Probing radiative solar neutrinos decays.

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Abstract

Motivated by a pilot experiment conducted by F. Vannucci et al. during a solar eclipse, we work out the geometry governing the radiative decays of solar neutrinos. Surprisingly, although a smaller proportion of the photons can be detected, the case of strongly non-degenerate neutrinos brings better limits in terms of the fundamental couplings. We advocate satellite-based experiments to improve the sensitivity.

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1 Introduction.

The group led by F. Vannucci has recently published the results of a search for radiative decays $\nu_1 \rightarrow \nu_2 \gamma$, where $\nu_1$ is (in principle) any neutrino present in the solar flux, either directly produced or arising from oscillations. To suppress obvious background, the experiment was conducted during a total solar eclipse and was in practice sensitive to radiative decays taking place between the Moon and the Earth. Their analysis of the data was conducted in the specific case where $\nu_1$ and $\nu_2$ are nearly degenerate ($\delta m^2 \equiv m_{\nu_1}^2 - m_{\nu_2}^2 \simeq 10^{-5} \text{eV}^2 / c^4$).

In the present note, we present the kinematics for the general case, and work out in details the limit $m_{\nu_2} \simeq 0$.

We interpret the decay in terms of a dipole (magnetic or electric) transition, taking into account the polarization of the neutrinos. Results are then confronted to earth-based limits on transition magnetic moments. The current sensitivity (assuming only one photon in the decay) falls way below current limits if either $\nu_1$ or $\nu_2$ is one of the known neutrinos, but the bounds apply here independently of the neutrino species. Possible improvements of the experiment involve satellite-based observations.

In section 2 and 3, we discuss the geometry and the kinematics leading to the observation of a visible photon, as a function, notably, of the mass difference and the telescope aperture. We discuss the possible signal configurations, in particular for the monochromatic $\nu_1$ from $^7$Be, using standard solar model data. We consider also the angular distribution of neutrino decays for the
case of polarized neutrinos, and we relate the observed $\gamma$ flux to the transition magnetic moment.

In section 4 to 6, we discuss the relation to existing bounds from earth-based experiments, and present some prospects for a satellite-bound extension of the experimental set-up.

2 Kinematics.

Let us consider the effective lagrangian (Pauli term) describing the reaction $\nu_1 \rightarrow \nu_2 \gamma$:

$$\mathcal{L} = \mu \bar{\psi}_{\nu_2} \sigma^{\alpha\beta} \psi_{\nu_1} q_{\beta} A_{\alpha}$$

where $\mu$ is the transition magnetic moment $\mu_{\nu_1\nu_2}$ and $q$ the momentum of the photon.

The correspondent differential decay probability in the neutrino rest frame (CM) reads:

$$d\Gamma^* = \frac{\mu^2}{(2\pi)^2} \frac{\delta^4(k_1 - k_2 - k_{\gamma})}{2\omega_1 2\omega_2 2\omega_{\gamma}} (8 m_1 k_{\gamma}^2 (\omega_2 + k_2) A(\theta^*)) d^3 k_2 d^3 k_{\gamma}$$

where $k_1, k_2, k_{\gamma}$ are the 4-momenta of $\nu_1, \nu_2, \gamma$ respectively;

$\omega_1, \omega_2, \omega_{\gamma}$, their energies;

and $\theta^*$, the angle between the photon and the boost direction (defined in the CM as the direction opposite to the sun recoil).

$A(\theta^*)$ is found to be:
1 for an unpolarized neutrino flux;
1 − cos θ∗ for a left-handed neutrino flux;
1 + cos θ∗ for a right-handed neutrino flux.

In the rest frame, we have:

\[ \frac{d\Gamma^*}{d\Omega^*} = \frac{\mu^2}{(2\pi)^2} A(\theta^*) \left( \frac{\delta m^2}{2m_1} \right)^3 \]

where Ω∗ is the solid angle of the photon in the rest frame.

A last integration gives us the decay probability and the mean lifetime:

\[ \Gamma^* = \frac{\mu^2}{\pi} \left( \frac{\delta m^2}{2m_1} \right)^3 \]
\[ \tau_{\nu_1} = K \cdot \left( \frac{\delta m^2}{eV^2} \right)^{-3} \cdot \left( \frac{m_1}{1eV} \right)^3 \cdot \left( \frac{\mu}{10^{-10} \mu_B} \right)^{-2} \]

where \( K \) is given by \( \frac{8}{\alpha} m_e^2 h \simeq 1.00 \times 10^{15} \text{s}. \)

### 3 Geometry.

We calculate first the image produced by a point-like source (taking advantage of the decay symmetry) at C, the center of the Sun. A point-like source located in Earth sky at angles \((\theta, \varphi)\) produces a translated image, centered at \((\theta, \varphi)\) instead of \((0, 0)\), and the full image results of the sum of the contributions from all pointlike sources inside the Sun.

