Dynamic response analysis of isolated curved beam bridge under multi-dimensional non-stationary random excitation considering the torsion component

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Abstract. The coupling effect of bending and torsion caused by plane irregularity of the isolated curved beam bridge makes its seismic response more complex than that of the isolated straight beam bridge. In order to study the influence of multi-dimensional non-stationary random excitation considering torsion component on seismic response of isolated curved beam bridge. The classical Bouc-Wen model was used to simulate the hysteretic characteristics of the curved beam bridge, and the nonlinear dynamic equation considering the eccentricity of superstructure was established. The pseudo excitation was transformed into harmonic external load by Euler formula, and combining with the precise integration method, the precise integration forms of the special solution under different modulation functions were derived. By using these integral forms to solve the response at each time, the time-varying variance of displacement of the isolated curved beam bridge under multi-dimensional non-stationary random excitation was obtained, and the law of time-varying variance of displacement, including the upper and lower structures of the isolated curved beam bridge, was analyzed with different deck width, curvature radius and yield ratio of bearing. The results show that the multi-dimensional non-stationary random excitation with torsion component has significant influence on the seismic response of the isolated curved beam bridge. Under the rare non-stationary random excitation, the dynamic response of the isolated curved beam bridge presents strong non-stationarity, and the time-lag phenomenon is apparent. For the isolated curved beam bridge with large deck width and small yield ratio of bearing, the influence of non-stationary random excitation on the structure is particularly obvious. The influence of non-stationary random excitation considering torsion component on dynamic response of the isolated curved beam bridge with large deck width, small curvature radius and large yield ratio of bearing is more significant.

Keywords. Isolated curved beam bridge; Bouc-Wen model; Multi-dimensional non-stationary random excitation; Torsion component; The precise integration method

1. Introduction

Many scholars have found that the ground motion is a complex multi-dimensional motion when an earthquake occurs, in addition to the three translational components we are generally familiar with, there
are also three rotational components (one torsional component rotating around the vertical axis and two rocking components rotating around the horizontal axis). As a matter of fact, the response of both regular and irregular structures under multi-dimensional earthquake is greater than that of single-dimensional earthquake, especially for the irregular structures [1]-[2]. Moreover, since the practical ground motion is a non-stationary random process, the non-stationary random vibration analysis of the structure is more in line with the practical ground motion process.

For this kind of irregular structure like the isolated curved beam bridge, it is widely used in urban road interchange systems, because it is not restricted by the surrounding environment and the traffic route. However, the bending torsion coupling effect will appear in the curved beam bridge due to the inconsistency between the mass center and the stiffness center of the upper structure, so the joint action of multi-dimensional non-stationary ground motion should be considered. Zhou Xuhong et al. [3]-[4] respectively established the finite element model of the isolated curved bridge, and analyzed the influence of different factors on the nonlinear response of structure under multi-dimensional ground motion. Chen Yanjiang et al. [5] carried out random vibration analysis on seismic performance of the curved beam bridge under multi-dimensional and multi-point earthquake through the Pseudo-Excitation Method, and the results show that the spatial variation of ground motion has a significant impact on the seismic response of the curved beam bridge. But the above process of the finite element modeling and analysis is complex, the workload is very heavy, and the computational efficiency is relatively low. In order to simplify the analysis and improve the calculation efficiency, Wang Li et al. [6]-[7] established a simplified model, suitable for the analysis of the isolated curved beam bridge on the basis of the mechanical characteristics of it, and verified the correctness and accuracy of the simplified model. By using a simplified linear model with two particles and six degrees of freedom, Li Ximei et al. [8] analyzed the influence of different factors on the dynamic response of the isolated curved beam bridge under the stationary and non-stationary ground motions. However, the torsional component of multi-dimensional non-stationary ground motion is not considered in the above study.

In this paper, on the basis of the existing researches, the classical Bouc-Wen model is used to establish the elastic-plastic model of the isolated curved beam bridge with considering the eccentricity of the superstructure, and the stochastic equivalent linearization is carried out. Then, considering the non-stationarity of multi-dimensional random excitation of torsional component, the time-varying variance of displacement of the isolated curved beam bridge is obtained by combining the pseudo excitation deformed by Euler formula with the precise integration method. Finally, with different deck width, curvature radius and yield ratio of isolation bearing, the influence of non-stationary random excitation considering the torsion component on dynamic response of the isolated curved beam bridge is analyzed.

