ON MAKING PREDICTIONS WITH EFFECTIVE FIELD THEORIES IN NUCLEAR PHYSICS *

Tae-Sun Park
Theory Group, TRIUMF,
Vancouver, B.C., Canada V6T 2A3
E-mail: park@alpha02.triumf.ca

Kuniharu Kubodera
Department of Physics and Astronomy, University of South Carolina
Columbia, SC 29208, USA
E-mail: kubodera@nuc003.psc.sc.edu

Dong-Pil Min
Department of Physics, Seoul National University,
Seoul 151-742, Korea
E-mail: dpmi@phys.snu.ac.kr

Mannque Rho
Service de Physique Théorique, CE Saclay
91191 Gif-sur-Yvette, France
and
School of Physics, Korea Institute for Advanced Study
Seoul 130-012, Korea
E-mail: rho@spht.saclay.cea.fr

Based on the effective field theory previously formulated by us to accurately postdict all low-energy two-nucleon properties as well as predict certain electroweak transitions in heavy nuclei, we make parameter-free predictions for the polarized np capture process \( \vec{n} + \vec{p} \rightarrow d + \gamma \) presently being measured at the Institut Laue-Langevin in Grenoble. Other participants of this meeting are invited to make their own predictions using their preferred approaches and join the bet for the best prediction to confront the forthcoming experiment.

1 Introduction

This talk was initially meant to be given by Tae-Sun Park since he is the one doing most of the work – with a little help from three of us – but the

*Talk given by MR at the Workshop on “Nuclear Physics with Effective Field Theories,” Institute for Nuclear Theory, University of Washington, February 25-26, 1999, to appear in World Scientific.
organizers asked me (MR) to present the paper instead. In giving this talk, I would like to distinguish between the statements I am making for which the other members of the collaboration should not be held responsible and those that are endorsed by all of us. The former will be addressed (mostly in footnotes) by “I” and the latter by “We.”

Since the early attempt to apply effective field theories to nuclear physics, there have been many papers written on the subject, the most recent development of which is being summarized in this meeting. In confronting Nature with effective field theories in nuclei – one of the main themes of this meeting, one has had to be content mostly with postdictions, genuine predictions being harder to come by. The reason is simply that effective field theories involve, at each order of power counting, a certain number of parameters in the effective Lagrangian. It is believed that those parameters are in principle calculable from first principles for a given scale (e.g., lattice QCD) but in practice, they have to be fixed by experimental data. Once the parameters are so fixed, the Lagrangian can then be used to make predictions for other processes that involve the same parameters. Up to date, however, most of the calculations involved fixing of the parameters and only rarely could one predict and check the prediction by experiments.

In this talk, we would like to present a genuine prediction based on the formulation of an effective field theory that we have developed during the last few years. Since our formulation is available in the literature, we will not dwell on the details of the formalism but present the essence of the arguments while stressing the possible caveats involved. We illustrate how well postdictions can be made and then how to make predictions for two specific processes, one for which data are available and hence the theory can be tested immediately and the other for which data are not yet available but will be forthcoming, offering a marvelous possibility for an honest prediction totally unbiased by available experiments.

2 The Chiral Filter

An early attempt predating the advent of QCD and effective field theories in nuclear physics was made in 1978 under a conjecture called the “chiral filter hypothesis”. Based on current algebras, it was argued that corrections to the single-particle transitions in nuclei for the isovector M1 operator and the weak axial charge operator should be dominated by one-soft-pion-exchange two-body currents. As a corollary, if the soft-pion exchange two-body currents are suppressed either by symmetry or kinematics as for instance in the isoscalar EM current or the Gamow-Teller operator (i.e., the space component of the axial
the chiral filter says that corrections to the single-particle operator cannot be given by a few controlled terms, thereby making the calculations highly model-dependent. This hypothesis that appeared to be somewhat ad hoc at the time it was proposed turned out to be justified in the context of chiral perturbation theory\(^2\): The soft-pion exchange term is indeed the leading order correction in the chiral counting with the next-to-leading correction calculable but strongly suppressed.

There are two consequences of this chiral filter that survive in the context of modern effective field theories. One is a nontrivial postdiction and the other is an interesting prediction that has been largely confirmed.

2.1 A strategy for effective field theory (EFT)

As it stands, there are effectively two “alternative” ways of power counting in setting up effective field theories for two-nucleon systems. One is the original Weinberg scheme\(^1\) in which the leading four-Fermi contact interaction and a pion-exchange are treated on the same footing in calculating the “irreducible” graphs for a potential that is to be iterated to all orders in the “reducible” channel. The power counting is done only for the irreducible vertex. The other is the “power divergence subtraction” (PDS) scheme\(^7\) in which only the leading (nonderivative) four-Fermi contact interaction is iterated to all orders with the higher-order contact interactions and the pion exchange treated perturbatively. While the PDS scheme is perhaps more systematic in the power counting, we believe that the Weinberg scheme is not only consistent with the strategy of EFT but also, in the sense developed below, more flexible and predictive with possible errors committed due to potential inconsistency in the power counting generically suppressed. In our work, this scheme is adopted.

