Statistical analysis of imperfection effect on cylindrical buckling response

M S Ismail\textsuperscript{1,2}, J Purbolaksono\textsuperscript{1,2}, N Muhammad\textsuperscript{1}, A Andriyana\textsuperscript{2} and H L Liew\textsuperscript{2}
\textsuperscript{1}Department of Mechanical Engineering, Politeknik Sultan Salahuddin Abdul Aziz Shah, Shah Alam 40150, Selangor, Malaysia
\textsuperscript{2}Centre of Advanced Manufacturing and Materials Processing, Department of Mechanical Engineering, Faculty of Engineering, University of Malaya, Kuala Lumpur 50603, Malaysia

E-mail: judha@um.edu.my

Abstract. It is widely reported that no efficient guidelines for modelling imperfections in composite structures are available. In response, this work evaluates the imperfection factors of axially compressed Carbon Fibre Reinforced Polymer (CFRP) cylinder with different ply angles through finite element (FE) analysis. The sensitivity of imperfection factors were analysed using design of experiment: factorial design approach. From the analysis it identified three critical factors that sensitively reacted towards buckling load. Furthermore empirical equation is proposed according to each type of cylinder. Eventually, critical buckling loads estimated by empirical equation showed good agreements with FE analysis. The design of experiment methodology is useful in identifying parameters that lead to structures imperfection tolerance.

1. Introduction
Thin-walled cylindrical shells are susceptible to buckling failures caused by the axial compressive loading. During the design process or the buckling failure evaluation of axially compressed cylindrical shells, initial geometric and loading imperfections are of important parameters for the analyses. Therefore, the engineers/designers are expected to well understand the physical behaviours of shell buckling to prevent unexpected serious failure in structures. At the same time it is believe that most of cylinder imperfection is caused by defect and uncertainties that can be traced from the manufacturing process. As this process involved with a lot of variables and parameters to control consequently lead to manufacturing defects. These defects involved with various factors such as micro dented on the shell surface, fibre degree disorientations and fibre volume misalignment. As a result it can probably say that these aforementioned factors give large contribution towards structural imperfection.

To date, the issue corresponding to the discrepancies between the manufactured and computationally design finite element (FE) cylinder has become a vital point on establishing better design guideline. For example, in reality during curing process, the composite cylinder can be highly exposed to the distributed of residual stresses while this process is hardly replicate through FE model. As the FE model can be perfectly designed through finite element environment without considering the related issues of uncontrollable process variables [1]. As reported by Lee [2], initial geometry is one of a parameter that gives a significant different towards imperfection. This can be connected to the curing process that led the cylinder shell into variation of nominal radii and curvature and eventually changes the cylinder to be into slightly oval-shaped geometry. Indeed the deviated geometry somehow would affect the cylinder buckling response and its characteristic in final post bucking collapse.
The design of experiment (DOE) is a statistical based technique that used on conducting and analysing the sensitivity of parameters to the response output. Initially developed by Fisher [3], it has been promoted by various researchers for the use of experimental design strategy and sampling planning. Simultaneously, the interaction of single or combined parameter to the output response is then evaluated by analysis of variance (ANOVA). As this method is ideally used to reduce and statistically signify the number of necessary experiments and parameters. Up until now there is still unanswered question about to the precision knowledge of structure and material properties for composite cylinder. In response, the DOE technique is a suitable method to randomly vary the suspected variables to depict the output response thus link with its sensitivity. For instance, deviations in material properties, geometric correction factors and loading and boundary conditions are one of a few candidates that attribute towards the issue. While according to the manufacture, a typical composite fibre angles can be scatter deviated around ±2º.

In previous work, random imperfection based stochastic eigen-affine modes buckling is conducted by [4, 5]. It followed with the study conducted by [6-8] to generate eigen mode imperfection randomly using DOE method. While a study by [9-12] demonstrated a probabilistic imperfection analysis towards cylinders shells buckling through FE analysis. From their results it directs a possibility to improve the cylinder knockdown factor. However some limitation was found from their investigation, where the geometry and composite material is identical with a little different on fibres angle.

In response, new investigation comprised of different cylinder geometry and material characteristic is needed. As it is essential to re-evaluated the factors that affected the cylinder buckling response is more or less to be identical as previous reported in [9]. So far, it found that there is still shortage of study has been done on cylinder imperfection by means of design of experiment (DOE) method. The proposed area of investigation may confirmed of what been found from the previous studies with some extended work for different cylinders. Thus a better design guideline on predicting the buckling load that describes the imperfection from manufacture defect can be withdrawn.

