A note on the Declarative reading(s) of Logic Programming

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Abstract
This paper analyses the declarative readings of logic programming. Logic programming - and negation as failure - has no unique declarative reading. One common view is that logic programming is a logic for default reasoning, a sub-formalism of default logic or autoepistemic logic. In this view, negation as failure is a modal operator. In an alternative view, a logic program is interpreted as a definition. In this view, negation as failure is classical objective negation.

From a commonsense point of view, there is definitely a difference between these views. Surprisingly though, both types of declarative readings lead to grosso modo the same model semantics. This note investigates the causes for this.

Introduction
Logic’s fundamental role in the area of computing and artificial intelligence, is its use for knowledge representation. There may be innumerable ways in which some domain knowledge can be encoded in a logic theory; however, there is one principle which most consider as the canonical way of using logic. Declarative knowledge representation operates according to the following principle:

the expert represents his domain knowledge by a set of formal statements that are true in the problem domain.

This principle relies on the ability of the expert to interpret a logical axiom as a clear and precise statement about the domain of discourse. The ability of interpreting the formulas of a logic as meaningful statements about the problem domain -given some interpretation of the user-defined symbols- is the declarative reading of the logic. It is based on a clear understanding of the connectors and quantifiers and of how composed axioms combine these meanings.

A declarative semantics of the logic can be defined as a formal study of its -intuitive- declarative reading. As such it should contain the following parts:

• a clear account of some declarative reading of the formulas and theories in a logic;
• a mathematical characterisation of a formal semantics;
• a justification why and how this semantics characterises this declarative reading.

In a sense, a declarative semantics should relate a logic to some part of the human cognition and understanding. Note that a simple way of defining a declarative semantics of a logic is by providing an embedding in another logic with a well-established declarative semantics. Such semantics are sometimes called transformational semantics.

The declarative reading of logical axioms gives logic a decisive advantage over other languages. It allows the expert to compare a formal statement with his or her knowledge and to evaluate its truth, without going through the painful process of explicitly constructing the mathematical semantics of the axioms. This ideal is reached most clearly in classical logic. So, our common expert knowledge allows us to recognize the statement

\[ \forall x. \text{person}(x) \rightarrow \text{male}(x) \lor \text{female}(x) \]

as true (given the obvious intended interpretation of the predicate symbols), and the statement

\[ \forall x. \text{male}(x) \leftrightarrow \text{female}(x) \]

as false. We can do so on the basis of our intuitive understanding, without having to construct the semantics of these sentences, i.e. their class of models.

It is a fact that to be able to represent knowledge in a declarative way and to benefit the potential advantages of this type of knowledge representation, a human expert must have acquired a deep and precise understanding of the declarative reading of the logic that he is using. Ambiguities and unclarities on the level of declarative reading cause ambiguities and unclarities on the level of knowledge representation. For this reason, studies of declarative reading are of key importance in the development of knowledge representation methodologies.

This paper is a study of the declarative reading(s) of logic programming. The history of the declarative
semantics of logic programming is well-known. Originally, the picture was simple and clear: the declarative reading of a Horn logic program was the declarative reading of a set of classical logic implications. The introduction of negation as failure blurred this simple view. On the one hand, negation as failure derived conclusions with a strong commonsense appeal and turned out to be very useful and natural in many practical situations. On the other hand, the negation as failure inference rule was unsound with respect to the declarative reading of a program as a set of classical implications. As expressed by Przymusinski (Przymusinski 1988), “we really do not want classical logic semantics for logic programs. .. We want the semantics of a logic program to be determined more by its commonsense meaning.”. All main semantic investigations since the end of the seventies (least model semantics (van Emde & Kowalski 1976), completion (Clark 1978), perfect model semantics (Apt, Blair, & Walker 1988; Przymusinski 1988), stable model semantics (Gelfond & Lifschitz 1988), well-founded model semantics (Van Gelder, Ross, & Schlipf 1991) attempted to formalise and explicitate this commonsense meaning of logic programs.

