Yang-Type Monopoles In 5 Dimensional Curved Space-Time

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Abstract: Motivated by Gibbons and Townsend’s recent work, we construct Yang-Type monopoles in maximally symmetric space-time. We then analyze the dependence of horizon structure of the space-times around the 5-dimensional monopoles on the relative strength of gravitations to Yang-Mills interactions. We also analyze the stability of the monopoles against tensor type perturbations on metrics.
1. Introductions

Recently, based on Yang’s work \cite{yang}, Gibbons and Townsend\cite{gibbons} construct a class of solutions to Einstein-Yang-Mills system, the Yang type monopoles of gauge group $SO(2k+2)$. When the first version of their paper appears in the arXive, we are considering a related question in four dimensions—we are searching for numerical solutions to Einstein-Yang-Mills-Higgs-Kibble system \cite{higgs}, so we realized that their results can be generalized to asymptotically maximal symmetric space-times. Very quickly, Gibbons and Townsend revised their papers and made this generalizations. But in the 5-dimension case, the structure of space-time around the Yang-type monopoles has properties which are different from that of other dimensions. Besides this point, there is a free parameter of mass dimension, see eq(2.16) in the following, which is unconstrained in Gibbons and Townsend’s work. However, if the monopole solutions are to be stable against metric perturbations, this parameter are not totally free. Considering this two points, we write this paper to discuss the relevant questions. We also noticed an earlier work discussing Einstein-Yang-Mills systems in curved space-time \cite{curved}.

2. General Formalisms

We start with the following Einstein-Yang-Mills action,

$$S = -\frac{1}{2\kappa^2} \int d^Dx \sqrt{-g} (R + 2\Lambda) - \frac{1}{4g_Y^2} \int d^Dx \sqrt{-g} F_{\alpha \beta} F^{\alpha \beta}$$

where $D$ is the space-time dimension. Although our title contains key-words “5-dimensional”, most of our formalisms apply to general dimensions. In the above actions, $\Lambda$ is cosmological
constants, it can be greater than, or equal to or less than 0, while $\kappa^2$ is the gravitational coupling constant. $F^a_{mn}$ fills adjoint representation of group $SO(D - 2)$

$$F^a_{mn} = \partial_m A^a_n - \partial_n A^a_m + f^{abc} A^b_m A^c_n \tag{2.2}$$

Relative to Gibbons’ and Townsend’s works [1], version 1, we include a cosmological constant in the Einstein-Hilbert action. In the second version of Gibbons’s work, they have included cosmological constants in their basic actions. Because we begin to write this paper when their first version appears, we preserve this section in the current work as an adaptations for the relevant symbols.

From the above action, we get the system’s equation of motion

$$\frac{1}{\sqrt{-g}} \partial_m (\sqrt{-g} F^{mn}) - i [A_m, F^{mn}] = 0 \tag{2.3a}$$

$$R_{mn} - \frac{1}{2} (R + 2\Lambda) g_{mn} + \frac{\kappa^2}{2g_{YM}} T_{mn} = 0 \tag{2.3b}$$

where we have used the abbreviations $A_m = A^a_m T^a$, $F_{mn} = F^a_{mn} T^a$, remember that

$$T_{mn} = \text{tr} (F_m F^n) - \frac{1}{4} g_{mn} \text{tr} (F_p F^p) \tag{2.4}$$

$$[T^a, T^b] = f^{abc} T^c, \quad \text{tr}(T^a T^b) = \frac{1}{2} \delta^{ab} \tag{2.5}$$

Based on Gibbons’s and Townsend’s work [1], we seek the solution of eqs 2.3 which has the form

$$ds^2 = - f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Omega^2_{D-2} \tag{2.6a}$$

$$A_t = 0, \quad A_r = 0, \quad A_i = \frac{\Sigma_{ij} n^i}{1 + \sqrt{1 - n^2}} \tag{2.6b}$$

Where $\Sigma_{ij}$s are constant matrix which span the Lie algebra $so(D - 2)$, i.e.

$$[\Sigma_{ij}, \Sigma_{kl}] = 2i (\delta_{li} \Sigma_{kj} - \delta_{lj} \Sigma_{ki}) \tag{2.7}$$

