Structural Damping by Layers of Fibrous Media Applied to a Periodically-Constrained Vibrating Panel

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Abstract. It has recently been demonstrated that layers of fibrous, “acoustical” material can effectively damp structural vibration in the sub-critical frequency range. In that frequency range, the acoustical near-field of a panel consists of oscillatory flow oriented primarily parallel with the panel surface. When a fibrous layer occupies that region, energy is dissipated by the viscous interaction of the near-field and the fibrous medium, and the result is a damping of the panel motion. Previously, the damping effect has been demonstrated to occur for line-driven, infinite panels and panels with isolated constraints. In this article, the focus is instead on periodically-constrained panels driven into motion by a convective pressure distribution. The constraints are allowed to have translational and rotational inertias and stiffnesses. This arrangement is intended to represent a very simple model of an aircraft fuselage structure. By considering the power flows in this system, it is possible to compute an equivalent loss factor, and then to identify the fibrous layer macroscopic parameters that result in optimal damping at a given mass per unit area. Finally, given that information, it is possible to identify the microstructural details, e.g., fiber size, that would be required to achieve that damping in practice.

1. Introduction
Subject to considerations of cost-effectiveness and fuel-economy, automotive and aerospace industries are actively seeking acoustical solutions that reduce vehicle or aircraft interior noise without the addition of substantial weight. The main objective of this study was to examine the vibrational damping performance of the types of fibrous treatments that are conventionally used for sound absorption, so that a layer of properly designed fibers could serve as both a structural damper and a sound absorber at the same time, thus reducing the number of acoustical elements required in a sound package for a vehicle or an aircraft. To be more specific, when a layer of multi-functional fibrous material is placed adjacent to a substrate (e.g., vehicle roof/door panel, aircraft fuselage panel, etc.) as shown in Figure 1, it will attenuate both airborne noise and panel structural vibration, with the result that the cost and total weight of the sound package can be reduced, if, for example, the damping performance of the fibrous layer renders the addition of a separate damping treatment unnecessary.
Previously, an analytical model was developed for evaluating a fibrous layer’s near-field damping (NFD) performance when resting on an infinitely-extended panel driven by a harmonic line force at the origin. That evaluation was made by predicting the power dissipation within the layer given the layer’s bulk properties including its airflow resistivity [1]. Note that the damping results from the viscous interaction between the non-propagating near-field created by the panel vibration and the fibrous medium. The NFD model was also used for a parametric study based on the relation between the airflow resistivity and the microstructure of the fibers [2]. The understanding of this relation made it possible to optimize the fiber size to achieve the largest possible damping for a certain frequency and panel of interest, based on which, design concepts were summarized for different kinds of fibrous media (e.g., polymeric fiber and glass fiber) used in conjunction with different panel structures (e.g., an unconstrained infinite panel, or a partially-constrained panel) [3]–[5].

Inspired by Nadeau et al.’s study [6] on the fibrous material used for aircraft trim and floor structures, the NFD model has been extended in the current study to focus on analyzing and designing fibrous damping treatments for a fuselage-like structure: i.e., a panel with frames as shown in Figure 2 [6]. To model the fuselage-like structure in the NFD model, identical discontinuities (also referred to as constraints in this article) with finite translational and rotational inertias and stiffnesses were uniformly distributed along the panel to create a periodically-constrained structure, as shown in Figure 3. Note that previous studies [7]–[10] have relied on modeling a periodic structure by using various different methods, such as the
principle of virtual work, for example, while here the periodic structure is modeled based on
the equation of motion of the panel in order both to conveniently incorporate the NFD model
and to allow the use of the Inverse Discrete Fourier Transform (IDFT) in the analysis [1], which
has already proven to be an efficient tool when analyzing the spatial and frequency domain
responses of layered vibrating structures. The periodic structure modeling process used here is
introduced in Sections 2.1–2.3. Further, the fibrous layer damping was evaluated by calculating
the system equivalent damping loss factor based on a power analysis in the NFD model and
by using the power injection method (PIM) [11], which is introduced in Section 2.4. Finally,
the damping effectiveness and the microstructural design results for the fibrous layer (e.g., fiber
size) are introduced in Section 3.

