Dynamical Supersymmetry Breaking and Low Energy Gauge Mediation

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Abstract

Dynamical breaking of supersymmetry was long thought to be an exceptional phenomenon, but recent developments have altered this view. A question of great interest in the current framework is the value of the underlying scale of supersymmetry breaking. The “little hierarchy” problem suggests that supersymmetry should be broken at low energies. Within one class of models, low energy breaking be achieved as a consequence of symmetries, without requiring odd coincidences. The low energy theories are distinguished by the presence or absence of $R$ symmetries; in either case, and especially the latter one often finds modifications of the minimal gauge-mediated spectrum which can further ameliorate problems of fine tuning. Various natural mechanisms exist to solve the $\mu$ problem in this framework.
1 Introduction

If supersymmetry is relevant to low energy physics, supersymmetry breaking is probably dynamical; indeed, the possibility of dynamical breaking (DSB) is a principal reason to consider low energy supersymmetry\[1\].

In the past, this was a complicated topic. Models with gravity mediation were problematic; it was hard to suppress flavor changing neutral currents and to obtain a suitable gaugino spectrum. Models with low energy gauge mediation did not suffer from these problems, and lead to certain generic predictions. However, these early models were so complicated as to appear contrived. It was challenging to generate $\mu$ and $B_\mu$ terms for Higgs fields, and the theories often exhibited approximate $R$ symmetries. Moreover, as the limits on the Higgs mass and the masses of various superparticles have improved, problems of fine tuning have become severe.

Recent developments in dynamical supersymmetry breaking\[2, 3, 4, 5, 6, 7\] reopen these issues. As we will discuss, they permit the construction of simpler models. In this richer set of theories, the predictions of the simplest models of gauge mediation (Minimal Gauge Mediation, or MGM) do not necessarily hold\[8, 9\]. This has the potential to reduce fine tuning. Lowering the scale of supersymmetry breaking to energies of order 10’s to 100’s of TeV also reduces the amount of fine tuning. While challenging in earlier theories, this can readily be achieved with the present understanding. The issues of $\mu$ and $B_\mu$, and gaugino masses, take on a different character in these new theories.

This paper provides a perspective on these developments. Our focus will be on models in which supersymmetry is broken at the lowest scales commonly contemplated for gauge mediation, of order 100 TeV. This is a quite restrictive requirement. The ISS models, for example, involve two mass scales, one the scale of the new gauge interactions, $\Lambda_N$, the other the scale of the masses of the hidden-sector matter fields, $m_f$. The scale of supersymmetry breaking is suppressed relative to $\Lambda$ by a power of $m_f/\Lambda$. So unless $m_f$ is comparable to $\Lambda$, the scales $\Lambda$ and $m_f$ must be much larger than 100 TeV. Similar issues arise in many other models (in the "Pentagon" models of Banks and collaborators\[10\], this coincidence is argued to arise from an underlying principle of theories of quantum gravity).

More generally, we will see that:

1. In many of the new models, achieving DSB at a low scale still requires surprising coinci-
dences or baroque constructions. But in a broad class of models (the “retrofitted” models of [3] and generalizations[6]), one can achieve low energy breaking in a rather simple way, consistent with conventional notions of naturalness.

2. Many of the new models possess, at low energies, approximate $U(1)_R$ symmetries. These must be spontaneously broken. This requires the presence of gauge symmetries[5, 11], or fields $R$ charge different than 0 or 2[4]. With spontaneous breaking, suppression of CP violation is sometimes automatic[8]. However, in these cases, it is necessary to suppress dangerous one loop contributions to squark and slepton masses.

3. Explicit breaking of $R$ symmetry can be obtained by retrofitting. In the retrofitted models, the $R$ symmetry breaking scale can naturally be of order the supersymmetry breaking scale, allowing a low scale for supersymmetry breaking. To obtain metastable breaking naturally, a small parameter is required, which can arise through a Froggatt-Nielsen (FN) type mechanism; the underlying scale of supersymmetry breaking is then two orders of magnitude or more larger than the messenger scale. With explicit breaking, suppression of electric dipole moments is not automatic; additional features are required to suppress CP-violating phases.

4. A suitable $\mu$ term can be generated in the retrofitted models, consistent with conventional notions of naturalness, but we will see that additional degrees of freedom are required. We will consider two classes of models, one in which $\tan(\beta)$ is of order one[12, 13, 3]; another in which $\tan(\beta)$ is automatically large.

