Are There $\nu_\mu$ or $\nu_\tau$ in the Flux of Solar Neutrinos on Earth?

C. Giunti

INFN, Sezione di Torino,
and
Dipartimento di Fisica Teorica, Università di Torino,
Via P. Giuria 1, I–10125 Torino, Italy

(Dated: 10 October 2001)

Using the model independent method of Villante, Fiorentini, Lisi, Fogli, Palazzo, and the rates measured in the SNO and Super-Kamiokande solar neutrino experiment, we calculate the amount of active $\nu_\mu$ or $\nu_\tau$ present in the flux of solar neutrinos on Earth. We show that the probability of $\nu_e \rightarrow \nu_{\mu,\tau}$ transitions is larger than zero at 99.89% CL. We find that the averaged flux of $\nu_{\mu,\tau}$ on Earth is larger than 0.17 times the $^8\text{B}$ $\nu_e$ flux predicted by the BP2000 Standard Solar Model at 99% CL. We discuss also the consequences of possible $\nu_e \rightarrow \nu_{\mu,\tau}$ or $\nu_e \rightarrow \bar{\nu}_e$ transitions of solar neutrinos. We derive a model-independent lower limit of 0.52 at 99% CL for the ratio of the $^8\text{B}$ $\nu_e$ flux produced in the Sun and its value in the BP2000 Standard Solar Model.

PACS numbers: 26.65.+t, 14.60.Pq, 14.60.Lm

Keywords: Solar Neutrinos, Neutrino Physics, Statistical Methods

The first results of the SNO solar neutrino experiment \[1\] have beautifully confirmed the existence of the solar neutrino problem. A comparison of the neutrino flux measured through charged-current interactions in the SNO experiment with the flux measured through elastic scattering interactions in the Super-Kamiokande experiment \[2\] shows an evidence of the presence of active $\nu_\mu$ or $^1\nu_\tau$ in the solar neutrino flux measured by the Super-Kamiokande experiment \[1, 3\]. Such a presence represents a very interesting indication in favor of neutrino physics beyond the Standard Model, most likely neutrino mixing that generates oscillations between different flavors (see \[4\]).

The purpose of this paper is to quantify the amount of this flux of active $\nu_\mu$ or $\nu_\tau$ in a model-independent way in the framework of Frequentist Statistics\[2\].

The authors of Refs. \[6, 7\] have noted that the response functions of the SNO and Super-Kamiokande (SK) experiments to solar neutrinos can be made approximately equal with a proper choice of the energy thresholds of the detected electrons. It turns out that given the threshold $T_S^{\text{SNO}} = 6.75\text{ MeV}$, the two response functions are approximately equal for $T_{\text{SK}}^{\nu_e} = 8.60\text{ MeV}$ \[3\]. In this case the SNO and Super-Kamiokande event rates normalized to the BP2000 Standard Solar Model (SSM) prediction \[8\] can be written in a model-independent way as \[3\]

\[
R_{\text{SNO}} = f_B \langle P_{\nu_e \rightarrow \nu_e} \rangle, \tag{1}
\]

\[
R_{\text{SK}} = f_B \langle P_{\nu_e \rightarrow \nu_e} \rangle + f_B \frac{\langle \sigma_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e \rightarrow \nu_e} \rangle} \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle, \tag{2}
\]

where $f_B$ is the ratio of the $^8\text{B}$ $\nu_e$ flux produced in the Sun and its value in the SSM \[8\], $\langle P_{\nu_e \rightarrow \nu_e} \rangle$ is the survival probability of solar $\nu_e$’s averaged over the common SNO and Super-Kamiokande response functions,

\[
\frac{\langle \sigma_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e \rightarrow \nu_e} \rangle} = 0.152 \tag{3}
\]

is the ratio of the averaged $\nu_{\mu,\tau}$ and $\nu_e$ cross sections in the Super-Kamiokande experiment, and $\langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle$ is the averaged probability of $\nu_e \rightarrow \nu_{\mu,\tau}$ transitions.

