Using the method of canonical discriminant functions for a qualitative assessment of the response degree of producing wells to water injection during the development of carbonate deposits

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Abstract. The article proposes an algorithm for qualitative assessment of the response degree of producing wells to water injection into wells for the conditions of oil deposits of the Tournaisian stage of the Volga-Ural oil and gas province, based on the use of the method of canonical discriminant functions (CDF). The obtained CDF equations and distributions make it possible to quickly solve the issues of assessing success of a particular waterflooding system, select wells for transferring them to injection, evaluate the density of the grid of production and injection wells, pressure and volumes of water injection at the stage of drawing up the first design documents.

1. Introduction

An increase in the degree of development of hard-to-recover oil reserves is largely determined by the efficiency of the applied development system [1–8]. At the same time, it is important to determine not only during the development process, but also after the field leaves exploration, the degree of response of producing wells to water injection into injection wells [9–11].

The research presents [12] a method for determining the interaction of wells based on the use of the values of the total diagnostic coefficients (TDC) and the values of cross-correlation functions (R) for deposits in the carbonate reservoirs of the Volga-Ural oil and gas province.

One of the weak points of the results obtained in [12] is the presence of zones of uncertainty, which do not allow to give an unambiguous answer to the question of whether the producing well will react to water injection under certain conditions or not. The percentage of wells according to the calculation options that fell into the zones of uncertainty varies from 21 to 51%, averaging 34.3%, and the zones of uncertainty occupy from 8 to 68% of the total interval of changes in the values of the total diagnostic coefficients, averaging 53.4%, which is quite essential. In order to eliminate these shortcomings, namely, to increase the resolution of the method, simplify the calculations, reduce their labor intensity, as well as to control the results obtained and more clearly separate the wells, the article used the method of canonical discriminant functions (CDF).
2. Materials and methods

Calculations using this method were carried out according to six options:

1. 2 - using all parameters and parameters that are significant by the Kullback criterion, respectively;
2. 3, 4 - using all parameters and parameters that are significant according to the Kullback criterion, respectively, and the ones that are characterizing geological and physical properties of formations in wells, physicochemical properties of formation fluids, initial conditions of occurrence and intensity of the waterflooding system;
3. 5, 6 - using all parameters and parameters that are significant according to the Kullback criterion, respectively, characterizing technological features of the operation of wells and deposits, as well as intensity of the waterflooding system.

Variants with the use of parameters that characterize only the intensity of the waterflooding system were not considered, since their separate use without parameters reflecting features of the geological structure of objects and technological features of the operation of wells and deposits is not justified due to low resolution.

At the first stage, two groups of wells were considered:
1. wells that reacted to water injection \( R \geq 0.5 \);
2. wells that did not respond to water injection \( R < 0.5 \).

3. Results and Discussion

The distribution of wells shown in the figure in the axes of the first two canonical discriminant functions by calculation options showed a clearer division of wells into groups compared to the results that used values of the total diagnostic coefficients [12]. A comparison of the resolution capabilities of the TDC and CDF methods is presented in Table 1. Parameter \( Z_1 \) represents the percentage of wells that fell into the zone of uncertainty to the total number of wells that participated in the analysis in the TDC method, and the percentage of wells from the total number that fell into the zone of uncertainty and when this is not correctly assigned to a particular group in the CDF method. Parameter \( Z_2 \) is the ratio of the length of the interval of change in TDC values, to which the zone of uncertainty is confined, to the total interval of change in TDC values. In the CDF method, the ratio of the area of the zone of uncertainty to the area within which all wells are concentrated.

Table 1. Relative values of intervals and areas of uncertainty zones, the number of wells in them by calculation options using TDC and CDF methods

| Parameter \( Z_1 \) | Method | Parameter values by variant |
|---------------------|--------|----------------------------|
|                     | TDC    | 1  | 2  | 3  | 4  | 5  | 6  |
|                     | CDF    | 21 | 23 | 28 | 30 | 44 | 40 |
| \( Z_2 \) \%        | TDC    | 8  | 57 | 60 | 59 | 61 | 46 |
|                     | CDF    | 8  | 10 | 9  | 11 | 25 | 30 |

Table 1 shows that when using the method of canonical discriminant functions, the number of wells in the zone of uncertainty compared with the method of total diagnostic coefficients, according to the options presented, decreases by 4-7.5 times, and on average - 5.6 times, and the zone of uncertainty decreases by 3.1 times.
Δ1, ■2 – centroid and number of a group of wells that reacted (R ≥ 0.5) and did not respond (R < 0.5) to water injection, respectively; ●, ○ – wells of groups 1 and 2, respectively; ⋯ – boundary dividing groups of wells; ○, ⋆ – zone of concentration of all wells and zone of uncertainty, respectively.

