Evidence of a higher nodal band $\alpha^{+44}$Ca cluster state in fusion reactions and $\alpha$ clustering in $^{48}$Ti

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In the nucleus $^{48}$Ti, whose structure is essential in evaluating the half-life of the neutrinoless double-$\beta$ decay ($0\nu\beta\beta$) of $^{48}$Ca, the existence of the $\alpha$ cluster structure is shown for the first time. A unified description of scattering and structure is performed for the $\alpha^{+44}$Ca system. By using a global potential, which reproduces experimental $\alpha^{+44}$Ca scattering over a wide range of incident energies, $E_\alpha=18$ - 100 MeV, it is shown that the observed $\alpha^{+44}$Ca fusion excitation function at $E_\alpha=9$ - 18 MeV is described well. The bump at $E_\alpha=10.2$ MeV is found to be due to a resonance which is a $7^-$ state of the higher nodal band with the $\alpha^{+44}$Ca cluster structure in $^{48}$Ti. The local potential $\alpha^{+44}$Ca cluster model locates the ground band of $^{48}$Ti in agreement with experiment and reproduces the enhanced $B(E2)$ values in the ground band well. This shows that collectivity due to $\alpha$ clustering in $^{48}$Ti should be taken into account in the evaluation of the nuclear matrix element in the $0\nu\beta\beta$ double-$\beta$ decay of $^{48}$Ca.

I. INTRODUCTION

$\alpha$ clustering is essential not only in the light $0p$-shell and $sd$ shell regions [1,2] but also in the medium-weight $fp$ shell region [3,4] as evidenced typically in the $^{44}$Ti region [5,14]. Recent evidence of the higher nodal band states with the $\alpha$ cluster structure in $^{52}$Ti [15], in which the intercluster relative motion is excited, suggests that $\alpha$ clustering may persist in nuclei in-between such as $^{48}$Ti and $^{46}$Ti. In fact, the $\alpha$ spectroscopic factors of $^{48}$Ti and $^{46}$Ti in the ($^5$Li, d) $\alpha$-transfer reactions [16,17] are larger than those of $^{52}$Ti, which is the minimum in the $A=36$-64 mass region [16]. This is also confirmed in the $(p,p\alpha)$ reactions [18,20]. Reference [21] reports that the excess neutrons outside the core work as covalent bonding between the clusters.

The structure of $^{48}$Ti is crucial in determining the nature of neutrino, Dirac or Majorana particle in the measurements of neutrinoless double-$\beta$ decay $0\nu\beta\beta$ of $^{48}$Ca [22]. The study of $0\nu\beta\beta$ [23], which violates lepton number conservation, serves to solve the longstanding fundamental questions beyond the standard model. The inverse half-life of $0\nu\beta\beta$ of $^{48}$Ca (0+) $\rightarrow$ $^{48}$Ti (0+) is given by $[T_{1/2}^{0\nu}]^{-1} = G_{0\nu} |<m_\beta|/m_e|^2 |M_{0\nu}|^2$, where $<m_\beta|$ is the effective Majorana neutrino mass, $m_e$ is the electron mass, and $G_{0\nu} \approx 10^{-14}$ yr$^{-1}$ is a phase-space factor. For the evaluation of the nuclear matrix element of the transition $M_{0\nu}$ [24], it is essential to know the ground state wave function of $^{48}$Ti accurately.

Several approaches such as the shell model [25,28], ab initio calculations [29,30], quasi-particle random phase approximation [31,32], the projected Hartree-Fock Bogoliubov model [34], the generator coordinate method (GCM) [35,36], the energy density functional [37,38] and the interacting boson model [39] have been reported. As for the $\alpha$ clustering aspects of $^{48}$Ti, a microscopic $\alpha^{+44}$Ca cluster model calculation in the GCM was performed [40,41]. However, no state corresponding to the ground state of $^{48}$Ti was obtained. Experimentally in the recent challenges [42,43], no $\alpha$ cluster states have been observed in $^{48}$Ti. A discovery of a typical $\alpha$ cluster state, such as the higher nodal band states observed in $^{52}$Ti [15], would shed light on the $\alpha$ clustering of the ground state of $^{48}$Ti.

