Fradkin-Shenker Continuity and “Instead-of-Confinement” Phase

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Abstract

In 1979 Fradkin and Shenker observed \cite{FradkinShenker79} that if one considers a Yang-Mills theory fully Higgsed by virtue of scalar fields in the fundamental representation of the SU($N$) gauge group there is no phase transition in passing from the Higgs regime (weak coupling) to the “QCD confinement” regime at strong coupling. The above two regimes are continuously connected. We combine this observations with lessons from supersymmetric gauge theories which show that the Higgs phase is continuously connected to what is called “instead-of-confinement” phase rather than the phase with quark confinement. In the “instead-of-confinement” phase monopoles are confined and play a role of “constituent” quarks inside hadrons. In contrast, the Seiberg-Witten phase of quark confinement is not analytically connected to the Higgs phase.

We propose dedicated lattice studies of Yang-Mills theories with scalar quarks.
1 Introduction

Yang-Mills theory and its QCD versions are strongly coupled; their large distance dynamics defies analytic solutions for four decays. One of possible strategies of obtaining certain insights in the strong coupling regime is to introduce an extra adjustable parameter which one could continuously vary from weak coupling to strong coupling.

In this paper we re-visit the so-called Fradkin-Shenker continuity [1] – a continuity between the strong and weak coupling (Higgsed) regimes in Yang-Mills theory in four dimensions. We will confront this picture with lessons from recent supersymmetry-based results [2,3], more exactly, some qualitative aspects ensuing from these results. To this end non-supersymmetric Yang-Mills theory will be Higgsed in precisely the same way as the supersymmetric theory from which we draw our inspiration.

Needless to say, in non-supersymmetric Yang-Mills models powerful tools for obtaining exact results based on holomorphy are lost and we cannot make fully quantitative predictions. However, we expect the overall qualitative picture emerging in supersymmetric QCD to be preserved when supersymmetry is lifted. This will allow us to get a better understanding of QCD-like theories at strong coupling. This understanding turns out to be rather surprising.

Fradkin-Shenker picture motivated by lattice theory is as follows: if one introduces Higgs fields in the fundamental representation in such a way that all gauge fields are Higgsed, and then studies the theory as a function of the vacuum expectation value \( v^2 \), then at positive \( v^2 \gg \Lambda^2 \) (\( \Lambda \) is the dynamical scale of the theory) one has weak coupling regime, while diminishing \( v^2 \) and crossing the line \( v^2 < \sim \Lambda^2 \) one attains a strong coupling regime similar to “QCD confinement,” i.e. confinement with dynamical light (scalar) quarks. Then further downward evolution of \( v^2 \) (i.e., making \( v^2 \) vanishing or negative with \( |v^2| \ll \Lambda^2 \)) will change nothing. No phase transition is supposed to occur; there are no two distinct phases – Higgs vs. confinement – because there is no order parameter to distinguish them. One always has perimeter law rather than area law. Fradkin and Shenker suggested a picture of a single Higgs-confinement phase, with the perimeter law for the Wilson loop on both sides of the weak-strong coupling regimes.

Let us explain what is meant above by “QCD confinement.” With light scalar quarks in the fundamental representation even at strong coupling there is no confinement in the strict sense of this word: large contours are screened and no Wilson area law emerges. The quark pair creation breaks bona fide strings. The strings are not infinitely long. Nevertheless we can guess that they are there because if quarks have flavor quantum numbers, in addition to color, we do not observe these quantum
numbers in the mesonic spectrum, only those which are inherent to quark-antiquark bound pairs are asymptotic states. Even if the quarks are not light, strictly speaking the area law is recovered only in the limit $m \to \infty$, where $m$ is the quark mass term.

It is universally believed that replacing the light scalar quarks by light Dirac quarks in the same fundamental representation does not change the essential aspects of the above picture.

If solution of supersymmetric QCD can be used as an indication for non-supersymmetric version, this general belief may not be correct. “QCD confinement” regimes for theories with scalar and spinor quarks could be different, as is clearly different the confinement regime in pure Yang-Mills in which strings are unbreakable and the area law applies.

