Implications of LEP/SLD Data for New Physics in $Z\bar{b}b$ Couplings

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Abstract

The combined LEP/SLD data on the $b\bar{b}$ forward-backward asymmetry from the Z-pole measurements may imply the presence of new physics in the $Z\bar{b}b$ couplings. In general, the effect of new physics can be parameterized by $SU_C(3) \times SU_L(2) \times U_Y(1)$ invariant higher dimensional operators. By fitting the recently announced LEP/SLD data on $A_b$ and $R_b$, the size of the coupling strengths of these operators can be determined. We also found that the new physics operators can be divided into two types, depending on their Higgs field content. The ones involving the Higgs have very mild effects at higher energy colliders, while the other type which do not contain the Higgs field can show significantly large effects on $b\bar{b}$ production at LEP II, $t\bar{t}$ production at the NLC and single top production at the Tevatron. The preliminary data from the LEP II measurements disfavors the second type of operators.
I. INTRODUCTION

The continuing agreement between experimental data and theoretical predictions on almost all the electroweak variables has further solidified the standard model (SM). However, there are signs of perhaps something non-standard from the investigation of non-electroweak variables and the indication of possibilities of new physics from electroweak variables themselves as well. On the one hand, the accumulative and, especially, the recent data on neutrino oscillations have strongly suggested the fact of finite neutrino masses [1] and, therefore, the SM has to be modified. On the other hand, the Z-pole measurements at LEP/SLD on the \( b \bar{b} \) forward-backward asymmetry gives a value of \( A_b = (g_L^L(b) - g_R^L(b)) / (g_L^L(b) + g_R^L(b)) \) which deviates from the SM prediction by 2.7\( \sigma \) [2]. If this \( A_b \) anomaly is not a statistical or systematic effect, it signals the presence of new physics in association with the \( Zb\bar{b} \) coupling. Meanwhile, the experimental value of the related quantity \( R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons}) \), after showing a deviation from the Standard Model (SM) value for a few years, now agrees well with the SM prediction [2]. To explain the current experimental values of both \( A_b \) and \( R_b \), the required new physics contribution has to shift the left- and right-handed \( Zb\bar{b} \) couplings by \( \sim -1\% \) and \( \sim +30\% \), respectively [3,4]. Recently efforts have been made in exploring the \( A_b \) anomaly, either in explaining the effect in specified models [4,5] or in examining its implications for low energy decay processes [6]. The popular low energy SUSY and the models in which the third generation feels a different gauge dynamics from the usual weak interaction cannot yield such large anomalous \( Zb\bar{b} \) coupling, as showed in [7] and [8], respectively. These studies are focused on the theme that has been emerged since the discovery of the heavy top quark, that new physics is most likely related to the gauge symmetry breaking sector and, therefore, may affect the interactions of the third family quarks most significantly.

From the overwhelming success of the SM at the electroweak scale, one can conclude that the underlying theory of the new physics can openly manifest only at a higher energy scale. One can envisage in the scheme of a larger symmetry that encompasses the new physics, after integrating out the heavy degrees of freedom which lie outside the SM spectrum, higher dimensional terms will be present at energies not too far above the electroweak scale and the induced effective terms should preserve the basic SM structure. Then the new physics effects on the \( Zb\bar{b} \) couplings can be parameterized by a set of higher dimensional operators [9,10] which are \( SU_C(3) \times SU_L(2) \times U_Y(1) \) symmetric before the electroweak symmetry breaking becomes explicit. From the above argument we see that before the electroweak symmetry breaking, the \( Zb\bar{b} \) part of the effective Lagrangian can take the general form

\[
\mathcal{L}_{Zb\bar{b}}^{\text{eff}} = \mathcal{L}_{Zb\bar{b}}^{\text{SM}} + \frac{1}{\Lambda^2} \sum_i C_i O_i + O(\frac{1}{\Lambda^4}) \quad (1)
\]

where \( \mathcal{L}_{Zb\bar{b}}^{\text{SM}} \) is the SM part, \( \Lambda \) is the new physics scale, \( O_i \) are dimension-six SM gauge invariant operators, and \( C_i \) are constants which represent the coupling strengths of \( O_i \). We have assumed that operators of higher dimensions than 6 are suppressed by powers of \( 1/\Lambda^2 \), so Eq. (1) is a quite general parameterization of new physics.

Since there are many possible higher dimensional operators \( O_i \), it is important to use various experimental data to constrain the operator form in order to narrow down the directions of the underlying theory. Because the operator \( O_i \) can contribute to several
vertices after $SU_L(2) \times U_Y(1)$ symmetry breaking, these operators can contribute to and will show correlated effects on several different physical observables. It will become clear later that the new physics effects required to explain $A_b$ will also show up in other observables, such as $b\bar{b}$ production at LEP II, $t\bar{t}$ production at NLC, and $t\bar{t}$ production at the Tevatron. These effects help to distinguish the possible operator forms required.

