Homogeneous Rolling Tachyons 

in 

Boundary String Field Theory 

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Abstract 

We study decay of a flat unstable $D_p$-brane in the context of boundary string field theory action. Three types of homogeneous rolling tachyons are obtained without and with Born-Infeld type electromagnetic field.
1 Introduction

Rolling tachyons have been studied in the various contexts including boundary conformal field theory (BCFT), Dirac-Born-Infeld (DBI) type effective field theory (EFT), noncommutative field theory (NCFT), boundary string field theory (BSFT or background-independent string field theory), cubic string field theory (SFT), and matrix models. Various meaningful results were achieved in both homogeneous and inhomogeneous rolling tachyon configurations and have been contributed to understand real-time evolution of an unstable D-brane even in terms of closed string degrees through the bridge of duality.

A cornerstone knowledge about the homogeneous rolling tachyons is on their species, i.e., they are classified by three. In terms of BCFT, they are hyperbolic cosine and sine type rolling tachyon configurations called as full S-branes, and exponential type rolling tachyon called as half S-brane. All of them share the same late-time behavior of vanishing pressure so that classification is made by their different initial-time behaviors. This classification was also confirmed in DBI type EFT and NCFT even in a form of exact solutions. Though the late-time behavior of the rolling tachyon represented by vanishing pressure is reproduced in BSFT, the simple observation on the three species of rolling tachyons was not made in BSFT which is appropriate to describe the open string tachyon physics, a representative off-shell dynamics in string theory. This may be related with complicated form of the tachyon kinetic term in BSFT.

In this paper, we will find all the three types of homogeneous rolling tachyons as classical solutions of BSFT equations of motion. Comparison with other languages, particularly with BCFT and EFT, shows complete agreement in spectrum of the homogeneous rolling tachyons. The rolling tachyons with DBI type electromagnetic coupling are also taken into account. We show that the gauge field equation forces its homogeneous field strength to be constant, and thus species of the homogeneous rolling tachyons are the same as those without electromagnetic coupling.

In section 2 we write down equations of motion from the BSFT tachyon action, describing dynamics of an unstable Dp-brane. Section 3 devotes to the detailed analysis on homogeneous rolling tachyon solutions for the case of the pure tachyon field, and reproduce all the three types of homogeneous rolling tachyon solutions. In section 4, we consider coupling of the DBI electromagnetic field which stands for living of fundamental strings on the unstable Dp-brane. It is proved that every component of the homogeneous field strength is constant for arbitrary p, and then inclusion of electromagnetism does not change the spectrum composed of the three types of rolling tachyons. We conclude in section 5 with brief discussions on further studies.
2 Tachyon Effective Action from BSFT

We begin our discussion by recapitulating briefly derivation of an effective action of a real tachyon field [13, 14] and abelian gauge field [15, 16] in the context of BSFT [11], and then read equations of motion. In superstring theory, worldsheet partition function is believed to be identified with off-shell BSFT action, $S = Z$ [17].

For an unstable $Dp$-brane, the partition function is given by

$$Z = \int [DX] [D\psi][D\bar{\psi}] \exp\left[-(I_{\text{bulk}} + I_{\text{bdry}})\right], \quad (2.1)$$

where $I_{\text{bulk}}$ is the bulk action and $I_{\text{bdry}}$ is the boundary action of the form;

$$I_{\text{bdry}} = \int_{\partial\Sigma} d\tau \left[\frac{1}{4}T^2 - \frac{1}{2} \psi^\mu \partial_\mu T \partial_\tau^{-1}(\psi^\nu \partial_\nu T) + \frac{i}{2} F_{\mu\nu} \psi^\mu \psi^\nu - \frac{i}{2} A_\mu \partial_\tau^{-1}(A_\mu dX^\mu)\right]. \quad (2.2)$$

We consider a relevant linear perturbation for the tachyon $T(X) = u^\mu X^\mu$ and constant electromagnetic field $F_{\mu\nu}$ with symmetric gauge $A_\mu = -F_{\mu\nu} X^\nu/2$ on the boundary.

