Hallmarks of Majorana mode leaking into a hybrid double quantum dot

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(Dated: February 21, 2022)

We investigate the spectral and transport properties of a double quantum dot laterally attached to a topological superconducting nanowire, hosting the Majorana zero-energy modes. Specifically, we consider a geometry, in which the outer quantum dot is embedded between the external normal and superconducting leads, forming a circuit. First, we derive analytical expressions for the bound states in the case of an uncorrelated system and discuss their signatures in the tunneling spectroscopy. Then, we explore the case of strongly correlated quantum dots by performing the numerical renormalization group calculations, focusing on the interplay and relationship between the leaking Majorana mode and the Kondo states on both quantum dots. Finally, we discuss feasible means to experimentally probe the in-gap quasiparticles by using the Andreev spectroscopy based on the particle-to-hole scattering mechanism.

I. INTRODUCTION

Two quantum dots contacted in various arrangements with external macroscopic reservoirs have been proposed as promising building blocks of future nanoelectronic, spintronic and quantum information technologies [1, 2]. For instance, double quantum dot (DQD) configurations provide a versatile platform for the implementation of spin-based quantum information processing systems [3]. Moreover, a rapid progress in materials science of superconducting hybrid nanostructures [4, 5] stimulated vivid interest in constructing quantum bits out of the Andreev bound states [4, 6–9]. Further perspectives for the realization of topological superconducting qubits are related to nanostructures involving DQDs coupled to topological superconducting wires, hosting the Majorana zero-energy modes (MZM) at their ends, the so-called Majorana wires (MW) [10–15]. Such platforms allow for the implementation of fault-tolerant quantum computing protocols, which are in the center of interest of quantum information research [16].

The main motivation for studying the hybrid nanostructures composed of quantum dots and Majorana wires is associated with a tendency of such end-modes to leak into the neighboring objects [17]. Effectively, this gives rise to fractional values of the differential conductance, which serve as fingerprints of the exotic character of MZM [18]. Various situations have been investigated so far, considering mainly single quantum dots and exploring the interplay of correlation effects with the Majorana quasiparticles [19, 20]. In this regard, much less attention has been paid to hybrid systems composed of double quantum dots hybridized with topological superconducting wires. Therefore, this work aims to shed some light on the transport properties of DQD-MW hybrid structures, focusing specifically on the setup displayed in Fig. 1.

It is important to note that the properties of double quantum dots proximitized with conventional superconductors have been studied experimentally by the tunneling spectroscopy, using InAs [5, 21–24], InSb [25], Ge/Si [26], carbon nanotubes [27, 28] and by the scanning tunneling microscopy (STM) applied to magnetic dimers deposited on superconducting surfaces [29–32]. In-gap bound states of the double quantum dots (dimers) have been thoroughly analyzed by a number groups [33–50], in particular predicting quantum phase transitions, in which the total spin could vary between the singlet, doublet and triplet states [51, 52]. However, the properties of proximitized DQDs, additionally interacting with Majorana modes, are much less explored [53].

Recently, a fusion of individual bound states into their molecular (hydrogen atom-like) structure has been re-
alized by a controllable change of the hybridization between the quantum dots contacted with superconducting reservoirs [54]. A similar process for topological bound states would be highly desirable, therefore this issue triggers intensive activities [55]. Motivated by such achievements and trends, we study the transport behavior of a hybrid double quantum dot setup, where the central quantum dot (QD1) is embedded between the superconducting (S) and the normal metallic (N) lead (provided by e.g. an STM tip), forming a circuit. We assume QD1 to be connected through the second quantum dot (QD2) to the topological superconducting nanowire, see Fig. 1. Practically, such quantum dots can be considered as being a piece of a non-topological segment of the nanowire consisting of two sites, in analogy to the experimental hybrid system that provided the first evidence for leakage of the Majorana quasiparticles [17, 19].

To obtain the most reliable predictions for the behavior of the considered system when correlations play an important role, we resort to the density-matrix numerical renormalization group (NRG) calculations [56–60]. Moreover, to provide the complete picture and deepen the understanding of transport properties of the considered DQD-MW hybrid structure, we also perform analytical calculations for an uncorrelated case.

Our study reveals that: (i) besides the conventional in-gap quasiparticle branches (originating from the hybridization of both quantum dots) there appear additional structures induced by the Majorana mode in the form of constructive/destructive interference pattern imprinted on the spin-down/spin-up sectors of QD1; (ii) in the Kondo regime (when the Coulomb repulsion prevails over the superconducting proximity effect) the spin-resolved spectral functions indicate the detrimental/constructive influence of the Majorana mode on ↑/↓ spin sectors of QD1. For such strongly correlated system, we predict the optimal Andreev conductance near a crossover from the doublet to the BCS-type singlet configurations of proximitized QD1.

The paper is organized as follows. In Sec. II we introduce the microscopic model of the studied DQD-MW system, describing our setup. Next, in Sec. III we discuss the properties of the considered hybrid structure, neglecting the Coulomb repulsion on both quantum dots. In Sec. IV we address the correlation effects in the Kondo regime, using the numerical renormalization group approach. Finally, we summarize our findings in Sec. V.

