Quorum Sensing for Regenerating Codes in Distributed Storage

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Abstract—Distributed storage systems with replication are well known for storing large amount of data. A large number of replication is done in order to provide reliability. This makes the system expensive. Various methods have been proposed over time to reduce the degree of replication and yet provide same level of reliability. One recently suggested scheme is of Regenerating codes, where a file is divided into parts which are then processed by a coding mechanism and network coding to provide large number of parts. These are stored at various nodes with more than one part at each node. These codes can generate whole file and can repair a failed node by contacting some out of total existing nodes. This property ensures reliability in case of node failure and uses clever replication. This also optimizes bandwidth usage. In a practical scenario, the original file will be read and updated many times. With every update, we will have to update the data stored at many nodes. Handling multiple requests at the same time will bring a lot of complexity. Reading and writing or multiple writing on the same data at the same time should also be prevented. In this paper, we propose an algorithm that manages and executes all the requests from the users which reduces the update complexity. We also try to keep an adequate amount of availability at the same time. We use a voting based mechanism and form read, write and repair quorums. We have also done probabilistic analysis of regenerating codes.

I. INTRODUCTION

Cloud data storage is a challenging field posing many new problems for researchers since new companies such as Facebook and Google etc want to store several terabytes of data of the users and still provide the smooth user experience of services. Many companies such as Amazon etc. are providing cloud data storage services and in doing so one has to solve multi-dimensional optimization problem. For such a cloud storage, in order to make the data reliable one can use the classical technique of replication, in which the data on the nodes is replicated many times. This technique of storing data is very easy but may consume a lot of space. It is used in popular distributed systems [2], where data is divided into parts, and stored at various nodes. One advantage of distributed storage is reliability. Even if one of the node gets corrupted or fails, then the other nodes still remain alive. Most modern day distributed systems use replication [2], [3], [5]. While using distributed systems, there is a high chance for the nodes to get damaged or to leave after some amount of time. That new node will be directly copied from its replication as per the current storing scenario. The amount of space consumed in storing the data in replicated manner is huge, for example, Gmail uses around 21 times replication [5]. One aim to use the already existing nodes to make the new node instead of the replicated nodes and thereby reduce the storage consumption. In such systems a file of size $M$ is divided into $n$ parts with each part of size $\frac{M}{n}$ bytes and stored at different nodes in such a way that any node can be reconstructed using a subset of total $n$ nodes. These codes are called Erasure codes [3]. Maximum distance separable (MDS) erasure codes described in [4], [3] are optimal erasure codes and can generate a node using $k$ nodes where $k < n$. The main concern using MDS codes is the bandwidth consumption as compared to replication. We need to download data from $k$ nodes in MDS codes instead of just one in replication. In a seminal paper Dimakis et al. [3] has shown that repair bandwidth can be reduced if we apply network coding [2] on parts of the file and store $\alpha$ such linearly combined parts on every node. In order to repair a failed node, we now download data only from $d$ nodes instead of $k$, where $d < n$, and from each node instead of downloading the whole data, we download only $\beta$ packets out of $\alpha$, where $\beta < \alpha$. These codes are called Regenerating codes [3]. The repair bandwidth for the code is $d/\beta$. There is well known tradeoff between storage and repair bandwidth [3]. These codes can decrease the data that needs to be downloaded from the nodes to repair or construct the new node. Each file is divided, encoded, and stored at various nodes. Considering a practical scenario, there will be constant requests to read or write the file. Some nodes might fail in between, so there will be repair procedures going on. Any small change in the file will change its associated parts at various nodes. There has to be a proper method to do this, so that the multiple read’s, repair’s and write’s on the parts of the file are controlled in a proper way and do not interfere with each other creating wrong results. Also this method should not decrease the availability of the nodes and should be able to perform many requests in parallel. This motivated us to develop an algorithm by which the read, write and repair requests are scheduled in a way such that we do not encounter any inconsistency in the results and get the fastest results possible by processing as many requests as possible in parallel. We use the basic concept of simple voting mechanism with coding as in [1] where each node is given one vote to describe its state.

