MOTION OF A SINGLE HOLE IN A DISORDERED MAGNETIC BACKGROUND

A. BELKASRI and J.L. RICHARD

Abstract

The spectrum of a single hole is calculated within the spin-hole model using a variational method. This calculation is done for any rotational invariant magnetic background. We have found that when the magnetic background changes from a disordered to a locally ordered state, the spectrum changes qualitatively. We have also found that the spin pattern around the hole is polarized. This problem is related to the study of copper oxide planes CuO$_2$ doped with a small number of holes.

Key-Words: Strongly Correlated Systems, HTC Superconductivity.

Number of figures: 3

July 1994
CPT-94/P.3058

anonymous ftp or gopher: cpt.univ-mrs.fr

---

* Unité Propre de Recherche 7061
1 and Université d’Aix-Marseille II
1 Introduction

The study of the motion of one hole in strongly correlated systems is very important because, in principle, it may contain some features of the properties of the CuO$_2$ planes doped with a small number of holes. The one hole motion in a quantum antiferromagnetic (AFM) background has been studied by several authors[1, 2, 3, 4] in the framework of the $t$-$J$ model. These studies were carried out within an AFM ordered state. The main result of these analysis is the existence of a well-defined quasiparticle description for a coherent hole propagation with the band minima at $(\pm \pi/2, \pm \pi/2)$ and the bandwidth $W$ of the order of $J$ and not $t$. These results are in good agreement with numerical exact diagonalization for small systems[5][6]. All these investigations concern the magnetic ordered case, and they were carried out in the framework of the $t$-$J$ model which has the difficulty of the constraint of no double occupancy at the same site. In this paper we return to the Hamiltonian $H_{pd}^{(2)}$ derived in, e.g., Refs.[7, 8, 9, 10]. This Hamiltonian is obtained from the Emery model by means of a perturbative expansion up to the second order in $t_{pd}$ (the hopping parameter between Cu and O sites). We point out that the $t$ term of the $t$-$J$ Hamiltonian is obtained from $H_{pd}^{(2)}$ by projection on local singlet states[12]. We will not consider here the fourth order term which gives the super-exchange energy between spins located on the Cu sites. This term will not be relevant in our calculation since we want to study essentially the motion of a hole in a disordered magnetic background. In the framework of the $H_{pd}^{(2)}$ Hamiltonian we will compute the dispersion relation $E(k)$ by applying a trial wave function. The results agrees with previous calculation for the $t$-$J$ model when we consider the limit of an ordered magnetic background.

Our starting point is the $H_{pd}^{(2)}$ Hamiltonian (for shortness we set $H$ as $H_{pd}^{(2)}$ ) which can be written as

$$H = \sum_{i,j,\sigma} T_{ij} f_{i\sigma}^\dagger f_{j\sigma} + g \sum_{i,j,j',\sigma,\sigma'} w_{ij} w_{ij'} f_{j\sigma}^\dagger \vec{\tau}_{\sigma\sigma'} f_{j'\sigma'} \vec{S}_i \tag{1}$$

where $(f_{i\sigma}^\dagger, f_{i\sigma})$ is a fermion field with the spin index $\sigma$. $\vec{S}_i$ is the spin operator on the Cu site $i$, $\vec{\tau} = (\tau_x, \tau_y, \tau_z)$ are the Pauli matrices and

$$T_{ij} = T \frac{1}{N} \sum_k \varepsilon_k e^{ik(i-j)} \tag{2}$$
where \( \varepsilon_k = 4(1 - \gamma_k) \), \( w_k = \sqrt{\varepsilon_k} \) and \( \gamma_k = (\cos k_x + \cos k_y)/2 \). The parameters \( T \) and \( g \) are related to the parameters \( \Delta = \varepsilon_p - \varepsilon_d \), \( U_d \) and \( t_{pd} \) by the following relations

\[
T \equiv \frac{1}{2} t_{pd}^2 \left[ \frac{1}{\Delta} - \frac{1}{U_d - \Delta} \right] \\
g \equiv t_{pd}^2 \left[ \frac{1}{\Delta} + \frac{1}{U_d - \Delta} \right]
\]

The Hamiltonian (1) describes a system of itinerant holes interacting with a magnetic background. We notice that if we restrict the \( g \) term to a local spin-spin interaction (i.e. take \( w_{ij} = w_0 \delta_{ij} \)) we get the phenomenological Hamiltonian considered by Monthoux and Pines[13].