$\{\Sigma_{ij}\}$ can be thought as some linear transformation of $\{T^a\}$ defined in eq 2.3; $\Sigma_{ij} = \eta_{ij}^a T^a$. Note that in $SO(N)$ group, the number of independent generators is $\frac{1}{2} N(N - 1)$, we just have the same number of matrices $\Sigma_{ij}$ and $T^a$. In eqs 2.6, $n^i$ is a coordinate system which parametrizes the sphere $S^{D-2}$, they are defined through the following relations

$$d\Omega^2_{D-2} = (\delta_{ij} + \frac{n^i n^j}{1 - n^2}) dn^i dn^j =: h_{ij} dn^i dn^j, \tag{2.8}$$

where $n^2 = \delta_{ij} n^i n^j$. It should be noted that coordinate system $\{n^i\}$ covers only one half of the sphere $S^{D-2}$, because on the equator of this sphere, $n^2 = 1$ so the metric there is singular.
Obviously, the only non-zero Yang-Mills field strength components are $F_{ij}$. Starting from eq (2.6b), using (2.7) it can be shown that

$$\begin{align}
F_{ij} &= \Sigma_{ij} - \frac{2\delta_{[ij}}\Sigma_{j]k}h^k n^k}{1 - n^2 + \sqrt{1 - n^2}} \quad \text{(2.9a)} \\
F^{ij} &= r^{-4}h^{ik}h^{jl}F_{kl} \\
&= \frac{1}{r^4}(\Sigma_{ij} + \frac{2\delta_{[ij}}\Sigma_{j]k}n^k n^k}{1 - n^2 + \sqrt{1 - n^2}) \quad \text{(2.9b)}
\end{align}$$

As a result of eqs (2.6b) and (2.9), Yang-Mills equation of motion (2.3a) are satisfied automatically. So to solve the system governed by eq (2.3), we only need to consider its Einstein part. Note that, for the metric ansatz (2.6a),

$$G_{mn} = R_{mn} - \frac{1}{2}g_{mn}(R + 2\Lambda)$$

$$= \text{diag.} \left[ \frac{(D - 2)((D - 3)(f - 1) + rf') + 2\Lambda r^2}{2r^2f}, \quad \begin{array}{c}
(D - 2)((D - 3)(f - 1) + rf') + 2\Lambda r^2 \\
(D - 3)((D - 4)(f - 1) + 2rf') + 2rf'' + 2\Lambda r^2
\end{array}\right]$$

while

$$T_{mn} = \text{tr}(F_{mp}F_{pn}) - \frac{1}{4}g_{mn}\text{tr}(F_{pq}F^{pq})$$

$$= \text{diag.} \left[ \frac{1}{4f} \left( \text{tr}F_{ij}F^{ij} - \frac{1}{4f} \text{tr}F_{ij}F^{ij} \right), \quad \text{tr}(F_{ik}F_{jk}) - \frac{1}{4}g_{ij}\text{tr}(F_{kl}F^{kl}) \right]$$

$$= \text{diag.} \left[ \frac{1}{4f} \left( \text{tr}F_{ij}F^{ij} - \frac{1}{4f} \text{tr}F_{ij}F^{ij} \right), \quad -\frac{1}{4f} \text{tr}F_{ij}F^{ij} \right]$$

According to eqs (2.7) and (2.9), it can be verified that

$$\text{tr}(F_{ij}F^{ij}) = \frac{1}{r^4}\text{tr}(\Sigma_{ij}\Sigma_{ij}) = \frac{(D - 2)(D - 3)}{r^4} \quad \text{(2.12)}$$

Substituting eqs (2.10), (2.11) and (2.12) into eq (2.3b), take its first component, we obtain

$$\frac{(D - 2)(D - 3)(f - 1) + rf'}{2r^2} + \frac{\kappa^2}{2g_{YM}^2} \frac{(D - 2)(D - 3)}{4r^4} = 0 \quad \text{(2.13)}$$

