DYNAMICAL SUPERSYMMETRY BREAKING—
WHY AND HOW*

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This theoretical review is intended to give non-theorists a flavor of the ideas driving the current efforts to experimentally find supersymmetry. We discuss the main reasons behind the expectation that supersymmetry may be “just around the corner” and may be discovered in the near future. We use simple quantum-mechanical examples to illustrate the concept—and the power—of supersymmetry, the possible ways to break supersymmetry, and the dynamical generation of small scales. We then describe how this theoretical machinery helps shape our perception of what physics beyond the electroweak scale might be.

1. Introduction

Since the invention of supersymmetry more than 25 years ago, physicists have been fascinated by the possibility that this new fundamental space-time symmetry might govern physics at short distances. Theorists have devoted their research to both the mathematical aspects and the applications of supersymmetry to elementary particle physics. At the same time, experimentalists (not without the encouragement of their theoretical colleagues) have been continually searching for supersymmetry at increasingly higher energy scales, with enduring negative results. Will this process of continually probing new energy scales, looking for supersymmetry, and not finding it (and consequently setting new, higher benchmarks for its discovery) continue indefinitely? At which point are we willing to give up the idea that supersymmetry has anything to do with experimentally accessible particle physics? We believe that currently the search for supersymmetry has reached an im-

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*As of the date of submission of this article, 8447 publications on supersymmetry—both theoretical and experimental—were listed in the SLAC SPIRES-HEP database.
important threshold. Within the next 10 years—with the advent of the Large Hadron Collider—we will have the answer to the question: “Is supersymmetry relevant for physics at the electroweak scale?”

In this article, we review the main properties of supersymmetry that make it an attractive possibility for physics beyond the standard model (Sections 2 and 3). We explain why we believe that if supersymmetry is relevant for electroweak scale physics, it must be dynamically broken. Some quantum-mechanical examples illustrating how supersymmetry can break are discussed in Section 4. We point out that the central theoretical problem of extending what has become known as the Minimal Supersymmetric Standard Model (MSSM) to a consistent theoretical framework is the mechanism of supersymmetry breaking. We describe the current theoretical ideas of how supersymmetry breaks in Section 5. We conclude by stressing the importance of experimental input in sharpening these ideas: if supersymmetry is found and the spectrum of superparticles measured, we will get clues pointing towards the likely mechanism of supersymmetry breaking.

2. What is supersymmetry?

Supersymmetry is a new space-time symmetry interchanging bosons and fermions. In order to clarify this concise definition, we will introduce supersymmetry via the simplest supersymmetric quantum-mechanical system, the “supersymmetric oscillator.” The supersymmetric oscillator is a simple generalization of the one dimensional harmonic oscillator. In this section, we will discuss its properties in some detail (as an aside, we will see that the supersymmetric oscillator describes a well-known physical system: that of an electron moving in a constant, homogeneous magnetic field). Many of the properties of the supersymmetric oscillator do, in fact, generalize to quantum field theory and are behind the reasons that make supersymmetry an attractive possibility for physics beyond the electroweak scale. We will end this section by concluding that supersymmetry, if it is relevant for elementary particle physics, can not be realized in its simplest form—as it is in the supersymmetric oscillator—but rather has to be broken at an energy scale at order or above the electroweak scale.

Supersymmetry is a symmetry that relates bosons and fermions. The simplest quantum-mechanical system that leads to the introduction of bosons is the harmonic oscillator. Recall that the harmonic oscillator describes the motion of a particle in one spatial dimension (with coordinate denoted by $x$) in a quadratic potential $V(x) = \omega^2 x^2/2$. The stationary states of the particle in the harmonic well is described by its wave function, $\Psi(x)$, which obeys the Schrödinger equation $\hat{H}\Psi(x) = E\Psi(x)$. The Hamiltonian has the form (we use units where the mass of the particle equals one):

$$\hat{H}_B = -\frac{\hbar^2}{2} \frac{d^2}{dx^2} + V(x) = \hbar \omega \left( b^\dagger b + \frac{1}{2} \right).$$

(1)

The second equality follows from the substitution $b = \sqrt{\frac{\hbar}{2\omega}} \frac{d}{dx} + \sqrt{2\hbar} x$, $b^\dagger = \sqrt{\frac{\hbar}{2\omega}} \frac{d}{dx} - \sqrt{2\hbar} x$. \[\]
The commutation relation \( [\frac{d}{dx}, x] = 1 \) implies that the operators \( b^\dagger, b \) obey the commutation relation:

\[
[ b, b^\dagger ] = b b^\dagger - b^\dagger b = 1 .
\tag{2}
\]

The spectrum of allowed energy levels is labeled by a single quantum number, \( n = 0, 1, 2, \ldots \), and is given by the well-known formula:

\[
E_n^B = \hbar \omega \left( n + \frac{1}{2} \right), n = 0, 1, 2, \ldots .
\tag{3}
\]

The spectrum (3) of the bosonic oscillator is shown on Fig. 1.

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Fig. 1. The energy levels of the bosonic oscillator (in units of \( \hbar \omega \)).

The commutation relation (2) is typical for systems of bosons; the operators \( b^\dagger \) and \( b \) are called bosonic creation and annihilation operators, respectively. One can interpret the ground state state \( |0\rangle \) of the harmonic oscillator as a state with no quanta, the state \( |1\rangle \sim b^\dagger |0\rangle \) as a state with one quantum, and, generally, the state \( |n\rangle \sim (b^\dagger)^n |0\rangle \) as a state with \( n \) quanta. The operator \( b^\dagger \) is called a bosonic creation operator, because its action of a state with \( n \) quanta creates an additional quantum, i.e. a state with \( n + 1 \) quanta. Similarly, the operator \( b \) decreases the occupation number and annihilates a quantum from the state upon which it acts.

The interpretation of the harmonic oscillator in this “second quantized” representation is very useful when describing the quantization of, e.g., the Maxwell field. Later in this section, we will use this interpretation to generalize the supersymmetric oscillator to quantum field theory.

Now we can go on, and, by analogy with the bosonic oscillator, introduce what can be called the “fermionic oscillator.” The definition below may initially seem somewhat formal, but later in this section we will discuss a physical example. It will become clear that the fermionic oscillator describes many quantum mechanical physical systems, in particular systems with two energy levels (spin up/spin down).
operator is given by (1), with the bosonic creation and annihilation operators obeying the commutation relation (2). Similarly one can define the fermionic oscillator by its Hamiltonian:

$$\hat{H}_F = \hbar \omega \left( f^\dagger f - \frac{1}{2} \right),$$  \hspace{1cm} (4)

where the fermion creation ($f^\dagger$) and annihilation ($f$) operators obey the *anticommutation* relation

$$\{f, f^\dagger\} = f f^\dagger + f^\dagger f = 1,$$  \hspace{1cm} (5)

as well as

$$f^2 = (f^\dagger)^2 = 0.$$  \hspace{1cm} (6)

The latter property (called nilpotence) should be reminiscent of the Pauli principle—no two fermions can occupy the same quantum state. One can be even more explicit and use the following representation for the fermionic creation and annihilation operators by two by two matrices:

$$f = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad f^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}. \hspace{1cm} (7)$$

By matrix manipulation, it is easy to see that the operators (7) obey (5) and (6). The representation (7) also makes it easy to see that the Hamiltonian of the fermionic oscillator, (4), describes a two-level system with energy levels

$$E^F_k = \hbar \omega \left( k - \frac{1}{2} \right), \quad k = 0, 1.$$  \hspace{1cm} (8)

The spectrum (8) of the fermionic oscillator is shown on Fig. 2.

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**Fig. 2.** The energy levels of the fermionic oscillator (in units of $\hbar \omega$).

The operators $f^\dagger$ and $f$ can be interpreted as creation and annihilation operators, similar to the interpretation given to $b^\dagger$ and $b$ in the bosonic oscillator. The difference is that the anticommuting nature of $f^\dagger, f$, and their nilpotence (6) imply
that it is impossible to create two fermion quanta in the same state by applying \( f^\dagger \) twice. Thus (5) and (6) ensure that the fermionic oscillator obeys the Pauli principle. Correspondingly, the quantum number \( k = 0, 1 \) can be called the fermion occupation number.

