INELASTIC PROCESSES IN THE COLLISION OF RELATIVISTIC HIGHLY CHARGED IONS WITH ATOMS

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Abstract

A general expression for the cross sections of inelastic collisions of fast (including relativistic) multicharged ions with atoms which is based on the generalization of the eikonal approximation is derived. This expression is applicable for wide range of collision energy and has the standard nonrelativistic limit and in the ultrarelativistic limit coincides with the Baltz’s exact solution [1] of the Dirac equation. As an application of the obtained result the following processes are calculated: the excitation and ionization cross sections of hydrogenlike atom; the single and double excitation and ionization of heliumlike atom; the multiply ionization of neon and argon atoms; the probability and cross section of K-vacancy production in the relativistic $U^{92+} - U^{91+}$ collision. The simple analytic formulae for the cross sections of inelastic collisions and the recurrence relations between the ionization cross sections of different multiplicities are also obtained. Comparison of our results with the experimental data and the results of other calculations are given.

PACS numbers: 25.75.-q, 34.90.+q.

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Introduction

Study of inelastic processes in the collision of atoms with fast (including relativistic) highly charged ions has been a subject of considerable interest recently. One of motivation for this interest is the anticipated large effective field strength creating by highly charged ions which exceeds internal atomic field strength. Creating such a fields by the another methods is very complicated problem presently. Thus up to now, collision experiments with highly charged ions are the only way for investigating atoms and molecules in superstrong fields. From the fundamental viewpoint study of behavior of matter in superstrong electromagnetic fields is one of the important problems of modern physics. Furthermore, a number of applied problems such as measuring of energy spectrum of nuclear fission, synthesis of superheavy elements, interpretation of data on superheavy cosmic rays, ion diagnostics and spectroscopy of plasma, etc., stimulate the extensive study of collisions of highly charged ions with atoms. Among such a processes collisions of fast and relativistic highly charged ions with atoms are of great interest due to the anticipated large cross sections. Therefore these processes are also of applied importance.

The theoretical methods used for relativistic collisions are very similar to those applied for nonrelativistic collision energies and have mainly the perturbative character. The relativistic treatment within the plan wave Born approximation dates back to Möller \[\text{[2]}\] and has been further developed by Jamnikand Zupancic \[\text{[3]}\], Davidovich et.al \[\text{[4]}\] and by Anholt \[\text{[5]}\]. As is well known \[\text{[6]}\] in a relativistic ion -atom collision perturbation theory begins to break down for large projectile charges (for \(Z \geq 75\)). For example the well known Born approximation leads to a result in which (for small impact parameters) the ionization probability exceeds unity. For
this reason nonperturbative methods for calculation of such processes are needed. Presently a few nonperturbative results are available. Becker et.al. [7] used a finite difference method to solve the Dirac equation for $U^{92+} - U^{91+}$ collisions at 1 GeV/u on a discretized grid. Recently Baltz [1] have obtained exact solution of the Dirac equation for relativistic heavy ion collisions in the ultrarelativistic limit. Another nonperturbative methods of calculation of cross section such inelastic processes are the Glauber approximation [8, 9, 10] and sudden approximation [11, 12, 13]. In this work using the eikonal approximation we obtain a general expression for the cross section of inelastic collision of fast as well as relativistic highly charged ion with complex atom. This expression is applicable for wide range of collision energies and has the standard nonrelativistic limit and in the ultrarelativistic limit coincides with the Baltz’s exact solution [1] of the time-dependent Dirac equation. As an application of the obtained result excitation and ionization cross sections of hydrogenlike atom, single and double excitation and ionization of heliumlike atom, multiply ionization (up to eightfold) of neon atom and (up to eighteen) of argon atom, probability and cross section of the K-vacancy production in the relativistic $U^{92+} - U^{91+}$ collision are calculated. The simple analytic formulae for the cross sections of inelastic collisions and the recurrence relations between the ionization cross sections of different multiplicities are also obtained. Comparison of our results with the experimental data as well as the results of other calculations are given.
Eikonal approximation for relativistic ion atom collisions

A general expression for the inelastic transition amplitude from state $|\Phi_i\rangle$ to state $|\Phi_f\rangle$ for the collision of relativistic highly charged ion with light (nonrelativistic before and after collision) atom in the Glauber approximation has been obtained previously [14] (following [15]):

$$f_{if}(q) = \frac{ik_i}{2\pi} \int e^{-iqb} < \Phi_f | 1 - exp\{-\frac{i}{v} \int Udx\} | \Phi_i > d^2b,$$

where $q = k_f - k_i$ is the momentum transfer. The scattering potential $U = U(x, b; \{r_a\})$ is a function of ion’s coordinates $R = (x, b)$ as well as coordinates of atomic electrons, which we denote as $\{r_a\}, \ a = 1, 2, ... N$, where $N$ is the number of electrons.

