INTERACTION OF FOUNDATIONS OF VARIOUS BOTTOM OUTLINES WITH GROUNDS

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**Introduction.** Despite significant numbers of Ukrainian and foreign studies on the pressures of solid bodies of various shapes onto a yielding base [1], [2] their implementation into the foundation engineering still remains quite limited. As an exception, we can give an example of foundation stamp with rounded corners solution, offered by I. Shtaermann [4], as well as foundation structure with a projection in the central part, developed by E. Sorochan [5].

It is worth mentioning that the idea of I. Shtaermann allows us to actually decrease the marginal contact tensions, whereas implementation of numerical methods allows us to calculate this decrease in various software packages.

**Results and discussion.**

Just as before, we record the consistent pattern of the changing of coefficient of soil reaction as \( C_z = C_0 \cdot \left[ 1 - 4 \cdot \alpha \cdot \bar{z} \cdot (1 - \bar{z}) \right] \), and the relation between the resistance and the given extreme depths of the buried part of foundation \( y_z \) – according to Winkler’s hypothesis \( \sigma_z = C_z \cdot y_z \).

Therefore, when we set any function \( y_z \), it is quite easy to obtain the distribution of resistances under the foundation bottom.

1.a) Parabolic outline of the bottom

![Foundation with parabolic bottom outline](image)

**Fig. 1. Foundation with parabolic bottom outline**
Figure 1 shows the position of foundation under maximum external load $\sum N_{\text{max}}$ and the sinking $y_{\text{max}}$, and this can be represented by the function of the shape of buried part of foundation $y_z = 4 \cdot y_{\text{max}} \cdot \bar{z} \cdot (1 - \bar{z})$, whereby $\bar{z} = \frac{z}{L}$.

Having substituted the coefficient of soil reaction and the function of the shape of buried part into Wincler’s equation, we get the distribution of resistances along the bottom if $0 \leq \bar{z} \leq 1$

$$\sigma_z = 4 \cdot \bar{z} \cdot (1 - \bar{z}) \cdot [1 - 4 \cdot \alpha \cdot \bar{z} \cdot (1 - \bar{z})],$$

(1)

whereby $\sigma_z = \frac{\sigma_z}{C_0 \cdot y_{\text{max}}}$.

### Table 1

| $\bar{z}$ | 0   | 0.05 | 0.1  | 0.2  | 0.25 | 0.3  | 0.4  | 0.5  |
|-----------|-----|------|------|------|------|------|------|------|
| $\sigma_z$ | 0   | 0.165| 0.269| 0.333| 0.356| 0.346| 0.315| 0.3  |

For the flat deformation if $\alpha \approx 0.7$ we obtain the values $\sigma_z$, represented above in (table 1).

Having integrated the formula (1), we obtain the sum resultant value of vertical load $\sum N_{\text{max}} = \frac{2}{3} \cdot (1 - 0.8 \cdot \alpha) \cdot C_0 \cdot l \cdot y_{\text{max}}$.

If $\alpha = 0.7$: $\sum N_{\text{max}} = 0.2933 \cdot C_0 \cdot l \cdot y_{\text{max}}$.

Having set $y_{\text{max}}$, we further find out the ultimate load, or if we already know $\sum N_{\text{max}}$, we calculate $y_{\text{max}}$.

The distribution of resistances shown in the fig.2 clearly shows that the accepted bottom shape (3) is efficient if the two loads $N$ at the distance of approximately $0.25 \cdot l$ from the ends of foundation, which significantly decreases the value of maximum bending moments.

1.b) Sinusoidal bottom outline

The function of foundation bottom shape is set as (fig. 1)

$$y_z = y_{\text{max}} \cdot \sin(\pi \cdot \bar{z}),$$

(2)

whereas the resistance of the base along the bottom is

$$\sigma_z = \sin(\pi \cdot \bar{z}) \cdot [1 - 4 \cdot \alpha \cdot \bar{z} \cdot (1 - \bar{z})].$$

(3)

Accepting, for example, that $\alpha = 0.7$, we get the values $\sigma_z$, represented in (table 1) and (fig. 2)

### Table 2

| $\bar{z}$ | 0   | 0.05 | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  |
|-----------|-----|------|------|------|------|------|------|
| $\sigma_z$ | 0   | 0.1356| 0.2311| 0.3245| 0.3333| 0.312| 0.3  |

Having integrated the formula (3), we obtain the resultant value of external load

$$\sum N_{\text{max}} = \left(\frac{2}{\pi} - 0.516 \cdot \alpha\right) \cdot C_0 \cdot l \cdot y_{\text{max}}.$$  

If $\alpha = 0.7$, we obtain $\sum N_{\text{max}} = 0.2754 \cdot C_0 \cdot l \cdot y_{\text{max}}$.

