Polarization mode interaction equations in optical fibers with Kerr effect

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Abstract

We derive coupled nonlinear Schrödinger equation (CNLSE) for arbitrary polarized light propagation in a single-mode fiber. We introduce a basis of transverse eigen modes with the appropriate projecting hence solutions depend on the waveguide geometry. Considering a weak nonlinearity which is connected with Kerr effect, we give explicit expressions for nonlinear constants via integrals of Bessel functions. We compare numerical results for the nonlinear constant extracted from experimental observations of a soliton for the nonlinear Schrödinger equation (NLSE) (single-mode one). The method of projecting we use allows a direct generalization to multi-mode fiber case.

1 Introduction

There are lot of publications devoted to the propagation and interaction of polarized electromagnetic wave pulses in optical fibers (see the book [1]). Almost most of them exploit the results of [2], that is going up to [3]. It is claimed that one can consider the fiber as made of isotropic material and the birefringence is originated from the third order nonlinearity (Kerr effect [4]). The result of the derivation is achieved by means of averaging across the fiber section and gives the following evolution along $z$ axis.

\begin{align*}
&iX_+ - ik''_N X_+ + \frac{k''}{2}X_{tt}^+ + (\gamma |X_+|^2 + \eta |X_-|^2) X_+ + \ldots = 0, \quad (1a) \\
&iX_- - ik''_N X_- + \frac{k''}{2}X_{tt}^- + (\gamma |X_-|^2 + \eta |X_+|^2) X_- + \ldots = 0, \quad (1b)
\end{align*}

where $X_+, X_-$ are the envelopes components of electric fields (polarizations), $k''$ is the dispersion constants respectively, $\gamma$ correspond to SPM (self phase modulation) and $\eta$ corresponds to XPM (cross phase modulation). The computation of the $\gamma/\eta$ relation (ratio) for Kerr medium which is generally elliptically
birefringent, depends on birefringent ellipse axis choice \[2, 5, 6, 7\] and have value \(2/3\) (linear case), \(2\) (circular case), generally (for example \(\gamma/\eta = 2\)) we can say that the XPM is twice as effective as SPM. \(V_{\pm}^N = 1/k_N^2\) is the nonlinear group velocity of polarization components. Here we accept that the origin of a birefringence comes from nonlinear effects, but it can be also descent from random defects of fiber or special structure of waveguide (polarization maintaining fibers).

The averaging procedure looks reasonable from a physical scope but in many cases leads to significant deviations from experiments \[8\]. The transition from a three-dimensional to one-dimensional picture by the averaging is quite impossible in the case of multi-mode field: it leads to the only equation while the modes should be described by independent variables.

In this paper we base on a projecting procedure to the mode subspaces in a functional space of a multi-mode field \[8\]. In the nonlinear theory it leads to the important difference between results for nonlinear constants obtained by the projecting and averaging procedures already in the case of one-mode fiber.

The general plan of the paper is following. In the section 2 we briefly overview (fix) the notation that are chosen maximally close to standard books \[4\]. In the next section 3 we show how to build the general representation of the overall electromagnetic field as the mode superposition. We also define the transverse orthogonal eigenmodes together with the projecting procedure by the appropriate scalar product. The section 4 contains numerical results for a single mode waveguide and the evaluation of nonlinear coefficients for the NLS equation.

## 2 Basic equations

We describe polarization modes interaction in the cylindrical optical fibers. We start from the Maxwell electromagnetic field equations

\[
\begin{align*}
\nabla \cdot \mathbf{B} &= 0, \quad (2a) \\
\nabla \cdot \mathbf{D} &= 0, \quad (2b) \\
\n\nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t}, \quad (2c) \\
\n\nabla \times \mathbf{H} &= \frac{\partial \mathbf{D}}{\partial t}, \quad (2d)
\end{align*}
\]

in the system in the cylindrical polar coordinate \((r, \varphi, z)\) and materials equations

\[
\begin{align*}
\mathbf{H} &= \frac{1}{\mu_0} \mathbf{B}, \quad (3a) \\
\mathbf{D} &= \varepsilon_0 \mathbf{E} + \mathbf{P}. \quad (3b)
\end{align*}
\]