To generalize this eikonal approximation for the case of relativistic ion-heavy (relativistic) atom collision one should account the followings:

a) behavior of atomic electrons are described by the Dirac equation;

b) in the Glauber approximation $U(x, b; \{r'_a\})$ is the static Coulomb potential which is induced by the atomic nuclear and the electrons which are in fixed (and simultaneous from the projectile viewpoint) points $r'_a = (x'_a, y'_a, z'_a)$. Then we have

$$\frac{1}{v} \int_{-\infty}^{+\infty} Udx = \sum_{a=1}^{N} \chi_a(b, s'_a), \ \chi_a(b, s'_a) = \frac{2Z}{v} ln \left| \frac{b - s'_a}{b} \right|,$$

axis $x$ is directed on $k_i$, $s'_a = (y'_a, z'_a)$ is the two-dimensional vector. Let us, for the definiteness, consider electrons in the instantaneous positions $r'_a$ at the moment $t' = 0$ in the rest frame of the ion and corresponding wave function is $\Psi'(r'_a, t')$. Then from (1) we have

$$f_{if}(q) = \frac{ik_i}{2\pi} \int \Psi'_f (r'_a, t' = 0) \left[ 1 - exp\{-\frac{i}{v} \int U(x, b; \{r'_a\})dx\} \right] \times$$
\[ \times \Psi'_i(r'_a, t' = 0) \exp(-i \mathbf{q} \cdot \mathbf{b}) d^2 b \prod_{a=1}^{N} d^3 r'_a. \]

In the rest frame of the atom we have for \( t' = 0 \):

\[ x_a = \gamma x'_a, \quad s_a = s'_a, \quad t = -x_a \frac{v}{c^2}; \]

\[ \Psi(r_a, t) = \psi(r_a) \exp(-i Et) = \psi(r_a) \exp(iE x_a \frac{v}{c^2}) = S_a^{-1}\Psi(r'_a, t' = 0); \]

\[ d^3 r_a = dx_a dy_a dz_a = \gamma d^3 r'_a = \gamma dx'_a dy'_a dz'_a; \]

where \( \gamma = 1/\sqrt{1 - v^2/c^2} \), \( S_a^{-1} \) is the Lorentz matrix which transforms the wave function from ion-to-atom rest frame. It acts only to by spinor indices of the atomic electron with index \( a \) (corresponding Dirac matrices are \( \alpha_a \)), \( S_a^{-2} \) is the matrix which is written as [16]

\[ S_a^{-2} = \gamma(1 - v \alpha_a/c). \]

Thus in the rest frame of the atom the transition amplitude from state \(| \psi_i >\) with energy \( E_i \) to state \(| \psi_f >\) with energy \( E_f \) can be written, in the Glauber approximation, as the following:

\[ f_{if}(\mathbf{q}) = \frac{ik_i}{2\pi} \int < \psi_f | [1 - \exp\{-i \sum_{a=1}^{N} \chi_a(b, s_a)\}] \times \]

\[ \times \gamma^{-N} S^{-2} \exp[i \sum_{a=1}^{N} \frac{v x_a}{c^2} (E_f - E_i)] | \psi_i > \exp(-i \mathbf{q} \cdot \mathbf{b}) d^2 b, \quad (2) \]

where \( S^{-2} = \prod_{a=1}^{N} S_a^{-2} \). This is the final expression for the transition amplitude which could be used in the collision of relativistic ion with complex (relativistic or nonrelativistic) atom. The formula (2) (as well as (1)) is applicable when the collision time is considerably less than characteristic atomic time.

If we are not interesting in ion’s scattering angles one can perform integrating over these angles. So, for small angles one has

\[ d\Omega \approx d^2 q/(k_i k_f) \approx d^2 q/k^2. \quad (3) \]
Representing $|f_{if}(q)|^2$ from (4) in the form of product of integrals over $d^2b'$ and $d^2b$ and performing integration over these variables by using eq. (3) and integral representation of the $\delta$-function we obtain the cross section of transition from state $|\psi_i\rangle$ to state $|\psi_f\rangle$ for the relativistic ion-atom collision:

$$
\sigma = \int d^2b |<\psi_f| [1 - e^{i \sum_{a}^{N} \chi_{a}(b,s_{a})}]| \times \\
\times \gamma^{-N}S^{-2}e^{i \sum_{a}^{N} \frac{v_{x}a}{c^2}(E_f - E_i)}| \psi_i\rangle|^2 .
$$

In this expression integrand is interpreted as the transition probability from state $|\psi_i\rangle$ to state $|\psi_f\rangle$ in the collision with the impact parameter $b$. One should note that in this form this probability coincides with exact one obtained by Baltz [1] for the ultrarelativistic case and has a standard nonrelativistic limit [13]. For long-range potentials integral in (4) diverges for large impact parameters. However, as is known, such a divergence is not considerable [9, 17], since for large impact parameters the Born approximation can be applied. There is a region in which the regions of applicability of Born and eikonal approximations overlap one which other. This allows a correct matching of cross sections over the impact parameter.

Consider this matching in the case of K-vacancy production in the collision of relativistic heavy ion with heavy atoms when the transition of the atomic K-shell-electron from the state $|i\rangle$ to the continuum state $|k\rangle$ with momentum $k$ will occurs. Let’s denote via $b_0$ upper integration limit over the impact parameter $b$ in (4). For $b \gg s$ and orthogonal $|k\rangle$ and $|i\rangle$ the generalized inelastic formfactor

$$
<f|1 - e^{i \frac{2Z}{v}ln|\frac{b-s}{b}|}|i\rangle \approx <f|exp{iqr}|i\rangle
$$

tends (for small $q$) to $iq < f|r|i\rangle$, where $q = 2Zb/(vb^2)$. Therefore
integral in (4) over $d^2b$ depends on $b_0$ logarithmically and for this reason the contribution of the region $b < b_0$ to the cross section can be written as

