Optimal performances of a global allocation of radio resources in heterogeneous networks system

Déthié Dione

dethiedione79@gmail.com

1- Cheikh Anta Diop University, Dakar, Senegal

Abstract

The evolution of wireless and mobile networks becomes faster and faster, so the optimal allocation of radio resources is a problem which is imperative. This development of telecommunication networks is accompanied with an efficient deployment of wireless networks such Wireless Fidelity and networks mobile as LongTerm Evolution (LTE). In this paper, we propose an algorithm improving the global allocation of radio resources within the framework of a heterogeneous mobile and wireless networks system by using the dynamic programming in particular the Bellman principle of optimality. We took into account the mobility of users by using the 2D model Fluid Flow to obtain the best performances which are numerically tested by the Network Simulator 3 (NS3).

Key word:
Optimization, LTE, Wimax, Handover, Bellman principle of optimality.

1 Introduction

We are witnessing an increasing growth of wireless and mobile networks. This development of networks has led to the deployment of various types of networks such as Wireless Fidelity (Wi-Fi), Wimax, Global System Mobile (GSM), Universal Mobile Telecommunications System (UMTS) and most recently LongTerm Evolution (LTE).

These different technologies have contributed to the improvement of the communication of the users who become more and more demanding as regards the quality of the service (Qos). We take into account the mobility of users and the types of services to which they are connected for their satisfaction. To model the mobility of the users we used the 2D Fluid Flow. Indeed, in this model of mobility, users move in all directions in the Service area with the same probabilities. It is often used to represent characteristics of traffic on major routes and other similar situations with a constant flow of mobile terminals.

The integration of wireless and mobile networks is nowadays a necessity for the satisfaction of users whose demand is increasingly strong. The global mobility of services and users makes this task difficult. However, these networks have features to support different and mobile services
and users. New methods of backing up, transmitting and allocating radio resources are required. We have noted in the literature the most widely used allocation techniques. The efficient use of radio resources has been the subject of much research because efficiency of these resources makes it possible to improve the performance of telecommunication networks. We present here some algorithms of scheduling radio resources:

**The classical algorithms:** They are simple and old derivatives of the wire. This is the Round Robin and the random allocation. The Round Robin is best known in wireless networks [2] [3]. Its principle is to allocate the same quantity of resource units to all users. The random allocation, the Random Access (RA) [4], consists of allocating the radio resource randomly between the users.

**Equitable algorithms:** To correct the deficiencies of the previous algorithms, they provide an improvement by taking into account the notion of equity or service difference [1]. These are Fair Queuing (FQ), Max-Min Fair (FQ), and Weighted Fair Queuing (WFQ). In the Fair Queuing (FQ) algorithm [5] [10], for a rate $D$ transmission, each of the $N$ active users will be served at $\frac{D}{N}$. It is therefore more equitable than the Round Robin. In the Max-Min Fair [11] [15], the allocation of the resource units consists of allocating them in an iterative manner, so that the overall throughput for each user increases gradually and identically until no resource unit is assigned to a user who has reached the requested bit rate. Weighted Fair Queuing is an improvement in Fair Queuing [13] [14]. It uses a weighting system that allows more units of resources to be allocated to certain transmissions. It therefore takes into account the quality of service.

**Opportunistic algorithms:** They have been developed to take advantage of the frequency diversity and several users to allocate resource units by favoring the most favorable transmission / reception conditions (better signal/noise ratio). These are the Maximum Signal Noise Ratio (MaxSNR) and the Proportional Fair (PF). The MaxSNR also known as Maximum Carrier to Interference ratio (MaxC/I). In this algorithm priority is given to the transmission with the highest signal/noise ratio [17] [18]. The principle of the Proportional Fair is to allocate a time interval of a given sub-carrier to the user who has the most favorable transmission conditions mean (transmissions in the sub-carrier) [15] [16]. In this principle, the mobiles are only taken when their radio conditions are good and allow to make the best use of the units of resources.

