Superluminality in beyond Horndeski theory with extra scalar field

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Abstract
We study the superluminality issue in beyond Horndeski theory with additional scalar field, which is minimally coupled to gravity and has no second derivatives in the Lagrangian. We present the quadratic action for perturbations in cosmological backgrounds, stability conditions and expressions for sound speeds. We find that in the case of conventional additional scalar whose flat-space propagation speed is that of light, one of the modes in interacting theory is necessarily superluminal when this scalar rolls, even arbitrarily slowly. This result holds in any theory of the beyond Horndeski class (with 6 arbitrary functions in the Lagrangian) and for any stable rolling background. More generally, the requirement of the absence of superluminality imposes non-trivial constraints on the structure of the theory.

Keywords: beyond Horndeski theories, stability conditions, superluminality

(Some figures may appear in colour only in the online journal)

1. Introduction

A class of scalar-tensor theories of gravity—Horndeski theories [1] and their extensions [2–4]—has proved itself promising candidate for supporting various cosmological scenarios including those without the initial singularity. What makes (beyond) Horndeski theories and more general DHOST theories [5] suitable for constructing non-singular cosmological solutions is their ability to violate the Null Energy Condition (NEC)/Null Convergence Condition (NCC) while leaving the stability of the background intact (for a review see, e.g., [6]).

Even though the NEC/NCC can be safely violated in unextended Horndeski theories, the latter do not enable one to construct non-singular spatially flat cosmological solutions which are stable during the entire evolution [7, 8]. On the contrary, beyond Horndeski and DHOST theories admit completely stable cosmologies with a bouncing or Genesis stage, see [9–14] for specific examples and [4, 15] for topical reviews.

Another characteristic feature of modified gravities is potential appearance of superluminal perturbations. The issue of superluminality in Horndeski theories has been addressed from different viewpoints, see [16–20] and references therein. One of the most striking findings is that at least in a pure Horndeski Genesis model of [18], addition of even tiny amount of external matter (ideal fluid) inevitably induces superluminality in some otherwise healthy region of phase space [19]. The latter fact is troublesome (provided one would like to avoid superluminality altogether in view of arguments of [21]), since nothing appears to prevent adding extra fluid to Horndeski theory. Likewise, superluminality has been shown...
to occur in other stable non-singular cosmological backgrounds: in Cuscuton gravity [22] and in DHOST theory [23].

A step forward has been recently made in [24], where a beyond Horndeski model admitting a completely stable bouncing solution has been analyzed from the viewpoint of potential superluminality. As opposed to Genesis-supporting unextended Horndeski model with external matter [19], it has been shown that a specifically designed beyond Horndeski Lagrangian, which on its own admits a stable and subluminal bouncing solution, remains free of superluminalities upon adding extra matter in the form of perfect fluid with equation of state parameter \( w \leq 1/3 \) (or even somewhat larger).

On the other hand, by analysing the general expressions for the sound speeds of scalar modes in the system ‘beyond Horndeski + perfect fluid’, it has been found that for \( w \) equal or close to 1, one of the scalar propagation speeds inevitably becomes superluminal. The latter statement holds irrespectively of the cosmological scenario one considers, and is true for the most general beyond Horndeski theory [24]. This has to do with the fact, already noticed in [3, 25], that due to specific structure of beyond Horndeski Lagrangian, there is kinetic mixing between matter and Galileon perturbations, and hence the sound speeds of both scalar modes get modified (the superluminal one is predominantly sound wave in matter). The results of [24] imply that in beyond Horndeski theory with an additional minimally coupled conventional scalar field, whose flat-space propagation speed is that of light, one of the scalar modes is superluminal when this extra field has small but non-zero background kinetic energy. The main purpose of this note is to derive this property explicitly. We emphasize that superluminality is generic for beyond Horndeski theory of the most general form.

