Radiatively driven rotating pair-plasma jets from two-component accretion flows

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ABSTRACT

Centrifugal pressure of matter spiralling on to black holes has long been known to produce standing or oscillating shocks. The post-shock disc puffs up in the form of a torus, which intercepts soft photons from the outer Keplerian disc and inverse-Comptonizes them to produce hard photons. The post-shock region also produces jets. We study the interaction of both hard photons and soft photons, with rotating electron–positron jets. We show that hard photons from the post-shock torus are instrumental in acceleration of jets, while soft photons from the Keplerian disc is a better collimating agent. We also show that if the jets are launched closer to the black hole, relativistic and collimated jets are produced; if they are launched at larger distances both collimation and acceleration are less. We also show that if the shock location is at relatively larger distances from the black hole, collimation is better.

Key words: accretion, accretion discs – black hole physics – radiation mechanisms: general – radiative transfer – ISM: jets and outflows.

1 INTRODUCTION

Jets in microquasars as well as in quasars show relativistic terminal speed [e.g. GRS 1915+105, Mirabel & Rodriguez (1994); 3C 273, 3C 345, Zensus, Cohen & Unwin (1995); M87, Biretta (1993)], though the actual acceleration process is an enigma. It is well accepted in the scientific community that jets around compact objects originate from the accretion disc accompanying such compact objects. The study of the interaction of radiation from the disc with outflowing jets is quite extensive. The radiation field produced by a disc depends on the geometry as well as the dominant physical processes of the disc model, and hence the study of the interaction of jets with the radiation field produced by different disc models will, in general, draw different conclusions.

Icke (1980) studied the effect of radiative acceleration of gas flow above a Keplerian disc, but did not consider the effect of radiation drag. Piran (1982), while calculating the radiative acceleration of outflows about the rotation axis of thick accretion discs, found that in order to accelerate outflows to $\gamma > 1.5$ (where $\gamma$ is the bulk Lorentz factor), the funnels must be short and steep, but such funnels are found to be unstable. In a very important paper, Icke (1989) considered blobby jets about the axis of symmetry of thin discs (Novikov & Thorne 1973, hereafter NT73; Shakura & Sunyaev 1973) and he obtained the ‘magic speed’ of $v_m = 0.451c$ ($c$ is the velocity of light), $v_m$ being the upper limit of the terminal speed. Fukue (1996) extended this study for rotating flow above a thin disc and drew similar conclusions, although for rotating flow, away from the axis of symmetry the terminal speed was found to be a little less than the magic speed of Icke. It was shown that rotating winds above the thin disc generally spread as the radiation force along the azimuthal direction (generated because of disc rotation) increases the angular momentum of the flow, and are difficult to accelerate because of the presence of the radiation drag term in the vertical direction (Tajima & Fukue 1996, 1998). Radiatively driven winds were also studied for radiation coming from a slim disc (Watarai & Fukue 1999). In spite of the different disc temperature profile from the thin disc, and also the inclusion of the advection term, radiatively driven outflows from a slim disc mean that these radiation fields will spread the outflows and will suppress the vertical motion. Later Fukue and his collaborators achieved a relativistic terminal speed and collimation for pair-plasma jets from a disc model which contains an inner advection-dominated accretion flow (ADAF) region (non-luminous) and an outer slim disc (luminous), for a non-rotating black hole (Fukue, Tojyo & Hirai 2001), and repeated the same scheme for rotating black holes (Orihara & Fukue 2003). We are working in a different regime. We consider the TCAF (two-component accretion flow) disc model (see Chakrabarti & Titarchuk 1995, hereafter CT95), which consists of a less luminous outer Keplerian disc and a more luminous post-shock torus in the hard spectral state of the disc. Chattopadhyay & Chakrabarti (2000) calculated the radiative force on optically thin jets in and around the axis of symmetry for radiation coming from the post-shock
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Chattopadhyay & Chakrabarti (2002a) investigated the issue of radiative acceleration of normal plasma jets, by radiation only from the post-shock torus of the TCAF disc, and concluded that normal plasma jets are indeed accelerated to mildly relativistic terminal speed. Chattopadhyay & Chakrabarti (2002b) also reported that hard radiation from the post-shock torus does not impose any upper limit on the terminal speed. Chattopadhyay, Das & Chakrabarti (2004, Paper I), solved the equations of photohydrodynamics for on-axis pair-dominated jets for radiation coming from the whole of the TCAF disc model. Paper I concluded that the terminal speed $\vartheta \sim 0.9c$ is easily achieved if the location of the shock in the accretion is $x_s \sim \text{a few } \times 10r_g$ ($r_g = \text{Schwarzschild radius}$), and the luminosity of the post-shock torus ($\equiv L_C$) is $\sim \text{a few } \times 10$ per cent of the Eddington luminosity. Chattopadhyay et al. (2004) also showed that the special geometry of the post-shock torus ensures that its contribution to various radiative moments dominates over that due to a Keplerian disc. Thus radiation from a Keplerian disc (of luminosity $L_K$) has a very marginal effect on determining the terminal speed of jets. Having shown this, it is only natural to study how rotating jets behave in the presence of the radiation field of the disc. The azimuthal component of the radiative force generated by the disc rotation will try to spin up the jet, while the radiative drag force in the same direction will try to carry away the angular momentum from the jet. So it is particularly interesting to study which of these two phenomena wins.

In this paper we study the interaction of radiation from the whole of the TCAF disc and rotating jets. We show that the radiation from the CENBOL (centrifugal pressure-dominated boundary layer; see Chakrabarti et al. 1996, hereafter CTKE96) is the chief accelerating agent, while radiation from the Keplerian disc has a greater degree of collimation. Both collimation and acceleration are much higher when the jet materials are launched closer to the axis of symmetry. We show that, if radiative processes are the dominant accelerating agent, then we can get highly relativistic and collimated jets only in moderate hard states (i.e. $L_C \sim L_K$) and not in extreme hard states ($L_C \gg L_K$).

In the next section, we present a brief account of the TCAF disc model. In Section 3, we present the model assumptions and the equations of motion. We give an account of the streamline coordinates which we use to solve the governing equations. We also compute 10 independent moments of the radiation field from such a disc. In Section 4, we present our solutions, and finally in Section 5, we draw our conclusions.

2 TCAF DISC

A detailed account of a TCAF disc is given in Paper I (see also CT95; CTKE96; Chakrabarti 1997, hereafter C97; Chattopadhyay et al. 2003). For the sake of completeness, let us now give a brief account of the TCAF disc.

The inner boundary conditions for matter accreting on to black holes are (i) supersonic and (ii) sub-Keplerian. Accretion topologies for sub-Keplerian matter admit two X-type critical points (Liang & Thompson 1980), and supersonic matter after crossing the outer critical point may undergo shock due to centrifugal pressure (Fukue 1987; Chakrabarti 1989, 1990, 1996). The post-shock matter then enters the black hole through the inner critical point. Matter is slowed down in the immediate post-shock region and, as entropy is generated, it makes the post-shock region hot. This hot, slowed-down post-shock flow evaporates the Keplerian disc and falls to the black hole as a single component, in other words the shock location ($x_s$) is the outer boundary of the CENBOL and the inner boundary of the KD. A schematic diagram of such a disc structure is shown in Fig. 1, where

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**Figure 1.** Cross-sectional view of two-component accretion disc model.

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the KD is shown to be sandwiched by the SKH, and the position of $x$, as well as the central black hole, are also shown. Although the SKH (pre-shock) is optically thin for the radiation from the Keplerian disc, the post-shock torus is optically slim, in other words the optical depth of the CENBOL is around unity (i.e. $\tau \sim 1$; see Chakrabarti & Titarchuk 1995; Chakrabarti et al. 1996; Chakrabarti 1997 for details).

Chakrabarti & Titarchuk (1995) showed that, if the Keplerian accretion rate ($M_k$) is higher than the sub-Keplerian accretion rate ($M_s$), then it supplies more soft photons to cool down the CENBOL. This results in more power to the soft end of the accretion disc spectrum – a state known as the soft state. If, on the other hand, $M_s > M_k$, then the SKH supplies more hot electrons to the CENBOL than the soft photons supplied by the KD. The dearth of soft photons cannot cool down the CENBOL significantly. Thus the CENBOL remaining puffed up and hot, can intercept a significant fraction of the soft photons produced by the KD, and can inverse-Comptonize them to produce the hard power-law tail of the accretion disc spectrum – a state called the hard state. This kind of hybrid disc structure is known as the two-component accretion flow or the TCAF disc (Chakrabarti & Titarchuk 1995; Chakrabarti et al. 1996; Ebisawa, Titarchuk & Chakrabarti 1996; Chakrabarti 1997), and has observational support (Smith et al. 2001, 2002).

Chakrabarti and his collaborators have also shown that the unbalanced thermal gradient force of the CENBOL, in the $z$ direction, drives part of the infalling matter along the axis of symmetry to form jets (Chakrabarti 1998, 1999; Das & Chakrabarti 1999; Das et al. 2001). There is wide support for the idea that the jets are indeed coming out from a region within 50–100 Schwarzschild radii of the black hole (Junor, Biretta & Livio 1999). Similarly, it is believed that jets are produced only in hard states (see Gallo et al. 2003, and references therein). Thus it is natural to study the interaction of hard radiation from the CENBOL and the outflowing jets, with the particular interest of studying whether momentum deposited on the jet material by these hard photons can accelerate them to ultrarelativistic speeds.

It is to be remembered that we are not considering the generation mechanism of the jets self-consistently. Since hard radiation is expected to emerge from the CENBOL, the hard photons ‘look’ directly into the jet vertically above and hence eventually deposit their momentum into the latter. Furthermore, radiation from a hot CENBOL is a likely source of pair production and hence the possibility of radiative momentum deposition is likely to be higher even for radiation from the CENBOL hitting the outflow at an angle (see, e.g. Yamasaki et al. 1999 for the production mechanism of pairs from hot accretion flows). Thus we consider radiative momentum deposition on pair-dominated jets, which are generated above the inner surface of the CENBOL, i.e. within the funnel-like region.

3 ASSUMPTIONS, GOVERNING EQUATIONS AND COMPUTATION OF RADIATIVE MOMENTS FROM THE TCAF DISC

We ignore the curvature effects due to the presence of the central black hole mass. The metric is given by

$$\text{d}s^2 = c^2 \text{d}t^2 - \text{d}r^2 - r^2 \text{d}\phi^2 - \text{d}z^2,$$

where $r$, $\phi$ and $z$ are the usual coordinates in cylindrical geometry and $\text{d}s$ is the metric in four-space. The four-velocities are $u^{\mu}$. The convention we follow is that Greek indices signify all four components and Latin indices represent only the spatial ones. The black hole is assumed to be non-rotating and hence the strong gravity is taken care of by the so-called Paczynski–Wiita potential (Paczynski & Wiita 1980).

