Attractor neural networks
A ‘puzzle picture’
Dallenbach (1951)
Andy Goldsworthy artwork
micro-electrode
Primary visual cortex
Optic radiations
Optic tract
Lateral geniculate nucleus

an absolute depth judgment with respect to fixation, while fine stereopsis requires the judgment of relative depth, i.e., comparing depth across space; (2) the particular coarse stereopsis task used requires the monkey to discriminate a signal in noise, while the fine task does not; (3) the range of disparities is quite different. Chowdhury and DeAngelis (2008) replicate the finding that monkeys initially trained on coarse stereopsis show impaired coarse depth discrimination when muscimol is injected into MT. Remarkably, the same animals, after a second round of training on fine stereopsis, are unimpaired at either fine or coarse depth discrimination by similar injections. Moreover, recordings in MT show that neuronal responses are not altered by learning the fine stereopsis task. Given the differences between the tasks and the large number of visual areas containing disparity-sensitive neurons, one might not be surprised to find different areas involved in the two tasks. But it is quite unexpected that merely learning one task would change the contribution of areas previously involved in the other. Chowdhury and DeAngelis conclude that the change in outcome reflects a change in neural decoding—decision centers that decode signals to render judgments of depth, finding MT signals unreliable for the fine stereopsis task, switch their inputs to select some better source of disparity information. Candidates include ventral stream areas V4 or IT, where relative disparity signals have been reported (Orban, 2008) and which contain far more neurons (Figure 1). When challenged afresh with the coarse depth task, these same decision centers may now find that their new sources of information can solve the coarse task as well as the old ones. MT is no longer critical. Perhaps in other monkeys MT would never have a role in stereopsis at all. Chowdhury and DeAngelis' monkeys were trained simultaneously or previously to discriminate motion, which engages MT. Faced with a qualitatively similar random dot stimulus, it might make sense for the cortex to try to solve the new problem of stereopsis with existing decoding strategies. But if the animals were initially trained on a different task—say, a texture discrimination—MT might never be engaged at all. It would also be interesting to see the outcome if monkeys were trained on depth tasks that were less different and could be interleaved in the same sessions, for example noise-limited depth judgments using similar absolute or relative disparity.
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![Diagram of the Cortical Visual Areas of the Macaque](image)

**Figure 1.** A Scaled Representation of the Cortical Visual Areas of the Macaque. Each colored rectangle represents a visual area, for the most part following the names and definitions used by Felleman and Van Essen (1991). The gray bands connecting the areas represent the connections between them. Areas above the equator of the figure (reds, browns) belong to the dorsal stream. Areas below the equator (blues, greens) belong to the ventral stream. Following Lennie (1998), each area is drawn with a size proportional to its cortical surface area, and the lines connecting the areas each have a thickness proportional to the estimated number of fibers in the connection. The estimate is derived by assuming that each area has a number of output fibers proportional to its surface area and that these fibers are divided among the target areas in proportion to their surface areas. The connection strengths represented are therefore not derived from quantitative anatomy and furthermore represent only feedforward pathways, though most or all of the pathways shown are bidirectional. The original version of this figure was prepared in 1998 by John Maunsell.
Recurrent neuronal circuits in the neocortex
(Douglas & Martin 2007)

(Binzegger, Douglas & Martin, 2004)

(Douglas and Martin, 2007)
Energy function:

\[ E = -\frac{1}{2} \sum_{i,j \neq i} T_{ij} V_i V_j \]

Dynamics:

\[ U_i = \sum_j T_{ij} V_j \]
\[ V_i = \text{sign}(U_i) \]
Basins of attraction
Outer-product (Hebb) rule

\[ T_{ij} = \sum_{\alpha} P_i^{(\alpha)} P_j^{(\alpha)} \]
\[ = P_i^{(1)} P_j^{(1)} + P_i^{(2)} P_j^{(2)} + P_i^{(3)} P_j^{(3)} + \ldots \]

or

\[ T = P^{(1)} P^{(1)T} + P^{(2)} P^{(2)T} + P^{(3)} P^{(3)T} + \ldots \]

Thus

\[ U \equiv (P^{(1)} P^{(1)T} + P^{(2)} P^{(2)T} + P^{(3)} P^{(3)T} + \ldots) V \]
\[ = P^{(1)} (P^{(1)} \cdot V) + P^{(2)} (P^{(2)} \cdot V) + P^{(3)} (P^{(3)} \cdot V) + \ldots \]

\[ V = \text{sgn}(U) \]
Capacity vs. error rate

\[ P(C_i^\nu) \]

\[ \sigma = \sqrt{p/N} \]

\[ C_i^\nu \]

\[ P_{\text{error}} \]

**TABLE 2.1 Capacities**

| \( P_{\text{error}} \) | \( p_{\text{max}}/N \) |
|------------------------|-------------------------|
| 0.001                  | 0.105                   |
| 0.0036                 | 0.138                   |
| 0.01                   | 0.185                   |
| 0.05                   | 0.37                    |
| 0.1                    | 0.61                    |
Hopfield network with analog units

\[ \tau \dot{u}_i + u_i = \sum_{j \neq i} T_{ij} V_j + I_i \]

\[ V_i = g(u_i) \]
Lyapunov function

\[ E = -\frac{1}{2} \sum_{i} \sum_{j \neq i} T_{ij} V_i V_j + \sum_{i} \int_{0}^{V_i} g^{-1}(V) dV - \sum_{i} V_i I_i \]
From Lyapunov function to dynamics

\[ E = -\frac{1}{2} \sum_i \sum_{j \neq i} T_{ij} V_i V_j + \sum_i \int_0^{V_i} g^{-1}(V) dV - \sum_i V_i I_i \]

\[ \frac{\partial E}{\partial V_k} = -\sum_{j \neq k} T_{kj} V_j + g^{-1}(V_k) - I_k \]

Let \( u_i = g^{-1}(V_i) \) \( \Rightarrow \) \( V_i = g(u_i) \)

\[ \dot{u_i} \propto -\frac{\partial E}{\partial V_i} = \sum_{j \neq i} T_{ij} V_j + I_i - u_i \]

Thus \( \dot{E} = \frac{\partial E}{\partial V} \frac{\partial V}{\partial u} \dot{u} < 0 \)
State space
Place cells, grid cells, head-direction cells and continuous attractor neural networks

O’Keefe & Dostrovsky (1978) “The Hippocampus as a Cognitive Map”

1970

O’Keefe & Nadal (1978) Continuous attractor model of head-direction cells

1980

Zhang (1996) Continuous attractor model of grid cells

1990

Ranck (1985); Taube et al. (1990)

2000

Burak & Fiete (2009) Continuous attractor model of grid cells

2010

Hafting, Fyhn, Molden, Moser & Moser (2005)

2020

Chaudhuri et al. (2019) Ring topology

Gardner et al. (2022) Toroidal topology

“Place cells”

“Head-direction cells”

“Grid cells”

Seelig & Jayaraman (2015) - fly head-direction cells