Investigation of lubricant supply in rolling point contacts under starved conditions using CFD simulations

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Abstract. High friction and wear in rolling bearings are prevented by a lubricating film, which separates the contacting surfaces. In rolling contacts with limited lubricant supply, the film thickness decreases at high rolling speeds, as more lubricant is displaced than replenished. This lubricant depletion is known as starvation and may induce mixed lubrication due to a low film thickness. When estimating the bearing lifetime, a fully flooded lubrication, i.e. an unlimited lubricant supply, is usually assumed. This leads to an overestimation of film thickness in operating ranges where starvation occurs. Thus, considering the onset of starvation is crucial to reduce the risk of premature bearing failures due to high friction and wear in the mixed lubrication regime. Therefore, this contribution presents a method, based on computational fluid dynamics (CFD) simulations, to calculate the onset of starvation in oil lubricated point contacts.

Keywords. Starvation, Lubricant Supply, Wettability, CFD Simulation

Nomenclature

| Latin characters | Greek characters |
|------------------|------------------|
| $a_{Hertz}$       | $\alpha_p$       | Pressure-viscosity coefficient [GPa] |
| $Cu$              | $\eta$           | Dynamic viscosity [mPa s] $^{-1}$ |
| $E$               | $\theta$         | Contact angle [$^\circ$] |
| $F$               | $\nu$            | Poisson’s ratio [-] |
| $G$               | $\sigma$         | Surface tension coefficient [N m$^{-1}$] |
| $h$               |                  | |
| $k$               |                  | |
| $L$               |                  | |
| $m$               |                  | |
| $m_*$             |                  | |
| $p$               |                  | |
| $R$               |                  | |
| $u$               |                  | |
| $U$               |                  | |
| $V$               |                  | |
| $W$               |                  | |

| Subscripts     |
|----------------|
| $c$            | Central         |
| $ff$           | Fully flooded   |
| $i$            | Initial         |
| $s$            | Starved         |
| red.           | Reduced         |
| $x$            | x-direction      |
| $y$            | y-direction      |
1. Introduction
In machine elements, such as rolling bearings, oil or grease lubrication is applied to reduce friction and wear. For an efficient operation with long service life, the contacting surfaces of rolling elements and raceways must be separated by a lubricating film. The surface separation for oil lubricated contacts is ensured in the hydrodynamic lubrication regime, if the viscosity and the rotational speed are sufficiently high to form a complete separating film. However, if the amount of lubricant is limited, a complete film formation and thus, an operation in the hydrodynamic lubrication regime cannot be guaranteed. Mostly, grease lubricated rolling bearings operate under limited lubricant supply, since the amount of base oil, bleeding from the grease bulk, is limited and the bearing is not re-lubricated [1–3]. This can lead to insufficient lubricant supply and to a film thickness decrease, known as starvation, which might induce boundary or mixed lubrication under operating conditions. Consequently, premature bearing failures due to high friction and wear are likely to occur.

As presented in the authors’ previous work [4], the lubricant supply by replenishment in front of a rolling contact is significantly affected by the wettability, the available amount of oil as well as operating and lubricant properties, like rolling speed and viscosity. As mentioned in [5,6], the lubricant supply can be correlated to the inlet meniscus distance between contact centre and the boundary between oil and air in front of the contact. If the meniscus approaches the contact and the meniscus distance is reduced below a critical value, the film thickness decreases due to starvation [7–11]. The effects of speed and viscosity on the lubricant supply as well as on the onset of starvation were examined by several authors using experimental setups [9,12–21] and recently also by a few authors in numerical simulations [5,11,22,23]. HAMROCK and DOWSON derived a correlation (equation (1) to (4)) between inlet meniscus distance and the film thickness for oil lubricated contacts, which can be used to calculate the onset of starvation [24]. However, a major challenge in using their model is to determine the actual inlet meniscus distance in dependence of the operating conditions and the lubricant properties as well as the available oil amount [8].

