Interaction and ablation of fall-back disks in isolated neutron stars

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ABSTRACT

An analysis of ablation processes is made for a fall-back disk with inner and outer radii external to the neutron-star light cylinder. The calculated ablation rate leads, with certain other assumptions, to a simple expression relating the inner radius and mean mass per unit area of any long-lived fall-back disk. Expressions for the torque components generated by interaction with the pulsar wind are obtained. It is not impossible that these could be responsible for small observable variations in pulse shape and spin-down rate but they are unlikely to be the source of the periodic changes seen in several pulsars.

Key words: accretion, accretion disks - stars: neutron - pulsars: general

1 INTRODUCTION

A simple model for neutron-star disk formation from supernova fall-back was described nearly twenty years ago (Michel 1988). More recently, there has been a revival of interest in the subject in connection with topics such as planetary formation (Lin, Woosley & Bodenheimer 1991; Wolszczan & Frail 1992; Miller & Hamilton 2001), pulsar spin evolution (Menou, Perna & Hernquist 2001a; Blackman & Perna 2004), the anomalous X-ray pulsars (Chatterjee, Hernquist & Narayan 2000; Marsden et al 2001; Eksi & Alpar 2005; Eksi, Hernquist & Narayan 2005), and a common framework for the anomalous X-ray pulsars, soft gamma-ray repeaters and dim isolated thermal neutron stars (Alpar 2001).

Fall-back disks in isolated neutron stars may differ from the accretion disks (α-disks) of binary systems in a number of respects. It is possible for them to be at radii beyond the light cylinder radius $R_{LC}$, with temperatures observable in the infra-red resulting from heating by X-rays and by the pulsar wind rather than mass transfer and viscous evolution. A significant fraction of the mass may be in the form of dust grains. Computations of the optical and infra-red emission expected from fall-back disks have been made by a number of authors (Foster & Fischer 1996; Perna, Hernquist & Narayan 2000), and there have also been several experimental searches for the infra-red excess characteristic of dust grains (see Bryden et al 2006). In this way, upper limits of the order of $10^{-2} M_{\odot}$ have been obtained for disk masses in a number of radio pulsars (Phillips & Chandler 1994; Löhmer, Wolszczan & Wielebinski 2004).

The recent observation of an excess in the case of the anomalous X-ray pulsar 4U 0142+61 (Wang, Chakrabarty & Kaplan 2006) has been interpreted by these authors as evidence for an X-ray heated fall-back disk. The estimated mass of the disk is quite small, $M_d \sim 10^{-5} M_{\odot}$, and a factor favouring its observation is the high X-ray luminosity ($\sim 10^{36} \text{erg s}^{-1}$) of the central neutron star relative to the typical radio pulsar. Its estimated present inner radius is well outside the light cylinder, $r_i \approx 4.7 R_{LC}$. An alternative analysis by Ertan et al (2007) fits the complete infra-red and optical spectrum of 4U 0142+61 by the emission of a viscous gaseous disk with $r_i < R_{LC}$ and an extinction to that object identical with the estimate made independently by Durant & van Kerkwijk (2006a). This type of disk is much more complex than the one considered in the present paper and should properly be regarded as forming a boundary condition for the magnetosphere (see, for example, Cheng & Ruderman, 1991). The ablation processes considered in this paper would remain relevant at $r > R_{LC}$ for this type of disk but their use to define the inner radius $r_i$, as in Section 2.4, would not be possible.

Details of the fall-back disk formation are not described here. Initially, the disk must have been internally ionized and viscous, with mass and angular momentum transfer rates intrinsic to the formation process. We assume that the essential ideas of the evolutionary path described by Menou, Perna & Hernquist (2001b) are valid. The thermal ionization instability, in which free electron recombination causes a sudden decrease in opacity, occurs first at the outer disk radius but then propagates very rapidly inward. We accept the conclusion of Menou et al that a neutral, passive, gaseous disk is then formed, with no obvious source of internal viscosity. The power input to the disk, considered here in Sections 2.1 and 2.4, is not adequate to maintain temperatures

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above the thermal ionization instability temperature and so does not materially affect the onset and propagation of instability except, possibly, to delay it.

To summarize, the present paper assumes that the fallback disks with which it is concerned are passive, thin and have internal temperatures well below the ionization instability temperature. Hence, following Menou et al, there is negligible internal ionization and the viscosity of $\alpha$-disks is not present. We shall be concerned only with the interaction of the pulsar wind with the disk surface regions so that it will be possible to neglect the complex evolutionary dynamics of neutral dust and debris disks (see, for example, Frank, King & Raine 2002). There is certainly interaction between neutral dust grains but to a good approximation we can treat the disk as a set of annuli each having a radius $r$ and Kepler velocity $v_K$. Warping of the disk then occurs, in general, if torque components are $r$-dependent. It is assumed here that, in the general case, the disk plane at formation has a finite uniform tilt angle $\beta$, but that the disk angular momentum is more nearly parallel rather than antiparallel with the neutron-star spin. We consider only disks that are external to the light cylinder, $r_1 > R_{\text{LC}}$, and regard their effect on the pulsar magnetosphere as a partial termination of the pulsar wind rather than a boundary condition to be imposed on the solution for the magnetospheric fields. This limitation on $r_1$ means that the equivalent force on the disk derived from the Lense-Thirring effect (see, for example, Wilkins 1972) is negligible in comparison with those found in Section 3.1. The Robertson-Poynting effect can also be neglected. The reason is that the equivalent force on the disk depends on its Kepler velocity, $v_K \ll c$, but the torque components derived in Section 3.1 from the azimuthal part of the pulsar wind are independent of this factor.

The present paper is addressed principally to three problems. These are: (i) an analysis of disk ablation processes and estimates of the ablation rate and of the infra-red luminosity; (ii) calculation of the alignment and precessional torques acting between disk and neutron star; (iii) the question of whether or not disk alignment, counter-alignment or precession could give rise to time-variable phenomena that are observable over intervals of no more than several years.

