Chaotic inflation with curvaton induced running

Martin S. Sloth

CP³-Origins, Center for Cosmology and Particle Physics Phenomenology
University of Southern Denmark, Campusvej 55, 5230 Odense M, Denmark

Abstract

The apparent tension between the the recent BICEP2 data and the Planck data might be removed by allowing for a large running in the spectral index as suggested by the BICEP2 collaboration, but in disagreement with prediction of the simplest model of chaotic inflation. The large field chaotic model is sensitive to UV physics, and the non-trivial running of the spectral index hinted by the BICEP2 data could therefore be telling us some additional new information about the UV completion of inflation. However, before we can draw such strong conclusions with confidence, we might first have to carefully exclude the alternatives. Assuming monomial chaotic inflation is the right theory of inflation, we therefore explore the possibility that the running could be due to some other less UV sensitive degree of freedom. As an example, we ask if it is possible that the curvature perturbation spectrum has a contribution from a curvaton, which makes up for the large running in the spectrum. We find that this effect could mask the information we can extract about the UV physics.
1 Introduction

The recent data from the BICEP2 experiment has been interpreted by the BICEP2 collaboration as the discovery of primordial tensor modes, with a tensor-to-scalar ratio, $r$, measured to be $r = 0.2^{+0.07}_{-0.05}$ \cite{1}. While such a large value for the tensor-to-scalar ratio would fit well the predictions of chaotic inflation with a quadratic potential \cite{2}, this model is in some tension with the Planck data \cite{3}.

The source of the tension of quadratic chaotic inflation model with the Planck data is the prediction of a large value of $r \approx 0.15$ together with a small value of the running of the spectral index $\alpha$. If the assumption of a small running of the spectral index is relaxed, the value of $r$ found by BICEP2 is not tension with Planck, and assuming $r = 0.2$, as found by BICEP2, the Planck data appear to imply $\alpha = -0.02$. A running of the spectral index of this size was already favored by the WMAP7+SPT data \cite{4}.

While the measured value of $r$ might become lower with better foreground subtraction or better understanding of other systematical issues, it is interesting meanwhile to ask the question, what would it require from a theoretical point of view to bring BICEP2 and Planck in better agreement? Naively, the most straightforward approach would be to modify inflationary sector itself by allowing for a more non-trivial inflation potential \cite{5} or perhaps a modification of the initial state \cite{6,7}. If this is the correct interpretation of the data, it would be striking because in order to achieve the large tensor-to-scalar ratio in the first place, we would need a large field model of inflation, which is sensitive to UV physics. Therefore, any modification of the inflationary sector would carry information about the UV completion of inflation. However, assuming that the source of the tension between BICEP2 and Planck is not an issue of estimating systematic errors in either of experiments but actually contains physical information, one might want to exclude other possible alternatives before drawing the strong conclusion that this is new additional information about the UV completion of inflation. Therefore we will follow a simple logic. We will assume that the data is telling us that the minimal monomial chaotic inflation is the right theory of inflation, and the running is coming from some other light and less UV sensitive degree of freedom. As a
model, we will as a starting point consider the case where the curvature perturbation spectrum has a contribution from a curvaton [8–10], which makes up for the large running in the spectrum.

Models of mixed inflaton-curvaton scenarios has been considered before in the literature (see f.ex. [11–13]). However in [11] it was assumed that both the inflaton and the curvaton has a simple power law spectrum. This is a good approximation when there is no running in neither the inflaton or the curvaton spectrum, but will always lead to a positive running in the final spectrum $\alpha > 0$. Below we will therefore generalize the setup of [11] to the case where there is a large intrinsic negative running in the curvaton spectrum.

Curvaton models with a large negative running has been discussed before in [14,15] as a possible way of accommodating the claimed running of $\alpha = -0.024$ in the WMAP7+SPT data. But while a curvaton model in its pure form would lead to a vanishing tensor-to-scalar ratio and therefore be ruled out by BICEP2, one might speculate that it could be a component contributing to the total curvature perturbation, which however is responsible for all of the large running implied by the new data, as we will discuss below.

