In this work the relaxation processes of dense plasmas were studied. The relaxation rate of a Maxwellian velocity distribution function that has an initially anisotropic temperature \(T_\parallel \neq T_\perp\) is an important physical process in inertial confinement fusion plasmas. Relaxation characteristics in dense plasmas were studied on the basis of the effective potentials using the Coulomb logarithm. The effective potential is derived using the long wavelength expansion of the polarization function and quantum potential which takes into account the finite value of the interaction potential at close distance. In presented work temperature anisotropy relaxation processes in dense, non-isothermal plasma are considered. These interaction potential between particles take into account such collective effects as the ionization energy depression (reduction) and exchange-correlation effects. Therefore, this allowed us to examine the sensitivity of the computed relaxation time and the corresponding equilibrium plasma temperature on the quality of the description of the screening effect in dense plasmas.

**Key words:** Coulomb logarithm, dense plasma, effective potentials, inertial confinement fusion, temperature anisotropy relaxation.

**TEMPERATURE ANISOTROPY RELAXATION PROCESSES IN DENSE PLASMA**

...
Процессы релаксации температурной анизотропии в плотной плазме

В данной работе рассмотрены и изучены релаксационные процессы плотной плазмы. Скорость релаксации максвелловской функции распределения по скоростям, которая имеет изначально анизотропную температуру \( T_{\parallel} \neq T_{\perp} \), является важным физическим процессом в плазме инерционного термоядерного синтеза. Релаксационные характеристики в плотной плазме мы изучали на основе эффективных потенциалов с использованием кулоновского логарифма. Эффективный потенциал выводится с использованием длинноволнового разложения поляризационной функции и квантового потенциала, которое учитывает конечное значение потенциала взаимодействия на близком расстоянии. В представленной работе рассмотрены процессы релаксации анизотропии температуры в плотной неизотермической плазме. Этот потенциал взаимодействия между частицами учитывает такие коллективные эффекты, как снижение (уменьшение) энергии ионизации и обменно-корреляционные эффекты. Таким образом, это позволило нам изучить чувствительность вычисленного времени релаксации к параметрам взаимодействия на близком расстоянии. В работе приводится краткое описание модели и результаты расчетов процессов релаксации анизотропии температуры в плотной плазме. Чтобы показать правильность модели, ее результаты сравниваются с результатами \( \text{MD} \) - моделирования.

Ключевые слова: Кулоновский логарифм, плотная плазма, эффективные потенциалы, инерционный термоядерный синтез удержанием, релаксация температурной анизотропии.

Introduction

Intensive studies of the properties of dense non-ideal plasma were triggered by the idea of realization of inertial confinement fusion. It should be noted that it is especially important to study relaxation times of electrons and ions. In particular, during compression of a target by the flow of high-energy electrons the non-isothermal plasma with heated electrons and cold ions is created [1-2]. Non-isothermal plasma also appears during interaction of heavy ion beams with a target [3-4].

The temperature equalizes much faster within subsystems of electrons and ions than between electrons and ions. This is explained by a large mass difference between ions and electrons. Different methods are used to study relaxation processes in plasma, and among them there are the method of molecular dynamics (MD) [5-7] and quantum kinetic theory [8-14].

The relaxation of a temperature anisotropy is a canonical example of energy transport in dense plasmas [15-17]. For instance, plasmas that are preferentially heated, or cooled, in one direction will form a temperature anisotropy. Magnetized plasmas often have different energy confinement times either along or against the magnetic field, and can form a temperature anisotropy as a result [18].

In presented work temperature anisotropy relaxation processes in dense, non-isothermal plasma are considered. The relaxation characteristics are obtained on the basis of effective potentials. These interaction potential between particles take into account such collective effects as the ionization energy depression (reduction) and exchange-correlation effects [19]. Below we present a brief description of the model and the results of calculation of temperature anisotropy relaxation processes. To show the correctness of the model, its results are compared with the results of \( \text{MD} \) simulations.

