Abstract. Lo, Sargent, and Young (1993) have recently published an analysis of the HI kinematics of nine faint dwarf galaxies. Among other things, they conclude that the masses of these systems, as deduced by the modified dynamics (MOND) from the observed velocity dispersions, are systematically smaller than even the HI masses that are observed in these systems, by a factor of ten or more. Such a state of things would speak strongly against MOND. We show here that the MOND mass estimator used by Lo et al. is smaller than the proper expression, by a factor of about twenty. We derive the proper mass estimator as an exact virial-like relation between the 3-D rms velocity, \( \langle v^2 \rangle \), and the total mass, \( M \), of an arbitrary, self-gravitating system, made of light constituents, that is everywhere in the very-low-acceleration regime of MOND. This reads \( M = (9/4)\langle v^2 \rangle^2/Ga_0 \). (For a system that is not stationary, \( \langle v^2 \rangle \) involves also an average over time.) We do this in the Bekenstein-Milgrom formulation of MOND as a modification of gravity. This relation has been known before for the special case of a stationary, spherical system. We further generalize this relation to cases with constituent masses that are not small compared with that of the whole system. We discuss various applications of the \( M - v \) relation; inter alia, we derive an expression for the two-body force law in the large-distance limit. With the correct estimator the predictions of MOND are, by and large, in good agreement with the total observed masses (the observed gas mass plus a stellar mass corresponding to an M/L of order one solar unit).

I. Introduction

Lo, Sargent, and Young (1993) (hereafter LSY) have recently published the results of HI observations of nine intrinsically faint dwarf galaxies. Among other things, they determine total HI masses. They also estimate and discuss the total masses implied by the modified dynamics (MOND). They find that the latter are systematically smaller than the HI masses “by a factor of ten or more”—an unacceptable state of affairs (for the theory).

LSY have used a MOND mass estimator that is a mistaken adaptation
of the relation
\[ M = \frac{V_\infty^4}{G a_0}, \tag{1} \]
between the total mass, \( M \), of a body, and the asymptotic rotational velocity of test particles around it, \( V_\infty \) (Milgrom 1983a,b); \( a_0 \) is the acceleration constant of the theory.

Equation (1) is exact in MOND, but LSY have used an analogous relation with \( V_\infty \) replaced by \( v_h \), the half width at half maximum of the integrated HI line profile—which is a measure of the one-dimensional rms velocity for the whole galaxy. LSY thus use
\[ M^* = \frac{v_h^4}{G a_0}, \tag{2} \]

We show below that \( M^* \) gives a far underestimate of the mass. We derive a general mass-velocity relation that holds in the non-relativistic, Bekenstein-Milgrom (1984) formulation of MOND as a modification of gravity; this relation is
\[ M = \frac{9}{4} \langle v^2 \rangle^2 / G a_0. \tag{3} \]
Here, \( \langle v^2 \rangle \) is the three-dimensional mean-square velocity of the whole system (averaged over time if the system is not stationary), and \( M \) is its total mass. Relation (3) applies for an arbitrary, self-gravitating system, whereby the acceleration is much smaller than \( a_0 \) everywhere. Beside the factor of 9/4 missing in expression (2), a large factor of disparity (9 in the isotropic case) is present because of the difference between 3-D and 1-D velocities, leading to an underestimation of the mass by a factor of about twenty, in the isotropic case.

Gerhard and Spergel (1992) have derived relation (3) for the special case of a stationary, spherical system; it was derived, for the yet more special case of a sphere with constant radial and tangential velocity dispersions, by Milgrom (1984).

