Entanglement discrimination in multi-rail electron–hole currents

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Abstract

We propose a quantum-Hall interferometer that integrates an electron–hole entangler with an analyzer working as an entanglement witness by implementing a multi-rail encoding. The witness has the ability to discriminate (and quantify) spatial-mode and occupancy entanglement. This represents a feasible alternative to limited approaches based on the violation of Bell-like inequalities.

Keywords: electronic transport, mesoscopic systems, quantum Hall effects, entanglement production and manipulation, witnesses

(Some figures may appear in colour only in the online journal)

I. Introduction

The reliable production and detection of quantum-entangled electron currents is a relevant issue in the roadmap towards solid-state quantum information based upon flying qubits. To this aim, several schemes have been settled along the last years. For the production, the most noticeable proposals rely on Cooper pair emission from superconducting contacts [1, 2], correlated electron–hole pair production in tunnel barriers [3], and integrated single-particle emitters [4]. It is a widespread belief that these mechanisms are likely to produce highly entangled electron currents. Unfortunately, serious difficulties arise for the detection and quantification of the entanglement produced by those means. Ideal approaches are based on the violation of Bell-like inequalities [5] in terms of zero-frequency noise correlators [6], sometimes including postselection mechanisms. However, corresponding efforts have been unsuccessful so far, most probably due to technological limitations for the controlled manipulation of a relatively large amount of parameters (only indirect signatures of entanglement as the two-particle Aharonov–Bohm effect [7] have been found in the laboratories [8]). In this situation, alternative approaches were developed with a more pragmatic viewpoint in the form of entanglement witnesses, namely, specific observables that can detect entangled states belonging to a certain subspace of interest by introducing a limited amount of controlled parameters [9]. In particular, a series of works has addressed the possibility of bounding the entanglement of electronic currents via single observables by mapping the probe states into Werner states [10–12] (a concept recently extended to isotropic states [13]). Moreover, Giovannetti et al [12] introduced a procedure allowing the discrimination of different types of entanglement, viz. the conventional mode entanglement (focused on the internal degrees of freedom of given particles) and the less appraised occupancy entanglement (relying on fluctuating local particle numbers) [14].

Here, we propose an integer quantum-Hall interferometer that integrates in a single circuit an electronic entangler with an entanglement-witness analyzer. The entangler is based on a previous scheme by Frustaglia and Cabello [15] that employs the multi-rail encoding (i.e. spatially separated transmission channels or modes) of electron–hole pairs produced at a tunnel barrier, originally designed for exploiting only spatial-mode qubit entanglement (also referred to as path- or orbital-mode entanglement) by postselection, i.e. by filtering out any contribution to occupancy entanglement. The interferometer introduced in [15], sketched in figure 1(a), is the electronic version of the optical interferometer proposed by Cabello et al [17] to solve a fundamental deficiency present in the original Franson’s Bell-like proposal with energy-time entanglement [18] due to the actual existence of a local hidden variable model reproducing the observed results [19]. Our detection strategy, instead, implements the results obtained in [12] for the discrimination of spatial-mode and occupancy entanglement. This relies on
II. Electronic interferometer

Let us start by reviewing the electronic interferometer introduced in [15], schematically depicted in figure 1(a). A source emits an electron–hole pair, with the electron and hole traveling in opposite directions towards beam splitters BS-1 and BS-2, respectively. After meeting the corresponding beam splitter, each member of the electron–hole pair splits independently into two paths $\Gamma_{e1}/\Gamma_{e2}$ (electron) and $\Gamma_{h1}/\Gamma_{h2}$ (hole). Path $\Gamma_{e1}$ ($\Gamma_{h2}$) takes the electron (hole) to the right (R) side of the interferometer for detection, while path $\Gamma_{e2}$ ($\Gamma_{h1}$) does likewise in the left (L) side.