In this paper we give an algorithm based on quorum sensing to carry out the operations of read, write and repair on the nodes in a way such that the availability of the nodes also remain high and that the operations are properly ordered. The algorithm is generic and can be applied to any variant of regenerating codes. The rest of the paper is organized as follows. Background material and origin of the problem is given in Section 2. Our proposed algorithm is described in
Section 3, Section 4 discusses the Hadoop implementation and Section 5 gives the probabilistic analysis of regenerating codes. Finally Section 6 concludes the paper with some general remarks.

II. Preliminaries

When we store data in a distributed system, we have to coordinate the access to the data on various nodes in such a way such that the operations are fast and their results are free from error. One has to optimize two main parameters viz. update complexity and repair bandwidth. Network coding has shown a way to reduce the repair bandwidth and provide better reliability. In network coding, one performs mathematical operations on packets instead of just forwarding them. On the other hand quorum sensing has been used by researchers to give efficient algorithms that reduces update complexity in such a scenario. One kind of voting mechanism, suggested by Gifford, assigns votes to the various nodes and they communicate with each other using these votes. The read and write quorums are defined to provide a read-write and a write-write exclusion. The read quorum $r$ and write quorum $w$ are such that $r + w > N$ and $2 \times w > N$, where $N$ is the total number of votes assigned to all nodes. An approach called simple voting with coding (SVWC), as described in [11], defines read and write quorums for a replication scheme using coding to store files. The bounds on read and write quorum values were studied in this paper. These bounds were calculated in a way that you need at least votes equal to the lower bound to complete the read/write quorum and if equal to upper bound will always complete the read/write quorum. We use the basic concept from these algorithms about how votes can be used in managing a big network of nodes and communications with them. Throughout the whole discussion we assume that no votes are lost in the process and the server can fail only if it is stopped voluntarily. We also assume that all the votes of one request move at same speed in the network.

III. Proposed Algorithm

In this section, we consider a code with MDS property and discuss the basic file operations in a practical scenario. We define various terms as we go on describing the algorithm. We take specific cases in the beginning and generalize them one after other. Each time we generalize by some amount, we derive new read, write and repair quorums. In the algorithm we design conditions that will prevent both read-write and write-write from executing at the same node at the same time. Consider a Maximum Distance Separable (MDS) code for a file $F$. The file is initially divided into $k$ parts called as native chunks. Now, these $k$ chunks are encoded by linear combination to form $n$ code chunks, with $n > k$, and stored at $N$ nodes. Considering the MDS property, any $k$ nodes can be contacted to reconstruct the whole file again. The maximum number of failures which can be tolerated by the system is $n - k$. Every node in the cloud stores $\alpha$ chunks making $n\alpha$ chunks in total. We can contact any $d$ nodes and download $\beta$ out of $\alpha$ packets from each node in order to construct a new node which can replace the corrupted or failed node. For any given regenerating code, let us say any change in file $F$ updates $q$ chunks in total. There are three operations that can be executed on the file.

1) Read/download/reconstruct the whole file- There will be a lot of users who would perform these requests. In this case, we will have to generate the complete file, and to do that, we will have to contact any $k$ nodes out of total $N$ nodes.

2) Write/update- Whenever any file is changed, the corresponding data in the nodes will also change. There will again be a lot of users who would try to perform simultaneous write requests, which will update the information stored in the nodes.

3) Node Repair- If a node gets corrupted or fails, we need to generate a new similar node. For this, we need to connect to $d$ different nodes and download $\beta$ packets from each of them. The node making this request might be a specific node who is given the task of node repair.

We have divided the user requests into above three types. Everything can happen in a smoothly if requests do not interfere with each other. In a practical scenario, there will be constantly two or more requests which might be requested at the same time. Such simultaneous requests might result in some problem. There would be some inconsistency when we are reading from a certain node and suddenly a write operation tries to write on it. One way to operate and avoid inconsistency is to keep the incoming operations waiting and that write operation will keep trying till the read is completed. Following this approach, if another read request comes after the write request, it will also be put in the waiting queue behind write. There would be a lot of polling which is a waste of resources because two reads can occur at a same time. There will be many such requests trying to read or write a large number of nodes. If these requests are not properly controlled and managed, the whole network might get congested and may take too much time to process user requests. As a solution to the given problem, we propose a voting-locking approach to manage such large number of requests efficiently. We define quorum requirements to achieve this.

