Natural supersymmetry and dynamical flavour with meta-stable vacua

Steven Abel\textsuperscript{a} and Moritz McGarrie\textsuperscript{b}

\textsuperscript{a}Institute for Particle Physics Phenomenology, Durham University,
South Road, Durham, DH1 3LE, U.K.

\textsuperscript{b}School of Physics and Centre for Theoretical Physics, University of the Witwatersrand,
Johannesburg, WITS 2050, South Africa

E-mail: s.a.abel@durham.ac.uk, moritz.mcgarrie@wits.ac.za

Abstract: We show how gauged flavour breaking and $\mathcal{N} = 1$ supersymmetry breaking can be dynamically aligned to produce natural models. Supersymmetry is broken in a metastable vacuum, while a weakly gauged flavour symmetry is identified with an $SU(3)_F$ subgroup of the global symmetry of the SQCD model. We find that alignment can easily occur either through strongly coupled flavour models, with SQCD bound states playing the role of flavons, or entirely weakly coupled and renormalizable models with elementary $SU(3)_F$ adjoint flavons. In both cases it is essential that the $SU(3)_F$ breaking, and hence all the flavour structure, is driven dynamically by the SUSY breaking. The resulting flavour gauge mediation in conjunction with the usual gauge mediation contribution leads to naturally light third generations squarks, with first and second generations above exclusions.

Keywords: Supersymmetry and Duality, Supersymmetry Breaking

ArXiv ePrint: 1404.1318
1 Introduction

In its original format, gauge mediation had the great advantage that flavour universality provided an explanation for the absence of flavour changing neutral currents. However flavour universality (and gauge mediation in general) is becoming less attractive, because at the time of writing the LHC has excluded first and second generation squarks in the TeV range [1–8], while naturalness of the Higgs sector (and the much weaker bounds on 3rd generation squarks of around 300-400 GeV [9–20]), favours a sub-TeV 3rd generation.

Such large splittings must be imprinted in the high scale boundary conditions of the soft mass-terms in an inverted hierarchy or effective SUSY model. This observation was the starting point of a number of models of flavour; see for instance [21–30].

An inverted hierarchy suggests that the dynamics of supersymmetry breaking are, at least in part, related to the dynamics of flavour. Recently ref. [31] proposed that the dynamics of (weakly gauged) flavour breaking \( SU(3)_F \to SU(2)_F \) are actually \textit{intimately} related to supersymmetry breaking, with some of the supersymmetry breaking fields being charged under \( SU(3)_F \). This results in gauge (vector multiplet) messengers [32], and can
Figure 1. A representation of gauge mediation of $SU(3) \times SU(2) \times U(1)$ combined with flavour-gauge messengers in $SU(3)_F$. This paper extends the framework by giving a dynamical explanation of $SU(3)_F$ SUSY breaking and the generation of the leading Yukawa structure in terms of magnetic SQCD.

generate the required inverted hierarchies amongst the soft masses. At the messenger scale, integrating out the gauge (vector multiplet) messengers generates negative mass-squared terms for stops, which after renormalisation group running obtain positive but small values at the electroweak scale. Such boundary conditions are strikingly natural as they significantly reduce the radiative effects of the stop soft term on $\delta m^2_{H_u}$.

In this work we wish to show that such alignment can be readily generated dynamically within an $N_f$-flavour ISS type configuration [33], with an $SU(3)_F$ subgroup of the global flavour symmetry, namely

$$SU(N_f)_L \times SU(N_f)_R \supset SU(3)_L \times SU(3)_R \supset [SU(3)_L \times SU(3)_R]_V = SU(3)_F$$

becoming the weakly gauged SM flavour symmetry. The remaining (anomalous) global symmetry, namely $SU(N_f - 3)_L \times SU(N_f - 3)_R \times [SU(3)_L \times SU(3)_R]_A$ can be explicitly broken to avoid physical Goldstone modes, while the spontaneous breaking of $SU(3)_F$ induced by the ISS mechanism (rank condition) gives calculable models with inverted hierarchies.

Although the discussion concerns flavour, we would like to avoid making very restrictive assumptions about how the hierarchical structure comes about. Therefore we will consider both strong and weak models of flavour. In both cases the flavour symmetry is weakly gauged, but in the former case one generally supposes that the flavour sector comprises bound states of some SQCD theory [34, 35]. The flavour hierarchies would then be governed by the dynamical scale(s) of that theory. In the context of ISS one obviously works in the perturbative (magnetic dual) description. The weak model of flavour by contrast requires elementary flavon fields that merely couple to the ISS sector. Here we shall assume these states to transform in the adjoint of flavour. While adjoints are not normally considered as flavons, it is natural in the context of ISS (because of the bound-state mesons), and bring advantages of their own to do with the strong CP problem, as discussed in [36–38].

In detail, the models we will propose have the following properties:
An ISS sector, with a weakly gauged $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V \equiv SU(3)_F$ subgroup of the full flavour symmetry. The fields that obtain $F$-term vevs contribute to the mass of the flavour gauge vector multiplet, which play the role of gauge messengers.

Fields charged under the flavour symmetry that play the role of flavons (these may or may not be part of the ISS sector itself).

The flavon and F-term vevs and hence the $SU(3)_F$ breaking all have a dynamical origin driven by the ISS sector.

Flavon couplings to matter fields to generate Yukawa couplings.

With this general structure, a hierarchy can emerge between the $F$-terms and the one-loop induced vevs of many of the classical moduli including the flavons and other magnetic meson operators; consequently one can find an explanation of the Yukawa hierarchies that is closely aligned with a source of flavour mediation of SUSY breaking. The effects from the flavour gauge mediation then combine with the standard gauge mediation of the SSM gauge groups $SU(3)_c \times SU(2)_L \times U(1)_Y$ as in figure 1, yielding phenomenology with a number of interesting features:

One loop tachyonic soft mass terms from flavour gauge mediation that compete with the usual positive gauge mediated contribution.

Typically, this leads to nett tachyonic UV boundary conditions for stops, and hence lighter 3rd generation squarks at the weak scale, with improved naturalness properties.

Mild flavour changing neutral currents from subleading off-diagonal mass terms, that decouple as $\alpha_F \rightarrow 0$.

For a weakly gauged system, $\text{tr} M^2_{\text{Vector}} = -\text{tr} M^2_{\text{Higgs}} - \Phi$ (i.e. the $\text{Str} M^2 = 0$ relation still holds).

The second entry in the above list is particularly important: although the stops may be tachyonic in the UV, the RGE contribution from the gluino mass $M_3$ yields a nett positive stop mass-squared at the weak scale. In terms of stability (or lack thereof) this indicates the possibility of charge and colour breaking minima along $F$- and $D$-flat directions. However such minima can be avoided. For example the $U_3 D_3 D_2$ direction has a potential of the form $V(\phi) \sim \phi^2 (m^2_{tR} + m^2_{bR} + m^2_{sR})$. As long as there are enough positive mass-squareds in the UV this need never become unstable. Note that there is relative freedom in the choice of parameters as the gluino mass itself arises by the usual gauge mediation. The nett stop mass $m^2_{\tilde{t}}$ is therefore quite independent, and can be much lower than the gluino mass. This is an improvement on usual gauge mediation (with respect to naturalness)

---

1One might argue that morally speaking they are in any case no different from the global supersymmetric minimum of ISS and decay to them is likewise negligible.
as current bounds on gluinos (i.e. 1600 GeV [39, 40]), fix a supersymmetry breaking scale which would usually force stops to be at least this heavy. Note that the Higgs soft terms are unaffected by flavour mediation as the Higgs doublets are $SU(3)_F$ singlets. Indeed the Higgs soft terms are mediated by $SU(2)_L$ and $U(1)_Y$ and set at the messenger scale. If one chooses the leptons to be charged under $SU(3)_F$, then the splitting extends to the sleptons as well, allowing for a light Stau NLSP. Such a setup is consistent and compatible with supersymmetric Grand Unified Theories and indeed (if one wants to avoid additional SM charged states) mixed $SU(3)_F \times G_{SM}$ anomaly cancellation strongly suggests it.

We should add that $SU(3)_F$ symmetries have been considered in the past as a good starting point for flavour structure [37, 38, 41–44]. Usually one requires a quasi-degeneracy between the first and second generation to avoid flavour changing effects. This may arise from an approximate remaining $SU(2)_F$ and we build this into some of our examples. In other examples this degeneracy will be broken mildly and therefore the discovery of (small) FCNC processes is a prediction of our framework. Indeed an important advantage of combined flavour+standard gauge-mediation is that, with the first and second generation masses in the TeV range and dominated by the standard mediation contribution, the non-degeneracy required in the flavour-mediation contribution is significantly relaxed, to the extent that it can be consistent to have no apparent $SU(2)_F$ symmetry for the first two generations, but just the correct general pattern of flavour dependence.