In this paper we compare the Fradkin-Shenker continuity with the picture obtained in $\mathcal{N} = 2$ supersymmetric QCD (SQCD). First we briefly review the Fradkin-Shenker description of non-supersymmetric version of Yang-Mills theory endowed with the Higgs sector described in detail in Sec. 2. This theory has the action similar to the bosonic part of the action of $\mathcal{N} = 2$ SQCD.

In the Higgs regime (positive $v^2$) at weak coupling ($v^2 \gg \Lambda^2$) color charges are screened through Higgsing. The residual flavor symmetry of the model is $\text{SU}(N)_{\text{global}}$. There is no long-range forces in the theory and no bound states. Asymptotic states can be read off from the Lagrangian. All gauge bosons become massive and are in the adjoint of the $\text{SU}(N)_{\text{global}}$. The scalar quarks which used to be fundamentals of the global group before Higgsing become $\text{SU}(N)_{\text{global}}$ adjoints after Higgsing.

As we diminish $v^2$ it approaches $v^2 \lesssim \Lambda^2$, and may cross zero and become negative. Let us assume that $v^2$ freezes at a value below $\Lambda^2$. Then we are in the strong coupling regime.

At this point we use the picture obtained in $\mathcal{N} = 2$ SQCD. Exact results obtained from supersymmetry show that at strong coupling this theory does not go into the confinement phase. Instead it ends up in a novel phase, which we called “instead-of-confinement” [4, 5], see [3] for a review. In this phase there is no confinement of quarks – quarks and gluons screened at weak coupling evolve at strong coupling into monopole-antimonopole pairs confined by strings. The role of constituent quarks inside hadrons is played by monopoles which carry appropriate flavor quantum numbers. This phase is qualitatively rather similar to what we observe in real-world QCD. At the same time, we show that the Seiberg-Witten phase of quark confinement is not analytically connected to the Higgs-“instead-of-confinement” phase in SQCD.

We believe that qualitatively the above picture may be generic rather than spe-

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1Baryons are not seen at large $N$. 
pecific to SQCD. Supersymmetry just helps us to solve the theory. We suggest that non-supersymmetric scalar QCD and $\mathcal{N} = 2$ supersymmetric QCD are in the same universality class. In particular, we expect that non-supersymmetric scalar QCD continuously evolve from the Higgs phase at large $v^2$ to the “instead-of-confinement” phase rather than to the quark confinement phase at small $v^2$. The discussion below may be interpreted as an example that light scalar quarks vs. light Dirac quarks coupled to Yang-Mills at strong coupling are not as close theories as they were thought of.

The presence of a parameter which interpolates between weak and strong coupling regimes quite often is considered as a tool to reveal physics at strong coupling. One may hope that if it is independently known that there is no phase transitions on the way, one can carry out analytic analysis at weak coupling and then extrapolate – qualitatively or semi-quantitatively – the picture thus obtained to strong coupling. However, one must clearly understand what particular theory is obtained after the analytic continuation.

We stress that although there is no phase transition between weak and strong coupling regimes in $\mathcal{N} = 2$ SQCD (in accordance with the Fradkin-Shenker continuity) there is a crossover. In particular, light “fundamental” fields of the dual theory describing physics at small $v^2$ – “dual quarks” – are represented by solitonic states in terms of quarks and gluons of the original theory. In this sense the hope that if there is no phase transition on the way then one can straightforwardly extrapolate the weak coupling picture to strong coupling may turn out an illusion.

The paper is organized as follows. In Sec. 2 we review the scalar QCD at weak coupling and in Sec. 3 comment on the large $N$ limit. In Sec. 4 we discuss the picture obtained from $\mathcal{N} = 2$ supersymmetric QCD and in Sec. 5 we comment on the Seiberg-Witten confinement phase. Sec. 7 contains our conclusions.