In the next section, we will explain why the little hierarchy problem, in the framework of gauge mediation, suggests that the supersymmetry breaking scale should be low, and the superparticle spectrum compressed or squashed[14, 15, 8, 9]. In section 3, we will review the classification developed in [7] of theories exhibiting supersymmetry breaking, with a view to low energy gauge mediation. In most, significant additional structure is required in order to mediate supersymmetry breaking, especially if the supersymmetry breaking is to arise at a low scale. Additional scales must be introduced, and the scale of supersymmetry breaking at low energies is typically a ratio of various more microscopic mass scales. The situation is different in the class of models developed in[3, 6]. Here one begins with a (generalized) O’Raifeartaigh model, and understands the coefficients of relevant operators as dynamically generated by physics at some much higher energy scale. In section 4, we will discuss (elaborating ideas in [5]) how coefficients of relevant operators of different dimensions can all be given by powers of the same mass scale; this is necessary if the supersymmetry breaking scale is to be low. We explain why these models
frequently possess continuous R symmetries at low energies, and discuss both spontaneous and explicit breaking of these symmetries. In the resulting models, we explain why the spectrum need not be that of minimal gauge mediation (MGM), and can be compressed. Finally, we discuss mechanisms for generating suitable $\mu$ and $B_\mu$. A proposal of Giudice and Rattazi is readily implemented in this framework and can lead to moderate $\tan \beta$; another mechanism, closer to the spirit of the retrofitted models, leads to a large value of $\tan(\beta)$.

2 Implications of the Little Hierarchy

At present, the MSSM appears to be fine-tuned at the one percent level. While there are some regions of the parameter space where the difficulties may not be so serious\cite{16}, and additional degrees of freedom may ameliorate the problem, the tuning is even more severe in gauge mediated models. In general supersymmetric models, there are two sources of difficulty. First, in order to enhance the Higgs quartic coupling so as to be compatible with the LEP bound, one requires $\tilde{m}_t > 800$ GeV, and/or substantial stop mixing. This, in turn, implies a large correction to the Higgs mass from stop loops:

$$\delta m^2_H = \frac{-12y_t^2}{16\pi^2} m_t^2 \ln(M/m_t).$$

(1)

If the cutoff, $M$, is of order the Planck scale, and $m_t \sim 800$ GeV, then

$$\delta m^2_H/m^2_Z \approx 189$$

(2)

which implies a severe fine tuning. This problem can be ameliorated if $m_t$ is smaller. This requires the presence of additional dynamics beyond the MSSM\cite{17}. If, for example, $\tilde{m}_t = 300$ GeV, $\delta m^2_H/m^2_Z \approx 27$, suggesting a fine tuning of order 4%.

The situation is perhaps worse in MGM models, since even if there are extra degrees of freedom which can explain the Higgs mass, the mass of the stop is large. In MGM, the messengers lie in 5 and $\bar{5}$ representations of the usual $SU(5)$, and couple to a gauge singlet, $S$, with $\langle S \rangle = s + \theta^2 F_S$. Then

$$\tilde{m}^2 = 2\Lambda^2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 + \frac{5}{3} \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \right],$$

(3)

where $\Lambda = F_S/s$. $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets. This formula predicts definite ratios of squark, slepton and gaugino masses. Coupled with the current limits on the lightest sleptons (approximately 100 GeV),
it implies that slepton doublets have masses greater than 215 GeV, while squark masses are larger than 715 GeV. The cutoff, \( M \), in gauge-mediated models, is of order the messenger scale. Assuming a value of this scale of order the GUT scale, as in many models,
\[
\delta m_h^2/M_Z^2 \approx 130; \tag{4}
\]
if \( M = 10^2 \) TeV, 130 is reduced to 21, suggestive of a 5% fine tuning. So from the perspective of fine tuning, a low value of this scale is preferable. If the stop quark mass were not much above the current limit (say 300 GeV), and \( M \) were of this order, the fine tuning would be insignificant.

The challenge for gauge-mediated model building, then, is to obtain slepton singlets not much above the current experimental bound, doublet masses not much larger, and triplet masses not much above 300 GeV, while breaking supersymmetry at a low scale. This compression of the spectrum requires a structure different from that of minimal gauge mediation, which we will refer to as “General Gauge Mediation”, or GGM. Examples of such structures have appeared in the literature (see, e.g., [4]).\(^1\) This can be achieved in a model with multiple gauge singlets, \( S_i \). For example, with a single 5 and \( \bar{5} \) of messengers, with 5 = \( q + \ell \), \( \bar{5} = \bar{q} + \bar{\ell} \),
\[
W = \lambda_i S_i \bar{q}q + \gamma_i S_i \ell \ell \tag{5}
\]
where
\[
\langle S_i \rangle = s_i + \theta^2 F_i, \tag{6}
\]
we can define:
\[
\Lambda_q = \frac{\lambda_i F_i}{\lambda_i s_i} \quad \Lambda_\ell = \frac{\gamma_i F_i}{\gamma_i s_i} \tag{7}
\]
The masses of the gluinos are given by
\[
m_\lambda = \frac{1}{2} \frac{\alpha_3}{4\pi} \Lambda_q \quad m_w = \frac{1}{2} \frac{\alpha_2}{4\pi} \Lambda_\ell \quad m_b = \frac{1}{2} \frac{\alpha_1}{4\pi} \left[ \frac{2}{3} \Lambda_q + \Lambda_\ell \right]. \tag{8}
\]
Similarly, for the squark and slepton masses we have:
\[
\bar{m}^2 = 2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 \Lambda_q^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 \Lambda_\ell^2 + \left( \frac{Y}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \left( \frac{2}{3} \Lambda_q^2 + \Lambda_\ell^2 \right) \right], \tag{9}
\]
\(^1\)Such compression of the spectrum has been considered by many; N. Arkani-Hamed, N. Seiberg, and S. Thomas have all mentioned this possibility to us. Indeed, even the structure of the earliest gauge-mediated model[18] is not that of MGM.
The breakdown of the simple relation (“unification relation”) for gaugino masses is potentially problematic for electric dipole moments, since the phases of the various gaugino masses are no longer identical. As we will discuss shortly, some suppression of CP violating phases is almost certainly required in such a situation.

