Residual finiteness for central pushouts

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Abstract

We prove that pushouts of residually finite-dimensional (RFD) \( C^* \)-algebras over central subalgebras are always residually finite-dimensional, recovering and generalizing results by Korchagin and Courtney-Shulman. This then allows us to prove that certain central pushouts of amenable groups have RFD group \( C^* \)-algebras. Along the way, we discuss the problem of when, given a central group embedding \( H \leq G \), the resulting \( C^* \)-algebra morphism is a continuous field: this is always the case for amenable \( G \) but not in general.

Key words: \( C^* \)-algebra, amenable group, pushout, residually finite, residually finite-dimensional, Fell topology

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Introduction

Residual finiteness properties have elicited considerable interest, both in the operator algebra literature and in group theory. On the operator-algebraic side one typically considers residually finite-dimensional (henceforth RFD) \( C^* \)-algebras, i.e. those whose elements are separated by representations on finite-dimensional Hilbert spaces. The group-theoretic analogue is the concept of a residually finite (or RF) group, i.e. one whose finite-index normal subgroups intersect trivially (i.e. having “enough” finite quotients).

The literature on RFD \( C^* \)-algebras is rather substantial, as is that on RF groups; so much so, in fact, that it would be impossible to do it justice. For samplings (the best a short note such as this one can do) we direct the reader to, say, [14, 2, 10, 3, 17, 19, 20, 13, 24] (for RFD \( C^* \)-algebras) and [5, 4, 6] or [21, §6.5], [23, Chapters 6, 14, 15], [12, Chapter 2] (for RF groups), and references therein.

We are concerned here with the types of “permanence” properties for residual finiteness as studied, say, in [10, 3, 19, 20, 13, 24, 5], to the effect that various types of pushouts (also known as amalgamated free products) of RFD or RF objects are again such. Specifically, the main result of [19] is that pushouts of separable commutative \( C^* \)-algebras are RFD. More generally, the main theorem of [13] proves this for central pushouts \( A *_C B \) with \( A \) and \( B \) separable and strongly RFD, i.e. such that all of their quotients are RFD.

The present note is motivated in part by the desire to recover these results without the separability and strong RFD-ness assumptions.

The preliminary Section 1 recalls some background and sets conventions.

In the short Section 2 we prove Theorem 2.1, stating that central pushouts of RFD \( C^* \)-algebras are RFD.

Section 3 is devoted to the problem of when (or whether), given a central group embedding \( H \leq G \), the resulting embedding \( C^*(H) \to C^*(G) \) is a continuous field over \( \text{spec} \ C^*(H) \) in the...
Section 4 centers around the variant of [24, Theorem 6.9] obtained in Theorem 4.1. The latter proves that (the full group $C^\ast$-algebra of) $G_1 *_H G_2$ is RFD provided $G_i$ are amenable residually finite (RF) and $H \leq G_i$ is a common central subgroup such that the quotients $G_i/H$ are RF. The former result, on the other hand, assumes that $G_1 *_H G_2$ itself is RF.

Although Theorem 4.1 is formally stronger for that reason, we nevertheless show in Theorem 4.6 that its hypotheses imply the residual finiteness of the pushout $G_1 *_H G_2$. [24, Theorem 6.9] and Theorem 4.1 are thus equivalent, albeit non-obviously.

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1 Preliminaries

1.1 Fields of $C^\ast$-algebras

All $C^\ast$-algebras and pushouts are assumed unital. Pushouts over $C$ are undecorated, i.e. $A * B$ denotes what in the literature (e.g. [14]) is sometimes referred to as $A *_C B$. Given a $C^\ast$ morphism $C \to A$ with $C = C(X)$ commutative, we denote by $A_p$, the fiber of $A$ at the point $p$ of the spectrum $X$ of $C$:

$$A_p := A / (\ker p),$$

where $p \in X$ is regarded as a character $p : C(X) \to \mathbb{C}$ and angled brackets denote the ideal generated by the respective set. Similarly, for $a \in A$ we denote by $a_p$ its image through the surjection $A \to A_p$.

