Numerical simulation and stability analysis of pumping transients in the pumped-storage power plant

Yang Zheng, Qijuan Chen, Wanying Liu, Donglin Yan
School of Power and Mechanical Engineering, Wuhan University, No.8 South Donghu Road, Wuhan, 430072, P R China
E-mail: zhengyang@whu.edu.cn

Abstract. The pumped-storage unit (PSU) are of great significance in modern power system for its reversible operating mechanism in both power generation and water pumping directions. Aiming at the precise numerical simulation of pumping transient processes in PSU, this paper establishes a nonlinear and characteristic method-based mathematical model of the governing system of PSU in which the hydraulic behaviours of conduit pipes, surge tanks and the pump turbine as well as the mechanical-electrical characteristics of the generator-motor are taken into consideration. In this model, local hydraulic losses in model boundaries are specifically introduced to compensate for the unmodeled errors. To verify the modelling accuracy, simulation curves under both pump failure and pump shutdown conditions are compared with the practical data. Furthermore, the hydraulic oscillations in the two transient processes have been discussed specifically.

1. Introduction
Over the past few decades, pumped-storage unit (PSU) has become one of the most practical peak and frequency modulation sources in modern power systems. It has attached great significance to the balance of the supply and demand in power grids, for the dramatically increasing large-scale penetration of intermittent renewable energies like solar energy and wind power has greatly influenced the system operation nowadays [1]. Compared with the traditional hydropower unit, PSU boasts much more operating modes such as the pumping mode and the phase modulation states at both generation and pumping directions, bringing new challenges to its nonlinear system modeling. Therefore, improving modeling precision of the PSU is an important and urgent task because it consequently contributes to the real-time control as well as the short and medium-term operation of the whole power system.

Many scholars worldwide have made their efforts in modeling and computer-aided simulation of the PSU’s dynamic behaviors [2-6]. In General, the modeling methods can be divided into two approaches: one is state-space model-based and the other is characteristics model-based. The former approach is developed along with the advancing of modern control theory. In the previous related literature [4, 6-8], the complicated water hammer effect caused by water-delivery pipelines and hydraulic structures are often simplified as a single upstream penstock and expressed with a first-order non-minimum phase transfer function, thus to inevitably bring unnegligible and tremendous modeling errors to PSU. To describe the hydraulic characteristics in long diversion tunnels, surge tanks and tailrace pipes, some modified state-space models for PSU or hydraulic turbine unit have been
proposed over the past decade to improve the modeling precision [3, 9, 10]. These models, established on higher-order state-space functions or differential equations, are usually somewhat linearized and limited to certain working conditions. When deviating from the expected states, the models fail to trace transient processes and deteriorate sharply in the modeling precision. However, PSU is famed for its flexibility in wide range operating condition transitions. Its coupled hydraulic-mechanical-electrical characteristics should be described in a more detailed and distributed way. Thus, the characteristics method applied to hydraulic computation [2, 3, 5] has been extensively studied. To precisely reflect the complex hydraulic transients in the conduit system, much more information about the hydraulic structures and PSU parameters is needed in the characteristic-based modeling. Many computer-aided tools based on this method have been developed for hydraulic analysis as well as the adjustment and regulation calculation of the hydropower units and PSUs. In addition, the related researches have fully demonstrated its modeling accuracy and superiority in numerical simulations, especially for PSU’s dynamic processes of condition transitions.

Focused on the precise simulation of pumping transients in PSU, this paper establishes a nonlinear and distributed-parameter mathematical model of PSU based on the characteristics method. In the proposed model, hydraulic behaviors of the system components including conduit pipelines, surge tanks, inlet valve, pump-turbine and reservoirs are discussed. The local hydraulic loss coefficients in different parts of the conduit system are specifically introduced as the auxiliary compensation terms for unmodeled uncertainties. To verify the modeling precision, modeling errors between the proposed model and the practical data under both pump failure and pump shutdown conditions are measured. Furthermore, stability analysis of the two conditions have been conducted.