Recently, Cheung et al[8] have discussed models with multiple messengers and $R$ symmetries, with a single field, $S$, coupling to the messengers. In this case, they show that the squark and slepton masses can be compressed, while, perhaps surprisingly, the gaugino masses are still unified. So this class of models has less difficulty with questions of CP violation, then general gauge-mediated models. We will discuss later how these models can emerge naturally with low scale supersymmetry breaking. There is another potential difficulty with these models, however. In models such as that of eqn. 5, there is an a approximate left-right symmetry under which the hypercharge $D$ term is odd. But in the models with multiple messengers, this is not automatically the case. As a result, there can be one loop contributions to squark and slepton masses, of both positive and negative sign[18]. On the other hand, as we will explain elsewhere, such an approximate messenger parity symmetry can arise as an accident even in more complicated models.

In retrofitted models with explicit breaking of the $R$ symmetry, the formulas of minimal gauge mediation also do not necessarily hold. As in the case of multiple singlets, some suppression of CP violating phases is generally required to suppress electric dipole moments.

3 Survey of DSB Models

Recently, there has been an appreciation that metastable supersymmetry breaking is a rather generic phenomenon in supersymmetric theories, occurring even in theories which are non-chiral and posses non-vanishing Witten index. It has been noted that these models open many new possibilities for high energy supersymmetry breaking[19, 20, 3]. Here our focus is on the possibility of using such models for low energy breaking. Ref. [7] distinguishes four classes of theories which exhibit dynamical supersymmetry breaking (DSB):

1. The 3−2 class: these were the earliest theories of DSB which were well understood. These models are distinguished by a chiral structure, an absence of pseudomoduli (flat directions) and $R$ symmetries, at the level of non-renormalizable operators (more generally, operators up to a given dimension). In known examples, the $R$ symmetry is spontaneously broken.
The model with gauge group $SU(3) \times SU(2)$ and fields with the quantum numbers of a single generation, is the simplest example in this class. If one writes general, higher dimension operators in these theories, one finds supersymmetric minima at large fields. So one expects that even in these models, the vacuum is (highly) metastable.

2. The ISS class: The simplest model in this class is the $SU(N)$ gauge theory with $3/2N > N_f > N + 1$ studied in [2]. The models possess, in their simplest versions, distinct features: they are non-chiral, they possess no classical flat directions, and they have non-vanishing Witten index. Known examples possess parameters with dimensions of mass. These masses can be given a dynamical explanation if further gauge interactions are introduced (as in the retrofitted models below)[19, 20]. In simple examples, the susy-breaking vacua of these models exhibit an unbroken $R$ symmetry. This feature is probably not fundamental. It is also possible to break the symmetries of the models explicitly, again through retrofitting. In simple models, achieving a low scale of supersymmetry breaking requires coincidences, as we will explain.

3. The ITIY class: Models in this class have been constructed by coupling gauge singlets to a theory with a quantum moduli space[21]. More generally, these theories are characterized by coupling of singlets to a theory with flat directions (and, in particular, no mass gap); the resulting construction has a pseudomoduli space of vacua. The models possess a continuous $R$ symmetry at the level of low dimension operators. This symmetry may be spontaneously broken in the presence of additional weak gauge interactions. Recently, Ibe and Kitano have speculated on the existence of metastable $R$-breaking minima in the strong coupling region[22]. If this assumption is correct, this opens additional model-building strategies. Explicit breaking, as explained below, may require coincidences.

4. The retrofitted class: These are (generalized) O’Raifeartaigh models, in which some or all dimensionful parameters arise as a consequence of non-perturbative dynamics at a high energy scale. In the original proposal, the non-perturbative dynamics was associated with some new strong interactions. However, as pointed out in [6], string-scale instantons (and presumably other very high energy effects) could generate such couplings.

In the following subsections, we consider the first three classes in turn as models of low energy DSB. We address each of the phenomenological issues we enumerated above: the scales of new dynamics, $R$ symmetry breaking, and the origin of the $\mu$ and $B_\mu$ terms. The next section is devoted to the retrofitted models.
3.1 The 3 – 2 Class

Many models in this class admit the possibility of large global symmetry groups. One can try to identify a subgroup of this global symmetry group with the symmetry of the Standard Model. This provides a model of direct mediation: there is no limit of such a theory in which supersymmetry remains broken and the messengers decouple. Such a theory would yield a spectrum of gauginos, squarks and sleptons like that of gauge mediation, and in the simplest cases like that of minimal gauge mediation. The scale of supersymmetry breaking can well be very low. There are, however, two difficulties with such a scheme:

1. Asymptotic freedom: because the underlying strong groups are typically quite large, asymptotic freedom of the various gauge groups is rapidly lost.

