Talk presented at the International Workshop “On the Physics of the Quark-Gluon Plasma”, Palaiseau, France, Sept. 2001.

Strange Particle Production from SIS to LHC

H. Oeschler\textsuperscript{1}, J. Cleymans\textsuperscript{2}, and K. Redlich\textsuperscript{3,4,5}
\textsuperscript{1}Institut f"{u}r Kernphysik, Darmstadt University of Technology, D-64289 Darmstadt, Germany
\textsuperscript{2}Department of Physics, University of Cape Town, Rondebosch 7701, South Africa
\textsuperscript{3}Gesellschaft f"{u}r Schwerionenforschung, D-64291 Darmstadt, Germany
\textsuperscript{4}Institute for Theoretical Physics, University of Wroc\l{}aw, PL-50204 Wroc\l{}aw, Poland
\textsuperscript{4}now: Theory Division, CERN, CH-1211 Geneva 23, Switzerland

Abstract. A review of meson emission in heavy ion collisions at incident energies from SIS up to collider energies is presented. A statistical model assuming chemical equilibrium and local strangeness conservation (i.e. strangeness conservation per collision) explains most of the observed features.

Emphasis is put onto the study of $K^+$ and $K^-$ emission at low incident energies. In the framework of this statistical model it is shown that the experimentally observed equality of $K^+$ and $K^-$ rates at “threshold-corrected” energies $\sqrt{s} - \sqrt{s}_{th}$ is due to a crossing of two excitation functions. Furthermore, the independence of the $K^+$ to $K^-$ ratio on the number of participating nucleons observed between SIS and RHIC is consistent with this model.

It is demonstrated that the $K^-$ production at SIS energies occurs predominantly via strangeness exchange and this channel is approaching chemical equilibrium. The observed maximum in the $K^+/\pi^+$ excitation function is also seen in the ratio of strange to non-strange particle production. The appearance of this maximum around 30 $A\cdot$GeV is due to the energy dependence of the chemical freeze-out parameters $T$ and $\mu_B$.

1. Introduction

Central heavy ion collisions at relativistic incident energies represent an ideal tool to study nuclear matter at high temperatures. Particle production is – at all incident energies – a key quantity to extract information on the properties of nuclear matter under these extreme conditions. Particles carrying strangeness have turned out to be very valuable messengers.

A specific purpose of this paper is the presentation of the evolution of strange particle production over a large range of incident energies. The data at low incident energies, i.e. at and below the production threshold in $NN$ collisions will be shown in more detail as these results are less presented in other works. Many results are shown together with a theoretical interpretation. The attempts to describe particle production yields with statistical models \cite{1, 2, 3, 4, 5, 6, 7, 8} have turned out to be very successful over this large domain of incident energies.
Strange Particle Production from SIS to LHC

2. General Trends

2.1. Production of $K^+$ and $K^-$ from SIS to RHIC

At incident energies around 1 $A$-GeV pion and kaon production is very different: Pions can be produced by direct $NN$ collisions but kaons not. The threshold for $K^+$ production in $NN$ collisions is 1.58 $A$-GeV and only collective effects can accumulate the energy needed to produce a $K^+$ together with a $\Lambda$ (or another strange particle) for strangeness conservation.

The measured multiplicities for pions and $K^+$ differ evidently by orders of magnitude. They exhibit a further very pronounced contrast: While the pion multiplicity per number of participating nucleons $A_{\text{part}}$ remains constant with $A_{\text{part}}$, the $K^+$ multiplicity per $A_{\text{part}}$ rises strongly (Fig. 1). The latter observation seems to be in conflict with a thermal interpretation, which – in a naive view – should give multiplicities per mass number $A$ being constant.

Usually, the particle number densities or the multiplicities per $A_{\text{part}}$, here for pions, are described in a simplified way by a Boltzmann factor

$$M_{\pi}/A_{\text{part}} \sim \exp \left( -\frac{<E_{\pi}>}{T} \right),$$

with the temperature $T$ and the total energy $<E_{\pi}>$.