$$\sigma(b < b_0) = 8\pi \frac{Z^2}{v^2} \lambda_i \ln \frac{2\alpha_i}{q_0}, \quad q_0 = 2Z/(vb_0),$$

where

$$\lambda_i = \int d^3k | < \mathbf{k}| \mathbf{r}| i > |^2/3,$$

$$\alpha_i = \lim_{b_0 \to \infty} \frac{Z}{vb_0} \exp \left( \frac{1}{\lambda_i} \frac{v^2}{8\pi Z^2} \int_0^{b_0} 2\pi b db \int d^3k \times \right)$$

$$\times | < \mathbf{k}| [1 - \exp(-i \frac{2Z}{v} \ln \frac{|\mathbf{b} - \mathbf{s}|}{b})] \gamma^{-N} S^{-2} \exp\left[i \frac{v v_i}{c^2} (E_k - E_i) \right] | > |^2 \right \}.$$

In the region $b > b_0$ the field of the ion is a weak perturbation and one can use the so-called Bethe asymptotic:

$$\sigma_i(b > b_0) = 8\pi \frac{Z^2}{v^2} \lambda_i \left( \ln \frac{2v}{\eta b_0 \omega_i \sqrt{1 - \beta^2}} - \frac{\beta^2}{2} \right).$$

where $\eta = e^B = 1.781$ (B=0.5772 is the Euler constant), $\omega_i$ is the ”average” ionization frequency:

$$\ln \omega_i = \frac{\int d^3k | < \mathbf{k}| \mathbf{r}| i > |^2 \ln \Omega_{ki}}{\int d^3k | < \mathbf{k}| \mathbf{r}| i > |^2},$$

where $\Omega_{ki} = \epsilon_k - \epsilon_i$ - transition frequency. Summing (3) and (9) we obtain the total K-shell ionization cross section:

$$\sigma_i = 8\pi \frac{Z^2}{v^2} \lambda_i \left( \ln \frac{2\alpha_i v^2}{\eta Z \omega_i \sqrt{1 - \beta^2}} - \frac{\beta^2}{2} \right).$$

Quantities $\lambda_i, \alpha_i$ and $\omega_i$ are calculated numerically using the formulae (7), (8) and (10). Note that the dependence on the cut-off parameter $b_0$ disappears after matching.

If the states of more than one electron change after the collision, or the dipole transitions are forbidden, integration over the impact parameter in
(4) can be extended to the hole plan of impact parameters (since integrand guarantees convergence) and there is no need in matching.

Obtained formulae are of the general character and can be applied to the collisions of atoms with ions of arbitrary charges. Specificity of the collisions of highly charged ions with atoms is the fact that the cross sections of such collisions are large enough (considerably exceeding atomic sizes). This enables one to use for the calculations of the cross section the large impact parameter approximation (4). In this case the procedure of matching is most simply applicable for the collisions of relativistic ions with nonrelativistic (before and after collision\(^1\)) atoms. In this case in \(4\) \(\psi_i\) \(\psi_f\) are the two-component spinors and \(\gamma^{-n}S^{-2} = 1\). Besides that
\[
\exp[i \sum_a x_a v(E_f - E_i)/c^2] = 1,
\]
\[
\alpha_i = \lim_{q_0 \to 0} \frac{q_0}{2} \exp \left\{ \frac{1}{\lambda_i} \int_{q_0}^{\infty} \frac{dq}{q^3} \int d^3k |<k|\exp(-i qr)|i>|^2 \right\}, q_0 = \frac{2Z}{vb_0} \tag{12}
\]
and \(\omega_i\) depend on only the atomic characteristics but not depend on the impact parameter, projectile charge and velocity:
\[
\omega_i(Z_a) = \omega_i(Z_a = 1)Z_a^2,
\]
\[
\alpha_i(Z_a) = \alpha_i(Z_a = 1),
\]
\[
\lambda_i(Z_a) = \lambda_i(Z_a = 1)/Z_a^2. \tag{13}
\]

These facts allow us to obtain simple analytical expressions for the cross sections by matching.

\(^1\)Strictly speaking, atomic electrons appearing in the continuum in the result of ionization by the impact of relativistic ion can take relativistic velocities. However, as is shown in (4) such a processes occur for the small impact parameters and corresponding contributions to the full ionization cross section by the impact of highly charged ion can be neglected.
Collisions with hydrogenlike atoms

Here we give the transitions cross sections (obtained by matching using the large impact parameter approximation \(^5\)) of nonrelativistic hydrogenlike atom (with the nuclear charge \(Z\)) from the ground state to the state with principal quantum number \(n\) in the collision with relativistic highly charged ion

\[
\sigma_n = \frac{2^{11} \pi Z^2}{3} \frac{n}{v^2 (n^2 - 1)^5} \left( \frac{n - 1}{n + 1} \right) \frac{1}{Z^2_a} \ln \left( \frac{\gamma_n v^2 Z_a}{Z \Omega_n \sqrt{1 - \beta^2}} \right) - \frac{\beta^2}{2},
\]

where \(\Omega_n = \epsilon_n - \epsilon_1\), some of \(\gamma_n\) equal

\[
\gamma_2 = 0.30; \quad \gamma_3 = 0.44; \quad \gamma_4 = 0.49; \quad \gamma_5 = 0.53; \quad \gamma_6 = 0.54; \quad \gamma_7 = 0.55; \quad \gamma_8 = 0.56; \quad \gamma_9 = 0.57; \quad \gamma_{10} = 0.57; \quad \gamma_{11} = 0.57.
\]