Performing functional integration of the fields on the worldsheet and using zeta function regularization, we reach an action of the tachyon and the DBI electromagnetic field

$$S(T, A_\mu) = -T_p \int d^{p+1}x V(T) \sqrt{-\text{det}(\eta_{\mu\nu} + F_{\mu\nu})} F(y), \quad (2.3)$$

where the overall normalization was chosen by tension $T_p$ of the unstable $Dp$-brane, and

$$V(T) = e^{-\frac{1}{4}T^2}, \quad (2.4)$$

$$F(y) = \frac{4^y \Gamma(2y)^2}{2\Gamma(2y)}, \quad y = G^{\mu\nu} \partial_\mu T \partial_\nu T \quad (2.5)$$

with open string metric $G^{\mu\nu} = (\frac{1}{\eta+F})^{\mu\nu}_A$.

From the action (2.3) we read the equations of motion for both the tachyon and the gauge field. Then the obtained equations can be expressed in terms of the open string metric $G^{\mu\nu}$ and noncommutativity parameter $\theta^{\mu\nu} = (\frac{1}{\eta+F})^{\mu\nu}_A$ as

$$\partial_\mu \left[V \sqrt{-G} F^{\mu\nu} \partial_\nu T\right] = \frac{1}{2} \sqrt{-G} F dV \frac{dT}{dT}, \quad (2.6)$$

$$\partial_\mu \Pi^{\mu\nu} = 0, \quad (2.7)$$

where

$$\Pi^{\mu\nu} \equiv \frac{\delta S}{\delta \partial_\mu A_\nu} = \tilde{T}_p V \sqrt{-G} \left[\theta^{\mu\nu} F + 2 \left(\theta^{\rho\mu} G^{\sigma\nu} - \theta^{\rho\nu} G^{\sigma\mu}\right) \partial_{\rho} T \partial_{\sigma} T \right]. \quad (2.8)$$

Note that, in the previous expressions, the DBI Lagrangian with string coupling $g_s$ and slowly varying $F_{\mu\nu}$ in [23] is equivalent to NCFT Lagrangian with open string coupling $G_s$. 

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and nonconstant piece of noncommutative field strength tensor \( \hat{F}_{\mu\nu} \), up to total derivative and \( O(\partial F) \) term \[ \text{[18]} \]

\[-\frac{1}{g_s(2\pi)^{\frac{d+1}{2}}} \sqrt{-\det(\eta_{\mu\nu} + F_{\mu\nu})} = -\frac{1}{G_s(2\pi)^{\frac{d+1}{2}}} \sqrt{-\det(G_{\mu\nu} + \hat{F}_{\mu\nu})} \] (2.9)

with \( \det(G_{\mu\nu} + \hat{F}_{\mu\nu}) \equiv G, T_p \equiv 1/g_s(2\pi)^{\frac{d+1}{2}} \), and \( \tilde{T}_p \equiv 1/G_s(2\pi)^{\frac{d+1}{2}} \).

Instead of the tachyon equation (2.6), it is economic to examine conservation of energy-momentum tensor

\[ \partial_\mu T^{\mu\nu} = 0, \] (2.10)

where the energy-momentum tensor is also read from the action (2.9)

\[ T^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}} \bigg|_{g_{\mu\nu} = \eta_{\mu\nu}} = -\tilde{T}_p V(T) \sqrt{-G} \left[ F(y)G^{\mu\nu} - 2F'(y)(G^{\rho\mu}G^{\nu\sigma} + \theta^{\rho\mu}\theta^{\nu\sigma})\partial_\rho T\partial_\sigma T \right]. \] (2.11)

3 Homogeneous Rolling Tachyons

In this section let us consider homogeneous configuration of the tachyon field

\[ T = T(t), \] (3.1)

without DBI electromagnetism, \( F_{\mu\nu} = 0 \). Then the equation of gauge field (2.7) becomes trivially satisfied so that the only nontrivial equation is the tachyon equation (2.6) or equivalently the conservation of energy-momentum tensor (2.10). Here we use the simpler conservation of energy-momentum tensor, which reduces to \( dT^{00}/dt = \dot{T}^{00} = 0 \) and it becomes

\[ H \equiv T^{00} = T_p V(T) \left[ F(y) - 2yF'(y) \right], \] (3.2)

where \( y = -\dot{T}^2 \leq 0 \) from (2.5).

