The model of the positive column of a glow discharge with the influence of the acoustic oscillations

N F Kashapov, A I Saifutdinov and S A Fadeev
Kazan Federal University, 18 Kremllovskaya str., Kazan 420008, Russian Federation
E-mail: Sergey_Fadeev@kpfu.ru

Abstract. In article the model of the positive column of a glow discharge with the influence of acoustic waves is developed. It is shown that the radial convective flow caused by the acoustic streaming leads to a redistribution of the gas temperature, and reduces its gradient between the axis and the wall of the discharge chamber.

1. Introduction
The low temperature plasma of glow discharge is a partially ionized gas with a large electron temperature and has a wide range of practical applications. The plasma of this type used in MHD generators [1] and magnetron sputtering system [2], in the processes of plasma modification of surface properties [3-6] and in the plasma antennas [7]. We should also mention the gas CO\textsubscript{2} lasers [1, 8], which are the most widely used lasers in material processing. The most powerful lasers have a wide range of technological applications. The most reasonable way to improve the efficiency of CO\textsubscript{2} lasers is to increase the pressure in the discharge chamber and the power into the active medium of the laser. However, in the process of improving these parameters glow discharge of the CO\textsubscript{2} laser becomes unstable. He loses uniformity and constricted. This leads to a complete cessation of radiation generation. Overcoming this kind of instability is the main problem in creating high-power lasers.

In each of the kinds of practical application of low temperature plasma can identify processes which are the result of the interaction of plasma and external factors. Most detailed investigated the influence of electromagnetic waves, electric fields, magnetic fields, electron beams, ion beams, neutral beams, laminar and turbulent flows on the plasma properties. During the last decade intensively investigated issues of interaction of acoustic waves with plasma. Influence of acoustic waves on the low-temperature plasma is accompanied by a variety of processes [9, 10, 11]: lowering the temperature of the gas, reduction of the radial gradient of the gas temperature, the increase of the longitudinal electric field, raising the temperature of the electrons, modulation of the discharge current and voltage. It can be effectively used in the development of high-power CO\textsubscript{2} lasers. For a description of these phenomena is necessary to develop a mathematical model of the positive column of a glow discharge taking into account influence of acoustic waves.

2. The model of the positive column of a glow discharge with the influence of acoustic streaming
We know that the acoustic low-frequency oscillations cannot influence the transitions between electronic levels. The phenomena occurring in the discharge chamber at excitation of acoustic waves associated with the hydrodynamic flows, which affect the structure and stability of the positive column of glow discharge. This leads to a change the processes of ionization and recombination.
In this case, acoustic oscillations in a glow discharge plasma can be accounted through the vortex the so-called acoustic streaming. Acoustic streaming excited, firstly, due to the boundary layer at the wall where the velocity is reduced from the value in the acoustic waves in the middle of the tube to zero at the wall of the tube, and secondly, because of the strong temperature gradient along the radius of the tube.

As shown in [9], acoustic flows generated in the field of the acoustic wave and field of velocities can be expressed as follows (in the cylindrical coordinate system)

\[
    u_r = -\frac{u_a^2}{10c_0\delta^2\lambda}(1 - \frac{r^2}{R^2})\cos 2kx,
\]

\[
    u_x = -\frac{u_a^2}{c_0}120\frac{\alpha}{\delta^2\lambda}(1 - \frac{r^2}{R^2})(1 - \frac{5r^2}{R^2})\sin 2kx,
\]

where \(u_a\) is the parameter characterizing the temperature inhomogeneity of the medium

\[
    u_a = -\frac{p_A}{\rho c_0}(1 + \frac{\alpha \omega^2}{c_0^2}r^2(1 + \frac{r^2}{2R^2})).
\]

Here \(R\) is the tube’s radius, \(\alpha\) is the size of the inhomogeneity in the acoustic field, \(\rho\) is the gas density, \(\omega\) is the frequency of the acoustic wave, \(c_0\) is the speed at which the acoustic wave travels in an undistorted medium, \(\delta\) is the size of the boundary layer, \(\lambda\) is wavelength, \(P_A\) is pressure amplitude of the acoustic oscillations in the tube, \(k\) is wave vector.

A simplified model of the spatial structure of the positive column of a glow discharge with cylindrical symmetry (from the axis to the wall of the discharge chamber) with the influence of acoustic flows leading to decontraction discharge can be formulated on the basis of the following system of equations

1) Balance equation for the concentration of charged particles (for electrically neutral plasma)

\[
    \frac{u_x}{R}\frac{\partial n}{\partial x} + \frac{u_r}{R}\frac{\partial n}{\partial r} = \frac{D_a}{r^2\frac{\partial}{\partial r}}(r\frac{\partial n}{\partial r}) + \nu_i n - \beta n^2,
\]

where \(\nu_i\) – is the ionization frequency, which is a function of the electric field \(E\), \(\beta\) is the recombination coefficient, \(D_a\) is the ambipolar diffusion coefficient. The boundary conditions for equation (4) are given on the chamber wall and the discharge axis

\[
    n(R) = n'(0) = 0.
\]