II. The MOND mass estimator

Consider a self-gravitating system that is composed of various particle species with masses \( m_k \) and distribution functions \( f_k(\vec{r}, \vec{v}, t) \). As usual, take the time derivative of the quantity
\[ Q \equiv \sum_k \int d^3 r \, d^3 v \, m_k \, f_k(\vec{r}, \vec{v}, t) \, \vec{r} \cdot \vec{v} \tag{4} \]
(itself, the time derivative of the trace of the moment-of-inertia tensor).
\[ \dot{Q} = \sum_k \int d^3 r \, d^3 v \, m_k \, f_k(\vec{r}, \vec{v}, t) \, v^2 + \sum_k \int d^3 r \, d^3 v \, m_k \, f_k(\vec{r}, \vec{v}, t) \, \vec{r} \cdot \vec{a}. \tag{5} \]
(By Liouville’s theorem the time derivative of a quantity of the form \( \int d^3 r d^3 v f q(\vec{r}, \vec{v}, t) \) is \( \int d^3 r d^3 v f \dot{q} \).) The first term in eq. (5) is the momentary, mass-weighted, 3-D, mean-square velocity, \( \langle v^2 \rangle \), multiplied by the total
mass, $M$. In the second term we put $\ddot{\mathbf{a}} = -\nabla \phi$ (here it is assumed that gravity is the only important force), where $\phi$ is the (MOND) gravitational potential, to obtain

$$\dot{Q} = M \langle v^2 \rangle (t) - \sum_k \int d^3 r \, d^3 v \, m_k \, f_k (\mathbf{r}, \mathbf{v}, t) \, \mathbf{r} \cdot \nabla \phi. \quad (6)$$

As $\ddot{\mathbf{a}} = -\nabla \phi$ depends only on $\mathbf{r}$, the $v$ integration, and the sum over species can now be performed to give the standard result

$$\dot{Q} = M \langle v^2 \rangle (t) - \int d^3 r \, \rho (\mathbf{r}, t) \mathbf{r} \cdot \nabla \phi. \quad (7)$$

We now specialize to MOND, and here enters the assumption of self gravity: the density $\rho$ is the source of the gravitational potential. We substitute $\rho (\mathbf{r}, t)$ from the field equation of Bekenstein and Milgrom (1984),

$$\nabla \cdot [\mu (x) \nabla \phi] = 4 \pi G \rho, \quad x \equiv |\nabla \phi|/a_o, \quad (8)$$

in eq. (7); then, integrating by parts, we get

$$\dot{Q} = M \langle v^2 \rangle (t) - \frac{1}{4 \pi G} \int_\Sigma \mu (x) \ddot{\mathbf{r}} \cdot \nabla \phi \ddot{\nabla} \phi \cdot d s + \frac{1}{4 \pi G} \int \mu (x) \ddot{\mathbf{r}} \cdot \ddot{\nabla} \phi d^3 r. \quad (9)$$

The first integral is over any surface, $\Sigma$, encompassing all the mass; we take it at infinity. As $r$ goes to infinity $\ddot{\nabla} \phi$ becomes $\left( M G a_o \right)^{1/2} \ddot{\mathbf{r}}/r^2$ (see Bekenstein and Milgrom 1984), and $\mu (x)$ becomes $x$, so the second term becomes $-M \left( M G a_o \right)^{1/2}$. The second integral can be calculated thus

$$I_2 \equiv \int \mu (x) \ddot{\nabla} \phi \cdot \ddot{\mathbf{r}} \cdot \ddot{\nabla} \phi \, d^3 r = \int \mu (x) \ddot{\mathbf{r}} \cdot \ddot{\nabla} \phi \cdot \ddot{\nabla} \phi + (\ddot{\nabla} \phi \cdot \ddot{\mathbf{r}}) \, d^3 r =$$

$$= \int \frac{1}{2} \mu (x) \ddot{\mathbf{r}} \cdot \ddot{\nabla} [(\ddot{\nabla} \phi)^2] + \mu (x) (\ddot{\nabla} \phi)^2 \, d^3 r. \quad (10)$$

