Signatures of an intermediate 2d Coulomb phase at low temperatures

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Abstract. – The study of the ground state of spinless fermions in 2d disordered clusters (Phys. Rev. Lett. 83, 1826 (1999)) has suggested the existence of a new quantum phase for intermediate Coulomb energy to kinetic energy ratios $r_s$. Exact diagonalization of the same small clusters show that its low energy excitations (quantum ergodicity above a few “hexatic” excitations characterized by oriented currents) significantly differ from those occuring in the Fermi glass (weak $r_s$) and in the pinned Wigner crystal (large $r_s$). The “hexatic” excitations vanish for temperatures of order of the Fermi temperature.

An insulator-metal transition (IMT) has been observed when the carrier density $n_s$ is increased in dilute two dimensional (2d) gases of electrons and holes having in common very low Fermi temperatures ($T_F \approx 1$ K). The critical Coulomb energy to Fermi energy ratios $r_s \propto 1/\sqrt{n_s}$ are typically around 10 – 35 at the IMT, dropping as a function of the elastic scattering rate $1/\tau$ from $r_s \approx 35$ for a very clean heterostructure to a value of order 10 when $\tau \leq 10^{-11}s$. This coincides with the critical values $r_s^W$ found in numerical simulations for the melting of the Wigner crystal, in the clean limit ($r_s^W \approx 37$) and in the disordered case ($r_s^W \approx 10$, see Refs. [3, 4]). In the insulating phase ($r_s > r_s^W$), $I – V$ characteristics consistent with the existence of a pinned crystal have been observed, accompanied by a sharp drop of the compressibility. Therefore, it has been suggested that the insulating character of the phase is due to the pinning of a Wigner crystal, and not to Anderson localization. At higher densities ($r_s \leq r_s^F \approx 6$), the kinetic terms become more important, giving rise to a re-entry where the metallic behavior disappears. In this limit, the insulating behavior predicted by the scaling theory of localization is recovered (Fermi glass). The metallic behavior characterizing intermediate ratios $r_s^F < r_s < r_s^W$ persists down to $T \rightarrow 35$ mK ($T/T_F \rightarrow 10^{-2}$), but vanishes when the temperature $T$ exceeds the Fermi temperature $T_F$. Eventually, the IMT occurs when the elastic mean free path $l$ and the Fermi wave vector $k_F$ are such that $k_F l \approx 1$. In summary, the interaction, disorder and kinetic effects are a priori relevant and non pertubative in those dilute gases and the observation of the IMT in different
materials leads us to suspect the existence of a rather general mechanism.

Although a generalization up to $r_s \to 10$ of the non-interacting theory where the disorder depends on temperature has been proposed to partly explain the metallic behavior, we numerically explore another alternative: Would the IMT be one of the signatures of a new quantum phase (Coulomb metal) driven by the interplay between kinetic energy and Coulomb repulsion in a random substrate? A first numerical study was focused on the ground state of spinless fermions with Coulomb repulsion in disordered clusters forming a 2d torus enclosing an Aharonov-Bohm flux $\phi$ along the longitudinal direction. The topology of the pattern of the driven currents provides the signature of a new intermediate quantum regime. At weak $r_s$ (Fermi glass) one has a 2d pattern where the local current flows in every direction. At intermediate $r_s$ (new regime), the pattern becomes essentially 1d (no transverse currents and enhanced 1d longitudinal flows enclosing the flux, see also Ref. [4]). At large $r_s$ (pinned Wigner crystal), the persisting 1d flows disappear and charge crystallization sets in. This suggests the possible existence of a new quantum phase characterized by an orientational order of the persistent currents without charge crystallization. This phase could be a quantum analog in a disordered system of the hexatic phase supposed to occur at the melting of the classical Wigner crystal above the Kosterlitz-Thouless temperature. Characteristic values $r_s^F$ and $r_s^W$ delimiting the new intermediate regime were obtained in agreement with the observed re-entry ($r_s^F \approx 4$) and IMT ($r_s^W \approx 10$).

