A quantum processor based on coherent transport of entangled atom arrays

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The ability to engineer parallel, programmable operations between desired qubits within a quantum processor is central for building scalable quantum information systems [1, 2]. In most state-of-the-art approaches, qubits interact locally, constrained by the connectivity associated with their fixed spatial layout. Here, we demonstrate a quantum processor with dynamic, nonlocal connectivity, in which entangled qubits are coherently transported in a highly parallel manner across two spatial dimensions, in between layers of single- and two-qubit operations. Our approach makes use of neutral atom arrays trapped and transported by optical tweezers; hyperfine states are used for robust quantum information storage, and excitation into Rydberg states is used for entanglement generation [3–5]. We use this architecture to realize programmable generation of entangled graph states such as cluster states and a 7-qubit Steane code state [6, 7]. Furthermore, we shuttle entangled ancilla arrays to realize a surface code with 19 qubits [8] and a toric code state on a torus with 24 qubits [9]. Finally, we use this architecture to realize a hybrid analog-digital evolution [2] and employ it for measuring entanglement entropy in quantum simulations [10–12], experimentally observing non-monotonic entanglement dynamics associated with quantum many-body scars [13, 14]. Realizing a long-standing goal, these results pave the way toward scalable quantum processing and enable new applications ranging from simulation to metrology.

Quantum information systems derive their power from controllable interactions that generate quantum entanglement. However, the natural, local character of interactions limits the connectivity of quantum circuits and simulations. Nonlocal connectivity can be engineered via a global shared quantum data bus [15–18], but in practice these approaches have been limited in either control or size. A number of visionary architectures to address this challenge have been proposed theoretically over the past two decades. Based on coherent, dynamical transport of quantum information using movable traps or photonic links, these techniques have been the subject of intensive experimental explorations across different platforms [1, 19–24]. However, so far progress has been limited to small-scale, few-qubit systems lacking either full connectivity, programmability or true parallelism.

Our approach to address this long-standing challenge utilizes dynamically reconfigurable arrays of entangled neutral atoms, shuttled by optical tweezers in two spatial dimensions (Fig. 1a). Hyperfine states are used for storing and transporting quantum information in between quantum operations, and excitation into Rydberg states is used for generating entanglement. Highly parallel operations are enabled via selective qubit operations in distinct zones that qubits are dynamically shuttled between. Taken together, these ingredients enable a powerful quantum information architecture, which we employ to realize applications including entangled state generation, creation of topological surface and toric code states, and a hybrid analog-digital approach for quantum simulations.

Entanglement transport in atom arrays

Our experiments utilize a two-dimensional atom array system described previously [25], with key upgrades to enable coherent transport and multiple layers of single-qubit and two-qubit gates. We store quantum information in magnetically insensitive clock states within the ground state hyperfine manifold of $^{87}$Rb atoms [20], and implement robust single-qubit Raman rotations [26], realized by composite pulses which are insensitive to pulse errors (Extended Data Fig. 3) [27]. High-fidelity controlled-Z (CZ) entangling gates in the hyperfine basis $\{|0\rangle, |1\rangle\}$ (Fig. 1a) are implemented in parallel using global Rydberg excitation pulses on the $|1\rangle \leftrightarrow |r\rangle$ transition [5]. For dynamic reconfiguration, we initialize atoms into two sets of traps: static traps generated by a spatial light modulator (SLM) and mobile traps generated by a crossed 2D acousto-optic deflector (AOD). To execute a specific circuit, we arrange qubits into desired pairs, perform Rydberg-mediated CZ gates on each pair simultaneously, and then move all mobile traps in parallel to dynamically change the connectivity into the next
FIG. 1. Quantum information architecture enabled by coherent transport of neutral atoms. a, In our approach, qubits are transported to perform entangling gates with distant qubits, enabling programmable and nonlocal connectivity. Atom shuttling is performed using optical tweezers, with high parallelism in two dimensions and between multiple zones allowing selective manipulations. Inset shows the atomic levels used: the \(|0\rangle, |1\rangle\) qubit states refer to the \(m_F = 0\) clock states of \(^{87}\)Rb, and \(|r\rangle\) is a Rydberg state used for generating entanglement between qubits (Extended Data Fig. 1b). b, Atom images illustrating coherent transport of entangled qubits. Using a sequence of single-qubit and two-qubit gates, atom pairs are each prepared in the \(|\Phi^+\rangle\) Bell state (Methods), and are then separated by \(110 \, \mu m\) over a span of \(300 \, \mu s\). c, Parity oscillations indicate that movement does not observably affect entanglement or coherence. For both the moving and stationary measurements, qubit coherence is preserved using an XY8 dynamical decoupling sequence for \(300 \, \mu s\) (Methods). d, Measured Bell state fidelity as a function of separation speed over the \(110 \, \mu m\), showing that fidelity is unaffected for a move slower than \(200 \, \mu s\) (average separation speed of \(0.55 \, \mu m/\mu s\)). Inset: normalizing by atom loss during the move results in constant fidelity, indicating that atom loss is the dominant error mechanism (see Methods for details).

Programmable circuits and graph states

To exemplify the ability to generate nonlocal connectivity between qubit arrays in parallel, we carry out preparation of entangled graph states: a large class of useful quantum information states, with examples ranging from GHZ states and cluster states to quantum error correction codes [30]. Graph states are defined by ini-

desired qubit arrangement.

Figure 1 demonstrates our ability to transport qubits across large distances while preserving entanglement and coherence [20]. We initialize pairs at an atom-atom distance of \(3 \, \mu m\) (Fig. 1b) and then create a Bell state \(|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)\) in the hyperfine basis (Methods) [5]. To probe the resulting entangled-state fidelity, we apply an additional \(\pi/2\) pulse with a variable phase that results in oscillations of the two-atom parity \(|\sigma_1^Z \sigma_2^Z\rangle\) (Fig. 1c) [5]. We then repeat this experiment, but now move the atoms apart by \(110 \, \mu m\) before applying the final \(\pi/2\) pulse. Our transport protocol is optimized to suppress heating and loss by implementing cubic-interpolated atom trajectories (Methods), and is further accompanied by an 8-pulse XY8 robust dynamical decoupling sequence [28] to suppress dephasing. The resulting parity oscillations indicate that two-atom entanglement is unaffected by the transport process [20, 29]. Performing this experiment as a function of movement speed shows that fidelity remains unchanged until the total separation speed becomes \(> 0.55 \, \mu m/\mu s\), corresponding to the onset of atom loss (Fig. 1d). We note that the entanglement transport in Figure 1b corresponds to moving quantum information across a region of space that can in principle host \(\sim 2000\) qubits (at an atom separation of \(3 \, \mu m\)), on a timescale corresponding to \(< 10^{-3} T_2\) (Extended Data Fig. 3), directly enabling applications in large-scale quantum information systems.
a 1D cluster state graph

1st parallel layer

2nd parallel layer

b 1D and 2D graph states using dynamic entanglement transport. a, Generation of a 12-atom 1D cluster state graph, created by initializing all qubits (vertices) in \( |+\rangle \) and applying controlled-Z gates on the links (edges) between qubits. Atom images show the configuration for the first and second gate layers. b, Quantum circuit representation of the 1D cluster state preparation and measurement. Dynamical decoupling is applied throughout all quantum circuits (see Methods). c, Raw measured stabilizers of the resulting 1D cluster state, given by \( S_i = Z_{i-1}X_iZ_{i+1} \) for the edge qubits. d, Graph state representation of the 7-qubit Steane code (colors represent stabilizer plaquettes). e, Circuit for preparing the Steane code logical \( |+\rangle_L \) state, performed in four parallel gate layers. f, Measured stabilizers and logical operators after preparing \( |+\rangle_L \). Error detection is done by postselecting on measurements where all stabilizers are \( +1 \). For both the 1D cluster state and Steane code, the stabilizers and logical operators are measured with two measurement settings (see text). Error bars represent 68% confidence intervals.

tializing all qubits, located on the vertices of a geometric graph, \( \ell = 0^{12} \) and then performing CZ gates on the links between qubits (corresponding to the edges of the graph) \[30\]. N-qubit graph states \( G \) are associated with a set of \( N \) stabilizers, defined by \( S_i = X_i \prod_{j \in u_i} Z_j \), where \( u_i \) is the set of qubits (vertices) connected by a edge to qubit \( i \) \[30\]. The stabilizers each have \(+1\) eigenvalue for the graph state \( G \). Measuring these operators and their expectation values can be used to characterize preparation of the target state.

