Is there evidence for cosmic anisotropy in the polarization of distant radio sources?

Sean M. Carroll  
*Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA*  
E-mail: carroll@itp.ucsb.edu

George B. Field  
*Harvard-Smithsonian Center for Astrophysics, 60 Garden St., Cambridge, Massachusetts 02138, USA*  
E-mail: field@cfa.harvard.edu

**Abstract**

Measurements of the polarization angle and orientation of cosmological radio sources may be used to search for unusual effects in the propagation of light through the universe. Recently, Nodland and Ralston have claimed to find evidence for a redshift- and direction-dependent rotation effect in existing data. We re-examine these data and argue that there is no statistically significant signal present. We are able to place stringent limits on hypothetical chiral interactions of photons propagating through spacetime.
1 Introduction

The polarization of radiation emitted by distant radio galaxies and quasars offers a way to search for chiral effects in the propagation of electromagnetic radiation. Such objects are often elongated in one direction, so that one may define a position angle $\psi$ which describes the orientation of the object on the sky. Synchrotron radiation can lead to a significant linear polarization of the source, and the angle $\chi$ of the plane of polarization may also be measured [1]. (The angle of polarization will typically undergo Faraday rotation, but this effect can be removed by using the fact that Faraday rotation is proportional to the square of the wavelength.) One can therefore study the relative angle $\chi - \psi$ between the position and polarization vectors, keeping in mind that this quantity is only defined modulo $180^\circ$. It has been found [2, 3] that $\chi - \psi$ is not distributed randomly; there is a large peak at $\chi - \psi \approx 90^\circ$, and a smaller enhancement at $\chi - \psi \approx 0^\circ$. Since many of these sources are at significant redshifts, and therefore very far away, testing whether this relationship is maintained for distant sources provides constraints on possible chiral effects on the propagation of light through the universe, which could rotate $\chi - \psi$ away from the intrinsic value (that which would be measured at the source).

From a field theory point of view, the simplest such chiral effect arises from a Lagrange density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \phi F_{\mu\nu}\tilde{F}^{\mu\nu}. \quad (1)$$

Here, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength for the electromagnetic field $A_\mu$, $\tilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual field strength, and $\phi$ is a pseudoscalar field which need not be fundamental (it can be a function of other fields in the theory). We set $c = 1$ throughout. This Lagrangian is the simplest way to couple a neutral pseudoscalar to electromagnetism in a parity-invariant way, and often describes the effective coupling of pseudoscalar particles (such as pions or axions) to photons.

Our interest here is in the case where $\phi$ varies only very slowly over extremely large distances. In that case an electromagnetic wave travelling through the background $\phi$ field will undergo a rotation in its polarization state which depends on the change in $\phi$; such an effect arises in a variety of contexts [4]-[18]. In the WKB limit where the length scale for variations in $\phi$ is much larger than the wavelength of the photon, the polarization angle $\chi$ obeys the simple relation

$$\Delta \chi = \Delta \phi, \quad (2)$$

where $\Delta$ indicates the change between source and observer. (Here, and in Eq. (3) below, $\Delta \chi$
is measured in radians; elsewhere we measure all angles in degrees; no confusion should arise.)

This effect is independent of wavelength, and can therefore be distinguished from ordinary
Faraday rotation. Carroll, Field and Jackiw [10] suggested that observations of polarized
radio sources provide a stringent test of such an effect, since they afford an opportunity to
constrain $\Delta \phi$ over a large interval in space and time (see also [19, 20]).

The specific model investigated in [10] set $\partial_{\mu} \phi = -\frac{1}{2} p_{\mu}$, where $p_{\mu}$ is a 4-vector whose
expectation value parameterizes violation of Lorentz invariance (as well as CPT [21]). It
was hypothesized that there exists a preferred coordinate frame, close to the background
Robertson-Walker frame of our universe, in which $\partial_{\mu} p_{\nu} = 0$. This implies that the predicted
rotation of the polarization angle for a source at redshift $z$ is given in terms of the timelike
component $p_0$ and the spacelike vector $\vec{p}$ by

$$\Delta \chi = -\frac{1}{2} r (p_0 - p \cos \theta) ,$$

where $p = |\vec{p}| = (\delta^{ij} p_i p_j)^{1/2}$, $\theta$ is the angle between $\vec{p}$ and the direction toward the source,
and $r$ is the proper spacelike distance traveled. If we take a flat ($k = 1$) universe as a
reasonable approximation, we have

$$r = \frac{2}{3 H_0} \left[ 1 - (1 + z)^{-3/2} \right] ,$$

where $H_0$ is the Hubble constant today. Regardless of whether or not one is interested in
tests of Lorentz invariance, Eq. (3) is a useful parameterization of potentially observable
chiral effects.

