Analysis of Factors Affecting Productivity of Horizontal Wells Based on Grey Relational Theory

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Abstract. Taking Bp block of the Middle East as the target, on the basis of Joshi’s formula of horizontal well productivity prediction and the application of grey relevancy analysis on the average permeability, effective thickness, production pressure difference, oil saturation of reservoir and other major factors for the impact of the initial productivity of the horizontal wells, by means of obtaining the grey relational grade of influencing factors and the production of single well and optimal to optimize the combination of differential pressure, horizontal section length and other parameters, the effective exploitation of the oil field is achieved.

1. Introduction

Horizontal well technology which can significantly improve the production of oil and gas has been widely used in the development of the new oil fields and the potential tapping of the old oil fields, therefore, the accurate prediction of horizontal well productivity is an important basis for perfecting the design of horizontal well and optimizing the oilfield development program. The research of production prediction method of horizontal wells began in the 1950s. It was first developed by Soviet scholars Merkulov and Borisov and laid the theoretical foundation of the study. Since then, Giger and other scholars put forward their own horizontal well production calculation formula. In the late 80s, according to Borisov method, Joshi derived the static production formula of horizontal well in homogeneous reservoir [1-2], which have a better application in the field application, then the productivity of horizontal wells many authors deduced formula and Joshi formula with high similarity. Block Bp is in the initial stage of development now. As a pore type carbonate reservoir, it has the characteristics of high permeability, thin effective reservoir thickness and so on. So it is suitable for horizontal well development. For the same reservoir, however, there are many reasons affecting the productivity of horizontal well, including the reservoir itself and drilling engineering factors. To develop the reservoir efficiently and rationally, as well as clear the parameters influence of the productivity of horizontal wells, this dissertation compares the main factors that may affect the productivity based on the Joshi formula and with the help of the evaluation method of gray correlation in the study area.

2. Introduction of horizontal well formula

In 1988, by using electric field flow theory and assuming the oil drain of horizontal well is a ellipsoid which the focus are both ends of a horizontal well, Joshi divided the three-dimensional seepage problem of the horizontal well into the internal vertical radial fluid flow and the external horizontal
elliptic fluid flow. He also using three-dimensional solution theory and potential energy theory equivalent seepage resistance method to derive the steady-state productivity equation of horizontal well in homogeneous reservoir [2].

\[
Q = \frac{0.543K_h h (P_e - P_wf)}{\mu_o \bar{\rho}_o \left( a + \frac{a^2 - \frac{1}{4} \beta h}{L} \right) \ln \left( \frac{\beta h}{a + \frac{1}{4} \beta h} \right)}
\]

(1)

\[
a = \frac{L}{2} \sqrt{\frac{1}{2} + \left( \frac{2\pi}{L} \right)^4 + \frac{1}{4}}, \quad \beta = \sqrt{\frac{K_h}{K_p}}
\]

(2)

\[
\text{Figure 1. Sketch map of horizontal well section}
\]

Parameter introduction:
- a: Half long axis of oil drain radius (m);
- b: Half short axis of oil drain radius (m);
- h: Formation thickness (m);
- K: Permeability (mD);
- Q: Initial oil production (m³/d);
- L: Length of horizontal section (m);
- P_e: Marginal pressure of supply (MPa);
- P_wf: Bottom hole flow pressure (MPa);
- \( r_e \): Marginal radius of supply (m);
- \( r_w \): Oil well radius (m);
- \( \mu \): Fluid viscosity (mPa·s)
3. The principle of grey relational analysis

Grey Correlation Analysis (GCA) is a multi-factor statistical analysis method [3-4]. By solving the main relationship among the factors in the system, the main influencing factors are identified and the important factors affecting the target value are found out. It takes the sample data of each factor as the basis, using the grey relation degree to describe the strength, the magnitude and the order of the relations among the factors. If the sample data series reflects the trend of the any two factors (direction, size, speed, etc.) that basically are the same, the correlation between them is greater; otherwise, the smaller (Table 1).

Table 1. Sketch map of horizontal well seepage field

| Numble | Weak | Middle | Strong | pole-strength |
|--------|------|--------|--------|--------------|
| Range of correlation degree | 0~0.35 | 0.35~0.65 | 0.65~0.85 | >0.85 |

By introducing the grey relational theory, the relationship and degree of correlation between the productivity of single horizontal well and the parameters which effect the productivity can be established, and the greater the value, the more effect of the correlation parameters on horizontal well initial production have [5].

3.1. Determine the analysis sequence

On the basis of the analysis of the target variables of the research questions, a dependent variable and a number of independent variables can be determined. The reference sequence (mother sequence) is formed by dependent variable data, and the independent variable data form a comparative sequence can be expressed separately as [6]:

$$\bar{Y}_0 = \{Y_0(1), Y_0(2), Y_0(3), ..., Y_0(n)\}$$  
$$\bar{Y}_i = \{Y_i(1), Y_i(2), Y_i(3), ..., Y_i(n)\}$$  
$$\bar{Y} = \{Y_1(1), Y_2(2), Y_3(3), ..., Y_i(n)\}$$ (3)

Among them, $\bar{Y}_0$, $\bar{Y}_i$ are reference sequence and comparative sequence respectively. $n$ is the sequence length. $i=1,2, 3, m$, and $i$ is the comparison of the number of sequences.

