Detection of area under potential threat via an advanced aggregation operator on generalized triangular fuzzy number

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ABSTRACT

From last few decades criminal activities and terror attacks have been increased. In a big city or in highly populated city, it is very difficult to keep under surveillance each and every area of that city throughout whole day and night. Since, it will require huge amount of deployment of police duty and such type of deployment can cause heavy work load on the enforcement agencies. It has been seen that, some types of areas are often targeted by terrorist than others, mainly train junctions, bus stoppages, market places, near temples etc. In this paper, a new aggregation operator on generalized triangular fuzzy number has been defined. Through the newly developed aggregation operator, a fuzzy multi criteria decision making approach has been constructed which enables us to determine the area of a city which may be the potentially under threat, during any interval of time on a regular working day.

1. Introduction

In recent time prevention of terror attacks in a city has become a challenging task for some countries. The terror activities in the form of bombing or gunshots are done in an area of a city which is populated at that time, important from the business point of view or has important administrative offices. In short, a certain area is targeted if the attack maximizes the effect or kills more people. As far as security concerns, enforcement agencies may deploy a huge amount of police in a city to secure each and every area the city, but this will not be an efficient method. Since, deploying too much persons for the prevention of terror attacks is wastage of man power and money. Also, it is to be noted that in a city or a town some areas are not dense with people all the time during a day. For example, an area which has school and college only will be crowded only during the arrival time and departure time (approximately from 9 AM. to 10 A.M. and 2 P.M. to 3 P.M) on a working day of the school and colleges, and rest of the day, the area will not be crowded if any other factors are not there which will cause crowded. Thus, it can be seen that some area need not required extra attention throughout all the day. For this reason, it is quite necessary to determine which area is important than other for deployment of extra police duty or to keep under surveillance during a given interval of time.

In this paper, a new aggregation operator on generalized triangular fuzzy number is proposed and a fuzzy multi criteria decision-making methodology is developed to rank the areas that are under potential threat of being targeted attacks during any time of the day. Also, a case study is carried out to show the utility of the methodology. In “section 1” an overall idea about the current scenario of cities regarding the surveillance and detection of under potential threat area is elaborated. In “section 2” the need and scope for research on this particular topic is discussed. In “section 3” it is explained what motivates the author to study this particular topic. The contribution of the paper is illustrated in the “section 4”. “Section 5” reviews some studies carried out by different researches about this particular topic. In “section 6” a brief review on fuzzy set, generalized fuzzy numbers is given. A review on aggregation operator is given in the “section 7”. In “section 8” a new aggregation operator on triangular fuzzy number is defined and illustrated how it is more efficient than the earlier aggregation methods. In “section 9” the fuzzy multi-criteria decision-making approach has been proposed for identification of area that may be under potential threat in a specific time. In “section 10” a case study in Dibrugarh city (India) is carried out to show the usability of this methodology. Finally, “section 11” discusses about the key finding of the paper, its limitation and suggested future work.

2. Problem statement

It is already mentioned in the introduction section how and why it is important to have efficient methods for prioritization of areas that should be under surveillance during a certain time. The factors, that may attract any
attack mostly depends upon crowd, caused by markets, schools, institutes, administrative offices, etc. The crowds always vary with time throughout the day and this variation in crowds are not certain and specific most of the time. Also, an area may be crowded more than one factor at a certain time. In addition, the factors that causes crowd are always expressed in linguistic expressions, such as huge, big, less, etc. For this reason, prioritization of areas of a city to take under surveillance becomes a decision-making task under uncertainty, where the factors that causes crowd can be considered as the criteria and the areas of the city can be considered as the objective of the decision-making process. In short, depending upon the criteria (factors that cause crowds) we need to select an alternative (an area of the city) that should gain the first priority for surveillance.

3. The motivation

Crime prevention and crime prediction is not a new study to this world. Many researchers studied about crime prediction and crime prevention and gave some methodologies from the back ground of statistics and fuzzy mathematics. In a certain area of a city the density of crowded during a given time mainly caused by, markets, residential areas, educational institutes, social events, etc. As a result, any area does not remain crowded whole day and it varies with time throughout the day. For this reason, uncertainty arises in prioritize the different areas of a city on the basis of under potential threats. This motivates us to study in this particular topic.

4. Contribution

The main contribution of this paper is summarized below point wise:

(i) A new aggregation operator on generalized triangular fuzzy number is defined.
(ii) A fuzzy multi-criteria decision-making approach has been proposed to prioritize or rank the areas to be protected and to keep under strict surveillance at a specific time.
(iii) Finally, a case study is carried out for “Dirbrugarh (India)” city to show the utility of the methodology.