After scattering at BS-1 and BS-2, the resulting electron–hole excitation consists of a multi-rail superposition containing different number of excitations in the L and R arms of the interferometer. More precisely, two contributions are found in which one excitation flies off to the right and the other one to the left, together with two contributions in which both particles fly off to the same side of the interferometer [15]. When written in the left–right (L–R) bipartition basis, this state displays hyper-entanglement, viz., standard spatial-mode entanglement and occupancy entanglement. The first one corresponds to the two-qubit entanglement between L and R propagating channels, with exactly one particle occupying one of the L and R channels, while the second one results in the coherent superposition of terms with different local particle number: two particles occupying both of the L propagating channels (and no particles on the R channels) or vice versa [14] (see also [12]).

In [15], postselected spatial-mode qubit entanglement is detected via the violation of a Bell-CHSH inequality, with BS-L and BS-R playing the role of controllable local operators acting on L and R propagating channels. To this end, coincidence measurements used in the optical version of the interferometer [17] are replaced by zero-frequency current-noise cross correlations between terminals placed at different sides of the electronic interferometer: one on the left (terminals L1/L2) and the other one on the right (terminals R1/R2). By construction, this procedure postselects the components contributing to spatial-mode entanglement only, disregarding those contributions carrying more than one excitation to the L or R terminals [15]. Moreover, the use of Bell-CHSH inequalities to detect occupancy entanglement would require local mixing of quantum states with different number of fermions, which is forbidden by SSRs.

For this reason, we reconfigure the interferometer depicted in figure 1(a) to implement the witness scheme introduced in [12] in order to address both the spatial-mode and occupancy entanglement of two-particle probe states. This is based on
the study of current cross correlations at the output ports of a BS as a function of controllable phase shifts. To this aim, we identify the L and R propagating paths in figure 1(a), $\Gamma_L/\Gamma_{L2}$ and $\Gamma_R/\Gamma_{R2}$, with input ports 1 and 2 in the entanglement analyzer of [12] (see figure 2). It is most important to realize that the analyzer introduced in [12] consists in a single BS which does not produce channel mixing in the scattering process. In our case, this is equivalent to introduce two spatially separated BSs: one for electrons (BS-e) and one for holes (BS-h), as depicted in figure 2. This results in the design sketched in figure 1(b). It is worth noting that the topology of this setup coincides with that of Franson’s interferometer [18]. However, the correlations considered by Franson are essentially different from those needed here. Franson’s setup, as demanded by Bell-CHSH tests, would require the study of noise cross correlations between L1/R1 and L2/R2 terminals in figure 1(b) (namely, between electron and hole excitations). Here, instead, in order to discriminate mode from occupancy entanglement by following the protocol described in [12], one needs the current cross correlations between output ports 3 and 4 in the analyzer of figure 2 corresponding to terminals L1/L2 and R1/R2 (see figure 1(b)), respectively. This procedure implements nonlocal operations in the original L–R bipartition necessary for the entanglement discrimination.

To complete our device, we need to insert additional voltage gates (e.g. top or side gates) along paths $\Gamma_{R1}$ and $\Gamma_{L2}$ allowing the introduction of controllable phases $\varphi_1$ and $\varphi_2$ as demanded by the analyzer of figure 2. The resulting quantum-Hall setup is shown in figure 3. Electrons propagate from sources 1 and 2 (subject to equal voltages $V$) to grounded drains L1, L2, R1 and R2, along single-mode edge channels. Electrically controlled quantum point contacts labelled as BS-n, with $n = 0, 1, 2, e, h$, act as beam splitters. As discussed in [15], the production of entanglement involves only beam splitters BS-0, BS-1 and BS-2 together with the primary source 1. The BS-0 is set to be low transmitting (tunneling regime). Thus, an electron traveling from primary source 1 can tunnel through BS-0 to the right side of the interferometer, leaving a hole in the Fermi sea traveling towards the left side. After emission at BS-0, each member of the electron–hole pair splits into a pair of paths at BS-1 and BS-2, respectively, which results in a spatial-mode and occupancy entangled electron–hole excitation in the original L–R bipartition, as discussed above1. The secondary source 2 is not directly involved in the production of entanglement itself. Its role is to avoid contamination of the signal generated at BS-0 with the undesired current-noise correlations that would originate at BS-2 in the absence of this secondary source. Entanglement is discriminated by current cross-correlations between the output terminals of BS-e and BS-h, as mentioned above. Details are presented in the following sections.