2 The model

We will assume that inflationary part is given by a model of chaotic inflation with the potential [2]

$$V(\phi) = \lambda m_{pl}^4 \left( \frac{\phi}{m_{pl}} \right)^n.$$ (2.1)

Large field models of inflation, like the model above, are protected by an approximate shift symmetry, however it is a challenge to build UV complete models that leads to this softly broken shift symmetry, since quantum gravity does not appear to respect continuous global symmetries (for a detailed review of these arguments see [16] and see [17,18] for some recent related comments). One can consider models relying on an axion where the effect of the symmetry breaking is attempted to be kept under control by the periodicity of the axion potential. However, it does not appear that a super-Planckian axion decay constant, as would be required in a large field model of inflation,
is consistent with string theory [19]. Some model building efforts has been diverted to resolve this problem, and at present monodromy type models seems to be a promising direction [20]. It has been argued that quadratic chaotic inflation can be naturally realized from a low energy effective point of view in these type of setups [16, 21]. In addition to these ideas, it has been argued that within a pure supergravity framework, that essentially all models of chaotic inflation with \( n > 0 \) can be constructed [23–25], and models with \( n > 2 \) motivated by a superconformal approach to supergravity was put forward in [23–25].

The BICEP2 result for the measured value of \( r \) fits with the simple quadratic model of chaotic inflation, but a large running was suggested to obtain better agreement with Planck. While it may be that the corrections to the quadratic potential could become important, and carry non-trivial information about the UV completion, it may be important to check if other physics, separate from the UV sensitive large field inflaton potential, can affect such conclusions.

This motivates our study of whether it is possible that the large running of the spectral index can be induced by a curvaton with sub-Planckian field values. In the axiverse string landscape [26], one may generically expect many axion type particles around, and it may not be completely unexpected if an axion type particle with sub-Planckian decay constant could behave as a curvaton component.

We therefore consider the case where the scalar curvature perturbation spectrum is formed as a linear combination of two uncorrelated contributions from the the inflaton and the curvaton respectively. This can be parametrized as [11]

\[
P(k) = A_s \left[ (1 - f) \left( \frac{k}{k_*} \right)^{(n_{inf} - 1)} + f \left( \frac{k}{k_*} \right)^{(n_{\sigma(k)} - 1)} \right],
\]

where we have allowed for an intrinsic running in the curvaton spectrum, by making \( n_{\sigma} \) scale dependent. As mentioned above, it was shown in [11], that if we do not allow for an intrinsic running of either the inflaton or the curvaton spectrum, the induced running of the total spectrum is always positive. Here will relax the assumption of ignoring the intrinsic running in the curvaton, and consider the possibility that an intrinsic running of the curvaton can induce a negative running in the final spectrum.
If the fraction of the curvaton component is small $f << 1$, the prediction of the tensor-scalar ratio from chaotic inflation remains almost intact, and would have

$$r = (1 - f)16\epsilon$$

(2.3)

where $\epsilon = n/(4N_*)$ and $N_* \approx 55$ is e-foldings left of inflation when the observable modes exits the horizon.

The spectral index of the total curvature perturbation is given by

$$\frac{d\ln P(k)}{d\ln k} = n_s - 1$$

(2.4)

and the running is defined as

$$\frac{dn_s(k)}{d\ln k} = \alpha.$$  

(2.5)

For convenience we will also define the intrinsic running of the curvaton as

$$\frac{dn_\sigma(k)}{d\ln k} = \alpha_\sigma,$$  

(2.6)

while we will ignore the intrinsic running of the inflaton.