For the total ionization cross section we have

\[
\sigma_i = 8 \pi \frac{Z^2}{v^2} 0.283 \frac{1}{Z^2_a} \ln \left( \frac{0.84 v^2}{Z Z_a \sqrt{1 - \beta^2}} \right) - \frac{\beta^2}{2}.
\]

Summing \((14)\) over all \(n\) one obtains the total cross section of excitation of discrete states

\[
\sigma_{ex} = \sum_{n=2}^{\infty} \sigma_n = 8 \pi \frac{Z^2}{v^2} 0.717 \frac{1}{Z^2_a} \ln \left( \frac{1.4 v^2}{Z Z_a \sqrt{1 - \beta^2}} \right) - \frac{\beta^2}{2},
\]

and the total inelastic cross section

\[
\sigma_r = \sigma_{ex} + \sigma_i = 8 \pi \frac{Z^2}{v^2} \frac{1}{Z^2_a} \ln \left( \frac{1.4 v^2}{Z Z_a \sqrt{1 - \beta^2}} \right) - \frac{\beta^2}{2}.
\]

Obtained formulae \((14)\) and \((15)\) can be used for the estimation the cross sections of excitation and ionization of K-shell by the impact of relativistic highly charged ion when the K-shell electrons can be described by the hydrogenlike wave functions with the effective charge \(Z_a\). For the estimation of L-shell excitation or ionization cross sections one can use excitation or
ionization cross sections of hydrogenlike 2s and 2p states. For the hydrogenlike atom in the initial 2s-state we have

$$\sigma_i = 8\pi \frac{Z^2}{v^2} 0.82 \left\{ \ln \left( \frac{17.1v^2}{ZZ_a\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right\},$$

(18)

$$\sigma_n = 8\pi \frac{Z^2 2^{17}}{v^2} 3^{-n} n \left( n^2 - 1 \right) \left\{ \ln \left( \frac{\beta_n v^2 Z_a}{Z\Omega_n\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right\},$$

(19)

where \( n \geq 3 \), \( \Omega_n = \epsilon_n - \epsilon_2 \), the numbers \( \beta_n \) are

\[
\begin{align*}
\beta_3 &= 0.18; & \beta_4 &= 0.28; & \beta_5 &= 0.34; & \beta_6 &= 0.39; \\
\beta_7 &= 0.41; & \beta_8 &= 0.42; & \beta_9 &= 0.44; & \beta_{10} &= 0.45; & \beta_{11} &= 0.46.
\end{align*}
\]

For the hydrogenlike atoms initially being in the 2p-states (after averaging over the projection of the angular moment of this state) we have

$$\sigma_i = 8\pi \frac{Z^2}{v^2} 0.53 \left\{ \ln \left( \frac{271v^2}{ZZ_a\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right\},$$

(20)

$$\sigma_n = 8\pi \frac{Z^2 2^{15}}{v^2} 3^{-n} n 11 \left( n^2 - 1 \right) \left( \frac{11}{3} - \frac{4}{n^2} \right) \left\{ \ln \left( \frac{\beta_n v^2 Z_a}{Z\Omega_n\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right\},$$

(21)

where \( n \neq 2 \), the numbers \( \beta_n \) are

\[
\begin{align*}
\beta_1 &= 0.27; & \beta_3 &= 0.13; & \beta_4 &= 0.30; & \beta_5 &= 0.46; & \beta_6 &= 0.58; \\
\beta_7 &= 0.67; & \beta_8 &= 0.73; & \beta_9 &= 0.79; & \beta_{10} &= 0.82; & \beta_{11} &= 0.85.
\end{align*}
\]

To show the character of obtaining by the matching results we give in fig.1 the ionization cross sections obtained: in the Born approximation - 1; by matching (formula (15)) - 2; in the Glauber approximation [8] - 3; in the sudden approximation [17] - 4. As is seen from this figure formula (15) in the region of its applicability (\( v \sim Z \)) is close enough to one obtained in the Glauber approximation and tends to the Born approximation by increasing
of $v$. Note that obtained in this section expressions for the cross section not coincide (for $Z \sim v \ll 1$) with the results of Born approximation $\sigma_B$ (this fact is the general property of the approximation (4)), the ratio $(\sigma_B - \sigma)/\sigma \to 0$ for $v \to$. In the region $Z \sim v \sim 1$ our results based on unitary approximation (4) as the Glauber approximation give the better accordance with the experiment in comparison with the Born approximation which, as is well known, is not unitary and exceeds (approximately 1.5 times) inelastic cross section. Differing from the Glauber approximation in the form (2), [8, 18] which requires considerable numerical computations, our results are obtained in the analytical form.

**Excitation and ionization of nonrelativistic heliumlike atom**

When one uses the perturbation theory for the calculations of inelastic cross sections of collisions of fast charged particles with complex atom the one-electron excitation and ionization are the first-order effects. The two-electron transitions are the second order effects of perturbation theory, when the interaction of projectile with atomic electrons and interelectron interaction are taken into account once.