The contribution in our work is based on our model of allocation of radio resources to a user. Moreover, unlike other works, we have taken dynamic programming as an important technique for solving the optimization problem, which consists in obtaining an optimal allocation of radio resources to a user.

We have organized our article as follows: Section 2 introduces the model of the heterogeneous system studied where all the parameters of the system are defined. The allocation of radio resources to a user is established in section 3. By section 4, we have developed the mathematical model of resource allocation to users, which is based on dynamic programming. The method of solving the mathematical model is given in Section 5. The results obtained are simulated and analyzed in Section 6. We finished our study in Section 7 with a conclusion and perspectives.
2 Heterogeneous model of mobile networks and wireless

The model of a heterogeneous network system is given in figure 1. Indeed, we have a circular service area $Z_1$ fully covered by a mobile network. This service area is divided into several homogeneous circular subfields $(Z_i)_{2 \leq i \leq m}$ each of which is covered by a wireless network. Thus the mobile and wireless networks overlap in the $Z_i$ sub-zones and the wireless networks are disjoint to each other. We denote by $Z_0$ that part of the service area not covered by a wireless network. Users with multiple access devices have the ability to switch connections in areas where networks overlap by automatically choosing the network with the highest number of resources.

$$Z_O = Z_1 - \bigcup_{j=1}^{m} Z_j$$

Figure 1: Model cluster service zone
3 Allocation of resources

If a user is in a zone $Z_i$ where mobile and wireless networks overlap, he has the ability to connect either newly or through handover to the mobile or wireless network. If the number of resources is higher for the mobile network then the user connects to it otherwise the resources are blocked to connect to the wireless network as indicated in the anagram of radio resource allocation technique, below:

![Resource Allocation Technique Diagram]

Figure 2: Resource Allocation Technique
4 Mathematical model

4.1 Sets, parameters and variables

\( Z \): Set of zones, \( z \in Z \);
\( I \): Set of users, \( i \in I \);
\( K \): Set of services, \( k \in K \);
\( R \): Set of wireless, \( r \in R \);
\( \tau_{z,z'} \): Exit rate of zone \( z \) to cell \( z' \);
\( \tau_{i,k}^z \): Average rate of request for access to a service \( k \) by a user \( i \) in the zone \( z \);
\( P^i_z \): Probability of finding an user \( i \) in an area \( z \);
\( x_{z,r}^t \): Available resources of network \( r \) in zone \( z \) in step \( t \);

The variable of the optimization problem used is the number of resource units that must be allocated to allow a user to connect.

\( y_{z,r}^{i,t} \): Resources allocated to user \( i \) by network \( r \) in zone \( z \) at step \( t \). These resources depend directly on those available, the probability of finding a user and the average rate of handover. They are calculated by the formula:

\[
y_{z,r}^{i,t} = \sum_{k \in K} x_{z,r}^t (P^i_z + \tau_{z,H}) \tau_{i,k}^z
\]  

Where the average rate of handover (H) is given by:

\[
\tau_{z,H} = U_z^k \times \tau_{z,z'}
\]

According to the 2D Fluid Flow mobility model, the exit rate of the service area \( Z \) to the cell \( Z_i \) is defined by:

\[
\tau_{z,z'} = \frac{E(v) \times L_{Z_i}}{\pi \times A(z)}
\]

\( E(v) \) is the average velocity of mobile users in the service area, \( L_{Z_i} \) is the Length of the perimeter of the cell \( Z_i \) and \( A(z) \) is the area of the service zone. The probability of finding a user in a sub-zone \( A(z) \) is:

\[
P^i_z = \frac{A(z)}{A(z_{tot})}
\]

With:

\( A(z) \): Area of the sub-zone \( z \);
\( A(z_{tot}) \): Area of total zone of service.

