Signatures of unconventional pairing
in spin-imbalanced one-dimensional few-fermion systems

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(Dated: November 12, 2019)

A system of a few attractively interacting fermionic \(^{6}\)Li atoms in one-dimensional harmonic confinement is investigated. Non-trivial inter-particle correlations induced by interactions in a particle-imbalanced system are studied in the framework of the noise correlation. In this way, it is shown that evident signatures of strongly correlated fermionic pairs in the Fulde-Ferrell-Larkin-Ovchinnikov state are present in the system and they can be detected by measurements directly accessible within state-of-the-art techniques. The results convincingly show that the exotic pairing mechanism is a very universal phenomenon and can be captured in systems being essentially non-uniform and far from the many-body limit.

![Graph](image-url)  
**FIG. 1.** The most probable FFLO momentum \(q_0\) as a function of the Fermi momenta mismatch \(\Delta p_F\). Different points correspond to different number of particles and different imbalances. Exact description of each point is given in the supplementary material [25]. For clarity we do not show the point \(q_0 = \Delta p_F = 0\) corresponding to the balanced scenario \(N_+ = N_-\). The gray dashed straight line guiding the eye corresponds to phenomenologically predicted relation \(q_0 = \Delta p_F\). Visible deviations from this prediction are ramifications of the finite number of particles and simplifications explained in the main text.

One of the cornerstones of our understanding of strongly correlated states of quantum matter is based on the theory of superconductivity by Bardeen, Cooper, and Schrieffer [1]. In this theory, the existence of the superconducting phase is explained following the fundamental observation by Cooper [2] that the ground-state energy of an attractively interacting system is significantly decreased by the collective formation of Cooper pairs — non-trivially correlated states of two fermions with exactly opposite momenta. Based on this idea of collective pairing, a plethora of other pairing mechanisms have been proposed and investigated [3–5]. One of the most influential extensions of the Cooper’s idea comes from the observation that in the case of imbalanced systems, due to the mismatch of Fermi spheres of different components, the formation of correlated pairs forced by attractive mutual interactions is inseparably connected with resulting non-zero net momentum of the pair [6, 7]. This unconventional pairing mechanism named after Fulde, Ferrell, Larkin, and Ovchinnikov (FFLO) has been extensively examined theoretically, mostly in the case of various solid-state systems like iron-based superconductors [8–11], heavy-fermion compounds [11–15], or organic conductors [16–18]. However, it is also viewed as one of the possible ways to understand fundamental properties of neutron stars [19–21], specific quantum chromodynamics models [22], or fermionic ultra-cold gases [23]. The latter example is of high importance since ultra-cold atomic systems, due to their tremendous tunability, are believed to be the best candidates for the first experimental observation of the FFLO state. Unfortunately, up to this day, the FFLO state is ephemeral and there are only indirect signs of this state of matter (see [24] for a recent review).

In this Letter, we show that the many-body ground-state of a few \(^{6}\)Li atoms confined in a harmonic trap (in the presence of mutual attractions) possesses many characteristic properties of the FFLO state which can be experimentally captured. For example, if one would combine recent progress in preparing spin-imbalanced few-fermion systems [26] with recently achieved development in measuring correlations between opposite spin fermions [27], and perform the theoretical analysis of obtained data along the recipe described here, then the most notable hallmark of the FFLO phase can be observed — the direct linear relation between the most probable net momentum of the pair \(q_0\) and the momentum mismatch.
between Fermi surfaces $\Delta p_F$ (see Fig. 1 with predictions for different numbers of particles and different spin-imbalance). Concurrently it should be emphasized that, in contrast to recent proposal [28], our approach is based on quantities which can be directly measured with nowadays techniques and does not require any significant modifications of experimental setups.

Although our approach is very general and can be adopted to different fermionic systems confined in one-dimensional traps, we focus on particular experimental realization — the few-fermion mixture of $^6$Li atoms achieved currently almost on demand in Heidelberg [26]. From theoretical point of view, the system can be well described with the second-quantized Hamiltonian of the form

$$H = \sum_\sigma \int dx \left( \hat{\Psi}_\sigma^\dagger(x) \left( -\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + \frac{1}{2} m \omega^2 x^2 \right) \hat{\Psi}_\sigma(x) + g \int dx \hat{\Psi}_\sigma^\dagger(x) \hat{\Psi}_\sigma^\dagger(x) \hat{\Psi}_{\bar{\sigma}}(x) \hat{\Psi}_{\bar{\sigma}}(x) \right),$$