The outline of the paper is as follows. In section 2 we demonstrate a simple example in which the top coupling is generated through a meson vev and supersymmetry breaking in the $SU(3)_F$ is generated dynamically. In section 3 we give a derivation of the leading effect of the soft terms and discuss its limitations. We extend this in section 4 in which magnetic quark vevs generate a prototype Yukawa matrix. In section 5 we explore an entirely perturbative model of flavour in which the flavons are adjoint chiral superfields and the flavour breaking scale is generated dynamically. In section 6 we emphasise some of the features of the model’s SUSY spectrum. In section 7 we look at possible flavour contraints on the model and ideas to address the strong CP problem as well as anomalies. Finally in section 8 we discuss how such a programme may be refined and we conclude.

2 SUSY breaking & flavour alignment with mesonic flavons

We start with a simple case in which $SU(3)_F$ is broken to $SU(2)_F$ dynamically, and flavour and SUSY breaking ($F$-terms) are aligned. As described in the Introduction, we base the flavour/SUSY breaking sector on the magnetic description of Seiberg duality [34, 35] with $N_f > 3$ flavours. For simplicity we assume a degenerate dynamical scale for the magnetic and electric theories, denoted by $\Lambda$. The additional $N_f - 3$ flavours are necessary for the system to be in the metastable phase which requires $N_c + 1 \leq N_f < 3N_c/2$. The obvious choice of choice $N_f = 3$ is not within this window, and it may or may not result in a metastable minimum since it would require $N_c = 2$ and hence has $N_f = 3N_c/3$. This is a marginal case and establishing whether or not it has metastable minima requires a two-
Table 1. Matter content of the $SU(3)_F$ gauged flavour model based on the MSSM. By inclusion of the right-handed neutrinos, all anomalies including mixed $G_{SM}-SU(3)_F$ anomalies are cancelled by this matter content.

| Field     | $G_{SM}$ | $SU(3)_L \times SU(3)_R \rightarrow SU(3)_F$ |
|-----------|----------|---------------------------------------------|
| $\hat{Q}^f$ | $(2, \frac{1}{3}, 3)$ | $(3, 1)$ |
| $\hat{L}^f$ | $(2, -\frac{1}{2}, 1)$ | $(3, 1)$ |
| $\hat{H}_d$ | $(2, -\frac{1}{2}, 1)$ | $(1, 1)$ |
| $\hat{H}_u$ | $(2, \frac{1}{2}, 1)$ | $(1, 1)$ |
| $\hat{D}^f$ | $(1, \frac{1}{3}, 3)$ | $(1, 3)$ |
| $\hat{U}^f$ | $(1, -\frac{2}{3}, 3)$ | $(1, 3)$ |
| $\hat{E}^f$ | $(1, 1, 1)$ | $(1, 3)$ |
| $\hat{\nu}^f$ | $(0, 1, 1)$ | $(1, 3)$ |

Certainly, since the non-perturbative potential has no dependence on the dynamical scale $\Lambda$, the decay rate would not be parametrically suppressed. In addition we assume that the dynamical scale of the flavour sector is too large to disturb the metastability [45].
where $\mu^2$ is a diagonal $(N_f - 3) \times (N_f - 3)$ matrix, with generic entries. They could for this entire discussion be zero, but in order to break flavour correctly we must assume that they obey $\tilde{\mu}^2_{ii} < \mu^2 \forall i$.

Due to the ISS rank condition\(^3\) supersymmetry is broken when $N_f > \tilde{N}$. Under our assumption that $\mu^2$ are the largest entries in the $\tilde{\mu}^2$ matrix, $\tilde{N}$ of the $SU(3)_F$ flavours get vevs becoming heavy along with the off-diagonal $SU(3)_L \times SU(N_f - 3)_R$ as well as the $SU(N_f - 3)_L \times SU(3)_R$ components of $\tilde{\Phi}$. It is worth displaying the remaining relevant $SU(3)_L \times SU(3)_R$ block of the tree-level scalar potential in detail:

$$V = \sum_{i,j} |(\varphi \tilde{\varphi})_{ij} - \mu^2 \delta_{ij}|^2 + \sum_{i,\alpha} |(\varphi \Phi)_{i,\alpha}|^2 + \sum_{i,\alpha} |(\Phi \tilde{\varphi})_{\alpha,i}|^2,$$

where $i,j$ are flavour indices and $\alpha,\beta$ are $SU(\tilde{N})$ indices.

As the ISS breaking should also break $SU(3)_F$ spontaneously to $SU(2)_F$, we have an additional requirement that $\tilde{N} < 3$, or $N_f < N_c + 3$, on top of the requirement that $N_c + 1 \leq N_f < 3N_c/2$. Therefore the minimal case would have $N_c = 3$ and $N_f = 4$ (with $\tilde{N} = 1$) and would actually be s-confining with no magnetic gauge group. The case $N_c = 4$ allows $N_f = 5$ (also s-confining), and $N_c = 5$ allows $N_f = 6$ (s-confining) or 7 ($SU(\tilde{N} = 2)$ magnetic gauge group). Higher $N_c$ obviously allow $\tilde{N} = 3$ as well, although because this would be universal spontaneous breaking of the gauged $SU(3)_F$ symmetry it is not interesting for the present discussion.

Thus $\tilde{N} = 1$ or 2 flavours of quark in the $SU(3)_F$ block get vevs, and the $SU(3)_F$ symmetry is broken spontaneously. Meanwhile the extra $N_f - 3$ flavours remain vevless

---

\(^3\)The matrix $(\varphi \tilde{\varphi})_{ij}$ can only have maximum rank $\tilde{N}$ or $N_f$ whichever is smaller.
and indeed the $SU(N_f - 3)_V$ symmetry is broken only by the explicit $\tilde{\mu}_{ii}^2$ non-degeneracy in this model.

Let us take $\tilde{N} = 2$ as an example. The vevs of the $SU(3)_F$ flavoured quarks may be written as

$$\varphi^T = \check{\varphi} = \begin{pmatrix} \mu \\ \mu \\ 0 \end{pmatrix}. \quad (2.3)$$

Since $SU(3)_F$ is weakly gauged, the spontaneous breaking provides the goldstone modes to generate massive flavour gauge bosons, by the supersymmetric Higgs effect. The minimum of the potential still contains an $SU(2)$ symmetry. The $F$-terms in this block are given by

$$F_\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & h\mu^2 \end{pmatrix} \quad \text{such that} \quad V_{\text{min}} = |h^2 \mu^4|. \quad (2.4)$$

Note that there are no other $F$-terms in the $SU(N_f - 3)_L \times SU(N_f - 3)_R$ block.

We find that the flavour mediated contribution to the soft terms due to the gauging of $SU(3)_F$ is (see section 3)

$$\delta m_{Q,U,D}^2 = -\frac{g_F^2}{16\pi^2} |h^2 \mu^2| \begin{pmatrix} \frac{8}{9} & 0 & 0 \\ 0 & \frac{8}{9} & 0 \\ 0 & 0 & \frac{20}{9} \end{pmatrix} + ... \quad (2.5)$$

plus subleading off-diagonal terms. Note the minus sign: this contribution clearly suppresses third generation scalars relative to 1st and 2nd (see figure 2) and when combined with usual gauge mediation already achieves our first aim of dynamically generating a split squark spectrum suitable for a natural model of supersymmetry breaking.

### 2.1 Flavour from magnetic mesons

Of course to complete the programme we need a flavour model. As the mesons are composite objects the model of flavour inevitably has to have a strong coupling element to it. In models such as this one where the goldstino partly resides in the flavon, the main additional requirement is a vev for the scalar component, which unavoidably breaks the (accidental) $R$-symmetry of the model spontaneously. Breaking $R$-symmetry was the subject of a number of studies in the past, due to its central role in the generation of a Majorana gaugino mass. Here we seek a suitable deformation of the theory that can achieve this without explicitly breaking the $SU(3)_F$ gauge symmetry of the Lagrangian. We choose $[49, 50, 53, 54]^4$

$$\delta W = \frac{1}{2} h^2 \mu_\Phi (\text{Tr}\Phi^2 + \gamma \text{Tr}(\Phi)^2). \quad (2.6)$$

It is then expected that the one loop Coleman-Weinberg potential will minimise the classical modulus as

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & \langle X \rangle \end{pmatrix}, \quad (2.7)$$

---

$^4$Note that we cannot for example use the operator $\delta W \supset h^2 m_Y \text{Tr}(\check{Y}Y)$ of ref. [46] which generates $\langle X \rangle = \frac{1}{2} h m_Y$, because it breaks the $SU(3)_F$ explicitly.
so that the non-zero top Yukawa is aligned with the lightest squark generation as required. Indeed considering the charges of the fields in tables 1 and 2, and matching the magnetic meson with its electric composite $\Phi \Lambda = \tilde{q}q$, the allowed Yukawas are

$$W_{\text{Yuk}} = \lambda_u \frac{\Lambda}{\Lambda_0^2} H_u Q \Phi U + \lambda_d \frac{\Lambda}{\Lambda_0^2} H_d Q \Phi D,$$

(2.8)

where $\Lambda_0$ is some physical scale in the electric theory while recall that $\Lambda$ is identified with the dynamical transmutation scale of the ISS theory.

