Determining energy relaxation length scales in two-dimensional electron gases

Jordan Billiauld\textsuperscript{1}, Dirk Backes\textsuperscript{1}, Jürgen König\textsuperscript{2}, Ian Farrer\textsuperscript{1}, David Ritchie\textsuperscript{1}, Vijay Narayan\textsuperscript{1}

\textsuperscript{1}Cavendish Laboratory, University of Cambridge, J. J. Thomson Avenue, Cambridge, CB3 0HE, United Kingdom.
\textsuperscript{2}Theoretische Physik and CENIDE, Universität Duisburg-Essen, 47048 Duisburg, Germany.
(Dated: April 27, 2015)

We present measurements of the energy relaxation length scale $\ell$ in two-dimensional electron gases (2DEGs). We establish a temperature gradient in the 2DEG by means of a heating current and then estimate the elevated electron temperature $T_e$ by monitoring the resultant thermovoltage between a pair of differentially biased bar-gates. We adopt a model by Rojek \textit{et al.} \cite{1} to analyse the thermovoltage signal and as a result extract $\ell$, $T_e$ and the power-law exponent $\alpha$, for inelastic scattering events in the 2DEG. We show that in high-mobility 2DEGs, $\ell$ can attain macroscopic values of several $100 \mu$m, but decreases rapidly as the carrier density $n$ is decreased. Our work demonstrates a versatile low-temperature thermometry scheme, and the results provide crucial insights into heat transport mechanisms in low-dimensional systems and nanostructures. These, in turn, will be vital towards practical design considerations for nanoelectronic circuits \cite{2}.

There currently exist mature and well-established methods to probe the low-temperature (low-$T$) electrical and thermolectric properties of two-dimensional electron gases (2DEGs). However, probing heat transport mechanisms in these systems has proven more challenging, primarily due to the lack of convenient low-$T$ thermometers that couple directly to the electron gas. Conventional low-$T$ thermometers such as germanium or ruthenium-oxide films are sensitive only to the lattice temperature $T_L$, the temperature of the bulk material that hosts the 2DEG. At $T_L \lesssim 1$ K the coupling between electrons and phonons becomes relatively weak, because of which $T_e$ can differ significantly from $T_L$. Furthermore, the 2DEG resistance itself becomes insensitive to $T_e$ at these temperatures since the majority of scattering events are from static impurities. Therefore these both become ineffective at measuring $T_e$ in this regime. Accurate measures of $T_e$ can be obtained from the Coulomb-blockade characteristics of quantum dots \cite{3} which are broadened at a finite $T_e$. The weak localization characteristics of 2DEGs \cite{4} can also be useful as they depend sensitively on the phase-coherence length which is $T_e$-dependent. However, neither of these methods lend themselves easily to the measurement of spatial temperature gradients, which is required in order to measure the thermal conductivity $\kappa$ \cite{5,6} or the energy relaxation length scale $\ell$ in 2DEGs.

Appleyard \textit{et al.} \cite{7} demonstrated that instead of the electrical resistance, the diffusive component of the thermopower $S$ can serve as a tool to measure temperatures $T_e$ of 2DEGs that were elevated as compared to the lattice temperature $T_L$. Here $S \equiv V_{th}/\Delta T$, where $V_{th}$ is the thermovoltage developed in response to a temperature difference $\Delta T$. For this, they detected, similarly as in Ref. \cite{8} and, more recently, Ref. \cite{9}, the thermovoltage generated across a pair of quantum point contacts (QPCs) with a heated electron gas between them. Then $\Delta T$ was estimated as $V_{th}/S$, where $S$ was obtained using the Mott relation \cite{10}:

\begin{equation}
S = \frac{\pi^2 k_B^2 T}{3e\hbar^2} \left( \frac{d \ln \sigma}{dE} \right)_{E=\mu}.
\end{equation}