5. Literature review

In 1965, the concept of fuzzy set theory [1] was first introduced by Zadeh. The concept of generalized trapezoidal fuzzy number presented [2] by Chen in 1985, also he proposed the ranking method for generalized trapezoidal fuzzy number. Further, Chen et al., in 2006 proposed the concept of generalized interval-valued trapezoidal fuzzy number [3] based on the generalized trapezoidal fuzzy number proposed by Chen in 1985 and interval-valued fuzzy number [4] proposed by Wang and Li in 1998. Also, Wang and Li proposed a new method for ranking the generalized interval-valued trapezoidal fuzzy numbers. Generalized trapezoidal fuzzy numbers have been applied in various fields of decision-making such as reliability analysis, risk analysis, pattern recognition, rotor fault diagnosis, maximal flow problems, etc. Liu in 2011 proposed some aggregation operators [5] on generalized interval-valued trapezoidal fuzzy number such as the generalized interval-valued trapezoidal fuzzy number weighted aggregation operator, the generalized interval-valued trapezoidal fuzzy number ordered weighted aggregation operator and the generalized interval-valued trapezoidal fuzzy numbers hybrid aggregation operator. Also, a group decision methods based on these operators are also illustrated in his study. Liu and Jin in 2011 proposed the generalized interval-valued trapezoidal fuzzy numbers weighted geometric aggregation (GITFNWGA) operator [6], the generalized interval-valued trapezoidal fuzzy numbers ordered weighted geometric aggregation (GITFNOWGA) operator and the generalized interval-valued trapezoidal fuzzy numbers hybrid geometric aggregation (GITFNGHA) operator. In addition, they presented a group decision method based on these operators proposed.

Fuzzy multi-criteria decision-making (FMCDM) is a modelling and methodological tool for designing decision-making to reduce the uncertainty in the presence of various conflicting criteria, preferences, and information. Bellman and Zadeh in 1970 and Zimmermann in 1978 introduced fuzzy sets into the MCDM field [7,8]. Mainly there are two approaches to MCDM – Multi-objective decision-making (MODM) which concentrates on continuous decision space aimed at the realization of the best solution in which several objective function is to be achieved simultaneously. The Multi-attribute decision-making (MADM) refers towards decision-making under discrete decision spaces and focuses on how to select different alternatives from existing alternatives. Some important MADM approaches are:– analytical hierarchy process (AHP) [9], analytical network process (ANP) [10], technique for ordered preference by Similarity to Ideal Solution (TOPSIS) [11], VIKOR [12]. Some mathematical programming technique such as linear programming (LP), goal programming (GP) and mixed integer programming (MIP) are typically associated with MODM approaches.

In 2006, Grubesic [13] applied fuzzy clustering to detect the pattern of criminal activities and find crime hot-spot areas in an area. Also Compared with median and k-means clustering problem and the result implied that fuzzy clustering is better in handling crime pattern detection. Li et al., in 2010 proposed an intelligent decision-support model based on a fuzzy self-organizing map (FSOM) network [14] to analyse and detect crime pattern from temporal crime activity data.
Criteria Decision-Making (MCDM) technique, the Analytical Hierarchy Process (AHP) to identify the crime potential location with the help of Geographical Information System (GIS) [16].

6. Preliminaries

In this section preliminary concept of fuzzy set, generalized fuzzy numbers and aggregation of generalized fuzzy numbers are introduced.

Definition 6.1: Fuzzy Set:

A fuzzy set is one which assigns grades of membership between 0 and 1 to objects within its universe of discourse. If \( X \) is a universal set then a fuzzy set \( A \) is defined by its membership function

\[
\mu_A : X \rightarrow [0, 1].
\]

Definition 6.2: Generalized fuzzy number:

The membership function of a Generalized fuzzy number \( A = (a, b, c, d; w) \), where \( a \leq b \leq c \leq d \) and \( w < 0 \leq 1 \) is given by:

\[
\mu_A(x) = \begin{cases} 
0, x < a \\
\frac{x-a}{b-a}, a \leq x \leq b \\
w, b \leq x \leq c \\
\frac{d-x}{d-c}, c \leq x \leq d \\
0, x > d.
\end{cases}
\]

If \( w = 1 \) then the generalized fuzzy number \( A \) is called normal trapezoidal fuzzy number and is written as \( A = (a, b, c, d) \). For \( a = b \) and \( c = d \), \( A \) becomes a crisp interval. For \( b = c \), \( A \) becomes a generalized triangular fuzzy number. Similarly, for \( w = 1 \) the generalized fuzzy number \( A \) is called normalized triangular fuzzy number.

