A new logarithmic dielectric constant model of soils

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ABSTRACT

A series of experimental tests were conducted to analyze the change of dielectric constant with varied particle composition and moisture content, and the experimental data were compared with several commonly used dielectric constant models. Results show that both the empirical models and the volumetric mixing models are more applicable to sandy soils and will overestimate the dielectric constant of cohesive soils due to the neglect of the effect of bound water. Actually there are a large number of strong and weak bound water in cohesive soils and the dielectric constant of bound water is much smaller than that of free water. To describe the effect of mesoscopic particle composition and bound water, a new logarithmic dielectric constant model is put forward. The application of this model is verified by others’ experimental data and the calculation of dielectric constant of cohesive soils can be more accurate with this model. The moisture condition of soils can be judged roughly according to the ratio of the increment of dielectric constant and volumetric water content.

Keywords: dielectric constant, particle composition, bound water, volumetric mixing model

1 INTRODUCTION

The dielectric constant of a soil is affected by many factors, such as moisture condition, temperature, density and so on, among which the moisture condition plays the most important role (Roth et al., 1990; Harmsen et al., 2003). Since the dielectric constant of water is significantly higher than that of other materials commonly found in soils, the dielectric properties of soils will be dominated by the presence of water and the measurement of dielectric constant can be used to predict the moisture content of soils. With the rapid development of non-destructive technology such as capacitance sensors, time domain reflectometry(TDR) probes, and ground penetrating radar(GPR) devices, the relationship between soil’s moisture content and dielectric constant has been the hot issue of geotechnical engineering (Friedman, 1998).

During the several decades, a large number of soil’s dielectric constant models have been proposed. Most of them can be classified into 2 typical types: empirical models and volumetric mixing models (Roth et al., 1990; Chen et al., 2008). However, because of the special particle structure, bound water may exist in soil, whose dielectric constant is obviously smaller than that of free water. Thus, soil with different particle structure has different moisture condition, which will lead to a lot of problems in the application of dielectric constant models. For example, according to experimental data, Topp model, a classic empirical model, is not suitable to describe the dielectric properties of pure clays and clayed and organic soils (Topp et al., 1980; Dirksen and Dasberg, 1993). If all the water in soil is considered to be free water, volumetric mixture model only includes three parts and it will overrate the dielectric constant of cohesive soil. If bound water is taken into consideration, volumetric mixture model will include at least 4 or 5 parts and the model will be much too complex with so many unknown terms.

The studies of Park et al. (2017) show that the dielectric constant of soil is not only affected by the moisture content but also affected by the amount of bound water. The amount of bound water is influenced by soil’s particle composition. Thus, more studies about the relationship between soil’s moisture condition and dielectric constant should be carried out based on its
particle composition.

The relationship between dielectric constant and moisture condition of soils with different gradation is discussed based on the laboratory experiment, and the experimental results are compared with several common dielectric constant models in this paper. Furthermore, a logarithmic model is developed, which can take the mesoscopic particle composition and bound water content into consideration. The calculation of dielectric constant of cohesive soils can be more accurate with this model.

2 THEORETICAL CONSIDERATIONS

Currently, the common dielectric constant models can be divided into two typical types: empirical models and volumetric mixing models. The empirical models are based on a large number of experimental data, and the functional relationships are selected purely by their mathematical flexibility to fit the experimental data. The third-order polynomial empirical equation Topp et al. (1980) proposed is

\[ \varepsilon_r = -5.3 \times 10^{-2} + 2.92 \times 10^{-1} \varepsilon_w - 5.5 \times 10^{-4} \varepsilon_r^2 + 4.3 \times 10^{-6} \varepsilon_r^3 \]  

(1)

where \( \varepsilon_r \) is the dielectric constant of soil and \( \theta_w \) is its volumetric water content.

Roth et al. (1992) revised the above equation as follows,

\[ \theta_w = -7.8 \times 10^{-2} + 4.48 \times 10^{-2} \varepsilon_r - 1.95 \times 10^{-4} \varepsilon_r^2 + 3.61 \times 10^{-6} \varepsilon_r^3 \]  

(2)

The main advantage of an empirical model is that it does not require any additional soil parameters to determine \( \theta_w \). However, due to the fact that the coefficients of an empirical model are derived from the fittings of experimental data, the physical meaning of these coefficients is not clear enough.

