The Dry Friction Damping Model and Vibration Characteristics of a Cylindrical Shell Structure with Damping-Ring

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Abstract. To effectively solve the design problem of the damping ring used in the rotating thin cylindrical shell structure of aeroengine, a dynamic analysis model of the combined structure is established, based on its natural modal analysis. The analytical expression of the friction force and the critical slipping angle is derived with reference to the basic theory of mechanics of materials, and also the predicting method of damping work and damping ratio is demonstrated based on the macro sliding model. For the given structure model, the dry friction damping characteristics of the damping ring are studied. The results show that there exist a critical speed and if exceeding it, the damping ring cannot work functionally. Under an identical nodal diameter and vibration stress, the damping ratio increases first and then decreases with the increasing speed with the increase of rotating speed. When the speed is constant, the lower the nodal diameter, the higher the damping ratio it provided. For a given nodal diameter and speed, as the allowable vibration stress increases, the damping ratio rises first and then decreases. In addition, the damping ratio has a linear relationship with the cross-section width of the damping ring.

1. Introduction
The thin cylindrical shell structure is widely used in aero-engine, such as casing and combustion chamber flame tube. During the design and use of aircraft engines, high cycle fatigue problems in such thin cylindrical shell structure often occur. The labyrinth Seal Structure is a typical thin cylindrical shell structure which has the advantage of bearing large load and light weight, however at the same time, it has a weak resistance to internal excitation and is easy to vibration. Therefore, the thin cylindrical shell structure and the high-speed rotating shaft often fail due to violent vibration of the labyrinth Seal Structure in the aero-engine. There are various methods to control vibration in the thin cylindrical shell structure [1], mainly including: Rigid structure, demodulation or decoupling of resonance system, vibration isolation, vibration absorption and damping. Because dry friction damping has the advantages of simple structure, suitable for high temperature and high load, and remarkable effect of vibration control, it is widely used in the thin cylindrical shell structure of aero-engine for vibration controlling.

In the guideline for structural design of aircraft engines published in 2001 [2], it is stipulated that vibration control should be carried out according to needs for rotating thin-walled parts in low-pressure
shafts, high-pressure compressors and high-pressure turbines. In the guidelines, several vibration controlling structures such as opening damping ring/sleeve and multi-slit finger-shaped sleeve are also recommended. However, there is no detailed theoretical guidance for the design of this kind of vibration reduction structure. At present, the design is only based on the experience gained from experiments. Alford J S [3, 4] proposed a structural scheme of dry friction damping to reduce the self-excited vibration of labyrinth seals, and designed an experimental scheme to verify the effect of vibration reduction. For the additional damping ring structure of labyrinth seals, Niemotka and Ziegert [5] obtained the dry friction damping ratio expression of the damping ring through analysing the vibration characteristic equation and friction energy dissipation equation of the whole structure. At present, for the problem of dry friction damping, it can be said that it has entered an era of unified calculation [6], that is, there are already frequency-domain analysis models that can include local separation, local slip and even local partial slip, and Petrov [7, 8] has developed relatively stable calculation programs. The use of the dry friction damper in labyrinth seal structure is very common, and conducting experimental study and numerical simulation of a typical labyrinth seal damper are necessary [9]. In the actual process, in addition to sliding with the seal, the damper will stick for a short time when it vibrates, so the damping model actually becomes a nonlinear model. Domestic scholars have also carried out research on this issue. Li Hui [10] respectively studied the damping effect of the damping ring and the damping sleeve for the labyrinth seals by using the method of aeroelastic stability analysis, and on this basis proposed the engineering design method of the damping damper. Li Lin [11, 12] proposed the dynamic response analysis method of the labyrinth seal device with damper, and conducted an experimental study on the dynamic response characteristics of the cantilever cylindrical shell/damping ring (sleeve) combined structure.

Because the current vibration control theory of the dry friction damping model for the thin cylindrical shell structure/damping-ring combination structure is not very perfect, this paper established a thin cylindrical shell structure/opening damping ring dry friction damping model used in aviation engine’s rotating thin cylindrical shell structure, the contact strain of combined structures was derived and the theory formula of friction. The contact strain of combined structures and the theoretical formula of friction force are derived. Based on the theory of macro slip model, we also get the theoretical formula of the friction energy dissipation and the damping ratio. Using the model established in this paper, the vibration rule of such combined structure is analysed.