In addition, due to the physical meaning of each factor in the system is different, resulting in the data dimension is not necessarily the same, it is not easy to compare so that it’s hard to get the correct conclusion. To overcome these unreasonable factors, the grey relational analysis needs the quantitative data to make indexes being dimensionless such like standardization and normalization of the dimensionless [7], and transformed variables into the same scale of standardized variables. In this paper, the normalized initial productivity of a horizontal well is taken as a reference sequence, denoted as $X_0$. At the same time, the parameters that may affect the productivity of the horizontal well are taken as comparison sequences, denoted as $X_i$, ($i=1,2,3, m$, successively representing the effective reservoir thickness, effective permeability, horizontal section length and so on). Therefore, after the normalization, the $m+1$ data points sequence can form the following matrix:

$$(X_0, X_1, X_2, ..., X_m) = \begin{pmatrix}
X_0(1), & X_1(1), & \cdots, & X_m(1) \\
X_0(2), & X_1(2), & \cdots, & X_m(2) \\
\vdots & \vdots & \ddots & \vdots \\
X_0(n), & X_1(n), & \cdots, & X_m(n)
\end{pmatrix}_{n(m+1)}$$ (4)
In the formula 4, the first column is the reference sequence, and the remains are comparative sequences.

3.2. Calculates the absolute difference between the comparison and the reference sequence

That is, absolute difference between the first column and the other column corresponding period is obtained in the formula 4, and the absolute difference matrix is formed (formula 5). The maximum number and the minimum number in the matrix are the maximum difference and the minimum difference.

\[
\begin{pmatrix}
\Delta_{01}(1) & \Delta_{02}(1) & \cdots & \Delta_{0m}(1) \\
\Delta_{01}(2) & \Delta_{02}(2) & \cdots & \Delta_{0m}(2) \\
\vdots & \vdots & \ddots & \vdots \\
\Delta_{01}(n) & \Delta_{02}(n) & \cdots & \Delta_{0m}(n)
\end{pmatrix}_{n \times m}
\] (5)

3.3. Calculating grey relational coefficient

The correlation coefficient represents the degree of association between the I comparison sequence and the reference sequence at the K data point, the expression could be expressed by:

\[
r_{0i}(k) = \frac{\Delta_{min} + \rho \Delta_{max}}{\Delta_{0i}(k) + \rho \Delta_{max}}
\] (6)

\[
\Delta_{0i}(k) = |x_0(k) - x_i(k)|
\] (7)

\[
\Delta_{max} = \max_{1 \leq i \leq n} \max_{1 \leq k \leq m} |x_0(k) - x_i(k)|
\] (8)

\[
\Delta_{min} = \min_{1 \leq i \leq n} \min_{1 \leq k \leq m} |x_0(k) - x_i(k)|
\] (9)

In the equation above:

- \( r_{0i}(k) \): Correlation coefficients, \( i=0, 1, 2, 3, m; k=1, 2, 3, n. \)
- \( \Delta_{0i}(k) \): The absolute value of the difference between the reference sequence and the comparison sequence at each moment.
- \( \Delta_{min} \): The minimum of the absolute value of the difference, that is, the minimum difference between the two levels
- \( \Delta_{max} \): The maximum of the absolute value of the difference, that is, the maximum difference between the two levels
- \( \rho \): Resolution coefficient, also called resolution factor, is used to improve the resolution of association degree to improve the significance of the difference among correlation coefficients, it is among 0 to 1. The smaller the value is, the higher the resolution will be, and the value is among 0.1 to 0.5. According to experience, the value is 0.3.

Therefore, the correlation coefficient matrix is expressed as follows:

\[
\begin{pmatrix}
r_{01}(1) & r_{02}(1) & \cdots & r_{0m}(1) \\
r_{01}(2) & r_{02}(2) & \cdots & r_{0m}(2) \\
\vdots & \vdots & \ddots & \vdots \\
r_{01}(n) & r_{02}(n) & \cdots & r_{0m}(n)
\end{pmatrix}_{n \times m}
\] (10)
3.4. Calculate the correlation degree of each index

Due to the number of associated numbers is large and the information is scattered, to compare easily, the relational data is represented by an average value related expression (formula.10) and compared. The correlation degree value is among 0 to 1 and the correlation degree is greater, then the influence degree of the index on the reference sequence is greater; the smaller the other is:

$$R_{0r}(k) = \frac{1}{n} \sum_{k=1}^{n} r_{0r}(k)$$  \hspace{1cm} (11)

4. Analysis of factors affecting grey relation in block Bp production

Through the analysis of the variation characteristics of E oil field production, as well as the combination of reservoir properties and reservoir characteristics, select the reference sequence for horizontal wells in the initial capacity, the evaluation index for the 6 factors, respectively: $Y_1$: reservoir effective thickness (H); $Y_2$: effective permeability (K); $Y_3$: anisotropy coefficient ($\delta$); $Y_4$: production pressure difference ($\Delta P$); $Y_5$: Length of horizontal section(m); $Y_6$: oil saturation ($S_o$).

