A model for giant flares in soft gamma repeaters

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We argue that giant flares in SGRs can be associated to the core conversion of an isolated neutron star having a subcritical magnetic field \( \sim 10^{12} \) G and a fallback disk around it. We show that, in a timescale of \( \lesssim 10^5 \) yrs, accretion from the fallback disk can increase the mass of the central object up to the critical mass for the conversion of the core into quark matter. A small fraction of the neutrino-antineutrino emission from the just-converted quark-matter hot core annihilates into \( e^+ e^- \) pairs above the neutron star surface originating the gamma emission of the spike while the further cooling of the heated neutron star envelope originates the tail of the burst. We show that several characteristics of the giant flare of the SGR 1806-20 of 27 December 2004 (spike and tail energies, timescales, and spectra) can be explained by this mechanism.

Soft \( \gamma \)-ray repeaters (SGRs) are persistent X-ray emitters that sporadically emit short bursts of soft \( \gamma \)-rays. In the quiescent state they have an X-ray luminosity of \( \sim 10^{35} \) erg/s, while during the short \( \gamma \)-bursts they release up to \( 10^{42} \) erg/s in episodes of about 0.1 s. Only four SGRs are known at present, 3 in our Galaxy, and one in the Large Magellanic Cloud. Exceptionally, three of them have emitted very energetic giant flares which commenced with brief \( \gamma \)-ray spikes of \( \sim 0.2 \) s, followed by tails lasting hundreds of seconds. Hard spectra (up to 1 MeV) were observed during the spike and the hard X-ray emission of the tail gradually faded modulated at the neutron star (NS) rotation period.

The powerful giant flare of the SGR 1806-20 of December 27, 2004 is particularly important because it was observed by many satellites \cite{1, 2, 3, 4, 5, 6}. The duration of the initial spike was \( \sim 0.25 \) s. For the first 500 ms the emission was so strong as to saturate almost all the detectors on \( \gamma \)-rays satellites, though there are unique measurements of the initial flare activity \cite{3}. The luminosity of the spike was \( \sim 10^{47} \) erg/s (assuming a distance of 15 kpc and isotropic radiation), hundreds of times brighter than the giant flares previously observed from other SGRs. A long tail lasting \( \sim 400 \) s followed the initial spike with a thermal component with decreasing temperatures in the \( \sim \) few keV range \cite{1}. There is a clear modulation with the same period of 7.56s already known from previous studies of the quiescent emission.

While several characteristics of SGRs are often explained in terms of the magnetar model, assuming that the object is a NS with an unusually strong magnetic field \( (B \sim 10^{15} \) G) \cite{7}, there is alternative theoretical work trying to explain the behavior of SGRs and Anomalous X-ray Pulsars (AXPs) by assuming the existence of fallback disks around isolated NSs with subcritical magnetic fields of \( \sim 10^{12} \) G \cite{4, 7, 8, 9, 10} and Refs. therein). NSs are believed to be born from core–collapse supernovae with masses near the Chandrasekhar limit of the Fe core \( (\sim 1.4M_\odot) \). During the supernova explosion, masses typically as large as \( \sim 0.2M_\odot \) may fall back \cite{11} and although most of this material will be directly accreted onto the NS, part of it can form an accretion disk within a few hours of the initial explosion \cite{7}. In fact, a fallback disk around an AXP has been recently detected for the first time \cite{12}. In this work we will show that accretion from a fallback disk can increase the mass of the NS up to the critical value for the conversion of the core into quark matter and this will trigger a giant \( \gamma \)-flare.

The disk of SGRs: The inner disk radius \( R_m \) of a fallback disk can be approximately evaluated from the balance between the stellar magnetosphere stress and the ram pressure of the inflow matter

\[
R_m = \left( \frac{\mu^2}{(2GM)^{1/2}M^{3/2}} \right)^{1/7},
\]

where \( \mu = R^3 \) is the magnetic moment, and \( M \) is the mass inflow rate through the disk. If the NS magnetosphere rotates faster than the inner disk, i.e., if \( R_m > R_c \), where \( R_c = (GM/\beta \Omega^2)^{1/3} \) is the corotation radius (defined as the radius where the disk rotates with the angular velocity of the stellar magnetosphere), then the system is in the so-called propeller regime in which there is an angular momentum flux from the NS to the disk (that will lead to strong stellar spin-down as required by the observations of SGRs and AXPs). In the case that the NS rotates only a little faster than the inner disk \( R_m \gtrsim R_c \), a significant fraction of the mass lost by the disk can accrete onto the NS, while at the same time the NS remains spinning down in a "tracking" regime \cite{2, 11}. In recent work, Eksi et al. \cite{8} identified these different regimes in a stellar period \( P \) versus mass inflow rate \( M \) diagram.