\[ S_\chi = \int d^4x \sqrt{-g} \left( \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 \right), \]

\[ \mathcal{L}_2 = F(\pi, X), \]

\[ \mathcal{L}_3 = K(\pi, X) \Box \pi, \]

\[ \mathcal{L}_4 = -G_4(\pi, X) X \Box + 2G_{4X}(\pi, X) \]

\[ \times \left( [(\Box \pi)^2 - \pi_{,\mu \nu} \pi^{,\mu \nu}] + F_5(\pi, X) e^{\mu \nu \rho \sigma} e^{,\mu \nu \rho \sigma} \pi_{,\mu \nu} \pi_{,\rho \sigma} \right), \]

\[ \mathcal{L}_5 = G_5(\pi, X) G^{\mu \nu} \pi_{,\mu \nu} + \frac{1}{3} G_{5X} \]

\[ \times \left( \Box \pi^3 - 3 \Box \pi \pi_{,\mu \nu} \pi^{,\mu \nu} + 2 \pi_{,\mu \pi} \pi_{,\nu \pi} \pi_{,\rho \pi} \right) + F_5(\pi, X) e^{\mu \nu \rho \sigma} e^{,\mu \nu \rho \sigma} \pi_{,\mu \pi} \pi_{,\nu \rho} \pi_{,\rho \sigma} \pi_{,\sigma \pi}, \]

where \( \pi \) is a scalar field sometimes dubbed Galileon, \( X = g^{\mu \nu} \pi_{,\mu} \pi_{,\nu}, \pi_\mu = \partial_\mu \pi, \pi_{,\mu \nu} = \partial_\mu \partial_\nu \pi, \Box \pi = g^{\mu \nu} \partial_\mu \partial_\nu \pi, \]

\( G_{4X} = \partial G_4 / \partial X \), etc. The functions \( F, K, G_4 \) and \( G_5 \) are characteristic of unextended Horndeski theories, while non-vanishing \( F_4 \) and \( F_5 \) extend the theory to beyond Horndeski type. Along with the scalar field of beyond Horndeski type we consider another scalar field \( \chi \) in the form of k-essence

\[ S_\chi = \int d^4x \sqrt{-g} P(\chi, Y), \quad Y = g^{\mu \nu} \chi_{,\mu} \chi_{,\nu}. \]

The Lagrangian in equation (2) describes a minimally coupled scalar field \( \chi \) of the most general type (assuming the absence of second derivatives in the Lagrangian).

In flat space-time and for spatially homogeneous background (possibly rolling, \( Y = \chi^2 = 0 \), the stability conditions for the scalar field \( \chi \) have standard form

\[ P_Y > 0, \quad R \equiv P_Y + 2Y P_{YY} > 0, \]

while flat-space propagation speed of perturbations is

\[ c_{\text{lin}}^2 = \frac{P_Y}{R}. \]

Our main result on superluminality in section 3 applies most straightforwardly to the conventional scalar field with

\[ P = \frac{1}{2} Y - V(\chi), \]

but in this section we proceed in full generality and do not make any assumptions on the form of the function \( P(\chi, Y) \).

In what follows we consider cosmological setting with spatially flat FLRW metric and homogeneous background scalar fields \( \pi = \pi(t) \) and \( \chi = \chi(t) \) (\( t \) is cosmic time). Then

