A general derivation of differential cross section in quark-quark scatterings at fixed impact parameter

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We propose a general derivation of differential cross section in quark-quark scatterings at fixed impact parameters. The derivation is well defined and free of ambiguity in the conventional one. The approach can be applied to a variety of partonic and hadronic scatterings in low or high energy particle collisions.

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I. INTRODUCTION

The global polarization effect in quark scatterings in non-central heavy-ion collisions has been predicted due to orbital angular momenta resides in the system as a result of longitudinal flow shear [1, 2, 3]. The effect is defined with respect to the direction orthogonal to the reaction plane determined by the vector of impact-parameter and the beam momentum. The polarization of quarks can be partially carried by hadrons containing these quarks via hadronization. For example, as a consequence of this global polarization, vector mesons can have spin alignments in non-central heavy-ion collisions [4]. Polarized photons are recently proposed as a good probe to the polarized quarks [5].

In Ref. [4], the polarization in the quark-quark scatterings has been estimated in a hot medium. The differential cross-section with respect to the partonic impact parameter is derived by inserting into the differential cross-section in momentum space a delta function for transverse momenta, which can in turn be written as an integral over the partonic impact parameter. This derivation has an ambiguity. In this note we will give an alternative and a general way of deriving the differential cross-section with respect to the partonic impact parameter. We will show that the differential cross section with respect to impact parameter in our framework is well defined and free of ambiguity.

II. KINEMATICS SETUP

The geometry of a nucleus-nucleus collision at impact parameter $b$ is illustrated in Fig. 1 of Ref. [1]. We assume that two nuclei move along $\pm z$ directions and collide at $z = 0$. The reaction plane is in the $y$ direction. We consider the polarization along the $y$ direction.

Let us consider the scattering of quarks with different flavors ($P_1, \lambda_1$) $\rightarrow (P_3, \lambda_3) + (P_4, \lambda_4)$ through the Hard-Thermal-Loop (HTL) resummed gluon propagators, where $\lambda_i$ with ($i = 1, 2, 3, 4$) are spin states and $P_i = (E_i, p_{i\perp}, p_{iT})$ are 4-momenta for colliding quarks, with longitudinal and transverse momenta $p_{i\perp}$ and $p_{iT}$ and energies $E_i = \sqrt{p_{i\perp}^2 + p_{iT}^2}$. Note that we treat all quarks massless. We assume the initial momenta are along the $z$ direction, i.e. $p_{1z} = -p_{2z} > 0$ and $p_{1T} = p_{2T} = 0$. We will use the shorthand notation $p_T \equiv p_{3T}$. The total energy in the center-of-mass frame is denoted by $\sqrt{s} = \sqrt{(P_1^2 + P_2^2)^2}$.

III. AMBIGUITY OF DIFFERENTIAL CROSS SECTION WITH RESPECT TO IMPACT PARAMETER IN CONVENTIONAL TREATMENT

In a conventional derivation of the differential cross section at impact parameter $x_T$, one starts with the one in momentum space,

$$d\sigma = \frac{1}{16|P_1 \cdot P_2|} \sum_{\lambda_1, \lambda_2, \lambda_4} |M(P_1 \lambda_1, P_2 \lambda_2, P_3 \lambda_3, P_4 \lambda_4)|^2 \times (2\pi)^4 \delta^{(4)}(P_1 + P_2 - P_3 - P_4) \frac{d^3p_3}{(2\pi)^3 2E_3} \frac{d^3p_4}{(2\pi)^3 2E_4},$$

where a sum over the spin states $\lambda_1, \lambda_2, \lambda_4$ except $\lambda_3$ and an average (with a factor $1/4$) over the initial spin states have been taken. One can integrate out $p_4$ by consuming the delta function $\delta^{(3)}(p_1 + p_2 - p_3 - p_4)$ for 3-momentum...
arises when one calculates quantities like the polarization when the integral over $p_{3z}$ to remove $\delta(E_1 + E_2 - E_3 - E_4)$ for the energy conservation which gives a factor

$$
\int dp_{3z} \delta(E_1 + E_2 - E_3 - E_4) = \sum_{i=1,2} \frac{E_3 E_4}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|},
$$

where $p_{4z}^{(i)} = p_{1z} + p_{2z} - p_{3z}^{(i)}$ and $p_{3z}^{(i)}$ are roots of the energy conservation equation and given by

$$
p_{3z}^{(1/2)} = \pm \frac{E_1 + E_2}{2} \left[ 1 - \frac{2p_T^2}{|p_{1z}p_{2z} - p_{1z}p_{2z}|} \right]^{1/2} + \frac{p_{1z} + p_{2z}}{2}.
$$

