Finite element modeling of fluid filtration in a deformable porous medium

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Abstract. The scientific work considers the finite element modeling of fluid filtration in a deformable porous medium. A transversely isotropic medium has been taken as a deformable medium. Fluid filtration process to horizontal wells by drift type direction occurred in this transversely isotropic medium is studied. As filtration process goes, the stress layers are associated with the pressure, accordingly, equations have been shown regarding this pressure. Numerical solution of the problem is carried out using the finite element modeling.

1. Introduction
Fluid filtration process in a porous or cracked medium is an object of underground hydromechanics. Hydrodynamic modeling of the development of fluids is based on the use of mathematical equations which describe the real state of groundwater hydrodynamic filtration process. In order to define and develop filtration characteristics it is necessary to research the layers and wells. In the work the medium where filtration process goes is heterogeneous and transversely isotropic. In nature filtration process in heterogeneous media can be met a lot and it has caused scientists’ interests in research.

Modeling of fluid flow in a deformable porous medium is actual. It is important to determine the deformation state of fluid filtration going through the wells in a heterogeneous medium. Therefore, some information about several research works is shown: P.N.Vabishchevich, A.V.Grigoriev [1] considered models of anisotropic cracked and porous media; D.N.Nizamov, A.A. Khodjiboev, A.A. Khodjiboev [2] studied mathematical model of stress-strain state of rocks in a cracked medium in different ways; K.S Basniev, N.M.Dmitriev, G.D. Rosenberg [3] presented hydrodynamic models of fluid filtration theory in isotropic, cracked, homogeneous and heterogeneous porous media in their research work; Lin B., Chen S., Jin Y. [4] in order to extract oil from a horizontal well through the water injection to this well, taking into account different properties of the anisotropic medium, studied the deformation effect on the strata; F.M.Kadyrov, A.V.Kosterin [5] obtained Laplace equation in general for fluid pressure in an elastic medium and taking into account initial conditions considered filtration problems of pore-elastic media; B.Amaziane, M.E.Ossmani, Ch.Serres [6] used numerical methods for modeling of fluid flow in heterogeneous porous media, likewise methods of finite element modeling and triangle discrete methods; T.M.Vu, D.Subrin, J.Sulem, N.Monin [7] analyzed analytical solutions of displacements and stresses in transversely isotropic strata in their article; in the article of R.R.Siraevti [8] theoretical researches to incompressible fluid filtration of a heterogeneous porous medium were carried out; O.C.Zienkiewicz, R.L.Taylor, P.Nithiarasu [9] offered a full application of
the finite element modeling for fluid mechanics. In addition, there are differential equations and numerical solutions of hydrodynamics; in the research of R.W.Lewis, B.A.Schrefler and P.Nithiarasu, K.Seetharamu [10] heat-hydro-mechanics problems are revealed in numerical solutions. Moreover, the finite element modeling applied to solve heat transfer and advanced computational methods which are used for complex fluid motions; D.Deb and J.Hervouet [11] demonstrated concepts of the finite element modeling and programs for hydromechanics.

2. Problem statement
Fluid filtration in a deformable transversely isotropic layer can be studied as a complex multi-phase system. Assessment of filtration problem and stress-strain in the layer is complicated.

Transversely isotropic medium – a medium that consists of isotropic planes, has tiny pleats and properties are the same by horizontal directions. The filtration properties of a medium through the isotropic plane are the same by the whole direction, but filtration properties laid perpendicularly on the isotropic plane are different by the whole direction. The medium composed of these planes is called transversely isotropic medium (Figure 1).

![Figure 1. The scheme of the computing area](image)

Horizontal underground mining is carried out into heterogeneous, fractured inclined-layered rocks is divided into three types depending on the location of production at different depths: drift - in the direction of layers, crosscut - perpendicular to the direction of drift, horizontal - longitudinal axis makes any angle in the direction of drift. Stress-strain state of the aforementioned underground structures depends on the depth of location, the cross section of layers and the elasticity of surrounding mass.

In general plane deformation condition of the laws of distributing stresses and displacements by the drift type direction through the multi-bore wells which are in the transversely isotropic medium were studied by numerical researches with the finite element methods. Horizontal wells conducted by the drift type direction that have long wellbores in the transversally isotropic medium consist of the transversal \( \phi \) angular isotropic plane layers (Figure 2).
The pressure will be arisen when fluid is extracted from the multi-bore horizontal well in the transversely isotropic medium and the equation of movement through this pressure $P$ is characterized:

$$
\frac{1}{\eta} \frac{\partial P}{\partial t} - \frac{\alpha}{2} \left( \frac{\partial^2 u_x}{\partial t \partial x} + \frac{\partial^2 u_z}{\partial t \partial z} \right) = \frac{k_x}{\mu} \frac{\partial^2 P}{\partial x^2} + 2 \frac{k_x}{\mu} \frac{\partial^2 P}{\partial x \partial z} + \frac{k_z}{\mu} \frac{\partial^2 P}{\partial z^2}
$$

(1)

here $\alpha$ Biot coefficient; $\eta$ fluid dynamic viscosity; $\mu$ shear modulus, fluid viscosity coefficient.