In this paper, the generation mechanism of jets is not considered. We assume axis-symmetric, steady, rotating jets, i.e. \( \partial / \partial t = 0 \). The derivation of the equations of motion of radiation hydrodynamics for an optically thin plasma was investigated by a number of workers. A detailed account of this derivation has been presented by Mihalas & Mihalas (1984) and Fukue (1996), and references therein, and is not presented here. The equations of motion are:

\[
(\epsilon + p) \left( u^\mu \frac{\partial u^\nu}{\partial x^\mu} + \frac{GM u^\mu}{R(R - r_s^2)} - ru^\mu u^\nu \right) = -\frac{\partial p}{\partial r} - u^\mu u^\nu \frac{\partial p}{\partial x^\mu} + \rho \frac{\sigma_T}{mc} \mathcal{N}^\nu,
\]

\[
(\epsilon + p) \left( u^\mu \frac{\partial u^\phi}{\partial x^\mu} + \frac{2}{r} u^\mu u^\phi \right) = -u^\mu u^\phi \frac{\partial p}{\partial x^\mu} + \rho \frac{\sigma_T}{mc} \mathcal{N}^\phi,
\]

\[
(\epsilon + p) \left( u^\mu \frac{\partial u^\zeta}{\partial x^\mu} + \frac{GM u^\zeta}{R(R - r_s^2)} \right) = -\frac{\partial p}{\partial z} - u^\mu u^\zeta \frac{\partial p}{\partial x^\mu} + \rho \frac{\sigma_T}{mc} \mathcal{N}^\zeta.
\]

In the above equations, $\epsilon$, $p$ and $\rho$ are the internal energy, isotropic gas pressure and density measured in the comoving frame of the fluid and $R = (r^2 + z^2)^{1/2}$. $G$, $M_b$, $\sigma_T$, $m$ and $r_s(2GM_b/c^2)$ are the universal gravitational constant, the mass of the central black hole, the Thomson scattering cross-section, the mass of the gas particles and the Schwarzschild radius, respectively. $\mathcal{N}^\nu$ signifies radiative contributions along the respective components of the momentum balance equation.

The radiative contributions are given by

\[
\frac{\sigma_T}{m c} \mathcal{N}^\nu = \frac{\sigma_T}{m} \left[ \gamma \frac{F^\nu}{c} - \gamma^2 u^\nu E - u^\nu P^\nu \right] + u^\nu \left( \frac{2}{c} u^j F^j - u^i u^k g^{ik} \right).
\]
Similarly,
\[
\frac{\sigma T}{m} \frac{\gamma \phi}{c} = \frac{\sigma T}{m} \left[ \gamma \frac{F^\phi}{c} - \gamma^2 \rho u^\phi E - u^i P^{\phi j} + ru^\phi \left( 2 \gamma u^i F^j - u^i u^k P^{jk} \right) \right]
\]
\[
= \left[ \gamma f^\phi - \gamma^2 \rho u^\phi \varepsilon - u^i g^{\phi j} \right] + ru^\phi \left( 2 \gamma u^i f^j - u^i u^k g^{jk} \right) \quad (3b)
\]
and
\[
\frac{\sigma T}{m} \frac{\gamma z}{c} = \frac{\sigma T}{m} \left[ \gamma \frac{F^z}{c} - \gamma^2 \rho u^z E - u^i P^{z j} + ru^z \left( 2 \gamma u^i F^j - u^i u^k P^{jk} \right) \right]
\]
\[
= \left[ \gamma f^z - \gamma^2 \rho u^z \varepsilon - u^i g^{z j} \right] + ru^z \left( 2 \gamma u^i f^j - u^i u^k g^{jk} \right) \quad (3c)
\]
In equations (3a)–(3c), \(E(r, \phi, z), F^i(r, \phi, z)\) and \(P^{ij}(r, \phi, z)\) are the radiative energy density, three components of the radiative flux and six components of the radiative pressure tensors measured in the observer frame, while
\[
\varepsilon = \frac{\sigma T}{m} E, \quad f^i = \frac{\sigma T}{mc} F^i, \quad \text{and} \quad g^{ij} = \frac{\sigma T}{m} P^{ij}.
\]
Furthermore, \(\gamma(\gamma = u_t)\) is the Lorentz factor. We assume the gas pressure to be negligible compared to the radiation pressure terms.

3.0.1 Streamlines
In streamline coordinates \((s, \phi, t)\), the \(s\)-axis \((t\)-axis) is parallel (perpendicular) to the streamlines, where \(ds^2 = dx^2 + dz^2\), such that \(u^t\) is zero. In Fig. 2 we draw a schematic diagram of the streamline coordinates. The basic equations can then be written as follows (Fukue 1996; Fukue et al. 2001).

Momentum balance along the streamline:
\[
\frac{du^t}{ds} = - \frac{GM_B}{R(R - r_g)^2} \frac{r dr + z dz}{ds} + \left( \frac{r^2 u^t}{c} \right)^2 \frac{dr}{ds} + \frac{\sigma T}{m} \frac{\gamma \phi}{c}; \quad (4a)
\]
angular momentum conservation:
\[
\frac{d(r^2 u^\phi)}{ds} = \frac{\sigma T}{m} \frac{\gamma z}{c}; \quad (4b)
\]
and the streamline equation:
\[
- \frac{GM_B}{R(R - 2 r_g)^2} \frac{dz}{ds} + ru^\phi \frac{dz}{ds} + \left( f^r \frac{dz}{ds} - f^z \frac{dr}{ds} \right) = 0,
\]
where
\[
\frac{\sigma T}{m} \frac{\gamma z}{c} = \gamma f^z - \gamma^2 \rho u^z \varepsilon - (g^{\phi i} u^i + r g^{\phi} u^\phi) + u^i (2 \gamma u^j f^j - u^i u^k g^{jk}); \quad (4c)
\]
In equation (4d),
\[
f^s = f^t \frac{dr}{ds} + f^z \frac{dz}{ds}
\]

![Figure 2](https://academic.oup.com/mnras/article-abstract/356/1/145/1050456)

Figure 2. Schematic diagram of streamline coordinates \((s, \phi, t)\) in the meridional plane.

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\[ \dot{\gamma}^{\mu} = \gamma^{\nu} \left( \frac{d \gamma^{\nu}}{d \xi} \right)^2 + 2 \gamma^{\nu} \frac{dz}{d \xi} \frac{d \gamma^{\nu}}{d \xi} + \gamma^{\nu} \left( \frac{d \gamma^{\nu}}{d \xi} \right)^2 \]

and

\[ \dot{\gamma}^{\nu} = \gamma^{\nu} \frac{d \lambda}{d \xi} + \gamma^{\nu} \frac{dz}{d \xi}. \]

Equations (4a)–(4c) can be simplified in terms of \( v \) and \( \lambda \) and can be expressed in geometric units \( (r_g = c = M_\bullet = 1) \); for simplicity we will keep the same symbols representing various quantities defined so far.

We now define a three-velocity measured by the static observer in geometrical units defined above:

\[ v^2 = \frac{u^2}{u^1 u^1}, \quad (5a) \]

while the angular momentum and angular velocity are being defined as

\[ \lambda = -\frac{u^\phi}{u^1} \quad \text{and} \quad \omega = \frac{u^\phi}{u^1}. \quad (5b) \]

We further define the three-velocity measured by a corotating observer:

\[ v^2 = \frac{u^2}{1 - \omega \lambda}, \quad (5c) \]

so that

\[ \gamma^2 = \frac{1}{1 - v^2 - \omega \lambda} = \frac{1}{(1 - v^2)(1 - \omega \lambda)} = \gamma^2 \gamma^2, \quad u^1 = \gamma v, \quad \text{and} \quad u^\phi = \frac{\gamma \lambda}{r^2}. \]

Equations (4a)–(4c) can be rewritten as

\[ \frac{d \gamma}{d \xi} = \frac{N_1}{D_1}, \quad (6a) \]

where

\[ N_1 = \left[ \gamma \mathcal{F}^{\nu} - \gamma^2 \gamma v E - \gamma v E \mathcal{P}^{11} - \gamma^2 \mathcal{P}^{11} + 2 \gamma \left( \gamma v^2 \mathcal{F}^1 + \gamma \gamma v \frac{\lambda}{r} \mathcal{F}^{11} \right) - \gamma^2 v^2 \mathcal{P}^{11} \right. \]

\[ - 2 \gamma \gamma v^2 \frac{\lambda}{r} \mathcal{P}^{11} - \gamma^2 \gamma v \frac{\lambda}{r^2} \mathcal{P}^{11} + \mathcal{L} \left( \frac{d \mathcal{F}^{\nu}}{d \xi} + \frac{\mathcal{L}}{r^3} \right) \right] - \frac{1}{2R(R - 1)^2} \left( \frac{d r}{d \xi} + \frac{z}{r^3} \right) + \frac{(\gamma \lambda)^2}{r^3} \frac{d r}{d \xi}, \quad (6b) \]

\[ D_1 = \gamma^4 v \]

\[ \frac{d \lambda}{d \xi} = \frac{N_2}{D_2}, \quad (6d) \]

\[ N_2 = \left[ \gamma \mathcal{F}^{\phi} - \gamma^3 \frac{\lambda}{r} \mathcal{E} - \gamma v \mathcal{P}^{1 \phi} - \gamma \frac{\lambda}{r} \mathcal{P}^{1 \phi} - \gamma \left( \gamma v \mathcal{F}^{\phi} + \gamma \frac{\lambda}{r} \mathcal{F}^{1 \phi} \right) - \left( \gamma^2 v^2 \mathcal{P}^{1 \phi} \right) \right. \]

\[ + 2 \gamma \gamma v \frac{\lambda}{r} \mathcal{P}^{1 \phi} + \gamma \frac{\lambda^2}{r^2} \mathcal{P}^{1 \phi} \left. \right] \left( 1 + \frac{\gamma \gamma \lambda}{r^2} \right) \frac{d u^1}{d \xi} + \gamma u^1 \frac{\lambda}{r^3} \frac{d \gamma}{d \xi}, \quad (6c) \]

\[ D_2 = \gamma \gamma \left( 1 + \frac{\gamma \gamma \lambda}{r^2} \right) \frac{d \gamma}{d \xi}, \quad (6f) \]

and

\[ \frac{d r}{d \xi} = -\frac{r^2 [2R(R - 1)^2]}{1 + \gamma f^r (1 - 1)} \quad (6g) \]

The quantities defined in equations (6b) and (6c) are:

\[ \mathcal{F}^{\nu} = f^r \frac{d r}{d \xi} + f^z \quad (7a) \]

\[ \mathcal{F}^{\phi} = f^r \frac{d r}{d \xi} \quad (7b) \]

\[ \mathcal{E} = \frac{d v}{d \xi} \quad (7c) \]
\[ \mathcal{P}^{\mu} = g^{\mu r} \frac{dr}{dz} \frac{dr}{dz} + 2 g^{\nu r} \frac{dr}{dz} + g^{\nu z} \frac{dz}{ds} \]  

(7d)

\[ \mathcal{P}^{\phi} = g^{\phi r} \frac{dr}{dz} + g^{\phi \phi} \]  

(7e)

\[ \mathcal{P}^{\phi \phi} = g^{\phi \phi} \frac{ds}{dz} \]  

(7f)

We now have to solve equations (6a), (6d) and (6g), for a given radiation field \((\epsilon, f, h)\), specified by the disc parameters. It is to be noted that for motion along the axis with \(\lambda = 0\) and \(dr/dz = d\lambda/dz = 0\), then equation (6a) reduces to equation (6) of Chattopadhyay et al. (2004).

### 3.1 Computation of radiative moments from the TCAF disc

The radiation reaching each point within the funnel-like region and the region above it is coming from two parts of the disc, namely the CENBOL and the Keplerian disc, hence all the radiative moments should have both contributions.

In Fig. 3 a schematic diagram of the CENBOL is presented. The inner surface of the CENBOL is assumed conical (described by rotating \(OH\) about the axis of symmetry), and the outer surface of the CENBOL is cylindrical \((HH')\). \(XOY\) describes the equatorial plane. \(O\) is the position of the black hole. \(B(r, 0, z)\) is the field point where the various radiative moments are to be calculated. \(D(x, \phi, y)\) is the source point on the CENBOL inner surface. \(ON = OH' = x_s\) is the location of the shock, and \(HH' = h_s\) is the shock height. The local normal at \(D\) is \(DC/|DC|\). The differential area about \(D\) is marked as \(dA\). The shock height \(h_s\) depends on the shock location, and is expressed as \(h_s \sim 0.6 (x_s - 1)\) (see Chattopadhyay et al. 2004).