An important class of graph states are quantum error correcting (QEC) codes, where the graph state stabilizers manifest as the stabilizers of the QEC code and can be measured to correct errors on an encoded logical qubit. All stabilizer QEC states are equivalent to some graph state up to single-qubit Clifford rotations \[30\], hence the ability to generate arbitrary graph states allows one to readily prepare a wide variety of QEC states. As an example we prepare the 7-qubit Steane code \[6, 7\], a topological color code depicted by the graph in Fig. 2d, in the logical state \( |+\rangle_L \). To prepare this state, we initialize all qubits in \( |+\rangle \), apply CZs on the links between qubits (in four parallel layers, see Extended Data Fig. 5), and then rotate either of the two sublattices for measuring stabilizers (Fig. 2e). After sublattice rotation, six of the graph state stabilizers transform into the six Steane code stabilizers, given by four-body products of \( X_i \) or \( Z_i \). Figure 2f shows the raw measured expectation values of these six stabilizers. The seventh graph state stabilizer transforms into the logical qubit operator \( X_L \) and has eigenvalue \(+1\) for the graph state \( G \), while anticommuting with logical \( Z_L \). Accordingly, in Fig. 2f we find \( \langle X_L \rangle = 0.71(2) \) and \( \langle Z_L \rangle = -0.02(3) \), demonstrating preparation of the logical qubit state \( |+\rangle_L \). Moreover,
Topological states with ancilla arrays

We next make use of transportable ancillary qubit arrays to mediate quantum operations between remote qubits [1]. Due to the ability to quickly move arrays of atoms across the entire system, the use of ancillary qubits naturally complements our movement capabilities. Specifically, we employ ancillas for state preparation by mediating entanglement between physical qubits that never directly interact, followed by projective measurement of the ancilla array (performed simultaneously that never directly interact, followed by projective measurement of the ancilla qubits), a form of measurement-based quantum computation [31, 36]. In particular, we prepare topological surface code and toric code states [8, 9, 37, 38], whose states are more difficult to construct by direct CZ gates between physical qubits (requiring an extensive number of layers [8, 39]).

Figure 3a demonstrates preparation of a 19-qubit graph state creating the $|+\rangle_L$ logical state of the surface code [8, 37]. The surface code is defined by $X$ plaquette and $Z$-star stabilizers, and logical operators $X_L$ ($Z_L$) are defined as strings of $X$ ($Z$) products across the height (width) of the graph. To prepare this state, ancillas are moved to perform CZ gates with each of their four neighbors and are then measured, projecting the data qubits into the surface code state. The graph state stabilizers now transform into the $X$-plaquettes, the $Z$-stars (with value ±1 for a measurement outcome of ±1 of the central ancilla), and the logical $X_L$ operator [36, 40]. Figure 3b shows the measured expectation values of the twelve resulting stabilizers, as well as the logical operator expectation values with / without error detection. We find a raw value of $\langle X_L \rangle = 0.64(3)$, and a corrected value of $\langle X_L \rangle = 1^{+0}_{-0.01}$ using error detection (with 35(1)% probability of no detected errors), demonstrating preparation of this topological QEC state (see also Extended Data...
Fig. 7).

While surface code states have previously been prepared with other methods, our transport capabilities allow us to use the full range of motion of ancilla qubits across the entire qubit array to enable periodic boundary conditions and realize the toric code state on a torus [9]. To this end, we create the 24-qubit graph state shown in Fig. 3c by performing five layers of parallel gates and moving the ancillae to their separate zone for readout in a separate basis (see also the supplementary movie showing the full atom trajectory). The state we prepare in a separate basis (see also the supplementary movie in Fig. 3c by performing five layers of parallel gates and thus yields the second-order Renyi entanglement entropy $S_2(A) = -\log_2 \text{Tr} [\rho^2_A]$ (Methods). This measurement circuit provides the Renyi entropy of any constituent subsystem of our whole closed quantum system, where the calculation over any desired subsystem $A$ is simply performed in data processing [10, 11].

We use this method to probe the growth of entanglement entropy produced by many-body dynamics (see Methods for additional benchmarking of the technique, including data on small systems). Specifically, we study the evolution of two eight-atom copies under the Rydberg Hamiltonian, subject to the nearest-neighbor blockade constraint [4, 13]. Upon a rapid quench from an initial state with all atoms in the ground state ($|g\rangle = |1\rangle$), we observe that the half-chain Renyi entanglement entropy quickly grows and saturates (Fig. 4c), a process corresponding to quantum thermalization [12]. By analyzing the Renyi mutual information $I_{AB} = S_2(A) + S_2(B) - S_2(AB)$ between the leftmost $n$ atoms in the chain ($A$) and the complement subsystem of the rightmost $8 - n$ atoms ($B$), we find a volume-law scaling in the resulting state (Fig. 4d) [11, 12].

While such thermalizing dynamics is generically expected in strongly interacting many-body systems, remarkably, this is not always the case. In particular, it was demonstrated previously that for certain initial states this system can evade thermalization for relatively long times. Underpinned by special, non-thermal eigenstates called quantum many-body scars [13, 14, 42], these states were theoretically predicted to feature dynamics associated with a slow, non-monotonic entanglement growth. Figure 4 reports the measurement of entanglement properties of many-body scars following a rapid quench from the initial state $|Z_2\rangle \equiv |rrgrg...\rangle$, initialized by applying local light shifts within one sublattice and performing a global Rydberg $\pi$ pulse (Methods). We find that the rate of entropy growth for this initial state is significantly suppressed, and the mutual information reveals an area-law scaling (in contrast to the volume-law of the $|ggrgg...\rangle$ quench) (Fig. 4d). Furthermore, Fig. 4e shows the single-site entropy in the middle of the chain, demonstrating rapid growth and saturation of entropy for the thermalizing $|ggrgg...\rangle$ state but large oscillations in entropy for the $|Z_2\rangle$ state [13, 14]. Remarkably, the data show that when sites of one sublattice return to low entropy, the other sublattice goes to high entropy; this reveals that the scar dynamics entangle distant atoms (of the same

Hybrid analog-digital circuits

Having established atom movement for realizing digital circuits, we now explore the applications to quantum simulation. In particular, we perform hybrid, modular quantum circuits composed of analog Hamiltonian evolution, reconfiguration, and digital gates (Figure 4a). Together, these tools open a wide variety of new possibilities in quantum simulation and many-body physics. As a specific example, we measure the Renyi entanglement entropy after a quantum quench by effectively interfering two copies of a many-body system [10, 11].

Figure 4b illustrates the experimental procedure. After initializing both copies with all qubits in $|1\rangle$, we independently evolve each copy under the Rydberg Hamiltonian $H_{\text{Ryd}}$ for a time $t$, generating an entangled many-body state in the $\{|1\rangle, |r\rangle\}$ basis (Methods) [13]. Raman and Rydberg $\pi$ pulses then map $|1\rangle \rightarrow |0\rangle$ and $|r\rangle \rightarrow |1\rangle$, transferring the entangled many-body state into the long-lived and non-interacting $\{|0\rangle, |1\rangle\}$ basis [41]. Finally, we measure entanglement entropy by rearranging the system and interfering each qubit in the first copy with its identical twin in the second copy, by use of a Bell measurement circuit. Measuring twins in the Bell basis detects occurrences of the antisymmetric singlet state $|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$, whose presence indicates that subsystems of the two copies were in different states due to entanglement with the rest of the many-body system (and entanglement with the environment) [10, 11]. Quantitatively, analyzing the number parity of observed singlets within subsystem $A$ yields the purity $\text{Tr}[\rho^2_A]$ of reduced density matrix $\rho_A$, and thus yields the second-order Renyi entanglement entropy $S_2(A) = -\log_2 \text{Tr} [\rho^2_A]$ (Methods). This measurement circuit provides the Renyi entropy of any constituent subsystem of our whole closed quantum system, where the calculation over any desired subsystem $A$ is simply performed in data processing [10, 11].

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sublattice) while disentangling nearest neighbors, even with only nearest-neighbor interactions (see Methods). These measurements reveal nontrivial aspects of quantum many-body scars, and constitute the direct observation of exotic entanglement phenomena in a many-body system.

These observations are in excellent agreement with results of exact numerical simulations of the quantum dynamics in the isolated system (lines plotted in Figs. 4c,e and Extended Data Fig. 10). Moreover, whereas the single-site purity approaches that of a fully mixed state, our global purity (a 16-body observable composed of three-level systems) remains $>100\times$ that of a fully mixed state (see Extended Data Fig. 9), altogether demonstrating the high accuracy and fidelity of our circuit-based technique. These results demonstrate that combining atom movement, many-body Hamiltonian evolution, and digital quantum circuits yields powerful new tools for simulating and probing quantum physics of complex systems.

Discussion and outlook

Our experiments demonstrate highly parallel coherent qubit transport and entanglement enabling a powerful quantum information architecture. The present techniques can be extended along a number of directions. Local Rydberg excitation on subsets of qubit pairs would eliminate residual interactions from unintended atoms, allowing parallel, independent operations on arrays with significantly higher qubit densities. Two-qubit gate fidelity can be improved using higher Rydberg laser power or more efficient delivery methods, as well as more advanced atom cooling [43]. These technical improvements should allow for direct scaling to deep quantum circuits operating on thousands of neutral atom qubits. These upgrades can be additionally supplemented by more sophisticated local single-qubit control employing, for example, parallel Raman excitation through AOM arrays [16]. Mid-circuit readout can be implemented by moving ancillas into a separate zone and imaging using e.g. avalanche photodiode arrays within a few hundred microseconds [44].

