Sequential Operations in LogicWeb

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Abstract Sequential tasks cannot be effectively handled in logic programming based on classical logic or linear logic. This limitation can be addressed by using a fragment of Japaridze’s computability logic. We propose SeqWeb, an extension to LogicWeb with sequential goal formulas. SeqWeb extends the LogicWeb by allowing goals of the form $G \land^s G$ and $G \lor^s G$ where $G$ is a goal. These goals allow us to specify both sequential-conjunctive and sequential-disjunctive tasks.

Keywords Prolog · sequentiality · LogicWeb

1 Introduction

1.1 LogicWeb

Logic languages such as Prolog support nondeterminism (and automatic backtracking) and thus is at a higher level of abstraction than imperative programming languages. Unfortunately, logic programming has been popular only in limited applications including AI, expert systems, and compilers. In particular, logic programming languages has been little used for Web programming. Only recently has the work called LogicWeb [1] begun on the relationship between logic programming and the Web. LogicWeb views the Web as a collection of logic programs.

LogicWeb thus allows us to build a web of information that machines can understand and reason about. This includes the ability to answer complex queries. The standard language that has been introduced so far is a subset of first-order logic called Horn clauses with embedded implications.
Implementing LogicWeb, however, requires more than Horn clauses with embedded implications. It must be able to deal with name scoping or local constants to support information hiding. A language called first-order hereditary Harrop formulas (fohh) [12,13,11] has been extensively studied. This language offers modularity abstraction with local constants.

1.2 First-order hereditary Harrop formulas

The syntax of goals (or queries) permitted in Prolog is the following: they can be atomic formulas or conjunctions or disjunctions of goals. In the context of Prolog, conjunctions are written using commas, while disjunctions use semicolons.

The language of fohh extends this set of logical symbols to include implications and universal quantifiers. In this language, formulas such as \( D \supset G \) and \( \forall x G \) will be permitted as goals, provided that \( D \) is a program and \( G \) is itself a goal. The intended semantics of these two new operations is the following. A goal such as \( D \supset G \) is to be solved by adding \( D \) to the current program. Hence, \( D \) is to be available only during the course of solving \( G \). As for a goal such as \( \forall x G \), it is intended to be solved by instantiating \( x \) in \( G \) by a new constant \( c \) and then solving the resulting goal. Interpreted in this fashion, the universal quantifier provides a means for limiting the availability of names.

We illustrate the problem mentioned above by considering the definition of the fibonacci relation in Prolog. An efficient fibonacci program can be written by using the idea of an accumulator. Implementing this idea in Prolog requires an auxiliary predicate, called \( \text{fib}_\text{aux} \) below, to be defined. The following definition is a traditional way of realizing this:

```
module fib.
    fib(N, F) :- fib_aux(2, N, 1, 1, F).
end.
```

```
module fiba.
    fib_aux(M, N, F1, F2, F) :- M \geq N. % Nth fibonacci number reached
    fib_aux(M, N, F1, F2, F) :- M < N, fib_aux(M + 1, N, F2, F1 + F2, F).
end.
```

The declarative interpretation of \( \text{fib}_\text{aux} \) here is that it is true if \( F1 \) is the \((M-1)\)th, \( F2 \) is the \( M \)th fibonacci number and \( F \) is the \( N \)th fibonacci number. In this example, it is worth noting that \( \text{fib}_\text{aux} \) is a specialized predicate whose only purpose for existence is its usefulness in defining \( \text{fib} \).

The addition of implication to goals provides a means for solving at least part of this problem. Thus, the definition of \( \text{fib}_\text{aux} \) can be made “local” to that of \( \text{fib} \), as indicated below:

```
fib(N, F) :-
    \( \text{(module fiba)} \supset \text{fib}_\text{aux}(2, N, 1, 1, F). \)
```

where

```
module fiba.
    fib_aux(M, N, F1, F2, F) :- M \geq N. % Nth fibonacci number reached
    fib_aux(M, N, F1, F2, F) :- M < N, fib_aux(M + 1, N, F2, F1 + F2, F).
end.
```
Given this definition of \( fib \), the following points might be observed. First, the clauses defining \( fib_{aux} \) are not available at the top-level. Secondly, while the definition of \( fib_{aux} \) is not available at the top-level, we see that it will become available when solving the body of \( fib \).