Figure 1. Distribution of wells in the axes of canonical discriminant functions
However, some of the conclusions obtained using the CDF method support the conclusions obtained using the TDC method. Table 1 shows that there is no need to use all the parameters under consideration when diagnosing. It is enough to restrict ourselves to only the parameters that are significant according to the Kullback criterion, since there is no significant loss of information in options 2, 4, 6 compared to options 1, 3, 5. At the same time, the parameters characterizing the intensity of the waterflooding system and the parameters reflecting features of the geological structure of the deposits have a prevailing influence on the separation of wells by interaction. The parameters characterizing technological features of wells and reservoirs have a lesser impact on the separation of wells. The resulting equations of the canonical discriminant functions have the following form:

- according to option 1:

\[ y_1 = -0.31\mu_0 + 0.08\rho_N + 0.30K_{mix} + 0.80K_{prod} - 0.02F + 2.67H_E^N + 1.30H_P^N + 0.55n^N - \
0.54M_G^N + 0.42H_E^D + 0.87H_P^D - 0.13n^D + 0.05M_G^D - 3.07K_P^N - 1.46H_{tot}^N + 0.16K_P^D + \
+0.09H_{tot}^N - 0.48P_{inj}/P_{rock} - 0.16Q_{inj} + 0.33Q_{mc} + 0.31Q_{v} - 0.30f_i - 0.002Q_{adj} - \
-0.57t - 0.26Q_{N_{max}}^D , \]

(1)

\[ y_2 = -1.18\mu_0 - 0.78\rho_N + 0.26K_{mix} - 0.26K_{prod} + 0.41F + 0.22H_E^N - 2.70H_P^N - 2.29n^N + \
+0.42M_G^N - 0.49H_E^D - 0.57H_P^D - 0.77n^D + 0.30M_G^D + 2.13K_P^N + 2.37H_{tot}^N + 0.98K_P^D + \
+1.21H_{tot}^N + 1.31P_{inj}/P_{rock} + 0.46Q_{inj} - 0.27Q_{mc} + 0.18Q_{v} - 0.06f_i + 0.11Q_{adj} + 1.28t - \
-0.79Q_{N_{max}}^D , \]

(2)

\[ y'_1 = -9.23 - 0.14\mu_0 + 0.02\rho_N + 0.05K_{mix} + 1.12K_{prod} + 0.0001F + 0.84H_E^N + 1.17H_P^N + \
+0.37n^N - 0.41M_G^N + 0.13H_E^D + 0.84H_P^D - 0.08n^D + 0.03M_G^D - 21.1K_P^N - 0.28H_{tot}^N + \
+1.02K_P^D + 0.02H_{tot}^N - 5.54P_{inj}/P_{rock} - 0.001Q_{inj} + 0.0001Q_{mc} + 0.003Q_{v} - 1.33f_i + \
+0.0001Q_{adj} - 0.006t - 0.001Q_{N_{max}}^D , \]

(3)

\[ y'_2 = 174 - 0.54\mu_0 - 0.21\rho_N + 0.04K_{mix} - 0.37K_{prod} + 0.002F + 0.07H_E^N - 2.42H_P^N - \
-1.53n^N + 0.32M_G^N - 0.15H_E^D - 0.55H_P^D - 0.47n^D + 0.21M_G^D + 14.6K_P^N + 0.45H_{tot}^N + \
+6.43K_P^D + 0.22H_{tot}^N + 15.1P_{inj}/P_{rock} + 0.004Q_{inj} + 0.0001Q_{mc} + 0.002Q_{v} - 0.25f_i + \
+0.0001Q_{adj} + 0.01t - 0.004Q_{N_{max}}^D ; \]

(4)

