Nonlinear dynamic response analysis of localized damaged laminated composite structures in the context of component mode synthesis

S Mahmoudi, F Trivaudey and N Bouhaddi

Applied Mechanics Department, FEMTO-ST institute - UMR 6174, University of Franche-Comté-24, chemin de l’Épitaphe, 25000 Besançon, France
E-mail: (∗) noureddine.bouhaddi@univ-fcomte.fr

Abstract. The aim of this study is the prediction of the dynamic response of damaged laminated composite structures in the context of component mode synthesis. Hence, a method of damage localization of complex structures is proposed. The dynamic behavior of transversely isotropic layers is expressed through elasticity coupled with damage based on an existing macro model for cracked structures. The damage is located only in some regions of the whole structure, which is decomposed on substructures. The incremental linear dynamic governing equations are obtained by using the classical linear Kirchhoff-Love theory of plates. Then, considering the damage-induced nonlinearity, the obtained nonlinear dynamic equations are solved in time domain. However, a detailed finite element modelling of such structure on the scale of localized damage would generate very high computational costs. To reduce this cost, Component Mode Synthesis method (CMS) is used for modelling a nonlinear fine-scale substructure damaged, connected to linear dynamic models of the remaining substructures, which can be condensed and not updated at each iteration. Numerical results show that the mechanical properties of the structure highly change when damage is taken into account. Under an impact load, damage increases and reaches its highest value with the maximum of the applied load and then remains unchanged. Besides, the eigenfrequencies of the damaged structure decrease comparing with those of an undamaged one. This methodology can be used for monitoring strategies and lifetime estimations of hybrid complex structures due to the damage state is known in space and time.

Keywords: nonlinear dynamic, damage prediction, structural monitoring, composite structures.

1. Introduction

Over the last decades, composite materials have become increasingly used in industry, especially in high-tech products such as the defense and aerospace industries. This wide use of these materials is due to their excellent mechanical characteristics, particularly the high values of rigidity/weight and strength/weight ratios. As an example, in the new project of Airbus, about 53% of the total weight of the A-350 aircraft is made of composite materials. Comparing to metallic materials, composite structures are characterized by a sufficient strength with low density. In addition, they have other characteristics such as their resistance to corrosion and to chemical attack. The heterogeneous nature of these composite materials can provide a source of damage mechanism growing if the composite structure is subjected to a sufficient load. Indeed, the ruin of composite materials is not due to a single mechanism but is the consequence of the accumulation of several modes of degradation (matrix cracking, delamination, fiber breakage...).
So, the knowledge and the understanding of these mechanisms become a necessity during the design phase. It is then necessary to investigate the dynamic behavior of these composite structures taking into account the damage phenomenon and the effect of this mechanism on the overall behavior of the system. Thus, detection of the location and especially the degree of damage severity is of great importance in order to ensure the reliability and safety of service structures since the static or the dynamic responses can be modified by the damage evolution. To characterize this change in mechanical properties and in the dynamic characteristics, various damage indicators are used such as natural frequencies [1]. Kulkarni et al [2] have studied the presence and the detection of delamination in a composite structure based on the decrease of eigenfrequencies. This idea was further investigated and deepened by Cawley et al [3] suggesting that the shift of eigenfrequencies can be considered as a basis for a new nondestructive control technique. Moreover, there are other indices which can be used to study this relationship between the damage and the decrease in mechanical properties, such as flexibility matrix [4] and mode shapes [5].

The main objective of this paper is the construction of a new numerical modeling in order to predict the dynamic response and the damage evolution of laminated composites structures. Hence, a method of damage localization of complex structures is proposed in context of component mode synthesis. Based on a phenomenological macro model for cracked composites structures, the dynamic behavior of transversely isotropic layers is expressed through elasticity coupled with damage. The studied structure is made of transversely isotropic layers of polymer matrix reinforced with long fibers and it is decomposed in undamaged and damaged substructures. The damage is expressed by a single scalar variable and its evolution is determined by the principal of maximum dissipation. Then, using the classical Kirchhoff-Love theory, and assuming that damage induces nonlinearity, the resulting nonlinear formulation is implemented in MATLAB® software. Component Mode Synthesis (CMS) is introduced for modelling a nonlinear fine-scale damaged substructure which should be updated at each iteration due to the damage evolution. This nonlinear modelling is then connected to a linear dynamic models of the remaining substructures which can be condensed without updating in order to reduce the computational cost. Numerical results show that the mechanical properties of the structure highly change when damage is taken into account. Under an impact load, damage increases and reaches its highest value with the maximum of the applied load and then remains unchanged. Besides, the eigenfrequencies of the damaged structure decrease comparing with those of an undamaged one. This methodology can be used for monitoring strategies and lifetime estimations of hybrid complex structures.

