Aspects of finite field-dependent symmetry in SU(2) Cho–Faddeev–Niemi decomposition

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In this Letter we consider SU(2) Yang-Mills theory analysed in Cho–Faddeev–Niemi variables which remains invariant under local gauge transformations. The BRST symmetries of this theory is generalized by making the infinitesimal parameter finite and field-dependent. Further, we show that under appropriate choices of finite and field-dependent parameter, the gauge-fixing and ghost terms corresponding to Landau as well as maximal Abelian gauge for such Cho–Faddeev–Niemi decomposed theory appear naturally within functional integral through Jacobian calculation.

I. INTRODUCTION

The original formulation of Yang-Mills (YM) theory is suitable to explain the theory in the high-energy limit. In the high-energy limit, YM theory is asymptotically free and can be solved perturbatively. However, in the low-energy region, it fails to describe the dynamics correctly due to strong coupling. In studying the low-energy dynamics of YM theory and quantum chromodynamics (QCD), it is important to extract the most relevant (the dynamical or topological) degrees of freedom.

The Cho–Duan–Ge–Faddeev–Niemi decomposition, also called as Cho–Faddeev–Niemi (CFN) decomposition in literature, of the non-Abelian connection (gauge potential) of YM theory, was originally proposed by Cho [1], Duan and Ge [2], and Faddeev and Niemi [3] independently. CFN decomposition enables us to explain and understand some of the low-energy phenomena by separating the topological defects in a gauge-invariant manner. This has been done by introducing a color vector field to extract explicitly the magnetic monopole as a topological degree of freedom from the gauge potential without introducing the fundamental scalar field in YM theory.

Quark (color) confinement is a phenomenon in the low-energy dynamics due to strong interactions. Quark confinement is believed to be explained by topological defects including magnetic monopoles, vortices and merons. The monopole condensation provides indispensable description of the quark confinement through a dual Meissner effect [4]. It is conjectured that the restricted part of QCD which comes from the “Abelian projection” of the theory to its maximal Abelian subgroup is responsible for the dynamics of the dual Meissner effect [1]. The basic idea behind Abelian projection is that the partial gauge-fixing can extract the physical degrees of freedom relevant to the long distance structure of QCD [5]. A magnetic monopole appears as a defect (singularity) of the partial gauge-fixing at degenerate points of the operator to be diagonalized through the Abelian projection. The most efficient partial gauge-fixing from this point of view is known to be the maximally Abelian gauge (MAG) [6], although there are many candidates for Abelian gauge [7]. The numerical simulations [8] have confirmed that only MAG leads to the Abelian dominance in the theory at a long-distance scale [9] and also the magnetic monopole dominance for the string tension [10].

The BRST transformation plays a very important role in the proof of renormalizability and unitarity of the gauge theories [11–14]. The BRST quantization of the CFN decomposed YM theory in the continuum formulation has been studied thoroughly [15]. To determine the nilpotent BRST transformations for all the fields the nilpotency property has been utilized [15]. At this place, one needs more ghost and antighost fields than the usual YM theory reflecting the enlarged local gauge symmetry of the CFN decomposed

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YM theory. It has also been discussed that in order to remove the redundancy of the gauge degrees of freedom of this theory, it is necessary to incorporate the Landau covariant gauge condition also in addition to the new MAG.

The generalization of BRST transformation by making the infinitesimal parameter finite and field-dependent is known as finite field-dependent BRST (FFBRST) transformation [16]. Such generalizations have found various applications in gauge field theories [16–27]. For example, a correct prescription for poles in the gauge field propagators in noncovariant gauges has been derived by connecting covariant gauges and noncovariant gauges of the theory by using FFBRST transformation [22]. The Gribov-Zwanziger theory [28, 29] which is free from Gribov copies plays a crucial role in the non-perturbative low-energy region while it can be neglected in the perturbative high-energy region, has also been related to the YM theory in Euclidean space through FFBRST transformation [17, 18]. However, such formulation has not been investigated for YM theory explaining low-energy dynamics. With this motivation we implement this formulation in the CFN decomposed SU(2) YM theory.

