Coset approach to the N=2 supersymmetric matrix GNLS hierarchies

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Abstract

We discuss a large class of coset constructions of the N=2 sl(n|n−1) affine superalgebra. We select admissible subalgebras, i.e. subalgebras that induce linear chiral/antichiral constraints on the coset supercurrents. We show that all the corresponding coset constructions lead to N=2 matrix GNLS hierarchies. We develop an algorithm to compute the relative Hamiltonians and flows. We spell out completely the case of the N=2 \( \hat{sl}(3|2) \), which possesses four admissible subalgebras. The non–local second Hamiltonian structure of the N=2 matrix GNLS hierarchies is obtained via Dirac procedure from the local N=2 \( sl(n|n−1) \) affine superalgebra. We observe that to any second Hamiltonian structure with pure bosonic or pure fermionic superfield content there correspond two different N=2 matrix GNLS hierarchies.

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1 Introduction

In the last ten years there has been a great progress in the construction of new integrable hierarchies with extended \( N \geq 2 \) supersymmetry [1-16]. One of the main reasons of this success lies in the fact that the second Hamiltonian structures of the KdV–type hierarchies are extended superconformal algebras. This provides a way to classify supersymmetric KdV–type hierarchies by means of these superalgebras. The main steps in this direction include the construction of the three different \( N=2 \) supersymmetric KdV hierarchies [1-5] which possess the \( N=2 \) superconformal algebra as their second Hamiltonian structures. Subsequently the \( N = 3 \) \( \mathfrak{sl} \) , \( N = 4 \) KdV [7-10] and \( N=2 \) Boussinesq hierarchies [11], related with \( N = 3, 4 \) super–Virasoro and \( N=2 \) \( W_3 \) algebras, respectively, were found. The generalization to the case of the hierarchies with \( N=2 \) \( W_S \) \( (S \geq 4) \) superalgebras as the second Hamiltonian structures was initiated in [12] for the case of \( N=2 \) \( W_4 \). Finally the two different Lax operators for two (out of three) integrable hierarchies associated with \( N=2 \) \( W_S \) have been proposed in [9] and [13].

At the same time another series of integrable hierarchies with \( N=2 \) supersymmetry, the so called \((n, m)\)-GNLS ones [14], have been constructed. These hierarchies include only spin 1/2 chiral/antichiral superfields and possess non–local \( N=2 \) superalgebras as their second Hamiltonian structures [16]. One can try to construct new hierarchies by ‘joining’ Lax operators of this class with Lax operators of the previous (KdV) type. The extensions of the KdV and Boussinesq hierarchies by junctions of their Lax operators with GNLS one indeed have been found in [8, 9]. In [15] it has been proved that the former extension is in fact gauge related to \( N=2 \) GNLS hierarchies. Finally in [13] we have constructed the matrix GNLS (MGNLS) \( N=2 \) supersymmetric hierarchies and proposed a large class of different reductions of the \( N=2 \) matrix KP hierarchy.

As we said before, the \( N=2 \) \((n, m)\)-GNLS hierarchies are generically characterized by a non–local second Hamiltonian structure. The appearance of systems with the non–local second Hamiltonian structures seems ruin our hopes to classify the integrable hierarchies via their second Hamiltonian structures, due to lack of the classification of non–local algebras. However, as was shown for the bosonic case in [17], it is possible to relate the non–local second Hamiltonian structure to a local one via a coset construction. The first examples in the case of \( N=2 \) supersymmetric systems were elaborated in [18]. The aim of this letter is to show that all \((k|l, m)\)-MGNLS hierarchies (and \((l, m)\)-GNLS as a particular case for \( k = 1 \)) can be reproduced via the coset approach, starting from the local \( N=2 \) \( \mathfrak{sl}(n|n−1) \) superalgebra. The corresponding non–local superalgebras [10, 13] naturally appear in this scheme after applying Dirac reduction procedure. Thus, the classification problem reduces to the classification of \( admissible \) (see below) cosets of \( N=2 \) \( \mathfrak{sl}(n|n−1) \) superalgebra.

The letter is organized as follows. In section 2 we briefly recall the structure of \( N=2 \) \( \mathfrak{sl}(n|n−1) \) superalgebra [19]. The \( N=2 \) supercurrents \( \mathcal{J} \) which span the \( \mathfrak{sl}(n|n−1) \) superalgebra, satisfy nonlinear chirality constraints. To construct the cosets of \( N=2 \) \( \mathfrak{sl}(n|n−1) \) with the chiral/antichiral superfields, we find the subalgebras \( \mathcal{H}_s \) for which the nonlinear chirality constraints on the coset currents \( \mathcal{J}_{\mathfrak{sl}(n|n−1)/\mathcal{H}_s} \) reduce to the linear chiral/antichiral ones on the shell of constraints \( \mathcal{J}_{\mathcal{H}_s} = 0. \) We call these \( admissible \) cosets. In Section 3 we demonstrate how the coset approach works in the simplest non-trivial case of \( N=2 \) \( \mathfrak{sl}(3|2) \) superalgebra. We derive the non–local superalgebras for the coset supercurrents using the Dirac procedure. Moreover, we propose the general form of the admissible cosets of the \( N=2 \) \( \mathfrak{sl}(n|n−1) \) superalgebra for generic \( n \).
2 The N=2 affine superalgebra $\hat{sl}(n|n-1)$ and its admissible subalgebras

Here, we give a short account of the N=2 superalgebra $\hat{sl}(n|n-1)$ \cite{19}, which will be needed in the construction of the N=2 super $(k|l,m)$–MGNLS hierarchies and their second Hamiltonian structures.

The N=2 superalgebra $\hat{sl}(n|n-1)$ contains an equal number, precisely $2(n-1)$, of fermionic and bosonic N=2 supercurrents obeying the proper covariant nonlinear chirality conditions. In reference to the superalgebra $\hat{sl}(n|n-1)$, we use the following convention for the fundamental representation: we divide any supermatrix $J$ into four blocks; the top-left $n \times n$ and bottom-right $(n-1) \times (n-1)$ blocks are made of fermionic supercurrents, while the bosonic supercurrents are the entries of the top-right $n \times (n-1)$ and bottom-left $(n-1) \times n$ blocks.

