What You See is What You Get: Distributional Generalization for Algorithm Design in Deep Learning

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Abstract

We investigate and leverage a connection between Differential Privacy (DP) and the recently proposed notion of Distributional Generalization (DG). Applying this connection, we introduce new conceptual tools for designing deep-learning methods that bypass “pathologies” of standard stochastic gradient descent (SGD). First, we prove that differentially private methods satisfy a “What You See Is What You Get (WYSIWYG)” generalization guarantee: whatever a model does on its train data is almost exactly what it will do at test time. This guarantee is formally captured by distributional generalization. WYSIWYG enables principled algorithm design in deep learning by reducing generalization concerns to optimization ones: in order to mitigate unwanted behavior at test time, it is provably sufficient to mitigate this behavior on the train data. This is notably false for standard (non-DP) methods, hence this observation has applications even when privacy is not required. For example, importance sampling is known to fail for standard SGD, but we show that it has exactly the intended effect for DP-trained models. Thus, with DP-SGD, unlike with SGD, we can influence test-time behavior by making principled train-time interventions. We use these insights to construct simple algorithms which match or outperform SOTA in several distributional robustness applications, and to significantly improve the privacy vs. disparate impact tradeoff of DP-SGD. Finally, we also improve on known theoretical bounds relating differential privacy, stability, and distributional generalization.

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Figure 1. Differential privacy ensures the desired behavior of importance sampling on test data. The train and test accuracy of ResNets on CelebA, evaluated on the worst-performing (“male, blond”) subgroup. Left: Standard SGD has a large generalization gap on this subgroup, and Importance Sampling (IS) has little effect. Right: DP-SGD provably has small generalization gap on all subgroups, and IS improves subgroup performance as intended. See Section 7 for details.

1. Introduction

Much of machine learning (ML), both in theory and in practice, operates under two assumptions. First, we have i.i.d. samples from a distribution. Second, we care only about a single averaged scalar (e.g. error/loss/risk) over this distribution. Under these assumptions, we have relatively mature methods and theory: modern learning methods, including deep learning, excel when trained on i.i.d. data to directly optimize a scalar loss. On the theoretical side, many tools have been developed to reason about generalization, explaining when does optimization of a scalar on the train dataset translates to similar values of this scalar at test time. However, focusing on scalar metrics such as average error misses many theoretically, practically, and socially relevant aspects of model performance. For example, models with small average error often have high error on salient minority subgroups, such as certain races and genders (Buolamwini and Gebru, 2018; Koenecke et al., 2020). In general, ML models are applied to the heterogeneous and long-tailed data distributions of the real world (Van Horn and Perona, 2017). Attempting to summarize their complex behavior in such a setting with only a single scalar misses many rich and important aspects of learning.
These issues are compounded for modern overparameterized networks, which can be large enough to fit their train datasets exactly. For these, not only does test performance vary on different subsets of the input, but this variation is not apparent at train time: all train samples will have 0% error, but some test samples may have much higher error than others. Thus, for overparameterized models, “what you see” (on the train dataset) is not “what you get” (at test time). This pathology presents a serious obstruction to algorithm design in certain deep-learning settings. For example, consider the setting of importance sampling: suppose we know that a certain subgroup of inputs is underrepresented in the train dataset compared to the test distribution (breaking the “i.i.d. train/test” assumption). For underparameterized models, we can simply up-sample this underrepresented group to account for the distribution shift (see, e.g., Gretton et al., 2009). This approach, however, is known to empirically fail for overparameterized models (Byrd and Lipton, 2019). Because “what you see” (on the train dataset) is not “what you get” (at test time), we cannot make principled train-time interventions to affect test-time behaviors. This issue extends beyond importance sampling: for example, theoretically principled methods for distributionally robust optimization (e.g. Namkoong and Duchi (2016)) fail for overparameterized models, and require ad-hoc modifications (Sagawa et al., 2019).

We develop a theoretical framework which both sheds light on these existing issues, and leads to improved practical methods in privacy, fairness, and distributional robustness. The core object in our framework is a formalization of what we call the “What You See Is What You Get” (WYSIWYG) property. A training procedure with the WYSIWYG property does not exhibit the pathologies of standard stochastic gradient descent (SGD): all test-time behaviors will be expressed on the train dataset as well, and there will be “no surprises” in generalization. Thus, such procedures can be benefit from algorithmic advances in optimization, without worrying about generalization.

**What You See Is What You Get (WYSIWYG).** To formalize the idea of the “WYSIWYG” property, we use the notion of Distributional Generalization (DG), as introduced by Nakkiran and Bansal (2020); Kulychen et al. (2022). A training algorithm generalizes in expectation in the classical sense if the values of loss on the training dataset and at test time are close on average (Shalev-Shwartz et al., 2010):

$$\mathbb{E}_{\theta, S, z \sim S} \ell(z; \theta_S) \approx \mathbb{E}_{\theta, S, z \sim \mathcal{D}} \ell(z; \theta_S),$$

where $\theta_S$ is the parameter vector of the model obtained by training on the dataset $S \sim \mathcal{D}^n$, i.i.d. sampled from the data distribution $\mathcal{D}$. Distributional generalization is an extension of this standard concept that considers not only loss, but any other bounded test function $\phi(z; \theta) \in [0, 1]$. Specifically, by saying that a model distributionally generalizes we mean that for all such test functions $\phi$, their values in training and test are close on average:

$$\forall \phi : \mathbb{E}_{\theta, S, z \sim S} \phi(z; \theta_S) \approx \mathbb{E}_{\theta, S, z \sim \mathcal{D}} \phi(z; \theta_S).$$

For example, consider the test function $\phi((x, y); \theta) \equiv \mathbb{1}\{x \in G \land f_\theta(x) = y\}$ which measures accuracy of the model $f_\theta$ on subgroup $G$ of the input space weighted by the relative size of this subgroup. Then, Equation (2) states that this subgroup accuracy is close between the train and test time. Equation (2) is equivalent to stating that the distributions of $(z, \theta)$ for $z \sim S$ and $z \sim \mathcal{D}$ are close in total-variation (TV) distance:

$$(z, \theta)_{z \sim S, \theta \sim \mathcal{D}^n} \approx_{TV} (z, \theta)_{z \sim \mathcal{D}^n}.$$}

Thus, if we can achieve DG, then we have a “WYSIWYG” guarantee: whatever the model does on its train dataset is guaranteed to occur on the test distribution. This holds for much more than subgroup errors: for example, if a DG model has small small error with respect to certain corruptions (e.g., Gaussian noise) on its train dataset, it will have small respective error at test time as well. If DG holds, essentially any property of the model on its train dataset will transfer to test time. We describe DG formally in Section 4, including its relations to other notions of generalization.

**DG as a Design Principle.** The WYSIWYG property is desirable for two complementary reasons. The first reason is diagnostic: there are “no surprises” at test time — all properties of a model at test time are already evident on the train dataset. It cannot be the case, for example, that a DG model has small disparate impact on the train dataset, but large disparate impact at test time. The second reason is algorithmic: to mitigate any unwanted test-time behavior, it is sufficient to mitigate this behavior on the train dataset. This means that algorithm designers can be concerned only with achieving “good behavior” on the train dataset, and the WYSIWYG property guarantees good behavior at test time. In practice, this enables the usage of many theoretically principled algorithms which were developed in the underparameterized (small model) regime to also apply in the modern overparameterized setting. For example, we find that interventions such as importance sampling, or algorithms for distributionally robust optimization, which fail without additional regularization, work exactly as intended with DG (See Figure 1 for an illustration). We thus propose to use optimizers which satisfy distributional generalization (either provably or heuristically), and build algorithmic advances on top of them.

**Achieving DG via Differential Privacy (DP).** Our key observation is that distributional generalization is formally implied by differential privacy (Dwork et al., 2006; 2014).
We demonstrate how DG can be a useful design principle and show that a similar connection holds for the notion of ways of achieving DG. Inspired by this, we show how ideas from DP can be used to construct heuristic optimizers, which do not formally satisfy DP, yet empirically satisfy DG. Our heuristics lead to competitive results with SOTA algorithms.

In particular, we show that if a training procedure satisfies $\epsilon$-DP, it also satisfies DG as in Equation (3) with TV distance at most $\epsilon^2/2 + O(\epsilon^3)$. This guarantees the WYSIWYG property for any method that is differentially-private, including DP-SGD on deep neural networks (Abadi et al., 2016). We focus on our notion of DG for both conceptual and theoretical simplicity: DG formulated as in Equation (3) enables us to prove tight theoretical bounds that are independent of sample size. A more comprehensive discussion of relations to robust generalization is in Appendix A. It can also be useful to consider intermediary definitions, varying in strength from DG to robust generalization. In Appendix B, we introduce such an intermediary notion (“strong DG”) and show its connections to DP.

### 2. Related Work

**DP and Strong Forms of Generalization.** DP is known to imply a stronger than standard notion of generalization, called robust generalization (Dwork et al., 2015a; Cummings et al., 2016; Bassily et al., 2016). Robust generalization can be thought of as a high-probability counterpart of DG: generalization holds with high probability over the train dataset, not only on average over datasets. In particular, it is possible to recover our result that DP implies DG as a consequence of these prior works, although with looser bounds.

**Disparate Impact of DP.** Bagdasaryan et al. (2019); Pujol et al. (2020) have shown that ensuring DP in algorithmic systems can cause error disparity across population groups. Concurrently, Xu et al. (2021) have proposed DP-SGD-F, a variant of DP-SGD (Abadi et al., 2016) that dynamically adapts gradient-clipping bounds for different groups to reduce such disparate impact. We show that a simpler approach based on importance sampling is also an effective way to reduce disparities, and we provide a more general blueprint for designing algorithms with similar mitigations.

**Group-Distributional Robustness.** Group-distributional robustness aims to improve the worst-case group performance. Existing approaches include using worst-case group loss (Mohri et al., 2019; Sagawa et al., 2019; Zhang et al., 2020), balancing majority and minority groups by reweighting or subsampling (Byrd and Lipton, 2019; Sagawa et al., 2019; see Section 2 for more details) and stability than standard SGD (Dwork et al., 2015a; Cummings et al., 2016; Bassily et al., 2016). We show that a similar connection holds for the notion of distributional generalization, and prove (and improve) tight bounds relating DP, stability, and DG.

Taken together, our results emphasize the central role of distributional generalization — and the WYSIWYG property it implies — in designing machine learning algorithms which avoid the “pathologies” of standard SGD. We also establish differential privacy as a useful tool for achieving DG, and thus extend its applications further beyond privacy.