In this Letter, we generalize the BRST transformations for CFN decomposed YM theory. This is achieved, first by making all the fields \( \kappa \) (a parameter) dependent through continuous interpolation. Then, the infinitesimal constant parameter characterizing the BRST symmetry is made field-dependent. After that, we integrate such infinitesimal field-dependent parameter and therefore the finite field-dependent BRST parameter is obtained. Further, we construct the FFBRST transformations for CFN decomposed YM theory characterized by an arbitrary finite field-dependent parameter. Such FFBRST transformations are symmetry of the effective action only but not of the partition function because the path integral measure is not covariant under these. The Jacobian of the path integral measure in the expression of partition function changes non-trivially under such FFBRST transformation. We show that for an appropriate choice of this parameter the Jacobian of path integral measure leads to gauge-fixing and corresponding ghost terms in the theory naturally. Therefore we claim that the gauge-fixing and ghost terms are contribution of Jacobian of path integral measure under FFBRST transformation.

The Letter is presented as follows. In Sec. II, we study the preliminaries about CFN decomposition of YM theory. The nilpotent BRST symmetry is also discussed in this section. Sec. III is devoted to study the methodology used in generalizing BRST symmetry under which Jacobian changes non-trivially. The discussion about the evaluation of the non-trivial Jacobian is made in section IV. Further, in Sec. V, we show the emergence of gauge-fixing and ghost terms of CFN decomposed theory naturally. Last section is reserved for discussions and conclusions.

II. THE CFN DECOMPOSITION OF YANG-MILLS THEORY

In this section, we discuss the Cho–Faddeev–Niemi (CFN) decomposition of SU(2) YM theory, which explains the infra-red limit of the theory, and its BRST invariance. To achieve the CFN decomposition of connection \( A_\mu(x) \) of YM theory, the connection is separated in Abelian projection (called as restricted potential) part and in remaining gauge covariant potential part because Abelian projection dominates in infra-red limit. The Abelian projection of connection is achieved by introducing a three component vector \( \hat{n}(x) \) of unit length, i.e.,

\[
\hat{n}(x) \cdot \hat{n}(x) = 1
\]

Now, the CFN decomposition of the connection \( A_\mu \) in terms of component vector \( \hat{n} \) is given by,

\[
A_\mu(x) = \hat{A}_\mu(x)\hat{n}(x) + g^{-1}\partial_\mu\hat{n}(x) \times \hat{n}(x) + X_\mu(x),
\]

where \( g \) is coupling constant, \( \hat{A}_\mu(x) \) is the Abelian component (the electric potential) of the connection and \( X_\mu(x) \) is gauge covariant potential orthogonal to \( \hat{n}(x) \), i.e.,

\[
\hat{n}(x) \cdot X_\mu(x) = 0.
\]
The connection $A_\mu$ given in Eq. (2) is further written in terms of Abelian projected vector field $V_\mu$ for later convenience as

$$A_\mu(x) = V_\mu(x) + X_\mu(x),$$

where the Abelian projection $V_\mu(x)$ is defined by

$$V_\mu(x) = \hat{A}_\mu(x)\hat{n}(x) + g^{-1}\partial_\mu\hat{n}(x)\times\hat{n}(x).$$

