Semi-independent resampling for particle filtering

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Abstract—Among Sequential Monte Carlo (SMC) methods, Sampling Importance Resampling (SIR) algorithms are based on Importance Sampling (IS) and on some (resampling-based) rejuvenation algorithm which aims at fighting against weight degeneracy. However this mechanism tends to be insufficient when applied to informative or high-dimensional models. In this paper we revisit the rejuvenation mechanism and propose a class of parameterized SIR-based solutions which enable to adjust the tradeoff between computational cost and statistical performances.

I. INTRODUCTION AND BACKGROUND

Bayesian filtering consists in estimating some variable \( x_t \) from noisy measurements \( y_{0:t} = \{ y_0, \ldots, y_t \} \). We assume that \( \{(x_t, y_t)\}_{t=0}^T \) is a Hidden Markov Chain, i.e. that the joint density of \( (x_0, y_{0:t}) \) reads

\[
p(x_0,y_{0:t}) = p(x_0) \prod_{t=1}^T f_s(x_t|x_{t-1}) \prod_{t=0}^T g(y_t|x_t).
\]

The problem can be traced back to Kalman [1] in the context of linear and Gaussian state space models. Approximate solutions for non linear and/or non Gaussian state space models can be performed whatever the discrepancy between the target and importance densities; densities are assigned to the resampled particles the same weight \( \delta_{x_t} \) of the posterior pdf \( p(x_t|y_{0:t}) \).

A. The classical SIR algorithm

Let \( \Theta_t = \int \varphi(x_t)p(x_t|y_{0:t})dx_t \) be a moment of interest of \( p(x_t|y_{0:t}) \). One iteration of an SMC algorithm can be decomposed in three steps.

Starting at time \( t \) and from \( \{ w_{i-1}^t, x_{i-1}^t \}_{i=1}^N \), the first two steps consist in sampling (S). \( N \) particles \( x_i^t \) from importance densities \( q_{i} \) and weighting (W) them so as to take into account the discrepancy between the target and importance densities; then \( \Theta_t \) is estimated as \( \Theta_t^{SIS,N} = \sum_{i=1}^N \tilde{w}_i^t \varphi(\tilde{x}_i^t) \) (superscript SIS will be justified below). Finally a (optional) step consists in re-sampling (R) the weighted particles, i.e. in re-drawing each particle with a probability equal to its weight and assigning to the resampled particles the same weight \( \frac{1}{N} \). This yields the class of SIR algorithms [11] [12] [10] [17] described by Algorithm 1.

Let us comment this algorithm. If resampling is totally absent, each time iteration reduces to the first two steps, i.e. is based on IS only. However such a sequential IS (SIS) algorithm is well known to fail in practice after a few iterations most weights get close to zero. The third step (which can be performed whatever \( t \) or depending on some criterion such as the number of effective particles [12] [13] [14] [15]) which in turn affects the variance of \( \Theta_t^{SIS,N} \) at subsequent iterations. It has thus been proposed to control this extra variance term via alternative resampling schemes (see e.g. [17] [18] [19] \cdots). Yet despite many proposed refinements this generic SIR mechanism remains inefficient in informative models featuring very sharp likelihood functions (i.e., when \( g_{i}(y_t|x_t) \) is very small for most values of \( x_t \), and in particular in high-dimensional space-state models [20] [21].

B. The independent SIR algorithm

Recently it has thus been proposed to revisit the SIR algorithm [22] [23] [24] and more precisely to come back to the rejuvenation mechanism (R). The counterpart of this (R.) step is that it duplicates particles with high weights, which results in support degeneracy. Moreover given \( \{ w_{i-1}^t, x_{i-1}^t \}_{i=1}^N \) the samples \( \{ x_i^t \} \) produced by Algorithm 1 are marginally distributed from some compound pdf \( q_{i}^{N} \) which takes into account the effects of the three elementary (S.), (W) and (R.) steps, but are obviously dependent [22] (a single particle can be resampled more than once); by contrast, given \( \{ w_{i-1}^t, x_{i-1}^t \}_{i=1}^N \) the independent SIR Algorithm 2 [22] [24] produces \( N \) i.i.d. draws from \( q_{i}^{N} \). Note that Algorithm 2 below only describes the rejuvenation step of the independent SIR algorithm, and replaces the “if R. then” part of Algorithm 1.

