HADRONIC STRUCTURE FUNCTIONS OF THE PHOTON
MEASURED AT LEP

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The measurements of hadronic structure functions of the photon based on the reaction \( ee \to ee\gamma(\gamma^*) (P^2)\gamma^* (Q^2) \to ee \) hadrons are discussed. This review covers the latest developments in the analysis and the most recent measurements at LEP.

1 Introduction

One of the most powerful tools to investigate the structure of quasi-real photons, \( \gamma \), is the measurement of photon structure functions in deep inelastic electron-photon scattering at electron-positron colliders. A recent review on this topic along with the references to the published results can be found in [1].

The main idea is that by measuring the differential cross-section

\[
\frac{d^2\sigma_{ee\to eX}}{dx dQ^2} = \frac{2\pi\alpha^2}{x Q^4} \left[ (1 + (1 - y)^2) F_2^\gamma (x, Q^2) - y^2 F_L^\gamma (x, Q^2) \right],
\]

the photon structure function \( F_2^\gamma \) is obtained. Here \( Q^2 \) is the absolute value of the four momentum squared of the virtual photon, \( \gamma^* \); \( x \) and \( y \) are the usual dimensionless variables of deep inelastic scattering and \( \alpha \) is the fine structure constant.

In the region of small \( y \) studied \( (y \ll 1) \) the contribution of the term proportional to the longitudinal structure function \( F_L^\gamma \) is small and it is usually neglected. In leading order \( F_2^\gamma \) is proportional to the parton content, \( f_{i,\gamma} \), of the photon, \( F_2^\gamma = x \sum q^2 (f_{q,\gamma} + f_{\bar{q},\gamma}) \), where the sum runs over quarks \( q \) and antiquarks \( \bar{q} \) of charge \( e_q \). The hadronic structure function \( F_2^\gamma \) receives contributions both from the point-like and the hadron-like part of the photon structure. The point-like part can be calculated in perturbative QCD, whereas the hadron-like part is usually described based on the Vector Meson Dominance model.

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Because the energy of the quasi-real photon is not known, the value of $x = Q^2/(Q^2 + P^2 + W^2)$ has to be derived by measuring the invariant mass $W$ of the hadronic final state, which is a source of significant uncertainties, especially at low values of $x$, and makes measurements of $F_2^\gamma$ mainly systematics limited.

If both photons are virtual, Eq. (1) gets more complicated, however, in the region $Q^2 \gg P^2 \gg \Lambda^2$, where $\Lambda$ is the QCD scale, an effective structure function $F_\text{eff}^\gamma$ of virtual photons can be determined.

This review concentrates on the recent developments to reduce the systematic uncertainties, the newly evaluated radiative corrections to $F_2^\gamma$, and the latest measurements of $F_2^\gamma$ and $F_\text{eff}^\gamma$.

2 New developments in the analysis

In previous analyses it had been seen that the dominant error in the measurement of $F_2^\gamma$ at low $x$ is the imperfect modelling of the hadronic final state by the Monte Carlo programs. To reduce this error two approaches have been taken. Firstly, the LEP experiments have measured distributions of the hadronic final state corrected for detector effects. For large regions in most of the distributions studied the results of the different experiments agree with one another, and consequently the results have been combined while using the spread of the measurements as an estimate of the systematic uncertainty. Significant differences are found between the combined data and the predictions of the HERWIG and PHOJET Monte Carlo models. Therefore the combined LEP data serve as an important input to improve on the Monte Carlo models.

Secondly, several experiments have used improved unfolding methods to reduce the sensitivity of the result to the different predictions. The main idea is the following. If one assumes that the structure function $F_2^\gamma$ is independent of the fragmentation of the hadronic final state, then, in the one-dimensional unfolding, using the variable $x$, the result is independent of the actual shape of the input distribution function $f_{\text{part}}(x)$ used in the unfolding, and only depends on the transformation $A(x_{\text{vis}}, x)$ between the value of $x$ and the measured value $x_{\text{vis}}$. This transformation partly depends on the Monte Carlo model used, but also to a large extend on the detector capabilities which are independent of the chosen model. By using a second variable, $v$, the same argument applies to this variable. Now the result is largely independent of the joint input distribution function $f_{\text{part}}(x, v)$ and only the transformation $A(x_{\text{vis}}, v_{\text{vis}}, x, v)$ matters. Because only the transformation of $v$ but not its predicted distribution affects the unfolding result, part of the dependence on the Monte Carlo model is removed.

