ACCELERATING AUXILIARY FUNCTION-BASED INDEPENDENT VECTOR ANALYSIS

Andreas Brendel and Walter Kellermann

Multimedia Communications and Signal Processing, Friedrich-Alexander-Universität Erlangen-Nürnberg, Cauerstr. 7, D-91058 Erlangen, Germany, e-mail: Andreas.Brendel@FAU.de

ABSTRACT

Independent Vector Analysis (IVA) is an effective approach for Blind Source Separation (BSS) of convolutive mixtures of audio signals. As a practical realization of an IVA-based BSS algorithm, the so-called AuxIVA update rules based on the Majorize-Minimize (MM) principle have been proposed which allow for fast and computationally efficient optimization of the IVA cost function. For many real-time applications, however, update rules for IVA exhibiting even faster convergence are highly desirable. To this end, we investigate techniques which accelerate the convergence of the AuxIVA update rules without extra computational cost. The efficacy of the proposed methods is verified in experiments representing real-world acoustic scenarios.

Index Terms— Independent Vector Analysis, MM Algorithm, Convergence Acceleration

1. INTRODUCTION

In daily-life situations, acoustic sources are usually observed as a mixture, e.g., multiple simultaneously active speakers in the muchquoted cocktail party scenario or a desired acoustic source mixed with interferers and background noise such as, e.g., street noise. Blind Source Separation (BSS) methods aim at separating such mixtures while using only very little information about the given scenario. As typical acoustic scenes within enclosures involve multipath propagation, Independent Component Analysis (ICA)-based approaches relying on instantaneous demixing models [3] have been extended to demixing models that represent a circular convolution by solving the instantaneous BSS problem in individual Discrete Fourier Transform (DFT) bins [4]. However, the performance of such narrow-band methods strongly relies on effective solutions for the well-known internal permutation ambiguity [5]. As a state-of-the-art method to cope with the internal permutation problem, Independent Vector Analysis (IVA) which uses a multivariate Probability Density Function (PDF) as a source model for jointly describing all DFT bins has been proposed [6].

Real-time applicability of IVA calls for fast and efficient optimization and a large variety of methods has been developed since IVA has been proposed: Starting with simple gradient and natural gradient algorithms [6], step size control mechanisms have been considered to obtain fast and stable convergence [7]. A fast fixed-point algorithm, following the ideas of FastICA [3] has been proposed in [8]. An Expectation Maximization (EM)-based optimization scheme has been proposed for IVA considering additive noise [9]. Based on the Majorize-Minimize (MM) principle [10], fast and stable update rules have been proposed using the iterative projection principle under the name Auxiliary Function IVA (AuxIVA) [11], which do not require tuning parameters such as a step size. The latter can be considered as the gold standard for optimizing the IVA cost function. For the special case of two sources and two microphones, even faster update rules based on a generalized eigenvalue decomposition have been developed [12].

In this paper, we investigate three methods for further acceleration of the AuxIVA update rules. The first method considered here is a Quasi-Newton scheme [13], which approximates the differential of the AuxIVA update rules using previous MM iterates [14]. The second approach uses a gradient-type scheme also called Overrelaxed Bound Optimization [15], which is motivated by the intuition that extending the update of the algorithm into the direction of the current MM update may provide accelerated convergence [16]. As a third approach, we use the Squared Iterative Methods (SQUAREM) technique [17, 18], which has been developed for the acceleration of EM algorithms and is based on ideas of extrapolation for increasing the convergence speed of sequences [19]. All investigated acceleration methods are shown to provide faster convergence in experiments with measured Room Impulse Responses (RIRs) than the original AuxIVA update rules at the same computational cost.

2. INDEPENDENT VECTOR ANALYSIS

In the following, we consider an array of $K$ microphones recording the convolutive mixture of $K$ acoustic sources, i.e., a determined scenario. Using the observed microphone signals in the Short-Time Fourier Transform (STFT) domain with frequency bin $f \in \{1, \ldots, F\}$ and time frame index $n \in \{1, \ldots, N\}$

