I review recent progress in lattice QCD at finite temperature. Results on the transition temperature will be summarized. Recent progress in understanding in-medium modifications of interquark forces and quarkonia spectral functions at finite temperatures is discussed.

1 Introduction

It is expected that strongly interacting matter shows qualitatively new behavior at temperatures and/or densities which are comparable or larger than the typical hadronic scale. It has been argued that under such extreme conditions deconfinement of quarks and gluons should take place, i.e. thermodynamics of strongly interacting matter could be understood in terms of this elementary degrees of freedom and this new form of matter was called Quark Gluon Plasma. On the lattice the existence of the deconfinement transition at finite temperature was first shown in the strong coupling limit of QCD, followed by numerical Monte-Carlo studies of lattice SU(2) gauge theory which confirmed it. Since these pioneering studies QCD at finite temperature became quite a large subfield of lattice QCD (for recent reviews on the subject see Refs. [1,5,6]). One of the obvious reasons for this is that phase transitions can be studied only non-perturbatively. But even at high temperatures the physics is non-perturbative beyond the length scales $1/(g^2 T)$ ($g^2(T)$ being the gauge coupling). Therefore lattice QCD remains the only tool for theoretical understanding of the properties of strongly interacting matter under extreme condition which is important for the physics of the early universe as well as heavy ion collisions.
One of the basic questions we are interested in is what is the nature of the transition to the new state of matter and what is the temperature where it happens. In the case of QCD without dynamical quarks, i.e. SU(3) gauge theory these questions have been answered. It is well established that the phase transition is 1st order. Using standard and improved actions the corresponding transition temperature was estimated to be $T_c/\sqrt{\sigma} = 0.632(2)$ (σ is the string tension). The situation for QCD with dynamical quarks is much more difficult. Not only because the inclusion of dynamical quarks increases the computational costs by at least two orders of magnitude but also because the transition is very sensitive to the quark masses. Conventional lattice fermion formulations break global symmetries of continuum QCD (e.g. staggered fermion violate the flavor symmetry) which also introduces additional complications. Current lattice calculations suggest that transition in QCD for physical quark masses is not a true phase transition but a crossover, recent lattice results for the transition temperature $T_c$ from Wilson fermions, improved and unimproved staggered fermions with 2 and 2+1 flavors of dynamical quarks are summarized in Fig. 1. The errors shown in Fig. 1 are only statistical with the exception of the data point from the MILC collaboration, where the large error partly comes from the continuum extrapolation and also includes systematic error in scale setting. Since the “critical” energy density $\epsilon_c = \epsilon(T_c)$ (i.e. the energy density at the transition) scales as $T_c^4$ the error in $T_c$ is the dominant source of error in $\epsilon_c$.

3 Heavy quarks at finite temperature

In this section I am going to summarize some recent progress made in understanding the interaction of heavy quarks at finite temperature. Apart from being an interesting problem from a theoretical perspective understanding the interaction of heavy quarks at finite temperature also is very important for phenomenology. It has been suggested that quarkonium suppression due to color screening at high temperatures can serve as signature of Quark Gluon Plasma formation in heavy ion collisions. For static quarks one can calculate the free energy difference for the system with static quark anti-quark pair and the system without static charges. This quantity is often referred to as finite temperature potential, though it should be emphasized that it is a free energy and thus contains an entropy contribution. In Fig. 2 I show the free energy
of static $Q\bar{Q}$ in the singlet state calculated in three flavor QCD. As one can see from the Figure the free energy goes to a constant at distances $r > 0.9$ fm at low temperatures. This happens because once enough energy is accumulated the string can break due to creation of a light quark-antiquark pair. As the temperature increases the distance where the free energy levels off becomes temperature dependent and decreases with increasing temperature. This reflects the onset of chromo-electric screening. Similar results have been obtained in two flavor QCD,\textsuperscript{19,20}

It should be noticed that at short distances ($r < 0.4$ fm) the free energy of static $Q\bar{Q}$ is temperature independent. As expected at short distance medium effects are not important. This is also confirmed by studies of the coupling constant at finite temperature,\textsuperscript{21} which I also show in Fig. 2. The running of the coupling constant at finite temperature is controlled by the distance between the static quarks and its value is never larger than at zero temperature.\textsuperscript{21} This disfavors the picture of strongly coupled plasma where $\alpha_s$ runs to large value above the transition temperature.\textsuperscript{22}

Though the study of the free energy of a static quark anti-quark pair gives some useful insight into the problem of quarkonium binding at high temperatures (for a recent review on this subject see Ref.\textsuperscript{23}), it is not sufficient for detailed understanding quarkonium properties in this regime. To gain quantitative information on this problem quarkonium correlators and
spectral functions should be studied at finite temperature. Such studies became possible only recently and still are restricted to the quenched approximation. The results of these studies for charmonia are summarized in Fig. 3. The 1S states ($J/\psi, \eta_c$) seem to survive to temperatures as high as $1.6T_c$ (maybe even higher, cf. the figure) while the 1P states ($\chi_{c0}, \chi_{c1}$) are dissolved at $1.1T_c$. The survival of the 1S state is also confirmed by Umeda et al. As for bottomonia only preliminary results exists showing that $\Upsilon$ can exist in the plasma up to much higher temperatures but surprisingly enough the $\chi_b$ state is dissociated at temperatures smaller than $1.5T_c$.

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