In this Letter, we study the low energy excitations of the same clusters which were studied in Ref. [4], and their statistics when the microscopic configurations of the random substrate are changed. We find that the characteristic low energy excitations of the new intermediate regime disappear for excitation energies $\epsilon$ of order of the Fermi energy $\epsilon_F$. Inside the intermediate regime, the low energy excitations ($\epsilon \ll \epsilon_F$) are characterized by a non random orientation of the local currents and deviations from Wigner-Dyson (W-D) spectral statistics. In analogy with the classical Coulomb problem, we refer to these oriented non chaotic excitations as “hexatic”. For excitation energies $\epsilon \geq \epsilon_F$, the local currents become randomly oriented and the levels obey W-D statistics (quantum ergodicity). In the Fermi glass and in the Wigner crystal, ergodicity is expected for energies much larger than $\epsilon_F$ and the levels remain uncorrelated for $\epsilon \approx \epsilon_F$. Quantum ergodicity with correlated energy levels for $\epsilon \geq \epsilon_F$ is then another characteristic of the intermediate regime, in addition to “hexatic” excitations for $\epsilon \ll \epsilon_F$. Moreover, the sensitivity of the levels under a change of the threaded flux has a remarkable temperature dependence. When $T \to T_F$, the enhancement observed in the intermediate regime is suppressed while the weak responses of the Fermi and Wigner limits are enhanced. Eventually, assuming that this intermediate regime observed in small clusters is the signature of a new quantum phase, we propose the local current $\vec{j}$ driven by a flux $\phi$ as a possible order parameter for the successive quantum transitions. The first transition would be associated to the angle $\theta$ of $\vec{j}$ with the direction enclosing the flux and the second one to its amplitude $\pi$.

We consider a disordered square lattice with $N = 4$ spinless fermions on $L^2 = 36$ sites. The Hamiltonian reads

$$H = -t \sum_{<i,j>} c_i^\dagger c_j + \sum_i v_i n_i + U \sum_{i \neq j} \frac{\epsilon_i n_i n_j}{2r_{ij}},$$

where $c_i^\dagger$ ($c_i$) creates (destroys) an electron on site $i = (ix, iy)$, the hopping term $t$ between nearest neighbours characterizes the kinetic energy, the site potentials $v_i$ are taken from a box distribution of width $W$, $n_i = c_i^\dagger c_i$ is the occupation number at the site $i$ and $U$ measures the strength of the Coulomb repulsion. $r_{ij}$ is the inter-particle distance for a 2d torus. In our units, the factor $r_s = U(2t\sqrt{\pi n_e})^{-1}$ at a filling factor $n_e = N/L^2$. The Fermi energy is defined by $\epsilon_F = 4\pi n_e t$ and a Fermi golden rule approximation gives $k_F l \approx 192\pi n_e (t/W)^2$. We consider
At $W/t = 5$ corresponding to $k_F l = 2.7$. The boundary conditions (BCs) are always taken periodic in the transverse $y$-direction, and such that the system encloses an Aharonov-Bohm flux $\phi = \pi/2$ in the longitudinal $x$-direction ($\phi = \pi$ corresponds to anti-periodic BCs). The Hamiltonian is diagonalized using the Lanczos algorithm for a statistical ensemble of $10^3$ clusters. The first 20 energy levels $E_n$ ($n = 0, 1, 2, \ldots$) are considered for values of $r_s$ chosen inside the three phases ($r_s = 0.8$ for the Fermi glass, $r_s = 6.3$ for the conjectured intermediate phase and $r_s = 30$ for the pinned Wigner crystal).

We begin the presentation of our results by discussing the spectral fluctuations. For the one body spectra, localized wavefunctions yield uncorrelated spectra with Poisson statistics, characterized by a distribution $P(s)$ of the energy spacings between successive levels going to $P_P(s) = \exp(-s)$ when $L \gg L_1$. Delocalized wavefunctions yield correlated spectra with W-D statistics. With time-reversal symmetry ($\phi = 0$), $P(s) \rightarrow P_W^O(s) = (\pi s/2) \exp(-\pi s^2/4)$ as $L$ increases, and without time-reversal symmetry ($\phi = \pi/2$), $P(s) \rightarrow P_W^I(s) = (32s^2/\pi^4) \exp(-4s^2/\pi)$. The statistical properties of many body spectra are more complex. Without interaction, the levels are uncorrelated, and a sufficient two body interaction may restore W-D statistics. This is at the basis of the success of random matrix theory to describe complex nuclei, where low energy quasi-particle or collective excitations are followed by chaotic high energy excitations. The low energy modes being close to integrability display Poisson statistics as non-generic integrable models, whereas in the absence of integrals of motion quantum ergodicity and W-D statistics set in \cite{11}. The question is to know at what energy threshold the Poisson-Wigner crossover takes place. An answer was proposed in Ref. \cite{12,13} based on the breakdown of a perturbation theory valid for weak energy threshold the Poisson-Wigner crossover takes place. An answer was proposed in Ref. \cite{14,15} for the Hamiltonian which we consider, showing as expected quantum ergocity at sufficient excitation energies, but failing to check the proposed criterion was recently addressed \cite{14,15} for the Hamiltonian which we consider, showing as expected quantum ergocity at sufficient excitation energies, but failing to check the proposed criterion (the estimation of the matrix elements deserving a particular study). Another question is to know the factors $r_s$ delimiting the regime of quantum ergocity for a given excitation energy $\epsilon$. The qualitative answer can be guessed from a simpler problem \cite{14,15} (two interacting particles in 1d) where a duality $t/U \leftrightarrow U/t$ implies that a maximum spectral rigidity occurs for $U \approx t$. This leads us to a general picture where Fermi glass and Wigner crystal should be associated to integrability and Poisson statistics, in contrast to an intermediate regime displaying quantum ergocity at low excitation energies.