$$x_{f,n} = [x_{1,f,n}, \ldots, x_{K,f,n}]^T \in \mathbb{C}^K \quad (1)$$

the demixed signals $y_{f,n} \in \mathbb{C}^K$ are obtained according to

$$y_{f,n} = [y_{1,f,n}, \ldots, y_{K,f,n}]^T = W_f x_{f,n}, \quad (2)$$

by the demixing matrix

$$W_f = [w_{1,f}, \ldots, w_{K,f}]^H \in \mathbb{C}^{K \times K}, \quad (3)$$

with $w_{k,f}$ capturing the weights of the $K$-channel MISO system producing the $f$-th DFT bin of the $k$-th demixed signal. For notational convenience, we introduce also the broadband demixed signal vector of output channel $k$

$$y_{k,n} = [y_{k,1,n}, \ldots, y_{k,F,n}]^T \in \mathbb{C}^F. \quad (4)$$

Using a broadband source model $G(x_{k,n}) = -\log p(x_{k,n})$, where $p(\cdot)$ is the multivariate PDF capturing all complex-valued STFT bins of the $k$-th output channel at time frame $n$, IVA aims at separating

\[This work was funded by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) – 282835863.\]
the sources using the demixing matrices $W_f$ of all frequency bins determined by minimizing the cost function

$$J(w) = \sum_{k=1}^{K} \hat{E} \left\{ G \left( y_{k,n} \right) \right\} - 2 \sum_{f=1}^{F} \log |\det W_f|,$$

where $\hat{E} \{ \cdot \} = \frac{1}{N} \sum_{n=1}^{N} (\cdot)$ denotes the averaging operator and

$$w = [w_{f,1}^1, \ldots, w_{f,F}^F]^T \in \mathbb{C}^{KF}$$

the concatenation of demixing vectors of all channels and frequency bins. For minimizing the cost function $J$, the MM principle is used in [11]. Hereby, an upper bound $Q$ for the cost function $J$ is constructed which is easier to optimize and fulfills the properties of majorization and tangency, i.e.,

$$J(w) \leq Q(w|w(l)) \text{ and } J(w(l)) = Q(w(l)|w(l)),$$

where $w(l)$ denotes the concatenation of all demixing vectors determined in iteration $l \in \{1, \ldots, L\}$.

The MM algorithm iterates between two steps: construction of the upper bound $Q(w|w(l))$ by the recent update $w(l)$ to ensure (7) and optimization of this upper bound to obtain $w(l+1)$. To construct the upper bound for supergaussian source models $G(\cdot)$ the following inequality has been proposed [11]

$$\hat{E} \left\{ G \left( y_{k,n} \right) \right\} \leq \frac{1}{2} \sum_{f=1}^{F} \left( w_{f,k}^H C_f^{k,l} w_{f,k}^f \right) + \text{const}.$$  

Hereby, $C_f^{k,l}$ denotes a covariance matrix of the observed signals

$$C_f^{k,l} = \hat{E} \left\{ G_r^{l,k}(r_{k,n}) x_{f,n} x_{f,n}^H \right\},$$

weighted by a factor dependent on the short-time broadband signal magnitude of source $k$:

$$r_{k,n} = \left\| y_{k,n} \right\|_2 = \left\| \sum_{f=1}^{F} (w_{f,k}^l)^H x_{f,n} \right\|_2.$$  

Application of inequality (8) to the cost function $J$ yields the upper bound $Q$, which can be minimized using the iterative projection technique [11] stipulating the following update

$$w_{k,f}^{(l+1)} = \frac{\left( W_f^{(l)} C_f^{k,l} \right)^{-1} e_k}{\sqrt{\left( e_k W_f^{(l)} C_f^{k,l} W_f^{(l)} \right)^H e_k}},$$

where $e_k$ is the canonical basis vector with a one at the $k$th position. A complete iteration for the AuxIVA update is summarized in Alg. [11]

**Algorithm 1 AuxIVA: $w^{(l+1)} = f(w^{(l)})$**

**INPUT:** $w^{(l)}$

for $k = 1$ to $K$

for $f = 1$ to $F$

$$r_{k,n} = \sqrt{\sum_{f=1}^{F} |\{ w_{f,k}^{(l)} \}^H x_{f,n} |^2} \forall n$$

$$C_f^{k,l} = \hat{E} \left\{ G_r^{(l)}(r_{k,n}) x_{f,n} x_{f,n}^H \right\}$$

$$w_{k,f}^{(l+1)} = \frac{\left( w_{f,k}^{(l)} C_f^{k,l} \right)^{-1} e_k}{\sqrt{\left( e_k W_f^{(l)} C_f^{k,l} W_f^{(l)} \right)^H e_k}}$$

