1. INTRODUCTION

Transmit beampattern shaping by controlling the spatial distribution of the transmit power can play an important role in improving the radar performance through enhanced power efficiency, better detection probability, target identification, and improved interference mitigation, among others. The goal is to focus the transmit power on desired angles while minimizing it at undesired angles [1]. Recently, the beampattern shaping via waveform design in multiple-input multiple-output (MIMO) radar systems has been widely studied. From a waveform design perspective, there are two methods for beampattern shaping: indirect and direct methods [2], [3]. In the indirect (two-step) method, the waveform correlation matrix is first designed, and the waveform matrix is subsequently obtained through one of the decomposition methods [4]–[12], while in the direct method, the waveform is designed in one step [2], [3], [13]–[19]. On the other hand, there are several metrics (objective functions) to obtain the optimum beampattern, such as beampattern matching, spatial integrated sidelobe level ratio (ISLR)/peak sidelobe level ratio (PSLR) minimization, and signal-to-interference-plus-noise ratio (SINR) maximization.

1) Beampattern matching: In beampattern matching, the aim is to minimize the difference between the desired and designed beampatterns. For instance, the following papers have worked on designing the waveform covariance matrix employing beampattern matching. Stoica et al. [4] devised a method to address the joint beampattern shaping and the cross-correlation minimization in spatial domain through semidefinite quadratic programming technique. In [5], the cyclic algorithm is presented to shape the beampattern under low peak-to-average ratio (PAR) constraint. Lipor et al. [10] and Bouchoucha et al. [11] propose a covariance matrix design method based on discrete Fourier transform (DFT) coefficients and Toeplitz matrices. The DFT-based technique provides a well-match transmit beampattern at low complexity. However, the drawback of the DFT-based method is that, for a small number of antennas, the performance of the DFT-based method is slightly poorer. On the other hand, several papers focus on designing directly the transmit waveforms for beampattern shaping. For example, in [2], two optimization algorithms based on the alternating direction method of multipliers (ADMM) are proposed under constant modulus (CM) constraint for the probing waveform. In [3], a method based on the ADMM is proposed to design a beampattern in wideband systems. In [17], a method for beampattern matching is addressed based on gradient descent which they term it projection, descent, and retraction (PDR). Fan et al. [18] propose a method based on majorization–minimization (MM) for beampattern matching under the PAR constraint in two cases of wideband and narrowband.
2) Spatial ISLR and PSLR minimization: In spatial ISLR and PSLR minimization approaches, the aim is to minimize the ratio of summation of beampattern response on undesired over desired angles and to minimize the ratio of maximum beampattern response on undesired over minimum beampattern response on desired angles, respectively. In [8], a method based on semidefinite relaxation (SDR) under constant energy and 3-dB main beamwidth constraint is proposed to minimize the spatial ISLR. In [20], the robust designs of the waveform covariance matrix through optimizing the worst-case transmit beampattern are considered to minimize the spatial ISLR and PSLR of beampatterns, respectively. Unlike two aforementioned methods, a direct waveform design solution is proposed in [13], [16], and [19]. Raei et al. [13] propose the efficient UNI-modular set of seQUEnce design (UNIQUE) algorithm based on coordinate descent (CD) method [21] to minimize spatial and range ISLR under four different constraints, namely, limited energy, PAR, continuous-phase, and discrete-phase constraints. The method proposed in [19] is similar to UNIQUE without considering range ISLR metric and PAR and limited energy constraints. A method based on the ADMM is proposed in [16] to minimize the spatial PSLR under CM constraint.

3) SINR maximization: In SINR optimization approaches, the problem does not deal with the beampattern directly. However, the beampattern is implicitly shaped as a result of transmit waveform optimization. For example, Cui et al. [6], [7] address the problem of waveform design in the presence of signal-dependent clutter. In these works, an iterative approach is presented to jointly optimize the transmit waveform and the receive filter to maximize the output SINR. Wu et al. [22] propose majorized iterative algorithm (MIA) based on MM method for joint waveform and filter design under similarity, CM (MIA—CM constraint), and PAR (MIA—PAR constraint) constraints. Space–time transmitting code [23] is proposed based on CD to solve the problem under similarity, uncertain steering matrices, and continuous- or discrete-phase constraints. In [23], a Dinkelbach method and exhaustive search is proposed for continuous- and discrete-phase constraints, respectively.

In order to form the virtual array and enhancing the angular resolution, the received signal in the MIMO radar system should be separable (orthogonal) in the receiver, while a set of arbitrary waveforms are adopted in the transmit side. In order to obtain the orthogonality, the waveform should have small cross correlation [24]. In addition, small autocorrelation sidelobes is a requirement, to avoid masking weak targets within the range sidelobes of a strong target, and to mitigate the harmful effects of distributed clutter

returns close to the target of interest. Recently, many optimization techniques, e.g., multicyclic algorithm new [25], [26], iterative direct search [27], integrated sidelobe level (ISL) new [28], [29], MM-Corr [30], binary sequence sets [31], UNIQUE [13], and weighted BSUM sequence set [32] are proposed to design orthogonal sets of sequences, minimizing the ISL/peak sidelobe level metrics. However, beampattern shaping in MIMO radar systems yields a correlated waveform, which is in contradiction with orthogonality [13], [33]. In this context, there are few papers that addressed these two aspects in MIMO radar systems. For instance, Fuhrmann and Antonio [33] propose a beampattern matching problem under particular constraints on the waveform cross-correlation matrix. Deng et al. [34] minimize the difference between desired and undesired beampattern responses for one subpulse. Then, the quasi-orthogonality of other subpulses are obtained by random permutation. Alhujaili et al. [35] combine a beampattern matching by orthogonality requirement as a penalty in the objective function and use the PDR approach for the solution. Deng et al. [36] propose a method based on the ADMM to design a beampattern with good cross correlation. In [13], UNIQUE is proposed as a unified framework to minimize the spatial and range ISLR using the weighted sum technique under limited energy, PAR, continuous-phase, and discrete-phase constraints. By choosing an appropriate value for the regularization parameter, UNIQUE is able to make a trade-off between having a good orthogonality and beampattern shaping.

On the other hand, spectral shaping is an important aspect of resource management in cognitive MIMO radar systems. Using this approach, the cognitive radar system is able to utilize effectively the available bandwidth. One attractive application of spectral shaping is the coexistence of communications and cognitive MIMO radar systems, in which the whole bandwidth is shared between these two systems based on the priorities [38]. There are several methods for spectral shaping. For instance, in [39]–[44], the spectral matching approach is proposed to shape the spectral of the transmit waveform. Tang and Li [45] and Aubry et al. [46] consider a waveform design approach to maximize the SINR, while the spectral behavior is considered as a constraint. In [47] and [48], the ratio of the maximum stopband level to the minimum passband level is considered as the objective function to shape the spectrum. The spectral integrated level ratio minimization approach is considered under continuous- and discrete-phase constraints in [38]. The design of CM waveform for beampattern matching under the spectral constraint is addressed in [43] and [49]. To tackle the nonconvex optimization problem, Alhujaili et al. [43] and Kang et al. [49] propose iterative beampattern with spectral design and beampattern optimization with spectral interference control methods, respectively.