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7 This generalizes the formulas given in [4]; similar results have been obtained in ADM formalism in [3, 25].
the background gravitational equations following from the action $S_\phi + S_1$ read
\[
\delta g^{(0)}: F = 2 F_\perp X - 6 H K X \Pi X + K X + 6 H^2 G_4
+ 6 H G_4 X \Pi - 24 H^4 X (G_{4x} + G_{4xx} X)
+ 12 H G_{4xx} X \Pi - 2 H^2 X (5 G_{5x} + 2 G_{5xx} X)
+ 3 H^3 X (3 G_{6x} + 2 G_{5xx} X)
+ 6 H^2 F_3 (S_3 + 2 F_3 X) + 6 H^2 X^2 \Pi (7 F_3 + 2 F_3 X) + P - 2 \Pi Y = 0, \tag{6a}
\]
\[
\delta g^{\mu \nu}: F - X (2 K_\perp X + K_\perp) + 2 (\Pi^2 + 2 H) G_4
- 12 H G_{4x} X - 8 H G_{4xx} X - 8 H G_{4xxx} X
- 16 H G_{4xx} X \Pi + 2 (\Pi^2 + 2 H) G_4
+ 4 X G_{4xx} (\Pi - 2 H \Pi) + 2 X G_{4xx}
- 2 X G_{3x} (2 H^2 + 2 H \Pi + 3 H^2 \Pi)
+ G_{5x} (3 H^2 X + 2 H X + 4 H^2 \Pi) - 4 H^2 G_{5xx} X \Pi
+ 2 H G_{5xx} X (2 \Pi - H X) + 2 H G_{5xxx} X
+ F_3 (5 H^2 X + 2 H X + 8 H \Pi)
+ 8 H F_3 X^2 \Pi + 4 H F_3 X^2 \Pi + 6 H F_3 X^2
+ (2 H^2 + 2 H \Pi + 5 H \Pi) + 12 H^2 F_3 X \Pi^2
+ 6 H^2 F_3 X^3 + P = 0, \tag{6b}
\]
where $F_\perp \equiv \partial F / \partial Y$, and $H \equiv \dot{a} / a$ is the Hubble parameter. The field equation for the additional scalar field $\chi$ is:
\[
\ddot{\chi} + 3 c_m^2 H \dot{\chi} - \frac{P_y - 2 Y P_v}{2 R} = 0. \tag{7}
\]
The field equation for Galileon $\pi$ follows from the gravitational equations (6), their derivatives and equation (7), so we do not give it here for brevity.