2. Establishment of vibration equation of isolated curved beam bridge

Taking the isolated curved beam bridge as the shear type, and the superstructure and substructure of it are marked as the upper and lower layers respectively. The mass center of the upper structure of the isolated curved beam bridge is taken as the coordinate origin, and the nonlinear motion equation of the isolated curved beam bridge is established, which can be expressed as follows:

\[ M \ddot{U} + C \dot{U} + F_e(U, \dot{U}) = -MEU_s \]

(1)

where \( M, C \) and \( E \) are the mass matrix, damping matrix and seismic impact matrix of the curved beam bridge model respectively; \( \dot{U}, \ddot{U} \) and \( U \) are the acceleration, velocity and displacement of each particle relative to the ground; Seismic excitation \( U_s \) generally consists of six components, based on the research needs, two translational components in the horizontal plane and a torsional component rotating around the vertical axis are used for analysis and research. \( U_s = [\dot{U}_x, \dot{U}_y, \dot{U}_z, \ddot{U}_x, \ddot{U}_y, \ddot{U}_z]^T \) is the seismic acceleration vector. \( F_e(U, \dot{U}) \) is the hysteresis recovery force of the curved beam bridge, and both substructure and isolation layer of the bridge are the classical Bouc-Wen model, which can be written as follows:

\[ F_e(U, \dot{U}) = K_e U + K_v \dot{U} \]

(2)
\[
\dot{\mathbf{u}} = \begin{bmatrix} \dot{u}_x \\ \dot{u}_y \end{bmatrix} = \begin{bmatrix} A\dot{u}_x - \beta [u] [\dot{u}_x]^{n-1} [\dot{u}_x - \gamma \dot{u}_y] [\dot{u}_y]^{n-1} \\ A\dot{u}_y - \beta [u] [\dot{u}_y]^{n-1} [\dot{u}_y - \gamma \dot{u}_x] [\dot{u}_x]^{n-1} \end{bmatrix}
\]

(3)

where \( \mathbf{U} = \{x, y, \theta\}^T = \{x_i, y_i, \theta_i\}^T \); \( x_i, y_i \) and \( \theta_i \) are the translational displacement in the \( x \) direction and \( y \) direction and the angle of the \( i \) layer respectively. \( \mathbf{v} \) is the hysteretic displacement vector of the structure; \( \dot{u}_x, \dot{u}_y \) are respectively the hysteretic displacement vector in the \( x \) direction and \( y \) direction of the structure; \( \dot{u}_x, \dot{u}_y \) are the relative velocity between layers of the structure, \( \dot{u}_x = \ddot{x}_i - \ddot{x}_{i-1}, \dot{u}_y = \ddot{y}_i - \ddot{y}_{i-1} \); \( A, \beta, \gamma, \mu_i \) are the parameter of hysteretic curve. \( \mathbf{K}_e, \mathbf{K}_p \) are the elastic stiffness matrix and plastic stiffness matrix of the structure respectively.

When the structure is made up of reinforced concrete, it can be seen from the actual measurement that: \( \alpha = 0.02 \sim 0.1, \ A_i = 1, \ \beta = -3\gamma_i, \ \gamma_i = \frac{1}{2} \left(1 - \alpha_i\right) \frac{K_i^2}{f_i} \), \( f_i \) is the shear bearing capacity of each layer of the structure[9]. The parameters of the Bouc-Wen model for isolation layer are as follows [10]: \( \alpha = 0.1, \ \mu = 2, A = 1, \ \beta = \gamma = 0.5 \).

Thus, the equivalent linearization equation of the isolated curved beam bridge can be written:

\[
\mathbf{M} \ddot{\mathbf{U}} + \mathbf{C}_e \dot{\mathbf{U}} + \mathbf{K}_e \mathbf{U} = -\mathbf{M} \ddot{\mathbf{U}}_g
\]

(4)

\[
\dot{\mathbf{u}} = \mathbf{C}_e \ddot{\mathbf{u}} + \mathbf{K}_e \dot{\mathbf{u}} - \mathbf{M} \ddot{\mathbf{U}}_g
\]

(5)

where \( \mathbf{C}_e, \mathbf{K}_e \) are the equivalent damping matrix and equivalent stiffness matrix of the Bouc-Wen model. For the specific equivalent linearization process of the Bouc-Wen model, please refer to literature [11].

