The effects of thermal radiation and chemical reaction on MHD flow of a casson fluid over and exponentially inclined stretching surface

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Abstract. The present paper is concerned with the study of flow, heat and mass transfer behavior of Casson fluid past an exponentially permeable stretching surface in the presence of magnetic field, thermal radiation and chemical reaction. We presented dual solutions by comparing the results of the Non-Newtonian fluids with the Newtonian fluid. If the partial differential equation is nonlinear, numerical methods have been developed to solve linear and nonlinear differential equations. A widely used algorithm is the shooting method. The influence of involved parameters on dimensionless velocity, temperature and concentration profiles are displayed graphically coupled with comprehensive discussions. Also, to verify the numerical results, a virtual analysis is carried out that ensures the authenticity of the results. Variation of skin friction coefficient, the rate of heat transfer and Sherwood number are also performed. Some already existing solutions of the particular cases of the same problem are also verified as the special cases of the solutions obtained here.

1. Introduction
The study of non-Newtonian fluids has a multiplicity of applications in various industries like petroleum products, design of solid matrix heat, nuclear waste disposal, ground water hydrology, transpiration cooling, etc. Casson fluid is a non-Newtonian fluid behavior is characterized by the existence of a threshold stress. Human blood can also be treated as a Casson fluid due to the blood cells, rouleaux, synovial fluid, fibrinogen etc. The Casson model also called rheological model, developed by Casson [1]. Common examples of Casson fluid are honey, concentrated fruit juices, ketchup, custard, toothpaste, starch suspensions, maizena, paint, frog saliva, shampoo etc. Also, it is the most appropriate rheological model for blood and chocolate. MHD Casson of a three-dimensional fluid flow over a stretching surface in porous medium was examined by Nadeem et al. [2]. The steady stagnation point of a Casson nanofluid flow in the presence of conceive boundary conditions. The fluid strikes the wall in an
oblique manner was illustrated by Nadeem et al. [3]. Recently, Oyelakin et al. [4] investigated heat and mass transfer in a Casson nanofluid flow over an unsteady stretching sheet with heat generation and thermal radiation. Reddy [5] investigated the theoretical study of MHD convective boundary layer flow of a Casson fluid flow over an exponentially inclined permeable stretching surface in the presence of thermal radiation and chemical reaction.

An unsteady two-dimensional Casson fluid of past a stretching surface through porous medium was illustrated by Kirubhashankar et al. [6]. The Reynolds number effects on pressure distribution are examined by using the averaged inertia method was discussed by Walicka and Falicki [7]. Malik et al. [8] discussed the steady boundary layer flow and heat transfer of a Casson nanofluid flowing over a vertical cylinder which is stretching exponentially along its radial direction. The influence of thermal radiation on unsteady flow of Casson fluid caused by stretching sheet subjected to suction/blowing was presented by Mukhopadhyay [9].

The impact of magnetohydrodynamic and chemical reaction in heat and mass transfer flow has huge importance in many areas of engineering and industries. This phenomenon plays an important role in chemical industry, drug delivery, magnetic cell separation, underground energy transport and in treatment of some arterial diseases and hyperthermia. Takhar et al. [10] An analysis has been carried out to obtain the flow and mass transfer characteristics of a viscous electrically conducting fluid on a continuously stretching surface with non-zero slot velocity influence of magnetic field. Hsiao [11] analysed hydromagnetic mixed convection flow along the wedge in the presence of suction and injection. Sheikholeslami [12] worked on two phase models with thermal radiation and magnetohydrodynamic nanofluid flow. Reddy and Vijaya Sekhar [13] studied the effects of heat generation/absorption and a transverse magnetic field on MHD mixed convection flow and heat transfer of a micropolar fluid over a vertical stretching surface with radiation. Hayat et al. [14] analyzed the Cu-water nanofluid flow due to a rotating disk with magnetohydrodynamics (MHD) and partial slip. Hayat et al. [15] discussed the magnetohydrodynamic flow of second grade nanofluid past a nonlinear stretched surface. Reddy et al. [16] was focused on the effect of thermal radiation on the natural convective heat and mass transfer of a viscous, incompressible, electrically conducting and chemically reacting fluid with time dependent suction. Makinde and Aziz [17] examined the convective heat and mass transfer conditions with a boundary layer flow of viscous fluid past a stretched surface. Manjula Jonnadula et al. [18] focussed on the influence of Thermal Radiation and Chemical Reaction on MHD Flow. Effects of variable viscosity and thermal diffusivity on MHD free convection flow along a moving vertical plate embedding in a porous medium with heat generation was investigated by Reddy [19]. Ellahi et al. [20] discussed the numerical study of magnetohydrodynamics generalized Couette flow of Eyring-Powell fluid with heat transfer and slip condition. The effects of thermal radiation over a stretching sheet under several flow conditions have been announced by several researchers [21-25].

