Mon. Not. R. Astron. Soc. 326, 274–282 (2001)

Precision timing measurements of PSR J1012+5307

Ch. Lange,1 F. Camilo,2 N. Wex,1 M. Kramer,3* D. C. Backer,4 A. G. Lyne3 and O. Doroshenko1

1Max Planck Institut für Radioastronomie, Auf dem Hügel 69, D-53121 Bonn, Germany
2Columbia Astrophysics Laboratory, Columbia University, 550 West 120th Street, New York, NY 10027, USA
3University of Manchester, Jodrell Bank Observatory, Macclesfield, Cheshire SK11 9DL
4Astronomy Department, 601 Campbell Hall, University of California, Berkeley, CA 94720-3411, USA

Accepted 2001 April 11. Received 2001 March 27; in original form 2001 February 2

ABSTRACT

We present results and applications of high-precision timing measurements of the binary millisecond pulsar J1012+5307. Combining our radio timing measurements with results based on optical observations, we derive complete 3D velocity information for this system. Correcting for Doppler effects, we derive the intrinsic spin parameters of this pulsar and a characteristic age of 8.6 ± 1.9 Gyr. Our upper limit for the orbital eccentricity of only 8 × 10^{-7} (68 per cent confidence level) is the smallest ever measured for a binary system. We demonstrate that this makes the pulsar an ideal laboratory in which to test certain aspects of alternative theories of gravitation. Our precision measurements suggest deviations from a simple pulsar spin-down timing model, which are consistent with timing noise and the extrapolation of the known behaviour of slowly rotating pulsars.

Key words: gravitation – relativity – time – binaries: general – pulsars: general – pulsars: individual: J1012+5307.

1 INTRODUCTION

The 5.3-ms pulsar J1012+5307 was discovered during a survey for short-period pulsars with the 76-m Lovell radio telescope at Jodrell Bank. Nicastro et al. (1995) showed that the pulsar is in a binary system with an orbital period of 14.5 h and a companion mass between 0.11 and 0.25 M\(_{\odot}\) (90 per cent confidence level, hereafter C.L.). Optical observations reported by Lorimer et al. (1995) revealed an optical counterpart within 0.2 ± 0.5 arcsec of the pulsar timing position, consistent with its being a white dwarf (WD) companion of mass 0.15 M\(_{\odot}\).

Indeed, PSR J1012+5307 is one of the few examples of binary millisecond pulsars where optical observations of the WD companion considerably enhance our knowledge about the radio pulsar and its binary system (van Kerkwijk, Bergeron & Kulkarni 1996). Callanan, Garnavich & Koester (1998), for instance, used their results from optical observations to obtain a number of essential pieces of information for this system. Comparing the measured optical luminosity to the value expected from WD models, they determined the distance to be d = 840 ± 90 pc. The Doppler shift of the measured H spectrum of the WD companion gives an additional radial velocity component of 44 ± 8 km s\(^{-1}\) relative to the Solar system barycentre, SSB. From the radial velocity (semi-)amplitude and the orbital parameters known from pulsar timing, the mass ratio between the pulsar and its companion is measured to be m\(_{\text{p}}\)/m\(_{\text{c}}\) = 10.5 ± 0.5.

In order to derive the intrinsic spin-down age of millisecond pulsars, one needs to have accurate distance estimates in order to correct for apparent acceleration effects (Shklovskii 1970). A proper motion obtained by pulsar timing is usually converted into a transverse velocity by using a distance estimate derived from the dispersion in the interstellar medium, and applying a model of the free electron density distribution (Taylor & Cordes 1993). However, this model yields a high level of systematic uncertainty (e.g. Toscano et al. 1999b). The dispersion measure (DM) distance of 520 pc is indeed somewhat smaller than the distance derived from luminosity models.

Applying classical cooling models (Iben & Tutukov 1986) for low-mass WDs and the DM distance, Lorimer et al. derived an age of only 0.3 Gyr for the WD companion which was in apparent contradiction with the inferred spin-down age of the pulsar of 7 Gyr. However, Alberts et al. (1996) were able to model the binary parameters if they assumed that the cooling age of the WD was similar to the spin-down age of the pulsar. More recently, Driebe et al. (1998) modelled the cooling process of WDs, obtaining an age of 6 ± 1 Gyr for the companion of PSR J1012+5307. Ergma, Sarna & Antipova (1998) modelled evolutionary sequences of neutron stars with close low-mass binary companions and short orbital periods, whilst Sarna, Antipova & Muslimov (1998) developed an evolutionary scenario for PSR J1012+5307,
reproducing the observed binary period and companion mass. They modelled the age of the neutron star to be between 4.5 and 6.0 Gyr, whilst the mass of the WD was found to be $0.16 \pm 0.02 \, M_\odot$, consistent with mass estimates derived by other models based on different observations and evolutionary models (Callanan et al. 1998; Driebe et al. 1998).