II. MODEL

In what follows, we analyze the spectroscopic and transport properties of \( N \)-QD1-S branch (see Fig. 1), focusing on the subgap energy region. The second quantum dot QD2 transmits the Majorana mode(s), which in turn affects the transport by interferometric effects. We study those effects in detail, considering the fully polarized case, \( V_{M\downarrow} = V_M \) and \( V_{M\uparrow} = 0 \), where \( V_{M\sigma} \) is the coupling between the second dot and Majorana wire for spin \( \sigma \). Some results for the arbitrary spin-dependent couplings \( V_{M\sigma} \) [19, 61–63] are presented in Appendix A.

Our hybrid structure (Fig. 1) can be modeled by the following Hamiltonian

\[
H = H_N + H_{DQD} + H_{MW} + H_{SC},
\]

where

\[
H_N = \sum_{k\sigma} \varepsilon_{Nk} c_{Nk\sigma}^\dagger c_{Nk\sigma}
\]

describes the metallic lead with the operators \( c_{Nk\sigma}^\dagger \) creating electrons with spin \( \sigma \), momentum \( k \) and energy \( \varepsilon_{Nk} \). The second term in Eq. (1) describes the tunneling processes between the metallic lead and the first quantum dot

\[
T_N = \sum_{k\sigma} V_N \left( d_{1\sigma}^\dagger c_{Nk\sigma} + c_{Nk\sigma} d_{1\sigma} \right),
\]

where \( V_N \) is the momentum independent tunneling matrix element and \( d_{1\sigma}^\dagger \) operator creates electrons with spin \( \sigma \) at the central quantum dot. When this quantum dot is coupled to the external contacts, it leads to the broadening of the dot level described by \( \Gamma_N = \pi \rho_N V_N^2 \), where \( V_N \) is assumed to be real, while \( \rho_N \) is the density of states of the metallic lead. For calculations we use the bandwidth of metallic lead as a convenient energy unit (\( D \equiv 1 \)).

The double quantum dot part is given by

\[
H_{DQD} = \sum_{j\sigma} \varepsilon_j d_{j\sigma}^\dagger d_{j\sigma} + \sum_{j} U_j d_{j\uparrow}^\dagger d_{j\uparrow} d_{j\downarrow}^\dagger d_{j\downarrow} + \sum_{\sigma} t \left( d_{1\sigma}^\dagger d_{2\sigma} + \text{h.c.} \right),
\]

where \( d_{j\sigma}^\dagger \) creates a spin-\( \sigma \) electron on the \( j \)-th quantum dot with energy \( \varepsilon_j \). The repulsive Coulomb potential \( U \) between the opposite spin electrons on individual quantum dots shall be assumed equal \( U_1 = U_2 = U \). These quantum dots are interconnected through the hybridization, denoted by \( t \).

The low-energy quasiparticles of the topological nanowire can be described by the following term

\[
H_{MW} = \sqrt{2} V_M \left( d_{2\downarrow}^\dagger \gamma_1 + \gamma_1 d_{2\downarrow} \right) + i \varepsilon_M \gamma_1 \gamma_2.
\]

The first part couples the spin-\( \downarrow \) electrons of the second quantum dot with the Majorana mode described by the operator \( \gamma_1 \) through the tunneling element \( V_M \). The role of the coupling of Majorana quasiparticles to both spins of QD2 is briefly discussed in Appendix. The Majorana operators can be rewritten in terms of an auxiliary fermion operator \( f \) as \( \gamma_1 = (f^\dagger + f)/\sqrt{2} \) and \( \gamma_2 = i(f^\dagger - f)/\sqrt{2} \). In the case of a short nanowire, we should assume the overlap \( \varepsilon_M \) between the wave functions of the Majorana modes. However, here we focus on the long wire case, i.e. when the Majorana quasiparticles do not overlap and \( \varepsilon_M = 0 \).
The last part of the Hamiltonian (1) refers to superconducting substrate and its coupling to QD1

\[ H_{SC} = \sum_{k\sigma} \varepsilon_{kS} c^\dagger_{k\sigma} c_{k\sigma} - \left( \Delta_{SC} \sum_{k} c^\dagger_{k\sigma} c_{-k\sigma} + \text{h.c.} \right) + \sum_{k\sigma} V_S \left( d^\dagger_{1\sigma} c_{k\sigma} + c^\dagger_{k\sigma} d_{1\sigma} \right). \] (6)

In the limit of large pairing gap, \( \Delta_{SC} \to \infty \), these terms give rise to the proximity induced on-dot pairing

\[ H_{SC} \approx -\Delta_1 (d^\dagger_{1\uparrow} d_{1\downarrow} + d_{1\downarrow} d^\dagger_{1\uparrow}). \] (7)

with the effective pairing potential \( \Delta_1 = \Gamma_S \), and \( \Gamma_S \) denoting the coupling strength between QD1 and the superconductor.

\[ \mathcal{G}^{-1}(\omega) = \omega \hat{I} + \begin{pmatrix} -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_1 + i\Gamma_N & -\Gamma_S & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 \\ 0 & -\Gamma_S & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & 0 & -\varepsilon_2 & 0 & 0 & 0 \\ 0 & t & 0 & 0 & 0 & -\varepsilon_2 & 0 & 0 \\ 0 & 0 & -t & 0 & 0 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 \end{pmatrix}, \] (8)

where \( \hat{I} \) stands for the identity matrix.

