DISSECTING THE RED SEQUENCE. I. STAR-FORMATION HISTORIES OF QUIESCENT GALAXIES: THE COLOR–MAGNITUDE VERSUS THE COLOR–$\sigma$ RELATION

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ABSTRACT

We use a sample of $\sim$16,000 non-emission-line galaxies from the Sloan Digital Sky Survey to investigate the physical parameters underlying the well known color–magnitude and color–$\sigma$ relations. Galaxies are sorted in terms of velocity dispersions ($\sigma$), luminosity ($L$), and color, and their spectra are stacked to obtain very high S/N mean spectra for stellar population analysis. This allows us to map mean luminosity-weighted ages, [Fe/H], [Mg/H], and [Mg/Fe] in $\sigma$–$L$–color space. Our first result is that there are many different red sequences, with age, [Fe/H], [Mg/H], and [Mg/Fe] showing different amounts of slope and scatter when plotted versus $\sigma$, $L$, or color. These behaviors are explained if the star formation histories of the galaxies populate a two-dimensional parameter space. One parameter is the previously well known increase in age, [Fe/H], [Mg/H], and [Mg/Fe] with $\sigma$. In addition to this, we find systematic variations at fixed $\sigma$, such that more luminous galaxies are younger, more Fe-rich, but have lower [Mg/Fe] than their fainter counterparts. The main $\sigma$ trends support a paradigm in which more-massive galaxies form their stars more rapidly and at earlier times than less-massive galaxies. The trends at fixed $\sigma$ are consistent with scatter in the duration of star formation for galaxies at a given $\sigma$. The covariation of stellar population properties and $L$ residuals at fixed $\sigma$ that we present here has a number of implications: it explains the differing behavior of stellar population indicators when investigated versus $\sigma$ as compared to $L$, and it reveals that $L$ is not as efficient as $\sigma$ for indicating galaxy “size” in stellar population studies.

Key words: galaxies: abundances – galaxies: elliptical and lenticular, cD

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1. INTRODUCTION

Early-type galaxies are observed to obey many scaling relations between their structural and spectral properties. Early work identified a number of one-dimensional relations, including a color–magnitude relation (Faber 1973; Sandage & Visvanathan 1978; Bower et al. 1992), color–line strength relations (Faber 1973), the Faber–Jackson relation between galaxy luminosity ($L$) and velocity dispersion ($\sigma$) (Faber & Jackson 1976), variations in galaxy mass-to-light ratio ($M/L$) versus $L$ (Tinsley 1981; Faber et al. 1987), and various correlations of $\sigma$ and $L$ with galaxy effective radius ($R_e$) and effective surface brightness ($\mu_e$) (Kormendy 1985), or with galaxy core radius ($r_c$) and central surface brightness ($I_0$) (Lauer 1985). It is clear however that early-type galaxies actually comprise a two-parameter population in terms of their structural and spectral properties. Early-type galaxies are observed to obey many scaling relations, despite the evidence that early-type galaxies form a two-parameter population, at least structurally. The stellar populations of early-type galaxies are known to vary with galaxy “size,” where the size is typically parameterized as either $\sigma$ or $L$, but sometimes also as stellar mass ($M_*$). Such relations include the color–magnitude and color–$\sigma$ relations (where it is assumed that the galaxy color is indicative of its stellar population) as well as absorption line strength–$\sigma$ relations (e.g., Burstein et al. 1984; Bender et al. 1993; Trager et al. 1998; Colless et al. 1999; Kuntschner et al. 2001; Bernardi et al. 2003a), which give potentially more detailed information about the stellar populations than do broadband colors. In general, colors are observed to redden and metal line strengths to increase with increasing $\sigma$ or $L$.

The question of whether these variations are primarily due to variations in metallicity, or whether age variations may also play a role was first posed by Faber (1973) and is still debated in the literature. The age–metallicity degeneracy is such that differences in population age and differences in population metallicity affect colors and metal absorption lines in nearly indistinguishable ways. This can be loosely quantified by the “3/2” rule, that a fractional change in metallicity is roughly equivalent to a 1.5 times larger fractional change in age in terms of its effect on colors and metal lines (Worthey 1994). Many authors (e.g., Bower et al. 1992; Kodama & Arimoto 1997) have since claimed that the color–magnitude relation is driven primarily by metallicity variations rather than age variations, although Faber et al. (1995) argued that age effects were also important. Recently, Gallazzi et al. (2006) showed that galaxy metallicities increase by 100% along the red sequence, while ages increase by only 50%. Applying the 3/2 rule, metallicity variations should therefore account for $\sim 75\%$ of the observed color variation along the color–magnitude relation, while the age variation provides the remaining $\sim 25\%$.

Other recent work studying stellar population trends with $\sigma$ (rather than $L$) have demonstrated age variations with $\sigma$ that make larger contributions to the color variation. Applying the 3/2 rule to these studies, age appears to contribute from $\sim 33\%$ (Thomas et al. 2005; Graves et al. 2007) to $\sim 40\%$ (Trager et al. 2000; Nelan et al. 2005; Smith et al. 2007a) of the observed color variation along the color–$\sigma$ relation.

In this paper, we tackle these subtle differences head-on, comparing the color–magnitude and color–$\sigma$ relations directly.
We explore in detail the related stellar population variations, focusing on age, metallicity, and abundance ratio variations with $\sigma$ and separately with $L$. We then look at how stellar populations correlate with residuals from the mean $\sigma$–$L$ relation. This analysis explains how these systematic residual relations conspire to produce the similar yet subtly and confusingly different observed trends with $\sigma$ and $L$.

This is the first in a series of papers presenting a detailed exploration of stellar population properties of quiescent (non-star-forming) galaxies, as well as the dependence of these properties on the structural parameters of the galaxies. The ultimate goal is to match up information about the star formation histories of quiescent galaxies (derived from stellar population analysis) to the mass assembly and structural evolution of these galaxies (as inferred from galaxy structure and morphology).

We use a sample of $\sim$16,000 quiescent galaxies from the Sloan Digital Sky Survey (SDSS), stacking dozens or hundreds of similar galaxies to get very high signal-to-noise (S/N) spectra for detailed and accurate stellar population work. For the purpose of this analysis, we differentiate between “global variables,” which we use to identify similar galaxies for stacking, and “stellar population variables,” which are the mean ages, metal abundances, and abundance ratios derived from the stacked spectra.

This first paper focuses on the color–magnitude and color–$\sigma$ relations of quiescent galaxies; the global variables considered are therefore $\sigma$, $L$, and color. The goal is to understand where these relations come from, what causes scatter around the relations, and how and why the color–magnitude and color–$\sigma$ relations differ from one another. The color–magnitude relation is a natural place to begin this series of papers, as is the most accepted and most widely used of the early-type galaxy scaling laws, to the extent that it is even sometimes used to identify galaxy clusters in photometric surveys where no redshifts are available (e.g., Gladders & Yee 2000). The stellar population variables presented here include mean stellar age, total Fe abundance ([Fe/H]), total Mg abundance ([Mg/H]), and the relative abundance ratio of Mg and Fe ([Mg/Fe]).

Much work has already been done on these topics and not all of the results presented in this paper are new. However, the origin of the color–magnitude relation is fundamental in understanding the formation and evolution of early-type galaxies, yet as discussed above, there is considerable confusion in the literature as to how the various scaling laws fit together. By directly investigating the various relations between $\sigma$, $L$, color, and stellar population properties simultaneously, we show that the various stellar population variables correlate quite strongly with some of the global variables but only very weakly with other global variables. In particular, $\sigma$ and $L$ are not interchangeable. For instance, age correlates strongly with $\sigma$ but rather weakly with $L$, while [Fe/H] shows the opposite behavior.

Briefly looking forward to the subsequent papers in this series, Paper II maps stellar population variations throughout the Fundamental Plane (FP). In Paper II, we show that the stellar populations of galaxies on the FP vary only with $\sigma$; the second dimension of the face-on FP appears to be uncorrelated with the galaxy star formation histories. However, variations are observed through the thickness of the FP. Paper III demonstrates that the two-dimensional family of quiescent galaxy stellar populations can be mapped onto a two-dimensional cross section through the FP and presents a toy model of galaxy star formation histories. Finally, Paper IV compares various stellar ($M_*/L$) and dynamical ($M_{\text{dyn}}/L$) mass-to-light ratios for the galaxy sample. Paper IV demonstrates that the variations in $M_*/L$ due to stellar population effects are inadequate to explain the observed variation in $M_{\text{dyn}}/L$, both along the FP and perpendicular to the plane.

In Section 2, we describe the sample of galaxies used in this analysis. Section 3 reviews known properties of the color–magnitude relation for early-type galaxies and the distribution of these galaxies in the three-dimensional $\sigma$–$L$–color space as a motivation for the subsequent analysis. In Section 4, we discuss the process by which we deconstruct the color–magnitude relation and measure stellar population properties across the red sequence. Section 5 presents the results of the analysis and maps stellar population ages, [Fe/H], [Mg/H] and $\alpha$-enhancement in the three-dimensional $\sigma$–$L$–color space. This is followed by a brief discussion of the results in Section 6, and a summary of our conclusions in Section 7.

2. THE QUIESCENT GALAXY SAMPLE

The goal of this analysis is to achieve a fundamental understanding of the correlations between $\sigma$, $L$, and color for typical early-type galaxies and their relation with stellar population properties. However, from the very outset we face challenges in defining a reasonable sample with which to explore these relations. Although there is a broad correspondence between galaxies with early-type morphologies, galaxies that lie on the red sequence, and galaxies that are not actively forming stars, not all early-type galaxies are on the red sequence, not all red sequence galaxies have early-type morphology, and some early-type galaxies are still actively forming stars.