This article is organized as follows. In Sec. II we list the contributing dimension-6 operators and derive their induced vertices. In Sec. III we constrain the coupling strengths of these effects and the conclusion.

II. THE OPERATORS AND THEIR INDUCED VERTICES

Assuming CP conservation and ignoring those operators which only lead to anomalous dipole-moment couplings which are suppressed, we have two dimension-six gauge invariant operators which give rise to anomalous right-handed $Z\bar{b}b$ coupling [3],

$$O_{bb} = \left[ \bar{b}_R\gamma^\mu D^\nu b_R + \bar{D}^\nu b_R\gamma^\mu b_R \right] B_{\mu\nu},$$

$$O_{\Phi b} = i \left[ \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right] \bar{b}_R\gamma^\mu b_R,$$

and four operators affecting the left-handed $Zb_L\bar{b}_L$ coupling,

$$O_{qW} = \left[ \bar{q}_L\gamma^\mu \frac{\sigma^I}{2} D^\nu q_L + \bar{D}^\nu q_L\gamma^\mu \frac{\sigma^I}{2} q_L \right] W_{\mu\nu}^I,$$

$$O_{qB} = \left[ \bar{q}_L\gamma^\mu D^\nu q_L + \bar{D}^\nu q_L\gamma^\mu q_L \right] B_{\mu\nu},$$

$$O_{\Phi q}^{(1)} = i \left[ \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi \right] \bar{q}_L\gamma^\mu q_L,$$

$$O_{\Phi q}^{(3)} = i \left[ \Phi^\dagger \frac{\sigma^I}{2} D_\mu \Phi - (D_\mu \Phi)^\dagger \frac{\sigma^I}{2} \Phi \right] \bar{q}_L\gamma^\mu \frac{\sigma^I}{2} q_L.$$

At the order of $C_4/A^2$, the contributions to $R_b$ and $A_b$ from those operators that only lead to anomalous dipole-moment couplings for $Z\bar{b}b$ are suppressed by a factor $m_b/m_Z$ relative to the contributions of the operators we are considering. We have used the conventional notation: $q_L = (t_L, b_L)$ is $SU_L(2)$ doublet of the third family of quarks; $\Phi$ the Higgs boson doublet; $B_\mu$ the $U_Y(1)$ gauge field and $B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$; $W_{\mu}^I$ ($I = 1, 2, 3$) are the $SU_L(2)$ gauge fields and $W_{\mu\nu} = \partial_\mu W_{\nu}^I - \partial_\nu W_{\mu}^I + g_2 \epsilon_{IJK} W_{\mu}^J W_{\nu}^K$, and $\sigma^I$ are the Pauli matrices.

After the electroweak symmetry breaking each operator can give rise to a set of vertices which may affect more than one electroweak observable involving the bottom quark and lead to correlated effects among these observables. The two right-handed operators, $O_{bb}$ and $O_{\Phi b}$ in Eqs. (2) and (3), will give rise to anomalous $Zb_R\bar{b}_R$ and $\gamma b_R\bar{b}_R$ couplings. The four left-handed operators given in Eqs. (4)-(7) also involve the top quark and therefore will contribute to both bottom quark and top quark anomalous couplings. The correlated effects of these operators at the various colliders are depicted in Fig.1.

Denoting the $V^0 q\bar{q}$ ($V^0 = Z, \gamma; q = t, b$) and $Wt\bar{b}$ vertices by
\[ \Gamma_{\nu \eta \eta}^\mu = -ieg^V \gamma^\mu \left[ P_L(g_L^Y + \delta g_L^Y) + P_R(g_R^Y + \delta g_R^Y) \right], \]
\[ \Gamma_{W \bar{b} b}^\mu = -\frac{g}{\sqrt{2}} \gamma^\mu P_L(1 + \delta g_W^V), \]

the SM couplings are \( g_L^V, g_R^V, \) and \( g_L^W, \) while the anomalous couplings are denoted by \( \delta g_L^V, \delta g_R^V \) and \( \delta g_L^W. \) Here \( P_{L,R} = (1 \mp \gamma_5)/2, \) \( g^V = 1, \) \( g^Z = 1/(4s_Wc_W) \) with \( s_W \equiv \sin \theta_W \) and \( c_W \equiv \cos \theta_W. \) The SM couplings are given by \( g_L^V(q) = g_R^V(q) = e_q, \) \( g_L^Z(q) = 4T_q^3 - 4s_W^2c_q \) and \( g_R^Z(q) = -4s_W^2e_q \) with \( e_q \) being the electric charge of the quark in units of \( e \) and \( T_q^3 = \pm 1/2, \) the weak isospin component. The new physics contributions to \( \delta g^V_L \) from the right-handed operators are given in Table I, and those to \( \delta g^V_L \) and \( \delta g^W_L \) from the left-handed operators in Table II. The three operators, \( O_{bb}, O_{qW} \) and \( O_{qB}, \) induce momentum dependent anomalous couplings, and, therefore, their effects will generally be enhanced in higher energy processes.