2. Problem formulation: finite element modeling
The finite element method is currently a very reliable engineering tool as a result of its flexibility and its relative ease in numerical implementation. There are several alternative techniques of modeling by finite elements of the composites structures. In this context, several theories were extended from modeling isotropic structures towards modeling composite structures. These formulations are essentially divided into two categories; those which are based on the Equivalent Single Layer Theory and those which are based on the Layer-Wise theory. The first category includes mainly the Classical Laminate Theory, the First-order (FSDT) and High-order (HSDT) Shear Deformation Theories. While the second category includes the Independent and Dependent layers theories. These formulations are more detailed in several works including [6, 7]. The primary difference between FSDT and HSDT theories is the order the polynomial function adopted for approximating the displacement field variables of the structure. The choice of one or other of these two formulations is mainly conditioned by the thickness of the structure to be modeled. For thin composite structures, it is recommended to use the FSDT theory which has a lower computational cost comparing to the HSDT theory. So, the displacement field of a
composite plate can be written as follow:

\[
\{u(x, y, z, t)\} = \begin{cases} u(x, y, z, t) \\ v(x, y, z, t) \\ w(x, y, z, t) \end{cases} + z \begin{cases} \varphi_x(x, y, t) \\ \varphi_y(x, y, t) \end{cases} = 0
\]

(1)

where \(\varphi_x(x, y, t)\) and \(\varphi_y(x, y, t)\) are the rotation, respectively, with respect to the \(\vec{x}\) and \(\vec{y}\) axes. Equation (1) shows that the finite element model should include the degrees of freedom in translations \((u, v, w)\) as well as rotations \(\varphi_x(x, y, t)\) and \(\varphi_y(x, y, t)\), i.e., five degrees of freedom per node. The finite element approximation of the global displacements can be expressed in terms of nodal displacements \(\{u^e_n\}\) by introducing the matrix \([N(\xi, \eta)]\) of interpolation function such that:

\[
\{u\} = [N(\xi, \eta)] \{u^e_n\}
\]

(2)

The quadrangular element Serendip Q8 with eight nodes is used to mesh the structure. Using virtual work principle where \(E_p\) and \(E_c\) are respectively the total potential and kinetic energies of the structure, then:

\[
\delta \int_{t_1}^{t_2} (E_p + E_c) dt = 0
\]

(3)

The Gauss integration method with 4 integration points and a single point in the thickness direction, is used to obtain the global mass and stiffness matrices \([M]\) and \([K]\). After admitting the assumption of proportionality of the damping expressed by \([B] = \alpha_1[K] + \alpha_2[M]\), the dynamic equation of motion is obtained as shown in (4) where \(D\) is a scalar variable representing the relative reduction of the transverse Young’s modulus and it will be characterized later.

\[
[M] \ddot{u} + [B] \dot{u} + [K(D)] u = F(t)
\]

(4)

Damage plays an important role in dynamic response modification of composite materials. The elastic behavior of the matrix is modified by the formation and evolution of micro cracks and cavities during the loading \(F\). These defects cause primarily the degradation of the stiffness through the decrease of three elastic modulus \(E_2, G_{12}\) and \(G_{23}\), according to the stress

\[
\begin{align*}
\varv_1 (M1) & \quad \varv_2 (M2) \\
\varv_2 & \quad \varv_1 (M1) \\
\varv_3 (M1) & \quad \varv_3 (M2) \\
\end{align*}
\]

Figure 1. Micro-cracks orientation in the matrix [8]
In order to characterise the reduction in the transverse Young’s modulus $E_2$ and the shear module $G_{12}$ and $G_{23}$, three parameters $D_I$, $D_{II}$ and $D_{III}$ are introduced such as:

$$D_I = \frac{\Delta E_2}{E_2} ; \quad D_{II} = \frac{\Delta G_{12}}{G_{12}} ; \quad D_{III} = \frac{\Delta G_{23}}{G_{23}}$$

(5)

Using a self-consistent method [9] which permits to express $D_{II}$ and $D_{III}$ as a function of $D_I$, a damage matrix $[H(D)]$ is fully expressed by a single scalar variable $D$ representing the relative reduction of the transverse Young’s modulus:

$$[H(D)] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & H_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & H_{14} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & H_{66} & 0 \end{bmatrix} ; \quad \begin{cases} H_{22} = \frac{D}{1 - D} S_{22} \\ H_{44} = \frac{D}{\sqrt{1 - D}} \sqrt{S_{11} S_{22}} \\ H_{66} = \frac{D}{\sqrt{1 - D}} S_{22} \end{cases}$$

(6)

Where $S_{ii}$ depict the flexibility components of $[S]$ of the undamaged material. The change in the mechanical properties is visible in equation (7) where the $[H(D)]$ is added to the flexibility matrix to describe the damaged elastic behavior of the composite structure.

$$\varepsilon = [S + H(D)] : \sigma$$

(7)

The damage evolution law is expressed through the thermodynamics of irreversible processes. A thermodynamic force $Y$ associated to the damage is obtained, $Y = \frac{1}{2} \sigma^T \left( \frac{\partial H}{\partial D} \right) \sigma$, where $\sigma^T$ is the transpose of the stress tensor $\sigma$ and $\left( \frac{\partial H}{\partial D} \right)$ is the derivative of $[H(D)]$ with respect to $D$.