The aim of the current model is to analyze the study of flow, heat and mass transfer behavior of Casson fluid past of an exponentially permeable stretching surface in the presence of magnetic field, thermal radiation and chemical reaction. The governing partial differential equation is nonlinear, the successively higher derivatives of the function will involve more and more combinations of the lower derivatives. This process can become cumbersome and time consuming. In an attempt to speed up the solution process, other numerical methods have been developed to solve linear and nonlinear differential equations. A widely used algorithm is the shooting method. The effects of various pertinent parameters on the momentum, heat and mass transfer characteristics have been studied and numerical results are presented graphically and in the tabular form.
2. Mathematical formulation

Consider two-dimensional flow of an incompressible viscous electrically conducting Casson fluid over an exponentially permeable stretching sheet which is inclined at an acute angle $\alpha$ to the vertical. The $x$-axis is taken along the stretching surface in the direction of the motion while the $y$-axis is perpendicular to the surface. The stretching surface has the velocity $U = U_0 e^{x/L}$, the temperature distribution $T_w = T_\infty + T_0 e^{x/L}$, and the concentration distribution $C_w = C_w + C_0 e^{x/L}$ where $U_0$ is the reference velocity, $T_0$ is the reference temperature, $C_0$ is the reference concentration and $L$ is the reference length. A variable magnetic field $B = B_0 e^{x/L}$ is applied normal to the sheet, where $B_0$ is a constant. We assume that the rheological equation of state for an isotropic and incompressible flow of a Casson fluid is as,

$$
\tau_{ij} = \begin{cases} 
2 \left( \mu_b + \frac{P}{\sqrt{2\pi}} \right) e_{ij}, & \pi > \pi_c \\
2 \left( \mu_b + \frac{P}{\sqrt{2\pi}} \right) e_{ij}, & \pi < \pi_c
\end{cases}
$$

where $\mu_b$ is the plastic dynamic viscosity of the non-Newtonian fluid, $P$ is the yield stress of the fluid, $\pi = e_{ij} e_{ij}$, $e_{ij}$ is the $(i, j)\text{th}$ component of the deformation rate and $\pi_c$ is the critical value of this product based on the non-Newtonian model.

The continuity, momentum, energy and concentration equations governing such type of flow can be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( 1 + \frac{1}{\beta} \right) \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right] \pm g \beta_t (T - T_\infty) \cos \alpha + g \beta' (C - C_\infty) \cos \alpha - \frac{\sigma B_0^2}{\rho} u - \frac{v}{k} u, \quad (2)$$

$$\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left( 1 + \frac{1}{\beta} \right) \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right], \quad (3)$$

$$\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} + \tau \left[ \frac{D_B}{\rho c_p} \frac{\partial C}{\partial y} + D_c \frac{\partial T}{\partial y} \frac{\partial T}{\partial y} \right] - \frac{1}{\rho c_p} \frac{\partial q_y}{\partial y}, \quad (4)$$

$$\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + D_c \frac{\partial^2 C}{\partial y^2} - \Gamma (C - C_\infty). \quad (5)$$

The boundary conditions:

$$u = U + N \mu \frac{\partial u}{\partial y}, \quad v = -V(x), \quad T = T_\infty + M \frac{\partial T}{\partial y}, \quad C = C_w + P \frac{\partial C}{\partial y} \quad \text{at} \quad y = 0$$

$$u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \quad (6)$$
Here \( N = N_i e^{-x/L} \) is the velocity slip factor, \( M = M_i e^{-x/L} \) is the thermal slip factor and \( P = P_i e^{-x/L} \) is the solutal slip factor. The no-slip conditions can be recovered, by considering \( N = M = P = 0 \). It is assumed that the permeability is in the form of \( k' = k_i e^{-x/L} \) and the reaction rate is in the form of \( \Gamma = k_0 e^{-x/L} \).

