Model- and calibration-independent test of cosmic acceleration

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Abstract. We present a calibration-independent test of the accelerated expansion of the universe using supernova type Ia data. The test is also model-independent in the sense that no assumptions about the content of the universe or about the parameterization of the deceleration parameter are made and that it does not assume any dynamical equations of motion. Yet, the test assumes the universe and the distribution of supernovae to be statistically homogeneous and isotropic. A significant reduction of systematic effects, as compared to our previous, calibration-dependent test, is achieved. Accelerated expansion is detected at significant level (4.3\(\sigma\) in the 2007 Gold sample, 7.2\(\sigma\) in the 2008 Union sample) if the universe is spatially flat. This result depends, however, crucially on supernovae with a redshift smaller than 0.1, for which the assumption of statistical isotropy and homogeneity is less well established.

Keywords: classical tests of cosmology, dark energy theory, supernova type Ia

1. Introduction

The concordance model of cosmology is very successful in describing observations. It states that the universe consists of baryonic and cold dark matter as well as a cosmological constant, where the baryonic matter makes up only 5\% of the content of the universe. The cosmological constant contributes 72\% and thus causes an accelerated expansion at the present epoch. A large variety of other cosmological models have been proposed that come to similar results. Especially as soon as homogeneity and isotropy are assumed, each model states accelerated expansion of the universe when confronted with observational data.

If we are only interested in the question, whether the universe really expands accelerated, but not in the specific content of the universe, proposing and testing a certain cosmological model is not the appropriate approach. This question rather needs to be answered as model-independent as possible, i.e. without making any assumptions about the matter and energy content of the universe. The so-called kinematical approach does so by using special parameterizations of the deceleration parameter \(q(z)\) \cite{1,2,3}, the scale parameter \(a(t)\) \cite{4}, the Hubble rate \(H(z)\) \cite{5}, the dimensionless coordinate distance \(y(z)\) \cite{6,7} or different distance scales \cite{8}. Other published methods are to
expand $q$ into principle components [9] or to expand the jerk parameter $j$ into a series of orthonormal functions [10].

It is, however, even possible to avoid these kinds of parameterizations. In [11] we presented a model-independent test to quantify the evidence for accelerated expansion with the only assumption that the universe is homogeneous and isotropic. While versions of this test have already been proposed [12] and applied [13, 14] by other groups, we additionally considered calibration effects, quantified the evidence for accelerated expansion and studied systematic effects.

The crucial assumption in our test is the Copernican principle. It states that we are typical observers in the universe. Together with the observed isotropy of the cosmic microwave background, the Copernican principle implies that the universe is also homogeneous, the statement of the cosmological principle.

Although it seems to be consequent to adopt the cosmological principle for the analysis of supernovae, a critical reflection on it is in order. Most probes of the statistical isotropy use objects at rather high redshift, while nearby probes show significantly less evidence for statistical isotropy. For supernovae, this issue has been investigated recently in [15, 16]. Statistically significant violation of isotropy was found for supernovae at redshift $z < 0.2$ [16]: The fluctuation of the Hubble rate $\Delta H/H$ on opposite hemispheres on the sky is about 5%. This corresponds to an anisotropy of distance moduli of 0.1 mag, which has to be compared to the effect of acceleration, which is about 0.2 mag.

The statistical homogeneity of the universe is less well established than the statistical isotropy at high redshifts. Several observations indicate that the scale of statistical homogeneity is of the order of 100 Mpc (e.g. from luminous red galaxies [17], but see also [18]). For supernovae it has been pointed out already many years ago that dropping the assumption of homogeneity, but keeping that of isotropy around one point, allows for excellent fits of the Hubble diagram without invoking dark energy [19, 20]. This does not come as a surprise, because spherically symmetric dust models (also called Lemaitre-Tolman-Bondi models) have two arbitrary free functions and if only supernovae are considered a perfect fit can be obtained. Thus instead of interpreting the supernovae Hubble diagrams as evidence for dark energy, one could question the Copernican principle. In our test, we assume statistical homogeneity and isotropy. We somewhat relax that assumption in a second step by excluding nearby supernovae from the test. We will demonstrate below, that the nearby supernovae at $z < 0.1$ are crucial in the detection of cosmic acceleration.

