Time reversal Aharonov–Casher effect using Rashba spin–orbit interaction

Junsaku Nitta\(^1,2,3\) and Tobias Bergsten\(^2,3,4\)

\(^1\) Graduate School of Engineering, Tohoku University, 6-6-02 Aramaki-Aza Aoba, Aoba-ku, Sendai 980-8579, Japan
\(^2\) CREST-JST, Kawaguchi Center Building, 4-1-8 Honcho Kawaguchi-shi, Saitama 332-0012, Japan
\(^3\) NTT Basic Research Laboratories, 3-1 Morinosato-Wakamiya, Atsugi-shi, 243-0198, Japan
E-mail: nitta@material.tohoku.ac.jp

New Journal of Physics 9 (2007) 341
Received 27 April 2007
Published 28 September 2007
Online at http://www.njp.org/
doi:10.1088/1367-2630/9/9/341

Abstract. We propose a spin interferometer using Rashba spin–orbit interaction. A spin interference effect is demonstrated in small arrays of mesoscopic InGaAs rings. This spin interference is the time reversal Aharonov–Casher (AC) effect. The AC interference oscillations are controlled over several periods. This result shows evidence for electrical manipulation of the spin precession angle in an InGaAs two-dimensional electron gas channel. We control the precession rate in a precise and predictable way with an electrostatic gate.

Contents

1. Introduction 2
2. Rashba SOI 3
3. Spin interferometer and electronic AC effect 4
4. AC spin interference experiment 6
5. Conclusion 10
Acknowledgments 10
References 10

\(^4\) Present address: Cavendish Laboratory, Cambridge University, J J Thomson Avenue, CB3 0HE Cambridge, UK.
1. Introduction

Spintronics, as the name suggests, is the art of controlling the spin degree of freedom of electrons and their movement in order to perform specific operations [1]–[4]. The spin–orbit interaction (SOI) couples the orbital motion of electrons with the orientation of electron spins. The spin Hall effect caused by SOI predicts that the orbital motion of electron spins depends on the spin orientation without any external magnetic field [5]–[8]. Optical detection experiments of the spin Hall effect have been performed and have demonstrated that spin-up and spin-down electrons are accumulated at the right and left edges of a semiconductor channel [9, 10]. On the other hand, a momentum-dependent effective magnetic field due to the SOI randomizes spin orientations after scattering events. A key ingredient for spintronics is to control the strength of SOI since the SOI plays a double-edged role for spin manipulation.

It is known that an inversion asymmetry lifts the spin degeneracy. The Rashba SOI [11] caused by a structural inversion asymmetry (SIA) is of importance for spintronics since the strength of SOI can be controlled by applying a gate voltage on top of a two-dimensional electron gas (2DEG) [12]–[14]. The Datta–Das spin field effect transistor (FET) [15] requires gate controlled spin orientation in the 2DEG channel. A mesoscopic Stern–Gerlach spin filter [16] has been proposed by a spatial distribution of the Rashba SOI strength. The Rashba SOI is of crucial importance not only to spintronics device applications but also to spin related transport and physics. In quantum interference experiments, crossover from weak anti-localization (WAL) to weak localization (WL) was observed by controlling the Rashba SOI strength [17] and Zeeman energy [18]. Furthermore, the universal behaviour of dephasing time as a function of the ratio between Rashba SOI and Zeeman energies was observed from systematic analysis of WAL under an in-plane magnetic field [19].

An electron acquires a phase around magnetic flux due to the vector potential leading to the Aharonov–Bohm (AB) effect in an interference loop. From the viewpoint of inherent symmetries between magnetic field and electric field in the Maxwell equations, Aharonov and Casher [20] have predicted that a magnetic moment acquires a phase around a charge flux line. It should be noted that the original Aharonov–Casher (AC) effect was proposed for charge neutral particles since the electric field modifies the trajectory of a charged particle in the same sense as the original AB effect was predicted to in the situation where magnetic flux should not exist in an electron path. Cimmino et al [21] managed to perform the AC interference experiment in a neutron (having spin-1/2 but no charge) beam loop using a voltage of 45 kV to create the electric field. However, the modified precession angle of neutron spin was only 2.2 mrad since the SOI (the interaction between magnetic moment and electric field) is not strong in a vacuum. Mathur and Stone [22] have proposed the electronic AC effect in a mesoscopic interference loop made of GaAs 2DEG with the Dresselhaus SOI [23]. It is emphasized that a thousand-fold improvement in its experiment can be expected in the electronic AC effect since the SOI in semiconductors is much enhanced compared with that in a vacuum.