The spirit of this observation is not novel: DP training methods are known to satisfy much stronger notions of generalization (e.g., robust generalization, see Section 2 for more details) and stability than standard SGD (Dwork et al., 2015a; Cummings et al., 2016; Bassily et al., 2016). We show that a similar connection holds for the notion of distributional generalization, and prove (and improve) tight bounds relating DP, stability, and DG. In particular, we show that if a training procedure satisfies $\epsilon$-DP, it also satisfies DG as in Equation (3) with TV distance at most $\epsilon^2/2 + O(\epsilon^3)$. This guarantees the WYSIWYG property for any method that is differentially-private, including DP-SGD on deep neural networks (Abadi et al., 2016). We focus on our notion of DG for both conceptual and theoretical simplicity: DG formulated as in Equation (3) enables us to prove tight theoretical bounds that are independent of sample size. A more comprehensive discussion of relations to robust generalization is in Appendix A. It can also be useful to consider intermediary definitions, varying in strength from DG to robust generalization. In Appendix B, we introduce such an intermediary notion (“strong DG”) and show its connections to DP.
We consider the following notion, which has been studied in training algorithms (Goel et al., 2020), and applying various regularization techniques (Sagawa et al., 2019; Cao et al., 2019). Although some work (Sagawa et al., 2019; Cao et al., 2019) discusses the importance of regularization in distributional robustness, they have not explored potential reasons for this importance (e.g., via the connection to distributional generalization). Another line of work studies how to improve group performance without group annotations (Duchi et al., 2021; Liu et al., 2021; Creager et al., 2021), which is a different setting from ours as we assume the group annotations are known.

3. Background

In this section, we overview the existing connections between differential privacy, stability, and generalization. These connections are well-known. In particular, stability of the learning algorithm — its non-sensitivity to limited changes in the training data — is known to imply generalization (Bousquet and Elisseeff, 2002; Shalev-Shwartz et al., 2010). In turn, differential privacy implies strong forms of stability, thus also ensuring generalization through the chain Privacy ⇒ Stability ⇒ Generalization (Raskhodnikova et al., 2008; Dwork et al., 2015b;a; Wang et al., 2016; Bassily et al., 2016; Feldman and Vondrák, 2018).

Notation. We consider a learning task with a set of examples $X$ and labels $Y$. We assume that a source distribution of labeled examples $z \sim (x, y) \sim D$ is defined over $D = X \times Y$. Given an i.i.d.-sampled dataset $S \sim D^n$ of size $n$, we use a randomized training algorithm $\theta(S)$ that outputs a model’s parameter vector from the set $\Theta$.

First, let us formally define differential privacy.

Definition 3.1 (Differential Privacy (Dwork et al., 2006; 2014)). An algorithm $\theta(S)$ is $(\epsilon, \delta)$-differentially private (DP) if for any two neighbouring datasets — differing by one example — $S$, $S'$ of size $n$, for any subset $T \subseteq \text{range}(\theta)$:

$$\Pr[\theta(S) \in T] \leq \exp(\epsilon) \Pr[\theta(S') \in T] + \delta.$$  (4)

Differential privacy mathematically encodes a notion of plausible deniability of the inclusion of an example in the dataset. However, it can also be thought as a strong form of stability (Dwork et al., 2015b). As such, DP implies other notions of stability.

We consider the following notion, which has been studied in the literature under multiple names and contexts. In the context of privacy, it is equivalent to $(0, \delta)$-differential privacy, and has been called additive differential privacy (Geng et al., 2019), and total-variation privacy (Duchi et al., 2013). In the context of learning, it has been called total-variation (TV) stability (Bassily et al., 2016). We take this last approach and refer to it as TV stability:

Definition 3.2 (TV Stability). An algorithm $\theta(S)$ is $\delta$-TV stable if for any two neighbouring datasets $S, S'$ of size $n$, for any subset $T \subseteq \text{range}(\theta)$:

$$\Pr[\theta(S) \in T] \leq \Pr[\theta(S') \in T] + \delta.$$  (5)

Equivalently, $d_{TV}(\theta(S), \theta(S')) \leq \delta$, where $d_{TV}(P, Q) \triangleq \sup_T |P(T) - Q(T)|$ is the total variation distance between probability distributions $P$ and $Q$.

It is easy to see that $(\epsilon, \delta)$-DP immediately implies $(\exp(\epsilon) - 1 + \delta)$-TV stability (we show a strengthening of this inequality in Proposition 7). By $(\epsilon, \delta)$-DP, we have:

$$\Pr[\theta(S) \in T] - \Pr[\theta(S') \in T] \leq \Pr[\theta(S') \in T] (\exp(\epsilon) - 1 + \delta) \leq \exp(\epsilon) - 1 + \delta.$$  (5)

In particular, TV-stability implies that for a $[0, 1]$-bounded loss function $\ell(z; \theta)$, for any $z$ and any two neighbouring datasets $S, S'$ we have $|\mathbb{E} \ell(z; \theta(S)) - \mathbb{E} \ell(z; \theta(S'))| \leq \delta$, where the expectation is taken over the internal randomness of the algorithm — a property called $\delta$-uniform stability$^2$ (Bousquet and Elisseeff, 2002). Moreover, Bousquet and Elisseeff (2002); Shalev-Shwartz et al. (2010) show that $\delta$-uniform stability implies generalization in expectation as in Equation (1), as well as stronger notions of generalization.

4. Theory of “What You See is What You Get” Generalization

We first overview the notion of distributional generalization and demonstrate its implication: the WYSIWYG property. Second, we show that stability implies distributional generalization. Finally, we improve on the known stability guarantees of differential privacy. As a result, we extend the connections between privacy, stability, and generalization to distributional generalization, showing that stability and privacy imply the WYSIWYG property.

4.1. Distributional Generalization and WYSIWYG

If on-average generalization — Equation (1) — guarantees closeness only of loss values on train and test data, distributional generalization (DG) also guarantees closeness of values of all test functions $\phi(z; \theta) \in [0, 1]$:

Definition 4.1 (Based on Nakkiran and Bansal (2020)). An algorithm $\theta(S)$ satisfies $\delta$-distributional generalization if for all $\phi : D \times \Theta \rightarrow [0, 1]$:

$$\mathbb{E}_{S \sim D^n} \phi(z; \theta(S)) - \mathbb{E}_{S \sim D^n} \phi(z; \theta(S)) \leq \delta.$$  (6)

$^2$This is a variant for randomized training algorithms, called strong uniform RO-stability by Shalev-Shwartz et al. (2010)
By the variational characterization of the total-variation distance, Equation (6) is equivalent to the following bound:

$$d_{TV}(P_1, P_0) \leq \delta,$$

where $P_1$ and $P_0$ are both distributions of $(z, \theta(S))$ over the randomness of $S \sim \mathcal{D}^{k}$ and the training algorithm $\theta(\cdot)$, with the difference that $z \sim S$ in the case of $P_1$ (train distribution), and $z \sim \mathcal{D}$ in the case of $P_0$ (test distribution).

Taking the definition of DG in Equation (6) at face value, it might seem that the guarantee only ensures average closeness of bounded tests on train and test data. This is not, however, the full picture. Consider generalization in terms of a broader class of functions:

**Definition 4.2** (Kulynych et al. (2022)). An algorithm $\theta(S)$ satisfies $(\delta, \pi)$-distributional generalization if for a given property function $\pi : \mathbb{D} \times \Theta \rightarrow \mathbb{R}^k$:

$$d_{TV}(\pi_{\pi}^*P_1, \pi_{\pi}^*P_0) \leq \delta,$$

where $\pi_{\pi}^*P$ is the distribution of $\pi(Z)$ for $Z \sim P$.

We can see that $\delta$-distributional generalization implies $(\delta, \pi)$-distributional generalization for all property functions, as TV distance is preserved under post-processing:

$$\forall \pi : \quad d_{TV}(P_1, P_0) \leq \delta \implies d_{TV}(\pi_{\pi}^*P_1, \pi_{\pi}^*P_0) \leq \delta.$$

Informally, $\delta$-DG means that for all numeric property functions $\pi(z; \theta)$ of a model, the distributions of the property values are close on the train and test data, on average. We call this fact the “What You See is What You Get” (WYSIWYG) guarantee. Some example property functions:

- **Subgroup loss**: $\pi(z; \theta) = \mathbb{I}\{z \in G\} \cdot \ell(z; \theta)$, for some subgroup $G \subset \mathbb{D}$.
- **Counterfactual fairness**: $\pi((x, y); \theta) = f_0(CF(x)) - f_0(x)$, where $CF(x)$ is a counterfactual version of $x$ had it had a different value of a sensitive attribute (Kusner et al., 2017).
- **Robustness to corruptions**: $\pi(z; \theta) = \ell(A(z); \theta)$, where $A(x)$ is a possibly randomized transformation that distorts the example, e.g., by rotating the image, or adding Gaussian noise.
- **Adversarial robustness**: $\pi(z; \theta) = \ell(A_{\theta}(z); \theta)$, where $A_{\theta}(z)$ is an adversarial example, e.g. generated using the PGD attack (Madry et al., 2018).

In the next sections, we show how a training algorithm can provably satisfy DG and therefore provide WYSIWYG guarantees for all properties, including the ones above.

**4.2. Distributional Generalization from Stability and Differential Privacy**

Similarly to the classical generalization, one way to achieve distributional generalization is through stability:

**Theorem 4.3.** Suppose that the training algorithm is $\delta$-TV stable. Then, the algorithm satisfies $\delta$-DG.

We refer to Appendix C for the proofs of this and all other formal statements in the rest of the paper.

As DP implies TV-stability, by Theorem 4.3 we have that DP also implies DG. Next, we show that DP algorithms enjoy a significantly stronger stability guarantee than the previously known one in Equation (5), which means that the DG guarantee that one obtains from DP is also stronger.

**Proposition 4.4.** Suppose that the training algorithm is $(\epsilon, \delta)$-DP. Then, the algorithm satisfies $\delta'$-TV stability with:

$$\delta' = \frac{\exp(\epsilon) - 1 + 2 \delta}{\exp(\epsilon) + 1}.$$  \hspace{1cm} (7)

Figure 2 shows that the known bound from Equation (5) quickly becomes vacuous, unlike ours. Moreover, in Appendix C we show that this bound is tight.

We note that bounds equivalent or similar to both the loose Equation (5) and our tight version in Equation (7) also appear when the DP guarantees are related to the advantage of an adversary that aims to distinguish between either $\theta(S)$ and $\theta(S')$ (Kasiviswanathan and Smith, 2014), or between examples coming from the training dataset and the examples from the data distribution (Yeom et al., 2018; Chatzikokolakis et al., 2020; Humphries et al., 2020).