Reshuffling the terms of (3.2), we obtain

\[ \mathcal{E} = K(y) + U(T), \] (3.3)

where \( \mathcal{E} = 1 \), and

\[ K(y) = 1 - \frac{1}{[F - 2yF']^2}, \] (3.4)

\[ U(T) = \left( \frac{T_p V}{H} \right)^2 = \left( \frac{T_p}{H} \right)^2 e^{-T^2/2}. \] (3.5)

Though \( F(y) \) becomes divergent at each negative integer \( y \), the function \( K(y) \) is bounded and behaves smoothly as shown in Fig. [1]. Since we will use the range of \( y \) from \(-1\) to
K(y) is monotonically decreasing from max(K(−1)) = 1 to min(K(0)) = 0 in the y-range of our interest. U(T) is a runaway function having its maximum value \((T_p/\mathcal{H})^2\) at \(T = 0\) and its minimum value “zero” at \(T = \pm\infty\) as shown in Fig. 1. As given in Fig. 1

Figure 1: The graphs of \(K(y)\) (left) and \(U(T)\) (right). For \(U(T)\), three representative cases of \((T_p/\mathcal{H})^2 = 2\) (dashed curve), 1 (solid curve), 1/2 (dotted-dashed curve) from top to bottom are shown. \(\mathcal{E} = 1\) is given by the straight dotted line.

homogeneous rolling tachyon solutions in BSFT are classified by three cases. From now on we investigate detailed properties of three types of rolling tachyons by examining the equation (3.3). Since the energy density \(T^{00}\) is a constant of motion and every momentum density \(T^{0i}\) and stress \(T^{ij}\) \((i \neq j)\) vanishes, physical property of each homogeneous rolling tachyon solution is expressed by pressure component

\[
P \equiv T^{ii} = -T_p V(T) \mathcal{F}(-\mathcal{T}^2). \tag{3.6}
\]

To simplify our discussion without distorting physics, we use time translation invariance so that the value of time \(t\) has meaning up to an overall constant like \(t - t_0\).

### 3.1 Hyperbolic cosine type rolling tachyon \((T_p > \mathcal{H})\)

When \(T_p > \mathcal{H}\), it corresponds to the case of dashed curve in Fig. 1. The allowed range of the tachyon field is either \(T \geq T_{\text{min}} = 2\sqrt{\ln(T_p/\mathcal{H})}\) or \(T \leq -T_{\text{min}}\). Since the rolling tachyon of negative \(T\) is signature flipped copy of positive rolling tachyon, it is enough to describe the positive case in what follows and the negative flipped copy will be given only in figures without detailed explanation. Possible homogeneous rolling tachyon is to start from \(T(-\infty) = \infty\), decreases monotonically to \(T(0) = T_{\text{min}}\) with \(\mathcal{T}(0) = 0\), and then return to \(T(\infty) = \infty\). No analytic solution is obtained but numerical solution is given in

\[
\begin{align*}
\begin{array}{c}
\mathcal{T}_p > \mathcal{H} \\
\mathcal{T}_p = \mathcal{H} \\
\mathcal{T}_p < \mathcal{H}
\end{array}
\end{align*}
\]
Fig. 2. Shape of this rolling tachyon corresponds to the full S-brane of hyperbolic cosine function.

