for our universe is called the flatness (or age) problem. It may be neatly solved by assuming that there is, at very early times, a brief epoch during which the energy density is dominated by vacuum energy density and, as a result, the universe expands at an exponential rate. After this “inflation”, \( \Omega = 1 \) with tremendous precision. The inflationary cosmology has many other attractive features as well\(^{[11]} \) So there are compelling reasons to believe that \( \Omega = 1 \). Whether observations support this prejudice is not obvious, although it seems fair to say that they are in rough agreement with it. Luminous matter contributes \( \Omega_{\text{lum}} \simeq 3.10^{-3} \) to \( 6.10^{-3} \). Dark matter in galactic halos and in clusters of galaxies contributes \( \Omega_{\text{gal}} \simeq 0.02 \) to 0.2. Finally, as Eq. (1.9) and the results of other observations on the largest scales studied suggest, there may be enough dark matter not associated with galaxies or clusters of galaxies (in voids, say) to yield \( \Omega = 1 \).

The success of nucleosynthesis\(^{[12]} \) in producing the primordial abundances of light elements requires that the contribution \( \Omega_B \) of baryons satisfies:

\[
0.011 \leq 0.011h^{-2} \leq \Omega_B \leq 0.019h^{-2} \leq 0.12
\]

(1.10)

where \( h \) parametrizes the present Hubble expansion rate

\[
H_0 = 100h \cdot \frac{km}{s \ Mpc}.
\]

(1.11)
Measurements of $H_0$ are in the range of $0.4 \leq h \leq 1$. Since $\Omega_{lum} < 0.006$, Eq. (1.10) implies that some baryons are dark. Recall that there is dark matter associated with the disk of our galaxy. Because it is in the disk rather than in a halo, this dark matter must be dissipative which presumably means that it is baryonic. (I am assuming that we have necessarily discovered the existence of any form of matter sufficiently abundant and sufficiently strongly interacting to be the disk dark matter. Disk matter must have sufficiently strong interactions to have concentrated in a disk by dissipating its energy while conserving its angular momentum.) Moreover, the recent discovery of microlensing in the direction of the galactic bulge indicates that our disk has a population of low mass ($\sim 0.1 M_\odot$) compact objects. These may be ‘brown dwarfs’, i.e. stars too low in mass to shine by nuclear burning, which are the most likely hiding place for dark baryons.$^{[13]}$

Eq. (1.10) is compatible with the assumption that all the dark matter in galaxies and clusters of galaxies is in baryons. However, we will see in the next section that this economical hypothesis runs into difficulties in most scenarios of galaxy formation. Eq. (1.10) is not compatible with $\Omega_B = 1$. Hence, if one believes in inflation and in standard nucleosynthesis—and both of these are very well motivated—one must conclude that our universe is dominated, at the 90% level, by a form of dark matter which is not baryonic.

2. DARK MATTER CANDIDATES

2.1. Baryons

They are known to exist. Moreover, they are known to be a form of dark matter, and the nucleosynthesis constraints allow all dark matter associated with galaxies to be in baryons. Hence, a conservative hypothesis may be that $\Omega \simeq \Omega_B \simeq 0.2$. (Inflation is given up then.) However, that particular scenario has serious difficulties with galaxy formation. The point is that the density perturbations in the baryon distribution that should produce galaxies cannot grow by gravitational instability till after the epoch of recombination at a temperature $T_{rec} \simeq 4.10^3$ K. Recombination is when electrons combine with ions to form neutral atoms. Before recombination, the baryons are in close thermal contact with the photon gas. Because the latter has pressure, the Jean’s mass is large:

$$M_J \sim 1.8 \cdot 10^{16} M_\odot (\Omega_B h^2)^{-2} \quad \text{for} \quad T > T_{rec}$$  \hspace{1cm} (2.1)

in this scenario. The Jean’s mass sets the critical scale below which density perturbations do not grow. After recombination, $M_J \sim 0$ and density perturbations in the matter distribution grow on all scales at the rate $\frac{\delta \rho}{\rho} \sim R$ where $R$ is the cosmological scale factor. The temperature drops as $T \sim R^{-1}$. Galaxies form when $\frac{\delta \rho}{\rho} \sim 1$ on the appropriate mass scale, about $10^{12} M_\odot$. For this to happen before the present, the density perturbations in the baryon distribution at recombination must therefore have a minimum amplitude:

$$\left. \frac{\delta \rho}{\rho} \right|_{rec} > \frac{R_{rec}}{R_0} = \frac{T_0}{T_{rec}} = \frac{2.73K}{4.10^3 K} = 0.7 \cdot 10^{-3}.$$  \hspace{1cm} (2.2)
One should expect accompanying photon temperature fluctuations of the same order of magnitude. These would contradict the upper limits ($\frac{\delta T}{T} < 10^{-5}$) on the microwave background anisotropy. The scenario has additional difficulties due to the diffusion of photons, which tends to erase adiabatic fluctuations in the baryon number density.\[15\]

The above model ($\Omega \simeq \Omega_B \simeq 0.2$) came first historically and, as we just saw, it led to the expectation of large CMBR anisotropies ($\frac{\delta T}{T} \simeq 10^{-3}$ or $10^{-4}$) which got into more and more severe disagreement with the observations. This outcome provided a strong impetus for the development of models with cold dark matter (CDM). Indeed, the CDM candidates decouple from photons and baryons. As a result, the density perturbations in CDM start to grow by gravitational instability as soon as $t > t_{eq}$ where $t_{eq}$ is the time of equality of the radiation and matter energy densities ($\rho_{rad.} \sim R^{-4}$, $\rho_{matt.} \sim R^{-3}$, $\rho_{rad.} = \rho_{matt.}$ at $t_{eq}$). In models where $\Omega_{CDM}$ is close to one, $t_{eq}$ comes well before $t_{rec}$. Because there is more time for their growth, the primordial density perturbations in these models are smaller than the RHS of Eq. (2.2).

At any rate, as we saw, some baryonic dark matter is known to exist and there may be large amounts of it, up to $\Omega_B \simeq 0.2$. As already mentioned, a likely hiding place\[14\] for these dark baryons is "brown dwarfs", i.e., stars too low in mass to burn by nuclear fusion. Paczynski\[16\] pointed out that objects of this kind, generically called MACHOs for massive compact halo objects, can be searched for by looking for the gravitational lensing of background stars by MACHOs that happen to pass close to the line of sight. Three collaborations\[13\] have reported compelling candidates for such microlensing events. This very exciting development is reviewed by Ansari\[17\] at this meeting.

### 2.2. Neutrinos

Neutrinos decouple in the early universe at a temperature $T_D$ of order a few MeV. After that, each neutrino moves freely and hence its momentum decreases with the universe’s expansion according to: $p_\nu \sim R^{-1}$. Thus, neglecting inhomogeneities, the neutrino phase-space density is given by:

$$N(\vec{r}, \vec{p}) = \frac{g_\nu}{(2\pi)^3} \frac{1}{e^{\frac{p}{T_{\nu}(t)}} + 1}$$  \hspace{1cm} (2.3)

where $T_{\nu}(t) \equiv T_D \frac{R(t)}{R_D}$ and where $g_\nu$ is the number of neutrino spin degrees of freedom. In the standard model, each neutrino flavor contributes $g_\nu = 2$. $T_{\nu}(t)$ is usually called the ‘neutrino temperature’ although the distribution (2.3) will deviate from a thermal one if the neutrino is massive. The photon temperature also decreases according to $T_{\gamma} \sim R^{-1}$ most of the time. If this were always true, $T_{\gamma}$ and $T_{\nu}$ would remain equal. However, at a temperature of order 1 MeV, electrons and positrons annihilate, thus reheating the photon gas. Because $e^+ e^-$ annihilation is adiabatic (the process goes back and forth very rapidly compared to the Hubble expansion rate), conservation of entropy allows one to relate the photon temperature after annihilation to the temperature the photons would have had if there had been no annihilation, which is the neutrino temperature.
This yields the famous result:

$$T_{\nu} = \left( \frac{4}{11} \right)^{\frac{1}{3}} T_\gamma$$

(2.4)

after $e^+e^-$ annihilation. Since $T_\gamma = 2.73$ K today, $T_{\nu} = 1.95$ K. This implies in particular that the number density of neutrinos and anti-neutrinos today $n_{\nu+\bar{\nu}}(t_0) = 113/\text{cm}^3$ per neutrino flavor. From this one readily finds that, in extensions of the standard model where the neutrinos have small Majorana masses, their contribution to the cosmological energy density is:

$$\Omega_{\nu} h^2 = \sum_i \frac{m_{\nu_i}}{94 eV},$$

(2.5)

where the sum is over flavors. Although none of these neutrinos have been observed directly or indirectly, we are confident that they are there because the theoretical arguments for their existence are very simple and conservative.

So the next assumption we will consider is that neutrinos constitute most of the dark matter and hence that they dominate the cosmological energy density. This scenario also tends to run into trouble with galaxy formation. As was already mentioned, perturbations in the matter density only start to grow after matter-radiation equality. In the present scenario, equality occurs when the temperature is of order the neutrino mass. Before that the neutrinos are relativistic and their “free-streaming” erases all density perturbations in the neutrino fluid on length scales less than the free-streaming distance, i.e., the distance a typical neutrino travels from the Big Bang till the time of equality. The corresponding mass scale is:

$$M_{\nu} = 4 \cdot 10^{15} M_\odot \left( \frac{30 eV}{m_{\nu}} \right)^2,$$

(2.6)

which is of order the mass in large galactic clusters. The resulting spectrum of primordial density perturbations is heavily suppressed on all mass scales less than $M_{\nu}$, including the mass scale ($\sim 10^{12} M_\odot$) of individual galaxies. If such a spectrum is used as input in computer simulations of large scale structure formation, a poor fit to the observations results. The difficulties neutrinos have with large scale structure formation may be eased if topological defects, such as cosmic strings, are the source of the density perturbations because in this case, the density perturbations continue to be created after $t_{eq}$. Liouville’s theorem tells us that the phase-space density is constant following the motion. Eq. (2.3) therefore implies that the neutrino phase-space density can nowhere be larger than $\frac{1}{2} g_{\nu}$. On the other hand, for neutrinos to constitute a galactic halo, their velocities must be less than the escape velocity, which for our own galaxy is of order $10^{-3}c$. The upper limits on the phase-space density and on the velocity imply the following upper limit on the physical density:

$$\rho_{\nu,\text{max}} = \frac{110^{-24} gr}{cm^3} \left( \frac{v_{\text{max}}}{10^{-3}c} \right)^3 \left( \frac{m_{\nu}}{19eV} \right)^4.$$

(2.7)
Eq. (1.3), (1.4), (2.5) and (2.7) tell us that neutrinos can only barely be packed tightly enough to constitute the Milky Way halo. Dwarf galaxies also have dark matter halos but smaller escape velocities. For these galaxies, the neutrino phase-space constraint is severely violated.\textsuperscript{[23]}

2.3. WIMPs

WIMPs is an acronym for Weakly Interacting Massive Particles. To be specific, consider a massive neutral lepton \( L \). If its mass \( m_L \) is less than the temperature \( T_D \) at which it decouples from the thermal bath, then \( L \) behaves like a neutrino and its cosmological energy density is given by Eq. (2.5) or something very similar to it. However,\textsuperscript{[24]} if \( m_L \) exceeds \( T_D \), then for \( m_L > T > T_D \), the number density of \( L \) particles:

\[
n_L(T) = n_{eq}(T) = \frac{g_L}{(2\pi)^3} \int_0^\infty 4\pi p^2 dp \frac{1}{e^{\frac{m_L^2 + p^2}{T}} + 1}
\]

falls off exponentially, as \( e^{-\frac{m_L}{T}} \). The number density \( n_L(T) \) tracks its equilibrium value \( n_{eq}(T) \) as long as the annihilation rate of \( L \) particles exceeds the Hubble rate. Thus \( T_D \) is given by:

\[
\langle \sigma_{ann} v \rangle n_{eq}(T_D) \approx H(T_D)
\]

where \( \sigma_{ann} \) is the annihilation cross-section of \( L \) particles. In Eq. (2.9), the dominant dependence upon \( T_D \) is the exponential \( e^{-\frac{m_L}{T}} \) behaviour of \( n_{eq}(T_D) \). As a consequence, \( T_D \) is proportional to \( m_L \) up to logarithmic corrections. For cross-sections typical of weakly interacting particles, one finds:

\[
T_D \approx \frac{1}{20} m_L.
\]