The rest of the paper is organized as follows. First, structure of the governing system of PSU and the functions of the auxiliary compensation terms are briefly introduced in Section 2. Then, mathematical models considering the local hydraulic loss compensation are introduced in Section 3. Section 4 presents numerical simulation results under pump failure and pump shutdown conditions and their comparison analysis. Finally, conclusions are given in Section 5.

2. Background knowledge of the governing system of PSU

The structural schematic of the governing system of PSU is shown in figure 1, where the whole control system is composed of the upper and lower reservoirs, water diversion tunnel, surge tanks, penstock, pump-turbine, tailrace tunnel, generator-motor, and governor. The major tasks of its governing control are to regulate the frequency and power of the unit and to achieve sequence control processes under different operating conditions.

![Figure 1. The structural schematic of the governing system of PSU](image-url)

In this study, we focused on the transient simulation in the pumping direction of PSU. For the sake of precisely describe the hydraulic characteristics of the distributed components in the whole system, several auxiliary parameters that compensate the unmodeled local hydraulic losses are introduced to
the dynamic model of some hydraulic structures. The locations of typical local hydraulic losses in this study have been marked with red circles in figure 1.

The parameter settings of PSU, conduit system and surge tanks in the system plant in this study are given in table 1-3, respectively.

Table 1. Parameters of the PSU

| Parameter                              | Value                  |
|----------------------------------------|------------------------|
| Rated rotation speed (r·min⁻¹)         | 500                    |
| Discharge of minimum head (m³·s⁻¹)     | 53.35                  |
| Power frequency (Hz)                   | 50                     |
| Maximum head (m)                       | 570                    |
| Rated head (m)                         | 540                    |
| Minimum head (m)                       | 535                    |
| 100% GVO (°)                           | 20.47                  |
| Rated mechanical torque (kN · m)       | 5.845×10³              |
| Generator-motor time constant (s)      | 8.503                  |
| Self-regulation coefficient (p.u.)     | 0.01                   |

Table 2. Parameters of equivalent pipes in the conduit system

| No | From location  | To location    | Equiv. length (m) | Equiv. diameter (m) | Loss coeffi. |
|----|----------------|----------------|-------------------|---------------------|--------------|
| 1  | Upper reservoir| Upper surge tank| 444.23            | 6.20                | 0.018        |
| 2  | Upper surge tank | Volute inlet | 983.55            | 4.36                | 0.027        |
| 3  | Draft tube inlet | Tailrace surge tank | 170.40  | 4.30                | 0.010        |
| 4  | Tailrace surge tank | Lower reservoir | 1065.20       | 6.58                | 0.015        |

Table 3. Parameters of the surge tanks

| Name               | Altitude (m) | Sectional area (m²) | Impedance hole area (m²) | Inflow loss coeffi. | Outflow loss coeffi. |
|--------------------|--------------|---------------------|--------------------------|---------------------|-----------------------|
| Upper surge tank   | 688-757      | 63.62               | 12.57                    | 0.00148             | 0.00108               |
| Tailrace surge tank| 130-189      | 95.03               | 15.90                    | 0.00092             | 0.00068               |

3. Back ground knowledge of the governing system of PSU

Characteristics method is used in this study to describe the hydraulic characteristics of components in the governing system of PSU. This modeling approach treats the system plant as a coupling of different distributed components.

3.1. Characteristic equations of the conduit pipelines

By discretizing the partial differential Saint-Venant equations [11], the hydraulic dynamic behavior in the pipelines can be expressed with the simple algebraic equations as equation (1),

\[
\begin{align*}
    H_i &= C_i - B_{p,i} \cdot Q_i , \\
    H_i &= C_{m,i} + B_{m,i} \cdot Q_i ,
\end{align*}
\]

where, \( i \) represents the node number; \( H_i , Q_i \) denote the water head and discharge at the \( i \)th node of the pipe. \( C_i = H_{i,0} + BQ_{i,0} \), \( B_{p,i} = B + R \left| Q_{i,1} \right| \), \( C_{m,i} = H_{m,0} - BQ_{m,0} \), \( B_{m,i} = B + R \left| Q_{m,1} \right| \); the constants \( B = aI/Ag \), \( R = fa\Delta t/2g\Delta t^2 \); and the superscript 0 denotes the state at last sample.

