Effective field theory approach to quasi-single field inflation
and effects of heavy fields

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We apply the effective field theory approach to quasi-single field inflation, which contains an additional scalar field with Hubble scale mass other than inflaton. Based on the time-dependent spatial diffeomorphism, which is not broken by the time-dependent background evolution, the most generic action of quasi-single field inflation is constructed up to third order fluctuations. Using the obtained action, the effects of the additional massive scalar field on the primordial curvature perturbations are discussed. In particular, we calculate the power spectrum and discuss the momentum-dependence of three point functions in the squeezed limit for general settings of quasi-single field inflation. Our framework can be also applied to inflation models with heavy particles. We make a qualitative discussion on the effects of heavy particles during inflation and those of sudden turning trajectory in our framework.

I. INTRODUCTION

Inflation gives the most natural solution to the horizon and the flatness problems of the big-bang theory as well as generates the primordial perturbations [1, 2], whose properties well coincide with the recent observations of cosmic microwave background anisotropies like the Wilkinson Microwave Anisotropy Probe [3]. Models of inflation can be classified into two categories with respect to relevant degrees of freedom during inflation: single-field models and multiple field models. Recently, the most general single field inflation model with the second order equations of motion [4] has been invented in the context of Horndeski [5, 6] and Galileon theories [7, 8]. Then, the bispectra of primordial curvature [9, 10] and tensor perturbations [11] are obtained as well as their powerspectra [4].

Effective scalar fields are ubiquitous in the extensions of the standard model of particle physics such as supergravity and superstring. Then, it is well motivated to consider multiple field models of inflation. Such multiple field models are roughly divided into three classes: (i) only one field is light, while the other fields are very heavy compared to the Hubble scale during inflation. Generically, this class virtually falls into the single field category [12]. However, it is recently discussed that heavy modes can affect the dynamics of light mode in some particular cases [13–21]. (ii) there are multiple light fields, in which isocurvature perturbations are generated in addition to curvature perturbations. (iii) only one field is light, while the masses of other fields are comparable to the Hubble scale during inflation. This class is called quasi-single field inflation model [22, 23].

In supergravity, inflation necessarily involves supersymmetry (SUSY) breaking, whose effects are transmitted into other scalar fields as Hubble induced masses [24–26]. Therefore, quasi-single field inflation is naturally realized in supergravity and it is well motivated by the model building based on supergravity or inspired by superstring. Furthermore, it is known that massive isocurvature modes which couple to the inflaton and have Hubble scale masses can give significant impacts on primordial curvature perturbations. In the original paper [22] by Chen and Wang, it was shown that, for example, scalar three point functions take the intermediate shapes between local and equilateral types. Based on these backgrounds, we would like to discuss quasi-single field inflation model in general settings.

Recently, effective field theory approach to inflation has been invented in [28–30], which is based on the symmetry breaking during inflation: Time diffeomorphism is broken by the time-dependent background evolution during inflation. Then, based on the unbroken time-dependent spatial diffeomorphism, the effective action for inflation can be constructed systematically in unitary gauge, where inflaton is eaten by graviton and there are no perturbations of inflaton. By use of the Stückelberg trick, the curvature perturbation can be associated with the Goldstone boson $\pi$, which non-linearly realizes the time diffeomorphism. The key observation is that the Goldstone $\pi$ could decouple

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1 The methods to keep inflaton flat against such SUSY breaking effects are reviewed in Refs. [27].
where we can construct the effective action for inflation. In the decoupling regime, the dynamics of Goldstone $\pi$ is described by a simplified action, which does not contain metric perturbations. As a consequence, calculations of scalar perturbations are also simplified and seeds of non-Gaussianities become clear.

In this paper, we apply this effective field theory approach to quasi-single field inflation. First, in unitary gauge, we write down the most general action invariant under the time-dependent spatial diffeomorphism and constructed from graviton and the massive isocurvature mode. The obtained action is expanded systematically in fluctuations and derivatives around the FRW background. By the Stückelberg trick, we introduce the action for Goldstone boson and carefully discuss its decoupling regime. Using the action in the decoupling regime, the power spectrum is calculated in the general setting of quasi-single field inflation. The momentum dependence of scalar three-point function is also discussed in the general setting. Our framework can be also applied to inflation models with heavy particles. As an application, we make a qualitative discussion on the effects of heavy particles during inflation and that of sudden turning trajectory.

The organization of this paper is as follows. In the next section, we briefly review the effective field theory approach and quasi-single field inflation. In Sec. [III] the most general action for quasi-single field inflation is constructed via effective field theory approach. The decoupling regime of the obtained action is also discussed. In Sec. [IV] the power spectrum is calculated first in the general setting of quasi-single field inflation with constant mixing couplings. Then, the effects of sudden turning trajectory on the power spectrum is qualitatively discussed. In Sec. [V] the momentum dependence of scalar three-point functions are discussed. Final section is devoted to summary and discussions. Technical details of the calculation of the power spectrum are summarized in Appendices.

II. EFFECTIVE FIELD THEORY APPROACH AND QUASI-SINGLE FIELD INFLATION

In this section we briefly review the effective field theory approach to inflation developed in [28] and the quasi-single field inflation model proposed in [22].

A. Effective field theory approach to inflation

Inflation is an accelerated cosmic expansion with an approximately constant Hubble parameter:

$$ds^2 = -dt^2 + a^2(t)dx^idx^i \quad \text{with} \quad H(t) = \frac{\dot{a}}{a}, \quad \epsilon = -\frac{\dot{H}}{H^2} \ll 1.$$  \hspace{1cm} (1)

It is characterized by the spontaneous breaking of the time-diffeomorphism:

$$\langle \phi(t,x) \rangle = \phi_0(t),$$  \hspace{1cm} (2)

where $\phi(t,x)$ is a certain scalar operator. Here we chose the frame in which the vacuum expectation value of $\phi(t,x)$ is spatially uniform. Assuming the degrees of freedom relevant to the cosmological perturbation and invariance under the time-dependent spatial diffeomorphism, $x^j \to x'^j = x^j + \xi^j(t,x^i)$, which is not broken by the condensation $\phi_0(t)$, we can construct the effective action for inflation.

In the simplest case, relevant degrees of freedom are three physical modes of graviton: two transverse modes and one longitudinal mode related to the inflaton. As discussed in [28], any action of graviton invariant under the time-dependent spatial diffeomorphism can be written in terms of the Riemann tensor $R_{\mu\nu\rho\sigma}$, the time-like component of the metric $g^{00}$, the extrinsic curvature $K_{\mu\nu}$ on constant-$t$ surfaces, the covariant derivative $\nabla_{\mu}$, and the time coordinate $t$:

$$S = \int d^4x\sqrt{-g} F(R_{\mu\nu\rho\sigma},g^{00},K_{\mu\nu},\nabla_{\mu},t),$$  \hspace{1cm} (3)

where all the free indices inside the function $F$ must be upper 0's. Note that $g^{00}$ should be treated as a scalar when considering its covariant derivative, and we can use $\partial_{\mu}g^{00}$ for example. The explicit form of the extrinsic curvature $K_{\mu\nu}$ is

$$K_{\mu\nu} = h_\sigma^\sigma \nabla_\sigma n_\nu = -\frac{\delta^0_\mu \delta^0_\nu g^{00} + \delta^0_\mu \partial_{\nu} g^{00}}{2(-g^{00})^{3/2}} - \frac{\delta^0_\mu \delta^0_\rho \partial_{\rho} g^{00}}{2(-g^{00})^{5/2}} + \frac{g^{00}(\partial_{\mu}g_{\nu\rho} + \partial_{\nu}g_{\mu\rho} - \partial_{\rho}g_{\mu\nu})}{2(-g^{00})^{1/2}},$$  \hspace{1cm} (4)
where \( n_\mu = - \frac{\delta_{\mu}^0}{\sqrt{-g}} \) is a unit vector perpendicular to constant \( t \) surfaces and \( h_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu \) is the induced spatial metric on constant \( t \) surfaces. In [28], it was shown that the action (3) can be expanded around a given FRW background as

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{\mathrm{Pl}}^2 R + M_{\mathrm{Pl}}^2 \dot{H} g^{00} - M_{\mathrm{Pl}}^2 (3 H^2 + \dot{H}) + F^{(2)}(\delta g^{00}, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \delta_{\mu}^0, g_{\mu\nu}, g^{\mu\nu}, \nabla_\mu, t) \right],
\]

(5)

where the function \( F^{(2)} \) starts with quadratic terms of the arguments \( \delta g^{00}, \delta K_{\mu\nu}, \) and \( \delta R_{\mu\nu\rho\sigma} \) and all the free indices must be upper 0's. The arguments \( \delta g^{00}, \delta K_{\mu\nu}, \) and \( \delta R_{\mu\nu\rho\sigma} \) are defined by

\[
\begin{align*}
\delta g^{00} &= g^{00} + 1, \\
\delta K_{\mu\nu} &= K_{\mu\nu} - H h_{\mu\nu}, \\
\delta R_{\mu\nu\rho\sigma} &= R_{\mu\nu\rho\sigma} - 2 H^2 h_{\mu[j} h_{\nu]0} + (\dot{H} + H^2)(h_{\mu\rho} \delta_{\nu}^0 \delta_{\sigma}^0 + (3 \text{ permutations})).
\end{align*}
\]

(6) \hspace{1cm} (7) \hspace{1cm} (8)

They are covariant under time-dependent spatial-diffeomorphism and vanish on the FRW background. Notice that the action of single field inflation in the uniform inflaton gauge can be reproduced by gauge-fixing the time-dependent spatial diffeomorphism as

\[
\pi_i(x) = a^2(t) e^{2\zeta(x)} (e^{\gamma(x)})_{ij} \quad \text{with} \quad \gamma_{ii} = \partial_i \gamma_{ij} = 0,
\]

(9)

where \( \zeta(x) \) is the scalar perturbation.

For the calculation of correlation functions of the scalar perturbation \( \zeta \), it is convenient to introduce the action for the Goldstone boson \( \pi \) by the St"uckelberg method. We perform the following time-diffeomorphism on the action in the unitary gauge:

\[
t \to \tilde{t}, \quad x^i \to \tilde{x}^i \quad \text{with} \quad \tilde{t} + \tilde{\pi}(\tilde{t}, \tilde{x}) = t, \quad \tilde{x}^i = x^i.
\]

(10)

In general, the transformation is realized by the following replacement:

\[
\begin{align*}
\delta_\mu^0 &\to \delta_\mu^0 + \partial_\mu \pi, \\
f(t) &\to f(t + \pi), \\
\int d^4x \sqrt{-g} &\to \int d^4x \sqrt{-g}, \\
\nabla_\mu &\to \nabla_\mu, \\
g_{\mu\nu} &\to g_{\mu\nu}, \\
g^{\mu\nu} &\to g^{\mu\nu}, \\
R_{\mu\nu\rho\sigma} &\to R_{\mu\nu\rho\sigma},
\end{align*}
\]

(11)

where we dropped the tilde for simplicity and \( g^{00} \) transforms, for example, as

\[
g^{00} \to g^{00} + 2 g^{0\nu} \partial_\mu \pi + g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi.
\]

(12)

The transformation rules of \( K_{\mu\nu} \) and \( h_{\mu\nu} \) also follow from straightforwardly. These procedures lead to the following action for the Goldstone boson \( \pi \):

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{\mathrm{Pl}}^2 R + M_{\mathrm{Pl}}^2 \dot{H} (t + \pi) (g^{00} + 2 g^{0\nu} \partial_\mu \pi + g^{\mu\nu} \partial_\mu \pi \partial_\nu \pi) - M_{\mathrm{Pl}}^2 \left( 3 H^2 (t + \pi) + \dot{H} (t + \pi) \right) + \ldots \right],
\]

(13)

where the dots stand for the terms corresponding to \( F^{(2)} \). The obtained action enjoys the time-diffeomorphism by assigning to \( \pi \) the non-linear transformation rule

\[
\pi(x) \to \tilde{\pi}(\tilde{x}) = \pi(x) - \xi^0(x) \quad \text{with} \quad t \to \tilde{t} = t + \xi^0(x), \quad x^i \to \tilde{x}^i = x^i,
\]

(14)

and the action in the unitary gauge can be reproduced by gauge-fixing the time-diffeomorphism as \( \pi(x) = 0 \).

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2 Here and in what follows, we concentrate on the spatially flat FRW background.

3 Here it should be noted that while the action is invariant under the time-diffeomorphism, it does not have the shift symmetry \( \pi \to \pi + \text{constant} \) because of the time-dependent free parameters such as \( H(t + \pi) \) or \( \dot{H}(t + \pi) \). Expanding the parameters in \( \pi \), we find, for example, that \( \pi \) has a mass term \( \sim M_{\mathrm{Pl}}^2 H^2 \pi^2 \), which is sub-leading in the slow-roll approximation.
It is important to recognize that, in the action \[ S \sim M_{Pl}^2 \hat{H}(t + \pi) \left( \frac{1}{2} \left( \partial_\mu \pi \partial^\mu \pi + \frac{1}{2} g^{\mu \nu} \partial_\mu \pi \partial_\nu \pi \right) - M_{Pl}^2 \left( 3\dot{H}^2 + \dot{H}(t + \pi) \right) \right) , \] (15)

In the canonical normalization \( \pi_c \sim M_{Pl} (-\dot{H})^{1/2} \pi , \ \delta g^{\mu \nu} \sim M_{Pl} \delta g^{\mu \nu} \), the mixing term \( M_{Pl}^2 \hat{H} \delta g^{\mu \nu} \partial_\mu \pi_c \) can be written as

\[ M_{Pl}^2 \hat{H} \delta g^{\mu 0} \partial_\mu \pi_c \sim (-\dot{H})^{1/2} \delta g^{\mu 0} \partial_\mu \pi_c \sim \epsilon^{1/2} H \delta g^{\mu 0} \partial_\mu \pi_c , \] (16)

and it can be neglected in the energy scale \( E \gg \epsilon^{1/2} H \). In other words, when the slow-roll parameter \( \epsilon \) is small, the mixing becomes irrelevant inside the horizon. The dynamics of \( \pi \) inside the horizon are then determined by the following action in the decoupling limit:

\[ S \sim \int d^4 x a^3 \left[ \frac{1}{2} \left( \partial_\mu \pi \partial^\mu \pi + \frac{1}{2} g^{\mu \nu} \partial_\mu \pi \partial_\nu \pi \right) - M_{Pl}^2 \left( (3H^2 + \dot{H}) + \dot{H}(t + \pi) \right) \right] , \] (17)

where the metric reduces to the FRW background. More generally, the mixing of the Goldstone boson and graviton becomes irrelevant inside the horizon when the free parameters of the action are in some regime (decoupling regime) as in the slow-roll regime for the above simplest case. In the decoupling regime, the calculation of scalar correlation functions also becomes tractable. Taking the spatially flat gauge \( g_{ij}(x) = a^2(t) (e^\gamma(x))_{ij} \) with \( \gamma_{ii} = \partial_i \gamma_{ij} = 0 \), (18)

the scalar perturbation \( \zeta(x) \) is given by \( \zeta(x) = -H(t) \pi(x) \) at the linear order, and the calculation of correlation functions of \( \zeta \) reduces to those of \( \pi \), which can be obtained using the simplified action in the decoupling limit. This kind of simplification in the decoupling regime is one of the advantages to use the effective field theory approach. In the next section we extend this approach to quasi-single field inflation.

### B. Quasi-single field inflation

The original model [22] of quasi-single field inflation is described by the following matter action:

\[ S_{\text{matter}} = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} \left( \hat{R} + \chi \right)^2 g^{\mu \nu} \partial_\mu \theta \partial_\nu \theta - \frac{1}{2} g^{\mu \nu} \partial_\mu \chi \partial_\nu \chi - V_{sr}(\theta) - V(\chi) \right] , \] (19)

where \( \theta \) and \( \chi \) are the tangential and radial directions of a circle with radius \( R \) and the potential \( V_{sr}(\theta) \) along the tangential direction is of slow-roll type. The homogeneous backgrounds and their equations of motion are given by

\[ \theta = \theta_0(t) , \ \chi = \chi_0 \text{ (constant)} , \]

\[ 3M_{Pl}^2 H^2 = \frac{1}{2} R^2 \dot{\theta}^2_0 + V(\chi_0) + V_{sr}(\theta_0) , \]

\[ -2M_{Pl}^2 H = R^2 \dot{\theta}^2_0 , \ \dot{V}(\chi_0) = R \dot{\theta}^2_0 , \ \ddot{R} \dot{\theta}_0 + 3R^2 H \dot{\theta}_0 + V_{sr}'(\theta_0) = 0 \] ,

where \( R = \dot{R} + \chi_0 \). Expanding the action around the homogeneous background, it yields the following second order action of the fluctuations \( \delta \theta = \theta - \theta_0 \) and \( \sigma = \chi - \chi_0 \):

\[ S_{\text{matter}}^{(2)} = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} R^2 g^{\mu \nu} \partial_\mu \delta \theta \partial_\nu \delta \theta - \frac{1}{2} g^{\mu \nu} \partial_\mu \sigma \partial_\nu \sigma - R \dot{\theta}_0 \sigma \partial^\mu \delta \theta - \frac{1}{2} (\dot{V}(\chi_0) - \dot{\theta}^2_0) \sigma^2 \right] . \] (21)

The mixing coupling \( \sigma \dot{\theta} \) converts the \( \sigma^4 \) coupling, for example, into three point functions of \( \delta \theta \), and hence this model can potentially give a large non-Gaussianities. Furthermore, it is known that the squeezed limit of scalar three point functions is sensitive to the mass of \( \sigma \):

\[ \lim_{k_3/k_1 = k_3/k_2 = \kappa \to 0} \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \propto \kappa^{-3/2} \mu k_1^{-6} , \] (22)

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4 Strictly speaking, the name of this gauge may be inadequate because there are still tensor fluctuations and hence the spatial hypersurface is not exactly flat.
where $\nu = \sqrt{\frac{9}{4} - \frac{m_\sigma^2}{H^2}}$ and $m_\sigma^2 = V''(\chi_0) - R\delta_0^2 < \frac{2}{3}H$. As this simple model implies, massive scalar fields of Hubble scale mass can cause a non-trivial behavior of non-Gaussianities. In the following sections we discuss more general setting for quasi-single field inflation using the effective field theory approach.

### III. MOST GENERIC ACTION OF QUASI-SINGLE FIELD INFLATION

In this section, we construct the most generic action of quasi-single field inflation using the effective field theory approach. After constructing the action in the unitary gauge first, we derive the action for the Goldstone boson $\pi$ and discuss its decoupling regime. Relations between our approach and models in the literatures are also discussed.

#### A. Action in the unitary gauge

In the unitary gauge, the relevant degrees of freedom in quasi-single field inflation are three physical modes of graviton and an additional scalar field $\sigma$. The typical mass of $\sigma$ is supposed to be of the order of the Hubble scale during the inflationary era. In this subsection, we construct the most generic action invariant under the time-dependent spatial diffeomorphism from graviton and the scalar field $\sigma$ up to the third order fluctuations. Here it should be noticed that the action constructed in this section can be applied to any two field models because no conditions on $\sigma$ are imposed.\(^5\)

Extending the procedures in [28] to our case, the most general action invariant under the time-dependent spatial diffeomorphism is given by

$$
S = \int d^4x \sqrt{-g} F(R_{\mu\nu\rho\sigma}, g^{00}, K_{\mu\nu}, \nabla_\mu, t, \sigma),
$$

(23)

and it is expanded around the given FRW background as

$$
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_\text{Pl}^2 R + M_\text{Pl}^2 \dot{H} g^{00} - M_\text{Pl}^2 (3H^2 + \dot{H}) + F^{(2)}(\delta g^{00}, \sigma, \delta K_{\mu\nu}, \delta R_{\mu\nu\rho\sigma}; \delta g^{00}, g^{0\mu
u}, g^{\mu\nu}, \nabla_\mu, t) \right],
$$

(24)

where all the free indices inside the functions $F$ and $F^{(2)}$ must be again upper 0’s and $F^{(2)}$ starts with quadratic terms of the arguments $\delta g^{00}$, $\sigma$, $\delta K_{\mu\nu}$, and $\delta R_{\mu\nu\rho\sigma}$.\(^6\) Then, let us write down possible terms in the action up to the third order fluctuations. Schematically, we write the action in the following way:

$$
S = S_{\text{grav}} + S_{\sigma} + S_{\text{mix}},
$$

(26)

where the first term $S_{\text{grav}}$, in the right hand side denotes terms constructed from $\delta g^{00}$, $\delta K_{\mu\nu}$, and $\delta R_{\mu\nu\rho\sigma}$, the second term $S_{\sigma}$ denotes those only from $\sigma$, and the last term $S_{\text{mix}}$ denotes those mixing the graviton fluctuations and $\sigma$. As discussed in [28], the first term $S_{\text{grav}}$ can be expanded as

$$
S_{\text{grav}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_\text{Pl}^2 R + M_\text{Pl}^2 \dot{H} (t) g^{00} - M_\text{Pl}^2 \left( 3H^2 (t) + \dot{H} (t) \right) + \frac{M_\text{Pl}^2 (t)}{2} (\delta g^{00})^2 + \frac{M_\text{Pl}^2 (t)}{3!} (\delta g^{00})^3 + \ldots \right],
$$

(27)

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\(^5\) For example, we do not require the shift symmetry of $\sigma$, $\sigma \to \sigma + \text{constant}$, which is assumed in multi-field inflation [28]. The mass of $\sigma$ is not necessarily of order Hubble scale so that the action constructed in this section can be applied not only for quasi-single field inflation but also inflation models with an additional heavy scalar.