We can make a completely general observation that will apply to all the models of strongly coupled flavour that we consider, which arises from the fact that the Yukawa coupling for the top is $\lambda_u \langle X \rangle \Lambda/\Lambda_0^2 \lesssim \mu \Lambda/\Lambda_0^2$. Since phenomenology requires this to be $O(1)$ we require $\Lambda_0 \ll \Lambda$. In other words the operators present in $W_{\text{Yuk}}$ creating the Yukawa couplings are generated by integrating out degrees of freedom at a relatively light scale $\Lambda_0$ that lies somewhere between $\Lambda$ and $\mu$. These degrees of freedom must mix with the composite states but it is easy to see that they can be charged only under flavour and hence are simply spectators of the ISS sector, so they need not interfere with the metastability there. When we come to consider weakly coupled flavour models later we shall consider the complete picture.

The model above not only gives the spectrum of natural supersymmetry but also generates the top coupling dynamically; indeed dynamical alignment of flavour with SUSY breaking demands it. Unfortunately it does not naturally yield further couplings. This is because the only other fields with vevs are the quarks, but since $\langle X \rangle \lesssim \mu$ is loop suppressed, operators involving them are likely to be larger than the top coupling itself. We would actually have to suppress all couplings between SSM matter and the magnetic quarks if we were to pursue this as a model of flavour. In the next sections we will explore examples that give a more realistic flavour model.

3 Calculation of the soft masses

Before continuing, we briefly recall the calculation of the soft mass contribution from flavour gauge messengers, in the flavour basis — this was the result that was used in the previous section. The sfermion soft masses of the visible sector, at one loop, are given by [31, 32]

$$\delta m_{\tilde{Q}_I}^2 = -T^{ab}_{IJ} \delta_{IJ} Z^{ab} |_{\theta^4}, \quad T^{ab} = \{ T^a, T^b \}.$$  

(3.1)

where $a,b$ are generator indices running over 1, ..., dim$G$ and $I$ is the sum over all charged Chiral superfields. This term arises from the one-loop Kähler potential

$$K^{(1)}_{\text{eff}} = \sum_{I,J} Q_I^\dagger T^{ab}_{IJ} Q_J \delta_{IJ} Z^{ab} + ...$$  

(3.2)

where the $Q_I$’s are visible sector Chiral superfields and $\Phi_I$ will denote the hidden sector fields. The matrix $Z^{ab}$ is defined as

$$Z^{ab} = \left( \frac{\alpha_F}{4\pi} \right) \log \left( g_F^2 \sum_i \Phi_i^\dagger T^{ab}_i \Phi_i \right).$$  

(3.3)
The combination $\Phi_i^\dagger T_i^{ab} \Phi_i$ is traced over flavour indices where $i$ runs over each field: it is a set of matrices in the $\theta$-expansion that terminate at $\theta^4$ with $(\bar{F}, F)^{ab}$. The inner product is defined as

$$\langle \bar{\phi}, \phi \rangle^{ab} \equiv \sum_i \phi_i^\dagger \{ T^a, T^b \} \phi_i. \quad (3.4)$$

Typically (and in our case) these matrices are not diagonal and one cannot assume $(\bar{\phi}, \phi)^{ab} = (\bar{\phi}, \phi) \delta^{ab}$. The result is that whilst one may expand the logarithm as

$$\text{Log}[ (\bar{\phi}, \phi) + \theta^2 (\bar{\phi}, F) + \theta^2 (\bar{F}, \phi) + \theta^2 \bar{\theta} (\bar{F}, F) ]^{ab} \quad (3.5)$$

one cannot take the $\text{Log}[ A^{ab} (I^{ab} + B^{ab}/A^{ab}) ]$ and carry out an expansion in $F/M$. One must instead take the matrix logarithm of eq. (3.3) and give an approximate expansion, typically in the off-diagonal elements of $(\bar{\phi}, \phi)$. If one can carry out the expansion, then the soft mass matrix can be represented as

$$\left( m_{1}^{2(1)} \right)_{cd} = - \left( \frac{\alpha_F}{4\pi} \right) \sum_{a,b} T^{ab}_{cd} \left( \frac{\bar{F}, F)^{ab}(\bar{\phi}, \phi)^{ab} - (\bar{\phi}, \phi)^{ab}(\bar{\phi}, F)^{ab}}{(\bar{\phi}, \phi)^{ab}(\bar{\phi}, \phi)^{ab}} \right) + O(|F|^4). \quad (3.6)$$

The result above is found from the usual small $F/M$ expansion and generalising $X = M + \theta^2 F$, where $\alpha_F = g_F^2/4\pi$ is the $SU(3)_F$ fine-structure constant. This will typically give the leading contribution, with subleading off-diagonal entries, and so we can use this formula to describe the essential feature of the model. More details may be found in [32].

4 The magnetic quark case

As we have seen, typically the magnetic quark vevs are larger than the one loop induced meson vev, so a natural way to generate Yukawa structure in a strongly coupled theory of flavour is to begin with the magnetic quark vevs. In this section we give two examples of dynamical supersymmetry breaking in which the magnetic quark vevs are used to generate a prototypical Yukawa structure.

4.1 $\tilde{N} = 1$ example

The case $\tilde{N} = 1$ allows for the vevs

$$\varphi^T = \bar{\varphi} = \begin{pmatrix} 0 \\ 0 \\ \mu \end{pmatrix} \quad (4.1)$$

which breaks $SU(3)_F \rightarrow SU(2)$. The leading Yukawa terms can be inferred by baryon matching. In this case (assuming for simplicity equal dynamical scales in the electric and magnetic theories) we have

$$\Lambda^{\tilde{N}} q^{N_c} \sim \Lambda^{N_c} \varphi^{\tilde{N}}, \quad (4.2)$$

so when $\tilde{N} = 1$ the natural object appearing in the superpotential is $\left( \frac{\Lambda}{\Lambda_0} \right)^{N_c} \varphi/\Lambda$ and similar for $\bar{\varphi}$. Taking the minimum case of $N_c = 3$ would give a leading contribution to the up-Yukawas of the form

$$W \supset \frac{\Lambda^4}{\Lambda_0^4} (\varphi U)(\bar{\varphi} Q) H_u. \quad (4.3)$$

– 9 –
Now an order one top-Yukawa requires $\mu \Lambda^2 / \Lambda_0^3 \sim O(1)$, but our previous comments apply: we require $\mu < \Lambda_0 < \Lambda$ for the picture to be valid.

The classical moduli space has maximal symmetry when $\langle \Phi \rangle = 0$ and the supersymmetry breaking is therefore anti-aligned with flavour:

$$F_\Phi = \begin{pmatrix} h \mu^2 & 0 & 0 \\ 0 & h \mu^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

such that $V_{\text{min}} = 2 |h^2 \mu^4|$. (4.4)

The soft mass formula is

$$\delta m^2_{Q,U,D} = -\frac{g_F^2}{16 \pi^2} |h\mu|^2 \begin{pmatrix} 11 & 0 & 0 \\ 0 & 14 & 0 \\ 0 & 0 & 29 \end{pmatrix} + ...$$ (4.5)

Thus this model also generates a mild splitting between 1st and 2nd generation squarks. An example low-energy spectrum is shown in figure 3.

### 4.2 Flavour in the $\tilde{N} = 1$ model

As we saw above the third generation is anti-aligned with supersymmetry breaking. In this model the magnetic meson may contribute subleading effects. The Coleman-Weinberg potential generates vevs for the diagonal components of the magnetic meson of the form

$$\Phi = \begin{pmatrix} \langle x \rangle & 0 & 0 \\ 0 & \langle x \rangle & 0 \\ 0 & 0 & \langle y \rangle \end{pmatrix}.$$ (4.6)

As before $\langle x \rangle$ may be generated by additional mesonic operators in the superpotential, and it is typically suppressed with respect to the leading term. The term $\langle y \rangle$ may also arise due to additional operators of magnetic quarks in the superpotential and is typically smaller than $\langle x \rangle$. Defining

$$\epsilon_x = \frac{\langle x \rangle}{\Lambda_0^2} \quad \epsilon_y = \frac{\langle y \rangle}{\Lambda_0^2},$$ (4.7)

eq. (2.8) leads to a contribution to the up-Yukawas of

$$\delta Y_u \sim \begin{pmatrix} \epsilon_x & 0 & 0 \\ 0 & \epsilon_x & 0 \\ 0 & 0 & \epsilon_y \end{pmatrix}.$$ (4.8)

All scales have been generated dynamically either from the vev of an electric quark mass or a one-loop Coleman-Weinberg minimisation of the scalar potential. However the model does not yet break the residual $SU(2)_F$ flavour symmetry. In addition the up and down Yukawas are still diagonal in a common basis. The former problem is quite difficult to solve in the $\tilde{N} = 1$ model. Moreover off-diagonal components can also not easily be generated. For example one might consider additional operators involving the magnetic quarks. Assuming that $U$ is in a fundamental triplet of flavour, while $Q$ is in an antifundamental (in order
to for example avoid mixed $SU(3)_F$ Standard Model anomalies), then $SU(3)_F$ baryonic operators such as
\[ W \supset \hat{O}_b = \frac{\Lambda^6}{\Lambda_0^8} \hat{\varphi} U(\varepsilon \hat{\varphi} \hat{Q}) H_u \]
are bound to give zero contribution to the Yukawa couplings since $\varphi$ and $\hat{\varphi}$ have only one non-zero component. To summarise the $\tilde{N} = 1$ model, the operator eq. (4.3) nicely generates a natural Yukawa coupling, and eq. (4.8) may contribute some subleading effects but leaves the flavour structure diagonal and still $SU(2)_F$ symmetric.

