Research on Power Quality Acquisition and Reconstruction Method Based on Compressed Sensing

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Abstract. The power quality acquisition method based on compressed sensing (CS) has become an inevitable trend to solve the problem of explosive growth in power quality data. In order to solve the problems that the signal type is not comprehensively considered and the method of power quality acquisition is not universal, which are difficult to adapt to the actual system construction, this paper studies the method of power quality information acquisition and reconstruction method based on CS from a new perspective. Based on power quality acquisition and reconstruction framework using CS, this paper deeply studies the sparse basis, measurement matrix and reconstruction algorithm, classifying and analyzing all kinds of power quality signals as the result. Finally, through the experimental analysis, the construction scheme for three-element of CS is given, which is suitable for power quality acquisition. All the work done in this paper is intended to provide a reference for follow-up research and actual system construction.

1. Introduction

With the access of new “source-load” resources such as electric vehicles and renewable energy, power quality acquisition faces two problems which are increasing collection points and dense collection intervals. Under the dual pressure of power quality data’s explosive growth and equipments’ efficient use, compressed sensing is widely used in power quality acquisition and analysis[1-5]. CS completes the two processes of data acquisition and data compression at the same time. Compared with the traditional compression method, sampling-end device has very low complexity in compression and saves storage space, which is very suitable for power quality acquisition systems. Reference [1] uses CS in power quality acquisition and conducted preliminary exploration. Reference [2] studies the identification of various power quality signals. Reference [3] summarizes the research of CS in power quality, and points out that the adaptive three-element of CS is key issue in system building of power quality acquisition. Reference [4] improves the two compression sensing elements which are measurement matrix and reconstruction algorithm according to the characteristics of harmonic signals. Reference [5] optimizes the element of sparse representation method for the power quality signal.

With the deepening of research, the optimization methods for three-element of CS are continuously proposed[1-5], but the following drawbacks are existing. Firstly, the improved methods are developed for a specific and classic method, which is not universal. Secondly, in the actual system construction in the future, mature and classic methods are easier to adopt as core algorithms, but there are less comprehensive and comparative researches on classical algorithms. Thirdly, in the research of power quality acquisition, CS is mostly directed to specific signals and lacks comprehensiveness.
In order to overcome the shortcomings of the above research, this paper uses a comprehensive perspective to study the power quality acquisition and reconstruction method based on CS. Based on the classification and modeling of various power quality signals, the three-element of CS (sparse basis, measurement matrix and reconstruction algorithm) are deeply studied. According to different signal characteristics, we analyze the effects of CS using different three-element, and then formulate the three-element construction scheme of CS in order to be suitable for future power quality acquisition systems.

2. Related Models

2.1 Mathematical model of power quality signal

The IEEE Standards Coordination Committee classifies power quality issues into two categories: steady-state power quality (including harmonics/inter-harmonics, voltage fluctuation/flicker, notch and noise, 4 categories in total) and transient power quality (including voltage dip, voltage swell, voltage interruption, pulse transient, oscillating transient, voltage cut, voltage spike, 7 categories in total). According to the power quality IEC series standards IEC-61000-1~IEC-61000-6, we can get of all 11 kinds of power quality data, which can be found in IEC-61000-1~IEC-61000-6 and we are no longer giving formulas of mathematical models here.

| Model type                        | Model formula | Explanation                                      |
|-----------------------------------|---------------|-------------------------------------------------|
| Model of Compressed Measurement   | \[ y = \Phi x \; \Phi \Psi s = c \] | Vector \( x \) is the \( N \times 1 \) dimensional original signal and \( y \) is the \( M \times 1 \) dimensional compressed sample vector (\( M \ll N \)); \( x \) needs to satisfy that coefficient \( s \) (sparse vector) is \( K \)-sparse under the sparse basis. |
| Model of Signal Reconstruction    | \[ \min_{s} \|s\|_{p} \quad \text{s.t.} \quad y = \Phi \Psi s \] | When \( 0 \leq p < 1 \), the Model of Signal Reconstruction becomes an NP-hard problem. When \( p \geq 1 \), it translates into a convex optimization problem, and bringing infinite solutions. The reconstruction model is mainly divided into 2 types: the minimized \( l_{0} \) norm and the minimized \( l_{1} \) norm. |

2.2 Mathematical model of compressed sensing

Sparse basis, measurement matrix and reconstruction algorithm constitute the three-element of CS. The mathematical models of CS\(^{[6-7]}\) are shown as Table 1.

