Cosmological Implications of the Second Parameter of Type Ia Supernovae

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\textbf{ABSTRACT}

Theoretical models predict that the initial metallicity of the progenitor of a Type Ia supernova (SN Ia) affects the peak of the supernova light curve. This can cause a deviation from the standard light curve calibration employed when using SNe Ia as standardizable distance candles and, if there is a systematic evolution of the metallicity of SN Ia progenitors, could affect the determination of cosmological parameters. Here we show that this metallicity effect can be substantially larger than has been estimated previously, when the neutronisation in the immediate pre-explosion phase in the CO white dwarf is taken into account, and quantitatively assess the importance of metallicity evolution for determining cosmological parameters. We show that, in principle, a moderate and plausible amount of metallicity evolution could mimic a $\Lambda$-dominated, flat Universe in an open, $\Lambda$-free Universe. However, the effect of metallicity evolution appears not large enough to explain the high-z SN Ia data in a flat Universe, for which there is strong independent evidence, without a cosmological constant. We also estimate the systematic uncertainties introduced by metallicity evolution in a $\Lambda$-dominated, flat Universe. We find that metallicity evolution may limit the precision with which $\Omega_m$ and $w$ can be measured and that it will be difficult to distinguish evolution of the equation of state of dark energy from metallicity evolution, at least from SN Ia data alone.

\textbf{Key words:} cosmological parameters – distance scale – supernovae: general – supernovae: Type Ia – galaxies: evolution

1 INTRODUCTION

The use of Type Ia supernovae (SNe Ia) as standardizable cosmological distance candles (Riess et al. 1998; Perlmutter et al. 1999; Tonry et al. 2003; Riess et al. 2004; Astier et al. 2006) relies on the empirical fact that there is a tight correlation between the supernova peak brightness and the width of the supernova light curve (Phillips 1993), i.e. the fact that, to lowest order, SN Ia light curves form a one-parameter family of curves. The driving parameter that determines the relation, assuming that the mass of the progenitor white dwarf at explosion is constant, has been shown to be the opacity in the ejecta (Khokhlov, Müller & Höflich 1993; Höflich et al. 1996, Mazzali et al. 2001). This is closely related to the quantity of radioactive $^{56}$Ni synthesised in the explosion, which is responsible for the SN luminosity. In recent years it has become apparent from a larger sample of observed supernovae that not all SN Ia light curves fit into this one-parameter family, producing an intrinsic, scatter in the Phillips relation (see the discussion in Mazzali & Podsiadlowski [2006] and Benetti et al. [2004]). This immediately implies that there must be more than one parameter controlling SN Ia light curves. The physical property in the progenitor that determines the dominating (first) parameter still has not been clearly identified. On the other hand, from a theoretical point of view it seems unavoidable that the metallicity of the original supernova progenitor must at least in part be responsible for the intrinsic scatter about the mean Phillips relation. Timmes, Brown & Truran (2003) showed, using straightforward and uncontroversial nuclear physics arguments, that the neutron excess in the immediate progenitor white dwarf, which is a direct function of the ini-
tial metallicity of the progenitor star, controls the fraction of radioactive to non-radioactive Ni produced in the exploding white dwarf and hence affects the peak supernova luminosity, an effect subsequently confirmed by detailed explosion calculations (Röpke et al. 2005; Travaglio, Hillebrandt & Reinecke 2005). On the other hand, the width of the supernova light curve is ultimately determined by the opacity of the ejecta which is mainly a function of the total amount of iron-peak elements, i.e. is independent of the ratio of radioactive to non-radioactive material (Mazzali et al. 2001). Mazzali & Podsiadlowski (2006) demonstrated that this produces an intrinsic scatter around the Phillips relation comparable to the observed scatter. An intrinsic scatter in the Phillips relation itself does not necessarily limit the usefulness of SNe Ia as cosmological probes, as long as the scatter is non-systematic. However, a physical parameter such as metallicity can in principle introduce subtle evolutionary effects that could affect the determination of cosmological parameters. It is the purpose of this paper to quantify the magnitude of such evolutionary effects and to demonstrate that these need to be taken into account, in particular when trying to constrain higher-order effects such as the cosmological evolution of the equation of state of dark energy or its variants.

In Section 2 we show that the effect of metallicity on the supernova light curves may actually be significantly larger than originally estimated by Timmes et al. (2003), since these authors did not consider the additional neutronisation in the white dwarf core in the immediate pre-explosion phase. In Section 3 we investigate quantitatively the conditions under which the apparently observed deviation in the Hubble relation from a Λ-free cosmology could be explained by evolutionary effects, and in Section 4 we estimate the metallicity-dependent corrections when determining the equation of state of dark energy in a Λ-dominated cosmology, as presently favoured by various experiments. Finally in Section 5 we discuss these results and possible observational tests.

