Breathe before Speaking: Efficient Information Dissemination despite Noisy, Limited and Anonymous Communication

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Inspiration

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Some mysteries from the experiment

- Why is the recruiting process so slow and seemingly inefficient?
- Why is it only the recruiting ant that is doing all the work?
Possible difficulties that ants may face

- Communication is very limited in its vocabulary
  (what do you mean when you bump into me?)
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Want to study:
Limited, Noisy and Stochastic communication
Distributed computing and Noise in communication

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Why?
When bandwidth is not a big issue, employ error correction.
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- When message size is restricted, redundancy comes at a price of limiting vocabulary.

- Repeatedly talking to the same person is difficult in stochastic and anonymous meeting patterns.
Rumor spreading in Computer science: a classic setting

Initially
A complete network with $n$ nodes. One source node $s$ has a message $m$ to be delivered to all nodes.
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The push model

- Synchronous model
- At each round, each node with the message \( m \) contacts another node, chosen uniformly at random, and delivers it the message.
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Complexities

- Time: \( \Theta(\log n) \) rounds
- Total number of messages sent: \( \Theta(n \log n) \)
- Good against crash faults.
The noisy rumor spreading problem

The problem
A source node $s$ has a bit $B \in \{0, 1\}$ that needs to be delivered to all nodes with high probability.

The Flip model of communication

- At each round, each agent $u$ contacts another agent $v$, chosen uniformly at random, and chooses whether or not to deliver it a bit message $b \in \{0, 1\}$.
- With probability at most $1/2 - \epsilon$, the bit $b$ is flipped and $v$ receives $\overline{b}$.

Synchronization assumptions

- Each agent can count rounds.
- Global clock: all agents start with their clock set to zero (assumption can be removed with some price).
Some basic strategies: what to do when you receive a message?

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**The problem:** The quality of messages quickly deteriorates
A closer look

In time $T$ at most $T$ agents heard directly from the source.
A closer look

- In time $T$ at most $T$ agents heard directly from the source.
- Most agents received a second hand rumor (at least). Hence the agents on level 2 will dominate the spreading.
What happens at level 2?

Probability of correct

\[(1/2 + \epsilon)(1/2 + \epsilon) + (1/2 - \epsilon)(1/2 - \epsilon) = 1/2 + 2\epsilon^2.\]
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For level \(i\)
Probability of correct is roughly \(1/2 + \epsilon^i\).

So quality of messages quickly deteriorates
Observation

The exists a simple protocol that runs in $O(\log n)$ rounds no matter how small is $\epsilon$. 
Our results

**Theorem**

∃ a simple symmetric protocol running in $O\left(\frac{1}{\epsilon^2} \log n\right)$ rounds using $O\left(\frac{1}{\epsilon^2} n \log n\right)$ messages in total.
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Observe: Each agent should receive \( \Omega\left(\frac{1}{\epsilon^2} \log n\right) \) messages to be convinced even if these messages come directly from the source. Hence:

- \( \Omega\left(\frac{1}{\epsilon^2} \log n\right) \) rounds are required even to convince 1 agent, directly informed from the source.
- \( \Omega\left(\frac{1}{\epsilon^2} n \log n\right) \) messages in total are required.
Phase 1: Spreading the information

**Goal:** Inform all agents, such that the fraction of agents with the correct opinion is at least $1/2 + 1/\sqrt{n}$. 
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**We want a good balance between:**

- Slow deterioration of messages (short depth of tree), and
- Fast rumor spread (high depth of tree).
We want to have: Level $i$ agents do not spread their opinion before sufficiently many level $i$ agents were informed.
A first idea: delay messaging to control synchronization of levels

We want to have: **Level $i$ agents do not spread their opinion before sufficiently many level $i$ agents were informed.**

▶ Divide the time into *phases*. Phase $i$ takes time $[T_i, T_i + \beta_i)$.
▶ If you receive a message (for the first time) in Phase $i$, wait until Phase $i + 1$ starts and only then start sending your opinion repeatedly.
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**Property**

If we have $L_i$ agents awake when Phase $i$ starts then we have $\beta_i \cdot L_i$ agents awake when Phase $i + 1$ starts.
Setting $\beta_i$

**Level 1:** We want at least $O\left(\frac{1}{\epsilon^2} \log n\right)$ agents of level 1, to make sure that w.h.p, the majority of those have the correct opinion. So let $\beta_1 \approx \frac{1}{\epsilon^2} \log n$. 
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**Level $i$, $i > 1$:** Recall, if $1/2 + \delta_i$ fraction is correct on level $i$ then $1/2 + \delta_i \cdot \epsilon$ fraction is correct on level $i + 1$. Set $\beta_i = \beta = O\left(\frac{1}{\epsilon^2}\right)$ (degree $\approx \frac{1}{\epsilon^2} \gg$ inverse of the deterioration factor $\approx \epsilon$).
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**Time complexity:** Total number of phases is $O(\log_\beta n)$. So total # rounds is $\beta_1 + \beta \log_\beta n = O(\frac{1}{\epsilon^2} \log n)$. 