The last of these has been prompted by the observation of periodic changes in timing residuals and in average-pulse characteristics (Stairs, Lyne & Shemar 2000; Shabanova, Lyne & Urama 2001; Haberl et al 2006) of several pulsars or isolated neutron stars that appear to be most simply explained by Eulerian precession (for a recent review of this interpretation, we refer to Link 2006). If the existence of Eulerian precession were established unambiguously, there would be far-reaching consequences for our present understanding of neutron-star internal structure. For this reason, we attempt in Section 4 to see if residues of fall-back disks could exist in these cases with inner radii small enough to give variability on the observed time-scales.

2 DISK IONIZATION AND ABLATION

2.1 Momentum densities in the vacuum solution

In order to study the interaction between a disk and the pulsar wind we shall use inertial frame cartesian coordinates in which the neutron-star spin angular velocity $\Omega$ is parallel with the $z$-axis. Where convenient, this frame is also represented by spherical polar coordinates $r, \theta, \phi$. The magnetic dipole moment is $\mu = B_z R^3$, where $R$ is the neutron star radius and $B_z$ the equatorial surface field. It is at an angle $\xi$ with $\Omega$. There has been much computational work on the aligned neutron-star magnetosphere, but in the general case with $\xi \neq 0$, the work of Spitkovsky (2006), based on relativistic force-free electrodynamics, seems to be the only published solution. Thus our present investigation is based on the Deutsch vacuum solution for the electromagnetic fields of a rotating neutron star in the form given in recent papers by Michel & Li (1999) and Eksi & Alpar (2005). We note that there appears to be some disagreement about a small number of near-field terms in this solution (see also Ferrari & Trussoni 1973; Good & Ng 1985) but these are not of primary concern for the torque calculations made here. Michel & Li also observe that the static radial electric field term, which remains significant at the light cylinder, is in principle undetermined because the total electric charge of the star and magnetosphere is an unknown quantity dependent on the details of formation. From the electric displacement $D$ and magnetic flux density $B$ given by this solution in the inertial frame at $r > R_{\text{LC}}$ it is simple to write down the spherical polar components of the momentum density,

$$p = \frac{1}{4\pi c} D \times B. \quad (1)$$

For the torque calculation described in Section 3.1, we require them averaged over the pulsar rotation period. They are:

$$\langle p_r \rangle = \frac{\mu^2 \Omega^4 \sin^2 \xi}{8\pi c^5 r^2} \left(1 + \cos^2 \theta \right), \quad (2)$$

$$\langle p_\theta \rangle = 0, \quad (3)$$

$$\langle p_\phi \rangle = \frac{\mu^2 \Omega^2 \cos^2 \xi \sin \theta}{6\pi c^5 r^2} + \frac{\mu^2 \Omega}{4\pi c^5 r^3} \left(1 + \left(\frac{r \Omega}{c}\right)^2 \right) \sin^2 \xi \sin \theta. \quad (4)$$

The first term in equation (4) is derived from the product of two static irrotational field components and hence gives a solenoidal contribution to $\langle p_\phi \rangle$. Taken at face value, this represents a purely azimuthal component of momentum density whose presence in the near field of an aligned rotating magnetic dipole appears not implausible. Integration to obtain the rate of outward transfer of angular momentum across a sphere of large radius $r$ gives the vacuum torque,

$$\langle \Gamma_v \rangle = cr^2 \int_{-1}^{1} d(\cos \theta) \int_0^{2\pi} d\phi (r \times p), \quad (5)$$

of which the $z$-component is the spin-down torque,

$$\langle \Gamma_{\nu z} \rangle = \frac{2\mu^2 \Omega^3}{3c^3} \sin^2 \xi, \quad (6)$$

with $\langle \Gamma_{\nu x} \rangle = \langle \Gamma_{\nu y} \rangle = 0$, as expected in the inertial frame. Naturally, identical results are obtained by integrating the time-averaged torque, formed directly from the Maxwell tensor, over the same surface. (In a frame of reference corotating with the star, the time-averaged components of $\Gamma_{\nu v}$ are all finite and $\Gamma_{\nu v} \propto (\Omega \times \mu) \times \mu$; see Davis & Goldstein 1970.
Thus the direction of $\Omega$ changes in this frame. Therefore, in general, it also changes relative to the dipole moment $\mu$ which may move in this frame during pulsar evolution in an uncertain manner.)

The relation between wind momentum density and torque expressed by equation (5) is no more than classical mechanics and remains valid, with $(T_c)$ replaced by the true torque $(T)$, whatever the degrees of freedom contributing to $p_\phi$. Thus $(p_\phi)$ is necessarily finite in a physical pulsar wind, in which the electromagnetic fields are loaded by a density of relativistic charged particles. However, it is unclear whether or not $(p_\phi)$ would remain zero in a real system: a finite value would not contribute to $(T)$, whose $x$ and $y$ components would necessarily be zero in the inertial frame. The effect of these terms on the interaction between disk and pulsar wind does not seem to have been considered previously and is significant in the case of thin dust and debris disks similar to those found in the major planets or in pre-main-sequence stars (see, for example, Beckwith et al 1990) which present a very small profile to $p_r$.

2.2 Composition of the pulsar wind

A pulsar luminosity, observed outside the light cylinder, can be broadly divided into a wind component $L_w$ and black-body radiation from the neutron-star surface in the form of an X-ray component $L_{bol}$. The wind luminosity, which can be estimated from $\Omega$ and the observed spin-down rate $\dot{\Omega}$, is further divided into a Poynting flux of electromagnetic fields and a relativistic particle component, $L_w = L_{cm} + L_p$. In previous work on disk ablation by the pulsar wind, Miller & Hamilton (2001) assumed $L_p$ to be the larger wind component, and of baryonic composition.