The concrete forms of each matrix in Eq. (4) and Eq. (5) are shown as follows:

\[
\mathbf{M} = \begin{bmatrix} \mathbf{M}_x \\ \mathbf{M}_y \end{bmatrix}, \quad \mathbf{J} = \begin{bmatrix} \mathbf{J}_1 \\ \mathbf{J}_2 \end{bmatrix},
\]

\[
\mathbf{M}_x = \mathbf{M}_y = \begin{bmatrix} m_1 \\ m_2 \end{bmatrix}, \quad \mathbf{J}_i = m_i (r_i^2 + X_i^2 + Y_i^2)
\]

where \( m_1, m_2 \) respectively are the mass of the substructure and superstructure of the isolated curved beam bridge; \( \mathbf{J}_1, \mathbf{J}_2 \) are the rotational inertia of the substructure and superstructure respectively[12]; \( r_i \) is the radius of gyration of the structure; \( X_i, Y_i \) are the coordinates of the mass center of the substructure and superstructure relative to the reference axis.

\[
\mathbf{K}_e = \begin{bmatrix} \mathbf{K}_{xx} & 0 & \mathbf{K}_{x\theta} \\ 0 & \mathbf{K}_{yy} & \mathbf{K}_{y\theta} \\ \mathbf{K}_{x\theta} & \mathbf{K}_{y\theta} & \mathbf{K}_{\theta\theta} \end{bmatrix}
\]

where \( \mathbf{K}_{xx}, \mathbf{K}_{yy} \) are the elastic translational stiffness of the structure in \( x \) direction and \( y \) direction respectively, and the isolated curved bridge is taken as the shear type, therefore:

\[
\mathbf{K}_{xx} = \begin{bmatrix} \alpha_1 K_{x1} + \alpha_2 K_{x2} & -\alpha_2 K_{x2} \\ -\alpha_2 K_{x2} & \alpha_1 K_{x1} \end{bmatrix},
\]

\[
\mathbf{K}_i = \sum_{i=1}^{j} k_{o,i}, \quad \mathbf{K}_i = \sum_{i=1}^{j} k_{p,i}
\]
where \( k_{xi} \), \( k_{yi} \) represent the lateral stiffness in \( x \) and \( y \) directions at the position of the \( r \) pier on the \( i \) layer; \( l \) represents the number of piers (or bearings) of the curved beam bridge; \( K_{xi} \) represents the total horizontal stiffness in \( x \) direction of the \( i \) layer before yielding; \( \alpha_i \) is the ratio of the horizontal stiffness after and before yielding on the \( i \) layer, note that when \( \alpha_i = 0 \), the \( i \) layer of the structure is in a completely nonlinear state, when \( \alpha_i = 1 \), the \( i \) layer of the structure is in an elastic state. \( K_{y,i} \) and \( K_{xx} \) are identical in form, except that the matrix \( K_{y,i} \) is replaced by \( K_{yi} \).

\( K_{ex} \), \( K_{ey} \) and \( K_{o,i} \) are respectively the elastic flat-torsional stiffness in \( x \) direction and in \( y \) direction and torsional stiffness of the isolated curved beam bridge, where

\[
K_{ex} = \begin{bmatrix}
 K_{x01} & K_{x02} \\
 K_{y01} & K_{y02}
\end{bmatrix}, \quad K_{ey} = \begin{bmatrix}
 K_{01} & K_{02} \\
 K_{03} & K_{04}
\end{bmatrix}, \quad K_{o,i} = K_{ex}^T, \quad K_{o,y} = K_{ey}^T
\]

where \( K_{o,i} \) represents the moment needed to be applied to the \( i \) layer when the \( i \) layer is stable and only the \( j \) layer has an unit displacement in \( x \) direction; Similarly, \( K_{o,y} \) and \( K_{o,i} \) are completely identical in form. \( K_{x0} \) represents the torque needed to be applied to the \( i \) layer when the \( i \) layer is stable and only the \( j \) layer has an unit displacement in \( y \) direction; \( K_{o,y} \) represents the torque needed to be applied to the \( i \) layer when the \( i \) layer is stable and only the \( j \) layer has an unit rotational angle in \( x \) direction [13].

\[
K_x = \begin{bmatrix}
 K_{sh} & 0 \\
 0 & K_{sh}
\end{bmatrix}
\]