where $\omega \approx 2\pi \cdot 1.488$ kHz is the frequency of the external harmonic trap, $m$ is the mass of a $^6$Li atom, and $g$ is the effective one-dimensional interaction strength [29]. The latter can be experimentally tuned by changing an external magnetic field and particularly it can become negative (effectively attractive interactions) [30]. In the following we assume that $g$ is fixed by the external magnetic field $B = 1202$ G (see Table III in [30]). If one expresses all quantities in natural units of harmonic oscillator, i.e., energies in $\hbar \omega = 9.86 \cdot 10^{-31}$ J, positions in $\sqrt{\hbar/m \omega} = 1.06 \mu$m, and wave vectors in $\sqrt{m \omega/\hbar} = 0.95 \mu$m$^{-1}$ then the assumed interaction strength corresponds to $g = -1$. The fermionic field operator $\hat{\Psi}_\sigma(x)$ annihilates a $\sigma$-component fermion at a position $x$ and obeys standard anti-commutation relations $\{\hat{\Psi}_\sigma(x), \hat{\Psi}_{\bar{\sigma}}^\dagger(x')\} = \delta(x - x') \delta_{\sigma \bar{\sigma}}$. For further convenience, we introduce density operators in position and momentum representations, $\hat{\rho}_\sigma(x) = \hat{\Psi}_\sigma^\dagger(x) \hat{\Psi}_\sigma(x)$ and $\hat{\pi}_\sigma(p) = \hat{\Psi}_\sigma^\dagger(p) \hat{\Psi}_\sigma(p)$, where $\hat{\Psi}_\sigma(p) = \int dx \hat{\Psi}_\sigma(x) \exp(-ipx/\hbar)$ is a Fourier transform of the field operator $\hat{\Psi}_\sigma(x)$.

To perform appropriate calculations for a given number of particles $N_\uparrow$ and $N_\downarrow$, we express the Hamiltonian (1) as a matrix in the Fock basis of many-body eigenstates of the non-interacting system $\{\{F_i\}\}$. The basis is given as a set of products of different Slater determinants of $N_\uparrow$ and $N_\downarrow$ harmonic potential orbitals chosen appropriately for each component. Since the Hilbert space grows exponentially along with the number of particles and number of single-particle orbitals, we restrict ourselves only to these Fock states which have the non-interacting energy lower than some properly chosen cutoff. As shown recently, as long as we are interested in the many-body ground state of the system, this approach can be applied effectively for any trapping potential and any number of particles [31–35]. Resulting matrix representation of the many-body Hamiltonian (1) is diagonalized using Arnoldi method [36] and the many-body ground-state $\{G_0\}$ is found as its decomposition coefficients in the non-interacting basis $\{\{F_i\}\}$.

Pairing between opposite component fermions, even if actually present in the system, is very resistant to detection. In principle, it requires experimental access to all possible two-particle measurements, i.e., a complete two-particle reduced density matrix (2RDM) is needed. Recently there were many attempts to measure inter-particle correlations in different scenario [37–42] but all of them give access only to the diagonal parts (two-particle density profiles) rather than complete 2RDM. Therefore some other theoretical framework is needed to capture mutual correlations. Fortunately, it was argued [43–46] that some subtle part of correlations induced by interactions can be picked up from pure diagonal parts of 2RDM provided that accidental correlations (encoded mostly in products of single-particle densities) are appropriately subtracted. In the case of two-component mixture of distinguishable fermions, the so-called two-point noise correlation $G$ is a convenient tool to unravel quantum correlations from the trivial background [35, 47–49]. It is defined respectively in the position and momentum representation straightforwardly as:

$G_\uparrow(x_1; x_2) = \langle \hat{\rho}_\uparrow(x_1) \hat{\rho}_\uparrow(x_2) \rangle - \langle \hat{\rho}_\uparrow(x_1) \rangle \langle \hat{\rho}_\uparrow(x_2) \rangle,$

$G_\downarrow(p_1; p_2) = \langle \hat{\pi}_\downarrow(p_1) \hat{\pi}_\downarrow(p_2) \rangle - \langle \hat{\pi}_\downarrow(p_1) \rangle \langle \hat{\pi}_\downarrow(p_2) \rangle.$

Note that in the non-interacting limit ($g \to 0$) the noise correlations (2) identically vanish. Therefore they can be interpreted as quantities measuring the amount of two-body correlation in the system forced purely by interactions. Importantly, it should be emphasized at this point that exactly this kind of correlations was captured experimentally very recently [27].