end for

end for

**OUTPUT:** $w^{(l+1)}$

Hence, determining this final value $w^{(\infty)}$ corresponds to finding a root of

$$\Delta f (w) = f (w) - w = 0_{KF \times 1}.$$  

This problem can be solved by Newton’s method [14]

$$w^{(l+1)} = w^{(l)} - \Delta \Delta f (w^{(l)})^{-1} \Delta f (w^{(l)})$$

where the differential of $\Delta f (w^{(l)})$ is denoted by $d\Delta f (w^{(l)}) = df (w^{(l)}) - I_{KF}$. In the following, we present three acceleration methods which can be derived from the Newton-type update [14].

### 3. ACCELERATION SCHEMES

In the following, we present three methods for accelerating the convergence of AuxIVA. For convenience, we denote one MM map in accordance with Alg. [11] by $w^{(l+1)} = f(w^{(l)})$.

After convergence, the MM algorithm attains a fixed point

$$f \left( w^{(\infty)} \right) = w^{(\infty)}.$$  

#### 3.1. Quasi-Newton

As a first acceleration scheme, we apply the Quasi-Newton approximation of (see, e.g., [14]) to [14]. Here, the differential of the MM map $df (w^{(l)})$ is approximated by a matrix $M$

$$df (w^{(l)}) \approx M \in \mathbb{C}^{KF \times KF},$$

which is constructed by so-called secant approximations [13]

$$M \Delta f (w^{(l)}) = \Delta^2 f (w^{(l)}) \cdot$$

Hereby, we introduced the following abbreviation

$$\Delta^2 f (w^{(l)}) = f \circ f (w^{(l)}) - f (w^{(l)}),$$

and $(\cdot) \circ (\cdot)$ denotes the concatenation of functions. Multiple secant approximations, we denote their number by $q$, have to be chosen in order to obtain decent results. This can be conveniently expressed in matrix notation as

$$MU = V \quad \text{where} \quad U, V \in \mathbb{C}^{KF \times q},$$

i.e., we would obtain, e.g., $U = [f(w^{(l)}), f(w^{(l-1)})] \text{ for } q = 2$. As a solution for $M$ which minimizes its Frobenius norm and obeys (18), the following expression has been derived [14]

$$M = V \left( U^H U \right)^{-1} U^H.$$  

Insertion into (14) and application of the matrix inversion lemma yields [14]

$$w^{(l+1)} = f (w^{(l)}) - V \left( U^H U - U^H V \right)^{-1} U^H \Delta f (w^{(l)}).$$  

(20)
Note that the matrix to be inverted here is of dimension \( q \times q \), i.e., small relative to the length of \( w \), and hence the inversion is computationally cheap. One update of the Quasi-Newton algorithm is summarized in Alg. 2.

**Algorithm 2 Quasi-Newton**

**INPUT:** \( w^{(l)} \)
\[
\Delta f \left( w^{(l)} \right) = f \left( w^{(l)} \right) - f \left( w^{(l-1)} \right)
\]
Constrain \( V \) and \( U \)
\[
w^{(l+1)} = f \left( w^{(l)} \right) - U^H(UU^H)^{-1}U^H\Delta f \left( w^{(l)} \right)
\]
**OUTPUT:** \( w^{(l+1)} \)

### 3.2. Gradient Approximation

By approximating the differential of (13) by a scaled identity matrix
\[
d\Delta f \left( w^{(l)} \right) \approx \frac{1}{\mu} I_{KF}
\]
we obtain with (14) a gradient-type algorithm with step size \( \mu \leq -1 \)
\[
w^{(l+1)} = w^{(l)} - \mu \Delta f \left( w^{(l)} \right)
\]
which operates on the results of the MM iterations. Note that a step size of \( \mu = -1 \) corresponds to the original MM algorithm and values above \(-1\) will slow down convergence. There are many options for the choice of \( \mu \) (see, e.g., [18]), where line search methods [13] would be a natural choice. However, the calculation of an adaptive step size adds significant computational load to the algorithm, e.g., caused by the evaluation of the cost function \( f \) for line search approaches. Hence, we will use a fixed step size here.

