Massive neutrinos in a grounds-up approach

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We examine neutrino oscillations in a two Higgs doublet model (2HDM) in which the second doublet couples only to the third generation right-handed up-fermions, i.e., to $t_R$ and $N_3$ which is the heaviest right-handed Majorana neutrino. The inherently large $\tan \beta$ of this model can naturally account for the large top-quark mass and, based on a quark-lepton similarity ansatz, when embedded into a seesaw mechanism it can also account for the observed neutrino masses and mixing angles giving a very small $\theta_{13}$: $-0.96^0 \lesssim \theta_{13} \lesssim 1.36^0$ at 99% CL, and a very restrictive prediction for the atmospheric mixing angle: $42.9^0 \lesssim \theta_{\text{atm}} \lesssim 45.2^0$ at 99% CL. The large value of $\tan \beta$ also sets the mass scale of the heaviest right-handed Majorana neutrino $N_3$ and triggers successful leptogenesis.
1. Introduction

In the past decade we have witnessed two remarkable findings: (i) the discovery of the top-quark which turned out to be enormously heavy compared to all the other fermions: $m_t \sim 175$ GeV, i.e., “weighing” almost as much as a Gold atom!, and (ii) the discovery of neutrino oscillations implying that neutrinos are massive with a typical mass in the sub-eV range, i.e., $m_\nu$ is more than 12 orders of magnitudes smaller than $m_t$. This two monumental discoveries of the 90’s present us with the pressing challenge of reconciling the apparent enormous hierarchy in the masses of fundamental fermions.

A possible resolution to this huge hierarchy between $m_\nu$ and $m_t$ may be encoded within the following triple-relation between $m_\nu$, $m_t$ (or the Electroweak scale) and the GUT mass-scale $M_{GUT} \sim 10^{16}$ GeV:

$$m_\nu \sim \frac{m_{2D}^2}{M_{GUT}}.$$  \hspace{1cm} (1.1)

Indeed, the beautiful seesaw mechanism dictates that (see also next sections):

$$m_\nu \sim \frac{m_{2D}^2}{M_{\nu_R}},$$  \hspace{1cm} (1.2)

where $m_{2D}$ is a Dirac neutrino mass term and $M_{\nu_R}$ is the mass of heavy right-handed Majorana neutrinos. Thus, based on the seesaw formula in Eq. 1.2, the triple-relation in Eq. 1.1 stands as a very strong hint for Dirac neutrino masses of $\mathcal{O}(m_t)$ and for the existence of super-heavy right-handed Majorana neutrinos with a typical mass of $M_{\nu_R} \sim \mathcal{O}(M_{GUT})$.

In this work [1] we seriously take the triple-relation in Eq. 1.1 at “face-value”, suggesting that the impressive findings in the neutrino sector are closely related to the heaviness of the top-quark. In particular, we construct a model that, based on a grounds-up approach, explicitly yields the triple-relation between the large $m_t$, the observed $\nu$-oscillation data (i.e., masses and mixing angles) and the super-heavy mass scale of the right-handed Majorana neutrinos. In addition, our model can rigger successful leptogenesis which can account for the observed Baryon asymmetry in the universe.

Our model is a two Higgs doublet model (2HDM) which treats the 3rd generation neutrino in a completely analogous manner to the top-quark. We have, therefore, named our model “the 2HDM for the 3rd generation” (3g2HDM).

2. The two Higgs doublet model for the 3rd generation (3g2HDM)

The 3g2HDM extends the idea of the so called “2HDM for the top-quark” (t2HDM) [2] to the leptonic sector. In particular, as in the t2HDM, we assume that $\phi_t$ [the Higgs doublet with a much larger vacuum expectation value (VEV)] couples only to the top-quark and to the 3rd generation right-handed Majorana neutrino, while the other Higgs doublet $\phi_f$ (with a much smaller VEV) couples to all the other fermions. The large mass hierarchy between the top-quark and all other fermions is then viewed as a consequence of $\nu_t/\nu_f \equiv \tan \beta >> \mathcal{O}(1)$, which, therefore, becomes the “working assumption” of our 3g2HDM.
The Yukawa interaction Lagrangian of our 3g2HDM takes the form:

\[
\mathcal{L}_Y = -Y^d \bar{Q}_L \phi_d d_R - Y^u \bar{Q}_L \phi_u u_R - Y^e \bar{Q}_L \phi_e e_R - Y^\nu \bar{L}_L \phi_\nu \nu_R - Y^\nu \bar{L}_L \phi_\nu N - N^\nu \bar{L}_L \phi_\nu N + h.c. ,
\]

where \( N \) are right-handed Majorana neutrinos with a mass \( M_N^i N_i N_j/2 \), \( Q \) and \( L \) are the usual quark and lepton doublets and the following Yukawa textures are assumed [1]

\[
Y_1^{\mu, \nu} = \begin{pmatrix} a^{\mu, \nu} & b^{\mu, \nu} & 0 \\ a^{\mu, \nu} & b^{\mu, \nu} & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad Y_2^{\mu, \nu} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},
\]

such that, in both the quark and leptonic sectors, \( \phi_\nu \) couples only to the third generation right-handed up-fermions. Note also that \( m_D, m_u = v_f (Y_1^{\nu} a + \tan \beta Y_2^{\nu})/\sqrt{2} \), where \( m_D, m_u \) are the Dirac mass matrices of the neutrinos and up-quarks, respectively.

3. Neutrino oscillations in the 3g2HDM

In the basis where \( M_N \) is diagonal, \( M_N = M \cdot \text{diag}(\epsilon_{M1}, \epsilon_{M2}, \epsilon_{M3}) \), we obtain from the seesaw mechanism formula \( m_\nu = -m_D M_N^{-1} m_D^T \):

\[
m_\nu = m_\nu^0 \begin{pmatrix} \epsilon & \epsilon & \delta \bar{\epsilon} \\ \epsilon & \epsilon & \delta \bar{\epsilon} \\ \delta \bar{\epsilon} & \delta \bar{\epsilon} & \omega \end{pmatrix},
\]

where

\[
m_\nu^0 \equiv (v_f)^2 / 2M, \quad \epsilon \equiv a^2 / \epsilon_{M1}, \quad \delta \bar{\epsilon} \equiv \epsilon - a^2 / \epsilon_{M1}, \quad \omega \equiv c^2 \beta \epsilon_{M3}.
\]

In the following we will adopt a quark-lepton similarity Ansatz (perhaps motivated by GUT scenarios): \( a^\mu \sim a^\nu \equiv a, b^\mu \sim b^\nu \equiv b \) and \( c^\mu \sim c^\nu \equiv c \), with \( a \sim O(10^{-3}), b \sim O(10^{-1}), c \sim O(1) \) which, in our model, follows from the up-quark sector since \( a^\mu v_f \sim O(m_u), b^\mu v_f \sim O(m_c) \) and \( m_1 \sim O(c^\mu v_f \tan \beta) \). Then, diagonalizing the light-neutrinos mass matrix in Eq. 3.1, we find that in the normal mass-hierarchy scheme, i.e., \( m_1 < < m_2 < < m_3 \), [1]:

- The mass of the heaviest-light-neutrino follows the triple relation in Eq. 1.1:

\[
m_3 \sim \frac{m_2^2}{M_{N_1}}, \quad \tag{3.3}
\]

where \( M_{N_1} \sim \text{M}_{\text{GUT}} \) is the mass of the 3rd and heaviest right-handed Majorana neutrino.

- Performing a minimum \( \chi^2 \) analysis with respect to each of the oscillation parameters \( \theta_{13}, \theta_{\text{atm}} \equiv \theta_{23}, \theta_{\text{sol}} \equiv \theta_{12} \) and \( \Delta m^2_{\text{atm}}, \Delta m^2_{\text{sol}} \), our 3g2HDM yields the following 99% CL allowed ranges for the mixing parameters:

\[
\begin{align*}
28.0^0 & \lesssim \theta_{\text{sol}} \lesssim 36.0^0 \quad 99\% \ \text{CL} , \\
1.0 \cdot 10^{-3} \ (\text{eV})^2 & \lesssim \Delta m^2_{\text{atm}} \lesssim 3.7 \cdot 10^{-3} \ (\text{eV})^2 \quad 99\% \ \text{CL} , \\
7.3 \cdot 10^{-5} \ (\text{eV})^2 & \lesssim \Delta m^2_{\text{sol}} \lesssim 9.1 \cdot 10^{-5} \ (\text{eV})^2 \quad 99\% \ \text{CL} ,
\end{align*}
\]