2. Dry Friction Damping Model of Damping Ring

When bending vibration of the thin cylindrical shell structure/damping ring combined structure occurs, the combined structure undergoes bending deformation. The thin cylindrical shell structure is stretched on the outside and compressed on the inside, and the deformation of the damping ring is the same with it. Therefore, due to the uncoordinated deformation of the two structures on the contact surface, relative slipping may occur, leading to dry friction.

The theoretical analysis of combined structures needs to be simplified. This article makes the following assumptions regarding its structure and vibration characteristics:

1. Both the thin cylindrical shell structure and the damper are thin-walled pieces;
2. The combined structure only produces elastic deformation;
3. During the vibration process, the thin cylindrical shell structure and the damper do not separate in the radial direction;
4. The friction coefficient of the contact surface is constant, and the positive pressure of the contact surface is uniform along the circumferential unit arc length.

2.1. Macro Sliding Dry Friction Model

In this paper, a macro-sliding friction model is used to derive the expression of nonlinear friction. The macro sliding friction model is equivalent to the dry friction interface between the contact areas as the spring series friction pair structure shown in figure 1a below, where the spring stiffness coefficient is the tangential contact stiffness of the friction surface and is the friction coefficient. Under a constant
contact surface pressure $P$, the hysteresis curve of the non-linear friction force and the tangential relative displacement is shown in figure 1b.

The relationship between friction and tangential displacement of a linear hysteretic spring model is

$$ f = \begin{cases} K_s \chi & |f| < \mu P \\ -\text{sgn}(\chi)\mu P & |f| \geq \mu P \\ 0 & N \leq 0 \end{cases} $$

(1)

where $\chi$ is the tangential relative displacement of the contact surface.

![Physical model](image1.png) ![Hysteresis loop](image2.png)

(a) Physical model (b) Hysteresis loop

**Figure 1.** Linear hysteretic spring friction model.

2.2. Analysis of Friction Force and Contact Surface State

2.2.1. Static Friction Force in the Viscous Region. From the assumption of elastic deformation, when the $N$-diameter vibration occurs in the combined structure, the radial displacement [13] of the thin cylindrical shell structure and the damping ring at the installation position of the damping ring is

$$ W = B \cos N \theta \cos \omega t $$

(2)

where $B$ is the amplitude of the radial vibration.

According to Ref. [14], the strains of the thin cylindrical shell structure and the damping ring at the contact surface are

$$ \varepsilon_s = -\frac{C_s B}{R_s} \left[ N^2 - 1 \right] \cos(N\theta) $$

$$ \varepsilon_d = \frac{C_d B}{R_d} \left[ N^2 - 1 \right] \cos(N\theta) $$

(3)

In the formula, $C_s$ and $R_s$ are the half-thickness and mid-surface radius of the thin cylindrical shell structure, $C_d$ and $R_d$ are the cross-section half-thickness and mid-surface radius of the damping ring.

The bending strain difference at any point on the contact surface between the thin cylindrical shell structure and the damping ring during vibration is

$$ \Delta \varepsilon_{\text{bend}} = \varepsilon_s - \varepsilon_d $$

(4)

When there is no macroscopic relative slip between the contact point of the thin cylindrical shell structure and the opening damping ring, in another word, when the contact surface is in a viscous state, the friction force is a static friction force. It can be considered that the friction strain $\varepsilon_{d,\text{friction}}$ is the elastic deformation corresponding to the friction force. Assuming $\varepsilon_{d,\text{friction}}$ all occur on the contact surface of the damping ring.

In the non-slip state region, the strain values of the thin cylindrical shell structure and the damping ring at the contact surface are equal. The friction strain is...
\[ \varepsilon_{d, friction} = \Delta \varepsilon_{\text{bend}} \]  \hspace{1cm} (5)

Known from Ref. [14],

\[ \varepsilon_{d, friction} = \frac{R_f}{A_d E} \int f \, d\theta \]  \hspace{1cm} (6)

In the formula, \( A_d \) is the cross-sectional area of the damping ring, \( R_f \) is the radius at the contact surface, and \( f \) is the friction force distribution function.