2 Construction of the trial state for the one hole motion

The most general state for a single hole is

\[
\psi = \sum_{i,\sigma} f_{i\sigma}^\dagger A(i\sigma) \Phi
\]

where \( \Phi \) is some magnetic background and \( A(i\sigma) \) is some combination of spin operators. The meaning of Eq.(6) is clear: the motion of the hole will perturb the magnetic background \( \Phi \), generating all sorts of excited magnetic states provided by the operator \( A(i\sigma) \).

In what follows, we will suppose that the magnetic background \( \Phi \) is rotational invariant, i.e.

\[
\vec{S}_{tot} \Phi = 0
\]

For the Hamiltonian under investigation Eq.(1), if we set \( g = 0 \), the following state for one hole with spin up

\[
\sum_i \psi_o(i) f_{i\uparrow}^\dagger \Phi
\]
is an eigenstate for the $T$ term of the Hamiltonian Eq.(1), if $\psi_o(i) = e^{ik \cdot i}$ for a given $k$. On the other hand the $g$ term can be rewritten in the following form

$$\begin{align*}
-\frac{3}{4} g \sum_{i,j,j'} w_{ij} w_{ij'} [d_{i\uparrow}^t f_{j\downarrow}^t - d_{i\downarrow}^t f_{j\uparrow}^t] [f_{j\uparrow}^t d_{i\uparrow} - f_{j\downarrow}^t d_{i\downarrow}] + \\
\frac{1}{2} g \sum_{i,j,j'} w_{ij} w_{ij'} \left\{ d_{i\uparrow}^t f_{j\uparrow}^t f_{j\downarrow}^t d_{i\uparrow} + d_{i\downarrow}^t f_{j\downarrow}^t f_{j\uparrow}^t d_{i\downarrow} + \\
\frac{1}{2} [d_{i\uparrow}^t f_{j\uparrow}^t + d_{i\downarrow}^t f_{j\downarrow}^t] [f_{j\uparrow}^t d_{i\uparrow} + f_{j\downarrow}^t d_{i\downarrow}] \right\} \end{align*}$$

(8)

In this form we have split the Hamiltonian into two parts, one which contains singlet operators and the other one which contains triplet operators. These singlet and triplet operators are more general than those considered by Zhang and Rice[12], because here they are non-local.

From Eq.(8) we see also that the singlet states and the triplet states are separated by an energy $2g$. If we want to look for low energy excitations and we neglect mixing between the two bands, we can write the following state for the one hole with spin up

$$\sum_{i,j} \psi_1(i,j) \left[ d_{i\uparrow}^t f_{j\uparrow}^t - d_{i\downarrow}^t f_{j\downarrow}^t \right] d_{i\downarrow} \Phi$$

(9)

Then a natural trial state for the full Hamiltonian Eq.(1) can be written as

$$\sum_i \psi_o(i) f_{i\uparrow}^t \Phi + \sum_{i,j} \psi_1(i,j) \left[ d_{i\uparrow}^t f_{j\uparrow}^t - d_{i\downarrow}^t f_{j\downarrow}^t \right] d_{i\downarrow} \Phi$$

(10)

To simplify calculation we take $\psi_1(i,j) = \varphi_1(j) w_{ji}$. Then the state (10) can be rewritten in a more transparent way

$$\psi = \sum_i \left\{ \varphi_o(i) f_{i\uparrow}^t + \sum_j \varphi_1(j) w_{ij} \left[ f_{i\uparrow}^t S_{j\uparrow}^z + f_{i\downarrow}^t S_{j\downarrow}^+ \right] \right\} \Phi$$

(11)

where $\varphi_o(i) = \psi_o(i) - \frac{1}{2} \sum_j \varphi_1(j) w_{ij}$.

Examples for the magnetic state $\Phi$ are the singlet Resonating-Valence-Bond states considered by Anderson, Douçot and Liang[14] which may be long ranged or short ranged ordered.
3 Dispersion relation for a single hole