2. Modeling process
The general modeling process is summarized in the flow chart shown in Figure 4.

![Flow Chart](image)

**Figure 4.** General Process of Modeling.

2.1. Governing equation for the fuselage-like structure
The top left block shows a typical configuration for this study, illustrating a single channel of
the periodically-constrained panel, a layer of limp-framed fibrous medium, and a half-space of
air above the layer. Instead of a harmonic line force as used previously, a convective pressure
was considered to exist below the panel to simulate a boundary layer excitation of the outer
surface of the fuselage. The constraints were modeled as having identical translational inertias
per unit length in the \( y \)-direction, \( m_l \), translational stiffnesses per unit length in the \( y \)-direction,
\( k_l \), rotational inertias per unit length in the \( y \)-direction, \( J_l \), and rotational stiffnesses per unit
length in the \( y \)-direction, \( s_l \). Based on the Euler-Bernoulli beam theory and the acoustical
theory applied in the NFD model [1], the governing equation of motion for the panel transverse
displacement, \( w_t \), can be expressed as

\[
D \frac{\partial^4 w_t}{\partial x^4} + m_s \frac{\partial^2 w_t}{\partial t^2} = -p_1(x, t) + f_{in}(x, t) + f_{re}(x, t),
\]

where \( D = \left[ h_p^3 E_0 (1 + i\eta_p) \right] / \left[ 12(1 - \nu^2) \right] \) is the flexural stiffness per unit length in the \( y \)-direction of the panel based on thickness, \( h_p \), Young’s modulus, \( E_0 \), loss factor, \( \eta_p \), and Poisson’s
ratio, \( \nu \), and \( m_s = \rho_p h_p \) is the mass per unit area of the panel based on density, \( \rho_p \) and thickness.
In addition, \( p_1 \) is the pressure in the limp fibrous layer at \( z = 0 \) that is acting on the panel, \( f_{in} \)
is the input forcing term, and \( f_{re} \) represents the reaction forces generated at the constraints.

The input forcing term, \( f_{in} \), due to the convective pressure excitation can be expressed as

\[
f_{in}(x, t) = P e^{+i\omega t} e^{-ik_v x},
\]
where $P$ is the forcing magnitude per unit material area, $\omega$ is the angular frequency, and $k_v$ the convective wavenumber, which is expressed as

$$k_v = \frac{\omega}{Ma_0},$$  \hfill (3)$$

indicating a subsonic excitation when the Mach number, $M < 1$, and a supersonic excitation when $M > 1$. The reaction force term, $f_{re}$, can be expressed as a summation of the reaction forces due to the translational and rotational inertias and stiffnesses of the constraints as

$$f_{re}(x,t) = \sum_{j=1}^{N_l} \left[ -m_l \frac{\partial^2 w_l(x,t)}{\partial t^2} - k_l w_l(x,t) \right] \delta(x - x_{l,j})$$

$$+ \sum_{j=1}^{N_l} \left[ -J_l \frac{\partial^2 \theta(x,t)}{\partial t^2} - s_l \theta(x,t) \right] \delta'(x - x_{l,j}),$$  \hfill (4)$$

where $N_l$ is the total number of the constraints, $x_{l,j}$ is the location of each constraint, and $\theta = \partial w_t / \partial x = -ik_x w_t$.

By substituting equations (2) and (4) into equation (1), the governing equation can then be rewritten as

$$D \frac{\partial^4 w_t(x,t)}{\partial x^4} + m_s \frac{\partial^2 w_t(x,t)}{\partial t^2} = -p_1(x,t) + Pe^{i\omega t} e^{-ik_x x}$$

$$\quad + \sum_{j=1}^{N_l} \left[ -m_l \frac{\partial^2 w_t(x,t)}{\partial t^2} - k_l w_t(x,t) \right] \delta(x - x_{l,j})$$

$$\quad + \sum_{j=1}^{N_l} \left[ -J_l \frac{\partial^2 \theta(x,t)}{\partial t^2} - s_l \theta(x,t) \right] \delta'(x - x_{l,j}).$$  \hfill (5)$$