Schawinski et al. (2007) identify a sample of morphologically selected SDSS early-type galaxies and classify their emission-line properties using the emission-line ratio diagrams pioneered by Baldwin et al. (1981, hereafter BPT). Their early-type sample consists of 81.5% quiescent galaxies (i.e., $[$H$\alpha]> 3$σ emission detections in H$\alpha$, H$\beta$, [O iii]λ3727, or [N ii]λ6583), 1.5% Seyferts, 5.7% low ionization nuclear emission-line regions (LINERS), and 11.2% star-forming or transition objects (which host both star formation and AGN activity). The majority of their star-forming galaxies, transition objects, and Seyferts are not on the red sequence; clearly it would not make sense to include such objects in an analysis of the color–magnitude and color–$\sigma$ relations for passive galaxies. Schawinski et al. (2007) require $>3$σ detections in all four emission lines listed above and therefore likely identify as “quiescent” numerous galaxies with low-level emission that may be detected in the strongest emission line but is below the 3σ threshold in weaker lines such as H$\beta$ or [N ii].

In a parallel study, Yan et al. (2006) classify a volume-limited color-selected sample of SDSS red sequence galaxies based on their emission-line ratios, also using BPT diagrams. Only galaxies that have no emission lines detected at the $3\sigma$ level qualify as quiescent galaxies under their criteria. In practice, the strong H$\alpha$ and [O iii]λ3727 emission lines drive this definition. They find that only 47.8% of red sequence galaxies are quiescent, with a further 28.8% exhibiting LINER-like emission and 23.4% likely hosting Seyfert activity, star formation, or a combination of the two. Their red sequence
galaxies almost certainly include dust-reddened star-forming galaxies, whose dust-corrected colors would lie off of the red sequence.

In light of these studies, we have chosen to study the correlations between $\sigma$, $L$, color, and stellar populations for a sample of quiescent galaxies. Excluding galaxies with active star formation removes potential contamination from dust-reddened star-forming objects whose colors do not reflect their underlying stellar populations. The quiescent criterion also excludes Seyferts and LINERs. Many Seyferts have early-type morphologies but typically have substantially younger stellar populations (Kauffmann et al. 2003), to the extent that they mostly do not fall on the red sequence (Schawinski et al. 2007) and are not relevant to the typical early-type galaxy color–magnitude and color–$\sigma$ relations. Furthermore, a strong point source may contaminate global galaxy color determinations.

A substantially stronger case could be made for including galaxies with LINER emission because they are common on the red sequence and typically reside in early-type hosts with red colors. In Graves et al. (2007), we analyzed the stellar populations of non-star-forming red sequence galaxies, comparing quiescent galaxies with those hosting LINERs. We found that the LINER hosts had systematically younger ages, but the same metallicities and abundance patterns as quiescent red sequence galaxies at the same $\sigma$. They also have similar or slightly redder colors than their quiescent counterparts, suggesting that the color effects of their younger populations are offset by larger mean dust content. Excluding them from the sample in this analysis will remove many of the most recent arrivals on the red sequence.

However, we have decided to limit the analysis in this work to quiescent galaxies only, saving the LINERs for a future separate analysis. It can be difficult to distinguish between LINERs and transition objects (Schawinski et al. 2007). There is no real consensus in the literature as to the true nature of LINER sources, which may well represent a heterogeneous populations. Furthermore, there is some indication that the LINER sample of Graves et al. (2007) in fact contains two separate subpopulations. A preliminary analysis suggests that these subpopulations have different stellar population properties from one another and from the quiescent galaxy population, as well as different luminosity and mass distributions at fixed $\sigma$. As such, the LINER population warrant a separate and detailed analysis.

The sample discussed in this paper is therefore a sample of quiescent galaxies, chosen spectroscopically to be free of emission lines of any kind. It is not a representative sample of “early-type galaxies” or “red sequence galaxies,” but instead provides a window onto the stellar population properties and star formation histories of galaxies that are no longer forming stars. The reader should bear in mind that the exclusion of LINERs from the sample may remove many galaxies whose star formation ended very recently and that the mean ages of the galaxies in this analysis are therefore biased toward older values. If the stellar population trends of the LINER host galaxies follow the same behavior as quiescent galaxies, the relative properties presented here should hold for the non-star-forming galaxy population as a whole even if the zero point of the age scale is uncertain. However, the properties of the LINER population must be explored in future work to verify this possibility.

2.1. SDSS Data

The galaxies used in this analysis are drawn from the SDSS (York et al. 2000), which was conducted with a dedicated 2.5 m telescope at Apache Point Observatory (Gunn et al. 2006). Images are taken in drift-scanning mode with a mosaic CCD camera (Gunn et al. 1998), processed (Lupton et al. 2001; Stoughton et al. 2002; Pier et al. 2003), and calibrated (Hogg et al. 2001; Smith et al. 2002; Ivezić et al. 2004; Tucker et al. 2006). All galaxies used in this work are part of the spectroscopic Main Galaxy Sample (Strauss et al. 2002).

Redshifts ($z$), de Vaucouleurs model photometry ($ugriz$ apparent magnitudes; Fukugita et al. 1996), Galactic extinctions, and Petroshian radii enclosing 50% and 90% of the total galaxy light ($R_{50}$ and $R_{90}$, respectively) are taken from the NYU Value-Added Galaxy Catalog (Blanton et al. 2005) version of SDSS Data Release Four (Adelman-McCarthy et al. 2006). These are supplemented with measurements from the SDSS catalog archive server of velocity dispersions ($\sigma_{\text{fiber}}$, as measured in the SDSS fiber aperture), r-band de Vaucouleurs radii ($R_{\text{deV}}$), and de Vaucouleurs profile axis ratios ($a/b$), as well as the likelihoods of de Vaucouleurs and exponential fits to the galaxy light profiles. The catalog archive server also provides separate magnitudes of the de Vaucouleur and exponential components of each galaxy’s light (essentially a bulge-disk decomposition), along with the relative contribution of each, which can be used to construct model bulge + disk magnitudes and colors for each galaxy.

The algorithm for measuring velocity dispersions has been updated in Data Release Six (DR6) to correct for small systematic biases present in the pre-DR6 measurements (Bernardi 2007). They are computed using the “direct-fitting” method (Burbidge et al. 1961; Rix & White 1992), based on fitting stellar templates convolved with Gaussian broadening functions. Only $\sigma$ values above the $\sim 70$ km s$^{-1}$ instrumental resolution are considered reliable. We therefore only use galaxies with $\log \sigma > 1.86$. Velocity dispersion measurement errors are typically $< 10\%$ (0.04 dex), based on repeat measurements of individual SDSS galaxies.

There is a known problem with SDSS photometry of bright galaxies caused by scattered light that biases the sky background high. Because of this, the total luminosities and radii of bright galaxies are underestimated by the SDSS pipeline, as noted by Lauer et al. (2007), Bernardi et al. (2007), and Lisker et al. (2007). To correct for this effect, we use the results of the simulations reported on the SDSS Web site to compute magnitude and radius corrections as a function of apparent magnitude, which we apply to the galaxies in our sample. We additionally limit our sample to galaxies with $15 < r < 18$ to avoid the worst of the photometry errors caused by this effect. The Galactic extinction corrections from the NYU Value-Added Catalog are applied to all magnitudes.

The galaxy sample presented here is limited to a modest range in redshift (0.04 $< z < 0.08$). This has a number of advantages. The SDSS Main Galaxy Sample is limited to apparent magnitude $r < 17.77$. For 0.04 $< z < 0.08$, our galaxy sample is complete down to absolute magnitude $M_r \approx -20.12$, while the lower end of our redshift range provides data down to $M_r > -19$. The relatively small range in redshifts means that the range in look-back times over the entire sample is only $\Delta t \sim 0.5$ Gyr, which essentially eliminates the effects of evolution within our sample. At these redshifts, the SDSS spectral fibers (1.5 radius) cover a significant portion of the galaxy, typically $\sim R_{\text{deV}}/2$. These spectra are therefore not

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4. http://cas.sdss.org/dr6/en/

5. http://www.sdss.org/dr6/algorithms/veldisp.html

6. http://www.sdss.org/dr6/start/aboutdr6.html
nuclear spectra but sample a substantial fraction of the galaxy light.

Throughout the rest of this paper, we will use velocity dispersion measurements that have been corrected to 1/8 effective radius. Following Bernardi et al. (2003a) and Jørgensen et al. (1995), we use \[ \sigma = r_{50}/(r_e)^0.04, \] where \( r_e \) is the circular galaxy radius in arcseconds, computed as \( r_e = R_{200}/(\Omega/\Lambda)^0.5 \), and \( r_{50} \) is the spectral fiber radius, 1.5" for SDSS spectra. This allows us to provide consistent comparisons with past stellar population work, which typically uses central velocity dispersion measurements. In practice, velocity dispersion gradients are shallow, and this correction is small (of order \( \sim 6\% \)). The \( r \)-band and \( g \)-band magnitudes used throughout this paper have been \( K \)-corrected to \( z = 0 \) using the IDL code kcorrect v4.1.4 (Blanton et al. 2003) and converted to absolute magnitudes \( (M_r \) and \( M_g \)) assuming a standard \( \Lambda \)CDM cosmology with \( \Lambda = 0.7, \Omega_M = 0.3, \) and \( h = 0.7 \).

Spectra of the galaxies in our final sample were downloaded from the SDSS DR6 (Adelman-McCarthy 2008) version of the SDSS data archive server.