Here $k_B$ is the Boltzmann constant, $e$ is the electronic charge, $\hbar \equiv \hbar/2\pi$ with $\hbar$ being Planck’s constant, $\sigma$ is

![Device geometry](image)

**FIG. 1.** Device geometry. (a) False-colour SEM micrograph of the device used in our experiments. The blue region shows ungated conducting mesa, whereas the yellow region indicates a mesa covered by a top-gate. A single bar-gate thermometer (BGT) is highlighted by white dashed lines. (b) Schematic representation of the BGT which consists of gated (II and III) and ungated (I and IV) regions of 2DEG, terminated by an ohmic contact. The red section between the gates illustrates a region of ‘hot’ electrons.
the electrical conductivity, \(E\) is the total energy and \(\mu\) is the chemical potential. This technique has been used to measure \(\kappa\) in quantum wires \([5,11]\) and energy-loss rates in 2DEGs \([12]\). Chickering et al. \([13]\) showed Eq. 1 to be broadly valid in gated regions of a 2DEG between 0.8 K and 2 K and therein suggested the possibility of using a symmetric pair of bar-gates as a thermometer for the local \(T_c\). This method was recently employed to measure \(S\) in mesoscopic 2DEGs \([14,15]\). Usefully, the relatively large size of the bar-gates eliminates the need for electron beam lithography which simplifies the fabrication process. However, it was noted in Ref. \([13]\) that the data systematically deviated from the Mott prediction. Rojek et al. \([11]\) attributed these deviations to the spatial extent of the BGTs being comparable to the energy relaxation length \(\ell\) in the 2DEG and developed a model to account for this. In this manuscript, we adapt the model developed by Rojek et al. to refine the analysis of the signal produced by a BGT, and to make an accurate measurement of \(\ell\). To the best of our knowledge this is the first experimental determination of \(\ell\) in such a system.

The 2DEGs used in this study had a mobility of 172 m²/\(\text{Vs}\) at a carrier density of \(n_0 = 1.45 \times 10^{15} \text{m}^{-2}\). An optical image of the device is shown in Fig. 1a. A wet etch is used to define a conductive mesa, and Au-Ge-Ni ohmic contacts and Ti-Au top-gates are then deposited by thermal evaporation. The device consists of a heating element and a longitudinal strip (of width 100 μm and length \(L_{\text{strip}} = 1\) mm) which together form a ‘T’ shape, and three BGTs spaced along the strip. Figure 1b shows the heating channel, a section of the 2DEG strip and the first BGT which is at a distance \(L_{T1} = 200\) μm from the heating element. A top-gate which sits over the heating element and the strip is used to tune the electron density \(n\) in these regions. This design minimized any power reflection at the interface between the heating channel and 2DEG strip. The strip terminates in a large ohmic contact.

Measurements on the devices were performed in a dilution refrigerator with a base temperature of 170 mK. \(T_L\) was measured using a ruthenium oxide thermometer attached to the mixing chamber. Throughout the experiment all ohmic contacts through which no current is passed were assumed to be at \(T_L\) due to being in direct thermal contact to the mixing chamber via the measurement wiring. Figure 2 shows a single BGT. It is a symmetric structure consisting of two arms flanking the hot 2DEG strip. Each arm is formed by a gated region labelled II (III), followed by an ungated region labelled I (IV), and terminated by an ohmic contact. Note, the ohmic contacts are not shown in the figure. The lengths of the gated and ungated regions of the thermometer arms are \(L_g = 150\) μm and \(L_u = 455\) μm respectively.

The experiment involves passing a heating current \(I_h\) at frequency \(f < 10\) Hz through the heating element. This current Joule heats the electron gas with a power of \(I_h^2R_h\), where \(R_h\) is the resistance of the heating element, establishing a temperature gradient along the 2DEG strip. The thermovoltage \(V_{th}\) generated across a thermometer arm in response to \(I_h\) is detected at 2 Hz using a lock-in amplifier. Figure 2 shows the best fit to the experimental data by assuming that hot electrons entirely relax in \(T_L\), within the gated region, i.e., over a distance \(L_g\). In this scheme, the only unknown is \(T_c\) between the BGT arms, but the fit is seen to systematically deviate from the experimental data.

