The Nonperturbative Regime in QCD Resummation

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ABSTRACT

We study the nonperturbative functions in the Collins–Soper–Sterman resummation formalism by examining Drell–Yan data in both fixed target and collider experiments and then predict the transverse momentum distributions of the $W^\pm$ and $Z^0$ bosons at the Tevatron. Our results differ from that in the literature and agree better with published CDF data. Using statistical arguments, we find a 1 fb$^{-1}$ luminosity at the Fermilab Tevatron should be able to provide useful constraints on the nonperturbative functions.
1. Introduction

The measurement of the mass ($M_W$) of the $W$ boson is very important in testing the Standard Model (SM), which has proven to be extremely successful. This measurement is currently done at the Tevatron by both the CDF and the D0 groups. In the framework of the SM, the mass of the Higgs boson can be known to within a couple of hundred GeV if $M_W$ is measured to within 100 MeV and the mass of the top quark within 10 GeV.\[1\]

Since the $W^\pm$ boson decays into a charged lepton and a neutrino, it is important to know the kinematics of $W^\pm$ boson production for measuring $M_W$, in particular, the transverse momentum distribution of the $W^\pm$ bosons. The measurement of the transverse momentum of the gauge bosons $V$ (either $W^\pm$, $Z^0$ or virtual photon in the Drell–Yan process) also provides a test of QCD in processes involving two–scale problems which are beyond the usual framework of perturbative calculations in the expansion of the strong coupling $\alpha_s$. The measurement of the rapidity asymmetry in the production of $W^\pm$ bosons provides a handle on the $u/d$ ratio in parton distribution functions such that quantities like $\Gamma_W/\Gamma_Z$ can be determined to good precision.\[2\]

To obtain the kinematics of the gauge boson we must resum the multiple soft and collinear gluon effects. We adopt the Collins–Soper–Sterman (CSS) formalism\[3,4\] to resum these multiple gluon effects and closely follow the notation used in Ref. 4. In this paper we study the nonperturbative part of this resummation formalism and show the importance of its contribution to the transverse momentum distribution of the gauge bosons.

To describe the kinematics of the gauge boson for all transverse momenta $Q_T$, we use the resummed formula differential in the mass ($\sqrt{Q^2}$), rapidity ($y$), and $Q_T$ of the boson $V$,\[4\]

$$\frac{d\sigma(AB \rightarrow V)}{dQ^2 dy dQ^2_T} = \frac{\pi}{S} \frac{\sigma_0}{\delta(Q^2 - M_V^2)} \left\{\frac{1}{(2\pi)^2} \int d^2 b e^{i\vec{Q}_T \cdot \vec{b}} \sum_j \tilde{W}_j(b_\ast; Q, x_A, x_B) \right. $$

$$ \times \exp \left[-\ln \left(\frac{Q}{2Q_0}\right) h_Q(b) - h_{j/A}(x_A, b) - h_{j/B}(x_B, b) \right] $$

$$ + Y(Q_T; Q, x_A, x_B) \right\}, \tag{1.1}$$

where $\sigma_0$ provides the process–dependent normalization and $\tilde{W}$ and $Y$ are derived from perturbative calculations.\[5\]
The functions $h_{j/A}$ and $h_{j/B}$, which carry a flavor dependence as well as their respective dependence on the momentum fractions $x_A$ and $x_B$, handle the nonperturbative behaviour at large $b$ along with $h_Q$. These functions are to be obtained by a fit to data, subject to the constraint that they must vanish when $b \rightarrow 0$. The constant $Q_0$, in Eq. (1.1), is completely arbitrary.

Among the nonperturbative functions used previously in the CSS formalism are those of Davies, Stirling and Webber.[6,7] They selected the functional form that provided a gaussian smearing of the transverse momentum,

$$h_Q(b) = \tilde{g}_2 b^2 \quad \text{and} \quad h_{j/A}(x_A, b) + h_{j/B}(x_B, b) = \tilde{g}_1 b^2,$$  \hspace{1cm} (1.2)

where $\tilde{g}_1, \tilde{g}_2$ are phenomenological constants. For simplicity the nonperturbative functions are assumed to be independent of flavor and momentum fractions $x_A, x_B$. Using the parton distribution functions of Duke and Owens,[8] their favored values after fitting to E288[9] and R209[10] Drell–Yan data with $Q_0 = 2$ GeV and $b_{max} = 0.5$ GeV$^{-1}$ were

$$\tilde{g}_1 = 0.15 \text{ GeV}^2 \quad \text{and} \quad \tilde{g}_2 = 0.4 \text{ GeV}^2.$$  \hspace{1cm} (1.3)

The interest at that time was the production of $W^\pm$ bosons in hadron collisions at $\sqrt{s} = 540$ GeV.