We introduce the grading $d_{\alpha \beta}$ for the supercurrents $J_{\alpha \beta}$ which is equal to 1 for the fermionic and 0 for the bosonic currents. Following \cite{19} we will also use the complex basis for N=2 $\hat{sl}(n|n-1)$ superalgebra, labeling its generators by $a$ and $\bar{a}$; $a, \bar{a} = 1, 2, \ldots, \frac{1}{2}((2n-1)^2 - 1)$. The explicit relation between these two bases is given by

\[
J_{\alpha \beta} = \begin{pmatrix}
    h_1 & h_2 \bar{\tau} & h_3 & \cdots & h_n \tau & h_{n+1} & \cdots & h_{2n-1} \\
    h_2 \tau & h_{n+1} & h_{n+2} & \cdots & h_{2n-1} & h_{n+1} & \cdots & h_{2n-1} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    h_n \tau & h_{n-1} & h_{n-2} & \cdots & h_1 & h_1 & \cdots & h_1 \\
    \bar{h}_1 & \bar{h}_2 \bar{\tau} & \bar{h}_3 & \cdots & \bar{h}_n \bar{\tau} & \bar{h}_{n+1} & \cdots & \bar{h}_{2n-1} \\
    \bar{h}_2 \bar{\tau} & \bar{h}_{n+1} & \bar{h}_{n+2} & \cdots & \bar{h}_{2n-1} & \bar{h}_{n+1} & \cdots & \bar{h}_{2n-1} \\
    \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\
    \bar{h}_n \bar{\tau} & \bar{h}_{n-1} & \bar{h}_{n-2} & \cdots & \bar{h}_1 & \bar{h}_1 & \cdots & \bar{h}_1
\end{pmatrix},
\]

(2.1)

Barred (unbarred) indices stand for $J_{\bar{a}}$ ($J_a$), these generators being written in a prescribed order. The Poisson brackets\cite{19} between the supercurrents $J_a, \bar{J}_\bar{a}$ form the N=2 superalgebra $\hat{sl}(n|n-1)$ at level $K$

\[
\{J_a(Z_1), J_b(Z_2)\} = - \left( f_{ab}^c D J_c + \frac{1}{K} (-1)^{(d_a+d_b)} f_{ac}^d f_b^{\bar{c} \bar{e}} J_d \bar{J}_\bar{c} \right) \delta(1,2),
\]

(2.2)

\[
\{J_a(Z_1), \bar{J}_{\bar{b}}(Z_2)\} = \left( - f_{\bar{a}\bar{b}} \bar{D} \bar{J}_\bar{c} + \frac{1}{K} (-1)^{(d_a+d_b)} f_{\bar{a}c}^d f_\bar{b}^{\bar{c} \bar{e}} \bar{J}_\bar{d} \bar{J}_\bar{c} \right) \delta(1,2),
\]

\[
\{J_a(Z_1), \bar{J}_{\bar{b}}(Z_2)\} = - \left( K g_{ab} \bar{D} \bar{D} + (-1)^{d_c} f_{ab}^c J_c \bar{D} + f_{ab}^{\bar{c} \bar{e}} \bar{D} J_\bar{c} \right. \\
\left. + \frac{1}{K} (-1)^{(d_a+d_b)} f_{ac}^d f_\bar{b}^{\bar{e} \bar{c}} J_d \bar{J}_\bar{c} \right) \delta(1,2),
\]

where $Z = (z, \theta, \bar{\theta})$ is a coordinate of the N=2 superspace, $\delta(1,2) = (\theta_1 - \bar{\theta}_2)(\bar{\theta}_1 - \theta_2)\delta(z_1 - z_2)$ is the N=2 superspace delta–function and the nonlinear chirality constraints on the supercurrents have the following form:

\[
D J_a - \frac{1}{2K} (-1)^{d_a} f_a^{bc} J_b \bar{J}_\bar{c} = 0, \quad \bar{D} \bar{J}_\bar{a} + \frac{1}{2K} (-1)^{d_{\bar{a}}} f_{\bar{a}}^{\bar{b} \bar{c}} \bar{J}_\bar{b} J_\bar{c} = 0
\]

(2.3)

\footnote{In what follows all the operators appearing in the r.h.s. of the Poisson brackets are evaluated at the point $Z_1$ and the derivatives are assumed to act freely to the right.}
(the summation over repeated indices is assumed). Here, $D$ and $\overline{D}$ are the N=2 supersymmetric fermionic covariant derivatives

$$D = \frac{\partial}{\partial \theta} - \frac{1}{2} \bar{\theta} \frac{\partial}{\partial z}, \quad \overline{D} = \frac{\partial}{\partial \bar{\theta}} - \frac{1}{2} \theta \frac{\partial}{\partial z}, \quad \{D, \overline{D}\} = -\frac{\partial}{\partial z}, \quad \{D, D\} = \{\overline{D}, \overline{D}\} = 0. \quad (2.4)$$

The structure constants $f_{abc}, f_{\bar{a}\bar{b}\bar{c}}, f_{ab\bar{c}}$ and Killing metric $g_{ab}$ are defined in [19].

Before going further, let us remark that the choice of the Cartan currents $h_a, h_{\bar{a}}$ (2.1) is imposed by the requirement that the nonlinear covariant chirality conditions (2.3) become for them standard linear chiral/antichiral ones: $Dh_a = \overline{D}h_{\bar{a}} = 0$.

The main aim of these letter is to demonstrate that the N=2 $(k|l, m)$–MGNLS hierarchies and their second Hamiltonian structures [13] can be obtained via coset approach from the N=2 $\hat{sl}(n|n-1)$ superalgebra.

The basic step in the construction of any coset $G/H$ is the choice of the stability subalgebra $H$. Once this is done, one has to find the Hamiltonians $H_p$ which belong to the coset space, i.e. commute with the currents $J_{G/H}$ of the subalgebra $H$ and therefore give the trivial equations of motion for them. Just these Hamiltonians will produce the equations of motion for the coset currents $J_{G/H}$ on the constraint shell $J_{G/H} = 0$:

$$\frac{\partial}{\partial t_p} J_{G/H} = \{H_p, J_{G/H}\} \bigg|_{J_{G/H} = 0}. \quad (2.5)$$

If the constraints $J_{G/H} = 0$ are second class, then one has to find the Dirac brackets for the coset currents. They will provide the second Hamiltonian structure for the system (2.5).