1Cognitive MIMO radar systems are smart sensors that interact with the environment to adapt the properties of the waveform with the environment to enhance their performance [37].
A. Contribution

In this article, we consider the spatial ISLR as the design metric similar to [19]. Raei et al. [19] proposed the CD-based method to enhance the performance of the radar in the spatial domain. However, in this article, we deal with the design of waveform considering the features in three domains: ISLR in the spatial and range domains and masking in the spectral domain. Particularly, we propose a waveform design framework to shape the beampattern with practical constraints, namely, spectral masking, 3-dB beamwidth, CM, and similarity constraints. The spectral masking constraint plays an important role in cognitive MIMO radar systems in several scenarios, such as spectral sharing in the coexistence of MIMO radar and MIMO communication. The 3-dB beamwidth constraint ensures that the beampattern has a good response at the main lobe. This constraint can be used in the emerging 4-D imaging automotive MIMO radar systems, wherein the short-range radar, mid-range radar, and long-range radar configurations are merged to provide unique and high angular resolution in the entire radar detection range [13], [19]. CM waveforms are attractive for radar system designers due to the efficient utilization of the limited transmitter power. Besides, since CM is a kind of only phase-modulated sequence, the implementation of CM waveform is simple. As to the orthogonality, we incorporate the beampattern shaping optimization problem with similarity constraints to make a tradeoff between having a good beampattern response and orthogonality [13]. This constraint imposes that the optimize waveform inherits some properties of a reference waveform. In fact, we consider the designed waveform to be similar to a specific waveform, which have a good orthogonality to form the virtual array in receivers and enhance its angular resolution.

It is desirable to include all these properties to improve radar performance in emerging applications and in the emerging scenario of crowded environments with interference from other radars or communication systems. In this context, the contributions of this article are listed as follows:

1) Incorporation of metrics from multiple domains: Radar tasks are influenced by parameters in the spatial, temporal, and spectral domains. Hence, it is pertinent to consider quality metrics in all these domains to improve performance. Thus, while it is highly interesting to consider all the metrics in the waveform design, the existing works consider only a selection of these performance metrics. A problem setup involving these key performance indicators in different domains is lacking in literature. In this context, the proposed framework incorporating all the metrics is the most comprehensive approach for MIMO radar waveform design focusing on beampattern shaping; it subsumes existing works as special cases. The gains obtained by incorporating these metrics over the existing works bear testimony to their impact.

2) Novel optimization framework: The incorporation of all the aforementioned quality metrics adds further complexity to the waveform optimization, and these cannot be handled by the existing frameworks. In this context, this article also offers a novel optimization framework to obtain a local optimum solution of the nonconvex multivariable and NP-hard problem. In an attempt to solve this problem, we propose an indirect method based on semidefinite programming (SDP). We first recast the waveform-design problem as a rank-1 constrained optimization problem. Then, unlike the conventional methods that drop the rank-1 constraint, we propose a new iterative algorithm for efficiently solving the resulting rank-1 constrained optimization problem. Each iteration of the proposed iterative algorithm is composed of an SDP followed by an eigenvalue decomposition (ED). In every iteration, we force the second largest eigenvalue toward zero to obtain the rank-1 solution. We prove that the proposed iterative algorithm converges to a local minima of the rank-1 constrained optimization problem. Furthermore, we compute the computational complexity of the proposed iterative algorithm. In addition, the proposed framework can be extended to apply other convex constraints.

B. Organization and Notation

The rest of this article is organized as follows. In Section II, the system model and design problem for minimizing the spatial ISLR under CM, 3-dB beamwidth, similarity, and spectral masking constraints is formulated. We develop the iterative Waveform design for beampattern shaping and SpEctral masking (WISE) framework based on SDP to obtain a rank-1 solution in Section III. We provide numerical experiments to verify the effectiveness of proposed algorithm in Section IV. Finally, Section V concludes this article.

Notations: This article uses lowercase and uppercase boldface for vectors (a) and matrices (A), respectively. The conjugate, transpose, and conjugate transpose operators are denoted by (·)∗, (·)T, and (·)† symbols, respectively. Besides, the Frobenius norm, ℓ2 and ℓp norms, absolute value, and round operator are denoted by ∥·∥F, ∥·∥2, ∥·∥p, |·|, and ⌊·⌋, respectively. For any matrix A, Tr(A), Diag(A), and Rank(A) stand for the trace, diagonal vector, and the rank of A, respectively. The letter j represents the imaginary unit (i.e., j = √−1), while the letter (i) is used as the step of a procedure. Finally, 1 and 0 denote a matrix and a vector with proper size, which all the elements are equal to 1 and 0, respectively.

II. SYSTEM MODEL

We consider a colocated narrowband MIMO radar system, with M transmit antennas, each transmitting a sequence of length N in the fast-time domain. Let the matrix...
\( S \subseteq \mathbb{C}^{M \times N} \) denote the transmitted set of sequences as

\[
S \triangleq \begin{bmatrix}
s_{1,1} & s_{1,2} & \cdots & s_{1,N} \\
s_{2,1} & s_{2,2} & \cdots & s_{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
s_{M,1} & s_{M,2} & \cdots & s_{M,N}
\end{bmatrix}.
\]

Let us denote that \( S \triangleq [s_1, \ldots, s_N] \triangleq [s_{1,1}, s_{1,2}, \ldots, s_{1,N}]^T \), where the vector \( s_m \triangleq [s_{m,1}, s_{m,2}, \ldots, s_{m,N}]^T \in \mathbb{C}^N \) denotes the angle with maximum desired and undesired discrete frequency bands are given by (4e). \( \Theta_1 \cap \Theta_2 = \emptyset \) and is \( \cup \Theta_1 \cup \Theta_2 \subseteq [-90^\circ, 90^\circ] \). In this regard, the spatial ISLR is given by the following expression [13]:

\[
P(S, \theta) = \frac{1}{N} \sum_{n=1}^{N} |a^T(\theta) s_n|^2 = \frac{1}{N} \sum_{n=1}^{N} s_n^\dagger A(\theta) s_n
\]

where, \( A(\theta) = a(\theta)^* a(\theta) \). Let \( \Theta_d \) and \( \Theta_u \) be the sets of desired and undesired angles in the spatial domain, respectively. These two sets satisfy \( \Theta_d \cap \Theta_u = \emptyset \) and \( \Theta_d \cup \Theta_u \subseteq [-90^\circ, 90^\circ] \). In this regard, the spatial ISLR is given by the following expression [13]:

\[
f(S) = \frac{\sum_{\theta \in \Theta_d} P(S, \theta)}{\sum_{\theta \in \Theta_d} P(S, \theta)} = \frac{\sum_{n=1}^{N} s_n^\dagger A_s s_n}{\sum_{n=1}^{N} s_n^\dagger A_u s_n}
\]

where \( A_u = \frac{1}{N} \sum_{\theta \in \Theta_u} A(\theta) \) and \( A_d = \frac{1}{N} \sum_{\theta \in \Theta_d} A(\theta) \).