Our method has a clear potential for realizing scalable quantum error correction [45]. For example, the procedure demonstrated in Fig. 3c can be used for syndrome extraction in a practical QEC sequence, wherein ancillas are entangled with their data qubit neighbors and then moved to a separate zone for mid-circuit readout. We estimate an entire QEC round can be implemented within a millisecond, much faster than the measured $T_2 > 1$ s, and with projected fidelity improvements theoretically surpassing the surface code threshold (Methods). Fur-
thermore, the ability to reconfigure and interlace our arrays will allow efficient, parallel execution of transversal entangling gates between many logical qubits [37, 46]. In addition, these techniques also enable implementation of higher-dimensional or nonlocal error correcting codes with more favorable properties [47, 48]. Together, these ingredients could enable a new approach to universal, fault-tolerant quantum computing with thousands of physical qubits.

Our dynamically reconfigurable architecture also opens many new opportunities for digital and analog quantum simulations. For example, our hybrid approach can be extended to probing the entire entanglement spectrum [49], simulating wormhole creation [50], performing many-body purification [51], and engineering novel non-equilibrium states [52]. Entanglement transport could also find use in metrological applications such as creating distributed states for probing gravitational gradients [53]. Finally, our approach can help facilitate quantum networking between separated arrays, paving the way toward large-scale quantum information systems [29, 54] and distributed quantum metrology [53, 55].

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METHODS

Dynamic reconfiguration in 2D tweezer arrays
Our experiments utilize the same apparatus described previously in Ref. [25]. Inside our vacuum cell, $^{87}$Rb atoms are loaded from a magneto-optical trap into a backbone array of programmable optical tweezers generated by a spatial light modulator (SLM) [56]. Atoms are rearranged in parallel into defect-free target positions in this SLM backbone [56] by additional optical tweezers generated from a crossed 2D acousto-optic deflector (AOD). Following the rearrangement procedure, we transfer selected atoms from the static SLM traps back into the mobile AOD traps, and then move these mobile atoms to their starting positions in the quantum circuit. During this entire process, the atoms are cooled with polarization gradient cooling. Before running the quantum circuit, we take a camera image of the atoms in their initial starting positions, and following the circuit we take a final camera image to detect qubit states $|0\rangle$ (atom presence) and $|1\rangle$ (atom loss, following resonant pushout). We postselect all data on finding perfect rearrangement of the AOD and SLM atoms before running the circuit. In all experiments here, each atom remains in a single static or single mobile trap throughout the duration of the quantum circuit [20, 57, 58].

The crossed AOD system is composed of two independently controlled AODs (AA Opto Electronic DTSX-400) for $x$ and $y$ control of the beam positions. Both AODs are driven by independent arbitrary waveforms which are generated by a dual-channel arbitrary waveform generator (AWG) (M4i.6631-x8 by Spectrum Instrumentation) and then amplified through independent MW amplifiers (Minicircuits ZHL-5W-1). The time-domain arbitrary waveforms are composed of multiple frequency tones corresponding to the $x$ and $y$ positions of columns and rows, which are independently changed as a function of time for steering around the AOD-trapped atoms dynamically; the full $x$ and $y$ waveforms are calculated by adding together the time-domain profile of all frequency components with a given amplitude and phase for each component. For running quantum circuits, we program the positions of the AOD atoms at each gate location and then smoothly interpolate (with a cubic profile) the AOD frequencies as a function of time between gate positions. The cubic profile enacts a constant jerk onto the atoms, which allows us to move roughly 5-10x faster (without heating and loss) than if we move at a constant velocity (linear profile). In our movement protocol, we only do stretches, compressions, and translations of the AOD trap array: i.e., the AOD rows and columns never cross each other in order to avoid atom loss and heating associated with two frequency components crossing each other.

We homogenize the AOD tweezer intensity throughout the whole atom trajectory in order to minimize dephasing induced by a time-varying magnitude of differential light shifts. To this end, we use a reference camera in the image plane to gauge the intensity of each AOD tweezer at each gate location and homogenize by varying the amplitude of each frequency component; during motion between two locations we interpolate the amplitude of each individual frequency component.

The SLM tweezer light (830 nm) and the AOD tweezer light (828 nm) are generated by two separate, free-running Ti:sapphire lasers (M Squared, 18-W pump). Projected through a 0.5 NA objective, the SLM tweezers have a waist of roughly $\sim 900$ nm ($\sim 1000$ nm for AODs). When loading the atoms, the trap depths are $\sim 2\pi \times 16$ MHz, with radial trap frequencies of $\sim 2\pi \times 80$ kHz, and when running quantum circuits the trap depths are $\sim 2\pi \times 4$ MHz, with radial trap frequencies of $\sim 2\pi \times 40$ kHz.

Raman laser system
Fast, high-fidelity single-qubit manipulations are critical ingredients of the quantum circuits demonstrated in this work. To this end, we use a high-power 795-nm Raman laser system for driving global single-qubit rotations between $m_F = 0$ clock states. This Raman laser system is based on dispersive optics, developed and described in Ref. [26]. 795-nm light (Toptica TA pro, 1.8W) is phase-modulated by an electro-optic modulator (Qubig), which is driven by microwaves at 3.4 GHz (Stanford Research Systems SRS SG384) that are doubled to 6.8 GHz and amplified. The laser phase modulation is converted to amplitude modulation for driving Raman transitions through use of a Chirped Bragg Grating (Optigrate) [26]. IQ control of the SG384 is used for frequency and phase control of the microwaves, which are imprinted onto the laser amplitude modulation and thus give us direct frequency and phase control over the hyperfine qubit drive.

The Raman laser illuminates the atom plane from the side in a circularly polarized elliptical beam with waists of 40 $\mu$m and 560 $\mu$m on the thin axis and the tall axis, respectively, with a total average optical power of 150 mW on the atoms. The large vertical extent ensures $< 1\%$ inhomogeneity across the atoms, and shot-to-shot fluctuations in the laser intensity are also $< 1\%$. For main text Figs. 1-3, we operate our Raman laser at a blue-detuned intermediate-state detuning of 180 GHz, resulting in two-photon Rabi frequencies of 1 MHz and an estimated scattering error per $\pi$ pulse of $7 \times 10^{-5}$ (i.e., 1 scattering event per 15000 $\pi$ pulses) [26]. For main text Fig. 4, in order to shorten the duration of the coherent mapping pulse sequence, we increase the Raman laser power and operate at a smaller blue-detuned intermediate-state detuning of 63 GHz, with a corresponding two-photon Rabi frequency of 3.2 MHz and an estimated scattering error per $\pi$ pulse of $2 \times 10^{-4}$. 

Robust single-qubit rotations
For almost all single-qubit rotations in this work (other than XY8 / XY16 self-correcting sequences) we implement robust single-qubit rotations in the form of composite pulse sequences. These composite pulse sequences are well-known in the NMR community [27, 59] and can be highly insensitive to pulse errors such as amplitude or detuning miscalibrations. Our dominant source of coherent single-qubit errors arise from ≲ 1% amplitude drifts and inhomogeneity across the array; as such, we primarily use the “BB1” (broadband 1) pulse sequence, which is a sequence of four pulses that implements an arbitrary rotation on the Bloch sphere while being insensitive to amplitude errors to 6th order [27, 59]. We benchmark the performance of these robust pulses in Extended Data Fig. 3a. Furthermore, by applying a train of BB1 pulses we find an accumulated error consistent with the estimated scattering limit (not plotted here), suggesting that the scattering limit roughly represents our single-qubit rotation infidelities (~ 3 × 10^{-4} error per BB1 pulse due to the increased length of the composite pulse sequence). Randomized benchmarking [60] can be applied in future studies to further study single-qubit rotation fidelity.

Qubit coherence and dynamical decoupling
In our 830-nm traps, hyperfine qubit coherence is characterized by $T_2^\ast = 4$ ms (not plotted here), $T_2 = 1.5$ s (XY16 with 128 total π pulses), and $T_1 = 4$ s (including atom loss) (Extended Data Fig. 3 b,c). All of our experiments in this work are performed in a DC magnetic field of 8.5 Gauss. Coherence can be further improved by using further-detuned optical tweezers (with trap depth held constant, the tweezer differential lightshifts decrease as $1/\Delta$ and $1/T_1$ decreases as $1/\Delta^3$ [61]) and shielding against magnetic field fluctuations.

All of our transport sequences [20, 57, 58] are accompanied with dynamical decoupling sequences. The number of pulses we use is a tradeoff between preserving qubit coherence while minimizing pulse errors. We interchange between two types of dynamical decoupling sequences: XY8 / XY16 sequences, composed of phase-alternated individual π-pulses which are self-correcting for amplitude and detuning errors [28, 62], and CPMG-type dynamical decoupling sequences composed of robust BB1 pulses. The CPMG-BB1 sequence is more robust to amplitude errors but incurs more scattering error. We empirically optimize for any given experiment by choosing between these different sequences and a variable number of decoupling π pulses, optimizing on either single-qubit coherence (including the movement) or the final signal. Typically, our decoupling sequences are composed of a total 12-18 π pulses.