The rate requesting access to a service \( k \) \( \tau_{i,k}^z \) is given by:

\[
\tau_{i,k}^z = P^i_z \times \tau_{z,k}^z
\]

Since resource units are limited then they depend on the number of sub-carriers and the spacing between them as they are subdivided. The purpose of the OFDM is to distribute the established radio transmissions on all the sub-carriers. A transmitted signal is distributed over all the sub-carriers of the network (which improves the throughput). The modulation in the OFDM system
adjusts the remote user problem because the modulation type allows the distance parameter to be taken into account, it also ensures the transmission power according to the transport distance. It acts as a cover that protects the signal in its routing.

| Table 1: Parameters Modulation |
|--------------------------------|
| $N_{\text{of},r}^{i,t}$ | The number of OFDM symbol of the network $r$ occupied by the user $i$ at step $t$ |
| $N_{\text{bit},r}^{i,t}$ | The number of bits of the modulation of the network $r$ occupied by the user $i$ at step $t$ |
| $K_r$ | Number of sub-carriers of the network $r$ |

4.2 Objective function and constraints

The available resources are limited, allocated taking into account the maximum to be achieved. This inequality shows that radio resource units can not be allocated to a user at beyond what is available.

We have the parameters that allow us to determine the occupancy function of a user in heterogeneous networks.

$$g^{i,t}(y_{i,t}^{z,r}) = \max(f^{i,t}_1(y_{i,t}^{z,r}), f^{i,t}_2(y_{i,t}^{z,r}), ..., f^{i,t}_{r}(y_{i,t}^{z,r}))$$ (3)

Under the following constraints:

$$\sum_{i \in I} y_{i,t}^{z,r} \leq x_{z,r}^{t}$$ (4)

$$y_{i,t}^{z,r} \geq 0$$ (5)

$$x_{z,r}^{t} \geq 0$$ (6)

Where we are:

$$f^{i,t}_{r}(y_{i,t}^{z,r}) = K_r \times N_{\text{of},r}^{i,t} \times N_{\text{bit},r}^{i,t} \times y_{i,t}^{z,r}$$

We assume that the numbers of sub-carriers of the networks $(K_r)_{r \in \mathbb{N}}$ following an increasing suite, $(K_r \leq K_{r+1})$.

$$x_{z,r}^{t+1} = x_{z,r}^{t} - \sum_{i \in I} y_{i,t}^{z,r}$$

$$= x_{z,r}^{t} - \sum_{i \in I} \sum_{k \in K} x_{z,r}^{t} (P_i^z + \tau_i^z H_z)^{i,k}$$

$$= x_{z,r}^{t} (1 - \sum_{i \in I} \sum_{k \in K} (P_i^z + \tau_i^z H_z)^{i,k})$$

$$= x_{z,r}^{0} [1 - \sum_{i \in I} \sum_{k \in K} (P_i^z + \tau_i^z H_z)^{i,k}]^{n+1}$$
5  Solving method based on Bellman’s equations in dynamic programming

Given the dynamic aspect of resource allocation in heterogeneous networks, we use dynamic programming to solve the mathematical model obtained in the previous section.

5.1 Dynamic programming

Dynamic programming is an exact method of optimization. It involves steps in which optimal decisions or policies are to be made. It decomposes an initial problem into a series of sub-problems of smaller size, which can themselves be further decomposed to reach sub-problems of elementary size whose resolution is simple, which is an advantage of the resolution by programming dynamic. The sequential nature of a problem can therefore lead to the choice of dynamic programming to solve it. The optimization problem presents itself as a decision to be taken for each possible state of the system. Indeed, depending on the availability of resources and the need of a user, we must allocate radio resources for its connection. This similarity means that dynamic programming is adapted to the problem.

5.1.1 Bellman principle of optimality:

In solving problems of optimization by dynamic programming, one obtains the solution of the initial problem by means of those of the subproblems. This method of resolution is based on a principle called Bellman principle of optimality: whose statement is:

"An optimal policy has the property that, whatever the initial state and the initial decision, the remaining decisions must constitute an optimal policy with respect to the state resulting from the first decision."