### 3.3. SQUAREM

In the following, we discuss the SQUAREM method, which has been introduced and extensively used for the acceleration of EM algorithms [17][18]. Let denote \( z^{(l)} \) the outcome of one gradient update according to (23) with step size \( \alpha \)
\[
z^{(l)} = w^{(l)} - \alpha \Delta f \left( w^{(l)} \right)
\]
The main idea of SQUAREM is to square this update, i.e., to subsequently perform another gradient update to obtain the next iterate
\[
w^{(l+1)} = z^{(l)} - \alpha \Delta f \left( z^{(l)} \right)
\]
\[
w^{(l+1)} = w^{(l)} - \alpha \Delta f \left( w^{(l)} \right) - \alpha \left[ f \left( w^{(l)} \right) - \alpha \Delta^2 f \left( w^{(l)} \right) \right] \ldots
\]
\[
\ldots \left[ f \left( w^{(l)} \right) - \alpha \Delta f \left( w^{(l)} \right) \right]
\]
\[
w^{(l+1)} = w^{(l)} - 2\alpha \Delta f \left( w^{(l)} \right) + \alpha^2 \Delta g \left( w^{(l)} \right)
\]
where we introduced the term
\[
\Delta g \left( w^{(l)} \right) = \Delta^2 f \left( w^{(l)} \right) - \Delta f \left( w^{(l)} \right)
\]
One iteration of the SQUAREM algorithm is summarized in Alg. 3

**Algorithm 3 SQUAREM**

**INPUT:** \( w^{(l)} \)
\[
\Delta f \left( w^{(l)} \right) = f \left( w^{(l)} \right) - f \left( w^{(l-1)} \right)
\]
\[
\Delta g \left( w^{(l)} \right) = \Delta^2 f \left( w^{(l)} \right) - \Delta f \left( w^{(l)} \right)
\]
\[
\alpha = -\frac{\|\Delta g \left( w^{(l)} \right) \|^2}{\|\Delta f \left( w^{(l)} \right) \|^2}
\]
\[
w^{(l+1)} = w^{(l)} - \alpha \Delta f \left( w^{(l)} \right) + \alpha^2 \Delta g \left( w^{(l)} \right)
\]
**OUTPUT:** \( w^{(l+1)} \)

### 4. EXPERIMENTS

In the following, we discuss the practical realization of the acceleration methods introduced above and present experimental results. For the Quasi-Newton method, we constructed the matrices \( U \) and \( V \) representing the secant constraints by using three values for \( \Delta f \left( w^{(l)} \right) \) and two of \( \Delta^2 f \left( w^{(l)} \right) \) prior to the current iteration, i.e., we computed only one MM update in each iteration. Using two MM updates per iteration as suggested in [14] did not yield better results in our experiments. The step size \( \mu \) of the gradient algorithm is chosen to be constant for simplicity. For the choice of a step size, convergence speed has to be traded off against stability. Here, a value of \( \mu = -1.8 \) showed good results in our experiments. The step size \( \alpha \) for the SQUAREM algorithm is chosen to be [18]
\[
\alpha = -\frac{\|\Delta g \left( w^{(l)} \right) \|^2}{\|\Delta f \left( w^{(l)} \right) \|^2}
\]
which is a quite common choice for the SQUAREM algorithm [20]. This expression for the step size compares the relative change in the cost function \( f \) by applying the MM map once with the corresponding change by applying it twice and weighted the first-order \( \Delta f \left( w^{(l)} \right) \) and second-order update \( \Delta g \left( w^{(l)} \right) \) accordingly. For the experimental evaluation, we simulated microphone signals by convolving speech signals randomly chosen from a set of 4 male and 4 female speech signals of about 10 sec duration with RIRs measured in three different rooms: two meeting rooms \((T_{60} = 0.2 \text{ s} \text{ and } T_{60} = 0.4 \text{ s}) \) and a seminar room \((T_{60} = 0.9 \text{ s}) \). The RIRs are measured with a linear microphone array with 4.2 cm spacing between the microphones. Two configurations of RIRs have been measured in the mentioned enclosures at 1 m and 2 m distance from the microphone array: 40°/140° and 40°/90°/140° w.r.t. the array axis. As we consider only determined scenarios, the number of sources and microphones was equal in all measurements. White Gaussian noise was added to obtain an Signal-to-Noise Ratio (SNR) of 30 dB at the microphones.

