$B_{(s)} \to S$ transitions in the light cone sum rules with the chiral current

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Abstract

We make a QCD light-cone sum rule (LCSR) assessment of $B_{(s)}$ semi-leptonic decays to a light scalar meson, $B_{(s)} \to S\ell\bar{\nu}_l, S\ell$ ($l = e, \mu, \tau$). Chiral current correlators are used and calculations are performed at leading order in $\alpha_s$. Having little knowledge of ingredients of the scalar mesons, we confine ourself to the two quark picture for them and work with the two possible scenarios. The resulting sum rules for the form factors receive no contributions from the twist-3 distribution amplitudes (DA’s), in comparison with the calculation of the conventional LCSR approach where the twist-3 parts play usually an important role. We specify the range of the squared momentum transfer $q^2$, in which the operator product expansion (OPE) for the correlators remains valid approximately. It is found that the form factors satisfy a relation consistent with the prediction of soft collinear effective theory (SCET). In the effective range we investigate behaviors of the form factors and differential decay widths and compare our calculations with the observations from other approaches.

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I. INTRODUCTION

With numerous scalar meson states being discovered experimentally, most of efforts have been devoted to a study about their inner structure and how they are classified. However, much controversy persists regarding the underlying components of them. Currently one of our main concerns is that these scalar particles can whether or not be described consistently in a quark picture. Recently, from a survey of the accumulated experimental data the two possible scenarios are suggested \[1, 2\], where the scalar mesons below and above 1 GeV are assumed to enter their respective nonets in two different ways. In scenario 1, there are the two scalar nonets formed by the two quark bound states. One contains, as the lowest lying scalar states, the isoscalars $\sigma(600)$ and $f_0(980)$, isodoublets ($\kappa^+(800), \kappa^0(800)$) and ($\kappa^0(800), \kappa^-(800)$) and isovector ($a_0^+(980), a_0^0(980), a_0^-(980)$). The other is made up of the corresponding first excited states: the isoscalars $f(1370)$ and $f_0(1500)$, isodoublets ($K^{*+}_0(1430), K^{*0}_0(1430)$) and ($K^{*0}_0(1430), K^{*-}_0(1430)$) and isovector ($a_0^+(1450), a_0^0(1450), a_0^-(1450)$). In scenario 2, those scalar states below 1 GeV are taken to be the members of a four-quark nonet, while $f(1370), f_0(1500), a_0(1450)$ and $K^*_0(1430)$ are treated as the lowest lying two-quark resonances and arranged into another nonet, with the corresponding first excited states between $2.0 \sim 2.3$ GeV.

Although now we are not able to discriminate among all the existing schemes for the scalar mesons, the above two are intriguing in that they can provide us with a ground to make a systematic study on the scalar mesons. In such assignment scenarios, an investigation has been made into the related decay constants and light-cone distribution amplitudes (DA’s)\[1\]. More importantly, to gain insight into the scalar mesons some of the B decays involving them have been explored in the same context. In Ref.\[1, 2\], the hadronic decays with a scalar final state are discussed in detail in the framework of QCD factorization, important implications being drawn for the properties of the scalar particles. More attentions are paid to the semileptonic decays with a potential interest $B \to Sl\bar{\nu}_l, S\bar{l}l$. Especially, one shows a great interest in the knowledge of their differential rates, since it is critical, as confronted with the coming experimental observations, for acquiring valuable information on ingredients of the scalar particles. Unfortunately, among the existing approaches no one can afford the task to understand the underlying form factors in the whole regions of $q^2$, with $q$ being the momentum transfers. An effective range of $q^2$, in which the calculations are believable, has
even not been specified in literature, the computations being carried out in just a small or intermediate kinematical region arbitrarily selected. So the results are less persuasive.

Superior to the three-point QCD sum rules in evaluating heavy to light meson transitions, the LCSR approach, which starts with a two-point correlation function, adopts the operator product expansion (OPE) near the light cone $x^2 = 0$ in terms of nonlocal operators, whose matrix elements are parameterized as the hadronic DA’s of increasing twist. Such that the resulting LCSR for form factors, in addition to having an estimable effective region of $q^2$, can embody as many long-distance effects as possible involved in the decaying processes. However, a better understanding of these DA’s is critical to have the calculation more reliable. Together with the leading twist-2 DA, in general, the twist-3 ones enter and play an important role in a LCSR calculation on the form factors. In the case of the scalar mesons, the probe into the twist-2 and -3 DA’s has been conducted in the framework of QCD sum rules and a DA model, in an expansion form in the Gegenbauer polynomials, has been formulated, but with a sizable error in some of the model parameters. To try our best to reduce uncertainty in LCSR calculation from the long distance parameters, a practical improvement scenario has been worked out with its validity examined and confirmed, in which a chiral correlator is so chosen that the twist-3 DA’s make no contribution \[3\]. In the present work, we intend to apply the same trick to revaluate the semileptonic transitions $B_{(s)} \rightarrow Sl\bar{\nu}_l, S\bar{l}l$, in the two quark picture for the scalar mesons. We will work in the effective regions required by the OPE validity and with the two different scenarios aforementioned, and calculation is to be performed at leading order in $\alpha_s$.

The paper is organized as follows: In the following section, we present the correlation functions with a chiral current and use them to derive the LCSR for the form factors for the $B_{(s)} \rightarrow S$ transitions. The discussion and comment are made on the important inputs—the DA’s and decay constants of the scalar mesons, in Sec.III. Sec.VI is devoted to a detailed numerical discussion about the form factors and differential widths for $B_{(s)} \rightarrow Sl\bar{\nu}_l, S\bar{l}l$, including a numerical comparison with the estimates of some other approaches. The final section is reserved for a summary.
II. THE LCSR FOR THE $B(s) \to S$ FORM FACTORS

In the standard model (SM), the semileptonic decays $B(s) \to Sl\bar{u}, S\bar{t}$ are induced by the following effective Hamiltonian:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ub}\bar{u}\gamma_\mu(1 - \gamma_5)b\bar{\ell}\gamma_\mu(1 - \gamma_5)\nu_\ell$$

$$+ \frac{G_F\alpha}{\sqrt{2}\pi} V_{ts}\bar{s}\gamma_\mu(1 - \gamma_5)b\bar{\ell}\gamma_\mu\gamma_5\nu_\ell$$

$$+ C_{10}\bar{s}\gamma_\mu(1 - \gamma_5)b\bar{\ell}\gamma_\mu\gamma_5\ell$$

$$- \frac{2m_b C_{7}^{\text{eff}}(m_b)}{q^2} \bar{s}\sigma_{\mu\nu}q' (1 + \gamma_5)b\bar{\ell}\gamma_\mu\ell. \quad (1)$$