The production of strange particles has to fulfil strangeness conservation. The attempt to describe the measured particle ratios including strange hadrons at AGS and SPS using a strangeness chemical potential $\mu_S$ is quite successful. However, this grand-canonical treatment is not sufficient, if the number of produced strange particles is small. Then, a statistical model has to take care of exact strangeness conservation in each reaction as introduced in [10]. This is done by taking into account that e.g. together with each $K^+$ a $\Lambda$ or another strange particle is produced:

$$M_{K^+}/A_{\text{part}} \sim \exp \left( -\frac{<E_{K^+}>}{T} \right) [g_\Lambda V \int \frac{d^3p}{(2\pi)^3} \exp \left( -\frac{(E_\Lambda - \mu_B)}{T} \right)],$$

Figure 1. The multiplicity of $K^+/A_{\text{part}}$ rises strongly with $A_{\text{part}}$ in contrast to the pion multiplicity. This rise can be described by the statistical model including local strangeness conservation (see text).
where \( T \) is the temperature, \( \mu_B \) the baryo-chemical potential, \( g_i \) the degeneracy factors, \( V \) the production volume for making the associate pair (see [4, 5]) and \( E_i \) the total energies. We note that this volume is not identical to the volume of the system at freeze out. The volume parameter \( V \) is taken as \( r_0^3A_{\text{part}} \) with a common \( r_0 \) for all systems and all incident energies.

This formula, simplified for demonstration purposes, neglects other combinations leading to the production of \( K^+ \) as well as the use of Bose-Fermi distributions, which are all included in the computation. The corresponding formula for \( K^- \) production

\[
\frac{M_{K^-}}{A_{\text{part}}} \sim \exp \left( -\frac{<E_{K^-}>}{T} \right) \left[ g_{K^+}V \int \frac{d^3p}{(2\pi)^3} \exp \left( -\frac{E_{K^-}}{T} \right) \right]
\]

is similar, but does not depend on \( \mu_B \). This point will become important later on.

These formulae lead to a reduction of \( K^+ \) and \( K^- \) yields as compared to the numbers calculated without exact strangeness conservation [4, 5]. Two extreme conditions can be seen from these equations. In the limit of a small number of strange particles, the additional term (due to the parameter \( V \)) leads to a linear rise of \( M_{K^+}/A_{\text{part}} \) while \( M_{K^-}/A_{\text{part}} \) remains constant. This is in remarkable agreement with the experimental observations shown in Fig. 1. For very high temperatures or very large volumina, the terms in brackets approach unity (see Ref. [4]) and the formulae coincide with the grand-canonical procedure. This is much better seen in the exact formulae using modified Bessel functions [4, 5, 11].

At low incident energies, the particle ratios (except \( \eta/\pi_0 \)) are well described using the canonical approach [4]. Surprisingly, even the measured \( K^+/K^- \) ratio is described and this ratio does not depend on the choice of the volume term \( V \). It should be noted that the statistical model uses normal masses of the particles while many transport calculation [12] have to reduce the \( K^- \) mass (as expected for kaon in the nuclear medium) in order to describe the measured yields. It is therefore of interest to see how the results of the statistical model appear in a representation where the \( K^+ \) and \( K^- \) multiplicities are given as a function of \( \sqrt{s} - \sqrt{s_{\text{th}}} \) as shown in Fig. 2. The choice of the x-axis follows the idea to correct for the different energy thresholds to produce \( K^+ \) in \( NN \) collisions (\( s_{\text{th}} = 2.548 \) GeV) and \( K^- \) (\( s_{\text{th}} = 2.87 \) GeV). It turned out that in this representation the measured yields of \( K^+ \) and \( K^- \) close to threshold in heavy ion collisions are about equal while they differ in pp collisions by factors of 10 – 100 [13].