However, this view of wind composition appears to be at variance with theories of particle acceleration at $r > R_{LC}$ that are widely accepted to be at least qualitatively valid. In the corotating frame of reference, there may exist finite electric-field gaps in the open magnetic flux-line regions of the magnetosphere. The total current flow through these gaps is limited by the Goldreich-Julian charge density, $\sigma_{GJ} = -\Omega \cdot \mathbf{B}/2\pi c$, where $\mathbf{B}$ is the magnetic flux density. Thus the total rate of baryon loss estimated from the polar-cap area $\pi R^2 \Omega/c$ intersected by open magnetic flux lines and from the polar-cap Goldreich-Julian charge density can be at most of the order of

$$R_b = \frac{\mu \Omega^2}{ec},$$

(7)

where $\mu$ is the neutron-star magnetic dipole moment. But the size of the finite-field gaps is limited by intense electron-positron pair formation with the consequence that the Goldreich-Julian current density $\sigma_{GJ}$ must contain both ion and positron components. In polar-cap models, particles are typically accelerated to energies of the order of $10^3$ GeV by unit charge, though greater energies may be reached in outer-gap models. It is also widely accepted that, as a consequence of the gap-limitation process, large numbers of electron-positron pairs are formed in open magnetic flux-line regions external to the gaps. Thus the total rate of pair formation can be given as $R_{\text{pair}} = k R_b$, but even its order of magnitude, $\kappa \sim 10^{-3}$, is not well known. The energies of these pairs are several orders of magnitude lower than the typical gap energy of $10^3$ GeV. It is obvious that the particle luminosity, estimated in the region of the light cylinder from equation (7) and from the gap energy, must be $L_p \ll L_w$ in most cases. For example, from the ATNF Pulsar Catalogue parameters given for PSR 1828-11 (Manchester et al 2005), we find $R_b = 8 \times 10^{21}$ s$^{-1}$ giving an estimated $L_p \approx 1.3 \times 10^{32}$ erg s$^{-1}$, to be compared with $L_w = 3.6 \times 10^{34}$ erg s$^{-1}$. This is consistent with the usual assumption (see, for example, Melatos & Melrose 1996) that the Poynting flux luminosity $L_{cm}$ is the larger component of the wind at the light cylinder. We shall assume that this is maintained at disk radii.

2.3 Ablation processes

In this paper, we assume that the presence of a partially ionized disk at $r > R_{LC}$, well outside any possible Alfvén radius, has little effect on fields and particle fluxes at $r < R_{LC}$ and can be seen as a partial wind termination rather than a modified boundary condition. The sequence of processes in disk ablation is described in this Section. We show first that the thermal X-ray flux $L_{bol}$ ionizes the outer regions of the disk surface and so makes possible the conversion of the Poynting flux $L_{cm}$ to ion or proton kinetic energy. Ablation is then the result of several possible secondary processes, including neutron production in the interaction of the accelerated ions and protons with disk nuclei.

It is necessary to find the conditions under which a disk consisting of neutral hydrogen, atoms of mass $m_{A,Z}$, and dust grains of radius $a$ has significant ionization. We shall assume that the disk has small thickness $h$ and that it is internally neutral, with a temperature well below the ionization instability temperatures found by Menou, Perna & Hernquist (2001b) for metal-rich compositions. But the surface of a thin disk must have a finite temperature owing to its interaction with the particle component of the wind momentum density $p_\phi$. As a result, a diffuse low-density surface region is present which is exposed to the X-ray component $L_{bol}$, principally photons of average energy $\bar{E} \approx 2k_B T_s$ determined by the neutron-star surface temperature $T_s$. The gravitational restoring force on an atom, acting toward the plane of a thin disk, in terms of the Kepler angular frequency $\Omega_K$, is $m_{A,Z} \Omega_K^2 w$ at height $w$ above the plane of the disk. Thus the depth of the diffuse surface must be of the order of

$$\left(\frac{w_{A,Z}}{m_{A,Z} \Omega_K^2}\right)^{1/2} = \sqrt{\frac{k_B T_s}{m_{A,Z} \Omega_K^2}},$$

(8)

typically $\sim 10^7$ cm for hydrogen at $T = 10^3$ K and for $\Omega_K = 10^{-2}$ rad s$^{-1}$. The mean life of a neutral hydrogen atom in this region against photoelectric ionization is

$$\tau_{pe} = \frac{\lambda}{m_H} \frac{4\pi r^2 \bar{E}}{L_{bol}},$$

(9)

where $\lambda$ is the mass attenuation length for low-energy photons in hydrogen and $m_H$ is the hydrogen atom mass. For a typical radio pulsar, $\bar{E} = 10^6$ eV equivalent to $L_{bol} = 8 \times 10^{31}$ erg s$^{-1}$ for a neutron star radius $R = 10^6$ cm. The Particle Data Group (Yao et al 2006) give $\lambda = 1.0 \times 10^{-4}$ g cm$^{-2}$ so that even at a radius $r = 10^{11}$ cm, the mean life is very short, $\tau_{pe} = 15$ s. For higher atomic numbers $Z$, lifetimes against ionization are of the same order.

The extent of dust grain sublimation is uncertain. Dust
grains exposed to $L_{\text{bol}}$ have an estimated equilibrium temperature $T_{\text{bol}} = T_{\text{b}} \sqrt{R/2r} \approx 1300$ K, provided details of grain composition, emissivity and shape are neglected. Sublimation rates may be significant at this temperature which is also approximately equal to the K-band Wien’s displacement law temperature. But its dependence on $T_{\text{b}}$ and $r$ make it impossible to give any general statement. This estimate assumes a grain radius $a \geq 10^{-4}$ cm such that almost all incident photons interact. The mass stopping power for low energy electrons has been tabulated by Seltzer & Berger (1982). It is so large that there is no doubt that the energy of the emitted photo-electrons is contained within dust grains of these radii. Smaller grains, with incomplete retention of photo-electron energy, have lower radiative equilibrium temperatures. On the basis of the Dulong and Petit law, the thermal equilibrium temperature is reached within times short compared with the Kepler period of the disk.

