Gauge Mediation from Emergent Supersymmetry

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Abstract

We explore the possibility of gauge mediation in a paradigm whereby supersymmetry is posited to be an accidental symmetry of Nature and the Standard Model fields are composite bound states that emerge from a conformal field theory. The resultant effective theory can, through sequestering and conformal dynamics, exhibit most of the properties of low energy supersymmetry breaking while averting a number of cosmological and astrophysical constraints of the traditional framework of gauge mediation via dynamical supersymmetry breaking. Of particular phenomenological interest is that in our scenario, the gravitino is superheavy, the neutralino LSP is a viable candidate for cold dark matter and the flavor changing neutral currents are constrained to be, at the very minimum, only an order of magnitude below current experimental bounds.

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1 Introduction

One of the most pressing and immediate concerns confronting theoretical high energy physics today is the question of what lies beyond the Standard Model (SM). Arguably, the most attractive scenario of physics beyond the SM is the existence of supersymmetry (SUSY). As is clear from the absence of supersymmetry in our world, it is manifestly broken. The breaking of supersymmetry in our sector, which shows itself in the sparticle spectrum, is dictated by the mediation mechanism or mechanisms as the case may be.

Gauge mediation\(^1\), by which our sector feels SUSY breaking through messenger fields charged under the SM gauge groups, ranks among the leading candidates for transmission of supersymmetry breaking. A simple and predictive mechanism, it gives flavor-independent contributions to soft masses that is proportional to the order parameter \( \frac{F}{M} \) where \( F \) and \( M \) are respectively the F-term and the scalar vacuum expectation value (vev) of some non-SM field that couples to the messenger fields. The question then arises as to how one might obtain \( F \) and \( M \). In the conventional picture of gauge mediation, \( F \) and \( M \) are obtained when SUSY is spontaneously broken by non-perturbative dynamical effects in a theory that possesses tree-level supersymmetric vacua, i.e. by dimensional transmutation. Dynamical supersymmetry breaking (DSB)\(^2\), as this mechanism is known, ensures the stability of the gauge hierarchy while at the same time evading the supertrace theorem that rules out communication of SUSY breaking through tree-level renormalizable couplings. Unfortunately, the phenomenology arising from this scenario or any other within the conventional framework usually leads to a gravitino lightest supersymmetric particle (LSP) as well as other light moduli fields\(^1\) that are plagued by a plethora of cosmological problems\(^3\).

Perhaps we need to take a step back and look at the theoretical assumptions of the above scenario, and see if there is a more compelling and natural framework in which to consider gauge mediation. One intriguing possibility is the “Supersymmetry without Supersymmetry” paradigm\(^4\) where SUSY is posited to be an accidental symmetry of Nature(see also Refs.\(^5\)\(^,\)\(^6\)). This scenario, which constitutes the theoretical underpinnings of the present paper, starts off with completely non-supersymmetric theories that flow, in the Wilsonian sense, to more supersymmetric ones. Though possessing the interesting property that the gravitino and potential cosmological moduli

\(^1\)In string or higher-dimensional supergravity theories, there generically exists a large number of flat directions which typically take on masses of the order of the gravitino mass, \( m_{3/2} \), once supersymmetry is broken.
fields are superheavy in this scenario, ultimately the theory does not solve the supersymmetric flavor problem. This seems to be a generic problem with previously considered theories of this class, i.e. the “Almost No-scale Supergravity” scenario. In this paper, we shall present an explicit model based on gauge mediation within the “Supersymmetry without Supersymmetry” paradigm that addresses the issues and shortcomings we have considered above. Before we give a summary of the results, a short discussion of the general paradigm is warranted. We shall, in the course of our discussion, be utilizing the AdS-CFT correspondence to extract useful insights from both perspectives.

Let us begin our discussion by questioning the implicit assumption that lies at the heart of the conventional SUSY picture. What if SUSY is not a fundamental symmetry of Nature? Even if it were, let us break it at the Planck scale. The fermion-boson splitting, and by extension the natural mass of the gravitino and the moduli fields, would then be Planckian in magnitude. Ostensibly, it would seem that we are re-introducing the hierarchy problem and we are no better off than when we started. However, this could be evaded if the Higgs fields as well as the rest of the SM are composite particles that “emerge” from the conformal sector. The $M$ in the order parameter is then related to the compositeness scale of the SM while $F$ would be the amount of SUSY breaking experienced by the SM fields. As we are considering Planck-sized fundamental SUSY breaking, $\sqrt{F}$ could in principle be much larger than $M$ and aside from not giving the correct observed low-energy SUSY breaking, would also break the SM gauge symmetries. However, the conformal sector fields can naturally conspire, by the sequestering mechanism, to systematically ameliorate the effects of high-energy supersymmetry breaking to give a theory that for most intents and purposes, exhibits the appearance of low-energy SUSY breaking, i.e. Higgs, gaugino and squark masses of $\sim O(100\text{GeV})$.

What kind of conformal sector do we need to achieve sequestering? First and foremost, it has to be strongly coupled such that the operators of the conformal field theory (CFT) have order one corrections to the anomalous dimensions which render all supersymmetry breaking operators sufficiently irrelevant such that their effects will become suppressed at low energies. The other ingredient that must be added is that there is a spontaneous breaking of the conformal symmetry at some intermediate scale so that most of the composite bound states (“mesons” and “baryons”)

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2Note that the sense in which we are employing the word sequestering is that the SUSY breaking effects are highly suppressed (which is crucial as we have Planckian breaking of SUSY) in our sector. It does not necessarily mean that anomaly mediation is the dominant contributor to SUSY breaking in the sparticle spectrum. Indeed as we shall see, anomaly mediation remains a sub-dominant effect in this scenario.
acquire intermediate masses and hence can be integrated out. What remains are the vestigial light composite (or emergent, if you like) degrees of freedom that form an effective low energy theory that has a gauge mediation messenger sector plus a nearly supersymmetric extension of the Standard Model, with either MSSM or the NMSSM being possibilities. This would then lead to a theory that is free of the cosmological problems of traditional gauge mediation while preserving the solution to the hierarchy problem.

Obviously, a glaring omission of the above discussion is how we are to explicitly realize this gauge mediation from emergent supersymmetry scenario. It is a highly non-trivial process to construct a strongly-coupled CFT possessing all the above properties. We could, however, exploit the AdS-CFT correspondence \cite{8} to construct a weakly-coupled fully-calculable five-dimensional AdS dual of the strongly-coupled four-dimensional CFT that would have exactly the same physics as described above. The basic picture is that we have a Randall-Sundrum (RS) type setup \cite{11} where the UV brane (identified as the Planck scale on the CFT side) has $\sim O(1)$ SUSY breaking while the bulk and IR brane (the compositeness scale of the CFT) are supersymmetric, insofar as the classical action is concern. The fact that we have a non-supersymmetric UV brane leads to SUSY breaking throughout the entire volume but at tree-level, the above conditions have to be chosen so as to match the superconformal limit of the CFT which would correspond to a restoration of SUSY on the IR brane as it is taken to infinity. This physical separation of the two sectors can suppress the SUSY breaking seen by the visible sector but it is an insufficient, albeit necessary, condition for sequestering. To accomplish sequestering on the AdS side, we require the absence of light bulk scalars which, on the CFT side, correspond to an absence of relevant operators(see Ref.\cite{10} for possible string-theoretic realizations). The SUSY breaking thus transmitted to the IR brane by the massive bulk scalars would be exponentially suppressed. The conformal symmetry breaking can then be realized on the AdS through bulk scalar dynamics that stabilizes the radius of the extra dimension. In the language of the CFT, the irrelevant operators dynamically break the conformal symmetry. For the present paper, we will be employing the racetrack mechanism in Ref.\cite{4} whereby the effective potential arising from two bulk scalars of nearly the same mass can terminate the conformal dynamics. The final step is the usual inclusion of a SM-charged messenger sector to couple to the bulk scalars and, as we shall show, through it impart the largest contribution of SUSY breaking to the visible sector.

