Varification of Electron’s Mass Deriving from Interactive Dark Sector and Cosmological Observation

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Abstract

We investigate the model where electrons and dark matter interact with dark energy through the rolling of a scalar field which comes from extra dimensional theories such as the braneworld theory and Brans-Dicke theory. In this model, dark energy couples to dark matter and electrons, which leads to larger values of the mass energies of dark matter and electrons in the early universe. We also fit our model to the cosmological data. By analyzing the data from Planck, baryon acoustic oscillation (BAO), light curves (Pantheon), and type-Ia supernovae (SH0ES), it can be seen that the Hubble tension is relieved in our model and the coupling parameter prefers a non-zero value with a significance of over 2σ.

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I. INTRODUCTION

The ΛCDM model has been successful for explaining the properties and observations of our universe. However, the discrepancy of the Hubble constant is reported between indirect measurements and direct measurements. As for indirect observations, the Planck [1] and the Atacama Cosmology Telescope [2] which observed Cosmic Microwave Background (CMB) reported the value of the Hubble constant $H_0$ as $H_0 = 67.36 \pm 0.54$ km/s/Mpc, $H_0 = 67.9 \pm 1.5$ km/s/Mpc, respectively. An analysis [3], which is independent of CMB and which combined the Dark Energy Survey (DES), baryon acoustic oscillation (BAO), and Big Bang Nucleosynthesis (BBN), reported the value of the Hubble constant as $H_0 = 67.2^{+1.2}_{-1.0}$ km/s/Mpc. On the other hand, as for local measurements of $H_0$, the observations of supernovae [4, 5] reported as $H_0 = 74.03 \pm 1.42$ km/s/Mpc (2019), $H_0 = 73.2 \pm 1.3$ km/s/Mpc (2020), respectively. Also, the H0LiCOW collaboration with lensed quasars [6] reported as $H_0 = 73.3$ km/s/Mpc and the observation using the Tip of the Red Giant Branch (TRGB) as distance ladders [7] reported as $H_0 = 69.6 \pm 0.8$ (stat) $\pm 1.7$ (sys) km/s/Mpc. The difference between the values of the Hubble constant measured by indirect and the ones by local observations (Hubble tension) is a significant problem, although the difference might be caused by statistical errors of the Planck [8–11].

In this paper, we explore the possibility of verification of electron’s mass deriving from interactive dark sector to solve this tension. In our model, which is inspired by the braneworld theory [12, 14–16] or Brans-Dicke theory [17], matter components couple to the scalar field $\phi$ through the mass like $m_0 e^{\beta \phi}$. Once the scalar field is rolled by the interaction, energy of elementary particles and dark matter (DM) flows into that of dark energy (DE). As a result masses of the elementary particles and the DM become lighter. Particularly, among the elementary particles, electron mass crucially contributes to the cosmological evolution. Therefore we investigate the coupling dependence of the CMB power spectrum and cosmological parameters. Through the investigation, we conclude that this scenario relieves the Hubble tension through the electron mass reduction as is described in the previous studies [18–22].
Here, we should emphasize the worth of our model or the significance of adding DM-DE interaction. Our model is inspired by cosmological models with extra dimensions such as heterotic M-theory [13] and the Randall-Sundrum-I (RS I) model [12] (review papers are [24, 25]). In the five-dimensional effective theory of these models, DE interacts with not only elementary particles but also DM. Therefore, we investigate the model which includes the dark sector interaction whose contribution is widely discussed in the previous works [26–48].

In addition to the theoretical interest that our model is motivated by the theory of extra dimensions, there is a phenomenological motivation. There have been some works which studied models with interactions between DE and baryons [49, 50] and they relieve the Hubble tension a little (the DE-baryon interaction is also discussed in the context of the direct detection of DE [51, 52]). In this paper we focus on electrons instead of the baryons as the matter which interacts with DE and to explore the efficacy of our model as a method of Hubble tension relief.

This paper is organized as follows. In section II, we present our model setting. In section III, we will see the method of our analysis and datasets which we use. In section IV, we give our result and analysis. In the section V, we summarize this paper.

