Using a fluid dynamical scenario for pp scattering at 7 TeV, we compute correlation functions for \( \pi^+ \pi^- \) pairs. Femtoscopic radii are extracted based on three-dimensional parametrizations of the correlation functions. We study the radii as a function of the transverse momenta of the pairs, for different multiplicity classes, corresponding to recent experimental results from ALICE. We find the same decrease of the radii with \( k_T \), more and more pronounced with increasing multiplicity, but absent for the lowest multiplicities. In the model we understand this as transition from string expansion (low multiplicity) towards a three-dimensional hydrodynamical expansion (high multiplicity).

I. INTRODUCTION

Bose-Einstein correlations have proven to be a very useful tool to provide space-time information about colliding systems at relativistic energies. Different kinds of reactions have been considered, elementary ones like electron-positron annihilation \([1]\), or more complex systems like proton-proton and nucleus-nucleus collisions \([2]\). Sophisticated methods have been developed in particular for heavy ion collisions at 200 GeV, to interpret the dependence of the two particle correlation function \( C \) on the relative pair momentum \( \mathbf{q} \). A simple way to summarize the results amounts to parametrize the correlation function as

\[
C(\mathbf{q}) = 1 + \lambda \exp \left( -R_{\text{out}}^2 q_{\text{out}}^2 - R_{\text{side}}^2 q_{\text{side}}^2 - R_{\text{long}}^2 q_{\text{long}}^2 \right),
\]

where "long" refers to the beam direction, "out" is parallel to the projection of the pair momentum \( \mathbf{P} \) perpendicular to the beam, and "side" is the direction orthogonal to "long" and "out" \([3,4]\). The fit parameters \( R_{\text{out}}, R_{\text{side}}, \) and \( R_{\text{long}} \) will be referred to as "femtoscopic radii" in the following. In this way, one can study for example the dependence of these radii on the centrality of the reaction, which is interesting because for central collisions we expect higher energy densities and finally more collective flow compared to peripheral collisions. A very general feature in heavy ion collisions seems to be the fact that all the radii increase with centrality. It is also found that for a given centrality all radii decrease substantially with increasing transverse momentum \( k_T = |\mathbf{P}_T|/2 \). This is compatible with a scenario of collective flow with a strong space-momentum correlation, as we are going to discuss later.

Recently first results have been shown concerning the \( k_T \) dependence of the femtoscopic radii, for different multiplicity classes \([5]\). The amazing result: with increasing multiplicity, one observes a more and more visible decrease of the radii with \( k_T \), as in heavy ions. And the radii increase with multiplicity, similar to the increase of the radii with centrality in heavy ion collisions. So do we have the same dynamics which governs proton-proton and heavy ion collisions? Is there a collective expansion driven by hydrodynamics in pp scattering as well?

To contribute to the answer to these questions, we will postulate a scenario of hydrodynamic evolutions based on flux-tube initial conditions, and compute correlation function, make the same fitting procedures as in the experiment, and analyze the \( k_T \) dependence of the femtoscopic radii.

II. HYDRODYNAMIC SCENARIO

Originally hydrodynamics was only thought to present a valid description for almost central collisions of heavy nuclei, where the volume is (relatively) big. But it seems that this approach works very well for all centralities. There is also no fundamental difference seen between CuCu and AuAu, although the copper system is much smaller. So it seems that systems much smaller than central AuAu fit well into this fluid picture. Finally it is more and more accepted that the famous ridge structure observed in angle-rapidity dihadron correlation \([2]\) is due to fluctuating initial conditions, which are subsequently transformed into collective flow \([5]\). Here, the relevant scale for applying hydrodynamics is not the nuclear size, but the size of the fluctuations, which is typically 1-2 fm.

Hydrodynamics is derived from a gradient expansion. In order to justify a hydrodynamical treatment one has to relate the length scale with the viscosity \( \eta \). There are many estimates of numerical values for the \( \eta/S \) (\( S \) being the entropy density), however, all are based on unproven model assumptions. As shown in \([5]\), the variation of the ratio of elliptical flow to eccentricity can be perfectly explained based on ideal hydrodynamics, whereas other authors extract a non-vanishing viscosity from the same observable. It should also be mentioned that in \([5]\), the ideal hydrodynamical partonic phase is followed by
a highly viscous hadronic one, so in the average the system is viscous. By talking about ideal hydrodynamics, we mean an ideal hydrodynamical partonic phase, and as shown in [8], at least all RHIC AuAu data on soft physics are best described with viscosity zero. From a theoretical point of view, the viscosity of a QGP is unknown, and even a lower limit for $\eta/S$ as $1/4\pi$ is not a mathematical limit, but rather an estimate with unknown error compared to the QCD value.

Is QGP formation a nuclear phenomenon? Or can it be formed in pp scattering, as already proposed in [9–12, 18, 19]? Based on the above discussion, there is no reason not to treat proton-proton scattering in the same way as heavy ions, namely incorporating a hydrodynamical evolution. This approach makes clear predictions for many variables, so the Nature will tell us whether the approach is justified or not. Therefore it will be extremely interesting to think about the implications of such a mini QGP, how such a small system can equilibrate so quickly, and so on. It would be an enormous waste of opportunities, not to consider this possibility, since a vast amount of proton-proton data will be available very soon, concerning all kinds of observables.