2.2. Selection Criteria

To create a sample of passively evolving galaxies, we include only those galaxies with no emission from ionized gas. Specifically, we require that the measured equivalent widths of H\(\alpha \) and [O \(\text{ii} \)]\(\lambda3727 \) emission be below a 2\(\sigma \) detection threshold, as determined by Yan et al. (2006). In this sample, the typical 2\(\sigma \) detection threshold corresponds to H\(\alpha \) and [O \(\text{ii} \)]\(\lambda3727 \) equivalent widths of \( \sim 0.3 \) Å and \( \sim 1.7 \) Å, respectively. This criterion excludes galaxies with significant ongoing star formation within the central portion of the galaxy covered by the spectral fiber, as well as active galactic nuclei (AGNs) with optical emission lines (Yan et al. 2006).

Because the SDSS spectra only sample galaxy light with the 3\(\prime \) spectral fiber radius, selecting quiescent galaxies in this way does not exclude disk galaxies with quiescent central bulge populations which harbor star formation in their outer regions. These galaxies are not truly quiescent galaxies, although they may have quiescent bulges. To limit contamination by these galaxies, we follow Bernardi et al. (2003a) in excluding galaxies with \( R_{90}/R_{50} < 2.5 \) in the \( i \)-band (eliminating galaxies without centrally concentrated light profiles) and for which the likelihood of a de Vaucouleurs light profile fit is less than 1.03 times the likelihood of an exponential fit. These further cuts reduce the sample size by \( < 10\% \), leaving a total sample of 16,008 galaxies.

In summary, the galaxies in this study have been selected from the SDSS Main Galaxy Sample using the following criteria:

1. Redshift range: 0.04 \( < z < 0.08 \).
2. Apparent magnitude range: 15 \( < r < 18 \).
3. No detected emission in H\(\alpha \) and [O \(\text{ii} \)]\(\lambda3727 \).
4. Concentrated light profiles: \( R_{90}/R_{50} > 2.5 \) in the \( i \)-band.
5. Likelihood of a de Vaucouleurs light profile \( \geq 1.03 \) times larger than the likelihood of an exponential light profile.

Throughout the rest of this paper, we refer to the galaxies in our sample as quiescent or passive galaxies, with the understanding that this categorization refers to galaxies that are not actively forming stars.

In Figure 1, the galaxies used in this analysis are shown with respect to the color–magnitude distribution of all galaxies in this redshift range. The gray points show the galaxies included in the sample, while the contour lines show the underlying color–magnitude distribution of all galaxies with \( 0.04 < z < 0.08 \) from the SDSS Main Galaxy spectroscopic sample. In choosing the sample presented here, no explicit color selection was applied, yet the resulting sample of galaxies populates a narrow red sequence relation between galaxy \( M_r \) and color. Figure 2 shows thumbnail images for a randomly selected set of galaxies from our final sample. The galaxies all have smooth morphologies. About 20% of the galaxies have significant disks, although all show evidence for prominent bulges. In Appendix A, we show that the use of magnitudes and colors from de Vaucouleur fits to the galaxy light profiles (even in galaxies which have a disk component) has a negligible effect on our results.

Of our sample galaxies, 85% have bulge fractions above 0.8 while only 3% have bulge fractions below 0.5, making the sample galaxies highly bulge dominated. Figure 2 shows that the sample contains a modest fraction of face-on and edge-on galaxy disks, however these do not dominate the light of the galaxies. Furthermore, the disk contribution to the measured spectrum will be substantially smaller than its contribution to the total galaxy light. The 3\(\prime \) SDSS spectral fibers (shown as circles in Figure 2) sample roughly 0.3–0.6 of the typical galaxy effective radius at these redshifts. In face-on disks, the SDSS 3\(\prime \) spectral fiber predominantly samples the galaxy bulge, while the spectra of inclined disks will contain a slightly increased light contribution from the galaxy disks. Edge-on disks in the sample must have negligible ongoing star formation in order to satisfy the stringent emission-line cuts.

3. THE COLOR–MAGNITUDE RELATION

The selection criteria defined above result in a sample of quiescent galaxies which populate the red sequence in the color–magnitude diagram. These galaxies show a clear color–magnitude relation (Figure 3, top panel), such that more luminous galaxies have redder colors. Data points are color coded by bins in \( \sigma \) as labeled in the lower figure panels. In the lower panels, the total galaxy sample is shown as a cloud of gray
data points, with the total color–magnitude relation overplotted as the gray dotted line. In each panel, contours indicate the color–magnitude distribution of galaxies within a single bin in $\sigma$, as labeled. These $\sigma$ bins are used throughout this paper. The vertical dashed line in Figure 3 indicates the completeness limit of the SDSS Main Galaxy Sample in the redshift range used for this analysis. The solid line at $M_r = -19.7$ shows where we expect the sample is missing more than 50% of the galaxies at these magnitudes. Thus the lack of galaxies at fainter magnitudes is likely a selection effect rather than a genuine lack of galaxies at $M_r > -19.7$.

What is notable about the contours in Figure 3 is that they show no color–magnitude relation at fixed $\sigma$, in agreement with the work of Bernardi et al. (2005). In each panel, which illustrates a narrow range in $\sigma$, the contours in the color–magnitude diagram are horizontal. However, galaxies with larger $\sigma$ (red points) are generally more luminous and have redder colors than those with smaller $\sigma$ (blue and purple points), so that superposing the various $\sigma$ bins results in a net color–magnitude relation.

In contrast, the color–$\sigma$ relation is readily apparent at fixed $M_r$ (Figure 4, top). Here, colored data points indicate bins in luminosity, with dark green, light green, yellow, orange, pink, and purple data points corresponding to $-18.8 > M_r > -20.0$, $-20.0 > M_r > -20.4$, $-20.4 > M_r > -20.8$, $-20.8 > M_r > -21.2$, $-21.2 > M_r > -21.6$, and $-21.6 > M_r > -23$, respectively. A clear color–$\sigma$ relation is apparent at fixed $M_r$ (e.g., purple data points), with roughly the same slope at the total color–$\sigma$ relation. This color–$\sigma$ correlation is stronger and tighter than the color–magnitude relation shown in Figure 3.

Finally, the bottom panel of Figure 4 shows the $\sigma$–magnitude or Faber–Jackson relation, color coded by galaxy $g - r$ color. Dark blue, medium blue, light blue, pale pink, medium pink, and dark pink data points represent $g - r < 0.74$, $0.74 < g - r < 0.76$, $0.76 < g - r < 0.78$, $0.78 < g - r < 0.80$, $0.80 < g - r < 0.82$, and $0.82 < g - r$, respectively. The
The color–magnitude relation is tight for the most luminous galaxies, but shows considerable spread for faint galaxies. The spread is correlated with galaxy color, such that bluer galaxies tend to have lower $\sigma$ at fixed luminosity. This may indicate that fainter, bluer galaxies have a larger degree of rotational support, resulting in lower values of $\sigma$ for a given $M_*$.

It is clear that $\sigma$, luminosity, and color are all correlated in early-type galaxies. Bernardi et al. (2005) argue that the fundamental relations are $\sigma$–color and $\sigma$–magnitude and that the color–magnitude relation results from a combination of these two correlations. The color–$\sigma$ relation (Figure 4, top) exists independent of galaxy luminosity (i.e., at fixed $M_*$). Likewise, the $\sigma$–magnitude relation (Figure 4, bottom) exists independent of galaxy color. However, the color–magnitude relation does not exist at fixed $\sigma$ (Figure 3), indicating that the $\sigma$ dependence of both color and magnitude is what leads to the observed color–magnitude relation.

From Figure 3, it is also clear that galaxies with very similar values of $\sigma$ show a substantial spread in $M_*$. The low-$\sigma$ bins suffer from incompleteness effects, but in the higher $\sigma$ bins (e.g., those with $\log \sigma > 2.18$), it is clear that galaxies with a range of only 0.09 dex in $\log \sigma$ span roughly 2 mag (0.8 dex) in $M_*$. Similarly, the substantial overlap of the $\sigma$ bins in the color–magnitude diagram means that galaxies at fixed $M_*$ have a range of $\sigma$.

These arguments illustrate that optical luminosity $L$ is not a good proxy for $\sigma$. This is expected from the fact that galaxies populate the FP and the Faber–Jackson relation is not an edge-on projection of the FP (Dressler et al. 1987). The two parameters are, however, often used interchangeably in the literature as measures of galaxy size or mass. We will see in Section 5.1 that using $L$ versus using $\sigma$ as the controlling variable in stellar population studies can lead to substantially different stellar population scaling relations, because stellar population properties vary systematically with $L$ at fixed $\sigma$.

### 4. DISSECTING THE COLOR–MAGNITUDE RELATION

Accurate stellar population parameters can only be derived from high-S/N spectra ($S/N > 100 \, \AA^{-1}$; Cardiel et al. 1998). As individual SDSS galaxy spectra in our sample typically have
S/N ≈ 20 Å⁻¹, dozens or more spectra must be combined to achieve the necessary S/N. If galaxies are combined randomly, the average spectra would all look very similar. The challenge in this method is to identify useful ways of sorting the galaxies in advance, in order to do the best possible job of populating the full range of galaxy star formation histories, without averaging out all the interesting variations.

In this section, we define a three-dimensional parameter space of σ (which is not sensitive to stellar population effects) along with L and color (which are) and sort galaxies into bins in this space. We then stack the spectra of galaxies within each bin and measure absorption line strengths. In Section 5, we combine the observed line strengths with stellar population models to obtain the fundamental stellar population parameters of galaxies along and across the color–magnitude relation.

4.1. Binning by σ, L, and Color

In Section 3, we divided our sample of quiescent galaxies into six bins based on σ. To study how stellar populations vary at fixed σ, we further divide those same σ bins into three bins in color by three bins in L for a total of 54 bins. Figure 5 illustrates this process for the log σ = 2.18–2.27 bin. Cuts in color and L are defined with fixed width, such that each L bin is 0.8 mag wide in M_r and each color bin is 0.032 mag wide in g−r. The 3 × 3 grid of sub-bins is centered on the median M_r and g−r for all the galaxies in the given σ bin. The median values of σ, M_r, and g−r for each of the 54 bins are listed in Table 1, along with the total number of galaxies in each bin.

