Analysis of 0-9 kHz Current Harmonics in a Three-Phase Power Converter Under Unbalanced-Load Conditions

ARASH MORADI1, (Member, IEEE), JALIL YAGHOOBI1, (Member, IEEE), FIRUZ ZARE1,2, (Fellow, IEEE), AND DINESH KUMAR3, (Senior Member, IEEE)

1School of Information Technology and Electrical Engineering, The University of Queensland (UQ), Brisbane, QLD 4072, Australia
2School of Electrical Engineering and Robotics, Queensland University of Technology, Brisbane, QLD 4000, Australia
3Global Research and Development Centre, Danfoss Drives A/S, 6300 Gråsten, Denmark

Corresponding author: Arash Moradi (arash.moradi@uq.net.au)

This work was supported by the Australian Research Council under Grant LP170100902.

ABSTRACT The unbalanced condition of typical loads in industrial networks could affect the current harmonics across the dc-link in Adjustable Speed Drives (ASDs). These current harmonics could pass through the dc-link and create new current harmonics at the grid side. Thus, the effect of unbalanced-load condition on the current harmonics across the dc-link needs to be considered in the design process. Moreover, new compatibility levels and emission limits will be developed for the 2-9 kHz current harmonics, so drives need to meet these new requirements in future. These current harmonics could also be significantly affected by the load power factor at the unbalanced-load condition. Due to these reasons, analysing the effect of unbalanced-load and load power factor on the current harmonics on the current harmonics between 0 to 9 kHz across the dc-link is vital. Thus, this paper presents an analytical approach for estimating the current harmonics of the inverter at the dc side considering unbalanced-load currents. The proposed mathematical equations are then validated with experimental results. These analyses reveal that the negative-sequence current generates new harmonics across the dc-link. Additionally, the relationship between unbalanced currents at the load side and dc-link are found as a function of positive- and negative-sequence currents.

INDEX TERMS Power quality, ASD, motor drive, inverter, PWM, unbalanced-load, 2-9 kHz current harmonic.

I. INTRODUCTION

Inverters have been applied in many applications with the significant advances in the power electronic technologies. Among these applications, motor drives have become increasingly popular due to their advantageous [1]. Fig. 1 illustrates the configuration of a three-phase Adjustable Speed Drive (ASD). As can be seen from this figure, a diode rectifier is used to generate dc voltage at the input of the inverter. The inverter then generates the required AC voltage waveform at the load side in this configuration. To develop a voltage waveform with the desirable amplitude and frequency at the load side of this inverter, Pulse Width Modulation (PWM) techniques are commonly applied on the inverter’s switches. However, the voltage generated by these PWM techniques is not pure sinusoidal and is distorted with harmonics around the inverter switching frequency $\omega_c$ and its multiples. Besides, current harmonics are generated according to the switching pattern determined by PWM in the dc-side current of inverter, $i_{\text{inv}}$ [2]. Consequently, $i_{\text{inv}}$ has a dc value and some harmonics due to the switching function of inverter. Although in some researches [3]–[7] PWM techniques have been developed to improve the generated voltage of inverter, the current harmonics at the dc-side have not been considered in these techniques. These current harmonics could affect the grid current harmonics and the lifetime of dc-link capacitor and filter components. For these reasons, analysing the current harmonics of $i_{\text{inv}}$ at different conditions is vital.

Among the conditions that affect the current harmonics in an ASD, an unbalanced-load condition occurs commonly in the practical networks. The unbalanced-load condition might happen because the parameters of motor windings could be different due to the possible errors in the
Manufacturing process and faults during the motor operation. In addition, this condition can be originated from the variations in the parameters of a motor due to aging effect, which affects these parameters at the different phases unequally [8].

Authors in [8] have provided a comprehensive review about the reasons that can make unbalance current or voltage in a power system. They have found that the main reason for the unbalanced currents or voltage is an unbalanced load, such as when the winding parameters of a motor are not similar at phases a to c. They have also shown that the models of system components need to be revised considering an unbalanced network feature. Besides, they have discussed that motors and ASDs are sensitive to imbalance voltage and current. The reason for this is that the unbalance current might cause overheating in the rotor conductors of a motor and overstressing in the ASD elements. The current generated by the unbalanced load in addition to switching pattern impact \( i_{\text{inv}} \) harmonics in the frequency range 0-9 kHz [9]. It is worth mentioning that the frequency range of 0-9 kHz has been divided into 0-2 and 2-9 kHz by standardisation committees [10]. In this frequency range, 2-9 kHz current harmonics have become more important recently by the advances in the switching devices and the lack of standards. The existing standards only cover the harmonics between 0-2 kHz and above 150 kHz based on IEC 61000-3-2, IEC 61000-3-12, and CISPR (International Special Committee on Radio Interference). Consequently, 2-9 kHz harmonics have become a crucial topic of standardisation committees, such as IEC SC77A WG1, WG8, and WG9, for being considered in the new regulations and standards [11].

The impact of grid impedance, grid unbalanced voltage, filter topology, and PWM technique of an ASD on the 0-9 kHz current harmonics have been studied in [12]–[17]. Authors in [12] have investigated the current harmonics at the input of an ASD under unbalanced grid voltage. In addition, they have studied distribution transformer k-factors connected to an ASD under input voltage unbalance and sag conditions. This analysis could help with the selection of proper transformers in terms of cost and safety in distribution systems that feed multiple ASDs. They have also shown that the amplitude of the current harmonic at 150 Hz is increased in all three phases at an unbalanced-load condition. This could bring about serious problems in many field applications where typical power quality filters are used. The presented analysis, however, only considered grid unbalanced voltage as a source of unbalance current and ignored the load side unbalance currents. Authors in [13] have indicated that 2-9 kHz current harmonics of an ASD in a weak grid are reduced more due to the damping effect of the grid inductance. Authors in [14], [15] have found that when grid voltage is unbalanced, the current harmonics as well as the Total Harmonic Distortion (THD) of input current are increased. It has also been pointed out that voltage dip can affect the conduction interval of diode switches in the rectifier. This changes 0-9 kHz current harmonics at the grid side of ASDs, as these current harmonics are generated with respect to the switching of the rectifier. In [16], the impact of applying ac and dc-choke topologies in an ASD on the current harmonics at the grid side have been investigated. Furthermore, that study has shown that the effect of grid inductance on the function of rectifier switches might change the amplitude of generated current harmonics. Authors in [17] have studied the impact of using different PWM strategies in ASDs on the dc-link current harmonics. They have found that reducing the fluctuation of inverter’s dc-link current could decrease the degradation of dc-link capacitors. In [13], it has been pointed out that the dc-current across dc-link is generated by inverter with respect to the active power of the motor. Besides, the reduction in \( i_{\text{inv}} \) dc-value can change the operating mode of rectifier in the ASD from Continuous Conduction Mode (CCM) to Discontinuous Conduction Mode (DCM).

Holmes et al. have found in [18] that the switching function of inverter switches can be employed to estimate the current harmonics of \( i_{\text{inv}} \). Then, these switching functions, which are determined based on the double Fourier analysis, could be multiplied by the load-side currents in the time domain. This approach has been employed in [19] to find the current harmonics of \( i_{\text{inv}} \) at a balanced-load condition. Moreover, the accuracy of estimating \( i_{\text{inv}} \) has been improved in [20] by considering load-side current harmonics. An analytical analysis of the ac and dc side harmonics in three-level inverters has been presented in [21]. In that study, an analytical model to calculate the harmonics of three-level converter with Space Vector Modulation technique has been developed at first. Then, the ac output voltage harmonics and dc side input current harmonics have been studied using the analytical model. Finally, the impacts of interleaving on load side voltage harmonics and the current RMS value at dc side of an inverter have been investigated. Although these research works have analysed and estimated \( i_{\text{inv}} \), they have not considered the impact of the unbalanced-load condition in their analysis.

