Fusion Power Density and Radiation Losses Characteristics for Tritium Fusion Reactions

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Abstract. A new data for Fusion power density has been obtained for T-\(^3\)He and T-T fusion reactions, power density is a substantial term in the researches related to the fusion energy generation and ignition calculations of magnetic confined systems. In the current work, thermal nuclear reactivities, power densities of a fusion reactors and the ignition condition inquiry are achieved by using a new and accurate formula of cross section, the maximum values of fusion power density for T-\(^3\)He and TT reaction are 1.1\( \times 10^7\) W/m\(^3\) at \(T=700\) KeV and 4.7\( \times 10^6\) W/m\(^3\) at \(T=500\) KeV respectively, While \(Z_{\text{eff}}\) suggested to be 1.44 for the two reactions. Bremsstrahlung radiation has also been determined to reaching self- sustaining reactors, Bremsstrahlung values are 4.5\( \times 10^6\) W/m\(^3\) at \(T=700\) MeV and 3.8\( \times 10^6\) W/m\(^3\) at \(T=500\) MeV for T-\(^3\)He and TT reaction respectively, ignition values then are 136 KeV for T-T and 155 KeV for T-\(^3\)He. So small \(Z_{\text{eff}}\) mean small ignition and large fusion power. Tritium Fusion Reactions have large ignition temperature than deuterium reactions.

Keywords: Tritium, fusion power density, ignition temperature, Bremsstrahlung, reaction rate.

1. Introduction
The acquisition of an important amount of energy from the fusion reactions requires that the fusion plasma is heated to a very high temperatures and confined to sufficient time [1]. Achieving controlled fusion on the earth as a possible energy source meets the world's growing energy needs.

In this paper, we calculate the fusion reaction rates among identical and non-identical nuclei in table (1) for the diverse nuclear burning systems in plasma, and then investigate the impact on the energy production by calculating the fusion power density and radiation loses to determined ignition temperatures.

The plasma is formed at high temperature by the ionized atoms, the fusion power system applicability is decided by the power density where lower power density can be related with higher cost of a proposed fusion power plant [2].
All systems will suffer radiation losses especially through bremsstrahlung. Impurities can be found in most plasmas, these high-Z impurities participate to further losses by increasing bremsstrahlung emissions [3].

It can be considered the thermonuclear reaction is an extraordinary energy amplifier. Energy has to be provided with a view to give ions and the associated electrons a mean energy congruous to the T. On the generation part, the energy liberated by the fusion reaction is hundreds times bigger. At a magnetically confined plasma, the projectile would always interact (100% efficiency) if it remains confined for a time τ equivalent to the mean free path divided by the thermal velocity, with respect to the velocity distribution.

For the reacting nuclei, in fusion, must beat the Coulomb barrier repulsion among them. Because of the Coulomb energy barrier develops higher with growing atomic number, fusion reactions at low Z nuclei can occur easier than of heavier nuclei. In the explanation and simulation of nuclear fusion experimentations, it is oftentimes of regard to deduce the energy spectrum of particles created in various nuclear reactions happening in the fusion plasma, relies on the masses of the particles took part in the reaction, the velocities of the reactants and reaction angle.[4]

Fusion reactivity, reaction rate for CTF reactions of the hydrogen isotopes are attained to arrive promising results in measuring factors that covered the design and construction of a given fusion system or reactor. They are intensely relying upon their operating fuels, the reaction rate, which in turn, reveals the physical behavior of all other parameters characterization of the arrangement design.

Previous studies by researchers such as Karabi Ghosh and S. V. G. Menon have found the ignition temperature from a nuclear thermal fusion reaction for DT pellets [5]. The higher cross section efficacious fusion, mean the bigger the probability of fusion. In general, the probability of fusion rises with kinetic energies of the nuclei. Kinetic energies of nuclei are relative to their temperatures in keV, so the reactions under investigation have the largest cross section nearly peaks at keV temperature revealed in reference [4].