When one study boundary conditions the polarization vector \(\mathbf{P}\) is considered as a linear function of \(\mathbf{E}\), we take the simplest form for the isotropic medium

\[
\mathbf{P} = \varepsilon_0 \chi_{\text{linear}} \mathbf{E} \quad (4)
\]
and a wave equation for the electric field in fiber is

$$\Delta \mathbf{E} - \mu_0 \varepsilon_0 \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0.$$ (5)

Boundary conditions for our waveguide are

$$D_{r1} - D_{r2} = 0,$$  \hspace{0.2cm} (6a)
$$B_{r1} - B_{r2} = 0,$$  \hspace{0.2cm} (6b)
$$\mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0,$$ \hspace{0.2cm} (6c)
$$\mathbf{n} \times (\mathbf{H}_2 - \mathbf{H}_1) = 0.$$ \hspace{0.2cm} (6d)

Conditions (6c) for electric field yield

$$\varepsilon_1 E_r(r_0+) = \varepsilon_2 E_r(r_0-),$$ \hspace{0.2cm} (7a)
$$E_\varphi(r_0+) = E_\varphi(r_0-),$$ \hspace{0.2cm} (7b)
$$E_z(r_0+, \varphi, z) = E_z(r_0-, \varphi, z) \quad (r_0 - \text{waveguide radius}).$$ \hspace{0.2cm} (7c)

Wave number $k$ must be the same inside and outside a waveguide. To perform boundary conditions, we defined two parameters $\alpha$ and $\beta$

$$\alpha^2 = \omega^2 \varepsilon_0 \mu_0 \varepsilon_1 - k^2, \quad r \leq r_0,$$ \hspace{0.2cm} (8a)
$$\beta^2 = k^2 - \omega^2 \varepsilon_0 \mu_0 \varepsilon_2, \quad r > r_0,$$ \hspace{0.2cm} (8b)

where $\omega$ is the frequency of a light wave.

Now if we use solution inside and outside then waveguide for linear polarization and all of boundary conditions we get equation (known as Hondros-Debye equation) for the eigenvalues $\alpha_{ln}$ (see equations 8). From this equation (which is well known in linear theory of waveguides [4]) we can numerically evaluate eigenvalues. The linearized ME defined the basis of eigenfunctions that are

$$J_l(\alpha_{ln}r)e^{il\varphi}$$ \hspace{0.2cm} (9)

where $l = 0, \pm 1, \pm 2, \ldots$ and $n = 1, 2, \ldots$ where $n$ is numbering following eigenvalues (following solutions for fixed $l$) and $\alpha_{ln}$ is connected with eigenvalues $k_{ln}$.

In general the polarization vector should be written as

$$\mathbf{P} = \varepsilon_0 \left( \chi^{(1)} \mathbf{E} + \chi^{(2)} \mathbf{E} \mathbf{E} + \chi^{(3)} \mathbf{E} \mathbf{E} \mathbf{E} + \cdots \right),$$ \hspace{0.2cm} (10)

where $\chi^{(1)}$ is linear dielectric susceptibility and corresponds to the refraction of light. In a case of the second order dielectric susceptibility $\chi^{(2)}$, we could omit it because it is equal zero in materials construct with symmetrical molecule. From higher order dielectric susceptibility we save only third order susceptibility because rest of orders are negligible. The third order susceptibility is responsible for nonlinear refraction of light, self phase modulation (SPM) and cross phase modulation (XPM).
If we take into consideration that for impulses longer then 0.1ps one can treat a response of a medium as instantaneous and we can write

\[ P_{NL}(t) = \varepsilon_0 \chi^{(3)}(t, t, t) \mathbf{E}(t) \mathbf{E}(t) \mathbf{E}(t). \]  