The source S emits a neutrino which eventually oscillates (but holds it initial flight direction) and decays at P. There, it emits a photon with an angle \(\theta\) with respect to its initial flight direction. This photon is collected in
the telescope T on earth. This simplified geometry (source and telescope are assumed point-like) is plane (cylindrical symmetry).

Four elements will help to solve our problem.

1. The energy $E$ of the solar neutrinos is typically of order 1 MeV. With a typical neutrino mass $m_1$ of order 1 eV, we get the following Lorentz parameters, connect the CM and the laboratory (LAB) frames: $\beta = 1 - \epsilon$, $\gamma = \frac{E}{m_1} \approx 10^6$, with $\epsilon \approx 10^{-12}$.

Therefore, if $k, k^*$ denote the energies of the photon respectively in the LAB and the CM,

$$k = k^* \gamma (1 + \beta \cos \theta^*) \approx k^* \gamma (1 + \cos \theta^*)$$ (5)

valid as long as the Lorentz parameter $\beta$ is close to 1.

2. Eq.(5) shows that the energy of the emitted photon is related to the emission angle $\theta$ in the LAB by:

$$\theta \approx \tan \theta \approx \frac{1}{\gamma} \sqrt{\frac{2k^*\gamma}{k} - 1} \approx \frac{m}{E} \sqrt{\frac{\delta m^2 E}{k m^2} - 1} \approx \frac{m}{E} \sqrt{\frac{k_{\text{max}}}{k} - 1}$$ (6)

where $k_{\text{max}} \equiv 2k^*\gamma$ is the maximum photon energy reachable in the LAB.

3. The set of points P with S,T and $\theta$ (thus $k$) fixed is an arc of a circle.
Since we consider only decays between the Moon and the Earth, this arc may be treated in a good approximation as a straight line. Moreover, this line is tangent to the arc at T and makes with the Sun-Earth direction an angle $\theta$ of the same amplitude than the emission angle.

4. The approximation above, valid as long as $d_{ST} \gg d_{PT}$, also implies that the results don’t depend on the distance of the source S, so that only its angular position, as seen on Earth sky is relevant. In other terms, all neutrino sources located in a small rod of the sun, defined by the angles $(\theta, \varphi)$ on earth, give superimposed images. From now on, such a rod will be called "rod-source" and its image, $\text{Image}(\theta, \varphi)$.

If the telescope of angular aperture $\theta_{\text{tel.}}$ (corresponding to an energy $k_{\text{tel.}}$), is sensitive in a given energy spectrum, for example $[k_{\text{red}}; k_{\text{violet}}]$ for the visible spectrum, only the photons between two angles $\theta_{k_+}$ and $\theta_{k_-}$ will be detected, where $k_- = \max(k_{\text{red}}; k_{\text{tel.}})$ and $k_+ = \min(k_{\text{violet}}; k_{\text{max}})$. We see that a rod-source will produce a crown on the telescope (centered on (0,0) for a source at C). That crown may be so small that it degenerates into a single

\[2\text{ It is in principle conceivable that neutrinos further than the Moon, but off-center, contribute by their decay. In practice however, the limited aperture of the telescope used in [1] forbids their observation.}\]
pixel (“pixel case”) or can be larger than the telescope aperture (“out case”).

![Expected Shape of the Image](image_url)

Figure 1: Expected image shapes for various values of $m_{\nu_1}$ ($m$, in eV) and $\delta m^2$ ($dm^2$, in eV$^2$). $N$ is in arbitrary units.

To restore the full geometry, we have to sum the different contributions from rod-sources. The data of the standard solar model (SSM) list usually the production rates as a function of the distance $R$ from the sun’s center. We have to reparametrize the data to work out the total emission in a rod-source, i.e. a thin cylinder in the Sun, approximately parallel to the Sun-Earth direction.
Let $f(R)$ be the solar neutrino flux produced per unit volume at a distance $R$ from the centre of the Sun ($f(R) = \frac{d\phi(R)}{4\pi R^2 dR}$ where $d\phi(R)$ are the usual form of the SSM data, see [2]). The emission in a thin rod in a solid angle $d\psi$ oriented in a direction $(\theta, \varphi)$ from Earth sky is independent of $\varphi$ and given by [3]:

$$
\epsilon(x) = 2 \left( \frac{d_{ST}}{R_{\odot}} \right)^2 d\psi \int_0^1 f(x') \frac{x' dx'}{\sqrt{x'^2 - x^2}}
$$

where $R_{\odot}$ is the radius of the Sun and $d_{ST}$ is the distance between the source S and the telescope T (approx. distance Sun-Earth).