Movement effects on atom heating and loss
We study here the effects of movement on atom loss and heating in the harmonic oscillator potential given by the tweezer trap. Motion of the trap potential is equivalent to the non-inertial frame of reference where the harmonic oscillator potential is stationary but the atom experiences a fictitious force given by $F(t) = -m\ddot{a}(t)$, where $m$ is the mass of the particle and $\dot{a}(t)$ is the acceleration of the trap as a function of time [63, 64]. By following Ref. [65] (Eq. 5.4), we find the average vibrational quantum number increase $\Delta N$ is given by

$$\Delta N = \frac{|\tilde{a}(\omega_0)|^2}{(2x_{zp}^2\omega_0^2)^2},$$

where $\tilde{a}(\omega_0)$ is the Fourier transform of $a(t)$ evaluated at the trap frequency $\omega_0$, and the zero point size of the particle $x_{zp} \equiv \sqrt{\hbar/(2m\omega_0)}$. $\Delta N$ is the same for all initial levels of the oscillator [65]. Experimentally, we apply an acceleration profile $a(t) = j t$ to the atom, from time $-T/2$ to $+T/2$ to move a distance $D$ with constant jerk $j$. We calculate $|\tilde{a}(\omega)|^2$, simplify using $\omega_0T \gg 1$, and assume a small range of trap frequencies to average the oscillatory terms, resulting in

$$\Delta N = \frac{1}{2} \left( \frac{6D/x_{zp}}{\omega_0^2T^2} \right)^2.$$  

Several relevant insights can be gleaned from this formula. First, this expression indicates our ability to move large distances $D$ with comparably small increases in time $T$. Furthermore, to maintain a constant $\Delta N$, the movement time $T \propto \omega_0^{-3/4}$. Moreover, to perform a large number of moves $k$ for a deep circuit, we can estimate $\Delta N \propto \omega_0^{-2k/T^2}$, suggesting that we can increase our number of moves from e.g. 5 to 80 by slowing each move from 200 μs to 400 μs.

We now compare Eq. 2 to our experimental observations. In Fig. 1d we start to observe atom loss when we move 55 μm in 200 μs under a constant negative jerk. This speed limit is consistent with our above estimates: using $\omega_0 = 2\pi \times 40$ kHz and $x_{zp} = 38$ nm, we predict $\Delta N \approx 6$ for this move, corresponding to the onset of tangible heating at this move speed. More quantitatively, we assume a Poisson distribution with mean $N$ and variance $N$ and integrate the population above some critical $N_{max}$ upon which the atom will leave the trap. From this analysis we find atom retention is given by

$$\frac{1}{2} \left( 1 - \text{erf} \left( (N_{max} - N)/\sqrt{2N} \right) \right).$$

ED Figs. 2a and b measure atom retention as a function of move time $T$ and trap frequency $\omega_0/2\pi$. Using the functional form above, for both sets of measurements, we extract an $N_{max}$ of $\approx 30$, corresponding to adding $\approx 30$ excitations before exciting the atom out of the trap. Such a limit is physically reasonable as the absolute trap depth of 4 MHz implies only $\approx 100$ levels, the atom starts at
finite temperature, and moreover the effective trap frequency reduces once the anharmonicity of the trap starts to play a role. We note that these estimates are only approximate (we roughly estimate \( \omega_0 \) for the trap depths used during the motion), but nonetheless suggests our motion limit is consistent with physical limits for our chosen \( a(t) \). Our analysis here also neglects the acoustic lensing effects associated with ramping the AOD frequency, which causes astigmatism by focusing one axis to a different plane and thus deforms the trap and reduces the peak trap intensity (and \( \omega_0 \)) as given by the Strehl ratio.

Additional heating and loss during the circuit can also be caused by repeated short drops for performing two-qubit gates, where the tweezers are briefly turned off to avoid anti-trapping of the Rydberg state and light shifts of the ground-Rydb erg transition. However, drop-recapture measurements in Extended Data Fig. 2c suggest the 500-ns drops we use experimentally have a negligible effect until hundreds of drops per atom (corresponding to hundreds of CZ gates). We find that atom loss and heating as a function of number of drops are well-described by a diffusion model, which would then predict that reducing atom temperature by a factor of 2\( \times \) (reducing thermal velocity by \( \sqrt{2} \times \)) and reducing drop time \( t_{\text{drop}} \) by \( 2 \times \), together would increase the number of possible CZ gates per atom to thousands.

Two-qubit CZ gates implementation

We implement our two-qubit gates and calibrations following Ref. [5]. Specifically, the two-qubit CZ gate is implemented by two global Rydberg pulses, with each pulse at detuning \( \Delta \) and length \( \tau \), and with a phase jump \( \xi \) between the two pulses. The pulse parameters are chosen such that qubit pairs, adjacent and under the Rydberg blockade constraint, will return from the Rydberg state back to the hyperfine qubit manifold with a phase depending on the state of the other qubit. The numerical values for these pulse parameters are:

\[
\Delta = -0.377371 \Omega \\
\xi = -0.621089 \times (2\pi) \\
\tau = 0.683201/[\Omega/(2\pi)]
\]

For our experiments in Figs 1-3, we operate with a two-photon Rydberg Rabi frequency of \( \Omega/2\pi = 3.6 \) MHz, giving a theoretical \( \tau = 190 \) ns and a theoretical \( \Delta/(2\pi) = -1.36 \) MHz. We choose the negative detuning sign to help minimize excitation into the \( m_j = +\frac{1}{2} \) Rydberg state which is detuned by about 24 MHz under the field of 8.5 G (and experiences a \( 3\times \) lower coupling to the Rydberg laser than the desired \( m_j = -\frac{1}{2} \) state due to reduced Clebsch-Gordan coefficients). In this work we operate with strong blockade between adjacent qubits, with Rydberg-Rydberg interactions \( V_0/2\pi \) ranging from 200 MHz to 1 GHz. In Fig. 4, we operate with \( \Omega/2\pi = 4.45 \) MHz for the two-qubit gates.

Managing spurious phases during CZ gates

The two-qubit gate from Ref. [5] induces both an intrinsic single-qubit phase, as well as spurious phases which are primarily induced by the differential light shift from the 420-nm laser. Under certain configurations, the 420-nm-induced differential light shift on the hyperfine qubit can be exceedingly large (> 8 MHz), yielding phase accumulations on the hyperfine qubit of \( \approx 6\pi \). Small, percent-level variations of the 420-nm intensity can thus lead to significant qubit dephasing.

Ref. [5] addresses this 420-induced-phase issue by performing an echo sequence: after the CZ gate, the 1013-nm Rydberg laser is turned off, a Raman \( \pi \) pulse is applied, and then the 420-nm laser is pulsed again to cancel the phase induced by the 420 light during the CZ gate. This method echoes out the 420-induced phase, but comes at a cost of a factor of two increase in the 420-induced scattering error, which is the dominant source of error in our two-qubit CZ gates.

Echo between CZ gates. To address these various issues, here we perform a Raman \( \pi \) pulse between each CZ gate to echo out spurious gate-induced phases on the hyperfine qubit (Extended Data Fig. 1). This approach has several advantages. The 420-induced phase is now cancelled by pairs of CZ gates, without explicitly applying additional 420-nm pulses to echo each individual CZ gate, thereby reducing the scattering error of the CZ gate in this work by a factor of approximately two. We estimate that this echo technique, having reduced the scattering error incurred during each gate, roughly compensates the increased scattering rate incurred by spreading our optical power over more space in 2D, thereby giving us comparable gate fidelities to the two-qubit CZ gate fidelities of \( \geq 97.4(2)\% \) reported in Ref. [5]. Further, the echo between CZ gates also cancels the intrinsic single-qubit phase of the CZ gate, removing errors in the calibration of this parameter, as well as canceling any other gate-induced spurious single-qubit phases such as \( a \approx 0.01 \) rad phase induced by pulsing the traps off for 500 ns for the two-qubit gate (Extended Data Fig. 1). In instances where the number of CZ gates we apply is odd, we perform the echo for the final CZ gate.

Sign of intermediate-state detuning. To further suppress the effect of the spurious, 420-induced phase, we operate our 420-nm laser to be red-detuned (by 2 GHz) from the \( 6P_{3/2} \) transition. For red detunings, the light shift on the \( |0\rangle \) state and the \( |1\rangle \) state are of the same sign, minimizing the differential light shift, while for blue detunings < 6.8 GHz, the light shift on the \( |0\rangle \) state and the \( |1\rangle \) state have opposite signs and amplify the differential light shift.
**Sensitivity to axial trap oscillations**

In typical Rydberg excitation timescales with optical tweezers, the axial trap oscillation frequencies of several kHz are inconsequential. Here with our circuits running as long as 1.2 ms, with Rydberg pulses throughout, we find that the axial trap oscillations can have important effects. In particular, the axial oscillations cause the atoms to make oscillations in/out of the Rydberg beams: at our estimated axial temperature of $\sim 25 \, \mu$K and axial oscillation frequency of 6 kHz, we estimate an axial spread $\sqrt{\langle z^2 \rangle} \approx 1.3 \, \mu$m. For our 20-micron-waist beams, the effect of this positional spread is relatively small on the pulse parameters of the CZ gate, but can be significant on the sensitive 420-induced phase we seek to cancel by echoing out the phase induced by CZ gates separated by $\sim 200 \, \mu$s (see previous section). When using 20-micron-waist beams, and a 2.5-GHz blue detuning of our 420-nm laser, we find that the dephasing due to the axial trap oscillations is significant (see Extended Data Fig. 4). To remedy this deleterious effect, we increase the beam waist of our 420-nm laser to 35 microns (while maintaining constant intensity) and change the laser frequency to be 2-GHz red-detuned, together resulting in a significant reduction in the dephasing associated with improper echoing of the 420-nm pulse.