\( K_{sh} \), \( K_{sh} \) are the plastic translational stiffness of the structure in the \( x \) and \( y \) directions respectively; \( K_{sxy} \), \( K_{sxy} \) are the plastic torsional stiffness in the \( x \) and \( y \) directions respectively.

where

\[
K_x = \begin{bmatrix}
 (1 - \alpha_i) K_{x,i} & -(1 - \alpha_i) K_{x,j} \\
 0 & (1 - \alpha_i) K_{x,j}
\end{bmatrix}
\]

\( K_{x,i} \), \( K_{x,j} \) and \( K_{sxy} \) are identical in form, except that the \( K_{x,j} \) in the matrix is replaced by \( K_{yi} \), \(-K_{sxy} e_i \) and \( K_{sxy} e_j \) respectively. Here, \( e_i \) and \( e_j \) respectively represent the distance between the centroid and the stiffness center along the \( x \) and \( y \) directions on the \( i \) layer, which can be expressed as: \( e_i = x_i - x_{ci} \), \( e_j = y_i - y_{cj} \). \( x_i \) and \( y_i \) are the \( x \)-direction and \( y \)-direction coordinates of the \( r \) pier on the \( i \) layer; \( x_{ci} \) and \( y_{cj} \) are the \( x \)-direction and \( y \)-direction coordinates of the centroid of the \( i \) layer.

The damping matrix \( C \) adopts the partitioned Rayleigh damping

\[
C = C_x + C_y, \quad C_x = \alpha_i M + \beta_i K_x
\]

\[
C_y = \begin{bmatrix}
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & C_{b,xx} & 0 & 0 & 0 & C_{b,xy} \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & C_{b,xy} & 0 & C_{b,yi} & 0 & C_{b,yy}
\end{bmatrix}
\]

where \( C_x \) is the classical Rayleigh damping matrix; \( C_y \) is the residual damping matrix of non-proportional damping; the calculation of each element in \( C_y \) can refer to the reference[14]. \( \alpha_i \), \( \beta_i \) are the proportional coefficients of Rayleigh damping of the substructure.
\[
\begin{pmatrix}
\alpha_i \\
\beta_j
\end{pmatrix} = \frac{2\xi_i}{\omega_i + \omega_j} \begin{pmatrix} \omega_i \omega_j \\ 1 \end{pmatrix}
\]

where \( \xi_i \) is the damping ratio of the lower structure, \( \omega_i \) and \( \omega_j \) are the \( i \) order frequency and the \( j \) order frequency of the structure.

Seismic impact matrix \( E = [I_s, I_s, I_s] \), where \( I_s = [1_{2n}, 0_{2n}, 0_{2n}]^T \), \( I_s = [0_{2n}, 1_{2n}, 0_{2n}]^T \), \( I_s = [0_{2n}, 0_{2n}, 1_{2n}]^T \).

Equivalent damping matrix and equivalent stiffness matrix
\[
C_e = \begin{bmatrix}
c_{eq} & 0 \\
-c_{eq} & c_{eq}
\end{bmatrix},
K_e = \begin{bmatrix}
k_{eq} & 0 \\
-k_{eq} & k_{eq}
\end{bmatrix}
\]

Eq. (4) and Eq. (5) are converted into state space expressions as follows:
\[
\dot{Z} = A_sZ + F(t)
\]
where \( Z = \begin{bmatrix} U \\ \dot{U} \\ 0 \end{bmatrix}, A_s = -M^{-1}K - M^{-1}C - \begin{bmatrix} 0 & 0 & 0 \\
-I & 0 & 0 \\
0 & C_e & K_e
\end{bmatrix}, F(t) = \begin{bmatrix} 0 \\
0 \\
-\xi \dot{U}_e
\end{bmatrix}; 0 \) is the zero matrix; \( I \) is the unit matrix.