Figure 2: The graphs of hyperbolic cosine function type rolling tachyon (a full S-brane) for \( T_p > \mathcal{H} \) (\( T_p/\mathcal{H} = 1.284 \)): tachyon field \( T(t) \) (left) and pressure \( P(t)/\mathcal{H} \) (right). The dashed line stands for the pressure in BCFT.

Near the minimum value of the tachyon \( T \approx T_{\text{min}} = 2\sqrt{\ln(T_p/\mathcal{H})} \), \( \dot{T} \) flips its signature and has relatively small magnitude. and the kinetic term (3.4) is approximated as

\[
K(-\dot{T}^2) \approx (4 \log 2) \dot{T}^2 + \mathcal{O}(\dot{T}^4),
\]

(3.8)

Therefore, as the tachyon increases near \( t = 0 \),

\[
T(t) \approx T_{\text{min}} \left( 1 + \frac{t^2}{16 \ln 2} + \ldots \right),
\]

(3.9)

absolute value of the pressure (3.7) decreases monotonically

\[
\frac{P}{\mathcal{H}} \approx -1 + \frac{T_{\text{min}}^2}{16 \ln 2} t^2 + \ldots.
\]

(3.10)

The leading unity comes from the “cosmological constant” at the turning point and the second positive term seems natural to reach tachyon matter with vanishing pressure.

For sufficiently large \( T \to \infty \), the tachyon potential \( V(T) \) and \( U(T) \) decrease exponentially zero, and thereby the equation (3.3) with the help of the shape of \( K(y) \) in Fig. 1 dictates a universal behavior. Time derivative of the tachyon arrives rapidly at unity

\[
\dot{T}(t) \to 1 - \frac{1}{2} \sqrt{\frac{T_p}{\mathcal{H}}} e^{-\frac{t^2}{8}},
\]

(3.11)

and, though \( \mathcal{F}(y) \) diverges in this limit, character of the pressure (3.7) vanishes, dictated by the vanishing tachyon potential;

\[
P \sim \frac{\sqrt{T_p \mathcal{H}}}{2} e^{-\frac{t^2}{8}}.
\]

(3.12)
The late-time behavior is not a character of the hyperbolic cosine type rolling tachyon but a universal character of all the homogeneous rolling tachyons in BSFT action \[10\], shown by the solid lines in Figs. 2–4. Thus the late-time behavior is different from that of pressure in BCFT, which increases monotonically from \[P(t = 0) = -\mathcal{H}/(2 - \mathcal{H}/T_p) < 0\] to zero \[1, 4\] as shown by the dashed lines in Figs. 2–4. This late-time behavior of the rolling tachyon solutions, represented by vanishing pressure, is universal irrespective of the detailed models and types of S-branes. This so-called tachyon matter \[6\], however it actually reflects the matter behavior of vacuum of massive closed string degrees \[19\].

Since the pressure \(P\) flips its signature as in (3.10) and (3.12), there should exist a point of vanishing pressure \(P = 0\) at a time \(t = t_*\). From the expression of pressure (3.7), we read \(\mathcal{F}(y = -T_*^2) = 0\) and thereby slope of the tachyon is fixed by \(T_* = \pm \sqrt{1/2}\). Since \(\mathcal{F}'(-1/2) = \pi\), value of the tachyon field at the vanishing pressure is \(T_* \equiv T(t_*) = \pm 2\sqrt{\ln(T_p\pi/\mathcal{H})}\) from the equation (3.2). The pressure near \(t = t_*\) is given by

\[
\frac{P}{\mathcal{H}} \approx \frac{1}{1 + 4 \ln 2} \sqrt{\frac{1}{2} \ln \left(\frac{\pi T_p}{\mathcal{H}}\right)} (t - t_*). \tag{3.13}
\]

### 3.2 Exponential type rolling tachyon \((T_p = \mathcal{H})\)

For the critical value of Hamiltonian density \(\mathcal{H} = T_p\), it corresponds to the solid curve in Fig. 1. Since \(T_{min} \to 0\), the rolling tachyon configuration is a monotonically-increasing function from \(T(t = -\infty) = 0\) to \(T(t = \infty) = \infty\) and a solution through numerical analysis is given in Fig. 3. Shape of the obtained rolling tachyon is identified by exponential type rolling tachyon in BCFT so-called half S-brane.

