Rate Maximization in Vehicular uRLLC with Optical Camera Communications

Amirul Islam, Leila Musavian, and Nikolaos Thomos

Abstract

Optical camera communication (OCC) has emerged as a key enabling technology for the seamless operation of future autonomous vehicles. By leveraging the supreme performance of OCC, we can meet the stringent requirements of ultra-reliable and low-latency communication (uRLLC) in vehicular OCC. In this paper, we introduce a rate optimization approach in vehicular OCC through optimal power allocation while respecting uRLLC requirements. We first formulate a discrete-rate optimization problem as a mixed-integer programming (MIP) subject to average transmit power and uRLLC constraints for a given set of modulation schemes. To reduce the complexity in solving the MIP problem, we convert the discrete-rate problem into a continuous-rate optimization scheme. Then, we present an algorithm based on Lagrangian relaxation and Bisection method to solve the optimization problem. Considering the proposed algorithm, we drive the rate optimization and power allocation scheme for both discrete-rate and continuous-rate optimization schemes while satisfying uRLLC constraints. We first analyze the performance of the proposed system model through simulations. We then investigate the impact of proposed power allocation and rate optimization schemes on average rate and latency for different target bit error rates. The results show that increasing the transmit power allocation improves the average rate and latency performance.

Index Terms

Ultra-reliable and low-latency communication (uRLLC), vehicular communication, optical camera communication (OCC), optimal power allocation, rate maximization.

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I. Introduction

Driven by vehicular networks, the automotive industry is undergoing key technological transformations and vehicles are being connected to share information. Automotive vehicles (AV) are driving as the revolution in future smart cities and are considered as the leading transformative technologies in intelligent transportation system (ITS). In the modern world, the number of vehicles and vehicle-assisting infrastructures are increasing rapidly, making transportation systems more vulnerable than ever, resulting in more traffic congestions, road causalities, and overall less road safety. These rapid growths of AVs will open a significantly challenging, but profitable market, for the future ITS [1]. To cope with the current evergrowing and complex vehicular networks, the practice of sharing information and cooperative driving on the road is extensively increasing. However, allowing AV communication can help to ensure traffic safety and enhance the overall driving experience by facilitating new services, such as collision avoidance and autonomous driving [2], [3]. Therefore, providing efficient vehicle-to-vehicle (V2V) communications is necessary, and the performance of the growing transportation systems depends on the availability of V2V communication links at extreme low latency and ultra-reliability [4].

The requirement to meet both latency and reliability requirements simultaneously makes the vehicular communication a very challenging problem. For enabling ultra-reliable and low-latency communication (uRLLC) in ITSs, several methods are examined in literature, such as delay minimization [5], reliability guarantee [6], vehicle clustering [7], and excess queue length evaluation [8]. Specifically, in [5], the vehicular network transmission power is minimized by grouping vehicles into clusters and modelling reliability as queuing delay violation probability. In [6], a joint resource allocation and power control algorithm is proposed to maximize the V2V rate considering latency and reliability constraints. In [7], the authors study the impact of transmission time interval on the performance of low-latency vehicular communications. Recently, several principles for supporting uRLLC from the perspective of traditional assumptions and models applied in communication theory are discussed in [8]. Moreover, edge computing is also considered as an attractive solution to minimize latency. This is done by processing the requested tasks locally, without relying on remote servers [9], [10].

The methods as mentioned above, enable uRLLC by either using radio frequency (RF) or cellular-based communication systems with central base stations (BS), remote or edge servers, mostly relying on centralized resource management. However, these servers have limited compu-
tational resources, and therefore, they can easily be overloaded with the frequently requested AV tasks. Most importantly, the security can be violated as information is exchanged between the servers and vehicles or infrastructures during communication. Besides, RF channels are prone to channel fading, noise, or interference.

On the contrary, in recent years, visible light communications (VLC) is anticipated to be an essential alternative for next-generation communication services [11], [12]. VLC uses LEDs as transmitter and camera or photodiode (PD) as the receiver. VLC systems are employing cameras as the receiver is known as optical camera communication (OCC), whereas those use PD are called light-fidelity. PDs are generally small non-imaging devices, which produce a quick response. But, it offers restrictions due to the trade-off between transmission range and signal reception. The use of cameras mitigates the challenges in PD [13], [14]. The revolutionary advancements and potential advantages of OCC have rendered this technology to be a promising mechanism for the future AV communication [13], [15]. To the best of our knowledge, although several vehicular OCC systems have tried to improve the bit error rate (BER) and capacity performances, none of them addresses uRLLC challenges until now.

In our earlier results published in [16], we analyzed the performance of vehicular OCC with fixed power allocation to justify whether OCC will be suitable for satisfying uRLLC requirements in AV communications. In this current paper, we propose a new rate optimization scheme in vehicular OCC through optimal power allocation while respecting uRLLC requirements. In our formulation, BER is used to characterize the reliability requirement, and the end-to-end latency is defined by transmission latency. We neglect computational latency as a small amount of data (related to action or safety information) is communicated in our system. To improve the efficiency of vehicular OCC, we employ adaptive modulation. We note that higher spectral efficiency and lower latency can be achieved by increasing the modulation order. This performance gain, however, is achieved at the expense of higher transmission power. In this paper, we investigate the problem of rate optimization through optimal power allocation while respecting uRLLC constraints. The major contributions of this paper are summarized as follows:

- We mathematically model the vehicular OCC channel in order to analyze the performance of BER, achievable spectral efficiency, and transmission latency at different inter-vehicular distances and angle of incidence (AoI) while considering adaptive modulation.
- We then formulate a discrete-rate optimization scheme as a mixed-integer programming (MIP) subject to average transmit power constraint and a predefined set of discrete mod-
ulation schemes while respecting uRLLC constraints. Due to the entailed computational and time complexity in solving the proposed MIP problem, we transform the discrete-rate optimization problem into a continuous-rate optimization problem by considering continuous modulation to make the solution traceable. We find the condition for the new problem to be concave with respect to the transmit power. The problem then can be solved using Lagrange relaxation method. We solve the relaxed problem by employing an iterative Bisection algorithm to obtain the optimal transmit power allocation.

- Then, we propose two algorithms to solve joint power allocation and rate optimization problem for both continuous-rate and discrete-rate optimization schemes. These algorithms permit us to find the optimal average rate and transmission latency while satisfying uRLLC constraints.
- Finally, we investigate the performance of the proposed discrete-rate optimization scheme and continuous-rate optimization scheme through numerous simulations.

The remainder of this paper is organized as follows. Section II outlines the vehicular OCC, while the OCC channel model and mathematical representation of the performance parameters of the proposed V2V system is presented in Section III. Section IV proposes the formulation of discrete-rate optimization scheme subject to the reliability and latency constraints. The problem transformation to continuous-rate optimization scheme and the solution to the optimization problem is presented in Section V. Section VI provides the simulation results of different performance parameters and comparison between the proposed rate optimization schemes. Finally, our conclusions are given in Section VII.

II. OVERVIEW OF VEHICULAR OCC

In this section, we first discuss the advantages of OCC over other existing communication systems. Then, we illustrate the general architecture of the vehicular OCC. Finally, we review several existing studies on vehicular OCC.

