Estimation of scour depth below free overfall spillways using multivariate adaptive regression splines and artificial neural networks

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Erosion and scouring caused by the outflow jets of hydraulic structures is one of the most important topics in hydraulic engineering. In free overfall spillways, a water jet impacts the erodible downstream bed almost vertically and creates a scour hole. The scour hole can affect the safety and stability of the dam. In the present paper, the multivariate adaptive regression splines (MARS) approach has been adopted as a new soft computing tool for estimating the equilibrium scour depth below free overfall spillways. Using experimental data and dimensionless parameters, the MARS model has been developed to predict scour depth. Results obtained from the MARS approach were compared with those from the artificial neural network (ANN) and decision-tree algorithm. Statistical indicators demonstrated that the MARS approach had a good performance and presented competitive results, slightly better than ANN, for the prediction of this phenomenon.

Keywords: artificial neural network; free overfall spillway; multivariate adaptive regression splines; scour depth; decision tree

1. Introduction

In the structure of the dams, spillways are constructed to discharge the water excess to the reservoir capacity. In free overfall spillways, waterfalls over the crown of the spillway almost vertically and impacts the downstream bed of the dams. Due to the high velocity and energy of the flow which impacts the erodible downstream bed, it may cause scouring close to the foundation of the dam and consequently threaten the stability of the dam. Thus, an accurate estimation of the scour depth is one of the important topics in hydraulic engineering. In order to estimate the scour depth in downstream spillways, empirical equations based on regression methods have been proposed by many researchers (Mason & Arumugam, 1985). Figure 1 displays a schematic view of the local scour downstream of a free overfall spillway.

At present, the development of soft computing models and the use of these methods in modeling complex and non-linear issues has increased considerably. A review of previous studies shows the application of soft computing techniques in many fields of civil engineering, such as water resources engineering, and in the simulation of engineering problems (Chau, Wu, & Li, 2005; Chen & Chau, 2006; Cheng, Chau, Sun, & Lin, 2005; Taormina, Chau, & Rajandrea, 2012; Wu, Chau, & Li, 2009), geotechnical engineering (Alavi & Gandomi, 2011a, 2011b), earthquake engineering (Gandomi & Alavi, 2012a, 2012b), transportation engineering (Haleem, Abdel-Aty, & Santos, 2010; Yang, Gandomi, Talatahari, & Alavi, 2012), structures and infrastructures (Gandomi, Yang, Talatahari, & Alavi, 2013), coastal and ocean engineering (Chau, 2010; Etemad-Shahidi & Bali, 2012; Jabbari & Talebi, 2011; Kamranzad, Jabbari, & Samadi, 2013), construction engineering (Afshar & Amir, 2010; Eshtehardian, Afshar, & Abbasnia, 2009), and environmental engineering (Etehaj & Bonakdari, 2013; Muttil & Chau, 2006; Saadatpour, Afshar, & Afshar, 2011).

In the last two decades, a new modeling technique called multivariate adaptive regression splines (MARS) has been introduced (Friedman, 1991). This technique has been applied successfully in various scientific fields. Among the notable applications of this method are the prediction of maximum shear modulus and minimum dam ratio (Samui & Kothari, 2012), the prediction of the maximum magnitude of a reservoir-induced earthquake (Pijush & Dookie, 2012), rainfall-runoff modeling (Sharda, Prasher, Patel, Ojasvi, & Chandra, 2008), the prediction of the friction capacity of driven piles in clay (Samui, 2011), the determination of the uplift capacity of suction caisson in clay (Samui, Das, & Kim, 2011), stream flow forecasting (Coulibaly & Baldwin, 2005), the prediction of sub-daily rainfall (Beuchat, Schaepli, Soutter, & Mermod, 2011).

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downscaling daily precipitation (Nasser, Tavakol-Davani, & Zahrarie, 2013), and pavement performance modeling (Attoh-Okine, Cooger, & Mensah, 2009).