2.2. Quadratic action and stability conditions

To address stability and superluminarity issues, we calculate the quadratic action for perturbations about homogeneous background in terms of propagating degrees of freedom (DOFs). We make use of the standard ADM parametrization of the metric perturbations,
\[
ds^2 = N_0^2 dr^2 - \gamma_{ij}(dx^i + N^i dr)(dx^j + N^j dr), \tag{8}
\]
where
\[
N = 1 + \alpha, \quad N_i = \partial_i \beta, \quad \gamma_{ij} = a^2(t) e^{2 \left( \delta_{ij} + h_{ij}^T + \frac{1}{2} h_{ii}^T h_{jj}^T \right)}, \tag{9}
\]
and we have already used some part of gauge freedom by setting the longitudinal part of $\partial \gamma_{ij}$ equal to zero, $\partial_i \beta, \partial_i \gamma = 0$. Here the scalar sector consists of $\alpha, \beta, \zeta$ from equation (8) and scalar field perturbations $\delta \nu$ and
\[
\delta \chi \equiv \omega,
\]
while $h_{ij}$ denote tensor modes ($h_{ii} = 0, \partial_i h_{ij}^T = 0$). Like in [24] we adopt the unitary gauge where $\delta \nu = 0$. Then the quadratic action for beyond Horndeski theory (1) reads [11]:
\[
S_\phi^{(2)} = \int dt \, d^3 x \, a^2 \left[ \frac{G_T(h_{ij}^T)^2}{8} - \frac{\mathcal{F}_T}{8a^2} (\partial_i h_{ij}^T)^2 \right] + \left( -3 \mathcal{G}_T \zeta^2 + \mathcal{F}_T \nabla^2 \zeta^2 \right)
+ \alpha^2 - 2 (G_T + D \xi) \nabla^2 \zeta^2 \right]
+ 6 \Theta \alpha^2 - 2 \Theta \nabla^2 \zeta^2 + 2 \mathcal{G}_T \zeta^2 \nabla^2 \alpha^2 \right], \tag{10}
\]
with $(\nabla \zeta)^2 = \delta \partial \zeta \partial \zeta$, $\nabla^2 \zeta = \delta \partial \zeta \partial \zeta$ and
\[
\mathcal{G}_T = 2 G_4 - 4 G_{4x} X + 2 G_{4xx} X
- 2 H G_{5x} X + 2 F_3 X^2 + 6 H F_3 X^2 \Pi, \tag{11a}
\]
\[
\mathcal{F}_T = 2 G_4 - 2 G_{5x} X - G_5 X, \tag{11b}
\]
\[
D = -2 F_3 X - 6 H F_3 X^3, \tag{11c}
\]
\[
\Theta = - K_\perp X + 2 G_4 H - 8 G_{4xx} X
- 8 H G_{4xx} X + 2 G_{4xx} X \Pi - 5 H^2 G_5 X \Pi
- 2 H G_{5xxx} X^2 + 3 H G_{5xx} X^2
+ 10 H F_3 X^2 + 4 H F_3 X^2 + 2 H^2 F_3 X^2 \Pi
+ 6 H^2 F_3 X^3 \Pi, \tag{11d}
\]
\[
\Sigma = F_3 X + 2 F_3 X^2 + 12 H K X \Pi
+ 6 H G_{4xx} X \Pi - K_\perp X + 2 K_\perp X^2 - 6 H^2 G_4
+ 4 H^2 G_{4xx} X + 9 H^2 G_{4xxx} X^4 + 24 H^4 G_{4xxx} X^5
- 6 H G_{4xx} X \Pi - 30 H G_{4xx} X \Pi
- 12 H G_{4xxx} X^2 \Pi + 30 H G_{5x} X \Pi
+ 26 H G_{5xxx} X^2 \Pi
+ 4 H^3 G_{5xxx} X^2 \Pi - 18 H^2 G_5 X \Pi
- 27 H^2 G_{5xx} X^2 - 6 H^2 G_{5xxx} X^3 - 90 H F_3 X^2
- 78 H F_3 X^3 - 12 H^2 F_3 X^4
- 168 H^3 F_3 X^2 \Pi - 102 H^2 F_3 X^3 \Pi
- 12 H^2 F_3 X^3 \Pi. \tag{11e}
\]
The first round brackets in equation (10) describe tensor sector, while the second ones refer to scalar modes. The quadratic action for $k$-essence (2) is as follows:
\[
S_\chi^{(2)} = \int dt \, d^3 x \, a^2 \left[ Y R \alpha^2 - 2 \chi \sigma R \alpha \omega
+ 2 \chi \sigma \nu \dot{\omega} \right]
+ \sigma \nabla^2 \beta - P_v \left( \nabla^2 \alpha \right)
- 6 \sigma \nu \zeta \dot{\omega} + (P_v - 2 Y P_v) \alpha \omega \right], \tag{12}
\]
where $\Omega = P_v / 2 - 3 H(\dot{P}_v - Y P_v) - \chi (P_v + 2 Y P_v)$. When deriving the actions (10) and (12) we used background equations (6a), which made the terms with $\alpha \zeta, \zeta^2$ and $\omega$ vanish. Let us now focus a moment concentrate on the scalar sector. According to the form of actions (10) and (12), $\alpha$ and $\beta$ are non-dynamical variables, so varying $S_\phi^{(2)} + S_\chi^{(2)}$ with respect to $\alpha$ and $\beta$ gives the following constraint equations, respectively:
\[
\Sigma_\alpha = - (\mathcal{G}_T + D \xi) \frac{\nabla \zeta^2}{a^2} + 30 \zeta - \Theta \frac{\nabla \zeta^2}{a^2} + Y R \alpha - \chi R \alpha \omega + \frac{1}{2} (P_v - 2 Y P_v) \omega = 0, \tag{13a}
\]
\[
\Theta_\alpha = - \mathcal{G}_T \zeta - \chi P_v \omega = 0. \tag{13b}
\]
By solving equations (13a) and (13b) for \((\nabla^2 \beta)/a^2\) and \(\alpha\) and substituting the result back into actions (10) and (12), one arrives at the quadratic action for scalar DOFs in terms of dynamical curvature perturbation \(\zeta\) and scalar field perturbation \(\omega^2\):

\[
S_{\zeta + \chi}^{(2)} = \int dt d^3x \frac{a^2}{2} \left[ \mathcal{G}_{AB} \nabla^A \zeta \nabla^B \chi + \Psi_1 \zeta \omega + \Psi_2 \omega^2 \right],
\]

where \(A, B = 1, 2\) and \(\nabla^A \chi = \zeta, \nabla^2 \chi = \omega^2\). Even though coefficients \(\Psi_1\) and \(\Psi_2\) are irrelevant for kinetic stability (absence of ghosts and gradient instabilities) as well as propagation speeds of \(\zeta\) and \(\omega\), they are given in Appendix for completeness. Kinetic matrices \(G_{AB}\) and \(F_{AB}\) have the following forms:

\[
G_{AB} = \begin{pmatrix} G_S + \frac{\partial}{\partial \Theta} YR - \frac{\partial}{\partial \Theta} \chi \hat{R} \\ -\frac{\partial}{\partial \Theta} \chi \hat{R} \\ R \\ R \end{pmatrix},
\]

\[
F_{AB} = \begin{pmatrix} F_S & -\frac{G_T + D\pi}{\Theta} \chi P_Y \\ -\frac{G_T + D\pi}{\Theta} \chi P_Y & P_Y \end{pmatrix},
\]

where

\[
G_S = \frac{\Sigma G_T^2}{\Theta^2} + 3G_T, \quad F_S = \frac{1}{a} \frac{d}{dt} \left[ \frac{G_T(G_T + 2D\pi)}{\Theta} \right] - \mathcal{F}_T.
\]

It is worth noting that both \(G_S\) and \(F_S\) are generally singular at \(\Theta = 0\), \(\Theta\)-crossing, or \(\gamma\)-crossing in terminology of [26, 27]). However, no singularity exists at \(\Theta = 0\) in the Newtonian gauge [27], and the perturbations are non-singular in the unitary gauge as well [28]. Thus, the system is well behaved at the moment of time when \(\Theta = 0\).

Now we can formulate the stability conditions for beyond Horndeski theories with additional scalar field in the ghosts and gradient instabilities provided that

\[
G_T > 0, \quad \mathcal{F}_T > 0.
\]

Let us note here that stability conditions (17) have retained their form as compared to the case of pure beyond Horndeski, see e.g. [15]. However, since generally the coefficient \(G_T\) involves the Hubble parameter, the stability of gravitational waves gets affected by the additional k-essence through the Friedmann equation (6a).

As for the scalar modes, it follows from action (14) that scalar sector is free of ghosts and gradient instabilities iff both kinetic matrices are positive definite \((G_{11}, G_{22} > 0, \det G > 0\) and \(F_{11}, F_{22} > 0, \det F > 0\):

\[
G_S > 0, \quad \mathcal{F}_S > 0, \quad R > 0, \quad P_Y > 0,
\]

\[
\times \mathcal{F}_S = YP_Y \left(\frac{G_T + D\pi}{\Theta^2}\right)^2 > 0.
\]

The first four conditions are formally the same as the stability conditions in pure beyond Horndeski theory and pure k-essence theory (extra scalar field affects \(G_S\) and \(\mathcal{F}_S\) through the Hubble parameter only) while the last condition is specific to the interacting theory.

### 3. Superluminality due to conventional scalar field

Let us now turn to the propagation speeds of perturbations. The sound speed squared for tensor perturbations follows immediately from action (10):

\[
c_T^2 = \frac{\mathcal{F}_T}{G_T},
\]

Again, \(c_T^2\) has a standard form, but in fact the tensor sound speed changes upon introducing additional k-essence due to new contributions in equation (6a) and, hence, the modified Hubble parameter.

In the scalar sector, the propagation speeds of \(\zeta\) and \(\omega\) are given by eigenvalues of matrix \(G_{AB}F_{AB}\):

\[
G_{AB}^{-1}F_{AB} = \left( \begin{array}{ccc} \frac{F_S}{G_S} - \frac{G_T + D\pi}{\Theta^2} \frac{Y P_Y}{G_S} & -\frac{\partial \chi \hat{P}_Y}{\Theta} \\ -\frac{\partial \chi \hat{P}_Y}{\Theta} & \frac{1}{2} \frac{1}{\Theta^2} \left( \frac{2}{\Theta^2} \frac{\partial \chi \hat{P}_Y}{\Theta} - \frac{G_T + D\pi}{\Theta^2} \frac{Y P_Y}{G_S} \right) \end{array} \right).
\]

