Study of Penguin Pollution in the $B^0 \rightarrow J/\psi K_S$ Decay *

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We study the penguin pollution in the $B^0 \rightarrow J/\psi K_S$ decay up to leading power in $1/m_b$ and to next-to-leading order in $\alpha_s$, $m_b$ being the $b$ quark mass and $\alpha_s$ the strong coupling constant. The deviation $\Delta S_{J/\psi K_S}$ of the mixing-induced CP asymmetry from $\sin(2\phi_1)$ and the direct CP asymmetry $A_{J/\psi K_S}$ are both found to be of $O(10^{-3})$ in a formalism that combines the QCD-improved factorization and perturbative QCD approaches.

I. INTRODUCTION

The $B^0 \rightarrow J/\psi K_S$ decay is known to be the golden mode for extracting $\sin(2\phi_1)$, $\phi_1$ being the weak phase of the Cabbibo-Kobayashi-Maskawa (CKM) matrix element $V_{td}$, through the time-dependent CP asymmetry

$$a(t) = \frac{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) - \Gamma(B^0(t) \rightarrow J/\psi K_S)}{\Gamma(\bar{B}^0(t) \rightarrow J/\psi K_S) + \Gamma(B^0(t) \rightarrow J/\psi K_S)} = S_{J/\psi K_S} \sin(\Delta M t) + A_{J/\psi K_S} \cos(\Delta M t),$$

where $\Delta M$ is the mass difference of the two $B_d$-meson mass eigenstates. The mixing-induced CP asymmetry $S_{J/\psi K_S}$ is naively equal to $\sin(2\phi_1)$ in the standard model. At the same time, the direct CP asymmetry $A_{J/\psi K_S}$ is expected to be vanishingly small. The penguin pollution, which would change the above naive expectations, is believed to be negligible in this mode. A complete estimation of its effect is, however, essential for future precision measurements, where the experimental error of $S_{J/\psi K_S}$ will be $\delta S_{J/\psi K_S} \approx \pm 0.2$ for 5 ab$^{-1}$ data at the Super $B$ factory.

The deviation $\Delta S_{J/\psi K_S} \equiv S_{J/\psi K_S} - \sin(2\phi_1)$ was found to be of $O(10^{-3})$ in the previous estimation by Boos et al. [2], taking into account corrections to the $B - \bar{B}$ mixing and to the $B^0 \rightarrow J/\psi K^0$ decay amplitude. A part of penguin-operator contributions, however, were overlooked in the estimation.

A model-independent approach to estimate the size of the penguin pollution proposed by Ciuchini et al. [3]. They extracted the range of the penguin corrections from the data of the branching ratio and the CP asymmetries for the $B^0 \rightarrow J/\psi \pi^0$ decay, and used the range to evaluate $\Delta S_{J/\psi K_S}$ with the flavor SU(3) symmetry. The current data lead to

$$\Delta S_{J/\psi K_S}^{\text{decay}} = 0.000 \pm 0.012,$$

where the large error comes from the experimental uncertainty of the $B^0 \rightarrow J/\psi \pi^0$ data.

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II. CORRECTIONS TO THE $B - \bar{B}$ MIXING

The mixing-induced CP asymmetry $S_{J/\psi K_S}$ and the direct CP asymmetry $A_{J/\psi K_S}$ in Eq. (1) are given by

$$S_{J/\psi K_S} = \frac{2 \Im \lambda_{J/\psi K_S}}{1 + |\lambda_{J/\psi K_S}|^2},$$

$$A_{J/\psi K_S} = \frac{|\lambda_{J/\psi K_S}|^2 - 1}{1 + |\lambda_{J/\psi K_S}|^2},$$

with the associated factor

$$\lambda_{J/\psi K_S} = \frac{q}{p} \frac{\mathcal{A}(\bar{B}^0 \rightarrow J/\psi K_S)}{\mathcal{A}(B^0 \rightarrow J/\psi K_S)},$$

where the ratio $q/p$ is related to the $B - \bar{B}$ mixing and $\mathcal{A}(B^0(\bar{B}^0) \rightarrow J/\psi K_S)$ is the decay amplitude. The corrections to the $B - \bar{B}$ mixing alters $q/p$ from $\exp(-2i\phi_1)$. In Ref. [3], non-local contributions to the $B - \bar{B}$ mixing, shown in Fig. (1) were calculated. The non-local operators