\(^6\) In general, sums of terms linear in the fluctuations can be practically second order. For example, let us consider the term $\int d^4x \sqrt{-g} f(t) \sigma \partial^0 \sigma$. Although this kind of action seems to be first order apparently, it turns out to be second order after taking into account the equation of motion for $\sigma$: $f_1 (t) + f_2 (t) + 3H f_2 (t) = 0$. Then, the function $F^{(2)}$ seems to contain such a combination of linear order terms. However, using the relation

$$
\int d^4x \sqrt{-g} f(t) \partial^0 (\ldots) = - \int d^4x \sqrt{-g} f(t) \sqrt{-g^{00}} \partial_\mu (\ldots) = \int d^4x \sqrt{-g} \left( - g^{00} \dot{f}(t) - \frac{1}{2} f(t) \partial^0 \ln (g^{00}) + f(t) \sqrt{-g^{00}} K^{\mu}_\mu \right) (\ldots)
$$

$$
= \int d^4x \sqrt{-g} \left( \dot{f}(t) + 3H f(t) + O(\delta g^{00}, \delta K^{\mu}_\mu) \right) (\ldots),
$$

(25)

we can rewrite it into the second and higher order terms in $\sigma$, $\delta g^{00}$, and $\delta K^{\mu}_\mu$. Similar discussions hold for more general cases and we conclude that $F^{(2)}$ starts with quadratic terms of $\delta g^{00}$, $\sigma$, $\delta K_{\mu\nu}$, and $\delta R_{\mu\nu\rho\sigma}$. See also appendix B in [28].
where the dots stand for terms of higher order in the fluctuations or with more derivatives. When we rewrite the action in the unitary gauge in terms of the Goldstone boson \( \pi \), the terms displayed in (27) are described by \( \pi \) and its first order derivatives. In this paper, we consider the action up to the same order in derivatives of \( \pi \) and \( \sigma \).

Let us first construct the second order action. The second order action \( S^{(2)}_\sigma \) containing \( \sigma \) and its first order derivative can be written generally as

\[
S^{(2)}_\sigma = \int d^4x \sqrt{-g} \left[ -\frac{\alpha_1(t)}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma + \frac{\alpha_2(t)}{2} (\partial^\mu \sigma)^2 - \frac{\alpha_3(t)}{2} \sigma^2 + \alpha_4(t) \sigma \partial^\mu \sigma \right],
\]

where we note that terms such as \( \sigma(\partial^\mu \sigma)^2 \) can be absorbed into other terms up to higher order fluctuations by integrating by parts. As discussed in (29), the second term leads to a non-trivial sound speed \( c_\sigma \) of \( \sigma \) given by

\[
c_\sigma^2 = \frac{\alpha_1}{(\alpha_1 + \alpha_2)}.
\]

The second order mixing \( S^{(2)}_{\text{mix}} \) is generally given by

\[
S^{(2)}_{\text{mix}} = \int d^4x \sqrt{-g} \left[ \beta_1(t) \delta g^{00} \sigma + \beta_2(t) \delta g^{00} \partial^\mu \sigma + \beta_3(t) \delta K^\mu_{\mu'} \sigma \right].
\]

It is convenient to note the relation

\[
\int d^4x \sqrt{-g} f(t) \delta K^\mu_{\mu'}(\ldots) = \int d^4x \sqrt{-g} \left[ \left( f(t) \partial^\mu(\ldots) - \left( \frac{f(t)}{2} + 3H f(t) \right)(\ldots) + \frac{f(t)}{2} \delta g^{00}(\ldots) + \frac{f(t)}{2} \delta g^{00} \partial^\mu(\ldots) + \ldots \right),
\]

which can be obtained using (25) in footnote [6] twice. Here the last dots stand for higher order terms in the fluctuations, which can be written using \( \delta g^{00}, \partial^\mu \delta g^{00}, \) and \( \delta K^\mu_{\mu'} \). Using this relation (30), \( S^{(2)}_{\text{mix}} \) in (29) is rewritten as

\[
S^{(2)}_{\text{mix}} = \int d^4x \sqrt{-g} \left[ \beta_1(t) \delta g^{00} \sigma + \beta_2(t) \delta g^{00} \partial^\mu \sigma + \beta_3(t) \delta K^\mu_{\mu'} \sigma - \left( \beta_5(t) + 3H \beta_3(t) \right) \sigma \right],
\]

where \( \beta_1 = \beta_1 + \beta_3/2, \beta_2 = \beta_2 + \beta_3/2, \) and we dropped higher order terms constructed from \( \sigma, \delta g^{00}, \partial^\mu \delta g^{00}, \) and \( \delta K^\mu_{\mu'} \). In the following, we employ Eq. (31) as a definition of the second order mixing action \( S^{(2)}_{\text{mix}} \).

The third order action \( S^{(3)}_\sigma \) of \( \sigma \) is generally given by

\[
S^{(3)}_\sigma = \int d^4x \sqrt{-g} \left[ \gamma_1(t) \sigma^3 + \gamma_2(t) \sigma^2 \partial^\mu \sigma + \gamma_3(t) \sigma (\partial^\mu \sigma)^2 + \gamma_4(t) (\partial^\mu \sigma)^3 + \gamma_5(t) \sigma \partial^\mu \sigma \partial^\nu \sigma + \gamma_6(t) \partial^\mu \sigma \partial^\nu \sigma \partial^\rho \sigma \right],
\]

and the third order mixing \( S^{(3)}_{\text{mix}} \) is given by

\[
S^{(3)}_{\text{mix}} = \int d^4x \sqrt{-g} \left[ \gamma_1(t) \delta g^{00} \sigma^2 + \gamma_2(t) \delta g^{00} \sigma \partial^\mu \sigma + \gamma_3(t) \delta g^{00} (\partial^\mu \sigma)^2 + \gamma_4(t) \delta g^{00} \sigma (\partial^\mu \sigma)^2 + \gamma_5(t) \delta g^{00} \partial^\mu \sigma \partial^\nu \sigma \partial^\rho \sigma + \gamma_6(t) (\delta g^{00})^2 \sigma + \gamma_7(t) \delta g^{00} (\partial^\mu \sigma)^2 \partial^\nu \sigma \right].
\]

Here, it may be wondered if the terms such as \( \delta K^\mu_{\mu'} \sigma^2, \delta K^\mu_{\mu'} \sigma \partial^\rho \sigma, \delta R^{00} \sigma^2, \) and \( \delta K^\mu_{\mu'} \delta g^{00} \sigma \) can appear at the same order in derivatives. However, they can be absorbed into other third order terms in (33) and the second order terms in (28) and (31) by integrating by parts as we did to rewrite (29) into (31). The term proportional to \( \delta R \sigma^2 \) can also appear but such a term vanishes in the decoupling limit, so we do not consider it here for simplicity.

To summarize, the most generic action in the unitary gauge can be written up to the third order fluctuations as follows:

\[
S = S_{\text{grav}} + S^{(2)}_\sigma + S^{(2)}_{\text{mix}} + S^{(3)}_\sigma + S^{(3)}_{\text{mix}},
\]

where \( S_{\text{grav}}, S^{(2)}_\sigma, S^{(2)}_{\text{mix}}, S^{(3)}_\sigma, \) and \( S^{(3)}_{\text{mix}} \) are defined in (27), (28), (31), (32), and (33).

### B. Ambiguity of the action in the unitary gauge

In multiple field inflation models, there are some ambiguities of the action in the unitary gauge: there are degrees of freedom of the field redefinition of \( \sigma \) and time coordinate transformations vanishing on the background trajectory.
\( \sigma = 0 \). Using these degrees of freedom, it is possible to drop some terms and simplify the action. Without loss of generalities, the action can be written into the following three normalizations using these ambiguities.\(^7\)

a. Normalization 1 : \( \alpha_1 + \alpha_2 = 1 \). Let us first consider the kinetic term of \( \sigma \). The second order action \( S_\sigma^{(2)} \) of \( \sigma \) can be expanded up to the second order fluctuations as

\[
\int d^4x \sqrt{-g} \left[ \frac{\alpha_1^2}{2} \left( \dot{\sigma}^2 - \frac{\partial(\sigma^2)}{\dot{\sigma}} - \frac{\alpha_3 - 3H\alpha_4 - \dot{\alpha}_4}{\alpha_1 + \alpha_2} \sigma^2 \right) \right],
\]

where the normalization factor \( \alpha_\sigma \) is defined as \( \alpha_\sigma^2 = \alpha_1 + \alpha_2 \). Although the factor \( \alpha_\sigma \) is time-dependent in general, it can be taken unity by redefining \( \sigma \) as \( \hat{\sigma} = \alpha_\sigma \sigma \). Since the derivative of \( \sigma \) can be written as

\[
\partial_\mu \sigma = \alpha_\sigma^{-1} \partial_\mu \hat{\sigma} - \delta_\mu^0 \hat{\dot{\sigma}},
\]

the action still takes the form \( \text{[28]} \) after the redefinition.

b. Normalization 2 : \( \alpha_4 = \beta_3 = \gamma_2 = 0 \). Using the time coordinate transformation vanishing on the background trajectory \( \sigma = 0 \), the following form of interaction terms can be eliminated:

\[
\int d^4x \sqrt{-g} \left[ f(t, \sigma) \partial^0 \sigma \right],
\]

where \( f(t, \sigma) \) is a function of \( t \) and \( \sigma \) but does not contain derivatives of \( \sigma \). As a simple example, let us consider the action

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R + M_{Pl}^2 \dot{H}(t) \right] + f(\sigma, t) \partial^0 \sigma \].
\]

Under the time coordinate transformation

\[
t \to \tilde{t} \quad \text{with} \quad t = \tilde{t} - \epsilon(\tilde{t}, \sigma),
\]

the action \( \text{[38]} \) is transformed into

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R + M_{Pl}^2 \dot{H}(t - \epsilon) \left( g^{00}(1 - \partial_\epsilon \epsilon)^2 - 2(1 - \partial_\epsilon \epsilon) \partial_\sigma \epsilon \partial^0 \sigma + (\partial_\sigma \epsilon)^2 \partial_\mu \sigma \partial^\mu \sigma \right) \right.
\]

\[
\left. - M_{Pl}^2 \left( 3H^2(t - \epsilon) + \dot{H}(t - \epsilon) \right) + f(\sigma, t - \epsilon) (1 - \partial_\epsilon \epsilon) \partial^0 \sigma - f(\sigma, t - \epsilon) \partial_\sigma \epsilon \partial_\mu \sigma \partial^\mu \sigma \right].
\]

Therefore, if we take \( \epsilon \) such that

\[
\partial_\sigma \epsilon = \frac{f(\sigma, t - \epsilon)}{2M_{Pl}^2 \dot{H}(t - \epsilon)}, \quad \epsilon(t, \sigma = 0) = 0,
\]

the action \( \text{[40]} \) reduces to

\[
S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{Pl}^2 R - M_{Pl}^2 \left( 3H^2(t - \epsilon) + \dot{H}(t - \epsilon) \right) \right.
\]

\[
\left. + M_{Pl}^2 \dot{H}(t - \epsilon) \left( g^{00}(1 - \partial_\epsilon \epsilon)^2 - (\partial_\sigma \epsilon)^2 \partial_\mu \sigma \partial^\mu \sigma \right) - f(\sigma, t - \epsilon) \partial_\sigma \epsilon \partial_\mu \sigma \partial^\mu \sigma \right],
\]

which does not contain interaction terms in the form \( f(t, \sigma) \partial^0 \sigma \). Note that the conditions \( \text{[41]} \) can be always solved at least as an expansion in \( \sigma \). It is straightforward to extend this discussion to general cases and we conclude that interaction terms in the form of \( \text{[37]} \) can be eliminated using the time coordinate transformation vanishing on the background trajectory \( \sigma = 0 \). In particular, we can set \( \alpha_4 = \beta_3 = \gamma_2 = 0 \) without loss of generalities.

\(^7\) Note that it is not possible in general to impose some of the three conditions at the same time.
c. Normalization 3: $\alpha_1 = 1$, $\gamma_5 = 0$. The action $S$ in Eq. (34) contains the following term:

$$
\int d^4x \sqrt{-g} \left[ -\frac{1}{2} f^2(t, \sigma) g^{\mu \nu} \partial_\mu \sigma \partial_\nu \sigma \right],
$$

where $f(t, \sigma)$ is a function of $t$ and $\sigma$, and does not contain derivatives of $\sigma$. The function is expanded in $\sigma$ as $f^2(t, \sigma) = \alpha_1(t) - 2\gamma_5(t) \sigma + \mathcal{O}(\sigma^2)$. By the field redefinition, it is possible to eliminate this kind of derivative couplings and to rewrite (43) into the canonical form of the kinetic term. Let us define $\tilde{\sigma}$ as $\tilde{\sigma} = F(t, \sigma)$ such that

$$
\partial_\sigma F(t, \sigma) = f(t, \sigma), \quad F(t, \sigma = 0) = 0.
$$

Since the derivative of $\tilde{\sigma}$ is given by

$$
\partial_\mu \tilde{\sigma} = f \partial_\mu \sigma + \delta^0_\mu \partial_0 F,
$$

the action can be rewritten as

$$
\int d^4x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu \nu} \partial_\mu \tilde{\sigma} \partial_\nu \tilde{\sigma} + \partial_\mu F \partial^\mu \tilde{\sigma} - \frac{1}{2} g^{00} (\partial_0 F)^2 \right],
$$

which still takes the form (34). Here $F$ should be regarded as a function of $\tilde{\sigma}$ and $t$. It also should be noticed that, though we can set the function $f$ to be unity, the terms proportional to $\partial^0 \sigma$ and $g^{00}$ appear.

### C. Action for the Goldstone boson and the decoupling regime

In this subsection we construct the action for the Goldstone boson $\pi$ and discuss its decoupling regime. As in the last section, we perform the time diffeomorphism (10). Practically, it is realized by the following replacements:

$$
g^{00} \rightarrow g^{00} + 2 \partial^0 \pi + \partial_\mu \pi \partial^\mu \pi, \quad \partial^0 \sigma \rightarrow \partial^0 \sigma + \partial_\mu \pi \partial^\mu \sigma, \quad \sigma \rightarrow \sigma, \quad f(t) \rightarrow f(t + \pi), \quad \int d^4x \sqrt{-g} \rightarrow \int d^4x \sqrt{-g}.
$$

With these replacements, $S_{\text{grav}}$, $S^{(2)}_\sigma$, and $S^{(2)}_{\text{mix}}$ are rewritten as

$$
S_{\text{grav}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_{\text{pl}}^2 R + M_{\text{pl}}^2 \dot{H}(t + \pi) (g^{00} + 2 \partial^0 \pi + \partial_\mu \pi \partial^\mu \pi) - M_{\text{pl}}^2 \left( 3H^2(t + \pi) + \dot{H}(t + \pi) \right) 
\right.
\left. + \frac{M_{\text{pl}}^2(t + \pi)}{2!} \left( \delta g^{00} + 2 \partial^0 \pi + \partial_\mu \pi \partial^\mu \pi \right)^2 + \frac{M_{\text{pl}}^2(t + \pi)}{3!} \left( \delta g^{00} + 2 \partial^0 \pi + \partial_\mu \pi \partial^\mu \pi \right)^3 \right],
$$

$$
S^{(2)}_\sigma = \int d^4x \sqrt{-g} \left[ -\frac{\alpha_1(t + \pi)}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{\alpha_3(t + \pi)}{2} (\partial^0 \sigma + \partial_\mu \pi \partial^\mu \sigma)^2 - \frac{\alpha_3(t + \pi)}{2} \sigma^2 + \alpha_4(t + \pi) (\partial^0 \sigma + \partial_\mu \pi \partial^\mu \sigma) \right],
$$

$$
S^{(2)}_{\text{mix}} = \int d^4x \sqrt{-g} \left[ \beta_1(t + \pi) \left( \delta g^{00} + 2 \partial^0 \pi + \partial_\mu \pi \partial^\mu \pi \right) \sigma + \beta_2(t + \pi) \left( \delta g^{00} + 2 \partial^0 \pi + \partial_\mu \pi \partial^\mu \pi \right) \left( \partial^0 \sigma + \partial_\mu \pi \partial^\mu \sigma \right) 
\right.
\left. + \beta_3(t + \pi) \left( \partial^0 \sigma + \partial_\mu \pi \partial^\mu \sigma \right) - \left( \beta_3(t + \pi) + 3H(t + \pi) \beta_3(t + \pi) \right) \sigma \right].
$$

The third order actions $S^{(3)}_\sigma$ and $S^{(3)}_{\text{mix}}$ can be obtained in a similar way.

