Effective chiral theory of mesons, coefficients of ChPT, axial-vector symmetry breaking

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Abstract

A phenomenological successful effective chiral theory of pseudoscalar, vector, and axial-vector mesons is introduced. Based on this theory all the 10 coefficients of ChPT are predicted. It has been found that a new symmetry breaking-axial-vector symmetry breaking is responsible for the mass difference between $\rho$ and $a_1$ mesons. It is shown that in an EW theory without Higgs $m_W$ and $m_Z$ are dynamically generated by the combination of the fermion masses and the axial-vector symmetry breaking: $m_W = \frac{1}{\sqrt{2}} g m_t$, $m_Z = \rho m_W / \cos \theta_W$ with $\rho \approx 1$, and $G_F = 1/(2\sqrt{2} m_t^2)$. Two gauge fixing terms of W and Z fields are dynamically generated too.
1 Effective chiral theory of mesons

The chiral perturbation theory is rigorous and phenomenologically successful in describing the physics of the pseudoscalar mesons at low energies ($E < m_\rho$). Models attempt to deal with the two main frustrations that the ChPT is limited to pseudoscalar mesons at low energy and contains many coupling constants which must be measured.

I have proposed an effective chiral theory of pseudoscalar, vector, and axial-vector mesons[1]. It provides a unified study of pseudoscalar, vector, and axial-vector meson physics at low energies.

The ansatz made in this theory is that the meson fields are simulated by quark operators. For example,

$$\rho^\mu = -\frac{1}{g_\rho m_\rho^2} \bar{\psi} \gamma_\mu \gamma_5 \psi.$$  \hspace{1cm} (1)

The ansatz can be tested. Applying PCAC, current algebra, and this expression to the decay $\rho \to \pi\pi$, under the soft pion approximation it is derived

$$\frac{1}{2} f_{\rho\pi\pi} g_\rho = 1$$  \hspace{1cm} (2)

which is just the result of the VMD.

As a matter of fact, using fermion operator to simulate a meson field has a long history:
1. More than six decades ago, Jordan et al observed

\[ \frac{1}{\sqrt{\pi}} \partial_\mu \phi = \bar{\psi} \gamma_5 \gamma_\mu \psi \] (3)

in 1+1 field theory.

2. A similar relation is proved in the bosonization of 1+1 field theory.

3. Use of quark operators to simulate meson fields has been already exploited in the

Nambu-Jona-Lasinio (NJL) model, a model of four quark interactions.

4. Quark operators have been taken as interpolating fields in lattice gauge calculations.

The simulations of the meson fields by quark operators are realized by a Lagrangian

which is constructed by chiral symmetry

\[ \mathcal{L} = \bar{\psi}(x)(i \gamma \cdot \partial + \gamma \cdot v + \gamma \cdot a \gamma_5 - m u(x))\psi(x) \]

\[ -\bar{\psi}(x)M\psi(x) + \frac{1}{2} m_0^2 (\rho_\mu \rho_\mu + K^{*\mu} K^*_\mu + \omega^\mu \omega_\mu + \phi^\mu \phi_\mu + a_\mu a^\mu + K_1^{\mu} K_1^\mu + f_\mu f^\mu + f_{1s}^\mu f_{1s\mu}) \] (4)

\( u = exp\{i \gamma_5 (\tau_\pi + \lambda_8 K_a + \lambda_8 \eta_8 + \eta_0)\} \). To avoid double counting, there are no kinetic terms of
mesons, which are generated dynamically. Using the least action principle, the relationships
between meson fields and quark operators are found from the Lagrangian. Taking the case
of two flavors as an example, from the least action principle we obtain

\[
\frac{\Pi_i}{\sigma} = i(\bar{\psi}\tau_i\gamma_5\psi + ix\bar{\psi}\tau_i\psi)/(\bar{\psi}\psi + ix\bar{\psi}\gamma_5\psi),
\]

\[
x = (i\bar{\psi}\gamma_5\psi - \frac{\Pi_i}{\sigma}\bar{\psi}\tau_i\psi)/(\bar{\psi}\psi + i\frac{\Pi_i}{\sigma}\bar{\psi}\tau_i\gamma_5\psi),
\]

\[
\rho^i_\mu = -\frac{1}{m_0^2}\bar{\psi}\tau_i\gamma_\mu\psi, \quad a^i_\mu = -\frac{1}{m_0^2}\bar{\psi}\tau_i\gamma_\mu\gamma_5\psi,
\]

\[
\omega_\mu = -\frac{1}{m_0^2}\bar{\psi}\gamma_\mu\psi, \quad f_\mu = -\frac{1}{m_0^2}\bar{\psi}\gamma_\mu\gamma_5\psi,
\]