### A. Resonant bound states for \( \Gamma_N = 0 \)

Consider first the conventional bound states of the proximitized double quantum dot in the absence of coupling to the Majorana nanowire [64]. In the limit of \( \Gamma_N \to 0 \) such in-gap resonant states are formed at

\[ \varepsilon_{AD1}^\pm = \pm \frac{1}{\sqrt{2}} \sqrt{A + \sqrt{A^2 - 4B}}, \]

\[ \varepsilon_{AD2}^\pm = \pm \frac{1}{\sqrt{2}} \sqrt{A - \sqrt{A^2 - 4B}}, \] (9)

where \( A = \varepsilon_1^2 + \varepsilon_2^2 + \Gamma_S^2 + 2t^2 \) and \( B = (\varepsilon_1 \varepsilon_2 - t^2)^2 + (\varepsilon_2 \Gamma_S)^2 \). The quasiparticles \( \varepsilon_{AD1}^\pm \) represent the Andreev bound states of QD1, \( E_{AD}^\pm = \pm \sqrt{\varepsilon_1^2 + \Gamma_S^2} \), now slightly modified by the hybridization \( t \) to QD2. The other quasiparticles \( \varepsilon_{AD2}^\pm \) originate from the energy level of QD2 owing to the induced electron pairing (via its coupling to QD1).

The attachment of the topological superconductor nanowire to the double quantum dot substantially affects the spectrum of this setup, revealing signatures of the zero-energy Majorana mode. Now, besides the initial quasiparticles \( \varepsilon_{AD1}^\pm \) and \( \varepsilon_{AD2}^\pm \) there emerge additional

### III. THE CASE OF UNCORRELATED SYSTEM

Let us start our analysis by investigating the spectral and transport properties in the case of uncorrelated quantum dots, i.e. when \( U = 0 \). For this purpose, we use the Green’s function method. The influence of the topological nanowire on the quantum dots can be captured by the matrix Green’s function defined in the particle-hole (Nambu) notation \( \hat{G}(\omega) = \langle \langle \Psi; \Psi^\dagger \rangle \rangle_\omega \), where \( \Psi = (d_{1\uparrow},d_{1\downarrow},d_{2\uparrow},d_{2\downarrow},d^\dagger_{2\uparrow},d^\dagger_{2\downarrow}, f, f^\dagger) \). For \( U = 0 \), the matrix equation for the Green’s function takes the following form

\[ \begin{pmatrix} -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \varepsilon_1 + i\Gamma_N & -\Gamma_S & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 \\ 0 & -\Gamma_S & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 \\ -t & 0 & 0 & 0 & -\varepsilon_2 & 0 & 0 & 0 \\ 0 & t & 0 & 0 & 0 & -\varepsilon_2 & 0 & 0 \\ 0 & 0 & -t & 0 & 0 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_2 \end{pmatrix} \rightarrow \begin{pmatrix} \Gamma_S & -t & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \Gamma_S & -t & 0 & 0 & 0 & 0 & 0 \\ -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & t & 0 & 0 & 0 \\ 0 & -\varepsilon_1 + i\Gamma_N & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\varepsilon_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\varepsilon_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \varepsilon_2 & V_M & V_M & 0 \\ 0 & 0 & 0 & 0 & V_M & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & V_M & 0 \end{pmatrix}, \] (8)

states at

\[ \varepsilon_{MD1}^\pm = \pm \frac{1}{\sqrt{2}} \sqrt{A_M + \sqrt{A_M^2 - 4B_M}}, \]

\[ \varepsilon_{MD2}^\pm = \pm \frac{1}{\sqrt{2}} \sqrt{A_M - \sqrt{A_M^2 - 4B_M}}, \] (10)

where \( A_M = A + 4V_M^2 \) and \( B_M = B + 4V_M^2 (\varepsilon_1^2 + \varepsilon_2^2 + t^2) \).

Let us notice that such new features depend on the coupling strength \( V_M \). We recognize that the quasiparticle states \( \varepsilon_{MD1}^\pm \) and \( \varepsilon_{MD2}^\pm \) are driven by a direct coupling of the QD2 and an indirect coupling of the QD1 (via QD2) to the Majorana mode. The quasiparticle energies \( \varepsilon_{MDi}^\pm \) (\( i = 1, 2 \)) are displayed in Figs. 2 and 3 by dashed lines.

### B. Quasiparticle spectrum for \( \Gamma_N \neq 0 \)

To discuss the empirically measurable properties of our setup (Fig. 1), let us now examine the effective spectrum for a finite (yet small) coupling \( \Gamma_N \). The continuous electronic spectrum of the normal lead broadens the subgap quasiparticle states, which acquire finite life-times. Because the transport properties of the considered system can be related to the spectral function of the first quantum dot, in the following we shall focus on its behavior.
We have computed this spectral function
\[ A_\sigma(\omega) = -\frac{1}{\pi} \text{Im} \langle d_{1\sigma}^\dagger d_{1\sigma} \rangle_{\omega+i0^+}, \] (11)
choosing such model parameters that allow for clear identification of the role played by the Majorana mode. For specific numerical calculations we use: \( \Gamma_N = 0.002 \), \( \Gamma_S = 0.02 \), \( t = 0.01 \), and \( \varepsilon_1 = 0 \) in units of bandwidth \( D = 1 \).