In this paper, we discuss the enhancement of SOI and the origin of Rashba SOI in section 2. A spin interferometer utilizing the Rashba SOI [24] is discussed in section 3. The conductance of the spin interferometer shows an oscillatory behaviour as a function of the Rashba SOI. This is very similar to the Datta–Das spin FET in which ferromagnetic source and drain electrodes are necessary. The spin interference device works without ferromagnetic electrodes. We present an experimental demonstration of the spin interference using small arrays of mesoscopic InGaAs 2DEG rings in section 4. This spin interference is the AC effect of time reversal symmetric
paths and is the electromagnetic dual to the Al’tshuler–Aronov–Spivak (AAS) effect [25]. We demonstrate that we can control the spin precession angle with an electrostatic gate and modulate the interference pattern over several periods [26]. This gate controlled spin precession angle is of importance for spintronics.

2. Rashba SOI

Not only an external magnetic field but also an electric field lifts spin degeneracy when SOI plays a role. The SOI is a relativistic effect on a particle with spin which is moving with a velocity \( \vec{v} \) through an electric field \( \vec{E} \). In the particle’s frame of reference, \( \vec{E} \) is Lorentz transformed into the magnetic field \( \vec{B} \) which is perpendicular to the electric field and to the direction of movement

\[
\vec{B} = -\frac{\vec{v} \times \vec{E}}{c^2} = -\frac{\vec{p} \times \vec{E}}{m_0 c^2}.
\]

(1)

Here \( c \) is the light velocity, \( m_0 \) is mass of electron, and \( \vec{v} \) is an electron velocity. In analogy to the Zeeman effect, the SOI is given by an inner product of magnetic moment and magnetic field, and is written as

\[
H_{SO} = -\mu_B \vec{\sigma} \cdot \left( \frac{\vec{p} \times \vec{E}}{2m_0 c^2} \right).
\]

(2)

Here \( \mu_B \) and \( \vec{\sigma} \) are the Bohr magneton and the Pauli spin matrix, respectively. A factor \( 1/2 \) difference stems from the relativistic effect. The SOI effect is small and negligible for non-relativistic momentum \( \vec{p} = \hbar \vec{k} \ll m_0 c \). However, it is drastically enhanced in semiconductors with energy gap \( E_g \). The Dirac gap \( E_D = 2m_0 c^2 \approx 1 \text{ MeV} \) in the denominator of equation (2) is replaced by the energy gap \( E_g \approx 1 \text{ eV} \) according to the \( k \cdot p \) perturbation theory [14, 27]. In solids, wavefunctions of electrons are described by a product of an envelope function and a Bloch function. The enhancement of SOI is due to the contribution from a quickly oscillating Bloch function which has large momentum and feels a strong electric field near atomic cores [27]. This enhancement of SOI is of critical importance for electrical manipulation of spins.

The confinement of electrons in a 2DEG is generally due to a quantum well (QW). Band off-sets of the boundary materials or applied voltages lead to an asymmetry of the QW. The breaking of spatial inversion symmetry results in the lift of spin degeneracy. This is the so called Rashba SOI [11]. The asymmetry of the QW yields a macroscopic electric field. To have a finite spin splitting, we need both a macroscopic electric field and a microscopic electric field from the atomic core. The SIA spin splitting energy is proportional to the macroscopic electric field times a prefactor that depends on the matrix element of the microscopic SOI.

