Stepwise Chirping of Chiral Sculptured Thin Films
for Bragg Bandwidth Enhancement

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ABSTRACT: A stepwise chirping of the periodicity of a chiral sculptured thin film is shown to considerably enhance the bandwidth of the Bragg regime, thereby extending the frequency range of operation as a circular–polarization filter.

Key words: Bragg regime; chirped filter; circular–polarization filter; sculptured thin film

1 Introduction

Theoretical as well as experimental studies have shown that chiral sculptured thin films (STFs) display the so–called Bragg phenomenon on axial excitation, in consequence of their periodically and unidirectionally nonhomogeneous constitution [1, 2, 3]. Let the direction of nonhomogeneity of a chiral STF be parallel to the z axis, while the film completely occupies the region $0 \leq z \leq L$.

When circularly polarized, monochromatic light falls normally on this film of sufficient thickness, then it is (i) almost perfectly reflected if the handedness of the incident light coincides with the structural handedness of the film, and (ii) almost perfectly transmitted if otherwise — provided absorption within the film is negligible and the free–space wavelength $\lambda_0$ of the incident light lies within the so–called Bragg regime. Optical devices such as circular–polarization filters and spectral–hole filters exploiting the circular Bragg phenomenon have been fabricated using
STF technology, delivering performance comparable to commercially available non–STF analogs [2, 4, 5].

The bandwidth of the Bragg regime is directly proportional to the local birefringence of the chiral STF. Although the serial bideposition technique [3] yields highly birefringent chiral STFs, further enhancement of the bandwidth can be desirable for certain applications. The objective of this communication is to show that stepwise chirping of the periodicity of a chiral STF can easily deliver a considerable enhancement of the bandwidth.

The stepwise chirped chiral STF is described in Section 2, and its response to normally incident plane waves is presented in Section 3. Comparison is made with the response of an unchirped chiral STF. An \( \exp(-i\omega t) \) time–dependence is implicit in this communication, with \( \omega \) as the angular frequency; vectors are underlined and dyadics are double–underlined; \( \mathbf{r} = xu_x + yu_y + zu_z \) is the position vector with \( u_x, u_y \) and \( u_z \) as the cartesian unit vectors; and \( \lambda_0 \) is the free–space wavelength.

2 Constitutive Relations

The nonhomogeneous permittivity dyadic of a chiral STF is expressed as follows [1]:

\[
\mathbf{\varepsilon}(\mathbf{r}) = \varepsilon_0 \mathbf{S}_z(z, h) \cdot \mathbf{S}_y(\chi) \cdot \left[ \varepsilon_a u_xu_z + \varepsilon_b u_yu_z + \varepsilon_c u_yu_y \right] \cdot \mathbf{S}_y^{-1}(\chi) \cdot \mathbf{S}_z^{-1}(z, h), \quad 0 \leq z \leq L.
\]

In this equation, \( \varepsilon_{a,b,c} \) are three relative permittivity scalars; \( \varepsilon_0 = 8.854 \times 10^{-12} \) F m\(^{-1} \) is the free–space permittivity; the tilt dyadic

\[
\mathbf{S}_y(\chi) = \mathbf{u}_y \mathbf{u}_y + (\mathbf{u}_x \mathbf{u}_x + \mathbf{u}_z \mathbf{u}_z) \cos \chi + (\mathbf{u}_z \mathbf{u}_x - \mathbf{u}_x \mathbf{u}_z) \sin \chi
\]

represents the locally columnar microstructure of any chiral STF with \( \chi > 0^\circ \); while the rotation dyadic

\[
\mathbf{S}_z(z, h) = \mathbf{u}_z \mathbf{u}_z + (\mathbf{u}_x \mathbf{u}_x + \mathbf{u}_y \mathbf{u}_y) \cos \left( \frac{\pi z}{\Omega} \right) + h (\mathbf{u}_y \mathbf{u}_x - \mathbf{u}_x \mathbf{u}_y) \sin \left( \frac{\pi z}{\Omega} \right)
\]

