Bihamiltonity as origin of T-duality of the closed string model

V. D. Gershun\textsuperscript{* a}

\textsuperscript{a}Institute of Theoretical Physics, NSC Kharkov Institute of Physics and Technology, P.O. Box 310108, 1 Akademicheskaya St., Kharkov, Ukraine

In assumption, that string model is the integrable model for particular kind of the background fields, the closed string model in the background gravity field and the antisymmetric B-field is considered as the bihamiltonian system. It is shown, that bihamiltonity is origin of T-duality of the string models. The new Poisson brackets, depending of the background fields and of their derivatives, are obtained. The integrability condition is obtained as the compatibility of the bihamiltony condition and the Jacobi identity of the new Poisson bracket. The B-chiral string model is dual to the chiral string model for the constant background fields.

1. Introduction

The first kind constraints in a gauge theory are the generators of the gauge transformations. All of the Fourier components of constraints are the integrals of motion and does not zero component only, on the constraints surface. Then, algebra of the integrals of motion can be non Abelian. Two dynamical systems are dual, if they are described by their sets of the integrals of motion $H_i, F^k$ the same dimension, which are in involution between themselves.

$$\{H_i, H_k\} = 0, \{F^a, F^b\} = 0, \ i, a = 1...N$$

(1)

bihamiltonian system \cite{1-3} is dynamical system with N integral of motions $H_1,...H_N$ in involution, if any of the integrals of motion can be considered as hamiltonian, and the following condition is be satisfied

$$\dot{x}^a = \{x^a, H_1\}_1 = ... = \{x^a, H_N\}_N$$

(2)

Also, the new hierarchies of the Poisson brackets (PB) are arised. It is more interesting, if the new hierarchies of the equation of motion arise under the new time coordinates $t_k$.

$$\frac{dx^a}{dt_{n+k}} = \{x^a, H_n\}_{k+1}$$

(3)

where $n, k = 1...N$. The new equations of motion are describe the new dynamical systems with the same set of the integrals of motion, which are dual to the original system. The dual set of the integrals of motion can be obtained from the original it by the mirror transformations and by the contraction of the constraints algebra. The contraction of the constraints algebra means, that dynamical system is belong to the orbits of corresponding generators and is describe the invariant subspace. The set of involutive integrals of motion is belong to Cartan subalgebra of the constraints algebra. Consequently, duality is property of the integrable models. T-duality in integrable models have considered in Ref. \cite{4,5} by the separation of variables method. This duality is correspond to the canonical transformations from one dynamical system to other it. Duality in the string models is appeared in analisis of toroidal compactifications \cite{6,7}. It was shown, that theories defined on the circles of radii $R$ and $R'$ are equivalent. In Ref. \cite{8} was given general prescription to find dual $\sigma$-models on the manifolds with Abelian isometries. T-duality of the compactified string is result of the symmetry of constraints and of the (PB)

$$H(\tau, \sigma) = \frac{1}{2}[2\pi\alpha' R p_a^2 + \frac{1}{2\pi\alpha'} R' x_a'^2]$$

$$P(\tau, \sigma) = p_a x_a'$$

$$\{x_a(\sigma), p_b(\sigma')\}_1 = \delta_{ab}\delta(\sigma - \sigma')$$

(4)
under transformations \( p_a \to x'_a, 2\pi\alpha' R \to \frac{1}{2\pi\alpha' R} \).

The nonlocal coordinates \( x_a(\sigma) = \int_{0}^{\sigma} p_a(\sigma')d\sigma' \) (5)

\([x_a(\sigma), x_b(\sigma')] \approx \theta(\sigma - \sigma')\)

arise. This kind of T-duality is described by canonical transformations \([11,12]\) and it is transform \( H \to H, P \to P \). Thus, there are two essentially different kinds of T-duality. First it is described by canonical transformations and it is transform \( H_k \to H_k, \{\} \to \{\} \). Second it is described by noncanonical transformations \([13]\) and it is transform \( H_k \to H_l, \{\} \to \{\}, k \neq l \).

