Optimization of the helically-winding structure required for the helical fusion reactor utilizing the high temperature superconductor (HTS) tape is studied by numerical calculation. When the HTS tapes are wound to be the helical structure without the torsion, a significant edgewise strain appears. This might lead to the degradation of the performance of the HTS tape. Therefore, the edgewise strain must be minimized. It is found that the edgewise strain can be suppressed by the proper determination of the torsion by the optimization utilizing the Fourier decomposition of the distribution of the edgewise strain. The HTS tapes with edgewise strain are intrinsically deformed due to the torsion to reduce the edgewise strain so to be an “edgewise-strain-free” helical shape.

1. Introduction

As one of the methods to realize the nuclear fusion reactor, helical plasmas are confined by the magnetic field produced by the helically wound coils which are in a three-dimensional complex shape. For the effective production of the high magnetic field without using helium, the high temperature superconductor (HTS) using the Rare-Earth Barium Copper Oxide (REBCO) coated conductor tapes can be a candidate. The next generation material for the fusion reactor, the HTS can be an alternative option to the cable-in-conduit conductor using low-temperature superconductor. In the previous study, the application of the HTS to the conceptual design of the helical fusion reactor FFHR has been intensively considered [1], in which the configuration optimization using the sub-helical coils called NITA coils [2] and the “joint-winding” method of the STARS coils [3] have been explained. Generally, the HTS is the flat tape, and is fragile because the HTS is made of ceramic REBCO. Therefore, the edgewise strain should be avoided whereas the flatwise bending and torsion are acceptable. In actual usage, when the HTS tape is wound along the torus surface for the helical winding, the edgewise strain should appear. As the result, the forced additional flatwise bending or torsion appears to reduce the edgewise strain.

Many experimental and theoretical studies regarding the deformation of the HTS have been performed under the many applications of not only the fusion reactor but also SEMS, etc. [4–12]. Those studies mainly reported the characteristics of the flatwise bending and torsion. Because the HTS tape is the thin and flat shape, those deformations are more likely to deform than the edgewise bending. The permissible strain $\varepsilon$ of the deformation of the HTS tape is less than $\varepsilon = 0.5\%$. The edgewise strain becomes larger than that of the flatwise bending if these deformations occur in the same curvature due to the certain width of the tape. Therefore, to minimize the edgewise strain in the helical winding is one of the important issues. This article is composed as follows. The following section shows the assumed helical coil shape (trajectories of the HTS tape), definition of the trajectories of the HTS tape, and the calculation method. Section 3 describes the result. Calculation result is discussed and summarized in Sec. 4.

2. Helical Winding HTS Tape and Calculation Method for Optimization

2.1 The trajectory of the helical coil and its expression

The trajectory of the helical coils considered here is sketched in Fig. 1, which is the similar figure of the Large Helical Device (LHD). The trajectory in the Cartesian coordinate is expressed by

$$x = (R_{ax} + r \cos \theta) \cos \phi,$$

$$y = (R_{ax} + r \cos \theta) \sin \phi,$$

$$z = r \sin \theta.$$

Here, $R_{ax}$, $r$, $\theta$, $\phi$, $m$, $l$, and $\alpha$ are the major radius, the minor radius, the poloidal angle defined by $\theta =$
Fig. 1 Trajectory of helical coil. (a) Bird’s eye view of helical coil. (b) Top view of helical coil in which the toroidal angle $\phi$ is defined.

\[(m/l)\phi + \alpha \sin((m/l)\phi)\], the toroidal angle, the toroidal pitch number ($m = 10$), the poloidal pole number ($l = 2$), and the pitch modulation parameter, respectively. The parameters for Fig. 1 are $R_{ax} = 3.3$ m, $\gamma = 1.254$, and $\alpha = 0.1$. Here, the helical pitch parameter is defined by $\gamma = (m/l)(r/R)$. We use these parameters as the basic configuration. Top view of the coil is plotted in the experimental coordinate of $X$ - $Y$ space in Fig. 1 (b), in which the position of $Z$ is indicated by the color bar. The toroidal angle $\phi$ is defined as positive directed toward the CCW, in which the coil sets at outboard side at $\phi = 0, 0.4, 0.8, 1.2, 1.6 \pi$ rad.

