Relic gravitons and viscous cosmologies

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(Dated: March 27, 2022)

Abstract: Previously it was shown that there exists a class of viscous cosmological models which violate the dominant energy condition for a limited amount of time after which they are smoothly connected to the ordinary radiation era (which preserves the dominant energy conditions). This violation of the dominant energy condition at an early cosmological epoch may influence the slopes of energy spectra of relic gravitons that might be of experimental relevance. However, the bulk viscosity coefficient of these cosmologies became negative during the ordinary radiation era, and then the entropy of the sources driving the geometry decreases with time.

We show that in the presence of viscous sources with a linear barotropic equation of state $p = \gamma \rho$ we get viscous cosmological models with positive bulk viscous stress during all their evolution, and hence the matter entropy increases with the expansion time. In other words, in the framework of viscous cosmologies, there exist isotropic models compatible with the standard second law of thermodynamics which also may influence the slopes of energy spectra of relic gravitons.

PACS numbers: 98.80.Cq, 04.30.Nk, 98.70.Vc

Our universe can be viewed as containing a sea of stochastically distributed gravitational waves of primordial origin. Among all observational cosmological evidences (present and future), primordial gravitational waves should have a sufficiently enlightened character in order to better understand the very early universe. The gravitational waves of cosmological origin are nothing but squeezed states of many gravitons produced from the vacuum fluctuations of the background metric. A qualitative analysis can be performed in the context of different physical frameworks, since all models for the very early universe predict the formation of stochastic gravitational wave backgrounds. As examples we can mention inflationary quintessential models, inflationary models in Brans-Dicke theory of gravity, cosmological models in the Brane-world scenario, and superstring theories. The shape of the stochastic graviton background spectrum is affected by the variations of the background dynamics.

In this context, Giovannini has considered the interesting possibility of constructing flat Friedmann–Robertson–Walker (FRW) cosmologies endowed with a bulk viscous stress which induces a violation of the dominant energy condition (DEC) for a limited amount of time at an early cosmological epoch. This kind of cosmological models may be connected to some of the recent remarks of Grishchuk concerning the detectability of stochastic gravitational wave background by forthcoming interferometric detectors, such as LIGO, VIRGO, GEO600, LISA. Effectively, bulk viscous dissipative processes may influence the slopes of the energy spectra of relic gravitons (generated at the time of violation of the DEC) producing an increasing with frequency in a calculable way. These slopes are crucially related to the sign of the $\rho + p$, where $\rho$ and $p$ are, respectively, the energy density and the pressure density of the cosmic fluid. The requirement that one wants expanding and inflationary universes implies that the energy density of the created gravitons cannot increase with frequency if $\rho + p \geq 0$, i.e. if the DEC is not violated. Unfortunately, previous models which exploit this idea have a phase in their evolution where the matter entropy decreases. Specifically was considered a class of solutions which correspond to a viscous fluid with an equation of state given by $p = -\rho$.

In this model the early phase (where the DEC is violated) is smoothly connected to a radiation dominated evolution. Depending upon the sign of the bulk viscosity coefficient, the entropy of the sources driving the geometry can very well decrease.

Let us discuss the class of viscous cosmologies considered in Ref. more in detail. In a flat FRW background the Einstein field equations in the presence of the bulk viscosity coefficient $\xi$ can be written as

$$H^2 = \frac{\kappa}{3} \rho,$$

(1)
$$H^2 + \dot{H} = -\frac{\kappa}{6} \left(\rho + 3P_{\text{eff}}\right),$$  
where the effective pressure is given by
$$P_{\text{eff}} = p - 3\xi H.$$  
In this case $\kappa = 8\pi G$, $H = \dot{a}/a$, $a(t)$ is the scale factor of the flat FRW metric and the overdot represents derivation with respect to the cosmic time coordinate. In order to have the notation of the paper [5] we must identify $M_0^2 = 3/\kappa$ and $p' = P_{\text{eff}}$.

The equations (11)–(3) imply the energy balance
$$\dot{\rho} + 3H(\rho + P_{\text{eff}}) = 0.$$  
In order to have a cosmological model whose evolution violates the DEC only for a finite amount of time, in Ref. [5] it is assumed that
$$\kappa \xi = \frac{2\dot{H}}{3H}. \quad (5)$$

This parametrization is very reasonable since the amount of violation of DEC is proportional to $\dot{H}$. Effectively, from Eqs. (11) and (4) we have that for any solution
$$\rho + P_{\text{eff}} = -\frac{2}{\kappa} \dot{H}, \quad (6)$$
and then a violation of the DEC implies that $\dot{H} > 0$.

