Anisotropic cosmological reconstruction in $f(R,T)$ gravity

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Anisotropic cosmological models are constructed in $f(R,T)$ gravity theory to investigate the dynamics of universe concerning the late time cosmic acceleration. Using a more general and simple approach, the effect of the coupling constant and anisotropy on the cosmic dynamics have been investigated. Cosmic anisotropy is found aect substantially the cosmic dynamics.

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I. INTRODUCTION

Cosmological models either incorporation of possible dark energy candidates or of geometrical modification of action are constructed in recent times to account for the predicted late time cosmic acceleration. Amidst the debate that, whether dark energy exists or whether there really occurs a substantial cosmic acceleration [1–3], researchers have devoted a lot of time in proposing different dark energy models. These models are also tested against the observational data accumulated over a long period of time. Among all these constructed models, modified gravity theories have attracted a lot of research attention. In this context, Harko et al. have proposed the geometry part of the action has been modified in such a manner that, the usual Ricci Scalar $R$ replaced by a function $f(R)$ of Ricci Scalar $R$ and the trace of the energy momentum tensor $T$. In that work, Harko et al. have suggested three different possible forms of the functional $f(R)$ such as $f(R) = R + 2f(T)$, $f(R,T) = f_1(R) + f_2(T)$ and $f(R,T) = g_1(R) + g_2(R)g_3(T)$, where $f(T), f_1(R), f_2(T), g_1(R), g_2(R)$ and $g_3(T)$ are some arbitrary plausible functions or $R$ and $T$. Many workers have used different forms of these functionals to address the issue of mysterious dark energy and the late time cosmic phenomena [5–16].

With the advent of recent observations regarding the cosmic anisotropy, there has an increase in the belief of the breakdown of the standard cosmology based on cosmic isotropy. In view of this, anisotropic cosmological models that bear a similarity to Bianchi morphology has gained importance. In the context of geometry modification to explain the late time cosmic dynamics and to take into account the cosmic anisotropy, many workers have constructed some Bianchi type cosmological models in $f(R,T)$ gravity [17–20]. However, a lot remain to be explored in this modified gravity theory in the context of different unanswered issues concerning the late time cosmic acceleration and cosmic anisotropy.

In this work, we have constructed some anisotropic cosmological models in $f(R,T)$ gravity. We have adopted a simple approach to the cosmic anisotropy to investigate the effect of anisotropy on cosmic anisotropy. In order to provide some anisotropic directional pressure, we have considered an anisotropic source along x-direction such as the presence of one dimensional cosmic strings. The effect of the coupling constant in determination of the cosmic evolution has been investigated. We organise the work as follows: In Sect-2, some basic equations concerning different properties of the universe are derived for Bianchi $VI_h$ model in the framework of the modified $f(R,T)$ gravity. The dynamical features of the models are discussed in Sect-3. Considering the dominance of quark matter that have not yielded to the hadronization process, we have derived the quark energy density and pressure and their evolutionary behaviour in Sect-4. We conclude in Sect-5.

II. BASIC EQUATIONS

The field equation in $f(R,T)$ gravity for the choice of the functional $f(R,T) = f(R) + f(T)$ is given by [3] [21]

$$f_R(R)R_{ij} - \frac{1}{2} f(R)g_{ij} - (\nabla_i \nabla_j - g_{ij} \Box) f_R(R) = [8\pi + f_T(T)]T_{ij} + \left[ f_T(T)p + \frac{1}{2} f(T) \right] g_{ij}$$  \hspace{1cm} (1)
where \( f_R = \frac{\partial f(R, T)}{\partial R} \) and \( f_T = \frac{\partial f(R, T)}{\partial T} \). We wish to consider a functional form of \( f(R, T) \) in such a way that it can be reduced to the usual field equations in General Relativity (GR). A popular choice is \( f(R, T) = R + 2\Lambda_0 + 2\beta T \). \( \Lambda_0 \) is the cosmological constant and \( \beta \) is coupling constant. The field equation in the modified theory of gravity becomes,

\[
R_{ij} - \frac{1}{2} R g_{ij} = [8\pi + 2\beta] T_{ij} + [(2p + T) \beta + \Lambda_0] g_{ij}
\]

which can also be written as

\[
R_{ij} - \frac{1}{2} R g_{ij} = [8\pi + 2\beta] T_{ij} + \Lambda(T) g_{ij}.
\]