The membership function of the generalized triangular fuzzy number \( A = (a, b, c; w) \) is given by

\[
\mu_A(x) = \begin{cases} 
0, x < a \\
\frac{x-a}{b-a}, a \leq x \leq b \\
w, b \leq x \leq c \\
\frac{c-x}{c-b}, b \leq x \leq c \\
0, x > c.
\end{cases}
\]

6.1. Operations on the generalized fuzzy number

Let \( A = (a_1, b_1, c_1, d_1; w_A) \) and \( B = (a_2, b_2, c_2, d_2; w_B) \) be any two generalized fuzzy numbers then its operations can be expressed as follows:

\[
\tilde{A} \oplus \tilde{B} = (a_1, b_1, c_1, d_1; w_A) + (a_2, b_2, c_2, d_2; w_B)
\]

\[
= (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2; \min\{w_A, w_B\}),
\]

\[
\tilde{A} - \tilde{B} = (a_1, b_1, c_1, d_1; w_A) - (a_2, b_2, c_2, d_2; w_B)
\]

\[
= (\min(a_1 - a_2, 0), \min(b_1 - b_2, 0), \min(c_1 - c_2, 0), \min(d_1 - d_2, 0); \min\{w_A, w_B\}),
\]

\[
\tilde{A} \otimes \tilde{B} = (a_1, b_1, c_1, d_1; w_A) \times (a_2, b_2, c_2, d_2; w_B)
\]

\[
= (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2, d_1 \times d_2; \min\{w_A, w_B\}),
\]

\[
A / B = (a_1, b_1, c_1, d_1; w_A) / (a_2, b_2, c_2, d_2; w_B)
\]

\[
= (a_1/a_2, b_1/b_2, c_1/c_2, d_1/d_2; \min\{w_A, w_B\}).
\]

Similarly, let \( A = (a_1, b_1, c_1; w_A) \) and \( B = (a_2, b_2, c_2; w_B) \) be any two generalized triangular fuzzy numbers then its operations can be expressed as follows:

\[
\tilde{A} \oplus \tilde{B} = (a_1, b_1, c_1; w_A) + (a_2, b_2, c_2; w_B)
\]

\[
= (a_1 + a_2, b_1 + b_2, c_1 + c_2; \min\{w_A, w_B\}),
\]

\[
A - B = (a_1, b_1, c_1; w_A) - (a_2, b_2, c_2; w_B)
\]

\[
= (\min(a_1 - a_2, 0), \min(b_1 - b_2, 0), \min(c_1 - c_2, 0); \min\{w_A, w_B\}),
\]

\[
\tilde{A} \otimes \tilde{B} = (a_1, b_1, c_1; w_A) \otimes (a_2, b_2, c_2; w_B)
\]

\[
= (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2; \min\{w_A, w_B\}),
\]

\[
A / B = (a_1, b_1, c_1; w_A) / (a_2, b_2, c_2; w_B)
\]

\[
= (a_1/a_2, b_1/b_2, c_1/c_2; \min\{w_A, w_B\}).
\]

7. Aggregation operator

Aggregation operations are the process by which several fuzzy numbers are combined to form a new one. Generally, an aggregation operator can be defined by the following function

\[
h : [0, 1]^n \rightarrow [0, 1]
\]

for some \( n \geq 2 \).

Let us consider \( n \) fuzzy sets \( A_1, A_2, \ldots, A_n \) defined on \( X \) is aggregated by the aggregation operator \( h \) and get a new fuzzy set \( A \) then for any \( x \in X \) membership grade in \( A \) can be written as

\[
\mu_A(x) = h(\mu_{A_1}(x), \mu_{A_2}(x), \ldots, \mu_{A_n}(x)).
\]
From the aggregation defined on generalized interval-valued trapezoidal fuzzy numbers \([5]\) one can easily apply the results on generalized trapezoidal fuzzy numbers.

