THE THEORETICAL AGENDA IN CMB RESEARCH

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Abstract

The terrain that theorists cover in this CMB golden age is described. We ponder early universe physics in quest of the fluctuation generator. We extoll the virtues of inflation and defects. We transport fields, matter and radiation into the linear (primary anisotropies) and nonlinear (secondary anisotropies) regimes. We validate our linear codes to deliver accurate predictions for experimentalists to shoot at. We struggle at the computing edge to push our nonlinear simulations from only illustrative to fully predictive. We are now phenomenologists, optimizing statistical techniques for extracting truths and their errors from current and future experiments. We begin to clean foregrounds. We join CMB experimental teams. We combine the CMB with large scale structure, galaxy and other cosmological observations in search of current concordance. The brave use all topical data. Others carefully craft their prior probabilities to downweight data sets. We are always unbiased. We declare theories sick, dead, ugly. Sometimes we cure them, resurrect them, rarely beautify them. Our goal is to understand how all cosmic structure we see arose and what the Universe is made of, and to use this to discover the laws of ultrahigh energy physics. Theorists are humble, without hubris.

1 Theoretical Theory

Early Universe Physics is the likely source of fluctuations to input into the cosmic structure formation problem (e.g., [1]). We want to measure the CMB (and large scale structure) response to these initial fluctuations. The goal is the lofty one of peering into the physical mechanism by which the fluctuations were generated, by learning: the statistics of the fluctuations, whether Gaussian or non-Gaussian; the mode, whether adiabatic or isocurvature scalar perturbations, and whether there is a significant component in gravitational wave tensor perturbations; the power spectra for these modes (albeit over only 3 decades in wavenumber for CMB probes, Fig. 2, and fewer for large scale structure (LSS) probes).

Fluctuation Generation Mechanisms: The contenders are (1) “zero point” oscillations in scalar and tensor fields that must be there in the early universe if quantum mechanics is applicable and (2) topological defects which may arise in the inevitable phase transitions expected in the early universe.

Inflation: For quantum oscillations to be important, a period of accelerated expansion seems essential, for then the comoving Hubble length \((Ha)^{-1}\) shrinks in size, freezing the time-incoherent noise into time-coherent patterns for structure formation.\(^1\) A major challenge for

\(^1\)Acceleration (i.e., inflation) seems to be generic unless it is explicitly forbidden by some law of physics unknown at this time. It may be extremely improbable \(a\ priori\) that a bit of space would accelerate and yet be highly probable \(a\ posteriori\) that we would find ourselves in a region that once inflated because of the vast space created.
Many variants of the basic inflation theme have been proposed, sometimes with radically different consequences for the appearance of the CMB sky, which is used in fact to highly constrain the more baroque models. A rank-ordering of inflation possibilities: (1) adiabatic curvature fluctuations with nearly uniform scalar tilt over the observable range, slightly more power to large scales \((n_s < 1)\) than “scale invariance” \((n_s = 1)\) gives, a predictable nonzero gravity wave contribution, with tilt similar to the scalar one, and tiny mean curvature \((\Omega_{tot} \approx 1)\); (2) same as (1), but with a tiny gravity wave contribution; (3) same as (1) but with a subdominant isocurvature component of nearly scale invariant tilt (the case in which isocurvature dominates is ruled out by \(\Delta T/T\)); (4) radically broken scale invariance with weak to moderate features (ramps, mountains, valleys) in the fluctuation spectrum (strong ones are largely ruled out by \(\Delta T/T\)); (5) radical breaking with non-Gaussian features as well; (6) “open” inflation, with quantum tunneling producing a negatively-curved (hyperbolic) space which inflates, but not so much as to flatten the mean curvature \((d_c \sim (Ha)^{-1}, \text{ not } \gg (Ha)^{-1}, \text{ where } d_c \equiv H_0^{-1}|1 - \Omega_{tot}|^{-1/2})\); (7) quantum creation of compact hyperbolic space from “nothing” with volume \(d^2\) which inflates, with \(d_T \sim (Ha)^{-1}, \text{ not } \gg (Ha)^{-1}\), and \(d_T\) of order \(d_c\); (8) flat \((d_c = \infty)\) inflating models which are small tori of scale \(d_T\) with \(d_T\) a few \((Ha)^{-1}\) in size. It is quite debatable of which the cases beyond (2) are more or less plausible, with some claims that (4) is supersymmetry-inspired, others that (6) is not as improbable as it sounds (see Cohn, these proceedings, for a nice discussion). It is the theorists’ job to push out the boundaries of the inflation idea and use the data to select what is allowed.