In this paper, we combine a set of high-precision timing data with the information derived for the binary system from radio pulsar and optical WD observations as well as from model calculations. After a short description of the timing observations and data reduction techniques, we describe a new binary timing model that is important for the low-eccentricity case, discuss the resulting solution using statistical tests, and present the derived timing parameters. These results are analysed in the following sections. We show that proper motion and Galactic acceleration of PSR J1012+5307 have a significant influence on the derived characteristic age. Furthermore, we obtain strong limits on the true eccentricity of the system and discuss its meaning for evolutionary scenarios. As a result of the stringent limits obtained both on the change in orbital period and on the true eccentricity of the system, we demonstrate that this binary system is highly suitable for use in testing different theories of gravitation.

2 OBSERVATIONS

2.1 Effelsberg timing

We have made regular high-precision timing observations of PSR J1012+5307 since 1996 October, using the 100-m radio telescope of the Max Planck Institut für Radioastronomie in Effelsberg near Bonn, Germany. The typical observing rate has been once per month with a usual observing time of about 1 h. The overall post-fit rms of the timing model applied to the 1213 resulting times-of-arrival (TOAs) is 3.1 \mu s. Sometimes, however, the source shows strong intensity fluctuations on time-scales of typically 2 h caused by scintillation effects in the ionized component of the interstellar medium. We made use of these intensity maxima by optimizing our observing strategy, i.e. staying ‘on source’ until the pulsar became weaker again. As a consequence of this highly successful ‘scintillation hopping’, many TOAs are of much higher quality, with errors frequently smaller than 1 \mu s. Fig. 1 shows a typical example for the development of TOA uncertainties during a scintillation maximum. Since we have many such high-quality TOAs covering several orbits, they proved to be extremely useful for the precise determination of orbital parameters.

2.1.1 Receiving systems

Most of the data were obtained with a 1.3–1.7 GHz tunable High Electron Mobility Transfer (HEMT) receiver installed in the primary focus of the telescope. The noise temperature of this system is 25 K, resulting in a system temperature from 30 to 50 K on cold sky, depending on elevation. The antenna gain at these frequencies is 1.5 K Jy$^{-1}$. An intermediate frequency (IF) centred on 150 MHz for left-hand (LHC) and right-hand (RHC) circularly polarized signals was obtained after down-conversion from a central radio frequency (RF) of usually 1410 MHz.

In order to monitor DM changes, occasionally we also obtained data at 860 and 2700 MHz. For the 860-MHz observations, we used an uncooled HEMT receiver operating in the primary focus with a bandwidth of 30 MHz. The system temperature during the observations was about 65 K on cold sky with a telescope gain of about 1.5 K Jy$^{-1}$. The 2700-MHz data were obtained with a cooled HEMT receiver in the secondary focus, with a system temperature of typically 50 K on cold sky. As for the other receivers, LHC and RHC signals were mixed down to an IF of 150 MHz.

2.1.2 Coherent de-disperser

The signals received from the telescope were acquired and processed with the Effelsberg-Berkeley Pulsar Processor (EBPP), which removed the dispersive effects of the interstellar medium online using the technique of ‘coherent de-dispersion’ (Hankins & Rickett 1975). Before entering the EBPP, the two IF LHC and RHC signals are converted to an internal IF of 440 MHz. A maximum bandwidth of 112 MHz$^1$ is split into four independent portions for both circular polarizations. Each portion is mixed down to baseband and subdivided into eight narrow channels via a set of digital filters (Backer et al. 1997) for coherent online de-dispersion by convolution using a custom Very Large Scale Integration (VLSI) device. In total, 64 output signals were detected and integrated in phase with the predicted topocentric pulse period.

2.1.3 Measurements of arrival times

A TOA was calculated for each average profile obtained during a 5–10 min observation. During this process, the observed time-stamped profile was compared to a synthetic template, which was constructed out of 12 Gaussian components fitted to a high signal-to-noise ratio standard profile (see Kramer et al. 1998, 1999). This template matching was performed by a least-squares fitting of the Fourier-transformed data (Taylor 1991). The final TOA was obtained by using the measured time delay between the actual profile and the template; the accurate time stamp of the data provided by a local H maser and corrected off-line to the coordinated Universal Time of the National Institute of Science and Technology, denoted UTC(NIST), using recorded information from the satellites of the Global Positioning System (GPS). The uncertainty of each TOA was estimated using a method described by Downs & Reichley (1983).

\[1\] The bandwidth available depends on the observing frequency and the DM of the pulsar. At 1.41 GHz, we used a bandwidth of 56 MHz for PSR J1012+5307.

