We study the asymptotic behavior of the bulk spacetimes with the negative cosmological constant in the context of the brane-world scenario. We show that, in Euclidean bulk, or in Lorentzian static bulk, some sequences of hypersurfaces with the positive Ricci scalar evolve to the warped geometries like the anti-de Sitter spacetime. Based on the AdS/CFT correspondence, we discuss that the positivity of the Ricci scalar is related to the stability of CFT on the brane. In addition, the brane-world is described from the holographic point of view. The asymptotic local structure of the conformal infinity is also investigated.

I. INTRODUCTION

The recent progress of M/String theory suggests a new picture that our universe is described by a domain wall in a higher dimensional spacetime. The simplest and comprehensive model have been proposed by Randall and Sundrum [1,2]. Therein our universe is realised as the three-brane (four-dimensional Minkowski spacetime) embedded in the five-dimensional anti-de Sitter (AdS) bulk spacetime.

The brane has the fine-tuned non-zero tension to cancel the bulk cosmological constant. One might expect that the brane inherits the gravity from the higher-dimensional bulk spacetime. We can, however, check that the gravity is indeed localised on the brane; namely the usual Newton law of gravitation is realised at least on the low energy scales [1–6]. There is also the exact cosmological solution based on the brane-world scenario, which is given by the domain wall motion in the five-dimensional Schwarzschild–AdS spacetime [7,8]. The warped compactification is also used in the holographic renormalisation [9]. In the same sense as Witten suggested, the brane-world contains the aspect of the AdS/CFT correspondence [10], in which the bulk graviton is identical to CFT with the cut-off on the brane [11,12].

In this brane-world scenario, the warped geometry is essential for the mass hierarchy problem [1] and the localisation of gravity [2]. This is also preferred for the cosmological constant problem [4]. This mechanism can be contrasted with that of the inflationary scenario, in which the rapid cosmological expansion in the early universe resolves the homogeneity, horizon and flatness problems [5]. It is believed that an initially inhomogeneous universe evolves to a homogeneous isotropic state during the inflation. In other words, there is the so-called cosmic no-hair (baldness) conjecture, which states that all expanding universes approach at least locally the de Sitter spacetime, if there is a positive cosmological constant. The cosmic no-hair conjecture is confirmed in several cases [16,17].

Bearing the inflation in mind, one might expect that the warped geometry of the brane-world model is the general effect of the negative cosmological constant; namely, the bulk spacetime might be governed by the AdS no-hair theorem in some sense, and this is the subject we consider here. We will adopt the following geometrical approach; the brane-world is just the slicing of hypersurfaces in asymptotically AdS spacetimes. The slice or several slices are regarded as the branes where we or hidden matters are living. The extrinsic curvature determines the matter distribution including the tension on the brane. In this geometrical picture, it is important to study the general features of asymptotic structure.

This paper is organised as follows. In section II we show that the bulk geometry foliated by probe branes with positive or zero Ricci curvature approaches the standard AdS form asymptotically in the cases of the static bulk and the Euclidean bulk. Then, we consider the validity of the assumption \( R \geq 0 \), and discuss the relation to the AdS/CFT correspondence. In section III, we also show the asymptotic local structure of the conformal infinity. The features obtained there are appropriate for the mass hierarchy problem and so on.

II. NO-HAIR AND ADS/CFT

Our strategy is as follows: First, we consider the full geometry regardless of the branes. Then, choosing slices appropriately, we may be able to identify them with the branes where we or hidden matters are living. After some cutting and pasting, this results in the brane-world model.
A. The No-Hair

First of all, we show that the bulk geometry “evolve” to the AdS spacetime. Here we consider the “evolution” of the bulk in the Euclidean case and the Lorentzian static case. The discussion in this subsection is reminiscent of the Wald’s cosmic no-hair theorem for homogeneous universe with negative Ricci spatial curvature (namely, for the Bianchi IX model) \[17\].