This Abelian projection (restricted potential) leaves the component vector $\hat{n}$ invariant under parallel transport

$$D[V]\hat{n} := \partial_\mu\hat{n} + gA_\mu(x)\times\hat{n} = 0.$$  

Now, exploiting equations (2) and (3), the electric potential $\hat{A}_\mu(x)$ is expressed in terms of $\hat{n}$ and $A_\mu$ as follows:

$$\hat{A}_\mu(x) = \hat{n}(x)\cdot A_\mu(x).$$

Further, relation (6) reflects that the gauge covariant potential $X_\mu$ also depends only on $\hat{n}$ and $A_\mu$ as follows:

$$X_\mu(x) = g^{-1}\hat{n}(x)\times D_\mu[A]\hat{n}(x).$$

Incorporating these CFN variables, the classical Lagrangian density for YM theory is defined by

$$\mathcal{L}_{YM} = -\frac{1}{4}\hat{F}^{\mu\nu},$$

where the field-strength tensor $\hat{F}_{\mu\nu}$ has the following expression:

$$\hat{F}_{\mu\nu} = \left[\partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu - g^{-1}\hat{n}\cdot(\partial_\mu\hat{n} \times \partial_\nu\hat{n})\right]\hat{n} + D_\mu[V]X_\nu - D_\nu[V]X_\mu + gX_\mu \times X_\nu.$$  

The above Lagrangian density described in terms of CFN variables remains invariant under following local gauge symmetry:

$$\delta A_\mu(x) = D_\mu[A]\omega(x),$$

$$\delta\hat{n}(x) = g\hat{n}(x)\times\theta(x) = g\hat{n}(x)\times\theta_\perp(x),$$

where $\omega(x)$ is an arbitrary local parameter and $\theta(x)$ is an angle of local rotation with component $\theta_\perp(x)$ which is orthogonal to $\hat{n}$, i.e., $\hat{n}\cdot\theta_\perp(x) = 0$. Now, using equations (7), (8) and (11), we are able to write the gauge transformations of variables $\hat{A}_\mu$ and $X_\mu$ as follows:

$$\delta\hat{A}_\mu(x) = g(\hat{n}(x)\times A_\mu(x))\cdot(\omega_\perp(x) - \theta_\perp(x)) + \hat{n}(x)\cdot\partial_\mu\omega(x),$$

$$\delta X_\mu(x) = gX_\mu(x)\times(\omega_\parallel(x) + \theta_\perp(x)) + D_\mu[V](\omega_\perp(x) - \theta_\perp(x)), $$

where $\parallel$ and $\perp$ denote the components of variables parallel and perpendicular to $\hat{n}$. In this context, two types of local gauge transformations, the active (background) gauge transformation and the passive (quantum) gauge transformation have been studied [31]. The gauge transformations in Eqs. (11) and (12) are identified as the active gauge transformation. For particular case: $\omega_\perp(x) = \theta_\perp(x)$, these active transformations given (11) and (12) reduce to

$$\delta_\omega\hat{n}(x) = g\hat{n}(x)\times\omega'(x),$$

$$\delta_\omega\hat{A}_\mu(x) = \hat{n}(x)\cdot\partial_\mu\omega'(x),$$

$$\delta_\omega X_\mu(x) = gX_\mu(x)\times\omega'(x),$$

$$\delta_\omega V_\mu(x) = D_\mu[V]\omega'(x),$$

(13)
with the local parameter \( \omega'(x) = (\omega_\parallel(x), \omega_\perp(x) = \theta_\perp(x)) \). However, the passive gauge transformation are defined by \[31\]

\[
\delta \omega \hat{n} = 0, \\
\delta \omega A_\mu = \hat{n} \cdot D_\mu [A] \omega, \\
\delta \omega X_\mu = D_\mu [A] \omega - \hat{n} (\hat{n} \cdot D_\mu [A] \omega), \\
\delta \omega V_\mu = \hat{n} (\hat{n} \cdot D_\mu [A] \omega).
\]

(14)

Here we note that the above two gauge transformations given in equations (13) and (14) are not independent.