Definition 1: The read, write and repair quorums are defined as the minimum number of votes required to initiate the execution of that request. We denote the read, write and repair quorum by $r$, $w$ and $rep$.

Our main concern is to define a systematic approach which will try to reduce as many inconsistencies as possible and give the result of the requests in least possible time. Following are the six possible simultaneous request cases. Even if the simultaneous requests are more than two they will be made up of any such two pairs of requests only.

1) read-read
2) read-write
3) read-repair
4) write-write
5) write-repair
6) repair-repair

In order to describe the algorithm, let us first take some assumptions and derive the read, write and repair quorums under these assumptions. We then gradually go on generalizing after removing the assumptions one by one.
A. Single chunk node with simultaneous updates

In this subsection we find out the quorum values for single chunk node with simultaneous updates possible. More precisely, we have the following:

1) Each node has only one code chunk stored. So number of code chunks and number of cloud nodes are same i.e. \( \alpha = 1, n = N \)

2) Any update in file \( F \) changes every code chunk associated with it. So as per assumption, data in every node changes with any change in the file.

Under these two assumptions, it is easy to see that \( N = n = q \), so we will call one code chunk as one node for now. With these assumptions, let us see which requests can execute simultaneously.

- read-read - Yes. Two simultaneous reads will never be a problem. They do not create any inconsistency.
- read-write- No. As every write updates every node currently, there will be nodes on which both operations would be executed simultaneously. This will bring an inconsistency in read operation.
- read-repair- Yes, because repair is just another kind of a read request contacting lesser nodes.
- write-write- No. Simultaneous write’s will definitely create inconsistency in as both of them will try to update every node at the same time giving incorrect results in the end.
- write-repair- No. It is another kind of read-write repair. Even though the number of nodes that we are contact- ing for repair operation is less, the write operation is updating all the nodes and so there will definitely be some common nodes and so the inconsistency.
- repair-repair- Yes. It is an another kind of read-read case and so it is possible.

We define a unique vote-bit and lock-bit of one bit on every chunk. When the vote bit is 1, the node can send votes and when it is 0, they cannot. Lock-bit for a node is a one bit value describing what lock is on it, read lock or write lock. If the lock bit is 0 then there is no lock on it. We call the server a super-user from which the read, write and repair requests will be executed and end nodes as end users from where the requests are coming. This is shown in Figure 1. Whenever the super user gets any kind of request, it will check if the nodes are free to execute that request on them. We will discuss later the pattern in which the super-user will send requests. We now define two kinds of locks.

1) Read lock- Whenever any node is free to be read, we apply a read lock on it keeping its vote bit unchanged i.e. equal to 1 and lock bit to 1. When the read lock is set, it cannot surpass write requests but can allow multiple read and repair requests. Once read lock is set, the super user starts reading from that node and when the read operation is done, the super user node frees it from the read lock and the lock bit is changed to 0.

We define a lock table at the super user, which enlists if there is any lock at a node, and if there is one, which lock is it. When the super user gets a request from end users, it tries to know the availability of nodes for that request by sending the request to the nodes. Every request from the super user to the nodes has a unique request id. Nodes will be available if there is no lock on them, and if there is a lock, then the availability will depend on the type of lock on them. If they are available for a certain request, they will send a vote to the super user in form of a two bit (node number, request id) number only if their vote bit is 1. When this vote reaches super user, it checks in the lock table if that node can be granted that lock. If it can be granted, then it updates its lock table with the corresponding lock on the request id, and then updates the vote bit and lock bit on the node, if required. It would not be necessary to update the vote bit and lock bit if the node on which the request was granted was already locked, because these requests will be read-read, read-repair or repair-repair and all these have same vote bit and lock bit. The node structure can be seen in Figure 2 and the lock table can be seen in Figure 3.

Remark 1: We use a read lock for both, the read and repair request and only a read lock can be granted in the presence of another read lock but no other lock can be granted in the presence of a write lock. A read operation can be done from any set of nodes, so we are not restricted to read from specific nodes after putting a read lock on them. However, write’s for a request are specific. They need to be done on specific set of nodes. So, if a write lock is not obtained on specific chunks, then it waits for the current lock on it to release. If one more write request arrives for the same chunk, then it will also go
in the waiting list.

