Urban transport phenomena in the street canyon

Maciej M. Duras *
Institute of Physics, Cracow University of Technology, ulica Podchorążych 1, PL-30-084 Cracow, Poland
"The Proceedings of the Comphy02 Computational Physics of Transport and Interface Dynamics conference", M. Schreckenberg et al Eds., Springer-Verlag, Berlin, Germany (2002).
(AD 2002, 13th March)

Abstract

A field deterministic model of the vehicular dynamics in a generic urban street canyon with two neighboring canyons is considered. The assumed hydrodynamical model of vehicular movement is coupled to the gasdynamical model of the air and emitted pollutants. The vehicles are assumed to move on distinct left lanes and right ones. At the upstream and downstream ends there are coordinated traffic lights which introduce control parameters to the field model. The model of optimal control of street canyon dynamics is based on the two optimal multi-criteria control problems. The problems consist of minimization of the dimensionless functionals of the cumulative total travel time, global emissions of pollutants, and global concentrations of pollutants, both in the studied street canyon, as well as together with the two nearest neighbor substitute canyons, respectively.

47.10.+g, 47.62.+q, 89.60.-k, 89.40.-a

*Electronic address: mduras @ riad.usk.pk.edu.pl
I. THE FIELD MODELS OF VEHICLES AND POLLUTANTS.

In the present article we consider two coupled deterministic field model of the urban street canyon. The vehicular fields are one-dimensional in spatial variable $x$ and they depend on time variable $t$. The considered fields are the vehicular number density $k_{l,v}(x, t)$, vehicular velocity $w_{l,v}(x, t)$, emissivity of exhaust gases $e_{l,v}(x, t)$, and vehicular heat emissivity $\sigma_{l,v}(x, t)$, where $vt$ is emission type’s number, $ct$ is number of the constituent of emitted exhaust gases. It is assumed that there are $n_1 = n_L$ left lanes $s = 1$ and $l$ is the left lane’s number, $l = 1, ..., n_L$, whereas for $n_2 = n_R$ right lanes $s = 2$ and $l$ is the right lane’s number, $l = 1, ..., n_R$, $vt = 1, ..., VT$, $ct = 1, ..., CT$. $VT$ is total number of types of vehicular emissions, and $CT$ is total number of emitted exhaust gases. The abovementioned model consists of the equations of balances of vehicular numbers [5] and vehicular equations of state (Greenshields’ equilibrium speed-density $u-k$ model [6]) as well as exhaust gas emissivities and heat emissivities. The vehicular flow in the canyon is multilane bidirectional one-level rectilinear, and it is considered with two systems of coordinated signalized junctions at the upstream and downstream ends of lanes [1], [2], [3], [4]. The considered vehicles belong to distinct vehicular classes: passenger cars, and trucks. Emissions from the vehicles are based on technical measurements of the following types of pollutants: carbon monoxide CO, hydrocarbons HC, nitrogen oxides NO$_x$. The field model of pollutants is gasdynamical. The gases are Newtonian viscous perfect and noninteracting. The considered fields are three-dimensional in spatial variables $(x, y, z)$ and one-dimensional in temporal variable $t$ and they are mixture’s density $\rho(x, y, z, t)$, velocity $v(x, y, z, t)$, temperature $T(x, y, z, t)$, pressure $p(x, y, z, t)$, as well as constituents’ mass concentrations $c_i(x, y, z, t)$, and partial pressures $p_i(x, y, z, t)$, $i = 1, ..., N$, where $N$ is total number of mixture’s elements. $(N = N_E - 1 + N_A)$. The first $N_E - 1 = 3$ gases are the exhaust gases emitted by vehicle engines during combustion CO, CH, NO$_x$. The remaining $N_A = 9$ gases are the constituents of air: O$_2$, N$_2$, Ar, CO$_2$, Ne, He, Kr, Xe, H$_2$. The field equations are the balances of mixture’s mass, linear momentum, energy, and equation of state (Clapeyron’s law) as well as the balances of masses of constituents and constituents’ equations of states (Dalton’s law). The studied sources are mixture mass source $S(x, y, z, t)$, mixture energy source $\sigma(x, y, z, t)$, and constituents’ mass sources $S_i^E(x, y, z, t)$ (exhaust gases and consumed oxygen). The boundary-initial problems are of mixed types (Dirichlet’s and von Neumann’s types). The equations of dynamics are solved by the finite difference scheme. The two separate multi-criteria optimization problems are posed by defining the dimensionless functionals of cumulative total travel time, global emissions of pollutants, and global concentrations of pollutants, either in the studied street canyon or in the canyon and its two nearest neighbor substitute canyons. The vector of control is a five-tuple composed of two cycle times, two green times, and one offset time between the traffic lights. The optimal control problems consists of minimization of the two functionals over the admissible control domain manifold.

