Methodical uncertainty in the two-point measurement of parts prismatic elements linear dimensions

V I Glukhov, V V Shalay

Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia

Abstract. The article is devoted to the topical problem of the reliability of measurements of geometrical specifications of technical products. The purpose of this article is to improve the quality control of the linear dimensions of the parts by the two-point measurement method. The goal of the article is the study of advanced methodical uncertainties of measurement of linear dimensions of plane-parallel prismatic elements. The method of research is geometric modeling of deviations of the element surfaces form and position in the rectangular coordinate system. The research was carried out for various executive elements with regard to their information content corresponding to the classes of kinematic pairs in the theory of mechanics and the number of constrained degrees of freedom in function of the basic elements. Prismatic elements with informative content 3, 2, 1 and 0 (zero) were studied. Uncertainty assessment of two-point measurements were made by comparing the measurements results of linear dimensions and functional dimensions of the element maximum and minimum material constrained by the limits of maximum and minimum. As a result of studies of the two-point measurements accuracy, it is found that the methodical uncertainty is formed when measuring the average size of elements for all types of form deviations. The two-point measurement method cannot account for deviations of the location dimension of the element and therefore its use for items with less than the maximum informativeness creates unacceptably large methodical uncertainties of measurement the maximum, minimum and medium linear sizes. The same methodical uncertainties occur under the arbitration control of the parts linear dimensions by the limit two-point gauges.

Keywords – Technical measurements, linear sizes, elements informativeness, methodical uncertainty.

1. Introduction
The International Organization for Standardization technical Committee TC213 “Dimensional and geometrical product specifications and their control” updated all terminology to linear and angular values at the level of international standards [1-5] in the first decade of the 21st century without convincing justification. Millions of engineers – users of standards in all processes of the product life cycle – designers, technologists, metrologists, control masters, university and college teachers found themselves in a difficult position. New terms and definitions did not specify or improve old terms that served for 25-50 years. The new terms abolished the old engineering ones and replaced them by more complex terms [6]. Still, between the new and old terminology there must be continuity in the development and improvement of terms, including on the following unresolved problems:
– there is no classification of geometric specifications for a functional purpose, coordinating dimensions of the elements of parts are not introduced; geometric and kinematic specifications of parts connections are not analyzed;
– coordinate systems for measuring the geometrical specifications of parts, parts elements, and assembled products are not applied; despite the fact that the design of electronic products in the computer-aided design is carried out in a three-dimensional rectangular 3d coordinate systems, the production of components of products provided by multi-axis CNC machines and the control - by coordinate measuring machines;
– the engineering principles for ensuring the quality of products in terms of the geometric characteristics accuracy are not used: the principle of inversion, the principle of the unity of design, technological and measuring datums, the principle of linear-angular two-dimensionality of geometric specifications, and the principle of geometric elements informativeness [7];
– systematic functional deviations of the component surfaces shape parts are considered as random variables and are estimated by statistical characteristics.

2. Statement of the problem
The possibility of reliable measurements of the size and shape of the parts is essential for the precise products manufacturers [8]. Know-how – to know how to measure – is a basis of knowledge about the measurements required by a designer, a technologist and a metrologist [9]. The evaluation of the uncertainty in measuring geometric characteristics is a prerequisite for products quality control [10]. Technical measurements make it possible to evaluate the part quality by the accuracy of geometric specifications in the production process. The quality evaluation is carried out by controlling the dimensional and geometric accuracy of the component parts to meet the requirements of the design project. The transition of international GPS standards to the evaluation of the dimensional accuracy of parts based on the average size of elements using the two-point method requires estimation of the measurements uncertainty, which is the aim of this work.

The specific problem of evaluating the influence of form and an joints deviations of cylindrical dimensional elements surfaces of various informative content is stated and solved on the expanded uncertainty of measurements of average, maximum and minimum linear sizes of elements by the two-point method.