In order to discuss the decoupling regime of the action, we first clarify in which regime graviton fluctuations become irrelevant to tree-level three point functions of $\pi$. For this purpose, let us take the spatially flat gauge (18) and use the ADM decomposition:

$$
ds^2 = -(N^2 - N_i N^i) dt^2 + 2N_i dx^i dt + a^2(e^\gamma)_{ij} dx^i dx^j \quad \text{with} \quad \gamma_{ii} = \partial_\gamma \gamma_{ij} = 0.
$$

Here and in what follows we use the spatial metric $h_{ij} = a^2(e^\gamma)_{ij}$ and its inverse $h^{ij} = a^{-2}(e^{-\gamma})_{ij}$ to raise or lower the indices of $N^i$. The inverse metric $g^{\mu \nu}$ are written in terms of $N$, $N^i$, and $h_{ij}$ as

$$
g^{00} = -\frac{1}{N^2}, \quad g^{0i} = g^{i0} = \frac{N^i}{N^2}, \quad g^{ij} = h^{ij} - \frac{N^i N^j}{N^2}.
$$
In this gauge, there are no second order mixing terms of $\pi$ and $\gamma_{ij}$ because $\gamma_{ij}$ has two spatial indices and is transverse-traceless. Then, the tensor fluctuation $\gamma_{ij}$ does not contribute to tree-level three point functions of $\pi$. Therefore, possible contributions of graviton fluctuations come only from the auxiliary fields $\delta N = N - 1$ and $N^i$. As discussed in Ref. 31, it is sufficient for the calculation of three-point functions to solve the constraints up to first order. Expanding the actions (48) and (50) up to the second order in $\pi$, $\delta N$, and $N^i$,

$$S_{\text{grav}} = \int d^4x a^3 \left[ - M_{Pl}^2 (3H^2 + c_\pi^2 \dot{H}) \delta N^2 - 2M_{Pl}^2 H \delta N \partial_i N^i + M_{Pl}^2 \frac{1}{4} N^i \partial_i \rho_j N^j - M_{Pl}^2 \frac{1}{4} N^i \partial^2 N^i \right] - \frac{M_{Pl}^2 \dot{H}}{c_\pi^2} \left( \dot{\sigma}^2 - c_\pi^2 \frac{(\partial_i \sigma)^2}{a^2} \right) - 3M_{Pl}^2 H^2 \pi^2 + M_{Pl}^2 (2c_\pi^2 \dot{H} \dot{\pi} - 6H \dot{H} \pi) \delta N + 2M_{Pl}^2 \dot{H} N^i \partial_i \pi \right],$$

$$(53)$$

$$S_{\sigma}^{(2)} = \int d^4x a^3 \left[ \frac{\sigma}{2} \left( \dot{\sigma} - c_\sigma^2 \frac{(\partial_i \sigma)^2}{a^2} \right) - \frac{\alpha_3}{2} \sigma^2 - \alpha_4 \sigma \dot{\sigma} \right],$$

$$(54)$$

$$S_{\text{mix}}^{(2)} = \int d^4x a^3 \left[ 2\beta_1 (\delta N - \dot{\pi}) \sigma - 2\beta_2 (\delta N - \dot{\pi}) \dot{\sigma} - \delta N \left( - \beta_3 \sigma + \dot{\beta}_3 \sigma + 2H \beta_3 \sigma \right) + \beta_3 N^i \partial_i \sigma \right] + \beta_3 \left( - \frac{\dot{\pi} \sigma + \dot{\beta}_3 \partial_i \sigma}{a^2} \right) + \beta_3 \dot{\pi} \sigma - 3H \beta_3 \pi \sigma \right],$$

$$(55)$$

the constraints are solved up to first order as follows:

$$\delta N = - \frac{\dot{H}}{H} \pi + \frac{\beta_3}{2M_{Pl}^2 H} \sigma, \quad N^i = a^{-2} \partial_i \psi$$

with

$$\psi = a^2 \partial^2 \left( c_\pi^2 \frac{\dot{H}}{H^2} (\dot{H} \pi + \pi \dot{H}) + \frac{\beta_1}{M_{Pl}^2 H} \sigma - \frac{\beta_2}{M_{Pl}^2 H} \dot{\sigma} + \frac{\beta_3}{2M_{Pl}^2 H} \left( c_\pi^2 \frac{\dot{H}}{H} \sigma + \dot{\sigma} \right) - \frac{\dot{\beta}_3}{2M_{Pl}^2 H} \sigma \right).$$

$$(56)$$

Here the sound speed $c_\pi^2$ of $\pi$ are defined as $c_\pi^2 = \dot{H}/M_{Pl}^2/(H^2 M_{Pl}^4 - 2M_{Pl}^2)$. The factors $c_\pi^2 = \alpha_1/(\alpha_1 + \alpha_2)$ and $\alpha_2 = \alpha_1 + \alpha_2$ are the sound speed and the normalization factor of $\sigma$, respectively. Using the canonical normalization

$$\pi_c \sim M_{Pl} (\dot{\pi}) \left( c_\pi^{-1} \right), \quad \sigma_c \sim \alpha_3 \sigma_c, \quad \delta N_c \sim M_{Pl} \delta N_c, \quad N^i_c \sim M_{Pl} N^i,$$

$$(57)$$

and redefining the coupling constants $\beta_1$, $\beta_2$, and $\beta_3$ correspondingly as

$$\beta_1' \sim \frac{c_\pi}{\alpha_3 M_{Pl} (\dot{\pi})^{1/2} \beta_1}, \quad \beta_2' \sim \frac{c_\pi}{\alpha_3 M_{Pl} (\dot{\pi})^{1/2} \beta_2}, \quad \beta_3' \sim \frac{c_\pi}{\alpha_3 M_{Pl} (\dot{\pi})^{1/2} \beta_3},$$

$$(58)$$

we rewrite the constraints (56) as

$$\delta N_c \sim \frac{\dot{\epsilon}^{1/2}}{2} \left( c_\pi^2 \pi_c - \frac{1}{2} \beta_3' \sigma_c \right),$$

$$(59)$$

$$N^i_c \sim \frac{\dot{\epsilon}^{1/2}}{H^2} \partial_i \left( - \pi_c + \frac{1}{2} \dot{\eta} \pi_c + \beta_1' \sigma_c - \beta_2' \left( \sigma_c - \frac{\dot{\alpha}_3}{a} \right) - \frac{1}{2} \beta_3' \sigma_c + \frac{1}{2} \beta_3^2 \sigma_c \left( - \frac{\dot{\alpha}_3}{\alpha_3} + (c_\pi^2 - 1) \dot{\pi} \pi \right) - \frac{1}{2} \beta_3' \sigma_c \right),$$

$$(60)$$

where we have defined $\dot{\epsilon} = -c_\pi^2 \frac{\dot{H}}{H^2}$ and $\dot{\eta} = \dot{\epsilon} / \dot{H}$ in analogy with usual slow-roll parameters $\epsilon$ and $\eta$. It is manifest that $\delta N_c$ and $N^i_c$ are suppressed by the parameter $\dot{\epsilon}^{1/2}$ and contributions from $\delta N_c$ and $N^i_c$ become irrelevant in the limit $\dot{\epsilon} \rightarrow 0$. In this limit, tree-level three point functions of $\pi$ are determined by the following action in the

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8 To be precise, we need to assume that $\dot{\epsilon}^2 H^2 \ll 1$. 
decoupling limit:

\[
S_{\text{grav}} = \int d^{4}x a^{3} \left[ -\frac{M_{\text{Pl}}^{2} \dot{H}}{c_{\pi}^{2}} \left( \dot{x}^{2} - c_{\pi}^{2} \left( \partial_{\tau} \sigma \right)^{2} \right) - M_{\text{Pl}}^{2} \dot{H} \left( c_{\pi}^{-2} - 1 \right) \left( \dot{x}^{3} - \dot{x} \left( \partial_{\tau} \sigma \right)^{2} \right) - \frac{4 M_{\text{Pl}}^{4}}{3} \dot{x}^{3} \right.
\]

\[
- 3 M_{\text{Pl}}^{2} \dot{H}^{2} \dot{x}^{2} - \partial_{\tau} \left( \frac{M_{\text{Pl}}^{2} \dot{H}}{c_{\pi}^{2}} \right) \dot{x}^{2} + M_{\text{Pl}}^{2} \dot{H} \left( c_{\pi}^{-2} \dot{x}^{2} - 3 M_{\text{Pl}}^{3} \ddot{H} \dot{x}^{3} \right),
\]

\[
S^{(2)}_{\sigma} = \int d^{4}x a^{3} \left[ \alpha_{1}^{2} \left( \dot{x}^{2} - 2 \alpha_{1} \theta_{1} \ddot{x} + \theta_{1} \dddot{x} \right) + (\alpha_{2}^{2} \dot{x}^{2} - \alpha_{2} \theta_{1} \ddot{x} + \theta_{1} \dddot{x}) \right],
\]

\[
S^{(3)}_{\sigma} = \int d^{4}x a^{3} \left[ \alpha_{3} \left( \dot{x}^{2} - 2 \alpha_{3} \theta_{1} \ddot{x} + \theta_{1} \dddot{x} \right) + (\alpha_{4}^{2} \dot{x}^{2} - \alpha_{4} \theta_{1} \ddot{x} + \theta_{1} \dddot{x}) \right],
\]

It should be noticed that non-trivial cubic interactions appear generically when the sound speed \( c_{\sigma} \) of \( \sigma \) is small, \( \alpha_{4} \) is non-zero, or mixing couplings \( \beta_{1} \) and \( \beta_{2} \) exist as well as the sound speed \( c_{\pi} \) of \( \pi \) is small.

\[A.2\]  

Before closing this section, we clarify the relation between our approach and models in the literatures. For this purpose, we first discuss the original model of quasi-single field inflation [22], and then, we investigate the effects of heavy particles during inflation. At the end of this subsection, a class of two field models will be considered.

1. Original model discussed by Chen and Wang

As was reviewed in section [11], the original model [22] of quasi-single field inflation is described by the following matter action:

\[
S_{\text{matter}} = \int d^{4}x \sqrt{-g} \left[ -\frac{1}{2} \left( \dddot{R} + \chi \right)^{2} g^{\mu \nu} \partial_{\mu} \theta \partial_{\nu} \theta - \frac{1}{2} g^{\mu \nu} \partial_{\mu} \chi \partial_{\nu} \chi - V_{sr}(\theta) - V(\chi) \right].
\]

The homogeneous backgrounds are given by \( \theta = \theta_{0}(t) \) and \( \chi = \chi_{0} \) (constant), which leads to the action in the unitary gauge \( \delta \theta = \theta - \theta_{0} = 0 \),

\[
S_{\text{matter}} = \int d^{4}x \sqrt{-g} \left[ -\frac{1}{2} \left( \dddot{R} + \sigma \right)^{2} g^{\mu \nu} \partial_{\mu} \sigma \partial_{\nu} \sigma - V_{sr}(\theta_{0}) - V(\chi_{0} + \sigma) \right]
\]

\[
= \int d^{4}x \sqrt{-g} \left[ -\frac{1}{2} R^{2} \ddot{\sigma}^{2} g^{\mu \nu} - \left( V_{sr}(\theta_{0}) + V(\chi_{0}) \right) - \frac{1}{2} g^{\mu \nu} \partial_{\mu} \sigma \partial_{\nu} \sigma - \frac{1}{2} \left( V(\chi_{0}) + \frac{2 M_{\text{Pl}}^{2} \dot{H}}{R} \right) \dot{\sigma}^{2}
\]

\[
- R \ddot{\sigma}^{2} \ddot{\sigma}^{2} g^{\mu \nu} - \frac{V''(\chi_{0})}{3!} \dot{\sigma}^{3} - \frac{1}{2} \ddot{\sigma}^{2} \dot{\sigma}^{2} \ddot{\sigma} + O(\dot{\sigma}^{4}) \right],
\]

where \( R = \dddot{R} + \chi_{0} \), \( \sigma = \chi - \chi_{0} \) and we used the background equations of motion. Using the equations of motion [20], the action [27] can be written in terms of the Hubble parameter \( H \) as

\[
S_{\text{matter}} = \int d^{4}x \sqrt{-g} \left[ M_{\text{Pl}}^{2} \dot{H} g^{\mu \nu} - M_{\text{Pl}}^{2} \left( 3 H^{2} + \dot{H} \right) - \frac{1}{2} g^{\mu \nu} \partial_{\mu} \sigma \partial_{\nu} \sigma - \frac{1}{2} \left( V''(\chi_{0}) + \frac{2 M_{\text{Pl}}^{2} \dot{H}}{R^{2}} \right) \dot{\sigma}^{2}
\]

\[
+ \frac{2 M_{\text{Pl}}^{2} \dot{H}}{R} \delta g^{\mu \nu} - \frac{V''(\chi_{0})}{3!} \dot{\sigma}^{3} + \frac{M_{\text{Pl}}^{2} \dot{H}}{R^{2}} \delta g^{\mu \nu} \ddot{\sigma} + O(\dot{\sigma}^{4}) \right],
\]

(68)
which corresponds to the following parameters in our framework,
\begin{align*}
\alpha_1 &= 1, \quad \alpha_3 = V''(\chi_0) + \frac{2M_P^2 \dot{H}}{R^2}, \quad \beta_1 = \frac{2M_P^2 \dot{H}}{R}, \quad \gamma_1 = -\frac{1}{3!} V'''(\chi_0), \quad \gamma_2 = \frac{M_P^2 \dot{H}}{R^2}, \quad (\text{others}) = 0.
\end{align*}

2. Effects of heavy particles

Recently, it is argued that the existence of heavy particles can cause a non-trivial sound speed of effective single field inflation \cite{13-21}. As was mentioned earlier, our framework is also applicable for such inflation models with heavy particles. In the following, we give a simple explanation for the effects of heavy fields.

Let us start from the following simplest case:
\begin{align*}
S &= \int d^4 x \sqrt{-g} \left[ \frac{1}{2} M_P^2 R + M_P^2 \dot{H} g^{00} - M_P^2 (3H^2 + \dot{H}) - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{m^2}{2} \sigma^2 + \beta \delta g^{00} \sigma \right]. \tag{70}
\end{align*}

Here \( m \) is the mass of \( \sigma \) and \( \beta \) is the mixing coupling between the adiabatic mode and the massive particle \( \sigma \). We assume that the mass of \( \sigma \) is much larger than the Hubble scale during inflation, \( m \gg H \), and the time-dependence of \( \beta \) is negligible compared to the mass \( m \). In such a regime, the kinetic term of \( \sigma \) becomes irrelevant and the dynamics is determined by
\begin{align*}
S &\sim \int d^4 x \sqrt{-g} \left[ \frac{1}{2} M_P^2 R + M_P^2 \dot{H} g^{00} - M_P^2 (3H^2 + \dot{H}) - \frac{m^2}{2} \left( \sigma - \frac{\beta}{m^2} \delta g^{00} \right)^2 + \frac{\beta^2}{2m^2} (\delta g^{00})^2 \right], \tag{71}
\end{align*}

which implies that the perturbation \( \sigma \) quickly responds to the variation of the adiabatic mode \( \delta g^{00} \). Integrating out the massive particle \( \sigma \), we obtain the following effective action for single-field inflation:
\begin{align*}
S_{\text{eff}} &= \int d^4 x \sqrt{-g} \left[ \frac{1}{2} M_P^2 R + M_P^2 \dot{H} g^{00} - M_P^2 (3H^2 + \dot{H}) + \frac{\beta^2}{2m^2} (\delta g^{00})^2 \right]. \tag{72}
\end{align*}

In particular, the last term gives the following non-trivial sound speed:
\begin{align*}
\frac{c_s^2}{\beta} &= \frac{-\dot{H} M_P^2}{-\dot{H} M_P^2 + 2\beta^2 / m^2}, \tag{73}
\end{align*}

which reproduces the result in \cite{13-21}. Note that it is obvious in our approach that the effective action contains the \( (\delta g^{00})^2 \) interaction: our result explains not only the effective sound speed but also non-trivial cubic effective interactions of the Goldstone boson \( \pi \) associated with the \( (\delta g^{00})^2 \) term.

The above discussions can be extended to more general settings. Let us consider the following action with more generic mixing couplings:
\begin{align*}
S &= \int d^4 x \sqrt{-g} \left[ \frac{1}{2} M_P^2 R + M_P^2 \dot{H} g^{00} - M_P^2 (3H^2 + \dot{H}) \\
&\quad - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{m^2}{2} \sigma^2 + \beta_1 \delta g^{00} \sigma + \beta_2 \delta g^{00} \partial^0 \sigma + \beta_3 \partial^0 \sigma - (\beta_3 + 3H\beta_3) \sigma \right]. \tag{74}
\end{align*}

When the mass of \( \sigma \) is much larger than the Hubble scale during inflation, \( m \gg H \), and the time-dependence of \( \beta_i \)'s is negligible compared to the mass \( m \), the low energy effective action can be obtained via the following procedure (see appendix A for more detailed discussions):

1. Drop the kinetic term of heavy fields.
2. Eliminate derivatives of heavy fields by partial integrals.
3. Complete square the Lagrangian and integrate out heavy fields.
To perform the second step, it is convenient to introduce the following relations, which follow from the formulae (26) and (30):

\[
\int d^4x \sqrt{-g} f(t) \delta g^{00} \partial^0 \sigma = \int d^4x \sqrt{-g} \left[ (\dot{f}(t) + 3H f(t)) \delta g^{00} \sigma - f(t) \delta^0 \delta g^{00} \sigma + \ldots \right],
\]

(75)

\[
\int d^4x \sqrt{-g} \left[ f(t) \partial^0 \sigma - (\dot{f}(t) + 3H f(t)) \sigma \right] = \int d^4x \sqrt{-g} \left[ f(t) \delta K_{\mu}^\mu - \frac{\dot{f}(t)}{2} \delta g^{00} \sigma - \frac{f(t)}{2} \delta g^{00} \partial^0 \sigma + \ldots \right],
\]

(76)

where dots stand for higher order terms in \(\delta g^{00}, \delta K_{\mu}^\mu\), and their derivatives. From these relations, it follows that

\[
\int d^4x \sqrt{-g} \left[ \beta_1 \delta g^{00} \sigma + \beta_2 \delta g^{00} \partial^0 \sigma + \beta_3 \partial^0 \sigma - (\dot{\beta}_3 + 3H \beta_3) \sigma \right] = \int d^4x \sqrt{-g} \left[ \left( \beta_1 - \frac{\dot{\beta}_3}{2} \right) \delta g^{00} \sigma + \left( \beta_2 - \frac{\dot{\beta}_3}{2} \right) \delta g^{00} \partial^0 \sigma + \beta_3 \delta K_{\mu}^\mu \sigma + \ldots \right]
\]

= \int d^4x \sqrt{-g} \left[ \left( \beta_1 - \beta_2 + 3H \beta_3 - \frac{3}{2} H \beta_3 \right) \delta g^{00} \sigma - \left( \beta_2 - \frac{\dot{\beta}_3}{2} \right) \partial^0 \delta g^{00} \sigma + \beta_3 \delta K_{\mu}^\mu \sigma + \ldots \right].
\]

(77)

Note that the first equality is the same as the relation used to rewrite (29) into the form of (31) plus higher order terms. Then, it is straightforward to obtain the following effective action for single-field inflation using the above prescription:

\[
S_{\text{eff}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_P^2 R + M_P^2 \dot{H} \delta g^{00} - M_P^2 (3H^2 + \dot{H}) + \frac{1}{2m^2} \left( \beta_1 - \beta_2 + 3H \beta_3 - \frac{3}{2} H \beta_3 \right) \delta g^{00} - \left( \beta_2 - \frac{\dot{\beta}_3}{2} \right) \partial^0 \delta g^{00} + \beta_3 \delta K_{\mu}^\mu \right]^2 + \ldots \]

(78)

where dots stand for higher order terms in \(\delta g^{00}, \delta K_{\mu}^\mu\), and their derivatives. It turns out that interactions such as \((\partial^0 \delta g^{00})^2\) and \((\delta K_{\mu}^\mu)^2\) appear in the effective action as well as the \((\delta g^{00})^2\) interaction.

