Consistency Analysis of Accelerated Degradation Mechanism of Smart Meter Based on Grey Theory

Zheng-mao WU1, Si-hai Li1, Yong WU1 and Yin-jing FENG2

1Sanya Aviation & Tourism College, Sanya, Hainan 572000, China
2Civil Aviation University of China, Tianjin, 300300, China

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Abstract. This paper proposed a new method to analyze accelerated degradation mechanical consistency of smart meter based on gray theory. Firstly, the existing methods are compared with each other especially on their weakness. And then a new degradation mechanism consistency analysis method was revealed based on gray prediction within gray theory. Finally, two gray prediction models, which are GM (1,1) and equal dimension new information, are elaborated. What follows are the procedure and specific process of this new consistency analysis method. An enhancement test of smart meter was applied with the grey prediction method, which verifies this method.

Introduction

With the increasing reliability of smart meter, it is difficult to obtain life data and effective degradation data under normal working environment, which makes reliability evaluation more and more difficult. Accelerated Life Test (ALT) and Accelerated Degradation Test (ADT) can effectively solve this problem. The basic premise of ALT and ADT is that the failure mechanism of products can’t be changed during the test, especially at the highest stress level[1]. Therefore, it is very important to ensure the consistency of product failure (degradation) mechanism in accelerated test.

At present, there are three main methods to determine the consistency of failure mechanism: the method based on parameter invariance of accelerated model, the method based on statistics and the method based on experimental observation. The method based on parameter invariance of acceleration model is simple and feasible, but its degradation law must conform to physical acceleration model or empirical acceleration model, and the consistency determination method of inverse power law model has not been strictly explained physically. The other method to determine mechanism consistency of acceleration model is whether the activation energy has changed or not, and the concept of activation energy has not been unified among different theories[2-4]. Statistical methods have a wide range of applications, but the relationship between coefficient of variation and mechanism is not given from a physical point of view; constant coefficient of variation is only a necessary condition for accelerating the consistency of degradation mechanism, so statistical methods may not find the first condition for mechanism change; moreover, statistical methods are suitable for large samples and post-test, and have their limitations[5,6]. The advantage of the experimental observation method is intuitive, but because the failure mechanism of some products is not easy to be observed, the failure mode and failure mechanism can’t correspond to each other, and the experimental observation brings some empirical judgment, so this method often brings some subjectivity, and is not suitable for all products[7,8]. It can be seen that the existing methods for identifying the consistency of degradation mechanism have great shortcomings.

In this paper, the grey theory of fuzzy mathematics is introduced into the consistency judgement of failure by using the idea of complexity, and the consistency of accelerated degradation mechanism is analyzed from the data point of view.
Grey Prediction Model

Grey system theory is a theory dealing with uncertainties such as few data, small samples, incomplete new information and lack of experience. Grey prediction is based on grey dynamic model (GM), which can reveal the dynamic development process of things and predict the future development trend. At present, the most commonly used grey prediction new information models are GM (1,1) and equal dimensional model.

Grey Prediction Method Based on GM(1,1)

Let \( n \) be the number of samples in the sequence \( x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \).

For first-order cumulative generation \( x^{(i)}(k) = \sum_{i=1}^{k} x^{(0)}(i) \quad k=1, 2, \ldots, n \) \hspace{2cm} (1)

\[ x^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)) \] \hspace{2cm} (2)

Let \( Z^{(1)} \) be a sequence of adjacent mean values of \( x^{(1)} \), that is:

\[ Z^{(1)}(k) = 0.5 \left[ x^{(1)}(k) + x^{(1)}(k-1) \right] \quad k=2, 3, \ldots, n \] \hspace{2cm} (3)

\[ Z^{(1)} = (Z^{(1)}(2), Z^{(1)}(3), \ldots, Z^{(1)}(n)) \] \hspace{2cm} (4)

Then the grey differential equation model of GM(1,1) is as follows:

\[ \frac{dZ^{(1)}(k)}{dk} + aZ^{(1)}(k) = b \] \hspace{2cm} (5)

Among them, \( a \) is the coefficient of development and \( b \) is the grey action. The analytic form of \( Z^{(1)} \) is:

\[ Z^{(1)}(k) = \left[ \frac{x^{(0)}(1) - \frac{b}{a}}{e^{-a} + \frac{b}{a}} \right] \] \hspace{2cm} (6)

Set

\[ B = [-Z^{(1)}, 1] \]

\[ Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} \]

Least squares estimation is

\[ \hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y \] \hspace{2cm} (9)

The final predicted value of \( x^{(0)} \)
\[ \hat{x}^{(0)}(k+1) = (1 - e^a)(x^{(0)}(1) - \frac{b}{a})e^{-ak} \]  

(10)

GM(1,1) model has a certain scope of application\(^{[12]}\). In this paper, we generally take, \(-a \leq 0.8\).