3. The theory of elements functional linear sizes
Any part of a technical product consists of geometric elements, which are parts of the material of the part, constrained by one or several geometric surfaces, primarily flat and cylindrical. Every part element performs a specific function. The set of main design datums performs the functions of positioning the part in the product and identifying its generalized coordinate system[7]. sliding joints, the set of main datums performs the third function - it provides the kinematics of rotation and (or) the translational movement of the product part with the fit gap. The set of auxiliary design datums performs the functions of positioning the attached part to the considered one and identifying the auxiliary coordinate system for the attached part. The actuating elements perform the working functions of the part. Dimensional elements determine the overall dimensions of the part, and free elements with dimensions without tolerances perform the functions of connecting elements that unite all the geometric elements into a single solid body of the part.

The geometric element datum may constrain the detail one to six degrees of freedom (up to three linear "t" and three angular "r") that in total is element informativeness specification "c" and coincides with the class of kinematic pairs in theoretical mechanics. Maximum informativeness of the flat datum of the prismatic element is equal to three (3c) and consists of a single linear constraint (1t) normal datum-wise and two angular (2r) – around the two perpendicular directions in the plane of the datum (3c=1t+2r).

Datums are combined into sets of datums to constrain the part of the six degrees of freedom (6c = 3t + 3r) and form a generalized or auxiliary coordinate system in which the dimensions and coordinates of the elements should be measured. The parts are joined on sets of datums in order to combine the
generalized coordinate system of the attached part with the auxiliary coordinate system of the base part.

The surfaces of all geometric elements have shape deviations in the form of alternating smooth harmonic protrusions and cavities. In addition, elements with informative content less than the maximum have protrusions and cavities of surfaces in the form of location deviations. Surface protrusions form the elements datums. Fits in the joints of the shafts and holes are formed by protrusions of the elements real surfaces. Consequently, the dimensions of the elements along the projections of real surfaces are the functional maximum material dimensions of the elements.

The cavities of the elements real surfaces constrain the shape and location deviations, and together with the micro-roughness, they are the containers for the joint lubrication. Therefore, the size of the element along the cavities of the real surface is the functional minimum material size.

Thus, each real dimension of the geometric element has two boundary functional sizes: maximum material size and minimum material size. According to the principle of two-dimensionality of linear-angular values [7], these are the sizes of two regular (ideal) elements of the same type as the dimensional element, nominally located relative to the coordinate system of the datums set, in which the dimensional element fulfills its executive function, and covering the real surfaces of the dimension element with a tangent: the first - from the material element - forms the maximum material size; the second - from the cavities of the element material - forms the minimum material size.

![Figure 1](image)

**Figure 1.** The model of the interference formation (b) along the height maximum material sizes in the fit with a gap (a) along the average sizes of the prismatic hole and shaft with informativeness 3

The sizes of the maximum and minimum material are constrained by the upper and lower limits of the element size. Their reliable measurement guarantees the quality of the connections in the assembly by ensuring the required fits.

The average element size should be calculated as half the sum of the sizes of the maximum and minimum material. However, the connection with the gap between the average sizes of the height of the hole and shaft \( H_m \) and shaft \( h_m \) (Fig.1,a) can only be provided by positive differences of the hole minimum material size \( H_{LM} \) and the shaft maximum material size \( h_{MM} \) that can take place in a single phase distortions of the executive surfaces J and K opposite to the elements datums A and B, but leads to the undesirable mismatch of the coordinate systems of the hole and the shaft. When there are anti-phase distortions of the executive surfaces (Fig.1,b), in the connection interference \( N_{MM} \) can be formed between the dimensions of maximum material of holes height \( N_{MM} \) and shaft \( h_{MM} \) that will preclude forward movement of the prismatic shaft.

Hereinafter, note that the maximum material size of the element is the largest size of the element (shaft) external dimension and the minimum size of the internal dimension element (holes). The minimum material size of the element is the smallest size of the external dimension element (shaft)
and the maximum dimension size of the internal element (holes). It is for these functional sizes a system of tolerances and fits is developed [2]. According to the new terminology [4], the element maximum material size is the minimum described size of the outer element and the maximum inscribed size of the inner element. The minimum material size is the maximum inscribed size of the outer element and the minimum described size of the inner element. Fitting in the compound is formed by the maximum inscribed size of the internal element (hole) and minimum described size of the outer element (shaft).