Analogously other many-electron transitions can be calculated: i.e., the interaction of projectile with atomic electrons accounts only once and all other interactions correspond to the interelectron correlation which should be accounted necessary times. However situation changes when the interaction of atomic electrons with projectile is considerably large than the correlation of atomic electrons. In this case the many-electron transition should be considered [26, 22, 9] as a result of direct interaction of projec-
tile’s strong field. Formulae (1, 2) and (4) correspond to the such a mechanism of direct interaction. Below we give the expressions for the total cross sections of one- and two-electron transitions from ground state of nonrelativistic heliumlike atom in the collision with relativistic highly charged ion, obtained in the large impact parameter approximation (5). In all the cases two-electron states of the heliumlike atom are described in the form of symmetrized product of hydrogenlike one-electron wave functions. In order to avoid orthogonolization procedure (which is not simply defined) we choose the one-electron hydrogenlike wave functions with effective charge equal: \( Z_1 \) - for one-electron transitions, \( Z_2 \) - for two-electron transitions. Let’s denote via \( |n_1, n_2> \) two-electron states of heliumlike atom with two set of one-electron hydrogenlike quantum numbers \( n_1 \) and \( n_2 \). Then the cross section (4) of transition from the ground state \( |0, 0> \) to state \( |n_1, n_2> \) in the large impact parameter approximation (5) is

\[
\sigma = \int d^2b \langle n_1, n_2 | exp\{-i\mathbf{q}(\mathbf{r}_1 + \mathbf{r}_2)\} | 0, 0>^2.
\]

Thus the cross section is expressed by the integral (over the impact parameter) from the product of the well known hydrogenlike formfactors [13]. The cross sections of two-electron transitions (when the states of both electron change) can be obtained directly from (22) by integrating over the hole plan of impact parameters. The cross sections of inelastic processes which include one-electron transitions (for example, single excitation or ionization, total inelastic cross sections) can be obtained by matching with perturbation theory. Therefore these formulae logarithmically depend on the projectile velocity and relativistic factor \( \gamma = 1/\sqrt{1 - v^2/c^2} \).

The single ionization cross section when one of the electrons get into
the continuum and another one get into one of the discrete states is

$$\sigma^{1+} = 16\pi \frac{Z^2}{v^2} 0.283 \left\{ \frac{1}{Z_1^2} \left[ \ln \left( \frac{5.08v^2}{ZZ_1\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right] - \frac{1}{Z_2^2} \ln 3.72 \right\}. \quad (23)$$

The total cross section of the one-electron excitations of discrete spectrum when one of the atomic electrons excites to the one of states of the one-electron discrete spectrum, another one remains in the ground state is

$$\sigma^{1*} = 16\pi \frac{Z^2}{v^2} 0.375 \left\{ \ln \left( \frac{0.256v^2}{ZZ_1\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right\}. \quad (24)$$

The total double ionization cross section can be obtained by summing (22) over all \(n_1\) and \(n_2\) corresponding to the two-electron continuum:

$$\sigma^{2+} = 16\pi \frac{Z^2}{v^2} 0.283 \frac{1}{Z_2^2} \ln 3.72 = 9.36 \frac{Z^2}{(Z_2)^2v^2}. \quad (25)$$

The total cross section of transition of heliumlike atom to any doubly excited state, after summing over \(n_1\ \ n_2\) (belonging to the two-electron discrete spectrum) is

$$\sigma^{2*} = 16\pi \frac{Z^2}{v^2} 0.283 \frac{1}{Z_2^2} \ln 1.15 = 2.03 \frac{Z^2}{v^2Z_2^2}. \quad (26)$$

The above cross sections are connected by the general relation

$$\sigma_r = \sigma^{1+} + \sigma^{1*} + \sigma^{2+} + \sigma^{2*}, \quad (27)$$

where the total inelastic cross section \(\sigma_r\), corresponds to the arbitrary excitation of heliumlike atom and is given by

$$\sigma_r = 16\pi \frac{Z^2}{v^2} 0.717 \frac{1}{Z_a^2} \left\{ \ln \left( \frac{1.03v^2}{ZZ_a\sqrt{1-\beta^2}} \right) - \frac{\beta^2}{2} \right\}, \quad (28)$$

where \(Z_a\) is the effective charge of heliumlike atom in the ground state \((1S^2)\) and equal to the charge of bare nucleus minus \(5/16\).

As an another example of two-electron transition to the discrete state we give the excitation cross sections of autoionization states of heliumlike atom
with principal quantum number \( n = 2 \) (\( L \)-shell). Since in the considered collisions the spins of the electrons can not change, the excitations of the following autoionization states are possible: \( 2s^2 \, {}^1S \), \( 2s2p \, {}^1P \), \( 2p^2 \, {}^1S \), \( 2p^2 \, {}^1D \).

The corresponding cross sections are
\[
\sigma(2p^2 \, {}^1D) = 2\sigma(2p^2 \, {}^1S) = \frac{10}{3}\sigma(2s2p \, {}^1P) = 30\sigma(2s^2 \, {}^1S) = \pi \frac{Z^2}{v^2Z_2^2} \frac{2^{31}}{3^{19}} \frac{1}{11}.
\]

In the table 1 the experimental data (for excitation of autoionization states), the results of our calculations and the results of calculations from [19] are compared. The results are given for the sum of \((2s2p + 2p^2)\) excitation cross sections for helium atom; Column 1 is the projectile energy per nucleon; column 2 is the projectile charge; column 3 is the experimental result; column 4 is the our result \((Z_2 = 1.97)\); column 5 is the result of numerical calculations from [19].