**Defects:** Gradients in the disordered field energy left by a phase transition are smoothed on the growing scale over which causal communication can occur. Topological knot-like or string-like field configurations disappear slowly enough that they can act as isocurvature seed perturbations to drive the growth of fluctuations in the total mass density. The inherent nonlinearity of these defects in the field implies many issues can only be answered with numerical simulations, and these are subject to the same computational-size-to-resolution limitations that plague the rest of nonlinear cosmology. Much analytic progress in understanding the basic observable features of defect models is being made and renewed effort is being put into doing large enough simulations to predict CMB anisotropies in both texture and string theories.

**Hydro:** Although hydrodynamic and radiative processes are expected to play important roles around collapsed objects and may bias the galaxy distribution relative to the mass, a global role in obscuring the early universe fluctuations by late time generation on large scales now seems unlikely.\(^2\)

**Transport:** Cosmological radiative transfer is on a firm theoretical footing. Together with a gravity theory\(^3\) and the transport theory for the other fields and particles present (baryons, hot, warm and cold dark matter, coherent fields, i.e., “dynamical” cosmological “constants”, etc.), we propagate initial fluctuations from the early universe through photon decoupling into

\(^2\)Not too long ago it seemed perfectly reasonable based on extrapolation from the physics of the interstellar medium to the pregalactic and intergalactic medium to suppose hydrodynamical amplification of seed cosmic structure would obscure primordial fluctuations from the early Universe. The strong limits on Compton cooling from FIRAS, in energy \(\delta E_{\text{Compton}} < 4y < 6.0 \times 10^{-5} \text{ (95% CL)}\), constrain the product \(f_{\text{exp}}R_{\text{exp}}^2\) of filling factor \(f_{\text{exp}}\) and bubble formation scale \(R_{\text{exp}}\) to values too small for a purely hydrodynamic origin. If supernovae were responsible for the blasts, the accompanying presupernova light radiated would have been much in excess of the explosive energy (more than a hundred-fold), leading to much stronger restrictions.

\(^3\)Einstein’s theory is invariably assumed, but deviations are expected and indeed necessary at very high energy, with potential impact on the fluctuation generation process, and, if exotic enough, on transport through decoupling to now. Eventually, as we understand the CMB sky better, the data will undoubtedly be turned to constraining or discovering modified theories of gravity.
the (very) weakly nonlinear phase, and predict primary anisotropies, those calculated using either linear perturbation theory (e.g., for inflation-generated fluctuations), or, in the case of defects, linear response theory. The sources driving their development are listed in the Table.

**Cosmic Parameters:** Even simple Gaussian inflation-generated fluctuations for structure formation have a large number of early universe parameters we would wish to determine: power spectrum amplitudes at some normalization wavenumber $k_n$, for the modes present, $\{P_{aa}(k_n), P_{ss}(k_n), P_{GW}(k_n)\}$; “tilt” shape functions $\{n_s(k_n), n_{is}(k_n), n_{t}(k_n)\}$, usually chosen to be constant or with a logarithmic correction, e.g., $n_s(k_n), d n_s(k_n)/d \ln k$. The transport problem is dependent upon physical processes, and hence on physical parameters. A partial list includes various mean energy densities $\{\Omega_{tot}, \Omega_B, \Omega_{vac}, \Omega_{cdm}, \Omega_{hdm}\}$, the Hubble parameter $h$, the number of relativistic neutrinos, the abundance of primordial helium, and parameters characterizing the ionization history of the Universe, e.g., the Compton optical depth from a reheating redshift $z_{reh}$ to the present. For a given model, the early universe power spectrum amplitude measures are uniquely related to late-time power spectrum measures of relevance for the CMB, such as the quadrupole or bandpowers for various experiments, or to large scale structure observations, such as the $\sigma_8$ density fluctuation level on the $8 \, h^{-1} \, \text{Mpc}$ (cluster) scale.