Here $V_{ij}$ are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, and $C_i^{\text{eff}}$ the Wilson coefficients, among which $C_9^{\text{eff}}$ and $C_{10}$ are scale-independent for the corresponding operators have a vanishing anomalous dimension. $C_7^{\text{eff}}$ and $C_9^{\text{eff}}$ are expressed as

$$C_7^{\text{eff}}(\mu) = C_7(\mu) + C_{b\to s\gamma}(\mu), \quad (2)$$

$$C_9^{\text{eff}} = C_9(\mu) + Y_{\text{pert}}(s') + Y_{\text{LD}}(s'), \quad (3)$$

where $C_{b\to s\gamma}(\mu)$ stems from the absorptive part of $b \to sc\bar{c} \to s\gamma$ rescattering which will be neglected here, $Y_{\text{pert}}$ and $Y_{\text{LD}}$ stand for, respectively, the short-and long-distance contributions from the four quark operators \cite{4}, with

$$Y_{\text{pert}}(s') = h(z, s')C_0 - \frac{1}{2}h(1, s')(4C_3 + 4C_4 + 3C_5 + C_6)$$

$$- \frac{1}{2}h(0, s')(3C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6), \quad (4)$$

$C_0 = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$, and

$$h(z, s') = -\frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2 + x)|1 - x|^{1/2} \left\{ \ln \left|\frac{\sqrt{1 - x} + 1}{\sqrt{1 - x} - 1}\right| - i\pi \quad \text{for } x \equiv 4z^2/s' < 1 \right.$$\n
$$2 \arctan \frac{1}{\sqrt{x - 1}} \quad \text{for } x \equiv 4z^2/s' > 1 ,$$

$$h(0, s') = \frac{8}{27} - \frac{8}{9}\ln\frac{m_b}{\mu} - \frac{4}{9}\ln s' + \frac{4}{9}i\pi , \quad (5)$$

| TABLE I: The values of Wilson coefficients $C_i(m_b)$ in the leading logarithmic approximation in Standard Model, with $m_W = 80.4\text{GeV}$, $m_t = 173.8\text{GeV}$, $m_b = 4.8\text{GeV}$ \cite{5}. |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $C_1$           | $C_2$           | $C_3$           | $C_4$           | $C_5$           | $C_6$           | $C_7$           | $C_9$           | $C_{10}$        |
| 1.119           | -0.270          | 0.013           | -0.027          | 0.009           | -0.033          | -0.322          | 4.344           | -4.669          |
where \( z = m_c/m_b \) and \( s' = q^2/m_b^2 \). The Wilson coefficients \( C_i(m_b) \), listed in Tab.\( \[ \[ \) are given in the leading logarithmic accuracy.

Aiming at an evaluation of the semileptonic decays \( B_{(s)} \to Sl \bar{\nu}_l, Sl \bar{\nu}_l \), we need to confront the hadronic matrix elements \( \langle S(p)|\bar{q}_2\gamma_\mu\gamma_5b|B_{(s)}(p + q) \rangle \) and \( \langle S(p)|\bar{q}_2\sigma_{\mu\nu}\gamma_5q^\nu b|B_{(s)}(p + q) \rangle \). They can be parameterized, in terms of the form factors \( f_+(q^2), f_-(q^2) \) and \( f_T(q^2) \), as

\[
\langle S(p)|\bar{q}_2\gamma_\mu\gamma_5b|B(p + q) \rangle = -2ip_\mu f_+(q^2) - i[f_+(q^2) + f_-(q^2)]q_\mu,
\]

\[
\langle S(p)|\bar{q}_2\sigma_{\mu\nu}\gamma_5q^\nu b|B(p + q) \rangle = [2p_\mu q^2 - 2q_\mu(p \cdot q)]\frac{-f_T(q^2)}{m_B + m_S},
\]

where the \( B_{(s)} \) mesons are signified by \( B \) for short. The relative form factors could be calculated in the LCSR. Instead of the correlation functions used in Ref.\( \[ \[ \), we would like to consider the following two correlators, with the T product of chiral current operators sandwiched between the vacuum and one on-shell scalar meson state\( \[ \[ \) \( q_1 \):

\[
\Pi_\mu(p, q) = i \int d^4xe^{iqx}\langle S(p)|T\{ \bar{q}_2(x)\gamma_\mu(1 - \gamma_5)b(x), \bar{b}(0)i(1 - \gamma_5)q_1(0)\}|0\rangle,
\]

\[
\tilde{\Pi}_\mu(p, q) = i \int d^4xe^{iqx}\langle S(p)|T\{ \bar{q}_2(x)\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b(x), \bar{b}(0)i(1 - \gamma_5)q_1(0)\}|0\rangle,
\]

where \( q_1, q_2 \) denotes the light quark field.

The hadronic representations for them are easy to achieve, by inserting between the currents a complete set of resonance states with the same quantum numbers as the operator \( \bar{b}(0)i(1 - \gamma_5)q_1(0) \). On the desired pole contributions due to the lowest pseudoscalar \( B \)-meson are insolated, we obtain the hadronic representations:

\[
\Pi^h_\mu(p, q) = \frac{\langle S(p)|\bar{q}_2\gamma_\mu\gamma_5b|B(p + q)\rangle\langle B(p + q)|\bar{b}i\gamma_5q_1|0\rangle}{m_B^2 - (p + q)^2} \frac{m_B^2 - (p + q)^2}{m_B^2 - (p + q)^2} + \sum_h \frac{\langle S(p)|\bar{q}_2\gamma_\mu(1 - \gamma_5)b|B^h(p + q)\rangle\langle B^h(p + q)|\bar{b}i(1 - \gamma_5)q_1|0\rangle}{m_B^2 - (p + q)^2},
\]

\[
\tilde{\Pi}^h_\mu(p, q) = -\frac{\langle S(p)|\bar{q}_2\gamma_\mu(1 + \gamma_5)q^\nu b|B^h(p + q)\rangle\langle B^h(p + q)|\bar{b}i\gamma_5q_1|0\rangle}{m_B^2 - (p + q)^2} \frac{m_B^2 - (p + q)^2}{m_B^2 - (p + q)^2} + \sum_h \frac{\langle S(p)|\bar{q}_2\sigma_{\mu\nu}(1 + \gamma_5)q^\nu b|B^h(p + q)\rangle\langle B^h(p + q)|\bar{b}i(1 - \gamma_5)q_1|0\rangle}{m_B^2 - (p + q)^2}.\]

