Dynamic surface control for nonlinear fractional-order chaotic systems with time delays

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Abstract. Due to the uncertainty and randomness, the chaotic behavior of nonlinear system has become an important factor affecting the stability of the system. Therefore, it is very necessary to ensure that the nonlinear system has strong robustness against the external stochastic interference. This paper presents a new dynamic surface controller for nonlinear systems with time delays. In order to be closer to the actual engineering complex operation environment, the fractional calculus operator is added in the system. The dynamic surface control and time-delay effect is first applied to the second-order nonlinear fractional-order system. To deal with the problem of number explosion and time-delay in traditional inversion methods, a fractional-order filter and an alpha-order Padé approximation method are designed. After fully considering the tracking error, virtual control error and filtering error, the dynamic surface controller is established and its stability is verified. It can be seen from the simulation results that the system tracks the set function trajectory in finite time, and the controller has a good effect.

1. Introduction

In the past few decades, the research on robust control of nonlinear system with inversion design method has been favored by the researchers of control theory and control engineering. However, the derivation of the inversion controller is very complicated, and there is no good solution to the differential explosion caused by the derivation of virtual variables in the implementation process. With the increase of the order of the system studied, the defects of the inversion method are more and more obvious. By introducing a first-order filter, Jinkun Liu[1] and Xu Fang[2] proposed a dynamic surface control method to eliminate the expansion of differential terms and overcome the shortcomings of inversion design. Huan Shen[3] and Guofa Sun[4] applied dynamic surface control to electro-hydraulic servo nonlinear system to realize synchronous stability tracking of the system.

However, the relevant results of the design of nonlinear fractional-order time-delay system combined with dynamic surface control are still relatively few. In this paper, a dynamic surface control strategy is proposed to realize the finite-time synchronization control of fractional-order chaotic systems. Two numerical examples are utilized to demonstrate the effectiveness of the proposed control scheme.

2. Establishment of second-order nonlinear system

The second-order nonlinear fractional order chaotic systems with time-delay are modeled by Eqs. (1).
\[
\begin{cases}
D^\alpha x_1 = x_2 \\
D^\alpha x_2 = f(X, t) + b(X, t)u(t - \tau) \\
y = x_1
\end{cases}
\]  
(1)

where \(\alpha \in (0, 1)\) represents the order of time-fractional derivative. \(X = [x_1, x_2] \in R^2\) and \(f(X, t) \in R\) denote state vector and known nonlinear functions of \(X\) and \(t\) respectively. \(b(X, t) \neq 0\) denotes constant term. \(\tau \in R\) and \(u(t - \tau)\) denote time-delay term and the input of the system with time-delay effect.

According to Hongyi Li’s Pade approximation method[5], an alpha-order Pade approximation method based on fractional order system is proposed as shown to solve the problem of time-delay in system input control.

\[
\ell \{u(t - \tau)\} = \exp(-\tau s^\alpha) \ell \{u(t)\} = \exp(-\frac{\tau s^\alpha}{2}) \ell \{u(t)\} \approx \frac{(1 - \tau s^\alpha / 2)}{(1 + \tau s^\alpha / 2)} \ell \{u(t)\}
\]  
(2)

Taking the simplification of Eq. (2), one can get

\[
u(t - \tau) = x_3 - u(t)
\]  
(3)

Where \(\ell \{u(t)\}\) denotes the Laplace transform of the input controller.

Inserting Eq. (3) into Eq. (2) yields

\[
\ell \{u(t)\} = \ell \{x_3(t)\} - \ell \{u(t)\}
\]  
(4)

Taking the inverse Laplace transform of Eq. (4), one obtains

\[
u - \frac{\tau D^\alpha u}{2} = x_3 + \frac{\tau D^\alpha x_3}{2} - u - \frac{\tau D^\alpha u}{2}
\]  
(5)

After simplification, the Eq. (5) has been rewritten as

\[
D^\alpha x_3 = -\frac{2}{\tau} x_3 + \frac{4}{\tau} u(t)
\]  
(6)

From inequality Eq. (3) and Eq. (6), Eq. (1) has been converted into the non-time-delay system shown in Eq. (7).

\[
\begin{cases}
D^\alpha x_1 = x_2 \\
D^\alpha x_2 = f(X, t) + b(X, t)(x_3 - u(t)) \\
D^\alpha x_3 = -\frac{2}{\tau} x_3 + \frac{4}{\tau} u(t) \\
y = x_1
\end{cases}
\]  
(7)

In the above equations, \(x_3\) is not the real state variable name in the system. but the virtual state quantity that must be assumed in order to transform the system into a non-delay system. \(x_3\) is the virtual state quantity that must be assumed in order to convert the system into a non-time-delay system.