In recent years, OCC has gained new momentum as a promising complementary technology over existing communication systems, e.g., RF or PD-based communication, [14], [17]. The advantages of the license-free unlimited spectrum, longer lifespans, lower implementation cost, negligible interference, less power consumption, and enhanced security have prompted the OCC technology as a viable candidate for future wireless communication applications [15]. Also, OCC does not harm human body or eyes and is not affected by electromagnetic interference. It is, in
Fig. 1. An illustration of vehicular optical camera communication operation.

fact, very easy to integrate OCC to the existing vehicular systems without making any significant changes. This is because LEDs already exist in vehicles, traffic lights, or road infrastructures. Besides, achieving a low data rate, OCC can be a better alternative to the congested and saturated RF systems due to its negligible noise and interference and higher security [14]. However, OCC can face challenges due to its LoS requirements for communication. Readers interested in a detailed comparison among OCC, PD, and RF-based systems are referred to [16].

In general vehicular OCC architecture, LED arrays located on the rear side of a vehicle or other light sources act as transmitters, and cameras act as receivers (see Fig. 1). As shown in Fig. 1, Vehicle 1 and Vehicle 2, communicate information through LED lights, which are called hereafter transmitters. Other light sources, e.g., Sunlight, ambient lights, traffic lights, and digital signages, are considered as noise sources. Meanwhile, the camera at the receiver vehicle captures the video frames within its field-of-view (FoV), which then passes it through an imaging lens. The captured images are fed into the image processor, which identifies the LEDs pattern from the captured images. After processing the image, the signal is passed to the data extraction unit to recover the communicated information. In vehicular OCC, no complex signal processing algorithm is required to filter out the light sources that do not convey information. The noise and data sources can easily be distinguished and captured on the image plane of the image sensor because the cameras only focus on the pixels in which the LED lights strike. In this manner, interference-free and secure communications can be achieved using an image sensor.

M-ary quadrature amplitude modulation (M-QAM) is used to modulate the signal in VLC [18] as it is a multi-level, high-order, and spectrally-efficient modulation technique that is relatively easy to implement and offers very low BER, high-speed, and flicker-free communication. For employing M-QAM, at the transmitter, the data bit-streams are first mapped into a symbol
consisting of amplitude and phase by splitting into in-phase and quadrature parts. The symbol is modulated to a square wave signal with an amplitude, period and a shifting phase, i.e., period × phase/2π. A preamble is inserted with the data bits for synchronization and modulation scheme estimation at the receiver. The resulting signal is then transmitted through the optical wireless channel by modulating the intensity of LEDs. At the receiver, the camera captures the modulated light waveform within its exposure time. During this time, the image sensor captures the intensity of the light coming in as different LED states, e.g., on, off, mid. The camera integrates the signal during its exposure time, which is recorded as the pixels of the image. Finally, we can extract the correct signal information from this recorded intensity in these pixels using an efficient M-QAM demodulation scheme [19]. At the receiver, frame synchronization, modulation scheme estimation, and post-equalization are carried out to eliminate the effect of the channel with the help of the preamble. In [19], a simple mathematical formulation for encoding and decoding of the amplitude and phase of transmitted symbols is proposed, where the modulated symbol is sampled in three consecutive frames by the image sensor. From the captured frames, the LED states, e.g., on, off, mid, is identified, and a lookup table is developed. Then, the phase position is retrieved using the lookup table, and the reconstructed phase is converted to radian so that it can be mapped with the M-QAM. Finally, the original signal is perfectly reconstructed from the detected amplitude and phase.

Besides OCC, vehicular ad-hoc networks (VANETs) created immense opportunities in the ITS at lower operational cost [20], [21]. But, VANETs have shortcomings, such as lower accuracy, unreliable internet service, and lack of pure network architecture [22]. Alternatively, AV communication uses wireless access in vehicular environments, i.e., IEEE 802.11p standard [23]. However, OCC has several advantages over IEEE 802.11p, including unlicensed frequency spectrum access, BSs independency, and simultaneous lighting and communication. Moreover, there are several existing experimental methods to improve the performance of vehicular OCC systems. Specifically, in [15], the authors achieved 10 Mbps data rate based on the LEDs intensity variation by generating a flag image from the communication image pixels in which the high-intensity light sources appear. In [24], the authors proposed an image sensor based VLC system, which achieved a 20 Mb/s/pixel data rate without LED detection and 15 Mb/s/pixel data rate with 16.6 ms real-time LED detection. In [25], the transmission rate was improved to 45 Mbps without bit errors and to 55 Mbps with BER < 10⁻⁵.

The OCC schemes mentioned above investigated the data rate enhancement through experi-
mental study, and none of them examined the uRLLC aspects of vehicular OCC and optimization of system parameters, e.g., power and rate. In this paper, we propose a novel rate optimization scheme for vehicular OCC, which is done through optimal power allocation while satisfying uRLLC requirements. To the best of our knowledge, this is the first time where uRLLC in vehicular OCC is examined, and rate optimization scheme is analyzed.

III. SYSTEM MODEL

In this section, we present the considered system model and parameters of vehicular OCC. Then, we specify the performance defining metrics of OCC in terms of the BER, the achievable rate, and the observed transmission latency.\textsuperscript{1}

A. System Modelling

Fig. 2 outlines our proposed vehicular OCC system model, where vehicles communicate with each other. In this scenario, the vehicle conveying information is denoted as “Transmitter Vehicle (TV)”. Whereas the vehicle which follows TV and receives the transmitted information is defined as “Receiver Vehicle (RV)”. The vehicle located at the back of the RV is termed as “Backward Vehicle”. As shown in Fig. 2, the TV sends information, which is mainly the vehicle’s internal information, e.g., speed, next action, position, and safety or action-related information from other vehicles. The RV detects signals from the LED transmitters using a camera. In our system, each vehicle has two camera sets, one in the front and another in the back. The function of the camera in the front, i.e., high-speed camera (1000 frame per second (fps)), is twofold. Firstly, it measures forward distance, $D$, between the TV and RV using the distance between the LEDs and pixel information on the image sensor, which we will discuss later in the next sub-section. Secondly, the camera acts as the receiver, which decodes transmitted data from the LED transmitters. The camera at the back, i.e., vision camera, measures the backward distance, $d_b$, between the vehicles using a stereo-vision camera similar to [26].

\textsuperscript{1}In this paper, we use the terms image sensor and camera interchangeably.
B. Optical Channel Model

We assume an uninterrupted LoS link between the transmitter and camera receiver, ensuring the vehicles are free from obstruction to communicate with each other continuously. Depending on the channel conditions, OCC has either a flat-fading or diffuse channel. Generally, OCC channel has two types of light propagation components: (i) LoS component resulting from direct light propagation from the transmitter to the receiver and (ii) diffuse components resulting from the reflected lights from other vehicles or reflective surfaces. Usually, the diffuse propagation has much lower energy than the LoS component, and therefore, the diffuse light component is neglected in this paper.

We assume that the LED follows a Lambertian radiation pattern and has wider directivity. Therefore, the light emission from the LED transmitters can be modelled using a generalized Lambertian radiant intensity [27], [28] and following the link geometry, as shown in Fig. 3(a). Accordingly, the direct current (DC) channel gain for visible light LoS link is derived by [27]

\[
H(\theta, t) = \begin{cases} 
\frac{(m+1)A_{\text{eff}}(\theta)}{2\pi D^2(t)} \cos^m(\phi), & 0 \leq \theta \leq \theta_l \\
0, & \theta > \theta_l 
\end{cases}
\]  

(1)

where \( m \) is the order of Lambertian radiation pattern, which is derived from the LED semi-angle at half luminance, \( \Phi_{1/2} \), as \( m = \frac{-\ln(2)}{\ln(\cos(\Phi_{1/2}))} \); \( A_{\text{eff}}(\theta) \) is the effective signal collection area of the image sensor; \( \theta \) is the AoI with respect to the receiver; \( \phi \) is the angle of irradiance with respect to the emitter; \( D(t) \) is the distance between the transmitter and receiver; \( t \) is the time-frame index; and \( \theta_l \) denotes the FoV of image sensor lens. The \( D(t) \) is expressed as [15]:

\[
D(t) = \frac{f}{a} \cdot \frac{\delta}{\eta(t)},
\]

(2)

where \( \delta \) is the distance between the left and right LED array units, \( f \) is the lens focal length, \( \eta(t) \) is the distance in terms of number of pixels between the left and right LED array units on the captured image, and \( a \) is the image pixel size. The inter-relation between the distance...
measurement parameters is illustrated in Fig. 3(b). Whereas, the backward distance, \( d_b \) can be estimated with a stereo vision camera using a similar method to the one in [26].