II. MODELING

A. Background evolution

The model which we discuss is based on the Randall-Sundrum-I (RS I) model [12], in which there are two branes. It is known that this model implies the existence of two scalar fields, $\phi_1$ and $\phi_2$, in the low energy region. One of the two fields corresponds to a bulk scalar field, which can propagate in the bulk spaces between the two branes, while the other field is related to the physical distance between the two branes. These scalar fields couple to matter on the branes differently. In this paper, we will focus on one of them which can evolve in time, or in the evolution of the universe, which is denoted by $\phi$ from now on. We also focus on one of the two branes, the visible brane, for simplicity.

Using this idea we will see a possibility that in addition to masses of dark matter (DM), masses of elementary particles (e.g. electrons) can be varied through the interaction with
the bulk scalar field $\phi$. We assume, however, that masses of baryons are not changed, since
the masses of the elementary particles which compose a baryon is in general very small than
the mass of the baryon and we can ignore the contribution of the varying masses of the
elementary particles.

With these ideas in mind, the action which we discuss has the form in the Einstein frame

$$S = \int d^4x \sqrt{-g} \left[ \frac{R}{2\kappa^2} - \frac{1}{2} g^{\mu\nu} (\partial_\mu \phi)(\partial_\nu \phi) - V(\phi) \right] + S_{\text{matter}}(\psi, A(\phi)g_{\mu\nu}) ,$$

where $S_{\text{matter}}$ is the Lagrangian for matters on the visible brane, $R$ is the Ricci scalar, $\psi$ is
the matter field on the brane. Also the quantity $A$ is written as

$$A = \exp (2\beta \phi) ,$$

where $\beta$ is a negative constant. Henceforth, we assume the derivative of the potential $V$ to
be negligible in order to compare our theroy with the $\Lambda$CDM model.

Since we assume that the universe is homogeneous, isotropic, and flat, we have the line
segment of the form

$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j).$$

Then we obtain the field equations

$$\mathcal{H}^2 = \frac{1}{3} a^2 \left( \rho_{\text{total}} + \frac{1}{2a^2} \dot{\phi}^2 + V(\phi) \right) ;$$

$$\ddot{\phi} + 2\mathcal{H} \dot{\phi} = - \sum_{(i)} \beta (\rho_{(i)} - 3p_{(i)}) a^2 ;$$

$$\dot{\rho}_{(i)} + 3\mathcal{H} (\rho_{(i)} + p_{(i)}) = \beta (\rho_{(i)} - 3p_{(i)}) \dot{\phi} ,$$

where the dot denotes the derivative with respect to the conformal time $\tau$ and $\mathcal{H} := \dot{a}/a$. Note that $\rho_{(i)}$ runs over density of cold dark matter $\rho_c$ and density of elementary particles $\rho_{ep}$, while the $\rho_{\text{total}}$ contains all densities (including density of the baryons $\rho_b$). Hence we
can rewrite the equation (6) as

$$\dot{\rho}_{(i)} + 3\mathcal{H} \rho_{(i)} = \beta \rho_{(i)} \dot{\phi} ,$$

1 We may include density of radiations as well. However the energy-stress tensor of the radiations is traceless
and the right hand sides of the equations (4) and (5) vanish, which are not interesting.
where the subscript $i$ can be either “c” or “ep”. The solution to this equation has the form

$$\rho^{(i)} = \rho^{(i)0} a^{-3} e^{\beta \phi},$$  \hspace{1cm} (8)

where $\rho^{(i)0}$ is a constant and this implies that we can express the variation of the masses as

$$\frac{m^{(i)}}{m^{(i)0}} \propto \sqrt{A} = e^{\beta \phi}.\hspace{1cm} (9)$$

This formula shows us the explicit relation between the evolution of the bulk scalar field $\phi$ and the evolution of the masses of matters which interact with the scalar field $\phi$.