**Claim 1:** Quorum values for single chunk node with simultaneous updates are given below.

1) \( r = k \), because we need at least \( k \) nodes from out of \( N \) to read to download/read the file.
2) \( w = n = N = q \), because every write is updating every node, so it requires all nodes to be available for writing at a time.
3) \( rep = d \), because it requires any \( d \) nodes to read from to reconstruct the failed node.

We take the NCCloud system [7] to explain this scenario. If we look at the encoding mechanism of NCCloud from \( k \) native chunks to \( n \) code chunks, we will see that each code chunk is dependent on every native chunk. Thus, even a small change in the file will change all the \( n \) chunks. Now, if the \( 8 \) code chunks of NCCloud were stored on \( 8 \) nodes instead of \( 4 \), then this would be a perfect example as per our assumptions. So, assuming \( N = 8 \), and each chunk stored on individual node, we can say that for this system, the read and write quorums will be \( r = 2 \), and \( w = 8 \).

**Example 1:** Consider a different scenario with \( N = 10 \), \( r=6 \) and \( w=6 \). What if 5 out of 10 nodes allow read operation on them and other 5 allows write operation on them. Neither of the quorums will get satisfied and both of them will wait for one extra vote which they will never get in this condition. This is a deadlock situation. In order to solve this we define two terms, timeout time \( t_0 \) and a request queue. A request queue is a queue which distributes the incoming requests in to slots, where all the requests of one slot are handled together and every request has its own priority. The priority for every request \([\text{read}_0, \text{read}_1, \text{write}_0, \text{write}_1, \text{rep}_0, \text{rep}_1]\) is zero initially. Therequests with higher priority than others in the same slot are handled first. The division of the slots can be done on the basis of time or number of requests i.e. one slot can be of all the requests that arrived in time \( t \) or one slot can be a slot till \( r_s \) requests arrive at super user. Here \( r_s \) can be a read or write or repair request. The request queue is handled at the super-user level and once a request comes in the running slot, it does not wait for the slot to fill, it just starts asking for votes. We define request-vote ratio for a request as the ratio of the votes obtained to the quorum votes needed by that request. If the quorum requirements in the running slot are not completed for any of the requests for a continuous time of \( t_0 \), then we assume it as a dead lock situation and pick one-fourth of the requests with minimum request-vote ratio, decrease their priority and then release their acquired locks on nodes. If the situation remains same even after another interval of \( t_0 \), this is repeated again and then the priority of the requests whose priority were reduced earlier will be further reduced. We now further limit our assumptions as below.

**B. Single chunk node with relative updates**

We now remove the assumption (2) that we took in Section A and say that any update in file \( F \) changes \( q \) code chunks out of \( n \). We are now left with following assumption.

1) Each node has only one code chunk stored. So number of code chunks and number of cloud nodes are same i.e. \( \alpha = 1 \), \( n = N \)

Let us now see which of the consecutive requests can hold.

- **read-read -** Yes, because two simultaneous reads are never a problem.
- **read-write -** May be. Here every write updates \( q \) node(chunks), hence there might be a possibility when there will be no common nodes among both operations.
- **read-repair -** Yes, because repair is just another kind of a read request contacting lesser nodes.
- **write-write -** May be. Simultaneous write will definitely create inconsistency in write operations but if both the write’s are writing on completely different set of \( q \) nodes, then this would be possible.
- **write-repair -** Yes, if the intersection set of nodes for both requests is zero.
- **repair-repair -** Yes, this is also another kind of read-read and so it is possible.

In this case, quorums are

**Claim 2:** Quorum values for single chunk node with relative updates are given as:

1) \( r = k \), because we need at least \( k \) nodes to read from to download/read the file.
2) \( w = q \), because every write is updating \( q \) nodes at a time
3) \( rep = d \), because it requires any \( d \) nodes to read from to reconstruct the failed node.