In the absence of topological nanowire (see the orange curves in Figs. 2 and 3), we notice two pairs of the Andreev peaks centered at \( \varepsilon_{AD1}^\pm \) and \( \varepsilon_{AD2}^\pm \) accompanied by the interferometric (Fano-type) structure at \( \omega = \varepsilon_2 \). The direct coupling of QD2 to the topological nanowire indirectly affects the electronic states of QD1. Such influence is transmitted via spin-\( \downarrow \) electrons of the second quantum dot. In consequence, the leakage of Majorana mode shows up at zero energy of the spin-down spectral function \( A_{\downarrow}(\omega) \). For \( \varepsilon_1 = \varepsilon_2 \) and arbitrary couplings \( V_M \neq 0 \), we obtain the universal values \( A_{\downarrow}(0) = 1/2 \pi \Gamma_N \) and \( A_{\uparrow}(0) = 0 \), see Fig. 2, in analogy to the previously reported behavior of single dot configurations [20].

The coupling \( V_M \) is responsible for additional features, appearing at \( \varepsilon_{M1}^\pm \) and \( \varepsilon_{M2}^\pm \), whereas the quasiparticle states \( \varepsilon_{AD1}^\pm \) and \( \varepsilon_{AD2}^\pm \) practically do not change their energies (see green and red dashed lines), except for their spectral weights. For the weak coupling \( V_M \), the quasiparticle states \( \varepsilon_{M1}^\pm \) coincide with \( \varepsilon_{AD1}^\pm \), while the other states \( \varepsilon_{M2}^\pm \) are pushed to higher energies. On the other hand, for stronger couplings \( V_M \), we observe the development of a molecular structure, in which \( \varepsilon_{M2}^\pm \) are mixed with \( \varepsilon_{AD1}^\pm \). Figures 2 and 3 show that the quasiparticles \( \varepsilon_{M1}^\pm \) have the dominant spectral weights in spin-\( \downarrow \) sector, whereas the states \( \varepsilon_{M2}^\pm \) gain their spectral weights mainly in the spin-\( \uparrow \) sector.

Let us now inspect the role of QD2 detuning from the particle-hole symmetry point (\( \varepsilon_2 \neq 0 \)). Under such conditions, the Majorana peak is observable in both spin components of the spectral function \( A_\sigma(\omega) \), see Fig. 3. The width of the zero-energy peak depends on the coupling \( V_M \), while the height ratio \( A_{\downarrow}(0)/A_{\uparrow}(0) \) is controlled.

**FIG. 2.** The normalized spectral function \( \pi \Gamma_N A_\sigma(\omega) \) of the first quantum dot for (a) spin-\( \uparrow \) and (b) spin-\( \downarrow \) electrons obtained for \( \varepsilon_1 = \varepsilon_2 = 0 \), \( \Gamma_S = 0.02 \), \( \Gamma_N = 0.002 \) and \( t = 0.01 \). The quasiparticle energies \( \varepsilon_{AD1}^\pm \) and \( \varepsilon_{M1}^\pm \) given by Eqs. (9) and (10), are marked by the dashed lines \( \varepsilon_{AD1}^\pm \) (green), \( \varepsilon_{AD2}^\pm \) (red), \( \varepsilon_{M1}^\pm \) (blue), \( \varepsilon_{M2}^\pm \) (magenta). All energies are expressed in units of the bandwidth \( D = 1 \).

**FIG. 3.** (a) The spin-up and (b) spin-down normalized spectral function obtained for detuned energies of the quantum dots \( \varepsilon_1 = 0 \) and \( \varepsilon_2 = 0.005 \), using the same model parameters as in Fig. 2.
by the energy level $\varepsilon_2$, keeping the total spectral function $A_1(0) + A_1(0) \approx 1/2\pi T_N$. For the case of strong coupling $V_M$, we obtain $A_1(0) > A_1(0)$, and additionally there emerges a Fano-type dip in $A_1(\omega)$ at $\omega = \varepsilon_2$. Such interferometric feature is absent in $A_1(\omega)$ because of the hybridization of spin-$\downarrow$ electrons of the second quantum dot with the Majorana nanowire.

The second quantum dot (QD$_2$) is indirectly affected by the superconducting reservoir, absorbing the electron pairing [64–68]. Figure 4 shows the variation of the pairing correlations $\langle d_{1\downarrow}d_{1\uparrow} \rangle$ induced on the first ($i = 1$) and the second ($i = 2$) quantum dot with respect to the coupling $V_M$ for several values of energy level of QD$_2$. For $\varepsilon_2 = 0$ and $V_M = 0$, we reproduce the standard result $\langle d_{1\downarrow}d_{1\uparrow} \rangle = \langle d_{2\downarrow}d_{2\uparrow} \rangle$ reported in Ref. [65]. Upon detuning the quantum dot energy levels, we observe that pairing correlations of QD$_2$ are gradually suppressed, at the expense of enhancing the effective pairing on the central quantum dot (QD$_1$). The coupling of QD$_2$ to Majorana nanowire has rather minor influence on all pairing sectors, it merely weakens the pairing correlations $\langle d_{2\downarrow}d_{2\uparrow} \rangle$ on the second quantum dot.