4.3 The $\tilde{N} = 2$ example

These problems may be partially solved if one also includes an $\tilde{N} = 2$ model. First consider the $\tilde{N} = 2$ model in isolation. For $SU(\tilde{N}) = SU(2)$ the rank condition gives (recalling that this is for just the $SU(3)_F$ charged degrees of freedom)
\[
\varphi = e^{-i\theta} \begin{pmatrix} 0 \\ \mu \\ \mu \end{pmatrix}, \quad \hat{\varphi}^T = e^{i\theta} \begin{pmatrix} 0 \\ \mu \\ \mu \end{pmatrix},
\]
spontaneously breaking $SU(3)_F \to U(1)$, where we have assumed a relative phase between the magnetic quark vevs.

As we saw earlier the minimal case has $N_c = 5$ so that the natural scaling in the superpotential is $\left( \frac{\Lambda}{\Lambda_0} \right)^{5/2} \varphi/\Lambda$, and the Yukawa operator is now
\[ W \supset \frac{\Lambda^3}{\Lambda_0^5} (\varphi U)(\hat{\varphi} Q) H_u, \]
which gives
\[ Y_u \sim \frac{\Lambda^3}{\Lambda_0^5} \mu^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \epsilon & \epsilon \\ 0 & \epsilon & \epsilon \end{pmatrix}. \]

The Coleman-Weinberg potential will set
\[
\Phi = \begin{pmatrix} \langle x \rangle & 0 & 0 \\ 0 & \langle y \rangle & 0 \\ 0 & 0 & \langle y \rangle \end{pmatrix}, \quad F_\Phi = \begin{pmatrix} h\mu^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]
such that $V_{\text{min}} = |h^2 \mu^4|$ and the soft masses are now found to be
\[ \delta m^2_{Q,U,D} = -\frac{g_F^2}{16\pi^2} |h\mu|^2 \begin{pmatrix} 0 & 0 & 0 \\ 48 + \sqrt{29} & 0 & 0 \\ 0 & 2\sqrt{29} & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 42 - 2\sqrt{29} & 0 & 0 \\ 0 & 0 & 66 + \sqrt{29} \end{pmatrix} + \ldots \]
An example spectrum is shown in figure 4. The soft terms do not preserve an $SU(2)$ at leading order and so it would likely lead to larger FCNCs than the previous two models. As we later discuss in more detail, to some extent this is alleviated by reducing the value of $g_F$ but at the cost of lessening the effect of reduced stop masses and therefore decreasing the naturalness effect, as pictured in figure 4. Thus there is an interesting interplay between FCNCs and naturalness in this model.
4.4 Two ISS sectors

A more complete model of flavour is then to include two ISS sectors: the first \( \tilde{N} = 1 \) component (referred to as ISS(1)) generates the leading soft breaking of eq. (4.5) and top Yukawa of eq. (4.3), while the second \( \tilde{N} = 2 \) component (referred to as ISS(2)) generates subleading Yukawas, such as eq. (4.11). The subleading soft-terms from ISS(2) can be ignored and the resulting spectrum is represented in figure 3.

The magnetic quarks of ISS(2) \( \varphi_2, \tilde{\varphi}_2 \) give operators

\[
W \supset O_2 = \frac{\lambda_2^3 \Lambda_0^3}{\Lambda_0^5} (\varphi_2 U)(\tilde{\varphi}_2 Q)H_u. \tag{4.14}
\]

In addition one can incorporate all other operators including those coupling ISS(1) to ISS(2), such as (for example) \( (\varphi_i \varphi_j), (\tilde{\varphi}_1 Q), (\varphi_1 U), (\varphi_1 \varphi_2 Q) \) and \( (\tilde{\varphi}_1 \tilde{\varphi}_2 U) \). Thus one has operators

\[
O_3 \sim (\varphi_i \varphi_j)(\tilde{\varphi}_1 Q)(\varphi_1 U)H_u \tag{4.15}
\]

\[
O_4 \sim (\varphi_i \varphi_j)(\tilde{\varphi}_1 Q)(\varphi_1 U)H_u \tag{4.16}
\]

\[
O_5 \sim (\varphi_i \varphi_j)(\tilde{\varphi}_2 Q)(\varphi_1 U)H_u \tag{4.17}
\]

\[
O_6 \sim (\varphi_i \varphi_j)(\tilde{\varphi}_2 Q)(\varphi_2 U)H_u \tag{4.18}
\]

and further operators suppressed by powers of \( \mu^2/\Lambda_0^5 \) such as

\[
O_7 \sim (\varphi_i \varphi_j)(\varphi_1 \varphi_2 Q)(\varphi_k U)H_u \tag{4.19}
\]

\[
O_8 \sim (\varphi_i \varphi_j)(\tilde{\varphi}_k Q)(\tilde{\varphi}_1 \tilde{\varphi}_2 U)H_u. \tag{4.20}
\]

Additional discrete symmetries may be applied here, judiciously, and clearly there is enough freedom here to generate realistic looking Yukawa matrices.

5 Alignment with UV complete models: a dynamical flavour scale

As we mentioned earlier, the fact that for example the combination \( \mu \Lambda/\Lambda_0^2 \) appears in the Yukawa couplings, where \( \Lambda_0 \) is a mass scale in the UV completion of the theory, while \( \Lambda \) is the strong coupling scale with \( \mu < \Lambda \), inevitably means that there must be additional physics below the strong coupling scale. It is therefore of interest to present a model that is entirely perturbative apart from any ISS sectors that we add to break gauged \( SU(3)_F \) and SUSY, that has no non-renormalizable operators, and in which all the additional modes are explicit.

5.1 The perturbative flavour structure

The idea we focus on here is that the flavons correspond to elementary flavour adjoints \( \phi_{u,d} \).

As well as providing the necessary flavour structure these can ameliorate or even eliminate the strong CP problem \([36–38]\). In addition we will require additional fields of mass \( O(\Lambda_0) \) charged under the Standard-Model gauge group. These are denoted \( D \) and \( \tilde{D} \), and carry
the same hypercharge as the usual down multiplets and its conjugate respectively. (In order to have correct flavour structure we would obviously repeat the following procedure for the ups.)

Let us begin this time with the generation of flavour structure, which appears in the superpotential as

$$ W \supset qhD + \Lambda_0 \bar{D}^T D + \bar{D}^T \phi_d d, \quad (5.1) $$

where $\Lambda_0$ is our dimensionful parameter, and as promised all the operators are now renormalizable. For one generation, upon integrating out the heavy $D, \bar{D}$ we would have light and heavy quarks given by

$$ \begin{pmatrix} d_e \\ d_h \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} d \\ D \end{pmatrix}, \quad (5.2) $$

where (taking a classical background for $\phi_d$)

$$ \tan \theta = -\phi_d/\Lambda_0. \quad (5.3) $$

More generally, with multiple generations, the heavy state is given by

$$ d_h = \frac{1}{\sqrt{\Lambda_0^2 + \frac{1}{3} \text{Tr}(\phi_d^2)}} (\Lambda_0 D + \phi_d d) \quad (5.4) $$

where we normalize as $d_i^id_j = \delta_{ij}/3$. The orthogonal light state is then

$$ d_e = \frac{1}{\sqrt{\Lambda_0^2 + \frac{1}{3} \text{Tr}(\phi_d^2)}} (-\phi_d D + \Lambda_0 d). \quad (5.5) $$

Then the low-energy superpotential becomes

$$ W \supset qh\sqrt{\Lambda_0^2 + \frac{1}{3} \text{Tr}(\phi_d^2)}(\Lambda_0^2 - \phi_d^2)^{-1}\phi_d d_e. \quad (5.6) $$

Obviously in the limit that $\phi_d/\Lambda_0 \ll 1$ this can be approximated as

$$ W \supset \frac{1}{\Lambda_0} qh\phi_d d_e, \quad (5.7) $$

but interestingly one can have a natural expansion in flavour structure.