**C. Multiple chunk node with relative updates**

Let us now remove the assumption in section B also and say that each node has \( \alpha \) code chunks stored in it. Every write updates \( q \) out of \( N\alpha \) code chunks. We cannot take votes from individual chunks, we have to take them from nodes only. We give \( \alpha \) votes to each node instead of one. Every vote is distinct, even on the same node. Node \( i \) will have votes from \( \alpha_{i1}, \alpha_{i2},...\alpha_{i\alpha} \) for its \( \alpha \) number of packets, where \( 1 \leq i \leq N \). We will now treat one node and one chunk differently. This can be seen in Figure 3. The conditions of the consecutive requests will be same as described under Section B. Due to generalization, we do a little modification in the mechanism.
add one more condition. We are not allowed to perform any waiting to be. This might result into an incorrect read, so we need to read from the chunks that got updated and those who are locked on 2 chunks and a waiting in their operations as explained below.

If a request and a write request and both have two chunks common in their operations as explained below. If the chunks are able to allow that request, it sends a 3 bit vote $[i,j,\text{requestid}]$ where $i=$node number and $j=$ chunk number of that node and requested is the unique id of the request that came to that chunk. Quorum values in this case are given below.

**Claim 3:** Quorum values for multiple chunk node with relative updates are given as:

1. $r = \sum \alpha_{ij}$, where $i=$any $k$ values from 1 to $N$ and $j = \alpha$.
2. $w = q$, because every write is updating $q$ chunks at a time
3. $\text{rep} = \sum \alpha_{ij}$, where $i=$any $d$ values from 1 to $N$ and $j =$any $\beta$ values from 1 to $\alpha$.

and $\sum \alpha_{ij}$indicates the total number of votes, with $\alpha_{ij}$as one vote.

**Example 2:** Consider a case when we try to perform a read request and a write request and both have two chunks common in their operations as explained below. If $r = 4$, $w = 4$ and we have 6 chunks. Suppose the vote for both read and write lock goes to the super user but read lock is formed first and then the write lock. So currently we have read locks on 4 chunks, write lock on 2 chunks and a waiting write lock on 2 chunks that are being read currently. Meanwhile, suppose the other 2 chunks get updated and lock is removed from them. Now, suppose a new read request is made and it tries to read from the two recently updated chunks and the two on which the write locks are waiting. Now, this is not supposed to happen, we are trying to read from the chunks that got updated and those who are waiting to be. This might result into an incorrect read, so we add one more condition. We are not allowed to perform any new request $r_i$on a chunk with a waiting write/read/repair lock of request $r_k$on it, when operations of request $r_i$are supposed to involve both, the chunks updated by and the chunks with waiting locks from the same request $r_k$.

So, as per the Figure 4, read lock will be enabled on nodes $v_1, v_2$ and $v_4$, write lock on nodes $v_1$ and $v_2$ and a waiting write lock on node $v_3$. If the operation of write on $v_1$ and $v_2$ is completed then they would be freed from existing write locks. Suppose a new read request comes with the current read request still going on. This new request will be able to form a lock on $v_1$ and $v_2$ easily. As per our current algorithm, $v_3$ and $v_4$ should also be allowed to be read under the new read request, but if we do so, the constructed file might be incorrect as the MDS property might not hold for nodes with only half of an operation done. Thus, we restrict ourselves from choosing one or more of the both the sets $\{v_1, v_2\}$, $\{v_3, v_4\}$. We can still select one or more of $\{v_1, v_2\}$ with some other nodes or one or more of $\{v_3, v_4\}$ with other nodes. This will prevent from any possible inconsistency in the network. Let us discuss the way in which the super user should send the request for votes to the nodes. There should be some proper way for the super-user to carry out the process efficiently. Multicasting is one way but it will increase the overhead by a large amount. We try form some groups which can reduce the overhead. After we form groups, instead of multicasting, write requests are sent group wise and read requests are sent depending upon the existing write requests to create least possible interference. One such approach is as follows. We have $k$ native chunks and any change in the file will be replicated in any of these $k$ native chunks. We can look at encoding mechanism of constructing $n$ coded chunks from $k$ native chunks and can exactly say which out of the $n$ coded chunks will change for a corresponding change in some native chunk. We can look further and know...
which $q$ chunks out of $\alpha N$ will change for a corresponding change in native chunk. There are $k$ native chunks, so there will be $k$ groups overall. We make a group of all the chunks which are changing due to change in some native chunk. There will be $k$ groups formed each containing $q$ chunks. We have a total of $N$ each with a capacity of $\alpha$ chunks. We try to store the first group of $q$ chunks in first node. There wont be any problem if $q < \alpha$ but if $q > \alpha$, then we store the remaining $\alpha - q$ chunks on second node. We now search other groups and find which group has highest number of chunks that are common with the $\alpha - q$ chunks. We store all the chunks of that group other than the chunks that were common with $\alpha - q$ chunks on second node. We keep on making groups in this way till every chunk gets placed somewhere. If a read request comes first, it is sent to random $k$ nodes which are adjacent to one another. When a write request comes, the super user first checks which group will be affected most by that change and sends request to that group. We know the chunks that needs to be updated and the group associated. If that group corresponds to nodes $N_i, N_{i+1},$ etc., the write request is sent to those corresponding chunks on the corresponding nodes and the next incoming read request is sent to the $k$ nodes which has higher write distance.