We are now ready to define the supersymmetric oscillator. It is simply the sum of the bosonic and fermionic oscillators with equal spacing \( \hbar \omega \) of energy levels, zero-point energy \( \hbar \omega / 2 \) for the bosonic, and \(-\hbar \omega / 2 \) for the fermionic oscillator (for any other choice of the fermion zero point energy, the resulting oscillator will not be supersymmetric; see the discussion below). The Hamiltonian of the supersymmetric oscillator will therefore be:

\[
\hat{H}_{\text{SUSY}} = \hat{H}_F + \hat{H}_B = \hbar \omega \left( b^\dagger b + f^\dagger f \right).
\]  

(9)

The bosonic and fermionic creation and annihilation operators obey the commutation (2) and anticommutation (5) relations, respectively (while all bosonic operators commute with all fermionic operators), and the fermionic operators obey (6). The Hamiltonian (9) can be written in several equivalent forms using the (anti-)commutation relation (5) and (6):

\[
\hat{H}_{\text{SUSY}} = \hbar \omega \left( b^\dagger b + f^\dagger f \right)
= \hbar \omega \left( b^\dagger f + f^\dagger b \right)^2
= \hbar \omega \left( Q^\dagger + Q \right)^2
= \hbar \omega \{ Q^\dagger, Q \}.
\]  

(10)

Here we have defined the operators

\[
Q = b^\dagger f, \quad Q^\dagger = f^\dagger b.
\]  

(11)

These operators are called, for reasons to become clear below, supersymmetry generators. It is easy to see from (11) that they obey the same kind of property (nilpotence) as \( f^\dagger \) and \( f \): \( Q^2 = (Q^\dagger)^2 = 0 \) (this was essential for going from the third to the last line in (10)). The generators of supersymmetry \( Q, Q^\dagger \) can be easily seen, using (10) and (6), to commute with the Hamiltonian:

\[
[Q, H_{\text{SUSY}}] = [Q^\dagger, H_{\text{SUSY}}] = 0.
\]  

(12)

By absorbing the factor \( \hbar \omega \) in a redefinition of the Hamiltonian, their anticommutation relation can be written as

\[
\{ Q^\dagger, Q \} = \hat{H}.
\]  

(13)

The energy spectrum of \( H_{\text{SUSY}} \) is simply the sum of the energy spectra of the bosonic (Fig. 1) and fermionic (Fig. 2) oscillators. The energy levels are labeled by the two quantum numbers, \( (n, k) \), with \( n = 0, 1, 2, \ldots \) and \( k = 0, 1 \) (the bosonic and fermionic occupation numbers inherited from the bosonic and fermionic oscillator):

\[
E_{n,k}^{\text{SUSY}} = \hbar \omega \left( n + k \right).
\]  

(14)
Dynamical Supersymmetry Breaking

The spectrum (14) of the supersymmetric oscillator is shown on Fig. 3. All energy levels in the spectrum are doubly degenerate—an energy level with \( n + 1 \) bosons and no fermions is degenerate with the level with \( n \) bosons and one fermion. This Bose-Fermi degeneracy is due to supersymmetry: we noted above that the operators \( Q \) and \( Q^\dagger \) (13) commute with the Hamiltonian (12). As usual, the existence of operators that commute with the Hamiltonian indicates that the energy levels are degenerate. The novelty in the case of supersymmetry is that the operators that generate the symmetry (i.e. commute with the Hamiltonian) themselves obey anticommutation (rather than the usual commutation) relations and thus relate bosons to fermions: we see that the operator \( Q^\dagger = f^\dagger b \) (which creates a fermion and annihilates a boson) brings us from an energy level labeled by the bosonic and fermionic occupation numbers \( (n+1,0) \) to the \( (n,1) \) level; similarly, the operator \( Q = b^\dagger f \) creates a boson and annihilates a fermion and relates the \( (n,1) \) level to the \( (n+1,0) \) level. The degenerate levels related by the action of the supersymmetry generators are said to form supermultiplets.

In addition to the degeneracy of the energy levels of bosons and fermions, supersymmetric systems have other general properties that are important in applications of supersymmetry to elementary particle physics. Since the Hamiltonian is a total square (see the second equation in (10)) of a hermitean operator, all its states have nonnegative energies. In particular, the ground state (if supersymmetry is unbroken) always has vanishing energy (see Fig. 3). The vanishing of the zero point energy occurs because of the particular value of the zero point energy of the fermionic oscillator (1) which was forced upon us by supersymmetry—if the zero point energies of the bosons and fermions did not cancel, we could not have written \( \hat{H}_B + \hat{H}_F \) in the form (14). We will see in the next section that the cancellation of the zero point energies of the bosons and fermions due to supersymmetry is one of the central reasons to believe that supersymmetry may be present at energy scales of the order of the electroweak scale.
The discussion in this section has had so far a rather formal character. To illustrate the fact that supersymmetry exists in the physical world, we note that a simple physical system has energy levels that precisely match those of the supersymmetric oscillator. One can recognize on Fig. 3 the Landau levels of an electron moving in a constant, homogeneous magnetic field $H$, upon identifying $\omega = |e|H/(me)$ (here $e, m_e$ are the electron charge and mass, and $c$ is the speed of light). It is well known that upon quantizing the classical Larmor orbits of an electron in a magnetic field, only a discrete set of radii is allowed, labeled by an integer radial quantum number $n = 0, 1, 2, ...$. The quantum number $k = 0, 1$, on the other hand, denotes the projection of the electron’s spin on the direction of the magnetic field. Supersymmetry, therefore, relates the energy of an electron in the $n$-th Landau level with spin along the magnetic field to the energy of an electron in the $n + 1$-st level with spin opposite the magnetic field (see Fig. 4). We should also note that this nonrelativistic supersymmetry is only approximate: the degeneracy of the electron Landau levels due to supersymmetry is spoiled by many effects (e.g. the electron’s anomalous magnetic moment) that are not taken into account in the nonrelativistic Schrödinger approach.

![Fig. 4. The allowed orbits of an electron in a constant homogeneous magnetic field $H$. The degeneracy of the two neighboring levels—the $n-1$-st Landau level with electron spin along ($k = 1$) the field and the $n$-th level with spin opposite ($k = 0$) the field—is due to supersymmetry. The action of the supersymmetry generators $Q$ and $Q^\dagger$ is shown by the corresponding arrows.](image)

As emphasized in the beginning, the supersymmetric oscillator has most of the elements needed to generalize supersymmetry to quantum field theory. To this end, recall that the theory of photons—the quantized free Maxwell field—can be considered as an infinite collection of simple harmonic oscillators. Upon putting classical Maxwell electrodynamics in a large box and going to a Fourier representation, one observes that the field equations of the various Fourier (i.e. momentum) modes decouple. The equation for each momentum mode describes a simple harmonic oscillator (with momentum playing the role of the coordinate). The quantum state
of the momentum mode with momentum $\vec{k}$ and frequency $\omega(\vec{k}, \lambda) = \sqrt{\vec{k}^2 + m^2}$ is described by a single quantum number $n_{\omega(\vec{k}, \lambda)}$—the number of photons with momentum $\vec{k}$ and polarization $\lambda$.

We can now repeat this quantization procedure word for word starting with the supersymmetric oscillator. A supersymmetric field theory will be just the infinite sum of supersymmetric oscillators. The spectrum of the theory is given by the sum of the spectra of Fig. 3 for each momentum mode and polarization—in Fig. 3 we have to replace the frequency $\omega$ with the frequency appropriate for the given momentum mode $\omega \to \omega(\vec{k}, \lambda)$. Thus, in supersymmetric Maxwell electrodynamics, as in the supersymmetric oscillator, a single-photon state with a given momentum and polarization $(1, \omega(\vec{k}, \lambda); 0, \omega(\vec{k}, \lambda))$ is, due to supersymmetry, degenerate with the state $(0, \omega(\vec{k}, \lambda); 1, \omega(\vec{k}, \lambda))$ of a fermion of spin 1/2 in a state with given momentum and polarization. More generally, a state with $n$ photons of momentum $\vec{k}$ and polarization $\lambda$, $(n, \omega(\vec{k}, \lambda); 0, \omega(\vec{k}, \lambda))$ is degenerate with the state with $n - 1$ photons and one spin-1/2 fermion $(n - 1, \omega(\vec{k}, \lambda); 1, \omega(\vec{k}, \lambda))$. This new spin-1/2 state is the supersymmetric partner of the photon, known in the supersymmetric nomenclature as the photino. In supersymmetric Maxwell electrodynamics the photino has the same mass and quantum numbers as the photon, but half integer spin.