A. Gauge-fixed action and BRST symmetry

The presence of gauge symmetries in the YM theory decomposed in CFN variables reflects that theory is enriched with some redundant degrees of freedom. It is also obvious from LHS and RHS of relation \[2\] that RHS has two extra degrees of freedom introduced by \( \hat{n} \). To fix these two redundant degrees of freedom we put an extra constraints on \( X_\mu \) as

\[
D_\mu [V] X^\mu = 0,
\]

(15)

which is called as new MAG \[15, 32\]. The nilpotent BRST symmetry for YM theory decomposed in CFN variables has been discussed in great details \[15\]. To write the BRST symmetry transformations, we introduce two ghost fields \( C_\omega \) and \( C_\theta \) corresponding to the parameters \( \omega \) and \( \theta \), respectively, characterising gauge transformations. Then, the BRST transformations for \( A_\mu \) and \( \hat{n} \), by replacing \( \omega \) and \( \theta \) with \( C_\omega \) and \( C_\theta \) respectively in (11), is defined as

\[
s_b A_\mu = -D_\mu [A] C_\omega, \quad s_b \hat{n} = -g \hat{n} \times C_\theta = -g \hat{n} \times C_\theta^\perp,
\]

(16)

where we have acquired the fact that \( \hat{n} \cdot C_\theta = 0 \).

BRST symmetry of \( \omega \) sector:

To analyse the BRST transformation for fields in \( \omega \) sector we use mainly the nilpotency property of BRST operator. The nilpotency of the BRST symmetry for \( A_\mu \) reflects the BRST of ghost field \( C_\omega \) as follows \[15\]:

\[
s_b C_\omega = -\frac{g}{2} C_\omega \times C_\omega,
\]

(17)

One can easily check the nilpotency of \( C_\omega \), i.e., \( s_b^2 C_\omega = 0 \). Making analogy to standard YM case, we write the BRST symmetry for the antighost field \( \bar{C}_\omega \) by

\[
s_b \bar{C}_\omega = iB_\omega, \quad s_b B_\omega = 0,
\]

(18)

where \( B_\omega \) is Nakanishi-Lautrup type auxiliary field. The BRST transformation of \( B_\omega \) is an outcome of nilpotency property of BRST transformation for \( \bar{C}_\omega \).

BRST symmetry of \( \theta \) sector:

The BRST symmetry of the fields in \( \theta \) sector can be written as follows:

\[
s_b C_\theta = g C_\theta \times C_\theta, \\
s_b \bar{C}_\theta = iB_\theta, \\
s_b B_\theta = 0,
\]

(19)

where \( B_\theta \) is Nakanishi-Lautrup type auxiliary field for \( \theta \) sector.
With the help of BRST transformations given in Eqs. \((16), (17), (18), (19)\) and \((20)\), we are able to to write the BRST transformations for CFN variables \(A_\mu\) and \(X_\mu\):

\[
\begin{align*}
  s_b A_\mu &= -g (\hat{n} \times A_\mu) \cdot (C^\perp_\omega - C^\parallel_\phi) - \hat{n} \cdot \partial_\mu C_\omega, \\
  s_b X_\mu &= -g X_\mu \times (C^\perp_\omega + C^\parallel_\phi) - \partial_\mu [V](C^\perp_\omega - C^\parallel_\phi),
\end{align*}
\]

(20)

where \(\parallel\) and \(\perp\) denote the components which are parallel and perpendicular to \(\hat{n}\) respectively. Now, the gauge-fixing and ghost terms of the YM Lagrangian density described in CFN variables is given by [15]

\[
\mathcal{L}_{GF+FP} = \mathcal{L}^\omega_{GF+FP} + \mathcal{L}^\theta_{GF+FP},
\]

(21)

where the gauge-fixing and ghost terms for \(\omega\) and \(\theta\) sectors are defined, respectively, as

\[
\mathcal{L}^\omega_{GF+FP} = B_\omega \cdot \partial^\mu A_\mu + i \tilde{C}_\omega \partial^\mu D_\mu [A] C_\omega,
\]

\[
\mathcal{L}^\theta_{GF+FP} = B_\theta \cdot D^\mu [V] X_\mu - i \tilde{C}_\theta \cdot D^\mu [V - X] D_\mu [V + X] (C_\theta - C_\omega).
\]

