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Distributed State Estimation of a Non-linear process system with interconnected subsystems

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Abstract. In this paper authors propose a distributed state estimation scheme for the hybrid system with interconnected subsystems to estimate the states. The system considered in this work has different subsystems which can interact with each other via their states over a communication network. The objective is to implement the distributed state estimation scheme for interconnected subsystems in which each subsystem sensors are connected to the communication network, the estimator has been used for each subsystem to estimate the current states by utilizing the states of the neighboring subsystems over the communication network. The proposed state estimation framework utilizes unscented Kalman filter algorithm. Unscented Kalman filter has been designed for each subsystem to estimate the current state estimates which corrupted by state and measurement noise. The benchmark system taken to implement distributed state estimation is continuous stirred tank reactor units which are strongly interconnected via their neighboring states. The reactors temperature is maintained at unstable operating point by a decentralized proportional integral controller for each subsystem by utilizing the measurements of the sensors from the network. The estimation framework has been verified in the absence of the network failure to stabilize the plant at unstable operating point. The estimate of the corresponding subsystems is used to compute the controller output in the absence of the network failure with minimal sharing over the network.

Keywords: Distributed state estimation, interconnected systems, network estimation.

1. INTRODUCTION

In [1] the authors have proposed a robust unknown input observer for state estimation and fault detection using linear parameter varying model is proposed. The parameters of the Unknown Input Observer (UIO) are obtained by solving the linear matrix inequalities (LMIs) and linear matrix equalities (LMEs) and also convergence of the UIO is analyzed through Lyapunov theory. The state of a nonlinear dynamical system is estimated through consensus-based networked estimation. It is mainly focus on a family of distributed state estimation algorithms which relies on the extended Kalman filter linearization paradigm. The effectiveness of the nonlinear consensus filter is analyzed with target tracking applications [2]. A mathematical model of distributed state estimation is constructed for nonlinear networked systems against denial-of-service attacks. The feasibility of the distributed state estimation is confirmed and a sufficient condition of the proposed estimation method is tested [3]. An
estimation of a state of discrete nonlinear systems with uncertainties and sensor delays is examined in [4]. A distributed state estimation method is applied for power system applications [5], continuous-time stochastic process [6] and stochastic non-linear systems with multi-step transmission delays [7]. Industrial systems consists of complex systems which contain Distributed Model Predictive Control (DMPC) scheme is emerging as one of the most effective control schemes for the control of interconnected subsystems. The distributed state estimation scheme [8 - 11] has attracted the attention of the researchers and distributed Kalman filter for sensor networks are available in the literature [12, 13]. Recently a series of work, related to DMHE have been proposed by Zhang and Liu [11].

The non-linear dynamic system can be decomposed into ‘m’ interconnected subsystems, where $i^{th}$ subsystem is described by the non-linear state (Eq. 1) and measurement equations (Eq. 2) as shown below:

$$x^{(i)}(k) = x^{(i)}(k-1) + \int_{k-1}^{k} F^{(i)}(\tau, x^{(i)}(\tau), u^{(i)}(k-1), d^{(i)}(k-1)) d\tau + w^{(i)}(k)$$  \hspace{1cm} (1)

$$y^{(i)}(k) = H^{(i)}(x^{(i)}(k)) + v^{(i)}(k)$$  \hspace{1cm} (2)

where $i=1,2,3$, $x^{(i)}(k) \subset x(k)$ denotes the $i^{th}$ subsystem state variables, $u^{(i)} \subset u$ denotes the known subsystem inputs, and $d^{(i)} \subset x(k)$ denotes the neighboring subsystems state variables, where $I_i$ denotes the set of subsystem indices whose state variables are involved in $d^{(i)}$; for example, if $d^{(i)}$ contains state variables of subsystems 2 and 4, then $I_3 = \{2, 4\}$. In this work, it is assumed that subsystems are interconnected through system state variables only and $F^{(i,j)}$ & $H^{(i)}$ are the known $i^{th}$ subsystem state transition function and $i^{th}$ subsystem measurement model respectively. $w^{(i)}(k)$ denotes the random disturbances associated with $i^{th}$ mode of the $i^{th}$ subsystem and $v^{(i)}(k)$ denotes errors due to measurements of the $i^{th}$ subsystem. The vector $y^{(i)} \subset y$ is the measured outputs of the $i^{th}$ subsystem.