The construction of supersymmetric field theory can be generalized along the above lines to include interactions, as well as particles of various spins. Doing this in any detail requires the introduction of new techniques and would take us far from the objective of this article (the interested reader can consult refs. 2,5). However, it should be clear by now that, similar to the construction of the supersymmetric oscillator, one can “supersymmetrize” the theory that describes all known elementary particles: the standard model of elementary particle theory. Similar to the photon and photino, all known elementary particles acquire supersymmetric partners, which have the same quantum numbers (charges, masses, etc.) as the ordinary particles. The superpartner of the electron is the spin-0 boson called the selectron; the quarks and the other leptons acquire spin-0 partners called squarks and sleptons, respectively; the gluons—the spin-1/2 gluinos, the $W$ and $Z$ bosons—the spin-1/2 winos and zinos, etc. The whole nomenclature of what has become known as the MSSM (the “minimal supersymmetric standard model”) can be found in refs. 3,4,5.

Once we include interactions, the supersymmetric partners of the ordinary quarks and leptons, the squarks and sleptons introduced above, will acquire interactions similar to those of the quarks and leptons (since now not only the harmonic terms in the Hamiltonian, but the nonlinear interaction terms of the bosons and fermions will be related by supersymmetry). For example, the spin-0 squarks and sleptons couple to the photon and the $Z$-boson in the same way as the quarks and leptons: the corresponding Feynman graphs for electrons and selectrons are shown on Fig. 5. If supersymmetry was exact, the squarks and sleptons would have the same mass as $m$.

\[\text{Hereafter we put } \hbar = c = 1; \text{ for Maxwell electrodynamics we take } m = 0.\]
their quark and lepton superpartners. They would contribute (through the upper graph in Fig. 5) to the decay width of the $Z$ an amount similar to the contribution of the nonsupersymmetric particles. The precision measurement at LEP determining the width of the $Z$, however, could not have agreed so spectacularly with the prediction of the nonsupersymmetric standard model (for $\Gamma_Z^{\text{exp}} = 2.4946 \pm 0.0027$ GeV, while the theoretical prediction without supersymmetric particles is $\Gamma_Z^{\text{th}} = 2.4972$ GeV).

![Diagram of electron supermultiplet](image)

Fig. 5. Contributions of the electron supermultiplet to the decay width of the $Z$ boson in the supersymmetrized standard model (the supersymmetric partners of the electrons are denoted by $\tilde{e}$).

The considerations from the previous paragraph force us to conclude that supersymmetry, when applied to the known elementary particles, can not be realized as in the supersymmetric oscillator—the spectrum of the known particles does not look anything like Fig. 3. From our everyday experience (and from many low-energy experiments) we can conclude that the superpartners, if they exist, can not be degenerate in mass with the ordinary particles (the strongest bound on their mass comes from the LEP experiments mentioned above). Does this mean that one must right away abandon the idea of supersymmetry? It is certainly possible that supersymmetry is a mathematical construction, remarkable for its inherent beauty but irrelevant for the physics of the elementary particles at energies near the electroweak scale. It is also possible, however, that supersymmetry is just around the corner, and a rich new spectrum of supersymmetric particles is waiting to be discovered just above the presently attainable energy. In the next section, we will review the theoretical arguments that make one believe that this might be the case.

Before going on to that, recall the definition of supersymmetry that we gave at the beginning of this section. We hope to have elucidated the second part of the definition—that supersymmetry relates bosons to fermions—by means of our quantum mechanical example. However, we did not give any evidence that super-
symmetry is a space-time symmetry. To see that this is the case, consider the defining relation (13) of the supersymmetry generators: \( \{ Q, Q^\dagger \} = \hat{H} \). To generalize this relation to elementary particle theory, one has to promote it to a relativistically invariant relation. In a relativistically invariant theory, the Hamiltonian is the zero component of the four-momentum vector, while an anticommuting object (such as the Dirac spinor field) has to carry spinor indices under the Lorentz group. Heuristically, one expects to make the following replacements in (13):

\[
H = P_0 \rightarrow P_\mu \\
Q \rightarrow Q_\alpha \\
Q^\dagger \rightarrow Q^\dagger_\dot{\alpha}.
\]

In (15), \( P_\mu \) denotes the four-momentum vector, while \( \alpha, \dot{\alpha} \) denote spinor indices (under the \((0, \frac{1}{2}), (\frac{1}{2}, 0)\) representations of the Lorentz group, respectively). After these replacements, the anticommutation relation (13) can be written in a Lorentz covariant form

\[
\{ Q_\alpha, Q^\dagger_\dot{\alpha} \} = -2i \sigma^\mu_{\alpha\dot{\alpha}} P_\mu,
\]

known as (part of) the \( N = 1 \) supersymmetric algebra in 3 + 1 dimensions. (We stress again that our goal here is to give a flavor of the subject and familiarize the reader with the main ideas; we only note that \( \sigma^\mu_{\alpha\dot{\alpha}} \) are related to the \( \gamma \) matrices, for details see e.g. 2, 5.)

The relation (16) of the supersymmetry algebra implies that supersymmetry is a space-time symmetry—the anticommutator of two supersymmetry transformations (those generated by \( Q \) and \( Q^\dagger \)) is a translation in space time generated by the momentum \( P_\mu \) (eq. 16) is often interpreted by saying that supersymmetry is a “square root of momentum”). The supersymmetry algebra (16) also implies that if we want to construct a supersymmetric theory where supersymmetry is a gauge symmetry (i.e. the parameter of the transformation depends on the space-time point), we will necessarily have to gauge space-time translation (since, for consistency, both sides of (16) will need to represent local transformations). Since gauging space time translations is equivalent to constructing general relativity, gauging supersymmetry therefore implies general relativity. This is a hint that there might be some deep connection between the structure of space time and supersymmetry.

3. Why is (broken) supersymmetry relevant in high-energy physics?

From the discussion in the previous section, we learned that supersymmetry, at least at energies below \( \sim 100 \) GeV, cannot be linearly realized (i.e. lead to spectra like that on Fig. 3). Such a spectrum, with the superpartners degenerate in mass with the ordinary particles, is in blatant contradiction with all known data. All is not lost, however, and there is still hope that supersymmetry might be relevant in one form or another to physics at energy scales accessible to the next generation of colliders. This hope rests mainly on various theoretical ideas, which we review in this section.
At the end of the previous section, we saw that supersymmetry is a space-time symmetry: the generators of supersymmetry can in a loose sense be described as square roots of the translation generators. We also saw that the anticommutation relation (16) of the \( N = 1 \) supersymmetry algebra means that local supersymmetry implies gravity (or rather, its supersymmetric extension known as supergravity, which will come into play later on). Moreover, the only known consistent theory of gravity, string theory, often implies the existence of space time supersymmetry (all known consistent vacua of string theory have space time supersymmetry). The last statement needs some qualification: strictly speaking, superstring theory would only imply the existence of supersymmetry at energy scales above or of order of the Planck scale, \( M_{Pl} = \sqrt{\hbar c/(8\pi G_N)} = 2.4 \cdot 10^{18} \text{ GeV} \) \( (G_N \text{ is Newton's gravitational constant}) \). On the other hand, low energy experiments (and our everyday experience) teach us that the supersymmetric partners and the ordinary elementary particles are not degenerate in mass: the agreement of \( \Gamma_Z \) between theory and experiment implies that the masses of the superpartners of the ordinary quarks and leptons that couple to the \( Z \)-boson have to be at least \( m_Z/2 \), so that they do not contribute to the width \( \Gamma_Z \) of the \( Z \)-boson. Therefore, we are forced to conclude that supersymmetry has has to be “lost” somewhere between \( M_{Pl} \sim 10^{18} \text{ GeV} \) and the electroweak scale, \( M_{W,Z} \sim 10^{2} \text{ GeV} \).

String theory, in fact, provides only one of the motivations for supersymmetry at or above the electroweak scale. Another motivation, which historically preceded string theory, stems from considering electroweak symmetry breaking and the ensuing hierarchy problem. In the following, we will discuss this motivation (which can be called the “naturalness” motivation).

Recall that in the standard model, the weak and electromagnetic interactions are described by a nonabelian gauge theory, with gauge group \( SU(2) \times U(1)_Y \). This nonabelian gauge symmetry is, however, not linearly realized at energies below 100 GeV (i.e. it does not give rise to only massless gauge bosons). As is well known, the electroweak gauge group \( SU(2) \times U(1)_Y \) is broken down to \( U(1)_{\text{e,m.}} \), describing Maxwell electrodynamics, at a scale of order \( 10^2 \text{ GeV} \). The only massless gauge boson is thus the photon. The other three gauge bosons, which have been observed experimentally, of \( SU(2) \times U(1)_Y \)—the \( W^\pm \) and the \( Z \)-bosons—obtain mass of the same order of magnitude \( \sim 100 \text{ GeV} \). The theory of the electromagnetic and weak interactions has been subjected to numerous experimental tests and the agreement between theory and experiment is spectacular.

