Calculation of field of spectrometric magnet with 0.13 m gap

Ivan Yudin¹,*

¹Joint Institute for Nuclear Research, Dubna, Moscow region, Russia

Abstract. The spectrometric magnet is widely used in experiments on high-energy physics and a number of other fields. In this report, we present the results of calculating the magnetic field for a variant of a magnet with an interpolar gap of 0.13 m. Our spectrometer magnet has external dimensions of 2.95 x 2.12 x 1.62 m and an aperture (occupied by the beam) of 0.30 x 0.13 m. The simulation was carried out using an integral method of magnetostatics. The results of the calculated field distribution of the analyzing magnet with interpolar 0.13 m gap are given. The calculation was performed for the volume 0.33 x 0.0645 x 1.5 m, the volume of overlapping dimensions. The calculation of the spatial distribution of the three components of the magnetic field of the magnet described in the work is performed to obtain the information about the magnitude and uniformity of the magnetic field for different modes of operation of the spectrometer. The obtained results are planned for using in the processing of physical data.

1 Introduction

In mathematical modeling of the field distribution of various magnetic systems of electro-physical installations, differential formulations of the magnetostatic problem are actively used [1]. In this paper, we consider the problem of magnetostatics in the integral formulation. The integrated formulation is chosen because of the ability to perform calculations on the grid only inside the ferromagnetic core. Fig. 1 shows the magnetic system of the spectrometer. The magnet has external dimensions of 2.95 m x 2.12 m x 1.62 m and an aperture of 0.30 m x 0.13 m. The interpolar gap of 0.13 m is new for installation. Previously, a gap of 0.09 m was used.

The OZ axis of the coordinate system is chosen in the direction of movement of particles of the primary beam, parallel to the plane of the lower pole tip; OY axis - vertically "upwards" along the normal to the plane of the same pole tip; OX axis - so as to get the right coordinate system. The center of the spectrometric magnet is taken as the beginning of the Cartesian coordinate system (the "right" triple XYZ) of the spectrometer. The center of the lower pole tip has the coordinates x = 0, y = -0.065 m, z = 0.

The calculation was carried out for a volume of 0.35 m x 0.10 m x 2.18 m. The described calculation of the spatial distribution of the three components of the magnetic field of the magnet was carried out in order to obtain the information on the magnitude and homogeneity of the magnetic field and to build a working field map for various operating modes of the spectrometer. It is planned to use these results in the processing of physical data.

*e-mail: yudin@jinr.ru
2 Mathematical formulation of the magnetostatic problem

Let us consider a physical system consisting of a ferromagnetic ($\Omega_f$ region) and a vacuum ($\Omega_v$ region) with closed current windings ($\Omega_c$ region). The problem of finding the distribution of the magnetic field created by stationary currents and the magnetization of isotropic ferromagnetic is solved. We will assume the absence of surface currents and currents flowing through a ferromagnetic. Then the Maxwell equations for a stationary magnetic field take the form:

\[
\text{rot} \, H(\rho) = J(\rho),
\]
\[
\text{div} \, B(\rho) = 0,
\]
\[
B(\rho) = \mu_0 H(\rho),
\]
and conditions at the interface of media and at infinity
\[
n (B_f - B_v) = 0, \quad n \times (H_f - H_v) = 0, \quad H(\rho)_{\rho \to \infty} \to 0.
\]

Here, the following notation is used: $\rho$ is the point of the three-dimensional space $\mathbb{R}^3$, the indices $f$ and $v$ correspond to the ferromagnetic and vacuum regions; $H$ is the vector of magnetic field strength; $B$ is the magnetic induction vector; $J$ is the known vector of the volume of the current density, which is different from zero in the limited domain $\Omega_c$ and satisfies the relation $\int_{\Omega_c} J \, d\Omega = 0$; $\mu(|H|)$ is known in a bounded simply connected domain; $\Omega_f$ is a nonlinear function of the magnetic permeability of a ferromagnetic (for a nonmagnetic medium, $\mu = 1$); $\mu_0$ - magnetic permeability of vacuum; $n$ is the unit normal vector to the ferromagnetic / vacuum interface.

