The effect of the synthetic fibre reinforcement on the fracture energy of the concrete

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Abstract. Fibre reinforced concrete belongs to the group of composite materials, where matrix means the concrete, and fibres mean the various shaped (straight, hooked, wavy, etc.) and material (steel, polypropylene, glass, etc.) fibres, which are short compared to the size of concrete elements. Fibres are mixed in fresh concrete which supposedly provides a perfect and uniform mix. Although material of the fibres can be different (steel and synthetic polypropylene are the most widespread), material models in literature are primarily for steel fibres. Elastic modulus of steel and synthetic fibres differs from each other in terms of scale (steel 210 GPa, synthetic 8-12 GPa), despite this, however, similar results are received frequently at bending beam tests. The different result lies in the energy absorption of the fibres pulling out from the concrete matrix. In the present paper, this different behaviour is examined and a suggested material model for fibre reinforced concrete is presented. This material model is based on the modification of the fracture energy.

1. Introduction
Synthetic macro fibre reinforced concrete is becoming more and more widespread in the industry and more and more accepted among engineers as well [1]. Their applicability was questioned primarily because of their low elastic modulus (steel: 210 GPa, synthetic: 8-12 GPa) [2]. There are numerous other arguments against their applicability (their low melting point, the creeping of the synthetic polypropylene material), but they have one significant advantage over steel fibres, which is their corrosion resistance. When a structure is exposed to a corrosive environment, it is worth considering the use of synthetic macro fibres.

Material model of fibre reinforced concrete has plenty of recommendations in the literature, however in most cases these only apply to steel fibres [3,4,5], or do not distinguish between steel and synthetic materials [6]. The difference is apparent at long-term loading due to the creeping of synthetic material [7], or at temperature loading due to the low melting point. These qualities of the material are not manifested at normal temperature short-term static loading. Examining the energy absorption of steel and synthetic fibres, a new and simplified material model is presented in the current paper, which models fibre reinforced concrete material by modifying fracture energy of the concrete. As fibres have a low impact on the flexural tensile strength of plain concrete, material model of plain concrete has to be modified only after cracking. Fracture energy of concrete is estimated based on the formulas in the literature, which then is modified by added fracture energy depending on fibre type and fibre dosage. With this approach, a simple yet accurate material model can be defined.
2. Fracture energy of the concrete

Concrete is a quasi-brittle material, which means that after the cracking of the material it is capable of taking decreasing stress depending on the opening of the crack. Due to the tensile stresses in the concrete a thin band with micro-cracks will appear: this is called the fracture process zone [8]. Increasing the stress the concrete reaches its tensile strength, and leads to merging of the micro-cracks, and finally a macro crack is created and is visible. After this point the tensile capacity of the concrete will decrease, the cracks will bypass or cross the aggregates and then slowly the entire section will be crossed by the crack, see figure 1.

Figure 1. Crack propagation in the concrete

Measuring the residual tension stress and crack width a stress-crack width (σ-w) diagram is obtained, area below this curve is the fracture energy of concrete: $G_F$ (Nmm), see figure 2.

Figure 2. Fracture energy of the plain concrete

The fracture energy of the concrete is influenced by a number of factors which are clearly not related to the concrete strength class. Most of the existing design methods neglect the fracture energy of the concrete and do not pay much attention to the tensile strength. There are numerous references in the literature about how to specify the fracture energy of concrete, most frequently in function of strength class and maximum aggregate size [9,10,11]. According to CEB-FIP Model Code the fracture energy can be estimated as following:

$$G_F = G_{F0} (f_{cm}/f_{cm0})^{0.7}$$

(1)

where:

$f_{cm0} = 10$ MPa

$G_{F0}$ is base value of fracture energy, depends on the maximum aggregate size, according to table 1.
Table 1. Base values of fracture energy $G_{F0}$

| $d_{max}$ (maximum aggregate size) [mm] | $G_{F0}$ [N/mm] |
|----------------------------------------|-----------------|
| 8                                      | 0.025           |
| 16                                     | 0.030           |
| 32                                     | 0.058           |

Some recommendations of the stress-crack width diagram can be seen in figure 3, and some formulas in table 2.