2. Higgs: unless the Higgs fields somehow emerge from the strong dynamics (along the lines of little higgs theories), they do not couple through renormalizable operators to the fields which break supersymmetry. As a result, \( \mu \) and \( B_\mu \) are protected by approximate symmetries. It is necessary to add additional degrees of freedom, such as singlets or additional \( U(1) \) gauge fields, which couple to the susy breaking sector.

An alternative approach is to take for messengers additional degrees of freedom with Standard Model quantum numbers. It is then necessary that there be additional degrees of freedom which couple both to messengers and to the supersymmetry breaking sector. Workable models of this type have been constructed, but they are typically quite complicated, and the underlying SUSY-breaking sector tends to be at a quite high energy scale, because of the extra loop factors involved\[23\].

3.2 The ISS Class

The ISS models greatly broaden the class of susy-breaking theories. However, in models constructed to date, in addition to the scale, \( \Lambda \), of the strong dynamics, another scale is required (\( m \), the quark mass in the simplest models). To achieve low energy supersymmetry breaking, the scales \( \Lambda \) and \( m \) must be similar. Even if \( m \) arises dynamically, a rather remarkable coincidence is required.

Putting this concern aside, one can explore different approaches to model building. The ISS models often admit large global symmetries, as in the 3 – 2 case. Constructing models of
direct mediation runs into difficulties with asymptotic freedom and with $\mu$ and $B_\mu$ terms, as above. The simplest models have the further difficulty that they possess approximate unbroken $R$ symmetries. To date, most model building with these theories has involved the introduction of $R$ symmetry breaking dynamics at very high scales, with $\mu$ and $B_\mu$ terms generated by similar dynamics. The scale of supersymmetry breaking in these models is quite large, typically intermediate between the 100 TeV scale and $10^8$ TeV. Seiberg and Shih[11] and Shih[4] have explored alternative possibilities for breaking the $R$ symmetry. Banks and collaborators have followed a different approach. They assume that in a model with non-calculable strong dynamics (the “pentagon model”), the $R$ symmetry is broken. The required coincidence of scales is assumed to originate from principles of an underlying theory of gravity (“cosmological supersymmetry breaking”). The pentagon model has five additional vector-like 5 and $\bar{5}$’s in the sense of ordinary $SU(5)$, so the unified coupling is perhaps just barely small enough to account for perturbative unification.

3.3 The ITIY Class

Many of the comments of the previous sections apply to the ITIY models. Again, it is possible to obtain large global symmetries which can be gauged, but this leads to difficulties with asymptotic freedom. Similar to the ISS case, the simplest models do not break $R$ symmetry at the scale of the underlying strong dynamics, and again, most model building along these lines[24] assumes $R$ symmetry and supersymmetry breaking at a very high energy scale. Recently, conjecturing that there is a metastable vacuum in the strongly coupled region, Ito and Kitano[22] have constructed a potentially viable model with low scale messengers. However because there are five additional vector-like fields, unification may be problematic, as in other instances we have discussed.

4 Model Building in Retrofitted Models

What has been called retrofitting is a simple realization of Witten’s original vision for dynamical supersymmetry breaking[1]: an exponentially small coefficient of a relevant operator is generated by dynamical effects, precipitating supersymmetry breaking.

The basic ideas of retrofitted models can be illustrated with the simplest O’Raifeartaigh
model:

\[ W = Z(A^2 - \mu^2) + mYA. \]  

(10)

Here we have not explicitly indicated a dimensionless number (the coefficient of \(ZA^2\)) in the superpotential; we will frequently do this to avoid introducing names for parameters not germane to our main arguments. The idea is to replace the parameters \(\mu\) and/or \(M\) with dynamically generated scales. One simple way to do this is to replace \(\mu^2\) in the superpotential by \(W^2_\alpha/M\), where \(W_\alpha\) is the field strength of some new, strongly interacting group (which does not break susy), i.e. to replace \(W\) by:

\[
\int d^2\theta \left( Z(A^2 - \frac{W^2_\alpha}{M}) + mYA \right). 
\]  

(11)

Then \(\mu^2 = \Lambda^3/M\), where \(\Lambda\) is the scale of the new gauge group.

An even simpler version of this idea, in the sense that the number of degrees of freedom at low energies is smaller, is provided by the work of [6]. These authors exhibit string theory constructions where parameters such as \(\mu^2\) or \(m\) are generated by string scale instantons.

For low scale supersymmetry breaking, one needs \(\mu \sim m\). As explained in [5], this can arise if the underlying theory possesses a discrete \(R\) symmetry, with the dynamical source of the parameters \(\mu\) and \(m\) transforming under the symmetry. The idea that the dimensionful parameter, \(\Lambda\), arising in strong dynamics or instanton computations can transform under a discrete (gauge) symmetry is a familiar one. Consider, for example,

\[
\int d^2\theta(Z(A^2 - \frac{W^4}{M^4}) + \frac{W^2_\alpha}{M^2}YA). 
\]  

(12)

With \(W^2_\alpha \rightarrow e^{2\pi ik/N}W^2_\alpha\), \(A \rightarrow e^{2\pi ik/N}A\), and \(Y\) and \(Z\) are neutral (it is also necessary to impose a \(Z_2\), under which \(A\) and \(Y\) are odd), this is the most general structure consistent with symmetries.