III. Entanglement production

We start by writing an explicit expression of the state produced at the entangler, to be probed by the analyzer. To this end, we follow the steps detailed in [15] adapted to the setup of figure 3. Electrons are injected from sources 1 and 2 with energy $\epsilon$ on an energy window $eV$ above the Fermi sea ($0$). Upon tunneling of electrons from source 1 (transmission probability $T_0 = |t_0|^2 \ll 1$), an electron–hole pair packet is generated at BS-0. After scattering at BS-1 and BS-2 (with amplitudes $t_1, r_1$ and $t_2, r_2$, respectively), the pair state evolves into

1 For the sake of direct comparison with the interferometer introduced in [15], we keep the terminology in terms of the original left and right elements of the interferometer.
where

\[ |\Psi\rangle = |\bar{0}\rangle + |\bar{\Psi}\rangle, \]  

(1)

\[ |\Psi\rangle = i e^{i \phi_0} \int_{0}^{V} \text{d} \varepsilon \left[ r_{12}^{a} e^{i \varepsilon \gamma_{-}} a_{1}^{+}(\varepsilon) a_{2}(\varepsilon) - r_{12}^{a} e^{i \varepsilon \gamma_{-}} a_{2}(\varepsilon) a_{1}^{+}(\varepsilon) + r_{21}^{a} e^{i \varepsilon \gamma_{+}} a_{2}(\varepsilon) a_{1}(\varepsilon) + r_{21}^{a} e^{i \varepsilon \gamma_{+}} a_{1}(\varepsilon) a_{2}^{+}(\varepsilon) \right] |\bar{0}\rangle \]  

(2)

describes an electron–hole wavepacket out of a redefined vacuum \(|\bar{0}\rangle = \prod_{\varepsilon=0}^{\infty} a_{1}^{+}(\varepsilon) a_{2}(\varepsilon) |0\rangle\). Here, \(a_{\alpha}^{+} (a_{\alpha})\) creates an electron (hole) propagating towards terminal \(n = L1, L2, R1, R2\) when BS-e and BS-h are closed. In addition, \(\phi_{\alpha m}, \gamma_{\alpha m} (m = 1, 2)\) are the phases acquired by an electron travelling along paths \(\Gamma_{\alpha m}\), \(\Gamma_{\alpha i}\), respectively, when the controllable phases \(\phi_{\alpha} \) and \(\phi_{\alpha} \) are set to 0. The redefined vacuum \(|\bar{0}\rangle\) consists of a noiseless stream of electrons emitted from BS-2 towards terminals L2 and R2. This is possible by virtue of secondary source 2. Otherwise, electrons entering BS-2 from primary source 1 alone would scatter as correlated noisy currents that mask the signatures of the electron–hole excitations emitted from BS-0 [15].

Notice that the first two terms within brackets in equation (2) correspond to a coherent superposition of an electron and a hole entering the analyzer of figure 2 from different ports (two spatial-mode entangled qubits), while the last two terms describe an electron and a hole occupying the same port when entering the analyzer (occupancy entangled excitations). In the next section, we examine both types of entanglement by following the lines established in [12] via current cross correlators.

IV. Entanglement detection

According to [12], both BS-e and BS-h are set to be 50% beam splitters with scattering matrices given by

\[ S_{\alpha}(\varepsilon) = \sqrt{1/2} \begin{pmatrix} 1 & e^{i \phi_{\alpha}} \\ 1 & -e^{i \phi_{\alpha}} \end{pmatrix} \]  

(3)

with \(\alpha = 1, 2\) for BS-e and BS-h, respectively. These matrices relate the annihilation operators on both sides of the analyzer of figure 2 as \(b_{\alpha o}(\varepsilon) = S_{\alpha} a_{\alpha o}(\varepsilon)\), with incoming \(a_{\alpha o}(\varepsilon) = (a_{\alpha o}, a_{\alpha L})\) and outgoing \(b_{\alpha o}(\varepsilon) = (b_{\alpha o}, b_{\alpha L})\).