## 2 The basic set up

The basic non-supersymmetric model has the form

$$\mathcal{L}_0 = -\frac{1}{4g^2} F_{\mu\nu}^p F_{\mu\nu}^p - \frac{1}{4g_1^2} F_{\mu\nu} F_{\mu\nu} + \sum_A |D_\mu \phi_A|^2 - V(\phi),$$

where $p$ is the adjoint index of the color SU($N$), $(p = 1, 2, ..., N^2 - 1)$, $k$ is the fundamental index of the color SU($N$), $k = 1, 2, ..., N$, and the subscript $A$ labels the fields of the scalar sector, $A = 1, 2, ..., N$. The gauge symmetry in (1) is $\text{U}(N)$ =
The covariant derivative is
\[ D_\mu = \partial_\mu - iA_\mu - iT^p A^p_\mu, \] (2)
where \( A_\mu \) and \( A^p_\mu \) are U(1) and SU(N) gauge potentials, while \( T^p \) are generators of the color SU(N). To make our discussion simpler we assume that U(1) and SU(N) coupling constants \( g_1^2 \) and \( g_2^2 \) are related
\[ g_2^2 = \sqrt{\frac{N}{2}} g_1^2 \] (3)
(although this assumption could be lifted). The scalar sector is assumed to have a global (flavor) SU(N) symmetry labeled by the index \( A \) (see Eq. (1)) so that the potential \( V(\phi) \) must be chosen appropriately. Our choice will be motivated by supersymmetric field theory (see e.g. [2]),
\[ V = \lambda_1 \sum_p \left( \sum_A \bar{\phi}^A T^p \phi_A \right)^2 + \lambda_2 \left( \sum_A \bar{\phi}^A \phi_A - Nv^2 \right)^2. \] (4)
Here \( \lambda_{1,2} \) are constants. If \( \lambda_{1,2} \sim g^2 \), then (4) represents a (somewhat reduced) bosonic sector of the supersymmetric theory. For what follows it is convenient to introduce \( N \times N \) matrix \( \Phi = \{ \phi^i_A \} \) which is constructed of \( N \) columns
\[ \Phi \leftrightarrow \{ \phi^k_1, \phi^k_2, \ldots, \phi^k_N \} \] (5)
where the subscript marks flavor while the superscript \( k \) refers to color (in the fundamental representation). In this notation
\[ L_0 = -\frac{1}{4} F^\mu_{\nu, p} F^{\mu, \nu, p} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \text{Tr} (D_\mu \Phi)^\dagger (D^\mu \Phi) - V(\phi), \]
\[ V = \lambda_1 \sum_p (\text{Tr} \Phi^\dagger T^p \Phi)^2 + \lambda_2 \left[ \text{Tr} (\Phi^\dagger \Phi - Nv^2) \right]^2. \] (6)
Now, it is perfectly clear that if \( v^2 \) is positive \( \Phi \) develops the diagonal expectation value of the form
\[ \Phi_{\text{vac}} = \begin{pmatrix} v & 0 & \cdots & 0 \\ 0 & v & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & v \end{pmatrix} \] (7)
(up to irrelevant gauge transformations). This form of the vacuum expectation values (VEVs) was suggested long ago [6].

All \( N^2 \) gauge bosons of the original \( U(N) \) gauge theory are Higgsed, acquiring one and the same mass \( M_W = g v \). All \( N^2 \) real physical Higgses acquire masses, \( N^2 \) “phases” are eaten up by gauge bosons.

Equation (7) shows that in the Higgs regime both the local gauge and global flavor groups are spontaneously broken, but the diagonal \( SU(N)_{C+F} \) survives as the exact global symmetry of the model. The \( M_W = g v \) vector bosons form the singlet and adjoint representations of the above global group, so we will call them \( W \) bosons. They can be defined in the gauge invariant form as follows:

\[
g(W_\mu)^A_B = i c \left( \bar{\phi}^A_i \overset{\leftrightarrow}{D}_\mu \phi^i_B \right)
\]

where \( c \) is a normalization factor. Their effective low-energy self-interaction of dimension three and four is similar to that of gluons in the microscopic Lagrangian. The physical Higgs bosons

\[
H^B_A = \tilde{c} (\bar{\phi}^B_i \phi^i_A - v^2 \delta^B_A)
\]

form a singlet and adjoint representations with respect to the exact global symmetry \( (N^2 \) real fields altogether). Here \( \tilde{c} \) is another normalization factor. The interaction vertices between the physical Higgses in the adjoint representation of \( SU(N)_{C+F} \) and the massive \( W \) bosons are proportional to \( \text{Tr}(\partial^\mu H_\mu H + \ldots) \). Overall, the effective low-energy theory in the Higgs regime looks like Yang-Mills with massive gauge bosons. Of course, it is not meant to calculate loops.

