From Anomalous Hall Effect to the Quantum Anomalous Hall Effect

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Abstract

A short review paper for the quantum anomalous Hall effect. A substantially extended one is published as Adv. Phys. 64, 227 (2015).
In 1879, Edwin H. Hall discovered that when a conductor carrying longitudinal current was placed in a vertical magnetic field, the carrier would be pressed to against the transverse side of the conductor, which led to the observed transverse voltage. This is called Hall effect (HE) [1], which is a remarkable discovery, in spite of that the understanding of it was difficult at that time since electron was to be discovered 18 years later. After that, as we know, HE is well understood and it is the Lorentz force experienced by the moving electrons in magnetic field pressing them against the transverse side. The HE is now widely used as to measure the carrier density or the strength of magnetic fields.

In 1880, Edwin H. Hall further found that this ”pressing electricity” effect can be larger in ferromagnetic (FM) iron than in non-magnetic conductors. The enhanced Hall effect comes from the additional contribution of the spontaneous long range magnetic ordering, which can be observed even without applying external magnetic field. To be distinguished from the former one, this is termed as anomalous Hall effect (AHE) [2]. Though HE and AHE is quit similar to each other, the underlying physics are much different since there is no orbital effect (Lorentz force) when external magnetic field is absent in AHE.

The mechanism of AHE has been a enigmatic problem since its discovery, and it lasted for almost a century. The AHE problem involves concepts deeply related with topology and geometry that have been formulated only in recent years after the Berry phase being recognized in 1984 [3]. In hindsight, Karplus and Luttinger [4] provided a curtail step in unraveling this problem as early as in 1954. They showed that moving electrons can have an additional contribution to its group velocity when an external electric field is applied. This additional term, dubbed ”anomalous velocity”, which is contributed by all occupied band states in FM conductors with spin-orbit coupling (SOC), can be non-zero and leads to the AHE. Therefore, this contribution depends only on the band structure of perfect periodic Hamiltonian and is completely independent of scattering from impurities or defects (therefore called intrinsic AHE), which makes it hard to be widely accepted before the concept of Berry phase being well established. In a long time, two other ”extrinsic” contributions had been thought as the dominant mechanisms that give rise to an AHE. One is the skew scattering [5] from impurities caused by effective SOC and the other one is the side jump [6] of carriers due to different electric field experienced when approaching and leaving an impurity. The controversy arises also because it is hard to make quantitative comparison with experimental measurements. The unavoidable defects or domains in samples are complex and hard to be
treated quantitively within any extrinsic model and the contributions from both ”intrinsic” and ”extrinsic” mechanisms co-exist.

In 1980, K. von Klitzing et al. made remarkable discovery of quantum Hall effect (QHE) [7]. He found that, with the increase of external magnetic filed, the Hall conductivity exhibits a serials of quantized plateau, and at the same time, the longitudinal conductivity becomes zero, i.e., the sample bulk becomes insulating when Hall conductivity is quantized. In the QHE, electrons constrained in two-dimensional (2D) samples, are enforced to change its quantum states into new ones, namely the Landau energy levels, under highly intensive magnetic field. The originally free-like conducting electrons start to make cyclotron motion. If the magnetic field is strong enough, such cyclotron motions will form full circles, which makes electrons localized in the bulk and the sample becomes insulating. While along the edges of the 2D system, the circular motion enforced by the magnetic field can not be completed due to the presence of the edges, which enforce the electrons to travel in one way forming so called edge states. The electrons in such state can circumambulate defects or impurities on their way ”smartly”. Therefore, current carried by these electrons is dissipationless and conductance is quantized into unit of $e^2/h$ with quantum number corresponding to the number of edge states. Such fascinating quantum state and physical phenomena are highly interesting and impact the whole field of physics, because this is a new state of condensed matters and it should be characterized by the topology of electronic wave-function [8]. This topological number is given by D. J. Thouless, M. Kohmoto, M. P. Nightingale and M. den Nijs [9] in 1982 and is called as the TKNN number or the first Chern number, which has direct physical meaning as the number of edge states or the quantum number of Hall conductance.

After reaching this point, one may immediately ask the question: Can we have the quantum version of AHE, similar to the QHE? Unfortunately, this question was irrelevant at that historical moment, because we even did not know yet the fundamental mechanism of AHE then. Nevertheless, in 1988, F. D. M. Haldane proposed [10] that a QHE without any external magnetic field is in principle possible. Although this proposal is very simple and has nothing to do with either the AHE or the SOC, his idea play important roles for many of our nowadays studies, such as the topological insulators and the quantum anomalous Hall effect (QAHE), because he pointed out the possibility to have a novel electronic band structure of perfect crystal, which carry non-zero Chern number even in the absence of external magnetic
FIG. 1: (Color online) (a) Hall effect. The longitudinal current $I_x$ under vertical external magnetic filed $H_z$ contributes to the transversal voltage $V_y$ due to the Lorentz force experienced by carriers. (b) Quantum Hall effect. The strong magnetic field $H_z$ enforces electrons into Landau level with cyclotron motion and become localized in bulk while conducting at edges. (c) Anomalous Hall effect. The electrons with majority and minority spin (due to spontaneous magnetization $M_z$) having opposite ”anomalous velocity” due to spin-orbit coupling, which causes unbalanced electron concentration at two transversal sides and leads to finite voltage $V_y$. (d) Quantum Anomalous Hall effect. The nontrivial quantum state satisfies all necessary conditions and leads to insulating bulk while topologically protected conducting edge state with spontaneous magnetization. (e) Spin Hall effect. In nonmagnetic conductor, equivalent currents in both spin channels with opposite ”anomalous velocity” leads to balanced electron concentration at both sides while net spin current in transversal direction. (f) Quantum Spin Hall effect. The 2D nontrivial $Z_2$ insulator has conducting edge states with opposite spin in different direction, which can be viewed as two time reversal symmetrical copies of QAHE.