Let us recall that the first and second flow equations of the N=2 supersymmetric $(k|l, m)$–MGNLS hierarchy have the following form [13]

$$\frac{\partial}{\partial t_1} F = F', \quad \frac{\partial}{\partial t_1} \overline{F} = \overline{F}', \quad \frac{\partial}{\partial t_2} F = F'' + \frac{1}{2K^2} D(\overline{F} F \overline{DF}), \quad \frac{\partial}{\partial t_2} \overline{F} = -\overline{F}'' + \frac{1}{2K^2} \overline{D}((D\overline{F}) F \overline{F}), \quad (2.6)$$

where $F \equiv F_{Aa}(Z)$ and $\overline{F} \equiv F_{bB}(Z)$ ($A, B = 1, \ldots, k; a, b = 1, \ldots, l + m$) are chiral and antichiral rectangular matrix-valued N=2 superfields,

$$DF = 0, \quad \overline{D} \overline{F} = 0, \quad (2.7)$$

respectively. In (2.6) the matrix product is understood. The matrix entries are bosonic superfields for $a = 1, \ldots, l$ and fermionic superfields for $a = l + 1, \ldots, l + m$. The parameter $K$ in the flow equations (2.6) could be set equal to 1 by suitably rescaling the superfield matrices $F$ and $\overline{F}$. But, as we will see in the next section, in the coset approach it coincides with the level of the superalgebra we start from, therefore we prefer to write it explicitly.

Thus, all the supercurrents in the N=2 $(k|l, m)$–MGNLS hierarchies satisfy linear chiral/antichiral conditions (2.7), while in the N=2 $\hat{sl}(n|n-1)$ superalgebra the supercurrents are nonlinearly constrained (2.3). Therefore our first aim is to find the admissible subalgebras $H$ of the N=2 $\hat{sl}(n|n-1)$ superalgebra, i.e. those subalgebras for which the nonlinear chirality constraints (2.3) for the coset supercurrents $J_{G/H}$ reduce to the linear chiral/antichiral ones on the shell $J_{G/H} = 0$. It is somewhat surprising that we can list (a large class of) such subalgebras of N=2 $\hat{sl}(n|n-1)$ with a very simple recipe. In fact there are $2n - 2$ admissible subalgebras. They are identified by the
supermatrices formed by extracting the top–left \( s \times s \) and bottom–right \((2n - 1 - s) \times (2n - 1 - s)\) blocks out of the supermatrix \( \mathcal{J}_{\alpha\beta} \) (2.1):

\[
\mathcal{J}_{\alpha\beta} = \begin{pmatrix}
\mathcal{J}_{\mathcal{H}_s} & \mathcal{J}_{\mathcal{G}/\mathcal{H}_s} \\
\mathcal{J}_{\mathcal{G}/\mathcal{H}_s} & \mathcal{J}_{\mathcal{H}_s}
\end{pmatrix}_{s \times s \times (2n - s - 1) \times (2n - s - 1)}
\]

(2.8)

The currents \( \mathcal{J}_{\mathcal{H}_s} \) span the following N=2 superalgebras as \( s \) runs from 1 to 2\( n - 2 \):

\[
\begin{align*}
\widehat{gl}(s) & \oplus \widehat{sl}(n - s|n - 1), \quad s = 2r \leq n, \\
\widehat{gl}(n - s|n) & \oplus \widehat{sl}(2n - s - 1), \quad s = 2r > n,
\end{align*}
\]

(2.9)

and

\[
\begin{align*}
\widehat{sl}(s - n) & \oplus \widehat{gl}(n - s|n - 1), \quad s = 2r + 1 \leq n, \\
\widehat{sl}(n - s|n) & \oplus \widehat{gl}(2n - s - 1), \quad s = 2r + 1 > n.
\end{align*}
\]

(2.10)

Now we can make three statements. First, the supercurrents in the top–left \( s \times s \) and bottom–right \((2n - s - 1) \times (2n - s - 1)\) blocks (2.8) do not form N=2 subalgebras separately. In fact in order to expose the structure (2.9),(2.10), a suitable recombination of the Cartan currents is necessary. Second, the general formulae for Poisson brackets (2.2) and constraints (2.3) on the currents of the N=2 \( sl(n|n - 1) \) superalgebra provide the corresponding expressions for the subalgebras (2.9) and (2.10) without any modification. All we have to do to get the Poisson brackets and the constraints on the currents for the N=2 superalgebras from the list (2.9), (2.10) at a given value of \( s \), is to restrict the indices of the supercurrent \( \mathcal{J}_{\alpha\beta} \) (2.1) to run within the corresponding top–left and bottom–right blocks. For example, the currents \( \mathcal{J}_{11}, \mathcal{J}_{\alpha\beta} \) (\( \alpha, \beta \geq 2 \)) form the superalgebra \( \widehat{gl}(n - 1|n - 1) \), while the currents \( \mathcal{J}_{(2n - 1)(2n - 1)}, \mathcal{J}_{\alpha\beta} \) (\( \alpha, \beta \leq (2n - 2) \)) span \( \widehat{gl}(n|n - 2) \) (with constraints (2.3) taken into account). The last statement we have to make is that the constraints (2.3) on the coset currents \( \mathcal{J}_{\mathcal{G}/\mathcal{H}_s} \) are reduced to the pure chiral/antichiral ones if we put \( \mathcal{J}_{\mathcal{H}_s} = 0 \) for all subalgebras from (2.9), (2.10). All these statements can be easily proved after rewriting the Poisson brackets (2.2) and constraints (2.3) for the supercurrents (2.1) in the fundamental representation. We will not present the proof here.

After identifying the admissible subalgebras, in order to construct the cosets of N=2 \( \widehat{sl}(n|n - 1) \) with respect to any given subalgebra \( \mathcal{H}_s \) (2.9) or (2.10), we have to find the Hamiltonian densities which commute with all supercurrents from \( \mathcal{H}_s \). They will include in general arbitrary parameters. Next, using (2.3), we find the equations of motion for the coset supercurrents \( \mathcal{J}_{\mathcal{G}/\mathcal{H}_s} \) (with the condition \( \mathcal{J}_{\mathcal{H}_s} = 0 \)) and then fix the parameters in the Hamiltonians in order to recover integrability. The first two steps are purely technical ones, but in practice the straightforward calculations become quickly very cumbersome because a lot supercurrents are involved. A useful way to simplify the calculations is to define new fermionic derivatives for all the coset supercurrents, \( \mathcal{D}\mathcal{J}_{\mathcal{G}/\mathcal{H}_s} \) and \( \overline{\mathcal{D}}\mathcal{J}_{\mathcal{G}/\mathcal{H}_s} \), which are covariant with respect to \( \mathcal{H}_s \) (i.e. they transform to themselves with respect to the adjoint action of the subalgebra \( \mathcal{H}_s \) at \( \mathcal{J}_{\mathcal{H}_s} = 0 \)). By means of covariant derivatives it is therefore easy to construct \( \mathcal{H}_s \)-invariants. In fact, as we will see, all currents from \( \mathcal{H}_s \) appear in the Hamiltonians only through these derivatives.
On the other hand proving integrability is a more difficult task. As usual, either we are able to construct the relevant Lax operators or we are able to show that the constructed systems coincide with known integrable ones. Our aim here is not to prove integrability in general, but to demonstrate, for all admissible subalgebras of the N=2 $\hat{sl}(3|2)$ superalgebra, that we can construct the corresponding cosets and that they do reproduce the integrable N=2 supersymmetric ($k|l, m$)–MGNLS hierarchies. We also suggest that the same construction holds for the generic case of N=2 $\hat{sl}(n|n−1)$ superalgebra.