Calling

\[
R_A \equiv R_{\text{SK}} - R_{\text{SNO}}, \tag{4}
\]

\[*
\text{Electronic address: giunti@to.infn.it; URL: } \text{http://www.to.infn.it/~giunti}
\]

1 In this paper the conjunction “or” is used as a logical inclusive disjunction (the sentence is true when either or both of its constituent propositions are true).

2 Since the results that we obtain are not too close to physical boundaries for the quantities under discussion and we assume a normal distribution for the errors, the numerical values in the framework of Bayesian Probability Theory with a flat prior are close to those obtained here, but their meaning is different (see, for example, Ref. \[8\]).
from Eqs. (1) and (2) we have

\[ R_A = f_B \frac{\langle \sigma_{\nu_e} \rangle}{\sigma_{\nu_e}} \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle. \]  

(5)

Therefore, \( R_A \) is the rate of \( \nu_{\mu,\tau} \)-induced events in the Super-Kamiokande experiment, relative to the \( \nu_e \)-induced rate predicted by the SSM.

Considering the data of the Super-Kamiokande experiment above the energy threshold \( T_{SK}^e = 8.60 \text{ MeV} \) and the BP2000 Standard Solar Model \([8]\), the measured values of \( R_{\text{SNO}} \) and \( R_{\text{SK}} \) are:

\[ R_{\text{exp}}^{\text{SNO}} = 0.347 \pm 0.029 \quad [1], \]

(6)

\[ R_{\text{exp}}^{\text{SK}} = 0.451 \pm 0.017 \quad [2, 3]. \]

(7)

Adding in quadrature the uncertainties of \( R_{\text{SNO}} \) and \( R_{\text{SK}} \), for \( R_A \) we obtain:

\[ R_{\text{exp}}^{\text{A}} = 0.104 \pm 0.034. \]

(8)

The standard deviation of \( R_{\text{exp}}^{\text{A}} \) is

\[ \sigma_{\text{exp}}^{\text{A}} = 0.034, \]

(9)

and we have

\[ \frac{R_{\text{exp}}^{\text{A}}}{\sigma_{\text{exp}}^{\text{A}}} = 3.06 \pm 1. \]

(10)

Hence, the central value of \( R_A \) is 3.06\( \sigma \) away from zero, implying an evidence of solar \( \nu_e \rightarrow \nu_{\mu,\tau} \) transitions [1, 3]. Our purpose is to quantify the probability of these transitions and possibly derive a lower limit.

The authors of Ref. [1] calculate the probability of a fluctuation larger than the observed one assuming \( R_A = 0 \): for normally distributed errors the probability of a fluctuation larger than 3.06\( \sigma \) from the mean is 0.11%.

Recently some frequentist methods have been proposed that allow to obtain always meaningful confidence intervals with correct coverage for quantities like \( R_A \) that are bound to be positive by definition [9, 10, 11, 12]. In particular, the Unified Approach proposed in Ref. [9] has been widely publicized by the Particle Data Group [13] and used by several experimental collaborations.

Using the Unified Approach we can derive confidence intervals for \( R_A \). Figure 1 shows the confidence belts in the Unified Approach for a normal distribution with unit standard deviation for 90\% (1.64\( \sigma \)), 99\% (2.58\( \sigma \)), 99.73\% (3\( \sigma \)) and 99.89\% (3.06\( \sigma \)) CL. One can see that the measured value (10) of \( \frac{R_{\text{exp}}^{\text{A}}}{\sigma_{\text{exp}}^{\text{A}}} \) implies that

\[ 0 < \frac{R_A}{\sigma_A} < 6.32 \quad \text{at} \quad 99.89\% \text{ CL}, \]

(11)

i.e. active \( \nu_\mu \) or \( \nu_\tau \) are present in the solar neutrino flux on Earth at 99.89\% CL. Equation (11) implies that there is a 0.11\% probability that the true value of \( \frac{R_A}{\sigma_A} \) is zero or larger than 6.32. This probability is the same as the probability of a fluctuation larger than 3.06\( \sigma \) calculated in Ref. [1] assuming \( R_A = 0 \). However, our result have been derived without making any assumption on the true unknown value of \( R_A \) and has a well defined meaning in the framework of Frequentist Statistics: whatever the true value of \( R_A \), the interval (11) belongs to a set of intervals that could be obtained in the same way from repeated measurements and have the property that 99.89\% of these intervals cover the true value of \( \frac{R_A}{\sigma_A} \).