2 RELATION BETWEEN $M_{\text{PEAK}}$ AND $Z$

As Timmes et al. (2003) showed, the neutron excess in the core of a CO white dwarf at the time of the thermonuclear runaway determines the ratio of stable to unstable Ni, which in turn determines the supernova peak luminosity. The neutron excess depends on the abundance of elements that have an excess of neutrons, such as 56Fe and 22Ne. The main source of excess neutrons in the interstellar medium from which a star is born is 56Fe which has four extra neutron in each nucleus. The neutron excess $\eta$ for 56Fe therefore is

$$\eta_{56Fe} = 4 \frac{X(56Fe)}{56},$$

where we use the notation $X(E)$ to denote the mass fraction of a given element E. The abundance of 56Fe is not modified until the explosion (any gravitational settling will be removed by the growth of the convective core during the carbon flash [C-flash]).

In contrast, 22Ne is formed predominantly during the progenitor’s nuclear evolution: during hydrogen burning, the CNO cycle converts most of the initial nuclei of 12C, 16O into 14N, which subsequently is converted into 22Ne by two α captures during helium burning. Thus, essentially all of the initial abundances of C, N and O are converted into 22Ne in the white-dwarf core, making it the most abundant element after 12C and 16O. Hence there is a direct relation between the 22Ne contained in the C+O WD and the amount of C, N and O nuclei initially present in the star:

$$\frac{1}{22} X(22Ne) = \frac{1}{12} X(12C) + \frac{1}{14} X(14N) + \frac{1}{16} X(16O).$$

Since 16O is usually the most abundant element by number in star-forming regions, it is the initial 16O abundance that is the most important element for determining the neutron excess in the white-dwarf core.

Timmes et al. (2003) assumed that the neutron excess at the time of the explosion was entirely determined by the initial abundances of C, N, O and Fe. However, before carbon burning runs away and initiates the supernova explosion, there is a long phase of low-level carbon burning, typically lasting several thousand years, during which the excess energy produced by carbon burning is efficiently transported away from the burning region by convection, delaying the thermonuclear runaway and leading to a gradually increasing convective core. In this phase, the densities are high enough that electron captures are very efficient and further neutronise the core material. To model this realistically, one needs detailed stellar evolution calculations with a detailed nuclear reaction work and a proper treatment of the convective Urca process (see Lesaffre, Podsiadlowski & Tout 2005). However, with some approximations, we can estimate the effect of additional electron captures onto 22Ne prior to the explosion phase. Each electron capture (or inverse beta decay) increases the number of excess neutrons per nucleus by two (since one proton is converted into one neutron). On the other hand, at these high densities the inverse reactions (beta decays, positron captures) are strongly disfavoured energetically. Therefore, the change in the overall neutronisation of the matter depends only on the total number of electron captures/inverse beta decays.

For this analysis, we consider the reduced network of the main nuclear reactions during carbon burning shown in table 6 of Arnett & Thielemann (1985). In this network, we further neglect neutron-capture reactions compared to α and proton captures. This is reasonable since the main basic reactions are 12C(12C, α)20Ne and 12C(12C, p)13Na which produce α particles and protons in approximately equal amounts, whereas neutron sources come from much weaker reactions.

We now consider the fate of 22Ne in this network and record the net number of electron captures involved in the subsequent chain of reactions. Each electron capture will

1 The neutron excess, $\eta$, for a nucleus of species $i$ can be written as $\Delta n Y_i$, where $\Delta n$ is the number of extra neutrons per nucleus (i.e. the number of neutrons minus the number of protons) and $Y_i$ is the number fraction for this species. The total $\eta$ summed over all species, is directly related to the electron number fraction, $Y_e$, according to $\eta = 1 - 2Y_e$.

2 Note that the 12C and 16O nuclei that exist at the time of the explosion have been exclusively produced in the progenitor star by the triple-α reaction and α captures during helium burning.
Indeed, add two extra neutrons/nucleus. Neglecting the neutron capture reaction, $^{22}\text{Ne}$ has one of two choices:

- it can either capture an $\alpha$ particle and be locked in $^{26}\text{Mg}$ via the chain $^{22}\text{Ne}(\alpha,n)^{25}\text{Mg}(n,\gamma)^{26}\text{Mg}$,
- or it can capture a proton via $^{22}\text{Ne}(p,\gamma)^{23}\text{Na}$.

Using analytical rates from the REACLIB (Rauscher & Thielemann 2002) data base, we observe that the branching ratio of these two reactions is more than 97\% in favour of the proton capture for all temperatures below $2 \times 10^9$ K, a temperature which is well below the maximum temperature that can be expected during the C-flash (Lesaffre et al. 2006). We can hence safely neglect the alpha-capture reaction.

The $^{23}\text{Na}$ nucleus produced by the proton capture can now either capture an electron with a probability $q$ to become $^{23}\text{Ne}$ or capture a proton with a probability $1 - q$. In the latter case, the chain of reactions is $^{23}\text{Na}(p,\gamma)^{24}\text{Mg}(p,\gamma)^{25}\text{Al}(\beta^+)^{25}\text{Mg}$ with one inverse beta decay. $^{25}\text{Mg}$ can capture either an electron or a proton, but in both cases it leads to one electron capture/inverse beta decay since $^{25}\text{Mg}(p,\gamma)^{26}\text{Al}(\beta^+)^{26}\text{Mg}$. The probability $q$ is not easy to estimate since it depends on how the proton-capture rate compares to the convective Urca process on $^{23}\text{Mg}$.