The underlying physics of AHE, in particular, the topological nature of intrinsic AHE was not fully appreciated only until the early years of 21 century. In a series of papers, (i.e, Jungwirth, et.al (2002) [11]; Onoda, et.al. (2002) [12]; Fang, et.al (2003) [13]; Yao, et.al. (2004) [14]), it was discovered that the intrinsic AHE can be related to the Berry phase of the occupied Bloch states. The so-called ”anomalous velocity” is originated from the Berry-phase curvature, which can be regarded as effectively magnetic field in the momentum space,
and thus modifies the equation of motion of electrons, leading to the AHE. This effective magnetic field can be also traced back to the band crossings and the magnetic monopoles in the momentum space \[13\], which is now called Weyl nodes in the Weyl semimetals \[15, 16\]. The quantitative first-principles calculations for SrRuO\(_3\) \[13\] and Fe \[14\], in comparison with experiments, suggested that intrinsic AHE actually dominates over extrinsic ones. The presence of SOC and the breaking of time reversal symmetry (due to the FM ordering) are crucially important, because otherwise the Berry phase contribution will be prohibited by symmetry. What makes those understandings unique is that, like the QHE, the intrinsic AHE is now directly linked to the topological properties of the Bloch states. Up to this point, it is now proper to ask the question: Can the AHE also be quantized like its cousin HE? If it is realized, we will have a kind of QHE without magnetic field, although it is already much different from Haldane’s original speculation in the sense that SOC must play important roles here. Nevertheless, from the applicational point of view, the realization of QAHE will certainly stimulate the wide usage of such novel quantum phenomena in future technology, in particular, the dissipationless edge transport without magnetic field.

To reach this goal, there is still a big step to be overcome. The QAHE requires four necessary conditions: (1) the system must be 2D; (2) Insulating bulk; (3) FM ordering; and (4) non-zero Chern number. It may be easy to satisfy one or two conditions, but hard to realize all of them simultaneously. Fortunately, the recently rapid progresses in the studies of topological insulators (TI) \[17, 18\] make the challenge of realizing QAHE possible. The TI is a new state of quantum matter, which is characterized by the \(Z_2\) topological number \[19\] and is protected by the time-reversal symmetry (TRS), in contrast to the QHE and QAHE, where the TRS must be broken. However, an important view is that the 2D topological insulator and the quantum spin Hall effect (QSHE) \[20, 21\] can be effectively viewed as two copies of distinct QAHE states which are related by the TRS, in other words the 2D QSHE state can be viewed as the time-reversal invariant version of QAHE state. Given the known material realizations of 2D and 3D TI \[17, 21, 22\], it is natural to start from those systems and try to break the TRS in order to achieve the QAHE. Following this strategy, several possibilities are proposed theoretically. Qi, et.al. \[26\] first pointed out that gapping the Dirac type surface states of 3D TI by FM insulating cap-layers may produce the QAHE, although their arguments are not concrete. Later on, the ”band inversion” picture and the experimental observation of QSHE \[21, 22\] inspired the idea of realizing QAHE by transition
metal (Mn) doped HgTe quantum well structure [27]. Unfortunately, in such case, the Mn local moments do not order ferromagnetically. In 2010, Yu, et.al. [28] predicted that when a topological insulator Bi$_2$Se$_3$ or Bi$_2$Te$_3$ is made thin and magnetically doped (by Cr or Fe), the system should order ferromagnetically through the van Vleck mechanism, and exhibit the QAHE with a quantized Hall resistance of $h/e^2$ — a proposal that was finally achieved experimentally by Chang, et.al. [29, 30] after great efforts.

While this is not the end of the story, instead, we believe this success will inspire more extensive researches on QAHE. Two important issues become the natural directions for the future studies: (1) how to increase the temperature range of QAHE (it is now observed only in the tens mK range); (2) how to realize the higher plateau with Chern number larger than 1. There are several other proposals worth trying. HgCr$_2$Se$_4$ has been predicted to be a Weyl semimetal, and its quantum well structure with proper thickness would exhibit QAHE with Chern number being 2 [16], different from the Cr-doped Bi$_2$Te$_3$ family thin film. The advantage of this proposal is that HgCr$_2$Se$_4$ is a chemically pure and stable compound with known bulk Curie temperature higher than 100K. One similar proposal is GdBiTe$_3$ [31], the thin film of which have one edge state contributing to QAHE. Garrity and Vanderbilt [32] proposed that Au, Pb, Bi, Tl, I, and etc heavy-element layers on the surface of magnetic insulators may contribute to large AHE and even QAHE. With the further material breakthroughs, we have strong reason to expect that the QAHE may someday find its places in our electronic devices.

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