A quantity which has proven to be very useful as the second unfolding variable is the forward energy. As an example from ALEPH, Figure(left)
shows the correlation between $x$ and $x_{vis}$ at $\langle Q^2 \rangle = 56.5 \text{ GeV}^2$ for several bins of the energy observed in the forward region below a polar angle of 17 degrees, denoted with $E_{17}$. If very little energy is observed under small polar angles, which means the hadronic system is well contained in the central detector, there is a good correlation between $x$ and $x_{vis}$. In contrast, for large values of the forward energy the correlation severely deteriorates. Using two-dimensional unfolding the result for $F_2$ is almost independent of the largely different predicted distributions of the forward energy.

At large $Q^2$ more transverse momentum is transferred to the hadronic system which therefore is better contained in the detector. It has been shown by L3 that in this case already a kinematic fit using energy momentum conservation gives a good correlation between the generated and measured hadronic invariant mass. Based on a generator simulating the quark parton model (QPM) prediction, this is demonstrated in Figure 1 for $\langle Q^2 \rangle = 120 \text{ GeV}^2$, for quasi-real target photons and also for virtual target photons with $\langle P^2 \rangle = 3.7 \text{ GeV}^2$ (lower right). Already without the kinematic fit an acceptable resolution is found for quasi-real target photons (lower left), which improves after the fit procedure has been applied leading to a good correlation (upper). For the virtual target photons the initial resolution is somewhat worse and consequently the fit yields a bigger improvement (lower right).
Usually QED radiative corrections to Eq. (1) are neglected in the analysis of $F_2^\gamma$. For this conference, OPAL for the first time presented a measurement corrected for this effect based on the prediction of the RADEG program, which takes into account initial state radiation from the deep-inelastically scattered electron and the Compton scattering process. It has been found that the radiative corrections are $x$-dependent and largest at small values of $x$, such that the shape of $F_2^\gamma$ is changed when these corrections are applied. An example of the size of the corrections is shown in Figure 2, where the arrows connect the values of $F_2^\gamma$ before and after the correction. With the present level of accuracy of the measurements the corrections are comparable to the statistical precision of the OPAL data.

A second effect, which is usually not corrected for is the predicted suppression of $F_2^\gamma$ due to the fact that the quasi-real target photon is slightly off-shell. This $P^2$ suppression is theoretically uncertain and the predictions vary by as much as a factor of two at low values of $x$. Therefore, at present, this correction should not be applied to the data, in order not to bias the experimental result towards a particular theoretical model.

3 Recent Measurements

Several new or recently finalised measurements have been presented at this conference. OPAL has updated and extended the measurements concentrating on the low $x$ behaviour of $F_2^\gamma$ for $\langle Q^2 \rangle$ values ranging from 1.9 to 17.8 GeV$^2$, examples of which are shown in Figure 3. By using improved Monte Carlo models, improved reconstruction techniques to measure the visible invariant
hadronic mass, and two-dimensional unfolding, the measurement errors have been considerably reduced compared to the published analysis.

In general the shape of the GRV LO parameterisation is consistent with the OPAL data in all the accessible $x$ and $Q^2$ regions. The normalisation is also consistent with the data, except at the lowest scale, $\langle Q^2 \rangle = 1.9$ GeV$^2$ Figure 3(a), where GRV is too low. Within the precision of the OPAL measurement, the description of the data by SaS1D LO is of similar quality as for GRV LO. Also for L3 the shape of the GRV LO parameterisation is consistent, however, in this case GRV lies below the data at $\langle Q^2 \rangle = 5$ GeV$^2$ Figure 3(b). The LEP results extend the reach at low $x$ compared to measurements of $F_2^\gamma$ performed at lower $e^+e^-$ centre-of-mass energies. The results from PLUTO nicely agree with the LEP data at high values of $x$, whereas the shape of TPC/2$\gamma$ is largely different from all other measurements.

