Mathematical Modelling of Ice-Structure Interaction

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Abstract. Determination of the force effect of ice formations on the marine structure is based on a model of interaction with the edge of the ice field. The development of a model of ice failure accounting for the results of studies of the failure mechanism of the ice field edge with the surface of the vertical shape structure is presented. The processes of cracking and shear of different nature with the formation of large and small fragments are described with mathematical model. A criterion of the limit value of the accumulated potential energy of elastic compression of the ice volume before structure is proposed as specific energy of mechanical failure of sea ice. The values of this criterion, which takes into account all types of ice failure, are obtained experimentally by testing large samples of ice on dynamic crushing.

1. Introduction

The ice failure at the contact of the edge of drifting ice field and marine structure can lead to dangerous vibrations and high dynamic loads. This phenomenon leads to reduction the reliability of the structure. The solution of this task focuses on the quantification of physical and mechanical characteristics of sea ice, that are the key parameters calculation of ice force. The approach simplifies the description of the mechanisms of development the failure process at the edge of the ice field during its impact on structure and also gives the opportunity to implement a new approach to solving this problem by mathematical modeling using Mohr-Coulomb criterion.

Possible reason for the lack correct formulas for ice forces, which adequate description of unique failure mechanism of ice is the use of force criterion for calculations of stress state of the ice in the contact area of the structure. These characteristics are the specific energy of failure ε_cr, the internal friction angle φ and cohesion C that can be obtained from the full scale field measurements.

2. Background

Analysis of the failure mechanisms shows that the main reason is compression in the contact zone. The first radial cracks are formed during the interaction dividing the ice feature on the number of prismatic fragments. Experimental studies have shown that the angle of internal friction of ice in its turn greatly depends on the time of impact force (Zanegin et al., 1990). Therefore, when cutting the ice field by structure, in addition to the geometric characteristics of the aspect ratio d/h on its value is affected by the ice velocity (Kim et al., 1992).
At the beginning of active investigations devoted to this problem, factors of ice failure mechanism identification were defined as follows: velocity, temperature and contact area. The combination of these factors has defined ice failure process: spall of free surface of cold ice subject to high velocity of loading due to significant variation of contact forces values; crushing of "warm" ice subject to relatively low velocity of loading due to smooth changing of load (Sodhi and Morris, 1986).

The mechanism of ice failure have the complex character. The part of ice movement energy at a contact zone is spent for failure, i.e. for compressive and shear cracking, crushing and displacement of fragments out of the contact zone. The sizes, number and orientation of cracks defines the volume of compressed area in front of the structure. Contact force due to ice penetration is usually greater than in case of ice field stability loss, so calculation can be referred to penetration of structure. The contact pressure decreases due to increasing of d.

3. Mathematical model of ice failure
Initially the radial crack passing through the axis of symmetry of the pier in the direction of ice movement. The cracks appearance and are in a horizontal plane (Tstrupik, 2012; Tstrupik et al., 2017). The next step is compression of prismatic ice fragments formed as a result of crack formation. The ice in this state substantially saturated brine. Its failure occurs the liquid phase until finely ground fractions which redistribute the resulting stress in all directions with decreasing of temperature and increasing the cracking velocity. For sea ice such behaviour is favoured by low speed increasing the force in combination with increasing temperature, when the saturation of the brine increases considering the intergranular interaction (Jamashita et al., 1985).

Figure 1 shows the ice failure with periodic crushing mode and formation ice spalls on ice field edge reducing the ice thickness up to effective thickness $h'$. 

![Figure 1. The scheme of ice-structure interaction.](image)

The value $h'$ decreases with velocity to be higher, and the size of spalls grows in the zone of ice failure. The alternating process of splitting-crushing a saw-tooth force dependence can be observed (Michel and Blanchet, 1983). These models have been developed since the late 1960's and this complex interaction process was effectively described by the model of system with one degree of freedom. The force concept of periodic ice fragmentation was proposed by Peyton (1966).
The solution of the problem of cylindrical body indentation into the half-space (Smirnov-Alyayev, 1978) gives the empirical relationship for radius of the zone of plastic deformation. The stress decreases with increasing distance from the contact surface of a pier. All stress deposits near the contact are compressive. At the outer boundary of the plastic region of the radial $\sigma_r$ and axial $\sigma_z$ stresses are compressive, the $\sigma_\theta$ are tensile. The radius from which will take place the origin of the radial cracks is at a distance $2.5r$ from the contact surface. On the outer boundary of the plastic zone $\sigma_z$ absent, and the values of the circumferential stresses $\sigma_\theta$ reach the radial. This means that all points lying on the surface, are in a state of pure shear. The distribution of stresses in the ice box before pier suggests that contact ice about pier these stresses can increase the tensile strength of ice. We consider the case of distributed force on the edge of the circular ice field. The radial and circumferential stresses on the axis of symmetry are we propose to use the Coulomb-Mohr criteria