A $^{22}\text{Ne}$ nucleus can hence either capture one electron with a probability $q$ or capture two electrons with a probability $(1 - q)$. Finally, if we assume that a fraction $f$ of the $^{22}\text{Ne}$ is burnt during the C-flash, this leads to an increase of the neutron excess of $2f(2 - q)$ for each initial $^{22}\text{Ne}$ nucleus which already has two extra neutrons. Summing up these contributions, we obtain the total neutron excess before the C-flash due to $^{22}\text{Ne}$

$$ \eta_{22} = 2[1 + f(2 - q)]X(22\text{Ne})/22. $$

Based on our present C-flash calculations (Lesaffre et al. 2006), we estimate that at least 0.02 of the mass of carbon is burnt in the C-flash phase, implying that most of the $^{22}\text{Ne}$ may be consumed (i.e. $f \simeq 1$). However, this will generally depend on the initial abundances and also the C/O ratio and needs to be confirmed with a full reaction network (both $f$ and $q$ could in fact depend on the first parameter!).

Summing up all contributions of metallicity to the neutronisation from the birth of the star until the start of the explosion, we get:

$$ \eta = \eta_{22} + \eta_0 = 4 \frac{X(56\text{Fe})}{56} + 2[1 + f(2 - q)] \left[ \frac{X(12\text{C})}{12} + \frac{X(14\text{N})}{14} + \frac{X(16\text{O})}{16} \right]. $$

Finally, the neutron excess might change according to parameters other than the metallicity, and we write the net neutronisation at the time of the explosion as

$$ \eta = \eta_0 + \eta_Z, $$

where the subscript 0 denotes the neutronisation for zero metallicity.

Following Timmes et al. (2003), we assume that the combustion is fast enough compared to weak interactions so that the neutron excess does not change in the process\(^3\).

Allowing only $^{56}\text{Ni}$ and $^{58}\text{Ni}$ as products of the burning, we get the mass fraction of unstable $^{56}\text{Ni}$

$$ X(56\text{Ni}) = 1 - 29\eta f. $$

Since the peak luminosity $L$ of a SN Ia is roughly proportional to $X(56\text{Ni})$ for a given value of the primary parameter (e.g. Arnett 1982), we can write

$$ L = l_0 X(56\text{Ni}). $$

From this we compute the magnitude difference due to metallicity effects

$$ M - M_0 = -2.5 \log_{10} \left( 1 - \frac{29}{1 - 29\eta_0} \eta_Z \right), $$

where $M$ and $M_0$ are the peak magnitudes for the light curves of SNe Ia of a given primary parameter with ($M$) and without ($M_0$) metals.

Setting $\eta_0 = 0$ (i.e. assuming that the primary parameter does not affect the neutronisation) and adopting the solar abundance ratios from Asplund, Grevesse & Sauval (2005), we can write equation (6) as

$$ X(56\text{Ni}) = 1 - \alpha Z/Z_0. $$

For $Z_0 = 0.02$, we obtain $\alpha = 0.165$ (for $q = 0$ and $f = 1$) and $\alpha = 0.111$ (for $q = 1$ and $f = 1$)\(^4\). For comparison, with $f = 0$ (i.e. neglecting the neutronisation during the C-flash), we recover $\alpha = 0.058$, the value obtained by Timmes et al. (2003).

The metallicity dependence of the supernova peak magnitude relative to a reference model $M_{\text{ref}}$ can then be written as

$$ M - M_{\text{ref}} = -2.5 \log_{10} \left( [1 - \alpha Z/Z_0]/(1 - \alpha Z_{\text{ref}}/Z_0) \right), $$

where $Z_{\text{ref}}$ could, e.g., represent the typical metallicity of the SN Ia sample used in the calibration of the Phillips relation. Figure 1 shows this relation for different values of $\alpha$ for zero metallicity. Expanding this relation to second order in $\alpha$, we can approximate this relation as

$$ M - M_{\text{ref}} \simeq 1.086\alpha \frac{Z - Z_{\text{ref}}}{Z_0} \left[ 1 + \frac{2}{3} \alpha \frac{Z + Z_{\text{ref}}}{Z_0} \right]. $$

Since the factor in square brackets is not much larger than 1, this shows that, to lowest order, the metallicity-dependent correction is proportional to both $\alpha$ and $(Z - Z_{\text{ref}})$. In the following sections, we generally adopt an intermediate value

\(^3\) This will generally not be correct for the central region of the white dwarf ($\sim 0.1 - 0.2 M_\odot$) where the main nucleosynthesis products are $^{54}\text{Fe}$, $^{56}\text{Fe}$ and $^{58}\text{Ni}$ (e.g. Nomoto et al. 1984). However, this radioactively inert region is not expected to contribute significantly to the lightcurve shape (e.g. Mazzali & Podsiadlowski 2006).