It should be stressed that the correlation functions receive contributions from the scalar resonances included in the intermediate states\( B^h \), in addition to the higher pseudoscalar ones, and the ground-state scalar meson is a bit lighter than the pseudoscalar resonance lying in the first excited state.
With the definitions of $B$-meson decay constant $\langle B|\bar{b}i\gamma_5q_1|0\rangle = \frac{m_B^2f_B}{m_{q_1}+m_b}$ and Eqs.\(\text{(6)}\) and \(\text{(7)}\), the phenomenological representations of the correlation functions read

\[
\Pi^h_\mu(p, q) = \frac{-i}{m_B^2 - (p+q)^2} \frac{m_B^2f_B}{m_{q_1}+m_b} \left[2f_+ (q^2)p_\mu + (f_+ (q^2) + f_- (q^2))q_\mu\right]
- \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{2\rho^h_+(s)p_\mu + (\rho^h_+(s) + \rho^h_-(s))q_\mu}{s - (p+q)^2},
\]

\[
\bar{\Pi}^h_\mu(p, q) = \frac{1}{m_B^2 - (p+q)^2} \frac{m_B^2f_B}{m_{q_1}+m_b} f_T[b] [2p_\mu q^2 - 2q_\mu (q \cdot p)]
- \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\rho^h_T(s)[2p_\mu q^2 - 2q_\mu (q \cdot p)]}{s - (p+q)^2}.
\]

Here we have replaced the summations in \(\text{(10)}\) and \(\text{(11)}\) with the dispersion integrations starting with the threshold $s_0$ near the squared mass of the lowest scalar $B$-meson. The spectral densities can be approximated as, by invoking the quark-hadron duality ansatz

\[
\rho^h_{+,\gamma}(s) = \rho^{QCD}_{+,\gamma}(s) \theta(s - s_0).
\]

The QCD spectral densities $\rho^{QCD}_{+,\gamma}(s)$ can be derived by calculating the correctors in QCD theory. To this end, we work in the large space-like momentum regions $(p+q)^2 << m_b^2$ for the $b\bar{q}_1$ channel and a larger recoil region of the decaying $B$-meson as given later, which correspond to the small light-cone distance $x^2 \approx 0$ and are required by the validity of the OPE. Considering the effect of the background gluon field, we can write down a full $b$-quark propagator

\[
\langle 0|Tb(x)b(0)|0\rangle = iS_b(x, 0) - ig_s \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \int dv \frac{k + m_b}{(m_b^2 - k^2)^2} G^{\mu\nu}(vx)\sigma_{\mu\nu} + \frac{1}{m_b^2 - k^2} vx_\mu G^{\mu\nu}(vx)\gamma_\nu.
\]

Here $G^{\mu\nu}$ is the gluonic field strength, $g_s$ denotes the strong coupling constant and $S_b(x, 0)$ expresses a free $b$-quark propagator

\[
iS_b(x, 0) = -i \int \frac{d^4k}{(2\pi)^4} e^{-ikx} \frac{k + m_b}{m_b^2 - k^2}.
\]

The large virtuality of the underlying heavy quarks makes it sound to neglect the contributions of soft gluon emission from the heavy quarks, which, in fact, is just a twist-4 effect. In this accuracy and leading order in $\alpha_s$, we find that as contrasted with the results of the traditional LCSR, only the nonlocal matrix element $\langle S(p)|\bar{q}_2(x)\gamma_\mu q_1(0)|0\rangle$ remains, while those concerning the nonlocal operators $\bar{q}_2(x)q_1(0)$ and $\bar{q}_2(x)\sigma_{\mu\nu}q_1(0)$ cancel out. As
usual, applying the light-cone OPE to the matrix element \( \langle S(p)|\bar{q}_2(x)\gamma_\mu q_1(0)|0 \rangle \), we could be led to the leading twist-2 DA’s of the scalar mesons \( \Phi_S(u, \mu) \) as defined in [1]. We are going to return to this point in the following section. Now the light-cone OPE forms for the correlators can be written as follows:

\[
\Pi^{QCD}_\mu (p, q) = \frac{2ip_\mu m_b}{m_b^2 - (q + up)^2} \int_0^1 du \frac{\Phi_S(u)}{m_b^2 - (q + up)^2},
\]

(17)

\[
\tilde{\Pi}^{QCD}_\mu (p, q) = -2(p_\mu q^2 - q_\mu (q \cdot p)) \int_0^1 du \frac{\Phi_S(u)}{m_b^2 - (q + up)^2}.
\]

(18)

We would like to convert them into a form of dispersion integration in order to facilitate the ensuing subtraction of the effect of the higher resonances and continuum states in the phenomenological representations (12) and (13). To this end, invoking the relation \( m_b^2 - (q + up)^2 = u(s - (p + q)^2) \) we make a replacement of \( u \) with \( s \). Matching both the forms of the correlators, subtracting continuum contributions and making Borel transformation \([11]\) with respect to the variable \( (p + q)^2 \),

\[
B M^2 \frac{1}{m_B^2 - (q + p)^2} = \frac{1}{M^2} e^{-\frac{m_B^2}{M^2}},
\]

(19)

\[
B M^2 \frac{1}{m_b^2 - (q + up)^2} = \frac{1}{uM^2} e^{\frac{1}{uM^2} \left[ m_b^2 + u(1-u)p^2 - (1-u)q^2 \right]},
\]

with \( M^2 \) being the Borel parameter and \( m_S \) the scalar meson mass, we get the sum rules for the form factors:

\[
f_+(q^2) = -\frac{m_{q_1} + m_b}{m_B^2 f_B m_b} m_b \int_\Delta^{1} du \frac{\Phi_S(u)}{u} e^\Lambda,
\]

(20)

\[
f_-(q^2) = \frac{m_{q_1} + m_b}{m_B^2 f_B m_b} m_b \int_\Delta^{1} du \frac{\Phi_S(u)}{u} e^\Lambda,
\]

(21)

\[
f_T(q^2) = -\frac{m_{q_1} + m_b}{m_B^2 f_B} (m_B + m_S) \int_\Delta^{1} du \frac{\Phi_S(u)}{u} e^\Lambda,
\]

(22)

where

\[
\Delta = \frac{1}{2m_S^2} \left[ \sqrt{(s_0 - m_S^2 - q^2)^2 + 4(m_b^2 - q^2)m_S^2} - (s_0 - m_S^2 - q^2) \right],
\]

\[
\Lambda = -\frac{1}{uM^2} \left[ m_b^2 + u(1-u)m_S^2 - (1-u)q^2 \right] + \frac{m_B^2}{M^2}.
\]

(23)

We find, as a by-product, that the form factors in question respect the following LCSR relations:

\[
f_+(q^2) = -f_-(q^2),
\]

(24)

\[
f_T(q^2) = \frac{(m_B + m_S)}{m_b} f_+(q^2).
\]

(25)
Actually, apart from that the same is observed in the LCSR involving a pseudoscalar meson, a simple relation is obtained also for the form factors in the vector meson case \cite{13}. All these observations, up to the hard-exchange corrections, are consistent with the results of soft collinear effective theory (SCET) \cite{12}. Having these relations at hand, in the numerical discussion we will put our focus on the form factor $f_+(q^2)$.

III. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES OF SCALAR MESONS

In this section, we give a brief review and discussion on the decay constants and DA’s of the related scalar mesons, which are the basic inputs for the LCSR calculation.

For a light scalar meson in the two quark picture, it could couple to the corresponding vector and scalar quark current operators thus we can define its decay constants as \cite{1},

\[
\langle S(p) | \bar{q}_2(0) \gamma_\mu q_1(0) | 0 \rangle = p_\mu f_S, \\
\langle S(p) | \bar{q}_2(0) q_1(0) | 0 \rangle = m_S \bar{f}_S.
\]

It is observed readily that the decay constants $f_S$ and $\bar{f}_S$ are scale independent and dependent, respectively. The neutral scalar mesons like $a_0^0$ and $f_0$ (if considered purely a $s\bar{s}$ bound state) cannot couple with a vector current operator owing to the charge conjugation invariance or conservation of the vector current and thus we have

\[
f_{f_0} = f_{a_0^0} = 0.
\]

For the other scalar mesons, the decay constants $f_S$ and $\bar{f}_S$ are connected by equation of motion

\[
\bar{f}_S = \mu_S f_S,
\]

where

\[
\mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)},
\]

the running quark masses $m_i(\mu)$ respect the renormalization group equation (RGE):

\[
m_i(\mu) = m_i(\mu_0) \left( \frac{\alpha_s(\mu_0)}{\alpha_s(\mu)} \right)^{-4/b},
\]
with \( b = (33 - 2n_f)/3 \), \( n_f \) being the number of active quark flavors. The decay constants \( f_S \) hence are either zero or small of order \( m_2 - m_1 \).

Similar to the case of pseudoscalar mesons, the twist-2 DA \( \Phi_S(u, \mu) \) of the scalar meson is defined as

\[
\langle S(p)|q_2(x)\gamma_\mu q_1(y)|0\rangle = p_\mu \int_0^1 du e^{iup \cdot x + \bar{u}p \cdot y} \Phi_S(u, \mu),
\]

with \( u \) being the fraction of the light-cone momentum of the scalar meson carried by \( q_2 \) and \( \bar{u} = 1 - u \), and obeys the normalization

\[
\int_0^1 du \Phi_S(u, \mu) = f_S.
\]

With reference to the DA’s of scalar mesons, a few words should be given. From the definition of \( \Phi_S(u, \mu) \), the corresponding scalar mesons have to carry a large light-cone momentum \( p_0 + p_3 \). Along with the requirement of the OPE validity, such a constrain condition demands that we work in a region assigned as,

\[
0 \leq q^2 < (m_b - m_S)^2 - 2(m_b - m_S)\Lambda_{QCD},
\]

which, to be specific, is \( 0 \leq q^2 < 11\text{GeV}^2 \) for a scalar meson below 1 GeV and \( 0 \leq q^2 < 8\text{GeV}^2 \) for one above 1 GeV. Also, it is important to realize that the DA’s of scalar meson, strictly speaking, become meaningful just at a scale \( \mu \geq m_S \), since the constituent quark of the scalar meson is in essence off-shell and in particular, it is far from its mass shell by the virtuality of \( m_S^2 \) as carrying the total momentum of the scalar meson. Considering the DA’s at a scale below \( m_S \) means that we are dealing with the situation that these off-shell modes are in part or in full integrated out, however, which is meaningless.

Based on the conformal symmetry hidden in the QCD Lagrangian, \( \Phi_S(u, \mu) \) can be expanded in a series of Gegenbauer polynomials \( C_{3/2}^{3/2}(x) \) with increasing conformal spin as

\[
\Phi_S(u, \mu) = \bar{f}_S(\mu)6u\bar{u} \left[ B_0(\mu) + \sum_{m=1} B_m(\mu)C_{3/2}^{3/2}(2u - 1) \right],
\]

where Gegenbauer moments \( B_m(\mu) \), which are scale dependent, are given as

\[
B_m(\mu) = \frac{1}{f_S} \frac{2(2m + 1)}{3(m + 1)(m + 2)} \int_0^1 C_{3/2}^{3/2}(2u - 1)\Phi_S(u, \mu)du.
\]
The scale evolutions of $\Phi_S(u, \mu)$ are determined using the following RGE:

$$\bar{f}_S(\mu) = \bar{f}_S(\mu_0) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{4/b},$$

$$B_m(\mu) = B_m(\mu_0) \left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\gamma(m)+4/b},$$

(36)

where the one-loop anomalous dimensions is [14]

$$\gamma(m) = C_F \left(1 - \frac{2}{(m+1)(m+2)} + 4 \sum_{j=2}^{m+1} \frac{1}{j}\right),$$

with $C_F = 4/3$. The conservation of charge parity demands an antisymmetric $\Phi_S(u, \mu)$ under the interchange $u \leftrightarrow 1-u$, namely, $\Phi_S(u, \mu) = -\Phi_S(1-u, \mu)$, for the neutral scalar mesons of a $q\bar{q}$ content. Accordingly, for the scalar mesons $a_0^0$ and $f_0$ we could write down their leading twist DA’s as

$$\Phi_S(u, \mu) = \bar{f}_S(\mu) 6u\bar{u} \sum_{m=0} B_{2m+1}(\mu) C_{2m+1}^{3/2}(2u-1).$$

(37)

In the two quark picture, it is concluded that the twist-2 DA’s of all the light scalar mesons are antisymmetric under the interchange $u \leftrightarrow 1-u$ in the flavor $SU(3)$ limit, thus the odd Gegenbauer moments dominate in the DA’s, forming a striking contrast to the corresponding situations of the pseudoscalar mesons where the leading DA of the pion, for instance, covers no odd Gegenbauer moments and so is symmetric. Indeed the zeroth Gegenbauer moment $B_0$, which is equal to $\mu_S^{1}$, vanishes in the $SU(3)$ limit. In the following, we will neglect the contributions of the even Gegenbauer moments and take only into account the first two odd moments.

