Study on the Influence of Reinforcement on Creep and Shrinkage Effects of Composite Beams

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Abstract. To obtain time-dependent behaviours of composite beams, the system of differential equations is established by Rüsch constitutive equation of creep. Rigorous solutions of the system are obtained by computer programs of the authors. Creep and shrinkage effects involve internal force redistribution and deflection increase, and these are significantly large for composite beam. Parameter analyse of the reinforcement of concrete slab on these effects is conducted. Among them the reinforcement aggravates significantly the reduction of axial force between individual sections and alliviates slightly the increase of deflection of the beam.

1. Introductions
A simple factor considering deflection of beam induced by creep and shrinkage of concrete structures is recommended in many Codes [1]. For composite beams creep and shrinkage effects are far more significant than that of concrete structures because of the large ratio of bending stiffness contributed by the steel beam [2-5]. Therefore more refined analysis must be conducted to provide an insight of its time-dependent behaviour, compared with concrete structures [6-8]. Normally the reinforcement of concrete slab is neglected and the slab is treated as plain concrete by a creep-transformed elastic modular of concrete. Alternately here the influences of reinforcement of on the internal forces induced by creep and shrinkage of uncrack composite beams in service time are presented by analytical method.

2. System of differential equations
Composite beam involves two different materials. For rigorous analysis they must be considered individually. As shown in Figure 1 concrete is represented by its centre of gravity C and steel is by S that is the combination of steel beam and reinforcement in the slab. So long as plan section remains plan the reinforcement can be considerd with steel beam together. Under sustained loading (bending moment \(M\)) the internal forces undergo large changes over time and the redistributed internal forces are unknown. In order to obtain analytical solutions of them, their differential increments are considered from time \(t\) to \(t+\Delta t\). The corresponding deformations (axial strain and curvature) are shown in Figure 1, in which subscript ‘c’, ‘sb’, ‘st’, ‘0’, ‘r’ and ‘t’ denote concrete, reinforcement, steel, initial time (\(t_0\)), redistribute and final time (\(t\)) respectively. For sign convention, the coordinate z is positive downwards, axial force is positive in tension, and moment that produces tension at the bottom fibre is positive.
Assuming that shrinkage develops in the same manner as creep, the system of equations can then be obtained according to equilibrium and compatibility conditions, and by Rüsch constitutive equation of creep the equations [9] can be expressed as

\[
\begin{align*}
\frac{d\varepsilon_{sh}}{dt} + \frac{N_{cr}}{E_{cr}A_{cr}} \frac{d\varphi_t}{dt} + \frac{1}{E_{cr}A_{cr}} \frac{dN_{cr}}{dt} - \frac{1}{E_sA_s} \frac{dN_{ss}}{dt} + \frac{dM_{cr}}{dt} = 0 \\
\frac{M_{cr} + M_{ss}}{E_{cr}I_{cr}} \frac{d\varphi_t}{dt} + \frac{1}{E_sI_s} \frac{dM_{ss}}{dt} = 0 \\
\frac{dM_{ss}}{dt} + \frac{dN_{ss}}{dt} = 0 \\
\frac{dN_{cr}}{dt} = 0 \\
\varphi_t = \frac{1}{1.4} \left[ \varphi(t, t_0) - 0.4 \right]
\end{align*}
\]

where \(\varepsilon_{sh}\) is shrinkage, \(\varphi_t\) is creep coefficient only including irreversible flow (that of delayed creep is assumed as 0.4), \(A_{cr}\) and \(I_{cr}\) are the transformed area and second moment of area of concrete slab respectively, and \(A_s\) and \(I_s\) are the area and second moment of area of combination of reinforcement and steel beam respectively.

\[
A_{cr} = A_c \frac{E_c}{1.4E_s}, \quad I_{cr} = I_c \frac{E_c}{1.4E_s}
\]

where \(E_c\) and \(E_s\) are the elastic modulus of concrete and steel respectively.

3. Example
One high and large composite beam (Figure 2) is considered.

![Cross-section](image-url)
Rigorously solving of Equation 1 is conducted by MATLAB programs. Firstly for plain concrete slab ($\rho=0$) time-dependent internal forces are analysed, shown in Figure 3.

![Figure 3. Internal forces](image)

Figure 3a shows that creep and shrinkage deduce the axial force between the two individual sections (the final is the 62% of the initial), which means that composite action can be deduced be creep and shrinkage significantly.

Figure 3b shows that the moment of slab ($M_{ct}$) is far smaller than that of the beam ($M_{st}$) and can be neglected in high steel beam such as in this example. Most importantly the final moment of steel beam ($M_{st}$) reaches 1.95 times the initial ($M_{s0}$), which means that the deflection of the beam will be increased by creep and shrinkage greatly, because the increase ratio of the deflection ($f$) is equal to that of the moment of steel beam according to the following relationship,

$$f = \frac{\int M_{st} \, dl}{E_{s} f_{s}} \Rightarrow f_t = \frac{f_{st}}{f_{s0}} = \frac{M_{st}}{M_{s0}}$$  \hspace{1cm} (3)

The symmetric reinforcement of concrete slab is considered, and the corresponding dates are given in table 1

| Reinforcement ratio $\rho$ (%) | Axial force $N$ (kN) | Bending moment $M$ (kN·m) |
|-------------------------------|----------------------|---------------------------|
| $N_{c0}$ | $N_{cr}$ | $N_{ct}$ | $M_{s0}$ | $M_{sr}$ | $M_{st}$ | $M_{st}/M_{s0}$ |
| 0  | -1848 | 380 | 316 | -1152 | 856 | 451 | 360 | 1666 | 1.95 |
| 0.5 | -1606 | 486 | 461 | -660 | 1272 | 531 | 487 | 2290 | 1.80 |
| 1  | -1390 | 526 | 574 | -290 | 1605 | 535 | 566 | 2706 | 1.69 |
| 1.5 | -1207 | 528 | 661 | -18 | 1865 | 502 | 611 | 2978 | 1.60 |

Table 1 shows that the influence of the reinforcement on the reduced composite action by creep and shrinkage is great, the reduction tends larger with the increase of $\rho$, and the axial force $N_{ct}$ almost closes to zero under $\rho=1.5\%$ (also shown in Figure 4).

On the other side the deflection can be decreased slightly by the increase of the reinforcement because it increases axial constraint effect on the concrete.

Moreover, both Figure 4a and 4b show that the curves are almost parallel to each other, which means that a linear relationship exists between them.
4. Conclusions

With the development of computer technique it is more easier to solve the system of coupled differential equations than before. Analytical solutions provide mathematical convenience for parameter analyse compared with FEM. Rüsch constitutive equation of creep is chosen to establish the equations. For composite beam the results show a linear relationship of the influence of the reinforcement, and:

(a) Composite action can be reduced be creep and shrinkage significantly and with the increase of the reinforce ratio the reduction tends larger.

(b) Deflection of the beam will be increased by creep and shrinkage significantly and with the increase of the reinforce ratio the increase is alleviated slightly.

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