By definition, the radiative moments (in natural units) at \(B\) are:

\[ E = \frac{1}{c} \int I \, d\Omega = \frac{1}{c} \left( \int_C I_C \, d\Omega_C + \int_K I_K \, d\Omega_K \right) \],

(8a)

\[ F^i = \frac{1}{c} \int I^i \, d\Omega = \frac{1}{c} \left( \int_C I^i_C \, d\Omega_C + \int_K I^i_K \, d\Omega_K \right) \],

(8b)

**Figure 3.** Cartoon diagram of the CENBOL. The outer KD and SKH are not shown. \(B(r, 0, z)\) is the field point and \(D(x, \phi, y)\) is the source point on the inner surface of the CENBOL. The position of the black hole is at \(O\). The inner surface of the CENBOL is assumed conical. The position of the black hole is \(O\). ON = \(x_s\), and \(HH' = h_s\). \(z'\) \(OD' = \phi\) and \(z'D'OY' = d\phi\), and \(dA\) is the differential area at \(D\). Only the top half is shown.
and

\[ P^{ij} = \frac{1}{c} \int I^i l^j \, d\Omega = \frac{1}{c} \int_c \left( \int_{-\pi/2}^{\pi/2} \cos \phi \, d\phi \, I^i \right) \, d\Omega_c + \int_{-\pi/2}^{\pi/2} \cos \phi \, d\phi \, I^i \, d\Omega_k. \]  \hspace{1cm} (8c)

In equations (8a)–(8c), \( I, d\Omega \) and \( l^i \) are the frequency integrated intensity from the disc, the differential solid angle at \( B \), and the direction cosines at \( B \) with respect to \( D \), respectively. The suffixes ‘\( C \)’ and ‘\( K \)’ represent quantities linked to the CENBOL and the KD, respectively. The expressions for the solid angles subtended at \( B \) from \( D \) are given by

\[ d\Omega_c = \left( \frac{dA \cos \angle CDB}{BD^2} \right) = \frac{x \cos \theta \, dx \, d\phi \, \cos \angle CDB}{BD^2}, \]  \hspace{1cm} (8d)

where

\[ \cos \angle CDB = \frac{BD^2 + CD^2 - BC^2}{2(BD)(CD)}, \quad BD^2 = r^2 + x^2 + (z - y)^2 - 2rx\cos \phi, \]

and \( CD = x \sec \theta \), and \( BC^2 = r^2 + (z - x \tan \theta - y)^2 \)

and \( \theta = \tan^{-1}(x/h_s) \) is the semivertical angle of the CENBOL inner surface. Similarly it is easy to see from Fig. 2 that the direction cosines are given by

\[ l_c^i = \frac{r - x \cos \phi}{BD}; \quad l^i_c = \frac{-x \sin \phi}{BD}; \quad \text{and} \quad l^i_c = \frac{z - y}{BD}. \]

The frequency-averaged CENBOL intensity is given by

\[ I_C = I_{C0} \left( \frac{1}{1 + z_{red}} \right)^4 = \frac{A \pi L_C}{4(1 + z_{red})^4}, \]  \hspace{1cm} (8e)

where \( L_C \) and \( A \) are the total CENBOL luminosity and total CENBOL area, and \( z_{red} \) is the redshift factor taken up to the first order of the disc velocity. The inner edge of the CENBOL, i.e. also the TCAF disc, is taken up to \( x_{in} = 2r_s \), within which the general relativistic effect has to be considered. \( I_{C0} \) is assumed uniform for simplicity, as was explained in Chattopadhyay et al. (2004).

We are not solving the disc equations simultaneously, so in principle we cannot consider the Doppler shift of the photons coming out of the CENBOL. None the less, not considering the disc motion robs us of a vital element of the physics. The rotational velocity of the disc generates the \( \phi \) component of the radiative flux. So we make an estimate of the post-shock disc motion. At the shock, the infalling matter is virtually stopped. We solve the geodesic equation, starting from \( x_s \), with a very small velocity (\( \approx 0.01c \)). We assume the solution (\( \tilde{u}_s \)) (Appendix A) to be the three-velocity of matter along the surface of the CENBOL. The angular momentum of the post-shock matter is sub-Keplerian and almost constant (e.g. Chakrabarti & Titarchuk 1995; Chakrabarti 1996), as the infall time-scale is much smaller than the viscous time-scale. So it is assumed to be constant, and is also the initial specific angular momentum (\( \lambda_{in} = 1.7 \) in the geometrical units defined above) of the jet. Under such assumptions, the radial, azimuthal and axial three-velocity of the matter on the CENBOL surface is given by (expressed in dimensionless units described above) \( \tilde{u}_s = \tilde{u}_s \sin \theta, \tilde{u}_\phi = (1 - 1/x)^{1/2}(\lambda_{in}/x) \) and \( \tilde{u}_z = u_z \cos \theta \). Therefore (1 + \( z_{red} \) = 1 + \( \tilde{u}_s \)), where

\[ \tilde{u}_s l^i = \left( \frac{\tilde{u}_s \cos \phi - \tilde{u}_s \sin \phi(r - x \cos \phi)}{BD} \right) \quad \text{and} \quad \tilde{u}_s l^i = \frac{\tilde{u}_s \sin \phi + \tilde{u}_s \cos \phi(x \sin \phi)}{BD} + \frac{\tilde{u}_s (z - y)}{BD}. \]  \hspace{1cm} (8f)

In Fig. 4, we represent a cartoon diagram of the CENBOL and Keplerian disc (KD). In Fig. 4(a), \( D_K(x_K, \phi, 0) \) is the source point on the KD, and as in the previous figure, \( B(r, 0, z) \) is the field point where the various moments are computed. In Fig. 4(b), only one half of the CENBOL/KD geometry above the equatorial plane is shown. \( B \) is still the field point, but \( D_K(x_K, \phi, 0) \) is the limit up to which \( B \) can see the

![Figure 4](https://example.com/figure4.png)

**Figure 4.** (a) Cartoon diagram of the CENBOL. The outer Keplerian disc and sub-Keplerian halo are not shown. \( B(r, 0, z) \) is the field point and \( D(x, \phi, y) \) is the source point on the inner surface of the CENBOL. The inner surface of the CENBOL is assumed conical. The position of the black hole is also shown. \( ON = x_s, \text{ and } HH' = h_s, \text{ z } B'OD' = \phi \text{ and } \zeta' D'OD' = dp, \text{ and } dA \) is the differential area. (b) Cartoon diagram of the CENBOL and Keplerian disc (KD) geometry. The SKH is not shown. \( B(r, 0, z) \) is the field point and \( D_K(x_K, \phi, 0) \) is the source point on the KD. The CENBOL is represented by the cylinder. The black spot represents \( O \), the position of the black hole. \( OH' = x_s, \text{ and } HH' = h_s, \text{ and } dA_K \) is the differential area at \( D_K \) (shaded).
annulus on the KD defined by radius $x_K$, in the positive $\phi$ direction. So $\phi_f$ is the limit of integration for $\phi$. From Fig. 4(a), one can easily find

$$d\Omega_K = \frac{dA_K}{BD_K} \cos \zeta CD B = \frac{z x_K \, d\phi \, dx_K}{BD_K},$$

where $BD_K^2 = r^2 + z^2 + x_K^2 - 2 \, r \, x_K \, \cos \phi$, and the direction cosines are given by

$$l'_K = \frac{r - x_K \, \cos \phi}{BD_K}; \quad l''_K = \frac{-x_K \, \sin \phi}{BD_K}; \quad \text{and} \quad l'''_K = \frac{z}{BD_K}.

The frequency-averaged KD intensity (see NT73) is given by

$$I_K = \frac{I_{KD}}{(1 + \xi_{red})^2} = \frac{(G M_B M_K)/(8\pi^2 r_s^3)}{(1 + \xi_{red})^2} \left(\frac{x_K^2 - \sqrt{3}x_K^{-5/2}}{2x_K}\right).$$

Now, the Doppler shift term is given by $1 + \xi_{red} = 1 - \sigma_i l'_K$, where

$$\sigma_i l'_K = -\ddot{u}_K \sin \phi \frac{r - x_K \, \cos \phi}{BD_K} - \ddot{u}_K \cos \phi \frac{x_K \, \sin \phi}{BD_K}.

In the above equation $\ddot{u}_K = \sqrt{1/(2|x_K - 1|)}$ is the Keplerian velocity around a non-rotating black hole (in geometrical units).

**Shadow effect of the CENBOL on the jets**

As the jets are produced within the funnel-like region of the TCAF disc, up to certain $z$ the radiation from the KD to the jet material is blocked by the presence of the CENBOL. It is easy to find from Figs 3 and 4 that for material at a particular $r (<x_s$, as jets are produced from the CENBOL region), the radiation from the KD is completely blocked for $z \leq h_s(x_s - r)/(x_o - x_s)$. But even for $z > h_s(x_s - r)/(x_o - x_s)$, the jet cannot 'see' the entire Keplerian disc. From Fig. 4(b), it is clear that the jet material at $B$ cannot see the whole of the annular area defined by radius $x_K$, as part of it is blocked by the CENBOL. If for a particular $r, h_s(x_s - r)/(x_o - x_s) \leq z \leq h_s(x_K + r)/(x_K - x_s)$, then, from Fig. 4(b), one can find an expression for $\phi_f$:

$$\phi_f = \cos^{-1} \left(\frac{x_s^2 + x_o^2 - F D_K^2}{2x_s}\right) + \cos^{-1} \left(\frac{r^2 + x_s^2 - (B'D_K - F D_K)^2}{2r_s}\right),$$

where $B'D_K^2 = r^2 + x_o^2 - 2r x_K \cos \phi$ and $F D_K = (h_s B)/r$. $\phi_f$ is to be computed numerically from equation (8j). For $z > h_s(x_K + r)/(x_K - x_s)$, $\phi_f = 2\pi$. Equations (8a)–(8c) are integrated with the help of equations (8d)–(8j), to get the expressions of various radiative moments of a TCAF disc.