### 7.1. Aggregation on generalized trapezoidal fuzzy numbers

Suppose \(\tilde{A}_i (i = 1, 2, \ldots, n)\) be a set of generalized trapezoidal fuzzy numbers such that \(\tilde{A}_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i}; w_{\tilde{A}_i})\) and \(h_{GFN}: \Omega^n \to \Omega\), then

\[
h_{GFN}(\tilde{w})(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{i=1}^{n} \tilde{w}_i \tilde{A}_i
\]

where \(\Omega\) is the set of all aggregation operators of GFN, and \(\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)\) be the weighted vector of \(\tilde{A}_i\)'s and \(\tilde{w}_i = (w_{\tilde{A}_1}, w_{\tilde{A}_2}, w_{\tilde{A}_3}, w_{\tilde{A}_4}; \eta_i)\).

If the respective weights become real numbers say, \(w = (w_1, w_2, \ldots, w_n)\) such that \(\sum_{i=1}^{n} w_i = 1\), then the aggregation operation can be written as

\[
h_{GFN(w)}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{i=1}^{n} w_i \tilde{A}_i
\]

Taking \(a_{2i} = a_{3i}\) and \(w_{2i} = w_{3i}\) for all \(i\) we can get the set of generalized triangular fuzzy numbers and aggregation operations similarly.

### 7.2. Aggregation on generalized triangular fuzzy numbers

Suppose \(\tilde{A}_i (i = 1, 2, \ldots, n)\) be a set of generalized triangular fuzzy numbers, such that \(\tilde{A}_i = (a_{1i}, a_{2i}, a_{3i}; w_{\tilde{A}_i})\) and \(h_{GFN}: \Omega^n \to \Omega\), then

\[
h_{GFN}(\tilde{w})(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{i=1}^{n} \tilde{w}_i \tilde{A}_i
\]

where \(\Omega\) is the set of all aggregation operators of GFN, and \(\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)\) be the weighted vector of \(\tilde{A}_i\)'s and \(\tilde{w}_i = (w_{\tilde{A}_1}, w_{\tilde{A}_2}, w_{\tilde{A}_3}; \eta_i)\).

If the respective weights become real numbers say, \(w = (w_1, w_2, \ldots, w_n)\) such that \(\sum_{i=1}^{n} w_i = 1\), then the aggregation operation can be written as

\[
h_{GFN(w)}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n) = \sum_{i=1}^{n} w_i \tilde{A}_i
\]

It is to be noted that this aggregation operations may give sometimes illogical results. Since knowingly or unknowingly in this method, one has to reduce the aggregated height to the minimum of all the heights of respective GFNNs. For example:

Let us consider \(A = (5, 1, 2; 8), B = (1, 2, 3; 9)\), and \(C = (1, 2, 3; 1)\) be three triangular fuzzy numbers with different heights and same weights i.e. Now aggregating \(A\) and \(B\) and \(C\) with same weights we get:

\[
h_{GFN(w)}(A, B) = \frac{1}{2} (2.5, 1, 2; 8) + \frac{1}{2} (1, 2, 3; 9)
\]

\[
= (2.5 + 5.5 + 1, 1 + 1.5; \min(8,9))
\]

\[
= (7.5, 6, 2.5; 8),
\]

\[
h_{GFN(w)}(A, C) = \frac{1}{2} (5, 1, 2; 8) + \frac{1}{2} (1, 2, 3; 1)
\]

\[
= (2.5 + 5, 1; 8) + (5, 1, 1.5; 1)
\]

\[
= (5.5 + 5 + 1.1 + 1.5; \min(8,1))
\]

\[
= (7.5, 6, 2.5; 8).
\]

From the above example it can be concluded if aggregation is done for same support but different heights, it gives same results, which is illogical.

Dutta in 2016 proposed an arithmetic operation \([17]\), in which arithmetic operation between two generalized triangular fuzzy numbers gives a generalized trapezoidal fuzzy number. Inspired by this idea and to eliminate the drawback that arises in earlier aggregation methods that explained above, a new aggregation operator is defined on the generalized triangular fuzzy number which aggregates a set of triangular number to a generalized trapezoidal fuzzy number.