The keystone for entanglement detection are the correlations of electron and hole excitations at output ports 3 and 4 in figure 2. These are properly accounted by the dimensionless current cross correlator [12]

\[ C_{44} = \frac{\hbar^{2} \nu^{2}}{2 e^{2}} \lim_{T \to \infty} \int_{0}^{T} d t_{1} d t_{2} I_{3}(t_{1}) I_{4}(t_{2}) \frac{(t_{1} - t_{2})}{T^{2}}. \]  

(4)

This is a measurable quantity, where \(I_{j}(t)\) are current operators at ports \(j = 3, 4\). These are given by the sum of electron (e) and hole (h) current operators along the corresponding terminals

\[ I_{3}(t) = I_{2}^{(e)}(t) + I_{2}^{(h)}(t), \]  

(5)

\[ I_{4}(t) = I_{1}^{(e)}(t) + I_{1}^{(h)}(t). \]  

(6)

More precisely, the hole currents are defined as

\[ I_{2}^{(h)}(t) = -I_{2}^{(e)}(t) + I_{0}^{(e)}, \]  

(7)

\[ I_{1}^{(h)}(t) = -I_{1}^{(e)}(t) + I_{0}^{(e)}, \]  

(8)

where \(I_{0}^{(e)}\) is the mean electronic current in either terminal L2 or R2 when BS-0 is closed (see figure 3), namely, when no electron–hole pairs are emitted. Moreover, the electron current operators at terminals \(n = L1, L2, R1, R2\) are defined as

\[ I_{n}^{(e)}(t) = \frac{e}{h \nu} \sum_{E} e^{-i \omega t} \langle n | b_{n}^{+}(E) b_{n}(E + h \omega) \rangle. \]  

(9)

In equation (4), \(T\) is the measurement time and \(\nu\) is the density of the leads (where we consider a discrete spectrum to ensure a proper regularization of the current correlations). The expectation value \(\langle \cdot \cdot \cdot \rangle \) is taken over the probe state \(|\Psi\rangle\) given in equation (1). Notice that, since the redefined vacuum \(|\bar{0}\rangle\) correspond to a noiseless stream of electrons emitted from BS-2, \(\langle \bar{0} | I_{3}(t_{1}) I_{4}(t_{2}) |\bar{0}\rangle = 0\). In addition, it can be shown that \(\langle \bar{0} | I_{3}(t_{1}) I_{4}(t_{2}) |\bar{0}\rangle = 0\). As a consequence, we find

\[ \langle \Psi | I_{3}(t_{1}) I_{4}(t_{2}) |\Psi\rangle = \langle \bar{0} | I_{3}(t_{1}) I_{4}(t_{2}) |\bar{0}\rangle, \]  

(10)

which brings us to focus our attention on the state \(|\Psi\rangle\) of equation (2). This is normalized by calculating \(\langle \Psi | \Psi \rangle\), which involves a double energy integral. Fermionic algebra reduces the expression to

\[ \langle \Psi | \Psi \rangle = |t_{0}|^{2} \int_{0}^{V} d \varepsilon \int_{0}^{V} d \varepsilon' e^{i \xi_{\varepsilon}(\varepsilon' - \varepsilon)} \times \left( |\xi_{\varepsilon}|^{2} |\xi_{\varepsilon'}|^{2} + |\xi_{\varepsilon'}|^{2} |\xi_{\varepsilon}|^{2} + |\xi_{\varepsilon}|^{2} |\xi_{\varepsilon'}|^{2} + |\xi_{\varepsilon'}|^{2} |\xi_{\varepsilon}|^{2} \right). \]  

(11)

The terms within brackets sum to one by unitarity. Moreover, \(\int_{0}^{V} d \varepsilon = N = eV \nu\) is the number of energy states in the leads. This leaves \(\langle \Psi | \Psi \rangle = N |t_{0}|^{2}\), which is nothing but the tunnel current through BS-0 in units of eVh.\(^{2}\) We now introduce the normalized state

\[ |\Psi'\rangle = \frac{1}{\sqrt{N |t_{0}|}} |\Psi\rangle \]  