The dispersion relation \( E(k) \) will be calculated by using the variational method. We will minimize \((\psi, H\psi)\) keeping the norm \((\psi, \psi)\) fixed. Straightforward calculation, using the property:

\[
\sum_{l,l'} w_{il} w_{jl'} \varphi_1(l') \varphi_1(l) < \vec{S}_l \vec{S}_l' >
\]

leads to

\[
(\psi, H\psi) = \sum_{i,j} T_{ij} \left\{ \overline{\varphi_o(i) \varphi_o(j)} + \sum_{l,l'} w_{il} w_{jl'} \overline{\varphi_1(l') \varphi_1(l)} < \vec{S}_l \vec{S}_l' > \right\} +
\]

\[+ g \sum_{j,l,l'} \hat{T}_{ll'} w_{jl} \left\{ \overline{\varphi_1(l') \varphi_o(j)} < \vec{S}_l \vec{S}_l' > + h.c \right\} -
\]

\[- g \sum_{l,l'} \hat{T}_{ll'} \left\{ \overline{\varphi_o(l) \varphi_1(l')} < \vec{S}_l \vec{S}_l' > + h.c \right\} +
\]

\[+ g \sum_{l,l'} \hat{T}_{ll'} \hat{T}_{ll'} \overline{\varphi_1(l') \varphi_1(l)} < \vec{S}_l \vec{S}_l' >
\]

\[+ ig \sum_{l_1,l_2,l_3 \ (l_1 \neq l_2 \neq l_3)} \hat{T}_{l_1 l_3} \hat{T}_{l_1 l_2} \overline{\varphi_1(l_3) \varphi_1(l_2)} < \vec{S}_{l_3} (\vec{S}_{l_1} \times \vec{S}_{l_2}) >
\]

with \( \hat{T}_{ij} = T_{ij}/T \) and the average \(< ... >\) defined as

\[< S^a ... S^b > \equiv (\Phi, S^a ... S^b \Phi)
\]

Now if we suppose that the magnetic background \( \Phi \) [in Eq.(11)] is invariant under the time reversal operation \( K \), we will have

\[
(K \Phi, S^x_{l_1} S^y_{l_2} S^z_{l_3} K \Phi) = (\Phi, S^x_{l_1} S^y_{l_2} S^z_{l_3} \Phi)
\]

\[= - (\Phi, S^x_{l_1} S^y_{l_2} S^z_{l_3} \Phi)
\]

for any \( l_1, l_2, l_3 \) (all different), since \( K \vec{S} K^{-1} = - \vec{S} \). And finally we have

\[< \vec{S}_{l_3} (\vec{S}_{l_1} \times \vec{S}_{l_2}) > = 0
\]

Consequently the last term of Eq.(12) vanishes.

The energy will be calculated by minimizing the energy functional

\[\mathcal{W}(\psi, \lambda) = (\psi, H\psi) - \lambda(\psi, \psi)
\]
Using Eq.(12) and

\[(\psi,\psi) = \sum_i |\varphi_o(i)|^2 + \sum_{j,j'} \hat{T}_{jj'}\varphi_1(j)\varphi_1(j') < \vec{S}_j\vec{S}_{j'} >\]  

(17)

we get the following equation for the energy \(E\)

\[
\begin{pmatrix}
T\varepsilon_k - E & gw_ka_k \\
gw_ka_k & Tc_k + gd_k - Ea_k
\end{pmatrix}
\begin{pmatrix}
\varphi_o(k) \\
\varphi_1(k)
\end{pmatrix} = 0
\]  

(18)

where \(\varphi_o(k)\) and \(\varphi_1(k)\) are the Fourier components of \(\varphi_o(i)\) and \(\varphi_1(i)\) respectively. On the other hand

\[
\begin{align*}
\begin{cases}
  a_k = 3 - 4\chi_{o1}\gamma_k; \\
  c_k = 15 - 32\chi_{o1}\gamma_k + 4\chi_{o2}(4\gamma_k^2 - 1) + 4\gamma_{2k}(\chi_{11} - \chi_{o2}) \\
  d_k = 12 + \chi_{o1} - 8a_k
\end{cases}
\end{align*}
\]  

(19)

with

\[
\chi_{o1} = -<\vec{S}_o\vec{S}_{e_1}>; \quad \chi_{11} = <\vec{S}_o\vec{S}_{e_1+e_2}> \quad\text{and} \quad \chi_{o2} = -<\vec{S}_o\vec{S}_{2e_1}>
\]  

(20)

e_1 and e_2 denoting the two unit vectors of the square lattice. From Eq.(18), the lowest energy band is given by

\[
E_k/g = \frac{1}{2} \left\{ \eta(\varepsilon_k + c_k/a_k) + \frac{d_k}{a_k} - \sqrt{[\eta(\varepsilon_k - c_k/a_k) - \frac{d_k}{a_k}]^2 + 4\varepsilon_k a_k} \right\}
\]  

