A mathematical modelling and numerical simulation of the SIRS political fanaticism figure voters model with presence of media in Indonesian presidential election

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Abstract. In presidential election, the presidential candidates who will be standing in it should try to get as much support among the voters. Understanding the dynamics of voters spread help presidential candidates or parties which supporting the candidates in designing appropriate strategies to win presidential election. In this paper, the dynamics of voters is described by using a mathematical model of the SIRS deterministic model. The model is constructed as a three-dimensional form of the non-linear differential equation. The spread of voters can move from neutral voters to supportive voters, neutral voters to apathetic voters, supportive voters to apathetic voters, or apathetic voters to neutral voters. Since presidential candidates need media to get the exposure they need to win the elections, we also consider its dynamics with the presence of media to estimate the number of votes in Indonesian presidential election. From the mathematical analysis of the model, we find the political fanaticism figure voters free equilibrium state explicitly and the political fanaticism figure voters endemic equilibrium state implicitly. Sufficient conditions for their existence and local stability of both equilibrium states are derived. Numerical simulation using R based on data from The General Election Commission are carried out to verify the analytical results.

Keywords: Indonesian presidential election, SIRS deterministic model, presence of media, political fanaticism figure voters.

1. Introduction
In 2019, there are 53 countries with elections. Nearly two billion voters will head the polls to elect their leaders. Some of the biggest elections include India – the world’s largest democratic elections with 900 million Indians are eligible to vote, Indonesia with more than 190 million Indonesians registered to vote, and Nigeria with 84 million registered voters.

There are three main types of elections to elect a country’s leader; presidential election, parliamentary election, and general election [1]. Two of the most popular types of democratic governments are the presidential and parliamentary systems. In presidential election, voters elect the country’s head of state or president. The president holds the highest political position and officially represents the country. In parliamentary election, the legislative body is elected by the people and, in turn, chooses the head of the government or prime minister. The legislative body implements laws and runs the government on a day-
to-day basis. In countries without prime minister, such as South Africa, Brazil, and the United States, the president is both the head of state and head of government. In general election, voters elect the head of state as well as federal and local representatives.

In the 21st century or digital era, the main sources of information are media. Media plays an important role in democracy. The media act as watchdogs checking government actions, but sometimes media used as proxies in the battle between rival political groups, hate speech, and channel in hoax distribution. In recent years, the use of media (especially social media) has emerged as an important electoral campaigning tool. Media not only inform but also shape public opinion.

Usually, epidemiological models are aimed at studying the dynamics of infectious diseases, for example in [2]. In the epidemiology study, there are several transmission models with different states, such as $SIR, SIRS, SEIR$, depending on the level of detail that is wished to introduce in the model [3]. All models of transmission are continuous time compartment level models. It is important to note that these models do not work with all diseases. In [4], the authors have applied the $SIR$ transmission model to understand the spread of voters in a population. In this paper, we will discuss the $SIRS$ model for the spread of political fanaticism figure voters considering the presence of media on the transmission model. These variables; $S(t), I(t)$ and $R(t)$ denote their values at a given time $t$ and they change over time. We apply this model to estimate the number of votes in 2014 Indonesian presidential election. This study use two survey institutions which conducted by Litbang KOMPAS and Social Mention for estimating the initial values and parameters of the models. The next section will discuss the $SIRS$ mathematical model with mathematical analysis and numerical simulation will be conducted in the following section. In the last section, some conclusions will be given.

2. Mathematical model for political fanaticism figure voters with presence of media

The $SIR$ transmission model in epidemiology is introduced for the first time in 1927 by Kermack and McKendrick [5]. They formulated a model with the population divided into three compartments; susceptible $S(t)$, infected $I(t)$, and removed $R(t)$. This model has had a major influence on the development of mathematical models for disease spread and still relevant in many epidemic situations. This paper will discuss the $SIRS$ deterministic model to observe the spread of voters in a closed population. In this model, the voters population is divided into three distinct categories using $S(t), I(t)$, and $R(t)$ to describe the total number of susceptible (neutral) voters, the infected (supportive) voters by political figure fever, and the recovered (apathetic) voters, respectively. Susceptible voters class means neutral to the political figure fever, infected voters class represents supporters who have interest in political figure, and recovered voters class are those who have lost interest in political figure. Let $V(t) = S(t) + I(t) + R(t)$ denote the total voters population. The complete transfer process between all categories of $SIRS$ compartmental political figure voters model is illustrated in Figure 1. To simplify, let $V(t) = V, S(t) = S, I(t) = I$, and $R(t) = R$.