B. System Model in the Spectrum Domain

Let \( \mathbf{F} \triangleq [\mathbf{f}_0, \ldots, \mathbf{f}_{K-1}] \subseteq \mathbb{C}^{N \times N} \) be the DFT matrix \( (N \geq N) \), where \( \mathbf{f}_k \triangleq [1, e^{-j2\pi k}, \ldots, e^{-j2\pi (N-1)k}]^T \in \mathbb{C}^N, \ k = 0, \ldots, N-1 \). Let \( \mathcal{U} = \bigcup_{k=1}^{K} (u_{k,1}, u_{k,2}) \) be the \( K \) number of normalized frequency stopbands, where \( 0 \leq u_{k,1} < u_{k,2} \leq 1 \) and \( \bigcup_{k=1}^{K} (u_{k,1}, u_{k,2}) = \mathcal{V} \). Thus, the undesired discrete frequency bands are given by \( \mathcal{V} \subseteq \bigcup_{k=1}^{K} ([N u_{k,1}], [N u_{k,2}]) \). In this regard, the absolute value of \( \mathbf{F} \) at undesired frequency bins can be expressed as \( |\mathbf{F} s_m| \), where \( s_m \) is \( \mathbf{F} \) zero-pad version of \( s_m \) and is defined as

\[
s_m \triangleq [s_m; 0; \ldots; 0] \quad (3)
\]

and, \( \mathbf{G} \in \mathbb{C}^{K \times N} \) contains the rows of \( \mathbf{F} \) corresponding to the frequencies in \( \mathcal{V} \), and \( K \) is the number of undesired frequency bins [38].

C. Problem Formulation

We aim to design a set of CM sequences for MIMO radar such that the transmit beampattern is steered toward desired directions and has nulls at undesired directions simultaneously, with spectrum compatibility and similarity constraints. To this end, we can formulate the following optimization problem:

\[
\begin{align*}
\min_{\mathbf{S}} ~ & \sum_{m=1}^{M} \sum_{n=1}^{N} |s_m^\dagger A_m s_n|^2 \\
\text{s.t.} ~ & 0.5 \leq \sum_{m=1}^{M} \sum_{n=1}^{N} |s_m^\dagger A_m s_n| \leq 1 \\
& |s_m| = 1 \\
& \max \{|G s_m|^2| \leq \gamma, m \in \{1, \ldots, M\} \quad (4d) \\
& \frac{1}{\sqrt{MN}}||S - \hat{S}_0||_F \leq \delta \quad (4e)
\end{align*}
\]

where (4b) indicates the 3-dB beamwidth constraint to guarantee that the beampattern response at all the desired angles is at least half the maximum power. In (4b), \( \theta_d = \{\forall \theta \in \Theta_d \} \) and \( \theta_o \) denotes the angle with maximum power, which is usually chosen to be at the center point of \( \Theta_d \). Constraint (4c) indicates the CM property; this is attractive for radar system designers since it allows for the efficient utilization of the limited transmitter power. Constraint (4d) indicates the spectrum masking and guarantees the power of spectrum in undesired frequencies not to be greater than \( \gamma \). Finally, constraint (4e) has been imposed on the waveform to control properties of the optimized code such as orthogonality similar to the reference waveform \( \hat{S}_0 \); for instance, this helps controlling ISLR in the range domain. If we consider \( \mathbf{S} \) and \( \hat{S}_0 \) to be CM waveforms, the maximum admissible value of the similarity constraint parameter would be \( \delta = \sqrt{2} (0 \leq \delta \leq \sqrt{2}) \).

In (4), the objective function (4a) is a fractional quadratic function; also, (4b) is a nonconvex inequality constraint. Equation (4c) is a nonaffine equality constraint, while the inequality constraint (4e) yields a convex set. Therefore, we encounter a nonconvex, multivariable, and NP-hard optimization problem [13]. In the following, we propose an iterative method based on SDP to obtain an efficient local optimum solution.

III. PROPOSED METHOD

The maximum of \( P(S, \theta) \) is clearly \( M^2 \) and occurs when \( s_n = a(\theta) n \in \{1, \ldots, N\} \). Therefore, the denominator of (4a) satisfies \( \sum_{n=1}^{N} s_n^\dagger A_m s_n \leq K_d M^2 \), where \( K_d \) is the number of desired angles. Thus, problem (4) can be equivalently
written as [50]

\[
\begin{align*}
\min_{s_n} & \sum_{n=1}^{N} \sum_{j=1}^{s_n} A_n A_n^* \\
\text{s.t.} & \quad \sum_{n=1}^{N} \sum_{j=1}^{s_n} A_n A_n^* \leq K_d M^2 \\
& \quad \sum_{n=1}^{N} \sum_{j=1}^{s_n} A_n A_n^* \leq \sum_{n=1}^{N} \sum_{j=1}^{s_n} A_n A_n^* \\
& \quad \sum_{n=1}^{N} \sum_{j=1}^{s_n} A_n A_n^* \leq 2 \sum_{n=1}^{N} \sum_{j=1}^{s_n} A_n A_n^* \\
& \quad |s_n| = 1 \\
& \quad \|G_{\delta m}^{k}\|_{p-\infty} \leq \gamma, \ m \in \{1, \ldots, M\} \\
& \quad \frac{1}{\sqrt{M}}\|S - S_0\|_F \leq \delta.
\end{align*}
\]

In (5), constraints (5c) and (5d) are obtained by expanding constraint (4b). Besides, we replace the constraint \(\max|G_{\delta m}^{k}| (4d)\) with \(\|G_{\delta m}^{k}\|_{p-\infty} (5f)\), which is a convex constraint for each finite \(p\).

**REMARK 1** Another possible solution to consider the constraint (4d) is direct implementation by individually bounding each frequency response at each undesired frequency bin. This reformulation makes the problem convex, but needs to consider \(M \times K\) constraints in total, which increases the complexity of the algorithm. As the alternative, in this article, we replace this constraint with \(\ell_p\)-norm and, leveraging the stability of the algorithm, choose a large \(p\) value to solve the problem.

