Finite volume effect of nucleons and the formation of the quark-gluon plasma

Bo-Qiang Ma, Qi-Ren Zhang, D. H. Rischke, and W. Greiner

1 Institut für Theoretische Physik der Universität Frankfurt am Main, Postfach 11 19 32, D-6000 Frankfurt, Germany
2 Center of Theoretical Physics, CCAST(World Laboratory), Beijing, China and Department of Technical Physics, Peking University, Beijing 100871, China

Abstract We study a thermodynamically consistent implementation of the nucleon volume in the mean field theory, and find that this volume has large consequences on the properties of hadronic matter under extreme conditions such as in astrophysical objects and high energy heavy-ion collisions. It is shown that we can reproduce the critical temperature $T_c \simeq 200$ MeV predicted by lattice QCD calculations for the phase transition from hadronic matter to quark-gluon plasma, by using parameters which are adjusted to fit all empirical data for normal nuclear matter.

To be published in Phys.Lett.B
One of the main goals of nuclear physics is to study the properties of hadronic matter under extreme conditions, such as in high density and high temperature astrophysical objects and high energy heavy-ion collisions [1], based on our knowledge of normal nuclear matter properties. There are some difficulties to reproduce simultaneously all known properties for nuclear matter under normal and extreme conditions by using the naive conventional scenario of point-like hadrons. It has been observed [2] that in the framework of the local relativistic mean field theory the mass limit for neutron stars is too low when using a nuclear equation of state with a realistic compression modulus $K$ around 200 MeV. Also, there is no reasonable phase transition to the quark-gluon plasma for vanishing net baryon number at large $T$ due to the possible excitation of a vast number of hadronic resonances [3, 4]. The purpose of this letter is to show that the above difficulties can be removed simultaneously by simply taking into account the effect from the finite volumes of nucleons. That such a consideration is reasonable can be motivated by the success of the Van der Waals equation of state to describe properties of real gases. The limits of the validity of the effective field theory of point-like mesons and baryons are also indicated by the off-shell behaviors of hadronic structure functions due to the internal quark and gluon structure of hadrons [5].

By adopting a thermodynamically consistent procedure in the mean field theory, we have successfully produced [6] a mass limit for neutron stars which is compatible with the observation, while still kept the compression modulus for normal nuclear matter around its empirical value $K = 240$ MeV. In this letter we will focus our attention on the phase transition to the quark-gluon plasma which is expected to occur in high energy heavy-ion collisions [7]. We will show that we can reproduce the critical temperature $T_c \approx 200$ MeV predicted by lattice calculations of pure SU(3) gauge theory [1, 8, 9] using the parameters which are adjusted to fit all empirical data for normal nuclear matter [10], namely, the equilibrium density $\rho_0 = (4\pi r^3)^{-1}$ with $r = 1.175$ fm, the binding energy per nucleon $B/A = 15.986$ MeV, the asymmetry energy $E_{as} = 36.5$ MeV, and the compression modulus $K = 240$ MeV, with the bag constant for the quark-gluon plasma phase ranging from
In our approach hadronic matter and the quark-gluon plasma are described separately by different equations of state and a transition is obtained via Gibbs’ phase coexistence conditions. The effect of the finite volume has been extensively discussed in the ideal hadronic gas model [11] which cannot reproduce the nuclear ground state properties. We introduce the finite volume of nucleons in a thermodynamically self-consistent way [4, 6, 12, 13] in the mean field theory framework, which has been widely used for the description of nuclear matter, finite nuclei, and nuclear dynamics [15]. One new aspect of our approach is that the equation of state for the hadronic phase is applicable to hadronic matter under extreme conditions as well as normal conditions, although there are problems with causality at very high density [4, 6]. The quark-gluon plasma is treated as a gas of relativistic massless quarks, antiquarks, and gluons, including first-order QCD corrections and non-perturbative effects by an overall bag constant $B$.

