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Binding energy and absorption of donor impurity in spherical GaAs/Al_{x}Ga_{1-x}As quantum dots with Konwent potential

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In this study, the electronic and optical properties of single or core/shell quantum dots, which are formed depending on the parameters in the selected Konwent potential, are investigated. Namely, the effects of the size and geometric shapes of quantum dots on the binding energy of the on-center donor impurity, the total absorption coefficient and refractive index which are including transitions between the some confined states, and the electromagnetically induced transparency between the lowest six confined states related to the donor impurity are investigated. We have used the diagonalization method by choosing a wave function based on the Bessel and Spherical Harmonics orthonormal function to find the eigenvalues and eigenfunctions of the electron confined within the quantum dots which have different types mentioned above. To calculate the optical absorption coefficients and electromagnetically induced transparency related to shallow-donor impurity, a two- and three-level approach in the density matrix expansion is used, respectively.

Keywords: Konwent potential, quantum dot, binding energy, absorption coefficient

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I. INTRODUCTION

Quantum dots (QDs) are one of the most important structures of low dimensional quantum systems which are used to manufacturing high-performance microelectronic and optoelectronic devices [1–3]. Due to advances in material growth technologies such as molecular beam epitaxy and metal-organic chemical vapor deposition, the production of low-dimensional semiconductor systems which have varied shapes and sizes is became possible. The dependence of the electronic energy spectrum on the geometric shape of the confinement potential and size of QD is very strong. This dependency allows us to manipulate the electronic and hence, optical characteristics of QD. For this reason, QDs are very important topics for researchers both theoretically and experimentally. As well as the geometric form and size of the confinement potential of QDs [4–9], the externally applied electric, magnetic and intense laser fields [10–19] affect the electronic properties, which influences the optical properties of QD. In devices based on QDs, the effects caused by impurities are also quite important and thus there are a lot of researches involving impurity-related studies with a special focus on QDs [20–25]. In addition to the shape, size, and externally applied fields, the presence of impurities causes radical changes in the energy spectrum of the quantum dots and provides the desired optical transitions to be obtained with respect to the purpose. As known, controlled optical transitions are very important in the design of optoelectronic devices with tunable emission or transmission characteristics and ultra-narrow spectral line width. The confinement of charge carriers in QDs causes to increase in the energy states, dipole matrix elements and the oscillator strength, which are related to a red or blue shift at absorption peak positions and an increase in peak amplitudes. Therefore, QDs are materials with great optical response [26]. This case prompted researchers to investigate optical phenomena such as optical absorption [27, 28], optical rectification [29–31], second and third-order harmonic generations [32–34] and refractive index changes [35–37]. Optical absorption coefficients (OACs) of QDs which including impurity have different confinement potentials have been previously studied by many authors [38–50]. Optical properties of QDs are investigated for: a cylindrical core-shell QD by Kria et al [38], the spherical sector-shaped QD by Mora-Ramos et al [39], a laser-driven two-dimensional disc-shaped QD by Kilic et al [40], a spherical QD by Stevanović et al [41], a typical ring-shaped elliptical QD by Ghajarpour and Karimi [42], core-shell ellipsoidal QDs by Shi and Yan [43], a mul-
tiers of aims to examine the electronic and optical proper-
investigating the EIT [63, 64].

In addition to the electronic structure in semiconductor nanostructures, optical absorption at inter-subband transitions, especially nonlinear absorption, is a very attractive subject. Electromagnetically induced transparency (EIT) has potential applications in many fields from nonlinear optics to quantum information science, namely nonlinear optics, optical memories, quantum optics, quantum computation, quantum information sciences and optical switches by controlling the light speed in low-dimensional semiconductors and researches about EIT continue intensively because of the potential applications mentioned above [51–57]. EIT which provides the coherent control of the optical properties of materials is one of the most interesting effects of quantum optics and it is a coherent optical nonlinearity that makes a medium transparent in a narrow spectral range around an absorption line. This transparency window leads to slow down and/or stop light that is one of the most striking EIT effects [58]. As known quantum confinement is mainly dealt with the energy of confined charge carriers (electrons and/or holes) in low dimensional systems. The energy levels of electrons in these systems are discrete, not continuum as in the bulk materials. QDs have perfect linear and nonlinear optical properties due to discrete energy levels which result from strong confinement that making them excellent for optoelectronic applications [59–62], and thus QDs are excellent structures for investigating the EIT [63, 64].