3 N=2 $\hat{sl}(3|2)$ superalgebra and MGNLS hierarchies

In this section we consider the cosets of the N=2 $\hat{sl}(3|2)$ superalgebra with respect to its four admissible subalgebras from the list (2.9), (2.10) and show that these cosets give rise to the N=2 MGNLS hierarchies.

3.1 N=2 $\hat{sl}(3|2)/\hat{gl}(2|2)$ coset construction

The first subalgebra $\mathcal{H}_1$ from the list (2.9), (2.10) for the N=2 $\hat{sl}(3|2)$ superalgebra is the N=2 superalgebra $\hat{gl}(2|2)$. Its currents and the N=2 $\hat{sl}(3|2)/\hat{gl}(2|2)$ coset currents are extracted from $\mathcal{J}_{\alpha\beta}$ (2.1),

$$\mathcal{J}_{\mathcal{H}_1} = \begin{pmatrix} h_1 \\ h_2 + h_1 & j_{23} & j_{24} & j_{25} \\ j_{32} & h_2 & j_{34} & j_{35} \\ j_{42} & j_{43} & h_1 + h_1 & j_{45} \\ j_{52} & j_{53} & j_{54} & h_2 + h_2 \end{pmatrix}, \quad \mathcal{J}_{\hat{sl}(3|2)/\mathcal{H}_1} = \begin{pmatrix} j_{12} & j_{13} & j_{14} & j_{15} \\ j_{21} \\ j_{31} \\ j_{41} \\ j_{51} \end{pmatrix}, \quad (3.1)$$

respectively. The coset currents $j_{12}, j_{13}, j_{21}, j_{31}$ are fermionic while the remaining ones are bosonic.

It is very convenient to group these supercurrents into a row $F$ and a column $\overline{F}$:

$$F = \{j_{14}, j_{15}, j_{12}, j_{13}\}, \quad \overline{F} = \begin{Bmatrix} j_{41} \\ j_{41} \\ -j_{21} \\ -j_{31} \end{Bmatrix} \quad (3.2)$$

To construct the first two Hamiltonians $H_{1,2}$ which commute with all the currents of N=2 $\hat{gl}(2|2)$, and therefore belong to the coset space $\hat{sl}(3|2)/\hat{gl}(2|2)$ (and provide the trivial equations for all the currents of $\hat{gl}(2|2) : \partial \mathcal{J}_{\hat{gl}(2|2)} / \partial t_{1,2} = 0$), we need to introduce the covariant derivatives for the coset currents. They can be defined as follows:

$$\overline{D}_j j_{12} = \overline{D}_j j_{12} - \frac{1}{K} (j_{12} h_1 - j_{13} j_{32} + j_{14} j_{42} + j_{15} j_{52}), \quad \overline{D}_j j_{14} = \overline{D}_j j_{14} + \frac{1}{K} (j_{14} h_1 + j_{15} j_{54}),$$

$$\overline{D}_j j_{13} = \overline{D}_j j_{13} - \frac{1}{K} (j_{13} h_1 + j_{13} h_2 + j_{14} j_{43} + j_{15} j_{53}), \quad \overline{D}_j j_{15} = \overline{D}_j j_{15} + \frac{1}{K} (j_{15} h_1 + j_{15} h_2),$$

$$\overline{D}_j j_{21} = \overline{D}_j j_{21} - \frac{1}{K} (h_1 j_{21} + j_{23} j_{31} - j_{24} j_{41} - j_{25} j_{51}), \quad \overline{D}_j j_{41} = \overline{D}_j j_{41} + \frac{1}{K} j_{45} j_{51},$$

$$\overline{D}_j j_{31} = \overline{D}_j j_{31} - \frac{1}{K} (h_1 j_{31} + h_2 j_{31} - j_{34} j_{41} - j_{35} j_{51}), \quad \overline{D}_j j_{51} = \overline{D}_j j_{51} - \frac{1}{K} h_{1,51}. \quad (3.3)$$
The expressions for the covariant derivatives (3.3) look complicated, but their construction is straightforward. The first term is always the fermionic derivative of the given current; then we have some additional terms, which can be easily constructed using the basic requirement that the Poisson brackets of covariant derivatives with the currents of \( \hat{gl}(2|2) \) contain only these derivatives and \( \hat{gl}(2|2) \) currents.

Now we are ready to construct the first two Hamiltonians:

\[
H_1 = -\frac{1}{K} \int dZ \text{tr} \left( F \mathcal{F} \right), \\
H_2 = -\frac{1}{K} \int dZ \text{tr} \left( (D \mathcal{F})D \mathcal{F} - \frac{1}{2K^2} F \mathcal{F} F \mathcal{F} \right),
\]

(here the matrix multiplication is assumed). To get the equations of motion for the coset superfields we use eq.(2.5). One can check that the resulting first and second flow equations coincide with the corresponding equations of the \( (1|2, 2) \)-MGNLS hierarchy (2.6). For the reader’s convenience we present eq.(2.3) in explicit form for the coset supercurrents (3.2), in this particular case,

\[
Dj_{12} = -\frac{1}{K} h_1 j_{12}, \\
Dj_{13} = -\frac{1}{K} ((h_1 + h_2)j_{13} - j_{12}j_{23}), \\
Dj_{14} = \frac{1}{K} (j_{12}j_{24} + j_{13}j_{34}), \\
Dj_{15} = -\frac{1}{K} (h_1j_{15} - j_{12}j_{25} - j_{13}j_{35} - j_{14}j_{45}), \\
\mathcal{D}j_{21} = \frac{1}{K} h_1 j_{21}, \\
\mathcal{D}j_{31} = \frac{1}{K} ((h_1 + h_2)j_{31} + j_{21}j_{32}), \\
\mathcal{D}j_{41} = \frac{1}{K} (h_1 j_{41} + j_{21}j_{42} + j_{31}j_{43}), \\
\mathcal{D}j_{51} = \frac{1}{K} ((h_1 + h_2)j_{51} + j_{21}j_{52} + j_{31}j_{53} + j_{41}j_{54}).
\]

(3.5)

After putting all supercurrents \( J_{H_1}, (3.1) \), equal to zero, they become the linear chiral/antichiral ones, \( DF = D \mathcal{F} = 0 \). Thus we end up with the chiral/antichiral matrix superfields \( F, \mathcal{F} (3.2) \) which obey the equations (2.6). Therefore we have succeeded in reproducing the first two flows of the \( (1|2, 2) \)-MGNLS hierarchy in the framework of the \( N=2 \) coset space \( \hat{sl}(3|2)/\hat{gl}(2|2) \).