In order to model scouring, a variety of soft computing schemes have been used in recent years, such as an artificial neural network (ANN) to predict scour below a ski-jump bucket spillway (Azamathullah, Deo, & Deolalikar, 2005) and culvert outlets (Azamathullah & Haque, 2012), classification and regression trees to estimate wave-induced scour around a circular pile (Ayoubloo, Etemad-Shahidi, & Mahjoobi, 2010), an M5’ model tree to estimate scour downstream of a ski-jump bucket spillway (Goyal & Ojha, 2011), an adaptive network based on fuzzy systems to determine scour around an arch-shaped bed sill (Keshavarzi, Gazni, & Homayoon, 2012), linear genetic programming to predict scour around a circular pile (Guven, Azamathulla, & Zakaria, 2009), and the group method of data handling to predict scour downstream of a ski-jump bucket spillway (Najafzadeh, Barani, & Hessami Kermani, 2014).

The above-mentioned studies reflect the successful application of soft computing techniques in scour modeling.

Recently, Samadi et al. (2014) employed two decision-tree algorithms, namely classification and regression trees (CART) and the M5’ model tree, for the prediction of equilibrium scour depth below free overfall spillways. These two algorithms were employed in order to discover and extract knowledge as a set of rules from the database used in order to provide mathematical expressions for the prediction of scour depth below free overfall spillways.

Equations derived from M5’ and CART can be employed in scour calculations as easily as empirical and traditional methods. In addition, a previous study (Samadi et al., 2014) showed that soft computing approaches are more accurate than empirical and traditional methods for the prediction of scour depth below free overfall spillways.

The CART and M5’ models expand in the form of a hierarchical inverted binary tree to predict scour depth. The main difference between these two methods, however, is that the M5’ model tree approach provides practical equations for the prediction of scour, while the regression trees give values for output parameters. Hence, it can be said that the M5’ model tree is more interpretable compared with CART. Furthermore, it seems that the equations resulting from M5’ can give better engineering judgment about the relation between the parameters and their importance (Samadi et al., 2014).

ANNs are one of the most widely used soft computing models applied to solve engineering problems. One of the weaknesses that usually has to be considered for the ANN model is that it acts like a black-box in that finding a relationship between independent variables and output parameters is difficult and it therefore does not have the ability to interpret and understand the results, although there have been attempts in some studies to convert the ANN from a black-box model into an explicit formulation (Alavi & Gandomi, 2011a; Alavi, Gandomi, Mollahasani, Heshmati, & Rashed, 2010). However, it seems that the method of extracting knowledge and obtaining equations from the M5’ model tree and CART is easier to understand, more convenient to use and provides a clearer relationship between input variables and output parameters compared with ANN.

The MARS approach is a novel technique that uses a large number of piece-wise regression equations for the prediction of output parameters based on input parameters. It is notable that some studies have shown that the MARS approach can provide competitive or even better results compared to those of ANN (Abraham, Steinberg, & Philip, 2001; Samui et al., 2011; Yang, Prasher, Lacroix, & Kim, 2004).

The primary aims of the present study are to develop practical equations for the prediction of scour depth below free overfall spillways using the MARS method and to evaluate the performance of this method. Moreover, the accuracy of the MARS and ANN models developed in the present study are compared with the results obtained from two decision-tree algorithm methods, namely M5’ and CART, which have already been performed by Samadi et al. (2014).

2. Multivariate adaptive regression splines (MARS)

Developed in the early 1990s by statistician Jerry Friedman, MARS is a fast, flexible and precise technique for solving regression and classification problems (Friedman, 1991). MARS is a new soft computing technique which combines several simple linear regression schemes. This technique divides the solution domain into multiple ranges of predictive variables (input). Moreover, it can automatically search for the non-linear input/output relationship hidden in a large database and create an explicit model. It can also determine the importance of the weights of the inputs with regard to the corresponding output (Yang, Prasher, Lacroix, & Kim, 2003). It makes basis functions for each input variable and then generates flexible
regression models for the prediction of the output parameters by combining the obtained basis functions.

The method is applicable for large or even small datasets with a free distribution type. MARS divides the space of the predictors into multiple knots and then fits a spline function between these knots. The elements used to fit a MARS model are called basis functions, each of which can be a main effect or an interaction of variables (Haleem, Gan, & Lu, 2013).

