A new approach to fuzzy evaluation of Metro operation safety based on entropy-weight and analytic hierarchy process

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Abstract. Accurate evaluation of Metro operational safety state plays critical role in cites stability and public personal safety. Here, we developed a novel fuzzy evaluation method to evaluate Metro operational safety status more scientifically and accurately, and it systematically integrated expert investigation, grading method and fuzzy comprehensive evaluation method based on entropy weight and analytic hierarchy process (AHP). We used a case data to validate Metro operational safety state, and the result shows that our method evaluates Metro operational safety state, and real dealing ability to safety accidents effectively. This method provides an important theoretical reference for the safe operation mode of the Metro.

1. Introduction

The Metro has brought great convenience to people's life with the advantage of large carrying capacity, fast, convenience, low energy consumption and less pollution. Nevertheless, it also has some disadvantages such as: huge management system, multi-source employee, enclosed environment, densely populated passengers, and difficulty of emergency evacuation, which makes the consequences hard to predict in the event of safety issues, and it would cause potentially serious outcomes. Therefore, accurate evaluation of the Metro operation safety status will be important for the stability and safety of the city [1].

Metro operation is a very complex system encompassing many heterogeneous factors, which could cause the different effects on the evaluation results. The expert system was often used to evaluate each factor, but there is a lot of subjectivity here. It is necessary to propose a new quantitative evaluation system for Metro operation status to make a more efficient, objective and accurate evaluation of Metro operational status. Now, researches have attempted to quantify the evaluation of Metro safety operation using mathematical methods, as Bayesian network [3], analytic hierarchy process [4], fuzzy evaluation and principal component analysis. These methods have greatly improved the effectiveness of evaluation system, but factors’ weight is hard to decide by these methods. Entropy-weight method is an objective and comprehensive evaluation method based on the consideration of the multi-type factors’ information. However, as the correlation and overlap among the various factors’ information, it produces the problem of weighting same factor repeatedly in the application of the entropy-weight method in the evaluation system. While, the AHP method can divide factors into different factor layers according to different factor levels, which can reduce the overlap among the various factors largely. Here, we proposed a hierarchy analysis method based on entropy-weight to build a comprehensive fuzzy evaluation system of Metro operation safety status, and the factors leading to subway safety...
accidents were analyzed systematically and the hierarchy evaluation index system was determined. Based on entropy-weight AHP Model, the fuzzy comprehensive evaluation system was established, and we illustrated the method by a case research. Our method provided a scientific basis for the subway system management and an operation form from point of view of the subway safety.

2. Fuzzy Evaluation of Metro Operation Safety

2.1. Establish Evaluation-factor System of Metro Operation Safety

One of the factors of Metro operation safety is the management of Metro operation; the other is the ability of handling accidents timely. Through the analysis of the data, we found out the influencing factors which lead to various accidents and security incident handling procedure effectively. Then we systematically constructed a 3-level safety status evaluation system for the operation status of the Metro. Based on the four first-level indicators, such as the early-warning capability of the former safety accident, the accident emergency response capability, the rescue capability after the accident and the support of the government, we have systematically constructed 15 second-level indicators. This indicators system was shown in Figure 1. Here, all of the indicators that influence the safety of the Metro were defined as influencing factors set, and the Metro safety status results were defined as status set. The value of first level of indicators was expressed as: \( U = \{ U_1, U_2, U_3, U_4 \} \), The value of second level of indicators was expressed as: \( U_1 = \{ U_{11}, U_{12}, U_{13}, U_{14} \} \), \( U_2 = \{ U_{21}, U_{22}, U_{23}, U_{24} \} \), \( U_3 = \{ U_{31}, U_{32}, U_{33}, U_{34} \} \), \( U_4 = \{ U_{41}, U_{42}, U_{43}, U_{44} \} \). The status set was expressed as: \( V = \{ V_1, V_2, V_3, V_4 \} \). If the value of \( V_1 \) was between 75 and 100, the status result was excellent, and if the value of \( V_2 \) was between 50 and 75, the status result was good, and if the value of \( V_3 \) was between 25 and 50, the status result was moderate, and the value of \( V_4 \) was between 0 and 25, the status result was poor.

![Figure 1](image.png)

Figure 1. The evaluation-factor system of Metro Operation Safetys.