To proceed, we must add that the LCSR for the form factor $f_+(q^2)$ would have a distinct scale dependence, due to the absence of the QCD radiative corrections. In such a case, it should be in order that we work at the scale $\mu_b = \sqrt{m_{B_s}^2 - m_b^2}$, which denotes the typical virtuality of the underlying $b$ quark. At this scale, the related parameters can be evaluated making use of the RGE (36) with an initial scale $\mu_0 \geq m_S$. As the initial conditions we prefer using the QCD sum rule estimates at $\mu = 1$ GeV [1], which though is a bit inadequate for the situation involving the scalar mesons above 1GeV. The numerical results for $\bar{f}_S(\mu)$ and $B_{1,3}$ are collected in Tab.II and III and the shapes of the DA’s in the two scenarios are illustrated in Fig.III.
TABLE II: Decay constants $\bar{f}_s$ and Gegenbauer moments $B_{1,3}$ of the twist-2 DA’s $\Phi_S$ at the scales $\mu = 1$ GeV [1] and 2.4 GeV (shown in parentheses) in scenario 1.

| State $\bar{f}_s$ (GeV) | $B_1$       | $B_3$       |
|-------------------------|-------------|-------------|
| $a_0(980)$              | -0.93±0.10  | 0.14±0.08   |
| $a_0(1450)$             | 0.89±0.20   | -1.38±0.18  |
| $f_0(980)$              | -0.78±0.08  | 0.02±0.07   |
| $f_0(1500)$             | 0.80±0.40   | -1.32±0.14  |
| $\kappa(800)$          | -0.92±0.11  | 0.15±0.09   |
| $K_0^*(1430)$           | 0.58±0.07   | -1.20±0.08  |

TABLE III: Decay constants $\bar{f}_s$ and Gegenbauer moments $B_{1,3}$ of the twist-2 DA’s $\Phi_S$ at the scales $\mu = 1$ GeV [1] and 2.4 GeV (shown in parentheses) in scenario 2.

| State $\bar{f}_s$ (GeV) | $B_1$       | $B_3$       |
|-------------------------|-------------|-------------|
| $a_0(1450)$             | -0.58±0.12  | -0.49±0.15  |
| $f_0(1500)$             | -0.48±0.11  | -0.37±0.20  |
| $K_0^*(1430)$           | -0.57±0.13  | -0.42±0.22  |

IV. NUMERICAL CALCULATION AND DISCUSSION

We proceed to do the LCSR calculation in the two scenarios with the scalar mesons in the two quark picture. For illustrative purpose it is sufficient to take, as a case study, the processes: $\bar{B}^0 \to a_0^+(980)/a_0^+(1450) \ l\bar{\nu}_l$, $\bar{B}_s^0 \to \kappa^+(800)/K_0^{*(+)}(1430) \ l\bar{\nu}_l$, $\bar{B}^0 \to \kappa^0(800)/K_0^{*(+)}(1430) \ l\bar{\nu}_l$ and $\bar{B}_s^0 \to f_0(980)/f_0(1500) \ l\bar{\nu}_l$.

The following inputs [6, 15, 16] will be taken in the numerical analysis:
FIG. 1: Leading twist distribution amplitudes $\Phi_S$ of the scalar mesons in scenario 1 and scenario 2 at the scale $\mu = 2.4$ GeV. It can be seen that $\Phi_S$ is antisymmetric under the replacement of $u \leftrightarrow 1 - u$ in the SU(3) limit owing to the conservation of $C$ parity.

$$G_F = 1.166 \times 10^{-2} \text{GeV}^{-2}, \quad |V_{ub}| = 3.96^{+0.09}_{-0.09} \times 10^{-3},$$

$$|V_{tb}| = 0.9991, \quad |V_{ts}| = 41.61^{+0.10}_{-0.06} \times 10^{-3},$$

$$m_u(1 \text{ GeV}) = 2.8 \text{ MeV}, \quad m_d(1 \text{ GeV}) = 6.8 \text{ MeV},$$

$$m_s(1 \text{ GeV}) = 142 \text{ MeV}, \quad m_b = (4.8 \pm 0.1) \text{ GeV},$$

$$m_{e,\mu} = 0 \text{ MeV}, \quad m_\tau = 1776.82 \text{ MeV},$$

$$m_{B_0} = 5.279 \text{ GeV}, \quad m_{B_s} = 5.368 \text{ GeV},$$

$$f_{B_0} = (0.19 \pm 0.02) \text{ GeV}, \quad f_{B_s} = (0.23 \pm 0.02) \text{ GeV}. \quad (38)$$

In the first place, let us make investigation in the context of scenario 1. The numerical discussions of the form factors $f^+(q^2)$ can proceed in terms of the standard procedure for sum rule calculations. The threshold parameters $s_0$, which correspond to the masses $m_{B_S}^2$ of the lowest scalar $B_{(s)}$ mesons [7], need to be estimated in a certain nonperturbative approach. Using the QCD sum rule result [8] for the binding energy difference between the scalar and pseudoscalar B mesons in the heavy quark effective theory, we could give reasonably $s_{B_0}^{B_0} = s_{B_0}^{B_0} = 33 \pm 1 \text{ GeV}^2$, which is smaller than the threshold values in the corresponding conventional sum rule calculations, with the experimental values of the pseudoscalar B mesons. Also, it is possible to determine the threshold parameters in other approaches, among which the scenario suggested in [9] is more effective. The range of the Borel parameter $M^2$, which is shared by all the sum rules in question, is determined as $10 \text{ GeV}^2 \leq M^2 \leq
TABLE IV: Form factors $f_+$ and $f_-$ at zero momentum transfer $q^2 = 0$ GeV$^2$ in scenario 1(S1) and scenario 2(S2) for semileptonic decays $B_s \to Sl\bar{\nu}_l$ with light-cone sum rules(LCSR)\[6\], sum rules(SR)\[17\] and perturbative QCD(pQCD)\[18\] approaches.