![Figure 1. Transfer diagram for the SIRS compartmental political fanaticism figure voters model with presence of media.](image-url)
This model describes the transitions of voters from $S$ to $I$ to $R$ to $S$. This model is called an SIRS model since recovered voters can return to class $S$ and susceptible voters can move directly to class $R$. Assume that the number of voters enters into the susceptible voters class is $\pi = \mu V$, voters exit from the system (for example by migration or die) from each class with the rate $\mu$. The members of infected voters class contact the individuals of the susceptible voters class and try to convince them to support their political figure, with transmission rate $\beta$. Alternatively, individuals can become infected through positive media rate $P$ with the proportion $\epsilon$, from susceptible voters into infected votes or recovered voters open to infection again into susceptible voters. Otherwise, infected voters can become susceptible voters or susceptible voters can become recovered voters because of negative media rate $N$. Voters can become bored of political figure at rate $b$.

According to transfer diagram in Figure 1, the differential equations that describe the compartmental model for political figure voters including presence of media is given by:

$$\frac{dS}{dt} = \pi - \mu S - \beta S I V - \epsilon PS + (1 - \epsilon)NI - (1 - \epsilon)NS + \epsilon PR$$

$$\frac{dI}{dt} = \beta S I V + \epsilon PS - (1 - \epsilon)NI - \mu I - bI$$

$$\frac{dR}{dt} = bI + (1 - \epsilon)NS - \mu R - \epsilon PR$$

with nonnegative initial conditions for all categories.

By adding three differential equations of Equation (1), we have $\frac{dV}{dt} = 0$, and thus $V(t) = V(0) = S(0) + I(0) + R(0)$ is a constant. Since the total voters population is constant, we obviously have $V = \frac{\pi}{\mu}$. The domain of model in Equation (1) is

$$\Omega = \{(S, I, R) \in \mathbb{R}_+^3 | 0 \leq S + I + R \leq V\}$$

The initial values of the model based on data from The General Election Commission are displayed in Table 1.

| Variable          | Symbol | Initial value Prabowo-Hatta | Initial value Jokowi-Kalla |
|-------------------|--------|------------------------------|---------------------------|
| Susceptible voters| $S$    | 113,272,270 (people)        | 113,272,270 (people)      |
| Infected voters   | $I$    | 26,498,620 (people)         | 46,839,110 (people)       |
| Recovered voters  | $R$    | 46,839,110 (people)         | 26,498,620 (people)       |

The parameter values of the model in Equation (1) are presented in Table 2.

| Parameter              | Symbol | Parameter value Prabowo-Hatta | Parameter value Jokowi-Kalla |
|------------------------|--------|------------------------------|------------------------------|
| Maturation rate        | $\mu$  | 1/58 (per years)             | 1/58 (per years)             |
| Recruitment rate       | $\pi$  | 3,217,413 (people per years) | 3,217,413 (people per years) |
| Transmission rate      | $\beta$| 0.033883447 (per people per years) | 0.051897431 (per people per years) |
| Boredom rate           | $b$    | 1/5 (per years)              | 1/5 (per years)              |
| Positive media rate    | $P$    | 504 (per years)              | 780 (per years)              |
| Negative media rate    | $N$    | 36 (per years)               | 60 (per years)               |
| Positive media proportion| $\epsilon$| 0.095                      | 0.12                         |
In this model, we take the boredom rate based on the best result from numerical simulation of the first model, that is \( b = 0.2 \). The best model here means the difference of number of voters by simulation with this boredom rate and the polling result of number of voters from corresponding survey institution is small. A report from Social Mention from May 4, 2014 to June 3, 2014 showed that the ratio between positive media rate and negative media rate of Prabowo-Hatta is 42 to 3 per months (504 to 36 per years) while Jokowi-Kalla had 65 to 5 per months (780 to 60 per years). Jokowi-Kalla seemed to be the topic of more number of positive conversations than Prabowo-Hatta. Social Mention is a social media search and analysis platform that aggregates user generated content from across the universe into a single stream of information. Based on results from the numerical simulation, we take positive media proportion for Prabowo-Hatta is 0.095 while positive media proportion for Jokowi-Kalla is 0.12.