Figure 2 demonstrates that at values of \( \sqrt{s} - \sqrt{s_{\text{th}}} \) less than zero, the excitation functions for \( K^+ \) and \( K^- \) cross leading to the observed equality of \( K^+ \) and \( K^- \) at SIS energies. The yields differ at AGS energies by a factor of five. The difference in the rise of the two excitation functions can be understood by the formulae given above. The one for \( K^+ \) production contains \( (E_A - \mu_B) \) while the other has \( E_{K^+} \) in the exponent of the second term. As these two values are different, the excitation functions, i.e. the variation with \( T \), exhibit a different rise.

Furthermore, the two formulae predict that the \( K^+/K^- \) ratio for a given collision should not vary with centrality as \( V \) cancels in the ratio. Indeed, this has been observed in \( Au+Au/Pb+Pb \) collisions between 1.5 \( A \)-GeV and RHIC energies [12, 10, 11, 18, 13] as shown in Fig. 3. This independence of centrality is most astonishing as one expects at low incident energies an influence of the different thresholds and the density variation with centrality. At 1.93 \( A \)-GeV the \( K^+ \) production is above and the \( K^- \) production below their respective \( NN \) thresholds.
Strange Particle Production from SIS to LHC

Figure 2. Calculated $K^+/A_{\text{part}}$ and $K^-/A_{\text{part}}$ ratios in the statistical model as a function of $\sqrt{s} - \sqrt{s_{\text{th}}}$ for Ni+Ni collisions. The points are results for Ni+Ni collisions at SIS energies [14, 15] and Au+Au at 10.2 A-GeV (AGS) [16]. At AGS energies the influence of the system mass is negligible.

Figure 3. The $K^+/K^-$ ratio appears to be constant of a function of centrality from SIS up to RHIC energies. The dotted lines represent the predictions of the statistical model. Data from [14, 16, 18].

Transport-model calculations show clearly that strangeness equilibration requires a time interval of 40 – 80 fm/c [20, 21]. On the other hand, the statistical models assuming chemical equilibration are quite successful in describing the particle yields, including strange particles.

In case of the $K^+$ production, no strong absorptive channel seems to be available which could lead to chemical equilibration. For $K^-$ production the situation is quite different. At low incident energies strange quarks are found only in a few hadrons. The
Strange Particle Production from SIS to LHC

**Figure 4.** Measured $K^{-}/K^{+}$ ratio, $M(\pi^{0} + \pi^{-})/A_{\text{part}}$ and the double ratio ($[K^{-}]/[K^{+}])/((M(\pi^{0} + \pi^{-}))/A_{\text{part}})$ as a function of $A_{\text{part}}$) both for Ni+Ni and Au+Au collisions at 1.5 $A\cdot$GeV. Preliminary results.

$s$ quark is essentially only in $K^{+}$, while the $s$ quark will be shared between $K^{-}$ and $\Lambda$ (or other hyperons). This sharing of the $s$ quark might be in chemical equilibrium as the reactions

$$\pi^{0} + \Lambda \leftrightarrow p + K^{-} \quad \text{or} \quad \pi^{-} + \Lambda \leftrightarrow n + K^{-}$$

are strong and have only slightly negative Q-values of -176 MeV.

The idea that the $K^{-}$ yield is dominated by strangeness exchange via the $\pi^{-} + \Lambda$ channel has been suggested by [22] and has been demonstrated quantitatively in a recent theoretical study [23]. The direct $K^{+}K^{-}$ pair production via baryon-baryon collisions has negligible influence as these $K^{-}$ are absorbed entirely. In these transport-model calculations the strangeness exchange is approaching equilibrium but does not fully reach it [23].

If these reactions are the dominating channel, the law of mass action might be applied giving for the respective concentrations [24]

$$\frac{[\pi] \cdot [\Lambda]}{[K^{-}] \cdot N} = \kappa.$$  

As the number of $K^{-}$ relative to $\Lambda$ is small, $[\Lambda]$ can be approximated by $[K^{+}]$ and rewriting gives

$$\frac{[K^{-}]}{[K^{+}]} \propto M(\pi^{0} + \pi^{-})/A_{\text{part}}.$$  

This relation also explains the measured constant ratio of $K^{-}/K^{+}$ with centrality (Fig. 3) as the pion multiplicity does not vary with centrality. Figure 3 demonstrates the constancy of the $K^{-}/K^{+}$ ratio and of the pion multiplicity with $A_{\text{part}}$ for Ni+Ni and Au+Au collisions at 1.5 $A\cdot$GeV [15, 17, 25]. It turns out that these ratios do not even depend on the choice of the collision system. The right part of this figure exhibits the double ratio ($[K^{-}]/[K^{+}])/((M(\pi^{0} + \pi^{-}))/A_{\text{part}}$) which shows only a minor deviation from a horizontal line. This result can be taken as an argument that this specific channel might be not far from chemical equilibrium.