(5)

where \( \sigma + i\gamma_5\tau \cdot \Pi = ue^{-i\eta\gamma_5} \), \( \sigma = \sqrt{1 - \Pi^2} \), and \( x = \tan\eta \). The pseudoscalar fields have very complicated quark structures. Substituting these expressions into the Lagrangian of two flavors, a Lagrangian of quarks is obtained. It is no longer a theory of four quarks. This model is different from NJL model.

Using path interal to integrate out the quark fields, the effective Lagrangian of mesons is derived.

\[
\mathcal{L}_E = \ln det \mathcal{D},
\]

\[
\mathcal{L}_{re} = \frac{1}{2}\ln det (\mathcal{D}^\dagger\mathcal{D}),
\]

\[
\mathcal{L}_{im} = \frac{1}{2}\ln det (\mathcal{D}/\mathcal{D}^\dagger).
\]

The question is whether the masses, decay widths, and interactions of mesons can be described by the quark operators correctly. Taking masses as an example,
1. Pseudoscalar mesons

\[ m_{\pi^\pm}^2 = \frac{4}{f_{\pi^\pm}^2} \left\{ -\frac{1}{3} < \bar{\psi}\psi > (m_u + m_d) - \frac{F^2}{4} (m_u + m_d)^2 \right\}, \]

\[ m_{\pi^0}^2 = \frac{4}{f_{\pi^0}^2} \left\{ -\frac{1}{3} < \bar{\psi}\psi > (m_u + m_d) - \frac{F^2}{2} (m_u^2 + m_d^2) \right\}, \]

\[ m_{K^+}^2 = \frac{4}{f_{K^+}^2} \left\{ -\frac{1}{3} < \bar{\psi}\psi > (m_u + m_s) - \frac{F^2}{4} (m_u + m_s)^2 \right\}, \]

\[ m_{K^0}^2 = \frac{4}{f_{K^0}^2} \left\{ -\frac{1}{3} < \bar{\psi}\psi > (m_d + m_s) - \frac{F^2}{4} (m_d + m_s)^2 \right\}, \]

\[ m_{\eta_8}^2 = \frac{4}{f_{\eta_8}^2} \left\{ -\frac{1}{3} < \bar{\psi}\psi > \frac{1}{3} (m_u + m_d + 4m_s) - \frac{F^2}{6} (m_u^2 + m_d^2 + 4m_s^2) \right\}, \] (6)

The formulas at the first order in quark masses are Gell-Mann, Oakes, and Renner chiral perturbation theory.

2. Vector mesons

\[ m_\rho^2 = m_\omega^2 = 6m^2 [\pi]. \]

3. Axial-vector mesons

\[ (1 - \frac{1}{2\pi^2 g^2})m_a^2 = 6m^2 + m_\rho^2, \]

\[ (1 - \frac{1}{2\pi^2 g^2})m_f^2 = m_\rho^2 + m_\omega^2. \]

The masses of vector originate in dynamical chiral symmetry breaking. This is the reason why they are much heavier than pseudoscalars. The masses of the axial-vector mesons are
resulted by a new symmetry breaking-axial-vector symmetry breaking.