(21)

with \(\eta = T/g\) and for a given \(k\) we have the following solutions for \(\varphi_o(i)\) and \(\varphi_1(i)\)

\[
\begin{align*}
\varphi_o(i; k) &= w_k a_k U(k)e^{ik.i} \\
\varphi_1(i; k) &= -(\eta\varepsilon_k - E_k/g)U(k)e^{ik.i}
\end{align*}
\]  

(22)

where the function \(U(k)\) is determined by writing \((\psi,\psi) = 1\) and we get

\[
U(k)^2 = \frac{1}{\varepsilon_k a_k^2 + \frac{3}{4}[\eta\varepsilon_k - E_k/g]^2}
\]  

(23)
The dispersion relation $E_k$ depends only on the the spin-spin correlation function for nearest and next nearest neighbors. This is due to our expression for the trial state. Introducing more and more spin excitations would essentially produce new spin correlation functions at larger distance. However since we are interested essentially in a background with short correlation functions, this would merely not change our result at least qualitatively. Knowing $\chi_{o1}$, $\chi_{11}$ and $\chi_{o2}$ we evaluate then the energy $E_k$.

If we set $\eta = 0$, we can have a qualitative idea about the position of the minimum of $E_k$.

$$\frac{E_k}{g} = -\frac{1}{2} \left\{ \left| \frac{d_k}{a_k} \right| + \sqrt{\left| \frac{d_k}{a_k} \right|^2 + 4\varepsilon_k a_k} \right\}$$

(24)

It is easy to see that $|d_k/a_k|$ is maximal for $k = (0,0)$ and $4\varepsilon_k a_k$ is maximal for $k = (\pi,\pi)$. Then since to lower $E_k$ both of the quantities should be maximal, there will be a competition between them. For $\chi_{o1} = 0$, $|d_k/a_k| = 4$ and consequently the minimum will be at $k = (\pi,\pi)$. For $\chi_{o1} \neq 0$ the position of the minimum will be in between $(0,0)$ and $(\pi,\pi)$.

### 3.1 Paramagnetic case

For a completely disordered magnetic system, we have

$$\langle \vec{S}_i \cdot \vec{S}_j \rangle = \frac{3}{4} \delta_{ij}$$

(25)

Therefore $\chi_{o1}$, $\chi_{11}$ and $\chi_{o2}$ vanish and we obtain for the energy

$$\frac{E_k}{g} = \eta \left( \frac{9}{2} - 2\gamma_k \right) - 2 - \sqrt{12(1 - \gamma_k) + [2 - \eta \left( \frac{1}{2} + 2\gamma_k \right)]^2}$$

(26)

The dispersion relation Eq.(26) is plotted in Fig.(1) for the physical value $\eta = 1/6$. The band minimum is reached at points $(\pm \pi, \pm \pi)$. The center of the Brillouin zone $\Gamma(0,0)$ is a maximum. The bandwidth is given by

$$W = E(0,0) - E(\pi,\pi)$$

$$= 2g \left\{ \sqrt{6 + \left[ \frac{3}{4} \eta \right]^2} - 1 - \frac{3}{4} \eta \right\}$$

(27)
for the physical value $\eta = 1/6$, we have $W = 3.14g$. From Ref. [1] we can get the relation between $g$ and the $t$ parameter of the $t$-$J$ model, and we have $t = g(3 - 2\eta)/4$. We end then with a bandwidth $W \simeq 4.71t$. We obtain a bandwidth of the order of $t$. We notice that even in this disordered case the bandwidth was reduced from $8t$ (for the $t$-$J$ model with $J = 0$: the free motion) to $4.71t$, which express the strong correlation in the system. We expect that in the ordered state the bandwidth will be reduced further.

Now we want to examine the spin pattern around the hole. We started from a state without order and we want to calculate the local magnetization at site $j$, if the hole is at site $i$ for the trial wave function $\psi^{(k)}$ with energy $E_k$. Algebraic calculation leads to

$$
\left(\frac{\psi^{(k)}, n_i S_j^z \psi^{(k)}}{\psi^{(k)}, \psi^{(k)}}\right) = -w_{ij} \cos k(i - j) \frac{w_k a_k (\eta \varepsilon_k - E_k/g)}{\varepsilon_k a_k^2 + 3[\eta \varepsilon_k - E_k/g]^2}
$$

where we have used Eqs.(22),(23). Since the minimum of energy is at $Q = (\pi, \pi)$, the hole will have the lowest energy $E_Q$ and then we get the following correlation function between the hole and surrounding spins

$$
<n_i S_j^z> = \frac{\psi^{(k)}, n_i S_j^z \psi^{(k)}}{\psi^{(k)}, \psi^{(k)}} = \frac{1}{\sqrt{2}} \left\{ \frac{E_Q/g - 8\eta}{6 + \frac{1}{4}[E_Q/g - 8\eta]^2} \right\} w_{ij} \cos Q(i - j)
$$

For $\eta = 1/6$ we obtain

$$
<n_i S_j^z> \simeq -\frac{1}{4} w_{ij} \cos Q(i - j)
$$

Eq.(30) means that the hole polarizes the spins in its vicinity. The spin pattern around the hole is antiferromagnetic and the correlation decreases like $w_{ij}$ when the distance between the hole and neighbor spin increases (see Fig.(2)).