The basis function represents the information about the independent variable (input) and its relation with the output parameter and is defined by the following equations (Friedman, 1991):

\[ h_m(x) = \max(0, c - x), \]  

(1)

or

\[ h_m(x) = \max(0, x - c), \]  

(2)

where \( x \) is the input variable, \( c \) is the threshold value for input variable of \( x \), and \( h_m(x) \) is the basis function.

The general form of the MARS model is introduced as:

\[ Y = f(x) = \beta_0 + \sum_{m=1}^{M} \beta_m h_m(x), \]  

(3)

where \( Y \) is the output parameter, \( \beta_0 \) is the constant value, \( M \) is the number of basis functions, \( h_m(x) \) is the \( m \)th basis function and \( \beta_m \) is the corresponding coefficient of \( h_m(x) \) (Friedman, 1991).

Two important steps are taken in the development of MARS in order to produce the optimal structure for the prediction of phenomena. In the first step, all the possible basis functions are introduced to define Eq. (3). As a result, certain overfitted models with many basis functions are developed (forward step). In the second step, to prevent overfitting, the basis functions of less importance are discarded via the generalized cross-validation (GCV) measure and omitted from Eq. (3).

GCV was introduced by Craven and Wahba (1979) and extended by Friedman (1991) for MARS.

The expression of the GCV measure is described as follows:

\[ \text{GCV} = \frac{(1/n) \sum_{i=1}^{n} [y_i - f(x_i)]^2}{[1 - (C(B)/n)]^2}. \]  

(4)

In the above equation, \( n \) denotes the number of observations and \( C(B) \) denotes a complexity penalty that increases with the number of basis functions in the model. It is defined as:

\[ C(B) = (B + 1) + dB, \]  

(5)

where \( d \) denotes a penalty for each basis function included in the model and \( B \) denotes the number of basis functions.

More information about \( d \) and the details of MARS has been provided by Friedman (1991).

3. Artificial neural networks (ANNs)

The ANN scheme is one of the most popular soft computing tools and has been used in a broad spectrum of engineering problems. The ANN is a nonlinear mathematical model with the ability to simulate the biological neurons of the human brain (Kamranzad, Etemad-Shahidi, & Kazeminezhad, 2011). Each neuron input in hidden layers or output layer is multiplied by a corresponding interconnection weight, and then a bias (threshold) is added to the sum of the products:

\[ I_j = \sum_i W_{ij} x_i + \theta_j. \]  

(6)

Next, the result is passed through a transfer function which is usually a sigmoidal function \( F \) to determine an output, \( O_j \), defined respectively by:

\[ F(x) = \frac{1}{1 + \exp(-x)}, \]  

(7)

\[ O_j = F(I_j), \]  

(8)

where \( O_j \) is the output from the \( j \)th neuron, \( W_{ij} \) is the weight of the connection from neuron \( j \) in the previous layer to neuron \( i \), \( x_i \) is the output of the neuron from the previous layer, and \( \theta_j \) is the bias (threshold) of the \( j \)th neuron (see Figure 2).

The most common neural network is a multilayer perceptron (MLP). An MLP consists of an input layer, one or more hidden layers and an output layer. Each layer contains neurons (nodes) and the number of nodes in the input and output layers represents the input variables (predictor variables) and dependent variables (target variables), respectively. The number of neurons in the hidden layers corresponds to the complexity of a given problem and is determined in the training stage via a trial-and-error approach.

In a feed-forward network, input flows are unidirectional from the input to the output layer and each node is connected solely to one node in the next layer; there are no links among the nodes in the same layer.

Once the configuration of the network is specified and the network is trained, the training process searches for the smallest error function (mean sum squared) as defined by:

\[ E = \frac{1}{2} \sum (O_n - t_n)^2, \]  

(9)

where \( O_n \) and \( t_n \) are the network output and target output of the \( n \)th output node, respectively.

