SQUEEZED GLUON CONDENSATE AND QUARK CONFINEMENT
IN THE GLOBAL COLOR MODEL OF QCD

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We discuss how the presence of a squeezed gluon vacuum might lead to quark confinement in the framework of the global colour model of QCD. Using reduced phase space quantization of massive vector theory we construct a Lorentz invariant and colourless squeezed gluon condensate and show that it induces a permanent, nonlocal quark interaction (delta-function in 4-momentum space), which according to Munczek and Nemirovsky might lead to quark confinement. Our approach makes it possible to relate the strength of this effective confining quark interaction to the strength of the physical gluon condensate.

Keywords: gluon propagator, squeezed condensate, quark confinement, global colour model

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1. Introduction

Phenomenological models have been developed for the low energy sector of QCD which are rather successful in describing hadrons and their properties. Here the aspect of chiral symmetry is understood much better than that of quark and gluon confinement. The mechanism for dynamical chiral symmetry breaking is identified fairly well since the work by Nambu and Jona-Lasinio (NJL)\(^1\). For recent reviews on the application of the NJL-model to low energy QCD, see\(^2\). For the explanation of confinement many different models have been discussed. We will refer in the present work to approaches based on the global colour model\(^3, 4, 5, 6\) with nonlocal confining quark interactions. These approaches address quark and gluon confinement via the criterion of absence of real \(q^2\) poles for the propagators\(^7\) for a review see\(^5\). Such confining quark models use effective gluon propagators, which have infrared singularities like \(1/k^4\) or a delta function \(\delta(k)\) at low energy\(^7, 3, 4, 8, 9\). These phenomenological approaches have proven successful in explaining low energy hadronic observables\(^9\) and in addressing the problem of chiral vs. deconfinement transition at finite temperatures\(^6\). However, the question for the mechanism which leads to the infrared singularities of the gluon propagator remains open. There is evidence from detailed studies\(^11, 12\) of the Schwinger-Dyson equations of QCD for a strong infrared enhancement of the gluon propagator due to the non-Abelian character of the theory and in particular due to the gluon-gluon self coupling. On the other hand there are several approaches which relate the confinement problem to the question of the true physical vacuum of QCD.

In more rigorous treatments of QCD, the problem of the vacuum is well known. For instance, the simple perturbative vacuum is unstable\(^13\), and there is no stable (gauge invariant) coherent vacuum in Minkowski space\(^14\). Therefore, in the context of the construction of a gauge invariant, stable QCD vacuum in Minkowski space the squeezed condensate of gluons has become a topic of great interest\(^15\)-\(^20\). From the physical point of view the squeezed state differs from the coherent one by the condensation of colour singlet gluon pairs rather than condensation of single gluons. It has been discussed before that a squeezed vacuum in form of Gaussian fields\(^16, 17\) leads to confinement of quarks via the criterion of the linearly rising static potential\(^17\) and via the area law behaviour of the Wilson loop\(^19\).

The present paper is devoted to the study of confinement as a possible consequence of the squeezed vacuum. Our approach differs from the previous ones addressing the squeezed vacuum\(^15, 16, 17\) in the following ways. We will not try to generate the squeezed vacuum by treating selfinteractions of gluons but we will use the squeezed condensate as a semi-phenomenological input. In difference to e.g.\(^19\) where all gauge degrees of freedom are squeezed and only afterwards a projection to the gauge invariant sector is performed, we shall apply the squeezing transformation only on the physical degrees of freedom. Furthermore, in contrast to previous work we discuss confinement by using the criterion of the absence of poles for the propagators\(^7\) as used in the approaches based on the global colour model\(^3\). We will prepare the squeezed vacuum by macroscopically populating it with zero momentum gluon pairs and study the changes in the analytical properties of the propagators caused by the change of the vacuum structure. This procedure parallels the original idea by Bogoliubov\(^21\) for the explanation of the Landau sound in a superfluid liquid according to which the change in the excitation spectrum of the theory at low energies is due to the condensation of a macroscopic number of particles in the zero momentum state. As a first step we shall assume, inspired by the success of the approaches based on the global colour model, that the gluon selfinteractions in the low energy region only lead to the formation of a squeezed gluon condensate and include it into an otherwise Abelian approximation to QCD. For simplicity we shall ignore here the effects of the gluon selfinteractions in the...
high energy region leading to asymptotic freedom. In order to control the infrared divergence of massless gluons we start in a large finite volume and give the gluons a small mass inversely proportional to the volume. Finally we take the infinite volume limit which leads to a Lorentz invariant squeezed condensate of massless gluons and at the same time to a 4-momentum δ-function interaction between the quarks. A type of delta function interaction has been considered for the computation of the meson spectra on the level of Schwinger-Dyson and Bethe-Salpeter equations in both Minkowski space and Euclidean space. Munczek and Nemirovsky have shown that such a delta function interaction removes the poles of the effective quark propagator constructed by using the Schwinger-Dyson equation. The corresponding Bethe-Salpeter equation on the other hand has bound state solutions.

A well-defined massless limit is not straightforward because the covariant propagator of massive vector bosons has a singularity at mass $M = 0$. Although the singular term drops out when the massive vector field is coupled to a conserved current as in QED, for the explicit construction of the squeezed vacuum it is necessary to have a physical representation of the vector field such that it smoothly turns into the gauge field in the massless limit. We shall use the results of where it has been shown that massive vector theory can be quantized in such a way that its massless limit agrees with the theory of photons. In contrast to the approach by St"uckelberg where ghost fields are introduced in order to maintain manifest Lorentz covariance, in a massive QED theory with a good massless limit is obtained in the framework of reduced phase space quantization without introducing any additional unphysical degrees of freedom, but by choosing certain nonlocal dynamical degrees of freedom.

The present paper is organized as follows: In Section 2 we briefly review how quark confinement in the framework of the global colour model has been discussed by Munczek and Nemirovsky. In Section 3 the squeezed condensate of gluons is constructed. In Section 4 we derive the effective quark Lagrangian in the presence of the squeezed condensate. We show that the effective quark interaction induced by the squeezed condensate is a momentum delta function and relate the strength of the δ-function interaction to the value of the physical gluon condensate. Section 5 finally contains our conclusions.

