Modeling of Non-Stationary Electrokinetic Effect in a Conductive Crust

O. MAJAEVA¹, Y. FUJINAWA¹, and M. E. ZHITOMIRSKY²

¹National Research Institute for Earth Science and Disaster Prevention, Tsukuba, Japan       ²Institute for Solid State Physics, University of Tokyo, Tokyo 106, Japan

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The magnetic field, generated by the electrokinetic effect is calculated for a spherical, time-varying pressure source in a layered half-space chosen due to its exact solvability. In addition to a widely discussed steady-state phase, electromagnetic signals of electrokinetic origin have a transient phase which corresponds to their propagation through a conductive crust. The duration of the transient phase depends on a distance from there pressure gradient source and on the crustal conductivity. For characteristic conductivities of $0.1 - 10^{-3}$ S/m and distances of several kilometers the transient phase lasts for 10–100 sec. Although in the layered model both transient and steady state external magnetic fields are zero, this is not so in the general case. Therefore, if the electrokinetic effect occurs in a water filled fault, the transient magnetic field can appear at the surface as an ULF pulse.

1. Introduction

Electric currents of electrokinetic (EK) origin are induced when fluid flows through a porous medium in the presence of an electrical double layer at the solid-liquid interface. Such an electrical layer appears as a result of microscopic charge separation if two different chemical phases are in contact (Overbeek, 1952). Fluid flow convects ions in the double layer and produces electric currents whose magnitude depends on fluid conductivity, characteristics of the porous medium and on the $\zeta$-potential of the pore wall. Evaluation of coupling coefficients between electric current and fluid flow is a subject of the microscopic theory of the EK effect. The experimental investigation of the microscopic aspects of the EK phenomena has been undertaken by Ishido and Mizutani (1981), who studied $\zeta$-potentials of minerals and rocks in solutions under various physicochemical conditions.

The subject of the macroscopic theory of the EK effect is the calculation of electromagnetic fields produced by an arbitrary pore pressure source using the relation between electric currents and pressure gradients found from the microscopic consideration. Fitterman (1978) developed a theoretical approach to the stationary electrotelluric and geomagnetic potential anomalies near the Earth's surface. He has shown that magnetic field of EK origin appears only if there is a discontinuity of conductivity or electrokinetic coupling coefficients and pore fluid flows parallel to the boundary of regions with different characteristics. The stationary theory of the EK effect has been applied for description of self-potential changes in hydrothermal areas (Ishido, 1989) and for calculation of time-varying magnetic fields generated in a water filled fault zone before strong earthquakes as well (Dobrovolsky et al., 1989; Fenoglio et al., 1995). However, limitations of such an approach have not been discussed in the literature and remain unclear.

In this paper we generalize the stationary macroscopic theory of the EK effect by Fitterman (1978) to the case of a non-stationary pore pressure source and show that there are two characteristic time scales of generated electromagnetic fields. One of them is associated directly with water flow and is assumed to be slow. The second time scale is determined by propagation of electromagnetic fields in the conductive medium. This transient phase in crustal systems lasts up to several tens of seconds and depends on conductivity and on a distance from the source of
pressure gradients. Next we report on a calculation of the time-varying magnetic fields of EK origin, made for a spherical pressure source in a layered half-space.

2. Time-Scales of Electrokinetic Effect

The mineral grains lining pore walls are assumed to adsorb electrolytic ions on their surfaces. As a result, fluid flow forces uncompensated ions of opposite charge in a diffusive layer into motion, producing an electric current the capillary:

\[ j_{ek} = -\sigma \nabla \phi - \frac{\varepsilon \zeta}{\eta} \nabla P, \]  

where \( \varepsilon, \sigma \) and \( \eta \) are the dielectric constant, conductivity and dynamic viscosity of the fluid respectively, \( \phi \) is the electric potential, and \( P \) is the water pressure. The first term in Eq. (1) is often called a back current (Fenoglio et al., 1995), since it describes an effect opposite to the driving pressure term and is caused by redistributed charges. According to the charge conservation law, the equilibrium microscopic charge density on a pore-length scale is reached in time \( \tau \sim \varepsilon/\sigma \). For the fluid conductivity of \( \sigma \sim 1 \text{ S/m} \) this time interval is extremely short \( \tau \sim 10^{-11} \text{ sec} \). Hence, one can consider redistribution of charges as an instantaneous process and write down the charge conservation law in the following form:

\[ \nabla \cdot j_{ek} = 0. \]  