The new results on $F_2$ from ALEPH are shown in Figure 4. Based on
52.9 pb\(^{-1}\) of data, taken at \(e^+e^-\) centre-of-mass energy of 183 GeV. \(F_2^\gamma\) has been obtained in two \(Q^2\) ranges of \(7 \leq Q^2 \leq 24\) GeV\(^2\) and \(17 \leq Q^2 \leq 200\) GeV\(^2\) with average values as indicated in the figure. The inner error bars represent the statistical errors while the outer ones include systematical uncertainties, mainly coming from the remaining model dependence and the details of the smoothing and regularisation technique used in the unfolding procedure. The new ALEPH result clearly disfavours the strongly rising \(F_2^\gamma\) prediction from LAC1. The very same effect, namely that predictions of steeply rising \(F_2^\gamma\) at low \(x\), driven by large gluon distribution functions of the photon are disfavoured by the data, had been seen previously\(^{[1]}\).

L3 has finalised the results for \(\langle Q^2 \rangle = 120\) GeV\(^2\) based on LEP data taken at \(e^+e^-\) centre-of-mass energies around the mass of the Z boson\(^{[2]}\). The
structure function $F_2^\gamma$ is not described by the quark parton model for $x < 0.4$, where the hadron-like component is expected to be largest. In this region the data are even higher than the predictions of several parametrisations of $F_2^\gamma$ which contain a hadron-like contribution $\overline{\text{a}}$.

In addition, the structure function $F_{\text{eff}}^\gamma$ was measured for $\langle Q^2 \rangle = 120 \text{ GeV}^2$ and $\langle P^2 \rangle = 3.7 \text{ GeV}^2$, thereby ensuring $Q^2 \gg P^2 \gg \Lambda^2$, Figure $\overline{\text{b}}$. As in the case of the PLUTO result $\overline{\text{c}}$, the QPM prediction is too low compared to the data. Also the GRS prediction falls short with respect to the data. However, this may be expected, because the GRS prediction only contains the contribution from transverse virtual target photons. The QPM prediction of the $P^2$ evolution of $F_{\text{eff}}^\gamma$ is consistent in shape with the data, but too low, with the most significant difference stemming from $F_2^\gamma$ at $P^2 = 0$ and for $x < 0.4$ $\overline{\text{d}}$. The measurement at $P^2 > 0$ cannot rule out the quark parton model prediction, although the data are consistently higher. For more detailed comparisons to be made the full statistics of the LEP2 programme has to be explored.

4 The present status of the measurements

In the present investigations of the photon structure function $F_2^\gamma$ two distinct features of the photon structure are studied. Firstly, the shape of $F_2^\gamma$ is measured as a function of $x$ at fixed $Q^2$. Particular emphasis is put on measuring the low $x$ behaviour of $F_2^\gamma$ in comparison to the proton structure function obtained at HERA. Predictions of strongly rising $F_2^\gamma$ at low $x$ are disfavoured by the data which show a rather flat behaviour of $F_2^\gamma$ at low $x$.

Secondly, the evolution of $F_2^\gamma$ with $Q^2$ is investigated. The present status of the measurements, including those discussed above, is shown in Figure $\overline{\text{e}}$. The positive scaling violation predicted by QCD for all values of $x$ is clearly seen.
In addition, it has been seen that the slope of $F_2^\gamma$ increases for increasing values of $x$, and that the measurements can be described by an augmented asymptotic prediction (ASYM) for $F_2^\gamma$ as described in more detail in [1].

5 Conclusions
The measurement of the hadronic structure of the photon is an active field of research. The newly available results indicate that the systematic error of the measurements can be largely decreased when two-dimensional unfolding and improved Monte Carlo models are used. With the analysis of the full data taken within the LEP2 programme, considerable improvement of our understanding of the hadronic structure of the photon can be expected.

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Figure 6: The $Q^2$ evolution of $F_2^\gamma$ in bins of $x$. 