C. Scope of the paper

Algorithm 2 has displayed good results in severe situations [22] and can be combined with a post-resampling, second-stage reweighting scheme due to its auxiliary particle filtering.

Algorithm 1 The classical SIR algorithm

\[ \text{Data: } q_{i}(x_t|x_{t-1}), y_t, \{ w_{i-1}^t, x_{i-1}^t \}_{i=1}^N \]

\[ \text{for } 1 \leq i \leq N \text{ do} \]

\[ \text{S. } x_i^t \sim q_{i}(x_t|x_{t-1}^t); \]

\[ \text{W. } \tilde{w}_i^t \propto w_i^{t-1}, \frac{f_s(x_t|x_{t-1}^t)}{q_{i}(x_t|x_{t-1}^t)}, \quad \sum_{i=1}^N \tilde{w}_i^t = 1; \]

\[ \text{end for} \]

\[ \Theta_t^{SIS,N} = \sum_{i=1}^N \tilde{w}_i^t \varphi(\tilde{x}_i^t); \]

\[ \text{if } R. \text{ then} \]

\[ \text{for } 1 \leq i \leq N \text{ do} \]

\[ l_i \sim \Pr(L = l) = \tilde{w}_i^t, \quad 1 \leq l \leq N; \]

\[ \text{end for} \]

\[ \text{Set } \{ \tilde{w}_i^t, x_i^t \}_{i=1}^N = \{ \tilde{w}_i^l, x_i^l \}_{i=1}^N; \]

\[ \text{else} \]

\[ \text{Set } \{ \tilde{w}_i^t, x_i^t \}_{i=1}^N = \{ \tilde{w}_i^t, x_i^t \}_{i=1}^N; \]

\[ \text{end if} \]
Algorithm 2 Indep. SIR algorithm (resampling step only)

Data: $q(x_t|x_{t-1}), y_t, \{w_{t-1,i}, x_{t-1}\}_{i=1}^N$

for $1 \leq j \leq N$
\[ x_t^{1:j} \leftarrow x_t^j, \quad \tilde{w}_t^j \leftarrow \tilde{w}_t^j. \]
end for

for $1 \leq i \leq N$
\begin{align*}
& \mathbf{R.} \quad Y_t \sim \text{Pr}(L=l) = \tilde{w}_t^{l,i}, \quad 1 \leq l \leq L; \\
& \text{Rejuvenation of the support for iteration } i + 1 \\
& \quad \text{if } (i < N) \text{ then} \\
& \quad \quad \text{for } 1 \leq j \leq N \\
& \quad \quad \quad \tilde{x}_t^{i+1:j} \sim q(\tilde{x}_t|x_t^i); \\
& \quad \quad \quad \tilde{w}_t^{i+1:j} = w_t^{-1} f(x_t^{i+1:j}|x_t^i) g(y_t|x_t^{i+1:j}) q(\tilde{x}_t^{i+1:j}|x_t^i), \\
& \quad \quad \end{align*}
end for

\[ \tilde{w}_t^{i+1} \propto \tilde{w}_t^{i+1:1}; \quad \sum_{j=1}^N \tilde{w}_t^{i+1,j} = 1; \]
end if

end for

Set $\{w_{t,i}^j, x_{t,i}^j\}_{i=1}^N = \{\frac{1}{N}, x_{t,i}^{1:j}\}_{i=1}^N$.

Algorithm 3 Semi-ind. SIR algorithm (resampling step only)

Data: $q(x_t|x_{t-1}), y_t, \{w_{t-1,i}, x_{t-1}\}_{i=1}^N$

for $1 \leq j \leq N$
\[ x_t^{1:j} \leftarrow x_t^j, \quad \tilde{w}_t^{1:j} \leftarrow \tilde{w}_t^j. \]
end for