Problem (5) is still nonconvex with respect to \(S\) due to (5c)–(5e). To cope with this problem, defining \(X_n \triangleq s_n s_n^T\), we recast (5) as follows:

\[
\begin{align*}
\min_{S, X_n} & \sum_{n=1}^{N} \text{Tr}(A_n X_n) \\
\text{s.t.} & \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \leq K_d M^2 \\
& \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \leq \sum_{n=1}^{N} \text{Tr}(A_n X_n) \\
& \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \leq 2 \sum_{n=1}^{N} \text{Tr}(A_n X_n) \\
& \quad \text{Diag}(X_n) = I_M \\
& \quad (5f), (5g) \\
& \quad X_n \succeq 0 \\
& \quad X_n = s_n s_n^T.
\end{align*}
\]

It is readily observed that, in (6), the objective function and all the constraints but (6h) are convex in \(X_n\) and \(S\). In the following, we first present an equivalent reformulation for (6), which paves the way for iteratively solving this nonconvex optimization problem.

**THEOREM 3.1** Defining \(Q_n \triangleq \begin{bmatrix} 1 & s_n^T \\ s_n & X_n \end{bmatrix} \in \mathbb{C}^{(M+1) \times (M+1)}\) and considering slack variables \(V_n \in \mathbb{C}^{(M+1) \times M}\) and \(b_n \in \mathbb{R}\), the optimization problem (6) is equivalent to

\[
\begin{align*}
\min_{S, X_n, b_n} & \sum_{n=1}^{N} \text{Tr}(A_n X_n) + \eta \sum_{n=1}^{N} b_n \\
\text{s.t.} & \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \leq K_d M^2 \\
& \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \leq \sum_{n=1}^{N} \text{Tr}(A_n X_n) \\
& \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \leq 2 \sum_{n=1}^{N} \text{Tr}(A_n X_n) \\
& \quad \text{Diag}(X_n) = I_M \\
& \quad (5f), (5g) \\
& \quad X_n \succeq 0 \\
& \quad X_n = s_n s_n^T. \\
\end{align*}
\]

Once \(X_n^{(i)}, S_n^{(i)}, b_n^{(i)}\) are found by solving (8), \(V_n^{(i)}\) can be obtained by seeking an \((M+1) \times M\) matrix with orthonormal columns such that \(b_n^{(i)} I_M \succeq Q_n^{(i)} V_n^{(i)} V_n^{(i)-1}\). Choosing \(V_n^{(i)}\) to be equal to the matrix composed of the eigenvectors of \(Q_n^{(i)}\) corresponding to its \(M\) smallest eigenvalues, and following similar arguments provided after (12) in the Appendixes, we have [51, Corollary 4.3.16]

\[
V_n^{(i)} D_n^{(i)} V_n^{(i)-1} = \text{Diag} \left( \left[ \rho_1^{(i)}, \rho_2^{(i)}, \ldots, \rho_M^{(i)} \right]^T \right) \\
\leq \text{Diag} \left( \left[ v_1^{(i)-1}, v_2^{(i)-1}, \ldots, v_M^{(i)-1} \right]^T \right) \leq b_n^{(i)} I_M
\]

where \(\rho_1^{(i)} \leq \rho_2^{(i)} \leq \cdots \leq \rho_M^{(i)}\) and \(v_1^{(i)-1} \leq v_2^{(i)-1} \leq \cdots \leq v_M^{(i)-1}\) denote the eigenvalues of \(Q_n^{(i)}\) and \(V_n^{(i)-1} D_n^{(i)} V_n^{(i)-1}\), respectively. It follows from (9) that the matrix composed of the eigenvectors of \(Q_n^{(i)}\) corresponding to its \(M\) smallest eigenvalues is the appropriate choice for \(V_n^{(i)}\).

Accordingly, at each iteration of the proposed algorithm, which we term as WISE, we need to solve a SDP followed...
by an ED. Algorithm 1 summarizes the steps of the WISE approach for solving (4). In order to initialize the algorithm, $V_n^{(0)}$ can be found through the ED of $Q_n^{(0)}$ obtained from solving (8) without considering (8j) and (8k) constraints. Furthermore, we terminate the algorithm when $\xi_n, s_n |_{\infty}$ converges to $X_n$. In this regard, let us assume that

$$\xi_{n,1} \geq \xi_{n,2} \geq \cdots \geq \xi_{n,m} \geq \cdots \geq \xi_{n, M} \geq 0$$

are the eigenvalues of $X_n$; we consider $\xi = \max\{\xi_{n,1}, \min\{\xi_{n, m}\}\} < e_1$ ($e_1 > 0$) as the termination condition. In this case, the second largest eigenvalue of $X_n$ is negligible compared to its largest eigenvalue and can be concluded that the solution is rank 1. In addition, we consider $\max\{\frac{|s_n| - X_n e_i}{\sqrt{MN}}\} < e_2$ ($e_2 > 0$) as the second termination condition.

We note that the proposed algorithm, which is based on alternating optimization method, is guaranteed that the objective function converges to at least a local minimum of (7) [52].

A. Convergence

It readily follows from (8k) that $\lim_{k \to \infty} \frac{\|Q_n^{(k)}\|}{\|\nu_n^{(k)}\|} \leq 1$. This implies that $b_n^{(i)}$ converges at least sublinearly to zero [53]. Hence, there exists some $I$ such that $b_n^{(i)} \leq \epsilon$ ($\epsilon \to 0$) for $i \geq I$. Making use of this fact, we can deduce from (8) that

$$V_{n, i-1}^{(i-1)} Q_n^{(0)} V_{n, i-1}^{(i-1)} \leq \epsilon I_M, \quad \epsilon \to 0$$

(10)

for $i \geq I$. Then, it follows from (10) and (9) that $\text{Rank}(Q_n^{(i)}) \leq 1$ for $i \geq I$; thereby, $X_n^{(i)} = s_n^{(i)} b_n^{(i)}$ for $i \geq I$. This implies that $X_n^{(i)}$ for any $i \geq I$, is a feasible point for the optimization problem (7). On the other hand, considering the fact that $b_n^{(i)} \leq \epsilon$ for $i \geq I$, we conclude that $X_n^{(i)}$ for $i \geq I$ is also a minimizer of the function $\sum_{n=1}^{\infty} \text{Tr}(A_n X_n)$. These imply that $X_n^{(i)}$ for $i \geq I$ is at least a local minimizer of the optimization problem (7). The proves the convergence of the proposed iterative algorithm.

B. Computational Complexity

In each iteration, Algorithm 1 needs to perform the following steps.