Recently, NOGI et al. [11] presented an equation based on numerical and experimental investigations, to determine the meniscus distance (equation (5)). To derive the equation, a numerical model was used under the assumption of a single-phase oil flow and the available oil amount is represented by the height of the initial oil film in front of the contact. However, the oil film height is mostly unknown in an application and can hardly be determined [2,18,25]. Thus, following the idea by NOGI et al. [11] but using the two-phase CFD model presented in [26], this work demonstrates the derivation of an equation to calculate the meniscus distance considering the available oil volume. Moreover, on the basis of published work [7,12,20,27], a significant impact of the oil wettability on the meniscus distance is expected. The work by HUANG et al. [28] points out, that in particular for grease lubricated contacts, the contact angle between bled oil and a thickener layer on the contacting surfaces significantly changes in comparison to oil on a steel surface. Hence, the presented model in this contribution additionally considers the effect of wettability, i.e. contact angle, on the meniscus distance.

The following contribution describes the derivation of an equation to calculate the inlet meniscus distance. Therefore, the CFD model presented in [26] is used for a parametric study to identify the effect of capillary number (Ca), contact angle and oil volume on the inlet meniscus distance. In accordance to [11], a least square approximation is applied on the simulation results to derive a general equation for the meniscus distance. Applying this equation to the HAMROCK and DOWSON model (equation (1) to (4)), the effects of contact angle and available oil volume on the onset of starvation are examined and discussed.

2. Governing Equations
The equation used to calculate the central (c) film thickness for fully flooded (ff) point contacts is given by HAMROCK and DOWSON [29] as:

\[ h_{c,ff} = 2.69 \times U^{0.67} \times G^{0.53} \times W^{-0.067} \times (1 - 0.61e^{-0.73k}) \times R_x \]  \hspace{1cm} (1)

with the dimensionless parameters for speed \( U \), load \( W \), material \( G \) and ellipticity \( k \):


\[ U = \frac{\eta \cdot u_{\text{mid}}}{E_{\text{red}} \cdot R_x}; \quad G = E_{\text{red}} \cdot \alpha_p; \quad W = \frac{F}{E_{\text{red}} \cdot R_x^2}; \quad k = 1.03 \left( \frac{R_y}{R_x} \right)^{0.64} \]

with \( \eta \) being the dynamic viscosity, \( u_{\text{mid}} \) the rolling speed, \( E_{\text{red}} \) the reduced modulus of elasticity, \( R_x/R_y = 1 \) as \( R_x = R_y \) is valid for point contacts, \( F \) the applied load and \( \alpha_p \) the pressure-viscosity coefficient. However, under starved lubrication, equation (1) is extended by a film thickness reduction factor, which depends on the inlet meniscus distance \( m \) and the critical meniscus distance \( m^* \). The critical inlet meniscus distance \( m^* \) describes the meniscus distance, which leads to the onset of starvation and is formulated in [24] as

\[ m^* = \left[ 1 + 3.06 \left( \frac{h_{c,ff} \cdot R_x}{a_{\text{Hertz}}^2} \right)^0.58 \right] \cdot a_{\text{Hertz}} \]

with \( a_{\text{Hertz}} \) as the HERTZian contact radius. For \( m \leq m^* \), the central \( (c) \) film thickness in the range of starvation \((s)\) can be calculated using

\[ h_{c,s} = h_{c,ff} \left( \frac{m - a_{\text{Hertz}}}{m^* - a_{\text{Hertz}}} \right)^0.29 \]

Finally, the application of equation (4) requires the knowledge of the actual inlet meniscus distance \( m \), which depends on lubricant and operating properties. As described in section 1, NOGI et al. [11] recently published an equation for the meniscus distance:

\[ m_{\text{NOGI}} = 1.94 \left( \frac{h_i \cdot R_x}{a_{\text{Hertz}}^2} \right)^{0.59} \cdot \left( \frac{h_{c,ff} \cdot R_x}{a_{\text{Hertz}}^2} \right)^{-0.061} \cdot e^{-1.95 \cdot Ca^{0.45}} \cdot a_{\text{Hertz}} \]

where \( h_i \) is the initial film height, \( a_{\text{Hertz}} \) the HERTZian radius and \( Ca \) the capillary number

\[ Ca = \left( \frac{\eta \cdot u_{\text{mid}}}{\sigma_{\text{oil,air}}} \right) \]

with \( \sigma_{\text{oil,air}} \) as the surface tension coefficient. However, in order to consider the wetting behaviour as well as the actual oil volume, another equation for the calculation of the inlet meniscus distance is presented in this work.