This study that we thought had not been studied before aims to examine the electronic and optical properties of GaAs/Al$_x$Ga$_{1-x}$As QDs with Konwent potential, which transform into single or core/shell quantum dots depending on the changes in the structure parameters. The organization of the paper is the following: Section II contains the presentation of the theoretical framework, in Section III, we discuss the obtained results, and in Section IV the conclusions are given.

II. THEORY

A. Energies and wavefunctions

In this study we are interested in investigating the states of an electron-impurity system confined in a spherical GaAs/Al$_x$Ga$_{1-x}$As QD with confining potential described by the Konwent-like model. It is considered a shallow donor impurity located on the center of the structure and the effects of spatial variation of the effective mass are neglected. Once the wavefunctions and energies for the system have been obtained, without and with impurity, we proceed to calculate, among others, the binding energy, the total absorption coefficient between the lowest three confined states, and the EIT between the lowest six confined states.

In the effective mass approach, the Hamiltonian for confined electron in KQD without impurity can be written as follows

$$H^{(0)} = \frac{\vec{p}^2}{2m^*} + V_{KP}(r),$$

where $\vec{p}$ and $m^*$ are the momentum and the electron effective mass, $V_{KP}(r)$ is the confinement potential proposed by Konwent [65] and defined as

$$V_{KP}(r) = V_0 \left[A \cosh \left(\frac{r}{k}\right) - 1 \right]^2,$$

where $k$ is the parameter related to the dot-size in the radial direction, $A$ is the structure parameter used to get with different QD-shapes, and $V_0 = Q_e \Delta E_g$ is the potential depth. $Q_e = 0.6$ is the offset parameter of conduction band and $\Delta E_g = 1.247 \times 10^{-4}$ eV [66] is the difference between the band gaps of Al$_x$Ga$_{1-x}$As and GaAs materials, where the aluminum concentration is taken as $x = 0.3$.

![FIG. 1. (color online) The $r$-dependence of the 3D-confinement potential in a GaAs-Al$_{0.3}$Ga$_{0.7}$As Konwent-like spherical quantum dot. Results are for three values of the $A$-parameter with $k = 2.5$ nm and $V_0 = 0.225$ eV (see Eq. (2)).](image)

The $r$-dependence of the 3D-confinement potential in a GaAs-Al$_{0.3}$Ga$_{0.7}$As KQD for three values of the $A$-parameter is depicted in Fig. 1 (see Eq. (2)). As seen, the value of the $A$-parameter determines the shape of the potential. For $A < 1$, the system behaves as a core/shell QD where the potential height in the shell is regulated by the value of the $A$-parameter, reaching its maximum value when $A \to 0$. In the limit $A = 0$, $V_{KP}(r) = V_0$. As the $A$-parameter presents a finite value, but close to zero, the core/shell-like behavior appears with a core where the potential is constant and a shell whose distance from the QD-center systematically tends to infinity. As the $A$-parameter increases, approaching $A = 1$, a combination of two factors occurs: $i$) a systematic reduction of
the potential barrier in the core region and ii) a systematic reduction of the core radius. The central barrier in the core disappears when \( A = 1 \) and its null value remains nearly constant over space up to a certain extent of the radial coordinate that depends on the value of the \( k \)-parameter, which controls the effective size of the QD. For \( A > 1 \) and \( k \)-decreasing, the system turns into a QD with parabolic potential in which it is possible to observe a shift of the potential bottom towards positive energies. It is important to highlight that the height of the central barrier in the core is definitely controlled by the \( A \)-parameter.

In the presence of central donor impurity, the Hamiltonian is written as

\[
H = H^{(0)} - \frac{e^2 Z}{\varepsilon r},
\]

where \( Z = 0 \) (\( Z = 1 \)) denotes the case without (with) the impurity, \( \varepsilon \) is the GaAs dielectric constant and \( r \) is the electron position. If the lengths are normalized to the effective Bohr radius \( \alpha_B = h^2 / (m^* e^2) \) and the energies to the effective Rydberg \( |R^*| = h^2 / (2m^* \alpha_B^2) \), the dimensionless Hamiltonian is found as follows

\[
H = -\nabla^2 + V_{K\Phi}(r) - \frac{2Z}{r},
\]

where \( \nabla^2 \) is the 3D-Laplace operator.