We assume that the bulk is foliated by the geodesic slices\[5\]:
\[
(5) \quad g_{ab} dx^a dx^b = dy^2 + g_{\mu\nu}(y, x) dx^\mu dx^\nu,
\]
where \( \{x^{\mu}\} = \{x^0, \cdots, x^3\} \) and \( y = x^4 \) is the coordinate of the extra dimension. We have two basic equations: One is the equation for the trace of the extrinsic curvature, \( \dot{K} \):
\[
\dot{K} = \frac{4}{\ell^2} - \frac{1}{4} K^2 - \sigma_{\mu\nu} \sigma^{\mu\nu}, \tag{2}
\]
where the dot denotes the derivative with respect to \( y \), and
\[
\sigma_{\mu\nu} = K_{\mu\nu} - \frac{1}{2} g_{\mu\nu} K
\]
is the trace-free part of the extrinsic curvature. Another is the Hamiltonian constraint:
\[
R - \frac{3}{4} K^2 + \sigma_{\mu\nu} \sigma^{\mu\nu} = -\frac{1}{\ell^2}, \tag{4}
\]
In the above derivations, we have assumed the five-dimensional Einstein equation for the bulk:
\[
(5) \quad R_{ab} - \frac{1}{2} g_{ab} R = -\frac{6}{\ell^2} (5) g_{ab}. \tag{5}
\]
In the Euclidean, and the Lorentzian static case, we can see that the inequality
\[
\sigma_{\mu\nu} \sigma^{\mu\nu} \geq 0 \tag{6}
\]
holds. Using Eq \[2\] together with the above inequality and assuming \( R \geq 0 \) and \( K > 0 \) (\( y = y_0 \) chosen arbitrarily), we obtain
\[
K \rightarrow \frac{4}{\ell}. \quad (y \rightarrow +\infty) \tag{7}
\]
The Hamiltonian constraint \[3\] therefore implies
\[
\sigma_{\mu\nu} \rightarrow 0. \quad (y \rightarrow +\infty) \tag{8}
\]
From Eqs \[2\] and \[3\], the metric of slices behaves as
\[
g_{\mu\nu}(y, x) \rightarrow e^{(y-y_0)/\ell} h_{\mu\nu}(y_0, x). \quad (y \rightarrow +\infty) \tag{9}
\]
Then we naively conclude that the bulk geometry evolve toward the warped geometries like the AdS spacetime. Since the metric of the terminal hypersurface \( y = y_0 \) is not fixed in the above argument, the bulk geometry does not necessarily evolve to the exact AdS.

This theorem can be easily extended to the cases with bulk fields: Eqs \[2\] and \[3\] are modified as
\[
\dot{K} = \frac{4}{\ell^2} - \frac{1}{4} K^2 - \sigma_{\mu\nu} \sigma^{\mu\nu} - \kappa_3^2 \left( T^{(\text{bulk})}_{ab} - \frac{1}{3} g_{ab} T^{(\text{bulk})} \right) n^a n^b, \tag{10}
\]
and
\[
R - \frac{3}{4} K^2 + \sigma_{\mu\nu} \sigma^{\mu\nu} + 2\kappa_3^2 T^{(\text{bulk})}_{ab} n^a n^b = -\frac{12}{\ell^2}, \tag{11}
\]
where \( n = \partial_y \), and we have used
\[
(5) \quad R_{ab} - \frac{1}{2} (5) g_{ab} R = -\frac{6}{\ell^2} g_{ab} + \kappa_3^2 T^{(\text{bulk})}_{ab}. \tag{12}
\]
If we suppose the additional conditions
\[
T^{(\text{bulk})}_{ab} n^a n^b \geq 0 \tag{13}
\]
and
\[
\left( T^{(\text{bulk})}_{ab} - \frac{1}{3} g_{ab} T^{(\text{bulk})} \right) n^a n^b \geq 0, \tag{14}
\]
then we obtain the similar theorem. The condition Eq \[13\] is naively identical with the reasonable condition that the effective pressure is positive in the bulk. On the other hand, the condition Eq \[14\] requires the condition \( \rho - P \geq 0 \) for the effective pressure \( P \) and energy density \( \rho \) in the bulk.

We have assumed that \( R \geq 0 \) holds to show the AdS no-hair theorem. In the next subsection, we examine this assumption in several aspects.

Finally we describes two examples for no-hair argument. (i) five-dimensional Schwarzschild-AdS spacetime: The metric is
\[
d s_5^2 = -f(r) dt^2 + f^{-1}(r) dr^2 + r^2 d\Sigma_3^2, \tag{15}
\]
where \( f(r) = k - \mu/r^2 + (r/\ell)^2 \) and \( k = 0, \pm 1 \). Let us consider the foliation of \( r = \) constant hypersurfaces having the Ricci scalar:
\[
R = 6k/r^2. \tag{16}
\]

*For example, the holographic renormalisation group and the AdS/CFT correspondence are often formulated in the Euclidean spacetime which offers the regular boundary condition. In this sense, the consideration of the Euclidean bulk is significant. In general, we may expect that the consideration of the Euclidean region gives us the relevant boundary condition \[13\].