3. Numerical and Experimental Methods

In the following, the CFD model is described, which is used for the simulations of lubricant supply to determine the meniscus distance \( m \). As mentioned before, the simulation results for \( m \) are applied to equation (4) to calculate the film thickness. Furthermore, the ball-on-disc tribometer as well as lubricant properties and operating conditions used for film thickness measurements are presented. In section 4.2 the measurement results are compared to the calculated film thickness in order to verify the application of the inlet meniscus equation.

3.1. CFD Model for Simulation

The geometry in the vicinity of a point contact of a ball over a plane was modelled using the open source software OPENFOAM [30]. The setup of this CFD model is described in detail in the authors’ previous work [26] and will be briefly presented in the following. Figure 1 shows the simulation domain with a flow field in characteristic butterfly shape in the vicinity of the contact. The surfaces of plane and ball are partially wetted with a thin oil film, while the ambient free volume is filled with air. Both, oil and air, were modelled as incompressible, isothermal and isoviscous NEWTONian fluids. The body surfaces were defined as rigid walls with no-slip boundary condition and constant velocity. The wetting behaviour was considered by the boundary condition of a static contact angle between oil and the solid
surfaces. Pure rolling condition was achieved by setting the circumferential velocity equal to the plane velocity. To simulate the effect of oil displacement and replenishment due to over rolling of two balls in series, a periodic boundary condition was applied. Thus, the flow fields, including pressure, velocity and oil/air distribution, at the outlet of the simulation domain were projected onto the inlet. This method allows to consider the influence of lubricant displacement and replenishment on the meniscus distance between two contacts. The resulting meniscus distance was determined as the distance between the center of the contact and the boundary between oil and air in front of the contact, see Figure 1. For further information on the CFD model, the simulation process and the meniscus distance evaluation, the reader is referred to [26].

![Figure 1. Simulation domain and flow field in the CFD model [26].](image)

This CFD model is used in this work to simulate the fluid flow of oil around a point contact and thereby, determine the meniscus distance in dependence of Ca-number, contact angle and oil volume. These parameters were varied in 46 simulations, so that the effect of each parameter on the meniscus distance can be identified. The parameter combinations and results for the meniscus distance are listed in Table 2 in the appendix.

### 3.2. Experimental Setup

In order to verify the results of film thickness calculation presented in this contribution, the film thickness of a point contact was measured on a ball-on-disc tribometer EHD2 from PCS Instruments by the principle of interferometry, which is described in detail in [31]. For the experiments the contact was lubricated with a limited amount of polyalphaolefin (PAO) oil containing no additives. Therefore, the weight of disc, ball and ball carriage were determined in dry condition before applying 10 µl oil to the disc. A running in phase of 24 disc revolutions at constant speed 50 mm/s was set before the actual measurement to distribute the oil droplets along the track, so that a continuous oil band was formed on the track. During measurement, the polished steel ball was loaded with 47 N against the glass disc at a track radius of 37.5 mm and the rolling speed was increased in the range from 20 mm/s to 1000 mm/s. Both, ball and disc, were driven to ensure pure rolling and the measurements were performed for two temperatures at 42 °C and 24 °C. Table 1 gives material, lubricant and load parameters.

| Parameter | Value         | Parameter | Value         |
|-----------|---------------|-----------|---------------|
| $F$       | 47 N          | $\alpha_p$| 15 GPa$^{-1}$ |
| $p$       | 0.7 GPa       | $R_e$     | 9,525 mm      |
| $u_{mid}$ | 20 – 1000 mm/s| $E_{steel}$| 207 GPa       |
| $V_{oil}$ | 10 µl         | $v_{steel}$| 0.29          |
| $\eta_{1,24 \degree C}$ | 73.30 mPas | $E_{glass}$ | 75 GPa        |
| $\eta_{2,12 \degree C}$ | 195.34 mPas | $v_{glass}$ | 0.22          |
For each temperature two tests were performed and after each measurement the weight of disc, ball and ball carriage was determined. From the weight difference of the components before and after the measurement the amount of oil attached to the components was identified.