Then by Fourier transforming equation (5) from the time domain to the frequency domain, and forward wavenumber Fourier transforming from the spatial domain to the wavenumber domain [1], the governing equation can be expressed as

$$Z_m(k_x,\omega)v_t(k_x,\omega) = -p_1(k_x,\omega) + 2\pi Pe^{i\omega t} e^{-ik_x x}$$

$$\quad - i \left( \omega m_l - \frac{k_l}{\omega} \right) \sum_{j=1}^{N_l} v_t(x_{l,j},\omega)e^{ik_x x_{l,j}} - k_x \left( \omega J_l - \frac{s_l}{\omega} \right) \sum_{j=1}^{N_l} \theta(x_{l,j},\omega)e^{ik_x x_{l,j}},$$  \hfill (6)$$

where $k_x$ is the trace wavenumber, which is shared by all the components in the system, and $Z_m$ is the mechanical impedance of the panel, which can be expressed as

$$Z_m(k_x,\omega) = i \left[ \omega m_s - (D/\omega)k_x^4 \right].$$  \hfill (7)$$

For the harmonic case considered here, the relation between the linear transverse displacement and the velocity is $v_t = i\omega w_t$, and the relation between the angular displacement and the velocity is $\dot{\theta} = i\omega \theta$.

At $z = 0$, the acoustic pressure, $p_1$, can be expressed as

$$p_1(k_x,\omega) = Z_{a1}(k_x,\omega)v_{z1}(k_x,\omega),$$  \hfill (8)$$
where \( v_z \) is the acoustic particle velocity at \( z = 0 \), which is equal to \( v_t \) based on continuity considerations \([1]\), and \( Z_{a1} \) is the normal acoustic impedance at \( z = 0 \) looking into the acoustic medium, and it can be expressed as

\[
Z_{a1}(k_x, \omega) = \frac{T_{11}Z_{a2}(k_x, \omega) + T_{12}}{T_{21}Z_{a2}(k_x, \omega) + T_{22}},
\]

where \( Z_{a2} \) is the normal acoustic impedance looking into the acoustic half-space: \( i.e. \), air, above the fibrous layer, which can be expressed as

\[
Z_{a2}(k_x, \omega) = \frac{\omega \rho_0}{\sqrt{[\omega(1 - i\eta_a)/c_0]^2 - k_x^2}},
\]

and it can be evaluated given the air density, \( \rho_0 = 1.21 \text{ kg/m}^3 \), air loss factor, \( \eta_a = 0.0005 \), and the speed of sound in the air, \( c_0 = 343 \text{ m/s} \). The transfer matrix of the limp fibrous layer, \([T_{11}, T_{12}, T_{21}, T_{22}]\), can be evaluated at each frequency based on the Johnson-Champoux-Allard (JCA) model \([12]\) and the Biot limp porous medium theory \([13, 14]\), given the properties that include the fibrous layer thickness, \( d \), airflow resistivity, \( \sigma \), porosity, \( \phi \), tortuosity, \( \alpha_{\infty} \), bulk density, \( \rho_b \), Prandtl number, \( B^2 = 0.71 \), specific heat ratio, \( \gamma = 1.402 \), and dynamic viscosity of air, \( \eta = 1.846 \times 10^{-5} \text{ kg/(s-m)} \).

At \( z = d \), a similar relation can be found for the acoustic pressure, \( p_2 \), and the acoustic particle velocity, \( v_{z2} \): \( i.e. \)

\[
p_2(k_x, \omega) = Z_{a2}(k_x, \omega)v_{z2}(k_x, \omega),
\]

where \( v_{z2} \) can be expressed as

\[
v_{z2}(k_x, \omega) = \frac{v_{z1}(k_x, \omega)}{T_{21}Z_{a2}(k_x, \omega) + T_{22}}.
\]

Note that \( Z_{a1} = Z_{a2} \) when there is no fibrous layer but only a bare panel loaded by the acoustic half-space \([1]\).