![Diagram](image-url)
A (slow) deterioration of opinions

At phase $i$ fraction of correct agents is at least $\approx 1/2 + \epsilon^i$. 
A (slow) deterioration of opinions

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The number of phases is $m \leq \log_{\beta} n = \log \frac{1}{\epsilon^2} n = \log \epsilon (1/\sqrt{n})$,

so the final fraction of correct agents is:

$$\geq \frac{1}{2} + \epsilon^m \geq \frac{1}{2} + \epsilon^{\log_\epsilon (1/\sqrt{n})} = \frac{1}{2} + 1/\sqrt{n},$$

as desired.
Second stage: boosting the faction of correct agents

Note: we start with a very small bias towards the correct opinion: \(1/2 + 1/\sqrt{n}\).

In such a case, even without noise, the task of boosting the majority opinion is non-trivial. E.g., # of samples each agent should get from such a population should be higher than \(n\).
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An $O(\log n)$ time majority boosting algorithm exists [Angluin, Aspnes, and Eisenstat, DISC 2007]. However, this algorithm uses messages of size 2 bits (rather than 1) and does NOT account for noise in messages.

[Doerr et al SPAA 2011] show that a method based on gradual boosting the majority can achieve consensus in $O(\log n)$ time. We show that a similar approach works, also in the presence of noise.
The idea: gradual boosting

- Divide the time into phases. Up to the last phase, all phases consist of $\gamma = O(1/\epsilon^2)$ rounds. In each phase:
  - Send $\gamma$ times your current opinion,
  - Receive $\gamma$ opinions. Set your opinion to the majority opinion among those $\gamma$ opinions.

After $O(\log n)$ of such phases, fraction of correct agents is $1/2 + \text{constant}$.

Then, one last phase of length $O(1/\epsilon^2 \cdot \log n)$ where each agent is sending its opinion in each round, and at the end taking majority guarantees that all agents have the correct opinion with high probability.
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Gradual boosting- a closer look

Let $\delta_i$ be such that $1/2 + \delta_i$ is the fraction of correct agents at phase $i$. (Note $\delta_1 > 1/\sqrt{n}$).

**Theorem**

- As long as $\delta_i$ is smaller than some constant $c_1$, we have $\delta_{i+1} \geq 2\delta_i$.
- If $\delta_i > c_1$, then $\delta_{i+1}$ is greater than another constant $c_2 < c_1$.

Note that since $\delta_i$ maybe very small we cannot use Chernoff directly to obtain the theorem!
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Corollary

After $O(\log n)$ phases (which is $O(\frac{1}{\epsilon^2} \log n)$ time), the fraction of correct agents is at least $1/2 + c_2$. 
Removing the global clock assumption

So far we assumed all agents wake up at time 0. What about if agents do not have the same starting time?
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First note, if all clocks are initially in the range $[0, D]$, We can use the synchronized push model to synchronize agents:
Conclusion

Delaying propagation of messages, relying on synchronizing, and taking majority of samples, allows to overcome highly stochastic, anonymous, and noisy settings.
Open problems

▶ What about if the synchronization is very bad?
▶ Our time complexity is polylogarithmic. In case an adversary controls the content of the faulty message, can we prove a polynomial lower bound?
▶ Different graph families...
Adversary model: What happens at level 2?

Assume that an adversary controls the content of faulty messages. Assume $p = 1 - \frac{1}{\sqrt{2}} \approx 0.3$.

With prob $1/2$ the first message at $u$ is "clean" (u receives B)

With prob $1/2$ at least 1 fault. Adversary makes u receive $\overline{B}$

Messages received at level 2 nodes are uniformly spread between 0 and 1
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- **Discrete noise**: E.g., the flip model of communication.

- **Continuous distortion**: A message is a real number. If a message is sent as $x$ then the received message is $x + n$, where $n$ is sampled from some continuous noise distribution.
Thank you!