| Methods       | $\bar{B}^0 \to K^{+*}$ (1430) | $\bar{B}^0 \to a_0^+$ (1450) | $\bar{B}_s^0 \to \kappa^+$ (800) | $\bar{B}^0 \to a_0^+$ (980) |
|---------------|-------------------------------|-------------------------------|---------------------------------|-------------------------------|
| This work(S1) | $+0.10$                       | $+0.26$                      | $+0.53$                        | $+0.56$                       |
| This work(S2) | $+0.44$                       | $-0.26$                      | $-0.53$                        | $-0.56$                       |
| SR \[17\]    | $+0.24$                       | $-0.34$                      | $+0.52$                        | $-0.44$                       |
| LCSR(S2) \[6\]| $+0.42$                       | $+0.31$                      | $-0.29$                        | $+0.39$                       |
| pQCD(S1) \[18\]| $-0.32$                       | $-0.68$                      | $-0.32$                        | $-0.68$                       |
| pQCD(S2) \[18\]| $+0.56$                       | $-0.56$                      | $+0.56$                        | $-0.56$                       |

FIG. 2: Dependence of form factors $f_+(q^2 = 0)$ for $\bar{B}^0 \to a_0^+(980)$ and $\bar{B}_s^0 \to f_0(1500)$ on the Borel parameter $M^2$ in scenario 1 within the LCSR approach at the scale $\mu = 2.4$ GeV. We take the threshold $s_0 = 32, 33, 34$ GeV$^2$ and $b$ quark mass $m_b = 4.8$ GeV.

15 GeV$^2$. In this interval, the higher states and continuum contribute less than 30% and the sum rule results vary by 13 ~ 30% around the central values, depending on the decay modes.

To elucidate our findings for the form factors, we can consider typically the case of the $B \to a_0(980)$ and $B_s \to f_0(1500)$ transitions. The LCSR for form factors, $f^B_{+ \to a_0^+(980)}(0)$ and $f^B_{+ \to f_0(1500)}(0)$, are of a good stability against $M^2$-varying, as shown in Fig2. For
TABLE V: Form factors $f_+,$ $f_-$ and $f_T$ for rare decays $B_\pm \rightarrow S\bar{t}$ at $q^2 = 0$ GeV$^2$ in scenario 1 (S1) and scenario 2 (S2), with light cone sum rules (LCSR)\textsuperscript{[6, 19]}, sum rules (SR)\textsuperscript{[20, 21]}, light front quark model (LFQM)\textsuperscript{[22]}, minimal supersymmetric standard model (MSSM)\textsuperscript{[23]}, covariant light-front (CLF)\textsuperscript{[24]}, covariant quark model (CQM)\textsuperscript{[25]} and perturbative QCD (pQCD)\textsuperscript{[18]} approaches.

| Methods       | $\bar{B}^0 \rightarrow K^+_0(1430)$ | $\bar{B}^0_s \rightarrow f_0(1500)$ | $\bar{B}^0 \rightarrow \bar{K}^0(800)$ | $\bar{B}^0 \rightarrow f_0(980)$ |
|---------------|------------------------------------|-------------------------------------|--------------------------------------|----------------------------------|
|               | $f_+$  | $f_-$  | $f_T$ | $f_+$  | $f_-$  | $f_T$ | $f_+$  | $f_-$  | $f_T$ |
| This work (S1)| +0.17  | −0.17  | +0.24 | +0.14  | −0.14  | +0.20 | +0.46  | −0.46  | +0.58 |
| This work (S2)| +0.49  | −0.49  | +0.69 | +0.41  | −0.41  | +0.59 | −       | −       | −       |
| LFQM\textsuperscript{[22]} | −0.26  | +0.21  | −0.34 | −       | −       | −       | −       | −       | −       |
| CLF\textsuperscript{[24]}   | +0.26  | −       | −       | −       | −       | −       | −       | −       | −       |
| SR (S2)\textsuperscript{[20]}| +0.31  | −0.31  | −0.26 | −       | −       | −       | −       | −       | −       |
| SR\textsuperscript{[21]}   | −       | −       | −       | −       | −       | −       | −       | −       | +0.12  |
| LCSR (S2)\textsuperscript{[6]}| +0.49  | −0.41  | +0.60 | +0.43  | −0.37  | +0.56 | −       | −       | −       |
| LCSR\textsuperscript{[19]} | −       | −       | −       | −       | −       | −       | −       | −       | +0.19  |
| pQCD (S1)\textsuperscript{[18]} | −0.34  | −       | −0.44 | −0.26  | −       | −0.34 | +0.27  | −       | +0.29  |
| pQCD (S2)\textsuperscript{[18]} | +0.60  | −       | +0.78 | +0.60  | −       | +0.82 | −       | −       | −       |
| CQM\textsuperscript{[25]}   | −       | −       | −       | −       | −       | −       | −       | −       | +0.40  |
| MSSM\textsuperscript{[23]}  | +0.49  | −0.41  | +0.60 | −       | −       | −       | −       | −       | −       |

Simplicity, throughout the numerical investigation we give only the central values of the sum rule results, corresponding to $M^2 = 12$ GeV$^2$ and $s_{B^0} = s_{B^0_s} = 33$ GeV$^2$. Then we have the observations $f_{B^0 \rightarrow a_0^+(980)}(0) = 0.56$ and $f_{B^0_s \rightarrow f_0(1500)}(0) = 0.14$. Furthermore, use of the relations (24) and (25) leads to $f_{-}^{B^0 \rightarrow a_0^+(980)}(0) = -0.56, f_{B^0_s \rightarrow f_0(1500)}(0) = -0.14$ and $f_{T}^{B^0 \rightarrow f_0(1500)}(0) = 0.20$. Within the LCSR allowed kinematical regions, $f_{+}^{B^0 \rightarrow a_0^+(980)}(q^2)$ and $f_{T}^{B^0_s \rightarrow f_0(1500)}(q^2)$ as a function of $q^2$ are depicted in Fig\textsuperscript{[3]} along with those corresponding to the other modes. The behaviors of $f_{B \rightarrow S}(q^2)$ and $f_{T}^{B \rightarrow S}(q^2)$ are understandable likewise with the relations (24) and (25). Additionally, for a complete understanding of dynamical behaviors of the $B \rightarrow S$ transitions at the largest recoils, one can be referred to Tab\textsuperscript{[IV]} and Tab\textsuperscript{[V]} where we collect the present LCSR results for the form factors $f_{B^0 \rightarrow S}(0)$ in all the cases and the predictions of other approaches for comparison.
FIG. 3: Dependence of $B_s \to S$ form factors on the transfer momentum $q^2$ in scenario 1 within the LCSR approach with the scale $\mu = 2.4$ GeV, threshold parameter $s_0 = 33$ GeV$^2$ and Borel parameter $M^2 = 12$ GeV$^2$.