However, there is one problem that still remains. The meaning of the predicate \( fib_{aux} \) inside the body of \( fib \) is not insulated from definitions in existence outside the body. This problem may be solved by using a universal quantifier.

\[
\text{fib}(N, F) : - \forall fib_{aux} ((\text{module fib}) \supset fib_{aux}(2, N, 1, 1, F)).
\]

The indicated semantics of the universal quantifier dictates picking a new name for \( fib_{aux} \) and then solving the instantiation of the given query with this name.

The examples presented here have been of a very simple nature. They are, however, sufficient for understanding the intended semantics of the new logical symbols and also for appreciating some of their value from a programming perspective. We note that the usefulness of the scoping mechanisms provided by these symbols have been extensively studied and we point the interested reader to, e.g. [12].

1.3 Contributions of this paper

Although the class of \( fooh \) is quite attractive, sequential tasks cannot be naturally handled in it. This limitation can be addressed by using a fragment of Japaridze’s Computability Logic (CL) ([2] – [9]). CL is a new semantic platform for reinterpreting logic as a theory of computable tasks. Formulas in CL stand for instructions that can carry out some computational tasks. Queries stand for actually executing these instructions.

This paper proposes Sequential LogicWeb (SeqWeb), an extension of LogicWeb [11] with sequentiality in goal formulas. Traditionally, LogicWeb has the problem of expressing sequential tasks. The class of sequential goal formulas enables the programmer to express these sequential tasks. They offer the possibility to combine imperative programming with declarative programming.

We need to define some terminology. A move (or a choice) is a string over the keyboard alphabet. To be specific, executing \( \exists x G \) where \( G \) is a goal requires a move by the machine. Similarly, executing \( \forall x C \) where \( C \) is a clause also requires a move by the machine.

– A sequential-multiplicative-conjunctive (SMC) goal is of the form \( G_1 \land^* G_2 \) where \( G_1, G_2 \) are goals. Executing this goal has the following intended semantics: execute both \( G_1 \) and \( G_2 \) sequentially. In other words, all the moves in \( G_1 \) must precede all the moves in \( G_2 \). Both executions must succeed for executing \( G_1 \land^* G_2 \) to succeed. It can be seen as a restricted version of the parallel-conjunctive goal \( G_1 \land G_2 \) where all the moves in \( G_1 \) must precede all the moves in \( G_2 \).
A sequential-multiplicative-disjunctive (SMD) goal is of the form $G_1 \vee^s G_2$ where $G_1, G_2$ are goals. Executing this goal has the following intended semantics: execute both $G_1$ and $G_2$ sequentially. At least one of these two executions must succeed for executing $G_1 \vee^s G_2$ to succeed. A SMD goal can be used to express success with a degree. That is, new successes are sought, even after a success (solution) was found, as long as there are more successes to come. Imagine Kim decides to buy a used BMW from the local car dealers. Everytime he finds one at a local dealer (i.e., a success), he will probably go to another dealer for a cheaper price (i.e., for another success). Coming up with multiple successes does not affect the successfulness of the effort, but the behaviors of the subsequent tasks such as how much Kim has to pay for the car (Remember that we now have the notion of sequential tasks, hence the notion of subsequent tasks.).

In this paper we present the syntax and semantics of this extended language, show some examples of its use and study the interactions among the newly added constructs.

The remainder of this paper is structured as follows. We describe SeqWeb based on a first-order sequential hereditary Harrop formulas in the next section. In Section 3 we present some examples of SeqWeb. Section 4 concludes the paper.