The purpose of this paper is to show that the existence of a higher nodal band $\alpha^{+44}$Ca cluster structure in $^{48}$Ti is evidenced for the first time by investigating the observed $\alpha^{+44}$Ca scattering over a wide range of incident energies, $E_\alpha=18$ - 100 MeV. By using the local potential $\alpha^{+44}$Ca cluster model the observed enhanced $B(E2)$ values of the ground band of $^{48}$Ti are reproduced well. It is shown that the enhanced $B(E2)$ values are caused by $\alpha$ clustering in $^{48}$Ti.

The paper is organized as follows. Section II is devoted to the analysis of $\alpha^{+44}$Ca scattering and fusion reactions using a global potential. In Sec. III $\alpha^{+44}$Ca cluster structure in $^{48}$Ti is studied. In Sec. IV discussions and a summary are given.

II. ANALYSIS OF $\alpha^{+44}$Ca SCATTERING AND FUSION REACTIONS

In exploring the $\alpha$ cluster structure in medium-weight mass region where the level density is high, a unified description of $\alpha$ scattering including rainbow scattering, prerainbows, backward angle anomaly (BAA) or anomalous large angle scattering and the $\alpha$ cluster structure in the bound and quasi-bound energy region has been very powerful [4–7,13,15]. In fact, the $\alpha$ cluster structure in the $^{44}$Ti region was successfully explored from this viewpoint and the predicted $\alpha$ cluster $K=0^-$ band with the $\alpha^{+40}$Ca cluster structure [6,7], which is a parity-doublet partner of the ground band of $^{44}$Ti, was observed in experiment [8,9]. Systematic theoretical and experimental studies in the $^{44}$Ti region [8,10,14] confirmed the existence of the $\alpha$ cluster in the beginning of the $fp$-shell.
The BAA in α particle scattering, which was first observed for α+40Ca \(^{[17, 35]}\), was systematically investigated in comparison with the isotopes \(^{42}\)Ca and \(^{44}\)Ca \(^{[49]}\) at low energies, \(E_\alpha=18-29\) MeV. It was concluded \(^{[49]}\) that α+44Ca scattering with no backward enhancement, which is described well by the so-called standard optical with a Wood-Saxon form factor, is normal. The normal behavior of the angular distributions was attributed to strong absorption due to the excess neutrons outside the \(^{40}\)Ca core \(^{[40, 41, 49, 50]}\) and little attention has been paid to the \(^{16}\)O+\(^{12}\)C cluster structure in \(^{44}\)Ca \(^{[47, 48]}\), was systematically investigated \(^{[49]}\) for \(\alpha+^{44}\)Ca scattering at the lowest energy \(E_\alpha=18\) MeV is reproduced well by the global potential. In Fig. 1 the calculated angular distributions at 18 MeV and 24.1 MeV are compared with the experimental data. The potential parameters \(V, W\) and \(W_s\) are searched with other parameters fixed as in Ref. \(^{[50]}\), which are listed in Table I. The fits to the experimental data are much better than the ones calculated with the standard optical model in Ref. \(^{[49]}\) and the improvements are entirely due to the Laneburg lens-like shape of the real potential \(^{[44]}\) \(^{[46]}\). To see why BAA of the cross sections is absent, the calculated angular distributions are decomposed using the technique of Ref.\(^{[51]}\) into the barrier-wave (dotted lines) and the internal-wave (dashed lines) contributions. At 18 MeV the solid line overlaps with the dotted line and the dashed line is negligibly small and is not drawn.