5.1.2 Bellman’s equation:

Bellman’s equation governs a dynamic system \( t = 0, 1, \ldots, T - 1, T \) discrete time in a finished horizon \( T \).

- \( E_t \) State of the system at the moment \( t \);
- \( \Gamma(E_t) \) set of action at the state \( E_t \);
- \( a_t \in \Gamma(E_t) \) action among the possible ones when the system is at the state \( E_t \);
  
  If \( \Phi \) is the transfer function then we have:

\[
E_{t+1} = \Phi(E_t, a_t)
\]

- Let \( F(E_t, a_t) \) the cost function to choose action \( a_t \) if the system is at the state \( E_t \);
The total cost after $T$ step when it becomes the state $E_{T+1}$ is given so by:

$$Z^*_T(E_{T+1}) = \min_{a_t} \{ \sum_{t=0}^{T} F(E_t, a_t) \}$$

Then:

$$Z^*_T(E_{T+1}) = \min_{a_t \in \Gamma(E_t) ; E_{t+1} = \Phi(E_t, a_t)} \{ \sum_{t=0}^{T} F(E_T, a_T) + Z^*_T(E_{T-1}) \}$$

5.2 Resolution of the resource allocation model:

We noted by:

$t$: A discrete step of the finite-horizon dynamic system $T$, $t = 0, 1, ..., T$

$x^t_{z,r}$: State of the system at the beginning of the step $t$.

$\Gamma(x^t_{z,r})$: Set of the actions if the system is in State $x^t_{z,r}$. It’s defined by:

$$\Gamma(x^t_{z,r}) = \{ w^t : w^t = (\mathbb{P}^i_z + \tau^H_z \tau^{i,k}_z) \}$$

(7)

Noting by $\Phi$ the transfer function then there:

$$x^{t+1}_{z,r} = \Phi(x^t_{z,r}, w^t)$$

(8)

The function cost $F(x^t_{z,r}, w^t)$ to choose the action $w^t$ if the system is at State $x^t_{z,r}$ which corresponds to the occupation function of the user $i$ is thus defined:

$$F(x^t_{z,r}, w^t) = K_r \times N_{of,r} \times N_{bit,r} \times y^{i,t}_{z,r}$$

$$= K_r \times N_{of,r} \times N_{bit,r} \times (x^t_{z,r} - x^{t+1}_{z,r})$$

$$= K_r \times N_{of,r} \times N_{bit,r} \times (x^t_{z,r} \times 1 - w^t)$$

The global number of resource units allocated to a user after $T$ step when arriving at the $x^{T+1}$ state is thus given by:

$$Z^*_T(x^{T+1}) = \max_{w^t} \{ \sum_{t=0}^{T} F(x^t, w^t) \}$$

Then:

$$Z^*_T(x^{T+1}) = \max_{w^t \in \Gamma(x^t) ; x^{t+1} = \Phi(x^t, w^t)} \{ \sum_{t=0}^{T} F(x^T, w^T) + Z^*_T(x^{T-1}) \}$$

The number of resource units allocated is the maximum that allows users flow. Each resource unit carries a quantity of data determined by the division of the bandwidth and the unit of time considered.
Algorithm 1 Principle of optimal allocation of finite-time resources

Require:
Z: Set of zones;
z: A random area;
I: Set of users;
i: A given user;
R: Set of networks;
r: A network selected;
T: Horizon finished;
t: A given step;
K: Set of services;
k: Un service selected;
τ_{z,z'}: Exit rate of zone z to cell z';
τ_{i,k}^{z}: Average rate of request for access to a k service by a user i in zone z;
P_{i}^{z}: Probability of finding a user i in a zone z;
x_{t}^{z,r}: Available network resources r in the area z at step t;
y_{i,t}^{z,r}: Resources allocated to user i by network r in zone z at step t;
Γ: Set of Optimal policy;