Then, we took a questionnaire survey of five emergencies experts and five mangers and they judged the level of excellent, good, moderate, and poor of each indicator according to the actual situation of the factors, and we standardized the evaluation results of each factor by using the number of experts in a given judgment divided by the total number of experts, as the degree of membership factors to the state results. The larger the ratio, the more likely the influencing factor affects the state result. Then, the second-level indicators’ membership matrix was constructed using evaluation results, and the second-level indicators were judged one by one for determining the degree of membership factors to the state results.
2.2. Construction of Fuzzy Comprehensive Evaluation Model

By Saaty’s basic thoughts about the scale-selection [5], the Judgment Matrix P of second-level indicators in the evaluation system of Metro operation safety is constructed, Pij was the measure of the importance of the i indicator for the j indicator as shown in table 1.

| Numerical rating | Judgments of preferences between factor i and j |
|------------------|-----------------------------------------------|
| 1                | Equally important                             |
| 3                | Slightly important                            |
| 5                | Obviously important                           |
| 7                | Strongly important                            |
| 9                | Extremely important                           |
| 2, 4, 6, 8       | Between the upper and lower levels            |

We next calculate the product Ni for each row of the matrix P, A is the n-th power root of Ni, and the formula was shown as 1:

\[ A_i = \frac{A_i}{\sum A_i} \]  

We get the eigenvector \( A=(A_1,A_2,...,A_n)^T \), and calculate the value of \( \lambda_{max} \), \( (PA) \), is the i-th element in vector PA. The consistency check is made by formula 2.

\[ \lambda_{max} = \frac{\sum (PA_i)}{nA}, CI = \frac{\lambda_{max} - n}{n-1}, CR = \frac{CI}{RI} \]

From formula 2, we can see that judgment matrix is the most consistent when \( \lambda_{max} - n = 0 \), \( CI = 0 \), \( CR = 0 \), \( CR < 0.1 \), and then the consistency of the judgment matrix would be acceptable; otherwise, it should be modified. The values the average random consistency index RI were shown in table 2.

| N    | 4   | 5   | 6   | 7   | 8   | 9   |
|------|-----|-----|-----|-----|-----|-----|
| RI   | 0.89| 1.12| 1.26| 1.36| 1.41| 1.46|

Next, the weight of the index is calculated using the information entropy of evaluation index. First, the entropy of the evaluation index is obtained using the formula 3. Here, assuming when \( f_{ij}=0 \), \( f_{ij}\ln f_{ij}=0 \).

\[ H_i = -\frac{1}{\ln m} \sum_{j=1}^{m} f_{ij} \ln f_{ij}, f_{ij} = \frac{y_{ij}}{\sum y_{ij}} \]  

Where the sum of \( W_i \) is 1, and the range of i is from 1 to m, and then we get the second-level index entropy-weight vector of the \( W_i=(W_{i1},...,W_{im}) \), and combined weight of the i-th second-level indicator \( d_i \) is obtained with the formula 4.

\[ W_i = \frac{1-H_i}{m-\sum H_i}, d_i = \frac{A W_i}{\sum A W_i} \]

The second-level index combined weight vector is obtained: \( T^T = (d_1, d_2, ..., d_m)^T \), and the comprehensive safety score is calculated as fellows: we define the first level indicator \( D, D=TR \), where \( D_i \) is the membership index vector of the i-th first-level, and obtain membership vector of evaluation results using the formula of \( A(D^T, ..., D^T) = B = (B_1, ..., B_m) \). Finally, we judge the safety status of Metro operation by calculating the scores of C with the formula 5, and the larger the value of C is, the more secure the operation status of the Metro is. Here, \( C_i \) is the median of the scores for the first rating.

\[ C = (B_1, B_2, ..., B_m) \]

\[ \left( \begin{array}{c} C_1 \\
C_2 \\
. \\
C_n \end{array} \right) \]
3. A Case Research
Here, we used a case research to validate the effectiveness of model. Five Metro managers and five emergencies experts were selected to fill out the questionnaire survey of Metro, and they evaluated fifteen second-level indicators of the safety-state model. Then we used entropy-weight analytic hierarchy process to assess the status of Metro security as fellows.

(1) Determining Fuzzy Evaluation Matrix of Evaluation Index. 10 experts evaluated each individual index in the factor set $U$ respectively. Using the method of fuzzy comprehensive evaluation, the membership matrix of evaluation index was built as follows:

$$
R_1 = \begin{bmatrix}
0.6 & 0.2 & 0.1 & 0.1 \\
0.7 & 0.2 & 0.1 & 0 \\
0.45 & 0.2 & 0.1 & 0 \\
0.3 & 0.4 & 0.3 & 0 \\
\end{bmatrix},
R_2 = \begin{bmatrix}
0.5 & 0.3 & 0.2 & 0 \\
0.3 & 0.4 & 0.3 & 0 \\
0.45 & 0.2 & 0.3 & 0.05 \\
0.3 & 0.3 & 0.2 & 0 \\
\end{bmatrix},
R_3 = \begin{bmatrix}
0.3 & 0.4 & 0.2 & 0.1 \\
0.45 & 0.3 & 0.25 & 0 \\
0.3 & 0.4 & 0.3 & 0 \\
0.4 & 0.2 & 0.3 & 0.1 \\
\end{bmatrix},
R_4 = \begin{bmatrix}
0.45 & 0.3 & 0.2 & 0.05 \\
0.3 & 0.4 & 0.3 & 0 \\
0.4 & 0.2 & 0.4 & 0 \\
\end{bmatrix}
$$