Let us close this subsection with some comments which will also apply to the next subsections.

First of all we would like to repeat that here and in the coming subsections, we quotient only with admissible subalgebras \( \mathcal{H}_s \) (2.9), (2.10) of the \( N=2 \) \( \hat{sl}(3|2) \) superalgebra. This is what guarantees that in the corresponding coset spaces the non–linear chirality constraints (2.3) are reduced to the linear chiral/antichiral ones.

Second, we remark that using the matrix \( F \) and \( \mathcal{F} \) (3.3) together with the covariant derivatives (3.3) greatly reduces the number of possible terms in the Hamiltonians (3.4). Without introducing these covariant derivatives even \( H_2 \) would contain a huge number of terms – remember that the initial system has 24 supercurrents!

Third, it immediately follows from our construction that the non–local second Hamiltonian structure for the \( N=2 \) \( (1|2, 2) \)-MGNLS \((2, 2)\)-GNLS hierarchy [16] can be obtained from the local \( N=2 \) \( \hat{sl}(3|2) \) superalgebra through the Dirac procedure. This can be done following the lines of [13].

Finally, let us remark that the two terms in the Hamiltonian \( H_2 \) (3.4) commute separately with the all currents of the \( N=2 \) superalgebra \( \hat{gl}(2|2) \), therefore they would allow for two arbitrary coefficients in the equations of motion. We have fixed these coefficients in order to reproduce the known \( (1|2, 2) \)-MGNLS hierarchy.
3.2 $\mathcal{N}=2 \hat{sl}(3|2)/\hat{gl}(2) \oplus \hat{sl}(2|1)$ coset construction

The currents spanned the next $\mathcal{N}=2$ subalgebra $\mathcal{H}_2 = \hat{gl}(2) \oplus \hat{sl}(2|1)$ from the list \(2.9, 2.10\) and the currents of the coset space $\hat{sl}(3|2)/\hat{gl}(2) \oplus \hat{sl}(2|1)$ fill the following blocks in $\mathcal{J}_{\alpha\beta} (2.1)$:

\[
\mathcal{J}_{\mathcal{H}_2} = \begin{pmatrix}
  h_1 & j_{12} \\
  j_{21} & h_2 + h_1
\end{pmatrix}, \quad \mathcal{J}_{\hat{sl}(3|2)/\mathcal{H}_2} = \begin{pmatrix}
  j_{13} & j_{14} & j_{15} \\
  j_{23} & j_{24} & j_{25}
\end{pmatrix},
\]

respectively. Now, among the coset currents, the fermionic ones are $j_{13}, j_{23}, j_{31}, j_{32}$, while the eight remaining ones are bosonic. Let us again arrange these supercurrents into two matrices $-F$ and $\bar{F}$:

\[
F = \begin{pmatrix}
  j_{14} & j_{15} & j_{13} \\
  j_{24} & j_{25} & j_{23}
\end{pmatrix}, \quad \bar{F} = \begin{pmatrix}
  j_{41} & j_{42} \\
  j_{51} & j_{52} \\
  -j_{31} & -j_{32}
\end{pmatrix}.
\]

After introducing the covariant derivatives for the coset currents, which we define as:

\[
\begin{align*}
\bar{\mathcal{D}} j_{13} &= \bar{\mathcal{D}} j_{13} - \frac{1}{K} (j_{13} h_2 + j_{13} h_1 + j_{14} j_{43} + j_{15} j_{53}) , \\
\bar{\mathcal{D}} j_{15} &= \bar{\mathcal{D}} j_{15} + \frac{1}{K} (j_{15} h_1 + j_{15} h_2) , \\
\bar{\mathcal{D}} j_{23} &= \bar{\mathcal{D}} j_{23} - \frac{1}{K} (j_{23} h_2 + j_{23} j_{21} + j_{24} j_{43} + j_{25} j_{53}) , \\
\bar{\mathcal{D}} j_{25} &= \bar{\mathcal{D}} j_{25} + \frac{1}{K} (j_{15} j_{21} + j_{25} h_2) , \\
\mathcal{D} j_{31} &= \mathcal{D} j_{31} - \frac{1}{K} (h_1 j_{31} + h_2 j_{31} - j_{34} j_{41} - j_{35} j_{51}) , \\
\mathcal{D} j_{41} &= \mathcal{D} j_{41} + \frac{1}{K} j_{45} j_{51} , \\
\mathcal{D} j_{51} &= \mathcal{D} j_{51} - \frac{1}{K} h_1 j_{51} , \\
\mathcal{D} j_{42} &= \mathcal{D} j_{42} + \frac{1}{K} (h_1 j_{42} + h_2 j_{41} + j_{45} j_{52}) , \\
\mathcal{D} j_{52} &= \mathcal{D} j_{52} + \frac{1}{K} j_{12} j_{51} ,
\end{align*}
\]

one can check that the first two Hamiltonians (3.4) give rise to the first two flows, (2.6), of the $\mathcal{N}=2 (2|2, 1)$–MGNLS hierarchy. Once again, the matrix supercurrents $F, \bar{F}$ become chiral/antichiral ones on the shell $\mathcal{J}_{\hat{gl}(2) \oplus \hat{sl}(2|1)} = 0$.

Thus our second $\mathcal{N}=2$ coset construction, $\hat{sl}(3|2)/\hat{gl}(2) \oplus \hat{sl}(2|1)$, give rise to the $\mathcal{N}=2 (2|2, 1)$–MGNLS hierarchy.