In order to have a cosmology whose early phase (where the DEC is violated) is smoothly connected to a radiation dominated era, Giovannini considers the scale factor given by
$$a(t) = \left(t + \sqrt{t^2 + t_1^2}\right)^{1/2}, \quad (7)$$
and then the self-consistent solution takes the form (see Fig. 1)
$$H = \frac{1}{2\sqrt{t^2 + t_1^2}} \quad (8)$$
and
$$\kappa \xi(t) = -\frac{2t}{3(t^2 + t_1^2)}, \quad \kappa \rho(t) = \frac{3}{4(t^2 + t_1^2)} \quad (9)$$

One can immediately see by taking the limit $t \to \pm \infty$ that $\xi^-\infty(t) > 0$, $\xi^+\infty(t) < 0$ and $a_{\pm}\infty(t) \to (\pm t)^{1/2}$. So $a(t)$ at the final phase of the whole evolution has the behavior of the radiation dominated era.

Now, if we put the expression (5) into Eq. (6) we conclude that the Giovannini parametrization implies that the local equilibrium pressure is given by $p = -\rho (10)$. So for $t \to +\infty$ the asymptotic solution to Eqs. (4) and (3) is the following exact solution of the field equations (11)–(4):
$$a(t) = t^{1/2}, \quad \kappa \xi = -\frac{2}{3t}, \quad \kappa \rho = -\kappa \rho = -\frac{3}{4t^2}. \quad (10)$$

Notice that Eqs. (7) and (8) imply that
$$\kappa P_{\text{eff}} = \frac{\left(4t - 3\sqrt{t^2 + t_1^2}\right)}{4(t^2 + t_1^2)^{3/2}}, \quad (11)$$
and then $P_{\text{eff}} \to \rho/3$ for $t \to +\infty$, while the local equilibrium pressure behaves always as $p = -\rho$. This is illustrated in Fig. 1.

Let us now consider some physical aspects of the discussed viscous cosmological model. The thermodynamical entropy associated with the bulk viscosity (9) decreases for $t > 0$. This implies that, for large positive cosmic time values, the second law of thermodynamics would be violated (11). However, as was discussed in Ref. [8], this statement might be not justified since it only takes into account the matter entropy but not the entropy of the geometry itself. This implies that, for a FRW background, in the framework of a well defined extension of the second law of thermodynamics, one should include both the entropy connected with matter and the entropy connected with the FRW background. In this context, for positive cosmic time values, the decrease in the entropy of the sources may be compensated by the growth of the entropy of the FRW background.

Unfortunately, at the present such an extension of the second law of thermodynamics is ambiguous even for cosmological models which do not violate the DEC (see for example [11]). So in this paper we are interested in finding viscous cosmological models which do not need such
FIG. 2: We show the behavior of the local equilibrium pressure $p = -\rho$ (see Eq. 9), effective pressure $P_{\text{eff}}$ (see Eq. 11) and the radiation pressure $\kappa p_{\text{rad}} = 1/4t^2$ for $t \geq 0$. We have chosen $t_1 = 1$. We can see that $p$ is always negative and $P_{\text{eff}}$ behaves like $p_{\text{rad}}$ for $t \to \infty$.

FIG. 3: Some curves for the bulk viscous stress given in Eq. 15 are plotted. We set $t_1 = 1$ and $\gamma = -3/2, -1, 0$ as illustrative values. We can see that each curve has a phase where it is positive, and then it transits to a negative phase.

an extension of the second law of thermodynamics.

In the following we shall generalize the Giovannini class of solutions discussed above in order to include cosmological scenarios which always have a positive viscous coefficient and thus satisfy the standard second law of thermodynamics.

We will search for wider classes of solutions in the presence of viscous sources with a linear barotropic equation of state $p = \gamma \rho$ for the local pressure. Specifically, we are interested in studying solutions which preserve the form of the scale factor (11) (in order to keep all physical properties explaining the slopes of the energy spectra of relic gravitons and the smooth transition to the radiation dominated era) but having a positive bulk viscous stress in order to have an increasing matter entropy during all evolution of the cosmic time.

From Eqs. 1 and 2 we obtain that the bulk viscous coefficient for this kind of fluid may be written as

$$\kappa \xi = \frac{2 \dot{H}}{3H} + (\gamma + 1)H.$$  

(12)

Notice that Eq. (12) implies that the parametrization (5) has a state parameter $\gamma = -1$. So we can consider viscous cosmological models for which the state parameter $\gamma \neq -1$, in other words we shall find a self–consistent solution for the full set of Einstein field equations. The second term of (12) may be positive and then we can have a non–negative bulk viscosity for all cosmic evolution. We can see that this is possible for expanding universes (for which $H > 0$) with state parameter $\gamma > -1$.

Now from the field equations (1) and (2) we have that

$$P_{\text{eff}} = -\frac{3}{2}H^2 - 2\dot{H},$$  

(13)

and then this new class of viscous models will have the same effective pressure (11), which behaves as $P_{\text{eff}} \to \rho/3$ for $t \to +\infty$.

The new class of solutions can be written as

$$a(t) = \left( t + \sqrt{t^2 + t_1^2} \right)^{1/2}, \quad \kappa \rho(t) = \frac{3}{4(t^2 + t_1^2)},$$

(14)

$$p = \gamma \rho, \quad \kappa \xi(t) = \frac{3(\gamma + 1)\sqrt{t^2 + t_1^2} - 4t}{6(t^2 + t_1^2)}.$$  

(15)

From here we get that the viscous pressure is given by

$$\kappa \Pi = -3\kappa H \xi = \frac{4t - 3(\gamma + 1)\sqrt{t^2 + t_1^2}}{4(t^2 + t_1^2)^{3/2}},$$  

(16)

and then $P_{\text{eff}}$ takes the form of Eq. (11).