3. A class of two-field models

Let us then consider a class of two-field models described by the following matter action:

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \gamma_{\mu\nu}(\phi^a) g^{\mu\nu} \partial_\mu \phi^a \partial_\nu \phi^b - V(\phi^a) \right] \quad (a, b = 1, 2),
\]

(79)

where \(\gamma_{\mu\nu}(\phi^a)\) is the metric on the field space. This class of models were carefully studied in [32] and recently discussed in [13] to investigate effects of massive particles during inflation. Suppose that the trajectory of the homogeneous background fields \(\phi^a(t)\) is on a curve \(\phi^a = \bar{\phi}^a(\lambda)\), and the background fields are given by \(\phi^a_0(t) = \bar{\phi}(\lambda_0(t))\). We also assume that \(\lambda > 0\). Defining the coordinates \((\lambda, \sigma)\) of the field space such that the curve \(\sigma = 0\) coincides with the trajectory curve, the fields \(\lambda(x)\) and \(\sigma(x)\) describe the adiabatic mode and the isocurvature mode, respectively. There are still many choices of the coordinates or degrees of freedom of the field redefinition. In the following, we consider two types of basis of the fields and discuss their properties in our framework.

a. Orthogonal basis

We first consider the orthogonal-basis. We can always take the coordinate \((\lambda, \sigma)\) such that

\[
\gamma_{\lambda\sigma} = \gamma_{\sigma\lambda} = 0.
\]

(80)

By the field redefinition of \(\lambda\), we further require \(\gamma_{\lambda\lambda}(\lambda, \sigma = 0) = 1\). In this basis, the matter action (79) is given by

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \gamma_{\mu\nu}(\lambda, \sigma) g^{\mu\nu} \partial_\mu \lambda \partial_\nu \lambda - \frac{1}{2} \gamma_{\sigma\sigma}(\lambda, \sigma) g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\lambda, \sigma) \right],
\]

(81)

and it can be expanded in the unitary gauge \(\delta\lambda = \lambda - \lambda_0 = 0\) up to the third order fluctuations as

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \lambda_0^2 \delta g^{00} - \frac{1}{2} \lambda_0^2 (\gamma_{\lambda\lambda})' \delta g^{00} \sigma + \frac{1}{4} \lambda_0^2 (\gamma_{\lambda\lambda})'' g^{00} \sigma^2 - \frac{1}{4} \lambda_0^2 (\gamma_{\lambda\lambda})'' \delta g^{00} \sigma^2 + \frac{1}{12} \lambda_0^2 (\gamma_{\lambda\lambda})''' \sigma^3 - \frac{1}{2} \gamma_{\sigma\sigma} \delta g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{6} \gamma_{\sigma\sigma} \delta g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} \gamma_{\sigma\sigma} \delta g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V_0 - \frac{1}{2} V_0'' \sigma^2 - \frac{1}{6} V_0''' \sigma^3 \right].
\]

(82)
where we have used the equation of motion for \( \sigma \). In this subsection, we write derivatives of the metric and the potential evaluated at the classical value, for example, as \( (\gamma_{\lambda\lambda})'_0 = \partial_{\sigma} \gamma_{\lambda\lambda}(\lambda, \sigma)|_{\lambda=\lambda_0, \sigma=0} \) and \( V'_0 = \partial_{\sigma} V(\lambda, \sigma)|_{\lambda=\lambda_0, \sigma=0} \). Using the equations of motion for graviton,

\[
M_P^2 \dot{H} = -\frac{1}{2} \hat{\lambda}\sigma^2, \quad M_P^2(3H^2 + \dot{H}) = V_0, \tag{83}
\]

we can rewrite \([82]\) as

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ M_P^2 g^{00} \dot{H} - M_P^2(3H^2 + \dot{H}) - \frac{1}{2}(\gamma_{\sigma\sigma})_0 g^{\mu\nu} \partial_\mu \gamma_\sigma \partial_\nu \gamma_\sigma - \frac{1}{2} \left( V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0 \right) \sigma^2 \right. \\
+ M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0 \delta g^{00} \sigma - \frac{1}{2}(\gamma_{\sigma\sigma})_0 \sigma g^{\mu\nu} \partial_\mu \gamma_\sigma \partial_\nu \sigma - \frac{1}{6} \left( V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0 \right) \sigma^3 + \frac{1}{2} M_P^2 \dot{H}(\gamma_{\lambda\lambda})'' \delta g^{00} \sigma^2 \right], \tag{84}
\]

which is described in our framework as

\[
\alpha_1 = (\gamma_{\sigma\sigma})_0, \quad \alpha_3 = V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0, \quad \beta_1 = M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0, \\
\gamma_1 = -\frac{1}{6} \left( V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0 \right), \quad \gamma_5 = -\frac{1}{2}(\gamma_{\sigma\sigma})_0, \quad \tau_1 = \frac{1}{2} M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0, \quad (\text{others}) = 0. \tag{85}
\]

We notice that this basis corresponds to the second normalization of \( \sigma \) in section [III A]

\[b. \quad \text{Canonical } \sigma \text{ basis} \]

We then choose the coordinate \((\lambda, \sigma)\) such that

\[
(\gamma_{\sigma\sigma})(\lambda, \sigma) = 1, \quad \gamma_{\lambda\lambda}(\lambda, \sigma = 0) = 1, \tag{86}
\]

where the normalization of \( \sigma \) is always canonical and that of \( \lambda \) is canonical only on the trajectory curve. In this basis, the matter action \([79]\) is given by

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ \frac{1}{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) g^{\mu\nu} \partial_\mu \lambda \partial_\nu \lambda - \gamma_{\lambda\lambda}(\lambda, \sigma) g^{\mu\nu} \partial_\mu \lambda \partial_\nu \lambda - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - V(\lambda, \sigma) \right], \tag{87}
\]

and it can be expanded in the unitary gauge \( \delta \lambda = \lambda - \lambda_0 = 0 \) up to the third order fluctuation as

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \hat{\lambda}\sigma^2 \right. \\
+ \frac{1}{2} (\gamma_{\lambda\lambda})'_0 \sigma \left. - \frac{1}{2} \hat{\lambda}\sigma^2 (\gamma_{\lambda\lambda})'_0 \sigma^2 - \frac{1}{2} \hat{\lambda}\sigma^2 (\gamma_{\lambda\lambda})'_0 \sigma^2 \right. \\
- \frac{1}{4} \hat{\lambda}\sigma^2 (\gamma_{\lambda\lambda})'_0 \sigma^2 \left. - \frac{1}{4} \hat{\lambda}\sigma^2 (\gamma_{\lambda\lambda})'_0 \sigma^2 \right. \\
- \frac{1}{2} \lambda'_0 (\gamma_{\lambda\lambda})'_0 \sigma - \lambda'_0 (\gamma_{\lambda\lambda})'_0 \sigma \partial^0 \sigma - \frac{1}{2} \lambda'_0 (\gamma_{\lambda\lambda})'_0 \sigma^2 \partial^0 \sigma - \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma \\
- V_0 - V'_0 \sigma - \frac{1}{2} V''_0 \sigma^2 - \frac{1}{6} V'''_0 \sigma^3 \left. \right], \tag{88}
\]

where the equation of motion for \( \sigma \) implies \( V'_0 = \frac{1}{2} \hat{\lambda}\sigma^2 (\gamma_{\lambda\lambda})'_0 - \left( 3H \lambda'_0 (\gamma_{\lambda\lambda})_0 + \lambda'_0 (\gamma_{\lambda\lambda})_0 + \lambda'_0 \partial_1 (\gamma_{\lambda\lambda})_0 \right) \). In terms of the Hubble parameter \( H \), we can rewrite it as

\[
S_{\text{matter}} = \int d^4x \sqrt{-g} \left[ M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) g^{\mu\nu} \partial_\mu \hat{H} \partial_\nu \lambda - \sqrt{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) \partial^0 \sigma - (V'_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0) \sigma \right. \\
- \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{2} \left( V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0 \right) \sigma^2 - \sqrt{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) \sigma^0 \partial^0 \sigma + M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0 \sigma \sigma^0 \sigma \left. \right] \\
+ \frac{1}{2} \left( V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0 \right) \sigma^3 - \frac{\sqrt{2}}{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) \sigma^0 \sigma^0 \sigma + \frac{1}{2} M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0 \sigma^0 \sigma^0 \sigma, \tag{89}
\]

which is described in our framework as

\[
\alpha_1 = 1, \quad \alpha_3 = V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0, \quad \alpha_4 = -\sqrt{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) \sigma^0 \partial^0 \sigma, \quad \beta_1 = M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0, \\
\beta_3 = -\sqrt{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) \sigma^0 \partial^0 \sigma, \quad \gamma_1 = -\frac{1}{6} \left( V''_0 + M_P^2 \dot{H}(\gamma_{\lambda\lambda})''_0 \right), \quad \gamma_2 = -\frac{\sqrt{2}}{2} M_P^2 \gamma_{\lambda\lambda}(\lambda, \sigma) \sigma^0 \sigma^0 \sigma, \\
\tau_1 = \frac{1}{2} M_P^2 \dot{H}(\gamma_{\lambda\lambda})'_0, \quad (\text{others}) = 0. \tag{90}
\]

The above action apparently includes terms linear in \( \sigma \). However, it can be easily shown that such terms turn out to be more than second order after integration by parts. We notice that this basis corresponds to the third normalization of \( \sigma \) in section [III A].
IV. POWER SPECTRUM

In this section we calculate the power spectrum for a class of quasi-single field inflation models in our framework. In the following, we take the decoupling limit $\dot{\epsilon} \to 0$ and use the action for the Goldstone boson $\pi$. It is also assumed that the background trajectory satisfies $\epsilon \ll 1$. Up to the second order fluctuations, the action for $\pi$ and $\sigma$ takes the following form in the decoupling limit:

$$S = \int d^4x \left[ \frac{\alpha^2}{2} (\dot{\pi}^2 - c^2 (\partial_i \pi)^2) + \frac{\alpha^2}{2} (\dot{\sigma}^2 - c^2 (\partial_i \sigma)^2) - m^2 \sigma^2 \right] + \beta_1 \dot{\pi} \sigma + \beta_2 \dot{\pi} \dot{\sigma} + \beta_3 c^2 \sigma \partial_i \pi \partial_i \sigma ,$$  \hspace{1cm} (91)

where we have dropped sub-leading terms $\sim M^2_{Pl} \dot{H} \pi^2$ and $\sim \dot{H} \pi \sigma$ in the regime $\epsilon \ll 1$, and have defined

$$\alpha^2 = -\frac{2 M^2_{Pl} \dot{H}}{c^2}, \hspace{0.5cm} m^2 = \frac{\alpha^3 - 3 H \alpha_4 - \alpha_4}{\alpha^2}, \hspace{0.5cm} \beta_1 = -2 \beta_1 + \beta_3, \hspace{0.5cm} \beta_2 = 2 \beta_2 - \beta_3, \hspace{0.5cm} \beta_3 = c^{-2} \beta_3 .$$  \hspace{1cm} (92)

The corresponding Hamiltonian is given by

$$H = \int d^4x \left[ \frac{1}{2a^3} (\alpha^2 P^2_\pi + \alpha^2 P^2_\sigma) + \frac{a^3}{2} \left( \alpha^2 c^2 \pi (\partial_i \pi)^2 \right) \right] + \frac{a^3}{2} \left( \alpha^2 c^2 \sigma (\partial_i \sigma)^2 \right) + \frac{1}{2a^3} \alpha^4 \alpha^2 \beta^2 \beta^2 P^2_\pi + \ldots ,$$  \hspace{1cm} (93)

where $P_\pi$ and $P_\sigma$ are canonical momentum variables conjugate to $\pi$ and $\sigma$, respectively. In this paper, we assume that the mixing couplings can be treated as perturbations and calculate the power spectrum up to the second order in the mixing couplings $\beta_i$'s. The dots in (93) stand for term irrelevant to the power spectrum at this order, and therefore, we drop them in the following.

Let us then calculate the power spectrum using the in-in formalism. Before going to concrete models, we first introduce general expressions for the power spectrum. Choosing the free part $H_{\text{free}}$ and the interaction part $H_{\text{int}}$ of the Hamiltonian as

$$H = H_{\text{free}} + H_{\text{int}} ,$$  \hspace{1cm} (94)

$$H_{\text{free}} = \int d^4x H_{\text{free}} = \int d^3x \left[ \frac{1}{2a^3} (\alpha^2 P^2_\pi + \alpha^2 P^2_\sigma) + \frac{a^3}{2} \left( \alpha^2 c^2 \pi (\partial_i \pi)^2 \right) \right] ,$$  \hspace{1cm} (95)

$$H_{\text{int}} = \int d^4x H_{\text{int}} = \int d^3x \left[ -\alpha^2 \beta_1 P_\pi \sigma - a^{-3} \alpha^2 \alpha^2 \beta_2 P_\pi P_\sigma - \beta_3 a^2 \partial_i \pi \partial_i \sigma + \frac{1}{2a^3} \alpha^4 \alpha^2 \beta^2 \beta^2 P^2_\pi \right] ,$$  \hspace{1cm} (96)

the dynamics of canonical variables in the interaction picture are determined by

$$\dot{\pi} = \frac{\partial H_{\text{free}}}{\partial P_\pi} = a^{-3} \alpha^2 P_\pi , \hspace{0.5cm} \dot{\sigma} = \frac{\partial H_{\text{free}}}{\partial P_\sigma} = a^{-3} \alpha^2 P_\sigma ,$$  \hspace{1cm} (97)

$$\dot{P}_\pi = -\frac{\partial H_{\text{free}}}{\partial \pi} = a^2 c^2 a \partial_i \pi , \hspace{0.5cm} \dot{P}_\sigma = -\frac{\partial H_{\text{free}}}{\partial \sigma} = a^2 c^2 \partial_i \pi \partial_i \sigma .$$  \hspace{1cm} (98)

The fields $\pi$ and $\sigma$ are then expanded in the Fourier space as

$$\pi_k = u_k a_k + u_k^* a_k^\dagger , \hspace{0.5cm} \sigma_k = v_k b_k + v_k^* b_k^\dagger ,$$  \hspace{1cm} (99)

with the standard commutation relations

$$[a_k, a_k^\dagger] = (2\pi)^3 \delta^{(3)}(k - k') , \hspace{0.5cm} [b_k, b_k^\dagger] = (2\pi)^3 \delta^{(3)}(k - k') .$$  \hspace{1cm} (100)

Here the mode functions $u_k$ and $v_k$ satisfy the equations of motion in the free theory and depend on $k = |k|$: \hspace{1cm} (101)

$$\ddot{u}_k + H(3 - 2\epsilon + \dot{\eta})\dot{u}_k + \frac{c^2 k^2}{a^2} u_k = 0 , \hspace{0.5cm} \ddot{v}_k + H(3 + 2\delta_\alpha)\dot{v}_k + \left( m^2_\sigma + \frac{c^2 k^2}{a^2} \right) v_k = 0 \hspace{0.5cm} \text{with} \hspace{0.5cm} \delta_\alpha = \frac{\dot{\alpha}}{H \alpha} .$$  \hspace{1cm} (101)

Their normalization follows from

$$\alpha^2 a^3 (u_k \dot{u}_k^* - u_k^* \dot{u}_k) = 2M^2_{Pl} H^2 \epsilon a^3 (u_k \dot{u}_k^* - u_k^* \dot{u}_k) = i , \hspace{0.5cm} \alpha^2 a^3 (v_k \dot{v}_k^* - v_k^* \dot{v}_k) = i .$$  \hspace{1cm} (102)
Using these expressions, the Hamiltonian in the interaction picture can be written as

$$H_{\text{int}}(t) = \int \frac{d^3k}{(2\pi)^3} a^3(t) \left[ -\tilde{\beta}_1 \pi_k \sigma_k - \tilde{\beta}_2 \pi_k \sigma_k - \tilde{\beta}_3 c_\pi^2 \frac{k^2}{a^2} \pi_k \sigma_k + \frac{1}{2\alpha_\sigma} \tilde{\beta}_2^2 \pi_k \pi_k \right](t) . \tag{103}$$

Then, the expectation value of $\pi_k(t)\pi_{k'}(t)$ is calculated as

$$\langle \pi_k(t)\pi_{k'}(t) \rangle = \langle 0 | \left[ T \exp \left( -i \int_{t_0}^t dt' H_{\text{int}}(t') \right) \right] \pi_k(t)\pi_{k'}(t) \left[ T \exp \left( i \int_{t_0}^t dt' H_{\text{int}}(t') \right) \right] |0 \rangle$$

$$= \langle 0 | \pi_k(t)\pi_{k'}(t) |0 \rangle - 2\text{Re} \left[ \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \langle 0 | \pi_k(t)\pi_{k'}(t) H_{\text{int}}(t_1) |0 \rangle \right]$$

$$- 2\text{Re} \left[ \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \langle 0 | \pi_k(t)\pi_{k'}(t) H_{\text{int}}(t_1) H_{\text{int}}(t_2) |0 \rangle \right] + \ldots , \tag{104}$$

where the dots stand for the higher order terms in the couplings. In terms of the mode functions and the couplings, the general form of the two point function is given up to the leading order corrections from the mixing couplings by

$$\langle \pi_k(t)\pi_{k'}(t) \rangle = (2\pi)^3 \delta^{(3)}(k + k') u_k^*(t) u_k(t) \left[ 1 + C_1 + C_2 + C_3 \right] , \tag{105}$$

where $C_1$, $C_2$, and $C_3$ are defined by

$$C_1 = 2 \left| \int_{t_0}^t dt_1 a^3 \left[ \beta_1 \dot{u}_k v_k + \beta_2 \dot{u}_k \dot{v}_k + \beta_3 c_\pi^2 \frac{k^2}{a^2} u_k v_k \right](t_1) \right|^2 , \tag{106}$$

$$C_2 = -4\text{Re} \left[ \frac{u_k^2(t)}{|u_k(t)|^2} \int_{t_0}^t dt_1 a^3 \left[ \beta_1 \dot{u}_k v_k + \beta_2 \dot{u}_k \dot{v}_k + \beta_3 c_\pi^2 \frac{k^2}{a^2} u_k v_k \right](t_1) \right]$$

$$\times \left[ \int_{t_0}^{t_1} dt_2 a^3 \left[ \beta_1 \dot{u}_k^* v_k^* + \beta_2 \dot{u}_k^* \dot{v}_k^* + \beta_3 c_\pi^2 \frac{k^2}{a^2} u_k^* v_k^* \right](t_2) \right] , \tag{107}$$

$$C_3 = 2\text{Re} \left[ -i \frac{u_k^2(t)}{|u_k(t)|^2} \int_{t_0}^t dt_1 a^3 \alpha_{\sigma}^{-2} \beta_2^2 \dot{u}_k^2(t_1) \right] . \tag{108}$$

It is also convenient to rewrite $C = C_1 + C_2 + C_3$ as follows:

$$C = 4\text{Re} \left[ \int_{t_0}^t dt_1 a^3 \left[ (\beta_1 \dot{u}_k v_k + \beta_2 \dot{u}_k \dot{v}_k + \beta_3 c_\pi^2 \frac{k^2}{a^2} u_k v_k) \right] \right.$$

$$\times \left[ \int_{t_0}^{t_1} dt_2 a^3 \left[ \beta_1 \dot{u}_k^* v_k^* + \beta_2 \dot{u}_k^* \dot{v}_k^* + \beta_3 c_\pi^2 \frac{k^2}{a^2} u_k^* v_k^* \right](t_2) \right]$$

$$+ 2\text{Re} \left[ -i \frac{u_k^2(t)}{|u_k(t)|^2} \int_{t_0}^t dt_1 a^3 \alpha_{\sigma}^{-2} \beta_2^2 \dot{u}_k^2(t_1) \right] . \tag{109}$$

Since the scalar perturbation $\zeta$ is given at the linear order by $\zeta = -H\pi$, we obtain the expectation value of $\zeta_k(t)\zeta_{k'}(t)$ as

$$\langle \zeta_k(t)\zeta_{k'}(t) \rangle = (2\pi)^3 \delta^{(3)}(k + k') \frac{2\pi^2}{k^3} P_\zeta(k) , \tag{110}$$

where the power spectrum $P_\zeta(k)$ is given by

$$P_\zeta(k) = \frac{H^2(t)k^3}{2\pi^2} u_k^2(t) u_k(t) (1 + C) . \tag{111}$$

The factor $C$ can be considered as a deviation factor from single field inflation. Here it should be noticed that in the derivation of the above general expression we assumed only $\epsilon \ll 1$, $\dot{\epsilon} \ll 1$, and the perturbativity of the mixing.
ponents. In principle, we can calculate the power spectrum for any models satisfying those three conditions. Note that, as pointed out in \[23\], the perturbativity of mixing couplings is justified even in the case when they are large only in a sufficiently short period of time, which is realized for example by the sudden turning background trajectory \[14–17, 19, 20\]. In the rest of this section, we first perform the calculation concretely for the case when the time-dependence of mixing couplings is irrelevant (constant turning trajectory), and then we discuss the qualitative features of the case when the mixing couplings are large in a sufficiently short period of time (sudden turning trajectory).