(12)

which can be written, up to a global phase factor, as

\[ |\Psi'\rangle = |\xi_{\varepsilon}| |\xi_{\varepsilon'}| \left( |\Phi_{LL}\rangle + |\xi_{\varepsilon'}| |\Phi_{RR}\rangle \right) + \sqrt{|\xi_{\varepsilon}|^{2} |\xi_{\varepsilon'}|^{2} + |\xi_{\varepsilon'}|^{2} |\xi_{\varepsilon}|^{2} + |\xi_{\varepsilon}|^{2} |\xi_{\varepsilon'}|^{2} + |\xi_{\varepsilon'}|^{2} |\xi_{\varepsilon}|^{2} \right) \]  

(13)

Here, \(|\Phi_{j}\rangle (i, j = L, R)\) are normalized states defined as

\(^{2}\) Despite the state \(|\Psi\rangle\) is a two-particle (electron–hole) excitation, its norm \(\langle \Psi | \Psi \rangle\) is proportional to a current (and not to a square current) because the electron–hole pair is created in a single tunneling event.
\[ |\Phi_{LL}\rangle = \frac{t_0}{\sqrt{N} |t_0|} \int_{0}^{\pi} d\theta' r_1 e^{i(\theta' - \theta)} a_L(\theta)(e) a_L(\theta)|0\rangle \]

\[ |\Phi_{RR}\rangle = \frac{t_0}{\sqrt{N} |t_0|} \int_{0}^{\pi} d\theta' r_2 e^{i(\theta' - \theta)} a_R(\theta)(e) a_R(\theta)|0\rangle \]

\[ |\Phi_{LR}\rangle = \frac{t_0}{\sqrt{N} |t_0|} \int_{0}^{\pi} d\theta' \frac{1}{\sqrt{|r_1|^2 |r_2|^2 + |r_2|^2}} \left[ i r_1^* e^{-i(\gamma - \delta)} a_L(\theta)(e) a_R(\theta)|0\rangle \right. \\
\left. - r_2^* e^{-i(\gamma - \delta)} a_L(\theta)(e) a_R(\theta)|0\rangle \right]. \]

The \( |\Phi_{LR}\rangle \) corresponds to a delocalized L-R component of the electron-hole excitation contributing to spatial-mode qubit entanglement (addressed by postselection in [15]) while \( |\Phi_{LL}\rangle \) and \( |\Phi_{RR}\rangle \) contribute to occupancy entanglement. By introducing \( \theta, \phi \in [0, \pi/2] \) such that

\[
\sin \theta = \sqrt{|r_1|^2 |r_2|^2 + |r_2|^2 |t_0|^2}, \\
\sin \phi = \frac{|r_1|}{\sqrt{|r_1|^2 |r_2|^2 + |r_2|^2 |t_0|^2}},
\]

the state in equation (13) reduces to

\[ |\Psi'\rangle = \sin \theta (\cos \phi |\Phi_{LL}\rangle + \sin \phi |\Phi_{RR}\rangle) + \cos \theta |\Phi_{LR}\rangle. \]

This has the exact form of the generic two-particle input state considered in [12]. By following this reference, we can write

\[ C_{34} = [1 + w \cos^2 \theta + v \sin^2 \theta \sin(2\phi)]/4, \]

where \( w \) and \( v \) real quantities satisfying \(|w|, |v| \leq 1\) such that \( 0 \leq C_{34} \leq 1/2 \). These depend on the controllable phases \( \phi_1 \) and the numerical coefficients appearing in states \( |\Psi_i\rangle \) (14)

denoted as \( \Phi_{AB}^{(i)} \), where \( \alpha, \beta = 1, 2 \) refer to the electron/hole propagating channels. More precisely, in this case we arrive to

\[
w = \int_{0}^{\pi} d\theta' \left[ \Phi_{12}^{(LR)}(\theta') e^{i(\phi_1 - \phi_2)} + \Phi_{21}^{(LR)}(\theta') e^{i(\phi_2 - \phi_1)} \right], \]

\[ v = -2 \text{Re} \left[ \int_{0}^{\pi} e^{i\phi_1} \left[ \Phi_{12}^{(LL)}(\theta') \Phi_{21}^{(RR)}(\theta') e^{i(\phi_1 + \phi_2)} \right] \right]. \]