The end result is that we have a phenomenologically viable theory that has among other things, a superheavy gravitino and moduli fields that decouple entirely from the
low energy physics and thus avoid cosmological and astrophysical constraints. In our model, the lightest neutralino is the LSP and assuming R-parity is conserved, is a candidate for cold dark matter. Because of the nature of the bulk (in CFT language, CFT states) SUSY breaking mediation mechanism, the FCNC contributions are not highly suppressed. The ratio of flavor non-diagonal to flavor diagonal contributions is at the very minimum only $\sim 10^{-3}$ and could be ruled out in future experiments. This is a completely different prediction from normal gauge mediation models where the FCNCs are negligible due to the extremely tiny ratio between the scale of SUSY breaking to $M_P$.

The rest of the paper is organized as follows. In Section 2, we present the setup and the equations of motion. Details of the general solutions are available in the Appendix. In Section 3, we calculate the various contributions to SUSY breaking in the low energy theory thus enabling us to eliminate the models from Section 2 that are non-viable. In Section 4, we construct an explicit model realizing gauge mediation from emergent SUSY. We also consider radius stabilization and the 4-D effective low energy theory. This culminates in a discussion of the sparticle spectrum and phenomenology arising from this class of theories as well as its differences with conventional gauge mediation. Section 5 contains our conclusions.

2 Setup and General Solutions

We follow the framework of Ref.[4] by having a Randall-Sundrum (RS) model [11] with a 5-D spacetime is compactified on a $S^1/Z_2 \times Z_2$ orbifold, with metric

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

(2.1)

where $y$ is a periodic variable with period $4\ell$, and $\sigma(y)$ is $(+,+)$ under the $Z_2 \times Z_2$.

With the addition of a hypermultiplet, the action is then given by [14],

$$S = -\frac{M_5^3}{k} \int d^4x \int d^4\theta (\omega^\dagger \omega - \varphi^\dagger \varphi) + \int d^4x \int_0^\ell dy \mathcal{L}_{5d-hyp},$$

(2.2)

where the radion chiral multiplet and the conformal compensator respectively contain

$$\omega = e^{-k\ell} + \cdots + \theta^2 F_\omega,$$

(2.3)

$$\varphi = 1 + \theta^2 F_\varphi.$$  

(2.4)
and the hypermultiplet Lagrangian\(^3\) is given by \([12]\),

\[
\mathcal{L}_{5d-\text{hyp}} = \int d^4 \theta e^{-2\sigma} (\Phi^\dagger \Phi + \tilde{\Phi}^\dagger \tilde{\Phi}) + \left[ \int d^2 \theta e^{-3\sigma} \left( \frac{1}{2} \tilde{\Phi} \partial_y \Phi + c\sigma' \tilde{\Phi} \Phi \right) + \text{h.c.} \right] - \delta(y) U(\Phi, \tilde{\Phi}, F, \tilde{F}) + \delta(y - \ell) \omega^3 \left[ \int d^4 \theta W(\Phi, \tilde{\Phi}) + \text{h.c.} \right].
\]

(2.5)

It is useful at this point to recall the AdS-CFT correspondence \([8]\) which relates the mass of the bulk scalars, \(m\), with the dimension of the CFT operator, \(d\), through the following equation, \(d = 2 + \sqrt{4 + m^2/k^2}\). The bulk masses of the scalars of the hypermultiplet above are given by

\[
m_{\Phi, \tilde{\Phi}}^2 = k^2 (c \mp \frac{3}{2}) (c \pm \frac{5}{2})
\]

(2.6)

Hence, the dimensions of the operators associated with the scalar components of \(\Phi\) and \(\tilde{\Phi}\) are

\[
\text{dim}(O_{\Phi, \tilde{\Phi}}) = 2 + \lvert c \pm \frac{1}{2} \rvert.
\]

(2.7)

As we require the operators associated with both \(\Phi\) and \(\tilde{\Phi}\) be irrelevant to achieve sequestering, we are therefore constrained to \(\lvert c \rvert > \frac{5}{2}\). This, as we shall see, does not mean we have to analyze a greater number of unique models as our orbifold parity conditions are capable of absorbing either sign of \(c\).

We can now obtain the equations of motion which we have explicitly written out in component form for clarity\(^4\).

\[
e^{-3\sigma} \left[ \partial_y + (c - \frac{3}{2}) \sigma' \right] F = \delta(y) \frac{\partial U}{\partial \Phi} - \delta(y - \ell) \omega^3 \frac{\partial^2 W}{\partial \Phi^2} \tilde{F}
\]

\[
e^{-3\sigma} \left[ \partial_y - (c + \frac{3}{2}) \sigma' \right] \tilde{F} = -\delta(y) \frac{\partial U}{\partial \tilde{\Phi}} + \delta(y - \ell) \omega^3 \frac{\partial^2 W}{\partial \tilde{\Phi}^2} F
\]

\[
e^{-3\sigma} \left[ \partial_y + (c - \frac{3}{2}) \sigma' \right] \Phi + e^{-2\sigma} \tilde{F} = \delta(y) \frac{\partial U}{\partial F} - \delta(y - \ell) \omega^3 \frac{\partial W}{\partial \Phi}
\]

\[
e^{-3\sigma} \left[ \partial_y - (c + \frac{3}{2}) \sigma' \right] \tilde{\Phi} - e^{-2\sigma} F = -\delta(y) \frac{\partial U}{\partial \tilde{F}} + \delta(y - \ell) \omega^3 \frac{\partial W}{\partial \tilde{\Phi}}
\]

(2.8)

\(^3\)We have written the action in terms of the two-sided derivative \(\tilde{\Phi} \partial_y \Phi = \Phi \partial_y \Phi - (\partial_y \tilde{\Phi}) \Phi\). Also, our \(y\)-integration is defined as \(\int^\ell_\epsilon = \int^\ell_{-\epsilon} + \frac{1}{2}(\int^\ell_{-\epsilon} + \int_{-\epsilon}^{\ell+\epsilon})\).

\(^4\)One may also write them in terms of superfields for a more compact form.
The general solution (for $0 < y < \ell$) is then given by

$$F = F_0 e^{-(c-\frac{3}{2}) \sigma}$$

$$\tilde{F} = \tilde{F}_0 \frac{\sigma'}{k} e^{(c+\frac{3}{2}) \sigma}$$

$$\Phi = \Phi_0 e^{-(c-\frac{3}{2}) \sigma} - \frac{\tilde{F}_0^\dagger}{(2c+1)k} e^{(c+\frac{3}{2}) \sigma}$$

$$\tilde{\Phi} = \tilde{\Phi}_0 e^{(c+\frac{3}{2}) \sigma} - \frac{F_0^\dagger}{(2c-1)k} e^{-(c-\frac{3}{2}) \sigma}$$

The jump conditions at the UV and IR branes dictate the following relations between bulk and brane quantities.

$$F^+_{UV} - F^-_{UV} = \frac{\partial U}{\partial \Phi_{UV}}$$

$$F^+_{IR} - F^-_{IR} = -\frac{\partial^2 W}{\partial \Phi^2_{IR}} \tilde{F}_{IR}$$

$$\tilde{F}^+_{UV} - \tilde{F}^-_{UV} = -\frac{\partial U}{\partial \Phi_{UV}}$$

$$\tilde{F}^+_{IR} - \tilde{F}^-_{IR} = \frac{\partial^2 W}{\partial \Phi^2_{IR}} F_{IR}$$

and

$$\Phi^+_{UV} - \Phi^-_{UV} = \frac{\partial U}{\partial F_{UV}}$$

$$\Phi^+_{IR} - \Phi^-_{IR} = -\frac{\partial W}{\partial \Phi_{IR}}$$

$$\tilde{\Phi}^+_{UV} - \tilde{\Phi}^-_{UV} = -\frac{\partial U}{\partial F_{UV}}$$

$$\tilde{\Phi}^+_{IR} - \tilde{\Phi}^-_{IR} = \frac{\partial W}{\partial \Phi_{IR}}$$

where we have defined $\lim_{\epsilon \to 0} f(\pm \epsilon) \equiv f^\pm_{UV}$, $\lim_{\epsilon \to 0} f(\ell \pm \epsilon) \equiv f^\pm_{IR}$, $f(0) \equiv f_{UV}$ and $f(\ell) \equiv f_{IR}$.