Figure 4. Top panel: color–σ relation, with data points color coded by galaxy luminosity. Dark green (purple) points show the faintest (brightest) luminosity bins. A color–σ relation exists at fixed M_r (e.g., purple data points alone) with roughly the same slope as the total color–σ relation. The color–σ trend is stronger and narrower than the color–magnitude relation in Figure 3. Bottom panel: the σ–magnitude relation, color coded by galaxy g−r color. Dark blue (dark pink) points show the bluest (reddest) galaxies. The σ–magnitude relation is broader for fainter galaxies. This is correlated with galaxy color, such that bluer galaxies tend to have lower σ at fixed M_r. This may be due to a greater degree of rotational support in fainter and bluer early-type galaxies, showing lower σ for the same M_∗.

(A color version of this figure is available in the online journal.)

Figure 5. Method used to bin galaxies by L and color at fixed σ. The color–magnitude relation for the total quiescent galaxy sample in this typical σ bin is indicated by the gray contours. Overplotted as gray points are all galaxies with 2.18 < log σ < 2.27. Within this σ bin, there is no color–magnitude relation. The median point in M_r and g−r is shown as the black cross in the center. Galaxies in this σ bin are divided into a 3 × 3 grid in color–magnitude space, with the grid centered on the median point and with fixed grid spacing of 0.8 mag in M_r and 0.032 mag in g−r. Defining the grid in this way eliminates color and L outliers which may bias the average properties of the bins. Galaxies in each of the six σ bins are sorted in this way into nine sub-bins in color–L space, and the galaxies in each sub-bin are stacked as described in Section 4.2 to produce high-S/N mean spectra for all 54 bins in σ–L–color space.

The central sub-bin in color–L space contains more galaxies than the outer bins, but all bins are sufficiently well populated to produce high-S/N stacked spectra. Defining the color–L grid in this way excludes color and L outliers from each σ bin, which ensures that the stacked spectra will not be biased by very deviant or misidentified objects.
In the lowest $\sigma$ bins, the magnitude limit of the sample presented here means that the fainter galaxies at that $\sigma$ are under-represented. Under the assumption that the missing faint galaxies at $z \sim 0.08$ are not substantially different from those at $z \sim 0.04$ which are included in the sample, the fainter bins are not strongly biased but are less populated than they
ought to be. Comparisons between galaxies at different $L$ in the same $\sigma$ bin should therefore be robust to completeness effects. However, because these $\sigma$ bins miss the faintest galaxies, the median $M_r$ used to locate the color–$L$ grid may be biased brighter than the underlying galaxy population at that $\sigma$.

### 4.2. Constructing the Stacked Spectra

In each of the 54 bins in $\sigma$–$\log L$–color space, we combine spectra of the galaxies in the bin in order to produce a very high S/N average spectrum. Regions around the bright skylines at 5577 Å, 6300 Å, and 6363 Å are masked in the individual galaxy spectra, which are then shifted to the rest frame. The individual spectra are normalized so that all galaxies in the bin contribute equally to the averaged spectrum. The normalization uses the median flux in the 4100–5000 Å range without modifying the continuum shape, as the flux-calibrated spectral shape affects the Lick index measurements. In practice, because the galaxies in a given bin have very similar colors, the shape of the continuum is nearly identical for all galaxies in the bin.

The stellar absorption features in higher-$\sigma$ galaxies are effectively observed at lower resolution than those in lower-$\sigma$ galaxies, due to the increased intrinsic Doppler smoothing of the lines from stellar motions within the galaxy. This intrinsic smoothing has non-negligible effects on the measured absorption line strengths. In order to compare all galaxies at the same effective resolution, we smooth lower-$\sigma$ galaxies up to match the highest-$\sigma$ galaxies in our sample, $\sigma \sim 300$ km s$^{-1}$.

The algorithm we use to stack the galaxy spectra first averages the unsmoothed spectra of galaxies within narrow $\sigma$ sub-bins, with a routine that rejects very deviant pixels. The average clean sub-bin spectra are then smoothed to the maximum $\sigma$ of the sample ($\sim 300$ km s$^{-1}$) and coadded. The result is a stacked spectrum for each $\sigma$–$\log L$–color bin from which outlier pixels have been rejected and which has been smoothed to an effective resolution identical to all the other bins. The error spectrum is also computed from the individual SDSS galaxy error spectra by adding the individual pixel errors in quadrature to produce an error spectrum for each stacked spectrum.

### 4.3. Lick Index Absorption Strengths

We measure the full set of Lick indices in each of the stacked spectra. These include the Balmer lines H$\beta$, H$\gamma_F$, and H$\delta_F$ (also broad definitions of H$\gamma_A$ and H$\delta_A$), and a set of Fe-dominated lines (Fe4383, Fe4531, Fe5015, Fe5270, Fe5335, Fe5406, Fe5709, and Fe5782), as well as numerous lines that are sensitive to abundances of elements other than Fe (Mg$\alpha_1$, Mg$\alpha_2$, Mg b, CN$\gamma_1$, CN$\gamma_2$, C$\gamma_2$, 6468 Å, 4427 Å, 4455 Å, NaD, TiO$\gamma_1$, and TiO$\gamma_2$). The index definitions are taken from Worthey et al. (1994) and Worthey & Ottaviani (1997), and line strengths are measured using the Lick$_{EW}$ code that is available online as part of the EZ$\_Ages$ code package (Graves & Schiavon 2008). The Lick$_{EW}$ code also computes statistical errors for each Lick index based on the error spectra, using Equations (33) and (37) of Cardiel et al. (1998). Because the Lick indices are defined as equivalent widths, they are insensitive to dust extinction effects.

All the stacked spectra have been smoothed to $\sigma = 300$ km s$^{-1}$. Combined with the intrinsic resolution of the SDSS spectrograph, the spectra are at lower resolution than the Lick/IDS resolution at which the Lick indices are defined. A correction must be applied to the measured line strengths to bring them back onto the Lick index system. These corrections are included in the Lick$_{EW}$ computation, based upon smooth single stellar population spectra, as given in Schiavon (2007) Table A2a. All of the spectra have been smoothed to the same $\sigma$ and corrected in the same way so that uncertainties in the velocity dispersion correction should not affect relative line strength measurements. The corrected Lick index measurements and statistical errors for each of the 54 stacked spectra are given in Table 2. We do not attempt to match the zero points of the SDSS spectra to those of the Lick system as defined by Schiavon (2007). This should only introduce minor zero-point uncertainties because the Schiavon models are based on flux-calibrated spectra and the SDSS spectra are also flux calibrated (see Schiavon 2007, Section 2.2.2).

### 4.4. Stellar Population Modeling

We compare the Lick index measurements for each of the stacked spectra to the stellar population models of Schiavon (2007) to derive fundamental stellar population parameters from the line strengths. The Schiavon (2007) models are single burst models which include the effect of variable abundance ratios by combining theoretical stellar isochrones (Girardi et al. 2000) with a library of empirical stellar spectra (Jones 1999) and individual line strength sensitivities to elemental abundance variations computed from theoretical stellar atmospheres (Korn et al. 2005). With these models, a set of Lick index measurements can be used to determine the mean luminosity-weighted stellar population age, iron abundance ([Fe/H]), and abundance ratios for the elements Mg, C, N, and Ca ([Mg/Fe], [C/Fe], [N/Fe], and [Ca/Fe]) using the algorithm described in Graves & Schiavon (2008) and implemented in the publicly available code EZ$\_Ages$. Briefly, the Graves & Schiavon (2008) method determines a fiducial mean age and [Fe/H] from a combination of Balmer and Fe-dominated indices, then uses other indices that are sensitive to Mg, C, N, and Ca to adjust the element abundance ratios until the model index predictions match the ensemble of Lick index data. The basic modeling process has not changed since the original analysis of red sequence galaxies with LINER-like emission in Graves et al. (2007); subsequent minor updates to the EZ$\_Ages$ code have improved the robustness of the code at runtime and produce results that match those of Graves et al. (2007) within the quoted errors. EZ$\_Ages$ estimates statistical errors for each of the stellar population parameters, based on the measurement errors in the Lick indices. The errors in these various parameters are not independent; an analysis of the correlated errors is presented in Figure 3 of Graves & Schiavon (2008). The reader is referred to that article for additional details on the age- and abundance-fitting process.

Graves & Schiavon (2008) have shown that this fitting method is robust to the choice of Lick indices used in the analysis. Here we use the “standard set” of Lick indices from Graves & Schiavon (2008)—H$\beta$, (Fe) (which is an average of the Fe5270 and Fe5335 indices), Mg b, C$\gamma_2$, CN$\gamma_1$, and Ca4227—with the EZ$\_Ages$ code to fit a mean luminosity-weighted age, [Fe/H], [Mg/Fe], [C/Fe], [N/Fe], and [Ca/Fe] for each of the 54 average galaxy spectra. We can thus track each of these stellar population parameters as a function of $\sigma, L$, and color for the galaxies in our sample.

In this analysis, we focus on the stellar population age, [Fe/H], [Mg/H], and [Mg/Fe], leaving the analysis of other...
purposes of this type of stellar population work, it is typically straightforward to measure in optical galaxy spectra. For the abundance ratios for future work. The results of the stellar population analysis for the stacked spectra in Table 2. Of the $\alpha$-elements, Mg is the only one that is relatively straightforward to measure in optical galaxy spectra. For the purposes of this type of stellar population work, it is typically assumed that all $\alpha$-elements scale together and thus that [Mg/Fe] is equivalent to [$\alpha$/Fe]. There is some observational evidence that $\alpha$ elements may not always vary in lock-step (e.g., Fulbright et al. 2007; Humphrey & Buote 2006) but, lacking an alternative, we will use [Mg/Fe] as an estimate of the total [$\alpha$/Fe]. The Schiavon (2007) models are computed at fixed [Fe/H] rather than at fixed total metallicity ([Z/H]), so that enhancements in individual elements also change the total metallicity of the galaxy.