Comparing the graphs 1a and 1b in the (fig. 2), we notice their closeness, as was to be expected.
2. Bottom in the shape of a flat wedge

The position of a foundation with a maximum load is shown in (fig. 3). The function of sinking for the left half of buried part of foundation looks like $y_z = 2 \cdot y_{\text{max}} \cdot \bar{z}$, whereas resistance distribution, if $0 \leq z / \bar{z}$, is equal to

$$\bar{\sigma}_z = 2 \cdot \bar{z} \cdot \left[1 - 4 \cdot \alpha \cdot \bar{z} \cdot (1 - \bar{z})\right]. \quad (4)$$

If $\alpha = 0.7$, the value $\bar{\sigma}_z$ is represented in (table 3) and (fig. 2).

The resulting sum load on the foundation is obtained through integrating of formula (4) and it amounts to $N_{\text{max}} = 0.5 \cdot \left(1 - \frac{\alpha}{6}\right) \cdot C_0 \cdot l \cdot y_{\text{max}}$.

If $\alpha = 0.7$, then $N_{\text{max}} = 0.2083 \cdot C_0 \cdot l \cdot y_{\text{max}}$. 

Fig. 2. 1a – parabolic; 1b– sinusoidal; 2 – flat wedge; 3 – sinusoidal wedge

Fig. 3. Foundation with a flat wedge bottom outline
Table 3

| \( \bar{z} \) | 0   | 0.05 | 0.1  | 0.2  | 0.3  | 0.4  | 0.45 | 0.5  |
|----------------|-----|------|------|------|------|------|------|------|
| \( \bar{\sigma}_z \) | 0   | 0.087| 0.150| 0.221| 0.247| 0.262| 0.276| 0.3  |

3. Sinusoidal wedge bottom shape

![Fig. 4. Foundation with a sinusoidal wedge bottom outline](image)

We accept the function \( y_z \) (for half of the foundation) as

\[
y_z = y_{\text{max}} \cdot \left(1 - \cos \left( \pi \cdot \bar{z} \right) \right),
\]

then the resistance distribution, if \( 0 \leq \bar{z} \), looks like

\[
\bar{\sigma}_z = \left(1 - \cos(\pi \cdot \bar{z})\right) \cdot \left[1 - 4 \cdot \alpha \cdot \bar{z} \cdot (1 - \bar{z})\right].
\]

If \( \alpha = 0.7 \), the value \( \bar{\sigma}_z \) can be shown in (table 4) and (fig. 2).

Table 4

| \( \bar{z} \) | 0   | 0.05 | 0.1  | 0.2  | 0.3  | 0.4  | 0.45 | 0.5  |
|----------------|-----|------|------|------|------|------|------|------|
| \( \bar{\sigma}_z \) | 0   | 0.0107| 0.0366| 0.105 | 0.167 | 0.227 | 0.3  |

Through integrating the formula (6), we obtain

\[
N_{\text{max}} = (0.3634 - 0.3246 \cdot \alpha) \cdot C_0 \cdot l \cdot y_{\text{max}}.
\]

If \( \alpha = 0.7 \), we get for the flat task \( N_{\text{max}} = 0.13617 \cdot C_0 \cdot l \cdot y_{\text{max}} \).

Conclusions.

Many scientists have been trying to decrease the concentration of marginal contact strain under the bottom of foundation through modeling of the base as a finite elastic layer. However, as I. Shtayermann has demonstrated, as long as the angle of the border of foundation and its bottom is 90º, it is impossible to avoid resistance concentration for any layer thickness.

We directly solved the task of foundation design through the suggested “deformation method”. We determine the distribution of ground resistance for various foundation sizes with given surface shape containing the bottom of a foundation, and this will help to obtain the efficient structure of a foundation bottom.

References:

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