In order to find the energies and their corresponding wavefunctions, we have to solve the Schrödinger equation

\[
H\psi_{nlm}(r, \theta, \phi) = E_{nlm}\psi_{nlm}(r, \theta, \phi),
\]

where, \( n, l, \) and \( m \) are the principal, angular momentum, and magnetic momentum quantum numbers. To solve this equation, the Hamiltonian matrix is diagonalized by a series expansion using the electronic wavefunctions of the infinite spherical QD as the basis. This series expansion is given in the form \([67-69]\)

\[
\psi_{nlm}(r, \theta, \phi) = \sum_j c_{nlj} \psi_{nlj}^{(0)}(r, \theta, \phi),
\]

where \( c_{nlj} \) are the expansion coefficients and \( \psi_{nlj}^{(0)}(r, \theta, \phi) \) is the total wavefunction describing the motion of the electron without impurity and it is expressed as follows in the form of product of the radial and angular functions by virtue of the QD symmetry

\[
\psi_{nlj}^{(0)}(r, \theta, \phi) = \varphi_{nl}^{(0)}(r) Y_{lm}(\theta, \phi),
\]

where \( \varphi_{nl}^{(0)}(r) \) and \( Y_{lm}(\theta, \phi) \) are the radial and spherical harmonics wavefunctions, respectively. \( \varphi_{nl}^{(0)}(r) \) is defined in terms of the \( a \)-radius of the infinite spherical QD by the following equation

\[
\varphi_{nl}^{(0)}(r) = \begin{cases} 
N j_l(k_{nl} r), & r < a \\
0, & r \geq a
\end{cases},
\]

Here \( N \) is a normalization constant and \( j_l \) are the spherical Bessel functions. Additionally, \( k_{nl} \) is the \( n \)-th zero of the \( j_l \) function.

The binding energy \( (E_{nlm}^b) \) is obtained from the difference between the energies in any \( nlm \)-states of the electron in the absence and presence of donor impurity and it is as defined by

\[
E_{nlm}^b = E_{nlm}(Z = 0) - E_{nlm}(Z = 1),
\]

where, \( E_{nlm}(Z = 0) \) and \( E_{nlm}(Z = 1) \) are the energies without and with impurity in the KQD, respectively.

We want to emphasize that in this study, we will focus our attention on states with \( n = 1 \) and \( m = 0 \). For the study of the OAC, we will use the first three states with \( l = 0, 1, 2 \). In the case of EIT, and due to the polarization of the two incident radiations, it will be necessary to extend the set to \( l = 0, 1, 2, 3, 4, 5 \).
In Figs. 2 and 3 we present the wavefunction projections in the \( y = 0 \) plane for a confined electron in a GaAs/Al\(_{0.3}\)Ga\(_{0.7}\)As QD. In particular, Fig. 2 compares the ground state wavefunctions in the absence and presence of the on-center impurity for three values of the \( A \)-parameter and two values of the \( k \)-parameter. In Fig. 3 the ground state and the first two excited states wavefunctions in the presence of impurity are shown for the same combinations of the \( A \) - and \( k \)-parameters. Note that for \( k = 2.5 \text{ nm} \), we have used \( a = 15 \text{ nm} \) while for \( k = 5.0 \text{ nm} \) the corresponding value is \( a = 25 \text{ nm} \). This ensures that the effects of the infinite barrier in the outer region of the structure; this despite the presence of the on-center impurity for three values of the \( k \)-parameter, the first excited state has \( p_z \)-like symmetry, corresponding to two antinodes located symmetrically on the z-axis and having the opposite sign; in this case the wave function is null at \( r = 0 \) (or \( z = 0 \)), and iii) the second excited state has \( d \)-like symmetry with two antinodes of the same sign located symmetrically on the z-axis and a torus region of maximum amplitude, but opposite sign, which has axial symmetry with respect to the z-axis. Compared to the ground state, the first and second excited states have a greater spatial extension, and this extension increases with the increase (decrease) of the \( k \)-parameter (\( A \)-parameter). Considering \( z \)-polarized resonant incident radiation, and taking into account the symmetries of the wave functions, clearly the \( \psi_{100} \rightarrow \psi_{110} \) and \( \psi_{110} \rightarrow \psi_{120} \) transitions are allowed while the \( \psi_{100} \rightarrow \psi_{120} \) transition is forbidden.