![Graphs of exponential function type rolling tachyon (half S-brane) for \(T_p = \mathcal{H}\): tachyon field \(T(t)\) (left) and pressure \(P(t)/\mathcal{H}\) (right). The dashed line stands for the pressure in BCFT.](image)

Even at initial stage near top of the tachyon potential at \(T = 0\) for \(t \to -\infty\), the tachyon field is already increasing exponentially like the exact half S-brane configuration.
in BCFT
\[ T(t) \approx \lambda e^{\frac{t}{\sqrt{2 \log 2}}} + \ldots, \quad (3.14) \]
where \( \lambda \) should be fixed by the boundary condition at \( t = \infty \). Insertion of it into the pressure \( P \) \((3.7)\) provides leading negative contribution of the cosmological constant at the top of tachyon potential and subleading tachyon matter contribution
\[ P \approx -\mathcal{T}_p \left( 1 - \frac{\lambda^2}{2} e^{\frac{t}{\sqrt{2 \log 2}}} + \ldots \right). \quad (3.15) \]

At late time, leading behavior of this exponential type rolling tachyon shares the same late-time behavior \((3.11)-(3.12)\) with the hyperbolic cosine type rolling tachyon in the previous subsection \((3.11)-(3.12)\) as expected. The pressure \( P(t) \) is also shown in Fig. 3.

### 3.3 Hyperbolic sine type rolling tachyon (\( \mathcal{T}_p < \mathcal{H} \))

When \( \mathcal{H} > \mathcal{T}_p \), \( \dot{T}^2 \) and the kinetic term \( K(-\dot{T}^2) \) are always positive so that the rolling tachyon profile connects smoothly and monotonically one vacuum at \( T(t = -\infty) = \mp \infty \) and the other vacuum at \( T(t = \infty) = \pm \infty \), respectively, as shown by the numerical solution in Fig. 4. It is identified as hyperbolic sine type rolling tachyon corresponding to the other full S-brane.

![Figure 4: The graphs of hyperbolic sine function type rolling tachyon (full S-brane) for \( \mathcal{T}_p < \mathcal{H} \) (\( \mathcal{T}_p/\mathcal{H} = 0.944 \)): tachyon field \( T(t) \) (left) and pressure \( \frac{P(t)}{\mathcal{H}} \) (right). The dashed line stands for the pressure in BCFT.](image)

Its late-time behavior is also universal and its initial-time behavior at \( t = -\infty \) is the reflected image of the late-time behavior. When the tachyon field rolls over the top of tachyon potential, expansion of \( T(t) \) near \( T(t = 0) = 0 \) and \( \dot{T}(t = 0) = \dot{T}_0 \) provides
\[ T(t) \approx \dot{T}_0 \left( t + \frac{1}{24C_0} t^3 \right) + \mathcal{O}(t^5), \quad (3.16) \]
which is consistent with hyperbolic sine function near zero, and

$$C_0 \equiv \frac{T_p}{\mathcal{H}} \left[ \mathcal{F}'(-\dot{T}_0^2) - 2\dot{T}_0^2 \mathcal{F}''(-\dot{T}_0^2) \right] > 0.$$  \hspace{1cm} (3.17)

Then the pressure $P$ becomes

$$P \approx -T_p \left[ \mathcal{F}(-\dot{T}_0^2) - \frac{\dot{T}_0^2}{4} \left( \mathcal{F}(-\dot{T}_0^2) - \frac{1}{C_0} \mathcal{F}'(-\dot{T}_0^2) \right) t^2 \right] + \mathcal{O}(t^4).$$  \hspace{1cm} (3.18)

Among the homogeneous rolling tachyon configurations obtained in the previous subsections, two of them in 3.1 and 3.3 are identified with full S-branes $[3]$ and one in 3.2 is half S-brane $[5]$. These completely coincide with the results in BCFT $[1]$ and DBI type EFT $[7]$. If all of these theories are a variety of languages describing superstring theory with an unstable D$p$-brane, there should exist a direct field redefinition among the tachyon fields. In the subsequent section, we analyze the cases with electromagnetic field turned on.