**Forecasting Method Based on Equal Dimension New Information Model**

Equal dimension new information model is a grey forecasting model established by GM (1,1), which keeps the data dimension equal while removing the oldest data and adding the latest actual data. Equal dimension new information compensation model is based on the equal dimension new information model. It is generally used in the case of insufficient original data. The predicted data is added to the series used for prediction, and the dimension of the data is equal, so as to make further data prediction. Generally speaking, the prediction effect of the equal dimensional new information model is better.

**Method for Determining Dimension \( m \) of Equal Dimension New Information Model**

Although the validity of the equal dimensional new information model has been proved in practice, there is no detailed study on how to determine the dimension \( m \) by scholars at present. Generally, the value of \( m \) is selected according to experience. Because the purpose of the equal dimensional new information grey model applied to mechanism consistency determination is to find the change outliers in the existing data by predicting the trend of data development, the number of predictable data should be considered in addition to the accuracy of the model.

It is necessary to formulate a rule, which not only satisfies the prediction accuracy requirements, but also satisfies the requirement of finding the test points of mechanism change points. Firstly, we need to determine an acceptable range of \( m \) values, and then verify the minimum \( m \) values within the acceptable range of accuracy. It can be expressed by the following equation:

\[
\begin{align*}
\min m \\
\text{s.t.} & \quad m \geq m_{\text{low}} \\
& \quad m \leq m_{\text{up}} \\
\sigma^2(m) - \min \{ \sigma^2(i), i \in [m_{\text{low}}, m_{\text{up}}] \} \leq C
\end{align*}
\]

(11)

If the data obey normal distribution or lognormal distribution (the degraded data is logarithmically operated), the consistency test of variance of residuals can be used, that is:

\[
\begin{align*}
\min m \\
\text{s.t.} & \quad m \geq m_{\text{low}} \\
& \quad m \leq m_{\text{up}} \\
\sigma^2(n) = \min \{ \sigma^2(i), i \in [m_{\text{low}}, m_{\text{up}}] \} \\
\frac{\sigma^2(m)}{\sigma^2(n)} & \in [F_{\alpha/2}(m-2,n-2), F_{1-\alpha/2}(m-2,n-2)]
\end{align*}
\]

(12)

The variance of the predicted residuals \( \sigma^2(i) \) is used in the formula.

The \( m \) value satisfying Formula (11) is the best \( m \) value. Because Equation (10) is used for least squares estimation, in order to test its fitting accuracy, \( F \) testing is needed, \( F \left( \frac{F_{\alpha/2}(n-2), F_{1-\alpha/2}(n-2)}{l} \right) \). The sample size should be greater than or equal to 4, otherwise the scope of \( F_{\alpha/2}(l) \) will be very large and the significance of the test will be lost. So \( m_{\text{low}} (m_{\text{low}} \geq 4) \) can be determined.

In the trend analysis of residual value by comparing the predicted data with the original data, at least three sets of values are needed to detect the fluctuation of the data, and at least four sets of values are needed to determine the specific fluctuation points (outliers), \( m_{\text{up}} \leq n-4 \).
Consistency Judgment Principle of Smart Meter Accelerating Degradation Mechanism Based on Grey Prediction

In order to establish the stress level of accelerated test, it is necessary to analyze the strengthened test data and find the consistent boundary conditions. However, in engineering practice, in order to carry out the accelerated test smoothly, it is necessary to collect data through short-term step stress intensification test, and to judge the consistency of the data of the intensified test (also known as bottom-finding test in literature) so as to make the test profile of the accelerated test for a long time. Generally speaking, the time of intensive test is very little, so it is difficult to deal with it by general methods. This paper chooses grey prediction to study the consistency judgment, because grey prediction can discover and grasp the development law of the system through less data, so it can also judge when the law of the system changes.