2. A New Study

For this study we chose $Q_0 = 1.6$ GeV and $b_{max} = 0.5$ GeV$^{-1}$ with a different functional form for the nonperturbative part,

$$h_Q(b) = g_2 b^2 \quad \text{and} \quad h_{j/A}(x_A, b) + h_{j/B}(x_B, b) = g_1 [b + g_3 \ln(100 x_A x_B)],$$  \hspace{1cm} (2.1)

where $g_1, g_2, g_3$ are phenomenological constants. Since we want to investigate the production of gauge bosons at the Fermilab Tevatron with $\sqrt{s} = 1.8$ TeV, our kinematic region includes values of $\tau = x_A x_B$ that are significantly lower than those relevant to Ref. 6. For this reason we refit for the nonperturbative functions using lower mass ranges in the R209 ($pp \rightarrow \mu^+ \mu^- + X$ at $\sqrt{s} = 62$ GeV) and E288 ($pN \rightarrow \mu^+ \mu^- + X$ at $\sqrt{s} = 27.4$ GeV) data in addition to the data from CDF ($pp$ collisions at $\sqrt{s} = 1.8$ TeV) for $Z^0$ production. These lower $\tau$ values used from the R209 and E288 data, however, are still roughly a factor of two larger than the typical $\tau$ value of $W^\pm$ and $Z^0$ physics at the Tevatron. To improve these nonperturbative functions in the CSS formalism, we postulate the $x_A, x_B$ dependence for the nonperturbative
functions $h_{j/A}$ and $h_{j/B}$ as in Eq. (2.1). Our choice of $\ln(x_A x_B)$ is inspired by the fact that the average transverse momentum of the Drell–Yan pair grows slowly with $\tau$. When better statistics becomes available, these nonperturbative functional forms also can be extracted for the lower $\tau$ kinematics through the study of Drell–Yan pairs and $Z^0$ boson production at the Tevatron collider.

For $W^\pm$ boson studies at the Tevatron, it would be excellent if one had very high statistics for the $Z^0$ data, since this would enable a determination of the nonperturbative functions in a kinematic region that overlaps greatly with the kinematics for $W^\pm$ boson production, but such high statistics data do not exist. Using the R209 data over the mass range $5 < Q < 8$ GeV in conjunction with the $Z^0$ boson data from CDF, $g_2$ was determined. We note that $g_2$ is associated with the $\ln(Q^2/Q_0^2)$ factor in the nonperturbative functions. With that $g_2$, the same R209 data were taken in combination with the E288 data over $6 < Q < 8$ GeV to obtain an $x_A, x_B$ dependence by determining $g_1$ and $g_3$. Using the CTEQ2M parton distribution functions (PDFs) with $Q_0 = 1.6$ GeV and $b_{\text{max}} = 0.5$ GeV$^{-1}$, the nonperturbative parameters are

\[
g_1 = 0.11^{+0.04}_{-0.03} \text{ GeV}^2, \quad g_2 = 0.58^{+0.1}_{-0.2} \text{ GeV}^2, \quad g_3 = -1.5^{+0.1}_{-0.1} \text{ GeV}^{-1}.
\]