4.1 R Symmetries: Spontaneous Breaking

In eqn. 12, we have exhibited only the lowest dimension terms (relevant and marginal operators constructed from \(A, Z, Y\)). At this level, the theory has an accidental, continuous \(R\) symmetry. The appearance of such symmetries is typical; in the next section, we will explore ways to avoid them. In building realistic models, with messengers, (spontaneous) breaking of the \(R\) symmetry is crucial. It is well known that in this simple model, there is no vacuum close to the origin.
which breaks the $R$ symmetry. As shown by Shih[4], this result is general for theories without
gauge interactions, and with the restriction that all $R$ charges are 0 or 2; with more general $R$
charges, spontaneous breaking can occur. With gauge interactions, or with fields with negative
$R$ charge, the symmetry may be spontaneously broken[5, 11]. In [5], for example, a model
was constructed with additional gauge interactions, a single neutral field, $Z$, and charged fields
$Z^\pm, \phi^\pm$. The superpotential was taken to be:

$$W = M(Z^+\phi^- + Z^-\phi^+) + \lambda Z^0(\phi^+\phi^- - \mu^2).$$  \hspace{1cm} (13)

Here $Z^\pm, Z^0$, at tree level, have non-zero $F$-components; at one loop, for a range of parameters,
they have non-zero scalar components as well, breaking the $R$ symmetry. $Z^0$, for example, can
then be coupled to messengers neutral under the $U(1)$. As in eqn. 12, the scales $\mu$ and $M$
can naturally be of the same order, so low energy breaking can be natural[5].

In simple models, with only a small number of susy breaking fields and a single pair of
messengers, one obtains the spectrum of minimal gauge mediation. With more fields, the pre-
dictions may be modified (and one might obtain a compressed spectrum). As we will explain
later, there are a variety of ways to obtain a $\mu$ term with a suitably small $B_\mu$ in these cir-
cumstances. While natural, $R$ symmetry breaking requires that the new gauge coupling to be
similar in value to the Yukawa coupling[11]. So it is interesting to consider mechanisms which
might give rise to an explicit breaking of the $R$ symmetry (i.e. for which there is no accidental
$R$ symmetry in the low energy theory).

As we will discuss in the next section, it is possible, in the retrofitted framework, to
formulate theories without $R$ symmetries at low energies. But first, we note that Cheung et.
al.[8] have considered a larger class of theories with $R$ symmetries at low energies. They consider
theories with a single supersymmetry-breaking field, $X$, coupled to multiple messengers, with
the quantum numbers of 5 and $\bar{5}$'s of $SU(5)$, $5_i$ and $\bar{5}_i$. They take the general superpotential

$$W = \lambda_{ij}X5_i\bar{5}_j + m_{ij}5_i\bar{5}_j$$  \hspace{1cm} (14)

and assume that the couplings preserve a continuous $R$ symmetry. While the couplings and
fields we have written above are shorthand for couplings to fields $q_i, \bar{q}_i$ and $\ell_i, \bar{\ell}_i$, and the
couplings to triplets and doublets need not be identical, it is important in their analysis that
the $R$ charges of doublets and triplets are the same. They then show, as we indicated before,
that the formulas of MGM for the scalar masses are modified, but that the gaugino masses do
obey unification relations. This has promise for ameliorating the little hierarchy problem while
possibly solving the problems of CP violation in supersymmetric theories. It is straightforward
to produce the soft breaking parameters in many of his models through retrofitting, in a way which naturally yields the required approximate R symmetries.

While these models have virtues for understanding CP violation, they possess another potential difficulty. With multiple messengers, supersymmetry breaking can induce a non-zero $D$-term for hypercharge at one loop (for a discussion, see [18, 15]). Additional special features are required to suppress these. Typically one invokes a “Messenger parity” symmetry. By itself, this symmetry is necessarily violated by the gauge couplings of the MSSM fields, but such an approximate symmetry can arise as an accident of other symmetries, as will be discussed elsewhere. In addition, from our earlier discussion, it is clear that $R$ symmetry, by itself, is not enough to guarantee suppression of CP violation. If there are several singlets, $X_i$, of $R$ charge 2, then the unified formula for the gaugino masses does not hold.

4.1.1 Retrofitting Cheung et al’s Model

Rewriting the model above in the notation of [8]:

$$W = \lambda X(\tilde{\phi}_1 \phi_2 + \phi_2 \phi_2) + m\tilde{\phi}_2 \phi_1 + FX$$

Such a model has an R-symmetry. Following the analysis of [4], the Coleman Weinberg potential for this model can lead to $\langle X \rangle \neq 0$, spontaneously breaking the R-symmetry and making such models phenomenologically viable. In the case that $F$ and $m_i$ are generated dynamically from gaugino condensation we have a discrete R-symmetry under which $R(F) = 2$ and $R(m_i) = 1$. The Superpotential can be constrained to be of this form if, for example, $R(W) = 2, R(X) = -2, R(\phi_1) = 1, R(\phi_1) = 3, R(\phi_2) = 5$, and $R(\tilde{\phi}_2) = -1$. (and there are no $Z_N$ equivalences).

