Josephson threshold calorimeter

Claudio Guarcello,1,∗ Alessandro Braggio,1 Paolo Solinas,2 Giovanni Piero Pepe,3,4 and Francesco Giazotto1

1NEST, Istituto Nanoscienze-CNR and Scuola Normale Superiore, Piazza San Silvestro 12, I-56127 Pisa, Italy
2SPIN-CNR, Via Dodecaneso 33, 16146 Genova, Italy
3Dipartimento di Fisica Ettore Pancini, Università degli Studi di Napoli Federico II, Napoli, Italy
4CNR-SPIN, Complesso Monte Sant’Angelo via Cinthia, I-80126 Napoli, Italy

We suggest a single-photon thermal detector based on the abrupt jump of the critical current of a temperature-biased tunnel Josephson junction formed by different superconductors, working in the dissipationless regime. The electrode with the lower critical temperature is used as radiation sensing element, so it is kept thermally floating and is connected to an antenna. The warming up resulting from the absorption of a photon can induce a drastic measurable enhancement of the critical current of the junction. We propose a detection scheme based on a threshold mechanism for single- or multi-photon detection. This Josephson threshold detector has indeed calorimetric capabilities being able to discriminate the energy of the incident photon. So, for the realistic setup that we discuss, our detector can efficiently work as a calorimeter for photons from the mid infrared, through the optical, into the ultraviolet, specifically, for photons with frequencies in the range \([30 − 9 \times 10^4]\) THz. In the whole range of detectable frequencies a resolving power significantly larger than one results. In order to reveal the signal, we suggest the fast measurement of the Josephson kinetic inductance. Indeed, the photon-induced change in the critical current affects the Josephson kinetic inductance of the junction, which can be non-invasively read through a LC tank circuit, inductively coupled to the junction. Finally, this readout scheme shows remarkable multiplexing capabilities.

I. INTRODUCTION

Superconducting electronics is nowadays efficiently employed for developing detectors for (single-) photon and particle calorimetry. These sensors are particularly appealing in view of their high detection efficiency, low energy threshold, and high energy resolution [1]. Actually, the panorama of superconducting radiation detectors offers different approaches, that are quite different from each other and take advantages of peculiar features of the superconducting materials used for the devices. Among these, transition-edge sensors (TESs) [2] are demonstrated high sensitivity for X-ray and γ-ray spectrometry. Instead, superconducting nanowire single photon detectors (SNSPDs) [3] are specifically used for near-infrared single photon detection in quantum communication, according to their peculiar characteristics such as high speed, large detection efficiency, reduced timing jitter, and small dark count rates. Finally, ultrasensitive Microwave Kinetic Inductance Detectors (MKIDs) are expressly recommended for high resolution far-infrared spectroscopy applications [4].

Low-temperature detectors provide a drastic thermal noise suppression and pave the way to quantum-mechanical phenomena at cryogenic temperatures [1]. The suppression of thermal noise allows energy deposited by photons or particles to be detected, in view of the high resolution attainable in typical working conditions. Nevertheless, the geometrical efficiency of superconducting sensors can be a limitation, and thus the progress in this field concentrate on providing wide collecting areas, large absorption efficiency, and high energy acceptance.

Thermal detectors rely on the conversion of a temperature rise in a measurable variation of an electric signal. In particular, “quantum” calorimetry permits to detect single photons and discriminate their energy [5–7]. These detectors are characterized by shorter photon-induced energy deposition and internal equilibration times as compared to the characteristic thermal relaxation time. They require fast and ultra-sensitive thermometry [8–12] and cryogenic operating temperatures. Time-resolved photon detection finds applications in many research fields including quantum communication [13], security [14], and quantum thermodynamics [5, 15–19], and is nowadays recently receiving increasing attention.

Here, we suggest a dissipationless Josephson-based quantum calorimeter working on a thermal detection approach. This device is based on the peculiar sharp temperature dependence of the critical current in a Josephson tunnel junction formed by different superconductors [20], see Fig. 1. To make the junction working as a detector, we need to maintain a temperature gradient along the device. This means that the electrodes have to reside at specific temperatures. By properly choosing the working temperatures of the junction, small photon-induced temperature increases could be easily revealed. Hereafter, we discuss a proposal based on a threshold mechanism, which gives us the possibility to develop a single photon detector with calorimetric capabilities. This Josephson threshold calorimeter (JTC) may be a new element in the panorama of recently designed superconducting devices that effectively take advantage of a thermal gradient imposed across the system. Specifically, we are dealing with solid-state applications in the realm of phase-coherent caloritronics [1, 21, 22], a research field that promises new ways to coherently master, store, and
transport heat at the meso and nanoscopic scale. In fact, different kinds of temperature-based devices, such as heat interferometers [23], diffractors [21 24], diodes [25], transistors [27], memories [28 30], logic elements [31], switches [32], routers [33 34], and circulators [35] were recently conceived.

The paper is organized as follows. In Sec. II, we introduce the working principles of a radiation detector based on a temperature-biased asymmetric Josephson junction, looking also the influence of the thermodynamics fluctuations. In Sec. III we focus on the calorimeter detection modes of our device, discussing the idle state of the detector, proper figure of merits, the time evolution of both the temperature and the critical current, and finally a readout scheme based on the measure of the Josephson kinetic inductance, including a proposal of a multiplexing scheme. In Sec. IV the conclusions are drawn.

II. DETECTION OPERATING PRINCIPLES

In this section we discuss the operating principles of a radiation detector based on the discontinuous thermal response of the critical current, $I_c$, of an asymmetric tunnel Josephson junction (JJ) [20]. This peculiar behavior of $I_c$ derives from the matching of the superconducting gaps at specific temperatures. Essentially, we briefly recollect the main mechanisms discussed in Ref. [20], developing a careful investigation of the sensor design in order to achieve the best detection performances. The Josephson junction is asymmetric in the sense that the two superconducting leads $S_1$ and $S_2$, with energy gaps $\Delta_1$ and $\Delta_2$, and residing at temperatures $T_1$ and $T_2$, are made up of different BCS superconductors. We can define the asymmetry parameter

$$r = \frac{T_{c_1}}{T_{c_2}} = \frac{\Delta_{10}}{\Delta_{20}},$$

(1)

where $T_{c_1}$ is the critical temperature and $\Delta_{j0} = 1.764k_BT_{c_1}$ is the zero-temperature superconducting BCS gap [36] of the $j$-th superconductor (with $k_B$ being the Boltzmann constant). A film with the desirable critical temperature can be obtained by using a proximity-coupled bilayer, as it is usually done in TESs [2]. In a thermally biased Josephson tunnel junction the critical current reads [20 37 39]

$$I_c(T_1, T_2) = \frac{1}{2eR} \int_{-\infty}^{\infty} \left\{ F(\varepsilon, T_1) \text{Re} \left[ \tilde{F}_1(\varepsilon, T_1) \right] \text{Im} \left[ \tilde{F}_2(\varepsilon, T_2) \right] + F(\varepsilon, T_2) \text{Re} \left[ \tilde{F}_2(\varepsilon, T_2) \right] \text{Im} \left[ \tilde{F}_1(\varepsilon, T_1) \right] \right\} d\varepsilon. \quad (2)$$

Here, $R$ is the normal-state resistance of the junction, $e$ is the electron charge, $F(\varepsilon, T_j) = \tanh(\varepsilon/2k_BT_j)$, and

$$\tilde{F}_j(\varepsilon, T_j) = \frac{\Delta_j(T_j)}{\sqrt{(\varepsilon + i\Gamma_j)^2 - \Delta_j^2(T_j)}} \quad (3)$$

is the anomalous Green’s function of the $j$-th superconductor [40], with $\Gamma_j = \gamma_j\Delta_{j0}$ being the Dynes parameter [41]. In this work, we set $\gamma_j = 10^{-5}$, a value often used to describe realistic superconducting tunnel junctions [41 42].

