Analysis of Collar Roof Considering Deflection of Reinforced Concrete Rim with Cracks

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Abstract. This paper deals with the interaction between collar roofs and the bottom support structure made as a reinforced concrete rim. In present, the collar roofs are widely used for their ability to omit internal vertical support structures (columns, support walls), which might be sometimes a complication during a design of the layout of the building. Collar roof acts on the bottom support structure of the building by quite big lateral horizontal forces. For structure analysis of collar roof is often assumed the infinity stiffness of the reinforced concrete rim. The correctness of this assumption depends of the constructional solution of the concrete rim anchoring. When the concrete rim is anchored to the transverse wall only, the assumption of infinity stiffness of the reinforced concrete rim is not fulfilled properly. The paper shows the example of the collar roof supported on the reinforced concrete rim. There is made a comparison of internal forces in two frames. One frame has rigid lateral support; the second one has a flexible lateral support. The flexibility of the lateral support is due to the deflection of the reinforced concrete rim. The occurrence of cracks in the reinforced concrete rim is taken into account. This paper shows the possibility how to deal with the considering cracks in reinforced concrete beam in calculation without using software, which is able to calculate reinforcement concrete beams with steel reinforcement and cracks in concrete.

1. Introduction
Collar roofs are one of the oldest systems that have been used throughout all periods and since their expansion in the Middle Ages they are one of the most widespread construction systems. Collar roofs are very often used for roofing of simple roofs of smaller objects. Collar roofs allow to free up ground floor layout because they do not require internal vertical support elements. However, the collar roofs cause a significant lateral horizontal force on the lower support structure in the area of anchoring.

One way to capture these transverse horizontal forces is to design a reinforced concrete rim to be carried out at the top of the wall. This reinforced concrete rim then acts as a horizontal continuous beam that transmits horizontal forces from the roof. Horizontal support of this beam can be made of the front walls which the rim is sufficiently anchored to. The inner supports can be formed either by pulling the rim into the inner transverse load-bearing wall or, for example, by using a steel rod anchored to the supporting structure of the ceiling, which may optionally be hidden, for example, in a non-load-bearing wall. This reinforced concrete rim has to be dimensioned to transfer all of horizontal load. However, there is a question how far the deflection and flexibility of this reinforced concrete rim affects the behaviour of the individual frames, it means the internal forces in them and the individual horizontal forces on the reinforced concrete rim. On the example of a concrete roof and the calculation will be made which will try to take into account the flexibility of the reinforced concrete rim.
2. Description of collar roof for calculation

Layout and the section of the roof is shown on the Figure 1.

![Figure 1. Layout and the section of the roof](image)

It is a roof of a typical family house of a rectangular ground plan. In the middle of the house there is one transverse wall into which a reinforced concrete rim is pulled. The rim is further anchored to the front walls. Concrete of the rim is C20 / 25. B500B is the reinforcement material. Diameter of the reinforcement bars is 14mm. The reinforcement scheme is in Figure 2.
It is assumed the reinforced concrete rim is designed according to the present valid standards, for example [1], or [2] and the timber structure is designed for example according to [3] or [4]. The dimensions of rafters and collars are 120 / 140mm. The roof load is determined in accordance with the set of European Standards. In the calculation, the permanent load and the snow load is consider. Wind load is not considered to simplify the task. Simplified design combination for the ultimate limit state considered in the calculation is: 1.35 x permanent load + 1.5 x snow load.

### 3. Computational model

The computational model on Figure 3 considers a reinforced concrete rim, which is simulated as a beam of C20/25 isotropic concrete. The reinforcement of the rim cross section is not taken into account in this model. This computational model also does not consider the fact that concrete can’t transfer the tension (practically is transmitted by the tension reinforcement) and also does not consider reducing the bending stiffness of the rim due to cracks. The influence of steel reinforcement and the influence of cracks in concrete will be taken into account by manual calculation presented in further chapters.
Figure 4 shows the shape and size of the bending moment from the considered load combination on the rafters in row no. 7. Row no. 7 is located at the transverse load bearing wall which supply the rigid transverse support for the rim. The horizontal displacement at this point is zero, as is commonly contemplated in the ideal collar roof system. So it is possible to consider this row as ideal situation for collar roof frame, where the stiffness of lateral support is infinity.

![Figure 4. Shape and size of the bending moment on the rafters in the row no. 7 [kNm]](image)

The computational model presented above doesn’t take into account the real reinforcement and the cracks in the concrete rim. To consider these facts, it can be used software, which can involve reinforcement and cracks in the computational model. However, the structure engineer doesn’t have always such software available. In [1] there is presented a method how to calculate the deflection of reinforced concrete beam considering cracks and reinforcement. For our calculation we neglect the effect of creep and shrinkage of concrete. In the following a procedure is shown how to use the method mentioned above for including reinforcement and cracks inside the concrete rim into our computational model. First of all, we determine the shape and value of the horizontal bending moment on the concrete rim. For this purpose, we can use the results from the computational model above. The bending moment is visible on Figure 5.

Further, we will find the cross-sectional characteristics of the reinforced concrete wreath. These cross sectional characteristics will be determined:

- for the cross-section intact by the crack - the so-called ideal cross-section,
- for cracked cross-section.