Then Eq. (1) is simplified as

$$
\frac{d\sigma}{d^2p_T} = \frac{1}{64|P_1 \cdot P_2|} \sum_{i=1,2} \sum_{\lambda_1, \lambda_2, \lambda_4} |M(P_3\lambda_3, P_4\lambda_4)|^2 \frac{1}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|} d^2p_T,
$$

where we have suppressed the $(P_1\lambda_1, P_2\lambda_2)$ dependence of the amplitude. Note that $p_{4T} = -p_{3T} = -p_T$ is implied in the above expression due to momentum conservation and the assumption that the initial state momenta are along the $z$-axis, $p_{1T} = p_{2T} = 0$. One can rewrite it by inserting a delta function for transverse momenta,

$$
\frac{d\sigma}{d^2x_T} = \frac{1}{64|P_1 \cdot P_2|} \int d^2p_T \frac{d^2p_T'}{(2\pi)^2 (2\pi)^2} \delta^{(2)}(p_T - p_T')
$$

$$
\times \sum_{i=1,2} \sum_{\lambda_1, \lambda_2, \lambda_4} \frac{1}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|} M(P_3\lambda_3, P_4\lambda_4) M^*(P_3'\lambda_3, P_4'\lambda_4),
$$

where $P_3' = (E_3', p_{3z}, P_T')$ and $P_4' = (E_4', p_{4z}, -P_T')$. Then we obtain the differential cross section at impact parameter $x_T$,

$$
\frac{d^2\sigma}{d^2x_T} = \frac{1}{64|P_1 \cdot P_2|} \int d^2p_T \frac{d^2p_T'}{(2\pi)^2 (2\pi)^2} \delta^{(2)}(p_T - p_T') x_T
$$

$$
\times \sum_{i=1,2} \sum_{\lambda_1, \lambda_2, \lambda_4} \frac{1}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|} M(P_3\lambda_3, P_4\lambda_4) M^*(P_3'\lambda_3, P_4'\lambda_4).
$$

Note that the above expression of $d^2\sigma/d^2x_T$ is not unique, since one can make the replacement in Eq. (5), for example,

$$
\frac{1}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|} \rightarrow \frac{1}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|^a} \frac{1}{|E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)}|^a},
$$

with $a_1 + a_2 = 1$, while keeping the total cross section unchanged. Of course one can make many other choices which conserve the total cross section. In Ref. [4], $a_1 = a_2 = 1/2$ is used, while Eq. (5) implies $a_1 = 1, a_2 = 0$. So we see that there is an ambiguity in $d^2\sigma/d^2x_T$, or in other word, $d^2\sigma/d^2x_T$ is not unique by this definition. The problem arises when one calculates quantities like the polarization when the integral over $x_T$ is not made in the whole space (therefore the delta function $\delta^{(2)}(p_T - p_T')$ is not recovered), which would lead to different or inconsistent results.

IV. DIFFERENTIAL CROSS SECTION AT FIXED IMPACT PARAMETER

In order to solve the ambiguity in the previous section, we will derive in this section the cross sections of parton-parton scatterings at fixed impact parameter in a general approach. To this end, we need to introduce particle states labeled by transverse positions and longitudinal momenta, which we call states in the mixed representation. They
are connected with states in momentum space by Fourier transform in transverse sector. We express in the mixed representation the final states in the scatterings as

\[ |p_{3z}, \lambda_3, x_{3T} \rangle = \int \frac{A_T d^2 p_{3T}}{(2\pi)^2} e^{i p_{3T} \cdot x_{3T}} |p_3, \lambda_3\rangle, \]

\[ |p_{4z}, \lambda_4, x_{4T} \rangle = \int \frac{A_T d^2 p_{4T}}{(2\pi)^2} e^{i p_{4T} \cdot x_{4T}} |p_4, \lambda_4\rangle, \]

where \( A_T \) is the area in the transverse plane. The S-matrix element from the initial to final states then reads