It should be pointed out that there are no contributions from anomalous \( t_R \) couplings in the present considerations. The reason is that the operators which contribute to the \( t_R \) coupling are of the form of the operators given in Eqs. (2) and (3), \( b_R \) replaced by \( t_R, \) and do not involve the bottom quark; we therefore ignore all these operators.

### III. Constraints from the Z-pole Data

The non-standard contributions to \( R_b \) and \( A_b \) at the Z-pole from the anomalous couplings in Eq. (3) can be written as

\[
\delta R_b = 2R_b^{SM} (1 - R_b^{SM}) \left[ \frac{g_L^Z(b)\delta g_L^Z(b) + g_R^Z(b)\delta g_R^Z(b)}{(g_L^Z(b))^2 + (g_R^Z(b))^2} \right], \tag{10}
\]
\[
\delta A_b = 2A_b^{SM} \left[ \frac{g_L^Z(b)\delta g_L^Z(b) - g_R^Z(b)\delta g_R^Z(b)}{(g_L^Z(b))^2 - (g_R^Z(b))^2} - \frac{g_L^Z(b)\delta g_L^Z(b) + g_R^Z(b)\delta g_R^Z(b)}{(g_L^Z(b))^2 + (g_R^Z(b))^2} \right]. \tag{11}
\]

Here we only keep the lowest order effects of new physics; i.e., \( \mathcal{O}(C_i/\Lambda^2), \) which is the interference of new physics terms with the SM contributions, and neglect the interference terms proportional to \( m_b/m_Z. \) The SM values of \( R_b^{SM} \) and \( A_b^{SM} \) are taken to be [3]

\[
R_b^{SM} = 0.2158 \pm 0.0002, \quad A_b^{SM} = 0.9347 \pm 0.0001, \tag{12}
\]

which are the predictions in the SM including radiative corrections. The anomalous couplings \( \delta g_L^Z(b) \) and \( \delta g_R^Z(b) \) in Eqs. (10) and (11) are obtained from Tables 1 and 2 with the Z boson being on mass-shell \( (k^2 = m_Z^2). \)

The experimental values for \( A_b \) and \( R_b \) reported in [3] are

\[
R_b^{exp}(\text{LEP + SLD}) = 0.21642 \pm 0.00073, \quad \tag{13}
\]
\[
A_b^{exp}(\text{LEP}) = 0.881 \pm 0.020, \quad A_b^{exp}(\text{SLD}) = 0.905 \pm 0.026, \quad \tag{14}
\]

where the LEP value of \( A_b \) is obtained from the measured quantities \( A_{FB}(b) = \frac{3}{4} A_e A_b \) using \( A_e = 0.1496 \pm 0.0016, \) the combined average of \( A_e \) from LEP and SLD. The combined value of \( A_b \) from LEP and SLD is then given by \( A_b^{exp}(\text{LEP + SLD}) = 0.8902 \pm 0.0158, \) which is 2.8\( \sigma \) below the SM prediction. To fit the data on both \( R_b \) and \( A_b \) the Z\( b \bar{b} \) couplings are required.
to be $g^Z_L(b)/4 = -0.4163 \pm 0.0020$ and $g^Z_R(b)/4 = 0.0996 \pm 0.0076$. Comparing with the SM values $g^Z_L(b)/4 = -0.4208$ and $g^Z_R(b)/4 = 0.0774$, obtained by including radiative corrections and taking $m_t = 174$ GeV and $m_H = 100$ GeV, we find that new physics effects are needed in both $Zb_R b_R$ and $Zb_L b_L$, which indicates both right- and left-handed higher dimension operators are necessary. If we use only one right- and one left-handed operator at a time to obtain the required modifications to the $Zb_R b_R$ and $Zb_L b_L$ couplings, we obtain the ranges of strengths of the operators at the 1σ (2σ) level:

$$2.72 \ (1.30) \lesssim \frac{|C_{bb}|}{(\Lambda/\text{TeV})^2} \lesssim 5.54 \ (6.96), \quad 0.48 \ (0.23) \lesssim \frac{|C_{\Phi b}|}{(\Lambda/\text{TeV})^2} \lesssim 0.99 \ (1.24), \quad (15)$$

$$0.51 \ (0.10) \lesssim \frac{|C_{qW}|}{(\Lambda/\text{TeV})^2} \lesssim 1.33 \ (1.73), \quad 0.47 \ (0.09) \lesssim \frac{|C_{qB}|}{(\Lambda/\text{TeV})^2} \lesssim 1.21 \ (1.58), \quad (16)$$

$$0.08 \ (0.02) \lesssim \frac{|C^{(1)}_{\Phi q}|}{(\Lambda/\text{TeV})^2} \lesssim 0.22 \ (0.28), \quad 0.08 \ (0.02) \lesssim \frac{|C^{(3)}_{\Phi q}|}{(\Lambda/\text{TeV})^2} \lesssim 0.22 \ (0.28). \quad (17)$$