It is important to keep in mind that the stellar population parameters based on modeling absorption line indices give
ages and abundances averaged over the ensemble of stars in the galaxy. The contribution of each star to the average age or abundance measurement is proportional to the light contributed at the wavelength of the absorption feature in question, and also to the equivalent width of the particular absorption feature of that star. When comparing ages of galaxies, a younger mean luminosity-weighted age in one galaxy compared to another does not necessarily mean that all of the stars in the galaxy are younger: a galaxy may contain a fraction of young stars that skew the average age to lower values. The luminosity-weighted ages presented here are also averaged over the ensemble of galaxies in the bin and should thus be treated as a statistical description of galaxies in that bin rather than as “true” ages for all of the stars in an individual galaxy.

5. STELLAR POPULATIONS ON THE RED SEQUENCE

5.1. Age, [Fe/H], [Mg/H], and [Mg/Fe] as Functions of \( \sigma_L \), and Color

Figure 6 shows the age, [Fe/H], [Mg/H], and [Mg/Fe] results of the stellar population analysis as a function of galaxy \( \sigma_L \). The stellar population parameters derived from the stacked spectra are plotted against the median value of \( \sigma_L \) for the ensemble of galaxies in each of the 54 bins. The median statistical errors in the derived population parameters are shown in the lower-right corner of each panel. The Spearman rank correlation coefficient \( \tau_s \) is shown for each pair of parameters, indicating the strength of the correlation. Age, [Fe/H], [Mg/H], and [Mg/Fe] all show correlations with \( \sigma_L \), such that higher-\( \sigma_L \) galaxies have older mean ages, higher iron abundances, higher magnesium abundances, and higher levels of Mg-enhancement, which strongly suggests that higher-\( \sigma_L \) galaxies also have higher total metallicities. These results are in agreement with numerous previous authors (e.g., Bernardi et al. 2003b; Thomas et al. 2005; Nelan et al. 2005; Smith et al. 2007a; see the summary in Table 8 of Graves et al. 2007), although these trends are not universally observed in all early-type galaxy samples (e.g., Kelson et al. 2006; Trager et al. 2008).

The [Mg/H]-\( \sigma_L \) and [Mg/Fe]-\( \sigma_L \) relations are particularly tight compared to the typical measurement errors. This implies that at fixed \( \sigma_L \) [Mg/H] and [Mg/Fe] vary only weakly with the other two global parameters studied here \( (L \) and color). At high \( \sigma_L \), [Mg/H] shows almost no variation at fixed \( \sigma_L \). A linear least-squares fit of [Mg/Fe] onto \( \sigma_L \) gives a slope of 0.34, which is consistent with previous results.

Unlike [Mg/H] and [Mg/Fe], both age and [Fe/H] show scatter at fixed \( \sigma_L \) that is large compared to the statistical error bars. For [Fe/H], the scatter is roughly the same at all \( \sigma_L \), whereas the spread in mean ages at fixed \( \sigma_L \) appears to be larger for lower-\( \sigma_L \) galaxies. It is noteworthy that some low-\( \sigma_L \) galaxies have mean ages nearly as old as some of the highest-\( \sigma_L \) galaxy bins. Thus any model for galaxy formation that predicts a correlation between

\[ \tau_s = 0.73 \]

\[ \tau_s = 0.64 \]

\[ \tau_s = 0.77 \]

\[ \tau_s = 0.93 \]

Figure 6. Mean stellar population parameters for each of the 54 stacked galaxy spectra, shown as a function of median \( \sigma_L \) for the galaxies in each bin. The error bars in the lower right corner of each panel show the median statistical error in the derived population parameters, based on the uncertainties in the Lick index measurements. The data have been separated into those with small statistical errors (filled black circles) and those with larger statistical errors (open gray circles), as indicated by the corresponding error bars. “Large” statistical errors are defined as those more than two standard deviations larger than the median error. Mean luminosity-weighted stellar age, [Fe/H], [Mg/H], and [Mg/Fe] all increase with increasing \( \sigma_L \). Dashed lines show linear least squares fits of the stellar population properties onto log \( \sigma_L \), computed using the black data points only. The Spearman rank correlation coefficients \( \tau_s \) of the correlations are specified in each panel. The scatter in mean age and [Fe/H] is significantly larger than that expected due to measurement errors, implying genuine variation in these population parameters at fixed \( \sigma_L \). The spread in age is small for the highest-\( \sigma_L \) galaxies, all of which show old ages, while the lowest-\( \sigma_L \) galaxies cover a substantial factor in mean age. There is only modest scatter in either [Mg/Fe] or [Mg/H] at fixed \( \sigma_L \). The correlation between \( \sigma_L \) and [Mg/H] is particulary strong, as indicated by the very high value of \( \tau_s \).

\[ \begin{align*}
\tau_s &= 0.73 \\
\tau_s &= 0.64 \\
\tau_s &= 0.77 \\
\tau_s &= 0.93 
\end{align*} \]

\[ \sigma \] indicates perfect correlation. The sign of \( \tau_s \) indicates whether the correlation is positive or negative. The definition of \( \tau_s \) makes no assumptions about the functional form of the correlation and allows robust comparison of correlation strengths between different correlations.

\[^{10} \]For a given pair of parameters, \( \tau_s \) indicates the extent to which they produce the same rank ordering of objects, with 0 indicating no correlation and 1 indicating perfect correlation.
Figure 7. Mean stellar population parameters as a function of galaxy $L$. The results of the stellar population analysis are plotted against the median $M_r$ for the galaxies in each bin. Statistical errors in the derived population parameters are shown in the lower right corner of each panel. The data have been separated into those with small statistical errors (filled black circles) and those with larger statistical errors (open gray circles), as indicated by the corresponding error bars. The Spearman rank correlation coefficient ($r_s$) is given for each relation. The strong increases in mean age and $[\text{Mg/Fe}]$ visible in Figure 6 when plotted against $\sigma$ are considerably washed out when plotted against $M_r$, while the trend between $[\text{Fe/H}]$ and $M_r$ is much tighter than the $\sigma$–$[\text{Fe/H}]$ relation. The $[\text{Mg/H}]$–$M_r$ is still strong, although $r_s$ indicates it is slightly weaker than the $\sigma$–$[\text{Mg/H}]$ correlation.

5.2. Mapping Stellar Populations Across the Red Sequence

Figure 9 shows contours of stellar population age as a function of $\sigma$, $L$, and color. The six panels contain data for our six standard $\sigma$ slices corresponding to the panels in Figure 3. Black and gray data points in each panel show the color–magnitude relation at that $\sigma$, with gray points indicating galaxies in the $\sigma$ range which were excluded from the stacked spectra. Plotted over the color–magnitude relations in each bin are color contours representing the mean luminosity-weighted ages derived from the stacked spectra. The age contours were constructed as follows: age values from the stacked spectra in each $\sigma$ range were plotted at the median values of $M_r$ and $g-r$ for each bin, forming a 3 × 3 grid of age values in color–magnitude space. The values of age were linearly interpolated (in log age) between the nine grid points to produce age contours across the color–magnitude relation. The color bar in the lower right panel indicates the scaling of the age contours, with the lowest and highest values indicated. The dashed lines at $M_r = -21$ and $g-r = 0.75$ are the same in all panels and exist merely to guide the eye. As $\sigma$ increases, galaxies become systematically both brighter and redder with respect to the dashed lines.

The age contours in Figure 9 illustrate the same trends that were visible in Figure 6. As $\sigma$ increases from low (upper left panel) to high values (lower right panel), the average age indicated by the contours increases. However, in each slice in $\sigma$, a range of ages exists. The spread in ages at fixed $\sigma$ is large in the low-$\sigma$ bins and becomes increasingly smaller toward the highest-$\sigma$ bins.

The contours also illustrate the behavior of stellar population age at fixed $\sigma$ across the color–magnitude relation. Lines of constant age thus run roughly diagonally in color–magnitude space at fixed $\sigma$, with young ages in the lower right corner (bright and blue) and old ages in the upper left corner (faint and red). This trend is approximately repeated in each $\sigma$ slice, although at high $\sigma$ the dynamic range is less. The variation in the low-$\sigma$ bins.
at fixed \( \sigma \) cannot be explained by errors in \( \sigma \) that result in galaxies being assigned to the wrong stacked spectrum because this would result in the opposite effect: galaxy contamination from higher-\( \sigma \) bins would tend to make the more luminous galaxies appear older, not younger.

Figure 10 shows the same slices through the color–magnitude relation, this time with color contours indicating \([Fe/H]\). Again, the trends of Figure 6 are visible: \([Fe/H]\) increases with \( \sigma \) from the lowest-\( \sigma \) bins to the highest, and a range in \([Fe/H]\) exists within each slice in \( \sigma \). However, the variation in \([Fe/H]\) at fixed \( \sigma \) is markedly different from the age variation seen in Figure 9. Lines of constant \([Fe/H]\) run nearly vertically in the color–magnitude diagram for fixed \( \sigma \), such that more luminous galaxies have higher \([Fe/H]\).

