NANOGrav signal from MHD turbulence at QCD phase transition in the early universe

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NANOGrav collaboration has recently reported evidence for existence of stochastic gravitational wave background in the 1-100 nHz frequency range. We argue that such background could have been produced by magneto-hydrodynamic (MHD) turbulence at the first-order cosmological QCD phase transition. NANOGrav measurements suggest the magnetic field parameters: the comoving field strength close to microGauss and the correlation length close to 10% of the Hubble radius at the QCD phase transition epoch. We notice that turbulent decay of non-helical magnetic field with such parameters leads to the sufficiently strong magnetic field at recombination epoch which has been previously proposed as a solution to the Hubble tension problem. We show that the MHD turbulence model of the NANOGrav signal can be tested via measurement of the relic magnetic field in the voids of the Large Scale Structure with gamma-ray telescopes like CTA.

I. INTRODUCTION

The problem of the origin of cosmic magnetic fields is one of the long-standing problems of astrophysics and cosmology \cite{1}. Magnetic fields in galaxies and galaxy clusters are produced via dynamo amplification of pre-existing weak seed magnetic fields which should have been produced before the galaxy formation, possibly in the Early Universe. Those seed fields may still be found in their original form in the intergalactic medium \cite{1–3}.

The techniques of observations of Faraday rotation in distant radio sources \cite{1}, analysis of anisotropies and polarization of Cosmic Microwave Background (CMB) \cite{4, 5}, search for emission from electromagnetic cascades developing along the line of sight toward distant $\gamma$-ray sources \cite{6–8} have been previously used to constrain the strength and correlation length of the cosmological magnetic fields and provide information on their origin.

New techniques of measurement of cosmological magnetic fields are emerging with the appearance of new generation of gravitational wave (GW) detectors. Magnetic fields that might have existed in the Early Universe are expected to have highly turbulent structure and produce spatially variable stress-energy tensor that forces production of GWs. GWs generated in this way at cosmological phase transitions are expected to have frequencies in the nHz to mHz range accessible to LISA \cite{9} and pulsar timing arrays (PTA) \cite{10, 11}.

NANOGrav collaboration has recently reported detection of a signal in the frequency range between 3 and 100 nHz, that might be consistent with the stochastic gravitational wave background (SGWB) \cite{12}. Fig. 1 shows the NANOGrav sensitivity \cite{10} and the detection suggested in Ref. \cite{12} expressed in terms of the frequency spectrum of the density fraction of the SGWB $d\Omega_{GW}/d\log f$ (we have used Eq. (17) from Ref. \cite{13} to convert the result of Ref. \cite{12} in this format). Green and orange wedges in Fig. 1 show the envelope of the powerlaw type spectra falling within the 90% confidence contour of the $A_{CP}, \gamma_{CP}$ parameter space of Ref. \cite{12}, for the powerlaw and broken powerlaw fits of the cross-power spectral density of the GW signal.

NANOGrav SGWB can potentially be produced by conventional astrophysical source, the population of merging supermassive black holes \cite{12, 17, 18}. Alternative “new physics” model interpretations of the signal, including cosmic strings \cite{19–21}, primordial black holes \cite{22–25} or dark phase transition in the Early Universe \cite{26, 27} have also been considered.

In this paper we discuss a possibility that SGWB in the NANOGrav frequency range can be produced by Magneto-Hydro-Dynamic (MHD) processes in the Early Universe during the epoch of Quantum Chromo-Dynamic (QCD) phase transition \cite{28–33}. We study implications...
of such an interpretation of the NANOGrav signal for the physics of cosmological magnetic fields. We also discuss possible “multi-messenger” tests of the model of the production of this SGWB at the QCD phase transition using CMB and gamma-ray data. We show that the estimate of a magnetic field produced at the QCD phase transition derived from the NANOGrav detection is consistent with that obtained for the recombination epoch magnetic field based on the CMB data. The post-recombination magnetic field relic from the QCD epoch can still reside in the voids of the Large scale structure where it is detectable with gamma-ray telescopes.

II. GRAVITATIONAL WAVE PRODUCTION BY PRIMORDIAL MAGNETIC FIELDS

GWs can be described as transverse traceless perturbations \( h_{ij} \) of the metric of expanding Universe \( ds^2 = a^2(\delta_{ij} + h_{ij})dx^idx^j \), where \( a(\eta) \) is the scale factor and \( \eta \) is the conformal time. The Fourier components of \( h_{ij} = ah_{ij} \) satisfy the wave equation (we drop the indices \( ij \) in calculations below)

\[
\partial_\eta^2 h + k^2 h = \frac{16\pi G}{a} \hat{T}^{TT}
\]

where \( G \) is the Newton’s constant, \( k \) is the comoving wave number, and \( \hat{T}^{TT} = a^4T^{TT} \) are the transverse traceless components of the matter stress-energy tensor. Equation (1) describes the dynamics of forced oscillator with the source term scaling with magnetic field strength, \( \hat{T}^{TT} \sim B^2/2 + \hat{\rho}(1 + 4v^2)/3 \), with \( B = a^2B \) and \( \hat{\rho} = a^4\rho \) being the comoving magnetic field strength and energy density, and \( v \) being the plasma velocity. If the energy density of magnetic field is larger than the kinetic energy density of the plasma, the solution of the forced oscillator equation with initial conditions \( h = \partial_\eta h = 0 \) describes oscillations around an equilibrium point \( h \sim 8\pi G B^2/(ak^2) \).