Apart from the limits that can be obtained from the electroweak variables the strengths of the operators can also be constrained by the partial wave unitarity condition of the appropriate 2-to-2 scattering processes $b\bar{b} \leftrightarrow b\bar{b}$, $b\bar{b} \leftrightarrow t\bar{t}$ and $t\bar{t} \leftrightarrow t\bar{t}$, which involve the helicity channels $b_+ \bar{b}_- + b_- \bar{b}_+ + t_+ \bar{t}_- + t_- \bar{t}_+$, with + and − denoting positive and negative helicities [13]. For the three operators, $O_{bb}$, $O_{qW}$, and $O_{qB}$, which give rise to momentum dependent couplings, the unitarity constraints are found to be significant [13], and are given by

$$\frac{|C_{bb}|}{\Lambda^2} < \frac{\sqrt{8\pi}}{s}, \quad (18)$$

$$\frac{|C_{qW}|}{\Lambda^2} < \frac{\sqrt{4\pi}}{s}, \quad (19)$$

and

$$\frac{|C_{qB}|}{\Lambda^2} < \frac{\sqrt{4\pi}}{s}. \quad (20)$$

Here $s$ is the center-of-mass energy squared for the relevant process. Requiring the unitarity condition to be satisfied for the processes with center-of-mass energy up to the new physics scale; i.e., $\sqrt{s} \approx \Lambda$, we obtain the upper limits on the coupling strengths, which are $|C_{bb}| < \sqrt{8\pi}$, $|C_{qW}| < \sqrt{4\pi}$, and $|C_{qB}| < \sqrt{4\pi}$. They imply that to give the minimal contribution required by the $A_b$ and $R_b$ data the new physics scale cannot be too high. The 1σ (2σ) upper bounds are found to be

$$\Lambda \lesssim 1.4\ (2.0)\text{ TeV} \quad \text{(for } O_{bb}), \quad (21)$$

$$\Lambda \lesssim 2.6\ (5.9)\text{ TeV} \quad \text{(for } O_{qW}), \quad (22)$$

and

$$\Lambda \lesssim 2.8\ (6.2)\text{ TeV} \quad \text{(for } O_{qB}). \quad (23)$$

The upper bounds on the new physics scales for the three momentum-independent operators are much weaker; e.g., $\Lambda \lesssim 10 \ (14)\text{ TeV}$ at the 1σ (2σ) level for the operator $O_{\Phi b}$. 

5
IV. CORRELATED EFFECTS AT LEP II, THE NLC AND THE TEVATRON

The anomalous couplings in Eqs. (8) and (9) also contribute to the production cross sections for $e^+e^- \rightarrow Z^*, \gamma^* \rightarrow q\bar{q}$ $(q = b, t)$ and $u\bar{d} \rightarrow W^* \rightarrow t\bar{b}$ as follows:

$$
\delta\sigma(e^+e^- \rightarrow q\bar{q}) = \frac{3}{2} \beta_q \left\{ D_{\gamma\gamma} e^2 \left[ (3 - \beta_q^2) e_q(\delta g_L + \delta g_R) \right]
\right.
$$

$$
+ D_{ZZ} (v_e^2 + a_e^2) \left[ (3 - \beta_q^2) u_q(\delta g_L^Z + \delta g_R^Z) + 2\beta_q^2 a_q(\delta g_L^Z - \delta g_R^Z) \right]
$$

$$
+ D_{Z\gamma} e_e v_e \left[ \frac{3 - \beta_q^2}{2} \left( e_q(\delta g_L^Z + \delta g_R^Z) + u_q(\delta g_L^Z + \delta g_R^Z) \right) \right. 
$$

$$
\left. + \beta_q^2 a_q(\delta g_L^Z - \delta g_R^Z) \right\},
$$

(24)

and

$$
\delta\hat{\sigma}(u\bar{d} \rightarrow t\bar{b}) = \frac{g^4}{384\pi \bar{s}^2 (s - m_W^2)^2} \left[ 2(2\hat{s} + m_t^2)\delta g_L^W \right].
$$

(25)