### 8. Newly proposed aggregation operation

Suppose \(\tilde{A}_i (i = 1, 2, \ldots, n)\) be a set GFN such that \(\tilde{A}_i = (a_{1i}, a_{2i}, a_{3i}; w_{\tilde{A}_i})\) and \(\tilde{w} = (\tilde{w}_1, \tilde{w}_2, \ldots, \tilde{w}_n)\) be the
weighted vector of $\tilde{A}_i$s and $\tilde{w}_i = (w_1, w_2, w_3; \eta_j)$. $h_{GFN} : \Omega^n \rightarrow \Omega$ such that

$$h_{GFN}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$$

$$= \sum_{i=1}^{n} \left( w_1 a_{1i}, a_{1i} + w_2 a_{2i}, \ldots, w_n a_{ni} \right)$$

$$\tilde{a}_{3i} = w_3 \tilde{w}_A(\tilde{a}_{3i} - \tilde{a}_{2i}), \tilde{w}_3 a_{3i} = \frac{1}{n} (\eta_i \times \tilde{w}_A).$$

If the respective weights become real numbers say, $w = (w_1, w_2, \ldots, w_n)$ such that $\sum_{i=1}^{n} w_i = 1$, then the aggregation operation can be written as:

$$h_{GFN}(\tilde{A}_1, \tilde{A}_2, \ldots, \tilde{A}_n)$$

$$= \sum_{i=1}^{n} (w_1 a_{1i}, a_{1i} + w_2 a_{2i}, \ldots, w_n a_{ni}).$$

$$w_i a_{3i} = w_i a_{2i} (w_i a_{3i} - w_j a_{2j}), w_i a_{3i} = w_i a_{j} (\eta_i \times w_A).$$

It is to be noted that in this proposed aggregation operation, a collection of generalized triangular fuzzy numbers are aggregated to form a generalized trapezoidal fuzzy number. Also, an aggregated height is taken rather than taking the minimum of the collection of the heights.

Applying the proposed aggregation operation on the earlier example from section 7.2, we get:

$$h_{GFN}(w)(A, B) = (0.5, 1, 2; 0.8) \oplus (1, 2, 3; 0.9)$$

$$= (0.75, 1.075, 1.075, 2.5; 0.85),$$

$$h_{GFN}(w)(A, C) = (0.5, 1, 2; 0.8) \oplus (1, 2, 3; 0.9)$$

$$= (0.75, 1.1, 2.05, 2.5; 0.9).$$

Thus, we can observe that the new aggregation operation gives reasonably different generalized trapezoidal fuzzy number. Also, an aggregated height can be obtained rather than taking the minimum of heights, which will be more logical in dealing with uncertainties.

9. Proposed methodology

Let us consider a collection of the continuous interval of time which divides a given duration of time into $m$ sub-intervals, say $T_1, T_2, \ldots, T_m$. Now, $C_1, C_2, \ldots, C_n$ be the different factors that cause an area to be under potential threat in a city and is taken as the criteria of our multi-criteria decision-making approach. City which will be under study are divided into $K$ parts by the decision maker, say $Z_1, Z_2, \ldots, Z_K$, which will be called “zone” in our study later on. Now, as mentioned earlier any zone or any area of a city does not deserve constant surveillance throughout 24 h of the day. For example, a market area at 1 A.M. is less likely to be attacked, as at that time effect of the attack will be minimal compared to the attack if done at day time when the place will be crowded. For this reason, the factors that can attract a terror attack have different degree of relationship during different time of a day in a different zone. The whole idea can be described with the help of the following table: Table 1.

Here $\tilde{A}_i^j$ is a generalized triangular fuzzy number and $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, K$. Here, $\tilde{A}^j_k$ gives the degree of relationship with the factors that can attract a terror attack $C_j$ during time interval $T_i$, in Zone-$k$.

Now, $A_i^j$ are aggregated to form a single degree of relationship $\tilde{A}^i_k$ which will give the degree of relationship to what extent during the time interval $T_i$ the Zone-$k$ will be under potential threat of terror attack.

And can be written as Ranking of $\tilde{A}^i_k = h_{GFN}(\tilde{A}^1_k, \tilde{A}^2_k, \ldots, \tilde{A}^m_k)$. Thus, the zones of the city can be ranked with the help of aggregated degree of relationship $\tilde{A}^i_k$ for each interval of time $T_i$. The decision situation for ranking of zone can be shown as follows:

$$T_1 \begin{bmatrix} \tilde{A}_1^1 & \tilde{A}_1^2 & \ldots & \tilde{A}_1^K \\ \tilde{A}_2^1 & \tilde{A}_2^2 & \ldots & \tilde{A}_2^K \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{A}_m^1 & \tilde{A}_m^2 & \ldots & \tilde{A}_m^K \\ \end{bmatrix} \times \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_K \\ \end{bmatrix}$$

Table 1. Degrees of relationships during different time of a day and factors that cause an area to be under potential threat in different zone.