The third order dielectric susceptibility \( \chi^{(3)} \) for isotropic media is discussed in the papers [4, 9]. Basing on it we write

\[ \chi_{ijkl} = \chi_{xxxx} \delta_{ij} \delta_{kl} + \chi_{xyxy} \delta_{ik} \delta_{jl} + \chi_{xyyx} \delta_{il} \delta_{kj}, \]  

\[ \chi_{xxxx} = \chi_{yyyy} = \chi_{zzzz} = \chi_{xyyy} + \chi_{xxyy} + \chi_{xyyx}, \]  

all components of electric field are in standard form

\[ E_i = \frac{1}{2} A_i e^{i\omega t} + c.c., \]  

inserting this relation into equation (11), we get nonlinear polarization as (non-resonant terms are removed)

\[ P_i = \frac{1}{8} \chi_{xxxx} \varepsilon_0 \sum_j \left( 2 A_i |A_j|^2 + A_j^2 A_i \right) e^{i\omega t} + c.c., \]  

where \( i, j = x, y, z \)

For example, for \( z \) component we have

\[ P_z = \frac{3}{8} \chi_{xxxx} \varepsilon_0 \left\{ \left[ |A_z|^2 + \frac{2}{3} (|A_x|^2 + |A_y|^2) \right] A_z + \frac{1}{3} A_z \left( A_x^2 + A_y^2 \right) \right\} e^{i\omega t} + c.c.. \]  

We have same equation as in paper [4, 2]. Next we put it into the Maxwell equations (We introduce nonlinearity into the Maxwell equations in the form of the Kerr effect [4], with assumption of small nonlinearity.)

Let us rewrite the wave equation system as

\[ \Box E_i = -\mu_0 \frac{\partial^2}{\partial t^2} P_i, \]  

where \( \Box \) is defined by

\[ \Box = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} - \nabla. \]
3 General solution, main results

We write solution for electromagnetic field with amplitude depend on time and propagation coordinate in form \( S \):

\[
E_z(r, \varphi, z, t) = \frac{1}{2} \sum_{p, l, n} \left[ A_{ln}^p (z, t) J_l(\alpha_n r) e^{i \varphi} e^{i(\omega t - kz)} + c.c. \right],
\]

(20a)

\[
E_r(r, \varphi, z, t) = -\frac{1}{2} \sum_{p, l, n} \left\{ \frac{i}{\alpha_{nl}} \left[ \tilde{G}_{ln}^p (z, t) \frac{i l \omega}{r} J_l(\alpha_n r) + \tilde{C}_{ln}^p (z, t) k \partial_r J_l(\alpha_n r) \right] e^{i \varphi} e^{i(\omega t - kz)} + c.c. \right\},
\]

(20b)

\[
E_\varphi(r, \varphi, z, t) = \frac{1}{2} \sum_{p, l, n} \left\{ \frac{i}{\alpha_{nl}} \left[ \tilde{D}_{ln}^p (z, t) \omega \partial_r J_l(\alpha_n r) - \tilde{E}_{ln}^p (z, t) \frac{i l k}{r} J_l(\alpha_n r) \right] e^{i \varphi} e^{i(\omega t - kz)} + c.c. \right\},
\]

(20c)

\[
B_z(r, \varphi, z, t) = \frac{1}{2} \sum_{p, l, n} \left[ F_{ln}^p (z, t) J_l(\alpha_n r) e^{i \varphi} e^{i(\omega t - kz)} + c.c. \right],
\]

(20d)

\[
B_r(r, \varphi, z, t) = \frac{1}{2} \sum_{p, l, n} \left\{ \frac{i}{\alpha_{nl}} \left[ \tilde{H}_{ln}^p (z, t) \omega \mu_0 \varepsilon_0 \varepsilon \frac{i l \omega}{r} J_l(\alpha_n r) - \tilde{F}_{ln}^p (z, t) k \partial_r J_l(\alpha_n r) \right] e^{i \varphi} e^{i(\omega t - kz)} + c.c. \right\},
\]

(20e)

\[
B_\varphi(r, \varphi, z, t) = -\frac{1}{2} \sum_{p, l, n} \left\{ \frac{i}{\alpha_{nl}} \left[ \tilde{S}_{ln}^p (z, t) \omega \mu_0 \varepsilon_0 \varepsilon \partial_r J_l(\alpha_n r) + \tilde{S}_{ln}^p (z, t) \frac{i l k}{r} J_l(\alpha_n r) \right] e^{i \varphi} e^{i(\omega t - kz)} + c.c. \right\}.
\]

(20f)

