Oscillatory instability in a closed cylinder with rotating top and bottom

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Abstract. A numerical investigation of oscillatory instability is presented for axisymmetric swirling flow in a closed cylinder with rotating top and bottom. The critical Reynolds number and frequency of the oscillations are evaluated as function of the ratio of angular velocities of the bottom and the top (\( \xi = \frac{\Omega_{\text{bottom}}}{\Omega_{\text{top}}} \)). Earlier Linear Stability Analysis (LSA) using the Galerkin spectral method by Gelfgat et al. [Phys. Fluids, 8, 2614-2625 (1996)] revealed that the curve of the critical Reynolds number behaves like an “S” around \( \xi = 0.54 \) in the co-rotation branch and around \( \xi = -0.63 \) in the counter-rotation branch. Additional finite volume computations, however, did not show a clear “S” behaviour. In order to check the existence of the “S” shape, computations are performed using an axisymmetric finite volume Navier-Stokes code at aspect ratios (\( \lambda = H / R \)) 1.5 and 2.0. Comparisons with LSA at \( \lambda = 1.5 \) show that the “S” shape does exist. At an aspect ratio \( \lambda \) of 2, our results show that the critical Reynolds number curve has a “beak” shape in the counter-rotation region and a much wider “S” shape in the co-rotation region. This transformation of the “S” shape is caused by the change in aspect ratio from 1.5 to 2 and therefore the corresponding topological behaviour of the transition is different.

1. Introduction
The flow in a closed cylindrical container with a rotating lid has been studied both experimentally and computationally for more than three decades. The influence of co- and counter-rotation of the other end wall of the cylinder on vortex breakdown was studied experimentally by Bar-Yoseph et al. [1], Gautier et al. [2] and Fujimura et al. [3]. In computations, Valentine & Jahnke [4] and Lopez [5] studied the case of co-rotating end walls with the same angular velocity for steady and unsteady swirl flow. Nore et al. [6] studied the case of counter-rotating end walls for steady and unsteady swirl flow and determined that onset of three-dimensionality for exactly counter-rotating end walls takes place at a Reynolds numbers of about 350.

Gelfgat et al. [7] performed a parametric investigation of the oscillatory instability for the axisymmetric flow case at an aspect ratio of 1.5. From their linear stability analysis (see Figs. 10 and 13 in [7]), they found, at a ratio of angular velocities \( \xi \) about 0.54 in the co-rotation branch, the existence of three distinct critical points with different critical frequencies. The curve of the critical Reynolds number around \( \xi = 0.54 \) looks like an “S”. In the counter-rotation branch, the same phenomenon was observed at a ratio of angular velocity \( \xi \) about -0.63. In the same paper, finite
volume computations were also performed. From the computations, using three different meshes, no such “S” shape of bifurcation could be seen. What happens here? Is it the linear stability method or the finite volume method that fails to predict the physics?

2. Formulation of the problem and numerical method

Consider a circular cylinder of radius $R$ and height $H$ filled with an incompressible Newtonian fluid of constant density $\rho$ and kinematical viscosity $\nu$. The top and the bottom of the cylinder are rotating with angular velocities $\Omega_{\text{top}}$ and $\Omega_{\text{bottom}}$, respectively (see Fig. 1). The various flow regimes are determined by the Reynolds number, $Re=\frac{\Omega_{\text{top}}R^2}{\nu}$, the aspect ratio, $\lambda=H/R$, and the ratio of angular velocities, $\xi=\frac{\Omega_{\text{bottom}}}{\Omega_{\text{top}}}$.

![Figure 1: Schematic representation of the flow geometry.](image)

The flow is governed by the unsteady incompressible Navier-Stokes equations. Because of the simple geometry of the cylinder, it is natural to formulate the equations in cylindrical coordinates. The resulting Navier-Stokes equations are solved by a predictor-corrector method based on a cell-centred finite-volume / multi-block strategy [8] [9]. In the predictor step, the momentum equations are discretized using a second-order backward differentiation scheme in time and second-order central differences in space, except for the convective terms that are discretized by the QUICK upwind scheme. Since all variables are defined at cell centres, no special treatment is needed for the singularity problem at the centre axis. In the corrector step, the new Rhie-Chow interpolation developed by Shen et al. [10] and the new SIMPLE-C scheme on collocated grids [11] are used in order to avoid numerical oscillations from pressure decoupling. The obtained pressure Poisson equation is solved by a five-level multi-grid technique. For more details about the numerical technique, the reader is referred to the references [8]-[11].

3. Comparison with linear stability analysis

In order to check the existence of the “S” shape, computations are first performed at an aspect ratio $\lambda=1.5$, corresponding to the case considered by Gelfgat et al. [7]. Our attention, however, is mainly focused on the ratio of angular velocities $\xi$ in the regions where the “S” shape appears. Fig. 2 shows stability diagrams of the critical Reynolds number in the counter-rotating region computed by the present finite volume code and the LSA of Gelfgat et al. [7]. From the figure, excellent agreement between the present method and linear stability analysis of Gelfgat is found and the “S” shape is very well captured by our finite volume method.
Fig. 3 shows stability diagrams of the critical Reynolds number in the co-rotating region. From the figure, the “S” shape is seen to be captured by our finite volume code, but the width of the “S” shape is smaller than that of LSA. Before reaching the “S”, the two stability curves are in good agreement. After the “S” shape, the critical Reynolds number computed by the present code is smaller than that predicted by LSA. One interesting point found during the computations is that a stability spot is found at $\xi = 0.58$. The flow changes state from steady to unsteady when the Reynolds number reaches 4404. The flow again becomes steady when the Reynolds number is increased to 4408, and stays steady until
the Reynolds number exceeds 4413.5. After this point, the flow becomes unsteady. This phenomenon can be seen from velocity fluctuations at different positions (see Fig. 4).