### 2.2. Solution for the wavenumber domain response

The relation between the linear and angular velocities of the panel, \( \dot{\theta} = -ik_xv_t \), and the motion continuity condition, \( v_{z1} = v_t \), can then be applied to equation (6), and the wavenumber domain linear and angular velocity responses of the panel can be expressed as

\[
v_t(k_x, \omega) = \frac{2\pi P\delta(k_x - k_0) + f_{re}(k_x, \omega)}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)},
\]

\[
\dot{\theta}(k_x, \omega) = -ik_x\frac{2\pi P\delta(k_x - k_0) + f_{re}(k_x, \omega)}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)},
\]

with

\[
f_{re}(k_x, \omega) = -i \left( \omega m_1 - \frac{k_1}{\omega} \right) \sum_{j=1}^{N_1} v_t(x_{1,j}, \omega)e^{ik_xx_{1,j}}
- k_x \left( \omega J_1 - \frac{s_1}{\omega} \right) \sum_{j=1}^{N_1} \dot{\theta}(x_{1,j}, \omega)e^{ik_xx_{1,j}}.
\]
In order to evaluate \( v_t(k_x, \omega) \), all the constraint responses, \( v_t(x_{i,j}, \omega) \) and \( \dot{\theta}(x_{i,j}, \omega) \), need to be evaluated first. Since the periodic structure has uniformly-spaced, identical constraints, it can be assumed that the responses of all the constraints must have the same amplitude, but any two adjacent constraint responses differ in phase by \( e^{-ik_x l} \), with \( l \) being the constraint separation [7]. Equation (14) can then be rewritten as

\[
f_{re}(k_x, \omega) = -i \left( \omega m_l - \frac{k_l}{\omega} \right) v_t(x_{i,1}, \omega) - k_x \left( \omega J_l - \frac{s_l}{\omega} \right) \dot{\theta}(x_{i,1}, \omega) \sum_{j=1}^{N_i} e^{ik_x (x_{i,j} - x_{i,1})},
\]

with the constraint at \( x = x_{i,1} \) being the base point to which all the other constraints are referred: thus, there are just two unknown constraint responses, \( v_t(x_{i,1}, \omega) \) and \( \dot{\theta}(x_{i,1}, \omega) \).

Further, equations (13a) and (13b) can be inverse Fourier transformed at \( x = x_{i,1} \) to obtain

\[
v_t(x_{i,1}, \omega) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{2\pi P \delta(k_x - k_v) + f_{re}(k_x, \omega)}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} e^{-ik_x x_{i,1}} dk_x,
\]

\[
\dot{\theta}(x_{i,1}, \omega) = -i \frac{1}{2\pi} \int_{-\infty}^{+\infty} k_x \frac{2\pi P \delta(k_x - k_v) + f_{re}(k_x, \omega)}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} e^{-ik_x x_{i,1}} dk_x.
\]

Then a system of equations can be formed by substituting equation (15) into equations (16) to obtain

\[
M_{11} v_t(x_{i,1}, \omega) + M_{12} \dot{\theta}(x_{i,1}, \omega) = J_1,
\]

\[
M_{21} v_t(x_{i,1}, \omega) + M_{22} \dot{\theta}(x_{i,1}, \omega) = J_2,
\]

with

\[
M_{11} = -i \left( \omega m_l - \frac{k_l}{\omega} \right) \int_{-\infty}^{+\infty} \sum_{j=1}^{N_i} e^{i(k_x - k_v)(x_{i,j} - x_{i,1})} \frac{1}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} dk_x - 2\pi,
\]

\[
M_{12} = -\left( \omega J_l - \frac{s_l}{\omega} \right) \int_{-\infty}^{+\infty} k_x \sum_{j=1}^{N_i} e^{i(k_x - k_v)(x_{i,j} - x_{i,1})} \frac{1}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} dk_x,
\]

\[
M_{21} = -\left( \omega m_l - \frac{k_l}{\omega} \right) \int_{-\infty}^{+\infty} k_x \sum_{j=1}^{N_i} e^{i(k_x - k_v)(x_{i,j} - x_{i,1})} \frac{1}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} dk_x,
\]

\[
M_{22} = i \left( \omega J_l - \frac{s_l}{\omega} \right) \int_{-\infty}^{+\infty} k_x^2 \sum_{j=1}^{N_i} e^{i(k_x - k_v)(x_{i,j} - x_{i,1})} \frac{1}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} dk_x - 2\pi,
\]

\[
J_1 = -2\pi P \int_{-\infty}^{+\infty} \frac{e^{-ik_x x_{i,1}}}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} \delta(k_x - k_v) dk_x,
\]

\[
J_2 = i2\pi P \int_{-\infty}^{+\infty} k_x e^{-ik_x x_{i,1}} \frac{1}{Z_m(k_x, \omega) + Z_{a1}(k_x, \omega)} \delta(k_x - k_v) dk_x.
\]