C. Linear Andreev conductance

The Majorana mode features leaking into the quantum dots could be detected by the charge transport measurements [17, 69–72]. In our setup, the central quantum dot (QD$_1$) is embedded between the normal and superconducting electrodes, so its subgap states should be observable in the Andreev current measurements [73–76]. From theoretical side, the Andreev current can be calculated from

$$I_A(V) = \frac{e}{h} \int d\omega \ T_A(\omega) \left[ f(\omega - eV) - f(\omega + eV) \right], \quad (12)$$

where $f(x) = [1 + \exp(x/k_B T)]^{-1}$ denotes the Fermi-Dirac distribution function and the energy-dependent Andreev transmittance is given by

$$T_A(\omega) = \Gamma_N^2 \sum_\sigma |\langle d_{1\sigma}:d_{1\bar{\sigma}} \rangle_{\omega+i0^+}|^2. \quad (13)$$

For $t = 0$, the optimal value of the low-temperature Andreev conductance $G_A(V) = dI_A(V)/dV$ is $4e^2/h$ [73, 77–79]. Finite hopping between the dots, however, lowers this maximum value. Moreover, the side-attached Majorana nanowire can result in a further reduction of the Andreev conductance down to $e^2/h$ [20, 80, 81].

In what follows, we briefly investigate the influence of the Majorana mode on the Andreev conductance measured in the N-QD$_1$-S circuit. Figure 5 presents the linear (zero-bias) Andreev conductance $G_A$ as a function of $\varepsilon_2$ calculated for a fixed value of the first dot level...
position $\varepsilon_1 = 0$. For nonzero $V_M$ the optimal Andreev conductance is equal to $e^2/h$, reflecting the fractional nature of the Majorana mode leaking to DQD. Exactly for $\varepsilon_2 = 0$, the conductance $G_A$ vanishes for all values of $t$ and $V_M$ because of the destructive quantum interference associated with the Fano effect. For a given interdot coupling $t$, the linear Andreev conductance rather weakly depends on the coupling $V_M$ [see Fig. 5(a)]. However, for a fixed value of coupling $V_M$, $G_A$ is very sensitive to interdot hybridization $t$ [see Fig. 5(b)]. In the weak inter-dot hopping limit, the conductance $G_A$ achieves its optimal value at small values of the second quantum dot energy level $\varepsilon_2$. On the other hand, when the inter-dot coupling becomes stronger, the position of the energy level $\varepsilon_2$, at which the Andreev conductance is optimal, substantially increases.

The linear Andreev conductance is also dependent on the first quantum dot energy level. This situation is shown in Fig. 6, which displays the Andreev conductance as a function of both quantum dot energy levels for three selected values of $t$ and $V_M$. As can be seen, no matter what the value of $\varepsilon_1$ is, $G_A$ always vanishes for $\varepsilon_2 = 0$. This characteristic feature is caused by the destructive quantum interference. Figure 6 displays the typical Fano line-shape of such entirely suppressed linear conductance of the considered setup.

### IV. THE CASE OF THE KONDO REGIME

We now address the situation when the Coulomb correlations in the quantum dots are relevant, $U \neq 0$. In this case new effects emerge that are exclusively due to electron correlations. One of such effects is the Kondo phenomenon [82], in which a correlated state is formed between the quantum dots and the conduction band electrons of the normal lead [83]. The main goal of this section is to examine the system’s transport properties in the Kondo regime, and to shed light on the interplay of Kondo correlations, electron pairing and Majorana modes.

In the large pairing gap limit of the superconductor the Kondo effect can be induced by the spin-exchange coupling between QD$_1$ and the metallic lead [84]. Efficiency of such interaction is sensitive to the competition between the on-dot pairing and the Coulomb repulsion. The Kondo state would arise if QD$_1$ is in the singly occupied configuration $|\sigma\rangle$, which takes place when the Coulomb repulsion dominates over the superconducting proximity effect, $\Gamma_S < U/2$. Furthermore, the magnitude of the exchange interaction is substantially enhanced near a transition between the spinful $|\sigma\rangle$ and the spinless (BCS-type) $u|0\rangle - v|\uparrow\downarrow\rangle$ configurations [85, 86]. Since this parity change of QD$_1$ is manifested by a crossing of the in-gap bound states, therefore the influence of the zero-energy Majorana mode on this crossing would be very important.

The relationship between the subgap Kondo effect and the Majorana physics has been previously investigated in heterostructures with a single correlated quantum dot [20, 81]. Here, we extend these studies to the double quantum dot system (Fig. 1), where the Majorana mode can affect QD$_1$ only indirectly, via the second quantum dot. For efficiency of this two-stage Majorana leakage, one should also take into account the Kondo effect developed in QD$_2$ [15, 87, 88]. With lowering the energy scale below the Kondo temperature $T_K$, the Kondo effect first develops in the first quantum dot, giving rise
to enhanced conductance [89]. However, for even lower energies, the spin on the second quantum dot becomes screened and, because this dot is not directly coupled to the superconductor-normal lead circuit, cf. Fig. 1, the conductance becomes in turn suppressed. This is known as the two-stage Kondo effect, which is characterized by a dip in the spectral function of width proportional to the second-stage Kondo temperature $T^*$ [90–93]. In what follows, we analyze a subtle interplay of these effects and discuss their signatures observable in the charge transport properties of N-QD$_1$-S circuit.

To reliably treat the low and high energy features originating from the correlation effects and the proximity-induced pairing we perform the density-matrix numerical renormalization group calculations [56–60]. We impose the band discretization parameter $\Lambda = 2 - 2.2$, keeping at least 2000 states during the iterative diagonalization. The microscopic model (1) satisfies the $Z(2)$ parity, which we implement to facilitate the numerical calculations. We perform the computations for the particle-hole symmetric case, $\varepsilon_1 = \varepsilon_2 = -U/2$, assuming $U/D = 0.1$, $\Gamma_N/D = 0.01$.