| Time intervals | Zone-1 | Zone-2 | Zone-K |
|----------------|--------|--------|--------|
|                | $C_1$  | $C_2$  | $C_n$  |
| $T_1$          | $\tilde{A}^1_1$ | $\tilde{A}^1_2$ | $\tilde{A}^1_n$ |
| $T_2$          | $\tilde{A}^2_1$ | $\tilde{A}^2_2$ | $\tilde{A}^2_n$ |
| $\vdots$       | $\tilde{A}^m_1$ | $\tilde{A}^m_2$ | $\tilde{A}^m_n$ |
| $T_m$          | $A_i^1$  | $A_i^2$  | $A_i^K$  |
| Ranking of (Zone - 1, Zone - 2, L, Zone - K) | | | |
| Ranking of (Zone - 1, Zone - 2, L, Zone - K) | | | |
| Ranking of (Zone - 1, Zone - 2, L, Zone - K) | | | |

10. A case example

A case study is done on Dibrugarh city (Dist: Dibrugarh, State: Assam, country: India, Area: 66.14 km², garh, State: Assam, country: India, Area: 66.14 km²,
27.4728°North, 94.9120°East). In this study, an assumption has been taken that the study will be on a regular working day officially and no socially gathering event like festival, functions etc. are taken under consideration. The study will be done during the time from 8 A.M. to 10 P.M., a total of 14 h. 14 h is divided into seven equal time intervals each consisting of 2 h.

The city is divided into five Zones as shown in the figure (the city is divided by major roads for our convenience):

The Zone-1 has a bus stoppage at NH-37, a road junction of two ways where traffic arises, The Dibrugarh University market complex and a few number of schools and colleges. The Zone-2 has The Dibrugarh University, Some branches of national banks, a railway colony, The Dibrugarh Municipality Board and few temples. The Zone-3 includes most of the hotels, some branches of national banks, Fish Market, 10–12 number of school–colleges, Police stations, Major road junctions and Administrative offices. The Zone-4 consists of a rail junction, some school–colleges, a few temples and musks and a children park. Zone 5 consists of the Assam Medical College, some private hospitals, pharmacies, diagnosis centres, medical laboratories, lodges, ATMs, a large market, some road junction’s cause heavy traffics and a bus stoppage.

The following five criteria are taken for the case study:

(a) Market places/Business places $C_1$
(b) Hospitals $C_2$
(c) School/Colleges/Educational institutions $C_3$
(d) Bus stoppages/Rail Junctions/Airports/Important Road Junction $C_4$
(e) Administrative offices/Other offices $C_5$

The following linguistic variables as shown in the Table 2 and their corresponding values in triangular fuzzy number will be used to assign the degree of

| Linguistic variable | Generalized triangular fuzzy number |
|---------------------|-------------------------------------|
| Absolutely low (AL) | (0, 0, 0.1; 1)                      |
| Very low (VL)       | (0, 0.1, 0.2; 0.9)                  |
| Low (L)             | (0.1, 0.2, 0.3; 0.9)                |
| Above low (ABL)     | (0.2, 0.3, 0.4; 0.8)                |
| Below medium (BM)   | (0.3, 0.4, 0.5; 0.7)                |
| Medium (M)          | (0.4, 0.5, 0.6; 0.7)                |
| Above medium (AM)   | (0.5, 0.6, 0.7; 0.7)                |
| Below high (BH)     | (0.6, 0.7, 0.8; 0.8)                |
| High (H)            | (0.7, 0.8, 0.9; 0.9)                |
| Very high (VH)      | (0.8, 0.9, 1; 0.9)                  |
| Absolutely high (AH)| (0.9, 1, 1; 1)                      |

Table 3. Table that shows degree of relationship between time intervals and criterion for Zone-1.

| Time interval       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---------------------|-------|-------|-------|-------|-------|
| 8 A.M.–10 A.M.      | L     | VL    | H     | AM    | M     |
| 10 A.M.–12 P.M.     | AM    | VL    | M     | BH    | BM    |
| 12 P.M.–2 P.M.      | AM    | VL    | AM    | BH    | BM    |
| 2 P.M.–4 P.M.       | M     | VL    | BH    | H     | M     |
| 4 P.M.–6 P.M.       | H     | AL    | L     | H     | L     |
| 6 P.M.–8 P.M.       | BH    | AL    | M     | L     | AL    |
| 8 P.M.–10 P.M.      | L     | AL    | L     | AL    | AL    |

Table 4. Table that shows degree of relationship between time intervals and criterion for Zone-2.