In order to derive a lower limit for the averaged flux of \( \nu_{\mu,\tau} \) on Earth, we consider in the following 99\% confidence intervals. From Fig. 1 we obtain

\[ 0.74 < \frac{R_A}{\sigma_A} < 5.63 \quad (99\% \text{ CL}), \]

(12)

whose meaning is that there is a 99\% probability that the interval (12) covers the true unknown value of \( \frac{R_A}{\sigma_A} \).

For \( f_B \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle \), that gives the flux of active \( \nu_{\mu,\tau} \) averaged over the common Super-Kamiokande and SNO response function, relative to the SSM \( ^8B \nu_e \) flux, we find

\[ 0.17 < f_B \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle < 1.26 \quad (99\% \text{ CL}). \]

(13)
Hence, we can say that the averaged flux of $\nu_{\mu,\tau}$ on Earth is larger than 0.17 times the $^8$B $\nu_e$ flux predicted by the Standard Solar Model at 99% CL. This is an evidence in favor of relatively large $\nu_e \rightarrow \nu_{\mu,\tau}$ transitions if $f_B$ is not too large.

One could argue that it is possible to derive a more stringent lower limit for $f_B \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle$ by calculating a confidence belt without left edge, instead of the one in Figure 1 calculated in the Unified Approach. Such a procedure is not acceptable, because it would lead to undercoverage if not chosen a priori, independently from the data, as shown in Ref. [2] for the case of upper limits. The correct procedure is to choose a priori a method like the Unified Approach that gives always sensible results and apply it to the data, as we have done here. A priori one could have chosen another method, as those presented in Refs. [10, 11, 12], that may have even better properties than the Unified Approach [14, 15], but we have verified that the intervals [11]–[13] do not change significantly.

Unfortunately, we cannot derive a model independent lower limit for the averaged $\nu_e \rightarrow \nu_{\mu,\tau}$ probability $\langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle$, because $f_B$ could be large. However, from Figure 1 we can say that $R_A/\sigma_A^{\exp} > 0$ at 99.89% CL (see Eq. (11)), and hence

$$P_{\nu_e \rightarrow \nu_{\mu,\tau}} > 0 \text{ at } 99.89\% \text{ CL} \quad (14)$$

in the range of neutrino energies covered by the common SNO and Super-Kamiokande response function presented in Ref. [3].

On the other hand, it is interesting to note that the relations (1) and (2) allow to derive a model-independent lower limit for $f_B$, taking into account that

$$\langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle \leq 1 - \langle P_{\nu_e \rightarrow \nu_e} \rangle. \quad (15)$$

Using this inequality, from Eqs. (1) and (2) we obtain

$$f_B \geq \frac{\langle \sigma_{\nu_e} \rangle}{\langle \sigma_{\nu_{\mu,\tau}} \rangle} R_{SK} - \left( \frac{\langle \sigma_{\nu_e} \rangle}{\langle \sigma_{\nu_{\mu,\tau}} \rangle} - 1 \right) R_{SK} = f_{B,\text{min}}. \quad (16)$$

From Eqs. (3), (4) and (5), the experimental value of $f_{B,\text{min}}$ is

$$f_{B,\text{exp}} = 1.031 \pm 0.197. \quad (17)$$

Since the central value of $f_{B,\text{min}}$ is 5.2$\sigma$ away from zero, we can calculate the resulting 99% CL interval for $f_{B,\text{min}}$ using the Central Intervals method (see [3]), that gives the same result as the Unified Approach far from the physical boundary $f_{B,\text{min}} > 0$. Since in the Central Intervals method 99% CL corresponds to 2.58$\sigma$, we obtain the confidence interval

$$0.52 < f_{B,\text{min}} < 1.54 \quad (99\% \text{ CL}). \quad (18)$$

Therefore, we can conclude that the SNO and Super-Kamiokande data imply the model-independent lower limit

$$f_B > 0.52 \quad (99\% \text{ CL}). \quad (19)$$

This is a very interesting information for the physics of the Sun.