Regarding the above parameters: \( \delta \) is sent from TV to RV through LEDs, and \( f \) and \( a \) are known values for any system, such as 15 mm and 7.5 \( \mu \)m, respectively, as we considered in this paper. The value of \( \eta(t) \) can be obtained via simple image processing techniques or by calculating the pixel values using data pointer. In this way, using both the received data and the captured image, the RV can estimate the inter-vehicular distance, \( D(t) \) and the channel gain.

Also, \( A_{\text{eff}}(\theta) \) can be expressed as [27]

\[
A_{\text{eff}}(\theta) = \begin{cases} 
A T_s(\theta) g \cos(\theta), & 0 \leq \theta \leq \theta_l \\
0, & \theta > \theta_l 
\end{cases}
\]  

(3)

where \( A \) is the area of the entrance pupil of the camera lens, \( T_s(\theta) \) is the signal transmittance of the optical filter, and \( g \) is the gain of the lens. An ideal lens has a gain: \( g = \nu^2/\sin^2(\theta_l) \), where \( \nu \) is the internal refractive index of the lens.

Based on the above definitions and considering (3), finally, \( H(\theta,t) \) can be formulated as

\[
H(\theta,t) = \begin{cases} 
\frac{(m+1)A}{2\pi D^2(t)} \cos^m(\phi) T_s(\theta) g \cos(\theta), & 0 \leq \theta \leq \theta_l \\
0, & \theta > \theta_l 
\end{cases}
\]  

(4)

The received optical power, \( P_r(\theta,t) \), can hence be derived for the optical transmit power \( P \) from the LED lights according to

\[
P_r(\theta,t) = P \cdot H(\theta,t).
\]  

(5)

We note that in our paper, we neglect the signal detection overhead of recognizing the desired light sources under mobile scenarios. This is motivated by [17], where authors proposed a statistical vehicle motion model in an image plane and showed that the vehicle motion along the vertical and horizontal axes of the image plane is limited to within one pixel in most cases, which is very small compared to entire image pixels on the captured image. Moreover, the DC gain, and as a result, the signal-to-noise ratio (SNR) at a pixel remains constant as long as the projected image of the transmitter LED occupies several pixels. Further, a simple design of an LED detection and tracking system is proposed using the result and the vehicle motion model of [17], which limits the tracking area of the VLC transmitter and reduces computational cost. Thus, the vehicle motion and the pixel illumination model is used as a guideline for our system to overlook the overhead of recognizing the desired light sources for the mobile environment.
C. Performance Analysis

The presence of noise in the VLC system is mainly specified by the ambient-induced shot noise in the receiver as well as preamplifier noise. The outdoor VLC applications are highly susceptible to the visible background light because of daylight and other light sources, e.g., digital signage and advertisement board, which in turn, lead to the generation of high-intensity ambient-induced shot noise at the receiver. Generally, the received background light can be minimized by optical filtering though it still adds shot noise. If no or little ambient lights are present, the dominant noise will be the receiver preamplifier noise.

In order to analyze the system performance, we first formulate the SNR of the optical link.\(^2\) We consider SNR as a measure of communication link quality of the signal transmission. Therefore, according to [29], the received SNR, \(\gamma(\theta, D)\) of visible light link can be expressed by

\[
\gamma(\theta, D) = \frac{\zeta^2 P_r^2(\theta, D)}{\sigma(\theta, D)} = \frac{\zeta^2 P^2 H^2(\theta, D)}{\sigma(\theta, D)},
\]

(6)

where \(\zeta\) is receiver responsivity and \(\sigma(\theta, D)\) denotes the total noise variance in \(A^2\). The \(\sigma(\theta, D)\) can be expressed as

\[
\sigma(\theta, D) = \sigma_{\text{shot}}^2(\theta, D) + \sigma_{\text{ther}}^2,
\]

(7)

where \(\sigma_{\text{shot}}^2(\theta, D)\) is the shot-noise variance [30] and \(\sigma_{\text{ther}}^2\) is the thermal noise variance [29]. \(\sigma_{\text{shot}}^2(\theta, D)\) can be calculated from

\[
\sigma_{\text{shot}}^2(\theta, D) = 2q\zeta BP_r(\theta, D) + 2qBI_{\text{bg}}I_2,
\]

(8)

where \(q\) is the electronic charge, \(B\) is the equivalent noise bandwidth, \(I_{\text{bg}}\) is the background current, and \(I_2\) is the noise bandwidth factor for a rectangular transmitter pulse [31]. The thermal noise variance is given by:

\[
\sigma_{\text{ther}}^2 = \frac{8\pi k T_A}{G} I_2 B^2 C_f A + \frac{16\pi^2 k T_A \Gamma}{g_m} I_3 B^3 C_f^2 A^2,
\]

(9)

where \(k\) is Boltzmanns constant, \(T_A\) is absolute temperature of the environment, \(G\) is the open-loop voltage gain, \(C_f\) is the fixed capacitance of the image sensor per unit area, \(g_m\) is the FET trans-conductance, \(\Gamma\) is the FET channel noise factor, and \(I_3\) is the noise bandwidth factor [30].

\(^2\)From (4), we see that the channel gain depends on \(\theta\) and \(t\), where \(t\) represents the changes in inter-vehicular distance, i.e., \(D\) over time. So, the only changing variable is the distance, and from now we can drop the variable \(t\) by only keeping \(D\). Therefore, for formulating the SNR, we use \(D\) by leaving \(t\).
We can now approximate the shot noise as $\sigma_{\text{shot}}^2 = 2qBI_{\text{bg}}I_2$, because the second part of (8) is dominant. So, the approximated total noise variance can be expressed as $\sigma_s^2 = \sigma_{\text{shot}}^2 + \sigma_{\text{ther}}^2$. As a result, the simplified SNR, $\gamma_s(\theta, D)$ is given by
\[
\gamma_s(\theta, D) = \frac{P^2H^2(\theta, D)}{\sigma_s} = P^2S(\theta, D),
\]
where $S(\theta, D) = \frac{H^2(\theta, D)}{\sigma_s}$ is channel-to-noise ratio (CNR). Here, we ignore the detector responsivity as it is considered $1 \text{ AW}^{-1}$, most of the case.