The comoving energy density of GWs is \( \rho_{GW} = k^2h^2/(32\pi G) \simeq 2\pi GB^2/(a^2k^2) \). Dividing by the overall comoving density of the Universe \( \bar{\rho} \) we find the density fraction of GWs \( \Omega_{GW} = \rho_{GW}/\bar{\rho} \simeq 3\Omega_B/k^2 \) where we have introduced the physical wave number \( k/a \) and expressed it in the units of the Hubble rate \( H \), \( (k/a) = \kappa H \). We have also introduced the magnetic energy density fraction \( \Omega_B = B^2/2\bar{\rho} \).

Re-scaling the GW density fraction to the present day Universe, we find \( \Omega_{GW,0} = (a^4H^2/H_0^2)\Omega_{GW} \) where \( H_0 \) is the present day expansion rate of the Universe. Numerically,

\[
\Omega_{GW,0} \simeq 2 \times 10^{-4} \left[ \frac{N_{eff}}{10} \right]^{-1/3} \frac{\Omega_B^2}{\kappa^2}
\]

for the effective number of relativistic degrees of freedom \( N_{eff} \sim 10 \) at the temperature \( T \sim 100 \text{ MeV} \).

The characteristic scale of MHD turbulence at a given moment of Hubble time \( t_H = H^{-1} \) is that of the largest processed eddies, i.e. the eddies for which the turnover time scale is equal \( t_H \). Their length scale and the wave number are

\[
l_{LPE} = \frac{v_A}{H} = \sqrt{\frac{2\Omega_B}{H}}; \quad \kappa_{LPE} = \frac{1}{l_{LPE}H} = \frac{1}{\sqrt{2\Omega_B}}
\]

where \( v_A \) is the Alfvén velocity.

The source term in the wave equation (1) is quadratic in stress-energy tensor. This means that the gravitational waves sourced by the largest processed eddies have the wave numbers \( \kappa = 2\kappa_{LPE} \). For this special case, we find

\[
\Omega_{GW,0} = 10^{-4} \left[ \frac{N_{eff}}{10} \right]^{-1/3} \Omega_B^2
\]

This relation provides a convenient estimate of sensitivities of gravitational wave detectors for the measurement of the primordial magnetic field at the moment of cosmological magnetogenesis. A detector sensitive at the frequency \( f \) can measure the energy fraction of gravitational waves produced by magnetic field modes variable on the comoving distance scale \( l_B = 2/(2\pi f) \). Converting the sensitivity limits of LISA [15] and NANOGrav [10] for \( d\Omega_{GW}/d\log f \) into the limit on \( \Omega_B \) using Eq. (4), we arrive at the sensitivity curves shown in Fig. 2.

From this figure one can see that NANOGrav is sensitive for fields with comoving correlation length in the 0.1 pc \(< l_B < 10 \text{ pc} \) which contains the Hubble scale of the QCD phase transition at \( T \sim 100 \text{ MeV} \)

\[
\tilde{l}_H = (aH)^{-1} \simeq 1 \left[ \frac{N_{eff}}{10} \right]^{-1/6} [T/100 \text{ MeV}]^{-1} \text{ pc.}
\]

III. NANOGRAV SIGNAL FROM MHD TURBULENCE AT QCD PHASE TRANSITION

Figure 1 shows the range of \( \Omega_{GW,0} \) values suggested by the NANOGrav measurements for a range of frequencies covered by the experiment. We compare this measurement with the spectrum \( \Omega_{GW,f} = d\Omega_{GW}/d\log f \) expected for the QCD phase transition.

This spectrum depends on parameters of the phase transition like the bubble nucleation rate, the coherence time scale of the bubble collision process, among others [29] [31]. It is expected to behave as a broken powerlaw following \( f^\beta \) scaling at small (super-horizon) frequencies \( \tilde{f} \ll f_H = aH/(2\pi) [37] \), breaking to \( f^\alpha \), with \( \alpha \simeq 1 \) [16] or \( 0 < \alpha < 2 \) [30] in the frequency range between \( 2f_H \) and the largest processed eddy scale \( 2l_{LPE} = aH/\sqrt{\Omega_B} \), or possibly to still higher frequency scale \( \tilde{f}_* \) at which the magnetic field is forced. In the frequency range \( f > \tilde{f}_* \) the powerlaw \( f^\beta \) is expected to follow from freely decaying turbulence laws. The slope \( \beta \) depends on the type

1 We are using the system of units in which the speed of light \( c = 1 \).
of turbulence, especially on temporal coherence properties of turbulent velocity and magnetic field modes \[16, 29, 31\]. Figure 1 shows examples of broken powerlaw spectra of \(\Omega_{GW,f} \) for different values of magnetic field forcing scale \(f_\ast\), derived from numerical modelling of Ref. \[16\].