2. PROCESS DESCRIPTION

The two interconnected continuous stirred tank reactor (CSTR’s) [14] with recycle has been taken as the benchmark system, to demonstrate the proposed distributed state estimation scheme. The well mixed non-isothermal interconnected CSTR is shown in Figure 1. Three irreversible exothermic chemical reactions has been taking place inside the chemical reactors of the form $A \rightarrow B$, $A \rightarrow U$ and $A \rightarrow R$ takes place, where $A$ is the fresh reactant species, $B$ the desired product, and $U$ and $R$ undesired byproducts as shown in Figure 1. The CSTR1 has two input streams one is having fresh reactant species with flow rate $F_0$, molar concentration $C_{A0}$ and temperature $T_0$, and the second input stream recycled from the CSTR2 with flow rate $F_1$, molar concentration $C_{A2}$ and temperature $T_2$. The CSTR 2 has another input stream having fresh species $A$ with flow rate of $F_3$, molar concentration $C_{A0}$ and temperature $T_0$. The output of the CSTR 2 is recycled to CSTR1. Both the reactors are provided with a outer jacket to remove or to provide heat to the reactors as the chemical reaction is exothermic. The mass and energy balance governing the reactors is given below. The dynamics of the reactors has been treated as two separate subsystems.
Figure 1. Process flow diagram of interacting CSTR

**Subsystem – 1**

The dynamics of the reactor 1 is given by the mathematical model given below

\[
\frac{dT_1}{dt} = \frac{F_1}{V_1}(T_0 - T_1) + \frac{F_1}{V_1}(T_2 - T_1) + \sum_{i=1}^{3} G_1(T_i)C_{A1} + \frac{Q_1}{\rho c_p V_1}
\]

(3)

\[
\frac{dC_{A1}}{dt} = \frac{F_1}{V_1}(C_{A0} - C_{A1}) + \frac{F_1}{V_1}(C_{A2} - C_{A1}) - \sum_{i=1}^{3} R_i(T_i)C_{A1}
\]

(4)

**Subsystem – 2**

The dynamics of the reactor 2 is given by the mathematical model given below

\[
\frac{dT_2}{dt} = \frac{F_2}{V_2}(T_1 - T_2) + \frac{F_2}{V_2}(T_3 - T_2) + \sum_{i=1}^{3} G_2(T_2)C_{A2} + \frac{Q_2}{\rho c_p V_2}
\]

(5)

\[
\frac{dC_{A2}}{dt} = \frac{F_2}{V_2}(C_{A1} - C_{A2}) + \frac{F_2}{V_2}(C_{A03} - C_{A2}) - \sum_{i=1}^{3} R_i(T_2)C_{A2}
\]

(6)

Where, \( R_i(T_j) = k_{j0} \exp\left(-\frac{E_{j}}{RT_{j}}\right), G_i(T_j) = \left(-\frac{\Delta H_i}{\rho c_p}\right)R_i(T_j), j = 1, 2. \)

\( T_j \) temperature of the \( j^{th} \) reactor, \( C_{A0} \) concentration of the reactor and \( Q_j \) is the heat input to the reactor and \( V_j \) reactor volume, subscript 1 and 2 represents CSTR 1 and CSTR 2 respectively.