On the other hand, the baryons do not interact with the scalar field $\phi$, since as we have mentioned at the beginning of this section, the mass of the baryons is assumed to be invariant and this means that the energy density of the baryons $\rho_b$ is also invariant due to the fact that the baryons are non-relativistic particles. Therefore, the counterpart of the equation (6) for them becomes

$$\dot{\rho}_b + 3H \rho_b = 0,\hspace{1cm} (10)$$

which leads to the solution

$$\rho_b = \rho_{b0} a^{-3}, \hspace{1cm} (11)$$

where $\rho_{b0}$ is an arbitrary constant. The numerical solutions to the equations (4), (5), and (6) are given in Fig. 1. Note that in matter dominant era, the scalar field $\phi$ evolves in logarithmic way while in the radiation dominant era, that behaves as almost constant.

![Fig. 1](image-url)

(a) Evolution of the scalar field $\phi$  \hspace{1cm} (b) Evolution of the mass  

FIG. 1. We have set the initial value of $\phi$ as zero and set the ratio of the baryon density $\rho_b$ over the all matter densities as 0.2.
The mass variation is also shown in Fig. 1. Note that the electron mass contributes to the energy levels of a hydrogen atom ($\propto m_e$) and Thomson scattering cross-section $\sigma_T$ ($\propto m_e^{-2}$) [18–22].

Substituting the solutions (8) and (11) to the rest equations (4) and (5) and exploiting the fact that $\rho_c > \rho_{ep}$, in the matter-dominant we have

$$\mathcal{H}^2 = \frac{1}{3a} \left( \rho_{b0} + \rho_{c0} e^{\beta \phi} \right); \quad \text{(12)}$$

$$\ddot{\phi} + 2\mathcal{H} \dot{\phi} = -\beta \rho_{c0} a^{-1} e^{\beta \phi}. \quad \text{(13)}$$

Here we set the today’s energy ratio $\omega$ of matters and CDM as

$$\omega = \frac{\rho_{c0}}{\rho_{b0} + \rho_{c0}}. \quad \text{(14)}$$

As discussed in the previous work [15], the solution of the scalar field $\phi$ to the system of equations has the form of $-2\beta \ln a$ in the matter-dominant and in the case that the matter consists of only dark matter. To take into account the contribution from the baryons, we have exploited a fitting formula for the numerical solution:

$$\phi = -2\beta \omega \left( \ln(a + a_{eq}) - \ln(a_0 + a) \right) + \phi_0. \quad \text{(15)}$$

Note that this formula also fits to the numerical solutions in radiation-dominant and matter-DE-equality eras as well as in the matter-dominant era. Here $a_{eq}$ and $a_0$ are the scale factors at the matter-radiation equality and matter-DE-equality, respectively, and $\phi_0$ is the initial value. Note that the $\ln(a + a_{eq})$ behaves like $\ln a$ and $\ln a_{eq}$ when $a > a_{eq}$ and $a < a_{eq}$, respectively.

To understand the evolution of the dark energy, we consider evolution of the effective equation of state $w_{DE}^{\text{eff}}$ (Fig. 2b), which is defined as

$$w_{DE}^{\text{eff}} = -1 - \frac{1}{3 \rho_{DE}} \frac{\partial \rho_{DE}}{\partial (\ln a)}; \quad \text{(16)}$$

where $\rho_{DE} = \dot{\phi}^2/(2a^2) + V$ is the energy density of the dark energy.

We set the critical point $a_{\text{crit}}$ as the transition point when dark energy changes from kination dominant to potential dominant. Before the critical point $a_{\text{crit}}$, since the potential $V$ has only less contribution, we can write down the $w_{DE}^{\text{eff}}$ as

$$w_{DE}^{\text{eff}} \approx -1 + \frac{2}{3} \left( 1 - \frac{\phi''}{\phi'} - \frac{\mathcal{H}'}{\mathcal{H}} \right), \quad \text{(17)}$$
FIG. 2. Evolution of the density of DE $\rho_{DE}$ (a) and density of the effective equation of state $w_{DE}^{\text{eff}}$ (b). We have set the value of $a$ as 1 at the present.

where the prime denote the derivative with respect to $\ln a$. Using equation (15), $\phi''/\phi'$ can be calculated as

$$\frac{\phi''}{\phi'} = \frac{-a^2 + a_0a_{eq}}{(a + a_0)(a + a_{eq})}. \quad (18)$$

In the radiation dominant era, the value of $\phi''/\phi'$ becomes 1, while the value of $H'/H$ becomes $-1$ so that the $w_{DE}^{\text{eff}}$ has the values asymptotically going to $-1/3$. In the matter dominant era, $\phi''/\phi'$ has the value 0 at a moment, while $H'/H$ has the value $-1/2$. As a result the maximum value of the $w_{DE}^{\text{eff}}$ close to 0 and it decreases with only small rate for a while as time goes back.