Most volumetric mixing models are based on the volume fraction of the components in the mixture. They relate the composite dielectric constant of a multiphase mixture to its constituents’ volumetric fractions and single-phase dielectric constant. Birchak et al. (1974) derived a mixing model based on the index of refraction, and the equation is given as

\[ \varepsilon_r = \sum_i \theta_i \varepsilon_i^a \]  

(3)

where \( a = 0.5 \) for an isotropic two-phase medium.

Considering that soil is a mixture of solid, water and air, the general form of a volumetric mixing model can be written as

\[ \varepsilon_r = [\theta_w \sqrt{\varepsilon_{rw}} + (1-n) \sqrt{\varepsilon_{rs}} + (n-\theta_w) \sqrt{\varepsilon_{ra}}]^2 \]  

(4)

where \( \theta_s = 1-n, \theta_w, \theta_a = n-\theta_w \) are the volumetric fractions, and \( \varepsilon_{rs}, \varepsilon_{rw} \) and \( \varepsilon_{ra} \) are the dielectric constant of the solid, aqueous and gaseous phase, respectively. \( n \) is the soil’s porosity.

Eq. (4), the three-phase CRIM (complex refractive index model), is frequently used to predict the effect of volumetric water content upon dielectric properties of soils as it is simple to simple to apply and accurate over the GPR frequency range (Greaves et al., 1996; Tsui and Matthews, 1997; Loeffler and Bano, 2004; Cassidy, 2007).

Alharthi and Lange (1987) considered the application of the mixing law and developed an empirical equation as below, whose major benefit is its ease of application.

\[ \theta_w = -0.204 + 0.128 \varepsilon_r^{0.5} \]  

(5)

Chen et al. (2008) concluded a volumetric mixing model based on the polarization mechanisms, and the equation is

\[ (2 \sum_i A_i - 3) \varepsilon_r^2 + (\sum_i A_i \varepsilon_r + 4 \sum_i A_i + 3) + 2 \sum_i A_i \varepsilon_r = 0 \]  

(6)

where \( A_i = \theta_i(\varepsilon_{ri} - 1)/\varepsilon_{ri} + 2 \) and \( i \) represents the constituents of the solid, aqueous and gaseous phase, respectively. To simplify this equation, the dielectric constant of solid phase of soils can be approximately taken as \( \varepsilon_{rs} = 5 \) (Roth et al., 1990; Friedman, 1998), and the corresponding coefficient is \( A_s = 0.570 \varepsilon_r \). The dielectric constant of free water at room temperature generally equals to 81, and the corresponding coefficient of liquid phase is \( A_w = 0.964 \varepsilon_r \). The dielectric constant for air is 1 and \( A_a = 0 \). Eq. (6) can be reduced to the following form.

\[ (1.14 \theta_s + 1.92 \theta_w - 3) \varepsilon_r^2 + (5.15 \theta_s + 81.90 \theta_w + 3) \varepsilon_r + 5.72 \theta_s + 156 \theta_w = 0 \]  

(7)

3 EXPERIMENTAL INVESTIGATION AND RESULTS

3.1 Experimental procedures

Experimental research was carried out to analyze the influence of particle composition and volumetric water content on the composite dielectric content. The soil used is silty clay taken from the North slope mountain of Bailuyuan in Xi’an Shaanxi, China. An index sand content is defined as

\[ S_c = \frac{m_{sand}}{m} \]  

(8)

where \( S_c \) is the sand content, \( m \) the total mass of soil and \( m_{sand} \) the mass of sand particles whose size is bigger than 0.075mm in soil.

The samples were prepared according to the sand content of 0, 20%, 40%, 60%, 80% and 100% respectively. For each sand content, ten groups of soil samples were prepared with different water content.