![Figure 2](image)

**FIG. 2.** Fits to the experimental data are not satisfactory when the contribution to \(V_{th}\) from the ungated 2DEG regions are not taken into account. See text for details.
FIG. 3. Fits to the data using Eqs. 2, 3 and 4. The graphs show $V_{th}$ as a function of the swept gate voltage $V_{tg}$. The top axis shows $n_p$, the corresponding carrier density under the swept gate. The different traces in (a) and (b) represent different heating currents $I_h$ and different top-gate voltages $V_{tg}$ respectively.

FIG. 4. (a) The elevated electron temperature $\Delta T_e \equiv T_e - T_L$ extracted from the model is seen to increase in a power-law fashion as a function of $I_h$. (b) As $V_{tg}$ becomes more negative, $T_e$ at the first thermometer decreases rapidly, indicating that the electrons thermalize over a much shorter distance.

We first describe why it is essential to consider the contribution from all the regions $p = I$ to $IV$ towards $V_{th}$. At low-$T$ and especially in high-mobility 2DEGs, the energy relaxation length $\ell$ over which hot electrons relax through inelastic processes can significantly exceed the mean-free path of electrons [1, 18]. The dependence of $\ell$ on the inelastic scattering time $\tau_i$ is given by $\ell = \sqrt{D\tau_i}$, where $D = v_F^2\tau_e/2$ is the diffusion constant of the electrons, and $v_F$ is the Fermi velocity. $\tau_i$ depends on $n$ in a power-law fashion: $\tau_i = \tau_{i,0}(n/n_0)^{\alpha_i}$, where the subscript 0 denotes the values when the 2DEG is ungated. Therefore, the distance over which electrons lose their excess energy in the BGT arm is crucially dependent on $n_p$. Furthermore, if $\ell > L_g + L_{ug}$, the boundary condition imposed by the ohmic contact enforces $T_e = T_L$ at the electron gas to ohmic boundary. Thus in all the above situations both the gated and ungated regions contribute to $V_{th}$ in a manner dependent on $T_e$ in central hot 2DEG, $T_e$ at the interface of the gated and ungated 2DEGs, $\ell$ and $\alpha_i$, all of which need to be estimated self-consistently. We address this problem by adapting the model used in Ref. [1] (described in the Supplementary Material) and the resulting expression for $\Delta T_e \equiv T_e - T_L$ at the junction of the gated and ungated 2DEG regions $\Delta T_e(L_g)$, reads:

$$\Delta T_e(L_g) = \Delta T_e \frac{z \sinh(L_{ug}/\ell_0)}{\cosh(L_{ug}/\ell_0) \sinh(L_g/\ell) + z \sinh(L_{ug}/\ell_0) \cosh(L_g/\ell)}.$$ (4)
Here $\ell_0$ is the energy relaxation length at $n = n_0$, and $z \equiv (n/n_0)^{(1+\alpha_e-\alpha_i)/2}$. Within the framework of Rojek et al.’s model\(^1\) $T_\theta = T_L$ such that $V_{th}$ is linear in $\Delta T_p$. Equation 4 provides an expression for $\Delta T_p$, which when substituted together with Eq. 3 in Eq. 2 results in an expression for $V_{th}$ as a function of $n_0$, $n$ in each gated region, $R_h$, $\alpha_e$, $\alpha_i$, and $T_c$. We measure $n_0$ and $n(V_{tg})$ in the device by observing $V_{tg}$-dependent edge-state reflections in the quantum Hall regime \cite{17}, and $\alpha_e$ is extracted from the dependence of conductivity $\sigma$ on $n$ and turns out to be $\approx 0.89$ over the relevant range of $n$. This leaves three unknowns, namely $T_c$, $\alpha_i$ and $\ell_0$ which are used as fitting parameters. Importantly though, by fitting to several complementary data sets whilst varying different tuning parameters, namely the dependence of conductivity $\sigma$ on $n$ and the top-gate voltage $V_{tg}$, we are able to considerably reduce the uncertainty in these three fitting parameters.