### B. Optical absorption coefficient

Photoabsorption is the phenomenon of transition from low energy level to high energy level by absorption of a photon in any quantum system. When a photon with energy \( h \omega \geq E_j - E_i \) is absorbed, a transition takes place between levels with energy \( E_j \) and \( E_i \). The linear, third-order nonlinear, and total optical absorption coefficient (OAC) corresponding to these transitions are described as follows, respectively:

\[
\alpha^{(1)}(\omega) = \sqrt{\frac{\mu}{\varepsilon_r}} \cdot \frac{\sigma_s h \omega |M_{ij}|^2 \Gamma_{ij}}{(E_{ij} - h \omega)^2 + (h \Gamma_{ij})^2},
\]

and

\[
\alpha(\omega, I) = \alpha^{(1)}(\omega) + \alpha^{(3)}(\omega, I),
\]

where \( \omega \) is the angular frequency of the incident photon, \( \mu \) is the vacuum magnetic permeability, \( \varepsilon_r \) is the real part of the GaAs susceptibility, \( \sigma_s \) is the electron density, \( \Gamma_{ij} = 1/T_{ij} \) (where \( T_{ij} \) is the relaxation time) is the non-diagonal matrix element known as the relaxation rate of the final and initial states, \( E_{ij} = E_j - E_i \), is the transition energy between the initial and final states, \( I = 2 \varepsilon_0 n_r c |E|^2 \) (where \( |E| \) is amplitude) is the intensity of the incident resonant field, \( n_r \) is the GaAs refraction index, \( c \) is the vacuum light speed, and the following characteristics are observed: i) as mentioned above, the ground state has spherical symmetry, that is, it corresponds to \( s \)-like states, where the wavefunctions are more spread out in space as \( k \)-increases and \( A \)-decreases, ii) in all considered cases of \( A \)- and \( k \)-parameters, the first excited state has \( p_z \)-like symmetry, and iii) the second excited state has \( d \)-like symmetry with two antinodes of the same sign located symmetrically on the z-axis and a torus region of maximum amplitude, but opposite sign, which has axial symmetry with respect to the z-axis.

\[\varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m} \] is the vacuum permittivity. \( M_{ij} \) is the dipole transition matrix element, and for incident resonant radiation polarized in the \( z \)-direction it is given as

\[
M_{ij} = \langle \psi_j | e r \cos \theta | \psi_j \rangle,
\]

where \( \psi_i \) and \( \psi_j \) are the wavefunctions representing the initial and final states, respectively.

In accordance with the energy conservation law, every possible transition in the absorption spectrum cannot be observed. In addition to the energy conservation, the angular momentum conservation and several symmetry rules have to also be provided. All permit-
ted transitions are added in the dipole transition elements, and the cases where the dipole transition element is nonzero are sought. So, for a linearly polarized light interacting with the electric field polarized in the $z$-direction, only the $z$-component of the dipole transition element is nonzero, while the other components are zero. According to the property of spherical harmonics, only dipole transitions are allowed between cases that supply the rule $\Delta l = \pm 1$. Also $M_{jj} - M_{ii}$ is zero for the donor impurity at the center of QD. The resonance conditions are $\hbar \omega_1 = \sqrt{E_{ij}^2 + (\hbar \Gamma_i)^2}$ and $\hbar \omega_3 = \frac{3}{2} \left( E_{ij} + \sqrt{4 E_{ij}^2 + 3 (\hbar \Gamma_i)^2} \right)$ for linear and third-order nonlinear OACs, respectively. We define the following two-functions which are associated to the resonant conditions

$$\Omega = e^{-2} \sqrt{E_{ij}^2 + (\hbar \Gamma_i)^2} |M_{ij}|^2$$  \hspace{1cm} (14)

and

$$\Gamma = \frac{e^{-4}}{3} \left( E_{ij} + \sqrt{4 E_{ij}^2 + 3 (\hbar \Gamma_i)^2} \right) |M_{ij}|^4.$$  \hspace{1cm} (15)

C. Electromagnetically induced transparency

\[
|c\rangle \quad \Delta_p \quad \Delta_c \quad |b\rangle \\
\omega_p, \vec{E}_p \\
\omega_c, \vec{E}_c
\]

FIG. 4. (color online) The three-energy levels in the \Lambda-configuration for the electromagnetically induced transparency occurrence.

In Fig. 4 we present the three energy level system in the \Lambda-configuration to study the impurity related EIT in KQD. The interlevel transition between the $|a\rangle$ and $|c\rangle$ ($|b\rangle$ and $|c\rangle$) states is dipolarly allowed through the probe (control) field while the transition between the $|a\rangle$ and $|b\rangle$ states is dipolarly prohibited ($M_{ac} \neq 0, M_{bc} \neq 0$, and $M_{ab} = 0$). The parameters $\omega_p$ and $E_p$ ($\omega_c$ and $E_c$) are the frequency and strength of the probe (control) field. The probe and control lasers are detuned from the resonance frequencies by $\Delta_p = \omega_{ca} - \omega_p$ and $\Delta_c = \omega_{cb} - \omega_c$.