Motivated by the trade-off among modulation order, achieved BER, and improved spectral efficiency, we consider adaptive modulation that permits us to adapt the modulation order by satisfying the minimum BER requirement of the system. Adaptive modulation is a powerful method to maintain the desired quality of service and to maximize the rate for the given channel conditions [32], [33]. However, in practice, a particular discrete modulation set is used to examine how the performance varies after limiting the system to a small modulation set. For the considered modulation set, we study binary phase-shift keying (BPSK) and uncoded M-QAM with the square constellation as an example. Still, other modulation schemes can also be used. The BER of the optical wireless channel at the receiver using BPSK, M-QAM can be expressed as [34] and [35]:
\[
\text{BER}(\theta, D) = \begin{cases} 
Q\left(\sqrt{2\gamma_s(\theta, D)}\right), & \text{for BPSK} \\
\frac{4}{\log_2(M_n(\theta, D))} \cdot Q\left(\sqrt{\frac{3\gamma_s(\theta, D) \log_2(M_n(\theta, D))}{M_n(\theta, D)-1}}\right), & \text{for M-QAM}
\end{cases}
\]
where $M_n = 2^n$ is the constellation size ($n = 2, 3, \cdots$) and $Q(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right)$ stands for the $Q$-function. So, the spectral efficiency of the BPSK and M-QAM modulation schemes can be expressed as $\text{SE}_{\text{BPSK}} = 1$ and $\text{SE}_{\text{M-QAM}} = \log_2(M_n(\theta, D))$, respectively.

It is worth to note that the adaptive modulation in our system is adjusted as follows. Suppose there is any change in the modulation scheme during communication. In that case, the transmitter informs the receiver regarding the employed modulation by appending a small overhead, e.g., some extra bits, in each transmitted packet. This overhead can be neglected because, in practice, as a small set of modulation scheme is used, e.g., 6, in our system. We require only 3 bits to be appended in the transmitted data for the receiver. Hence, the overhead will be minimal compared to the transmitted packet size, i.e., 5 kbits, in our system.

We should note that the transmission rate (measured in bit per second (bps)) of a camera based-communication system depends on the received SNR as shown in [36] and is given by
\[
C(\theta, D) = \frac{W_{\text{fps}}}{3}W_s(D) \cdot \log_2(1 + \gamma_s(\theta, D)),
\]
where $W_{\text{fps}}$ is the camera-frame rate, $W_s(D)$ is the spatial-bandwidth, which denotes the number of information-carrying pixels per image frame. We divide $W_{\text{fps}}$ by three as the modulated signal must be sampled three times of sampling frames by the camera, which is sufficient for decoding the original M-QAM signal [19]. In other words, for reconstructing the amplitude and phase perfectly, a modulated symbol is sampled in three consecutive frames. The $W_s(D)$ is defined as

$$W_s(D) = N_{\text{LEDs}} \cdot N_{\text{row}}(D),$$  \hspace{1cm} (13)

where $N_{\text{LEDs}}$ is the number of LEDs at each row of the transmitter and $N_{\text{row}}(D)$ represents the captured number of row pixel lines in each frame. Considering a rolling shutter camera, the actual number of samples, $N_{\text{row}}(D)$ acquired from the captured image at $D$ can be expressed as

$$N_{\text{row}}(D) = \frac{w L}{2 \tan \left( \frac{\theta_l}{2} \right) D},$$  \hspace{1cm} (14)

where $w$ is the image resolution and $L$ is the normalized length (diameter) of the LEDs along the width. Taking into account (13) and (14), $C(\theta, D)$ is re-written as:

$$C(\theta, D) = \frac{W_{\text{fps}} N_{\text{LEDs}} w L}{6 \tan \left( \frac{\theta_l}{2} \right) D} \cdot \log_2(1 + \gamma_s(\theta, D)) = \frac{l_0}{D} \cdot \log_2(1 + \gamma_s(\theta, D)), $$  \hspace{1cm} (15)

where $l_0 = \frac{W_{\text{fps}} N_{\text{LEDs}} w L}{6 \tan \left( \frac{\theta_l}{2} \right)}$. Considering the communications between the vehicles, the overall end-to-end latency, $\tau_c(\theta, D)$ can be found as [9]

$$\tau_c(\theta, D) = \frac{L_p}{C(\theta, D)},$$  \hspace{1cm} (16)

where $L_p$ is the packet size in bits. Recall that in our system, we consider that the end-to-end latency is dominated by transmission latency, and therefore, we neglect the computational latency. This is because we process a small amount of data, i.e., the decision information from TVs to the RVs, and hence, the computational time will be short.

Similar to (15) and (16), the discrete rate, $R(\theta, D)$ and the latency, $\tau_d(\theta, D)$ for the considered discrete modulation set can be written, respectively, as follows:

$$R(\theta, D) = \frac{l_0}{D} \cdot \log_2(M_n(\theta, D)), $$  \hspace{1cm} (17)

$$\tau_d(\theta, D) = \frac{L}{R(\theta, D)}. $$  \hspace{1cm} (18)

In our system, we analyze the performance of our proposed OCC system by varying both the inter-vehicular distances and AoIs. However, as it is challenging to change AoI sharply in a realistic scenario, which would introduce additional delays in changing the AoI inside the vehicle mechanically, in the next section, we consider distance as the only free variable and fix AoI in a value that guarantees system requirements.
IV. CONSTRAINED PROBLEM FORMULATION

Considering the proposed framework and system requirements, we formulate a rate optimization scheme that aims at maximizing the average rate through adjusting transmit power allocation. To maintain uRLLC, the BER and latency requirements are constrained so that they respect the constraints imposed by uRLLC system. In our studied model, the SNR changes with distance, and hence we have to adjust the transmit power to the condition of the channel variation, which is basically a function of distance. Therefore, power is expressed as a function of distance. However, since our objective is to maximize the rate through optimal power allocation, the rate is characterized by distance and transmit power, which itself changes over distance. So, from this section, we express our rate optimization scheme as a function of both power and distance. As we see that our rate is discrete and power is continuous, thus our problem is a MIP problem.

Mathematically, the considered constrained rate-maximization problem is formulated as:

\[
\max_{P(D), M_n} \mathbb{E}[R(P(D), D)]
\]

\[
= \max_{P(D), M_n} \mathbb{E}\left[\frac{l_0}{D} \cdot \log_2(M_n(D))\right] \quad (19)
\]

s.t. \(\mathbb{E}[P(D)] \leq P_{\text{max}}\), \(\text{BER}(D) \leq \text{BER}_{\text{tgt}}\), \(\tau_d(P(D), D) \leq \tau_{\text{max}}\), \(M_n \in \{\text{BPSK, M-QAM}\}\), \(\text{BER}_{\text{tgt}}\) is the maximum target BER, and \(\tau_{\text{max}}\) is the maximum latency. Constraint (20) specifies that the average power is limited by \(P_{\text{max}}\). Constraint (21) indicates that the reliability is satisfied by maintaining a target BER, and the latency requirement is respected by (22), for ensuring uRLLC. The modulation scheme is chosen from a small set of available modulation schemes, as shown in (23). Although we did not consider peak power constraint, the peak power will not be significantly bigger than the average power for our considered model. If we consider peak power constraint, the optimal power transmission policy will be to transmit at the peak power, and the optimization will be straightforward. From the simulation analysis, we can conclude that if we only consider the average power constraint by ignoring the peak power, the system performance will not be affected considerably. Therefore, we only study the average power constraint in our formulation.
For the considered maximization problem (19), the distance is maintained such that \( D \geq d_{\text{stop}} \), where \( d_{\text{stop}} \) is the stopping distance, which is equal to the sum of covered distance by the vehicle to travel after the brakes are activated, i.e., braking distance, and the covered distance to travel due to driver’s reaction time, i.e., reaction distance, after observing a situation [37]. We, however, assume that in the unlikely event that distance may drop to below \( d_{\text{stop}} \), the uRLLC communication will not be interrupted and will continue. Although the maximization problem does not include the backward distance, \( d_b \), it is constrained by \( d_b \geq d_{\text{stop}} \), in order to avoid collision with the vehicle behind. Besides, the vehicle behind also runs the same algorithm, and hence, the constraint is met.