The widths are

\[ \Gamma_\rho = 142 \text{MeV} (\text{Exp.} 150 \text{MeV}), \]

\[ \Gamma_\omega = 7.7 \text{MeV} (\text{Exp.} 7.49 \text{MeV}), \]

\[ \Gamma_{f \to \rho \pi} = 6.01 \text{MeV} (\text{Exp.} 6.96(1 \pm 0.33) \text{MeV}) \]

This theory has following features:

1. The theory is chiral symmetric in the limit of \( m_q \to 0 \). The theory has dynamically chiral symmetry breaking (m),

2. VMD is a natural result

\[ \frac{e}{f_v} \{-\frac{1}{2} F^{\mu\nu}(\partial_\mu \rho_\nu - \partial_\nu \rho_\mu) + A^\mu j^\mu \}. \]

3. Axial-vector currents are bosonized

\[ -\frac{g_W}{4 f_a f_a} \{-\frac{1}{2} F^{i\mu\nu}(\partial_\mu a^i_\nu - \partial_\nu a^i_\mu) + A^{i\mu} j^i_\mu \} \]

\[ -\frac{g_W}{4} \Delta m^2 f_a A^i_\mu a^{i\mu} - \frac{g_W}{4} f_\pi A^{i\mu} \partial_\mu \pi^i, \]

Axial-vector symmetry breaking is taken part in.
4. The Wess-Zumino-Witten anomalous action is the leading term of the imaginary part of the effective Lagrangian,

5. Weinberg’s first sum rule is satisfied analytically,

6. The constituent quark mass is introduced as $m,$

7. Theoretical results of the masses and strong, E&M, and weak decay widths of mesons agree well with data,

8. The form factors of pion, $\pi_{l3}, K_{l3}, \pi \rightarrow e\gamma\nu,$ and $K \rightarrow e\gamma\nu$ are obtained and agree with data. For example,

\[
< r^2 >_\pi = 0.445 \text{ fm}^2,
\]

\[
Exp. = 0.44 \pm 0.01 \text{ fm}^2,
\]

\[
\rho - pole = 0.39 \text{ fm}^2.
\]

9. The theory has been applied to $\tau$ mesonic decays\textsuperscript{[3]} successfully. For example,

\[
B(\tau \rightarrow \eta 3\pi \nu) = 3.4 \times 10^{-4},
\]

\[
Exp. = (4.1 \pm 0.7 \pm 0.7) \times 10^{-4},
\]

\[
ChPT = 1.2 \times 10^{-6}.
\]
10. $\pi\pi$ and $\pi K$ scatterings are studied. Theory agrees with data,

11. The parameters of this theory are: $m$ (quark condensate), $g$ (universal coupling constant), and three current quark masses,

12. Large $N_C$ expansion is natural in this theory. All loop diagrams of mesons are at higher orders in $N_C$ expansion. So far all calculations are done at the three level,

13. A cut-off has been determined to be 1.6GeV. All the masses of mesons are below the cut-off. The theory is self consistent,

2 Coefficients of chiral perturbation theory

Chiral perturbation theory is the low energy limit of any successful effective meson theory

$$
\mathcal{L} = \frac{f_\pi^2}{16} Tr D_\mu U D_\mu U^\dagger + \frac{f_\pi^2}{16} Tr \chi (U + U^\dagger) \\
+ L_1 [Tr(D_\mu U D_\mu U^\dagger)]^2 + L_2 (Tr D_\mu U D_\nu U^\dagger)^2 \\
+ L_5 Tr (D_\mu U D_\mu U^\dagger)^2 + L_4 Tr (D_\mu U D_\mu U^\dagger) Tr \chi (U + U^\dagger) \\
+ L_5 Tr D_\mu U D_\mu U^\dagger (\chi U^\dagger + U \chi) + L_6 [Tr \chi (U + U^\dagger)]^2 \\
+ L_7 [Tr \chi (U - U^\dagger)]^2 + L_8 Tr (\chi U \chi U + \chi U^\dagger \chi U^\dagger) \\
- i L_9 Tr (F^L_{\mu \nu} D_\mu U D_\nu U^\dagger + F^R_{\mu \nu} D_\mu U^\dagger D_\nu U)$$
Many models try to predict the coefficients of ChPT (see Table I).

The effective chiral theory of mesons is used to predict all the 10 coefficients.