The contours of \([Mg/H]\) shown in Figure 11 are similar to those in \([Fe/H]\) for the same \( \sigma \) slices through the color–magnitude relation. In addition to the total increase in \([Mg/H]\) with \( \sigma \), the brightest galaxies at fixed \( \sigma \) have the highest values of \([Mg/H]\). However, \([Mg/H]\) shows less variation at fixed \( \sigma \) than does \([Fe/H]\), particularly in the high-\( \sigma \) bins. This is as expected from Figure 6. Although the spread in \([Mg/H]\) at fixed \( \sigma \) seen here and in Figure 6 is small, Figure 11 strongly suggests that these variations are in fact real because they are consistent across the \( \sigma \) bins.

Finally, Figure 12 shows \( \sigma \) slices overlaid with contours of constant \([Mg/Fe]\), directly comparing the differences in Fe and Mg enrichment. Again, \([Mg/Fe]\) increases with increasing \( \sigma \), as expected from Figure 6. Like \([Mg/H]\), the variation at fixed \( \sigma \) is mild and therefore somewhat more difficult to categorize than the variation in age and \([Fe/H]\), but broadly \([Mg/Fe]\) behaves similarly to age, showing larger enhancements in fainter galaxies at fixed \( \sigma \). Because Mg is produced by supernovae Type II (SNe II) on short timescales while Fe is produced by supernovae Type Ia (SNe Ia) on longer timescales, the observed overabundance of Mg relative to the solar abundance pattern is typically interpreted to indicate a short timescale for star formation, leaving the stellar population of the galaxy enhanced in SN II products but not in SN Ia products.

These maps of age, \([Fe/H]\), \([Mg/H]\), and \([Mg/Fe]\) across the color–magnitude relation confirm that quiescent galaxies form a multiparameter family in stellar population properties. Age, \([Fe/H]\), \([Mg/H]\), and \([Mg/Fe]\) all increase with \( \sigma \), but show variations at fixed \( \sigma \) depending on their \( L \). Of the four stellar population parameters presented in Figures 9–12, only age shows a strong dependence on color as well as \( L \). Variations in age and \([Mg/Fe]\) with \( L \) at fixed \( \sigma \) are similar (both increase toward fainter galaxies) and are opposite to the variations in \([Fe/H]\) and \([Mg/H]\) (both of which increase toward brighter galaxies). This suggests that the oldest galaxies in the universe formed their stars over short timescales, while younger quiescent galaxies experienced more extended star formation.

Using these stellar population maps, the differing trends with \( \sigma \) and with \( L \) illustrated in Figures 6 and 7 can be understood by superposing the various \( \sigma \) slices of the color–magnitude relation. Although we have seen that age and \([Fe/H]\) both increase with \( \sigma \), Figures 9 and 10 show that age and \([Fe/H]\) exhibit opposite behavior from one another at fixed \( \sigma \), namely that age decreases and \([Fe/H]\) increases for brighter galaxies at fixed \( \sigma \). Because of this difference, superposing the various \( \sigma \) slices to form the total color–magnitude relation acts to reinforce the \( \sigma \)--\([Fe/H]\) relation into an even tighter \( L \)--\([Fe/H]\) relation. At the same time, the age trends at fixed \( \sigma \) counteract the \( \sigma \)--age relation,
resulting in almost no \( L \)-age relation. This explains why stellar population studies as a function of \( L \) or \( M_\ast \)\(^{11}\) find weak or nonexistent \( L \)-age relations (e.g., Kuntschner & Davies 1998; Terlevich & Forbes 2002; Gallazzi et al. 2005), while stellar population studies as a function of \( \sigma \) turn up significant \( \sigma \)-age correlations (e.g., Bernardi et al. 2003b; Thomas et al. 2005; Nelan et al. 2005; Smith et al. 2007a; Graves et al. 2007).

In this scenario, although the \( L \)-[Fe/H] correlation is stronger than the \( \sigma \)-[Fe/H] correlation, we interpret the \( \sigma \)-[Fe/H] trend as the primary relation, with the increased tightness of the \( L \)-[Fe/H] relation being caused by correlated residuals from the \( \sigma \)-\( L \) and \( \sigma \)-[Fe/H] relations. In the next section, we use principal components analysis to show that the family of stellar populations in quiescent galaxies is nearly two dimensional, with the first dimension parameterized by \( \sigma \) and the second dimension parameterized by correlated residuals from the various \( \sigma \)-\( L \), \( \sigma \)-color, and \( \sigma \)-stellar population trends.

### 5.3. Principal Components Analysis and Stellar Population Residuals

To quantitatively explore the multidimensional space of global and stellar population parameters illustrated in the previous section, we have performed a principal components analysis (PCA; see, e.g., Faber 1973; Trager et al. 2000) in the seven-dimensional space parameterized by \( \sigma \), \( L \), color, [Fe/H], [Mg/H], and [Mg/Fe]. All parameters are “standardized” to have a mean of 0 and a variance of 1 before performing the PCA. The results are shown in Table 3. Although the parameter space is nominally seven dimensional, there are only six principal components (PCs) because \([\text{Mg/H}]=[\text{Mg/Fe}]+[\text{Fe/H}]\). The first two PCs account for 91% of the variance in the population.

\(^{11}\) \( M_\ast \) is closely related to \( L \) because red sequence galaxies span only a very limited range in \( M_\ast /L \).

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**Table 3**

Principal Component Analysis

| Parameter     | PC1 | PC2 | PC3 | PC4 | PC5 | PC6 |
|---------------|-----|-----|-----|-----|-----|-----|
| log \( \sigma \) | 0.43 | 0.03 | 0.31 | 0.37 | -0.53 | 0.55 |
| log \( L \)    | 0.37 | -0.40 | 0.04 | 0.59 | 0.58 | -0.14 |
| \( \sigma \)-[Fe/H] | 0.40 | 0.23 | -0.62 | -0.25 | 0.32 | 0.50 |
| log age       | 0.32 | 0.53 | -0.33 | 0.32 | -0.27 | -0.57 |
| [Fe/H]        | 0.32 | -0.55 | -0.22 | -0.29 | -0.33 | -0.21 |
| [Mg/H]        | 0.43 | -0.21 | 0.14 | -0.40 | -0.08 | -0.21 |
| [Mg/Fe]       | 0.36 | 0.39 | 0.59 | -0.34 | 0.31 | -0.11 |
| Eigenvalue    | 4.88 | 1.49 | 0.35 | 0.18 | 0.07 | 0.03 |
| Percentage of variance | 69.7 | 21.3 | 5.0 | 2.6 | 1.0 | 0.4 |
| Cumulative percentage | 69.7 | 91.0 | 96.0 | 98.6 | 99.6 | 100.0 |

**Notes.** Parameters are “standardized” to have a mean of 0 and a variance of 1 before performing PCA. Although seven parameters are reported here, there are only six principal components because \([\text{Mg/H}]=[\text{Mg/Fe}]+[\text{Fe/H}]\).

\( ^{\ast} \)PCA is performed using log \( L \) rather than \( M_\ast \) to eliminate confusion due to the sign convention of the magnitude scale.
indicating that the quiescent galaxy population can be well described by a two-dimensional hyperplane.

The first PC (PC1) accounts for a full 70% of the observed variation and is comprised of positive contributions from all of the input parameters. This illustrates quantitatively that all the parameters presented here are correlated with one another. We saw in Section 3 that the color–magnitude relation is the result of fundamental \( \sigma \)–color and \( \sigma \)–magnitude relations. It therefore seems reasonable to treat \( \sigma \) as the primary global parameter behind PC1.

The second PC (PC2) encompasses another 21% of the variance and includes significant contributions from all parameters except for \( \sigma \). PC2 therefore represents the correlated residuals from the mean relations between \( \sigma \) and the other parameters. The contributions of \( g-r \), age, and \([\text{Mg}/\text{Fe}]\) to PC2 are positive while those of \( L \), \([\text{Fe}/\text{H}]\), and \([\text{Mg}/\text{H}]\) are negative. This indicates that at fixed \( \sigma \), \( g-r \), age, and \([\text{Mg}/\text{Fe}]\) are correlated with one another and are anticorrelated with \( L \), \([\text{Fe}/\text{H}]\), and \([\text{Mg}/\text{H}]\). Of the global parameters, \( L \) contributes the most to PC2.

5.4. Stellar Population Residuals from \( \sigma \) Relations

Residuals in the four stellar population parameters (\( \Delta \log \text{age} \), \( \Delta [\text{Fe}/\text{H}] \), \( \Delta [\text{Mg}/\text{H}] \), and \( \Delta [\text{Mg}/\text{Fe}] \)) are defined as the difference between the observed values and the mean relations with \( \sigma \), as indicated by the dashed lines in Figure 6. The correlated stellar population residuals associated with PC2 are shown in Figure 13. In each panel, high and low-S/N data are indicated by filled and open circles, respectively. The error ellipses in the lower left corner of each panel indicate the statistical errors from the stellar population modeling process, with solid black (dashed gray) lines showing the median statistical errors for high (low) S/N data. Measurement errors in the index absorption line strengths produce correlated errors in the derived stellar population parameters. These are determined from Monte Carlo simulations in Graves & Schiavon (2008) and are indicated by the error ellipse orientation. In all panels, residuals are color coded by luminosity residuals; the green circles represent the central bin in \( L \) at fixed \( \sigma \) (centered on the median \( L \) for that \( \sigma \), as in Figure 5) while blue and red circles indicate brighter (\( \Delta L \approx -0.8 \) mag in \( r \)) and fainter (\( \Delta L \approx +0.8 \) mag in \( r \)) luminosity bins.