Authors in [22] have studied the effects of ASD passive filters as well as drive’s output frequencies and
different load torque values on the input current interharmonics. They also have investigated the interharmonics at the input of the ASD at unbalanced and balanced operating modes of ASD. As a result, they have found that the small filter components in an ASD reduce the interharmonic distortions more than the larger ones in the unbalanced-load condition. Besides, they have found that the harmonic distortions at the input of an ASD are higher at the lower load values. However, they have not considered the harmonics due to the switching of the inverter at the rear end of the drive in the analysis. This approximation might increase the error of analysis with regards to the dc-link filter configuration and grid impedance although it simplifies the mathematical calculations. The impact of an unbalanced load with different degrees on the harmonics injected by a 6-pulse converter has been studied in [23]. In that work, the current harmonics at the input of a 6-pulse converter have been categorized into the characteristic (e.g. harmonics with an order equal to \((6n \pm 1)\)) and non-characteristic current harmonics. It has been found by [23] that the magnitude of non-characteristic harmonics is increased at unbalanced-load conditions, whereas the amplitude of the characteristic harmonics is decreased. Although the paper presents a comprehensive analysis, it only considers the front-end diode bridge of an ASD, so neglects the impact of the inverter connected to the load at the rear end. The effect of an unbalance load on the current harmonics of \(i_{\text{INV}}\) has also been investigated in [24], [25]. Input current interharmonics of Variable-Speed Drives due to motor current imbalance have been investigated in [24]. Although that work has investigated the interharmonics at the input of an ASD, its analysis might have some errors, especially at high frequencies. The reason is that the harmonics due to the switching function of inverter around the switching frequency and its multiples have been neglected. Hence, the switching function of inverter at phase a has been only modelled by \(0.5 + 0.5M \cos(\omega_f t)\) in which \(M\) is the modulation index. However, the current harmonics due to the switching of inverter are significant around the switching frequency and its multiples in practice. These current harmonics could be transferred to the grid side with regards to the dc-link filter configuration and grid impedance. As a result, the approximation employed in [24] could increase the estimation error of current harmonics at both across dc-link and the input of an ASD although it simplifies the calculation. Rather that the interharmonics investigated in [24], dc-link current and voltage ripples for a three-phase inverter connected to an unbalance load have been analyzed in [26]. The main focus of that work was finding the voltage ripples across dc-link of an inverter rather than the current harmonics at unbalanced-load condition. In that study also, the current harmonics of \(i_{\text{INV}}\) due to inverter’s switching function were recognized insignificant for dc voltage ripples and they were ignored. So, the analysis in [26] does not include the study of the current harmonic of inverter across dc link at unbalanced-load condition. Additionally, it is worth mentioning that only a three-phase inverter was considered in [26] rather than an ASD. Besides, they have not considered the impact of load power factor variations and positive- and negative-sequence currents on the 2-9 kHz current harmonics of \(i_{\text{INV}}\).

This article comprehensively explores the impact of unbalanced-load condition on the 0-9 kHz current harmonics of \(i_{\text{INV}}\). For that purpose, the analytical approach of finding \(i_{\text{INV}}\) at the unbalanced-load condition is first presented. After validating the accuracy of this analytical approach with simulation and practical results, the effect of different parameters on the current harmonics of \(i_{\text{INV}}\) is investigated. These analyses reveal that a new current harmonic at double the fundamental frequency is generated between 0-2 kHz when the load is unbalanced. It is also found that the amplitude of this current harmonic can be determined based on the negative-sequence load current. On the other hand, it is proved that the current harmonics of \(i_{\text{INV}}\) that are affected by negative-sequence currents are independent from the positive-sequence current. Moreover, the current harmonics at 2-9 kHz that are affected by the negative-sequence current are determined.

The main contributions of this work are given as follows,

- Proposing an analytical equation for estimating \(i_{\text{INV}}\) at unbalanced-load condition.
- Developing equations for the current harmonic orders of \(i_{\text{INV}}\) that are affected by negative-sequence current.
- Studying the impact of positive- and negative-sequence currents on the 2-9 kHz current harmonics of \(i_{\text{INV}}\).

The contribution of the proposed equation is that the current harmonics across dc link can be estimated up to 9 kHz at an unbalanced-load condition. This helps to estimate the current harmonics at the input of an ASD with different filter configurations and grid conditions when the load is unbalanced. On the other hand, the analysis provided in this paper includes the current harmonics in the frequency range of 2 to 9 kHz. The standardization committees have been working to establish new standards for this frequency range, which has not been covered in the existing standards. Hence, the outcomes of the proposed article could be very helpful for both the standardization committees and drive manufacturers in future.

This article is organised as follows. The analytical approach for finding \(i_{\text{INV}}\) at unbalanced-load condition is presented in the next section. Section III then verifies the accuracy of the proposed analytical approach to calculate \(i_{\text{INV}}\). In section IV, the impacts of positive- and negative-sequence currents on the 0-9 kHz current harmonics of \(i_{\text{INV}}\) are evaluated. The proposed mathematical formulation is then validated further through experimental results in section V. Section VI analyzes voltage harmonics across dc-link and the propagation of the current harmonics to the grid at an unbalanced-load condition. Finally, the conclusions are drawn in section VII.

II. ESTIMATING THE DC-LINK CURRENT OF AN INVERTER AT UNBALANCED-LOAD CONDITION

In this section, an analytical formula is proposed for the dc-side current of the three-phase inverter, \(i_{\text{INV}}\), (see Fig. 1)
under unbalanced-load condition. In this circuit, $i_{\text{inv}}$ can be found as given in (1).

$$i_{\text{inv}} = s_a i_a + s_b i_b + s_c i_c$$ (1)

where $s_a$, $s_b$, and $s_c$ are the switching functions of inverter at phases a to c as given by (2) to (4) derived based on the sine-triangular PWM [19].

$$s_a = \frac{1}{2} + \frac{M}{2} \cos(\omega_0 t) + \sum_{m=1}^{+\infty} \sum_{k=-\infty}^{+\infty} (C_{m,k} \cos(m \omega_0 t + k \omega_0 t))$$ (2)

$$s_b = \frac{1}{2} + \frac{M}{2} \cos(\omega_0 t - \frac{2\pi}{3}) + \sum_{m=1}^{+\infty} \sum_{k=-\infty}^{+\infty} (C_{m,k} \cos(m \omega_0 t + k \omega_0 t - \frac{2\pi}{3}))$$ (3)

$$s_c = \frac{1}{2} + \frac{M}{2} \cos(\omega_0 t + \frac{2\pi}{3}) + \sum_{m=1}^{+\infty} \sum_{k=-\infty}^{+\infty} (C_{m,k} \cos(m \omega_0 t + k \omega_0 t + \frac{2\pi}{3}))$$ (4)

where $\omega_0$ is the angular frequency of the motor-side voltage fundamental component, and $\omega_c$ is the angular frequency of carrier waveform. In addition, $m$ is the switching frequency gain, which its maximum value determines the highest frequency of harmonics considered in the analysis due to the switching at $\omega_c$. Also, $M$ is the modulation index and $C_{m,k}$ can be found as given in (5).

$$C_{m,k} = \frac{1}{m} J_k(m \pi M) \sin((m + k) \pi)$$ (5)

where $J_k$ is the Bessel function with order $k$. On the contrary to the switching functions, $i_a$, $i_b$, and $i_c$ need to be defined at the unbalanced-load condition. Three-phase currents at the load side of an ASD are considered as follows.

$$\begin{align*}
i_a &= I_{m_a} \cos(\omega_0 t - \theta_a) \\
i_b &= I_{m_b} \cos(\omega_0 t - \theta_b - \frac{2\pi}{3}) \\
i_c &= I_{m_c} \cos(\omega_0 t - \theta_c + \frac{2\pi}{3})
\end{align*}$$ (6)

where $I_{m_a}$, $I_{m_b}$, and $I_{m_c}$ are the amplitude of $i_a$, $i_b$, and $i_c$, and $\theta_a$, $\theta_b$, and $\theta_c$ are the phase angle of $i_a$, $i_b$, and $i_c$. According to the symmetrical component method, these currents can be converted into three symmetrical components: positive sequence $I_+$, negative sequence $I_-$ and zero sequence $I_0$ as given by (7) assuming that $I_a$, $I_b$ and $I_c$ are the vectors of $i_a$, $i_b$, and $i_c$, respectively.

$$\begin{bmatrix} I_0 \cos \theta_0 \\
I_+ \cos \theta_+ \\
I_- \cos \theta_-
\end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix} I_a \\
I_b \\
I_c
\end{bmatrix}$$ (7)

where $a = \cos \frac{2\pi}{3}$, $I_0$, $I_+$ and $I_-$ are the amplitudes of zero, positive and negative-sequence currents, and $\theta_0$, $\theta_+$ and $\theta_-$ are the phase angle of zero, positive and negative-sequence currents. It must be noted that the zero-sequence current $I_0$ is zero here, as the load is considered Y-connected with floating neutral point, thus $i_a + i_b + i_c = 0$. Similarly, $i_0 = 0$ when a load is connected to the ASD with Delta connection, so the outcomes for Y-connected with floating neutral point connection is valid for a load with Delta connection. Therefore, $i_a$, $i_b$, and $i_c$ at the unbalanced-load condition can be found according to (7) as given by (8) to (10) [26].