The question considered in this paper is to what extent the commonsense meaning referred to by Przymusinski has been identified: what is or are the declarative reading(s) of logic programming and what are the corresponding meanings of its symbols not, ←. What are the semantics that correspond best to these declarative readings?

The analysis of the declarative reading of logic programming is complicated by at least two factors. One complication is that different formal semantics exist (in 2-, 3- and 4-valued versions). In particular, stable and well-founded semantics are generally considered as the main ones. However, there is a second complication which is more subtle and much more dangerous. Let me try to pinpoint this problem.

It is well-known that logic programming can be used for representing many different sorts of knowledge (Baral & Gelfond 1994); (inductive) definitions, defaults, reflective knowledge of experts, etc., both under well-founded semantics and stable model semantics. Remarkable though is that the same logic program or rule in a logic program can be used to represent knowledge with a decidedly distinct commonsense flavor. For example, we might represent the definition that dead means not alive by the rule:

\[ \text{dead} \leftarrow \text{not alive} \]

The interpretation of this rule as a definition of dead is explicitated by the completion semantics (Clark 1978). Clark’s embedding of logic programming in classical logic. If the rule defining dead is the only rule with dead in the head, the completion will contain the equivalence

\[ \text{dead} \leftrightarrow \neg \text{alive} \]

Since stable and well-founded models are models of the completion, this rule is satisfied also in these models.

On the other hand, consider the reflective knowledge of the distrustful man who is unhappy if he does not know that his wife is faithful to him. His reflective knowledge can be represented by the same rule (modulo renaming) as in the above scenario:

\[ \text{unhappy} \leftarrow \text{not wife.f\_faithful} \]

Indeed, Gelfond’s embedding of logic programs in autoepistemic logic (Gelfond 1987) maps this rule to the autoepistemic formula:

\[ \text{unhappy} \leftarrow \neg \text{K\_wife\_faithful} \]

which directly represents the knowledge of the distrustful man.

Gelfond’s embedding defines a transformational declarative semantics for logic programming. It formed the knowledge theoretical foundation for stable model semantics (Gelfond & Lifschitz 1988); therefore we are entitled to assume that stable semantics is conceived to represent this sort of knowledge. Note here that negation as failure is interpreted as a modal operator and the implication operator as classical implication. This is a common feature of all embeddings of logic programming in autoepistemic logic (AEL) and in default logic (DL). Another important embedding is Marek and Truszczyński’s one to DL (Marek & Truszczyński 1989). It maps the above rule to:

\[ \neg \text{wife.f\_faithful} \]

\[ \text{unhappy} \]

For an overview of different embeddings see (Marek & Truszczyński 1993).

From the commonsense point of view, there is a definite distinction between both pieces of knowledge. The definition of dead expresses that in the actual state of the world, dead and alive are mutually exclusive: the exclusive “or” holds between them. If the expert does not know whether alive is true or false, then he does not know whether dead is true or false. On the other hand, the knowledge of the distrustful man does not imply any relationship between the unhappiness of the distrustful man and the loyalty of the spouse in the actual state of the world. It is perfectly possible that in the actual state of the world, she is not loyal but he does not know and he is happy.

Consider the logic program program \( \{ p \leftarrow \neg q \} \). All main semantics coincide for it; its formal semantics (i.e. the set of its models) is the unique model \( \{ p \} \). Yet, as illustrated above, Clark’s embedding and Gelfond’s embedding assign two different commonsense meanings to this program:

- In the completion, the program states that \( q \) is false (because of the empty definition) and that \( p \) is true iff \( q \) is false.

The model represents the state of the world in which \( p \) is true and \( q \) is false. According to the completion, this is the only possible state of the world.
The program in Gelfond’s embedding states that $p$ holds if $q$ is not known to be true.