The only sector of the theory that has eluded experimental tests is the one that is responsible for electroweak symmetry breaking and the generation of mass. That such a sector has to exist follows from considering the theory of massive spin-1 bosons, the \( W^\pm \) and the \( Z \)-bosons. It is well known that in theories of spin-1 massive particles the scattering amplitudes grow with energy. On Fig. 6, we have shown the Feynman diagrams responsible for the scattering process \( ZZ \rightarrow W^+W^- \). The amplitude of this scattering process grows with the center of mass energy \( E \)
Fig. 6. The graphs contributing to the $ZZ \to WW$ scattering process. The contribution of these graphs to the scattering amplitude violates unitarity at energies $\sim 1000$ GeV.

like

$$A (Z Z \to W^+ W^-) \sim \left(\frac{E}{4\pi v}\right)^2,$$

(17)

where $v$ is a scale of order the mass of the $W$ boson. The growth of the amplitude with energy is disastrous—it implies that at energies of order 1000 GeV some probabilities become larger than one and unitarity is lost! This growth of the amplitude can be stopped (and hence unitarity restored) by an exchange of, for example, a spin-0 boson—the Higgs boson—via the graph of Fig. 7. Moreover, the introduction of the Higgs boson sector is required for consistency of the theory (it supplies the longitudinal components of the $W,Z$ bosons). Its expectation value breaks electroweak symmetry and generates the mass of the $W^{\pm}$- and $Z$-bosons.

Fig. 7. Higgs boson contribution to $ZZ \to WW$ scattering.

So far, the Higgs boson (or more generally, the Higgs sector of the theory that is responsible for electroweak symmetry breaking) has eluded experimental searches. Its mass is not fixed by the theory—it is an additional free parameter in the Lagrangian. However, if the Higgs boson is elementary, both its mass and expectation
value have to be of the same order of magnitude, i.e. one expects \( m_H \sim 10^2 - 10^3 \) GeV.

This is where we first encounter the hierarchy problem. This face of the hierarchy problem stems from the fact that theories of elementary scalar particles suffer severe naturalness problems. To elucidate, recall that elementary scalar fields can have pointlike four-particle interactions like the one on Fig. 8. Since the Higgs boson is a spin-0 particle, it can have pointlike quartic interactions both with itself or with other, possibly heavier, spin-0 particles (in the latter case, two of the lines in Fig. 8 would represent the Higgs boson, while the other two would correspond to the other spin-0 particle, denoted by \( H' \)). This interaction generates, via the vacuum fluctuation graph shown on Fig. 9, a contribution to the Higgs mass

\[
\delta m_H^2 \sim \left( \frac{\Lambda}{4\pi} \right)^2.
\]  

(18)

Here, \( \Lambda \) denotes the ultraviolet cutoff, or more physically, it is a scale of order of magnitude of the mass of the scalar particle running inside the loop. In most theoretical models that attempt to describe the physics beyond the standard model such heavy scalar particles abound—this could be a particle responsible for the breaking of the Grand Unification symmetry (and hence of mass \( 10^{15} \) GeV), or it could simply be one of the many Planck-mass particles left over from string compactification. The point here is that, once elementary scalar particles are allowed into the theory, there is no reason why some of them would not be as heavy as \( 10^{18} \) GeV. There is also absolutely no symmetry reason why they would not couple to the Higgs via quartic pointlike interactions like the one on Fig. 8.

Couplings like the one on Fig. 8 lie at the heart of the hierarchy problem, for the correction to the Higgs mass from the graph on Fig. 9 is many orders of magnitude larger than the value of the Higgs mass, the electroweak scale. In order to achieve a Higgs mass of order 100 GeV, we would have to fine tune both the bare value,
$m_{H,0}^2$ and $\delta m_H^2$, so that
\[ m_{H,0}^2 + \delta m_H^2 = m_H^2 \sim (100 \text{ GeV})^2. \] (19)

Since $\delta m_H^2 \sim \Lambda^2 \sim (10^{15-18} \text{ GeV})^2$, in order to satisfy (19) we need to take $m_{H,0}$ of the same order of magnitude as $\delta m_H$ and achieve a cancellation between the bare value $m_{H,0}$ and the correction $\delta m_H$ to order $10^{-13} - 10^{-17}$! We would have to perform the same fine tuning not only at one loop, but in every order of perturbation theory. This very unnatural situation leads to the puzzle of why the electroweak scale—and hence the masses of the $W$, $Z$ bosons and the Fermi constant—is so tiny compared to the Planck or Grand Unified scale. This puzzle is called the “hierarchy problem.”

The hierarchy problem, phrased above as the stability of the electroweak scale against large radiative corrections, arises because there is no symmetry that protects scalar particle masses from receiving huge radiative corrections. Massless fermions, on the other hand, carry a conserved quantum number that protects them from obtaining mass through radiative corrections: the projection of the spin onto the direction of motion, known as chirality. Since supersymmetry interchanges bosons and fermions, in a theory that is supersymmetric, chirality will also protect the bosonic superpartners of the fermions from acquiring large corrections to their masses. Thus supersymmetry in effect communicates chirality to the fermions. More simply put, the graph of Fig. 9, the consideration of which led us to the hierarchy problem, will have a supersymmetric partner in a theory with supersymmetry. The scalar running inside the loop would have a fermionic superpartner, which would contribute through the second graph on Fig. 10. Since supersymmetry relates the couplings of the two graphs, and since the fermion contributes, due to the Pauli principle, with the opposite sign, the leading contribution between the two graphs cancels, and only a small correction to the Higgs mass is left. We note that this cancellation is akin to the cancellation of the zero point energies of the fermions and the bosons in the supersymmetric oscillator discussed in the previous section (see eqs. 1, 4, 9).

This is the way supersymmetry ensures the stability of the electroweak scale against large radiative corrections. Technically, the stability of the small electroweak scale against large radiative corrections (i.e. its naturalness) is due to
the cancellation of quadratic divergences in supersymmetric theories. Due to these cancellations, no infinite number of fine tunings is required to keep the mass of the Higgs and the electroweak scale light.

We described the way supersymmetry would solve the hierarchy problem in a theory in which supersymmetry is realized linearly, and the spectrum is like that on Fig. 3. But the spectrum of elementary particles is not supersymmetric. So how are we going to take advantage of these wonderful cancellations, brought by supersymmetry, to solve the hierarchy problem? Remarkably (we will not be able to discuss details here), the cancellation of quadratic divergences (i.e., the absence of huge radiative corrections to scalar masses) persists even if supersymmetry is not exact (i.e., the spectrum is not like that on Fig. 3), but is softly broken. In the context of application to elementary particle physics, the Lagrangian that describes the theory of the softly broken supersymmetric standard model consists of two parts:

\[ \mathcal{L} = \mathcal{L}_{SUSY \ SM} + \mathcal{L}_{soft}. \]  

In (20), \( \mathcal{L}_{SUSY \ SM} \) is the supersymmetric Lagrangian of the standard model. It describes the known elementary particles and their superpartners, contains no dimensionful (mass) parameters; and can be constructed by the procedure outlined in the previous section for the supersymmetric oscillator.

In the absence of the second term in (20), the spectrum of the elementary particles looks schematically like the one on Fig. 3. The second term, the soft-breaking Lagrangian, \( \mathcal{L}_{soft} \), is responsible for lifting the degeneracy of the spectrum and making (20) a Lagrangian consistent with experiment—it contains the soft-breaking terms that include mass terms for the superpartners of all known elementary particles. These mass terms, of order of magnitude \( m_{soft} \), explain why the superpartners evade the bounds from LEP on \( \Gamma_Z \) and have eluded observation (so far). The mass of the Higgs boson is also of order \( m_{soft} \). If supersymmetry is

*We prefer to list the \( \mu \)-parameter, which is formally supersymmetric, in the soft-breaking Lagrangian. For successful phenomenology, \( \mu \sim m_{soft} \sim m_W \) is required, and the only natural solution is to assume that the appearance of the \( \mu \) term is related to supersymmetry breaking.
responsible for protecting the electroweak scale from large radiative corrections, the natural value of the soft mass parameters is \( m_{\text{soft}} \sim m_W \sim 10^2 \) – \( 10^3 \) GeV, for if all superpartners were much heavier than \( m_W \), we would have to explain another hierarchy problem: why \( m_W \ll m_{\text{soft}} \). The physics below the scale of the soft breaking masses \( m_{\text{soft}} \) is the physics of the nonsupersymmetric standard model—the Lagrangian reduces to that of the nonsupersymmetric standard model, as the effect of the heavy supersymmetric particles is negligible at lower energy scales.