Here \( p \) numbering two orthogonal polarization and have values "+" and "−". Coefficients with tilde includes all constants to simplify notation.
Inserting these solutions into the Maxwell equations yields

\begin{align}
- \mathcal{B}_l^p + il \mathcal{D}_l^p &= 0, \\
- C_l^p \ln \alpha^2 + \partial_z \mathcal{A}_l^p &= 0, \\
- C_l^p \ln il - \mathcal{E}_l^p il &= 0, \\
il \mathcal{A}_l^p + \partial_z \mathcal{E}_l^p - \partial_t \mathcal{H}_l^p &= 0, \\
- \partial_z \mathcal{D}_l^p + \partial_t \mathcal{G}_l^p &= 0, \\
- \partial_t \mathcal{C}_l^p - \mathcal{A}_l^p - \partial_t \mathcal{P}_l^p &= 0, \\
- \partial_z \mathcal{B}_l^p - \mathcal{G}_l^p &= 0, \\
- \partial_z \mathcal{D}_l^p \alpha^2 - \partial_t \mathcal{F}_l^p &= 0, \\
- \partial_z \mathcal{E}_l^p + il \mathcal{G}_l^p &= 0, \\
- \partial_z \mathcal{F}_l^p - \partial_t \mathcal{G}_l^p &= 0, \\
- \mathcal{H}_l^p \ln \alpha^2 - \partial_z \mathcal{F}_l^p &= 0, \\
- \mathcal{H}_l^p \ln il &= 0.
\end{align}

Using (21b), (21r) and (21f) we can verify that the amplitudes \( \mathcal{A}_l^p \) satisfy equation (similarly we can get equation for \( \mathcal{F}_l^p \))

\begin{equation}
\partial_{zz} \mathcal{A}_l^p - \mu_0 \varepsilon_0 \varepsilon \partial_t \mathcal{A}_l^p = \alpha^2 \mathcal{A}_l^p
\end{equation}

\begin{equation}
\partial_{zz} \mathcal{A}_l^p - \mu_0 \varepsilon_0 \varepsilon \partial_t \mathcal{F}_l^p = \alpha^2 \mathcal{F}_l^p
\end{equation}

It can be proved that the Bessel functions satisfy orthogonality in form \[10\]

\begin{equation}
\int_0^{r_0} r J_l(\alpha r) J_l(\alpha l) r dr = \frac{r_0^2}{2} \left[ J_{l+1}^2(\alpha r_0) - J_{l-1}(\alpha r_0) J_{l+1}(\alpha r_0) \right] \delta_{l' l} = N_{nl} \delta_{l' l},
\end{equation}

(23)

taking into account boundary condition for optical waveguide.

Let us now exploit orthogonal relation, we can show that the equation for \( z \) coordinate are \[8\]

\begin{equation}
(\square_z + \alpha^2) \mathcal{A}_l^p = \frac{2 \varepsilon_0 \mu_0}{\pi N_{nl}} \int_0^{r_0} r J_l(\alpha r) e^{-il\phi} \frac{\partial^2}{\partial l^2} \sum_{klm} \chi_{klm} E_k E_{lm} d\phi dr,
\end{equation}

(24)
where $\Box_z$ is defined by

$$
\Box_z = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2}.
$$

We choose relation between $A_{ln}^p$ and $F_{ln}^p$ in form

$$
\partial_t F_{ln}^p = \pm i \partial_z A_{ln}^p,
$$

this yield to the two orthogonal polarization which can be write as

$$
E_{\varphi}^\pm (r, \varphi, z, t) = \frac{1}{2} \sum_{l,n} \frac{i}{\alpha_{ln}} \left[ \partial_z A_{ln}^\pm J_{l\pm 1}(\alpha_{ln} r) \right] e^{il\varphi} e^{i\omega t - ikz} + \text{c.c.}, \quad (27a)
$$

$$
E_{\varphi}^\pm (r, \varphi, z, t) = \frac{1}{2} \sum_{l,n} \frac{i}{\alpha_{ln}} \left[ \partial_z A_{ln}^\pm J_{l\pm 1}(\alpha_{ln} r) \right] e^{il\varphi} e^{i\omega t - ikz} + \text{c.c.}, \quad (27b)
$$