Fig. 1 shows a possible experimental realization of the detector. The device includes also the thermal contact through a tunnel junction with a normal metal lead, $N$, having a large heat capacity and residing at the bath temperature. As we will discuss later, this “cooling finger” can play a predominant role in the thermal balance of the floating $S_2$ lead. It allows to control the working temperatures of the device, to extend the range of detectable photon frequencies, and to master the thermal response time of the device. Moreover, the resistance of this cooling finger, namely, the resistance $R_{S_2N}$ of the junction between $S_2$ and $N$, determines both the position and the steepness of the abrupt variation of the critical current. Finally, for low $R_{S_2N}$’s, the cooling finger becomes the predominant thermal relaxation channel in the system and the thermalization process in $S_2$ becomes faster.

In Fig. 1 we also indicate how to control the phase difference $\varphi$ across the thermally-biased Josephson junction, which is enclosed, through clean contacts, within a superconducting ring pierced by a magnetic flux $\Phi$. In

![FIG. 1. Schematic illustration of a Josephson threshold calorimeter, namely, a temperature-biased Josephson tunnel junction formed by the superconducting leads $S_1$ and $S_2$, with critical temperatures $T_{c_1} \neq T_{c_2}$, and residing at temperatures $T_1$ and $T_2$. The junction is enclosed in a superconducting ring pierced by a magnetic flux $\Phi$ which allows phase biasing of the weak link. The ring is supposed to be made by a third superconductor $S_3$ with energy gap $\Delta_3 \gg \Delta_1, \Delta_2$, so to suppress the heat losses. The main thermal pathways in the JJ are also depicted. In detail, the phase-dependent thermal current from $S_1$ to $S_2$, $P_{\phi,12}(T_1, T_2, \varphi)$, the outgoing thermal currents from $S_2$ to the phonon bath and the cooling finger, $P_{\phi,2}(T_2, T_{\text{bath}})$ and $P_{\phi,2N}(T_1, T_2, \varphi)$, respectively, and the photon-induced power diffusion in the absorber, $P_{\text{rad}}$, are also represented, for $T_1 > T_2 > T_{\text{bath}}$.](image-url)
this way, the phase-biasing can be achieved via the external flux, since, by neglecting the ring inductance, the phase-flux relation is given by \( \varphi = 2\pi \Phi / \Phi_0 \). Here, \( \Phi_0 = h/2e \approx 2 \times 10^{-15} \text{ Wb} \) is the magnetic flux quantum, with \( h \) being the Planck constant. Therefore, the phase drop across the junction can be varied within the whole phase space, i.e., \(-\pi \leq \varphi \leq \pi\). The ring is supposed to be made by a third superconductor \( S_3 \) with energy gap \( \Delta_3 \gg \Delta_1, \Delta_2 \), which is well thermalized with the phonon bath. Under these conditions we neglect the heat transfer between \( S_1, S_2 \) and \( S_3 \) due to Andreev reflection heat mirroring effect [43]. Thus, only the temperature gradient between \( S_1 \) and \( S_2 \) is important for the junction.

The JTC that we are conceiving is based on the critical current steeper behavior, which stems from the matching of the superconducting gaps. This phenomenon is illustrated in the next section.

### A. The superconducting gap constraints

With the aim to understand the conditions at which \( I_c \) abruptly jumps, we show in Fig. 2 the temperature dependence of the superconducting gaps, assuming \( r > 1 \), namely, \( \Delta_{10} > \Delta_{20} \). The peculiar step-like behavior of the critical current, which is clearly shown in the inset of this figure where we show \( I_c(T_2) \) at a fixed \( T_1 \), stems from the alignment of the singularities in the Green’s functions \( \delta_j \) at \( \varepsilon = \Delta_j \), see Eq. (3), when the superconducting gaps coincide [20] [40]

\[
\Delta_1(T_1) = \Delta_2(T_2).
\]  

We observe that for \( r > 1 \) the superconducting gaps can be equal if and only if the temperature \( T_1 \) assumes a value higher than a threshold \( T_1^{\text{th}} \), namely, for \( T_1 \)'s within the shaded region in Fig. 2. The value of \( T_1^{\text{th}} \) depends on \( r \) and can be calculated as the temperature at which \( \Delta_1(T_1^{\text{th}}) = \Delta_{20} \). Then, once the temperature \( T_1 \in [T_1^{\text{th}}, T_{c1}] \) is settled, the critical current jumps at a specific \( T_2 = T_2^j \), see the dashed line in the inset of Fig. 2 at which \( \Delta_2(T_2^j) = \Delta_1(T_1) \). Therefore, \( T_2^j \) depends on the operating value of \( T_1 \), but it can in principle assume every value in the whole range \([0 - T_{c2}]\).

Following the previous discussions, it is worthwhile to choose as sensing element the electrode with the lower \( T_c \). In fact, this choice makes it larger the range of temperatures available as a result of a photonic energy absorption, so to better protect the system against thermal fluctuations (see later). Then, since we label the absorbing lead with \( S_2 \), this electrode has the lower critical temperature, that is \( T_{c2} < T_{c1} \) and, consequently, \( r > 1 \). In Ref. [20] it was observed that the case with \( r > 1 \) corresponds to a sudden increase of \( I_c \) at \( T_2 = T_2^j \). Conversely, for \( r < 1 \) the critical current suddenly reduces at \( T_2 = T_2^j \). Thus, our choice for the sensing element is supported by the fact that a detection scheme in which a photonic event causes \( I_c \) to increase provides a stronger Josephson coupling. Furthermore, an abrupt increase of the critical current will certainly result from an energy absorption in the floating lead, since a photon eventually absorbed in \( S_1 \) would not determine any steeper critical current variation. This configuration is also more robust against thermal fluctuations, in comparison to the case in which \( I_c \) decreases.

We also note that by reducing the asymmetry of the junction, i.e., \( r \to 1 \), the threshold temperature \( T_1^{\text{th}} \) decreases. This could be beneficial in order to choose an operating point for \( T_1 \) not too close to the critical temperature. On the other side, the height of \( I_c \) jump, and therefore the sensor sensitivity, is expected to decrease when \( r \to 1 \), up to vanish for \( r = 1 \). Conversely, by increasing the asymmetry between the gaps, namely, for \( r \gg 1 \), we are suppressing one superconducting gap with respect to the other. In these cases, \( T_1^{\text{th}} \to T_{c1} \). This implies that the range of \( T_1 \) values suitable for the detection is reduced and the requirement on the stability of the temperature of \( S_1 \) becomes critical. Furthermore, since in this case \( I_c \to 0 \), we expect that the height of the \( I_c \) jumps will tend to diminish.