**4.3. Stronger Forms of DG**

While we focus on the notion of $\delta$-DG for conceptual simplicity, it can be helpful to work with slightly stronger notions of DG for particular applications. For example, in Appendix B we introduce the notion of “$\delta$-strong-DG”, which is conceptually similar, but slightly stronger in a certain technical sense. Like standard DG, strong-DG is also satisfied by differentially private or TV-stable algorithms. This lets us show generalization of other interesting metrics, like the calibration gap. In Appendix D, we prove that for stable or private training algorithms, the Expected Calibration
We informally propose that as preliminary evidence, in Section 7 we show experiments we are concerned with accuracy on subgroups, then directly (Shalev-Shwartz and Ben-David, 2014). We posit that this connection is an important open problem for future work. As preliminary evidence, in Section 7 we show experiments which suggest that various kinds of regularizers used in practice induce at least weak forms of DG.

Moreover, while δ-DG implies generalization bounds for all properties in a black-box way, it is sometimes possible to obtain tighter bounds for specific properties. For example, if we are concerned with accuracy on subgroups, then directly applying δ-DG to the subgroup loss will yield a bound that decays with the size of the subgroup: accuracy on very small subgroups is not guaranteed to generalize well. However, it is possible to show that TV-stability also implies “subgroup DG”, which guarantees that the accuracy on even arbitrarily small subgroups will generalize well. We describe this extension in Appendix B.1.

4.4. Can We Achieve DG Without DP?

Differential privacy is one way to achieve distributional generalization, but not the only way. It is easy to see that there exist training methods which satisfy DG, but are not differentially private—because DP requires a worst-case guarantee over all possible train sets, while DG only considers the “expected” behavior of train sets. This opens the possibility of improved distributionally-generalizing algorithms that bypass the well-known performance degradation of DP training (Chaudhuri et al., 2011).

We informally propose that regularization should be thought of as a generic way of achieving DG. In theory, some forms of capacity control (e.g. description length, VC dimension) are already known to provably imply DG (Cummings et al., 2016). Meanwhile, regularization is classically known to induce stability, which implies standard generalization (Shalev-Shwartz and Ben-David, 2014). We posit that this connection should extend to DG: strong forms of regularization should imply distributional generalization, not just standard generalization. Theoretically characterizing this connection is an important open problem for future work. As preliminary evidence, in Section 7 we show experiments which suggest that various kinds of regularizers used in practice induce at least weak forms of DG.

5. Example Applications

To demonstrate that DG is a useful property in algorithm design, in the remainder of this paper we use it to construct simple and high-performing algorithms for two example applications: mitigation of disparate impact of DP and ensuring group-distributional robustness.

Mitigating Disparate Impact of DP. First, we consider applications in which learning presents privacy concerns, e.g., in the case that the training data contains sensitive information about individuals. Using training procedures that satisfy DP is a standard way to guarantee privacy in such settings. Training with DP, however, is known to incur disparate impact on the model accuracy: some subgroups of inputs can have worse test accuracy than others. For example, Bagdasaryan et al. (2019) show that using DP-SGD — a standard algorithm for satisfying DP (Abadi et al., 2016) — in place of regular SGD causes a significant accuracy drop on “darker skin” faces in models trained on the CelebA dataset of celebrity faces (Liu et al., 2015), but a less severe drop on “lighter skin” faces. Our goal is to mitigate such disparate impact. This issue — a quality-of-service harm (Madaio et al., 2020) — is but one of many possible harms due to ML systems. We do not aim to mitigate any other fairness-related issues, nor claim this is possible within our framework, or with technical solutions in general.

Formally, we assume the data distribution D is a mixture of m groups indexed by set G = {1, ..., m}, such that D = ∑m i=1 q D i. The vector (q1, ..., q m) ∈ [0, 1] m represents the group probabilities, with ∑m i=1 q i = 1. For given parameters (ε, δ), we want to learn a model θ that simultaneously satisfies (ε, δ)-DP, has high overall accuracy, and incurs small loss disparity:

\[
\max_{g, g' \in G} \mathbb{E}_{z \sim D_g} [\ell(z; \theta)] - \mathbb{E}_{z \sim D_{g'}} [\ell(z; \theta)] = \text{loss disparity}.
\]

Group-Distributional Robustness. Next, we consider a setting of group-distributionally robust optimization (e.g., Sagawa et al., 2019; Hu et al., 2018). If in the standard ERM we want to train a model that minimizes average loss, in this setting, we want to minimize the worst-case (highest) group loss. This objective can be used to mitigate fairness concerns such as those discussed previously, as well as to avoid learning spurious correlations (Sagawa et al., 2019).

Formally, we want to learn a model θ that minimizes the worst-case group loss:

\[
\max_{g \in G} \mathbb{E}_{z \sim D_g} [\ell(z; \theta)]
\]

Unlike the previous application, in this setting, we do not require privacy of the training data. We do, however, use training with DP as a tool to ensure the generalization of the worst-case group loss, as described next.

6. Algorithms which Distributionally Generalize

In this section, we construct algorithms for the applications in Section 5. Our approach follows the blueprint: First, we apply a principled algorithmic intervention that ensures
Algorithm 1 DP-IS-SGD (DP Importance Sampling SGD)

Input: Training dataset $S$, model $f_{\theta}$ parameterized by $\theta$, loss function $\ell$, max gradient norm $C$, noise parameter $\sigma$, number of epochs $T$, sampling rate $\bar{\rho}$; group probabilities $\vec{q} = (q_1, \ldots, q_m)$.

Output: $\theta_T$

1: for $t = 1, \ldots, T$ do
2: \hspace{1em} Batch$_t \leftarrow \text{Sample}_{p(\cdot)}(S)$, with $p(z) \triangleq \bar{\rho}/m \cdot q_g(z)$
3: \hspace{1em} $\Delta_t \theta \leftarrow 0$
4: \hspace{1em} for $(x_i, y_i) \in \text{Batch}_t$ do
5: \hspace{2em} $\Delta_t \theta \leftarrow \Delta_t \theta + \text{Clip}(\nabla_{\theta} \ell(f_{\theta}(x_i), y_i), C)$
6: \hspace{1em} end for
7: \hspace{1em} Sample $\xi_t \sim \sigma C \cdot N(0, I_{d \times d})$
8: \hspace{1em} $\Delta_t \theta \leftarrow \frac{1}{|\text{Batch}_t|} \Delta_t \theta + \xi_t$
9: \hspace{1em} $\theta_t \leftarrow \text{Optimizer}(\theta_{t-1}, \Delta_t \theta)$
10: end for

The highlighted parts indicate the differences with respect to DP-SGD. We obtain DP-SGD as a special case when we have a single group with $q = 1$ (implying $p(z) = \bar{\rho}$).

desired behavior on the training dataset (e.g., importance sampling). Second, we modify the resulting algorithm to additionally ensure DG, which guarantees that the desired behavior generalizes to the test data.

6.1. DP Training with Importance Sampling

Our first algorithm, DP-IS-SGD (Algorithm 1), is a version of DP-SGD (Abadi et al., 2016) that performs importance sampling. DP-IS-SGD is designed to mitigate disparate impact while retaining DP guarantees.

The standard DP-SGD samples batches from the training set using uniform Poisson subsampling: Each example in the training set is chosen into the batch according to the outcome of a Bernoulli trial with probability $\bar{\rho} \in [0, 1]$. To correct for unequal representation and the resulting disparate impact, we use non-uniform Poisson subsampling: Each example $z \in S$ has a possibly different probability $p(z)$ of being selected into the batch, where $p(z)$ does not depend on the dataset $S$ otherwise, and is bounded: $0 \leq p(z) \leq \bar{\rho}^* \leq 1$. We denote this subsampling procedure as $\text{Sample}_{p(\cdot)}(S)$.

We assume that we know to which group any $z = (x, y)$ belongs, denoted as $g(z)$, e.g., the group is one of the features in $x$. We choose $p(z)$ to satisfy two properties. First, to increase the sampling probability for examples in minority groups: $p(z) \propto 1/q_g(z)$. Second, to keep the average batch size equal to $\bar{\rho} \cdot n$ as in standard DP-SGD. In the rest of the paper, we assume that the group probabilities $(q_1, \ldots, q_m)$ are known, but it is possible to estimate them in a private way if needed. We present DP-IS-SGD in Algorithm 1, along with its differences to the standard DP-SGD.

DP Properties of DP-IS-SGD. Uniform Poisson subsampling is well-known to amplify the privacy guarantees of an algorithm (Chaudhuri and Misra, 2006; Li et al., 2012; Abadi et al., 2016; Ball et al., 2018; Bu et al., 2020). For example, Li et al. (2012) show that if an algorithm $\theta(S)$ satisfies $(\epsilon, \delta)$-DP, then $\theta \circ \text{Sample}_p(S)$ provides approximately $(O(\bar{\rho}^*), \bar{\rho})$-DP for small values of $\epsilon$. We show in Appendix C that non-uniform Poisson subsampling provides the same amplification guarantee, with $\bar{\rho} = p^*$, where $p^*$ is the maximum value of $p(\cdot)$.

As this guarantee is independent of the internal workings of $\theta(S)$, it is loose. In the particular case of DP-SGD, one way of computing tight privacy guarantees of Poisson subsampling is using the notion of Gaussian differential privacy (GDP) (Dong et al., 2019). GDP is parameterized by a single parameter $\mu$. If an algorithm $\theta(S)$ satisfies $\mu$-GDP, we can efficiently compute a set of $(\epsilon, \delta)$-DP guarantees also satisfied by $\theta(S)$ (Dong et al., 2019).

We show that we can use any GDP-based mechanism for computing the privacy guarantee of DP-SGD to obtain the privacy guarantees of DP-IS-SGD in a black-box manner:

Proposition 6.1. Let us denote by $\mu(\bar{\rho}, \sigma, T)$ (see Algorithm 1 for the interpretation of parameters) a function that returns a $\mu$-GDP guarantee of DP-SGD. Then, DP-IS-SGD satisfies a GDP guarantee $\mu(p^*, \sigma, T)$.

6.2. Group-DRO with Noisy Gradients

From the previous section, DP-IS-SGD enjoys theoretical guarantees for both DP and DG. However, DP models often have lower test accuracy compared to standard training (Chaudhuri et al., 2011). This can be an unnecessary disadvantage in settings where privacy is not required, such as in group-distributional robustness. Thus, we now explore algorithms which do not satisfy DP, but which still empirically satisfy DG. These algorithms do not come with theoretical guarantees, but are inspired by our theory, and perform well in practice.