\[ S_{fi} = \langle p_{3z}, \lambda_3, x_{3T}; p_{4z}, \lambda_4, x_{4T} | S | p_1, \lambda_1, p_2, \lambda_2 \rangle \]

\[ = \int \frac{A_T d^2 p_{3T}}{(2\pi)^2} \frac{A_T d^2 p_{4T}}{(2\pi)^2} e^{-i p_{3T} \cdot x_{3T}} e^{-i p_{4T} \cdot x_{4T}} (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \]

\[ \times \frac{1}{\sqrt{2E_1V}} \frac{1}{\sqrt{2E_2V}} \frac{1}{\sqrt{2E_3V}} \frac{1}{\sqrt{2E_4V}} M(p_3, p_4, \lambda_3, \lambda_4), \]

where the final state momenta are \( p_3 = (E_3, p_{3z}, p_{3T}) \) and \( p_4 = (E_4, p_{4z}, p_{4T}) \). The squared matrix element becomes

\[ |S_{fi}|^2 = \int \frac{A_T d^2 p_{3T}}{(2\pi)^2} \frac{A_T d^2 p_{4T}}{(2\pi)^2} \frac{A_T d^2 p_{3T}'}{(2\pi)^2} \frac{A_T d^2 p_{4T}'}{(2\pi)^2} e^{-i (p_{3T} - p_{3T}') \cdot x_{3T}} e^{-i (p_{4T} - p_{4T}') \cdot x_{4T}} \]

\[ \times (2\pi)^6 \delta^{(4)}(p_1 + p_2 - p_3 - p_4) \delta^{(4)}(p_1 + p_2 - p_3' - p_4') \]

\[ \times \frac{1}{V^4} \frac{1}{16 E_1 E_2 \sqrt{E_3 E_4 E_3 E_4}} M(p_3, p_4, \lambda_3, \lambda_4) M^*(p_3', p_4', \lambda_3, \lambda_4), \]

where \( \delta^{(0, z)} \) denote the delta functions for the energy and the \( z \) component of momenta. The differential cross section is then

\[ d\sigma = \frac{d^2\chi_T}{A_T \cdot 4\pi v_{rel}^2} \sum_{\lambda_1, \lambda_2, \lambda_4} \int \frac{L dp_{3z}}{2\pi} \frac{L dp_{4z}}{2\pi} |S_{fi}|^2 \]

\[ = d^2\chi_T \frac{1}{\tau} \frac{1}{64(2\pi)^2} \int d^2 p_T dp_{3z} d^2 p_{3T} dp_{4z} e^{-i (p_{3T} - p_{4T}) \cdot x_T} \]

\[ \times \frac{1}{v_{rel} E_1 E_2 \sqrt{E_3 E_4 E_3 E_4}} \delta^{(0, z)}(p_1 + p_2 - p_3 - p_4) \delta(E_1 + E_2 - E_3' - E_4') \]

\[ \times \sum_{\lambda_3, \lambda_4} M(p_3, p_4, \lambda_3, \lambda_4) M^*(p_3', p_4', \lambda_3, \lambda_4), \]

where we have defined \( \chi_T = x_{3T} - x_{4T} \). In the first line we have used that the differential cross section is proportional to the fraction \( d^2\chi_T/A_T \) with \( A_T = \int d^2\chi_T \). Here \( L \) is the length along the \( z \) direction and \( v_{rel} = |p_1 \cdot p_2|/(E_1 E_2) \) the relative velocity of incident partons. We also used \( V = A_T L \) and \( 2\pi \delta^{(z)}(0) = L \) (because two delta functions for \( z \)-momenta are identical). Note that \( \tau \) is the time period of the scattering. It is obvious to see \( d^2\sigma/d^2\chi_T > 0 \) from Eq. (12).