Let $F(y)$ be such that $\mu (x) = F^1 (y)|_{y=x^2}$ [ $F(x^2)$ is the Lagrangian density for the potential (see Bekenstein and Milgrom 1984)]. The first term in the integrand of eq. (10) can then be written as

$$\frac{a_o^2}{2} \ddot{\mathbf{r}} \cdot \ddot{\nabla} F = \frac{a_o^2}{2} \ddot{\nabla} \cdot (F \ddot{\mathbf{r}}) - \frac{3 a_o^2}{2} F. \quad (11)$$

The first divergence term may again be written as a surface integral at infinity that contributes $\left( 1/3 \right) M \left( M G a_o \right)^{1/2}$ to the right-hand side of eq. (9). We thus end up with the relation

$$\dot{Q}/M = \langle v^2 \rangle (t) - \frac{2}{3} \left( M G a_o \right)^{1/2} - \frac{a_o^2}{4 \pi G M} \int \frac{3}{2} F (x^2) - \mu (x) x^2 \, d^3 r \quad (12)$$
Take now the long-time average of eq. (12). That of the left-hand side vanishes, as $Q$ is finite at both ends of time, and we get

$$\overline{\langle v^2 \rangle} = \frac{2}{3} (MGa_o)^{1/2} + \frac{a_o^2}{4\pi GM} \int \frac{3}{2} F(x^2) - \mu(x)x^2 \, d^3r,$$  

(13)

where an overline signifies the long-time average. This relation is exact and general (i.e. independent on how close to the MOND regime we are). In the Newtonian limit ($a_o$ goes to 0) it gives the usual virial theorem. In the very limit where the accelerations are always and everywhere in the system much smaller than $a_o$, the integrands in eqs. (12) (13) vanish [because in the limit $x \to 0$ we have $F(x^2) \to (2/3)x^3$, and $\mu(x) \to x$], and we obtain

$$\overline{\langle v^2 \rangle} = \frac{2}{3} (MGa_o)^{1/2}.$$

(14)

When the system is stationary (and sometimes in more general cases–such as systems that are stationary in some rotating frame) the momentary value of $\langle v^2 \rangle$ remains constant and it can then be used in eq. (14). As we cannot determine time averages for relevant astrophysical systems, we usually make the simplifying assumption that the system is stationary and use relation (14) with the observed momentary value of $\langle v^2 \rangle$.

Equation (14) is the trace of the following relation, which holds under the same conditions, and which can be derived along the same lines:

$$\overline{\langle v_i v_j \rangle} = \frac{2}{9} (MGa_o)^{1/2} \delta_{ij} + \frac{a_o^2}{8\pi GM} \int \frac{9}{2} \delta_{ij} - 3 \phi_{ij} \phi_{ij} / |\vec{\nabla}\phi|^2 F \, d^3r.$$  

(15)

There are possible formulations of MOND other than that of Bekenstein and Milgrom (1984). The present $M - v$ relation is probably not exact in all of them. It is, however, a good indicative estimator, and, in any case, the best we have at the moment. We note, for example, that in the class of MOND theories based on modification of the law of inertia, discussed by Milgrom (1993), the $M - v$ relation is still exact for stationary, spherical systems, whose constituents move on circular trajectories (in the deep MOND limit).

In many instances, what we would like to use as test particles are themselves sub-systems with internal structure, or with masses that are not small compared with $M$. We must then be careful in treating the subsystems. Let $m$ be the mass of a body we want to treat as a structureless constituent, and let $\ell$ be its size. The assumptions underlying our derivation of the $M - v$ relation seem to require the following: (i) The constituent itself is kept together by gravitational forces balancing internal motions (say of some perfect fluid). (ii) In order to have $|\vec{\nabla}\phi| \ll a_o$ everywhere (within the body as well as outside) we must have $mG/\ell^2 \ll a_o$. (iii) If $L$ is the length scale over which the field varies appreciably, in the vicinity of the body, we must have $\ell \ll L$. (iv) As we want to use only the centre-of-mass velocity of the body in calculating $\langle v^2 \rangle$, and neglect the internal velocities, we must have $m \ll M$.