In the following section, we will shortly summarize the basic concept and results of our test. In the present work, we slightly modify the test in order to avoid systematics due to calibration. The reason for those systematics is also explained in the next section. In section 3 the modified test is presented and applied to different data sets, including the recently published Union set [21], assuming a spatially flat universe. The cases of open and closed universes will be considered in section 4.
2. Calibration-dependent test

We consider the distance modulus
\[ \mu = m - M = 5 \log d_L + 25 \]  
(1)
of supernovae type Ia (SN Ia), where the luminosity distance \( d_L \) is given in units of Mpc. \( m \) and \( M \) are the apparent and absolute magnitudes, respectively. If \( \mu_i \) is the distance modulus of the \( i \)-th SN with redshift \( z_i \), one can define a new quantity
\[ \Delta \mu_i = \mu_i - \mu_{q=0}(z_i) = \mu_i - 5 \log \left[ \frac{1}{H_0} (1 + z_i) \ln(1 + z_i) \right] - 25, \]  
(2)
where \( \mu_{q=0}(z) \) is the distance modulus of a universe that neither accelerates nor decelerates, i.e. with deceleration \( q(z) = 0 \).

The weighted average of the \( \Delta \mu_i \) is given by
\[ \Delta \mu = \frac{\sum_{i=1}^{N} g_i \Delta \mu_i}{\sum_{i=1}^{N} g_i}, \]  
(3)
where \( g_i = 1/\sigma_i^2 \). The \( \sigma_i \) include measurement errors and errors due to peculiar velocities. The standard deviation of this average is calculated by
\[ \sigma = \left[ \frac{\sum_{i=1}^{N} g_i (\Delta \mu_i - \Delta \mu)^2}{(N - 1) \sum_{i=1}^{N} g_i} \right]^{\frac{1}{2}}, \]  
(4)
with \( N \) being the number of SNe that is averaged over.

Our null hypothesis is that the universe never expanded accelerated which implies
\[ \Delta \mu \leq 0. \]  
(5)
Note that this holds independently of the content of the universe and does not depend on the validity of Einstein’s equations. Thus if the observed value of \( \Delta \mu \) is significantly larger than zero, the null hypothesis can be rejected. In that case, one can state that there must have been a phase of acceleration. However, this does not exclude a phase of deceleration. From (1) and (2) it follows that we need \( M \) and \( H_0 \) in order to test the null hypothesis (5). There always exist values of \( M \) and \( H_0 \) such that (5) is satisfied. Thus, these values have to be fixed by an independent calibration measurement. This is in contrast to model-dependent tests, where a combination of \( M \) and \( H_0 \) is used as a fitting parameter.

In [11], we considered two SN Ia data sets (the Gold sample [22] and the ESSENCE set [23]), two different light-curve fitters (MLCS2k2 [24] and SALT [25]) and two different calibrations (the calibration presented by Riess et al. [26] and that given by Sandage et al. [27], which in the following will be referred to as Riess calibration and Sandage calibration, respectively). We calculated the averaged value \( \Delta \mu \) over all SNe of a data set with a redshift \( z \geq 0.2 \) and divided the result by its standard deviation \( \sigma \), thus obtaining the evidence for accelerated expansion. Assuming a flat universe, the ESSENCE set using the MLCS2k2 fitter and the Sandage calibration gave the weakest evidence, namely \( 5.2\sigma \). In the other cases the evidence is much larger and goes up to \( 17\sigma \)
for ESSENCE (SALT) in the Riess calibration. Thus, we observe enormous systematic effects for the different data sets, fitting methods and calibrations.

Consequently, we have to make an attempt to reduce these large systematics. It turns out that they are largely due to systematics in the calibration. This can be understood by the following considerations.

The absolute magnitude $M$ and the Hubble constant $H_0$ cannot be determined independently by only considering SNe. Thus, the absolute magnitude of the SNe Ia has to be calibrated by measurements of the distance moduli of cepheids in the host galaxies. Then SNe can be used to determine $H_0$. As there is still some controversy between different groups about the correct calibration, we considered two very discrepant calibrations for our test of accelerated expansion. This test, however, does not depend on $M$ and $H_0$ independently, but on the quantity $M = M - 5 \log(H_0) + 25$, which can be seen when we rewrite the null hypothesis $\Delta \mu \leq 0$ as

$$m - 5 \log [(1 + z) \ln(1 + z)] \leq M.$$  

(6)

The fact that we observe huge systematic errors depending on the considered calibration seems somewhat strange: Assume we have found two different values $M_1$ and $M_2$ for the absolute magnitude of SNe Ia by cepheid measurements. Using these results, two values $H_{01}$ and $H_{02}$ for the Hubble constant can be obtained by observations of nearby SNe. Although $M_1 \neq M_2$ and $H_{01} \neq H_{02}$, the resulting values $M_1$ and $M_2$ are equal by definition, if the same set of low redshift SNe and the same analysis is used for the determination of the Hubble constant.