Iterating to all orders in the reducible channel with the irreducible vertex is equivalent to solving the Schrödinger equation with a corresponding potential. This then suggests that we use, in calculating response functions to slowly varying electroweak fields that we are interested in, those wave functions computed with so-called “realistic potentials,” the prime example being the Argonne \(v_{18}\) potential\(^8\) (called in short Av18). In fact this procedure of mapping effective field theory to realistic wave functions — a hybrid approach\(^9\) — for two-nucleon response functions employed previously by us\(^2\) has recently been justified by means of a cutoff regularization\(^10\). van Kolck has presented a similar argument in support of such a hybrid procedure\(^11\). Now in this framework, the power counting reduces simply to a chiral counting in the irreducible vertex for the current as the current appears only once in the graphs. We are then allowed to separate the current matrix element of interest into the single-
particle and exchange-current terms, with the single-particle matrix element given entirely by that with the Av18 wave function and the exchange current contribution — given by the matrix element with the same wave function — from operators computed in standard baryon chiral perturbation theory. The old soft-pion exchange term figuring in the chiral filter — if not suppressed — is just the leading order contribution in this series.

2.2 Unpolarized np capture

A good example of postdictions that follow from the above scheme and that will be a basis for a genuine prediction described below is the process

\[ n + p \rightarrow d + \gamma \]  

where both nucleons are unpolarized and the incoming neutron has a thermal energy. This process has been computed to the next-to-next-to-leading order (NNLO) in the chiral counting in the scheme described above for the current. The theoretical cross section \( \sigma_{th} = 334 \pm 3 \text{ mb} \) which agrees with the experiment within the error bar consists of the leading contribution, 305.6 mb, coming from the single-particle matrix element given by the Av18 wave function and the remainder from the exchange current dominated by the soft one-pion exchange according to the chiral filter.

Two points are worth noting in this result. First, this is a bona-fide calculation and \textit{not} a fit: Within the scheme adopted here, there are no free parameters. Secondly as shown in\(^\text{[12]}\), the single-particle matrix element has a negligible uncertainty, so that the error in the theory is entirely attributed to the uncertainty in the exchange-current operator associated with the short-distance part of the interactions that cannot be accessed by chiral perturbation theory. This part introduces a scale and renormalization-scheme dependence and only those results that are not sensitive to this short-distance uncertainty can be trusted. In the framework in which the realistic wave functions figure, the short-distance scale is set by the cutoff proportional to \( r_C^{-1} \) where \( r_C \) is the “hard-core radius” that removes the part of the wave function for \( r \lesssim r_c \neq 0 \). The net effect of this cutoff is that in addition to cutting the radial integrals in the coordinate space, it removes \textit{all} zero-range terms in the current operator including zero-ranged counter terms. The 1% error bar assigned to the theory for \(\text{[12]}\) represents the uncertainty in this cutoff procedure. This procedure can be justified for the process in question by using a cutoff \( \sim r_C^{-1} \) and showing that the counter term “killed” by the hard core is in that uncertainty range. Given that the four-Fermi counter term is removed by the hard core which may

\(^1\text{The relative momentum in the center of mass is } \sim 3.4 \times 10^{-3} \text{ MeV.}\)
be viewed as exploiting a scheme dependence, there are no more parameters in the theory. This procedure, familiar to nuclear physicists, will be referred to as “hard-core cutoff scheme” (or HCCS in short). Below we will see that when the chiral filter does not apply, the removal of the zero-range counter terms by the simple hard-core cutoff may be suspect.

2.3 Prediction for the axial-charge transitions in nuclei

The EFT scheme described in section 2.1 makes a rather clean prediction which has not yet been adequately appreciated in nuclear physics community. The chiral filter idea, by now validated in chiral perturbation theory, predicts that in the nonrelativistic regime, the axial charge transition matrix element for the $\beta$ decay process in nuclei

$$A(J^\pm) \rightarrow A'(J^\mp) + e + \nu; \quad \Delta T = 1$$

should receive a huge one-soft-pion exchange current correction, amounting to more than 50% of the single-particle matrix element. The prediction for this is quite robust and firm with very little nuclear model dependence: possible corrections from higher chiral order terms – which are calculable a priori – are estimated to be less than 10% of the leading soft-pion terms.

How do we go about checking this prediction against experiments?

It is not possible to test this in the two-nucleon systems for which the effective field theory is most extensively developed: the matrix element for (2) cannot be measured in few-nucleon systems. The measurement can only be made in heavier nuclei. This means that to compare with experiments, we would have to account for the effect of density or multi-nucleon interactions on the transition. This introduces a subtlety which is interesting in its own right. We will show below that there is rather compelling evidence that both the chiral filter and density effect are confirmed.

Experimentalists extract the axial charge matrix element $M$ from their experiments and then write

$$\epsilon_{MEC} = \frac{M_{\text{measured}}}{M_{\text{th}} - 1b}$$

---

Footnotes:

1. See appendix B in the second reference of for the zero-ranged counter term in question.
2. The issue of effective field theory for hadrons immersed in a hadronic medium with baryon density $\rho$ was one I (MR) would have liked to discuss in this meeting but it is out of the scope of this presentation. For a recent discussion on this subject, we simply refer to the article.
3. For the present problem, the net effect is the presence of what is known as “Brown-Rho (BR) scaling” $\Phi(\rho)$. 