DP-SGD uses gradient clipping (line 5 in Algorithm 1) and gradient noise (lines 7–8). Individually, these are used as regularization methods for improving stability and generalization (Hardt et al., 2016; Neelakantan et al., 2015), thus possibly improving DG in practice. Following this, we relax DP-IS-SGD to only use addition of noise to the gradient as a regularizer. This sacrifices privacy in exchange for practical performance. Specifically, we apply gradient noise to three standard algorithms for achieving group-distributional robustness: importance sampling (IS-SGD), importance weighting (IW-SGD) (Gretton et al., 2009), and gDRO (Sagawa et al., 2019). This results in the following variations: IS-SGD with noisy gradient (IS-SGD-n), IW-SGD with noisy gradient (IW-SGD-n), and gDRO with noisy gradient (gDRO-n). See Appendix E for more details.
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Remark on Regularization and DG. Regularization mechanisms such as $\ell_2$ and early stopping are already used to improve group-distributional robustness, but without theoretical justification (Sagawa et al., 2019). Our framework suggests a unifying reason why strong regularization is helpful in distributional robustness: because it enforces DG. Following this theoretically-inspired intuition, other methods such as dropout (Srivastava et al., 2014), label smoothing (Müller et al., 2019), and mixup (Zhang et al., 2018) could also be potentially used for achieving distributional robustness. We leave this for future work.

7. Experiments

In this section, we empirically study the distributional generalization in real-world applications. The code for the experiments is available on Github.\(^3\)

Datasets. We consider five datasets, studied in the distributional robustness and fairness literature: CelebA (Liu et al., 2015), UTKFace (Zhang et al., 2017), iNaturalist2017 (iNat) (Horn et al., 2018), CivilComments (Borkan et al., 2019), and MultiNLI (Williams et al., 2018; Sagawa et al., 2019). For every dataset, each example belongs to one group (e.g., CelebA) or multiple groups (e.g., CivilComments). For example, in the CelebA dataset, there are four groups: “blond male”, “male with other hair color”, “blond female”, and “female with other hair color”. We present more details on the datasets and their groups in Appendix G.

7.1. Enforcing DG in Practice

In Section 4, we proved that a training procedure with DP guarantees also has a bounded DG gap. In Section 4.4, we also informally discussed that other kinds of regularization besides DP should also be expected to induce DG in practice. In this section, we empirically verify our formal claims, and provide evidence for our informal claims.

In Figure 3, we train a neural network on CelebA using DP-SGD, and decrease the “regularization strength” in several different ways: by increasing privacy budget $\epsilon$ (Figure 3a), decreasing the $\ell_2$ regularization (Figure 3b), or increasing the number of training iterations (Figure 3c).\(^4\) We then measure the gap in worst-group accuracy on train vs. test, as a proxy for the DG gap. We observe that for all regularizers, the gap between training and testing worst-group accuracy increases as the regularization is weakened. Further, when using privacy budget as a regularizer (Figure 3a), the observed gap is consistent with our theoretical bounds.

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We acknowledge that measuring the true DG gap (as in Definition 4.1) is experimentally infeasible, as it involves checking the generalization gap of all possible test functions $\phi$. In these experiments, we use worst-group accuracy as a proxy for the worst-case test function, but this proxy can be loose. Finding the worst-case test function is an object of study in the literature on membership inference attacks (Shokri et al., 2017), as DG and vulnerability to such attacks are equivalent (Kulynych et al., 2022). Alternatively, it is possible to state weaker (and finer-grained) versions of DG, by explicitly parameterizing the set of test functions. This finer-grained definition is presented in Nakkiran and Bansal (2020), and we leave further investigation as an important topic for future work.

7.2. Disparate Impact of Differentially Private Models

In this section, we evaluate DP-IS-SGD (Algorithm 1), and demonstrate that it can mitigate the disparate impact in realistic settings where both privacy and fairness are required.

Figure 6 shows the accuracy disparity, test accuracy and worst-group accuracy, computed as in Equation (8), as a function of the privacy budget $\epsilon$. The models are trained with DP-SGD and DP-IS-SGD. When comparing DP-SGD and DP-IS-SGD with the same or similar $\epsilon$, we observe that DP-IS-SGD achieves lower disparity on all datasets. However, this comes with a drop in average accuracy. On CelebA, for example, with $\epsilon \in [2, 12]$, DP-IS-SGD has around 8 p.p. lower test accuracy than DP-SGD. At the same time, the disparity drop ranges from 40 p.p. to 60 p.p., which is significantly higher than the accuracy drop. We observe similar results on UTKFace, as shown in Appendix G. On iNat, however, although DP-IS-SGD decreases disparity, the overall test accuracy suffers a significant hit. This is likely because the minority subgroup is extremely small, and importance-sampling are poorly behaved for very small groups. Details for UTKFace and iNat appear in Appendix G.

In summary, we find that DP-IS-SGD achieves lower disparity at the same privacy budget compared to standard DP-SGD, with only mild impact on test accuracy.

7.3. Group-DRO

In this section, we investigate whether our proposed versions of standard algorithms with Gaussian gradient noise (Section 6.2) can improve group-distributional robustness. We evaluate distributional robustness through worst-case accuracy as in Equation (9), following the evaluation criteria in prior work (Sagawa et al., 2019; Idrissi et al., 2021). SOTA methods apply $\ell_2$ regularization and early-stopping to achieve the best performance. We compare three baselines with $\ell_2$ regularization, IS-SGD-$\ell_2$, IW-SGD-$\ell_2$, and gDRO-$\ell_2$ to our three noisy-gradient variations: IS-SGD-n, IW-SGD-n, and gDRO-n, as well as DP-IS-SGD. We use the validation set to select the best-performing regularization

\(^3\)https://github.com/yangarbitere/dp-dg

\(^4\)Train time can be considered a regularizer, since decreasing train time induces stability (e.g. Hardt et al. (2016)).
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Figure 3. **Regularization induces DG.** Train/test worst-group accuracies as a function of regularization strength for SGD on CelebA, with different types of regularizers: differential privacy budget $\epsilon$, weight decay, and train time. For DP-SGD, $\epsilon = \infty$ represents standard SGD. For all types of regularizers, increasing the strength (left on x-axis) corresponds to a smaller generalization gap in worst-group accuracy.

Figure 4. **Importance Sampling helps Disparate Impact of DP-SGD.** The accuracy disparity of the models trained with DP-SGD and DP-IS-SGD on CelebA. Adding importance sampling (IS) improves disparate impact at most privacy budgets in this setting. We set $\delta = 1/2n$, where $n$ is the number of training examples. We use GDP accountant to compute the privacy budget $\epsilon$.

**Results.** Table 1 shows the worst-group accuracy of each algorithm on five datasets. When comparing IS-SGD, IW-SGD, and gDRO with their noisy counterparts, we observe that the noisy versions in general have similar or slightly better performance compared to non-noisy counterparts. For instance, IS-SGD-n improves state-of-the-art results on CivilComments dataset. This showcases that in terms of learning distributionally robust models, noisy gradient can be potentially a more effective regularizer than the current-standard $\ell_2$ regularizer.

We also find that DP-IS-SGD improves on baseline methods or even achieves similar SOTA performance on several datasets. For instance, on CelebA and MNLI, DP-IS-SGD achieves better performance than IS-SGD-$\ell_2$, and achieves comparable performance to SOTA. This is surprising, as DP methods tend to have worse performance than non-DP counterparts. This suggests that distributional robustness and privacy may not be incompatible goals. Moreover, DP can be a useful tool even when privacy is not required.

**Statistical Concerns.** Although our results appear to be comparable to or better than SOTA, we caution readers about the exact ordering of methods due to high estimation variance: these benchmarks have small validation and test sets (e.g., CelebA has 182 validation examples), and so hyperparameter tuning is subject to both overfitting and estimation error. For example, we observe validation accuracies which differ from their test accuracies by up to 5% in our experiments. We attempt to mitigate this using three random train/val/test splits on CelebA, and avoid large hyperparameter sweeps, but this is not done in prior work.

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5The IW-SGD numbers in Table 1 are different from Figure 1, as in the figure we do not apply any regularization.

6For example, we do not tune the “group adjustments” parameter for gDRO, using the default from Koh et al. (2021) instead.
Table 1. The worst-group accuracy of each algorithm on 5 datasets. gDRO-$\ell_2$-SOTA shows the average and standard error over five runs from Idrissi et al. (2021). Baselines are in the top block of rows; our algorithms (noisy versions) are in the bottom. For CelebA, we show mean and standard error over three random splits. We find that our noisy gradient produces competitive results compared to their counterparts with $\ell_2$ regularization.

| Algorithm | CelebA | UTKFace | iNat. | Civil. | MNLI |
|-----------|--------|---------|-------|-------|------|
| SGD-$\ell_2$ | 73.0 ± 2.2 | 86.3 | 41.8 | 57.4 | 67.9 |
| IS-SGD-$\ell_2$ | 82.4 ± 0.5 | 85.8 | 70.6 | 64.3 | 70.4 |
| IW-SGD-$\ell_2$ | **89.0 ± 0.9** | 86.5 | 67.6 | 65.7 | 68.1 |
| gDRO-$\ell_2$ | 84.5 ± 0.8 | 85.2 | 67.3 | 67.3 | 75.9 |
| gDRO-$\ell_2$-SOTA | 86.9 ± 0.5 | — | 69.9 ± 0.5 | **78.0 ± 0.3** |
| DP-IS-SGD | 86.0 ± 0.8 | 82.5 | 51.4 | 70.4 | 72.3 |
| IS-SGD-n | 84.9 ± 1.0 | 85.5 | **71.0** | **71.9** | 70.8 |
| IW-SGD-n | **88.5 ± 0.4** | **88.5** | 70.9 | 69.9 | 69.7 |
| gDRO-n | 83.3 ± 0.5 | 87.5 | 56.4 | 71.3 | **78.0** |

8. Conclusions

We argue that distributional generalization (DG) is a desirable property for learning algorithms, enabling more principled algorithm design in settings including deep learning. DG captures a “What You See is What You Get” (WYSIWYG) property, which ensures that any training-time behavior is reproduced at test time. We show that this property is possible to achieve, by proving it is implied by differential privacy. This allows us to leverage advances in differential privacy (such as DP-SGD for neural networks) to enforce DG in many applications.

We propose enforcing DG as a general design principle, and we use it to construct simple yet effective algorithms in two settings. In certain fairness settings, we largely mitigate the disparate impact of differential privacy by using importance sampling and enforcing DG in our new algorithm, DP-IS-SGD. In certain distributional robustness settings, inspired by DP-SGD, we propose using a noisy gradient regularizer instead of the traditional $\ell_2$ regularizer, to enforce DG. Compared to SOTA algorithms, noisy gradient regularization can achieve competitive results across many standard benchmarks. Moreover, we argue that certain ad-hoc regularizers in the existing distributional robustness literature are perhaps best seen as alternate ways to enforce distributional generalization.