What makes pp scattering at LHC energies interesting in this respect, is the fact that at this high energy multiple scattering becomes very important, where a large number of scatterings amounts to a large multiplicity. In such cases, very large energy densities occur, even bigger than the values obtained in heavy ion collisions at RHIC – but in a smaller volume. Several authors discussed already the possibility of a hydrodynamical phase in pp collisions at the LHC, to explain the ridge correlation or to predict elliptical flow [13–18].

We are going to employ a sophisticated hydrodynamical scenario, first presented in ref. [8] where many details can be found, with the following main features:

- initial conditions obtained from a flux tube approach (EPOS), compatible with the string model used since many years for elementary collisions (electron-positron, proton proton), and the color glass condensate picture [20];
- event-by-event procedure, taking into the account the highly irregular space structure of single events, being experimentally visible via so-called ridge structures in two-particle correlations;
- core-corona separation, considering the fact that only a part of the matter thermalizes [21]; only in the core region, the energy density from the strings is considered for the hydrodynamical evolution;
- use of an efficient code for solving the hydrodynamic equations in 3+1 dimensions, including the conservation of baryon number, strangeness, and electric charge;
- employment of a realistic equation-of-state, compatible with lattice gauge results – with a cross-over transition from the hadronic to the plasma phase [22, 23];
- use of a complete hadron resonance table, making our calculations compatible with the results from statistical models;
- hadronic cascade procedure after hadronization from the thermal system at an early stage [24, 25].

In ref. [8], we test the approach by investigating all soft observables of heavy ion physics, in case of AuAu scattering at 200 GeV. In refs. [18, 19] we investigate first proton-proton results, with among other things “the ridge”.

### III. RESULTS

In the following, we will consider several multiplicity classes, named $\text{mult} \, 1$, $\text{mult} \, 4$, $\text{mult} \, 7$ and $\text{mult} \, 8$, corresponding to four out of the eight multiplicity classes used in ref. [6], going from low multiplicity ($\text{mult} \, 1$, less than minimum bias) to high multiplicity ($\text{mult} \, 8$, five time minimum bias). Our core-corona procedure will find no core for $\text{mult} \, 1$, then increasing core fraction, and for $\text{mult} \, 4$ to $\text{mult} \, 8$ essentially core, with increasing energy densities. So the $\text{mult} \, 1$ events are just ordinary strings which expand longitudinally (see fig. 1(a)), whereas $\text{mult} \, 4$ to $\text{mult} \, 8$ show a hydrodynamical expansion, also in transverse direction (see fig. 1(b)). In fig. 2 we show the evolution of the energy density for the different multiplicity classes. Obviously the energy density increases with multiplicity, values of more than 100 GeV/fm$^3$ are achieved.
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Figure 2: (Color online) Evolution of the energy density (average over many events) for the three multiplicity classes Milt 4, Milt 7 and Milt 8. The full red lines are the initial conditions, the other curves are respectively 1 and 2 fm/c later.

We consider two options for the equation of state, one being a parametrization of the results of [22], the other one referring to [23]. There is a big difference between the two, the transition temperature is much lower and the transition is much smoother in [22], compared to [23]. The corresponding “freeze out” radii (where the transition hydro / cascade is done) are very different, see fig. 3. The final results will differ less, because the early freeze out for the EoS from [22] is followed by an intense hadronic rescattering.

Based on roughly ten million simulations of the hydrodynamical evolution, we compute in the usual way the correlation functions for $\pi^+\pi^+$ pairs, taking into account Bose-Einstein statistics, as discussed in [8, 19, 26]. Whereas in the data Pythias has to be used as “baseline”, we stay consistently within our scenario and use simply a calculation without Bose-Einstein statistics as baseline. We then fit the correlation functions to obtain the radii.

Before showing the results, let us discuss in a qualitative way why we expect a decrease of the radii with $k_T$. Let us consider the freeze out curves of fig. 3. Most particle production occurs in the region where the radii drop to zero. Comparing the two points “1” and “2” in the figure, we have to recall that the collective flow at a large radius (1) is much bigger compared to small radius (2). In fig. 4 we sketch the corresponding momentum vectors of pairs, emitted at large and small radii (which finally amounts to large and small $k_T$), where the vectors are such that their difference is the same. The space distances in case 2 are then bigger than the ones in case 1 (and these distances are essentially the femtoscopic radii).

Let us discuss our main result: As seen in fig. 5 all radii indeed show a more and more pronounced decrease with increasing $k_T$, for data and simulations, which can – in the calculations – clearly be attributed to collective flow. For the case Milt 1 the radii $R_{\text{out}}$ and $R_{\text{side}}$ are essentially flat, only $R_{\text{long}}$ has already some $k_T$ dependence. So we see here nicely the transition from a longitudinal expansion (string) towards a three-dimensional hydrodynamical expansion for higher multiplicities.

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Figure 5: (Color online) Femtoscopic radii for three different multiplicity classes, using an EoS compatible with ref. [22] (solid curves) or compatible with ref. [23] (dotted curves). The curves are the same for Mult 1 because there is no fluid phase. The points are data. The curves are absolute predictions, the parameters of the model are obtained from other comparisons (yields, \(p_T\) spectra).

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