Rapid Secular Radial Mass Accretion in N-Body Simulated Galaxy Disks

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ABSTRACT

Recent studies have indicated that large mass radial inflow rates are possible in observed galaxies with strong density wave patterns. Yet, numerical simulations have generally failed to account for such high rates. Here it is shown that the reason for the discrepancy is the treatment of “softening,” the artificial parameter inserted by numerical simulators into the formula for gravitational potential, to control the magnitude of relaxation in simulations with small numbers of particles compared to a real galaxy. Excess softening reduces the collective effects underlying the significant secular evolution inferred for physical galaxies. Less softening, coupled with an increase in number of particles, allow the N-body simulations to reveal significant morphological transformation of galaxies over a Hubble time.

Key words: galaxies: evolution; galaxies: structure

1 INTRODUCTION

The prospect of significant transformation of the Hubble type of a galaxy during its lifetime has important implications on our understanding of the origins of the Hubble sequence and on broader issues in cosmology. Despite the coming of age of this field of research, there is as of now no consensus on how important this process is, and through what dynamical mechanism this transformation is mainly achieved (see the review of Kormendy & Kennicutt 2004).

Zhang (1996, 1998, 1999) proposed and demonstrated a collective dissipation process in galaxies that could serve as the underlying dynamical mechanism for secular evolution of galaxies along the entire Hubble sequence, by incorporating the stellar component in the mass inflow process rather than just the interstellar medium as proposed in the pseudo-bulge formation scenario (Kormendy 1979). Despite the confirmation of the theoretically derived mass flow rates in the accompanying N-body simulations, which established the viability of the analytical approach, the simulated mass flow rates were small, far from being adequate to lead to significant morphological transformation of a galaxy within a Hubble time. This small mass flow rate was attributed to the small amplitudes of the spiral patterns formed in these simulations, with the state-of-the-art in simulating spontaneously formed modes far from being adequate to produce the extremely nonlinear density wave modes observed in physical galaxies.

In order to circumvent this difficulty, Zhang & Buta (2007; 2012) used near-and-mid-infrared images of galaxies, coupled with the analytical rate equations derived in Zhang (1996, 1998, 1999) to calculate the torques and mass flow rates in these physical galaxies, and found that mass flow rates from a few $M_\odot$ per year to over one hundred $M_\odot$ per year were obtained, depending on the Hubble types of galaxies and their interacting environment, which together determined the density wave amplitudes and pitch angles in these galaxies. These mass flow rates for real galaxies far exceed those obtained from the past N-body simulations, partly as a result of the fact that the mass flow rate is proportional to the effective wave amplitude squared (Zhang 1998), and the amplitudes in physical galaxies are oftentimes a factor of 3-8 times higher than obtained in N-body simulations, implying a difference in evolution rate of a factor of 10 - 100 between physical and previously-simulated galaxies.

Despite the confirmation of the importance of secular evolution process in physical galaxies in works such as Zhang & Buta (2007, 2012), one could not help but wonder what exactly were the factors that have prohibited the simulated disk galaxies from obtaining the level of mass flow rates and the magnitude of wave amplitudes in physical galaxies. It is to this question that the result of the current Letter partly addresses. Incidentally, the main discovery of this work was made in a totally serendipitous fashion – i.e. the result was stumbled upon while the author was searching in a different direction for the cures of the low mass flow rates in N-body simulations[1]. The lesson learned is that one has to be cautious in taking the results of N-body simulations literally.
without questioning sacred "rules of thumb" and accepted wisdom, especially in circumstances where new physical processes are encountered – in our situation, that is when the collective effects become important.

2 ROLES OF SOFTENING: THEORETICAL AND NUMERICAL

To come to the core result of this Letter immediately: the main culprit to the low mass accretion rate in N-body simulations of disk galaxies turned out to be the improper choice of softening parameter used to control relaxation effects in the simulation of these nominally collisionless systems.

The softening parameter is generally defined as follows: The gravitational potential in an N-body system is calculated as

$$\Phi = \frac{-Gm}{\sqrt{r^2 + a_{soft}^2}} \quad (1)$$

where $a_{soft}$ is the so-called softening parameter. This parameter is introduced into the N-body simulations due to the fact that the numerical setups usually have significant less number of particles than the physical systems they try to emulate. A finite softening parameter reduces the amount of unrealistic close encounters which would be more often for a small N system, and thus reduces the level of artificial relaxation in these small N systems. The hope is that a collisionless configuration would then result from a well-matched N and $a_{soft}$. In particle-mesh based N-body approaches, the grid size serves as an additional softening parameter that acts in consort with $a_{soft}$ in controlling relaxation (see Sellwood 1987 and the references therein).