It is known that MIP problem has high computational complexity [38], [39] and can be solved by methods, such as dynamic programming and exhaustive search techniques, which are either too time-consuming or computationally demanding for finding optimal solutions. For large-scale, time-critical MIP problems, it is challenging to generate a dedicated solution within limited computational time. To cope with the complexity, we transform the MIP problem into a continuous-rate optimization problem by considering continuous modulation. This allows us to solve the problem with reduced complexity and permits practical deployment of our system. We present the continuous approximation of the MIP problem in Section V.

V. Problem Transformation and Proposed Solution

As we mentioned in Section IV, the rate optimization problem in (19) entails high complexity. In this section, we, therefore, convert the problem into a continuous-rate optimization problem, that renders the problem solvable and mathematically tractable. To solve the continuous problem, we devise an iterative algorithm combining Lagrangian optimization with Bisection algorithm. Finally, the solutions to the rate optimization problems are presented in two separate algorithms.

A. Problem Transformation

To evaluate the target BER requirement, the \( Q \)-function of (11) should be approximated by a closed-form expression. The closed-form expression may involve some mathematical complexities [40], e.g., the infinite integration that is needed while evaluating the BER includes \( Q(\sqrt{x}) \) rather than \( Q(x) \). In some cases, conversion of \( Q(x) \) complicates the closed-form derivation. The main objective is to find the closed-form approximation that is simple and accurate though there is a trade-off between the accuracy and complexity of the \( Q \)-function approximation. The
BER approximation for the M-QAM square constellation over an additive white Gaussian noise channel is presented in [35], [41], and can be approximated for each $D$ as

$$\text{BER}(D) \leq 0.2 \exp\left(-\frac{1.5 P^2(D) S(D)}{M_n - 1}\right).$$  \hfill (24)

The approximation used in [35], [41] and in our work is invertible in the sense that it provides a simple closed-form expression for the transmission rate as a function of the distance. By inverting (24), the modulation order for continuous-rate M-QAM to maintain a target BER can be approximated as

$$M_n(D) = 1 + \frac{-1.5 S(D)}{\ln(5\text{BER}_{tgt})} \cdot P^2(D) = 1 + K P^2(D) S(D),$$  \hfill (25)

where $K = -1.5/\ln(5\text{BER}_{tgt})$. So, the rate expressed by MIP in (19) can be rewritten for continuous modulation as

$$\frac{l_0}{D} \cdot \log_2(M_n(D)) = \frac{l_0}{D} \cdot \log_2 (1 + KP^2(D)S(D)) = C(P(D), D).$$  \hfill (26)

The transmission latency for continuous-rate should satisfy the constraint (22) and is given by

$$\tau_c(P(D), D) = \frac{L_p}{C(P(D), D)} \leq \tau_{\text{max}}, \Rightarrow P(D) \geq \sqrt{\frac{2l_0\tau_{\text{max}}}{K S(D)}} - 1.$$  \hfill (27)

Finally, considering the uRLLC constraints, the converted rate optimization problem is summarized as:

$$\max_{P(D)} E\left[\frac{l_0}{D} \cdot \log_2 (1 + KP^2(D)S(D))\right]$$

s.t. $E[P(D)] \leq P_{\text{max}},$  \hfill (28) \hfill (29)

$$P(D) \geq \sqrt{\frac{2l_0\tau_{\text{max}}}{K S(D)}} - 1.$$  \hfill (30)

Lemma 1: The objective function described in (28) is a concave function with respect to $P(D)$, under the condition, $P(D) \geq \sqrt{\frac{1}{K S(D)}}$.

Proof: Please see Appendix A.

B. Solution of the Problem with Lagrangian Formulation

In Appendix A, we have proved that the objective function (28) is concave under a certain condition, i.e., $P(D) \geq \sqrt{\frac{1}{K S(D)}}$. Hence, the solution to the problem (28)-(30) may be found
with the aid of the Lagrangian formulation [42] and Karush-Kuhn-Tucker (KKT) conditions. The Lagrangian for the optimization problem (28) - (30) can be expressed as:

$$L(P(D), \lambda, \mu) = \int_{0}^{d_{\text{max}}} \frac{1}{D} \cdot \log_{2} \left( 1 + KP^2(D)S(D) \right) f_D(D) dD - \lambda \left[ \int_{0}^{d_{\text{max}}} P(D) f_D(D) dD - P_{\text{max}} \right] - \mu \left[ P(D) - \sqrt{\frac{D_{\text{Lp}}}{2\tau_{\text{max}}}} - 1 \right], \quad (31)$$

where \(\lambda\) and \(\mu\) are Lagrange multipliers associated with (29) and (30) for average power and latency constraint, \(f_D(D)\) is the probability distribution function (PDF) of \(D\), and \(d_{\text{max}}\) is the maximum inter-vehicular distance that we can have uRLLC. We assume \(f_D(D)\) to be log-normal distribution for both our numerical and simulation analysis [43], although our formulation is valid for any distribution of \(D\). The PDF of log-normal distribution is given by\( f_D(D) = \frac{1}{D \varsigma \sqrt{2\pi}} e^{-\frac{(\ln(D) - \alpha)^2}{2\varsigma^2}} \), where \(\alpha\) and \(\varsigma\) is the mean and standard deviation of log-normal random variable. In a practical scenario, if \(D \leq d_{\text{stop}}\), the system will automatically enable emergency braking system so that accident is avoided, which is already implemented in the vehicles. But in our case, the regular communication will continue with normal power, even though the distance condition, i.e., \(D \geq d_{\text{stop}}\) is violated. As a result, we integrate between the interval of 0 to \(d_{\text{max}}\) though the probability of having a distance with \(D \leq d_{\text{stop}}\) is very low in our PDF formulation.

The KKT conditions of (31) can be written as follows:

$$\frac{\partial L}{\partial (P(D))} = \int_{0}^{d_{\text{max}}} \frac{1}{D} \cdot \frac{2K P(D) S(D)}{1 + KP^2(D)S(D)} f_D(D) dD - \lambda \int_{0}^{d_{\text{max}}} f_D(D) dD - \mu = 0, \quad (32)$$

$$\lambda \left[ \int_{0}^{d_{\text{max}}} P(D) f_D(D) dD - P_{\text{max}} \right] = 0, \quad (33)$$

$$\mu \left[ P(D) - \sqrt{\frac{D_{\text{Lp}}}{2\tau_{\text{max}}}} - 1 \right] = 0, \quad (34)$$

$$\lambda \geq 0, \quad \mu \geq 0. \quad (35)$$

Solving (32), we can get an expression of the power allocation. However, this does not provide a closed-form solution because (32) does not have closed-form expression for the Lagrange multipliers. Now we relax one of the complementary slackness conditions in (33) and (34) to have a closed-form expression for the power allocation. We set the average power constraint to equality at the optimal power allocation, whereas the latency constraint is set to inequality. We have already mentioned earlier in Section III that average power constraint is more stringent in
Algorithm 1 Optimal power allocation and rate optimization algorithm for continuous-rate problem (28)

Initialization: Initialize $\lambda_{lb}$, $\lambda_{ub}$, tolerance ($\epsilon$), $P_{max}$, BER$_{tgt}$, $i_{max}$, and $i$ according to system requirements.

Output: Optimal power, $P_{opt}$, average rate, $\mathbb{E}[C(P(D), D)]$, and latency, $\tau_c(P(D), D)$.