Where \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions respectively, \( \nu = \frac{\mu}{\rho} \) is the kinematic viscosity, \( \mu \) is dynamic viscosity, \( \rho \) is the density of the fluid, \( \beta \) is the Casson parameter, \( \sigma \) is the electrical conductivity, \( g \) acceleration due to gravity, \( \beta_T \) coefficient of thermal expansion, \( \beta_s \) coefficient of solutal expansion, \( T \) is the temperature, \( T_o \) is the temperature of the ambient fluid, \( C \) is the concentration, \( C_\alpha \) is the concentration of the ambient fluid, \( c_p \) is the specific heat at constant pressure, \( k \) is the thermal conductivity, \( q_r \) is the radiative heat flux, \( \tau = (\rho c)_p l (\rho c)_i \) is the ratio of the nanoparticle heat capacity and base fluid heat capacity, \( D_B \) is the Brownian motion diffusion coefficient and \( D_T \) is the thermophoretic diffusion coefficient.

Thermal radiation is simulated using the Rosseland diffusion calculation and in accordance with this, the radiative heat flux \( q_r \) is given by

\[
q_r = -\frac{4\sigma^* \partial T^4}{3k^* \partial y}. \tag{7}
\]

Where \( \sigma^* \) is the Stefan–Boltzmann constant and \( k^* \) is the Rosseland mean absorption coefficient. If the temperature differences within the mass are sufficiently small, then Equation (7) can be linearized by expanding \( T^4 \) into the Taylor’s series about \( T_o \) and neglecting higher order terms, we get

\[
T^4 \approx 4T_o^3T - 3T_o^4. \tag{8}
\]

Using Equations (7) and (8), Equation (4) can be written as

\[
\left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \alpha \left( 1 + \frac{16T_o^2 \sigma^*}{3k^*} \right) \frac{\partial^2 T}{\partial y^2} + \tau \left( D_B \frac{\partial C}{\partial x} \frac{\partial T}{\partial y} + \frac{T_D}{T_o} \left( \frac{\partial T}{\partial y} \right)^2 \right) \tag{9}
\]

We introduce the similarity variables as

\[
\eta = \left( \frac{U_0}{2\alpha L} \right)^{\frac{1}{2}} \frac{x}{2L}, \quad u = U_0 e^{\frac{x}{2L}} f'(\eta), \quad v = -\sqrt{\frac{vU_0}{2L}} e^{\frac{x}{2L}} \left( f(\eta) + \eta f'(\eta) \right), \quad T = T_o + T_0 e^{\frac{x}{2L}} \theta(\eta),
\]

\[
C = C_\alpha + C_0 e^{2\eta} \phi(\eta). \tag{10}
\]

The pressure outside the boundary layer in quiescent part of flow is constant and the flow occurs only due to the stretching of the sheet and hence the pressure gradient can be neglected. Considering the usual boundary layer approximations, \( u >> v, \frac{\partial u}{\partial y} >> \frac{\partial u}{\partial x}, \frac{\partial v}{\partial x} \), the momentum equation in \( y \)-direction reduces to

\[
\frac{\partial p}{\partial y} = 0. \quad \text{Now substituting (10) into the Equations (2), (5) and (9), we get}
\]
\[
\begin{align*}
\left(1 + \frac{1}{\beta}\right) f'' + f f'' - 2(f')^2 &\pm \lambda \theta \cos \alpha + \delta \phi \cos \alpha -(H + K) f' = 0, \\
\left(1 + \frac{4}{3} R\right) \theta'' + Pr \left( f \theta' - f' \theta' + Nb \theta' \phi' + Nt \theta'^2\right) &= 0, \\
\phi'' + Sc \left( \frac{f' - f' \phi}{\phi} \right) + \frac{Nt}{Nb} \theta'' - Sc \gamma \phi &= 0.
\end{align*}
\]  