Definition 2: The distance between two nodes is equal to the number of hops from node to node in between those two nodes.

Definition 3: The write distance for a node is defined as the sum of the distance between the that node and the nodes on which the write operation is being performed at the present time.

Remark 2: This write distance for all the nodes should be kept regularly updated in a table called write distance table and stored at super-user. Whenever a read request comes, we pick out the $k$ nodes with highest write distance and send read requests to them. This helps in keeping read and write operation as far as possible.

Example 3: Consider a scenario with $N = 5$ nodes with write operation being performed on any number of chunks at nodes $v_1$ and $v_3$. The current write distance for nodes $v_1, v_2, v_3, v_4$ and $v_5$ will be $2, 2, 2, 4$ and $6$ respectively. In a case if we have to pick any one node for the read request, we will pick the node $v_5$ first and if we have to pick one more node then we will pick node $v_4$ after $v_5$.

IV. IMPLEMENTATION IN HADOOP

We have discussed the algorithm. Let us try to apply this to current storage systems like Hadoop. The Hadoop Data File Structure (HDFS) is comprised of interconnected clusters of data in it. Every cluster has one single node called Name Node and many number of servers. Each server has Data Node with it which contains the data. Name Node is the one who handles all the read and write requests from the HDFS clients. Data Nodes perform action on data as per the instructions given by the Name Node. Data Nodes continuously loop and ask the name node for instructions. Following changes in the Hadoop system are proposed to make it more efficient.

1) The Name node of the each cluster should be modified to act like the super user node described in

| Algorithm 1 Read Algorithm for any request $r_i$ |
|-----------------------------------------------|
| 1) Divide in groups $G_1, G_2, \ldots, G_k$ such that $|G_i| = N_i, 1 \leq i \leq k$ |
| 2) Send vote requests to all the chunks in the nodes $N_1, N_2, \ldots, N_i$, with maximum write distance |
| 3) Define $V = \{ \text{set of nodes whose all chunks have sent ready votes} \}$ |
| 4) if $|V| = k$, Form a read lock on these nodes and start read operation |
| 5) Else, Form a read lock on the nodes in $V$ |
| 6) Do |
| 7) Contact $k-n(V)$ out of $N-n(V)$ with highest write distances. |
| 8) Add those nodes to $V$ whose every chunk has sent read vote and form a read lock on these nodes. |
| 9) WHILE $(n(V)! = k)$ |
| 10) Contact these $k$ nodes present in $V$ to perform read operation. |

| Algorithm 2 Write Algorithm for any request $w_i$ |
|-----------------------------------------------|
| 1) Find the chunks which will be changed for the write request $w_i$. |
| 2) Find the nodes on which these chunks are stored in groups. |
| 3) Send vote requests to all the chunks in the corresponding nodes. |
| 4) If any other locks are already present on the nodes, then wait for those locks to release. |

our system. The data on the data nodes will be stored in form of regenerating codes instead of normal distributed storage. The benefits of storing as regenerating codes over normal distributed storage have already been described. Once this is established the Name node will work on its own and the data nodes wont have to continuously ask the name node for instructions. This will reduce the communication overhead in an interaction between name node and data node. Millions of such interactions take place in small interval of time. Due to this, even a very small improvement in the overhead will lead to a countable decrease in the overall overhead.

