Parity Violation in Graviton Non-Gaussianity

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based on JHEP 1108 067 (2011)
**Parity violation**

**Origin of chirality**

- CPT invariance is fundamental
- CP & T violations may be transmitted to gravity sector via field eqs.
- In GR, this is generally suppressed

**String theory**

- candidate for unified theory & quantum gravity
- imples P-violating gravitational interactions

\[ S = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g}e^{-2\phi} \left[ R + 4(\nabla \phi)^2 - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right] + S_{PV} \]

\[ S_{PV} = \frac{\alpha'}{8\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( F_{\mu\nu}^a \star F_{\mu\nu}^a - \mathcal{R}_{\mu\nu\rho\sigma} \star \mathcal{R}^{\mu\nu\rho\sigma} \right) \]

Chern-Simons interactions are ubiquitous in string theory

⇒ detection of P-violation of gravity will shed light on ultimate theory
Gravitational waves

Inflationary universe as HEP laboratory

• best testbed to explore parity violation
• primordial gravitational waves during inflation

⇒ P-violation encoded in power spectrum

➤ have different GW amplitudes b/w +ve & -ve helicity (circular polarizations)
➤ detectable through correlations of CMB

Our work

Focus on non-Gaussianity of gravitons

• gives another useful measure for the Planck scale physics

Seto 05, Seto-Taruya 06, Saito-Ichiki-Taruya 07, Sato & Soda 08, Takahashi-Soda 09
Correlators via dual CFT

Maldacena-Pimentel 2011

• discussed graviton non-Gaussianity in dS by studying 3D CFT

• shapes of bispectrum are constrained by dS isometry $SO(4,1)$

\[
S = \int d^4x \sqrt{-g} \left( \frac{1}{2} M_{pl}^2 R + a W^{\mu \nu \rho \sigma} W^{\rho \sigma \tau \lambda} W^{\tau \lambda \mu \nu} - b \ast W^{\mu \nu \rho \sigma} W^{\rho \sigma \tau \lambda} W^{\tau \lambda \mu \nu} \right)
\]

$W_{\mu \nu \rho \sigma}$: Weyl tensor \hspace{1cm} $\ast W_{\mu \nu \rho \sigma} = \frac{1}{2} \epsilon_{\mu \nu \lambda \tau} W^{\lambda \tau \rho \sigma}$

• valid in arbitrary order in derivative expansion

this does not imply that parity violation appears in graviton bispectrum

What we would like to do

• reveal conditions under which the parity-violation arises in bispectrum

• develop a new formalism for computing higher-order graviton correlators
(I) New formalism for graviton correlators

(II) Graviton bispectrum during inflation

(III) Concluding remarks
GWs in Minkowski spacetime

**Polarization of GWs**

\[ ds^2 = -d\eta^2 + (\delta_{ij} + \gamma_{ij})dx^i dx^j \]

\[ \gamma_{ii} = \partial_i \gamma_{ij} = 0 \]

\[ \gamma''_{ij} - \Delta \gamma_{ij} = 0 \]

- **linear polarization**
  \[ \gamma_{ij} = \gamma^{\text{plus}} e^{\text{plus}}_{ij} + \gamma^{\text{cross}} e^{\text{cross}}_{ij} \]

  - **plus mode** \( \gamma_{+} \)
  - **cross mode** \( \gamma_{\times} \)

- **circular polarization**
  \[ \gamma_{ij} = \gamma^L e^{\text{(+)}}_{ij} + \gamma^R e^{\text{(-)}}_{ij} \]

  - **left-handed**
    \[ \gamma^L = \frac{1}{\sqrt{2}} (\gamma_{+} - i \gamma_{\times}) \]

  - **right-handed**
    \[ \gamma^R = \frac{1}{\sqrt{2}} (\gamma_{+} + i \gamma_{\times}) \]

\[ k_j e^{(s)}_{ij}(k) = 0 \]

\[ e^{(s)}_{ii}(k) = 0 \]

\[ e^{(s)}_{ij}(k) = e^{(-s)}_{ij}(-k) = e^{(s)}_{ij}(-k) \]

\[ \varepsilon_{ijl} \frac{\partial}{\partial x_l} [e^{(s)}_{mj}(k) e^{ik \cdot x}] = sk e^{(s)}_{im}(k) e^{ik \cdot x} \]

\[ e^{(s)}_{ij}(k) e^{(s')}_{ij}(k) = \delta_{ss'} \]
Helicity variables

Duality operator

Define a map \( \sim: \gamma_{ij} \mapsto \tilde{\gamma}_{ij} = \epsilon_{ikl} \partial_l \gamma_{kj} \)

\[
\tilde{\gamma}_{ij} = \tilde{\gamma}_{(ij)} \,, \quad \tilde{\gamma}_{ii} = \partial_i \tilde{\gamma}_{ij} = 0 \,, \quad \tilde{\gamma}_{ij} = -\Delta \gamma_{ij} \,, \quad \gamma_{ij}'' - \Delta \gamma_{ij} = 0
\]

\[
\gamma_{ij}^\pm := \frac{1}{2} (\gamma'_{ij} \mp i\tilde{\gamma}_{ij})
\]