This set of requirements is too restrictive for the $M - v$ relation to be of much application: Condition ii, for instance, would bar stars from being legitimate constituents; condition i would bar atoms and elementary particles, etc. (condition iii is quite benign). Fortunately, it is possible to weaken
conditions i, ii greatly, or, indeed, to eliminate them altogether. Also, the $M - v$ relation may be generalized to circumvent condition iv.

We note first that when all the would-be constituents have $m \ll M$ requirements i, ii may be disregarded. To see this, replace all bodies that do not satisfy i and/or ii by ones that do; i.e., by ones having the same mass, but a size $\ell_*$ satisfying $mG/\ell_*^2 \ll a_o$, and that are made of a perfect fluid held by gravity. Because $m \ll M$, $\ell_*$ can be chosen small enough so that $\ell_* \ll L$. The new system does satisfy all the assumptions, and the $M - v$ relation holds for it. However, if $m$ is small enough there exist a length $a$ such that $a \ll L$; on one hand, and such that at a distance $a$ from $m$ the contribution of the latter to $\vec{\nabla}\phi$ is small. Under these conditions, Bekenstein and Milgrom (1984) showed that the centre-of-mass motion of the body is oblivious to its internal structure and mass, and it behaves like a test particle. The centre-of-mass motions of our replacement bodies are, then, like that of the original. The correction on this very-small-mass case is of order $(m/M)^{1/2}$. In addition, we want to be able to neglect the internal velocities in the replacement constituent bodies; again this is permitted because $m \ll M$, and the higher order correction is here also of order $(m/M)^{1/2}$. It is thus valid to apply the $M - v$ relation to the original system without having to worry about the internal makeup of the constituents.

When at least some of the sub-system’s masses are not very small compared with $M$, the $M - v$ relation, as given by eq. (14) is not valid. Given the masses and sizes of the constituents, we consider the limit where the distances between them are very large—in keeping with our working in the extreme MOND limit. We can, as described before, replace the masses with “live”, self-gravitating ones made of a perfect fluid. Relation (14) can now be applied but we must reckon with the internal velocities of the “live” replacements, which does not necessarily have a counterpart in the original constituent. In the limit of large distances, each replacement body itself satisfies eq. (14), as can be shown. (We assume that the internal acceleration produced by the constituent dominates over that of the rest, at a distance where the asymptotic form of the isolated-mass potential—as described below eq. (9)—already holds. This is valid in the limit that we consider here.) The prescription is thus as follows: if $m_i$ is the mass of the $i$th particle, and $v_i$ its centre-of-mass velocity, then the mean square velocity in the $M - v$ relation is to be taken as

$$M^{-1} \sum_i m_i(v_i^2 + \frac{2}{3} \sqrt{m_iGa_o})$$

where the second term in parentheses is the internal rms velocity within $m_i$. This leads to the relation

$$\frac{2}{3}(MGa_o)^{1/2}(1 - M^{-3/2} \sum m_i^{3/2}) = M^{-1} \sum m_i v_i^2 = \langle v^2 \rangle,$$

where $\langle v^2 \rangle$ now includes only centre-of-mass velocities.

To demonstrate the necessity of this crucial modification, consider a system made of two masses $m$ and $M \gg m$, in a circular orbit around each other. A blind application of relation (14) will give $MGa_o \approx (9/4)(m/M)^2 v^4$, where $v$ is the speed of the smaller mass. This, however, is quite wrong: MOND tells us that, in fact, $MGa_o \approx v^4$. The reason for the failure of eq (14), in this...
case, is that most of the contribution to $\langle v^2 \rangle$ should have come from intrinsic motions within $M$, which are neglected when we treat it as a test particle. If we apply eq. (17) instead, we get the correct result (as the first order term in $m/M$).

The relative correction introduced by eq. (17) is bounded by $(m_x/M)^{1/2}$, where $m_x$ is the maximum mass of a constituent. When all the masses are equal the relative correction is $(m/M)^{1/2}$.

