CONDENSED MATTER

Splitting of the Superfluid Transition in Liquid $^3$He Induced by a Nematic Aerogel

I. A. Fomin*

Kapitza Institute for Physical Problems, Russian Academy of Sciences, Moscow, 119334 Russia

*e-mail: fomin@kapitza.ras.ru

Received July 9, 2021; revised July 14, 2021; accepted July 14, 2021

The effect of inhomogeneous perturbations created by a nematic aerogel in liquid $^3$He on the form of the order parameter arising at the transition to the superfluid state is studied. It is shown that the stability boundary of the distorted polar Anderson–Brinkman–Morel phase can arise just on the superfluid transition line at certain characteristics of the nematic aerogel. A symmetry-based argumentation for the choice of aerogels most suitable for stabilizing the polar phase is given.

DOI: 10.1134/S0021364021160049

1. INTRODUCTION

In recent years, new superfluid phases have been successfully stabilized in liquid $^3$He [1] (see also review [2]). The possibility of forming different phases is a typical manifestation of the unconventional Cooper pairing. In the case of $^3$He, this pairing corresponds to the state with orbital angular momentum $l = 1$ and spin $s = 1$. The corresponding order parameter is proportional to the complex $3 \times 3$ matrix $A_{ij}^\dagger$: its first subscript enumerates three possible spin projections, and the second one, three possible projections of the orbital angular momentum. According to the Landau theory, the phase transition occurs when the coefficient at the second-order invariant in the expansion of the free energy gain in powers of the order parameter changes its sign. In $^3$He, the second-order term in this expansion is $\delta f \sim a(P, T)A_{ij}^\dagger A_{ij}$. The transition temperature $T_c$ at a given pressure $P$ is determined by the condition $a(P, T_c) = 0$. The temperature thus found is “degenerate”; i.e., the normal phase at this temperature is unstable with respect to the formation of Cooper pairs with three possible spin projections $s_z = 0$, $+1$, and $-1$ and with three projections of the orbital angular momentum $l_z = 0$, $+1$, and $-1$. The proper combination of these basis functions is found by minimizing the contribution of fourth-order invariants to the energy gain. In $^3$He, there are five such invariants:

$I_1 = A_{ij} A_{ij}^\dagger A_{ij}^\ast A_{ij}^\ast$, $I_2 = A_{ij} A_{ij}^\ast A_{ij} A_{ij}^\ast$, $I_3 = A_{ij} A_{ij}^\ast A_{ij} A_{ij}^\ast$, $I_4 = A_{ij}^\ast A_{ij} A_{ij} A_{ij}^\ast$, and $I_5 = A_{ij}^\ast A_{ij}^\ast A_{ij} A_{ij}^\ast$. They enter the free energy in the form of the combination $\sum_{l_z=0}^3 \beta_l I_l$, where $\beta_0$, ..., $\beta_4$ are phenomenological coefficients. In the general case, this combination has many extrema, including several minima [3, 4]. For those values of the coefficients $\beta_0$, ..., $\beta_4$ that are implemented in $^3$He, only two minima are actually used. They correspond to the Anderson–Brinkman–Morel and Balian–Werthamer phases [4].

Among other minima, there is a polar phase corresponding to the Cooper pairing with the orbital angular momentum $l_z = 0$ and the order parameter $A_{ij}^\dagger = \Delta \exp(i\varphi)d_{ij}m_j$, where $\Delta$ is the common amplitude, $d_{ij}$ is the unit spin vector, and $m_j$ is the unit vector in the space of wave vectors. In the impurity-free volume of liquid $^3$He, this minimum is much less favorable in energy than the other two and cannot be actually implemented. At the same time, the corresponding phase has interesting physical properties, such as a topologically stable line of zeros in the spectrum of Fermi excitations [5] and the possibility of the existence of half-quantum vortices in this phase (see [6] and references therein).