**Bell state preparation and fidelity**

In Figure 1, we prepare the $|\Phi^+\rangle$ Bell state in the same way that is done in Ref. [5]: after initializing a pair of qubits in $|00\rangle$, we apply $X(\pi/2)$ pulse - CZ gate - $X(\pi/4)$ pulse. We calculate and plot the raw resulting fidelity of this $|\Phi^+\rangle$ Bell state as the sum of populations in $|00\rangle$ and $|11\rangle$, averaged with the fitted amplitude of parity oscillations (example in Fig. 1c) which measures the off-diagonal coherences. In Fig. 1d, upon significant loss from movement, this fidelity estimate becomes skewed because we begin measuring an artificially large population in $|11\rangle$ (since state $|1\rangle$ is detected as loss); accordingly, we estimate the $|\Phi^+\rangle$ population as $2\times$ the population of $|00\rangle$ once the population difference between $|11\rangle$ and $|00\rangle$ becomes greater than 0.1 (an arbitrary cutoff where the effects of loss start to become significant). In Fig. 1d, for moves slower than $300 \, \mu$s we achieve an average raw Bell state fidelity after the moving of 94.8(2)%. If we do not move or attempt to preserve coherence for $500 \, \mu$s (i.e. if we measure immediately after preparing the Bell state) then we measure a raw Bell state fidelity of 95.2(1)% (not plotted here), consistent with the results in Ref. [5].

**Analysis of error sources**

We detail here some of our measured and estimated sources of error for an entire sequence (toric code preparation in particular, our deepest circuit). We find the total single-qubit fidelity after performing the entire sequence is roughly 96.5% for the toric code circuit, which we measure by embedding the entire experiment in a Ramsey sequence: i.e., we perform a Raman $\pi/2$ pulse, do all motion and decoupling, and then do a final $\pi/2$ pulse with variable phase to measure total contrast. We are able to account for our single-qubit fidelity quantitatively as being composed of our known single-qubit errors in Extended Data Fig. 6c.

Estimated contributions to two-qubit gate error are summarized in Extended Data Fig. 6c. These estimates come from numerical simulations in QuTiP with experimental parameters. The effects of intermediate state scattering and Rydberg decay are included via collapse operators in the Lindblad master equation solver. Other error contributions include finite temperature random Doppler shifts and position fluctuations, as well as laser pulse-to-pulse fluctuations, all of which are simulated using classical Monte Carlo sampling of experiment parameters. Experimental parameters used for the simulations are as follows: blue and red Rabi frequencies $(\Omega_b, \Omega_r) = 2\pi \times (160, 90) \, MHz$, $6P_{3/2}$ intermediate state detuning = 2 GHz, intermediate state lifetime = 110 ns, $70S_{1/2}$ Rydberg state lifetime = 150 $\mu$s, Rydberg blockade energy = 500 MHz, splitting to second Rydberg state = 24 MHz, radial and axial trap frequencies $(\omega_r, \omega_z) = 2\pi \times (40, 6) \, kHz$, and temperature $T = 20 \, \mu$K. We can also use this modeling to project for future performance; by assuming a 10x increase in available 1013-nm intensity and that atoms are cooled to 2 $\mu$K temperature, we project a possible CZ gate fidelity of 99.7%, beyond the surface code threshold [37, 66]. Alkaline-earth atoms could also offer other routes to high fidelity operations for quantum error correction [67–69].

To understand how our various single-qubit and two-qubit errors contribute to our graph state fidelities, we perform a stochastic simulation of the quantum circuit used for graph state preparation (Extended Data Fig. 6a,b). We utilize the Clifford properties of our circuit, allowing for efficient numerical evaluation and random sampling of many possible error realizations. The simulation is performed under a realistic error model, where the rates of ambient depolarizing noise and atom loss are measured in the experiment (see Extended Data Fig. 6c). The resulting stabilizer and logical qubit expectation values agree well with those measured experimentally.

**Rydberg beam shaping and homogeneity**

We shape our Rydberg beams into tophats of variable size through wavefront control using the phase profile on a spatial light modulator (SLM) [25]. This ability allows us to match the height of our beam profile to the experiment zone size of any given experiment, thereby maximizing our 1013-nm light intensity and CZ gate fidelities. We optimize our Rydberg beam homogeneity until peak-to-peak inhomogeneities are below < 1%. To
this end, we correct all aberrations up to the window of our vacuum chamber, as done in [25], which yields an inhomogeneity on the atoms of several percent that we attribute to imperfections of the final window. To further optimize the homogeneity, we empirically tune aberration corrections on the tophat through Zernike polynomial corrections to the phase profile in the SLM plane (Fourier plane). With this procedure we reduce peak-to-peak inhomogeneities to < 1% over a range of 40-50 µm in the atom plane.

Creation and optimization of graph layouts
We outline here a description of how we optimize our graph layouts for the cluster state, Steane code, surface code, and toric code preparation. Our optimization in this work is heuristic, and future work can develop appropriate algorithms for designing optimal circuits through atom spatial arrangement and AOD trajectories. Extended Data Figure 5 shows all of the graphs we create and the process for creating them. There are several parameters we optimize for:

1. Minimize number of parallel two-qubit gate layers.
2. Minimize total control distance for the moving atoms.
3. Have all moving atoms in one sublattice (all graphs realized here are bipartite) to facilitate the final local rotation of one sublattice.
4. Minimize the vertical extent of the array and the number of distinct rows (to maximize 1013 intensity and minimize sensitivity to beam inhomogeneity between the rows).
5. When ordering gates, apply two-qubit gates as early as possible in the circuit. If a gate layer induces a bit-flip (X error) then that error can propagate during subsequent gates (becoming a Z error on the other qubit), so gates should be in the earliest layer possible.

Local (sublattice) hyperfine rotations
We perform local rotations in the hyperfine basis by use of our horizontally propagating 420-nm beam, which imposes a differential light of several MHz on the hyperfine qubit and can thus be used for realizing a fast Z rotation. To realize the local $Y(\pi/2)$ rotation used throughout this work, we move one sublattice of atoms out of the 420-nm beam, then apply $[\text{global } Y(\pi/4)] - [\text{local } Z(\pi)] - [\text{global } Y(\pi/4)]$. This realizes a $Y(\pi/2)$ rotation on one sublattice and a $Z(\pi)$ rotation on the other sublattice (which is inconsequential as it then commutes with the immediately following measurement in the Z-basis). To apply a $Y(\pi/2)$ on the other sublattice of atoms, we add an additional global $Z(\pi)$ (implemented by jumping the Raman laser phase) between the two $Y(\pi/4)$ pulses. Future experiments will benefit from an additional set of locally focused beams for performing local Raman control of hyperfine qubit states, but we find that moving atoms works so efficiently (even for moving > 50 µm) to move out of the 420-nm beam that this approach is well-suited for producing a high-fidelity, homogeneous rotation on roughly half the qubits.

Local Rydberg initialization
We perform local Rydberg control in order to initialize the $|Z_2\rangle = |rgrg...\rangle = |111\ldots\rangle$ state for studying the dynamics of many-body scars. We achieve this local initialization by applying $\sim 50$ MHz light shifts between $|1\rangle$ and $|r\rangle$ using 810-nm tweezers generated by an SLM onto a desired subset of sites, and then apply a global Rydberg $\pi$ pulse which excites the non-lightshifted atoms. We use this approach here to prepare every other atom in each chain into $|r\rangle$, but emphasize that since the locations of the SLM tweezers are fully programmable, this technique can be used to prepare any initial blockade-satisfying configuration of atoms in $|1\rangle$ and $|r\rangle$.

The 50 MHz biasing light shift is significantly larger than the Rydberg Rabi frequency $\Omega/2\pi = 4.45$ MHz, leading to a Rydberg population on undesired sites of < 1%. The $t = 0$ time point of Extended Data Fig. 10b shows the high-fidelity preparation of the $|Z_2\rangle$ state using this approach. We note that with 810-nm light, even though the achieved biasing light shift is significant, the Raman-scattering-induced $T_1$ (of the hyperfine qubit) is still $\sim 50$ ms and thus leads to a scattering error $\lesssim 4 \times 10^{-6}$ during the 200-ns pulse of the light-shifting tweezers. There can also be a motional effect from the biasing tweezers, with an estimated radial trapping frequency of 150 kHz, which we also deem to be negligible during the 200-ns pulse.