It should be noted that the Dresselhaus SOI [23] is independent of any macroscopic electric fields. The Rashba Hamiltonian for a 2DEG in an asymmetric QW is given by

\[
H_R = \frac{\alpha}{\hbar} (\vec{p} \times \vec{\sigma}) \cdot \hat{z}.
\]

(3)

Here \( \hat{x} \)- and \( \hat{y} \)-axes are in the 2DEG plane, and \( \alpha \) is the Rashba SOI parameter. The spin splitting energy at the Fermi energy is given by \( \Delta_R = 2\alpha k_F \) where \( k_F \) is the Fermi momentum. According
to the $k \cdot p$ perturbation theory \cite{14, 27}, the Rashba SOI parameter $\alpha$ is given by the following equation

$$\alpha = \frac{\hbar^2 E_p}{6m_0} \langle \Psi(z) | \frac{d}{dz} \left( \frac{1}{E_F - E_{\Gamma_7}(z)} - \frac{1}{E_F - E_{\Gamma_8}(z)} \right) | \Psi(z) \rangle,$$

where $\Psi(z)$ is the wavefunction for the confined electron, $E_p$ is the inter-band matrix element, $E_F$ is the Fermi energy in the conduction band, and $E_{\Gamma_7}(z)$ and $E_{\Gamma_8}(z)$ are positions of the band edge energies for $\Gamma_7$ (spin split off band) and $\Gamma_8$ (the highest valence band), respectively. Contribution to equation (4) can be split into two parts: (i) the field part, which is related the electric field in the QW and (ii) interface part, which is related to band discontinuities at the heterointerface.

For many years, there has been an intense discussion about the Rashba SOI. It was thought that the Rashba SOI should be very small because the average electric field for the bound state is zero i.e. $\langle E \rangle = 0$ in order to satisfy the condition that there is no force acting on a bound state. In fact this controversy is resolved by equation (4). It is clear that the Rashba spin splitting in the conduction band originates from the electric field in the valence band \cite{14, 24}. Equation (4) also shows that the strength of Rashba SOI can be controlled by the gate electric field on top of the 2DEG \cite{12}–\cite{14}.

3. Spin interferometer and electronic AC effect

A rotation operator for spin-$1/2$ produces a minus sign under $2\pi$ rotation \cite{28}. Neutron spin interference experiments performed by two groups have verified this extraordinary prediction of quantum mechanics \cite{29, 30}. In solids, an electron spin interference experiment in an n-GaAs interference loop has been conducted using optical pump and probe methods \cite{31}. A local magnetic field due to dynamic nuclear spin polarization caused spin precession of the wavepacket in one of the interference paths. In the above spin interference experiments, spin precession was controlled by a local magnetic field. To confirm the AC effect, we need to control spin rotation by an electric field. Mathur and Stone \cite{22} have theoretically shown that the effects of SOI in disordered conductors are manifestations of the AC effect in the same sense as the effects of weak magnetic fields are manifestations of the AB effect. Qian and Su \cite{32} have obtained the AC phase, which is the sum of spin–orbit Berry phase and spin dynamical phase in a 1D ring with SOI.

In the presence of an external magnetic field and an electric field, the one-particle Hamiltonian can be expressed as \cite{33}

$$H = \frac{(\vec{p} - e\vec{A} - \mu_B \vec{\sigma} \times \vec{E}/2e^2)^2}{2m_0}. \tag{5}$$

The contribution from the vector potential $\vec{A}$ corresponds to the AB phase and the contribution from the SOI to the AC phase. In this paper, we assume that the Rashba SOI is stronger than the Dresselhaus SOI \cite{23}, and we only take the Rashba SOI into account. We have proposed a spin-interference device based on the Rashba SOI as shown in figure 1(a) \cite{24}. The AC phases acquired by the spin wavefunctions during a cyclic evolution are calculated in a mesoscopic ring in the presence of the Rashba SOI \cite{24, 34, 35}. The origin of the spin interference is the spin precession angle difference between the left and right branches. The phases acquired in
Figure 1. (a) Schematic structure of the proposed spin interference device. The Rashba SOI is tunable by a gate voltage. (b) The conductance of a 1D ring as a function of the Rashba SOI strength $\alpha$.

The left and right branches are not the same: they have an opposite sign because the precession orientation is opposite. It is interesting that the obtained AC dynamical phase is very similar to that of the spin-FET. The origin of the phase difference in both cases is related to the spin precession. By using the spin interference device, we can investigate how much the spin precession angle can be controlled by a gate voltage. Furthermore, an observation of spin interference in a mesoscopic ring is an experimental demonstration of the electronic AC effect.

