Realistic Hybrid Inflation in 5D Orbifold SO(10) GUT

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In collaboration with

- A series of renormalizable minimal SO(10) GUT. N.Okada, T.Kikuchi, S.Meljanac, and A.Ilacovac
  - Phys.Rev.D78:015005,2008 N.Okada
  - JCAP 0809:024,2008 N.Okada and T.Osaka
  - A Simple 5D SO(10) GUT and sparticle masses. To appear in Phys.Rev.
N.Okada
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1. Problems of naïve inflation scenario

- Inflation model with single inflaton (scalar)

\[
\frac{\delta \rho}{\rho} = \frac{4 \sqrt{6\pi}}{5} \lambda^{1/2} \left( \frac{N}{\pi} \right)^{3/2}
\]

\[\lambda \approx 6 \times 10^{-14} \text{ for } N = 55\]

- requires Unnatural fine tuning
2. Hybrid Inflation (Non SUSY)

\[ V(\chi, \sigma) = \kappa^2 \left( M^2 - \frac{\chi^2}{4} \right)^2 + \frac{\chi^2 \sigma^2}{4} + \frac{m^2 \sigma^2}{2} \]

\[ \frac{\delta T}{T} = \left( \frac{32\pi}{45} \right)^{1/2} \frac{V^{3/2}}{M_P^3 V'} \approx \left( \frac{16\pi}{45} \right)^{1/2} \frac{\lambda \kappa^2 M^5}{M_P^3 m^2} \]

So if \( M = M_{GUT}, m \approx \kappa \sqrt{\lambda} \times 10^{15} \approx 10^{12} \text{ GeV} \).

This indicates that \( \lambda \approx \kappa \approx O(10^{-2}) \).

but leads us to monopole and other problems
3. Hybrid Inflation Model (SUSY)

\[ W = \begin{cases} 
\kappa S \left( \Phi \Phi - M^2 \right) \\
\kappa S \left( \Phi \Phi - M^2 \right) - S\frac{(\Phi \Phi)^2}{M_S^2} \\
S \left( \frac{(\Phi \Phi)^2}{M_S^2} - \mu_S^2 \right) 
\end{cases} \]  \hspace{1cm} (1)

Here from the top, Standard F-term Hybrid Inflation (FHI), Shifted FHI, Smooth FHI. For Standard FHI,

\[ V(\Phi, \phi, S) = \kappa^2 |M^2 - \Phi \phi|^2 + \kappa^2 |S|^2 (|\Phi|^2 + |\phi|^2) \]

\[ + \quad D - terms \]  \hspace{1cm} (2)
• We consider hybrid inflation in GUT scheme, where S is G singlet and Phi is of some representation.

We consider G is the Pati-Salam $SU(4) \otimes SU(2)_L \otimes SU(2)_R$.

In the following, we explain why we consider the PS group.
3.1 Problems of SO(10) GUT in 4D

- Renormalizable minimal SUSY SO(10) GUT
  \[ 16 \times 16 = 10 + 120 + 126 \]
- So Yukawa coupling is made by the Higgs
  \[ 10, \, 126, \, 120 \]

We use only 10 and 126 Higgs -renormalizable minimum SO(10) GUT
Renormalizable Minimal SUSY SO(10) model

(Babu-Mohapatra (93'); Fukuyama-Okada (01'))

- Two kinds of symmetric Yukawa couplings

\[ W_Y = Y_{10}^{ij} 16_i H_{10} 16_j + Y_{126}^{ij} 16_i H_{126} 16_j \]

- Two Higgs fields are decomposed to

\[ 10 \rightarrow (6,1,1) + (1,2,2) \]
\[ 126 \rightarrow (6,1,1) + (15,2,2) \]
\[ \quad + (10,3,1) + (10,1,3) \]

- SU(4) adjoint 15 have a basis, \( \text{diag}(1,1,1,-3) \) so as to satisfy the traceless condition. Putting leptons into the 4\(^{\text{th}}\) color, we get, so called, ‘Georgi-Jarlskog’ factor, \(-3\) for leptons.
Yukawa couplings

- After the symmetry breakings, we have

\[
W_Y = (u^c_R)_i \left( Y^{ij}_{10} H^u_{10} + Y^{ij}_{126} H^u_{126} \right) q_j \\
+ (d^c_R)_i \left( Y^{ij}_{10} H^d_{10} + Y^{ij}_{126} H^d_{126} \right) q_j \\
+ (\nu^c_R)_i \left( Y^{ij}_{10} H^u_{10} - 3Y^{ij}_{126} H^u_{126} \right) \ell_j \\
+ (e^c_R)_i \left( Y^{ij}_{10} H^d_{10} - 3Y^{ij}_{126} H^d_{126} \right) \ell_j \\
+ (\nu^c_R)_i \left( Y^{ij}_{126} v_R \right) (\nu^c_R)_j
\]

