Fractional data-driven control for a rotary flexible joint system

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Abstract
As one of the most promising topics in complex control processes, data-driven techniques have been widely used in numerous industrial sectors and have developed over the past two decades. In addition, the fractional-order controller has become more attractive in applied studies. In this article, a fractional integral control is implemented for a rotary flexible joint system. Moreover, an adjusted virtual reference feedback tuning (VRFT) technique is used to tune the fractional-order integrator. In this method, fractional integral control is designed based on state feedback control. Then, VRFT is adjusted and applied to the fractional integral controller. The effectiveness of the proposed adjusted VRFT method is discussed and presented through simulation and experimental results. The tracking performance of the rotary arm and the minimization of the vibration tip is evaluated based on the proposed method. In this article, the comparison of our proposed VRFT fractional scheme is made with the classical state feedback as well as a recently developed state feedback-based fractional order integral (SF-FOI) controller. The current investigations determine the performance improvement of our proposed scheme of comparable structure to the recent SF-FOI, with the introduction of the VRFT to the SF-FOI scheme.

Keywords
Data-driven control, virtual reference feedback tuning, fractional order controller, rotary flexible joint, state feedback control

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Introduction
Over the last few years, flexible joint manipulators have received extensive attention in fractional control studies. They have more advantages than traditional rigid manipulators because flexible joint materials are lighter and less expensive. These flexible joint materials are safe to operate and have low power with high performance. Flexible joint manipulators are used in different applications, such as space robots, automatic cranes, and underwater robotic.

Two things that must be controlled in a rotary flexible joint (RFJ) are the minimization of the vibration of the flexible joint and the tracking of its setpoint. Many control...
techniques are used to control the tip setpoint of a flexible joint system. Iterative learning control scheme is proposed by Wang \(^1\) to improve the tracking accuracy of the RFJ. The sliding-mode method was applied to achieve robust control for flexible arms. \(^2\) Disturbances were compensated by introducing a disturbance observer. A Lyapunov-type controller was proposed by Tso et al. \(^3\) and constructed with deflection feedback. The control approach regulated the endpoint of a flexible robot and damped out the tip oscillations. Adaptive control with sliding control was proposed by Huang and Chen. \(^4\) A control algorithm for double nested feedback loops was proposed by Saitou et al. \(^5\) The position of the motor was controlled by an inner loop, and the tip setpoint was controlled by an outer loop. A hybrid control scheme for vibration and tip deflection control of a single-link flexible manipulator system was presented by Abdullahi et al. \(^6\) The control scheme combined dual feedback loops consisting of a resonant controller and a fuzzy logic controller. An adaptive control technique with a non-linear approach was also investigated by combining adaptive backstepping and dynamic surface control for a single flexible link manipulator. \(^7\) A fuzzy logic control technique was proposed by Jalani and Jayaraman. \(^8\) Another control scheme for a flexible single link based on Duffing oscillator dynamics was proposed by Chen et al. \(^9\) It is difficult to obtain an accurate model of a plant in many cases. Therefore, a data-driven or model-free design approach is an attractive alternative technique, because such approaches only rely on the available restrained input/output information. One promising data-driven control technique is virtual reference feedback tuning (VRFT), developed by Guardabassi and Savaresi. \(^10\) VRFT is used to solve the reference control problem for one-shot data. \(^11\) Recently, the VRFT technique has been shown to have a wide variety of applications. VRFT was introduced for robot force control in the literature. \(^12,13\) Several investigations are accomplished to develop VRFT for an integer controller. \(^14,15\) In this article, VRFT for fractional-order control is applied for a RFJ system. Fractional-order controllers provide more flexibility than integer controllers to track the performance of the flexible tip and minimize the vibration of the system. Fractional-order controllers based on state feedback were investigated in several published works. \(^16–19\) A fractional order-based optimization algorithm was proposed to enhance the robustness in the literature. \(^20\) In this article, VRFT is applied to tune fractional integral control based on state feedback. The proposed method is implemented on Quanser’s RFJ system. The comparison of our proposed VRFT fractional scheme is made with the classical state feedback and a recently developed state feedback-based fractional order integral (SF-FOI) control scheme. Our aim in the article is to determine the performance improvement of our proposed scheme of comparable structure to the recent SF-FOI with the introduction of the VRFT to the SF-FOI scheme. Simulation and experimental results show the effectiveness of the tracking performance of the rotary arm and minimization of the vibration tip.