Therefore, one has to find an optimal value of the asymmetry parameter \( r \gtrsim 1 \) which maximizes the sensitivity without requiring an operating temperature \( T_1 \) too close to \( T_{c1} \). In Ref. [20] we found that the maximum of the critical current jump is observed at \( r \sim 3 \). Hereafter, we set a slightly smaller value of the asymmetry parameter, specifically, we choose \( r = 1,5 \), in order not to restrict the operating value of \( T_1 \) within a range too close to \( T_{c1} \). Then, assuming for the electrode \( S_1 \) the critical temperature \( T_{c1} = 1.2 \text{ K} \), i.e., a lead made by Al, the critical temperature of \( S_2 \) has to be \( T_{c2} = 0.8 \text{ K} \). This choice fixes the threshold value of \( T_1 \) to the value \( T_1^{\text{th}} \approx 0.992 \text{ K} \), so to maintain the temperature of \( S_1 \) in the range of \(~83\% \) of the critical temperature.

Our aim is to induce through the absorbed radiation heating of the electrode \( S_2 \), and a following sudden in-
crease of $I_c$. In our detection scheme we assume that the electrode $S_1$ has a large enough volume and is well connected to heating probes, so that it resides at a stable temperature. So, its temperature is weakly affected by the changes of $T_2$. In this condition, we can neglect thermal fluctuations of $S_1$, treating it as a thermal reservoir.

To allow a photon to be detected, in the idle state, namely, in the absence of incident radiation, the absorbing element has to reside at a temperature, $T_2^0$, which is just below the temperature $T_2^1$, at which an $I_c$ jump occurs. Consequently, due to a photonic event the temperature $T_2$ can increase enough to exceed the threshold value, inducing an abrupt increment of the critical current. However, if $T_2^1$ is not close enough to $T_2^0$, lower energy photons may not trigger an appreciable $I_c$ enhancement. Conversely, if $T_2^0$ is too close to $T_2^1$, thermal fluctuations could induce undesired critical current jumps. Therefore, we need first of all to estimate the amplitude of the temperature fluctuations in $S_2$, in order to set a suitable detection threshold temperature, $\Delta T_2$, namely, the distance between $T_2^0$ and $T_2^1$. Choosing an optimal detection threshold $\Delta T_2$ is essential for the proper functioning of the JTC and for minimizing the dark counts rate, i.e., the probability of false positive detections.

**B. The detection threshold temperature**

Thermodynamic fluctuations can be estimated through the root-mean-square fluctuations in energy

\[
\delta E_j(T_j) = \sqrt{C_j(T_j)k_B T_j^2}.
\]

Here, $C_j$ is the heat capacity of the $j$-th superconductor reading

\[
C_j(T) = T \frac{\partial S_j}{\partial T}.
\]

In this equation, $S_j(T)$ is the electronic entropy of $S_j$, which is given by

\[
S_j(T) = -4k_B N_F j V_j \int_{-\infty}^{\infty} f(\varepsilon, T) \ln[f(\varepsilon, T)] N_j(\varepsilon, T) d\varepsilon,
\]

where $N_{F,j}$ is the density of states at the Fermi energy and $V_j$ is the volume of $S_j$. Then, we can evaluate the temperature fluctuation as

\[
\delta T_j = \frac{\delta E_j(T_j)}{C_j(T_j)} = \sqrt{\frac{k_B T_j^2}{C_j(T_j)}}.
\]

The behavior of thermal fluctuations, $\delta T_j$, in the absorbing lead $S_2$, setting $V_2 = 1 \mu m^3$, as a function of $T_2$ is shown in Fig. 3. According to the exponential suppression of the heat capacity in a superconductor at a low temperature, we observe that $\delta T_2$ diverges for $T_2 \to 0$, and it monotonically decreases by increasing $T_2$ approaching the value $\delta T_2 \simeq 0.2$ mK at $T_2 = T_{c_2}$.

With the aim to avoid unwanted transitions and to minimize the dark counts one need to set an idle temperature $T_2^0$ which is distant more than $\delta T_2$ from the

![FIG. 3. Temperature fluctuation in $S_2$, see Eq. (7), as a function of $T_2$. In the inset: Critical current, $I_c(T_1, T_2)$, as a function of $T_2$ near the jump, at $T_1 = 994.8$ mK. The values of other parameters are: $T_{c_1} = 1.2$ K, $T_{c_2} = 0.8$ K, $R = 1$ kΩ, and $V_2 = 1 \mu m^3$.](image)

**C. The heat exchanges**

In Fig. 1 the thermal pathways in our device for $T_1 > T_2 > T_{bath}$ are depicted. Once a thermal gradient along the system is imposed, a phase-dependent heat flux, $P_{S_1IS_2}(T_1, T_2, \varphi)$, flows through the junction from $S_1$ to $S_2$. According to the exponential suppression of the heat capacity in a superconductor at a low temperature, we observe that $\delta T_2$ diverges for $T_2 \to 0$, and it monotonically decreases by increasing $T_2$ approaching the value $\delta T_2 \simeq 0.2$ mK at $T_2 = T_{c_2}$.

The stationary phase-dependent total thermal power flowing from $S_1$ to $S_2$, see Fig. 1 reads

\[
P_{S_1IS_2}(T_1, T_2, \varphi) = P_{qp}(T_1, T_2) - \cos \varphi P_{\cos}(T_1, T_2).
\]

In the adiabatic regime, the terms in this
and reads
\[ P_{\text{qp}}(T_1, T_2) = \frac{1}{e^2 R} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d\varepsilon \frac{z N_1(\varepsilon, T_1) N_2(\varepsilon, T_2)}{f(\varepsilon, T_1) - f(\varepsilon, T_2)}, \] (9)

\[ P_{\text{cos}}(T_1, T_2) = \frac{1}{e^2 R} \int_{-\infty}^{\infty} \frac{d\varepsilon N_1(\varepsilon, T_1) N_2(\varepsilon, T_2)}{\delta(T_i) \delta(T_j)} [f(\varepsilon, T_1) - f(\varepsilon, T_2)], \] (10)

where \( f(\varepsilon, T_j) = 1/\left(1 + e^{\varepsilon/k_B T_j}\right) \) is the Fermi distribution function, and \( N_j(\varepsilon, T_j) = \left| \text{Re} \left( \frac{e^{i\pi/2}}{\sqrt{(\varepsilon + i\Gamma_j)(\varepsilon + i\Gamma_j)}} \right) \right| \) is the smeared BCS density of states of the \( j \)-th superconducting lead.

Eq. (8) derives from processes involving both Cooper pairs and quasiparticles in tunnelling through a JJ as predicted by Maki and Griffin. \( P_{\text{qp}} \) is the heat current carried by quasiparticles, namely, an incoherent power flow through the junction from the hot to the cold electrode. \( P_{\text{cos}} \) determines the phase-dependent part of the heat transport originating from the energy-carrying tunnelling processes involving recombination/destruction of Cooper pairs on both sides of the junction.

The thermal current flowing through the S2IN junction, see Fig. 1 is
\[ P_{\text{S2IN}}(T_2, T_{\text{bath}}) = \frac{1}{e^2 R_{\text{S2IN}}} \int_{-\infty}^{\infty} \frac{d\varepsilon \varepsilon N_2(\varepsilon, T_2)}{f(\varepsilon, T_2) - f(\varepsilon, T_{\text{bath}})}. \] (11)

For low \( R_{\text{S2IN}} \), the cooling finger can become the predominant thermal relaxation channel. In the opposite case, it competes with the heat exchanged between electrons and phonons, the latter being thermalized at \( T_{\text{bath}} \). Indeed, thanks to the vanishing Kapitza resistance between thin metallic films and the substrate at low temperatures, we can assume that the lattice phonons in the electrodes are very well thermalized with the substrate, residing at the temperature \( T_{\text{bath}} \).