The arena in which theory battles observation is the anisotropy power spectrum figure. Fig. 1 illustrates how primary $C_\ell$’s vary with cosmic parameters. They are normalized to the 4-year $dmr(53+90+31)(A+B)$ data. A “standard” CDM model, with $n_s=1, \Omega_{tot}=1, H_0=50$, $\Omega_B=0.05$, and a 13 Gyr age, is the upper solid curve. It has $\sigma_8 = 1.20 \pm 0.08$, far from the $\sim 0.6$ target value derived from cluster abundance observations. An (almost indistinguishable) dotted curve has the same parameters except that it includes a light neutrino with $\Omega_{\nu e}=0.2$ (and $\Omega_{cdm}=0.75$). It has $\sigma_8 = 0.83 \pm 0.06$. The upper dashed curve is a vacuum-dominated model with $H_0=75$ and the 13 Gyr age (and $\Omega_{\Lambda} = 0.73, \Omega_B = 0.02, \Omega_{cdm} = 0.24$). It has $\sigma_8 = 1.03 \pm 0.07$, which is OK for cluster abundances. An open CDM model has the $C_\ell$ peak shifted to higher $\ell$; the one shown has $H_0 = 60$ and the 13 Gyr age, with $\Omega_{tot}=0.33, \Omega_{cdm}=0.30$, $\Omega_B = 0.035$, and $\sigma_8 = 0.50 \pm 0.04$. By $H_0 = 70$, $\Omega_{tot}$ is down to 0.055 at this age. The lower solid curve is a CDM model with reionization at $z_{reh} = 30$, and almost degenerate with it is a tilted CDM model ($n_s=0.95$ but otherwise standard). Even the nearly degenerate hot/cold and CDM models shown should be distinguishable by COBRAS/SAMBA.

**COMBA:** Spurred on by the promise of percent-level precision in cosmic parameters from CMB satellites, a considerable fraction of the CMB theoretical community with Boltzmann transport codes compared their approaches and validated the results to ensure percent-level accuracy up to $\ell \sim 3000$ [4]. The arena shifted from figures like Fig. 1 to $\Delta C_\ell/C_\ell$ figures with tiny vertical range. We look forward to the happy day when such a relative difference figure will be used to reveal the remaining tiny residuals in the best fit theoretical model. An important goal for COMBA was speed, since the parameter space we wish to constrain has many dimensions. Most groups have solved cosmological radiative transport by evolving a hierarchy of coupled moment equations, one for each $\ell$. Although the equations and techniques were in place prior to the COBE discovery for scalar modes, and shortly after for tensor modes, to get the high accuracy with speed has been somewhat of a challenge. There are alternatives to the moment hierarchy for the transport of photons and neutrinos. In particular the entire problem of photon transport reduces to integral equations in which the multipoles with $\ell > 2$ are expressed as history-integrals of metric variables, photon-bunching, Doppler and polarization sources, as in the Table. The fastest COMBA-validated code uses this method (Seljak, these proceedings).

**Secondary:** Secondary anisotropies, with sources listed in the Table, arise from nonlinear structures. They are a nuisance foreground to be subtracted to get at the primary primary ones, but also invaluable probes of shorter-distance aspects of structure formation theories, full
of important cosmological information. The effect of lensing is to smooth slightly the Doppler peaks and troughs of Fig. 1. \( \ell \)’s from quadratic nonlinearities in the gas at high redshift are concentrated at high \( \ell \), but for most viable models are expected to be a small contaminant. Scattering from gas in moving clusters also has a small effect on \( \ell \), although is measurable in individual clusters. Power spectra for the thermal SZ effect from clusters are larger; examples in Fig. 1 are for a cluster-normalized \( \sigma_8 = 0.7 \) hot/cold hybrid model (solid, \( \Omega_{\text{hdm}} = 0.3 \)) and an \( n_s = 0.8 \) tilted CDM model. Although \( \ell^{(\text{SZ})} \) may be small, because the power for such non-Gaussian sources is concentrated in hot or cold spots the signal is detectable, and often has been. \( \ell \) for a typical dusty primeval galaxy model is also shown, the larger (arbitrarily normalized) part a shot-noise effect for galaxies with dust distributed over 10 kpc, the smaller a contribution associated with clustering. Similar spectra are expected for other extragalactic point sources, e.g., radio galaxies.