2. Material and Methods
All the analyses were carried by using a finite element software package of ABAQUS v6.10. The schematic of the axially compressed cylindrical shell is shown in figure 1. As shown in figure 1, the displacement and rotation constraints are applied on one end of the cylindrical shell, and the load-controlled displacement $\Delta U$ with slow quasi-static compression is applied on the other end of the cylinder.

The cylinder geometries used in this study referring to those experimentally conducted by [13, 14] for the CFRP laminated shell, and the details are presented in table 1. The material properties of the monolithic laminates of carbon fibre composites reported by [14] were also summarised in table 1. Nevertheless two cases of cylinders with different layers are performed in this work which is four-ply with stacking sequences of [0/45/-45/0] and [45/-45] CFRP laminated shell.
Figure 1. Boundary condition of FE modelling.

Table 1. Nominal cylinder geometries and material properties of CFRP.

| Description                        | Unit   | Bisagni [13] |
|------------------------------------|--------|--------------|
| Cylinder designation               |        | CFRP         |
| Radius, \( r \)                    | [mm]   | 350          |
| Free length, \( L \)               | [mm]   | 520          |
| Nominal thickness, \( t \)         | [mm]   | 1.32         |
| \( r/t \)                          |        | 265          |
| \( L/r \)                          |        | 1.45         |
| Controlled displacement, \( \Delta U \) | [mm] | 2            |

**Stiffness**

| Property                           | Unit       | Value      |
|------------------------------------|------------|------------|
| Elastic modulus \( E_{11} \)       | [N/mm\(^2\)] | 52000      |
| Elastic modulus \( E_{22} \)       | [N/mm\(^2\)] | 52000      |
| Shear modulus \( G_{12} \)         | [N/mm\(^2\)] | 2350       |
| Poisson’s ratio \( \nu \)          |            | 0.302      |
| Density, \( \rho \)                | [kg/mm\(^3\)] | 1.32 x 10\(^{-8}\) |
| Ply thickness \( t_{ply} \)        | [mm]       | 0.33       |

3. Design of Experiment: Theoretical Background

The response surface method is statistical based mathematical method that gives an effective and practical way on optimizing design [15]. The theory is based on the output response function, \( y \) which react in the function of designs variables, \( x_i \). The function of designs variables, \( x_i \) is mostly designed as feasible domain. On the other hand, the polynomial approximation expressed by, \( y = f(x_i) \) is usually described to the function on the basis of observation data. However in case of quadratic response function with two variables the polynomial reads as equation (1).

\[
y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 + \varepsilon
\]  

(1)
These terms is defined as $\beta_i$ are the regression coefficient and $\varepsilon$ is error approximate function. Consequently when equation (1) is substitute with such variables as, $x_1^2 = x_3$, $x_2^2 = x_4$ and $x_1x_2 = x_5$ it yields to be multivariable equation as shows in equation (2). As this substitution shows the regression model that contained higher terms can often reduce to a linear regression model. Then by taking into account of $k$ regression coefficient, the following equation obtained:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_i x_i + \beta_k x_k + \varepsilon$$  \hspace{1cm}(2)

Let says if $n$ sets of observation data and the $n$ corresponding to combination of design variables are known. Then the response can be expressed by the representation of matrix as shows in equation (3).

\[
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n \\
\end{bmatrix} = \begin{bmatrix}
1 & x_{11} & x_{12} & \cdots & x_{1k} \\
1 & x_{21} & x_{22} & \cdots & x_{2k} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1 & x_{n1} & x_{n2} & \cdots & x_{nk} \\
\end{bmatrix} \begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_k \\
\end{bmatrix} + \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\vdots \\
\varepsilon_n \\
\end{bmatrix} \hspace{1cm}(3)
\]

Then equation (3) can be rewrite in matrix form (4):

$$y = X\beta + \varepsilon$$  \hspace{1cm}(4)