The term \( P_{\text{e-ph,2}} \) in Eqs. (14) and (21) represents the energy exchange between electrons and phonons in \( S_2 \) and reads
\[ P_{\text{e-ph,2}} = -\frac{\Sigma_2 V_2}{96\zeta(5)k_B^3} \int_{-\infty}^{\infty} dE \int_{-\infty}^{\infty} d\varepsilon \varepsilon^2 \text{sign}(\varepsilon) M^2_{E,E+\varepsilon} \times \left\{ \coth \left( \frac{\varepsilon}{2k_B T_{\text{bath}}} \right) \left[ F(E, T_2) - F(E + \varepsilon, T_2) \right] \right\}. \] (12)

In the heat exchange analysis we neglect any contribution from the photonic radiative channel. Indeed, the superconductors \( S_1 \) and \( S_2 \) are electrically connected thoroughly the JJ and the \( S_3 \) superconducting ring. In this configuration one could speculate if a pure radiative contribution has to be considered too. Anyway, this contribution can be neglected for two main reasons. Firstly, this contribution, as evaluated in Ref. [58], is several orders of magnitude lower than the quasiparticle galvanic contribution. Then, the radiative term to be effective requires an efficient impedance matching, as a result of the circuital configuration [58]. In our setup, the impedance matching is not satisfied, since both the JJ and the superconducting ring have quite different lumped element schematization leading to a strong impedance mismatch for photonic transport.

### III. CALORIMETER

Depending on the characteristic timescales of the process, the JJ can operate as a bolometer or as a calorimeter. Specifically, the detector works as a bolometer if the mean time between the arrival of incident photons is much shorter than the characteristic thermal relaxation time of the device. In the opposite regime, as the photonic arrival time exceeds the thermal relaxation time, the detector operates as a calorimeter. Therefore, a bolometer measures the total amount of radiation incident on an active area, whereas a calorimeter measures the energy of each single-photon absorption event.

Hereafter we propose a detection scheme based on a threshold mechanism for single- or multi–photon detection that takes advantages of the discussed very sharp variation of the critical current. During the design stage of this detector we realized the possibility of developing a different type of detection scheme, suitable for bolometric sensing, which is still based on the same physical mechanism. It will be thus considered elsewhere.

#### A. The operating temperatures

In order to evaluate the thermal evolution following the absorption of a photon in \( S_2 \), we need firstly to select in the absence of photonic excitation the stationary operating temperatures, \( T_{0}^{1} \) and \( T_{0}^{2} \), suitable for our detection scheme. So, once we have chosen the value of the detection threshold \( \Delta T_2 \), we need to identify the stationary operating temperatures \( T_{0}^{1} \) and \( T_{0}^{2} \) yielding an idle state of the detector close enough to a jump in the critical current, namely, the temperatures at which
\[ \Delta_{1}(T_{0}^{1}) = \Delta_{2}(T_{0}^{2} + \Delta T_2). \] (13)

Here, we are labelling these temperatures with \( T_{0}^{1} \) and \( T_{0}^{2} \), since in next sections we will use them as the initial values for the time-dependent analysis. Now, in order to tune the detector at a specific operating point \( (T_{0}^{1}, T_{0}^{2}) \),
the temperature $T_2^0$ of the floating lead has to be obtained by numerically solving a steady-state heat balance equation for $S_2$,

$$P_{\text{S}_1\text{S}_2}(T_1^0, T_2^0, \varphi) - P_{\text{e-ph},2}(T_2^0, T_{\text{bath}}) - P_{\text{S}_2\text{IN}}(T_2^0, T_{\text{bath}}) = 0,$$  

(14)

in the presence of the gap constraint Eq. (13) and for fixed values of $T_{\text{bath}}$ and $\varphi$. The key quantities for setting properly the idle temperatures of the detector are the volume $V_2$ and the resistances $R$ and $R_{\text{S}_2\text{IN}}$ of the junctions. The steady temperature-bias established between the electrodes strongly depends on these quantities, since the higher $R$ and $R_{\text{S}_2\text{IN}}$, the narrower the thermal channels towards and from $S_2$, since $P_{\text{S}_1\text{S}_2} \propto R^{-1}$ and $P_{\text{S}_2\text{IN}} \propto R_{\text{S}_2\text{IN}}^{-1}$. Whereas the larger $V_2$, the better the thermal coupling of the absorber with the phonon bath, since $P_{\text{e-ph},2} \propto V_2$.

The selection of a specific device configuration (namely, the values of $R$, $R_{\text{S}_2\text{IN}}$, and $V_2$) is a crucial point to best set the operation mode, i.e., bolometer or calorimeter [59–61], of the detector. So, in order to choose the values of these quantities one needs to find a compromise. We can assume to use the resistance of the cooling finger as a knob to set the operating point of the device. This means, at this stage, to properly choose the values of $R$ and $V_2$ and to study how $R_{\text{S}_2\text{IN}}$ affects the operations of the detector. In fact, in order to reduce the operative value of $T_2^0$ one can make the thermal contact with the normal lead more transparent, by lowering the resistance $R_{\text{S}_2\text{IN}}$. Reducing the temperature $T_2^0$ can be advantageous since it leads both to the enhancement of the amplitude of the $I_c$ jumps, as it is discussed in Ref. [20], and to a larger range of detectable photonic frequencies.

We observe that the higher the Josephson junction normal-state resistance $R$, the lower the maximum value of the critical current, see Eq. (3). To choose the value of $R$, we require that the Josephson energy $E_J = \frac{\Phi_0}{2\pi} I_c$ is much larger than the thermal energy $k_B T_c$. For instance, by assuming the value $R = 1 \Omega$, we obtain a maximum critical current of $\sim 0.2 \, \mu A$, corresponding to a Josephson energy in units of $k_B$ of $\sim 5.5 \, K$, that is a value well above the working temperatures that we will discuss later.

Then, one can assume to keep fixed the resistance $R = 1 \, \Omega$ and the volume $V_2 = 1 \, \mu m^3$, and to find for any values of $R_{\text{S}_2\text{IN}}$ the idle temperatures $T_1^0$ and $T_2^0$. This is exactly displayed in Fig. 4(a), where we show the behavior of $T_1^0$ and $T_2^0$ as a function of $R_{\text{S}_2\text{IN}}$ at a fixed $R = 1 \, \Omega$ and $V_2 = 1 \, \mu m^3$, for $T_{\text{bath}} = 0.01 \, K$ and $\varphi = 0$.

The thermal bath is kept to a low temperature, reducing the strength of the phononic thermalization channel and to enhance at the same time the effectiveness of the metallic cooling finger. The need for an accurate phase regulation, which is performed through the external flux $\Phi$, is significantly relaxed, since we observe that at low values of $R_{\text{S}_2\text{IN}}$ the idle temperatures are only slightly affected by modifications in $\varphi$. Instead, the value we choose for $\varphi$ will be more important later, when we will discuss about how to reveal the detector state via a kinetic inductance readout.