2 The Language

The language we use is an expanded version of hereditary Harrop formulas. It is described by $G$- and $D$-formulas given by the syntax rules below:

$$G ::= A \mid G \wedge G \mid G \vee G \mid D \supset G \mid \forall x \ G \mid \exists x \ G \mid G \wedge^s G \mid G \vee^s G$$

$$D ::= A \mid G \supset A \mid \forall x \ D \mid D \wedge D$$

In the rules above, $A$ represents an atomic formula. A $D$-formula is called a clause or an instruction.

In the transition system to be considered, $G$-formulas will function as queries and a set of $D$-formulas will constitute a set of available instructions. For this reason, we refer to a $G$-formula as a a query, to a set of $D$-formula as an instruction set. Our language is an extension to first-order Hereditary Harrop formulas with the main difference that new sequential constructs are added in $G$-formulas.

We will present an operational semantics for this language. These rules in fact depend on the top-level constructor in the expression and have the effect of producing a new program and a new available instruction set.

The rules for executing queries in our language are based on “goal-directness” in the sense that the next rule to be used depends on the top-level construct of the goal formula. This property is known as uniform provability[13]. This
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property guarantees that the programmer knows exactly how execution will proceed.

**Definition 1.** Let $G$ be a goal and let $\mathcal{P}$ be a finite set of available instructions. Then the notion of executing $\langle \mathcal{P}, G_1 \rangle$ — executing $G$ relative to $\mathcal{P}$ — is defined as follows:

1. If $G$ is an atom and is identical to an instance of a program clause in $\mathcal{P}$, then the current execution terminates with a success.
2. If $G$ is an atom and an instance of a program clause in $\mathcal{P}$ is of the form $G_1 \supset G$, execute $\langle \mathcal{P}, G_1 \rangle$. This execution must succeed for the current execution to succeed.
3. If $G$ is $G_1 \land G_2$, then execute both $\langle \mathcal{P}, G_1 \rangle$ and $\langle \mathcal{P}, G_2 \rangle$ in parallel. Both executions must succeed for the current execution to succeed.
4. If $G$ is $G_1 \lor G_2$, then execute both $\langle \mathcal{P}, G_1 \rangle$ and $\langle \mathcal{P}, G_2 \rangle$ in parallel. At least one of these two executions must succeed for the current execution to succeed.
5. If $G$ is $\exists x G_1$, then execute $\langle \mathcal{P}, [t/x]G_1 \rangle$ where $t$ is a term. This execution must succeed for the current execution to succeed.
6. If $G$ is $\forall x G_1$, then execute $\langle \mathcal{P}, [a/x]G_1 \rangle$ where $a$ is a new constant. This execution must succeed for the current execution to succeed.
7. If $G$ is $D \supset G_1$, then execute $\langle \{D\} \cup \mathcal{P}, G_1 \rangle$. This execution must succeed for the current execution to succeed.
8. If $G$ is $G_1 \land^s G_2$, then execute both $\langle \mathcal{P}, G_1 \rangle$ and $\langle \mathcal{P}, G_2 \rangle$ in sequence. Both executions must succeed for the current execution to succeed.
9. If $G$ is $G_1 \lor^s G_2$, then execute both $\langle \mathcal{P}, G_1 \rangle$ and $\langle \mathcal{P}, G_2 \rangle$ in sequence. At least one of these executions must succeed for the current execution to succeed.

The symbols $\land^s$ and $\lor^s$ provide sequential executions of instructions: they allow, respectively, for the sequential conjunctive execution of the instructions and the sequential disjunctive execution of instructions.

### 3 SeqWeb

In our context, a SeqWeb page corresponds simply to a set of $D$-formulas or a $G$-formula with a URL. The module construct $\text{mod}$ allows a URL to be associated to a set of $D$-formulas or a $G$-formula. An example of the use of this construct is provided by the following “arithmetic” module which contains some basic arithmetic-handling instructions written in conventional Prolog.

```prolog
mod(www.dau.com/arith).

cube(X, Y) :- Y = X * X * X.
fib(0, 1). % first Fibonacci number
fib(1, 1). % second Fibonacci number
fib(X, Y) :- X > 1 \land X1 is X - 1
```

\[ X_2 \text{ is } X - 2 \land \text{fib}(X_1, Z) \land \text{fib}(X_2, W) \land Y \text{ is } W + Z. \]

\[ \text{prime}(X) : = \ldots. \]