II. THE GOVERNING EQUATIONS.

Under the abovementioned specifications, the set of governing equations is formulated as follows [7], [8]:

2
\[
\frac{\partial k_{i,vt}}{\partial t} + \text{div}(k_{i,vt} w_{i,vt}) = 0. \tag{1}
\]

\[
w_{i,vt}(x, t) = (w_{i,vt,f} \cdot (1 - \frac{k_{i,vt}(x, t)}{k_{i,vt,jam}}), 0, 0). \tag{2}
\]

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{v}) = S. \tag{3}
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \eta \Delta \mathbf{v} + \left(\xi + \frac{\eta}{3}\right) \nabla (\text{div} \mathbf{v}) + \mathbf{F}. \tag{4}
\]

\[
\rho \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = \left(-\frac{1}{2} \mathbf{v}^2 + \epsilon\right) S + \mathbf{T} : \nabla \mathbf{v} + \text{div}(-\mathbf{q}) + \sigma. \tag{5}
\]

\[
p = \frac{R}{\rho} \cdot m_{\text{air}} \cdot T. \tag{6}
\]

\[
\rho \frac{\partial c_i}{\partial t} + \mathbf{v} \cdot \nabla c_i = S_i^E - c_i S + \sum_{m=1}^{N-1} \{(D_{im} - D_{iN}) \cdot \text{div}[\rho \nabla (c_m + \frac{k_{T,m}}{T} \nabla T)]\}. \tag{7}
\]

\[
p_i = c_i \cdot \frac{m_{\text{air}}}{m_i} \cdot p, \tag{8}
\]

where \(w_{i,vt,f}\) is maximum free flow speed, \(k_{i,vt,jam}\) is jam vehicular density, \(\eta\) is the first viscosity coefficient for the air, \(\xi\) is the second viscosity coefficient, \(\mathbf{F} = \rho \mathbf{g}\) is the gravitational body force density, \(\mathbf{g}\) is the gravitational acceleration, \(D_{im} = D_{mi}\) is the mutual diffusivity coefficient from the \(i\)-th constituent to \(m\)-th one, and \(D_{ii}\) is the autodiffusivity coefficient of the \(i\)-th constituent, and \(k_{T,m}\) is the thermodiffusion ratio of the \(m\)-th constituent, \(\epsilon\) is the mass density of intrinsic (internal) energy of the air mixture, \(\mathbf{T}\) is the stress tensor, symbol : denotes the contraction operation, \(\mathbf{q}\) is the vector of flux of heat. We assume that [2]:

\[
\epsilon = \sum_{i=1}^{N} \epsilon_i, \tag{9}
\]

\[
\epsilon_i = \frac{1}{m_i} \left[ c_i k_B T \exp\left(\frac{m_i |\mathbf{g}| z}{k_B T}\right) \cdot \left(\frac{z}{c}\right) \cdot \left(1 - \exp\left(-\frac{m_i |\mathbf{g}| c}{k_B T}\right)\right) - \exp\left(-\frac{m_i |\mathbf{g}| c}{k_B T}\right)\right] + \tilde{\mu}_i c_i, \tag{10}
\]

\[
\tilde{\mu}_i = \frac{\mu_i}{m_i}, \tag{11}
\]

\[
\mu_i = k_B T \cdot \left[ \ln[(c_i p)(k_B T)^{\frac{z e_i}{2}} \cdot \left(m_{\text{air}} \cdot (2\pi k_B^2)^{\frac{3}{2}}\right)] + m_i |\mathbf{g}| z, \tag{12}
\]

\[
T_{mk} = -p \delta_{mk} + \eta \cdot \left[\left(\frac{\partial v_m}{\partial x_k} + \frac{\partial v_k}{\partial x_m} - \frac{2}{3} \delta_{mk} \text{div}(\mathbf{v}) \frac{\partial v_k}{\partial x_m}\right) + \xi \cdot \left[(\delta_{mk} \text{div}(\mathbf{v}))^2\right], \right. \tag{13}
\]