Here \( \Lambda(T) = (2p + T) \beta + \Lambda_0 \) can be identified as the effective time dependent cosmological constant. If \( \beta = 0 \), the above modified field equation reduces to the Einstein field equation in GR with a cosmological constant. One can note that, the effective cosmological constant \( \Lambda(T) \) picks up its time dependence through the matter field. For a given matter field described through an energy momentum tensor, the effective cosmological constant can be expressed in terms of the matter components. In the present work, we consider the energy momentum tensor as the metric \( g_{ij} = \text{constant} \). The direction of the cosmic strings is represented through \( x^i \) that are orthogonal to \( u^i \).

The field equations (3) of the modified \( f(R, T) \) gravity theory, for Bianchi type \( I_h \) space-time described through the metric \( ds^2 = dt^2 - A^2 dx^2 - B^2 e^{2\beta} dy^2 - C^2 e^{2\alpha} dz^2 \) now have the explicit forms

\[
\frac{\dot{B}}{B} + \frac{\dot{C}}{C} + \frac{\dot{B}C}{BC} - \frac{h}{A^2} = -\alpha(p - \xi) + \rho \beta + \Lambda_0
\]

\[
\frac{\ddot{A}}{A} + \frac{\dot{C}}{AC} - \frac{h^2}{A^2} = -\alpha p + (\rho + \xi) \beta + \Lambda_0
\]

\[
\frac{\ddot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{A}B}{AB} - \frac{1}{A^2} = -\alpha p + (\rho + \xi) \beta + \Lambda_0
\]

\[
\frac{\dot{A}B}{AB} + \frac{\dot{B}C}{BC} + \frac{\dot{C}A}{CA} - \frac{1 + h + h^2}{A^2} = \alpha \rho - (p - \xi) \beta + \Lambda_0
\]

\[
\frac{\dot{B}}{B} + \frac{\dot{C}}{C} - (1 + h) \frac{\dot{A}}{A} = 0.
\]

An over dot over a field variable denotes ordinary differentiation with respect to the cosmic time. Here \( \alpha = 8\pi + 3\beta \) and \( A = A(t), B = B(t), C = C(t) \). A peculiar part in this space time is the constant exponent \( h \), which takes integral values \(-1, 0, 1\). These three integral values decide the behaviour of the model. However, Tripathy et al. [22] and Mishra et al. [23] have shown from the calculation of the energy and momentum of diagonal Bianchi type universes that, the value \( h = -1 \) is favoured compared to other values. Moreover, only in this value of the exponent \( h \), the total energy of an isolated universe vanishes. In view of this, in the present work, we have considered the exponent to be \( h = -1 \). In view of this, in the present work, we assume this value of \( h \) and study the dynamics of the anisotropic universe in presence of anisotropic energy sources. The directional Hubble rates may be considered as \( H_x = \frac{\dot{A}}{A}, \ H_u = \frac{\dot{B}}{B} \) and \( H_z = \frac{\dot{C}}{C} \). With \( h = -1 \), it is straightforward to get \( H_y = H_z \) from (8). The mean Hubble parameter becomes, \( H = \frac{1}{3}(H_x + 2H_z) \). The set of field equations can be reduced to

\[
2\dot{H}_z + 3H_z^2 + \frac{1}{A^2} = -\alpha(p - \xi) + \rho \beta + \Lambda_0,
\]

\[
\dot{H}_x + \dot{H}_z + H_x^2 + H_z^2 + H_x H_z - \frac{1}{A^2} = -\alpha p + (\rho + \xi) \beta + \Lambda_0,
\]

\[
2H_x H_z + H_z^2 - \frac{1}{A^2} = \alpha \rho - (p - \xi) \beta + \Lambda_0.
\]