5
where the numerator is the total “measured” value or more precisely the value extracted from experiments and the denominator is the theoretical single-particle matrix element of the single-particle current whose constants are unrenormalized by medium. The model dependence of the denominator – and to some extent the numerator – makes this quantity not entirely empirical, thus open to controversy among theorists: It would seem that the “experimental” $\epsilon_{\text{MEC}}$ would in practice depend upon the model for the wave function used for the sing-particle matrix element. On the other hand, our claim is that the theoretical expression for this quantity is well-defined and free of model dependence \footnote{Several people in the audience voiced doubt as to whether the experimental test I discussed is truly valid. One of the reasons given is the model dependence of the so-called empirical information in $\epsilon_{\text{MEC}}$. This objection is to some extent valid and needs to be carefully examined. The presently employed procedure is to calculate the single-particle matrix element with the “best” nuclear wave functions of the single-particle axial charge operator whose coupling constants are unrenormalized by medium. The question then is what does one mean by “best” wave function? Here we should stress that in the case in question, there is a reasonably satisfactory answer. Let’s take Warburton’s analyses \cite{Warburton2001}. Although his analyses do involve shell-model wavefunctions, the semi-empirical nature of the effective transition operator method he adopted gives his results much more robustness than people normally expect from shell-model calculations. That is, after including the core polarization effects in the form of rescaling the single-particle matrix elements, there is in fact very little room for changing nuclear physics input in his analyses. As a measure of the basic soundness of his approach, we mention the following fact. As far as nuclear dynamics is concerned, the rank-zero and rank-one first-forbidden transitions share the same feature, but the exchange currents for them have very different behavior. The rank-one operators coming from the space component of the axial current has only small exchange currents whereas the time component of the axial current that contributes to the rank-zero operator is expected to have a huge exchange current. Now, Warburton’s calculations reproduce very well the strengths of all the rank-one matrix elements with no further adjustments whereas, for the rank-zero matrix elements, he found it clearly necessary to introduce the “empirical” enhancement factor $\epsilon_{\text{MEC}}$. The other reason (which is theoretical) is that the BR scaling is so far implemented in the limit of infinite nuclear matter and the question arises regarding the finite size effect of the nuclei measured. My answer to this objection is that whereas experimentally the quantity $\epsilon_{\text{MEC}}$ may perhaps be somewhat model-dependent, theoretically, however, it is very well defined: It involves the BR scaling factor $\Phi$ which is smoothly varying for density up to $\rho = \rho_0$, so what matters is the average density involved and the quantity $R$, the ratio of the matrix element of the exchange current over that of the single-particle operator, which is quite insensitive to nuclear models provided the same wave functions are used for both.}

Let $M_{\text{total}}$ denote the total theoretical axial-charge matrix element and $M_n$ with $n = 1, 2$ be the matrix element of the $n$-body operator effective in medium. Then the prediction is that

\begin{align}
\epsilon_{\text{MEC}}^{\text{th}} &= \frac{M_{\text{total}}}{M_1} = \Phi^{-1}(1 + R), \quad (4) \\
M_{\text{total}} &= M_1 + M_2 \quad (5)
\end{align}
with

\[
\mathcal{R} = \mathcal{M}_2 / \mathcal{M}_1 = (M_2 / M_1) (1 + \mathcal{O}(10^{-1}))
\]

(6)

where \( M_n \) is the matrix element of the \( n \)-body current with its basic coupling constants (e.g., \( f_\pi, g_A \) etc) unrenormalized by medium. This relation follows within our HCCS (hard-core cutoff scheme) provided one assumes that all light-quark hadron mass \( M \) except for the pion mass scales in medium as

\[
M^*/M \approx f^*_\pi / f_\pi \approx \Phi(\rho). 
\]

(7)

The expression (6) was first derived by Kubodera and Rho\[18\] and later corrected\[14,15\]. Now what we need is the value for \( \Phi \) and \( \mathcal{R} \) for a range of nuclei that have been measured. The range of nuclear density involved is \( 1/2 \lesssim \rho/\rho_0 \lesssim 1 \) where \( \rho_0 \) is the nuclear matter density. For this range, we know from information gotten from giant dipole resonances and QCD sum rules in medium\[15\] that \( \Phi \) goes as \( \sim 1/(1 + 0.28(\rho/\rho_0)) \),

\[
0.78 \lesssim \Phi \lesssim 0.88 \quad \text{for} \quad 1/2 \lesssim \rho/\rho_0 \lesssim 1. 
\]

(8)

The two-body current in the ratio \( \mathcal{R} \) is given within 10% by a soft-pion exchange which is completely fixed by chiral symmetry. The ratio turns out to be extremely insensitive to nuclear models used to compute the matrix elements and depends only slightly on density. It comes out to be\[7\]

\[
0.43 \lesssim \mathcal{R} \lesssim 0.61 \quad \text{for} \quad 1/2 \lesssim \rho/\rho_0 \lesssim 1. 
\]

(9)

This gives the range

\[
1.63 \lesssim \epsilon^{th}_{MEC} \lesssim 2.06 \quad \text{for} \quad 1/2 \lesssim \rho/\rho_0 \lesssim 1. 
\]

(10)

A formula as simple as (6) must have a simple and clean test. It must be readily confirmed or infirmed.