After the critical point $a_{\text{crit}}$, since the potential $V$ has significant contribution, the value of $w_{DE}^{\text{eff}}$ becomes $-1$.

B. perturbative equation

The interaction changes the perturbative equation as well. In the synchronous gauge, we have the line segment corresponding to the scalar perturbation of the metric

$$\text{d}s^2 = a^2(\tau) \left\{ -\text{d}\tau^2 + \left[ \left( 1 + \frac{b}{3} \right) \delta_{ij} + \left( \partial_i \partial_j - \frac{1}{3} \delta_{ij} \nabla^2 \right) 6\eta \right] \text{d}x^i \text{d}x^j \right\}. \quad (19)$$

Before the scalar field begin to roll, dark energy behaves as constant and we set the dark energy perturbation $\delta_{\text{de}}$, $\theta_{\text{de}} = 0$. The equation of motion for the perturbed scalar field $\delta \phi$ is given in [15].
\[ \ddot{\delta} + 2\mathcal{H}\delta + \left( k^2 + a^2 \frac{\partial^2 V}{\partial \phi^2} \right) \delta \phi + \frac{1}{2} \dot{h}\phi = -\beta \rho_c \delta_c a^2 \tag{20} \]

With this equation and the relations \( \rho_{de}\delta_{de} = a^{-2}\dot{\delta}\delta + V_{,\phi}\delta \phi \) and \( (\rho_{de} + p_{de})v_{de} = a^{-2}k\dot{\phi}\delta \phi \), where \( \delta \) is the density fluctuation and \( v \) is the velocity, we modify the perturbative equation of dark energy as follows,

\[ \dot{\delta}_{de} = -3\mathcal{H}(1 - w_{de})\delta_{de} - (1 + w_{de})kv_{de} - (1 + w_{de})\frac{\dot{h}}{2} \]
\[ - 9\mathcal{H}^2(1 - c_s^2)(1 + w_{de})\frac{v_{de}}{k} - \beta \delta_{de} a^2 \frac{\rho_c}{\phi} + \beta \frac{\rho_c}{\rho_{de}} \dot{\phi}(\delta_{de} - \delta_c) \tag{21} \]

\[ \dot{v}_{de} = 2\mathcal{H}v_{de} + \frac{\delta_{de}}{1 + w_{de}}k + \beta \frac{\dot{\phi}}{\rho_{de}} v_{de} \frac{c_s^2}{1 + w_{de}} \tag{22} \]

where \( c_s^2 \equiv p_{de}/\rho_{de} \) is the adiabatic sound speed and we substituted 1 for the sound speed \( c_s \). The non-perturbative part and the last term of eq. (21) are the same as the previous work [29] and the other terms are changed due to the different treatments of the perturbations.

We also change the perturbative equation of DM as follows [15],

\[ \ddot{\delta}_{c} = -\left( kv_c + \frac{\dot{h}}{2} \right) + \beta \dot{\phi} \tag{23} \]

\[ \dot{v}_c = -\mathcal{H}v_c + \beta k\delta \phi - \beta \dot{\phi}v_c \tag{24} \]

We should note that even in the synchronous gauge, the velocity of DM \( v_c \) is not zero and evolves due to the DM-DE interaction, while we set \( v_c = 0 \) initially to reduce the degrees of freedom of the gauge. We do not change the perturbative equations of the baryon, because baryons barely interact with the scalar field.