Ensure: x_{0}^{z,r} ← 0

for t = 0:T do
if w^{t} ∈ Γ then
\quad w^{t} = (P_{i}^{z} + τ_{z}^{H})τ_{i,k}^{z};
\quad x_{t+1}^{z,r} = \Phi(x_{t}^{z,r}, w^{t});
\quad \Phi(x_{t}^{z,r}, w^{t}) = x_{t}^{z,r}(1 - w^{t});
\quad F(x_{t}^{z,r}, w^{t}) = K_{r} \times N_{of,r}^{dt} \times N_{bit,r}^{dt} \times (x_{t}^{z,r}w^{t});
\quad Z_{T}^{t}(x^{T+1}) = \max_{w^{t}}\{\sum_{t=0}^{T} F(x^{t}, w^{t})\};
\quad Z_{T}^{t}(x^{T+1}) = \max_{w^{t} ∈ Γ(x^{t}), x^{t}+1 = \Phi(x^{t}, w^{t})} \{\sum_{t=0}^{T} F(x^{t}, w^{t}) + Z_{T}^{t}(x^{T-1})\};
end if
end for

6 Application to resource allocation of heterogeneous LTE and WIMAX networks

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique that supports user mobility. The operational part of the OFDM is the OFDMA, which is a frequency division multiple access type of radio encoding technology which is used in particular in the LongTerm Evolution (LTE) network. It is used in other networks such as UMTS.

The aim is therefore to share a common radio resource (frequency band) and dynamically assign parts to the users. The principle of the OFDMA is to distribute on a large number of subcarriers the digital signal that it is desired to transmit. This modulation consists in carrying out a multiple access by assigning users sub-parts of the set of sub-carriers constituting the free
bandwidth. This technique allows to support a large number of users with varying requirements in terms of quality of services.

6.1 Mathematical Model

Considering any user in communication, he occupies either the resources of the LTE mobile network or WIMAX depending on whether one of them gives him full satisfaction.

| \( N_{of,r}^{i,t} \) | The number OFDM symbol of the LTE or WIMAX network occupied by the user \( i \) in step \( t \) with \( r \in \{1,2\} \). |
| \( N_{bit,r}^{i,t} \) | The number of bits of LTE or WIMAX network modulation occupied by the user \( i \) at step \( t \) with \( r \in \{1,2\} \). |
| \( (K_r)_{r \in \{1,2\}} \) | Number of sub-carriers of the network (denoted \( K_1 \) or \( K_2 \) depending on whether the user is connected to LTE or WIMAX). |

The occupation function of a user \( i \) located only in the LTE network which allocated him \( y_{z,1}^{i,t} \) resources in an area \( z \) at step \( t \) is given by:

\[
g_1^{i,t}(y_{z,1}^{i,t}) = K_1 \times N_{of,1}^{i,t} \times N_{bit,1}^{i,t} \times y_{z,1}^{i,t}
\]  

(9)

If the user is connected to the WIMAX wireless network then the busy function becomes:

\[
h_2^{i,t}(y_{z,2}^{i,t}) = K_2 \times N_{of,2}^{i,t} \times N_{bit,2}^{i,t} \times y_{z,2}^{i,t}
\]  

(10)

We find that the difference lies in the number of sub-carriers. Indeed, the LTE network having the largest bandwidth that the WIMAX we have:

\[ K_1 \leq K_2 \]

The occupation function depends on the distance and the amount of data to be transferred through the modulation. Thus, it is not the same depending on whether you connect to the LTE mobile network or the WIMAX wireless network. The heterogeneity of the LTE and WIMAX networks allows the user to choose the best network according to his position in terms of available resources.