Since in this interpretation, the program has no knowledge about $q$, $q$ is unknown, hence $p$ is entailed. Note that contrary to the first reading, here $q$ is not known to be false.

What is proven in (Gelfond & Lifschitz 1988), is that the stable model represents the set of believed atoms of the above theory: $\{p\}$ means that $p$ is believed, and that the truth of $q$ is not believed.

Based on our intuition, what can be said about the possible states of the world in this reading? Obviously, since $q$ is unknown, there should be states in which $q$ is true and states in which $q$ is false; $p$ should be true always. Hence, the two possible states are $\{p\}$ and $\{q, p\}$.

Moore (Moore 1984) defined a possible world semantics for AEL. Intuitively, a possible world model is a set of possible states of the world according to the expert’s knowledge. The unique possible world model of the theory $\{p \leftarrow \neg Kq\}$ is exactly the set $\{\{p\}, \{q, p\}\}$; this confirms our intuition.

The above discussion leads to a key point of this paper: even if we know the models of a logic program, we still cannot decide the intended declarative reading. We can only know – to some extent – what is the declarative reading if we know also what is the role of the model.

A mathematical definition of some collection of models of theories in a logic cannot define a declarative reading. In this sense, a model theory is not a declarative semantics in its own right.

In the sequel, a model semantics will be called a possible state semantics for some declarative reading if it characterises the possible states of the world; a model theory will be called an atomic belief set semantics of some declarative reading if it characterises the sets of believed atoms in this declarative reading.

It is not a simple task to search for the declarative reading(s) of logic programming. In the first place, many of the early semantical studies in the context of logic programming are not primarily concerned with finding and formalising a commonsense meaning, but are more concerned with finding a mathematical justification for the reasoning techniques in Prolog.

Other studies are more focussed on the commonsense meaning, but fail to give a clear account of the formalised declarative reading and the role of the models. There is an enormous amount of mathematical results on the relationships between different model semantics. However, because atomic belief sets and possible states are incomparable objects, it is a priori not clear what these relations mean on the level of declarative reading. Moreover, in many knowledge representation examples, one can observe that the same semantics is used once as an atomic belief set semantics, once as an possible state semantics.

In order to clarify the role of logic programming for Knowledge representation, the question of declarative reading of logic programming cannot be circumvented. In the rest of the paper I will try to pinpoint the main ideas on declarative reading and the confusions on this topic.

### Declarative readings of logic programming

An investigating of the transformational semantics that have been proposed for logic programming, gives some insight in the possible declarative readings of logic programming.

It seems to me that at least in the non-monotonic and A.I.-oriented part of the logic programming community, logic programming is now routinely seen as a sub-logic of default logic or autoepistemic logic. In this view, the negation as failure is interpreted as a modal negation. This view has a natural motivation: a Prolog system is said to infer a negative literal not $p$ when it is unable to prove $p$. A natural way of modeling failure to prove in semantics is as not knowing. From here, it was natural to interpret not $p$ as $\neg Kp$.

All main logic programming semantics - least model, supported model, 3-valued supported model, perfect model, stable model, 3-valued stable model and well-founded semantics - have been justified as atomic belief set semantics of diverse modal interpretations of logic programming. The methods that have been used are analogous as Gelfond and Lifschitz’s justification for stable semantics: one defines an embedding of logic programming to some non-monotone modal logic and the models of the logic program are shown to be the set of believed atoms. In these transformational semantics, models of logic programming semantics systematically play the role of a set of believed atoms. For an overview of these results, I refer to (Marek & Truszczyński 1992).

On the other hand, I believe that there is also a persistent and strong intuition that among all classical models of a logic program, there is a canonical one (or at most a small number of canonical ones) which represents the unique possible state of the world. The Clark completion semantics was an early, weak attempt to identify this canonical model. In this view, negation as failure does not need to be interpreted as a modal operator: it is classical objective negation denoting falsity in the canonical model.