The $\mathcal{N}=2 (2|2, 1)$–MGNLS hierarchy we constructed here in the framework of the coset approach is the unique example of matrix hierarchy with both fermionic and bosonic superfields among the four coset constructions of the $\mathcal{N}=2 \hat{sl}(3|2)$ superalgebra. Therefore it is instructive to explicitly construct the Poisson brackets for the coset matrix superfields $F$ and $\bar{F}$, (3.7). However, this cannot be done straightforwardly because the constraints $\mathcal{J}_{\hat{gl}(2) \oplus \hat{sl}(2|1)} = 0$ are mixed first and second class. This can be immediately seen from their Poisson brackets on the constraint shell:

\[
\begin{align*}
\Delta_{aa}(1, 2) &\equiv \left. \left\{ \mathcal{J}_a^\mathcal{H}(Z_1), \mathcal{J}_a^\mathcal{H}(Z_2) \right\} \right|_{\mathcal{J}_{\hat{gl}(2) \oplus \hat{sl}(2|1)} = 0} = K\delta_{aa} D\bar{D}\delta(1, 2), \\
\bar{\Delta}_{aa}(1, 2) &\equiv \left. \left\{ \bar{\mathcal{J}}_a^\mathcal{H}(Z_1), \bar{\mathcal{J}}_a^\mathcal{H}(Z_2) \right\} \right|_{\mathcal{J}_{\hat{gl}(2) \oplus \hat{sl}(2|1)} = 0} = -K\delta_{aa} \bar{D}D\delta(1, 2),
\end{align*}
\]

which contain the non–invertible chiral/antichiral projectors $D\bar{D}$ and $\bar{D}D$ acting on the delta-function (in the above equation, we use the compact notation $\mathcal{J}_a^\mathcal{H} = \{h_1, h_2, -j_{12}, -j_{34}, -j_{35}, j_{45}\}$.
and $\mathcal{J}_a^H = \{h_1, h_1 + h_2, j_{21}, j_{43}, j_{53}, j_{54}\}$. Nevertheless, one can define the following brackets for any supercurrent $\mathcal{J}$ from the N=2 $\mathfrak{sl}(3|2)$ superalgebra:

$$\{\mathcal{J}(Z_1), \mathcal{J}(Z_2)\}^* \equiv \left[\{\mathcal{J}(1), \mathcal{J}(2)\} - \int dZ_3 dZ_4 \left(\{\mathcal{J}(1), \mathcal{J}_a^H(3)\} \Delta^a(3, 4) \{\mathcal{J}_a^H(4), \mathcal{J}(2)\} - \{\mathcal{J}(1), \mathcal{J}_a^H(3)\} \bar{\Delta}^a(3, 4) \{\mathcal{J}_a^H(4), \mathcal{J}(2)\}\right]\right]_{\mathcal{J}_a^H(2) = \mathcal{J}(2)} = 0,$$  \hspace{1cm} (3.10)

where we introduced the operators $\Delta^a(1, 2)$ and $\bar{\Delta}^a(1, 2)$ which are required to satisfy the following equations

$$\int dZ_3 \Delta_{aa}(1, 3) \Delta^{ab}(3, 2) = -\delta^b_a D \bar{D} \partial^{-1} \delta(1, 2), \hspace{1cm} \int dZ_3 \Delta^{ba}(1, 3) \Delta_{aa}(3, 2) = -\delta^b_a D \bar{D} \partial^{-1} \delta(1, 2),$$

$$\int dZ_3 \bar{\Delta}_{aa}(1, 3) \bar{\Delta}^{ab}(3, 2) = -\delta^b_a D \bar{D} \partial^{-1} \delta(1, 2), \hspace{1cm} \int dZ_3 \bar{\Delta}^{ba}(1, 3) \bar{\Delta}_{aa}(3, 2) = -\delta^b_a D \bar{D} \partial^{-1} \delta(1, 2).$$

\hspace{1cm} (3.11)

The brackets (3.10) possess the main property of the Dirac brackets:

$$\{\mathcal{J}, \mathcal{J}_a^H\}^* = 0,$$  \hspace{1cm} (3.12)

so we will call them simply Dirac brackets (to check (3.12) one should use eqs. (3.11), the algebra (2.4) for the fermionic derivatives and take into account that on the constraint shell $\mathcal{J}_a^H(2) = \mathcal{J}(2) = 0$ the Poisson brackets $\{\mathcal{J}(Z_1), \mathcal{J}_a^H(Z_2)\}$ become chiral on the second point while the brackets $\{\mathcal{J}_a^H(Z_1), \mathcal{J}(Z_2)\}$ are antichiral on the first point). For the case under consideration the explicit solutions of eqs. (3.11) are

$$\Delta^a(1, 2) = -\bar{\Delta}^a(1, 2) = -\frac{1}{K} \delta^{ab} D \bar{D} \partial^{-1} \delta(1, 2),$$

which are defined up to obvious irrelevant terms which do not change the brackets (3.10). Now we can calculate the Dirac brackets between the matrix superfields $F, \mathcal{F}$ (3.7). They form the following closed non–local algebra

$$\{F_{Aa}(Z_1), F_{Bb}(Z_2)\}^* = \frac{1}{K} \left(F_{Ba} D \bar{D} \partial^{-1} F_{Ab} - (-1)^{d_a d_b} F_{Ab} D \bar{D} \partial^{-1} F_{Ba}\right) \delta(1, 2),$$

$$\{\mathcal{F}_{aA}(Z_1), \mathcal{F}_{bB}(Z_2)\}^* = \frac{1}{K} \left((-1)^{d_a d_b} \mathcal{F}_{bA} D \bar{D} \partial^{-1} \mathcal{F}_{aB} - \mathcal{F}_{aB} D \bar{D} \partial^{-1} \mathcal{F}_{bA}\right) \delta(1, 2),$$

$$\{F_{Aa}(Z_1), \mathcal{F}_{bB}(Z_2)\}^* = \left(-K(-1)^{d_b} \delta_{ab} \delta_{AB} D \bar{D} + \frac{(-1)^{d_b}}{K} \delta_{ab} F_{Ac} D \bar{D} \partial^{-1} \mathcal{F}_{cB}ight) \delta(1, 2)$$

$$\hspace{5cm} - \frac{1}{K} \delta_{AB} F_{Ca} D \bar{D} \partial^{-1} \mathcal{F}_{cB}\right) \delta(1, 2).$$

\hspace{1cm} (3.14)

The algebra (3.14) is a particular case of the non–local algebras proposed in [13]. Let us remark that validity of the Jacobi identities for it is guaranteed since we constructed it by applying Dirac procedure to the local N=2 superalgebra $\mathfrak{sl}(3|2)$. Now one can easily check that the Hamiltonians (3.4) with the covariant derivatives replaced by the fermionic ones (2.4) produces the correct flow equations (2.6) if we use the brackets (3.14). Let us stress that generically the non–local superalgebra (3.14) is valid for the all cosets of N=2 $\mathfrak{sl}(3|2)$ we are considering in this section.
3.3 \( \mathbb{N}=2 \) \( \hat{sl}(3|2)/\hat{gl}(3) \oplus \hat{sl}(2) \) coset construction