3.2. Border node treatment
The boundary between different hydraulic structures and pipelines is considered as the border node in the system’s characteristic model. The typical border nodes taken into consideration in this study include the upper/lower reservoir border, the surge tank border, the inlet valve border and the pumped-turbine border. The auxiliary local hydraulic loss coefficients are introduced to some of the border node equations here.

3.2.1. **Upper reservoir.** The upper reservoir is next to the first node on the upstream diversion tunnel. The mathematical dynamic description of this border is composed of an energy conservation equation and a characteristic equation. In order to compensate modeling errors caused by the unmodeled sections and local hydraulic loss at the water inlet, the local hydraulic loss coefficient $k_1$ is introduced to the dynamic equations. Therefore, the relationship water head and discharge of the upper reservoir node is expressed in equation (2),

$$\begin{align*}
H_u - H_1 - (1 + k_1)Q_i^2 / 2gA_1^2 &= 0 \\
H_1 - C_{M1} - B_{M1}Q_i &= 0
\end{align*}$$

where, $H_u$, $H_1$ denote the water levels of upper reservoir and water-inlet node, respectively; $Q_i$ the discharge of the inlet node; and $B_{M1} = B_1 + R_1 |Q_i'|$, $C_{M1} = H_2^0 - B_1 \cdot Q_{2,j}$. The subscript 2 denotes the number of neighboring node on the right of the boundary.

3.2.2. **Lower reservoir.** The treatment of lower reservoir node is similar to that of the upper reservoir. The boundary between downstream reservoir and tailrace tunnel can be taken as another border node numbered $i = N + 1$. To improve the modeling accuracy, the local hydraulic loss coefficient $k_1$ is added to the energy conservation equation of the outlet node. The water level and discharge of the lower reservoir node are stated in equation (3),

$$\begin{align*}
H_{N+1} - H_1 + (1 - k_2)Q_{N+1}^2 / 2gA_1^2 &= 0 \\
H_{N+1} - C_{PN+1} + B_{PN+1}Q_{N+1} &= 0
\end{align*}$$

where, $N$ is the total number of the subsection pipes in tailrace tunnel. $H_d$ denotes the water level of lower reservoir. $H_{N+1}$ and $Q_{N+1}$ represent the water head and discharge of the outlet node on the tailrace tunnel, respectively. and $C_{PN+1} = H_N^0 + B_1 \cdot Q_N^0$, $B_{PN+1} = B_1 + R_1 |Q_N'|$.

3.2.3. **Surge tank.** Surge tank is an important hydraulic structure in pumped-storage plant which helps attenuate the heavy water hammer effect in long pressurized pipes or tunnels. In this study, dynamic behavior of the throttled surge tank is investigated and its structural schematic is shown in figure 2.

The local hydraulic loss coefficients $k_1$ and $k_2$ are introduced to Node 1 and Node 2, respectively. These two additional loss coefficient can effectively reflect the unmeasured local water head losses at two ports of the surge tank. The relationship between the water pressure and discharge of the impedance hole is expressed by the formula of orifice outflow in hydraulics. The both sides of surge tank connect to the border node of adjacent pipelines, respectively. Boundary equations of the surge tank can be formed as equation (4),
\[ \begin{aligned}
H_1 - C_{p_i} + B_{p_i} Q_i &= 0 \\
H_2 - C_{M_2} - B_{M_2} Q_2 &= 0 \\
Q_i - Q_2 &= 0 \\
H_i + \left( Q_i^2 - k_i Q_i \right) / 2 g A_i^2 - H_2 - \left( Q_2^2 + k_i Q_2 \right) / 2 g A_2^2 &= 0 \\
H_i - H_s - Q_i \left( Q_i \right) / 2 g A_s^2 &= 0 \\
H_i - H^0_s - (Q_i + Q_m) \Delta T / 2 A_s &= 0
\end{aligned} \]  \hspace{1cm} (4)