A threshold function $\tilde{Y}$ is defined in order to record the damage level and adapt the threshold of damage activation to the damage level. Thereby, the following charge function is established:

$$f_d = Y - \tilde{Y} = \frac{1}{2} \sigma^T \left( \frac{\partial H}{\partial D} \right) \sigma - (Y_c + q D^p)$$

(8)

where the constants $Y_c$, $q$ and $p$ are damage parameters.

3. Component mode synthesis (CMS): Craig-Bampton method

The numerical costs related to computing time and the important data storage for complex systems are important. So, looking for models with a small number of degrees of freedom, called Reduced Order Model, is a major issue which reduces the computation time and the storage space. CMS approach is commonly used in complex finite element analysis models whose consist of an assembly of reduced models. In this paper, the studied structure is composed of damaged and undamaged components and its nonlinear finite element modelling has a large size leading to an important computational costs. For these reasons, an approach by substructuring of the whole complex structure using the Craig-Bampton method [10] is used as a technique of model reduction where the structure is decomposed in two substructures as shown is figure 2. Indeed, this method permits to limit the impact of a modification to a considered component. The method requires first to treat each component separately and determine its reduction basis. In order to perform the Craig-Bampton method, the model is divided into two sets of degrees of freedom (DOFs). The DOFs of interface with subscript ”j” and the interior DOFs with subscript ”i”, i.e. $q^k = \{ q_{ik}^k, q_{jk}^k \}$ where the exponent $k$ denotes the number of the substructure. Then, a
Figure 2. Real composite structure and its decomposition for the 2D finite element reduced model

The component fixed-interface normal modes $\varphi_{ii}^k$ are obtained by solving the generalized eigenvalue problem:

$$[K_{ii}^k - w^2 M_{ii}^k] \varphi_{ii}^k = 0 \quad (10)$$

The interface constrained modes matrix $\psi_c^k$ is:

$$\psi_c^k = \left[\begin{array}{c} \psi_{ij}^k \\ I_{jj} \end{array} \right] = \left[\begin{array}{c} -K_{ii}^{-1} K_{ij}^k \\ I_{jj} \end{array} \right] \quad (11)$$

So, the Craig-Bampton transformation matrix $T_{CB}^k$ is defined as follow where the truncated basis $\Phi_{ii}$ contains the first retained normal modes $\varphi_{ii}^k$:

$$T_{CB}^k = \left[\begin{array}{c} \Phi_{ii} \\ 0 \\ I_{jj} \end{array} \right]^k \quad (12)$$

Then, the reduced component mass, stiffness and damping matrices are:

$$M_{CB}^k = T_{CB}^k M^k T_{CB}^k ; \quad K_{CB}^k = T_{CB}^k K^k T_{CB}^k ; \quad B_{CB}^k = T_{CB}^k B^k T_{CB}^k \quad (13)$$

Finally, the reduced model is formed of the assembly of the components models which are condensed separately.
4. Solving

Noting that the damage phenomenon is irreversible, the damaged state is kept in memory when the structure is subjected to a load. For this reason an incremental way in solving process is recommended. So, at each time iteration, a stress increment $\Delta \sigma$ is applied. Then, the accumulation of stress continues until the function $f_d$ becomes positive which implies the creation of a damage increment $\Delta D$ in order to kept the function $\dot{f}_d = 0$. It will be obtained by solving the consistence equation $\dot{f}_d$ using the Newton-Raphson method. Expressing $\dot{f}_d$ as a function of $\Delta D$ and knowing the damage $D_i$ at $(i)$th iteration, the solution of $\dot{f}_d = 0$ is reduced to seek the damage increment $\Delta D$ which cancels the function $f_{d,i+1}$ for the $(i+1)$th iteration i.e Eq(14):

$$ f_{d,i+1} = \frac{1}{2} \left[ \sigma_{i+1} H'(D_i + \Delta D) \sigma_{i+1} \right] - \left[ Y_c + q(D_i + \Delta D)^p \right] $$

(14)

With $H'(D_i + \Delta D)$ denotes the derivative of the damage matrix $[H(D)]$ with respect to $D$ and expressed at $(D_i + \Delta D)$. Once the damage increment is calculated, the Craig-Bampton reduction method is applied. The undamaged substructure is condensed and keeps its initial properties. Whereas, the stiffness of the damaged substructure depends on the damage level $D$ obtained at each time iteration. The number of retained modes of the reduced substructures is chosen so that the frequency band is twice of the frequency band of interest. The Craig-Bampton reduction is applied and only the stiffness matrix of the damaged substructure is updated to take into account of this change of material properties. The Newmark implicit predictor-corrector scheme [11] is used to solve the resulting dynamic nonlinear problem in time domain.