3. Solution of non-stationary random response

3.1. The construction of virtual excitation

The non-stationary random seismic excitation process \( \dot{U}_e(t) \) can be described as the product of the stationary random process \( r(t) \) and the modulation function \( g(t) \) [15], which is as follows:
\[
\dot{U}_e = g(t)r(t)
\]

The process of pseudo random excitation can be expressed as the product of unit harmonic excitation \( e^{j\omega t} \) and square root of auto-spectral density function of the seismic excitation \( \sqrt{S_{\eta e}(\omega)} \) [16], namely:
\[
\dot{U}_e(t) = g(t)\sqrt{S_{\eta e}(\omega)} e^{j\omega t}
\]
The Eq. (8) is modified by Euler equation
\[
\dot{U}_e(t) = g(t)\sqrt{S_{\eta e}(\omega)} (\cos \omega t + i \sin \omega t)
\]

3.2. The precise integration method

For the general solution of the first order ordinary differential equation(6), which is the sum of the corresponding homogeneous solution \( Z_h \) and the special solution \( Z_p \), namely
\[
Z(t) = Z_h(t) + Z_p(t)
\]
In an integral step \( t \in [t_k, t_{k+1}] \), the homogeneous solution \( V_h(t) \) of the differential equations is
\[
Z_h(t) = T(t)c
\]
where
\[
T(t) = e^{\tau}, \quad \tau = t - t_k
\]
where \( c \) is the integral vector determined by the initial state \( t=t_k \), and \( T(t) \) can be based on 2\(^N\) algorithm to get the solution of computer precision [17].
Assuming that the expression of the special solution \( Z_p(t) \) has been obtained, and it is noted that \( T(0) = I \), then \( c = Z(t_k) - Z_p(t_k) \) can be obtained from Eq. (11) and Eq. (12), thus obtaining
\[ Z(t) = T(t) \left[ Z(t) - Z_p(t) \right] + Z_p(t) \] (13)

Let \( t = t_{i+1} \) to get the state at the end of the integration step
\[ Z(t_{i+1}) = T(t) \left[ Z(t) - Z_p(t) \right] + Z_p(t) \] (14)

For the three-segment uniform modulation function:
\[ g(t) = \begin{cases} 
(t / t_i)^2 & 0 \leq t < t_i \\
1 & t_i \leq t < t_2 \\
e^{-i(t-t_i)} & t \geq t_2 
\end{cases} \] (15)

Replace the expressions of different time periods in Eq. (15) into Eq. (9), by using the special solutions of different simple harmonic external load forms derived in reference [18], the special solution forms under different loads are derived.

(1) when \( g(t) = (t / t_i)^2 \)
\[ F(t) = (r_i + r_i t + r_i t^2) (\alpha \sin \omega t + \beta \cos \omega t) \] (16)
where \( r_i = 0, r_i = 0, r_i = E \sqrt{S_{\omega}} \), \( \alpha = i, \beta = 1 \).

Its particular solution is:
\[ Z_p(t) = (a_1 + a_1 t + a_1 t^2) \sin \omega t + (b_1 + b_1 t + b_1 t^2) \cos \omega t \] (17)
where
\[ a_1 = (\omega^2 I + A_i)^{-1} (-A_i P_{0} + \omega P_{0}) \] (18)
\[ b_1 = (\omega^2 I + A_i)^{-1} (-A_i P_{0} - \omega P_{0}) \]
while
\[ P_a = \alpha r_i, \quad P_a = \alpha r_i, \quad P_{a1} = \beta r_i - 2b_i, \quad P_{a1} = \beta r_i - b_i, i = 2, 1, 0 \] (19)

In this case, the corresponding integral format is
\[ Z_{t_{i+1}} = T(t) \left[ Z(t) - (a_1 + a_1 t + a_1 t^2) \sin \omega t_i \\
- (b_1 + b_1 t + b_1 t^2) \cos \omega t_i \right] + \]
\[ (a_0 t_{i+1} + a_{i+1} t_{i+1}^2) \sin \omega t_{i+1} \\
- (b_0 + b_{i+1} t_{i+1} + b_{i+1} t_{i+1}^2) \cos \omega t_{i+1} \] (20)

(2) when \( g(t) = 1 \)
\[ F(t) = r_i \sin \omega t + r_2 \cos \omega t \] (21)
where \( r_i = E \sqrt{S_{\omega}}, r_i = E \sqrt{S_{\omega}} \).