4 Homogeneous Rolling Tachyons with Electromagnetic Field

When we derive the BSFT action (2.3), we assumed constant electromagnetic field on the worldsheet. On the unstable D$p$-brane we take into account homogeneous configurations of the electromagnetic field, consistent with the homogeneous tachyon field (3.1),

$$F_{\mu\nu} = F_{\mu\nu}(t).$$  \hspace{1cm} (4.1)

In the subsequent subsections we analyze homogeneous rolling tachyon solutions by examining the classical equations by dividing the $p = 1$ case and $p \geq 2$ cases, since Bianchi identity

$$\partial_{\mu} F_{\nu\rho} + \partial_{\nu} F_{\rho\mu} + \partial_{\rho} F_{\mu\nu} = 0,$$  \hspace{1cm} (4.2)

is trivial for $p = 1$. The main results will be constancy of every component of the electromagnetic field strength tensor, and then spectrum of the three species of homogeneous rolling tachyon configurations will also be kept to be the same as that of the pure tachyon case, irrespective of the value of constant $F_{\mu\nu}$.

4.1 $p = 1$

On an unstable D1-brane there exists only one electric component of the field strength tensor $E(t) \equiv -F_{01}(t)$ and then we have $-\det(\eta_{\mu\nu} + F_{\mu\nu}) = 1 - E^2$. Because of the
homogeneity of the fields, (3.11) and (4.1), momentum density $T^{01}$ vanishes and then spatial component of the energy-momentum conservation (2.10) is automatically satisfied. The remaining time component of it leads to constancy of the energy density (Hamiltonian density), $T_{00} = \mathcal{H} = \text{constant}$. Since $\Pi^\mu_\nu$ (2.8) is antisymmetric due to antisymmetricity of $F^\mu_\nu$, only the conjugate momentum $\Pi = \Pi^{01}$ survives. Thus, time component of the gauge field equation (2.7) is trivially satisfied and its spatial component results in constant conjugate momentum, $\Pi = \text{constant}$. Specific expressions of the Hamiltonian $\mathcal{H} = T_{00}$ (2.11) and the conjugate momentum $\Pi = \Pi^{01}$ tell us that ratio of two quantity is negative electric field

$$E = \frac{\mathcal{H}}{\Pi} = \frac{\sqrt{1 - E^2} \mathcal{F} - 2yF'}{2yF'},$$

which forces constant electric field.

When $1 - E^2 > 0$, rescaling of the time as $\tilde{t} = t\sqrt{1 - E^2}$ and the Hamiltonian density $\tilde{\mathcal{H}} = \sqrt{1 - E^2} \mathcal{H}$ lets the equation (4.3) the same equation (3.2) of pure tachyon case. Since the rescaling preserves the boundary conditions, this proves that spectrum of the homogeneous rolling tachyon solutions with homogeneous electric field coincides exactly with that without electromagnetic field in the previous section 3 through the rescaling of time $\tilde{t}$ and Hamiltonian density $\tilde{\mathcal{H}}$.