We will refer to a central $C^\ast$ morphism $C \to A$ as a $C$-algebra $A$ (e.g. [18, Definition 1.5] or [9, Définition 2.6]). Following standard practice (see [9, Définition 3.1] for instance), we have

Definition 1.1 A $C$-algebra $A$ constitutes a continuous field of $C^\ast$-algebras if for every $A$ the norm function

$$\text{spec } C \ni p \mapsto \|a\|_{A_p}$$

is continuous.

The function is known to always be upper semicontinuous, for example by [22, Proposition 1.2].

1.2 Group filtrations

Following [5, §2.2], we need the following notion applicable to a residually finite group $G$.

Definition 1.2 A filtration on $G$ is a family of finite-index normal subgroups $N_\alpha \leq G$ with trivial intersection.

Let $H \leq G$ be a subgroup. A filtration $\{N_\alpha\}$ of $G$ is an $H$-filtration provided

$$\bigcap_\alpha HN_\alpha = H.$$
2 Pushouts over central subalgebras

The main result of this section is

**Theorem 2.1** Let $A$ and $B$ be two $C^*$-algebras, $C \subseteq A, B$ central $C^*$-embeddings, and assume all corresponding fibers $A_p$ and $B_p$ are RFD for $p \in X := \text{spec } C$.

Then, $M := A \ast C B$ is RFD.

**Proof** Because $C$ is central in $A$ and $B$ it is central in $M$. Note the isomorphism $M_p \cong A_p \ast C_p B_p \cong A_p \ast B_p$.

For $m \in M$ we have

$$\|m\| = \sup_{p \in X} \|m_p\|_{M_p}$$

(2-1) by [9, Proposition 2.8] and hence we can approximate the norm of $m$ arbitrarily well with the norms of its images through representations of $M_p \cong A_p \ast B_p$ as $p$ ranges over $X$. By the RFD-ness assumption on $A_p$ and $B_p$, their coproduct $A_p \ast B_p$ is again RFD by [14, Theorem 3.2]. This finishes the proof. ■

**Remark 2.2** Note that in fact, in the proof above one does not need the precise norm estimate (2-1): all that is needed is that every $0 \neq m \in M$ be non-zero in some quotient $M \rightarrow M_p, p \in X$. ♦

3 Fibers over dense sets

It will be useful, in working with group $C^*$-algebras, to allow the points $p \in X$ from Theorem 2.1 to range only over a dense subset $Y \subseteq X := \text{spec } C$ rather than the entire spectrum. We cannot do this in full generality, as (2-1) does not hold for the supremum over only a dense subset $Y \subseteq X$:

**Example 3.1** Let $I = [0, 1]$, $M = C_b(I)$ (all bounded functions) and $C = C(I) \subset M$ (continuous functions), both equipped with the supremum norm. The indicator function $m$ of $\{0\} \subset I$ is non-zero, even though its image in every fiber $M_p, p \in Y := (0, 1] \subset I$ vanishes. For that reason, we do not have

$$\|m\| = \sup_{p \in Y} \|m_p\|_{M_p}.$$ ♦

**Proposition 3.2** Let $G_i, i = 1, 2$ be two amenable groups and $H \leq G_i$ a common central subgroup. Then, for every dense subset $Y \subseteq \hat{H}$ of the spectrum of $C := C^*(H)$ and every element

$$m \in M := C^*(G_1) \ast C C^*(G_2)$$

we have

$$\|m\| = \sup_{p \in Y} \|m_p\|_{M_p}.$$
**Proof** First, fix an arbitrary unitary representation \( \rho_2 \) of \( G_2 \) where \( H \) acts by scalars, with central character \( p_0 \in \hat{H} \). Let

\[
Y \ni p_\lambda \to p_0
\]

be a net of characters from \( Y \) converging to \( p_0 \) and set

\[
q_\lambda := p_\lambda p_0^{-1} \to 1 \in \hat{H}
\]

be the corresponding “error”.