3. Power Quality Acquisition and Reconstruction Method Based on CS

3.1 Power quality acquisition based on CS

Power quality acquisition system constructed by CS can divide into 2 processes: compressed measurement and signal reconstruction. Power quality acquisition process based on CS is shown in Figure 1.

As shown in the figure, there are three key problems in the whole process—sparse basis, measurement matrix and reconstruction algorithm, which are the three-element of CS. In addition to considering the requirements of CS theory, the design of these three key issues should also consider the characteristics of power quality data. \( y \) is the front-end sampled value, the process for obtaining \( y \) is implemented in the analog signal, that is, compression is integrated into the sample.
3.2 Research on sparse basis

Commonly-used sparse basis include fixed orthogonal transform (FOT) basis, multi-scale geometric analysis, and redundant dictionary, among which FOT basis is more suitable for one-dimensional signals. Compared with the redundant dictionary, FOT has simple structure and its implementation is more flexible. Therefore, this paper firstly determines the FOT as the applicable sparse basis, including Discrete Fourier Transform (DFT) basis, Discrete Cosine Transform (DCT) basis and sparse basis based on Wavelet Transform (WT). The signal in power system is a sinusoidal signal based on power frequency 50Hz. Various power quality data models can be considered as superimposing certain interfering signals on the basis of the sinusoidal signal, whether the interfering signal has short-term characteristics (such as pulse, oscillating) or low energy characteristics (such as harmonics). DFT basis and DCT basis are best suited for sinusoidal signals while DCT is better than DFT in implementation effect and degree of difficulty. Therefore, we considered that DCT basis should be selected as the sparse basis.

3.3 Research on measurement matrix

In order to accurately reconstruct signal, the measurement matrix must satisfy Restricted Isometry Property (RIP), and the random matrix can satisfy the RIP with great probability\(^6\). At present, typical random matrices are mostly dense matrices, including Gaussian matrix, partial Hadamard matrix, Bernoulli matrix, partial orthogonal matrix (such as partial fast Fourier transform, PFFT), and Toeplitz matrix. Gaussian measurement is the most widely used in power quality because its incoherence with orthogonal sparse matrices. Compared with dense measurement matrix, one kind of matrices called sparse matrix can greatly save sampling cost in sampling-end equipments through reducing complexity of compression, and can solve implementation difficulty of hardware\(^8\). This paper proposes to select the Binary Sparse matrix as the measurement matrix in power quality acquisition\(^8\).

**Definition 2.1 (Binary Sparse matrix)** An \(M \times N\)-dimensional matrix \(\Phi\) if each element of its column contains only \(\mu M\) positions with an element value of 1, and remaining elements are all zero, and \(\mu M\) non-zero element-positions are random, the matrix \(\Phi\) is called Binary Sparse matrix. \(\mu\) is the sparsity rate, \(\mu \ll 1\).

Compared to dense measurement matrices, Binary Sparse matrix have the following advantages: (1) Great sparsity reduce complexity of measurement in sampling-end, which see Table 2 for details. Due to \(\mu \ll 1\), the complexity is greatly reduced. (2) The hardware of sampling-end is easy to implement, and its method can adopt as Analog Information Converter (AIC).

|                  | Binary Sparse | Dense |
|------------------|---------------|-------|
| Compression complexity | \(O(\mu MN)\) | \(O(\mu MN)\) |
| Storage data of matrix | \(2\mu MN\) | \(MN\) |

3.4 Research on reconstruction algorithm

The reconstruction algorithms of CS can be divided into 3 categories, which are greedy algorithm based on \(l_0\) norm, convex optimization algorithm based on \(l_1\) norm and some other combined reconstruction algorithms such as iterative threshold algorithm. The greedy algorithm gradually approximates the signal through multiple iterative searches, including Orthogonal Matching Pursuit (OMP), Compressive Sampling Matching Pursuit (CoSaMP), Subspace Pursuit (SP), Gradient Pursuit (GP), and so on. The convex optimization algorithm is based on the Basis Pursuit problem and has high complexity, including Gradient Projection for Sparse Reconstruction (GPSR), Spectral Projected Gradient (SPG), and so on. The Iteration Hard Threshold (IHT) and Fast Iteration Shrink Threshold Algorithm (FISTA) are representative of the iterative threshold algorithm.