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3$x$ is a reduced coordinate ($\frac{R}{R_{\odot}}$ or $\frac{\theta}{\theta_{\odot}}$). Here $x$ means the reduced distance between the rod and C, while $x'$ in (7,8) is the distance between the source S and C.
We sum now the contributions from all solar sources

\[ 2 \left( \frac{dS_T}{R_\odot} \right)^2 \int_0^{\theta_\odot} \sin \theta \, d\theta \int_0^{2\pi} d\varphi \, I(\theta, \varphi) \cdot \int_x^1 f(x') \frac{x' dx'}{\sqrt{x'^2 - x^2}} \]  

We also calculate the expected number of photons landing in the telescope (assuming an ideal efficiency of 100%).

\[ N_\gamma = \frac{T_{\text{exp}}}{\hbar} \cdot \frac{F}{\beta c} \cdot \int dv \cdot \frac{d\Gamma}{d\Omega} \cdot \Delta\Omega \]  

where

\[ T_{\text{exp}} \] is the exposition time, and we have restored the dimensional factor \( \hbar \).

\[ \frac{F}{\beta c} \] is the number of neutrinos present in a unit volume between Moon and Earth.

\[ \frac{d\Gamma}{d\Omega} \cdot \Delta\Omega \] is the probability of a decay detection. \( \Delta\Omega \) is the solid angle under which the telescope is seen from the decay point \( P \); it’s value is \( \pi \frac{r^2}{r_{\text{tel}}^2} \) and varies in space.

The coordinates \( \theta \) in \( dv = r^2 dr d\varphi d\cos \theta \) and in \( \Delta\Omega \) are in principle different. The first one locates \( P \) from Earth sky. The second one locates the photon from \( P \). But we already pointed out that those angle have the same value.

Substituting \( f dv \) by \( f r^2 dr f d\Omega \) and \( \frac{d\Gamma}{d\Omega} \) by \( \frac{1}{\gamma} \frac{d\Gamma^{*}}{d\Omega^{*}} \frac{d\Omega^{*}}{d\Omega} \), we obtain

\[ N_\gamma = \frac{T_{\text{exp}}}{\hbar} \cdot \frac{F}{\beta c} \cdot \frac{m}{E} \cdot \frac{\mu^2}{4\pi} \left( \frac{\delta m^2}{2m} \right)^3 \cdot \int_0^{2\pi} d\varphi \cdot \int_A^B d\cos \theta^* A(\theta^*) \int_{d_{\text{ME}}}^{d_{\text{tel}}} r^2 dr \cdot \frac{r_{\text{tel}}^2}{r^2} \]
where $d_{ME}$ is the distance between the Moon and the Earth.

The first integral gives us a factor $2\pi$; the third one, $r^2_{tel.} \cdot d_{EM}$; the second one depend on $A(\theta^*)$: it gives $\frac{2\Delta k}{k_{max}} \cdot H$. Putting into (10):

$$N_{\gamma} = \frac{1}{8\hbar c} \cdot (\Delta t \Delta k r^2_{tel.} d_{ME}) \cdot \left( \frac{F}{E^2} \right) \cdot ((\delta m^2)^2 \mu^2 \cdot H)$$

(11)

where $H$ is, for unpolarized, left-handed and right-handed neutrinos respectively, $\frac{2\Delta k}{k_{max}}$, $\frac{2\Delta k}{k_{max}} \cdot (2 - \frac{k_- + k_+}{k_{max}})$, and $\frac{2\Delta k}{k_{max}} \cdot (\frac{k_- + k_+}{k_{max}})$. The first bracket gathers the experimental parameters; the second, the SSM; and the third describes the neutrino physics.

Note the independence on $m$, and the somewhat unexpected dependence on $E$: the lower energies are favoured. A limit on $\delta m^2 \cdot \mu$ is obtained from the observed $N_{\gamma}$ and at fixed $N_{\gamma}$, the limit on $\mu$ varies like the inverse of $\delta m^2$.

It is easy to generalize all this results to a spectrum of neutrino energies. We only need to substitute $F$ by $\int dE f(E)$ where $f(E)$ is the flux per energy unit.

4 Results.

An analysis of experimental data should involve two aspects: the shape of the expected signal and the number of photons collected.

As already discussed, the shape of the signal is a centered crown, which may however be concentrated into a single pixel, or, as another extreme case, fall

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4This has some unexpected consequences. For example, in the $pp$ neutrinos, however the flux at low energy (a few keV) is very small, we expect a more intensive emission at low energy than at high ($> 100keV$)
out of the telescope aperture. A full analysis must also consider the entirety of the neutrino spectrum (and not only those produced by the $^7\text{Be}$). The informations obtained from the shape above concern especially $\delta m^2$. Indeed $\mu$ doesn’t appear in (9), and $m$ is a less sensitive parameter.

In the “pixel” or “out” cases, only limits can be obtained. The pixel case provides also informations about the SSM, because the shape of the signal reproduces $e(x)$.