1) Solving (8): It needs the solution of an SDP, whose computational complexity is $O(M^3)$ [54].
2) Obtaining $V_n^{(i)}$ and $b_n^{(i)}$: It needs the implementation of a singular value decomposition, whose computational complexity is $O(M^3)$ [55].

Since we have $N$ summation, the computational complexity of solving (8) is $O(N(M^3 + M^3))$. Let us assume that $I$ iterations are required for convergence of Algorithm 1. Therefore, the overall computational complexity of Algorithm 1 is $O(IN(M^3 + M^3))$.

IV. NUMERICAL RESULTS

In this section, numerical results are provided for assessing the performance of the proposed algorithm for beampattern shaping and spectral matching under CM constraint. Toward this end, unless otherwise explicitly stated, we consider the following setup. For transmit parameters, we consider the ULA configuration with $M = 8$ transmitters, with the spacing of $d = \lambda/2$ and each antenna transmits $N = 64$ samples. We consider a uniform sampling of regions $\theta = [-90^\circ, 90^\circ]$ with a grid size of $5^\circ$ and the desired and undesired angels for beampattern shaping are $\Theta_d = [-55^\circ, -35^\circ] \cup [-30^\circ, 30^\circ]$ and $\Theta_u = [-90^\circ, -60^\circ] \cup [30^\circ, 90^\circ]$, respectively. The DFT point number is set as $N = 5N$, the normalized frequency stopband is set at $U = [0.2, 0.3] \cup [0.62, 0.64] \cup [0.8, 0.85]$, and the absolute spectral mask level is set as $\gamma = 0.01\sqrt{N}$. As to the reference signal for the similarity constraint, we consider that $S_0$ is a set of sequences with a good range ISLR property, which is obtained by the method in [13]. For the optimization problem, we set $n = 0.1$ and $p = 1000$ to approximate constraint (5f). The convex optimization problems are solved via the CVX toolbox [58], and the stop condition for Algorithm 1 is set at $e_1 = 10^{-5}$ and $e_2 = 10^{-4}$, with the maximum iteration of 1000.

A. Convergence

Fig. 1 depicts the convergence behavior of the proposed method in different aspects. In this figure, we consider the maximum admissible value for the similarity parameter, i.e., $\delta = \sqrt{2}$. Fig. 1(a) shows the convergence of $\xi$ to zero. This indicates that the second largest eigenvalue of $X_n$ is

\textsuperscript{2}In order to implement the convex or concave branches of the power function $x^p$ and $e^{-x}$ norms $\|x\|_p$, for general values of $p$, CVX uses an enhanced version of the SDP/SOCP conversion method [56] described by Alizadeh and Goldfarb [57].
negligible in comparison with the largest eigenvalue, therefore resulting in a rank-1 solution for $\tilde{s}_n$. Fig. 1(b) shows that the solution of $X_n$ converges to $\tilde{s}_n$, which confirms our claim about obtaining a rank-1 solution.

To indicate the performance of the proposed method under CM constraint, by defining

$$s_{\max} \triangleq \max_{m \in \{1, \ldots, M\}} \{ |s_{m,n}| \} , \quad s_{\min} \triangleq \min_{m \in \{1, \ldots, M\}} \{ |s_{m,n}| \}$$

(11)

for $m \in \{1, \ldots, M\}$, and $n \in \{1, \ldots, N\}$, in Fig. 1(c), we evaluate the maximum/minimum absolute values of the code entries in $S$. The results indicate that the values of $s_{\max}$ and $s_{\min}$ are converging to a fixed value, which indicates the CM solution of WISE.

In addition, Fig. 1(d) depicts the proposed method’s PAR convergence. In the first step, we have a high PAR value, and as the number of iterations increases, the PAR value converges to 1, indicating the CM solution.

Note that the first iteration in Fig. 1(a)–(c) shows the SDR solution of (8) by dropping (8j) and (8k). As can be seen, the SDR method offers neither rank-1 nor CM solution. Since in the initial step (SDR) we drop the constraints (8j) and (8k), we do not have lower bound or equality energy constraints on variable $S$. By considering those two constraints (which are equivalent to (6h) constraint) in the next steps of the algorithm, we indeed impose the $X_n = \tilde{s}_n \tilde{s}_n^H$ constraints. On the other hand, we have the Diag $X_n = I_M$ constraint, which forces the $S$ variable to be CM. Therefore, in the first iteration, the magnitude of the sequence in Fig. 1(c) is close to zero.

B. Performance

Fig. 2 compares the performance of the proposed method in terms of beampattern shaping and spectral masking with the UNIQUE [13] method in the spatial ISLR minimization mode ($\eta = 1$) as a benchmark. Fig. 2(a) shows the beampattern response of the proposed method and UNIQUE. In this figure, for fair comparison, we drop the spectral masking (8f) and 3-dB main beamwidth (8c) and (8d) constraints. As can be seen, in this case, the proposed method offers almost similar performance (in some undesired angles deeper nulls) as compared to the UNIQUE method. However, considering the spectral masking (8f) and 3-dB main beamwidth (8c) and (8d) constraints, the proposed method is able to steer the beam toward the desired and steer the nulls at undesired angles simultaneously.

The beampattern response of WISE at the desired angle region and the spectrum response of the proposed method has better performance compared to the UNIQUE method. Fig. 2(b) shows the main beamwidth response of the proposed method and UNIQUE. Since UNIQUE does not have the 3-dB main beamwidth constraint, it does not have a good main beamwidth response. However, the 3-dB main beamwidth constraint incorporated in our framework improves the main beamwidth response. Besides, the maximum beampattern response is located at $-45^\circ$ in the proposed method, while there is a deviation in the UNIQUE method. On the other hand, Fig. 2(c) shows the spectrum response of the proposed method. Observe that the waveform obtained by WISE masks the spectral power in the stopband region ($\mathcal{U}$) below the $\gamma$ value. However, since the UNIQUE method is not spectral compatible, it is unable to
put notches on the stopbands. Furthermore, as can be seen, the spectral of the transmitters in UNIQUE are the same. On other words, in order to obtain a good beampattern response, the UNIQUE method offers high correlated waveforms. This shows the contradiction of the steering beampattern with orthogonality.

C. Impact of Similarity Parameter

In this subsection, we evaluate the impact of choosing the similarity parameter $\delta$ on the performance of the proposed method. When we consider the maximum admissible value for the similarity parameter, i.e., $\delta = \sqrt{2}$, we do not include the similarity constraint, and by decreasing $\delta$, we have the degree of freedom to enforce properties similar to the reference waveform on the optimal waveform. As mentioned in Section IV, we consider $\mathcal{S}_0$ to be a set of sequences with a good range ISLR property as the reference signal for the similarity constraint, which is obtained by the UNIQUE method [13]. Therefore, by decreasing $\delta$, we obtain a waveform with good orthogonality, which leads to omnidirectional beampattern.