We study the distribution of the normalized $N$-body energy spacings $s_n = (E_{n+1} - E_n)/<E_{n+1} - E_n>$ (the brackets denote ensemble average). In Fig. 4 A, one can see that $P(s)$ is given by the Wigner surmises, with an orthogonal-unitary crossover when $\phi = 0 \rightarrow \pi/2$, in the intermediate regime for $1.4 < \epsilon/\epsilon_F < 1.9$ ($n = 10, \ldots, 20$). We define a parameter

$$ \eta = \frac{\text{var}(P(s)) - \text{var}(P_W^I(s))}{\text{var}(P_P(s)) - \text{var}(P_W^I(s))}, \tag{2} $$

($\text{var}(P(s))$ denoting the variance of $P(s)$). $\eta = 1$ when $P(s) = P_P(s)$ and $\eta = 0$ when $P(s) = P_W^I(s)$. Taking $\phi = \pi/2$, the variation of $\eta$ as a function of $\epsilon/\epsilon_F$ is given in Fig. 4 B in the three phases. For the Fermi glass with $r_s = 0.8$, the two first spacings display $\eta$ values compatible with the unitary ensemble, while the following spacings become more and more Poissonian. This is what is expected for $r_s = 0$, the two first excitations being single-electron transitions with W-D statistics for $L < L_1$, while the following excitations being the sum of more than one single-electron excitation, give uncorrelated levels. For a small $r_s$, this still remains true, the system being composed of weakly interacting quasi-particles. For the Wigner crystal ($r_s = 30$), $\eta$ does not depend on $\epsilon$ up to $3\epsilon_F$ and keeps an intermediate value, $\eta = 1$.\[\]
Fig. 1. – A: Spacing distribution $P(s)$ for $r_s = 6.3$ when $\phi = 0$ (empty circles) and $\phi = \pi/2$ (full circles) at excitation energies $1.4 < \epsilon/\epsilon_F < 1.9$, compared to $P^W_0(s)$ (dashed line) and $P^W_U(s)$ (continuous line). B: parameter $\eta$ as a function of $\epsilon/\epsilon_F$ for $\phi = \pi/2$, $r_s = 0.8$ (Fermi glass, squares), $r_s = 6.3$ (intermediate regime, full circles) and $r_s = 30$ (Wigner crystal, diamonds). C (Ensemble average $<|\theta|>$) and D (typical total longitudinal current $I_l$) as a function of $\epsilon/\epsilon_F$ (same cases and symbols than in B). Inset of C: angular distribution $P^{(\theta)}_{|\theta|}$ in the intermediate regime ($r_s = 6.3$) for $\epsilon/\epsilon_F = 0.4$ (dashed line), 1 (dotted line) and 1.4 − 1.9 (solid line), respectively.

being expected in the thermodynamic limit. For the intermediate regime ($r_s = 6.3$) one has quantum ergodicity and W-D statistics above a (low) excitation energy $\approx \epsilon_F$, but the first excitations exhibit deviations from W-D statistics, indicating a break-down of a description in terms of weakly interacting quasi-particles and suggesting possible non chaotic collective excitations.