$$\begin{align*}
i_a &= I_+ \cos(\omega_0 t - \theta_+) + I_- \cos(\omega_0 t - \theta_-) \\
i_b &= I_+ \cos(\omega_0 t - \theta_+ - \frac{2\pi}{3}) + I_- \cos(\omega_0 t - \theta_- + \frac{2\pi}{3}) \\
i_c &= I_+ \cos(\omega_0 t - \theta_+ + \frac{2\pi}{3}) + I_- \cos(\omega_0 t - \theta_- - \frac{2\pi}{3})
\end{align*}$$ (8)

The equations (2) to (10) need to be employed in (1) to find $i_{\text{inv}}$ at the unbalanced-load condition. As can be observed in (2) to (4), $s_a$, $s_b$, and $s_c$ include three terms. The first term of $s_a$, $s_b$, and $s_c$ is $\frac{1}{2} M$, which needs to be multiplied by $i_a$, $i_b$, and $i_c$. However, this term can be ignored in the analysis, as both $\frac{1}{4} I_+ (\cos(\omega_0 t - \theta_+) + \cos(\omega_0 t - \theta_- + \frac{2\pi}{3}) + \cos(\omega_0 t - \theta_+ + \frac{2\pi}{3}))$ and $\frac{1}{4} I_- (\cos(\omega_0 t - \theta_- - \frac{2\pi}{3}) + \cos(\omega_0 t - \theta_- + \frac{2\pi}{3}) + \cos(\omega_0 t - \theta_+ - \frac{2\pi}{3}))$ are equal to zero.

The second term in (2), (3), and (4) is $\frac{M}{2} \cos(\omega_0 t)$, $\frac{M}{2} \cos(\omega_0 t - \frac{2\pi}{3})$, and $\frac{M}{2} \cos(\omega_0 t + \frac{2\pi}{3})$, respectively. These terms denoted by $s_{a,\omega_0}$, $s_{b,\omega_0}$, and $s_{c,\omega_0}$ need to be multiplied by the positive-sequence current of their corresponding phases in (8) to (10). By manipulating the results of these multiplications, the equations to consider the impact of positive-sequence current of load at phases a to c are obtained as given in (11) to (13).

$$\begin{align*}
s_{a,\omega_0} I_+(\omega_0 t) &= \frac{M}{4} I_+ (\cos(2 \omega_0 t - \theta_+) + \cos(\theta_+)) \\
s_{b,\omega_0} I_(\omega_0 t - \frac{2\pi}{3}) &= \frac{M}{4} I_+ (\cos(2 \omega_0 t - \theta_- + \frac{4\pi}{3}) + \cos(\theta_+)) \\
s_{c,\omega_0} I_(\omega_0 t + \frac{2\pi}{3}) &= \frac{M}{4} I_+ (\cos(2 \omega_0 t - \theta_+ + \frac{4\pi}{3}) + \cos(\theta_-))
\end{align*}$$ (11)

As $\cos(2 \omega_0 t - \theta_-) + \cos(2 \omega_0 t - \theta_+ + \frac{4\pi}{3}) + \cos(2 \omega_0 t - \theta_+ + \frac{4\pi}{3}) = 0$, the summation of (11) to (13) is found as given in (14).

$$\begin{align*}
s_{a,\omega_0} I_+(\omega_0 t) + s_{b,\omega_0} I_(\omega_0 t - \frac{2\pi}{3}) + s_{c,\omega_0} I_(\omega_0 t + \frac{2\pi}{3}) &= \frac{3}{4} M I_+ \cos(\theta_-)
\end{align*}$$ (14)

In the same way, the negative-sequence currents of $i_a$, $i_b$, and $i_c$ are multiplied by the second term of switching functions. By doing this and expanding the results, the equations for considering the negative-sequence currents at phases a to c are found as given in (15) to (17).

$$\begin{align*}
s_{a,\omega_0} I_-(\omega_0 t) &= \frac{M}{4} I_- (\cos(2 \omega_0 t - \theta_-) + \cos(\theta_-))
\end{align*}$$ (15)
By employing (14), (18), (21), and (22), the equation of \( i_{inv} \) at the unbalanced-load condition is determined as given by (23), as shown at the bottom of the page, where

\[
A_+ \, k = 1 + 2 \cos \left( \frac{(1 + k)2\pi}{3} \right) \\
A_- \, k = 1 + 2 \cos \left( \frac{(1 - k)2\pi}{3} \right)
\]

The proposed equation (23) can be applied to analyse and estimate \( i_{inv} \) at unbalanced-load conditions for different applications, such as motor drives. This equation can be used to estimate the current harmonics below and above 9 kHz by manipulating the values of \( m \) and \( k \). However, the value of circuit parameters above 9 kHz varies significantly due to switching transient effects on PWM pulses, the saturation of magnetic elements, and the high-frequency characteristic of electrolytic capacitors. Therefore, the model of the system needs to be modified comprehensively, as described in [11], to study the current harmonics above 9 kHz. This is the main reason that the standardization committees separate the 2-150 kHz frequency range to 2-9 kHz and 9-150 kHz, considering different measurement methods based on International Electrotechnical Commission (IEC) and International Special Committee on Radio Interference (CISPR) standards [11]. In this article, the target is estimating and analysing the current harmonics across dc-link below 9 kHz at an unbalanced-load condition.

In the next section, the accuracy of the proposed equation is examined using simulation results.

### III. VALIDATION OF THE PROPOSED EQUATION FOR ESTIMATING INVERTER CURRENT ACROSS THE DC-LINK AT UNBALANCED-LOAD CONDITION

To validate the proposed equation for \( i_{inv} \) at the unbalanced-load condition, the circuit shown in Fig. 1 has been modelled in MATLAB. Fig. 2 compares the simulation results of \( i_{inv} \)
and the waveform estimated by (23) at similar conditions. Although the figure shows slight differences between simulation and estimated results of $i_{inv}$, they are quite close. These differences that are similar to small current spikes at the peak of estimated $i_{inv}$ can be reduced by increasing the highest considered value for $m$ and $k$. It is worth mentioning that the load current has been modelled by fundamental current in (23) as explained in (6). However, $i_a$, $i_b$, and $i_c$ usually include other harmonics with regards to the load characteristics and the inverter switching pattern. This approximation has small error when load is not pure resistive and has been applied in the previous research works, such as [26]. To investigate the accuracy of the proposed equation further, the waveforms of Fig. 2 are compared in the frequency domain in Table 1.

![Comparison of simulated and estimated waveforms.](image)

**FIGURE 2.** Comparing the estimated values of $i_{inv}$ with simulation results in the time domain.

The main 0 to 9 kHz current harmonics estimated by (23) are compared with simulation results in Table 1. It is worth mentioning that the switching frequency was adjusted to 3 kHz, so the main 2-9 kHz current harmonics are around 3 kHz and its multiples as can be seen in this table. This table shows that the dc value of $i_{inv}$ is estimated quite accurately as well as its current harmonics. The estimation error at 2850 Hz is almost 1%, which fluctuates between 0 % and 2 % at different frequencies and reaches to almost 2% near 9 kHz. These comparisons verify the accuracy of (23) for analysing and estimating the current harmonic in the frequency range of 0-9 kHz. Using the proposed equation, the dc link current harmonics can be found based on the amplitude and phase angle of currents at the phases a to c when load is unbalanced. This equation only needs positive and negative sequences of the load current to estimate these current harmonics. Besides, the provided analysis for the 2-9 kHz current harmonics across dc-link remains valid at the different degrees of unbalanced load. The reason for this is that this degree can only affect the amplitude or phase angle of these current harmonics rather than their frequency. Therefore, this equation can be employed for analysing the impact of different parameters on the current harmonics at the dc side of the inverter in the next step.

**IV. ANALYSING THE CURRENT HARMONICS INJECTED BY THE INVERTER INTO THE DC-LINK IN UNBALANCED-LOAD CONDITION**

The 0-9 kHz current harmonics of $i_{inv}$ at the unbalanced-load condition are studied in this section using the proposed equation (23). Then, the impact of load power factor on these current harmonics is explored.

**A. ANALYSING THE IMPACT OF $I_a$ AND $I_c$ ON 0 TO 9 kHz $i_{inv}$ CURRENT HARMONICS**

As it can be concluded from (23), the load power draws a dc component equal to $\frac{3}{2}MI_+\cos(\theta_+)$. This equation depends on $M$, $I_+$, and $\theta_+$ similar to the balanced condition.