Fig. 3 shows the beampattern response of the proposed method with different values of $\delta$. As can be seen, with $\delta = \sqrt{2}$, we obtain an optimized beampattern, and by decreasing $\delta$, the beampattern gradually tends to be omnidirectional.

On the other hand, Fig. 4(a), (c), and (e) shows the correlation level of the fourth transmitter of the proposed method with other transmitters with different values of $\delta$. Observe that with $\delta = \sqrt{2}$, we obtain a fully correlated waveform, and by decreasing $\delta$, the waveform gradually becomes uncorrelated. Thus, having orthogonality and beampattern shaping at the same time is incompatible, and the choice of $\delta$ effects a tradeoff between the two and enhance the performance of radar system [13]. Besides, Fig. 4(b), (d), and (f) shows the spectrum of the proposed method with different values of $\delta$. As can be seen in all the cases, the proposed method is able to perform the spectral masking. Furthermore, observe that, in the desired frequency region (for instance, in the region $[0.36, 0.69]$), when $\delta = \sqrt{2}$, the spectral responses of the transmitter waveforms are more similar to the lower values of $\delta$. This observation verifies
that more similar spectral response leads to more correlated waveforms.

D. Impact of $\hat{N}$

Fig. 5 shows the impact of choosing $\hat{N}$ on the spectral response of the proposed method. In this figure, we indicate the DFT points as $N_{fft}$. Fig. 5(a) shows the spectrum response of WISE when we do not have zero padding ($\hat{N} = N$) and $N_{fft} = N$. In this case, the proposed method is able to mask the spectral response on undesired frequencies. When we increase the DFT point to $N_{fft} = 5N$, some spikes are appeared on the $U$ region [see Fig. 5(b)]. However, as can be seen from Fig. 5(c) and (d), when we apply zero padding to $\hat{N} = 5N$, the proposed method is able to mask the spectral response on undesired frequencies for both $N_{fft} = 5N$ and $N_{fft} = 10N$, respectively.

E. Impact of CM in the Intermediate Frequency (IF) Band

In general, in order to transmit the baseband waveform in radar system, it needs to be translated to IF and radio frequency (RF) band, respectively, and subsequently transmit through antennas. Digital upconverter (DUC) is the conventional method to translate the baseband signal to IF in the digital domain, which increases the sample rate and provides a real signal. The DUC consists of three main blocks: interpolator, low-pass finite impulse response (FIR) filter, and complex mixer. The DUC output passes through the digital-to-analog converter to generate the analog IF signal. Then, that signal passes through the bandpass filter, the amplifier, and the mixer to translate to the RF band. Finally, the RF signal passes the power amplifier (PA) for transmitting through antennas.

In this subsection, we evaluate the impact of the DUC on the output power with constant and non-CM baseband
Fig. 5. Impact of choosing $\hat{V}$ and $N_{ff}$ on spectral response ($M = 8$, $N = 64$, and $\Theta_d = [-55^\circ, -35^\circ]$ and $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, $\U = [0.12, 0.14] \cup [0.3, 0.35] \cup [0.7, 0.8]$, and $\gamma = 0.01\sqrt{N}$). (a) ($\hat{N} = N$ and $N_{ff} = N$). (b) $\hat{N} = N$ and $N_{ff} = 5N$. (c) $\hat{N} = 5N$ and $N_{ff} = 5N$. (d) $\hat{N} = 5N$ and $N_{ff} = 10N$.

Fig. 6. Comparing the constellation of WISE and SDR methods in baseband ($M = 8$, $N = 64$, $\hat{N} = N$, $\delta = \sqrt{2}$, $\Theta_d = [-55^\circ, -35^\circ]$, $\Theta_u = [-90^\circ, -60^\circ] \cup [-30^\circ, 90^\circ]$, $\U = [0.3, 0.4]$, and $\gamma = 0.01\sqrt{N}$).

signal. In this regard, Fig. 6 depicts the normalized constellation of the optimized waveforms obtained by two proposed methods in this article, say WISE and SDR. As can be seen, unlike the SDR method, WISE offers CM waveform.

We pass these two optimized waveforms through the DUC to obtain the IF band signal in the digital domain. As a simulation setup, we consider the baseband signal bandwidth to be $b_w = 1$ MHz, and it is sampled with a rate of $f_s = 6$ MHz. We use upsampling as kernel for interpolation with factor of $Q = 8$. We design a low-pass FIR filter with length, cutoff frequency, and window of $F_l = 100$, $f_{cut} = 3.5$ MHz, and “Hamming,” respectively.

Fig. 7 compares the power level of the optimized waveforms obtained by WISE and SDR after the DUC when the reference signal in the similarity constraint is randomly chosen in 100 trials. In this figure, $P_{\text{WISE}}$, $\bar{P}_{\text{WISE}}$, $\sigma_{\text{WISE}}$, $P_{\text{SDR}}$, $\bar{P}_{\text{SDR}}$, and $\sigma_{\text{SDR}}$ denote the instant, mean, and standard deviation of the power of WISE and SDR, respectively. In addition, we normalized the maximum output power of the DUC of both the waveforms to 1. Based on Fig. 7, two important facts can be concluded.

1) The average power of SDR is $\bar{P}_{\text{SDR}} = 0.7742$, whereas the average power of WISE is $\bar{P}_{\text{WISE}} = 0.9960$. Therefore, WISE offers higher average power rather than the SDR waveform (almost more than 28%).

2) The standard deviation of SDR is $\sigma_{\text{SDR}} = 0.0803$, whereas the standard deviation of WISE is $\sigma_{\text{WISE}} = 0.0015$. Therefore, WISE offers lower deviation rather than the SDR waveform (almost 53 times lower).

In order to utilize the maximum power efficiency, the input power of the PA should be constant as much as possible [59] (see Appendix B). Therefore, the WISE (CM waveform) is attractive for radar designers since they can drive the PAs at their maximum efficiency.
Fig. 7. Comparing the IF power of (a) SDR and (b) SDR methods after the DUC for 100 trials $(M = 8, N = 64, \hat{N} = N, \delta = \sqrt{2}, \Theta_1 = [-55°, -35°], \Theta_2 = [-90°, -60°] \cup [-30°, 90°], \mu = [0.3, 0.4], \gamma = 0.01\sqrt{\hat{N}}, \beta_w = 1$ MHz, $f_s = 6$ MHz, $f_c = 12$ MHz, $N = 1024$, and $Q = 8)$. (a) SDR. (b) WISE.

Fig. 8. Nonlinearity behavior of the PA [61].