where, the subscript 3 and 4 denote the neighboring nodes left to Node 1 and right to Node 2, respectively. \(Q_i\), \(H_i\) denote the discharge and water head of the \(i\)th node (\(i=1, 2, 3, 4\)) in figure 2, respectively; \(C_{p_i} = H_3^0 + B_3 \cdot Q_i^0\), \(B_{p_i} = B_3 + R_3 \cdot |Q_i^0|\), \(C_{M_2} = H_4^0 + B_2 \cdot Q_2^0\), \(B_{M_2} = B_2 + R_2 \cdot |Q_2^0|\).

![Figure 2](image-url)

**Figure 2.** The structural schematic of throttled surge tank

3.2.4. **Inlet Valve.** The inlet valve near the volute is also an important border node for the dynamic system. The dynamic behavior of the inlet valve is equivalent to that of the orifice outflow in hydraulics and can be described as equation (5),

\[ \begin{aligned}
H_1 - C_{p_i} + B_{p_i} Q_i &= 0 \\
H_2 - C_{M_2} - B_{M_2} Q_2 &= 0 \\
Q_i - Q_2 &= 0 \\
Q_i |Q_i| - \mu^2 A_i^2 2g(H_i - H_s) &= 0
\end{aligned} \]  \hspace{1cm} (5)

where, \(H_1, H_2\) denote the front and back water pressure of the inlet valve; \(Q_1, Q_2\) the front and back discharge of the inlet valve; \(\mu\) and \(A_i\) the discharge coefficient and the open area of the inlet valve; \(C_{p_i} = H_3^0 + B_3 \cdot Q_i^0\), \(B_{p_i} = B_3 + R_3 \cdot |Q_i^0|\), \(C_{M_2} = H_4^0 + B_2 \cdot Q_2^0\), \(B_{M_2} = B_2 + R_2 \cdot |Q_2^0|\).

3.2.5. **Pump-Turbine.** The two border nodes of the pump-turbine are at the inlet of the volute and the inlet of the draft tube. Ignoring the flow loss in the wicket gate and volute, the turbine discharge and water head satisfy the following equalities in equation (6),

\[ \begin{aligned}
H_1 - C_{p_i} + B_{p_i} Q_i &= 0 \\
H_2 - C_{M_2} - B_{M_2} Q_2 &= 0 \\
Q_i - Q_2 &= 0
\end{aligned} \]  \hspace{1cm} (6)

5
where, \( C_{m1} = H_{i}^0 + B_1 \cdot Q_i^0 \), \( B_{pi} = B_1 + R_1 \cdot |Q_i^0| \), \( C_{m2} = H_{i}^0 + B_2 \cdot Q_i^0 \), \( B_{m2} + R_2 = B_2 + R_2 \cdot |Q_i^0| \).

To consider the changing flow velocity and the kinetic energy recovery phenomenon in the draft tube, the correction coefficient \( k_5 \) is introduced to modify the empirical formula of the energy \( \Delta H \) recovered from the draft tube diffusion because the route loss and modeling error by equivalent draft tube pipes can greatly influence the value of \( \Delta H \). The modified equation for \( \Delta H \) is stated in equation (7),

\[
\Delta H = k_5 Q_i^2 \left( 1/ A_n^2 - 1/ A_{in}^2 \right) / 2g
\]

where, \( A_n \) and \( A_{in} \) denote the area of draft tube’s inlet and outlet, respectively.

Therefore, the working head \( H_r \) can be expressed as equation (8),

\[
H_r = H_i - H_2 + \Delta H
\]

4. Case study
Two typical transient processes in the pumping direction, i.e., the pump failure condition and the pump shutdown condition, are chosen in our study to validate the modeling effectiveness and precision. The system parameters of numerical simulations are given in table 1-3, and the simulation results in both cases are compared with the practical data under corresponding conditions.