An expression for the static friction force in the viscous zone can be obtained by combining the above formulas.

\[ f = \frac{NA_d EC_d B}{R^2 R_f} \left[ \frac{C_s}{C_d} \left( \frac{R_s}{R_f} \right)^2 + 1 \right] (N^2 - 1) \sin(N\theta) \]  \hspace{1cm} (7)

2.2.2. Analysis of Contact Surface Slip-Viscous State

If the contact surface of the damping ring and the thin cylindrical shell structure is in a viscous state as a whole, it is known from equation (7) that the maximum friction force is at the nodal line position, \( \theta = \pi / 2N \). When slip occurs at the position of the nodal line, there is \( f = f_{\text{max}} = \mu P \) here, and the critical modal amplitude \( B_i \) under the slip-free state is

\[ B_i = \frac{\mu P R_s^2 R_f}{NA_d EC_d} \left[ \frac{C_s}{C_d} \left( \frac{R_s}{R_f} \right)^2 + 1 \right] \sin(N\theta) \]  \hspace{1cm} (8)

In the same way, the critical positive pressure of the contact surface in the viscous state at a given modal amplitude is

\[ P_i = \frac{NA_d EC_d B}{\mu R^2 R_f} \left[ \frac{C_s}{C_d} \left( \frac{R_s}{R_f} \right)^2 + 1 \right] \sin(N\theta) \]  \hspace{1cm} (9)

In a vibration period, assuming that the slip region and the sticky region in a substructure sector \([0, \pi/2N]\) remain unchanged, and also \([0, \theta_o]\) is set as the sticky region, \([\theta_o, \pi/2N]\) is the slip area.

At a given positive contact surface pressure \( P_i \), if \( B > B_i \), slipping will occur in the \([\theta_o, \pi/2N]\) region. At the slipping critical point \( \theta_o \), we can get the following equation from equations (8) and (9).

\[ \sin(N\theta_o) = \frac{P_i}{P_i} = \frac{B_i}{B} \]  \hspace{1cm} (10)

The friction strain caused by friction in the slip zone damping ring is

\[ \varepsilon_{d, friction} = -\frac{\mu P R_f}{A_d E} \left( \frac{\pi}{2N} - \theta \right) \]  \hspace{1cm} (11)

After sliding occurs, at \([\theta_o = \pi/2N]\), there is \( \varepsilon_{d, friction} = 0 \) due to the anti-symmetric relationship.

Assume that at one moment \( t \) (\( t \in [0, T / 4]\), \( T \) is the vibration period of the structure) in the vibration process, the frictional force in the slip zone is constant, which is \( f = \mu P_i \). The schematic diagram of the circumferential viscous-slip region of the damping ring is shown in figure 2.

If there is no initial positive pressure on the contact surface, the contact pressure is caused only by centrifugal force, and the circumferential strain caused by the centrifugal force of the thin cylindrical shell structure does not change during a vibration period, so the centrifugal force term don’t need to be included in the expression of relative displacement.

2.3. Friction Energy Dissipation of Combined Structures

In this paper, the energy method is used to calculate the friction energy dissipation. The friction energy dissipation is defined as the work done by a friction force in a period. Given the hysteresis curve of the
non-linear friction force with relative displacement, the area of the hysteresis curve can be calculated. Then we can get friction energy dissipation, the formula is as following:

\[ W_{\text{friction}} = \int f(x)dx \] (12)

In the slip region, the circumferential displacements of the damping ring and the thin cylindrical shell structure at the contact surface are

\[
\begin{align*}
\varepsilon_i &= \int \varepsilon_{c}R_{i}d\theta \\
\varepsilon_{0} &= \int \varepsilon_{c}R_{0}d\theta
\end{align*}
\] (13)

Let \( s(\theta, t) \) be the difference between the circumferential displacement of the tooth structure and the damping ring at the contact surface, which can be obtained from equations (2) and (13) and \( s(\theta_0, t) \).

\[
s(\theta, t) = -\frac{\mu PR_i}{A_{c}E} \left\{ \frac{1}{N^2} \left[ \sin\left(\frac{N\theta}{\theta_0}\right) - 1 \right] + \frac{1}{2} \left[ \left( \frac{\pi}{2N} - \theta \right)^2 - \left( \frac{\pi}{2N} - \frac{\pi}{\theta_0} \right)^2 \right] \right\} \cos \omega t
\] (14)