Adopting an approach similar to Ref. [21] we obtain the expressions for pressure, energy density and the string tension density as
respectively as cosmic evolution. This behaviour of ξ also decreases from a large value in the initial epoch to small values at late phase of cosmic evolution. It is interesting to note that, also we assume that θ also decreases from a large value in the initial epoch to small values at late phase of cosmic evolution. Consequently, the equation of state parameter ω and the effective cosmological constant Λ can be expressed as

\[ \omega = -1 + \frac{\beta}{\alpha^2 - \beta^2} \frac{s_2 - s_3}{s_1\beta - (\alpha - \beta)\Lambda}, \]

\[ \Lambda = \frac{\beta}{\alpha^2 - \beta^2} [s_1s_2\alpha - (2s_2 - s_3)\beta - (\alpha + \beta)(s_2 - s_1) - 2(\alpha - \beta)\Lambda] + \Lambda_0. \]

In the above equations, s₁, s₂ and s₃ are functions of the directional Hubble parameters and scale factor: s₁ = 2Hₓ + 3Hₙ + \frac{1}{t^{\frac{3}{2}}}, s₂ = Hₓ + Hₙ + H₂ + HₙHₚ - \frac{1}{t} and 2HₓHₙ + H₂ - \frac{1}{t^2}. Eqns (12)-(16) describe the dynamical behaviour of the model. Once the evolutionary behaviour of the functions s₁, s₂ and s₃ are redefined constants. These physical quantities evolve with the cosmic expansion. Their evolution is governed by two time dependent factors: one behaving like t⁻² and the other behaving as t⁻\frac{3}{2}. Consequently, the equation of state parameter ω can be expressed as A = t^{km/(k+2)}, B = C = t^{m/(k+2)}. For such an assumption, the functions s₁, s₂ and s₃ reduce to

\[ s_1 = \left[ \frac{3m^2 - 2(k + 2)m}{(k + 2)^2} \right] \frac{1}{t^2} + \frac{1}{t^{\frac{3}{2}}} \]

\[ s_2 = \left[ \frac{(k^2 + k + 1)m^2 - (k + 1)(k + 2)m}{(k + 2)^2} \right] \frac{1}{t^2} - \frac{1}{t^{\frac{3}{2}}} \]

\[ s_3 = \left[ \frac{(2k + 1)m^2}{(k + 2)^2} \right] \frac{1}{t^2} - \frac{1}{t^{\frac{3}{2}}}. \]

From the field eqns. (9)-(11), the pressure, energy density and string tension density can be obtained as:

\[ p = \frac{1}{\alpha^2 - \beta^2} \left[ \left( \frac{\phi_1}{(k + 2)^2} \right) \frac{1}{t^2} + \frac{(\alpha + \beta)}{t^{3/(k+2)}} + (\alpha - \beta)\Lambda \right], \]

\[ \rho = \frac{1}{\alpha^2 - \beta^2} \left[ \left( \frac{\phi_2}{(k + 2)^2} \right) \frac{1}{t^2} + \frac{(\beta - \alpha)}{t^{3/(k+2)}} - (\alpha - \beta)\Lambda \right], \]

\[ \xi = \frac{1}{\alpha^2 - \beta^2} \left[ \frac{(k - 1)(m^2 - m)}{(k + 2)^2t^2} - \frac{2}{t^{3/(k+2)}} \right], \]