FIG. 4: Differential decay widths of the semileptonic $B \to S l\bar{\nu}_l$ decays as functions of $q^2$ in scenario 1. Here $l = e, \mu$ in the left diagram.
FIG. 5: Differential decay widths of the rare $B_{(s)} \rightarrow S\bar{\ell} \ell$ ($l = e, \mu$) decays as functions of $q^2$ in scenario 1.

FIG. 6: Dependence of $B_{(s)} \rightarrow S$ form factors on the transfer momentum $q^2$ in scenario 2 within the LCSR approach with the scale $\mu = 2.4$ GeV, threshold parameter $s_0 = 33$ GeV$^2$ and Borel parameter $M^2 = 12$ GeV$^2$.

It is manifest that there is a sizable numerical difference in the form factors between the transitions to the ground states and to the excited ones. To make it clear, we go back to the LCSR expressions for the form factors. We observe that the DA’s $\Phi_S(u, \mu)$ make contribution only in a smaller region of the momentum fraction $u$ ranging approximately from $0.8 \sim 1$ at $q^2 = 0$. The light quark from the heavy quark decays prefers transferring to the region close to its kinematical end-point to build a bound state with the spectator quark of the decaying heavy meson, which is the so-called Feynman mechanism, that is,
FIG. 7: Differential decay widths of the semileptonic $B \to Sl\bar{\nu}_l$ decays as functions of $q^2$ in scenario 2. Here $l = e, \mu$ in the left diagram.

FIG. 8: Differential decay widths of the rare $B_{(s)} \to Sl\bar{\nu}_l$ ($l = e, \mu$) decays as functions of $q^2$ in scenario 2.

soft-exchanges predominate over hard-ones in the decay process. Referring to Fig. one finds that in that subregion the DA’s behave quite differently between the scalar objects below and above 1 GeV. For the scalar mesons below 1 GeV, in the whole subregion their DA’s turn out to be negative and hence make a constructive contribution to the sum rules. A different situation manifests itself as scalar mesons involved are heavier ones: the DA’s contribute constructively in one part of the subrange but do destructively in the other. That the two effects cancel out to a large degree leads to a form factor in magnitude much smaller than those for the ground states. Physically, this indicates that for a given $q^2$, as with the
former situation the decaying B mesons have a larger energy release in the latter one.

In the same picture the $B \to S$ transitions have been explored in the several approaches, such as the pQCD \cite{18}, QCD sum rules \cite{17,20,21} and LCSR \cite{6,19}. It is interesting to confront our results with some of the previous studies. In what follows, wherever a result of any other approach is referred, it should be understood that we have, if necessary and possible, converted it into that in the present convention. Application of the LCSR is enforced to B decays to a scalar final state by taking the $B_s \to f_0(980)$ semileptonic processes as a study case in Ref.\cite{19}. The sum rules for the form factors, with the asymptotic forms used for twist-3 DA’s, give $f^{B_s\to f_0(980)}_+ (0) = 0.19$ and $f^{B_s\to f_0(980)}_T (0) = 0.23$ subject to an uncertainty estimate omitted here. Counting QCD next-to-leading corrections, which is estimated roughly based on the observation of the LCSR calculation for the $B \to \pi$ transitions, the above results are modified to $f^{B_s\to f_0}_+ (0) = 0.24$ and $f^{B_s\to f_0}_T (0) = 0.31$, about 45% less than our calculations. The reason for the sizable differences is mainly use of the different inputs for the decay constant $\bar{f}_{f_0(980)}$. The different scales are taken for the leading and the subleading twist DA’s as important inputs, which would have, of course, an impact on accuracy of result. Using the same inputs, the two evaluations are found to be consistent with each other and that of QCD sum rules. The pQCD approach predicts, for the decay modes to the scalar ground states, that the form factors are a bit smaller in magnitude but within error comparable with the present calculations, and have the approximately same value as in the case of the first excited states, a result quite other than our predictions. It is not difficult to understand for heavy-to-light transitions, because the pQCD approach accords with the hard-exchanges mechanism, and the resulting form factors rely on the behaviors of the DA of light meson in the whole momentum region accessible for the constitute quarks.

All the approaches mentioned above are no doubt applicable in the kinematical region near the largest recoil for calculation of the form factors. Nevertheless, no decisive region of $q^2$, in which these approaches work well, has been provided in the existing applications to the $B \to S$ transitions. In the LCSR calculation \cite{6}, the form factors are artificially limited to the range $0 < q^2 < 15$ GeV$^2$, which seem somewhat large against our estimate, and then the results are fitted to a dipole model for having an understanding of the behaviors of the form factors in the whole kinematically accessible region. The same way is adopted in the pQCD calculation \cite{18} to extrapolate the results for the form factors from small $q^2$ range to large one. Although such a extrapolation manner is phenomenologically extensively assumed,
caution should be taken when one applies it to the present case. First of all, we have no theoretical justification for doing so. The pole models are believed to be suitable merely for description of those form factors corresponding to \( q^2 \) near the squared pole masses \( m^2_{\text{pole}} \), however, for the present \( B \to S \) transitions the \( m^2_{\text{pole}} \) are far away from their kinematical regions. On the other hand, if the work region for an approach can not be assigned effectively, choosing different fitting regions would lead to different results. Hence, it is questionable to use a pole description to get an all-around understanding of \( q^2 \) dependence of the form factors for \( B \to S \) transitions. Taking this into account, we prefer calculating in the effective regions rather than in the whole kinematical range.