At the commencement of training, the weights are randomly initialized, then fixed by the following process:

\[ \Delta W(k + 1) = \alpha \Delta w(k) - \eta \partial E/\partial W, \]  

(10)

where \( \Delta w(k) \) indicates the change in the weight at the \( k \)th iteration and \( \Delta w(k + 1) \) is the change in the weight at the \((k + 1)\)th iteration; \( \alpha \) indicates the momentum factor and \( \eta \) indicates the learning rate.
4. Relevant parameters and dimensional analysis

The equilibrium scour depth below a free overfall spillway can be a function \((f)\) of certain parameters expressed by the following relations (Azar, 1998; Ghodsian & Azar, 2002):

\[
D_s = f(q, H, Y_t, d_{50}, g, \rho_w, \rho_s),
\]  
(11)

where \(D_s\) is the depth of scour measured from the surface of tail water, \(q\) is the unit discharge, \(H\) is the total head or the difference between the upstream and downstream elevations, \(Y_t\) is the depth of the tail water, \(d_{50}\) is the mean diameter of the sediment grains in the downstream bed, \(g\) is the gravitational acceleration and \(\rho_w\) and \(\rho_s\) are the mass density of the water and sediment, respectively.

Applying dimensional analysis, the governing dimensionless equation is written as:

\[
\frac{D_s}{H} = f \left( \frac{q}{\sqrt{gH^3}}, \frac{d_{50}}{H}, \frac{H}{Y_t}, \frac{\rho_w}{\rho_s} \right).
\]  
(12)

Furthermore, by discarding the constant parameter \(\frac{\rho_w}{\rho_s}\) and combining \(\frac{q}{\sqrt{gH^3}}\) and \(\frac{d_{50}}{H}\), it is possible to rewrite Eq. (12) as (Azar, 1998; Ghodsian & Azar, 2002):

\[
\frac{D_s}{H} = f \left( F_1, \frac{H}{Y_t} \right),
\]  
(13)

where \(F_1 = \frac{q}{d_{50} \sqrt{gH}}\) is a densimetric Froude number (Ghodsian & Azar, 2002).

5. Data collection

In the present study, in order to estimate the depth of scour below a free overfall spillway, a database comprised of 104 data entries from laboratory experiments was taken from the work of Samadi et al. (2014). A brief description of the data used for development of the MARS and ANN models is given below.

5.1. Mahboobi experiments

Mahboobi (1997) conducted experiments to measure the maximum scour depth downstream of a free overfall spillway. He applied two different discharges per width of the spillway, 0.01 and 0.015 m³/s, three drop heights of 45 cm, 60 cm, and 75 cm, and a constant tailwater depth of 10 cm. Uniform sediment with a mean size of 1 mm, 2.57 mm, 7.15 mm, and 11 mm was used below the spillways bed material. Mahboobi obtained 24 sets of data from his experiments.

5.2. Azar experiments

Azar (1998) measured scour downstream of a free overfall spillway. The total discharge of the spillway he applied ranged between 5 and 20 L/s. Drop heights ranged between 26 cm and 51 cm and tailwater depth varied from 5 to 25 cm. Azar employed uniform sediment with a mean size of 2.9 mm, 5 mm, 8 mm, and 15.6 mm downstream of the spillway. He obtained 80 sets of data from his experiments.

6. Development of the models

All of the 104 laboratory data entries that were collected were randomly divided into two groups – the training set and the testing set. Two thirds of them (69 data entries) were selected for training and the rest (35 data entries) were used to test the models. Table 1 shows the statistical parameters of the input and output variables of the datasets used in this study.

6.1. MARS model development

In order to develop the MARS model, with regard to Eq. (13), two parameters of \(F_1\) and \(\frac{H}{Y_t}\) were used as input parameters for the estimation of the relative scour depth \(\frac{D_s}{H}\).
To develop the MARS model, in the first step (forward step) thirteen basis functions were obtained with this method. In the second step (backward step), four basis functions was removed. In the end, the optimal MARS model was obtained with nine basic functions, as described below:

\[
D_s = 0.383 + \sum_{\mu=1}^{9} \beta_m h_m(x). \quad (14)
\]

The expression for \( h_m(x) \) and values of \( \beta_m \) are presented in Table 2.