Rydberg Hamiltonian
In Fig. 4, we study dynamics under the many-body Rydberg Hamiltonian

$$\frac{H_{\text{Ryd}}}{\hbar} = \frac{\Omega}{2} \sum_i \sigma_i^x - \Delta \sum_i n_i + \sum_{i<j} V_{ij} n_i n_j,$$

where $\hbar$ is the reduced Planck constant, $\Omega$ is the Rabi frequency, $\Delta$ is the laser frequency detuning, $n_i = |r_i\rangle\langle r_i|$ is the projector onto the Rydberg state at site $i$ and $\sigma_i^x = |1_i\rangle\langle r_i| + |r_i\rangle\langle 1_i|$ flips the atomic state. For the entanglement entropy measurements in this work, we choose lattice spacings where the nearest-neighbor (NN) interaction $V_0 > \Omega$ results in the Rydberg blockade, preventing adjacent atoms from simultaneously occupying $|r\rangle$. In particular, the many-body experiments are performed on 8-atom chains, quenching to a time-independent $H_{\text{Ryd}}$ with $V_0/2\pi = 20$ MHz, $\Omega/2\pi = 3.1$ MHz, $\Delta/2\pi = 0.3$ MHz. Quenching to small, positive $\Delta = 0.0173$ $V_0$ partially suppresses the always-positive long-range interactions and thereby is optimal for scar lifetime, as derived and shown experimentally in Ref. [70].
Coherent mapping protocol
As described in the text, we implement a coherent mapping protocol to transfer a generic many-body state in the \{\{1\rangle, \{r\rangle\} basis to the long-lived and non-interacting \{\{0\rangle, \{1\rangle\} basis. To achieve this mapping, immediately following the Rydberg dynamics we apply a Raman \(\pi\) pulse to map \(|1\rangle \rightarrow |0\rangle\), and then a subsequent Rydberg \(\pi\)-pulse to map \(|r\rangle \rightarrow |1\rangle\) [71].

Even for perfect Raman and Rydberg \(\pi\) pulses (on isolated atoms), there are three key sources of infidelity associated with this mapping process:

1. Any population in blockade-violating states (i.e., two adjacent atoms both in \(|r\rangle\)) will be strongly shifted off-resonance for the final Rydberg \(\pi\) pulse. As such, this atomic population will be left in the Rydberg state and lost.

2. Long-range interactions, e.g., from next-nearest-neighbors, will detune the final Rydberg \(\pi\) pulse from resonance and thus reduce pulse fidelity. Since the long-range interactions are not the same for all many-body microstates, this effect cannot be mitigated by a simple shift of the detuning.

3. Dephasing of the state occurs throughout the duration of the Raman \(\pi\) pulse, predominantly from Doppler shifts between the ground states \(|0\rangle, |1\rangle\) and the Rydberg state \(|r\rangle\). Although these random on-site detunings are also present during the many-body dynamics, turning the Rydberg drive \(\Omega\) off allows the system to freely accumulate phase and makes us particularly sensitive to dephasing errors.

We now detail our mitigation of the above error mechanisms. To minimize errors from (1), we perform our many-body dynamics with \(\Omega^2/(2V^2) \approx 0.01\). This minimizes the probability of an atom to violate blockade to be of order 1%. To help minimize errors from (2), we increase the amplitude of the 420-nm laser for the final \(\pi\) pulse by a factor of 2x, such that \((V_{\text{NNN}}/\Omega)^2 \approx 0.005\) (where \(V_{\text{NNN}}\) are the interactions with next-nearest neighbors), reducing pulse errors from long-range interactions to order 1%. Finally, to reduce errors from (3), we perform a fast Raman \(\pi\) pulse and leave only 150 ns between ending the many-body Rydberg dynamics and beginning the Rydberg \(\pi\) pulse. The 150-ns gap is comparably short relative to the \(T_2^* \approx 3-4\) ms of the \(|g\rangle, |r\rangle\) basis, leading to a random phase accumulation of order \(\sim 0.02 \times 2\pi\) rad per particle, but is further compounded by having entangled states of N particles in one copy accumulating a random phase relative to entangled states of N particles in the second copy. We study these various effects numerically in Extended Data Fig. 9c.

Finally, we note that the global Raman beam induces a light-shift-induced phase shift of \(\approx \pi\) on \(|0\rangle, |1\rangle\) relative to \(|r\rangle\) during the Raman \(\pi\) pulse. Similarly, the global 420-nm laser also induces a light-shift-induced phase shift of \(\approx \pi\) between \(|0\rangle\) and \(|1\rangle\) during the Rydberg \(\pi\) pulse. While the measurements we perform here are interferometric (in other words, the singlet state we measure is invariant under global rotations) and thus not affected by these global phase shifts, in future work these phase shifts can be measured and accounted for where relevant.

Measuring entanglement entropy
The second-order Renyi entanglement entropy is given by \(S_2(A) = -\log_2 \text{Tr} [\rho^2_A]\), where \(\text{Tr} [\rho^2_A]\) is the state purity of reduced density matrix \(\rho_A\) on subsystem \(A\). The purity can be measured with two copies by noticing that \(\text{Tr} [\rho^2_A] = \text{Tr} [\hat{S}_A \otimes \rho_A]\) is the expectation value of the many-body SWAP operator \(\hat{S}\) [10, 11]. The many-body SWAP operator is composed of individual SWAP operators \(\hat{s}_i\) on each twin pair, i.e. \(\hat{S} = \Pi_i \hat{s}_i\) (with \(i \in A\)). Measuring this expectation value amounts to probing occurrences of the singlet state |01⟩−⟨00⟩√2 (with eigenvalue -1 under \(\hat{s}_i\)), as all other \(\hat{s}_i\) eigenstates have eigenvalue +1. Occurrences of the singlet state in each twin pair, i.e. the Bell state |Ψ−⟩, is extracted by a Bell measurement circuit (with an additional local \(Z(\pi)\), see next paragraph) which maps |Ψ−⟩ → |00⟩ and can thereafter be measured in the computational basis. As such, after performing the Bell measurement circuit we analyze the resulting bit string outputs and calculate the purity of any subsystem \(A\) by calculating \((\Pi_i \hat{s}_i)|\Psi\rangle\): i.e., we measure purity as the average parity = \((-1)^{\text{observed (00) pairs}}\) within \(A\). In the absence of experimental imperfections, the purity will equal 1 for the whole system, and be less than 1 for subsystems which are entangled with the rest of the system.

A Bell measurement circuit can be decomposed into applying an \(X(\pi/2)\) rotation on one atom of the twin pair, then applying a CZ gate, and then a global \(X(\pi/2)\) rotation. In other measurements we realize a local \(X(\pi/2)\) by doing a global \(X(\pi/4)\) rotation, then local \(Z(\pi)\) rotation, and then global \(X(\pi/4)\). However, we note that for this singlet measurement circuit, the first \(X(\pi/4)\) is redundant as the singlet state is invariant under global rotations, and so for the local \(X(\pi/2)\) we only apply the local \(Z(\pi)\) and then the second global \(X(\pi/4)\). This effectively realizes the \(X(\pi/2)\) on one qubit, up to a \(Z(\pi)\) on the other qubit (not shown in circuit diagram in Fig. 4). Under this simplification, the Bell measurement circuit to map |Ψ−⟩ → |00⟩ can be roughly understood as the reverse of the Bell state preparation circuit in Ref. [5], which is precisely how we calibrate the parameters of the Bell measurement.

Calibrating and benchmarking the interferometry. To validate the interferometry measurement (and check for proper calibration), we benchmark it separately from the many-body dynamics and coherent mapping protocol. We perform this benchmarking by preparing independent qubits in identical, variable single-qubit superpositions (through a global Raman pulse of variable time) and ensuring that the interferometry rarely
results in \( |00\rangle \) for all the variable initial product states (Extended Data Fig. 8a). We find this is an important benchmarking step, because we find that small miscalibrations of the Bell measurement can lead to lower fidelity (i.e. higher entropy) for different initial product states and thereby result in additional spurious signals in an entanglement entropy measurement. We note that this measurement is particularly sensitive to the single-qubit phase immediately before the final \( X(\pi/2) \) pulse (induced by the CZ gate and cancelled by a global \( Z(\theta) \) pulse).

Additional many-body data and details
To benchmark our method of measuring entanglement entropy in a many-body system, in Extended Data Figure 8b we study the entanglement dynamics after initializing two proximal atoms in \( |1\rangle \) and resonantly exciting to the Rydberg state for a variable time \( t \). Under conditions of Rydberg blockade, this excitation results in two-particle Rabi oscillations between \( |11\rangle \) and the entangled state \( |W\rangle = \frac{1}{\sqrt{2}} (|1r\rangle + |r1\rangle) \) (top panel of Extended Data Figure 8b) [3, 13, 71]. The state purity of this two-particle system is measured by performing Bell measurements on atom pairs from two identical copies. Locally, the measured purity of the one-particle subsystem reduces to a value of \( \approx 0.5 \) when the system enters the maximally entangled \( |W\rangle \) state, at which point the reduced density matrix of each individual atom is maximally mixed. In contrast, the purity of the global, two-particle state remains high. The observation that the global state purity is higher than the local subsystem purity is a distinct signature of quantum entanglement [11, 12].