Taking into account the relationship of the vectors $B$, $H$ and the magnetization of the substance $M$

\[
B = \mu_0 (H + M),
\]
we introduce a scalar potential as follows:

\[
H(p) = -\nabla \phi,
\]
\[
\nabla \phi = \nabla M.
\]
Then the solution of the problem can be written in terms of integrals over the surface and volume, i.e. in the integral formulation:

\[
\phi(r) = -\frac{1}{4\pi} \int \int \int \frac{\nabla M}{|r' - r|} dV' - \frac{1}{4\pi} \oint \frac{M_n}{|r' - r|} dS',
\]

\[
H(r) = \frac{1}{4\pi} \int \int \int \frac{(r' - r) \nabla M}{|r' - r|} dV' - \frac{1}{4\pi} \oint \frac{(r' - r) M_n}{|r' - r|} dS'.
\]

3 Algorithm for the numerical solution of the boundary value problem in the integral formulation

Let us consider the spectrometric magnet (Figs. 1(a,b)).

For the difference approximation of the boundary value problem, we construct a non-uniform grid with elementary cells in the form of parallelepipeds. We will integrate over the volume of the parallelepiped with the coordinates of the center of the block \((x_c, y_c, z_c)\) and the size of the block \((w_x, w_y, w_z)\).

We will assume that \((x, y, z)\) are coordinates of the observed (current) points, or otherwise points on the line of integration, \((v_x, v_y, v_z)\) is a unit vector parallel to the line of integration.

The problem of calculating the magnetic field of a spectrometric magnet will be solved relative to the magnetization vector \([2]\). We assume that the magnetization vector in the unit cell is constant and equal to the value at the center of the cell \(M_i\). The magnetic field strength produced by \(N\) objects of arbitrary shape, with a magnetization vector \(M_i\), \(i = 1, 2, N\), at the point with the radius vector \(r\) can be expressed as follows:

\[
H(r) = \sum_{i=1}^{N} Q_i(r) M_i,
\]

where \(Q_i(r)\) is a 3 x 3 matrix, which depends on the geometry of the object and the radius of the vector \(r\). Its components can be expressed in terms of integrals over the \(S_i\) surface:

\[
Q_i(r) = \frac{1}{4\pi} \oint \frac{(r' - r) n_s}{|r' - r|} dS', \quad (a \otimes b) c \equiv a(bc).
\]

Surface integrals for the geometric factor \(Q_i(r)\) can be calculated analytically for various geometric shapes, including the parallelepiped.

Another important magnetic characteristic is the integral field along a straight line.

\[
I(r_0, v) = \int_{-\infty}^{+\infty} H(r + vs) ds = \sum_{i=1}^{N} G_i(r, v) \cdot M_i,
\]

\[
G_i(r_0, v) = \frac{1}{2\pi} \oint \frac{[v \times [(r_0 - r') \times v]] \otimes n_s}{|(r_0 - r') \times v|^2} dS'.
\]

The magnetic field at the center of the \(i\)-th cell is

\[
H_i = \sum_{k=1}^{N} Q_{ik} M_k + H_{ext}, \quad i = 1, 2, ..., N.
\]
4 Results of calculations

In the center of coordinates for the current \( I = 225 \) A, the magnetic induction \( B_y(0, 0, 0) \) is equal to 0.824 T, for the current \( I = 328 \) A it is equal to 0.999 T, for the current \( I = 460 \) A it is equal to 1.266 T and for the current of \( I = 635 \) A it is equal to 1.5 T.

Fig. 2 shows the results of calculations of the magnetic field for the current 328 A for the median plane (Figs. 2(a,b)) and for the plane \( y = 4 \) cm (Figs. 2(c,d)).