![Fig. 1: Non-local corrections to the $B - \bar{B}$ mixing, where the dots represent local operators.](image-url)
are defined as
\[ T_1 = -\frac{i}{2} \int d^4x \mathcal{T} \left[ \left[ \bar{b} c \right] \left( \bar{c} d \right)^\mu (x) \left[ \bar{b} c \right] \left( \bar{c} d \right)^\nu (0) \right], \]
\[ T_2 = -\frac{i}{2} \int d^4x \mathcal{T} \left[ \left[ \bar{b} c \right] \left( \bar{c} d \right)^\mu (x) \left[ \bar{b} u \right] \left( \bar{u} d \right)^\nu (0) \right], \]
\[ T_3 = -\frac{i}{2} \int d^4x \mathcal{T} \left[ \left[ \bar{b} u \right] \left( \bar{u} d \right)^\mu (x) \left[ \bar{b} c \right] \left( \bar{c} d \right)^\nu (0) \right], \]
where the left-handed current is shortened into \( \left( \bar{b} q \right) \mu \equiv \left( \bar{b} \gamma_\mu q \right) \), and \( \mathcal{T} \) denotes the time-ordered product. As the scale evolves down, the non-local operators mix into the \( \Delta B = 2 \) local operators of dimension 8,
\[
\begin{align*}
Q_1 &= \left( \bar{b} d \right) \mu \left( \bar{d} b \right)^\mu , \\
Q_2 &= \delta^\mu \delta^\nu \left( \bar{b} d \right) \mu \left( \bar{d} b \right) \nu , \\
Q_3 &= m_\alpha^2 \left( \bar{b} d \right) \mu \left( \bar{d} b \right) \mu ,
\end{align*}
\]
and it is unlikely to further factorize the form factor. The factorizable contributions to \( A_{J/\psi K^0}^{(c)} \) and \( A_{J/\psi K^0}^{(t)} \) displayed in Fig. 2 are given by
\[
\begin{align*}
A_{J/\psi K^0}^{(c)} &= 2\sqrt{6} \int_0^1 dx \Psi(x) F_{J/\psi K^0}^{2B} (m_{J/\psi}) a_2(x_c, t), \\
A_{J/\psi K^0}^{(t)} &= 2\sqrt{6} \int_0^1 dx \Psi(x) F_{J/\psi K^0}^{2B} (m_{J/\psi}) \times [a_3(x_c, t) + a_5(x_c, t)],
\end{align*}
\]
respectively, where \( x_c \) is the momentum fraction carried by the \( c \) quark, and \( \Psi(x) \) the twist-2 \( J/\psi \) meson distribution amplitude. The explicit expressions of \( \Psi(x) \) is referred to Ref. 1. The hard scale \( t \) has been chosen as
\[
t = \max \left( \sqrt{x_c (m_B^2 - m_{J/\psi}^2)}, \sqrt{x_c (m_B^2 - m_{J/\psi}^2)} \right),
\]
with the notation \( x_c = 1 - x_c \) and \( m_B \) being the \( B \) meson mass. The effective Wilson coefficients, including the \( O(\alpha_s) \) vertex corrections in Figs. 2(b)-(d), are defined as
\[
a_2(x, \mu) = C_1(\mu) + \frac{C_2(\mu)}{N_c} + \frac{C_3(\mu)}{4\pi} \ln \frac{m_B}{\mu} + f_1(x),
\]
where the unitarity relation of the CKM matrix elements \( V_{cb}^\ast V_{us} \approx -V_{cb}^\ast V_{us} \approx V_{cb}^\ast V_{cs} \) is used. The term with \( V_{cb}^\ast V_{cs} \) dominates the decay amplitude, whereas that without \( V_{cb}^\ast V_{us} \) causes the penguin pollution, since it carries the CKM phase \( \delta_3 \), which is defined by \( V_{ub} = \left| V_{ub} \right| \exp(-i\delta_3) \). Note that the amplitude \( V_{ub}^\ast V_{us} A_{J/\psi K^0} \) was missed in Ref. 3.