contains \( 2\Omega \) as the structural period, with \( h = 1 \) for structural right–handedness and \( h = -1 \) for structural left–handedness. Taking a whole number of periods to encompass the film thickness, we choose the ratio \( L/2\Omega \) to be a non–prime integer for filtering applications.
The stepwise chirped chiral STF comprises \( N_p \) steps, each step having \( \ell_p \) complete periods, with \( N_p \ell_p = L / 2 \Omega \). The half-period of the \( n^{th} \) step is denoted by

\[
\Omega_n = \Omega + \frac{2n - N_p - 1}{2} \delta \Omega, \quad n \in [1, N_p],
\]

wherein the chirping step \( \delta \Omega \) is selected to be a small fraction of \( \Omega \) to assure that \( \Omega_1 > 0 \). Thus, the expression of the permittivity dyadic of the stepwise chirped chiral STF is the same as (1), except that the dyadic \( \hat{S}_z (z, h) \) has to be replaced by \( \hat{S}_{chirp}^z (z, h) \), where

\[
\hat{S}_{chirp}^z (z, h) = u_z u_z + (u_x u_x + u_y u_y) \cos \left( \frac{\pi (z - \zeta_n)}{\Omega_n} \right) + h (u_y u_x - u_x u_y) \sin \left( \frac{\pi (z - \zeta_n)}{\Omega_n} \right), \quad \zeta_n \leq z \leq \zeta_{n+1}, \quad n \in [1, N_p],
\]

and

\[
\zeta_n = 2 \ell_p (n - 1) \left( \Omega + \frac{n - N_p - 1}{2} \delta \Omega \right)
\]

is the total thickness of the first \( n - 1 \) steps. Note that the thickness \( \zeta_{N_p+1} \) of the chirped chiral STF equals \( L \).

3 Numerical Results and Discussion

Let an arbitrarily polarized plane wave be normally incident from the lower half-space \( z \leq 0 \) on either of the two films described in the previous section. As a result, a plane wave is reflected into the lower half-space and another is transmitted into the upper half-space \( z \geq L \). The electric field phasors associated with the two plane waves in the lower half-space are stated as

\[
\hat{E}_{inc}(r, \lambda_0) = (a_L u_+ + a_R u_-) \exp (i k_0 z); \quad z \leq 0,
\]

\[
\hat{E}_{ref}(r, \lambda_0) = (r_L u_- + r_R u_+) \exp (-i k_0 z); \quad z \leq 0,
\]

where \( u_\pm = (u_x \pm i u_y) / \sqrt{2} \) and \( k_0 = 2\pi / \lambda_0 \) is the free-space wavenumber. Likewise, the electric field phasor in the upper half-space is represented as

\[
\hat{E}_{trs}(r, \lambda_0) = (t_L u_+ + t_R u_-) \exp [i k_0 (z - L)]; \quad z \geq L.
\]
Here, $a_L$ and $a_R$ are the known amplitudes of the left- and the right-circularly polarized (LCP & RCP) components of the incident plane wave; $r_L$ and $r_R$ are the unknown amplitudes of the reflected plane wave components; while $t_L$ and $t_R$ are the unknown amplitudes of the transmitted plane wave components.

The reflection amplitudes $r_{L,R}$ and the transmission amplitudes $t_{L,R}$ can be computed for specified incident amplitudes ($a_L$ and $a_R$) by following standard procedures described elsewhere; see, e.g., [1]. Reflection and transmission coefficients are thus defined as the entries in the 2x2 matrixes in the following two relations:

\[
\begin{bmatrix}
    r_L \\
    r_R
\end{bmatrix} =
\begin{bmatrix}
    r_{LL} & r_{LR} \\
    r_{RL} & r_{RR}
\end{bmatrix}
\begin{bmatrix}
    a_L \\
    a_R
\end{bmatrix},
\]

(10)

\[
\begin{bmatrix}
    t_L \\
    t_R
\end{bmatrix} =
\begin{bmatrix}
    t_{LL} & t_{LR} \\
    t_{RL} & t_{RR}
\end{bmatrix}
\begin{bmatrix}
    a_L \\
    a_R
\end{bmatrix}.
\]

(11)

The co-polarized transmission coefficients are denoted by $t_{LL}$ and $t_{RR}$, and the cross-polarized ones by $t_{LR}$ and $t_{RL}$; and similarly for the reflection coefficients in (10).