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5In the interest of generality, we have not yet specified a specific set boundary conditions.
The above results can then be used to write down the effective potential purely in terms of brane-localized quantities.

\[
V_{\text{eff}} = \omega^3 \left( -\Phi_{\text{IR}}^- \tilde{F}_{\text{IR}}^- + \tilde{\Phi}_{\text{IR}}^- F_{\text{IR}}^- + \text{h.c.} \right) + U \\
+ \left( \Phi_{\text{UV}} F_{\text{UV}}^+ - \Phi_{\text{UV}}^+ F_{\text{UV}}^- - \tilde{\Phi}_{\text{UV}} \Phi_{\text{UV}}^+ - F_{\text{UV}}^+ \tilde{\Phi}_{\text{UV}} + \text{h.c.} \right)
\] (2.12)

Consistent orbifolding requires that the orbifold parity of \( \tilde{\Phi} \) be opposite to that of \( \Phi \) under both \( Z_2 \) and we have used this to simplify the general potential to the above form. We can further simplify the form of the potential by assigning to \( \Phi \) either \((+,+), (-,+), (+,-) \) or \((-,-)\) under the \( Z_{2,UV} \times Z_{2,IR} \). Note that we can perform the orbifold parity reversal for all the above jump conditions, \(+ \leftrightarrow -\), and also the following transformations \( c \to -c \), \( \Phi \to -\tilde{\Phi} \) and \( \tilde{\Phi} \to \Phi \) to maintain the same equations of motion and potential. This essentially reduces the number of cases to the following four: (i) \((+,+)\) and \( c > 0 \), (ii) \((+,+)\) and \( c < 0 \), (iii) \((-,+\)) and \( c > 0 \) and (iv) \((-,+\)) and \( c < 0 \).

The full analysis and classification of these cases, which are somewhat tortuous, are presented in the Appendix. The main results for the viable cases are summarized in the Table I. The opposite orbifold parities of \( \Phi \) and \( \tilde{\Phi} \) and the fact that we can always define \( \Phi \) to have even orbifold parity at the IR brane permit the following simplification.

\[
V_{\text{eff}} = U - \left[ \Phi_{\text{UV}} F_{\text{UV}}^+ - \Phi_{\text{UV}} F_{\text{UV}}^- + \tilde{\Phi}_{\text{UV}} \Phi_{\text{UV}}^+ - F_{\text{UV}} \tilde{\Phi}_{\text{UV}}^+ \right] + \text{h.c.}
\] (2.13)

3 Supersymmetry Breaking

In this section, we consider the various supersymmetry breaking contributions to the visible sector and calculate their effects. There is a variety of ways by which high-energy supersymmetry breaking is transmitted to us; 5-d gravity loops, gauge mediation, direct mediation from unknown UV physics and anomaly mediation. The 5-d gravity loop calculations is given by [13] and the soft masses induced by 1-loop gravity effect are estimated to be,

\[
m_{\text{soft,gravity}} \sim \omega^2
\] (3.1)

where for estimation purposes, we do not display the 1-loop factor but we do take it into account when we compare relative strengths of SUSY breaking mechanisms.
Also, we will be using the normalization $M_5 = 1$ and will do so for the rest of the paper. $M_5$ can easily be restored by dimensional analysis.

For the gauge mediation sector, we add a pair of vector-like messenger fields $Q_M$ and $\bar{Q}_M$ of $5 + 5^*$ representation under the SM gauge group $SU(5)_SM \supset SU(3)_c \times SU(2)_L \times U(1)_Y$ that will couple to $\Phi$ through the IR-brane-localized superpotential term, $\Delta W = \Phi Q_M \bar{Q}_M$. From the point of view of the CFT, this corresponds to composite bound states of the CFT that acquire intermediate masses and convey supersymmetry breaking to our SM fields through their $F$-terms.

The effective 4-d Lagrangian that characterizes the soft SUSY breaking masses from the various mechanisms\(^6\) can then be parameterized as

$$\mathcal{L}_{4d} = -V_{\text{eff},\omega} + \int d^4 \theta \omega^\dagger \omega \left[ 1 + (1 + \phi_{\text{IR}}^\dagger \phi_{\text{IR}})(Q^\dagger Q + Q_M^\dagger Q_M + \bar{Q}_M^\dagger \bar{Q}_M) \right]$$

$$+ \int d^2 \theta \beta \omega^3 \phi_{\text{IR}} Q_M Q_M + \text{h.c.} \quad (3.2)$$

where $Q$ denote SM chiral superfields and we have set most of the coefficients (except $\beta$, which though also of order one, is essential for us to see that we have viable gauge mediation) to be one for simplicity. The effective potential of the radion, $V_{\text{eff},\omega}$, can be found by redoing the entire calculation thus far in terms of superfields and then through the method of spurion analysis\(^{[14]}\), promote the radius into a superfield and finally extracting the requisite potential by singling out the $F$-term of the radion, $F_\omega$.

We have only outlined the above procedure as the cases of unique or bifold parities and $c$ lead us to the same result as Ref. \([4]\).

$$V_{\text{eff},\omega} = \omega^2 \left[ 3 \left( -W + \frac{1}{2} \phi \frac{\partial W}{\partial \phi} \right) \right.$$  

$$\left. + \frac{1}{2} \omega \frac{\partial \phi_{\text{IR}}}{\partial \omega} \left( - \frac{\partial W}{\partial \phi} + \phi \frac{\partial^2 W}{\partial \phi^2} \right) \right]_{\text{IR}} F_\omega + \text{h.c.} + \cdots \quad (3.3)$$

With the effective radion potential, we can determine the scale of anomaly mediation.

$$m_{\text{soft,anomaly}} \sim \frac{F_\omega}{\omega} \sim \frac{1}{\omega} \frac{\partial V_{\text{eff,}\omega}}{\partial \omega} \sim \omega W \sim \omega \phi_{\text{IR}}^n \sim \Lambda_{\text{IR}} \omega^{n \frac{d-5}{m-1}} \quad (3.4)$$

where $\Lambda_{\text{IR}} = \omega M_5$ is the cutoff of the 4-d effective theory localized on the IR brane.

\(^6\)With the exception of gravity which we have estimated earlier.
After canonical normalization, the direct mediation from the Lagrangian above gives the following direct contribution to the SM chiral superfields.

\[ m_{\text{soft, direct}}^2 \sim F_{\text{IR}} \Phi_{\text{IR}} \]  

(3.5)

This is generally flavor non-diagonal and causes SUSY FCNC processes that are strongly constrained by experiments.