V. CONCLUSION

In this article, we discussed about the problem of beam-pattern shaping with practical constraints in MIMO radar systems, namely, spectral masking, 3-dB beamwidth, CM, and similarity constraints. Solving this problem, not considered hitherto, enabled us to control the performance of MIMO radar in three domains, namely, spatial, spectral, and orthogonality (by similarity constraints). Accordingly, we considered a waveform design approach for the beampattern shaping optimization problem. The aforementioned problem led to a nonconvex and NP-hard optimization problem. In order to obtain a local optimum solution of the problem, first, by introducing a slack variable, we converted the optimization problem to a linear problem with a rank-1 constraint. Then, to tackle the problem, we proposed an iterative method to obtain the rank-1 solution. Numerical results showed that the proposed method is able to manage the resources efficiently to obtain the best performance.

APPENDIX A

It is readily confirmed that the constraint $X_n = \bar{s}_n\bar{s}_n^H$ is equivalent to $\text{Rank}(X_n - \bar{s}_n\bar{s}_n^H) = 0$. Furthermore, it can be equivalently expressed as $1 + \text{Rank}(X_n - \bar{s}_n\bar{s}_n^H) = 1$. Since 1 is positive definite, it follows from the Guttman rank
additivity formula \[60\] that \(1 + \text{Rank}(X_n - \bar{s}_n \bar{s}_n^T) = \text{Rank}(Q_n)\). Moreover, it follows from \(X_n = \bar{s}_n \bar{s}_n^T\) and \(1 > 0\) that \(Q_n\) has to be positive semidefinite. These imply that the constraint \(X_n = \bar{s}_n \bar{s}_n^T\) in (6) can be replaced with a rank and semidefinite constraints on matrix \(Q_n\). Hence, the optimization problem (6) can be recast as follows:

\[
\begin{align}
\min_{X_n} & \quad \sum_{n=1}^{N} \text{Tr}(A_n X_n) \\
\text{s.t.} & \quad (6b) - (6g) \\
& \quad Q_n \succ 0 \\
& \quad \text{Rank}(Q_n) = 1.
\end{align}
\]

Now, we show that the optimization problem (7) is equivalent to (12). Let \(\rho_{n,1} \leq \rho_{n,2} \leq \cdots \leq \rho_{n,M+1}\) and \(v_{n,1} \leq v_{n,2} \leq \cdots \leq v_{n,M}\) denote the eigenvalues of \(Q_n\) and \(V_n^T Q_n V_n\), respectively. From the constraint \(t_{n} I_M - V_n^T Q_n V_n \succ 0\), we have \(v_{n,i} \leq \rho_{n,i} \leq v_{n,i}, 1, 2, \ldots, M\) for any \(V_n\) and \(Q_n\) in the feasible set of (7). In addition, it follows from [51, Corollary 4.3.16] that \(0 \leq \rho_{n,i} \leq v_{n,i}, i = 1, 2, \ldots, M\) for any \(V_n\) and \(Q_n\) in the feasible set of (7). Hence, we observe that

\[
0 \preceq \text{Diag} \left( \left[ \rho_{n,1}, \ldots, \rho_{n,M} \right]^T \right) \preceq \text{Diag} \left( v_{n,1}, \ldots, v_{n,M} \right) \preceq b_n I_M
\]

for any \(V_n\) and \(Q_n\) in the feasible set of (7). It is easily observed from (7) and (13) that, by properly selecting \(\eta\), the optimum value of \(V_n\) will be equal to the eigenvectors of \(Q_n\), corresponding to its \(M\) smallest eigenvalues, and the optimum values of \(b_n, \rho_{n,1}, \ldots, \rho_{n,M}, v_{n,1}, \ldots, v_{n,M}\) will be equal to zero. This implies that the optimum value of \(Q_n\) in (13) possesses one nonzero and \(M\) zero eigenvalues. This completes the proof.

APPENDIX B

Fig. 8 shows the nonlinearity behavior of a PA. Based on this figure, the PA curve is linear when the input power is between a sensitivity level and IP1 dB value. Beyond that, by increasing the input level, the output level will be entered to the nonlinear region and converges to a saturation power. Therefore, the maximum power that can be obtained by the PA without distortion is OP1 dB, corresponding to the IP1 dB input level. It means that the deviation of the input power should not be large near the IP1 dB value.

REFERENCES

[1] T. Aittomaki and V. Koivunen, “Low-complexity method for transmit beamforming in MIMO radars,” in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process., 2007, pp. II-305–II-308.
[2] Z. Cheng, Z. He, S. Zhang, and J. Li, “Constant modulus waveform design for MIMO radar transmit beampattern,” IEEE Trans. Signal Process., vol. 65, no. 18, pp. 4912–4923, Sep. 2017.
[23] X. Yu, G. Cui, L. Kong, J. Li, and G. Gui, “Constrained waveform design for colocated MIMO radar with uncertain steering matrices,” IEEE Trans. Aerosp. Electron. Syst., vol. 55, no. 1, pp. 356–370, Feb. 2019.

[24] J. Li and P. Stoica, MIMO Radar Signal Processing. Hoboken, NJ: USA:Wiley, 2009.

[25] H. He, P. Stoica, and J. Li, “Designing unimodular sequence sets with good correlations—Including an application to MIMO radar,” IEEE Trans. Signal Process., vol. 57, no. 11, pp. 4391–4405, Nov. 2009.

[26] H. He, J. Li, and P. Stoica, Waveform Design for Active Sensing Systems: A Computational Approach. Cambridge, U.K.: Cambridge Univ. Press, 2012.

[27] G. Cui, X. Yu, M. Piezzo, and L. Kong, “Constant modulus sequence set design with good correlation properties,” Signal Process., vol. 139, pp. 75–85, 2017.

[28] Y. Li, S. A. Vorobyov, and Z. He, “Design of multiple unimodular waveforms with low auto- and cross-correlations for radar via majorization-minimization,” in Proc. 24th Eur. Signal Process. Conf., 2016, pp. 2235–2239.

[29] Y. Li and S. A. Vorobyov, “Fast algorithms for designing unimodular waveform(s) with good correlation properties.” IEEE Trans. Signal Process., vol. 66, no. 5, pp. 1197–1212, Mar. 2018.

[30] J. Song, P. Babu, and D. P. Palomar, “Sequence set design with good correlation properties via majorization-minimization,” IEEE Trans. Signal Process., vol. 64, no. 11, pp. 2866–2879, Jun. 2016.

[31] M. Alae-Kerahroodi, M. Modarres-Hashemi, and M. M. Naghsh, “Designing sets of binary sequences for MIMO radar systems,” IEEE Trans. Signal Process., vol. 67, no. 13, pp. 3347–3360, Jul. 2019.

[32] E. Raei, M. Alae-Kerahroodi, P. Babu, and M. R. B. Shankar, “Design of MIMO radar waveforms based on lp-norm criteria,” 2021, arXiv:2104.03190.