2.2 Shape of the HTS tape wound in helical structure

We assume that the single HTS tape having finite width of 10 mm is wound along the trajectory shown in Fig. 1 and the HTS tape is placed on the surface of the torus. For this situation, the edgewise strain spontaneously appears. Figure 2 is the image of the geometry of the wound HTS tape not in scale. At the cross section, the open circle indicates the neutral point, and the blue and the red circles mean the deviated position of the right and the left, respectively. The trajectories of the HTS tape are described by the three dimensional coordinate based on the Frenet – Serret formulas [13]. The definition of the edgewise strain $\varepsilon$ is determined by $\varepsilon \equiv (\Delta S_1 - \Delta S_0)/(\Delta S_0)$, here, $\Delta S_0$ and $\Delta S_1$ are the minute path at the neutral line and that at the deviated position, respectively, as shown in Fig. 3. The $\varepsilon > (<)$ indicates the tension (the compression). This definition is equivalent to the definition using the local curvature.

The analytical calculation of the $\varepsilon$ is derived in the Eq. (21) in the Ref. [8] with the toroidal effect assumption and the direction to the tape width is assumed to be perpendicular to the curvature. Substituting the parameters used here (major radius $R_{ax} = 3.3$ m, minor radius $r = 0.828$ m, coil pitch number $N = 5$, tape width $h = 0.005$ m, and the averaged torsion $\tau_{ave} = 0.59$), the equation derives $\varepsilon = 4.3 \times 10^{-4}$ ($\%$) which can be thought to be almost edgewise strain free. On the other hand, in the geometry with the finite toroidal effect, considerable edgewise strain appears. The profile of the edgewise strain is shown in Fig. 4.
in which the maximum edgewise strain is $|\varepsilon| = 0.13\%$.

Even though this value is less than the critical value, it should be important for suppressing the edgewise strain in order to ensure the performance of the HTS tape. The red and the blue lines indicate the deviated position of right and left. Both of the profiles show almost symmetry. Each profile has the toroidal mode number $n = 5$ which corresponds to the ratio of the toroidal pitch parameter to the poloidal pole number $m/l$ of the helical wound coils. The toroidal angle $\phi = 0$ ($0.2\pi$) rad corresponds to the HTS tape that is set at the outboard (inboard) side of the equatorial plane.

2.3 How to decrease edgewise strain

The fundamental idea to decrease the edgewise strain is to make the HTS tape deform to the flatwise and/or to the torsion. This idea can be imagined from Fig. 5. When the edgewise strain indicates the tension $\varepsilon > 0$ (compression $\varepsilon < 0$), the trajectory should deviate from the original position by the modification of the shortcut (detour). For a more detailed explanation, the trajectory at the right side (blue) and the trajectory at the left side (red) are shown in Fig. 5. As shown in Fig. 5 (a), at $0 < \phi < 0.2\pi$ rad, the edgewise strain $\varepsilon$ indicates negative (positive) in the right (left) side, which means the compression (tension). To release the compression (tension), the trajectory is modified as the detour (shortcut) (Fig. 5 (b)). Similarly, at $0.2 > \phi > 0.4\pi$ rad, the edgewise strain $\varepsilon$ indicates positive (negative) in the left (right) side, which means the tension (compression). Resulting deformation is shown in Fig. 5 (c). Here, if one gives the torsion angle $\theta_{\text{tens}}$ as a proper function of $\phi$, the edgewise strain originated from the tension and/or compression is decreased to obtain the optimized shape of the HTS tape.

2.4 Method for optimization of edgewise strain free configuration

The flowchart used in this study is shown in Fig. 6. At first, according to the Eqs. (1) - (3), the center of the trajectory is defined as a reference as shown in Fig. 1, which corresponds to the condition of $H = 0$ and $W = 0$ (see Fig. 5). Subsequently, the shape of the HTS tape wound helically without torsion ($H = 0$ and $W = 5$ mm) is calculated and the finite edgewise strain $\varepsilon$ should appear as shown in Fig. 4. The edgewise strain $\varepsilon$ is treated by the Fourier decomposition to obtain the amplitude of the edgewise strain of the $n = 5$ component. The amplitude is represented by the following equation.

$$\varepsilon_n = \sqrt{\varepsilon_s^2 + \varepsilon_c^2}. \quad (4)$$

Values of $\varepsilon_s$ and $\varepsilon_c$ are derived by the sine and cosine weighted integrals of $\varepsilon(\lambda)$ with the path along the coil trajectory $\lambda$ and with the toroidal angle $\phi(\lambda)$ (rad) as follows.

$$\varepsilon_s = \int \varepsilon(\lambda) \sin(n\phi(\lambda)) d\lambda,$$

$$\int \sin^2(n\phi(\lambda)) d\lambda.$$ \quad (5)

$$\varepsilon_c = \int \varepsilon(\lambda) \cos(n\phi(\lambda)) d\lambda,$$

$$\int \cos^2(n\phi(\lambda)) d\lambda.$$ \quad (6)

Fig. 5 Image showing how to avoid edgewise strain. (a) Example of edgewise strain $\varepsilon$ profile. (b) Torsion to minimize $\varepsilon$ at $\phi = 0 \sim 0.2\pi$ rad. (c) Torsion to minimize $\varepsilon$ at $\phi = 0.2 \sim 0.4\pi$ rad.

