Motion control for an underactuated brachiation robot based on Lyapunov direct method

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Abstract. This paper studies a control strategy to drive a brachiation robot to realize the movement from an arbitrary initial posture to the desired target position imitating a gibbon to move from branch to branch. The main control aim is to drive an underactuated 2-DOF brachiation robot to reach the target in two action modes of swing motion and locomotion motion. Then, a brachiation motion controller is designed to consist of two control processes in which one is used to drive the brachiation robot to move from the suspend posture to the target position via a Lyapunov method, and the other is to ensure that the end of the link of the robot can reach the target bar through direction feedback control. Finally, some experimental results show that the control method can succeed to control a brachiation robot to achieve successfully the movement from one bar to another adjacent bar in two different cases.

1. Introduction

In the past three decades, many researchers have studied on how to realize the dynamic and dexterous movements performed by living creatures by means of the underactuated robots. The main representatives of these studies focused on the movement mechanism and control strategies of the acrobot [1-3], the horizontal bar gymnastic robot [4-7], the brachiation robot [8-11], etc.

The brachiation robot is considered to be a kind of underactuated mechanism which can be used to imitates a gibbon to move from branch to branch using the potential energy of gravity by the body swinging [12]. Gravity is often considered an unfavorable factor needed to be diminished by compensation in robot control, but it is a good available factor for the motion of a brachiation robot due to inertia.

Fukuda et al.[8] developed a heuristic learning method which can generat feedforward control inputs for the locomotion and exhibited that the proposed method enable the robot to catch the goal. Nakanishi et al. demonstrated a new motion control algorithm using a target dynamics method to solve some problems such as the ladder and swing up [13]. To realize continuous brachiation motion effectively, a kind of energy-based method for swing movement was developed [14-16]. Some researchers proposed some new schemes based on fuzzy control architecture, which have the advantages of transforming easily human knowledge into fuzzy rules [17-19]. Recently, Cheng et al. [20] proposed a systematic trajectory planning method to solve the problems of motion flexibility and controller robustness for ricochetal brachiation locomotion. Unlike other studies which mainly focused
on model simulations, Yamakawa et al. developed a 2-DOF robot consisting of links, servo motors and end effectors [21].

Despite a lot of research achievements in brachiation robot control, it is still a challenging task to realize the flexible movement of ape-like. This paper focuses on the motion control based on Lyapunov direct method for an underactuated 2-DOF brachiation robot to realize the locomotion movement.

The rest of this paper is organized as follows. In Sec. 2, the analysis model is shown in brief. Section 3 discusses a control strategy which consists of a motion control designed for the brachiation robot to move from an initial posture to the target position based on Lyapunov direct method and a direction feedback control ensuring the end of the robot link to catch the target bar. In section 4, the effectiveness of the proposed method has been proved by some simulation results. Section 5 gives a brief conclusion.

2. Model of brachiation robot
Brachiation robot imitates the swing movement of gibbon. Figure 1 shows a simplified robot model which has two arms connected by one actuator. Each arm has a grip to catch horizontal bars and the robot moves by hand-over-hand swinging from one bar to the neighboring bar. It is considered to be an underactuated mechanical system since it has fewer actuators than its DOF. Assume the arm holding a bar to be the 1st-arm and another one to be the 2nd-arm. Define \( q_1 \) as the rotation angle of the 1st-arm relative to the vertical line and \( q_2 \) as that of the 2nd-arm relative to the 1st-arm. For each arm, its mass, length, centroid location and inertia moment are denoted as \( m_i, l_i, a_i, I_i \) respectively. Assume the robot motion be constrained to the vertical plane.

![Figure 1. Simplified brachiation robot model.](image)

Define the Lagrangian of the system as the following

\[
L = K - U = \frac{1}{2} \dot{q}^T M(q) \dot{q} - U(q),
\]

where \( K \) is the kinetic energy and \( U \) is the potential energy of the system. \( M(q) \) respects the inertia matrix and \( q = [q_1, q_2]^T \) is the generalized coordinate vector.