Its particular solution is:
\[ Z_p(t) = a \sin \omega t + b \cos \omega t \] (22)
where
\[ a = (\omega^2 I + A_i)^{-1} (r_i \omega - A_i r_i) \] (23)
\[ b = (\omega^2 I + A_i)^{-1} (-r_i \omega - A_i r_i) \]

Therefore, the corresponding integral format is
\[ Z_{t_{i+1}} = T(t) \left[ Z(t) - a \sin \omega t_i - b \cos \omega t_i \right] \\
+ a \sin \omega t_{i+1} + b \cos \omega t_{i+1} \] (24)

(3) when \( g(t) = e^{-i(t-t_i)} \)
\[ F(t) = e^{i\omega t}(r_1 \sin \alpha t + r_2 \cos \alpha t) \]  

where \( \alpha = -c, \quad r_1 = -E \sqrt{\sigma} e^{i\xi_1}, \quad r_2 = -E \sqrt{\sigma} e^{i\xi_2} \)

Its particular solution is:

\[ Z_p(t) = e^{i\omega t}(a \sin \alpha t + b \cos \alpha t) \]  

where

\[
a = \left[ (\alpha I - A_1)^2 + \omega^2 I \right]^{-1} \left[ (\alpha I - A_1) r_1 - \omega r_1 \right] \\
b = \left[ (\alpha I - A_1)^2 + \omega^2 I \right]^{-1} \left[ (\alpha I - A_1) r_2 - \omega r_2 \right] 
\]

At the moment, the corresponding integral format is

\[ Z_{e1} = T(t) \left[ Z_k - e^{i\omega t} (a \sin \alpha t_k + b \cos \alpha t_k) \right] + e^{i\omega t} (a \sin \alpha t_1 + b \cos \alpha t_1) \]

The dynamic response of the isolated curved beam bridge under the three-segment uniform modulation function can be obtained by using the recurrence formula of Eq. (20), Eq. (24) and Eq. (28), the power spectrum matrix of the structure is obtained as follows

\[ S_{Z_2}(\omega,t) = Z(\omega,t)^* \cdot Z(\omega,t)^T \]  

The time-varying variance of the structure is

\[ D_z(t) = \int_{-\infty}^{+\infty} S_{Z_2}(\omega,t)d\omega \]  

4. Examples and discussion

4.1. Engineering background

A circular curve continuous beam bridge on the ramp of an interchange has 3 spans, each span is 20m, the radius of curvature is \( R = 50 \)m, and the center angle of circle is \( \alpha = 69^\circ \). The main girder adopts single-box and single-chamber box girder, the deck width is 8m, the piers are cylindrical with a diameter of 1.6m and a pier height of 7m, and the damping ratio of substructure is \( \zeta_s = 0.05 \), a lead rubber bearing with diameter of 500mm is installed on each pier top, and horizontal damping ratio of isolation layer is \( \zeta_b = 0.15 \).

The mass center of the superstructure of the curved bridge is taken as the origin point of the overall coordinate system, and the plane layout is shown in Fig. 1. The corresponding calculation parameters of model of the curved beam bridge are shown in Table 1.

### Table 1. Calculation Parameters of Corresponding Model of Curved Bridge

|          | \( m_1 \) | \( m_2 \) |
|----------|-----------|-----------|
| Mass(kg) | 144681.6  | 839265    |
### 4.2. Stochastic model of multi-dimensional ground motion

The double filtered white noise seismic power spectrum with low frequency filter is more suitable for the random response analysis of the isolated structures [19]. Considering the advantages and disadvantages of several common ground motion acceleration power spectra, the Clough-Pension model is selected as the power spectrum model of horizontal component in random vibration analysis, the auto-spectral density function is as follows:

\[
S(\omega) = \frac{1 + 4\xi_s^2(\omega/\omega_s)^2}{\left[1 - (\omega/\omega_s)^2\right] + 4\xi_s^2(\omega/\omega_s)^2} \times \frac{(\omega/\omega_f)^2}{\left[1 - (\omega/\omega_f)^2\right] + 4\xi_f^2(\omega/\omega_f)^2} S_h
\]

where \( \omega_s \) and \( \xi_s \) are the dominant circular frequency and damping ratio of the site soil respectively; \( S_h \) is the self-spectral density of bedrock acceleration (white noise). The combination of the \( \xi_f \) and \( \omega_f \) parameters can simulate the change of low-frequency energy of ground motion, usually taking \( \xi_f = \xi_s \) and \( \omega_f = 0.1 - 0.2\omega_s \) [20].