Finally, in this work we only applied our WYSIWYG-based blueprint for algorithms in two settings. We hope future work can explore extending this algorithmic design principle to other settings, such as adversarial robustness, counterfactual fairness, and calibration.

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Author Contributions

BK initially noticed the connection between differential privacy and distribution generalization, and led the theory developments and algorithm analysis. Yao-Yuan Yang led the experiments, designed the experimental settings, and implemented the algorithms. Yaodong Yu contributed to the experiments, especially in the NLP settings, and helped benchmark baselines. The noisy-gradient algorithm is proposed by Yao-Yuan Yang and Yaodong Yu. JB contributed to the theory, in particular the algorithmic analysis, proof of the tightness of the bound, and formal connections to robust generalization and calibration. PN organized the team and managed the project, contributing in parts to the theory and experiments. All authors participated in framing the results and writing the paper.

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A. Related Work Details: Differential Privacy and Robust Generalization

DP is known to imply a stronger notion of generalization, called robust generalization, which is a kind of “tail bound” version of DG (Dwork et al., 2015a; Cummings et al., 2016; Bassily et al., 2016). The original motivations for robust generalization are slightly different, but in our notation, a training procedure $\theta$ is said to satisfy $(\gamma, \eta)$-Robust Generalization if and only if for any test $\phi : \mathbb{R} \times \Theta \rightarrow [0, 1]$, we have

$$\Pr_{z \sim \mathcal{D}} \left( | \mathbb{E}_{z \sim S} \phi(z; \theta(S)) - \mathbb{E}_{z \sim \mathcal{D}} \phi(z; \theta(S)) | > \gamma \right) \leq \eta.$$

Comparing this to the definition of DG (Definition 4.1), it is immediate that any procedure that satisfies $(\gamma, \eta)$-Robust Generalization, also satisfies $(\gamma + \eta)$-DG. Any training method satisfying $(\epsilon, \delta)$-DP also satisfies $O(\epsilon), O(\delta)$-robust generalization, as long as the sample size $n$ is of size $\Omega((\log(1/\delta))/\epsilon^2)$ (Bassily et al., 2016, Theorem 7.2), therefore it satisfies $O(\epsilon + \delta)$-DG by the previous implication. Thus, it is possible to recover the result that DP implies DG as a consequence of these previous works, although with looser bounds.

The difference between Distributional Generalization and Robust Generalization is that DG considers all quantities in expectation, while robust generalization considers tail bounds with respect to the train dataset. We focus on DG for two reasons: First, we believe DG is conceptually simpler, as it can be seen as simply the TV-distance between two natural distributions (Equation (3)), and does not involve additional parameters. This simplicity is conceptually useful to the algorithm designer, but also allows us to prove simpler tight theoretical bounds that are independent of sample size. Second, it is often possible to lift results about DG to the stronger setting of robust generalization, with additional bookkeeping. Thus, we focus on DG in this paper, with the understanding that stronger guarantees can be obtained for these methods if desired.

B. Stronger Formalizations of Distributional Generalization

The main notion discussed in this paper is that of $\delta$-Distributional Generalization. To recap, a learning algorithm $\theta(S)$ satisfies it if for any bounded test function $\phi$ we have

$$\mathbb{E}_{S \sim \mathcal{D}^n, \theta} | \mathbb{E}_{z \sim S} \phi(z; \theta(S)) - \mathbb{E}_{z \sim \mathcal{D}} \phi(z; \theta(S)) | \leq \delta.$$

In Appendix A we have compared it to a stronger property called $(\gamma, \eta)$-Robust Generalization; namely learning algorithm $\theta(S)$ satisfies it if for any bounded $\phi$ we have:

$$\Pr_{S \sim \mathcal{D}^n} \left( | \mathbb{E}_{z \sim S} \phi(z; \theta(S)) - \mathbb{E}_{z \sim \mathcal{D}} \phi(z; \theta(S)) | > \gamma \right) \leq \eta.$$

A natural middle ground between these two is the following notion.

**Definition B.1.** We say that a learning algorithm $\theta(S)$ satisfies $\delta$-strong Distributional Generalization if and only if

$$\mathbb{E}_{S \sim \mathcal{D}^n, \theta} | \mathbb{E}_{z \sim S} \phi(z; \theta(S)) - \mathbb{E}_{z \sim \mathcal{D}} \phi(z; \theta(S)) | \leq \delta.$$

Clearly any algorithm satisfying $(\eta, \gamma)$-Robust Generalization satisfies also $(\eta + \gamma)$-strong DG, and any algorithm satisfying $\delta$-strong-DG satisfies $\delta$-DG — so in fact the $\delta$-strong DG can be thought of as a strengthening of Distributional Generalization as well as Robust Generalization.

One of the motivations behind the definition of $\delta$-strong DG is the following issue of the Distributional Generalization itself. As it turns out, it is possible to artificially construct a mechanism for binary classification which satisfies 0-DG, always outputs a classifier with test error $\frac{1}{2} + o(1)$ (almost completely useless), but with probability $1/2$ outputs a classifier with training error 0 (and with probability $1/2$ the training error is 1). That is problematic: it is natural to expect from a training procedure with strong generalization guarantee, that if we used it to train a model up to a small training error, we should have high confidence that the test error is also small — but this example shows that our confidence cannot be larger than $1/2$.

The $\delta$-strong DG solves this problem — the following proposition is a formal statement of the desired property, and can be
Proposition B.2. If the learning procedure \( \theta \) satisfies \( \delta \)-strong-DG, then for any bounded property \( \phi \) we have
\[
\Pr_{S,\theta}(|\mathbb{E}_{z \sim S} \phi(z, \theta(S)) - \mathbb{E}_{z \sim D} \phi(z, \theta(S))| > \delta) < \lambda^{-1}.
\]
Equivalently, if a learning procedure satisfies \( \delta \)-strong-DG it also satisfies \((\delta \lambda, \lambda^{-1})\)-Robust Generalization for any \( \lambda > 1 \).

The \( \delta \)-strong DG defined this way is also a more direct strengthening of the classical notions of generalization gap — which are usually defined as \( \mathbb{E}_{S \sim \mathcal{D}^n, \theta}(|\mathbb{E}_{z \sim S} \ell(z, \theta(S)) - \mathbb{E}_{z \sim D} \ell(z, \theta(S))|\) where \( \ell \) is the loss function.

We can follow the same proof strategy as in (Bassily et al., 2016, Theorem 7.2), simplifying it significantly, to show that TV-stability (and in turn Differential Privacy) implies strong Distributional Generalization, as soon as the training set is large enough.

Lemma B.3. If a learning algorithm \( \theta(S) \) satisfies \( \delta \)-TV stability, and the sample size \( |S| \geq 2^{\frac{2}{\delta}} \), then \( \theta(S) \) satisfies \( 4\delta \)-strong-DG.

Proof. Let us assume first that we have a \( \delta \)-TV stable mechanism \( \mathcal{M} : \mathcal{D}^n \rightarrow \mathcal{Q} \) where \( \mathcal{Q} \) is a set of bounded queries \( q : \mathcal{D} \rightarrow \mathbb{R} \) satisfying \( \sup_{x}(q(x)) - \inf_{x}(q(x)) \leq 1 \).

We wish to find another \( 2\delta \)-TV stable mechanism \( \tilde{\mathcal{M}} : \mathcal{D}^n \rightarrow \mathcal{Q} \) such that if we use notation \( q_S := \mathcal{M}(S) \) and \( \tilde{q}_S := \tilde{\mathcal{M}}(S) \), we have
\[
\mathbb{E}_{S,\mathcal{M}} \left| \mathbb{E}_{z \sim \mathcal{D}} q_S(z) - \mathbb{E}_{z \sim S} q_S(z) \right| \leq 2 \left( \mathbb{E}_{S,\tilde{\mathcal{M}},z \sim S} \tilde{q}_S(z) - \mathbb{E}_{S,\mathcal{M},z \sim D} \tilde{q}_S(z) \right).
\]

Indeed, consider the following mechanism \( \tilde{\mathcal{M}}(S) \): on a sample \( S \), get \( q_S := \mathcal{M}(S) \) and evaluate \( \tilde{\mu} := \mathbb{E}_{z \sim S} q_S(z) + \frac{1}{\sigma n} U \) where \( U \) is a uniform variable in \([-1/2, 1/2] \). Finally if \( \tilde{\mu} \geq \mathbb{E}_{z \sim D} q_S(z) \) output \( q_S \), otherwise output \( -q_S + 2 \mathbb{E}_{z \sim D} q_S(z) \).

Since \( d_{TV}(\eta U, \eta U + \gamma) \leq 2 \frac{\eta}{\gamma} \), given \( q \) the mechanism \( \mu_q(S) := \mathbb{E}_{z \sim S} q(z) + \frac{1}{\sigma n} U \) is \( \delta \)-stable. Hence, by composition and post-processing this mechanism \( \tilde{\mathcal{M}} \) is \( 2\delta \)-TV stable.

For \( n \geq 2\delta^{-2} \) the inequality (10) follows from Lemma B.4. Indeed, let \( \tilde{Z} := \mathbb{E}_{z \sim S} q_S(z) - \mathbb{E}_{z \sim D} q_S(z) \), \( Z = \tilde{Z} \), and \( \sigma = \text{sign}(\tilde{Z} + \frac{1}{\sigma n} U) \). Note that, expanding the chosen notation, we have \( \mathbb{E}_{z \sim S} \tilde{q}_S(z) - \mathbb{E}_{z \sim D} \tilde{q}_S(z) = \sigma Z \), and \( \mathbb{E}[\sigma|Z] = \max(\frac{2Z}{\delta}, 1) \). The conclusion of the Lemma B.4 is \( \mathbb{E} Z \leq 2 \mathbb{E}[Z\sigma] \), which is exactly (10).

Since the mechanism \( \tilde{\mathcal{M}} \) satisfies \( 2\delta \)-TV stability, by Theorem 4.3 applied to the mechanism \( \tilde{\mathcal{M}} \), we can bound the right hand side of the inequality (10) by \( 4\delta \).

Finally, for an arbitrary learning algorithm \( \theta \) satisfying \( \delta \)-TV stability, and any bounded test function \( \phi(z, \theta) \), we can describe mechanism \( \tilde{\mathcal{M}} \) which given a sample \( S \in \mathcal{D}^n \) outputs a function \( q_S(z) := \phi(z, \theta(S)) \). By post-processing such a mechanism \( \tilde{\mathcal{M}} \) also satisfies \( \delta \)-TV stability, and the inequality (10) becomes
\[
\mathbb{E}_{S,\tilde{\mathcal{M}}} \left| \mathbb{E}_{z \sim S} \phi(z, \theta(S)) - \mathbb{E}_{z \sim D} \phi(z, \theta(S)) \right| \leq 4\delta.
\]

Since \( \phi \) was chosen in an arbitrary way, this proves that mechanism \( \tilde{\mathcal{M}} \) indeed satisfies the condition of \( 4\delta \)-strong-DG. \( \square \)

Lemma B.4. Let \( Z \) be a non-negative random variable satisfying \( \mathbb{E} Z \geq \delta \) and \( \sigma \) be a \( \{\pm 1\} \) valued random variable with \( \mathbb{E}[\sigma|Z] = \max(\frac{2Z}{\delta}, 1) \). Then \( \mathbb{E} Z \leq 2 \mathbb{E}[Z\sigma] \).