Fig. 6 Flowchart of optimization method to minimize edgewise strain.
Then the HTS tape is deformed by the periodic torsion with toroidal Fourier mode \( n = 5 \) following the idea to make \( \epsilon_{n=5} \) be small as shown in Fig. 5 in Sec. 2.3. The trajectory is made to be a detour (shortcut) at the negative (positive) edgewise strain. The proper torsion angle \( \theta_{\text{tors}} \) is given as the periodic function with \( n = 5 \). From the comparison of the \( \epsilon_{n=5} \) between the no-deformed and deformed HTS tape, the difference of the edgewise strain \( \Delta \epsilon \) can be obtained. The torsion is scanned to make the \( \Delta \epsilon \) minimum obtaining the optimized deformed HTS tape. The above procedure is operated with each 5 steps of Fourier modes from \( n = 5 \) to 50.

3. Optimized Helical Winding of HTS Tape with Edgewise Strain Free

3.1 Edgewise strain free winding

Figure 7 shows the toroidal profile of the edgewise strain \( \epsilon \) and the torsion angle \( \theta_{\text{tors}} \) in the range of the one toroidal period \( \phi = 0 \sim 0.4 \pi \text{ rad} \). The maximum edgewise strain \( \epsilon \) of the HTS tape without the torsion (no-optimization) indicates the \( |\epsilon| = 0.13\% \) strain shown by dashed line in Fig. 7 (a) whereas the almost edgewise strain free structure can be realized in the optimized case shown by solid line in Fig. 7 (a). The torsion angle for the optimized HTS case is shown by solid line in Fig. 7 (b), in which the angle is \( \theta_{\text{tors}} = 20 \) degrees at the bottom \( (\phi = 0.1 \pi \text{ rad}) \) and \( \theta_{\text{tors}} = -20 \) degrees at the top \( (\phi = 0.3 \pi \text{ rad}) \) of the torus. The torsion angle \( \theta_{\text{tors}} = 0 \) at the inboard side \( (\phi = 0.2 \pi \text{ rad}) \) and the outboard side \( (\phi = 0) \); the image figures are shown in Fig. 7 (c) - (f). These results mean that the edgewise strain free winding of the HTS tape can be realized by the optimization of the torsion of the HTS tape.

Here, we can investigate the influence of other toroidal mode numbers except for the \( n = 5 \). Figure 8 shows the edgewise strain of each toroidal mode number from 5 to 50 in the step of 5. The no-optimization and optimization cases are plotted by the gray and the red circles, respectively. The dominant component is \( n = 5 \) mode \( \epsilon_{5} \) which is reduced from \( \epsilon_{5} = 0.131\% \) to 0.0008\%. Secondary dominant mode is \( n = 10 \) which is reduced from \( \epsilon_{10} = 0.0054\% \) to 0.00026\%. As for larger modes, \( n > 10 \) can be thought to be below the negligible level less than 0.0001\% even in the no-optimization case.

3.2 Application to various helical coil configuration

The method to optimize the HTS tape winding can be applied to other configurations of helical coils. We have attempted to scan the helical pitch parameter \( \gamma \) and the pitch modulation parameter \( \alpha \). In this study, the \( \gamma \) defines the minor radius \( r \) because the toroidal pitch number \( m \), the poloidal pole number \( l \), and the major radius \( R \) are fixed.

Figure 9 shows the edgewise strain \( \epsilon_{5} \) and torsion angle \( \theta_{\text{tors}5} \) of the \( n = 5 \) mode in the various helical pitch parameters \( \gamma = 1.1 \sim 1.3 \). The optimized \( \epsilon_{5} \) shown by red in Fig. 9 (a) slightly decreases with \( \gamma \), and is reduced to the order of 1/100 compared to the no-optimization cases shown by gray in Fig. 9 (a). The torsion angles in the optimization case are almost constant \( \theta_{\text{tors}5} \sim 20 \) degrees as shown in Fig. 9 (b).