Below the GUT scale, we assume MSSM is realized, and we have two Higgs doublet which are linear combinations of original fields.
Superpotential was fully analyzed

- Fukuyama et al. hep-ph/0401213 v1 gave the symmetry breaking pattern from minimal SO(10) to Standard Model, starting from

\[
W = m_1 \Phi^2 + m_2 \Delta \bar{\Delta} + m_3 H^2 + \lambda_1 \Phi^3 + \lambda_2 \Phi \Delta \bar{\Delta} + \lambda_3 \Phi \Delta H + \lambda_4 \Phi \bar{\Delta} H.
\]

\[
H = 10, \; \Delta = 126, \; \Phi = 210
\]

All intermediate energy scales have been determined unambiguously and …
Many vacua (SM singlets)

- High dimensional rep. has many SM singlet (vacua)
- (1234)
- (5678+5690+7890)  210
- (1256+1278+1290+3456+3478+3490)
- (13579)  126
- (24680)  126
The gauge coupling unification is spoiled by the threshold effects

From Bertolini-Scwetz-Malinsky hep-ph/0605006
3.3 Hybrid inflation in 5D orbifold
SO(10) GUT-second solution

- Kawamura('01), Hall-Nomura('01), Dermisek-Mafi('01)

\[
S^1 / (Z_2 \times Z'_2)
\]

Proton decay is suppressed by B.C.

B.C. breaks SU(5) into MSSM with direct no coupling with triplet, whereas SO(10) into Pati-Salam included it in general, inducing large proton decay ratio.
Model set up

SO(10) inv

PS inv

y = 0

y = \pi/2

Invisible

Visible
Set up

N=1 SUSY SO(10) gauge theory in 5D. All matter and Higgs in PS brane, while gauge multiplet reside in bulk. Assuming parity \((P, P')\) as Table, PS gauge multiplet has zero mode. and 5D N=1 SUSY SO(10) is broken to 4D N=1 PS symmetry.


\((P, P')\) assignment and masses of fields in the bulk SO(10) gauge multiplet \((V, \Phi)\)

| \((P, P')\) | bulk field | mass       |
|-------------|------------|------------|
| (+, +)      | \(V(15, 1, 1), V(1, 3, 1), V(1, 1, 3)\) | \(\frac{2n}{R}\) |
| (+, −)      | \(V(6, 2, 2)\) | \(\frac{(2n+1)}{R}\) |
| (−, +)      | \(\Phi(6, 2, 2)\) | \(\frac{(2n+1)}{R}\) |
| (−, −)      | \(\Phi(15, 1, 1), \Phi(1, 3, 1), \Phi(1, 1, 3)\) | \(\frac{(2n+2)}{R}\) |
Pati-Salam inv. Model (SUSY)

\[ W_Y = Y_{ij}^{i} F_{Lj}^c F_{Rj}^c H_1 + \frac{Y_{15}^{ij}}{M_5} F_{Li}^c F_{Rj}^c \left( H'_1 H_{15} \right) \]

\[ + \frac{Y_{R}^{ij}}{M_5} F_{Ri}^c F_{Rj}^c (\phi\phi) + \frac{Y_{L}^{ij}}{M_5} F_{Li} F_{Lj} \left( \overline{H_L H_L} \right) \]

\[ H_1 = (1, 2, 2)_H, \quad H'_1 = (1, 2, 2)'_H, \]

\[ H_6 = (6, 1, 1)_H, \quad H_{15} = (15, 1, 1)_H, \]

\[ H_L = (4, 2, 1)_H, \quad \overline{H_L} = (\overline{4}, 2, 1)_H, \]

\[ \phi = (4, 1, 2)_H, \quad \overline{\phi} = (\overline{4}, 1, 2)_H. \]  \hspace{1cm} (1)
Particle contents on the PS brane.

| Matter Multiplets          | $\psi_i = F_{Li} \oplus F_{Ri}^c \quad (i = 1, 2, 3)$ |
|----------------------------|------------------------------------------------------|
| Higgs Multiplets           | $(1, 2, 2)_H, (1, 2, 2)'_H, (15, 1, 1)_H, (6, 1, 1)_H$ |
|                            | $(4, 1, 2)_H, (\overline{4}, 1, 2)_H, (4, 2, 1)_H, (\overline{4}, 2, 1)_H$ |

$F_{Li}$ and $F_{Ri}^c$ are matter multiplets of $(4, 2, 1)$ and $(\overline{4}, 1, 2)$
Smooth Inflation Model

\[ W = S \left( -\mu^2 + \frac{(\bar{\phi}\phi)^2}{M^2} \right), \]

and

\[ v_{PS} = \sqrt{\mu M}. \]