By using the rotating-wave approximation, the OAC ($\alpha(\omega)$) and refraction index ($n_T, RI$) for a three-level system in the \Lambda-configuration interacting with two applied laser pulses (probe and control) are, respectively

$$\alpha(\omega) = \frac{\omega \Im(\chi)}{e n_T}$$  \hspace{1cm} (16)

and

$$n_T = \sqrt{\frac{n_t^2 + \Re(\chi) + \sqrt{(n_t^2 + \Re(\chi))^2 + \Im(\chi)^2}}{2}}$$  \hspace{1cm} (17)

where

$$\Im(\chi) = \sigma_s M_{ca}^2 \frac{\gamma_{ca} \left[ \gamma_{ca}^2 + (\Delta_p - \Delta_c)^2 \right] + \gamma_{ca} \Omega_c^2}{\Theta}$$  \hspace{1cm} (18)

$$\Re(\chi) = \sigma_s M_{ca}^2 \frac{\Delta_p - \Delta_c \left[ (\Delta_p - \Delta_c) \Delta_p - \Omega_c^2 \right] + \Delta_p \gamma_{ba}^2}{\Theta}$$  \hspace{1cm} (19)

and

$$\Theta = [\Omega_c^2 + \gamma_{ca} \gamma_{ba} - \Delta_p (\Delta_p - \Delta_c)]^2 + [\Delta_p \gamma_{ca}^2 + (\Delta_p - \Delta_c) \gamma_{ca}]^2.$$  \hspace{1cm} (20)

Here $\Omega_c = M_{ca} E_p / (2 \hbar)$ is the half of the Rabi frequency and $\gamma_{ij}$ is the decay rate between the $i$ and $j$ states composed from the radiative decay rate due to the spontaneous emission and the dephasing rate. For details, see Ref. [70].

III. RESULTS AND DISCUSSION

In this section, the effects of potential parameters on the binding energies, OACs, and EIT in the GaAs/Al$_{0.3}$Ga$_{0.7}$As Konwent-like spherical quantum dot is analyzed. The physical parameters used in our calculations are: $\epsilon = 13.18, \ V_0 = 0.225 \text{ eV}, \ m^* = 0.067 \ m_0$ (where $m_0$ is the free electron mass), $T_{ij} = 1.0 \mu s$, $\mu = 4 \pi \times 10^{-7} \text{ H/m}, \ n_e = 3.63, \ \sigma_s = 3.0 \times 10^{23} \text{ m}^{-3}$, and $I = 100 \text{ MW/m}^2$ [28]. Other parameters are: $\Omega_c = 40 \text{ THz}, \ \gamma_{ba} = 0.1 \text{ THz}, \ \text{and } \gamma_{ca} = 5 \text{ THz}$ [70].
The Fig. 5 shows the changes of energy without impurity ($Z = 0$) and with impurity ($Z = 1$) and binding energy as a function of the $A$-parameter for two different $k$-values in a KQD. Since the electron confinement increases due to the transformation of the structure from core/shell KQD towards a single KQD with increasing the $A$-parameter, the uncorrelated and correlated energies are increasing functions of the $A$-parameter. Furthermore, all energies mentioned above decreases with the $k$-parameter since the electron confinement decreases. Furthermore, with the addition of impurity to the structure, the negative Coulomb potential between the electron and the impurity causes a reduction in energy. Finally, it is seen that the impurity-free and impurity-containing energies increase gradually towards the higher states ($E_{100} < E_{110} < E_{120}$). We can see that the effect of impurity is systematically reduced as one passes from the ground state to the first excited state and subsequently to the second excited state. It is clear that the excited states are more extended in space and thus the interaction with the impurity becomes weaker. This explains the reason why for the binding energy of the three states shown, $E_B^{100} > E_B^{110} > E_B^{120}$ is satisfied. Note the trend exhibited by the binding energy for the three states shown. They are increasing functions of the $A$-parameter. However, it can be seen that for large values of $A$-parameter there is a tendency to reach constant values. This is explained by the fact that for a sufficiently large $A$ value, the system has the tendency to reach the limit of a strongly confined zero-dimensional hydrogen system. The binding energy grows as $A$ increases due to the fact that the infinite potential barrier, located on the outside of the KQD, approaches towards the central region of the structure, pushing the electronic wavefunction towards the impurity position. With this, there is a reduction in the average electron-impurity distance, which leads to a gradual increase in the Coulomb interaction and consequently to an increase in the binding energy.
The energy transitions (a1, a2), the reduced dipole matrix elements (b1, b2), the Ω-function (c1, c2), and Γ-function (d1, d2) in the GaAs/Al0.3Ga0.7As KQD as a function of the A-parameter. The results are for \( k = 2.5 \text{ nm} \) (a1, b1, c1, d1) and \( k = 5.0 \text{ nm} \) (a2, b2, c2, d2) with the shallow-donor impurity localized at the center of the dot. Solid lines are considering transitions from the ground state to the first excited state (1 → 2) whereas dashed lines are considering transitions from the first excited state to the second excited state (2 → 3).