Next we test in Fig. 4 the validity of the law of mass action by plotting the $K^{-}/K^{+}$ ratio as a function of the pion multiplicity $M(\pi^{0} + \pi^{-})/A_{\text{part}}$ at incident energies from SIS up to RHIC. At SIS and AGS energies the direct relation holds,
Strange Particle Production from SIS to LHC

2.2. Maximum relative strangeness content in heavy ion collisions around 30 A·GeV

The experimental data from heavy ion collisions show that the $K^+/\pi^+$ ratio rises from SIS up to AGS but it is larger for AGS than at the highest CERN-SPS energies\cite{3, 26, 27, 28, 29} and even at RHIC\cite{19} as shown in Fig. 6.

This behavior is of particular interest as it could signal the appearance of new dynamics for strangeness production in high energy collisions. It was even conjectured\cite{30} that this property could indicate an energy threshold for quark-gluon plasma formation in relativistic heavy ion collisions.

In the following we analyze the energy dependence of strange to non-strange particle ratios in the framework of a hadronic statistical model. In the whole energy range, the hadronic yields observed in heavy ion collisions resemble those of a population in chemical equilibrium along a unified freeze-out curve determined by the condition of fixed energy/particle $\simeq 1$ GeV\cite{3} providing a relation between the temperature $T$ and the baryon chemical potential $\mu_B$. As the beam energy increases $T$ rises and $\mu_B$ is slightly reduced. Above AGS energies, $T$ exhibits only a moderate change and converges to its maximal value in the range of 160 to 180 MeV, while $\mu_B$ is strongly decreasing.

Instead of studying the $K^+/\pi^+$ ratio we use the ratios of strange to non-strange particle multiplicities (Wroblewski factor)\cite{31} defined as

$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

where the quantities in angular brackets refer to the number of newly formed quark-antiquark pairs, i.e. it excludes all quarks that were present in the target and the projectile.

**Figure 5.** The $K^-/K^+$ ratio as a function of the pion multiplicity $M(\pi^0 + \pi^-)/A_{part}$ as a test of the law of mass action. Preliminary data. The dashed line shows the prediction of the statistical model.
Applying the statistical model to particle production in heavy ion collisions calls for the use of the canonical ensemble to treat the number of strange particles particularly for data in the energy range from SIS up to AGS [4, 32] as mentioned before. The calculations for Au-Au and Pb-Pb collisions are performed using a canonical correlation volume defined above and given by a radius of \( \sim 7 \text{ fm} \) determined in [4]. The quark content used in the Wroblewski factor is determined at the moment of chemical freeze-out, i.e. from the hadrons and especially, hadronic resonances, before they decay. This ratio is thus not an easily measurable observable unless one can reconstruct all resonances from the final-state particles. The results are shown in Fig. 7 as a function of \( \sqrt{s} \).

The solid line (marked “sum”) in Fig. 7 describes the statistical-model calculations in complete equilibrium along the unified freeze-out curve [3] and with the energy-dependent parameters \( T \) and \( \mu_B \). From Fig. 7 we conclude that around 30 A·GeV laboratory energy the relative strangeness content in heavy ion collisions reaches a clear and well pronounced maximum. The Wroblewski factor decreases towards higher incident energies and reaches a limiting value of about 0.43. For details see Ref. [33].