The second term of (23), $\frac{3}{2}MI_-\cos(2\omega_0t - \theta_-)$, shows that at unbalanced-load condition a new current harmonic at $2\omega_0$ is generated in the dc side. This current harmonic is slightly damped by dc-link capacitor, so it can significantly affect the current harmonics of power converter at the grid side as explained in [24]. As it can be observed, the amplitude of the current harmonic at $2\omega_0$ is determined according to $I_-$ multiplied by $\cos(2\omega_0t - \theta_-)$. Therefore, this current harmonic affects the harmonics in the ASD in the frequency range below 2 kHz according to $f_0$. The amplitude of this current harmonic also increases when the load has a higher unbalance degree, which leads to an increase in $I_-$ value.

The first two terms in (23), as aforementioned, affect the dc value and low-frequency-range harmonic contents of $i_{inv}$ assuming that $f_0$ is adjusted to a low-frequency value, such as 50 Hz. The third term, however, mainly controls the harmonic of $i_{inv}$ around $f_c$ and its multiples, $m_2f_c$. Hence, this term mainly determines the harmonics of $i_{inv}$ at higher frequency ranges because $f_c$ is usually adjusted to a high value, such as 3 kHz, with regards to the load requirements. These current harmonics are generated with regards to the $\omega_0$ and $\omega_c$. It is worth mentioning that 2-9 kHz current harmonics of $i_{inv}$ depend on both $I_+$ and $I_-$ in the unbalanced-load condition. In addition to these variables, the 2-9 kHz current harmonics of $i_{inv}$ are determined by the $\theta_+$ and $\theta_-$. As it can be seen

**TABLE 1.** The main 0-9 kHz current harmonics of $i_{inv}$ calculated from simulation and estimated by the proposed equation when ASD’s load is unbalanced and $f_c = 3$ kHz.

| Frequency (Hz) | Simulation (A) | Estimation (A) | Error (A) |
|---------------|----------------|----------------|-----------|
| 0             | 8.02           | 7.97           | 0.05      |
| 100           | 1.05           | 1.05           | 0         |
| 2850          | 2.42           | 2.44           | 0.02      |
| 2950          | 0.33           | 0.33           | 0         |
| 3050          | 0.34           | 0.33           | 0.01      |
| 3150          | 2.42           | 2.44           | 0.02      |
| 5700          | 0.25           | 0.25           | 0         |
| 5900          | 0.19           | 0.19           | 0         |
| 6000          | 2.89           | 2.90           | 0.01      |
| 6100          | 0.19           | 0.19           | 0         |
| 6300          | 0.25           | 0.25           | 0         |
| 8550          | 0.16           | 0.17           | 0.01      |
| 8850          | 0.80           | 0.81           | 0.01      |
in (23) also, the values of constants $C_+, C_-, A_+, k$, and $A_- k$ affect the current harmonics of $i_{\text{inv}}$. To investigate the impact of these constants, their functions similar to (23) are modelled by two constants as given in (26) and (27).

$$C_+[m, k] = C_{m,k} A_+, k$$

$$C_-[m, k] = C_{m,k} A_-, k$$

The values of these constants with regards to $m$ and $k$ are as shown in Fig. 3 (a) and (b). Fig. 3 (a) reveals that $C_+$ is not zero only at $k=3n+1$. It is worth mentioning that $n$ in $k$ denotes a number from the whole set of integer numbers ($Z$), and the order of a current harmonic is determined according to the value of both $m$ and $k$. Besides, the absolute amplitude of $C_+$ decreases gradually from almost 0.8 to zero when $m$ is increased. $C_-$ is also reduced by increasing $m$ similar to $C_+$ as it can be realised from Figs. 3 (a) and (b). It is worth mentioning that $C_-$ is not zero at $k=3n+1$, and $m + k$ needs to be other than an even number since $m + k = 2n$ make $\sin((m + k)\frac{\pi}{k})$ in $C_{m,k}$ zero (see (5)).

After investigating the value of $C_+$ and $C_-$ with regards to the $m$ and $k$, the next step is determining the current harmonics due to the switching function of inverter. As it can be observed in (23), the current harmonics affected by $I_c$ and $I_r$ due to the switching frequency of inverter are at $m\omega_c + (1 + k)\omega_0$ and $m\omega_c - (1 - k)\omega_0$ for $k=3n+1$. Besides, the frequency of these current harmonics are changed to $m\omega_c - (1 - k)\omega_0$ and $m\omega_c + (1 + k)\omega_0$ for $k=3n-1$. To study this, the current harmonics of $i_{\text{inv}}$, around 3 kHz and 6 kHz are illustrated in Figs. 4 (a)-(b). Notably, as the current harmonics are almost 0.8 to zero when $m$ is increased. $C_-$ is also reduced by increasing $m$ similar to $C_+$ as it can be realised from Figs. 3 (a) and (b). It is worth mentioning that $C_-$ is not zero at $k=3n+1$, and $m + k$ needs to be other than an even number since $m + k = 2n$ make $\sin((m + k)\frac{\pi}{k})$ in $C_{m,k}$ zero (see (5)).

After investigating the value of $C_+$ and $C_-$ with regards to the $m$ and $k$, the next step is determining the current harmonics due to the switching function of inverter. As it can be observed in (23), the current harmonics affected by $I_c$ and $I_r$ due to the switching frequency of inverter are at $m\omega_c + (1 + k)\omega_0$ and $m\omega_c - (1 - k)\omega_0$ for $k=3n+1$. Besides, the frequency of these current harmonics are changed to $m\omega_c - (1 - k)\omega_0$ and $m\omega_c + (1 + k)\omega_0$ for $k=3n-1$. To study this, the current harmonics of $i_{\text{inv}}$ around 3 kHz and 6 kHz are affected by $I_+$ and $I_-$ in the same way as the current harmonics around 3 kHz, they are not depicted in this section. In this figure, $I_+$ has been increased from 5 to 15 A while $I_-$ has been kept constant 1.5 A. As can be observed, only the 2-9 kHz current harmonics with frequencies $m\omega_c - (1 - k)\omega_0$ for $k=3n+1$ and $m\omega_c + (1 + k)\omega_0$ when $k$ is $3n-1$ are affected by $I_+$. To investigate this further, the $I_+$ has been kept constant 10 A, whereas $I_-$ has been set to 1, 1.5, and 2 A as shown in Fig. 5 (a) and (b). These figures depict that $I_-$ affects different current harmonic orders compared to $I_+$ in Fig. 4. The frequency of these current harmonics can be $m\omega_c - (1 - k)\omega_0$ at $k=3n-1$ and $m\omega_c + (1 + k)\omega_0$ for $k=3n+1$.

As mentioned earlier, $\theta_+$ and $\theta_-$ affect the current harmonics of $i_{\text{inv}}$. The impact of variation in the value of these parameters on the 0-9 kHz current harmonics is investigated in the next step.

**B. THE EFFECT OF $\theta_+$ AND $\theta_-$ ON THE 0 TO 9 kHz CURRENT HARMONICS OF $i_{\text{inv}}**

As it can be observed in (23), $\theta_+$ impacts both the dc-component and harmonics due to switching in (23). The dc-component current of $i_{\text{inv}}, I_{\text{inv},0}$, is maximum at $\theta_+ = 0^\circ$, whereas it falls to zero when $\theta_+ = 90^\circ$. It is interesting to note that, although $I_{\text{inv},0}$ is zero at $\theta_+ = 90^\circ$, the current harmonics

![Figure 3](image-url)

**FIGURE 3. Investigating the values of parameters that affect the harmonics of $i_{\text{inv}}$ around $\omega_c$, considering $m$ is increased from 1 to 5 and $k$ between -5 and 5. (a) $C_+$, (b) $C_-$.**

![Figure 4](image-url)

**FIGURE 4. Investigating the impact of variation in $I_+$ on the current harmonics of $i_{\text{inv}}$. (a) Around switching frequency, 3 kHz. (b) Near $2\omega_c$, 6 kHz.**
for orders related to dc-component. To clarify the impact of a current harmonic, it affects the amplitude of current harmonics between 0-2 kHz. So the summation of these current harmonics determines the $n\omega$ whereas they are equal to 3 and 0 when $m$ equals $A$ and $c$, respectively.