The general form of the MARS model that can be used to estimate scour depth downstream of a free overfall spillway is obtained from Eq. (14). As can be seen, the basis functions of MARS are determined by considering the range and values of the independent variables \( F_1 \) and \( H/H' \) and substituting in the general form of the scour depth equation, Eq. (14), which is transformed into the simple mathematical expression Eq. (15), which can easily be used for computation of the scour.

\[
D_s = \frac{H}{H'} = \frac{0.383 + 0.794 \times (2.547 - \frac{F_1}{Y'}) - 0.210 \times (1.434 - F_1)}{1}. \quad (15)
\]

As can be seen, the basis functions of MARS are determined by considering the range and values of the independent variables \( F_1 \) and \( H/H' \) and substituting in the general form of the scour depth equation, Eq. (14), which is transformed into the simple mathematical expression Eq. (15), which can easily be used for computation of the scour.

The following explanation can be given of how to use the MARS model obtained in this study to estimate scour depth (Eq. (14)) according to the range of the input parameters \( F_1 \) and \( H/H' \) for estimation of \( D_s/H' \).

For example, if \( H/H' \) becomes smaller than 1.66, the values of basis functions \( h_1, h_5, h_6, h_8, \) and \( h_9 \) will be zero. Moreover, if \( F_1 \) becomes smaller than 0.758, the values of basis functions \( h_3 \) and \( h_7 \) will be zero. Finally, using these values of the basis functions in Eq. (14), the following equation is obtained to estimate scour depth:

\[
D_s = \frac{H}{H'} = \frac{0.383 + 0.794 \times (2.547 - \frac{F_1}{Y'}) - 0.210 \times (1.434 - F_1)}{1}.
\]

As can be seen, the basis functions of MARS are determined by considering the range and values of the independent variables \( F_1 \) and \( H/H' \) and substituting in the general form of the scour depth equation, Eq. (14), which is transformed into the simple mathematical expression Eq. (15), which can easily be used for computation of the scour.

As was illustrated, in order to estimate \( D_s/H' \) with regard to the range of input variables and basis functions, the proposed MARS model could be transformed into simpler equations that can easily be employed to estimate scour depth.

### 6.2. ANN model development

The same training and testing data sets employed for the development of MARS were used for the development of ANN.

For the ANN technique in the present study, a simple MLP with three layers (input, hidden and output) with a back propagation (BP) learning rule was used to train the network. Also, the number of hidden-layer neurons was chosen based on the following criterion presented by Hecht-Nielsen (1987) and Rogers and Dowla (1994):

\[
N^H \leq 2N^I + 1 \quad (16)
\]

\[
N^H \leq \frac{N^{TR}}{N^I} + 1 \quad (17)
\]
where \( N^H \) is the number of nodes in the hidden layer, \( N^I \) is the number of inputs and \( N^{TR} \) is the number of training samples.

Recommendations have been made for the selection of the learning rate and the momentum factor (Basheer & Hajmeer, 2000). For example, Zupan and Gasteiger (1993) suggest that the summation of the selected learning rate and the momentum factor should be approximately equal to 1. Fu (1995) recommends a learning rate and momentum factor between 0 and 1. Furthermore, Swingler (1996) uses a learning rate of 0.25 and a momentum factor of 0.9 in order to solve all problems.

During the development of the ANN, different numbers of hidden layer neurons can be taken, using the Hecht-Nielsen (1987) method via a trial-and-error procedure, until the eventual optimum number of nodes in the hidden layer is found, which was four. The network training was terminated when 97% of the outputs were successfully matched with the actual values. In addition, we chose a learning rate and momentum coefficient of 0.25 and 0.9, respectively, as Swingler (1996) proposed. The illustration in Figure 3 shows the proposed ANN configuration used in this study.

The corresponding weights and biases extracted from the trained network, given in Table 3, could be beneficial and practical for the prediction of scour depth in the future.

| Input parameter | CC    | RMSE  |
|-----------------|-------|-------|
| \( H_Yt \)      | 0.9244| 0.1610|
| \( F_1 \)       | 0.4551| 0.3849|

Weights obtained from the optimal ANN in this study could be useful for other researchers seeking to reproduce the network on their own (Ayoubloo, Azamathulla, Jabbari, & Zanganeh, 2011).