Our language in Section 2 permits sequential conjunction in goals. This allows for sequential interactions between the user and the system. An example of the use of this construct is provided by the page which does the following sequential tasks: read a number from the user, output the number, and then output its fibonacci number:

\[
\begin{align*}
\text{mod(www.dau.com/query1).} \\
\text{read}(N) \land^s \\
\text{write}(N) \land^s \\
\text{fib}(N, O) \land^s \\
\text{write('fib :')} \land^s \\
\text{write}(O)
\end{align*}
\]

These pages can be made available in specific contexts by explicitly mentioning the URL via a hyperlink. For example, consider a goal \( \text{www.dau.com/arith} \supset \text{www.dau.com/query1} \). Solving this goal has the effect of executing \( \text{query1} \) with respect to the instructions in \( \text{arith} \).

Imagine the number typed in is twin prime in the sense that both the number and its successor by two are prime. In such cases we want to indicate in addition that the number is twin prime. One way of displaying this extra information is through the sequential disjunctive goals. The following is a modification of the \( \text{query1} \) page based on this idea.

\[
\begin{align*}
\text{mod(www.dau.com/query2).} \\
\text{read}(N) \land^s \\
\text{write}(N) \land^s \\
( \\
\text{fib}(N, O) \land^s \\
\text{write('its fib :')} \land^s \\
\text{write}(O)) \\
\lor^s \\
(\text{prime}(N) \land \text{prime}(N + 2)) \land^s \\
\text{write('twin prime:')} \\
\end{align*}
\]

It is interesting to note that the above task requires the parallel-AND, the sequential-AND, and the sequential-OR executions of instructions altogether.

Consider the task of measuring the blood pressure of a patient, i.e., \( \text{bp}(X, Y) \), where \( \text{bp}(X, Y) \) means “patient \( X \) has blood pressure \( Y \)”. Successfully solving this task requires performing the measurement by a nurse dynamically and, in addition, the outcome to be less than 180. The following is a program based on this idea.
Now imagine a system that diagnoses the blood pressure of a patient by having him taken the measurements twice in sequence. One way of expressing this system is the following query.

\begin{align*}
\text{mod}(\text{www.dau.com/query3}). \\
\text{read}(X) \land^s X \text{ is a patient} \\
((\text{bp}(X,Y1) \land^s \text{write}(Y1)) \lor^s (\text{bp}(X,Y2) \land^s \text{write}(Y2)))
\end{align*}

Consider a goal \( \text{www.dau.com/nurse} \supset \text{www.dau.com/query3} \). Solving this goal has the effect of executing \text{query3} with respect to the instructions in \text{nurse}. At the beginning, the system waits until the user specifies a patient \( X \). After \( X = \text{Kim} \) is selected, the system requests the nurse to measure Kim’s blood pressure. If this is a success, the nurse reports the outcome to the system. Once the outcome – say \( Y1 = 150 \) – is received, the system requests the nurse to measure Kim’s blood pressure once more. Again once the outcome – say \( Y2 = 130 \) – is received, the system reports \( Z = 130 \).

4 Conclusion

In this paper, we have considered an extension to LogicWeb with sequential hereditary Harrop formulas. This extension allows goals of the form \( G \land^s G \) and \( G \lor^s G \) where \( G \) is a goal. These goals are particularly useful for sequential executions of instructions.

In [3], Japaridze introduced two sequential operators \( \triangle \) and \( \triangledown \). These are additive versions of our \( \land^s \), \( \lor^s \): For goals \( A \triangle B \) or \( A \triangledown B \) to succeed, it is required to succeed in only one of the tasks. These features in goal formulas are probably desirable in terms of expressiveness. Our future interest is in a language which extends sequential hereditary Harrop formulas with these features.

Acknowledgements This work was supported by Dong-A University Research Fund.

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