Figure 4a shows $\Delta T_e$ as a function of $I_h$ on a log-log scale and we find that $\Delta T_e \propto I_h^\beta$ with $\beta \approx 1.65$. This sub-squared dependence is presumably due primarily to a fraction of the power being dissipated via phonon emission. The value of $\alpha_i$ and $\ell_0$ are found to be $\approx 3.76$ and $\approx 280\,\mu m$, respectively. Figure 4a shows $\Delta T_e$ as a function of $V_{tg}$ for $\Delta T_e$ has been scaled for the changing voltage of $R_h$, i.e., a corrective factor of $R_h,0/R_h(V_{tg})$ has been applied so that any change in $\Delta T_e$ is now solely due to a change in $\ell$. Here $R_h(V_{tg})$ is the resistance of the heating channel as a function of $V_{tg}$. The graph suggests, therefore, that for $V_{tg} < -0.11\, V$, i.e., $n < n^* = 0.6 \times 10^{15}\, m^{-2}$, the hot electrons completely relax within a distance $L_{TT1}$, or in other words, $\ell(n^*) < L_{TT1}$.

Thus our data suggests that hot electrons thermalize over macroscopic length scales ($\approx 300\, \mu m$) in the ungated 2DEG, but that this length scale rapidly decreases with $n$. This strongly justifies the need to account for the ungated 2DEG arms when using BGTs at high $n_g$. While $n_g$ can certainly be lowered till $\ell < L_{e}$ such that the entire $\Delta T_e$ drop is across the gated region, care must be taken to ensure that Eq. 3 remains valid. Indeed, we have observed that if $n_g$ is lowered significantly below $n = 0.7 \times 10^{15}\, m^{-2}$, the model is unable to fit the data satisfactorily. This is likely due to localization and/or strong inter-electron interaction effects that become significant as $n$ is lowered \cite{14}.

Importantly, the model only provides the value of $\ell_0$ and $\alpha_i$ from which $\ell(n)$ can be reconstructed. However, as argued in the previous two paragraphs, this information is independently contained in Fig. 4b. The $T_c$-profile in the 2DEG strip is given by (see Supplementary Material):

$$
\Delta T_e(y) = \Delta T_{e,0} \sinh \left[ (L_{\text{strip}} - y)/\ell \right] / \sinh (L_{\text{strip}}/\ell)
$$

Here $y$ is the spatial coordinate along the 2DEG strip (see Fig. 4b) and $\Delta T_{e,0} \equiv \Delta T_e(y = 0)$, the temperature elevation in the heating channel. Thus, since the heating channel is effectively at a constant temperature between pairs of points in Fig. 4b, the ratio $\Delta T_e(n_1)/\Delta T_e(n_2)$, taken at $y = L_{TT1}$, provides an implicit relation between $\ell(n_1)$ and $\ell(n_2)$, where $n_1$ and $n_2$ are the respective carrier densities. This therefore allows us to infer $\ell$ from $T_c$, and reconstruct the dependence of $\ell$ on $n$ as shown in Fig. 5. This is the principal result of this study showing the dependence of energy relaxation length scale on carrier density. We stress that this is a direct measurement of $\ell(n)$ using the pre-calibrated BGT, which we find to be in striking agreement with indirectly obtained dependence $\ell(n) = \ell_0(n/n_0^{1+(\alpha_e-\alpha_i)/2})$, using the model-based values of $\ell_0$ and $\alpha_i$.