The Lagrangian \( \mathcal{L} = \mathcal{L}_{\text{SUSY SM}} + \mathcal{L}_{\text{soft}} \) is the main object of study of supersymmetric phenomenology. The problem with using this Lagrangian is that \( \mathcal{L}_{\text{soft}} \) contains an enormous number of parameters (> 100). These are somewhat constrained by low-energy measurements (see e.g. ref. 6, and the discussion of FCNC in Section 5), but still a large degree of arbitrariness remains. How can we reduce (short of measuring the spectrum of superparticles) this large number of arbitrary parameters?

In order to attempt to answer this question, we note that the hierarchy problem has another face, in addition to the stability of the small electroweak scale from large radiative corrections. Although in a theory described by \( \mathcal{L} = \mathcal{L}_{\text{SUSY SM}} + \mathcal{L}_{\text{soft}} \), with \( m_{\text{soft}} \sim m_W \), the stability of the electroweak scale against large radiative corrections is ensured, it is clear that the Lagrangian cannot be the ultimate Lagrangian of the universe, since it does not describe physics at arbitrarily small distances (for example, it does not include gravity). In fact, \( \mathcal{L} \) still contains an enormous amount of fine tuning, albeit only at tree level: the only scale that appears in \( \mathcal{L} \) is of order \( m_W \sim m_{\text{soft}} \sim 10^{-16} M_{\text{Planck}} \). From the point of view of a (more) fundamental theory (string theory) that describes the physics at short distances and has only one fundamental scale (\( M_{\text{Planck}} \)), the appearance and order of magnitude of the parameters in \( \mathcal{L}_{\text{SUSY SM}} + \mathcal{L}_{\text{soft}} \) is as big a puzzle as it was before the introduction of supersymmetry. A successful theory leading to a solution of the hierarchy problem should provide a dynamical explanation of the origin and magnitude of the soft parameters \( m_{\text{soft}} \) in the low-energy Lagrangian. This means that dynamics at some higher scale, \( M_{\text{SUSY}} \), should be responsible for the breaking of supersymmetry. The supersymmetry breaking dynamics should therefore generate the soft breaking terms \( \mathcal{L}_{\text{soft}} \) in the low-energy Lagrangian with the right order of magnitude \( m_{\text{soft}} \sim m_W \). The dynamics that break supersymmetry will then naturally imply definite relations between the different soft parameters, reducing thus the large arbitrariness. It is thus the dynamics of supersymmetry breaking that is responsible for the smallness of the electroweak scale. Conversely, we are hopeful that measuring the soft parameters (i.e. finding the superpartners and measuring their masses) will allow us to gain insight into the higher-scale dynamics responsible for the breaking of supersymmetry.

Before proceeding in the next section with a more technical (though still based on quantum-mechanical examples) discussion of the possible ways to break supersymmetry, let us summarize the main points of the previous two sections:

- Supersymmetry is a space-time symmetry that unifies bosons and fermions.
Although not exact at low energies, supersymmetry can be relevant for high-energy physics, because:

- Consistent string theory vacua have space-time supersymmetry.
- Supersymmetry can explain the smallness of the electroweak scale.

Supersymmetry must be dynamically broken and the masses of the superpartners (selectron, photino...) should result from that breaking. The breaking of supersymmetry should also trigger electroweak symmetry breaking.

The idea of supersymmetry at the electroweak scale is falsifiable. If superpartners are found, measuring their masses can give us a glimpse upon dynamics at higher energy scales.

It is therefore interesting to understand how supersymmetry can break.

Finally, we would like to mention the possibility that electroweak symmetry breaking is due to other, non-supersymmetric dynamics without elementary scalar fields, such as technicolor. However, until the existence of supersymmetry at the TeV scale is firmly disproved, we should not give up on theoretical investigations of, and experimental searches for, supersymmetry—we would be missing a great opportunity to learn about physics at high-energy scales and fundamental space-time symmetries.

4. How does supersymmetry breaking occur?

In this section, we will discuss some possible ways to break supersymmetry. To this end, we will return to our discussion of supersymmetric quantum mechanics. In order to allow for symmetry breaking, we will have to include interactions and slightly generalize the discussion of Section 2.

The quantum mechanical system we will use as an example is that of a spin-1/2 particle moving on the line. The state of the spin-1/2 particle is described by a two-component wave function (a Pauli spinor), \( \Psi(x) = \begin{pmatrix} \psi_1(x) \\ \psi_2(x) \end{pmatrix} \). The two components of \( \Psi \) describe the wave functions of the particle with spin projections +1/2 and −1/2 respectively. The Hamiltonian of our spin-1/2 particle on the line is:

\[
\hat{H} = -\frac{1}{2} \frac{d^2}{dx^2} + \frac{1}{2} \left( \frac{d W(x)}{d x} \right)^2 + \frac{1}{2} \sigma_3 \left( \frac{d^2 W(x)}{d x^2} \right). \tag{21}
\]

Here and below \( \sigma_{1,2,3} \) denote the Pauli matrices. That this Hamiltonian is supersymmetric is reflected in the fact that it can be represented as the square of a hermitean supersymmetry generator, just like (10). The hermitean supersymmetry generator (this would be the analog of \( Q + Q^\dagger \) in the supersymmetric oscillator, see eq. (11)) is

\[
Q = -\frac{i}{\sqrt{2}} \sigma_1 \frac{d}{dx} + \frac{1}{\sqrt{2}} \sigma_2 \frac{d W(x)}{d x}. \tag{22}
\]
Using (22) it is easy to check explicitly that $Q^2$ yields the Hamiltonian (21): $\hat{H} = Q^2$. The function $W(x)$ is called the superpotential and completely determines the interactions. The various terms in the Hamiltonian (21) have the following interpretation. The first term is the kinetic energy of the particle; we put its mass equal to 1. The second term is the potential energy—the particle moves in a potential well $V(x) = (W')^2/2$. The third term describes the “spin-orbit interaction” $\sim \sigma_3 W''$. The superpotential, $W(x)$, determines both the potential and spin-orbit terms in the Hamiltonian (since they are related by supersymmetry).

We are interested in the issue of supersymmetry breaking. In order to be able to discuss it, we need to find an order parameter: a quantity that signals whether supersymmetry is broken or not. In general, the spontaneous breaking of any symmetry means that although the dynamics is invariant under the symmetry, the ground state is not (a common example of spontaneous symmetry breaking is, e.g. the spontaneous magnetization of a ferromagnet, which breaks the rotational symmetry). The noninvariance of the ground state $|0\rangle$ under supersymmetry transformations would mean that the supersymmetry generator $Q$ does not annihilate the ground state, e.g. $Q|0\rangle \neq 0$. Consider now the following chain of equalities:

$$E_0 \equiv \langle 0 | \hat{H} | 0 \rangle = \langle 0 | Q Q | 0 \rangle = \| Q | 0 \rangle \|^2 > 0 , \text{ iff } Q | 0 \rangle \neq 0 .$$

Here, $E_0$ is the ground state energy and we used the fact that the Hamiltonian is the square of $Q$. The inequality in the last line is true whenever supersymmetry is broken, i.e. $Q|0\rangle \neq 0$. We thus see that the ground state energy of a supersymmetric system is positive if and only if supersymmetry is broken, and zero if and only if supersymmetry is unbroken. The ground state energy is thus the order parameter for supersymmetry breaking. Hence answering the question of whether supersymmetry is manifest or broken is equivalent to finding whether the ground state energy vanishes or not.

At the classical level—ignoring the spin-orbit interaction and the zero-point energies—this question is easy to answer. We only have to look at the graph of the potential energy $V(x)$. We have shown three possibilities on Fig. 11. Fig. 11a shows a potential which is everywhere positive. Thus, classically, the ground state energy is positive and supersymmetry is broken. The potentials on Fig. 11b,c both allow for classical states of zero energy, hence, classically, supersymmetry is unbroken.