For our test in [11], we adopted the values of $M$ and $H_0$ given by Riess et al. [26] and Sandage et al. [27]. As the two groups analysed different SNe and used different analysis pipelines, they obtained $H_{01}$ and $H_{02}$ which (combined with $M_1$ and $M_2$) did not lead to the same $M$, but different values $M_1$ and $M_2$.

Thus, the observed systematics are not due to a different determination of the absolute magnitude $M$, but are caused by the systematic errors and the different SN data sets used in the measurement of $M$.

While our approach is to test a null hypothesis, model-dependent tests fit special cosmological models or parameterizations to observational data. An essential difference between these two approaches is that we test an inequality, whereas model-dependent tests use an equality, which requires different analyses. Model-dependent tests are typically based on the minimization of $\chi^2(p_i)$, where $M$ is one of the model parameters.
Then one can estimate the likelihood as a function of the parameters, which allows marginalization over $\mathcal{M}$. As in our test an inequality is tested, this kind of analysis is not suitable. We cannot use $\mathcal{M}$ as a free parameter, since the null hypothesis (6) is always fulfilled for large enough $\mathcal{M}$. Thus for the test summarized in this section, $\mathcal{M}$ needs to be calibrated.

### 3. Calibration-independent test

It is, however, easy to modify our test in such a way that we can avoid using a certain calibration of $\mathcal{M}$ when testing the accelerated expansion. We just need to consider relative values of $\Delta \mu$ instead of absolute values, i.e. we use $\Delta \mu - \Delta \mu_{\text{nearby}}$ rather than $\Delta \mu$, where $\Delta \mu_{\text{nearby}}$ is the average of $\Delta \mu_i$ using only nearby SNe of a data set. Thus, the null hypothesis now reads

$$\Delta \mu - \Delta \mu_{\text{nearby}} \leq 0 .$$

The standard deviation $\sigma$ is obtained by adding the standard deviations of $\Delta \mu$ and $\Delta \mu_{\text{nearby}}$ in quadrature. In the following, we will assume a spatially flat universe. Open and closed universes will be considered in the next section.

In figure 1a, $\Delta \mu(z)$ is plotted for different spatially flat cosmological models: a $\Lambda$CDM model with $\Omega_m = 0.28$ and $\Omega_\Lambda = 0.72$ (WMAP 5yr + BAO + SN best fit [28]), a de Sitter universe and models with constant deceleration ($q = 0.5$, which corresponds to the Einstein-de Sitter model) and constant acceleration ($q = -0.5$). Although the transition redshift between acceleration and deceleration in the $\Lambda$CDM model is about $0.73$, the curve still slightly increases beyond that redshift and has its maximum at $z = 1.3$. Thus, an increase of $\Delta \mu$ with redshift does not necessarily correspond to a phase of acceleration at that specific redshift. Note that for each value of the spatial curvature $\Omega_k$ a different plot is necessary as in general $\Delta \mu_{q=0}(z)$ depends on $\Omega_k$ (see section 4).

Figure 1b shows $\Delta \mu - \Delta \mu_{\text{nearby}}$ averaged over redshift bins with width 0.2 for the Gold sample [22] (fitted with MLCS2k2), the ESSENCE set [23] (once fitted with MLCS2k2 and once with SALT) and the Union set [21] (fitted with SALT). As the first bin corresponds to the nearby SNe, its value is per definition equal to zero. The values in all the other bins are significantly above zero which indicates accelerated expansion. It is kind of a natural choice to define the nearby SNe as all SNe having a redshift smaller than 0.2 since at this point there is a gap in all presently available data sets. Doing so, there is a strong evidence for accelerated expansion.