\[
(3.4)
\]
with a very restrictive prediction for $\theta_{13}$ and the atmospheric mixing angle:

$$-0.96^0 \lesssim \theta_{13} \lesssim 1.36^0 \quad 99\% \text{ CL},$$

$$42.9^0 \lesssim \theta_{\text{atm}} \lesssim 45.2^0 \quad 99\% \text{ CL}.$$  
(3.5)

- The mass-spectrum of the heavy Majorana neutrinos (subject to the constraints coming from oscillation data) becomes:

$$M_{N_1} \sim 100M, \quad M_{N_2} \sim 0.01M, \quad M_{N_3} \gg 10^{-6}M,$$

with $M \sim 10^{13}$ GeV.

4. Leptogenesis in the 3g2HDM

A CP-asymmetry, $\varepsilon_{N_i}$, in the decay $N_i \rightarrow \ell \phi_j$ can generate the lepton asymmetry [3]:

$$n_L/s = \varepsilon_{N_i} Y_{N_i}(T >> M_{N_i}) \eta,$$

where $Y_{N_i}(T >> M_{N_i}) = 135\zeta(3)/(4\pi^4 g_s)$ and $g_s$ being the effective number of spin-degrees of freedom in thermal equilibrium. Also, $\eta$ is the “washout” parameter (efficiency factor) that measures the amount of deviation from the out-of-equilibrium condition at the time of the $N_i$ decay. This lepton asymmetry can then be converted into a baryon asymmetry through nonperturbative sphaleron processes. In our case (i.e., two scalar doublets) we obtain:

$$n_B/s \sim -1.4 \times 10^{-3} \varepsilon_{N_i} \eta.$$  
(4.2)

As seen from Eq. 3.6, our 3g2HDM can lead to a hierarchical mass spectrum for the heavy Majorana neutrinos, $M_{N_1} << M_{N_2} << M_{N_3}$. In this case, only the CP-asymmetry produced by the decay of $N_1$ survives, i.e., $\varepsilon_{N_1} \rightarrow \varepsilon_{N_1}$. Calculating the CP-asymmetry $\varepsilon_{N_1}$ and the corresponding washout factor $\eta$ we obtain [1]:

$$\frac{n_B}{s} \sim 10^{-17} \tan^2 \beta \frac{\Delta m_{32}^2}{2 m_1^2} \varepsilon_{M_{N_1}} \left( \frac{M_{N_1}}{\text{GeV}} \right)^{1.2} \sin 2(\theta_b - \theta_a).$$  
(4.3)

where the CP-phases arise from the possible complex entries in $Y_{\ell}^\dagger$: $a = |a|e^{i\theta_a}$ and $b = |b|e^{i\theta_b}$. Eq. 4.3 has to be compared with the observed baryon to photon number ratio $n_B/n_\gamma \sim 6 \times 10^{-10}$, implying $n_B/s \sim 8.5 \times 10^{-11}$. For example, taking $\varepsilon \sim 0.5$ and $\Delta m_{32}^2 \sim 8.2 \times 10^{-5} \text{ eV}^2$ (these values are consistent with the observed oscillation data), along with $\tan \beta \sim 10$ and $m_1 \sim 170 \text{ GeV}$, Eq. 4.3 reproduces the observed baryon asymmetry for e.g., $M_{N_1} \sim 10^{10}$ GeV and $\sin 2(\theta_b - \theta_a) \sim 0.1$, or for $M_{N_1} \sim 3.6 \times 10^9$ GeV if CP is maximally violated in the sense that $\sin 2(\theta_b - \theta_a) \sim 1$.

References

[1] For more details see, D. Atwood, S. Bar-Shalom and A. Soni, [hep-ph/0502234].
[2] A.K. Das and C. Kao, Phys. Lett. B372, (1996) 106.
[3] See e.g., P. Di Bari, [hep-ph/0406115]; T. Hambye, [hep-ph/0412053].