![Figure 3. Recommendations of the stress-crack width diagrams](image)

a) Linear [8] b) Bilinear [21] c) Power curve [22]

Table 2. Recommendations of $\sigma$-$w$ diagram

| Name                | Expression                                                                 |
|---------------------|---------------------------------------------------------------------------|
| Linear curve [8]    | $\sigma = f_t \left(1 - \frac{w}{w_c}\right)$                           |
|                     | $w_c = \frac{2G_t}{f_t}$                                                 |
| Bilinear curve [21,23,24] | $\sigma = \begin{cases} f_t - (f_t - \sigma_1) \frac{w}{w_i} & \text{if } w \leq w_i \\ \sigma_1 - \sigma_1 \frac{w - w_i}{w_c - w_i} & \text{if } w_i > w \end{cases}$ |
|                     | $w_i = \frac{2G_t}{f_t}$                                                 |
|                     | $w_i = \frac{2}{\psi f_t} \left(G_F - (1 - \psi)G_t \right)$             |
| Power curve [22]    | $\sigma = f_t \left[1 + \left(\frac{c_1}{w_c}\right)^{-1}\right] \exp\left(\frac{c_2 - w}{w_c}\right) - \frac{w}{w_c} \left(1 + c_1\right) \exp\left(-c_1\right)$ |
|                     | $c_1 = 3$                                                                 |
|                     | $c_2 = 6.93$                                                              |
|                     | $w_c = 5.14 \frac{G_F}{f_t}$                                              |

(2) (3) (4)
3. Energy-absorbing fibre-matrix mechanisms of steel and synthetic fibres

The fibres start to work effectively after the crack of the matrix and increase the residual strength and the ductility of the material, thus increasing fracture energy of the concrete. According to Zollo [12] the energy-absorbing function of the fibres could be 1) fibre bridging, 2) fibre pull-out and 3) fibre failure (figure 4). The bridging and pull-out of fibres could give the highest added ductility for the concrete, whilst fibre failure will decrease it. For steel fibres, the fibre pull-out will have a major role even from small crack opening, while synthetic fibres function more through a bridging mechanism.

![Energy-absorbing fibre/matrix mechanism](image)

**Figure 4.** Energy-absorbing fibre/matrix mechanism [12]

The tensile strength of the fibre reinforced concrete will be the same as plain concrete, the effect of the fibres in the linear phase is limited. The fibres start to work when the micro cracks start appearing, first the fibres detaching from the concrete matrix. This is an important phase of the mechanism, because if this cannot happen the fibres will rupture. This may justify a reduction in ductility of steel fibre reinforced concrete in time, called as embrittlement according to Bernard [13]. After the detachment there is the pull-out phase (figure 5).

![Mechanics of the fibre](image)

**Figure 5.** Mechanics of the fibre
Thus the following formula for crack opening can be written as:

\[ w = l_w \varepsilon_t + l_s \]  

where:

- \( l_f \) = length of the fibre
- \( l_w \) = working length of the fibre \((l_w \leq l_f)\) as given in equation (6)
- \( l_s \) = slip distance as given in equation (7)

\[ l_w = \begin{cases} \frac{w}{\varepsilon_t} = \frac{wE_t}{\sigma_t} & \text{if } \frac{wE_t}{\sigma_t} \leq l_f \\ l_f & \text{if } \frac{wE_t}{\sigma_t} > l_f \end{cases} \]  

which means that it is assumed that the fibres will pull-out if the working length \((l_w)\) reaches the fibre length \((l_f)\).

\[ l_s = w - l_w \varepsilon_t \]  

which will give zero if \( l_w < l_s \), i.e. only some parts of the fibre are working.

In this model the influence of the unsymmetrical anchoring and different orientation of the fibre were neglected. This model is made only for comparing the steel- and synthetic fibre mechanics during the crack opening. The fibres give added fracture energy via detachment, bridging and pull-out. The detachment depends on the connection of the mortar and the surface of the fibre, the bridging process on the detachment, while the pull-out phase depends on the surface of the fibre and the end-hook configuration. Fibre reinforced concrete beams were tested according to RILEM recommendations [14] by three point notched beam test and the load and the CMOD were measured. The beams were reinforced with 20 kg/m\(^3\) steel and 5 kg/m\(^3\) synthetic macro fibres. From these series steel and synthetic fibre reinforced concrete beam test results were selected where the load-CMOD results are close to each other (figure 6).

![Figure 6. Load-CMOD diagrams of plain concrete, steel and synthetic fibre reinforced concrete](image_url)
The stress-strain and stress-crack width material law was defined according to the RILEM recommendation [4]. Due to this it was possible to find out the stress distribution on the section. If location of the fibres is known on the section then the stresses in the fibres ($\sigma_f$), the working length of the fibre ($l_w$) and the slip distance ($l_s$) could be calculated. As the beams were reinforced with different fibres, steel and synthetic, but the load-CMOD diagrams were almost the same, it can be estimated that the stress-crack width material law is also affined for both materials. Having the same stress-crack opening law for the same load-CMOD curve can be estimated (figure 7). The method of calculation is the following: the value of the CMOD and the stress distribution on the section is known. From this the crack opening (with linear estimation) and the stress in the individual fibres (according to the distribution of the fibres) can be calculated. The working length and the slip can be calculated according to equation (5) and (6).