Here $s$ and $\hat{s}$ are the squared center-of-mass energies for $e^+e^- \rightarrow q\bar{q}$ and $u\bar{d} \rightarrow t\bar{b}$, respectively. $v_f$ and $a_f$ represent, respectively, the vector and axial-vector $Zf\bar{f}$ couplings in the SM; i.e., $v_f \equiv (g_L^Z + g_R^Z)/2$ and $a_f \equiv (g_L^Z - g_R^Z)/2$. In Eq. (24) we have defined

$$
\beta_q = \sqrt{1 - 4m_q^2/s},
$$

(26)

$$
D_{\gamma\gamma} = \frac{4\pi\alpha^2(s)}{3s},
$$

(27)

$$
D_{ZZ} = \frac{G_F^2}{96\pi} \frac{sm_Z^4}{(s - m_Z^2)^2 + (s\Gamma_Z/m_Z)^2},
$$

(28)

and

$$
D_{Z\gamma} = \frac{G_F\alpha(s)}{3\sqrt{2}} \frac{m_Z^2(s - m_Z^2)}{(s - m_Z^2)^2 + (s\Gamma_Z/m_Z)^2}.
$$

(29)

The anomalous contribution to $R_b \equiv \sigma(e^+e^- \rightarrow b\bar{b})/\sigma(e^+e^- \rightarrow q\bar{q})$ and the forward-backward asymmetry $A_{FB}^b$ at LEP II are given by

$$
\frac{\delta R_b}{R_b^{SM}} = (1 - R_b^{SM}) \frac{\delta\sigma(e^+e^- \rightarrow b\bar{b})}{\sigma^{SM}(e^+e^- \rightarrow b\bar{b})},
$$

(30)

and

$$
\frac{\delta A_{FB}^b}{A_{FB}^{SM}} = \frac{D_{Z\gamma} e_e v_e (e_b\delta g_L^Z - e_b\delta g_R^Z + g_L^Z\delta g_L^Z - g_R^Z\delta g_R^Z) + 4D_{ZZ} v_e a_b (g_L^Z\delta g_L^Z - g_R^Z\delta g_R^Z)}{2D_{Z\gamma} e_e v_e e_b a_b + 8D_{ZZ} a_e v_e a_b a_b}
$$

$$
- \frac{\delta\sigma(e^+e^- \rightarrow b\bar{b})}{\sigma^{SM}(e^+e^- \rightarrow b\bar{b})}. 
$$

(31)

The total hadronic cross section for $p\bar{p} \rightarrow t\bar{b} + X$ is evaluated by the convolution of the parton cross section and parton distribution functions. Here we use the CTEQ3L parton
distribution functions [12] with $\mu = \sqrt{s}$. The top quark mass is taken to be 175 GeV. The values of the other parameters are taken to be $m_Z = 91.187$, $m_W = 80.33$, $G_F = 1.16639 \times 10^{-5}\text{GeV}^{-2}$ and $\alpha = 1/128$.

Since the existence of both new right- and left-handed operators is necessary to explain the LEP I data, we again assume only one pair of the operators given in Eqs. (3) - (6) contributes at a time in obtaining the limits on their contributions. Subject to the limits derived from the data on $A_b$ and $R_b$, their effect on $R_b$ and $A_{FB}^b$ at LEP II, the $t\bar{t}$ production rate at the NLC, and the single top production cross section $\sigma(p\bar{p} \rightarrow t\bar{b} + X)$ at the Tevatron can be evaluated. These effects are summarized in Table III.

A few remarks regarding to the results in Table III are appropriate:

(a) The right-handed operator $O_{bB}$ has large effects on $R_b$ and $A_{FB}^b$ at LEP II due to two reasons: One is that it contributes to the $\gamma^*b_R\bar{b}_R$ coupling and the $\gamma^*$ intermediate state gives the dominant contribution to $\sigma(e^+e^- \rightarrow b\bar{b})$ at LEP II. The other is that the effects of $O_{bB}$ are enhanced by a factor $s/m_Z^2$, which is 4.3 for $\sqrt{s} = 189$ GeV at LEP II, due to its momentum dependent couplings. The preliminary LEP II data at $\sqrt{s} = 189$ GeV give [14]

$$R_b^{exp} = 0.167 \pm 0.011(\text{stat.}) \pm 0.008(\text{syst.}), \quad R_b^{SM} = 0.162,$$

$$A_{FB}^{b,exp} = 0.68 \pm 0.21(\text{stat.}) \pm 0.04(\text{syst.}), \quad A_{FB}^{b,SM} = 0.56.$$  \hspace{1cm} (32)

Note the large statistical error in $A_{FB}^{b,exp}$. At the 1$\sigma$ (2$\sigma$) level the new physics effects are limited to the following ranges

$$-5\%(-14\%) \leq \frac{\delta R_b}{R_b^{SM}} \lesssim 11\% (20\%),$$

$$-17\%(-55\%) \leq \frac{\delta A_{FB}^b}{A_{FB}^{b,SM}} \lesssim 60\% (98\%).$$  \hspace{1cm} (35)