At any rate, when the accelerations internal to the subsystems are small compared with $a_o$, it is best not to consider them as test particles, but include in $\langle v^2 \rangle$ the full rms velocity for the system.

III. Some applications of the mass-velocity relation

First, and perhaps foremost, the general $M - v$ relation puts on firmer and wider footings the MOND prediction of a Faber-Jackson like relation for all self-gravitating low-acceleration objects. This includes disk galaxies, as we saw; for these, the $M - v$ relation comes in addition to the (different) relation between total mass and asymptotic rotational velocity (1). These relations tell us that we can use in the Tully-Fisher relation either the rms rotational velocity (for low-$a$ galaxies) or the asymptotic, rotational velocity (for all disks). We then expect relations with the same slope, but different intercepts. In fact, almost all known astronomical objects, on the scale of galaxies and up, have internal, mean accelerations that are about $a_o$ or smaller, and the $M - v$ relation should approximately hold universally. In applying it to observed objects we note again the uncertainties in the deduction of the 3-D velocities, and also the error introduced if we assume that a system is already stationary when, in fact, they are not. (and take its momentary value of $\langle v^2 \rangle$ to be the time-average value).

The $M - v$ relations affords significant shortcuts to the derivation of the MOND forces acting on bodies in a few configurations. If the gravitational forces can be balanced by rotating a configuration, rigidly, with some frequency $\omega$, about some known centre, then, $\omega$ may be determined from relation (17), and from it the forces on all the masses may be gotten. Perhaps the most interesting application is a derivation of an analytic expression for the two-body force in the Bekenstein-Milgrom formulation, in the long distance limit: Consider two masses $m_1$, and $m_2$, at a distance $\ell$ from each other: a system that can be supported by rotation about the centre of mass. In the limit of large $\ell$ an expression for the force, $F$, can be conveniently put in the form

$$F(m_1, m_2, \ell) = \frac{m_1 m_2}{\ell} \left( \frac{G a_0}{m_1 + m_2} \right)^{1/2} A(m_1/m_2),$$

with

$$A(q) = \frac{2}{3} q^{-1} (1 + q)^{1/2} [(1 + q)^{3/2} - q^{3/2} - 1].$$

Numerical results for the dimensionless function $A(q)$, for a few values of the mass ratio, $q$, are given by Milgrom (1986), and agree with those determined from the analytic expression (19). We note that $A(q)$ varies rather little across its full range, from $A = 1$ at $q = 0$ to $A \approx 0.8$, at $q = 1$. ($A$ is, of course, symmetric under the exchange of the two masses $q \leftrightarrow q^{-1}$, because the forces on the two masses are equal.)
Similarly, we can calculate the force (of a more academic interest) on any of \( n \) equal masses \( M/n \), on the vertices of a regular polygon, of diameter \( 2r \), with a mass \( m \) at its centre.

\[
F(M, m, n, r) = \frac{2}{3n}(M + m)^{3/2}(Ga_o)^{1/2} \left[ 1 - \frac{m^{3/2} + n^{-1/2}M^{3/2}}{(M + m)^{3/2}} \right] r^{-1}. \tag{20}
\]

This was checked numerically, by R. Brada (private communication), for some configurations.

It is useful to specialize the \( M - v \) relation to rotationally supported disks (e.g. a very low acceleration disk galaxies). If \( \Sigma(r) \) is the surface density, and \( v(r) \) the rotation curve, then eq. (14) implies the relation

\[
\frac{2}{3}M^{3/2}(Ga_o)^{1/2} = \int_0^\infty 2\pi r\Sigma(r)v^2(r) \, dr, \tag{21}
\]

where \( M = \int_0^\infty 2\pi r\Sigma(r) \, dr \), is the disk’s mass. If we add a point mass \( m \) at the centre, representing a bulge, say (within a radius that is small compared with the dimensions of the disk). eq. (17) tells us that the relation is now

\[
\frac{2}{3}[(M + m)^{3/2} - m^{3/2}](Ga_o)^{1/2} = \int_0^\infty 2\pi r\Sigma(r)v^2(r) \, dr. \tag{22}
\]