Panel (a) shows an anticorrelation between \( \Delta \log \text{age} \) and \( \Delta [\text{Fe}/\text{H}] \), such that older galaxies are typically Fe-poor compared to younger galaxies at the same \( \sigma \). This anticorrelation at fixed \( \sigma \) has been reported by numerous previous authors (e.g., Worthey et al. 1995; Colless et al. 1999; Jørgensen 1999; Trager et al. 2000) and was quantified by Trager et al. (2000) as the “metallicity hyperplane.” The anticorrelation conspires to keep the color–\( \sigma \) relation tight (see Figure 4). In contrast to the observed age–[Fe/H] anticorrelation at fixed \( \sigma \), we saw in Figure 7 that at fixed \( L \), there is significant scatter in age but almost no scatter in [Fe/H] (although see Poggianti et al. 200112). This

12 Unlike the results shown in Figure 7, Poggianti et al. (2001) find a range of metallicity at fixed \( L \), which they observe to be anticorrelated with age. Their results may differ from ours because we have averaged over a large number of galaxies in each stacked spectrum, washing out genuine [Fe/H] variation at fixed \( L \) and \( \sigma \), or the observed age–metallicity correlation in their data may be due to an underestimate of the correlated errors in the stellar population modeling process.
Figure 11. [Mg/H] as a function of $\sigma$, $L$, and color. Panels show the color–magnitude diagram for slices of fixed $\sigma$. Black points show galaxies included in the stacked spectra, while gray points show color and $M_r$ outliers that have been excluded from this analysis. Contours of [Mg/H] derived from the stellar population modeling of Section 4.4 are overplotted. Mg is used to trace $\alpha$-elements, thus contours of [Mg/H] here are a proxy for contours of [$\alpha$/H]. [Mg/H] increases with increasing $\sigma$, showing the least variation at fixed $\sigma$ of any of the stellar population parameters. In the lower-$\sigma$ bins, [Mg/H] behaves similarly to [Fe/H], with lines of constant [Mg/H] running nearly vertically in color–magnitude space. (A color version of this figure is available in the online journal.)

implies that age differences will produce color variations at fixed $L$ which cannot be offset by correlated variations in [Fe/H]. Thus, the color–magnitude relation should show more color variation at fixed $L$ than is observed in the color–$\sigma$ relation at fixed $\sigma$, consistent with Figures 3 and 4. Furthermore, the observed color spread in the color–magnitude relation should correlate with age, such that bluer galaxies are younger than redder galaxies at fixed $L$. This is consistent with Figure 9 and also with the results of Cool et al. (2006).

In addition to the age–[Fe/H] anticorrelation evident in Figure 13(a), a similar anticorrelation is present between age and [Mg/H] (Figure 13(c)), although it is significantly weaker. A further anticorrelation is observed between [Fe/H] and [Mg/Fe] (Figure 13(b)), such that Fe-poor galaxies are more Mg-enhanced than their comparatively Fe-rich counterparts at fixed $\sigma$. Finally, Figure 13(d) shows a positive correlation between age and [Mg/Fe], such that older galaxies are more Mg-enhanced with respect to the solar abundance pattern than their younger counterparts at fixed $\sigma$.

In Figure 13(a)–(c), the slopes of the observed anticorrelations are similar to those expected from correlated errors in stellar population modeling, as indicated by the error ellipses. However, the observed anticorrelations cannot be due to correlated statistical errors for several reasons. Firstly, the typical statistical errors are too small to produce the observed spread in stellar population properties. More conclusively, if the observed anticorrelations were due to measurement errors, they would not be systematically correlated with $\Delta L$. The observed $L$ dependence of stellar population properties at fixed $\sigma$ argues strongly that the anticorrelation is real. In Appendix B, we verify that

the observed anticorrelations are not due to systematic effects in the stellar population modeling process caused by using single burst models to fit integrated galaxy spectra, which are almost certainly not single burst populations.

It is evident from Figure 13 that, not only are the various stellar population residuals correlated with one another, they are also strongly correlated with $\Delta L$, as quantified by the PCA analysis in the previous section. Collecting all parameters together, the residual trends can be summarized as follows: at fixed $\sigma$, galaxies fainter than the median are older, Fe-poor, somewhat Mg-poor, and more Mg-enhanced than typical galaxies at that $\sigma$, while galaxies brighter than the median are younger, Fe-rich, somewhat Mg-rich, and less Mg-enhanced.

6. DISCUSSION

We have shown that (1) stellar population age, [Fe/H], [Mg/H], and [Mg/Fe] all increase with increasing galaxy $\sigma$ (Figure 6), and (2) the star formation histories of galaxies vary systematically at fixed $\sigma$ such that fainter galaxies are older, Fe-poor, and Mg-enhanced compared to their bright counterparts at the same $\sigma$ (Figure 13). The first of these results is in agreement with a substantial number of earlier works (e.g., Bernardi et al. 2003b; Thomas et al. 2005; Nelan et al. 2005; Smith et al. 2007a; Graves et al. 2007). In particular, the increase in age with $\sigma$ is consistent with “archaeological downsizing” (Thomas et al. 2005), in which massive galaxies form their stars predominantly at early times, while lower mass galaxies show evidence of younger stellar populations.
Figure 12. [Mg/Fe] as a function of σ, L, and color. Panels show the color–magnitude diagram for slices of fixed σ. Black points show galaxies included in the stacked spectra, while gray points show color and Mσ outliers that have been excluded from this analysis. Contours of [Mg/Fe] derived from the stellar population modeling of Section 4.4 are overplotted. Mg is used to trace α-elements, thus contours of [Mg/Fe] here are a proxy for contours of [α/Fe]. [Mg/Fe] increases with increasing σ. There is limited variation in [Mg/Fe] at fixed σ. A comparison with Figure 9 shows that [Mg/Fe] varies similarly to age, such that fainter galaxies at fixed σ are older and have higher [Mg/Fe]. This variation runs in the opposite sense of variation in [Fe/H] at fixed σ, which increases with increasing luminosity. When the various σ slices are stacked together in the observed red sequence, the decrease of [Mg/Fe] with increasing L at fixed σ counteracts the trend of increasing [Mg/Fe] with increasing σ, thus the σ–[Mg/Fe] relation is substantially weakened when L is used to parameterize galaxies instead of σ.

(A color version of this figure is available in the online journal.)

The anticorrelation of age and [Fe/H] at fixed σ shown in this analysis is not a new result and is in agreement with the metallicity hyperplane of Trager et al. (2000) and with the recent work of Smith et al. (2007b) for individual galaxies. However, an important new result of this work is that this anticorrelation is correlated with galaxy luminosity, such that the more luminous galaxies at fixed σ are the younger, Fe-rich galaxies while the fainter galaxies are older and more Fe-poor. This confirms that the observed anticorrelation of age and [Fe/H] must be genuine, rather than an effect of correlated stellar population modeling errors.

The covariation of age, [Fe/H], and [Mg/Fe] we observe at fixed σ cannot be driven by large-scale environment. The age–[Fe/H] anticorrelation is observed separately in the Trager et al. (2000) data, which include only field galaxies, and in the Smith et al. (2007b) data, which include only cluster galaxies. It is also observed in our stacked spectra, which average over all environments.

The observed covariation of age, [Fe/H], and [Mg/Fe] is qualitatively consistent with a scenario in which all galaxies at the same σ start forming stars at the same z but the duration of star formation varies between galaxies. At fixed σ, galaxies with short star formation timescales end star formation early and consequently have older mean stellar ages. They also have less time for enrichment in SN Ia products, resulting in lower [Fe/H] and higher [Mg/Fe]. To the extent that σ measures the depth of the gravitational potential, the loss of metals through winds should be consistent for all galaxies at the same σ if they experience comparable galactic winds. Thus early truncation of star formation should have little effect on [Mg/Fe] but should result in a genuine underabundance of [Fe/H], as observed. To also be consistent with the general trends of Figure 6, these variations in the duration of star formation must be superimposed on a staged star formation model, similar to the Noeske et al. (2007) model, where the redshift at which star-formation peaks and the duration of star formation depend on σ. Unlike the Noeske model, the data presented here additionally require a spread in star formation duration at fixed σ. A quantitative test of such a model requires the construction of chemical evolution models, which we defer to future work. Furthermore, this scenario postulates a range of star formation timescales at fixed σ but does not propose a physical mechanism to produce the observed variation.

A key result of this analysis is that the covariation of age, [Fe/H], and [Mg/Fe] at fixed σ also correlates with galaxy luminosity, such that the brighter galaxies are younger, Fe-rich, and less α-enhanced than the fainter galaxies at the same σ. If the above scenario is correct, the more luminous galaxies at fixed σ would be those which experienced more extended star formation histories. This at first appears to be a trivial result, since galaxies with more extended star formation and younger mean ages are expected to have lower M/L and therefore higher L at fixed σ. However, the differences in M/L predicted from stellar population models are too small to account for the observed variation in L. This will be shown in detail in future papers in this series, but can also be inferred from Figure 3. This figure shows that the range in galaxy colors at fixed σ is small, suggesting that the range of M/L is likewise small. In fact, the variation in
L at fixed σ turns out to be dominated by variation in total stellar mass at fixed σ, while M/L variations contribute only a modest amount. Thus any physical mechanism invoked to explain the observed covariation in stellar population properties at fixed σ must also reproduce a differing total stellar mass content in the galaxies.

7. CONCLUSIONS

This analysis has used very high S/N, stacked spectra of \( \sim 16,000 \) SDSS early-type galaxies to map out variations in stellar population age, \([\text{Fe/H}],[\text{Mg/H}],\) and \([\text{Mg/Fe}]\) with galaxy σ, L, and color. In addition to studying the mean trends in stellar populations with each of these three global properties individually, we have mapped out variations in age, \([\text{Fe/H}],[\text{Mg/H}],\) and \([\text{Mg/Fe}]\) in the three-dimensional σ–L–color parameter space. This allows us to understand the differing behavior of stellar populations as a function of σ versus L, and to explore the systematic variations in stellar populations at fixed σ.