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The optical potential parameters used in Fig. 1 and the volume integrals per nucleon pair, \(J_0\), in unit of MeVfm\(^3\) for the real potentials. \(E_\alpha\), \(V\), \(W\) and \(W_s\) are in units of MeV and \(r_\alpha\), \(a_\alpha\), \(r_w\), \(a_w\), \(r_s\) and \(a_s\) are in units of femtometers.

| \(E_\alpha\) | \(J_0\) | \(V\) | \(r_\alpha\) | \(a_\alpha\) | \(W\) | \(r_w\) | \(a_w\) | \(W_s\) | \(r_s\) | \(a_s\) |
|---|---|---|---|---|---|---|---|---|---|---|
| 18 | 388.181 | 1.42 1.25 | 14.0 1.75 | 0.934 53.2 | 1.36 0.378 |
| 24.1 | 356.166 | 1.42 1.25 | 10.6 1.75 | 0.934 17.0 | 1.36 0.378 |

\[ U(r) = -Vf^2(\mathbf{r}; R_w, a_w) + V_Coul(\mathbf{r}) - iWf^2(\mathbf{r}; R_w, a_w) - i4\alpha_\alpha W_s f^2(\mathbf{r}, R_s, a_s) \quad \text{with} \quad f(\mathbf{r}; R_s, a_s) = 1/\{1 + \exp[(r - R_s)/a_s]\}. \]

The Coulomb potential is assumed to be a uniformly charged sphere with a reduced radius \(r_\alpha=1.3\) fm. First, I show that the experimental angular distribution in α+44Ca scattering at the lowest energy \(E_\alpha=18\) MeV is reproduced well by the global potential. In Fig. 1 the calculated angular distributions at 18 MeV and 24.1 MeV are compared with the experimental data. The potential parameters \(V, W\) and \(W_s\) are searched with other parameters fixed as in Ref. \(^{[50]}\), which are listed in Table I. The fits to the experimental data are much better than the ones calculated with the standard optical model in Ref. \(^{[49]}\) and the improvements are entirely due to the Laneburg lens-like shape of the real potential \(^{[44]}\) \(^{[46]}\). To see why BAA of the cross sections is absent, the calculated angular distributions are decomposed using the technique of Ref.\(^{[51]}\) into the barrier-wave component reflected at the surface and the internal-wave component, which penetrates deep into the internal region of the potential \(^{[52]}\). In Fig. 1, the barrier-waves dominate and the internal-waves hardly contribute, not visible at 18 MeV. The absence of the BAA in α+44Ca scattering is due to no internal-wave contributions under strong absorption, and not due to the real part of the potential since it is similar to the α+40Ca system as will be shown in Fig. 6.

Second, I investigate the α+44Ca fusion cross sections below \(E_\alpha=18\) MeV. In contrast to α+40Ca \(^{[53, 55]}\), the α+44Ca fusion excitation function with no pronounced oscillations \(^{[54]}\) has never been paid attention to in the past decades. However, one note in Fig. 2 that the observed excitation function is not monotonic with a bump at around \(E_\alpha=10.2\) MeV where absorption is relatively small near the threshold. The origin of this bump, which may be a remnant of the fusion oscillations seen typically for α+40Ca \(^{[53, 55]}\), \(^{12}\)C+\(^{12}\)C \(^{[56]}\), \(^{16}\)O+\(^{12}\)C and \(^{16}\)O+\(^{16}\)O \(^{[57]}\), seems to be related to the α+44Ca molecular resonance.

In calculating the fusion cross sections using the optical potential, I followed the prescription used for α+40Ca in Ref.\(^{[53]}\) where the fusion cross section is defined as a total reaction cross section due to the short-ranged imaginary potential. The fusion cross sections at \(E_\alpha\le18\) MeV are calculated using the optical potential at \(E_\alpha=18\) MeV in Table I with \(r_w=1.3\) for the volume imaginary potential and \(W_s=0\) for the surface imaginary potential due to direct reactions. The energy dependence of the strength parameter, \(W\), which was originally determined at \(E_\alpha>24\) MeV in Ref. \(^{[50]}\) is modified for \(E_\alpha<18\) MeV to decrease linearly toward the barrier top energy due to the dispersion relation of the threshold anomaly \(^{[53]}\), \(W=aE_\alpha + b\) for \(E_\alpha \ge 8.4\) MeV with \(a=1.281\) and \(b=-10.74\).

In Fig. 2 the calculated fusion excitation function is compared with the experimental data \(^{[54]}\). It is to be noted that the bump at \(E_\alpha=10.2\) MeV, which was not reproduced in Ref. \(^{[54]}\), is reproduced well by the cal-
FIG. 2. (Color online) Calculated excitation function of the $\alpha + ^{44}\text{Ca}$ fusion cross sections (solid lines) is compared with the experimental data (filled circles) [54].