We now proceed to discuss the comparison of the $Q_T$ distributions obtained from the previous values of $\bar{g}_1, \bar{g}_2$ with that given by $g_1, g_2, g_3$. For the high $\tau$ kinematics of the R209 data with $11 < Q < 25$ GeV, the $d\sigma/dQ_T^2$ distribution given by the nonperturbative form of Eq. (1.3), where we naively carry over the $\bar{g}$ to the CTEQ2M PDF, lies very close to the results provided by using the CTEQ2M PDF with Eq. (2.2). The results provided by Eq. (2.2) also are consistent with the E288 Drell–Yan data, whose typical $\tau$ values range from about $1/5$ to $1/2$. For the $5 < Q < 8$ GeV range in the R209 data, where the $\tau$ values probed are much smaller and more relevant to $W^\pm$ and $Z^0$ boson physics at the Fermilab Tevatron, Fig. 1 shows that the old form of Eq. (1.3) with the CTEQ2M PDF is quite different from the result of Eq. (2.2), deviating from the experimental result. This is a clear demonstration that for precision measurements (like the determination of $M_W$ or $\Gamma_W$) or simply for theoretical consistency, it is necessary, particularly when entering a new kinematic region or changing the PDF used, to have a nonperturbative parametrization consistent with that PDF and kinematics. In Fig. 1 we also show the result obtained when the favored values of Ref. 6 are used in conjunction with the HMRSB PDF, which has been used in previous publications. This too does not agree with the data, clearly discriminating between the two nonperturbative functions of Eq. (1.3) and Eq. (2.2). The former parametrization is invalid in this kinematic region independent of the PDF used.
Having a better nonperturbative dependence for describing the low $Q_T$ kinematics of the $W^\pm$, $Z^0$ boson production at the Fermilab Tevatron, we can see if there is any change from previous expectations. In Fig. 2a (2b) we display three calculations against the results for $Z^0 (W^\pm)$ production obtained by the CDF collaboration. (We use $M_W = 80$ GeV and $M_Z = 91.17$ GeV.) The dashed (dashed-dotted) curve is where we took the previous values of Eq. (1.3) with the CTEQ2M (HMRSB) PDF, while the solid curve represents the results of the new fit with the CTEQ2M PDF. Simply comparing the new set with the old set as applied to the CTEQ2M PDF, it is apparent that the peak of the $Q_T$ spectrum has dropped and shifted to a higher $Q_T$ value. The HMRSB result has a lower peak height, but the peak remains at the lower $Q_T$, just as the dashed curve. It is not due to the comparison of these theoretical results with the low statistics data that favor one result over the other, rather, it is the inability of the Eq. (1.3) to account for the Drell-Yan results with the more pertinent $\tau$ kinematics that decides. Recall that at very low $Q_T$ the $d\sigma/dQ_T$ experimental results$^{[13]}$ for the $W^\pm$ boson were below theoretical expectations with the old fits and that as the $Q_T$ rose the data quickly gave experimental results above theoretical expectations.$^{[14]}$ The new fit not only shifts the peak in the $d\sigma/dQ_T$ distribution to higher $Q_T$, it also makes the peak broader, thereby yielding an improvement between the data and the theory in the region of $5 \leq Q_T \leq 20$ GeV. In Fig. 3a (3b) we show the $Q_T$ distribution at $y = 0$ for the $Z^0 (W^\pm)$ boson relevant to the CERN collider.$^{[15]}$

We observe that the results with the old parameters are similar between the CTEQ2M and HMRSB PDFs for the $\tau$ region of Fig. 1, yet as we enter the lower $\tau$ kinematics of the CDF data, the difference appears to grow. In this regard, we note that the CTEQ2M PDF fits with the low–$x F_2$ data from HERA,$^{[16]}$ while HMRSB does not. Using the new nonperturbative parameters with the MRSD–′ PDFs$^{[18]}$ gave results that differed only negligibly when compared against the CTEQ2M results for $Q_T < 20$ GeV.
3. Measuring the Nonperturbative functions at the Fermilab Tevatron

Measuring the nonperturbative functions in the CSS resummation formalism provides information about the nonperturbative nature of QCD theory. For instance, $h_Q(b)$ in Eq. (1.2) might have an operator definition as the vacuum expectation of the gluon condensate.$^{19}$

We explore the necessary luminosity and energy of the upgraded Tevatron to perform this kind of study. To isolate the problem interested, we assume that the parton distribution functions are known in the relevant kinematic region for producing $W^\pm$ and $Z^0$ bosons at the Tevatron. In Fig. 4 we show the transverse momentum distributions for the $W^\pm$ and $Z^0$ bosons produced at a $p\bar{p}$ collider with $\sqrt{s} = 1.8, 3.5$ TeV using the CTEQ2M PDFs.

For this study we chose $Q_0 = 1.6$ GeV, fixed $g_3 = -1.5$ GeV$^{-1}$ and pretended that nature wants $g_1 = 0.11$ GeV$^2$ and $g_2 = 0.58$ GeV$^2$. To estimate the accuracy for the measurement of the nonperturbative parameters as a function of luminosity, fake data was generated for the $d\sigma/dp_T$ distribution describing the production of $Z^0$ bosons, where a branching fraction of 0.06 was included to focus only on the $Z \to e^-e^+$ and $Z \to \mu^-\mu^+$ decay channels. This fake data was then fit to theory using MINUIT$^{20}$ to both determine the best values for the parameters $g_1$ and $g_2$ and to estimate the errors.