Then, we can obtain the dynamical equations as

\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) = u,
\]

where \( M(q) \in \mathbb{R}^{2 \times 2}, C(q, \dot{q}) \in \mathbb{R}^{2 \times 1}, G(q) \in \mathbb{R}^{2 \times 1} \) denote respectively the inertia, Coriolis, gravitational matrix. The joint torque vector has the form of \( u = (u_1, u_2)^T = (0, \tau)^T \in \mathbb{R}^2 \). Here,

\[
M(q) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad C(q, \dot{q}) = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}, \quad G(q) = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix}.
\]
The details of the above matrices are as follows.

\[ M_{11} = l_1 + l_2 + m_1 a_1^2 + m_2 [l_1^2 + a_2^2 + 2l_1a_2 \cos(q_2)], \]
\[ M_{12} = l_2 + m_2 [a_2^2 + l_1 a_2 \cos(q_2)], \]
\[ M_{21} = M_{12}, \quad M_{22} = l_2 + m_2 a_2^2, \quad C_1 = -\ddot{q}_2 (l_1 + 2l_2 l_1) a_2 \sin(q_2), \quad C_2 = \ddot{q}_1 l_2 a_2 \sin(q_2), \]
\[ G_1 = (m_1 q_1 + m_2 l_1 g \sin(q_1)) + m_2 a_2 g \sin(q_1 + q_2), \quad G_2 = m_2 a_2 g \sin(q_1 + q_2). \]

3. Design of the controller

3.1. Problem formulation

The aim of this study is to control the brachiation robot to move from one bar to another adjacent bar. Generally, the motion of an underactuated brachiation robot can be considered to consist of two action modes: one is swing action and another is the so-called locomotion action as shown in Fig. 2. It is considered that the robot can succeed in locomotion from an arbitrary initial condition if it is able to continue the above two motions. In the swing action case as shown in Fig. 2(a), the robot starts from a suspended posture and needs to be pumped periodically the energy. In the locomotion action case as shown in Fig. 2(b), the robot moves after releasing the initial bar.

![Figure 2. Motion mode of the robot.](image)

3.2. Control strategy

This section derives a method in order to control the brachiation robot to realize the movement from an initial posture to the target. To achieve this, a motion control method using Lyapunov direct method is taken into consideration.

Note that Eq. (2) can be rewritten as

\[ M_{11} \ddot{q}_1 + M_{12} \ddot{q}_2 + C_1 \dot{q}_1 + G_1 = 0, \tag{4} \]
\[ M_{21} \ddot{q}_1 + M_{22} \ddot{q}_2 + C_2 \dot{q}_2 + G_2 = \tau, \tag{5} \]

It is supposed that Eq. (4) is solved for \( \ddot{q}_1 \) and Eq. (5) uses the resulting expression. Thus, Eq. (5) will be a feedback linearizable equation involving only \( \ddot{q}_2 \). From Eqs. (4) and (5), it can be obtained

\[ \ddot{q}_2 + \dddot{q}_2 + \dddot{q}_2 + \dddot{q}_2 = \tau, \tag{6} \]

where \( \dddot{q}_2, \dddot{q}_2, \dddot{q}_2 \) are given by

\[ M_{22} = M_{22} - M_{21} M_{11}^{-1} M_{12}, \quad \dddot{q}_2 = C_2 - M_{21} M_{11}^{-1} C_1, \quad \dddot{q}_2 = G_2 - M_{21} M_{11}^{-1} G_1. \tag{7} \]

A swing motion controller is defined for Eq. (6) according to

\[ \tau = \dddot{q}_2 + \dddot{q}_2 + \dddot{q}_2, \tag{8} \]

where \( v_2 \) is an additional control term which is used to achieve the generation of the locomotion control law.

Thus, the equations (4) and (5) can be rewritten as
\[
\begin{align*}
\begin{cases}
M_{11} \ddot{q}_1 + C_1 + G_1 &= -M_{12} \dot{v}_2, \\
M_{12} \ddot{q}_2 &= v_2.
\end{cases}
\end{align*}
\] (9)

Let \(v_2\) be determined so that the second joint angle converges to the desired angles \(q_{2d}\) as
\[
v_2 = -k_{d2} \dot{q}_2 + k_{p2} (q_{2d} - q_2),
\] (10)
where \(k_{d2}\) and \(k_{p2}\) are gains.