As in \cite{2, 8}, we shall assume here that SGRs are NSs with moderate \( B \)-fields that spin down to \( P \approx 5 - 8 \) s, as observed, in timescales \( \lesssim 10^5 \) yrs due to a fallback disk. Notice that, if attributed to electron resonance, the observed 5 keV cyclotron lines of 1806-20 \cite{12} suggest

\[
\]
$B \sim 5 \times 10^{12}$ G. After a brief accretion phase that follows its formation, the disk enters the propeller (spindown) regime and later, it reaches the tracking regime. During this phase, the system will lie at the boundary between the propeller and the X-rays luminous accretor (shaded) zone of the $P-M$ diagram of Ref. [3]. According to that diagram, a NS with $B \approx 5 \times 10^{12}$ G can enter the tracking phase only if its period is $\gtrsim 0.2$ s and $M < 10^{30}$ g/s. Once reaching this point, we assume that the disk will evolve along the tracking path until the present observed period of 7.56 s (for SGR 1806-20) which corresponds to a disk mass inflow rate $\dot{M} \approx 5 \times 10^{16}$ g/s [8].

**Quasi-steady luminosity and frequent soft $\gamma$-flares:** The disk mass accretion rate onto the NS during the tracking regime may consist of two contributions. One of them is a quasi-steadily evolving sub-Eddington component, $\dot{M}_{qs}$, whose value is of the order of $\dot{M}$ above. In the framework of the fallback disk model, the persistent X-ray luminosity of $\sim 10^{35}$ erg/s produced during the more quiescent states of the SGR 1806-20 is attributed to this component. The maximum luminosity that a NS being presently powered by an accretion rate $\dot{M}_{qs} \approx \dot{M} \approx 5 \times 10^{16}$ g/s can produce is $L \approx GM\dot{M}/R \approx 2.7 \times 10^{36}$ erg/s, so that 4% of the accretion power will be sufficient to explain the observed persistent X-ray luminosity. The other disk accretion rate component may be associated to the observed soft $\gamma$-ray pulses of SGRs with luminosities $L \sim 10^{42}$ erg/s. A number of mechanisms have already been suggested to explain these pulses. Among them, a hybrid model (proposed, e.g., in [8]) assumes the existence of local very strong multipole components of the B-field with $B \sim 10^{14} - 10^{15}$ G that act on the NS crust to produce the flares in a similar way to the magnetar models, while the large scale dipole component is $\sim 10^{12} - 10^{13}$ G, as required in the fallback disk scenario. Alternatively, we could speculate here that a sporadic super-Eddington disk mass accretion rate $\dot{M}_{sp} \sim 5 \times 10^{21}$ g/s onto the NS surface could produce the observed flares with $L \sim GM\dot{M}/R \sim 10^{42}$ erg/s (cf. [14]). Violent magnetic reconnection events occurring between the NS magnetosphere and the inner disk lines could trigger these events with the observed rise times $\sim 10$ ms [15].

**Accretion and conversion of the core:** The total accreted mass onto the NS during the source lifetime will be the sum of both accreting components above, and may be written as $\int \dot{M}_{qs} dt + N_{\tau sp} \dot{M}_{sp}$, where $N$ is the number of soft bursts during the object’s lifetime (several hundreds [8]) and the integral is taken along the tracking path between $\dot{M}_{qs} \sim 10^{20}$ g/s and $5 \times 10^{16}$ g/s. Using a self-similar evolution for $\dot{M}_{qs} \propto t^{-\alpha}$, with $\alpha \approx 1.2$ [8], we obtain a total accreted mass $\Delta M \sim 10^{-3} M_{\odot}$ in a time interval $\lesssim 10^5$ yr, most of which comes from the first component. This increase of the NS mass is essential for triggering the conversion of the NS core into quark matter (QM).

In this paper we are not assuming the QM to be absolutely stable (i.e. we assume that it has an energy per baryon at zero pressure and temperature that is larger than the neutron mass). Thus, stars containing QM are hybrid, i.e. contain $\beta$-stable QM only at their interior and not up to the surface (as in strange stars). The critical mass $M_{cr}$ for the formation of a QM core in a cold hadronic NS has been calculated recently exploring the effect of surface tension and color superconductivity [10]. If the parameters of the hadronic and QM equations of state are such that QM is not absolutely stable, $M_{cr}$ is very close to (but smaller than) the maximum mass of hadronic stars for a large range of the parameter space [10]. We emphasize that the conclusions of the present paper are not sensitive to the exact value of $M_{cr}$ provided that it is larger than the NS mass short after the supernova explosion and smaller than the maximum mass of NSs. Thus, if right after the initial supernova fallback, the NS acquires a mass that is not too far from $M_{cr}$, then the small accreted $\Delta M$ above will be sufficient to trigger the conversion of the core of the NS into QM at anytime after the beginning of the tracking phase.