### A. Superconducting pairing correlations

Before analyzing the spectral and transport properties of the system, let us examine the behavior of the superconducting pairing correlations induced in the quantum dots. These correlations are presented in Fig. 7 as a function of the coupling to superconductor $\Gamma_S$ for selected values of the coupling to Majorana wire $V_M$. First of all, we note that the electron correlations ($U > 0$) strongly modify the behavior of $\langle d_{1\uparrow}d_{1\uparrow} \rangle$ as compared to the uncorrelated case, cf. Figs. 4 and 7. In the absence of coupling to Majorana wire the pairing in the first dot exhibits the behavior, which is typical for superconductor-proximized quantum dots [86]: $\langle d_{1\downarrow}d_{1\uparrow} \rangle$ is suppressed in the doublet state, $\Gamma_S < U/2$, and approaches $\langle d_{1\downarrow}d_{1\uparrow} \rangle = 1/2$ on the singlet side of the transition, i.e. for $\Gamma_S > U/2$. Moreover, we find that in the strongly correlated case $\langle d_{1\downarrow}d_{1\uparrow} \rangle$ very weakly depends on $V_M$, see Fig. 7(a). This is contrary to the behavior of $\langle d_{2\uparrow}d_{2\uparrow} \rangle$, which becomes enhanced when the coupling to Majorana wire is turned on, see Fig. 7(b). Furthermore, the dependence of the pairing induced on the second dot on $\Gamma_S$ and $V_M$ is more subtle. For $\Gamma_S < U/2$, $\langle d_{2\uparrow}d_{2\uparrow} \rangle < 0$, while for $\Gamma_S > U/2$, $\langle d_{2\uparrow}d_{2\uparrow} \rangle > 0$. The negative value on the doublet side of the transition can be associated with the quantum interference and the polarization of the second quantum dot that is induced by the Majorana mode.

### B. Majorana features in spectral functions

To elucidate the role of the Majorana mode on the spectral density, we first analyze the dependence of the spectral function of the half-filled quantum dots on the strength of coupling to the superconductor in the absence of coupling to Majorana wire. The normalized spectral function for spin $\sigma$ (note that for $V_M = 0$ the spin degeneracy is restored) is presented in Fig. 8. Let us recall that in the absence of the interdot coupling ($t = 0$) the parity change of QD$_1$ from $|\sigma\rangle$ to $u |0\rangle - v |\uparrow\downarrow\rangle$ configuration occurs at $\Gamma_S = U/2$. The subgap Kondo effect can

![Image](https://via.placeholder.com/150)

**FIG. 7.** Superconducting pairing correlations of (a) the first and (b) the second quantum dot calculated as a function of the coupling to superconductor $\Gamma_S$ for different values of $V_M$, as indicated. The other parameters are: $t/U = 0.025$, $\Gamma_N/U = 0.1$, $\varepsilon_1 = \varepsilon_2 = -U/2$ and $U = 0.1$ in units of bandwidth.

**FIG. 8.** The normalized spectral function $\pi \Gamma_N A_\sigma(\omega)$ of the half-filled central quantum dot obtained for various couplings to superconductor $\Gamma_S$, as indicated, while $V_M = 0$. The other parameters are the same as in Fig. 7. The spectral function is symmetric with respect to the Fermi energy, therefore only positive energies are shown. Note also the logarithmic energy scale. The dashed curves correspond to the values of $\Gamma_S$ analyzed in Figs. 9 and 10.
function. The proximity induced pairing affects the two-stage Kondo effect [91–93], with suppressed spectral density for $\omega < T$. A corresponding spin-resolved spectral functions calculated for different couplings $V_M$ to the topological wire, as indicated. The top, middle and bottom panels refer to $\Gamma_S/U = 0.4$, 0.5 and 0.6, respectively. The other parameters are the same as in Fig. 7.

thus be realized for $\Gamma_S < U/2$. On the other hand, for finite $t$ but $\Gamma_S = 0$, one observes the behavior typical for the two-stage Kondo effect [91–93], with suppressed spectral density for $\omega < T^*$. In the case of finite $t$ and $\Gamma_S$, the interplay of the on-dot pairing and Kondo correlations gives rise to an interesting behavior of the spectral function. The proximity induced pairing affects $A_s(\omega)$ by lifting the second-stage of screening and giving rise to a finite value of $A_s(0)$, which is the largest around the transition between the doublet and BCS singlet state, $\Gamma_S \approx U/2$, see Fig. 8.

Let us now examine the behavior of the spectral functions in the case of finite coupling to topological nanowire, focusing on values of $\Gamma_S$ around the singlet-doublet transition, cf. the dashed lines in Fig. 8. The corresponding spin-resolved spectral functions $A_s(\omega)$ obtained for various couplings $V_M$ to the topological nanowire are presented in Figs. 9 and 10. First of all, we note that the numerical results reveal qualitative differences between the spectra of the spin-up and spin-down electrons, which should be attributed to the presence of Majorana mode. Such influence is predominantly manifested in the low-energy sector, therefore our plots are presented in a logarithmic scale. The upper panels of the figures correspond to the situation when $\Gamma_S < U/2$ and the subgap Kondo effect can be realized, therefore $A_s(\omega)$ has a relatively large weight at the Fermi energy for $V_M = 0$. On the other hand, the lower panels display the case of $\Gamma_S > U/2$, where the weight is much reduced. However, once $V_M$ is finite, these features may be completely changed.