### Table 1. The fusion reactions [3].

| Thermonuclear fusion reactions | Q MeV     |
|-------------------------------|----------|
| $ ^1T + ^2T \rightarrow ^4He + 2nT $ | 11.332   |
| $ T + ^3He \rightarrow D + ^4He $ | 14.320   |
| $ T + ^3He \rightarrow p + ^5He $ | 14.320   |

2. Calculation and results

2.1. Fusion collision cross section and reactivity

The cross section is a very important quantity to assess the nuclear reactions, so we must use a modern formula to get a very accurate results, in which the other fusion energy parameters calculations depends on. This formula is given by equation (1), which should be used with constants ($C_1$, $C_2$, $C_3$) listed in the references [6, 7]. The results of these calculations is depicted as a function of the kinetic temperature.
\[
\sigma(E) = -16389 \, C_3 \left(1 + \frac{M_a}{M_b}\right)^2 \left\{ M_a E_{\text{lab}} \left[ \exp \left(31.4Z_1 Z_2 \left(\frac{M_a}{E_{\text{lab}}}\right)^2\right) - 1 \right] \left[(C_1 + C_2 E_{\text{lab}})^2 + (C_3 - \exp(31.4Z_1 Z_2 \left(\frac{M_a}{E_{\text{lab}}}\right)^2)\right)^{-1}\right\}^{-1}
\]

(1)

\(M_a\) and \(M_b\) are reactant masses, \(Z_1, Z_2\) reactant charges \(E_{\text{lab}}\) projectile energy in the laboratory system. The (three parameter formula) of cross section equation (1) have a good results conforming standard published data. As the cross sections of some reactions shows the required energies up \(\sim 10\) keV conforming to a temperature of \(\sim 10^8\) K.

The cross sections fusion of the reactions that set in table (1) are defined by using equation (1) for range of energies from \(1\) to \(10^3\) KeV, and the calculations are evidently tell us that: The cross section of the \(T^3\)He reaction amount to an extreme (0.24 barn) near \(10^3\) KeV at value bigger than T-T, this climax is because of a resonance influence (excited state of the compound nucleus) in nuclear procedure, so \(T^3\)-He reaction here is obviously favorable [4]

Fusion reaction rate is determined to reach promising data for the useful factors for design and construction of a fusion systems or reactors. If there are two kinds of reactants 1 and 2, then the number of densities \(n_1\) and \(n_2\), so reaction rate is [8]:

\[
R = \frac{1}{1 + \delta_{12}} n_1 n_2 < \sigma v >
\]

(2)

The term \(\delta_{12}\) (known as \(\delta_{12}= 1\) otherwise \(\delta_{12}= 0\)). To calculate the thermonuclear reactivity \(< \sigma v >\) for a given reactions, we have [9]:

\[
<\sigma v> = (8/\pi mc^2T^3)^{1/2}c \int_0^\infty E \sigma(E_{C.M}) e^{(-E/T)}dE
\]

(3)

where \(c\) is light velocity, we can use the reduced mass and express the cross section in terms of centre of mass (\(C.M\)) coordinate. The fusion reactivity curves shown in the following figure (1a,b) and table (2).

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**Figure 1.** Fusion reactivity for (T-\(^3\)He two branch) and (T-T) reactions.
Table 2. \( <\sigma v> \) for T - \(^3\)He and T-T.

| T (KeV) | \( <\sigma v> \) | T - \(^3\)He Two Branch | T-T |
|---------|----------------|-------------------------|------|
| 1       | 4.9924e-027    | 9.1731e-026             |      |
| 10      | 2.8847e-026    | 4.7628e-025             |      |
| 30      | 2.3171e-025    | 2.2968e-024             |      |
| 50      | 1.0085e-024    | 5.6616e-024             |      |
| 70      | 2.6914e-024    | 9.9000e-024             |      |
| 100     | 7.0908e-024    | 1.6882e-023             |      |
| 200     | 3.2997e-023    | 3.9209e-023             |      |
| 400     | 7.4556e-023    | 6.1539e-023             |      |
| 500     | 8.1688e-023    | 6.3493e-023             |      |
| 700     | 8.2054e-023    | 6.0049e-023             |      |
| 800     | 7.9120e-023    | 5.6936e-023             |      |
| 1000    | 7.1480e-023    | 5.0328e-023             |      |