However, with the realization that galaxies which possess large scale density wave modes are governed by collective dissipation processes (Zhang 1996), one soon faces the fact that it is impossible to completely avoid collision-like behavior, no matter how large the particle numbers are used: After all, the collective effects in these systems depend on the near-collision or scattering of particles in the global instabilities to set up long-range correlations to achieve self-organization and to induce secular evolution. In some sense, these galactic systems are in a forced relaxation configuration, with the forcing accomplished by the density wave collisionless shocks, which force the effective Q in the spiral arms to be less than one and lead to inter-particle interactions and correlations. Thus, in hindsight, it is also obvious that excess softening reduces the very inter-particle interaction that is the backbone support of collective effects. In essence, overly-softened gravity is modified gravity, and thus is modified Newtonian interaction (not to be confused with the Modified Newtonian Dynamics proposed to account for the flat rotation curves). The system one simulates with softened gravity is no longer exactly the same system governed by Newton’s gravitational law, but rather a more sluggishly-interacting system. This might not be a serious concern if one’s interest is in obtaining a modal morphology that mimics the observed galaxy morphology (as shown in the work of Donner & Thomasson 1994, hereafter DT94, which is the forerunner of long-lasting N-body spiral modes), but as we will show here, it is detrimental to determining the realistic secular mass flow rates that are relevant to physical galaxies.

In Fig. 1 we show the result of a set of test runs of N-body simulations using the basic state first explored in DT94, and subsequently used for the study of collective effects and secular evolution in Zhang (1996,1998,1999). The simulations are done in a 2D configuration, using a particle-mesh approach on a polar grid, with an active disk which has a modified exponential surface density profile, as well as a rigid bulge and a rigid halo. The grid size is 220 by 256 in the radial and azimuthal direction, respectively. The mass in the active disk, the inert halo and inert bulge are 0.4, 0.5, 0.1, respectively, in the normalized unit, similar to that used in Zhang (1998). The scaling of the time unit is such that 1256 time steps represent one rotation period at r=20. Both the particle numbers and the time resolution used are higher than that in Zhang (1998). These grid and time step resolutions are used for the rest of the simulations presented in this Letter as well, only particle numbers and the softening parameters are changed and will be noted in each case.

The curves in Fig.1 are for the evolution of enclosed mass within the central $r = 3.5$ region (in the scale of this disk which has corotation radius around 20-25 in normalized unit, this corresponds to the central bulge region). The three curves, from bottom to top, correspond to softening parameter value of 1.5 (as used in DT94 and Zhang 1998), 0.75, and 0.25 in the unit of the smallest grid size of 1. As we can see, the choice of softening of 1.5 produced a barely noticeable mass inflow (to really see the small amount of mass inflow for this choice of softening, one is referred to Figure 10 of Zhang 1998 which zoomed in on the vertical scale, even though that was for the enclosed mass within central $r = 13.5$). For that particular choice of softening, which is of standard usage in the N-body simulation community as being adequate in suppressing unwanted relaxation effects as well as in matching the grid softening, the central mass growth of only 6.5% was observed over about 25 pattern rotation periods (Zhang 1998), which is obviously quite inadequate in transforming the Hubble type of a galaxy. Gradually reducing softening is seen to systematically lift up the mass inflow rate. For the range of softening tried by the current author (1.5 - 0.1), the trend of increasing mass inflow with reduced softening continues. It is possible that realistic galaxies, which have zero softening in the form of gravitational law but do have much larger numbers of particles, as well as the finite thickness of the disk to keep relaxation in check, the mass flow rates much exceed those we have seen in these recent simulations are possible – as we indeed observed in the mass flow rates calculated for the sample in Zhang & Buta (2007, 2012).
3 A NEW SET OF N-BODY SIMULATIONS WITH SMALL SOFTENING:
ACCELERATED BULGE BUILDING

The price one pays for reduced softening in small-N simulations is the increase in relaxation rate. To compensate, one needs to use a correspondingly increased number of particles, for otherwise the collision-induced heating will swamp the density wave activity and reduce the wave amplitude. How much the particle number needs to be increased for a given choice of softening can be found empirically (recalling the fact that the traditional rules of thumb in cases with collective effects may not always apply, since we do not want to kill the right kind of collisional relaxation due to collective effects).

