The perspective prospective professional teachers toward (specific) pedagogical content knowledge on derivative concept

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Abstract. This study is aimed to investigate the pedagogical content knowledge of prospective professional teachers who participated in the In-service Teacher Professional Program (Program Profesi Guru Dalam Jabatan or PPG Daljab) for the topic of Derivative. The investigation into teachers’ knowledge was conducted during the phase of developing lesson plan. For this purpose, 24 in-service teachers were observed during their participation in the PPG program. The data analysis was performed for the interview results, the teachers’ lesson plan (RPP), and observation note from the peer teaching activities. The data analysis reveal that the teachers already possessed a correct conceptual understanding of derivative as indicated from their lesson plan. Nevertheless, the concepts that teachers presented in the lesson plan was still partial and did not represent a comprehensive conceptual understanding. This partial conceptual understanding was also observed during peer teaching activities. When performed a teaching simulation, teachers tended to deliver the concept in a procedural way so that the classroom activities did not fully support the development of students’ higher order thinking skills.

1. Introduction
A professional teacher must have competences to create dynamical classroom activities that support meaningful, joyful, creative, and dialogic learning (UUD Sistem Pendidikan Nasional). To be a professional teacher, every pre-service and in-service teacher must take a Teacher Professional Program (PPG). As mandated by the National Education Act or UU No 20/2003 about the national education system, professional program is a higher education program after bachelor program that is aimed to prepare its graduates for occupations that require specific competences. In-service Teacher Professional Program or PPG Dalam Jabatan is a program that integrates theoretical classes, workshop or training, and fieldwork. The program is designed for teachers to acquire the four teacher competences – i.e. personality, social, professional, and pedagogical competences – comprehensively as required by the national standard for education. Teachers who possess comprehensive teacher competences will be granted with a professional teacher certificate (Sertifikat Pendidik).

The pedagogic content in PPG program concerns on empowering teachers with the basic of educational science and main principles of professional teachers. The professional or subject matter content in PPG program covers not only mathematical concept, but also the strategy to teach the concept and the principle of Technological Pedagogical Content Knowledge (TPACK). The principles of TPACK require professional teachers to possess not only content knowledge and pedagogical...
knowledge and technology, but also the integration of these two or three types of knowledge. Professional teachers are not enough deliver content and pedagogy as separated entities, the continuity of a good learning process requires integrity and a complex balance of the two needs integrity and a complex balance of both [1]. A good teaching and learning process needs the integration and balance between content and pedagogy, known as the term Pedagogical Content Knowledge (PCK).

Many studies discuss the material contained in PCK, detailing the points contained in it. Specific content knowledge (SCK) is one of the material proposed as a component in PCK [2]. SCK is specific mathematics knowledge and skills required for teaching. SCK is a content knowledge required for teaching mathematics as an addition to general knowledge about the content [3]. SCK includes knowledge on determining essential mathematics content, knowledge on providing explanation about general and procedural mathematics, and knowledge on non-routine problems. In addition to the aforementioned component of SCK, one of the knowledge in SCK that one of competences in the scope of is interpreting mathematical results [4]. Possessing SCK enable teachers to: provide appropriate response for students’ ‘why’ questions, select and define proper definition, and adjust or modify the complexity of problems in accordance with students’ ability level.

Specific meaning in SCK does not mean different knowledge or knowledge specifically mastered by the teacher. Specific content knowledge for mathematics teacher is mathematical knowledge and specific skills required for teaching [5]. Mathematics teachers should not only master mathematical material, but understand school mathematics and have the skills to teach mathematics in this case is to facilitate students learning mathematics so that students easily understand it. SCK deals with specific content related to configuration of the relationships between object and nature of knowledge [6]. Consequently, teachers need to systematically identify the content to be taught and pay attention to its nature.

Derivatives are one of the mathematical material in high schools, this material is new material for students in high school. Generally the initial material is considered difficult material, therefore teachers must be able to systematically identify objects associated with derivatives so students can understand easily. To understand derivatives, students are introduced to the understanding of limits. In general the notation for limits is written as \( \lim_{x \to c} f(x) = L \), the notion of limits for high school students is described intuitively, with the statement "if \( x \) approaches \( c \), then \( f(x) \) approaches \( L \).

The topic of Continuation is material that is learned after studying the limit. In this case, it is limited to the continuity of algebraic functions. To recognize continuity in an interval for algebraic function begins with recognizing continuity at one point. The continuity at a point is generally defined as “\( f \) is continuous at point \( c \in (a,b) \) if \( f \) is defined at \( c \) and \( \lim_{x \to c} f(x) = f(c) \)” [7]. This definition requires three main requirements which are mentioned in a long sentence without a clear indication so that it might not be easy for most students to notice and grasp these three requirements. Therefore, it will be easier for most students if the definition is defined as: “function \( f \) is continuous at point \( c \in (a,b) \), if the following three requirements are fulfilled: (i) function \( f \) is defined at \( c \), (ii) \( \lim_{x \to c} f(x) \) exists, and (iii) \( \lim_{x \to c} f(x) = f(c) \). The later definition of continuity clearly indicates the requirements for a function to be continuous. Furthermore, this definition indicates prerequisite or prior knowledge that must be acquired by students before learning continuity.