We can evaluate Eq. (12) as follows. First we integrate out \( p_{4z} \) to remove \( \delta(p_{1z} + p_{2z} - p_{3z} - p_{4z}) \). The remaining two delta functions enforce energy conservation, which can be removed by carrying out integrals over the magnitudes of transverse momenta \( p_{3T}' \) and \( p_{4T}' \). We end up with

\[ \frac{d^2\sigma}{d^2\chi_T} = \frac{1}{\tau} \frac{1}{64(2\pi)^3} \frac{1}{|P_1 \cdot P_2|} \int_0^{2\pi} d\varphi \int_0^{2\pi} d\varphi' \int_{p_{4T}^{min}}^{p_{4T}^{max}} dp_{3z} e^{-i p_T (\cos \varphi - \cos \varphi') x_T} \frac{E_3 E_4}{(E_3 + E_4)^2} \]

\[ \times \sum_{\lambda_3, \lambda_4} M(p_3, p_4, \lambda_3, \lambda_4) M^*(p_3', p_4', \lambda_3, \lambda_4), \]
where $\varphi$ and $\varphi'$ are azimuthal angles of $p_T$ and $p_T'$ relative to the direction of $x_T$ respectively. The magnitude of transverse momentum satisfying the energy conservation for $P_{3,4}$ and $P_{3,4}'$ is proved to be the same and given by

$$p_T = \left\{ \frac{(E_1 + E_2)^2 + p_{3z}^2 - (p_{1z} + p_{2z} - p_{3z})^2}{2(E_1 + E_2)} \right\}^{1/2}.$$  

(14)

The energies are

$$E_3 = E_3' = \sqrt{p_{3z}^2 + p_T^2},$$

$$E_4 = E_4' = \sqrt{(p_{1z} + p_{2z} - p_{3z})^2 + p_T^2}.$$  

(15)

The integral $p_{3z}$ is in the range $[p_{3z}^{\text{min}}, p_{3z}^{\text{max}}]$ where

$$p_{3z}^{\text{max}} = \frac{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}{2(E_1 + E_2)} \left[ 1 - \frac{p_{1z} + p_{2z}}{E_1 + E_2} \right]^{-1},$$

$$p_{3z}^{\text{min}} = -\frac{(E_1 + E_2)^2 - (p_{1z} + p_{2z})^2}{2(E_1 + E_2)} \left[ 1 + \frac{p_{1z} + p_{2z}}{E_1 + E_2} \right]^{-1}.$$  

(16)

The amplitude square is given by

$$M(P_3\lambda_3, P_4\lambda_4)M^\ast(P_3'\lambda_3, P_4'\lambda_4) = g^4 c_{qg} E_1 E_2 \sqrt{E_3 E_4 E_3' E_4'} J_1^{\mu
u}(E_1 - P_3) \Delta_{\mu
u}(P_1 - P_3) \Delta_{\mu
u}'(P_1 - P_3)$$

(17)

Here $g$ is the quark-gluon coupling constant and $\alpha_s = g^2/(4\pi)$. The color factor for $qq$ scatterings is $c_{qq} = (1/N_c^2)\delta^{ab}\delta^{ab}/4 = 2/9$, where $1/N_c^2$ is from the average over the initial state colors. $J_1^{\mu
u}$ and $J_2^{\mu
u}$ are tensors only dependent on momentum directions,

$$J_1^{\mu
u} \equiv \frac{1}{E_1 \sqrt{E_3 E_3'}} \text{Tr}[u(P_1, \lambda_1)\bar{u}(P_3, \lambda_3)\gamma^\mu P_1\gamma^\nu]$$

$$J_2^{\mu
u} \equiv \frac{1}{E_2 \sqrt{E_4 E_4'}} \sum_{\lambda_4} \text{Tr}[u(P_1, \lambda_1)\bar{u}(P_4, \lambda_4)\gamma^\mu P_2\gamma^\nu]$$

(18)

where $u(P_1, \lambda_1)$ denotes the spinor for the parton $i$ with the spin state $\lambda_i$ along the reference direction $n \equiv e_y$, and $\bar{u}_{i,\lambda_i} = u^\dagger(P_1, \lambda_i)\gamma_0$ is its conjugate. $\Delta_{\mu
u}$ is the HTL resummed gluon propagator. Here we only take the magnetic gluon exchange into account which involves the magnetic mass $\mu_m$ is introduced to regulate the divergence arising from the soft gluon exchange. The magnetic mass $\mu_m$ is proportional to the temperature $T$, $\mu_m = 0.255(N_c/2)^{1/2}g^2T$. The numerical result from Eq. (13) is given in Fig. 1 for collisions in the center-of-mass system of the colliding quarks. The polarized part is much less than unpolarized one. Both the unpolarized and polarized differential cross sections show an oscillation feature.