The density \( \rho \) and mass water content \( w \) of each soil sample were measured under different sand content. The dielectric constant of each sample was measured by 0932F surface probe dielectric constant meter. The volumetric water content can be calculated by Eq. (9).

\[ \theta_w = \frac{w \rho}{(1+w)\rho_w} \]  

(9)

According to the GB5-007-2011(2012), soil samples with sand content of 60%, 80% and 100% can be regarded as sandy soil. For soil samples with sand
content of 0, 20% and 40%, the liquid limit and plastic limit are measured and the results are shown in Table 1. We can see that the plasticity indexes are greater than 10 and soil samples with 0, 20% and 40% sand content can be attributed to cohesive soil.

Table 1. Results of liquid and plastic limit tests.

| Sand content/% | 0       | 20      | 40      |
|----------------|---------|---------|---------|
| \( w_p/\% \)  | 18.4    | 18.64   | 18.79   |
| \( w_L/\% \)  | 34.75   | 32.63   | 31.97   |
| \( I_p \)      | 16.35   | 13.99   | 13.18   |

3.2 Experimental results and comparison

Fig. 1 and Fig. 2 show the comparisons of dielectric constant \( \varepsilon_r \) and volumetric water content \( \theta_w \) between experimental results and predictions of empirical models using Eq. (1), (2) and (5), respectively.

As can be seen from Fig. 1, there is a significant difference between the experimental results of 0, 20%, and 40% sand content samples. Under the same volumetric water content, the dielectric constant increases with the increase of sand content. However, when the sand content increases to a certain extent, such as causing sandy soil to become cohesive soil, the effect of sand content is negligible. In Fig. 2, the experimental results of sandy soil of different sand content are very close.

For cohesive soil, the experimental data has a large deviation from the Topp model and the Roth model, but the deviation gradually decreases with the increase of the sand content. When the sand content reaches 60% or more, the experimental data is close to the Roth model, and basically consistent with the Topp model. Therefore, these two empirical dielectric models are more suitable for sandy soil, and they will produce large deviations when used in cohesive soil. For sandy soil samples used in this paper, Topp model is more suitable than Roth model, and Alharthi model more closely matches well with soils of 40% sand content.

Compared with empirical models, volumetric mixing models in Eq. (4) and (7) have an additional parameter, soil’s porosity \( n \). In order to analyze the applicability of volumetric mixing models, the experimental data are compared with these two volumetric mixing models in the three-dimensional coordinate system of volumetric moisture content \( \theta_w \), porosity \( n \) and dielectric constant \( \varepsilon_r \) in Fig. 3 and Fig. 4.

As is shown in Fig. 3, when sand content is less than 40%, the experimental data are all distributed under the surface of the CRIM model. When sand content is higher than 60%, the experimental data are basically distributed on the surface of the CRIM model, and are close to the model surface. It indicates that the CRIM model is very suitable to predict the dielectric constant of sandy soil, but it will overestimate the dielectric constant when used in cohesive soil.

In Fig. 4, all the experimental data are distributed under the surface of Chen’s equation, which means that
Chen’s equation overestimates the dielectric constant of soil. However, the experimental data gradually approaches the surface of Chen’s equation as the sand content increases. The two volumetric mixing models are both more suitable for sandy soil.

From what has been discussed, we can know that both empirical models and volume mixing models are more suitable for sandy soil, and the dielectric constant will be overestimated when used in cohesive soil. This is because there are a large number of strong and weak bound water in cohesive soil while empirical models and volume mixing models above consider all the water in the soils as free water. For sandy soil, due to the smaller specific surface area of soil particles and less content of bound water, the experimental results can be predicted well with these models. However, for cohesive soil, the dielectric constant will be overestimated if the strong and weak bound water are considered as free water.