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2. Method
Twenty-four in-service teachers participating in PPG Dalam Jabatan involved in the present study. These teachers had been teaching for at least five years. Thirteen teachers worked in General Senior High School (SMA) and 11 teachers worked in Vocational Senior High School (SMK). The teachers were interviewed in semi-structured way. They were asked about essential points or aspects for teaching derivative, contexts which were usually used to teach derivative, how they introduced derivative to their students, how they identify and know students’ way of thinking in understanding the concept of limit, continuity, and derivative (see Figure 1 for examples of questions during interview). The interviews were recorded and transcribed for data analysis. In addition to the interview, the present study also conducted content analysis on teachers’ lesson plans. Furthermore, observation during peer teaching activities was also conducted.

| Interview Guide |
|-----------------|
| 1. Experiences in teaching derivative |
| a. Have you ever taught derivative? |
| b. If the first topic is derivative, what will be the next topic? |
| c. What are the important factors you need to devise a plan to teach derivative? |
| d. Explain how you (usually) teach derivative? |
| e. What contexts did you use to introduce the concept of limit? Continuity? Derivative? Why did you consider those contexts are important? |
| 2. The teaching of derivative |
| a. In your opinion, what students should know about derivative? |
| b. What are the main ideas that must be learned by students? |
| c. What skills are important for students to acquire? |
| d. What kinds of concepts or ideas do students need to master before learning derivative? |
| e. In your opinion, how can students understand the concept of limit, continuity, and derivative through classroom activities? |
| f. What are important factors that contribute to students’ conceptual understanding of the derivative concept? |
| g. How can we assess students’ understanding of derivative? |

Figure 1. Interview guide

3. Results and discussion
Data obtained from interviews, content analysis (or lesson plans), and peer teaching observation were summarized and categorized to seek answers for the following issues: prerequisite or prior knowledge required to learn derivative, contexts which were used to teach derivative, and approaches or strategies to teach derivative.

All teachers agreed that there are prerequisites that must be acquired by students before they learn new concept. With respect to the concept of derivative, all 24 teachers mentioned that limit is the prerequisite. Moving backward, the teachers stated that students must understand about function before they learn the concept of limit, with an emphasis on the value of the function, not various functions. Some teachers explain the function by displaying pictures to show that the function is a relationship that has exactly one pair for each point in the domain area, but does not provide further explanation on the graph regarding the nature of the graph, for example the gradient of the graph pointing up, or vice versa, the graph pointing to the bottom, and the possibility of various kinds of function descriptions. Provides explanations of two functions with almost the same graphic images but different function formulas. For example in the rational function and linear function. The graphs presented in Figure 2 and Figure 3 below.
The topic of limits is new material for high school students, so the limits are given intuitively by using the statement "if \( x \) approaches \( c \), then \( f(x) \) approaches \( L \)." The context given at the beginning of the introduction of the term limit is to give the meaning of the word "limit" as an "approach" for a certain value, for example, the percentage of batteries at HP of 5% is said to be a limit or close to being used up. Although this context is not very appropriate to give a description related to the limit, but limited to providing an appropriate equivalent word.

All teachers present an intuitive understanding of limits in the form of changes in values through the table to show changes in the value of \( x \) approaching \( c \) and the value of \( f(x) \) which approaches a value, for example \( L \), but do not give emphasis to the concept of the left-handed limit and the right-handed limit. This is more evident in the selection of functions in the examples and exercises that are functions that are defined in \( R \), so that the problem of the left-handed limit and right-handed limit which have different values becomes difficult to materialize. The emphasis on limit learning in the learning design emphasizes the determination of the limit value at a point not the idea of a limit because of the special case of a function, for example the function \( f(x) = \frac{x^2-9}{x-3} \) at \( x=3 \) or the function presented as in Figure 1 at \( x=-1 \) will be better and vice versa the function as defined in Figure 2 will not provide a complete understanding. The focus of the material on the topic of limits is generally on the properties of the limit that will be used to calculate the limit value. The exercises given are mostly about calculating the limit value using substitution and factorization.