If we consider a special case of a thin ring of mass \( M \) and radius \( r \), with a point mass \( m \) in the centre, we can use eq. (22) to calculate the gravitational force on the ring (\( M \) times the force on a unit mass):

\[
F = \frac{2}{3}(M + m)^{3/2}(Ga_o)^{1/2} \left[ 1 - \left( \frac{m}{M + m} \right)^{3/2} \right] r^{-1}. \tag{23}
\]

(This is also a special case of eq. (20), taking there the limit of \( nF \) for \( n \to \infty \).)

IV. A revisitation of the masses of the dwarfs

The 3-D rms velocity, which enters the \( M - v \) relation is not, in itself, directly observable, and we have to express \( \langle v^2 \rangle \) in terms of the observed, integrated, 1-D rms velocity, \( v_\parallel \). This, as usual, involves assumptions on the velocity structure of the system. If \( v_\parallel \) is independent of the direction of the line of sight—as when the velocity distribution is isotropic—we have \( \langle v^2 \rangle = 3v_\parallel^2 \), and the mass estimator for this case is

\[
M = \frac{81}{4}(v_\parallel^4/Ga_o). \tag{24}
\]

If the motions in the system are confined to a plane that is at an inclination \( i \) to the line of sight, and if, in that plane, the overall velocity distribution
is isotropic—as is the case for any axisymmetric systems, such as a rotation-supported disk—then \( \langle v^2 \rangle = (2/sin^2 i)v_\|^2 \), and

\[
M = \frac{9}{sin^4 i}(v_\|^4/Ga_o).
\]  

We see that the LSY estimator could, in the case of flat systems, be even more discrepant with the correct one. For example for \( i < 45^\circ \) we get a factor of discrepancy \( > 36 \) (for \( i < 30^\circ \) the factor is \( > 144 \)).

Two factors conjoin to give the large disparity between the naive mass estimator used by LSY, and the correct expression. The estimator given in eq (1), which LSY mimic, uses the asymptotic circular velocity around the system. However, the (3-D) rms velocity in a system is smaller than the asymptotic rotational velocity corresponding to system’s mass. Also, the measured line-of-sight rms velocity is of course smaller then the 3-D one. Since these ratios enter in the fourth power, the resulting disparity is large. As \( v_\| \) is approximately \( v_h \) we see that the mass estimator used by LSY differs by a large factor from the correct one. [In fact, \( v_\| \) may be somewhat smaller than \( v_h \)—according to LSY \( v_h \approx 1.18v_\| \)—but the correction introduced by this difference is compensated by the fact that LSY use the outdated pristine estimate of \( a_o \) given by Milgrom 1983b, when the best value today, which is based on detailed studies of rotation curves, is about a factor of two smaller (Begeman, Broeils, and Sanders 1991)].

In default of more detailed information on the galaxies studied by LSY, the best we can do to estimate their MOND masses is to use relation (24). This means that we simply have to multiply LSY’s MOND masses by a factor of twenty. Because deduced MOND masses are subject to large errors, as they depend on the fourth power of the velocities, we feel it is more appropriate to use the “luminous” masses in relation (24) in order to calculate from them the velocity dispersions that MOND implies, and compare these with the observed line-of-sight velocities \( v_\| \) (designated \( \sigma_T \) by LSY). We do this in Table 1 where we give \( v_\|| = \sigma_T \) along with the total line-of-sight velocity dispersions predicted by MOND on the basis of the stationary \( M - v_\| \) relation (24), for the isotropic case. These are calculated for three values of the stellar \( M/L : \alpha = 0.5, 1., 4 \) solar units, and are designated \( v_\| (\alpha) \). (We multiply the HI mass by a factor of 1.3 to account for He.) We use the value \( a_o = 1.2 \ 10^{-8} cm/s^2 \) (Begeman, Broeils, and Sanders 1991). We see that the observed \( v_\| \) falls within the predicted range of velocities corresponding to the range of \( M/L \) values we use, in all but two galaxies, where MOND predict higher velocities than observed (the quoted errors in \( v_\| \) are about \( \pm 20\% \)).