Proof. This is just a calculation.
\[
\mathbb{E} Z\sigma = \mathbb{E} 1_{Z \geq \delta/2} Z + \mathbb{E} 1_{Z < \delta/2} \frac{2Z^2}{\delta} \geq \mathbb{E} 1_{Z \geq \delta/2} Z = \mathbb{E} Z - \mathbb{E} 1_{Z < \delta/2} Z \geq \mathbb{E} Z - \delta/2 \geq \frac{1}{2} \mathbb{E} Z.
\]

\( \square \)

B.1. Subgroup DG

Let us introduce a more fine-grained notion of DG:
We now show that TV-stability implies this stronger notion of DG as well: where the last equality is simply renaming of the variables for convenience. Note that analogously we also can obtain a symmetric bound:

Subgroup DG is a stronger notion of DG which says that the model’s behavior on examples from each group in \( \mathcal{G} \) distributionally generalizes in expectation, as long as the model encounters at least one representative of the group in training. We now show that TV-stability implies this stronger notion of DG as well:

**Proposition B.6.** \( \delta \)-TV stability implies \((\delta, \mathcal{G})\)-subgroup-DG for any group partitioning \( \mathcal{G} \).

**Proof.** Observe that the following distributions are equivalent:

\[
\Pr_{S \sim D^n} [\phi(z; \theta(S))] = \Pr_{S \sim D^{n-1}} [\phi(z; \theta(S \cup \{z\})],
\]

\[
\Pr_{z \sim D^n} [\phi(z; \theta(S))] = \Pr_{z \sim D^{n-1}} [\phi(z'; \theta(S \cup \{z\})].
\]

The statement follows by applying each step from the proof of Theorem 4.3 to the equivalent distributions in Equation (11). Thus, the difference with the proof of Theorem 4.3 is that \( z, z' \sim D_g \), not \( z, z' \sim D \).

### C. Proofs

#### C.1. TV-Stability implies Distributional Generalization

**Proof of Theorem 4.3.** First, observe that the following distributions are equivalent as the dataset is an i.i.d. sample:

\[
\Pr_{S \sim D^n} [\phi(z; \theta(S))] = \Pr_{S \sim D^{n-1}} [\phi(z; \theta(S \cup \{z\})],
\]

\[
\Pr_{z \sim D^n} [\phi(z; \theta(S))] = \Pr_{z \sim D^{n-1}} [\phi(z'; \theta(S \cup \{z\})].
\]

It is thus sufficient to analyze the equivalent distributions instead. By the post-processing property of differential privacy, for any dataset \( S \in \mathcal{D}^{n-1} \), any two examples \( z, z' \in \mathcal{D} \), and any set \( K \subseteq \{0, 1\} \):

\[
\Pr[\phi(z; \theta(S \cup \{z\})) \in K] \leq \Pr[\phi(z; \theta(S \cup \{z'\})) \in K] + \delta,
\]

as datasets \( S \cup \{z\} \) and \( S \cup \{z'\} \) are neighbouring. Taking the expectation of both sides over \( z, z' \sim \mathcal{D} \) and \( S \sim \mathcal{D}^{n-1} \), we get:

\[
\Pr_{S \sim \mathcal{D}^{n-1}} [\phi(z; \theta(S \cup \{z\})) \in K] \leq \Pr_{S \sim \mathcal{D}^{n-1}} [\phi(z; \theta(S \cup \{z'\})) \in K] + \delta
\]

\[
= \Pr_{S \sim \mathcal{D}^{n-1}} [\phi(z'; \theta(S \cup \{z\})) \in K] + \delta,
\]  

where the last equality is simply renaming of the variables for convenience. Note that analogously we also can obtain a symmetric bound:

\[
\Pr_{S \sim \mathcal{D}^{n-1}} [\phi(z'; \theta(S \cup \{z\})) \in K] \leq \Pr_{S \sim \mathcal{D}^{n-1}} [\phi(z; \theta(S \cup \{z\})) \in K] + \delta
\]
We can now prove Proposition 4.4: A well-known result due to Le Cam provides the following relationship between the trade-off between the two types of errors in hypothesis testing:

\[
d_{TV}\left( \Pr_{\mathcal{S} \sim D^n, z \sim D} [\phi(z; \theta(S \cup \{z\}))], \Pr_{\mathcal{S} \sim D^n, z \sim D} [\phi(z', \theta(S \cup \{z\}))] \right) = \sup_{K \subseteq \text{range}(\phi)} \left| \Pr_{\mathcal{S} \sim D^n, z \sim D} [\phi(z; \theta(S \cup \{z\})) \in K] - \Pr_{\mathcal{S} \sim D^n, z \sim D} [\phi(z', \theta(S \cup \{z\})) \in K] \right| \leq \delta,
\]

where the last inequality is by Equation (14). Using the equivalences in Equation (12) we can see that:

\[
d_{TV}\left( \Pr_{\mathcal{S} \sim D^n} [\phi(z; \theta(S))], \Pr_{\mathcal{S} \sim D^n} [\phi(z'; \theta(S))] \right) = \left| \mathbb{E}_{\mathcal{S} \sim D^n} [\phi(z; \theta(S))] - \mathbb{E}_{\mathcal{S} \sim D^n} [\phi(z'; \theta(S))] \right| \leq \delta,
\]

which is the sought result. \(\square\)

### C.2. Tight Bound on TV-Stability from DP

To prove Proposition 4.4, we make use of the hypothesis-testing interpretation of DP (Wasserman and Zhou, 2010). Let us define the hypothesis-testing setup and the two types of errors in hypothesis testing. For any two probability distributions \(P\) and \(Q\) defined over \(\mathbb{D}\), let \(\phi : \mathbb{D} \to \{0, 1\}\) be a hypothesis-testing decision rule that aims to tell whether a given observation from the domain \(\mathbb{D}\) comes from \(P\) or \(Q\).

**Definition C.1** (Hypothesis-testing FPR and FNR). Without loss of generality, the **false-positive error rate** \(\alpha_\phi\) (FPR, or type I error rate), and the **false-negative error rate** \(\beta_\phi\) (FNR, or type II error rate) of the decision rule \(\phi : \mathbb{D} \to \{0, 1\}\) are defined as the following probabilities:

\[
\begin{align*}
\alpha_\phi & \triangleq \Pr_{z \sim P} [\phi(z) = 1] = \mathbb{E}_P[\phi], \\
\beta_\phi & \triangleq \Pr_{z \sim Q} [\phi(z) = 0] = 1 - \mathbb{E}_Q[\phi].
\end{align*}
\]

A well-known result due to Le Cam provides the following relationship between the trade-off between the two types of errors and the total variation between the probability distributions:

\[
\alpha_\phi + \beta_\phi \geq 1 - d_{TV}(P, Q).
\]

DP is known to provide the following relationship between FPR and FNR of any decision rule:

**Proposition C.2** (Kairouz et al. (2015)). Suppose that an algorithm \(\theta(S)\) satisfies \((\epsilon, \delta)\)-DP. Then, for any decision rule \(\phi : \mathbb{D} \to \{0, 1\}\):

\[
\begin{align*}
\alpha_\phi + \exp(\epsilon) \beta_\phi & \geq 1 - \delta, \\
\exp(\epsilon) \alpha_\phi + \beta_\phi & \geq 1 - \delta.
\end{align*}
\]

We can now prove Proposition 4.4:

**Proof.** Consider a hypothesis-testing setup in which we want to distinguish between the distributions \(\theta(S)\) and \(\theta(S')\). Let us sum the two bounds in Equation (17):

\[
(\exp(\epsilon) + 1)(\alpha_\phi + \beta_\phi) \geq 2(1 - \delta) \implies \alpha_\phi + \beta_\phi \geq \frac{2 - 2\delta}{\exp(\epsilon) + 1}.
\]

Let us take the optimal decision rule \(\phi^*\). In this case, the bound in Equation (16) holds exactly:

\[
d_{TV}(\theta(S), \theta(S')) = 1 - (\alpha_{\phi^*} + \beta_{\phi^*}).
\]

Combining this with Equation (18), we get:

\[
d_{TV}(\theta(S), \theta(S')) \leq 1 - \frac{2 - 2\delta}{\exp(\epsilon) + 1} = \frac{\exp(\epsilon) - 1 + 2\delta}{\exp(\epsilon) + 1}.
\]
Next, we show that the upper bound is tight up to \( \delta \):

**Proposition C.3.** There is an algorithm \( \theta(S) \) satisfying \((\varepsilon, 0)\)-DP, such that \( d_{TV}(\theta(S), \theta(S')) = \frac{\exp(\varepsilon)-1}{\exp(\varepsilon)+1} \) for two neighbouring datasets \( S \) and \( S' \).

**Proof.** Consider two distributions \( P_0 \) and \( P_1 \) on a set \( \{0, 1\} \), with \( P_0(\{0\}) = P_1(\{1\}) = \gamma \) for some \( \gamma \) to be chosen later, and \( P_0(\{1\}) = P_1(\{0\}) = 1 - \gamma \). Those two distributions satisfy \( d_{TV}(P_0, P_1) = 1 - 2\gamma \), as well as the closeness condition appearing in the definition of \((\varepsilon, 0)\)-DP

\[
\forall T, \Pr_{z \sim P_0}(z \in T) \leq \exp(\varepsilon) \Pr_{z \sim P_1}(z \in T),
\]

with \( \exp(\varepsilon) = \frac{1-\gamma}{\gamma} \). Expressing now TV-distance in terms of \( \varepsilon \), we get \( d_{TV}(P_0, P_1) = \frac{\exp(\varepsilon)-1}{\exp(\varepsilon)+1} \). With those distributions in hand, it is easy to provide a mechanism \( \theta : \{0, 1\} \to \{0, 1\} \) satisfying the desired property: on the input 0, it generates output according to distribution \( P_0 \), and on the input 1, it generates output according to distribution \( P_1 \).

**C.3. Privacy Analysis of DP-IS-SGD**

First, we present a loose analysis of the privacy guarantees of non-uniform Poisson subsampling.