A. Constant turning trajectory

In this subsection, the power spectrum is calculated in the case that the time-dependence of mixing couplings is irrelevant. To make the calculation tractable, we assume that the time-dependence of \(\epsilon, \tilde{\epsilon}, \alpha, \sigma, \) and \(m_{\sigma}\) is negligible and we use the de-Sitter approximation. In this approximation, the equations of motion \(101\) for the mode functions \(u_k\) and \(v_k\) can be written as

\[
\begin{align*}
u'' - & \frac{2}{\tau} \nu' + \frac{c^2}{\tau} k^2 \nu = 0, \quad \frac{H^2}{2} \tau^2 v_k = 0, \tag{112}
\end{align*}
\]

where the conformal time \(d\tau = a^{-1}dt\) is given by \(\tau = -1/(aH)\) in the de-Sitter approximation and the primes denote derivatives with respect to \(\tau\). The equations \(112\) can be solved as follows:

\[
\begin{align*}
u_k &= \frac{1}{2M_{\text{Pl}}^2(c_{\sigma}k)^{3/2}} (1 + ic_{\pi}k\tau) e^{-ic_{\pi}k\tau}, \quad \nu_k = \frac{1}{2M_{\text{Pl}}^2(c_{\pi}k)^{3/2}} (1 - ix)e^{ix}, \\
v_k &= -ie^{\frac{\pi}{2} + \frac{x}{2}} \frac{\sqrt{\pi H}}{2\alpha_{\sigma}} \left( -\tau \right)^{3/2} H_{\nu}^{(1)}(c_{\sigma}k\tau) = -ie^{\frac{\pi}{2} + \frac{x}{2}} \frac{\sqrt{\pi H}}{2\alpha_{\sigma}(c_{\pi}k)^{3/2}} \left( 3/2 - \nu \right) H_{\nu}^{(1)}(r_x x) + (r_x x) H_{\nu}^{(1)}(r_x x), \tag{114}
\end{align*}
\]

where \(x = -c_{\pi}k\tau, r_x = c_{\sigma}/c_{\pi}\), and we chose the Bunch-Davies vacuum for \(\pi\) and \(\sigma\). The function \(H_{\nu}^{(1)} = J_\nu + i Y_\nu\) is the Hankel function and \(\nu\) is defined as

\[
\nu = \sqrt{\frac{9}{4} - \frac{m_{\sigma}^2}{H^2}} \quad \text{for} \quad m_{\sigma} < \frac{3}{2} H, \quad \nu = i \sqrt{\frac{m_{\sigma}^2}{H^2} - \frac{9}{4}} \quad \text{for} \quad m_{\sigma} > \frac{3}{2} H. \tag{115}
\]

The time derivatives of \(u_k\) and \(v_k\) are given by

\[
\begin{align*}
\dot{u}_k &= -H \tau u_k' = -\frac{H}{2M_{\text{Pl}}^2(c_{\pi}k)^{3/2}} \frac{e^{ix}}{\tau^2}, \\
\dot{v}_k &= -H \tau v_k' = ie^{\frac{\pi}{2} + \frac{x}{2}} \frac{\sqrt{\pi H^2}}{2\alpha_{\sigma}(c_{\pi}k)^{3/2}} e^{3/2} \left( (3/2 - \nu) H_{\nu}^{(1)}(r_x x) + (r_x x) H_{\nu}^{(1)}(r_x x) \right), \tag{117}
\end{align*}
\]

where we used the identity \(z \partial_z H_{\nu}^{(1)} = z H_{\nu - 1}^{(1)} - \nu H_{\nu}^{(1)}\). Therefore, the factor \(C\) defined in \(109\) takes the form

\[
C = e^{\frac{\pi}{2} (\nu - \nu^*)} \frac{\pi^2}{4 \alpha_{\sigma}^2 M_{\text{Pl}}^2(-H)} \times \Re \left[ \int_0^\infty dx_1 \left[ \left( \frac{\beta_1}{H} + (\nu - 3/2)\beta_2 - \beta_3 \right) \left( A_1(x)^* - A_1(x_1) \right) - \beta_2 \left( A_2(x)^* - A_2(x_1) \right) + \beta_3 \left( A_3(x)^* - A_3(x_1) \right) \right] \right]
\]

\[
\times \int_0^\infty dx_2 \left[ \left( \frac{\tilde{\beta}_1}{H} + (\nu - 3/2)\tilde{\beta}_2 - \tilde{\beta}_3 \right) A_1(x_2) - \tilde{\beta}_2 A_2(x_2) + \tilde{\beta}_3 A_3(x_2) \right]
\]

\[
- \frac{c^2}{2M_{\text{Pl}}^2(-H)} \Re \left[ i \int_0^\infty dx \alpha_{\sigma}^2 \beta^2 \frac{e^{-2ix}}{2} \right], \tag{118}
\]

where we set \(t_0 = -\infty\) and \(t = \infty\), and we defined

\[
\begin{align*}
A_1(x) &= x^{-1/2} e^{ix} H_{\nu}^{(1)}(r_x x), \quad A_2(x) = r_x x^{1/2} e^{ix} H_{\nu - 1}^{(1)}(r_x x), \quad A_3(x) = i x^{1/2} e^{ix} H_{\nu}^{(1)}(r_x x), \\
\tilde{A}_1(x) &= x^{-1/2} e^{ix} H_{\nu}^{(2)}(r_x x), \quad \tilde{A}_2(x) = r_x x^{1/2} e^{ix} H_{\nu - 1}^{(2)}(r_x x), \quad \tilde{A}_3(x) = i x^{1/2} e^{ix} H_{\nu}^{(2)}(r_x x). \tag{119}
\end{align*}
\]
When the time-dependence of mixing couplings is irrelevant, mixing couplings $\tilde{\beta}$'s can be evaluated at the time of horizon-crossing $\tau = -(c_s k)^{-1}$. In such a case, $C$ is given by

$$C = \frac{c^2}{\alpha^2 M^2_{Pl}(-H)} \text{Re} \left[ \left| \tilde{\beta}_1/H + (\nu - 3/2)\tilde{\beta}_2 - \tilde{\beta}_3 \right|^2 I_{11} + (\tilde{\beta}_1/H + (\nu^* - 3/2)\tilde{\beta}_2 - \tilde{\beta}_3)(-\tilde{\beta}_2 I_{12} + \tilde{\beta}_3 I_{13}) \\
- \tilde{\beta}_2 \left( (\tilde{\beta}_1/H + (\nu - 3/2)\tilde{\beta}_2 - \tilde{\beta}_3) I_{21} - \tilde{\beta}_2 I_{22} + \tilde{\beta}_3 I_{23} \right) \\
+ \tilde{\beta}_3 \left( (\tilde{\beta}_1/H + (\nu - 3/2)\tilde{\beta}_2 - \tilde{\beta}_3) I_{31} - \tilde{\beta}_2 I_{32} + \tilde{\beta}_3 I_{33} \right) - \frac{1}{4} \tilde{\beta}_2^2 \right], \quad (120)$$

where $I_{ij}$'s are integrals defined by

$$I_{ij} = \frac{\pi}{4} e^{\pi(\nu-\nu^*)} \int_0^\infty dx_1 \left( A_i(x_1)^* - A_i(x_1) \right) \int_{x_1}^\infty dx_2 A_j(x_2). \quad (121)$$

The last term in (118) was calculated using the $i\epsilon$-prescription as

$$-\frac{c^2}{2M^2_{Pl}(-H)} \text{Re} \left[ \int_0^\infty dx \alpha^2 \tilde{\beta}_2^2 c^{-2ix} \right] = \frac{c^2}{4 \alpha^2 M^2_{Pl}(-H)} \tilde{\beta}_2^2 \text{Re} \left[ e^{-2ix} \right] = -\frac{c^2}{4 \alpha^2 M^2_{Pl}(-H)} \tilde{\beta}_2^2. \quad (122)$$

We then have

$$C = \frac{c^2}{\alpha^2 M^2_{Pl}(-H)} \left( \frac{\tilde{\beta}_1^2}{H^2} C_{11} + \frac{\tilde{\beta}_2^2}{H^2} C_{22} + \frac{\tilde{\beta}_3^2}{H^2} C_{33} + \frac{\tilde{\beta}_1}{H} \tilde{\beta}_2 C_{12} + \frac{\tilde{\beta}_1}{H} \tilde{\beta}_3 C_{13} + \frac{\tilde{\beta}_2}{H} \tilde{\beta}_3 C_{23} \right), \quad (123)$$

and $C_{ij}$'s are given by

$$C_{11} = \text{Re} \left[ I_{11} \right], \quad C_{22} = \text{Re} \left[ |\nu - 3/2|^2 I_{11} - (\nu^* - 3/2) I_{12} - (\nu - 3/2) I_{21} + I_{22} - \frac{1}{4} \right], \quad (124)$$

$$C_{33} = \text{Re} \left[ I_{11} + I_{33} - I_{13} - I_{31} \right], \quad C_{12} = \text{Re} \left[ (\nu + \nu^*) - 3 I_{11} - I_{12} - I_{21} \right], \quad C_{13} = \text{Re} \left[ -2 I_{11} + I_{13} + I_{31} \right], \quad (124)$$

$$C_{23} = \text{Re} \left[ -(\nu + \nu^*) - 3 I_{11} + I_{12} + I_{21} + (\nu^* - 3/2) I_{31} + (\nu - 3/2) I_{31} - I_{32} \right].$$

Here the explicit form of the power spectrum is

$$P_\zeta(k) = \frac{H^2}{8\pi^2 M^2_{Pl} c_s} \left[ 1 + \frac{c^2}{\alpha^2 M^2_{Pl}(-H)} \left( \frac{\tilde{\beta}_1^2}{H^2} C_{11} + \frac{\tilde{\beta}_2^2}{H^2} C_{22} + \frac{\tilde{\beta}_3^2}{H^2} C_{33} + \frac{\tilde{\beta}_1}{H} \tilde{\beta}_2 C_{12} + \frac{\tilde{\beta}_1}{H} \tilde{\beta}_3 C_{13} + \frac{\tilde{\beta}_2}{H} \tilde{\beta}_3 C_{23} \right) \right], \quad (125)$$

where $H$, $c_s$, $\alpha_s$, $\tilde{\beta}_1$, and $r_s$ are evaluated at the time of horizon-crossing. Then, the calculation of the power spectrum reduces to the evaluation of $\text{Re}[I_{ij} + I_{ji}]$, $\text{Im}[I_{12} - I_{21}]$, and $\text{Im}[I_{13} - I_{31}]$.

As is understood from the definition, $I_{ij}$'s and $C_{ij}$'s are functions of $m_\sigma$ and $r_s = c_s/c_\sigma$. For general value of $r_s$, it is difficult to perform the integrals analytically and we performed numerical calculations by contour deformations (see appendix B for details). For the special case $r_s = 1$, however, it is possible to perform the integrals $I_{ij}$'s analytically by extending the results in [22], and the results are summarized in appendix C. In such a way, $C_{ij}$'s are calculated and the obtained results are summarized in figure 1 and figure 2. We find that they monotonically decrease in $m_\sigma$ for fixed $r_s$, but they are not monotonic for fixed $m_\sigma$. As is discussed in section II D 2, the effects of mixing interactions appear in the form of $\tilde{\beta}_i^2/m_\sigma^2$ in the heavy mass limit and it is implied that $C_{ij} \sim 1/m_\sigma^2$ for large $m_\sigma$, which is consistent with our results in this subsection. Therefore, the power spectrum is not affected by heavy particles unless the mixing couplings are comparable to the mass of heavy particles [13 21].

**B. Qualitative features of sudden turning trajectory**

In this subsection, we discuss qualitative features of the case when the mixing couplings are large in a sufficiently short period of time (sudden turning trajectory). As a simplest example, let us first consider the case when the $\tilde{\beta}_1$ coupling is the only relevant mixing coupling and it is proportional to a delta function:

$$\tilde{\beta}_1 = \beta_s (t - t_s) = \beta_s a^{-1}(t_s) \delta(t - t_s), \quad \tilde{\beta}_2 = \tilde{\beta}_3 = 0, \quad (126)$$
where $t_*$ is the time of sudden turning, $\tau_*$ is the corresponding conformal time, and the mode $k_*$ crossing the horizon at $t = t_*$ is given by $k_* = -1/(c_\pi \tau_*)$. We assume that $\beta_*$ is small enough to be treated perturbatively. In this case, the deviation of the power spectrum from that of single field inflation can be written as

$$\mathcal{C} = 2\beta_*^2 \Re \left[ a^6 (\dot{u}_k - \dot{\bar{u}}_k) u_k v_k(t_*) \right],$$

which is related to the power spectrum by \ref{eq:power_spectrum}. For simplicity, suppose that time-dependence of $m_{\pi}$, $\alpha_\sigma$, $c_{\sigma}$, and $c_\pi$ is negligible at the time of sudden turning and the mode functions $u_k(t_*)$ and $v_k(t_*)$ are given by

$$u_k(t_*) = \frac{1}{2M_{\text{Pl}} \xi^{1/2}(c_\pi k)^{3/2}} (1 - ix_*) e^{i x_*},$$

$$v_k(t_*) = -i e^{\frac{\pi \nu + \frac{1}{2}}{2\alpha_\pi (c_\pi k)^{3/2}}} \sqrt{\pi H} \frac{\xi^{3/2} H^{(1)}(r_\pi x_*)}{2\alpha_\pi (c_\pi k)^{3/2} x_*^{3/2}}.$$

\footnote{In our calculation we have treated the mixing coupling $\beta_*$ as an interaction. In such an interaction picture, it is manifest that the deviation from single field inflation starts from the second order in $\beta_*$. On the other hand, it is also possible to treat the mixing coupling as a part of the kinetic and mass terms. In that picture, the commutation relations of creation and annihilation operators are affected by the mixing as well as the mode functions $u_k$ and $v_k$ are modified. However, in some literatures, these modifications are not taken into account adequately and the deviation from single field inflation is calculated to start from the first order in $\beta_*$.}
where \( x_* = k/k_* \) and parameters such as \( m_\sigma \) are evaluated at the time of sudden turning. Then, (127) is given by

\[
\begin{align*}
C = \beta_*^2 \frac{\pi}{4\alpha_*^2} \frac{\epsilon_*^2}{M_\epsilon^2} F_{\nu}(x_*) \quad \text{with} \quad F_{\nu}(x_*) = e^{\pi(\nu-\nu^*)} x_s^2 x_*^2 |H_\nu^{(2)}(r_s x_*)|^2.
\end{align*}
\]  

(130)

For \( k \ll k_* \) or \( x_* \ll 1 \), the asymptotic behavior of \( F_{\nu}(x_*) \) is

\[
F_{\nu}(x_*) \sim \begin{cases} 
\frac{4^{\nu}}{\pi^2} \Gamma(\nu)^2 r_s^{-2\nu} x_*^{3-2\nu} & \text{for } m_\sigma(t_*) < \frac{3}{2} H, \\
\frac{1}{\pi^2} \left| e^{\pi(\nu-\nu^*)} x_s^2 x_*^2 |H_\nu^{(2)}(r_s x_*)|^2 \right|^{\nu} x_*^3 & \text{for } m_\sigma(t_*) > \frac{3}{2} H,
\end{cases}
\]  

(131)

which is consistent to the intuition that modes outside the horizon at \( t = t_* \) are not much affected by the sudden turning. For \( k \gg k_* \) or \( x_* \gg 1 \), it reduces to

\[
F_{\nu}(x_*) \sim \frac{2}{\pi} r_s^{-1} \sin^2 x_* = \frac{1}{\pi} r_s^{-1} (1 - \cos 2x_*).
\]  

(132)

This kind of oscillating behavior was also found in [15, 17, 18]. The turning trajectory generically oscillates around the turning point and the mixing couplings at the turning point become more regular than delta functions. In such a case, it is expected that the oscillating behavior of short modes \( c_* k \geq 1/\tau_* \) begins damping at some scale characterized by the oscillation of the trajectory. Let us next confirm such a behavior explicitly for a concrete example with a finite
The second order action is given in (91) and (92), and three point vertices are given by \(C\) decoupling limit and assume that time dependence of \(\beta\) define \(F\) expect to find a similar behavior of \(C\) since it is difficult to calculate \(F\) the turning point. We consider the following \(\tilde{\beta}_1\) profile:

\[
\tilde{\beta}_1 = \begin{cases} 
\beta_\ast (-H \tau_\ast) (\Delta \tau)^{-1} & \text{for } \tau_\ast - \frac{\Delta \tau}{2} < \tau < \tau_\ast + \frac{\Delta \tau}{2}, \\
0 & \text{otherwise},
\end{cases}
\]

(133)

where we normalized \(\beta_\ast\) so that it reproduces the coupling \([126]\) in the limit \(\Delta \tau \to 0\). For this class of couplings, we define \(F_\nu\) by

\[
C = \beta^2_\ast \frac{\pi}{4 a^3} \frac{c^2_\pi}{M^2_{Pl}(-H)} F_\nu(x_\ast).
\]

(134)

Since it is difficult to calculate \(F_\nu\) analytically, we can confirm the expected damping behavior of \(F_\nu\) by numerical calculations and the results are given in figure 3. To summarize, \(C\) vanishes in the long mode limit \(c_\pi k \ll -1/\tau_\ast\), oscillates for short modes \(c_\pi k \gtrsim -1/\tau_\ast\), and damps at some scale characterized by the oscillating trajectory around the turning point.

It is straightforward to extend the above discussions to the case with non-vanishing \(\tilde{\beta}_2\) and \(\tilde{\beta}_3\). We generically expect to find a similar behavior of \(C\): vanishing in the long mode limit \(c_\pi k \ll -1/\tau_\ast\), oscillating for short modes \(c_\pi k \gtrsim -1/\tau_\ast\), and damping at some scale characterized by the oscillating trajectory around the turning point.

V. THREE POINT FUNCTIONS IN THE SQUEEZED LIMIT

In this section, we discuss the momentum dependence of three point functions in the squeezed limit. We take the decoupling limit and assume that time dependence of \(\alpha_\ast, \beta_\ast, \gamma_\ast\)'s, and \(H\) is negligible. Under these assumptions, the second order action is given in \([91]\) and \([92]\), and three point vertices are given by

\[
S^{(3)} = \int d^4x a^3 \left[ -(M^2_{Pl} \dot{H}(c_\pi^2 - 1) + \frac{4 M^4_{Pl}}{3}) \dot{\sigma}^3 + M^2_{Pl} \dot{H}(c_\pi^2 - 1) \dot{\pi} \frac{(\partial_i \pi)^2}{a^2} + (-\beta_1 + 4 \tau_6) \check{\sigma}^2 + (3 \beta_2 - 4 \tau_7) \check{\sigma} \dot{\sigma} 
- 2 \beta_3 \pi \sigma \frac{\partial_i \pi \sigma}{a^2} + \beta_1 \frac{(\partial_i \pi)^2}{a^2} \dot{\sigma} - \beta_2 \frac{\partial_i \pi^2}{a^2} \check{\sigma} - 2 \tau_4 \check{\sigma} \dot{\sigma} + (\alpha_4 \dot{\sigma} + 2 \tau_2) \check{\sigma} \dot{\sigma} 
+ \left( \alpha_2^2 \left( 1 - c_\pi^2 \right) - 2 \tau_3 + 2 \tau_5 \right) \check{\sigma} \dot{\sigma}^2 - 2 \tau_5 \check{\sigma} \frac{(\partial_i \sigma)^2}{a^2} + \alpha_3 \dot{\sigma} \frac{\partial_i \sigma \partial_i \sigma}{a^2} \dot{\sigma} - \alpha_2^2 \left( 1 - c_\pi^2 \right) \frac{(\partial_i \pi \partial_i \sigma)^2}{a^2} \dot{\sigma} - 2 \tau_4 \check{\sigma} \dot{\sigma} 
+ \gamma_1 \sigma^3 - \gamma_2 \sigma^2 \dot{\sigma} + (\gamma_3 - \gamma_5) \sigma \dot{\sigma}^2 + \gamma_5 \frac{(\dot{\partial}_i \sigma)^2}{a^2} + (-\gamma_4 + \gamma_6) \sigma^3 - \gamma_6 \frac{(\partial_i \sigma)^2}{a^2} \right].
\]

(135)
For the calculation of three point functions using the in-in formalism, it is necessary to obtain the Hamiltonian description of the system. The relevant contribution arises from the three point vertex. Then, the momentum dependence of the first term in (136) is given by

\[ \langle \pi_{k_1}(t) \pi_{k_2}(t) \pi_{k_3}(t) \rangle = \frac{\bar{\beta} \bar{g}_0 \gamma(2\pi)^3S^{(3)}(k_1 + k_2 + k_3)}{M_{\text{Pl}}^3} \text{Re} \left[ \int_{-\infty}^{\infty} dt_1 a^3 \dot{u}_{k_1}(t_1) \dot{u}_{k_2}(t_1) \right. \]

\[ \times \left\{ v_{k_3}^*(t_1) \int_{-\infty}^{\infty} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) - v_{k_3}^*(t_1) \int_{t_1}^{\infty} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) - v_{k_3}(t_1) \int_{-\infty}^{t_1} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) \right\} \]

\[ + (2 \text{ permutations}). \quad (136) \]

In the following, we discuss what kind of momentum dependence in the squeezed limit appears from the above three point vertices.\(^\text{10}\)