The ionization time $\tau_{\text{io}}$ is also smaller than the Kepler period leading to the conclusion that, for interesting intervals of $L_{\text{bol}}$ and $r$, matter in the very diffuse surface regions of a disk has a high degree of ionization independent of the state of the dust component. Neutron star blackbody photons therefore have an important effect on the state of the disk but their energies, $E \sim 10^2$ eV, are several orders of magnitude too small to eject protons (a proton with $10^2$ eV/c momentum has negligible kinetic energy; see also Miller & Hamilton 2001).

Following the discussion of the particle component of $L_{w}$ in Section 2.2, we shall assume that the fields at the disk radius, though not necessarily at radii that are many orders of magnitude greater, satisfy the ideal magnetohydrodynamic condition $\mathbf{v} \times \mathbf{E} = -\mathbf{v}_0 \times \mathbf{B}$, where $\mathbf{v}_0$ is the particle velocity (see, for example, Melatos & Melrose 1996; for a review of pulsar wind nebulae, see Gaensler & Slane 2006). To the extent that this is true, the further conditions $E < B$ and $\mathbf{E} \cdot \mathbf{B} = 0$ are also satisfied. In this case, the qualitative details of particle acceleration as a result of the interaction of the Poynting flux in the wind with the diffuse ionized disk surface can be analyzed easily by making a Lorentz transformation of velocity

$$\mathbf{v}' = \frac{\mathbf{E} \times \mathbf{B}}{B^2}$$

from the inertial frame of Section 2.1 to a frame with fields $\mathbf{E}', \mathbf{B}'$ in which $\mathbf{E}' = 0$ (see, for example, Landau & Lifshitz 1962). In this frame, $\mathbf{B}'$ is parallel with $\mathbf{B}$ and of magnitude $B' = \sqrt{B^2 - E^2}$. A proton formed by photoelectric dissociation of neutral hydrogen has an initial velocity (neglecting Kepler and thermal velocities) in this frame of $-\mathbf{v}'$ and its subsequent orbit is a circle of radius $v' / \omega_{\text{B}}$ in a plane perpendicular to $\mathbf{B}'$, with angular frequency $\omega_{\text{B}} = ev' B'/cq'$ where $q'$ is its momentum in this frame. Thus its time-averaged velocity in the original inertial frame is simply $\mathbf{v}'$. Although $\mathbf{v}'$ and $\mathbf{v}_0$ are not, in general, exactly parallel, it is broadly correct to say that the proton is transported forward, with the existing particle component of $L_{w}$, into the body of the disk. As this process continues, the Poynting flux is necessarily converted to kinetic energy. The energy transfer is linearly dependent on rest mass and thus is almost entirely to protons or partially ionized atoms. The penetration depth is very roughly defined by equating $L_{\text{bol}} / 4\pi r^2 c$ with the particle pressure, as for the collisionless shock present in pulsar wind nebulae (Rees & Gunn 1974).

In this elementary analysis, the time-dependence of the $E$ and $B$ fields in the inertial frame has been ignored. This appears to be a reasonable approximation because, at disk radii $r \sim R_{\text{LC}}$, the proton $\omega_{\text{B}}$ is several orders of magnitude greater than $\Omega$. We emphasize that the diffuseness of the disk surface is the important factor in its interaction with the Poynting flux component of the pulsar wind.

Disk ablation, which we define as baryon loss, can occur principally through three processes. The initial assumption made here is that the most important is the production of neutrons in strong interactions between the accelerated protons or with heavier nuclei. The nuclear interaction cross-section for baryons is defined primarily by the nuclear radius. Thus the mass attenuation length is $\lambda_{\text{b}} \approx 100$ g cm$^{-2}$. A second source of loss is the possibility that, with the reflection of $E$ and $B$ fields that is neglected here, protons and ions may be transported outward and away from the surface. Finally, the secondary protons formed in relativistic proton interactions inside the disk can also acquire a momentum component parallel or antiparallel with $\mathbf{B}$ which allows them to move out of the disk in a time small compared with the neutron star rotation period. These processes are all independent of the state of the dust component in the disk. The order of magnitude ablation rates obtained from the first of them are very different from those of Miller & Hamilton (2001) which, apart from a solid angle factor, were simply equal to $L_{w}$ divided by the gravitational potential energy of a disk proton.

Neutron production in strong interactions in the body of the disk has characteristics similar to those found by Agosteo et al (2005) who have measured the neutron spectra given by the interaction of 40 GeV/c protons with a copper target. (That the nuclear charge $Z = 29$ exceeds the disk average is unimportant for this purpose.) There are two groups of neutrons. Qualitatively, forward-directed neutrons interact with disk nuclei, as do secondary protons and mesons, to produce a hadronic cascade with length scale determined by $\lambda_{\text{b}}$. The more numerous neutrons are from decay of target nuclei and are emitted isotropically with average energy $\sim 3$ MeV. These neutrons interacting with disk nuclei are either moderated by elastic scattering or undergo $(n, \gamma)$ capture. The photon component of the cascades, mostly from $\pi^0$ meson decay, can be a source of neutrons through excitation of the giant dipole state in nuclei, but this is unimportant compared with direct strong-interaction production. Provided the disk is optically thick, the photon energy passes to thermal degrees of freedom through Compton scattering and photoelectric absorption. Very qualitatively, we can assume that, in a high-energy cascade, protons of momentum less than 1 GeV/c do not interact further but come to rest after losing energy by ionization in accordance with the Bethe-Bloch formula (see Yao et al 2006). For these considerations, it does not matter whether the target nuclei in the body of the disk, where the $E$ and $B$ fields are much reduced by conversion of the Poynting flux to kinetic energy, are free or still bound as atoms in dust grains.

For disks that are optically thick for high energy protons, satisfying $\rho h \gg \lambda_{\text{b}}$, where $\rho$ is the matter density, diffusion of the low-energy group of neutrons backward to the surface is the important mechanism of ablation. Obviously,
this process is dependent on the angle $\chi$ between $p_\phi$ and the disk outward normal unit vector $n_\perp$. The neutron production rate grows approximately exponentially with depth, the scale length being $\sim \lambda_\psi \sin \chi$, but the probability of diffusion to the surface without capture decreases more rapidly. It appears that only the primary and a small number of secondary target nuclei in a cascade are effective sources of neutrons. On this basis, and the neutron production measurements of Agosteo et al, we shall adopt a number $N$, in the interval $1 < N < 10$, as the mean number of neutrons escaping the disk per proton accelerated by the incident Poynting flux as described above. We shall also assume a constant value $\mathcal{E}_p = 10$ GeV for the accelerated energy.