The total Hamiltonian of a 1D ring with the Rashba SOI in cylindrical coordinates reads [36]

$$
H(\phi) = \frac{\hbar^2}{2m^*r^2} \left( -i \frac{\partial}{\partial \phi} + \frac{\Phi}{\Phi_0} \right)^2 + \frac{\alpha}{r} \left( \cos \phi \sigma_x + \sin \phi \sigma_y \right) \left( -i \frac{\partial}{\partial \phi} + \frac{\Phi}{\Phi_0} \right) - \frac{i \alpha}{2r} \left( \cos \phi \sigma_y - \sin \phi \sigma_x \right) + \frac{\hbar \omega_B}{2} \sigma_z.
$$

(6)

Here an external magnetic field $B_z$ is applied in the $z$-direction which is perpendicular to the ring plane, and magnetic flux through the ring is given by $\Phi = B_z \pi r^2$ with ring radius $r$, $\Phi_0 = \hbar/e$ is the flux quantum. The polar angle is given by $\phi$, $\omega_B = 2\mu_B B_z/\hbar$ is the Larmor frequency. In an isolated ring, the wavefunction is given by the following form

$$
\Psi = \frac{1}{\sqrt{2\pi}} \begin{pmatrix} C_n^+ e^{in\phi} \\ C_n e^{in\phi} \end{pmatrix},
$$

(7)
where $C^+_n$ and $C^-_n$ are coefficients of spin-up and spin-down eigenstates, respectively. When the Zeeman term is negligible, the energy eigenvalues can be written as [33]

$$E_{n,s} = \hbar \omega_0 \left[ n + \frac{\Phi}{\Phi_0} - \frac{\Phi_{AC}^s}{2\pi} \right]^2,$$

(8)

with $\omega_0 = \hbar / 2m^*r^2$, $n$ integer, and the AC phase $\Phi_{AC}^s$. The AC phase is given by [24], [33]–[37]

$$\Phi_{AC}^s = -\pi \left[ 1 + s \sqrt{\left( \frac{2mr^*\alpha}{\hbar^2} \right)^2 + 1} \right], \quad s = \pm.$$

(9)

This AC phase can be viewed as an effective spin dependent magnetic flux through the ring which modulates the conductance of the ring. Here $s = \pm$ corresponds to spin-up and spin-down along the effective magnetic field. From the above calculation, the conductance when electrons travel halfway around the ring at $B_z = 0$ is written as [24], [33]–[37]

$$G = \frac{e^2}{\hbar} \left[ 1 - \cos \left\{ \pi \sqrt{1 + \left( \frac{2mr^*\alpha}{\hbar^2} \right)^2} \right\} \right].$$

(10)

This equation shows that the conductance of the ring oscillates as a function of the strength $\alpha$ of Rashba SOI as shown in figure 1(b). This SOI dependence is very similar to the conductance of the spin-FET proposed by Datta and Das [15], in which they need ferromagnetic electrodes for spin injection and detection. This proposed spin interferometer works without ferromagnetic electrodes.

4. AC spin interference experiment

The resistance of a mesoscopic ring is affected by several quantum interference effects. The well-known AB effect results in a resistance oscillation with a magnetic flux period of $\hbar/e$. The AB effect is sample specific and very sensitive to the Fermi wavelength, therefore, the interference pattern is rapidly changed by the gate voltage. In order to detect the AC effect we used another quantum interference phenomenon, the AAS effect [25, 38]. The AAS effect is an AB effect of time reversal symmetric paths, where the two wavefunction parts go all around back to the origin on identical paths, but in opposite directions. In this situation any phase which is due to path geometry will be identical and will not affect the interference. This also means that it is independent of the Fermi energy $E_F$ (and consequently of the carrier density $n_e$). However, the AAS effect is sensitive to the spin phase when the SOI plays a role. If there is magnetic flux inside the paths the resistance will oscillate with the period of $\hbar/2e$. When the flux is increased the resistance oscillates with the period $\hbar/2e$, but the AAS oscillation amplitude decays after a few periods because of averaging between different paths in the ring, with different areas. If there is SOI in the ring, the electron spin will start precessing around the effective magnetic field and change the interference at the entry point. Note that the effective magnetic field due to the SOI is much stronger than the external magnetic field to pick up AAS oscillations. The precession axes for the two parts of the wavefunction are opposite and therefore the relative precession angle
is twice the angle of each part. If the relative precession angle is $\pi$ the spins of the two parts are opposite and cannot interfere, and the AAS oscillations disappear. If the relative angle is $2\pi$ the two parts will have the same spin but opposite signs because of the 1/2 spin quantum laws (a $4\pi$ rotation is required to return to the original wavefunction), effectively changing the phase of the AAS oscillations by $\pi$, which we interpret as a negative amplitude.