2. Quark confinement in the global colour model of QCD

The global colour model is a dynamical quark model with an effective current-current interaction, which includes highly nonperturbative gluon dynamics. It can describe many features of hadronic physics quite successfully. In order to briefly review how it results from full QCD, we start with the QCD generating functional $Z_{QCD}$ for Green functions in the absence of external sources

$$Z_{QCD} = \frac{1}{N} \int DAD\Psi D\bar{\Psi} \exp \left[ i \int d^4x \left( L_\Psi + L_A + L_{A\Psi} \right) \right].$$

Here

$$L_\Psi = \bar{\Psi}(x) \left( i \not{\partial} - \hat{m} \right) \Psi(x)$$

is the quark Lagrangian with the diagonal current mass matrix $\hat{m}$ and

$$L_A = -\frac{1}{4} G^{a\mu\nu} G_{a\mu\nu}$$

is the Yang-Mills Lagrangian with the non-Abelian field strength tensor $G_{a\mu\nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + gf^{abc} A_\mu^b A_\nu^c$. We have not included gauge-fixing terms and associated
quantities explicitly since they are not needed for the purpose of our discussion. The quark currents $J_\mu^a \equiv g \bar{\Psi} \gamma_\mu \frac{\partial}{\partial x^\mu} \Psi$ are minimally coupled to the gluon fields

$$\mathcal{L}_{A\Psi} = A_\mu^a J_\mu^a.$$ \hspace{1cm} (4)

After integration over the gluon field $Z_{QCD}$ takes the form

$$Z_{QCD} = \frac{1}{N} \int D\psi D\bar{\psi} \exp \left[ i \int d^4x \mathcal{L}_\Psi + iW(J_\mu^a) \right],$$ \hspace{1cm} (5)

where the gluon action functional

$$W(J_\mu^a) = \sum_{m=2}^{\infty} \frac{1}{m!} \int dx_1 dx_2 \cdots dx_m G_{a_1 \cdots a_m}^{\mu_1 \cdots \mu_m}(x_1, \ldots, x_m) J_{\mu_1}^{a_1}(x_1) \cdots J_{\mu_m}^{a_m}(x_m)$$ \hspace{1cm} (6)

is written as a functional Taylor expansion in terms of the quark currents thus defining $G_{a_1 \cdots a_m}^{\mu_1 \cdots \mu_m}(x_1, \ldots, x_m)$ as the connected $m$-point Green functions for the gluon field. In the global colour model the expansion of $W(J_\mu^a)$ is truncated at the level of the gluon two-point function $G_{\mu\nu}^{ab}(x-y)$. Thus the effective current-current coupling in the global colour model is

$$W_{GCM}[J_\mu^a] = -\frac{1}{2} \int d^4x \int d^4y J_\mu^a(x) G_{\mu\nu}^{ab}(x-y) J_\nu^b(y),$$ \hspace{1cm} (7)

which represents a four-fermion interaction term. In the Feynman gauge the gluon propagator simplifies to

$$G_{\mu\nu}^{ab}(x-y) = -i\delta^{ab}g_{\mu\nu}G(x-y).$$ \hspace{1cm} (8)

Note that for QED the truncation at the level of the two-point function is exact and $G(x-y)$ corresponds to perturbative one-photon exchange. In QCD, on the other hand, the two-point function contains highly nonperturbative contributions due to the gluon selfinteractions, such as the influence of a gluon condensate in the low energy region and asymptotic freedom in the high momentum limit.

At present it is impossible yet to calculate the nonperturbative gluon propagator from QCD. In the global colour model, several ansätze have been used for the nonperturbative gluon propagator. The simplest one, which may lead to quark confinement in the sense of absence of poles on the real $q^2$ axis in the quark propagator has been introduced by Munczek and Nemirovsky in the form

$$i\frac{4}{3}g^2 \frac{G(q)}{(2\pi)^4} = -\frac{1}{4}q^2 \delta^4(q) - U(q)$$ \hspace{1cm} (9)

Using just the $\delta$- function part of the interaction and the rainbow approximation for the vertex function, the Schwinger–Dyson equation for the quark self energy

$$i\Sigma(q) = \frac{4}{3}g^2 \int \frac{dk}{(2\pi)^4} G(k-q)\gamma_\mu S(k)\gamma_\mu,$$ \hspace{1cm} (10)

reduces to an algebraic equation. The solution for the inverse dressed quark propagator is given by

$$iS^{-1}(q) \equiv q - m - \Sigma(q) \equiv qA(q) + B(q),$$ \hspace{1cm} (11)

The term with $m = 1$ does not occur because of the colour neutrality of the vacuum.
where the solution functions \( A(q) \) and \( B(q) \) have been found by Muncek and Nemirovsky to conspire in such a way that \( S^{-1}(q) \) has no zeros for real \( q^2 > 0 \), which according to Refs. is signalling confinement. The coefficient \( \eta^2 = 1.14 \text{ GeV}^2 \) of the \( \delta \)-function ansatz has been fixed in by the rho meson mass value obtained from the solution of the corresponding Bethe-Salpeter equation in the ladder approximation.

A form of the residual interaction \( U(q) \) which reproduces the meson decay constants and provides asymptotic freedom has been given by. However it is not yet known, which approximations have to be made in calculating the effective gluon action \( W[J] \) of QCD such that a successful phenomenological model with confinement and asymptotic freedom like in Ref. can be derived.

We shall show in the present paper that it is possible to obtain the delta function interaction under the assumption that a squeezed condensate of zero-momentum gluon pairs is present in the QCD vacuum.

3. Construction of a squeezed gluon vacuum

It is a generally believed that the gluon selfinteractions in QCD lead to highly nontrivial infrared effects such as physical gluon condensation.

The gluon condensate is defined as the expectation value of the local gluonic operator \( \alpha_s G^{\mu \nu}(x)G_{\mu \nu}(x) \) in the nonperturbative QCD vacuum.

\[
\langle \alpha_s G^2 \rangle \equiv \langle \alpha_s G^{\mu \nu}(0)G_{\mu \nu}(0) \rangle \quad .
\]

Approximate empirical values for the physical gluon condensate are the estimate \( \langle \alpha_s G^2 \rangle \approx 0.04 \text{ GeV}^4 \) by Shifman, Vainshtein and Zakharov and the update average value \( \langle \alpha_s G^2 \rangle = (0.071 \pm 0.009) \text{ GeV}^4 \) obtained by Narison in a recent analysis of heavy quarkonia mass-splittings in QCD.

In the following we shall construct a squeezed gluon vacuum in an explicit way as a possible approximate model for the true QCD vacuum. The infrared singularity of the gluons as well as the their selfinteraction are generally believed to be the origin of the existence of a physical gluon condensate in the QCD vacuum. In phenomenological approaches based on the global colour model, where full QCD is truncated at the level of the gluon two-point function, which is correct only for Abelian theories, the gluon selfinteractions are included in the low energy region via some confinement term and in the high energy region via a term leading to asymptotic freedom. Inspired by their success to explain many features of hadronic physics we shall assume as a first step that the only effect of the gluon selfinteraction is to ensure the existence and stability of the gluon condensate, and shall consider an otherwise Abelian approximation to QCD. For simplicity we shall also neglect the high energy effects due to the gluon selfcoupling such as asymptotic freedom. In order to isolate the infrared singular zero momentum component of the gluon field from the nonzero ones we put the theory into a large but finite volume \( V \).