- according to option 2:

\[ y_1 = 2.81H_E^N + 0.66H_P^N - 0.03F - 0.42M_G^N + 0.33H_E^D + 0.84H_P^D - 2.57K_P^N - 1.15H_{tot}^N + \
+0.25K_P^D + 1.48t_{melt} + 0.73K_{prod} - 0.27P_{inj}/P_{rock} - 0.04Q_{inj} - 0.72H_{bed} + 0.04M_G^D - \
-0.16Q_{N_{max}}^D - 0.08t , \]

(5)

\[ y_2 = -2.57H_E^N - 0.23H_P^N + 0.36F + 1.03M_G^N - 0.19H_E^D + 0.10H_P^D + 1.98K_P^N + 2.27H_{tot}^N + \
+0.32K_P^D + 5.99t_{melt} - 0.38K_{prod} + 1.08P_{inj}/P_{rock} - 0.30Q_{inj} - 5.31H_{bed} - 0.20M_G^D - \
-0.22Q_{N_{max}}^D + 0.35t ; \]

(6)
\[ y'_{1} = -164 + 0.89 H_{E}^{N} + 0.60 H_{P}^{N} + 0.0001 F - 0.32 M_{G}^{N} + 0.10 H_{E}^{D} + 0.81 H_{P}^{D} - 17.6 K_{P}^{N} - 0.22 H_{int}^{N} + 1.63 K_{P}^{D} + 0.60 t_{melt} + 1.03 K_{prod} - 3.08 P_{inj} / P_{rock} + 0.0001 Q_{inj} - 0.007 H_{bed} + 0.03 M_{G}^{D} - 0.001 Q_{N_{max}}^{D} - 0.001 t, \] (7)

\[ y'_{2} = -670 - 0.81 H_{E}^{N} - 0.21 H_{P}^{N} + 0.002 F + 0.78 M_{G}^{N} - 0.06 H_{E}^{D} + 0.09 H_{P}^{D} + 13.6 K_{P}^{N} + 0.43 H_{int}^{N} + 2.08 K_{P}^{D} + 2.45 t_{melt} - 0.53 K_{prod} + 12.5 P_{inj} / P_{rock} + 0.0001 Q_{inj} - 0.05 H_{bed} - 0.14 M_{G}^{D} - 0.001 Q_{N_{max}}^{D} + 0.004 t, \] (8)

- according to option 3:

\[ y_{1} = -0.51 \mu_{0} + 0.11 \rho_{N} + 0.21 K_{mix} + 0.79 K_{prod} - 0.12 F + 2.25 H_{E}^{N} + 0.94 H_{P}^{N} + 0.42 n^{N} - 0.44 M_{G}^{N} + 0.64 H_{E}^{D} + 0.64 H_{P}^{D} - 0.15 n^{D} - 0.13 M_{G}^{D} - 2.18 K_{P}^{N} - 1.17 H_{int}^{N} - 0.03 K_{P}^{D} + 0.05 H_{tot}^{D} - 0.48 P_{inj} / P_{rock} - 0.004 Q_{inj}, \] (9)

\[ y_{2} = -0.67 \mu_{0} - 0.98 \rho_{N} + 0.28 K_{mix} - 0.29 K_{prod} + 0.33 F + 0.62 H_{E}^{N} - 1.89 H_{P}^{N} - 2.03 n^{N} + 0.31 M_{G}^{N} - 0.64 H_{E}^{D} - 0.10 H_{P}^{D} - 0.36 n^{D} + 0.12 M_{G}^{D} + 0.65 K_{P}^{N} + 2.00 H_{int}^{N} + 0.95 K_{P}^{D} + 0.95 H_{tot}^{D} + 1.31 P_{inj} / P_{rock} - 0.03 Q_{inj}, \] (10)

\[ y'_{1} = -17.8 - 0.23 \mu_{0} + 0.03 \rho_{N} + 0.04 K_{mix} + 0.11 K_{prod} - 0.001 F + 0.71 H_{E}^{N} + 0.84 H_{P}^{N} + 0.28 n^{N} - 0.33 M_{G}^{N} + 0.20 H_{E}^{D} + 0.62 H_{P}^{D} - 0.09 n^{D} - 0.009 M_{G}^{D} - 15.0 K_{P}^{N} - 0.22 H_{int}^{N} - 0.17 K_{P}^{D} + 0.009 H_{tot}^{D} - 5.53 P_{inj} / P_{rock} + 0.004 Q_{inj}, \] (11)