In Fig.2 the results of experiments from [20], the results of calculations obtained using formulae (23) and (23) for the cross section of double \((Z_2 = 1.97)\) and single \((Z_1 = 1.37)\) ionization of helium atom in the collision with \( U^{90+} \) (with energies 60, 120, 420 Mev/n) as well as the ratio \( \sigma^{2+}/\sigma^{1+} \) are given. Correctness of the choice for values of the effective charges \( Z_1 = 1.37 \) and \( Z_2 = 1.97 \) is confirmed by the good accordance of our results given in table 2 with the experimental ones from [21, 22].

The above data for the single ionization cross section \((\sigma^{1+})\) as well as the sum of cross sections \( \sigma^{1+} \) and \( \sigma^{2*} \) can be used for the estimations of total cross section of formation of one-electron helium ions as a result of direct ionization and Auger decay of various doubly excited states of helium atom, since for light atoms Auger decay is the dominating decay channel of doubly excited states (excluding comparatively small number of doubly excited states for which Auger decay is forbidden by selection rules.
Excitation and ionization of nonrelativistic complex atoms

Though strong field of highly charged ion leads to high ionization probabilities, in the case of multiple ionization and excitation the large impact parameter approximation (5) can break down, due to the fact that corresponding cross section may become comparable with atomic sizes. Therefore more general consideration on the ground of formula (4) is needed. Let’s consider the electrons of nonrelativistic (before and after collision) multielectron atom as distinguishable and each electron described by hydrogenlike wave function. Then the initial wave function. \[ \Psi_0(r_1, \ldots, r_N) = \prod_{i=1}^{N_0} \phi_i(r_i), \] and final one is \[ \Psi_f(r_1, \ldots, r_{N_0}) = \prod_{i=1}^{N_0} \psi_i(r_i). \] Therefore for the total \((N_0 - N)\) - fold ionization probability of nonrelativistic \(N_0\) - electron atom, corresponding to the ionization of \((N_0 - N)\) electrons by simultaneous transition of other \(N\) electrons to any of states of discrete spectrum, with account of unitarity, according to (4) we have

\[
W^{(N_0-N)+}(b) = \frac{N_0!}{(N_0 - N)!N!} \prod_{i=1}^{N_0-N} p_i(b) \prod_{j=N_0-N+1}^{N_0} (1 - p_j(b)),
\]

where \(\Pi_{j=N_0-N+1}^{N_0} \ldots = 1\), for \(N = 0\);

\[
p_i(b) = \int d^3k_i \mid d^3r_i \psi^*_k (r_i) \exp \{-i \chi_i(b, r_i)\} \phi_i(r_i) \mid^2,
\]

\(k_i\) - is the momentum of \(i\) - th electron in the continuum. This probability depends on the vector \(b\) but after averaging over the orbital moment of the initial state of atom it will depend on only \(|b|\). Let’s introduce averaged over moment \(l\) and its projection \(m\) inelastic formfactor for each electron,
which also averaged over all atomic shells:

\[
p(b) = \frac{1}{n_0} \sum_{n=1}^{n_0} \frac{1}{M_n \sum_{l,m}} \int d^3k |\int d^3r \psi_k^*(r) \exp\{-i\chi(b,r)\} \phi_{nlm}(r)|^2,
\]

(31)

where summation carried out over the all values of \(l\) and \(m\) of \(n\)-th shell, \(M_n\) - is the number of such values, \(n\) - is the principal quantum number, \(n_0\) - is the number of atomic shells. It is obvious that \(p(b) = p(|b|)\) not depends on the angles of vector \(b\), i.e. \(p(b)\) has the sense of average one-electron ionization probability. Then replacing in (29) each one-electron formfactor with average value from (31) we get, for the ionization probability of \((N_0 - N)\) electrons, the usual expression of independent electron approximation [26, 27]. However the effective charge \(Z^*\) of the atomic nucleus depends on the ionization degree. In order to account this fact we make in (31) the following substitutions \(k = k/Z^*, b = bZ^*, r = rZ^*\), corresponding to the transition to Coulomb units [15]. Then left side of (31) can be calculated using the wave functions of atomic hydrogen with unit charge. Below \(p(b)\) will mean the averaged by (31) hydrogen atomic formfactor. Such a replacement enables to calculate the ionization cross section for general (than independent electron approximation) cases. Consider ionization of high multiplicities \(N_0 \gg 1, N_0 - N \gg 1\). For the ionization of \(N_0\) electrons, \((N = 0\text{ in } (29))\) \(W\) can be reduced to the product of \(N_0\) one-electron formfactors. We introduce the effective charge \(Z_{N_0}^*\) of the nucleus corresponding to the total ionization of the atom. Replacing each one-electron formfactor with the average (31) one obtains the total ionization probability \(W^{N_0+} = [p(b)]^{N_0}\), where \(b = bZ_{N_0}^*\). Integration in (4) over \(d^2b\) can be performed asymptotically \((N_0 \gg 1)\) by the Laplace method assuming that \(p(b)\) has only one maximum for the \(b = b_1 = 0\). Existence of this maximum follows from [14, 17]. As a result the total \(N_0\)
- fold ionization cross section is

\[
\sigma^{N_0+} = \pi \frac{1}{(Z_{N_0}^*)^2} \left[ \frac{-2\pi}{p''(b_1)N_0} \right]^{1/2} [p(b_1)]^{N_0+1/2},
\]  

(32)

here and below \(b_1\) is the maximum point of the function \(p(b)\), \(p''(b_1)\) is the second derivative of \(p(b)\) over \(b^2\).