4. Design of Experiment: Factorial Design

The number of samples can be derived by summed the 2 levels and 7 factors ($2^7$) of the designed numerical experiments that yields to a set of 128 samples. However to minimised the numbers of runs, only a fraction of one-eighth ($1/8$) from the whole samples is taken thus yields to 16 samples. Then for accuracy purposed, the 16 factorial designs is augmented by 5 centre points that lead to in total of 21 cases was used to study the effect of factors for each cylinder. Yet from the minimised work the samples is still adequate for design of experiment (DOE) prospect. These factors namely Young’s modulus (longitudinal), $E_{11}$, cylinder thickness, $t$, load imperfection, $\theta_{load}$ and each of fibres degree angle orientation ($\theta_1$, $\theta_2$, $\theta_3$ and $\theta_4$) are consider as a primary candidate that would give a significant effect on cylinder buckling load response. The selected investigated factors are based on the report by Degenhardt et al. [9] which identified these four parameters having high sensitivity towards buckling response compared to the others. Therefore the other parameters were set to be constant throughout the study. Moreover the replicates run at centre point are design to take into account of quadratic effect. Thus lead to an independent estimate of error to be obtained.

Table 2 summarised the factors and range applied for the 21 factorial designs. In this study, the possibility range of the factors was determined by varying in the range of ±5% from the nominal value. Also the statistical significant of each parameters and their combinations were evaluated by 5% of their level of significant. On the other hand a multiple regression analysis was performed based on the regression model to obtain the second-order response function [16]. From equation (1) $y$ is the output response while $\beta_0, \beta_i, \beta_{ii}$ and $\beta_{ij}$ terms are the coefficients to intercept, linear, quadratic and interaction variables respectively. In additional $x_i$ and $x_j$ are the independent variables in coded unit and $\varepsilon$ is the error terms that account the effect of excluded parameters. During analysis, the least squared method is been used to generate the prediction model equation (1) for a best fit of collected response data.
Table 2. Factors and range applied of $2^7$ factorial designs.

| Parameters                  | Symbols | Units   | Range       |
|-----------------------------|---------|---------|-------------|
| Load imperfection, $\theta$ | A       | °       | $0 \leq 0.33$ |
| Shell thickness, $t$        | B       | mm      | $1.254 \leq 1.38$ |
| Elastic modulus, $E_{11}$   | C       | N/mm$^2$| $49.4 \leq 54.6$ |

| Ply orientation | [0/45/-45/0] | [45/-45]$_S$ |
|-----------------|-------------|--------------|
| Fibre, $\theta_1$ | D           | -1 $\leq 1$ | 44 $\leq 46$ |
| Fibre, $\theta_2$ | E           | 44 $\leq 46$ | -46 $\leq -44$ |
| Fibre, $\theta_3$ | F           | -46 $\leq -44$ | -46 $\leq -44$ |
| Fibre, $\theta_4$ | G           | -1 $\leq 1$ | 44 $\leq 46$ |

5. Result and discussion

In this section the response of load-end shortening from FE analyses is presented and focus is highlighted to $[0/45/-45/0]$ ply stack-up. The CFRP of $[0/45/-45/0]$ ply with its load shortening response, modeshape and final collapse deformation is presented in figure 2 (a)-(c). The load-displacement curve is given in figure 2 (a) indicated as a guide of shell compressive response. The axial load, $P$ and end-shortening is normalized to the linear eigenvalue critical buckling load, $P_{crit}$ and nominal shell wall thickness, $t$ of 240 kN and 1.32 mm respectively. General instability occurs at normalized axial load of 0.75, labelled by letter A. The instability response at point A is followed by a sudden reduction of axial load at point B - D that associated with shell transient collapse. Then it reached to steady state value at point D - E. The reduction of shell axial stiffener is associated with the slope at postbuckling region. The shell reached to it stable-equilibrium postbuckling condition after approximately 0.007 seconds. Furthermore the buckles wave pattern illustrates in figure 2 (c) and (c) shows different characteristic of its buckles bay occurred around the cylinder circumferential by comparing both eigenvalue linear and non-linear analyses. It should point out that the difference of buckling characteristic is due to the dissimilar of solver method. The eigenvalue solve the problem based on bifurcation manner while the non-linear analysis solves the problem based on incremental time until the solution reaches its equilibrium path.
The significant effect of each parameter was evaluated by standardized effect of 5% significance level. In particular the interaction analysis of variance (ANOVA) is conducted and summarised in Table 3 for the case of cylinder [0/45/-45/0]. In which the approximated regressed equation throughout the factorial design analysis is derived from the second order polynomial. Herein the unknown parameters, \( \beta \) has been identified from the numerical experiment and solved by least square method. Therefore from 21 sampling points of each case, the cylinder buckling load prediction can be estimated by the following empirical expression;