Furthermore, in order to regularize the infrared singularity we shall give the gluons a small mass. Hence we shall start with the simpler theory of massive Abelian QCD in a large finite volume and shall show that it is possible to include a squeezed gluon condensate of zero momentum gluon pairs in the infinite volume limit by explicit construction. For this we shall let the small mass vanish like \( 1/V \) in the large volume limit after quantization of the theory. By calculating the value of the gluon condensate in the constructed squeezed vacuum in the zero momentum...
approximation we shall then find an estimate for the squeezing strength due to the selfinteraction of the gluons.

### 3.1. Construction of a squeezed gluon vacuum in Abelian QCD

For the construction of the squeezed vacuum we shall first consider the Lagrangian of massive Abelian QCD given by

\[
L_{\text{Abel}}[M, V_\mu, \Psi] = -\frac{1}{4} F^{a}_{\mu\nu} F^{a}_{\mu\nu} + \frac{1}{2} M^{2} V^{a}_{\mu}^{2} + \bar{\Psi}(i\partial - m)\Psi - J^{a}_{\mu} V^{a}_{\mu},
\]

with the Abelian field strengths \( F^{a}_{\mu\nu} = \partial_{\mu} V^{a}_{\nu} - \partial_{\nu} V^{a}_{\mu} \). In order to construct the squeezed vacuum as described above, it is necessary to quantize massive Abelian QCD in such a way that it has a good massless limit. Following the work of 22 we shall use reduced phase space quantization, where this can be achieved without the introduction of additional unphysical ghost degrees of freedom.

#### 3.1.1. Reduction to physical variables

The Euler-Lagrange equation corresponding to \( V^{a}_{0} \) is not an equation of motion but a constraint. \( V^{a}_{0} \) is therefore not a dynamical variable and has to be eliminated using the constraint equation before the remaining reduced phase space can be quantized. As shown in 22 and briefly reviewed in the Appendix, the reduced Lagrangian obtained from (13) by eliminating \( V^{a}_{0} \) can be expressed in terms of new nonlocal variables \( \tilde{V}^{a}_{k} \) and \( \tilde{\Psi} \) as

\[
L_{\text{red}}^{\text{Abel}}[\tilde{V}^{a}_{k}, \tilde{\Psi}] = \frac{1}{2} \left( \dot{\tilde{V}}^{a}_{i} P^{-1}_{ij} \dot{\tilde{V}}^{a}_{j} + \tilde{V}^{a}_{i} (\partial^{2} - M^{2}) P^{-1}_{ij} \tilde{V}^{a}_{j} \right) - J^{a}_{i} \tilde{V}^{a}_{i} + \frac{1}{2} \frac{J^{a}_{0}}{\partial^{2} - M^{2}} J^{a}_{0} + \bar{\tilde{\Psi}}(i\partial - m)\tilde{\Psi}.
\]

The nonlocal fields \( \tilde{V}^{a}_{k} \) are defined as

\[
\tilde{V}^{a}_{k} = P_{kj} V^{a}_{j}
\]

with \( P_{ij} \) given by

\[
P_{ij} = \delta_{ij} - \frac{\partial_{i} \partial_{j}}{\partial^{2} - M^{2}} = \delta_{ij}^{T} - \frac{M^{2}}{\partial^{2} - M^{2}} \delta_{ij}^{||}
\]

where we have used the longitudinal and transverse projection operators \( \delta_{ij}^{||} \equiv \partial_{i} \partial_{j} / \partial^{2} \) and \( \delta_{ij}^{T} \equiv \delta_{ij} - \delta_{ij}^{||} \). In contrast to the massless case, \( P_{ij} \) is invertible and \( P^{2} \neq P \). The nonlocal fermionic field \( \tilde{\Psi} \) is defined as

\[
\tilde{\Psi} \equiv \exp \left( ig \lambda^{a} \frac{1}{2} \frac{1}{\partial^{2} - M^{2}} \partial_{i} V^{a}_{i} \right) \Psi.
\]

Since this is only a phase transformation, the corresponding currents \( J^{a}_{\mu} \) remain unchanged.

The nonlocal fields \( \tilde{V}^{a}_{i} \) differ from the standard local ones \( V^{a}_{i} \) in that their longitudinal components are shortened relative to those of \( \tilde{V}^{a}_{i} \). In the massless limit they are shortened to zero i.e. projected out and hence reduce to the corresponding
transverse gauge fields. The zero momentum modes of the nonlocal fields on the other hand coincide with those of the local vector field, since $P_{ij}(q = 0) = \delta_{ij}$:

\[
\lim_{M \to 0} \left( \tilde{V}_i^a(t, \tilde{p} = 0), \tilde{\Pi}_i^a(t, \tilde{p} = 0) \right) = (A_i^a(t), \xi_i^a(t))
\]

\[
\lim_{M \to 0} \left( \tilde{V}_i^a(t, \tilde{p}), \tilde{\Pi}_i^a(t, \tilde{p}) \right) = (A_i^{T,a}(t, \tilde{p}), E_i^{T,a}(t, \tilde{p}) \quad \tilde{p} \neq 0
\]

with the three spatial zero momentum components $A_i^a$ and $\xi_i^a$ and the two transverse nonzero momentum components $A_i^{T,a}$ and $E_i^{T,a}$ of the gauge fields and the corresponding canonically conjugate electric fields. For a small mass we can regard the massive nonlocal field as a smooth interpolation between the three gauge invariant zero momentum components and the two transverse components of the high momentum limit. In this scenario one could therefore conclude that the zero momentum component of the Abelian gauge field has three dynamical degrees of freedom in contrast to the non zero momentum components which have only two transverse degrees of freedom.

In order to construct the squeezed gluon vacuum via a canonical transformation we shall now pass to the Hamiltonian formalism. The Hamiltonian corresponding to the theory is now quantized in terms of the nonlocal $\tilde{V}_i^a(x)$ and the canonical conjugate momenta $\tilde{\Pi}_i^a$, $\tilde{\Pi}$ by

\[
H_{\text{free}} = H_0 + H',
\]

where the free and interacting parts read

\[
H_0 = \int d^3 \vec{x} \left\{ \frac{1}{2} \tilde{\Pi}_i^a \tilde{P}_i^a - \tilde{V}_i^a(\tilde{\partial}^2 - M^2) P_{ij}^{-1} V_j^a + \tilde{\Pi}_i^a \gamma_0 (\vec{\gamma} \cdot \tilde{\partial} + m) \bar{\Psi} \right\}
\]

\[
H' = \int d^3 \vec{x} \left( \tilde{V}_i J_i - \frac{1}{2} J_0 \frac{1}{\tilde{p}^2 - M^2} J_0 \right).
\]

Note that the second term in $H'$ corresponds to a Yukawa potential.