(22)

\(\mathcal{L}_{GF+FP}\) remains invariant under above mentioned sets of BRST transformations. Now, we are able to define the partition function (vacuum to vacuum transition amplitude) for CFN decomposed YM theory in Euclidean space as follows:

\[
Z(0) = \int D\phi \ e^{-S_{eff}},
\]

(23)

where \(D\phi\) is path integral measure written compactly in terms of generic field \(\phi(\equiv \hat{n}, A_\mu, C_\omega, \tilde{C}_\omega, B_\omega, \tilde{C}_\theta, B_\theta, X_\mu)\). The effective action for YM theory decomposed in CFN variables, \(S_{eff}\), is defined as a sum of classical part, gauge-fixing part and ghost part, i.e.,

\[
S_{eff} = \int d^4 x \left[ L_{YM} + \mathcal{L}_{GF+FP} \right].
\]

(24)

These partition function and effective action are invariant under BRST transformations given in equations \((16), (17), (18), (19)\) and \((20)\).

III. GENERALIZED BRST SYMMETRY IN EUCLIDEAN SPACE

In this section, we generalize the sets of BRST transformations (obtained in previous section) written for CFN variables by making the infinitesimal Grassmann parameter finite and field-dependent. For this purpose, we first define the infinitesimal BRST transformations \((\delta_b)\) written, compactly, for generic field \(\phi(\equiv \hat{n}, A_\mu, C_\omega, \tilde{C}_\omega, B_\omega, \tilde{C}_\theta, B_\theta, X_\mu)\),

\[
\delta_b \phi = s_b \phi \ \delta \Lambda,
\]

(25)

where \(\delta \Lambda\) is an infinitesimal, space-time independent parameter which belongs to Fermi-Dirac statistics. Now, we make \(\delta \Lambda\) finite and field-dependent without affecting its properties. Such a generalization of BRST transformation is known as finite field-dependent BRST (FFBRST) transformation [16]. The mechanisms of FFBRST transformation are as follows: first of all we start by making the infinitesimal parameter \((\delta \Lambda)\) field-dependent with introduction of an arbitrary parameter \(\kappa(0 \leq \kappa \leq 1)\), i.e., \(\delta \Lambda[\phi(x, \kappa)]\), where all the fields generically \(\phi(x)\) of the theory depend on \(\kappa\) such that \(\phi(x, \kappa = 0) = \phi(x)\) are initial fields and \(\phi(x, \kappa = 1) = \phi'(x)\) are FFBRST transformed fields.

Now, the infinitesimal field-dependent BRST transformations for generic fields \(\phi\) are defined explicitly as [16],

\[
\frac{d \phi(x, \kappa)}{d \kappa} = s_b \phi(x) \Lambda'[\phi(x, \kappa)],
\]

(26)
with infinitesimal field-dependent parameter $\Lambda'[\phi(x, \kappa)] = d(\delta\Lambda[\phi(x, \kappa)])/dk$. The FFBRST transformation $(\delta_f)$ is then constructed by integrating the above infinitesimal field-dependent transformation from $\kappa = 0$ to $\kappa = 1$, i.e.,

$$\delta_f \phi(x) := \phi'(x, \kappa = 1) - \phi(x, \kappa = 0) = s_b \phi(x) \Lambda[\phi(x)],$$

where

$$\Lambda[\phi(x)] = \int_0^1 d\kappa \, \Lambda'[\phi(x, \kappa)],$$

is finite field-dependent parameter.