Considering the effect of free water and strong and weak bound water separately, Eq. (4) can be rewritten as

\[ \varepsilon = \left[ \theta_{w1} \sqrt{\varepsilon_{w1}} + \theta_{w2} \sqrt{\varepsilon_{w2}} + \theta_{w3} \sqrt{\varepsilon_{w3}} + (1 - n) \sqrt{\varepsilon_{\text{ra}}} + (n - \theta_w) \sqrt{\varepsilon_{\text{ra}}} \right] \]

(10)

where \( \theta_{w1}, \theta_{w2} \) and \( \theta_{w3} \) are the volume fractions, and \( \varepsilon_{w1}, \varepsilon_{w2}, \varepsilon_{w3} \) are the dielectric constant of free water, strong bound water and weak bound water, respectively. However, \( \theta_{w1}, \theta_{w2} \) and \( \theta_{w3} \) are difficult to measure, which brings inconvenience to the application of Eq. 10.

## 4 A NEW LOGARITHMIC DIELECTRIC CONSTANT MODEL

### 4.1 Model proposal

In order to establish a dielectric constant model in logarithmic form, which takes the particle composition and bound water into consideration, several assumptions are made to simplify the model:

1. the density of particles in soil is the same;
2. the content of cohesive particles determines the content of bound water;
3. the total volume of soil remains unchanged when moisture content is rising, which means the increased water content accounts the volume of air pores in soil.

Fig. 5 is the schematic representation of our model. Before the volumetric water content reaches the liquid limit \( \theta_{\text{LL}} \), all the cohesive particles are wrapped by bound water and all sand particles are surrounded by free water, which means the content of free and bound water is determined by sand content. When the volumetric water content adds as \( d\theta_w \), the increment of free water content and bound water content can be expressed as

\[ d\theta_{\text{w}} = (1 - S) d\theta_w \]

(11)

\[ d\theta_{\text{wb}} = S d\theta_w \]

(12)

where \( d\theta_w \) is the incremental water content, \( d\theta_{\text{w}} \) the incremental bound water content and \( d\theta_{\text{wb}} \) the incremental free water content.

Based on the mixing law, the increment of logarithmic dielectric constant is

\[ d\log \varepsilon = d\theta_{\text{w}} \log \varepsilon_{\text{w}} + d\theta_{\text{wb}} \log \varepsilon_{\text{wb}} + d\theta_{\text{ra}} \log \varepsilon_{\text{ra}} \]

(13)

where \( \varepsilon_\text{w} \) is the dielectric constant of soil, \( \varepsilon_{\text{w}} \) the dielectric constant of free water (generally equals to 80 at room temperature), \( \varepsilon_{\text{wb}} \) the dielectric constant of bound water (less than 81) and \( \varepsilon_{\text{ra}} \) the dielectric constant of air. Since \( \varepsilon_{\text{ra}} \) generally equals to 1 and its logarithmic value is 0, so the third item can be ignored.

The dielectric constant of bound water \( \varepsilon_{\text{wb}} \) varies exponentially with the distance \( L \) from the surface of particles (Gui et al., 1978), and the equation is as follows.

\[ \varepsilon_{\text{wb}} = A e^{-nL} \]

(14)

The dielectric constant of strongly bound water in the innermost layer is approximately 3.2 (Dirksen and Dasberg, 1993). The thickness of bound water film increases with the increase of water content, and \( L \) can be considered as a function of volumetric water content \( L = f(\theta_w) \). Substituting Eq. (11), Eq. (12) and Eq. (14) into Eq. (13) results in
\[ \text{dlg} \varepsilon_r = d\theta_w S_c \text{lng81} + d\theta_w (1 - S_c) \text{lng25} \]  \hspace{0.5cm} (15)

In Eq.(15), we can know that the slope of the curve \( \text{lng} \varepsilon_r - \theta_w \) is increasing with the increase of water content. However, experiment results in Fig. 6 indicate that the relationship between logarithmic dielectric constant and volumetric water content is approximately linear, which means the influence of \( f(\theta_w) \) is finite. Many experiments results show that the dielectric constant of bound water is distributed between 25 and 40. To simplify our model, take \( \varepsilon_{r wb} = 25 \). Then Eq. 15 can be simplified as

\[ \eta = \frac{\text{dlg} \varepsilon_r}{d\theta_w} = S_c \text{lng81} + (1 - S_c) \text{lng25} \]  \hspace{0.5cm} (16)

where \( \eta \) is the slope of \( \text{lng} \varepsilon_r - \theta_w \).