The classical approximation is of course not the whole story. It is natural to ask whether quantum corrections can change the classical answer. Fortunately, in supersymmetric systems, it is often easy to give the exact answers to questions about the ground state. The reason behind this power supersymmetry has can be

\footnote{We note that these formulas are quite similar to the ones that are obtained in 3 + 1 dimensional renormalizable supersymmetric field theory—all interactions are derived by the derivatives of a single function, the superpotential $W(x)$. In the field theory case, the “spin-orbit” term would correspond to the Yukawa interaction between the bosons and fermions in the supermultiplet.}
traced to the fact that the Hamiltonian is a total square (see eqn. (10)). Answering
the question of whether supersymmetry is broken is equivalent to finding whether
the Hamiltonian has a normalizable eigenstate of zero energy. In a supersymmetric
system, however, in order to find the zero-eigenvalue state, we do not have to solve
the second order Schrödinger equation
\[ \hat{H} |0\rangle = 0. \]  
Since eq. (23) shows that \( E_0 = 0 \) if and only if \( Q |0\rangle = 0 \), it suffices, instead of (24), to solve the first order equation
\[ Q |0\rangle = \left( -\frac{i}{\sqrt{2}} \sigma_1 \frac{d}{dx} + \frac{1}{\sqrt{2}} \sigma_2 \frac{d}{dx} W(x) \right) \Psi_0(x) = 0. \]  
Compared to the second order equation (24), which, for a general superpotential
can only be solved numerically, the first order equation (25) can be solved for an
arbitrary superpotential \( W(x) \). Using simple Pauli matrix algebra, it is easy to
check that
\[ \Psi_0(x) = e^{\sigma_3 W(x)} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} e^{W(x)} c_1 \\ e^{-W(x)} c_2 \end{pmatrix} \]  
is the general solution of the zero-eigenvalue equation (25). The solution for the
ground state wave function \( \Psi_0 \) depends on two integration constants \( c_1, c_2 \). From
eq. (26) it follows that \( \Psi_0 \) is normalizable only in two cases:
\[ c_1 = 0, \quad W(x) \to +\infty \text{ as } x \to \pm \infty, \text{ or} \]
\[ c_2 = 0, \quad W(x) \to -\infty \text{ as } x \to \pm \infty. \]  
Thus a normalizable ground state of zero energy exists only if the superpotential
\( W(x) \) is “even at infinity”, i.e. it has the same limit at both \( x = \pm \infty \) (+ or −\( \infty \),
since we assume here that the spectrum is discrete). A smooth function \( W(x) \) with
this property will necessarily have an odd number of extrema (and its derivative
\( W' \)—an odd number of zeros). But since \( V(x) = (W')^2/2 \), this means that the
criterion for unbroken supersymmetry is that the potential has an odd
number of zeros.

We can now revisit the three potentials on Fig. 11 and find whether supersymmetry
is broken or not at the exact quantum-mechanical level. The potential on
Fig. 11a has no zeros, hence according to our criterion from the last paragraph,
supersymmetry, being broken at the tree level, remains broken once quantum
corrections are included. The potential on Fig. 11b has one minimum, hence super-
symmetry remains unbroken in the quantum theory. Finally, in the case of Fig. 11c,
the potential has an even number of zeros. Therefore, even though supersymmetry
is unbroken at the classical level, it is broken by quantum effects.

It is the case depicted on Fig. 11c that will be of most interest for us. The
reason is that the breaking of supersymmetry in the supersymmetric system with
a double-well potential is due to nonperturbative effects—it occurs because of tun-
neling between the two wells. We found earlier that in the classical approximation
Fig. 11. Three possible potentials \( V(x) \): (a) breaks supersymmetry both at the classical and quantum level; (b) has unbroken supersymmetry both classically and quantum-mechanically; (c) has manifest supersymmetry at the classical level, but quantum-mechanical nonperturbative effects (tunneling) break supersymmetry. All three possibilities have counterparts in quantum field theory.

(and, even though we did not show this, also in perturbation theory, including the zero-point energy and the spin-orbit interaction) the ground state energy vanishes and supersymmetry is unbroken. The effect of tunneling can be evaluated in the semiclassical approximation. The WKB formula for the ground state energy splitting gives for the vacuum energy:

\[
E_{0}^{WKB} = \langle 0 | \hat{H} | 0 \rangle \\
\sim \hbar \omega e^{\frac{1}{\hbar} \int dx \sqrt{2V(x)}} \ll \hbar \omega ,
\]

where \( \omega \) is the frequency of classical motion near the bottom of the well, and the integral is over the classically forbidden region of \( x \). Since, for appropriate parameters of the potential (or, in the formal semiclassical \( \hbar \to 0 \) limit), the tunneling probability is exponentially suppressed, the scale of supersymmetry breaking—the ground state energy—is much smaller than the characteristic frequency of motion inside the wells.

The breaking of supersymmetry due to nonperturbative effects (similar to the tunneling described here) has a counterpart in quantum field theory. The typical expression one obtains for the scale of supersymmetry breaking, \( M_{SUSY} \), in field theory is very similar to the WKB formula (28):

\[
M_{SUSY} \sim e^{-O(1)} \frac{\alpha^2}{\pi^2} M_{Planck} ,
\]

where \( g \) is a gauge coupling. The scale of supersymmetry breaking is thus exponentially suppressed compared to the Planck scale and can be many orders of magnitude smaller. Since the electroweak scale is generated as a result of supersymmetry breaking, it will also be much smaller than the Planck scale (and, usually, smaller then \( M_{SUSY} \)).
There are two main features of the discussion of our quantum-mechanical example that survive generalization to supersymmetric quantum field theory.

- Supersymmetry breaking is controlled by the extrema of the superpotential.
- Supersymmetry breaking can occur nonperturbatively and generate exponentially small scales, see eq. (29).

The quantum mechanical example we gave was intended to illustrate these two points. We also hope to have given a hint of the “power of supersymmetry”: in supersymmetric theories various aspects of the dynamics, especially questions about the ground state, that are usually difficult to analyze (and are often intractable), can be understood exactly.

The realization of this fact in 3 + 1-dimensional supersymmetric field theory came after the work, in 1994, of Seiberg, and Seiberg and Witten. It initiated what can be called a “supersymmetric revolution,” which still continues. A tremendous progress has been made in understanding, even nonperturbatively, the low-energy dynamics of supersymmetric field theories. It was realized that, in supersymmetric field theory, the superpotential (i.e. the appropriate generalization of \( W(x) \) of (21)) of the lowest-energy excitations can be determined exactly, including all nonperturbative effects. As we argued above, it is the superpotential that determines the phase structure of the theory. Thus, one gains a powerful tool to study the exact phase structure of supersymmetric field theories, and in particular, to answer the question: which supersymmetric field theories break supersymmetry? While a simple criterion, as in our quantum-mechanical example (see discussion after eq. (26)), is still absent, given a supersymmetric field theory, this question can be answered with certainty in most cases. Indeed, since 1994, the list of theories for which the answer is known has grown dramatically. Many new theories and nonperturbative mechanisms of supersymmetry breaking have been found (for short reviews of this development and a list of references, we refer the reader to [8]).

In this section, we illustrated the present theoretical ideas of how supersymmetry can break nonperturbatively and thereby generate small scales. The better understanding of the physics of supersymmetry breaking, gained in the last several years, makes one hopeful that similar mechanisms could be used to explain the smallness of the electroweak scale and generate soft masses \( m_{soft} \) for the superpartners of the quarks, leptons, and gauge bosons. In the next section, we will describe the main ideas of how to use our understanding of supersymmetry breaking to construct phenomenological models that lead at low energies to the Lagrangian of the supersymmetric extension of the standard model (20).

5. How is SUSY breaking communicated?

In this section, we describe the main current theoretical ideas of how supersymmetry is broken and how the breaking manifests itself in the softly broken Lagrangian (21) of the supersymmetric standard model.
The first attempts to apply supersymmetry to elementary particle physics date back to the early and mid 1970s. It was then quickly realized that the supersymmetrized version of the standard model (i.e. $L_{\text{SUSY SM}}$ of (20), constructed along the lines described in Section 1), does not break supersymmetry. This meant that dynamics in addition to that of the standard model was required to break supersymmetry. The existence of an additional sector of the theory was thus postulated: the \textit{supersymmetry breaking sector}.