By Taylor expanding (2.2) above around some pivot scale $k_*$, we obtain

$$n_s - 1 = n_{inf} - 1 + f(n_\sigma - n_{inf}).$$

(2.7)

On the other hand, assuming $(n_{inf} - 1)^2 \sim (n_\sigma - 1)^2 << \alpha_\sigma$, we find for the running

$$\alpha \approx f\alpha_\sigma$$

(2.8)

Thus with $f \approx 0.4$, then on the scale $k_*$ with $\alpha_\sigma \sim -0.02$, one would find $\alpha \approx -0.01$ and for chaotic inflation with the quartic potential $V(\phi) = \lambda\phi^4$ one would obtain $r \approx 0.2$. Thus these
parameters naively seems to be in simultaneous agreement with both BICEP and Planck.

The amplitude of the curvaton perturbation is

$$P_\zeta^c = r_{dec}^2 \frac{1}{9\pi^2} \frac{H_*^2}{\sigma_*^2}$$

(2.9)

where $\ast$ denotes the time at which CMB scales left the horizon, and $r_{dec}$ is the curvaton fraction of the total energy density at time of decay. Thus in order for the curvaton perturbation to be a fraction $f$ of the total curvature perturbation as measured by Planck \[3\]

$$P_\zeta \approx 2.2 \times 10^{-9}$$

(2.10)

with $H_* = 1.1 \times 10^{14}$ GeV fixed by requiring $r = 0.2$ as measured by BICEP2, we need

$$\sigma_* \approx 0.02 \frac{r_{dec}}{f^{1/2}} m_{pl}.$$

(2.11)

Thus, if we take $r_{dec} \approx 0.1$, then with $0.1 \lesssim f \lesssim 1$, the curvaton field value stays two orders of magnitude below the Planck mass.

Assuming the curvaton is light, one may still worry how the curvaton gets displaced so high up in its potential initially compared with the Hubble scale. However, we note that the initial field value of the curvaton roughly corresponds to the GUT scale around $10^{16}$ GeV. Thus, it is natural to believe that a phase transition around the GUT scale might have initially displaced the curvaton in its potential. Since the observable part of inflation also happens near the GUT scale, the curvaton might have been displaced just before the last 55 e-folds of inflation, which could explain the relatively high value for the initial curvaton field.

Since the curvaton component is non-Gaussian, there is also a possibility that the total curvature perturbation will have a non-Gaussian component. The non-Gaussianity can be estimated by writing

$$f_{NL} \simeq \frac{\langle \zeta^3 \rangle}{\langle \zeta_{inf}^2 \rangle^2} = \frac{\langle \zeta^2 \rangle^2}{\langle \zeta_{inf}^2 \rangle^2} \frac{\langle \zeta^3 \rangle}{\langle \zeta_{inf}^2 \rangle^2}$$

(2.12)
where it is well known that for $r_{\text{dec}} << 1$ \[27\]

\[
\frac{\langle \zeta^2 \rangle}{\langle \zeta^2 \rangle^2} = \frac{5}{4r_{\text{dec}}} ,
\]

(2.13)

and one will obtain

\[
f_{\text{NL}} = \frac{5}{4} \frac{f^2}{r_{\text{dec}}} .
\]

(2.14)

Thus, with the numerical examples $0.5 < f < 0.1$ and $r_{\text{dec}} \approx 0.1$, we obtain $0.1 < f_{\text{NL}} < 2$ consistent with current upper bounds.

As a consequence of the above observations, it is interesting to consider if there are curvaton models, which self-consistently can lead to a large intrinsic negative running. In the light of the large negative running of the spectral index previously indicated by the SPT data \[4\], models of this type has already been explored to some extend in the literature \[14,15\]. Here we will for simplicity just discuss one of these models.