Moreover, in order to find the importance of different input parameters based on output parameters, new models of ANN with \( H_Yt \) and \( F_1 \) were constructed and evaluated individually to estimate \( D_s \).

The results of these models are shown in Table 4. As can be seen, the model made only on the basis of parameter \( H_Yt \) has the highest accuracy compared to the model made only on the basis of parameter \( F_1 \). This fact shows the higher importance of \( H_Yt \) in the estimation of scour depth compared to \( F_1 \). It is noteworthy that the obtained results are consistent with those obtained from the MARS model and previous studies conducted by Samadi et al. (2014).

### 7. Analysis of the development models

In this study, in order to assess the performance and accuracy of the models quantitatively, statistical indicators such as the correlation coefficient (CC), root mean square error (RMSE) and mean absolute error (MAE) were used. These parameters are defined as below:

\[
CC = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{i=1}^{n} (y_i - \bar{y})^2}},
\]

\[
RMSE = \sqrt{\frac{\sum_{i=1}^{n} (x_i - y_i)^2}{n}},
\]

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} |x_i - y_i|,
\]
where \( x_i \) and \( y_i \) are the measured and predicted parameters, respectively, \( \bar{x} \) and \( \bar{y} \) are the average of the measured and predicted values, respectively, and \( n \) is the total number of measurements. A model’s performance is good when the amount of CC is close to one and the values of RMSE and MAE are close to zero.

8. Results and discussion

The accuracy of the MARS model developed in this study is verified by the testing data set and presented in Table 5. The results obtained from the MARS are compared with the results of the ANN obtained in this study and the M5′ model tree and CART reported by Samadi et al. (2014).

Considering the contents of Table 5, it is clear that compared with the other models for estimation of scour downstream of a free overfall spillway, MARS is superior and has a higher accuracy. Moreover, although the results obtained with the MARS and ANN models are very close and more or less similar, the MARS model results are slightly better in determining the value of \( \frac{D_s}{H} \), as indicated by the high value of the CC and the low error indices (RMSE, MAE).

The MARS model provides a regression equation, employing many partial functions (basis functions) between the input variables and the output parameters, which results in a more transparent relation between the output and input parameters. This allows a straight estimation of the scour depth using only two predictor variables, i.e., \( \frac{H}{Y_t} \) and \( F_1 \). The regression equation obtained from MARS can readily and easily be implemented and used for scour calculation below free overfall spillways, while ANN does not provide a clear relationship between input and output parameters for the prediction of scour depth and it seems it is difficult to extract understandable information about the relation between the input and output parameters.

In addition, in order to train the ANN it is necessary for the user to determine and select the optimum values of the parameters required by the network for proper performance. The number of hidden layers, the number of the nodes in the hidden layers, the rate of learning, the momentum coefficient, and the transfer function can be mentioned among the parameters that should be determined by the user in order to train the ANN.

Therefore, optimum selection of the values of these parameters usually requires the application of a trial-and-error procedure, which makes the construction of the ANN time consuming, while MARS can automatically search a large dataset and extract knowledge in the form of an explicit model that clearly shows the relation between input variables and output parameters (Yang et al., 2003). In addition, the obtained regression equations from MARS indicate the importance of input variables in the estimation of the output parameters, while in order to highlight the importance of each input variable ANN requires new models to be built, trained and tested individually for sensitivity analysis. Hence, determination of the importance of the input variables in the estimation of the output parameters in ANN compared to MARS is time consuming.

Additionally, comparison of MARS with the M5′ model shows that MARS results are significantly better than M5′ considering the error measures of RMSE and MAE, such that RMSE and MAE decreased by 53% and 50.9% respectively (see Table 5).