** Rotary flexible joint

**System overview**

The Quanser RFJ system \(^21\) consists of a rigid arm connected with a flexible joint that rotates with the help of a DC motor. Figure 1 shows the main elements of the system. For control purposes, the speed and position of the arm are measured using an encoder sensor. This system is designed to fit on a servo plant (SRV02) developed by Quanser. The rotary flexible arm is connected through two springs anchored to a solid frame, ensuring that it is a flexible instrument. This plant is useful for evaluating flexible joint
A schematic diagram of the RFJ is shown in Figure 2. The angle $\theta$ is called the servo position angle, and the angle $\alpha$ is called the vibration angle of the rotary arm due to the flexible joint. It is challenging to control the servo position angle while minimizing the vibration angle.

**System modeling**

The linear state-space model of the RFJ system is

$$
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
$$

The manipulated input $u(t)$ is the supply voltage to the DC servo motor in volts. The output $y(t)$ consists of two components: the servo position angle $\theta(t)$ and the vibration angle $\alpha(t)$. The state vector includes $\theta(t)$, $\alpha(t)$, and their corresponding derivatives $\dot{x}^T = [\theta \, \dot{\theta} \, \dot{\alpha}]$. Matrices $A$, $B$, and $C$ are given by

$$
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & \frac{B_s}{J_{eq}} - \frac{B_{eq}}{J_{eq}} & 0 \\
0 & -\frac{B_s(J_l+J_{eq})}{J_{eq}} & \frac{B_{eq}}{J_{eq}} & 0
\end{bmatrix}
$$

(2)

$$
B^T = \begin{bmatrix}
0 & 0 & \frac{1}{J_{eq}} & -\frac{1}{J_{eq}}
\end{bmatrix}
$$

$$
C = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
$$

where $\tau$ is the torque, $B_{eq}$ is the viscous friction coefficient of the servo, $J_{eq}$ is the inertia of the rotary arm, $J_l$ is the inertia of the link, and $B_s$ is the linear spring stiffness.

Servo actuator saturation is a key factor in flexible joint control. A fractional-order integral control scheme is used to avoid actuator saturation. Simulation and experimental results investigate the tracking performance and fractional-order control using different types of numerical approximations.

**Model-based design of the fractional-order controller**

The fractional-order integral controller was introduced by Al-Saggaf et al. The state feedback technique was incorporated in these works. The proposed method in this article involves proportional and integral fractional control and a general structure, as shown in Figure 3. In the fractional scheme, the fractional integrator is expressed by $\frac{1}{s^\alpha}$, $(0 < \alpha < 2)$. The compensator $K(s)$ is cascaded with the fractional integrator as static gain that obtains a closed-loop response like Bode’s ideal transfer function. The state feedback $k_s$ is responsible for stabilizing the plant. The compensator $K(s)$ is given by

$$
K(s) = \frac{\Delta_f(s)}{\tau_s s N(s)(1+\tau_f s)^r}
$$

(3)

where $\Delta_f(s)$ is the inner-loop characteristic polynomial and $\tau_s$ is the time constant of the closed-loop reference model. The Bode’s ideal transfer function is used as a reference model. $N(s)$ is the numerator of the linear integer system transfer function. The low-pass filter $\frac{1}{(1+\tau_f s)^r}$ is cascaded with $K(s)$ to realize the transfer function of the compensator. The integer number $r$ and the time constant $\tau_f$ of the filter are chosen by the designer so that $K(s)$ is realizable. Details and proof of equation (3) can be found by Al-Saggaf et al.

**Feedback tuning of virtual reference**

The VRFT technique is one of the data-driven approaches proposed in the last two decades. In VRFT, a virtual reference is the input signal that should be applied to the reference model. It is called “virtual” because it is not used to generate an output signal. The plant-generated single set data can be used for the implementation of the VRFT method. The method does not need any specific experiments or iterations. The VRFT method is based on the
control scheme shown in Figure 4. The VRFT can be an ideal case or a nonideal case. Here, the VRFT is considered as an ideal case with the following properties:

1. The noise is not affecting the system.
2. An ideal controller \( C_d(z) \) is assumed; this is the transfer function of controller that achieves the required closed-loop transfer function \( T_d(z) \)

\[
C_d(z) = \frac{T_d(z)}{G(z)(1 - T_d(z))}
\]

(4)

3. \( C_d(z) \) belongs to the considered controller class as Assumption A.