Let us now multiply $\sigma_i/m$ with equations (8a)–(8c), to get

$$\varepsilon = \varepsilon_{KD} \int_{x_o}^{x_s} \int_{0}^{2\pi} \left[\frac{z(x_{K}^{-2} - \sqrt{3}x_{K}^{-5/2})}{(r^2 + z^2 + x_{K}^2 - 2r x_{K} \cos \phi^3)/(1 + \xi_{red})^2}\right] \, d\phi \, dx_K

+ \varepsilon_{KD} \int_{x_o}^{x_s} \int_{0}^{2\pi} \left[\frac{z(x_{K}^{-2} - \sqrt{3}x_{K}^{-5/2})}{r^2 + z^2 + x_{K}^2 - 2r x_{K} \cos \phi^3)/(1 + \xi_{red})^2}\right] \, d\phi \, dx_K

+ \varepsilon_{KD} \int_{x_o}^{x_s} \int_{0}^{2\pi} \left[\frac{(x^2 + y^2)^{1/2} \cos \zeta CD B}{(r^2 + z^2 + x_{K}^2 - 2r x_{K} \cos \phi^3)/(1 + \xi_{red})^2}\right] \, d\phi \, dx

= \varepsilon_{KD} E_K(r, z, x_s) + \varepsilon_{KD} E_C(r, z, x_s)

= \varepsilon_{K} + \varepsilon_{C}

(9a)

$$f_i = f_{KD} \int_{x_o}^{x_s} \int_{0}^{2\pi} \left[\frac{z(x_{K}^{-2} - \sqrt{3}x_{K}^{-5/2})}{r^2 + z^2 + x_{K}^2 - 2r x_{K} \cos \phi^3)/(1 + \xi_{red})^2}\right] \, d\phi \, dx_K

+ f_{KD} \int_{x_o}^{x_s} \int_{0}^{2\pi} \left[\frac{z(x_{K}^{-2} - \sqrt{3}x_{K}^{-5/2})}{r^2 + z^2 + x_{K}^2 - 2r x_{K} \cos \phi^3)/(1 + \xi_{red})^2}\right] \, d\phi \, dx_K

+ f_{KD} \int_{x_o}^{x_s} \int_{0}^{2\pi} \left[\frac{(x^2 + y^2)^{1/2} \cos \zeta CD B}{(r^2 + z^2 + x_{K}^2 - 2r x_{K} \cos \phi^3)/(1 + \xi_{red})^2}\right] \, d\phi \, dx

= f_{KD} F_K(r, z, x_s) + f_{KD} F_C(r, z, x_s)

= f'_K + f'_C

(9b)
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\[
\gamma^{ij} = \varphi_{K0} \int_{x_i}^{x_e} \left[ \int_{0}^{\phi_f} \frac{z(x^{2} - \sqrt{3}x_{K}^{5/2})}{(r^{2} + z^{2} + x_{K}^{2} - 2r x_{K} \cos \phi)^{3/2}} \left(1 + \xi_{\text{red}}\right)^4 \right. \\
\left. + \int_{\phi_f}^{2\pi} \frac{z(x^{2} - \sqrt{3}x_{K}^{5/2})}{(r^{2} + z^{2} + x_{K}^{2} - 2r x_{K} \cos \phi)^{3/2}} \left(1 + \xi_{\text{red}}\right)^4 \right] dx_{K} \\
+ \varphi_{C0} \int_{x_i}^{x_e} \int_{0}^{\phi_f} \frac{(x^{2} + y^{2})^{1/2} \cos C \, dB}{(r^{2} + z^{2} + x^{2} - 2r \cos \phi)^{3/2}(1 + z_{\text{red}})^2} \phi_{K} \, d\phi_{K} \\
= \varphi_{K0} \hat{P}^{ij}_{K}(r, z, x_{i}, x_{o}) + \varphi_{C0} \hat{P}^{ij}_{C}(r, z, x_{i}, x_{o}) \\
= \gamma^{ij} + \gamma^{ij}_{C}. \tag{9c}
\]

In equations (9a)–(9c), the space-dependent parts of \( \varepsilon, f^{i} \) and \( \gamma^{ij} \) are expressed as \( \hat{E}, \hat{F} \) and \( \hat{P}^{ij} \). The suffixes ‘K’ and ‘C’ signify Keplerian and CENBOL contributions. If the moments are expressed in dimensionless units then the constants in equations (9a)–(9c) are given by

\[\varepsilon_{K0} = f_{K0} = \varphi_{K0} = \frac{1.3 \times 10^{38} \ell_{c} \sigma_{T}}{2 \pi cmAG M_{\odot}} \tag{9d}\]

and

\[\varepsilon_{C0} = f_{C0} = \varphi_{C0} = \frac{4.32 \times 10^{37} m_{i} \sigma_{T} c}{32 \pi^{3} mG M_{\odot}} \tag{9e}\]

where \( \ell_{c} = L_{C}/L_{\text{Edd}} \) (\( L_{C} \) is the CENBOL luminosity and \( L_{\text{Edd}} \) is the Eddington luminosity), \( m_{i} = M_{K}/M_{\text{Edd}} \) (\( M_{K} \) is the Keplerian accretion rate and \( M_{\text{Edd}} \) is the Eddington luminosity), and \( A \) is the CENBOL surface.

For simplicity, we will not compute the shock location \( x_{i} \) or \( L_{K} \); instead, we will supply them as free parameters. They can be easily computed from accretion parameters (Chakrabarti 1989, 1990; Chakrabarti & Titarchuk 1995; Das et al. 2001; Chattopadhyay et al. 2003).

To obtain all the components of the radiation field, we supply the following disc parameters:

(i) the inner radius of the CENBOL \( x_{in} = 2r_{g} \), as explained in Section 3.1;
(ii) the shock location \( x_{s} \);
(iii) the CENBOL luminosity \( \ell_{c} \) (in units of \( L_{\text{Edd}} \));
(iv) the Keplerian accretion rate \( \dot{m}_{i} \).

The expression for the Keplerian luminosity was given in Chattopadhyay et al. (2004), and is

\[L_{K} = \frac{r_{s}^{2}}{2} \int_{x_{i}}^{x_{e}} \frac{2 \pi k_{B} T_{K} 2 \pi x_{K} d x_{K}}{\frac{3}{4} \dot{m}_{i}} \left[ \frac{1}{x_{K}} + \frac{2}{3x_{K}} \sqrt{\frac{3}{x_{K}}} \right] L_{\text{Edd}} = \ell_{K} L_{\text{Edd}} \tag{10}\]

It is to be remembered that as jets are only observed in intermediate-to-hard spectral states of the accretion disc, so we will constrain our analysis to the domain \( \ell_{K}/\ell_{c} \sim 1 \) to \( \ell_{K}/\ell_{c} < 1 \).

3.2 The components of the radiation field

In Fig. 5, we show the contour plots of the space-dependent parts of various moments of the radiation due to the CENBOL. The shock location is \( x_{s} = 20r_{g} \). The space variation of various moments due to the CENBOL is strongest within the funnel-like region of the disc; so we only plot them within and above the funnel-like region. Further, as the jets are likely to be produced in this region, so this is the region that matters for our purpose.

Contour plots of various moments are: \( \ell_{C}(r, z) \) (Fig 5a; max. value: 50.55), \( \hat{F}_{C}(r, z) \) (Fig 5b; max./min. value: 1.7/–5.42), \( \hat{F}_{C}^{\phi}(r, z) \) (Fig 5c; max. value: 28.94), \( \hat{F}_{C}^{\theta}(r, z) \) (Fig 5d; max./min. values: 5.2/–1.19), \( \hat{P}_{C}^{ij}(r, z) \) (Fig 5e; max. value: 14.62), \( \hat{P}_{C}^{\phi \theta}(r, z) \) (Fig 5f; max./min. values: 0.98/–1.66), \( \hat{P}_{C}^{\phi \phi}(r, z) \) (Fig 5g; max./min. values: 0.67/–1.39), \( \hat{P}_{C}^{\phi \theta}(r, z) \) (Fig 5h; max. value: 29.8), \( \hat{P}_{C}^{\phi \phi}(r, z) \) (Fig 5i; max. value: 4.4) and \( \hat{P}_{C}^{\theta \theta}(r, z) \) (Fig 5j; max. value: 3.88).

The following features should be noted from the various moments of the radiation, computed from the CENBOL.

(i) The radiation field is highly anisotropic within the funnel-like region of the CENBOL.
(ii) All the moments peak and have very sharp gradients around the inner edge \( x_{in} \).
(iii) Within the funnel (i.e. for \( r < x_{i} \), and \( z \gtrsim h_{s} \) \( \hat{F}_{C}^{\theta} < 0 \), but at moderate values of \( r \) and \( z \) away from the black hole, \( \hat{F}_{C}^{\theta} > 0 \).
(iv) \( \hat{F}_{C}^{\phi} < 0 \) very close to black hole.
(v) \( \hat{F}_{C}^{\theta} > 0 \) for \( r > 0 \) and \( z > 0 \).
(vi) \( \hat{F}_{C}^{\phi} = \hat{F}_{C}^{\theta} = 0 \) at \( r = 0 \).
(vii) \( \hat{F}_{C}^{\phi} > \hat{F}_{C}^{\theta} > \hat{F}_{C}^{\phi \phi} \) at \( r \sim x_{in} \) and as \( z \) becomes small.

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Figure 5. The space-dependent part of the 10 independent components of the radiation field due to the CENBOL. (a) $\tilde{E}_C(r, z)$, (b) $\tilde{F}_r^C(r, z)$, (c) $\tilde{F}_\phi^C(r, z)$, (d) $\tilde{F}_z^C(r, z)$, (e) $\tilde{P}_{rr}^C(r, z)$, (f) $\tilde{P}_{\phi r}^C(r, z)$, (g) $\tilde{P}_{rz}^C(r, z)$, (h) $\tilde{P}_{\phi\phi}^C(r, z)$, (i) $\tilde{P}_{\phi z}^C(r, z)$, (j) $\tilde{P}_{zz}^C(r, z)$. The disc parameters are $x_s = 20r_g$ and $x_{in} = 2r_g$. 

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(iii) $\tilde{F}_\phi \approx \tilde{F}_C$ but less than $\tilde{F}_C$ at $r \sim x_a$ and as $z$ becomes large.

(ix) $\tilde{E}_C$ is the most dominant of all the moments.

(x) $P^{\phi}_C \approx P^{\phi}_C$ and $P^{\phi}_C (P^{\phi}_C) > P^{\phi}_C$.

(xii) For $x_a \sim 10$–$20r_g$ and $R \rightarrow 100r_g$, the radiation field due to the CENBOL approaches that due to a point source (Chattopadhyay et al. 2004).

It is evident that, as $z$ becomes small, and for $\tilde{F}_C > \tilde{F}_C > \tilde{F}_C$, still higher values of $\tilde{E}_C$ and $\tilde{P}^{\phi}_C$ will ensure that the $\lambda$ gained by $\tilde{F}_C$ will be less than that reduced by the drag terms of equation (6d). It is also evident that as $\tilde{F}_C$ changes from negative to positive, as one goes away from the black hole and the axis of symmetry, which means closer to the axis and the black hole, radiation from the CENBOL would push the jet material towards the axis, and further away it will tend to spread the jet. This does not mean that the more luminous the CENBOL, the greater is the spreading, since radiation drag along the $r$-direction will limit the radial expansion of the jet.

In Fig. 6, contour maps of various moments of the radiation field due to the outer Keplerian disc are plotted. The shock location is the same as in Fig. 5. In contrast to the contribution due to the CENBOL, the Keplerian contribution is zero within the domain $r \leq x_a$, $z \leq h_s (x_a - r)/(x_o - x_s)$. The anisotropic nature of the radiation field extends to a region at much larger distances above the funnel-like region of the CENBOL. The contour maps of various moments from the Keplerian disc are: $E_k (r, z)$ (Fig. 6a; max. value: $2.23 \times 10^{-5}$), $\tilde{F}_k (r, z)$ (Fig. 6b; max./min. values: $1.77 \times 10^{-6}$–$1.12 \times 10^{-5}$), $P_k (r, z)$ (Fig. 6c; max. value: $8.62 \times 10^{-7}$), $\tilde{P}_k (r, z)$ (Fig. 6d; max. value: $1.7 \times 10^{-5}$), $P^{\phi}_k (r, z)$ (Fig. 6e; max. value: $5.75 \times 10^{-5}$), $\tilde{P}^{\phi}_k (r, z)$ (Fig. 6f; max./min. values: $9.49 \times 10^{-5}$–$1.37 \times 10^{-5}$), $\tilde{P}^{\phi}_k (r, z)$ (Fig. 6g; max./min. values: $1.25 \times 10^{-5}$–$8.6 \times 10^{-5}$), $\tilde{P}^{\phi}_k (r, z)$ (Fig. 6h; max. value: $5.75 \times 10^{-5}$), $\tilde{P}^{\phi}_k (r, z)$ (Fig. 6i; max. value: $5.9 \times 10^{-5}$) and $\tilde{P}^{\phi}_k (r, z)$ (Fig. 6j; max. value: $1.4 \times 10^{-5}$).