We must note that the comparison for individual cases is subject to large uncertainties due to the following factors: (i) We do not know that the systems under study are in a stationary state, so that their momentary, observed rms velocity equals the long-time average. In fact, LSY state that the velocity fields of some of the galaxies suggest radial motions such as expansion or contraction. Non-stationary systems spend more time in a state of lower-than-average rms velocity. If departure from stationarity is important we would expect that the velocities deduced from the \( M - v \) relation—representing long-time averages—would be, by and large, larger than the momentary, observed ones. (ii) Our \( M - v \) relation assumes that the HI is the dominant mass component (i.e. is self gravitating). This is clearly not so in some of the cases (for which LSY find \( M_H/L_B < 1 \)); notably in LGS-3, and DDO216, but also in DDO69,
and DDO155). In these cases, the deduced velocities depend strongly on the distributions of both the stellar, and the HI masses which we do not know. Here there is an additional uncertainty in the value of the observed masses due to that in the stellar $M/L$, which we try to cover in Table 1. (iii) The distances to the galaxies in the sample are poorly known according to LSY. The value of the deduced velocities is proportional to the square root of the observed distance. (iv) The observed line-of-sight velocities need not be $3^{-1/2}$ of the 3-D rms velocities as we assume in applying the $M - v_\parallel$ relation for the isotropic case.

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References
Begeman, K.G., Broeils, A.H., and Sanders, R.H., 1991, MNRAS, 249, 523
Bekenstein, J., and Milgrom, M., 1984, ApJ 286, 7
Gerhard, O.E., and Spergel, D.N. 1992, ApJ 397, 38
Lo, K.Y., Sargent, W.L.W., and Young, K. 1993, AJ 106, 507
Milgrom, M., 1983a, ApJ 270, 365
Milgrom, M., 1983b, ApJ 270, 371
Milgrom, M., 1984, ApJ 287, 571
Milgrom, M., 1986, ApJ 302, 617
Milgrom, M., 1993, Annals of Physics, In the press
| Name        | $M_{HI}$ | $L_B$ | $v_\parallel$ | $v_\parallel(0.5)$ | $v_\parallel(1)$ | $v_\parallel(4)$ |
|-------------|----------|-------|---------------|-------------------|-----------------|-----------------|
| LGS-3       | 0.02     | 0.07  | 7.5 ± 1.3     | 4.7               | 5.3             | 7.0             |
| UGC4483     | 3.7      | 2.3   | 13.0          | 14.5              | 15.4            | 18.2            |
| DDO69       | 3.6      | 3.6   | 8.0           | 15.0              | 16.0            | 19.7            |
| CVn dwA     | 8.1      | 1.1   | 9.5           | 17.2              | 17.4            | 18.5            |
| DDO155      | 0.2      | 0.23  | 10.5          | 7.4               | 7.9             | 9.9             |
| DDO187      | 5.0      | 2.3   | 13.0          | 15.6              | 16.2            | 18.7            |
| Sag D1G     | 0.8      | 0.25  | 8.0           | 7.8               | 10.1            | 11.3            |
| DDO210      | 0.3      | 0.13  | 8.0           | 7.7               | 8.0             | 9.2             |
| DDO216      | 1.3      | 6.9   | 10.0          | 14.2              | 16.1            | 21.9            |

Table 1. Observed masses, luminosities, and line-of-sight velocity dispersions for the dwarfs, along with velocity dispersions predicted by MOND $v_\parallel(\alpha)$, for assumed $M/L = \alpha$ in solar units. All masses in units of $10^7 M_\odot$, luminosities in units of $10^7 L_\odot$, and velocities in km/s.