Now from Eq. (15) we can derive the general behavior of $\xi$. Note that its numerator in general can be positive, negative, or zero. It can be shown that, if

$$t_{\text{root}} = \frac{(\gamma + 1)t_1}{\sqrt{\left(\frac{1}{3} - \gamma\right)(\gamma + \frac{5}{3})}}.$$  

(17)
the bulk viscous stress is zero. This root is a real one if $-7/3 \leq \gamma \leq 1/3$. This means that, in this range of the state parameter, the bulk viscous stress has a phase where $\xi$ is positive (the matter entropy increases with time) and another phase where it is negative (the matter entropy decreases with time). In this case, if $-7/3 < \gamma < -1$, the value of $t_{\text{root}}$ is negative and, if $-1 < \gamma < 1/3$, the value of $t_{\text{root}}$ is positive. This is shown in Fig. 4. Note that the solution studied in Ref. \[15\] lies in this range so, for $-7/3 \leq \gamma \leq 1/3$, we have a class of Giovannini–like solutions with $\gamma \neq -1$. The solutions for which the matter entropy always increases with cosmic time lie out of the range $-7/3 \leq \gamma \leq 1/3$. In this case the root \[15\] does not exist and then the bulk viscosity is always negative or always positive. Effectively one can show that, for $\gamma \leq -7/3$, the bulk viscous stress is always negative, and for $\gamma \geq 1/3$ the bulk viscous stress is always positive (see Fig. 4).

In conclusion, any viscous cosmology with a constant barotropic state parameter $\gamma \geq 1/3$ will have a positive bulk viscous stress, and thus an increasing matter entropy during all cosmic evolution. This new class of solutions has the following asymptotic behavior at $t \to +\infty$: $\kappa \rho \to 3/4t^2$, $\kappa P_{\text{eff}} \to 1/4t^2$, $\kappa \xi \to (3\gamma - 1)/6t$ and always $p = \gamma \rho$. From here we conclude that the radiative viscous fluid solution is more physically acceptable than other ones since, for it, at first order, we have $P_{\text{eff}} \to \rho$, $\xi \equiv 0$ and always $p = \rho/3$. So, it should be emphasized that if $t \to +\infty$ the scale factor expands as $a \to t^{1/2}$ while the equation of state of the local equilibrium pressure $p$ may be different from $\rho/3$. However, what counts is the effective pressure for which we have that $P_{\text{eff}} \to \rho/3$ as it should.

Notice also that the sign of the effective enthalpy, i.e. $\rho + P_{\text{eff}}$, plays an important role. Effectively, we can see that the DEC is associated with Eq. \[6\], which implies that for $\dot{H} > 0$ DEC is violated, and for $\dot{H} < 0$ DEC is preserved. On the other hand, as we have stated above, the slopes of the energy spectra of relic gravitons are crucially related to the sign of the effective enthalpy. The energy density of the created gravitons increases with frequency if $\rho + P_{\text{eff}} \leq 0$, i.e. if the DEC is violated ($\dot{H} > 0$). So the considered here viscous cosmological models with a constant barotropic state parameter $\gamma \geq 1/3$, violate the DEC only for a finite amount of time. In this early stage ($t < 0$) the relic gravitons are created, and when the DEC is restored ($t > 0$) the universe exits to the standard radiation dominated stage. During all evolution of the cosmic time the bulk viscosity is positive. In this way we have shown that the possible bulk viscous influence on the slopes of energy spectra of relic gravitons makes sense even without violation of the standard second law of thermodynamics.

Lastly, notice that we have considered the same form for the background metric \[7\]; thus all calculations reported in Ref. \[7\] are preserved since they rely mostly on the specific time dependence of the scale factor, rather than on the bulk viscosity coefficient. Thus we come to the same conclusions concerning the amplification induced by the background \[6\] in the proper amplitude of the gravitational waves reported in Ref. \[7\]. It is important to stress here that these results imply that the existence of tilted spectra of relic gravitons can be connected, in the framework of general relativity, with the violation of the DEC induced by a bulk viscosity compatible with the standard second law of thermodynamics.

Finally, going back to the original Giovannini motivation \[4\], we conclude that these results indicate the possibility of the existence of growing spectra of gravitational waves at frequencies of the interferometric devices which can put bounds on the possible violation of the DEC occurring in the early universe. It is interesting to note that growing energy spectra can also be obtained in the framework of non-Einsteinian theories \[4\], \[12\].

I. ACKNOWLEDGEMENTS

The authors thank Paul Minning for carefully reading this manuscript. This work was supported by CONICYT through Grant FONDECYT No. 1051086 (MC, PM) and by Dirección de Investigación de la Universidad del Bío-Bío (MC). The financial support of Escuela de Graduados of the Universidad de Concepción is acknowledged (PM).
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