There is an alternative way to do the integrals in Eq. (12). One can intergrate out $p_{4z}$ to remove $\delta(p_{1z} + p_{2z} - p_{3z} - p_{4z})$ and then $p_{3z}$ to remove $\delta(E_1 + E_2 - E_3 - E_4)$ as in Eq. (2). We finally obtain the differential cross section with respect to the impact parameter,

$$\frac{d^2\sigma}{d^2x_T} = \frac{1}{\tau} \frac{1}{64(2\pi)^3} \int d^2p_T d^2p_T' e^{-i(p_T-p_T')\cdot x_T} \delta(E_3 + E_4 - E_3' - E_4') \times \sum_{i=1,2} \sum_{\lambda_i, \lambda_{3',4'}} \frac{1}{|P_1\cdot P_2| \sqrt{E_3 E_4 E_3' E_4'}} \left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right| M(P_3\lambda_3, P_4\lambda_4)M^\ast(P_3'\lambda_3, P_4'\lambda_4).$$

(19)

Note that all final state energies are functions of $p_T$ and $p_T'$. The root $p_{3z}^{(i)}$ is given by Eq. (3) and $p_{3z}^{(i)}$ is given by $p_{3z}^{(i)} = p_{1z} + p_{2z} - p_{3z}^{(i)}$. We now rewrite the remaining delta-function into an integral over a time $t$ which is the conjugate variable of the uncertainty of the final state energies arising from the specified transverse positions in the final state,

$$2\pi \delta(E_3 + E_4 - E_3' - E_4') \approx \int_{-\tau/2}^{\tau/2} dt e^{i(E_3 + E_4 - E_3' - E_4)t},$$

(20)
where we have assumed $T \to \infty$. Inserting the above into Eq. (21), we obtain

$$
\frac{d^2\sigma}{d^2x_T} = \frac{1}{64(2\pi)^4|P_1 \cdot P_2|} \int d^2p_T d^2p_T' e^{-i(p_T-p_T') \cdot x_T} \sum_{i=1,2} \sum_{\lambda_1,\lambda_2,\lambda_4} \frac{1}{\sqrt{E_3 E_4 E'_3 E'_4}} \left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right| \times M(P_3 \lambda_3, P_4 \lambda_4) M^*(P'_3 \lambda_3, P'_4 \lambda_4),
$$

where the time average $\langle \cdots \rangle_t$ is defined by

$$
\langle \cdots \rangle_t \equiv \frac{1}{\tau} \int_{-\tau/2}^{\tau/2} dt \langle \cdots \rangle e^{i(E_3 + E_4 - E'_3 - E'_4)t}. \tag{22}
$$

At small transverse momenta in the center-of-mass system where $p_T \sim p_T' \ll \sqrt{s}$, we have $(E_3 + E_4 - E'_3 - E'_4) \sim p_T^2/\sqrt{s} \sim p_T'^2/\sqrt{s}$. If $p_T^2|t|/\sqrt{s} \ll p_T^2/\sqrt{s} \ll 1$ or $\tau \ll \sqrt{s}/p_T^2$, we can expand the phase factor in Eq. (21) as

$$
e^{i(E_3 + E_4 - E'_3 - E'_4)t} \approx 1 + i(E_3 + E_4 - E'_3 - E'_4)t - \frac{1}{2}(E_3 + E_4 - E'_3 - E'_4)^2 t^2 + \cdots. \tag{23}
$$

To the leading order, we have $E_3 \approx E'_3$, $E_4 \approx E'_4$ and $e^{i(E_3 + E_4 - E'_3 - E'_4)t} \approx 1$, Eq. (21) becomes

$$
\frac{d^2\sigma^{(0)}}{d^2x_T} = \frac{1}{64(2\pi)^4|P_1 \cdot P_2|} \int d^2p_T d^2p_T' e^{-i(p_T-p_T') \cdot x_T} \sum_{i=1,2} \sum_{\lambda_1,\lambda_2,\lambda_4} \frac{1}{\sqrt{E_3 E_4 E'_3 E'_4}} \left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right| \times M(P_3 \lambda_3, P_4 \lambda_4) M^*(P'_3 \lambda_3, P'_4 \lambda_4). \tag{24}
$$

It is interesting to see the above is unique to the leading order since

$$
\frac{1}{\left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right|} \approx \frac{1}{\left| E_3 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right|} \frac{1}{\left| E_4 p_{4z}^{(i)} - E_4 p_{3z}^{(i)} \right|} \tag{25}
$$