Step 1. The data after averaging are Table 2.

| Numble | BP3 | BP8 | BP9 | BP8-3 | BP8-11 | BP10-9 |
|--------|-----|-----|-----|-------|--------|--------|
| $X_0$  | 0.935 | 1.057 | 0.845 | 1.547 | 0.740 | 0.877 |
| $X_1$  | 1.000 | 1.045 | 0.866 | 1.157 | 0.970 | 0.963 |
| $X_2$  | 1.007 | 1.138 | 0.957 | 1.290 | 0.790 | 0.819 |
| $X_3$  | 1.058 | 0.890 | 0.993 | 0.769 | 1.391 | 0.899 |
| $X_4$  | 0.982 | 1.088 | 0.877 | 1.228 | 0.877 | 0.947 |
| $X_5$  | 0.999 | 1.044 | 1.012 | 1.061 | 0.897 | 0.987 |
| $X_6$  | 1.128 | 0.871 | 0.987 | 0.869 | 1.097 | 1.048 |

Step 2. Calculate the absolute difference between the sequence and reference sequence of any well. Therefore, the absolute difference between 6 horizontal wells can be expressed as Table 3.

| Numble | BP3 | BP8 | BP9 | BP8-3 | BP8-11 | BP10-9 |
|--------|-----|-----|-----|-------|--------|--------|
| $\Delta_{01}$ | 0.065 | 0.012 | 0.021 | 0.390 | 0.230 | 0.086 |
| $\Delta_{02}$ | 0.072 | 0.081 | 0.112 | 0.257 | 0.050 | 0.058 |
| $\Delta_{03}$ | 0.123 | 0.167 | 0.148 | 0.778 | 0.651 | 0.022 |
| $\Delta_{04}$ | 0.048 | 0.031 | 0.033 | 0.319 | 0.137 | 0.071 |
| $\Delta_{05}$ | 0.064 | 0.013 | 0.167 | 0.486 | 0.157 | 0.111 |
| $\Delta_{06}$ | 0.193 | 0.186 | 0.142 | 0.678 | 0.357 | 0.171 |

The absolute difference matrix is calculated by the dimensionless matrix, and the maximum difference and the minimum difference are obtained:

$$\Delta_{\text{max}} = 0.778; \hspace{1cm} \Delta_{\text{min}} = 0.012$$

Step 3. The correlation coefficient is calculated and the resolving factor is 0.3, Then the process of the calculation is:
The correlation coefficients of 6 wells are calculated by this method, as shown in Table 4:

| Number | BP2 | BP8 | BP9 | BP8-3 | BP8-11 | BP10-9 |
|--------|-----|-----|-----|-------|--------|--------|
| $r_{01}$ | 0.823 | 1.000 | 0.966 | 0.394 | 0.530 | 0.769 |
| $r_{02}$ | 0.804 | 0.783 | 0.712 | 0.501 | 0.868 | 0.844 |
| $r_{03}$ | 0.690 | 0.614 | 0.644 | 0.243 | 0.278 | 0.961 |
| $r_{04}$ | 0.875 | 0.931 | 0.924 | 0.445 | 0.664 | 0.808 |
| $r_{05}$ | 0.827 | 0.998 | 0.614 | 0.342 | 0.629 | 0.714 |
| $r_{06}$ | 0.576 | 0.585 | 0.654 | 0.270 | 0.416 | 0.607 |

Step 4. Computing Association degree. According to the results in Table 4, the average of the correlation coefficients of each oil well of each series is obtained, and the final correlation degree is obtained (Table 5).

$$
\bar{r}_{01} = \frac{1}{6} (0.823 + 1.000 + 0.966 + 0.394 + 0.530 + 0.769) = 0.747
$$

$$
\bar{r}_{02} = \frac{1}{6} (0.804 + 0.783 + 0.712 + 0.501 + 0.868 + 0.844) = 0.752
$$

| Parameter | $\bar{r}_{01}$ | $\bar{r}_{02}$ | $\bar{r}_{03}$ | $\bar{r}_{04}$ | $\bar{r}_{05}$ | $\bar{r}_{06}$ |
|-----------|----------------|----------------|----------------|----------------|----------------|----------------|
| Average Value | 0.747 | 0.752 | 0.572 | 0.774 | 0.687 | 0.518 |

By using the grey correlation method, calculate the correlation factors of production decline, finally selected six main factors affecting the production of single well oil were: $\Delta P > K > h > L > \delta > S_o$.

5. Conclusion
Grey relational analysis method is used to analyze the influence of many factors on the initial productivity of horizontal wells, this method is simple and quick to compute, and does not require a large amount of sample data. It has wide applicability and guidance in Oilfield field application, especially in incomplete development blocks. According to this method, the productivity of E oilfield horizontal wells is mainly affected by three factors, production pressure difference, permeability, and effective thickness.

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