Now, \( v_t(x_{i,1}, \omega) \) and \( \dot{\theta}(x_{i,1}, \omega) \) can be evaluated by calculating all the coefficients in equation (18), and by substituting those coefficients into equation (17). Finally, the wavenumber domain transverse velocity response, \( v_t(k_x, \omega) \), can be evaluated by using equations (13a) and (15) along with the constraint responses, \( v_t(x_{i,1}, \omega) \) and \( \dot{\theta}(x_{i,1}, \omega) \).
2.3. IDFT for the spatial domain response

The process of inverse wavenumber transforming the velocity response to the spatial domain by using the IDFT is introduced in this section through an example calculation.

Note that choosing the proper sampling rate and number of sample points is very important to ensure that the discrete Fourier transform is evaluated accurately. The frequency wavenumber domain velocity level, \(20 \log_{10} |v_t(k_x, \omega)|\), was evaluated for a 1 mm thick aluminum panel over a wide range of trace wavenumbers, \(k_x\) (i.e., evenly-discretized into 214 points from \(-6000\) to \(6000\) rad/m), and over a wide range of temporal frequencies, \(f\) (i.e., at 500 different frequencies equally-spaced from 10 to 10000 Hz), by following the process described in Sections 2.1 and 2.2 for the case of \(h_p = 1\) mm and \(M = 0.8\), in combination with the parameters shown in Table 1.

The constraint parameters were chosen to create a periodically-clamped panel with a constraint separation of 1 m, which will be further used for model validation in Section 2.5.

The results are shown in Figure 5. The original wavenumber domain sampling rates, \(\gamma_{or}'s\)

![Figure 5](image)

**Figure 5.** Wavenumber domain response of the velocity level for a fibrous layer-treated periodically-clamped structure, with the original \(\gamma_{or}'s\) based on the cutoff level marked as the black solid line, and the adjusted \(\gamma_s's\) marked as the magenta dashed line.

| Parameter | Value |
|-----------|-------|
| \(P\)     | 1 N/m² |
| \(E_0\)   | 7 \times 10^{10} Pa |
| \(\eta_p\) | 0.003 |
| \(\nu\)   | 0.33 |
| \(\rho_p\) | 2700 kg/m³ |
| \(d\)     | 3 cm |
| \(\sigma\) | 20000 Rayls/m |
| \(\phi\)  | 0.99 |
| \(\alpha_{\infty}\) | 1.2 |
| \(\rho_b\) | 10 kg/m³ |
| \(l\)     | 1 m |
| \(m_l\)   | 200 kg/m |
| \(k_l\)   | 0 |
| \(J_l\)   | 0.7083 kg·m |
| \(s_l\)   | 0 |

The results are shown in Figure 5. The original wavenumber domain sampling rates, \(\gamma_{or}'s\)
(black solid line), at each frequency were based on a fixed cutoff level, e.g., −300 dB, but the γs’s were just raw estimates of the sampling rates needed to minimize bias due to windowing and truncation effects. These sampling rates were then further adjusted to the closest sampling rates that corresponded to an integer spatial span, xspan (i.e., a multiple of the constraint separation), which ensures that the IDFT is conducted on a complete periodic structure. The adjusted sampling rates, γs’s, are marked as the magenta dashed line in Figure 5. According to the adjusted xspan, the constraint locations, xl,j, were assigned at ±0.5, ±1.5, ..., ±(|xspan|/2 − 0.5).

Note that in all the calculation routines used in this model, kx is always first evaluated by using equation (3), and is then approximated by the closest wavenumber from the sampled kx array. This approximation also ensures that the IDFT is conducted on a complete periodic structure by making xspan an integer multiple of the excitation wavelength.

Finally, the spatial domain velocity response, vt(x,ω), can be evaluated by the IDFT, given the γs’s and Ns = 214 at each frequency, as

\[ vt(m\Delta x,\omega) = \frac{1}{N_s \Delta x} \sum_{n=0}^{N_s-1} vt(n\Delta k_x,\omega)e^{-\frac{2\pi mn}{N_s}}, \]

where

\[ \Delta x = \frac{2\pi}{\gamma_s} = \frac{x_{\text{span}}}{N_s}, \]

and

\[ \Delta k_x = \frac{\gamma_s}{N_s} = \frac{2\pi}{x_{\text{span}}}. \]

Note that the spatial domain responses of vz1, vz2, p1 and p2 can be evaluated in the same way.