This paper considers various factors to determine the applicable reconstruction algorithm. In all typical algorithms, convex optimization algorithm has higher reconstruction precision while has slow convergence and poor anti-noise performance. Among them, SPG has higher reconstruction accuracy.
than other convex optimization algorithms. With the development of greedy algorithms, GP algorithm takes the convergence speed and reconstruction effect into account. Its computation time is better than the OMP which is typical fast algorithm, and the reconstruction accuracy is similar to SPG. Therefore, this paper believes that the GP is used as the main algorithm, and SPG is adopted as the reconstruction algorithm for power quality signal with poor sparsity.

4. Experimental Analysis Based on CS for all Power Quality Signals

4.1 Performance of sparse basis

We analyzed the reconstruction effects of all 11 power quality signals with different sparse basis, and give the reconstruction effects of partial steady-state and transient power quality data with DCT basis, DFT basis and DWT basis, shown as Figure 2. The reconstruction errors with the DCT basis and DFT basis are almost identical and they are significantly better than the reconstruction errors with DWT basis, which is consistent with the conclusions previously mentioned herein.

4.2 Performance of measurement matrix

The power quality data is measured by different measurement matrices, and reconstruction effects with different measurement matrices are compared. The signal noise of reconstruction (SNR) of various signals is shown in Figure 3 and Figure 4. Uniformly, the sparse basis is DCT and reconstruction algorithm is OMP.

It can be seen from the figure that the SNRs with Binary Sparse measurement and with Gaussian measurement are better than other measurement methods by about 3~20 dB. In the Gaussian matrix and Binary Sparse matrix which have better effects than others, the implementation and sampling complexity of the Binary Sparse are obviously better than Gaussian matrix, which is consistent with the conclusions mentioned earlier.

4.3 Performance of reconstruction algorithm

The power quality data is reconstructed by different algorithms, and different effects are compared. The SNR of various signals is shown in Figure 5 and Figure 6. Uniformly, the sparse basis is DCT and the measurement is Binary Sparse matrix. It can be seen from the figure that the convex optimization algorithm is better than others. In all kinds of power quality signals, SPG and GP are obviously superior to other algorithms while IHT and FISTA are the worst. In addition to 4 type transient signals (pulse transient, oscillating transient, voltage cut and voltage spike), reconstruction effect of GP in other power quality signals has similar effects to the SPG, SNR of which is within 1 dB compared with SPG (SPG is slightly better than GP), and GP is superior to some convex optimization algorithms such as GPR. Therefore, this paper considers that SPG algorithm is used to 4 transient power quality signals including oscillating transient, pulse transient, voltage cut and voltage spike. The other 7 power quality signals adopt GP algorithm.

4.4 Analysis of compression ratio

The smaller compression ratio ($M/N$) will bring the worse reconstruction effect. We analyzed various signal reconstruction effects using Binary Sparse measurement, GP algorithm and DCT basis. When
$M/N$ is less than 30%, the reconstruction error is greatly increased. And when the $M/N$ is 40%, the SNR is greater than 60 dB, which exceeds the requirement. Therefore, the proposed compression ratio is 30%–40%.

Figure 3. Reconstruction SNR with different measurement matrix in steady-state power quality

Figure 4. Reconstruction SNR with different measurement matrix in transient power quality

Figure 5. Reconstruction SNR with different reconstruction algorithm in steady-state power quality

Figure 6. Reconstruction SNR with different reconstruction algorithm in transient power quality

5. Summary
For the future trend of building power quality acquisition system based on CS, we systematically studied the construction of key elements in CS for power quality acquisition, and make up for the shortfall that current researches are lack of comprehensiveness and applicability. The recommended scheme of power quality acquisition and reconstruction based on CS in this paper is as follows.

Table 3. The recommended scheme of power quality acquisition and reconstruction based on CS

| Sparse basis | DCT |
|--------------|-----|
| Measurement matrix | Binary Sparse measurement matrix |
| Reconstruction algorithm | SPG: oscillating transient, pulse transient, voltage cut and voltage spike, GP: other kinds of power quality |
| Compression ratio | 30%–40% |
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