\( \Delta H_i \) represents enthalpies, \( k_j \) pre-exponential constants and \( E_j \) activation energies of the three reactions respectively, where \( i = 1, 2, 3 \). Heat capacity \( c_p \) and density (\( \rho \)) of the fluid in the reactor. The reactor has two steady states and one unstable steady state \( T_1^{ss}, C_{A1}^{ss}, T_2^{ss}, C_{A2}^{ss} = 457.9 \) K, 1.77 kmol/m³, 415.5 K, 1.75 kmol/m³ with heat inputs \( Q_1 = Q_2 = 0 \) and concentrations \( C_{A0} = C_{A0}^{ss}, C_{A03} = C_{A03}^{ss} \). The parameters associated with the mathematical model are given in Table 1. The objective is to implement the distributed state estimation scheme on the reactor.
subsystems by sharing the information of the subsystems in the network. In this scheme temperature of the subsystems is maintained at unstable steady states and different operating points closer to the unstable steady state by a decentralized proportional integral controller by manipulating heat input to the reactors.

| Parameter | Value       |
|-----------|-------------|
| $F_0$     | 4.998 m$^3$/h |
| $F_3$     | 39.996 m$^3$/h |
| $F_6$     | 30.0 m$^3$/h   |
| $V_1$     | 1.0 m$^3$     |
| $V_2$     | 3.0 m$^3$     |
| $R$       | 8.314 kJ/kmol K |
| $T_0$     | 300.0 K       |
| $T_{03}$  | 300.0 K       |
| $C_{A0}$  | 4.0 kmol/m$^3$ |
| $C_{A03}$ | 2.0 kmol/m$^3$ |
| $\Delta H_1$ | $-5.0 \times 10^4$ kJ/kmol |
| $\Delta H_2$ | $-5.2 \times 10^4$ kJ/kmol |
| $\Delta H_3$ | $-5.4 \times 10^4$ kJ/kmol |
| $k_{10}$  | 1/h          |
| $k_{20}$  | 1/h          |
| $k_{30}$  | 1/h          |

3. IMPLEMENTATION OF DISTRIBUTED STATE ESTIMATION

The dynamics of the interconnected system is decomposed into subsystems, the measurements from each subsystem is passed to the network. The objective of the distributed estimation scheme is to share the measurements of the each subsystem over the network so the local controller i.e decentralized controller is implemented by utilizing the measurements over the network.

![Figure 2](image-url)
Each local controller is implemented based on the measurements of the corresponding subsystem, if the communication network failure happens each controller is provided with an estimate of the corresponding subsystem. Once the communication network is recovered the measurements are shared among the subsystems over the communication network.

The control action to each subsystem is computed based on the sensor information shared over the network, each decentralized controller is provided with model based estimation of its own subsystem as shown in Figure 2.

4. SIMULATION RESULTS

From the simulation results of the implementation of distributed state estimation of interconnected subsystems temperature T1 of the CSTR1 is presented in Figure 3, the temperature is initially maintained at unstable steady state upto 400 samplings instants at 400th sampling instant a network failure is introduced the estimator is switched automatically to provide the estimate of the current subsystem.

**Figure 3.** Temperature (T1) profile of subsystem 1

**Figure 4.** Concentration (Ca1) profile of subsystem 1
The Concentration profile of both the reactors is shown in Figure 4 and Figure 6. The temperature profile of the CSTR 2 is shown in Figure 5.

![Temperature profile of subsystem 2](image1)

**Figure 5. Temperature (T2) profile of subsystem 2**

![Concentration profile of subsystem 2](image2)

**Figure 6. Concentration (Ca2) profile of subsystem 2**

5. CONCLUSION

The authors have proposed a distributed state estimation scheme of an interconnected subsystem. It has been implemented in an interacting CSTR process which is able to provide the state estimate fairly by utilizing the subsystem estimate.

Appendix

**Unscented Kalman filter**

Generation of sigma points is as follows:

A set of 2L+1 sigma points, $\chi(k-1|k-1,i)$ with the associated weights, $W(i)$ are chosen symmetrically about $\hat{x}(k-1|k-1)$ as given below.