It must be noted that the summation of these current harmonics is $2\omega c$, so the summation of these current harmonics determines the amplitude. The amplitudes of these two current harmonics are equally related to $I_{\text{in}}$, but their phase angles are different, $\theta_{\text{in}}$ and $-\theta_{\text{in}}$. Consequently, the amplitude of current harmonic at $2m\omega c$ is maximum when $\theta_{\text{in}}$ is zero, whereas it is zero at $\theta_{\text{in}} = 90^\circ$. To study the impact of $\theta_{\text{in}}$ variation further, the amplitude of current harmonics around $2n\omega c$, 6 kHz, is depicted in Fig. 8 at $\theta_{\text{in}} = 15^\circ$, $50^\circ$, and $75^\circ$. This figure indicates that the amplitude of $I_{\text{inv}}$ at 6 kHz is decreased from almost 2.5 A to 0.7 A by the $60^\circ$ variation of $\theta_{\text{in}}$.

Similar to $\theta_{\text{in}}$, $\theta_{\text{inv}}$ affects the current harmonics in the frequency range of 0-9 kHz. In fact, it only affects the phase angle of current harmonic at $2\omega 0$ rather than its amplitude as it could be seen in (23). Besides, this equation shows that $\theta_{\text{inv}}$ could affect the phase angle of current harmonics due to the switching of inverter. Similar to the previous case study, the values of $57^{\text{th}}$, $59^{\text{th}}$, $61^{\text{th}}$, and $63^{\text{th}}$ current harmonics are depicted in Fig. 9 to study the effect of $\theta_{\text{inv}}$ variation. In this figure, $\theta_{\text{inv}}$ has been changed from $15^\circ$ to $50^\circ$ and $75^\circ$, whereas $\theta_{\text{in}}$ has been kept constant $75^\circ$. Then, the amplitude and phase angle of current harmonics at these conditions have been calculated.

This figure reveals the importance of $\theta_{\text{inv}}$ to find the phase angle of current harmonics due to the switching of inverter. As can be seen in Fig. 9, changing $\theta_{\text{inv}}$ affects the phase angle of current harmonic orders 59 and 61 around $180^\circ$, whereas it does not have impact on the amplitude of these current harmonics. Besides, $57^{\text{th}}$ and $63^{\text{th}}$ current harmonics are independent from $\theta_{\text{inv}}$. It is noted that the phase angle of $I_{\text{inv}}$ is very important for the current harmonics at grid side. The reason for this is that the variation in the phase angle of $I_{\text{inv}}$ might cause harmonic cancellation across dc-link. It is worth mentioning that the amplitude of $I_{\text{inv}}$ current harmonics below 2 kHz remains constant at these cases, whereas their phase angle changes similar to the 2-9 kHz current harmonics.

The impact of $I_{\text{in}}$, $I_{\text{inv}}$, $\theta_{\text{in}}$, and $\theta_{\text{inv}}$ on the 2-9 kHz current harmonics as explained earlier is investigated further in Table 2. This table illustrates the variation of $I_{\text{inv}}$ around 3 kHz and 6 kHz when the parameters of unbalanced-load current are changed. To find this variation, both $I_{\text{in}}$ and $I_{\text{inv}}$ have been decreased 50%, and $\theta_{\text{in}}$ and $\theta_{\text{inv}}$ have been reduced 70%. As it can be seen in this table, $\Delta I_{\text{inv},57}=50\%$ when $I_{\text{in}}$ has been decreased 50%, whereas $\Delta I_{\text{inv},59}=0\%$ in this condition. On the other hand, $\Delta I_{\text{in},57}$ and $\Delta I_{\text{in},59}$ are found 0% and 33% by the 50% variation of $I_{\text{in}}$. Besides, the impact of $\theta_{\text{in}}$ on $I_{\text{inv}}$ is different from $I_{\text{in}}$ and $I_{\text{inv}}$ as it can be observed in Table 2. This table shows that $\Delta I_{\text{inv},120}$ is 44% higher than $\Delta I_{\text{inv},57}$ at $\Delta \theta_{\text{in}}=50\%$. Hence, it can be concluded from Table 2 that the harmonics affected by $I_{\text{in}}$ and $\theta_{\text{in}}$ are at $\omega c \pm 3\omega 0$, $2\omega c \pm 6\omega 0$, and $2\omega c$. These frequencies are different from the frequency of current harmonics at $\omega c \pm \omega 0$ and $2\omega c \pm 2\omega 0$ that are affected by $I_{\text{inv}}$. Moreover, $I_{\text{inv}}$ at $2\omega 0$ is much more sensitive to the variation of $\theta_{\text{inv}}$ than the other 2-9 kHz current harmonics. It is also realised that $\theta_{\text{inv}}$ does not affect the amplitude of the current harmonics between 2 to 9 kHz, as $\Delta I_{\text{inv},h} = 0\%$ when $\theta_{\text{inv}}$ varies 70%.

![FIGURE 5. Investigating the impact of variation in $I_{\text{in}}$ on the current harmonics of $I_{\text{inv}}$ (a) Around switching frequency, 3 kHz. (b) Near $2\omega c$, 6 kHz.](image)
In the next section, laboratory tests are performed for investigating the above-mentioned analytical study further.

V. EXPERIMENTAL TESTS FOR INVESTIGATING THE CURRENT HARMONICS OF $i_{inv}$ AT UNBALANCED-LOAD CONDITION

The proposed equation was validated using simulation results in section III, so the experimental results are compared with the simulation results in this section to further verify the analytical study. For that purpose, the setup shown in Fig. 10 was constructed by a 7.5 kW three-phase Adjustable Speed Drive (ASD) and a Chroma 61511 Programmable Grid Simulator. The $f_0$ and $f_c$ of the ASD were adjusted 50 Hz and 3 kHz, and the ASD was connected to the Grid Simulator. Two equal 80 Ω resistors were also connected to phases a and b at the output of this ASD, whereas a 100 Ω resistor was connected to its phase c. Then, three similar 40 mH inductors were put in series with these resistors to resemble a motor load. The higher resistance at phase c makes its current lower than the other two phases, so it creates unbalanced-load condition. To clarify this, the measured waveforms of load side currents: $i_a$, $i_b$, and $i_c$, and the current at the dc side $i_{inv}$ simultaneously captured from oscilloscopes are illustrated in Fig. 11. As can be seen in Fig. 11(a), the amplitudes of $i_a$ and $i_b$ are almost 0.5 A higher than the amplitude of $i_c$. These unbalanced currents affect $i_{inv}$ amplitude as can be realised from Fig. 11(b). It should be noted that the currents $I_+$ and $I_-$ are calculated 3.51 A and 0.25 A, and $\theta_+$ and $\theta_-$ are found 48° and 72° in this condition. It is also worth mentioning that that the degree of the unbalanced-load current is 9%, which is found based on the amplitude of current with maximum deviation from the average value of current at phases a to c.

The parameters of the experimental setup have been modelled in MATLAB Simulink in the next step. The simulation model is made based on the measured value of parameters in a practical 7.5 kW industrial ASD. Besides, the measured voltages at the grid side have been used to synchronize the simulation and experimental results and consider the amplitude and phase angle of grid voltage during the measurements. It is noted that the nonlinear characteristic of the circuit parameters in the frequency range of 2-9 kHz has been ignored to simplify the analysis. Moreover, the switching function of ASD’s inverter at phases a, b, and c has been defined similar to the switching function of the experimental setup.
Furthermore, resistors and inductors have been connected to the ASD’s output to model the unbalanced load in the simulations.

Fig. 12 compares the simulation and experimental results in the time domain. This figure reveals that $i_{\text{inv}}$ can be estimated with promising accuracy using the simulation platform. The small errors in this comparison can be categorised into two groups:

- The errors at the peak of waveforms. The reason for this error is that the load’s inductors have nonlinear characteristic with regards to the frequency, whereas they are simply modelled by linear inductors.
- The deviation of waveforms at the switching instants. This is because of the fact that the model of inverter switches in the simulation model is not exactly similar to the practical switches in the setup.

Fig. 13 indicates the 0-9 kHz harmonic components of measured and simulated $i_{\text{inv}}$ depicted in Fig. 12. This figure shows that the main current harmonic of $i_{\text{inv}}$ below 2 kHz is at 100 Hz, which is in line with the conclusion from (23). The value of this current harmonic is almost similar 0.2 A for both simulation and experimental results. As can be seen, both experimental and simulation results of $i_{\text{inv}}$ have similar 2-9 kHz harmonic distortions, especially at significant harmonics. Table 3 compares the main current harmonics measured from experimental tests and simulations between 0 and 9 kHz for further clarification. This table shows that the current harmonics from measurements and experimental tests are quite similar although the actual values of the current harmonics are low. The sources of these errors are mainly the difference between the model of switches in practice and
simulation models, nonlinear effects of magnetic elements, and accuracy of the measuring devices and noises.