The trial state $\psi$ corresponds to a state with a total spin $1/2$ and $S_{tot}^z = +1/2$. The state corresponding to $S_{tot}^z = -1/2$ is just given by $S_{tot}^- \psi$. And the correlation function for a hole with spin down is related to the case of spin up by

$$
<n_i S_j^z>^{(\downarrow)} = (S_{tot}^- \psi, n_i S_j^z S_{tot}^- \psi) = -<n_i S_j^z>^{(\uparrow)}
$$

7
This implies that a hole with spin down will polarize spins in its vicinity with opposite direction with respect to a hole with spin up. Naively we may say that when two holes with opposite spins are introduced in the system it will be preferable to have them close to each other in order to cancel their spin polarizations and obtain again a system without magnetic order which favours the motion of the holes. This scenario might be similar to the Cooper pairing.

3.2 Locally ordered case

Since experiments on copper oxides show that the spin susceptibility presents a sharp peak near the wave vector $Q[1]$ in the metallic phase, we shall take a lorentzian shape for the static susceptibility to describe the case of a locally ordered state. We write then

$$\chi(q) = \frac{C(\xi)}{1 + \xi^2[1 + \frac{1}{2}(\cos q_x + \cos q_y)]}$$

(32)

which may be correct only for short correlation length $\xi$. $C(\xi)$ is determined by writing $\langle \vec{S}_i \vec{S}_i \rangle = 3/4$ which gives

$$C(\xi) = \frac{3}{4} \left[ \frac{1}{N} \sum_q \frac{1}{1 + \xi^2[1 + \frac{1}{2}(\cos q_x + \cos q_y)]} \right]^{-1}$$

(33)

The inverse Fourier Transform of Eq.(32) gives

$$\chi(i-j) = \frac{1}{N} \sum_q \frac{C(\xi)e^{i(q(i-j)}}{1 + \xi^2[1 + \frac{1}{2}(\cos q_x + \cos q_y)]}$$

$$= \frac{4C(\xi)}{\xi^2} \chi_Q(i-j)e^{-iQ(i-j)}$$

(34)

and $\chi_Q(i-j)$ satisfies the equation

$$\sum_l (-\Delta)_{ll} \chi_Q(l-j) + \frac{4}{\xi^2}\chi_Q(i-j) = \delta_{ij}$$

(35)

with $\Delta$ the Laplacian on the square lattice. The asymptotic solution of this equation (i.e. for $|i-j| \to \infty$) is known[13] and we have
\[ \chi_Q(i - j) \sim e^{-|i-j|/\xi_L} \]  

(36)

where \( \xi_L \) is given by

\[ \frac{1}{\xi_L} = \tanh^{-1}(1 + \frac{1}{\xi^2}) \]  

(37)

For \( \xi \) sufficiently large we have \( \xi_L \simeq \xi/\sqrt{2} \).

Now having the phenomenological form of the static susceptibility (Eq.(32)), we can compute \( \chi_{o1} \), \( \chi_{11} \) and \( \chi_{o2} \) for different correlation length. The table (1) contains some values of these quantities. We notice that by increasing \( \xi \) we can have more and more ordered state. For example we almost recover the values of \( \chi_{o1} \), \( \chi_{11} \) and \( \chi_{o2} \) predicted by the linear spin wave approximation for \( \xi = 5 \).

| \( \xi \) | \( \chi_{o1} \) | \( \chi_{11} \) | \( \chi_{o2} \) |
|----------|----------|----------|----------|
| 1        | -0.105   | 0.0278   | 0.015    |
| 2        | -0.199   | 0.094    | 0.058    |
| 3        | -0.259   | 0.150    | 0.101    |
| 4        | -0.299   | 0.192    | 0.138    |
| 5        | -0.328   | 0.223    | 0.167    |

Table 1 : Correlation functions \( \chi_{o1} \), \( \chi_{11} \) and \( \chi_{o2} \) calculated from the phenomenological form of the static susceptibility Eq.(32), for different \( \xi \).