Figure 4 shows a comparison between the observed and predicted dimensionless scour depth by the MARS model for the testing data. Furthermore, a scatter plot of the ANN, illustrated in Figure 5, graphically displays the performance of the ANN in the prediction of the value of \( \frac{D_s}{H} \). As is shown in Figures 4 and 5, MARS and ANN have good performance in the prediction of scour depth.

Table 5. Statistical error measures of the MARS, ANN, M5′ and CART approaches proposed in this study for the estimation of \( \frac{D_s}{H} \).

| Approach                   | Training          | Testing           |
|----------------------------|-------------------|-------------------|
|                            | \( CC \) | \( RMSE \) | \( MAE \) | \( CC \) | \( RMSE \) | \( MAE \) |
| MARS                       | 0.9950 | 0.0425  | 0.0319  | 0.9965 | 0.0360  | 0.0302  |
| ANN                        | 0.9916 | 0.0559  | 0.0417  | 0.9942 | 0.0459  | 0.0369  |
| M5′ (Samadi et al., 2014)  | 0.9857 | 0.0716  | 0.0562  | 0.9833 | 0.0766  | 0.0615  |
| CART (Samadi et al., 2014) | 0.9260 | 0.1599  | 0.1321  | 0.9305 | 0.1546  | 0.1288  |
As can be seen in Figure 4, the MARS model predicts $D_s/H$ appropriately and predicts the trend of the observed data very well. This is confirmed by the values of the statistical parameters, such as CC, in Table 5.

9. Summary and conclusions

This work suggests the use of alternative soft computing tool approaches in order to improve the accuracy of the estimation of scour depth below free overfall spillways.

Two predictive models based on ANN and MARS were developed. For this purpose, the MARS technique was employed as a new soft computing tool capable of modeling complex non-linear phenomena.

Experimental data and dimensionless parameters were used to develop the MARS and ANN models. The performance of both models was comparable. Moreover, statistical indicators demonstrated that the results of MARS were better than other soft computing methods — namely ANN, M5’ and CART — when used for the prediction of scour depth below free overfall spillways.

In contrast to ANN, as a non-black-box model MARS is able to provide a clear and explicit relation between the input variables and output parameters for the prediction of scour depth.

Practical equations derived from MARS clearly showed the importance of input parameters in the phenomenon of scour depth. However, the same process in ANN needs sensitivity analysis and the building of new models for each input variable.

In the present study, it was demonstrated with reasonable accuracy that the MARS model can estimate scour depth using limited data and few inputs. Successful application of the MARS technique in the present study shows that MARS is a robust tool for the prediction of scour and suggests that the use of this technique is an appropriate practical method for researchers in other fields of hydraulic engineering.

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**Appendix**

**Nomenclature**

ANN: artificial neural network.

MARS: multivariate adaptive regression splines.

CART: classification and regression trees.

BP: back propagation.

N: number of observations.

$N_I$: the number of the inputs.

$N_{TR}$: the number of the training samples.

MLP: multilayer perceptron.

CC: correlation coefficient.

RMSE: root mean square error.

L/s: liter per second

MAE: mean absolute error.

X: independent variable.

Y: output parameter.

W: weight of neuron.

$D_s$: the depth of scour measured from the surface of tail water.

$q$: unit discharge.

$H$: total head or the difference between the upstream and downstream elevations.

$Y_t$: depth of tail water.

$d_{50}$: mean diameter of the sediment grain.

$g$: gravitational acceleration.

$F_1$: densimetric Froude number.

$c$: the threshold value.

$h_m(x)$: basis function.

$M$: the number of the basis functions.

$W_{ij}$: the weight of the connection from the neuron $i$ in the previous layer to the neuron $j$.

$\Delta w(k)$: the change in weight at the $k$th iteration.

$\Delta w(k+1)$: the change in weight at the $(k+1)$th iteration.

$m^3/s$: discharge per width of the spillway.

**Greek letters**

$\beta_0$: the constant value.

$\beta_m$: corresponding coefficient of $h_m(x)$.

$\theta_j$: the bias (threshold) of the $j$th neuron.

$\alpha$: momentum factor.

$\eta$: learning rate.

$\rho_s$: mass density of sediment.

$\rho_w$: mass density of water.