**Lemma C.4.** Suppose that \( \theta(S) \) satisfies \((\varepsilon, \delta)\)-DP and \( \text{Sample}(S) \) is a Poisson sampling procedure where each of the sampling probabilities \( p_i \) depend on the element \( z_i \) (but do not depend on the set \( S \) otherwise) and is guaranteed to satisfy \( p_i \leq p^* \). Then \( \theta \circ \text{Sample} \) satisfies \((\ln(1 - p^* + p^*e^\varepsilon), p^*\delta)\)-DP. For small \( \varepsilon \) this can be bounded by \((O(p^*\varepsilon), p^*\delta)\)-DP.

**Proof of Lemma C.4.** Consider two neighboring datasets \( S \) and \( S' = S \cup \{z_0\} \) for some \( z_0 \notin S \). We wish to show that for any set \( K \), we have

\[
\Pr(\theta(\text{Sample}(S')) \in K) \leq (1 - p + pe^\varepsilon) \Pr(\theta(\text{Sample}(S)) \in K) + p\delta
\]

and symmetrically for \( S \) and \( S' \). We will only prove first of those inequalities, as the second is analogous.

Note that with probability \( p_0 \leq p \) the element \( z_0 \) is included in \( \text{Sample}(S) \) and we have \( \text{Sample}(S') = \{z_0\} \cup \text{Sample}(S) \), otherwise the element \( z_0 \) is not included, and conditioned on \( z_0 \) not being included \( \text{Sample}(S') \) has the same distribution as \( \text{Sample}(S) \). Therefore,

\[
\Pr(\theta(\text{Sample}(S')) \in K) = p_0 \Pr(\theta(\{z_0\} \cup \text{Sample}(S)) \in K) + (1 - p_0) \Pr(\theta(\text{Sample}(S)) \in K). \tag{19}
\]

Now for each realization \( \text{Sample}(S) = \tilde{S} \), we have \( \Pr(\theta(\{z_0\} \cup \tilde{S}) \in K) \leq e^\varepsilon \Pr(\theta(\tilde{S}) \in K) + \delta \) by the assumed DP guarantee of the algorithm \( \theta(S) \). We can average over all possible subsets \( \tilde{S} \) to get

\[
\Pr(\theta(\{z_0\} \cup \text{Sample}(S)) \in K) = \sum_{\tilde{S}} \Pr(\text{Sample}(S) = \tilde{S}) \Pr(\theta(\{z_0\} \cup \tilde{S}) \in K)
\]

\[
\leq \sum_{\tilde{S}} \Pr(\text{Sample}(S) = \tilde{S})(e^\varepsilon \Pr(\theta(\tilde{S}) \in K) + \delta)
\]

\[
= e^\varepsilon \Pr(\theta(\text{Sample}(S)) \in K) + \delta.
\]

Plugging this back to the inequality (19), we get

\[
\Pr(\theta(\text{Sample}(S')) \in K) \leq p_0(e^\varepsilon \Pr(\theta(\text{Sample}(S)) \in K) + \delta) + (1 - p_0) \Pr(\theta(\text{Sample}(S)) \in K)
\]

\[
\leq (1 - p^* + p^*e^\varepsilon) \Pr(\theta(\text{Sample}(S)) \in K) + p^*\delta.
\]

Finally, when \( \varepsilon \leq 1 \) we have \( e^\varepsilon \leq (1 + 2\varepsilon) \), and therefore \( (1 - p^* + p^*e^\varepsilon) \leq 1 + 2p^*e^\varepsilon \leq e^{2p^*} \).

For the tight privacy analysis of non-uniform Poisson subsampling, we make use of the notion of \( f \)-privacy:
We denote the resulting subsample as $S$. Analogously, for a neighbouring dataset $S'$, the following holds:

$$\tau(\theta(S), \theta(S')) \geq f,$$

where $\tau(P, Q)$ is a trade-off function between the FPR and FNR of distinguishing tests (see Appendix C.2):

$$\tau(P, Q)(\alpha) = \inf_{\phi: \mathbb{B} \rightarrow [0, 1]} \{\beta_\phi : \alpha_\phi \leq \alpha\},$$  \hspace{1cm} (20)

and $f(\alpha) \in [0, 1]$ is a convex, continuous, non-increasing function.

Bu et al. (2020) show that uniform Poisson subsampling (see Section 6.1) provides the following privacy amplification:

**Proposition C.6** (Bu et al. (2020)). Suppose that $\theta(S)$ satisfies $f$-privacy, and $\text{Sample}(S)$ is a uniform Poisson sampling procedure with sampling probability $\bar{p}$. The composition $\theta \circ \text{Sample}(S)$ satisfies $f'$-privacy with $f' = \bar{p}f + (1 - \bar{p})\text{Id}$, where $\text{Id}(\alpha) = 1 - \alpha$ is the trade-off function that corresponds to perfect privacy.

To show this, we adapt the proof Proposition C.6, and make use of the following lemma:

**Lemma C.7.** Suppose that $\theta(S)$ satisfies $f$-privacy, and $\text{Sample}(S)$ is a non-uniform Poisson sampling procedure, where the sampling probabilities $p_i$ depend on the element $z_i$ (but do not depend on the set $S$ otherwise) and each is guaranteed to satisfy $p_i \leq p^*$. The composition $\theta \circ \text{Sample}(S)$ satisfies $f'$-privacy with $f' = p^* + (1 - p^*)\text{Id}$.

To show this, we adapt the proof Proposition C.6, and make use of the following lemma:

**Lemma C.8** (Bu et al. (2020)). Let $\{P_i\}_{i \in I}$ and $\{Q_i\}_{i \in I}$ be two collections of probability distributions on the same sample space for some index set $I$. Let $(\lambda_i)_{i \in I} \in [0, 1]^{|I|}$ be a collection of numbers such that $\sum_{i \in I} \lambda_i = 1$. If $\tau(P_i, Q_i) \geq f$ for all $i \in I$, then for any $p \in [0, 1]$:

$$\tau\left(\sum_{i} \lambda_i \cdot P_i, \sum_{i} (1 - p) \cdot \lambda_i \cdot P_i + \sum_{i} p \cdot \lambda_i \cdot Q_i\right) \geq pf + (1 - p)\text{Id}.$$

**Proof of Lemma C.7.** We can think of the result of the subsampling procedure as outputting a binary vector $\bar{b} = (b_1, \ldots, b_n) \in \{0, 1\}^n$, where each bit $b_i$ indicates whether an example $z_i \in S$ was chosen in the subsample or not. We denote the resulting subsample as $S' \subseteq S$. By definition of Poisson subsampling, each bit $b_i$ is an independent sample $b_i \sim \text{Bern}(p_i)$. Let us denote by $\lambda_{\bar{b}}$ the joint probability of $\bar{b}$. The composition $\theta(S) \circ \text{Sample}(S)$ can be expressed as a mixture distribution:

$$\theta(S) \circ \text{Sample}(S) = \sum_{\bar{b} \in \{0, 1\}^n} \lambda_{\bar{b}} \cdot \theta(S).$$

Analogously, for a neighbouring dataset $S' \equiv S \cup \{z_0\}$, with the sampling probability $p_0$ corresponding to $z_0$, we have:

$$\theta(S) \circ \text{Sample}(S) = \sum_{\bar{b} \in \{0, 1\}^n} p_0 \cdot \lambda_{\bar{b}} \cdot \theta(S' \cup \{z_0\}) + \sum_{\bar{b} \in \{0, 1\}^n} (1 - p_0) \cdot \lambda_{\bar{b}} \cdot \theta(S').$$

Applying Lemma C.8, we get $f_0$-privacy with $f_0 = p_0f + (1 - p_0)\text{Id}$. Applying to an arbitrary other $z_0 \in \mathbb{D}$, we potentially get the worst-case privacy guarantee for the highest sampling probability, i.e., $f = p^*f + (1 - p^*)\text{Id}$. \hfill $\square$

Proposition 6.1 is immediate from Lemma C.7 by the fact that GDP is a special case of $f$-privacy.

### D. Calibration Generalization via Strong Distributional Generalization

For simplicity we consider binary classifiers, with $\mathbb{Y} = \{0, 1\}$ and $f_\theta : \mathbb{X} \rightarrow [0, 1]$.
The calibration gap, or Expected Calibration Error (ECE), of a classifier $f_\theta$ is defined as:
\[
CGAP(f_\theta) := \mathbb{E}[|\mathbb{E}[y \mid f_\theta(x)] - f_\theta(x)|]
\]
\[
= \int_0^1 |\mathbb{E}[y - p \mid f_\theta(x) = p]| dF(p)
\]
(where $F(p)$ is the law of $f_\theta(x)$)

Where all expectations are taken w.r.t. $(x, y) \sim D$. To estimate this quantity from finite samples, we will discretize the interval $[0, 1]$. The $\tau$-binned calibration gap of a classifier $f_\theta$ is:
\[
CGAP_\tau(f_\theta; D) := \sum_{p \in \{0, \tau, 2\tau, \ldots, 1\}} |\mathbb{E}_{(x, y) \sim D} [\mathbb{1}\{f_\theta(x) \in (p, p+\tau)\} \cdot (y - p)]|
\]
So that $CGAP_\tau \rightarrow CGAP$ as the bin width $\tau \rightarrow 0$. The empirical version of this quantity, for train set $S$, is:
\[
CGAP_\tau(f_\theta; S) := \sum_{p \in \{0, \tau, 2\tau, \ldots, 1\}} |\mathbb{E}_{x, y \sim S} [\mathbb{1}\{f_\theta(x) \in (p, p+\tau)\} \cdot (y - p)]|
\]

We will now show that the strong Distributional Generalization (Definition B.1) implies that the expected binned calibration gap is similar between train and test.

**Theorem D.1.** If the training method $\theta$ satisfies $\delta$-strong DG, then
\[
\mathbb{E}_{\theta, S \sim D^n} [CGAP_\tau(f_\theta(S); S) - CGAP_\tau(f_\theta(S); D)] \leq \frac{\delta}{\tau},
\]

**Proof.** First, let us define the family of tests:
\[
\phi_p(x, y; \theta) := \mathbb{1}\{f_\theta(x) \in (p, p+\tau)\} \cdot (y - p)
\]
The assumed $\delta$-strong DG implies that each of these tests $\phi_p$ have similar expectations between train and test distributions:
\[
\forall p \left| \mathbb{E}_{S \sim D^n; (x, y) \sim S} [\phi_p(x, y; \theta(S))] - \mathbb{E}_{S \sim D^n; (x, y) \sim D} [\phi_p(x, y; \theta(S))] \right| \leq \delta
\]
Thus, the difference in expected calibration gaps between train and test is:
\[
\left| \mathbb{E}_{S \sim D^n} [CGAP_\tau(f_\theta; S) - CGAP_\tau(f_\theta; D)] \right|
\]
\[
= \left| \mathbb{E}_{S \sim D^n} \sum_{p \in \{0, \tau, 2\tau, \ldots, 1\}} \mathbb{E}_{x, y \sim S} [\phi_p(x, y; \theta)] - \mathbb{E}_{x, y \sim D} [\phi_p(x, y; \theta)] \right|
\]
\[
\leq \sum_{p \in \{0, \tau, 2\tau, \ldots, 1\}} \mathbb{E}_{S \sim D^n} \mathbb{E}_{x, y \sim S} [\phi_p(x, y; \theta)] - \mathbb{E}_{x, y \sim D} [\phi_p(x, y; \theta)]
\]
\[
\leq \frac{\delta}{\tau},
\]
where the last inequality follows from the definition of $\delta$-strong DG applied to functions $\phi_p$.