4. Results
As mentioned before, the CFD model presented in the authors’ previous work [26] was used in this contribution to determine the inlet meniscus distance in dependence of the parameters $Ca$-number, contact angle and available oil volume. In a parametric study the lubricant supply is simulated for 46 parameter variations (see Table 2 in appendix) to identify the impact of each parameter on the meniscus distance. Similar to [11,23], a least square approximation is applied on the results to derive an equation, which predicts the meniscus distance as a function of the mentioned parameters. In the following, the derived equation for the inlet meniscus distance is presented and compared to the equation for point contacts presented by NOGi et al. in [11] for verification. Furthermore, the equation is applied to the HAMROCK and DOWSON equation (4) to calculate the film thickness under starved lubrication and the results are compared to measurement results on the ball-on-disc tribometer.

4.1. Meniscus Distance Equation
To find a general formulation for the meniscus distance, which describes the combined effects of the $Ca$-number, the contact angle and the oil volume, a least square fit is applied on the results for the meniscus distance. The simulation results and the simulation parameters are listed in Table 2 in the appendix. Thus, the meniscus distance equation, which gives $m_{\text{CFD}}$ in metres, yields:

$$m_{\text{CFD}} = 4.533 \times e^{(-2.571 \times Ca^{0.384})} \times \cos(\theta)^{-0.249} \times \left(\frac{V_{\text{Oil}}}{L_{\text{Track}}} \times 10^{-6}\right)^{0.446}$$  \hspace{1cm} (7)

The oil volume factor $V_{\text{Oil}}/L_{\text{Track}}$ in µl/mm describes the available oil volume related to the track length in rolling direction. This factor is introduced since the available oil volume is distributed over the whole track length during operation. Hence, if the total amount of applied oil volume for a lubricated system is known, the oil volume factor allows to apply equation (7) to point contact systems with different track lengths. This will be shown exemplarily for a ball-on-disc tribometer in section 4.2. Figure 2 shows exemplarily simulation results (see appendix, case #7 and #41), which emphasis the meniscus distance at different contact angles $11^\circ$ and $66^\circ$.

![Figure 2. Simulation results of case #7 and #41.](image)

To verify the results for the meniscus distance equation (7), it is compared to results using equation (5) presented by NOGi et al. [11] in Figure 3. For the comparison, $m$ is presented as a function of the capillary number $Ca$, which is defined in equation (6). $Ca$ is varied by increasing the rolling speed, but the surface tension and the viscosity are kept constant to $\sigma_{\text{oil,air}} = 0.028$ N/m and $\eta = 73.30$ mPas, respectively. The contact angle is set to $\theta = 0^\circ$ in equation (7), since it is not considered in equation (5). Load, geometry, lubricant and material parameters, which are required for equation (5), are chosen exemplarily according to the experimental setup described in section 3.2. As already mentioned in
section 1, the amount of oil volume in equation (5) is represented by the initial oil film height \( h_i \) in front of the contact. According to [11] an exemplarily value of \( h_i = 20 \mu m \) is chosen for the comparison of \( m \). The oil volume for equation (7) is estimated according to [11,25] from the initial oil film height, the width and the track length. The oil film width is measured in the experiments, described in section 3.2, to a width of 2.0 to 2.5 mm, so an exemplarily width of 2.25 mm is chosen for the comparison in Figure 3. Since the oil volume in equation (7) is related to the track length, considered by the oil volume factor \( V_{oil}/L \), a track length is not required to compare both equations. From \( h_i = 20 \mu m \) and a width of 2.25 mm, the oil volume factor is determined to \( V_{oil}/L = 0.045 \mu l/mm \). In Figure 3 the critical meniscus distance \( m \) is additionally presented exemplarily for the chosen parameters, so that the onset of starvation can be determined at the intersection between the curves for \( m \) and \( m^* \).

![Figure 3. Comparison of derived equation (7) with equation (5) by NOGI et al. [11] for \( V_{oil}/L = 0.045 \mu l/mm, \eta = 73.3 \text{ mPas}, \theta = 0^\circ \).](image1)

The results for \( m \) and the onset of starvation at the intersection are in good agreement for both models. A slight deviation for \( Ca \)-numbers from 0 to 1 might be a result of an inaccurate estimation of \( V_{oil}/L \) from the initial oil film height and width. Moreover, both equations are based on different types of simulation methods with different boundary conditions and simplifications [11,26]. While equation (5) is dependent on the film thickness under fully flooded condition \((h_{c,f})\), the film thickness is not considered in the derived equation (7). As explained in [26], the influence of this factor is assumed to be negligibly small. This is confirmed by the low exponent of \( -0.061 \) at the film thickness term in equation (5).