The low energy excitations are characterized by a particular structure of the corresponding persistent currents when a flux $\phi$ is applied. The current $I^{(n)}_l$ of the $n^{th}$ many-body level has a total longitudinal component given by

$$I^{(n)}_l(\phi) = \frac{\partial E_n}{\partial \phi} \bigg|_{\phi = 0} = \frac{\sum_i J^{(n)}_{i,l}}{L}.$$
The local current \( j_{i,t}^{(n)} \) flowing at the site \( i \) in the longitudinal direction is defined as

\[
j_{i,t}^{(n)} = 2\text{Im} \langle \Psi_n | e_{x,i+1,y_i} e_{x,i_y} \exp(i\phi/L) | \Psi_n \rangle,
\]

where \( \Psi_n \) \((n = 0, 1, 2, \ldots)\) is the many-body wavefunction of energy \( E_n \). One defines the local transverse current \( j^{(n)} \) by a similar expression. The local current of the \( n \)th level flowing at the site \( i \) is a two-component vector \( \vec{j}^{(n)}(i) = (j_{i,t}^{(n)}, j_{i,t}^{(n)}) \) characterized by its angle \( \theta_i^{(n)} = \arctan(j_{i,t}^{(n)}/j_{i,t}^{(n)}) \) and its absolute value \( j_i^{(n)} = |\vec{j}^{(n)}(i)|\).

The inset of Fig. 1C gives the distribution \( P_\theta \) of \( |\theta| \) in the intermediate regime for increasing excitation energies \( (P_\theta \) is symmetric when \( \theta \rightarrow -\theta \). For \( \epsilon/\epsilon_F \geq 1.4 \) where W-D rigidity sets in, \( P_\theta \) is uniform, the flow is randomly oriented. For \( \epsilon/\epsilon_F \leq 1.4 \) where deviations from W-D statistics can be seen, \( P_\theta \) is non uniform, the flow being mainly paramagnetic at \( \epsilon/\epsilon_F \approx 0.4 \) and diamagnetic at \( \epsilon/\epsilon_F \approx 1 \). The average of \( |\theta| \) is shown in Fig. 1C: values \( < \pi/2 \) or \( > \pi/2 \) correspond to paramagnetic or diamagnetic responses respectively. In the Fermi glass, the flow is weakly paramagnetic and becomes randomly oriented for \( \epsilon > \epsilon_F \). In the pinned Wigner crystal, a strongly paramagnetic response becomes randomly oriented at \( \epsilon > 3\epsilon_F \).

The intermediate regime has a very specific behavior, the flow being mainly paramagnetic \( (\epsilon/\epsilon_F < 0.7) \) then diamagnetic \( (\epsilon/\epsilon_F \approx 1) \), before being randomly oriented at \( \epsilon/\epsilon_F \geq 1.4 \) when quantum ergodicity sets in. The amplitudes of the longitudinal paramagnetic \( (\text{probability } c_p) \) and diamagnetic \( (\text{probability } c_d) \) total current \( I(E_n \approx \epsilon) \) have log-normal distribution. The typical value \( I_l = c_p \exp < \ln |I_l^p| > - c_d \exp < \ln |I_l^d| > \) is given in Fig. 1D. Below the ergodicity threshold of the intermediate regime, \( I_l \) is non random, being paramagnetic then diamagnetic in contrast to the Fermi and Wigner limits. The sign of \( I_l \) can be explained \( [8] \) in the Wigner limit, where the \( \phi \) dependent parts of the energies \( E_n \) can be expanded in power of \( t/U \). The leading contribution comes from the shortest 1d paths enclosing \( \phi \) and the sign is given by Leggett’s theorem (see Ref. [4]) for 1d spinless fermions. The sign depends on the sample geometry and the parity of \( N \) when \( r_s \rightarrow \infty \), independently of the microscopic disorder, but remains the same as far as \( r_s \geq r_s^F \), becoming sample dependent in the Fermi glass only.