**E. Additional Details on Algorithms**
We define $q_g$ as the probability of group $g$, and $m$ as the number of groups.
**IS-SGD.** The weight for group $g$ is $w_g = \frac{1}{m_q g}$. Let $g_i$ be the group that the $i$-th example belongs to. We then sample (with replacement) from the training set with the $i$-th example having a $w_{g_i}$ chance of being sampled until we have $b$ examples, where $b$ is the batch size. Finally, for each mini-batch, we optimize the standard cross-entropy loss with the sampled examples.

**IW-SGD.** The weight for group $g$ is $w_g = \frac{1}{m_q g}$. We optimize on the following loss function,

$$w_g \cdot \ell(f_{\theta}(x), y),$$

where $\ell(\cdot, \cdot)$ is the cross-entropy loss and $(x, y) \in S$ drawn uniformly random drawn from the dataset, and $g$ is the group to which $(x, y)$ belongs.

**DP-IS-SGD.** The weight for group $g$ is $w_g = \frac{1}{m_q g}$. We perform importance weighted (non-uniform) random sampling with replacement to sample examples based on the weights. Let the sampling rate be $\bar{p}$ and $g_i$ be the group that the $i$-th example belongs to. For the sampling process, for each example $i$, we first sample a number uniformly at random $v_i \in [0, 1]$. Example $i$ is sampled into the batch if $v_i < \bar{p} \cdot w_{g_i}$, thus its probability to be in the batch is $p = \frac{\bar{p}}{m_q g}$.

**F. Other methods that achieve distributional generalization**

As mentioned in Section 6, many regularization methods can be used to improve different generalization gaps. For example, Sagawa et al. (2019) show that strong $\ell_2$ regularization helps with improving group-distributional generalization, and Yang et al. (2020) show that dropout helps with adversarial robustness generalization. However, these works do not have theoretical justification. In Figure 5, we show the training and testing worst-group accuracy with different strength of $\ell_2$ regularization and on different epochs (w/ and w/o $\ell_2$ regularization). We have three observations: (1) with properly tuned regularization parameter, the gap between training and testing worst-group accuracy can be narrowed, (2) the gap can start widening in very early stage of training, and (3) the testing worst-group accuracy can fluctuate largely, which highlights the importance of using validation set for early stopping in this task.

**G. Additional Experiment Details**

All algorithms are implemented in PyTorch (Paszke et al., 2019). For DP-related utilities, we use opacus (Yousefpour et al., 2021). For gDRO (Sagawa et al., 2019), we use the implementation from wilds (Koh et al., 2021).

**Datasets.** For CelebA and CivilComments, we follow the training/validation/testing split in Koh et al. (2021). For UTKFace and iNat, we randomly split the data into 17000/2000/4708 and 550000/50000/75170 for training/validation/testing. For MNLI, we use the same training/validation/testing split in Sagawa et al. (2019). Tables 2 to 6 show the dataset statistics on each group.

**Models.** Similar to previous work (Sagawa et al., 2019), we use the ImageNet-1k pretrained ResNet50 (He et al., 2016) from torchvision for CelebA, UTKFace, and iNat, and use the pretrained BERT-Base (Devlin et al., 2019) from huggingface (Wolf et al., 2019) for CivilComments and MNLI.

**Hyperparameters.** We run 50 epochs for CelebA, 100 epochs for UTKFace, 20 epochs for iNat, and 5 epochs for Civil-Comments and MNLI. For image datasets (CelebA, UTKFace, and iNat), we use the SGD optimizer and for NLP datasets (CivilComments and MNLI), we use the AdamW (Loshchilov and Hutter, 2019) optimizer. We use opacus’s (Yousefpour et al., 2021) implementation on DP-SGD and DP-AdamW to achieve DP guarantees.

We fix the batch size for none-DP algorithms to 64 for CelebA and UTKFace, 256 for iNat, 16 for CivilComments, and 32 for MNLI. For DP-SGD and DP-IS-SGD, we set the sample rate to 0.0001 for CelebA and iNat, 0.001 for UTKFace, and 0.00005 for CivilComments and MNLI.

**G.1. Additional details for Section 7.2**

Figure 6 shows the accuracy disparity, test accuracy, and worst-group accuracy for CelebA, UTKFace, and iNat on DP-SGD and DP-IS-SGD.
Figure 5. We show the training and testing worst-group accuracy with different strength of $\ell_2$ regularization and on different epochs (w/ and w/o $\ell_2$ regularization). The network is trained with IS-SGD on CelebA, UTKFace, and iNat. For (a), (b), and (c), we show the result of the last epoch. For (g), (h), and (i), we set weight decay to 0.01.

|               | training | validation | testing |
|---------------|----------|------------|---------|
| not blond, female | 71629    | 8535       | 9767    |
| not blond, male  | 66874    | 8276       | 7535    |
| blond, female   | 22880    | 2874       | 2480    |
| blond, male     | 1387     | 182        | 180     |

Table 2. The number of examples in each subgroup for CelebA.

The reason that UTKFace has a similar disparity between DP-SGD and DP-IS-SGD is possibly because UTKFace has a relatively small difference in the number of training examples between the largest group and the smallest group. In UTKFace, the majority group has around seven times more examples than in the minority group, whereas in CelebA, this difference is $52 \times$.  

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| Gender, Race | training | validation | testing |
|-------------|----------|------------|---------|
| male, White | 3919     | 454        | 1105    |
| male, Black | 1700     | 181        | 437     |
| male, Asian | 1115     | 157        | 303     |
| male, Indian| 1594     | 190        | 477     |
| male, Others| 563      | 61         | 136     |
| female, White| 3316    | 384        | 902     |
| female, Black| 1606    | 188        | 414     |
| female, Asian| 1302    | 158        | 399     |
| female, Indian| 1230   | 152        | 333     |
| female, Others| 655     | 75         | 202     |

Table 3. The number of examples in each subgroup for UTK-Face.

| Kingdom | training | validation | testing |
|---------|----------|------------|---------|
| Actinopterygii | 2112     | 195        | 312     |
| Amphibia   | 14531    | 1242       | 1930    |
| Animalia   | 5362     | 491        | 737     |
| Arachnida  | 4838     | 461        | 660     |
| Aves       | 191773   | 17497      | 26251   |
| Chromista  | 435      | 52         | 55      |
| Fungi      | 6148     | 575        | 883     |
| Insecta    | 96894    | 8648       | 13013   |
| Mammalia   | 26724    | 2475       | 3624    |
| Mollusca   | 7627     | 693        | 1057    |
| Plantae    | 159843   | 14653      | 22117   |
| Protozoa   | 309      | 25         | 37      |
| Reptilia   | 33404    | 2983       | 4494    |

Table 4. The number of examples in each subgroup for iNat.

| Category        | training | validation | testing |
|-----------------|----------|------------|---------|
| Non-toxic, Identity | 94895    | 15759      | 46185   |
| Non-toxic, Other  | 143628   | 24366      | 72373   |
| Toxic, Identity   | 18575    | 3088       | 9161    |
| Toxic, Other      | 11940    | 1967       | 6063    |

Table 5. The number of examples in each subgroup for CivilComments.

| Algorithm | blond female | male | not blond female | male |
|-----------|--------------|------|------------------|------|
| ERM       | 1.00         | 0.99 | 1.00             | 1.00 |
|           | 0.80         | 0.42 | 0.97             | 1.00 |
| IW-SGD    | 0.98         | 0.99 | 0.98             | 0.99 |
|           | 0.87         | 0.49 | 0.95             | 0.98 |
| IS-SGD    | 1.00         | 1.00 | 1.00             | 1.00 |
|           | 0.83         | 0.38 | 0.96             | 0.99 |
| DP ERM    | 0.80         | 0.41 | 0.96             | 0.99 |
|           | 0.74         | 0.29 | 0.98             | 1.00 |
| DP ERM IW | 0.94         | 0.96 | 0.88             | 0.90 |
|           | 0.92         | 0.85 | 0.91             | 0.92 |

Table 7. The accuracy for each subgroup on CelebA. These results are acquired without any regularization or early stopping (trained on full 50 epochs).

G.2. Experiment Setup for Section 7.3

We compare different algorithms, including SGD-$\ell_2$ and IW-SGD-$\ell_2$ as baselines, and two other algorithms, IS-SGD-$\ell_2$ (Idrissi et al., 2021) and gDRO-$\ell_2$ (Sagawa et al., 2019) in terms of the group robustness. We set the learning rate as 0.001 for CelebA, UTKFace, and iNat, 0.00002 for MNLI, and 0.00001 for CivilComments. We use the validation set to select the hyperparameters. More specifically:
Figure 6. The disparity (lower the better) and test accuracies of the models trained with DP-SGD and IW-SGD on three datasets. (If we care about privacy, DP-IS-SGD improves disparate impact at most privacy budgets). For CelebA, we train the model for 30 epochs, for UTKFace, we train for 100 epochs, and for iNat, we train for 20 epochs. The GDP accountant is used to compute the privacy budget.

1. For SGD-\ell_2, IW-SGD-\ell_2, IS-SGD-\ell_2, and gDRO-\ell_2, we select the weight decay from 0.0001, 0.01, 0.1, and 1.0.

2. For DP-IS-SGD, we fix the gradient clipping to 1.0 (except for iNat, where we set the value to 10.0 as 1.0 does not converge). We select the noise parameter from 1.0, 0.1, 0.01, 0.001 on CelebA and UTKFace, select the noise parameter from 0.00000001, 0.000001, 0.00001, and 0.0001 on iNat and select the noise parameter from 0.01 and 0.001 on CivilComments and MNLI.

3. For IW-SGD-n, IS-SGD-n, and gDRO-n, we select the standard deviation of the random noise from 0.001, 0.01, 0.1, and 1.0 on CelebA, UTKFace, and iNat, and we select standard deviation of the random noise from 0.000001, 0.0001, and 0.001 on CivilComments and MNLI.