The hypermultiplet-messenger interaction term in the superpotential completely fixes the parameters of our gauge mediation sector. This allows us to write down the mass matrix of the scalar messengers.

\[ m_{\text{messenger}}^2 = \left( \beta^2 \omega^\dagger \omega |\Phi_{\text{IR}}|^2 + |F_{\text{IR}}|^2, \beta \omega F_{\text{IR}} \right) \]

\[ \beta \omega^\dagger F_{\text{IR}} \beta^2 \omega^\dagger |\Phi_{\text{IR}}|^2 + |F_{\text{IR}}|^2 \]

(3.6)

Requiring that SM gauge symmetries (as the messenger superfields are charged under the SM) not be broken gives us the following constraint.

\[ \beta^2 \omega^\dagger |\Phi_{\text{IR}}|^2 + |F_{\text{IR}}|^2 \geq |\beta \omega F_{\text{IR}}| \]

(3.7)

Obviously, the \( |F_{\text{IR}}|^2 \) term should not dominate as we would otherwise have \( \Lambda_{\text{IR}} < |F_{\text{IR}}| \). Taking this into account, the constraint condition becomes \( \beta \omega |\Phi_{\text{IR}}|^2 \geq |F_{\text{IR}}| \).

The gauge mediation soft breaking masses are given by

\[ m_{\text{soft, gauge}} \sim \frac{F_{\text{IR}}}{\Phi_{\text{IR}}} \]

(3.8)

In order to have a viable gauge mediation scenario, we require that \( \Phi_{\text{IR}} \ll \omega^0 \) as well, so that the messenger scale is lower than \( \Lambda_{\text{IR}} \) and that gauge mediation dominates over direct mediation. As can be easily seen from the results given in the Appendix, there are only two cases which satisfy the above requirements: both orbifold parity assignments with \( c < 0, a_1 = 0 \) and \( n \geq 3 \). The solutions to the equations of motion for these viable models are summarized in Table 1.

In a nutshell, the soft contributions from different mechanisms for transmission of SUSY breaking are, to leading order, given below.

\[
\begin{align*}
F_{\text{IR}} & \sim \Lambda_{\text{IR}} \omega^{d-5\frac{c}{2}(n-2)} \quad \text{gauge} \\
F_{\text{IR}} & \sim \Lambda_{\text{IR}} \omega^{d-5\frac{c}{2}(n-1)} \quad \text{direct} \\
\frac{F_{\text{IR}}}{\omega} & \sim \Lambda_{\text{IR}} \omega^{d-5\frac{c}{2}n} \quad \text{anomaly} \\
& \sim \Lambda_{\text{IR}} \omega \quad \text{gravity}
\end{align*}
\]

(3.9)

We see that for \( n \geq 3 \) and \( c < -\frac{5}{2} (d > 5) \), gauge mediation always dominates over direct and anomaly mediation and will dominate over gravity in phenomenologically viable regions.
Table 1. Summary of the results from the Appendix for various brane quantities. We give the leading power \( p \), where \( Q = \omega^p + \) higher order term in \( \omega \), for two cases with \( c < 0 \). We have used \( d \equiv |c| + \frac{5}{2} \), \( \tilde{d} \equiv |c| + \frac{3}{2} \) and we parameterized the superpotential by \( W = a_n \Phi^n \) plus higher order terms with \( n \geq 2 \). As for the UV potential, \( U \) is taken to be, to leading order, a function of \( \Phi_{UV} \) for the \((+,+)\) case or a function of \( \tilde{\Phi}_{UV}^{q+1} \) for the \((-,+)_s\) case, \( (q \geq 1) \).

| Quantity (to leading order) | \((+,+)\) orbifold parity for \( \Phi \) | \((-,+)_s\) orbifold parity for \( \Phi \) |
|-----------------------------|---------------------------------|---------------------------------|
| \( F_{IR} \)               | \( \frac{d-5}{2n-3}(n-1)+1 \) | \( \frac{d-5}{2n-3}(n-1)+1 \) |
| \( \Phi_{IR} \)            | \( \frac{d-5}{2n-3} \)         | \( \frac{d-5}{2n-3} \)         |
| \( F_{UV} \)               | \( \frac{d-5}{2n-3}(n-1)+d \)  | \( \frac{d-5}{2n-3}(n-1)+d \)  |
| \( \Phi_{UV} \)            | 0                               | 0                               |
| \( \tilde{\Phi}_{IR} \)    | \( \frac{\partial U}{\partial F} \) | \( \frac{\partial U}{\partial F} \) |
| \( \tilde{F}_{UV} \)       | \( \frac{d-5}{2n-3}(n-1) \)    | \( \frac{d-5}{2n-3}(n-1) \)    |
| \( \tilde{F}_{IR} \)       | \( d-4 \)                       | \( d-4 \)                       |

The radion mass can be found from the scalar potential given in eq.(2.13) and the sub-dominant contribution is given by \( F^\dagger \omega F \). The radion mass from the latter can be estimated quickly as \( \Delta m_{\text{radion}} \sim \frac{E_n}{\omega} \) which is of order \( m_{\text{soft,anomaly}} \). For the other contributions, as we need all the possible \( \omega \)-dependent leading powers of \( \omega \), knowing that the leading order of \( \Phi_{UV} \) and \( \tilde{F}_{UV} \) are \( O(1) \) in Planckian units may not be enough. It is possible that the next order, which is the leading \( \omega \)-dependent order, may contribute. Although it turns out to be unimportant, we give the result here for completeness.

\[
\tilde{F}_{UV} \sim \Phi_{UV} \sim 1 + \Phi_{IR}\omega^\tilde{d} \tag{3.10}
\]

The potential is dominated by the first term in eq.(2.13) and we found

\[
V \sim \omega^4 \omega^{2(n-1)} \frac{d-5}{2n-3} \tag{3.11}
\]

This gives us the radion mass,

\[
m_{\text{radion}} \sim \Lambda_{IR}\omega^{(n-1)} \frac{d-5}{2n-3} \tag{3.12}
\]

which is the same order as the direct mediation soft masses. In our case, the radion mass is around 10 GeV. Due to the effective 4-d theory cutoff \( \text{TeV} \ll \Lambda_{IR} \ll M_{pl} \), a
10 GeV radion is not ruled out by either collider search or cosmological observation. We will discuss this in more detail in the next section. If the same potential is to be used to stabilize the radius, it has to be at least bigger than the potential due to the Casimir effect\(^7\) which is estimated to be \(\omega^6\) \[^{15}\]. From the results given above, we see that \(d\) should be \(d \leq 7 - 1/(n - 1)\).

### 4 Stabilization and Phenomenology

We now analyze a specific model and discuss the stabilization and phenomenological issues involved. We pick the \((+, +)\) orbifold parity condition as well as \(d > 5\) (which also means \(c < -\frac{5}{2}\)) and choose a potential on the UV brane and a superpotential on the IR brane of the following form.

\[
U = b(\Phi_{UV} + \Phi^\dagger_{UV}), \quad W = a\Phi^3_{IR}
\]

We discard the higher order terms of \(\Phi_{UV}\) in \(U\) for simplicity. Restoring these terms does not affect the results significantly as our equations contain only first derivatives in \(\Phi_{UV}\) and since \(b\) is of order one. Although \(F\) dependent term can be added to \(U\), this modification will only change the result of \(\tilde{F}_{UV}\) which is not relevant for our purpose.