### 5.2 The symmetry breaking

We now wish to induce the required flavour symmetry breaking via the ISS mechanism to generate the flavon vev at the same time as SUSY is broken. It is clear that we can never generate a satisfactory hierarchy with a traceless adjoint. Therefore instead we consider $\phi_d$ to have a trace component as well (i.e. there is an extra singlet). This could have its own coupling but for simplicity we assume it is degenerate with the other coupleings so the full global symmetry (and possibly even gauge symmetry) is $U(3)_F$.

Consider two ISS theories with $\tilde{N} = \tilde{N}' = 1$. The magnetic theories are the usual ISS theories with mesons $\Phi, \Phi'$, and we add additional couplings between the elementary
and mesonic adjoints. Note that further complication will be required because we are attempting to incorporate the correct breaking of the $U(3)^3$ flavour symmetry, and multiple ISS sectors are a rather efficient way to do this dynamically. However our aim is not to achieve a better model of flavour, but rather to focus on how the SUSY breaking can naturally be aligned with it.

The $SU(3)_F$ charged part of the superpotential is

$$ W_{ISS} = \left( \Phi \tilde{\varphi} - \mu^2 \Phi + m_\Phi \tilde{\varphi} \right) + \left( \Phi' \tilde{\varphi}' - \mu'^2 \Phi' + m_{\Phi'} \tilde{\varphi}' \right). \quad (5.8) $$

It contains the usual ISS terms, and in addition terms coupling the elementary adjoints to the meson of the original SQCD theory. Given that the latter correspond to renormalizable couplings in the electric theory, one expects $m \sim \Lambda$ and $m' \sim \Lambda'$, while $\mu = m Q \Lambda$ and $\mu' = m' Q' \Lambda'$ depend on the dimensionful mass parameters, so are not directly related. Note that as per the previous models we can more or less ignore the $SU(N_f - 3)_V$ part of the superpotential which simply provides extra quark supermultiplets, provided that their linear terms have $\tilde{\mu}^{2}_{ij} < \mu^2$.

The potential for this part of the theory is of the following form at tree-level;

$$ V_{ISS} = \sum_{b=1}^{3} \left| \Phi_{ab} \tilde{\varphi}^b \right|^2 + \left| \varphi^a \Phi_{ab} \right|^2 + \left| \Phi'_{ab} \tilde{\varphi}'^b \right|^2 + \left| \varphi'^a \Phi'_{ab} \right|^2 $$

$$ + \sum_{ab} \left| \varphi_{a} \tilde{\varphi}_{b} - \mu^2 \delta_{ab} + m_{\Phi_{ab}} \right|^2 + \left| \varphi'_{a} \tilde{\varphi}'_{b} - \mu'^2 \delta_{ab} + m'_{\Phi'_{ab}} \right|^2 $$

$$ + \sum_{ab} \left| m \Phi_{ab} + m' \Phi'_{ab} + \tilde{D}_{a} \delta_{b} \right|^2. \quad (5.9) $$

Since the same $\phi_d$ couples to both ISS sectors, it is not possible to satisfy both unbroken SUSY constraints at the same time. In order to see this we may write the $\phi_d$ VEV as

$$ \phi_d = \rho_a \delta_{ab}. \quad (5.10) $$

Only one $(\tilde{\varphi} \varphi)_{ab}$ composite meson VEV is available in each ISS sector for cancelling $F_\Phi$ terms. Hence we may without loss of generality rotate to a basis in which $(\tilde{\varphi} \varphi)_{ab} = \eta_{a1} \delta_{a1} \delta_{b1}$ and $(\tilde{\varphi}' \varphi')_{ab} = \eta_{a3} \delta_{a3} \delta_{b1}$, so that the ISS part of the potential is

$$ V_{ISS} = \left( \eta_{1}^2 - \mu^2 + m \rho_1 \right)^2 + \left( \mu^2 - m \rho_2 \right)^2 + \left( \mu^2 - m \rho_3 \right)^2 + \left( \mu'^2 - m' \rho_1 \right)^2 + \left( \mu'^2 - m' \rho_2 \right)^2 + \left( \eta_{3}^2 - \mu'^2 + m' \rho_3 \right)^2 \Rightarrow \left( \mu^2 - m \rho_2 \right)^2 + \left( \mu^2 - m \rho_3 \right)^2 + \left( \mu'^2 - m' \rho_1 \right)^2 + \left( \mu'^2 - m' \rho_2 \right)^2. \quad (5.11) $$

The minimum is at

$$ \rho_1 = \frac{\mu^2}{m'}, \quad \rho_3 = \frac{\mu^2}{m}, \quad \rho_2 = \frac{m \rho_3 + m' \rho_1}{m'^2 + m^2}, \quad (5.12) $$

with $F-$terms

$$ \frac{F_{\Phi_1}}{m'} = - \frac{F_{\Phi_2}}{m} = \frac{mm'}{m'^2 + m^2} (\rho_1 - \rho_3) \delta_{i2}. \quad (5.13) $$
This gives
\[ \eta_1^2 = m(\rho_3 - \rho_1) ; \eta_3^2 = m'(\rho_1 - \rho_3). \] (5.14)

In order to generate a Yukawa hierarchy, suppose \( \mu' = \xi^3 \mu \) and \( m' = m/\xi^2 \) with \( \xi \lesssim 1 \). Then \( \rho_1 = \xi^8 \rho_3 \) and \( \rho_2 \approx \xi^4 \rho_3 \). Here the parameter \( \xi \approx 0.4 \) to achieve the correct down hierarchy. (The ups would require \( \xi \to \xi^2 \approx 0.16 \).) Retaining the leading terms in \( \xi^2 \) only we find
\[ \phi_d = \frac{\mu^2}{m} O \begin{pmatrix} \xi^8 & 0 & 0 \\ 0 & \xi^4 & 0 \\ 0 & 0 & 1 \end{pmatrix} \] (5.15)

leading to the correct Yukawa structure and for the supersymmetry breaking and gauge symmetry breaking,
\[ F_\Phi = \mu^2 O \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; F_\Phi' = \mu^2 O \begin{pmatrix} 0 & 0 & 0 \\ 0 & \xi^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \] (5.16)
\[ \varphi' \tilde{\varphi} = \mu^2 O \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ; \varphi' \tilde{\varphi}' = \mu^2 O \begin{pmatrix} 0 & 0 & 0 \\ 0 & \xi^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \] (5.17)

Clearly by adding further \( \tilde{N} = 1 \) ISS sectors and adjoints \( \phi_u \) for the ups, one can fill up the entire flavour structure. (This method of flavour breaking is direct in the sense that there is a field associated with each layer of symmetry breaking.) It should be noted however that only one set of susy breaking is required as it will be mediated to all fields charged under \( SU(3)_F \). For example it could be that the up Yukawas are driven by an SQCD theory that has no metastability and does not contribute to supersymmetry breaking. Then the flavour mediated contribution to the soft-terms would all be aligned with the down Yukawas as described above.

For definiteness we consider this example; the soft-terms are then found to be
\[ \delta m_{Q,U,D}^2 = -\frac{g_F^2}{16\pi^2} |h\mu|^2 \begin{pmatrix} 1.6 & 0 & 0 \\ 0 & 3.8 & 0 \\ 0 & 0 & 5.5 \end{pmatrix}. \] (5.18)

The stop contribution is enhanced sufficiently but the flavour mediated contribution does not approximately preserve \( SU(2)_F \) symmetry. Nevertheless it has the right kind of structure, so we expect some alleviation of the naturalness problem at the expense of larger FCNCs.

### 6 Supersymmetry breaking and the low energy spectrum

As discussed before, we should combine gauge mediated supersymmetry breaking in the SM gauge groups \( SU(3)_c \times SU(2)_L \times U(1)_Y \) and gauge (vector multiplet) messengers in
the $SU(3)_F$ group pictured in figure 1. The soft masses from $SU(3)_c \times SU(2)_L \times U(1)_Y$, are given by

$$M_{\lambda_i} = k_i \frac{\alpha_i}{4\pi} \Lambda_{\lambda_i}$$

and

$$m_{\tilde{f}_{3 \times 3}}^2 = 2 \sum_{i=1}^{3} k_i C_i \frac{\alpha_i^2}{(4\pi)^2} \Lambda_{S_i}^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

where $k_i = (5/3, 1, 1)$ to GUT normalise the fine-structure constants. In the limit $\Lambda_{\lambda_i} \simeq \Lambda_S \simeq \Lambda_F$ where $\Lambda_F = F/M_{\text{mess}}$, the setup will represent minimal GMSB. We want to combine this with $SU(3)_F$ gauge messenger mediation

$$\delta m_{Q,U,D}^2 = - \left( \frac{\alpha_F}{4\pi} \right) \Lambda_{F}^2 N_{3 \times 3}$$

where $N_{3 \times 3}$ is a three by three matrix whose entries are a prediction of each particular model but which typically has a leading diagonal form. To generate the observed Higgs mass we assume the NMSSM. To analyse the spectrum we start from the NMSSM model file of SARAH 4.0.3 [55–58] in the super-CKM basis and implement the boundary conditions at the messenger scale. We set the model up to run the RGEs from the messenger scale $M_{\text{mess}}$ down to $M_{EWSB}$. After the SPheno model is built in SARAH we pass it over to SPheno 3.2.4 [59, 60] to carry out the running.

