Reflective thinking process of students in completing mathematical problems based on mathematical reasoning ability

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Abstract. This research aimed to describe the reflective thinking process of eighth-grade students of junior high schools in solving mathematical problems of circle material based on mathematical reasoning. It was a qualitative descriptive with a case study approach. There were 6 subjects selected by using purposive sampling technique based on mathematical reasoning levels. The methods used were test, interviews, and documentation techniques. The results showed that students with high mathematical reasoning level were able to fulfill all indicators of the reflective thinking process including identifying facts and questions, explaining the operations to be chosen, carrying out plans, and giving logical conclusions with indicators students wrote the correct final answers based on the order of the previous problem solving process; students with medium mathematical reasoning fulfilled two to three indicators of identifying facts and questions, explaining the operations to be chosen, carrying out a problem-solving plan; students with low mathematical reasoning fulfilled an indicator of identifying facts and questions; students tend to experience errors in the calculation aspects, lack of students' prior knowledge, understanding, and mathematical attitude. It was concluded that there is a connection between problem solving and mathematical reasoning with reflective thinking ability of students. It have implications for teachers to condition mathematics learning by paying attention to students' thinking skills.

1. Introduction

The rapid development of science and technology in the era of globalization is now very influential on all dimensions of human life, both in family life, social, economic, cultural and country because the progress of a country is seen from the process and educational products. Education is a human effort to foster his personality in accordance with the values in society and culture [1]. Therefore education plays a very important role in preparing human resources for life in the future. Education helps humans in developing their potential, among others through the learning process in schools, both elementary schools, junior high schools, senior high schools, and vocational high schools, as well as universities, each of which has a specific vision, mission and goals. In general, there is a content of
competency given to students in formal education institutions to develop student potential. One of these competencies is knowledge, attitudes, and skills in mathematics.

Mathematics is studied not only to provide the experience of memorizing formulas, but students are taught to be able to use mathematics as an alternative to solving problems. Problem solving is a process that every time becomes a mental activity of humans. In learning mathematics, mathematical problems are often interpreted as a question or complex problem that requires a solution or answer, where what is meant by a question is a problem that requires a solution or answer that meets two conditions, among others: the question can be understood by students and students can solve it or the question cannot be answered by students [2]. Polya (2004) suggests four steps in solving problems, namely: understanding the problem, planning problem solving, carrying out the plan, and checking the answers [3].

As written in the online daily edukasi.kompas.com [4] that most Indonesian students have low-level thinking skills. This was shown through a number of education surveys, one of which is the Program for International Student Assessment (PISA) where Indonesia ranks 72 out of 78 countries with a score of 379 based on an assessment of mathematical ability, or this has decreased from 2015 (386). This fact demands a learning strategy that can improve higher order thinking skills. It include critical, logical, reflective, metacognitive, and creative thinking. They are activated when individuals encounter unfamiliar problems, uncertainties, questions, or dilemmas [5]. In other words dimensions of learning support insight in both pattern recognition and reasoning [6].

Reasoning is an activity that involves the thought process carried out by each individual in the process of building and comparing ideas to solve problems. As the previous results it shown that one component of the culture of problem solving—the ability to use the existing knowledge—strongly correlates with three dimensions of the scientific reasoning structure: proportional reasoning, control of variables and probability reasoning [7].

Whereas mathematical reasoning is the ability to understand mathematical ideas more deeply, observe, explore implied ideas, arrange analogies and generalizations, reason logically. Reasoning and mathematics are two inseparable things. It is because mathematical material is understood through reasoning and reasoning is understood and trained through mathematical material. That is, mathematical reasoning ability is very important and is needed by students in learning mathematics. Mathematical reasoning is the ability to analyze, generalize, synthesize/integrate, and provide the right reasons in solving non-routine problems.

Mathematical reasoning involves creativity, discovery. Mathematical reasoning is a key element of mathematics and thus is central to learning mathematics in school [8]. Referring to this opinion, this thought process is a series of mental activities so that it can produce something new through the process of making decisions and solving problems. This means that students who have high reasoning will demonstrate the ability to think logically, critically, and creatively. This is like when students solve math problems well starting from the process of understanding the problems presented and being able to solve problems through a high-level thinking process. Creativity, basic knowledge, creative critical thinking skills, and communication skills become capital for students so they can succeed in achieving learning goals. There are several aspects of mathematical reasoning, as conveyed by Sumarmo (2010) seen in Table 1 below [9].

| No | Aspects                          | Indicators                                      |
|----|----------------------------------|------------------------------------------------|
| 1  | Provide explanations using models, facts and relationships | ✓ Able to analyze mathematical situations based on certain patterns / elements. |
| 2  | Estimating the answer or solution process | ✓ Able to estimate the answer and the solution process of the problem. |
| 3  | Giving a proof                   | ✓ Able to provide evidence of these elements directly or indirectly |
Able to solve problems and provide logical conclusions.