\(^4\) The metallicity normalisation was chosen as $Z_0 = 0.02$ since this is a typical value adopted in the literature to represent "solar metallicity". With the Asplund et al. (2005) composition, this implies a logarithmic oxygen abundance by number of log (O/H) $+ 12 = 8.87$. In contrast, the corresponding values for the Sun are 0.0122 and 8.66 for the Asplund et al. (2005) composition mixture, which has a hydrogen mass fraction $X = 0.7329$, and abundance ratios (by number) C/O= 0.537, N/O= 0.132 and Fe/O= 0.062.
Figure 1. The effect of metallicity on the peak magnitude of SNe Ia, relative to a reference peak magnitude $M_0$ for $Z = 0$, as a function of logarithmic oxygen abundance (by number). On this scale, a metallicity of $Z = 0.02$ corresponds to log(O/H) + 12 = 8.87 (Asplund et al. 2005).

for $\alpha$ of 0.111, but because of this linear relationship, it is easy to rescale all the metallicity effects for different values of $\alpha$.

Hamuy et al. (2000) and Gallagher et al. (2005) have previously looked for a correlation between SN Ia peak magnitudes and the metallicity of the host galaxy for nearby SNe Ia and found no significant correlation. This indeed confirms that metallicity cannot be the dominant first parameter that controls the variation of SN Ia light curves but can only be a second parameter. These authors suggested that the first parameter is related to age, which in turn could be related to the central ignition density (see, e.g., Lesaffre et al. 2006). Gallagher et al. (2005) also attempted to identify any systematic metallicity-dependent deviation from the mean Hubble relation and found a weak metallicity trend, but one that was not very statistically significant (also see Wang et al. 2006).

Finally, we note that metallicity can modify the properties of the exploding white dwarfs in other ways than just through the neutron excess; for the single-degenerate model, various theoretical studies have predicted or suggested that metallicity affects the initial mass range for white dwarfs that will ultimately explode (Langer et al. 2000), the C/O ratio in the white dwarf (Höflich, Wheeler & Thielemann 1998; Umeda et al. 1999) and the accretion efficiency in the progenitor phase (Kobayashi et al. 1998; Umeda et al. 1999). All of these factors could either enhance or reduce the effect of the neutron excess on its own (the possible evolutionary consequences of some of these effects have recently also been considered by Riess & Livio 2006).

3 THE EFFECTS OF METALLICITY ON THE DETERMINATION OF COSMOLOGICAL PARAMETERS

As is clear from Figure 1, a systematic variation in metallicity could in principle introduce a systematic effect on the observed SN Ia relation that is comparable in magnitude to the inferred effect that led to the first evidence for an accelerating Universe (Riess et al. 1998; Perlmutter et al. 1999). In this section we will therefore ask the question what metallicity evolution is required to mimic a $\Lambda$-dominated Universe in a more traditional $\Lambda$-free cosmology.

The Phillips relation and the first and second parameter

Before investigating metallicity effects, it is important to point out that, while the Phillips relation used to correct SN Ia light curves is a one-parameter relation, whether this is done using the $\Delta M_{15}$ method (Phillips et al. 1999) or an equivalent method such as the stretch method (Goldhaber et al. 2001), this parameter is unlikely to be completely independent of metallicity. The calibration of the relation uses a local sample of SNe Ia which includes supernovae over a wide range of metallicity. Therefore, the Phillips relation is a convolution of whatever the dominant parameter is that controls supernova light curves (e.g. central ignition density) and the metallicity dependence of the local sample. Hence, it is probably not surprising that the local sample does not show a clear, systematic metallicity-dependent deviation from the mean Phillips relation (Gallagher et al. 2005).

However, this does not imply that there should be no metallicity effect at an earlier epoch. As long as there is more than one parameter controlling SN Ia light curves, and as long as the supernova sample changes with redshift, systematic deviations from the nearby relation are expected. This is illustrated in Figure 2, which schematically shows how sampling effects can affect the correction function that needs to be applied. Note, in particular, that it is possible to have a similar distribution of the main observable parameter (e.g. $\Delta M_{15}$) for the local and the high-z sample, but still have a systematic bias in the high-z sample. The consequences of sampling different populations is less...
Intrinsic metallicity dependence in the Phillips relation and of the dominant first parameter, independent of metallicity. Assume instead that the Phillips relation is just a function of redshift mimicking any evolutionary effects due to metallicity.

In Figure 3, we illustrate the metallicity evolution that is the supernova sample as a function of redshift (z) required to mimic a (\( \Omega_m = 0.3; \ \Omega_\Lambda = 0.7 \)) Universe in a \( \Lambda \)-free Universe with \( \Omega_m = 0.0 \) (empty Universe), 0.3 (open Universe) and 1 (flat Universe), as indicated. \( \Omega_m \) and \( \Omega_\Lambda \) are the dimensionless densities of matter and “dark energy”, respectively, \( Z_0 = 0.02 \), and the present-day Hubble parameter is assumed to be 70 km/s/Mpc. Bottom panel: the effect of metallicity on the estimate of the distance modulus for the three cases in the top panel, based on equation (10) with \( \alpha = 0.111 \) and \( Z_{\text{ref}} = Z_0 \).