From Table III we see that the minimum contributions of $O_{bB}$, when it is used together with any one of the left-handed operators, lie outside the allowed ranges in Eqs. (34) and (35) at the 1$\sigma$ level. Hence the right-handed operator $O_{bB}$ is disfavored unless the central values of both $R_b^{exp}$ and $A_{FB}^{b,exp}$ are modified in the refined LEP II data analysis. The operator $O_{q_b}$, which is neither momentum dependent nor contributes to the $\gamma^*b_R\bar{b}_R$ coupling, makes much smaller contributions to $R_b$ and $A_{FB}^b$ at LEP II. As shown in Table III, when $O_{q_b}$ is used together with a left-handed operator the deviation from the SM is allowed by the preliminary LEP II data.

(b) The left-handed operator, $O_{qW}$, when used to fit the anomalous $Zb_L\bar{b}_L$ coupling at the $Z$-pole, can give rise to a sizable effect in the cross section for $p\bar{p} \rightarrow t\bar{b} + X$ at the Fermilab Tevatron because its effect on $W^*t\bar{b}$ is enhanced by a factor $s/m_Z^2$ ($\sqrt{s}$ is the center-of-mass energy of the parton-level process $ud \rightarrow t\bar{b}$). Moreover, both $O_{qW}$ and $O_{qB}$ can cause large effects in the production rate of $t\bar{t}$ pairs at the NLC because their anomalous $\gamma^*t_L\bar{t}_L$ coupling is enhanced by a factor $s/m_Z^2 \approx 30$ for $\sqrt{s} = 500$ GeV at the NLC. Note that the effect of $O_{qB}$ is about a factor two larger than that of $O_{qW}$.
(c) Due to the clean environment and anticipated large number of the top pair events at the NLC it is possible to measure the top pair production rate at the level of a few percent \[15\]. So the $t\bar{t}$ production rate decrease caused by $O_{qW}$ ($O_{qB}$), with the minimum 2σ limit of 3.7% (7.2%), should be observable at the 2σ level. In contrast, due to the backgrounds at hadron colliders, it will be challenging to measure the single top production rate at the level of a few percent \[16\] at the Tevatron. The decrease of the single top production rate caused by $O_{qW}$, with the minimum 1σ (2σ) limit of 15% (3%), should be observable at the 1σ level, but probably not at the 2σ level. A detailed Monte Carlo analysis, with the consideration of all possible backgrounds, showed \[17\] that the effects of $O_{qW}$ on the single top production rate are observable at Run 3, or Run 2b, (30 fb$^{-1}$ luminosity) at the 2σ level for $C_{qW}/(\Lambda/\text{TeV})^2 \gtrsim 0.5$. Consequently, the effects of $O_{qW}$, with coupling strength in the 2σ range, 1.73 $\gtrsim C_{qW}/(\Lambda/\text{TeV})^2 \gtrsim 0.1$ in Eq.(16), will only marginally be observable in Run 3, or Run 2b, at the Tevatron.

(d) Among the left-handed operators the two operators that involve the Higgs field, $O_{\Phi q}^{(1)}$ and $O_{\Phi q}^{(3)}$, together with the right-handed operator, $O_{\Phi b}$, which also involve the Higgs field, indeed, do provide the required contribution to $Zb_L\bar{b}_L$ coupling but cause only small effects at LEP II, the NLC, and the upgraded Tevatron; only a few percent deviation from the SM in the $b$ forward-backward asymmetry at LEP II.

V. DISCUSSION AND CONCLUSION

We have examined in some detail the possibility of new physics, characterized by higher dimension operators, if the current data on the $Zb\bar{b}$ couplings at the $Z$-pole are taken literally. The dimension six operators we considered consist of two right-handed operators and four left-handed ones. Both the right- and left-handed operators are needed to explain the data. We also examined the effects of the eight pairs of operator at high energy colliders, LEP II, NLC, and upgraded Tevatron. Since the operators have sufficiently different behaviors at these higher energy colliders, their effects can mostly be distinguished, as can be seen from Table [11]. In particular, the preliminary LEP II data disfavor four pairs of these operators, all of which involve the right-handed operator $O_{bB}$.

The four pairs which contain the operator $O_{\Phi b}$ also behave differently. The pair $O_{\Phi b}$ and $O_{qW}$ has only small effects on $\delta R_b$ and $\delta A_{FB}^b$ but observable effects on $t\bar{t}$ production at the NLC and single top production at the upgraded Tevatron. The pair $O_{\Phi b}$ and $O_{qB}$ has a strong effect on $t\bar{t}$ production at the NLC, and an observable effect on $\delta A_{FB}^b$, but a negligible effect on $\delta R_b$ and no observable effects at the Tevatron.