Explicitly, the speeds are (recall that \(c_m^2 = P_Y/R\)):

\[
c_S^2 = \frac{1}{2} c_m^2 + \frac{1}{2} \left( \frac{F_S}{G_S} - \frac{G_T + 2D\pi}{\Theta^2} \frac{Y P_Y}{G_S} \right) \pm \sqrt{\left( \frac{F_S}{G_S} - \frac{G_T + 2D\pi}{\Theta^2} \frac{Y P_Y}{G_S} \right)^2 - 4 c_m^2 \left( \frac{F_S}{G_S} - \frac{G_T + 2D\pi}{\Theta^2} \frac{Y P_Y}{G_S} + c_m^2 \right)^2}.
\]

In accordance with the above remark, there is no singularity in the sound speeds at \(\Theta = 0\). Indeed, the speeds are finite as
Now we see a considerable difference between the unextended Horndeski and beyond Horndeski theories. In the unextended Horndeski case, the coefficient $D$ vanishes (see equation (11c)), so the matrix (20) is triangular and the speed of perturbations in k-essence recovers its standard value $c_{S-}^2$, while the propagation speed of Galileon perturbations is modified. Indeed, for $D = 0$, equation (21) reduce to

$$c_{S-}^2 |_{D=0} = \frac{F_S}{G_S} - \frac{Y P_Y G_T^2}{Q_S} \Theta^2, \quad c_{S+}^2 + |_{D=0} = c_m^2,$$

(22)

and we restore the results for Horndeski theory with k-essence $P(Y)$ given in [4]. On the contrary, with $D \neq 0$, there is kinetic mixing between the scalars $\zeta$ and $\omega$, so both scalar speeds get modified, in general agreement with [3, 25].

The key observation is that equation (21) has the following form (cf [24]):

$$c_S^2 = \frac{1}{2}(c_m^2 + A) \pm \frac{1}{2}\sqrt{(c_m^2 - A)^2 + B},$$

(23)

where

$$A = \frac{F_S}{G_S} - \frac{Y P_Y G_T^2}{Q_S} \frac{G_T}{2\Theta^2},$$

$$B = 4c_m^2 \frac{Y P_Y}{G_S} \frac{(D\Phi)^2}{\Theta^2}.$$  

In stable and rolling background ($G_S, P_Y > 0, Y > 0$), the coefficient $B$ is positive ($D = 0$ unless the value of $Y$ and, hence, the Hubble parameter is fine-tuned, see equation (11c)). This gives immediately

$$c_{S+}^2 > c_m^2 \quad \text{for} \quad Y = 0.$$  

(24)

So, if the flat-space propagation of the scalar perturbation $\omega$ is luminal, $c_m = 1$, then it becomes superluminal in the ‘beyond Horndeski + scalar field’ system. Equations (21), (23) and (24) are our main results.

4. Discussion

The interpretation of the result (24) is most straightforward in the case of the conventional scalar field $\chi$ with the Lagrangian (5). In that case one has $c_m = 1$ for any $Y$, and even tiny kinetic energy of rolling scalar background $\chi(t)$ immediately yields superluminal propagation of one of the modes. It is suggested (see, e.g., [29]) that a covariant theory which is fundamentally Lorentz invariant should recover a sound speed equal to unity in the far UV limit ($k \rightarrow \infty$, where $k$ is spatial momentum), even though for smaller $k$ perturbation modes could be superluminal. Our result is independent of $k$, so this is not the case in theories we consider: superluminality would occur even as $k$ tends to infinity. Therefore, we decide to insist on Lorentz invariance of an underlying theory and hence to avoid superluminality for good, we have to conclude that in scalar-tensor theories with multiple scalar fields, none of these fields can be conventional and minimally coupled, as long as at least one of the scalar fields is of beyond Horndeski type.

More generally, if we insist on the absence of superluminality, the result (21), (23) implies a non-trivial constraint on the structure of ‘beyond Horndeski + minimal quintessence’ systems: it is required that $c_{S+} < 1$ everywhere in the part of the phase space $(\pi, \pi, \chi, \xi)$ where stability conditions (18) are satisfied. In particular, this constraint forbids luminal flat-space propagation, $c_m = 1$ (and, by continuity, $c_m$ close to 1), in any rolling background $Y = 0$, unless such a background is unstable for any $\pi$ and $\xi$. Viewed differently, the constraint that $c_{S+} \leq 1$ in the entire ‘stable’ part of phase space suggests intricate properties of the UV completion of the scalar-tensor theories considered in this note, if such a UV completion exists and is Lorentz-invariant.