The theory is now quantized in terms of the nonlocal $\tilde{V}_i^a(x)$ and the canonical conjugate momenta $\tilde{\Pi}_i^a$ by imposing the canonical commutation relations

\[
i[\tilde{\Pi}_i^a(\vec{x}, t), \tilde{V}_j^b(\vec{y}, t)] = \delta^{ab} \delta_{ij} \delta(\vec{x} - \vec{y}).
\]

The free Hamiltonian is diagonalized by the interaction picture fields

\[
\tilde{v}_i^a(x, t) = \frac{1}{V} \sum_{\vec{q}} \frac{1}{2\omega(q)} \sum_{\lambda = 1,2,3} \left( a^\lambda(\lambda, \vec{q}) \tilde{\epsilon}_i^\lambda(\lambda, \vec{q}) e^{-i(\omega(q)t - \vec{q} \cdot \vec{x})} + \text{h.c.} \right),
\]

\[
\tilde{n}_i^a(x, t) = -i \frac{1}{V} \sum_{\vec{q}} \sqrt{\frac{\omega(q)}{2}} \sum_{\lambda = 1,2,3} \left( q^\lambda(\lambda, \vec{q}) \tilde{\epsilon}_i^\lambda(\lambda, \vec{q}) e^{-i(\omega(q)t - \vec{q} \cdot \vec{x})} - \text{h.c.} \right),
\]

with the creation and annihilation operators satisfying

\[
[a_\alpha(\lambda, \vec{q}), a_\beta^+(\lambda', \vec{q}')] = \delta_{\alpha\beta} \delta_{\lambda\lambda'}(2\pi)^3 \delta(\vec{q} - \vec{q}') ,
\]

and with the real nonlocal polarization vectors $\tilde{\epsilon}_i^\lambda(\lambda, \vec{q})$ and $\epsilon_i^\lambda(\lambda, \vec{q})$ satisfying the completeness relations

\[
\sum_{\lambda} \tilde{\epsilon}_i^\lambda(\lambda, \vec{q}) \epsilon_j^\lambda(\lambda, \vec{q}) = \delta_{ij} - \frac{q_i q_j}{\vec{q}^2 + M^2},
\]

\[
\sum_{\lambda} \epsilon_i^\lambda(\lambda, \vec{q}) \epsilon_j^\lambda(\lambda, \vec{q}) = \delta_{ij} + \frac{q_i q_j}{M^2}.
\]
3.1.2. A simple model for the squeezed vacuum

From the representation (23) we obtain the following expressions for the vacuum expectation values of squares of the zero momentum components of the nonlocal vector fields and their conjugate momenta in the limit of small mass $M$

\[
\lim_{M \to 0} \langle 0 | \tilde{v}_i^a (\vec{q} = 0) | 0 \rangle^2 = \langle 0 | (A_i^a)^2 | 0 \rangle = \frac{12}{MV},
\]

\[
\lim_{M \to 0} \langle 0 | \tilde{\pi}_i^a (\vec{q} = 0) | 0 \rangle^2 = \langle 0 | (E_i^a)^2 | 0 \rangle = \frac{12 M}{V},
\]

using the massless fields $A_i^a$ and $E_i^a$ according to (18). Note that the vacuum expectation values of these zero momentum components are Lorentz invariant expressions as discussed in the Appendix. For $M = 0$ the first term is singular which corresponds to the well known infrared problem common to all massless theories. One way to regularize the infrared singularity is to have a volume dependent mass $M_{sq}(V)$ which vanishes in the infinite volume limit. One particular regularizing choice is to let $M_{sq}$ behave inversely proportional to the volume,

\[
M_{sq} = \frac{1}{2C_0 V},
\]

with some open parameter $C_0$, which will be fixed below. The corresponding Fock vacuum turns into the squeezed vacuum as will be shown in the following. We denote the vacuum corresponding to the mass (28) by $0_{sq}$. Using the mass dependence (28) in (27), one easily finds the following expectation values for the squares of $A_i^a$ and $E_i^a$ in the infinite volume limit

\[
\langle 0_{sq} | (A_i^a)^2 | 0_{sq} \rangle = 24 C_0,
\]

\[
\langle 0_{sq} | (E_i^a)^2 | 0_{sq} \rangle = O(V^{-2}).
\]

Such vacua are called squeezed vacua [14].

In order to understand the meaning of the squeezed vacuum better it is useful to consider for comparison the case of a perturbative vacuum $0$ corresponding to a very small but finite and volume independent mass $M$. For a large finite volume the squeezed vacuum $0_{sq}$ can be related to the perturbative vacuum $0$ by a unitary transformation

\[
0_{sq} = U_{sq}^{-1} 0,
\]

whose action on the zero momentum components of the fields is given by the unitary squeezing operator

\[
U_{sq}^{(0)} = \exp \left[ i f_0 \left( A_i^a E_i^a + E_i^a A_i^a \right) \right]
\]

with the squeezing parameter

\[
f_0 = \frac{1}{2} \ln (2MC_0V).
\]

We see that the coefficient $C_0$ is a measure of the squeezing strength and is an open parameter to be fixed below. The squeezing operation (31) corresponds to a Bogoliubov transformation. Bogoliubov [1] used such a transformation for nonzero momentum modes to rediagonalize the Hamiltonian after having macroscopically filled the zero-momentum mode with single particles. In difference to such a homogeneous coherent condensate which corresponds to a shift of the zero-momentum
field operator by a macroscopic c-number, the homogeneous squeezed condensate \(^{(29)}\) is obtained by the multiplicative operation on the zero momentum modes:

\[
U^{(0)}_{\text{sq}} a_i U^{(0)-1}_{\text{sq}} = e^{f_0 A_i},
\]

\[
U^{(0)}_{\text{sq}} e_i U^{(0)-1}_{\text{sq}} = e^{-f_0 E_i}.
\]

Like the coherent transformation, the squeezing transformation is a canonical one, since it leaves the canonical commutator invariant. Although it is possible in a finite volume to relate the squeezed vacuum \( |0_{\text{sq}}\rangle \) to the perturbative vacuum \( |0\rangle \) via the unitary squeezing operator, it is important to note that these two vacua become unitarily inequivalent in the infinite volume limit \(^{28}\).