Aoyama and Ikeda [7] theoretically proposed a method to stabilize the polar phase in a certain temperature range near $T_c$. The idea of the method is to reduce the symmetry of the normal phase from spherical to axial. Then, the possible superfluid phases and the corresponding transition temperatures will be classified by the projection $l_z$ of the orbital angular momentum $l$ on the symmetry axis rather than by the absolute value of this momentum. If the transition temperature is the highest for $l_z = 0$, the polar phase should be stable in a certain temperature range below this $T_c$. To reduce the orbital symmetry of liquid $^3$He, Aoyama and Ikeda proposed to use oriented anisotropic impurities. Specifically, they studied a set of impurities for which the average scattering cross...
section for fermionic quasiparticles has the form \(\sigma(\hat{k}) \sim [1 + \delta(\hat{k} \cdot \hat{\tau})^2]\). Here, \(\hat{\tau}\) is the direction of the anisotropy axis and \(\hat{k}\) is the momentum transfer at scattering. Following the argumentation of the theory of superconducting alloys [8], Aoyama and Ikeda arrived at the conclusion that the transition temperature should be split into two values: \(T_{c0}\) corresponding to the projection \(l_z = 0\) and \(T_{c1}\) corresponding to \(l_z = \pm 1\). If \(\delta < 0\), \(T_{c0} > T_{c1}\), and even at small \(\delta\), for each pressure value, there exists a nonzero temperature range starting from \(T_c\), where Cooper pairs have \(l = 1\) and \(l_z = 0\). This means that the polar phase of \(^3\)He should arise in this range. This result stimulated experiments on the stabilization of the polar phase (see review [2]). Global anisotropy in these experiments was created using various samples of so-called nematic aerogels. These aerogels are formed by long and nearly parallel filaments. In the first experiments, samples of the Obninsk aerogel were used [9]. The filaments of this aerogel have diameters \(d \approx 9\) nm and consist of amorphous AlOOH. According to the ratio of mean free paths of fermionic quasiparticles along and across the filaments \(l_c/l_z = 1.4\), the degree of anisotropy of this aerogel is much higher than that assumed in the calculations by Aoyama and Ikeda. The matrix \(\Lambda_{jl}\) depends on the position \(\mathbf{r}\), and must be Hermitian since the free energy density is real. It is assumed that aerogel filaments do not interact directly with the spins of quasiparticles. In the experiments reported in [2], such an interaction was excluded by covering the filaments with a thin \(^4\)He film. It is convenient to represent the matrix \(\Lambda_{jl}(\mathbf{r})\) as the sum of its average over the ensemble \(\tau_{jl} = \langle \Lambda_{jl} \rangle\) and fluctuating local anisotropy \(\eta_{jl}(\mathbf{r}) = \Lambda_{jl}(\mathbf{r}) - \tau_{jl}\), so that \(\langle \eta_{jl} \rangle = 0\). In what follows, it will be assumed that the aerogels under study, on average, do not have chirality; i.e., there is no difference between right and left orientations. In this case, the matrix \(\tau_{jl}\) is symmetric and real; its principal values \(\tau_{xx}, \tau_{yy},\) and \(\tau_{zz}\) are real functions of the temperature, and the principal directions, as explained above, can be taken as the directions of the coordinate axes \((x, y, z)\): \(\tau_{xx} = \tau_x\hat{x}\hat{x}, \tau_{yy} = \tau_y\hat{y}\hat{y},\) and \(\tau_{zz} = \tau_z\hat{z}\hat{z}\). In this notation, the expansion of the free energy density near the temperature-driven instability of the normal phase can be written as

\[
\frac{f_N - f_s}{N(0)} = \left[\tau_{xx} + \eta_{xx}(\mathbf{r})\right]A_{xx}^{\ast}A_{xx} + \xi_1^2 \left(\frac{\partial A_{xx}^{\ast}}{\partial x_n} \frac{\partial A_{xx}}{\partial x_n}\right) + \frac{1}{2} \sum_{s=1}^{5} \beta_s I_s.
\]