Step 1: Let $I(\lambda) = \int_{0}^{d_{max}} P_{opt}(D) f_D(D) dD - P_{max}$ from (40). Take initial $\lambda_{lb}$ and $\lambda_{ub}$ and solve for $I(\lambda)$.

\[
i = 0, \quad \text{while } I(\lambda_1) > \epsilon \text{ and } i \leq i_{max} \text{ do}
\]

\[
\text{Set } \lambda = \frac{\lambda_{lb} + \lambda_{ub}}{2} \text{ and calculate } I(\lambda).
\]

\[
\text{if } I(\lambda_{lb}) I(\lambda) < 0 \text{ then}
\]

\[
\lambda_{ub} = \lambda,
\]

\[
\text{else}
\]

\[
\lambda_{lb} = \lambda,
\]

\[
\text{end if}
\]

\[
i = i + 1,
\]

\[
\text{end while}
\]

\[
\text{return optimal } \lambda;
\]

Step 2: Compute the average rate and latency for continuous-rate optimization scheme.

Applying the optimal $\lambda$, allocate optimal power, $P_{opt}$ by (39) and compute optimal rate, $C(P(D), D)$ using (41) and latency using (27).

Our system. So, it is reasonable assumption to relax (34). This leads to a solution with $\lambda \neq 0$ and $\mu = 0$. Hence, (32) reduces to the following:

\[
\frac{\partial \mathcal{L}}{\partial (P(D))} = \int_{0}^{d_{max}} \frac{l_0}{D} \cdot \frac{2K P(D) S(D)}{1 + KP^2(D) S(D)} f_D(D) dD - \lambda \int_{0}^{d_{max}} f_D(D) dD = 0.
\]

(36)

Then, we can find

\[
P(D) = \frac{l_0}{\lambda D} \pm \sqrt{\frac{l_0^2}{\lambda^2 D^2} - \frac{1}{KS(D)}}.
\]

(37)

Lemma 2: The solution with the negative discriminant in (37) is not a valid solution for the power allocation problem.

Proof: Please see Appendix B.

In Appendix B, we prove that the solution with the negative discriminant of $P(D)$ is not a valid solution. Therefore, the solution of (36) is given by

\[
P(D) = \frac{l_0}{\lambda D} + \sqrt{\frac{l_0^2}{\lambda^2 D^2} - \frac{1}{KS(D)}}.
\]

(38)
Algorithm 2 Power allocation and rate optimization algorithm for discrete-rate problem (19)

**Initialization:** Initialize $\lambda_{lb}$, $\lambda_{ub}$, tolerance ($\epsilon$), $P_{\text{max}}$, BER$_{\text{tgt}}$, $M_n$, $i_{\text{max}}$, and $i$ according to system requirements.

**Output:** Optimal power, $P_{\text{opt}}$, average rate, $\mathbb{E}[R(P(D), D)]$, and latency, $\tau_d(P(D), D)$.

**Step 1:** Follow the same procedure as described in Algorithm 1 (Step 1) and find optimal $\lambda$.

**Step 3:** Compute the average rate and latency for discrete-rate optimization scheme.

Applying the optimal $\lambda$, allocate optimal power, $P_{\text{opt}}$ using (39). Compute instantaneous capacity, $C(P(D), D)$ using (26) with the optimal power allocation.

\[
\text{if } \frac{\lambda_0}{D} \cdot \log_2(M_n) \leq C(P(D), D) < \frac{\lambda_0}{D} \cdot \log_2(M_{n+1}) \text{ then}
\]

\[
R(P(D), D) = R(P(D), D) + \frac{\lambda_0}{D} \cdot \log_2(M_n)
\]

end if

where $\lambda$ is found such that (29) is satisfied. Furthermore, to make sure the latency constraint is satisfied, power adaptation should respect the latency constraint in (30). The optimal power should also satisfy the condition of (46) such that the concavity is fulfilled. Finally, considering (30), (38), and (46), the optimal power allocation, $P_{\text{opt}}(D)$, is expressed as

\[
P_{\text{opt}}(D) = \max \left( \frac{l_0}{\lambda D} + \sqrt{\frac{l_0^2}{\lambda^2 D^2} - \frac{1}{K S(D)}} , \sqrt{\frac{2 \frac{D_{\text{opt}}}{\tau_{\text{max}}} - 1}{K S(D)}} , \sqrt{\frac{1}{K S(D)}} \right). \tag{39}
\]

In order to obtain $P_{\text{opt}}(D)$ from (39), we need to find the value of $\lambda$, which satisfies the power constraint at the boundary. The power constraint is computed by substituting $P_{\text{opt}}(D)$ in (29) as:

\[
\int_0^{d_{\text{max}}} P_{\text{opt}}(D) f_D(D) dD = P_{\text{max}}. \tag{40}
\]

A classic Bisection method may be invoked to determine the optimal value of $\lambda$ such that (40) is fulfilled. The Bisection searching procedure of obtaining the optimal $\lambda$ is shown in Step 1 of Algorithm 1. Once the optimal $\lambda$ is found, the optimal power is allocated based on (39). When $P_{\text{opt}}(D)$ is found, the optimal rate is obtained by

\[
\mathbb{E}[C(P(D), D)] = \int_0^{d_{\text{max}}} \frac{l_0}{D} \cdot \log_2 \left( 1 + P_{\text{opt}}^2(D) K S(D) \right) f_D(D) dD. \tag{41}
\]

Since the power adaptation is continuous and the constellation size is discrete, we need to determine the optimum $M_n$ and the optimum power, $P_{\text{opt}}(D)$, in the discrete-rate optimization problem (19) while fulfilling the uRLLC constraints. For each value of $D$, we must decide the associated transmit power and the suitable constellation size to transmit. The choice of constellation size based on the transmit power is analyzed as follows. We determine the instantaneous rate using (26) through optimal power adaptation (39). Then, the discrete-rate is determined
| Parameter, Notation | Value          | Parameter, Notation | Value          |
|---------------------|----------------|---------------------|----------------|
| Angle of irradiance w.r.t. the emitter, $\phi$ | $70^\circ$ | Absolute temperature, $T_A$ | 298 K |
| Semi-angle at half luminance of LED, $\Phi_{1/2}$ | $60^\circ$ | Boltzmann constant, $k$ | $1.38 \times 10^{-23}$ JK$^{-1}$ |
| FoV of the camera lens, $\theta_l$ | $90^\circ$ | Open loop voltage gain, $G$ | 10 |
| Focal length of the camera lens, $f$ | 15 mm | Fixed capacitance per unit area, $C_f$ | $112 \text{ pF cm}^{-2}$ |
| Image pixel size, $a$ | 7.5 $\mu$m | FET channel noise factor, $\Gamma$ | 1.5 |
| Image sensor physical area, $A$ | 10 cm$^2$ | FET trans-conductance, $g_m$ | 30 mS |
| Transmission efficiency of optical filter, $T_s$ | 1 | Noise bandwidth factor, $I_3$ | 0.0868 |
| Refractive index of lens, $\nu$ | 1.5 | Constellation size, $M_n$ | 4, 8, 16, 32, and 64 |
| Number of LEDs at each row, $N_{\text{LEDs}}$ | 30 | Camera-frame rate, $W_{\text{fps}}$ | 1000 fps |
| Equivalent noise bandwidth, $B$ | 2 MHz | Electron charge, $q$ | $1.6 \times 10^{-19}$ C |
| Background current, $I_{\text{bg}}$ | 5100 $\mu$A | Resolution of image, $w$ | $512 \times 512$ pixels |
| Noise bandwidth factor for a rectangular pulse, $I_2$ | 0.562 | Packet size, $L_p$ | 5 kbits |

Fig. 4. BER versus SNR (dB) for various modulation schemes considering AoI = $60^\circ$ and fixed transmit power at 1.2 W, when $D$ is varying.