5. Results and discussion

Several numerical simulations have been performed in order to highlight the influence of damage on the dynamic response. The geometrical and mechanical properties of the considered laminated structure are given in table 1. The structure is made of three layers oriented as $(90^\circ/0^\circ/90^\circ)$, it is clamped at $(x = 0, y, z = 0)$ and at $(x = 0.3, y, z = -0.3)$. It is subjected to a distributed impact in $\vec{x}$ direction and along the side of junction between the two beams with $F = 1917.5 \text{ N}$ as magnitude and $\tau = 1 \text{ ms}$ as a duration.

In order to validate the chosen CMS method, a comparison in terms of eigenfrequencies was made as depicted in Figure 3. The black bars show the eigenfrequency difference in percent between the eigenfrequencies $f_{\text{red}}$ and $f_{\text{unred}}$, respectively, of the reduced and the unreduced

| Table 1. Geometrical and mechanical properties of the laminated beam |
|---------------------------------------------------------------|
| Elastic modulus $E_1 (\vec{e}_1)$ | 45680 MPa |
| Elastic modulus $E_2 (\vec{e}_2)$ | 16470 MPa |
| Shear modulus $G_{12}$ | 6760 MPa |
| Poisson ratio $\nu_{12}$ | 0.34 |
| Poisson ratio $\nu_{23}$ | 0.34 |
| $Y_c$ | 0.0027 MPa |
| $q$ | 1.246 MPa |
| $p$ | 0.816 |
| $L_1 : (\vec{x})$ | 0.3 m |
| $L_2 : (\vec{z})$ | $-0.3$ m |
| $b : (\vec{y})$ | 0.03 m |
| Thickness $(\vec{z}) : h_1$ | 0.001 m |
| Thickness $(\vec{x}) : h_2$ | 0.001 m |
Figure 3. Error in percent of the eigenfrequencies between reduced and unreduced (full) models (full) models. Figure 3 shows a larger eigenfrequency difference in the 7th and 8th modes which are respectively a torsion and a membrane vibration mode. The largest difference in these frequencies is lower than $3.5 \times 10^{-4}$% which permits to validate the CMS method.

The nodal displacement with respect to $\vec{x}$ direction of the junction side are depicted in figure 4 where the displacement amplitude is smaller in the undamaged case than in the damaged one. As a consequence, damage has an important impact on the dynamic response since it affects the structure stiffness. The damage state in the layer oriented as $90^\circ$ of the damaged substucture is depicted in figure 5-(b), where the damage is localized and quantified according to the (x,y) coordinates . According to figure 5-(a) and (b), it can be seen that there is a spatial correlation between the stress propagation and the damage state. The stress and the damage are maximum at ($x = 0.25 m$) where there is significant bending during impact load. The effect of damage is more visible in figure 6 when the eigenfrequencies $f_D$ and $f_0$ respectively on the damaged and undamaged cases are compared. As a result, it can be seen that when the damage is taken into account, the eigenfrequencies have a significant decrease which reaches up to 10% for some modes.

Figure 4. Dynamic impulse response in the $\vec{x}$ direction of the nodes of the junction side
6. Conclusion

The non-linear dynamic response of a locally damaged composite structure composed of two laminated beams was investigated including the material nonlinearity generated by damage. Thus, based on a phenomenological macro model for cracked composites structures, a method of damage prediction and localization of complex structures is proposed and investigated in the context of CMS. Hence, the Craig-Bampton reduction method is introduced as a CMS technique. The resulting nonlinear modelling of the damaged component is connected to the condensed linear modelling of the remaining substructure reducing the computational cost. Several numerical simulations have been performed to highlight the effect of the damage on the dynamic behavior, particularly, the eigenfrequencies and the displacement responses. Thereby, the damage modifies significantly the dynamic response by the increasing the displacement response amplitude. Indeed, the eigenfrequencies decrease when the damage is taken into account due to the modification of the material properties of the damaged substructure. These modifications are introduced into the dynamic analysis during loading. Therefore, this methodology can be used in life time estimations and monitoring strategies of the complex structures.
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