| Time interval       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---------------------|-------|-------|-------|-------|-------|
| 8 A.M.–10 A.M.      | M     | L     | H     | H     | H     |
| 10 A.M.–12 P.M.     | H     | L     | AM    | H     | H     |
| 12 P.M.–2 P.M.      | H     | L     | M     | L     | AM    |
| 2 P.M.–4 P.M.       | M     | L     | H     | H     | H     |
| 4 P.M.–6 P.M.       | H     | AL    | BM    | AM    | BH    |
| 6 P.M.–8 P.M.       | M     | AL    | L     | M     | L     |
| 8 P.M.–10 P.M.      | ABL   | AL    | AL    | L     | AL    |

Table 5. Table that shows degree of relationship between time intervals and criterion for Zone-3.

| Time interval       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---------------------|-------|-------|-------|-------|-------|
| 8 A.M.–10 A.M.      | M     | L     | H     | VH    | VH    |
| 10 A.M.–12 P.M.     | H     | ABL   | BH    | H     | VH    |
| 12 P.M.–2 P.M.      | H     | ABL   | BH    | H     | H     |
| 2 P.M.–4 P.M.       | H     | ABL   | VH    | VH    | H     |
| 4 P.M.–6 P.M.       | VH    | ABL   | AM    | BH    | BM    |
| 6 P.M.–8 P.M.       | H     | L     | L     | AM    | L     |
| 8 P.M.–10 P.M.      | M     | AL    | L     | BM    | L     |

Table 6. Table that shows degree of relationship between time intervals and criterion for Zone-4.

| Time interval       | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|---------------------|-------|-------|-------|-------|-------|
| 8 A.M.–10 A.M.      | L     | VL    | VH    | VH    | L     |
| 10 A.M.–12 P.M.     | M     | VL    | M     | H     | L     |
| 12 P.M.–2 P.M.      | M     | VL    | H     | H     | L     |
| 2 P.M.–4 P.M.       | M     | VL    | VH    | H     | L     |
| 4 P.M.–6 P.M.       | AM    | VL    | M     | BH    | AL    |
| 6 P.M.–8 P.M.       | M     | VL    | L     | M     | AL    |
| 8 P.M.–10 P.M.      | L     | VL    | L     | M     | AL    |
relationship between the criteria and any zone during a certain time interval.

Then the degree of relationship between the criterion and each interval of time taken under consideration during a time interval can be given by the following Tables 3–7.

Now, ranking is done with the help of aggregated generalized trapezoidal fuzzy numbers by the rule $V_{pr}(A) = \frac{a + d + 2(b + c)}{6}$ where $(a, b, c, d; w)$ generalized trapezoidal fuzzy numbers are.

Thus, from the ranking it can be seen that the zone-3 (includes most of the hotels, some branches of national banks, Fish Market, 10–12 number of school-colleges, Police stations, Major road junctions and Administrative offices) should get the priority for surveillance or extra duty deployment of polices throughout the given time. Then, zone-2 (includes The Dibrugarh University, Some branches of national banks, a railway colony, The Dibrugarh Municipality Board, and few temples) becomes the 2nd most important zone for surveillance and to be keeping under check during the time interval 8 A.M.–10 A.M., 10 A.M.–12 P.M., and 2 P.M.–4 P.M. and 4 P.M.–6 P.M. But, zone-5 (includes Assam Medical College, some

**Table 7.** Table that shows degree of relationship between time intervals and criterion for Zone-5.

| Time interval | C1       | C2       | C3       | C4       | C5       | Aggregation |
|---------------|----------|----------|----------|----------|----------|-------------|
| 8 A.M.–10 A.M.| 0.3608   | 0.5332   | 0.5676   | 0.414    | 0.432    |             |
| 10 A.M.–12 P.M.| 0.4966   | 0.3772   | 0.5684   | 0.4728   | 0.4368   |             |
| 12 P.M.–2 P.M. | 0.416    | 0.5332   | 0.6512   | 0.43     | 0.4056   |             |
| 2 P.M.–4 P.M.  | 0.3864   | 0.4264   | 0.4524   | 0.328    | 0.3772   |             |
| 4 P.M.–6 P.M.  | 0.1281   | 0.252    | 0.344    | 0.2352   | 0.3116   |             |
| 6 P.M.–8 P.M.  | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 8 P.M.–10 P.M. | 0.5332   | 0.5676   | 0.414    | 0.432    |          |             |

**Table 8.** Table for degree of relationship between the criterion and time interval and aggregated degrees in the zone-1.