$$A = \frac{1+\sin \varphi}{1-\sin \varphi}$$

After transformation

$$P = \frac{9.38d}{A-5}$$

or

$$P = \sigma_o \, d \, K$$

where

$$K = \frac{2.345(1-\sin \varphi)}{1.51 \sin \varphi-1}$$

If the force is distributed according to cosinus function then

$$K = \frac{4.69(1-\sin \varphi)}{1.51 \sin \varphi-1}$$

Thus, for uniformly distributed ice pressure on the ice thickness

$$P = K \frac{2c \cos \varphi}{1-\sin \varphi} \, d$$

The ice before the pier can be thought of as part of thick-walled pipes with internal radios $r$ under internal pressure. In this case the stress according to the Lamb’s solution

$$\sigma_{r\theta} = \pm P \frac{r^2}{R_n^2}$$

The decrease in stress with increasing $R_n$ is not unlimited. The stress at points located at a distance $R_n=5r$ are 0.04 of the maximum values occurring at the ice-structure interface. Therefore, content with an accuracy of about 4% can be considered that the stress at a distance $5r$ is small. Field observations (Peyton, 1966) indicated the cyclic failure of the ice.

It is of interest to consider the application of the deformation criterion. Experiments of the authors show uniaxial compressive state for the critical strain value $\varepsilon_{cr}$ without significant increasing of stress. Compression of ice, separated by a group of cracks, before structure can be thought of as a deformation of individual shape under uniaxial compression. Then the absolute deformation of one cycle can be defined by the expression

$$\varepsilon = \varepsilon_{cr} \, 2d$$
The ice velocity determines the time required for one cycle of ice failure

\[ t = \frac{e_{sp}2d}{V} \]  

(8)

So, one can evaluate the average strain rate for every cycle

\[ e = \frac{V}{2d} \]  

(9)

4. Conclusions
The existing standard methods of ice load calculation provides adequacy of calculations of ice load acting on marine vertical face structures that is justified by continuous investigations. However, in compliance with performed analysis of several aspects of its practical application, they still demonstrate several problems that cannot be settled on the basis of standard methods due to significant non-conformity of the applied force criterion (uniaxial strength) against tensile state of ice massive in contact with structural support. There is therefore a search of another ice strength criterion which should be more stable and easily determined in tests.

These facts provide the possibility of method development for the calculation of ice load acting upon the structure on the basis of application of the specific energy of ice failure of as an alternative to existing methods. The proposed approach not only can simplify the description of the mechanisms of development the fracture process of the edge of the ice plate during its impact on structure, but also gives the opportunity to implement a new approach to solving this challenge by methods of numerical modeling. The solution of this task focuses on the quantification of physical and mechanical characteristics of sea ice, that are the key parameters calculation of ice force.

Application of specific energy of ice \( e_{sp} \) for calculation of marine ice-resistant foundations on to cycle ice load, it seems completely justified and efficient because the energy has definite physical sense and better another parameters of strength correspond to essence of notions about it as about a complex of potential power barriers preventing the develop of kinetic processes in ice.

5. References
[1] Jamashita M, Katayama M, Taguchi Y, Nawata T, Kawasaki T, Kayo Y and Tozawa S 1985 Model test and analytical simulation of fracture mechanism ice Proc. International Conference on Port and Ocean Engineering under Arctic Conditions Conf. (Narssarsuq) pp 195-204
[2] Kim L V, Bekker A T and Khrapaty N G 1992 Kinematic study of pile-soil system Proc. International Offshore and Polar Engineering Conf. (San-Francisco) pp 688-91
[3] Michel B and Blanchet D 1983 Indentation of an S2 floating ice sheet in the brittle range Annals of Glaciology pp 180-7
[4] Peyton H R 1966 Sea ice forces. Ice pressures against structures Technical Memorandum, 92 National Research Council of Canada, (Ottawa) pp 117–23
[5] Smirnov-Alyayev A 1978 Resistance of materials to plastic deformation (Leningrad: Mashinostroeniye Publ. 365 p
[6] Sodhi D S and Morris C E 1986 Characteristic frequency of force variations in continuous crushing of sheet tee against rigid cylindrical structures Cold Regions Science and Technology 12 pp 1-12
[7] Tsuprik V G 2012 Theoretical and Experimental Studies of Specific Energy of Mechanical Failure of Sea Ice Proc. 22nd International Offshore and Polar Eng. Conf. (Rhodes) pp 1242-6
[8] Tsuprik V G, Bekker A T and Pomnikov E E 2017 Studies of Specific Energy Fracture of Ice Using Method Test Samples on Uniaxial Compression Proc. International Offshore and Polar Engineering Conf. (San Francisco) pp 1319-25
[9] Zanegin V G, Khrapaty N G and Lyubimov V S 1990 Investigation of Sea Ice Shear Properties Proc. European Offshore Mechanics Symp. (Trondheim) pp 550-5