Consider first the case of the spin-up spectrum of QD$_1$ shown in Fig. 9. Each panel of the figure clearly displays a substantial depletion of the low-energy states driven by the Majorana mode. In the weak coupling limit, $V_M \ll U$, such destructive influence appears in the form of interferometric dip formed on top of either the subgap
Kondo peak [in the strongly correlated limit $\Gamma_S/U = 0.4$, see Fig. 9(a)], or on a flat background between the Andreev peaks [corresponding to the BCS-type configuration $\Gamma_S/U = 0.6$, see Fig. 9(c)]. Upon increasing the coupling $V_M$, we observe the development of a molecular structure, where the Kondo effect gets suppressed. In other words, in the subgap Kondo regime, one can see a supporting influence of the coupling to the Majorana mode on the second-stage Kondo effect, i.e. there occurs a restoration of this effect for finite $V_M$. Moreover, the stronger the coupling $V_M$ is, the higher the second-stage Kondo temperature $T^*$ becomes. This picture resembles itself through the singlet-doublet transition, lowering the maximum of the Kondo peak, and finally suppressing the spectral function in the BCS regime, where the Kondo effect does not develop. Thus, the coupling to the topological nanowire has a rather destructive influence on the low-energy behavior of the spin-up spectral function.

In contrast to this tendency, the Majorana mode has a more complex impact on the spin-down spectral function shown in Fig. 10. In the subgap Kondo regime, finite coupling to the topological superconductor suppresses the low-energy spectral density by around 1/3 to the universal value of $A_\downarrow(0) = 1/2\pi\Gamma_N$. However, the picture changes when the BCS configuration is formed and the influence of $V_M$ becomes constructive. For weak couplings, $V_M \ll U$, we notice a buildup of the narrow peak at zero-energy due to the leaking Majorana mode. Upon increasing $V_M$, the value of the spectral function $A_\downarrow(\omega = 0)$ saturates, while the zero-energy peak gradually broadens. For larger values of $V_M$, this zero-energy quasiparticle state dominates over all other subgap features. The corresponding spectral function develops then its universal shape with the characteristic value $A_\downarrow(0) = 1/2\pi\Gamma_N$, regardless of the Coulomb potential $U$. Summarizing, the attachment of topological superconductor to the spin-$\downarrow$ electrons of QD$_2$ induces the zero-energy state in spin-$\downarrow$ sector of QD$_1$ with the fractional value of the low-energy spectral function.

FIG. 11. The Andreev transmittance $T_A(\omega)$ as a function of $\Gamma_S/U$ and energy $\omega$ calculated for several values of the coupling to Majorana wire $V_M$ and interdot hopping $t$ (as indicated) obtained by NRG. The other parameters are the same as in Fig. 7.
C. Subgap charge transport

Empirical detection of the quasiparticle spectra of QD$_1$ can be done with the use of the charge tunneling spectroscopy. In the subgap region the only transport channel is contributed by the particle-to-hole (Andreev) scattering mechanism, which combines the both spin components. The subgap tunneling spectroscopy would hence provide an important information about the convoluted spin-up and spin-down spectra.

At low temperatures the Andreev differential conductance $G_A(V) = dI_A(V)/dV$ can be approximated by the Andreev transmittance $T_A(\omega)$ taken at $\omega = eV$, $\lim_{V \to 0} G_A(V) = \frac{2e^2}{h} T_A(\omega = eV)$. We display $T_A(\omega)$ in Fig. 11 for several values of the inter-dot hopping $t$ (panels from left to right) and different couplings $V_M$ (panels from top to bottom). In general, we notice, that the Andreev conductance achieves its optimal value $4e^2/h$ near $\Gamma_S \approx U/2$ [79]. This situation corresponds to the ground state changeover of QD$_1$, whose in-gap bound states tend to cross each other and simultaneously the subgap Kondo peak is enhanced (on the doublet side).

As regards the Majorana mode, its influence shows up by suppression of the zero-bias conductance (see the middle and bottom panels in Fig. 11). This effect comes merely from the destructive quantum interference of the Majorana mode on the spin-$\uparrow$ sector of QD$_1$ spectrum. Additionally, upon increasing the inter-dot hopping $t$, we observe the signatures of emerging molecular bound states. In particular, this is visible by a dark region splitting the optimal conductance. We note that for a more precise and direct observability of all the spin-resolved spectra of QD$_1$ one could use the spin-polarized Andreev spectroscopy of bound states [94].

V. SUMMARY

We have studied the influence of the Majorana mode transmitted to the double quantum dot side-attached to a topological superconducting nanowire. This setup could be probed by tunneling spectroscopy through a circuit with the outer quantum dot (QD$_1$) sandwiched between the normal and superconducting leads. Proximity of DQD to the superconducting reservoir induces the in-gap bound states, whose complex structure depends on the inter-dot coupling, as recently revealed in Ref. [54]. Here, we have inspected the modification of these conventional bound states by the topological superconducting nanowire hosting the Majorana boundary mode. This issue might be important for designing the braiding protocols of Majorana quasiparticles.