We are suppose, that string model, in the background gravity field and antisymmetric B-field, is the integrable model for particular kind of the background fields and we are use bihamiltonian approach to the investigation of the T-duality and of the integrability of the closed string models. Any of generators of \( Vir \oplus Vir \) algebra can be considered as hamiltonian. But, there is only one model, dual to original, due to two elements of Cartan subalgebra. It is shown, that string model is bihamiltonian system for free string and for constant background fields. The free chiral string is dual string to free original string. The B-chiral string in the constant background fields is dual to chiral string also. In the case of the arbitrary background fields, it is obtained the new (PB), depending of the background fields and of their derivatives. The integrability condition is obtained as the compatibility of the bihamiltonity condition and of the Jacobi identity for the new (PB).

2. T-duality of the free closed string model

The free closed string model in conformal gauge is described by Lagrangian

\( L = \frac{1}{2}[\dot{x}_a^2 + x'_a^2] \) (6)

and by first kind constraints

\( H_1 = \frac{1}{2}(2\pi\alpha' p_a^2 + \frac{x'_a^2}{2\pi\alpha'}) \approx 0 \) (7)

\( H_2 = p_a x'_a \approx 0, \ x_a(\sigma + 2\pi) = x_a(\sigma) \)

These constraints form \( Vir \oplus Vir \) algebra under (PB) \( \{\} \).

\( \{L_n, L_m\}_1 = -i(n + m)L_{n-m} \)

\( \{L_n, \bar{L}_m\}_1 = -i(n - m)\bar{L}_{n-m} \)

\( \{L_n, \bar{L}_m\}_1 = 0 \) (8)

\( L_k = \frac{1}{4\pi} \int_{0}^{2\pi} (H_1 + H_2)e^{ik\sigma}d\sigma \) (9)

\( \bar{L}_k = \frac{1}{4\pi} \int_{0}^{2\pi} (H_1 - H_2)e^{ik\sigma}d\sigma \) (10)

Hamiltonian equations of motion under Hamiltonian \( H_1 \) are

\( \dot{x}_a(\sigma) = \int_{0}^{2\pi} d\sigma' \{x_a(\sigma), H_1(\sigma')\}_1 = 2\pi\alpha' p_a \)

\( \dot{p}_a = \frac{1}{2\pi\alpha'} x''_a \) (11)

The classical solution is

\( x^a(\tau, \sigma) = x^a(0) + \alpha' \int_{0}^{\tau} e^{-\tau'}d\tau' + \sqrt{\alpha' \frac{\pi}{2}} \sum \frac{\epsilon_n}{\sqrt{n}} e^{-in(\tau + \sigma)} + \frac{\epsilon_n^*}{\sqrt{n}} e^{-in(\tau - \sigma)} + c.c. \) (12)

The bihamiltonity conditions are modified

\( \dot{x}_a = \{x_a, \int_{0}^{2\pi} d\sigma' H_1\}_1 \approx \{x_a, \int_{0}^{2\pi} d\sigma' H_{\pm 2}\}_1 \)

\( = 2\pi\alpha' p_a - \alpha' P_a \) (13)

\( \dot{p}_a = \frac{1}{2\pi\alpha'} x''_a, \ P_a = \int_{0}^{2\pi} d\sigma p_a(\sigma) = const. \) (14)

The dual (PB) are

\( \{x_a, x_b\}_\pm = \pm \delta_{ab}[\pi \alpha' \epsilon(\sigma' - \sigma) - \alpha'(\sigma' - \sigma)] \) (15)

\( \{p_a(\sigma), p_b(\sigma')\}_\pm = \pm 2\pi\alpha' \delta_{ab} \frac{\partial}{\partial \sigma}(\sigma_1 - \sigma_2) \) (16)

The (PB) \( [14,15] \), without last term, was introduces in Ref. \([14,15]\). This term is reduce to absence.
of the total momentum in \([2]\). The \(Vir \oplus Vir\) algebra has wrong sign on comparison with \([8]\) under (PB) \{.,\}2. To satisfy the bihamiltonity condition, the following dual map is necessary:

\[
H_1 \to \pm H_2, \quad L_0 \to \pm L_0, \quad \tilde{L}_0 \to \pm \tilde{L}_0, \quad \tau \leftrightarrow \sigma \quad (17)
\]