2 Phenomenological Theory

Phenomenology: We have progressed from the tens of pixels of early \( \Delta T/T \) experiments through the thousands for DMR and SK, and will soon be dealing with tens of thousands for long duration balloon experiments and eventually millions for the MAP and COBRAS/SAMBA satellites. How best to analyze statistically these data sets is a subject being developed largely by CMB theorists. Theorists have also taken on an increasingly phenomenological role in LSS studies and many of the same techniques are being applied to both \( \Delta T/T \) and LSS data sets. This is opening up into a major subfield, a trend which we should strongly encourage. Finding nearly optimal strategies for data projection, compression and analysis which will allow us to disentangle the primary anisotropies from the Galactic and extragalactic foregrounds and from the secondary anisotropies induced by nonlinear effects will be the key to realizing the theoretically-possible precision on cosmic parameters and so to determine the winner (and losers) in theory space. Particularly powerful is to combine results from different CMB experiments and combine these with LSS and other observations. Almost as important as the end-product is the application of the same techniques to probing the self-consistency and cross-consistency of experimental results.

Current State: Current band-powers, shown in Fig. 1, broadly follow inflation-based expectations, but may still include residual signals. Consistency with the primary anisotropy frequency spectrum has been shown for DMR and for the smaller angle experiments, but over a limited range. That the level is \( \sim 10^{-5} \) provides strong support for the gravitational instability theory. To get the large scale structure of COBE-normalized fluctuations right provides encouraging support that the initial fluctuation spectrum was not far off the scale invariant form that inflation (and defect) models prefer. That there appears to be power at \( \ell \sim 400 \) suggests the universe could not have reionized too early.

Large Scale Structure: We have always combined CMB and LSS data in our quest for viable models. Fig. 2 shows how the two are connected. As we have seen, the DMR data precisely determines \( \sigma_8 \) for each model considered. For the COBE-normalized density power spectra to thread the “eye of the needle” associated with cluster abundances severely constrains the parameters determining them. Similar constrictions arise from galaxy-galaxy and cluster-cluster clustering observations. Smaller angle CMB data (e.g., SP94, SK95) are consistent with these models (e.g., Bond and Jaffe, these proceedings), and will soon be powerful enough for the CMB by itself to offer strong selection, but this will definitely not diminish the combined LSS-CMB phenomenology.

Ultra-large Scale Structure: The “beyond our horizon” land in Fig. 2 is actually partly
accessible because long waves contribute gentle gradients to our observables. Constraints on such “global parameters” as average curvature are an example, $d_c > 1.1H_0^{-1}$, though not yet very restrictive. One may also probe whether a huge bump or deficit in power exists just beyond $k^{-1} \sim H_0^{-1}$, but this has not been much explored. The remarkable non-Gaussian structure predicted by stochastic inflation theory would likely be too far beyond our horizon for its influence to be felt. The bubble boundary in hyperbolic inflation models may be closer and its influence shortly after tunneling occurred could have observable consequences for the CMB. Theorists have also constrained the scale of topology in simple models; e.g., we (Pogosyan, Sokolov and I) find $d_T/2 > 2H_0^{-1}$ for flat 3-tori and $> 1.5H_0^{-1}$ for flat 1-tori from DMR (see de Oliveira-Costa, these proceedings). A number of groups are now trying to constrain the compact hyperbolic topologies.

**Futures:** Let us look forward to the day phenomenological theorists will have optimally-analyzed LDBs/VSA/CBI/ChiSPI/MAP/COBRAS/SAMBA and we know the power spectrum and cosmic parameters to wonderful precision. What will it mean? It may not be clear. Take inflation as an example. There will be attempts, undoubtedly optimal ones, to reconstruct the inflaton’s potential, but all of our CMB and LSS observations actually access only a very small region of the potential surface, and even this will be fuzzily determined if we allow too much freedom in parameter space. Still even a fuzzy glimpse is worth the effort. Most fun will be when phenomenology teaches us that non-baroque inflation and defect models fail and howling packs of theorists go hunting for the elusive generator following trails well marked by the data.