Equations (1) and (2) are basic equations in the theory of stationary EK effect. They, however, should be modified for time-dependent pressure sources. The total electric current in a conductive medium is given by a sum of the potential part \(-\nabla \phi\) and the curl fields which describe of propagation of the electric signal. Hence, expression (1) for the total current is changed to

\[ j = j_{ek} - \sigma \frac{\partial A}{\partial t}, \]  

where \( A \) is the vector potential. In the Coulomb gauge \( \nabla \cdot A = 0 \) and the total current satisfies the same equation:

\[ \nabla \cdot j = 0. \]  

We will refer to the time interval during which the eddy current can not be neglected with respect to \( j_{ek} \) as to the non-stationary phase of the EK effect. In this case the magnetic field is not given by Biot-Savart law and has to be derived from Maxwell’s equations.

In a steady-state problem the eddy currents vanish and \( j = j_{ek} \). Then, distribution of the generated electric field is given by a modified equation of (2):

\[ \Delta \psi = 0 \]  

for the total potential \( \psi = \phi + CP \), where \( C = \varepsilon \zeta/\sigma \eta \), and \( j_{ek} = -\sigma \nabla \psi \). This expression was first introduced by Nourbehecht (1963). This equation has to be combined with boundary conditions of the continuity of the normal component of the electric current and the electric potential \( \phi \) at the interface \( S \):

\[ (\sigma_1 \nabla_n \psi_1)|_S = (\sigma_2 \nabla_n \psi_2)|_S, \quad (\psi_1 - C_1 P)|_S = (\psi_2 - C_2 P)|_S. \]  

In a homogeneous crust \( j_{ek} \equiv 0 \) and magnetic field is not generated. The while electric field is nonzero and is given by \( E = -\nabla \phi = C \nabla P \).
For time-varying pore pressure, the macroscopic charge density \( \rho(\mathbf{r}, t) \) found from Eqs. (1') and (2') is given by
\[
\rho(\mathbf{r}, t) = \frac{\varepsilon_0 \varepsilon_r}{\sigma \eta} \Delta P.
\] (5)

Thus, electromagnetic fields of EK origin have the same temporal characteristics as \( P \) and for most crustal systems are assumed to be slow \((\ll 10^{-3} \text{ Hz})\). If \( P \) does not depend on \( t \), charge appears only in the source (if there is one) and at the boundaries between layers with different \( C \) and \( \sigma \). If \( P \) is time-dependent, \( \Delta P = -\frac{\partial P}{\partial t} / \gamma \) and charge density is volumetric.

Skin-depth \( \lambda \) for such low-frequency fields is very large \((\lambda > 100 \text{ km for } \sigma = 0.1 \text{ S/m})\) and they can appear at long distances from a current source. However, the propagation of electromagnetic fields in a conductive medium is affected by the electromotive force, which creates an additional contribution in the total current resulting from a non-conservative electric field. At distance \( l \) from the EK source, the field appears only after a characteristic time \( T \). We will refer to this stage as a transient phase of the non-stationary EK effect and use for its description Maxwell's equations for a quasistationary electromagnetic field:
\[
\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}, \quad \nabla \times \mathbf{H} = \mathbf{j},
\] (6)

where \( \mu_0 = 4\pi \times 10^{-7} \text{ H/m} \). Combining them we obtain
\[
\Delta \mathbf{H} = \mu_0 \sigma \frac{\partial \mathbf{H}}{\partial t} - [\mathbf{n} \times (\mathbf{j}_2 - \mathbf{j}_1)] \delta_S.
\] (7)