By using arrays rather than single rings we get a stronger spin signal and we average out some of the universal conductance fluctuations (UCF) and sample specific AB oscillations [38]. Complex gate voltage dependence has been reported in an AB type AC experiment in a single ring fabricated from HgTe/HgCdTe QWs [39]. Therefore, a detailed analysis is necessary to compare with AC theory.

The ring arrays were etched out in an electron cyclotron resonance (ECR) dry-etching process from an InP/InGaAs/InAlAs based 2DEG, the same as used for the Shubnikov–de Haas (SdH) measurements. The electron mobility was 7–11 m$^2$ V$^{-1}$ s$^{-1}$ depending on the carrier density and the effective electron mass $m^*$ was 0.050 $m_0$ as determined from the temperature dependence of SdH oscillation amplitudes. Figure 2 shows an example of the ring array which consists of $4 \times 4$ rings of 1.0 $\mu$m radius. Note that the actually measured sample was a $5 \times 5$ ring array. The rings were covered with a 50 nm thick SiO$_2$ insulator layer, and an Au gate electrode, used to control the carrier density and the SOI parameter $\alpha$. In the present sample, we design the array with a small number of rings in order to avoid a gate tunnelling leakage problem. The advantage of using a small number of rings rather than a large array is that the gate tunnelling leakage is much smaller and we can use a relatively high gate voltage [26]. This makes it possible to see several oscillations of AC interference. Earlier experiments on square loop arrays with very large numbers of loops showed convincing spin interference results, but only up to one interference period [40, 41].

The experiment was carried out in a $^3$He cryostat at the base temperature which varied between 220 and 270 mK. The sample was put in the core of a superconducting magnet with the field $B$ perpendicular to the 2DEG plane. We measured the resistance $R$ of the ring array.
Figure 3. Magnetoresistance oscillations with period of $h/2e$ due to the AAS effect at three different gate voltages. The curves are shifted vertically for clarity. The oscillations in the top curve are reversed compared to the bottom one. The oscillation amplitude in the middle curves is suppressed. These gate voltage dependent AAS oscillations are due to the AC effect.

simultaneously with the Hall resistance $R_H$ of the Hall bar close to the rings, while stepping the magnetic field and the gate voltage $V_G$. Close to the arrays and in the same current path and under the same gate was a Hall bar, 5 $\mu$m wide and 20 $\mu$m long, used to measure the carrier density. We calculated the carrier density $n_e$ from the slope of the $R_H$ versus $B$ ($n_e^{-1} = e \frac{dR_H}{dB}$) and the carrier concentration is linearly increased with the gate voltage $V_G$.

In order to reduce noise and UCF effects we averaged ten resistance versus magnetic field ($R$ versus $B$) curves with slightly different gate voltages. This averaging preserves the AAS oscillations but the averaging of $M$ curves reduces the AB amplitude roughly as $M^{-1/2}$. We took the FFT spectrum (using an Exact Blackman window) of this average and got a spectrum with two peaks, corresponding to the AB oscillations and the AAS oscillations at twice the frequency. We integrated the area of the AAS peak to get the amplitude and determined the sign by analysing the phase of the central part of the filtered $R$ versus $B$ data.

In figure 3, we display the $h/2e$ magnetoresistance oscillations due to the AAS effect at three different gate voltages. The oscillations in the top curve are reversed compared to the bottom one because of the AC effect. The middle curve has almost no oscillations the spin precession rotates the spins of the two wavefunction parts to opposite directions. Figure 4 shows the colour scale plot after digital band-pass filtering of the AAS oscillations which are visible as vertical stripes in the figure. We can clearly see the oscillations switching phase as we increase the gate voltage. We then plotted the amplitude against the gate voltage as shown in figure 5. The AAS amplitude oscillates as a function of the gate voltage which changes the SOI parameter $\alpha$. As we discuss below using equations (11) and (12) the amplitude crosses zero, inverting the AAS oscillations. Each period represents one extra $2\pi$ spin precession of an electron moving around a ring.