1. The effective L of ππ scattering at low energy is derived. Two of the three coefficients are obtained

\[
2(L_1 + L_2) + L_3 = \frac{1}{4} \left( \frac{1}{(4\pi)^2} \right) \left(1 - \frac{2c}{g}\right)^4
\]

\[
L_2 = \frac{1}{4} \left( \frac{c^4}{g^2} \right) + \frac{1}{4} \left( \frac{1}{(4\pi)^2} \right) \left(1 - \frac{2c}{g}\right)^2 \left(1 - \frac{4c}{g} - \frac{4c^2}{g^2}\right)
\]

\[
+ \frac{1}{8} \left(1 - \frac{2c}{g}\right) c \{2gc + \frac{1}{\pi^2} \left(1 - \frac{2c}{g}\right) \}
\]

\[
c = \frac{f_\pi^2}{2g\rho^2}.
\]  
(8)

2. A complete determination of \( L_{1,2,3} \) is carried out from the effective L of πK scattering

\[-2L_1 + L_2 = 0,\]

\[
L_3 = -\frac{3}{16} \frac{2c}{g} \left(1 - \frac{2c}{g}\right) \{2gc + \frac{1}{\pi^2} \left(1 - \frac{2c}{g}\right) \}
\]

\[
- \frac{1}{2} \left( \frac{1}{(4\pi)^2} \right) \left(1 - \frac{2c}{g}\right)^2 \left(1 - \frac{4c}{g} - \frac{8c^2}{g^2}\right) - \frac{3}{4} \frac{c^4}{g^2},
\]

\[
L_1 = \frac{1}{32} \frac{2c}{g} \left(1 - \frac{2c}{g}\right) \{2gc + \frac{1}{\pi^2} \left(1 - \frac{2c}{g}\right) \}
\]

\[
+ \frac{1}{8} \left( \frac{1}{(4\pi)^2} \right) \left(1 - \frac{2c}{g}\right)^2 \left(1 - \frac{4c}{g} - \frac{4c^2}{g^2}\right) + \frac{1}{4} \frac{c^4}{g^2}.
\]  
(9)
3. The coefficients $L_{4-8}$ are determined from the quark mass expansions of $m^2_{\pi}$, $m^2_K$, $m^2_\eta$, $f_{\pi}$, $f_K$, and $f_\eta$

\begin{align}
L_4 &= 0, \quad L_6 = 0, \quad (10) \\
L_5 &= \frac{f_{\pi}^2 f}{8 m B_0}, \quad (11) \\
L_8 &= -\frac{F^2}{16 B_0^2}, \quad (12) \\
3L_7 + L_8 &= -\frac{F^2}{16 B_0^2}, \quad L_7 = 0, \quad (13)
\end{align}

where

\begin{align}
B_0 = \frac{4}{f_{\pi}^2} \left( -\frac{1}{3} \right) < \bar{\psi} \psi >. \quad (14)
\end{align}

$L_5$ and $L_8$ are written as

\begin{align}
L_5 &= \frac{1}{32Q} \left( 1 - \frac{2c}{g} \right) \left\{ \left( 1 - \frac{2c}{g} \right)^2 \left( 1 - \frac{1}{2\pi^2 g^2} \right) \right. \\
&\quad \left. - \left( 1 - \frac{2c}{g} \right) + \frac{4}{\pi^2} Q \left( 1 - \frac{c}{g} \right) \right\}, \quad (15) \\
L_8 &= -\frac{1}{1536g^2Q^2} \left( 1 - \frac{2c}{g} \right)^2, \quad (16)
\end{align}

where

\begin{align}
Q = -\frac{1}{108g^4} \frac{1}{m^3} < \bar{\psi} \psi >. \quad (17)
\end{align}

$Q$ is a function of the universal coupling constant $g$ only. By fitting $\rho \to e^+e^-$, $g$ is determined to be 0.39. The numerical value of $Q$ is 4.54.
4. \( L_9 \) and \( L_{10} \) are determined by \( < r^2 >_\pi \) and the amplitudes of pion radiative decay, \( \pi^- \rightarrow e^- \gamma \nu \)

\[
L_9 = \frac{f^2_\pi}{48} < r^2 >_\pi, \quad (18)
\]

\[
L_0 = \frac{1}{32\pi^2} \frac{R}{FV}, \quad (19)
\]

\[
L_{10} = \frac{1}{32\pi^2} \frac{F^A}{FV} - L_9, \quad (20)
\]

\[
L_9 = \frac{1}{4}cg + \frac{1}{16\pi^2} \left\{ \left(1 - \frac{2c}{g} \right)^2 - 4\pi^2c^2 \right\}, \quad (21)
\]

\[
L_{10} = -\frac{1}{4}cg + \frac{1}{4}c^2 - \frac{1}{32\pi^2} \left(1 - \frac{2c}{g} \right)^2. \quad (22)
\]

There are two parameters: \( g \) and \( f^2_\pi/m_\rho^2 \) in all the 10 coefficients, which have been determined already. The numerical values of the 10 coefficients predicted by present theory are shown in Table II.