Here, the problem becomes how to decide the value of the desired angle, \(q_{2d}\). Consider the swing control based on a single variable link pendulum model as shown in Fig. 3. Define \(F\) as a force which is tangential to the rotational direction of the link.

Then, the motion equation of the above variable link pendulum can be described by
\[
I \ddot{q}_v + mg l_v \sin(q_v) = l_v F.
\] (11)

Consider a Lyapunov function candidate as
\[
V(q_v, q_\dot{v}) = \frac{1}{2} \lambda l_v q_\dot{v}^2 + mg(1 - \cos(q_v)).
\] (12)

where \(\lambda\) is a positive constant. Note that Eq. (11) can be rewritten as
\[
\ddot{q}_v = -\frac{mg l_v}{I} \sin(q_v) + \frac{l_v}{I} F.
\] (13)

Therefore, the time derivative of Eq. (12) along the trajectory of the system is given by
\[
\dot{V}(q_v, q_\dot{v}) = q_\dot{v}[(1 - \lambda l_v) m g \sin(q_v) + l_v F].
\] (14)

In order for the robot to be swung, the following condition needs to be satisfied.
\[
\dot{V}(q_v, q_\dot{v}) > 0.
\] (15)

To do this, choose the length, \(l_v\), and the force, \(F\), as
\[
l_v = \lambda - |l_v| \cdot \text{sgn}(q_v) \cdot \text{sgn}(q_\dot{v}), \quad F = |F| \cdot \text{sgn}(q_\dot{v}),
\] (16)

where \(|l_v| < 1/\lambda\).

Furthermore, define \(\tilde{q}\) be an angle of the centroid location of the robot. Due to the fact that the position from the support point to the centroid location can be changed by straightening or bending its posture of the 2nd arm, the desired angles \(q_{2d}\) is given as follows.
\[
q_{2d} = \begin{cases}
\alpha, & \text{if } \tilde{q} \geq 0 \text{ and } \dot{\tilde{q}} \geq 0 \\
-\alpha, & \text{else if } \tilde{q} < 0 \text{ and } \dot{\tilde{q}} < 0 \\
0, & \text{otherwise}.
\end{cases}
\] (17)
Next, to ensure that the robot can reach the target, a direction feedback control strategy needs to be considered. It is assumed that the 2nd arm end can enter the region with the target point, B, as the center of the circle and \( r (\ll l_2) \) as the radius which is shown in Fig. 4.

When the end of the second arm reach the target point, the angle of the 2nd arm, \( q_{2E} \), satisfies

\[
 q_{2E} = \frac{\pi}{2} - q_{1E} + \angle OBA, \tag{18}
\]

It is obvious that

\[
 \angle OBA = \arctan(\frac{l_1 \cos(q_{1E})}{d - l_1 \sin(q_{1E})}), \tag{19}
\]

where \( d \) denotes the distance from the support bar till the target bar.

The equations (18) and (19) yield

\[
 q_{2E} = \frac{\pi}{2} - q_{1E} + \arctan(\frac{l_1 \cos(q_{1E})}{d - l_1 \sin(q_{1E})}), \tag{20}
\]

Thus, a direction feedback controller is designed to be

\[
 \tau = \begin{cases} 
 k_{e2}(q_2 - q_{2E}), & \text{if } q_{1E} \geq \beta, \\
 0, & \text{otherwise}.
\end{cases} \tag{21}
\]

where \( k_{e2} \) is a feedback gain and \( \beta \) is a control parameter.

4. Simulation experiments

This section presents some results obtained from simulation experiments comparing the two different actuation modes to show the effectiveness of the above method. Table 1 gives the robot parameters used in the experiments.