The microphone signals have been transformed into the STFT domain by employing a Hamming window of length 2048 and 50% overlap at a sampling frequency of 16 kHz. The performance of the algorithms has been measured by the Signal-to-Distortion Ratio (SDR), Signal-to-Interference Ratio (SIR) and Signal-to-Artifact Ratio (SAR) w.r.t. the unprocessed signals [21]. Note that these performance measures are indirect indicators for the convergence of the algorithm, as they do not express the costs to be minimized. However, they can be seen as a strong indicator for the separation quality as experienced by a user. We used a Laplacian source model, i.e., \( G(r_{h,n}(w_k)) = r_{h,n}(w_k) \), which is a common choice for IVA applied to audio signals [6][11]. The results of the experiments described above are shown in Fig. 4 in terms of SDR and SAR. Results...
for the improvement of the SAR are omitted due to space constraints. However, the SAR improvement was roughly the same for the investigated methods. Fig. 1 shows the results for scenarios comprising 2 sources and 2 microphones and 3 sources and 3 microphones. All three different rooms ($T_{60} = 0.2$ sec, 0.4 sec, 0.9 sec) and the two different source-array distances (1 m, 2 m) have been evaluated by repeating the experiment 20 times for each configuration, where the source signals are drawn randomly from a set of four male and four female speech signals. The mean performance values from these different acoustic conditions are shown for the discussed algorithms over runtime in Fig. 1.

The SQUAREM-based method converged after roughly 15 iterations, all other methods after about 30 iterations. To take into account additional computational cost of more advanced algorithms which increase the convergence rate per iteration the runtime per iteration has been considered in order to obtain a fair comparison. Here, it turned out that the runtime is dominated by the evaluation of the MM map and the additional runtime caused by operations added to the MM map was negligible. The runtime per iteration for AuxIVA, the gradient-based and the Quasi-Newton-based method was roughly $0.16$ sec for two sources and $0.27$ sec for three sources on average. Due to the second required MM map the SQUAREM method needed roughly twice as much runtime per iteration. These observations have been incorporated into Fig. 1 by showing the performance of the algorithms in terms of runtime of the algorithm. It can be observed that all algorithms converge to similar final values with a slight advantage for the acceleration methods. However, all acceleration schemes provide significantly faster convergence than AuxIVA itself. The gradient-type method and the Quasi-Newton method, both using only a single MM map, showed similar convergence speed. The SQUAREM method based on two MM maps outperforms these methods especially for the three-source case and provides SDR and SIR improvements in the early convergence phase which are higher by several dB compared to the AuxIVA results at the same runtime requirement.

5. CONCLUSIONS

We investigated the application of three different schemes for the acceleration of the convergence of the AuxIVA update rules. We showed that all three methods increased the convergence speed in terms of SDR and SIR improvement at the same runtime requirements as AuxIVA. The gradient-based approach represents a simple but effective modification of the original algorithm but requires the selection of a suitable step size. In our experiments, a fixed step size showed promising results, but future work should investigate mechanisms to choose this step size automatically. The Quasi-Newton method performed similarly as the gradient-based method and was slightly outperformed by the SQUAREM method.
Future work will include an in-depth investigation of other acceleration methods (e.g., [22]). Also the application of such acceleration schemes to other BSS algorithms, which suffer from slow convergence, e.g., Multichannel NMF (MNMF) [23] and TRIPLE-N Independent component analysis for CONvolutive mixtures (TRINICON) [24], will be part of future work.

6. REFERENCES

[1] E. Vincent, T. Virtanen, and S. Gannot, Eds., Audio source separation and speech enhancement, John Wiley & Sons, Hoboken, NJ, 2018.

[2] S. Makino, T.-W. Lee, and H. Sawada, Eds., Blind speech separation, Signals and communication technology. Springer, Dordrecht, 2007.

[3] A. Hyvärinen, J. Karhunen, and E. Oja, Independent component analysis, J. Wiley, New York, 2001.

[4] P. Smaragdis, “Blind Separation of Convolved Mixtures in the Frequency Domain,” Neurocomputing Journal, vol. 22, pp. 21–34, 1998.

[5] H. Sawada, R. Mukai, S. Araki, and S. Makino, “A Robust and Precise Method for Solving the Permutation Problem of Frequency-Domain Blind Source Separation,” IEEE Transactions on Speech and Audio Processing, vol. 12, no. 5, pp. 530–538, Sept. 2004.

[6] T. Kim, H. T. Attias, S.-Y. Lee, and T.-W. Lee, “Blind Source Separation Exploiting Higher-Order Frequency Dependencies,” IEEE Transactions on Audio, Speech and Language Processing, vol. 15, no. 1, pp. 70–79, Jan. 2007.