For case of cylinder [0/45/-45/0];

\[
Y = -307.875 - 28.4205A + 345.069B + 1.59591C + 0.353125D - 0.061875E \\
- 0.234375F + 0.301875G
\]

For case of cylinder [45/-45];

\[
Y = -135.494 - 12.2917A + 178.383B + 0.194471C - 0.175625D - 0.353125E \\
+ 0.351875F + 0.306525G
\]

Table 3. ANOVA for cylinder [0/45/-45/0].

| Source             | DF | Seq SS | F   | P    | Significant |
|--------------------|----|--------|-----|------|-------------|
| Load imperfection, | 7  | 351.84 | 378.61 | 0.000 | Significant |
| Shell thickness,   | 1  | 7561.61 | 8136.94 | 0.000 | Significant |
| Elastic modulus \( E_{11} \) | 1  | 275.48 | 296.44 | 0.000 | Significant |
| Fibre, \( \theta_1 \) | 1  | 2     | 2.15  | 0.169 |             |
| Fibre, \( \theta_2 \) | 1  | 0.06  | 0.07  | 0.802 |             |
| Fibre, \( \theta_3 \) | 1  | 0.88  | 0.95  | 0.35  |             |
| Fibre, \( \theta_4 \) | 1  | 1.46  | 1.57  | 0.234 |             |

The design matrix and results of 21 factorial designs are depicted in figure 3 (a) and (b) for each case of cylinder. Overall the cylinder buckling load are obtained in the range of 200 kN to 260 kN for
cylinder [0/45/-45/0] and 100 kN to 128 kN for cylinder [45/-45]. The results also provide the ratio of maximum to minimum equal to 1.3 and 1.28 respectively. The smaller ratio indicated no further requirement to apply any kind of data transformation. The ANOVA of both cylinder cases are then evaluated to determine the accuracy of the empirical models. Consequently the coefficient of determination, $R^2$ are found for the case of [0/45/-45/0] and [45/-45] to be 0.99 and 0.96. The $R^2$ value indeed showed the level of model accuracy. This is due to the fact that the closer value to 1 indicates that the regression model fits well with the data.

At the same time the Pareto’s chart shows in figure 4 (a) and (b) indicated the sensitivity of candidate factors that possibly affect the cylinder buckling response for each case. From the results, it observed that the main factors that affect the cylinder buckling response were identified as shell thickness, $t$ (A), load imperfection (B) and Young modulus, $E_{11}$ (C) for the case of [0/45/-45/0] plies. However for cylinder with ply angle of [45/-45] shows a different pattern, whereas the Young modulus, $E_{11}$ (C) is not taking effect on cylinder response as previous case. This outcome has been expected due to the axial stiffness of [45/-45] cylinder is considerably lower than [0/45/-45/0] cylinders. In fact they shared similar value of Young modulus and the rest of material properties. This behaviour is believed to be connected with direct consequences of the missing 0º of fibre angle that perpendicular to the loading axis. Eventually lead to the relatively low cylinder stiffness. Yet with this indication, it confirmed the finding of Degenhardt et al. [9], that these three factors posses high sensitivity towards cylinder buckling response as it with different material properties and fibre direction. In return, it can be assumed that these factors can be more consistent with the following isotropic material.

Figure 5 (a) and (b) illustrated the effect of selected factors toward buckling load for each cylinders. The relationship shows in the results indicate the level of lower and upper bound on the buckling response. While the straight horizontal line marked as the mean data distribution. A part from that it distinguished the relationships of shell thickness towards buckling load are linear for both cylinders accordingly. Interestingly the effect of loading imperfection and material stiffness are remarks to be less important. As this can be seen where a steady decreases was found by increasing the compression rotation angle. This behaviour is assumed that single factor like loading imperfection could not affect the cylinder buckling alone. Whereas a combination between either shell thickness or elastic stiffness should provide more reasonable effect on cylinder buckling load sensitivity.
Figure 3. Factorial design response of cylinders buckling load distribution.
Figure 4. Pareto chart of CFRP composite cylinder standardised effect for (a) [0/45/-45/0] and (b) [45/-45]_S attribute to each factors.