†We know that this foliation can work very well locally. But, in general cases, we cannot use it globally because the focusing point occurs due to the presence of inhomogeneities.

‡In the static case, \( \sigma_{0i} = 0 \) for \( i = 1, 2, 3 \).
The trace of extrinsic curvature is
\[ K = \frac{4}{\ell} \left[ 1 + \left( \frac{e}{r} \right)^2 + \cdots \right]. \tag{17} \]

The shear becomes
\[ \sigma_{tt} = -\frac{3k}{\ell} \left( \frac{e}{r} \right)^2 \tag{18} \]
and
\[ \sigma_{ij} = \frac{k}{4\ell} \left( \frac{e}{r} \right)^2 \delta_{ij}. \tag{19} \]

Regardless of the signature of \( R \), the geometry evolve to the AdS-like geometry.

(ii) Sol Bianchi VI-\(1\): The metric is \([19]\)
\[ ds^2 = -\frac{4}{3} \frac{r^2}{\ell^2} dt^2 + \frac{3}{4} \frac{r^2}{\ell^2} dr^2 + \frac{1}{2} \left( r^2 (e^{2\sigma} dx^2 + e^{-2\sigma} dy^2) + dz^2 \right). \tag{20} \]
Let us consider the foliation of \( r = \text{constant} \) hypersurfaces having the Ricci scalar:
\[ R = -\frac{4}{\ell^2}. \tag{21} \]

The trace of extrinsic curvature is
\[ K = 2\sqrt{3} \frac{1}{\ell}. \tag{22} \]

The Ricci scalar is negative and \( K \) does not approach to the value \( 4/\ell \). This is a case which will not evolve to AdS-like geometry.

B. AdS/CFT and the Ricci Curvature on the Brane

To see the signature of the Ricci scalar, \( R \), on each hypersurfaces, we use the gravitational equation on the brane \([13,22]\). It is derived using the Gauss equation, which connects the five-dimensional Riemann tensor with that in four dimensions, and the Israel’s junction condition \([20]\):
\[ G_{\mu\nu} = 8\pi G_4 T_{\mu\nu} + \kappa_5^2 \pi_{\mu\nu} - E_{\mu\nu}, \tag{23} \]
where
\[ \pi_{\mu\nu} = -\frac{1}{4} T_{\mu\alpha} T_{\nu}^{\alpha} + \frac{1}{12} T T_{\mu\nu} + \frac{1}{8} g_{\mu\nu} T_{\alpha\beta} T^{\alpha\beta} - \frac{1}{24} g_{\mu\nu} T^2 \tag{24} \]
and
\[ E_{\mu\nu} = (5) G_{\mu\nu\beta} \eta^\alpha_\beta. \tag{25} \]

We set the net cosmological constant on the brane to be zero. To evaluate \( E_{\mu\nu} \) we have to solve it globally \([13,22,24]\). The trace part can be written in terms of the four-dimensional quantities:
\[ R = -8\pi G_4 T - \kappa_5^2 \left( \frac{1}{4} T_{\mu\nu} T^{\mu\nu} - \frac{1}{3} T^2 \right) \tag{26} \]

On the other hand, from AdS/CFT, we can obtain the following effective equation on the brane \([11,12,13,22]\):
\[ G^{\mu\nu} = 8\pi G_4 T^{\mu\nu} + \frac{4}{\ell} \frac{1}{\sqrt{|g|}} \left( \delta S_{\text{cl}}^{(4)} \Gamma_{\text{CFT}} + \delta \Gamma_{\text{CFT}} \right), \tag{27} \]
where \( \Gamma_{\text{CFT}} \) is the effective action of CFT living on the boundary and has the trace anomaly \([12,13,22]\):
\[ \frac{\delta \Gamma_{\text{CFT}}}{\delta g_{\mu\nu}} g^{\mu\nu} = \frac{\ell^3}{16} \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right). \tag{28} \]
The quantity \( S_{\text{cl}}^{(4)} \) is \( R^2 \) terms of the counter-term which makes the action finite. For our purpose, we need not to write down the explicit form (See Ref. \([13]\)). What we will use is that \( \delta S_{\text{cl}}/\delta g_{\mu\nu} \) is traceless. Then, the trace part of Eq \((27)\) is
\[ R = -8\pi G_4 T - \frac{\ell^2}{4} \left( R_{\mu\nu} R^{\mu\nu} - \frac{1}{3} R^2 \right) \tag{29} \]