Hence, the cosmological energy density in \( L \) particles today:

\[
\rho_L(t_0) = m_L n_L(t_0) = m_L n_L(t_D) \left( \frac{R_D}{R_0} \right)^3 \approx m_L \frac{H(T_D)}{\langle \sigma_{ann} v \rangle} \frac{N_0}{N_D} \left( \frac{T_0}{T_D} \right)^3
\]

where \( N_0 \) and \( N_D \) are the effective numbers of thermal degrees of freedom today and at the decoupling of \( L \) particles, conservation of entropy from \( t_D \) till \( t_0 \) having been assumed. Remarkably, the \( T_D \) dependence on the RHS of Eq. (2.11) cancels out because of Eq. (2.10) and because \( H(T_D) = \sqrt{\frac{8\pi G}{3} \rho(T_D)} = \sqrt{\frac{8\pi G}{3} N_D \frac{\pi^2 T_4}{m_0^4} T_D} \). As a result, the contribution of \( L \) particles, and more generally that of WIMPs, to the cosmological energy density depends almost exclusively upon their annihilation cross-section. One finds:

\[
\Omega_{\text{WIMP}} h^2 \approx \frac{6 \cdot 10^{-27}}{\langle \sigma_{ann} v \rangle} \text{cm}^3 \text{sec}.
\]

For the particular case of a heavy neutral lepton \( L \), one has \( \sigma_{ann} \sim \frac{G_F m_L^2}{m_L^2} \) for \( m_L \lesssim m_Z = 91.2 \text{ GeV} \) and \( \sigma_{ann} \sim \frac{\alpha^2}{m^2} \) for \( m_L \gtrsim m_Z \). In that case, \( \Omega_L h^2 \approx \frac{1}{m_L^2} \) in the
range few MeV < \( m_L < m_Z \) with \( \Omega_L h^2 = 1 \) for \( m_L \approx 2 \) GeV, and \( \Omega_L h^2 \sim m_L^2 \) when \( m_L \gtrsim m_Z \) with \( \Omega_L h^2 = 1 \) for \( m_L \approx 10 \) TeV. Note that if the WIMP is not its own anti-particle, there may be a WIMP-antiWIMP asymmetry, similar to the baryon asymmetry. In that case there is an additional contribution to \( \Omega_{\text{WIMP}} h^2 \), aside from the one given by Eq. (2.12).

The best motivated WIMP candidate is the lightest supersymmetric partner (LSP) in supersymmetric extensions of the standard model. Typically, this is a linear combination of the photino, the zino and the Higgsino.

Because WIMPs are non-relativistic from the moment of their decoupling, their free-streaming distance is very small and hence their free-streaming does not erase density perturbations on any relevant scales. For this reason, WIMPs are called Cold Dark Matter (CDM). In contrast, neutrinos, because of their large free-streaming distance, are called Hot Dark Matter (HDM). The assumptions of CDM, with \( \Omega_{CDM} \approx 1 \), and of a flat (Zel’dovich-Harrison) spectrum of primordial density perturbations yield a model of large scale structure formation\(^{[25]}\) which has been thoroughly tested and which has been, by and large, very successful. However, in light of the COBE measurement of the cosmic microwave anisotropy which is also a measurement of the primordial density perturbations on the largest scales observable, some modification of the pure CDM model may be required.