Students can generate ideas or ideas, form an understanding, opinion, and draw conclusions that make sense when students are able to sort, consider, process, store information and data according to their needs. That is, students are expected to think that responds to problems by using information or data that comes from within (internal), can explain what has been done, improve errors found in solving problems, and communicate ideas or symbols instead of images or objects directly. This thought process is hereinafter referred to as reflective thinking.

Reflective thinking is a process of directed and precise activities in which individuals analyze, evaluate, gain deep meaning, and use appropriate learning strategies [10]. Reflective thinking is very important for teachers and students in the mathematics learning process which involves a lot of problem solving processes. There are four characteristics of reflective thinking, namely: reflection as retrospective analysis, reflection as problem solving, critical reflection of self, and reflection on beliefs about self and self-efficacy [11]. It was further explained that the retrospective analysis included the ability to assess oneself by combining experiences before reflection and how these experiences affect what is being done now. Reflection is considered as determining the steps needed to analyze the problem before action is taken. Critical reflection can be considered as a process of analysis, reconsidering and questioning experience in a broad context of issues such as ethical practice, theory, and the use of technology. This process needs confidence in one's own abilities or self-efficacy so that one's reflection process results in the right decision making.

Reflective thought process in solving problems involves brain activity to reflect back on every step of problem solving that has been implemented. Individuals look for what was done at the time of solving the problem and fix it again. For example, when students are given a mathematical problem in the form of a math problem, then the student looks for the answer, solves it then reflects on it or remembers whether the solution is correct, incorrect, or incorrect. This shows that reflective thinking can minimize mistakes made by an individual in solving problems.

Reflective thought processes in solving mathematical problems in this study were analyzed based on the characteristics that exist [3,7], seen in Table 2 below:

| Problem solving stage | Description of reflective thinking |
|-----------------------|-----------------------------------|
| understanding the problem | To explain identification of facts that have been carried out. (To explain how to connect the identification of facts, identification of questions, and the adequacy of the data with the information held.) |
| devising a plan | To explain how to organize and present data. To explain what operations will be selected. To explain how to solve problems that will be done. |
| carrying out the plan | To solve problems according to plans made previously. To explain the problem solving that has been done. |
| looking back | To explain whether the results obtained already answer the question. To explain whether the results obtained make sense. |

Based on the brief explanation above, there is a connection between students' mathematical reasoning in activists' reflective thinking of students when solving mathematical problems. If associated with gender differences, there was no significant difference of mathematical reasoning ability between male and female students after using a problem-based learning approach in their learning [12]. Previous researchers, reveals that a large percentage of the “reflections” were non-reflective rather than reflective.
actions. Reflective thinking is not spontaneous, but should be deliberately stimulated by the educational context [13].

If it is associated with differences in mathematical reasoning, students who have high mathematical reasoning can optimize their reflective thinking processes better in completing assignments and work. That is, students have the ability to learn to complete the task and these conditions can foster confidence and positive actions to achieve learning objectives optimally. Based on these descriptions, the purpose of this study is to determine the students’ reflective thinking processes in solving mathematical problems with high, medium, and low mathematical reasoning abilities as well as any factors that inhibit students’ reflective thinking processes in solving mathematical problems.

2. Method

It is qualitative research. The social situation under study is the reflective thought process of students in solving circle material problems based on the level of mathematical reasoning. The subjects were eighth grade students of SMP Negeri 2 Tulakan in Pacitan regency. The subject selection technique (data source / informant) in this study was purposive sampling. It is a technique for determining data source samples with a variety of specific considerations [14]. The selection of research subjects was based on consideration with the assumption that eighth grade has sufficient knowledge and learning experience so that it was expected to be able to solve mathematical problems and will be more easily interviewed to obtain more accurate data to support this research. The research subjects also who were considered capable of supporting researchers in achieving research objectives. The consideration of the subject sampling was based on a mathematical reasoning test as well as a written test for solving circle material problems. In addition, researchers also considered the advice of mathematics subjects and teachers to get research subjects.