At low $R_{\text{S}_2\text{IN}}$’s, $T_1^0$ and $T_2^0$ tend to the threshold value $T_1^{th}$ and the bath temperature, respectively, see Fig. 4(a). By increasing the resistance of the $S_2$ junction the idle temperatures monotonically rise. In the high resistances limit, i.e., $R_{\text{S}_2\text{IN}} \gtrsim 1 \, \Omega$, both temperatures reach a plateau, since this relaxation channel is too weak and does not affect the idle state of the device.

Therefore, each value of $R_{\text{S}_2\text{IN}}$ corresponds to specific operating temperatures $(T_1^0, T_2^0)$. This means that a given $R_{\text{S}_2\text{IN}}$ results in a specific critical current profile, $I_c(T_1^0, T_2^0)$, as it is shown in Fig. 4(b). In this figure we present the behavior of $I_c$ as a function of $T_2$. These

FIG. 4. (a) Idle temperatures, $T_1^0$ and $T_2^0$, as a function of the resistance $R_{\text{S}_2\text{IN}}$. (b) Critical current $I_c(T_1, T_2)$ as a function of the temperature $T_2$. The curves are obtained by imposing $T_1$ equal to the idle temperatures $T_1^0$ calculated in panel (a) changing $R_{\text{S}_2\text{IN}}$. The values of the other parameters are: $T_{c_1} = 1.2 \, K$, $T_{c_2} = 0.8 \, K$, $R = 1 \, \Omega$, $T_{\text{bath}} = 10 \, mK$, $\varphi = 0$, $\Sigma_2 = 0.3 \times 10^9 \, Wm^{-3}K^{-5}$, and $N_{F_2} = 10^{17} \, J^{-1}m^{-3}$. 

\begin{align*}
T_{c_1} &= 1.2 \, K, \\
T_{c_2} &= 0.8 \, K, \\
R &= 1 \, \Omega, \\
T_{\text{bath}} &= 10 \, mK, \\
\varphi &= 0, \\
\Sigma_2 &= 0.3 \times 10^9 \, Wm^{-3}K^{-5}, \\
N_{F_2} &= 10^{17} \, J^{-1}m^{-3}.
\end{align*}
curves are obtained by imposing the temperatures $T_1$ coinciding with the idle temperatures $T_{01}$ shown in Fig. 3(a) by changing $R_{S2,IN}$. By increasing the resistance, since $T_{01}$ increases, we observe that the $I_c$ jump shifts towards high $T_2$ values, becoming lower and steeper. Conversely, for low $R_{S2,IN}$’s the $I_c$ jump tends to become smoother. In this regard, we note that the sharpness of the jump depends usually on $\Gamma_j^{-1}$, as it is discussed in Ref. [20]. Anyway, here we are assuming to keep constant the value of the Dynes parameter, so that the smoothing of the $I_c$ jump in Fig. 3(b) can be ascribed instead to the peculiar temperature dependence of the gap. In fact, we notice that the jump in the $I_c$ profile tends to become smoother when the temperature $T_2^0$ is in the range of $T_2$ values in which the superconducting gap is roughly flat, see Fig. 2. In such a case, the variation of $T_2$ reflects on the gap behavior, and the $I_c$ variation at the jump is smoother.

In summary, the selection of $R_{S2,IN}$ makes it possible to engineer both the position and the sharpness of the jump in the critical current profile. A small value of $R_{S2,IN}$ ensures a low working temperature, and therefore a larger range of temperature attainable as a result of a photonic absorption, but it corresponds to a smooth $I_c$ variation at the jump. Conversely, for the threshold calorimetric mechanism that we are suggesting, it is more convenient to have a sharp $I_c$ transition. Nonetheless, by increasing $R_{S2,IN}$ the height of the $I_c$ jump tends to reduces. Then, we settle the value $R_{S2,IN} = 10 \, \Omega$ which guarantees a steep and sufficiently high $I_c$ jump. This resistance gives an idle temperature $T_{20}^0 = 0.258$ K.

Finally, we verify that the temperatures $T_{10}$ and $T_{20}$, obtained by solving Eqs. (13) and (14), at a given $R_{S2,IN}$ as shown in Fig. 3(a), give a critical current value just below a jump. In the inset of Fig. 3 we show the critical current profile as a function of the temperature $T_2$, of a JJ with $T_{c1} = 1.2$ K, $T_{c2} = 0.8$ K, and $R = 1$ kΩ, at a fixed temperature $T_1 = 994.8$ mK. This temperature corresponds to the $T_{10}$ value obtained for $R_{S2,IN} = 10 \, \Omega$, see Fig. 4(a). The dot-dashed line indicates the temperature $T_{20}^0 = 0.258$ K, and it allows to clearly highlight the closeness of the idle temperature to the temperature $T_{20}^0$ (see the dashed line) at which $I_c$ jumps.

Now, it is convenient to estimate through simple arguments, namely, without addressing yet the full time evolution of the temperature $T_2$, the dynamical ranges of the calorimeter, that is the photon-induced temperature rise and the range of detectable photonic frequencies.

B. Dynamical range of the calorimeter

By assuming in a first approximation the full conversion of the photon energy to internal energy of electrons, we can estimate the temperature $T_{peak}$ reached by $S_2$, as a result of the fast absorption of a photon with frequency $\nu$ when the electrode resides at the idle temperature $T_{20}^0$. The previous assumption corresponds to neglect any energy loss during the timing jitter of the detector. Then, $T_{peak}$ can be evaluated by equating the integrated internal energy of $S_2$ with the photon energy, that is

$$ \int_{T_2^0}^{T_{peak}} C_2(T) dT = h \nu. $$

(15)

In the low temperature limit, $T \ll T_c$, we can give an analytic expression of $T_{peak}$. In fact, in this temperature regime the entropy of a BCS superconductor behaves exponentially with the temperature as [51, 52]

$$ S(T) \simeq k_B N_F \Delta_0 \sqrt{2\pi} \left( \frac{\Delta_0}{k_B T} \right)^{3/2} e^{-\frac{\Delta_0}{k_B T}}, $$

(16)

from which the heat capacity at the leading order reads

$$ C(T) \simeq k_B N_F \Delta_0 \sqrt{2\pi} \left( \frac{\Delta_0}{k_B T} \right)^{3/2} e^{-\frac{\Delta_0}{k_B T}}. $$

(17)

By inserting Eq. (17) in Eq. (15), and using the following integral identity

$$ \int_0^a dx e^{-\frac{x^2}{2}} = \sqrt{\pi} \left[ 1 - \text{erf} \left( \frac{1}{\sqrt{a}} \right) \right], $$

where erf(x) is the error function, one finds

$$ h \nu \simeq \varepsilon V_2 \left[ \text{erf} \left( \sqrt{\frac{\Delta_{20}}{k_B T_{20}^0}} \right) - \text{erf} \left( \sqrt{\frac{\Delta_{20}}{k_B T_{peak}}} \right) \right], $$

(18)

with $\varepsilon = \sqrt{2\pi} N_F \Delta_{20}^2$. In the low temperature regime, i.e., $T_{20}^0 \ll T_{c2}$, where $\text{erf} \left( \sqrt{\frac{\Delta_{20}}{k_B T_{20}^0}} \right) \rightarrow 1$, we can determine from Eq. (18) the analytical dependence of the peak temperature on both the volume $V_2$ and the photon frequency $\nu$, that reads

$$ T_{peak} \simeq \frac{\Delta_{20}}{k_B \text{erf}^{-1} \left( 1 - \frac{h \nu}{\varepsilon V_2} \right)^2}. $$

(19)
FIG. 6. Maximum temperature, $T_{\text{peak}}$, (left axis) and $T_2$ rise, $\Delta T_2 = T_{\text{peak}} - T_2^0$, (right axis) as a function of the photon frequency, $\nu$, at the base temperature $T_2^0 = 0.258$ K.