If WIMPs constitute the halo of our galaxy, they may be searched for on earth by looking for WIMP + nucleus elastic scattering in a laboratory detector.\(^{[26]}\) The nuclear recoil can be put into evidence by low temperature calorimetry, by ionization detection or by the detection of ballistic phonons.\(^{[27]}\) WIMPs may also be searched for by looking for the decay products (photons, anti-protons . . .) of WIMP annihilation in the halo of our galaxy\(^{[28]}\) or by looking for neutrinos produced by the annihilation of WIMPs that have been captured by the sun.\(^{[29]}\)

### 2.4. Axions

The axion is a hypothetical particle whose existence would insure that the strong interactions conserve \( P \) and \( CP \) in spite of the fact that other interactions violate those symmetries.\(^{[30]}\) Indeed, the action density of the standard model of elementary particles contains in general a term:

\[
L_\theta = \frac{\theta g_s^2}{32\pi^2} G_{\mu\nu}^a \tilde{G}_{\mu\nu}^a
\]  \hspace{1cm} (2.13)

where \( G_{\mu\nu}^a \) is the gluonic field strength, \( \tilde{G}_{\mu\nu}^a \) is the dual of \( G_{\mu\nu}^a \), and \( g_s \) is the QCD gauge coupling. If \( \theta \neq 0 \), non-perturbative QCD effects induce violations of \( P \) and \( CP \) in the strong interactions. No such violation has been observed. In particular, the upper limit on the neutron electric dipole moment requires \( \theta < 10^{-9} \). But there is no reason in the standard model for the parameter \( \theta \) to be small. This shortcoming has been called the “strong CP problem”.

Peccei and Quinn modified the standard model in such a way that the parameter \( \theta \) in Eq. (2.13) gets replaced by \( \frac{a(x)}{f_a} \) where \( a(x) \) is a dynamical pseudo-scalar field whose quantum is called the axion; \( f_a \) is a quantity with dimension of energy called the axion
decay constant. By construction, the vacuum expectation value of \(a(x)\) is indifferent except for those non-perturbative effects that make QCD depend upon \(\theta\). The latter produce an effective potential \(V(\theta) = V(\frac{a(x)}{f_a})\) whose minimum is at \(\theta = 0\). Thus by postulating an axion, \(\theta\) is allowed to relax to zero dynamically and the strong CP problem is solved.

The properties of the axion can be derived using the methods of current algebra. The axion mass is related to \(f_a\) by:

\[
m_a \simeq 0.6\text{eV} \frac{10^7\text{GeV}}{f_a}.
\]

All the axion couplings are inversely proportional to \(f_a\). Thus, a very light axion is also very weakly coupled.\(^{[31]}\) A priori, the value of \(f_a\), and hence that of \(m_a\), is arbitrary. However, astrophysical considerations\(^{[32]}\) and searches for the axion in high-energy and nuclear physics experiments\(^{[33]}\) rule out \(m_a > 10^{-3}\text{ eV}\). On the other hand, cosmology places a lower limit on \(m_a\) of order \(10^{-6}\text{ eV}\) by requiring that axions do not overclose the universe.\(^{[34]}\)

Indeed, for small masses, axion production in the early universe is dominated by a novel mechanism. The point is that the non-perturbative QCD effects that produce the effective potential \(V(\frac{a(x)}{f_a})\) are strongly suppressed at temperatures high compared to \(\Lambda_{\text{QCD}}\). At these high temperatures, \(\langle a(x)\rangle\) has arbitrary value. At \(T \simeq 1\text{ GeV}\), the potential \(V\) turns on and the axion field starts to oscillate about its CP conserving minimum. These oscillations do not dissipate in other forms of energy because, in the relevant mass range, the axion is too weakly coupled for that to happen. The oscillations of the axion field may be described as a fluid of axions. The typical momentum of these axions is the inverse of the correlation length of the axion field at \(T \simeq 1\text{ GeV}\). Since that correlation length is of order the horizon then, we have \(p_a \sim \frac{1}{f_1\text{GeV}} \sim \frac{1}{10^{-9}\text{sec}} \sim 10^{-9}\text{ eV}\). Thus, the axion fluid is very cold compared to the ambient temperature. Its contribution to the present cosmological energy density is found to be of order

\[
\Omega_a h^2 \simeq 0.3 \left( \frac{10^{-6}\text{eV}}{m_a} \right)^{\frac{2}{3}} \left( \frac{200\text{MeV}}{\Lambda_{\text{QCD}}} \right)^{\frac{3}{4}}.
\]