The appearance of the maximum can be traced to the specific dependence of \( \mu_B \) and \( T \) on the beam energy. Figure 8 shows values of constant \( \lambda_s \) in the \( T - \mu_B \) plane. As expected \( \lambda_s \) rises with increasing \( T \) for fixed \( \mu_B \). Following the chemical freeze-out curve, shown as a full dashed line in Fig. 8, one can see that \( \lambda_s \) rises quickly from SIS to AGS energies, then reaches a maximum around \( \mu_B \approx 500 \text{ MeV} \) and \( T \approx 130 \text{ MeV} \). These freeze-out parameters correspond to 30 GeV laboratory energy. At higher incident energies the increase in \( T \) becomes negligible but \( \mu_B \) keeps on decreasing and as a consequence \( \lambda_s \) also decreases.

The importance of finite baryon density on the behavior of \( \lambda_s \) is demonstrated...
Strange Particle Production from SIS to LHC

Figure 7. Contributions to the Wroblewski factor $\lambda_s$ (for definition see text) from strange baryons, strange mesons, and mesons with hidden strangeness. The sum of all contributions is given by the full line.

Figure 8. Lines of constant Wroblewski factor $\lambda_s$ (for definition see text) in the $T - \mu_B$ plane (solid lines) together with the freeze-out curve (dashed line).

in Fig. 7 showing separately the contributions to $\langle s\bar{s} \rangle$ coming from strange baryons, from strange mesons and from hidden strangeness, i.e., from hadrons like $\phi$ and $\eta$. As can be seen in Fig. 7, the origin of the maximum in the Wroblewski ratio can be traced to the contribution of strange baryons. This channel dominates at low $\sqrt{s}$ and loses importance at high incident energies. Even strange mesons exhibit a broad maximum. This is due to the presence of associated production of e.g. kaons together with hyperons.

The energy dependence of the $K^+/\pi^+$ ratio measured at midrapidity is shown in Fig. 8. The model gives an excellent description of the data, showing a broad maximum at the same energy as the one seen in the Wroblewski factor. In general, of course, statistical-model calculations should be compared with $4\pi$-integrated results since strangeness does not have to be conserved in a limited portion of phase space. A drop in this ratio for $4\pi$ yields has been reported from preliminary results of the
NA49 collaboration at 158 AGeV. This decrease is, however, not reproduced by the statistical model without further modifications, e.g. by introducing an additional parameter $\gamma_s \sim 0.7$. This point might be clearer when data at other beam energies will become available.

3. Summary

Strange particle production in heavy ion collisions close to threshold can be described by a statistical model in canonical formulation, i.e. using local strangeness conservation. This approach is able to explain several features of $K^+$ and $K^-$ production at SIS energies.

While for $K^+$ production it remains open whether and how chemical equilibrium can be reached, the situation for $K^-$ is quite different. It is shown that the strangeness exchange process $\pi A \leftrightarrow N + K^-$ is the dominant channel for $K^-$ production at SIS and likely also at AGS energies. This is demonstrated by applying the corresponding law of mass action. Theoretical studies confirm this interpretation.

Using the energy dependence of the parameters $T$ and $\mu_B$ we have shown that the statistical-model description of relativistic heavy ion collisions predicts that the yields of strange to nonstrange particles reaches a well defined maximum near 30 GeV lab energy. It is demonstrated that this maximum is due to the specific shape of the freeze-out curve in the $T - \mu_B$ plane. In particular, a very steep decrease of the baryon chemical potential with increasing energy causes a corresponding decline of relative strangeness content in systems created in heavy ion collisions above lab energies of 30 GeV. The saturation in $T$, necessary for this result, might be connected to the fact that hadronic temperatures cannot exceed the critical temperature $T_c \approx 170$ MeV for the phase transition to the QGP as found in solutions of QCD on the lattice.