Inserting the values of \( \chi_{o1} \), \( \chi_{11} \) and \( \chi_{o2} \) in Eq.(21) we get the dispersion plotted in Fig.(3) for different values of \( \xi \). We see from Fig.(3) that the spectrum changes drastically when the magnetic background changes from a disordered state to an ordered one. The minimum of \( E_k \) shifts from \( (\pm \pi, \pm \pi) \) in the case of a disordered background \( (\xi = 0) \) to \( (\pm \pi/2, \pm \pi/2) \) for \( \xi = 3 \). We see clearly that the bandwidth has been strongly reduced in comparison with the disordered case. It was reduced from \( W \simeq 3.14g \) (for \( \xi = 0 \)) to \( W \simeq 1.3g \)
( for $\xi = 3$). In term of $t$ we get a bandwidth $W \simeq 1.8t$. But it is still of the order of $t$ and not $J$. This is a discrepancy with previous calculations done in the framework of the $t$-$J$ model for an AFM background\[1, 2, 3, 4\] which may be explained by the fact that no magnetization is present in our calculation since we considered our background as a singlet state. Other reason might be that with our variational approach we do not include the incoherent motion of the quasi-particle which is the consequence of retarded effects.

4 Conclusion

In this section we have computed the dispersion relation $E(k)$ for a single hole in the framework of the spin-hole model using a variational method. The calculation was done for any rotational invariant magnetic background. And we have shown how the hole spectrum changes when the magnetic environment goes from a disordered background to an ordered one. Especially we have found that the minimum energy shifts from $(\pm \pi, \pm \pi)$ for the completely disordered case to $(\pm \pi/2, \pm \pi/2)$ in the case of ordered magnetic background. The bandwidth is strongly reduced by a factor $1/3$, when the magnetic background changes from a disordered to an ordered state. An other interesting result that we have found is the spin polarization around the hole. Namely when a hole moves in a completely disordered background it will polarize spins in its vicinity. This polarization is decreasing with the distance as a power law and has an AFM ordering. We expect that two holes with opposite spins will tend to pair in order to remove their respective spin polarizations. This question deserves further studies and it is now under investigation.
References

[1] S. Schmilt-Rink, C. M. Varma and A. E. Ruckenstein, Phys. Rev. Lett. 60, 2792 (1988)

[2] C. L. Kane, P. A. Lee and N. Read, Phys. Rev. B 39, 6880 (1989)

[3] G. Martinez and P. Horsch, Phys. Rev.B 44, 317 (1991).

[4] J. L. Richard and V. Yu. Yushankhai, Phys. Rev. B 47, 1103(1993).

[5] D. Poilblanc, H. J. Schultz and T. Ziman, Phys. Rev. B 47, 3268(1993).

[6] W. Stephan and P. Horsch, Phys. Rev. Lett. 66, 2258(1991).

[7] J. Zaanen and A.M. Oleš, Phys. Rev. B 37, 9423(1988).

[8] P. Prelovsek, Phys. Lett. A 126, 287(1988).

[9] J.L. Shen and C.S. Ting, Phys. Rev. B 41, 1967(1990).

[10] P. Fulde, *Electron Correlation in Molecules and Solids*, Springer Series In Solid state Sciences 100 ( Springer, Berlin, 1991)pp. 353-354

[11] See for example the review paper of : Rossat-Mignod et al., Preprint, (1993).

[12] F.C. Zhang and T.M. Rice, Phys. Rev. B 37, 3759(1988).

[13] P. Monthoux and D. Pines, Phys. Rev. B 47, 6069(1993).

[14] S. Liang, B. Douçot, and P.W. Anderson, Phys. Rev. Lett. 61, 365(1988).

[15] O.A. MacBryan and T. Spencer, Commun. math. Phys. 53, 299(1977).
Figure 1: Dispersion relation for a single hole in disordered magnetic background (with $M = (\pi, \pi)$, $\Gamma = (0, 0)$ and $X = (\pi, 0)$). $E_k$ is in unit of $g$.

Figure 2: Polarization of spins surrounding the hole.
Figure 3: Dispersion relation for a single hole in different magnetic backgrounds: from a completely disordered state ($\xi = 0$) to an ordered state $\xi = 5$. $E_k$ is in unit of $g$ (with $M = (\pi, \pi)$, $\Gamma = (0, 0)$ and $X = (\pi, 0)$).