Solving the equations of motion yields the following relations.

\[
\Phi_{IR} = -\left(\frac{b}{(3a)^2(2d - 4)k}\right)^{\frac{1}{5}} \omega^{\frac{d-5}{3}} + ...
\]

\[
F_{IR} = -\left(\frac{b^2 k(d - 2)}{12a}\right)^{\frac{1}{5}} \omega^{\frac{d-1}{3}} + ...
\]

\[
\Phi_{UV} = -\frac{b}{2(2d - 4)k} + \Phi_{IR}\omega^d
\]

The jump conditions across the brane then determines the solution of the other fields.

\[
\tilde{F}_{UV} = -\frac{1}{2} b
\]

\[
\tilde{\Phi}_{IR} = -\frac{1}{2} \left(\frac{b^2}{3ak^2(2d - 4)^2}\right)^{\frac{1}{5}} \omega^2 \omega^{\frac{d-5}{3}} + ...
\]

\[
\tilde{\Phi}_{UV} = 0
\]

\(^7\)An interesting possibility for radius stabilization is through an interplay of bulk hypermultiplet and Casimir effects.
The final form of the effective potential, to leading order, can then be obtained.

\[ V_{\text{eff}} = \frac{3b^4}{4} \Phi \omega^{d-1} + \ldots = -\frac{1}{2} \left( \frac{3b^4}{2a^2k(d-2)} \right) \frac{1}{\omega^4} \omega^{d'} + \ldots \]  

Notice that the sign of the potential is always negative. In order to have a racetrack-type stabilization, we need to include another contribution to the effective potential which would give a leading order term with a positive sign. To do that, let us introduce another bulk hypermultiplet, \( \Psi \) (which corresponds to a CFT operator dimension \( d' \)), with the same \((+,+)^8\) orbifold parity conditions but we flip \( c \)'s sign so that \( c > 0 \). The brane-localized potential and superpotential for \( \Psi \) is,

\[ U = b'\Psi_{\text{UV}}^2 + b'_2 F + \text{h.c.}, \quad W = a'\Psi_{\text{IR}}^2 \]  

which leads to an effective potential due to \( \Psi \) of the form

\[ V_{\text{eff,}\Psi} = \left[ 1 - \frac{a'^2(d' - 3)}{d' - 2} \right] b'_2(d' - 3)k\omega^{2d' - 6} + \ldots \]  

where for small enough \( a' \) we can get a positive contribution to the effective potential. For racetrack stabilization to occur, we can compare the \( \omega \)-dependence of Eq.(4.4) with the above and conclude that

\[ d' = \frac{2d + 5}{3} + \epsilon \]  

where \( \epsilon \) is an \( \mathcal{O}(\frac{1}{10}) \) positive number. Now, we need to check the SUSY breaking to ensure our earlier discussion is not invalidated by the presence of this hypermultiplet. First, the direct mediation contribution is parameterized by

\[ F_{\text{IR,}\Psi} = b'_2(d' - 3)k\omega^{d' - 4} + \ldots \sim F_{\text{IR,}\Phi}\omega^\epsilon \]  

on substitution of Eq.(4.7), and so we have a subdominant contribution to FCNCs from this hypermultiplet. As for the anomaly-mediated contribution, plugging in our IR superpotential into Eq.(3.3), we obtain a vanishing coefficient for the \( F_\omega \) tadpole term which is to be expected as this term preserves conformal symmetry (\( a' \) is dimensionless). The gravity loop contributions are the same while we do not need to consider gauge mediation from this sector as we are not coupling \( \Psi \) to the messengers. It is clear that the effects of the \( \Psi \) field are subdominant and for the rest of the paper, we will only consider the contributions from the \( \Phi \) field.

\[ \text{Although the \((+,+)^8\) orbifold boundary condition with } c > 0 \text{ does not lead to viable gauge mediation, it can however be used for stabilization.} \]
Having the full effective potential would allow us to work out the radion mass in this scenario,

\[
m_{\text{radion}}^2 \sim \frac{20(d' - d)}{9} \left( \frac{b^4(d - 2)^2}{18a^2k} \right)^\frac{1}{3} \omega^4 A_{\text{IR}}^2 \tag{4.9}
\]

where we have expressed it in terms of \( \Lambda_{\text{IR}} = \omega M_5 \). To obtain actual values, one must note the coefficient in front of \( \Lambda_{\text{IR}} \) is still expressed in units where \( M_5 = 1 \).

We are now ready to discuss the phenomenology of our model. Assuming a low-energy MSSM content, the gauge mediation contribution to the sparticle masses is roughly given by (for \( n = 3 \) case) \cite{16}

\[
m_{\text{soft}} \sim \frac{\alpha_{\text{SM}} F_{\text{IR}}}{4\pi} \sim 10^{-2} \Lambda_{\text{IR}} \omega^\frac{d - 5}{3}, \tag{4.10}
\]

where \( \alpha_{\text{SM}} \) stands for the SM gauge coupling constants. Considering that the messenger scale is given by \( M_{\text{mess}} \sim \omega \Phi_{\text{IR}} \sim \Lambda_{\text{IR}} \omega^\frac{d - 5}{3} \), we find \( M_{\text{mess}} \sim 10^2 m_{\text{soft}} \sim 10 - 100 \) TeV for the natural scale of the sparticle mass \( m_{\text{soft}} = 100 - 1000 \) GeV, independent of the parameters of the model.

We also have the direct mediation contribution,

\[
m_{\text{direct}} \sim F_{\text{IR}} \sim \Lambda_{\text{IR}} \omega^\frac{d - 5}{3}. \tag{4.11}
\]

This is flavor-dependent and should be a sub-dominant contribution compared to the gauge mediation contribution being flavor blind. Define the ratio as

\[
\epsilon = \frac{m_{\text{direct}}}{m_{\text{soft}}} \sim 10^2 \omega^\frac{d - 5}{3}. \tag{4.12}
\]

Using this and the relation \( \Lambda_{\text{IR}} = M_5 \omega \), Eq. (4.10) leads to the relation between \( d \) and \( \epsilon \),

\[
d = 5 + 3 \log(10^{-2}\epsilon) - \log\left(\frac{10^4 m_{\text{soft}}}{\epsilon M_5}\right). \tag{4.13}
\]

FCNC processes induced by flavor dependent soft terms are strongly constrained by experiments, roughly \( \epsilon \lesssim 10^{-2} \) \cite{17}. This gives the lower bond on \( d \geq 6.16 \) when we take \( m_{\text{soft}} = 100 \) GeV and \( M_5 = 2.4 \times 10^{18} \) GeV (reduced Planck mass). Recall that there exists an upper bound on \( d \leq 6.5 \) as discussed in the previous section. Therefore, the parameter \( d \) (in the \( n = 3 \) case) should lie in the range \( 6.16 \leq d \leq 6.5 \) which corresponds to \( 1.6 \times 10^{-3} \leq \epsilon \leq 10^{-2} \) and a compositeness scale of \( 6.2 \times 10^8 \) GeV \( \geq \Lambda_{\text{IR}} \geq 1.0 \times 10^6 \) GeV. It is extremely interesting that the upper
bound on $d$ gives the lower bound on $\epsilon$ being an order of magnitude below current experimental bounds. We may expect that future experiments will reveal a sizable FCNC originating from flavor-dependent soft masses. In comparison, conventional gauge mediation models have negligibly small FCNC predictions.

There is another crucial difference between our model from conventional models. The gravitino and cosmological moduli, by virtue of SUSY being an accidental symmetry, have Planckian masses and therefore completely decouple from the low energy phenomenology. This is in contrast to the usual gauge mediation models where gravitino is always the LSP. The feature where gravitino is not the LSP in a gauge mediation model was first proposed in Ref. [18] in a different context. In their model, the gravitino mass lies in the range of 100 GeV to 1 TeV and consequently still suffers from the gravitino and cosmological moduli problems [19], [20]. In our model, there is no gravitino problem, because the superheavy gravitino cannot be produced in the early universe.