In Fig. 2 we plot noise correlations (2) for the system with $N_\uparrow + N_\downarrow = 10$ particles and different imbalances $\Delta N = N_\uparrow - N_\downarrow$. Without losing generality, in the following, we consider only non-negative imbalances $\Delta N \geq 0$. In the balanced case $\Delta N = 0$ (top panel) the Fermi spheres for both components are exactly the same and the standard Cooper-pairing mechanism occurs in the system [50]. Consequently, when the noise correlation is considered, the pairing mechanism is manifested by a strong anti-correlation of fermions’ momenta — strong enhancement of the probability of finding fermions with exactly opposite momenta $p_\uparrow = -p_\downarrow$ is clearly evident. This picture is substantially changed when the particle imbalance is introduced to the system (the second and subsequent rows in Fig. 2). It is quite evident that in these cases the anti-diagonal enhancement of correlations is split into two ridges which are pushed out from the line $p_\uparrow + p_\downarrow = 0$. It means that in contrast to the balanced scenario the most probable outcome of the two-point momentum measurement is that paired fermions
FIG. 2. The noise correlation $G_\rho (x^\uparrow; x^\downarrow)$ in position (momen-
tum) domain is presented in the left (right) column. Attractive
interactions enhance the probability of finding particles of
two species in the same position for different configurations
of $N = N^\uparrow + N^\downarrow = 10$ particles. Simultaneously,
the momenta are anti-correlated for the same number in both components
$N^\uparrow = N^\downarrow$. Whenever the particle imbalance $\Delta N \neq 0$,
there is visible shift in momentum that corresponds to the net mo-
mentum of a correlated pair.

have nonzero net momentum $q_0 = p^\uparrow + p^\downarrow \neq 0$. It is
also very clear that the total momentum $q_0$ monotonically increases with the imbalance $\Delta N$ which is one of
the clearest signatures of the FFLO-like pairing. Moreover,
in the balanced case ($\Delta N = 0$) all momenta (below
maximal Fermi momentum) are accessible for fermions and they almost equally contribute to the collective pair-
ing mechanism (notice an almost flat distribution along the
anti-diagonal for $\Delta N = 0$ in Fig. 2). However, when
the system is imbalanced, particles with smaller momenta
do not contribute to the formation of pairs with net mo-
mentum $q_0$ (the empty region in the middle of the noise
correlation $G_\pi$ for $\Delta N > 0$ in Fig. 2). This kind of formation
of correlated pairs is another well-known property of
the FFLO mechanism [24, 48].

In the next step we aim to find the most probable net
momentum of the pair $q_0$ as a function of the imbalance
$\Delta N$. For this purpose, we introduce the filtering pro-
cedure giving us possibility to quantify the occurrence of
different FFLO momenta $q$. In general, the filtering is
done by convoluting the noise correlation with appropri-
ately chosen filter function:

$$Q(q) = \int dp^\uparrow dp^\downarrow F(p^\uparrow + p^\downarrow + q)G_\pi(p^\uparrow; p^\downarrow). \quad (3)$$

In our approach we choose the simplest Gaussian filtering
function $F(\xi) = (\pi \kappa)^{-1/2} \exp(-\xi^2/2\kappa^2)$ with $\kappa$ being of
the order of the perpendicular width of the enhanced cor-
rrelations area. We checked that the final results are not
sensitive to the exact shape of the filtering function, since
for reasonable values of $\kappa$ the most probable momentum
$q_0$ (value for which the measure $Q(q)$ is maximum) does
not change. In Fig. 3 we plot resulting function $Q(q)$ for
the system of $N = 10$ particles and different imbalances
$\Delta N$. It is clear, that the balanced system ($\Delta N = 0$)
is characterized by vanishing $q_0$ (black curve). When the
particle imbalance is increased, the maximum moves to-
wards higher absolute values of momenta.

Finally, to make a whole picture comprehensive, we
make a connection of the imbalance $\Delta N$ with the dis-
crepancy between Fermi momenta of both components
$\Delta p_F$. In the case of an essentially non-homogenous sys-
tem of a few particles, the definition of the Fermi momen-
tum is obviously not straightforward since the system is
not translationally invariant. Particularly, it is no longer
valid that the Fermi momentum $p_F^\sigma$ is proportional to
the number of particles $N^\sigma$. Moreover, due to a small
number of particles, any approaches based on local den-
sity approximation being appropriate for a large number
of particles (see for example [48, 51, 52]) are also not
adequate. To overcome this difficulty, let us first notice
that the well-determined quantity is the Fermi energy in
the limit of weak interactions, $\epsilon_F^\sigma = \hbar \omega (N^\sigma - 1/2)$. This energy defines the maximal value of the momentum
which is accessible for the particle moving on the Fermi
surface, $p_F^\sigma = \sqrt{2m \epsilon_F^\sigma}$. In the semi-classical picture,
this is a momentum gained by a particle when it passes through the middle of the trap. If we associate the Fermi momentum with this quantity then we immediately find a phenomenological connection between the imbalance $\Delta N$ and the maximal discrepancy of the Fermi momenta $\Delta p_F$. When momenta are expressed in the natural unit $\sqrt{\hbar m \omega}$ then this relation reads