As in the previous figure, the following features may also be noted in Fig. 6.

(i) Each of the moments of the radiation field due to the KD is a few orders of magnitude less than the corresponding ones due to the CENBOL.

(ii) $\text{Max}(\tilde{F}_k) > \text{Max}(\tilde{F}_k) > \text{Max}(\tilde{F}_k)$ which means the spreading of jets will be less.

(iii) The gradients of the moments from the KD are smaller than those from the CENBOL.

(iv) $\tilde{F}_k > 0$.

(v) $\tilde{P}_k < 0$ in a larger domain results in pushing the jet materials towards the axis in a larger domain above the funnel of the CENBOL, and the angular momentum gained will also be less as $\tilde{P}_k$ is the smallest of the three components of the flux.

(vi) $P^{\phi}_k \approx \tilde{P}^{\phi}_k \approx \tilde{P}^{\phi}_k$.

It is quite evident that in the hard state the Keplerian contribution to the radiative momentum will be marginal compared to the CENBOL contribution. But as $\tilde{F}_C$ is weakest amongst all the components of the CENBOL flux, and $\tilde{F}_k$ directed towards the axis in a larger domain, then for $\ell_s/\ell_s \leq 1$ it is possible to observe greater collimation.

It is to be noted in Figs 5(a)–(j) that the various components of radiative moments computed in this paper are quantitatively different from earlier work on thin discs (Tajima & Fukue 1996, 1998), as well as on slim discs (Watarai & Fukue 1999). When comparing radiation fields from a thin disc and that from the TCAF disc model, the first point to be noticed is that the geometry of the two-disc model is different; secondly, the disc motions are different; and thirdly, the spatial variation of the disc intensity ($I_0$ and $I_K$) in the TCAF model is different from that of a purely thin disc. Even the radiative contribution from the KD of the TCAF model to various radiative moments (Figs 6a–j) differs from that computed by Tajima & Fukue (1998). The reasons are that (a) in Tajima & Fukue (1998), the inner radius of the thin disc is $3r_g$, while in this paper the inner radius of the KD is $x_s$ (~few $\times 10r_g$), so the jet does not ‘see’ the most luminous part of the KD, but instead ‘sees’ the luminous CENBOL, and (b) the shadow effect of the CENBOL. The shadow effect of the CENBOL on the jet material is extensively discussed in Section 3.1; we will just point out here that for jet material at $z \leq h_s (x_a - r)/(x_o - x_s)$, radiation from the KD is completely blocked and for $z > h_s (x_a - r)/(x_o - x_s)$, the jet material only ‘sees’ a fraction of the outer rim of the KD. With increasing $z$, the jet ‘sees’ more and more of the inner part of the KD. In Tajima & Fukue (1998), this shadow effect was not considered and hence the difference in spatial variation of various radiative components between these two papers, namely between Figs 6a–j of this paper and Figs 2–4 of Tajima & Fukue (1998).

In Watarai & Fukue (1999), radiative moments are calculated for the slim disc model, for three values of the height-to-disc-radius ratio (H/R): (a) H/R $\sim 0.05$, (b) H/R $\sim 0.4$ and (c) H/R $\sim 0.56$. In the present paper, the height-to-radius ratio of the CENBOL is $h_s/x_a \sim 0.57$ (for $x_s = 20r_g$, but is always <0.6 for any higher $x_s$), so one might be tempted to think that cases (b) and (c) of Watarai & Fukue (1999) should be similar to Figs 5a–(j) of this paper. The components of the radiative moments are not solely determined by the disc geometry, but also by the disc dynamics and its radiative properties, and the CENBOL and the slim disc models of Watarai & Fukue (1999) differ on both these counts. The radial velocity component of the slim disc considered was $v_r \sim c_1/r^{1/2}$ ($u_0$ in our case) and the azimuthal velocity found to be $v_\phi \sim c_1/r^{1/2}$ ($u_0$ in our case) and $c_1, c_2$ depends on the advection parameter, viscosity parameter, the ratio of specific heats, etc.). The radial velocity of the CENBOL has no explicit analytical expression (computed in Appendix A), and, is not proportional to $r^{-1/2}$. In the immediate post-shock region, the radial velocity ($u_0$) of matter in the CENBOL is much less than the radial velocity of matter in models (b) and (c) of Watarai & Fukue (1999), but close to the black hole it is higher. Even the nature of $u_0$ (in the CENBOL) and $v_\phi$ (in the slim disc) are different. The
Figure 6. The space-dependent part of the 10 independent components of the radiation field due to the CENBOL. (a) $\tilde{E}_K(r, z)$, (b) $\tilde{F}_K(r, z)$, (c) $\tilde{F}^\phi_K(r, z)$, (d) $\tilde{F}_zK(r, z)$, (e) $\tilde{P}_{rr}K(r, z)$, (f) $\tilde{P}_{r\phi}K(r, z)$, (g) $\tilde{P}_{rz}K(r, z)$, (h) $\tilde{P}_{\phi\phi}K(r, z)$, (i) $\tilde{P}_{\phi z}K(r, z)$, (j) $\tilde{P}_{zz}K(r, z)$. The disc parameters are $x_s = 20r_g$ and $x_o = 500r_g$. 

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higher velocity components close to $x_{in}$ enhance the CENBOL intensity close to the black hole, and ultimately enhance each of the radiative moments near the inner edge of the CENBOL compared to that achieved in the slim disc.

This implies that the streamline velocity ($\sim 10^{-4}$) at the base of the jet above the CENBOL, will experience a greater driving force, but because the jet velocity is small there, the jet will experience less drag force. The opposite will be true for the angular momentum: the drag forces will reduce the angular momentum more than the jet may gain from the radiation, as $\lambda_{in}$ is high at the jet base.

The other major difference in the two models is the difference in radiative property. The CENBOL radiation in the rest frame is uniform, while that of the slim disc model falls like $r^{-2}$. Thus the intensity at the outer edge of the CENBOL is comparatively more significant than the slim disc case. This makes the radial flux directed inwards in a larger part of the domain above the CENBOL, compared to the region above the slim disc. In spite of the quantitative differences between the moments computed by the CENBOL and the slim disc, the overall qualitative similarity between Figs 5(a)–(j) of this paper, and fig. 4 of Watarai & Fukue (1999) is quite evident.

4 RESULTS

The results are obtained by integrating equations (6a), (6d) and (6g) in the radiation field of the TCAF disc given in equations (9a)–(9e). Apart from the disc parameters supplied to calculate the radiation field from the disc, we also supply the injection radius of the jet $r_i$. As the jets are launched from the inner surface of the funnel-like region of the CENBOL, the injection height ($z_i$) should be just above the CENBOL inner surface. With no loss of generality we take $z_i = r_i(h_i/x_i) + 0.1$ in units of $r_g$. We assume that the outflow is made up of a purely electron–positron pair plasma. We are interested in studying the dependence of terminal speed as well as the relative spread of jets on disc parameters such as the $\ell_c$, $m_{in}$, $x_{in}$, $r_{in}$. We define $r_{\infty}/z_{\infty}$ as the relative spread of the jets, where $r_{\infty}$, $z_{\infty}$ are the cylindrical radial and axial coordinates at which $v \rightarrow v_{\infty}$. If $r_{\infty}/z_{\infty} < 0.1$, we define the jet as well collimated; $0.1 < r_{\infty}/z_{\infty} < 0.2$ as fairly collimated; $0.2 < r_{\infty}/z_{\infty} < 0.3$ as poorly collimated, and so on.

4.1 Dependence on $\ell_c$ and $m_{in}$

In Fig. 7(a), $v_r$ is plotted with log$(z)$, where the CENBOL luminosity is increased from $\ell_c = 0.2$ (long-dashed), $\ell_c = 0.3$ (dashed), $\ell_c = 0.4$ (solid) in units of $L_{Edd}$, while the Keplerian luminosity corresponds to $m_{in} = 6$ (in units of $M_{Edd}$) and $x_{in} = 20 r_g$. The parameters which are kept constant throughout the paper are: $v_{in} = 10^{-4}$, $\lambda_{in} = 1.7$, $x_{in} = 2r_g$ and $x_{in} = 500 r_g$. The injection radius for the jet is $r_{in} = 3r_g$ in Figs 7(a) and (b). We see that the streamline velocity $v_{\infty}$ increases with increasing $\ell_c$, but the amount of increase in $v_{\infty}$ for an equivalent increase in $\ell_c$ increases.

This is to be expected as it was well documented in Paper I that CENBOL radiation is a good accelerator. Let us look at Fig. 7(b), to see how the corresponding streamlines behave.

In Fig. 7(b), the streamlines ($z$ versus $r$) are plotted for $\ell_c = 0.2$ (long-dashed), $\ell_c = 0.3$ (dashed) and $\ell_c = 0.4$ (solid), while $m_{in} = 6$ is kept fixed. Other disc parameters are $x_{in} = 20 r_g$ and $r_{in} = 3r_g$. As the CENBOL luminosity is increased, the streamlines are spreading, though the spreading is decreasing for an equivalent increase in $\ell_c$, as in the case of streamline velocities. Before going into the reason for this let us probe into further details.

In Fig. 7(c), the terminal speed $v_{\infty}$ is plotted with $\ell_c$ for $m_{in} = 2$ (solid), $m_{in} = 5$ (dashed) and $m_{in} = 8$ (long-dashed), other disc parameters being $r_{in} = 2r_g$ and $x_{in} = 20 r_g$. Similarly to Fig. 7(a), we see that $v_{\infty}$ increases appreciably with $\ell_c$. For lower values of $\ell_c$, $v_{\infty}$ increases with $m_{in}$, but decreases with increasing $m_{in}$ for higher values of $\ell_c$, although the change in $v_{\infty}$ due to the change in $m_{in}$ is small compared to the change due to $\ell_c$. This phenomenon was also reported in Chattopadhyay et al. (2004). As this is a rotating jet, we would naturally try to see how the jet $\lambda$ behaves at large distances.

In Fig. 7(d), $\lambda_{\infty}$ is plotted with $\ell_c$ for $m_{in} = 2$ (solid), $m_{in} = 5$ (dashed) and $m_{in} = 8$ (long-dashed), other disc parameters being $r_{in} = 2r_g$ and $x_{in} = 20 r_g$. We see that generally $\lambda_{\infty}$ increases with $\ell_c$. Within the funnel-shape region of the CENBOL, the radiation field produces strong drag terms along the $\phi$ direction which removes $\lambda$ more than $\lambda$ increased by $f$, but further away from black hole $\lambda$ reduces so much that the drag terms become ineffective, and the jet gains angular momentum. So as the CENBOL becomes more and more luminous the jet gains angular momentum. Furthermore, for higher values of $\ell_c$, $\lambda_{\infty}$ increases with $m_{in}$. This shows that the KD radiation does not remove angular momentum any better than the CENBOL radiation.

In Fig. 7(e), $v_{\infty}$ is plotted with $m_{in}$, parametrized by $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed) and $\ell_c = 0.1$ (long-dashed). As reported in Chattopadhyay et al. (2004), $v_{\infty}$ has a very weak dependence on $m_{in}$. In Fig. 7(f), $\lambda_{\infty}$ is plotted with $m_{in}$, parametrized by $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed) and $\ell_c = 0.1$ (long-dashed). It is evident that $\lambda_{\infty}$ has a very weak dependence on $m_{in}$.