Vanishing electric field limit $E^2 = 0$ corresponds obviously to pure tachyon limit. For critical electric field $E^2 = 1$, the rescaling becomes singular and then we need careful analysis. If we rewrite the equation (4.3), then we have

$$\mathcal{E}_1 = K_1(y) + U_1(T),$$

where

$$\mathcal{E}_1 = 1, \quad K_1(y) = 1 - \frac{1}{[\mathcal{F}(y) - 2yF'(y)]^2}, \quad U_1 = \frac{T^2_pV^2}{(1 - E^2)\mathcal{H}^2}.$$  

In the critical limit $E^2 \to 1$, $U_1$ becomes divergent unless $T \to \pm \infty$ with $V(T = \pm \infty) = 0$. From the variable $y$ in (4.3), $y \to -\infty$ unless $T \to 0$. Thus there exists a trivial vacuum solution of $T = \pm \infty$ and $\dot{T} = 0$ in the critical limit of the electric field. If $\dot{T} \neq 0$, $y \to -\infty$ and then $\lim_{y \to -\infty} K_1(y) \to 1$. The equation (4.4) forces $V(T) = 0$ and subsequently the Hamiltonian in (4.3) can be finite. This nontrivial rolling vacuum solution attained due to the vacuum at infinity is not overruled. Correspondingly, for both the trivial vacuum solution and the rolling vacuum solution, the pressure becomes constant as

$$\lim_{E^2 \to 1} P = \lim_{E^2 \to 1} -\frac{T^2_pV}{\sqrt{1 - E^2}} [\mathcal{F} - 2E^2yF'] = -\mathcal{H}.  $$
4.2 \( p \geq 2 \)

The homogeneous electric field \( E \) on the unstable D1-brane is proven to be constant due to classical equations of motion, and a rescaling of the time variable led to the same spectrum of homogeneous rolling tachyon solutions, i.e., they are hyperbolic cosine and sine type rolling tachyons (two full S-branes), and exponential type rolling tachyon (a half S-brane). For higher-dimensional Dp-brane, number of components of the field strength tensor increases as \( p(p+1)/2 \) and the number of equations of motion with Bianchi identity also increase. In this subsection, through a careful analysis, all the components of homogeneous electromagnetic fields are proven to be constant and, through an appropriate rescaling of time variable, the single tachyon equation supports exactly the same three homogeneous rolling tachyon solutions.

We begin the analysis with homogeneity of the fields (3.1) and (4.1) for the cases of \( p \geq 2 \). For the homogeneous fields, the Bianchi identity (4.2) requires every component of the magnetic field \( F_{ij} \) to be constant, i.e., \( p(p-1)/2 \) components among \( p(p+1)/2 \) components are constants. In addition, the conservation of the energy-momentum (2.10) becomes simple, \( \dot{T}^{0\nu} = 0 \), which lets \( T^{0\nu} \) constants

\[
T^{0\nu} = \frac{T_p V}{\sqrt{\beta}} C_S^{0\nu} \left[ F(y) - 2yF'(y) \right] = \text{constant},
\]

where \( y \) is, from (2.5),

\[
y = -\alpha \dot{T}^2 / \beta.
\]

Throughout this subsection we use notations that \( \beta = -\det(\eta_{\mu\nu} + F_{\mu\nu}) \), \( C^{\mu\nu} \) denotes cofactor of the matrix \( (\eta + F)_{\mu\nu} \) with its symmetric part \( C_S^{\mu\nu} \) and antisymmetric part \( C_A^{\mu\nu} \), and \( \alpha = C^{00} \). Since \( C_S^{00} \) contains only the magnetic field \( F_{ij} \) which is constant, it is also a constant. Combining this with (4.7), we obtain

\[
\frac{T_p V}{\sqrt{\beta}} \left[ F(y) - 2yF'(y) \right] = \frac{T^{00}}{C^{00}} = \frac{\mathcal{H}}{\alpha},
\]

where the Hamiltonian density \( \mathcal{H} \) is a constant. Similarly, the equation of the gauge field (2.7) makes every charge density of the fundamental strings constant, i.e., \( \dot{\Pi}^{0\nu} = 0 \) results in constant \( \Pi^{0i} \)

\[
\Pi^{0i} = \frac{T_p V}{\sqrt{\beta}} C_A^{0i} \left[ F(y) - 2yF'(y) \right] = \frac{\mathcal{H}}{\alpha} C_A^{0i}
\]