2) Energy balance equation for neutral particles

\[
    \rho c_p\frac{u_x}{R}\frac{\partial T}{\partial x} + \rho c_p\frac{u_r}{R}\frac{\partial T}{\partial r} = \frac{\chi}{r^2\frac{\partial}{\partial r}}(r\frac{\partial T}{\partial r}) + jE,
\]

where \(\chi\) is coefficient of thermal conductivity, \(c_p\) is thermal capacity, \(j\) is current density. The equation describes the balance of Joule heating of gas and heat loss to the walls by thermal flow with a coefficient of thermal conductivity \(\chi\) and by acoustic streaming. Boundary conditions for (6) are given

\[
    T(R) = T_0, T'(0) = 0
\]

3) The equation of state of the gas in which the pressure is determined by the temperature of the neutral particles
where \( N \) is concentration of neutral particles, \( \kappa \) is Boltzmann constant. The equation describes the redistribution of neutral particles \( N \) over the radius caused by nonuniform heating of the gas.

4) The equation for the discharge current (current is determined by the electrons)

\[
i = e\mu_e E \int_0^R n(r)rdr,
\]

which binds the concentration of charged particles with external parameters. Here is the mobility coefficient.

3. Evaluation of the radial distribution parameters of a glow discharge with acoustic flows

To solve system of equations (4)-(9) with (1) and (2) requires the use of special numerical methods. However, approximate analytical evaluation of the radial distribution of the parameters of the positive column, which uncontracted by creating acoustic flows in some approximation can be determined, assuming that the radial component of velocity \( u_r \) is constant, and distribution of the plasma parameters along the \( x \) axis uniform. In addition, we assume that, under our pressure due to acoustic streaming, loss of charged particles at the chamber walls dominates the recombination in the plasma volume.

In this case, the system of equations can be written as

\[
\frac{u_r}{R} \frac{\partial n}{\partial r} = \frac{D_a}{R^2 r} \frac{\partial}{\partial r} \left( r \frac{\partial n}{\partial r} \right) + v_i n, \tag{10}
\]

\[
\rho c_p \frac{u_r}{R} \frac{\partial \tau}{\partial r} = \frac{X}{R^2 r} \frac{\partial}{\partial r} \left( r \frac{\partial \tau}{\partial r} \right) + jE, \tag{11}
\]

\[
p = N\kappa T \tag{12}
\]

\[
i = e\mu_e E \int_0^R n(r)rdr \tag{13}
\]

Equation (9) is the confluent hypergeometric equation and its solution has the form

\[
n(r) = n_0 \exp \left( -\frac{r \left( u_r^2 - 4v_r \rho_a - u_r \right)}{2D_a} \right) \Phi(a, b, \xi), \tag{14}
\]

where \( n_0 \) is electron density on the axis of symmetry,

\[
\Phi(a, b, \xi) = \frac{\Gamma(b)}{\Gamma(a)\Gamma(a-b)} \int_0^1 e^{\xi t} t^{a-1} (1 - t)^{b-a-1} dt,
\]

is special function of Kummer \([12]\),

\[
\Gamma(r) = \int_0^\infty e^{-t} t^{r-1} dt,
\]

is gamma function,
\[ a = \frac{\sqrt{u^2 - 4\nu D a - u r}}{2\sqrt{u^2 - 4\nu D a}}, \quad b = 1, \quad \xi = \frac{r\sqrt{u^2 - 4\nu D a}}{D a}. \]

The solution of equation (11) can be found by direct integration

\[ T(r) = -\int_{r_0}^{r} \left( \frac{R^2}{x} \exp \left( \frac{\rho c_p u_r r}{x} \right) \int_{r_0}^{r} j(r) E(r) \exp \left( \frac{\rho c_p u_r r}{x} \right) r \, dr \right) \, dr + T_0 \] (15)

Profile of the radial distribution of the electric field uncontracted column can be found by substituting the expression (14) in (13) and solving an integral equation relatively \( E, \) or by equating the sum diffusion and convection components with drift component of the radial flow of electrons (Boltzmann equilibrium)

\[ -D_e \frac{d n_e}{d r} + u_r \frac{d n_e}{d r} - \mu_e E_r n_e = 0, \] (16)

then

\[ E_r = -\frac{(D_e - u_r)}{\mu_e} \frac{d}{d r} \ln \left( n_0 \exp \left( -\frac{r\sqrt{u^2 - 4\nu D a - u r}}{2D a} \right) \Phi(a, b, \xi) \right). \] (17)

4. Conclusions

In a glow discharge plasma parameters are changed due to the presence of acoustic oscillations. To describe such processes developed a mathematical model of the positive column of a glow discharge with the influence of acoustic streaming. Revealed (equation 15) that the convective radial flow caused by acoustic streaming causes a redistribution of the gas temperature, to reduce its gradient between the axis and the wall of the discharge chamber, which entails a redistribution of the concentration of neutral particles (equation 12). The uniform distribution of the density of neutral particles, as well as convective concentration of charged particles to the walls of the chamber should lead to a discontraction of positive column.

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