For the sake of simplicity, we assume symmetric, homogeneous, and isotropic hadronic matter with $N_B$ baryon species (with equal properties) in a volume $V$, and restrict ourselves to non-strange degrees of freedom. In the ideal gas model the pressures of nucleons and antinucleons can be written as

\[ P^0_N(\mu, T) = \frac{kT}{V} \zeta(\mu, T), \]  
\[ P^0_{\bar{N}}(\mu, T) = \frac{kT}{V} \zeta(-\mu, T), \]  

with $\zeta(\mu, T)$ defined by

\[ \frac{1}{V} \zeta(\mu, T) = N_B \int \frac{d^3 \vec{p}}{(2\pi)^3} \ln[1 + e^{\beta(\mu - \epsilon)}], \]  

where $\beta = 1/kT$ with $T$ being the temperature, $\mu$ is the chemical potential and $\epsilon = \sqrt{\vec{p}^2 + \chi^2}$ is the energy of a nucleon of 3-momentum $\vec{p}$ and mass $\chi$. We now consider a system of nucleons and antinucleons with finite volume $\tau$. Assuming that the repulsive force due to the finite volume is operative between nucleons or antinucleons but not between a nucleon and an antinucleon due to the possibility of matter-antimatter annihilation, we can simply
discuss the finite volume effect for nucleons and antinucleons separately. The pressures can be written as \[4, 13\]

\[
P_N(\mu, T) = \mathcal{P}_N^0(\bar{\mu}, T) = \frac{kT}{V} \zeta(\bar{\mu}, T); \quad (4)
\]

\[
P_{\bar{N}}(\mu, T) = \mathcal{P}_{\bar{N}}^0(\bar{\mu}', T) = \frac{kT}{V} \zeta(-\bar{\mu}', T), \quad (5)
\]

with the effective chemical potentials \(\bar{\mu}\) for nucleons and \(\bar{\mu}'\) for antinucleons given by

\[
\bar{\mu} = \mu - \tau P_N(\mu, T); \quad \bar{\mu}' = \mu + \tau P_{\bar{N}}(\mu, T), \quad (6)
\]

as required to keep thermodynamical consistency \([4, 13]\). In the mean field theory of Bodmer and Boguta \([14]\), the energy density of the hadronic phase is

\[
\mathcal{E}_h = \mathcal{E}_N + \mathcal{E}_{\bar{N}} + U + 2\pi\alpha_\omega \frac{m^2}{m_\omega^2} \rho_h^2; \quad (7)
\]

with \(U\) given by

\[
U = \frac{1}{3\pi^2\alpha} (1 - \chi)^2 [1 + \alpha_1 (1 - \chi) + \alpha_2 (1 - \chi)^2], \quad (8)
\]

where \(\rho_h\) is the net baryon number density, \(m\) and \(m_\omega\) are the masses of free nucleon and \(\omega\) meson, and \(\alpha, \alpha_1, \alpha_2, \) and \(\alpha_\omega\) are the mean field theory parameters adjusted by fitting the normal ground state properties with the finite volume of the nucleon \(\tau = \frac{4\pi}{a^3}\) (with \(a = 0.62\) fm) \([4]\). The effective chemical potentials \(\nu\) for nucleons and \(\nu'\) for antinucleons are further modified to

\[
\nu = \bar{\mu} - 4\pi\alpha_\omega \frac{m^2}{m_\omega^2} \rho_h; \quad \nu' = \bar{\mu}' - 4\pi\alpha_\omega \frac{m^2}{m_\omega^2} \rho_h \quad (9)
\]

due to the effects from mean fields \([10]\). The baryon number and kinetic energy densities for nucleons and antinucleons are

\[
\rho_N = \int \frac{2N_B d^3 p}{(2\pi)^3} \frac{1 - \rho_N \tau}{e^{\beta(\sqrt{p^2 + \chi^2} - \nu)} + 1}; \quad (10)
\]

\[
\rho_{\bar{N}} = \int \frac{2N_B d^3 \bar{p}}{(2\pi)^3} \frac{1 - \rho_{\bar{N}} \tau}{e^{\beta(\sqrt{\bar{p}^2 + \chi^2} + \nu')} + 1}; \quad (11)
\]
The kinetic pressures for nucleons and antinucleons can be written as

\[ \mathcal{E}_N = \int \frac{2N_B d^3 \vec{p}}{(2\pi)^3} \frac{\sqrt{p^2 + \chi^2(1 - \rho_N \tau)}}{e^{\beta(\sqrt{p^2 + \chi^2} - \nu)} + 1}; \]  
\[ \mathcal{E}_{\bar{N}} = \int \frac{2N_B d^3 \vec{p}}{(2\pi)^3} \frac{\sqrt{p^2 + \chi^2(1 - \rho_{\bar{N}} \tau)}}{e^{\beta(\sqrt{p^2 + \chi^2} + \nu') + 1}}. \]  