In the absence of correlation effects we have derived analytical expressions for the resulting in-gap bound states, identifying the trivial Andreev quasiparticle branches and the additional structures induced by the Majorana mode. We have shown that these features manifest themselves differently in each spin sector. In particular, the zero-energy quasiparticle state induced at QD$_1$ appears in a form of destructive/constructive interference pattern imprinted on the $\uparrow/\downarrow$ spin sectors. Since the subgap charge transport mixes both spins through the particle-to-hole scattering, the resulting tunneling characteristics are predominantly affected by these destructive signatures. We have discussed them in detail, considering the linear subgap Andreev conductance.

We have also extended our considerations to the strongly correlated nanostructure, treating the competition of the superconducting proximity effect with the repulsive Coulomb interactions by the numerical renormalization group technique. Focusing on the half-filled quantum dots, we have studied both the Kondo regime, when the Coulomb repulsion dominates over the superconducting proximity effect, as well as the opposite limit, where the superconducting proximity-induced pairing surpasses the Kondo correlations. We have revealed qualitative differences in these two regimes, evidenced in the spin-resolved spectral functions. We have predicted that the optimal Andreev conductance would occur near a crossover between the singly-occupied doublet to the BCS-type configurations of QD$_1$. Moreover, our numerical results obtained for the subgap spectroscopy indicate that the Majorana mode strongly suppresses the zero-bias conductance, owing to its destructive influence on spin-$\uparrow$ sector of the outer quantum dot.

ACKNOWLEDGMENTS

This work was supported by the National Science Centre in Poland through the Project No. 2018/29/B/ST3/00937. The computing time at the Poznań Supercomputing and Networking Center is acknowledged.

Appendix A: Role of polarization

In this appendix we extend the discussion of the properties of our hybrid structure in the uncorrelated case by assuming that both spins of QD$_2$ are coupled to the Majorana nanowire though with different amplitudes $V_{M\sigma}$. Hoffman et al. [61] have shown that the spin-dependent tunneling amplitudes between Majorana modes and the quantum dot depend on the quantum dot distance to the topological section. Moreover, such finite polarization of quantum dot attached to the topological nanowire has been proposed as a suitable tool for probing the topology of Majorana wave function [19, 61–63]. To account for this effect, we use the following parametrization of the spin-dependent amplitudes

$$V_{M\uparrow} = V_M \sin \left( \frac{\theta}{2} \right)$$
$$V_{M\downarrow} = V_M \cos \left( \frac{\theta}{2} \right)$$
finite-energy quasiparticles. In the fully polarized cases, the spectral weights of the zero-energy peak, and also on the canting angle \( \theta \). One can clearly see the influence of the canting angle \( \theta \) on spectral weights of the zero-energy peak, and also on the finite-energy quasiparticles. In the fully polarized cases, \( V_{M\uparrow} = 0 \) and \( V_{M\downarrow} = V_M \) (\( V_{M\uparrow} = V_M \) and \( V_{M\downarrow} = 0 \)), corresponding to \( \theta = 0 \) (\( \theta = \pi \)), we obtain the Majorana peak of spin-\( \downarrow \) (spin-\( \uparrow \)) spectral function \( A_\uparrow(0) = 1/2\pi \Gamma_N \) \( [A_\downarrow(0) = 1/2\pi \Gamma_N] \). On the other hand, in the unpolarized case, i.e. \( \theta = \frac{\pi}{2} \), the zero-energy Majorana peak is identical in both spin sectors, \( A_\uparrow(0) = A_\downarrow(0) = 1/4\pi \Gamma_N \).

Additionally, we notice, that the canting angle neither affects the energies of the Andreev bound states \( \epsilon_{\pm}^{ADi} \) [cf. Eq. (9)] nor the quasiparticle states \( \epsilon_{\pm}^{MDi} \) [cf. Eq. (10)]. Its influence is visible merely in the spectral densities \( A_\sigma(\epsilon_{\pm}^{ADi}) \) and \( A_\sigma(\epsilon_{\pm}^{MDi}) \).

We have shown that \( \theta \) affects the quasiparticle weights of the diagonal spectral functions, but additionally it also strongly modifies the off-diagonal spectral functions. Such influence would be empirically observable in the subgap tunneling conductance. Figure 13 presents the linear Andreev conductance \( G_A \) as a function of the spin canting angle \( \theta \) obtained for \( \epsilon_1 = 0, V_M = 0.005, t = 0.01, \Gamma_S = 0.02, \Gamma_N = 0.002 \) and several values of \( \epsilon_2 \), as indicated.

![FIG. 12. The normalized spectral function \( \pi \Gamma \sigma A_\sigma(\omega) \) for (a) the spin-up and (b) spin-down components as a function of the canting angle \( \theta \). Results are obtained for the case of uncorrelated quantum dots using the following model parameters: \( \epsilon_1 = \epsilon_2 = 0, V_M = 0.005, t = 0.01, \Gamma_S = 0.02, \Gamma_N = 0.002 \).](image)

![FIG. 13. The linear Andreev conductance \( G_A \) as a function of the spin canting angle \( \theta \) obtained for \( \epsilon_1 = 0, V_M = 0.005, t = 0.01, \Gamma_S = 0.02, \Gamma_N = 0.002 \) and several values of \( \epsilon_2 \), as indicated.](image)

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