Assuming R-parity conservation, the LSP neutralino is the most reasonable candidate for dark matter. This case was first investigated in detail in Ref. [21] and it was found that in a wide range of parameter space, the neutralino LSP is primarily composed of the B-ino and it can constitute the dominant component of the dark matter through co-annihilation processes with the right-handed scalar leptons. Note that recent cosmological observations, especially the Wilkinson Microwave Anisotropy Probe (WMAP) satellite [22], have established the relic density of the cold dark matter with great accuracy (in the $2\sigma$ range),

$$\Omega_{\text{CDM}}h^2 = 0.1126^{+0.0161}_{-0.0181}.$$  

In addition, recent results of LEP-2 have pushed up the lower bound on the lightest Higgs mass, $m_h \geq 114$ GeV. These recent results will dramatically reduce the allowed parameter region previously obtained in [21], and updating the previous results would be a relevant and worthwhile exercise. We will give a full detailed phenomenological studies including additional experimental considerations as well as the latest cosmological and astrophysical observations in a forthcoming paper [23].

Before concluding, we briefly consider phenomenology and cosmology related to the radion. One might naively be tempted to conclude that we have exchanged the problems associated with the gravitino in conventional gauge mediation for ones associated with the radion in the present scenario. But actually, the radion behaves in a very different way from the gravitino. Although its precise value depends on parameters in the model including those in the brane potentials, the mass scale of the radion lies around 10 GeV. After electroweak symmetry breaking, through mixing
with neutral Higgs boson, the radion couples to the SM particles with strength of \( \sim y \frac{v}{\Lambda_{\text{IR}}} \), where \( y \) is the Yukawa coupling constant and \( v \) is the Higgs vev. This coupling is very much suppressed and so the radion totally decouples from the collider phenomenology.

The most stringent constraint actually comes from cosmological considerations. There is a possibility that the coherent oscillation of the radion to dominate the energy density of the early universe at a low temperature and its decay into SM particles to reheat the universe. In order not to change the successful predictions of Big Bang Nucleosynthesis (BBN), the reheating temperature should exceed the temperature of the BBN era, typically \( \mathcal{O}(1 \, \text{MeV}) \). For a radion mass of around 10 GeV, the radion decays mainly into \( \bar{b}b \) and \( \bar{\tau}\tau \) and its decay width can be estimated as

\[
\Gamma \sim \frac{m_{\text{radion}}^2}{\Lambda_{\text{IR}}} \sim 10^{-15} \text{GeV},
\]

which gives us a reheating temperature of the order

\[
T_{RH} \sim \sqrt{\frac{\Gamma}{M_{\text{Pl}}}} \sim 50 \text{GeV},
\]

which is high enough not to affect BBN. Note that this temperature is also sufficient for a neutralino dark matter with mass around 100 GeV (which we have discussed earlier) to be in thermal equilibrium.

5 Conclusions

We have considered various five-dimensional brane and bulk configurations to determine generic setups that would allow the implementation of the “Supersymmetry without Supersymmetry” paradigm in a gauge mediation setup as opposed to the traditional dynamical supersymmetry breaking. Requiring that the dominant contribution to the sparticle masses be through gauge mediation, instead of anomaly, direct or gravity mediation, and that flavor changing contributions from direct mediation be lower than current experimental bounds, we have established a region of parameter space whereby a class of these gauge mediation from emergent supersymmetry theories can naturally exist. In addition to having no small parameters in our theory, the parameters of the gauge mediation sector are completely fixed by the conformal dynamics.

We find that the racetrack stabilization of the extra dimension, or alternatively the spontaneous breaking of the conformal symmetry through irrelevant operators from the CFT viewpoint, necessarily dictates that the FCNCs are only suppressed to
the extent that it is, at the minimum, an order magnitude below current experimental bounds. This is very different from conventional GMSB models where the FCNC contributions are negligibly small. This setup not only solves the hierarchy problem and the SUSY flavor problem but also averts the gravitino problem by completely decoupling it from low-scale physics as it receives mass corrections of Planckian order. The radion is the only low-energy degree of freedom in this theory besides the SM fields and their supersymmetric partners. But it decouples from collider phenomenology and does not affect Big Bang Nucleosynthesis. Assuming R-parity conservation, the neutralino LSP (dominantly B-ino) from this class of gauge-mediated theories can also provide a cold dark matter candidate.

This class of models frees up large regions of parameter space for gauge mediation that were previously excluded in the conventional picture. Many interesting model-building possibilities and directions await further exploration with this explicit realization of gauge mediation from emergent supersymmetry.
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Appendix: Classification of General Solutions

In this section, we classify the general solutions that arise after the imposition of the orbifold parity and the selection of the sign of $c$. Before we go any further, we would like to introduce some notational changes that will help us understand the underlying conformal dynamics of this model. To this end, we can use the fact that we have large anomalous dimensions to re-express the dimensions of the operators corresponding to $\Phi$ and $\tilde{\Phi}$ respectively as

$$d \equiv |c| + \frac{5}{2}, \quad \tilde{d} \equiv |c| + \frac{3}{2}$$

and we parameterize the superpotential by $W = a_1 \Phi + a_n \Phi^n$ with $n \geq 2$.

A.2 Case (i): $(+, +)$ and $c > 0$

The equations that have to be solved are

$$\frac{\partial U}{\partial \Phi_{\text{UV}}} = \frac{\partial^2 W}{\partial \Phi_{\text{IR}}^2} F_{\text{UV}} \omega^{(d + \tilde{d} - 4)}$$

(A.2)

$$\frac{\partial U}{\partial F_{\text{UV}}} = \frac{\partial W}{\partial \Phi_{\text{IR}}} \omega^{\tilde{d}} + 2 F_{\text{UV}} \frac{1 - \omega^{2d - 4}}{(2d - 4)k}$$

(A.3)

$$\Phi_{\text{IR}} = \Phi_{\text{UV}} \omega^{(d - 4)} + \frac{F_{\text{UV}}}{2(2d - 4)k} \left( \frac{\partial^2 W}{\partial \Phi_{\text{IR}}^2} \right) \omega^{(d - 5)} (1 - \omega^{2d - 4})$$

(A.4)
The other parameters can be obtained from
\[
\tilde{\Phi}_{\text{IR}} = -\frac{1}{2} \frac{\partial W}{\partial \Phi_{\text{IR}}} \quad (A.5)
\]
\[
\tilde{F}_{\text{UV}} = \tilde{F}_{\text{IR}} \omega^d = -\frac{1}{2} \frac{\partial U}{\partial \Phi_{\text{UV}}} \quad (A.6)
\]
\[
F_{\text{IR}} = F_{\text{UV}} \omega^{d-4} \quad (A.7)
\]
\[
\tilde{\Phi}_{\text{UV}} = -\frac{1}{2} \frac{\partial U}{\partial F_{\text{UV}}} \quad (A.8)
\]

To estimate the size of these fields, we note that for a generic UV potential, both \(\Phi_{\text{UV}}\) and \(F_{\text{UV}}\) are of \(O(1)\) and satisfy,
\[
\frac{\partial U}{\partial \Phi_{\text{UV}}} = 0 \quad \text{and} \quad \frac{\partial U}{\partial F_{\text{UV}}} = 0 \quad (A.9)
\]

Although we can have a model with suppressed \(\Phi_{\text{UV}}\) or \(F_{\text{UV}}\), or both, we restrict ourselves to a class of model where \(F_{\text{UV}}\) is of order one as \(F\) is then UV dominated. A small \(\Phi_{\text{UV}}\) does not provide interesting models for gauge mediation by itself since \(\Phi_{\text{IR}}\) is smaller in this case and so would be harder to meet the requirements of gauge mediation model-building. As we will see below, even \(\Phi_{\text{UV}} \sim 1\) is insufficient for gauge mediation.