by mapping the rate associated with each modulation order region to the continuous-rate, i.e., $C(P(D), D)$. Specifically, the optimal discrete-rate for a certain $P_{\text{opt}}(D)$ is characterized as

$$R(P(D), D) = \frac{l_0}{D} \cdot \log_2(M_n), \quad \text{if} \quad \frac{l_0}{D} \cdot \log_2(M_n) \leq C(P(D), D) < \frac{l_0}{D} \cdot \log_2(M_{n+1}). \quad (42)$$

In this manner, we determine the optimal discrete-rate subject to transmit power allocation and a small set of modulation scheme while satisfying uRLLC constraints. Finally, we summarize the procedure of evaluating the optimal $\lambda$, $P_{\text{opt}}(D)$, and optimal rate for both continuous-rate and discrete-rate optimization schemes in **Algorithm 1** and **Algorithm 2**, respectively.
VI. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

In this section, simulations are conducted to investigate the performance of the proposed system model and rate optimization schemes in vehicular OCC. We start by evaluating different performance metrics of the proposed system model to get a better understanding of the interplay among the various parameters of our system. We propose an adaptive modulation scheme using BPSK and M-QAM with five different constellations, $M_n = \{4, 8, 16, 32, 64\}$. Finally, we examine the performance of the proposed joint power allocation and rate optimization schemes subject to average transmit power and uRLLC constraints for different transmission rate strategies and target BERs. Here, we consider a transmitter size of 300 ($30 \times 10$) LEDs with 0.5 cm spacing between each LED and a 1000 fps camera for the receiver, where the resolution of the received image is $512 \times 512$. The rest of the simulation-related parameters are summarized in Table I. We would like to note that the results in Sections VI-A and VI-B, are obtained for constant power allocation as we analyze the performance of the proposed OCC model. In contrast, the results in Section VI-C, are generated by considering optimal power allocation as computed by (39).

A. Performance of BER Modelling

In this sub-section and the following sub-section, we analyze the performance of the proposed system model for BER, spectral efficiency, and latency at different inter-vehicular distances and AoIs. We start by comparing the BER versus SNR (dB) for the chosen modulation set with a
fixed transmit power of 1.2 Watt. The results for different inter-vehicular distances and AoIs are illustrated in Fig. 4 and Fig. 5, respectively. The plots demonstrate that we achieve better BER performance at higher-order modulation, but this comes at the cost of higher SNR level. In our evaluation, we do not vary the distance and AoI at the same time. While varying distance, we change it from 0 m to 150 m by keeping the AoI at 60°, similarly we vary AoIs between 0° to 90° by keeping the distance to 50 m. In this manner, we justify that the same BER performance can be achieved at various distances and AoIs while using different modulation schemes.

In Fig. 6 and Fig. 7, we evaluate the achieved BER performance for the different modulation schemes at varying distances and AoIs, respectively. Fig. 6 shows that BPSK satisfies target BER ($10^{-4}$) up to 82 m, and for 64-QAM, it is satisfied at 52 m. Similarly, in Fig. 7, target BER ($10^{-4}$) is satisfied at 80° and 62° for BPSK and 64-QAM, respectively. The plots confirm that at a shorter distance and narrower AoI, the modulation order will be higher due to higher SNR at the receiver. Because at the narrower AoI, the strength of the light beam on the image sensor is strong, which increases channel gain. Alternatively, at the shorter distance, the SNR gets higher. Thus, the target BER can be achieved while maintaining the trade-off between the modulation order and distances or AoIs.

B. Spectral Efficiency and Latency Performance

The achieved spectral efficiency and observed latency improvements of the proposed system are presented in Fig. 8 for various distance values. In this evaluation, we consider, $10^{-4}$ and $10^{-5}$,
as the target BER for performance comparison. We determine the distance that satisfies the target BER, and then, adopt the highest modulation scheme from the available schemes using Fig. 6. Then, we calculate spectral efficiency at that corresponding modulation scheme and distance. We achieve spectral efficiency of 6 bps/Hz (Fig. 8) for distance until 48 m (for BER = $10^{-5}$) and 52 m (for BER = $10^{-4}$) using 64-QAM. Likewise, we notice a spectral efficiency of 2 bps/Hz from 74 m to 81 m and 69 m to 76 m at the target BER of $10^{-4}$ and $10^{-5}$, respectively. Please note that the above evaluation is ideal since the modulation level is perfectly adapted, and the target BER is known to both the transmitter and receiver in advance. As a result, the transmitter and receiver can choose the modulation order and target BER from the predefined sets.

For latency evaluation, we first calculate the achievable rate using (15), considering $w$ as 512 x 512 pixels. Then, we compute the transmission latency for a packet size of 5 kbits using (16). Here, we consider transmission latency to be equal to the end-to-end latency because we process small amount of data in our system. The results are presented in Fig. 8 for distances from 0 m to 90 m and two different target BERs, i.e., $10^{-4}$ and $10^{-5}$. This evaluation shows that our system can achieve the latency of 1ms at 52 m and 48 m at target BER of $10^{-4}$ and $10^{-5}$, respectively. From Fig. 8, it can be seen that we gain 1ms latency and 6 bps/Hz spectral efficiency using 64-QAM at constant power. Therefore, we note that both latency and BER requirements are satisfied at 60$^0$. As a result, we consider AoI as 60$^0$ for our optimization problem formulation to deal with the complexity of changing AoI in practice and its induced latency.
From Fig. 8, we can conclude that the use of adaptive modulation offers higher spectral efficiency and lower latency. Whereas, a single modulation scheme offers fixed-rate and latency having limitations in distance coverage and BER requirements. For example, 64-QAM can satisfy a target BER of $10^{-4}$ and a latency of 1.2 ms up to 52 m. Thus, beyond this distance, we need another modulation scheme for satisfying the target BER, i.e., 32-QAM, 16-QAM, and so on. Similarly, BPSK meets the target BER of $10^{-4}$ up to 83 m but offers a lower rate, i.e., 1 bps/Hz, and higher latency of 7.5 ms. Thus, it can be said that adaptive modulation provides better performance while satisfying the trade-off between reliability and latency requirements.

C. Optimal Power Allocation and Rate Optimization

In this sub-section, we compare the performance of the proposed continuous-rate optimization scheme (41) and discrete-rate optimization scheme (42) presented in Section V. For this evaluation, we consider two different target BERs, i.e., $10^{-4}$, $10^{-5}$, while we assume the maximum transmission latency as 10 ms. We also examine $K = 1$ for the sake of performance comparison. The continuous-rate optimization problem is solved by Algorithm 1, whereas the discrete-rate optimization problem is solved by Algorithm 2. We allocate the optimal power using (39). For all the presented results, we consider 100000 Monte-Carlo simulations to evaluate the average rate. The inter-vehicular distance is given by the log-normal distribution for our evaluation considering $\alpha = 3.78$ and $\varsigma = 0.21$. Communication distances are considered up to 90 m.
Fig. 9. Peak power versus average power under two different target BERs.

We first investigate the effect of our power allocation scheme, which is to satisfy an average power constraint on the consumed peak power. The peak power indeed depends on power allocation strategy of (39). In Fig. 9, we present the comparison of peak power versus average power limits for two different target BERs. From Fig. 9, we see that the peak power remains flat for average power from 5 dBW to 12 dBW for BER of $10^{-4}$ and 5 dBW to 12.5 dBW for BER of $10^{-5}$. This happens because the second term of power allocation of (39) becomes dominant when average power remains between these intervals. Beyond these intervals, the first part of (39) becomes dominant, and therefore, the peak power increases with the increase of average power limit. The peak power converges with each other for both target BERs because the contribution of BER on the power allocation is minimal from 13 dBW to 15 dBW. From Fig. 9, it is also seen that the peak power is slightly bigger than the average power limit. For example, at an average power of 10 dBW, the peak power is 15.55 dBW, and at an average power of 14 dBW, the peak power is 17.4 dBW. This makes clear that the difference between average and peak power is not significantly large.