The Gupta-Bleyler quantization of this model is reduce to the same mass spectrum and to the same wave functions, with regard to the dual mapping. The dual dynamical system is defined by equations

\[
\dot{x}_a = \{x_a, \pm \int_0^{2\pi} d\sigma H_2\}_1 = \pm x'_a,
\]

\[
\{x_a, \int_0^{2\pi} d\sigma H_1\}_2 = \pm x'_a, \quad \dot{p}_a = \pm p'_a \quad (18)
\]

and is describe the left(right)chiral string \(x_a(\tau, \sigma) = x_a(\tau \pm \sigma)\). It means, that \(L_n = 0, \) or \(\tilde{L}_n = 0, \) \(n \neq 0.\) The Gupta-Bleyler quantization of the chiral string is coincide with the quantization of the closed string with additional condition of "freezing" modes \(L_n = 0, \) or \(\tilde{L}_n = 0, \) \(n \neq 0 \) in the strong sense. Let me remember, that both dynamical systems, the original string and the chiral string, is described by the same Lagrangian. Also, it is possible to consider hamitlonians \(L_k + \tilde{L}_k\) together with (PB)

\[
\{x_a(\sigma), p_b(\sigma')\}_k = \epsilon^{ik\sigma'} \delta_{ab}(\sigma - \sigma') \quad (19)
\]

and hamitlonians \(L_k - \tilde{L}_k\) together with (PB)

\[
\{x_a(\sigma), x_b(\sigma')\}_k = \delta_{ab}[\pi \alpha' \epsilon(\sigma' - \sigma) - \alpha'(\sigma' - \sigma)]e^{ik\sigma'} \quad (20)
\]

\[
\{p_a(\sigma), p_b(\sigma')\}_k = \delta_{ab} \frac{1}{2\pi \alpha'} e^{ik\sigma'} \frac{\partial}{\partial \sigma} \delta(\sigma - \sigma')
\]

But, this hamitlonians do not generate the new dynamical systems.

### 3. Closed string in the background fields

The closed string in the background gravity field and the antisymmetric B-field is described by first kind constraints

\[
H_1 = \frac{1}{2} g^{ab}(x)[p_a - \alpha B_{ac}(x) x'c][p_b - \alpha B_{bd}(x) x'd] + \frac{1}{2} g_{ab}(x)x'^ax'^b, \quad H_2 = p_a x'^a \quad (21)
\]

where \(a, b = 0, 1, \ldots D - 1,\alpha\) -arbitrary parameter. The original (PB) are

\[
\begin{align*}
\{x^a(\sigma), p_b(\sigma')\}_1 &= \delta^a_b \delta(\sigma - \sigma') \quad (22) \\
\{x^a, x^b\}_1 &= \{p_a, p_b\}_1 = 0
\end{align*}
\]

The dual (PB) has following form

\[
\begin{align*}
\{A(\sigma), B(\sigma')\}_2 &= \frac{\partial A}{\partial x^a} \frac{\partial B}{\partial x^b} \left[\omega^a_b(\sigma) + \omega^b_a(\sigma')\right] \epsilon(\sigma' - \sigma) + \\
&+ \left[\Phi^a_b(\sigma) + \Phi^b_a(\sigma')\right] \frac{\partial}{\partial \sigma} \delta(\sigma' - \sigma) + \\
&+ \left[\Omega^a_b(\sigma) + \Omega^b_a(\sigma')\right] \delta(\sigma' - \sigma)
\end{align*}
\]

\[
\begin{align*}
&\frac{\partial A}{\partial p_a} \frac{\partial B}{\partial p_b} \left[\omega^a_b(\sigma) + \omega^b_a(\sigma')\right] \epsilon(\sigma' - \sigma) + \\
&+ \left[\Phi^a_b(\sigma) + \Phi^b_a(\sigma')\right] \frac{\partial}{\partial \sigma} \delta(\sigma' - \sigma) + \\
&+ \left[\Omega^a_b(\sigma) + \Omega^b_a(\sigma')\right] \delta(\sigma' - \sigma)
\end{align*}
\]

The arbitrary functions \(A, B, \omega, \Phi, \Omega\) are functions of \(x^a(\sigma), p_a(\sigma)\) and \(\omega^a_b, \omega^b_a, \Phi^a_b, \Phi^b_a\) are symmetric functions on \(a, b\) and \(\Omega^a_b, \Omega^b_a\) are antisymmetric functions.