Because

\[
G_i/H, \ i = 1, 2
\]

are amenable homogeneous spaces of \( G_i \) respectively in the sense of [8, §2], the trivial representation \( 1_{G_2} \) is weakly contained in the induced representation \( \text{Ind}_{H}^{G_2} 1_H \). Since \( q_\lambda \to 1_H \), we have

\[
\text{Ind}_{H}^{G_2} q_\lambda \to \text{Ind}_{H}^{G_2} 1_H \to 1_{G_2}
\]

in the Fell topology, and hence

\[
\rho_2 \otimes \text{Ind}_{H}^{G_2} q_\lambda \to \rho_2 \otimes 1_{G_2} \cong \rho_2.
\] (3-1)

Because \( H \) acts by \( p_0 \) in \( \rho_2 \), it acts via \( p_\lambda = q_\lambda p_0 \) in the left hand side of (3-1). In short, we can Fell-approximate \( \rho_2 \) with unitary representations where \( H \) acts by characters from \( Y \).

Now fix \( m \in M \). According to [9, Proposition 2.8], the norm of \( m \) is achieved in some unitary representation \( \rho \) of \( G := G_1 *_H G_2 \) where \( H \) acts by scalars, with central character \( p_0 \in \hat{H} \). Working only with non-degenerate (indeed, unital) representations isomorphic to their own \( \mathbb{R}_0 \)-multiples, [15, Lemma 2.4] implies that it will suffice to approximate \( \rho \) arbitrarily well in the Fell topology [7, Appendix F.2] with unitary representations where \( H \) operates with central characters belonging to the dense subset \( Y \subseteq \hat{H} \).

In turn, in order to achieve the above it is enough to approximate the restrictions \( \rho|_{G_i} \) with representations where \( H \) acts via elements of \( Y \). This, however, is what the first part of the proof does. ■

Over the course of the proof of Proposition 3.2 we have obtained

**Proposition 3.3** Let \( H \leq G \) be a central subgroup of an amenable group. Then, for every dense subset

\[
Y \subseteq \hat{H} = \text{spec } C^*(H)
\]

the canonical map

\[
C^*(G) \to \prod_{p \in Y} C^*(G)_p
\] (3-2)

is one-to-one. ■

In other words, a variant of [9, Proposition 2.8] with only a dense subset of the spectrum rather the entirety of it. Despite Example 3.1, one might hope that this is at least always possible for group \( C^* \)-algebras. This is not the case, as Example 3.5 below shows. In the same context of amenable groups though, one can do better. Recall (e.g. [9, Definition 3.1]) that the \( C^* \)-algebra \( M \) constitutes a continuous field over the spectrum \( X \) of its central \( C^* \)-subalgebra \( C \cong C(X) \) if the map

\[
X \ni x \mapsto \|m_x\|
\] (3-3)

is continuous. We then have
**Theorem 3.4** For a central subgroup $H \leq G$ of an amenable group $C^*(G)$ is a continuous field of $C^*$-algebras over $\hat{H} = \text{spec } C^*(H)$.

**Proof** Indeed, the first part of the proof of Proposition 3.2 shows that for every net $p_\lambda \to p$ in $\hat{H}$ we have
\[
\|m_p\| \leq \limsup_\lambda \|m_{p_\lambda}\|.
\]
This means precisely that (3-3) is lower semicontinuous. Since, as noted in §1.1, it is in any case upper semicontinuous, it must be continuous. ■

To prepare the ground for Example 3.5, note that by [7, Theorem F.4.4] (3-2) is an embedding if and only if the $G$-representations where $H$ acts by characters in $Y$ form a dense set in the Fell topology. This cannot possibly happen if

- $G$ has the Kazhdan property (T) (e.g. [7, §1.1]), and hence the trivial representation is isolated in the unitary dual;
- the dense subset $Y \subset \hat{H}$ does not contain the trivial element.

To construct such examples all we need is a property-(T) group $G$ with an infinite central subgroup $H$, whereupon we can simply take
\[
Y = \hat{H} \setminus \{1\}.
\]

**Example 3.5** Let $G$ be the universal cover $\tilde{Sp}_4(\mathbb{R})$ of the $4 \times 4$ real symplectic group. It is shown in [16, Theorem 6.8] that $G$ has property (T) (indeed, even the stronger property $(T^*)$; note that the authors of that paper denote $Sp_4$ by $Sp_2$).

The fundamental group of $Sp_4(\mathbb{R})$ is $\mathbb{Z}$, so we can simply take that copy of $\mathbb{Z}$ as the infinite central subgroup $H < G$.