The second term on the right-hand side of Eq. (7) which is proportional to the Dirac \( \delta \)-function, vanishes over all space except at the boundaries, where \( \sigma \) or \( C \) have discontinuities. \( \mathbf{n} \) is a unit vector normal to the boundary. This equation is basic in our calculations of the electromagnetic fields generated by the non-stationary EK effect. The initial condition for Eq. (7) is the absence of the magnetic field at \( t = 0 \): \( \mathbf{H}_1|_{t=0} = 0 \). The boundary condition is the continuity of \( \mathbf{H} \) across the interfaces between different media.

For example, at the Earth's surface magnetic field in the air is determined by the Laplace equation \( \nabla^2 \mathbf{H}_{\text{air}} = 0 \), i.e. without the time-derivative term. The extra condition at the ground surface comes from the second term on the right hand side of Eq. (7):
\[
(n \cdot \nabla)(\mathbf{H}_1 - \mathbf{H}_{\text{air}})|_S = (n \times \mathbf{j}_1)|_S,
\]
where \( \mathbf{H}_1 \) and \( \mathbf{j}_1 \) are the magnetic field and the electric current in the crust. Taking into account the continuity of magnetic field across \( S \) we conclude that its temporal characteristics in the air will be the same as within the crust in the close vicinity to the Earth's surface. At the same time the magnitude and polarization of the external magnetic field are defined by the overall geometry of the problem. In some cases, such as in a cylindrically symmetric model which we choose below due to its exact solvability, the magnetic field goes to zero when the surface is approached from below. Then in the air it is always zero, though this is not true in the general case.

At \( t > T \) the steady-state electric field coincides with the corresponding potential field \( -\nabla \varphi \), whose temporal characteristics are the same as for the pressure source. The total electric current has the form (1) and the steady-state magnetic field will be given by the Biot-Savart law:
\[
\mathbf{H}_0(\mathbf{r}) = \nabla \times \int \frac{\mathbf{j}_{\text{ek}}(\mathbf{r}')}{4\pi |\mathbf{r} - \mathbf{r}'|} dV.
\] (8)

The time interval \( T \), during which the electromagnetic field penetrates to a distance \( l \) from the EK source and reaches the steady-state value (8), can be easily estimated. For that one...
should compare both sides of Eq. (7): $\Delta H \approx H/l^2$ and $\mu_0 \sigma \partial H/\partial t \approx \mu_0 \sigma H/T$, and obtain:

$$T \sim l^2 \mu_0 \sigma.$$  

(9)

Only in the limit $t \gg T$ does the time-derivative term become small enough to be neglected. Then, solution of Eq. (7) is given by $H_0(r)$. In Section 3 we consider a model of the EK effect in a layered geometry for which this non-stationary behavior of the magnetic field can be found exactly.

3. Model of an Electrokinetic Half-Space

Let us consider the non-stationary EK effect in a simple geometry of the crust which allows analytical calculation of the electromagnetic fields from the EK source and confirms estimates for time-scales of the process. As a model we choose a two-layered, cylindrically symmetric half-space, proposed by Fitterman (1978) (Fig. 1(a)), with a spherical source of time-varying pressure gradient (EK source). We approximate it by a sphere with initial inhomogeneous pore pressure $P_0$ of radius $a$ situated at depth $h$. Pressure changes and the EK effect are assumed to occur within the lower highly porous layer with conductivity $\sigma_2$ and electrokinetic coupling coefficient $C$. We describe the upper layer as impermeable ($C = 0$) with rock conductivity $\sigma_1$. Taking into account that conductance in the ground rocks is mainly occurring through the pore fluid, we assume that $\sigma_2 > \sigma_1$. The boundary $S$ between layers is placed at a depth $d$.