(2) Calculation of Index’s Weight. Applying the fuzzy membership matrix $R$, we calculated the corresponding entropy weight vector with the formulas 3 and 4, the results were as followed:

$$
W_1 = (0.2769, 0.2724, 0.2238, 0.2267, 0.2414, 0.2333, 0.2514, 0.2737),
W_2 = (0.0465, 0.4534, 0.4490, 0.0509, 0.3347, 0.3246, 0.3),
W_3 = (0.3497, 0.3246, 0.3406)
$$

Then the weight is obtained according to analytic hierarchy process. Their judgment matrix $P$ for each level of indicators was built, and the consistency level of the judgment matrix was calculated by the formula. The index comparison matrix about the first index was obtained as follows:

$$
A = \begin{bmatrix}
1 & 3 & 5 & 7 \\
1/3 & 1 & 3 & 5 \\
1/5 & 1/3 & 1 & 3 \\
1/7 & 1/5 & 1/3 & 1 \\
\end{bmatrix}
$$

We obtained $CR=0.044<0.1$ according to formula 2. Therefore, the weight of the first level indicator is: $A=(0.57, 0.26, 0.12, 0.05)$. Using the same method, we got the values of $A_1, A_2, A_3, A_4$:

$$
A_1 = (0.16, 0.20, 0.41, 0.23), A_2 = (0.25, 0.56, 0.11, 0.08),
A_3 = (0.12, 0.10, 0.39, 0.39), A_4 = (0.31, 0.38, 0.31)
$$

(3) Determining indicator weight through entropy weight analytic hierarchy. Then, we calculated the combined weight of the second-level index as shown in formula 4:

$$
T_1 = (0.1825, 0.2244, 0.3780, 0.2148), T_2 = (0.2508, 0.5431, 0.1149, 0.0910),
T_3 = (0.0227, 0.1843, 0.7120, 0.0808), T_4 = (0.3118, 0.3707, 0.3173)
$$

(4) Fuzzy comprehensive evaluation. Through the fuzzy comprehensive evaluation on the second-level factors, we got the evaluation vector of the second-level factors as follows:

$$
D_1 = T_1R_1 = (0.5256, 0.2453, 0.2013, 0.0276), D_2 = T_2R_2 = (0.3860, 0.2981, 0.2484, 0.0125),
D_3 = T_3R_3 = (0.3731, 0.3444, 0.2726, 0.0097), D_4 = T_4R_4 = (0.4846, 0.3673, 0.1501, 0.4846)
$$

We obtained the comprehensive score of the evaluation results using the formula 5.

$$
B = AD = \begin{bmatrix}
0.5256 & 0.2453 & 0.2013 & 0.0276 \\
0.3860 & 0.2981 & 0.2484 & 0.0125 \\
0.3731 & 0.3444 & 0.2726 & 0.0097 \\
0.4846 & 0.3673 & 0.1501 & 0.4846 \\
\end{bmatrix} = \begin{bmatrix}
0.4689 & 0.2770 & 0.2195 & 0.0444 \\
0.3860 & 0.2981 & 0.2484 & 0.0125 \\
0.3731 & 0.3444 & 0.2726 & 0.0097 \\
0.4846 & 0.3673 & 0.1501 & 0.4846 \\
\end{bmatrix}
$$

Then we got the value of $CA$ by the formula 5.

$$
C_1 = BC = B_1C_1 + B_2C_2 + B_3C_3 + B_4C_4 = 0.4689 \times 87.5 + 0.2770 \times 62.5 + 0.2195 \times 37.5 + 0.0444 \times 12.5 = 64.0025
$$
4. Conclusion
Here, we provided a new method to fuzzily evaluate Metro operation safety using analytic hierarchy process based on entropy-weight. We built a fuzzy comprehensive evaluation system for Metro operation safety, and got the comprehensive safety score of Metro which was 64.0025, and judged the safety status of Metro was good. The result was consistent with the Metro operation actual state and showed that it could predict the operational safety state of the Metro and the actual handling capacity of the safety accident accurately.

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