### 3.2. Gluon selfinteraction and the stability of the squeezed vacuum

After having constructed a squeezed gluon vacuum in an explicit way we shall see that the gluon selfinteraction is a necessary condition for the stability of the squeezed vacuum as a candidate for the true QCD vacuum. For this reason we shall now relate the coefficient \( C_0 \) to the value of the gluon condensate in our model of the squeezed vacuum. Neglecting the nonzero momentum modes we find for the value of the gluon condensate in the squeezed vacuum

\[
\langle 0_{\text{sq}} | \alpha_s G_{\mu\nu} G^{\alpha\beta\mu\nu} | 0_{\text{sq}} \rangle_{\text{zero mom}} = 2 \langle 0_{\text{sq}} | \alpha_s (B_i^a)^2 | 0_{\text{sq}} \rangle - 2 \langle 0_{\text{sq}} | \alpha_s (E_i^a)^2 | 0_{\text{sq}} \rangle
\]

where we have written \( B_i^a \equiv g\epsilon_{ijk} f^{abc} A_j^b A_k^c \). Noting that the expectation value of the electric field in the squeezed condensate vanishes, see \(^{(29)}\), and using Wick’s theorem to express the expectation value of the magnetic field in terms of the contraction \( C_0 \) we obtain

\[
\langle 0_{\text{sq}} | \alpha_s G_{\mu\nu} G^{\alpha\mu\nu} | 0_{\text{sq}} \rangle_{\text{zero mom}} = 2\alpha_s^2 N_c^2 - \frac{1}{3} \frac{3}{N_c} (3N_c C_0)^2
\]

We see here that in our model of a squeezed vacuum of zero momentum gluon pairs the self interaction of the gluons is essential for the coefficient \( C_0 \) to be nonzero. In an Abelian model \( C_0 \) would be zero and the squeezed gluon vacuum constructed in the last paragraph unstable. One can regard the squeezed condensate as a tool for integrating out gluons including nonperturbative infrared effects.

### 4. Effective quark action in the squeezed gluon vacuum

In this section we shall derive the effective quark action of Abelian QCD in the presence of the squeezed condensate constructed in the last section.

#### 4.1. Integrating out the gluon fields

We write the generating functional for the massive Abelian QCD model defined in \(^{(13)}\) in the form

\[
Z_{\text{Abel}}[M] = \int D\bar{\Psi} D\Psi \exp \left[ i \int d^4x L_{\text{Abel}}[M] \right] = \int D\bar{\Psi} D\Psi Z[J, M] \exp \left[ i \int d^4x L_{\Psi} \right]
\]

with the generating functional \( Z[J, M] \) originating from integrating out the gluons. From the expression \(^{(13)}\) for the reduced Hamiltonian we obtain in a large but finite volume \( V \)

\[
Z_V[J_\mu, M] = Z_V^{(1)}[J_0] \cdot Z_V^{(2)}[J_1]
\]

\(^{36}\)
with the two parts

\[ Z^{(1)}_V[J_i, M] = \langle 0 | T \exp \left( \frac{i}{\hbar} \int d^3x \int dt \tilde{v}^a_i J^a_i \right) | 0 \rangle , \]  
\[ Z^{(2)}_V[J_0, M] = \exp \left[ -\frac{i}{2} \int dt \int d^3x \int d^3y J^a_0(\vec{x}, t) \right] e^{-M|x-y|} \frac{e^{-M|x-y|}}{4\pi|x-y|} J^b_0(\vec{y}, t) \right] . \]  

Using Wick’s theorem we can write

\[ Z^{(1)}_V[J_i, M] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4y J^a_0(x) \langle 0 | T \tilde{v}^a_i(x) \tilde{v}^b_j(y) | 0 \rangle J^b_0(y) \right\} \]  

with the causal Green function

\[ \langle 0 | T \tilde{v}^a_i(x) \tilde{v}^b_j(y) | 0 \rangle = \delta^{ab} \delta_{ij} \frac{e^{-iM|x-y|}}{2MV} + \delta^{ab} \frac{1}{V} \sum_{q \neq 0} e^{i\vec{q} \cdot (\vec{x}-\vec{y})} \frac{e^{-i\omega(q)|x-y|}}{2\omega(q)} \left( \delta_{ij} - \frac{q_i q_j}{q^2 + M^2} \right) \]  

where

\[ \omega(q) = \sqrt{q^2 + M^2} . \]  

With these results for a large finite volume in hand we shall now study the infinite volume limit \( Z[J] \equiv \lim_{V \to \infty} Z_V[J] \) for the two different cases, the perturbative and the squeezed vacuum. For the first case we keep the mass \( M \) finite while taking the infinite volume limit and only afterwards set \( M = 0 \). For the case of the squeezed vacuum on the other hand we vary the mass \( M = 1/2C_0V \) with the volume \( V \) according to (28).

### 4.2. Generating functional for the perturbative vacuum

For the case when \( M \) is nonvanishing and constant with respect to \( V \), we find in the infinite volume limit the standard result

\[ \lim_{V \to \infty} \langle 0 | T \tilde{v}^a_i(x) \tilde{v}^b_j(y) | 0 \rangle = \delta^{ab} \frac{1}{(2\pi)^3} \int \frac{d^3q}{2\omega(q)} e^{-i\omega(q)|x-y|} \left( \delta_{ij} - \frac{q_i q_j}{q^2 + M^2} \right) \]  

\[ = \delta^{ab} \frac{1}{(2\pi)^4} \int \frac{d^4q}{q^2 - M^2 - i\epsilon} \left( \delta_{ij} - \frac{q_i q_j}{q^2 + M^2} \right) \]  

\[ \equiv \delta^{ab} \hat{D}_{ij}(M ; x - y) . \]  

Formula (41), with (13) inserted, together with (28) yields the corresponding generating functional

\[ Z[J_\mu, M] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4y J^\mu_0 \hat{D}_{\mu\nu}(M ; x - y) J_{0'}^\nu \right\} \]  

with

\[ \hat{D}_{\mu\nu}(M ; x - y) \equiv \delta_{\mu0}\delta_{\nu0} \hat{D}_{00}(M ; x - y) + \delta_{\mu i} \delta_{\nu j} \hat{D}_{ij}(M ; x - y) , \]  

where the spatial components \( \hat{D}_{ij}(M ; x - y) \) are given by (33) and the time components are defined by

\[ \hat{D}_{00}(M ; x - y) \equiv \frac{e^{-M|x-y|}}{4\pi|x-y|} = \int \frac{d^4q}{(2\pi)^4} \frac{e^{-i\omega(q)(x-y)}}{q^2 + M^2} . \]
corresponding to the Yukawa potential in (39). Taking now $M = 0$, after the infinite volume limit, and noting that the Abelian $J^\mu_a$ are conserved, we find the standard generating functional for the perturbative gluon vacuum

$$Z_{\text{pert}}[J^\mu] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4y J^\mu_a(x) D(x-y) J^\mu_a(y) \right\}$$

(47)

with the Feynman gauge gluon propagator

$$D(x-y) \equiv -i \int \frac{d^4q}{(2\pi)^4} \frac{e^{-iq \cdot (x-y)}}{q^2 + i\epsilon}.$$  

(48)

4.3. Generating functional for the squeezed gluon vacuum

In the case of a squeezed vacuum with $M_{sq} = C_0/(2V)$, the two-point Green function (41) becomes in the infinite volume limit

$$\lim_{V \to \infty} \langle 0_{sq} | T \bar{v}_i^a(x) v^b_j(y) | 0_{sq} \rangle > = \delta^{ab} \left( \bar{D}_{ij}(M = 0; x-y) + \delta_{ij} C_0 \right).$$