3. Mathematical analysis and numerical simulation

In this section, the SIRS political fanaticism figure voters model will be applied in 2014 Indonesian presidential election. The model will be analysed and numerical simulation using R software is also performed to support the analytical results and to illustrate the dynamics of the model. Throughout this section we use the initial and parameter values as given in the preceding section.

Consider the following transformations:

\[
sl = \frac{S}{V}, i = \frac{I}{V}, r = \frac{R}{V}
\]

Calculating the proportion of these compartments, varying from 0 to 1, it is considered that the total population is constant over time, i.e., \( s + i + r = 1 \). Then the ODE system in Equation (1) is transformed to

\[
\begin{align*}
\frac{ds}{dt} &= \mu - \mu s - \beta s i - \varepsilon Ps + (1 - \varepsilon)Ni - (1 - \varepsilon)Ns + \varepsilon Pr \\
\frac{di}{dt} &= \beta si + \varepsilon Ps - (1 - \varepsilon)Ni - \mu i - bi \\
\frac{dr}{dt} &= bi + (1 - \varepsilon)Ns - \mu r - \varepsilon Pr
\end{align*}
\]

with \( s, i, r \) denotes the proportionate variables of our system. The domain of the system in Equation (2) is \((s, i, r) \in D = \{(s, i, r) : 0 < s \leq 1, 0 \leq i \leq 1, 0 \leq r \leq 1, 0 < s + i + r \leq 1\} \).

The system (2) has two different equilibrium points, which we called the political fanaticism figure voters free equilibrium point and the political fanaticism figure voters endemic equilibrium point. The political fanaticism figure voters free equilibrium point occurs when there is no infected voters in the population \((i = 0)\). In case presence of media, because there is positive media proportion \((P \neq 0)\), so \(i = 0\) occurs when \( s = 0 \) or \( \varepsilon = 0 \). Since \( s > 0 \), the political fanaticism figure voters free equilibrium point only occurs when \( \varepsilon = 0 \), that is at \( E_1 \left( \frac{\mu}{\mu+N}, 0, \frac{N}{\mu+N} \right) \). The political fanaticism figure voters endemic equilibrium point \( E_2 \) is given by

\[
\begin{align*}
s &= \frac{(1 - \varepsilon)Ni + bi + \mu i}{\beta i + \varepsilon P} \\
i &= \frac{-\mu + \mu s + \varepsilon Ps - \varepsilon Pr + (1 - \varepsilon)Ns}{-\beta s + (1 - \varepsilon)N}, \\
r &= \frac{bi + (1 - \varepsilon)Ns}{\varepsilon P + \mu}
\end{align*}
\]

The Jacobian matrix of system in Equation (2) for case presence of media is given by

\[
J(E_k) = \begin{pmatrix}
-\beta I - \varepsilon P - \mu - (1 - \varepsilon)N & -\beta S + (1 - \varepsilon)N & \varepsilon P \\
\beta I + \varepsilon P & \beta S + (1 - \varepsilon)N - b - \mu & 0 \\
(1 - \varepsilon)N & b & -\varepsilon P - \mu
\end{pmatrix}, \quad k = 1, 2
\]

The political fanaticism figure voters free equilibrium point is locally asymptotically stable if system in Equation (2) satisfy the condition