Nevertheless, we also consider the cases where the nearby SNe are defined as those with $z < 0.1$ and those with $0.1 \leq z < 0.2$ (figures 1c and 1d respectively). In both plots a bin width of 0.1 is used. In figure 1d the first bin is skipped and thus the second bin is fixed to zero. One can already see without any quantitative analysis that the

$\dagger$ Note that this hypothesis does not correspond to the hypothesis of a never accelerating universe tested by an observer at $z_{\text{nearby}}$. 
Figure 1: $\Delta \mu$ for different cosmological models (a) and $\Delta \mu - \Delta \mu_{\text{nearby}}$ for different data sets and fitting methods, where nearby SNe are defined as those SNe with redshifts fulfilling the given inequalities for $z_{\text{nearby}}$ (b-d).

Evidence for acceleration dramatically decreases if SNe with $z < 0.1$ are not used for the test. In particular for the two data sets that are fitted with MLCS2k2, the evidence completely vanishes since several data points become negative. However, one should not take that result very serious, as it is based on a very small number of SNe in the calibrating bin. Most reliable is the Union set with 6 SNe between redshift 0.1 and 0.2.

A quantitative value for the evidence of acceleration can be obtained by dividing $\Delta \mu - \Delta \mu_{\text{nearby}}$ by its standard deviation $\sigma$. The results corresponding to the bins plotted in figure 1 are given in table 1. A better statistics can be achieved when averaging $\Delta \mu$ over all SNe of a set with a redshift larger than that of the nearby ones. The results are listed in table 2. Although the test was only modified in order to avoid systematics from calibration, also the other systematics are reduced. In the test presented in [11] the calculated evidences varied from $11.9\sigma$ to $17.0\sigma$ in the Riess calibration and from $5.2\sigma$ to $10.4\sigma$ in the Sandage calibration. We then only used the Gold and ESSENCE sets, but not the Union set. For the same sets, using $z_{\text{nearby}} < 0.2$, we now obtain values
that lie much closer to each other, namely between $4.3\sigma$ and $5.7\sigma$.

Analysing the data sets using $0.0 \leq z_{\text{nearby}} < 0.1$ and $0.1 \leq z_{\text{nearby}} < 0.2$, respectively, a drawback of this test becomes evident: It crucially depends on the SNe in the first bin. (Note that this drawback is also implicitly included in calibration-dependent tests as nearby SNe are needed to determine $M$ and $H_0$.) Skipping the SNe with a redshift smaller than 0.1, the evidence for acceleration vanishes if the MLCS2k2 fitter is used, which could be partly due to the fact that there are only four SNe classified

### Table 1: Evidence for acceleration ($\Delta \mu - \Delta \mu_{\text{nearby}})/\sigma$ for different data sets and fitting methods using SNe in different redshift bins. Nearby SNe are defined as those SNe with redshifts fulfilling the given inequalities for $z_{\text{nearby}}$.