2) Files can be stored on the clusters as per our group forming mechanism with group 1 parts stored on data node 1 and so on. This will further reduce the communication overhead.

3) By default the replication factor in Hadoop system is 3. However, the replication factor can be increased by companies depending on the data they want to store. We can make the system work with 2 times replication using our algorithm with amost equal or more reliability than with replication factor as 3. For this, we store the $n$ parts of the file on all the clusters except the one where the file is stored. We divide the $n$ parts into various clusters in the way that every cluster has almost same and minimum number of parts. Our system will be able to tolerate many failures, cluster failure or data failure.

Example 4: If the total number of clusters are 15 and the
vale of \( n \) is 40, we have to divide these 40 parts in 14 clusters. For distributing, we put the first 14 parts on 14 clusters, then another 14 parts on 14 clusters making each cluster store 2 parts. We are still left with 12 parts. We divide this 12 parts on any 12 clusters out of 14 making 12 clusters to store 3 parts each and 2 clusters to store 2 parts each.

V. PROBABILISTIC ANALYSIS OF REGENERATING CODES

In this section, we look at the second aspect of the paper. We analyze the incoming request, completing request and calculate the probabilities of downloading a file and repairing a node in a regenerating code.

A. Probability Calculation

Let the probability to read and write a node be \( p_r \) and \( p_w \) respectively then it is clear that

\[
P (\text{download a file}) = \begin{cases} 0 & \text{if } i < k; \\ \sum_{i=k}^{N} \binom{N}{i} p_r^i (1 - p_r)^{N-i} & \text{if } i \geq k. 
\end{cases}
\]

\[
P (\text{repair a node}) = \begin{cases} 0 & \text{if } i < d; \\ \sum_{i=d}^{N-1} \binom{N}{i} p_r^i (1 - p_r)^{N-i} & \text{if } i \geq d. 
\end{cases}
\]

B. Analysis of Incoming Request

For regenerating codes we can make the following assumptions.

1) There are \( n \) requests of some operations (either read or write) in the system associated with a chunk \((i, j)\) at time \( t \) and the probability of exactly one arrival request is given by \( \lambda \Delta t + O(\Delta t) \) during a small interval \( \Delta t \) where \( \lambda \) is arrival rate of request independent of \( t \).

2) \( \Delta t \) is so small that the probability of arrival of more then one request is almost zero.

3) The incoming requests are independent of each other.

Under these assumptions using the Queueing theory [10], we get

*Theorem 1:* If the arrival of requests are completely random, then the number of arrival requests follows Poisson distribution in a fix time interval.

Because of read as well as write requests satisfy the above three assumptions so we can say that the read requests and the write requests follows the Poisson distribution separately with request arrival rate \( \lambda_r \) and \( \lambda_w \) respectively. Hence the probability of \( m \) read requests associated with a chunk \((i, j)\) at time \( t \) is given by

\[
P_m (t) = \frac{(\lambda_r t)^m e^{-\lambda_r t}}{m!}.
\]

Similarly the probability of \( l \) write requests associated with a chunk \((i, j)\) at time \( t \) is given by

\[
P_l (t) = \frac{(\lambda_w t)^l e^{-\lambda_w t}}{l!}.
\]

Now on a chunk \((i, j)\) writing and reading is not possible at the same time so we can say that after finishing writing request the chunk resets its time clock to \( t = 0 \) for rest of read requests. Hence let \( P_{0,m} (t) \) be the probability to perform \( m \) read operations at time \( t \) and \( P_{1,0} (T) \) be the probability to perform \( l \) write operations at time \( T \) then

\[
P_{0,m} (t) = \frac{(\lambda_r t)^m e^{-\lambda_r t}}{m!} \quad \text{and} \quad P_{1,0} (T) = \frac{(\lambda_w T)^l e^{-\lambda_w T}}{l!}.
\]

If \( P_{l,m} (T, t) \) is the probability to perform \( m \) read operations at time \( t \) after performing \( l \) write operations at time \( T \) then

\[
P_{l,m} (T, t) = P_{1,0} (T) P_{0,m} (t) = \frac{(\lambda_w T)^l e^{-\lambda_w T}}{l!} \frac{(\lambda_r t)^m e^{-\lambda_r t}}{m!}.
\]

*Remark 3:* Note that

\[
\sum_{t=0}^{\infty} \sum_{m=0}^{\infty} \frac{(\lambda_w T)^l e^{-\lambda_w T}}{l!} \frac{(\lambda_r t)^m e^{-\lambda_r t}}{m!} = 1
\]

Hence expected request for read and write for a chunk in \( T + t \) time is \( (\lambda_w \lambda_r T t) \).