\[
\frac{\varepsilon P + \mu}{(1 - \varepsilon)N} < 1
\]
The characteristic equation of system in Equation (2) for the political fanaticism figure voters endemic equilibrium state is
\[
\begin{align*}
N + b + \mu &< \frac{\mu \beta}{\mu + N} < 3\mu + 2N + b \\
\end{align*}
\]
where \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are eigenvalues of Jacobian matrix \( J(E_k) \). One of the eigenvalues is given by \( \lambda_1 = -\mu \) and two eigenvalues \( \lambda_2 \) and \( \lambda_3 \) can be found from the quadratic equation \( \lambda^2 + p\lambda + q = 0 \), where
\[
\begin{align*}
p &= 2\mu + (-2N + 2P)e + 2N + (-s + i)\beta + b \\
q &= \mu^2 + \left((-2N + 2P)e + 2N + (-s + i)\beta + b\right)\mu + (N^2 - NP + P^2)e^2 + \left(-2N^2 + (s\beta + P - b)N - ((s - i)\beta - 2b)P\right)e + N^2 + (-s\beta + b)N + ib\beta \\
\end{align*}
\]
The eigenvalues \( \lambda_2 \) and \( \lambda_3 \) are negatives when \( \lambda_2 + \lambda_3 = -p < 0 \) and \( \lambda_2\lambda_3 = q > 0 \). It will be satisfied when
\[
2N\epsilon + s\beta < 2\mu + 2N + b + 2P\epsilon + i\beta
\]
and
\[
\mu^2 + ib\beta + P^2e^2 + 2P\epsilon\mu + b\mu + s\beta N\epsilon + 2bP\epsilon > s\beta N
\]
If all of the eigenvalues \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) are negatives, then the political fanaticism figure voters endemic equilibrium point is locally asymptotically stable.
Figure 2. The dynamics of SIRS model for Prabowo-Hatta \((b = 0.2\) and \(\varepsilon = 0.095\)) and Jokowi-Kalla \((b = 0.2\) and \(\varepsilon = 0.12\)) as time series and in phase space.

Figure 2(a) shows the dynamics of the SIRS model for Prabowo-Hatta while Figure 2(b) shows the dynamics of the SIRS model for Jokowi-Kalla. For \(b = 0.2\) and \(\varepsilon = 0.095\) in one year the susceptible voters for Prabowo-Hatta is 31.78%, the infected voters is 46.41%, and the recovered voters is 21.81%. The \(S-I\) trajectory shows that \(S\) decreases from 60.7% to 31.78% and \(I\) increases from 14.2% to 46.41%. The \(S-R\) trajectory shows that \(S\) decreases from 60.7% to 31.78% and \(R\) also decreases from 25.1% to 21.81%. The \(I-R\) trajectory shows that \(I\) increases from 14.2% to 46.41% and \(R\) decreases from 25.1% to 21.81%. For \(b = 0.2\) and \(\varepsilon = 0.12\) in one year the susceptible voters for Jokowi-Kalla is 30%, the infected voters is 52.97%, and the recovered voters is 17.03%. The \(S-I\) trajectory shows that \(S\) decreases from 60.7% to 30% and \(I\) increases from 25.1% to 52.97%. The \(S-R\) trajectory shows that \(S\) decreases from 60.7% to 30% and \(R\) increases from 14.2% to 17.03%. The \(I-R\) trajectory shows that \(I\) increases from 25.1% to 52.97% and \(R\) also increases from 14.2% to 17.03%.

Table 3. Percentage number of votes from various models.

| Model       | Votes for Prabowo-Hatta | Votes for Jokowi-Kalla |
|-------------|-------------------------|------------------------|
| SIR \((b = 0.2)\) | 11.67%                  | 20.86%                 |
| SIR \((b = 1)\)   | 5.26%                   | 9.39%                  |
| SIR \((b = 4)\)   | 0.26%                   | 0.47%                  |
| SIRS          | 46.41%                  | 52.97%                 |
| Real data     | 46.85%                  | 53.15%                 |

Table 3 shows the percentage number of votes for both presidential candidates during 1 years (365 days) from two models; the SIR model which has been done in [4] and the SIRS model. Its results will be compared with real data in estimation of the percentage number of votes in Indonesian presidential election in 2014. The percentage number of votes of the SIRS model for Prabowo-Hatta and Jokowi-Kalla are 46.41% and 52.97%, respectively. The difference in the percentage number of votes between both candidates is 6.56% and the percentage number of swing voters is 0.62%.

4. Conclusions
We have studied the transmission model of political fanaticism figure voters with presence of media in a closed population through epidemiological approach. From analytical results, we find two equilibrium points; political figure fanaticism voters free point and political fanaticism figure voters endemic...
equilibrium point whose existence and local stability depend on such conditions. To assess the effect of presence of media, we conducted numerical simulation of the model. From numerical simulation, we conclude that the presence of media is potential in increasing the spread of political fanaticism figure voters among class of voters. Boredom rate and positive media proportion are two of the sensitive parameters in our models; they affect the number of votes significantly.

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