We claim that there is a strong empirical support for this prediction. Indeed this prediction can be compared with the presently available “empirical” values\[3,2,17\]

\[
\epsilon_{MEC}^{exp} = 1.64 \pm 0.05 \quad (A = 12) ; 1.62 \pm 0.05 \quad (A = 50) ; \\
1.95 \pm 0.05 \quad (A = 205) ; 2.05 \pm 0.05 \quad (A = 205 \sim 212). 
\]

(11)

vi. There was an error in the original derivation due to the fact that the non-scaling of the pion mass, indicated by both theory and experiment, was not properly taken into account.

vii. Owing to the chiral filter applicable to this process, the uncertainty in our hard-core cutoff scheme, i.e., “killing” zero-range counter terms, is reflected in the 10% uncertainty in \( \mathcal{R} \) mentioned above.
We further suggest that these constitute evidence for both (1) the gigantic enhancement predicted by the chiral filter, ranging from 46% to 61% and (2) the additional enhancement predicted by BR scaling, ranging from 20% to 45%. There is of course the caveat that individual transitions must be subject to some finite-size effects but the point is that what one is probing here is a generic bulk property of nuclear matter – which to us is the most interesting part of the story.

3 Polarized Neutron-Proton Capture: A New Probe

We will now go outside of those processes protected by the chiral filter and make a genuine prediction even though the process, unprotected by the chiral filter, could be highly suppressed. One would think that such a prediction is out of the scope of effective field theories but surprisingly, it turns out not to be the case in our EFT scheme.

3.1 Selection rules

Consider the process

\[ \vec{n} + \vec{p} \rightarrow d + \gamma \] (12)

where now both the target proton and the projectile neutron are polarized. This process is being measured at the Institut Laue-Langevin (ILL) in Grenoble by Müller et al.\cite{Mueller22}. The interest in this experiment is that with the polarized target and the polarized beam, one can measure small matrix elements that are overwhelmed by the dominant isovector M1 matrix element when averaged over polarization. To see this, look at the quantum numbers involved in the process. The initial state of (12) at very low energy we are dealing with can be in either \( ^1S_0 \) or \( ^3S_1 \) channel. The Fermi-Dirac statistics requires that the former must be in \( T = 1 \) and the latter in \( T = 0 \) where \( T \) is the isospin. The final nuclear state in (12) is the deuteron which is in \( ^3S_1 \) or \( ^3D_1 \) with \( T = 0 \). There are then three relevant transition matrix elements with the emission of a soft photon, i.e., isovector M1, isoscalar M1 (which we shall denote from now on as \( \overline{M1} \) to distinguish it from the isovector M1) and isoscalar E2. We shall adopt the convention of Müller et al.\cite{Mueller22} and write the transition amplitude as

\[
\langle \psi_d(M_d), \gamma(\vec{k}\lambda)|T|\psi_{np}(s_p, s_n) \rangle = \chi_{1M_d}^1 \mathcal{M}(\vec{k}, \lambda) \chi_{s_p} \chi_{s_n} \] (13)

with

\[
\mathcal{M}(\vec{k}\lambda) = \sqrt{4\pi} \frac{\sqrt{n_n}}{2\sqrt{\omega A_s}} \left[ i(\vec{k} \times \epsilon^*) \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) \mathcal{M}(^1S_0) \right]
\]
\[-i(\hat{k} \times \epsilon^*) \cdot (\hat{\sigma}_1 + \hat{\sigma}_2) \frac{M_1(3S_1)}{\sqrt{2}} + (\hat{\sigma}_1 \cdot \hat{k} \hat{\sigma}_2 \cdot \epsilon^* + \hat{\sigma}_2 \cdot \hat{k} \hat{\sigma}_1 \cdot \epsilon^*) \frac{E_2(3S_1)}{\sqrt{2}} \] (14)

where $M_d$ and $\lambda$ are respectively the polarizations of the deuteron and the photon, $\hat{k}$ is the unit momentum vector of the photon, $\omega$ its energy, $\hat{\epsilon} \equiv \hat{\epsilon}(\hat{k} \lambda)$, $v_n$ is the velocity of the neutron and $A_s$ is the deuteron normalization factor $A_s \approx 0.8850 \text{ fm}^{-1/2}$. In the way defined, the quantities $M_1(1S_0)$, $M_1(3S_1)$ and $E_2(3S_1)$ all have a dimension of fm and the cross section for the unpolarized $np$ system takes the form

\[\sigma_{\text{unpol}} = |M_1(1S_0)|^2 + |M_1(3S_1)|^2 + |E_2(3S_1)|^2.\] (15)

The first term is the isovector M1 contribution, the second the isoscalar M1 and the last the isoscalar E2. As we shall see below, the second and third terms are strongly suppressed compared to the first, $\sim O(10^{-6})$, so the unpolarized cross section cannot “see” these terms.

3.2 Polarization observables

In order to see those small isoscalar terms, one measures polarization observables, i.e., the photon circular polarization $P_\gamma$ and the photon anisotropy $\eta$. For an unpolarized proton and a polarized neutron with the polarization vector $\vec{P}_n$, $P_\gamma$ is given by

\[P_\gamma = \frac{|\vec{P}_n|}{2} \frac{2\sqrt{2}(R_{M1} - R_{E2}) + (R_{M1} + R_{E2})^2}{1 + R_{M1}^2 + R_{E2}^2} \] (16)

where we have defined the ratios

\[R_{M1} \equiv \frac{M_1(3S_1)}{M_1(1S_0)}, \quad R_{E2} \equiv \frac{E_2(3S_1)}{M_1(1S_0)}.\] (17)