The kinetic pressures for nucleons and antinucleons can be written as

\[ \mathcal{P}_N(\mu, T) = \mathcal{P}_N^0(\nu, T) = \frac{kT}{V} \zeta(\nu, T); \]  
\[ \mathcal{P}_{\bar{N}}(\mu, T) = \mathcal{P}_{\bar{N}}^0(\nu', T) = \frac{kT}{V} \zeta(-\nu', T), \]

Thus we obtain the total pressure and the net baryon number density for the hadronic phase:

\[ \mathcal{P}_h(\mu, T) = \mathcal{P}_N(\mu, T) + \mathcal{P}_{\bar{N}}(\mu, T) - U + 2\pi\alpha_s \frac{m^2}{m^2_{\omega}} \rho_h^2; \]
\[ \rho_h = \rho_N - \rho_{\bar{N}}. \]

The effective nucleon mass \( \chi \) is fixed by minimizing the energy density at constant baryon number density \( \rho_h \) and entropy density; which is equivalent to maximize the pressure at fixed \( T \) and \( \mu \). For the quark-gluon plasma phase, the pressure, energy density and net baryon number density are given by [3, 17, 18]:

\[ \mathcal{P}_{\text{qgp}}(\mu_q, T_q) = \frac{8\pi^2 T_q^4}{45} (1 - \frac{15\alpha_s}{4\pi}) + N_f \frac{7\pi^2 T_q^4}{60} (1 - \frac{50\alpha_s}{21\pi}) + \left( \frac{\mu_q^2 T_q^2}{2} + \frac{\mu_q^4}{4\pi^2} \right) \left( 1 - \frac{2\alpha_s}{\pi} \right) - B; \]
\[ \mathcal{E}_{\text{qgp}} = \frac{8\pi^2 T_q^4}{15} (1 - \frac{15\alpha_s}{4\pi}) + N_f \frac{7\pi^2 T_q^4}{20} (1 - \frac{50\alpha_s}{21\pi}) + 3 \left( \frac{\mu_q^2 T_q^2}{2} + \frac{\mu_q^4}{4\pi^2} \right) (1 - \frac{2\alpha_s}{\pi}) + B; \]
\[ \rho_{\text{qgp}} = \frac{N_f}{3} (\mu_q T_q^2 + \frac{\mu_q^3}{\pi}) (1 - \frac{2\alpha_s}{\pi}), \]

where \( B \) is the bag constant and \( \alpha_s \) is the QCD running coupling constant which depends on the quark-gluon plasma temperature \( T_q \) and the quark chemical potential \( \mu_q \) through [17]

\[ \alpha_s(\mu_q, T_q) = \frac{4\pi}{(11 - 2N_f/3) \ln[(0.8\mu_q^2 + 15.622T_q^2)/\Lambda^2]}. \]
with $\Lambda$ being the QCD scale parameter.