#### 3.1. Constant background fields

The equations of motion under (PB) \{.,\}1 are

\[
\begin{align*}
\dot{x}_a &= g^{ab} (p_b - \alpha B_{bc} x'^c) \\
\dot{p}_a &= \alpha B_{bc} g^{bd} p_c + \left[ g_{ab} - \alpha^2 B_{ac} g^{cd} B_{cd}\right] x'^b
\end{align*}
\]

The bihamiltonity condition is reduce to the following constraints on the phase space

\[
\begin{align*}
\dot{x}_a &= -2\omega^a_b x'^b + 4\omega^a_b p_b + 2\Phi^a_b p'_b - 2\Phi^b_a x'^b + 2\Omega^a_b x'^b, \quad \omega^a_b = 0 \\
\dot{p}_a &= -4\omega_a b x'^b - 2\Phi_{ab} x'^b + 2\Omega_{ab} x'^b + 4\omega^b_a p_b + 2\Phi^b_a p'_b + 2\Omega^b_a p_b = \alpha B_{bc} g^{bd} p_c + \\
&+ \left[ g_{ab} - \alpha^2 B_{ac} g^{cd} B_{cd}\right] x'^b
\end{align*}
\]
There is the unique solution without constraints
\[ \omega^{ab} = \frac{1}{4}g^{ab}, \quad 2\Omega_b^a = -\alpha g^{ac}B_{cb} = \alpha B_{bc}g^{ca}, \]
\[ -2\Phi_{ab} = g_{ab} - \alpha^2 B_{ac}g^{cd}B_{db} \]  
(26)
The rest structure functions are equal zero. The dual (PB) are
\[ \{x^a(\sigma), x^b(\sigma')\} = \frac{1}{2}g^{ab}(\sigma' - \sigma) \]  
(27)
\[ \{x^a(\sigma), p_b(\sigma')\} = -\alpha g^{ac}B_{cb}\delta(\sigma' - \sigma) \]
\[ \{p_a(\sigma), p_b(\sigma')\} = -\delta_{ab} \]
(28)
The dual dynamical system is the chiral string
\[ \dot{x}^a = \dot{x}^a, \quad \dot{p}_a = p_b' - \dot{x}_b \]
To find the nontrivial dual dynamical system, it needs to introduce some constraints (25) to the original model. At the beginning, we are consider the toy model.

3.2. Relativistic particle in the background electromagnetic field and noncommutativity

The relativistic particle in the constant background electromagnetic field is described by hamiltonian
\[ H = \frac{1}{2}[p_a + i\beta B_{ab}x_b]^2 + m^2] \]  
(29)
The electromagnetic field is \( A_a(x) = -2F_{ab}x_b \). The simplest constraint from (25) without derivatives is
\[ \varphi_a = p_a + i\alpha B_{ab}x_b \approx 0 \]  
(30)
The consistency condition
\[ \{\varphi_a, H\} = i\alpha(\alpha + \beta)B_{ab}\varphi_b \]  
(31)
shows, that there are unique set of constraints, if \( \alpha + \beta = 0 \). They are second kind constraints \( \{\varphi_a, \varphi_b\} = 2i\alpha B_{ab} \). There are the following algebra of the phase space coordinates under the Dirac bracket
\[ \{x_a, x_b\} = -\frac{i}{2\alpha}(B^{-1})_{ab}, \quad \{x_a, p_b\} = \frac{1}{2}\delta_{ab} \]
\[ \{p_a, p_b\} = \frac{-i\alpha}{2}B_{ab} \]  
(32)
The motion equation under Dirac bracket
\[ \dot{x}_a + 2i\alpha B_{ab}x_b = 0, \quad \dot{p}_a + 2i\alpha B_{ab}x_b = 0 \]  
(33)
have solution \( x_a(\tau) = \{\epsilon^{-2i\alpha B\tau}\}_{ab}x_b(0) \). The quantization of the Dirac bracket is reduce to the noncommutative space
\[ [x_a, x_b] = \frac{1}{2\alpha}(B^{-1})_{ab}, \quad [p_a, p_b] = \frac{\alpha}{2}B_{ab} \]
\[ [x_a, p_b] = \frac{i}{2}\delta_{ab} \]  
(34)
in the analogy to noncommutativity of the open string model [11]. But, there is the commutative solution under some restriction on the background field. We are suppose \( \alpha = -i \) for simplicity and we introduce "physical" coordinates \( y_a = p_a + x_a, q_a = p_a - x_a \). They form following algebra under constraint \( B_{ab}B_{cb} = \delta_{ab} \)
\[ \{y_a, y_b\} = 0, \quad \{q_a, q_b\} = 0 \]
\[ \{q_a, y_b\} = \delta_{ab} - B_{ab} \]  
(35)
We have after quantization
\[ [y_a, y_b] = [q_a, q_b] = 0, \quad [y_a, q_b] = i(\delta_{ab} - B_{ab}) \]
\[ q_a = -i(\delta_{ab} - B_{ab}) \frac{\partial}{\partial y_a} \]  
(36)
As is obvious from (34), the coordinates \( x_a \) are commute in the strong coupling regime \( \alpha \rightarrow \infty \). The commuting momenta \( p_a \) must to consider as the coordinates of position in the weak coupling regime \( \alpha \rightarrow 0 \). The similar condition on the B-field is arise in the closed string model in the constant background fields with the additional constraint.