Several sources of uncertainty affect the relationship between \(\Omega_a h^2\) and \(m_a\), amongst which are the nature of the QCD phase transition and the contribution to \(\Omega_a h^2\) from cosmic axion strings.\(^{[35,36]}\) Also, if inflation occurs and the post-inflation reheating temperature is less than \(f_a\), then the axion field gets homogenized and there may be an accidental suppression of \(\Omega_a h^2\) because the axion field happens to lie everywhere close to the CP conserving minimum of \(V\). From the point of view of large scale structure formation, axions are cold dark matter since they are non-relativistic from the moment of their production during the QCD phase transition, as was emphasized above.

Axion dark matter may be searched for by stimulating the conversion of galactic halo axions to photons in a laboratory magnetic field.\(^{[37]}\) The relevant coupling is

\[
L_{a\gamma\gamma} = g_{\alpha} \frac{\alpha}{\pi} \frac{a}{f_a} \vec{E} \cdot \vec{B}
\]
where $g_\gamma$ is a model-dependent coupling constant of order one. If an electromagnetic cavity is permeated by a static approximately homogeneous magnetic field $\vec{B}_0$ and the resonant frequency of the lowest TM (relative to the direction of $\vec{B}_0$) mode equals the axion mass, some galactic halo axions will convert to quanta of that cavity mode. For $B_0 \sim 10$ Tesla and cavity volumes of order $1\text{m}^3$, the power from these $\alpha \rightarrow \gamma$ conversions becomes detectable in a sufficiently short amount of time to allow a search over a large axion mass range. Two pilot experiments using this technique have been carried out. At present, a second generation experiment is under construction at Lawrence Livermore National Laboratory that will be able to detect dark matter axions if their local density is equal to the local halo density given in Eq. (1.4) or higher, if $g_\gamma \geq 1$ and if $m_a$ is in the range $1.3\mu\text{eV} \leq m_a \leq 13\mu\text{eV}$.

3. THE PHASE-SPACE STRUCTURE OF COLD DARK MATTER HALOS

If a signal is found in the cavity detector of galactic halo axions, it will be possible to measure the energy spectrum with great precision and resolution because all the time that was previously used in searching for the signal can now be used to accumulate data. Hence there is good motivation to ask what can be learned about our galaxy from analyzing such a signal.

In many past discussions of dark matter detection on earth, it has been assumed that the dark matter particles have an isothermal distribution. Thermalization has been argued to be the result of a period of “violent relaxation” following the collapse of the protogalaxy. If it is strictly true that the velocity distribution of dark matter particles is isothermal, which seems to be a strong assumption, then the only information that can be gained from its observation is the corresponding virial velocity and our own velocity relative to its standard of rest. If, on the other hand, the thermalization is incomplete, a signal in a dark matter detector may yield additional information.

J.R. Ipser and I discussed the extent to which the phase-space distribution of cold dark matter particles is thermalized in a galactic halo and concluded that there are substantial deviations from a thermal distribution in that the highest energy particles have discrete values of velocity. There is one velocity peak on earth due to dark matter particles falling onto the galaxy for the first time, one peak due to particles falling out of the galaxy for the first time, one peak due to particles falling out of the galaxy for the second time, etc. The peaks due to particles that have fallen in and out of the galaxy a large number of times in the past are washed out because of scattering in the gravitational wells of stars, globular clusters and large molecular clouds. But the peaks due to particles which have fallen in and out of the galaxy only a small number of times in the past are not washed out.

If the fraction of the local dark matter density which is in these velocity peaks is sufficiently large, a direct dark matter search, such as the LLNL experiment, may be made more sensitive by having it look specifically for velocity peaks. I. Tkachev, Y. Wang and I have been studying galactic halo formation with the purpose of obtaining estimates of the sizes and locations of the velocity peaks. To this end, we have generalized the secondary infall model of galactic halo formation to include angular momentum of the dark matter particles. This new model is still spherically symmetric and it has self-similar solutions. We find that the typical fraction of the local cold dark
matter density in any one of the highest energy velocity peaks is several percent. A forthcoming paper will give estimates of the highest energy peaks as a function of the amount of angular momentum and other model parameters.