**Table 1. Robot link parameters.**

|                  | 1st link | 2nd link |
|------------------|----------|----------|
| Mass             | \( m_i \) [kg] | 0.285 | 0.285 |
| Moment of inertia| \( I_i \) [kgm²] | 0.0059 | 0.0059 |
| Link length      | \( l_i \) [m] | 0.5 | 0.5 |
| Offset of mass center | \( a_i \) [m] | 0.25 | 0.25 |

Table 2 lists the control parameters. In each motion case, two patterns were studied by setting the distance of the target and the support bar to 0.6[m] and 0.8[m], respectively. The robot achieved brachiation motion between bars which was arranged at a fixed distance.

**Table 2. Control parameters.**

| Case 1      | \( d=0.6 \) | \( d=0.8 \) | \( d=2.5 \) |
|-------------|-------------|-------------|-------------|
| \( k_{e2} \) | 3.5         | 3.5         | 8.0         |
| \( k_{d2} \) | 1.5         | 2.5         | 1.2         |
| \( \alpha \) | 0.2         | 0.15        | 0.15        |
| \( \beta \) | 0.1         | 0.1         | 0.1         |

4.1. Case studies: Case 1

First, the motion case 1 in which the robot started from a suspended posture was studied. In this case, two kind of control targets were implemented. Figure 5 depicts the phase portrait \((x_2, y_2)\) and shows the trajectory of the end of the 2nd arm in x-y plane varying with the time. Here, Fig. 5(a) indicates the motion pattern of the target distance \( d = 0.6[m] \) and Fig. 5(b) indicates that of \( d = 0.8[m] \).
From Fig. 5, it can be observed that in each pattern, the robot succeed to achieve the movement goal. Meanwhile, the control torques applied on the second joint of both patterns were shown in Fig. 6.

**Figure 5.** Trajectory of the end of the 2nd link on x-y plane.

**Figure 6.** Applied torque on the 2nd joint.

4.2. Case studies: Case 2

Next, we studied the motion case 2 in which the robot started the movement after releasing the initial bar and stopped after approaching the target bar. Similar to those of case 1, two kind of control patterns were implemented by presetting respectively the target distance to be $d = 0.6 \, [m]$ and $d = 0.8 \, [m]$.

The trajectory of the end of the 2nd link was depicted in Fig. 7. Here, Fig. 7(a) depicts the phase portrait $(x_2, \dot{x}_2)$ of pattern 1 and Fig. 7(b) plots the phase portrait of pattern 2. It can be confirmed from Fig. 7 that the robot achieved two different movements successfully. Figure 8 shows the control torque applied on the second joint under the motion controller.

**Figure 7.** Trajectory of the end of the 2nd link on x-y plane.
Figure 8. Applied torque on the 2nd joint.

Comparing case 2 to case 1, it can be found in Fig. 5 and Fig. 7 that the time for the robot to achieve the movement target in case 2 is significantly less than that in case 1. This can be attributed to the fact that in case 2, the robot creates a certain kinetic potential energy from the movement after releasing from the initial bar. It was considered that the robot no longer needs to be pumped too much energy from the controller in the process of swing motion due to the action of initial motion inertia.

Figure 9 depicts the stick diagrams which are snapshots of the brachiation robot in motion. Here, the figures 9(a) and 9(b) depicts both motion patterns in case 1 and the figures 9(c) and 9(d) plots those in case 2. It can be observed from Fig. 9 that in each pattern of both the swing action and locomotion action, the brachiation robot succeed in approaching the target.

Figure 9. Stick diagrams.

5. Conclusions

This paper studied a motion control method for a 2-DOF brachiation robot to realize the movement imitating a gibbon to move by hand-over-hand swinging. Based on the analysis model of a 2-DOF brachiation robot, it was firstly introduced that the main purpose of this study is to control a brachiation robot to realize two action modes: swing action and locomotion action. Secondly, a controller consisting of two control processes was designed to drive the brachiation robot to move from a suspend posture or a state releasing from an initial bar to the target bar by composing a Lyapunov direct method and a direction feedback control method. Finally, we gave some simulation results to valid the effectiveness of control methods.

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