$$\chi(k-1|k-1,0) = \hat{x}(k-1|k-1);$$
\[
\chi(k-1|k-1,i) = \hat{x}(k-1|k-1) + \left(\sqrt{(L+\lambda)P(k-1|k-1)}\right) , \quad i = 1,\ldots,L
\]

\[
\chi(k-1|k-1,i) = \hat{x}(k-1|k-1) - \left(\sqrt{(L+\lambda)P(k-1|k-1)}\right) , \quad i = L+1,\ldots,2L
\]

\[
W^{m}(0) = \frac{\lambda}{L+\lambda};
\]

\[
W^{c}(0) = \frac{\lambda}{L+\lambda} + (1-\alpha^{2}+\beta) ;\quad \lambda = \alpha^{2}(L+\kappa) - L
\]

\[
W^{c}(i) = W^{m}(i) = \frac{1}{2(L+\lambda)}; \quad i = 1,\ldots,2L
\]

Where \( \kappa \) is a secondary scaling parameter, \( \alpha \) is a factor determining the spread of sigma points around \( \hat{x}(k-1|k-1) \) and is usually set between \( 1e-4 \) to 1. The parameter \( \beta \) is used to incorporate prior knowledge of distribution of \( x \) and for Gaussian distribution its optimum value is 2. The \( 2L+1 \) sigma points have been derived from the state \( \hat{x}(k-1|k-1) \) and covariance of the state vector \( P(k-1|k-1) \), where \( L \) is the dimension of the state vector.

**Implementation of UKF algorithm**

In the prediction step, the sigma points are propagated through the nonlinear process model to obtain the predicted set of sigma points as,

\[
\chi(k | k-1, i) = \chi(k-1 | k-1, i) + \int_{(k-1)\Delta t}^{k\Delta t} F[\chi(\tau, i)]d\tau; \quad i = 0,\ldots,2L
\]

Predicted Mean is given by

\[
\hat{x}(k | k-1) = \sum_{i=0}^{2L} W^{m}(i) \chi(k | k-1, i)
\]

Predicted covariance matrix is computed as follows

\[
P(k | k-1) = \sum_{i=0}^{2L} W^{c}(i)\{\chi(k | k-1, i) - \hat{x}(k | k-1)\}^T\{\chi(k | k-1, i) - \hat{x}(k | k-1)\} + Q
\]

Sigma points are redrawn using the predicted mean as given below

\[
\chi^*(k | k-1, 0) = \hat{x}(k | k-1);
\]

\[
\chi^*(k | k-1, i) = \hat{x}(k | k-1) + \left(\sqrt{(L+\lambda)P(k | k-1)}\right) , \quad i = 1,\ldots,L
\]

\[
\chi^*(k | k-1, i) = \hat{x}(k | k-1) - \left(\sqrt{(L+\lambda)P(k | k-1)}\right) , \quad i = L+1,\ldots,2L
\]

The Predicted observation is given by

\[
\hat{y}(k | k-1) = \sum_{i=0}^{2L} W^{m}(i) \chi^*(k | k-1, i)
\]

The computation of Innovation covariance and cross covariance is as follows:

\[
P_{yy}(k) = \sum_{i=0}^{2L} [W^{c}(i)[H[\chi^*(k | k-1, i)] - \hat{y}(k | k-1)]^T[\chi^*(k | k-1, i)] - \chi^*(k | k-1)]^T] + R
\[ P_{xy}(k) = \sum_{i=0}^{2L} W^i(i)\{\chi'(k | k - 1, i) - \hat{x}(k | k - 1)\}^* [C\{\chi'(k | k - 1, i)\} - \hat{y}(k | k - 1)]^T \]

The innovation is computed as follows:
\[ \gamma(k) = y(k) - \hat{y}(k | k - 1). \]

The Kalman gain matrix \( K(k) \) can be determined as follows
\[ K(k) = P_{xy}(k)P_{yy}^{-1}(k) \]

The updated State and Covariance matrix are computed using the following equations
\[ \hat{x}(k | k) = \hat{x}(k | k - 1) + K(k) \cdot \gamma(k) \]
\[ P(k | k) = P(k | k - 1) - K(k) \cdot P_{yy}(k)K^T(k) \]

The authors have proposed a distributed state estimation scheme of an interconnected subsystem. It has been implemented in an interacting CSTR process which is able to provide the state estimate fairly by utilizing the subsystem estimate.

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