Let us now see the effect of radiation on spreading the jet. In Fig. 8 we plot $r_{\infty}/z_{\infty}$. In Fig. 8(a), $r_{\infty}/z_{\infty}$ is plotted with $\ell_c$, parametrized for $m_{in} = 2$ (solid), $m_{in} = 5$ (dashed) and $m_{in} = 8$ (long-dashed) (same parameters as Figs 7c and d), while in Fig. 8(b), $r_{\infty}/z_{\infty}$ is plotted with $m_{in}$, parametrized by $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed) and 0.1 (long-dashed) (same parameters as Figs 7c and f). The most remarkable contrast is that the jets are more collimated with increasing KD luminosity, while the jet tends to spread with increasing CENBOL luminosity, though the spreading does not increase monotonically with $\ell_c$, but tends to decrease with equivalent increase in $\ell_c$.

There are two features which are quite intriguing: (i) the contrasting nature of the CENBOL and KD radiation field in terms of the collimation of the jet, and (ii) the KD radiation seems to have a nominal effect on determining $v_{\infty}$ and $\lambda_{\infty}$, but plays a relatively greater role in collimation.

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Figure 7. (a) Variation of $v_s$ with $\log(z)$. Each of the curves represents $\ell = 0.2$ (long-dashed), $\ell = 0.3$ (dashed) and $\ell = 0.4$ (solid) in units of $L_{\text{Edd}}$. (b) The streamlines of the jet solution in the $z-r$ plane, corresponding to $\ell = 0.2$ (long-dashed), $\ell = 0.3$ (dashed) and $\ell = 0.4$ (solid). For both figures $\dot{m}_k = 6$, $r_{\text{in}} = 3r_g$. (c) Variation of terminal speed $v_\infty$ with $\ell$. Each curve represents $\dot{m}_k = 2$ (solid), $\dot{m}_k = 5$ (dashed) and $\dot{m}_k = 8$ (long-dashed), where $r_{\text{in}} = 2r_g$. (d) Variation of terminal specific angular momentum $\lambda_\infty$ with $\ell$. Each curve represents $\dot{m}_k = 2$ (solid), $\dot{m}_k = 5$ (dashed) and $\dot{m}_k = 8$ (long-dashed), where $r_{\text{in}} = 2r_g$. Values of $v_{\text{in}}, \lambda_{\text{in}}, x_{\text{in}}$ and $x_0$ is kept constant throughout the paper.

The reason that radiation from the KD can collimate the jets better than radiation from the CENBOL can be understood from Figs 5 and 6 and equation (6g). We know that rotating matter tends to move away from the axis due to centrifugal force. If the angular momentum is reduced by the drag forces then the spreading may be arrested. Close to the CENBOL surface the radiation field is dominated by the CENBOL radiation and it produces large drag forces along $\phi$. Further out as the jet starts to ‘see’ the KD radiation, the spreading of jets becomes marginal and the jet starts to gain some angular momentum. From Figs 5 and 6, it is evident that the contribution from the KD to the total radiation field is much less than that of the CENBOL, so angular momentum removed from or added to the jet by the radiation from the KD is marginal (e.g. Fig. 7f). From equation (6g), it is clear that the spreading of jets depends as much on centrifugal force as on $f'$. From the discussion below Figs 5 and 6, we know that $\tilde{F}_C^K$ is weakest amongst all the $\tilde{F}_C^K$s, while $\tilde{F}_K^K < \tilde{F}_K^K$ but $\tilde{F}_K^K > \tilde{F}_K^K$, and $\tilde{F}_K^K$ is towards the axis in a larger domain. Thus when $\ell$ increases it does not help in collimation, but as $\dot{m}_k$ increases the higher negative values of $\tilde{F}_K^K$ make $f'$ more and more negative, and so it pushes the jet towards the axis of symmetry and helps in collimation, in spite of not removing angular momentum to the same extent as CENBOL radiation does.

4.2 Dependence on $r_i$

In Fig. 9, we show the effect of injection radius of the jets. In Fig. 9(a), streamlines are plotted for $r_i = 2r_g$ (solid), $r_i = 3r_g$ (dashed), $r_i = 4r_g$ (long-dashed), $r_i = 5r_g$ (dashed-dotted) and $r_i = 6r_g$ (long dashed-dotted), where $\ell = 0.4$, $\dot{m}_k = 6$ and $x_i = 20r_g$. In Fig. 9(b),
the corresponding $\lambda$ distribution is shown along the streamlines for $r_i = 2r_g$ (solid), $r_i = 3r_g$ (dashed), $r_i = 4r_g$ (long-dashed), $r_i = 5r_g$ (dashed-dotted) and $r_i = 6r_g$ (long dashed-dotted), where $\ell_c = 0.4, \lambda_1 = 6$ and $x_c = 20r_g$. From both the figures we see that the injection height $z_i$ changes with $r_i$ as has been discussed at the start of this section. It is evident that as the injection radius is varied, the angular momentum of the jets is higher and it spreads further. The angular momentum of the jets at different $r_i$ is the same, but the radiative moments just above the CENBOL surface decreases with $r$. In other words the jets will gain less $\lambda$ from the radiation field, but at the same time they will lose less $\lambda$ due to the drag terms. The net effect is that with increasing $r_i$ the jets are of higher angular momentum and thus these jets are less collimated.

On the other hand, jets are generated with very low velocity at the base ($\sim r_i$), so the drag term is negligible, but as all the radiative moments decrease with $r$ above the CENBOL surface, the driving that the jets get due to $f'$ and $f^\parallel$ is much less as $r_i$ is increased. In Figs 9(c)–(f), various terminal values are plotted with $r_i$, for $n_{\lambda} = 4$ (solid), $n_{\lambda} = 7$ (dashed) and $n_{\lambda} = 10$ (long-dashed), other parameters being $\ell_c = 0.4, x_c = 20r_g$. In Fig. 9(c), we see that $v_\infty$ decreases with $r_i$, but for fixed values of $r_i$ it increases with lower values of $n_{\lambda}$, or in other words higher values of $\ell_c/\ell_{\lambda}$. It has been observed by Chattopadhyay et al. (2004) and also in Fig. 7(c) of this paper, that for $\ell_c \gtrsim 0.22$, $v_\infty$ decreases with increasing $n_{\lambda}$, Fig. 9(d), on the other hand, shows that $\lambda_\infty$ increases with increasing $r_i$. In Fig. 9(e), the rotational velocity $(v_\phi^2 = -u_\phi u^\phi/\mu u^z)$ at infinity ($v_\phi^\infty$) is plotted with $r_i$. Similar to $\lambda_\infty$, $v_\phi^\infty$ also increases with $r_i$. An interesting feature is seen if one compares Fig. 9(f) with the previous two. In Fig. 9(f), the relative spread $r_\infty/z_\infty$ is shown to increase with $r_i$, but the interesting feature is, as one increases $n_{\lambda}, r_\infty/z_\infty$ decreases, but both $\lambda_\infty$ as well as $v_\phi^\infty$ increase. This vindicates equation (6g), which says $f'$ and $f^\parallel$ determine the streamlines along with the centrifugal force.

4.3 Dependence on $x_c$

Until now we have investigated the solutions for $x_c = 20r_g$. We now concentrate on the dependence of jet solutions on $x_c$. In Fig. 10(a), $v_\infty$ is plotted with $x_c$, for $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed) and $\ell_c = 0.1$ (long-dashed), where $n_{\lambda} = 3$ and $r_i = 2r_g$ are kept fixed. With increasing values of $x_c$, the CENBOL intensity decreases and the driving force goes down, producing a lower terminal speed. We plot $\lambda_\infty$, corresponding to the cases in Fig. 10(a), in Fig. 10(b). Similarly to $v_\infty$, $\lambda_\infty$ also decreases with $x_c$. The reason for this is same as in the previous case, i.e. the CENBOL intensity decreases with increasing $x_c$. Interestingly, as $\ell_c$ is increased, $v_\infty$ increases appreciably but $\lambda_\infty$ is increased marginally. Close to the injection radius, the $\lambda$ of the jet decreases rapidly (see Fig. 9b) and then, when at large distances from the disc $\lambda$ becomes too low so that the drag in the azimuthal direction becomes negligible, the jet starts to gain some angular momentum from the radiation. So increasing $\ell_c$ will produce higher $\lambda_\infty$, but as the gain in $\lambda$ occurs at distances farther away from the disc, the radiative moments fall off anyway, so the increase in $\lambda_\infty$ is small.

In Fig. 10(c), $v_\infty$ is plotted with $x_c$, for $n_{\lambda} = 2$ (solid), $n_{\lambda} = 5$ (dashed) and $n_{\lambda} = 8$ (long-dashed), with constant $\ell_c = 0.4$. The general conclusion that $v_\infty$ decreases with increasing $x_c$ is still valid, though increasing $n_{\lambda}$ decreases $v_\infty$. Another difference we notice is that the curve of $v_\infty$ widens with increasing $x_c$, for higher values of $\ell_c$ in Fig. 10(a), while in Fig. 10(c), the $v_\infty$ curves converge with increasing $x_c$. The reason is one and the same, i.e. with increasing $x_c$, the KD contribution to all the components of the radiative moments decreases and the
Figure 9. (a) Streamlines for $r_l = 2r_g$ (solid), $r_l = 3r_g$ (dashed), $r_l = 4r_g$ (long dashed), $r_l = 5r_g$ (dashed-dotted) and $r_l = 6r_g$ (long dashed-dotted); $\ell_c = 0.4$, $\dot{m}_k = 6$. (b) $\lambda$ versus $\log(z)$, corresponding to the previous figure. (c) $v_\infty$ versus $r_l$, (d) $\lambda_\infty$ versus $r_l$, (e) $v_{\phi\infty}$ versus $r_l$, and (f) $r_\infty/z_\infty$ versus $r_l$; for $\dot{m}_k = 4$ (solid), $\dot{m}_k = 7$ (dashed) and $\dot{m}_k = 10$ (long dashed). For all the figures $x_s = 20r_g$.

CENBOL contribution dominates. As CENBOL radiation is a good accelerator, so for higher $x_s$, increasing $\ell_c$ produces higher $v_\infty$ relative to the case where $\dot{m}_k$ is increased.

In Fig. 10(d), $\lambda_\infty$ is plotted with $x_s$ for $\dot{m}_k = 2$ (solid), $\dot{m}_k = 5$ (dashed) and $\dot{m}_k = 8$ (long-dashed), with constant $\ell_c = 0.4$. $\lambda_\infty$ decreases with $x_s$ but has almost no dependence on the KD radiation. This is because the azimuthal component of the radiative flux produced by the KD is smaller compared to the other components (see Section 3.2).
Figure 10. (a) Variation of $v_\infty$ with $x_s$, $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed), $\ell_c = 0.1$ (long-dashed), for constant $m_k = 3$. (b) Variation of $\lambda_\infty$ with $x_s$, $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed), $\ell_c = 0.1$ (long-dashed), for constant $m_k = 3$. (c) Variation of $v_\infty$ with $x_s$, $\ell_c = 0.4$, $\dot{m}_k = 2$ (solid), $\dot{m}_k = 5$ (dashed), $\dot{m}_k = 8$ (long-dashed) for $\ell_c = 0.4$. (d) Variation of $\lambda_\infty$ with $x_s$, $\dot{m}_k = 2$ (solid), $\dot{m}_k = 5$ (dashed), $\dot{m}_k = 8$ (long-dashed) for $\ell_c = 0.4$. For all the figures $r_i = 2r_g$.