The supersymmetry breaking sector is usually a supersymmetric field theory, the ground state of which breaks supersymmetry just as in our example of supersymmetric quantum mechanics with potentials of Figs. 11a and 11c. If we want to use supersymmetry not only to stabilize the electroweak scale against large radiative corrections, but also to explain its smallness compared to the Planck scale, the scenario of Fig. 11c is preferred to the one of Fig. 11a. This is because the theory with potential on Fig. 11a breaks supersymmetry already at the classical level. Since supersymmetry breaks at tree level, the scale of supersymmetry breaking is simply one of the parameters of the potential and needs to be put in by hand (i.e. fine tuned). But a fine tuning of the ratio $m_W/M_{\text{Pl}} \sim 10^{-16}$ is precisely what we wanted to avoid. Hence the theory like that of Fig. 11c is more suitable to us: the theory has unbroken supersymmetry at tree level, the scale of supersymmetry breaking is generated dynamically by nonperturbative effects and is exponentially smaller than the fundamental scale of the theory, $M_{\text{Pl}}$.

To summarize, we expect that the supersymmetry breaking sector is a theory that breaks supersymmetry due to some nonperturbative effects. It is characterized by the scale of supersymmetry breaking $M_{\text{SUSY}}$ (which is related to the vacuum energy density, as explained in the previous section). In a theory of dynamically broken supersymmetry, the scale of supersymmetry breaking is exponentially smaller than the Planck scale (29): $M_{\text{SUSY}} \sim e^{-O(1)} \frac{e}{g^2} M_{\text{Planck}}$.

By now, we have discussed two of the ingredients needed to construct a theory whose low-energy effective Lagrangian is $L_{\text{SUSY SM}} + L_{\text{soft}}$: the supersymmetrized standard model and the supersymmetry breaking sector. In order to generate the soft breaking parameters—the masses of the superpartners of the ordinary standard model particles—these two sectors need to couple to each other. The two main phenomenological frameworks for generating the soft masses ($L_{\text{soft}}$) are distinguished by the nature of the coupling between the supersymmetric standard model and the supersymmetry breaking sector. The coupling between these two sectors has come to be known as the \textit{messenger interaction}. Presently there are two main candidates to play the role of messenger interactions—gravity and gauge interactions—and we discuss them in turn.

\footnote{We note that obtaining an exponential suppression of $M_{\text{SUSY}}$ requires that the factor in the exponent is $\gg 1$, i.e. the coupling $g \leq O(1)$. This might look like another fine tuning problem. The gauge coupling $g$, however, requires much less adjustment than fine tuning the Higgs mass to 17 significant digits.}
5.1. **Supergravity mediated supersymmetry breaking.**

As explained in Section 2, supergravity arises naturally once supersymmetry is promoted to a local symmetry. Moreover, the low-energy effective description of string theory is precisely the theory of supergravity. For our purposes, it will be enough to state that in the theory of supergravity all elementary particles have their superpartners, as described earlier. In addition, the massless spin-2 graviton has a supersymmetric, massless spin-3/2 partner called the gravitino. Since gravity is a universal interaction (it couples to the energy-momentum tensor), it couples to both the supersymmetry breaking sector and the supersymmetric standard model. Thus it is a natural candidate to play the role of a messenger interaction. We will examine its consequences in what follows.

Once a supersymmetry-breaking theory is coupled to supergravity, an effect similar to the Higgs effect takes place (the “super-Higgs” effect). The important result of this effect is that the massless gravitino obtains a mass $m_{3/2}$, which generally scales like

$$m_{3/2} \sim \frac{M_{SUSY}^2}{M_{Pl}}.$$  \hspace{1cm} (30)

Furthermore, the coupling of supergravity to the supersymmetric standard model also leads, as a consequence of the classical equations of motion of supergravity, to the appearance of soft mass parameters for all scalar superpartners of the ordinary quarks, leptons, and gauge bosons (and for the Higgs). The natural order of magnitude of $m_{\text{soft}}$ is

$$m_{\text{soft}} \sim m_{3/2} \sim \frac{M_{SUSY}^2}{M_{Pl}}.$$  \hspace{1cm} (31)

This scaling can be obtained by dimensional reasoning. The scalar masses for the superpartners enter as $m_{\text{soft}}^2$ in the Lagrangian. Furthermore, since $m_{\text{soft}}^2$ arise as a consequence of the classical equations of motion of supergravity, they should be proportional to the Newton constant $m_{\text{soft}}^2 \sim G_N \sim 1/M_{Pl}^2$. Hence the scaling of $m_{\text{soft}}$ with $M_{Pl}$ in eq. (31). The dependence on $M_{SUSY}$ can be then recovered by dimensional analysis.

In order to obtain the desired $m_{\text{soft}} \sim m_W \sim 10^{2-3}$ GeV, taking into account $M_{Pl} \sim 10^{18}$ GeV, eq. (31) gives for the scale of supersymmetry breaking $M_{SUSY} \sim 10^{11-13}$ GeV. Therefore, to explain the smallness of the electroweak scale, the scale of supersymmetry breaking in supergravity-mediated models of supersymmetry breaking has to be of the order of an “intermediate” scale—the geometric average of $m_W$ and $M_{Pl}$. The supergravity-mediated models are often called intermediate scale models, or _hidden sector_ models: since the scale of supersymmetry breaking is so high, and since gravity is the only interaction between the standard model and the supersymmetry breaking sector, the supersymmetry breaking sector is “hidden” from observation, up to energies of order $M_{Pl}$. On Fig. 12, we have schematically shown the structure and relevant scales in hidden sector models.

Supergravity mediated models have been in the center of study of supersymmetric phenomenology since their appearance in the early 1980s. Among the reasons...
for their popularity is the fact that supersymmetry breaking is communicated at the classical (tree) level, and that the appearance of soft mass parameters is an automatic consequence of the coupling to gravity.

However, the hidden sector models suffer from a severe drawback: they lack predictive power. The reason behind the lack of predictivity is that supergravity theories are effective field theories, only valid up to energy scales of order $M_{Pl}$. The general Lagrangian of supergravity depends on three arbitrary functions (to boot, some of them have indices). If we possessed detailed knowledge of the theory beyond the Planck scale, we could compute these functions. However, in the absence of such detailed knowledge (the only candidate for such a theory, string theory, as of now does not allow matching to an effective supergravity theory with N=1 supersymmetry in 3 + 1 dimensions) the Lagrangian remains rather arbitrary.

As a consequence of this arbitrariness, the supergravity Lagrangian can lead to soft parameters that can be in conflict with experimental data. As mentioned before, there exist constraints from low-energy experiments on the soft parameters. One of these is the requirement of sufficient degeneracy of the masses of the
supersymmetric particles of the first two generations that carry the same gauge quantum numbers. This requirement arises from the absence of flavor changing neutral currents contributions to the $K^0 - \overline{K}^0$ mass difference; see ref. 5. The general supergravity Lagrangian allows for nondegenerate scalar soft masses and is thus in conflict with low-energy data. Various mechanisms have been proposed to remedy this situation, but in the absence of understanding of the short-distance physics, it is hard to decide whether any of these can be operative.

To summarize, we believe that supergravity is an attractive mechanism of communicating supersymmetry breaking and generating the soft scalar masses of the superpartners of the ordinary particles. Unfortunately, as of now, it lacks predictive power. Recent developments in nonperturbative string theory (string duality, M-theory) offer hope that this flaw may some day be remedied.

5.2. **Gauge mediated supersymmetry breaking.**

Historically, the first phenomenological models that coupled the supersymmetry breaking sector to the supersymmetric standard model were the models where a gauge force plays the role of messenger interaction. They were proposed also in the early 1980s, but before the idea of supergravity as the messenger of supersymmetry breaking made its appearance. There are several reasons that they were abandoned: supergravity, where the soft parameters are generated at the classical level, won out with its simplicity, and, in addition, at the time there was little understanding of the nonperturbative dynamics of supersymmetric gauge theories. The gauge-mediated models were resurrected after 1994, when many aspects of supersymmetric nonperturbative gauge dynamics were better understood.

It is natural to ask whether the standard model gauge interactions can play the role of the messenger of supersymmetry breaking. If this was the case, the usual color and electroweak gauge interactions would have to also couple to the supersymmetry breaking sector. The simplest possibility is that some of the ordinary quarks, leptons, Higgs, and their superpartners somehow participate in supersymmetry breaking. It was realized, however, that, at least in weakly coupled models, this direct coupling to the supersymmetry breaking sector was disastrous: it led to spectra that were in conflict with experiment (some of the superpartners were lighter than the ordinary particles). The possibility that some of the known particles and their superpartners couple to the supersymmetry breaking sector remains if the models are strongly coupled; however, at present, such models lack predictive power.