Using the Einstein equation, \( G_{\mu\nu} = 8\pi G_4 T_{\mu\nu} \), approximately, we can check that Eq \((24)\) is approximately identical with Eq \((29)\). It is remarkable that \( \rho^2 \) terms appeared in the cosmological solution of the brane-world can be regarded as the non-linear contribution from CFT. In the linear order we easily obtain the relation between the energy-momentum tensor of CFT and a part of Weyl tensor:
\[ E^\mu_\nu \approx -\frac{4}{\ell} \frac{1}{\sqrt{|g|}} \frac{\delta \Gamma_{\text{CFT}}}{\delta g_{\mu\nu}}. \tag{30} \]

This has been suggested by Witten and firstly confirmed by Gubser \([11]\). The elegant formulation has been done in Refs. \([11,13,22]\). This is applied to the cosmology with large N CFT \([22]\). The generalised Friedmann equation has the dark-radiation term, which comes from the mass term of the five-dimensional Schwarzschild-AdS spacetime. Indeed, we can check that the entropy of the dark radiation is exactly same as the black-hole entropy \([3]\). This is a realisation of the AdS/CFT in the brane-world.

From now on we consider the signature of \( R \). For simplicity, we consider the massless scalar field on the brane:
\[ T_{\mu\nu} = \partial_{\mu}\varphi \partial_{\nu}\varphi - \frac{1}{2} g_{\mu\nu} \partial_{\lambda}\varphi \partial^{\lambda}\varphi \tag{31} \]

\[ ^3 \text{Some class of the homogeneous-isotropic cosmological solution is described by the motion of the domain wall in the five-dimensional AdS spacetime. In this case, } E_{\mu\nu} = 0 \text{ and the equation for the scale factor can be derived from Eq (24).} \]
\[ T^\mu_\nu = -\partial^\mu \varphi \partial_\nu \varphi \leq 0 \]  

(32)

and

\[ \pi^\mu_\mu = \frac{1}{6} (\partial^\mu \varphi \partial^\mu \varphi)^2 \geq 0. \]  

(33)

In the linear order of \( T^\mu_\nu \) we see that \( R \geq 0 \) holds. However, \( \pi^\mu_\mu \sim \rho^2 \) term dominates in the very early universe. As stated before, \( \pi^\mu_\mu \) is related to the trace anomaly of CFT living on the brane. This means that \( \pi^\mu_\mu \) is quantum effect in some sense. To check the signature of \( T^\mu_\nu \) and \( \pi^\mu_\mu \), we must consider the concrete model of the matter on the brane. This seems to be hard task in general.

We may be able to use the Witten-Yau argument \[^26,27\], where they showed that the boundary with the positive Ricci scalar will be stable for large \( y \) (near the infinity). They discussed the fluctuation of the brane which the mass term of the fluctuation mode is proportional to the Ricci scalar on the brane. This therefore seems to justify the assumption of the positive Ricci scalar. At first glance, however, their argument on the stability seems inappropriate for the present purpose, because they do not include the higher curvature term, \( R^2 \). Fortunately, it suddenly turns out that this is not the case for four-dimensional branes. You can check that, in four-dimensions, the Witten-Yau argument does not depend on whether \( R^2 \) term is included.

One might want to consider only background geometry because the holographic argument is often concentrated on the probe brane in the fixed background. In this case, we can set \( T^\mu_\nu = 0 \), which automatically implies \( R = 0 \). Thus, we can anyway conclude that a no-hair theorem described here holds at least for the empty brane case.

III. THE CONFORMAL INFINITY AND THE NO-HAIR

In this section, we assume the existence of the infinity and we briefly consider the detail of the asymptotic structure of the conformal infinity. Note that we did not assume the existence in the previous section. To discuss the geometry near the infinity, we will use the so-called conformal completion formalism \[^28,31\]. We focus on the local differential geometrical structure near the infinity \[^2\]. We do not impose the positive Ricci scalar condition.