[33] D. R. Fuhrmann and G. S. Antonio, “Transmit beamforming for MIMO radar systems using signal cross-correlation,” IEEE Trans. Aerosp. Electron. Syst., vol. 44, no. 1, pp. 171–186, Jan. 2008.

[34] H. Deng, Z. Geng, and B. Himed, “MIMO radar waveform design for transmit beamforming and orthogonality,” IEEE Trans. Aerosp. Electron. Syst., vol. 52, no. 3, pp. 1421–1433, Jun. 2016.

[35] K. Alhujaili, V. Monga, and M. Rangaswamy, “MIMO radar beampattern design under joint constant modulus and orthogonality constraints,” in Proc. 52nd Asilomar Conf. Signals, Syst., Comput., 2018, pp. 1899–1904.

[36] M. Deng, Z. Cheng, Y. Lu, Z. He, and G. Ren, “Waveform design for MIMO radar transmit beampattern formation with good range sidelobes,” in Proc. IEEE Radar Conf., 2019, pp. 1–5.

[37] J. Guerci, Cognitive Radar: The Knowledge-Aided Fully Adaptive Approach (see. Artech House Radar Library). Norwood, MA, USA: Artech House, 2010.

[38] M. Alae-Kerahroodi, E. Raei, S. Kumar, and B. S. M. R. Rao, “Cognitive radar waveform design and prototype for coexistence with communications,” in IEEE Sensors J., vol. 22, no. 10, pp. 9787–9802, May 15, 2022, doi: 10.1109/JSEN.2022.3165348.

[39] H. He, P. Stoica, and J. Li, “Waveform design with stopband and correlation constraints for cognitive radar,” in Proc. 2nd Int. Workshop Cogn. Inf. Process., 2010, pp. 344–349.

[40] R. Rowe, P. Stoica, and J. Li, “Spectrally constrained waveform design [sp tips & tricks],” IEEE Signal Process. Mag., vol. 31, no. 3, pp. 157–162, May 2014.

[41] J. Liang, H. C. So, C. S. Leung, J. Li, and A. Farina, “Waveform design with unit modulus and spectral shape constraints via Lagrange programming neural network,” IEEE J. Sel. Topics Signal Process., vol. 9, no. 8, pp. 1377–1386, Dec. 2015.

[42] J. Liang, H. C. So, J. Li, and A. Farina, “Unimodular sequence design based on alternating direction method of multipliers,” IEEE Trans. Signal Process., vol. 64, no. 20, pp. 5367–5381, Oct. 2016.

[43] K. Alhujaili, X. Yu, G. Cui, and V. Monga, “Spectrally compatible MIMO radar beampattern design under constant modulus constraints,” IEEE Trans. Aerosp. Electron. Syst., vol. 56, no. 6, pp. 4749–4766, Dec. 2020.

[44] M. Alae-Kerahroodi, S. Kumar, M. R. B. Shankar, and K. V. Mishra, “Discrete-phase sequence design with stopband and PSL constraints for cognitive radar,” in Proc. 17th Eur. Radar Conf., 2021, pp. 17–20.

[45] B. Tang and J. Li, “Spectrally constrained MIMO radar waveform design based on mutual information,” IEEE Trans. Signal Process., vol. 67, no. 3, pp. 821–834, Feb. 2019.

[46] A. Aubry, A. De Maio, M. Govoni, and L. Martino, “On the design of multi-spectrally constrained constant modulus radar signals,” IEEE Trans. Signal Process., vol. 68, pp. 2231–2243, 2020.

[47] Y. Jing, J. Liang, D. Zhou, and H. C. So, “Spectrally constrained unimodular sequence design without spectral level mask,” IEEE Signal Process. Lett., vol. 25, no. 7, pp. 1004–1008, Jul. 2018.

[48] L. Wu and D. P. Palomar, “Sequence design for spectral shaping via minimization of regularized spectral level ratio,” IEEE Trans. Signal Process., vol. 67, no. 18, pp. 4683–4695, Sep. 2019.

[49] B. Kang, O. Aldayel, V. Monga, and M. Rangaswamy, “Spatio-spectral radar beampattern design for coexistence with wireless communication systems,” IEEE Trans. Aerosp. Electron. Syst., vol. 55, no. 2, pp. 644–657, Apr. 2019.

[50] K. Shen and W. Yu, “Fractional programming for communication systems—Part I: Power control and beamforming,” IEEE Trans. Signal Process., vol. 66, no. 10, pp. 2616–2630, May 2018.

[51] R. A. Horn and C. R. Johnson, Matrix Analysis. Cambridge, U.K.: Cambridge Univ. Press, 2012.

[52] J. C. Bezdek and R. J. Hathaway, “Convergence of alternating optimization,” Neural, Parallel Sci. Comput., vol. 11, no. 4, pp. 351–368, 2003.

[53] J. R. Senning, “Computing and estimating the rate of convergence,” Gordon College, Dept. Math. Comput. Sci., Accessed: Aug. 9, 2022. [Online]. Available: https://www.math-cs.gordon.edu/courses/mas342/handouts/rate.pdf.

[54] Z. Luo, W. Ma, A. M. So, Y. Ye, and S. Zhang, “Semidefinite relaxation of quadratic optimization problems,” IEEE Signal Process. Mag., vol. 27, no. 3, pp. 20–34, May 2010.

[55] G. H. Golub and C. F. Van Loan, Matrix Computations, 3rd ed.Baltimore, MD, USA: Johns Hopkins Univ. Press, 1996.

[56] Advanced Topics, Power Functions and p-Norms. Accessed: Jul. 6, 2022. [Online]. Available: http://cvxr.com/cvx/doc/advanced.html#powerfunc.

[57] F. Alizadeh and D. Goldfarb, “Second-order cone programming,” Math. Program., vol. 95, pp. 3–51, 2001.

[58] CVX Package. Accessed: Jun. 7, 2021. [Online]. Available: http://cvxopt.org/cvx/.

[59] A. Aubry, V. Carotenuto, A. De Maio, A. Farina, A. Izzo, and R. S. L. Morlello, “Assessing power amplifier nonlinearities on digital predistortion on radar waveforms for spectral coexistence,” IEEE Trans. Aerosp. Electron. Syst., vol. 58, no. 1, pp. 635–650, Feb. 2022.

[60] M. S. Gowda and R. Sznajder, “Schur complements, Schur decompositions, and their applications,” Linear Algebra Appl., vol. 30, pp. 589–601, 1992.

[61] RAQ Issue 195: A Guide for Choosing the Right RF Amplifier for Your Application. Accessed: Feb. 25, 2022. [Online]. Available: https://www.analog.com/en/analog-dialogue/raqs/raq-issue-195.html.

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