The energy transitions (a1, a2), the reduced dipole matrix elements (b1, b2), the Ω-function (c1, c2, see Eq. (14)), and the Γ-function (d1, d2, see Eq. (15)) in the GaAs/Al0.3Ga0.7As Konvent-like spherical QD as a function of the A-parameter are depicted in Fig. 6. The results are for \( k = 2.5 \text{ nm} \) (upper row) and \( k = 5.0 \text{ nm} \) (lower row) with on-center shallow-donor. Solid lines are considering transitions from the ground state to the first excited state (\( \psi_{100} \rightarrow \psi_{110} \)) whereas dashed lines are considering transitions from the first excited state to the second excited state (\( \psi_{110} \rightarrow \psi_{120} \)). From Figs. 4(a1, a2), it is observed that all the transition energies are increasing functions of the A-parameter. This is in perfect agreement with the fact that as the confinement of the carriers increases due to the effective reduction in the KQD-size, the separation between the energy levels also increases, as can be seen in Fig. 6, and hence the consequent increase of transition energies. It is clear that \( \Delta E_{12} > \Delta E_{10} \) because the presence of the impurity affects mainly the ground state with a reduction in energy. Comparing the first and second excited states, the impurity produces a more noticeable effect on the first one, hence the reason why \( \Delta E_{12} > \Delta E_{23} \) is explained. Comparing Figs. 6(a1) and 6(a2), it can be seen in the first case that the calculated variation of the A-parameter generates a change in the transition energy of 0.5 eV while in the second one the variation is of the order of 0.2 eV. The larger the structure, the closer the energy states and the smaller the transition energy. Taking into account that the increase in the A-parameter translates into an effective decrease in the KQD-size, this explains the reason why in panels 6(b1, b2) the reduced dipole matrix element is always a decreasing function of the A-parameter. The larger the original structure (which corresponds to the one whose \( k \)-parameter is the largest), the more noticeable is the variation of \( M_{ij}^0 \) and \( M_{ij} \). While in Fig. 6(b1) the variation exhibited by the matrix elements is of the order of 2.5 nm, in Fig. 6(b2) the changes exceed 5.0 nm. In the calculation of the Ω- and Γ-functions, according to Eqs. (14) and (15), it is evident that these are dominated by the behavior of the dipole matrix elements. Then, Fig. 6(d1) is in line with the decreasing behavior of the curves in Fig. 6(b1). The same can be seen in Figs. 6(c2) and 6(d2) with respect to that shown in Fig. 6(b2). However, the behavior of Fig. 6(c1) shows that the Ω-function comes from a competitive effect between the transition energy, which is always increasing with the A-parameter, and the dipole matrix elements, that are always a decreasing functions of the A-parameter. However, note that in Fig. 6(c1) the changes of the Ω-function in the entire range of the A-parameter do not exceed 20 meV nm$^2$. 

FIG. 6. (color online) Energy transitions (a1, a2), reduced dipole matrix elements (b1, b2), Ω-function (c1, c2), and Γ-function (d1, d2) in the GaAs/Al0.3Ga0.7As KQD as a function of the A-parameter. The results are for \( k = 2.5 \text{ nm} \) (a1, b1, c1, d1) and \( k = 5.0 \text{ nm} \) (a2, b2, c2, d2) with the shallow-donor impurity localized at the center of the dot. Solid lines are considering transitions from the ground state to the first excited state (1 → 2) whereas dashed lines are considering transitions from the first excited state to the second excited state (2 → 3).
FIG. 7. (color online) The shallow-donor impurity related total optical absorption coefficient in the GaAs/Al$_{0.3}$Ga$_{0.7}$As KQD as a function of the incident photon energy for several values of the $A$-parameter. The results are for $k = 2.5$ nm (a) and $k = 5.0$ nm (b) with the shallow-donor impurity localized at the center of the dot. Calculations are considering the $1 \rightarrow 2 = \psi_{100} \rightarrow \psi_{110}$ and $2 \rightarrow 3 = \psi_{110} \rightarrow \psi_{120}$ transitions.