To solve this equation, we set

$$f = 1 - \frac{\kappa^2 M(r)}{r^{D - 3}} \quad \text{(2.14)}$$

substituting it into eq (2.13), we will get

$$\frac{dM(r)}{dr} = \frac{D - 3}{4g_{YM}^2}r^{D - 6} + \frac{2\Lambda}{\kappa^2(D - 2)}r^{D - 2} \quad \text{(2.15)}$$
so

\[
f = \begin{cases}
1 - \frac{\kappa^2 m}{r} + \frac{\kappa^2}{4g^2_{YM}} \frac{1}{r^2} - \frac{\Lambda r^2}{3}, & D = 4 \\
1 - \frac{\kappa^2 m}{r^2} - \frac{2\kappa^2}{4g^2_{YM}} \ln|g_{YM}r| - \frac{\Lambda r^2}{6}, & D = 5 \\
1 - \frac{\kappa^2 m}{r^{D-3}} - \frac{(D-3)\kappa^2}{4(D-5)g^2_{YM}} \frac{1}{r^2} - \frac{2\Lambda r^2}{(D-1)(D-2)}, & D \geq 6
\end{cases}
\] (2.16)

where \( m \) is an integration constant of mass dimensions. From this expression, we see that in the 4-dimensional case, the space time is that of charged black-holes. Note that, the field produced by the charges contribute positively to gravitational potentials. While in 6 or greater-than-6 dimensional case, the field produced by the magnetic charges contribute negatively to gravitational potentials. Because, in the 4 dimensional case, our gauge group is \( SO(4-2) \), an abelian group, while in the 6 or greater-than-6 dimensional case, the gauge group is \( SO(D-2), D \geq 6 \), which are non-abelian groups. We suspect the positive or negative property of the magnetic monopoles’ contribution to the gravitational potentials has relevance to the non-abelian-ness of the underlying gauge theories. We know, in the higher dimensional Riessner-Nordström black holes, the abelian \( U(1) \) charge contribute to the gravitational potentials positively. So if we can construct magnetic monopoles out of some non-abelian gauge theories in 4-dimensional space-time and calculate the metrics around them exactly, we may find that contributions from the monopoles to gravitational potentials are negative instead of positive.

From our calculations, especially the ansatz eqs(2.6) and the final results (2.16), we see that, adding cosmological constant in the Eistein-Hilbert action does not affect Yang-Mills field’s configuration of the monopoles, i.e. in the asymptotically (Anti)de-Sitter space-time, Yang-Mills fields’ configuration is completely the same as that in the asymptotically flat space-time. This is because, the Yang-Mills field of the monopoles has only non-zero \( A_i \) or equivalently \( A_\theta, A_\phi \ldots \) components, its \( A_\ell \) and \( A_r \) components are all zero. As a result, as we add cosmological constants in the Einstein-Hilbert action, although the metric’s \( g_{tt} \) and \( g_{rr} \) are changed, its \( g_{\theta\theta}, g_{\phi\phi} \ldots \) components does not change. So Yang-Mills equations of motion (2.3a) does not change as we add cosmological constants in the Einstein-Hilbert action.

3. Five-Dimensional Yang-Type Monopoles

In the five dimensional case, our gauge group is \( SO(3) \), i.e. Yang-Mills field fills the adjoint representation of \( SO(3) \). While the metric function has the form given in the second line of eq(2.16). From the metric function, we see that space-time around the magnetic monopoles has very interesting singular structures depending on the relative strength of gravitations \( \kappa \) to Yang-Mills interactions \( g_{YM} \).

First, for simplicity, we set the mass dimensional constant \( m \) and the cosmological constant \( \Lambda \) in eqs(2.16) both to zero. In this case, we write \( f(r) \) as

\[
f(r) = 1 - \frac{\kappa^2}{2g^6_{YM}} \frac{\ln|g_{YM}r|}{g^4_{YM}r^2}
\] (3.1)
Figure 1: The metric function of 5-dimensional pure Yang-Type magnetic monopoles for five specific values of $\kappa^2 / g_{YM}^6$.

