Spin squeezing properties of an initial coherent state via a two-axis countertwisting Hamiltonian in the presence and absence of an external field

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Abstract. We consider a four-qubit system, initially in a coherent state along the z-axis, and study its time dependence under the influence of the well-known two-axis countertwisting Hamiltonian. Analyzing the time dependence of the spin operators, we observe its squeezing properties, according to the criterion given by Kitagawa et al. We also perform similar calculations for the same Hamiltonian in the presence of an external field and compare its squeezing properties with that of the former. It is observed that the details of the squeezing properties are different in the two cases. Plots of the squeezing parameters as a function of time and the rotation angle for both cases are presented.

1. Introduction
It is well known that spin squeezing may be achieved via nonlinear interactions [1, 2, 3, 4, 5, 6]. Kitagawa and Ueda have introduced two fundamental types of nonlinear Hamiltonians that lead to spin squeezing; namely, one-axis twisting and two-axis countertwisting Hamiltonians [7]. In this paper we focus on the two-axis countertwisting Hamiltonian

\[ H = \frac{\chi}{2i}(\hat{S}_x^2 - \hat{S}_y^2), \]  

(1)

where, \( \hat{S}_x \) and \( \hat{S}_y \) are the spin ladder operators. We shall also consider the Hamiltonian model

\[ H_f = \frac{\chi}{2i}(\hat{S}_x^2 - \hat{S}_y^2) + \Omega \hat{S}_z, \]  

(2)

where, the second linear term in \( H_f \), may be realized as an additional external field. The main purpose of this paper is to compare the squeezing property of a 4-qubit system, which is dynamically evolved via the Hamiltonians \( H \) and \( H_f \). We shall use the criterion of spin squeezing as has been advanced by Kitagawa and Ueda. They define the parameter \( \xi_K^2 \), as a measure of squeezing,

\[ \xi_K^2 = \frac{2(\Delta S_\perp)^2}{S}, \]  

(3)

where, \( \Delta S_\perp \) is the uncertainty of an angular momentum component perpendicular to the mean angular momentum direction \( \langle \hat{S} \rangle \). A spin state is said to be squeezed according to this criterion if \( \xi_K^2 < 1 \).
2. Squeezing in the absence of the field

Our initial coherent state is a collection of four spin-half systems, with all of them in the upward direction denoted by $|2, 2\rangle_z$ [8].

We calculate the following quantities in our initial state $|2, 2\rangle_z$

$$|\langle \hat{S} \rangle| = \langle \hat{S}_z \rangle = \tilde{z}(2, 2)\hat{S}_z|2, 2\rangle_z = 2,$$

$$\Delta S_x = S = \frac{s}{2} = 1.$$  

(4)

(5)

We note that the uncertainty relation

$$(\Delta S_x)^2(\Delta S_y)^2 \geq \frac{1}{4}|\langle \hat{S}_z \rangle|^2,$$

(6)

is satisfied here with equal sign, as we expect for a coherent state. We also have $\xi_K^2 = 1$; an indication of the lack of squeezing in a coherent state.

Now, it is assumed that the spin system is evolved via the countertwisting Hamiltonian $H$. To find the optimal squeezing direction, we also rotate the reference frame about the $z$-axis by an angle $\varphi$ and thus replace $x$ and $y$ by $\hat{x}$ and $\hat{y}$, respectively [9]. It is trivial to show that

$$\tilde{z}(2, 2)|\hat{S}_z|2, 2\rangle_z = \tilde{z}(2, 2)|\hat{S}_y|2, 2\rangle_z = 0.$$  

(7)

We work in the Schrödinger picture and the evolution is enforced by the unitary transformation operator $U(t) = e^{-iHt}$. Thus, the wave function at time $t = 0$, is evolved in time according to the transformation $\Psi(t) = e^{-iHt}\Psi(0)$. We can also show that

$$\langle \Psi(t)|\hat{S}_z|\Psi(t)\rangle = \langle \Psi(t)|\hat{S}_y|\Psi(t)\rangle = 0.$$  

(8)

Therefore, the average spin direction remains along the $z$-direction and we find

$$|\langle \hat{S} \rangle| = \langle \hat{S}_z \rangle = \langle \Psi(t)|\hat{S}_z|\Psi(t)\rangle = 2\cos(2\sqrt{3}\chi t),$$  

(9)

we also obtain $\xi_K^2$ as follows

$$\xi_K^2 = (\Delta \hat{S}_z)^2 = \frac{3}{2} - \frac{1}{2}\cos(4\sqrt{3}\chi t) + \sqrt{3}\cos(2\varphi)\sin(2\sqrt{3}\chi t);$$  

(10)

it is a function of the rotation angle $\varphi$, time $t$ and the Hamiltonian parameter $\chi$. Minimizing (10) with respect to these variables we find $\chi = -0.3781$, $t = 0.7994$ and $\varphi = 0$, which leads to $(\xi_K^2)_{min} = 0.25$.

We now study the squeezing property of the system along the $\varphi = 0$ direction implemented by $H$ at $\chi = -0.3781$. Substituting the above values for $\chi$ and $\varphi$ in (10), we have plotted $\xi_K^2$ versus $t$ in figure 1a. It is observed that the squeezing condition $\xi_K^2 < 1$ is satisfied during alternate time intervals. Also assuming $\chi = -0.3781$ and $t = 0.7994$ we have plotted $\xi_K^2$ versus $\varphi$ in figure 1b. We observe that the system is squeezed through alternate angle ranges at these conditions.

3. Squeezing in the presence of the field

We now consider the evolution of our initial coherent state via $H_f$. Again the average spin is in the $z$ direction and we may write

$$|\langle \hat{S} \rangle| = \langle \hat{S}_z \rangle = \langle \Psi(t)|\hat{S}_z|\Psi(t)\rangle = -\frac{2[\Omega^2 + 3\chi^2\cosh(2At)]}{A^2},$$  

(11)
Minimizing $\xi_{K,f}^2$ with respect to all its parameters, we find $\chi = 0.3445$, $t = 1.3162$, $\Omega = -1.0335$ and $\varphi = \pi/4$, leading to $\xi_{K,f}^2 = 0.25$.

Inspecting the $\xi_{K,f}^2$ plots in figure 2a, discloses an interesting phenomenon; the addition of the external field has increased the time span of the squeezing, in the deepest squeezing direction, substantially. In fact, barring some separate instances of time, the inequality $\xi_{K,f}^2 < 1$, at the given values of the parameters $\chi$, $\varphi$ and $\Omega$, is always satisfied and the system is always squeezed in the presence of the field.

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