**Energy release from the conversion:** The conversion into QM of a large part of the core releases a large amount of energy $\Delta E$ that can be estimated by the difference between the gravitational mass of the initial and final configurations (both having the same baryonic mass) [10]. As shown recently [10], $\Delta E \sim M_{53} - M_{3.6}$ ergs for the conversion of a hadronic star having the critical mass. We assume that most of this energy is thermalized inside the quark matter core, in the form of a trapped gas of neutrinos and antineutrinos of all flavors ($\nu_i\bar{\nu}_i$ pairs). To be sure, a small fraction can be used to excite vibrating modes [10, but we shall neglect this mechanical energy in the rest of the paper. Then, the initial temperature of the core is estimated from $\Delta E \sim C_q T$, where $C_q$ is the heat capacity of the quark core, resulting $T_{11} \approx 1.2 \times 10^9$ K.

**Neutrino annihilation:** Once they leave the opaque and hot inner core, the $\nu_i\bar{\nu}_i$ pairs can annihilate into $e^-e^+$ pairs in the outer layers of the NS, and also above the NS surface. Annihilation within the NS will heat matter significantly while above the NS surface it will create a $\gamma$-burst through $e^-e^- \rightarrow \gamma$. For simplicity, we shall assume a blackbody spectrum for the $\nu\bar{\nu}$’s in the core. Also, for annihilation above the NS surface, we can neglect blocking effects in the phase spaces of $e^-$ and $e^+$. Thus, the “unblocked” local energy deposition rate at a radial position $r$ due to $\nu_i\bar{\nu}_i \rightarrow e^-e^-$ can be written as $Q_{\nu e^-e^+}^{unb} = AT^{\nu e^-e^+}_{211} \chi(x)$, being $T^{\nu e^-e^+}_{211}$ the temperature at the $\nu$-sphere (i.e. the last scattering surface of the $\nu\bar{\nu}$’s), $A = 1.28 \times 10^{34}$ erg cm$^{-3}$s$^{-1}$ for $\nu_e\bar{\nu}_e$’s, and $A = 2.7 \times 10^{35}$ erg cm$^{-3}$s$^{-1}$ for $\nu_\mu\bar{\nu}_\mu$’s and $\nu_\tau\bar{\nu}_\tau$’s [17]. The angular factor $\chi(x) = \frac{1}{8}(1-x)^2(5+4x+x^2)$ depends on $x(r) = \left(1 - R_{n}^2/r^2\right)^{1/2}$, being $R_n$ the $\nu$-sphere radius. $\chi(x)$ is responsible for the rapid decrease in $Q_{\nu e^-e^+}^{unb}$ for
increasing \( r \), as the interacting neutrinos become more collinear. The total luminosity injected \textit{above} the NS surface is determined by \( L_{e^+e^-} = \int_R^\infty Q_{e^+e^-} \cdot 4\pi r^2 dr = \xi A\, T_r R_Q^3 \) where \( \xi = \frac{1}{\varepsilon(R)} \frac{4\pi}{(1 - x^2)^{5/2}} x dx \) is a function of the ratio between \( R_\nu \) (which we assume to be coincident with the quark-matter core radius \( R_Q \)) and the NS radius \( R \). Within the NS, electrons are degenerate and there is a strong blocking effect over \( Q_{e^+e^-} \). This can be accounted for by applying an average blocking factor to the unblocked results \cite{17}. Then, one writes \( Q_{e^+e^-} = B_e Q_{\text{unb}} \), where \( B_e \sim 1/[1 + \exp((\mu_e - T_\nu)/T)] \), \( \mu_e \) is the \( e^- \) chemical potential at the annihilation point, and \( T \) the corresponding temperature \cite{17}.