4.2 R Symmetries: Explicit Breaking

Achieving explicit breaking in a model where all scales are comparable, and in which the structure of the underlying lagrangian is dictated by symmetries, turns out to be challenging; it is not simple to achieve metastable supersymmetry breaking in a natural way. To illustrate the problems, consider adding messengers to the simple O’Raifeartaigh model. We can take these to be a 5 and $\bar{5}$ of $SU(5)$ ($q, \bar{q}, \ell, \bar{\ell}$), with couplings

$$W = Z\mu^2 + Z A^2 + mY A + (m_3 + \lambda_3 Z) q\bar{q} + (m_2 + \lambda_2 Z) \ell\bar{\ell}.$$ 

The terms $m_1, m_2$ as well as the supersymmetry breaking scale ($F_Z$) can be generated dynamically. Classically, supersymmetry is broken in this theory; provided $|m_i + \lambda_i Z| > |\mu|$, 

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the $q, \bar{q}, \ell, \bar{\ell}$ fields have positive mass-squared. $\langle Z \rangle$ is undetermined classically. At one loop, the standard Coleman Weinberg computation gives a minimum, whose precise value depends on the parameters in the superpotential. At the minimum, the fields $q, \bar{q}$ and $\ell, \bar{\ell}$ have supersymmetric masses,

$$m_q = m_3 + \lambda_3 Z \quad m_\ell = m_2 + \lambda_2 Z.$$  \hfill (17)

Instead of the usual gauge-mediated formula, we now have the formula of eqn. 9, with

$$\Lambda_q^2 = \frac{\lambda_3^2 F_Z}{m_q} \quad \Lambda_\ell^2 = \frac{\lambda_2^2 F_Z}{m_\ell}.$$  \hfill (18)

So we have a spectrum in which the mass formula of MGM does not hold, and in which compression of the spectrum is possible.

Not surprisingly, it is difficult to obtain this structure as a consequence of symmetries, if $\mu$ and $m$ are to be the same order. Examining equations 11,12, one sees that the $R$ charge of $Z$ and $m_i$ and $\mu$ must be the same. This necessarily means that $Z^3, mZ^2$ and $m^2 Z$ (and others) are allowed by the symmetry. In order that there be a metastable ground state within a distance of order $m_i$ from the origin, the coefficients of each of these terms must be extremely small, substantially smaller than the loop factors which give rise to the $Z$ potential near the origin.

On the other hand, we are used to the idea that there are very small Yukawa couplings in nature. A standard scenario for generating such small couplings is provided by the Froggatt-Nielsen mechanism. Here there is some field, $\phi$ (for convenience taken to be dimensionless), transforming under additional symmetries, such that $\phi$ is a small number (in our example below, we will need $\phi$ small compared to a loop factor). Adopting this approach will imply a hierarchy between the masses of the messengers and the fundamental supersymmetry breaking scale.

Introducing such a field, we can consider a superpotential of the form:

$$W = m^2 \phi^2 Z - Z A^2 + m_\phi Y A + m \phi^2 \bar{55} + Z \phi^2 \bar{55}$$  \hfill (19)

where we are being schematic in two ways:

1. We are again not indicating explicitly dimensionless numbers of order one.

2. Couplings and masses such as $m\bar{55}$ are shorthand for $m_1q\bar{q} + m_2\ell\bar{\ell}$
In this potential, the splittings of the scalar components of $5, \bar{5} (m^2\phi^4)$ are comparable to the masses-squared of the fields, necessary if the messenger masses are to be only a loop-factor removed from the masses of the squarks and sleptons. Note, however, that the messenger fields are lighter by a factor of $\phi$ than the fields $A$ and $Y$. The lagrangian, classically, exhibits, for general values of $Z$, supersymmetry breaking metastable ground states with $\langle 5 \rangle = \langle \bar{5} \rangle = 0$; there is no approximate $R$ symmetry. $Z$ will be determined by quantum mechanical effects, which we will discuss shortly.

The superpotential of eqn. 19 is not the most general consistent with symmetries. Under any would-be symmetry, $Z$ transforms as $m$. This means that $m\phi^2 Z^2$ and $\phi^2 Z^3$ are allowed, as well as similar terms involving $Y$. These additional terms are dangerous. They lead to supersymmetric minima, and they yield tadpoles and mass corrections which may destabilize the non-supersymmetric minima if $\phi$ is not sufficiently small. To understand the constraints on $\phi$, consider the Coleman-Weinberg calculation for the system represented by 19. The largest contribution to the potential comes from loops containing the field $A$. This calculation is standard, and gives a minimum for $Z$ at the origin. The curvature of the Coleman-Weinberg potential is of order

$$V'' \sim \frac{|F_Z|^2}{m^2\phi^4} \sim \alpha m^2.$$

(20)

Here $\alpha$ denotes a loop factor ($\frac{\lambda^2}{16\pi^2}$, where $\lambda$ is a typical dimensionless coupling in the superpotential). Including $5, \bar{5}$ loops gives a small $Z$ tadpole

$$\delta V'' \sim \alpha m^2 \phi^{12}.$$

(21)

These lead to tiny shifts in $Z$ ($O(\phi^6)$).