for $1 \leq i \leq N$
\begin{align*}
& \mathbf{R.} \quad Y_t \sim \text{Pr}(L=l) = \tilde{w}_t^{l,i}, \quad 1 \leq l \leq L; \\
& \text{Partial rejuvenation of the support for iteration } i + 1 \\
& \quad \text{if } (i < N) \text{ then} \\
& \quad \quad \text{for } 1 \leq j \leq k \text{ do} \\
& \quad \quad \quad m^j \sim \text{Pr}(m = n | n \in 1:N \setminus \{m^{j-1}\}); \\
& \quad \quad \quad \tilde{x}_t^{i+1:m^j} \sim q(\tilde{x}_t|x_t^i); \\
& \quad \quad \quad \tilde{w}_t^{i+1:m^j} = w_t^{-1} f(x_t^{i+1:m^j}|x_t^{i+1:m^j}) g(y_t|x_t^{i+1:m^j}) q(\tilde{x}_t^{i+1:m^j}|x_t^{i+1:m^j}), \\
& \quad \quad \end{align*}
end for
\[ \tilde{w}_t^{i+1} \propto \tilde{w}_t^{i+1:1}; \quad \sum_{j=1}^N \tilde{w}_t^{i+1,j} = 1; \]
end if

end for

Set $\{w_{t,i}^j, x_{t,i}^j\}_{i=1}^N = \{\frac{1}{N}, x_{t,i}^{1:j}\}_{i=1}^N$.

B. Performances vs. computational cost

We now evaluate the performance of this procedure by comparing the variances of the estimates computed after the resampling step because they affect the variances of the estimates at subsequent iterations [13]. So let $\tilde{\Theta}_i^N = \frac{1}{N} \sum_{i=1}^N \varphi(x_{t,i}^j)$, where the generic notation $x_{t,i}^j$ represents the points produced either by Algorithm [1][2] or [3] so we consider $\tilde{\Theta}_i^N, \tilde{\Theta}_i^\text{SIR}, N$, and $\tilde{\Theta}_i^\text{SIR, k}$, where SR stands for semi-resampling. We have the following proposition (the proof is given in the Appendix).
Proposition 1: Given the previous set of particles \( \{x_{0:t-1}\}_{t=1}^{N} \), for all \( 0 \leq k \leq N \), we have:

\[
\begin{align*}
E(\Theta_{t}^{\text{SR},N,k}) &= E(\Theta_{t}^{\text{SR},N}) = E(\Theta_{t}^{\text{SR}}), \\
\text{var}(\Theta_{t}^{\text{SR},N,k}) &\leq \text{var}(\Theta_{t}^{\text{SR},N}) \leq \text{var}(\Theta_{t}^{\text{SR}}), \\
\text{var}(\Theta_{t}^{\text{SR},N,k}) &\leq \text{var}(\Theta_{t}^{\text{SR},N,k-1}).
\end{align*}
\]

So as the number \( k \) of intermediate redrawings increases from 0 to \( N \), the conditional variance of the semi-independent resampling estimator \( \Theta_{t}^{\text{SR},N,k} \) decreases from the upper bound of inequality (1b) (if \( k = 0 \), \( \Theta_{t}^{\text{SR},N,0} \) reduces to \( \Theta_{t}^{\text{SR},N} \)) to its lower bound (if \( k = N \), \( \Theta_{t}^{\text{SR},N,N} \) reduces to \( \Theta_{t}^{\text{SR},N} \)). However remember from section II.B that \( N + (N - 1) \times k \) samples are needed for building \( \Theta_{t}^{\text{SR},N,k} \), so parameter \( k \) of the SR scheme enables to fix a compromise between variance reduction and computational budget.

C. A parallelized version

Finally Algorithm 3 can be transformed into a parallelized version, the non-sequential SR (NSSR) algorithm. At iteration \( i \), instead of duplicating the \( N - k \) surviving particles from the previous support \( \tilde{x}_{i}^{k-1} \) (see Fig. 1), we propose to duplicate the \( N - k \) surviving particles directly from the initial set \( \tilde{x}_{i}^{0} \) of particles. The \( N - 1 \) new supports can thus be produced in parallel, contrary to Algorithm 3 which by nature is sequential. Of course, this procedure alters the diversity of the final set of particles, as is illustrated by the following proposition.