\textit{Assumption A.} (Matched control) \( C_d(z) \in C \) or equivalently

\[
\exists \rho_d : C(z, \rho_d) = C_d(z)
\]

(5)

4. The controller is parameterized linearly as Assumption B.

\textit{Assumption B.} The controller is expressed as

\[
C(z, \rho) = \rho^T \bar{C}(z)
\]

(6)

where \( \rho \) is a vector containing the controller parameters. It is given by equation (12) in the ideal case and by equation (15) in the nonideal case. \( \bar{C}(z) \) being a vector of \( p \) rational transfer functions independent of \( \rho \).

**Ideal case (matched case)**

The VRFT principle is shown in Figure 5. Input and output data, that is, \( u(t) \) and \( y(t) \), are measured from the plant experiment. This experiment is performed for an open-loop or closed-loop architecture. \( r^*(t) \) is the virtual reference signal, which is defined by a given measured \( y(t) \) such that

\[
T_d(z)r^*(t) = y(t)
\]

(7)

where \( T_d(z) \) is the required reference model for the closed-loop response.

The tracking error is computed as

\[
e(t) = r^*(t) - y(t)
\]

(8)

The signal \( e(t) \) is fed to the ideal controller in this experiment, where \( e(t) \) is the controller’s input, \( C(z, \rho) \) is a model for the controller, and \( C(z, \rho)e(t) = u(t) \) is the expected model output of the controller.

**Figure 4.** Reference model-based control scheme.

**Figure 5.** VRFT principle. VRFT: virtual reference feedback tuning

The error identification criterion is taken as the \( H_2 \) norm of the anticipated error; the VRFT criterion \( J^{FR}(\rho) \) is

\[
J^{FR}(\rho) = \bar{E}[u(t) - C(z, \rho)\bar{e}(t)]^2
\]

(9)

where \( J^{FR}(\rho) \) is a linearly parameterized controller criterion as in Assumption B, which can be written as

\[
J^{FR}(\rho) = \bar{E}[u(t) - \rho^T \varphi(t)]^2
\]

(10)

where the regressor vector \( \varphi(t) \) is defined as

\[
\varphi(t) = \bar{C}(z)\bar{e}(t) = \bar{C}(z)\frac{1 - T_d(z)}{T_d(z)}y(t)
\]

(11)

In the end, we obtain the following

\[
\hat{\rho} = \left[ \sum_{i=1}^{N} \varphi(t)\varphi(t)^T \right]^{-1} \sum_{i=1}^{N} \varphi(t)u(t) = \rho^*
\]

(12)

Details and proof of equation (12) can be found by Li et al.\textsuperscript{22}

While there is no noise, the calculation will exactly result in the asymptotic solution \( \rho^* \).

**Nonideal case (mismatched case)**

The control is mismatched when Assumption A is not satisfied. In the matched case, \( \arg\min (J^{FR}(\rho)) = \arg\min (J_s(\rho)) \), but in the mismatched case, this will no longer be applicable when Assumption A is not satisfied.

Using a VRFT prefilter \( L(z) \), noise is treated. It suffices to choose \( L(e^{\omega}) \), \( (\forall \omega \in [-\pi; \pi]) \) as follows

\[
|L(e^{\omega})|^2 = |T_d(e^{\omega})|^2 |1 - T_d(e^{\omega})|^2 \Phi_L(e^{\omega}) \Phi_L(e^{\omega})^{-1}
\]

(13)

In the nonideal case, the filter is computed using equation (12) and the asymptotic value of the parameter vector \( \rho \) is given by

\[
\rho^* = \bar{E}\left[\varphi_L(t)\varphi_L(t)^T\right]^{-1} \bar{E}\left[\varphi_L(t)u_L(t)^T\right]
\]

(14)

where \( \varphi_L(t) = L(z)\varphi(t) \) and \( u_L(t) = L(z)u(t) \).

The optimal parameter value is computed as

\[
\hat{\rho} = \left[ \sum_{i=1}^{N} \varphi_L(t)\varphi_L(t)^T \right]^{-1} \left[ \sum_{i=1}^{N} \varphi_L(t)u_L(t)^T \right] = \rho^*
\]

(15)
Figure 6. VRFT for fractional integral control based on state feedback control. VRFT: virtual reference feedback tuning.