The remaining two pairs, $O_{\Phi b}$ and $O_{\Phi q}^{(1)}$ and $O_{\Phi b}$ and $O_{\Phi q}^{(3)}$, have very little effect on the high energy quantities under consideration, except for a few percent deviation from the SM in $A_{FB}^b$. However, since these operators all contain the Higgs field and, therefore, are related to the symmetry breaking sector, they may well have something to do with new physics; at least, our analysis indicates they are the most likely suspects. The $Ht\bar{t}$ production at the NLC, $e^+e^- \rightarrow t\bar{t}H$, or $e^+e^- \rightarrow b\bar{b}H$, are useful processes for detecting their existence, as these operators can induce anomalous four-point couplings: $Zt\bar{t}H$ and $Zb\bar{b}H$. However, the production rates for such processes depend on the Higgs mass, which awaits discover.
In our analyses we have considered one pair of right- and left-handed operators at a time. If the two right-handed operators $O_{bB}$ and $O_{Φb}$ are considered together $C_{bB}$ will be strongly constrained by the LEP II data for the reasons discussed in remark (a) of the previous section. Then the effects of $O_{bB}$ on the LEP I observables will be suppressed relative to these of $O_{Φb}$. Thus $O_{Φb}$ will remain to explain the LEP I data and our previous conclusion that $O_{Φb}$ is the favored candidate for the right-handed new physics operator in the $Zb\bar{b}$ couplings is not changed. Similarly, if we consider the four left-handed operators together we can obtain a bound on a linear combination of their coupling coefficients from the LEP I data. From top pair production at the NLC we can distinguish $O_{qW}$ and $O_{qB}$ from the other two. From single top production at the upgraded Tevatron we might also be able to single out $O_{qW}$. However, it will be difficult to distinguish $O_{Φq}^{(1)}$ from $O_{Φq}^{(3)}$.

In conclusion, if new physics effects exist in the $Zb\bar{b}$ vertex, as indicated by the current measurements at the $Z$-pole, then: (1) The responsible right-handed operator could only be $O_{Φb}$, to be consistent with the LEP II data. (2) The responsible left-handed operators can be distinguished by measuring $t\bar{t}$ production rate at the NLC and single top production at the Tevatron. (3) It is most likely that any new physics in $Zb\bar{b}$ couplings will involve the Higgs sector.

**ACKNOWLEDGMENTS**

JMY thanks Ken-ichi Hikasa for useful discussions. The work was supported in part by the Grant-in-Aid for Scientific Research (No. 10640243) and the Grant-in-Aid for JSPS Fellows (No. 97317) from the Japan Ministry of Education, Science, Sports, and Culture, and by the U.S. Department of Energy, Division of High Energy Physics, under Grant Nos. DE-FG02-91-ER4086, DE-FG02-94ER40817, and DE-FG02-92ER40730. BLY acknowledges the support by a NATO grant and the support and hospitality provided to him by Dr. T.K. Lee at the National Center for Theoretical Science, Hsin Chu, where part of the work has been done.
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TABLE I. The anomalous couplings induced by right-handed operators. $k$ is the momentum of the corresponding vector boson.

|                  | $O_{bB}$                         | $O_{\Phi_b}$                        |
|------------------|----------------------------------|-------------------------------------|
| $\delta g_R^Z(b)$| $\frac{4s_Wc_W}{e}\frac{k^2}{\Lambda^2}C_{bB}$ | $-\frac{4s_Wc_W}{e}\frac{v_m}{\Lambda^2}C_{\Phi_b}$ |
| $\delta g_R^Z(t)$| $-\frac{c_W}{e}\frac{k^2}{\Lambda^2}C_{bB}$           | 0                                   |
| $\delta g_R^Z(t)$| 0                               | 0                                   |

TABLE II. The anomalous couplings induced by left-handed operators. $k$ is the momentum of the corresponding vector boson.