For the data shown in main text Figs. 4c and 4e, we subtract the data by an extensive classical entropy as is done in Ref. [12]. This fixed, time-independent offset is given by the entropy-per-particle, i.e. \( \text{(global entropy at quench time } t = 0 ) \times \text{(subsystem size) } / \text{(global system size)} \). In Extended Data Fig. 9a we show the raw entanglement entropy measurements alongside numerics, to indicate the size of the extensive classical entropy contribution. In plotting, we also delay the theory curves by 10 ns to account for the fact that the Raman \( \pi \) pulse cuts off the final 10 ns of the Rydberg evolution, which is done to keep the coherent mapping gap as short as possible and minimize Doppler dephasing. Further, in Extended Data Fig. 9b we plot the measured global purity and compare it to numerical simulations incorporating experimental errors (Extended Data Fig. 9c).

In Extended Data Fig. 10 we show additional many-body data on the 8-atom chain system, with the same parameters as those used in the main text. We show the measured single-site entropy of each site in the 8-atom chain for the \( |Z_2\rangle \) quench in Extended Data Fig. 10a. Furthermore, in Extended Data Fig. 10b we plot the global Rydberg population, measured in both the \{\( |1\rangle, |r\rangle \)\} and \{\( |0\rangle, |1\rangle \)\} bases.

Data Availability
The data that supports the findings of this study are available from the corresponding author on reasonable request.

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Author contributions
D.B., H.L., G.S., T.T.W., S.E., M.K., and A.K. contributed to building the experimental setup, performed the measurements, and analyzed the data. M.K., N.M., and H.P. performed theoretical analysis. All work was supervised by M.G., V.V., and M.D.L. All authors discussed the results and contributed to the manuscript.

Competing interests
M.G., V.V. and M.D.L. are co-founders and shareholders of QuEra Computing. A.K. is an executive at and shareholder of QuEra Computing. All other authors declare no competing interests.

Additional information
A supplementary movie is available for this paper.

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Extended Data Fig. 1. CZ gate echo, atomic level structure, and typical pulse sequence. a, The two-qubit gates we apply, in addition to applying a controlled-Z operation between the two qubits, also induce a single-qubit phase $Z(\zeta)$ to both qubits, composed of the intrinsic phase of the CZ gate [5] and additional spurious phases from the 420-nm Rydberg laser and pulsing the traps off. Since we apply all gates in parallel by global pulses of the Rydberg laser, if a qubit is not adjacent to another qubit, it does not perform a CZ gate but still acquires the same $Z(\zeta)$ (identical to being adjacent to another qubit in state $|0\rangle$, which is dark to the Rydberg laser). As diagrammed, we cancel the additional, undesired $Z(\zeta)$ by applying a $\pi$ pulse between pairs of CZ gates. This echo procedure removes any need to calibrate the intrinsic phase from the CZ gate, and renders us insensitive to any spurious changes in $Z(\zeta)$ slower than $\sim 200$ $\mu$s. The additional $Y(\pi)$ propagates in a known way through the CZ gates and multiplies certain stabilizers by a -1 sign, which simply redefines the sign of stabilizers and logical qubits. b, Level diagram showing key $^{87}$Rb atomic levels used. Our Rydberg excitation scheme from $|1\rangle$ to $|r\rangle$ is composed of a two-photon transition driven by a 420-nm laser and a 1013-nm laser (see Ref. [25] for description of laser system). A DC magnetic field of $B = 8.5$ G is applied throughout this work. c, A typical pulse sequence for running a quantum circuit.
Extended Data Fig. 2. **Movement characterization and multiple drop-recaptures.**

**a,** Atom retention as a function of average separation speed \(2D/T\) (as is plotted in Fig. 1d of the main text for separating Bell pairs), with subtracted background loss of 0.7%. The inset in Figure 1d of the main text is normalized by \((\text{Atom retention})^2\) (without subtracting background loss). Dark curve is calculated using experimental parameters and Eq. 2, matched to the experimental data by setting \(N_{\text{max}} = 26\) and averaging over a range of \(\omega_0/2\pi\) of \(\pm 15\%\) around an average \(\omega_0/2\pi = 40\) kHz.

**b,** Atom retention as a function of inverse trap frequency \((2\pi/\omega_0)\) after the four moves of the surface code circuit. For calculating the atom loss here we set \(N_{\text{max}} = 33\) and average the trap frequencies over a range of \(\pm 15\%\). We note that these quantitative estimates are sensitive to \(\omega_0\) which we roughly estimate.

**c,** Atom loss as a function of drop time and number of drop loops, with 100 \(\mu\)s wait between each drop. When running quantum circuits we use 500-ns drops for each CZ gate (to avoid anti-trapping of the Rydberg state and light shifts of the transition), for which we observe here that hundreds of drops can be made (corresponding to hundreds of possible CZ gates per atom) before atom loss becomes significant.

**d,** By rescaling the x-axis of the data to \(t_{\text{drop}}\sqrt{N}\), we find the data of the various \(t_{\text{drop}}\) collapse onto a universal curve, suggesting a diffusion model for explaining the atom loss after a certain number of drops. By modeling such a diffusion process analytically we obtain the black curve with a temperature of 10 \(\mu\)K and a trapping radius of 1 \(\mu\)m.
Robust single-qubit control and qubit coherence. a, Robust BB1 single-qubit rotation in comparison to a normal single-qubit rotation, as a function of pulse area error. An arbitrary BB1(θ, φ) rotation on the Bloch sphere of angle θ about axis φ is realized with a sequence of four pulses: \((\pi)_{\phi} + \phi(2\pi)_{\phi} + \phi(\pi)_{\phi} + \phi\), where \(\phi = \cos^{-1}(-\theta/4\pi)\) [59]. Pulse fidelity is measured here for a π pulse, defined such that the fidelity is the probability of successful transfer from \(|0\rangle\rightarrow|1\rangle\), including SPAM correction. b, Preserving hyperfine qubit coherence using dynamical decoupling (XY16 with 128 total π pulses). Qubit coherence is observed on a timescale of seconds, with a fitted coherence time \(T_2 = 1.49(8)\) s. Data is measured with either a +π/2 or −π/2 pulse at the end of the sequence, and these curves are then subtracted to yield the coherence y-axis. c, Hyperfine qubit \(T_1\), measured by the difference of final \(F=2\) populations between measurements starting in \(|F=2,m_F=0\rangle\) and \(|F=1,m_F=0\rangle\). Atom loss without cooling is separately measured (predominantly arising from vacuum loss) and normalized to also measure the intrinsic spin relaxation time \(T_1^*\) in the absence of atom loss. All data here is measured in 830-nm traps.