Let us now discuss the results of the low energy spectrum of the models given in this paper, namely the case of magnetic meson vev in which $\tilde{N} = 2$, the two cases in which magnetic quark vevs are used to generate the Yukawa structure, and the perturbative $\tilde{N}, \tilde{N}' = 1$ flavour model.

[Model 1]. The mesonic case. The soft masses for the flavour gauge mediation are given by eq. (2.5) at leading order. The results are in figure 2. This model has an approximate $SU(2)$ and will have best compatibility with FCNC constraints.
Figure 3. A plot [Left] of the squark and gluino masses for model 2 with the NMSSM. [Right] a plot of Higgs mass versus $g_F$ for the same range. $\lambda = 0.8$, $\kappa = 0.8$, $v_s = 1000$, $m^2_{H_d} = m^2_{H_u} = 10^5$, $\Lambda_{\lambda_i} = \Lambda_S = \Lambda_F = 2.3 \times 10^5$, $M_{\text{mess}} = 10^7$, $\tan \beta = 1.5$.

Figure 4. A plot [Left] of the squark and gluino masses for model 3 with the NMSSM. [Right] a plot of Higgs mass versus $g_F$ for the same range. $\lambda = 0.8$, $\kappa = 0.8$, $v_s = 1000$, $m^2_{H_d} = m^2_{H_u} = 10^5$, $\Lambda_{\lambda_i} = \Lambda_S = \Lambda_F = 2.3 \times 10^5$, $M_{\text{mess}} = 10^7$, $\tan \beta = 1.5$.

[Model 2]. The magnetic quark case with $\tilde{N} = 1$. The $SU(3)$ symmetry is completely broken resulting in non degenerate first and second generation tachyonic contributions to the soft masses. The results are in figure 3. The behaviour is overall similar to model 1 but now each squark 1,2 pair are split for each generation and the effect increases with $g_F$ increasing.

[Model 3]. The magnetic quark case with $\tilde{N} = 2$. As before, the $SU(3)$ symmetry is completely broken resulting in non degenerate first and second generation tachyonic contributions to the soft masses. The results in figure 4 show a much slower reduction in the stop mass values as $g_F$ is increased and increased splitting between first and second generation, therefore increased FCNCs are likely in this model for larger $g_F$, and this model is almost certainly excluded for this range of $\Lambda_F$.

[Model 4]. The perturbative $\tilde{N} = \tilde{N}' = 1$ model. A typical example of the up-squark spectrum is shown in figure 5. The stop contribution is enhanced sufficiently in the spec-
Figure 5. A plot [Left] of the squark and gluino masses for the perturbative model of flavour in section 5 with the NMSSM. [Right] a plot of Higgs mass versus \( g_F \) for the same range. \( \lambda = 0.8, \kappa = 0.8, v_s = 1000, m_{\tilde{t}_d}^2 = m_{\tilde{t}_u}^2 = 10^4, \Lambda_\lambda = \Lambda_S = \Lambda_F = 2.3 \times 10^5, M = 10^7, \tan \beta = 1.7. \)

trum to achieve the correct Higgs mass with sub TeV stops. As the flavour mediated contribution does not approximately preserve \( SU(2)_F \) symmetry, there is relatively large first-second generation splitting which means that this model would be bound to show significant deviations for FCNCs; with the heavier squark masses however this is at the margins of exclusion. (We discuss this in more detail presently).

In this class of models some squarks, usually those of the third generation, have tachyonic UV boundary conditions. As long as \( m_{\tilde{t}} \) is not larger than \( M_{\tilde{g}} \) we should expect that the stops turn positive by the electroweak scale as per the figures. It is interesting also that in

\[
\delta m_{H_u}^2 \simeq -\frac{3}{4\pi^2} m_{\tilde{t}}^2 \log \frac{M_{\text{mess}}}{m_{\tilde{t}}},
\]

the contributions in the regime of negative and positive \( m_{\tilde{t}}^2 \) may cancel out, reducing the little hierarchy problem rather nicely \([61, 62]\). Larger stop mixing may be achieved at the electroweak scale also, from UV boundary conditions with no mixing (\( A_t(M_{\text{mess}}) = 0)\).

7 Constraints from flavour

The addition of flavour gauge messengers introduces some non-universal soft terms and additional massive vector states in the flavour sector. One must ensure that these do not introduce flavour changing neutral currents. Typically in these models the mass of the flavour gauge bosons may be computed from

\[
(m_C^2)^{AB} = (m_V^2)^{AB} + (\delta m_0^2)^{AB},
\]

where

\[
(m_V^2)^{AB} = g^2 (\bar{\phi}, \phi)^{AB}, \quad (m_\chi)^{AB} = \frac{(\bar{\phi}, F)^{AB}}{(\phi, \phi)^{AB}}, \quad (\delta m_0^2)^{AB} = \frac{(\bar{F}, F)^{AB}}{(\phi, \phi)^{AB}}.
\]
Figure 6. Maximum allowed splitting between first and second generation up squarks, for a given first generation up squark, for $\delta_{12} = 0.1$.

Essentially these states will have mass squared roughly of the order of $\sim g_F^2 M^2$ or a little smaller. As in [25] integrating out the gauge bosons will give dimensions 6 operators

$$L \supset -\frac{g_F^2}{2m_V} (\bar{f}_L^i R \gamma^\mu T_{ij}^a f_L^j L,R) (\bar{f}_R^k L \gamma^\mu T_{kl}^a f_R^l R,L).$$

(7.3)

In order to avoid FCNCs from this source, it then sufficient to set the messenger scale to be above $\sim 10^5$ TeV to protect against for example $K^0 - \bar{K}^0$ mixing, which is the most constraining (CP preserving) $\Delta F = 2$ FCNC process. In our examples (see figures 2, 3 and 4) we have chosen $M_{mess} = m_V \sim 10^7$ GeV or higher.

As we are introducing flavour splittings amongst the squarks, the other obvious source of flavour violation comes from radiative contributions. Our principle concern is the up (rather than down) squark sector since it is chiefly the stop that has to be made light for the model to be natural. Some of our models break the approximate $SU(2)_F$ degeneracy of the first and second generation and so it is useful to see qualitatively how non-degenerate these may be within the current flavour constraints. These bounds are usually expressed in terms of the parameter [30]

$$\delta_{ij} = \frac{m_{q_2}^2 - m_{q_1}^2}{\frac{1}{2} (m_{q_2}^2 + m_{q_1}^2)} K_{ij} K_{ii}^*, \quad (7.4)$$

where $K$ is the rotation matrix from the quark-mass basis to the squark mass-basis. As we are aligning supersymmetry breaking and flavour it is clearly possible within this context
to avoid some of the bounds that may be most severe. For example if the flavour/susy
breaking was aligned such that the down/sdown sector was entirely diagonal, then the
most stringent FCNC constraints ($\delta_{d,12} < 0.07$ [30]) are avoided. Nevertheless in that case
one has the constraint from the ups ($\delta_{u,12} < 0.1$ [30]) which is only slightly less severe.
Therefore it seems reasonable to take the latter as a representative bound. It is also
reasonable to suppose that the Cabibbo angle is representative of the order parameter that
breaks $SU(3)_F$, so one might expect mild suppression of $K_{ij}K_{ii}^* \simeq 0.22$.

To guide the eye, figure 6 shows the maximum allowed splitting between first and
second generation up squarks in this case for a given first generation squark mass, for
various values of $K_{11}^* K_{12}$. With $\sim 1.5$ TeV first and second generation one can allow as
much as 400 GeV mass-splitting if the maxing matrix $K$ has Cabibbo sized off-diagonal
entries. Obviously heavier first and second generation, in the $5 - 10$ TeV range, allow
500 GeV- 1 TeV splittings. Comparing with our models, we find that the models 1 & 2 in
figures 2 and 3 respectively are clearly allowed, whilst model 3 in figure 4 is likely to be
excluded by flavour constraints. The perturbative flavour model in figure 5 is a borderline
case for such low values of $\Lambda_F$ but by increasing $\Lambda_F$ to achieve $5 - 10$ TeV first and second
generation, the constraints from flavour may be alleviated substantially. As remarked above
constraints on the down sector are slightly more stringent but the discussion is qualitatively
the same.