References

1. J. Binney and S. Tremaine, *Galactic Dynamics*, Princeton U. Press, 1987; E.W. Kolb and M.S. Turner, *The Early Universe*, Addison-Wesley, 1988; P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton U. Press, 1993; V. Trimble, Ann. Rev. Astron. Astroph. 25 (1987) 425; C. Jones and A. Melissinos, editors, *Cosmic Axions*, World Scientific, 1989; M. Srednicki, Editor, *Particle Physics and Cosmology: Dark Matter*, North-Holland., 1990; M. Turner, Phys. Scripta T36 (1991) 167.

2. J. Oort, Bull. Astr. Inst. Netherlands 6 (1932) 249.

3. F. Zwicky, Helv. Phys. Acta 6 (1933) 110.

4. S. Smith, Ap. J. 83 (1936) 23.

5. J.P. Ostriker and P.J.E. Peebles, Ap. J. 186 (1973) 467.

6. V.C. Rubin and W.K. Ford, Ap. J. 159 (1970) 379; D.H. Rogstad and G.S. Shostak, Ap. J. 176 (1972) 315; V.C. Rubin, W.K. Ford and N. Thonnard, Ap. J. 238 (1980) 471.

7. J.N. Bahcall and R.M. Soneira, Ap. J. Suppl. 44 (1980) 73; J.A.R. Caldwell and J.P. Ostriker, Ap. J. 251 (1981) 61.

8. J. Einasto, A. Kaasik and E. Saar, Nature 250 (1974) 309.

9. P.J.E. Peebles, *Principles of Physical Cosmology*, Princeton U. Press, 1993.

10. A. Dressler et al., Ap. J. 313 (1987) L37; E. Bertschinger and A. Dekel, Ap. J. 336 (1989) L5; M. Rowan-Robinson et al., MNRAS 247 (1990) 1.

11. For a review, see for example: *Inflationary Cosmology*, edited by L. Abbott and S.-Y. Pi, World Scientific, 1986.

12. See the review by L. Krauss in these proceedings.

13. C. Alcock et al., Nature 365 (1993) 621; F. Aubourg et al., Nature 365 (1993) 623; A. Udalski et al., Acta Astr. 43 (1993) 289.

14. B. Carr, Ann. Rev. Astron. Astroph. 32 (1994) 531.

15. J. Silk, Ap. J. 211 (1977) 638; M.J. Rees and J.P. Ostriker, MNRAS 179 (1977) 541.

16. B. Paczynski, Ap. J. 304 (1986) 1.

17. R. Ansari, these proceedings.

18. S.S. Gershtein and Y.B. Zel’dovich, JETP Lett. 4 (1966) 120.

19. J.R. Bond, G. Efstathion and J. Silk, Phys. Rev. Lett. 45 (1980) 1980.

20. S.D.M. White, C.S. Frenk and M. Davis, Ap. J. 274 (1983) L1.
21. For a review, see A. Vilenkin and E.P.S. Shellard, *Cosmic Strings and other Topological Defects*, Cambridge U. Press, 1994.

22. S. Tremaine and J.E. Gunn, Phys. Rev. Lett. 42 (1979) 407.

23. M. Aaronson, Ap. J. 266 (1983) L11; D.N.C. Lin and S.M. Faber, Ap.J. 266 (1983) L21.

24. P. Hut, Phys. Lett. B69 (1977) 85; B.W. Lee and S. Weinberg, Phys. Rev. Lett. 39 (1977) 165; M.I. Vysotskii, A.D. Dolgov and Y.B. Zel’dovich, JETP Lett. 26 (1977) 188.