In the case \((N_0 - 1)\) - fold ionization the ionization probability is the difference of two terms one of which contains the product \(N_0 - 1\) one-electron formfactors (corresponding effective charge is \(Z_{N_0-1}^*\)); second term is the product of \(N_0\) one-electron formfactors and corresponding effective charge is \(Z_{N_0}^*\). Integrating each term by the Laplace method we obtain the cross section of \((N_0 - 1)\) - fold ionization.

\[
\sigma^{(N_0-1)+} = N_0\sigma^{N_0+} \left[ \left( \frac{Z_{N_0}^*}{Z_{N_0-1}^*} \right)^2 \left( \frac{N_0}{N_0 - 1} \right)^{1/2} \frac{1}{p(b_1)} - 1 \right].
\]  

(33)

Analogously in the general case of \((N_0 - N)\) - fold ionization acting analogous one obtains

\[
\sigma^{(N_0-N)+} = \frac{N_0!\sigma^{N_0+}}{(N_0 - N)!} \sum_{m=0}^{N} (-1)^m \left( \frac{Z_{N_0}^*}{Z_{N_0-N+m}^*} \right)^2 \times
\]

\[
\times \frac{N!\sqrt{N_0/(N_0 - N + m)}}{(N - m)!m!} \{p(b_1)\}^{-N+m},
\]  

(34)

where \(Z_{N_0-N+m}^*\) is the effective charge for \((N_0 - N + m)\) - fold ionization. Obtained formulae (32), (33) and (34) enables one to calculate the ionization cross section of any multiplicity (when \(N_0 \gg 1\), \((N_0 - N) \gg 1\)), or to construct other cross section using known two experimental values of cross section. It is simplest to find \(\sigma^{N_0+}\) and \(\sigma^{(N_0-1)+}\) using which one can obtain \(p(b_1)\) and substitute it into (34). Thus the cross section of \((N_0 - N)\) - fold ionization is expressed via \(\sigma^{N_0+}\) and \(\sigma^{(N_0-1)+}\). The results of such calculations for multiply (up to 8) ionization of \(Ne\) and \(Ar\) (up to 18) atoms are given in Figs. 3 - 5. In this calculations the effective charge

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taken equal to the ionization degree \( Z^*_N = N \). As is seen from these Figs. accordance of our calculations with the experimental data from \[28, 29\] is good enough even for the ionization of lower multiplicities, lying out of the region of applicablity \( (N_0 - N \gg 1) \) of formulae \( (32) \), \( (33) \) and \( (34) \).

**K-vacancy production in the collisions with heavy atoms**

In this section the above results on the eikonal approximation are applied for the calculation of probabability and cross section of K-vacancy production in the collision of relativistic highly charged ions with heavy (relativistic) atoms. As was mentioned above the integrand in \( (4) \) can be interpreted as the probability of transition from state \( | \psi_i > \) to state \( | \psi_f > \) in the collision with the impact parameter \( b \). Direct calculation of the probability can be performed only numerically. Therefore we assume the following simplifications:

a) We will use the large impact parameter approximation \( (3) \)

b) The target atom is considered as a quasirelativistic i.e.,
\[ \exp \left[ i \sum_a x_a v (E_f - E_i) / c^2 \right] \approx 1; \gamma^{-N} S^{-2} \approx 1, \]

that means as a wave functions \( | \psi_i > \) and \( | \psi_f > \) one can use the well known Darwin and Sommerfeld-Mau wave functions \( [6] \).

Then for the K-vacancy production probability we have
\[ I(b) = N \int a_k^2 dk, \]
where
\[ N = (1 + \frac{Z_a^2 \alpha^2}{4})^{-1}, (\alpha = 1/137) \]

\[ a_n^2 = N \frac{2^8 Z_a^6 k q^2 (q^2 + (Z_a^2 + k^2) / 3) \exp \left[ -\frac{2Z_a^2}{k} \right] \arctg \left[ \frac{2Z_a k}{q + Z_a^2 - k^2} \right] dk}{(1 - \exp (\frac{-2\pi Z_a}{k}))(q + k)^2 + Z_a^2} \]

\[ \frac{(q - k)^2 + Z_a^2}{3/(1 + k^2 \alpha^2)} \]
is the square of the absolute value of the well known relativistic hydrogen-
like formfactor $< \mathbf{k}|\exp\{i\mathbf{q}\cdot\mathbf{r}\}|i>$ integrated over the emission angles of
the electron [8, 9]. It differs from the nonrelativistic one by the presence
of the constant $N$ and the factor $(1 + k^2\alpha^2)^{-1}$, ($Z_a$ is the charge of target).
The ionization probability (as a function of impact parameter) calculated
using this formula for $U^{92+}-U^{91+}$ collision is given in Fig.6. As is seen
from this figure our approach based on the eikonal approximation gives,
for small impact parameters, an ionization probability which is less than
unity. Let’s calculate now the K-vacancy production cross section using
the formula (11). The above simplifications a) and b) leads to the formula
(8) which is of the same form as the formula (12) but in this formula as
the wave functions $|\psi_i>$ and $|\psi_f>$ are taken Darwin and Sommerfeld-
Mau wave functions. Therefore in the calculations of $\lambda_i$, $\alpha_i$ and $\omega_i$ the
dependence on $Z_a$ cannot be factorized and the quantities $\lambda_i$, $\alpha_i$ and $\omega_i$ as
a functions of $Z_a$ should be calculated numerically. As an example of such
calculation in Fig. 7 the dependence of K-vacancy production cross section
on the relativistic factor $\gamma$ is given for $U^{92+}-U^{91+}$ collision.