In the FFT spectra there is also a small peak at $h/4e$. This is due to the wavefunction parts going twice around the ring before interfering. If we do the same analysis on this peak we get an oscillating amplitude with half the period compared to the $h/2e$ amplitude. This is expected
because the distance is double and therefore the precession angle is also double. Both \( h/2e \) and \( h/4e \) oscillation amplitudes increase with increasing gate voltage \( V_G \). This is because the phase coherence length of ring becomes longer with increasing diffusion constant which depends on the carrier density.

The precession angle \( \theta \) of an electron moving along a straight narrow channel is given by [15]

\[
\theta = \frac{2\alpha m^*}{\hbar^2} L, \tag{11}
\]

with \( L \) being the travel distance. The modulation of the \( h/2e \) oscillation amplitude can be expressed as a function of \( \alpha \),

\[
\frac{\delta R_\alpha}{\delta R_{\alpha=0}} = \cos \left\{ 2\pi \sqrt{1 + \left( \frac{2m^*\alpha}{\hbar^2 r} \right)^2} \right\}, \tag{12}
\]

where \( \delta R_\alpha \) and \( \delta R_{\alpha=0} \) is the \( h/2e \) amplitude with and without SOI, respectively. In relating the result of the spin interference experiment to the spin precession angle, the cosine argument in equation (12) reduces to the spin precession angle \( \theta \) in the limit of strong SOI or large ring radius because the distance travelled around the ring is \( 2\pi r \).

We also measured SdH oscillations at different carrier densities in a separate Hall bar. The SOI strength \( \alpha \) obtained from the beating pattern of the SdH oscillations shows carrier density dependence because the Rashba SOI depends on the SIA and therefore on the shape of the QW which is modified by applying the gate voltage. The gate voltage sensitivity \( \Delta \alpha/\Delta V_G \) is about \( 0.51 \times 10^{-12} \text{ eV m V}^{-1} \) in the present heterostructure. From the gate voltage dependence of SOI \( \alpha \) we estimate the spin precession angles at several different gate voltages. The estimated spin precession angles at the peak and dips of the AC oscillations are shown in figure 5. It is found that the spin precession angle is controlled over the range of \( 4\pi \) by the gate electric
field. We could observe more than 20π spin precession angle. This clear demonstration of the AC interference controlled by the gate electric field can be attributed to the fact that the SOI is much enhanced in semiconductor heterostructures compared to the SOI in vacuum. The AC effect is of fundamental importance for quantum interference phenomena and quantum interactions.

5. Conclusion

We discuss the enhancement of SOI and the origin of Rashba SOI. A spin interferometer based on the Rashba SOI is proposed and discussed. The conductance oscillation of the spin interferometer as a function of the Rashba SOI is very similar to the Datta–Das spin FET, but ferromagnetic source and drain electrodes are not necessary in the spin interferometer. We have demonstrated the electronic AC effect dual to the magnetic AAS effect in small arrays of rings. The AC interference oscillations are controlled over several periods. This result shows that the spin precession rate can be controlled in a precise and predictable way by an electrostatic gate. This electrical manipulation of spin precession angle is of crucial importance in order to realize semiconductor spintronics devices based on SOI.

Acknowledgments

The authors thank T Koga, Y Sekine, T Kobayashi, F Meijer and M van Veenhuizen for their valuable discussions. This work is partly supported by Grant-in-Aids from JSPS and MEXT.

References

[1] Wolf S A, Awschalom D D, Buhrman R A, Daughton J M, Von Molnar S, Roukes M L, Chtcchelkanova A Y and Treger D M 2001 Science 294 1488
[2] Awschalom D D, Samarth N and Loss D 2002 Semiconductor Spintronics and Quantum Computation (Heidelberg: Springer)

New Journal of Physics 9 (2007) 341 (http://www.njp.org/)