Now including the coupling $m\phi^2 Z^2$ gives, classically, a tadpole for $Z$ of order $m^3\phi^4$; this leads to a shift in $Z$ of order $\alpha^{-1} m\phi^2$. However, the $Z$ potential varies (in field space) on scales $m\phi$, so for $\phi < \alpha$ this is a small shift. Similarly, the $Z^3$ coupling generates a mass term for $Z$ near the origin of order $m^2\phi^2$; this is larger than the Coleman Weinberg contribution by a factor of order $\alpha^{-1}$.

The basic structure of the model is technically natural; renormalizable couplings other than those we have mentioned can be forbidden by symmetries. A sample set of symmetries are listed in the table below:
| Field | R | R' | Z_2 |
|-------|---|----|-----|
| W     | 3 | 7  | 0   |
| m     | 1 | 1  | 0   |
| A     | 1 | 3  | -1  |
| Y     | 1 | 1  | -1  |
| Z     | 1 | 1  | 0   |
| φ     | 0 | 2  | 0   |
| 55    | 2 | 2  | 0   |

The first of these symmetries accounts for the absence of large mass terms. The second explains the suppression by various powers of φ. The third forbids certain potentially dangerous couplings, like YA^2.

It is interesting that to an observer at the scale m_i, the superpotential does not appear generic, and there is no relic of the symmetries responsible for this at low energies. The observer would simply note that the structure is preserved in perturbation theory, due to non-renormalization theorems.

In these models, while the messenger scale can be of order 100 TeV, the underlying scale of supersymmetry breaking is necessarily significantly larger. The messenger scale is φ^2m; the splittings among the messengers are of the same order, so this scale might be as low as 100 TeV or so. So, along with compression of the spectrum, these models can ameliorate the fine tuning problems of gauge mediated models. However, the susy-breaking F term, which determines the gravitino mass, is of order (φm)^2, and the gravitino mass correspondingly larger. This is potentially problematic for dark matter[25], requiring a non-standard cosmology above the weak scale.

### 4.3 CP Violation

Models with *spontaneous* breaking of R symmetry often have the virtue that CP violation can automatically be small or zero. If, for example, Z^0 in eqn. 13 is replaced by its expectation value, i.e. if fluctuations of Z are ignored, then through field redefinitions, all of the couplings of the lagrangian may be made real; CP is, at the very least, loop suppressed. As we will shortly see, this feature can be preserved in some cases by the dynamics which generates the supersymmetric Higgs mass, μ[12, 13]. Cheung et al[8] have shown that in a broad class of models with R symmetries at low energies, even when there are CP violating phases, gaugino masses all carry the same phase, and one loop contributions to electric dipole moments vanish.
But we have also seen that $R$ symmetries are not, by themselves, enough.

If the breaking of the symmetry is explicit, CP violation is more problematic. First, it should be noted that even if this framework is embedded in a conventional GUT, one does not expect GUT relations to hold for the messenger mass terms. These arise from couplings, for example, to $W^2_\alpha$, and, in general, these will be one loop effects in the underlying theory. Alternatively, these couplings may arise from instantons, and there is, in general, no reason to expect that they should obey GUT relations.

In theories without automatic suppression of CP violation, we need to suppose that CP-violation is inherently small. This can arise if CP is spontaneously broken by a small amount. This still permits an order one KM phase, while allowing suppression of CP-violating phases in the messenger and supersymmetry violating phases, much along lines suggested some time ago by Nir and Rattazzi[26]

4.4 $\mu$, $B_\mu$ and $A$

Various proposals have been put forth for generating $\mu$ in the framework of gauge mediation. Many of these involve additional degrees of freedom, beyond those of the MSSM. One of the most attractive approaches is that of [12, 13]. Here one considers two sets of messenger fields, $5_1, \bar{5}_1$ and $5_2, \bar{5}_2$. One has couplings

$$Z(5_1\bar{5}_1 + 5_2\bar{5}_2) + S5_1\bar{5}_2 + SH_UH_D + S^3. \tag{22}$$

One can readily check that the one loop contribution to the mass of $S$ vanishes, but the two loop contribution is negative, for suitable choice of parameters. As a result, $S$ obtains a vev of one loop order, while $F_S$ is two loop order. As a result, $B_\mu \sim \mu^2$. In the context of models with spontaneous breaking of the $R$ symmetries, such an approach was followed in [5]; in both this framework and the framework of explicit $R$ symmetry breaking, one can obtain models whose structure is enforced by symmetries. As in all multimessenger models, one requires some sort of messenger parity symmetry to suppress one loop D-term contributions to sfermion masses. In eqn. 22, this can be an accidental consequence of a discrete symmetry interchanging the fields $5_1, \bar{5}_1$ with $5_2, \bar{5}_2$. As discussed above, with spontaneous breaking, one can sometimes understand the suppression of CP violating effects. This is not true with explicit violation, as we have already seen even before including the Higgs fields.