Now, we are in a position to look into the differential decay rates for the \( B \to S \) semileptonic decays, which are expressed as

\[
\frac{d\Gamma}{dq^2}(B(s) \to Sl\bar{\nu}_l) = \frac{G_F^2 |V_{ub}|^2 q^2 - m_l^2}{192\pi^3 m_B^3 (q^2)^2} \sqrt{\frac{(q^2 - m_t^2)^2}{q^2}} \sqrt{\frac{(m_B^2 - m_S^2 - q^2)^2}{4q^2}} - m_S^2
\]

\[
\times \left[ (m_t^2 + 2q^2)(q^2 - (m_B - m_S)^2)(q^2 - (m_B + m_S)^2)f_+(q^2)\right.
\]

\[
+ 3m_t^2(m_B^2 - m_S^2)^2 \left( f_+(q^2) + \frac{q^2}{m_B^2 - m_S^2}f_-(q^2) \right)^2 \right], \quad (39)
\]

\[
\frac{d\Gamma}{dq^2}(B(s) \to S\bar{l}\bar{l}) = \frac{G_F^2 |V_{ub}V_{ts}|^2 m_B^3 \alpha_e^2}{1536\pi^5} \left( 1 - \frac{4r_l}{s} \right)^{1/2} \left[ \left( 1 + \frac{2r_l}{s} \right) \varphi_{S}^{3/2} \alpha_s + \varphi_{S}^{1/2} r_l \delta_s \right], \quad (40)
\]

where \( m_l \) denotes the mass of a final state lepton, and

\[
s = q^2/m_B^2, \quad r_l = m_t^2/m_B^2, \quad r_S = m_S^2/m_B^2,
\]

\[
\varphi_S = (1 - r_S)^2 - 2s(1 + r_S) + s^2,
\]

\[
\alpha_s = \left| C_{9\text{eff}} f_+(q^2) - 2 C_7 f_T(q^2) \frac{1}{1 + \sqrt{r_S}} \right|^2 + \left| C_{10} f_+(q^2) \right|^2,
\]

\[
\delta_s = 6 |C_{10}|^2 \left\{ [2(1 + r_S) - s] \left| f_+(q^2) \right|^2 + (1 - r_S) 2\text{Re}[f_+(q^2) f_+(q^2)] + s \left| f_-(q^2) \right|^2 \right\}.
\]

In the respective effective regions \( m_t^2 \leq q^2 \leq (m_B - m_S)^2 \) and \( 4m_t^2 \leq q^2 \leq (m_B - m_S)^2 \), we assess the distributions of the differential rates for the \( B(s) \to Sl\bar{\nu}_l \) and \( B(s) \to S\bar{l}\bar{l} \), with the results displayed in Fig[4] and Fig[5] where we have set \( m_e = m_\mu = 0 \). In the case of \( B(s) \to S\bar{l}\bar{l} \) there appears a discontinuity at \( q^2 = 4m_e^2 \) stemming from the function \( h(z, s') \). The differential decay rates for the \( B(s) \to S\tau^+\tau^- \) are incalculable in the present approach, for the dilepton threshold \( 4m_e^2 \) is beyond our work regions. It is shown that our calculations
and the predictions of pQCD \cite{18} are comparable with each other, although they are based on two different dynamical schemes.

As the scenario 2 is adopted, an analogous LCSR analysis can be made in principle, however a complete discussion is not practicable at present, due to little knowledge of the 4-quark scalar states below 1 GeV. Along the same line as above, we can assess the semileptonic decays of $B_{(s)}$ to a scalar above 1 GeV, which is viewed as a two quark ground state. The sum rules show the same Borel interval as in the case of scenario 1. The variations of the form factors $f^+(q^2)$ with $q^2$ are exhibited in Fig.6, and at the largest recoil, a summary of the numerical results for the form factors involved, including some of the previous estimates, is given in Tab.IV and Tab.V. Comparing the sum rule calculations between the scenarios 1 and 2, we see that in the latter case the form factors $f^+(q^2)$ have a central value between $0.40 \sim 0.70$ in the effective regions, depending on the decay modes, and hence are less sensitive to $q^2$ than in the former case in which there is a large numerical range from $0.10 \sim 0.60$. The present evaluations of $f^+(0)$, which show a better agreement with the conventional LCSR calculation \cite{6, 19}, are a bit smaller than the numerical observation in pQCD \cite{18}, and meanwhile are large numerically in comparison with the calculation of QCD sum rules in both the $B \to K^*$ and $B_s \to K^*$ situations, especially our result turning out to be about twice as large as that of QCD sum rules in the latter case.

The resulting differential decay rates, as exhibited in Fig.7 and Fig.8, have a behavior other significantly from what is observed in scenario 1, with the remarkably different QCD dynamics embedded in the form factors between the two scenarios. Once these scalar mesons above 1 GeV are clearly identified to be, purely or mainly, the two quark bound state, this result might help to distinguish between both the pictures for them, as the future experiments become accessible. In addition, the distribution shapes, which are demonstrated by the differential rates for $B_{(s)} \to Sl\bar{\nu}_l$ in Fig.7, are compatible with the LCSR calculation.

The decays to the scalar meson below 1 GeV, despite theoretically little accessible for the moment, could be discussed qualitatively. In the four quark final states there is a quark-antiquark component from the annihilations of emitted gluons in the decaying processes, which gets the transitions highly suppressed. Consequently we may deduce that in scenario 2 the related form factors are of a small numerical value with respect to the results in the two quark picture.

Finally, we should point out that all the above discussions can not be generalized to $D_{(s)}$
decays to a scalar meson, because of the fact that the decaying mesons have a recoil energy not large enough to make LCSR applicable, in their decaying processes.

V. SUMMARY

We have presented a LCSR computation on $B_{(s)} \rightarrow S \bar{l} \nu_l, S l \bar{l}$ at leading order in $\alpha_s$, in the two quark picture for the scalar mesons with the two different scenarios. A correlation function with chiral current operator is chosen such that the resulting LCSR the form factors can avoid the pollution with the twist-3 DA’s of the scalar mesons. Applicable regions of the LCSR approach are discussed and are assigned reasonably as $0 \leq q^2 < 11$ GeV$^2$ and $0 \leq q^2 < 8$ GeV$^2$, for the scalar final states below and above 1 GeV, respectively. Also, we investigate the properties of the DA’s of the scalar mesons, obtaining an observable difference from the case of the pseudoscalar mesons. In the effective regions, the form factors and differential decay rates are estimated, with the main findings summarized as the following: (1) There exist the relations among the the form factors for the $B \rightarrow S$ transitions, which are in accordance with the prediction of SCET. (2) For the decays to a scalar ground state, in the case of scenario 1 the form factors at $q^2 = 0$ show the numerical result much larger those for the first excited states, and as confronted with the corresponding observations in scenario 2, the former seem large in magnitude, but the latter are predicted to be small. (3) For the semileptonic processes with the scalar final state above 1 GeV$^2$, the resulting differential decay rates have a significantly different behavior for the different scenarios. Some of them might be beneficial to experimentally identify physical natures of the scalar mesons. The present results might be improved as the QCD radiative corrections are taken into account, and however they are not expected to change too much from the LCSR calculation on the $B \rightarrow \pi$ transition\[26].

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