2.4. System equivalent damping loss factor

Based on the spatial domain response and the power injection method (PIM) [11], the system equivalent damping loss factor can be calculated as

\[ \eta_e = \frac{E_{\text{inj}}}{E_{\text{tot}}}. \]

The energy injected into the panel per cycle, Einj, can be expressed as

\[ E_{\text{inj}} = \frac{P_{\text{inp}} - P_2}{\omega}, \]

where Pinp is the power input to the system by the convective excitation,

\[ P_{\text{inp}} = \frac{1}{2} \text{Re} \left( \sum_{j=1}^{N_s} P e^{-ik_x x_j} v_t^*(x_j,\omega) \right) \Delta x, \]

and where P2 is the power radiation into the acoustic half-space. The latter can be evaluated by using either spatial or wavenumber domain integrations as

\[ P_2 = \frac{1}{2} \text{Re} \left( \sum_{j=1}^{N_s} p_2(x_j,\omega) v_{z2}^*(x_j,\omega) \right) \Delta x, \]
\[ P_2 = \frac{1}{4\pi} \text{Re} \left( \sum_{j=1}^{N_s} Z_{a2}(k_{x,j}, \omega) |v_{z2}^*(k_{x,j}, \omega)|^2 \right) \Delta k_x. \]  

(25b)

The total energy of the panel, \( E_{\text{tot}} \), can be expressed as

\[ E_{\text{tot}} = \text{PE} + \text{KE} = 2\text{KE} = 2 \left( \frac{1}{2} \sum_{j=1}^{N_s} m_j |v_t(x_j, \omega)|^2 \right), \]

(26)

where \( m_j = \rho_p h_p \Delta x \) is the discretized mass of each panel element. The potential energy, \( \text{PE} \), and the kinetic energy, \( \text{KE} \), are assumed to be equal to each other.

### 2.5. Model validation

A related finite element model created by using COMSOL Multiphysics was implemented to help validate the NFD fuselage-structure model. The parameters as shown in Table 1 were input to the NFD model to create the case of a 1 mm thick periodically-constrained aluminum panel with a 3 cm thick limp fibrous layer. The finite element model consisted of a baffled 1 mm thick, 1 m long aluminum panel clamped (i.e., zero deflection and zero rotation) at both ends with the same limp fibrous layer on top as in the periodic case, along with the air half-space (see Figure 6). The comparison of the velocity response spectrum from 20 to 1000 Hz at \( x = 0 \) for the \( M = 0.8 \) case is shown in Figure 7, which shows reasonable agreement, especially given that the COMSOL model represents a stand-alone fully-clamped structure in contrast to the configuration in the NFD model in which the panel is one channel of a periodically-constrained structure with distributed discontinuities having large but finite inertias and stiffnesses that approximate fully-clamped boundary conditions at the constraints.

![Figure 6. Configuration in the (a) NFD fuselage model, (b) COMSOL finite element model.](image)

### 3. Results

The results to be presented in this section were primarily based on a fuselage-like configuration: i.e., a 3 mm thick periodically-constrained aluminum panel as shown in Figure 8. The input parameters for this configuration are shown in Table 2, which differs from Table 1 only in the constraint parameters. The wavenumber domain velocity level responses are presented in Section 3.1 for different \( M \)'s in order to illustrate the shift of the convective critical frequency. And a comparison of the frequency and spatial responses between a bare fuselage-like panel and a panel with a limp fibrous layer on top are presented in Sections 3.2 and 3.3, respectively. The damping loss factor evaluation is presented in Section 3.4. Finally, the fiber size design for optimal damping performance is introduced in Section 3.5.
Figure 7. Frequency domain velocity response from 100 to 1000 Hz at $x = 0$ by NFD model as the blue line, by COMSOL finite element model as the magenta line.

Figure 8. Configuration of the fuselage-like structure with the limp fibrous layer.