When the combined structure undergoes \( N \) nodal vibration under the external excitation, there are 2N substructures in the circumferential direction, and each substructure is divided into left and right two parts with \( \theta = 0 \) as the boundary, so the friction energy dissipation in a vibration cycle is

\[
W_{\text{friction}} = 16N\mu P \int_{\theta_0}^{\pi/2N} s(t)dt \int R_i d\theta
\] (15)

The vibration stress is related to the differential of the vibration displacement. For a given \( N \) diameter modal mode, the ratio of the vibration stress to the vibration displacement is a fixed value. Due to the small additional mass and additional stiffness of the damping ring, the influence of the damping ring on the vibration characteristics of the thin cylindrical shell structure can be ignored. The finite element method is used to analyse the natural vibration characteristics of the thin cylindrical shell structure at different speeds. Using the modal deformation and modal stress of the thin cylindrical shell structure as a reference, the relationship between the actual vibration stress and vibration displacement and the reference modal stress and modal displacement can be obtained

\[
\frac{\sigma_{\text{ref}}}{B_{\text{ref}}} = \frac{\sigma_{\text{e}}}{B_{\text{e}}}
\] (16)

Given the allowable vibration stress \( \sigma_{\text{e}} \), the vibration amplitude under the allowable vibration stress can be obtained:
The expressions of critical slipping angle $\theta_0$ can be obtained from equations (7) and (17).

$$\theta_0 = \frac{1}{N} \arcsin \left[ \frac{\mu \rho R^4 R_s \Omega^2}{NEB_i C_i} \cdot \frac{C_d R_s^2}{(C_s R_s^2 + C_d R_s^2)(N^2 - 1)} \right]$$

(18)

From (14) and (15), the friction energy dissipation is

$$W_{\text{friction}} = 16 \left( \frac{\mu \rho A_s R_s \Omega^2}{N^2 A_s E} \right)^2 R^2 \rho h \left[ \cot \left( N \theta_0 + N \theta_0 - \frac{\pi}{2} \right) - \frac{1}{3} \left( \frac{\pi}{2} - N \theta_0 \right)^3 \right]$$

(19)

2.4. Equivalent Damping Ratio

The vibration of the thin cylindrical shell structure is mainly radial vibration, neglecting the effects of axial and circumferential directions, the vibration kinetic energy is

$$W_{\text{max}} = \int \rho h \left( \frac{\partial \omega}{\partial t} \right)^2 dx dt = \frac{1}{4} \rho h R L \pi B^2 \omega^2$$

(20)

From Ref. [15], it is known that the friction energy is equal to the work done by the damping force in a vibration period. Therefore, after the damping energy dissipation and the maximum kinetic energy have been obtained, the damping ratio can be approximately expressed as following, and taking into account the equations (19) and (20) we can obtain the analytical expression of the damping ratio.

$$\zeta = \frac{W_{\text{friction}}}{4 \pi \cdot W_{\text{max}}} = 16 \left( \frac{\mu \rho A_s R_s \Omega^2}{N^2 A_s E} \right)^2 R^2 \rho h R L \pi B^2 \omega^2$$

(21)

3. The Thin Cylindrical Shell Structure/Damping Ring Structure Model

The finite element method is used to calculate the vibration characteristics of the rotating thin-walled tube. The geometric parameters and material parameters of the structure are shown in table 1.

| Parameters          | Value          |
|---------------------|----------------|
| Length (m)          | 2.56           |
| Thickness (m)       | $0.25 \times 10^2$ |
| Midplane radius (m) | 0.16           |
| E (GPa)             | $1.10 \times 10^2$ |
| Poisson’s ratio     | 0.31           |

According to the given parameters, a finite element model of a thin cylindrical shell structure is established. The mesh element type is the four-node shell element, with a total of 2048 elements and 112 nodes. The working rotating speed of the thin cylindrical shell structure is set as 10000 r/min. The boundary condition is that one end is fixed and the other is free. Only the mode with axial half wave number of 1 is examined.