4.1 Model validation

4.1.1 Case I: transient process of the pump failure. The transient process of the absolute rotation speed of PSU is depicted in figure 3. In this case, the guide vane of PSU linearly closes from the nominal opening to zero opening within 34.2 seconds, which is equivalent to the practical guide vane closure rule. From figure 3, it can be found that the rotation speed rapidly decreases from the nominal speed to zero after the pump failure at first. After it reaches zero, the speed starts to accelerate reversely in the generation direction until it reaches the crest \( n_c = 0.98 \). Then, the rotation speed gradually slow down to the static state. The simulated curve almost coincides with the practical curve and crests of the two curves in the reversed direction are also very close, so the proposed model can correctly reflect dynamic process of the rotation speed of PSU under this condition.

![Figure 3. The absolute rotation speed under pump failure condition](image-url)
draft tube pressure are very close and the oscillation period of the simulated curve are nearly the same with the practical one. The comparison results clearly show that the simulated water pressures are close to the practical data, indicating the effectiveness of the modeling accuracy on system’s hydraulic characteristics.

![Figure 4](image)

**Figure 4.** Water pressures at volute inlet and draft tube inlet under pump failure condition

4.1.2 Case II: transient process of the pump shutdown. During the pump shutdown process, the guide vane of PSU start to close linearly from the nominal opening to zero. In this case, the guide vane of PSU linearly closes from the nominal opening to zero opening in 38.4 seconds, which is equivalent to the practical guide vane closure rule. Different from the pump failure condition, the driving torque gradually reduces along with the closure of the guide vane, so the rotation speed of PSU remains the synchronous speed until the relative guide vane opening reaches 0. The comparison result of the rotation speed is shown in figure 5. The two curves are in high coincidence, which indicates the modeling superiority in the mechanical aspect.

![Figure 5](image)

**Figure 5.** The absolute rotation speed under pump shutdown condition

Concluded from figure 6 (a) and (b), it’s quite evident to find that the two simulated water pressure curves at both the front of inlet valve and the inlet of draft tube highly match their corresponding practical data as well. Both trends of the curves and the oscillation periods in two figures conform to the reality. Therefore, the model’s capability in hydraulic characteristic description is validated again under this condition.
4.2 Hydraulic oscillation comparison of the two conditions

The aforementioned pump failure and pump shutdown conditions are two important transient processes in pumping direction of PSU. In this subsection, the guide vane closing strategies are totally the same in the simulations of the two conditions. The closure duration is set as $T_0 = 40$ s here. The comparison of the water pressure curves at the draft tube inlet are illustrated in figure 7. The hydraulic oscillation crest of the pump failure process is about 110m in figure 7, while that of the pump shutdown process is only 90m. In addition, the highest pressure in the pump failure process happens at the occurrence of the pump failure, but the crest of the water pressure during the pump shutdown process happens much later in time span. The comparison results show that pump failure may cause a much more severe hydraulic oscillation to the conduit system compared with the pump shutdown and these results are exactly in agreement with engineering practice.

5. Conclusions

This paper presents a precise numerical simulation scheme for the pumping transients in the governing system of PSU. The modeling method is established on the basis of the characteristics method. In order to compensate the modeling error caused by unmodeled uncertainties in the system plant, several auxiliary local hydraulic loss coefficients are added to the model’s borders such as reservoirs, surge tanks and the pump turbine. Then, the modified nonlinear dynamic equations for each component in the PSU’s governing system has been stated in detail. To verify its effectiveness, the modeling precision in both mechanical and hydraulic aspects have been validated with the practical data under

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**Figure 6.** Water pressures at valve inlet and draft tube inlet under pump failure condition

**Figure 7.** Comparison of the draft tube inlet water pressure under two conditions
pump failure and pump shutdown conditions. Furthermore, the hydraulic oscillations under these two conditions are compared to investigate the stability issue in typical pumping transients.

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