\textit{Cooling evolution of the NS}: The conversion of the inner core region into quark matter is expected to be very fast (\( \sim 10^{-5} \text{ s} \))\cite{18}. We assume for simplicity that after the conversion, the NS settles reasonably fast into a nearly hydrostatic configuration. Though this assumption is not strictly correct, particularly during the first seconds of the evolution, we shall see below that the characteristics of the \( \gamma \)-ray tail over 400 s are determined by a thin shell near the surface of the NS which is much less sensitive to dynamical rearrangement than the core region. As we will also see, it is the neutrinos from the core that produce the spike. Even if the core is subject to dynamical variations these will hardly affect the order of magnitude of the released energy and the \( \nu \)-emission timescale in comparison to the hydrostatic case (cf. \cite{13}). Therefore, soon after the conversion we adopt a simplified model in which the NS (with \( M = M_{\text{cr}} \)) has a hot quark matter inner-core with \( R_Q \approx 0.6 R \) in equilibrium with a trapped \( \nu \bar{\nu} \) gas at \( T_{\nu11} \approx 1 \) surrounded by an \textit{initially cold} outer-core (composed of free nucleons, electrons and muons) and a crust (composed of nuclei, free neutrinos and electrons). We assume there is no significant lepton number gradient, since most of the \( \nu \)-s are produced in pairs, so that the lepton-number-transport equations will not dominate the evolution. Thus the cooling of the quark core is described by the energy-transport equation:

\[
\frac{\partial T}{\partial t} = \frac{\Gamma}{c_v} \frac{\partial}{\partial r} \left[ T \frac{\partial}{\partial r} \right] - \frac{Q}{c_v}
\]

where the neutrino energy density is \( c_v = 3 \times 10^6 \text{erg cm}^{-3} \text{K}^{-1} \), \( a = \pi^2 k_B/15c^3 \), and \( c_v \) is the specific heat of quark matter. The neutrino mean free path \( \lambda \approx 1.3 \times 10^9 \text{cm}/T_{\nu11} \) in the hot inner-core is very small and the \( \nu \bar{\nu} \)-s are released in a diffusion timescale, due to scattering with free quarks. At the surface of the quark core we assume a free streaming regime into vacuum. Solving numerically this equation we find the time evolution of the temperature \( T_\nu \) at the \( \nu \)-sphere. \( T_\nu \) is then used to evaluate the luminosities \( L_{e^+e^-} \) given above and \( L_\nu = 4\pi R_Q^2 \times 3 \frac{\Gamma}{\sigma SB} T_\nu^4 \) (Fig. 1). The cold outer-core and the crust are essentially transparent to \( \nu \)-s and therefore they will hardly heat due to the emission of the hot inner-core. Also, since \( e^- \)-s are strongly degenerate, heating of a given layer through \( e^- + e^- \rightarrow \nu \bar{\nu} \) will be significant only if the mean energy of the \( \nu \bar{\nu} \)-s (which is of the order of that of the outgoing \( e^- + e^- \)) is larger than the Fermi energy of the \( e^- \)-s at that layer, i.e. \( k_B T_\nu \gtrsim \mu_e \). Since \( T_\nu \lesssim 10 \text{ MeV} \), this is fulfilled only in the outer-crust/envelope. Using a simple relation between the depth \( z \) measured downward from the NS surface and \( \mu_e \), \( z \approx 25 \mu_e m_e - 1 \approx 25 [T_\nu (m_e - 1)] \approx 25 [T_{\nu11} (m_e - 1)] \), we find that initially only the outermost \( \sim 500 \text{ meters} \) will be significantly heated by this process. Since the calculations show that \( T_\nu (t) \) decreases exponentially (Fig. 1), heating ceases in less than \( \sim 1 \text{ s} \) and thereafter, the envelope evolves decoupled from the core. The cooling of the envelope can then be described in a plane parallel approximation by:

\[
\frac{\partial T}{\partial t} = \frac{\Gamma}{c_v} \frac{\partial}{\partial z} \left[ T \frac{\partial}{\partial z} \right] - \frac{Q}{c_v}
\]