The change of filtration coefficient in the isotropic plane layers is:

$$
k_{xy} = k_y \left( \cos^2 \varphi + 1 \right) + k_z \sin^2 \varphi,
$$

$$
k_{zz} = k_y \sin^2 \varphi + k_z \cos^2 \varphi,
$$

$$
k_{xz} = k_z \sin \varphi \cos \varphi - k_y \sin^2 \varphi.
$$

(2)

Hooke’s law changes:

$$
\{ \sigma \} = [D]\{ \varepsilon \} + [I]p
$$

(3)

here

$$
\{ \sigma \} = \{ \sigma_y \sigma_z \tau_{yz} \tau_{xz} \tau_{xy} \}^T, \quad \{ \varepsilon \} = \{ \varepsilon_y \varepsilon_z \gamma_{yz} \gamma_{xz} \gamma_{xy} \}^T
$$

$$
[I] = \text{diag} \{1 \ 1 \ 0 \ 0 \ 0 \}
$$

In the deformable porous medium general stress satisfies the next equilibrium equation:

$$
\sum_{j=1}^{2} \frac{\partial \sigma_{ij}}{\partial x_j} + \delta_{ij} \frac{\partial P}{\partial x_i} = 0, \quad i = 1, 2; \quad x_1 = x, \ x_2 = z
$$

(4)
The pressure in the contour of the drift type multi-bore horizontal well is given:

\[ p|_s = p_s \]  

(5)

The boundary conditions are given as follows:

\[ p|_{ABCD} = p_1, \quad p|_{A'B'C'D'} = p_2, \quad \frac{\partial p}{\partial n}_{AA'B'B'} = \frac{\partial p}{\partial n}_{CC'D'D'} = 0 \]  

(6)

Deformation coefficients in the drift type multi-bore horizontal well in the formula (3) and the elements of symmetric matrix \([D]\) are defined by these formulas [20-22]:

\[ d_{11} = \frac{E_1(E_1 - E_2v_2^2)}{1 + \nu_1(E_1(1 - \nu_1) - 2E_2v_2^2)}, \]

\[ d_{12} = \frac{E_1(E_1v_1 + E_2v_2^2)}{1 + \nu_1(E_1(1 - \nu_1) - 2E_2v_2^2)} \sin^2 \varphi + \frac{E_1E_2v_2}{E_1(1 - \nu_1) - 2E_2v_2^2} \cos^2 \varphi \]  

(7)

\[ d_{13} = \frac{E_1[(v_2 - 1)E_1v_2 + (E_1 - E_2v_2)v_1]}{2(1 + \nu_1)(E_1(1 - \nu_1) - 2E_2v_2^2)} \sin 2\varphi \]

\[ d_{55} = G_2 \sin^2 \varphi + \frac{E_1}{2(1 + \nu_1)} \cos^2 \varphi \]

Here \(E_1, E_2\) isotropic plane and elasticity modulus by perpendicular direction to it; \(\nu_1, \nu_2\) Poisson’s coefficient in the case of tension-compression; \(G_2\) shear modulus of normal planes to isotropic plane; \(\varphi\) an inclined angle by horizontal axis of isotropic plane.

3. Numerical solution of the problem.

The influence of the stress-strain state of the anisotropic medium and filtration coefficients on the flow rate of the horizontal well have been analyzed. Experiments were developed with the following initial data for the problem (1)-(4) in accordance with the real conditions of conducting the horizontal well [23-25]:

- 8m, 14m, 22m, 150m, 2.4cps, 10atom, , , 0.908 t/m.
- 0.106d, =1/m2, =0.25, =0.4t/m2.

The area that has two wellbores is divided into 2464 triangle elements and 1362 nodes. Stationary filtration problem is given to solve 1362 and 4046 consistent algebraic equation systems according to the pressure and displacement. It is solved, taking into consideration its boundary conditions, by Zeidel-Gauss iteration method which has high coefficient of relaxation.

Software package was created that divides the area automatically into finite elements with the given results. By the help of it the computational area is divided into triangle elements in accordance with the wells dimensions (Figure 3).
Figure 3. Division of the computational area
The pressure under the influence of the multi-bore horizontal wells is (Figure 4) described.

Figure 4. Change of the pressure
The developed algorithm and software package have been tested in a special task for determining the flow rate of the horizontal well in the isotropic planes. The gained results (Table 1) fluctuate just for 1-2% compared to the real solution.

On the basis of the computational results, when the vertical and horizontal permeability of the anisotropic transversely isotropic deformable and undeformable pleats changes, we can observe that its stress-strain state will significantly influence on the flow rate of the horizontal well, because the flow rate of the horizontal well in 45° inclined angle layer with tiny pleats has the lowest value.
Table 1. The values (t/day) of the flow rate of horizontal well that has different length in the isotropic pleat

| $l$ | $Q_{an}$ | $Q_{nd}$ | $Q_d$ | $|Q_{an}-Q_{nd}|$ | $|Q_{an}-Q_d|$ | $|Q_{nd}-Q_d|$ | $|Q_{an}-Q_{nd}|/Q_{nd}$ |
|-----|---------|---------|-------|------------------|----------------|----------------|-----------------|
| 1   | 2.91    | 2.91    | 2.25  | 0                | 0.66           | 0.66           | 0.0000          |
| 2.4 | 6.98    | 6.99    | 5.39  | 0.01             | 1.59           | 1.6            | 0.0014          |
| 13.7 | 39.87  | 39.9    | 30.76 | 0.03             | 9.11           | 9.14           | 0.0008          |
| 20  | 58.2    | 58.25   | 44.91 | 0.05             | 13.29          | 13.34          | 0.0009          |
| 50  | 145.5   | 145.62  | 112.26| 0.12             | 33.24          | 33.36          | 0.0008          |
| 100 | 291     | 291.25  | 224.53| 0.25             | 66.47          | 66.72          | 0.0009          |

4. Conclusion

In conclusion, by the gained results the production of fluid is calculated which flows in the horizontal multilayer medium to the drift type multi-bore horizontal well. The stress-strain state of the anisotropic medium and the effect of filtration coefficient on the flow rate of the horizontal well were considered. The production of the well in the horizontal layer was researched.

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