In \[14\] a modulation of the curvaton potential on the following form was considered

\[
V(\sigma) = V_0(\sigma) + \delta V(\sigma)
\]

(2.15)

where the modulation of the curvaton potential is given by

\[
\delta V(\sigma) = \Lambda(1 - \cos(\sigma/f_\sigma)) .
\]

(2.16)

Since the field value of the curvaton, $\sigma_*$, is at least two orders of magnitude below the Planck scale, even $V_0(\sigma)$ could be taken to have the periodic form $V_0(\sigma) = \Lambda_0(1 - \cos(\sigma/\tilde{f}_\sigma))$ with a sub-Planckian decay constant $f_\sigma << \tilde{f}_\sigma << m_{\text{pl}}$. As discussed in \[14\], such a potential could be generated by an axion\textsuperscript{1} coupling to two different gauge fields, or by a complex scalar field with an approximate global $U(1)$ symmetry that gets spontaneously broken in the context of SUSY with discrete $R$ symmetry. For generality we will here keep the potential $V_0$ unspecified, and using the

\textsuperscript{1}Some earlier examples where the string axion was considered as a possible curvaton candidate are \[8,28\].
equation of motion for the curvaton

\[ 3H\dot{\sigma} \simeq -V_0'(\sigma) \]  

it was shown that one can express the axion decay constant, \( f_{\sigma} \), in terms of number of e-foldings, \( \Delta N \), giving one period of the modulation induced by \( \delta V(\sigma) \), and \( V_0'(\sigma_*) \),

\[ f_{\sigma} = \frac{1}{2\pi} \frac{V_0'(\sigma_*)}{3H^2_*} \Delta N . \]  

We note that for a small curvaton mass \( m_{\sigma}^2 \equiv V_0'' << H_*^2 \) and using \( V_0'(\sigma_*) \sim V_0''(\sigma_*)\sigma_* = m_{\sigma}^2\sigma_* \), the axion decay constant is indeed small \( f_{\sigma} << m_{pl} \).

Using from [14]

\[ \alpha_{\sigma} \sim \frac{2\pi}{\Delta N} (n_{\sigma} - 1) \]  

where \( \Delta N/2 > 8 \) is needed to maintain a relatively constant running over the CMB scales, one obtains by taking \( n_{\sigma} - 1 \simeq 0.05 \) and \( \Delta N \approx 16 \)

\[ \alpha_{\sigma} \approx -0.02 . \]  

Since we have \( \alpha = f\alpha_{\sigma} \), then in order to have a large \( \alpha \), we do not want to choose too small a curvaton fraction. A large curvaton fraction however means decreased value of \( r \), so to keep \( r \) large enough to match BICEP, we might want to take as an example a quartic potential, \( n = 4 \), with a 50% curvaton mix, \( f = 0.5 \), which gives

\[ \alpha \approx -0.01 , \quad r = 0.15 \]  

while a slightly smaller curvaton mix would give slightly larger \( r \), but also slightly less running. For instance a 40% curvaton mix, \( f = 0.4 \) would give \( \alpha \approx -0.008 \) but \( r = 0.2 \). Likewise running can be increased by lowering \( r \) further from \( r = 0.15 \).
3 Conclusion

Since large field models are sensitive to UV physics, the non-trivial running of the spectral index hinted by the BICEP2 data could be telling us something new about the UV completion of inflation. However, before drawing such strong conclusions, we explored the possibility that the curvature perturbation spectrum has a contribution from a curvaton, which makes up for the large running in the spectrum. This could relax the tension of chaotic inflation with the Planck data while still be in accordance with the BICEP2 result, and it could be accommodated by a new degree of freedom, which is less sensitive to the UV physics compared to accommodating the running by a direct modification of the inflaton potential. It turns out that the possibility of a curvaton component might actually mask the information we can extract about UV physics by allowing $\lambda \phi^4$ in addition to $m^2 \phi^2$ as a monomial chaotic inflation model consistent with the data. On the other hand a non-Gaussian component consistent with observations, but with $f_{NL}$ of order one, could be a feature of such models. It would be interesting to explore this model in more details in order to obtain more precise constraints on this scenario, or if possible excluding it as a step towards purifying our knowledge about inflation.

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