So far we have not considered the possible existence of exotic mechanisms that produce $\nu_e \rightarrow \bar{\nu}_{\mu,\tau}$ or $\nu_e \rightarrow \bar{\nu}_e$ transitions (in addition or alternative to $\nu_e \rightarrow \nu_{\mu,\tau}$ transitions), such as resonant spin-flavor precession of Majorana neutrinos [16, 17]. In this case, Eq. (2) must be replaced with

$$R_{SK} = f_B \langle P_{\nu_e \rightarrow \nu_e} \rangle + f_B \left[ \frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle + \frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \rightarrow \bar{\nu}_{\mu,\tau}} \rangle + \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \rightarrow \bar{\nu}_e} \rangle \right]. \quad (20)$$

and Eq. (5) with

$$R_A = f_B \left[ \frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \rightarrow \nu_{\mu,\tau}} \rangle + \frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \rightarrow \bar{\nu}_{\mu,\tau}} \rangle + \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} \langle P_{\nu_e \rightarrow \bar{\nu}_e} \rangle \right]. \quad (21)$$

---

3 We would like to thank a referee of the first version of this paper for pointing out the possibility of $\nu_e \rightarrow \bar{\nu}_{\mu,\tau}$ transitions of solar neutrinos.

4 In the case of Majorana neutrinos the right-handed states are conventionally called antineutrinos.
Using the $^8$B neutrino spectrum given in Ref. [18], the neutrino-electron elastic scattering cross section calculated in Ref. [19] taking into account radiative corrections, and the Super-Kamiokande energy resolution given in Ref. [20], we obtain the following values for the ratios of the averaged cross sections in the Super-Kamiokande experiment for the threshold energy $T_e = 8.60$ MeV:

$$\frac{\langle \sigma_{\nu_e} \rangle}{\langle \sigma_{\nu_e} \rangle} = 0.114, \quad \frac{\langle \sigma_{\bar{\nu}_e} \rangle}{\langle \sigma_{\nu_e} \rangle} = 0.120.$$  \hspace{1cm} (22)

Hence, we have the useful inequalities

$$\frac{\langle \sigma_{\nu_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle} < \frac{\langle \sigma_{\nu_e} \rangle}{\langle \sigma_{\nu_e} \rangle} < \frac{\langle \sigma_{\bar{\nu}_{\mu,\tau}} \rangle}{\langle \sigma_{\nu_e} \rangle}.$$  \hspace{1cm} (23)

The lower bound in Eq. (11) implies the existence of solar $\nu_e \to \nu_{\mu,\tau}$ or $\nu_e \to \bar{\nu}_{\mu,\tau}$ or $\nu_e \to \bar{\nu}_e$ transitions at 99.89% CL. The inequalities in Eq. (23) imply that the quantity on the right-hand side of Eq. (21) is limited in the interval (0.025, 0.19) at 99% CL. Using the inequalities (23), we obtain

$$0.17 < f_B \left[ (P_{\nu_e \to \nu_{\mu,\tau}}) + (P_{\nu_e \to \bar{\nu}_{\mu,\tau}}) + (P_{\nu_e \to \bar{\nu}_e}) \right] < 1.67 \quad (99\% \text{ CL}).$$  \hspace{1cm} (24)

Therefore, the averaged flux of $\nu_{\mu,\tau}$, $\nu_\tau$, $\bar{\nu}_{\mu,\tau}$, $\bar{\nu}_\tau$ and $\bar{\nu}_e$ on Earth is larger than 0.17 times the $^8$B $\nu_e$ flux predicted by the BP2000 Standard Solar Model at 99% CL.

Let us derive now the most general model-independent lower limit for $f_B$ (assuming only that the Super-Kamiokande and SNO events are produced by neutrinos or antineutrinos generated as $\nu_e$ from $^8$B decay in the Sun). Using the inequality

$$\langle P_{\nu_e \to \nu_{\mu,\tau}} \rangle + \langle P_{\nu_e \to \bar{\nu}_{\mu,\tau}} \rangle + \langle P_{\nu_e \to \bar{\nu}_e} \rangle \leq 1 - \langle P_{\nu_e \to \nu_e} \rangle$$  \hspace{1cm} (25)

and those in Eq. (23), from Eqs. (1) and (20) we obtain again the limit in Eq. (16). Therefore, Eq. (19) gives the most general model-independent lower limit for $f_B$ following from the SNO and Super-Kamiokande data.