With \(F_{\text{UV}}\) and \(\Phi_{\text{UV}}\) set to one, Eq. (A.4) gives us,
\[
\Phi_{\text{IR}} \sim \begin{cases} \omega^{d-4}, & a_2 \neq 0 \\ \omega^{d-4}, & a_2 = 0 \end{cases} \quad (A.10)
\]
\[
F_{\text{IR}} \sim \omega^{d-4} \quad (A.11)
\]

### A.3 Case (ii): (+, +) and \(c < 0\)

The equations that have to be solved are
\[
\frac{\partial U}{\partial \Phi_{\text{UV}}} \omega^{d-4} = \frac{\partial^2 W}{\partial \Phi_{\text{IR}}^2} F_{\text{IR}} \quad (A.12)
\]
\[
\frac{\partial U}{\partial F_{\text{UV}}} \omega^{d-4} = \frac{\partial W}{\partial \Phi_{\text{IR}}} + 2 \frac{F_{\text{IR}}^\dagger (1 - \omega^{2d-4})}{(2d-4)k} \omega^{-1} \quad (A.13)
\]
\[
\Phi_{\text{UV}} \omega^{d-4} = \Phi_{\text{IR}} \omega^{d+\tilde{d}-4} - \frac{F_{\text{IR}}^\dagger}{2(2d-4)k} \left( \frac{\partial^2 W}{\partial \Phi_{\text{IR}}^2} \right)^\dagger (1 - \omega^{2\tilde{d}-4}) \quad (A.14)
\]
The other parameters are then obtained from

\[ \Phi_{IR} = -\frac{1}{2} \frac{\partial W}{\partial \Phi_{IR}} \]  
(A.15)

\[ \tilde{F}_{UV} = \tilde{F}_{IR} \omega^{4-d} = -\frac{1}{2} \frac{\partial U}{\partial \Phi_{UV}} \]  
(A.16)

\[ F_{IR} = F_{UV} \omega^{-d} \]  
(A.17)

\[ \Phi_{UV} = -\frac{1}{2} \frac{\partial U}{\partial F_{UV}} \]  
(A.18)

In this case, \( \tilde{F}_{UV} \) is UV dominated and so we consider only \( \tilde{F}_{UV} = O(1) \) which, from Eq. (A.16) and the fact that \( F \) is IR dominated, requires \( U = U_1(\Phi_{UV}) + \ldots \). From Eqs. (A.12) and (A.14), \( \Phi_{UV} \) is also \( O(1) \). Note that it seems we have to use a special UV potential in order to achieve our goal but all it requires is actually that there be at least one term in the potential which is \( F \)-independent. As \( F_{IR} \) is expected to be small, extra \( F \)-dependent terms will not change our conclusions and different \( \Phi \)-dependent terms will only change the minor details of the result without affecting our main conclusions. The equations we have to solve are then reduced to

\[ \frac{\partial U}{\partial F_{UV}} \omega^{d-4} \sim \frac{\partial W}{\partial \Phi_{IR}} + 2 F_{IR} \omega^{-1} \]  
(A.19)

\[ F_{IR} \frac{\partial^2 W}{\partial \Phi_{IR}^2} \sim \omega^{d-4} \]  
(A.20)

For \( a_1 \neq 0, \frac{\partial W}{\partial \Phi} \sim O(1) \). Hence from Eq. (A.19), \( F_{IR} \sim \omega^{d-4} \). It is then implied by Eq. (A.20) that \( \frac{\partial^2 W}{\partial \Phi_{IR}^2} \sim \omega^{d-5} \) and so \( \Phi_{IR} \sim \omega^{\frac{d-4}{2}} \) for \( n > 2 \) (this does not lead to viable gauge mediation). For the special case where \( n = 2 \) it is easy to see that the only solution is \( \Phi_{IR} \sim O(1) \) and \( F_{IR} \sim \omega^{d-4} \) (this again does not lead to viable gauge mediation as the messenger scale would be at Planck scale).

For \( a_1 = 0 \), the equations above can be solved by observing from Eq. (A.20) that \( F_{IR} \geq \omega^{d-4} \). So we can simply set the left-hand side of Eq. (A.19) to zero. The solutions are then found to be,

\[ F_{IR} = \omega^{\frac{d-5}{2n-2} (n-1)+1} \]  
(A.21)

\[ \Phi_{IR} = \omega^{\frac{d-5}{2n-2}} \]  
(A.22)

\( \text{A.4 Case (iii): } (-, +) \text{ and } c > 0 \)

We consider the case where \( F_{UV} \sim O(1) \). This implies \( \Phi_{UV} \sim O(1) \) and \( \frac{\partial U}{\partial \Phi_{UV}} = f(\Phi_{UV}) + \ldots \).
The equations that we need to solve are
\[
\Phi_{IR} \sim (\partial U / \partial \tilde{F}_{UV} + \frac{\partial^2 W}{\partial \Phi_{IR}^2} \omega^{-1}) \omega^{d-4} \tag{A.23}
\]
\[
\tilde{F}_{UV} \sim \frac{\partial^2 W}{\partial \Phi_{IR}^2} \omega^{d+\tilde{d}-4} \tag{A.24}
\]
The other parameters can be obtained from
\[
\tilde{\Phi}_{IR} = -\frac{1}{2} \frac{\partial W}{\partial \Phi_{IR}} \tag{A.25}
\]
\[
F_{IR} = F_{UV} \omega^{d-4} \tag{A.26}
\]
\[
\tilde{F}_{IR} = \tilde{F}_{UV} \omega^{-\tilde{d}} \tag{A.27}
\]
\[
\Phi_{UV} = \frac{1}{2} \frac{\partial U}{\partial \tilde{F}_{UV}} \tag{A.28}
\]
For \(a_2 \neq 0\), the second term on the RHS of (A.23) dominates and so we have \(\Phi_{IR} \sim \omega^{d-5}\). For \(a_2 = 0\), this term is always less than \(\Phi_{IR}\) and so can be discarded. The solution of \(\Phi_{IR}\) is determined from the \(\frac{\partial U}{\partial \Phi_{IR}}\) term. If \(U\) has a term linear in \(\tilde{F}\), \(\frac{\partial U}{\partial \Phi_{UV}} \sim O(1)\) and so \(\Phi_{IR} \sim \omega^{d-4}\). Otherwise from (A.24), \(\frac{\partial U}{\partial \Phi_{UV}}\) has to be less than \(\Phi_{IR}\) as well. In that case, \(\Phi_{IR}\) has only a trivial solution. We summarize the results for this case in the following,
\[
\Phi_{IR} = \begin{cases} 
\omega^{d-5}, & a_2 \neq 0 \\
\omega^{d-4}, & a_2 = 0, \frac{\partial U}{\partial \Phi_{UV}} \sim O(1) \\
0, & a_2 = 0, \frac{\partial U}{\partial \Phi_{UV}} \sim \text{otherwise}
\end{cases} \tag{A.29}
\]
\[
F_{IR} = \omega^{d-4} \tag{A.30}
\]
which are the same as that obtained in case (i).

A.5 **Case (iv):** \((-+,+)\) and \(c < 0\)

Again, we consider only the case where \(\tilde{F}_{UV} \sim 1\). This implies that \(\Phi_{UV} \sim 1\) and \(\frac{\partial U}{\partial \Phi_{UV}} = f(\tilde{F}_{UV}) + \ldots\)

The equations that we have to solve are
\[
F_{IR} \omega^{\tilde{d}} \sim \frac{\partial U}{\partial \Phi_{UV}} \tag{A.31}
\]
\[
\frac{\partial^2 W}{\partial \Phi_{IR}^2} F_{IR} = \omega^{d-4} \tag{A.32}
\]
\[
\frac{\partial W}{\partial \Phi_{IR}} = \tilde{\Phi}_{UV} \omega^{d-4} + F_{IR} \omega^{-1} \tag{A.33}
\]
The potential $U$ can be parameterized by $\frac{\partial U}{\partial \Phi} \sim \tilde{\Phi}^q$ where $q$ is some integer. In order to have a solution with $F_{IR} \ll 1$, we have to choose a potential $U$ with $q \neq 0$. We can also simplify Eq. (A.33) by keeping only the dominant term on the RHS of the equation. The second term always dominates as we can see from Eq. (A.32) that $F_{IR} \geq \omega^{d-4}$.