3.1. Ideal cross-section

When calculating the cross-sectional characteristics of an ideal reinforced concrete cross-section, it is assumed that the cross-sectional area of the cracks is intact. The cross-sectional characteristics of the cross-section include the effect of the reinforcement. For this purpose, the coefficient \( \alpha_e \), which takes into account the different material properties of concrete and reinforcement:

\[
\alpha_e = \frac{E_s}{E_{c,eff}}
\]  

where:

- \( E_s \) is the modulus of elasticity of steel
- \( E_{c,eff} \) is the modulus of elasticity of concrete, which we will consider \( E_{c,eff} = E_{cm} \)
Figure 5. Shape and size of the horizontal bending moment on the concrete rim [kNm]

Relationships for calculating the characteristics of the ideal cross-section can be found, for example, in [2]. After calculating the cross-sectional characteristics, we determine the flexural compliance of the cross-section not broken by the crack:

\[ C_I = \frac{1}{I_I \cdot E_{c, e f f}} \]  

(2)

where \( I_I \) is the moment of inertia of the ideal reinforced concrete cross section.

It is also necessary to check whether cracks are created in the rim. The bending moment of cracks can be calculated from the following relationship:

\[ M_{cr} = f_{ctm} \cdot \frac{I_I}{h - a_{gi}} \]  

(3)

where:
- \( f_{ctm} \) is the tensile strength of the concrete in the tightening
- \( h \) is the height of the rim (in our case \( h = 300\text{mm} \))
- \( a_{gi} \) is the position of the center of gravity of the ideal cross-section.

In our case, \( M_{cr} = 7.64 \text{ kNm} \), which is less than the horizontal bend moment on the \( M_d = 35.38 \text{ kNm} \). Cracks will occur.

3.2. Cracked cross-section

The relations for the calculation of cross-sectional characteristics for a cracked section can be found, for example, in [2]. After calculating these cross-sectional characteristics, we determine the flexural compliance of the cross-section that is broken by the crack:

\[ C_{II} = \frac{1}{I_{it} \cdot E_{c, e f f}} \]  

(4)

where \( I_{it} \) is the moment of inertia of the concrete cross section with the crack.
3.3. Calculation of horizontal deflection of the concrete rim

To calculate the deflection, it is necessary to determine the coefficient $\xi$:

$$\xi = 1 - \beta \cdot \left(\frac{M_{cr}}{M_d}\right)^2$$

(5)

where $\beta$ is the factor depending on the duration of the load duration. Given the fact that we consider the load combination for the ultimate limit state, we consider $\beta = 1$. After calculation we will obtain $\xi = 0.953$.

Now we can calculate the curvature $(1/r_m)$ from the transverse load:

$$\frac{1}{r_m} = M_d \cdot [(1 - \xi) \cdot C_l + \xi \cdot C_H]$$

(6)

The horizontal deflection of the ring $f$ in the middle of the field is obtained from the following relationship:

$$f = k \cdot \frac{1}{r_m} \cdot L^2$$

(7)

where:
- $L$ is the span of the rim
- $k$ is the coefficient which depends on the shape and magnitude of the bending moment on the beam. For our case the coefficient $k = 0.086$

For a calculated curvature value $(1/r_m) = 6.86 \times 10^{-3} m^{-1}$ and a span $L = 7 m$, we obtain a horizontal beam deflection in the field $f = 29.04 mm$. It is now necessary to put this obtained result into our computational model in Figure 3. We can make a simplified consideration that the modulus of elasticity of the concrete wreath, which we have modeled in the calculation in model, is reduced by the cracks. We have to find such a modulus of elasticity $E_z$, which will give in the model in Figure 3 the deflection in a field equal to $f$ (about 29mm). We can find the $E_z$ in the computational model by gradually reducing the elastic modulus of the wreath in the material library of software until we obtain the corresponding deflection $f$ in the model. After completing the above procedure, a new replacement modulus of elasticity $E_z = 7200 MPa$ has been found. When we replace the original modulus of elasticity of the concrete C20 / 25 by the new one ($E_z = 7200 MPa$), we can make a new calculation of internal forces in the model.

4. Results and discussions

Figure 6 shows the course of the bending moment on the rafters for the binding No. 3, which is located in the middle of the span in the field of the gilded wreath. Table 1 shows selected result gained from the calculation.

| Row | Normal force in collar beam (compression) [kN] | Transverse horizontal force into the rim [kN] | Transverse displacement of the rim [mm] |
|-----|---------------------------------------------|---------------------------------------------|----------------------------------------|
| 7   | 8.41                                       | 10.74                                       | 0.0                                    |
| 3   | 4.72                                       | 8.78                                        | 29.04                                  |
5. Conclusions
This paper presents how to include the horizontal flexibility of a reinforced concrete rim in the global analysis of the collar roof, including the reinforcement and cracks in the reinforced concrete rim.

On the example of a collar roof, a comparison of the size and shape of the internal forces was made in two different rows. One row corresponded to a collar roof with perfectly stiff lateral supports, and the second one was influenced by the horizontal flexibility of the reinforced concrete rim. The comparison shows that the horizontal flexibility of the concrete rim has a significant effect on the shape and size of the internal forces in the collar roof.

References
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