Fig. 7 Toroidal profile of (a) edgewise strain and (b) torsion angle. Dashed and solid lines indicate no-optimization case and optimization case, respectively. (c) - (f) Schematic images of the cross-section of twisted HTS tape in each toroidal angle.

Fig. 8 Fourier decomposed edgewise strain. Gray squares and red circles indicate no-optimization case and optimization case, respectively.
Figure 10 shows the pitch modulation parameter $\alpha$ dependence on the edgewise strain of $n = 5$ component $\varepsilon_5$ and torsion angle $\theta_{\text{tors}5}$. In the no-optimization case, the $\varepsilon_5$ monotonically decreases with increment of $\alpha$ around 0.1% shown by gray circles in Fig. 10 (a). On the other hand, in the optimization case, the $\varepsilon_5$ shows different behavior in three regions. At the lower $\alpha$ region ($\alpha = -0.6 \sim -0.2$), the $\varepsilon_5$ decreases with the $\alpha$. At the middle $\alpha$ region ($\alpha = -0.2 \sim 0.2$), the $\varepsilon_5$ is almost constant below 0.001% indicating the minimum value. At the higher $\alpha$ region ($\alpha = 0.2 \sim 0.4$), the $\varepsilon_5$ increases with the $\alpha$. Throughout the entire $\alpha$ region, the torsion angle $\theta_{\text{tors}5}$ shows the monotonic decreasing with $\alpha$ as shown in Fig. 10 (b).

4. Discussion and Summary

When the HTS tape is wound in helical structure on the toroidal geometry, the deformation of the flat bending and/or torsion arise to prevent the edgewise strain. Resulting deformed tape is the naturally achieved shape to release the edgewise strain. The quantitative estimation can be realized using the simple law to minimize the edgewise strain, which can show the optimized torsion angle whose maximum value is 20 degrees. HTS tape can be acceptable for the flatwise bending and torsion. These two kinds of deformation receive the strain from the edgewise strain. The deformation of the HTS tape considered here is not extremely serious, because the maximum torsion angle is 20 degrees, which may be acceptable for the HTS tape. In this study, we assume the single HTS tape for optimization. The real helical winding, however, contains the multiple tapes. Our model can be applied to the various coil configurations to estimate the shape of the coils composed of the HTS tape. To understand the realistic situation, modifications should be required for the future studies of the more complex helical winding structure.

The helical winding using single HTS tape is studied. The edgewise-strain-free configuration is found using the optimized method with Fourier decomposition. The edgewise strain can be suppressed from 0.13% to 0.0012%. This calculation can be applied to more complicated structures for a large-current conductor by combining a number of stacked HTS tapes.

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[1] N. Yanagi, T. Goto, J. Miyazawa, H. Tamura et al., J. Fusion Energy 38, 147 (2019).
[2] N. Yanagi, T. Goto, H. Tamura, J. Miyazawa et al., Plasma Fusion Res. 11, 2405034 (2016).
[3] Y. Terazaki et al., IEEE Trans. Appl. Supercond. 25, 4602905 (2015).
[4] H. Kitaguchi, J. Nishioka, T. Hasewaga et al., IEEE Trans. Appl. Supercond. 12, 1141 (2002).
[5] H.S. Shin, J.R.C. Dizon, T-H. Kim et al., IEEE Trans. Appl. Supercond. 17, 3274 (2007).
[6] N.C. Allen, L. Chiesa and M. Takayasu, IEEE Trans. Appl. Supercond. 25, 4800805 (2015).
[7] M. Takayasu and L. Chiesa, Mater. Sci. Eng. 102, 012023 (2015).
[8] H. Tsutsui, S. Tsuji-Iio, S. Nomura, T. Yagai et al., IEEE Trans. Appl. Supercond. 26, 4901704 (2016).
[9] T. Yagai, Y. Kimura, H. Kamada, S. Nomura et al., IEEE Trans. Appl. Supercond. 26, 8401905 (2016).
[10] T. Yagai, H. Akai, R. Dong Ryun et al., IEEE Trans. Appl. Supercond. 27, 6603105 (2017).
[11] W. Ta, Phys. Lett. A 382, 2395 (2018).
[12] S. Matsunaga, Y. Narushima, Y. Onodera et al., “HTS-WISE Conductor and Magnet Impregnated with Low-Melting Point Metal” MT26 Special Issue of the IEEE Trans. Appl. Supercond., to be published.
[13] K. Takahashi, N. Amemiya, T. Nakamura et al., IEEE Trans. Appl. Supercond. 22, 4901705 (2012).