4. Prismatic elements dimensions measurement uncertainty

Dimensional prismatic elements constrain a part of the material by two nominally parallel planes, with informativeness 3, 2, 1 and \( \theta \) (zero), and form, respectively, the dimensions of height, width, length and overall size of the element.

The outer prismatic element with the maximum informative value 3 (3s=2t+1r) has a flat datum D3 (Fig.2) and the executive plane J0 with zero informativeness, the distance between which form the size of the height h. Deviations of the element plane form are the following: a V-shape (a), a dual convexity (b) a dual concavity (c) and an incurvation (d) form two boundary height sizes – maximum height size \( h_{MM} \) and the size of the minimum height \( h_{LM} \). These dimensions are respectively equal to the sizes of the two right prismatic elements nominally positioned relative to the datum D3 of the dimension element, and covering with tangent the actual surface of the element. The average element size \( h_m \) is equal to the half-sum of the height maximum \( h_{MM} \) and minimum \( h_{LM} \) sizes. Measurements of maximum \( h_{MM} \) and minimum \( h_{LM} \) height of the prismatic element of the two-point method is carried out without methodical uncertainties in the deviation of the shape of flat surfaces such as in-curvature, dual convexity and the dual concavity. However, the estimation of the average size of the height \( h_m \) can differ from the maximum material size and minimum material size and will therefore have methodical uncertainties \( U_{LM} \) and \( U_{MM} \), numerically equal to half the deviation of the shape of each real flat surface.

![Figure 2](image)

**Figure 2.** The model of the measurements uncertainties when applying the two-point method of the average height outer prismatic element with informativeness 3
If the prismatic element is curve (Fig.2,d), D3 base surface has concavity EFS while the executive surface - convexity EFT. The two-point measurement method in this case allows to measure only the average size hm of the element height with uncertainties UMM and ULM, which are numerically equal to the sum of deviations of the two flat surfaces form. The sizes of the maximum and minimum material height are in this case measured by using two-point measurement method, thus, the evaluation of boundary sizes on the average size, their uncertainty is equal to the uncertainty of the average element size.

Prismatic elements with informative content 2 (Fig.3,a) have the dimensions of maximum and minimum wMM and wLM of width between the flat datum B2 with informative content 2 and executive nominally parallel plane Dθ. Datum B2 and executive surface Dθ have deviations of perpendicularity relative to datum A3. Datum B2 and executive surface Dθ have form deviations of flatness type with deviations from perpendicularity EPE form 2 width sizes - the size of the maximum material of width wMM and minimum material size of width wLM as the sizes of the two correct prismatic elements nominally positioned relative to the datum A3 and covering with tangent the actual surfaces of the dimensional element: the first – out of the element material, the second from the element material. The average size of the element width wmm is the half-sum of the maximum and minimum material sizes of the element width.

Figure 3. Uncertainty measurement model of the two-point method of average widths (a) and length (c) of prismatic elements with informativeness 2, 1 and θ (zero)

The two-point method can only measure the average size of width wmm with the methodical uncertainty equal to the half-sum of the deviations from perpendicularity EPE of the elements flat surfaces taking into account the form deviations of the surfaces (Fig. 2). The sizes of the maximum and minimum widths cannot be measured by the two-point method. Attributing a medium size to them gives the boundary widths the uncertainties of the average size U LM and U MM.

A similar approach to the dimensions measurements uncertainties of length l from a flat datum with informativeness 1 is shown in Fig.3.b.

Prismatic elements with zero informativeness θ perform an executive function of the executive or dimension elements, which form fittings with a gap in the connections of details or dimensions of parts (Fig.3 c).