3.3. B-chiral string

Let me consider following constraint from (25)
\[ \varphi_a = p_a + \beta B_{ab}x^b \approx 0 \]  
(37)
The consistency condition
\[ \{\varphi_a, \int_0^\pi d\sigma' H_1\} = (\alpha + \beta)B_{ab}g^{bc}\varphi'_c + \]
\[ + [g_{ab} - (\alpha + \beta)^2 B_{ac}g^{cd}B_{db}]x^c' \approx 0 \]  
(38)
If we suppose condition
\[ g_{ab} = (\alpha + \beta)^2 B_{ac}g^{cd}B_{db} \]  
(39)
we have first kind constraint \( \varphi_a \), which is describe the B-chiral string. The motion equations are

\[
\dot{x}^a = -(\alpha + \beta) g^{ab} B_{bc} x^c \tag{40}
\]

\[
\dot{p}_a = -(\alpha + \beta) B_{bc} g^{bc} p_c, \quad \ddot{x}^a = x''^a
\]

This model is the bihamiltonian model under (PB) (27) also. The B-chiral string model is dual to the chiral model also.

3.4. Closed string in the arbitrary background fields

The equations of motion of the closed string, in the arbitrary background gravity field and antisymmetric B-field under hamiltonian \( H_1 (21) \) and (PB) (11), are

\[
\dot{x}^a = g^{ab} [p_b - \alpha B_{bc} x^c] \tag{41}
\]

\[
\dot{p}_a = \alpha B_{bc} g^{bc} p_c + [g_{ab} - \alpha B_{ac} g^{cd} B_{db}] x''^b - \frac{1}{2} \frac{\partial g^{bc}}{\partial x^a} p_b p_c - \alpha \frac{\partial}{\partial x^a} (B_{bc} g^{de} x''^b) p_c + \alpha \frac{\partial}{\partial x^a} (B_{ad} g^{de} x''^b) p_b - \frac{1}{2} \frac{\partial g^{bc}}{\partial x^a} [g_{bc} - \alpha^2 B_{ad} g^{de} B_{eb}] x''^b x^c + \frac{\partial}{\partial x^a} [g_{ac} - \alpha^2 B_{ad} g^{de} B_{ec}] x''^b x^c
\]

The bihamitlony condition for coordinate \( x^a \) is

\[
\dot{x}^a = -2\omega_a x^b + 4\omega_a^b p_b + 2\Phi_a x''^b - 2\Phi_b x''^b + 2\Phi_a x''^b - 2\omega_a x^b - 2\omega_a x^b - \int_0^\pi d\sigma' [\omega_a x^b + \frac{\partial \Phi^{ac}}{\partial x^b} x^c p_c + \frac{\partial \Phi^{ac}}{\partial p_b} p_c] \epsilon (\sigma' - \sigma) + (\frac{\partial \Phi^{ac}}{\partial x^b} x^c + \frac{\partial \Phi^{ac}}{\partial p_b} p_c) p_c - (\frac{\partial \Phi^{ac}}{\partial x^b} x^c + \frac{\partial \Phi^{ac}}{\partial p_b} p_c) x^c = g^{ab} x^b - \alpha g^{ab} B_{bc} x^c
\]

The consistency condition of the equation (42) is

\[
\Phi^{ab} = \Phi^b_a = \Omega^{ab} = 0, \quad \omega^{ab}_{c} = 0, \quad \omega^{ab}_{c} = C g^{ab} \tag{43}
\]