**III. DATA AND ANALYSIS**

We perform a Markov-Chain Monte Carlo (MCMC) analysis on the braneworld model described in the previous section. We use the public MCMC code **CosmoMC-planck2018** [55] with implementing the above braneworld scenarios by modifying its equation file in **camb** [56].
To implement the scale-factor dependence of dark energy, we use the approximation formula (15). Note that we have defined the parameter \( \delta \equiv -2\beta^2 \omega \) just for convenience and varied \( \delta \in [-0.01, 0.00] \) in our model.

We analyze models with referring the following cosmological observation datasets:

- The CMB measurements from Planck (2018) \[1\]: temperature and polarization likelihoods for high \( l \) (\( l = 30 \) to 2508 in TT and \( l = 30 \) to 1997 in EE and TE) and low\( l \) Commander and lowE SimAll (\( l = 2 \) to 29)
- Gravitation lensing from Planck \[57\]
- BAO from 6dF \[58\], DR7 \[59\] and DR12 \[60\]
- the local measurement of light curves from Pantheon \[61\]
- the local measurement of the Hubble constant from the Hubble Space Telescope observation of Supernovae and Cepheid variables from SH0ES (R19) \[4\]

IV. RESULT AND DISCUSSION

![Fig. 3. CMB TT angular power spectra for different values of \( \delta \).](image)

Fig. 3 shows the CMB power spectrum which is computed using camb. We can find the two significant effects of our model. First, the electron mass contributes to the recombination scale factor \( a_* \) through the energy levels of a hydrogen atom and Thomson scattering cross-section \( \sigma_T \) \[18, 22\]. Such contributions vary the redshift of the recombination \( z_* \) and
FIG. 4. Posterior distributions of $\delta$, $H_0$, $S_8$, and $z_*$ on our model, which is called IDE-me. These posteriors have been derived for all datasets (CMB+BAO+JLA+R19).

sound horizon $r_*$, and shift the peak positions of the spectrum: lower $\delta = -\beta^2 \omega$ (or larger $m_e/m_{e,\text{today}}$) increases $z_*$ and shifts the peak positions to higher multipole $l$. Second, the dark sector interaction, where energy flows from DM into DE, leads to the compression of the first peak [35]. On the other hand, the second peak is a little amplified due to the earlier recombination which leads to the decrease of the Silk damping [18, 63].

Fig. 4 and Fig. 5 show the results of our Monte Carlo analysis with the full dataset and with the only distant datasets (CMB + BAO), respectively.

From these figures, you can find that the Hubble tension is relaxed in our model. This is due to the increment of the electron mass $m_e$, earlier recombination $z_*$, and shorter sound horizon $r_*$. We also show that the coupling parameter $\delta$ prefers a non-zero value with
FIG. 5. Posterior distributions of $\delta$, $H_0$, $S_8$, and $z_*$ on our model, which is called IDE-me. These posteriors have been derived for only distant datasets (CMB+BAO).

significance of over $2\sigma$ when we use the full datasets.

We obtain

$$\delta(= -\beta^2 \omega) = -1.4^{+1.1}_{-1.1} \times 10^{-3}, \quad H_0 = 69.9^{+1.6}_{-1.5} \text{ km/s/Mpc}, \quad (95\%, \text{Planck + BAO + Pantheon + SH0ES(R19)})$$

(25)

$$\delta(= -\beta^2 \omega) > -1.4 \times 10^{-3}, \quad H_0 = 68.5^{+1.5}_{-1.3} \text{ km/s/Mpc}, \quad (95\%, \text{Planck + BAO}).$$

(26)

We quote $H_0 = 67.65^{+1.52}_{-1.51}$ (95% Planck 2018 + CMB lensing + BAO + JLA +
CFHTLensS + Planck SZ) from the previous study of DE-baryon interaction \cite{50} and our model relieves the Hubble tension more than the previous study.