4.2 Film Thickness Measurement and Calculation

To calculate the film thickness under starved lubrication, the derived meniscus distance equation (7) is applied to the HAMROCK and DOWSON equation (4). To verify the calculated film thickness, the results are compared to film thickness measurement results under limited oil supply. The setup of the experiments on the ball-on-disc tribometer is described in section 3.2. For the measurements, an oil volume of \( V_{oil} = 10 \mu l \) is distributed on the disc at a track radius of 37.5 mm, i.e. on a track length of \( L_{\text{track}} = 235.62 \text{ mm} \). From the measured weight difference of the components before and after each measurement, no significant oil loss could be identified. Moreover, it is found, that about 90 % of the oil mass sticks to the disc. Therefore, it is assumed, that the entire supplied oil volume of 10 \( \mu l \) is present on disc and ball during the measurements. Figure 4 shows the measured central film thickness in comparison to the calculated central film thickness as a function of rolling speed for two different viscosities of PAO oil. The static contact angle between PAO oil and glass disc is measured to approx. 8° at room temperature and is applied to equation (7). The pressure-viscosity coefficient \( \alpha_p = 15 \text{ GPa}^{-1} \) is determined from high pressure measurements at \( p = 0.176 \text{ GPa} \) and 40 °C. The presented results show a good agreement for the onset of starvation for both viscosities.

![Figure 4. Comparison of calculated and measured film thickness at two different viscosities for \( V_{oil}/L = 0.042 \mu l/mm, \theta = 8^\circ \).](image2)
However, with increasing rolling speed, the calculation of film thickness in the range of starvation predicts a film thickness decrease with a high gradient. This behaviour differs significantly from the measurement results, which show a low gradient in the film thickness decrease in accordance to literature [32,33]. Moreover, the measured film thickness reaches a residual constant film thickness between 80 and 100 nm from approx. 600 mm/s. Both, the gradient in film thickness decrease as well as the residual film thickness cannot be predicted accurately by the HAMROCK and DOWSON equation (4). The minimum film thickness in the range of starvation is therefore artificially bounded for the calculation to 20 nm, as presented in [32,33]. This value is set constant for all calculations regardless of lubricant and operating properties, like viscosity and rolling speed.

To predict film thickness for oil lubricated contacts in the range of starvation, further work should focus on enhancing the HAMROCK and DOWSON equation (4) with a physics-based model, describing the gradient in film thickness decrease and formation of a residual constant film thickness. Nevertheless, the presented model can be applied to predict the onset of starvation in dependence of the Ca-number, contact angle and available amount of oil volume. In the following, the effects of the mentioned parameters on the film formation, with focus on the onset of starvation, are discussed more in detail.

5. Discussion
The presented measurement and simulation results in section 4 are focused on oil lubricated contacts. As mentioned in section 1, especially grease lubricated rolling bearings often operate under starved lubrication due to a limited amount of bled oil bleeding from the grease bulk. Moreover, as described in [28], the wettability of bled oil on the thickener material is significantly depended on the thickener type. In comparison to the measured contact angle of PAO oil on the glass disc with approx. 8°, the contact angle of oil on thickener material can reach values up to 57° [28]. As shown in the authors’ previous work [26] the contact angle significantly affects the meniscus distance and as emphasised in [20], the wettability determines the film formation. Hence, in the following, the derived meniscus distance equation is applied to calculate the film thickness and the effects of contact angle and available oil amount on the onset of starvation are discussed in the context of grease lubrication.