3 Axial-vector symmetry breaking and W and Z masses

Pion, \( \rho \) and \( a_1 \) are made of u and d quarks. Pions are Goldstone bosons. is light. \( m_\rho \) is resulted in dynamical chiral symmetry breaking (quark condensate) and is much heavier than pions. It is well known that \( a_1 \) meson is the chiral partner of \( \rho \) meson. However, \( a_1 \) is much heavier (1.26GeV) than \( \rho \) meson (0.77GeV) is.

Why?
What kind of symmetry breaking is responsible for the mass difference between $\rho$ and $a_1$ mesons?

There must be a new dynamical symmetry breaking which generates the mass difference between $a_1$ and $\rho$. It has been found out that the axial-vector coupling results in a new dynamical symmetry breaking - axial-vector symmetry breaking which leads to

$$\left(1 - \frac{1}{2\pi^2 g_0^2}\right)m_a^2 = 6m^2 + m_\rho^2.$$ 

Can this axial-vector symmetry breaking bring something to the EW theory?

A Lagrangian of EW interactions without Higgs is studied

$$\mathcal{L} = -\frac{1}{4}A_{\mu\nu}^i A^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{q}\{i\gamma \cdot \partial - M\}q$$

$$+\bar{q}_L\{\frac{g}{2}\tau_i\gamma \cdot A^i + g'Y^i \cdot B\}q_L + \bar{q}_Rg'Y^i \gamma \cdot Bq_R$$

$$+\bar{l}\{i\gamma \cdot \partial - M_f\}l + \bar{l}_L\{\frac{g}{2}\tau_i\gamma \cdot A^i - \frac{g'}{2}\gamma \cdot B\}l_L$$

$$-\bar{l}_Rg'\gamma \cdot Bl_R.$$ \hspace{1cm} (23)

There are no $m_{W,Z}$ and there are fermion mass terms which break the charged part of the $SU(2)_L \times U(1)$ symmetry. This explicit symmetry breaking makes charged bosons, W, massive. However, there are still two U(1) symmetries. Z and photon are massless. In EW interactions there are axial-vector couplings, therefore, there are axial-vector symmetry breaking which contribute mass to both W and Z bosons. Using
path integral to integrate out the fermion fields, the Lagrangian of boson fields is derived.

After multiplicative renormalization following results are obtained

1.

\[ m_W^2 = \frac{1}{2} g^2 \{ m_t^2 + m_b^2 + m_c^2 + m_s^2 + m_u^2 + m_d^2 \}
+ m_{\nu_e}^2 + m_{\nu_\mu}^2 + m_{\nu_\tau}^2 + m_{\tau}^2 \} \]  

\[ m_W = \frac{g}{\sqrt{2}} m_t. \]  

Using the values \( g = 0.642 \) and \( m_t = 180 \pm 12 \text{GeV} \), it is found

\[ m_W = 81.71(1 \pm 0.067) \text{GeV}, \]  

which is in excellent agreement with data \( 80.33 \pm 0.15 \text{GeV} \).

2.

\[ G_F = \frac{1}{2 \sqrt{2} m_t^2} = 0.96 \times 10^{-5} (1 \pm 0.13) m_N^{-2}. \]

3.

\[ m_Z^2 = \rho m_W^2 / \cos^2 \theta_W, \]  

\[ \rho = (1 - \frac{\alpha}{4\pi} f_4)^{-1}, \]

\[ f_4 = \frac{1}{3} N_G - \frac{2}{3} \sum_q f_3 + \frac{2}{3} \sum_l f_3. \]
\[ f_3 = \frac{1}{2} \left( \ln \frac{1 + \sqrt{x}}{1 - \sqrt{x}} \right) \]

\[ x = \left( \frac{m_2^2}{m_1^2} \right)^2, \quad m_{1,2} = \frac{1}{2} (m_t \pm m_b). \]

\[ \rho \sim 1, \]

\[ m_Z^2 = \frac{m_W^2}{\cos^2 \theta_W}. \]