FIG. 3. (Color online) Calculated excitation functions of the $\alpha + ^{44}\text{Ca}$ fusion cross sections (solid lines) are decomposed into the partial wave contributions; even $L$ (dotted lines) and odd $L$ (dashed lines). The inset is for $E_\alpha=8.4-9.0$ MeV.

culation. Also the calculated fusion cross sections do not decrease monotonically toward the threshold energy in accordance with the very smooth oscillatory behavior of the experimental data, which is a remnant of the pronounced oscillatory structure seen in the fusion excitation function for $\alpha + ^{40}\text{Ca}$ in the same energy region. The emergence of a bump at $E_\alpha=10.2$ MeV is due to the relatively weak absorption near the barrier top energy, $E=6.2$ MeV, which corresponds to $E_\alpha=7.2$ MeV.

In Fig. 3 the calculated fusion cross sections are decomposed into the partial cross sections. One notes that the bump peak is caused by the partial fusion cross sections with $L=7$ (dashed line). The broad resonance-like behavior of the $L=6$ and 8 partial fusion cross sections contributes to the non-monotonic behavior of the fusion excitation function. The calculated fusion excitation function at $E_\alpha=8.4-9$ MeV is displayed in the inset. Although the magnitude of the cross sections becomes smaller, one notices the appearance of a bump at $E_\alpha=8.5-8.6$ MeV where the absorption is much smaller than the bump at $E_\alpha=10.2$ MeV. It is found that this structure is caused by the $L=5$ partial cross sections (dashed line).

In order to reveal the origin of the bump structures in the fusion excitation function, in Fig. 4 the phase shifts calculated with $V=181$ MeV potential (referred to as D181 hereafter) at $E_\alpha=18$ MeV by switching off the imaginary potentials are displayed. One notes that the resonance at $E_\alpha=10.3$ MeV with a width $\Gamma_{\text{c.m.}}=0.18$ MeV is responsible for the peak of the partial fusion cross section for $L=7$ and therefore for the bump observed at around $E_\alpha=10.2$ MeV. The resonance at $E_\alpha=8.3$ MeV with $\Gamma_{\text{c.m.}}=0.22$ MeV is also responsible for the bump at $E_\alpha=8.3$ MeV in the inset of Fig. 3. There appear resonances for $L=9$ and 11 at $E_\alpha=12.96$ ($\Gamma_{\text{c.m.}}=0.11$ MeV) and $16.07$ ($\Gamma_{\text{c.m.}}=0.04$ MeV), respectively, however, the contributions to the fusion cross section are not seen clearly. The broad $L=6$ and 8 resonances at around $E_\alpha=12$ MeV and 16 MeV, respectively, are also the origin of the non-monotonic behavior of the fusion excitation function.

![Image](image_url)

FIG. 4. (Color online) The phase shifts for $\alpha + ^{44}\text{Ca}$ scattering calculated with the D181 real potential. The vertical axis is $\delta_L=m_L\pi$ where $m_L$ is the number of the Pauli-forbidden states. See the text. The resonances for odd $L$ and even $L$ correspond to $N=15$ and $N=16$, respectively.

III. $\alpha + ^{44}\text{Ca}$ CLUSTER STRUCTURE IN $^{48}\text{Ti}$

How these resonances are understood as the highly excited states with the $\alpha + ^{44}\text{Ca}$ cluster structure in $^{48}\text{Ti}$? In Fig. 5(a) the energy levels calculated with the D181 potential is displayed. The number of the Pauli-forbidden redundant states in the $\alpha + ^{44}\text{Ca}$ cluster model [40] is $m_L=(12-L)/2$ for even $L\leq12$ and $m_L=(13-L)/2$ for odd $L\leq13$. $m_L=0$ for $L \geq 14$. According to the generalized Levinson theorem [59], the phase shift $\delta_L$ in which the existence of the Pauli-forbidden states are taken into account should satisfy $\delta_L = m_L \pi$ at $E_\alpha=0$. The energy levels are calculated with the D181 potential for $E_\alpha=0$ up to $E_\alpha=20$ MeV.
TABLE II. Calculated intercluster rms radii (femtometers) and $B(E2)$ values (W.u.) for the $J \rightarrow (J-2)$ transitions of the ground band of $^{48}$Ti. The $B(E2)$ values calculated with the D171 potential (cal-1) and the $L$-dependent $V$ (MeV) (cal-2) are compared with the experimental data [72].