5.1. Effect of Contact Angle
With the derived meniscus distance equation (7) applied to the HAMROCK and DOWSON equation (4) for starved lubricated point contacts, the effect of contact angle on the onset of starvation is examined for the three cases of $\theta_1 = 11^\circ$, $\theta_2 = 55^\circ$ and $\theta_3 = 66^\circ$. The calculated film thickness for these contact angles is shown in Figure 5 and compared to the fully flooded results. For the load, geometry and material parameters given in Table 1, the onset of starvation for a contact angle of 11° can be determined at approx. 250 mm/s rolling speed and 131 nm film thickness. Increasing the contact angle to 55°, leads to shift of the onset of starvation to 290 mm/s (+16 %) and 143 nm (+10 %). For 66° starvation is shifted to 315 mm/s (+26 %) and 151 nm (+15 %).
5.2. Effect of Oil Volume

Another parameter, which significantly affects the lubricant supply in grease lubricated contacts is the available amount of bled oil, which is released from the grease bulk due to oil bleeding. As shown in [26], a higher available oil amount increases the meniscus distance at constant lubricant and operating properties. The effect of increasing the available oil volume on the onset of starvation is exemplarily shown in Figure 6 for three oil volume factors ($V_{Oil/L}$) 1 = 0.021 µl/mm, ($V_{Oil/L}$) 2 = 0.042 µl/mm and ($V_{Oil/L}$) 3 = 0.085 µl/mm. For the lowest oil volume factor, the onset of starvation can be determined to 210 mm/s rolling speed and 115 nm film thickness. Increasing the available oil volume by factor 2 shifts the onset of starvation to 287 mm/s (+37 %) and 143 nm (+24 %). Increasing the oil volume by factor 4 leads to a shift to 386 mm/s (+47 %) and 174 nm (+51 %). These results demonstrate that the available amount of oil has a significant impact on the onset of starvation and the maximum film thickness, which can be reached with increasing rolling speed. In comparison to the effect of contact angle, a change of the oil volume has a more pronounced impact on the onset of starvation. Thus, greases with a low bleeding behaviour increase the risk of the onset of starvation, which is in accordance with literature [34–36].

Furthermore, using the derived meniscus distance equation (7), the minimal required oil amount to ensure an operation in the hydrodynamic lubrication regime at a lambda ratio $\lambda \geq 3$ can be determined. If a composite surface roughness of 80 nm is assumed, a minimum film thickness of $h = 240$ nm must be reached to fulfil $\lambda \geq 3$. This film thickness is built up under fully flooded conditions at approx. 625 mm/s for a viscosity of $\eta = 73.30$ mPas as well as for material, lubricant and load parameters given in Table 1. Figure 7 shows exemplarily the minimal required oil volume factor for different contact angles to reach a film thickness of $h = 240$ nm in the range of starvation. With increasing speed $> 625$ mm/s, a higher oil amount is required to obtain a film thickness of $h = 240$ nm. Additionally, the results point out, that at higher contact angles less oil volume is needed to ensure an operation in the hydrodynamic lubrication regime.

**Figure 5.** Onset of starvation for different contact angles for $\eta = 73.3$ mPas, $V_{Oil/L} = 0.042$ µl/mm.

**Figure 6.** Onset of starvation for different oil volume factors for $\eta = 73.3$ mPas and $\theta = 55^\circ$.

This analysis shows, that the impact on the shift of the starvation speed and film thickness is more dominant at higher contact angles. Thus, greases with thickener/base oil combinations, that lead to high contact angles support the lubricant supply and reduce the risk of the onset of starvation.
5.3. Validity of the derived Meniscus Distance Equation

The presented meniscus distance equation is based on simulation results for a limited parameter range shown in Table 2 in the appendix. In accordance to the experiments, all simulations consider a ball radius of 9.525 mm. Applying the equation to extended parameter ranges, as demonstrated in the sections 4.1 and 4.2, the results show a good agreement with a similar model presented in [11] as well as experimental measurements. However, for an extensive validation, more experiments with a varying contact angle and varying oil volume should be performed. Moreover, for an accurate prediction of the film thickness in the range of starvation, both the HAMROCK and DOWSON equation (4) and the meniscus distance equation (7) should be enhanced with focus on an advanced description for the formation of the residual constant film thickness in the range of starvation.