Based on the research objectives, the data collection techniques used were the method of documentation, tests, and interviews. The problem solving test consists of 4 item description matter circle. Previously the content validation test was conducted by the validator without any reliability test. The test method in this study was used to describe students’ reflective thinking processes in solving circle problems and students’ mathematical reasoning. The interview technique was carried out after the researcher has tested the students. Interviews were conducted with students selected as subjects in this study to determine students’ reflective thinking processes in solving mathematical problems based on the mathematical reasoning. The steps in preparing this test include: arranging the material that will be used to make the test based on the purpose of the study, making the test grid, compiling the test, compiling the guideline for the screening. Before being used, the test instrument was validated with the consideration concerned that he had a position and profession in his field. This instrument was validated beforehand by expert judgment by referring to the validation sheet which contains a number of questions related to the content, sentence construction and suitability of the language used.

The flow of the first auxiliary instrument is illustrated in Figure 1 as follows:

![Figure 1. Flow chart of test instrument preparation](image)
The researcher used the validity of the data using triangulation techniques and sources. Triangulation technique, which compares the results of data obtained through the test method and the results of data obtained through the interview method. While triangulation of sources, namely checking data that has been obtained from various sources. Activities in data analysis include data reduction, data display, and conclusion drawing/verification. Data reduction for each instrument is done separately. For mathematical reasoning begins by providing a validated test, after obtaining the data then choose valid data and then categorize into high, medium and low groups.

Table 3. Results of grouping mathematical reasoning writing tests

| Mathematical Reasoning of Students | Total | Overall percentage |
|-----------------------------------|-------|--------------------|
| High                              | 4     | 17.39%             |
| Medium                            | 12    | 26.09%             |
| Low                               | 7     | 30.43%             |
| Total                             | 23    | 100%               |

Furthermore, each group of mathematical reasoning is given a problem solving test for circle material to analyze the reflective thinking process. The results of student work are corrected, then valid data are selected that show complete information according to the research objectives in each group of mathematical reasoning. The written test results of solving mathematical problems to find out the reflective thinking process can be seen in the following Table 4:

Table 4. Data on problem solving test results

| No | Category | Code of Student | Number test 1 | Number test 2 | Number test 3 | Number test 4 |
|----|----------|-----------------|---------------|---------------|---------------|---------------|
| 1  | Medium   | KAS             | R             | WR            | R             | W             |
|    |          | DAB             | WR            | R             | W             | R             |
|    |          | PSM             | WR            | R             | W             | R             |
|    |          | ESM             | R             | WR            | W             | W             |
| 2  | Low      | AAW             | R             | WR            | W             | W             |
|    |          | DS              | W             | W             | R             | WR            |
| 3  | High     | WNA             | R             | R             | WR            | R             |
|    |          | Y               | R             | WR            | R             | WR            |

Notes: Wrong =; R = Right; WR = 50% right

Based on the results of problem solving, 6 students were taken to be interviewed according to the research objectives, namely KAS and PSM subjects in the category of moderate mathematical reasoning, AAW and DS with low mathematical reasoning categories low, and subjects WNA and Y in high mathematical reasoning category. The initial stage after the interview transcript is obtained, then choose the right footage based on information extracted about the reflective thinking process. After all instruments have been reduced, the data is presented.

3. Result and Discussion
Reflective thought process analysis is performed on students with high, medium and low mathematical reasoning. In this section 1 respondent will be described in each group of mathematical reasoning as a sample, namely WNA, PSM, and AAW.

3.1. Result
A respondent with high reasoning, WNA, he was asked to solve a question number 1 as follow.
Problem number 1: Irfan rides a bicycle to school. The length of the spokes of the bicycle is 35 cm. a) Determine the length of the path Irfan traveled when the wheel of the bicycle rotated 2,500 times. b) how many times did the wheel rotate if the length of the track was 8.80 km?

Figure 2. The results of solving students' problems with high mathematical reasoning

Based on the WNA subject's answer from question number 1 above (Figure 2) compared with the interview, WNA has understood the problem by writing down what was known (a) and asked from the questions asked (b). WNA subject has already carried out the problem-solving step by searching around the circle first to calculate the patchwork base needed (c). WNA subject has explained whether the results obtained have answered the question. It was seen that the subject of WNA wrote the conclusions at the end of the answer to a question correctly (d). WNA was confident that his steps were correct without any streaks on a paper test. It showed that WNA did reflective process thinking.

Furthermore, PSM subjects who have moderate mathematical reasoning, were given problem number 2.

Problem number 2:
Irfan rides a bicycle to school. The length of the spokes of the bicycle is 35 cm. a) Determine the length of the path Irfan traveled when the wheel of the bicycle rotated 2,500 times. b) how many times did the wheel rotate if the length of the track was 8.80 km?