In this paper, taking the seismic intensity of 8 degree as an example, the unilateral power spectrum intensity \( S_h = 2.1774 \times 10^{-3} \text{m}^2/\text{s}^5 \) of rare earthquake under PGA=0.4g is calculated, and class II site is selected as the target site, and the design earthquake is divided into the second group, then the random model parameters of ground motion in Eq. (31) are taken as [21]: \( \omega_s = 15.71 \), \( \omega_f = 0.15\omega_s \), \( \xi_f = \xi_s = 0.72 \).

The input power spectral density curve is shown in Fig. 2, and the duration of earthquake motion is taken as \( T=20s \). The modulation function \( g(t) \) takes the form of Eq. (15), and its parameter is taken as \( c=0.35, t_1=0.8s, t_2=7.0s \).

The rotational power spectrum mathematical model proposed by Li Hongnan [22] is selected as the random model of torsional component of the ground motion

\[
S(\omega) = \frac{\omega^2}{\omega^2 + \gamma} \times \frac{1 + 4\xi^{2}_s(\omega/\omega_s)^2}{\left[1 - (\omega/\omega_s)^2\right] + 4\xi_s^2(\omega/\omega_s)^2} \times \frac{1 + 4\xi_f^2(\omega/\omega_f)^2}{\left[1 - (\omega/\omega_f)^2\right] + 4\xi_f^2(\omega/\omega_f)^2} S_h
\]
where $\gamma$ is the low frequency decrement coefficient; $\omega_1$ and $\xi_1$ are the frequency and damping ratio of the soil layer filter; $\omega_2$ and $\xi_2$ can be considered as frequency and damping ratio of the bedrock filter; $S_0$ is the spectrum strength of the bedrock. The parameter values of the model in Eq. (32) are shown in reference [22], and the input power spectral density curve is shown in Fig. 2.

![Input Power Spectral Density](image)

**Figure 2.** Acceleration power spectral density function

### 4.3. Analysis of non-stationary random dynamic response

In order to study the influence of non-stationary random excitation (considering the torsional component) on the dynamic response of the isolated curved beam bridge, with different deck width (B varies from 4m to 24m, taking a value every 4m, and taking 8m for other conditions), curvature radius (R is 30m, 50m, 90m, 130m, 170m and 210m respectively, and R is 50m for other conditions) and yield ratio of bearing ($\alpha$ is 0.02, 0.05, 0.07, 0.1, 0.12 and 0.15 respectively, and $\alpha$ is 0.05 for other conditions). In this paper, the substructure and superstructure are the research objects, the displacement time-varying variances of the upper and lower structures of the isolated curved girdr bridge in $x$ and $y$ directions under rare ground motions are obtained. The results are shown in Fig. 3 ~ Fig. 8.

#### 4.3.1. Influence on the isolated curved beam bridge with different deck width

![Displacement Time-Varying Variance Curves](image)

(a) The time-varying variance of the $x$-direction displacement  
(b) The time-varying variance of the $y$-direction displacement

**Figure 3.** Displacement time-varying variance curve of the substructure
As can be seen from Fig. 3 to Fig. 4, under the rare non-stationary random excitation, the displacement response of the isolated curved bridge with different deck widths presents strong non-stationary and time-delay phenomena, and the non-stationary characteristics and time-delay phenomena of the structural displacement response become more obvious when the bridge deck width \( B \) keeps increasing. When the torsion component of the non-stationary random excitation is taken into account, the peak value of \( x \)-direction displacement variance of the substructure with the deck width gradually increasing from 4m to 24m increases by 0.10cm\(^2\), 0.13cm\(^2\), 0.17cm\(^2\), 0.22cm\(^2\), 0.28cm\(^2\) and 0.4cm\(^2\), respectively, compared with that of without considering the torsion component. The peak value of \( x \)-direction peak displacement variance of the superstructure increases by 1.2cm\(^2\), 2.4cm\(^2\), 2.6cm\(^2\), 2.6cm\(^2\), 5cm\(^2\) and 6cm\(^2\), respectively. Since the isolated curved beam bridge only has eccentricity in \( y \) direction, the torsional component of nonstationary random excitation has no effect on the \( y \)-direction peak displacement of the upper and lower structures of the isolated curved beam bridge (For other factors, the torsion component of non-stationary random excitation has no effect on the time-varying variance of \( y \)-direction displacement of the isolated curved beam bridge, which is also caused by this reason). Based on the above analysis we can see that the torsional component of non-stationary random excitation has great influence on the dynamic response of the isolated curved girder bridge with different deck width, and with the bridge deck width increases from 4m to 24m, the time-varying variance of peak displacement of the isolated curved beam bridge is higher than that of horizontal two-way bridge, and with the increase of bridge deck width \( B \), the increment also gradually increases.