Since these ratios turn out to be $\sim 10^{-3}$, eq. (16) simplifies with high accuracy to

\[P_\gamma \approx |\vec{P}_n| \sqrt{2}(R_{M1} - R_{E2}).\] (18)

The anisotropy measures the fully polarized $np$ system involving the polarization of the target and projectile nucleons and is given by

\[\eta = \frac{I(90^\circ) - I(0^\circ)}{I(90^\circ) + I(0^\circ)} = pP \frac{R_{M1}^2 + R_{E2}^2 - 6R_{M1}R_{E2}}{4(1-pP) + (4+pP)(R_{M1}^2 + R_{E2}^2) + 2pP R_{M1} R_{E2}} \] (19)
where $I$ is the photon intensity,

$$pP \equiv \vec{P}_p \cdot \vec{P}_n$$

(20)

and the angle in $I$ measures the photon direction with respect to the spin polarization of the neutron and the proton. Note that unless the factor $(1 - pP) \sim 0$ the anisotropy $\eta$ will be quadratically suppressed while $P_\gamma$ is linear in the ratio. If however $(1 - pP) \sim 0$, then the anisotropy could be substantial, supplying an additional formula that would allow one to extract the two ratios (17). The purpose of the experiment is to determine these two ratios. The quantity $P_\gamma$ has already been measured before by Bazhenov et al (23), so the aim of the ILL experiment (22) is to measure the anisotropy $\eta$.

3.3 Doing EFT

As mentioned in section (2), the current involved here is not protected by the chiral filter. This means that the soft-pion exchange which is entirely given by chiral symmetry considerations cannot contribute. The corollary to the chiral filter hypothesis would then suggest that we might be opening Pandora’s box. To our surprise, this does not seem to be the case for the problem at hand.

- **Power Counting**

Since the isovector M1 operator is calculated very accurately to $O(Q^3)$ relative to the single-particle operator (i.e., $M1(^3S_0) = 5.78 \pm 0.03$ fm in the notation of (13) which comes at $O(Q^{-2})$ we will focus on the isoscalar operators here.

We assume that as in the case studied so far, given the accurate wave functions, the leading single-particle matrix elements are accurately given for both $M1$ (i.e., isoscalar M1) and $E2$ operators. The power counting that we adopt then shows that while the ratio $(2\text{-body})/(1\text{-body})$ goes like $O(Q^1)$ for the operator protected by the chiral filter (e.g., the isovector M1), the ratio for the isoscalar operators goes like $O(Q^n)$ with $n \geq 3$, so naively one would expect a suppression by two orders of power counting. Now the question is: How well can we pin down such two-body terms and higher-order corrections to them?

In our scheme, there are two classes of irreducible two-body terms in the current operator. The class A term consists of graphs with the one-pion exchange involving an $NN\pi\gamma$ vertex (see Fig.1a) and the class B term consists

\[\text{unless otherwise specified, we will always give counting relative to the leading single-particle operator.}\]
Figure 1: Generic diagrams for the two-body isoscalar current $\mathcal{B}_{2b}^\mu$. The solid circles include counter-term insertions and (one-particle irreducible) loop-corrections. The wiggly line stands for the external field (current) and the dashed line the pion. One-loop corrections for the pion propagator and the $\pi NN$ vertex are of course to be included at the same order.

Figure 2: One-loop graphs that contribute to the $\mathcal{B}\pi NN$ vertex where $\mathcal{B}$ is the isoscalar current. They give rise to $\mathcal{O}(Q^4)$ and higher corrections to the leading order (LO) one-body term.
of graphs with two- or more-pion exchanges (see Fig.3b). Since there is no chiral-filter-protected one-pion exchange in the isoscalar vector current, the class A term receives its leading contribution from one-loop corrections (see Fig.3b). The class B term is generically given by the loop graphs of Fig.3. The loop and derivatives (in Fig.3c) account for the additional power suppression.

The graphs in Figs.2 and 3 typically contribute to the isoscalar $M_1$ operator $(\overline{M}_1)$ and E2 operator at $O(Q^4)$. This is however not the whole story to $O(Q^3)$, for there are so-called “counter terms” that can come in at $O(Q^3)$. There are in fact two such terms in the case that we are concerned with. One such term is a one-pion exchange graph in Fig.3, with the $B\pi NN$ vertex given by a finite counter term. This term that contributes to the $\overline{M}_1$ operator but not to the E2 operator is dominated by the $\gamma\rho\pi$ coupling in the anomalous parity component of the effective chiral Lagrangian, i.e., the Wess-Zumino term, which is connected with the Adler-Bell-Jackiw triangle anomaly. This term is known and so brings in no unknown parameters. The other is a four-Fermi counter term, call it $g_4$, in Fig.3b, contributing only to $\overline{M}_1$. The coefficient of this counter term, not known a priori, needs to be fixed in the usual way.

Note that there are no $O(Q^3)$ counter term contributions to the E2.

![Figure 3: One-loop graphs that contribute to the two-body baryonic currents. They come at $O(Q^3)$ and higher order relative to the LO one-body term. All possible insertions of the external line are understood.](image)

- **There is No Parameter in the Theory**

We shall now argue that while there remains one parameter undetermined by the theory it can be gotten rid of in either of the two ways in which physics at short distance is treated.