Pressure gradients lead to water flow through a porous medium. The evolution of the pore pressure is given by the diffusion equation (Brace et al., 1968):

$$\Delta P = \frac{1}{\gamma} \frac{\partial P}{\partial t},$$

(10)

where $\gamma = k/\eta \beta \rho$ is the diffusion coefficient, $\beta$ is water compressibility, $\eta$ is dynamic viscosity, $k$ is permeability, and $\rho$ is porosity. The appropriate boundary condition at the interface of the

![Fig. 1. Model of cylindrically symmetric layered half-space, used for calculation of magnetic fields from the EK source. The EK source is approximated by a sphere with radius $a = 150$ m, situated at the depth of $h = 10$ km. Fluid flows and EK effect occurs in the lower layer with electrokinetic coupling coefficient $C$ and conductivity $\sigma_2$. The upper layer is supposed to be impermeable ($C = 0$) with conductivity $\sigma_1$. The boundary $S$ between the two layers is placed at the depth $d = 9700$ m.](image-url)
impermeable zone is $\nabla_z P|_{z=d} = 0$, since $k$ in the upper layer is zero and fluid does not flow their. Solution of Eq. (10) for a spherical source in cylindrical coordinates $(r, \theta, z)$ has the form:

$$P(r, z, t) = \frac{P_0a^3}{(\pi(a^2 + 4\gamma t))^{3/2}} \exp \left(-\frac{r^2}{a^2 + 4\gamma t}\right) \times \left[ \exp \left(-\frac{(z-h)^2}{a^2 + 4\gamma t}\right) + \exp \left(-\frac{(z-2d+h)^2}{a^2 + 4\gamma t}\right) \right]. \quad (11)$$

The boundary condition is satisfied by a superimposing two pressure sources symmetrically situated above the boundary at $z = d$: the first is a real source at $z = h$, and the second is a mirror image at $z = 2d - h$. Magnitudes of pore pressure gradients at the interface of the EK layer depend on the size of the source inhomogeneity and on its distance from the boundary.

We calculate first the steady-state electric and magnetic fields generated by the above pressure gradient. Our solution of the Laplace equation (3) for the EK potential $\psi$ subject to the boundary conditions (4) is analogous to that of Fitterman (1978):

$$\psi^u(r, z, t) = \int_0^\infty A_1(\lambda, t) (e^{-\lambda z} + e^{\lambda z}) J_0(\lambda r) d\lambda, \quad 0 \leq z \leq d,$$

$$\psi^l(r, z, t) = \int_0^\infty A_2(\lambda, t) e^{-\lambda z} J_0(\lambda r) d\lambda, \quad d < z, \quad (12)$$

where $J_0(x)$ is the zeroth-order Bessel function and superscripts $u$ and $l$ correspond to the upper and lower layers, respectively,

$$A_1(\lambda, t) = \frac{\sigma_2 C \lambda \tilde{P}_d(\lambda, t) e^{-\lambda d}}{(\sigma_1 + \sigma_2) + (\sigma_2 - \sigma_1) e^{-2\lambda d}}, \quad A_2(\lambda, t) = \frac{\sigma_1 C \lambda \tilde{P}_d(\lambda, t) e^{\lambda d}(1 - e^{-2\lambda d})}{(\sigma_1 + \sigma_2) + (\sigma_2 - \sigma_1) e^{-2\lambda d}},$$

and $\tilde{P}_d(\lambda, t)$ is Hankel transform of the pressure field at $z = d$:

$$\tilde{P}_d(\lambda, t) = \frac{P_0a^3}{(\pi^3(a^2 + 4\gamma t))^{1/2}} \exp \left(-\frac{\lambda^2(a^2 + 4\gamma t)}{4}\right) \exp \left(-\frac{(d-h)^2}{a^2 + 4\gamma t}\right).$$