8 Discussion

In this paper we have shown how various prototypes of flavour structure and supersymmetry
breaking may be given a common dynamical origin in a magnetic SQCD. The models we
presented are based on weakly gauging an $SU(3)_F$ subgroup of the SQCD global symmetry
to play the role of the SM flavour symmetry, as in table 1. The $SU(3)_F$ is broken at the
same time as supersymmetry by the ISS mechanism, resulting in gauge (vector multiplet)
messengers that generate tachyonic soft terms. The breaking of the $SU(3)_F$ is naturally
aligned with the supersymmetry breaking and these soft terms compete with the usual
GMSB soft terms. This tachyonic contribution affects mainly the third generation scalars,
and a realistic and natural spectrum is generated.

The underlying description of flavour can be weakly or strongly coupled. Either way,
thanks to the ISS mechanism one has a calculable and predictive framework. Implicitly,
the scale of the usual SM gauge mediation contributions and the flavour mediation contri-
butions are assumed to be of the same order, and set by the UV physics, so as to achieve
light stops.

For reasonable phenomenology the models we consider tend to contain negative soft
mass-squareds at the messenger scale for the third generation. It is worth mentioning
that this need not lead to tachyons because the first two generations can have positive
mass-squareds there. A tachyonic direction could potentially develop along $F$ and $D$ flat
directions such as the $UDD$ direction but as described in the Introduction, these can
be lifted by the positive contributions of higgs and sfermion mass-squareds along these
directions. Moreover even if the tachyonic mass-squareds do induce charge and colour
breaking minima, they appear at scales close to the mediation scale which is parametrically larger than $\sqrt{F}$. As such they would have a similar status to the SUSY restoring minima of ISS. (Moreover the EWSB vacua is closer to the origin in field space with higgsed states scaling as $m^2 \sim g^2 \langle \phi \rangle^2$, so at finite temperature we would expect the system to be driven into the vacuum at the origin with more light degrees of freedom, much as in [63, 64].)

These models are consistent with both LHC data and naturalness. Their “smoking gun” is that in contrast to minimal GMSB it does allow for FCNCs at (and violating) present limits, but avoiding them does not apparently require significant fine tuning at least within the limited scope of the flavour models we constructed for this analysis. It is obviously of interest to extend the analysis to complete models of flavour, to perform a more comprehensive study of these constraints. The parameter space of these models may also be excluded either by direct stop searches, which reduce the allowed values of $g_F^2 \Lambda_F^2$. Exclusions for the stau NLSP [65, 66] would also reduce the allowed values of $g_F^2 \Lambda_F^2$ and thereby reduce its effect on the stop sector.

These models are among the first that elevate the masses of the first and second generation squarks dynamically and in a purely gauge mediated framework. This may offer a natural way to achieve stop-scharm admixtures [67] which would further alleviate collider bounds. In extending this work, it may be interesting to consider realisations based on SO(N) or $Sp(N)$ groups.

Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

[1] ATLAS collaboration, Search for new phenomena in final states with large jet multiplicities and missing transverse momentum at $\sqrt{s}=8$ TeV proton-proton collisions using the ATLAS experiment, JHEP 10 (2013) 130 [arXiv:1308.1841] [insPIRE].

[2] ATLAS collaboration, Search for strongly produced supersymmetric particles in decays with two leptons at $\sqrt{s} = 8$ TeV, ATLAS-CONF-2013-089 (2013).

[3] ATLAS collaboration, Search for squarks and gluinos in events with isolated leptons, jets and missing transverse momentum at $\sqrt{s} = 8$ TeV with the ATLAS detector, ATLAS-CONF-2013-062 (2013).

[4] ATLAS collaboration, Search for squarks and gluinos with the ATLAS detector in final states with jets and missing transverse momentum and 20.3 $fb^{-1}$ of $\sqrt{s} = 8$ TeV proton-proton collision data, ATLAS-CONF-2013-047 (2013).

[5] CMS collaboration, Search for supersymmetry in hadronic final states with missing transverse energy using the variables $\alpha_T$ and $b$-quark multiplicity in $pp$ collisions at $\sqrt{s} = 8$ TeV, Eur. Phys. J. C 73 (2013) 2568 [arXiv:1303.2985] [insPIRE].

[6] CMS collaboration, Search for new physics in events with same-sign dileptons and jets in $pp$ collisions at 8 TeV, CMS-PAS-SUS-13-013 (2013).

[7] CMS collaboration, Search for New Physics in the Multijets and Missing Momentum Final State in Proton-Proton Collisions at 8 TeV, CMS-PAS-SUS-13-012 (2013).
[8] CMS collaboration, *Search for supersymmetry using razor variables in events with b-jets in pp collisions at 8 TeV*, CMS-PAS-SUS-13-004 (2013).

[9] ATLAS collaboration, *Search for direct third-generation squark pair production in final states with missing transverse momentum and two b-jets in $\sqrt{s} = 8$ TeV pp collisions with the ATLAS detector*, JHEP 10 (2013) 189 [arXiv:1308.2631] [inSPIRE].

[10] ATLAS collaboration, *Search for pair-produced top squarks decaying into a charm quark and the lightest neutralinos with 20.3 fb$^{-1}$ of pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector at the LHC*, ATLAS-CONF-2013-068 (2013).

[11] ATLAS collaboration, *Searches for direct scalar top pair production in final states with two leptons using the stransverse mass variable and a multivariate analysis technique in $\sqrt{s} = 8$ TeV pp collisions using 20.3 fb$^{-1}$ of ATLAS data*, ATLAS-CONF-2013-065 (2013).

[12] ATLAS collaboration, *Search for strong production of supersymmetric particles in final states with missing transverse momentum and at least three b-jets using 20.1 fb$^{-1}$ of pp collisions at $\sqrt{s} = 8$ TeV with the ATLAS Detector*, ATLAS-CONF-2013-061 (2013).

[13] ATLAS collaboration, *Search for direct top squark pair production in final states with two leptons in $\sqrt{s} = 8$ TeV pp collisions using 20fb$^{-1}$ of ATLAS data*, ATLAS-CONF-2013-048 (2013).

[14] ATLAS collaboration, *Search for direct top squark pair production in final states with one isolated lepton, jets and missing transverse momentum in $\sqrt{s} = 8$ TeV pp collisions using 21 fb$^{-1}$ of ATLAS data*, ATLAS-CONF-2013-037 (2013).

[15] ATLAS collaboration, *Search for direct stop pair production in events with a Z boson, b-jets and missing transverse energy with the ATLAS detector using 21 fb$^{-1}$ from proton-proton collision at $\sqrt{s} = 8$ TeV*, ATLAS-CONF-2013-025 (2013).

[16] ATLAS collaboration, *Search for direct production of the top squark in the all-hadronic $t\bar{t}$ + etmiss final state in 21 fb$^{-1}$ of p-p collisions at $\sqrt{s} = 8$ TeV with the ATLAS detector*, ATLAS-CONF-2013-024 (2013).

[17] CMS collaboration, *Search for top-squark pair production in the single-lepton final state in pp collisions at $\sqrt{s} = 8$ TeV*, Eur. Phys. J. C 73 (2013) 2677 [arXiv:1308.1586] [inSPIRE].

[18] CMS collaboration, *Search for SUSY Partners of Top and Higgs Using Diphoton Higgs Decays*, CMS-PAS-SUS-13-014 (Search for SUSY Partners of Top and Higgs Using Diphoton Higgs Decays).

[19] CMS collaboration, *Search for supersymmetry in pp collisions at $\sqrt{s} = 8$ TeV in events with three leptons and at least one b-tagged jet*, CMS-PAS-SUS-13-008 (Search for supersymmetry in pp collisions at $\sqrt{s} = 8$ TeV in events with three leptons and at least one b-tagged jet).

[20] CMS collaboration, *Search for direct top squark pair production in events with a single isolated lepton, jets and missing transverse energy at $\sqrt{s} = 8$ TeV*, CMS-PAS-SUS-12-023 (2012).

[21] Y. Shadmi and P.Z. Szabo, *Flavored Gauge-Mediation*, JHEP 06 (2012) 124 [arXiv:1103.0292] [inSPIRE].

[22] Z. Kang, T. Li, T. Liu, C. Tong and J.M. Yang, *A Heavy SM-like Higgs and a Light Stop from Yukawa-Deflected Gauge Mediation*, Phys. Rev. D 86 (2012) 095020 [arXiv:1203.2336] [inSPIRE].
[23] A. Albaid and K.S. Babu, Higgs boson of mass 125 GeV in GMSB models with messenger-matter mixing, Phys. Rev. D 88 (2013) 055007 [arXiv:1207.1014] [nSPIRE].

[24] M. Abdullah, I. Galon, Y. Shadmi and Y. Shirman, Flavored Gauge Mediation, A Heavy Higgs and Supersymmetric Alignment, JHEP 06 (2013) 057 [arXiv:1209.4904] [nSPIRE].