**Figure 4.** Displacement time-varying variance curve of the superstructure

4.3.2. **Influence on the isolated curved beam bridge with different curvature radius**
As can be seen from Fig. 5 to Fig. 6, under the rare non-stationary random excitation, the displacement response of the isolated curved bridge with different curvature radii presents strong non-stationary and time-delay phenomena, and the non-stationary characteristics and time-delay phenomena of the structural displacement response become apparent. When the torsion component of the non-stationary random excitation is taken into account, the peak value of $x$-direction displacement variance of the substructure with the curvature radius gradually increasing from 30m to 210m increases successively by $0.29cm^2$, $0.12cm^2$, $0.05cm^2$, $0.03cm^2$, $0.02cm^2$ and $0.02cm^2$, compared with that without the torsion component. The peak value of $x$-direction peak displacement variance of the superstructure increases by $2.97cm^2$, $2cm^2$, $1.04cm^2$, $0.64cm^2$, $0.42cm^2$ and $0.28cm^2$, respectively. Based on the above analysis we can see that the torsional component of non-stationary random excitation has great influence on the dynamic response of the isolated curved girder bridge with different curvature radius, and when the bridge curvature radius increases from 30m to 210m, the time-varying variance of peak displacement of the isolated curved beam bridge is higher than that of horizontal two-way bridge, and the peak value of displacement variance relatively decreases.

4.3.3. Influence on the isolated curved beam bridge with different bearing yield ratio
(a) The time-varying variance of the x-direction displacement  
(b) The time-varying variance of the y-direction displacement

**Figure 7.** Displacement time-varying variance curve of substructure

(a) The time-varying variance of the x-direction displacement  
(b) The time-varying variance of the y-direction displacement

**Figure 8.** Displacement time-varying variance curve of superstructure

As can be seen from Fig. 7 to Fig. 8, under the rare non-stationary random excitation, the displacement response of the isolated curved bridge with different bearing yield ratios presents strong non-stationary and time-delay phenomena, and the non-stationary characteristics and time-delay phenomena of the structural displacement response become more apparent. When the torsion component of the non-stationary random excitation is taken into account, the peak value of x-direction displacement variance of the substructure with the bearing yield ratios increasing from 0.02 to 0.15 increases successively by 0.07cm², 0.13cm², 0.15cm², 0.16cm², 0.16cm² and 0.16cm², compared with that with only considering the two translational components. The peak value of x-direction peak displacement variance of the superstructure increases by 1.24cm², 2.11cm², 2.47cm², 2.67cm², 2.71cm² and 2.8cm², respectively. It can be seen from the above analysis that the time-varying variance of peak displacement of the isolated curved beam bridge is higher than that of horizontal two-directional components, and with the increase of bridge bearing yield ratio, and the displacement variance gradually increases.

**5. Conclusions**

In order to study the influence of multi-dimensional non-stationary random excitation with torsion component on the isolated curved bridge, the classical Bouc-Wen model is adopted to establish the elastic-plastic model of isolated curved bridge. The influence of non-stationary random excitation with torsion component on seismic response of the isolated curved beam bridge with different deck width, curvature radius and bearing yield ratio is studied by using the precise integration method. By analyzing the variation rule of time-varying displacement variance of the superstructure and substructure of the isolated curved beam bridge, the following conclusions can be drawn:

1. After the pseudo excitation is transformed into a harmonic load by Euler formula, the hybrid fine time-history integral method is derived by combining the precise integration method. With a large time step, the numerical solution of time-varying power spectrum of the structural response with computer precision can be obtained. In addition, the method is easy to understand, easy to program, and can be used to analyze the dynamic response of structure under multi-dimensional non-stationary or stationary conditions.

2. The dynamic response of the curved bridge under multi-dimensional non-stationary random excitation with torsion component is greater than that of horizontal non-stationary random excitation.

3. Under the rare non-stationary random excitation, the displacement response of the isolated curved beam bridge presents strong non-stationarity, and the time-delay phenomenon is more obvious.
(4) The non-stationary random excitation with the torsion component has significant influence on the dynamic response of the curved bridge with larger deck width, smaller radius of curvature and larger bearing yield ratio.

6. References

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