Fig. 5 shows a comparison between the peak temperatures calculated analytically in the low temperature limit through Eq. (19) (dashed lines) and computed numerically through Eq. (15) (solid lines) taking the full temperature dependence of the entropy, see Eq (6), and assuming $T_2 = 0$. We observe that the analytic estimate of $T_{\text{peak}}$ is systematically higher than the numerical one. This is because at high temperatures Eq. (17) underestimates the full heat capacity. Nevertheless, the numerical calculation converges to the analytic result in the low frequency case, namely, the regime of low temperatures where Eq. (16) holds.

In Fig. 6 we plot the behavior of $T_{\text{peak}}$ as a function of $\nu$, calculated through Eq. (15) assuming a base temperature equal to $T_2^0 = 0.258$ K, namely, the working temperature obtained by choosing $R_{\text{S2IN}} = 10$ Ω. On the right vertical axis of this panel the photon-induced temperature rise, $\Delta T_{\text{peak}} = T_{\text{peak}} - T_2^0$, is shown. We observe that the proposed setup could effectively detect photons with frequencies up to $\nu_{\text{max}} \simeq 9 \times 10^4$ THz, since for frequencies higher than this threshold value the energy supplied by the photon is enough to bring the superconducting electrode in the normal state. Yet, to make the photon-induced heating $\Delta T_{\text{peak}}$ at least higher than the detection threshold temperature $\Delta T_2 = 2$ mK, it is necessary a photon with frequency above the threshold value $\nu_{\text{min}} \simeq 30$ THz.

We have shown that Eq. (15) allows also to directly estimate the detection frequency range of the device. We can derive both the minimum detectable photon energy, that is the energy giving a temperature rise just higher than the detection threshold $\Delta T_2$, and the maximum photon energy, that is the energy high enough to induce a superconducting to normal phase transition. Accordingly, we can calculate the threshold frequencies $\nu_{\text{max}}$ and $\nu_{\text{min}}$ by replacing in Eq. (15) the upper integration limit with $T_{c2}$ and $T_2^0 + \Delta T_2$, respectively.

Since the idle temperature $T_2^0$ depends on $R_{\text{S2IN}}$ [see Fig. 4(a)], the detection frequency range of the device changes by modifying the value of $R_{\text{S2IN}}$. Fig. 7 shows the behavior of the threshold frequencies $\nu_{\text{max}}$ and $\nu_{\text{min}}$ as a function of $R_{\text{S2IN}}$. Shaded regions indicate forbidden frequency values, since larger than $\nu_{\text{max}}$ or smaller than $\nu_{\text{min}}$. We observe that the range of permitted frequencies reduces by increasing $R_{\text{S2IN}}$. So, the choice of $R_{\text{S2IN}}$ significantly affects not only the working temperature, but also the detection frequency range of the sensor. Finally, we observe that at a photon can be absorbed if its energy is $h\nu \gtrsim 2\Delta_2(T_2)$. For $T_{c2} = 0.8$ K, we obtain $\nu \gtrsim 50$ GHz at $T_2 = 0.258$ K.

C. Resolving power

To estimate the performance of a calorimeter a relevant figure of merit is the resolving power, which is calculated in the idle state and represents the energy sensitivity for the case of low-energy photons absorption. The resolving power reads

$$h\nu \over \Delta E, 20$$

where $T_2$ is the steady temperature of the absorber and $\Delta E$ is the intrinsic energy resolution of full width at half maximum for a calorimeter, determined by Johnson-Nyquist thermodynamics fluctuations in energy $h\nu [62]$. Fig. 8(a) shows the resolving power as a function of the photon frequency, $\nu$, at a few temperatures of the electrode $S_2$ with volume $V_2 = 1 \mu$m$^3$. The horizontal dashed line indicates unitary resolving power. We observe that the resolving power increases linearly with the photon frequency. At the same time, by rising the temperature $T_2$, the increase of the heat capacity $C_2(T_2)$ will reduce the resolving power. We note that at 0.15 K a resolving power exceeding one results in almost the whole range of frequencies shown in Fig. 8(a) (infrared to UV light spectrum). The range of frequencies giving $h\nu/\Delta E > 1$ decreases by increasing the temperature. At $T_2 = 0.26$ K, that is roughly the working temperature previously discussed, we obtain $h\nu/\Delta E > 1$ only at frequencies above...
FIG. 8. (a) Resolving power as a function of the photon frequency at a few temperatures. The shaded regions indicate the frequency ranges corresponding to IR (red), visible (green), and UV (purple) light spectrum. (b) Resolving power as a function of the temperature at a few values of the photon frequency. We are considering an electrode with volume \( V_2 = 1 \mu m^3 \). The shaded areas indicate forbidden regions, since corresponding to frequencies below \( \nu_{\text{min}} \) (light red area) and above \( \nu_{\text{max}} \) (light blue area). In both panels, the dashed horizontal line marks the unitary value of the resolving power.

\(\sim 21 \text{ THz} \). This means that a detector, residing at this temperature and with the chosen detection volume \( V_2 \), could efficiently work as a calorimeter for frequencies above 21 THz. Then, in the whole detection frequency range discussed in the previous section we achieve a resolving power larger than 1.

The temperature dependence of the resolving power at a few values of the photon frequency is displayed in Fig. 8(b). The shaded areas in this figure indicate forbidden regions, since corresponding to frequencies below \( \nu_{\text{min}} \) (light red area) and above \( \nu_{\text{max}} \) (light blue area). We note that at a fixed \( \nu \) the resolving power monotonically reduces by increasing the temperature, and that the higher \( \nu \), the larger the range of temperatures giving \( h\nu/\Delta E > 1 \).

Finally, we note that the preliminary evaluations developed in these sections are performed by omitting both the phononic and the S\(_{2}\)IN thermal relaxation channels, that will be instead considered in the next section, where we deal with the full thermal dynamics of the device. In principle, these terms could modify the dynamical ranges of the calorimeter, but as we will see the estimate given before are still valid.

D. Temperature time evolution

In this section we study the time evolution of the electronic temperature of the electrode \( S_2 \), when a photon is absorbed, namely, when \( P_{\text{rad}} \neq 0 \). The time evolution of the temperature \( T_2 \) is governed by the equation

\[
P_{S_1,\text{IN}}(T_1, T_2, \varphi) + P_{\text{rad}} - P_{\text{e-ph}, 2}(T_2, T_{\text{bath}}) + P_{\text{e-IN}}(T_2, T_{\text{bath}}) = C_2(T_2) \frac{dT_2}{dt},
\]

which includes all the incoming and outgoing thermal powers in \( S_2 \).