| Nearby SNe | Bin | Gold (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) | Union (SALT) |
|------------|-----|----------------|-------------------|---------------|--------------|
| $0.0 \leq z_{\text{nearby}} < 0.2$ | 0.2 \leq z < 0.4 | 2.0 | 2.2 | 3.2 | 3.5 |
| | 0.4 \leq z < 0.6 | 4.4 | 4.2 | 5.2 | 6.8 |
| | 0.6 \leq z < 0.8 | 2.8 | 4.6 | 3.4 | 4.4 |
| | 0.8 \leq z < 1.0 | 2.6 | 4.9 | 3.7 | 4.2 |
| | 1.0 \leq z < 1.2 | 2.0 | 1.3 |
| | 1.2 \leq z < 1.4 | 1.6 | 2.1 |
| $0.0 \leq z_{\text{nearby}} < 0.1$ | 0.1 \leq z < 0.2 | 2.8 | 2.2 | 3.8 | 0.6 |
| | 0.2 \leq z < 0.3 | 1.8 | 2.2 | 3.4 | 2.9 |
| | 0.3 \leq z < 0.4 | 2.0 | 2.0 | 2.2 | 2.8 |
| | 0.4 \leq z < 0.5 | 3.9 | 2.9 | 4.0 | 5.3 |
| | 0.5 \leq z < 0.6 | 3.9 | 5.3 | 4.8 | 5.2 |
| | 0.6 \leq z < 0.7 | 3.7 | 4.2 | 2.5 | 3.3 |
| | 0.7 \leq z < 0.8 | 1.1 | 5.2 | 5.1 | 3.6 |
| | 0.8 \leq z < 0.9 | 2.8 | 3.7 | 3.7 | 3.8 |
| | 0.9 \leq z < 1.0 | 1.1 | 3.5 | 1.7 | 2.2 |
| | 1.0 \leq z < 1.1 | 1.9 | 1.8 |
| | 1.1 \leq z < 1.2 | 0.9 | \text{-0.7} |
| | 1.2 \leq z < 1.3 | 4.3 | 2.7 |
| | 1.3 \leq z < 1.4 | -0.1 | 0.6 |
| $0.1 \leq z_{\text{nearby}} < 0.2$ | 0.2 \leq z < 0.3 | -0.8 | -0.6 | 2.1 | 1.9 |
| | 0.3 \leq z < 0.4 | -0.2 | -0.8 | 0.5 | 1.7 |
| | 0.4 \leq z < 0.5 | 1.1 | -0.5 | 1.9 | 3.5 |
| | 0.5 \leq z < 0.6 | 1.1 | 0.7 | 3.3 | 3.8 |
| | 0.6 \leq z < 0.7 | 1.1 | 0.0 | 0.7 | 2.4 |
| | 0.7 \leq z < 0.8 | -0.8 | 1.1 | 3.3 | 2.7 |
| | 0.8 \leq z < 0.9 | 0.8 | 0.5 | 2.7 | 3.1 |
| | 0.9 \leq z < 1.0 | -0.8 | 0.9 | 0.9 | 1.9 |
| | 1.0 \leq z < 1.1 | 1.1 | 1.6 |
| | 1.1 \leq z < 1.2 | -0.1 | \text{-1.0} |
| | 1.2 \leq z < 1.3 | 2.6 | 2.3 |
| | 1.3 \leq z < 1.4 | -1.2 | 0.4 |
Table 2: Evidence for acceleration \((\Delta \mu - \Delta \mu_{\text{nearby}})/\sigma\) for different data sets and fitting methods, where nearby SNe are defined as those SNe with redshifts fulfilling the given inequalities for \(z_{\text{nearby}}\). Also given are the numbers of SNe in different redshift intervals.

| Redshift Inequality | Gold (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) | Union (SALT) |
|---------------------|----------------|------------------|----------------|--------------|
| \(0.0 \leq z_{\text{nearby}} < 0.2\) | 4.3 | 5.2 | 5.6 | 7.2 |
| \(0.0 \leq z_{\text{nearby}} < 0.1\) | 4.4 | 5.7 | 5.7 | 5.9 |
| \(0.1 \leq z_{\text{nearby}} < 0.2\) | 0.8 | 0.9 | 4.3 | 3.7 |
| #SNe at \(z < 0.1\) | 36 | 43 | 44 | 51 |
| #SNe at \(0.1 \leq z < 0.2\) | 4 | 4 | 2 | 6 |
| #SNe at \(z > 0.2\) | 142 | 115 | 132 | 250 |
| #SNe in total | 182 | 162 | 178 | 307 |

as nearby. The reason why there is still some evidence if we use ESSENCE (SALT) instead of ESSENCE (MLCS2k2) is not only due to the systematic effects that occur when different light-curve fitters are used. The main reason is that there are only two nearby SNe in ESSENCE (SALT), which by chance have almost the same value of \(\Delta \mu\) and thus a very small variance. The Union set contains 6 nearby SNe with \(z \geq 0.1\) and still states acceleration at 3.7\(\sigma\).

As large scale structures are observed up to \(\sim 400\) Mpc (the Sloan Great Wall \cite{29}) which corresponds to a redshift of about 0.1, it is questionable if the assumption of isotropy and homogeneity is still justified for the analysis of SNe at lower redshifts. Thus, one should prefer to make cosmological tests using only SNe with higher redshifts and average over bin widths of at least \(\Delta z \geq 0.1\). Unfortunately, this is not possible at the moment as there are very few SNe at redshifts between 0.1 and 0.3 in all presently available data sets. As soon as SN data at intermediate redshifts are published, repeating our analysis will show if better statistics gives rise to at least some evidence of cosmic acceleration and if there is still a significant difference in the evidences obtained by using MLCS2k2 and SALT, respectively.