The number of photons provides informations on the quantity $\delta m^2 \cdot \mu$. For a fixed value of $\delta m^2$, one can extract $\mu_{\nu_1\nu_2}$ and thus $\tau_{\nu_1}$.

For example, for a $\phi1m$ telescope and an exposition of 1s., assuming that the optics allow for collecting of all visible photons falling on the telescope mirror, considering the full $^7\text{Be}$ neutrino flux alone (all neutrinos assumed to be $\nu_1$), and an unpolarized neutrino with $m = 1\text{eV}$, the limit $N_\gamma < 1000$ provides:

$$\delta m^2 \cdot \mu < 1.44 \text{ eV}^2 \mu_B$$

For $\delta m^2 \simeq 1$ 
$$\mu_{\nu_1\nu_2} < 1.44 \mu_B$$
$$\tau_{\nu_1} > 0.482 \times 10^{-5} \text{ s.}$$

For $\delta m^2 \simeq 10^{-5}$ 
$$\mu_{\nu_1\nu_2} < 1.44 \times 10^5 \mu_B$$
$$\tau_{\nu_1} > 0.482 \text{ s.}$$

We see that the nearly degenerate masses give a better limit on $\tau_{\nu_1}$. The non degenerate masses offers less acceptance because the major part of the
Photons are now $\gamma$-rays; but the decay probability depends on $(\delta m^2)^3$. That case provides thus a better limit on $\mu_{\nu_1\nu_2}$.

The signal will fall out of the telescope aperture if $\theta_{tel.} < \theta_{violet}$, i.e. if $\delta m^2 > 17 \text{ eV}^2$ (for an aperture of $0.5 \times 10^{-2} \text{ rad}$). The photon emission will be entirely in the infrared region if $k_{max} < k_{red}$, i.e. if $\delta m^2 < 2.1 \times 10^{-6} \text{ eV}^2$.

Note such an experiment covers a large domain of $\delta m^2$ because $E$ varies from a few keV to about 10 MeV. For example, it is possible that the image from the $^7\text{Be}$ falls out of the telescope but that the $pp$ neutrinos produce a crown.

The detection of unpolarized photons at a given wavelength is not sensitive to the polarization of the incident neutrinos. To do this, the best way is to accumulate data at different wavelengths probing the $k$ dependence of $H$ in (11).

5 Comparison to earth-bound limits.

If indeed the radiative decay of sun-born neutrinos proceeds through a single photon emission, the limits on lifetimes are equivalent to limits on magnetic transition dipole moments (or a combination of electric and magnetic moments). As discussed in [4] strong limits on those transition moments exist, and can be derived directly from the searches for "diagonal" magnetic moments. These limits are for instance, for any decay involving an electronic left-handed neutrino $\mu_{\nu e_L} \leq 1.8 \times 10^{-10} \mu_B$, and are out of reach at least for
eclipse-linked experiments. This would be the case, either for the electron neutrinos originating from the sun and decaying into $\nu_i$, or for the neutrino of type $i$, ($i \neq e, \mu, \tau$) which would have already been created through oscillations, decaying to $\nu_{eL}$.

The limits obtained from eclipse or space-based experiments however cover the extra case of an hypothetical $\nu_i$ decaying into $\nu_j$, where neither $i$ nor $j$ refer to left-handed known neutrinos (for example, a new $\nu_i$ decaying to a right-handed $\nu_e$).

Reference [4] also raised the point of the momentum dependence of the magnetic transition form factor; indeed, accelerator or even reactor experiments measure $\mu$ at very large momentum transfer with respect to the real decay processes considered here (photon on shell), and the extrapolation may not be trivial; we will however not elaborate on this possibility here, since it was shown that extraordinary assumptions would be needed to bring any serious difference.

6 Prospects.

Earth-based experiments can obviously be improved through larger telescopes, airplane-borne equipment, but satellite-based experiments offer more possibilities: $X$ and $\gamma$-rays detection; longer and recurrent expositions; strong reduction of noise;...

For example, an earth-based experiment can involve more than one tele-
scope to increase the exposition time. If the eclipse occults an observation
centre, bigger mirrors can be used. The optical efficiency and the CCD resolu-
tion can be increased. But on Earth, the rarity of total solar eclipses and the
important atmospheric noise are intrinsic constraints.

On the contrary, satellite-based experiment provides the advantage of recur-
rent data taking, because the Earth occults the satellite every day. Moreover,
the allowed aperture of the telescope will be increased. For a geostationary
orbit (at 36 000 km), the decay volume is reduced by a factor $\sim 10$ but the
satellite is occulted during about half an hour per day and an aperture of 0.15
rad. is allowed. Further gain comes from the atmospheric noise reduction and
the wide accessible spectrum, including $X$, $\gamma$ and $IR$.

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