The logarithmic dielectric constant of dry soil can be expressed as \( \text{lng} \varepsilon_r = \theta_w \text{lng} \varepsilon_{r wb} = (1 - \eta) \text{lng} \varepsilon_{r wb} \). Taking this as a critical condition, Eq. (16) can be written as follows:

\[ \text{lng} \varepsilon_r - \theta_w \text{lng} \varepsilon_{r wb} = [S_c \text{lng81} + (1 - S_c) \text{lng25}] \theta_w \]  \hspace{0.5cm} (17)

As can be seen from Eq. (16), \( \eta \) is related to the sand content and it reflects the influence of particle composition and bound water on the logarithmic dielectric constant model. Therefore, \( \eta \) can be defined as the sensitivity coefficient of soil’s dielectric constant, which reflects the sensitivity of the change in dielectric constant at the unit volume water content increment. The sensitivity coefficient \( \eta \) increases with the increase of sand content, which indicates that the dielectric constant of sandy soil is more sensitive to the change of volumetric water content than that of cohesive soil. Obviously, for the same volumetric water content increment, the increase of free water in sandy soil is more than in cohesive soil. Since the dielectric constant of bound water is smaller than that of free water, the dielectric constant of sandy soil is more sensitive to the change of water content than that of cohesive soil.

When volumetric water content reaches or exceeds the liquid limit \( \theta_{wL} \), we assume that bound water in soil has reached saturation and the increased water exists as free water. Then the change of dielectric constant can be expressed as \( \text{lng} \varepsilon_r = d\theta_w \text{lng} \varepsilon_{rfw} \). When \( \theta_w > \theta_{wL} \), the logarithmic dielectric constant model is

\[ \text{lng} \varepsilon_r - \theta_w \text{lng} \varepsilon_{rfw} = S_c \theta_w \text{lng81} + (1 - S_c) \theta_w \text{lng25} + (\theta_w - \theta_{wL}) \text{lng81} \]  \hspace{0.5cm} (18)

4.2 Model verification

In order to verify the applicability of the proposed model, the experimental data of soil samples are compared with the calculated results using Eq. (17) in Fig. 6.

We can see that the logarithmic dielectric constant model which takes particle composition into consideration matches well with the experimental data. When the volumetric water content is close to the liquid limit, there are some deviations between the experimental data and the logarithmic model. This is because as the volumetric water content increases, the thickness of bound water increases rapidly and taking \( \varepsilon_{r wb} = 25 \) is not practical. The dielectric constant of the outer layer bound water is greater than that of the inner layer bound water. In addition, when the volumetric water content increases to a certain extent, the total volume of the soil sample may expand and assumption (2) will be no longer suitable.

![Graphs comparing logarithmic model and experimental data](Fig. 6. The comparison between the logarithmic model and the experimental data.)

Table 2. Coefficients of the logarithmic model.

| Sand content/% | 0  | 20 | 40 | 60 | 80 | 100 |
|----------------|----|----|----|----|----|-----|
| \( \eta \)      | 1.398 | 1.500 | 1.602 | 1.704 | 1.806 | 1.908 |

According to the sensitive coefficients with different sand content in Table 2, we propose that soil types can be classified based on the \( \eta \) value. For \( \eta \) in the range of 0.70 to 0.90, the soil is classified as sand. For \( \eta \) in the range of 1.60 to 1.90, the soil is classified as silt. For \( \eta \) less than 1.60, the soil is classified as clay.

Fig. 6. The proposed model is further investigated using Qin’s (2004) experimental data of Q3 loess. The Q3 loess used in Qin’s paper is determined as silty soil according to the GB50007-2011 (2012) and the volumetric water content does not exceed \( \theta_{wL} \).

As is shown in Fig. 7, the experimental data is basically distributed around the fitting equation using Eq. (17), which indicates that the proposed logarithmic model is reasonable and applicable. For \( \theta_w > \theta_{wL} \), the logarithmic dielectric constant model Eq. (18) needs further verification.
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