We thus define for a user \( i \) the occupancy function in a heterogeneous system WIMAX and LTE networks by:

\[
f_r^{i,t}(y_{z,r}^{i,t}) = \max(g_i(y_{z,1}^{i,t}), h_i(y_{z,2}^{i,t}))
\]  

(11)

As we try to allocate \( y_{z,r}^{i,t} \) radio resources to maximize the occupancy function, we establish the following optimization problem:

\[
\max \sum_{i=1}^{n} f_i(y_{z,r}^{i,t})
\]
\[
\begin{align*}
\sum_{i \in I} y_{z,r}^{i,t} & \leq x_{z,r}^t \quad r \in \{1, 2\} \\
y_{z,r}^{i,t} & \geq 0 \quad r \in \{1, 2\} \\
x_{z,r}^t & \geq 0 \quad r \in \{1, 2\}
\end{align*}
\]

6.2 Numerical results and interpretation

The purpose of the digital test is to determine the number of resources allocated to each user and also the state of the system at each step (throughput). To do this we need the different parameters related to the LTE and WIMAX networks such as modulation, the number of bits and especially the number of users that we have assigned in the table 6.2. The table 6.2 summarizes the results of our simulation. The last two columns obtained by our algorithm are respectively the number of radio resources units allocated to each user and the state of the system, that is to say the amount of data that transit during this time now.

| Number of users | Modulation | Number of Bits |
|-----------------|------------|---------------|
| 0               | 2          | 6             | 0              |
| 1               | 6          | 4             | 2500           |
| 2               | 6          | 2             | 3000           |
| 3               | 2          | 4             | 4000           |
| 4               | 4          | 6             | 2250           |
| 5               | 2          | 6             | 1500           |
| 6               | 4          | 2             | 3200           |
| 7               | 2          | 6             | 4500           |
| 8               | 2          | 4             | 6000           |
| 9               | 6          | 4             | 5000           |
| 10              | 4          | 6             | 1600           |
| 11              | 2          | 4             | 7000           |
| 12              | 6          | 2             | 1450           |
| 13              | 4          | 2             | 800            |
| 14              | 6          | 2             | 700            |
| Number of users | Modulation | Data Size | Number of Bits | Status of the system |
|----------------|------------|-----------|----------------|---------------------|
| 0              | 2          | 6         | 0              | 0                   |
| 1              | 6          | 4         | 2500           | 4                   | 3168                |
| 2              | 6          | 2         | 3000           | 5                   | 7128                |
| 3              | 2          | 4         | 4000           | 20                  | 11448               |
| 4              | 4          | 6         | 2250           | 7                   | 14220               |
| 5              | 2          | 6         | 1500           | 5                   | 16200               |
| 6              | 4          | 2         | 3200           | 8                   | 19656               |
| 7              | 2          | 6         | 6000           | 12                  | 24408               |
| 8              | 2          | 4         | 5000           | 29                  | 30672               |
| 9              | 6          | 4         | 1600           | 7                   | 36216               |
| 10             | 4          | 6         | 7000           | 5                   | 38376               |
| 11             | 2          | 4         | 1450           | 43                  | 45600               |
| 12             | 6          | 2         | 1450           | 3                   | 47976               |
| 13             | 4          | 2         | 3800           | 3                   | 49272               |
| 14             | 4          | 2         | 7000           | 2                   | 50852               |

7 Conclusion

The optimal allocation of available radio resources is an optimization problem because it should take into account the availability of resources and maximize the need of the users, we chose dynamic programming to bring a method allocation. The change of state every time we have a new user and the decision of how to allocate a number of resources according to their needs, justify the application of the problem.

The proposed algorithm assumes that established communication is continuous until there is a depletion of resources for new users. This algorithm takes into account the user’s actual need and the maximization of the flow of the heterogeneous network considered, contributes to the satisfaction of the user and the operator; which is an important contribution to telecommunications.

Taking into account the communication time (because when it ends its communication one User), mobility (path prediction), signaling costs, the actual shape of the cell, the propagation conditions of the radio signals, etc., are factors to be taken into account in Algorithm for the efficient management of radio resources, and provide interesting perspectives for research.

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