VRFT for fractional-order control

As shown in Figure 6, the state feedback control for a RFJ system is considered for a plant in this article. The VRFT scheme chooses the optimal parameter value for the fractional integral control \((K_l \) and \(a_l\)). Selecting the right reference model is important. However, there are constraints in the control design. We do not require the reference model more than necessary to provide a desired performance. Depending on equation (4), we observe that the first-order reference model yields the mismatched case of VRFT.

One idea to select a reference model is to use Bode’s ideal control loop. In this work, unit feedback control with Bode’s ideal transfer function is calculated inside the plant. To obtain a desired fractional-order reference model, the first-order reference model is selected, and the value of \(r\) is selected so that \(0 < r < 1\). Note that the fractional integral is approximated using CRONE approximation.

The implementation of the proposed VRFT for fractional integral control based on state feedback control is summarized as follows:

Step 1: Design a controller in which Bode’s ideal transfer function-based fractional integral action is introduced as in the previous work.

Step 2: Select the first-order reference model as an initial solution to the iteration process.

Step 3: Select the number of iterations to tune the fractional integral order \(r\). Then, compute the reference tracking performance criterion \(J_r\) for each \(r\). The value of \(r\) with a minimum \(J_r\) is the tuned fractional integral order.

Step 4: After choosing the fractional integral order \(r\), apply one-shot VRFT to tune \(K_l\).

Simulation results

As illustrated in the previous section, the implementation workflow for the VRFT of fractional integral control is as follows:

Step 1. State feedback with a fractional integral approach is applied to the Quanser RFJ system, where \(J_f = 0.02552 \text{ kg m}^2\), \(J_{eq} = 0.01625 \text{ kg m}^2\), \(B_{eq} = 65407 \text{ kg/s}\), and \(B_l = 10 \text{ kN/m}\). By substituting the system parameters, the system state-space equations are

\[
A = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 671.7 & -1.92 & 0 \\
0 & -1098.9 & 1.92 & 0
\end{bmatrix}
\]

\[
B^T = \begin{bmatrix}
0 & 0 & 479.81 & -479.81
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0
\end{bmatrix}
\]

The open-loop transfer functions for \(\theta\) and \(\alpha\) are

\[
G_\theta(s) = \frac{61.63(s + 396.6)}{s}\Delta(s)
\]

\[
G_\alpha(s) = \frac{-61.63s}{\Delta(s)}
\]

where \(\Delta(s) = (s + 25.07)(s^2 + 15.26s + 637.9)\).

The poles of the closed-loop system chosen as \(p_1 = -40\), \(p_2 = -30\), and \(p_{3,4}\) are the complex roots of the second-order polynomial \(s^2 + 2\zeta\omega_n s + \omega_n^2\), with \(\zeta = 0.7\) and \(\omega_n = 5\). Therefore, the state feedback gain is calculated as

\[
k_s = \begin{bmatrix}
1.23 & -10.04 & -0.24 & -0.83
\end{bmatrix}
\]

The closed-loop transfer functions between the reference signal \(r(s)\) and the outputs \(\theta\) and \(\alpha\) are

\[
\frac{\theta(s)}{r(s)} = \frac{75.65(s^2 + 396.6)}{(s + 40)(s + 30)(s^2 + 7s + 25)}
\]

\[
\frac{\alpha(s)}{r(s)} = \frac{-75.65(s^2 + 396.6)}{(s + 40)(s + 30)(s^2 + 7s + 25)}
\]

From the open-loop system, the gain crossover frequency is \(\omega_c = 1.78 \text{ rad/s}\) and the phase margin is \(\varphi_m = 64.56\). Then, from the following equation

\[
\lambda = \frac{\pi - \varphi_m}{\pi/2} \quad \text{and} \quad \tau_c = \frac{1}{\omega_c^{r+1}}
\]

to obtain, in closed-loop, the Bode’s ideal transfer function

\[
\frac{1}{1+\tau_c s^{r+1}},
\]

the numerical values \(\tau_c = 0.3 \text{ s}\) and \(\lambda = 0.1\) are chosen for the design of the fractional controller.

Now, the fractional compensator \(K(s)\) is calculated from equation (3) with \(r = 3\) and \(\tau_f = 0.005 \text{ s}\). The time constant of the closed loop \(\tau_c\) is larger than the \(\tau_f\), the time constant of the filter. Note that \(\tau_f\) must be greater than the time constant of the simulation or experiment (0.002 s). CRONE approximation method is used to implement the
fractional integral operator in the frequency range $[10^1, 10^4]$ with 10 cells.