(49)

The extra constant term $C_0$ is due to the presence of the squeezed condensate. Using (40) and (39), now with (49) instead of (43), we obtain

$$Z_{sq}[J^\mu] = Z_{\text{pert}}[J^\mu] \cdot \exp \left[ -\frac{C_0}{2} \left( \int d^4x J^\mu_a(x) \right)^2 \right]$$

(50)

with $Z_{\text{pert}}[J^\mu]$ given by (47). In order to obtain the corresponding effective quark action it is useful to write the squeezed generating functional (50) in the form

$$Z_{sq}[J^\mu] = \exp \left\{ -\frac{1}{2} \int d^4x \int d^4y J^\mu_a(x) D_{sq}(x-y) J^\mu_a(y) \right\}$$

(51)

with the covariant squeezed propagator

$$D_{sq}(x-y) \equiv D(x-y) + C_0,$$

(52)

where $D(x-y)$ is the Feynman gauge gluon propagator (48).

4.4. Effective quark-quark interaction

From (51) we obtain the effective quark action

$$W_{\text{sq}}^\text{eff}[\Psi, \bar{\Psi}] = \int d^4x \mathcal{L}_\Psi + W_{\text{sq}}[\Psi, \bar{\Psi}],$$

(53)

where

$$W_{\text{sq}}[\Psi, \bar{\Psi}] \equiv -i \ln Z_{sq} = \frac{1}{2} i \int d^4x \int d^4y J^\mu_a(x) D_{sq}(x-y) J^\mu_a(y),$$

(54)

*Due to the assumption of global colour neutrality of the vacuum in our model, we can make use of the identity $1 = \exp \left\{ -C_0 \left( \int d^4x J^\mu_0(x) \right)^2 / 2 \right\}$ in order to write $Z_{sq}[J^\mu]$ in a covariant form.*
with \( D_{sq}(x - y) \equiv D(x - y) + C_0 \). Eq. (54) represents a modified current-current interaction where in addition to the ordinary bilocal coupling a permanent and nonlocal coupling of currents with the strength \( C_0 \) occurs due to the presence of the squeezed condensate. It represents the zero momentum sector of the theory and is due to the presence of the squeezed condensate.

Fourier transformation of the four-quark interaction term leads to

\[
W_{sq}[\Psi, \overline{\Psi}] = \frac{g^2}{2} \int \frac{d^4k_1}{(2\pi)^4} \cdots \frac{d^4k_4}{(2\pi)^4} \overline{\Psi}(k_1) \frac{\lambda_\alpha}{2} \gamma^\mu \Psi(k_2) D_{sq}(k_1 - k_2) (2\pi)^4 \delta^4(k_1 - k_2 + k_3 - k_4) \overline{\Psi}(k_3) \gamma_\mu \lambda_\alpha \Psi(k_4),
\]

with

\[
D_{sq}(q) = \left[ \frac{i}{q^2 + i\epsilon} + (2\pi)^4 \delta^4(q) C_0 \right].
\]

The first term corresponds to the usual propagator of a massless boson. It is responsible for perturbative interactions at large momentum transfer. The second term is a delta function interaction which may lead to quark confinement as discussed by Munczek and Nemirovsky\textsuperscript{3}.

We shall now compare our coefficient of the delta function contribution to the quark interaction (56) with that by Munczek and Nemirovsky\textsuperscript{3}, see Eq. (9). This allows us to express the coefficient \( \eta^2 \) in terms of the squeezed condensate parameter \( C_0 \) as

\[
\eta^2 = \frac{16g^2}{3} C_0 = \frac{64\pi\alpha_s}{3} C_0.
\]

Using the relation \textsuperscript{35} which gives the parameter \( C_0 \) in terms of the gluon condensate \( \langle \alpha_s G^2 \rangle_{\text{no quarks}} \) in absence of quarks,

\[
C_0 = \alpha_s^{-1} \sqrt{\frac{1}{512\pi} \langle \alpha_s G^2 \rangle_{\text{no quarks}}}
\]

and taking into account the fact that the values of the gluon condensate with and without quarks are related via a suppression factor \( \gamma \)

\[
\langle \alpha_s G^2 \rangle_{\text{phys}} = \gamma \langle \alpha_s G^2 \rangle_{\text{no quarks}},
\]

we obtain the following relation between \( \eta^2 \) and the physical gluon condensate

\[
\eta^2 = \frac{8}{9} \sqrt{\frac{\pi}{\gamma}} \langle \alpha_s G^2 \rangle_{\text{phys}}.
\]

Using the SVZ value \( \alpha_s G^2 = 0.04 \) GeV\(^4\)\textsuperscript{2} and \( \gamma = 1/3 (1/2) \)\textsuperscript{2} we find the \( \eta^2 = 0.54 (0.44) \) GeV\(^2\) respectively. Recently, Narison\textsuperscript{27} has pointed out that the SVZ value might be too small. In his analysis of heavy quarkonia mass splittings in QCD he found an update average value \( \alpha_s G^2 = (0.071 \pm 0.009) \) GeV\(^4\). The corresponding values of \( \eta^2 \) according to (60) are \( \eta^2 = 0.73 (0.59) \) GeV\(^2\) for \( \gamma = 1/3 (1/2) \) respectively. Comparing these with the value \( \eta^2 = 1.14 \) GeV\(^2\) obtained by Munczek and Nemirovsky\textsuperscript{3} from the fit to experimental meson spectra, we find rather good agreement. The deviation by a factor of about two is not too bad in view of the simple models used.
5. Conclusions

The squeezed gluon condensate has become an attractive topic of research in the last years as an interesting alternative to existing models of the QCD gluon vacuum, see e.g. [19]. It is of interest to see which consequences the existence of a squeezed gluon condensate has for observable quantities. One possibility has already been discussed in the earlier work [20], where it has been found that a squeezed gluon vacuum can indeed explain the large mass of the \( \eta' \) quite successfully. In the present work we have considered another consequence, its influence on the quark-quark interaction. This would open the possibility to understand the phenomenological approach by Munczek and Nemirovsky. Including the gluon selfinteractions in the low energy region into an Abelian approximation to QCD in form of a squeezed gluon condensate we were able to obtain the \( \delta \)-function type confining part of the effective gluon two-point function. Furthermore we were able to estimate the strength \( \eta^2 \) of this interaction from the existing values of the physical gluon condensate. With the recent estimate of the value of the gluon condensate by Narison we have obtained a value of \( \eta^2 \) which is only a factor of about two smaller than that obtained by Munczek and Nemirovsky in their analysis of meson spectra, which is quite encouraging in view of the simple models used.