We find another significant result that baryon fraction $\Omega_b h^2$ does not increase, while in the simple electron mass varying model, $\Omega_b h^2$ increases. This means that our model does not get the baryon density discrepancy worse between the big-bang nucleosynthesis (BBN) analysis and CMB measurements. This discrepancy has appeared by focusing on the correlation between the baryon density and the deuterium abundance $D/H$ synthesized during BBN \cite{64,65}. In the $\Lambda$CDM model, using PRIMAT \cite{62}, it is reported that this tension is $1.7 \sigma$ \cite{1} and $1.8 \sigma$ \cite{66} and it is also reported that in some Hubble tension solutions, the increment of $\Omega_b h^2$ makes this tension more severe \cite{67}. However, our model does not. This result supports the idea that we introduce the DM-DE interaction. As we explained, DM-DE interaction suppresses the amplitude of the first peak of the CMB power spectrum, while the amplitude of the second peak is amplified due to the increment of the electron mass. Therefore, increment of $\Omega_b h^2$ is disfavored, which enlarges the difference between the first and second peak.

As for the $S_8$ tension, our model does not relieve the tension. We can find that our model does not change the $S_8$. Therefore, the tension remains with the Kilo-Degree Survey (KiDS) \cite{68} and Dark Energy Survey (DES) \cite{69}, which give $S_8 = 0.737^{+0.040}_{-0.037}$ and $S_8 = 0.773^{+0.026}_{-0.020}$, respectively.

The best-fit values of $\delta$, $H_0$, and $\chi^2$ from our model and $\Lambda$CDM model are summarized in Tab.II. The total $\chi^2$ of our model is less than the $\Lambda$CDM model. The reduction in the value of $\chi^2_{\text{total}}$ is mostly due to the improved fit of SH0ES measurement. In addition, the slightly improved fit of CMB high $l$ spectra also decrease the value of $\chi^2_{\text{total}}$. However, the value of $\chi^2_{\text{BAO}}$ increases in our model, which results from the modification of the low-$z$ cosmology through the DM-DE interaction. Therefore, we conclude that our model, which includes DE-DM & DE-electron interaction, is promising, although we should take care of BAO.

V. SUMMARY

In this paper, we have studied the model which varies the electron mass deriving from the interactive dark sector and researched the fit to cosmological observation. To sum up our model, the interaction rolls the scalar field $\phi$ of DE which couples to DM and elementary
\begin{table*}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Parameter & ΛCDM & our model \\
\hline
δ = −β²ω & 0 & -0.00104 \\
H₀ [km/s/Mpc] & 68.17 & 69.54 \\
χ²\text{CMB high} & 2346.31 & 2345.61 \\
χ²\text{CMB low} & 22.62 & 23.293 \\
χ²\text{CMB low E} & 398.180 & 398.760 \\
χ²\text{CMB lensing} & 8.595 & 8.852 \\
χ²\text{H₀74p03} & 16.983 & 9.980 \\
χ²\text{JLA} & 1034.80 & 1034.77 \\
χ²\text{prior} & 1.795 & 2.105 \\
χ²\text{BAO} & 5.200 & 6.386 \\
χ²\text{total} & 3834.47 & 3829.75 \\
\hline
\end{tabular}
\caption{The best-fit values of δ, H₀, and \( \chi^2 \) for ΛCDM model and our model.}
\end{table*}

particles. This leads to a phenomenon that the energy of DM and elementary particles (especially electrons, in this paper) transforms into that of DE. As a result, the masses of electrons and DM are larger than today.

We have performed the Monte Carlo analysis on our model with cosmological data. As Fig.4 and Fig.5 show, our model prefers larger Hubble constant. Even the analysis with the only distant data sets gives the upper limit (95%) of the Hubble constant as 70km/s/Mpc. Although this value does not reach the value which is measured by the SH0ES measurement, TRGB measurement is comparable with our model and we conclude that our model relives the Hubble tension.

The Tab.1 summarizes the \( \chi^2 \) of the each measurement. This table shows that the total \( \chi^2 \) of our model is reduced by about 5 from ΛCDM model due to the improvement of the Hubble constant (SH0ES) and the slight improvement of the high \( l \) CMB fit.

In our model, whose potential is clarified through the analysis, the scalar field of DE is rolled by the DE-DM interaction and the electron mass is varied. Of course, the rolling can be caused by the potential of the scalar field and such model of DE-electron coupling is also worth considering.
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