Proposition 2: Let \( \Theta_{t}^{\text{NSSR},R,k} \) be the estimate built from the non-sequential semi-independent resampling procedure. Then given the previous set of particles \( \{x_{0:t-1}\}_{t=1}^{N} \), for all \( 0 \leq k \leq N \), we have:

\[
\begin{align*}
E(\Theta_{t}^{\text{NSSR},R,N,k}) &= E(\Theta_{t}^{\text{SR},N}) = E(\Theta_{t}^{\text{SR}}), \\
\text{var}(\Theta_{t}^{\text{NSSR},R,N,k}) &\leq \text{var}(\Theta_{t}^{\text{NSSR},N,k}) \leq \text{var}(\Theta_{t}^{\text{SR}}), \\
\text{var}(\Theta_{t}^{\text{NSSR},N,k}) &\leq \text{var}(\Theta_{t}^{\text{NSSR},N,k-1}), \\
\text{var}(\Theta_{t}^{\text{NSSR},N,k}) &\leq \text{var}(\Theta_{t}^{\text{NSSR},N,k-1}).
\end{align*}
\]

So we see that \( \text{var}(\Theta_{t}^{\text{NSSR},N,k}) \) still decreases with \( k \), but is always larger than \( \text{var}(\Theta_{t}^{\text{SR},N,k}) \). As with Proposition 1 the variance inequalities still rely on Jensen’s inequality, and the proof is omitted.

III. Simulations

We consider a tracking problem based on range-bearing measurements. The hidden state-vector contains the position and velocity of the target in cartesian coordinates, \( x_{t} = [\tilde{c}_{x,t}, \tilde{c}_{y,t}, \tilde{c}_{v_{x},t}, \tilde{c}_{v_{y},t}]^T \). We set \( f_{t}(x_{t}|x_{t-1}) = N(x_{t}, F_{x_{t-1}}; Q) \),

\[
g_{t}(y_{t}|x_{t}) = N(y_{t}; [c_{x,t}^2 + c_{y,t}^2 \arctan \frac{\tilde{c}_{v_{y},t}}{\tilde{c}_{v_{x},t}}] ; R),
\]

where \( R = \text{diag}(\sigma_{r}^2, \sigma_{\theta}^2) \), \( F = I_{2} \otimes \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \) \( Q = 10 \times I_{2} \otimes \begin{bmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{4} \end{bmatrix} \)

where \( \otimes \) is the Kronecker product. We set \( q(x_{t}|x_{t-1}) = f_{t}(x_{t}|x_{t-1}) \) and we compare the RMSEs averaged over 1000 MC runs.

A. Variance of SR procedures

We first analyze the behaviour of our algorithms as a function of \( k \). We set \( N = 100, \sigma_{r} = 0.1 \) and \( \sigma_{\theta} = \frac{\pi}{400\pi} \), all MC runs use the same measurements. Fig. 2 displays the RMSE of \( \Theta_{t}^{\text{SR},N,k} \), \( \Theta_{t}^{\text{NSSR},N,k} \), \( \Theta_{t}^{\text{SIR},N} \) deduced from our resampling schemes and \( \Theta_{t}^{\text{SIS},N} \) (a resampling step is computed at each time step but the estimate is taken before this step). Of course, the performances of estimates based on the SR procedure improve when \( k \) increases. Even for small values of \( k \), the improvement is significant. It is also interesting to note that \( \Theta_{t}^{\text{NSSR},N,k} \) (resp. \( \Theta_{t}^{\text{NSSR},N} \)) has the same performance as \( \Theta_{t}^{\text{SIS},N} \) when \( k \geq N/2 \) (resp. \( k \geq 4N/5 \)).

B. RMSE in the informative case at equal cost

We now compare our estimates with existing improvements of the PF in informative models. In particular, the PF with MCMC resample move is a popular solution to introduce sample variety after resampling [25]. Roughly speaking, the \( N \) particles which follow the \( (R) \) step of Algorithm 1 are moved via an MCMC algorithm with \( k \) iterations (here an independent Metropolis-Hasting algorithm). Thus, our SR procedure has the same computational cost in terms of sampling steps as the SIR PF with MCMC moves. We also compare our estimates with those based on the classical SIR and I-SIR algorithms but with a given budget of total sampling (sampling + resampling) operations. We thus set \( N = 100 \) particles and \( k = N/2 \) for the computation of \( \Theta_{t}^{\text{SR},N,k} \) and the estimate based on the resample move PF, \( N = 72 \) for that of \( \Theta_{t}^{\text{SIS},N} \) and \( N + \frac{(N-1)k}{2} = 2575 \) particles for that of \( \Theta_{t}^{\text{SIS},N} \). The global sampling cost for all these algorithms is approximately \( (2N + Nk) \). We also compute \( \Theta_{t}^{\text{SIS},N,k} \) with \( N = 100 \) and \( k = 4N/5 \); its computation does not have the same computational cost but can be parallelized. The results are displayed in Fig. 3.