**References**

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[2] Bertschinger, E., Bode, P., Bond, J.R., Coulson, D., Crittenden, R., Dodelson, S., Efstathiou, G., Gorski, K., Hu, W., Knox, L., Lithwick, Y., Scott, D., Seljak, U., Stebbins, A., Steinhardt, P., Stompor, R., Souradeep, T., Sugiyama, N., Turok, N., Vittorio, N., White, M., Zaldarriaga, M. 1995, ITP workshop on *Cosmic Radiation Backgrounds and the Formation of Galaxies*, Santa Barbara.
### PHYSICAL PROCESSES FOR ANISOTROPY

#### PRIMARY SCALAR ANISOTROPIES

| Term | Description |
|------|-------------|
| $\Phi/3$ | "Naive" Sachs-Wolfe effect: Gravitational Potential |
| $\frac{1}{4} \frac{\delta \rho_c}{\rho_c}$ | photon bunching (acoustic): $\frac{1}{3} \frac{\delta \rho_c}{\rho_B}$ effect, isocurvature effect |
| $\sigma_T \bar{n}_e \mathbf{v}_e \cdot \hat{q}$ | Linear-order Thompson scattering (Doppler) |
| $2 \int_{l.o.s.} \dot{\phi}$ | Integrated Sachs-Wolfe effect |
| | subdominant anisotropic stress and polarization terms |

#### PRIMARY TENSOR ANISOTROPIES

| Term | Description |
|------|-------------|
| $\frac{1}{2} h_{+,x}$ | gravity waves (two polarizations) |
| | subdominant polarization terms |

#### SECONDARY ANISOTROPIES

| Term | Description |
|------|-------------|
| $\varepsilon_{AB}$ | Linear Weak Lensing: 2D shear tensor |
| $2 \int_{l.o.s.} \dot{\phi}_{NL}$ | Rees-Sciama effect: Linear Response to $\Phi$ of nonlinear structure |
| $\sigma_T \delta n_e \mathbf{v}_e \cdot \hat{q}$ | Nonlinear Thompson scattering: Quadratic-order (Vishniac) effect, "kinematic" SZ effect (moving cluster/galaxy) |
| $\ldots$ | thermal SZ effect: Compton cooling from nonlinear gas ($x = \frac{E_\gamma}{T_\gamma}$) |
| $\int_{l.o.s.} \psi_K(x) \delta(n_e T_e)$ | redshifted dust emission, pregalactic/protogalactic ($x_d = \frac{E_\gamma}{T_d}$) |

### FOREGRONDS

| Term | Description |
|------|-------------|
| extragalactic radio sources: falling, flat, rising |
| IRAS sources and extrapolations to moderate $z$ |
| Galactic bremsstrahlung, synchrotron |
| Galactic Dust: regular, cool, strange?? |
Figure 1: Sample primary and secondary power spectra, $C_\ell \equiv \ell(\ell+1)\langle|\Delta T/T|_{\ell m}|^2\rangle/(2\pi)$, are compared with the band-power estimates derived for the anisotropy data up to March 1996. The lower panel is a closeup of the first two ‘Doppler peaks’. Average filter functions for a variety of experiments are shown in the middle panel. The upper solid primary $C_\ell$ curve is a COBE-normalized “standard” untilted CDM model, and variants are shown with the same cosmological age and $\Omega_B h^2$, but nonzero tilt, $\Omega_{cdm}$ (i.e., $m_\nu > 0$), $\Omega_{vac}$ (i.e., $\Lambda > 0$), average curvature (i.e., $\Omega_{tot} < 1$) or weak reionization. They all broadly agree with the data. By contrast, the thermal SZ anisotropy power is way down, the kinematic SZ power is off-scale, and dusty emission power from early galaxies is concentrated at higher $\ell$ and higher frequency.
Figure 2: The bands in comoving wavenumber probed by CMB primary and secondary anisotropy experiments and by large scale structure observations are contrasted. Sample (linear) density power spectra are for the “standard” $n_s = 1$ CDM model (labelled $\Gamma = 0.5$), for a tilted ($n_s = 0.6$, $\Gamma = 0.5$) CDM model and for a model with the shape modified ($\Gamma = 0.25$) by changing the matter content of the Universe. A (uniform?) bias is allowed to raise the shapes into the hatched $w_{gg}$ region; only the latter two fit. The solid data point in the cluster-band denotes the constraint from the abundance of clusters, and the open data point at $10 \ h^{-1}$ Mpc a constraint from streaming velocities (for $\Omega_{tot} = 1, \Omega_{vac} = 0$).