In this paper, we will not attempt to correct for an intrinsic metallicity dependence in the Phillips relation and assume instead that the Phillips relation is just a function of the dominant first parameter, independent of metallicity. We suspect that this assumption will somewhat exaggerate any evolutionary effects due to metallicity.

Mimicking \( \Lambda \)-dominated cosmologies

In Figure 3, we illustrate the metallicity evolution that would mimic a flat, \( \Lambda \)-dominated Universe with \( \Omega_m = 0.3 \) and \( \Omega_\Lambda = 0.7 \) in different \( \Lambda \)-free cosmologies as indicated (here \( \Omega_m \) is the present matter density as a fraction of the critical density and \( \Omega_\Lambda \equiv \Lambda / 3H_0^2 \), where \( \Lambda \) is the cosmological constant and \( H_0 \) is the present-day Hubble parameter, assumed to be 70 km/s/Mpc). These curves were obtained by calculating the difference in distance modulus as a function of redshift \( z \) between the reference \( \Lambda \)-dominated cosmology and the respective \( \Lambda \)-free cosmologies (bottom panel of Fig. 3) and then solving equation (10) for \( Z \), where we adopted \( \alpha = 0.111 \) and assumed that the present-day reference metallicity is \( Z_0 \) (i.e. \( Z_{\text{ref}} = Z_0 \)). For example, if in an open Universe with \( \Omega_m = 0.3 \), the mean metallicity of the supernova sample increased from \( Z_0 \) to about 2.5 \( Z_0 \) at \( z = 1 \), as shown in the top panel, this would exactly mimic a \( \Lambda \)-dominated Universe with \( \Omega = 0.3 \) and \( \Omega_\Lambda = 0.7 \).

To relate this metallicity dependence of the supernova sample to the metallicity of stellar populations as a function of redshift, one also needs to consider that there may be a significant time delay between the formation of the supernova progenitor system and the actual supernova. While the typical time delay in one of the most popular single-degenerate models is only \( \sim 1 \) Gyr, the fact that SNe Ia also occur in old elliptical galaxies suggests that there should also be a population of SNe Ia with relatively long time delays (e.g. Cappellaro & Turatto 1988; Branch & van den Bergh 1993). This could be either due to the tail of the time-delay distribution if there is a single dominant progenitor channel (e.g. in the double-degenerate channel) or due to a second progenitor channel (e.g. Hachisu et al. 1996; for a detailed discussion, see Förster et al. 2006).

In Figure 4, we show the evolution of the metallicity of the star-forming population with redshift that produces the metallicity dependence in Figure 3, assuming a short time delay of 1 Gyr and a long time delay of 5 Gyr.

An open, \( \Lambda \)-free Universe with \( \Omega_m = 0.37 \)

As is clear from Figs 3 and 4, in order for the metallicity evolution to mimic a \( \Lambda \)-dominated Universe, the metallicity and the respective \( \Lambda \)-free cosmologies (bottom panel of Fig. 3) and then solving equation (10) for \( Z \), where we adopted \( \alpha = 0.111 \) and assumed that the present-day reference metallicity is \( Z_0 \) (i.e. \( Z_{\text{ref}} = Z_0 \)). For example, if in an open Universe with \( \Omega_m = 0.3 \), the mean metallicity of the supernova sample increased from \( Z_0 \) to about 2.5 \( Z_0 \) at \( z = 1 \), as shown in the top panel, this would exactly mimic a \( \Lambda \)-dominated Universe with \( \Omega = 0.3 \) and \( \Omega_\Lambda = 0.7 \).

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6 If the reference metallicity were 0.5 \( Z_0 \), the metallicity would have to increase to about 2 \( Z_0 \), since to lowest order it is change in \( \Delta Z \) that matters (see eq. 11).

7 Note that for a given time delay, no SN Ia would occur beyond a particular redshift \( z_{\text{max}} \) at which the time that has passed since the first star formation is equal to the time delay.
licity has to increase with redshift, since this makes observed supernovae intrinsically fainter than is assumed. This is at first sight rather counter-intuitive. After all, the global metallicity in baryonic matter (stars and gas) has to increase as the Universe evolves. However, this need not apply for the mean metallicity in star-forming regions. In the now widely accepted picture of galaxy down-sizing (e.g. Treu et al. 2005), the most massive galaxies form first and fast and have largely finished star formation at a redshift larger than 1 (e.g. Cowie 1996; Kauffmann et al. 2003). But the most massive galaxies are also the galaxies with the largest metallicities (e.g. Kudelnick & Kewley 2004). Thus, it is possible that, as the Universe evolves and star-formation predominantly occurs in galaxies of lower mass (with lower metallicities), the metallicity in the star-forming regions decreases while the global metallicity increases. Indeed, Panter, Heavens & Jimenez (2003) (also see Sheth et al. 2006) have claimed that the metallicity in star-forming regions has declined for the last 6 Gyr, based on their reconstruction of the metallicity history of the galaxies in the Sloan Digital Sky Survey. However, this method is not without its uncertainties (e.g., Mathis, Charlot & Brinchmann 2006).