To avoid unnecessary complications in finding the contribution of gradient terms, a model isotropic
expression is adopted here. The equilibrium order parameter is found from the equations
\[
[\tau_{jl} + \eta_{jl}(r)]A_{jl} - \xi_{jl}^2 \left( \frac{\partial^2 A_{jl}}{\partial x^2} \right) + \sum_{m=1}^{2} \beta_{jl} \frac{\partial J_{rl}}{\partial A_{jl}} = 0. \tag{2}
\]
When solving the system of equations (2), we consider only the case where random anisotropy \(\eta_{jl}(r)\) can be treated as a perturbation. We seek a solution of Eqs. (2) in the form \(A_{jl}(r) = \overline{A}_{jl} + a_{jl}(r)\), where \(\overline{A}_{jl}\) is the order parameter averaged over the ensemble. This average value should be treated as the order parameter for the \(^3\)He phase in the aerogel, which we discuss here. It describes the thermal properties of this liquid. The fluctuating part \(a_{jl}(r)\) of the order parameter disappears upon averaging. The condition of the instability of the normal phase of \(^3\)He should be directly expressed in terms of \(\overline{A}_{jl}\). For this, it is necessary to substitute \(A_{jl} = \overline{A}_{jl} + a_{jl}(r)\) into Eq. (2) and average it, retaining the terms not exceeding the second-order terms in the expansion in terms of \(a_{jl}\) and \(\eta_{jl}(r)\) and linear in \(\overline{A}_{jl}\). As a result, we find
\[
\tau_{jl} \overline{A}_{jl} + \langle \eta_{jl}(r)a_{jl}(r) \rangle = 0. \tag{3}
\]
Here, \(\mu\) is a parameter. For the phases discussed in relation to experiments reported in [2], the order parameter is factorized into the spin and orbital parts. The spin part is the real vector \(d_{jl}\). Orbital anisotropy affects the orientation of \(d_{jl}\) only via the weak dipole interaction, which is disregarded here. In what follows, only the orbital part \(A_{jl}\) will be considered and the subscript \(\mu\) will be omitted:
\[
\tau_{jl} \overline{A}_{jl} + \langle \eta_{jl}a_{jl} \rangle = 0. \tag{4}
\]
The fluctuating part \(a_{jl}\) is found from the solution of the linear equation
\[
\tau_{jl} a_{jl} - \xi_{jl}^2 \left( \frac{\partial^2 a_{jl}}{\partial x^2} \right) = -\eta_{jl} \overline{A}_{jl}, \tag{5}
\]
which is solved by passing to Fourier transforms
\[
a_{jl}(k) = -\left(\tau_{kj} + \delta_{jk} \xi^2 k^2 \right)^{-1} \eta_{lm}(k) \overline{A}_{jm}. \tag{6}
\]
The substitution of this solution into
\[
\langle \eta_{jl}a_{jl} \rangle = \int \eta_{jl}(-k) a_{jl}(k) \frac{d^3 k}{(2\pi)^3} \tag{7}
\]
leads to the linear equation for the order parameter at the point of instability of the normal phase
\[
(\tau_{jl} - V_{jl}) \overline{A}_{jl} = 0, \tag{8}
\]
where
\[
V_{jl} = \int \frac{d^3 k}{(2\pi)^3} \eta_{jl}^*(k)(\tau_{mn} + \delta_{mn} \xi^2 k^2)^{-1} \eta_{ml}(k). \tag{9}
\]
Here, we use the relation \(\eta_{jl}(-k) = \eta_{jl}^*(k)\). Equation (8) has a solution if
\[
\det (\tau_{jl} - V_{jl}) = 0. \tag{10}
\]
Thus, we have an eigenvalue problem with respect to the transition temperature \(T_c\), on which \(\tau_{jl}\) depends. To solve this problem, it is possible to use the conventional perturbation theory (see, e.g., [15]). In the zero order approximation, \(\tau_{jl} \overline{A}_{jl} = 0\), there are three eigenvalues \(\tau_x = 0\), \(\tau_y = 0\), and \(\tau_z = 0\) with the corresponding eigenvectors \((1,0,0)\), \((0,1,0)\), and \((0,0,1)\). Let \(\tau_x \geq \tau_y > \tau_z\), and the first transition upon cooling occur at \(\tau_z(T = T_c) = 0\). According to the Landau theory, \(\tau_z(T)\) can be expanded near \(T_c\). It is possible to choose the normalization in such a way that \(\tau_z = (T - T_c)/T_c\). The first-order correction to the transition temperature has the form \(T - T_c = TV_{zz}\). In this approximation, the order parameter arising at the transition has the following components: \(\overline{A}_x = V_{13}/\tau_x\), \(\overline{A}_y = V_{31}/\tau_y\), and \(\overline{A}_z = 1\). According to definition (9), \(V_{jl}\) is the Hermitian matrix. This means that its symmetric part \((V_{jl} + V_{jl})/2\) is real, whereas the antisymmetric one \((V_{jl} - V_{jl})/2\) is imaginary. If the matrix elements \(V_{13}\) and \(V_{23}\) are real, their appearance can be treated as a change in the direction of \(\overline{A}\). The arising phase is polar, but with another orientation. Imaginary additions to the transverse components cannot be included in the order parameter of the polar phase. Along with the \(z\) component \(\overline{A}_z = 1\), they form the order parameter of the distorted polar Anderson–Brinkman–Morel phase, which, with a further decrease in the temperature, transforms into the Anderson–Brinkman–Morel phase continuously, without additional phase transitions. A purely polar phase is formed if \(V_{13} = 0\) and \(V_{23} = 0\). This condition is satisfied for the model expressions for \(\eta_{jl}(k)\) discussed in [7, 16]. The models under discussion are based on generalizations of the theory of superconducting alloys [8], where the aerogel is treated as an ensemble of independent impurities, and the tensor \(\eta_{jl}(r)\) for them is real and symmetric. This requirement ensures that there is no imaginary part in \(V_{jl}\), but it is difficult to find a physical justification for such assumption. Symmetry reasons allow us to understand what kind of an ideal aerogel should exist to stabilize the polar phase. It is a nematic aerogel with infinitely long and smooth filaments. Such an aerogel is symmetric with respect to the reflection in the plane orthogonal to the filaments. In this case, under the \(z \rightarrow -z\) transformation, the \(V_{xx}\) and \(V_{yy}\) components of the tensor \(V_{jl}\) must change their sign, but they cannot change because of the assumed
symmetry; i.e., these components must vanish. According to [2], the structure of naphene and mullite aerogels is closer to ideal than that of the Obninsk aerogel. This may explain the fact that the polar phase is observed in the first two cases and was not observed in the last one. It is also useful to bear in mind that an ideal nematic aerogel should not reduce the temperature of the transition to the superfluid state for $^3$He [17] and the value of the decrease in $T_c$ for the aerogel can serve as an indicator of the sample quality.