The basic idea of the consistency judgment method of accelerated degradation mechanism based on grey prediction is: to predict the intensified test data of products with equal dimensional new information model, to compare the predicted value with the actual value, and to observe whether the residual has changed significantly. If the residual has changed significantly, it shows that the development trend of products has changed, and that the products have changed qualitatively, it can be recognized that the mechanism of the product has changed.

Consistency Judgment Process of Accelerated Degradation Mechanism Based on Grey Prediction

(1) Choosing appropriate dimension to build an equal dimensional new information model for the data of intensified experiments;

(2) According to the characteristics of the data, the appropriate grey model is selected to predict each segment of the data in the equal dimensional new information model, and a prediction value is obtained;

(3) Check whether the selected grey model is suitable or not. If not, the grey model will be re-selected for grey prediction;

(4) Comparing the predicted value with the corresponding true value and calculating the residual;

(5) Observe whether there is a significant change in the residual. If there is a first significant change point, the local mechanism corresponding to the point has changed. If there is no significant change, the mechanism of the product has not changed in the strengthening test, and the test stress of the strengthening test needs to be increased.

Specific Solution and Decision Method

Assuming that the length of the original data is n, the consistency of degradation mechanism is determined by using the equal m dimension new information model.

First, the original data is grouped: 1~ m, 2~ m+1, ..., n-(m-1)~ n, denoted as X_0(1), X_0(2), ..., X_0(n-m+1), the cumulative sequence X_1(1), X_1(2), ..., X_1(n-m+1), and the adjacent mean sequence Z_1(1), Z_1(2), ..., Z_1(n-m+1). The corresponding B and Y of each sequence can be obtained.

Then, the unknown parameters a and b of each sequence are estimated by \[
\hat{a} = [a, b]^T = (B^T B)^{-1} B^T Y .
\]

If the estimation of development coefficient a meets the requirement of grey prediction, the consistency of failure mechanism can be determined by grey prediction method.

According to the formula \[\tilde{x}^{(0)}(k+1) = (1-e^{-a})(x^{(0)}(1) - \frac{b}{a})e^{-ak},\] the predicted value of the original sequence is obtained. The difference between the predicted value and the real value \[\hat{x}^{(0)} - x^{(0)}\] after m+1 is analyzed, and the point where the residual fluctuation is more significant is identified as the change point of accelerated degradation mechanism. The corresponding stress intensity is the upper limit of stress which needs to be controlled, and can't exceed the limit in further test.
Example Analysis

High Temperature Intensification Test of Electric Energy Meter

Intensified test is a kind of excitation test, which uses accelerated stress environment to quickly excite potential defects of products and expose the main failure modes of products. The main purposes of the intensification test are as follows:

1) Determine the limit stress condition of the watt-hour meter;
2) Determine the typical fault mode, fault characteristics and performance degradation trend of the meter, and provide basis for degradation test.

The starting and ending temperature points of step-by-step high temperature intensification test of watt-hour meter are 40°C and 120°C, and a step at 5°C near the critical temperature point 70°C and 10°C at other temperature points. At each temperature step, the meter is tested online, and then the temperature is adjusted to a fixed temperature and humidity point (23°C, 45%) for off-line testing. Figure 1 shows the profile of high temperature hardening test.

![Figure 1. Enhancement test profile of temperature limit.](image)

Consistency Judgment Result of Failure Mechanism of Electric Energy Meter (Grey Theory)

Because the basic measurement errors of watt-hour meter and the variation of load current can represent the measurement accuracy of watt-hour meter, and other factors are found to have the same law with them through analysis, so the consistency of failure mechanism of watt-hour meter is determined by using the measurement data of basic error and variation of load current. Table 1 shows the basic errors and shifting variation of some meter intensification tests.

| Serial Number | Temperature (°C) | Basic Errors(%) | Load Current Fluctuation (%) |
|---------------|-----------------|-----------------|----------------------------|
| 1             | 40              | 0.326           | 0.296                      |
| 2             | 50              | 0.38            | 0.36                       |
| 3             | 60              | 0.44            | 0.424                      |
| 4             | 65              | 0.5             | 0.483                      |
| 5             | 70              | 0.561           | 0.548                      |
| 6             | 75              | 0.627           | 0.616                      |
| 7             | 80              | 0.698           | 0.684                      |
| 8             | 90              | 0.782           | 0.766                      |
| 9             | 100             | 0.947           | 0.926                      |
| 10            | 110             | 1.025           | 0.972                      |