From Eq. (2) the following conclusions can be drawn:
(1) All the reaction rates indicate steep temperature reliance among 102 and 103 KeV.
(2) The height of the Coulomb potential barrier is also a main factor in fixing the absolute magnitude of the reaction at a given temperature. It is clear from the calculated \( <\sigma v> \) that is described in equation (3) would seem or show compatible agreement with the published experimental results. We came to the fact that the mathematical results vary from experimental and semi-experimental results, particularly in certain conditions that happen in the experimentation and losses in all parts of the plasma devices and because of parameterizations that used for modeling \( \sigma(E) \) equation.

2.2. Fusion Power Density.

The core of the CTF is to confine the plasma of hydrogen isotopes for a long time so that the \( <\sigma v> \) becomes high sufficient to produce the preferred power. The fusion power density is a useful parameter to reach a best temperature for magnetic confinement, its proportional to \( n_1 \), so the plasma must confine in small space. Based on using the reactivity and the number densities of reacting kinds, one can measure the fusion power density \( P_{fus} \) multiplying the reaction rate given in equation (2) by the energy \( Q \) of the reaction, the produced power by thermonuclear reactions can be written as [10]:

\[
P_{fus} = RQ
\]

\[
P_{fus} = \frac{1}{1+\delta_{12}} n_1 n_2 <\sigma v > Q
\]

In equation (5) we supposed the reaction occurs between two kinds of reactants with identical numbers of density and electron density is identical the total ion density.

From above assumptions we have \( n_0 = n_T = 1/2n_e \). for T-T ignition state, we need an amount of energy to produce and storage energy equal to exiting energy \( E_{ext} \), the energy of the producing charged particles will contribute to heating plasma, produced neutrons don’t contribute in heating plasma, so \( E_{ext} > P_{loss} \). The reactor must produce energy more than need to heat plasma and operate device, but there is a many energy loses inside fusion reactors the most important one is bremsstrahlung losses equation (6), then, the reactor must overcome the energy loses to reach ignition point.

And in unit of watt/m\(^3\) bremsstrahlung losses can be written as [11]:

\[
P_{br} = 4.8 \times 10^{-37} n^2 Z_{eff} T^{4/2} \text{ W/m}^3
\]
Where

\[ Z_{\text{eff}} = \frac{\sum n_i z_i^2}{\sum n_i z_i} \]  

Table (3) show the fusion power density and Bremsstrahlung for T-T reaction.

**Table 3. Fusion power density and Bremsstrahlung for T-T.**

| T (KeV) | \( P_{\text{fus}} \times 1.0e+06 \) | \( P_{\text{br}} \times 1.0e+06 \) |
|---------|-----------------------------------|-----------------------------------|
| 1       | 0.0068                            | 0.1728                            |
| 10      | 0.0355                            | 0.5464                            |
| 30      | 0.1711                            | 0.9465                            |
| 50      | 0.4218                            | 1.2219                            |
| 70      | 0.7376                            | 1.4457                            |
| 100     | 1.2578                            | 1.728                             |
| 200     | 2.9213                            | 2.4438                            |
| 400     | 4.5851                            | 3.456                             |
| 500     | 4.7307                            | 3.8639                            |
| 700     | 4.4741                            | 4.5719                            |
| 800     | 4.2421                            | 4.8875                            |
| 1000    | 3.7498                            | 5.4644                            |

\( Z_{\text{eff}} \) is the effective ion charge which describes the pollution of plasma components. \( Z_{\text{eff}} \) very important parameter and must be under a certain value to attain a felicitous ignition since growth of plasma pollution increases bremsstrahlung losses [12]. So the net resulting energy will be influenced by the heat radiated from plasma. Figure (2 a, b) show the fusion power density and bremsstrahlung calculations as a function of kinetic energy, assumption that densities are about 5\( \times \)10^20 m^-3.