In Figs. 7 the total OAC is presented as a function of the incident photon energy in the presence of the impurity atom for $k = 2.5$ nm (a) and $k = 5.0$ nm (b) and several $A$-values. Here, since the third order nonlinear OAC is very small, the total OAC gives an equivalent result with the linear one, so we prefer to show only the total OAC. The peak positions of the total OACs for the transitions corresponding to the case with impurities shift to the blue with the increase of $A$-parameter. Since the energy difference between the levels of interest increases with the $A$-parameter, the peak positions of the total OAC shift to blue. We also see that while the peak amplitude of the total OAC is almost constant for single KQD ($A > 1$), it decreases for the core/shell KQD ($A = 0.1$). This is a result related to the $\Omega$-factor given in Figs. 6(c1, c2), where the resonant peak of linear OAC is directly proportional to the $\Omega$-factor (see Eqs. (10) and (14)). Although the wavefunctions overlap better in the core/shell structure (see Figs. 2 and 3), the $\Omega$-factor decreases due to the decrease in the transition energy. According with the results in Fig. 6, the presence and absence of the impurity does not create a noticeable change in absorption amplitudes. However, the presence of impurity causes an increase in transition energies as seen in Figs. 6(a1, a2), and thus the peak positions of the total OAC shift to the blue. Finally, it should be noted that the peak positions of the total OAC associated with the $\psi_{110} \rightarrow \psi_{120}$ transition always occur at higher energies than those associated with the $\psi_{100} \rightarrow \psi_{110}$ transition. Because, as seen in Figs. 6(a1, a2), $\Delta E_{23}$ transition energy is greater than $\Delta E_{12}$ transition energy for all parameter values. At the same time, the peak amplitudes of the total OAC corresponding to the $\psi_{110} \rightarrow \psi_{120}$ transitions are always greater than those of the $\psi_{100} \rightarrow \psi_{110}$ transitions. As a result of the better overlap of the wave function of the $\psi_{110}$ state with the wave function of the $\psi_{120}$ state, the $\Omega$-factor is larger for the $\psi_{110} \rightarrow \psi_{120}$ transition (see Figs. 6(c1, c2)).

Next, we turn our attention to the impurity-related EIT in a GaAs/Al$_{0.3}$Ga$_{0.7}$As KQD. We have chosen a QD with $k = 5.0$ nm and considering on-center impurity. Our results are presented as a function of the $A$-parameter and probe field energy.
of the energy levels in the GaAs/Al0.3Ga0.7As QKD as a function of the A-parameter. The results are for $k = 5.0$ nm with the shallow-donor impurity localized at the center of the dot. The inset shows the $1 \rightarrow 6$ transition energy. The probe field is associated to the $1 \rightarrow 6$ transition whereas the control field is associated to the $3 \rightarrow 6$ transition. In (b), the reduced dipole matrix elements for the $1 \rightarrow 6, 1 \rightarrow 3, \text{and } 3 \rightarrow 6$ transitions are presented. Note that $M_{13} = 0$.

In Fig. 8(a), the energies of the six lowest states are presented as a function of the A-parameter. In the inset of Fig. 8(a), we report the transition energy between the shallow-donor impurity localized at the center of the dot. The inset shows the $1 \rightarrow 6$ transition energy. The probe field is associated to the $1 \rightarrow 6$ transition whereas the control field is associated to the $3 \rightarrow 6$ transition. In (b), the reduced dipole matrix elements for the $1 \rightarrow 6, 1 \rightarrow 3, \text{and } 3 \rightarrow 6$ transitions are presented. Note that $M_{13} = 0$.

In Figs. 9 (a) and (b), our results for the on-center shallow-donor impurity related total OAC and RI in the KQD (with $k = 5.0 \text{ nm}$) are presented as a function of the incident probe photon energy for several values of the A-parameter taking into account the $\psi_1, \psi_3, \text{and } \psi_6$ states in the three-levels Λ configuration. Dashed lines are for $\Omega_c = 0$, in which case the OAC presents only one resonant structure located at $E_p \sim E_{61} = E_6 - E_1$ (where $E_p$ is the incident probe photon energy). The position of the resonant peak shows a blueshift as the A-strength increases in complete agreement with the dependence shown with said parameter on the transition energy reported in the inset of Fig. 8(a).