In what follows, we show a set of simulations with changing particle numbers while holding the softening parameter $a_{soft} = 0.1$. Fig. 2 shows the morphological evolution of a spontaneously formed spiral-bar mode for one of these runs using 20 million particles. Note that even though the duration of run for this mode is similar to that used in Zhang (1998) (note due to the different temporal resolution the 400 step here corresponds to 100 step in the older simulation), the modal morphology here appears to have changed much more that that seen in Zhang (1998), i.e. the pattern in the older simulation retained the spiral morphology throughout, whereas the pattern in the current simulation evolved from a spiral-like morphology to a more bar-like morphology. This is natural to expect, since the much-higher radial mass accretion rate in the current simulation resulted in a significant increase in central mass concentration at the end of the run, this further means that the basic state is change so much that the mode it supports needs also to change correspondingly from that of spiral to that of bar with possibly nested inner spiral (note that a heavy bar is the natural modal shape for a centrally-heavy disk - the observed early-types disks certainly contain many massive bars). The pattern speed, however, appears to be little affected by the change in morphology, as can be seen later in the clean concentration of the power spectrum around a single pattern speed throughout the duration of the run.

In Fig. 3 we present the enclosed mass within the central $r=3.5$ radius, with the different runs having a constant $a_{soft} = 0.1$ but varying the number of particle used. For the smaller particle number runs, the rapid mass inflow is seen to saturate at an earlier time step, which corresponds to the time when spiral activity is damped by heating due to the insufficient number of particles for this extremely small softening, especially in the outer disk region where the surface density is low and the particle numbers are small. For large particle runs, the rapid mass inflow is seen to remain at a constant rate all the way until the end of the run, which corresponds to 25 rotations.

Furthermore, we note that the mass inflow rate observed here corresponds to about 60% increase in enclosed mass within $r=3.5$ (the bulge region) over 10 rotation periods, or roughly 1/5 of a Hubble type. This level of mass accretion is more than sufficient to transform the Hubble type of a galaxy by several stages in a Hubble time. Plus, the size of the bar is roughly twice that of the spiral arms.

In Fig. 3 the disk surface density at the beginning of the run, and at the end of the four runs with different numbers of particles (as used in Fig 3 but with different line style due to the need to represent the “before” surface density) are presented. It can be seen that the large inflow of the large-particle-number runs indeed builds up a more substantial bulge. However, the 20 million particle run in fact produced more substantial bulge-building than the 40 million particle run. This is consistent with the trend observed
in Fig. 3 where it is seen that the 40 million particle run has somewhat reduced mass inflow rate near the end of the run. This may be related to the emergence of different kinds of local instabilities in the different particle runs (a small knot of globular-cluster-like morphology can be seen in the outer disk of the second and third frame of Fig. 2). This kind of cluster-forming activity in association with density wave modal activity may be a reflection of the actual process happening in physical galaxies. By step 32000 (not shown in Fig. 2), the 20 million particle run has developed two prominent dumb-bell like structures at the end of the massive bar which might account for the increased mass inflow for this run.

In Fig. 5, the $m=2$ power spectrum together with the $\Omega$, $\Omega \pm \kappa/2$ curves are presented for two runs with two different numbers of particles. These contour plots clearly show a single dominant mode throughout the time interval of the runs. These are the cleanest power spectra (albeit obtained using potential $m=2$ components rather than the more conventional surface density) yet observed for this particular mode, compared to those runs with larger softening and smaller number of particles (note that due to higher temporal resolution by a factor of 4 compared to that used in Zhang 1998, the power spectrum should be multiplied by the same factor of 4 to compare with older results. After taking this into account, the pattern speed still appears to be slightly higher than before – even the 40 million run appears to have a slightly higher pattern speed than the 20 million particle run).

In Fig. 5, we plot the mass inflow (positive part of the curve) and outflow (negative part of the curve) versus radius at time step 8000. This mass flow rate curve was calculated based on the volume torque prescription given in Zhang (1996, 1998). The major transition from inflow to outflow is close to $r=20$ for the inner disk, which is close to (but not exactly the same, due probably to the fast evolving nature of the surface density) the corotation radius predicted from pattern speed (Fig. 5). Note that the larger number-of-particle case the power spectrum predicts a slightly smaller corotation radius than the smaller number-of-particle run. The level of peak accretion, $2 - 4 \times 10^{-7}$ per time step for the inner disk region, matches quite well the average accretion rate shown from the slope of the Fig. 3 20 million particle curve, which corresponds to $2 \times 10^{-7}$ per time step, within the central $r=3.5$. It should be noted that the small peaks and valleys of the mass flow curve in the inner disk do change with time and in general the peaks move inward with time.

Thus this new set of simulations show that the broad modal nature of the pattern as well as the predictions of the collective dissipation formalism persisted even with the drastic reduction in softening length. The dominant mode morphology does evolve a lot faster in appearance, which is to be expected in a fast changing basic state.