Effect of axial trap oscillations on echo fidelity of 420-nm Rydberg pulse. a, Noise correlation measurement of the 420-nm Rydberg laser pulse intensity. In the blue-detuned configuration used in this figure only, the 420-nm laser induces an 8 MHz differential light shift on the hyperfine qubit, and consequently a phase accumulation of 32π during a 2-µs pulse (our CZ gates are 400-ns total). Small fluctuations of the 420-nm laser intensity lead to large fluctuations in phase accumulation of the hyperfine qubit, and thus cause significant dephasing. The echo sequence diagrammed here probes the correlation of the accumulated phase between two 420-nm pulses separated by a variable time \(\tau\), and thus informs how far-separated in time the 420-nm pulses can be while still properly echoing out fluctuations in the 420-nm intensity. b, Hyperfine coherence (a proxy for echo fidelity) versus gap time \(\tau\) between the two 420-nm pulses. The echo fidelity decreases initially due to a decorrelation of the 420-nm intensity, but then increases again, showing that the correlation of the 420-nm intensity is non-monotonic. The decaying oscillations are fit to a functional form of \(y = y_0 + A\cos^2(\pi f\tau) \exp\left[-\left(\tau/T\right)^2\right]\). c, The fitted oscillation frequency \(f\) of the correlation / decorrelation of the noise follows a square-root relationship with the trap power, and is consistent with the expected axial trap oscillation frequency. These observations indicate that a significant portion of the correlation / decorrelation of the 420-nm noise arises from the several-µm axial oscillations of the atom in the trap. For this measurement, we intentionally displace the 420-nm beam by several µm in order to place the atom on a slope of the beam, increasing our sensitivity to this phenomenon. For the other experiments in our work, we minimize sensitivity to these effects by operating in the center of a larger (35-micron-waist) 420-nm beam and operating red-detuned of the intermediate-state transition.
Extended Data Fig. 5. **Movement schematics.** Schematics showing the gate-by-gate creation of a the 1D cluster state, b the Steane code, c the surface code, and d the toric code (see also supplementary movie), in a side-by-side comparison. These various graph states are all generated in the same way, and encoding a desired circuit is a matter of positioning the atoms in different initial positions and applying an appropriate AOD waveform. To realize a desired circuit, atom layouts and trajectories are optimized heuristically in the way described in the Methods text. Panel c also shows the definition of surface code stabilizers as ordered in the main text.
Extended Data Fig. 6. Error simulations and tabulated single-qubit and two-qubit error estimates. We compare our measured graph state fidelities to those from a stochastic Monte Carlo simulation of our system for a, the surface code and b, the toric code. We find that the simulated stabilizers agree well with the experimental data for this empirical depolarizing noise model. In addition, for the surface code (toric code) in the experiment we find 35% (20%) of measurements detect no stabilizer errors, compared to 40% (26%) in the simulation. We assume two-qubit errors are described by rates of 0.2% Y error, 0.2% X error, 0.5% Z error, and 0.5% loss per qubit per parallel layer (4 layers for surface code, 5 layers for toric code), corresponding to a 97.2% CZ-gate fidelity. We also add ambient, single-qubit errors at a rate of 0.1% Y error, 0.1% X error, 0.4% Z error, and 0.2% loss per qubit per parallel layer, as well as an initial 1% loss before the circuit begins (empirically factoring in SPAM errors). c, Tabulation of single-qubit (SQ) and two-qubit (TQ) gate errors that are measured, estimated, and extrapolated. Simulated TQ fidelities include the 0.6% scattering error from the 420-nm echo pulse. The estimated TQ fidelities are given for the experiments of the surface code and toric code, but is an underestimate of the TQ fidelities for the cluster state and Steane code measurements where we increase the 1013-nm intensity by $2\times$ and reduce the 420-nm intensity by $2\times$, increasing gate fidelity. The Bell state estimate of CZ gate fidelity is similarly done with $2\times$ higher 1013 intensity, but includes the 420-nm echo pulse, and consequently yields a similar gate fidelity as the surface and toric code estimates.
Extended Data Fig. 7. **Additional graph state data.** **a,** Summary of logical error probabilities for the various error correcting graphs made in this work (all in logical state $|+\rangle_L$), for raw measurements as well as implementing error correction and error detection in postprocessing. Error correction for the Steane code is implemented with the Steane code decoder [34, 72] and is implemented with the minimum-weight-perfect-matching algorithm for the surface and toric codes [37]. For the even-distance toric code, when correction is ambiguous we do not flip the logical qubit, and accordingly the distance $d = 2$ logical qubit does not change under the correction procedure. **b,** Logical $\pi/2$ rotation on the Steane code to prepare logical qubit state $|0\rangle_L$. The Steane code, surface code, and toric code all have transversal single-qubit Clifford operations on the logical qubit [8, 34] (including in-software rotations of the lattice), which is a high-fidelity operation in our system since the transversal rotations are implemented in parallel with our global Raman laser and the physical single-qubit fidelities are high. We show a logical $\pi/2$ rotation here for the Steane code as an example but emphasize that we can readily realize the various basis states along the cardinal axes of the logical Bloch sphere for all of these codes.

Extended Data Fig. 8. **Benchmarking the interferometry measurement.** **a,** To benchmark our gate-based interferometry technique, we prepare variable single-particle pure states (by applying a variable-length resonant Raman pulse) and then reconfigure the system and apply the interferometry circuit on twin pairs. The interferometry circuit converts the antisymmetric singlet state $|\Psi^-\rangle$ to the computational basis state $|00\rangle$, while converting the symmetric triplet states to other computational states. We plot the resulting twin pair output states in the left panel. We rarely observe the $|00\rangle$ state (1.95(2)% of measurements), with a measurement fidelity independent of the initial state. This low probability $P_{00}$ of observing $|00\rangle$ corresponds to a high extracted single-particle purity of $2P_{00} - 1 = 0.961(3)$ (right panel). We find this measurement to be a useful benchmark, as interferometry miscalibrations can result in significant state-dependence of the observed purity that would then compromise the validity of the many-body entanglement entropy measurement. **b,** Benchmarking the entanglement entropy measurement with Bell state arrays. (Top) Microstate populations during two-particle oscillations between $|11\rangle$ and $\frac{1}{\sqrt{2}}(|1r\rangle + |r1\rangle)$ under a Rydberg pulse of variable duration. Faint lines show measurement results in the $\{|1\rangle, |r\rangle\}$ basis, and dark lines show results in the $\{|0\rangle, |1\rangle\}$ basis after the coherent mapping process. (Bottom) Measured local and global purities by analyzing the number parity of observed $|00\rangle$ twin pairs in each measurement. For this two-particle data we use a gap of 230 ns in the coherent mapping sequence as opposed to the 150-ns gap used in the many-body data.
Extended Data Fig. 9. **Raw many-body data and numerical modeling of errors.** a, Raw measured Renyi entropy without subtracting the extensive classical entropy, as a function of subsystem size for quenches from \(|rgrgrgrgr\rangle\) and \(|gggggggg\rangle\). The Renyi entropy of the 4-atom subsystem is the same underlying data as the half-chain entanglement entropy plotted in Fig. 4d of the main text. In the main text, we subtract the data by a fixed offset given by the classical entropy-per-particle, corresponding to the time = 0 offset for each subsystem size. The extensive, classical entropy offset is slightly larger for the \(|rgrgrgrgr\rangle\) quench due to non-unity fidelities both of preparing \(|r\rangle\) and mapping \(|r\rangle\rightarrow|1\rangle\). b, Raw global purity after the \(|gggggggg\rangle\) quench. The global purity is a sensitive proxy for the fidelity of our entire process. We find this 16-body observable, composed of three-level systems, remains > 100× the purity expected for a fully mixed state of 8 qubits (1/2^8) (see inset). For comparison of scale we also plot single-particle purity to the 8th power, to indicate what the global purity would be if the measurement results on each twin were uncorrelated. c, Global purity for the 8-atom quench calculated through numerical modeling of the three-level system \(
{|0\rangle, |1\rangle \equiv |g\rangle, |r\rangle\}
\) with a variety of simulated error sources. We model the experimentally measured purity by calculating the expectation value of the SWAP operator in the \({|0\rangle, |1\rangle}\) basis between two independent chains, taking into account that residual population in \(|r\rangle\) results in atom loss and measurement associated with the +1 eigenvalue of the SWAP operator (as the twin state \(|00\rangle\) can no longer be detected). The top curve includes only errors from population left in \(|r\rangle\) following the coherent mapping step (see methods text). The second-from-top curve includes single-site dephasing (\(T_2^*\)) during the Rydberg dynamics and the coherent mapping gap, modeled by a random on-site detuning which is Gaussian-distributed with zero mean and standard deviation of 100 kHz. The third and fourth curves include multiplication by the experimentally observed raw global purity at quench time \(t = 0\), and then further multiplying empirically by an exponential decay \(\exp[-16 \times t/(70 \mu s)]\) as a simple model for scattering and decay errors with an experimentally estimated rate of roughly 70 \(\mu s\) for each of the 16 atoms between the two chains.
Extended Data Fig. 10. **Local observables and entanglement entropy for quantum many-body scars.**

**a,** Experimentally measured single-site entropy for each site in the 8-atom chain when quenching from the scarred $|Z_2\rangle$ state, including the classical entropy subtraction. Solid curves plot exact, ideal (imperfection-free) numerics of $H_{Ryd}$ (Eq. 3); excellent agreement between data and numerics is found for every atom in the chain. **b,** (Top) Same data as Fig. 4f of the main text, showing single-site entropy of the middle two atoms in the chain, for two different initial states [73]. (Bottom) Measurements of the many-body state in the $Z$-basis with the interferometry circuit turned off. Characteristic of the scars from the $|Z_2\rangle = |rgrgrgrg\rangle$ state, the Rydberg excitation probability on the sublattices exhibits periodic oscillations [70]. In the bottom row, the dark data points are measured in the {$|1\rangle, |r\rangle$} basis, and the faint data points are measured in the {$|0\rangle, |1\rangle$} basis after the coherent mapping sequence. Measurements in both bases agree well with exact numerics (solid lines), which we emphasize has no free fit parameters and does not account for any experimental imperfections, such as detection infidelity. Moreover, the data indicate the high fidelity of preparation into the $|Z_2\rangle$ state by use of local Rydberg $\pi$ pulses. In plotting, we delay the theory curves and the {$|1\rangle, |r\rangle$} basis measurement by 10 ns to account for the fact that the Raman $\pi$ pulse we apply cuts off the final 10 ns of the Rydberg evolution, when measuring in the {$|0\rangle, |1\rangle$} basis. **c,** Numerical simulations of the single-site Renyi entropy on two adjacent sites in the idealized ‘PXP’ model of perfect nearest-neighbor blockade [14]. The system size is 24 atoms with periodic boundary conditions, showing the same out-of-phase oscillations in the entanglement entropy of the two sublattices. **d,** Diagram of the constrained Hilbert space of the system [14]. The early-time, out-of-phase entropy oscillations [73] of the scars can be understood in this constrained Hilbert space picture, where the scar dynamics are known to take the state from the left end ($|rgrgrgrg\rangle$) to the right end ($|grgrgrgr\rangle$) (dark circles represent $|r\rangle$ and white circles represent $|g\rangle$) [14]. Near these crystalline ends of this constrained Hilbert space, the Rydberg atoms can fluctuate (high entropy), but the ground state atoms are pinned (low entropy). Our analysis shows that entanglement between atoms on the same sublattice contributes to the eventual degradation of the revival fidelity of the $|Z_2\rangle$ state.