It was then that the *messenger quarks and leptons* were proposed as a remedy. These are just like the ordinary quarks and leptons but are expected to be heavier (and hence undetectable at present). They are part of the so-called messenger sector, which is coupled directly to the supersymmetry breaking sector (see Fig. 13). As a result of supersymmetry breaking, they obtain mass, of order $M_{mess}$—the *messenger scale*. Since supersymmetry is broken, their spectrum is not exactly supersymmetric (i.e., unlike that of Fig. 3); the degeneracy between the masses of the
messenger quarks and leptons and their superpartners is lifted by supersymmetry breaking. Now, the messenger quarks and leptons carry ordinary \( SU(3) \times SU(2) \times U(1) \) quantum numbers. Therefore, through quantum loop effects, they will also couple to the ordinary quarks and leptons (an example of such a radiative coupling that generates a soft mass for a squark is given on Fig. 14). This is the way the squarks and sleptons “learn” about supersymmetry breaking and obtain soft masses.

The magnitude of the soft masses is easy to estimate:

\[
m_{\text{soft}} \sim \frac{g^2}{16\pi^2} M_{\text{mess}}.
\]

The graph of Fig. 14 generates \( m_{\text{soft}}^2 \) at two loops, hence \( m_{\text{soft}} \) is only proportional to a single loop factor (each loop contributes a factor of \( g^2/(16\pi^2) \)). In (32) \( g \) is the relevant standard model gauge coupling.

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![Diagram](image-url)

**Fig. 13.** Gauge mediated (“visible sector”) supersymmetry breaking. We have assumed for simplicity that the scale of supersymmetry breaking, \( M_{\text{SUSY}} \), is of the same order of magnitude as the scale of the messenger quarks and leptons, \( M_{\text{mess}} \). In general, these two scales need not be of the same order of magnitude. For a recent review, see [4].

Similar to what we did in the case of hidden sector models, we can estimate the relevant scales. Demanding that \( m_{\text{soft}} \sim m_W \sim 10^{2-3} \) GeV, we obtain for the scale
of the messengers: $M_{\text{mess}} \sim 10^{4-5}$ GeV (the supersymmetry breaking scale $M_{\text{SUSY}}$ may or may not be of the same order of magnitude as the messenger scale; we crudely estimated the factor $g^2/(16\pi^2) \simeq 100$). We note that the scale relevant for the communication of supersymmetry breaking in the gauge-mediated case is many orders of magnitude smaller than that in supergravity. An important consequence of eq. (32) is that the soft scalar masses are proportional to the gauge couplings, therefore superpartners with the same gauge quantum numbers are automatically degenerate. Hence the flavor changing neutral currents are naturally absent in gauge mediated models. This fact, and their predictive power—because of their independence on the short-distance physics—are the main arguments in favor of these models.

We should also mention the possibility of additional signatures in gauge-mediated models, which do not naturally arise in supergravity theories. To explain these, note that the supergravity effects are still there, but are not the leading ones as far as the soft mass parameters are concerned (since their contributions are suppressed by $1/M_{Pl}$). However, supergravity effects are still the leading contribution to the gravitino mass: substituting $M_{\text{SUSY}} \sim 10^{4-5}$ GeV in the formula for the gravitino mass (30), we obtain $m_{3/2} \sim 10^{-10} - 10^{-8}$ GeV—a mass in the eV range. Such a light gravitino would lead to new signatures of supersymmetry (see 9 and references therein).

![Fig. 14. An example of a two-loop contribution to the soft mass of the squarks, sleptons and Higgs in gauge mediated models. The usual quarks, leptons and their superpartners couple to the messenger quarks and leptons (and to the supersymmetry breaking sector) only through standard model gauge interactions, hence two loop graphs represent the leading contribution.](image-url)

The explicit construction of models that realize the ideas discussed above, and are phenomenologically acceptable and elegant (admittedly the last criterion is rather subjective) has not been so successful, however. Nothing resembling a "standard model" of gauge mediated supersymmetry breaking has emerged quite yet. It is perhaps fair to say that the lack of predictive power of supergravity has been replaced with a multitude of models, which are successful in varying degrees (for a
recent review and a list of references see 9. The study of the possible experimental consequences of gauge-mediated models, perhaps in a model-independent way, is still a matter of importance. Once the superpartners are found and their spectrum measured it can help pin down the right mechanism of supersymmetry breaking.

6. Summary.

Here we summarize the main points discussed in this review:

- Supersymmetry is a space-time symmetry that interchanges bosons with fermions.
- Even though it is not exact, supersymmetry can be relevant for elementary particle physics, since:
  - Consistent string theory vacua have space time supersymmetry.
  - Dynamically broken supersymmetry can explain the smallness of the electroweak scale, $m_W \sim 10^{-16} M_{Pl}$, and the masses of the superpartners. The idea of supersymmetry breaking at the electroweak scale is falsifiable in the near future.
- Recent advances in the study of supersymmetric quantum field theory allow us to exactly study aspects of the supersymmetry breaking dynamics. Many new models and mechanisms of supersymmetry breaking have been found in the last several years.
- These advances allow us to gain better insight into the phenomenological possibilities for supersymmetry breaking, make predictions, and rule out models.

On the purely theoretical side, recent developments in string theory—string duality, D-branes, and M-theory—offer hope of more complete understanding of the dynamics of supersymmetry breaking, in both supergravity and gauge mediated frameworks. Perhaps some progress can be made towards solving some of the many outstanding problems of supersymmetric phenomenology (the stabilization of the dilaton, the cosmological constant problem, etc.).

As emphasized in the text, there is no “standard model” of supersymmetry breaking. There exist a variety of models, none of which is entirely phenomenologically satisfactory, or has a particular aesthetic appeal. In view of this, it appears that experimental input is important for the further development of the field. One’s hope is that a discovery of supersymmetry will yield important hints towards the true mechanism of supersymmetry breaking.

7. Acknowledgments

It is a pleasure to thank W. Skiba for comments on the manuscript. This work was supported by DOE contract no. DOE-FG03-97ER40506.

8. References
1. L.D. Landau and I.M. Lifshitz, *Quantum Mechanics*, Ch. XV, §111 (Pergamon Press, 1982).
2. A standard textbook on supersymmetry is:
   J. Wess and J. Bagger, *Supersymmetry and Supergravity*, (Princeton University Press, 1992);
3. H.-P. Nilles, *Supersymmetry, Supergravity and Particle Physics, Phys. Rep.* **110** (1984) 1.
4. H. Haber and G. Kane, *The Search for Supersymmetry: Probing Physics Beyond the Standard Model, Phys. Rep.* **117** (1985) 75.
5. A recent introduction to supersymmetry, the supersymmetric standard model, and its experimental signatures is:
   S.P. Martin, *A Supersymmetry Primer*, to appear in *Perspectives on Supersymmetry*, G. Kane (ed.), hep-ph/9709356.
6. E. Witten, *Dynamical Breaking of Supersymmetry, Nucl. Phys. B* **188** (1981) 513.
7. For reviews on the recent developments in supersymmetric field theory, see:
   K. Intriligator and N. Seiberg, *Lectures on Supersymmetric Gauge Theory and Electric-Magnetic Duality, Nucl. Phys. Proc. Suppl.* **45BC** (1996) 1, hep-th/9509066.
   M. Peskin, *Duality in Supersymmetric Yang-Mills Theory, Lectures at TASI-96*, hep-th/9702034.
   M. Shifman, *Nonperturbative Dynamics in Supersymmetric Gauge Theories, Lectures at the “Enrico Fermi” School, 1996*, hep-th/9704114.
8. W. Skiba, *Dynamical Supersymmetry Breaking*, Mod. Phys. Lett. **A12** (1997) 737, hep-th/9703159.
   A.E. Nelson, *Dynamical Supersymmetry Breaking*, Talk at SUSY-97, hep-ph/9707442.
9. C. Kolda, *Gauge Mediated Supersymmetry Breaking: Introduction, Review and Update*, Talk at SUSY-97, hep-ph/9707450.
10. For recent reviews of future experiments, their reach, and possible implications, see:
    J. Bagger, *Supersymmetry at the LHC and NLC*, Talk at SUSY-97, hep-ph/9709335.
    G. Kane, *Sphenomenology: An Overview, with a Focus on a Higgsino LSP world, and on Eventual Tests of String Theory*, Talk at SUSY-97, hep-ph/9709318.