Step 2. Reducing the fractional approximation order $N$ can significantly reduce the favorable reference model order. Therefore, the first-order reference model provides a global solution for the VRFT algorithm.

Step 3. We choose 10 iterations with an accuracy of 0.1 to tune the fractional integral order. The minimum reference tracking performance criterion $J_y$ is $\hat{\lambda} = 0.3$, and as shown in Figure 7, $\hat{\lambda} = 0.3$ has the best tracking performance for the angle $\theta(t)$.

Table 1 provides the different tuning iterations. It is observed that increasing the number of iterations minimizes the reference tracking performance criterion. However, a high iteration number increases the complexity in a real-time implementation. Figure 8 shows that 10 iterations are sufficient and that an increase in the number of iterations is not valuable in terms of implementation complexity.

Step 4. Now, using the VRFT algorithm, $k_i = 0.5016$ for $\hat{\lambda} = 0.3$. Figures 9 and 10 show a comparison between state feedback control, state feedback with a fractional-order integrator and VRFT for state feedback with a fractional-order integrator. Table 2 summarizes the main characteristics of the obtained results, where $M_p(\%)$ is the overshoot of the servo position angle $\theta(t)$, $t_s$ is the settling time, and $\Delta_{\alpha_{\text{max}}}$ is the maximum deviation of the vibration angle $\alpha(t)$. It is obvious that the proposed method has better performance.
tracking performance for $q(t)$ and a minimum vibration in $a(t)$. Indeed, the results summarized in Table 2 show that the performance of the SF control and the SF-FOI is worse than those obtained by the proposed SF-FOI-VRFT method. Note, however, that the settling time is greater for the SF-FOI-VRFT method.

### Experimental results

The experimental setup is part of Quanser’s lab systems and tools. The experimental setup shown in Figure 11 contains the following four parts:

- QUARC tool is a real-time control software based on MATLAB version: 2017a and Simulink.
- DAQ (Q2-USB) is a data acquisition device for the analog output to the command motor and the digital input to receive the encoder signals.
- The amplifier (VoltPAQ-X1) is used to amplify the command to the motor voltage level (24 V).
- The RFJ system is a flexible joint module mounted on a rotary servo base unit. A RFJ system is used as an experimental plant.

Figure 12 shows the experimental results for the tracking performance for $\theta(t)$, and Figure 13 shows the experimental results for the minimization of the vibration angle $a(t)$. In this case too, the performance achieved with the three control structures is summarized in Table 3. These results confirm those obtained by simulation.

### Table 2. Performance of the three controllers.

| Controller       | $M_p$ (%) | $t_s$ | $\Delta a_{\text{max}}$ |
|------------------|-----------|------|-------------------------|
| SF               | 20.0      | 0.4  | 12.4                    |
| SF-FOI           | 12.5      | 0.4  | 8.5                     |
| SF-FOI-VRFT      | 0.0       | 0.6  | 6.25                    |

SF: state feedback control; SF-FOI: state feedback with a fractional-order integrator; SF-FOI-VRFT: VRFT for state feedback with a fractional integrator; VRFT: virtual reference feedback tuning.

Figure 11. Experimental setup.

Figure 12. Experimental results. The tracking performance for the angle $\theta(t)$ using a square-wave reference. SF: state feedback control; SF-FOI: state feedback with a fractional-order integrator; SF-FOI-VRFT: VRFT for state feedback with a fractional integrator; VRFT: virtual reference feedback tuning.
The proposed method improved the control performance in terms of tracking error and overshoot. As shown in Figure 13, the proposed method minimizes the peak vibration of the angle $\alpha(t)$.

**Conclusion**

In this article, a data-driven approach is proposed for a fractional-order integrator based on state feedback control. Performance of three control structures is demonstrated in this investigation. It is well known that VRFT is a promising method in data-driven techniques. The proposed method improves the control performance, in which the simulation and an experiment have been performed with the Quanser RFJ system. In addition to the data-driven feature, it is obvious that the proposed method has better tracking performance for $\theta(t)$ and a minimum vibration in $\alpha(t)$. It is clear that the proposed VRFT method has better tracking performance with minimum vibration. Indeed, the simulation and experimental results show that the performance of the SF control and the SF-FOI is worse than that obtained by the proposed SF-FOI-VRFT method.

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