Physical model

The relaxation rate of the electron-ion temperature, i.e., the rate of energy exchange, is determined by the difference of the average energy or temperature [17]:

\[
\frac{dT_{\perp}}{dt} = -\frac{1}{2} \frac{dT_{\parallel}}{dt} = -\nu(T_{\perp} - T_{\parallel}),
\]

where \( T_{\perp} \) and \( T_{\parallel} \) are the perpendicular and parallel temperatures, which related to the total temperature by \( T = \left( T_{\perp} + T_{\parallel} \right)/3 \) the anisotropy relaxation rate is
Temperature anisotropy relaxation processes in dense plasma

\[ \nu = \frac{3\sqrt{\pi}}{16} \left( 1 + \frac{2}{3} A \right)^{3/2} \]

\[ \times \int d\xi \xi^2 e^{-\xi^2} \frac{\sigma(2)}{\sigma_0} \times \left[ \frac{2}{3} \xi^2 \alpha \text{Erf} \left( \xi \sqrt{\xi A} \right) - \psi \left( \xi^2 \alpha A \right) \right] \]

where \( \sigma_0 = \pi e^4 / \left( 2k_B T \right)^2 , \xi^2 = u^2 / \left( 4k_B T / m \right) , \alpha \equiv T / T_{\perp} = 1/3(1 + A) , A \equiv \frac{T_{\parallel}}{T_{\perp}} - 1 \) is a measure of the temperature anisotropy, and \( \psi (x) \equiv \text{erf} \left( \sqrt{x} \right) - \frac{2}{\sqrt{\pi}} \sqrt{x} e^{-x} \) is the Maxwell integral. \( p \) momentum scattering cross section is

\[ \sigma^{(1)} = 2\pi \int_0^\infty db \left[ 1 - \cos^2 (\pi - 2\Theta) \right] \]

and \( \Theta = \pi - 2\Theta \) is the scattering angle, where

\[ \Theta = b \int_0^\infty dr \left[ 1 - b^2 / r^2 - \varphi (r) / \xi^2 \right]^{1/2} . \]

The Coulomb logarithm on the basis of the effective interaction potential of the particles is determined by the scattering angle of the pair Coulomb collisions. Introducing the centre of mass in the collision process the Coulomb logarithm reads [20-23]:

\[ \lambda_{ab} = \frac{1}{b_{\perp}} \int_0^{b_{\perp}^{\prime}} \sin^2 \frac{\Theta}{2} \, db , \]

where \( b_{\perp} = Z_aZ_b e^2 / (m_{ab} b^2) \), \( b_{\min} = \max \{ b_{\perp}, \chi_{ab} \} \) describes the minimum impact parameter, where \( \chi_{ab} = \frac{h}{\sqrt{2\pi m_{ab} k_B T}} \) is the thermal de Broglie wave length.

In formula (4) \( \varphi (r) \) is the interaction potential and \( r_0 \) is the distance of the closest approach for a given impact parameter \( b \) :

\[ 1 - \frac{\varphi_{ab} (r_0)}{E_c} - \frac{b^2}{r_0^2} = 0. \]

It is known that in order to correctly describe static and dynamic properties of plasmas the collective screening effect is to be taken into account. In this work the dense plasma is considered for which quantum effects must be taken into account at short distances. Further, the effective interaction potential which including such collective effects as the ionization energy depression (reduction) and exchange-correlation effects [18]:

\[ \Phi_{ab}(r) = \frac{Z_aZ_b e^2}{r} \left\{ \frac{1}{r^2 - \left( 2k_B T / \lambda_{ee} \right)^2} \right\} \times \left[ \left( \frac{1 - \lambda_{ee}^2 B^2}{1 - \lambda_{ee}^2 B^2} \right) \exp(-rB) - \left( \frac{1 - \lambda_{ee}^2 A^2}{1 - \lambda_{ee}^2 A^2} \right) \exp(-rA) + \right. \]

\[ + \frac{Ze^2 \exp(-r / \kappa_a)}{r (1 + C_e)} , \]

where \( C_e = \left( k_0^2 \lambda_{ee}^2 - k_1^2 \lambda_{ee}^2 \right) \left( \lambda_{ee}^2 / \lambda_{ee}^2 - 1 \right) \), \( \gamma^2 = k_i^2 + 1 / \lambda_{ee}^2 \), \( \kappa_{ee} = -b_1 / 4k_i^2 \) (here \( b_1 = \theta^{-1} I_{-3/2} (\eta) / (3I_{-3/2} (\eta)) \), \( k_{le} = k_i \), \( k_i^2 = k_{le}^2 + k_i^2 \), and \( k_i^2 = 4m_i e^2 / k_B T_i \).