The third admissible subalgebra from the list (3.10), (2.11) \( \mathcal{H}_{3} = \hat{gl}(3) \oplus \hat{sl}(2) \) gives rise to the following decomposition for the supercurrents

\[
\mathcal{J}_{\mathcal{H}_{3}} = \begin{pmatrix}
h_{1} & j_{12} & j_{13} \\
j_{21} & h_{2} + h_{1} & j_{23} \\
j_{31} & j_{32} & h_{2} \\
h_{1} + h_{1} & j_{54} & h_{2} + h_{2}
\end{pmatrix}, \quad \mathcal{J}_{\hat{sl}(3|2)/\mathcal{H}_{3}} = \begin{pmatrix}
j_{14} & j_{15} \\
j_{24} & j_{25} \\
j_{34} & j_{35}
\end{pmatrix}
\]

(3.15)

We again arrange the coset supercurrents into two matrices – \( F \) and \( \overline{F} \):

\[
F = \begin{pmatrix} j_{14} & j_{15} \\
j_{24} & j_{25} \\
j_{34} & j_{35} \end{pmatrix}, \quad \overline{F} = \begin{pmatrix} j_{41} & j_{42} & j_{43} \\
j_{51} & j_{52} & j_{53} \end{pmatrix}
\]

(3.16)

The covariant derivatives for the coset currents (3.16) are

\[
\overline{\mathcal{D}}_{j_{14}} = \overline{\mathcal{D}}_{j_{14}} + \frac{1}{K} (j_{14} h_{1} + j_{15} j_{54}), \quad \overline{\mathcal{D}}_{j_{15}} = \overline{\mathcal{D}}_{j_{15}} + \frac{1}{K} (j_{15} h_{1} + j_{15} h_{2}),
\]

\[
\overline{\mathcal{D}}_{j_{24}} = \overline{\mathcal{D}}_{j_{24}} + \frac{1}{K} (j_{14} j_{21} + j_{25} j_{54}), \quad \overline{\mathcal{D}}_{j_{34}} = \overline{\mathcal{D}}_{j_{34}} - \frac{1}{K} (j_{34} h_{2} - j_{14} j_{31} - j_{24} j_{32} - j_{35} j_{54}),
\]

\[
\overline{\mathcal{D}}_{j_{25}} = \overline{\mathcal{D}}_{j_{25}} + \frac{1}{K} (j_{25} h_{2} + j_{15} j_{21}), \quad \overline{\mathcal{D}}_{j_{35}} = \overline{\mathcal{D}}_{j_{35}} + \frac{1}{K} (j_{15} j_{31} + j_{25} j_{32}),
\]

\[
\mathcal{D}_{j_{41}} = \mathcal{D}_{j_{41}} + \frac{1}{K} j_{45} j_{51}, \quad \mathcal{D}_{j_{42}} = \mathcal{D}_{j_{42}} + \frac{1}{K} (h_{1} j_{42} + j_{12} j_{41} + j_{45} j_{52}),
\]

\[
\mathcal{D}_{j_{43}} = \mathcal{D}_{j_{43}} + \frac{1}{K} (h_{1} j_{43} + h_{2} j_{43} + j_{13} j_{41} + j_{23} j_{42} + j_{45} j_{53}), \quad \mathcal{D}_{j_{51}} = \mathcal{D}_{j_{51}} - \frac{1}{K} h_{1} j_{51},
\]

\[
\mathcal{D}_{j_{52}} = \mathcal{D}_{j_{52}} + \frac{1}{K} j_{12} j_{51}, \quad \mathcal{D}_{j_{53}} = \mathcal{D}_{j_{53}} + \frac{1}{K} (h_{2} j_{53} + j_{13} j_{51} + j_{23} j_{52})
\]

(3.17)

Again one can check that the Hamiltonians defined in (3.4), applied to this case, produce the first and the second flows (2.7) of the \( \mathbb{N}=2 (3|2,0) \)–MGNLS hierarchy.

The coset construction we considered in this subsection contains only bosonic superfields which we arranged in a \( (3 \times 2) \) matrix \( F \) and a \( (2 \times 3) \) matrix \( \overline{F} \) (3.10). The Dirac brackets for them are given by the non–local superalgebra (3.14). But here comes an important observation. There exists another non–local superalgebra of the type (3.14) with the same number of degrees of freedom – 6 chiral and 6 antichiral bosonic superfields – but with a different matrix arrangement – a \( (2 \times 3) \) matrix \( F_{1} \) and a \( (3 \times 2) \) matrix \( \overline{F}_{1} \). The latter system describes a different hierarchy – the \( \mathbb{N}=2 (2|3,0) \)–MGNLS one. One can verify that actually these two algebras coincide after the redefinition \( F_{1} = i F^{T}, \overline{F}_{1} = i \overline{F}^{T} \) (the superscript \( T \) denotes matrix transposition). Therefore, in the framework of the same \( \mathbb{N}=2 \) coset construction \( \hat{sl}(3|2)/\hat{gl}(3) \oplus \hat{sl}(2) \) with the matrix superfields \( F, \overline{F} \) (3.16) there is room for two different hierarchies, the \( \mathbb{N}=2 (3|2,0) \)–MGNLS and the \( \mathbb{N}=2 (2|3,0) \)–MGNLS hierarchies. The Hamiltonians and the flows corresponding to the last hierarchy can be obtained from eqs.(3.4) and (2.6), respectively, via the following substitutions:

\[
F \rightarrow i F^{T}, \quad \overline{F} \rightarrow i \overline{F}^{T},
\]

(3.18)

where \( i \) is the imaginary unit.
What we just illustrated in the above example is actually completely general. One can show that the Hamiltonian structure for the N=2 (k|0, 0)–MGNLS hierarchy is the same as for the N=2 (l|k, 0)–MGNLS one. One Hamiltonian structure is mapped to the other by the transformation (3.18). Similarly the N=2 (k|0, m)–MGNLS and the N=2 (m|0, k)–MGNLS hierarchies, which involve pure fermionic matrix superfields, possess the same second Hamiltonian structure. In this case one Hamiltonian structure is mapped to the other by the transformation $F \rightarrow F^T$ and $\mathcal{T} \rightarrow \mathcal{T}^T$.

So one can state that to any non–local superalgebra (3.14) with either only bosonic or only fermionic matrices, there correspond two different N=2 MGNLS hierarchies. It is natural to remark the analogy with the case of the N=2 $W_S$ superalgebra which is the second Hamiltonian structure for three different supersymmetric KdV–type hierarchies.