Prismatic elements with the informativeness θ do not participate in the formation of coordinate system OX4Y2Zθ of the datums set of part A3B2C1. The position of the element in the coordinate system is
defined, generally, by using the center of the element symmetry plane by the linear coordinate \( \theta \pm EY \)
and two angular coordinates \( \theta^\circ \pm AEZ \) and \( 90^\circ \pm AEX \) symmetry plane.
The primary position deviation symmetry EPS and parallelism EPA in relation to the datum B2 and
perpendicularity relative to the datum A3 (not shown) of the prismatic element with informativeness \( \theta \)
influence the functional size of the element – increase the maximum material size \( g_{MM} \) and reduce
the minimum material size \( g_{LM} \).
These dimensions ensure the required clearance fit when assembling parts. However, the two-point
measurement method cannot account for deviations of the location of the dimension element in the
part coordinate system.

5. Results and discussion
The results of the modeling uncertainties were verified through experimental measurements of the
precision products parts prismatic elements linear dimensions: hydraulic motors, bearings, gear and
vane pumps, and compressors. As a result of studies of the accuracy of two-point measurements of the
linear dimensions it is established that methodical uncertainties do not exist only for the maximum and
minimum sizes at maximum informativeness 3 of prismatic elements having shape deviations of the
wedge-shape type, double convexity or concavity. Methodological uncertainty is formed while
measuring the boundary functional linear dimensions of the prismatic elements with maximum
informativeness at the deviation of the shape of the incurvation type. The methodical uncertainty is
formed when measuring the average size of elements for all types of geometric tolerances. The two-
point measurement method cannot account for deviations of the location dimension of the element and
therefore its use for items with less than the maximum informativeness creates unacceptably large
methodical uncertainty of measuring the maximum, minimum and medium linear sizes. The same
methodical uncertainties occur under the arbitration control of the parts linear dimensions by the limit
two-point gauges. The results of the research cover the outer and inner prismatic elements.

6. Conclusion
1. Each dimensional geometric element, constrained by a real surface, has two dimensions: two
permissible size covers for the normalization of the accuracy class and two functional sizes for the
operation of the part in the product: the maximum material size and the minimum material size.
2. According to the principle of sizes two-dimensionality, the size of the material maximum and the
size of the material minimum of the dimensional element are the dimensions of two regular (ideal)
elements of the same type as the dimension element, nominally located relative to the coordinate
system in which the dimensional element fulfills the executive purpose, with the tangentially
covering the real surface of the dimensional element: the first one - outside the element material -
materializes the size of the material maximum, the second - from the element material - materializes
the size of the material minimum.
To implement these definitions, when measuring two sizes of an element, we need a coordinate system
in which the dimensional element operates, and knowing the base informativeness of an element, i.e.
the number of degrees of freedom constrained by the element datum.
4. Measurement of local linear sizes according to ISO standards by the two-point method is carried out
by the shortest distance between opposite points of a dimensional element real surface in a set of pairs
of points. By the processing of the measurement results the maximum size, the minimum size and the
average size, which is preferred as an ordered one and having the biggest methodical uncertainty, are
found.
5. Scientific novelty of the work is the proof of the existence of two functional actual linear
dimensions of each of the prismatic elements of the shafts and the holes forming the fitting – the
maximum material size and minimum material size. The character of fitting in the connection is
determined by the dimensions of the mating elements maximum material. The average sizes are only
statistical specifications of prismatic elements real dimensions.
6. The greatest influence on the uncertainty of two-point measurements of prismatic elements linear sizes with maximum informativeness 3 has the elements form deviation of an incurvature (straightness) type.
7. The uncertainty in the two-point measurements of the linear dimensions of the prismatic elements having the informative value of the maximum deviation impact location of the type of parallelism, perpendicularity and symmetry bases of the measured elements, which leads to unacceptably large values of uncertainties and output sizes for the intervals of tolerances.
8. Only one way for solving this problem is offered: the transition of the ISO standards to the functional approach to geometrical specifications of the products based on the principle of two-dimensional size classification of elements in terms of information, with the introduction of coordinate systems to reference specifications, and with the inclusion of these applications in the strategic plan of the standards updating [5].

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