More direct evidence that the metallicity in star-forming regions may have been higher in the past than at the present time comes from the archaeological reconstruction of the metallicity history in early-type galaxies (Thomas et al. 2005). According to Thomas et al. (2005), the most massive early-type galaxies have a metallicity of 2 to 3 times solar and formed most of their stars between a redshift of 1.5 and 2 in low-density environments and a redshift up to 5 in the densest environments (see also di Serego Alighieri, Lanzoni & Jørgensen 2006). Thomas et al. (2005) argue that the increase in metallicity is mainly due to an increase in α elements rather than an increase in iron-peak elements. However, since the oxygen abundance is the most important abundance in determining the neutron excess, this is directly applicable to our analysis. In order to mimic a Λ-dominated Universe in an open Ω_m = 0.3 Universe, this only requires an increase in the metallicity from 1 Z_⊙ to ∼ 2.5 Z_⊙, well within the maximum plausible metallicity range in early-type galaxies. However, at a redshift of ∼ 0.8 where the SNe Ia with the largest metallicity are required, the most massive galaxies have already completed most of their star formation. Therefore understanding the time delay between the formation epoch of the progenitors of SNe Ia and the actual explosion becomes essential. If there is a systematic time delay of a few Gyr, the peak in the metallicity–redshift relation (the solid curves in Figs 3 and 4) can easily be shifted into the redshift range where massive early-type galaxies form most of their stars (also see Table 1). We note that even though the local sample of SNe Ia shows a strong correlation with star formation (e.g. Mannucci et al. 2005), implying a relatively short time delay of ≲ 1 Gyr (see Förster et al. 2006), this need not be the case for the high-z sample where a different population, even one with a longer time delay, could dominate; this depends entirely on the details of the star-formation and metallicity histories and their effects on the SN Ia rate. We conclude that considering the present uncertainties in the metallicity evolution of star-forming regions, a systematic metallicity trend as large as required for an Ω_m = 0.3, open Universe cannot be ruled out.

### A closed, Λ-free Universe with Ω_m = 1.0?

In contrast to the Ω_m = 0.3 case, for a flat, Λ-free Universe, the metallicity of the supernova sample has to increase from Z_⊙ to about 4 Z_⊙ at z = 0.8. This is a much larger effect than seems plausible, at least for α = 0.111. To produce such a large effect, one would probably require the maximum value for our estimated range of α (i.e. α ≳ 0.165) and that the time delays are fine-tuned so as to maximise the effect. This would essentially require that all SNe Ia at a redshift around z = 0.8 occurred in the most massive early-type galaxies or at least in the bulges of galaxies. This can almost certainly already be ruled out from a comparison of local and high-z host galaxies (e.g., Strolger et al. 2004; Sullivan et al. 2006), which does not show such a dramatic trend in host galaxy properties. However, we caution that there are still significant uncertainties in our estimate of the metallicity evolution effect, due to uncertainties in the value of α, the effective metallicity and its calibration to be used in high-z galaxies and other cumulative metallicity effects. We estimate that this uncertainty is about a factor of 2 in either direction.

### Table 1. The effects of time delays.

| cosmology | t₀(z = 0.8) (Gyr) | Δt = 1 Gyr | 3 Gyr | 5 Gyr |
|-----------|------------------|------------|-------|-------|
| no cosmological constant (Ω_Λ = 0) | |
| Ω_m = 0.0 | 6.2 | 1.1 | 1.9 | 4.1 |
| 0.3 | 5.9 | 1.1 | 2.3 | > 5 |
| 1.0 | 5.5 | 1.2 | 3.9 | – |
| variation of the equation of state for flat cosmologies: (Ω_m, w_0, w_a) | |
| (0.30, -1.0, 0) | 6.8 | 1.0 | 1.8 | 3.7 |
| (0.35, -1.0, 0) | 6.7 | 1.0 | 1.9 | 4.5 |
| (0.20, -1.0, 0) | 7.2 | 1.0 | 1.6 | 2.7 |
| (0.30, -1.4, 0) | 7.3 | 1.0 | 1.7 | 3.4 |
| (0.30, -0.8, 0) | 6.6 | 1.0 | 1.8 | 4.2 |
| (0.30, -1.0, 1) | 6.6 | 1.0 | 1.9 | 4.9 |
| (0.30, -1.0, -2) | 7.2 | 1.0 | 1.7 | 3.3 |

Note. — For the given cosmological parameters, t₀ is the look-back time at a redshift z = 0.8, z(t₀ + Δt) gives the redshift at lookback times t₀ plus time delays of 1, 3 and 5 Gyr, as indicated.