25. P.J.E. Peebles, Ap.J. 263 (1982) L1; J. Ipser and P. Sikivie, Phys. Rev. Lett. 50 (1983) 925; G.R. Blumenthal, S.M. Faber, J.R. Primack and M.J. Rees, Nature 311 (1984) 517; M. Davis, G. Efstdathiu, C.S. Frenk and S.D.M. White, Ap. J. 292 (1985) 371; S.D.M. White, C.S. Frenk, M. Davis and G. Efstdathiu, Ap. J. 313 (1987) 505.

26. M. Goodman and E. Witten, Phys. Rev. D31 (1985) 3059; I. Wasserman, Phys. Rev. D33 (1986) 2071; K. Griest, Phys. Rev. D38 (1988) 2357.

27. S.P. Ahlen et al., Phys. Lett. B195 (1987) 603; D.O. Caldwell et al., Phys. Rev. Lett. 61 (1988) 510; P.F. Smith and J.D. Lewin, Phys. Rep. 187 (1990) 203.

28. J. Silk and M. Srednicki, Phys. Rev. Lett. 53 (1984) 624.

29. W.H. Press and D.N. Spergel, Ap. J. 296 (1985) 679; J. Ellis, R. Flores and S. Ritz, Phys. Lett. B198 (1987) 393.

30. R.D. Peccei and H. Quinn, Phys. Rev. Lett. 38 (1977) 1440 and Phys. Rev. D16 (1977) 1791; S. Weinberg, Phys. Rev. Lett. 40 (1978) 223; F. Wilczek, Phys. Rev. Lett. 40 (1978) 279.

31. J.E. Kim, Phys. Rev. Lett. 43 (1979) 103; M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B166 (1980) 493; M. Dine, W. Fischler and M. Srednicki, Phys. Lett. 104B (1981) 199; A.P. Zhitnitskii, Sov. J. Nucl. 31 (1980) 260.

32. For a review, see: M.S. Turner, Phys. Rep. 197 (1990) 67; G.G. Raffelt, Phys. Rep. 198 (1990) 1.

33. For a review, see: J.E. Kim, Phys. Rep. 150 (1987) 1; H.-Y. Cheng, Phys. Rep. 158 (1988) 1; R.D. Peccei, in *CP-Violation*, edited by C. Jarlskog, World Scientific, 1989.

34. L. Abbott and P. Sikivie, Phys. Lett. 120B (1983) 133; J. Preskill, M. Wise and F. Wilczek, Phys. Lett. 120B (1983) 127; M. Dine and W. Fischler, Phys. Lett. 120B (1983) 137.

35. R. Davis, Phys. Rev. D32 (1985) 3172 and Phys. Lett. 180B (1986) 225; A. Vilenkin and T. Vachaspati, Phys. Rev. D35 (1987) 167; R.L. Davis and E.P.S. Shellard, Nucl. Phys. B324 (1989) 167; A. Dabholkar and J.M. Quashnock, Nucl. Phys. B333 (1990) 815; R.A. Battye and E.P.S. Shellard, Phys. Rev. Lett. 73 (1994) 2954.

36. D. Harari and P. Sikivie, Phys. Lett. B195 (1987) 361; C. Hagnann and P. Sikivie, Nucl. Phys. B363 (1991) 247.
37. P. Sikivie, Phys. Rev. Lett. **51** (1983) 1415 and Phys. Rev. **D32** (1985) 2988; L. Krauss, J. Moody, F. Wilczek and D. Morris, Phys. Rev. Lett. **55** (1985) 1797.

38. S. DePanfilis et al., Phys. Rev. Lett. **59** (1987) 839 and Phys. Rev. **D40** (1989) 3153; C. Hagmann et al., Phys. Rev. **D42** (1990) 1297.

39. K. van Bibber et al., “Status of the Large-Scale Dark-Matter Axion Search,” LLNL preprint UCRL-JC–118357, to be published in the Proceedings of the International Conference on *Critique of the Sources of Dark Matter in the Universe*, Bel Air, CA, February 16–18, 1994.

40. D. Lynden-Bell, MNRAS **136** (1967) 101.

41. P. Sikivie and J. Ipser, Phys. Lett. **B291** (1992) 288.

42. P. Sikivie, I. Tkachev and Y. Wang, to be published.