To summarize we have demonstrated that BGTs are a versatile tool with which to detect differences between $T_c$ and $T_L$. As remarked earlier, BGTs are a viable alternative to QPCs which require electron beam lithography and, moreover, come with the associated difficulties of sub-micrometre devices such as electrical sensitivity and susceptibility to disorder. The BGTs, in contrast, can be macroscopic and therefore robust to electrical noise and spikes, and relatively insensitive to disorder.

While our manuscript seems to suggest that the trade-off between QPCs and BGTs is that in the latter, detailed modelling is required to extract $T_c$, it is important to note that modifying the device design such that $L_{tg} > \ell$ significantly reduces the analysis complexity. On the other hand, the advantages of the employed device design
Determining energy relaxation length scales in two-dimensional electron gases
Supplementary Material

I. THERMOPOWER DIFFUSION MODEL

Rojek et al. consider a fixed electrical power input into a two-dimensional electron gas (2DEG) that results in an elevated electron temperature $T_e \equiv \Delta T_e + T_L$ with respect to the lattice temperature $T_L$. In the steady-state, the power input to the 2DEG is balanced by the heat loss to the lattice. Then, one can derive from the continuity equation for heat and charge flow a diffusion equation for $\Delta T_e(x)$,

$$\Delta T_e - \frac{\ell^2}{\kappa} \frac{\partial}{\partial x} \left( \kappa \frac{\partial \Delta T_e}{\partial x} \right) = 0. \quad (S1)$$

Here $\kappa$ is the thermal conductivity of the 2DEG, $\ell$ the energy relaxation length, and $x$ is the spatial coordinate along which the temperature varies (for the temperature variation within the BGT, relevant for Eq. 4, this is the $x$-direction while for the temperature variation within the 2DEG strip, relevant for Eq. 5, this is the $y$-direction, see Fig. 1 of the main text).

An applied gate voltage changes the electron density $n$ which, in turn, modifies the transport properties, in particular the elastic and inelastic scattering times $\tau_e$ and $\tau_i$, the thermal conductivity $\kappa$, and the energy relaxation length $\ell$. Their $n$-dependences are described by power laws with the following exponents:

$$\frac{d \ln \tau_e}{d \ln n} = \alpha_e,$$
$$\frac{d \ln \tau_i}{d \ln n} = \alpha_i,$$
$$\frac{d \ln \kappa}{d \ln n} = 1 + \alpha_e,$$
$$\frac{d \ln \ell}{d \ln n} = (1 + \alpha_e + \alpha_i)/2.$$

For each region with constant electron density (the gated region II(III) and the ungated region I(IV) of the BGT, relevant for Eq. 4, as well as the 2DEG strip, relevant for Eq. 5), the general solution of Eq. (S1) is

$$\Delta T_e(x) = ae^{-x/\ell} + be^{x/\ell}, \quad (S2)$$

where the coefficients $a$ and $b$ have to be determined by boundary conditions.

A. Derivation of Eq. 4

For the BGT, the spatial position $x$ is defined relative to the interface between the 2DEG strip and the gated region II. There are two regions of constant electron density (the gated and the ungated one, I and II) which requires four boundary conditions to fully determine the temperature profile. One is given by $\Delta T_e(0)$ at $x = 0$, the quantity to be measured by the BGT. Furthermore, the Ohmic contact at $x = L_g + L_{ug}$ fixes the electron temperature to the lattice temperature, $\Delta T_e(L_g + L_{ug}) = 0$. Finally, at $x = L_g$ (the interface between the gated (I)

We acknowledge funding from the Leverhulme Trust, UK and the Engineering and Physical Sciences Research Council (EPSRC), UK. JB and VN acknowledge helpful discussions with Chris Ford and Charles Smith.