For a generic potential, there is always the possibility that a solution with $\Phi_{IR} \sim 1$. Hence from Eq. (A.32), $F_{IR} \sim \omega^{d-4}$ exist. These solutions are not interesting and require higher order terms in the potential. We will ignore these solutions and concentrate on those with small $\Phi_{IR}$. Let us look at the case where both $a_1$ and $a_2$ are not vanishing. It is obvious from Eqs. (A.32) and (A.33) that these potential falls into the category above. If $a_1 \neq 0$ and $a_2 = 0$ (e.g. $n > 2$), Eq. (A.33) implies that $F_{IR} \sim \omega$ and Eq. (A.32) implies $\Phi_{IR} \sim \omega^{d-n-2}$. For the case $a_1 = 0$, Eq. (A.33) becomes

$$\Phi_{IR}^{n-1} = F_{IR} \omega^{-1}$$  \hspace{1cm} (A.34)

The solution is then found to be

$$\Phi_{IR} = \begin{cases} 
\omega^{\frac{d-5}{d-4}}, & a_1 \neq 0, a_2 = 0 \\
\omega^{\frac{d-4}{2n-3}}, & a_1 = 0 
\end{cases} \hspace{1cm} (A.35)$$

$$F_{IR} = \begin{cases} 
\omega^{\frac{d}{d(n-1)-(2n-1)}} & a_1 \neq 0, a_2 = 0 \\
\omega & a_1 = 0 
\end{cases} \hspace{1cm} (A.36)$$

References

[1] M. Dine, W. Fischler and M. Srednicki, Nucl. Phys. B 189, 575 (1981); S. Dimopoulos and S. Raby, Nucl. Phys. B 192, 353 (1981); L. Alvarez-Gaume, M. Claudson and M. B. Wise, Nucl. Phys. B 207, 96 (1982); M. Dine and A.E. Nelson, Phys. Rev. D 48, 1277 (1993) [hep-ph/9303230]; M. Dine, A.E. Nelson and Y. Shirman, Phys. Rev. D 51, 1362 (1995) [hep-ph/9408384]; M. Dine, A.E. Nelson, Y. Nir and Y. Shirman, Phys. Rev. D 53, 2658 (1996) [hep-ph/9507378]; H. Murayama, Phys. Rev. Lett. 79, 18 (1997) [hep-ph/9705271]; S. Dimopoulos, G.R. Dvali, R. Rattazzi and G.F. Giudice, Nucl. Phys. B 510, 12 (1998) [hep-ph/9705307]; M.A. Luty, Phys. Lett. B 414, 71 (1997) [hep-ph/9706554].

[2] K.A. Intriligator, N. Seiberg and S.H. Shenker, Phys. Lett. B 342, 152 (1995) [hep-ph/9410203]; H. Murayama, Phys. Lett. B 355, 187 (1995) [hep-th/9505082]; E. Poppitz and S.P. Trivedi, Phys. Lett. B 365, 125 (1996) [hep-th/9507169]; K.I. Izawa and T. Yanagida, Prog. Theor. Phys. 95, 829 (1996) [hep-th/9602180].
K.A. Intriligator and S. Thomas, Nucl. Phys. B 473, 121 (1996) [hep-th/9603158]. For a review, see E. Poppitz and S. P. Trivedi, Ann. Rev. Nucl. Part. Sci. 48, 307 (1998) [arXiv:hep-th/9803107] or Y. Shadmi and Y. Shirman, Rev. Mod. Phys. 72, 25 (2000) [arXiv:hep-th/9907225].

[3] T. Asaka, J. Hashiba, M. Kawasaki and T. Yanagida, Phys. Rev. D 58, 083509 (1998) [arXiv:hep-ph/9711501]; T. Moroi, Phys. Rev. D 58, 124008 (1998) [arXiv:hep-ph/9807265].

[4] H. S. Goh, M. A. Luty and S. P. Ng, JHEP 0501, 040 (2005) [arXiv:hep-th/0309103].

[5] T. Gherghetta and A. Pomarol, Phys. Rev. D 67, 085018 (2003) [arXiv:hep-ph/0302001].

[6] D. B. Kaplan, Phys. Lett. B 136, 162 (1984).

[7] M. A. Luty and N. Okada, JHEP 0304, 050 (2003) [arXiv:hep-th/0209178].

[8] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109].

[9] L. Randall and R. Sundrum, Nucl. Phys. B 557, 79 (1999) [arXiv:hep-th/9810155]; M. A. Luty and R. Sundrum, Phys. Rev. D 65, 066004 (2002) [arXiv:hep-th/0105137].

[10] M. J. Strassler, [arXiv:hep-th/0309122].

[11] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999) [arXiv:hep-ph/9905221].

[12] D. Marti and A. Pomarol, Phys. Rev. D 64, 105025 (2001) [arXiv:hep-th/0106256].

[13] T. Gregoire, R. Rattazzi, C. A. Scrucca, A. Strumia and E. Trincherini, Nucl. Phys. B 720, 3 (2005) [arXiv:hep-th/0411216]. For the flat case, see R. Rattazzi, C. A. Scrucca and A. Strumia, Nucl. Phys. B 674, 171 (2003) [arXiv:hep-th/0305184] as well as I. L. Buchbinder, S. J. J. Gates, H. S. Goh, W. D. . Linch, M. A. Luty, S. P. Ng and J. Phillips, Phys. Rev. D 70, 025008 (2004) [arXiv:hep-th/0305169].
[14] M. A. Luty and R. Sundrum, Phys. Rev. D 64, 065012 (2001) [arXiv:hep-th/0012158]. For the flat case, see M. A. Luty and R. Sundrum, Phys. Rev. D 62, 035008 (2000) [arXiv:hep-th/9910202].

[15] J. Garriga and A. Pomarol, Phys. Lett. B 560, 91 (2003) [arXiv:hep-th/0212227].

[16] For a review, see G. F. Giudice and R. Rattazzi, Phys. Rept. 322, 419 (1999) [arXiv:hep-ph/9801271].

[17] See, for example, F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477, 321 (1996) [arXiv:hep-ph/9604387], and references therein.

[18] Y. Nomura and T. Yanagida, Phys. Lett. B 487, 140 (2000) [arXiv:hep-ph/0005211].

[19] M. Y. Khlopov and A. D. Linde, Phys. Lett. B 138, 265 (1984); J. R. Ellis, J. E. Kim and D. V. Nanopoulos, Phys. Lett. B 145, 181 (1984).

[20] For the latest analysis, see K. Kohri, T. Moroi and A. Yotsuyanagi, [arXiv:hep-ph/0507245] and references therein.

[21] Y. Nomura and K. Suzuki, Phys. Rev. D 68, 075005 (2003) [arXiv:hep-ph/0110040].

[22] D. N. Spergel et al. [WMAP Collaboration], Astrophys. J. Suppl. 148, 175 (2003) [arXiv:astro-ph/0302209].

[23] H.S. Goh, S.P. Ng and N. Okada, in preparation.

[24] C. Csáki, M. Graesser, L. Randall and J. Terning, Phys. Rev. D 62, 045015 (2000) [arXiv:hep-ph/9911406].