Neutron diffusion is more probable from disks that are not optically thick to high-energy protons, for example, those with $p \phi \sim 10 \lambda_\phi$. Thus the ablation rates of these can be higher, as noted by Miller & Hamilton (2001). There is also the interesting possibility in these instances that the net momentum transfer normal to the disk may be antiparallel with to the normal component of $(p_\phi)$. The arguments for this will be given in Section 3.3 with a brief discussion of some consequences that might be observable.

It must be obvious that the analysis of ablation processes given here is intended to be no more than qualitative in the extreme. In particular, there is no rigorous solution for the interaction of the $E$ and $B$ fields with the diffuse outer regions of the disk. One difficulty is that the value of $B$ at the light cylinder varies by four orders of magnitude for the specific neutron stars considered here. Thus the description of Poynting flux conversion to particle kinetic energy given here is more obviously valid for the smaller values of the orbit radii $r'/\omega_B$ than for the larger values present in long-period neutron stars such as 4U 0142+61. But these failings can be viewed less seriously when it is remembered that the properties of the physical pulsar wind, even in the absence of a disk, are not well known. The present analysis attempts no more than to show that, on interaction with the disk, much of the Poynting flux is irreversibly converted to proton kinetic energy and that the average proton momentum is roughly parallel with the Poynting vector.

### 2.4 The ablation rate and disk luminosity

We consider first the effect of the time-averaged azimuthal pulsar wind component $(p_\phi)$ on the disk. The coordinate system adopted is that defined in Section 2.1. and the disk is assumed initially plane, with uniform tilt angle $\beta$ and rectilinear line of nodes in the $xy$-plane at an angle $\gamma$ with the $x$-axis. For integration over the area of the disk it is convenient to transform from spherical polar coordinates $\theta, \phi$ to an azimuthal angle $\psi$, defined in the disk plane and with respect to the line of nodes. The angular relations satisfied by points on the disk are:

\[
\begin{align*}
\cos \chi &= -\cos(\phi - \gamma) \sin \beta, \\
\cos \theta &= \sin \beta \sin \psi, \\
\sin(\phi - \gamma) &= \frac{\cos \beta \sin \psi}{\sqrt{1 - \sin^2 \beta \sin^2 \psi}}, \\
\cos(\phi - \gamma) &= \frac{\cos \psi}{\sqrt{1 - \sin^2 \beta \sin^2 \psi}}.
\end{align*}
\]  

The power input per unit area of disk is $-c^2(p_\phi \cdot n_\perp)$. For a disk annulus of radius $r$ and width $dr$, the total power input to both sides is,

\[
\delta W = r \delta r \int_0^{2\pi} d\psi c^2 |(p_\phi \cdot n_\perp)|. 
\]  

It will be convenient to eliminate $\mu$ in favour of $L_w = \Omega(\Gamma_{\psi})$ using equations (2), (4) and (6). Given that

\[
\int_0^{2\pi} d\psi \sin \theta |\cos(\phi - \gamma)| = 4, 
\]

equation (12) can be re-expressed as,

\[
\delta W = r \delta r \frac{3}{2\pi \Omega^2 \sin^2 \xi} \left( \frac{2}{3p^2} \cos^2 \xi + \left( \frac{1}{r^3} + \frac{\Omega^2}{r^2 c^2} \right) \sin^2 \xi \right). 
\]

The first term on the right-hand side of this equation derives from the solenoidal component of $(p_\phi)$ which was mentioned in Section 2.1. But its singularity at $\xi = 0$ is clearly an artefact of the unphysical vacuum solution for the $\mathbf{E}$ and $\mathbf{B}$ fields and for disks external to the light cylinder with radii $r_i < r < r_o$, we need retain only the final term in equation (14). Integration gives the total estimated power input to the disk from the azimuthal wind component,

\[
W = \frac{3}{2\pi} R_{LC} L_w \sin \beta \left( \frac{1}{r_i} - \frac{1}{r_o} \right). 
\]

The rate of baryon loss per unit area of disk at radius $r$ is,

\[
B = \frac{3R_{LC} N L_w \sin \beta}{4\pi r^2 \mathcal{E}_p}. 
\]

Evaluation of equations (15) and (16) for the estimated disk parameters of 4U 0142+61 (Wang et al 2006) is of interest. The wind luminosity derived from the ATNF parameters (Manchester et al 2005) is $1.2 \times 10^{32}$ erg s$^{-1}$ and the inner and outer radii are $4.7R_{LC}$ and $16R_{LC}$, respectively. The total power input to the disk is then $W = 9 \times 10^{30}$ erg s$^{-1}$. At the inner radius, the power input per unit area is $5 \times 10^{30}$ erg cm$^{-2}$ s$^{-1}$ which is large in relation to the gravitational potential energy per unit area of disk divided by the $t_0 \sim 10^5$ yr lifetime of the neutron star, $GM\Sigma/t_0 r_i \sim 5 \times 10^6$ erg cm$^{-2}$ s$^{-1}$, where $M$ is the neutron star mass and $\Sigma \approx 1.6 \times 10^4$ g cm$^{-2}$ is the mean mass per unit area found from the disk mass and radii given by Wang et al. Thus the inner edge power input from the wind is large compared with any possible rate of thermal energy release by viscous evolution of the disk. Assuming re-radiation exclusively in the infra-red, this power input corresponds with a blackbody temperature of $T \lesssim 960$ K, which is lower than the temperature of 1200 K found by Wang et al. This is not unreasonable owing to the very large value of $L_{bol} \gg L_w$ for this neutron star. We have assumed the disk thickness to be small and uniform. But it is possible that it has some radial dependence, possibly of the form $h \propto r^{1/7}$ which is characteristic of point-source illumination of an $\alpha$-disk (see, for example, Pringle 1996), and that some of the power input to the disk is from $L_{bol}$ as in the model fit of Wang et al. The model adopted by these authors must include some assumption about the absolute thickness of the disk, in effect, the solid angle it subtends at the neutron star. This
does not appear to be stated, but it is also of interest that their fit involves a very large X-ray albedo, \( \eta_H = 0.97 \). Finally, we have to accept that non-disk contributions to the the infra-red luminosity may also be significant. It is possible that the recent observation of short time-scale downward fluctuations in the K-band intensity of this object (Durant & van Kerkwijk 2006b), not readily accommodated within the passive disk model, may be evidence for these.