Firstly, the most suitable dimension m is determined according to the method and process of section 1.3. The dimension of the equal dimensional new information model is 4. According to the above method, the GM (1,1) is used to predict the basic errors and shifting variation with MATLAB. The coefficients of development for each data segment of the two sequences are shown in Tables 2 and 3.
Table 2. Development Coefficient of Basic Error.

| Raw Data Serial Number | 1~4   | 2~5   | 3~6   | 4~7   | 5~8   | 6~9   | 7~10  |
|------------------------|-------|-------|-------|-------|-------|-------|-------|
| a value                | -0.13615 | -0.12081 | -0.1129 | -0.109 | -0.11058 | -0.15618 | -0.1301 |

Table 3. Development Coefficient of Shifting Variation.

| Raw Data Serial Number | 1~4   | 2~5   | 3~6   | 4~7   | 5~8   | 6~9   | 7~10  |
|------------------------|-------|-------|-------|-------|-------|-------|-------|
| a value                | -0.14508 | -0.12792 | -0.12109 | -0.1103 | -0.1092 | -0.15501 | -0.1134 |

Because the value of the development coefficient a calculated accords with the range of medium and long term prediction, the method of grey prediction can be used to determine the consistency of failure mechanism. The predicted values and residual values of the two sequences are shown in tables 4 and 5 respectively.

Table 4. Predictive, True and Residual Value of Basic Error.

| Raw Data Serial Number | 1~4   | 2~5   | 3~6   | 4~7   | 5~8   | 6~9   | 7~10  |
|------------------------|-------|-------|-------|-------|-------|-------|-------|
| Forecast $\hat{x}^{(0)}$ | 0.57311 | 0.63309 | 0.70135 | 0.7778 | 0.871554 | 1.0936 | 1.182416 |
| True Value $x^{(0)}$    | 0.561  | 0.627  | 0.698  | 0.782  | 0.947  | 1.025  |
| Residual               | 0.01211 | 0.00609 | 0.00335 | -0.0042 | -0.07545 | 0.0686 |

Table 5. Predictive, True and Residual value of shifting variation

| Raw Data Serial Number | 1~4   | 2~5   | 3~6   | 4~7   | 5~8   | 6~9   | 7~10  |
|------------------------|-------|-------|-------|-------|-------|-------|-------|
| Forecast $\hat{x}^{(0)}$ | 0.55941 | 0.62198 | 0.695  | 0.76401 | 0.852305 | 1.0683 | 1.10822 |
| True Value $x^{(0)}$    | 0.548  | 0.616  | 0.684  | 0.766  | 0.926  | 0.972  |
| Residual               | 0.01141 | 0.00598 | 0.011  | -0.00199 | 0.926  | 0.972  |

The residual curve of basic error and shifting variation is shown in Fig. 2:

Figure 2 (a) residual value of basic error (b) residual value of shifting variation.

As can be seen from Fig. 3, the basic error changed significantly at point 9, so it can be considered that the basic error potential changed abruptly at point 9. The temperature corresponding to point 9 was 100°C, that is to say, the mechanism changed at 100°C. Shifting variation changed significantly at point 9. It can be concluded that the development trend of Shifting variation changed abruptly at point 9, and the temperature corresponding to point 9 was 100°C. Temperature, that is to say, the mechanism changes at 100°C. Generally speaking, when the accelerated degradation test of watt-hour meter is carried out, the test temperature can’t exceed 100°C, so the degradation data can be used for further reliability analysis.