![Figure 12](image-url) **FIGURE 12.** Comparing the experimentally measured \( i_{inv} \) at the 3 kHz switching frequency and the simulation results for verifying the accuracy of the simulation platform at unbalanced-load condition.

![Figure 13](image-url) **FIGURE 13.** Comparing the harmonic components of measured \( i_{inv} \) and the simulation results when \( f_c = 3 \text{ kHz} \) for investigating the accuracy of the analytical analysis at unbalanced-load condition.

It can be observed in Fig. 13 that the significant 2-9 kHz current harmonics around 3 kHz are at 2.55 kHz, 2.85 kHz, and 3.45 kHz. The frequency of these current harmonics around 6 kHz is 5.7 kHz, 6 kHz, and 6.3 kHz. Also, 8.55 kHz and 8.85 are the most significant 2-9 kHz current harmonics around 3 times \( f_c = 3 \text{ kHz} \). Consequently, it can be realised that the 2-9 kHz current harmonics with frequencies \( m\omega_c - (1 - k)\omega_0 \) when \( k \) denotes \( 3n+1 \) and \( m\omega_c + (1 + k)\omega_0 \) at \( k=3n-1 \) are affected by \( I_L \). On the other hand, it can be observed in Fig. 13 that the current harmonics are not zero at 2.95 kHz and 3.05 kHz around 3 kHz, and 5.9 kHz and 6.1 kHz around 6 kHz although the amplitude of these harmonics is small. These current harmonics are due to \( I_L \), so their values are increased when \( I_L \) increases. In general, these current harmonics have frequencies equal to \( m\omega_c - (1 - k)\omega_0 \) at \( k=3n+1 \) and \( m\omega_c + (1 + k)\omega_0 \) for \( k=3n-1 \). Thus, the experimental results confirm the current harmonic prediction using the analytical analysis presented for the unbalanced-load condition.

In the next section, the impact of voltage harmonics across dc-link on the current harmonics at the dc-link of an ASD considering an unbalanced-load condition is discussed. Besides, the propagation of these current harmonics to the grid-side of the ASD for different type of dc-link filters at the unbalanced-load condition is studied.

**TABLE 3.** The main 0-9 kHz current harmonics of \( i_{inv} \) calculated from simulation and experimentally recorded data when ASD’s load is unbalanced and \( f_c = 3 \text{ kHz} \).

| Frequency (Hz) | Experimental (A) | Simulation (A) | Error (A) |
|---------------|-----------------|---------------|-----------|
| 100           | 0.21            | 0.2           | 0.01      |
| 2850          | 0.45            | 0.46          | 0.01      |
| 2950          | 0.02            | 0.02          | 0         |
| 3050          | 0.07            | 0.06          | 0.01      |
| 3150          | 0.48            | 0.49          | 0.01      |
| 5700          | 0.21            | 0.22          | 0.01      |
| 5900          | 0.06            | 0.07          | 0.01      |
| 6000          | 0.44            | 0.48          | 0.04      |
| 6100          | 0.03            | 0.03          | 0         |
| 6300          | 0.25            | 0.26          | 0.01      |
| 8550          | 0.15            | 0.15          | 0         |
| 8850          | 0.07            | 0.08          | 0.01      |

**VI. DISCUSSION ON VOLTAGE HARMONICS ACROSS DC-LINK AND THE PROPAGATION OF CURRENT HARMONICS TO THE GRID CONSIDERING DIFFERENT FILTER TYPES**

The voltage harmonics across dc-link can generate current harmonics with regards to the different filter parameters and configurations. These current harmonics might affect the current harmonics caused by \( i_{inv} \) across dc-link, so they can increase the amplitude of input current harmonics of an ASD.

![Figure 14](image-url) **FIGURE 14.** The simplified model of the under-study ASD for discussing the voltage harmonics across dc link and the propagation of current harmonics to the grid side.

**A. VOLTAGE HARMONICS ACROSS DC-LINK**

Fig. 14 shows the simplified model of an ASD for discussing the voltage harmonics across dc link and the propagation of current harmonics to the grid side. In this figure, \( u_{dc} \) is the dc-link voltage, which is generated based on the voltage of the grid at phases \( u, v \) and \( w \), \( u_u, u_v \) and \( u_w \). The values of \( u_{dc}, u_{in}, i_{rec} \) and \( i_u \) in Fig. 14 for a typical 7.5 kW ASD connected to an unbalanced load are demonstrated in Fig. 15. This figure depicts that \( u_{dc} \) is a dc value with some ripples, which affects the \( i_{rec} \) at the output of the front-end rectifier. The ripples of \( i_{rec} \) then can be transferred to the grid side with regards to the rectifier switching function. The variable \( u_{dc} \) and its harmonics in this figure can be found according to the front-end rectifier’s function as given by (28).

\[
u_{dc} = u_{uN} s_u + u_{vN} s_v + u_{wN} s_w
\]  

(28)
It is noted that the switching function of the rectifier at phase \( u \) is the phase angle of the voltage waveform. Note that the equation for the voltage of other phases \( u_N \) and \( u_N \) can be found by replacing \( \omega_g t \) in (29) with \( \omega_g t \pm \frac{2\pi}{3} \). The switching function of this rectifier at phase \( u \) and the angular frequency of grid voltage, and \( \theta_{u,k} \) is as given below.

\[
\begin{align*}
  s_u & = \frac{4}{\pi} \sum_{k=0}^{+\infty} \cos \left( \frac{(6k+1)\pi t}{6} \right) \sin \left( (6k + 1)\omega_g t + \theta_{u,6k+1} \right) \\
  & + \frac{4}{\pi} \sum_{k=1}^{+\infty} \cos \left( \frac{(6k-1)\pi t}{6} \right) \sin \left( (6k - 1)\omega_g t + \theta_{u,6k-1} \right) 
\end{align*}
\] (30)

It is noted that the switching function of the rectifier at the other phases \( s_v \) and \( s_w \) can be derived by replacing \( \omega_g t \) in (30) with \( \omega_g t \pm \frac{2\pi}{3} \). Subsequently, the equation for \( u_{dc} \) can be derived by replacing the equations (29) and (30) in (28) as given below.

\[
U_{dc} = U_{dc,0} + \sum_{k=1}^{+\infty} U_{dc,6k} \sin(6k\omega_g t + \theta_{u,6k}) 
\] (31)

where \( U_{dc,0} \) and \( U_{dc,k} \) are the dc value of the voltage \( u_{dc} \) and the amplitude of its harmonics, and \( \theta_{u,6k} \) is the phase angle of the \( u_{dc} \) harmonics at an order equal to 6\( k \). This equation shows that the voltage harmonics of voltage across dc-link \( u_{dc} \) are at a frequency equal to \( 6k\omega_g \). To investigate this further, the voltage harmonics across the dc link for a 7.5 kW ASD at an unbalanced-load condition are represented in Fig. 16. This figure confirms that the voltage harmonics across dc-link are at \( 6k\omega_g \), which is in line with the (31).

These voltage harmonics are applied over the dc-link, so they can generate current harmonics at the frequency equal to their frequency. The current harmonics due to \( U_{dc,k} \) are then added to \( I_{inv,k} \) across dc-link at the output current of rectifier \( I_{rec} \) as represented by Fig. 15. However, the filter configurations and the value of filter parameters can significantly affect the current harmonics across dc-link. The reason for this is that the current harmonics of \( I_{rec} \) due to the \( U_{dc,h} \) depend on the dc-link impedance. Besides, the amplitudes of \( I_{inv} \) current harmonics that are transferred to the rectifier side across dc link are affected by the filter configurations, especially the damping effect of dc-link capacitor at high frequencies. To clarify the effect of ASD’s filter configurations, the filter configurations: ac choke, dc choke and slim dc capacitors have been considered in this analysis as illustrated in Fig. 17. These three are the common filter configurations used in the industry for different applications [16], [27]. The values of \( L_{dc} \) and \( C_{dc} \) for dc-choke filter configuration was set 2.5 mH and 500 \( \mu \)F as explained in [16]. Additionally, the value of parameters for ac choke have been adjusted to \( L_{ac} = 1.5 \) mH and \( \epsilon_{ac} = 500 \) F, and the slim dc capacitors \( C_{slim} \) was adjusted 30 \( \mu \)F.

**FIGURE 15.** Simulation results of \( u_{dc}, u_{ac}, I_{rec} \) and \( i_u \) of the under-study 7.5 kW ASD with the dc-choke filter configuration for clarifying the voltage harmonics across dc link and the propagation of current harmonics to the grid side when the load is unbalanced.