In conclusion, average uniaxial anisotropy induced in liquid $^3$He by an aerogel immersed in it does not yet guarantee the stabilization of the polar phase. Spatial fluctuations of local anisotropy, depending on the structure of the aerogel, can promote emergence of the imaginary components of the order parameter and the formation of the distorted polar Anderson–Brinkman–Morel phase at temperatures near $T_c$. The most reliable way to stabilize a purely polar phase is to use an ideal nematic aerogel.

ACKNOWLEDGMENTS

I am grateful to V.V. Dmitriev, A.A. Soldatov, and A.N. Yudin for helpful discussions and constructive criticism.

REFERENCES

1. V. V. Dmitriev, A. A. Senin, A. A. Soldatov, and A. N. Yudin, Phys. Rev. Lett. 115, 165304 (2015).
2. V. V. Dmitriev, A. A. Soldatov, and A. N. Yudin, J. Exp. Theor. Phys. 131, 2 (2020).
3. V. I. Marchenko, Sov. Phys. JETP 66, 79 (1987).
4. D. Vollhardt and P. Woelfle, The Superfluid Phases of Helium 3 (Taylor and Francis, London, 1990).
5. V. B. Eltsov, T. Kamprienen, J. Rysti, and G. E. Volovik, arXiv: 1908.01645.
6. S. Autti, V. V. Dmitriev, J. T. Mäkinen, A. A. Soldatov, G. E. Volovik, A. N. Yudin, V. V. Zavjalov, and V. B. Eltsov, Phys. Rev. Lett. 117, 255301 (2016).
7. K. Aoyama and R. Ikeda, Phys. Rev. B 73, 060504 (2006).
8. A. A. Abrikosov and L. P. Gor’kov, Sov. Phys. JETP 9, 220 (1959).
9. R. Sh. Askhadullin, V. V. Dmitriev, D. A. Krasnikhin, P. N. Martynov, A. A. Osipov, A. A. Senin, and A. N. Yudin, JETP Lett. 95, 326 (2012).
10. N. Zhelev, M. Reichl, T. S. Abhilash, E. N. Smith, K. X. Nguen, E. J. Mueller, and J. M. Parpia, Nat. Commun. 7, 12975 (2016).
11. J. A. Sauls, Phys. Rev. B 88, 214503 (2013).
12. I. A. Fomin and E. V. Surovtsev, JETP Lett. 97, 644 (2013).
13. I. A. Fomin, J. Exp. Theor. Phys. 118, 765 (2014).
14. D. Rainer and M. Vuorio, J. Phys. C: Solid State Phys. 10, 3093 (1977).
15. L. D. Landau and E. M. Lifshitz, Course of Theoretical Physics, Vol. 3: Quantum Mechanics: Non-Relativistic Theory (Nauka, Moscow, 1963; Pergamon, New York, 1977), Chap. 6.
16. R. C. Regan, J. J. Wiman, and J. A. Sauls, arXiv: 2105.01257v1.
17. I. A. Fomin, J. Exp. Theor. Phys. 127, 933 (2018).

Translated by K. Kugel