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8 We note that the underlying evolutionary tracks generally used in studies of this type tend to use scaled solar abundances, where iron-peak elements contribute significantly to the opacity. The effect of using α-enhanced (Fe-deficient) tracks should be to somewhat increase the inferred metallicity and effective oxygen abundance, i.e. further enhance the metallicity effect on SN Ia light curves.
Figure 5. The effects of metallicity evolution on the determination of cosmological parameters in a flat, \( \Lambda \)-dominated Universe. The underlying (true) cosmology is assumed to be a flat Universe with \( \Omega_m = 0.3, \Omega_\Lambda = 0.7 \) and no evolution of the equation of state (i.e., with \( w_0 = -1 \) and \( w_a = 0 \), where the equation of state is parametrized as \( w = P/(\rho c^2) = w_0 + w_a (1 - a) \), and \( a = 1/(1 + z) \) is the dimensionless scale factor of the Universe). The top panels show the mean metallicity evolution in the star-forming component that would mimic the indicated deviation from the reference model, while the bottom panels give the effect on the inferred distance modulus (the two families of curves in the top panels are for time delays of 1 Gyr [left] and 5 Gyr [right], respectively). For each of the three sets of models, two of the three cosmological parameters \( \Omega_m, w_0 \) and \( w_a \) are kept fixed, while the third is varied as indicated (\( \Omega_\Lambda = 1 - \Omega_m \) in all cases, and \( Z_0 = 0.02 \)).

4 MEASURING THE EQUATION OF STATE OF DARK ENERGY

Independent of the SN Ia data, there is now ample observational evidence for a globally flat Universe from measurements of anisotropies in the microwave background (WMAP; Spergel et al. 2003; 2006) which, when combined with other observational constraints (e.g. from galaxy clustering and weak lensing), provides strong independent evidence for a flat Universe dominated by dark energy (see the discussion and references in Spergel et al. 2006). It has been argued that one of the main challenges in modern cosmology is to understand the nature of this dark energy. Future surveys, now under consideration, will attempt to constrain the physical nature of this new form of energy by measuring its cosmological evolution. It is the purpose of this section to quantitatively estimate the systematic uncertainties metallicity evolution is likely to introduce when measuring the equation of dark energy in future supernova surveys such as SNAP (SuperNova/Acceleration Probe; Aldering et al. 2002; also see Riess & Livio 2006 for a complementary study of evolutionary effects).

To describe the time evolution of the equation of state of dark energy, we use the parametrization suggested by Linder (2003),

\[
    w(a) \equiv \frac{P}{\rho c^2} = w_0 + w_a (1 - a),
\]

where \( a = 1/(1 + z) \) is the dimensionless scale factor of the Universe, the parameter \( w_0 \) gives the present value of the equation of state, and \( w_a \) parametrizes its evolutionary history (for a cosmological constant, \( w_0 = -1 \) and \( w_a = 0 \)). For a flat Universe, the Friedmann equation then becomes

\[
    H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2 \left[ \Omega_m a^{-3} + (1 - \Omega_m) a^{-3(1+w_0+w_a)} e^{-3w_a (1-a)} \right],
\]

where \( H_0 \) is the present-day Hubble parameter (again assumed to be 70 km/s/Mpc). In this parametrization, the evolution is completely determined by three parameters \( \Omega_m, w_0 \) and \( w_a \) (keeping \( H_0 \) constant). In order to estimate the systematic uncertainties due to metallicity evolution, we adopt
The true underlying cosmology is a flat Λ-dominated Universe with that might be mimicked by metallicity evolution (see Fig. 5). The evolution of the scale factor as a function of lookback time (in units of $1/H_0$) for a variety of equations of state that might be mimicked by metallicity evolution (see Fig. 5). The true underlying cosmology is a flat Λ-dominated Universe with $\Omega_m = 0.3$, $w_0 = -1$, and $w_a = 0$ (dashed curve). For the individual solid curves, one of the three parameters was varied within a range that could plausibly be caused by metallicity evolution (as indicated).

![Figure 6](image)

Figure 6. The evolution of the scale factor as a function of lookback time (in units of $1/H_0$) for a variety of equations of state that might be mimicked by metallicity evolution (see Fig. 5). The true underlying cosmology is a flat Λ-dominated Universe with $\Omega_m = 0.3$, $w_0 = -1$, and $w_a = 0$ (dashed curve). For the individual solid curves, one of the three parameters was varied within a range that could plausibly be caused by metallicity evolution (as indicated).

In order to estimate the systematic errors introduced by metallicity evolution, one in principle needs an estimate for the range of metallicity variation that is “reasonable.” Following the discussions in the previous section, we make the simple and conservative assumption that a variation of $Z$ from a few tenth $Z_\odot$ to ~ $2 Z_\odot$ could be plausible. We can then directly read off rough estimates for the systematic errors for each parameter from Figure 5. We obtain $\Delta \Omega_m = -0.1/0.05$, $\Delta w_0 = -0.4/0.2$, and $\Delta w_a = -2/1$. With some prior assumptions about the metallicity evolution, one would obviously be able to constrain these errors further.

To illustrate the uncertainty this could introduce in determining the nature of dark energy through its time evolution, we plot in Figure 6 the scale factor as a function of lookback time (similar to Fig. 1 in Linder 2003). Here the dashed curve represents the evolution of the scale factor for the assumed underlying cosmology (a flat Universe with a cosmological constant), while the solid curves show the evolution of a for cosmologies that could plausibly be mimicked by metallicity evolution. Comparing this figure to Figure 1 in Linder (2003), we note that there is a large overlap of these curves with the curves produced by different physical models of dark energy. This implies that it will be difficult to distinguish these different models, unless metallicity evolution effects are taken into account and are corrected for.