3. Nonlinear fractional order dynamic surface controller design

In this section, in order to comprehensively improve the defects and inaccuracy of inversion method in practical application, the dynamic surface controller of fractional order system is designed and verified by lyapunov stability principle.

**Lemmas 1** [6]: For \(0 < \alpha \leq 1, p \in 2^n, n \in N\), the arbitrary continuous and differentiable function \(x(t)\) is satisfied:

\[
D_1^\alpha x^p(t) \leq px^{p-1}(t)D_1^\alpha x(t)
\]  
(8)
From the above **Lemmas 1**, we can get the following inference:

**Inference 1**: For the arbitrary continuous and differentiable function $x(t)$, then the following equation holds:

$$\frac{1}{2} D_t^\alpha x^2(t) \leq x(t) D_t^\alpha x(t)$$  \hspace{1cm} (9)

**Theorem 1**: For systems of fractional order, such as systems (1), if $x(t)$ is the equilibrium point of the system, there must be a lyapunov function satisfying the following inequality:

$$\dot{D}_t^\alpha V(t, x(t)) \leq 0$$  \hspace{1cm} (10)

The design steps of the dynamic surface controller in the fractional order system are as follows:

Firstly, supposing the trajectory of the bounded function expected to be tracked is $x_{id}$, the position error is defined as:

$$z_1 = x_1 - x_{id}$$  \hspace{1cm} (11)

Taking the fractional derivative of Eq. (11), one obtains

$$D^\alpha z_1 = D^\alpha x_1 - D^\alpha x_{id}$$  \hspace{1cm} (12)

After considering the influence of position error, the lyapunov function is designed as follows:

$$V_1 = \frac{1}{2} z_1^2$$  \hspace{1cm} (13)

Inserting Eq. (13) into Eq. (9) yields

$$D^\alpha V_1 \leq z_1 D^\alpha z_1 = z_1 (D^\alpha x_1 - D^\alpha x_{id})$$  \hspace{1cm} (14)

Inserting Eq. (1) into Eq. (14) yields

$$D^\alpha V_1 \leq z_1 D^\alpha z_1 = z_1 (D^\alpha x_1 - D^\alpha x_{id}) = z_1 (x_2 - D^\alpha x_{id})$$  \hspace{1cm} (15)

Introducing dummy control variable $z_2$, and setting $z_2 = x_2 - \alpha_1$, one obtains

$$D^\alpha V_1 \leq z_1 (z_2 + \alpha_1 - D^\alpha x_{id})$$  \hspace{1cm} (16)

Based on the **Theorem 1**, in the traditional inversion method, the function $\alpha_1$ is designed as follows:

$$\alpha_1 = D^\alpha x_{id} - c_1 z_1$$  \hspace{1cm} (17)

In this paper, setting the function $x_2 = D^\alpha x_{id} - c_1 z_1$, which is the input of fractional-order filter.

The output of fractional-order filter is the function $\alpha_1$. Then the following equation holds:

$$\begin{cases} \tau D^\alpha \alpha_1 + \alpha_1 = \overline{x_2} \\ \alpha_1(0) = x_2(0) \end{cases}$$  \hspace{1cm} (18)

According to Eq. (18), the error after passing the filter is defined as

$$y_2 = \alpha_1 - x_2$$  \hspace{1cm} (19)

After considering the influence of position error, filtering error and virtual control quantity, the lyapunov function is designed as follows:

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} y_2^2$$  \hspace{1cm} (20)

Based on Eq. (1), taking the alpha-order fractional derivative of Eq. (11), one obtains
Setting $B_2 = -D^{2\alpha}x_{id} + c_1D^\alpha z_1$ and taking the simplification of Eq. (21), one can get

$$D^\alpha V_2 \leq z_1(z_2 + y_2 - c_1z_1) + z_2(f(X,t) + b(X,t)(x_j - u(t)) - D^\alpha x_j) + y_2\left(-\frac{y_2}{\tau} - D^{\alpha}x_{id} + c_1D^\alpha z_1\right)$$

From Eq. (22), the dynamics of the designed dynamic surface controller can be obtained:

$$u(t) = x_3 + \frac{1}{b(X,t)}\left(f(X,t) - D^\alpha x_1 + c_2z_2\right)$$

where $c_2$ is a positive constant.