Following this methodology, we construct the FFBRST transformation for CFN decomposed YM theory as follows:

$$
\begin{align*}
\delta_f A_\mu & = -D_\mu[A]C_\omega \Lambda[\phi(x)], \\
\delta_f \dot{n} & = -g \dot{n} \times C_\theta \Lambda[\phi(x)] = -g \dot{n} \times C_\theta \Lambda[\phi(x)], \\
\delta_f C_\omega & = -\frac{g}{2} C_\omega \times C_\omega \Lambda[\phi(x)], \\
\delta_f \bar{C}_\omega & = iB_\omega \Lambda[\phi(x)], \quad \delta_f B_\omega = 0, \\
\delta_f C_\theta & = g C_\theta \times C_\theta \Lambda[\phi(x)], \\
\delta_f \bar{C}_\theta & = iB_\theta \Lambda[\phi(x)], \quad \delta_f B_\theta = 0, \\
\delta_f \dot{A}_\mu & = - (g \dot{n} \times A_\mu) \cdot (C_\omega^\perp - C_\theta^\perp) + \dot{n} \cdot \partial_\mu C_\omega \Lambda[\phi(x)], \\
\delta_f X_\mu & = - \left( g X_\mu \times (C_\omega^\perp + C_\theta^\perp) + D_\mu[V] (C_\omega^\perp - C_\theta^\perp) \right),
\end{align*}
$$

where $\Lambda[\phi(x)]$ is an arbitrary finite field-dependent parameter. The effective action given in Eq. (24) is invariant under this FFBRST transformations, however, the generating functional given in (23) is not because the Jacobian of path integral measure in the expression of partition function gives some non-trivial contribution to it (for detail see e.g. [16]).

### IV. Method for Evaluating the Jacobian

For the symmetry of the generating functional we need to calculate the Jacobian of the path integral measure in the definition of generating functional. The Jacobian of the path integral measure for FFBRST transformation $J$ can be evaluated for some particular choices of the finite field dependent parameters $\Lambda[\phi(x)]$. We start with the definition of path integral measure. [16]

$$
D\phi = J(\kappa) \, D\phi(\kappa) = J(\kappa + d\kappa) \, D\phi(\kappa + d\kappa).
$$

Now the transformation from $\phi(\kappa)$ to $\phi(\kappa + d\kappa)$ is infinitesimal in nature, thus the infinitesimal change in Jacobian can be calculated as [16]

$$
\frac{J(\kappa)}{J(\kappa + d\kappa)} = \Sigma_{\phi} \pm \frac{\delta \phi(x, \kappa)}{\delta \phi(x, \kappa + d\kappa)},
$$

where $\Sigma_{\phi}$ sums over all fields involved in the path integral measure and $\pm$ sign refers to whether $\phi$ is a bosonic or a fermionic field. Using the Taylor expansion we calculate the above expression as [16]

$$
\frac{1}{J(\kappa)} \frac{dJ(\kappa)}{dk} = - \int d^4 x \left[ \Sigma_{\phi}(\pm) s_b \phi(x, \kappa) \frac{\partial \Lambda'[\phi(x, \kappa)]}{\partial \phi(x, \kappa)} \right].
$$
The Jacobian, $J(\kappa)$, can be replaced (within the functional integral) in Euclidean space as

$$J(\kappa) \to e^{-(S_1[\phi(x, \kappa), \kappa])},$$

(33)

if and only if the following condition is satisfied

$$\int D\phi(x) \left[ \frac{1}{J(\kappa)} \frac{dJ(\kappa)}{d\kappa} + \frac{dS_1[\phi(x, \kappa)]}{d\kappa} \right] e^{-(S_1[\phi(x, \kappa), \kappa] + S_1)} = 0,$$

(34)

where $S_1[\phi]$ is some local functional of fields and satisfies the initial condition

$$S_1[\phi(\kappa = 0)] = 0.$$

(35)