**Figure 2.** The ignition temperature for the T-^3He and T-T fusion (higher fusion-born energy deposition in the fuel and radiation losses).
3. Discussion and conclusion

The power gains depend on densities, incident energy of reactants and calculated reaction rate with respect to velocity distribution function. Only charged products yield gains. Reaching to self-sustainability circumstances for T-3He (both branches) and T-T, when the temperature exceed a particular crossing point (ignition point), where power gains by charged particles (blue curves) greater than losses (red curves). It is noteworthy, the ignition point not a fixed point, so when the temperature overtake ignition, the reactor produce energy more than it used, the plasma gain and loss curves crossing again in the stable equilibrium point. After the temperature increases, the medium go towards equilibrium. So when we cooling down the plasma, the medium back to production zone and then we can controllable the reactor. The value of $Z_{eff}$ must kept all time at least. We suggest the $Z_{eff}$ value in the table (4) to get a best ignition.

Net fusion power and bremsstrahlung have the same dependence upon number density. Each nuclear reaction has a peak as in the figure (2 a,b) where we get the greatest energy. The threshold point (ignition point) is affected by $Z_{eff}$. The values in the table (4) are adopted in the ignition point calculations.

Table 4. The effective charge $Z_{eff}$ that used for calculations.

| Fusion reaction | The effective charge $Z_{eff}$ |
|----------------|-------------------------------|
| T-T            | 1.44                          |
| T-3He          | 1.44                          |

The ranges for which gains are exceeding losses are conclude from figure (2 a,b). Power and losses per unit volume proportional to the temperatures. The self-sustainability thresholds for fusion are revealed according to more precise calculations, T-3He (both branches) and T-T temperatures should exceed 100 keV as table (5).

Table 5. Calculated ignition temperatures for T-3He (both branches) and T-T reactions.

| Fusion reaction | Ignition temperature KeV |
|----------------|--------------------------|
| T-3He          | 136                      |
| T-T            | 155                      |

There are radiation losses through bremsstrahlung, synchrotron radiation, line and recombination radiation. In a pure plasma, line and recombination radiation do not play an essential role except for a lot cooler plasma boundary zone, because all ions are fully ionized and the central plasma is too hot for recombination. The status is diverse if the plasma is polluted by nuclei of bigger charge number. We rather crudely take into consideration such radiation only, using for it equation (6) with some active charge number for the impurities. This means that we must have to heat a T plasma to a temperature greater than $10^8$ K or we would have to begin with D-T mix. Otherwise, the bremsstrahlung losses would cool the plasma and the nuclear fire would go out.

In addition to radiation losses, many mechanisms partake to energy losses like connection with a wall and escaping particles. Losses may occur from the whole volume or through the surface. Radiative volume losses are basically bremsstrahlung, a sequel of electron-ion Coulomb collisions. In the case of fusion plasmas, bremsstrahlung losses are an inevitable obstacle.
Once the selection of fusion reactions have been made as the T-3He or T-T reaction, the significant technical problem is to confine a plasma at a temperature of various million degrees inside a fixed volume. No usual container like steel, glass or concrete could resist a big temperature like that. We would like to indicate out that all the objectives for this study have been achieved, starting with the calculations of the cross sections are the entry gate to deduce energy distribution function and reaction rate of fusion. The last parameter very important for power generation and radiation losses calculations. We suggested specific values for $Z_{\text{eff}}$ in fusion plasma. Finally, we calculate and introduce a novel calculations for T-T and T-3He fusion reactions which are considered a novel calculations.

One would like to find, in laboratory, the probability of preserving controlled thermonuclear reaction with least number of reacting nuclei. Special attention must be taken to avoid the loss of energy and particles from such systems. The farthest significant problem encountered is the design of systems where the reacting material is governed by the use of electromagnetic field. The second significant problem is that of discovering a fusion reaction that would occur at a low temperature sufficient for electromagnetic containment to be possible.

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