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**Figure 1.** Errors of the TOAs during a scintillation maximum.
2.2 Jodrell Bank timing

Since its discovery in 1993, PSR J1012+5307 has been regularly monitored using the 76-m Lovell telescope at Jodrell Bank with cryogenic receivers at 408, 606, and 1404 MHz. Both LHC and RHC signals were observed using a 2 x 64 x 0.125-MHz filter bank at 408 and 606 MHz and a 2 x 32 x 1.0-MHz filter bank at 1404 MHz. After detection, the signals from the two polarizations were filtered, digitized at appropriate sampling intervals, and incoherently de-dispersed in hardware before being folded on-line with the topocentric pulse period and written to disk. Each integration was typically of 1–3 min duration; 6 or 12 such integrations usually constituted an observation. In the analysis stage, the profiles were added in polarization pairs before being summed to produce a single total-intensity profile. A standard pulse template was fitted to the observed profiles at each frequency to determine a total of 400 pulse TOAs. Details of the observing system and the data reduction scheme can be found elsewhere (e.g. Bell et al. 1997).

Although the templates used in the Effelsberg and Jodrell Bank data reduction differed, resulting offsets were absorbed in a global least-squares fit. The typical measurement accuracy of the Jodrell Bank 1404 MHz timing data is about 9 μs.

3 PRECISION TIMING

The combined TOAs, corrected to UTC(NIST) and weighted by their individual uncertainties determined in the fitting process, were analysed with the TEMPO software package,2 using the DE200 ephemeris of the Jet Propulsion Laboratory (Standish 1990). Owing to the extremely low eccentricity of this system, we applied a binary timing model that uses the Laplace–Lagrange parameters h and k and the time of ascending node $T_{ASC}$ in place of the Keplerian parameters $e$, $\omega$ and $T_0$ (see Appendix A for details). The Keplerian parameters as a function of those used by the model can be calculated as

\[ e = \sqrt{k^2 + h^2} \]

\[ \omega = \arctan(h/k) \]

\[ T_0 = T_{ASC} + \frac{P h}{2\pi} \arctan(h/k) \].

TEMPO minimizes the sum of weighted squared timing residuals, i.e., the difference between observed and model TOAs, $r_i$, yielding a set of improved pulsar parameters and post-fit timing residuals. The timing residuals shown in Fig. 2 indicate deviations from the timing model which are slightly larger than the calculated TOA uncertainties, $\sigma_i$. This most likely stems from a small under-estimation of the error in the TOAs. The data uncertainties used in the following were scaled by an appropriate factor to achieve a uniform reduced $\chi^2 = 1$ for each data set. In order to test the statistical properties of our data we have generated histograms of the deviations of the timing residuals from the model. The final residuals show the expected Gaussian distribution (Fig. 3).

In order to search for systematic errors that are either small enough to be individually undetectable, or have a time dependence which approximates some linear combination of terms in the model, we have searched for tell-tale correlations among nearly adjacent post-fit residuals (cf. Taylor & Weisberg 1989). We combined 2, 4, 8 etc. consecutive TOAs and evaluated the average residual of the remaining data set. A ‘clean’ data set should exhibit

2http://pulsar.princeton.edu/tempo

Figure 2. Post-fit timing residuals as a function of observing year.

Figure 3. Distribution of all post-fit timing residuals normalized by their uncertainties.

Figure 4. Mean post-fit rms residuals of the data sets versus the number of consecutive averaged timing residuals. The symbols represent data from: EBPP 1.4GHz (△) and 2.7 GHz (□), and Jodrell Bank 410 MHz (○), 606 MHz (○), and 1.4 GHz (*). The 870-MHz data from Effelsberg are not included here owing to their small number.
Table 1. Timing parameters for the millisecond pulsar J1012+5307. Quoted errors correspond to twice the errors derived by TEMPO. Upper limits represent 95 per cent C.L.

| Parameter                        | Value             |
|----------------------------------|-------------------|
| Right ascension, \( \alpha \)    | 10^{5.12^{\pm0.33}}34370(3) |
| Declination, \( \delta \)        | 53.0702^{+0.5884}_{-0.5884}(4) |
| \( \mu_\alpha \) (mas yr\(^{-1}\)) | 2.41(2)          |
| \( \mu_\delta \) (mas yr\(^{-1}\)) | -25.2(2)         |
| \( \nu \) (Hz)                   | 190.267837621903(4) |
| \( \nu \) (s\(^{-1}\))         | -6.2029(4) \times 10^{-16} |
| \( \dot{\nu} \) (s\(^{-2}\))   | -9.8(2.1) \times 10^{-27} |
| \( P \) (ms)                    | 5.2557490141971(7) |
| \( \dot{P} \) (s\(^{-1}\))     | 5.1(1.1) \times 10^{-31} |
| Epoch (MJD)                     | 50700.0           |
| Dispersion Measure (DM) (cm\(^{-3}\) pc) | 9.0233(2) |
| Orbital period, \( P_0 \) (days) | 0.60867271355(8)  |
| Proj. semimajor axis, \( a \) (s) | 0.581187(2)      |
| \( \eta \)                      | 0.9(8) \times 10^{-6} |
| \( \kappa \)                    | 0.01(80) \times 10^{-6} |
| \( T_{\text{ASC}} \) (MJD)       | 50700.0816290(1)  |

Upper limits:

| Parameter       | Value             |
|-----------------|-------------------|
| Parallax, \( \pi \) (mas) | <\(1.3 \times 10^{-6}\) a |
| \( e \)         | <\(1.3 \times 10^{-6}\) a |
| \( e \) (s\(^{-1}\)) | <\(2 \times 10^{-14}\) |
| \( \dot{e} \)   | <\(1.4 \times 10^{-14}\) |
| \( P_0 \)       | <\(1 \times 10^{-13}\) |
| DM (cm\(^{-3}\) pc yr\(^{-1}\)) | <\(1.2 \times 10^{-4}\) |

a See Section 4.4 for details.