| $J^\pi$ | $<R^2>$ | $B(E2)$ | $V$ | $B(E2)$ |
|---------|----------|---------|-----|---------|
|         | cal-1    | cal-1 exp [72] | cal-2 | cal-2   |
| 0$^+$   | 4.41     | 171.0   |      |         |
| 2$^+$   | 4.37     | 13.6    | 15.0 | 169.4   | 13.8   |
| 4$^+$   | 4.33     | 17.9    | 18.4 | 167.7   | 18.8   |

and tends asymptotically to $\delta_L = 0$ at $E_\alpha = \infty$. Shown in Fig. 4 are the phase shifts, $\delta_L-m_L \pi$, to make easy to see the band structure of $N = 15$ and $N = 16$ where $N = 2n+L$ with $n$ being the number of nodes in the relative wave function with the $\alpha + ^{44}$Ca molecular structure. Surprisingly the calculated lowest Pauli-allowed $N = 12$ band, which satisfies the Wildermuth condition due to the Pauli principle, falls in correspondence to the experimental ground band of $^{48}$Ti. One finds that the $L = 7$ resonance that contributes to the bump of the fusion cross section at $E_\alpha = 10.2$ MeV is a member state of the higher nodal $N = 15$ band with the $\alpha + ^{44}$Ca cluster structure, in which the intercluster relative motion is one more excited compared with the $N = 13$ band. This $7^-$ of $N = 15$ is a second example of the higher nodal band with negative parity in addition to $^{44}$Ti [12, 13], which gives strong support to the persistence of the $\alpha$ cluster structures in $^{48}$Ti. It is highly desired to observe the $5^-$ state theoretically predicted at $E_\alpha = 8.3$ MeV as well as the $3^-$ and $1^-$ states of the $N = 15$ band in a precise experiment such as sub-barrier fusion reactions [60, 61] and transfer reactions etc., although the $\alpha$-strengths as well as the $N = 13$ and 14 bands, may be fragmented as in $^{40}$Ca and $^{44}$Ti [10, 13].

The reason why the $7^-$ state of $N = 15$ is observed in Fig. 2 is related to the considerably high $\alpha$ threshold energy of $^{48}$Ti, 9.45 MeV due to a non-$\alpha$ nucleus. Because of this, although the excitation energy $E_\alpha = 18.8$ MeV of the $7^-$ state is high, the energy from the $\alpha$ threshold becomes relatively small. The $N = 14$ band, which is a higher nodal band of the ground band $N = 12$, starts just above the $\alpha$ threshold. Its observation in addition to the known analog higher nodal bands in $^{20}$Ne [62, 65], $^{40}$Ca [10, 11], $^{44}$Ti [10, 12, 13] and $^{52}$Ti [15, 13] would also reinforce the $\alpha$ cluster structure in $^{48}$Ti.