Our main conclusion therefore is that the possible systematic uncertainties in $\Omega_m$ and $w_0$ caused by metallicity evolution could be quite significant, but may be manageable (in particular, if further constraints on the metallicity evolution can be employed). In contrast, the uncertainty in $w_a$ seems uncomfortably large, implying that it may be very difficult to differentiate time evolution in the equation of state from time evolution in $Z$ using SN Ia data alone.

5 DISCUSSION AND CONCLUSIONS

Our main conclusion is that metallicity should affect the SN Ia calibration method; this relies only on reasonably straightforward and well understood nuclear physics. As long as there is any significant cosmological evolution in the metallicity, which a priori is to be expected, this has to affect the determination of cosmological parameters at some level. The main remaining uncertainty is the level at which this becomes important.

In our estimates of systematic errors, we adopted $\alpha = 0.111$, roughly twice the estimate of Timmes et al. (2003), though $\alpha$ could possibly as large as 0.165. It still remains to be shown theoretically that the metallicity effect is as large as these estimates suggest. This will require detailed stellar evolution calculations of the pre-explosion phase that model the convective Urca process realistically (Lesaffre et al. 2005 and references therein) combined with up-to-date explosion modelling (e.g. Röpke et al. 2005), work that is presently in progress (F. Förster, et al.). In general, since the magnitude corrections scale linearly with $\alpha$, at least to lowest order (see eq. 11), all estimates of systematic errors in this paper can be rescaled with whatever the appropriate value for $\alpha$ will turn out to be.

It would be particularly useful to be able to constrain the value of $\alpha$ observationally. This is quite a challenging task even for the local SN Ia sample, since it is clear that metallicity cannot be the primary parameter controlling SN Ia light curves (Hamuy et al. 2000; Gallagher et al. 2005) and without a better understanding of the dominant parameter and how it affects the basic light curve correction method, it will be difficult to isolate the metallicity contribution. On the other hand, since the predicted metallicity-dependent correction factor is, to lowest order, a linear function of metallicity, with a sufficiently large sample of SNe Ia with well determined host galaxy metallicities, one could include a linear metallicity term in the Phillips relation and attempt a two-parameter calibration. This could in principle allow a measurement of the magnitude of the metallicity effect (i.e. constrain $\alpha$) and could then be used to correct for metallicity effects at high $z$.

Obtaining the metallicity of the SN Ia progenitor directly is probably not feasible, and therefore one has to rely on a statistical measure of the metallicity using the host galaxy metallicity. This itself is not without problems, considering, e.g., that there may be a substantial time delay between the formation of the SN
More progress is likely to come from a systematic comparison of the properties of nearby and high-z supernovae, one of the major objectives of both the ESSENCE project\(^\text{10}\) and the SuperNova Legacy Survey (SNLS)\(^\text{11}\). First results from the SNLS (Astier et al. 2006; Sullivan et al. 2006) indicate that the nearby and high-z samples are similar, but that there may be some differences in properties. While at present these are only of marginal statistical significance, this may change as the sample increases.

The evolution of galaxy properties, in particular the metallicity in the star-forming regions, clearly plays an important role in the theoretically predicted evolution of SN Ia properties. There is a general consensus that there has been a significant evolution of galaxy properties since a redshift of 2–3: star formation first occurred in the most massive galaxies and since then has moved increasingly towards lower-mass galaxies. Observationally, this is reflected in star formation predominantly occurring in ultraluminous infrared galaxies (ULIRGs) at a redshift \(z \gtrsim 2\), in luminous infrared galaxies (LIRGs) at a redshift \(z \approx 1\) and starburst galaxies in the local Universe (see, e.g., Fig 10 in Pérez-González et al. 2005). If this galaxy trend is also correlated with a trend in metallicity, it would exactly produce the type of evolutionary effect that could affect the determination of cosmological parameters. A better understanding of the metallicities in these different types of galaxies is therefore clearly needed.

In summary, metallicity effects should be expected and, at some level, will affect the determination of cosmological parameters. Uncertainties in the metallicity evolution in star-forming galaxies and in the intrinsic properties of SN Ia progenitors (in particular, their time delays) will limit the precision with which the equation of state can be measured; it will be particularly difficult to distinguish an evolution of the equation of state from evolution in SN Ia properties. As the statistical errors in SN Ia surveys and complementary surveys (CMB, galaxy clustering, weak lensing) decrease, this may first show up in statistical discrepancies in the determination of cosmological parameters using different methodologies.

Our estimates show that metallicity evolution could possibly be large enough to mimic a \(\Lambda\)-dominated Universe in an open Universe, but is unlikely to be large enough to mimic a preferred flat Universe. Nevertheless, considering the fundamental shift in our view of the physical world adoption of a cosmology dominated by dark energy would bring about, we emphasize the importance of taking such systematic effects into account. One can hope that with a better understanding of such effects, one will be able to correct for these and ultimately obtain a reliable calibration of cosmological parameters, where systematic uncertainties are minimal.

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\(^{10}\) See http://www.ctio.noao.edu/~wsne/index.html.

\(^{11}\) http://cfht.hawaii.edu/SNLS.
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