Conclusion

Thus we have derived general formulas for cross section which are applicable
in the case of collisions of atoms with ions of arbitrary charge. These
obtained formulae are applied to the (analytically and numerical) calcula-
tions of the following processes: 1) the excitation and ionization cross
sections of (nonrelativistic) hydrogenlike and heliumlike atoms in the coll-
sions with relativistic highly charged ions;
2) the probability and cross section of K-vacancy production in the rela-
tivistic $U^{92+}-U^{91+}$ collision;
3) the multiple (up to 8 for Ne and up to 18 for Ar) ionization cross sections in the collision of complex atoms with relativistic highly charged ions.

We also have obtained simple analytical expressions for inelastic cross sections and derived recurrence relations between the cross sections of various multiplicities. Obtained theoretical results are compared with the experimental data.

Besides that the above calculations of cross section and ionization probability, using the eikonal approximation, enables to avoid some difficulties appearing in the case of application of perturbation theory to the relativistic highly charged ion -heavy atom collisions and leads to the result coinciding, in the ultrarelativistic limit, with the known exact one.
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Table 1. The sum of the excitation cross sections of the autoionization states $2s2p^1P$ and $2p^2^1D \times 10^{19}$ for the helium atom.

| Energy, MeV/n. | Charge, ion | Experiment, $10^{-19} cm^2$ | Our results, $10^{-19} cm^2$ | Calculation $^{[19]}$, $10^{-19} cm^2$ |
|----------------|--------------|-----------------------------|------------------------------|------------------------------------------|
| 1.84           | 6            | 8.305 ± 1.744               | 18.45                        | 25.6                                     |
| 1.5            | 6            | 20.1 ± 7.20                 | 22.61                        | 31.8                                     |
| 1.5            | 9            | 48.99 ± 17.66               | 50.79                        | 111.6                                    |

Table 2. The cross sections of double and single ionization of the helium atom.

| Energy, MeV/n. | Charge, ion | $\sigma^{2+}$ experiment | $\sigma^{2+}$ theory | $\sigma^{1+}$ experiment | $\sigma^{1+}$ theory | $\sigma^{1+} + \sigma^{2+}$ theory |
|----------------|--------------|--------------------------|----------------------|--------------------------|---------------------|----------------------------------|
| 0.64           | 8            | 1.32                     | 1.687                | 7.9                      | 10.231              | 10.597                           |
| 1.00           | 8            | 1.06                     | 1.08                 | 6.7                      | 8.11                | 8.344                            |
| 1.44           | 8            | 0.45                     | 0.75                 | 5.9                      | 6.518               | 6.68                             |
| 1.4            | 15           | 2.91                     | 2.712                | 17.9                     | 17.798              | 18.385                           |
| 1.4            | 18           | 4.50                     | 3.905                | 22.4                     | 23.322              | 24.168                           |
| 1.4            | 20           | 5.41                     | 4.821                | 26.0                     | 27.146              | 28.191                           |
| 1.4            | 36           | 16.0                     | 15.621               | 57.2                     | 58.206              | 61.59                            |
| 1.4            | 37           | 16.8                     | 16.501               | 59.5                     | 60.02               | 63.594                           |
| 1.4            | 44           | 23.0                     | 23.335               | 72.1                     | 71.779              | 76.833                           |
Figure captions

Fig.1 The ionization cross section of hydrogen atom by $C^{6+}$ ions obtained using:
1 - The Born approximation,
2 - The method of matching (formulae (15)),
3 - The Glauber approximation [8],
4 - The sudden approximation [17].

Fig.2 Experimental results from [20] and results of calculations (by formulae (23) and (23)) for single and double ionization cross section of heliumlike atom by $U^{90+}$ ions with energy 60, 120, 420 Mev/n., as well as for the ratio $\sigma^{2+}/\sigma^{1+}$:
• - experiment, × - calculations.

Fig.3 The dependence of the multiply ionization cross section for $Ar$ atom colliding with $U^{75+}$ ions by energy 15 Mev/n:
□ - experiment from [28],
△ - our results.

Fig.4 The experimental results from [28, 29] and calculations (using (32) - (34)) for the multiply ionization cross sections of $Ne$ atoms in the collision with 120 Mev/n. $U^{90+}$ ions as a function of ionization degree $n$:
□ - experiment,
△ - our results.

Fig.5 Experimental results from [28, 29] and results of calculations (using (32) - (34)) for multiply ionization cross sections of $Ar$ atoms in the collisions with 120 Mev/n. relativistic $U^{90+}$ ions as a function of ionization degree $n$: 

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☐ - experiment,
\(\Delta\) - our results.

Fig.6 K-vacancy production probability as a function of the impact parameter \(b\) for \(U^{92+} U^{91+}\) collision. Solid line -our result, dashed line is the result of Valluri S.R. et.al \[30\].

Fig.7 K-vacancy production cross section as a function of relativistic factor \(\gamma\) for \(U^{92} + U^{91}\) collision. Solid line is the result of our calculations, dashed one is the result from \[6\]. The cross section is given in barns.