- \( C \)-valued, symmetric TT tensor
- Form an irrep. of \( E^3 \)
- Correspond to projections onto left & right circular polarizations

c.f. Higaki 1986

Imaginary (anti-)self dual Weyl tensor

\[
W_{\mu\nu\lambda\sigma}^\pm := W_{\mu\nu\lambda\sigma} \pm i * W_{\mu\nu\lambda\sigma}, \quad *W_{\mu\nu\lambda\rho} = \mp i W_{\mu\nu\lambda\rho}^\pm
\]

\[
(W^\pm)^3 = 64[(\gamma^\pm)']^3
\]

\[
W_{\mu\nu \rho \sigma} W^{\rho \sigma \tau \lambda} W^\tau_{\lambda \mu\nu} = \frac{1}{4} \text{Re}(W^+)^3
\]

\[
*W_{\mu\nu \rho \sigma} W^{\rho \sigma \tau \lambda} W^\tau_{\lambda \mu\nu} = \frac{1}{4} \text{Im}(W^+)^3
\]

\( W^3 \) & \( *WW^2 \) can be treated in a unified manner

\( W^3 \) & \( *WW^2 \) are interchanged into each other under

\[
\partial_\eta \leftrightarrow i \epsilon_{ijk} \partial_k
\]
GWs in FLRW

\[ ds^2 = a^2(\eta) \left[ -d\eta^2 + (\delta_{ij} + h_{ij}) dx^i dx^j \right] \]

\[ \partial_i h_{ij} = h_{ii} = 0 \]

\[ h_{ij}'' + 2 \frac{a'}{a} h_{ij}' - \Delta h_{ij} = 0 \]

\[ W_{\pm 0 i0 j}^+ = \frac{1}{2a} \left[ a (h_{ij}' \mp i \epsilon_{ikl} \partial_l h_{jk}) \right]' \]

(radiative parts \( \sim \Psi_0, \Psi_4 \))

Define \( \gamma_{ij}' \equiv ah_{ij}' \)

\[ W_{\mu \nu}^{\alpha \beta} (h) W_{\alpha \beta}^{\gamma \delta} (h) W_{\gamma \delta}^{\mu \nu} (h) = \frac{64}{a^9} \gamma_{ij}' \gamma_{jk}' \gamma_{ki}' \]

\[ W^3 = \frac{1}{4} \text{Re}(W^+)^3, \quad *WW^2 = \frac{1}{4} \text{Im}(W^+)^3 \]

\[ S_{BPV} = -b \int d^4 x \sqrt{-g} *W^{\mu \nu} \rho_{\sigma} W^{\rho \sigma} \tau \lambda W^{\tau \lambda} \mu \nu = 8ib \int d\eta d^3 x a^{-5} [(\gamma^+)^3 - (\gamma^-)^3] \]

\[ H_{\text{int}} = -8ib \int d\eta a^{-5} [(\gamma^+)^3 - (\gamma^-)^3] \]
Graviton bispectrum in dS

Pure de Sitter case

\[ h_{ij} = \frac{2}{M_{pl}} \int \frac{d^3k}{(2\pi)^{3/2}\sqrt{2k}} \sum_{s=\pm} \left[ e^{(s)}_{ij}(k) u_k(\eta) a_s(k) e^{ik \cdot x} + (\text{h.c.}) \right] \]

\[ h''_{ij} + \frac{2a'}{a} h'_{ij} - \Delta h_{ij} = 0 \]

mode function

\[ u_k = \frac{H}{k} \left( 1 + ik\eta \right) e^{-ik\eta}, \]

\[ \left[ a_s(k), a_{s'}^\dagger(k') \right] = \delta_{ss'} \delta(k - k') \]

\[ \gamma'_{ij} = a h'_{ij} \]

\[ \gamma^\pm_{ij} := \frac{1}{2} (\gamma'_{ij} \mp i\tilde{\gamma}_{ij}) \]