Finally, if a discrete group is desired then one can simply take the preimage through
\[
\tilde{Sp}_4(\mathbb{R}) \to Sp_4(\mathbb{R})
\]
of any lattice in the latter (it will again have property (T) by [7, Theorem 1.7.1]). See also [7, §1.7] for a discussion of property (T) permanence under passage to universal covering groups. ♦

## 4 Central pushouts of RF groups

The present section attempts to prove a slightly more general version of [24, Theorem 6.9]. The difference is that we only assume that $G_i$ are individually RF rather than assuming that $G_1 \ast_H G_2$ is.

**Theorem 4.1** Let $G_i, i = 1, 2$ be two amenable groups and $H \leq G_i$ a common central subgroup such that

- each $G_i$ is RF;
- each $G_i/H$ is RF.

Then, $G_1 \ast_H G_2$ is RFD.
Proof We will obtain the result as an application of Proposition 3.2, with
\[ A = C^*(G_1), \quad B = C^*(G_2) \text{ and } C = C^*(H). \]

According to that result, what we have to argue is that the fibers \( A_p \) and \( B_p \) are RFD for a dense set of points \( p \) in the spectrum
\[ \hat{H} = \text{Pontryagin dual of } H = \text{spec } C^*(H). \]

This is precisely what Corollary 4.5 does, finishing the proof. \( \blacksquare \)

**Lemma 4.2** Let \( H \trianglelefteq G \) be a central inclusion such that \( G \) and \( G/H \) are both RF. Then, the characters \( p : H \to \mathbb{S}^1 \) whose kernels
\[ N = \ker p \]
give rise to RF quotients \( G/N \) form a dense subset of the Pontryagin dual \( \hat{H} \).

**Proof** We know that the normal finite-index subgroups \( G_\alpha \trianglelefteq G \) have trivial intersection, and also that
\[ \bigcap_\alpha HG_\alpha = H. \quad (4-1) \]

Since
\[ H_\alpha := G_\alpha \cap H \leq H \]
have trivial intersection, the union of the duals
\[ \hat{H}/H_\alpha \subseteq \hat{H} \]
is a dense subgroup. We claim that any \( p \in \hat{H}/H_\alpha \) will meet the requirements in the statement; proving this will achieve the desired conclusion, so it is the task we turn to next.

Fix such a character \( p : H \to \mathbb{S}^1 \), factoring through some
\[ H \to H/H_{\alpha_0}, \]
and let \( N \leq H \) be its kernel. Then, by its very construction, \( N \) will contain \( H_{\alpha_0} = H \cap G_{\alpha_0} \). This means that
\[ NG_{\alpha_0} \cap H \subseteq N \]
(which is then an equality), and hence the intersection
\[ \bigcap_\alpha NG_\alpha \subseteq H \]
(where the latter inclusion uses (4-1)) cannot possibly contain \( N \) strictly. In conclusion we have
\[ \bigcap_\alpha NG_\alpha = N, \]
meaning that the filtration \( \{ G_\alpha \} \) is compatible with \( N \) (i.e. an \( N \)-filtration) and hence \( G/N \) is RF. \( \blacksquare \)
Remark 4.3 The RF requirements in Theorem 4.1 are both crucial:

On the one hand, [24, §8] recalls the example given by Abels in [1] of a central inclusion \( \mathbb{Z} < G \) into an RF amenable group such that \( G/\mathbb{Z} \) is not RF.

On the other hand, [11] contains an example (attributed there to C. Kanta Gupta) of a non-residually finite group \( G \) whose quotient by an order-two normal subgroup \( K \triangleleft G \) is residually finite. Centrality is easy to arrange, since the centralizer of \( K \) in \( G \) will have finite index in the latter.

Although Lemma 4.2 ensures that the quotients \( G/N \) for

\[
N = \ker(p : H \to S^1)
\]

are RF, we have yet to prove that the resulting fiber algebras \( C^*(G)/\langle h - p(h), \ h \in H \rangle \) are RFD.

**Lemma 4.4** Let \( H \leq G \) be a central subgroup, \( p \in \hat{H} \) a character of finite order, and \( N = \ker p \) its kernel in \( H \). If \( G/N \) is RFD then so is the fiber \( C^* \)-algebra

\[
C^*(G)_p := C^*(G)/\langle h - p(h), \ h \in H \rangle
\]

(4-2)
corresponding to \( p \) is RFD.