An alternative is to generate $\mu$ by “retrofitting”, just as we did the relevant parameters of
the O’Raifeartaigh models. We could include a term
\[ \gamma \frac{W^2}{M^2} H_U H_D \] (23)

in the superpotential. The structure, again, can be enforced by \( R \) symmetries. It is necessary that \( \gamma \) be small, of order a loop factor. We can simply postulate a small number, or suppose that this is small from a FN mechanism, just as we accounted for small, dimensionless numbers in the previous sections. Couplings of \( X \) or \( Z^0 \) can be forbidden by the same \( R \) symmetries, so classically, there is no \( B_\mu \) term. Instead, \( B_\mu \) is generated by one loop effects, so \( \tan(\beta) \) is large.

In the case of explicit breaking, one can again generate \( \mu \) by retrofitting, writing, e.g.
\[ \phi^3 \frac{W^2}{M^2} H_U H_D \] (24)

However, since \( Z \) transforms under any symmetry like \( m \), one cannot forbid a coupling \( Z \phi^3 H_U H_D \), so
\[ |B_\mu|^2 \gg |\mu|^2. \] (25)

Similar problems arise if we try to couple \( A \) or \( Y \) to \( H_U H_D \), and even if we allow hierarchies involving powers of \( \phi \) between different scales (this can easily be shown quite generally).

With additional fields, we can account for \( \mu \) in a natural way. As a simple example, we introduce fields \( X \) and \( B \), with superpotential:
\[ W_H = X(B^2 - \phi^4 m^2) + \phi B H_U H_D \] (26)

This leads to \( \mu \sim m \phi^3 \), which is the desired order of magnitude. Since \( B \) has no \( F \) component, classically, \( B_\mu \) vanishes. We can banish couplings of \( Z \) to \( H_U H_D \) by symmetries Under our symmetries, we can assign charges as follows:

| Field | \( R \) | \( R' \) | \( Z_3 \) |
|-------|-------|-------|------|
| \( X \) | 1 | -3 | 0 |
| \( B \) | 1 | 5 | -1 |
| \( H_U H_D \) | 2 | 0 | -1 |

This construction avoids the need for messenger parity. But, as before, CP is an issue in this framework. Again, one possible solution is that the field(s) \( \phi \) spontaneously break \( CP \). This can suppress the phases of gaugino masses. The phases associated with \( B, X \) and the Higgs fields can then be absorbed in field redefinitions.
If we assume that this is the origin of $\mu$, we can ask about the origin of $B_\mu$. In the low energy theory, there is a one loop diagram which contributes to an $H_UH_D$ term in the potential, containing an internal wino and Higgsino. This gives an $H_UH_D$ mass term of order

$$B_\mu \approx \frac{\alpha_w}{2\pi} \mu m_w |\ln(\phi)|.$$  \hspace{1cm} (27)

This leads, in turn, to a value of $\tan(\beta)$ of order

$$\tan \beta \sim \frac{2\pi}{\alpha_w |\ln(\phi)|},$$  \hspace{1cm} (28)

which, depending on the precise values of the masses, may or may not be acceptable.

Let us return briefly to models with spontaneous breaking of the $R$ symmetry and ask about alternative mechanisms for generating the $\mu$ and $B_\mu$ terms through retrofitting. Coupling a susy-breaking field, $X$, to $H_UH_D$ poses the usual $\mu - B_\mu$ problem; generating a small coupling by introducing a field $\phi$, by itself, does not help. So it is simplest to introduce, again, additional fields analogous to $B$ and $X$.

## 5 Conclusions

A priori, in speculating about the possibility of gauge-mediated supersymmetry breaking, one can contemplate phenomena over many decades of energy. We have motivated a possible role for very low energy gauge mediation by considerations of fine tuning. It is well-known, however, that the phenomenological possibilities of such low energy breaking are potentially quite interesting, including dramatic decay channels and signatures.

We have suggested that such low energy breaking arises most naturally in the framework of O’Raifeartaigh models whose dimensionful parameters all arise from non-perturbative effects in a theory which is weakly coupled at high energies. Particularly important is the question of $R$ symmetry breaking. In models with approximate $R$ symmetries (spontaneously broken), one can contemplate a compressed spectrum, approximate CP conservation, and interesting mechanisms for generating $\mu$ and $B_\mu$, in a natural framework (i.e. structures enforced by symmetry). For models with explicit breaking, many of these same features are readily obtained, though understanding CP violation is more challenging, and a hierarchy between the messenger scale and the underlying supersymmetry-breaking scale at least of order a loop factor seems required.
From a low energy point of view, these are simply O’Raifeartaigh models. In the high scale breaking models, what is surprising to a low energy observer is that the models do not appear natural (generic); one needs a microscopic understanding to see that they are the most general models consistent with symmetries. Indeed, this picture is a realization of a number of Witten’s original vision for dynamical supersymmetry breaking[1]:

1. There are small parameters, as a consequence of short distance non-perturbative effects.
2. There are couplings which vanish, for no apparent reason, in the low energy theory.
3. The consistency of the first two points is a consequence of the non-renormalization theorems for the couplings of the low energy theory. In the case of hierarchical breaking, the vanishing of certain superpotential terms can be understood in terms of the symmetries of the microscopic theory.

Surely more elegant models can be constructed than those in this paper, which are presented mainly to provide an existence proof of simple and sensible models with very low energy supersymmetry breaking.

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