At large \( v \) the spectrum of the theory consists of vector and scalar particles specified above. Of course, we can combine a number of these particles to create states with arbitrary spins, angular momentum, and belonging to various representations of \( SU(N)_{C+F} \). They will not be bound states, however, because bound states do not form for arbitrarily weak coupling constant unless there are long range forces, absent in our case.

One type of non-perturbative objects discussed at weak coupling (in the Euclidean space) are instantons [7,8]. Note that the instanton action as well as the mass of the confined monopole-antimonopole pair (see Sec. 4) scale as \( N \) at large \( N \). The Euclidean instanton is a reflection of the existence of non-perturbative quasiclassical objects in the Minkowski space.\(^2\)

\(^2\) A remark about renormalons in passing. Occurrence of infrared renormalons in Yang-Mills theories [9] (for a recent review [10]) is a formal indicator for \( N \)-independent nonperturbative
3 Large-\(N\) limit

The model introduced in Sec. 2 has a significant advantage over many other models in the Higgs regime. Namely, it allows one to consider a smooth large-\(N\) limit, analogous to the 't Hooft limit. More exactly, since the number of flavors in the given model is equal to the number of colors, here we deal with the Veneziano limit \[11\].

It is clear that the dynamical scale parameter \(\Lambda\) which is kept fixed as \(N \to \infty\) is now replaced by the mass \(M_W = g v\) of the vector bosons. Therefore the scaling law is

\[\sum_{n} a_n (\alpha(Q^2))^n, \quad a_n \sim n! \text{ at } n \gg 1\]

Under this scaling both the BPST part of the instanton action and the 't Hooft part scale as \(N\), and so does the sphaleron mass. Thus, we must be careful with interchanging limits of large energies and large \(N\). We may keep in mind a large value of \(N\) but \(N \neq \infty\). In principle, \(N\) does not have to be large in what follows.

4 Microscopic picture from supersymmetric QCD

QCD with \(N\) flavors of scalar quarks discussed above is not yet solved at strong coupling. Our goal now is to compare the Fradkin-Shenker continuity in the model at hand with the results obtained from our previous considerations of \(\mathcal{N} = 2\) supersymmetric models, in which the microscopic picture of the underlying phenomena can be revealed, see \[2, 3\] for reviews. The regime which may be closest to what we see in real-world QCD at strong coupling has been studied in \[4, 5\] where it is referred to as “instead-of-confinement.” Why “instead” will become clear shortly.

We will narrow down a class of theories considered in \[3, 4\] to a subclass with the bosonic sector coinciding with that of Sec. 2. Namely, we will discuss theories with the U(\(N\)) gauge group which have equal numbers of colors and quark hypermultiplets,
i.e., $N_f = N$, in the so-called quark vacuum. In this vacuum at large $v^2$ all $N$ squark flavors condense through the potential similar to (1), so that color-flavor locking takes place. The Fayet-Iliopoulos $D$-term is not introduced. Instead, we introduce equal mass terms $m$ to all flavors. Each flavor consists of complex scalar superfields $q^{kA}$ and $\bar{q}_{kA}$ where $A$ is the flavor index, while $k$ is the color index (in the fundamental representation); both run from 1 to $N$. Each complex scalar field has a Weyl superpartner. All quarks have the same mass $m$.

In addition, a mass term $\mu$ is introduced for the adjoint scalar superfield, generally speaking breaking $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. However, the deformation superpotential

$$W_{\text{def}} = \mu \text{Tr} \left( \frac{1}{2} A + T^a A^a \right)^2,$$

(11)
does not break $\mathcal{N} = 2$ supersymmetry in the small-$\mu$ limit, see [12–14]. The fields $A$ and $A^a$ in Eq. (11) are chiral superfields, the $\mathcal{N} = 2$ superpartners of the U(1) and SU($N$) gauge bosons. We will treat $\mu$ as a free parameter which can be taken arbitrarily small. At small $\mu$ the deformation superpotential reduces to the Fayet-Iliopoulos $F$-term.