Evaluation of equation (16) at the present value of \( \Omega \) gives an estimated baryon loss rate, at the inner radius, of \( 3 \times 10^{10} \) cm\(^{-2}\) s\(^{-1}\) for \( N = 10 \) and \( \mathcal{E}_p = 10 \) GeV, which is negligible in comparison with the mean disk density \( \Sigma \). This is an essential result because the ablation rate must have been considerably higher at earlier times following neutron-star formation. It is more interesting to integrate the ablation rate over the life of the star, assuming simple spin-down because its time-average vanishes. This may not be too unsatisfactory for torque estimates, as in Section 3.1, but it is obvious that the ablation rate must have been considerably higher at earlier times following neutron-star formation. It is more interesting to integrate the ablation rate over the life of the star, assuming simple spin-down because its time-average vanishes.

\[
\int_0^t \mathcal{B}dt = \frac{3cI}{4\pi\gamma^2} \left( \frac{N \sin \beta}{\mathcal{E}_p} \right) (\Omega_0 - \Omega),
\]

where \( I \) is the neutron star moment of inertia and \( \Omega_0 \) its spin angular frequency at formation. This assumes that, at earlier times, the disk extended inward to smaller radii but has since suffered ablation to its present value of \( r_i \). It provides an estimate of a combination of the unknown parameters \( \mathcal{E}_p \) and \( N \) in terms of quantities accessible to experimental measurement,

\[
\frac{N\Omega_0 \sin \beta}{\mathcal{E}_p} = \frac{4\pi^2r_i^7\Sigma}{3cIm_H}. \tag{18}
\]

The comparison for 4U 0142+61 gives a value for the right-hand side of equation (18) of \( 3.4 \times 10^{21} \) erg\(^{-1}\) s\(^{-1}\) or \( 5 \times 10^4 \) rad s\(^{-1}\) GeV\(^{-1}\). Insertion of the values we adopted in Section 2.3 for \( N \) and \( \mathcal{E}_p \) then leads to \( \Omega_0 \sin \beta = 5 \times 10^4 \) which is obviously about two orders of magnitude too large. But given the considerable uncertainties both in our parameter values and in those estimated by Wang et al., the discrepancy is not disturbing. If the disk parameters of Wang et al. are accepted, the comparison suggests smaller \( \mathcal{E}_p \) and larger \( N \) than the values adopted in Section 2.3. Possibly, the two processes of baryon loss listed but not considered there may be more important than we assumed. We have also neglected any increase or decrease in disk radii caused by angular momentum transfer from the neutron star via the torque component \( T_{\perp} \) given by equation (24) in Section 3.1.

Apart from the uncertainties in \( N \) and \( \mathcal{E}_p \), we emphasize that there are at least two principal sources of uncertainty in our calculation of \( \mathcal{W} \). Our treatment of the wind-disk interaction in Section 2.3 is elementary and does not give any estimate of the reflection of wind energy. There is also the uncertainty in our assumption that \( p_\phi \) makes no contribution because its time-average vanishes. This may not be too unsatisfactory for torque estimates, as in Section 3.1, but may nevertheless introduce an error in calculating the total power input.

\section{3 DISK-PULSAR TORQUES}

\subsection{3.1 Calculation of the torque}

Calculation of the torque components acting on the disk follows that for \( \delta \mathcal{W} \) given by equations (11) and (12). It will be convenient consider an annulus of width \( \delta r \) and to resolve the torque acting on it in the plane of the disk into components \( \delta \Gamma_p \) and \( \delta \Gamma_a \) that are, respectively, parallel with and perpendicular to the line of nodes. The components are,

\[
\delta \Gamma_p = -cr^2\delta r \int_0^{2\pi} d\psi \sin \psi (p_\phi) |\cos \chi| \cos \chi + \frac{2}{3} \text{sgn}(\cos \chi), \tag{19}
\]

and

\[
\delta \Gamma_a = cr^2\delta r \int_0^{2\pi} d\psi \cos \psi (p_\phi) |\cos \chi| \cos \chi + \frac{2}{3} \text{sgn}(\cos \chi). \tag{20}
\]

formed from the momentum flux perpendicular to the plane (neglecting any wind-energy albedo) and with the assumption that the disk is optically thick and re-radiates photons as a Lambertian surface. Use of the time-averaged azimuthal momentum density in these expressions is valid because the neutron star rotation period is several orders of magnitude smaller than the Kepler period of the disk. Thus it is permissible to treat the annulus, though not the whole disk, as a rigid body. Evaluation of the first of these integrals gives a zero disk precession torque, \( \delta \Gamma_p = 0 \). This cancellation of contributions from the four quadrants of \( \psi \) is a consequence of the vacuum-field form of \( (p_\phi) \), as given by equation (4), and it is by no means obvious that it would hold for a physical pulsar wind. Integration of equation (20) gives a finite torque which acts to align the angular momentum \( \delta \mathcal{L}_d \) of the disk annulus with the neutron-star spin,

\[
\delta \Gamma_a = cr^2\delta r \int_0^{2\pi} d\psi (p_\phi) |\cos \chi| \cos \chi + \frac{2}{3} \text{sgn}(\cos \chi).
\]

in which the integral,

\[
I_a = \int_0^{\pi/2} d\psi \sin^2 \psi, \tag{22}
\]

is a slowly varying function of \( \beta \), \( 0.66 < I_a < 0.79 \).