(11)  

(12)  

(13)  

with the boundary conditions

\[ f = S, \quad f' = 1 + S, \quad f''(0), \quad \theta = 1 + S, \quad \theta'(0), \quad \phi = 1 + S, \quad \phi'(0) \quad \text{at} \quad \eta = 0 \]

\[ f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \phi \rightarrow 0 \quad \text{as} \quad \eta \rightarrow \infty. \]  

(14)  

Where \( H; K; G\beta; G\delta; R; Pr; Nb; Nt; Sc; \gamma \) and \( S \) are the non-dimensional parameters called, respectively, the magnetic parameter, permeability parameter, local Grashof number, buoyancy parameter, local solutal Grashof number, solutal buoyancy parameter, radiation parameter, Prandtl number, Brownian motion parameter, thermophoresis parameter, Schmidt number, chemical reaction parameter and suction/blowing parameter are given by

\[
H = \frac{2\sigma R_0^2 L}{\rho \varepsilon U_0}, \quad K = \frac{2\varepsilon L}{k_0 U_0}, \quad Gr = \frac{2g \beta_f (T_w - T_{\infty}) L x^2}{\nu^2}, \quad \delta = \frac{Gr}{Re_x^2}, \quad Gc = \frac{2g \beta_f (C_w - C_\infty) L x^2}{\nu^2},
\]

\[
\lambda = \frac{4 \sigma T_\infty^3}{k k}, \quad Pr = \frac{\mu c_p}{k}, \quad Nb = \frac{D_h (C_w - C_\infty) (\rho c)_p}{\nu (\rho c)_f}, \quad Nt = \frac{D_f (T_w - T_{\infty}) (\rho c)_p}{T_{\infty} \nu (\rho c)_f},
\]

\[
Sc = \frac{v}{D_h}, \quad \gamma = \frac{2 L k_0}{U_0}, \quad S = \frac{V_0}{U_0 \sqrt{2L}}.
\]

The non-dimensional velocity slip \( S_v \), thermal slip \( S_t \) and solutal slip \( S_c \) are defined by

\[
S_v = N_t \rho \sqrt{\frac{\nu U_0}{2L}}, \quad S_t = M_t \sqrt{\frac{U_0}{2\nu L}} \quad \text{and} \quad S_c = P \sqrt{\frac{U_0}{2\nu L}}. \quad (15)
\]

The quantities of physical interest in this problem are the skin-friction coefficient, heat transfer rate and mass transfer, which are defined as

\[
C_f = \frac{2 \tau_w}{2\varepsilon^2 U_0^2}, \quad Nu_v = \frac{\nu q_w}{k (T_w - T_{\infty})} \quad \text{and} \quad Sh_t = \frac{\nu J_w}{D (C_w - C_\infty)}.
\]  

(16)  

The surface shear stress \( \tau_w \), surface heat flux \( q_w \) and mass flux \( J_w \) are given by

\[
\tau_w = \mu (1 + \frac{1}{\beta}) \left( \frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} \quad \text{and} \quad J_w = -D \left( \frac{\partial C}{\partial y} \right)_{y=0}.
\]  

(17)  

Substituting (10) and (17) into Equation (16), we get
\[
\frac{C_f \sqrt{\frac{\text{Re} \sqrt{x}}{2}}}{\sqrt{\frac{\text{Re} \sqrt{x}}{2}}} = (1 + \frac{1}{\beta}) f'(0), \quad \frac{\text{Nu}_x}{\sqrt{\frac{\text{Re} \sqrt{x}}{2}}} = -\theta'(0) \quad \text{and} \quad \frac{\text{Sh}_x}{\sqrt{\frac{\text{Re} \sqrt{x}}{2}}} = -\phi'(0).
\] (18)

where \( \text{Re} = \frac{xU_0 e^L}{\nu} \) is the local Reynolds number. The above Skin-friction coefficient, local Nusselt number and Sherwood number shows that its variation depends on the variation of the factors \( f'(0), -\theta'(0) \) and \( -\phi'(0) \) respectively. The governing equations (11) to (13) are nonlinear and cannot solved exactly. Apply shooting method, these equations are solved subject to the conditions (14).