It can be easily find that

$$f(r) \text{ has only one zero point } r = g_{YM}^2 e^2$$  \hspace{1cm} (3.2)

So, as $\kappa^2 / g_{YM}^6$ increases from zero to $4e$, the function $f$ varies from having no to having one zero point. The corresponding space-time varies from having no to having one horizons. Then as $\kappa^2 / g_{YM}^6$ increases from $4e$ to infinite, the space-time varies from having one to having two horizons. We displayed in Figure 1 $f(r)$’s dependence on $r$ for some specific $\kappa^2 / g_{YM}^6$ values. As long as $\kappa^2 / g_{YM}^6 \neq 0$, the point $r = 0$ is a time-like singular point.

Second, we consider the mass parameter $m$’s effects on the singular structure of space-time around the monopoles. In this case, we write the metric function $f(r)$ as

$$f(r) = 1 - \frac{\kappa^2}{2g_{YM}^6} \frac{2mg_{YM}^2 + \ln(g_{YM}^2 r)}{g_{YM}^{-4} r^2}$$  \hspace{1cm} (3.3)

$$= 1 - \frac{\kappa^2 \cdot e^{4mg_{YM}^2} \ln(e^{2mg_{YM}^2} \cdot g_{YM}^2 r)}{2g_{YM}^6 e^{4mg_{YM}^2} \cdot g_{YM}^{-4} r^2}$$  \hspace{1cm} (3.4)

so

$$as \quad \frac{\kappa^2 \cdot e^{4mg_{YM}^2}}{g_{YM}^6} = 4e, \quad e = 2.71828...$$

$$f(r) \text{ has only one zero point } r = e^{-2mg_{YM}^2} \cdot g_{YM}^2 e^2$$  \hspace{1cm} (3.5)

As a result, if $m > 0$, we will find that relative to the zero $m$ case, the minimum point of the function $f(r)$ moves to the bottom-left direction; while if $m < 0$, the minimum moves to
the top-right directions. The larger is $|m|$, the more the moving amount. We displayed in Figure 3 $f(r)$’s dependence on $r$ for three different values of $mg_{YM}^2$ and fixed $\kappa^2/g_{YM}^6 = 2$. From the figure, we can see that if for some too less value of $\kappa^2/g_{YM}^6$ and $m = 0$, the space time has no horizons, but as long as we change $m$ to large enough values, we will find horizons, behind which there is another horizon or a time-like singularity. Contrarily, if for some very large value of $\kappa^2/g_{YM}^6$ and $m = 0$, the space-time has two horizons, but as long as we change $m$ to appropriate values $m < 0$, we will find that the horizons become degenerate and even disappears at all.

![Figure 2](image_url)

**Figure 2**: The metric function of 5-dimensional non-pure Yang-Type magnetic monopoles of $\kappa^2/g_{YM}^6 = 2$ but with three different values of $mg_{YM}^2$

Third, let us consider the effects of cosmological constants on the singular structure of space-time. For this purpose, we write

$$f(r) = 1 - \frac{\kappa^2}{2g_{YM}^6} e^{4mg_{YM}^2} \ln[e^{2mg_{YM}^2} \cdot g_{YM}^{-2} e^{-r}] - \frac{\Lambda}{6} r^2$$

(3.6)

So, if the cosmological constants are small enough, concretely $|\Lambda| << e^{4mg_{YM}^2} \cdot g_{YM}^{-4} e^{-1}$, then except the asymptotical behavior, the singular structure of space-time around the monopoles keeps almost the same as that of eq(3.4). On the contrary, if $|\Lambda| >> e^{4mg_{YM}^2} \cdot g_{YM}^{-4} e^{-1}$, then the space time will only have cosmic horizon determined by $\Lambda > 0$, the Yang-Mills monopoles introduces no horizons to the space-time around. In the region, $|\Lambda|$ is comparable with $e^{4mg_{YM}^2} \cdot g_{YM}^{-4} e^{-1}$, the singular structure of space time will depend on the three parameters $\kappa^2 g_{YM}^{-6}$, $mg_{YM}^2$ and $\Lambda g_{YM}^4$ in more complicated patterns.