Table 2. Parameters for the fuselage-like structure and the limp fibrous layer.

|   | $P$ | $E_0$ | $\eta_p$ | $\nu$ | $\rho_p$ |
|---|-----|-------|---------|------|--------|
|   | 1 N/m² | $7 \times 10^{10}$ Pa | 0.003 | 0.33 | 2700 kg/m³ |
| $d$ | 3 cm | 20000 Rayls/m | 0.99 | 1.2 | 10 kg/m³ |
| $l$ | 1 m | 6.75 kg/m | $2.66 \times 10^8$ kg/(s²·m) | 0.0028 kg·m | $1.11 \times 10^5$ kg·m/s² |

3.1. Wavenumber domain response

The wavenumber domain velocity level was calculated over a range of $k_x$ from $-500$ rad/m to $500$ rad/m, and a range of $f$ from 10 to 10000 Hz for $M = 0.8$ (subsonic excitation), $M = 1.0$ (sonic excitation) and $M = 1.2$ (supersonic excitation). The results are plotted in Figure 9, and it can be observed that the excitation (black dashed line) and the panel dispersion curves intersects at the convective critical frequency, $f_{cv}$, which shifts with the increase of $M$, and which can be calculated using the equation

$$f_{cv} = \frac{(Mc_0)^2}{2\pi} \sqrt{\frac{m_s}{\text{Re}(D)}}.$$  (27)
Also, of course, for the sonic excitation case, $f_{cv}$ is the same as the conventional critical frequency, $f_c$, which is

$$f_c = \left(\frac{c_0}{2\pi}\right)^2 \sqrt{\frac{m_s}{\text{Re}(D)}}.$$  \hspace{1cm} (28)

Figure 9. Wavenumber domain velocity level for a fibrous layer-treated fuselage-like structure driven by (a) subsonic convective pressure ($M = 0.8$), (b) sonic convective pressure ($M = 1.0$), (c) supersonic convective pressure ($M = 1.2$), with the convective excitations marked as black dashed lines.

3.2. Velocity response spectrum at a certain point

A comparison of the velocity response spectrum from 10 Hz to 10000 Hz at $x = 0.11$ (a randomly-selected point between two adjacent constraints at $x = \pm 0.5$ m) between a bare fuselage-like structure and a fuselage-like structure treated with the 3 cm limp fibrous layer is shown in Figure 10 for both subsonic ($M = 0.8$) and supersonic ($M = 1.2$) convective pressure excitation cases. It can be observed that vibration peaks below $f_c$ were reduced by 5 to 15 dB by the fibrous layer. This observation is in-line with the previous finding that the fibrous near-field damping is primarily effective below the critical frequency, in which case that panel motion creates a strong, acoustical near-field [1].

3.3. Spatial response at a certain frequency

To show greater detail, Figure 10 was then replotted as Figure 11 over the frequency range from 100 Hz to 1200 Hz. The reduction in vibration level at the peaks is more easily discernible in this view. Two vibration peak frequencies, 420 Hz and 531 Hz, were selected for further observation of the spatial velocity response between the constraints at $x = \pm 0.5$ m for both subsonic ($M = 0.8$) and supersonic ($M = 1.2$) convective pressure excitation cases, respectively. The spatial responses are plotted in Figure 12, from which a 10 to 15 dB reduction of vibration can be observed at the peak frequencies (420 Hz, 531 Hz) across the observation area.

3.4. Damping loss factor

The system damping loss factor, $\eta_e$, was then calculated for the fuselage-like structure treated by a fibrous layer at three different input $\sigma$’s, which represent three different types of fiber microstructures; the results are plotted in Figure 13. A general trend that can be observed from both the subsonic and supersonic cases is that a fibrous layer with a relatively low airflow resistivity effectively reduces lower frequency vibration, while a layer with a relatively high
Figure 10. Velocity response spectrum from 10 Hz to 10000 Hz of the velocity at $x = 0.11$ for a bare fuselage-like structure (blue line) or a fibrous layer-treated fuselage-like structure (orange line) both driven by (a) subsonic convective pressure ($M = 0.8$), (b) supersonic convective pressure ($M = 1.2$).