$$\Delta p_F \approx p_F \uparrow - p_F \downarrow = \sqrt{2 \left( N \uparrow - \frac{1}{2} \right)^2 - 2 \left( N \downarrow - \frac{1}{2} \right)^2}. \quad (4)$$

Applying this definition, in the inset of Fig. 3 we plot the most probable net momentum of the pair $q_0$ as a function of the discrepancy $\Delta p_F$ for $N = 10$. It is clearly evident that all points lay almost exactly on the straight line. The situation is very similar if one repeats this procedure for a different number of particles. In Fig. 1 we show numerical results for $N = 3, \ldots, 14$ and different imbalances $\Delta N$. All these points almost ideally support relation $q_0 \approx \Delta p_F$ (dashed line) and display one of the fundamental consequences of the FFLO pairing mechanism — net momentum of the Cooper pair is equal to the Fermi momenta discrepancy, $q_0 = \Delta p_F$.

Finally, let us discuss evident deviations between numerical results and predictions of our phenomenological derivation. They can be explained on three levels. (i) It is clear that the definition of the momentum mismatch $\Delta p_F$ is very phenomenological and simplified. It focuses only on one momentum associated with the Fermi level. Therefore it may predict only a general trend rather than an exact relation. (ii) It is known that the relation between the FFLO momentum and the components’ Fermi momenta is, in fact, more complicated when interactions and an effective pairing potential are taken into account. For example, as discussed in [53], even simplified inclusion of an effective pairing potential immediately leads to increasing of the FFLO momentum. This effect is clearly seen in Fig. 1 (all points are shifted towards larger $q_0$). Moreover, as clearly evident in Fig. 3, the distribution of possible FFLO momenta becomes very wide for larger imbalances. Therefore, choosing a single $q_0$ to characterize a whole distribution is evidently oversimplified. (iii) The theory of FFLO pairing explains the appearance of correlations in terms of collective cooperation of all particles in the system. Therefore, it gives rigorous relations only for a large number of particles. From this point of view, the existence of some finite-size corrections is quite natural. They lead to small shifts of particular points in Fig. 1.

In summary, we showed — based on exact numerical calculations — that in the confined one-dimensional system of a few attractively interacting fermionic $^6$Li atoms the FFLO pairing mechanism is clearly manifested and can be detected with current experimental techniques. Taking into account tremendous tunability of ultra-cold systems, our proposal opens not only another route for the first direct experimental confirmation of unconventional pairing forced by broken symmetry between components, but also reveal an additional tool for studying the appearance of collectiveness when the quantum system undergoes a transition from few to the many-body limit. At this point, we want also to mention that the FFLO mechanism can be considered for spin-balanced few-body systems but with different mass atoms [54]. Taking into account huge experimental progress in controlling mass-imbalanced Fermi mixtures [55, 56], this possibility is also in-game. It should be noted however that the few-body regime for these kinds of systems has not been achieved yet.

ACKNOWLEDGEMENTS

The authors are very grateful to Konrad Kapcia, Maciej Lewenstein, Patrycja Lydżba, and Piotr Magierski for their fruitful comments and inspiring questions at different stages of this project. This work was supported by the (Polish) National Science Center Grants No. 2017/27/B/ST2/02792 (DP) and 2016/22/E/ST2/00555 (TS). Numerical calculations were partially carried out in the Interdisciplinary Centre for Mathematical and Computational Modeling, University of Warsaw (ICM) under the computational grant No. G75-6.
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SUPPLEMENTARY MATERIAL

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig_s1.png}
\caption{The most probable FFLO momentum $q_0$ as a function of the Fermi momenta mismatch $\Delta p_F$. Different points correspond to different number of particles and different imbalances. Labels linked to particular points denote the number of particles $(N_\uparrow, N_\downarrow)$. For clarity we do not show the point $q_0 = \Delta p_F = 0$ corresponding to the balanced scenario $(N_\uparrow = N_\downarrow)$.}
\end{figure}