The steady-state electric field is obtained from the above electric potential. Electric currents driven by these fields have a geometry of coaxial circles. It is easy to see that they generate only the azimuthal component of magnetic field in a cylindrical geometry (Fitterman, 1978). Using the Amperes law: $2\pi r H_\theta = \int j_z dS$ and condition of vanishing normal component of the currents at the Earth surface, we conclude that both the transient and the steady-state magnetic fields outside the region of current flow ($z < 0$) become zero. Therefore, for further calculations we use the azimuthal projection of Eq. (7):

$$\Delta H_\theta - \frac{1}{r^2} H_\theta = -(j_{r2} - j_{r1}) \delta(z - d) + \mu_0 \sigma \frac{\partial H_\theta}{\partial t}. \quad (13)$$

The steady-state magnetic field is calculated from Eq. (13) assuming that $\partial H_\theta / \partial t = 0$. The extra boundary condition in addition to continuity of $H$ is obtained by integration Eq. (13) over $z$ from $(d-0)$ to $(d+0)$:

$$\left(\frac{\partial H^u_\theta}{\partial z} - \frac{\partial H^l_\theta}{\partial z} \right)_{z=d} = -(j_{z2} - j_{z1})_{z=d}, \quad (14)$$

where the jump in the radial component of the electric current $(j_{r2} - j_{r1})$ at the boundary between different media for $t \gg T$ is calculated from Eqs. (4) and (12) as

$$(j_{r2} - j_{r1})_{z=d} = \int_0^\infty 2D(\lambda) J_1(\lambda r) d\lambda, \quad (15)$$
where

\[
D(\lambda, t) = \frac{\sigma_1 \sigma_2 C \lambda^2 \tilde{P}_d(\lambda, t)}{-(\sigma_1 + \sigma_2) + (\sigma_1 - \sigma_2) \exp(-2\lambda d)},
\]

and \( J_1(x) \) is the first-order Bessel function. Finally, the magnetic field is given by

\[
\begin{align*}
H^u(r, z, t) &= \int_0^\infty F_1(\lambda, t)(e^{\lambda z} - e^{-\lambda z})J_1(\lambda r) d\lambda, \quad 0 \leq z \leq d, \\
H^l(r, z, t) &= \int_0^\infty F_2(\lambda, t)e^{-\lambda z}J_1(\lambda r) d\lambda, \quad d < z,
\end{align*}
\]

(Note, that the result of zero magnetic field at \( z < 0 \) can be obtained directly from Eq. (13) without reference to the integral form of Maxwell's equations, if one takes into account the jump in transverse electric current at \( z = 0 \).)

During the non-stationary phase the time-derivative term in Eq. (13) is nonzero. Since the first term on the right-hand side of Eq. (13) is zero everywhere except at the boundary, we obtain for the magnetic field in each layer

\[
\Delta H_\theta - \frac{1}{r^2} H_\theta = \mu_0 \sigma \frac{\partial H_\theta}{\partial t},
\]

with the boundary condition (14) and the initial condition \( H_\theta|_{t=0} = 0 \). If \( \sigma_1 \neq \sigma_2 \) determination of a time-dependent magnetic field from Eq. (13) even for the simple pressure source (11) requires numerical calculations. However, the above estimate (9) of the characteristic time of the transient process can be checked for any solution of Eq. (13). We show this in the case \( \sigma_2 = \sigma_1 = \sigma \), when all results are obtained analytically. Using the Laplace transformation over the time variable we obtain the following expression for the field which is valid in both layers:

\[
H_\theta = \int_0^\infty D(\lambda)J_1(\lambda r) d\lambda \int_0^t \frac{dt'}{\sqrt{\pi t' \mu_0 \sigma}} \left[ \exp \left(-\frac{(d-z)^2}{4t'}\right) - \exp \left(-\frac{(d+z)^2}{4t'}\right) \right].
\]