which results in constant \( C_A^{0i} \). Therefore, as far as the determinant \( \beta \) is nonvanishing, the \( p \) independent relations, \( \left( \frac{1}{\eta + F} \right)^{0i}_A = C_A^{0i} / \beta \), require every electric field \( E^i = -F_{0i} \) is also constant. Summarized the results of \( p = 1 \) case in the subsection 4.1 and of \( p \geq 2 \) cases in the subsection 4.2, we now show that all the field strength components assumed to be homogeneous are constants on every unstable Dp-brane.
Now that the only remaining first-order equation (4.9) is equivalent to the tachyon equation (2.6), we rewrite it in a convenient form like (3.3)

\[ \mathcal{E}_p = K_p(y) + U_p(T), \]

(4.11)

where

\[ \mathcal{E}_p = 1, \quad K_p(y) = 1 - \frac{1}{[\mathcal{F}(y) - 2y\mathcal{F}'(y)]^2}, \quad U_p = \frac{\alpha^2 T_p^2 V^2}{\beta \mathcal{H}^2}. \]  

(4.12)

Since \( \alpha = C^{00} \) is positive and \( \beta = -\det(\eta_{\mu\nu} + F_{\mu\nu}) \) is nonnegative, the equation (4.11) supports the same three types of homogeneous rolling tachyon solutions. To be specific, they are hyperbolic cosine type rolling tachyon solution for \( \sqrt{\beta \mathcal{H}/\alpha} < T_p \), exponential type rolling tachyon solution for \( \sqrt{\beta \mathcal{H}/\alpha} = T_p \), and hyperbolic sine type rolling tachyon solution for \( \sqrt{\beta \mathcal{H}/\alpha} > T_p \). The obtained spectrum of rolling tachyons in BSFT action coincides with that in BCFT \[20\], DBI type EFT \[20, 21, 8\], and NCFT \[20, 9\]. Even functional form of the tachyon field for the homogeneous rolling tachyons is the same up to the rescaling of the time variable so that we omit detailed analysis of their physical properties.

5 Conclusion

We studied homogeneous rolling tachyons in the framework of the BSFT action and classified the obtained solutions into three cases, i.e., they are hyperbolic cosine, exponential, and hyperbolic sine type configurations. Analytic solutions were not obtained in closed form, but numerical solutions coincide with our analysis (refer to the Figs. 2-4). The late-time shapes of the pressure commonly approach to zero for sufficiently late time.

We also considered homogeneous rolling tachyon solutions in the presence of homogeneous electromagnetic field. Bianchi identity and the equation of motion for the gauge field require every component of the electromagnetic field to be constant. Then the nonzero electromagnetic field turns out to lead to the rescaling of the time variable. Therefore all of the homogeneous rolling solutions to the BSFT equations of motion are classified into three cases irrespective of existence of the DBI electromagnetic field.

Now the discussions for further studies are in order. A noteworthy difference between BCFT results and the obtained BSFT results is on behavior of the pressure. There exists a finite time where the pressure vanishes and sign of it flips in BSFT, however the pressure in BCFT keeps its negative signature for all the time. In relation with this discrepancy, it will be intriguing to consider the rolling tachyons by taking into account the correction from higher derivative terms.
Since all the perturbative open string modes on an unstable D-brane disappear in the late-time of its decay \cite{22}, production of nonperturbative soliton solutions through inhomogeneous rolling tachyons \cite{23} is worth tackling.

### Acknowledgements

We would like to thank Chanju Kim, O-Kab Kwon, and Ho-Ung Yee for helpful discussions. This work is the result of research activities (Astrophysical Research Center for the Structure and Evolution of the Cosmos (ARCSEC)) supported by Korea Science & Engineering Foundation and was supported by Samsung Research Fund, Sungkyunkwan University, 2005. The work of A.I. is also supported in part by the Postdoctoral Research Program of Sungkyunkwan University (2005).

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