[7] Y. Liang, S. M. Naqvi, and J. A. Chambers, “Adaptive step size independent vector analysis for blind source separation,” in 2011 17th International Conference on Digital Signal Processing (DSP), Corfu, Greece, July 2011.

[8] I. Lee, T. Kim, and T.-W. Lee, “Fast fixed-point independent vector analysis algorithms for convolutive blind source separation,” Signal Processing, vol. 87, no. 8, pp. 1859–1871, Aug. 2007.

[9] J. Hao, I. Lee, T.-W. Lee, and T. J. Sejnowski, “Independent Vector Analysis for Source Separation Using a Mixture of Gaussians Prior,” Neural Computation, vol. 22, no. 6, pp. 1646–1673, June 2010.

[10] D. R. Hunter and K. Lange, “A Tutorial on MM Algorithms,” The American Statistician, vol. 58, no. 1, pp. 30–37, Feb. 2004.

[11] N. Ono, “Stable and fast update rules for independent vector analysis based on auxiliary function technique,” in IEEE Workshop on Applications of Signal Processing to Audio and Acoustics (WASPAA), New Paltz, NY, USA, Oct. 2011, pp. 189–192.

[12] N. Ono, “Fast Stereo Independent Vector Analysis and its Implementation on Mobile Phone,” in International Workshop on Acoustic Signal Enhancement (IWAENC), Aachen, Germany, Sept. 2012.

[13] J. Nocedal and S. J. Wright, Numerical optimization, Springer series in operations research. Springer, New York, 2nd ed. edition, 2006.

[14] H. Zhou, D. Alexander, and K. Lange, “A quasi-Newton acceleration for high-dimensional optimization algorithms,” Statistics and Computing, vol. 21, no. 2, pp. 261–273, Apr. 2011.

[15] R. Salakhutdinov and Sam T. R., “Adaptive Overrelaxed Bound Optimization Methods,” in Proceedings of the Twentieth International Conference on Machine Learning (ICML-2003), Washington, DC, USA, 2003.

[16] K. Lange, “A gradient algorithm locally equivalent to the EM algorithm,” Journal of the Royal Statistical Society: Series B (Statistical Methodology), vol. 57, no. 2, pp. 425–437, Jan. 1995.

[17] R. Varadhan and Ch. Roland, “Squared Extrapolation Methods (SQUAREM): A New Class of Simple and Efficient Numerical Schemes for Accelerating the Convergence of the EM Algorithm,” Working Paper 63, Johns Hopkins University, Nov. 2004.

[18] R. Varadhan and C. Roland, “Simple and Globally Convergent Methods for Accelerating the Convergence of Any EM Algorithm,” Scandinavian Journal of Statistics, vol. 35, no. 2, pp. 335–353, June 2008.

[19] C. Brezinski and M. Redivo Zaglia, Extrapolation methods: theory and practice, Number 2 in Studies in computational mathematics. North-Holland, New York, N.Y., U.S.A, 1991.

[20] L. Zhao, J. Song, P. Babu, and D. P. Palomar, “A Unified Framework for Low Autocorrelation Sequence Design via Majorization-Minimization,” IEEE Transactions on Signal Processing, vol. 65, no. 2, pp. 438–453, Jan. 2017.

[21] E. Vincent, R. Gribonval, and C. Févotte, “Performance measurement in blind audio source separation,” IEEE Transactions on Audio, Speech and Language Processing, vol. 14, no. 4, pp. 1462–1469, July 2006.

[22] A. Berlinet and C. Roland, “Parabolic acceleration of the EM algorithm,” Statistics and Computing, vol. 19, no. 1, pp. 35–47, Mar. 2009.

[23] H. Sawada, H. Kameoka, S. Araki, and N. Ueda, “Multichannel Extensions of Non-Negative Matrix Factorization With Complex-Valued Data,” IEEE Transactions on Audio, Speech, and Language Processing, vol. 21, no. 5, pp. 971–982, May 2013.

[24] H. Buchner, R. Aichern, and W. Kellermann, “A generalization of blind source separation algorithms for convolutive mixtures based on second-order statistics,” IEEE Transactions on Speech and Audio Processing, vol. 13, no. 1, pp. 120–134, Jan. 2005.