Additional informations on the spectral statistics are given in Fig. 2A \((\eta \text{ as a function of } r_s \text{ for levels of increasing excitation energies})\). There are two values of \( r_s \) \((\approx 2 \text{ and } < 30)\) where \( P(s) \) is invariant when \( \epsilon/\epsilon_F \leq 3 \). Between those two values, the spectrum becomes more rigid as one increases \( \epsilon \), while the opposite behavior is observed otherwise, providing another signature of the intermediate regime. Assuming a Gibbs-Boltzmann population of the many-body levels, we show in Fig. 3B the variation of

\[
< \ln |I_l(kT)| > = \left( \sum_n \ln |I_l^p| \exp(-(E_n - E_0)/kT) \right) / \left( \sum_n \exp(-(E_n - E_0)/kT) \right)
\]

as a function of \( r_s \) at different temperatures. Another signature of the intermediate regime is obtained, the enhancement of the above expression disappearing as \( T \rightarrow T_F \), while there is a weak increase in the Fermi and Wigner limit. We point out that the metallic (insulating) phase is identified in the transport measurements from a decrease (increase) of the conductivity when \( T \) increases and that the low temperature metallic behaviour disappears for \( T \approx T_F \). In the inset, one can see that enhancement and opposite temperature dependence disappear for a larger disorder \((W/t = 15)\), indicating that the ratio \( W/t \) is another relevant parameter, a metallic behavior being not expected \([19]\) for too strong disorders.

The distribution \( P_\theta \) calculated for the ground state (Fig. 3A) shows how the local angles undergo a transition from a nearly uniform angular distribution (glass of currents randomly directed in the plane) towards a distribution strongly peaked in the longitudinal direction.
Fig. 2. – $W/t = 5$ and $\phi = \pi/2$. A: Parameter $\eta$ as a function of $r_s$ for the first excitation $s_0$ (circles), $s_2 - s_4$ (squares) and $s_{10} - s_{20}$ (diamonds). B: Typical longitudinal current for temperature in units of the Fermi temperature $T/T_F = 0$ (circles), 0.4 (squares), 0.7 (diamonds), 1.1 (triangles up). Inset: the same at $W/t = 15$.

Fig. 3. – $\phi = \pi/2$. A: distribution $P_\theta$ of the ground state local current angles for $r_s = 0$ (dotted line), $r_s = 6.3$ (full line) and $r_s = 42$ (dashed line). B: $\langle |\theta| \rangle$ (open symbols, left scale) and amplitudes $\langle \ln j \rangle$ (filled symbols, right scale) as a function of $r_s$ for the ground state (circles) and for $1.4 < \epsilon/\epsilon_F < 1.9$ (triangles).

(ordered flow of currents with a 1d topology). Fig. 3 B gives the average angle $\langle |\theta| \rangle$ and the local current amplitude $\langle \ln j \rangle$ when $r_s$ increases, both for the ground state and for $\epsilon > \epsilon_F$: The pattern of driven currents becomes ordered for $r_s$ values smaller than those at which $j$ is suppressed due to charge crystallization.

In summary, the local currents and the level statistics characterizing the low energy excitations confirm the existence of an intermediate regime in small clusters. The main issue is to know if this intermediate regime gives rise to a new quantum phase in the thermodynamic limit. A finite size scaling (FSS) study using exact diagonalization (larger $L$ at the same density $n_e$) is beyond the ability of the most powerful available computers. One alternative could be to use variational trial wavefunctions justified for small and large $r_s$ and to compare
their relative energies using Monte Carlo method [24]. However, such approaches depend on
the taken trial wavefunctions and can easily miss complex behaviors as the ones behind the
beautiful FQHE physics, where the exact numerical solution [22] of the three particle problem
have been instructive. To study larger sizes, we have also developed approximate methods
where the huge Hamiltonian is truncated in restricted sub-spaces using one body states [21] or
Hartree-Fock orbitals [19]. From those studies valid for small and large $r_s$, but which become
unreliable for intermediary values of $r_s$, we know that the obtained precisions do not allow us
to calculate very small quantities (as the local currents presented here). Numerical studies of
those Coulomb systems are limited to exact diagonalizations of small clusters combined with
approximate FSS approaches. For instance, a FSS study [21] of the length

$$\xi$$

of the local currents $\vec{j}$ this case, we propose the local current $\vec{j}$ from the intermediate phase to both the Fermi glass (via its angle $\theta$) and the pinned Wigner crystal (via its amplitude $j$). The melting of the Wigner crystal leads to an intermediate “hexatic” phase which has no translational order but still has orientational order (alignment of the local currents $j$). Shear modulus measurement [23] would be very useful, together with transport and compressibility measurements, to clarify these issues. We note that the disorder strength $W$ should play a role which has not been fully explored (the study is mainly centered on $k_F l \approx 2.7$). The role of the spin degrees of freedom will be discussed in details in a following study [24] which does not lead us to fundamentally reconsider our conclusions.

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