**Proof** Note that \( C^*(G)_p \) is precisely the fiber of \( C^*(G/N) \) at the character induced by \( p \) on the quotient (finite cyclic) group \( H/N \). For that reason, we may as well assume that

- \( N \) is trivial, and hence
- \( H \) is finite cyclic;
- \( G \) is RFD.

But now note that \( C^*(G)_p \) is a fiber of the RFD \( C^* \)-algebra over the **finite-dimensional** central subalgebra \( C^*(H) \leq C^*(G) \). In general, a \( C^* \)-algebra \( A \) will break up as a product of the fibers \( A_p \) over a finite-dimensional central \( C^* \)-subalgebra \( C \leq A \), by simply cutting \( A \) with the minimal projections in \( C \).

In particular, under our assumptions the fiber \( C^*(G)_p \) is a Cartesian factor of \( C^*(G) \), and hence the RFD-ness of the latter entails that of the former. ■

**Corollary 4.5** Under the hypotheses of Lemma 4.2 the characters \( p : H \to S^1 \) for which the fiber \( (4-2) \) is RFD form a dense subset of \( \hat{H} \).

**Proof** Indeed, the characters \( p \) in the proof of Lemma 4.2 are of finite order, and hence Lemma 4.4 applies. ■

### 4.1 Recovering residual finiteness for a pushout

Recall that [24, Theorem 6.9] assumes \( G_1 *_C G_2 \) is RF, whereas Theorem 4.1 only requires that \( G_i, \ i = 1, 2 \) be RF individually (along with \( G_i/H \)). We argue here that that distinction is only apparent:

**Theorem 4.6** Let \( H \leq G_i, \ i = 1, 2 \) be a common central subgroup such that \( G_i \) and \( G_i/H \) are all RF. Then, \( G_1 *_H G_2 \) is RF.
Proof Note first that the case $G_1 = G_2$ is clear: indeed, the assumptions that $G$ and $G/H$ are RF then show that $G$ has an $H$-filtration. The two copies of that filtration in the two copies of $G$ are then $(H, H, \text{id})$-compatible in the sense of [5, §2.2], and hence $G \ast_H G$ is RF by [5, Proposition 2].

It thus remains to reduce the problem to the case $G_1 = G_2$. To do this, consider the tensor product

$$G := G_1 \otimes_H G_2$$

defined by identifying the two copies of $H$ in $G_1 \times G_2$; in other words, $G_1 \otimes_H G_2$ is $G_1 \ast_H G_2$ modulo the relations making the elements of

$$G_1 \text{ and } G_2 \leq G_1 \ast_H G_2$$

commute.

We then have

$$G/H \cong (G_1/H) \times (G_2/H),$$

which is thus RF by assumption. On the other hand, if we show that $G \ast_H G$ itself is RF then so is

$$G_1 \ast_H G_2 \leq G \ast_H G$$

(where the inclusions $G_i \leq G = G_1 \otimes_H G_2$ are the obvious ones).

To summarize, we have thus far

- observed that the conclusion holds when $G_1 = G_2$ (equal to a common group $G$, say);
- reduced the problem to its instance for $G = G_1 \otimes_H G_2$, modulo the desired hypothesis that that $G$ is RF.

In conclusion, all that remains to be proven is that under our hypotheses $G := G_1 \otimes_H G_2$ is indeed RF; we relegate this to Lemma 4.7.

Lemma 4.7 Let $H \leq G_i$, $i = 1, 2$ be a common central subgroup such that $G_i$ and $G_i/H$ are all RF. Then, $g := G_1 \otimes_H G_2$ is RF.

Proof If $\{G_{i, \alpha}\}$ are $H$-filtrations of $G_i$ respectively for $i = 1, 2$ then the images $G_\alpha$ of

$$G_{1, \alpha} \times G_{2, \alpha} \leq G_1 \times G_2$$

through the surjection

$$G_1 \times G_2 \rightarrow G = G_1 \otimes_H G_2$$

identifying the two copies of $H$ will form an $H$-filtration for $G$.

References

[1] Herbert Abels. An example of a finitely presented solvable group. In *Homological group theory (Proc. Sympos., Durham, 1977)*, volume 36 of *London Math. Soc. Lecture Note Ser.*, pages 205–211. Cambridge Univ. Press, Cambridge-New York, 1979.