When the observations are very informative (\( \sigma_{r} \) and \( \sigma_{\theta} \) are small), the classical solution tends to degenerate (it starts working when \( (\sigma_{r}, \sigma_{\theta}) = (0.15, 0.15) \)), while our solutions are robust and present better performances. As the variance of the measurement noise increases, the different estimates tend to behave similarly; the classical SIR algorithm performs
slightly better, which is not surprising since in this case it no longer suffers from the degeneracy phenomenon and the number of final samples used is far superior to the other solutions. We also observe that the resample move which uses differently the k extra samples does not perform well when compared to the SR procedure in very informative models, and is outperformed by our solutions when the observations are not informative. Finally, our SR algorithm with multinomial resampling technique ($\Theta^S_N$) depends on $N$, $k$, and $t$ outperforms the (totally) independent resampling one when the budget is fixed.

![Graph](image)

Fig. 3: Tracking model, $\sigma_x \in [0.01, 0.3]$ and $\sigma_y \in [0.15, 0.3]$.  

**IV. Conclusion**

In this paper we revisited the resampling step of PF algorithms, and proposed a resampling scheme where each new final particle is resampled from a support which is partially rejuvenated with k new particles. This yields a class of parameterized solutions which encompasses the classical multinomial resampling technique ($k = 0$) and the independent resampling one ($k = N$), enabling to tune the balance between variance and computational cost. Simulations showed that choosing $k = N/2$ leads to similar performances to the fully independent resampling procedure. Moreover, in very informative models our algorithm is not affected by the degeneration phenomenon, contrary to the classical SIR algorithm.

**APPENDIX**

**Proof of Proposition 1**

Let us consider a PF with resampling at time t. First, [12] holds because the SIR, I-SIR and SR procedures all produce resampled particles which, given $\{x_{t-1}^{i,i-1}\}_{i=1}^N$, are (marginally) sampled from the same distribution $\hat{q}^N$; and [12] is straightforward from [12] and the fact that SR reduces to SIR (resp. I-SIR) when $k = 0$ (resp. $k = N$). Let us address [12]. Since $\hat{q}_{t}^{SIR,N,k} = \frac{1}{N} \sum_{i=1}^{N} \varphi(x_{t}^{i})$, given $\{x_{t-1}^{i,i-1}\}_{i=1}^N$:

$$N^2 \text{var}_{k}^{SIR,N,k} = \sum_{i=1}^{N} \text{var}_{k}(\varphi(x_{t}^{i})) + 2 \sum_{i_1,i_2=1 \atop i_1 < i_2}^{N} \text{cov}_{k}(\varphi(x_{t}^{i_1}),\varphi(x_{t}^{i_2}));$$

here index k in a (co)variance emphasizes the fact that it depends on k. The first term of the r.h.s. is independent of k (and coincides with $\text{var}(\hat{q}_{t}^{SIR,N})$), so the difference between different values of k stems from the covariance terms. Next:

$$\text{cov}_{k}(\varphi(x_{t}^{i_1}),\varphi(x_{t}^{i_2})) = E_{k}[\varphi(x_{t}^{i_1})\varphi(x_{t}^{i_2})] - E[\varphi(x_{t}^{i_1})]E[\varphi(x_{t}^{i_2})],$$

and again, the second term of the r.h.s. is independent of k. Finally for $i_1 < i_2$,

$$E_{k}[\varphi(x_{t}^{i_1})\varphi(x_{t}^{i_2})] = E[E_{k}[\varphi(x_{t}^{i_1})|\hat{x}_{t}^{i_1:2}]\varphi(x_{t}^{i_2})] = E[E[\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})|\Theta_{k}^{SIR}(\hat{x}_{t}^{i_2})] = E[E[\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})|\Theta_{k}^{SIR}(\hat{x}_{t}^{i_2})]|m_{i_1+1}^{i_2}(1:k)]$$