4.1 Weak coupling: Higgs regime

At weak coupling the gauge group is fully Higgsed, the squark VEVs are given by (7) with the identification

$$q^{kA} = \bar{q}^{kA} = \frac{1}{\sqrt{2}} \phi^{kA}.$$

(12)
The role of $\lambda_1, 2$ in Eq. (4) is played by $g^2$,

$$\lambda_1 = \frac{g^2}{2}, \quad \lambda_2 = \frac{g^2}{8} \sqrt{\frac{2}{N}},$$

(13)
while

$$v^2 = |\mu m|,$$

(14)
where $m$ is a quark mass. In SQCD $v^2$ is always positive. In non-supersymmetric theory of Sec. 2 the value of $v^2$ can be both positive and negative.

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3 Generally speaking the parameters $\mu$ and $m$ may have phases. The ansatz [12] assumes that this phases are rotated away, which one can always do.
The unbroken global symmetry of this model is

$$\text{SU}(N)_{C+F}.$$  \hfill (15)

The global SU($N$)$_{C+F}$ symmetry is responsible for the formation of the non-Abelian strings \cite{14\text{17}} with the world-sheet theory described by the $\mathcal{N} = 2$ CP($N-1$) model. In the limit $\mu \to 0$, to the first order in $\mu$, these strings are BPS saturated and their tension is

$$T = 2\pi v^2.$$  \hfill (16)

Note that at large $N$ the scaling of $v^2$ is given by Eq. (10).

What is the spectrum of this theory at weak coupling?

All states come in representations of the unbroken global group (15), namely, in the singlet and adjoint representations of SU($N$)$_{C+F}$, as was mentioned in Sec. 2. The perturbative states are screened quarks and gauge bosons, supermultiplets containing $H^B_A$ and $W^B_A$. There are also non-perturbative states, see \cite{2} for a detailed review.

Since the large-$v^2$ the theory is in the Higgs regime for squarks, non-Abelian strings confine monopoles. In fact in the U($N$) gauge theories non-Abelian strings are stable, a string cannot terminate on a monopole. Instead, the monopoles become junctions of two distinct non-Abelian strings. This leads to occurrence of mesons formed by monopole-antimonopole pairs confined by non-Abelian strings \cite{2}, see Fig. I.

The flavor quantum numbers of stringy monopole-antimonopole mesons were studied in \cite{18} in the framework of an appropriate two-dimensional CP($N-1$) model which describes world-sheet dynamics of the non-Abelian strings \cite{14\text{17}}. In particular, confined monopoles are seen as kinks in this world-sheet theory. The kinks belong to the fundamental representation of the global SU($N$)$_{C+F}$ \cite{19}. Therefore, if two strings in Fig. I are “neighboring” each meson is in the two-index representation $M^B_A$ of the flavor group, where the flavor indices are $A, B = 1, ..., N$. It splits into the singlet and adjoint representations of the global unbroken group SU($N$)$_{C+F}$.
At weak coupling the stringy mesons are heavy. Their masses are given by the product of the string tension times the length of the string \( M \sim v^2 L \). The string length is bounded from below by the string width, which is determined by masses of the bulk excitations forming the string, \( L \gtrsim 1/gv \). This gives

\[
M \sim \frac{v}{g}.
\]

The stringy mesons are highly unstable and decay into perturbative (screened) quarks and gluons \( H^B_A \) and \( W^B_A \) which are lighter at large \( v \), with masses \( \sim M_W = gv \). The stringy mesons may be related to the sphaleron solution [20].

4.2 Strong coupling: “instead-of-confinement” regime

Now let us pass to strong coupling, i.e. the domain of small \( v^2 \). In non-supersymmetric theories such as scalar QCD [13] this step cannot be carried out analytically. This is the point where supersymmetry becomes important. More exactly, we exploit the exact Seiberg-Witten solution on the Coulomb branch [21] in our theory. We start at large \( v^2 \sim |\mu m| \) (in the large-\( m \) limit; all quark masses are equal) and then go to the equal mass small-\( v^2 \) limit via the domain of large \( \Delta m \sim \Delta m_{AB} \equiv (m_A - m_B) \).

Now let us have a closer look at the spectrum and other features of the strong coupling (small \( v^2 \)) regime. We will deduce these features from the dual theory. In accordance with the general Fradkin-Shenker continuity there is no phase transition on the way from weak to strong coupling, just a crossover. Therefore the global symmetry group of the dual theory is the same as in the original one, \( SU(N)_{C+F} \), see Eq. (15).