**Apart from asymptotically flat spacetimes \[^2\], the spacetimes with the cosmological constant needs the extra assumption to derive the asymptotic symmetry and the conserved energy \[^23\]. We will not discuss this point.

\[ \Theta := \nabla_a \bar{n}^a \]

- \[ -(n - 1) \bar{N}_0 - \frac{n + 1}{2n(n - 1) \bar{N}_0} \Omega (\vec{R}^\theta + \Omega^2) \]  

(36)

and

\[ \Sigma^a_\mu = \Sigma_{ab} (e_\mu)^a (e_\nu)^b \]

- \[ = \left[ q_{ac} \nabla^c \bar{n}^a - \frac{1}{n - 1} q_{ab} \nabla^c \bar{n}^c \right] (e_\mu)^a (e_\nu)^b \]  

(37)

where \( \{ e_\mu \} \) are the orthonormal base vectors of \( (M, \hat{g}) \). \( \bar{n}^a \) is the unit normal vectors defined by

\[ \bar{n}^a = \frac{(\hat{g}^{ab} \nabla_b \Omega)}{(\nabla_c \Omega \nabla^c \Omega)^{1/2}} \]  

(38)

and \( q_{ab} = \hat{g}_{ab} - \bar{n}_a \bar{n}_b \). From the above formulas, we can conclude that the geometry near infinity has the similar structure to that of the AdS spacetime.

Finally, we investigate on the behavior of the Weyl tensor. To do so we use the useful equation derived from the Bianchi identity \[^24\].

\[^\dagger\dagger\]This definition is straightforward one from the definition of asymptotically 4-dimensional AdS spacetimes \[^28,30\]. It should be noted, however, that this definition is rather weak so that the black-string solutions \[^32\] are included.
\[ \Omega \hat{\nabla} [a \hat{S}_{bc}] + \Omega^{-2} \hat{g}_{[a} L_{b]c} \hat{a}^d \]
\[ + \frac{n - 2}{2} \hat{C}_{abc\rho} \hat{a}^d = \hat{\nabla} [a (\Omega^{-1} L_{b]c}), \]
where
\[ S_{ab} = (n) \hat{R}_{ab} - \frac{1}{2(n - 1)} \hat{g}_{ab} (n) \hat{R} \]
and
\[ L_{ab} = \Omega^2 (n) R_{ab} - \frac{1}{2(n - 1)} (n) g_{ab} (n) \hat{R}. \]
From Eq (39), we obtain
\[ (n) C_{\mu\nu\lambda\rho} \hat{a}^\sigma = 0, \]
and then
\[ (n) E_{\mu\nu} := (n) C_{\mu\nu\lambda\rho} \hat{a}^\lambda \hat{a}^\rho = 0, \]
\[ (n) B_{\mu\nu\lambda} := q_{\rho} \rho (n) C_{\mu\nu\lambda\rho} \hat{a}^\rho = 0, \]
on \mathcal{I}. Note however
\[ (n) C_{\mu\nu\lambda\rho} \neq 0, \]
in general even on \mathcal{I}. There is the substantial difference from the \((n = 4)\)-dimensions, in which \(^{(4)} C_{\mu\nu\lambda\rho} = 0\) on \mathcal{I} [24].

**IV. SUMMARY**

In this paper, we have shown that the bulk spacetimes with negative cosmological constant naively evolve to the anti-de Sitter spacetime. However, the induced structure of the brane cannot be fixed. What we can conclude is only that the local structure on the brane will be relatively spread due to the negative cosmological constant. This statement is useful to discuss the mass hierarchy problem, the localisation of gravity and the cosmological constant problem.

In order to show the no-hair theorem, we have used the positivity condition of the Ricci scalar of probe branes. This might be justified via the Witten-Yau’s stability argument [24]. It is stressed that the no-hair theorem is valid when we consider the foliation by empty probe branes.

