How to Build up the High Linearity SQIF Structure

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Abstract. The synthesis of SQIF structures capable of providing highly linear voltage response has been studied in detail. It has been shown that the high linearity SQIF structure ought to involve a set of the connected in series arrays which provide periodic triangular voltage responses with different periods. Such a SQIF shows voltage response characterized by a single high-amplitude triangular peak. One has proposed the basic array structures implementing interferometer cells with cosine-like voltage response. The array response linearity dependence on number of the cells has been studied by means of the spectrum analysis technique. The synthesized SQIF structure allows designing of wide-band high-linear amplifiers for gigahertz frequency range. A travelling wave amplifier design is suggested to avoid limitations resulting from distributive character of the long series array.

1. Introduction

Recently SQIF (Superconducting Quantum Interferometer Filter) structures, which are nonperiodic arrays of dc interferometers, where suggested [1-4]. SQIF voltage response shows single delta-like peak at zero magnetic field. In case of vanishing inductances the voltage response can be expressed analytically for both parallel and serial SQIFs. The parallel SQIF response to homogeneous magnetic field $B$ is as follows

$$V_{par}(B) = V_c \sqrt{(I_b / I_c)^2 - |S_K(B)|^2}, \quad (1)$$

where

$$S_K(B) = \frac{1}{K} \sum_{k=1}^{K} \exp \left( \frac{2\pi i B s_m}{\Phi_0} \right),$$

$I_b$ – bias current, $K$ – number of Josephson junctions, $I_c$ – total critical current, $s_m$ – area of the $m$-th interferometer cell.

Serial SQIF response to homogeneous magnetic field is sum of the interferometer cell responses:

$$V_{ser}(B) = V_c \sum_{n=1}^{N} \left( (I_b / I_c)^2 - \left| \cos(\pi B s_m / \Phi_0) \right|^2 \right), \quad (2)$$

where $N$ is number of dc interferometers. Different areas $s_k$ mean different magnetic fluxes coming into the cells and therefore different periods of the cell responses.

Analysis of both formulas (1) and (2) shows that it is impossible to provide highly linear response.

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by any set of the cell areas. To provide the linear response one should also change amplitudes of the cell responses.

Purpose of the work is to work out the composition law for serial array of dc interferometers capable of providing linear triangular voltage response.

2. Periodic Linear Response

If voltage response of serial array of dc interferometers to homogeneous magnetic field $B$ is periodic even function, we can develop the response in Fourier series as follows:

$$V(B) = \sum_k \alpha_k \cos(k\omega_0 B)$$ (3)

One can imagine the array cells providing sinusoidal voltage responses with frequencies proportionate to the cell areas; basic frequency $\omega_0$ can be expressed by basic area $s_0$ in the following way:

$$\omega_0 = (2\pi / \Phi_0) \cdot s_0.$$ (4)

Next we consider periodic triangular response. In this case harmonic amplitudes should show $sinc$-like behaviour:

$$V(B) = A \sum_k \frac{\sin^2(k\omega_0 \Delta B / 2)}{(k\omega_0 \Delta B / 2)^2} \cdot \cos(k\omega_0 B),$$ (5)

where $2\Delta B$ is width of the triangular pulse, $\omega_0 = 2\pi / B_T$, $B_T$ -period of the voltage response to magnetic field $B$. Fig. 1 presents spectra of the responses with the same triangular pulse width $\Delta B$ and different periods $B_T = 5\Delta B$ (dash-dotted lines) and $B_T = 2\Delta B$ (thick solid lines). Inset shows these periodic responses (b and c), as well as response with single triangular peak which corresponds to continues spectrum shown by solid line (a). It is significant that spectrum of the voltage response with minimum period $B_T = 2\Delta B$ contains only odd harmonics with amplitudes decreasing monotonically with harmonic number $k$ as $1/k^2$.

When dc interferometer with critical current $I_c$ is biased well above critical current ($I_b > 1.5I_c$), its voltage response to magnetic field becomes close enough to sinusoidal form. These interferometer cells may be used to compose serial array providing periodic linear voltage response. The array ought

![Figure 1](image_url). Spectrum of triangular voltage response on magnetic field. Solid line - continuous spectrum of the voltage response with single triangular peak. Dropped lines - spectrum of the periodic triangular voltage response. Inset shows the responses.
to consist of many groups of identical cells, i.e. with equal areas. The groups each are to provide corresponding spectral components. Amplitudes of the spectral components are achieved by number of cells in the groups. This approach to the spectral line formation by groups of interferometer cells with equal areas is illustrated in Fig. 2.

Fig. 3 shows degree of linearity of the triangular response versus cutoff frequency $\omega^*$ of the response spectrum. The cutoff frequency does not depend on period $B_T$ of the response, but total number $N^*$ of harmonic components needed does depend on $B_T$. In case of the response with minimum period $B_T = 2\Delta B$, total the number $N^*$ is half a normalized cutoff frequency $\omega^*\Delta B/\pi$.

Such an array providing highly linear triangular response can be called LRA - Linear Response Array.

3. Single Triangular-Peak Response

We may compose total array of several LRAs connected in series. If voltage responses of the LRAs are characterized by different periods $B_T$ and the same width $2\Delta B$ of triangular peak, the total array shows SQIF-like response with single triangular peak. This array can be called as LR SQIF - Linear Response SQIF. Spectrum of the LR SQIF response consists of a set of basic frequencies $\omega_0\Delta B/\pi \leq 1$ and their harmonics in the upper range $\omega\Delta B/\pi > 1$, and amplitudes of the spectral components have $\text{sinc}$-like frequency dependence. As for conventional SQIF, its response is characterized by quasi uniform discrete spectrum in a range specified by maximal and minimal cell areas.
4. Travelling Wave Amplifier

One should note that some limitations may result from large size of the array with great number of cells when the array becomes distributed. To overcome this problem we suggested travelling wave amplifier shown schematically in Fig. 4. The amplifier design includes two microwave lines coupled via interferometer cells of a serial array. Input wave signal propagates along the first line and then is absorbed by a matched load. The propagating signal acts magnetically on the interferometer cells inserted in second microwave line. Voltage responses of the cells induce output wave running in the second line. Equal velocities of both input and output waves provide just in-phase process and hence the effective amplification of the output signal. This approach allows using arbitrary number of the cells and thereby achieving of any desired linearity.

5. Conclusion

It is known that increase in number N of interferometer cells in series array provides increase of the array dynamic range as \( \sqrt{N} \) [4], and the developed design approach make the array the LRA with extremely high linearity of voltage response.

Experimental feasibility of LRA looks quite realistic. For example, if we need the linearity level of 80 db we have to provide \( N^* = 25 \) odd harmonic components (see Fig. 3) by means of 26 different groups with identical cells (see Fig. 2); numbers of the cells decrease as \( 1/(2k-1)^2 \), \( k =1, 2, \ldots 25 \). The effective area of the cell in each group must be proportional to the corresponding spectrum component frequency \( \delta(2k-1) \); the effective cell area which is responsible for the magnetic flux coming into the cell can be established by proper coupling between the cell and input signal line.

If we put 1000 cells in the first group, we can easily come to the total number of the cells in LRA \( N = 1250 \). The number seems quite optimistic; now we are going to test series array of more than four thousand cells fabricated on the base of Hypres technology.

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6. References

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