From this, together with equation (14), the current cross correlator \( C_{34} \) of equation (17) reduces to

\[ C_{34} = \frac{1}{4} \left[ 1 - 2 \text{Re}[r_1 r_2^* e^{i(\delta - \gamma - \theta)}] - 2 \text{Re}[r_1 r_2^* e^{i(\delta - \gamma + \theta + \phi_1 + \phi_2)}] \right]. \]

This correlator has been shown [12] to work as a hyper-entanglement witness: values of \( C_{34} \) smaller than \( 1/4 \) for some setting of the controllable phases \( \phi_1 \) and \( \phi_2 \) indicate the non-separability of \( |\Psi'\rangle \) in the L-R bipartition, representing a direct evidence of entanglement in the probe state without revealing its specific (either mode or occupancy) form. Moreover, entanglement-specific witnesses can be defined by appropriate

\[ \text{data processing as } C_{34}^{(+)} = [C_{34}(|\phi_1\rangle) + C_{34}(|\phi_2 + \pi/2\rangle)]/2, \]

finding [12]

\[ C_{34}^{(+)} = (1 + w \cos^2 \theta)/4, \]

\[ C_{34}^{(-)} = v \sin^2 \theta \sin(2\phi)/4, \]

such that \( 0 \leq C_{34}^{(+)} \leq 1/2 \) and \(-1/4 \leq C_{34}^{(-)} \leq 1/4 \). The particular expressions for the interferometer of figure 3, derived form equation (20), reduce to

\[ C_{34}^{(+)} = \frac{1}{4} - \frac{1}{2} [r_1 r_2^* e^{i(\delta_1 - \gamma_1 + \delta_2 - \gamma_2 + \phi_1 + \phi_2)}], \]

\[ C_{34}^{(-)} = -\frac{1}{2} [r_1 r_2^* e^{i(\delta_1 + \gamma_1 + \delta_2 + \gamma_2 + \phi_1 + \phi_2)}]. \]

The presence of spatial-mode qubit entanglement in the probe state is revealed whenever \( C_{34}^{(+)} < 1/4 \), corresponding to negative values of \( w \) [11, 12]. Occupancy entanglement, instead, manifests as a \( C_{34}^{(-)} \) different from zero. From equations (23) and (24) we notice that these conditions are satisfied for some values of the controllable phases \( \phi_1 \) and \( \phi_2 \) provided the transmission \( t_1, t_2 \) and reflection \( r_1, r_2 \) amplitudes of beam splitters BS-1 and BS-2 are non-vanishing (i.e. for partially open quantum points contacts). The witness signals are optimized for 50% BS-1 and BS-2. However, hyper-entanglement is a constraint impeding the saturation of the algebraic bounds allowed by equations (21) and (22): that would require either \( \cos^2 \theta = 1 \) in equation (21) (corresponding to a purely LR component in equation (16)) or \( \sin^2 \theta = 1 \) and \( \sin 2\phi = \pm 1 \) in equation (22) (no LR component and balanced LL and RR ones in equation (16)), something impossible to accomplish in our setup.

For illustration, in figure 4 we plot \( C_{34} \) (solid line), \( C_{34}^{(+)} \) (dashed line) and \( C_{34}^{(-)} \) (dash-dotted line) for two different BS-1 and BS-2 transparencies as a function of the controllable phase \( \phi_2 \). For simplicity, we set \( \gamma_1 = \gamma_2 = \delta_1 = \delta_2 \) and \( \phi_1 = 5\pi/12 \). The top panel corresponds to 50% BS-1 and BS-2, showing large-amplitude oscillations indicative of highly entangled spatial-mode and occupancy components. Still, the amplitudes do not saturate due to hyper-entanglement as pointed out above. The bottom panel shows results for highly transmitting (95% transparency) BS-1 and BS-2. Low-amplitude oscillations are a signal of partial entanglement, eventually undetectable depending on the experimental accuracy. Notice that similar results are obtained for either low transparencies or highly asymmetric settings of BS-1 and BS-2 since the oscillation amplitudes in figure 4 are fully determined by the products \( r_1 r_2^* \) and \( r_1 r_2^* \) according to equations (20), (23) and (24).