The torque component perpendicular to the plane of the disk is,

\[
\delta \Gamma_{\perp} = cr^2\delta r \int_0^{2\pi} d\psi (p_\phi) |\cos \chi| (\cos \beta \sin \theta + \sin \beta \cos \theta \sin(\phi - \gamma)). \tag{23}
\]

Evaluation of the angular integral using equations (11) gives,

\[
\delta \Gamma_{\perp} = cr^2\delta r \left( \frac{3cL_w}{8\pi\Omega^2 \sin^2 \xi} \right) \left( 2\beta \cos \beta (1 + \sin^2 \beta) \right) \cos \beta \sin \theta + \cos \beta \cos \theta \sin(\phi - \gamma). \tag{24}
\]

We shall assume that the angular momentum of a fall-back
disk is likely to be more nearly parallel, than anti-parallel, with the neutron star spin, in which case the interaction increases $\delta L_d$ and moves the disk outward to larger radii.

### 3.2 The effect of torques on the neutron star

The motivation for considering this aspect of the interaction is that the angular momentum $\mathbf{L}$ of the neutron star can be several orders of magnitude smaller than that of the disk. In the case of 4U 0142+61, for example, the disk parameters estimated by Wang et al, including its very small mass $M_d \sim 10^{-5} M_\odot$, give $L_d = 2.0 \times 10^{47}$ g cm$^2$ s$^{-1}$ which is between one and two orders of magnitude larger than $L$. Consequently, the changes in direction of both $\mathbf{L}$ and $\mathbf{L}_d$ can be significant. Thus the Euler equation for the neutron star angular momentum in the absence of a disk, $\dot{\mathbf{L}} = \mathbf{\Gamma}$, is replaced by,

$$\dot{\mathbf{L}} + \dot{\mathbf{L}}_d = (\mathbf{\Gamma} - \mathbf{\Gamma}_d) + (\mathbf{\Gamma}_d), \quad (25)$$

for the whole system, in which $\mathbf{\Gamma}_d = \mathbf{\Gamma}_a + \mathbf{\Gamma}_p + \mathbf{\Gamma}_\perp$ is the integrated torque acting on the disk, and $\mathbf{\Gamma}$ is the modified non-disk torque. For hypothetical disks with $r_i \gg R_{L\perp}$, the terms in equations (21) and (24) that are significant contributors to the torque would be those derived from the long-range component of $\mathbf{p}_d$ in equation (4). But the vacuum torque ($\mathbf{\Gamma}_v$) can also be obtained from this term, as could the true ($\mathbf{\Gamma}$) if the true $\mathbf{p}_d$ were known (see equation 5). This suggests, but does not prove, that the first, bracketed, term on the right-hand side of equation (25) approaches ($\mathbf{\Gamma}$) in this limit, the disk torque being cancelled by terms in ($\mathbf{\Gamma}$). However, there is no reason to suppose that this cancellation is exact at disk radii $r \sim R_{L\perp}$ of physical interest where the near-field terms in equation (4), (21) and (24) are not negligible.

The terms in $\mathbf{\Gamma}_d$, including any non-zero $\mathbf{\Gamma}_p$, are all finite in the inertial frame when averaged over the neutron star period. Thus the direction of $\mathbf{L}$ must change during the evolution of the system, but at a rate that is unlikely to be of observational interest. To confirm that this is so, we can consider PSR 1828-11 as an example and suppose that $\mathbf{L}$ is at a small angle $\alpha$ with the total angular momentum $\mathbf{L} + \mathbf{L}_d$ and precesses about it with angular frequency $\Omega_p$. The torque required is $\mathbf{\Gamma}_d \sim L_d \Omega_p = 2 \times 10^{47}$ g cm$^2$ s$^{-2}$ for a 500 day precession with amplitude $10^{-2}$, several orders of magnitude greater than the spin-down torque which is $L_w/\Omega = 2.3 \times 10^{33}$ g cm$^2$ s$^{-2}$ for this pulsar.

### 3.3 Alignment and precession of the disk

The initial evolution of a very thin disk with small $\Sigma$ and $L_d \ll L$ can be obtained directly from equation (25), to the extent that we can neglect the effect of changes in its geometrical form on calculations of $\mathbf{\Gamma}_d$. The solution is,

$$\dot{L}_d = \Gamma_\perp$$

$$L_d \dot{\beta} = -\Gamma_a$$

$$L_d \sin \beta \dot{\gamma} = \Gamma_p. \quad (26)$$

From a hypothetical initial state of a plane disk with $\beta \neq 0$, the alignment torque $\mathbf{\Gamma}_a$ reduces $\beta$ at a radius-dependent initial rate given by

$$\dot{\beta} = -\frac{\delta \Gamma_a}{\delta L_d} = -\frac{1}{2\pi \Sigma (G M)^{1/2} r^{5/2}} \left( \frac{3L_w}{8\pi \Omega} \right) \left( 4\lambda_a \sin^2 \beta + \frac{2}{3} \pi \sin \beta \right), \quad (27)$$

in which only the long-range term in equation (21) has been retained. The disk mass per unit area $\Sigma$ in equation (27) is the local value, not the mean $\Sigma$. There is initially no precession for the vacuum-field pulsar wind assumed in Section 3.1. But the $r$-dependent alignment rate destroys the planar form of the disk and therefore introduces a further torque contribution derived from the radial momentum flux $c(\mathbf{p}_r) + L_{bol}/4\pi r^2 c$. This is the torque concerned in the Pringle instability (Pringle 1996; see also Pettersson 1977, Frank et al 2002). Initially, it is a contribution to the precessional component $\mathbf{\Gamma}_p$, but further evolution of the shape and motion of the disk is more complex. It is worth adding that any finite torque component derived from equation (19) as the consequence of a momentum density $\langle \mathbf{p}_d \rangle \neq 0$ in a physical wind, rather than the vacuum field solution used in Section 2.1, would also induce precession.