4 INTRINSIC PULSAR PARAMETERS

As summarized in Section 1, the binary millisecond pulsar PSR J1012+5307 is unique since the combination of optical and radio measurements allows a full determination of the motion of the system through space and also the determination of several intrinsic parameters.

4.1 3D velocity

The full 3D motion of the pulsar relative to the SSB is determined by combining the timing proper motion with distance to the system obtained from luminosity models for the white dwarf companion, \( d = 840 \pm 90 \) pc. This distance is consistent with the observed upper limit for the timing parallax. Given the pulsar’s ecliptic latitude of 38\(^{\circ}\), it is not unexpected that a significant value for the timing parallax could not be obtained. We derive transverse velocities of

\[ v_\alpha = \mu_\alpha d = 8.8 \pm 1.2 \text{ km s}^{-1} \]  \tag{4}

and

\[ v_\delta = \mu_\delta d = -102 \pm 11 \text{ km s}^{-1}. \]  \tag{5}

Optical measurements (cf. Section 1) yield a radial velocity component of \(+44(8) \) km s\(^{-1}\). Hence, the total velocity of the system is \(11(28)\) km s\(^{-1}\). This value is consistent with the average space velocity of millisecond pulsars of \(130\) km s\(^{-1}\) estimated by Lyne et al. (1998) and Toscano et al. (1999a).

4.2 Doppler effects

As a result of the motion of PSR J1012+5307 relative to the Solar system, any period \( P^{\text{obs}} \) of the pulsar spin or binary system, measured at the SSB, differs from the period \( P^{\text{intr}} \) in the pulsar reference frame by a Doppler factor \( D \). Following Damour & Taylor (1991), we write the relation between these periods

\[ p^{\text{obs}} = D P^{\text{intr}} = (1 + \frac{n \cdot \vec{v}}{c}) P^{\text{intr}} + O(v^2/c^2) \]  \tag{6}

with \( n \) the unit vector to the pulsar and \( \vec{v} \) the relative velocity between the pulsar and the SSB (Damour & Taylor 1992). As \( v/c \) is typically less than 0.1 per cent for millisecond pulsars, this Doppler shift is for most purposes unimportant. Hence, we will for the rest of this paper denote \( P^{\text{obs}} \) as \( P \). However, the period derivative may be modified by the relative motion.

Calculating the time derivative of equation (6) and separating the effect of the proper motion of the system (Shklovskii 1970) from the influence of the Galactic acceleration, we derive as contributions to the period derivative

\[ \dot{p}^{\text{obs}} = \dot{P}^{\text{intr}} D + \dot{P}^{\text{Shk}} + \dot{P}^{\text{Gal}}, \]  \tag{7}

where the difference of the constant Doppler factor \( D \) from unity may be neglected. The contributions \( P^{\text{Shk}} \) and \( P^{\text{Gal}} = P^D \) are written explicitly

\[ P^{\text{Shk}} = \frac{1}{c} \mu^2 dP \]  \tag{8}

\[ P^{\text{Gal}} = \frac{1}{c} (n_{\text{psr}} \cdot (a_{\text{psr}} - a_{\text{s}})) P, \]  \tag{9}

where \( d \) is the distance to the pulsar. Here \( n_{\text{psr}} \) is the position vector to the pulsar, \( \mu \) is its proper motion and \( a_{\text{psr}} \) and \( a_{\text{s}} \) are the Galactic.
acceleration of the pulsar and the Earth, respectively. We measure \( n_{\text{agg}} \) and \( \mu \) from our timing observations. The small correction term \( \dot{P}^\text{corr}/P = -7 \times 10^{-20} \text{ s}^{-1} \) for the Galactic acceleration is obtained by applying a model for the Galactic potential (Carlberg & Innanen 1987; Kuijken & Gilmore 1989). Inserting these parameters into equations (8) and (9), we obtain a Doppler correction of

\[
\dot{P}^D/P = 1.4 \pm 0.3 \times 10^{-18} \text{ s}^{-1}.
\]

(10)

Its error is dominated by the uncertainty in the distance to the pulsar.