As an example, let us start from the case when the mixing coupling \( \bar{\beta} \dot{\pi} \sigma \) and the three point vertex \( 4\bar{g}_0 \dot{\pi}^2 \sigma \) are relevant. In this case, the three point function of \( \pi \) takes the form

\[ \langle \pi_{k_1}(t) \pi_{k_2}(t) \pi_{k_3}(t) \rangle \]

\[ \equiv \frac{\bar{\beta} \bar{g}_0 \gamma(2\pi)^3S^{(3)}(k_1 + k_2 + k_3)}{M_{\text{Pl}}^3} \text{Re} \left[ \int_{-\infty}^{\infty} dt_1 a^3 \dot{u}_{k_1}(t_1) \dot{u}_{k_2}(t_1) \right. \]

\[ \times \left\{ v_{k_3}^*(t_1) \int_{-\infty}^{\infty} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) - v_{k_3}^*(t_1) \int_{t_1}^{\infty} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) - v_{k_3}(t_1) \int_{-\infty}^{t_1} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) \right\} \]

\[ + (2 \text{ permutations}). \quad (136) \]

Here there are three terms in the curly brackets, and they correspond to the Feynman diagrams in figure 4, respectively. In the squeezed limit, \( k_1 = k_2 = k \) and \( k_3/k = \kappa \ll 1 \), the long mode \( k_3 \) crosses the horizon much earlier than the short modes \( k_1 \) and \( k_2 \). Since the interactions are considered to be relevant around the horizon, it is expected that the relevant contribution arises from \((\tau_1, \tau_2) \sim (-1/k_1, -1/k_3)\), and therefore, the middle term in the curly brackets becomes irrelevant in the squeezed limit. In fact, we can confirm this expectation explicitly from the expression (136), and the integrals in the curly brackets can be written at the leading order in \( \kappa \) as

\[ \int_{-\infty}^{\infty} dt_1 a^3 \dot{u}_{k_1}^*(t_1) \dot{u}_{k_2}(t_1) 2i \text{ Im} \left[ v_{k_3}^*(t_1) \int_{-\infty}^{\infty} dt_2 a^3 \dot{u}_{k_3}(t_2) v_{k_3}(t_2) \right]. \quad (137) \]

We notice that the \( t_2 \)-integral is \( k_3 \)-independent and \( k_3 \)-dependence of the total integral appears only via \( v_{k_3}^*(t_1) \) originated from the three-point vertex. Then, the momentum dependence of the first term in (136) is given by

\[ \text{the first term in (136) } \propto \begin{cases} \kappa^{-3/2} & \text{for } m_\sigma < \frac{3}{2} H, \\ \kappa^{-3/2} k^{-6} \sin[i\nu \log \kappa + \delta_\nu] & \text{for } m_\sigma > \frac{3}{2} H, \end{cases} \quad (138) \]

where \( \delta_\nu \) is a \( \nu \)-dependent phase factor. We note that the only information necessary to derive (138) was the fact that the field of momentum \( k_3 \) in the three point vertex takes the form \( \sigma \). For the other two permutation terms in (136), the \( k_3 \)-dependence of the integral is determined by \( u_{k_3} \) originated from the three point vertex, and their momentum dependence is given by

\[ \text{the other two permutation terms in (136) } \propto \kappa^{-1} k^{-6}. \quad (139) \]

\(^\text{10}\) For the calculation of three point functions using the in-in formalism, it is necessary to obtain the Hamiltonian description of the system. Since the \( \pi \sigma \sigma \) coupling contains the second order derivative of \( \pi \), careful discussions are required when it is relevant in the action (135) and calculate using the Hamiltonian formalism in the interaction picture. In this paper, we do not consider such a situation for simplicity and concentrate on other cubic couplings. It should be also noted that the form of interactions in the Hamiltonian formalism in the interaction picture does not coincide with minus that in the Lagrangian formalism because the action contains derivative interactions. Correspondingly, the coefficients of cubic couplings in the Hamiltonian are changed from minus those in the action (135).
Then, the first term dominates for small $\kappa$. Therefore, when the mixing coupling $\tilde{\beta}_1$ and three point coupling $T_6$ are relevant, the momentum dependence of scalar three point functions in the squeezed limit is given by

$$
\lim_{k_3/k_1=k_3/k_2=\kappa \to 0} \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3} \rangle \propto \begin{cases} 
\kappa^{-3/2-\nu} k^{-6} & \text{for } m_\sigma < \frac{3}{2} H, \\
\kappa^{-3/2-\nu} k^{-6} \sin[i\nu \log \kappa + \delta_\nu] & \text{for } m_\sigma > \frac{3}{2} H,
\end{cases}
$$

(140)

It is straightforward to extend the above discussion for more general cases. First, for general $\tilde{\beta}_1$'s, we can show that when the mixing couplings convert $\pi$ of momentum $k_3 = \kappa k$ to $\sigma$, the $i_2$-integral becomes $\kappa$-independent in the limit $\kappa \ll 1$ and the three point vertex determines the $\kappa$-dependence of the diagram. Second, the only information necessary to obtain the momentum dependence is the form of the field of momentum $k_3$ in the vertex. In the previous examples, when it takes the form $\sigma$ in the vertex the diagram was proportional to $\kappa^{-3/2-\nu}$ or $\kappa^{-3/2} \sin[i\nu \log \kappa + \delta_\nu]$, and it was proportional to $\kappa^{-1}$ for $\tilde{\pi}$. More generally, we can obtain the relations in Table I between momentum dependence of the diagram and the form of the field of momentum $k_3$ in the three point vertex. Here it should be noted that the $\nu$-dependent phase factor $\delta_\nu$ depends on the details of mixing couplings. Finally, as discussed in the previous example, momentum dependence of the contribution from each vertex is now identified for $\kappa \ll 1$ so that it is straightforward to obtain momentum dependence of the contribution to scalar three point functions from each three point vertex displayed in Table I. The results are summarized in Table II. Here note that although the contribution from the $\tilde{\pi} \frac{\partial^2 \pi}{a^2}$ vertex seems to be proportional to $\kappa^{-2} k^{-6}$ apparently, explicit calculations show that this kind of leading contribution vanishes and the three point functions begin with terms proportional to $\kappa^{-1} k^{-6}$.

As we have seen, the momentum dependence of scalar three point functions in the squeezed limit has robust information about mass of $\sigma$ and three point vertices.

### VI. SUMMARY AND DISCUSSION

In this paper we discussed quasi-single field inflation using the effective field theory approach. We first constructed the most generic action in the unitary gauge based on the unbroken time-dependent spatial diffeomorphism, and then constructed the action for the Goldstone boson by the St"uckelberg method. Its decoupling regime was also discussed carefully, and the action in the decoupling regime implies that non-trivial cubic interactions generically appear and non-negligible non-Gaussianities can arise when the sound speed $c_\sigma$ of $\sigma$ is small, $\alpha_4$ is non-zero, or mixing couplings $\tilde{\beta}_1$ and $\tilde{\beta}_2$ exist as well as the sound speed $c_\pi$ of $\pi$ is small. Using the obtained action, two classes of concrete models were discussed: the constant turning trajectory and the sudden turning trajectory.

In the constant turning case, we first calculated the power spectrum of scalar perturbations numerically for general values of $r_s = c_\sigma/c_\pi$ and analytically for the special case $r_s = 1$. We then discussed the momentum dependence of scalar three point functions in the squeezed limit for general settings of quasi-single field inflation. It was shown that
the momentum dependence is determined only from the cubic interactions and the cubic interactions were classified into five classes. The three point functions in the squeezed limit take the intermediate shapes between local and equilateral types when the mixing couplings are relevant, and this kind of momentum dependence characterizes quasi-single field inflation. Recently in [33], the detectability of such a momentum dependence was discussed for some cases. It would be interesting to discuss the detectability of the momentum dependence in the form of $\kappa^{-3/2} \sin[i \nu \log \kappa + \delta_\nu]$, which arises in the second class with $m_\sigma > \frac{3}{2}H$. It is also important to calculate the full bi-spectrum for general settings of quasi-single field inflation.

In the sudden turning case, we made a qualitative discussion of the power spectrum. It was found that the deviation factor $C$ from single field inflation vanishes for long modes $c_\pi k \ll -\frac{1}{\tau^*}$, oscillates for short modes $c_\pi k \gg -\frac{1}{\tau^*}$, and damps at some scale characterized by the oscillating trajectory around the turning point. Since our framework makes the contributions from the mixing couplings clear, it would be useful to discuss more on the sudden turning trajectory.

Our framework can be considered as a starting point for systematic discussions on multiple field models, and there would be a lot of applications such as those mentioned above. We hope to report our progress in these directions elsewhere.

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Appendix A: Integrating out heavy fields

In section III D 2, we discussed effects of heavy fields by the following procedure:

1. Drop the kinetic term of heavy fields.
2. Eliminate derivatives of heavy fields by partial integrals.
3. Complete square the Lagrangian and integrate out heavy fields.
In this appendix we make some comments and careful discussions on the first and second steps of this procedure.\footnote{See also [21] for related discussions.}

1. Role of kinetic term

We first discuss the procedure to integrate out heavy fields using the following simple model:

\[
S = \int dt \left[ \frac{1}{2} \left( \dot{\sigma}^2 - m^2 \sigma^2 \right) + \sigma f[\phi_i(t); t] \right],
\]

(A1)

where \( f[\phi_i(t); t] \) is a function of light fields \( \phi_i \)'s and time \( t \). Before discussing the cosmological perturbation, let us recall the case when we calculate correlation functions in the momentum space and the total energy of the system.

Integrating out the kinetic term because it can be written as \( \frac{1}{2} \dot{\sigma}^2 \sim \frac{1}{2} E^2 \sigma^2 \ll \frac{1}{2} m^2 \sigma^2 \). We therefore drop the kinetic term of the heavy field \( \sigma \) and obtain

\[
S = \int dt \left[ -\frac{m^2}{2} \sigma + \sigma f[\phi_i(t); t] \right] = \int dt \left[ -\frac{m^2}{2} \left( \sigma - \frac{1}{m^2} f[\phi_i(t); t] \right) + \frac{1}{2} m^2 \left( f[\phi_i(t); t] \right)^2 \right],
\]

(A2)

which reduces to the following effective action after integrating out \( \sigma \):

\[
S_{\text{eff}} = \frac{1}{2m^2} \int dt \left( f[\phi_i(t); t] \right)^2.
\]

(A3)

In this way, the first step of the procedure is justified in the momentum space when the momentum conservation holds.

In the cosmological perturbation, we calculate correlation functions in the real time coordinate. Furthermore, the time translation is broken by the time-dependent background. As an illustrative toy model for the cosmological perturbations, let us next consider to calculate correlation functions of the model (A1) in the real time coordinate space. In such a case, \( \dot{\sigma} \) may seem to behave as \( \dot{\sigma} \sim m \sigma \) because the heavy field \( \sigma \) oscillates like \( \sigma \sim e^{i m t} \) on shell: it may not be so obvious whether the kinetic term of \( \sigma \) can be neglected. To clarify this point, let us make a careful discussion without neglecting the kinetic term of \( \sigma \). We first rewrite the kinetic term and the mass term as

\[
\frac{1}{2} \int dt \left( \dot{\sigma}^2 - m^2 \sigma^2 \right) = -\frac{1}{2} \int dt_1 \int dt_2 K(t_1, t_2) \sigma(t_1) \sigma(t_2) \quad \text{with} \quad K(t_1, t_2) = m^2 \delta(t_1 - t_2) + \delta''(t_1 - t_2),
\]

(A4)

and introduce the inverse \( P(t_1, t_2) \) of the kinetic operator \( K(t_1, t_2) \) satisfying

\[
\int dt P(t_1, t) K(t, t_2) = \int dt K(t_1, t) P(t, t_2) = \delta(t_1 - t_2).
\]

(A5)

We then rewrite the action as

\[
S = -\frac{1}{2} \int dt_1 dt_2 K(t_1, t_2) \left( \sigma(t_1) - \int dt_1' P(t_1', t_1) f[\phi_i(t_1'); t_1'] \right) \left( \sigma(t_2) - \int dt_2' P(t_2, t_2') f[\phi_i(t_2'); t_2'] \right)
\]

\[
+ \frac{1}{2} \int dt_1 dt_2 P(t_1, t_2) f[\phi_i(t_1); t_1] f[\phi_i(t_2); t_2].
\]

(A6)

Integrating out \( \sigma \), the following effective action is obtained:

\[
S_{\text{eff}} = \frac{1}{2} \int dt_1 dt_2 P(t_1, t_2) f[\phi_i(t_1); t_1] f[\phi_i(t_2); t_2].
\]

(A7)
Here note that no approximations are used so far. Let us then consider the property of $\mathcal{P}(t_1, t_2)$ when $\sigma$ is heavy. Since the condition (A5) can be rephrased as

$$\partial_i^2 \mathcal{P}(t_1, t_2) + m^2 \mathcal{P}(t_1, t_2) = \partial_i^2 \mathcal{P}(t_1, t_2) + m^2 \mathcal{P}(t_1, t_2) = \delta(t_1 - t_2),$$  \hfill (A8)

$\mathcal{P}(t_1, t_2)$ can be expanded in $1/m^2$ as

$$\mathcal{P}(t_1, t_2) = \frac{1}{m^2} \delta(t_1 - t_2) - \frac{1}{m^2} \partial_i^2 \mathcal{P}(t_1, t_2)$$

$$= \frac{1}{m^2} \delta(t_1 - t_2) - \frac{1}{m^4} \partial''(t_1 - t_2) + \frac{1}{m^4} \partial_i^4 \mathcal{P}(t_1, t_2)$$

$$= \ldots$$

$$= \frac{1}{m^2} \delta(t_1 - t_2) + \frac{1}{m^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \delta^{(2n)}(t_1 - t_2).$$  \hfill (A9)

Using this expression, the effective action (A7) can be written as

$$S_{\text{eff}} = \frac{1}{2m^2} \int dt \left[ \left( f[\phi_i(t); t] \right)^2 + \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \frac{d^n}{dt^n} f[\phi_i(t); t] \right].$$  \hfill (A10)

Therefore, if the time-dependence of $f[\phi_i(t); t]$ is negligible compared to the mass $m$ of $\sigma$, or in other words, if the fields $\phi_i$'s are light and the explicit time-dependence of $f[\phi_i(t); t]$ is irrelevant compared to $m$, the second term in (A10) is negligible and the effective action reduces to

$$S_{\text{eff}} = \frac{1}{2m^2} \int dt \left( f[\phi_i(t); t] \right)^2,$$  \hfill (A11)

which coincides with the result (A3) obtained by dropping the kinetic term of $\sigma$.

To summarize, the kinetic term $\frac{1}{2} \dot{\sigma}^2$ can be neglected when the time-dependence of $f[\phi_i(t); t]$ is negligible compared to the mass $m$ of the heavy field $\sigma$. It would be notable that, when we neglect the kinetic term $\frac{1}{2} \dot{\sigma}^2$, the kinetic operator $K(t_1, t_2)$ takes the form $K(t_1, t_2) = m^2 \delta(t_1 - t_2)$ and its inverse $\mathcal{P}(t_1, t_2)$ is given by

$$\mathcal{P}(t_1, t_2) = \frac{1}{m^2} \delta(t_1 - t_2),$$  \hfill (A12)

which coincides with the first term in (A9). Therefore, it can be considered that the delta-function like behavior of $\mathcal{P}(t_1, t_2)$ originates from the mass term and the kinetic term plays a role to regularize the singular behavior by reproducing the second term in (A9).

2. Derivative coupling, partial integral, and Hamiltonian formalism

Let us next consider the second step of the procedure using the following two actions:

$$S = \int dt \left[ \frac{1}{2} \left( \dot{\sigma}^2 - m^2 \sigma^2 \right) + \dot{\sigma} g[\phi_i(t); t] \right],$$  \hfill (A13)

$$S' = \int dt \left[ \frac{1}{2} \left( \dot{\sigma}^2 - m^2 \sigma^2 \right) - \sigma \frac{d}{dt} g[\phi_i(t); t] \right],$$  \hfill (A14)

where $g[\phi_i(t); t]$ is a function of light fields $\phi_i$'s and time $t$. Since (A13) and (A14) are related to each other by partial integrals, they are expected to describe the same dynamics. In particular, they are expected to reproduce the same effective theory in the low energy regime. However, it may be wondered that the mixing term in (A13) becomes relevant when $\sigma$ is heavy because $\dot{\sigma} \sim m \sigma$ on shell and that the low energy dynamics can be different from those of (A14). In this subsection we would like to clarify this point and show that (A13) and (A14) describe the same dynamics as is expected from the partial integral perspective.
Similarly to the previous discussions, the action \( A_{13} \) can be written in terms of \( \mathcal{K}(t_1, t_2) \) and \( \mathcal{P}(t_1, t_2) \) as

\[
S = -\frac{1}{2} \int dt_1 dt_2 \mathcal{K}(t_1, t_2) \left( \sigma(t_1) - \int dt_1' \left[ \partial_{t_1'} \mathcal{P}(t_1', t_1) g[\phi(t_1'); t_1'] \right] \right) \left( \sigma(t_2) - \int dt_2' \left[ \partial_{t_2'} \mathcal{P}(t_2', t_2) g[\phi(t_2'); t_2'] \right] \right)
+ \frac{1}{2} \int dt_1 dt_2 \left[ \partial_{t_1} \partial_{t_2} \mathcal{P}(t_1, t_2) \right] g[\phi_i(t_1); t_1] g[\phi_i(t_2); t_2],
\]

and we obtain the following effective action after integrating out \( \sigma \):

\[
S_{\text{eff}} = \frac{1}{2} \int dt_1 dt_2 \left[ \partial_{t_1} \partial_{t_2} \mathcal{P}(t_1, t_2) \right] g[\phi_i(t_1); t_1] g[\phi_i(t_2); t_2].
\]

It follows from the expression \( A_{10} \) of \( \mathcal{P}(t_1, t_2) \) that

\[
\partial_{t_1} \partial_{t_2} \mathcal{P}(t_1, t_2) = \partial_{t_1} \partial_{t_2} \left[ \frac{1}{m^2} \delta(t_1 - t_2) + \frac{1}{m^2} \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \delta(2n)(t_1 - t_2) \right]
= \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \delta(2n)(t_1 - t_2),
\]

and therefore, the effective action \( A_{16} \) can be written as

\[
S_{\text{eff}} = \frac{1}{2} \int dt \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} g[t, \phi_i(t)] \frac{d^2}{dt^2} g[\phi_i(t); t]
= \frac{1}{2m^2} \int dt \left[ \left( \frac{d}{dt} g[\phi_i(t); t] \right)^2 + \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \left( \frac{d}{dt} g[\phi_i(t); t] \right) \frac{d^2}{dt^2} \left( \frac{d}{dt} g[\phi_i(t); t] \right) \right],
\]

which is nothing but the effective action for \( A_{16} \), obtained by applying the previous result \( A_{10} \). We therefore conclude that the actions \( A_{13} \) and \( A_{14} \) describe the same dynamics.