We first note that there are no unknown parameters in the class A diagrams: As mentioned, the $O(Q^3)$ counter term is fixed by the Wess-Zumino
term and the loop graphs are completely calculable to $O(Q^4)$ without any parameters. In the class B diagrams, there are two parameters, call them $V_i$ with $i = 1, 2$, associated with Fig. 3c. One of them, $V_1$, contributes to both $\mathcal{M}_1$ and $E_2$ and the other, $V_2$, only to $\mathcal{M}_1$. We find that the $V_1$ term plays no role in $\mathcal{M}_1$ or $E_2$, as it is suppressed by some power of $Qc \ll 1$.

The upshot of all this is that we are finally left with the four-Fermi counter term $g_4$ and $V_2$. Furthermore, both of them are associated with zero-range terms in the coordinate space. The combination that appears here will be called, in short, the $g_4$ " + " $V_2$ term. So we can combine them with the $O(Q^3)$ zero-ranged operator that comes from the one-pion exchange term with the Wess-Zumino vertex (we shall call this in short the $O(Q^3)$ WZ term), thereby reducing them effectively to only one parameter in $\mathcal{M}_1$. The $E_2$ operator is completely free of parameters to $O(Q^4)$.

Now in the HCCS, the single parameter of the theory does not figure since the contact operator that is multiplied by the parameter gets suppressed by the hard core. But there is a possible caveat here: Since the leading order of this term is $O(Q^3)$ whereas the finite loop corrections are of $O(Q^4)$, it is not obvious that the HCCS should be reliable in the present case. One might suspect that here short-distance physics could intervene more strongly than when the chiral filter is operative. There are two possible remedies to this problem. One is to implement the operator-product-expansion (OPE) factorization in the wave function suggested by Lepage and the other is to use a cutoff $\sim r^{-1}_c$ in Fourier-transforming the current operators into the coordinate-space form. We shall use the latter which is a lot simpler. We use an equivalent method which is to replace the delta function in all zero-ranged operators by the delta-shell form

$$\delta(r) \to \delta(r - r_c). \quad (21)$$

This procedure allows the $O(Q^3)$ contact operator to contribute in $\mathcal{M}_1$, hence allowing to fix the unknown $g_4$ + $"V_2$ parameter by fitting the deuteron magnetic moment. It turns out however that this contact term is dominated by the known $O(Q^3)$ WZ term, so in practice, the unknown parameter plays only a minor role here. We shall call this scheme the "modified hard core cutoff scheme."

3.4 Our predictions

We shall now make our predictions.

- Hard Core Cutoff Scheme (HCCS)
With the hard core in the wave function, the parameter-dependent term is killed, so we can now predict the deuteron magnetic moment \( \mu_d \), the quadrupole moment \( Q_d \), and the ratios \( R_{M1} \) and \( R_{E2} \). There must of course be some dependence on the hard core size which enters as a cutoff but the consistency of EFT requires that the cutoff dependence be small.

**Deuteron magnetic moment** \( \mu_d \): There is a strong cancellation between the \( O(Q^3) \) and \( O(Q^4) \) two-body terms, leaving the one-body term essentially uncorrected: The net two-body correction is found to be less than 0.7% of the one-body term. The predicted values for \( r_c = 0.01, 0.2, 0.4, 0.6, 0.8 \) fm are (in units of nuclear magneton)

\[
\mu_d = 0.8408, 0.8443, 0.8426, 0.8407, 0.8390. \tag{22}
\]

The experimental value is \( \mu_d^{exp} = 0.8574 \).

This small discrepancy will be exploited later to fix the one parameter that figures when the zero-range operator is not killed by the hard core as in MHCCS.

**Deuteron quadrupole moment** \( Q_d \): The two-body correction is equally tiny, less than 0.6% and more or less independently of \( r_c \) within the range \( 0.01 \lesssim r_c \lesssim 0.8 \) fm. The result is (in unit of \( \text{fm}^2 \))

\[
Q_d = 0.2710 \tag{23}
\]

This can be compared with the experiment \( Q_d^{exp} = 0.2859 \text{ fm}^2 \). It appears that the 5% discrepancy found here cannot be understood in low-order effective field theories.

\( R_{M1} \): The two-body terms of \( O(Q^3) \) and \( O(Q^4) \) in \( M_1 \), separately, are of the same magnitude as the one-body term, so although naively suppressed in the power counting, there is no genuine suppression according to the hierarchy of the order. However there is a considerable cancellation between the two higher-order terms, leaving the correction to be between 9% and 25% of the single-particle value. The predicted values for \( r_c = 0.01, 0.2, 0.4, 0.6, 0.8 \) fm are

\[
- R_{M1} \times 10^3 = 0.869, 0.788, 0.826, 0.871, 0.887. \tag{24}
\]

\( R_{E2} \): As in the quadrupole moment, the two-body correction to the \( E2 \) matrix element is small \( \lesssim 0.4\% \), so the result is essentially given by the one-body term. Thus within the \( r_c \) range considered, the result is, independently of \( r_c \),

\[
R_{E2} \times 10^3 = 0.242. \tag{25}
\]
**Photon circular polarization** \( P_\gamma \): With the above values for \( R_{M1} \) and \( R_{E2} \), the predicted values for \( r_c = 0.01, 0.2, 0.4, 0.6, 0.8 \) fm are (for \(|P_n| = 1\))

\[
- P_\gamma \times 10^3 = 1.57, 1.46, 1.51, 1.57, 1.60. \tag{26}
\]