We conclude by adding that it is certainly of interest to study the superluminality issue in more general DHOST theories coupled to conventional or k-essence scalar field(s), and also address phenomenological implications of our result, especially in models for dark energy in the late-time Universe.

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Appendix

In this appendix we give explicit expressions for coefficients $\Psi_1$ and $\Psi_2$ involved in the quadratic action (14) for beyond Horndeski + k-essence $P(\chi, Y)$:

$$\Psi_1 = \frac{\Theta}{\Theta^2}[2\dot{\chi}P_Y(\Sigma + YR) + \Theta(P_\chi - 2YPR)],$$

(25)

$$\Psi_2 = \Omega + \frac{\dot{\chi}}{\Theta}P_\chi(P_\chi - 2YPR) + \frac{Y P_Y^2}{G_S^2}(\Sigma + YR) + \frac{d}{dt}[2YP_Y R],$$

(26)

where

$$\Omega = \frac{P_{\chi\chi}}{2} - 3H\dot{\chi}P_{\chi Y} - \chi(P_{\chi Y} + 2YP_{YY}).$$

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References

[1] Horndeski G W 1974 Second-order scalar-tensor field equations in a four-dimensional space Int. J. Theor. Phys. 10 363
[2] Zumalacárregui M and García-Bellido J 2014 Transforming gravity: from derivative couplings to matter to second-order scalar-tensor theories beyond the horndeski lagrangian Phys. Rev. D 89 064046
[3] Gleyzes J, Langlois D, Piazza F and Vernizzi F 2015 Healthy theories beyond Horndeski Phys. Rev. Lett. 114 211101
[4] Kobayashi T 2019 Horndeski theory and beyond: a review Rept. Prog. Phys. 82 086901
[5] Langlois D 2019 Dark energy and modified gravity in degenerate higher-order scalar-tensor (DHOST) theories: a review Int. J. Mod. Phys. D 28 1942006
[6] Rubakov V A 2014 The null energy condition and its violation Phys. Usp. 57 128
Rubakov V A 2014 Usp. Fiz. Nauk. 184 137
[7] Libanov M, Mironov S and Rubakov V 2016 Generalized Galileons: instabilities of bouncing and Genesis cosmologies and modified Genesis JCAP 1608 037
[8] Langlois D 2019 Dark energy and modified gravity in degenerate higher-order scalar-tensor (DHOST) theories: a review Int. J. Mod. Phys. D 28 1942006
[9] Easson D A, Sawicki I and Vikman A 2013 When matter matters JCAP 1307 014
[10] Kolevatov R, Mironov S, Sukhov N and Volkova V 2017 Cosmological bounce and Genesis beyond horndeski JCAP 1705 038
[11] Cai Y, Wan Y, Li H G, Qiu T and Piao Y S 2017 The effective field theory of nonsingular cosmology: A no-go theorem Phys. Rev. D 94 043511
[12] Ilyas A, Zhu M, Zheng Y, Cai Y F and Saridakis E N DHOST Bounce arXiv:2002.08269
[13] Mironov S, Rubakov V and Volkova V 2015 Exploring gravitational theories beyond Horndeski JCAP 1502 018
[14] Dobre D A, Frolov A V, Gheris J T G, Ramazanov S and Vikman A 2018 Unbraiding the bounce: superluminality around the corner JCAP 1803 020
[15] Adams A, Arkani-Hamed N, Dubovsky S, Nicolis A and Rattazzi R 2006 Causality, analyticity and an IR obstruction to UV completion JHEP 0610 014
[16] Ijjas A and Steinhardt P J 2016 Classically stable nonsingular cosmological bounces Phys. Rev. Lett. 117 121304
[17] Ijjas A 2018 Space-time slicing in Horndeski theories and its implications for non-singular bouncing solutions JCAP 02 007