The reflectances $R_{LL} = |r_{LL}|^2$, etc., and the transmittances $T_{LL} = |t_{LL}|^2$, etc., were calculated as functions of the free-space wavelength $\lambda_0$ for a chiral STF described by the following parameters: $h = 1$, $\epsilon_a = 2.8$, $\epsilon_b = 3.4$, $\epsilon_c = 2.9$, $\chi = 20^\circ$, $\Omega = 155$ nm, and $L = 126 \Omega$. Graphs are presented in Figure 1, which clearly show the Bragg regime delineated by $\lambda_0 \in [528, 564]$ nm almost agrees with the theoretical range $\lambda_0 \in \left[2\Omega \epsilon_c^{1/2}, 2\Omega \epsilon_d^{1/2}\right]$ [1]. In this regime, $R_{RR}$ and $T_{LL}$ are very high, while $R_{LL}$ and $T_{RR}$ are very low, because the chosen film is structurally right-handed.

For the stepwise chirped chiral STF, we chose $N_p = 21$, $\ell_p = 3$ and $\delta_\Omega = 0.5$ nm. Thus, the smallest half-period $\Omega_1 = 150$ nm and the largest half-period $\Omega_2 = 160$ nm, while the average half-period is 155 nm (the same as $\Omega$ for Figure 1). The computed reflectances and transmittances, plotted in Figure 2 as functions of $\lambda_0$, show that the Bragg regime now is $\lambda_0 \in [514, 580]$ nm. The lower limit of the Bragg regime is somewhat higher than $2\Omega_1 \epsilon_c^{1/2}$, and the upper limit is somewhat lower than $2\Omega_N \epsilon_d^{1/2}$.

Relative to the 36 nm bandwidth in Figure 1, the Bragg regime in Figure 2 has a bandwidth of
66 nm — an enhancement by more than 80%. The enhancement comes without any noticeable reduction in the polarization-sensitivity of the film. Furthermore, calculations show that mild dispersion and absorption do not affect the enhancement significantly.

Stepwise changes in the periodicity of a chiral STF are likely to be accompanied by changes in the relative permittivity scalars $\epsilon_{a,b,c}$, which attribute of STFs was not accommodated in the theoretical study presented. Because the chirping step is very small compared with the mean half-period $\left(\sum_{n=0}^{N_p} \Omega_n\right) / N$ however, those changes are expected have insignificant effect.

To conclude, the presented theoretical study has demonstrated that a stepwise chirping of the periodicity of a chiral sculptured thin film considerably enhances the bandwidth of the Bragg regime, thereby extending the frequency range of operation as a circular-polarization filter.

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Figure 1: Computed spectrums of the reflectances $R_{RR}$, $R_{LL}$ and $R_{LR} = R_{RL}$ and the transmittances $T_{RR}$, $T_{LL}$ and $T_{LR} = T_{RL}$ of a chiral STF described by the following parameters: $h = 1$, $\epsilon_a = 2.8$, $\epsilon_b = 3.4$, $\epsilon_c = 2.9$, $\chi = 20^\circ$, $\Omega = 155$ nm, and $L = 126 \Omega$. 

Lakhtakia, Figure 1
Figure 2: Same as Figure 1, except for a stepwise chirped chiral STF described by the following parameters: $h = 1$, $\epsilon_a = 2.8$, $\epsilon_b = 3.4$, $\epsilon_c = 2.9$, $\chi = 20^\circ$, $N_p = 21$, $\ell_p = 3$, $\delta\Omega = 0.5$ nm, and $L = 126\ \Omega$. 