where $m_{i_1+1}^{i_2}(1:k)$ represents all the indices redrawn from iterations $i_1 + 1$ to $i_2$ (the third equality holds because $x_t^{i_1}$ is resampled from support $\hat{x}_{t}^{i_1:2}$; see [12], so $E(\varphi(x_{t}^{i_1})) = \Theta_{k}^{SIR,N}(\hat{x}_{t}^{i_1})$ where $\Theta_{k}^{SIR,N}$ was defined in section 3). The outer expectation in this last expression corresponds to a uniformly weighted sum over all possible values of $m_{i_1+1}^{i_2}(1:k)$, i.e. over $(A_{N})^{i_2-i_1}$ terms where $A_{N}$ is the number of arrangements of k among N. Given $m_{i_1+1}^{i_2}(1:k)$, the general term of this sum reads

$$E[E[\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})|\Theta_{k}^{SIR}(\hat{x}_{t}^{i_2:2})]|m_{i_1+1}^{i_2}(1:k)] = E[E[\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})|\Theta_{k}^{SIR}(\hat{x}_{t}^{i_2:2})]|x_{t}^{i_1:1:N\setminus m_{i_1+1}^{i_2}(1:k)}]|m_{i_1+1}^{i_2}(1:k)]$$

where $x_{t}^{i_1:1:N\setminus m_{i_1+1}^{i_2}(1:k)}$ are the particles shared by supports $\hat{x}_{t}^{i_1:2}$ and $\hat{x}_{t}^{i_2:2}$. Under this conditioning, $\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})$ and $\Theta_{k}^{SIR}(\hat{x}_{t}^{i_2:2})$ are independent so the general term is

$$E[\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})|\Theta_{k}^{SIR}(\hat{x}_{t}^{i_2:2})]|x_{t}^{i_1:1:N\setminus m_{i_1+1}^{i_2}(1:k)}] = E[\Theta_{k}^{SIR}(\hat{x}_{t}^{i_1:2})|x_{t}^{i_1:1:N\setminus m_{i_1+1}^{i_2}(1:k)}]|m_{i_1+1}^{i_2}(1:k)]$$

because given the trajectories from the previous time steps, particles from different supports are all marginally drawn from the same densities. Finally

$$E_{k}(\varphi(x_{t}^{i_1})\varphi(x_{t}^{i_2})) = \frac{1}{(A_{N})^{i_2-i_1}} \sum_{m_{i_1+1}^{i_2}(1:k)} h(m_{i_1+1}^{i_2}(1:k)).$$

It remains to compare (3) with the same expression with $k \rightarrow k - 1$. We observe that (3) can be rewritten as

$$E_{k}(\varphi(x_{t}^{i_1})\varphi(x_{t}^{i_2})) = \frac{1}{(A_{N})^{i_2-i_1+1}} \sum_{m_{i_1+1}^{i_2}(1:k-1)} h(m_{i_1+1}^{i_2}(1:k - 1), m_{i_1+1}^{i_2}(1:k)),$$

where the second line coincides with the conditional expectation $E[\hat{h}(m_{i_1+1}(1:k))|m_{i_1+1}(1:k-1)]$. Given $m_{i_1+1}(1:k - 1)$, the set $x_{t}^{i_1:1:N\setminus m_{i_1+1}(k-1):1}$ is included in $x_{t}^{i_1:1:N\setminus m_{i_1+1}(1:k-1)}$; consequently, the Rao-Blackwell decomposition ($E[E^2(X|Y)] \leq E(E^2(X|Y, Z))$) ensures that

$$h(m_{i_1+1}(1:k - 1), m_{i_1+1}(k)) \leq h(m_{i_1+1}(1:k - 1))$$
for all $m_{i+1}^{(1)}(k)$, and so that $E[h(m_{i+1}^{(1)}(1:k))|m_{i+1}^{(1)}(1:k-1)] \leq h(m_{i+1}^{(1)}(1:k-1))$, whence (13).

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