As for the $N = 12$ ground band, the D181 potential with $J_\alpha = 388$ MeVfm$^3$ determined at $E_\alpha = 18$ MeV locates the ground state $0^+$, which overbinds -4.27 MeV, compared with the experimental ground state. When discussing the ground band deep below the threshold, as was the case in $^{52}$Ti [15], the potential must be readjusted by taking into account that the volume integral of the real potential decreases toward the threshold due to the threshold anomaly [58]. With a decreased potential strength, $V = 171$ MeV (D171) with $J_\alpha = 367$ MeVfm$^3$, the calculated ground band in Fig. 5 is well in correspondence with the experimental ground band in Fig. 5(c). In Table II calculated rms intercluster radii and $B(E2)$ values are given. The $B(E2)$ values are calculated with the D171 potential and the $L$-dependent $V$ tuned to reproduce the experimental excitation energy of the ground band because the $L$-dependence of $V$ has been known widely, for example, in $^{20}$Ne [67], $^{44}$Ti [5, 6], $^{94}$Mo [68, 69], $^{212}$Po [68, 70] and $^{46, 50}$Cr [71]. A small effective charge $\Delta e = 0.1$e is introduced for protons and neutrons. The experimental $B(E2)$ values [72] are reproduced well. This small effective charge seems reasonable considering that core excitations are important in the $^{44}$Ti region [12, 73] and that the observed $B(E2) = 2^+ (1.157\text{MeV}) \rightarrow g.s.)$ of $^{44}$Ca, 10.9 W.u., is large. The rms charge radius $<r^2>^{1/2} = 3.71$ fm of the ground state calculated using the experimental values $<r^2>^{1/2} = 1.676$ fm and $<r^2>^{1/2} = 3.518$ fm is to be compared with the experimental value 3.59 fm [73]. The calculated intercluster distance of the ground state is about 85% of the sum of the experimental rms charge radii of the two clusters, which is only slightly small compared to 87% for the ground state of $^{44}$Ti [6]. For $J = 0^+$, the overlaps of the six deeply bound forbidden states supported by the potential with the harmonic oscillator wave functions with $\hbar \omega = 10.41$ MeV, which is nearly equal to $\hbar \omega = 10.5$ MeV used in the {	extit{ab initio}} cal-
means that \( \alpha \) clustering, i.e., sizable four-particle excitations from the \( fp \)-shell to the higher major shells, contributes not only to the large \( B(E2) \) values but also to the \( 0\beta \beta \) decay half-life of \( \text{^{48}Ca} \), performing longer than the shell model within the \( N_{HO} = 12 \) model space.

### IV. Discussion and Summary

In Fig. 6 the \( \text{D181} \) potential is compared with a Luneburg lens potential [40] potential, which is a truncated harmonic oscillator potential [41, 42] given by \( V(r) = V_0 \left( r^2/R_0^2 - 1 \right) \) for \( r \leq R_0 \) and \( V(r) = 0 \) for \( r > R_0 \). The potential resembles the Luneburg lens potential in the internal region, which explains why the potential embeds the deeply bound Pauli-forbidden states with \( N < 12 \) and locates the Pauli-allowed \( N = 12 \) band in correspondence to the experimental ground band. The energy dependence of \( J_r = 388 \text{ MeVfm}^3 \) at \( 18 \text{ MeV} \) and 356 MeVfm\(^3\) at 24.1 MeV are consistently in line with \( J_r = 340 \text{ MeVfm}^3 \) at \( E_0 = 29 \text{ MeV} \) in Ref. [47]. The \( \text{D181} \) potential in Fig. 6 is reasonable being in-between the potentials for \( \alpha + ^{40}\text{Ca} \) and \( \alpha + ^{48}\text{Ca} \). The equivalent local potential of the microscopic GCM cluster model calculation [40, 41] belongs to a shallower potential family, which explains why the \( N = 12 \) band appears above the \( \alpha \) threshold energy and there appears no state corresponding to the ground state of \( ^{48}\text{Ti} \).

To summarize, the \( \alpha + ^{44}\text{Ca} \) fusion excitation function at \( E_0 = 9-18 \text{ MeV} \) was reproduced well by a global Luneburg lens-like potential, which fits \( \alpha + ^{44}\text{Ca} \) scattering at \( E_0 = 18-100 \text{ MeV} \). The existence of a \( 7^- \) state with the \( \alpha + ^{44}\text{Ca} \) cluster structure of the higher nodal \( N = 15 \) band in \( ^{48}\text{Ti} \) was confirmed for the first time at \( E_x = 18.8 \text{ MeV} \) in the bump of the fusion excitation function. The first Pauli-allowed \( N = 12 \) band with the \( \alpha + ^{44}\text{Ca} \) cluster structure falls below the \( \alpha \) threshold in correspondence well with the experimental ground band of \( ^{48}\text{Ti} \). The experimental \( B(E2) \) values of the ground band were reproduced well by calculations and the enhancement is found to be due to \( \alpha \) clustering.

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