\[ \varepsilon_{ijl} \partial_k \left[ e^{(s)}_{mj}(k) e^{ik \cdot x} \right] = sk e^{(s)}_{im}(k) e^{ik \cdot x}, \]

\[ \gamma^\pm_{ij} = -2 \int \frac{k d^3k}{M_{pl}(2\pi)^{3/2}\sqrt{2k}} \left[ e^{(+)}_{ij}(k) e^{-ik\eta} a_+(k) + e^{(-)}_{ij}(-k) e^{ik\eta} a_-(k) \right] e^{ik \cdot x} \]

correlators can be calculable via in-in formalism

\[ \langle A(t) \rangle = -i \int_{t_0}^t dt' \langle [A(t), H_{\text{int}}(t')] \rangle \]

\[ H_{\text{int}} = b \int d^4x \sqrt{-g} * WW^2 \]

\[ \langle \gamma^\pm_{ij}(p) (\gamma^\mp_{kl}(\eta, k))' \rangle = 2iM^{-2}_{pl} k^2 \Pi^\pm_{ij,kl}(p) \delta^{(3)}(p + k) e^{ik\eta} \]

\[ \Pi^\pm_{ij,kl}(p) = e^{(\pm)}_{ij}(p) e^{*(\pm)}_{kl}(p) \]
No parity violation

\[ \langle \gamma^+_{i_1j_1}(0,p_1)\gamma^+_{i_2j_2}(0,p_2)\gamma^+_{i_3j_3}(0,p_3) \rangle \]

\[ = 384ibH^5M_p^{-6}(2\pi)^3(p_1p_2p_3)^2\delta^{(3)}(p)\frac{5!}{p^6} \left[ \Pi^+_{i_1j_1,kl}(p_1)\Pi^+_{i_2j_2,lm}(p_2)\Pi^+_{i_3j_3,mp}(p_3) + \Pi^-_{i_1j_1,kl}(p_1)\Pi^-_{i_2j_2,lm}(p_2)\Pi^-_{i_3j_3,mp}(p_3) \right] \]

\[ p = \sum_i p_i \]

• project onto left & right handed circular polarizations

\[ h^R := h_{ij}e^{i\phi}_{ij}^{(+)} \quad h^L := h_{ij}e^{i\phi}_{ij}^{(-)} \]

\[ h_{ij}(\eta = 0, p) = -\frac{H}{p^2}[\gamma^+_{ij}(p) + \gamma^-_{ij}(p)] \]

\[ \langle h^R(0,p_1)h^R(0,p_2)h^R(0,p_3) \rangle \]

\[ = -384i(2\pi)^3(bH)^2(H/M_p)^6F(p_2,p_2,p_3) \left[ \frac{5!}{(p_1+p_2+p_3)^2} - \frac{5!}{(p_1+p_2+p_3)^2} \right] \delta^{(3)}(p_1+p_2+p_3) = 0 \]

\[ F(k_1,k_2,k_3) = -\frac{(k_1+k_2+k_3)^3(k_1+k_2-k_3)(k_2+k_3-k_1)(k_3+k_1-k_2)}{64k_1^2k_2^2k_3^2} \]

\[ \langle h^L(0,p_1)h^L(0,p_2)h^L(0,p_3) \rangle = 0 \quad \langle h^R(0,p_1)h^L(0,p_2)h^L(0,p_3) \rangle = 0 \]

\[ \langle h^R(0,p_1)h^R(0,p_2)h^R(0,p_3) \rangle - \langle h^L(0,p_1)h^L(0,p_2)h^L(0,p_3) \rangle = 0 \]

\[ \langle h^R(0,p_1)h^R(0,p_2)h^L(0,p_3) \rangle - \langle h^L(0,p_1)h^L(0,p_2)h^R(0,p_3) \rangle = 0 \]

\[ \text{In dS universe, parity violation does not appear in graviton non-Gaussianity.} \]
Slow roll corrections

Consider deviations away from de Sitter

• expand terms by slow roll parameter \( \epsilon := -\dot{H}/H^2 \), \( a \sim (-H_*\eta)^{-1-\epsilon} \)

I. mode function

\[
u_k(\eta) = \sqrt{\frac{\pi k}{2H_*}} e^{i\pi \nu/2-i\pi/4} (-H_*\eta)^\nu H^{(1)}_\nu (-k\eta) \quad , \quad h_{ij}(0, p) = -\frac{H_*}{p^2} C [\gamma^+_i(0, p) + \gamma^-_{ij}(0, p)]
\]

\[
C = 1 + \epsilon(2 - \gamma - \log 2 + \log H_*/k)
\]