Finally, let us show that the near horizon geometry of the above first two cases is that of $AdS_2 \times S^3$ if the parameters of the system are chosen so that the two horizons degenerate. For this purpose we write the metric function $f(r)$ as

$$f(r) = 1 - 2e^{\frac{\ln r}{r^2}}$$

(3.7)
where we have translated all the relevant quantities dimensionless. Note that in this paper, $e = 2.71828\ldots$ is the base of natural logarithm, which has nothing to do with electro-magnetic coupling constant. Obviously, $f(r)|_{r = e} = 0$, $f'(r)|_{r = e} = 0$, $f''(r) = 4e^{-1}$, so if we let expand $f(r)$ around $r = e$, we have

$$f(r) = 4e^{-1}(r - e)^2 =: x^2$$

(3.8)

as a result

$$ds^2 = -x^2 dt^2 + x^{-2}dr^2 + e^2 d\Omega^2_3$$

(3.9)

which has the standard form of $AdS_2 \times S^3$.

### 4. Stability of The Monopoles Against Metric Perturbations of Tensor Type

In this section we study the stability of the Yang type monopoles under the perturbation on space-time metrics. The study of perturbations on the Yang-Mills field configurations are more complicated than that space-time metrics, we leave them for future works. For simplicity, we will focus on metric perturbations of tensor types.

So, start with

$$ds^2 = -f(r)dt^2 + f^{-1}(r)dr^2 + r^2 d\Omega^2_{D-2}$$

$$f = \begin{cases} 
1 - \frac{\kappa^2 m}{r} + \frac{\kappa^2}{4g^2_{YM}} \frac{1}{r^2} - \frac{\Delta r^2}{3}, & D = 4 \\
1 - \frac{\kappa^2 m}{r^2} - \frac{2\kappa^2}{4g^2_{YM}} \frac{ln[g^2_{YM}]}{r^2} - \frac{\Delta r^2}{6}, & D = 5 \\
1 - \frac{\kappa^2 m}{r^{D-3}} - \frac{4(D-3)\kappa^2}{4(D-5)g^2_{YM}} \frac{1}{r^2} - \frac{2\Delta r^2}{(D-1)(D-2)}, & D \geq 6
\end{cases}$$

(4.1)

we write the tensor type perturbations of metric as

$$\delta g_{ab} = 0, \delta g_{ai} = 0, \delta g_{ij} = r^2HT_{ij}$$

(4.2)

We will use index $a$, $b$ to denote $t$, $r$ coordinate, while $i$, $j$ to denote $\theta$, $\phi$ etc, $\hat{D}_i$ is the covariant grad operators on the transverse sphere, $D_a$ is the covariant grad operators in the $\{t, r\}$ space, $\hat{D}$. In the above ansaltz, $T_{ij}$ is tensor harmonic function on the transverse sphere $S^{D-2}$ and satisfy

$$[\hat{D}_j \hat{D}^j + l(l + D - 3)]T_{ij} = 0$$

(4.3)

where $l$ is the angular quantum number which takes integer values. Under the perturbations eq(4.2), the first order perturbed Einstein Equations leads to

$$\{D^a D_a H + (D - 2)r^{-1}fH' - l(l + D - 3)r^{-2}H\} = 0$$

(4.4)

Let $H = \sum_\omega e^{-i\omega t}r^{-\frac{D+2}{2}}\phi(\omega, r)$, eq(4.4) becomes

$$\partial_r(f\partial_r\phi) + [\omega^2 f^{-1} - \frac{(l + D - 2)}{r^2} + \frac{(D - 2)(D - 4)f}{4r^2} + \frac{(D - 2)f'}{2r}]\phi = 0$$

(4.5)
Introducing tortoise coordinate $dr_*=f^{-1}dr$, the above equation further simplifies to

$$\partial_\star \partial_\star \phi + [\omega^2 - V(r_*)]\phi = 0$$

$$V(r_*) = \left(\frac{l(l+D-3)}{r^2} + \frac{(D-2)(D-4)f}{4r^2} + \frac{D-2}{2r} f'f\right)$$  \hspace{1cm} (4.6)

In the stability analysis of black holes in higher dimensions \([5]\), we know that if the operator

$$A = -\partial_\star \partial_\star + V(r_*)$$  \hspace{1cm} (4.7)

appearing in eq(4.6) is positive and symmetric with respect to the inner product

$$<\phi_1, \phi_2> = \int_{-\infty}^{\infty} dr_\star \bar{\phi}_1(r_*) \phi_2(r_*),$$  \hspace{1cm} (4.8)

then the space-time configuration is stable under tensor perturbations. A statement which is more simple in operations about the stability of black holes against perturbations is, as long as the potential appears in eq(4.7) is positive (or although contain negative part but the minimum is finite) in the range $r_{\text{horizon}} < r < \infty$, the space-time around the black hole is stable, otherwise it is unstable.