With the help of a series expansion for the Bessel function the integral over \( \lambda \) can be calculated explicitly (\( \int_0^\infty \lambda^2 e^{-\lambda^2} J_1(\lambda b) d\lambda = b/(4a^2) e^{-b^2/4a} \)) and finally we find

\[
H_\theta = \frac{\sigma CP_0}{2 \pi^{3/2}} \exp \left(-\frac{(h-d)^2}{a^2}\right) \int_0^{t/\sigma} \frac{dt'}{\sqrt{\pi t' \sigma}} \frac{r}{4a(t' + \frac{1}{4})^2} \left[ \exp \left(-\frac{(d-z)^2}{4a^2 t'}\right) - \exp \left(-\frac{(d+z)^2}{4a^2 t'}\right) \right], \quad t_0^2 = \mu_0 \sigma a^2.
\]

The steady-state magnetic field in the model can be obtained by integration of the tangential component of the electric current over \( z \) from 0 to the depth of calculation (Fitterman, 1978). Both the steady state and the transient magnetic fields depend upon the product of \( \sigma \) and \( C \). This product does not change largely with increasing/decreasing conductivity under the typical crustal conditions.

The electric component of the transient electromagnetic field generated during the non-stationary phase of the EK process develops simultaneously with the magnetic field and also becomes steady-state in time \( T \). Evaluation of field amplitudes by Eqs. (16) and (19) using realistic crustal parameters is given in the Section below.
4. Results and Interpretation

We have done numerical calculations of macroscopic magnetic fields generated by the EK source of radius $a = 150$ m situated at the depth $h = 10$ km within a permeable layer. Parameters for this layer were taken from literature: $k = 10^{-12}$ m$^2$, $\eta = 10^{-4}$ Pa-sec, $\varrho = 0.5$, $\beta = 3 \cdot 10^{-10}$ Pa$^{-1}$, $\sigma = 0.1$ S/m, $C = 10^{-5}$ V/Pa, $\zeta \sim 0.1$ V (Fenoglio et al., 1995). The quite high conductivity level is in agreement with other reports of 3 to 10 Ohm-m resistivities exhibited by faults in the presence of normal meteoric water (Eberhart-Phillips et al., 1995). At this time there is no consensus on values of conductivity, porosity and permeability within and outside a fault zone. Therefore, we have chosen one of the typical sets of parameters appearing in the literature.

The initial pressure inhomogeneity in the source is $P_0 = 2 \cdot 10^7$ Pa which is, approximately 10% of hydrostatic pressure at a depth of 10 km. These parameters may correspond to the EK source generated in a water saturated shallow fault as a result of fracture processes. Pressure gradients might appear due to the crushing of sealed walls of a compartment containing over-pressured water (Byerlee, 1993) or as a result of changes of porosity around a new shear fracture (Scholz, 1990). The value of initial pressure gradient which can appear in the fault varies widely and is believed to reach 1.5 time the hydrostatic pressure at a depth of the EK source. This assumption was used by Fenoglio et al. (1995).

Calculations have been done for the cylindrically symmetric model with infinite EK layers (Fig. 1). The upper boundary $d$ of the EK layer is placed at a depth of 9700 m. In Fig. 3 we present the evolution of the non-stationary (19) and steady-state (16) magnetic fields corresponding to conductivity $\sigma = 0.1$ S/m. Curves (a) and (b) correspond to near surface magnetic fields at a

![Graph showing the evolution of horizontal pressure gradient from a spherical source with starting value of pressure $P = 2 \cdot 10^7$ Pa at the interface of EK layer, calculated as a function of time at distances a) 2 km; b) 3 km; c) 5 km.](image-url)
Fig. 3. Time variations of magnetic field at the depth $z = 10$ m for distances (a) 5 km and (b) 15 km. $\sigma_1 = \sigma_2 = 0.1$ S/m. The first 3 and 28 sec of curves (a) and (b) correspond to the transient phase. Dashed lines are the field calculated from Eq. (16) without taking into account the non-stationary effect.

depth of $z = 10$ m, calculated for distances of $r = 5$ and 15 km. We have chosen the 'observation' point very close to the surface, but still inside the layer with non-zero conductivity as in all cylindrical models no magnetic field is generated outside the crust. The time axis is a logarithmic scale. The left parts of the curves correspond to the non-stationary phase of the EK process. Duration of the transient phase is 3 sec and 28 sec respectively, which agrees with estimate (9). After that the magnetic field reaches a steady-state condition and varies slowly with water diffusion.