3. **Results and Discussion**

The theme of the section is to discuss the effects of various physical parameters on the velocity profile \( f'(\eta) \), temperature profile \( \theta(\eta) \) and concentration profiles \( \phi(\eta) \) for various physical parameters such as magnetic parameter \( H \), permeability parameter \( K \), buoyancy parameter \( \lambda \), inclined angle \( \alpha \), solutal buoyancy parameter \( \delta \), radiation parameter \( R \), Prandtl number \( Pr \), Brownian motion parameter \( Nb \), thermophoresis parameter \( Nt \), Schmidt number \( Sc \), chemical reaction parameter \( \gamma \), suction and blowing parameter \( S \), velocity slip \( S_v \), thermal slip \( S_t \) and solutal slip \( S_c \) for Newtonian and Non-Newtonian fluids. In this study, the default values of the various parameters which we considered are:

\[
\beta = 0.3, \quad H = 1.0, \quad K = 1.0, \quad \lambda = 3.0, \quad \delta = 3.0, \quad \alpha = \frac{\pi}{4}, \quad R = 1.0, \quad Nb = Nt = 0.5, \quad Pr = 0.72, \quad Sc = 0.60, \quad \gamma = 1.0, \quad S_v = S_t = S_c = 0.1 \quad \text{and} \quad S = 0.7.
\]

Effect of the magnetic parameter \( H \) on the velocity profile is displayed by Fig. 1. It is observed that an increasing in \( H \) decreases the velocity profile. Physically by increasing the magnetic field, it develops the opposite force which is called Lorentz force and it decreases the motion of the fluid hence the velocity of the fluid is reduced for both cases. Fig. 2 shows the effect of inclined angle \( \alpha \) on the velocity profile. The velocity profiles decrease with an increase in the \( \alpha \). This can be attributed to the fact that the angle of inclination decreases the effect of the buoyancy force due to thermal diffusion by a factor of \( \cos \alpha \). The effect of the buoyancy parameter \( \lambda \) on the velocity profile is shown in the Fig. 3. It reveals that the velocity profiles increases with an increase in the \( \lambda \) for Newtonian and Non-Newtonian fluids. Fig. 4 describes the effects of permeability parameter \( K \) on velocity profile. As expected, it is found that the velocity profile is decreases by increase in \( K \) for both cases. Some characteristic velocity profiles for different values of solutal buoyancy parameter \( \delta \) are presented in Figure 5. It can be seen that the velocity increases with an increase in solutal buoyancy parameter for Newtonian and Non-Newtonian fluids. Fig. 6 is drawn to discuss the influence of suction/blowing parameter \( S \) on velocity profile. It is observed that the velocity profile decreases with increase of \( S \) for both cases. The effect of the velocity slip \( S_v \) on the velocity profile is shown in the Fig. 7. From this figure, we see that the velocity profile decreases with increasing of \( S_v \), swiftly up to \( \eta = 2 \). After \( \eta = 2 \), the velocity increase.

Figs. 8 has been plotted to demonstrate the effects of the thermophoresis parameter \( Nt \) on the temperature profile. It is observed that as \( Nt \) increases, the temperature of the fluid decreases for Newtonian and Non-Newtonian fluids. It is interesting to mention here that the influence of...
thermophoresis is significantly high in Newtonian fluid than non-Newtonian fluid. The variation in the temperature profile for changing the in the Brownian motion parameter \( Nb \). It is noticed that the temperature increases with an increase in the \( Nb \) for both cases. Fig. 10 is plotted to examine the influence of Prandtl number \( Pr \) on temperature profile. It is clear that a fluid with large values of Prandtl number. Therefore, we observed that a rapid increase in \( Pr \) leads to decrease in the temperature and the temperature of a flow field is a monotonically decreasing function of \( Pr \). The temperature profiles are presented for several values of radiation parameter in Fig. 11. It is found that the temperature enhances as the radiation increases. This may be attributed to the physical fact that the thermal boundary layer thickness enhances with increasing radiation parameter for Newtonian and Non-Newtonian fluids. Fig. 12 is plotted to examine the influence of thermal slip \( S_t \) on temperature profile. We can see that the temperature profile decreases with increase in \( S_t \).