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**APPENDIX A: CALCULATIONS**

The conformal transformation of Ricci tensors gives us
\[ \Omega^{-2} \left( R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R + \Lambda \eta_{\mu\nu} \right) \]
\[ = \hat{R}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \hat{R} + (n - 2) \Omega^{-1} \hat{\nabla}_{\mu} \hat{\nabla}_{\nu} \Omega \]
\[ - (n - 2) \Omega^{-1} \eta_{\mu\nu} \hat{\nabla}_{\alpha} \hat{\nabla}^\alpha \Omega \]
\[ + \frac{(n - 1)(n - 2)}{2} \Omega^{-2} \hat{\nabla}_{\alpha} \Omega \hat{\nabla}^\alpha \Omega_{\eta_{\mu\nu}} + \Omega^{-2} \Lambda \eta_{\mu\nu}. \]  
\[ \text{(A1)} \]
Looking at the \( \Omega \)-dependence in Eq (A1), we see
\[ \hat{N}^2 := \hat{g}_{\mu\nu} \hat{\nabla}^\nu \hat{\nabla}^\mu = - \frac{2 \Lambda}{(n - 1)(n - 2)} + O(\Omega) \]  
\[ \text{(A2)} \]
\[ \hat{\nabla}_{\mu} \hat{\nabla}^\mu = \frac{1}{n - 2} \hat{\nabla}^\mu \hat{\nabla}_{\mu} = \frac{1}{n - 2} \Omega^{-1} \left( \hat{R}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \hat{R} \right) \]
\[ - \frac{\Omega}{n - 2} \left( \hat{R}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \hat{R} \right) \]  
\[ \text{(A3)} \]
and
\[ \hat{\nabla}_{\mu} \hat{\nabla}^\mu = - \frac{1}{2(n - 1)} \Omega \hat{R} - \frac{1}{(n - 1)(n - 2)} \hat{g}_{\mu\nu} \hat{g}^{\mu\nu} \Omega^{-1} \]
\[ + \frac{n}{2} \Omega^{-1} \left[ \hat{\nabla}_{\mu} \hat{\nabla}^\mu + \frac{2}{(n - 1)(n - 2)} \Lambda \right]. \]  
\[ \text{(A4)} \]
Using the conformal rescaling, \( \Omega \rightarrow \tilde{\Omega} = \Omega \omega \), we obtain the following transformation:
\[ \hat{\nabla}_{\omega} \hat{\nabla}^{\omega} = \frac{1}{\omega} \hat{\nabla}_{\omega} \hat{\nabla}^{\omega} + \frac{n}{\omega^2} \hat{\nabla}^\omega \hat{\nabla}_{\omega} \omega \]
\[ + (n - 2) \Omega^{-3} \hat{\nabla}_{\omega} \hat{\nabla}^\omega \omega + \frac{\Omega}{\omega^2} \hat{\nabla}_{\omega} \hat{\nabla}^\omega \omega. \]  
\[ \text{(A5)} \]
Let us choose \( \omega \) so that \( n \omega^{-1} \hat{\nabla}^\alpha \hat{\nabla}_{\alpha} \omega = - \hat{\nabla}_{\alpha} \hat{\nabla}^\alpha \omega = 0 \). Then we are resulted in \( \hat{\nabla}_{\omega} \hat{\nabla}^\omega = O(\Omega) \). Moreover, using of the rescaling, \( \omega = \omega_0 + \Omega^2 f(\omega) \) such that \( \hat{\nabla}^\omega \hat{\nabla}_{\omega} \omega_0 = 0 \), and choosing the function \( f(\omega) \) appropriately, we can show
\[ \hat{\nabla}_{\mu} \hat{\nabla}^\mu = O(\Omega^2). \]  
\[ \text{(A6)} \]
From Eq (A1) we then see
\[ \hat{\nabla}_{\omega} \hat{\nabla}^\omega + \frac{2}{(n - 1)(n - 2)} \Lambda = \frac{\Omega^2}{n(n - 1)} \hat{R} + O(\Omega^3). \]  
\[ \text{(A7)} \]
Since we can write \( \hat{\nabla}_{\omega} \hat{\nabla}^\omega \) as
\[ \hat{\nabla}_{\omega} \hat{\nabla}^\omega = -(n - 1) \hat{N} + \frac{\Omega}{N} \hat{\nabla}_{\alpha} \hat{\nabla}^\alpha \hat{N} - \frac{\Omega}{N^2} \hat{\nabla}^\alpha \hat{\nabla}_{\omega} \hat{N}, \]  
\[ \text{(A8)} \]
we obtain Eq (B2) in the text. In the same way, we can derive Eq (B5).
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