Finally we note that with the drastically-reduced $a_{soft}$ in the equation for the N-body gravitational potential in our new set of simulations, the corresponding grid softening has not been much reduced (the newer grid is only about a fac-

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**Figure 3.** Enclosed mass within central $r=3.5$ of the N-body disk for runs with different numbers of particles and $a_{soft} = 0.1$. Solid: 1 million particles. Dashed: 10 million particles. Dotted: 20 million particles. Dash-Dotted: 40 million particles.

**Figure 4.** Basic state surface densities at the beginning (solid), and the end of four N-Body runs with different number of Particles. Dashed: end surface density for 1 million particles. Dotted: end surface density for 10 million particles. Dash-and-Single-Dotted: end surface density for 20 million particles. Dash-and-Triple-Dotted: end surface density for 40 million particles. $a_{soft} = 0.1$

**Figure 5.** Power spectra from $m=2$ Potential at different radii during time interval step=0 and step=32768. Top: 20 million particle run. Bottom: 40 million particle run. The solid line indicates galactic rotation speed $\Omega$, the two dashed lines are $\Omega \pm \kappa/2$. 
tor of 4 finer in linear resolution than used in Zhang 1998, whereas the softening is reduced by a factor of 15). In fact Zhang 1996 already explored a grid twice as fine as used in DT94 (or in Zhang 1998) and the mass inflow rate was not found to have changed much. Thus it appears that particle softening is the main inhibitor to obtaining higher mass inflow rates in N-body disks containing collective modes, the grid softening has only marginal effect in this regard. This is reasonable as the main contributor to collective effects is inter-particle interactions, and the finite amount of grid softening perhaps also helped to maintain the desirable finite thickness effect in these 2D simulations.

4 DISCUSSION

4.1 Are the Increased Mass Flow Rates in Less-Softened N-Body Runs Due to the Collective Instabilities or Due to Few-Body Encounters?

The answer is that it is due to collective instabilities associated with the density wave modes. The hint for this conclusion can be found again in Fig. 3. It is seen there that when not enough number of particles are used for a given $a_{\text{soft}}$ (which generally increases the few-particle relaxation effect), the mass inflow tapers off at an earlier time step. Whereas for very large particle numbers (which generally decreases the few-particle relaxation effect), both the spiral/bar activity and the mass inflow continues until the end of the run. This correlation between the number of simulation particles and the longevity of effective mass inflow shows that the survivability of the density wave modes is key to the continued mass inflow. Few-particle effect causes negligible mass inflow once the spiral/bar activity ceases.

4.2 The Remaining Unrealistic Elements in the Secular Bulge Building Simulations

The simulations described in this Letter serve to highlight an important, and so far neglected, aspect of N-body simulations that has a huge impact on the ability of these simulations to reproduce the observed level of mass inflow for bulge building as inferred from the MIR observations (Zhang & Buta 2007, 2012). However, since the simulation is conducted in 2D, obvious physical effects that are missed include the continued growth of the disk through halo mass accretion, the role of active disk and halo (though as discussed in the footnote, we have a general idea of what this will do to the enhancement of the mass inflow rate). Furthermore, since this is a pure particle simulation, the role of gas is ignored, though the star-gas two component simulation conducted in Zhang (1998) shows that the general features of the collective effects are preserved when gas is added, and Zhang & Buta (2012) showed that the overall effect of stellar accretion is more important than gas accretion due to the larger stellar surface density, for all galaxies except the very late types.

5 CONCLUSIONS

The low mass inflow rates found in the past N-body simulations of disk galaxies had cast doubt on the effectiveness of secular evolution as an important dynamical process for the morphological transformation of galaxies along the Hubble sequence. In this work it is shown that such low numerical mass inflow rates are chiefly the result of the artificial “softening” of gravity which is a common practice in galaxy simulations to avoid rapid relaxation due to the small number of particles employed. By decreasing the amount of softening and simultaneously increasing the number of simulation particles, realistic level of mass inflow comparable to those inferred from observed galaxy images through torque calculations are achieved in N-body simulations of disk galaxies, which reinforce the importance of secular evolution as an extremely relevant process in transforming the morphologies of galaxies during the past Hubble time.

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The strong objections of J. Sellwood when serving as the referee to a separate manuscript coauthored by Zhang & Buta, which used mid-infrared galaxy images to calculate torques and mass flow rates, but did not contain new N-body simulations to corroborate these observationally-derived results, became the direct motivation for the author to carry out a new set of N-body simulations to explore possible means to improve the mass flow behavior of her old codes. She thanks both Sellwood and her collaborator on the observational study R. Buta for serving as the catalyst which made this breakthrough possible.

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