4.3 Characteristic age

We can now use our timing results to obtain the true spin-down age, the so-called characteristic age of PSR J1012+5307. With a spin period of \( P = 5.256 \text{ ms} \) and a Doppler factor as given in equation (10), we derive a Doppler correction of

\[
\dot{P}^D = 7.4 \pm 1.6 \times 10^{-21},
\]

(11)

which has to be subtracted from the measured \( \dot{P} \) before calculating the intrinsic characteristic age of the system:

\[
\dot{P}^\text{intr} = \frac{\dot{P}}{2\dot{P}^\text{corr}} = \frac{\dot{P}}{2(P_{\text{obs}} - P^D)} = 8.6 \pm 1.9 \text{ Gyr}.
\]

(12)

We note that the characteristic age assumes magnetic dipole braking (i.e., a braking index of 3) and is only a realistic estimate of the real pulsar age if the initial spin period was sufficiently small (Camilo, Thorsett & Kulkarni 1994). Although this may not apply to He-WD binaries (Backer 1998), the characteristic age is in reasonable agreement with a cooling age of the white dwarf as derived by evolutionary models (Alberts et al. 1996; Sarna et al. 1998; Driebe et al. 1998).

4.4 Orbital eccentricity

In Appendix A, below, we analyse the contribution of the Shapiro delay for binary orbits with moderate inclination angles and negligible intrinsic eccentricities to the apparent eccentricity of the system. As we show, the Shapiro delay cannot be separated from the Roemer delay, which leads to a small correction to the observed eccentricity. In order to investigate this effect for PSR J1012+5307 quantitatively and to obtain the true orbital eccentricity, we make use of independent determinations of the system parameters.

From the companion mass of 0.16(2) M\(_\odot\), the measured mass ratio of \( q = m_c/m_p = 10.5(5) \), and the mass function

\[
f_w = \frac{m_c^3 \sin i}{(m_c + m_p)^3} = 0.000578 \text{ M}_\odot
\]

obtained from the timing analysis, we derive the range \( r \) and shape \( s \) of the Shapiro delay in the system according to

\[
r[\mu\text{s}] = 4.9255(m_c/\text{M}_\odot)
\]

and

\[
s = \sin i = \left[ f_w(q + 1)^{2}/m_c \right]^{1/3}.
\]

(13)

(14)

(15)

The resulting shape parameter of \( s \approx 0.8 \) indicates a moderate inclination of the system, i.e., \( i = 52^\circ \) or 128\(^\circ\). The contribution of the Shapiro delay to the observed \( \eta \) is calculated according to equation (A22) and is subtracted from the measured value. In

Monte Carlo simulations, we have constructed artificial data sets for \( \eta \) and \( \kappa \) according to their measured values and uncertainties listed Table 1. After correcting \( \eta \) as described, we obtain upper limits on the true eccentricity by calculating its value using equation (1), i.e.

\[
\epsilon^\text{intr} < 0.8 \times 10^{-6} \quad (68 \text{ per cent C.L.)}
\]

\[
\epsilon^\text{intr} < 1.3 \times 10^{-6} \quad (95 \text{ per cent C.L.)}
\]

(16)

In these calculations, the results of which are displayed in Fig. 5, we also simulated the parameters \( q \) and \( m_c \) in accordance with the observational uncertainties.

5 TESTS OF THEORIES OF GRAVITATION

5.1 Dipole gravitational waves

Unlike general relativity, many alternative theories of gravity predict the presence of a radiative monopole and dipole as well as the quadrupole and higher-order multipoles (Will 1993). Scalar–tensor theories, for instance, predict for binary systems a loss of orbital energy which is at highest order dominated by scalar dipole radiation. As a result, the period \( P_b \) of a circular orbit will change with

\[
P_b^{\text{dipole}} = - \frac{4\pi^2 G_*}{c^4 P_b} \frac{m_p m_c}{m_p + m_c} (\alpha_p - \alpha_c)^2,
\]

(17)

where \( G_* \) is the bare gravitational constant, \( c \) is the speed of light and \( m_p \) and \( m_c \) the mass of the pulsar and its companion (Damour & Esposito-Farèse 1996). The parameters \( \alpha_p \) and \( \alpha_c \) represent the effective coupling strengths of the scalar field to the pulsar and its companion, respectively.

Damour & Esposito-Farèse (1996) show that \( \alpha_c \) can be neglected compared to \( \alpha_p \), since the gravitational binding energy per unit mass of a WD, \( E_{\text{WD}} = E_{\text{grav}}/m_c \approx 10^{-4} \), is very much smaller than that of a 1.4-M\(_\odot\) neutron star, \( E_{\text{p}} = 0.15 \).