Now we borrow this result into our analysis of Yang type monopoles. In the case of 4 dimensions, if the mass parameter $m \geq (\kappa g_{YM})^{-1}$, the space time around the monopole is a standard Reissner-Nordström black hole, and the potential $V(r_*)$ is positive in the range $r_{\text{horizon}} < r < \infty$, so the space-time configuration around it is stable. On the contrary, if $m < (\kappa g_{YM})^{-1}$, the space-time around the monopoles has no horizon, and the potential function of eq(4.7) $V(r_*) \to -\infty$ as $r \to 0$, so we conclude that the space-time configuration around the 4-dimensional Yang-type monopoles is unstable against tensor perturbations if the mass parameter $m < (\kappa g_{YM})^{-1}$. Considering the effects of cosmological constants does not alter this conclusion \([5]\).

In 6 or higher dimensional cases, the space-time around the monopoles are different from that of higher dimensional Reissner-Nordström black holes since the sign of the magnetic charge contributions to gravitational potentials are negative. Because of this, as long as the mass parameter $m > 0$, the space time around the monopoles has non-zero horizon radius and the potential function of eq(4.7) $V(r_*) > 0$ as $r_{\text{horizon}} < r < \infty$, so we conclude that the space-time configuration around the 6 or greater-than-6 dimensional Yang-type monopoles is stable against tensor perturbations if the mass parameter $m > 0$. If $m < 0$ but $|m|$ is small enough so that space-time has non-zero horizon radius, the space-time configurations around the monopoles will be still be stable. It can be shown that the sufficient condition that space-time has non-zero horizon radius is

$$m > -\frac{1}{(D-5)2^{D-4}} \left(\frac{\kappa}{g_{YM}}\right)^{D-3}$$  \hspace{1cm} (4.9)

However, if $m < 0$ but $|m|$ is so large that the space-time has only zero horizon radius, the potential function in eq(4.7) $V(r_*) \to -\infty$ as $r \to 0$, so the space-time configurations will
be unstable against tensor perturbations. Considering the effects of cosmological constants
does not alter this conclusion [5].

From the above analysis, we find that as long as the parameters of the system are
chosen so that non-zero horizon radius exist, the space-time configurations around the
monopoles will be stable. Otherwise, the space-time configuration will be unstable. We
verified that this is the case in 5 dimensions either.

5. Summaries

In the first section of this paper, we generalize Gibbons and Townsend’s construction of
Yang-type monopoles to maximally symmetric space time and find that the asymptotical
property of space-times does not change the configuration of the monopoles. In the second
section of the paper, we analyzed the singular structure of the space-time around the
five dimensional Yang-type monopoles. We find that, even the pure Yang-type monopoles,
which has no explicitly introduced masses, could have non-trivial horizon structures as long
as the relative strength of gravitations and Yang-Mills interactions is chosen appropriately.
This is a phenomenon which is not mentioned by Gibbons and Townsend. In the final
section of the paper, we analyze the stabilities of the Yang-Type monopoles under the
tensor type perturbations on space-time metrics. We find that as long as the parameters
of the system are chosen so that non-trivial horizons exists in the space-time around the
monopoles, the space-time configuration will be stable against the perturbations.

We wish our works will be relevant in the study of AMNV’s weak gravitation conjectures using Ads/CFT. Because, as we have seen in the five dimensional case, the relative
strength of gravitations and Yang-Mills interactions directly determines the singular struc-
ture of the space-time. While ANMV’s conjectures are just about the relative strength
of gravitations and some $U(1)$ gauge interactions. We know that the magnetic monopoles
and $U(1)$ gauge interactions usually occurs accompanying the spontaneous break down of
some larger non-abellian gauge symmetric theories.

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