\[ y'_{2} = 221 - 0.31 \mu_{0} - 0.26 \rho_{N} + 0.05 K_{mix} - 0.40 K_{prod} + 0.002 F + 0.20 H_{E}^{N} - 1.70 H_{P}^{N} - 1.36 n^{N} + 0.23 M_{G}^{N} + 0.20 H_{E}^{D} - 0.10 H_{P}^{D} - 0.22 n^{D} + 0.08 M_{G}^{D} + 4.46 K_{P}^{N} + 0.38 H_{int}^{N} + 6.21 K_{P}^{D} + 0.17 H_{tot}^{D} + 15.2 P_{inj} / P_{rock} + 0.0001 Q_{inj}, \] (12)

- according to option 4:

\[ y_{1} = 2.66 H_{E}^{N} + 0.58 H_{P}^{N} - 0.09 F - 0.40 M_{G}^{N} + 0.41 H_{E}^{D} + 0.74 H_{P}^{D} - 2.26 K_{P}^{N} - 1.08 H_{int}^{N} + 0.11 K_{P}^{D} + 1.40 t_{melt} + 0.74 K_{prod} - 0.30 P_{inj} / P_{rock} - 0.01 Q_{inj} - 0.67 H_{bed} + 0.004 M_{G}^{D}, \] (13)

\[ y_{2} = -2.12 H_{E}^{N} - 0.03 H_{P}^{N} + 0.32 F + 0.91 M_{G}^{N} - 0.16 H_{E}^{D} + 0.09 H_{P}^{D} + 1.21 K_{P}^{N} - 2.03 H_{int}^{N} + 0.33 K_{P}^{D} + 6.01 t_{melt} - 0.37 K_{prod} + 1.99 P_{inj} / P_{rock} - 0.27 Q_{inj} - 5.52 H_{bed} - 0.24 M_{G}^{D}, \] (14)

\[ y_{1} = -156 + 0.84 H_{E}^{N} + 0.52 H_{P}^{N} + 0.0001 F - 0.30 M_{G}^{N} + 0.13 H_{E}^{D} + 0.71 H_{P}^{D} - 15.5 K_{P}^{N} - 0.21 H_{int}^{N} + 0.73 K_{P}^{D} + 0.57 t_{melt} + 1.04 K_{prod} - 3.47 P_{inj} / P_{rock} + 0.0001 Q_{inj} - 0.007 H_{bed} + 0.002 M_{G}^{D}, \] (15)

\[ y_{2} = -666 - 0.67 H_{E}^{N} - 0.03 H_{P}^{N} + 0.002 F + 0.69 M_{G}^{N} - 0.05 H_{E}^{D} + 0.09 H_{P}^{D} + 8.32 K_{P}^{N} + 0.39 H_{int}^{N} + 2.14 K_{P}^{D} + 2.46 t_{melt} - 0.52 K_{prod} + 12.6 P_{inj} / P_{rock} + 0.0001 Q_{inj} - 0.06 H_{bed} - 0.17 M_{G}^{D}, \] (16)

- according to option 5:

\[ y_{1} = -0.17 F + 0.93 P_{inj} / P_{rock} + 0.21 Q_{N_{max}}^{D} + 0.12 Q_{inc}^{D} + 0.21 Q_{inj}^{D} - 0.31 f_{i} + 0.15 Q_{inj} + 0.18 t - 0.40 Q_{N_{max}}, \] (17)
\[ y_2 = 1.04F + 0.33P_{inj}/P_{rock} - 0.01Q_N^D + 0.40Q_{inc}^D + 0.40Q_V^D - 0.38f_1 + 0.07Q_{inj} - 0.1t - 0.52Q_{N,max}, \]  
(18)

\[ y'_2 = -4.77 - 0.001F + 10.8P_{inj}/P_{rock} + 0.002Q_N^D + 0.0001Q_{inc}^D + 0.002Q_V^D - 1.37f_1 + +0.0001Q_{inj} + 0.002t - 0.002Q_{N,max}, \]  
(19)

\[ y'_2 = -3.79 + 0.005F + 3.79P_{inj}/P_{rock} + 0.0001Q_N^D + 0.001Q_{inc}^D + 0.004Q_V^D - 1.67f_1 + +0.0001Q_{inj} - 0.001t - 0.003Q_{N,max}; \]  
(20)