In Fig. 11(a), the relative spread $r_\infty/z_\infty$ is plotted with $x_s$, for the same parameters as Figs 10(a) and (b), i.e. for $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed) and $\ell_c = 0.1$ (long-dashed), where $m_k = 3$ and $r_i = 2r_g$ are kept fixed. We find $r_\infty/z_\infty$ decreases with increasing $x_s$. As $\lambda_\infty$ decreases with increasing $x_s$ (see Fig. 10b), so it is not surprising that there would be greater collimation. Still for fixed values of $x_s$, the difference in $\lambda_\infty$ is marginal, while on the other hand the difference in $r_\infty/z_\infty$ is larger! If we turn our attention to Fig. 11(b), which is plotted for $\dot{m}_k = 2$ (solid), $\dot{m}_k = 5$ (dashed) and $\dot{m}_k = 8$ (long-dashed), with constant $\ell_c = 0.4$ (same case as Figs 10c and d), it is again seen that $r_\infty/z_\infty$ generally decreases with $x_s$, and at the same time, the decrement of $r_\infty/z_\infty$ for fixed values of $x_s$ with increasing $\dot{m}_k$ is larger, although $\lambda_\infty$ is almost indistinguishable. From Figs 10(a)–(d), we know why $v_\infty$ and $\lambda_\infty$ decrease with increasing $x_s$; but increasing collimation is partly due to decreasing $\lambda_\infty$; otherwise the variation of $r_\infty/z_\infty$ would have just mirrored the variation of $\lambda_\infty$. Increasing $x_s$ makes $f'_C$ negative in a larger part of the funnel-like region. Simultaneously, though radiative contributions by the KD becomes weaker, none the less increasing $x_s$ makes $f'_K$ bigger in a still larger part of the domain. Hence the combination of decreasing angular momentum as well as $f' < 0$ in a larger region around the axis collimates the jets to a greater degree.

Thus we conclude from Figs 10(a)–(d) and 11(a) and (b) that if $\ell_c > 0.2$, $x_s > 20r_g$ then jets with terminal properties $v_\infty \gtrsim 0.9c$ and $r_\infty/z_\infty < 0.1$ are possible.

It will be interesting to compare these results with earlier work. It is natural that these results would differ from earlier work, since the radiation field of the TCAF disc is different from either a thin (Tajima & Fukue 1998) or a slim disc (Watarai & Fukue 1999). For both the thin and slim disc models, the radiation field generally spreads the jet, and due to radiation drag the radiation field suppresses the motion along the $z$ direction. In another model (Fukue et al. 2001), where the authors considered an inner non-luminous disc and an outer luminous disc, the jets became collimated with increasing disc luminosity and the angular momentum was decreased too. The reason for the collimation is that the injection radius of the jet $r_i$ (in their nomenclature) is less than the inner boundary of the disc $x_n$ ($r_n$ in their nomenclature). This makes...
the radial flux $f'$ negative in a larger region above (and below) the disc surface, and it pushes the jet material towards the axis. Along with this, radiation drag reduces the angular momentum, so the jets are collimated. In both the earlier papers by Tajima & Fukue (1998) and Watarai & Fukue (1999), the injection radius is greater than the inner radius of the disc, which produces $f' > 0$ which pushes the jets outwards; at the same time the maximum of $f^\phi$ is in and around the injection radius, which manages to spin up the jet further. These two effects combine to increase the angular momentum of the jets even more. So the crucial point for collimation is whether or not $f' < 0$.

The TCAF disc model has two radiation sources – the CENBOL and the outer KD, and $r_i > x_{in}$. Jets from the TCAF model start with very low streamline velocity ($v_{in} = 10^{-4}$), and moderately high initial angular momentum ($\lambda_{in} = 1.7 \equiv$ angular momentum of the CENBOL). As the radiative energy density ($\varepsilon$) and the pressure components ($\varphi^\ell$) are quite intense close to the inner edge of the disc, the higher value of $\lambda_{in}$ ensures a very high drag force along $\phi$. As a result, the jet $\lambda$ is reduced appreciably, close to the injection radius (e.g. Fig. 9b). Apart from reduction of the angular momentum of the jets, the radial flux $f_r$ is negative, just above the CENBOL surface, and this collimates the jets. On the other hand, as $v_{in}$ is very small (i.e. the drag along the streamline is small), the jets are accelerated very quickly, shooting up to $30–40 r_g$ almost vertically, or sometimes initially pushed towards the axis (e.g. Figs 7b and 9a). At above $30–40 r_g$, above the CENBOL surface, $f_r > 0$, and the drag along $\phi$ is weakened (as discussed in Section 4.1) these facts tend to spread the jet, although the spreading is arrested by the KD luminosity, as well as by the weakening of the $f^\phi_r$.

One must also remember that in jets around the TCAF model, $r_i > x_{in}$. For $r_i \approx x_{in}$, radiation from the CENBOL region $x_{in} - r_i$ is negligible, and hence $f'$ is more negative near $r_i$, but for $r_i > x_{in}$, radiation from the CENBOL region $x_{in} - r_i$ cannot be neglected, making $f'$ less negative near $r_i$. Thus we see that for smaller values of $r_i$ the collimation and acceleration of jets are better (e.g. Figs 9a–4).

Although $\lambda_{in}$ increases with $\ell_c$, one must notice that $\lambda_{in} < \lambda_{in}$ (e.g. Figs 7d, and 10b and c), which points to the fact that indeed $\lambda$ is reduced by the drag force. One must also notice that jets are more collimated by increasing outer KD luminosity (e.g. Fig. 8b), or increasing $x_{se}$ (e.g. Fig. 11b); this means that jets are basically collimated by $f' < 0$, as was indicated by Fukue et al. (2001).

### 4.4 Dependence on $\ell_{ik}/\ell_c$

Until now in this section, we have seen that increasing $\ell_{ik}$ produces faster jets, while increasing $m_{ik}$ and $x_{se}$ makes the jets more collimated. We have also seen that increasing $r_i$ produces less $v_{in}$ and the collimation is worse. The information of CENBOL radiation is provided by its luminosity, and the information of the KD radiation is supplied by $m_{ik}$ and $x_{se}$, so to get a better understanding, we now study how the relative proportions of the CENBOL and KD luminosity affect the jet solutions.

In Fig. 12(a), $v_{in}$ is plotted with $\ell_{ik}/\ell_c$, for $\ell_c = 0.5$ (solid), $\ell_c = 0.3$ (dashed) and $\ell_c = 0.1$ (long-dashed), and $x_{se} = 20 r_g$, $r_i = 2 r_g$ are kept fixed. As in Fig. 7(e), we find that increasing $\ell_{ik}$ from 10 to 100 percent of $\ell_c$ has a marginal effect on $v_{in}$. In Fig. 12(b), the corresponding $r_{in}/z_{in}$ is plotted with $\ell_{ik}/\ell_c$, i.e. for $\ell_c = 0.5$ (solid), $\ell_c = 0.3$ (dashed) and $\ell_c = 0.1$ (long-dashed), and $x_{se} = 20 r_g$, $r_i = 2 r_g$ are kept fixed. Though $v_{in}$ is marginally dependent on $\ell_{ik}/\ell_c$, $r_{in}/z_{in}$ has a stronger dependence on $\ell_{ik}/\ell_c$. In other words, KD radiation is not a strong accelerator but is definitely a better collimator. As has been discussed in Section 4.1, the radial flux due to the KD is directed towards the axis in a larger domain, and as the ratio $\ell_{ik}/\ell_c \geq 0.5$ we find highly relativistic and collimated jets.

We have seen in the preceding subsection that we achieve better colimation with larger shock location. In Fig. 12(c), we plot $v_{in}$ with $\ell_{ik}/\ell_c$, for $x_{se} = 10 r_g$ (solid), $x_{se} = 20 r_g$ (dashed) and $x_{se} = 30 r_g$ (long-dashed), where $\ell_c = 0.4$ and $r_i = 2 r_g$ are kept fixed. We find for

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**Figure 11.** Variation of $r_{in}/z_{in}$ with $x_{se}$. (a) $\ell_c = 0.4$ (solid), $\ell_c = 0.25$ (dashed), $\ell_c = 0.1$ (long-dashed), for constant $m_{ik} = 3$. (b) $m_{ik} = 2$ (solid), $m_{ik} = 5$ (dashed), $m_{ik} = 8$ (long-dashed) for $\ell_c = 0.4$. Variation of $\lambda_{in}$ with $x_{se}$. For all the figures $r_i = 2 r_g$. © 2004 RAS, MNRAS 356, 145–166
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Figure 12. Variation of $v_\infty$ (a) and $r_\infty/z_\infty$ (b), with $\ell_k/\ell_c$, for $x_s = 0.5$ (solid), $x_s = 0.3$ (dashed) and $x_s = 0.1$ (long-dashed); $x_s = 10r_g$. Variation of $v_\infty$ (c) and $r_\infty/z_\infty$ (d), for $x_s = 10r_g$ (solid), $x_s = 20r_g$ (dashed) and $x_s = 30r_g$ (long-dashed); $j = 0.4$. For all the figures $r_i = 2r_g$.

$x_s = 10r_g$ (solid), $v_\infty$ varies from above 0.94$c$ to just above 0.93$c$; on the other hand, for $x_s = 30r_g$ (long-dashed), $v_\infty$ varies slightly and is always a little less than 0.91$c$. Interestingly, for smaller values of $x_s$, increasing $\ell_k/\ell_c$, or in other words proportionally increasing the KD radiation, has a relatively large effect (although much smaller than the equivalent increase in $\ell_c$). For larger $x_s$, the most luminous part of the KD ($\sim 4r_g$) is missing so its contribution to the components of radiative moments are quite low, and as a result increasing $\ell_k$ has a negligible effect on $v_\infty$. In Fig. 12(d), the corresponding $r_\infty/z_\infty$ is plotted with $\ell_k/\ell_c$, for $x_s = 10r_g$ (solid), $x_s = 20r_g$ (dashed) and $x_s = 30r_g$ (long-dashed), where $\ell_c = 0.4$ and $r_i = 2r_g$ are kept fixed. It is also seen that the decrease of $r_\infty/z_\infty$ with increasing $\ell_k/\ell_c$ is stronger for smaller values of $x_s$ (solid) than larger values (long-dashed).

Thus we see that, if $\ell_c > 0.2$, $x_s > 20r_g$, $1 > \ell_k/\ell_c < 0.5$, for $r_i \sim 2r_g$, we get relativistic ($v_\infty > 0.9c$) and collimated jets ($r_\infty/z_\infty < 0.1$), which means jets will be better collimated in intermediate hard states.

We already know that higher proportions of KD radiation, higher values of $x_s$, etc. are needed for collimation, while $\ell_c$ accelerates the jets. Still one should not forget about the injection radius of the jets, because we have seen that an increase in $r_i$ can disturb the collimation as well as produce slower jets. We now study the effect of $r_i$ on the jet solution for higher values of $\ell_k/\ell_c$ and increasing $x_s$.

In Fig. 13(a), $v_\infty$ is plotted with $x_s$ for $r_i = 2r_g$ (solid), $r_i = 4r_g$ (dashed) and $r_i = 6r_g$ (long-dashed), where $\ell_c = 0.4$ and $\ell_k = 0.2$, i.e. $\ell_k/\ell_c = 0.5$. The ratio of the CENBOL to KD luminosity is assumed to be around 0.5, in order to aid collimation. We now see a new feature: for lower $r_i$ (solid), $v_\infty$ decreases monotonically with $x_s$, but for higher $r_i$ (long-dashed), $v_\infty$ at first increases up to $x_s \sim 15r_g$ and then decreases. From Figs 5(a)–(j), we see that all the radiative moments peak around $\sim 2r_g$, just above the inner surface of the CENBOL.