The concentration profile for various values of Brownian motion parameter \( Nb \) as shown in Fig. 13. we noticed that the concentration profile decreases with an increase in \( Nb \) for both cases. Fig. 14 has been plotted to demonstrate the effects of the thermophoresis parameter \( Nt \) on the dimensionless concentration profile. We noticed that the concentration profile increase with an increasing in the value of \( Nt \) for Newtonian and Non-Newtonian fluids. Effect of Schmidt number \( Sc \) on concentration distribution is displayed in Fig. 15. Here concentration profile decreases when \( Sc \) increases. Physically the Schmidt number is dependent on mass diffusion \( D_B \) and an increase in Schmidt number corresponds to a decrease in mass diffusion and the concentration. Fig. 16 expose the variation of chemical reaction parameter \( \gamma \) on concentration profile. It is shows that concentration profile decreases with increase in the values of \( \gamma \). The effect of solutal slip \( S_c \) on the concentration profiles is plotted in Fig. 17. It reveals that the concentration profiles decrease with increase in \( S_c \).

To assess the validity and accuracy of the applied numerical scheme, numerical values for the heat transfer coefficient for various values of radiation and the Prandtl number in the absence of Casson fluid parameter, magnetic parameter, permeability parameter, buoyancy parameter, solutal buoyancy parameter, inclined angle, Schmidt number, suction/blowing parameter, velocity slip, thermal slip and solutal slip are compared with the available results and the outcome is shown in Table 1. The results are found in excellent agreement.

The values of skin friction coefficient, Nusselt number and the Sherwood number for various values of the involved pertinent parameters are presented in Table 2. It can be noted that the skin friction coefficient decreases with increasing values of buoyancy parameter, solutal buoyancy parameter, thermophoresis parameter and radiation parameter whereas the reverse trend is observed in the case of magnetic parameter, Casson parameter, Brownian motion parameter, chemical reaction parameter, thermal slip and solutal slip. It is found that the Nusselt number decreases with an increase in the magnetic parameter, Casson parameter, radiation parameter, Brownian motion parameter, thermophoresis parameter, chemical reaction parameter and thermal slip where as it increases with an increase in the buoyancy parameter, solutal buoyancy parameter and solutal slip. It is viewed that the Sherwood number decreases as the magnetic parameter, Casson parameter, thermophoresis parameter, and solutal slip are increased. It can be seen that the Sherwood number increases with increasing buoyancy parameter, solutal buoyancy parameter, radiation parameter, Brownian motion parameter, chemical reaction parameter and thermal slip are raised.
4. **Conclusions**

This paper presents the study of the effects of thermal radiation and chemical reaction on MHD flow of a Casson fluid over an exponentially inclined stretching surface. Appropriate similarity transformation is used to convert the governing partial differential equations into a system of coupled non-linear differential equations. The resulting coupled non-linear differential equations are solved numerically by using shooting method.

- Increasing Brownian motion parameter, increases heat transfer rate, skin friction and Sherwood number.
- Increasing thermophoresis parameter, decreases heat transfer rate, skin friction, Nusselt number and Sherwood number.
- The influence of thermophoresis is significantly high in Newtonian fluid than non-Newtonian fluid.
- Increasing magnetic parameter, decreases velocity, Nusselt number and Sherwood number, whereas it increases skin friction coefficient.

![Fig. 1 Velocity profile for various values of $H$](image1)

![Fig. 2 Velocity profile for various values of $\alpha$](image2)

![Fig. 3 Velocity profile for various values of $\lambda$](image3)

![Fig. 4 Velocity profile for various values of $K$](image4)
Fig. 5 Velocity profile for various values of $\delta$

Fig. 6 Velocity profile for various values of $S$

Fig. 7 Velocity profile for various values of $S_v$

Fig. 8 Temperature profile for various values of $Nt$

Fig. 9 Temperature profile for various values of $Nb$

Fig. 10 Temperature profile for various values of $Pr$
Fig.11 Temperature profile for various values of $R$

Fig.12 Temperature profile for various values of $S_t$

Fig.13 Concentration profile for various values of $R$

Fig.14 Concentration profile for various values of $N_t$

Fig.15 Concentration profile for various values of $Sc$

Fig.16 Concentration profile for various values of $\gamma$
Fig. 17 Concentration profile for various values of $S_c$.