where the specific heat is approximated that of a Dulong-Petit solid and a degenerate relativistic electron gas, \( c_v \approx (4.5 \times 10^6 \rho_{10} + 3.9 \times 10^6 \rho_{10}^{2/3} T_9) \text{ erg cm}^{-3} \text{K}^{-1} \). The thermal conductivity is \( K = 16\sigma SB T^3 / (3 \rho c) \) with \( \kappa^{-1} = \kappa_{\text{cond}}^{-1} + \kappa_{\text{rad}}^{-1} \) where the conductive opacity is \( \kappa_{\text{cond}} = 3.6 \times 10^{-2} \rho_6^{-1.2} T_9^{1.4} \text{ cm}^2 \text{g}^{-1} \) and the radiative opacity is \( \kappa_{\text{rad}} = 9.6 \times 10^{-2} \rho_6^{0.1} T_9^{-1.3} \text{ cm}^2 \text{g}^{-1} \) (within a factor of \( \sim 3 \)) \cite{21}. The density profile is given by \( \rho_6 \approx (z / 10^4 \text{cm})^3 \) \cite{20} and the effective surface is taken where the optical depth is \( \sim 2 \). For simplicity, the \( \nu \)-cooling sink is given approximately by \( Q_\nu = 1.03 \times 10^{57} T_9^{5.6} \text{ erg cm}^{-3} \text{ s}^{-1} \) for \( T_\nu > 1.3 \) and \( 10^{-2} \lesssim \rho_6 \lesssim 10^{-2} \) \cite{21}. The energy per unit volume injected by core-\( \nu \)-s in the envelope is \( U(z) \sim \int B_\nu A T_{\nu11}(t) \chi(z) dt \). Assuming that this energy is thermalized the initial temperature profile is given by
\[ \sim U(z)/c(z) > 10^{10} \text{ K}, \] but due to the efficient \( \nu \)-cooling sink it falls to values \( \lesssim 10^{10} \text{ K} \) in less than 1 s. Thus the later evolution of the envelope is rather insensitive to the initial energy injection.

**Results:** In the right panel of Fig. 1, we show the time evolution of the core neutrino luminosity and the luminosity \( L_{e^+e^-} \) of the pairs that annihilate above the NS’s surface. The timescale at which \( L_{e^+e^-} \) decays to \( \sim 1\% \) of its initial value is \( \sim 0.2 \text{ s} \), just as observed for the spike of giant flares in SGRs (this is a robust result that naturally arises also in models of GRBs involving the burning of quark stars; e.g. [22]). We also note that the efficiency of the \( \nu\overline{\nu} \rightarrow e^+e^- \) conversion is strongly temperature dependent, so that with variations within a factor of \( \sim 2 - 3 \) in the central temperature of the object, a wide range of observed spike-luminosities \( (10^{44} - 10^{47} \text{ erg/s}) \) can be explained. The spectrum of the spike has been calculated assuming that all pairs are converted into \( \gamma \)-rays (i.e. \( L_{\gamma} \approx L_{e^+e^-} \)). This results \( T_{\gamma}(t) \propto T_{e^+e^-}(t) \) which reproduces the observed best-fit blackbody temperature \( T_{\text{spike}} = 175 \pm 25 \text{ keV} \) of SGR 1806-20 [1] for an initial core temperature \( T_{11} \approx 0.5 \) (left panel of Fig. 1). The luminosity \( L_{e^+e^-} \) fades very fast and then is overwhelmed by photon radiation from the hot NS envelope. The surface temperature evolves according to Eq. (2) producing the tail emission as shown in Fig. 1.

**Discussion:** The model just described can explain the energy scale, spectrum and timescale of both the spike and the tail of giant flares in SGRs. Several features can be used to observationally distinguish between this and the magnetar model. For example, the concomitant detection of low energy neutrinos and anti-neutrinos \( (E_{\nu} \approx 10 \text{ MeV}) \) with the \( \gamma \)-emission will give strong support to our model. Unlike in the magnetar model, a huge amount of energy stored in the core is converted in \( \gamma \)-emission with just a tiny efficiency. The eventual observation of a giant flare with energy much larger than \( 10^{47} \text{ erg} \) will also give support to this model, while it would be hard to explain within the magnetar scenario. Also, all four SGRs are possibly associated with supernova remnants (SGR 1900+14 / G42.8+0.6, SGR 0526-66 / N49, SGR 1806-20 / G10.0-0.3, and SGR 1627-41 / G337.0-0.1) which makes it plausible that they still have a fallback disk. The recent identification of a fallback disk around the AXP 4U 0142+61 [13] points to this direction, provided that SGRs and AXPs belong to the same population. The release of energy in the core and the NS-structure rearrangement after phase transition, as well as the sporadic impact of accreted matter, may cause surface stresses and quakes just as those that in the magnetar scenario are caused by magnetic stresses. Therefore, quasi-periodic-oscillations (QPOs), for example, could be likewise explained by crust fracturing and release of elastic energy. Finally, since AXPs seem to have bursts but not giant flares, we can interpret them as accreting NSs whose mass is far from \( M_{\text{cr}} \). In this sense, the number of AXPs should be larger than that of SGRs. Also, our model predicts that all SGRs that had a giant flare should have essentially the same mass, i.e. \( M_{\text{cr}} \). Techniques for determination of NS-masses are rapidly improving and eventual measurements could probe the value of \( M_{\text{cr}} \). If so, this could also provide key information for the equation of state of dense matter.

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