The metric tensor \( g^{ab}(x) \) is homogeneous function of \( x^a \) order \( n \) and \( C \) is arbitrary constant. The bihamiltonity condition for coordinate \( p_a \) is

\[
\dot{p}_a = -2\omega_a x^b - 2\Phi_a x''^b + 2\Omega_{ab} x^b + 4\omega_a^b p_b + 2\Phi_a x''^b + \int_0^\pi d\sigma' [\omega_{ab} x^b] - \frac{\partial^2 \omega^{cd}}{\partial x^a \partial x^b} x''^b p_c - \frac{\partial \omega^{cd}}{\partial x^a} p_b p_c) \epsilon (\sigma' - \sigma) - \frac{\partial \Phi^{ac}}{\partial x^b} x^c + \frac{\partial \Phi^{ac}}{\partial p_b} p_c = 0
\]

The common consistency conditions are

\[
\Phi^{ab} = \Phi^b_a = \Omega^{ab} = 0, \quad \omega^{ab}_{c} = 0, \quad \omega^{ab}_{c} = C g^{ab} \tag{45}
\]

\[
\omega_{ab} = C \frac{\partial^2 g^{cd}}{\partial x^a \partial x^b} p_c p_d, \quad \omega^{a}_{b} = -C \frac{\partial g^{ac}}{\partial x^b} p_c,
\]

\[
\Phi_{ab} = -\frac{1}{2} [g_{ab} - \alpha^2 B_{ac} g^{cd} B_{db}], \quad \Omega_{ab} = \frac{1}{2} \frac{\partial \Phi^{bc}}{\partial x^b} x^c - \frac{\partial \Phi^{bc}}{\partial p_b} p_c
\]

\[
2\Omega_a = -\alpha g^{ac} B_{ca}, \quad \frac{\partial g^{ab}}{\partial x^c} x^c = n g^{ab}, \quad C = \frac{1}{2(n + 2)}
\]

The Jacoby identity

\[
\{A(\sigma), B(\sigma') \} C(\sigma'')_2 + \{C(\sigma''), A(\sigma) \} B(\sigma')_2 + \{B(\sigma'), C(\sigma'') \} A(\sigma)_2 = 0
\]
does not be satisfied for arbitrary background fields. The terms with derivatives of structure functions is reduce to integrability conditions of the closed string model in the background gravity field and the antisymmetric B-field. The principal of this conditions is condition on the structure function $\omega^{ab}(x)$.

$$\frac{\partial g^{ab}(\sigma)}{\partial x^d} \left[ g^{dc}(\sigma) + g^{dc}(\sigma'') \right] - \frac{\partial g^{ac}(\sigma)}{\partial x^d} \left[ g^{db}(\sigma) + g^{db}(\sigma') \right] \epsilon(\sigma' - \sigma) \epsilon(\sigma'' - \sigma) +$$

$$+ \frac{\partial g^{ab}(\sigma')}{\partial x^d} \left[ g^{da}(\sigma') + g^{da}(\sigma) \right] - \frac{\partial g^{ab}(\sigma)}{\partial x^d} \left[ g^{dc}(\sigma') + g^{dc}(\sigma') \right] \epsilon(\sigma - \sigma') \epsilon(\sigma'' - \sigma') +$$

$$\frac{\partial g^{ac}(\sigma'')}{\partial x^d} \left[ g^{db}(\sigma'') + g^{db}(\sigma') \right] -$$

$$- \frac{\partial g^{ab}(\sigma'')}{\partial x^d} \left[ g^{da}(\sigma'') + g^{da}(\sigma) \right] \epsilon(\sigma - \sigma'') \epsilon(\sigma' - \sigma) = 0$$

It is possible to reduce this condition to unique equation

$$\frac{\partial g^{ab}(\sigma)}{\partial x^d} \left[ g^{dc}(\sigma) + g^{dc}(\sigma'') \right] - \frac{\partial g^{ac}(\sigma)}{\partial x^d} \left[ g^{db}(\sigma) + g^{db}(\sigma') \right] \epsilon(\sigma' - \sigma) \epsilon(\sigma'' - \sigma) = 0$$

In assumption, that $g^{ab} = \frac{\partial^2 f}{\partial x^a \partial x^b}$, this equation is the matrix equation with the additional parametric dependence unknown function $f(x^s(\sigma))$ of parameter $\sigma$. The rest equations for structure functions $\omega, \Omega, \Phi$ together with corresponding generalized functions and crossed terms of the structure functions are similar to previous equation. We do not reduce tedious expression for this equations here.

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