where \( \phi_1 = m \{(k^2 + k - 2)\beta + (k^2 + 3k + 2)\alpha \} - m^2 \{(k^2 - k - 3)\beta - (k^2 + k + 1)\alpha \} \) and \( \phi_2 = (2k + 1)m^2\alpha - (3m^2 - 2km - 4m)\beta \) are redefined constants. These physical quantities evolve with the cosmic expansion. Their evolution is governed by two time dependent factors: one behaving like t⁻² and the other behaving as t⁻\frac{3}{2}. Since m and k are positive quantities, the magnitude of the physical quantities (neglecting their sign) decrease monotonically with cosmic time. It is interesting to note that, \( \xi \) also decreases from a large value in the initial epoch to small values at late phase of cosmic evolution. This behaviour of \( \xi \) implies that, at the initial phase, more anisotropic components are required than at late phase.

From eqs. (12)-(29), we obtain the equation of state parameter \( \omega = \frac{p}{\rho} \) and the effective cosmological constant \( \Lambda \) respectively as
\[\omega = -1 + (\alpha + \beta) \left[ \frac{\phi_3}{\phi_4 + (\alpha - \beta)(k + 2)^2 \left\{ \Lambda_0 t^2 - t^2 \left( \frac{4 - km + 2}{k + 2} \right) \right\} } \right], \tag{23}\]

\[\Lambda = \frac{\beta}{(\alpha^2 - \beta^2)} \left[ \frac{\phi_5}{(k + 2)^2 t^2} - \frac{2(\alpha + \beta)}{t^{\frac{4-3k}{k+2}}} - 2(\alpha - \beta)\Lambda_0 \right] - \frac{\phi_6}{(k + 2)^2 t^2} + \frac{\beta}{(\alpha - \beta)t^{\frac{4-3k}{k+2}}} + \Lambda_0, \tag{24}\]

where \(\phi_3 = (k^2 - 2k)m^2 - (k^2 + 2k + 3)m\), \(\phi_4 = (3m^2 - 2km - 4)\beta - (2k + 1)m^2\alpha\), \(\phi_5 = \{(k + 1)\alpha + (k - 3)\beta\}(m^2 - m)\) and \(\phi_6 = \frac{\beta (k - 1)(m^2 - m)}{(\alpha - \beta)}\) are some constants.

The dynamical nature of the model can be assessed through the evolution of the equation of state parameter \(\omega\). In Figure 1, \(\omega\) is plotted as function of redshift for four different values of the coupling constant \(\beta\) namely \(\beta = 0, 0.5, 1.0\) and 2.0. \(\beta = 0\) refers to the case in GR. The anisotropic parameter is considered to be \(k = 0.7\) and \(m\) is fixed from the observationally constrained value of deceleration parameter i.e. \(q = -0.598\). For all the cases considered here, \(\omega\) becomes a negative quantity and remains in the quintessence region through out the period of evolution considered in the work. It decreases from some higher value at the beginning to low values at late time. However, at late phase of cosmic evolution, \(\omega\) grows up a little bit.

The coupling constant \(\beta\) affects the dynamical behaviour of the equation of state parameter. In order to understand the affect of the \(\beta\) on \(\omega\), the equation of state at the present epoch is plotted as a function of \(\beta\) in Figure 2 for three different anisotropy. One can note that, \(\omega\) increases with the increase in the value of the coupling constant. In view of the recent observations predicting an accelerating universe, the value of coupling constant \(\beta\) should have a lower value close to 1.

In Figure 3, we have shown the effect of anisotropy on the equation of state parameter. In the figure, we assume three representative values of the anisotropy i.e \(k = 0.7, 0.8\) and 0.9 for a given coupling constant \(\beta = 0.5\). Anisotropy brings a substantial change in the magnitude as well as the behaviour of the equation of state parameter. There occurs a flipping behaviour of \(\omega\) at a redshift \(z_f \simeq 4\). At a cosmic time earlier to \(z_f\), with the increase in the anisotropy of the model, \(\omega\) assumes a higher value. In other words, prior to \(z_f\), higher the value of \(k\), higher is the \(\omega\). The opposite behaviour it displays at cosmic time later to \(z_f\). Also, rate of evolution of the equation of state parameter increases with the increase in the value of the anisotropic parameter. Also, at the redshift \(z_f\), curves corresponding to all \(k\) considered here cross each other.
IV. ANISOTROPIC UNIVERSE WITH QUARK MATTER