To conclude, we find that \( C_{34}^{(+)} \) sets a lower bound to the concurrence [21] \( C \) of the two spatial-mode qubits. We first notice from [12] that the entanglement of formation [22] \( E_1 \) of the state \( |\Phi_{LR}\rangle \) satisfies \( E_1(|\Phi_{LR}\rangle) \geq \mathcal{E}(1 - 2C_{34}(|\Phi_{LR}\rangle)) \), with \( \mathcal{E}(x) \) a monotonically increasing function for \( 1/2 \leq x \leq 1 \) (null
can be done about the entanglement under such particular circumstances.

V. Closing remarks

Our proposal integrates a reliable electronic entangler with a versatile analyzer, basis of an entanglement witness qualified to discriminate spatial-mode and occupancy entanglement with limited resources at reach [8]. This includes the ability to quantify the entanglement by appropriate lower bounds [12]. The witness is particularly suitable since it is optimized to detect the precise family of states that the entangler is able to produce. This means that, in ideal condition and in contrast to most witnesses, a negative signal (i.e. vanishing oscillation amplitudes in figure 4) is indicative of no entanglement. Moreover, the witness remains sound even in the presence of noisy inputs in the form of mixed states, as demonstrated in [12].

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References

[1] Recher P, Sukhorukov E V and Loss D 2001 Phys. Rev. B 63 165314
[2] Lesovik G B, Martin T and Blatter G 2001 Eur. Phys. J. B 24 287
[3] Beenakker C W J, Emary C, Kindermann M and van Velsen J L 2003 Phys. Rev. Lett. 91 147901
[4] Spletstoeßser J, Moskalets M and Büttiker M 2009 Phys. Rev. Lett. 103 076804
[5] Bell J S 1964 Physics 1 195
[6] Clauer F, Horne M A, Shimony A and Holt R A 1969 Phys. Rev. Lett. 23 880
[7] Samuelsson P, Sukhorukov E V and Büttiker M 2003 Phys. Rev. Lett. 91 157002
[8] Samuelsson P, Sukhorukov E V and Büttiker M 2004 Phys. Rev. Lett. 92 026805
[9] Neder I, Ofek N, Chung Y, Heiblum M, Mahalu D and Umansky V 2007 Nature 448 333
[10] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A 223 1
[11] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A 223 1
[12] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A 223 1
[13] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A 223 1
[14] Horodecki M, Horodecki P and Horodecki R 1996 Phys. Lett. A 223 1

More precisely, $C_{34} = H[1/2 + \sqrt{1-x}]$, with $H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$.
[15] Frustaglia D and Cabello A 2009 Phys. Rev. B 80 201312
[16] Beenakker C W J 2006 Quantum Computers, Algorithms and Chaos (Amsterdam: IOS Press) (Proc. of the Int. School of Physics ‘Enrico Fermi’ (Varenna, 2005))
[17] Cabello A, Rossi A, Vallone G, De Martini F and Mataloni P 2009 Phys. Rev. Lett. 102 040401
[18] Franson J D 1989 Phys. Rev. Lett. 62 2205
[19] Aerts S, Kwiat P G, Larsson J-Å and Żukowski M 1999 Phys. Rev. Lett. 83 2872

Aerts S, Kwiat P G, Larsson J-Å and Żukowski M 2001 Phys. Rev. Lett. 86 1909
[20] Büttiker M 1992 Phys. Rev. B 46 12485
[21] Wootters W K 1998 Phys. Rev. Lett. 80 2245
[22] Bennett C H, Brassard G, Popescu S, Schumacher B, Smolin J A and Wootters W K 1996 Phys. Rev. Lett. 76 722
Bennett C H, DiVincenzo D P, Smolin J A and Wootters W K 1996 Phys. Rev. A 54 3824