V. EMERGENCE OF GAUGE-FIXING AND GHOST TERMS IN THE THEORY

In this section, we calculate the Jacobian of path integral measure under FFBRST transformation given in Eq. (29) for a particular choice of finite field-dependent parameter. With this particular choice of finite parameter the gauge-fixing and Faddeev-Popov ghost terms emerge naturally through Jacobian calculation within functional integration written in $SU(2)$ CFN variables. For this purpose, we make an ansatz for finite field-dependent parameter obtainable from following infinitesimal field-dependent parameter

$$\Lambda'[\phi(y, \kappa)] = i \int d^4y \left[ \bar{\Theta}_\mu \cdot \partial^\mu A_\mu + \bar{C}_\theta \cdot D^\mu[V]X_\mu \right],$$

(36)

using relation (25). Now, exploiting relation (32) we calculate an infinitesimal change in Jacobian with respect to $\kappa$ for above $\Lambda'[\phi(y, \kappa)]$ as follows:

$$\frac{1}{J(\kappa)} \frac{dJ(\kappa)}{d\kappa} = \int d^4x \left[ -B_\omega \cdot \partial^\mu A_\mu - i \bar{C}_\omega \partial^\mu D_\mu[A]C_\omega - B_\theta \cdot D^\mu[V]X_\mu + i \bar{C}_\theta \cdot D^\mu[V - X]D_\mu[V + X](C_\theta - C_\omega) \right].$$

(37)

The expression of arbitrary functional $S_1[\phi(x, \kappa), \kappa]$, which appears in the exponential of Jacobian given in Eq. (33), is constructed as

$$S_1[\phi(x, \kappa), \kappa] = \int d^4x \left[ \chi_1(\kappa)B_\omega \cdot \partial^\mu A_\mu + \chi_2(\kappa)\bar{C}_\omega \partial^\mu D_\mu[A]C_\omega + \chi_3(\kappa)B_\theta \cdot D^\mu[V]X_\mu + \chi_4(\kappa)\bar{C}_\theta \cdot D^\mu[V - X]D_\mu[V + X](C_\theta - C_\omega) \right],$$

(38)

where $\chi_i(i = 1, 2, 3, 4)$ are arbitrary $\kappa$-dependent constants and satisfy the boundary conditions

$$\chi_i(\kappa = 0) = 0.$$  

(39)

To calculate the exact values of these constants, we first calculate the small change in above $S_1$ with respect to parameter $\kappa$ using equation (26) as,

$$\frac{dS_1[\phi(x, \kappa), \kappa]}{d\kappa} = \int d^4x \left[ \chi_1(\kappa)B_\omega \cdot \partial^\mu A_\mu + \chi_2(\kappa)\bar{C}_\omega \partial^\mu D_\mu[A]C_\omega + \chi_3(\kappa)B_\theta \cdot D^\mu[V]X_\mu + \chi_4(\kappa)\bar{C}_\theta \cdot D^\mu[V - X]D_\mu[V + X](C_\theta - C_\omega) \right] + \chi_3(\kappa)B_\theta \cdot D^\mu[V]X_\mu + \chi_4(\kappa)\bar{C}_\theta \cdot D^\mu[V - X]D_\mu[V + X](C_\theta - C_\omega),$$

(40)

where prime denotes the differentiation with respect to $\kappa$. Then we put the condition for existence of above functional $S_1$ as an exponent of Jacobian $J$, i.e., the Eqs. (37) and (38) must satisfy the condition given in (33). Therefore, this reflects

$$\int d^4x \left[ (\chi_1 - 1)B_\omega \cdot \partial^\mu A_\mu + (\chi_2 - i)\bar{C}_\omega \partial^\mu D_\mu[A]C_\omega + (\chi_3 - 1)B_\theta \cdot D^\mu[V]X_\mu + (\chi_4 + i)\bar{C}_\theta \cdot D^\mu[V - X]D_\mu[V + X](C_\theta - C_\omega) - (\chi_1 + i\chi_2)B_\omega \partial^\mu D_\mu[A]C_\omega \Lambda' + (\chi_3 - i\chi_4)B_\theta \cdot D^\mu[V - X]D_\mu[V + X](C_\theta - C_\omega) \Lambda' \right] = 0.$$  