The next-to-leading order non-vanishing contribution comes from the term $\sim t^2$ in Eq. (21) since the linear term is odd in $t$ whose integral gives zero in the range $[-\tau/2, \tau/2]$. It reads

$$
\frac{d^2\sigma^{(1)}}{d^2x_T} = -\frac{\tau^2}{1536(2\pi)^4|P_1 \cdot P_2|} \int d^2p_T d^2p_T' e^{-i(p_T-p_T') \cdot x_T}
$$

Figure 1: (color online) The polarized and unpolarized differential cross sections. The exact results from Eq. (19) is in the red solid line (unpolarized) and blue dashed line (polarized part multiplied by a factor of 10). The sum of the unpolarized part $d^2\sigma^{(0)}/d^2x_T$ from Eqs. (20,29,31) is in the red dotted line, while that of the polarized part $d^2\sigma^{(1)}/d^2x_T$ (multiplied by a factor of 10) is in the blue dash-dotted line. The parameters are chosen to be $\sqrt{s} = 20$ GeV, $T = 200$ MeV, $\tau = 1.1$ fm and $\alpha_s = 1$. In the perturbation approach to obtain $d^2\sigma^{(0)}/d^2x_T$ we use $p_T^{cut} = 3$ GeV.
Note that the above result is just Eqs. (40,41) in Ref. [4].

For a simple case in the center-of-mass frame of two colliding quarks, where \( p_{1z} = -p_{2z}, \ T_{p_{3z}} = -p_{4z} \) and \( E_1 = E_2 = E_3 = E_4 = \sqrt{s}/2 \). To the leading order, we have and \( E'_4 = E'_4 = \sqrt{s}/2 \), then Eq. (24) is evaluated as

\[
\frac{d^2\sigma^{(0)}}{d^2x_T} \approx \frac{\alpha_s^2}{30\pi^2} \int d^2p_T d^2p'_Te^{-i(p_T-p'_T)\cdot x_T} e^{-\frac{(p_T^2-p'_T^2)^2}{4(\frac{s}{2})^2}} \times \frac{1}{p_T^2 + \mu_m^2} \frac{1}{p'_T^2 + \mu_m^2} \left\{ 1 + i\lambda_3 n_1 \cdot \left[ n \times \frac{p_T - p'_T}{\sqrt{s}} \right] \right\}
\]

\[
= \frac{\alpha_s^2}{9} A^2(x_T) + \frac{2\alpha_s^2}{9\sqrt{s}} n \cdot (n_1 \times \hat{x}_T) A(x_T) \frac{dA(x_T)}{dx_T},
\]

where we have taken only the magnetic gluon exchange into account and the magnetic mass \( \mu_m \) is introduced to regulate the divergence arising from the soft gluon exchange. We denoted \( n_1 = e_z \) as the direction of \( p_1 \) and used

\[
A(x_T) = \int d^2p_T e^{\pm ip_T\cdot x_T} \frac{1}{p_T^2 + \mu_m^2} = 2\pi \int_{p_T^{cut}} dp_T \frac{p_T J_0(p_T x_T)}{p_T^2 + \mu_m^2},
\]

where \( J_0 \) is the Bessel function and \( p_T^{cut} \) is the cutoff to regulate the ultraviolet divergence. The polarized and unpolarized differential cross sections can be obtained

\[
\frac{d^2\sigma^{(0)}_{upol}}{d^2x_T} = \frac{\alpha_s^2}{9} A^2(x_T),
\]

\[
\frac{d^2\sigma^{(0)}_{pol}}{d^2x_T} = \frac{2\alpha_s^2}{9\sqrt{s}} n \cdot (n_1 \times \hat{x}_T) A(x_T) \frac{dA(x_T)}{dx_T}.
\]

Note that the above result is just Eqs. (40,41) in Ref. [4].