In conclusion, we have considered the model independent relations (1), (2) [3, 6, 7] (and (1), (20)) and the rates measured in the SNO [1] and Super-Kamiokande [2] solar neutrino experiment in the framework of Frequentist Statistics. We have shown that the probability of $\nu_e \to \nu_{\mu,\tau}$ (and $\nu_e \to \bar{\nu}_{\mu,\tau}$, $\nu_e \to \bar{\nu}_e$) transitions is larger than zero at 99.89% CL in the range of neutrino energies covered by the common SNO and Super-Kamiokande response function. We have found that the flux of $\nu_{\mu,\tau}$ (and $\nu_{\mu,\bar{\tau}}$, $\bar{\nu}_{\mu,\tau}$) on Earth averaged over the common SNO and Super-Kamiokande response functions is larger than 0.17 times the $^8$B $\nu_e$ flux predicted by the BP2000 Standard Solar Model at 99% CL. We have derived a model-independent lower limit of 0.52 at 99% CL for the ratio $f_B$ of the $^8$B $\nu_e$ flux produced in the Sun and its value in the BP2000 Standard Solar Model [3].

\[\text{References}\]

[1] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. 87, 071301 (2001), nucl-ex/0106015.
[2] S. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 86, 5651 (2001), hep-ex/0103032.
[3] G. L. Fogli, E. Lisi, D. Montanino, and A. Palazzo (2001), hep-ph/0106247.
[4] S. M. Bilenky, C. Giunti, and W. Grimus, Prog. Part. Nucl. Phys. 43, 1 (1999), hep-ph/9812360.
[5] G. D’Agostini, CERN Yellow Report 99-03 (1999).
[6] F. L. Villante, G. Fiorentini, and E. Lisi, Phys. Rev. D59, 013006 (1999), hep-ph/9807360.
[7] G. L. Fogli, E. Lisi, A. Palazzo, and F. L. Villante, Phys. Rev. D63, 113016 (2001), hep-ph/0102288.
[8] J. N. Bahcall, M. Pinsonneault, and S. Basu, Astrophys. J. 555, 990 (2001), astro-ph/0010346.
[9] G. J. Feldman and R. D. Cousins, Phys. Rev. D57, 3873 (1998), physics/9711021.
[10] S. Ciampolillo, Nuovo Cim. A111, 1415 (1998).
[11] C. Giunti, Phys. Rev. D59, 053001 (1999), hep-ph/9808240.
[12] M. Mandelkern and J. Schultz, J. Math. Phys. 41, 5701 (2000), hep-ex/9910041.
[13] D. E. Groom et al., Eur. Phys. J. C15, 1 (2000), wWw page: http://pdg.lbl.gov.
[14] C. Giunti and M. Lavender, Int. J. Mod. Phys. C, in press (2001), hep-ex/0002020.
[15] C. Giunti and M. Lavender, Nucl. Instrum. Meth. A, in press (2001), hep-ex/0011069.
[16] C.-S. Lim and W. J. Marciano, Phys. Rev. D37, 1368 (1988).
[17] E. K. Akhmedov, Phys. Lett. B213, 64 (1988).
[18] J. N. Bahcall et al., Phys. Rev. C54, 411 (1996), nucl-th/9601044.
[19] J. N. Bahcall, M. Kamionkowski, and A. Sirlin, Phys. Rev. D51, 6146 (1995), astro-ph/9503003.
[20] M. Nakahata et al. (Super-Kamiokande), Nucl. Instrum. Meth. A421, 113 (1999), hep-ex/9807027.
FIG. 1: Confidence belts in the Unified Approach for a normal distribution with unit standard deviation. The regions between the solid, long-dashed, dotted and dash-dotted lines correspond, respectively, to 90% (1.64σ), 99% (2.58σ), 99.73% (3σ) and 99.89% (3.06σ) CL. The thick solid vertical line represent the measured value of $R_A / \sigma_A$ (Eq. (10)).