(41)
The comparison of coefficients from the terms of LHS to RHS of the above equation gives the following restrictions on the parameters ($\chi_i$):

\[
\begin{align*}
\chi'_1 - 1 &= 0, & \chi'_2 - i &= 0, \\
\chi'_3 - 1 &= 0, & \chi'_4 + i &= 0, \\
\chi_1 + i\chi_2 &= 0, & \chi_3 - i\chi_4 &= 0.
\end{align*}
\]

(42)

The solutions of the above equations satisfying the boundary conditions given in (39) are

\[
\begin{align*}
\chi_1 &= +\kappa, & \chi_2 &= +i\kappa, & \chi_3 &= \kappa, & \chi_4 &= -i\kappa.
\end{align*}
\]

(43)

Now, with the help of these identifications of $\chi_i$, the expression of $S_1$ given in (38) reduces to the following:

\[
S_1[\phi(x, \kappa)] = \int d^4x \left[ \kappa B_\omega \cdot \partial^\mu A_\mu + i\kappa \bar{C}_\omega \partial^\mu D_\mu [A]C_\omega + \kappa B_\theta \cdot D^\mu [V]X_\mu \\
- i\kappa \bar{C}_\theta \cdot D^\mu [V - X]D_\mu [V + X](C_\theta - C_\omega) \right],
\]

(44)

which vanishes at $\kappa = 0$. However at $\kappa = 1$, it is nothing but the gauge-fixing and ghost parts of the action given in Eq. (24), i.e.,

\[
S_1[\phi(x, 1), 1] = \int d^4x \left[ B_\omega \cdot \partial^\mu A_\mu + i\bar{C}_\omega \partial^\mu D_\mu [A]C_\omega + B_\theta \cdot D^\mu [V]X_\mu \\
- i\bar{C}_\theta \cdot D^\mu [V - X]D_\mu [V + X](C_\theta - C_\omega) \right],
\]

\[
= \int d^4x \ L_{GF+FP}.
\]

(45)

This shows that the gauge-fixing and ghost terms appear naturally within functional integral through Jacobian calculation under FFBRST transformation with finite field-dependent parameter obtainable from (36) as

\[
\int D\phi \ e^{-S_1} \rightarrow \int D\phi \ e^{-S_1} = \int D\phi \ e^{-\int d^4x \ L_{GF+FP}}.
\]

(46)

VI. CONCLUDING REMARKS

In this Letter, we have considered the CFN decomposed $SU(2)$ YM theory having different ghost structures, which plays an important role in explaining low-energy limit of the theory, and analysed the BRST symmetry of this theory. Further, we have generalized the nilpotent BRST transformations of CFN variables in Euclidean space. The generalization has been made by making the infinitesimal Grassmannian parameter of BRST transformation finite and field-dependent which is known FFBRST transformation. We have shown that the FFBRST transformations are symmetry of the effective action however these do not leave the partition function (functional integral) invariant because the path integral measure is not invariant under these. The Jacobian of path integral measure in the expression of functional integral changes non-trivially under FFBRST transformations. Further, the method of Jacobian evaluation has been discussed. Utilizing this method, we have calculated the Jacobian for path integral measure under FFBRST transformations for a particular choice of finite field-dependent parameter. It has been found that Jacobian of path integral measure of YM theory analysed in CFN variables under FFBRST transformations for this particular choice of parameter produces the gauge-fixing and faddeev-popov ghost terms in the effective theory. However, for a different choices of finite field-dependent parameter the FFBRST
transformation will lead to connection between MAG and another Abelian gauge of the theory. From my points of view, these results will hold for any general gauge theory, even though, we have shown it for a CFN decomposed YM theory. It will be interesting to generalize the results for very general gauge theory, including those with open or reducible gauge algebras.

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