The main questions are what declarative reading of logic programming could explain the existence of a unique possible state and how could this unique model be mathematically characterized?

These questions were considered in (Denecker 1993, Denecker 1998). The idea is to read a logic program as an inductive definition. From the very start, logic programs were considered as definitions. This was Clark’s basic idea with the completion semantics. Note that Clark’s completed definitions are identical to the way (non-recursive) definitions are expressed, for example...
in Beth’s studies.

The evident problem with Clark completion semantics is that it does not deal well with inductive definitions. On the other hand, least model semantics is known to deal right with positive inductive definitions such as transitive closure. Using the above terminology, the least model semantics can be said to be possible state semantics for the reading of Horn programs as inductive definitions. In fact, as pointed out in [Denecker 1998], the methods that have been used to characterize monotone induction are identical to those that were used to characterize least model semantics of Horn programs.

Can we extend the view of Horn logic programs as inductive definitions to programs with negation? In [Denecker 1998], I have argued that the use of induction in mathematics is not restricted to positive induction. An example is the induction in a well-founded set. To some extent, a generalised form of non-monotone induction have been studied in the area of inductive definitions, the so called Iterated Inductive Definitions (Feferman 1977). As I showed, this formalism is isomorphic modulo syntactic sugar with stratified logic programs under perfect model semantics. Further on, I have pointed to several intolerable weaknesses of this stratified approach as a formalisation of generalised induction and have argued that these problems are solved by the well-founded semantics. Or, the argument there was that the well-founded semantics is a possible state semantics of the declarative reading of logic programs as generalised inductive definitions.

Consequently, logic programming has not a unique declarative reading. The modal view and the definition view are both consistent ways of interpreting logic programs; moreover they lead to very similar model theories, though these theories have different roles.

The above phenomenon, the existence of different consistent declarative views on the same formalism is a potential source of considerable confusion. In the remaining sections, I investigate possible confusions.

**Distinguishing between declarative readings**

**Comparing declarative semantics**

The mathematical relations between the least model, models of the completion, the perfect model, stable models and well-founded model are understood quite well. The question I consider here is what they tell about the relations between the two types of declarative readings.

A naive comparison of different semantics of logic programming is misleading. For example, the collection of stable models is known to be a subset of the collection of models of the completion. What does this result mean on the level of the declarative readings underlying both semantics?

Not much it seems. E.g. the possible world model of Gelfond’s embedding and Marek and Truszczyński’s embedding of the program \( \{ p \leftarrow not \ q \} \) is the set \( \{ \{ p \}, \{ q, p \} \} \); this is a proper superset of the (singleton) set of models of the completion. Consequently, in the case of this particular program, the completion is strictly stronger than the default or AEL reading.

**Expressing knowledge declaratively**

Consider a logic with an atomic belief set semantics for its declarative reading. Assume that the models of a logic theory are exactly the possible states of the world according to the expert’s knowledge. This theory encodes the expert’s knowledge but obviously, it is not necessarily a declarative representation of the expert’s knowledge. In general the declarative reading of the theory does not justify that the models are the only possible states of the world. In fact, it may well be that part of the axioms of the theory are false statements about the domain of discourse.

The above phenomenon can be illustrated in LP. Since stable semantics is an extension of least model semantics, it is suitable to encode positive inductive definitions such as the one of transitive closure:

\[
\begin{align*}
\text{p(a, a)} & \leftarrow \\
\text{p(b, c) } & \leftarrow \\
\text{tr(X, Y) } & \leftarrow \text{p(X, Y)} \\
\text{tr(X, Y) } & \leftarrow \text{p(X, Z)}, \text{tr(Z, Y)}
\end{align*}
\]

The unique stable model indeed represents the unique possible state of the graph and its transitive closure.

But this does not necessarily mean that inductive definitions can be expressed under the default reading of logic programs. For example, the meaning of this program under Gelfond’s embedding is identical to its classical logic meaning (since AEL is a conservative extension of classical propositional logic).