Figure 6 shows how well the empirical model prediction by comparing it with FE analyses for both cylinders. The data distribution is based on the ratio of empirical model over FE model prediction against number of test performs. According to the distribution it tells the accuracy of the results by closer it is to the line. In addition the marks below the benchmark line indicated that the empirical model is much lower than FE analysis. At the same time the marks over the line region show otherwise. However few observations for test number (11 and 13) gives a little high values of ratio to be nearly 5% for the case of cylinder [45/-45]_S. Finally most of the input values for both cylinders point out to have less than 5% of discrepancy to the unified value (benchmark line). Finally simple empirical formulation has been derived to predict the cylinder buckling load. Yet the proposed formulation is still bound with the range of considered variables to ensure it accuracy.
Figure 5. Main effect plot for cylinder (a) [0/45/-45/0] and (b) [45/-45].
Figure 6. Comparison result for both cylinders using empirical model with FE analysis.

6. Conclusion
Statistical based design of experiment analyses on axially compressed CFRP cylindrical shells with different ply angle were demonstrated through the finite element simulations. The finding of this work may be summarized as follows:

- The characteristic of the composite cylinder buckling event is briefly describe and identified and suggest that non-linear analysis is better approach on capturing the real buckling mechanism compared to the linear one.
- The design of experiment approach made a significant contribution on identifying the factors that sensitively reacted towards buckling load. As the sensitivity factors is drawn to be load imperfection, shell thickness and elastic modulus.
- From the statistical studies it may suggest that loading imperfection and material stiffness are less sensitive towards structural imperfection compared to shell thickness during the buckling event. However it delivers greater effect of structural imperfection by coupled of each other.
- Empirical equation is proposed according to each type of cylinder and critical buckling load estimated by empirical equation showed good agreements with FE analysis.

Acknowledgement
The authors would like to express sincere gratitude to Politeknik Sultan Salahuddin Abdul Aziz Shah and thank the Ministry of Higher Education, Malaysia, through the High Impact Research Grant (UM.C/625/1/HIR/MOHE/ENG/33).

References
[1] Lee M C, Kelly D W, Degenhardt R, Thomson R S 2010 Composite Structures 92 223-32
[2] Lee M C. Stochastic analysis and robust design of stiffened composite structures: The University of New South Wales; 2009.
[3] Fisher R A 1935 The Design of Experiment (Edinburgh: Oliver and Boyd)
[4] Chryssanthopoulos M, Poggi C 1995 Thin-walled structures 23 179-200
[5] Prabu B, Raviprakash A, Venkatraman A 2010 *Thin-Walled Structures* **48** 639-49
[6] Prabu B 2010
[7] Warren Jr J. Nonlinear stability analysis of frame-type structures with random geometric imperfections using a total-lagrangian finite element formulation: Virginia Polytechnic Institute and State University; 1997.
[8] Tran K, Douthe C, Sab K, Dallot J, Davaine L 2014 *Thin-Walled Structures* **79** 129-37
[9] Degenhardt R, Kling A, Bethge A, Orf J, Kärger L, Zimmermann R, et al. 2010 *Composite Structures* **92** 1939-46
[10] Degenhardt R, Bethge A, Kärger L. Probabilistic aspects of buckling knockdown factors—test and analysis. nal report. Technical report, ESA Contract 19709/06/NL/IA, 2007.
[11] J. Orf L K, R. Degenhardt, and A. Bethge. The Influence of imperfection on the buckling behaviour of unstiffened CFRD-cylinders. In proceeding of 2nd International Conference on Buckling and Postbuckling Behaviour of Composite Laminated Shell Structures2008. p. 2-5.
[12] J Kepple G P, G Pearce, D KElly, R Thomson and R Degenhardt, Influence of Imperfection on Axial Buckling Load of Composite Cylindrical Shells. 19th International Conference on Composite Materials; 2013.
[13] Bisagni C, Cordisco P 2003 *Composite Structures* **60** 391-402
[14] Bisagni C 2000 *Composites Part B: Engineering* **31** 655-67
[15] Bahloul R, Mkaddem A, Dal Santo P, Potiron A 2006 *International journal of mechanical sciences* **48** 991-1003
[16] Myers R H, Montgomery D C, Anderson-Cook C M 2009 *Response surface methodology: process and product optimization using designed experiments* **705** John Wiley & Sons)