In relation to disk alignment and precession, the interesting question is whether or not these processes are sufficiently rapid to produce time-varying phenomena that are observable within periods of no more than several years. For phenomena involving the body of a $10^{-3} M_\odot$ disk, the answer must be in the negative because $\mathbf{\Gamma}_d$ is perhaps an order of magnitude smaller than the spin-down torque $\mathbf{\Gamma}_v$ yet, as we have noted in Section 3.2, the disk angular momentum $L_d$ is at least of the same order as $L$.

But the inner edge of a disk must have small local values of $\Sigma$ owing to its continual ablation by baryon loss, given by equation (16). In principle, the alignment rate given by equation (27) for these radii could become observably fast. There is also the likelihood, mentioned briefly in the penultimate paragraph of Section 2.3, that for some intervals of small $\Sigma$, the net momentum transfer normal to the disk is antiparallel with the normal component of $\langle \mathbf{p}_d \rangle$.

We can see how this arises by referring back to both the description of hadronic cascade development in Section 2.3 and the torque calculations of Section 3.1. The integrands in equations (19) and (20) both contain the rate of momentum transfer normal to the disk and per unit area. Both expressions are based on two assumptions; that the disk radiates as a Lambertian surface and that it does so from the surface of incidence with which the wind interacts. The second assumption is certainly valid if the disk is optically thick for the photon spectrum emitted and also has sufficient thickness, at least of the order of $\Sigma \sim 10^3 \lambda_0 |\cos \chi|$, that the hadronic cascades terminate much nearer the surface of incidence rather than the far surface. But the transfer of energy in a cascade from the primary particle to thermal degrees of freedom and to low-energy photons of perhaps $10^2 - 10^3$ eV occurs predominantly in the late stages of its development. Thus for $\Sigma \sim 10\lambda_0 |\cos \chi|$, the wind heats the far surface of the disk, not the surface of incidence. The effect of this is to change the sign of the $s g n (\cos \chi)$ terms in equations (19) and (20) and, unless $\chi$ is relatively small, to change the sign of $\delta \Gamma_a$, transforming it from an alignment to a counter-alignment torque. There is a second factor having the same effect. The production of low-energy neutrons re-
ferred to in Section 2.3 also occurs mainly in the late stages of cascade development. For these \( \Sigma \), the baryon loss rate per unit area much exceeds that given by equation (16) and is mainly through the far surface of the disk. This is significant because massive low-energy particles are effective carriers of momentum.

Validation of these qualitative arguments would require very extensive Monte Carlo calculations of cascade development. But although these have not been made, we suggest that there is a strong likelihood that a counter-alignment torque acts on the inner edge of a disk. Again, we shall not attempt to consider the complex and detailed evolution of a disk under these torque components, but restrict our discussion to the question of whether or not they could cause, in principle, time-varying phenomena that are observable.

With a sign reversal of \( \delta \Gamma_a \), equation (27) gives a time constant \( t_a \) for exponential growth of \( \beta \).

\[
\frac{1}{t_a} \approx \frac{L_w}{8 \pi \Sigma (GM)^{1/2} r_i^{3/2}}. \tag{28}
\]

Evaluation for 4U 0142+61, with the parameters of Wang et al and a local density \( \Sigma = 300 \) g cm\(^{-2} \), gives \( t_a \approx 1.1 \times 10^{13} \) s, which is too long to be of observable interest. But the value of \( L_w \) is small for this neutron star and the inner radius of the disk is large. Under the assumption that, for example, PSR 1828-11 has a disk external to its light cylinder we find, given the ATNF parameters for this pulsar, that growth times could be quite short, \( t_a \approx 7 \times 10^6 (r_i/R_{LC})^{3/2} \) s.

4 CONCLUSIONS

We have found that the azimuthal component of the pulsar wind momentum-density exerts an \( r \)-dependent torque on a thin dust or debris disk external to the light cylinder. Both the Poynting and relativistic particle components of the wind are responsible for ablation of the disk. For disks that are optically thick to relativistic protons, the power input, which we assume determines the infra-red luminosity, is given by equation (15). This result neglects any albedo, which we assume determines the infra-red luminosity, \( \nu \) is mainly through the far surface of the disk. This is significant because massive low-energy particles are effective carriers of momentum.

It has not been possible to answer with any certainty the question of whether or not the torque components calculated here could be responsible for any observable time-varying phenomena. PSR 1828-11 is an interesting case of a pulsar that appears to have periodic variations in timing residuals and in pulse shape consistent with small-amplitude Eulerian precession of its spin axis relative to coordinates fixed in the star (Stairs, Lyne & Shemar 2000). If the inferred 500d period were equated with the growth time \( t_a \) estimated at the end of Section 3.3 for counter-alignment of the inner edge of a disk, the required radius would be \( r_i = 2.1 R_{LC} \), but the mean disk density would have to \( \Sigma \approx 2 \times 10^9 \) g cm\(^{-2} \). An almost identical value would be required in the radio pulsar, PSR B1642-03, which also appears to have periodic timing residuals (Shabanova, Lyne & Urama 2001). This might be thought a high density for a nominally thin disk but, assuming, for example, an outer radius \( r_o = 2r_i \), the mass is only \( M_d = 1.5 \times 10^{-4} M_\odot \), well below observed upper limits. In principle, movement of the ionized inner edge may have the potential to produce small changes in the observed radio-frequency emission pulse shape and in the pulsar spin-down rate. But it is not obvious why the changes should be periodic, as observed.

The more recent case of the isolated neutron star RX J0720.4-3125 in which the inferred blackbody temperature and certain other pulse characteristics have been interpreted as varying sinusoidally is rather different (Haberl et al. 2006; but see also van Kerkwijk et al. 2007 for an alternative analysis). This neutron star with a period of 8.39 s and a high magnetic field has some properties in common with 4U 0142+61. The small luminosity, \( L_w = 4.7 \times 10^{30} \) erg s\(^{-1} \) (Manchester et al. 2005), is such that, even for \( r_i = R_{LC} \), the variation-time estimate obtained from equation (28) is several orders of magnitude too long to be of interest.

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