Using this assumption, and taking \( G = G_* \), equation (17) can be approximated as

\[
P_b^{\text{dipole}} = - \frac{4\pi^2 G}{c^4 P_b} m_c \frac{q}{q + 1} \alpha_p^2.
\]

(18)

Since all terms except \( \alpha_p \) in this equation are measured, the effective coupling strength of PSR J1012+5307 can be restricted by limits on \( P_b \). We will show that these are the tightest bounds on \( \alpha_p^2 \) ever measured for a neutron star.
The observable rate of change of the orbital period of PSR J1012+5307 is the sum of several contributions, either intrinsic to the orbit or caused by projection effects:

\[ \dot{P}_b = \dot{P}_b^D + \dot{P}_b^{GR} + \dot{P}_b^G + \dot{P}_b^{\text{dipole}} \]  

(Damour & Taylor 1991). Here, the Doppler correction of \( \dot{P}_b \) is given by \( \dot{P}_b^D \) and the contribution \( \dot{P}_b^{\text{GR}} \) represents gravitational radiation as predicted by general relativity. The values of these effects can be estimated and may be subtracted from the measured value of \( P_b \). The additional contributions \( \dot{P}_b^G \) and \( \dot{P}_b^{\text{dipole}} \) represent the period derivative arising from a change of the gravitational constant and from gravitational dipole radiation, respectively. All terms of gravitational radiation not accounted for are of the order of \( c^{-5} \) or higher. We derive \( \dot{P}_b^D \) from equation (10), with the orbital period of 52.240 s, to be

\[ \dot{P}_b^D = 7.3 \pm 1.6 \times 10^{-14}. \]  

6 DISCUSSION

6.1 Timing noise

Over the observed time span of about 7 yr, PSR J1012+5307 has shown a significant deviation from a simple \( \nu \sim \nu^2 \) spin-down behaviour. In order to model this effect, the spin-down model can be extended by allowing for a non-zero second derivative of the spin frequency, \( \dot{\nu} \) (e.g. Cordes & Downes 1985). However, rather than being the result of rotational irregularities of the neutron star itself, a possible alternative explanation for a second spin-frequency derivative could be a third-order term of the proper motion,

\[ \ddot{\nu} = -\frac{3 V_r}{c} \mu^2 \nu_{\text{int}} - 2 \frac{d \mu^2}{c} \nu_{\text{int}}, \]  

where \( V_r \) is the radial velocity of the system (Phinney 1992). This term can be computed and it is of the order of \( 10^{-30} \) s\(^{-3} \), i.e., too small to explain the measured effect.

In order to investigate the possibility that the observed timing noise is caused by small changes in the DM, we modelled our multifrequency data with a simultaneous fit of \( \sigma \) and a second-order polynomial for DM. While DM and DM were not significant, the \( \dot{\nu} \) measurement was still a \( \sim 3 \sigma \) effect. The upper limit on DM given in Table 1 is interestingly much smaller than one would expect for DM = 9.02 cm\(^{-3}\) pc (Backer et al. 1993).

As a possible explanation, a second-period derivative in the timing residuals can also be the result of a second light companion in a wider orbit (Thorsett et al. 1999). Assuming a circular orbit and an intermediate orbital phase, the measured value for \( \dot{\nu} \) can be explained with, e.g., a planet of terrestrial mass at \( \sim 30 \) au distance or a Jupiter-like planet at \( \sim 170 \) au from the centre of mass. Future observations will show if the timing residuals can be explained by a second Keplerian orbit.

5.2 Local Lorentz invariance

Will & Nordtvedt (1972) pointed out that one expects the existence of a preferred cosmic rest frame for gravitational interaction if gravity is mediated in part by a long-range vector field or by a second tensor field. In the post-Newtonian limit, all the gravitational effects associated with such a preferred frame are phenomenologically describable by two parameters, \( \alpha_1 \) and \( \alpha_2 \). Only in gravity theories without a preferred frame, e.g. general relativity, are the values of these parameters equal to zero.

A consequence of a preferred cosmic rest frame would be the introduction of an eccentricity into orbits of gravitationally bound bodies of different masses which are in motion relative to this preferred frame. This effect is quantified by the parameter \( \alpha_1 \). Wex (2000) has applied a statistical analysis of all pulsar binary systems to find limits for this parameter. This analysis yields \( \alpha_1 \leq 1.4 \times 10^{-4} \) at 95 per cent C.L. As a result of its extremely low true eccentricity and the known 3D proper motion, the data presented here for PSR J1012+5307 and included in Wex’s analysis play a central role in this analysis. They might provide more stringent tests of local Lorentz invariance, if the real eccentricity of the system is even smaller and if better observational limits on it are found.
where $i$ is the observing span, which in their case was $10^8$ s. Despite noticeable scatter, they obtained a correlation of this parameter with the period derivative for slowly rotating pulsars. This correlation predicts a value of $\Delta \theta \approx -5.3$ for PSR J1012+5307. Using timing data over a time-span of $10^8$ s, we measure a value of $\Delta \theta \approx -5$. Given that the fit leading to Arzoumanian et al.’s relation was made by eye, both values are in excellent agreement, in particular given the large scatter seen for normal pulsars. Moreover, the value of the second spin-frequency derivative, $\dot{\nu} = -8.8(2.1) \times 10^{-27}$ s$^{-3}$, is similar to the value of $\dot{\nu}$ published for PSR B1937+21 (Kaspi et al. 1994).