| Time interval | C1       | C2       | C3       | C4       | C5       | Aggregation |
|---------------|----------|----------|----------|----------|----------|-------------|
| 8 A.M.–10 A.M.| 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 10 A.M.–12 P.M.| 0.416    | 0.5332   | 0.6512   | 0.43     | 0.4056   |             |
| 12 P.M.–2 P.M. | 0.3864   | 0.4264   | 0.4524   | 0.328    | 0.3772   |             |
| 2 P.M.–4 P.M.  | 0.1281   | 0.252    | 0.344    | 0.2352   | 0.3116   |             |
| 4 P.M.–6 P.M.  | 0.5332   | 0.5676   | 0.414    | 0.432    |          |             |
| 6 P.M.–8 P.M.  | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 8 P.M.–10 P.M. | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |

**Table 9.** Table for degree of relationship between the criterion and time interval and aggregated degrees in the zone-2.

| Time interval | C1       | C2       | C3       | C4       | C5       | Aggregation |
|---------------|----------|----------|----------|----------|----------|-------------|
| 8 A.M.–10 A.M.| 0.416    | 0.5332   | 0.6512   | 0.43     | 0.4056   |             |
| 10 A.M.–12 P.M.| 0.3864   | 0.4264   | 0.4524   | 0.328    | 0.3772   |             |
| 12 P.M.–2 P.M. | 0.1281   | 0.252    | 0.344    | 0.2352   | 0.3116   |             |
| 2 P.M.–4 P.M.  | 0.5332   | 0.5676   | 0.414    | 0.432    |          |             |
| 4 P.M.–6 P.M.  | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 6 P.M.–8 P.M.  | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 8 P.M.–10 P.M. | 0.5332   | 0.5676   | 0.414    | 0.432    |          |             |

**Table 10.** Table for degree of relationship between the criterion and time interval and aggregated degrees in the zone-3.

| Time interval | C1       | C2       | C3       | C4       | C5       | Aggregation |
|---------------|----------|----------|----------|----------|----------|-------------|
| 8 A.M.–10 A.M.| 0.416    | 0.5332   | 0.6512   | 0.43     | 0.4056   |             |
| 10 A.M.–12 P.M.| 0.3864   | 0.4264   | 0.4524   | 0.328    | 0.3772   |             |
| 12 P.M.–2 P.M. | 0.1281   | 0.252    | 0.344    | 0.2352   | 0.3116   |             |
| 2 P.M.–4 P.M.  | 0.5332   | 0.5676   | 0.414    | 0.432    |          |             |
| 4 P.M.–6 P.M.  | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 6 P.M.–8 P.M.  | 0.1344   | 0.1504   | 0.2352   | 0.1936   | 0.2064   |             |
| 8 P.M.–10 P.M. | 0.5332   | 0.5676   | 0.414    | 0.432    |          |             |
private hospitals, pharmacies, diagnosis centres, medical laboratories, lodges, ATMs, a large market, some road junction’s cause heavy traffic and a bus stoppage) becomes the 2nd most important zone for surveillance and to be keeping under check during the time interval 12 P.M.–2 P.M., 6 P.M.–8 P.M. and 8 P.M.–10 P.M.. Similarly, 3rd, 4th, and 5th priorities can be obtained during the given intervals of time.

11. Discussion and conclusion

In this paper, generalized triangular fuzzy numbers are used in multi-criteria decision-making methodology to rank the areas of a city that are under potential threat of being targeted attacks during any time of the day.

A new aggregation operator is defined on generalized triangular fuzzy number which eliminates the drawback of some earlier aggregation operators explained in section 7.2. The new aggregation operator is used in the proposed methodology to aggregate the triangular fuzzy numbers. Finally, a case study on “Dibrugarh (India)” city is carried out to show the utility of the methodology.

One important thing in this study is that the methodology purely based on the factors that can attract terror attack on a regular office working day only. Events like holidays when traffic becomes less during office hours and school-colleges are closed or during festivals when people are gathered at some events are not considered in this study. In this type of situations, the methodology cannot be applied properly. Therefore, future research may aim to develop the methodology using the different factors that cause crowd suddenly or in a non-regular day e.g.: festivals, sudden strikes, protests, traffic jams by accidents etc. Also, rather than taking the minimum of heights, which will not logical in dealing with uncertainties during aggregation, new and more methods of aggregation on triangular fuzzy numbers should be developed which can consider efficient consideration of different heights of different triangular fuzzy numbers.

Disclosure statement

No potential conflict of interest was reported by the authors.

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