We calculate the decay amplitude using a formalism that combines the QCD-improved factorization (QCDF) and the perturbative QCD approach (PQCD) up to leading power in \( 1/m_b \) and to NLO in \( \alpha_s \). We adopt QCDF to handle the factorizable amplitudes, since the energy release in the form factor \( F_{J/\psi K^0}^{2B} (m_{J/\psi}) \), \( m_{J/\psi} \) being the \( J/\psi \) meson mass, is small and it is unlikely to further factorize the form factor. The factorizable contributions to \( A_{J/\psi K^0}^{(c)} \) and \( A_{J/\psi K^0}^{(t)} \) displayed in Fig. 2 are given by
\[
\begin{align*}
A_{J/\psi K^0}^{(c)} &= 2\sqrt{6} \int_0^1 dx \Psi(x) F_{J/\psi K^0}^{2B} (m_{J/\psi}) a_2(x_c, t), \\
A_{J/\psi K^0}^{(t)} &= 2\sqrt{6} \int_0^1 dx \Psi(x) F_{J/\psi K^0}^{2B} (m_{J/\psi}) \times [a_3(x_c, t) + a_5(x_c, t)],
\end{align*}
\]
respectively, where \( x_c \) is the momentum fraction carried by the \( c \) quark, and \( \Psi(x) \) the twist-2 \( J/\psi \) meson distribution amplitude. The explicit expressions of \( \Psi(x) \) is referred to Ref. 1. The hard scale \( t \) has been chosen as
\[
t = \max \left( \sqrt{x_c (m_B^2 - m_{J/\psi}^2)}, \sqrt{x_c (m_B^2 - m_{J/\psi}^2)} \right),
\]
with the notation \( x_c = 1 - x_c \) and \( m_B \) being the \( B \) meson mass. The effective Wilson coefficients, including the \( O(\alpha_s) \) vertex corrections in Figs. 2(b)-(d), are defined as
\[
a_2(x, \mu) = C_1(\mu) + \frac{C_2(\mu)}{N_c} + \frac{C_3(\mu)}{4\pi} \ln \frac{m_B}{\mu} + f_1(x),
\]
where the unitarity relation of the CKM matrix elements \( V_{cb}^\ast V_{us} \approx -V_{cb}^\ast V_{us} \approx V_{cb}^\ast V_{cs} \) is used. The term with \( V_{cb}^\ast V_{cs} \) dominates the decay amplitude, whereas that without \( V_{cb}^\ast V_{us} \) causes the penguin pollution, since it carries the CKM phase \( \delta_3 \), which is defined by \( V_{ub} = \left| V_{ub} \right| \exp(-i\delta_3) \). Note that the amplitude \( V_{ub}^\ast V_{us} A_{J/\psi K^0} \) was missed in Ref. 3.
with the ratio $r_{J/\psi} = m_{J/\psi}/m_B$.

For the $O(\alpha_s)$ nonfactorizable spectator diagrams in Fig. 3 QCDF is not appropriate due to the end-point singularity from vanishing parton momenta. Note that the nonfactorizable contribution has a characteristic scale higher than in $F_{+s/\psi}^{B+K}(m_{J/\psi}^2)$ [10]. Therefore, we can employ PQCD based on $k_T$ factorization theorem, which is free of the end-point singularity. The nonfactorizable spectator amplitudes in PQCD are given by

\[ A^{(c)nf}_{J/\psi K^0} = M(a'_2), \]
\[ A^{(t)nf}_{J/\psi K^0} = M(a'_3 - a'_5), \]

where the explicit expression of the amplitude $M$ is referred to Refs. [11, 12], and the effective Wilson coefficients are defined by

\[ a'_2(\mu) = \frac{C_2(\mu)}{N_c}, \]
\[ a'_3(\mu) = \frac{1}{N_c} [C_4(\mu) + C_{10}(\mu)], \]
\[ a'_5(\mu) = \frac{1}{N_c} [C_6(\mu) + C_8(\mu)]. \]