Both sources exhibit a $\Delta \delta$ consistent with the correlation derived by Arzoumanian et al. Although we cannot yet completely rule out a different origin of the measured-period second derivative, the value determined is apparently consistent with what is known for timing noise from normal pulsars. Assuming that we are now indeed observing rotational irregularities in two Galactic field millisecond pulsars, being consistent with the behaviour of normal pulsars extrapolated to very small period derivatives, it may indicate that the empirical relation found by Arzoumanian et al. (1994) is generally valid for millisecond pulsars as well as for normal pulsars. As other millisecond pulsars are now being monitored with similar timing precision over similar time-spans, this conjecture can be tested. It may well be the case that we are finally exploring the ultimate accuracy of the pulsar clockwork, deciding over the suitability of millisecond pulsars as potential standards of time.

### 6.2 Orbital eccentricity – a relic of binary evolution

The convective fluctuation-dissipation theory of Phinney (1992) predicts a strong correlation between the orbital eccentricities and orbital periods of binary pulsars that have been recycled by stable mass-transfer from a Roche-lobe-filling, low-mass red giant. The orbital periods of binary pulsars that have been recycled by stable mass-transfer from a Roche-lobe-filling, low-mass red giant. The model of Phinney & Kulkarni (1994) points out that for extremely close binary systems, the orbital eccentricity is difficult to model. Therefore, it is interesting to note that the upper limit for the eccentricity of the 0.6-d binary pulsar J1012+5307 of only $e < 0.8 \times 10^{-6}$ (68 per cent C.L.) is significantly lower than that of PSR J0613−0200.

### 6.3 Future observations

The motion of the binary system relative to the SSB results in a change of the orbital inclination that causes a variation in the projected semimajor axis of the system (Arzoumanian et al. 1996; Kopeikin 1996; Bell et al. 1997; Sandhu et al. 1997). The amplitude of this effect is given by $\delta t = 1.54 \times 10^{-16} \times \cos^2 \theta \sin i / \mu_\alpha \cos \Omega + \mu_\delta \cos \Omega$, (27) where $\Omega$ is the position angle of the ascending node and $\mu_\alpha$ and $\mu_\delta$ are the components of the proper motion in mas yr$^{-1}$. Given the value of the orbital inclination $i \approx 0.8$ as determined from the white dwarf optical light curve, a measurement of $\delta t$ allows one to restrict the orbital orientation $\Omega$. Using our current data set, we derive a 3$\sigma$ uncertainty for $\delta t$ that is about twice the value expected for a perfect alignment between $\Omega$ and $\mu$. Continuing timing observation will permit the measurement of this effect. Simulations, assuming an accuracy and data-taking rate similar to those at present, show that a 3$\sigma$ detection of $\delta t$ should be possible within the next four or five years. This would finally complete our knowledge of the full orientation and motion of the PSR J1012+5307 system.

### 7 SUMMARY

We have performed long-term high-precision timing observations of PSR J1012+5307. Combining our measurements with results based on optical observations, we derived complete 3D velocity information for this system, permitting the correction of the measured spin parameters for Doppler effects. As a result of the precision of our measurements and the extremely small eccentricity of this binary pulsar, we could use it as an ideal laboratory with which to test certain aspects of theories of gravitation. The deviation of the timing data from a simple spin-down model is probably the result of rotational instabilities which are consistent with the extrapolation of the known spin-down behaviour of slowly rotating pulsars, suggesting that this phenomenon is a common property of most pulsars.

### ACKNOWLEDGMENTS

We are grateful to all staff at the Effelsberg and Jodrell Bank observatories for their help with the observations. FC is supported by NASA grant NAG 5-9095. OD acknowledges the receipt of an Alexander von Humboldt fellowship.

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APPENDIX A: TIMING OF SMALL-ECCENTRICITY BINARY PULSARS

Neglecting relativistic effects and parallax, the barycentric arrival time, $t_b$, of a signal emitted by a pulsar in an orbit around a compact companion is given by

$$t_b - t_0 = T + \Delta_0(T).$$  \hfill (A1)

$T$ denotes the time of emission of the pulse signal as measured in the reference frame of the pulsar, i.e., it is directly related to the rotational phase $\phi$ of the pulsar by

$$\phi = \phi_0 + \nu T + \frac{1}{2} \nu^2 T^2 + \ldots.$$  \hfill (A2)

The Roemer delay, $\Delta_0$, caused by the orbital motion of the pulsar, is given by (Blandford & Teukolsky 1976)

$$\Delta_0 = x (\cos U - e \sin \omega + \sin U (1 - e^2) \frac{1}{2} \cos \omega),$$  \hfill (A3)

where the eccentric anomaly, $U$, is determined by Kepler’s equation

$$U - e \sin U = n_b (T - T_0), \quad n_b = 2\pi/P_b,$$  \hfill (A4)

and $\omega$ denotes the longitude of periastron.