Figure 11. Velocity response spectrum from 100 Hz to 1200 Hz of the velocity at $x = 0.11$ for a bare fuselage-like structure (blue line) or a fibrous layer-treated fuselage-like structure (orange line) both driven by (a) subsonic convective pressure ($M = 0.8$), (b) supersonic convective pressure ($M = 1.2$).

airflow resistivity effectively reduces higher frequency vibration, which means that there must be an optimal $\sigma$ corresponding to an optimal fiber microstructure that can achieve the largest damping (also referred to as the “optimal damping”) at each frequency for a given structure.

3.5. Microstructure design

The optimal microstructure of the fibrous layer expressed in terms of the optimal fiber radius, $r_1$, is introduced in this section. Given fibrous layer microstructural parameters as shown in Table 3 and the bulk parameters as shown in Table 2, $r_1$ was optimized for both polymeric and
Figure 12. Spatial responses between the constraints at \(x = \pm 0.5\) m for a bare fuselage-like structure (blue line) or a fibrous layer-treated fuselage-like structure (orange line) both driven by (a) subsonic convective pressure (\(M = 0.8\)), (b) supersonic convective pressure (\(M = 1.2\)).

Figure 13. System equivalent damping loss factor for a fibrous layer-treated fuselage-like structure driven by (a) subsonic convective pressure (\(M = 0.8\)), (b) supersonic convective pressure (\(M = 1.2\)), with input fibrous layer’s airflow resistivity as 10000 Rayls/m (orange line), 20000 Rayls/m (magenta line), 50000 Rayls/m (blue line).

glass fibers at ten different equally-spaced frequencies between 100 and 1000 Hz through the optimal damping selection process [1]. The optimal damping here was the largest \(\eta_e\) selected from the \(\eta_e\)’s predicted for 100 equally-spaced input \(r_1\)’s within the range shown in Table 3. The largest \(\eta_e\) achieved by the two kinds of fibers’ optimal fiber radii at each frequency is shown in Figure 14, which shows that equivalent levels of damping could be achieved by the two kinds of fibers after the microstructural optimization. Note that the sudden rise of \(\eta_e\) corresponds to the ”dip” at 1000 Hz shown in Figures 10 and 11.

Further, the optimal fiber radii are shown in Figure 15, which generally shows that relatively
large fibers are required to achieve optimal damping at lower frequencies. Also, it can be observed that a larger fiber size is required for polymeric fibers than for glass fibers to achieve equivalent optimal damping. That finding could contribute to a potential benefit from using polymeric fibers (with fiber material density of 910 kg/m$^3$) instead of glass fibers (with fiber material density of 2730 kg/m$^3$) as a damping treatment for an aircraft fuselage, since it is generally easier and less costly to manufacture relatively larger fibers.

Table 3. Parameters for the fuselage-like structure and the limp fibrous layer.

| $r_1$ (µm) | $\rho_1$ (kg/m$^3$) |
|------------|---------------------|
| 0.01–11    | 910                 |
|            | 2730                |

Figure 14. Optimal damping in terms of $\eta_e$ achieved by a layer of microstructural-optimized polymeric fibers (blue solid line) or glass fibers (red dashed line) for a fuselage-like structure driven by (a) subsonic convective pressure ($M = 0.8$), (b) supersonic convective pressure ($M = 1.2$).

4. Conclusions

In this work, an aircraft fuselage-like structure was created and was combined with a previously-developed near-field damping (NFD) model. Based on the NFD fuselage model, spatial domain velocity responses for a periodically-clamped structure were first calculated and validated by comparison with finite element results. Then spatial and frequency domain velocity responses were evaluated for a bare fuselage-like structure and one treated with a limp fibrous layer. Significant levels of damping were achieved by the addition of the fibrous layer at the peak frequencies in the subsonic region below $f_c$. Finally, optimal fiber sizes were identified for either a glass fiber layer or a polymeric fiber layer that would achieve the largest damping at certain frequencies from 100 to 1000 Hz. Larger fiber sizes were found to be better for lower frequency damping for both polymeric and glass fibers. Also, polymeric fibers were found to have a potential manufacturing benefit over glass fibers due to the larger fiber size required by the former material to achieve the optimal damping for a fuselage-like structure.
Figure 15. Optimal fiber radii designed for a layer of polymeric fibers (orange line) or glass fibers (green line) to achieve the optimal damping for a fuselage-like structure driven by (a) subsonic convective pressure ($M = 0.8$), (b) supersonic convective pressure ($M = 1.2$).

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