![Figure 2](image)

**Figure 2.** For a current \( I = 328 \) A: distribution \( B_y(x = 0, y = 0, z) \) for the median plane \( x = 0, y = 0 \) (a); distribution \( B_y(x = 0, y = 0, z) \) for the median plane \( y = 0, z = 0 \) (b); distribution \( B_y(x, y = 0, z = 4 \text{ cm}) \) for the plane \( y = 4 \) cm, \( x = 0 \) (c); distribution \( B_y(x, y = 5 \text{ cm}, z = 0) \) for the plane \( y = 4 \) cm, \( z = 0 \) (d).

Let us dwell in more detail on the results obtained at \( I = 328 \) A.

Fig. 3 shows the three-dimensional distribution of the \( B_y \) component for the median plane \( B_y(x, 0, z) \) (a); and for the plane \( y = 5 \) cm \( B_y(x, 5 \text{ cm}, z) \) (b).

In the center of the magnet, the vertical component of the magnetic induction \( B_y(0, 0, 0) \) is equal to 0.999 T, and it practically remains along the \( x \) coordinate to the pole edge (\( x = 0.15 \) m). A sharp decline at these edges reduces the value of \( B_y(x, 0, 0) \) to 0.74 T at the point \( x = +0.17 \) m.

Similarly, along the \( z \) coordinate, the \( B_y(0, 0, z) \) component is practically preserved up to the pole edge (\( z = 0.65 \) m) (actually up to 0.6 m). A sharp drop at these edges reduces the value of \( B_y(0, 0, z) \) to 0.003 T at the point \( z = 1.09 \) m. The component \( B_x(0, 0, z) \) under the pole is equal to 0.006 T, the values at the edges \( z = 0.65 \) m practically do not change, but they
drop sharply to zero beyond the edges of the pole. The longitudinal component $B_z(0,0,z)$ here is zero everywhere but it is equal to 0.015 T at the edges of the pole at $z = +0.65$ m.

The three-dimensional distribution for the $B_y$ component is shown in Fig. 3(a) for the median plane, and near the upper pole of the $B_y(x,5cm,z)$ is shown in Fig. 3(b). Here $B_y(x;+0.05m,z)$ is equal to 0.977 T at the center ($x = 0$; $z = 0$) and 1.6 T at the edges (at the point $x = +0.13$ m, $z = 0$). Along the coordinate $z$ it is gradually decreases to 0.97 T near the pole edge ($z = +0.60$ m). A sharp decline at these edges reduces $B_y(0;0.05;0)$ to 0.002 T at the point $z = 1.09$ m.

The distribution of the magnetic field near the upper pole ($y = 5$ cm) is shown for the $B_y$ component in Fig. 4(a) and for the $B_z$ component - in Fig. 4(b).

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure3a.png} \hspace{0.5cm} \includegraphics[width=0.4\textwidth]{figure3b.png}
\caption{For a current $I = 328$ A: 3d-distribution $B_y(x,y = 0,z)$ for the median plane $y = 0$ (a); 3d-distribution $B_y(x,y = 5cm,z)$ for the plane $y = 5$ cm (b)}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure4a.png} \hspace{0.5cm} \includegraphics[width=0.4\textwidth]{figure4b.png}
\caption{For a current $I = 328$ A: 3d-distribution $B_y(x,y = 5cm,z)$ for the plane $y = 5$ cm (a); and 3d-distribution $B_z(x,y = 5cm,z)$ for the the plane $y = 5$ cm (b)}
\end{figure}
5 Conclusions

The magnetic field of the analyzing magnet was calculated in a spatial volume with dimensions of 0.35 x 0.10 x 2.18 m. A three-dimensional map of the distribution of all components of the magnetic field vector was compiled by volume. The graphs of the characteristic distribution of the field components were obtained at levels of 0.82, 1.0, 1.27 and 1.5 T. The formulas and algorithms for calculating the magnetic field using the method of a constant magnetization vector in an elementary volume are given. The results are used to carry out the computer simulation of the installation and to plan an experiment to measure a magnetic field and will subsequently be used in processing of the physical data obtained during the experiment.

References

[1] I.P. Yudin et al., PEPAN Letters 4(4), 614-627 (2007)
[2] E. Ayryan, E.P. Zhidkov, I.P. Yudin et al., PEPAN 21(1), 251-307 (1990)