We have here only considered a squeezed condensate of pairs of zero momentum gluons. An interesting extension of the present work would be to include nonzero momentum gluon modes also. We would then have to generalize the contraction \( C_0 \) to the momentum dependent \( C(q) \) to include pairs of nonzero relative momentum. One could then try to get more detailed information about the effective confining quark-quark interaction in the QCD vacuum beyond the momentum \( \delta \)-function originating from the zero momentum gluon pairs in the condensate. Of course the question of the gauge invariance and Lorentz invariance of the corresponding condensate of gluon pairs will be more difficult. Investigations to obtain a gauge invariant Hamiltonian in the low energy approximation are in progress [30].

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Appendix A Reduced phase space quantization of massive gauge theory

Appendix A.1. Reduction of the Lagrangian

In this Appendix we recall the method of reduced phase space quantization for the case of massive Abelian QCD following Ref. [22]. This approach is particularly useful for studies of the massless limit as performed in the main text. The classical action of massive Abelian QCD is

\[
W = \int d^4x \mathcal{L}(x) = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} M^2 V_a^2 + \bar{\Psi}(i \not\partial - m) \Psi - J^a_{\mu} V_{\mu}^a \right]
\]  

(A.1)

with the Lorentz, but not gauge invariant Lagrangian \( \mathcal{L}_{AQCD}(x) \) and the Abelian field strengths \( F_{\mu\nu}^a = \partial_\mu V_{\nu}^a - \partial_\nu V_{\mu}^a \).
The Euler-Lagrange equations for $V_0^a$

$$\frac{\delta W}{\delta V_0^a} = 0 \quad (A.2)$$

are not equations of motion but constraints

$$(\vec{\partial}^2 - M^2)V_0^a = -\partial_i \dot{V}_i^a + J_0^a. \quad (A.3)$$

The fields $V_0^a$ are therefore not dynamical variables and have to be eliminated using the constraint equations before the remaining reduced phase space can be quantized. The constraint equations correspond in the massless limit to the Gauss laws. They can be formally solved for $V_0^a$ by

$$V_0^a[\vec{V}, J_0] = \frac{1}{\partial^2 - M^2}(-\partial_i \dot{V}_i^a + J_0^a) \quad (A.4)$$

and inserted into the original Lagrangian. We have abbreviated

$$\frac{1}{\partial^2 - M^2}f(\vec{x}) = -\frac{1}{4\pi} \int d^3 y e^{-M|\vec{y} - \vec{x}|}f(\vec{y}). \quad (A.5)$$

The electric field strengths $E_k^a$ then become

$$E_k^a[\vec{V}] = -P_{kj} \dot{V}_j^a - \frac{1}{\partial^2 - M^2} \partial_k J_0^a \quad (A.6)$$

with

$$P_{ij} \equiv \delta_{ij} - \frac{\partial_i \partial_j}{\partial^2 - M^2} = \delta_{ij}^T - \frac{M^2}{\partial^2 - M^2} \delta_{ij}^{||}, \quad (A.7)$$

where we have used the longitudinal and transverse projection operators $\delta_{ij}^{||} \equiv \partial_i \partial_j / \partial^2$ and $\delta_{ij}^T \equiv \delta_{ij} - \delta_{ij}^{||}$. In contrast to the massless case, $P^2 \neq P$, and $P_{ij}$ is invertible

$$P_{ij}^{-1} = \delta_{ij}^T - \frac{\partial^2 - M^2}{M^2} \delta_{ij}^{||} = \delta_{ij} - \frac{\partial_i \partial_j}{M^2}. \quad (A.8)$$

Inserting this into $W$ we obtain the reduced $W_{\text{red}}$

$$W_{\text{red}}[\vec{V}, \Psi] = \int d^4 x L_{\text{red}} \equiv \int d^4 x (L_{\text{red}}^V + L_{\text{red}}^\Psi) \quad (A.9)$$

with

$$L_{\text{red}}^V = \frac{1}{2} \left( \dot{\bar{V}}_i^a P_{ij} V_j^a + V_i^a (\vec{\partial}^2 - M^2) P_{ij} V_j^a \right)$$

$$L_{\text{red}}^\Psi = \frac{1}{2} J_0^a \frac{1}{\partial^2 - M^2} J_0^a - J_0^a \left( \frac{1}{\partial^2 - M^2} \partial_i \dot{V}_i^a \right) - V_i^a J_0^a + \bar{\Psi}(i \bar{\partial} - m) \Psi,$$

with $P_{ij}$ given by (16).

We have several choices for the dynamical variables. Following we can eliminate the second term in $L_{\text{red}}^\Psi$ by introducing the new fermionic variable

$$\bar{\Psi} \equiv \exp \left( ig \frac{\lambda^a}{2} \frac{1}{\partial^2 - M^2} \partial_i \dot{V}_i^a \right) \Psi. \quad (A.10)$$
Since (A.10) is only a phase transformation, the corresponding current $J_\mu^a$ stay the same and $\mathcal{L}_\text{red}^\Psi$ becomes

$$
\mathcal{L}_\text{red}^\Psi = \frac{1}{2} J_0^a \frac{1}{\partial^2 - M^2} J_0^a - \frac{1}{2} J_i^a \left( \delta_{ij} - \frac{\partial_i \partial_j}{\partial^2 - M^2} \right) V_j^a + \bar{\tilde{\Psi}} (i \slashed{\partial} - m) \tilde{\Psi} .
$$

(A.11)

In terms of the nonlocal variables $\tilde{V}_k^a = P_{kj} V_j^a$ with $P_{ij}$ given by (A.7) the reduced Lagrangian can be written

$$
\mathcal{L}_\text{red} = \frac{1}{2} \left( \dot{\tilde{V}}_i^a \frac{1}{\partial^2 - M^2} \tilde{V}_i^a + \frac{1}{2} J_0^a \frac{1}{\partial^2 - M^2} J_0^a - J_i^a \frac{1}{\partial^2 - M^2} J_i^a \right) \tilde{\Psi} + \bar{\tilde{\Psi}} (i \slashed{\partial} - m) \tilde{\Psi} .
$$

(A.12)

The corresponding canonical conjugate momenta are

$$
\tilde{\Pi}_i^a \equiv \frac{\delta \mathcal{L}}{\delta \dot{\tilde{V}}_i^a} = P_{ij} \tilde{V}_j^a, \\
\tilde{\Pi} \equiv \frac{\delta \mathcal{L}}{\delta \tilde{\Psi}} = i \tilde{\Psi}^+ .
$$

(A.13)

The nonlocal variables $\tilde{V}_i^a$ and $\tilde{\Psi}$ smoothly turn into the photon field in the massless limit. The corresponding reduced Hamiltonian is found as

$$
H_\text{red} \equiv H_0 + H' \tag{A.14}
$$

with the free and interaction parts

$$
H_0 = \int d^3 \vec{x} \left\{ \frac{1}{2} \left( \tilde{\Pi}_i^a P_{ij} \tilde{V}_j^a - \tilde{V}_i^a (\partial^2 - M^2) P_{ij} \tilde{V}_j^a \right) + \tilde{\Pi}_i^a \gamma_0 (\vec{\gamma} \cdot \vec{\partial} + m) \tilde{\Psi} \right\}, \\
H' = \int d^3 \vec{x} \left( \tilde{V}_i J_i - \frac{1}{2} J_0 \frac{1}{\partial^2 - M^2} J_0 \right) .
$$

Note that the second term in $H'$ corresponds to a Yukawa potential.