4. The propagator of W field is derived

\[ \frac{i}{q^2 - m_W^2} \left\{ -g_{\mu\nu} + \frac{q_{\mu}q_{\nu}}{q^2} \right\} - \frac{i}{\xi_W q^2 - m_W^2} \frac{q_{\mu}q_{\nu}}{q^2}. \]  

(28)

Changing the index W to Z, the propagator of Z boson field is obtained.

\[ \xi_W = \frac{N_G g^2}{(4\pi)^2} \frac{1}{3}, \quad \xi_Z = \frac{N_G}{(4\pi)^2} (1 - \frac{\alpha}{4\pi f_4})^{-\frac{1}{3}} (g^2 + g'), \]

\[ N_G = 3N_C + 3. \]

1. \( m_{W,Z} \) are correctly dynamically generated by axial-vector symmetry bre

2. The propagators of W and Z fields have no problems with renormalization.

4 Conclusions

1. The effective chiral theory is phenomenologically successful,

2. All the 10 coefficients of ChPT are predicted,
3. What can we learned about QCD from this theory:

(a) Chiral symmetry,

(b) Dynamical chiral symmetry breaking,

(c) Large $N_C$ expansion,

(d) Simulations of meson fields by quark operators are working. It is worth to do theoretical study on whether the simulations are solutions of QCD at least in the limit of large $N_C$ expansion,

4. The Lagrangian is not closed. The quark condensate, $m$, should be related to gluons,

5. Axial-vector symmetry breaking provides an explanation to the mass difference between $\rho$ and $a_1$,

6. $m_{W,Z}$ are dynamically generated from the fermion masses and the axial-vector symmetry breaking.

References

[1] B.A.Li, Phys. Rev. D52,5165(1995);51841995.
[2] B.A.Li, Proc. of Intern. Europhys. Conf. on High Energy Phys., Brussels, Belgium, 27 Jul-2 Aug, 1995, edited by J.Lemonne et al., p.225.

[3] B.A.Li, Phys.Rev. D55, 1425(1997).

[4] B.A.Li, hep-ph/9711397.
Table 1: Coefficients obtained by various models

|                | Vectors | Quark | Nucleon loop | Linear $\sigma$ model | ENJL |
|----------------|---------|-------|--------------|-----------------------|------|
| $(L_1 + \frac{1}{2}L_3) \times 10^{-3}$ | -2.1    | -0.8  | 2.1          | -0.8                  | -0.5 |
| $L_1 \times 10^{-3}$        |         |       |              |                       | 0.8  |
| $L_2 \times 10^{-3}$        | 2.1     | 1.6   | 1.6          | 0.8                   | 1.6  |
| $L_3 \times 10^{-3}$        |         |       |              |                       | -4.1 |
| $L_4 \times 10^{-3}$        |         |       |              |                       | 0.0  |
| $L_5 \times 10^{-3}$        |         |       |              |                       | 1.5  |
| $L_6 \times 10^{-3}$        |         |       |              |                       | 0.0  |
| $L_7 \times 10^{-3}$        |         |       |              |                       |      |
| $L_8 \times 10^{-3}$        |         |       |              |                       | 0.8  |
| $L_9 \times 10^{-3}$        | 7.3     | 6.3   | 6.7          | 3.3                   | 0.9  |
| $L_{10} \times 10^{-3}$     | -5.8    | -3.2  | -5.8         | -1.7                  | -2.0 |
|                             |         |       |              |                       | -5.5 |
Table 2: Coefficients from this theory

| $10^3 L_1$ | $10^3 L_2$ | $10^3 L_3$ | $10^3 L_4$ | $10^3 L_5$ | $10^3 L_6$ | $10^3 L_7$ | $10^3 L_8$ | $10^3 L_9$ | $10^3 L_{10}$ |
|------------|------------|------------|------------|------------|------------|------------|------------|------------|--------------|
| .9 ± .3    | 1.7 ± .7   | -4.4 ± 2.5 | 0. ± .5    | 2.2 ± .5   | 0. ± 0.3   | -.4 ± .15  | 1.1 ± .3   | 7.4 ± .7   | -6. ± .7     |