In Figs. 9 (a) and (b), our results for the on-center shallow-donor impurity related total OAC and RI in the KQD (with $k = 5.0 \text{ nm}$) are presented as a function of the incident probe photon energy for several values of the A-parameter taking into account the $\psi_1, \psi_3, \text{and } \psi_6$ states in the three-levels Λ configuration. Dashed lines are for $\Omega_c = 0$, in which case the OAC presents only one resonant structure located at $E_p \sim E_{61} = E_6 - E_1$ (where $E_p$ is the incident probe photon energy). The position of the resonant peak shows a blueshift as the A-strength increases in complete agreement with the dependence shown with said parameter on the transition energy reported in the inset of Fig. 8(a). The systematic fall of $M_{16}$ in Fig. 8(b), explains the reason why there is an attenuation of the magnitude of the resonant peaks in Fig. 9(a) as A-increases. Consistent with only one resonant structure for the OAC when $\Omega_c = 0$, the results presented in Fig. 9(b) for the RI show that it becomes null at $E_p = E_{61}$ and that concerning this energy value, the RI shows two resonant structures that have inversion symmetry. For $\Omega_c = 0$ and taking a constant value of the A-parameter, the magnitudes of the resonant structures in the OAC and RI are controlled by the $\gamma_{61}$-parameter. When $\Omega_c \neq 0$, the control field is activated, and at that moment, the EIT effect appears. Precisely at the same value where the OAC presented the maximum when $\Omega_c = 0$, in the case of a finite $\Omega_c$ value, the OAC is now suppressed, and it is replaced by two resonant structures symmetrically located concerning zero absorption and whose separation is controlled by the $\Omega_c$ magnitude. The magnitude of these structures is not the same, the resonant peak located at higher energy is higher. This is associated with the linear term with the probe photon energy that controls the OAC. When the EIT is activated, two resonant structures appear in the OAC, in the same position as these, the RI replicates the results presented with $\Omega_c = 0$. In each resonant peak of the OAC, the RI becomes null with two structures that satisfy the inversion symmetry. It is important to note that for the case $\Omega_c \neq 0$, the magnitude of the resonant structures in the OAC and RI is controlled simultaneously by the $\gamma_{61}$ and $\gamma_{31}$ parameters, with the dominant effect associated with $\gamma_{31}$. For a high enough value of $\gamma_{31}$, the RI can go from four to just two resonant structures.
FIG. 9. (color online) The shallow-donor impurity related total optical absorption coefficient (a) and refraction index (b) in the GaAs/Al$_{0.3}$Ga$_{0.7}$As QD as a function of the incident probe photon energy for several values of the $A$-parameter. The results are for $k = 5.0$ nm with the shallow-donor impurity localized at the center of the dot. Calculations are considering the $1 \rightarrow 6$ transition excited by the probe laser, whereas the control field involves the $3 \rightarrow 6$ transition. Dashed lines are for $\Omega_c = 0$, whereas solid lines are for $\Omega_c = 40$ THz (in this case, the non-zero control field activates the electromagnetically induced transparency). Additionally, in these figures, $\Delta_c = 0$.

IV. CONCLUSIONS

In this study, the electronic and optical properties of GaAs/Al$_{0.3}$Ga$_{0.7}$As QD with Konwent potential, which transform into single or core/shell QDs depending on the changes in the structure parameters are investigated. Our results include: i) Uncorrelated ($Z = 0$) and correlated ($Z = 1$) energies of the electron as the functions of the shape ($A$-parameter) and size ($k$-parameter) of KQDs, ii) the binding energies for shallow donor impurity located on the center of the QDs, iii) the total OACs for the allowed transitions between the lowest three confined states, and iv) the impurity-related EIT between the lowest six confined states, respectively. Due to the strong quantum confinement caused by the change in the structure parameter in the potential, that is, the changes in size and geometric shape, it is seen that the QDs under investigation have a significant effect on the energy and optical spectrum. Moreover, it is seen that with selected realistic confinement potential and a special selection of the structure parameters, the optical response of the system can be tailored in a controllable manner. In this context, the results obtained indicate that quantum dots are highly important structures for optoelectronic applications.

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DECLARATIONS

Author contributions statement

The contributions of the authors are as follows:

**E. B. Al:** was responsible for numerical calculations, figures and writing of the manuscript.

**E. Kasapoglu:** proposed the problem and responsible for writing and editing the comments.

**H. Sari:** was responsible for the verification of the solution method.

**I. Sökmen:** proposed the solution method.

**C. A. Duque:** was responsible for writing and editing the comments.

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