The nonfactorizable contribution is essential for the $B^0 \to J/\psi K^0$ decay, since this mode is classified in the color-suppressed category of $B$ meson decays. In fact, the nonfactorizable contribution is comparable in size to the factorizable one.

The amplitude $A^{(u)}_{J/\psi K^0}$ receives the contribution from the $u$-quark loop shown in Figs. 4(a) and (b). The penguin corrections from the $u$-quark loops as well as from the $c$-quark ones are well-behaved in perturbation theory without any infrared singularity, which has been known as the Bander-Silverman-Soni mechanism [11]. It means that their long-distance contribution is unlikely to be large. The light-cone-sum-rules analysis has also suggested that these corrections are dominated by short-distance contribution [12]. The $u$-quark loop contribution was estimated na"ïvely as $\Delta \mathcal{S}_{J/\psi K^0} = -(4.24 \pm 1.94) \times 10^{-4}$ [3]. We shall reinvestigate this contribution below.

The $u$-quark loop contribution can be expressed in terms of the effective Hamiltonian

\[ H_{\text{eff}}^{(u)} = -\frac{G_F V_{us}^* V_{ub}}{\sqrt{2}} \left[ \frac{2}{3} - \ln \frac{l^2}{\mu^2} + i\pi \right] \times \left[ \alpha \frac{\alpha_s}{3\pi} e_u e_c (N_c C_1(\mu) + C_2(\mu)) (\bar{c}\gamma_\mu c) (\bar{s}\gamma^\mu(1 - \gamma_5)b) \right. \\
+ \left. \frac{\alpha_s}{3\pi} C_2(\mu) (\bar{c}\gamma_\mu T^a c) (\bar{s}\gamma^\mu(1 - \gamma_5)T^a b) \right], \]

where $G_F$ is the Fermi constant, $\alpha$ the fine-structure constant, $e_u(e_c) = 2/3$ the electric charge of the $u(c)$ quark, and $T^a$ the SU(3) generator. The first and second terms arise from the photon emission diagram in Fig. 4(a) and the gluon emission one in Fig. 4(b), respectively. $l^2$ denotes the invariant mass squared of the photon or gluon. The photon emission contribution is less than 5% of $A^{(t)}_{J/\psi K^0}$ due to the smallness of $\alpha$, and can be safely dropped. For the gluon emission, an additional gluon is necessary to form the color-singlet $J/\psi$ meson. If the additional gluon is hard, the resultant contribution in Fig. 4(a) is of next-to-next-to-leading order in $\alpha_s$ and beyond the scope of this work. If the additional gluon is soft, the corresponding nonperturbative input is the three-parton $J/\psi$ meson distribution amplitude as shown in Fig. 5(b). The twist-3 distribution amplitude, which contributes only to transversely polarized $J/\psi$ mesons, is irrelevant here. The twist-4 distribution amplitudes are antisymmetric under the exchange of the momentum fractions of the $c$ quark and of the $\bar{c}$ quark. Because the hard kernel associated with the $u$-quark loop is symmetric under the above exchange, its convolution with the distribution amplitudes diminishes. Therefore, the gluon emission contribution as well as the $J/\psi$ meson can be neglected. The $c$-quark loop corrections can also be neglected, since it modifies only the branching ratio slightly.
In summary, the amplitudes in Eq. (8) are given by
\[
A_{J/\psi K^0}^{(s)} \approx 0,
\]
\[
A_{J/\psi K^0}^{(c)} \approx A_{J/\psi K^0}^{(c)f} + A_{J/\psi K^0}^{(c)nf},
\]
\[
A_{J/\psi K^0}^{(t)} \approx A_{J/\psi K^0}^{(t)f} + A_{J/\psi K^0}^{(t)nf},
\]
where \(A_{J/\psi K^0}^{(c,t)f}\) and \(A_{J/\psi K^0}^{(c,t)nf}\) are found in Eqs. (9) and (13), respectively.