**Photon anisotropy** \( \eta \): Here the prediction is extremely sensitive to the value of the polarization \( pP \). We will quote for three cases: \( pP = 1 \) (the ideal case), \( pP = 0.96 \) (the highest polarization that may be reached), and \( pP = (0.5)^2 \) (a case most certainly accessible to the experiment). The predicted values for \( r_c = 0.01, 0.2, 0.4, 0.6, 0.8 \) fm are

\[
\eta[pP = 1] = 0.57, 0.61, 0.59, 0.57, 0.56, \\
\eta[pP = 0.96] \times 10^5 = 1.3, 1.1, 1.2, 1.3, 1.3, \\
\eta[pP = (0.5)^2] \times 10^7 = 1.7, 1.5, 1.6, 1.7, 1.8. \tag{27}
\]

- **Modified Hard Core Cutoff Scheme (MHCCS)**

  We shall apply the “smoothing” \([21]\) to the delta function and account for the term carrying information on the single parameter available in the theory. We have the possibility to fix the constant by “fine-tuning” it to the deuteron magnetic moment, that is, attributing the 5% discrepancy in \( \mu_d \) from the experimental value to the counter term containing the \( O(Q^3) \) WZ term and the \( g_4 + / V_2 \) term. As mentioned, in practice, this term is completely dominated by the former (in fitting the deuteron magnetic moment); therefore the unknown constant plays only a minor role in the resulting isoscalar M1 operator that is to be used to compute the \( M1 \) matrix element.

  While \( \mu_d \) is no longer predicted in this scheme (since it is used to pin down the small \( g_4 + / V_2 \) term), the deuteron quadrupole moment \( Q_d \) remains unmodified from eq.(23). The ratios \( R_{M1} \) and \( R_{E2} \) are of course predicted.

  \( R_{M1} \): There turns out be a remarkable \( r_c \) independence for this quantity. In fact, in the range \( 0.01 \leq r_c \leq 0.8 \) fm, the result is the same:

\[
- R_{M1} \times 10^3 = 0.500. \tag{28}
\]

  \( R_{E2} \): This quantity remains unchanged from the HCCS, \([23]\),

\[
R_{E2} \times 10^3 = 0.242. \tag{29}
\]

**\( P_\gamma \) and \( \eta \):** Independently of \( r_c \) in the range \( 0.01 \leq r_c \leq 0.8 \) fm, we find

\[
- P_\gamma \times 10^3 = 1.05 \tag{30}
\]
and

\[ \eta[pP = 1] = 0.80, \]
\[ \eta[pP = 0.96] = 0.62 \times 10^{-5}, \]
\[ \eta[pP = (0.5)^2] = 0.86 \times 10^{-7}. \] (31)

4 Conclusion: Call for a Bet

The values we have are not the final ones. First of all, they will have to be rechecked more thoroughly, and secondly, given that the isoscalar matrix elements are so suppressed relative to the isovector matrix element, it may be necessary to take into account the usually negligible isospin violation (both in the interaction and electromagnetic radiative corrections). In any event, we shall give our preliminary predictions here with the warning that they are subject to further changes. We expect to be able to publish a paper with our final numbers in the near future with the above caveats taken into account.

In Table 1 are summarized our predictions. Recall that there are two schemes for treating the zero-ranged counter terms. One scheme, referred to as hard core cutoff scheme (HCCS), is the usual nuclear physics practice to kill the delta function terms in the operator. The physics so purged from them is presumably shifted to the finite matrix elements affected by the “correlation hole.” In this case there are no more parameters left to account for the small deviation from the experimental data in \( \mu_d \) and \( Q_d \) and the results would depend on \( r_c \); the dependence would of course be weak if the procedure were consistent with the premise of EFT. The other scheme, called modified hard core cutoff scheme (MHCCS), exploits the non-vanishing of zero-range operators to fix one parameter available in the theory by fitting the deuteron magnetic moment, which then determines completely the isoscalar M1 operator to \( \mathcal{O}(Q^4) \). In this scheme, there is practically no \( r_c \) dependence and the convergence of the higher-order terms is assured by the procedure (specifically, there is no difference between the \( \mathcal{O}(Q^3) \) calculation and the \( \mathcal{O}(Q^4) \) calculation once \( \mu_d \) is fit).

There is only one polarization data available at the moment, namely, the photon circular polarization measured by the Russian group:

\[ P_\gamma^{exp} = (-1.5 \pm 0.3) \times 10^{-3}. \] (32)

As it stands, this experiment seems to favor the HCCS result. However the prediction in which we have more confidence is that of MHCCS, which puts it somewhat lower than the experimental value. Clearly additional measurements will be needed to confirm or improve on it.
Table 1: Predictions using two schemes for implementing the hard core: HCCS (hard-core cutoff scheme) in which zero-ranged operators are “killed” by the short-range correlation function in the wave function and MHCCS (modified hard core cutoff scheme) in which the delta function of zero-range operators is “smoothed” to the delta shell form. The “error bar” represents the variation over the range $0.01 \text{ fm} \leq r_c \leq 0.8 \text{ fm}$. No error bar means that there is no $r_c$ dependence.