6. Conclusion

To calculate the film thickness under starved lubrication and determine the onset of starvation, the model by HAMROCK and DOWSON, described in equations (1) to (4), can be used. However, to apply this model, the knowledge of the actual inlet meniscus distance in dependence on operating conditions is inevitable. Therefore, this contribution presents an equation (7) to calculate the inlet meniscus distance for point contacts. The derived equation is based on a least square approximation of CFD simulation results for the meniscus distance under variation of operating and lubricant properties. The results can be summarised as follows:

- A comparison between the derived equation (7) and equation (5) presented by NOGI et al. [11], shows good agreement of the calculated meniscus distance. However, in addition to equation (5), the derived equation (7) in this work considers the wettability of oil on the contacting surfaces and the available oil volume for lubrication.

- Applying the derived meniscus distance equation (7) to the HAMROCK and DOWSON equation (4) for film thickness calculation under starved lubrication, the onset of starvation agrees with film thickness measurement results. However, in the range of starvation the calculated film thickness results show significant deviations of the measured film thickness.

- The analysis of the effects by contact angle and amount of oil volume on the onset of starvation reveal, that with higher contact angle and oil volume, the onset of starvation is shifted to higher speeds. However, an increase of oil volume has a more pronounced impact on the onset of starvation than a higher contact angle.

- For a given film thickness, the meniscus distance equation (7) allows to determine the required oil amount depending on operating and lubricant properties.

To describe the film formation in the range of starvation more accurately, future work should focus on an enhancement of both, the HAMROCK and DOWSON model as well as the presented meniscus distance equation (7). In accordance to [23], extended geometries, like different ball radii and elliptical or line
contacts should be examined to enhance presented model by effects of geometry on the meniscus distance. Nevertheless, the derived meniscus distance equation can be used to determine the onset of starvation depending on contact angle and available oil volume, which is of significance describing the film formation in grease lubricated contacts. Thus, the presented equation can be combined with existing models, taking e.g. the thickener effect on the film formation into consideration, like recently developed in [37].

**Acknowledgements**

The authors would like to thank the Research Association for Drive Technology FVA (Forschungsvereinigung Antriebstechnik e. V.) as well as the participating member companies for the support of the IGF project N/1 19027, which is funded by the German Federation of Industrial Research Associations AiF (Arbeitsgemeinschaft industrieller Forschungsvereinigungen) within the framework of the program for the promotion of the Industrial Collective Research IGF (Industrielle Gemeinschaftsforschung) by the Federal Ministry for Economic Affairs and Energy BMWi (Bundesministerium für Wirtschaft und Energie) based on a resolution of the German Bundestag. Simulations were performed with computing resources granted by RWTH Aachen University under project ID rwth0628.
Appendix

Table 2. CFD simulation parameters and results for m.

| Case | θ (°) | Ca (µl/mm) | V_{oil}/L (µl/mm) | m (10^{-3} m) |
|------|-------|------------|-------------------|---------------|
| #1   | 11    | 0.090      | 0.173             | 1.592         |
| #2   | 11    | 0.098      | 0.174             | 1.559         |
| #3   | 11    | 0.100      | 0.173             | 1.545         |
| #4   | 11    | 0.111      | 0.173             | 1.480         |
| 5    | 11    | 0.120      | 0.173             | 1.461         |
| #6   | 11    | 0.127      | 0.173             | 1.395         |
| #7   | 11    | 0.166      | 0.173             | 1.257         |
| #8   | 11    | 0.249      | 0.173             | 1.035         |
| #9   | 11    | 0.332      | 0.174             | 0.886         |
| #10  | 11    | 0.100      | 0.222             | 1.831         |
| #11  | 11    | 0.100      | 0.274             | 1.860         |
| #12  | 11    | 0.100      | 0.318             | 1.968         |
| #13  | 11    | 0.127      | 0.274             | 1.730         |
| #14  | 11    | 0.127      | 0.332             | 1.873         |
| #15  | 11    | 0.127      | 0.452             | 2.094         |
| #16  | 11    | 0.127      | 0.531             | 2.222         |
| #17  | 44    | 0.100      | 0.174             | 1.555         |
| #18  | 44    | 0.107      | 0.174             | 1.521         |
| #19  | 44    | 0.111      | 0.174             | 1.496         |
| #20  | 44    | 0.120      | 0.174             | 1.533         |
| #21  | 44    | 0.127      | 0.174             | 1.467         |
| #22  | 44    | 0.166      | 0.174             | 1.199         |
| #23  | 44    | 0.249      | 0.174             | 0.957         |

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