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and also have very strong gradients as one moves along the CENBOL surface towards \( x_s \). For larger \( x_s \), the gradients are smoother, but for smaller \( x_s \), gradients become stronger. So when \( r_g = 6r_g \) is larger for small \( x_s \) (long-dashed), then the radiative moments received at \( r_s \) are small, because the presence of strong gradients in all the moments makes all of them peak around \( 2r_g \), and sharply decrease at \( 6r_g \). As \( x_s \) increases, for \( r_g \), the gradients become smoother, and \( r_g \) becomes relatively closer to the axis for larger \( x_s \). So, as \( x_s \) increases from \( 10r_g \) to \( 15r_g \), the jet material at \( r_g \) receives larger values of the radiative moments, resulting in an increase of \( v_\infty \). Increasing \( x_s \) further, decreases \( v_\infty \), because the CENBOL intensity decreases so much that the driving force of the radiation decreases anyway.

In Fig. 13(b), \( r_\infty/z_\infty \) is plotted with \( x_s \), for \( r_\infty = 2r_g \) (solid), \( r_\infty = 4r_g \) (dashed) and \( r_\infty = 6r_g \) (long-dashed), where \( \ell_s = 0.4 \) and \( \ell_k = 0.2 \). i.e. \( \ell_s/\ell_k = 0.5 \). The relative spread decreases monotonically with \( x_s \), as the collimation depends on whether \( f^t < 0 \) or \( f^t > 0 \). With increasing \( x_s \), \( f^t \) becomes negative (directed towards the axis) in a larger domain, thus helping in collimation. Thus we see that for \( x_s > 22r_g \), \( r_\infty < 4r_g \), \( \ell_s \sim 0.4 \) and \( \ell_s/\ell_k \gtrsim 0.5 \), we have jets with relativistic terminal speed \( \gtrsim 0.9c \) and \( r_\infty/z_\infty \gtrsim 0.2 \), in other words we have collimated and relativistic jets.

5 DISCUSSION AND CONCLUDING REMARKS

In this paper, we have studied the interaction of radiation with pair-dominated jets from the TCAF disc model. We have ignored the details of the mechanism of production of pair-dominated jets. High-energy photons can produce particle–antiparticle pairs close to the inner edge of a disc. It is well known that if the photon energy is \( h \nu \gtrsim 2mc^2 \), then an electron–positron pair may be created, where \( h \) is Planck’s constant, \( \nu \) is the photon frequency and \( m \) is the electron (or positron) mass. If, on the other hand, an electron and positron collide, they will annihilate each other to produce two gamma-ray photons, a process called pair annihilation. Evidently, to produce pair-dominated jets pair production has to dominate the pair annihilation process, as has been theoretically investigated by Mishra & Melia (1993) and Yamasaki et al. (1999). Observationally, electron–positron jets were suggested in the galactic black hole candidate Nova Muscae 1991/GS 1124–684 (Sunyaev et al. 1992), GRS 1915+105 (Mirabel & Rodriguez 1998), and in quasar 3C 279 (Wardle et al. 1998). Though there is little doubt about the existence of pair-dominated jets, radiative acceleration/collimation of such jets, on the other hand, is a different issue altogether. If the pairs are produced to such an extent that the jet medium is optically thick then the process discussed in this paper fails. For an optically thick medium, radiation drag terms are not present, but the disc intensity will fall off exponentially. We have not considered such details, because considering such details involves a self-consistent treatment of the inflow–outflow solutions around black holes. The present effort confines itself to extending our earlier work (Chattopadhyay et al. 2004) to rotating jets.

The post-shock torus of the TCAF model produces normal plasma jets and high-energy photons. In this paper we have considered that the electron–positron jets were produced within the funnel-like region of the post-shock torus. In contrast to Chattopadhyay et al. (2004), we have computed all the different moments of the radiation field for axial and off-axial points. Furthermore, in Chattopadhyay et al. (2004) we did not consider the Doppler shift of the radiation due to the disc motion. The rotational motion of matter on the disc surface will generate an azimuthal component of the radiative flux. The Doppler effect term also induces non-uniformity in the frequency integrated intensity of the CENBOL, while in Chattopadhyay et al. (2004) the CENBOL intensity was uniform. This non-uniformity of the CENBOL intensity resulted in strong gradients in the radiative moments around the inner edge of the CENBOL.

The motion of the Keplerian disc is primarily rotation dominated and its expression is known analytically even in general relativity, while the motion of matter in the post-shock torus has no simple analytical expression. Since we were not considering inflow–outflow solutions self-consistently, we needed to make a proper estimate of the motion of post-shock matter. Close to the black hole the infall time-scale is much smaller than the viscous time-scale so the angular momentum of infalling matter is almost constant near the black hole. Thus we assumed...
wedge flow for the motion of infalling matter along the inner surface of the post shock torus, and solved the geodesic equations. The post-shock surface motion may be a little overestimated, as pressure gradient terms are ignored. One could have used the Paczynski–Wiita potential, but as this potential blows up at the horizon, the geodesic equations are solved in the general relativistic realm. Another reason for estimating the inflow velocities in general relativity is to ensure that the rotational velocities of infalling matter should tend to zero as one comes closer to the central object.

The Doppler effect on the CENBOL intensity expression (equation 8e) is correct up to first order in \( \bar{u}/c \). We have chopped off the CENBOL at \( x_{in} = 2\gamma_{g} \), and the velocity field of the CENBOL is such that at \( x_{in} < x < x_{o}, \gamma_{g} \sim 1 \). At \( x < x_{in}, \gamma_{g} \) sharply increases. Thus taking first-order correct Doppler term in equation (8e) is consistent.

In this paper, we solved for three equations to find three variables \( \nu, \lambda \) and \( r \), for which three injection values corresponding to the three variables were supplied. All the other parameters like \( x_{in}, x_{o}, \ell_{s}, m_{K}, \) and the disc velocity components \( \bar{u} \) and \( \bar{u}_{K} \) were supplied to compute various independent components of the radiation stress tensor or, in other words, all the moments of the radiation field.

It has been noticed that the radiation from a Keplerian disc has marginal influence in determining \( \nu \) and \( \lambda \), while it has a greater role in determining \( r \). As has been explained above, collimation depends partly on reducing the angular momentum of the jet as well as pushing it towards the axis by the radial component of the radiative flux. Within the funnel-like region radiative flux from the Keplerian disc is negative because of the geometry of the TCAF model. Furthermore, the azimuthal component of the flux from it is also weak compared to the other components, hence the angular momentum gained by Keplerian radiation is small. In contrast, the radial flux from the CENBOL is weakest amongst all the other components, although, because of the special directiveness of the CENBOL, the radial flux is also towards the axis, close to the black hole. However, because of the small size of the CENBOL at \( z \sim 100r_{g} \), it approaches a point source and hence the radial flux is positive and spreads the jet. But drag terms along \( \phi \) and also the fact that radial flux from the CENBOL is weakest amongst all its other components, makes the spreading small. The situation dramatically changes if the injection radius is increased. As \( r_{i} \) is increased, the radial flux increases and is directed more and more away from the axis. On top of that, the drag terms along \( \phi \) become weaker, hence reduction of angular momentum decreases. The net effect is that the jet spreads. On top of that, with increasing \( r_{i} \), the CENBOL intensity goes down so the force driving the jets is less, resulting in slower jets. On the other hand, collimation is better with increasing \( x_{i} \). Increasing \( x_{i} \) makes the radial flux from the CENBOL become directed towards the axis in a greater part of the funnel-like region, which pushes the jet and helps in collimation, although as \( x_{i} \) is increased the radiation from the Keplerian disc has less influence in determining \( r \).

In the present paper we have restricted our analysis to non-rotating black holes only. It will be interesting to extend this study to a Kerr black hole, because the higher efficiency around a Kerr black hole will produce a more intense radiation field, and may produce terminal speeds higher than what we have observed in this paper. As the spectrum from the TCAF disc around a Kerr black hole has yet to be computed, so the issue of finding the spatial dependence of the disc intensity for a Kerr black hole is an open problem as yet.

We conclude that, if the radiative process is the main accelerating and collimating process then the following are true.

(i) Electron–positron jets are accelerated to highly relativistic terminal speed, and are collimated by the radiation from the two-component accretion disc model.

(ii) The space-dependent part of the radiative moments from the post-shock region dominates the corresponding moments from the Keplerian disc.

(iii) The CENBOL radiation is the main accelerating agent, but the Keplerian disc radiation has marginal influence on acceleration.

(iv) The drag terms in the azimuthal direction are greater than the radiative flux term in the same direction, hence the radiation removes angular momentum from the jet, near the jet base.

(v) Collimation is partly brought about by removal of angular momentum, and partly by the inward direction of the radial flux.

(vi) As the radial flux of the Keplerian disc is towards the axis, Keplerian radiation helps in collimation.

(vii) Collimation is better achieved for larger values of shock location and lower values of injection radius.

(viii) As the CENBOL radiation is the main accelerating agent and Keplerian disc radiation is a good collimator, we conclude that if this is the main process for acceleration and collimation then highly relativistic and collimated jets should be observed in intermediate hard states (\( \ell_{k}/\ell_{z} \lesssim 1 \)) and not in extreme hard states (\( \ell_{k}/\ell_{x} \ll 1 \)).

(ix) Drawing a concrete conclusion, our study shows that if shock in accretion is between 20\( r_{g} \sim 30r_{g} \), injection radius \( r_{i} \sim 4r_{g} \), CENBOL luminosity \( \ell_{c} \sim 0.2L_{Edd} \) and \( \ell_{k}/\ell_{x} \sim 0.5 \), then the jets will have a terminal speed greater than 90 per cent of the velocity of light, and the terminal relative spread will be less than 20 per cent.

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APPENDIX A: ESTIMATION OF POST-SHOCK INFLOW VELOCITY

The CENBOL is assumed conical, i.e. the motion of the post-shock flow is assumed to be wedge flow. Let us solve the geodesic equation along the radial coordinate $\tau$, such that $r^2 = x^2 + y^2$ where $x$ and $y$ define the cylindrical radial and axial coordinates of the CENBOL inner surface. The geodesic equation for conical inflow around a non-rotating black-hole is

$$\ddot{u} \frac{\partial \dot{u}}{\partial x^2} + \Gamma_{\tau \mu}^{\nu} \dot{u}^{\mu} \ddot{u}^{\nu} = 0. \quad (A1)$$

The radial velocity and azimuthal velocity are defined as

$$\dot{u}_r = -\frac{\ddot{u}x}{\dot{u}^2} \quad \text{and} \quad \dot{u}_\phi = \frac{(r - 1)\dot{u}^2}{\rho} \quad (A2).$$

Defining $\ddot{u} = \ddot{u}_r/(1 - \ddot{u}_\phi^2)^{1/2}$ reduces equation (A1) to

$$\frac{d\ddot{u}}{d\tau} = -\frac{1/2 + (\dot{u}^2/2 - \tau + 1)\dot{u}_\phi^2}{\rho(r - 1)\gamma \ddot{u}} \quad (A3),$$

where $\gamma \ddot{u} = 1/(1 - \ddot{u}_\phi^2)$. Equation (A3) is solved by supplying the value of $\ddot{u}_{\text{in}} = -0.01$ at $\tau_{\text{in}} = \sqrt{x^2 + y^2}$. As $\ddot{u}_r$ is the three-velocity of the inflow, so $\ddot{u}_r < 0$, i.e. at $\tau = 1\gamma_0$, $\ddot{u}_r = -1$, $\ddot{u}_\phi = 0$.

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