\[
\mathbf{q} = \sum_{i=1}^{N} \left[\left(\frac{T}{\alpha_{ii} + \tilde{\mu}_i} \mathbf{j}_i\right) + \left[(-\kappa) \nabla T\right], \right. \tag{14}
\]
\begin{align}
\mathbf{j}_i &= -\rho D_{ii}(c_i + \frac{k_{T,i}}{T}\nabla T), \\
\alpha_{ii} &= \frac{[\rho D_{ii}]}{\left(\frac{\partial n_i}{\partial c_i}(c_n)_{n=1,...,N,i\neq n,T,p}\right)}, \\
\beta_i &= [\rho D_{ii}] \cdot \left\{ \frac{k_{T,i}}{T} - \frac{\left(\frac{\partial n_i}{\partial T}(c_n)_{n=1,...,N,i\neq n,T,p}\right)}{\left(\frac{\partial n_i}{\partial c_i}(c_n)_{n=1,...,N,i\neq n,T,p}\right)} \right\},
\end{align}

where \(\epsilon_i\) is the mass density of intrinsic (internal) energy of the \(i\)th constituent of the air mixture, \(m_i\) the molecular mass of the \(i\)th constituent, \(k_B\) is Boltzmann’s constant, \(\mu_i\) is the complete partial chemical potential of the \(i\)th constituent of the air mixture (it is complete since it is composed of chemical potential without external force field and of external potential), \(m_{\text{air}} = 28.966 \ [u]\) is the molecular mass of air, \(\delta_{mk}\) is Kronecker’s delta, \(c_{p,i}\) is the specific heat at constant pressure of the \(i\)th constituent of air mixture, \(h\) is Planck’s constant, \(\mathbf{j}_i\) is the vector of flux of mass of the \(i\)th constituent of the air mixture, and \(\kappa\) is the coefficient of thermal conductivity of air. These magnitudes were derived from Grand Canonical ensemble with external gravitational Newtonian field.

### III. OPTIMAL CONTROL PROBLEMS.

Let us consider the measures of the total travel time (TTT): \(J_{\text{TTT}}\) [5], emissions (E): \(J_E\), and concentrations (C): \(J_C\) of exhaust gases in the street canyon as well as in the canyon and two its nearest neighbor substitute canyons, \(J_{\text{TTT,ext}}\) \(J_{E,\text{ext}}\) \(J_{C,\text{ext}}\), respectively. Therefore the appropriate optimization problems may be formulated as follows [2]. The measures depend on traffic lights at the upstream and downstream ends of the lanes. The vector of boundary control \(\mathbf{u}\) reads:

\[
\mathbf{u} = (g_1, C_1, g_2, C_2, F) \in U^{\text{adm}},
\]

\(g_m\) are green times, \(C_m\) are cycle times, \(F\) is offset time, and \(U^{\text{adm}}\) is a set of admissible control variables. The TTT, E, C functionals are given by:

\[
J_{\text{TTT}}(\mathbf{u}) = \sum_{s=1}^{2n_s} \sum_{l=1}^{n_s} \sum_{v=1}^{V_T} \int_0^{T_S} k_{l,v}^{s}(x,t)dx\,dt. 
\]

\[
J_E(\mathbf{u}) = \sum_{s=1}^{2n_s} \sum_{l=1}^{n_s} \sum_{v=1}^{C_T} \sum_{t=1}^{V_T} \int_0^{T_S} e_{l,v}^{s}(x,t)dx\,dt. 
\]

\[
J_C(\mathbf{u}) = \rho_{\text{STP}} \cdot \sum_{i=1}^{N_k-1} \int_0^{a} \int_0^{b} \int_0^{c} \int_0^{T_S} c_i(x,y,z,t)\,dx\,dy\,dz\,dt. 
\]

\[
J_{\text{TTT,ext}}(\mathbf{u}) = J_{\text{TTT}}(\mathbf{u}) + 
+ a \cdot \sum_{s=1}^{2n_s} \sum_{l=1}^{n_s} \sum_{v=1}^{V_T} k_{l,v,jam}^{s} \cdot (C_s - g_s). 
\]
\[ J_{E,\text{ext}}(u) = J_E(u) + \]
\[ + a \cdot \sum_{s=1}^{2} \sum_{i=1}^{n_s} \sum_{ct=1}^{CT} \sum_{vt=1}^{VT} e_{l,ct,vt,\text{jam}}^s \cdot (C_s - g_s). \]

\[ J_{C,\text{ext}}(u) = J_C(u) + \]
\[ + \rho_{\text{STP}} \cdot a \cdot b \cdot c \cdot \sum_{i=1}^{N_E-1} c_{i,\text{STP}} \cdot \sum_{s=1}^{2} (C_s - g_s). \]