- according to option 6:

\[ y_1 = -0.18F + 0.95P_{inj}/P_{rock} + 0.06Q_{inj} + 0.08t - 0.05Q_{inc}^D, \]  
(21)

\[ y_2 = 1.02F + 0.34P_{inj}/P_{rock} + 0.02Q_{inj} + 0.14t - 0.25Q_{inc}^D, \]  
(22)

\[ y'_1 = -4.97 - 0.001F + 11.0P_{inj}/P_{rock} + 0.0001Q_{inj} + 0.001t - 0.00001Q_{inc}^D, \]  
(23)

\[ y'_2 = -4.27 + 0.005F + 3.95P_{inj}/P_{rock} + 0.0001Q_{inj} + 0.002t - 0.00001Q_{inc}^D. \]  
(24)

The interpretation of the parameters of the equations is presented in [12].

The equations (1), (2), (5), (6), (9), (10), (13), (14), (17), (18), (21), (22) normalize the coefficients of canonical discriminant functions. The values of functions in centroids of the selected groups are presented in Table 2.

### Table 2. Values of discriminant functions in group centroids

| Option | Function | Well group number 1 | Well group number 2 |
|--------|----------|---------------------|---------------------|
| 1      | \[ y'_1 \] | 2.281               | -1.579              |
|        | \[ y'_2 \] | 0.025               | -0.114              |
|        | \[ y'_1 \] | 2.181               | -1.514              |
|        | \[ y'_2 \] | 0.011               | -0.073              |
|        | \[ y'_1 \] | 2.191               | -1.518              |
| 2      | \[ y'_2 \] | 0.021               | -0.102              |
|        | \[ y'_1 \] | 2.155               | -1.496              |
| 3      | \[ y'_2 \] | 0.011               | -0.072              |
|        | \[ y'_1 \] | 1.450               | -1.023              |
| 4      | \[ y'_2 \] | -0.005              | -0.012              |
|        | \[ y'_1 \] | 1.409               | -0.993              |
| 5      | \[ y'_2 \] | -0.005              | -0.011              |
| 6      | \[ y'_1 \] | 1.409               | -0.993              |

The algorithm for determining the belonging of wells to one of the two groups will be as follows:
- according to one of the options (depending on the availability of the initial geological and field information), the research determines the values of the canonical discriminant functions;
- belonging of the wells to any group in the axes of canonical discriminant functions in the figure is determined.

The values of indicators characterizing the intensity of the waterflooding system are set based on the wishes of the user.
If the well has fallen into the zone of uncertainty, by changing the adjustable parameters \( F, \ P_{inj} / P_{rock}, \ Q_{inj} \), it is necessary to transfer it to the zone where the wells are affected by injection.

If the well in the axes of the canonical discriminant functions is outside the zone of wells concentration shown in the figure, it is important to determine which group of wells it is closer to. The search for such a group should be done as follows:

- values of the canonical discriminant functions of the well should be determined, as well as its location on the plane in the axes \( y_i - y_j \);
- find the centroid of the group closest to the given object in the Euclidean space of two canonical discriminant functions based on the expression

\[
d_j = \sqrt{\sum_{i=1}^{m} (y_i - y_j')^2},
\]

where \( d_j \) is the Euclidean distance between the object (well) and the \( j \)-th centroid; \( y_i' \) – value of the \( i \)-th non-normalized canonical discriminant function of the well; \( y_j' \) – the value of the \( i \)-th unnormalized canonical discriminant function of the \( j \)-th centroid (see table 2); \( m=2 \) is the number of unnormalized canonical discriminant functions.

4. Conclusion

1. For the conditions of oil deposits of the Tournaisian stage of the Volga-Ural oil and gas province, based on the use of the method of canonical discriminant functions (CDF), an algorithm is proposed for the qualitative assessment of the degree of response of producing wells to water injection into injection wells.

2. The obtained CDF and distribution equations make it possible to quickly solve the issues of assessing the success of a particular waterflooding system, select wells for transferring them to injection, evaluate the density of the grid of production and injection wells, pressure and volumes of water injection at the stage of drawing up the first design documents.

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