Figure 3. Results of solving students' problems with moderate reasoning

Based on the results of the written test and the interview results can be obtained the results of the analysis that: PSM subjects were able to write what information was obtained from questions such as
writing what was known and asked of the questions, but PSM subjects did not write anything that it was known in full. The PSM subject planned a resolution step that will be used to solve the problem. The PSM subject worked on question number 2 with the predetermined settlement plan. The PSM subject has implemented a settlement plan along with the steps to solve the problem but the answer was still not quite right. The PSM subject did not conclude at the end of the answer to question number 2.

Next, the AAW subject which has low mathematical reasoning, he was given problem number 2. Figure 4. The results of problem solving students with low reasoning

Based on the results of the written test above (Figure 4) confirmed by interviews, an analysis can be obtained that the AAW subject was able to write down what information was obtained from the question such as writing what was known and asked from the question (a,b). The AAW subject planned a resolution step that will be used to solve the problem. AAW subjects worked on problems with a predetermined settlement plan (c). The AAW subject did not re-examine the answer and did not write a conclusion at the end of the answer. It was different result with WNA.

3.2. Discussion

Based on the results of identification in groups of students with high mathematical reasoning (WNA) generally showed a reflective thinking process at all stages of problem solving [3]. WNA subjects in question number 1 showed all indicators, number 2 showed 3 indicators, questions number 3 and 4 showed all indicators. This means that the WNA subject is able to show all the indicators of the reflective thinking process in problem solving according to the description in Table 2. These results are different from students with medium mathematical reasoning. The PSM subject in completing the math problem solving test questions only showed 2 to 3 indicators. Furthermore, AAW had low category mathematical reasoning that only showed 1 to 2 indicators on problem solving. The results of this study are in line with the results of previous studies that students with high mathematical reasoning could master the three indicators well. Students with moderate mathematical reasonable to master the mathematical reasoning 2 indicators well. Subjects with low mathematical reasoning abilities could only master one indicator well and complete the question by using Polya step which was understanding the planning and completing well but only the fourth stage about rechecking was still lacking. It was proved from the test sheets and interviews conducted, that some of the students could not explain their answers [15].

Students with high mathematical reasoning abilities tend to have no obstacles. When working on problem solving tests the subject WNA did not experience obstacles. While subject Y (data not described) experienced obstacles forgetting the formula when working on problem number 1 because he did not have a readiness to learn. Based on the description above, it is found that the readiness of learning affects students achievement [16]. While students with moderate mathematical reasoning, obstacles experienced by students was forgetting to use the appropriate formula, difficulties and errors
in the calculation aspects. Furthermore, students who have low mathematical reasoning experience obstacles in using formulas, cannot work on problems, assume that mathematical problems are quite difficult.

Based on the description above, it can be concluded that each student has certain characteristics in carrying out the reflective thinking process in solving a problem based on cognitive abilities possessed. In the initial stages, students need to understand the situation at hand and then think and determine whether the situation is a problem or not. Furthermore, with the ability to solve problems these problems can be overcome or resolved [17]. Each student also has different barriers to reflective thinking activities in solving mathematical problems, one of which is because they have different levels of mathematical reasoning. The way to overcome the difficulty factors in solving mathematics's problems is to provide training intensity, reinforce mathematics concepts of and improve students' mathematical communication [18].

4. Conclusion
Based on the results of data analysis, grade VIII students with high mathematical reasoning use all the reflective thinking processes in the four steps of problem solving (Polya) with the characteristics, the first stage of understanding the problem: identifying questions and facts, linking the identification of these facts with the information studied, reflecting on its own performance continuously; the two stages of planning problem solving: explaining how to organize and present data, explaining what operations to choose, explaining how problem solving is carried out; the three stages of implementing the problem solving plan, solving the problem according to the plan previously made, realizing mistakes when carrying out the problem solving and fixing it; the fourth stage checks back, explains the results already obtained, answers questions, and can draw conclusions. Grade VIII students with medium mathematical reasoning are using 3-4 steps in the reflective thinking process based on Polya's steps. Grade VIII students with low mathematical reasoning only use reflective thought processes at 1-2 steps of problem solving, that is understanding problems with characteristics: actively identifying facts, questions, and linking facts and questions with the information obtained. Students with high mathematical reasoning abilities tend to have no obstacles. While students with moderate and low mathematical reasoning tend to have obstacles in the form of forgetting formulas, errors in calculations, and not being able to solve problems because they do not understand the questions and the steps to solve them.

The results of this study have implications for teachers to condition mathematics learning by paying attention to student characteristics and providing positive support whatever the results obtained by students. According to the results of data analysis, it was concluded that there is a connection between problem solving and mathematical reasoning with reflective thinking ability. The factors examined in this study have limitations on aspects of qualitative data description, subject and object of study. The relationship between these factors can be further analyzed with a variety of research approaches by future researchers in order to obtain a broader contribution of research results.

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