NANOGrav measurement suggests the value \(\Omega_{GW,0} \sim 10^{-9}\) at \(f \sim 3 \times 10^{-9}\) Hz (see Fig. 2), with roughly order-of-magnitude uncertainty at the 90% confidence level. Assuming that the spectrum of \(\Omega_{GW,0}\) is a powerlaw the slope \(f^{\alpha}\) with \(0 < \alpha < 2\) in the frequency \(f_B < f < f_\ast\), as suggested by numerical modelling of GW from MHD turbulence \[16, 29, 30\], we can derive an order-of-magnitude relation from the measurement:

\[
\frac{\langle \Omega_{GW,0}/10^{-9} \rangle}{(f/3 \times 10^{-9} \text{ Hz})^{\alpha}} \approx 1
\]

This relation can be used to infer the suggested range of magnetic field strength and correlation lengths. The frequency of GWs is related to their wave number at generation as \(f = aH/(2\pi) \approx 2 \times 10^{-9}\kappa[T/100\text{ MeV}][N_{eff}/10]^{1/6}\text{ Hz}\). Substituting the frequency estimate into Eq. \(6\) and expressing \(\Omega_{GW,0}\) through \(\Omega_B\) using Eq. \(1\) we find for \(\alpha = 1\)

\[
\Omega_B \approx 2 \times 10^{-3}\kappa^{3/2} \left[ \frac{N_{eff}}{10} \right]^{1/4} \left[ \frac{T}{100\text{ MeV}} \right]^{1/2}
\]

This result is shown by the thick red line in Fig. 2. The red shading around the line shows the the uncertainty range of the estimates for \(0 < \alpha < 2\). For the special case of largest processed eddies \(\kappa = 2k_{LPE}\), the NANOGrav measurements suggest \(\Omega_B \approx 0.6\mu G, \tilde{B} \approx 0.2\text{ pc}\) shown by the red circle in Fig. 2.

**IV. DISCUSSION**

The strength and correlation length of a cosmological magnetic field derived from the NANOGrav measurements is well within the range of expectations of the models of magnetic field generation at the first-order QCD phase transition \[30, 33, 38\]. In these models magnetic field and turbulence are generated by collisions of bubbles of new phase on the distance scale which is a sizeable fraction of the cosmological horizon at the moment of phase transition. The bubble walls propagate with near relativistic velocities, so that the kinetic energy density released in bubble collisions can be comparable to the overall energy density of the Universe. This energy can be transferred to the magnetic field which tends to establish equipartition between its energy and the kinetic energy of plasma motions.

NANOGrav measurement points to the first-order nature of the QCD phase transition. The order of the QCD phase transition in the Early Universe depends on a number of unknown parameters, such as lepton asymmetry \[39\]. PTA measurements of SGWB originating from this phase transition can constrain those parameters and help to identify the beyond Standard Model physics effects converting a confinement cross-over into the first-order phase transition at the QCD temperature scale.

The NANOGrav indication of possible magnetic field generation at the QCD phase transition has an important implication in the context of the “Hubble tension” problem. Different measurements of the current expansion rate of the Universe, \(H_0\), based on the CMB probes \[8\]...
and measurements in the local Universe [40, 41], provide measurements of $H_0$ which are inconsistent at > 4σ level (see [34] and references therein). Ref. [34] has proposed a solution to this problem which invokes clumping effects of primordial plasma, introduced by the presence of magnetic field. This clumping influences the CMB signal if the magnetic field parameters at recombination epoch are within the range shown by the red interval superimposed on the green shaded line representing the end point of cosmological magnetic field evolution in Fig. 2.

Remarkably, this range is exactly the one suggested by the measurements of magnetic field at QCD phase transition derived from the NANOGrav data. The two field measurements are related by the evolutionary path of magnetic field strength and correlation length [2, 36]. If the magnetic field generated at the QCD phase transition is non-helical, its strength and correlation length evolve following $\tilde{B}_0 \propto \tilde{l}_B^{5/2}$ or $\tilde{B}_0 \propto \tilde{l}_B^{-3/2}$ line determined by the freely decaying turbulence [2, 36]. The evolutionary paths $\tilde{B} \propto \tilde{l}_B^{-3/2}$ shown by rose arrow in Fig. 2 correspond to compressible turbulence of the primordial plasma [2]. The range of possible endpoints of cosmological evolution of the field with parameters derived from the NANOGrav data coincides with the range of the CMB epoch magnetic field parameters derived in Ref. [34].

The model of magnetic field and GW production at the first order QCD phase transitions outlined above can be fully tested with next generation PTA with SKA [14], which can provide high-confidence measurement of GWs from the QCD epoch at the redshifts $z \sim 10^{-2}$ combined with intergalactic magnetic field measurements with next-generation gamma-ray telescope CTA 6 [35, 32], which has sufficient sensitivity for measurement of the relic field at $z = 0$ (see Fig. 2 and ref [33]).

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