One can believe that, quarks and gluons did not yield to hadronization and resisted as a perfect fluid that spread over the universe and may contribute to the accelerated expansion. Here we will reconstruct an anisotropic cosmological model with non interacting quarks that may well be dealt as a Fermi gas with an equation of state given by

\[ p_q = \frac{\rho_q}{3} - B_c, \]  

where \( p_q \) is the quark pressure, \( \rho_q \) is the quark energy density and \( B_c \) is the bag constant. We assume that quarks exist along with one dimensional cosmic strings without any interaction. The quark energy density can then be expressed as \( \rho_q = \rho - \xi - B_c \). Going in the same manner as described in the previous section, we can have the expression for the quark pressure and quark energy density as

\[ \rho_q = \frac{1}{\alpha^2 - \beta^2} \left[ (\alpha + \beta)s_2 + s_3\alpha - (\alpha + 2\beta)s_1 - (\alpha - \beta)\Lambda_0 \right] - B_c, \]  

\[ p_q = \frac{1}{3(\alpha^2 - \beta^2)} \left[ (\alpha + \beta)s_2 + s_3\alpha - (\alpha + 2\beta)s_1 - (\alpha - \beta)\Lambda_0 \right] - \frac{B_c}{3} \]

If we put \( \beta = 0 \), the model reduces to that in GR with a cosmological constant. In that case, the above equations reduce to

\[ \rho_q = \frac{1}{8\pi} \left[ s_2 + s_3 - s_1 - \Lambda_0 \right] - B_c, \]  

\[ p_q = \frac{1}{24\pi} \left[ s_2 + s_3 - s_1 - \Lambda_0 \right] - \frac{B_c}{3} \]

The quark matter energy density and quark pressure are obtained as

\[ \rho_q = \frac{1}{\alpha^2 - \beta^2} \left[ \frac{\phi}{(k+2)^2 t^2} + \frac{(\alpha + 3\beta)}{t^{\frac{4}{k+2}}} \right], \]  

\[ p = \frac{1}{3(\alpha^2 - \beta^2)} \left[ \frac{\phi}{(k+2)^2 t^2} + \frac{(\alpha + 3\beta)}{t^{\frac{4}{k+2}}} \right]. \]
where \( \phi_7 = \phi_2 - (k - 1)(m^2 - m)(\alpha + \beta) \). For some reasonable value of the coupling parameter \( \beta \) and the anisotropic parameter \( k \), the quark energy density and quark pressure decrease smoothly with the cosmic evolution. Bag constant certainly has a role to play at late times when the value of \( \rho_q \) and \( p_q \) are mostly dominated by this quantity.

V. CONCLUSION

This paper reports the investigation of the dynamical behaviour of an anisotropic Bianchi type \( VI_b \) universe in the presence of one dimensional cosmic strings and quark matter. Anisotropic cosmological models are reconstructed for a power assumption of the scale factor in the frame work of \( f(R,T) \) gravity. In the process of reconstruction and study of dynamical features of the model, we chose the functional \( f(R,T) \) as \( f(R,T) = R + 2\Lambda_0 + 2\beta T \). From some general expressions of the physical quantities, we derived the expression of the equation of state parameter and the effective cosmological constant. The effects of anisotropy \( k \) and the coupling constant \( \beta \) are investigated. It is observed that, with an increase in the coupling constant the equation of state parameter assumes a higher value. Anisotropy is observed to affect largely to the dynamics of the model. The equation of state parameter undergoes an increased rate of growth with an increase in the anisotropy. we hope, the present study, definitely put some light in the context of the uncertainty prevailing in the studies of the late time cosmic phenomena.

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