In order to analyse more quantitatively how much the evidence changes if the lowest redshifts are not considered, we split the SNe with \(z < 0.1\) of each set into two subsets containing an equal number of SNe. Subset 1 contains the SNe with the lowest redshifts, subset 2 those with the largest redshifts. \(\Delta \mu_{\text{nearby}}\) is calculated using subset 1 and 2, respectively. For the determination of \(\Delta \mu\) we use all SNe with \(z \geq 0.2\). The result for the evidence for acceleration is shown in table 3. There is a tendency of decreasing evidence when the lowest redshift SNe are dismissed, which is not surprising. Only the Gold sample does not show this trend. The amount by which the evidence is decreased is, however, unexpected. This can be seen in figure 2 for the Union set: \(\Delta \mu - \Delta \mu_{\text{nearby}}\) for a \(\Lambda\)CDM model \((\Omega_m = 0.28, \Omega_\Lambda = 0.72)\) using subset 2 drops only by 0.018 mag as compared to the case when subset 1 is used, whereas the data points drop by 0.066
Table 3: Evidence for acceleration \((\Delta \mu - \Delta \mu_{\text{nearby}})/\sigma\), where the SNe used to calculate \(\Delta \mu\) have \(z \geq 0.2\) and those to calculate \(\Delta \mu_{\text{nearby}}\) have \(z < 0.1\). The nearby SNe are split into two subsets for each data set, each containing an equal number of SNe. Subset 1 contains the SNe with the smallest redshift, subset 2 those with the largest redshifts. Also given is the weighted average redshift \(\overline{z}_{\text{nearby}}\) and the number of SNe in each subset.

|          | Gold (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) | Union (SALT) |
|----------|----------------|-------------------|----------------|---------------|
| subset 1 | evidence       | 3.1               | 6.6            | 5.6           | 6.3           |
|          | \(\overline{z}_{\text{nearby}}\) | 0.030             | 0.021          | 0.021         | 0.021         |
| subset 2 | evidence       | 3.4               | 3.2            | 3.9           | 4.8           |
|          | \(\overline{z}_{\text{nearby}}\) | 0.056             | 0.046          | 0.050         | 0.051         |
| # SNe    | 18             | 21                | 22             | 25            |

Figure 2: \(\Delta \mu - \Delta \mu_{\text{nearby}}\) for the Union set. The nearby SNe are those of subset 1 and 2, respectively, as given in table 3.

mag. Thus, the change in the data points is much larger than expected from \(\Lambda\)CDM. A model with a steeper increase of \(\Delta \mu(z)\) at small redshifts would be more consistent with these data. An alternative explanation could be the so called Hubble bubble or large void scenario, i.e. the local value of \(H_0\) could be different from the global one.

4. Open and closed universes

We now give up the assumption of spatial flatness. Then the distance modulus of a universe with deceleration parameter \(q = 0\) is given by

\[
\mu_{q=0}(z) = 5 \log \left[ \frac{1 + z}{H_0 \sqrt{|\Omega_k|}} S \left[ \sqrt{|\Omega_k|} \ln(1 + z) \right] \right] + 25 ,
\]  

(8)
Table 4: Evidence for acceleration assuming different values of the spatial curvature $\Omega_k$. The lower limits to the evidence for a flat/closed and an open universe are highlighted. Nearby SNe are defined as those with redshift $z < 0.2$.

| $\Omega_k$ | Gold (MLCS2k2) | ESSENCE (MLCS2k2) | ESSENCE (SALT) | Union (SALT) |
|------------|-----------------|--------------------|----------------|--------------|
| closed universe |               |                    |                |              |
| -1.0       | 6.7             | 7.0                | 7.5            | 10.2         |
| -0.8       | 6.3             | 6.6                | 7.1            | 9.6          |
| -0.6       | 5.8             | 6.3                | 6.8            | 9.0          |
| -0.4       | 5.3             | 5.9                | 6.4            | 8.4          |
| -0.2       | 4.8             | 5.6                | 6.0            | 7.8          |
| flat universe | 0.0             | 4.3                | 5.2            | 5.6          |
| open universe | 0.2             | 3.8                | 4.9            | 5.3          |
|             | 0.4             | 3.3                | 4.5            | 4.9          |
|             | 0.6             | 2.8                | 4.1            | 4.5          |
|             | 0.8             | 2.3                | 3.7            | 4.1          |
|             | 1.0             | 1.8                | 3.4            | 3.8          |

where $S(x) = \sin(x)$ for a closed and $S(x) = \sinh(x)$ for an open universe. Using this, $\Delta \mu_i = \mu_i - \mu_{z=0}(z_i)$ and subsequently $\Delta \mu - \Delta \mu_{\text{nearby}}$ can be easily calculated. Defining the nearby SNe as those with $z < 0.2$, the result for different values of the spatial curvature $\Omega_k$ is given in Table 4.