Only a few of the teachers who teach in SMA stated that the definition of limit is given (explained) to students so that students understand that \( x \) approaches a specified point can be from the left or from the right or in other words students knew the existence of left-handed and right-handed limits (\( \text{limit kiri and limit kanan} \)) before they learned the general definition of limit. Teachers usually use graphical approach to introduce the concept of limit. Teachers started from familiar function such as simple quadratic function, absolute value function \( f(x) = |x-1| \), floor function and ceiling function \( g(x) = \lfloor x \rfloor \), and rational function \( h(x) = \frac{2}{(x-1)} \). Unlike the SMA teachers, the teachers who worked in SMK took different approach to teach limit. SMK teachers emphasize on procedural knowledge on determining the value of limit function. The absence of conceptual understanding of the concept of function hindered SMK students in understanding and analysing the continuity of function.

All teachers state that gradients are a prerequisite material for teaching derivatives. However, it does not utilize this prerequisite material to understand the idea of derivatives as tangent gradients on curves. Generally the presentation of derivatives by giving \( f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} \). The derivative presentation will give more meaning to students, if it is preceded by the prerequisite material in the form of a gradient, and the idea of changes in the value of \( y \) and the value of \( x \), so that it is expressed in \( \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \). In the second presentation, the definition of derivative contains the necessary conditions, the existence of a limit value can change the value of a function to
the change in the value of x. Graphically it can be seen from the change in the value of the gradient at a point. Emphasizing the existence of subtle gradient changes on the graph can show a derivative idea at certain intervals, but this has never been done in learning. This can easily be presented by displaying the following two pictures. Figures 4 and Figure 5 show the difference in the existence of derivatives at intervals (-3.3). In Figure 3 tangent gradient before point (0,4) positive value moves towards point (0.4) where the gradient at this point is zero, and afterwards moves negative value, different from the gradient change in the graph in Figure 4 the gradient of the line tangent before point (0.4) is 2, at point (0.4) is 0 and after point (0.4) is -2.

In general, learning done by 24 teachers on the topic of derivation train students to memorize the derivative of function without interpreting the derivative itself. None of the 24 teachers who used functions that had several possibilities of limit values as the basis for analysing the continuity of the functions in the given intervals. Figure 6 below can be used as an alternative example problem in analysing derivatives at specific intervals, with the following function definition.

\[
f(x) = \begin{cases} 
2x + 11 & , -6 < x < -3 \\
|x^2 - 4| & , -3 < x < 3 \\
x^2 - 10x + 26 & , 3 < x < 6 \\
\frac{5x}{2} - \frac{9}{2} & , 6 < x < 10 
\end{cases}
\]

Function \( f(x) \) is continuous in the interval (-6,10) except at points (-3,6), (-2,0), (2,0), (3,6) and (6,3). It is expected that students could see that the function is discontinuous at points (-3,6), (-2,0), (2,0), (3,6) because limit does not exist at those points and the function is discontinuous at (6,3) because the value of limit is not equal to the value of function. Therefore the function is differentiable at certain intervals.
Measuring the success of student learning outcomes that have been developed by 24 teachers, actually in accordance with the design on which components and what skills are emphasized. In the draft prepared for the limit, because of the emphasis on the procedure for determining the limit, so the questions designed are problems related to the ability of substitution and simplification (factor) to determine the limit value. Similarly, for continuity and derivatives that are more emphasized on the procedure in algebra only, so that the measurement has not revealed the ability to think of high-level students.

4. Conclusions
The prospective professional teachers had different perspectives and approaches of teaching derivative. The different approaches of teaching are caused by teachers’ different conception and different emphasis about the main essential concept. All teachers have provided students with prerequisite knowledge, but they did not use familiar and proper contexts to introduce the concept of derivative. With regard to approaches for teaching derivative, SMA teachers used graphical approach because they emphasized on the conceptual understanding of derivative including gradient, rate of change, border, and also symbolical understanding. Although the teachers have delivered the concept in the correct order, but connection between concept was not yet emphasized. Therefore, the teachers still need more pedagogical knowledge so they can use a variety of approaches and also have a better competence in making connection between concepts. In this way, students do not merely learn limit and derivative from procedural perspective and, consequently, teachers can provide more opportunities for students to develop their higher order thinking skills.

References
[1] Shulman L S 1986 *Edu. Res.* **15** 4
[2] Hill H C, Ball D L, and Schilling S G 2008 *J. Res. Tea. Edu.* **9** 372
[3] Markworth K, Goodwin T, Glisson K 2009. *AMTE Monograph* 6 67
[4] Suzuka K, Sleep L, Ball D L, Bass H, Lewis J M, and Tames M H 2009 *AMTE Monograph* 6 7
[5] Ball D L, Thames M H, Phelps G 2008 *J. Tea. Edu.* **59** 389
[6] Godino J D, Ortiz J J, Roa R, Wilhelmi M R 2011 *Models for statistical pedagogical knowledge teaching statistics in school-mathematics challenges for teaching and teacher education* (New York: Springer)
[8] Purcell E and Varberg D 2019 *Kalkulus dan geometri analitis* (Bandung: Erlangga)