[2] R. J. Archbold. On residually finite-dimensional $C^*$-algebras. *Proc. Amer. Math. Soc.*, 123(9):2935–2937, 1995.
[3] Scott Armstrong, Ken Dykema, Ruy Exel, and Hanfeng Li. On embeddings of full amalgamated free product $C^*$-algebras. *Proc. Amer. Math. Soc.*, 132(7):2019–2030, 2004.

[4] Benjamin Baumslag and Marvin Tretkoff. Residually finite HNN extensions. *Comm. Algebra*, 6(2):179–194, 1978.

[5] Gilbert Baumslag. On the residual finiteness of generalised free products of nilpotent groups. *Trans. Amer. Math. Soc.*, 106:193–209, 1963.

[6] Gilbert Baumslag. Finitely generated cyclic extensions of free groups are residually finite. *Bull. Austral. Math. Soc.*, 5:87–94, 1971.

[7] Bachir Bekka, Pierre de la Harpe, and Alain Valette. *Kazhdan’s property (T)*, volume 11 of *New Mathematical Monographs*. Cambridge University Press, Cambridge, 2008.

[8] Mohammed E. B. Bekka. Amenable unitary representations of locally compact groups. *Invent. Math.*, 100(2):383–401, 1990.

[9] Étienne Blanchard. Déformations de $C^*$-algèbres de Hopf. *Bull. Soc. Math. France*, 124(1):141–215, 1996.

[10] Nathaniel P. Brown and Kenneth J. Dykema. Popa algebras in free group factors. *J. Reine Angew. Math.*, 573:157–180, 2004.

[11] Robert Campbell. [http://www.math.umbc.edu/~campbell/CombGpThy/RF_Thesis/2_RF_Results.html](http://www.math.umbc.edu/~campbell/CombGpThy/RF_Thesis/2_RF_Results.html). Accessed: 2020-01-21.

[12] Tullio Ceccherini-Silberstein and Michel Coornaert. *Cellular automata and groups*. Springer Monographs in Mathematics. Springer-Verlag, Berlin, 2010.

[13] Kristin Courtney and Tatiana Shulman. Free products with amalgamation over central $C^*$-subalgebras. *Proc. Amer. Math. Soc.*, 148(2):765–776, 2020.

[14] Ruy Exel and Terry A. Loring. Finite-dimensional representations of free product $C^*$-algebras. *Internat. J. Math.*, 3(4):469–476, 1992.

[15] J. M. G. Fell. Weak containment and induced representations of groups. *Canadian J. Math.*, 14:237–268, 1962.

[16] Uffe Haagerup, Sø ren Knudby, and Tim de Laat. A complete characterization of connected Lie groups with the approximation property. *Ann. Sci. Éc. Norm. Supér. (4)*, 49(4):927–946, 2016.

[17] Don Hadwin. A lifting characterization of RFD $C^*$-algebras. *Math. Scand.*, 115(1):85–95, 2014.

[18] G. G. Kasparov. Equivariant $KK$-theory and the Novikov conjecture. *Invent. Math.*, 91(1):147–201, 1988.

[19] Anton Korchagin. Amalgamated free products of commutative $C^*$-algebras are residually finite-dimensional. *J. Operator Theory*, 71(2):507–515, 2014.

[20] Qihui Li and Junhao Shen. A note on unital full amalgamated free products of RFD $C^*$-algebras. *Illinois J. Math.*, 56(2):647–659, 2012.
[21] Wilhelm Magnus, Abraham Karrass, and Donald Solitar. *Combinatorial group theory*. Dover Publications, Inc., Mineola, NY, second edition, 2004. Presentations of groups in terms of generators and relations.

[22] Marc A. Rieffel. Continuous fields of $C^*$-algebras coming from group cocycles and actions. *Math. Ann.*, 283(4):631–643, 1989.

[23] Derek J. S. Robinson. *A course in the theory of groups*, volume 80 of *Graduate Texts in Mathematics*. Springer-Verlag, New York, second edition, 1996.

[24] Tatiana Shulman. Central amalgamation of groups and the RFD property, 2019. arXiv:2001.00052.

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