We model the energy diffusion in the superconductor, following the photon absorption, by a Gaussian envelope centered in \( t_0 \) with standard deviation (in time) \( \sigma \), reading as follow

\[
P_{\text{rad}} = \frac{h\nu}{\sqrt{2\pi} \sigma} \exp \left[ -\frac{(t - t_0)^2}{2\sigma^2} \right].
\]

We fix the width \( \sigma = 2 \text{ ns} \) of the Gaussian envelope of the photonic energy diffusion, in order to describe effectively the timing jitter of the detector. Indeed, we can reasonably expect that the photon-induced energy deposition occurs in a very short timescale. However, we can assume that the superconductor responds to the photonic excitation on a timescale roughly given by the characteristic thermalization time \( \tau_2 \) of the superconductor.\[63\]\[65\]

In other words, if the photon energy is localized in time on a short timescale, at the photonic event one can associate an equilibrium distribution function to electronic states in the superconducting lead only after a time \( \tau_2 \). This time represents conservatively the intrinsic timing jitter of our detector, and anyway a quasiequilibrium analysis cannot address a dynamics shorter than this time.

To achieve the best detection performances, during the timing jitter the energy relaxations from \( S_2 \) should be negligible. This means that the diffusion process has to occur in a much shorter timescale with respect to the thermal relaxation process in the system. The characteristic time of the thermal relaxation processes in \( S_2 \) can be estimated from the thermal capacity \( C_2 \), as

\[
\tau_{th} = C_2/(G + K_{S_2,\text{IN}} - K_{S_1,\text{IN}}),
\]

where \( G \) and \( K \) indicate the electron-phonon and electron thermal conductances of the JJ, respectively. The full expressions of the thermal conductances are shown in Appendix \[\Lambda\]. At the idle temperature \( T_2 = 0.258 \text{ K} \), for the device parameters used previously, one obtains \( \tau_{th} \approx 26 \text{ ns} \). Interestingly, we observe that at low values of the resistance of the junction between \( S_2 \) and \( N \), the thermalization process is mainly ruled by this thermal relaxation channel. So, in principle, our detector can potentially react in a very short time for the lowest values of \( R_{S_2,\text{IN}} \). In order to describe the system with the quasiequilibrium approach that we developed, a characteristic relaxation time \( \tau_2 \) shorter than \( \tau_{th} \) is required. For instance, by inspecting Table 1 of Ref.\[64\], a superconducting electrode made by Pb, In, Sn, Tl, or Ta could be suitable for our detection scheme,
Here, we discuss the response of the detector to a photonic event, where the power of radiation is described by a Gaussian envelope as in Eq. (22). In Fig. 9 we show the behavior of the system when a photon with frequencies $\nu_1 = 750$ THz (violet light) is absorbed by the detector at the time $t_0 \sim 0.1 \mu$s, see Fig. 9(a). After an absorption, during a timing jitter the temperature $T_2$ increases reaching a maximum, $T_{2,\text{max}}$, and the critical current undergoes a first jump. Then, due to the thermal contact with the phonon bath and the cooling finger, the electrode $S_2$ recovers its initial steady temperature, see Fig. 9(b). During the thermal evolution following a photon absorption, the condition $T_2 = T_2^j$ is satisfied twice, namely, when the temperature is increasing and then when it is decreasing, see dashed line in Fig. 9(a). This means that a single photonic event causes two subsequent $I_c$ jumps of opposite sign. In fact, while the temperature reduces towards the idle value, the critical current goes through a second jump when $T_2 = T_2^j$.

The distance in time between the two subsequent $I_c$ jumps induced by a photonic event can be used to define the dead time, $\tau_d$, of the device, see Fig. 9(c). This is the time frame in which the detector cannot be used to reveal the arrival of a following incident photon. In fact, once a transition induced by a photon with enough energy has occurred, a further photon-induced temperature enhancement would not trigger another $I_c$ jump, unless the system has already switched back to its idle state [2].

In Fig. 9 we show also the response of the device due to a second photonic event with frequency $\nu_2 = 10^4$ THz (extreme UV), absorbed at $t_0 \sim 1 \mu$s, namely, as the system relaxed to its idle state after the first photonic event. We observe that the higher the photon energy, the higher the maximum temperature, $T_{2,\text{max}}$, reached by $S_2$. During the timing jitter, after the increase of $I_c$, we note a following dip, corresponding to $I_c(T_{2,\text{max}})$. In fact, during the timing jitter, the temperature $T_2$ increases reaching its maximum, and $I_c$ first suddenly increases and then it rapidly reduces, up to reach the value $L_c(T_{2,\text{max}})$, see Fig. 9(c). Notably, this value of the critical current is lower than its maximum value, see the current profile shown in the inset of Fig. 9(c), and this is the reason for the appearance of the tight peak in the $I_c$ response once the photon has been absorbed. Both $T_{2,\text{max}}$ and $I_c(T_{2,\text{max}})$ following the absorption of the photons with frequencies $\nu_1$ and $\nu_2$, are marked in Figs. 9(b) and (c) by a red circle and a pink diamond, respectively.

Interestingly, we observe also that the higher the photon energy, the higher $T_{2,\text{max}}$ and therefore the longer the time that the system takes to approach its idle state, namely, the longer the dead time $\tau_d$, see Figs. 9(b) and (c). Since $T_{2,\text{max}}$ depends on $\nu$, we can use the dead time as a probe to find the frequency of the absorbed photons. In Fig. 10(a) we show the behaviour of both the dead time, $\tau_d$, (green triangles) and the maximum temperature, $T_{2,\text{max}}$, (blue circles) as a function of the photon frequency $\nu$. We note that both quantities grow monoton-
physically by increasing the photon frequency. Interestingly, the dead time $\tau_d$ increases exponentially with the frequency, according to the exponential decay of the temperature after the absorption. At the same time, $T_{2,\text{max}}$ follows exactly the prediction as calculated through Eq. 15 and shown in Fig. 9. We observe also that there is no mismatch between the detection frequency range computed in this case and that one discussed in Sec. [IIIB](#) although we are now taking into account also the thermal relaxation channels. This is because we are assuming a much shorter jittering time with respect to the thermalization time.

Finally, according to the nonlinear behavior of $\tau_d$, we can define another adimensional figure of merit of our calorimeter, namely, the logarithmic derivative $\alpha_d = \frac{\nu}{\tau_d} \frac{d\tau_d}{d\nu}$. This quantity shows that the capability of the device to discern the photon frequency by measuring the dead time is stronger in the low part of the detection frequency range, see Fig. 10(b).

From the dead time shown in Fig. 10(a), one can derive the detection rate of photons for the device. In the case of wide spectrum detection, with the parameters considered in our work, the maximal detection rate is $\tau_d^{-1}(\nu_{\text{max}}) \simeq 2.5 \text{ MHz}$, whereas in the near infrared band we obtain $\tau_d^{-1}(430 \text{ THz}) \simeq 7 \text{ MHz}$.

Inasmuch as the dead time is proportional to the energy absorbed, in the case of monochromatic radiation our device can show unique photon-number-resolving detection capabilities.