**IV. NUMERICAL RESULTS**

We predict the branching ratio, \(\Delta S_{J/\psi K_S}\), and \(A_{J/\psi K_S}\), with the corrections to the decay amplitude,
\[
\text{Br}(B^0 \rightarrow J/\psi K^0) = (6.6^{+3.7(+3.7)}_{-2.3(-2.3)}) \times 10^{-4}, \\
\Delta_{J/\psi K_S}^{\text{decay}} = (7.2^{+2.4(+1.2)}_{-3.4(-1.1)}) \times 10^{-4}, \\
A_{J/\psi K_S}^{\text{decay}} = -\left(16.7^{+6.6(+3.8)}_{-8.7(-4.1)}\right) \times 10^{-4},
\]
where the choices of parameters used in the computation, including their allowed ranges, are referred to Ref. [1].

The errors are estimated from the variation of the CKM and hadronic parameters, while those in the parentheses arise only from the hadronic ones. The predicted branching ratio in agreement with the data \(\text{Br}(B^0 \rightarrow J/\psi K^0) = (8.63 \pm 0.35) \times 10^{-4}\) [13]. The form factor \(F_{B^0K}(m_{J/\psi}^2)\) increased by 15% and larger Gegenbauer moments in the kaon distribution amplitudes can easily account for the central value of the data. Our \(\Delta S_{J/\psi K_S}^{\text{decay}}\) (\(A_{J/\psi K_S}^{\text{decay}}\)) from the penguin corrections is about twice of the naïve estimation from the u-quark loops [2], and has an opposite (the same) sign. Both \(\Delta S_{J/\psi K_S}^{\text{decay}}\) and \(A_{J/\psi K_S}^{\text{decay}}\) in Eq. (17) indicate the \(O(10^{-3})\) penguin pollution in the \(B^0 \rightarrow J/\psi K_S\) decay, consistent with the conjecture made in Ref. [3].

Including the correction to the \(B-\bar{B}\) mixing in Eq. (7), we predict \(\Delta S_{J/\psi K_s}\) and \(A_{J/\psi K_S}\), up to leading-power in \(1/m_b\) and to NLO in \(\alpha_s\), as
\[
\Delta S_{J/\psi K_S} = \Delta S_{J/\psi K_s}^{\text{mix}} + \Delta S_{J/\psi K_S}^{\text{decay}}, \\
A_{J/\psi K_S} = A_{J/\psi K_s}^{\text{mix}} + A_{J/\psi K_S}^{\text{decay}},
\]
where \(\Delta S_{J/\psi K_S}^{\text{mix}}\) and \(\Delta S_{J/\psi K_S}^{\text{decay}}\) are found in Eqs. (9) and (13), respectively.

Taking into account the CP violation from the \(K-\bar{K}\) mixing [5], \(\Delta S_{J/\psi K_S}\) and \(A_{J/\psi K_S}\) remain \(O(10^{-3})\).

**V. CONCLUSION**

In this talk, we have presented the most complete analysis of the branching ratio and the CP asymmetries for the \(B^0 \rightarrow J/\psi K_S\) decay up to leading power in \(1/m_b\) and to NLO in \(\alpha_s\). We have found that both \(\Delta S_{J/\psi K_S}\) and \(A_{J/\psi K_S}\) are of \(O(10^{-3})\). Our prediction for \(\Delta S_{J/\psi K_S}\) is smaller than an expected systematic error in future data [14]. As for the direct CP asymmetry, our result supports the claim that \(A_{J/\psi K_S} \gtrsim 1\%\) would indicate new physics [15]. These predictions provide an important standard-model reference for verifying new physics from the \(B^0 \rightarrow J/\psi K_S\) data.

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