Magnitudes of pore pressure gradients at the interface of the EK layer depend on the size of the source inhomogeneity and on its distance to the boundary. Their values are very important for calculation of electromagnetic signals: the wider the area of significant pore pressure gradients at the interface of the EK layer, the stronger the electromagnetic effect. If the EK layer is approximated by a more realistic narrow porous band (such as a fault), pressure gradients at the interface will propagate greater distances and the area of significant gradient increases markedly. Up to now we have considered the EK effect from an ideal point instantly created pore pressure source. Realistic consideration must also take into account the time scale of mechanical process in the EK layer such as the frequency of crack generation or velocities of shear crack propagation. These mechanical processes determine another characteristic time scale of fluid motion of $P(t)$. It is clear from the above discussion that at any given point pulses of magnetic field relating to mechanical changes slower than time $T$ can be resolved. These signals are described in the frames of stationary theory (Fitterman, 1978). Signals associated with rapid changes in $P(t)$ appear as an ULF noise and has to be described by a non-stationary solution. Most of electromagnetic fields in the crust such as self-potential anomalies caused by ground-water flow or those generated by the EK effect within an active fault are expected to propagate over distances of up to several tens
of kilometers. For typical conductivities $\sigma = 10^{-3} - 10^{-1}$ the frequency lies in the ULF range.

Though a single pulse of transient magnetic fields from an opening crack is unlikely to be detected, an EK source with temporal characteristics, comparable with $T$ or slower might produce a measurable ULF noise. For example, Fraser-Smith et al. (1990) reported magnetic field variations in the ULF frequency range associated with the 17 October 1989 Loma Prieta $M_L = 7.1$ earthquake. Similar results were obtained prior to the devastating 7 December 1988 $M_S = 6.9$ Spitak event (Molchanov et al., 1992). Both reports present measurements of anomalous ULF magnetic fields with amplitudes of 0.2–1.5 nT prior to earthquakes. Since real fault geometries are not cylindrically symmetric, external magnetic fields are generated, and, therefore, can be measured at the surface. Fenoglio et al. (1995) have proposed the EK effect in the fault a possible source of ULF magnetic field pulses before the Loma Prieta earthquake. Temporal characteristics of the start and stop of fracture propagation within a low pressure compartment have periods from 0.1 to 100 sec. Since the duration of the transient phase is general for any geometry, we can estimate it for the distance of 7 km from the epicenter, where the precursory ULF signals have been observed by Fraser-Smith et al. (1990). For crustal conductivities of 0.1–0.01 S/m the transient phase lasts for 10–100 sec. Therefore, in the case of the EK nature of observed signals, the non-stationary theory of the EK effect has to be used for the description of the electromagnetic fields, associated with fracture propagation process (Fenoglio et al., 1995).

5. Conclusions

The present consideration of the non-stationary EK processes demonstrates that EK phenomena in the crust have two main phases. The non-stationary phase corresponds to the process of propagation of electromagnetic fields through the crust. Its duration $T$ depends on the conductive properties of the surrounding rock and on the distance from the EK source and is up to several tens seconds for characteristic distances of the problem. When the transient phase is finished the process reaches the steady state phase during which electromagnetic fields are described by the stationary theory of Fitterman (1978). The steady state phase lasts as long as there is a pressure gradient driving fluid flow that maintains electric charge redistribution. Amplitudes of the non-stationary magnetic field are of the order of 1 nT. During the steady state phase they can reach 10–15 nT. In the considered cylindrically symmetric model magnetic field outside the half-space is zero due to the symmetry, although that is not so in the general case. The transient phase has an important physical meaning. It allows ULF electromagnetic signals to be generated by the non-stationary electrokinetic effect.

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