The next-to-leading order contribution is evaluated as

\[
\frac{d^2\sigma^{(1)}}{d^2x_T} \approx -\frac{\tau^2 \alpha_s^2}{216\pi^2 s} \int d^2p_T d^2p'_Te^{-i(p_T-p'_T)\cdot x_T} (p_T^2 - p'_T^2)^2 \times \frac{1}{p_T^2 + \mu_m^2} \frac{1}{p'_T^2 + \mu_m^2} \left\{ 1 + i\lambda_3 n_1 \cdot \left[ n \times \frac{p_T - p'_T}{\sqrt{s}} \right] \right\}
\]

\[
= \frac{d^2\sigma^{(1)}_{upol}}{d^2x_T} + \lambda_3 \frac{d^2\sigma^{(1)}_{pol}}{d^2x_T},
\]

where we have used

\[
(E_3 + E_4 - E'_3 - E'_4)^2 = \left( \sqrt{s} - 2\sqrt{s/4 - p_T^2 + \mu_m^2} \right)^2 \approx \frac{(p_T^2 - p'_T^2)^2}{s/4}.
\]

The unpolarized part reads

\[
\frac{d^2\sigma^{(1)}_{upol}}{d^2x_T} = -\frac{\tau^2 \alpha_s^2}{216\pi^2 s} \int d^2p_T d^2p'_Te^{-i(p_T-p'_T)\cdot x_T} (p_T^4 + p'_T^4 - 2p_T^2p'_T) \times \frac{1}{p_T^2 + \mu_m^2} \frac{1}{p'_T^2 + \mu_m^2} \frac{\tau^2 \alpha_s^2}{27s} (A_0 A_1 - A_2^2),
\]

where we used

\[
A_i(x_T) = \int_{p_T^{cut}} dp_T \frac{p_T^n}{p_T^2 + \mu_m^2} J_0(p_T x_T).
\]
Figure 2: The polarized and unpolarized differential cross sections in the leading order from Eq. (29) (left panel) and the next-to-leading order from Eqs. (32,34) (right panel). The parameters are set to the same values as in Fig. 1.

For $i = 0, 1, 2$ with $n_i = 1, 5, 3$. The polarized part turns out to be

$$\frac{d^2\sigma_{pol}^{(1)}}{d^2x_T} = \frac{-\tau^2\alpha^2}{27s^{3/2}} n \cdot (n_1 \times \hat{x}_T) \frac{d}{dx_T}(A_0A_1 - A_2^2).$$

The numerical results of Eqs. (29,32,34) are shown in Fig. 2. We see that both the leading and next-to-leading parts are damped out above 0.4 fm. The next-to-leading contributions of the unpolarized and polarized parts are about 1/6 of the leading counterparts. Both the unpolarized and polarized differential cross sections show an oscillation feature. The sums of the leading and next-to-leading contributions are shown as the red dotted (unpolarized) and blue dash-dotted line (polarized) in Fig. 1. We already mentioned that a cutoff in transverse momentum $p_{cut}^t$ is needed to regulate the integrals in the leading and next-to-leading differential cross sections. The cross section results depend on $p_{cut}^t$ in the perturbation. The time scale $\tau$ is also a quantity to be determined. We can find the range of the $\tau$ by equating the exact result $d^2\sigma/d^2x_T$ from Eq. (13) and the sum $d^2(\sigma^{(0)} + \sigma^{(1)})/d^2x_T$ in the perturbation approach from Eqs. (29,32,34) at a specified value of $p_{cut}^t$.

V. SUMMARY AND DISCUSSION

We have proposed a general approach to the differential cross section with respect to impact parameters, which is well-defined and free of ambiguity existing in the conventional approach. The main difference of our approach from the conventional one is that (1) in the conventional approach the transfer of small transverse momenta is implied, while the general approach is valid for all transverse momenta; (2) there are two independent delta functions in the general approach for energy conservation in the cross section formula, which arises from fixing impact parameters in the final state partons making the total final state energy be uncertain. While in the conventional approach the two delta functions for energy conservation are identical turning the second one to be the infinite interaction time $\tau$.

As a simple illustration of our formalism, we evaluated the polarized and unpolarized differential cross sections at small angle quark-quark scatterings in the center-of-mass system of the colliding quarks. To smoothly connect the general approach and the conventional one, we propose an expansion in terms of $\Delta E = E_f - E_i \sim 1/\tau$ with $E_i$ and $E_f$ the initial and final state energies in collisions. The leading order contribution reproduced the conventional result, i.e. that of Ref. [4].

The general formulation in this paper can also be applied to many other parton-parton scatterings in heavy ion collisions or even proton-proton collisions [10].

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