In a closed universe, $\mu_{z=0}(z)$ decreases with increasing curvature, i.e. $\Delta \mu - \Delta \mu_{\text{nearby}}$ (and thus the evidence for accelerated expansion) increases. As we are interested in the lower limit of the evidence in a closed universe, we need to consider the smallest possible curvature, i.e. $\Omega_k \rightarrow 0$. Therefore, the results of the test are equal to those of a spatially flat universe. In an open universe the evidence decreases with increasing curvature. Thus, we need the largest possible value, $\Omega_k = 1$, to determine the lower limit to the evidence. Note that $\Omega_k = 1$ is the largest possible curvature only if the rule $\sum_i \Omega_i = 1$ holds, where $i = m, \Lambda, k, \ldots$. This rule is, however, only valid for general relativity and can be different in a modified gravity scenario. These weakest evidences for a flat/closed and an open universe are highlighted in Table 4.

5. Conclusion

We modified our model-independent test of accelerated expansion presented in [11] in such a way that systematics due to calibrations are avoided. This could be achieved by considering the SN data relative to the data in a bin of nearby SNe, where the most “natural” choice is to define the nearby SNe as those with redshift $z < 0.2$. Compared to the previous test, the evidence for acceleration is weaker, but all systematic effects are reduced. However, the evidence obtained from the ESSENCE set is still larger than that obtained from the Gold sample, and the SALT fitter gives a stronger evidence.
than MLCS2k2. Being conservative, one should take the lowest evidence obtained by using different data sets and light-curve fitters, i.e. $4.3\sigma$ in the case of a spatially flat universe using the Gold sample fitted with MLCS2k2. The Union set is however not an independent data set, but contains SNe from Gold, ESSENCE and other sets. Thus, the Union set seems to be a good choice in order to determine the evidence for accelerated expansion. Unfortunately, the published data of this set have only been fitted with SALT. As for the ESSENCE set MLCS2k2 gives a weaker evidence than SALT, it is quite probable that this is also the case for the Union set. Then MLCS2k2 would give a more conservative value than the obtained $7.2\sigma$ evidence.

By changing the set of nearby SNe, it becomes obvious that the test crucially depends on SNe with redshift $z \lesssim 0.1$. Skipping those data leads to vanishing evidence when the MLCS2k2 light-curve fitter is used. As a redshift of 0.1 corresponds approximately to a distance of 400Mpc, which is the size of the largest observed structures in the universe, the assumption of homogeneity and isotropy might not be justified for such low redshifts. For the Union set (based on 6 SNe between 0.1 and 0.2) still $3.7\sigma$ evidence is found. It remains to be seen if this is confirmed by larger data sets in the future.

We conclude that the largest publicly available supernova data sets show statistically significant evidence for cosmic acceleration if statistical isotropy and homogeneity are assumed at all redshifts. A violation of isotropy and/or homogeneity, e.g. by a local void, remains a viable alternative interpretation of supernova data (see e.g. [20]).

How to proceed? First of all we need more data at $0.1 < z < 0.3$. This would allow to anchor our test with supernovae at intermediate redshifts, which should not be affected by the details of the local environment. Thus such a test could detect cosmic acceleration in a way that relies only on the assumptions of isotropy and homogeneity on scales at which they can be established by observations directly. Secondly, in [16] a significant anisotropy of Hubble diagrams on opposite hemispheres of the sky has been found. The direction of maximal asymmetry was found to be close to the pole of the equatorial coordinate system, which points towards systematic errors in existing data sets. Thus with improved data sets, especially better control on systematics at all levels, one could try to use SN Ia at $z < 0.1$ to establish isotropy and homogeneity at even smaller redshifts. In both cases, ideally full sky surveys for nearby supernovae are required, while pencil beam like approaches would make it more difficult to disentangle a local void from dark energy.

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