Results and discussion

Figures 1-2 shows the parallel and perpendicular temperature profiles at two values of \( \Gamma \) in initial temperatures of \( T_{\parallel} = T_{\perp} / T = 1.15 \) and \( T_{\perp} = T_{\perp} / T = 0.923 (A = 0.2) \). Dashed lines show profiles predicted from the HNC approximation, circles - molecular dynamics simulations by S. Baalrud, J. Daligault [17]. The comparison shows good agreement with the predicted monotonically decreasing profiles at the \( \Gamma \) values. As the coupling strength increases, the rate of the relaxation remains well modeled.
Figures 1-2 demonstrate temperature relaxation profiles on the basis of the effective potentials and from MD simulations by S. Baalrud, J. Daligault [17] at $\Gamma = 0.1, 1$ conducted with a large initial temperature anisotropy of $T_{\perp}/T_{\parallel} = 10 (A = 9)$. The comparison with MD predictions is similar to what was observed at smaller initial anisotropy. The accuracy is, perhaps, slightly less than that observed from the small anisotropy cases in Figures 1-2 near $\Gamma \approx 1$, but the generally good agreement over this entire range of coupling strength provides strong evidence that the theory is robust even at a very large initial anisotropy. Dynamic screening apparently is not significant even at $T_{\perp}/T_{\parallel} = 10$.

Figures 3-4 illustrate the total temperature as a function of time for the same simulations. These show that, at weak coupling, the total temperature, and thus total kinetic energy, does not fluctuate more than approximately 0.1% throughout the evolution of the system. The higher $\Gamma$ values show that the temperature fluctuates in time by as much as 0.5%. These oscillations are not numerical artifacts, but rather represent oscillations in the exchange between kinetic and potential energy associated with correlations as the system relaxes. None of the theories discussed addresses the potential energy of the system, and thus does not model this effect.
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Figure 5 – Full temperature as a function of time at $\Gamma = 0.1$.

Figure 6 – Full temperature as a function of time at $\Gamma = 1$.

Figure 7 – The rate of anisotropy temperature relaxation in comparison with different theoretical approximations.

Figure 7 show the results of the relaxation rate on the basis of the effective potentials and obtained from the MD data using the fitting procedure, and theoretical approximations. The initial temperature anisotropy for this dataset was the same $A = -0.2$ as from the data shown in Figures 1-2.

As expected, the weakly coupled theories compare well with our results and the MD data at sufficiently small coupling strength ($\Gamma \leq 0.1$). There is apparently no significant advantage to accounting for dynamic screening at these conditions. For $\Gamma \leq 0.1$, the relaxation rate predicted by all of the theories is slightly larger than what is observed in the MD simulations (e.g., by approximately 20% at $\Gamma = 0.05$). This is likely associated with the assumption made in the theoretical analysis that the distributions maintain the anisotropic Maxwellian form. It is well known in other transport processes that distortions of the distribution function contribute an order unity correction to the transport rates at weak coupling. This is accounted for in hydrodynamic theories, such as Chapman–Enskog.

This comparison between two common approaches to plasma kinetic theory and MD simulations has suggested a few general conclusions with regard to the temperature anisotropy relaxation, and has also revealed a few
remaining gaps in current understanding. The data comparison suggests that dynamic screening, which is modeled in the Lenard–Balescu and Generalized Lenard–Balescu theories, does not significantly influence the relaxation rate. The Boltzmann-based approaches (Landau or Effective Potential Theory) can be accurately applied in most situations of practical interest, rather than Lenard–Balescu based theories which are comparatively difficult to evaluate. Comparison with MD revealed that Effective Potential Theory provides an accurate approach to modeling the anisotropy relaxation rate over a similar range of coupling strength as has been encountered for other processes, such as diffusion.

Conclusion

The relaxation processes in dense plasmas were studied on the basis of effective interaction potentials taking into account quantum effects of diffraction at short distances and screening at large distances. The results obtained for the Coulomb logarithm and temperature relaxation times for different plasma parameters are consistent with the results of other authors. Thus, knowledge of the values of the transport coefficients of heavy, charged particles in the plasma will help to more accurately calculate the design of thermonuclear target. The method of effective interaction potentials is important as it gives a deeper insight into physics of dense plasmas and can provide an effective tool for the fast and accurate calculation of various physical properties for future technological applications [24-25].

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