For small-eccentricity binary pulsars, however, the location of periastron is not a prominent feature in the TOAs. Therefore, modeling the timing data of small-eccentricity binary pulsars with equation (A3) leads to very high correlations between the parameters $\omega$ and $T_0$. As a result, $\omega$ and $T_0$ show unacceptably large uncertainties in the $\chi^2$ estimation of these parameters.

A1 A timing model for small-eccentricity binary pulsars

Neglecting terms of order $e^2$ the orbital motion $X = R(\cos \varphi, \sin \varphi, 0)$ of a pulsar in a small-eccentricity binary system, is given by (see, e.g., Roy 1988)

$$R = a_p (1 - e \cos M) = M = n_b (T - T_0),$$  \hfill (A5)

where $M = \varphi = 0$ corresponds to the location of periastron. To first order approximation in the small eccentricity $e$, the Roemer delay (equation A3) can therefore be written as

$$\Delta_0 \approx \pi (\sin \Phi + \frac{\kappa}{2} \sin 2\Phi - \frac{\eta}{2} \cos 2\Phi),$$  \hfill (A6)

$\Phi = n_b (T - T_{asc})$,

where terms which are constant in time are omitted. The three Keplerian parameters $T_{0b}, e,$ and $\omega$ are replaced by the time of ascending node which is defined by

$$T_{asc} = T_0 - \omega n_b,$$  \hfill (A7)

and the first and second Laplace–Lagrange parameter,

$$\eta = e \sin \omega \quad \text{and} \quad \kappa = e \cos \omega,$$  \hfill (A8)

respectively. Note that the actual time when the pulsar passes through the ascending node is given by $T_{asc} + 2\pi n_b$. The time of conjunction is given by $T_{asc} + P_b/4 - 2\pi n_b$.

Equation (A6) accounts only for first-order corrections in $e$. Therefore, the difference between the exact expression (A3) and equation (A6) can grow up to $e^2$. For most of the low-eccentricity binary pulsars, the error in the TOA measurements is much larger than $e^2$ and thus the linear-in-$e$ model is sufficient.
Secular changes in the parameters $n_b$, $x$, $e$, and $\omega$ in equation (A3) are accounted for in the new timing model by using

$$\Phi = \tilde{n}_b(T - T_{asc}) + \frac{1}{2} \tilde{h}_b(T - T_{asc})^2,$$

(A9)

and

$$x = x_0 + \delta(T - T_{asc}),$$

$$\eta = \eta_0 + \eta(T - T_{asc}),$$

$$\kappa = \kappa_0 + \kappa(T - T_{asc}),$$

(A10)

in equation (A6) where the relations

$$\tilde{n}_b = n_b + \dot{\omega} - \dot{n}_b(T_0 - T_{asc}),$$

(A11)

$$T_{asc} = T_0 - \frac{\alpha_0}{n_b + \dot{\omega}},$$

(A12)

$$\tilde{h}_b = \dot{n}_b,$$

(A13)

$$\delta = e \sin \omega + e \cos \omega \; \dot{\omega}$$

and

$$\kappa = e \cos \omega - e \sin \omega \; \dot{\omega},$$

(A14)

(A15)

hold.

**A2 Small-eccentricity orbits and Shapiro delay**

For small-eccentricity binary pulsars, the Shapiro delay can be written as

$$D_S = 2r \ln(1 - s \sin \Phi),$$

(A16)

where $r = Gm/c^3$ and $s = \sin i$. As a Fourier series, equation (A16) takes the form

$$\Delta \xi = 2r(a_0 + b_1 \sin \Phi - a_2 \cos 2\Phi + \ldots),$$

(A17)

where

$$a_0 = -\ln \left( \frac{1 + \sqrt{1 - s^2}}{2} \right),$$

(A18)

$$b_1 = 2 \frac{1 - \sqrt{1 - s^2}}{s},$$

(A19)

$$a_2 = 2 \frac{1 - \sqrt{1 - s^2}}{s^2} - 1.$$  

(A20)

Only for orbits where $\sqrt{1 - s^2} \ll 1$ (nearly edge-on) are higher harmonics, indicated as ... in equation (A17), significant. Otherwise, the Shapiro delay cannot be separated from the Roemer delay. Consequently the observed values for $x$ and $\eta$ differ from their intrinsic values by

$$x^{(\text{obs})} = x + 2rb_1,$$

(A21)

$$\eta^{(\text{obs})} = \eta + 4r\alpha_2/x,$$

(A22)

as can be seen by a comparison of equation (A17) with equation (A6). In terms of the Blandford–Teukolsky model, equation (A22) means that the observed value of the eccentricity $e$ and the observed value of the longitude of periastron $\omega$ are different from the intrinsic values of these parameters.

This new model has been implemented in the TEMPO timing software as binary model ELL1.

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