![Figure 7. Modelling the stress, strain and slip in the individual fibres](image)

With these approximations the strain and slip of the fibres were calculated at CMOD = 0.25 mm and 1 mm in the case of steel- or synthetic fibres. The results are shown in figure 8.

![Figure 8. Fibre bridging and fibre pull-out of steel and synthetic fibres](image)

As shown in figure 8, because of their small strain capacity, the steel fibres must start slipping at small crack opening, while the synthetic fibres are still in bridging phase. If the material is still in its
elastic range after unloading it will want to close the crack. At bridging, fibres are equal to the whole crack opening, i.e. they want to close the whole crack, while at slipping, fibres could close only with their strain energy and the slipping cannot be reversed.

4. Added fracture energy of the fibre reinforced concrete
Fracture energy of concrete can be estimated by many recommendations. Although the added fibre increases fracture energy, only a few instances occur when fibre reinforced concrete is modelled by considering modification of fracture energy [15] however it would be plausible. Added fibres increase this residual strength and the ductility of the material, thus increasing fracture energy of concrete. The various types of fibres exert their primary effect at different crack width [7]. Steel fibres with higher elastic modulus are able to take high stress even at low crack width, however by the increase of crack width fibres tear or pull out from the matrix, thus resulting in the decrease of residual stress. However, synthetic fibres of low elastic modulus can work even at higher strain than steel fibres, meaning that synthetic fibres behave efficiently in the case of higher crack width [13].

![Figure 9. $\sigma$-w diagram of steel- and synthetic fibre reinforced concrete and interpretation of fracture energy](image1)

![Figure 10. Fracture energy of concrete and fibre reinforced concrete, and added fracture energy](image2)

Crack width at which stress terminates between the crack surfaces of the concrete is called final crack opening width marked by $w_c$ [15]. In the case of fibre reinforced concrete this value is significantly higher, so it can be taken into account during calculation with a given value, according to guidelines. Thus, considered maximum crack opening width of fibre reinforced concrete is marked by $w_f$. Value of $w_f$ depends on the type and length of the fibre, its recommended baseline value is $w_f = 3$ mm. Fracture energy of the concrete is the area below the stress-crack width diagram of the concrete, marked by $G_f$. While for fibre reinforced concrete it is similarly the area below its curve, marked by $G_{FFRC}$. Fracture energy added by fibres is the difference of the two, thus: $G_f = G_{FFRC} - G_f$ (figure 10).

Based on both my own experimental results and the publications found in the literature [16,17,18,19] it can be stated that post-crack residual strength of synthetic fibres takes an approximately constant value. A simplified constitutive law is usually provided in the guidelines, where post-crack tension strength is modelled by a constant value [20,5]. Based on the fib model code 2010 value of the post-crack tension strength in the case of rigid-plastic model is the following:

$$f_{Pu} = \frac{f_{R3}}{3}$$  \hspace{1cm} (8)

where:

- $f_{Pu}$: post-crack residual tension strength
- $f_{R3}$: residual flexural tensile strength corresponding to CMOD = 3 mm, according to equation 9.
where:

\[ f_{R3} = \frac{3F_3 l}{2b h_{sp}} \]  (9)

where:
\( F_3 \) is the load corresponding to CMOD = 3 mm
\( l \) span length, 500 mm
\( b \) specimen width, 150 mm
\( h_{sp} \) distance between the notch tip and the top of the specimen, 125 mm

From the above the following constitutive law can be defined according to figure 11. Value of \( f_f \) can be defined based on the guidelines, thus \( f_f = f_{tu} \) based on the fib model code demonstrated above. The overlapping part of the \( G_F \) and \( G_{Ff} \) at the numerical diagram is negligible.

\[ G_{FFRC} = G_F + G_{Ff} \]

\[ G_{FFRC} = G_F + G_{Ff} \]

**Figure 11.** Experimental and numerical diagram of fibre reinforced concrete

Based on the above model added fracture energy \( (G_{Ff}) \) can be defined as follows:

\[ G_{Ff} = f_f w_f \]  (10)

5. Summary

Elastic modulus of the synthetic fibres (8-12 GPa) is much smaller than the elastic modulus of the steel fibres (201 GPa), despite this fact with the proper dosages similar load-CMOD diagram is received at beam tests. The reason for this is the different energy-absorption mechanism of the fibres: while steel fibres pull-out from the matrix, synthetic fibres bridge the crack. These two kinds of behaviours were analysed with beam tests.

Effect of fibres can be defined by analysing fracture energy of fibre reinforced concrete, which can be divided into two parts: fracture energy of the concrete and added fracture energy of fibres. There are numerous recommendations in literature defining the value of fracture energy of plain concrete, of which the most relevant models were introduced. By knowing the fracture energy of the concrete and the added fracture energy of the fibres the material model of the fibre reinforced concrete can be easily defined.

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