**FIGURE 16.** The voltage harmonics across dc-link at unbalanced-load conditions when \( f_c = 3 \) kHz.

**FIGURE 17.** Common filter configurations for an ASD: a) ac-choke, b) dc-choke and c) slim dc link capacitor to investigate the impact of dc-link filter configuration on the current harmonics across dc link.
Afterward, new simulations have been performed considering all these filter configurations with similar loading conditions. The current harmonics of \( i_{\text{rec}} \), \( I_{\text{rec},h} \) measured when either dc-choke, ac-choke or slim dc capacitor were used as filter at the unbalanced-load condition are illustrated in Fig. 18. This figure demonstrates that the slim dc capacitor configuration results in less current harmonics below 2 kHz than the other two filter configurations. However, this configuration does not properly damp the 2-9 kHz current harmonics generated by the inverter. On the other hand, although both ac-choke and dc-choke configurations result in higher low-order current harmonics (below 2 kHz), they damp a high portion of current harmonics due to the switching of the inverter. The main reason for this different pattern is that the value of the dc-link capacitor in ac-choke and dc-choke configurations is much higher than this value for the slim dc filter configuration.

![Figure 19](image)

**Figure 19.** \( i_{\text{rec}}, s_u \) and \( i_u \) at the same time for the under-study 7.5 kW ASD when the load is unbalanced to show how the current across dc-link propagates to the grid.

The amplitude of these current harmonics, however, depends on the dc-link filter types and filter parameters’ value. Fig. 20 illustrates the measured current harmonics at the grid side of the 7.5 kW ASD at the three filter configurations: (a) dc-choke, (b) ac-choke and (c) slim dc capacitor. It can be realized from this figure that in an unbalanced-load condition relatively high value of current harmonic is generated at 150 Hz. The value of this current harmonics is almost 2 A for when dc-choke or ac-choke is used, whereas it decreases to 0.8 A for slim dc capacitor configuration. The slim dc capacitor also has better performance at 250 and 350 Hz compared to the other two filter configurations. On the other hand, both the ac and dc choke configurations damp the current harmonics between 2-9 kHz much better than the slim dc capacitor.

The main contributions of this work are given as follows.

- Proposing an analytical equation for estimating \( i_{\text{inv}} \) at unbalanced-load condition. With this equation, the inverter at an ASD model can be simply replaced by a current source considering its load conditions. This can improve the accuracy of ASD model and decrease the simulation time, as considering the PWM and the operation of inverter switches increase the computational burden. In fact, the main advantage of applying this equation is in the modelling of multi-converter

\[
i_u = \frac{4}{\pi} \sum_{h=1}^{+\infty} \sum_{k=0}^{+\infty} \cos\left(\frac{(6k+1)\pi}{6k+1}\right) I_{\text{rec},h} \times \sin\left((6k+1)\omega_g t + \theta_{su,6k+1}\right) + \frac{4}{\pi} \sum_{h=1}^{+\infty} \sum_{k=0}^{+\infty} \cos\left(\frac{(6k+1)\pi}{6k+1}\right) \times I_{\text{rec},h} \sin\left((6k-1)\omega_g t + \theta_{su,6k-1}\right)
\]

at an unbalanced-load condition according to (5). Hence, the expansion \( \sin A \cos B = 0.5[\sin(A + B) + \sin(A - B)] \) can be employed to find the frequency of current harmonics at the grid side, as \( I_{\text{rec},h} \) is multiplied by \( \sin(6n\pm1)\omega_g t \). By doing this, the frequency of these harmonics transferred to the grid side can be found as represented in Table 4.

**TABLE 4.** The frequency of the current harmonics at the dc side and when they are transferred to the grid side.

| De-link harmonics frequency | Grid side harmonics frequency |
|-----------------------------|-------------------------------|
| \( \frac{2f_0}{6k \cdot f_g} \pm m \cdot f_e \pm (1 \pm 1)f_0 \) | \( \frac{(6n \pm 1)f_0 \pm 2f_0}{(6n \pm 6k \pm 1)f_g} \) |

The main advantages of applying this equation in the modelling of multi-converter
that are affected by positive-sequence current are different from the current harmonics determined by negative-sequence current. Moreover, the equation for determining the impact of positive- and negative-sequence currents on the current harmonic between 2-9 kHz has been found. Finally, experimental tests have been performed for verifying the proposed equation and the result of these analytical investigations. These experimental tests validated the accuracy of the analysis presented for the impact of unbalanced-load on the 0-9 kHz current harmonics at the dc side of the inverter.

VII. CONCLUSION

This article investigates the impact of an unbalanced-load condition on 0-9 kHz current harmonics across dc-link in an Adjustable Speed Drive (ASD). For that purpose, an analytical equation for estimating the current at the dc link of inverter is developed. Using this equation, the current at the dc link of inverter can be calculated based on the positive- and negative-sequence currents of load. Then, the impact of unbalanced currents on the current harmonics of inverter at the dc side is analysed by employing the proposed equation. These analyses reveal that new current harmonic orders are also realised that the amplitude of this current harmonic is significantly increases when the inverter of multiple converters is modelled in detail.

• Finding 2-9 kHz current harmonics generated by an ASD due to the inverter switching function. These current harmonics are the crucial topic of IEC standardisation IEC SC77A. The reason for this is that 2-9 kHz current harmonics have not been considered in the existing International Standards. Hence, the working groups IEC SC77A WG1, WG8, and WG9 have been investigating the 2-9 kHz current harmonics to propose the emission limits and compatibility levels for them.

• Estimating the impact of positive- and negative-sequence currents on the 2-9 kHz current harmonics of $i_{\text{inv}}$. This can help the drive manufacturer companies to easily find the current harmonics across dc link of ASD considering the unbalanced-load condition. Thus, the accurate value of these current harmonics can be considered in designing filter components. On the other hand, converters need to comply with the new emission limits and compatibility levels about the 2-9 kHz harmonics proposed by International Standardisation committees in future. Consequently, the proposed equation can be applied in optimisations to keep the 2-9 kHz current harmonics of an ASD below a certain level.

REFERENCES

[1] Y. Li, H. Lin, H. Huang, C. Chen, and H. Yang, “Analysis and performance evaluation of an efficient power-fed permanent magnet adjustable speed drive,” IEEE Trans. Ind. Electron., vol. 66, no. 1, pp. 784–794, Jan. 2019.
[2] D. G. Holmes and T. A. Lipo, Pulse Width Modulation for Power Converters: Principles and Practice, Hoboken, NJ, USA: Wiley, 2003.
[3] D. Wu, H. Qamar, H. Qamar, and R. Ayyanar, “Comprehensive analysis and experimental validation of 240°-clamped space vector PWM technique eliminating zero states for VE traction inverters with dynamic DC link,” IEEE Trans. Power Electron., vol. 35, no. 12, pp. 13295–13307, Dec. 2020.
[4] S. K. Sahoo and T. Bhattacharya, “Phase-shifted carrier-based synchronized sinusoidal PWM techniques for a cascaded H-bridge multilevel inverter,” IEEE Trans. Power Electron., vol. 33, no. 1, pp. 513–524, Jan. 2018.
[5] M. Ye, L. Chen, L. Kang, S. Li, J. Zhang, and H. Wu, “Hybrid multi-carrier PWM technique based on carrier reconstruction for cascaded H-bridge inverter,” IEEE Access, vol. 7, pp. 53152–53162, 2019.
[6] K. Shukla and R. Maheshwari, “Implementation of 3L DPWM techniques for parallel interleaved 2L VSIs,” IEEE Trans. Ind. Appl., vol. 55, no. 6, pp. 7604–7613, Nov. 2019.
[7] S. M. Dabour, A. S. Abdel-Khalik, A. M. Massoud, and S. Ahmed, “Analysis of scalar PWM approach with optimal common-mode voltage reduction technique for five-phase inverters,” IEEE J. Emerg. Sel. Topics Power Electron., vol. 7, no. 3, pp. 1854–1871, Sep. 2019.
[8] C. A. Reineri, J. C. G. Targaron, and N. G. Campetelli, “Unbalance on power systems: A general review,” in Proc. 8th Latin-Am. Congr. Electr. Gener. Transmiss., 2009, pp. 1–8.
[9] S. Jiao, K. R. Ramachandran Potri, K. Rajashekara, and S. K. Pramanick, “A novel DROGI-based detection scheme for power quality improvement using four-leg converter under unbalanced loads,” IEEE Trans. Ind. Appl., vol. 56, no. 1, pp. 815–825, Jan. 2020.
[10] D. Kumar and F. Zare, “A comprehensive review of maritime microgrids: System architectures, energy efficiency, power quality, and regulations,” IEEE Access, vol. 7, pp. 67249–67277, 2019.
[11] A. Ganjavi, H. Rathnayake, F. Zare, D. Kumar, J. Yaghoobi, P. Davari, and A. Abbosh, “Common-mode current prediction and analysis in motor drive systems for the new frequency range of 2–150 kHz,” IEEE J. Emerg. Sel. Topics Power Electron., early access, Jul. 3, 2020, doi: 10.1109/JESTPE.2020.3006878.
[12] K. Lee, G. Venkataramanan, and T. M. Jahns, “Source current harmonic analysis of adjustable speed drives under input voltage unbalance and sag conditions,” in Proc. 11th Int. Conf. Harmon. Qual. Power, Sep. 2004, pp. 579–587.
[13] F. Zare, H. Soltani, D. Kumar, P. Davari, H. A. M. Delpino, and F. Blaabjerg, “Harmonic emissions of three-phase diode rectifiers in distribution networks,” IEEE Access, vol. 5, pp. 2819–2833, 2017.
[14] D. Kumar, P. Davari, F. Zare, and F. Blaabjerg, “Analysis of three-phase rectifier systems with controlled DC-link current under unbalanced grids,” in Proc. IEEE Appl. Power Electron. Conf. Expo. (APEC), Mar. 2017, pp. 2179–2186.
[15] H. Wang, P. Davari, H. Wang, D. Kumar, F. Zare, and F. Blaabjerg, “Lifetime estimation of DC-link capacitors in adjustable speed drives under grid voltage unbalances,” IEEE Trans. Power Electron., vol. 34, no. 5, pp. 4064–4078, May 2019.
[16] D. Kumar and F. Zare, “Harmonic analysis of grid connected power electronic systems in low voltage distribution networks,” IEEE Trans. Emerg. Sel. Topics Power Electron., vol. 4, no. 1, pp. 70–79, Mar. 2016.
K. Nishizawa, J.-I. Itoh, A. Odaka, A. Toba, and H. Umida, “Current harmonic reduction based on space vector PWM for DC-link capacitors in three-phase VSIs operating over a wide range of power factor,” *IEEE Trans. Power Electron.*, vol. 34, no. 5, pp. 4853–4867, May 2019.