II. interaction Hamiltonian

\[
H_{\text{int}} = -8ib \int d\eta d^3x a^{-5} [(\gamma^+)^3 - (\gamma^-)^3] \quad , \quad (\gamma^\pm)_{ij}' \rightarrow (\gamma^\pm_{(dS)ij})' + \epsilon \chi^\pm_{ij}
\]

III. cosmic expansion

\[
\int_{-\infty}^{0} d\eta (-\eta)^{5+5\epsilon} e^{-i(p_1+p_2+p_3)\eta} \sim -\frac{5!}{(p_1+p_2+p_3)^6} \left( 1 + \frac{5\pi}{2i\epsilon} \right)
\]
Parity violation

\[ \langle h^R(0, p_1) h^R(0, p_2) h^R(0, p_3) \rangle = -\langle h^L(0, p_1) h^L(0, p_2) h^L(0, p_3) \rangle \]

\[ = -64(2\pi)^4 \epsilon (bH_*^2)(H_*/M_{pl})^6 \delta^{(3)}(p) F(p_1, p_2, p_3) \frac{6!}{p^6} \]

\[ F(k_1, k_2, k_3) = -\frac{(k_1 + k_2 + k_3)^3 (k_1 + k_2 - k_3) (k_2 + k_3 - k_1) (k_3 + k_1 - k_2)}{64k_1^2k_2^2k_3^2} \]

This sort of parity violation can be observed in CMB

Shiraishi-Nitta-Yokoyama 2011
Concluding remarks

Summary

• A new formalism for computing graviton correlators provides us with a unified treatment of $W^3$ & $*WW^2$.

• For Weyl cubic interactions, parity violation does not show up in exact de Sitter, but it does in the slow roll case.

Outlooks

• A single field effective field theory admits parity-odd interaction

\[ L_0 = R - \frac{1}{2}(\nabla \phi)^2 - V(\phi) \quad L_1 = f_1(\phi)(\nabla \phi)^4 + f_2(\phi)W_{abcd}W^{abcd} + f_3(\phi)W_{abcd} * W^{abcd} \]

\[ *W_{abcd} = \frac{1}{2} \epsilon_{aef} W^{ef}_{\quad cd} \quad W_{abcd}: \text{Weyl tensor} \]
(conformal) isometries in dS

Conformal Killing vectors
\[ \mathcal{L}_{\xi^{(i)}} g_{ab} = 2\phi^{(i)} g_{ab} , \quad \phi^{(i)} = \frac{1}{4} \nabla_{a} \xi^{a} \]

RW metric
\[ ds^2 = a^2(\eta) \left( -d\eta^2 + dx^2 + dy^2 + dz^2 \right) \]

admits maximal set of 15 CKVs

\[ P_\mu = \partial_\mu \quad D = x^\mu \partial_\mu \quad M_{\mu\nu} = x_\mu \partial_\nu - x_\nu \partial_\mu \]
\[ K_\mu = 2x_\mu D - (x_\nu x^\nu) P_\mu \]

\[
\begin{align*}
[D, K_\mu] &= K_\mu \quad [D, P_\mu] = P_\mu \\
[K_\mu, P_\mu] &= 2(\eta_{\mu\nu} D + 2M_{\mu\nu}) \quad \eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)
\end{align*}
\]

\[
\begin{align*}
[K_\mu, M_{\nu\rho}] &= \eta_{\mu\nu} K_\rho - \eta_{\mu\rho} K_\nu \\
[P_\mu, M_{\nu\rho}] &= \eta_{\mu\nu} P_\rho - \eta_{\mu\rho} P_\nu \\
[M_{\mu\nu}, M_{\rho\sigma}] &= -\eta_{\mu\rho} M_{\sigma\nu} + \cdots
\end{align*}
\]

\[ \phi^{(i)} \] are given by
\[
\begin{align*}
\phi_{P_0} &= \mathcal{H} \quad \phi_{K_0} = -2\eta - (\eta^2 + x^i x_i) \mathcal{H} \\
\phi_D &= 1 + \eta \mathcal{H} \\
\phi_{K_i} &= 2x_i (1 + \eta \mathcal{H})
\end{align*}
\]

\[ K_i \text{ generates special CT of } E^3 \text{ for late time (}\eta \sim 0) \]

\[ \Rightarrow \text{ restricts allowed correlators for tensors w/ conformal dimension 2 (}\approx\text{stress tensor}) \]