3.4 $N=2 \, \hat{sl}(3|2)/gl(3|1)$ coset construction

The last case from the list (2.9), (2.10) for the admissible N=2 $\hat{sl}(3|2)$ subalgebras is $\mathcal{H}_4 = \hat{gl}(3|1)$:

$$
\mathcal{J}_{\mathcal{H}_4} = \begin{pmatrix}
    h_1 & j_{12} & j_{13} & j_{14} \\
    j_{21} & h_2 + h_1 & j_{23} & j_{24} \\
    j_{31} & j_{32} & h_2 & j_{34} \\
    j_{41} & j_{42} & j_{43} & h_1 + h_2
\end{pmatrix},
\mathcal{J}_{\hat{sl}(3|2)/\mathcal{H}_4} = \begin{pmatrix}
    J_{15} \\
    J_{25} \\
    J_{35} \\
    J_{45}
\end{pmatrix}.
$$

(3.19)

Among the coset supercurrents only $j_{45}$ and $j_{54}$ are fermionic while the remaining ones are bosonic. We arrange these supercurrents into $F$ and $\mathcal{T}$:

$$
F = i \{ j_{15}, j_{25}, j_{35}, j_{45} \},\quad \mathcal{T} = i \begin{pmatrix}
    j_{51} \\
    j_{52} \\
    j_{53} \\
    j_{54}
\end{pmatrix}.
$$

(3.20)

The covariant derivatives for the coset currents (3.20) read:

$$
\bar{D} j_{15} = \bar{D} j_{15} + \frac{1}{K} (j_{15} h_1 + j_{15} h_2), \quad \bar{D} j_{25} = \bar{D} j_{25} + \frac{1}{K} (j_{25} h_2 + j_{15} j_{21}),
$$

$$
\bar{D} j_{35} = \bar{D} j_{35} + \frac{1}{K} (j_{15} j_{31} + j_{25} j_{32}), \quad \bar{D} j_{45} = \bar{D} j_{45} - \frac{1}{K} (j_{45} h_2 - j_{15} j_{41} - j_{25} j_{42} - j_{35} j_{43}),
$$

$$
D j_{51} = D j_{51} - \frac{1}{K} h_1 j_{51}, \quad D j_{53} = D j_{53} + \frac{1}{K} (h_2 j_{53} + j_{13} j_{51} + j_{23} j_{52}),
$$

$$
D j_{52} = D j_{52} + \frac{1}{K} j_{12} j_{51}, \quad D j_{54} = D j_{54} - \frac{1}{K} (h_1 j_{54} + j_{14} j_{51} + j_{24} j_{52} + j_{34} j_{53}).
$$

(3.21)

For this coset construction the Hamiltonians (3.4) again produce via (2.6) the flows which correspond to the N=2 (1|3, 1)–MGNLS hierarchy.

4 Conclusion and outlook

In the previous section we explicitly demonstrated that all the subalgebras of the N=2 $\hat{sl}(3|2)$ superalgebra from the list (2.9), (2.10), i.e. the admissible ones, give rise via coset construction to N=2 MGNLS hierarchies (GNLS ones, when the matrix superfields $F$ and $\mathcal{T}$ are simple row
and column matrices, respectively). We have checked that the same is true for the two admissible subalgebras of N=2 \( \hat{sl}(2|1) \). These examples suggest that for each N=2 \( \hat{sl}(n|n-1) \) superalgebra its admissible subalgebras \( (2.9),(2.10) \) give rise to the N=2 MGNLS hierarchies and the corresponding Dirac reductions of the local second Hamiltonian structure (N=2 \( \hat{sl}(n|n-1) \)) reproduce the corresponding non–local second Hamiltonian structures of the N=2 MGNLS hierarchies, [13].

We would like to point out that the coset construction can be applied to any N=2 subalgebra which appears as addenda in the list \( (2.9),(2.10) \), e.g. to \( \hat{gl}(2n|2m) \) and \( \hat{sl}(2n+1|2m) \). For example, for the case of N=2 \( \hat{sl}(2n+1) \) superalgebras this procedure leads to the N=2 GMNLS hierarchies with pure fermionic matrix superfields. We have explicitly checked this for the cases of N=2 \( \hat{sl}(3) \) and \( \hat{sl}(5) \) superalgebras. The coset constructions related to the N=2 \( \hat{sl}(2n+1) \) superalgebra involve only fermionic matrix superfields and we can construct two different integrable hierarchies for each admissible subalgebra – the N=2 \( (s|0,2n+1-s) \) and the N=2 \( (2n+1-s|0,s) \)–MGNLS ones. This is in fact a particular case of what was remarked at the end of subsection 3.3.

Now we should recall the original motivation of the present work, namely understanding the origin of the non–locality of the second Hamiltonian structure of all N=2 MGNLS hierarchies. In this sense the main result of our paper is that we have been able to ‘localize’ these non–local Hamiltonian structures. In fact, as we have already remarked, the original superalgebras we started from are local and we could make a list of admissible subalgebras, thus providing a sort of classification for the N=2 MGNLS (GNLS) hierarchies. Moreover these superalgebras, which localize the second Hamiltonian structures for N=2 MGNLS hierarchies, can give rise to new hierarchies with extended sets of superfields. For the case N=2 \( gl(2) \) this was demonstrated in [20].

Let us conclude our letter with a few remarks. One of the interesting consequences of our approach is the fact that some of the N=2 MGNLS (GNLS) hierarchies are more ”fundamental” than others. Indeed, in the framework of coset construction we were able to reproduce the matrix hierarchies only for some specific relations between the numbers of rows and columns – precisely, the supermatrix \( F \) is generically defined as an \( s \times (2n-1-s) \) rectangular matrix, with \( s = 1, \ldots, 2n-2 \), for the case of N=2 \( \hat{sl}(n|n-1) \). Of course, the resulting systems admit further reductions to the \( s_1 \times n_1 \) supermatrix, with \( s_1 \leq s, n_1 \leq 2n-1-s \), giving rise to all the N=2 MGNLS hierarchies [13], but it might not be possible to construct these ”secondary” reduced cases directly via the coset approach. It would be interesting to understand the reason of this ‘fundamental’ role. Another mysterious fact is that the number \( 4sn - 2s(s+1) \) of the N=2 superfields in all the admissible coset constructions is divisible by 4, thus providing the necessary condition for the existence of a hidden \( N = 4 \) supersymmetry. In [13] it has been verified that actually some of them do possess a hidden \( N = 4 \) supersymmetry.

Finally we would like to mention the possibility to use the coset approach for the superalgebras which come as a result of Hamiltonian reduction applied to the N=2 \( \hat{sl}(n|n-1) \) superalgebra [19] (e.g. N=2 \( W_S \) superalgebras, etc.). The first example of such construction has been elaborated in [18].

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