[1] J. Cleymans and H. Satz, Z. Phys. C57 (1993) 135.
[2] P. Braun-Munzinger, J. Stachel, J.P. Wessels and N. Xu, Phys. Lett. B 344 (1995) 43; Phys. Lett. B 365 (1996) 1.
[3] J. Cleymans and K. Redlich, Phys. Rev. Lett. 81 (1998) 5284; Phys. Rev. C60 (1999) 054908.
[4] J. Cleymans, H. Oeschler, K. Redlich, Phys. Rev. C59 (1999) 1663.
[5] J. Cleymans, H. Oeschler, K. Redlich, Phys. Lett. 485 (2000) 27.
[6] F. Becattini, J. Cleymans, A. Keränen, E. Suhonen and K. Redlich, Phys. Rev. C64 (2001) 024901.
[7] P. Braun-Munzinger, I. Heppe, J. Stachel, Phys. Lett. B 465 (1999) 15.
[8] P. Braun-Munzinger, D. Magestro, K. Redlich, J. Stachel, Phys. Lett. B 518 (2001) 41.
[9] M. Mang, Ph.D.thesis, University of Frankfurt, 1997.
[10] R. Hagedorn, CERN Yellow Report 71-12 (1971); E. Shuryak, Phys. Lett. B42 (1972) 357; J. Rafelski, Phys. Lett. B97 (1980) 297; K. Redlich and L. Turko, Z. Phys. C 5, 1980 (201); R. Hagedorn et al., Z. Phys. C27 (1985) 541.
[11] J. S. Hamieh, K. Redlich and A. Tounsi, Phys. Lett. B486 (2000) 61.
[12] W. Cassing et al., Nucl. Phys. A614 (1997) 415.
[13] F. Laue, C. Sturm et al., Phys. Rev. Lett. 82 (1999) 1640.
[14] R. Barth et al., Phys. Rev. Lett. 78 (1997) 4007.
[15] M. Menzel, Ph.D.Thesis, Universität Marburg, 2000.
[16] L. Ahle et al., (E-802 Collaboration), Phys. Rev. C58 (1998) 3523.
[17] A. Förster, Ph.D.Thesis, Technische Universität Darmstadt, in preparation.
[18] J.C. Dunlop and C.A. Ogilvie, nucl-th/9911012.
[19] J. Harris, STAR Collaboration. Talk presented at QM2001, Stony Brook, January 2001, Nucl. Phys. A (2001) (in print).
[20] P. Koch, B. Müller and J. Rafelski, Phys. Rep. 142 (1986) 167.
[21] E.L. Bratkovskaya et al., nucl-th/0001008.
[22] C. M. Ko, Phys. Lett. B138 (1984) 361.
[23] C. Hartnack, H. Oeschler, J. Aichelin, nucl-th/0109010.
[24] H. Oeschler, contribution to “Strangeness 2000”, Berkeley, July 2000; J. Phys. G: Nucl. Part. Phys. 27 (2001) 1.
[25] F. Uhlig, Ph.D. Thesis, Technische Universität Darmstadt, in preparation.
[26] L. Ahle et al., (E802 Collaboration), Phys. Rev. C60 (1999) 044904; L. Ahle et al., E866/E917 Collaboration, Phys. Lett. B476 (2000) 1.
[27] J. C. Dunlop and C. A. Ogilvie, Phys. Rev. C61 (2000) 031901 and references therein; C. A. Ogilvie, Talk presented at QM2001, Stony Brook, January 2001, Nucl. Phys. A (2001) (in print).
[28] Ch. Blume, NA49 Collaboration. Talk presented at QM2001, Stony Brook, January 2001, Nucl. Phys. A (2001) (in print).
[29] I. Bearden, NA44 Collaboration, Phys. Lett. B471 (1999) 6.
[30] M. Gaździcki and M. Gorenstein, Acta Phys. Pol. B30 (1999) 2705; M. Gaździcki and D. Röhrich, Z. Phys. C71 (1996) 55.
[31] A. Wroblewski, Acta Physica Polonica B16 (1985) 379.
[32] C.M. Ko, V. Koch, Z. Lin, K. Redlich, M. Stephanov and X.N. Wang, Phys. Rev. Lett. 86 (2001) 5438.
[33] P. Braun-Munzinger, J. Cleymans, H. Oeschler, K. Redlich, Nucl. Phys. A (in print), hep-ph/0106066.