**Appendix A.2. Quantization**

The theory is now quantized in terms of the nonlocal $\tilde{V}_i^a (x)$ by imposing the canonical commutation relations

$$
i [\tilde{\Pi}_i^{a}(\vec{x},t), \tilde{\Pi}_j^{a}(\vec{y},t)] = \delta^{ab} \delta_{ij} \delta(\vec{x} - \vec{y}).
$$

(A.15)

It has been shown in [2] that the corresponding Poincaré algebra is fulfilled on both the classical and the quantum level. The free Hamiltonian is diagonalized by the interaction picture fields

$$
\tilde{v}_i^a(\vec{x},t) = \frac{1}{V} \sum_q \sqrt{\frac{1}{2 \omega(q)}} \sum_{\lambda=1,2,3} \left( a^a(\lambda, q) \epsilon_i(\lambda, q) e^{-i(\omega(q)t - \vec{q} \cdot \vec{x})} + h.c. \right) , \\
\tilde{\pi}_i^a(\vec{x},t) = -i \frac{1}{V} \sum_q \sqrt{\frac{\omega(q)}{2}} \sum_{\lambda=1,2,3} \left( a^a(\lambda, q) \epsilon_i(\lambda, q) e^{-i(\omega(q)t - \vec{q} \cdot \vec{x})} - h.c. \right) ,
$$
with the creation and annihilation operators satisfying

\[ [a^\alpha(\lambda, \vec{q}), a^{b\dagger}(\lambda', \vec{q}')] = \delta^{ab} \delta_{\lambda\lambda'} (2\pi)^3 \delta(\vec{q} - \vec{q}') \]  

(A.16)

and with the real nonlocal polarization vectors

\[ \tilde{\epsilon}_i(\lambda, \vec{q}) \equiv P_{ij}(q)\epsilon_j(\lambda, \vec{q}) . \]  

(A.17)

Here

\[ P_{ij}(q) = \delta_{ij} - \frac{q_i q_j}{\vec{q}^2 + M^2} \]  

(A.18)

is the Fourier transform of the operator (A.7) and \( \epsilon_i(\lambda, \vec{q}) \) the spatial components of the real covariant orthonormal polarization vectors \( \epsilon_\mu(\lambda, \vec{q}) \) satisfying \( \epsilon_\mu(\lambda', \vec{q}) \cdot \epsilon_\nu(\lambda, \vec{q}) = 0 \) and the completeness relation \( \sum_\lambda \epsilon_\mu(\lambda, \vec{q}) \epsilon_\nu(\lambda, \vec{q}) = - \left( g_{\mu\nu} - q_\mu q_\nu / M^2 \right) \). For the nonlocal \( \tilde{\epsilon}_i(\lambda, \vec{q}) \) we therefore have the corresponding completeness relation

\[ \sum_\lambda \tilde{\epsilon}_i(\lambda, \vec{q}) \tilde{\epsilon}_j(\lambda, \vec{q}) = \delta_{ij} - \frac{q_i q_j}{\vec{q}^2 + M^2} . \]  

(A.19)

This leads to the free propagator

\[
\langle 0| T\tilde{\epsilon}_i^a(x, x_0)\tilde{\epsilon}_j^b(y, y_0)|0 \rangle = i\delta^{ab} \frac{1}{V} \sum_\vec{q} \int \frac{dq_0}{(2\pi)} \frac{e^{-i\vec{q}(x-y)}}{q^2 - M^2 + i\epsilon} \left( \delta_{ij} - \frac{q_i q_j}{\vec{q}^2 + M^2} \right)
\]

(A.20)

Combining this with the Yukawa term in \( H' \) defining

\[
\tilde{D}_{00}(M; x - y) \equiv \frac{1}{V} \sum_\vec{q} \int \frac{dq_0}{(2\pi)} \frac{e^{-i\vec{q}(x-y)}}{q^2 + M^2} ,
\]

(A.21)

we obtain the complete momentum space propagator

\[
\tilde{D}^{\mu\nu}(M; q) = \frac{\delta_{\mu0}\delta_{\nu0}}{(q^2 + M^2)} + \frac{\delta_{\mu\nu}\delta_{ij}}{q^2 - M^2 + i\epsilon} \left( \delta_{ij} - \frac{q_i q_j}{\vec{q}^2 + M^2} \right) .
\]  

(A.22)

It is the generalization of the photon propagator in Coulomb gauge QED. In difference to the conventional covariant massive vector propagator

\[
D_{\mu\nu}(M; q) = -\frac{\delta^{ab}}{q^2 - M^2 + i\epsilon} \left( g_{\mu\nu} - q_\mu q_\nu / M^2 \right) .
\]  

(A.23)

the reduced propagator (A.22) is regular in the limit \( M \to 0 \). When the vector field is coupled to a conserved fermion current \( q_\mu J^\nu = 0 \) and when no squeezed condensate is present as in QED, the two ways of quantization, local and nonlocal, coincide, since \( J_\mu^a \tilde{D}^{ab}_{\mu\nu} J_\nu^b = J_\mu D_{\mu\nu} J_\nu \). For the study of the infrared behaviour and in particular in order to construct a squeezed gluon vacuum it is necessary to have the regular form (A.22) of the vector field propagator as discussed in the main text.
Appendix A.3. Lorentz transformation properties

Finally we would like to mention that the following quantum Lorentz transformation properties of the field operators $\tilde{v}_k^\alpha$ and $\tilde{\pi}_k^\alpha$ have been derived:\footnote{22}

\begin{align}
\delta_L \tilde{v}_k^\alpha &= \epsilon_i(x_i \partial_t + t \partial_i)\tilde{v}_k^\alpha - \partial_k \frac{1}{\sqrt{\delta^2 - M^2}} \epsilon_i v_i^\alpha \quad (A.24) \\
\delta_L \tilde{\pi}_k^\alpha &= \epsilon_i(x_i \partial_t + t \partial_i)\tilde{\pi}_k^\alpha + \epsilon_i \partial_i(P^{-1}v_k^\alpha) + \epsilon_k \partial_i(P^{-1}_s v_s^\alpha) \quad (A.25)
\end{align}

This shows that the zero momentum components of the nonlocal fields and their canonical momenta are invariant under Lorentz boosts.

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