As was shown in [4, 5] (see also [3]) the domain of small \( |v^2| \) can be described by a weakly coupled dual theory with the gauge group \( U(1)^N \). Needless to say, this theory is infrared free. The matter fields are light “dual quarks” \( D^\gamma \). The index \( \gamma = 1, \ldots, N \) numbers \( D \)'s. Altogether there are \( N \) such fields, all are singlets with respect to the global \( SU(N)_{C+F} \), symmetry. The \( \gamma_i \)-th \( D \) is charged with respect to the \( \gamma_i \)-th \( U(1) \) gauge group from the set \( U(1)^N \) mentioned above.

At strong coupling the dual quarks condense [4]. The \( U(1)^N \) dual gauge group is completely Higgsed. This leads to the formation of confining strings of the Abelian Abrikosov-Nielsen-Olesen (ANO) type [24] in each of gauge \( U(1) \) factor. Since \( D^\gamma \)

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4The \( U(1)^N \) gauge group for the dual theory was first obtained by Seiberg and Witten [21, 22] in the monopole vacua, see also [23].

5In fact these strings can still be understood as vacua of the world sheet \( CP(N-1) \) model upon continuation to small \( v^2 \) [18].
color charges are identical to those of diagonal squarks from $q^{kA}$ these ANO strings confine monopoles [1].

The fact that $D^\gamma$’s are singlets with respect to the global SU($N$) group shows that the “dual quarks” are not the quarks of the weakly coupled theory: the latter are in the adjoint representation of the global SU($N$)$_{C+F}$.

Thus, the “dual quarks” and quarks are distinct states. At large $v^2$ “dual quarks” are heavy solitonic states. However below the crossover at small $v^2$ they become light and form the fundamental “elementary” states $D^\gamma$ of the dual theory.

This raises the question: what happens to quarks when we reduce $v^2$?

They find themselves in the “instead-of-confinement” phase. The Higgs-screened quarks $H_A^B$ and gauge bosons $W_A^B$ at small $v^2$ decay into the monopole-antimonopole pairs on the curves of marginal stability (the so-called wall crossing) [4, 18]. The latter are present in $\mathcal{N} = 2$ supersymmetric QCD [21, 22, 25].

At small non-vanishing values of $v^2$ the monopoles and antimonopoles produced in the “decay” of $H_A^B$ and $W_A^B$ cannot escape from each other because they are confined. Therefore, the (screened) quarks and gauge bosons evolve into stringy mesons similar to those which appear at weak coupling, see Fig. 1 namely monopole-antimonopole pairs connected by two confining strings [3, 4].

The picture of the crossover is schematically shown in Fig. 2. The left- and right-hand sides of this figure correspond to large and small values of $v^2$ respectively. In $\mathcal{N} = 2$ SQCD squarks are light at large $v^2$. They evolve into monopole-antimonopole stringy mesons at small $v^2$. Moreover, heavy (unstable) monopole-antimonopole stringy mesons present at large $v^2$ become light at small $v^2$ and form “fundamental” charged matter of the dual theory, namely “dual quarks.” Screened “dual quarks” (they are singlets with respect to global the SU($N$)$_{C+F}$) form glueballs.

To summarize at small $v^2$ the monopole-antimonopole stringy mesons in the two-index representations of SU($N$)$_{C+F}$ are descendants from (screened) quarks and gauge bosons of the original theory. We see that these mesons have “correct” adjoint or singlet quantum numbers with respect to the global group, like mesons in the real-world QCD. Singlets are interpreted as glueballs.

Moreover, because these mesons are formed by strings they lay on Regge trajectories.

Thus, the monopole-antimonopole mesons of the instead-of-confinement phase are qualitatively similar to mesons of the real-world QCD. The role of QCD constituent quarks is played by monopoles. If one could scatter external probes off such mesons one would see a gluon and scalar quarks in the deep inelastic limit.

Note also that baryons are represented by “monopole necklaces” formed by close string configurations with $N$ confined monopoles attached [2]. However, what is
Figure 2: A schematic picture of the crossover from large to small $v^2$. Big closed circle denotes light quarks, while big open circle denotes light “dual quarks”. Monopole-antimonopole mesons are shown as in Fig. 1. Large values of $v^2$ are represented by the left-hand side, while small values by the right-hand side.

usually called the baryonic U(1) symmetry is a part of the broken gauge group in our U($N$) SQCD. Therefore baryons are unstable and decay into mesons.