4. Simulation results

In this section, the following two illustrative examples are employed to demonstrate the effectiveness of the proposed control approach in finite time synchronization tracking control of fractional order chaotic systems.

4.1 Tracking control of the second-order nonlinear fractional order chaotic systems with certain time-delay

$$\begin{cases}
D^\alpha x_1 = x_2 \\
D^\alpha x_2 = -25x_2 + 100u(t - 0.2) \\
y = x_1
\end{cases}$$

where nonlinear functions $f(X,t) = -25x_2$, $b(X,t) = 100$, time-delay term $\tau = 0.2$.

From alpha-order Padé approximation method proposed above, the non-time-delay system is shown in Eq. (31).

$$\begin{cases}
D^\alpha x_1 = x_2 \\
D^\alpha x_2 = -25x_2 + 100(x_j - u(t)) \\
D^\alpha x_3 = -10x_3 + 20u(t) \\
v = x_1
\end{cases}$$

The initial conditions are selected as: $[x_1, x_2, x_3, x_4] = [0.5, 0, 1, -1.5]$. The controller parameters and tracking function are chosen as: $c_2 = 2$, $c_1 = 2.5$, $tol = 0.01$, $\alpha = 0.98$, $y_1 = x_{id}$. The dynamic surface controller is described as:

$$u(t) = x_3 + \frac{1}{100}\left(-25x_2 - D^\alpha x_1 + 2z_2\right)$$
4.2 Tracking control of the second-order nonlinear fractional order chaotic systems with uncertain time-delay

\[
\begin{aligned}
D^\alpha x_1 &= x_2 \\
D^\alpha x_2 &= -25x_2 + 100u(t - 0.2 - 0.2\cos(t)) \\
y &= x_1
\end{aligned}
\]

where nonlinear functions \( f(X,t) = -25x_1, b(X,t) = 100 \), time-delay term \( \tau = 0.2 + 0.2\cos(t) \).

From alpha-order Pade approximation method proposed above, the non-time-delay system is shown in Eq. (33).

\[
\begin{aligned}
D^\alpha x_1 &= x_2 \\
D^\alpha x_2 &= -25x_2 + 100(x_3 - u(t)) \\
D^\alpha x_3 &= \frac{2}{0.2 + 0.2\cos(t)} x_3 + \frac{4}{0.2 + 0.2\cos(t)} u(t) \\
y &= x_1
\end{aligned}
\]

The initial conditions are selected as: \( [x_1, x_2, x_3, \alpha_1] = [1.5, -0.5, 1, 1.5] \). The controller parameters and tracking function are chosen as: \( c_2 = 2, c_1 = 2.5, tol = 0.01, \alpha = 0.98, y_1 = x_{1d} \).
The simulation results are shown in FIG.1-FIG.8. The simulation results show that the designed dynamic plane controller has good synchronization tracking performance despite the existence of deterministic and uncertain delays in second-order nonlinear fractional-order systems. As can be seen from FIG. 4 and FIG. 8, this method ensures that the synchronization error rapidly converges to zero, and the convergence time is about 2.5s, which verifies the correctness of dynamic surface controller.

5. Conclusion

In this paper, fractional-order operator, dynamic plane control and time-delay effect are combined together, and a new alpha-order Pade approximation method is proposed to solve the problem of time-delay effect in nonlinear fractional-order system. In the design of dynamic surface, the virtual controller is introduced, and a fractional low pass filter is proposed to filter the virtual controller to avoid differential solution. After considering the tracking error, virtual control error and filtering error, the appropriately lyapunov function is designed and the corresponding dynamic surface controller is constructed, so that the nonlinear system can quickly track the set function trajectory within 2.5s.

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