To examine the phase transition between hadronic matter and the quark-gluon plasma, we apply Gibbs’ conditions of phase coexistence, i.e., $P_h = P_{qgp}$; $T = T_q$; and $\mu = 3\mu_q$. As a first approximation, we calculate the simplest case with $N_B = 2$ (i.e., only protons and neutrons) and $N_f = 2$ (i.e., only u- and d- quarks). The range of $B$ is chosen from $B^{1/4} = 125$ MeV to about 300 MeV which is consistent with the results from a bag model analysis of hadron spectroscopy [3, 11, 18, 19]. The QCD scale parameter is now quite well determined to be $\Lambda \simeq 200$ MeV [20]. Fig. 1 presents the numerical result for the $T - \mu$ phase diagram with $B^{1/4} = 200$ MeV. In Fig. 2, the dependence of the net baryon number densities on the temperature for the hadronic matter and the quark-gluon plasma phase boundaries are plotted. We see that we can reproduce a reasonable phase transition temperature of $T_c = 170$ MeV for vanishing net baryon number density. $T_c$ changes from about 120 to 240 MeV by changing the parameter $B^{1/4}$ from 125 MeV to 300 MeV. Changing $a = 0.62$ fm to $a = 0.74$ fm (and readjusting the mean field parameters) causes a very small change (within 1 MeV) in $T_c$. By turning off the $O(\alpha_s)$ correction we find $T_c = 140$ MeV, thus the first order perturbative QCD corrections change the ideal quark-gluon gas result for the phase transition temperature by 20%. The finite volume effect is very large for nucleons and negligible for antinucleons, as can be seen from Fig. 1 by comparing the effective chemical potentials $\nu$ for nucleons and $\nu'$ for antinucleons with the effective chemical potential

$$\tilde{\nu} = \mu - 4\pi\alpha_{\omega}m_{\omega}^2\rho_{h},$$

where the contribution from the finite volume is absent. We see that $\nu$ is reduced by about 40% in comparison with $\tilde{\nu}$ in the high density region whereas $\nu'$ coincides everywhere with $\tilde{\nu}$.

However, the finite volume effect should also play a non-trivial role for antibaryons if there is a large number of hadronic resonance excitations. It has been indicated [3, 4] that in a point-like hadronic scenario the pressure of hadronic matter will be larger than that of the quark-gluon plasma at large
T and vanishing $\rho_h$ due to the large number of hadronic resonances, thus one cannot reproduce a phase transition to the quark-gluon plasma as predicted by lattice QCD calculations. In our approach this situation will not occur due to the finite volume of baryons and antibaryons. To reflect this aspect of the approach, in a first rough estimate we simply use a large $N_B$, which is equivalent to assume a large number of baryons with the same properties as nucleons. We find that a phase transition still occurs with $T_c \simeq 260$ MeV, even if we use a very large $N_B$ of $2^7 = 128$. This means that by taking into account the finite volume of baryons we still have a transition to the quark-gluon plasma even if there are a large number of hadronic resonances, thus we overcome the difficulty of the point-like treatment of hadronic matter. From the $T-\mu$ phase diagram (Fig. 3), where the effective chemical potentials for baryons $\nu$ and antibaryons $\nu'$ are also plotted, we see that the finite volume effect is also large for antibaryons at large $T$, in contrast with the simple case of only considering nucleons and antinucleons. The necessity of considering other baryonic degrees of freedom can be also seen from the large difference between the two $T-\mu$ diagrams Figs. 1 and 3: the effective chemical potentials for baryons $\nu$ is much more reduced in the latter case than in the former.

From Fig. 2 we see that there are large differences between the baryon number densities for the hadronic matter and the quark-gluon plasma at the phase transition at given $T$. This aspect does not change in the case of a large baryon number $N_B = 128$, as can be seen from Fig. 4. Another interesting feature of our results is that the effective mass of the nucleon is non-zero in the whole hadronic region: we find $\chi \geq 0.6m$ for the case of Fig. 1 and $\chi \geq 0.5m$ for Fig. 3. This is different from previous expectations of a strong reduction of the effective baryon mass near $T_c$ [21] but consistent with the results of ref. [4]. Of course, the exact quantitative details depend on the specific couplings of other baryonic resonances to the $\sigma$ field and their effective masses. Such detailed investigations are out of the scope of the present letter.

In summary, we have studied the finite volume effect of nucleons by
using a thermodynamically consistent procedure in the mean field theory approach, and found that this volume effect has large consequences in high density and high temperature nuclear matter concerning the limit of neutron star masses and the quark-gluon plasma phase transition in high energy heavy-ion collisions. Further studies should include a realistic mass spectrum of baryonic resonances, instead of our simple estimate with $2^7$ equal resonance species, and extend the discussion to strange degrees of freedom.