In this case equations (27) have simple form in cartesian co-ordinate system

$$
E_x^\pm (x, y, z, t) = \frac{1}{2} \sum_{l,n} \frac{1}{\alpha_{ln}} \partial_z A_{ln}^\pm J_{l\pm 1}(\alpha_{ln} r) e^{i(l\pm 1)\varphi} e^{i\omega t - ikz} + \text{c.c.}, \quad (28a)
$$

$$
E_y^\pm (x, y, z, t) = \frac{1}{2} \sum_{l,n} \frac{i}{\alpha_{ln}} \partial_z A_{ln}^\pm J_{l\pm 1}(\alpha_{ln} r) e^{i(l\pm 1)\varphi} e^{i\omega t - ikz} + \text{c.c.}, \quad (28b)
$$

and we can use it to calculate $P_z$.

We take into computation electric field in form

$$
E_x = \frac{1}{2} E_x^+ + \frac{1}{2} E_x^-,
$$

$$
E_y = \frac{1}{2} E_y^+ + \frac{1}{2} E_y^-.
$$

For more generality considering about birefringent axis see [5].

Let us construct the only transversal mode with fixed $\alpha$ and $\beta$ which mean that we chose the simplest form by fixing $l = 0$ and $n = 1$ (we cut series to one term, this allow us to simplify calculation of the $P_z$). We introduce a slowly varying amplitude of the wave envelope in form

$$
\sigma X^\pm (\tau, \xi) e^{-ikz},
$$

where

$$
\xi = \sigma z, \quad (31a)
$$

$$
\tau = (t - k' z) \epsilon, \quad (31b)
$$

where $\sigma$ is nonlinearity parameter and $\epsilon$ is dispersion parameter.

We insert equations (28) into (30) and then we put the slowly varying amplitude in it and if the relation between parameters is $\epsilon^2 \sim \sigma$ and if we
use a new coordinate system which moves at group velocity \( \tilde{31} \), than to the second order in \( \epsilon \) we can obtain nonlinear Schrödinger equation in the form (we don’t get complex conjugate part)

\[
(i\partial_t X^+ + \frac{\epsilon^2 k''}{2\sigma}\partial_{\tau\tau} X^+ + i\partial_t X^- + \frac{\epsilon^2 k''}{2\sigma}\partial_{\tau\tau} X^-) e^{i(\omega t - k z)} =
\]

\[
= \frac{\varepsilon_0 \mu_0}{\sigma^2 \pi N_0 k} \int_0^{r_0} \int_0^{2\pi} r J_0(\alpha_{01} r) \frac{\partial^2}{\partial t^2} P_z(X^+, X^-) d\varphi dr.
\]

(32)

Let us now evaluate a right side of equations \( \tilde{32} \). If we keep terms up to the order of the third power of \( \epsilon \) and additional save expression with \( \partial_{\tau} X^\pm \) term, we have

\[
i\partial_t X^+ - ik_{N}^+ \partial_{\tau} X^+ + \frac{\epsilon^2 k''}{2\sigma}\partial_{\tau\tau} X^+ + P \left[ |X^+|^2 + Q |X^-|^2 \right] X^+ = 0, \quad (33a)
\]

\[
i\partial_t X^- - ik_{N}^- \partial_{\tau} X^- + \frac{\epsilon^2 k''}{2\sigma}\partial_{\tau\tau} X^- + P \left[ |X^-|^2 + Q |X^+|^2 \right] X^- = 0, \quad (33b)
\]

where

\[
P = \frac{3\omega^2 \mu_0 \varepsilon_0 \chi_{xxxx} \sigma}{8N_0 k} \int_0^{r_0} r \left( \frac{1}{4} J_0^4(\alpha_{01} r) + \frac{1}{3\alpha_{01}^2} J_1^2(\alpha_{01} r) J_0^2(\alpha_{01} r) k^2 \right) dr.
\]

(34)

where XPM coefficient (in this case self phase modulation (SPM) equal 1)

\[
Q = \frac{1}{P} \frac{3\omega^2 \mu_0 \varepsilon_0 \chi_{xxxx} \sigma}{8N_0 k} \int_0^{r_0} r \left( \frac{2}{4} J_0^4(\alpha_{01} r) \right) dr,
\]

(35)

and \( V_{gN}^\pm \) depends on amplitude \( X^\pm \).