Detection of photonic events can be done by reading the Josephson kinetic inductance, $L_\varphi$, of the junction. In fact, the behavior of $I_c$ reflects on the kinetic inductance according to 10, 35

$$L_\varphi = \frac{\Phi_0}{2\pi} \left( \frac{\partial I_c}{\partial \varphi} \right)^{-1} = \frac{\Phi_0}{2\pi \cos \varphi} I_c^2,$$  

as shown in Fig. 9(d). Here, $I_c$ is the typical current-phase relation of a tunnel JJ. Thus, the readout of the Josephson kinetic inductance can be performed dispersively through an LC resonator inductively coupled to the JJ 69, 70. In this readout scheme the modification of the inductance can be measured through a shift, and/or a broadening, of the circuit’s transmission or reflection resonance 4. Notably, multiplex readout is permitted, along the lines of the SQUID multiplexing scheme discussed in Refs. 2, 67. In this scheme, different rf-SQUIDs are inductively coupled to a common frequency line. The changes in their kinetic inductance affect the frequency response of each resonator. Then, all these resonators are coupled to a common feedline towards a high-bandwidth cryogenic HEMT amplifier 67. Each resonator is tuned at a different frequency, so that all the sensors can be read simultaneously by applying a wide spectrum of probing frequencies to the feedline. Unlike the scheme proposed in Refs. 2, 67, in the present case the rf-SQUID is exactly our radiation sensor, namely, the SQUID ring is formed by the temperature-biased asymmetric JJ used for the photon detection and the superconducting ring utilized for the phase-biasing, see Fig. 1. A schematic representation showing just three multiplexing channels is depicted in Fig. 11. Since the ring inductance of each detector can be made negligible, the kinetic inductance, $L_K$, of the SQUID is dominated by the Josephson inductance $L_\varphi$. Nevertheless, the investigation of the optimal multiplexing strategy is a topic for further researches.
IV. CONCLUSIONS

In summary, we propose a threshold calorimeter based on the peculiar behavior of the critical current, \( I_c \), of a temperature-biased tunnel Josephson junction made by different superconductors. In fact, the step-like variation with the temperature of \( I_c \), already discussed in Ref. [20], can allow us to design a single-photon threshold detector in which the sensing element is one lead of the junction. Then, the absorption of a photon produces an enhancement of the electronic temperature, which can induce a measurable sudden increment of the critical current.

The conceived device is inherently energy resolving, and can be also engineered to determine the photon number in the case of a monochromatic source of light. In order to prevent an unreliable absorber temperature readout and minimize dark counts, we investigated the thermodynamics fluctuations in the superconductor. This analysis allows us to settle the idle temperatures of the device. Then, we discussed the essential figure of merit of this type of detector, namely, the resolving power, in order to find the optimal detection design. With our choice of realistic parameters for the setup investigated, the proposed sensor can efficiently detect photons of frequencies \( \nu \in [30 \text{–} 9 \times 10^4] \text{THz} \). Our sensor is also able to determine the photon frequency by measuring the dead time of the detector. In fact, after a photonic event the critical current undergoes two subsequent abrupt jumps, as a consequence of the restoring of the temperature at the idle value due to losses mainly through the cooling finger. Then, the distance in time between these subsequent photon-induced \( I_c \) jumps can be used to define the dead time, whose value depends on the energy of the incident photon. Interestingly, we observe that both the capability to discriminate the photon frequency through the dead time and the detection rate of the sensor are stronger at low frequencies.

We propose a non-invasive readout scheme based on the modification of the Josephson kinetic inductance caused by a photon-induced temperature enhancement. This quantity can be dispersively read via a LC resonator inductively coupled to the detector. This readout scheme is suitable for a multiplexing configuration, in which several sensors are coupled to different resonators, which are connected to a common feedline towards a high-bandwidth cryogenic HEMT amplifier. Each resonator is tuned at a specific frequency, so that all the sensors can be read simultaneously by applying a wide spectrum of probing frequencies to the feedline.

We note that our proposal evinces some similarities with other single-photon detectors based on the critical current change due to photon absorption in a proximized nanowire [62, 63, 70, 71]. Anyway, there are also several qualifying differences. In fact, in our detection scheme the absorbing element is a superconducting lead of an asymmetric JJ and the phenomenon exploited for the detection is the peculiar steep variation of the critical current by changing the temperature, where its temperature variation is smoother in proximized sensors. Therefore, the strength of our device resides in a strong sensitivity due to the discontinuous response of \( I_c \) to a photonic event. Moreover, the fact that detection is not performed at extremely low temperature, could be advantageous for achieving a fast thermal response due to a better \( e-ph \) coupling, which results in a shorter dead time of the detector. Markedly, our detector represents an interesting combination between different types of superconducting single-photon and calorimetric devices. In particular, it has the potential sensitivity of superconducting tunnel junction (STJ) detectors, however without being affected by the Johnson-Nyquist noise, due to the dissipationless working regime. Besides, the proposed detector has potentially the energy sensitivity of proximity-based detectors, with reduced dead time at parity of photon energy, due to higher operating temperatures. Finally, it is characterized by a fast thermal response, due to energy absorption with a short timing jitter, similarly to TESs.

Finally, we observe that the characteristic timescales of the system, i.e., the timing jitter and the thermal relaxation time, depend on the choice of the superconductor used as absorbing element and on the characteristics of the metallic cooling finger. Moreover, the use of superconductors with higher \( T_c \)'s would permit higher working temperatures, resulting in a further reduction of both the thermal [28] and the quasiparticle [64, 65] relaxation time, the latter defining the timing jitter of the detector. So, a careful material selection could outperform the conservative estimate adopted in the above design study.

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Appendix A: Timescales of the thermalization process.

The characteristic time of the thermalization process in \( S_2 \) namely, the thermal relaxation time, \( \tau_{th} \), can be estimated by first-order expanding the heat current terms in Eq. (21) in the idle state of the system [29]. In doing so, one obtains

\[ \tau_{th} = \frac{C_2}{(G + \mathcal{K}_S_N - \mathcal{K}_{S_S^2})}, \]

where
\( G \) and \( K \) indicate the electron-phonon and electron thermal conductances of the JJ, respectively. These conductances can be obtained after the first derivatives of the heat power densities in Eqs. (14) and (21), calculated at a steady electronic temperature \( T_e \). Specifically the electron-phonon thermal conductance reads \([63]\)

\[
G_2(T_e) = \frac{\partial P_{\text{ph},2}}{\partial T_e} = \sum_{\Omega} \int_{-\infty}^{\infty} \frac{dE}{\sinh(\frac{\epsilon}{2k_BT_e})} \frac{dE}{\cosh(\frac{E}{2k_BT_e})} \frac{d\epsilon}{\cosh(\frac{E-\epsilon}{2k_BT_e})},
\]

while the electron thermal conductances read \([24]\)

\[
K_{\text{s1s2}}(T_e) = \frac{\partial P_{\text{s1s2}}}{\partial T_e} = \int_{-\infty}^{\infty} \frac{d\epsilon}{\cosh(\frac{\epsilon}{2k_BT_e})} \times (A2)
\]

\[
\left[ N_1(\epsilon, T_e) N_2(\epsilon, T_e) - M_1(\epsilon, T_e) M_2(\epsilon, T_e) \cos \varphi \right],
\]

and

\[
K_{\text{s2IN}}(T_e) = \frac{\partial P_{\text{s2IN}}}{\partial T_e} = \frac{1}{2\pi e^2 B^2 T_e^2} \times (A3)
\]

\[
\int_{0}^{\infty} \frac{d\epsilon}{\sinh^2(\frac{\epsilon}{2k_BT_e})} N_2(\epsilon, T_e),
\]

where \( N_j(\epsilon, T) = \text{Re} \left[ \frac{\epsilon + i \Gamma_j}{\sqrt{(\epsilon + i \Gamma_j)^2 - \Delta_j(T)^2}} \right] \) and

\[
M_j(\epsilon, T) = \text{Im} \left[ \frac{-i \Delta_j(T)}{\sqrt{(\epsilon + i \Gamma_j)^2 - \Delta_j(T)^2}} \right].
\]

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