Figure 4: Velocity signals at \((r, z) = (0.25, 0.375)\) for swirl flow in a closed cylinder with an aspect ratio 1.5 and co-rotating end walls of \(\xi = 0.58\). (a) Re=4403; (b) Re=4405; (c) Re=4410; (d) Re=4415.

4. Swirl flow at aspect ratio 2
To supplement the computations at \(\lambda = 1.5\), computations were performed at an aspect ratio \(\lambda = 2\). First the influence of counter-rotation on the transition from steady to unsteady flow is studied. The dependence of the critical Reynolds number, \(Re_{cr}\), on the ratio of angular velocities, \(\xi\), is plotted in Fig. 5 (a). From the figure, we can see that when increasing slightly the counter-rotating angular velocity, i.e. by letting \(\xi\) vary from zero to a small negative value, the critical Reynolds number decreases slightly. If counter-rotation increases more, the critical Reynolds number starts to increase rapidly, implying that the steady region for a moderate counter-rotation has been enlarged. Further increasing counter-rotation, the size of the steady region reaches a maximum and then decreases, forming a sharp “beak” around \(\xi = -0.5\). The critical Reynolds number stays flat after the “beak”. The reduced frequency is defined as \(\omega = 2\pi f / \Omega_{sp}\), where \(f\) [Hz] is the frequency of oscillation. The critical reduced frequency, \(\omega_{cr}\), as a function of the ratio of angular velocities is also plotted in Fig. \[\text{Figure 5}\]
5(b). From the figure, one can see that the frequency of oscillation changes at $\zeta = -0.02$ and $\zeta = -0.07$ from 0.23 to 0.16 and back from 0.16 to 0.22, respectively, signalling the appearance of a different disturbance mode in this range. Similar changes of the dominant mode were revealed by Gelfgat et al.\textsuperscript{25} Remark that at $\zeta = -0.02$, the dominant reduced frequency is 0.23, but that the signal also contains a lower frequency component of 0.16. At $\zeta = -0.07$, the same phenomenon is observed. The reduced frequency follows another branch in the region close to the “beak” of the critical Reynolds number ($-0.51 \leq \zeta < -0.4$). If the counter-rotating angular velocity is further increased, the frequency increases slightly and returns to the original branch.

Figure 5: Stability diagrams of oscillatory instability in a cylinder of aspect ratio 2 with rotating top and counter-rotating bottom, (a) critical Reynolds number; (b) critical frequency.

In the following, the influence of co-rotating on the transition from steady to unsteady flow will be studied. The dependence of the critical Reynolds number, $Re_{cr}$, on the ratio of angular velocities, $\zeta$, is plotted in Fig. 6 (a). From the figure, we can see that increasing the co-rotating angular velocity increases the steady Reynolds number region. At angular velocities about $\zeta = 0.7$, an “S” shaped diagram is found. This “S” shape of critical Reynolds numbers was first found by Gelfgat et al.\textsuperscript{[7]} using linear stability theory in the case of a swirl flow in a closed cylinder with an aspect ratio of 1.5. Unfortunately, the structure could not be reproduced by their finite volume computations. The phenomenon at $\zeta = 0.7$ is that the flow first becomes unsteady at a Reynolds number above 2870 and stays unsteady for Reynolds numbers up to 3418. The flow then becomes steady when the Reynolds number is further increased in the range from 3418 to 4398. The flow is fully unsteady at Reynolds numbers above 4398.

The critical frequency in Fig. 6 (b) increases continuously to $\omega_{cr} = 0.379$ for ratios of angular velocities up to 0.4 and thereafter it is shifted to another branch ($\omega_{cr} = 0.316$). It then stays on the same branch as long as the ratio $\zeta$ remains in the first half (lower side) of the “S”. The frequency changes at $\zeta = 0.67$ in the second half (upper side) of the “S” into a lower frequency ($\omega_{cr} = 0.273$). Shortly after $\zeta = 0.69$, a double harmonics appears in the signal. At $\zeta = 0.75$, this double harmonic frequency becomes the basic frequency. Thus, the lower frequency created at $\zeta = 0.67$ disappears at $\zeta = 0.75$. After that, another branch appears for $\zeta \geq 0.8$ with a slightly lower frequency.
At $\zeta = 0.9$, the critical frequency appears on a branch with a high value ($\omega_{cr} = 0.518$), where it stays continuously until $\zeta = 1$ ($\omega_{cr} = 0.542$).

Figure 6: Stability diagrams of oscillatory instability in a cylinder of aspect ratio 2 with rotating top and co-rotating bottom, (a) critical Reynolds number; (b) critical frequency.

5. Conclusions
The stability of steady flow and the onset of the oscillatory instability were studied by using a finite volume code for a closed cylinder with rotating top and bottom. From our computations at the same aspect ratio $\lambda = 1.5$ as the earlier investigations by Gelfgat et al., the “S” shape of the stability curve is confirmed in agreement with the linear stability analysis. At an aspect ratio $\lambda = 2$, our results show that the critical Reynolds number curve has a “beak” shape in the counter-rotation region and a much wider “S” shape in the co-rotation region. From linear stability theory for a cylinder at $\lambda = 1.5$ the “S” shape did exist in both co-rotation and counter-rotation branches of the stability curve of the critical Reynolds number but the width of the “S” shape in co-rotation branch is over-predicted. This transformation of the “S” shape is caused by the change in aspect ratio from 1.5 to 2 and therefore the corresponding topological behaviour of the transition is different.

The bifurcation from a steady to an unsteady regime is governed by Hopf bifurcations in most of the counter and co-rotating regions. In a region close to the top of the “S” shape, however, a discontinuous bifurcation has been detected.

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