Then, what was wrong in the naive expectation that the mixing term in \( A_{13} \) becomes relevant for heavy \( \sigma \)? To answer this question, it may be instructive to reconsider the above discussion using the following concrete form of \( \mathcal{P}(t_1, t_2) \):

\[
\mathcal{P}(t_1, t_2) = \frac{1}{2im} \left[ \theta(t_1 - t_2) e^{im(t_1 - t_2)} + \theta(t_2 - t_1) e^{-im(t_1 - t_2)} \right],
\]

which is essentially the same as the Feynman propagator. The property of the effective action \( A_{16} \) is determined by \( \partial_{t_1} \partial_{t_2} \mathcal{P}(t_1, t_2) \) and it is given for the choice \( A_{10} \) by

\[
\partial_{t_1} \partial_{t_2} \mathcal{P}(t_1, t_2) = m^2 \mathcal{P}(t_1, t_2) - \delta(t_1 - t_2).
\]

Here the first term is obtained by taking derivatives of \( e^{\pm imt} \) originated from mode functions of \( \sigma \) and the factor \( m^2 \) is the expected one from the observation that \( \dot{\sigma} \sim m\sigma \). An important point is that we also have the second term obtained by taking derivatives of both of step functions and \( e^{\pm imt} \). Because of this second term, the leading order term in the \( 1/m^2 \) expansions of \( m^2 \mathcal{P}(t_1, t_2) \) is canceled out as

\[
\partial_{t_1} \partial_{t_2} \mathcal{P}(t_1, t_2) = m^2 \frac{1}{m^2} \left[ \delta(t_1 - t_2) + \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \delta(2n)(t_1 - t_2) \right] - \delta(t_1 - t_2)
= \sum_{n=1}^{\infty} \frac{(-1)^n}{m^{2n}} \delta(2n)(t_1 - t_2),
\]

and the naive expectation that the interaction is enhanced by the mass of \( \sigma \) turns out to be wrong. The lesson is that it is important to take into account derivatives of step functions in the Feynman propagator appropriately when we discuss derivative interactions: the mass factor naively expected from time derivatives of massive fields can be canceled out.
It would be also notable that similar situations appear in the Hamiltonian formalism in the interaction picture. Let us consider the Hamiltonian system corresponding to (A.13). For simplicity, suppose that \( g[\phi_i(t); t] \) does not depend on time derivatives of \( \phi_i \)'s. Then, the momentum conjugate \( P_\sigma \) of \( \sigma \) and the Hamiltonian \( H \) are given by

\[
P_\sigma = \dot{\sigma} + g[\phi_i(t); t], \quad H = \dot{\sigma} P_\sigma - \left[ \frac{1}{2} \dot{\sigma}^2 - \frac{1}{2} m^2 \sigma^2 + \delta g[\phi_i(t); t] \right] = \frac{1}{2} P_\sigma^2 + \frac{1}{2} m^2 \sigma^2 - P_\sigma g[\phi_i(t); t] + \frac{1}{2} \left( g[\phi_i(t); t] \right)^2. \tag{A.23}
\]

Choosing the free part \( H_{\text{free}} \) and the interaction part \( H_{\text{int}} \) of the Hamiltonian as

\[
H_{\text{free}} = \frac{1}{2} P_\sigma^2 + \frac{1}{2} m^2 \sigma^2, \tag{A.24}
\]

\[
H_{\text{int}} = -P_\sigma g[\phi_i(t); t] + \frac{1}{2} \left( g[\phi_i(t); t] \right)^2, \tag{A.25}
\]

the dynamics of canonical variables in the interaction picture are determined by

\[
\dot{\sigma} = \frac{\partial H_{\text{free}}}{\partial P_\sigma} = P_\sigma, \quad \dot{P}_\sigma = -\frac{\partial H_{\text{free}}}{\partial \sigma} = -m \sigma, \tag{A.26}
\]

and we have

\[
H_{\text{free}} = \frac{1}{2} \dot{\sigma}^2 + \frac{1}{2} m^2 \sigma^2, \tag{A.27}
\]

\[
H_{\text{int}} = -\dot{\sigma} g[\phi_i(t); t] + \frac{1}{2} \left( g[\phi_i(t); t] \right)^2. \tag{A.28}
\]

Note that the first term in (A.28) corresponds to the interaction part of the Lagrangian and \( H_{\text{int}} \) has the additional second term \( \frac{1}{2} \left( g[\phi_i(t); t] \right)^2 \) as is usual in systems with derivative interactions. Since \( \dot{\sigma} \sim m \sigma \), the first term in \( (A.28) \) is enhanced by the mass \( m \) of \( \sigma \) and it may be wondered that the interaction of the system becomes relevant when \( \sigma \) is heavy. However, it turns out that the enhancement is canceled out by the second term \( \frac{1}{2} \left( g[\phi_i(t); t] \right)^2 \) just as the second term in (A.20) cancels out the leading order in the \( 1/m^2 \) expansions of the first term in (A.20): the additional second term in (A.28) plays an important role.

3. Extension to cosmological perturbation

The above discussions can be extended to the cosmological perturbation straightforwardly. Let us consider the following action:

\[
\int d^4 x \sqrt{-g} \left[ -\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{m^2}{2} \sigma^2 + \sigma f[\phi_i(x); x] \right]. \tag{A.29}
\]

The kinetic term and the mass term of \( \sigma \) can be written as

\[
S_{\text{free}} = -\frac{1}{2} \int d^4 x_1 \sqrt{-g(x_1)} \int d^4 x_2 \sqrt{-g(x_2)} \sigma(x_1) \mathcal{K}(x_1, x_2) \sigma(x_2) \quad \text{with} \quad \mathcal{K}(x_1, x_2) = (m^2 - \Box) \frac{\delta^4(x_1 - x_2)}{\sqrt{-g}}, \tag{A.30}
\]

where \( \Box = g^{\mu\nu} \nabla_\mu \nabla_\nu \) is the covariant d’Alembertian operator and \( \nabla_\mu \) is the covariant derivative. Introducing the inverse \( \mathcal{P}(x_1, x_2) \) of \( \mathcal{K}(x_1, x_2) \),

\[
\int d^4 x \sqrt{-g} \mathcal{K}(x_1, x) \mathcal{P}(x, x_2) = \int d^4 x \sqrt{-g} \mathcal{P}(x_1, x) \mathcal{K}(x, x_2) = \frac{\delta^4(x_1 - x_2)}{\sqrt{-g}}, \tag{A.31}
\]

the following effective action is obtained after integrating out \( \sigma \):\(^{12}\)

\[
S_{\text{eff}} = \frac{1}{2} \int d^4 x_1 \sqrt{-g} \int d^4 x_2 \sqrt{-g} \mathcal{P}(x_1, x_2) f[\phi_i(x_1); x_1] f[\phi_i(x_2); x_2]. \tag{A.32}
\]

\(^{12}\) Since the kinetic operator \( \mathcal{K}(x_1, x_2) \) contains metric perturbations, non-trivial log det \( \mathcal{K} \) contributions arise from the Gaussian path integrals of \( \sigma \). However, we do not consider these contributions for simplicity because they do not appear as long as tree-level perturbations are discussed around a given background spacetime.
which diverges for \( \nu > \) in \( I \) because of the oscillating behavior of the mode functions \( x_i \). Each integral diverges for some parameter region. However, we can show that such divergences cancel out in the cosmological perturbation around FRW backgrounds, the kinetic term of \( \sigma \) can be rephrased as

\[
(m^2 - \Box)P(x_1, x_2) = (m^2 - \Box_2)P(x_1, x_2) = \frac{\delta^4(x_1 - x_2)}{\sqrt{-g}},
\]

and \( P(x_1, x_2) \) can be expanded in \( 1/m^2 \) as

\[
P(x_1, x_2) = \frac{1}{m^2} \frac{\delta^4(x_1 - x_2)}{\sqrt{-g}} + \frac{\Box_1}{m^2} P(x_1, x_2)
\]

\[
= \ldots
\]

\[
= \frac{1}{m^2} \frac{\delta^4(x_1 - x_2)}{\sqrt{-g}} + \frac{1}{m^2} \sum_{n=1}^{\infty} \left( \frac{\Box_1}{m^2} \right)^n \frac{\delta^4(x_1 - x_2)}{\sqrt{-g}}.
\]

Then, the effective action can be written as

\[
S_{\text{eff}} = \frac{1}{2m^2} \int d^4x \sqrt{-g} \left[ \left( f[\phi_i(x); x] \right)^2 + \sum_{n=1}^{\infty} f[\phi_i(x); x] \left( \frac{\Box_1}{m^2} \right)^n f[\phi_i(x); x] \right],
\]

which reduces to the following form when the spacetime-dependence of \( f[\phi_i(x); x] \) is negligible compared to the mass \( m \) of \( \sigma \):

\[
S_{\text{eff}} = \frac{1}{2m^2} \int d^4x \sqrt{-g} (f[\phi_i(x); x])^2.
\]

Therefore, in the cosmological perturbation around FRW backgrounds, the kinetic term of \( \sigma \) can be neglected when the mass \( m \) of \( \sigma \) is much larger than the Hubble parameter \( H \) and the mass \( m_i \) of other fields \( \phi_i \): \( m \gg H, m_i \).

**Appendix B: Numerical calculations of power spectrum**

To perform the integral \([121]\) numerically, there are two technical obstacles \([22, 23]\): spurious divergences at \( x = 0 \) and oscillating behaviors at \( x = \infty \).

The integral \([121]\) contains two integrals, \( \int_0^\infty dx_1 A_i(x_1)^* \int_0^\infty dx_2 A_j(x_2) \) and \( \int_0^\infty dx_1 \tilde{A}_i(x_1) \int_0^\infty dx_2 A_j(x_2) \), and each integral diverges for some parameter region. However, we can show that such divergences cancel out in the calculation of \( C_{ij} \)'s. For example, let us consider \( C_{11} \) for real \( 0 < \nu < 3/2 \). The asymptotic behavior of each integral in \( I_{11} \) around \( x_1 = x_2 = 0 \) is given by

\[
\sim \int_0 dx_1 x_1^{-1/2 - \nu} \int_{x_1} dx_2 x_2^{-1/2 - \nu} \sim 0^{1 - 2\nu},
\]

which diverges for \( \nu > 1/2 \). We first notice that this kind of leading singularities cancel out between two terms and the asymptotic behavior of the total integral \( I_{11} \) is given by

\[
\sim i \int_0 dx_1 x_1^{1/2 - \nu} \int_{x_1} dx_2 x_2^{-1/2 - \nu} \sim i \, 0^{2 - 2\nu}
\]

up to a real constant number. Although it is still singular for \( \nu > 1 \), such singular contribution is pure-imaginary and does not contribute to \( \text{Re}[I_{11}] \). The higher order terms are finite for \( \nu < 3/2 \) and hence we conclude that \( C_{11} = \text{Re}[I_{11}] \) is finite. In a similar way, we can show that \( \text{Re}[I_{ij} + \bar{I}_{ij}], \text{Im}[I_{12} - I_{21}], \text{and Im}[I_{13} - I_{31}] \) are finite for \( 0 < \nu < 3/2 \) and \( \nu = \) pure-imaginary, and therefore all the \( C_{ij} \)'s are finite. To avoid this kind of spurious singularities in the numerical calculation, we introduce a IR cut off \( \epsilon_{IR} \):

\[
I_{ij} = \frac{\pi}{4} e^{\frac{\pi}{2}(\nu - \nu')} \int_{\epsilon_{IR}}^\infty dx_1 \left( A_i(x_1)^* - \tilde{A}_i(x_1) \right) \int_{x_1}^\infty dx_2 A_j(x_2),
\]

where we set \( \epsilon_{IR} = 10^{-10} \) in our calculation.

As is usual in the Feynman diagram calculation in the momentum space, the integral \( I_{ij} \) oscillates at \( x = \infty \) because of the oscillating behavior of the mode functions \( u_k \) and \( \tilde{u}_k \), which makes the numerical calculation difficult.
Following [22, 23], we perform contour deformations to avoid this kind of technical difficulties. Let us first consider the integral \( \text{Re}[I_{ij} + I_{ji}] \). It is convenient to rewrite it as follows:

\[
\text{Re}[I_{ij} + I_{ji}] = \frac{\pi}{4} e^{\frac{i}{2} \pi (\nu - \nu^*)} \text{Re} \left[ \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x_1) \int_{\epsilon_{\text{IR}}}^\infty dx A_j(x_2) + \int_{\epsilon_{\text{IR}}}^\infty dx A_j(x_1) \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x_2) \right. \\
- \left. \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x_1) \int_{\epsilon_{\text{IR}}}^\infty dx A_j(x_2) - \int_{\epsilon_{\text{IR}}}^\infty dx A_j(x_1) \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x_2) \right]
\]

\[
= \frac{\pi}{4} e^{\frac{i}{2} \pi (\nu - \nu^*)} \text{Re} \left[ \left( \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x) \right) * \int_{\epsilon_{\text{IR}}}^\infty dy A_j(y) \right. \\
- \left. \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x_1) \int_{\epsilon_{\text{IR}}}^\infty dx A_j(x_2) - \int_{\epsilon_{\text{IR}}}^\infty dx A_j(x_1) \int_{\epsilon_{\text{IR}}}^\infty dx A_i(x_2) \right]. \quad (B4)
\]

The first term in the bracket can be Wick rotated without crossing any poles as

\[
\left( \int_{0}^{\infty} dx A_i(\epsilon_{\text{IR}} + ix) \right) * \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy), \quad (B5)
\]

and the last two terms are Wick rotated as

\[
\int_{0}^{\infty} dx \tilde{A}_i(\epsilon_{\text{IR}} + ix) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy) + \int_{0}^{\infty} dx A_j(\epsilon_{\text{IR}} + ix) \int_{0}^{\infty} dy A_i(\epsilon_{\text{IR}} + iy). \quad (B6)
\]

Then, we obtain the following expression of \( \text{Re}[I_{ij} + I_{ji}] \):

\[
\text{Re}[I_{ij} + I_{ji}] = \frac{\pi}{4} e^{\frac{i}{2} \pi (\nu - \nu^*)} \text{Re} \left[ \int_{0}^{\infty} dx \left( A_i(\epsilon_{\text{IR}} + ix) + \tilde{A}_i(\epsilon_{\text{IR}} + ix) \right) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy) \right. \\
+ \left. \int_{0}^{\infty} dx A_j(\epsilon_{\text{IR}} + iy) + \int_{0}^{\infty} dx A_i(\epsilon_{\text{IR}} + iy) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy) \right] + (i \leftrightarrow j). \quad (B7)
\]

To avoid the singular behavior around \( x = 0 \) discussed above, we further modify the contour as follows:

\[
\text{Re}[I_{ij} + I_{ji}] = \frac{\pi}{4} e^{\frac{i}{2} \pi (\nu - \nu^*)} \text{Re} \left[ i \int_{0}^{1} dx \left( A_i(\epsilon_{\text{IR}} + x) + \tilde{A}_i(\epsilon_{\text{IR}} + x) \right) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy) \right. \\
+ \left. \int_{0}^{\infty} dx \left( A_i(\epsilon_{\text{IR}} + x) + A_i(\epsilon_{\text{IR}} + x) + \tilde{A}_i(\epsilon_{\text{IR}} + 1 + ix) \right) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + 1 + iy) \right] + (i \leftrightarrow j). \quad (B8)
\]

By performing similar contour deformations, \( \text{Im}[I_{ij} - I_{ji}] \) can be also expressed as follows:

\[
\text{Im}[I_{ij} - I_{ji}] = \frac{\pi}{4} e^{\frac{i}{2} \pi (\nu - \nu^*)} \text{Re} \left[ \int_{0}^{1} dx \left( A_i(\epsilon_{\text{IR}} + x) - \tilde{A}_i(\epsilon_{\text{IR}} + x) \right) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy) \right. \\
- \left. \int_{0}^{\infty} dx A_i(\epsilon_{\text{IR}} + iy) - \int_{0}^{\infty} dx A_i(\epsilon_{\text{IR}} + iy) \int_{0}^{\infty} dy A_j(\epsilon_{\text{IR}} + iy) \right] + (i \leftrightarrow j). \quad (B9)
\]

The expressions (B8) and (B9) are used in our numerical calculations of the power spectrum.

### Appendix C: Analytical calculation of power spectrum for \( c_\sigma = c_\sigma \)

In this appendix we calculate the power spectrum for the case \( r_\sigma = c_\sigma/c_\pi = 1 \). For this class of models, we can analytically calculate the integrals \( I_{ij} \)'s by extending the results in [23]. We first introduce a function \( A(\ell, \nu, x) \) defined by

\[
A(\ell, \nu, x) = x^{-\frac{\ell}{2}} e^{ix} H^{(1)}_{\nu}(x). \quad (C1)
\]

In terms of \( A \), \( A_i \) can be written as

\[
A_1(x) = A(0, \nu, x), \quad A_2(x) = A(1, \nu - 1, x), \quad A_3(x) = iA(1, \nu, x), \quad (C2)
\]
and hence the $x$-integrals in the first term and the $x_2$-integral in the second term of \cite{121} reduce to that of $A$. Similarly to the case in \cite{23}, the indefinite integral $D$ of $A$ can be expressed using hyper-geometric functions as

\[
D(\ell, \nu, x) = \int dx A(\ell, \nu, x) = \frac{2^\nu x^{\frac{1}{2} + -\nu} \Gamma(\nu)}{i\pi} \left( \frac{1}{2} - \nu, \frac{1}{2} + \ell - \nu, \frac{3}{2} + \ell + \nu, 1 + 2\nu; 2ix \right) + e^{-i\pi} \frac{2^{\nu+\ell} \pi \Gamma(1+\nu)}{i\pi(\ell + \nu + 1)} 2F_2 \left( \frac{1}{2} + \nu, \frac{1}{2} + \ell + \nu, \frac{3}{2} + \ell + \nu, 1 + 2\nu; 2ix \right).
\]

Then, the integral $I_{ij}$ can be written as

\[
I_{ij} = \frac{\pi}{8} e^{\frac{i\pi}{2}(\nu - \nu^*)} (-i)^{n_1-n_2} \left( D(n_1, \nu + n_2) - D(n_1, \nu + n_2, 0) \right) \left( D(n_1, \nu + n_2, 0) \right) + \cdots + \left( D(n_1, \nu + n_2, x) \right),
\]

where $(n_1, n_2), (\tilde{n}_1, \tilde{n}_2) = (0, 0), (1, -1), (1, 0)$ for $i, j = 1, 2, 3$, respectively. Here note that $D(\ell, \nu, x)$ gives a finite value in the limit $x = \infty$ (see appendix C for the derivation):

\[
D(\ell, \nu, \infty) = \frac{\Gamma(1/2 + \nu)}{\cos \pi \nu} 2^{\frac{1}{2} + \nu} \left( \frac{1}{2} + \nu \right) \left( \frac{1}{2} + \nu - 1 \right) \left( \frac{1}{2} + \nu - 2 \right) \left( \frac{1}{2} + \nu - 3 \right) \cdots (C5) \]

which is singular for $\text{Re}[1/2 + \nu] < 0$. However, as mentioned in appendix \cite{B}, this kind of singularities cancel out in the calculation of $C_{ij}$'s and we drop them in the following calculation. We therefore rewrite (C4) as follows:

\[
I_{ij} = \frac{\pi}{8} e^{\frac{i\pi}{2}(\nu - \nu^*)} (-i)^{n_1-n_2} \left( D(n_1, \nu + n_2) - D(n_1, \nu + n_2, 0) \right) + \cdots + \left( D(n_1, \nu + n_2, x) \right),
\]

with $(a)_m = \frac{\Gamma(a + m)}{\Gamma(a)}$. We next perform the $x_1$-integral in (C8). To perform this kind of integrals, we use a trick of resummation \cite{22}. Expanding $\tilde{A}_i$ in $x$ and using the identity for Bessel functions,

\[
x^{-\nu} J_{\nu}(x) e^{ix} = \sum_{m=0}^{\infty} a_m x^m \quad \text{with} \quad a_m = \frac{2^{m+\nu} \nu^m \Gamma(m + \nu + 1/2)}{m! \sqrt{\pi} \Gamma(m + 2\nu + 1)},
\]

$\tilde{A}_i$ is rewritten as

\[
\tilde{A}_i = i^{n_1 + n_2} x^{\frac{1}{2} + n_2} e^{ix} \sin \pi \nu \left( e^{i\nu n_2} J_{\nu}(x) - J_{-\nu}(x) \right) = -i^{n_1 + n_2} x^{\frac{1}{2} + n_2} \sum_{m=0}^{\infty} a_m x^m \left( e^{i\nu n_2} - \frac{1}{2} + n_2 \right) - (-1)^{n_2} \sum_{m=0}^{\infty} a_m x^m \left( e^{i\nu n_2} - \frac{1}{2} + n_2 \right),
\]

(10)
where again \((n_1, n_2) = (0, 0), (1, -1), (1, 0)\) for \(i = 1, 2, 3\), respectively. Using this expression, the \(x_1\)-integral in (C8) can be written as a sum of integrals in the form

\[
\int_0^\infty dx \, x^p \left( D(\ell, \nu, \infty) - D(\ell, \nu, x) \right) .
\] (C11)