B. P. McGrath and D. G. Holmes, “A general analytical method for calculating inverter DC-link current harmonics,” *IEEE Trans. Ind. Appl.*, vol. 45, no. 5, pp. 1851–1859, Sep./Oct. 2009.

J. Yaghoobi and F. Zare, “Impacts of three-phase power converter operating modes on harmonic emissions in distribution networks: harmonics emission within 2–9 kHz,” *IET Power Electron.*, vol. 13, no. 13, pp. 2935–2942, 2020.

J. Yaghoobi, F. Zare, and H. Rathnayake, “Current harmonics generated by motor-side converter: New standardizations,” *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 9, no. 3, pp. 2868–2880, Jun. 2021.

R. Chen, J. Niu, H. Gui, Z. Zhang, F. Wang, L. M. Tolbert, B. J. Blalock, D. J. Costinnet, and B. B. Choi, “Analytical analysis of AC and DC side harmonics of three-level active neutral point clamped inverter with space vector modulation,” in *Proc. IEEE Appl. Power Electron. Conf. Expo. (APEC)*, Mar. 2019, pp. 112–119.

H. Soltani, F. Blaabjerg, F. Zare, and P. C. Loh, “Effects of passive components on the input current interharmonics of adjustable-speed drives,” *IEEE J. Emerg. Sel. Topics Power Electron.*, vol. 4, no. 1, pp. 152–161, Mar. 2016.

D. P. Manjure and E. B. Makram, “Impact of unbalance on power system harmonics,” in *Proc. 10th Int. Conf. Harmon. Quality Power*, vol. 1, Oct. 2002, pp. 328–333.

D. Basic, “Input current interharmonics of variable-speed drives due to motor current imbalance,” *IEEE Trans. Power Del.*, vol. 25, no. 4, pp. 2797–2806, Oct. 2010.

S. Sakar, S. Ronnberg, and M. Bollen, “Interharmonic emission in AC–DC converters exposed to nonsynchronized high-frequency voltage above 2 kHz,” *IEEE Trans. Power Electron.*, vol. 36, no. 7, pp. 7705–7715, Jul. 2021.

X. Pei, W. Zhou, and Y. Kang, “Analysis and calculation of DC-link current and voltage ripples for three-phase inverter with unbalanced load,” *IEEE Trans. Power Electron.*, vol. 30, no. 10, pp. 5401–5412, Oct. 2015.

H. Soltani, P. Davari, D. Kumar, F. Zare, and F. Blaabjerg, “Effects of DC-link filter on harmonic and interharmonic generation in three-phase adjustable speed drive systems,” in *Proc. IEEE Energy Convers. Congr. Expo. (ECCE)*, Oct. 2017, pp. 675–681.

JALIL YAGHOOBI (Member, IEEE) was born in Zahedan, Iran, in 1985. He received the B.Sc. and M.Sc. degrees in electrical engineering from the Sharif University of Technology (SUT), Tehran, Iran, in 2008 and 2011, respectively, and the Ph.D. degree in power engineering from The University of Queensland (UQ), Brisbane, QLD, Australia, in 2016. Since 2017, he has been a Postdoctoral Research Fellow with the School of Information Technology and Electrical Engineering (ITEE), Faculty of Engineering, Architecture, and Information Technology (EAIT), UQ. He has authored 25 articles, including ten international journal articles and 15 international conference papers. His research interests include power quality with a focus on low- and high-frequency harmonics, power electronics, renewable energy integration in power systems, and energy efficiency. He has been an Active Member of the IEEE Power and Energy Society, since 2012.

FIRUZ ZARE (Fellow, IEEE) received the Ph.D. degree in power electronics from the Queensland University of Technology, Australia, in 2002. He has over 20 years of experience in academia, industry, and international standardization committees, including eight years in two large research and development centres working on power electronics and power quality projects. He is currently a Power Electronics Professor and the Head of the School of Electrical Engineering and Robotics, Queensland University of Technology, Australia. He is also the Task Force Leader (International Project Manager) of Active Infeed Converters to develop the first international standard IEC 61000-3-16 within the IEC standardization SC77A. He has published four books, over 280 journal articles and conference papers, five patents, and over 40 technical reports. His main research interests include power electronics topology, control and applications, power quality and regulations, and pulsed power applications.

Prof. Zare has received several awards, such as an Australian future fellowship, the John Madsen medal, a symposium fellowship, and the early career excellence research award. He is a Senior Editor of IEEE Access journal, and a Guest Editor and an Associate Editor of the IEEE JOURNAL OF EMERGING AND SELECTED TOPICS IN POWER ELECTRONICS, an Associate Editor of IET journal, and an editorial board member of several international journals.

DINESH KUMAR (Senior Member, IEEE) received the M.Tech. degree in power system engineering from the Indian Institute of Technology (IIT) Roorkee, Roorkee, India, in 2004, and the Ph.D. degree in power electronics from the University of Nottingham, U.K., in 2010. From 2004 to 2005, he served as a Lecturer for the Department of Electrical Engineering, National Institute of Technology, Kurukshetra, India. In 2006, he joined the Chemnitz University of Technology, Germany, as a Research Fellow in power electronics. From 2006 to 2010, he investigated and developed matrix converter-based multidevice system for aerospace applications. Since 2011, he has been with Danfoss Drives A/S, Denmark, where he is involved in many research and industrial projects. His current research interests include motor drive, harmonic analysis and mitigation techniques, power quality and electromagnetic interference in power electronics. He is a member of the IEC standardization Working Group in TC77A and SyC LVDC Committee. He is the Editor-in-Chief of International Journal of Power Electronics and an Associate Editor of IEEE TRANSACTIONS ON INDUSTRY APPLICATIONS, and IEEE Access journal, and a member of IEEE Transportation Electrification eNewsletter.

ARASH MORADI (Member, IEEE) received the B.S. degree in electrical engineering from the Shahid Chamran University of Ahvaz, Ahvaz, Iran, in 2014, and the M.Sc. degree (Hons.) in electrical power system engineering from the University of Isfahan, Isfahan, Iran, in 2016. He is currently pursuing the Ph.D. degree in electrical engineering with The University of Queensland, Brisbane, QLD, Australia.

From 2017 to 2019, he worked as a Design and Project Expert of high-power substations with Monenco Iran Consulting Company. His research interests include power quality, power electronics, and power system protection.