[1] S. Rojek and J. König, Phys. Rev. B 90, 115403 (2014).
[2] L. Shi Thermal and Thermoelectric Transport in Nanostructures and Low-Dimensional Systems, Nanoscale and Microscale Thermophysical Engineering 16, 79 (2012).
[3] J. Davies, The Physics of Low-Dimensional Semiconductors, Cambridge University Press (1998).
[4] A. Mittal, R.G. Wheeler, M.W. Keller, D.E. Prober, and R.N. Sacks, Surf. Sci. 361, 537 (1996).
[5] L. W. Molenkamp et al., Phys. Rev. Lett. 68, 3765 (1992).
[6] R. T. Syne, M. J. Kelly, and M Pepper, J. Phys.: Condens. Matter 1, 3375 (1989).
[7] N. J. Appleyard, J. T. Nicholls, M. Y. Simmons, W. R. Tribe, and M. Pepper, Phys. Rev. Lett. 81, 3491 (1998).
[8] L. W. Molenkamp et al., Phys. Rev. Lett. 65, 1052 (1990).
[9] M. Wiemann et al., Appl. Phys. Lett. 97, 062112 (2010).
[10] N. F. Mott and M. Jones, The Theory and the Properties of Metals and Alloys (Courier Dover, New York, 1958).
[11] O. Chiai et al., Phys. Rev. Lett. 97, 056601 (2006).
[12] Y. Proskuryakov, J. Nicholls, D. Hadjii-Ristic, A. Kristensens, and C. Sorensen, Phys. Rev. B 75, 045308 (2007).
[13] W. E. Chickering, J. P. Eisenstein, and J. L. Reno, Phys. Rev. Lett. 103, 046807 (2009).
[14] V. Narayan et al., Phys. Rev. B 86, 125406 (2012).
[15] V. Narayan et al., J. Low Temp. Phys. 171, 626 (2013).
[16] V. Narayan et al., New J. Phys. 16, 085009 (2014).
[17] M. Baenninger et al. Phys. Rev. B 72, 241311(R) (2005).
[18] A. Ganczarczyk et al., Phys. Rev. B 86, 085309 (2012).
and the ungated (II) region), both the temperature \( \Delta T_e \) and the heat current \(-\kappa(\partial \Delta T_e / \partial x)\) have to be continuous. It is, then, a matter of solving a set of four linear equations to express the coefficients of Eq. S2 in terms of \( \Delta T_e(0) \). However, the only relevant feature of the full temperature profile that enters the thermovoltage is the temperature at the interface between region I and II, \( \Delta T_e(L_g) \). Plugging \( x = L_g \) into the full solution immediately leads to the compact formula Eq. 4, where \( z \equiv (\kappa / \ell) / (\kappa_0 / \ell_0) = (n/n_0)^{(1+\alpha_e-\alpha_i)/2} \) originates from the boundary condition of the heat current to be continuous.

In the limit of a large gated region, \( L_g \gg \ell \), the coefficient \( b \) of the exponentially increasing term in Eq. S2 vanishes, \( \Delta T_e(L_g) \) goes to zero, and the ungated region does not influence the measure thermovoltage. For \( L_g \lesssim \ell \), however, the ungated region becomes important. Ignoring it leads to an inadequate fit to the data, as shown in Fig. 2 of the main text.

B. Derivation of Eq. 5

The modelling of the temperature profile along the \( y \)-direction in the 2DEG strip is analogous to that of the BGT. Since the electron density is constant in the 2DEG strip, only two boundary conditions are needed. They are given by fixing \( \Delta T_e \) to \( \Delta T_{e,0} \) at \( y = 0 \) and to 0 at the Ohmic contact \( y = L_{\text{strip}} \). Formally, determining the temperature profile along the 2DEG strip is just a special case of the calculation for the BGT. Therefore Eq. 5 follows from Eq. 4 by performing the replacements \( z \to 1, \ell_0 \to \ell, L_g \to y, L_{\text{ug}} \to L_{\text{strip}} - y, \) and \( \Delta T_e \to \Delta T_{e,0} \).