Integrating by parts and using \(\partial_x D(\ell, \nu, x) = A(\ell, \nu, x)\) and \(x_1^{1+p} A(\ell, \nu, x) = A(1 + \ell + p, \nu, x)\), we rewrite it as

\[
\int_0^\infty dx \, x^p \left( D(\ell, \nu, \infty) - D(\ell, \nu, x) \right) = \left[ \frac{x_1^{1+p}}{1+p} \left( D(\ell, \nu, \infty) - D(\ell, \nu, x) \right) \right]_0^\infty + \int_0^\infty dx \, \frac{1}{1+p} \left( D(1 + \ell + p, \nu, \infty) + \left[ x_1^{1+p} D(\ell, \nu, x) - D(1 + \ell + p, \nu, x) \right]_{x=0} \right)
\]

\[
= \frac{1}{1+p} \left( D(1 + \ell + p, \nu, \infty) \right)
\] (C12)

Here we again dropped the contribution from \(D\) at \(x = 0\), which vanishes or cancels out in our calculation as mentioned earlier. Then, the \(x_1\)-integral can be written as follows:

\[
- \frac{\pi}{4} i^{1+n_1+\tilde{n}_1+\tilde{n}_2} e^{\frac{i}{2} \pi (\nu^* - \nu)} \int_0^\infty dx_1 \tilde{A}_1(x_1) \left( D(\tilde{n}_1, \nu + \tilde{n}_2, \infty) - D(\tilde{n}_1, \nu + \tilde{n}_2, x_1) \right)
\]

\[
= \frac{\pi}{4} i^{1+n_1+\tilde{n}_1+\tilde{n}_2} e^{\frac{i}{2} \pi (\nu^* + \nu^* - \nu)} \sum_{m=0}^{\infty} a_m \left(\nu^* + n_2\right) \frac{a_m \left(\nu^* + n_2\right)}{2 + m + n_1 + n_2 + \nu^*} D(1/2 + m + n_1 + n_2 + \nu^* + \tilde{n}_1, \nu + \tilde{n}_2, \infty)
\]

\[
- e^{\frac{i}{2} \pi (\nu^* - n_2)} \sum_{m=0}^{\infty} a_m \left(-\nu^* - n_2\right) \frac{a_m \left(-\nu^* - n_2\right)}{2 + m + n_1 - n_2 - \nu^*} D(1/2 + m + n_1 - n_2 - \nu^* + \tilde{n}_1, \nu + \tilde{n}_2, \infty).
\] (C13)

After some lengthy calculations, we obtain

\[
\frac{\pi}{4} i^{1+n_1+\tilde{n}_1+\tilde{n}_2} e^{\frac{i}{2} \pi (\nu^* - \nu^* + \nu)} \sum_{m=0}^{\infty} a_m \left(\nu^* + n_2\right) \frac{a_m \left(\nu^* + n_2\right)}{2 + m + n_1 + n_2 + \nu^*} D(1/2 + m + n_1 + n_2 + \nu^* + \tilde{n}_1, \nu + \tilde{n}_2, \infty)
\]

\[
= i e^{\pm i \pi \nu^*} \left(\nu^* + n_2\right) \frac{a_m \left(\nu^* + n_2\right)}{2 + m + n_1 + n_2 + \nu^*} D(1/2 + m + n_1 + n_2 + \nu^* + \tilde{n}_1, \nu + \tilde{n}_2, \infty)
\]

\[
\times \begin{cases} 
(m + 1)_{n_1+\tilde{n}_2+\tilde{n}_1} & \text{for real } \nu, \\
(m + 1)_{n_1+\tilde{n}_2+\tilde{n}_1} & \text{for pure-imaginary } \nu,
\end{cases}
\] (C14)

where \(\Gamma(z) \Gamma(1 - z) = \frac{\pi}{\sin(\pi z)}\) was used. Finally, it is necessary to re-sum (C14) with respect to \(m\). In the case of \(\text{Re}[I_{11}]\) \((n_i = \tilde{n}_i = 0)\), for example, we perform the resummation as follows:

\[
- \frac{\pi}{4} e^{\frac{i}{2} \pi (\nu - \nu^*)} \int_0^\infty dx_1 \tilde{A}_1(x_1) \left( D(0, \nu, \infty) - D(0, \nu, x_1) \right)
\]

\[
= \frac{i}{4 \sin \pi \nu} \sum_{m=0}^{\infty} \left( \frac{(-1)^m e^{i \pi \nu}}{(m + \frac{1}{2} + \nu)^2} - \frac{(-1)^m e^{-i \pi \nu}}{(m + \frac{1}{2} - \nu)^2} \right)
\]

\[
= \frac{i}{4 \sin \pi \nu} \left[ e^{i \pi \nu} \Phi(-1, 2, \frac{1}{2} + \nu) - e^{-i \pi \nu} \Phi(-1, 2, \frac{1}{2} - \nu) \right]
\]

\[
= - \frac{i}{16 \sin \pi \nu} \left[ \psi^{(1)} \left(\frac{3}{4} + \nu \frac{1}{2} \right) - \psi^{(1)} \left(\frac{1}{4} + \nu \frac{1}{2} \right) \right] + \frac{i e^{-i \pi \nu}}{16 \sin \pi \nu} \left[ \psi^{(1)} \left(\frac{3}{4} - \nu \frac{1}{2} \right) - \psi^{(1)} \left(\frac{1}{4} - \nu \frac{1}{2} \right) \right].
\] Here \(\Phi(z, s, \alpha)\) and \(\psi^{(n)}(z)\) are the Lerch transcendent and the polygamma function, respectively:

\[
\Phi(z, s, \alpha) = \sum_{m=0}^{\infty} \frac{z^m}{(m + \alpha)^s}, \quad \psi^{(n)}(z) = (-1)^{n+1} n! \sum_{m=0}^{\infty} \frac{1}{(m + z)^{n+1}},
\] (C15)
which satisfy the following relation:

$$\Phi(-1, n + 1, z) = (-1)^n n! 2^{-n-1} \left[ \psi^{(n)} \left( \frac{z+1}{2} \right) - \psi^{(n)} \left( \frac{z}{2} \right) \right].$$  \hspace{1cm} (C16)

We then obtain

$$\text{Re}[\mathcal{I}_{11}] = \frac{\pi^2}{4 \cos^2 \pi \nu} + \text{Re} \left[ \frac{i e^{-i \pi \nu}}{16 \sin \pi \nu} \left( \psi^{(1)} \left( \frac{3}{4} + \frac{\nu}{2} \right) - \psi^{(1)} \left( \frac{1}{4} + \frac{\nu}{2} \right) \right) - \frac{i e^{i \pi \nu}}{16 \sin \pi \nu} \left( \psi^{(1)} \left( \frac{3}{4} - \frac{\nu}{2} \right) - \psi^{(1)} \left( \frac{1}{4} - \frac{\nu}{2} \right) \right) \right],$$  \hspace{1cm} (C17)

which reproduces the result in [23]. In a similar way, we can obtain analytic expression for \( \mathcal{I}_{ij} \)'s, and the results are summarized as follows:

$$\mathcal{I}_{22} = \frac{\pi^2 |\nu - \frac{1}{2} |^2 \nu - \frac{1}{2} |^2}{16 \cos^2 \pi \nu} + \frac{1}{16} - \frac{i}{128 \sin \pi \nu} \left( e^{i \pi \nu (2 \nu - 3)(2 \nu - 5) - e^{-i \pi \nu (2 \nu + 1)(2 \nu - 1)} \right)$$

$$+ \frac{1}{256 \sin \pi \nu} \left( (2 \nu - 1)^2 (2 \nu - 3)^2 \left( e^{i \pi \nu \Phi(-1, 2, \frac{1}{2} + \nu) - e^{-i \pi \nu \Phi(-1, 2, \frac{3}{2} - \nu) \right) \right) \right),$$  \hspace{1cm} (C18)

$$\mathcal{I}_{33} = \frac{\pi^2 (\nu - \frac{1}{2}) (\nu - \frac{3}{2})}{16 \cos^2 \pi \nu} - \frac{1}{16} + \frac{i}{128 \sin \pi \nu} \left( e^{i \pi \nu (2 \nu - 1)(2 \nu - 3) - e^{-i \pi \nu (2 \nu + 1)(2 \nu + 3)) \right)$$

$$- \frac{1}{256 \sin \pi \nu} \left( (2 \nu + 1)^2 (2 \nu - 1)^2 \left( e^{i \pi \nu \Phi(-1, 2, \frac{1}{2} + \nu) - e^{-i \pi \nu \Phi(-1, 2, \frac{3}{2} - \nu) \right) \right),$$  \hspace{1cm} (C19)

$$\mathcal{I}_{12} = -\frac{\pi^2 (\nu - \frac{1}{2}) (\nu - \frac{3}{2})}{16 \cos^2 \pi \nu} + \frac{i}{16 \sin \pi \nu} \left( e^{i \pi \nu (2 \nu - 1) + e^{-i \pi \nu (2 \nu + 1)) \right)$$

$$+ \frac{i}{8 \sin \pi \nu} \left( 3 + 4 \nu^2 \right) \left( e^{i \pi \nu \Phi(-1, 1, \frac{1}{2} + \nu) - e^{-i \pi \nu \Phi(-1, 1, \frac{3}{2} - \nu) \right) \right),$$  \hspace{1cm} (C20)

$$\mathcal{I}_{21} = -\frac{\pi^2 (\nu^* - \frac{1}{2}) (\nu^* - \frac{3}{2})}{8 \cos^2 \pi \nu} + \frac{i (-1 + 4 \nu^2)}{32 \sin \pi \nu} \left( e^{i \pi \nu \Phi(-1, 1, -\frac{1}{2} + \nu^*) - e^{-i \pi \nu \Phi(-1, 1, -\frac{3}{2} - \nu^*)) \right)$$

$$- \frac{i (4 \nu^2 - 5)}{32 \sin \pi \nu^*} \left( e^{i \pi \nu^* \Phi(-1, 1, \frac{1}{2} + \nu^*) - e^{-i \pi \nu^* \Phi(-1, 1, \frac{3}{2} - \nu^*) \right) \right),$$  \hspace{1cm} (C21)

$$\mathcal{I}_{13} = \frac{\pi^2 (\nu + \frac{1}{2}) (\nu + \frac{3}{2})}{8 \cos^2 \pi \nu} + \frac{i (1 + 4 \nu^2)}{16 \sin \pi \nu} \left( e^{i \pi \nu (2 \nu - 1) + e^{-i \pi \nu (2 \nu + 1)) \right)$$

$$- \frac{i (1 + 4 \nu^2)}{16 \sin \pi \nu} \left( e^{i \pi \nu \Phi(-1, 1, \frac{1}{2} + \nu) - e^{-i \pi \nu \Phi(-1, 1, \frac{1}{2} - \nu) \right) \right),$$  \hspace{1cm} (C22)

$$\mathcal{I}_{31} = \frac{\pi^2 (\nu^* + \frac{1}{2}) (\nu^* + \frac{3}{2})}{8 \cos^2 \pi \nu} + \frac{i (-1 + 4 \nu^2)}{32 \sin \pi \nu} \left( e^{i \pi \nu \Phi(-1, 1, \frac{1}{2} + \nu) - e^{-i \pi \nu \Phi(-1, 1, \frac{1}{2} - \nu) \right)$$

$$- \frac{i (3 + 4 \nu^2)}{32 \sin \pi \nu} \left( e^{i \pi \nu \Phi(-1, 1, -\frac{1}{2} + \nu) - e^{-i \pi \nu \Phi(-1, 1, -\frac{1}{2} - \nu) \right) \right),$$  \hspace{1cm} (C23)
\begin{align}
\mathcal{I}_{23} &= -\frac{\pi^2 |\nu - \frac{1}{2}|^2 (\nu + \frac{1}{2}) (\nu^* - \frac{3}{2})}{16 \cos^2 \pi \nu} + \frac{1}{16} \frac{i}{128 \sin \pi \nu^*} \left( e^{i \pi \nu^*} (-11 + 12 \nu^2) + e^{-i \pi \nu^*} -1 + 2 \nu (1 - 20 \nu^* + 20 \nu^2) \right), \\
&+ \frac{(1 - 2 \nu^*) (1 + 4 \nu^2 + 4 \nu)}{16 \cos^2 \pi \nu} \left( e^{i \pi \nu} \Phi(-1, 1, -1, \frac{1}{2}, \nu^*) - e^{-i \pi \nu} \Phi(-1, 1, -1, \frac{1}{2}, -\nu^*) \right), \\
&+ \frac{i(1 - 2 \nu^*)^2 (3 + 2 \nu^*) (1 + 2 \nu)}{256 \sin \pi \nu^*} \left( e^{i \pi \nu^*} \Phi(-1, 2, \frac{3}{2} + \nu^* - e^{-i \pi \nu^*} \Phi(-1, 2, \frac{3}{2} - \nu^*) \right), \quad (C24) \\
\mathcal{I}_{32} &= -\frac{\pi^2 |\nu - \frac{1}{2}|^2 (\nu - \frac{1}{2}) (\nu^* + \frac{1}{2})}{16 \cos^2 \pi \nu} - \frac{1}{16} \frac{i}{128 \sin \pi \nu} \left( e^{i \pi \nu -1 + 2 \nu (1 - 20 \nu^* + 20 \nu^2) + e^{-i \pi \nu} (1 - 24 \nu^* + 12 \nu^2) \right), \\
&+ \frac{(1 - 2 \nu) (-1 - 4 \nu + 4 \nu^2)}{32 \sin \pi \nu} \left( e^{i \pi \nu} \Phi(-1, 1, \frac{3}{2} + \nu) - e^{-i \pi \nu} \Phi(-1, 1, \frac{3}{2} - \nu) \right), \\
&+ \frac{i(3 + 2 \nu) (-1 + 2 \nu)^2 (1 + 2 \nu)}{256 \sin \pi \nu} \left( e^{i \pi \nu} \Phi(-1, 2, \frac{3}{2} + \nu) - e^{-i \pi \nu} \Phi(-1, 2, \frac{3}{2} - \nu) \right). \quad (C25)
\end{align}

Here we used $\sum_{m=0}^{\infty} (-1)^m = \frac{1}{1 + 1} = \frac{1}{2}$. As we displayed in figure 1, the analytic results obtained in this appendix and the numerical results for $r_s = 1$ well coincide with each other.

1. Asymptotic behavior of $D(\ell, \nu, x)$

In this subsection we derive the asymptotic behavior (C25) in the limit $x \to \infty$ of the function $D(\ell, \nu, x)$:

\begin{align}
D(\ell, \nu, x) &= \frac{2 \nu x^{\frac{1}{2} + \ell - \nu} \Gamma(\nu)}{i \pi \left( \frac{1}{2} + \ell - \nu \right)} 2F_2 \left( \frac{1}{2} - \nu, \frac{1}{2} + \ell - \nu; \frac{3}{2} + \ell - \nu, 1 - 2 \nu; 2i \nu \right) \\
&+ e^{-i \pi \nu} \frac{2 \nu x^{\frac{1}{2} + \ell + \nu} \Gamma(-\nu)}{i \pi \left( \frac{1}{2} + \ell + \nu \right)} 2F_2 \left( \frac{1}{2} + \nu, \frac{1}{2} + \ell + \nu; \frac{3}{2} + \ell + \nu, 1 + 2 \nu; 2i \nu \right). \quad (C26)
\end{align}

We use the following asymptotic expansion of hypergeometric functions:

\begin{align}
2F_2(a_1, a_2; b_1, b_2; z) &= \Gamma(b_1) \Gamma(b_2) \Gamma(a_1) \Gamma(a_2) \frac{e^{z}}{\Gamma(a_1 + a_2 - b_1 - b_2)} \sum_{k=0}^{\infty} c_k z^{-k} \\
&+ \frac{\Gamma(b_1) \Gamma(b_2) \Gamma(a_2 - a_1)}{\Gamma(a_2) \Gamma(b_1 - a_1) \Gamma(b_2 - a_1)} (-z)^{-a_1} 3F_1(a_1, a_1 - b_1 + 1, a_1 - b_2 + 1; a_1 - a_2 + 1; -1/z) \\
&+ \frac{\Gamma(b_1) \Gamma(b_2) \Gamma(a_1 - a_2)}{\Gamma(a_1) \Gamma(b_1 - a_2) \Gamma(b_2 - a_2)} (-z)^{-a_2} 3F_1(a_2, a_2 - b_1 + 1, a_2 - b_2 + 1; a_2 - a_1 + 1; -1/z), \quad (C27)
\end{align}

where $c_k$'s are numerical factors independent of $z$. The first term gives an oscillating term $\sim e^{2i \pi x}$, which can be dropped by an $i\epsilon$-prescription. Then, let us consider the contribution of the last two terms. They are respectively in the form

\begin{align}
&\frac{1}{i \sqrt{2 \pi} \ell \sin \pi \nu} \left( x^{\ell} e^{-i \pi (\nu - 1/2)} 3F_1(1/2 + \nu, 1/2 - \nu, -\ell; 1 - \ell; i/(2x)) \right) \\
&+ \frac{1}{i \sqrt{2 \pi} \ell \sin \pi \nu} \left( \Gamma(1/2 + \ell - \nu) \Gamma(-\ell)(-2i)^{-\ell} e^{-i \pi (\nu - 1/2)} 3F_1(1/2 + \ell + \nu, 1/2 + \ell - \nu, 0; 1 + \ell; i/(2x)) \right), \quad (C28)
\end{align}

and

\begin{align}
&\frac{1}{i \sqrt{2 \pi} \ell \sin \pi \nu} \left( x^{\ell} e^{-i \pi (\nu - 1/2)} 3F_1(1/2 + \nu, 1/2 - \nu, -\ell; 1 - \ell; i/(2x)) \right) \\
&- \frac{1}{i \sqrt{2 \pi} \ell \sin \pi \nu} \left( \Gamma(1/2 + \ell + \nu) \Gamma(-\ell)(-2i)^{-\ell} e^{-i \pi (\nu - 1/2)} 3F_1(1/2 + \ell + \nu, 1/2 + \ell - \nu, 0; 1 + \ell; i/(2x)) \right), \quad (C29)
\end{align}
where note that hypergeometric functions $pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ are symmetric under the permutations of $a_i$'s and those of $b_i$'s, respectively. The first term in (C28) and that in (C29) cancel out and we obtain

$$D(\ell, \nu, x) = \frac{1}{i\sqrt{2\pi}} \frac{1}{\sin \pi \nu} \left( \frac{\Gamma(1/2 + \ell - \nu)}{\Gamma(1/2 - \ell - \nu)} \right)^2 \frac{3F_1(1/2 + \ell + \nu, 1/2 + \ell - \nu, 0; 1 + \ell; i/(2x))}{\Gamma(-\ell)(-\ell)^{-\ell} e^{-\ell \pi(\nu-1/2)}}$$

$$- \frac{1}{i\sqrt{2\pi}} \frac{1}{\sin \pi \nu} \left( \frac{\Gamma(1/2 + \ell + \nu)}{\Gamma(1/2 - \ell + \nu)} \right) \Gamma(-\ell)(-\ell)^{-\ell} e^{-\ell \pi(\nu-1/2)} 3F_1(1/2 + \ell + \nu, 1/2 + \ell - \nu, 0; 1 + \ell; i/(2x)),$$

where note that we did not use any approximation so far. Finally, taking the limit $x \to \infty$, we conclude that

$$D(\ell, \nu, \infty) = \frac{1}{i\sqrt{2\pi}} \frac{1}{\sin \pi \nu} \left( \frac{\Gamma(1/2 + \ell - \nu)}{\Gamma(1/2 - \ell - \nu)} \right)^2 \frac{\Gamma(1/2 + \ell + \nu)\Gamma(1/2 + \ell - \nu)}{\Gamma(1/2 + \nu)\Gamma(1/2 - \nu)},$$

where we used $3F_1(a_1, a_2, a_3; b; 0) = 1$. Using the identity

$$\Gamma(z)\Gamma(1 - z) = \frac{\pi}{\sin \pi z},$$

we can also rewrite (C31) as follows:

$$D(\ell, \nu, \infty) = \frac{\sqrt{\pi}}{\cos \pi \nu} (2\pi)^{-\ell} \frac{1}{\Gamma(\ell + 1)} \frac{\Gamma(1/2 + \ell + \nu)\Gamma(1/2 + \ell - \nu)}{\Gamma(1/2 + \nu)\Gamma(1/2 - \nu)}$$

which reproduces the result in 23 for $\ell = 0$. 

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