The integrands \(k_{i,ct,vt}^s, e_{l,ct,vt}^s, c_i\) in the functionals depend on the control vector \(u\) through the boundary conditions, through the equations of dynamics, as well as, through the sources. The value of the vector of control \(u\) directly affects the boundary conditions for vehicular densities, velocities, and emissivities. It also affects the sources. Next, it propagates to the equations of dynamics and then it influences the values of functionals. \(\rho_{\text{STP}}\) is the density of air at standard temperature and pressure \(\text{STP}\), \(c_i,\text{STP}\) is concentration of the \(i\)th constituent of air at standard temperature and pressure, \(a, b, c\), are dimensions of the street canyon, \(T_S\) is time of simulation.

We define two additional functionals. The first one is the composite total travel time, emissions and concentrations of the pollutants in the single canyon, whereas the second one in the canyon and two its neighbors:

\[ H(a, u) = \alpha_{\text{TTT}} F_{\text{TTT}}(u) + \alpha_E F_E(u) + \alpha_C F_C(u). \]

\[ H_{\text{ext}}(a, u) = H(a, u) + \]
\[ + \alpha_{\text{TTT,ext}} J_{\text{TTT,ext}}(u) + \alpha_{E,\text{ext}} J_{E,\text{ext}}(u) + \alpha_{C,\text{ext}} J_{C,\text{ext}}(u). \]

The scaling parameters \(\alpha_{\text{TTT}}, \alpha_E, \alpha_C, \alpha_{\text{TTT,ext}}, \alpha_{E,\text{ext}}, \alpha_{C,\text{ext}}\) make the two functionals dimensionless, and

\[ a = (\alpha_{\text{TTT}}, \alpha_E, \alpha_C) \in A_{\text{adm}}, \]
\[ a_{\text{ext}} = (\alpha_{\text{TTT}}, \alpha_E, \alpha_C, \alpha_{\text{TTT,ext}}, \alpha_{E,\text{ext}}, \alpha_{C,\text{ext}}) \in A_{\text{adm,ext}}, \]

where \(A_{\text{adm}}, A_{\text{adm,ext}}\) are the sets of admissible scaling parameters. We formulate two separate multi-criteria optimization problems consisting in minimization of the functionals \(H, H_{\text{ext}}\) with respect to control vector \(u\) over admissible domain and with respect to scaling parameters, while the equations of dynamics are fulfilled:

\[ H^* = H(a^*, u^*) = \min\{a \in A_{\text{adm}}, u \in U_{\text{adm}} : H(a, u)\}. \]

\[ H_{\text{ext}}^* = H_{\text{ext}}(a_{\text{ext}}^*, u_{\text{ext}}^*) = \min\{a_{\text{ext}} \in A_{\text{ext}}, u \in U_{\text{ext}} : H_{\text{ext}}(a, u)\}, \]

where \(H^*, H_{\text{ext}}^*\) are the minimal values of the two functionals, \((a^*, u^*)\), and \((a_{\text{ext}}^*, u_{\text{ext}}^*)\), are scaling and control vectors at which the functionals reach the minima, respectively.
IV. ACKNOWLEDGEMENTS.

The author would like to gratefully thank the organizers of the Comphy02 Computational Physics of Transport and Interface Dynamics conference: Prof. Dr. Heike Emmerich, Britta Nestler, and Michael Schreckenberg, held in the Max-Planck-Institut für Physik komplexer Systeme, Dresden, Germany, for the invitation and creation of scientific atmosphere. The present work was partly covered by the Polish State Committee for Scientific Research KBN grant number F-1/63/BW/02.

V. CONCLUSIONS.

The proecological urban traffic control idea and the model of the street canyon have been developed. It is assumed that the proposed model represents the main features of complex air pollution phenomena.
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