Table 1 Similarity $\theta'(0)$ for particular values of Prandtl number and thermal radiation with $\beta = \infty$, $\lambda = \delta = H = K = Nt = Nb = S = 0$, $S_c = S_r = S_t = 0$.

| $Pr$ | $R$ | Nadeem et al [22] | Bidin and Nazar [26] | Ishak [27] | Reddy [5] | Present |
|------|-----|-----------------|-----------------------|------------|----------|---------|
| 1    | 0   | 0.9547          | 0.9548                | 0.95477    | 0.954783 |         |
| 2    | 0   | 1.4714          | 1.4715                | 1.47144    | 1.471460 |         |
| 3    | 0   | 1.8691          | 1.8691                | 1.86916    | 1.869074 |         |
| 5    | 0   | 1.1599          | 2.5001                | 2.50016    | 2.500132 |         |
| 10   | 0   | 3.6604          | 3.66038               | 3.660372   |          |         |
| 1    | 0.5 | 0.680           | 0.6765                | 0.67650    | 0.676500 |         |
| 1    | 1.0 | 0.534           | 0.5315                | 0.53150    | 0.53156  |         |
| 2    | 0.5 | 1.073           | 1.0735                | 1.07350    | 1.073519 |         |
| 2    | 1.0 | 0.863           | 0.8627                | 0.86270    | 0.862770 |         |
| 3    | 0.5 | 1.381           | 1.3807                | 1.38070    | 1.380752 |         |
| 3    | 1.0 | 1.121           | 1.1214                | 1.12140    | 1.121427 |         |
Table 2 The values of skin friction coefficient, Nusselt number and Sherwood number for various values of $\beta$, $H$, $\lambda$, $Nb$, $Nt$, $\delta$, $R$, $S_t$, $S_c$, $\gamma$.

| $\beta$ | $H$ | $\lambda$ | $\delta$ | $Nb$ | $Nt$ | $S_t$ | $S_c$ | $\gamma$ | $-f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
|---------|-----|---------|--------|------|------|-------|-------|---------|-----------|-------------|-------------|
| 0.3     | 1.0 | 3.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.3   | 0.7     | 1.0       | 0.509620   | 0.523109    | 0.589403    |
| 0.6     | 1.0 | 3.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.3   | 0.7     | 1.0       | 0.586838   | 0.512534    | 0.588060    |
| 0.3     | 1.5 | 3.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.3   | 0.7     | 1.0       | 0.569538   | 0.514774    | 0.588336    |
| 0.3     | 1.0 | 4.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.3   | 0.7     | 1.0       | 0.437570   | 0.532105    | 0.590692    |
| 0.3     | 1.0 | 3.0     | 4.0    | 0.5  | 0.5  | 1.0   | 0.3   | 0.7     | 1.0       | 0.466235   | 0.528102    | 0.590123    |
| 0.3     | 1.0 | 3.0     | 3.0    | 0.7  | 0.5  | 1.0   | 0.3   | 0.7     | 1.0       | 0.520879   | 0.513149    | 0.624506    |
| 0.3     | 1.0 | 3.0     | 3.0    | 0.5  | 0.7  | 1.0   | 0.3   | 0.7     | 1.0       | 0.491490   | 0.518046    | 0.548227    |
| 0.3     | 1.0 | 3.0     | 3.0    | 0.5  | 0.5  | 2.0   | 0.3   | 0.7     | 1.0       | 0.481859   | 0.416039    | 0.626616    |
| 0.3     | 1.0 | 3.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.5   | 0.7     | 1.0       | 0.534891   | 0.472264    | 0.599053    |
| 0.3     | 1.0 | 3.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.3   | 1.0     | 1.0       | 0.522721   | 0.524850    | 0.485385    |
| 0.3     | 1.0 | 3.0     | 3.0    | 0.5  | 0.5  | 1.0   | 0.3   | 0.7     | 2.0       | 0.535676   | 0.518942    | 0.672893    |

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