[25] N. Craig, M. McCullough and J. Thaler, Flavor Mediation Delivers Natural SUSY, JHEP 06 (2012) 046 [arXiv:1203.1622] [nSPIRE].

[26] R. Auzzi, A. Giveon, S.B. Gudnason and T. Shacham, A Light Stop with Flavor in Natural SUSY, JHEP 01 (2013) 169 [arXiv:1208.6263] [nSPIRE].

[27] A. Bharucha, A. Goudelis and M. McGarrie, En-gauging Naturalness, arXiv:1310.4500 [nSPIRE].

[28] E. Dudas, G. von Gersdorff, S. Pokorski and R. Ziegler, Linking Natural Supersymmetry to Flavour Physics, JHEP 01 (2014) 052 [arXiv:1304.1453] [nSPIRE].

[29] I. Galon, G. Perez and Y. Shadmi, Non-Degenerate Squarks from Flavored Gauge Mediation, JHEP 09 (2013) 117 [arXiv:1306.6631] [nSPIRE].

[30] K. Intriligator and M. Sudano, General Gauge Mediation with Gauge Messengers, JHEP 06 (2010) 047 [arXiv:1001.5443] [nSPIRE].

[31] K.A. Intriligator and N. Seiberg, Lectures on supersymmetric gauge theories and electric-magnetic duality, Nucl. Phys. Proc. Suppl. 45BC (1996) 1 [hep-th/9509066] [nSPIRE].

[32] A. Masiero and T. Yanagida, Real CP-violation, hep-ph/9812225 [nSPIRE].

[33] S. Abel, D. Bailin, S. Khalil and O. Lebedev, Flavor dependent CP-violation and natural suppression of the electric dipole moments, Phys. Lett. B 504 (2001) 241 [hep-ph/0012145] [nSPIRE].

[34] S. Abel and G. Servant, CP and flavor in effective type-I string models, Nucl. Phys. B 611 (2001) 43 [hep-ph/0105262] [nSPIRE].

[35] ATLAS collaboration, Search for squarks and gluinos with the ATLAS detector in final states with jets and missing transverse momentum and 20.3 fb$^{-1}$ of $\sqrt{s} = 8$ TeV proton-proton collision data, ATLAS-CONF-2013-047 (2013).

[36] CMS collaboration, Search for new physics in the multijet and missing transverse momentum final state in proton-proton collisions at $\sqrt{s} = 7$ TeV, Phys. Rev. Lett. 109 (2012) 171803 [arXiv:1207.1898] [nSPIRE].

[37] S.F. King and G.G. Ross, Fermion masses and mixing angles from SU(3) family symmetry, Phys. Lett. B 520 (2001) 243 [hep-ph/0108112] [nSPIRE].
S.F. King and G.G. Ross, *Fermion masses and mixing angles from SU(3) family symmetry and unification*, Phys. Lett. B 574 (2003) 239 [hep-ph/0307190] [SPIRE].

G.G. Ross, L. Velasco-Sevilla and O. Vives, *Spontaneous CP-violation and non-Abelian family symmetry in SUSY*, Nucl. Phys. B 692 (2004) 50 [hep-ph/0401064] [SPIRE].

S. Antusch, S.F. King, M. Malinsky and G.G. Ross, *Solving the SUSY Flavour and CP Problems with Non-Abelian Family Symmetry and Supergravity*, Phys. Lett. B 670 (2009) 383 [arXiv:0807.5047] [SPIRE].

G.G. Ross, L. Velasco-Sevilla and O. Vives, *Spontaneous CP-violation and nonAbelian family symmetry in SUSY*, Nucl. Phys. B 692 (2004) 50 [hep-ph/0401064] [SPIRE].

S. Antusch, S.F. King, M. Malinsky and G.G. Ross, *Solving the SUSY Flavour and CP Problems with Non-Abelian Family Symmetry and Supergravity*, Phys. Lett. B 670 (2009) 383 [arXiv:0807.5047] [SPIRE].

S. Förste, *Gauging Flavour in Meta-Stable SUSY Breaking Models*, Phys. Lett. B 642 (2006) 142 [hep-th/0608036] [SPIRE].

R. Kitano, H. Ooguri and Y. Ookouchi, *Direct Mediation of Meta-Stable Supersymmetry Breaking*, Phys. Rev. D 75 (2007) 045022 [hep-ph/0612139] [SPIRE].

S. Abel, C. Durnford, J. Jaeckel and V.V. Khoze, *Dynamical breaking of U(1)(R) and supersymmetry in a metastable vacuum*, Phys. Lett. B 661 (2008) 201 [arXiv:0707.2958] [SPIRE].

N. Haba and N. Maru, *A Simple Model of Direct Gauge Mediation of Metastable Supersymmetry Breaking*, Phys. Rev. D 76 (2007) 115019 [arXiv:0709.2945] [SPIRE].

A.Giveon and D. Kutasov, *Stable and Metastable Vacua in SQCD*, Nucl. Phys. B 796 (2008) 25 [arXiv:0710.0894] [SPIRE].

F.-q. Xu and J.M. Yang, *An Extension for Direct Gauge Mediation of Metastable Supersymmetry Breaking*, Europhys. Lett. 85 (2009) 61001 [arXiv:0712.4111] [SPIRE].

B. Keren-Zur, L. Mazzucato and Y. Oz, *Direct Mediation and a Visible Metastable Supersymmetry Breaking Sector*, JHEP 10 (2008) 099 [arXiv:0807.4543] [SPIRE].

S. Abel and V.V. Khoze, *Direct Mediation, Duality and Unification*, JHEP 11 (2008) 024 [arXiv:0809.5262] [SPIRE].

R. Essig, J.-F. Fortin, K. Sinha, G. Torroba and M.J. Strassler, *Metastable supersymmetry breaking and multitrace deformations of SQCD*, JHEP 03 (2009) 043 [arXiv:0812.3213] [SPIRE].

D. Koschade, M. McGarrie and S. Thomas, *Direct Mediation and Metastable Supersymmetry Breaking for SO(10)*, JHEP 02 (2010) 100 [arXiv:0909.0233] [SPIRE].

F. Staub, SARAH, arXiv:0806.0538 [SPIRE].

F. Staub, *Automatic Calculation of supersymmetric Renormalization Group Equations and Self Energies*, Comput. Phys. Commun. 182 (2011) 808 [arXiv:1002.0840] [SPIRE].

F. Staub, *SARAH 3.2: Dirac Gauginos, UFO output and more*, Computer Physics Communications 184 (2013) pp. 1792–1809 [arXiv:1207.0906] [SPIRE].

F. Staub, *SARAH 4: A tool for (not only SUSY) model builders*, Comput. Phys. Commun. 185 (2014) 1773 [arXiv:1309.7223] [SPIRE].

W. Porod, *SPheno, a program for calculating supersymmetric spectra, SUSY particle decays and SUSY particle production at $e^+e^-$ colliders*, Comput. Phys. Commun. 153 (2003) 275 [hep-ph/0301101] [SPIRE].

W. Porod and F. Staub, *SPheno 3.1: Extensions including flavour, CP-phases and models beyond the MSSM*, Comput. Phys. Commun. 183 (2012) 2458 [arXiv:1104.1573] [SPIRE].
[61] R. Dermisek and H.D. Kim, *Radiatively generated maximal mixing scenario for the Higgs mass and the least fine tuned minimal supersymmetric standard model*, Phys. Rev. Lett. 96 (2006) 211803 [hep-ph/0601036] [inSPIRE].

[62] R. Dermisek, H.D. Kim and I.-W. Kim, *Mediation of supersymmetry breaking in gauge messenger models*, JHEP 10 (2006) 001 [hep-ph/0607169] [inSPIRE].

[63] S.A. Abel, C.-S. Chu, J. Jaeckel and V.V. Khoze, *SUSY breaking by a metastable ground state: Why the early universe preferred the non-supersymmetric vacuum*, JHEP 01 (2007) 089 [hep-th/0610334] [inSPIRE].

[64] S.A. Abel, J. Jaeckel and V.V. Khoze, *Why the early universe preferred the non-supersymmetric vacuum: Part II*, JHEP 01 (2007) 015 [hep-th/0611130] [inSPIRE].

[65] CMS collaboration, *A search for anomalous production of events with three or more leptons using 19.5/fb of $\sqrt{s} = 8$ TeV LHC data*, CMS-PAS-SUS-13-002.

[66] ATLAS collaboration, *Search for Supersymmetry in Events with Large Missing Transverse Momentum, Jets, and at Least One Tau Lepton in 21 fb$^{-1}$ of $\sqrt{s} = 8$ TeV Proton-Proton Collision Data with the ATLAS Detector*, ATLAS-CONF-2013-026.

[67] M. Blanke, G.F. Giudice, P. Paradisi, G. Perez and J. Zupan, *Flavoured Naturalness*, JHEP 06 (2013) 022 [arXiv:1302.7232] [inSPIRE].