C. Analysis of Completing Request

For analyzing the availability of a chuck we need to analyze the requests (read or write) that has been completed. Assume that \( N_o^{(r)} \) \( (N_o^{(w)}) \) requests are associated with the chunk \((i, j)\) for read (write) operations at time \( t = 0 \) and no incoming requests are allowed from time \( t = 0 \) to \( t = T \). Let the rate to complete the request be \( \mu_r \) \( (\mu_w) \) per unit time. We make the following assumptions

1) Probability of completing one read (write) request is \( \mu_r \Delta t \) \( (\mu_w \Delta t) \).

2) Probability of completing more then one request is zero.

3) The completing requests are independent of each other.

Under the assumptions, by Queueing theory [10], the probability to have \( m(l) \) read(write) request at time \( t(T) \) i.e., \( P_m (t) \{ P_l (T) \} \) is given by

\[
P_m (t) = \begin{cases} (\mu_r t)^{N_o^{(r)} - m} e^{-\mu_r t} & \text{if } m = 1, 2, ..., N_o^{(r)}; \\ 1 - \sum_{n=1}^{N_o^{(r)}} \frac{(\mu_r t)^{N_o^{(r)} - n}}{(N_o^{(r)} - n)!} e^{-\mu_r t} & \text{if } m = 0. 
\end{cases}
\]

and

\[
P_l (T) = \begin{cases} (\mu_w T)^{N_o^{(w)} - l} e^{-\mu_w T} & \text{if } l = 1, 2, ..., N_o^{(w)}; \\ 1 - \sum_{n=1}^{N_o^{(w)}} \frac{(\mu_w T)^{N_o^{(w)} - n}}{(N_o^{(w)} - n)!} e^{-\mu_w T} & \text{if } l = 0. 
\end{cases}
\]
Fig. 5. Probability $P_0(t)$ of completing read requests with respect to time
with $\mu_r = 10$ requests / unit time.

Note that

$$P_0(t) = 1 - \sum_{n=1}^{N_0} \frac{(\mu_r t)^n}{N_0!} e^{-\mu_r t}.$$  

Thus

$$\lim_{N_0 \to \infty} P_0(t) = e^{-\mu_r t}.$$  

Figure 5 plots the probability $P_0(t)$ of completing requests with respect to time $\mu_r = 10$ requests / unit time. Similar analysis can be done for write requests.

D. Analysis of Incoming and Completing Requests

Let $\mu$ and $\lambda$ be the rates of completing and receiving requests in the system associated with the chunk. Then by queueing theory [10] the probability of $n$ requests (read or write) in system is given by

$$P_n(t) = \left( \frac{\lambda}{\mu} \right)^n \left( 1 - \frac{\lambda}{\mu} \right), \quad (\frac{\lambda}{\mu} < 1, n \geq 0).$$  

VI. CONCLUSION

We proposed an algorithm that can be used to manage the read, write and repair requests on data which is stored using regenerating codes. The algorithm proposed a solution to manage a large number of requests in a way such that they can be executed in least possible time. The algorithm also proposed a way to prevent inconsistencies that can happen due to read and write performed on the same node at the same time. It would be interesting in future to extend this algorithm further and solve other problems listed below:

- Some addition to the algorithm can be done to manage the system in case of loss of votes in the path and the case of sudden node failure while performing operations on it.
- The algorithm can be extended to a large number of servers involved in the network where single file will be connected to huge number of servers.
- Some other group formation technique can be formed which can decrease the communication overhead in the process.

- Cache technique can be applied to further decrease the communication overhead.
- The algorithm can be further extended and applied on Fractional Repetition (FR) codes.

To analyze the requests associated with regenerating code we have used queueing theory.

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