The damping ring is used for damping, and the damping ring is installed inside the free end of the thin cylindrical shell structure. The damping ring and the thin cylindrical shell structure are made of the
same material. The damping ring has a rectangular cross section with a thickness of $0.20 \times 10^{-2}$ m and a width of $0.04 \times 10^{-1}$ m. Take the coefficient of friction $\mu = 0.3$.

4. Analysis of Vibration Characteristics and Vibration Control Rule

4.1. Natural Vibration Characteristics

Using ABAQUS to calculate the natural vibration characteristics of the thin cylindrical shell structure under the condition of clamped-free boundary. Figure 3 shows the variation of the vibration frequency of the thin cylindrical shell structure with the nodal diameter at different speeds. At a given speed, the natural frequency of the thin cylindrical shell structure decreases first and then increases with the increase of the circumferential wave number (the nodal diameter). And because of the centrifugal stiffness effect, when the rotation speed increases the natural frequency also increases.

Figure 4 shows the equivalent vibration stress distribution of 2 to 5 nodal diameter modes, where the maximum vibration stress in the 2 and 3 nodal modes is at the root installation position, and the maximum vibration stress in other modes is at the free edge. The vibration pressure value extracts the maximum point of vibration displacement at the free edge.

![Figure 3. Natural vibration frequency of RCSS.](image)

![Figure 4. The modal stress of RCSS from 2-5 ND.](image)

4.2. Positive Pressure of Contact Surface and Maximum Speed

From equation (8), calculate the critical vibration amplitude $B_i$ at different speeds. The critical vibration amplitude increases with the increase of the rotational speed. When the rotational speed increases to a certain degree, the amplitude corresponding to a given allowable vibration stress is smaller than the critical amplitude. At this time, there is no slipping between the damping ring and the thin cylindrical shell structure, so the damping ring plays no role.
According to formula (9), the critical positive pressure $P_i$ is calculated under the condition that the allowable vibration stress is 50MPa, as shown in table 2. At the same time, the maximum rotational speed of the model at the corresponding vibration mode can be calculated based on the critical positive pressure. When exceeding this rotational speed, the damping ring does not work.

**Table 2.** The critical positive stress.

| Nodal diameter | $P_i$ (N/m) | Rotating speed (r/min) |
|---------------|-------------|------------------------|
| 2             | 42558.48    | 27080.36               |
| 3             | 66934.28    | 33961.41               |
| 4             | 73292.39    | 35537.83               |
| 5             | 89396.38    | 39248.35               |
| 6             | 107610.24   | 43061.41               |
| 7             | 126842.22   | 46751.26               |
| 8             | 146890.28   | 50310.42               |
| 9             | 167714.28   | 53758.42               |

![Figure 5](image). The damping ratio of 2-6 ND.

4.3. Change Law of Damping Ratio

4.3.1. The Allowable Vibration Stress Is 50 MPa. Figure 5 is the change law of the damping ratio provided by the damping ring with the rotation speed under the 2-6 nodal diameter mode. From the figure can be seen, as the working speed increases, the damping ratio at different nodal diameters increases first and then decreases. When the working speed exceeds the critical speed, the damping ratio approaches 0, that is, the damping ring no longer provides a damping function.

The analysis demonstrates that the damping effect of the damping ring is mainly affected by the positive pressure of the contact surface and the area of the sliding area in the contact surface. The positive pressure of the contact surface is related to the rotation speed. As the rotation speed increases, the positive pressure of the contact surface gradually increases, and the area of the slip area (critical slipping angle) decreases as the positive pressure increases. Therefore, under the same nodal diameter mode and
the same vibration stress, with the increase of the speed, the damping ratio increases firstly and then decreases.

For the model given in this example, the damping ring can play a good role in controlling vibration in the 100% working speed range. However, when the damping ring is designed in practical engineering, the structure needs to be optimized to ensure that it can provide an effective damping ratio within the working speed range.

When the rotation speed is constant, the damping ratio provided by the damping ring in the low nodal mode is smaller than that in the higher nodal mode. For example, for a two nodal diameter mode, the damping ring provides a damping ratio of less than 0.05% when the vibration stress is 50 MPa. However, in practical engineering applications, the low nodal diameter mode requires a high damping ratio, which is need to be considered in the design of the damping ring.