To create the fake data for the $d\sigma/dp_T$ distribution, a bin width of $\Delta p_T = 1$ GeV was assumed. Given the theoretical value for $d\sigma/dp_T$ and the luminosity ($\mathcal{L}$), a statistical error was evaluated for each value of $d\sigma/dp_T$,

$$\varepsilon_{\text{stat}} = \sqrt{\frac{d\sigma/dp_T}{\Delta p_T \mathcal{L}}}.$$ (3.1)

Assigning a detector uncertainty equivalent to the statistical error for the purposes of this estimate, the error for each of the fake data points in $d\sigma/dp_T$ becomes

$$\varepsilon = \sqrt{2}\varepsilon_{\text{stat}} = \sqrt{\frac{2d\sigma/dp_T}{\Delta p_T \mathcal{L}}}.$$ (3.2)

Taking eight values at $p_T = 1, 2, \ldots, 8$ GeV, each point of the $d\sigma/dp_T$ theory was randomized according to a gaussian distribution about its theoretical value using a width of $\varepsilon$. This provided a fake data sample to use for the fit of the nonperturbative functions.
When the fake data was fit to the theoretical values, the errors provided by MINUIT demonstrated that $g_1$ can be known to within about 6% (20%, 50%) and $g_2$ can be known to within about 2% (4%, 15%) at the 95% confidence level given a luminosity of 10 fb$^{-1}$ (1 fb$^{-1}$, 0.1 fb$^{-1}$).

The accuracy for the measurement of the nonperturbative functions in the CSS resummation formalism is relatively indifferent to the energy upgrade of the Tevatron (within a factor of two for 1.8 TeV and 3.5 TeV energies) when compared against an orders of magnitude increase in the luminosity, indicating that the luminosity of the machine is crucial. In this analysis, we only assume $Z \to e^+e^-$ and $Z \to \mu^+\mu^-$ data. If the mass of the $W$ boson is known, one can include the data from $W^\pm \to l^\pm + \text{neutrino}$ (for $l = e$ or $\mu$) to further improve the measurement by about a factor of two.

Although this study is merely a theoretical exercise, the nonperturbative functional forms as described in Eq. (2.1) might need to be revised when better data become available. It eventually may prove that the flavor dependence cannot be ignored. Nevertheless, our point is that data from Drell–Yan experiments and the production of $W^\pm$ and $Z^0$ bosons at hadron colliders and fixed target experiments like those at Fermilab will be instrumental in providing bounds on the nonperturbative structure at low $Q_T$.

4. Conclusion

For precision measurements (like the determination of $M_W$, $\Gamma_W$, or the charge asymmetries for $W^\pm$ bosons) or simply for theoretical consistency, it is necessary, particularly when entering a new kinematic region or changing the PDF used, to have a nonperturbative parametrization consistent with that PDF and kinematics. In this study we have demonstrated that in the CSS resummation formalism represented by Eq. (1.1), the contributions of the nonperturbative functions to the $Q_T$ distribution of Drell–Yan pairs and the production of $W^\pm$ and $Z^0$ bosons at fixed target and hadron colliders are important. When the physics of interest (e.g., low $Q_T$ boson production) probes different kinematics, such as lower momentum fractions $x_A$ and $x_B$, these nonperturbative functions have to be reevaluated, just as the PDFs have to be updated when new data probing smaller $x$ regions become available. Consistency between the data from Drell–Yan processes and the production of $W^\pm$ and $Z^0$ bosons will not only support QCD theory, but also will provide a tool which can facilitate our pursuits in physics beyond QCD through a better understanding of signal and background processes in this important kinematic region where event rates are large. As an example, we have shown that the theoretical $Q_T$ distributions for $W^\pm$ and $Z^0$ bosons at the Fermilab Tevatron agree better
with experimental data after a new fit for the nonperturbative dependence has been performed using Drell–Yan data proximate to the relevant kinematic regions in $\tau = x_A x_B$ for $W^\pm$ and $Z^0$ boson production at the Fermilab Tevatron.

For the precision measurement of $M_W$ and testing QCD theory in processes involving two–scale physics, such as the $W^\pm$, $Z^0$ and Drell–Yan pair production, it is important to know the theoretical errors due to the nonperturbative parametrization, the factorization scale dependence, and the parton distribution functions. All of these considerations are under study.\[17\]
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Figure Captions

1. Comparison of R209 Drell–Yan data with calculations using Eq. (2.2) and the CTEQ2M PDF or Eq. (1.3) using either the CTEQ2M or HMRSB PDFs.

2. Comparison of (a) $Z^0$ boson or (b) $W(= W^+ + W^-)$ boson production at CDF with calculations using Eq. (2.2) and the CTEQ2M PDF or Eq. (1.3) using either the CTEQ2M or HMRSB PDFs.

3. Calculations as in Fig. 2, except for $\sqrt{s} = 630$ GeV.

4. The transverse momentum distribution for the production of (a) $Z$ and (b) $W$ bosons in $p\bar{p}$ collisions at $\sqrt{s} = 1.8, 3.5$ TeV.