4 Numerical calculation

First we define normalized frequency as

\[
V = \frac{\omega}{c r_0 \sqrt{\varepsilon_1 - \varepsilon_2}}
\]

(36)

and coefficient which we calculate

\[
P = \chi_{xxxx} P_{coeff}
\]

(37)
Choosing a value for physical parameters

\[
\begin{align*}
\omega &= 12.2 \times 10^{14} \text{ Hz} \quad (\lambda \approx 1.54 \mu \text{m}) \quad \text{(38a)} \\
\varepsilon_1 &= 2.25 \quad \text{(ref. index 1.5)} \quad \text{(38b)} \\
\varepsilon_2 &= 1.96 \quad \text{(ref. index 1.4)} \quad \text{(38c)} \\
\varepsilon_0 &= 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}} \quad \text{(38d)} \\
\mu_0 &= 12.56 \times 10^{-7} \frac{\text{N}}{\text{A}^2} \quad \text{(38e)} \\
r_0 &= \text{from } 1.2 \times 10^{-6} \text{m to } 10 \times 10^{-6} \text{m} \quad \text{(38f)}
\end{align*}
\]

On picture 1 we show results for mode \(l = 0\) and \(n = 0\) (known as TE_{01}) also we show results for mode \(l = \pm 1\) and \(n = 1\) (known as HE_{11}) which can be calculate using above procedure (shows here for \(l = 0\) and \(n = 0\)). Picture 2 show XPM coefficient.

![Figure 1: Numerical results for \(P_{coef}\) for parameters \[35\]. Vertical line shows mode cut-off.](image)

If we consider only one mode with one polarization we can write Nonlinear Schrödinger Equation (NLS)

\[
\begin{align*}
i \partial_z U + \frac{k''}{2} \partial^4_{tt} U + \gamma |U|^2 X &= 0, \quad \text{(39)}
\end{align*}
\]

where \(\gamma\) is nonlinear coefficient. We compare \(\gamma\) with work \[3, 11, 12\] where it was defined as

\[
\frac{g \omega n_2}{c},
\]

(40)
where $g$ depend on variation of the electric field in the fiber cross section and in most cases takes a value of approximation $1/2$ and $n_2$ is defined by \[ n_2 = \frac{3}{4n} \chi_{xxxx}, \] \[ (41) \]

In our case $\gamma$ is define by (for $l = \pm 1$ and $n = 1$ with one polarization)

\[
\gamma = \frac{3\omega^2 \chi_{xxxx}}{16N_{11}} \int_0^{r_0} r \left[ \frac{1}{2} J_1^2(\alpha_{11}r) + \frac{k_1^2}{3\alpha_{11}^2} J_1^2(\alpha_{11}r) \left( J_0^2(\alpha_{11}r) + J_2^2(\alpha_{11}r) \right) \right] \text{d}r, \]
\[ (42) \]

We don’t have dependence on mode cross-section (radius of light beam and fiber) but it is included by the boundary condition. On picture 3 we make comparison our results to (40). On graph $G$ is defined as

\[
G = \frac{\gamma(\text{our numerical results})c}{g\omega n_2}, \]
\[ (43) \]

5 Conclusion

In this paper we shows a new approach to derive a formula for CNLS equations.

The main idea is to take into account multi-mode case. Here we show the simplest case for $l = 0, \pm 1$ and $n = 1$ (as single mode) but if we use equation (24) and take electromagnetic field (28) with more modes we could compute multi-mode case. The proceeding is the same as for single but it is more intricate.
and derive simple formula is more difficult (because we have additional terms corresponding to mode interaction).

In this paper we don’t show formula for $V_{gN}^{±}$ but it can be calculate.

Additionally we can allow for change birefringent axis (see eq. (29)) and take case with different grup velocity (introduce $k_+ \neq k_-)$.

The definition of the nonlinear coefficient is used in works relating to quantum effects in fibers [13], and the coefficient is defined likely in the non-quantum NLS equation.

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