Result Comparison

The results of the traditional methods for identifying mechanism consistency based on parameter invariance of accelerated model and statistics are as follows:

1. Constant parameters based on acceleration model

   The stress variable in the high temperature strengthening test of watt-hour meter is temperature, and Arrhenius model describes the relationship between degradation rate and temperature stress. Therefore, it is appropriate to choose Arrhenius model as the acceleration model of watt-hour meter. Arrhenius model is:
\[ \frac{\partial M}{\partial t} = A_e \exp\left(-\frac{E_a}{kT}\right) \]  

(13)

The mechanism determination method based on Arrhenius model considers that the same failure mechanism corresponds to the same activation energy regardless of the acceleration stress. If the failure mechanism remains unchanged, the activation energy will remain unchanged. Therefore, it is necessary to determine the consistency of the mechanism by determining the activation energy. It can be seen from equation (13) that when the activation energy does not change, the degradation rate of the sensitive parameter is exponentially related to the negative reciprocal of temperature - 1/T. Therefore, if constant stress is applied to the accelerated test, the logarithm of the degradation rate of sensitive parameters is determined during the test process, and whether the logarithm of the degradation rate is in exponential relation with the negative reciprocal of temperature-1/T, then the change of activation potential Ea can be determined, and the failure mechanism can be judged.

After taking logarithm of the basic error data of the meter, observe whether the negative reciprocal of \( \ln \frac{\partial M}{\partial t} \) and negative reciprocal of temperature - 1/T conform to a linear relationship. In this experiment, \( M \) is basic error, because this method needs to calculate the degradation rate through a series of degradation values, the activation energy \( E_a \) is calculated.

In order to compare, the data under different stresses in the intensified test were collected and the degradation data were collected. The data are shown in the figure 3.

![Figure 3. Basic error at different operation condition during test.](image)

It can be seen from the graph that the degradation trajectory obeys the power function and the data at 100°C are different from other temperatures. Using the degeneration-temperature relationship of Arrhenius model to determine the consistency, the graph can be obtained (As shown in figure 4):

![Figure 4. Consistency deterring method of activation energy.](image)

It can be seen from the graph that the logarithmic value of basic error is not in line with that of other temperatures at -1/T=0.00268, i.e. T= T=100°C (373K). Therefore, according to the activation energy of Arrhenius model, it is considered that the degradation mechanism has changed at T=100°C (373K).

(2) Statistical Methods

Because the pseudo-life of the watt-hour meter can’t be obtained by the enhanced test, the consistency of the test data can’t be determined directly. Instead, the accelerated degradation test is needed to obtain the degradation data, so that the pseudo-life under each stress level can be obtained.
According to engineering experience, the pseudo-life of degraded data of the watt-hour meter generally obeys lognormal distribution. The pseudo-life data of the meter are tested by lognormal distribution. The results are shown in the table 6.

Table 6. Test Statistics of Consistency Determine.

| $T_v/°C$ | $v_v$ | $v_{v+1}$ | $d^2_v$ | $d^2_{v+1}$ | $F_v$ | $F_v(v_v,v_{v+1})$ | $F_{v+1}(v_v,v_{v+1})$ | Is it consistent |
|---------|------|----------|---------|------------|-------|----------------|--------------------|-----------------|
| 50      | 2    | -        | 0.00414 | -          | -     | -              | -                  | -               |
| 60      | 2    | 2        | 0.00779 | 0.0041    | 1.880 | 0.0526         | 19                 | yes             |
| 70      | 2    | 4        | 0.00844 | 0.0060    | 1.414 | 0.052          | 6.94               | yes             |
| 80      | 2    | 6        | 0.00807 | 0.0068    | 1.188 | 0.0517         | 5.14               | yes             |
| 90      | 2    | 8        | 0.00581 | 0.0071    | 0.817 | 0.0516         | 4.46               | yes             |
| 100     | 2    | 10       | 0.13260 | 0.0068    | 19.358| 0.0516         | 4.1                | no              |

*The test rejection domain is: $W = \{ | F > F_{\alpha-v}(v_v,v_{v+1}) \} \cup \{ F < F_{\alpha-v}(v_v,v_{v+1}) \}$

It can be seen from the table that the F value of the basic error falls into the rejection domain at 100°C, and the consistency test of variance fails, which indicates that the mechanism of the meter has changed.

**Conclusion**

In summary, the grey prediction mentioned above can effectively test the mutation points of the enhanced test data, and then find out the boundary conditions for the consistency of accelerated degradation mechanism. The correctness of this method is verified by applying grey prediction to analyze the high temperature strengthening test data of electric energy meter. In the process of inspection, it is found that the amount of data needed by grey prediction is very small, and the consistency of fault (failure) of products can be judged by a small amount of data, so that the accelerated test conditions can be checked before accelerated test, and the effective maximum stress of accelerated test can be determined, which lays the foundation for the correct design of accelerated test.

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