4.3.2. Change Rule with Allowable Vibration Stress. Given different allowable vibration stresses, examine the variation of damping ratio with vibration stress. Examine the damping ratios at 60%, 80% and 100% rotating speed respectively. The critical vibration stress in the corresponding mode and speed is obtained through relative formula of the critical amplitude in Section 2 and formula (16), as shown in table 3. When the given vibration stress is less than this critical value, the corresponding amplitude is also less than the critical amplitude. At this moment, there is no slipping between the damping ring and the thin cylindrical shell structure, which cannot play the role of frictional vibration reduction.

The change law of the damping ring’s damping ratio with the allowable vibration stress at different nodal diameter modes is shown in figure 6, and the damping ratio curves at 60%, 80% and 100% rotating speed are given. It can be seen from the figure that at a given vibration stress value, the larger the number of vibration nodal diameters, the larger the damping ratio of the damping ring under the corresponding vibration mode. And the vibration stress ratio corresponding to the peak value of the low nodal vibration mode damping ratio is larger than the higher.

For a given nodal diameter and speed, as the allowable vibration stress increases, the damping ratio increases first and then decreases.

4.3.3. Change Rule with Rotating Speed. The variation of the damping ratio provided by the damping ring at different speeds with the allowable vibration stress under the same nodal diameter mode is shown in figure 7, and the damping ratio curves under the 2-5 nodal diameter are shown in the figure. As can be seen from the figure, when the nodal number is the same, the vibration stress corresponding to the peak value of the damping ratio at different speeds is different: the larger the rotating speed, the larger the vibration stress corresponding to the peak value of the damping ratio.

Table 3. The critical vibration stress.

| Nodal Diameter | Critical vibration stress (MPa) |
|----------------|--------------------------------|
|                | 6000 r/min | 8000 r/min | 10000 r/min |
| 3              | 38.32      | 47.60      | 9.82        |
| 4              | 39.52      | 50.65      | 9.72        |
| 5              | 56.78      | 72.35      | 14.07       |
| 6              | 69.03      | 87.82      | 17.14       |
| 7              | 81.33      | 103.41     | 20.21       |
| 8              | 93.28      | 118.58     | 23.18       |
| 9              | 102.27     | 129.99     | 25.42       |
|                | 106.43     | 135.22     | 26.47       |
When the number of nodal number changes, the peak value of the damping ratio at different speeds also shows a difference. The peak damping ratio of the 2 nodal diameter modes is basically the same. As the number of nodal diameters increases, the difference in the peak value of the damping ratio at different speeds becomes more and more obvious. And the larger the speed, the smaller the peak value of the damping ratio.

4.3.4. Change Rule with the Structure Size. From the expressions of the critical slip angles (18) and damping ratios (21), it can be seen that the damping ratio changes linearly with the section width of the damping ring. Without changing the section thickness of the damping ring, increasing the section width of the damping ring can improve the damping ratio of different nodal diameter modes. However, as the cross-section width of the damping ring increases, it will gradually affect the natural vibration characteristics of the thin-walled cylinder. When the cross-section width increases to a certain size, the radial displacement difference of points at different positions along the axis increases, so the damping ratio should be derived according to the damping sleeve model.
5. Conclusion
In this paper, a dry friction damping model of the thin cylindrical shell structure/open damping ring is established, and its vibration control rule is studied. The following conclusions can be obtained:

(1) There is a critical speed for the thin cylindrical shell structure/opening-damping ring structure. When the speed exceeds this critical value, the damping ring cannot play an effective role in controlling vibration.

(2) The damping ratio of the damping ring is related to the nodal modal shape, positive pressure, and vibration stress. Under the identical nodal diameter and the identical vibration stress, as the speed increases, the damping ratio increases first and then decreases. When the speed is constant, the damping ratio provided by the damping ring under the low nodal mode has a higher damping ratio, comparing with the high nodal diameter mode. For a given nodal diameter and speed, as the allowable vibration stress increases, the damping ratio increases first and then decreases.

(3) The damping ratio of the damping ring is related to its structural size. The damping ratio has a liner relationship with the width of the damping ring section. Increasing its width can increase the damping ratio of different nodal diameters. But when its width reaches a certain size, the damping ratio needs to be derived according to the damping sleeve model, and this model is not suitable.

(4) It is necessary to optimize the structure of the damping ring in engineering design to ensure a large damping ratio within the working speed range.
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