To further demonstrate the difference between peak and average power limit, we illustrate the comparison between peak-to-average power ratio (PAPR) and average power limit for two target BERs in Fig. 10. From Fig. 10, we see that the maximum PAPR is 3.22 for BER of $10^{-5}$ and 3.11 for BER of $10^{-4}$, and PAPR gradually falls to 1.22 for both BER values as the average power increases to 15 dBW. Similar to Fig. 9, the PAPR for both BERs converge to the same
values as the power reaches to 13 dBW because the signal becomes stronger when the transmit power increases. Therefore, the contribution of BER to power allocation becomes insignificant to change the allocated power. From the above observations, we can conclude that if we satisfy the average power constraint, the peak power will not be significantly higher than the average power limit, which is indeed tighter than the peak power to optimize the system performance. Therefore, we only study average power constraint in our proposed optimization problem.

We then examine the optimal power allocation for two different target BERs ($10^{-4}$, $10^{-5}$) at various distances by evaluating (39), and the results are presented in Fig. 11. For this evaluation, we consider the transmitter average power limit, $P_{\text{max}}$, as 5 dBW. From Fig. 11, we see that the allocated power decreases gradually as the distance increases. We also observe identical allocated power values for both BERs as far as 50 m distance. This happens because at smaller distance, the signal is stronger, and hence BERs can not alter the allocated power values. We can also note that once the power approaches to 2.7 dBW at 64 m (for BER=$10^{-5}$) and 68 m (for BER=$10^{-4}$), the power increases sharply. This is because we have a maximum operator of three power components at (39), and at this instance, the latency constraint (30) is activated into our power allocation formulation. Therefore, when the power reaches the lowest value, the allocated power increases due to the latency condition. Thus, we can conclude that the optimal power allocation is bounded by BER and latency requirements, in our optimization problem.

We then compare the average rate (in Mbps) of various schemes with respect to the average
power limit, and the results are shown in Fig. 12. As shown in Fig. 12, for continuous-rate problem, the plots show a similar pattern for both target BER, i.e., $10^{-4}$ and $10^{-5}$ compared to when $K = 1$. It is seen that with an increase in average power, the average rate of continuous-rate optimization scheme improves due to the higher available power. There is a constant gap between the plots due to the target BER requirement. However, for the discrete-rate optimization scheme, as the power approaches to 12 dBW and beyond that value, the average rate merges for both BERs. Since we choose the modulation order $M_n$ as $2^n$, the modulation order is close to each other at the smaller power values and smaller $n$. Therefore, a small change in power may lead to modulation order changes. But, as the power becomes higher, the difference between the modulation order also gets far away from each other. However, the difference between the BERs is not significant to lead to modulation order to switch into the next order. As a result, the modulation scheme is settling to the same range, and therefore, it confines the rate within.

Whereas the performance of average latency to average transmit power for continuous-rate and discrete-rate optimization schemes is illustrated in Fig. 13. We notice an improvement in achievable latency with the increase of power. The link quality improves when more power is allocated to the communication link, which leads to higher average latency consequently. However, for the discrete-rate problem, the average latency becomes flat and coincide with each other for both target BER when the power goes beyond 13 dBW. This is because, after this power value, the transmit power and target BER is not strong enough to promote an increase of
the modulation order toward the upper order.

VII. CONCLUSIONS

In this paper, we present a joint optimal power allocation and rate optimization scheme for vehicular OCC that respect uRLLC constraints. Firstly, we study the proposed channel model and several performance metrics where the latency is modelled as transmission latency and reliability is modelled by satisfying the target BER thresholds. Then, we formulate the original discrete-rate optimization problem as a MIP, considering a small set of modulation schemes and the uRLLC constraints. To reduce the complexity of the MIP problem, we convert the optimization problem into continuous-rate optimization scheme by considering continuous modulation, which enables us to find an analytical solution. To solve the problem, we propose an algorithm to determine the optimal power allocation, which is based on Lagrangian formulation and Bisection method. We also introduce an algorithm to solve the discrete-rate optimization problem. We verify our proposed scheme through simulation analysis. The performance of average rate and latency for continuous-rate problem shows a constant gap, which is just a function of the target BER. However, the average rate and latency for discrete-rate problem converge with each other for both BERs when the power becomes higher, and the gap between the constellation point increases.
APPENDIX A
PROOF OF LEMMA 1

In this appendix, we prove Lemma 1. For this purpose, we express our optimization problem in (28) as

\[ C(P(D), D) = \frac{I_0}{D} \cdot \log_2 \left( 1 + KP^2(D) S(D) \right). \]  

(43)

To ensure that \( C(P(D), D) \) is concave with respect to \( P(D) \), we need to satisfy that \( \frac{\partial^2 C(P(D), D)}{\partial P^2(D)} \leq 0 \). Therefore, the first derivative of \( C(P(D), D) \) is given by:

\[ \frac{\partial C(P(D), D)}{\partial P(D)} = \frac{2 Kl_0 S(D) P(D)}{D(1 + KS(D)P^2(D))}. \]  

(44)

Then, the second derivative of \( C(P(D), D) \) is given by:

\[ \frac{\partial^2 C(P(D), D)}{\partial P^2(D)} = \frac{2 Kl_0 S(D)}{D} \cdot \frac{1 - KS(D)P^2(D)}{(1 + KS(D)P^2(D))^2}. \]  

(45)

The denominator of (45) is always positive for any positive \( P(D) \). Applying the concavity condition, (45) can be re-expressed as

\[ 1 - KS(D)P^2(D) \leq 0, \quad \Rightarrow \quad P(D) \geq \sqrt{\frac{1}{KS(D)}}. \]  

(46)

We see that \( KS(D) \) is always greater than one for any \( D \), and \( P(D) \) is always greater than one in our system. As a result, for any positive value of \( P(D) \), the condition \( KS(D)P^2(D) > 1 \) is fulfilled. Thus, \( \frac{\partial^2 C(P(D), D)}{\partial P^2(D)} \) is always negative as it satisfies the concavity condition. Finally, we conclude that \( C(P(D), D) \) is concave function with respect to \( P(D) \).
APPENDIX B

PROOF OF LEMMA 2

We verify Lemma 2 by testing the negative root of $P(D)$ while satisfying the condition of (46). So, by substituting the negative root of $P(D)$ of (37) in (46), we find that

$$\frac{l_0}{\lambda D} - \sqrt{\frac{l_0^2}{\lambda^2 D^2} - \frac{1}{KS(D)}} > \sqrt{\frac{1}{KS(D)}}. \tag{47}$$

After further simplification, we get

$$\lambda^4 - l_0^2 KS(D)\lambda^2 D^{-2} > 0, \quad \Rightarrow \quad \lambda^2 - l_0^2 KS(D)D^{-2} > 0 \quad \text{as} \quad \lambda \neq 0,$$

$$\Rightarrow \quad \lambda > \sqrt{l_0^2 KS(D)D^{-2}}. \tag{48}$$

If we check (37) using the condition of (48), the value inside the square root becomes negative and the power contains the imaginary component. As a result, the solution with the negative discriminant is not acceptable for our power allocation.

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