This problem gives rise to unsoundness. For example, with the inductive definition, the expert knows that \( p(a, b) \) and \( tr(a, b) \) are false. Yet, the AEL embedding entails \( \neg Kp(a, b) \) and \( \neg Ktr(a, b) \).

In the case of the transitive closure, the modal declarative reading (as expressed under the above mentioned AEL and DL embeddings) of the axioms is true but too weak to justify the unique model. An example where the declarative reading would be plainly false can be given by a variant of the dead and alive example. Assume that the expert wants to represent the definition that dead means not alive, as represented by dead \( \iff \neg \text{ alive} \).

A possible way to do this using stable semantics is:

\[
\begin{align*}
\text{dead } & \leftarrow \text{ not alive} \\
\text{alive } & \leftarrow \text{ not alive*} \\
\text{alive* } & \leftarrow \text{ not alive}
\end{align*}
\]

The two stable models (after projection on the two atoms alive, dead) are identical to the models of the
equivalence. Yet, it is easy to see that both the AEL and DL embedding assigns \textit{false} meaning to the first axiom. Indeed, it is not true that \textit{dead} is true if one does not know that \textit{alive} is true.

**Mixing different declarative readings**

The presence of multiple declarative readings and multiple roles of models raises complications on the level of methodology. This becomes obvious when different roles are \textit{mixed}.

Consider what happens if the different sorts of declarative readings are used in the same program. Reconsider the example of the distrustful man. Assume that we want to add the definition that to be unhappy means not to be \textit{happy}. According to the completion, this knowledge is correctly represented by the rule:

\[
\text{happy} \leftarrow \neg \text{unhappy}
\]

What happens if we combine this rule with the reflective knowledge of the jealous husband. Consider the program:

\[
\{ \text{unhappy} \leftarrow \neg \text{wife} \cdot \text{faithful} \\
\text{happy} \leftarrow \neg \text{unhappy} \}
\]

This program consists now of two isomorphic statements which are both \textit{true}, but under different declarative readings. Note that again all semantics for this program coincide. The model is \{\text{unhappy}\}. What does this model mean? Consider the two options that arose earlier:

- If this model is to be interpreted that \textit{unhappy} is true and \textit{wife} \cdot \textit{faithful} and \textit{happy} are false, then there is a mismatch with our understanding because neither the truth of \textit{unhappy} nor even the knowledge that \textit{unhappy} is true is a sufficient condition for happy being false. I.e. \textit{happy} should be unknown.
- On the other hand, if the model is interpreted as a belief set, there is again mismatch with our understanding because since \textit{wife} \cdot \textit{faithful} is unknown, then also \textit{unhappy} should be unknown.

Whether the model is interpreted as a possible state or as the belief set, it contains an error.

**Extending Logic Programming**

This paper focuses on the original logic programming formalism with only the original negation as failure, and does not investigate its extensions with classical or strong negation. At this point, it is clear that there is natural reason for that. A point of this paper is exactly that in one commonsense view on logic programming, negation as failure is classical negation. It is clear then than in this view, the formalism cannot be further extended with another classical or strong negation. Extensions of logic programming with classical negation (including disjunctive logic programming) make sense only in the modal negation view on logic programs.

Logic programming extensions were designed as a way to solve a number of serious disadvantages of logic programming for knowledge representation. However, as argued in [Denecker 1997], the analysis of what are these disadvantages exactly, depends on the declarative reading one takes. Different declarative readings lead to different conclusions and more importantly they subsequently lead to different ways to different extensions of the formalism.

In the case of the modal views, the problem was the absence of classical negation. However, as argued in [Denecker 1995], the problem of logic programming under the definition view is that a logic program contains definitions for all predicates. For this reason, one should extend the formalism with the possibility of leaving certain predicates open. This view is further explored in [Denecker 2000].

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