We find the following results for quiescent galaxies:

1. Luminosity and σ both increase with galaxy mass, yet these “size” measures are not the same. Fundamental stellar population variables such as age, \([\text{Fe/H}],[\text{Mg/H}],\) and \([\text{Mg/Fe}]\) scale differently with respect to L and to σ, and also with respect to galaxy color.
2. Higher σ galaxies are typically more luminous and redder than lower σ galaxies. However, at fixed σ, there is no color–magnitude relation, despite the substantial variation in \( M_r \). The color–magnitude relation of passive galaxies is therefore a result of combining the σ–L and σ–color relations, in agreement with Bernardi et al. (2005).
3. Mean luminosity-weighted stellar population age, \([\text{Fe/H}],[\text{Mg/H}],\) and \([\text{Mg/Fe}]\) all correlate with σ such that galaxies with higher σ tend to be older and have higher \([\text{Fe/H}],[\text{Mg/H}],\) and \([\text{Mg/Fe}]\). At fixed σ, the spread in \([\text{Mg/H}]\) and \([\text{Mg/Fe}]\) is small, while both \([\text{Fe/H}]\) and age show significant spread.
4. At fixed σ, brighter galaxies have lower age, higher \([\text{Fe/H}]\), slightly higher \([\text{Mg/H}],\) and lower \([\text{Mg/Fe}]\) than their fainter counterparts. The anticorrelation of age and \([\text{Fe/H}]\) conspires to keep the color–magnitude relation flat at fixed σ and contributes to the overall narrowness of the color–σ relation.
5. At fixed σ, age is also correlated with color such that younger galaxies are both brighter and bluer at fixed σ than the fainter, redder, older galaxies. In contrast, \([\text{Fe/H}]\) and \([\text{Mg/H}]\) do not vary systematically with color, while \([\text{Mg/Fe}]\) varies only mildly with color.
6. The variation in stellar population properties with L at fixed σ acts to reinforce the σ–\([\text{Fe/H}]\) trend, resulting in a strong and tight L–\([\text{Fe/H}]\) correlation. In contrast, the σ–age and σ–\([\text{Mg/Fe}]\) correlations are opposed by the variations with L at fixed σ, leaving only weak L–age and L–\([\text{Mg/Fe}]\) correlations. In a similar way, residuals in color correlate with stellar population properties such that there is a strong...
color–age trend but only weak color–[Fe/H] and color–
[Mg/Fe] trends. Age correlates most closely with color, [Fe/H] correlates most closely with \( L \), and [Mg/H] and [Mg/Fe] correlate most closely with \( \sigma \). Only \( \sigma \) (and not \( L \) or color) correlates strongly with all four stellar population properties.

7. Trends in age, [Fe/H], [Mg/H], and [Mg/Fe] at fixed \( \sigma \) are likely not driven by environment. The age–[Fe/H] anticorrelation at fixed \( \sigma \) is detected in samples of individual galaxies which reside in similar environments (e.g., the field galaxies in Trager et al. 2000 and the cluster galaxies in Smith et al. 2007b) and also in our sample of stacked spectra, which average over all environments.

The variations in stellar population properties at fixed \( \sigma \) presented here illustrate that the narrow, seemingly one-dimensional color magnitude relation of quiescent galaxies conceals an underlying set of star formation histories that populate a two-parameter family. Not only do the star formation histories of galaxies vary systematically with their \( \sigma \), but there is clearly a range of processes at work in galaxies of the same \( \sigma \). These result in an anticorrelation between galaxy age and [Fe/H] at fixed \( \sigma \) and a weaker but positive age–[Mg/Fe] correlation. Moreover, these properties are linked to the present-day luminosities of the galaxies, such that brighter galaxies are younger, more metal-rich, and less \( \alpha \)-enhanced than fainter galaxies at the same \( \sigma \). The companion papers in this series will explore in more detail the connection between galaxy structure and galaxy stellar populations, providing evidence for the ways in which the star formation history of a galaxy is linked to its mass assembly and structural evolution.

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APPENDIX A

BULGE + DISK VERSUS de VAUCOULEURS MAGNITUDES AND COLORS

This analysis is based on magnitudes and colors measured from de Vaucouleurs fits to the galaxy light profiles. However, given that a fraction of the galaxies in the sample have a visible disk component it is important to verify that de Vaucouleurs magnitudes and colors are a reasonable approximation to the total galaxy magnitudes and colors.

Figure 14. Distribution of \( g - r \) color differences between composite bulge + disk and de Vaucouleurs model light profiles for our sample galaxies. In the bottom panel, the galaxies with color differences \(< 0.001 \) mag have been removed, showing the nearly Gaussian distribution of the remaining color differences. Overall, > 97% of the color differences are less than the 0.022 mag rms error in the \( g - r \) colors measured from SDSS photometry (dotted lines). We conclude that using magnitudes and colors derived from de Vaucouleurs fits to the galaxy light profiles does not substantially affect the results presented here.

The Petrosian magnitudes measured by the SDSS pipeline have larger errors (typically 5% in the \( r \) band) than the de Vaucouleurs model fits (typically 2% in the \( r \) band) as determined by repeat observations of targeted galaxies. However, they have the benefit of being independent of the galaxy light profile. We compare Petrosian \( r \)-band magnitudes to the de Vaucouleurs magnitudes used in our analysis and find that the de Vaucouleurs magnitudes are systematically brighter by \(< 0.1 \) mag (consistent with Blanton et al. 2001), with an rms scatter of 0.04 mag. This scatter is much smaller than the 0.8 mag bin widths used to sort and stack galaxy spectra and should therefore have little impact on the analysis presented here.

Color effects may be more important, both because the color differences observed in our sample galaxies are intrinsically small and because disk galaxies may exhibit significant color gradients. We combine the separate de Vaucouleurs and exponential fits to galaxy light profiles, weighting each component by the fractional contribution of each component (the SDSS pipeline value \( \text{fracDeV} \)) to produce composite bulge + disk galaxy magnitudes and colors. We compare the \( g - r \) colors from the bulge + disk models to the de Vaucouleurs profile colors in Figure 14. In the top panel, we show the distribution of color differences for all galaxies that go into our stacked spectra (i.e., after the color and magnitude outliers have been removed from each \( \sigma \) bin). The majority of galaxies show \(< 0.001 \) mag difference between the composite and de Vaucouleurs color values because the galaxies are highly bulge dominated. Removing these and examining the distribution of color differences for
galaxies with color differences larger than 0.001 mag (bottom panel), we find that the distribution is roughly Gaussian, with \( \mu = 0.0015 \) mag and \( \sigma = 0.0069 \) mag.

Given that the rms error for \( g-r \) color photometry in SDSS is \( \sim 2\% \) (0.022 mag, dotted lines in Figure 14), the color differences from the different model light profiles are within this error for \( > 97\% \) of our galaxy sample. We therefore conclude that the use of de Vaucouleurs model fits for the magnitudes and colors in this analysis does not have a significant effect on the results.

APPENDIX B

THE EFFECT OF YOUNG SUBPOPULATIONS

Because the observed age–[Fe/H] anticorrelation at fixed \( \sigma \) is in the same sense as the age–metallicity degeneracies that plague stellar population studies, it is necessary to confirm that systematic errors in the modeling process are not producing a spurious anticorrelation. We showed in Figure 13 that the age–[Fe/H] anticorrelation at fixed \( \sigma \) is not due to random observational errors. However, the models used in this work are single stellar population (SSP) models, whereas galaxies are composite populations with extended star-formation histories. The mean age measured from an SSP model is a mean luminosity-weighted age for all the stars contributing to the total light. Because young populations have lower mass-to-light ratios and stronger Balmer absorption lines, a subpopulation of intermediate age stars (either a younger subpopulation within individual galaxies or a subset of significantly younger galaxies in the bin) will contribute disproportionately to the total galaxy light, skewing the SSP age to low values. If the young subpopulation greatly strengthens the Balmer absorption lines without changing the Fe absorption lines much, the resulting model grid inversion may infer higher metallicities just due to the stronger Balmer lines rather than a genuine difference in metallicity.

This is in the correct sense to account for the anticorrelation we see between galaxy age and [Fe/H] at fixed \( \sigma \). However, it alone cannot account for metallicity differences at fixed \( \sigma \), as illustrated in Figure 15. This figure shows a stellar population model grid for the (Fe) and H\( \beta \) indices. The solid lines connect models of constant [Fe/H], while the dotted lines connect models of constant age. Triangles show the index measurements for stacked spectra in this work, with colors indicating the \( \sigma \) range represented (purple shows the lowest \( \sigma \) bin, red the highest \( \sigma \) bin, as in Figure 3). We have averaged over the three different color bins so the three points shown for each \( \sigma \) bin represent three different sub-bins in \( L \). The gray track with diamonds shows the effect of adding a young subpopulation to an underlying old population. The old population has age \( = 14\) Gyr and [Fe/H] = \(-0.4\), while the young subpopulation has the same [Fe/H] and age \( = 1.2\) Gyr. The gray diamonds indicate the fraction of the total luminosity at 5000 Å provided by the burst, starting at 0\% at the bottom and increasing in increments of 5\%.

It is clear that the trajectory in index–index space covered by composite burst models with fixed [Fe/H] is very different from the observed covariation in galaxy age and [Fe/H]. It is true that the [Fe/H] inferred for a composite burst would be skewed slightly high, since the gray track in Figure 15 lies consistently to the right of the [Fe/H] = \(-0.4\) grid line despite the fact that both subpopulations in the composite models have [Fe/H] = \(-0.4\). However, the effect is small compared to the observed variation in (Fe) at fixed \( \sigma \) in the galaxy sample. The data therefore require genuine variations in mean [Fe/H] at fixed \( \sigma \), in addition to age variations.

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