|                  | $O_{qW}$                         | $O_{qB}$                         | $O_{\Phi_q}^{(1)}$ | $O_{\Phi_q}^{(3)}$ |
|------------------|----------------------------------|----------------------------------|--------------------|--------------------|
| $\delta g_L^Z(b)$| $\frac{2s_Wc_W}{e}\frac{k^2}{\Lambda^2}C_{qW}$ | $\frac{4s_Wc_W}{e}\frac{k^2}{\Lambda^2}C_{qB}$ | $-\frac{4s_Wc_W}{e}\frac{v_m}{\Lambda^2}C_{\Phi_q}^{(1)}$ | $-\frac{4s_Wc_W}{e}\frac{v_m}{\Lambda^2}C_{\Phi_q}^{(3)}$ |
| $\delta g_L^Z(t)$| $\frac{8c_W}{2e}\frac{k^2}{\Lambda^2}C_{qW}$            | $-\frac{c_W}{e}\frac{k^2}{\Lambda^2}C_{qB}$           | $0$                | $0$                |
| $\delta g_L^Z(t)$| $-\frac{8s_Wc_W}{2e}\frac{k^2}{\Lambda^2}C_{qW}$       | $\frac{4s_Wc_W}{e}\frac{k^2}{\Lambda^2}C_{qB}$       | $-\frac{4s_Wc_W}{e}\frac{v_m}{\Lambda^2}C_{\Phi_q}^{(1)}$ | $-\frac{4s_Wc_W}{e}\frac{v_m}{\Lambda^2}C_{\Phi_q}^{(3)}$ |
| $\delta g_L^W(t)$| $\frac{8c_W}{2e}\frac{k^2}{\Lambda^2}C_{qW}$            | $-\frac{c_W}{e}\frac{k^2}{\Lambda^2}C_{qB}$           | $0$                | $0$                |
| $\delta g_L^W$   | $\frac{8s_Wc_W}{2e}\frac{k^2}{\Lambda^2}C_{qW}$       | $0$                               | 0                  | $\frac{8c_W^2}{e}\frac{k^2}{\Lambda^2}C_{\Phi_q}^{(3)}$ |
TABLE III. The ranges of correlated effects which are required by the data on $A_b$ and $R_b$ at the $Z$-pole. No contributions are indicated by ‘−’.

|                  | LEP II (189 GeV) | NLC (500 GeV) | Tevatron (2 TeV) |
|------------------|------------------|---------------|------------------|
|                  | $\frac{\delta R_b}{R_b^{SM}}$ (%) | $\frac{\delta A_{FB}^{b}}{A_{FB}^{SM}}$ (%) | $\frac{\delta \sigma(e^+e^-\rightarrow t\bar{t})}{\sigma^{SM}(e^+e^-\rightarrow t\bar{t})}$ (%) | $\frac{\delta \sigma(PP\rightarrow t\bar{b}+X)}{\sigma^{SM}(PP\rightarrow t\bar{b}+X)}$ (%) |
| $O_{bB}, O_{qW}$| 1σ 17 ~ 33 | -32 ~ −65 | -19 ~ −48 | -15 ~ −39 |
|                  | 2σ 9.4 ~ 41 | -15 ~ −81 | -3.7 ~ −63 | -3.0 ~ −51 |
| $O_{bB}, O_{QB}$| 1σ 21 ~ 44 | -36 ~ −77 | -36 ~ −93 | − |
|                  | 2σ 10 ~ 55 | -16 ~ −98 | -7.2 ~ −122 | − |
| $O_{bB}, O_{q}^{(1)}$| 1σ 21 ~ 42 | -31 ~ −64 | 0.2 ~ 0.5 | − |
|                  | 2σ 10 ~ 52 | -15 ~ −81 | 0.04 ~ 0.7 | − |
| $O_{bB}, O_{q}^{(3)}$| 1σ 21 ~ 42 | -31 ~ −64 | -0.2 ~ −0.5 | -1.0 ~ −2.6 |
|                  | 2σ 10 ~ 52 | -15 ~ −81 | -0.04 ~ −0.7 | -0.2 ~ −3.4 |
| $O_{bB}, O_{qW}$| 1σ -3.2 ~ −8.8 | -2.7 ~ −5.8 | -19 ~ −48 | -15 ~ −39 |
|                  | 2σ -0.4 ~ −12 | -1.2 ~ −7.4 | -3.7 ~ −63 | -3.0 ~ −51 |
| $O_{bB}, O_{qB}$| 1σ 0.8 ~ 1.7 | -7.5 ~ −18 | -36 ~ −93 | − |
|                  | 2σ 0.4 ~ 2.2 | -2.1 ~ −24 | -7.2 ~ −122 | − |
| $O_{bB}, O_{q}^{(1)}$| 1σ 0.04 ~ −0.3 | -2.6 ~ −5.5 | 0.2 ~ 0.5 | − |
|                  | 2σ 0.2 ~ −0.5 | -1.1 ~ −6.9 | 0.04 ~ 0.7 | − |
| $O_{bB}, O_{q}^{(3)}$| 1σ 0.04 ~ −0.3 | -2.6 ~ −5.5 | -0.2 ~ −0.5 | -1.0 ~ −2.6 |
|                  | 2σ 0.2 ~ −0.5 | -1.1 ~ −6.9 | -0.04 ~ −0.7 | -0.2 ~ −3.4 |
FIG. 1. The diagrams showing the correlated effects of new physics in $Zb\bar{b}$ couplings.