5 Seiberg-Witten confinement: no continuity

Now let us comment on the Seiberg-Witten confinement. It occurs in the so-called monopole vacua of $N = 2$ SQCD. In this vacua monopoles condense at non-zero $\mu$ and the ANO strings with color-electric fluxes are formed. These strings confine quarks \[21, 22\].

Let us stress that this happens in the monopole vacuum which is a different vacuum as compared to the quark vacuum which we considered in the previous sections. The quark vacuum is defined as the vacuum where at large quark masses $m \gg \Lambda$ all $N$ squark VEVs are large $v \sim \sqrt{|\mu m|}$, see \[7\]. In the monopole vacuum squarks classically do not condense, while the monopole VEVs are much smaller, $\sim \sqrt{\mu \Lambda}$. The monopole vacuum does not have a weak coupling description in terms of the original theory.

The monopole vacuum is not analytically connected to the quark vacuum. To see that this is the case note that the gluino condensate is identically zero in the quark vacuum while in the monopole vacuum it is a non-trivial function of the quark mass $m$ known exactly \[26\]. Thus to jump from the quark vacuum at weak coupling to the monopole vacuum at strong coupling we need to break analyticity, i.e. this passage involves a phase transition.
6 If SQCD results shed light on QCD

If the picture that was derived in the quark vacua of $\mathcal{N} = 2$ SQCD shares qualitative features with nonsupersymmetric QCD then we have to conclude that scalar and spinor QCD are not as close as was believed. The former has an obvious weak coupling regime. At strong coupling mesons of the scalar QCD are formed by “constituent scalar quarks” which are color-magnetically charged monopoles attached to each other by virtue of confining strings. On the contrary, in the spinor QCD we expect the Seiberg-Witten confinement scenario when quarks themselves are attached to each other by virtue of the confining strings.

Moreover, in the nonsupersymmetric theory discussed in Sec. 2, in contradistinction to SQCD, one can consider negative values of $v^2$ in Eq. (4). At negative $v^2$ the scalar quarks do not condense. When we increase $|v^2|$ keeping $v^2$ negative, the mass of the scalar quark increases. One can expect that its back reaction on the vacuum becomes suppressed. At a certain point the string that develops between the heavy scalar quark and its antiquark should be the same as that in pure Yang-Mills theory. Presumably, this would require a phase transition.

The distinction which our hypothesis outlines is hard to detect analytically. However, lattice studies could help verify it. We already know that spinor quarks undergo a chiral phase transition, which cannot happen in scalar QCD due to the absence of the chiral symmetry.

7 Conclusions

In this paper we revisited the Fradkin-Shenker continuity. We assumed that lessons from supersymmetry can prompt us how this continuity is implemented in scalar QCD. If so, we conclude that the Fradkin-Shenker continuity connects the Higgs and the “instead-of-confinement” regimes. In the latter regime the quarks and gauge bosons screened at weak coupling evolve at strong coupling into the monopole-antimonopole pairs confined by strings. This regime is qualitatively rather similar to what we observe in real-world QCD. Supersymmetry also teaches us that the Seiberg-Witten phase of quark confinement is not analytically connected to the Higgs phase.

Summarizing, we expect that non-supersymmetric scalar QCD continuously evolves from the Higgs regime at large $v^2$ to the “instead-of-confinement” regime at small $v^2$, rather than to the quark confinement phase. This passage occurs through a crossover.

The “instead-of-confinement” regime heavily relies on the presence of confined
monopoles. One may wonder how this phase can appear in the theory. Classically the 't Hooft-Polyakov monopoles are present in theories with the adjoint scalars which develop VEVs, as $\mathcal{N} = 2$ SQCD. However, at the quantum level the story becomes more subtle. It was shown that confined monopoles (seen as kinks in the CP($N - 1$) model on the world sheet of the non-Abelian string) can survive the limit when the adjoint scalars are decoupled, see the review paper [2] and the recent publication [27].

If we believe that SQCD gives us hints on the strong coupling dynamics in non-supersymmetric Yang-Mills theory, we can arrive at a general conclusion that the phase transition “perimeter law vs. area law” is not the only characterization of confinement vs. non-confinement; other crossover transitions and phase transitions exist too.

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