We would like to acknowledge many helpful discussions with Jürgen Schaffner, Debades Bandyopadhyay, Raffael Mattiello, and Horst Stöcker.
References

[1] See, e.g., The nuclear equation of state, edited by W. Greiner and H. Stöcker, (Plenum Press, New York, 1989) Part A and B.

[2] N. K. Glendenning, Phys. Rev. Lett. 57 (1986) 1120; Nucl. Phys. A 480 (1988) 597.

[3] U. Heinz, P. R. Subramanian, H. Stöcker, and W. Greiner, J. Phys. G 12 (1986) 1237.

[4] D. H. Rischke, M. I. Gorenstein, H. Stöcker, and W. Greiner, Z. Phys. C 51 (1991) 485.

[5] See, e.g., B.-Q. Ma, Phys. Rev. C 43 (1991) 2821; Int. J. Mod. Phys. E 1 (1992) 809; and references therein.

[6] Q.-R. Zhang, B.-Q. Ma, and W. Greiner, J. Phys. G 18 (1992) 2051.

[7] See, also, R. Hagedorn and J. Rafelski, Phys. Lett. B 97 (1980) 136.

[8] See, e.g., H. Stöcker and W. Greiner, Phys. Rep. 137 (1986) 277.

[9] See, e.g., Quark-gluon plasma, edited by R. C. Hwa, (World Scientific, Singapore, 1990).

[10] W. D. Myers and W. J. Swiatecki, Ann. Phys.(NY) 84 (1974) 186

[11] See, e.g., J. Cleymans and E. Suhonen, Z. Phys. C 37 (1987) 51; H. Kouro and F. Takagi, Z. Phys. C 42 (1989) 209.

[12] Q.-R. Zhang and X.-G. Li, J. Phys. G 18(1992) L111; Z. Phys. A 343 (1992) 337.

[13] Q.-R. Zhang, Acta Scient. Nat. Univ. Jilin, Suppl(Physics 1992) 1 (in Chinese).

[14] J. Bodmer and A. R. Boguta, Nucl. Phys. A 292 (1977) 413.
[15] For reviews, see, e.g., B.D.Serot and J.D.Walecka, in Adv. Nucl. Phys., vol.16, edited by J.W.Negele and E.Vogt, (Plenum Press, New York, 1986); and P.-G.Reinhard, Rep. Prog. Phys. 52 (1989) 439.

[16] D.H.Rischke, B.L.Friman, H.Stöcker, and W.Greiner, J. Phys. G 14 (1988) 191.

[17] H.Stöcker, Nucl. Phys. A 418 (1984) 587c.

[18] B.M.Waldhauser, D.H.Rischke, J.A.Maruhn, H.Stöcker, and W.Greiner, Z. Phys. A 43 (1989) 411.

[19] Q.-R.Zhang and H.-M.Liu, Phys. Rev. C 46 (1992) 2294.

[20] See, e.g., Particle Data Group, K.Hikasa et al., Phys. Rev. D 45 (1992) Part II, in page III. 54.

[21] See, e.g., J. Theis, G. F. Graebner, G. Buchwald, J. A. Maruhn, W. Greiner, H. St”ocker, and J.Polonyi, Phys. Rev. D 28 (1983) 2286.
Figure Captions

Fig. 1. The $T - \mu$ diagram for the phase equilibrium with $\Lambda = 200$ MeV and $B^{1/4} = 200$ MeV. The solid curve is the chemical potential $\mu$. The dashed and dot-dashed curves are the effective chemical potentials $\nu$ for nucleons and $\tilde{\nu}$ obtained by turning off finite volume corrections (see eq.(24)). The effective chemical potential $\nu'$ for antinucleons is in coincidence with $\tilde{\nu}$.

Fig. 2. The solid and dashed curves are the baryon number densities $\rho_b$ and $\rho_{qgp}$ for hadronic matter and the quark-gluon plasma under phase equilibrium with the same parameters as in Fig. 1.

Fig. 3. Same as Fig. 1, but with a very large $N_B = 128$. The dotted curve is the effective chemical potential $\nu'$ for antibaryons.

Fig. 4. Same as Fig. 2, but with the same parameters as in Fig. 3.