The potential is given by

\[ V = \left| -\mu^2 + \frac{(\bar{\phi}\phi)^2}{M^2} \right|^2 + 4S^2 \frac{|\phi|^2|\bar{\phi}|^2}{M^4} \left( |\phi|^2 + |\bar{\phi}|^2 \right). \]
Considering the D-flatness condition, we normalize

$$|\phi| = |\bar{\phi}| = \frac{\chi}{2}, \quad |S| = \frac{\sigma}{\sqrt{2}} \quad (1)$$

and then $V$ becomes

$$V = \left( \mu^2 - \frac{\chi^4}{16M^2} \right)^2 + \frac{\chi^6\sigma^2}{16M^4}. \quad (2)$$

For a fixed $\sigma$, $V$ has a minimum at

$$\chi^2 = -6\sigma^2 + \sqrt{36\sigma^4 + 16v_{PS}^4} \approx \frac{4v_{PS}^4}{3\sigma^2}, \quad (3)$$

we obtain the potential along this path

$$V \approx \mu^4 \left( 1 - \frac{2v_{PS}^4}{27\sigma^4} \right). \quad (4)$$
\[ n_s \simeq 1 - 6\epsilon + 2\eta, \]
\[ r \simeq 16\epsilon, \]
\[ \alpha_s = \frac{dn_s}{d\ln k} \simeq 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \]

and

\[ 0.963 \leq n_s \leq 0.968, \]
\[ 4.0 \times 10^{-7} \geq r \geq 3.1 \times 10^{-7}, \]
\[ -8.4 \times 10^{-4} \leq \alpha_s \leq -6.1 \times 10^{-4} \]
Gauge coupling unification in left-right symmetric case.
$M/10^{17}\text{GeV}$ or $\sigma_k/10^{17}\text{GeV}$ as a function of $T_{rh}$. 

$M$(solid) and $\sigma_k$ as a function of $T_{rh}$. 
The number of e-folds versus $T_{\text{rh}}$
The spectral index as a function of $T_{\text{rh}}$.
M as a function of $v_{PS}$

$v_{PS} \geq M \geq M_P$
• The number of e-folds versus $v_{\text{PS}}$
The tensor-to-scalar ratio versus $v_{\text{PS}}$
4. Dark Matters in this model

\[ \mathcal{L} = \delta(y) \int d^2 \theta \lambda \frac{X}{M_5^2} \text{tr} [\mathcal{W}^\alpha \mathcal{W}_\alpha] , \]

and

\[ M_\lambda = \frac{\lambda F_X M_c}{M_5^2} \sim \frac{\lambda F_X M_5}{M_5^2} , \]

which is smaller than

\[ m_{3/2} \sim \frac{F_X}{M_P} \]

by \( M_5 / M_P \).
To solve the problem of charged slepton (stau) LSP

If we consider the gaugino mediation scenario in our orbifold GUT model, right-handed stau is the LSP (charged DM)

**Gaugino mediation**

Boundary conditions at $M_c$ (compactification scale):

\[
\begin{align*}
\text{Non-zero gaugino mass} & \quad M_{1/2} \neq 0 \\
\text{Zero scalar mass}^2 & \quad m_0^2 = 0
\end{align*}
\]

In our simplest setup: $M_c = M_{PS} = 1.2 \times 10^{16}$ GeV

Approximate solution of soft mass RGE in MSSM

\[
\begin{align*}
\tilde{m}_\tau^2 & \approx m_0^2 + 0.15M_{1/2}^2 = 0.15M_{1/2}^2 \\
M_B^2 & \approx 0.2M_{1/2}^2 \quad @ \text{weak scale}
\end{align*}
\]

$\Rightarrow$ Therefore, $\tilde{m}_\tau < M_B$
To solve the problem → we take \( M_c > M_{PS} \)

\[ M_{\tilde{B}} \quad M_{1/2} \]

\[ M_Z \quad M_c = M_{PS} \]

\[ m_0 = 0 \]

\[ \tilde{m}_{\tilde{\tau}} \]

\[ M_{PS} \quad M_c \]

\[ m_0 = 0 \]

RGE running as PS model

PS running generates non-zero \( m_0^2 \) @ \( M_{PS} \)
Still gauge coupling unification? \(\rightarrow\) Yes

We impose the left-right symmetry and for the scale \(\mu > M_{PS}\) we have only two independent gauge coupling.

\(\rightarrow\) GCU is easily achieved with suitable \(b_4 - b_2 > 0\) (my notation)

For a given \(b_4 - b_2\), a suitable \(M_c\) realized the GCU

Sample results:

\[M_{PS} = 1.2 \times 10^{16} \text{ GeV}\]

(1) \(b_4 - b_2 = 1 \rightarrow M_{GUT} \sim M_c \sim M_{PS}\)

(2) \(b_4 - b_2 = 2 \rightarrow M_c = 7.74 \times 10^{16} \text{ GeV} > M_{PS}\)

\[M_{GUT} = 7.66 \times 10^{17} \text{ GeV}\]

\(b_4 - b_2 \geq 3 \rightarrow\) We can find a solution but \(M_{GUT} > M_{Pl}\)