| Hard-core scheme | HCCS   | MHCCS   |
|------------------|--------|---------|
| $10^3 \times P_\gamma$ | $-1.5 \pm 0.1$ | $-1.3$   |
| $\eta[pP = 1]$   | $0.58 \pm 0.03$ | $0.80$   |
| $10^5 \times \eta[pP = 0.96]$ | $1.2 \pm 0.1$ | $0.62$   |
| $10^7 \times \eta[pP = 0.25]$ | $1.6 \pm 0.1$ | $0.86$   |
| $10^3 \times R_{M1}$ | $-0.84 \pm 0.05$ | $-0.51$ |
| $10^3 \times R_{E2}$ | $0.24$ | $0.24$ |

The Bet

We would now like to invite other workers in the field – particularly those who are in this audience, who have worked out consistent and systematic power countings – to make similar predictions and participate in the bet for the best prediction, that is, the one that agrees best with the experimental result that is forthcoming. As for our prediction, the most interesting possibility would be that the experiment simply disagrees with our two versions of the hard-core scheme. That would sharpen our chiral filter conjecture and bring in totally new physics.

Acknowledgments

We would like to thank the organizers of this meeting for the invitation to participate in the exciting debate and Thomas Müller for discussions and correspondence on the on-going experiment at the ILL.

\*\*\* At the time of my talk, I made a specific proposal for the prize for the winner and I confirm that proposal here. I would propose that each participant should be willing to offer to the winner a bottle of one of the best wine of his/her country. As for me, I would offer a bottle of superb French wine, Chateau Mouton-Rothschild, a “premier grand cru classe.”
References

1. S. Weinberg, *Phys. Lett.* B 251, 288 (1990); Nucl. Phys. B 363, 3 (1991); *Phys. Lett.* B 295, 114 (1992)
2. M. Rho, *Phys. Rev. Lett.* 66, 1275 (1991); T.-S. Park, D.-P. Min and M. Rho, *Physics Reports* 233, 341 (1993); *Phys. Rev. Lett.* 74, 4153 (1995); Nucl. Phys. A 596, 515 (1996)
3. C. Ordonez and U. van Kolck, *Phys. Lett.* B 291, 459 (1992); C. Ordonez, L. Ray and U. van Kolck, *Phys. Rev. Lett.* 72, 1982 (1994); *Phys. Rev. C* 53, 2086 (1996); U. van Kolck, *Phys. Rev. C* 49, 2932 (1994)
4. S. Beane, T. Cohen, D. Kaplan, G.P. Lepage, M. Savage, J. Steele, U. van Kolck, M. Wise, ... talks in this meeting.
5. G.P. Lepage, this meeting.
6. K. Kubodera, J. Delorme and M. Rho, *Phys. Rev. Lett.* 40, 755 (1978)
7. D.B. Kaplan, M. Savage and M. Wise, Nucl. Phys. B 478, 629 (1995); *Phys. Lett. B* 424, 390 (1998); nucl-th/9801034, nucl-th/9802075, nucl-th/9804032
8. R.B. Wiringa, V.G.J. Stoks and R. Schiavilla, *Phys. Rev.* C 51, 38 (1995)
9. K. Kubodera, “Chiral symmetry in nuclei,” nucl-th/9903057
10. T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, *Phys. Rev. C* 58, R637 (1998); Nucl. Phys. A 646, 83 (1999)
11. U. van Kolck, “Effective field theory of nuclear forces,” nucl-th/9902015
12. T.-S. Park, D.-P. Min and M. Rho, *Phys. Rev. Lett.* 74, 4153 (1995); Nucl. Phys. A 596, 515 (1996)
13. M. Savage, K.A. Scaldeferri and M.B. Wise, nucl-th/9811029
14. T.-S. Park, K. Kubodera and I.S. Towner, Nucl. Phys. A 579, 381 (1994)
15. B. Friman, M. Rho and C. Song, nucl-th/9809088
16. G.E. Brown and M. Rho, *Phys. Rev. Lett.* 66, 2720 (1991)
17. E.K. Warburton, *Phys. Rev. Lett.* 66, 1823 (1991); E.K. Warburton, I.S. Towner and B.A. Brown, *Phys. Rev. C* 49, 824 (1994); E.K. Warburton and I.S. Towner, *Phys. Lett. B* 294, 1 (1992)
18. K. Kubodera and M. Rho, *Phys. Rev. Lett.* 67, 3479 (1991)
19. T. Minamisono et al, *Phys. Rev. Lett.* 82, 1644 (1999)
20. P. Baumann et al, *Phys. Rev. C* 58, 1970 (1998)
21. A. Van Geert et al, “Meson-exchange enhancement in the first-forbidden beta transition of 205Hg,” in Proceedings of Paris Conference on Nuclear Physics, Paris, France, August, 1998
22. T.M. Müller, private communication; T.M. Müller, D. Dubbers, P. Hautle and O. Zimmer, “Measurement of the $\gamma$ anisotropy in the $\vec{p}\vec{n}, \gamma d$-process,” in Proceedings of the “International Workshop on Particle Physics with Slow Neutrons,” ILL, Grenoble, France, 22-24 October 1998.

23. A.N. Bazhenov et al, *Phys. Lett.* B 289, 17 (1992)

24. G.P. Lepage, “How to renormalize the Schrödinger equation?”, [nucl-th/9706029](http://arxiv.org/abs/nucl-th/9706029)

25. T.-S. Park, K. Kubodera, D.-P. Min and M. Rho, in preparation.