In [10] it was shown that the radio galaxies at redshift greater than 0.4, with maximum
polarizations greater than 5%, were strongly clustered around $\chi - \psi \approx 90^\circ$, using a sample of
galaxies and redshifts obtained from the literature [2, 3, 22, 23]. Assuming that the timelike
component $p_0$ would be significantly larger than the spacelike part $\vec{p}$, the limit

$$p_0 \leq 1.7 \times 10^{-42} h_0 \text{ GeV}$$

was obtained, where $h_0 = H_0/100 \text{ km/sec/Mpc}$. Recently, Nodland and Ralston [24], using
the same set of data, searched for anisotropic effects such as those that would arise from
a nonzero spacelike part $\vec{p}$ in Eq. (3). Surprisingly, they claimed to find a significant signal

\footnote{As mentioned in [24], there were a handful of transcription errors in the table published in [10]. The
corrected values are: entry 9, coordinates 0106+72; entry 35, coordinates 0459+25; entry 84, $\psi = 39^\circ$; entry
124, coordinates 1626+27; entry 144, $z = 0.054$; and entry 153, $z = 0.0244$. We are grateful to Borge
Nodland for informing us of these corrections.}
in the data. Given the fundamental importance of such a result, we have undertaken a re-
examination of the data, and present our results in this paper. We conclude that the data
are most consistent with no effect, contrary to [24]. Our disagreement stems primarily from
the method used to disentangle the $180^\circ$ ambiguity in the quantity $\chi - \psi$, and the use of
randomly generated data for comparison purposes, as will be shown below.

As this manuscript was being completed we received a preprint by Eisenstein and Bunn
[25], who come to conclusions similar to those expressed in this paper.

2 Data

We consider the same set of data as was used in [10], with the corrections noted above. This
set includes 160 sources, with redshifts as high as 2.012. The first step is to establish the
existence of a reliable correlation between $\chi$ and $\psi$, so that we may test its behavior for
distant galaxies. Following [24], we have divided the data into distant and nearby objects,
with the division drawn at $z = 0.3$; there are 89 sources with $z < 0.3$, and 71 with $z \geq 0.3$.
In Figures One and Two, we have plotted histograms of $\chi - \psi$ for these two sets. We have for
the purposes of these figures defined $\chi - \psi$ so that it lies between $0^\circ$ and $180^\circ$, and grouped
the data into bins which are $10^\circ$ wide.

In Figure One, representing nearby galaxies, there appears to be evidence for a narrow
enhancement at $\chi - \psi \approx 0^\circ$, and a broader peak at $\chi - \psi \approx 90^\circ$. However, the correlation
is evidently not very strong. The most likely explanation for this fact is that many of these
galaxies are members of a different population than those at high redshift, which can be
observed only if they are of high luminosity, and the lower-luminosity galaxies demonstrate
a weaker correlation between polarization and position angle. For the more distant galaxies
shown in Figure Two, there is a very clear peak at $\chi - \psi \approx 90^\circ$. There is no noticeable peak
at $\chi - \psi \approx 0^\circ$ in this sample; again, this may be explained if the galaxies with $\chi - \psi \approx 0^\circ$
are members of a lower-luminosity population. (See [2, 3] for discussion of these correlations
and their interpretation in terms of models of the sources.)

It is reasonable to suppose that the galaxies with a higher degree of maximum polarization
would show any effect more strongly than those that are polarized only weakly. We therefore
show in Figures Three and Four the same plots as in Figures One and Two, this time limited
only to those sources with maximum polarization greater than or equal to 5%. These are
the sources that were analyzed in [10].

The peaks found in the previous plots, at $0^\circ$ and $90^\circ$ at low redshifts and more dramat-
Figure 1: Histogram of number of galaxies vs. $\chi - \psi$, for galaxies with $z < 0.3$.

Galaxies with redshift $z < 0.3$
and maximum polarization $\geq 0\%$
Figure 2: Histogram of number of galaxies vs. $\chi - \psi$, for galaxies with $z \geq 0.3$.
Figure 3: Histogram of number of galaxies vs. $\chi - \psi$, for galaxies with $z < 0.3$ and maximum polarization $\geq 5\%$. 

Galaxies with redshift $z < 0.3$

and maximum polarization $\geq 5\%$.
Figure 4: Histogram of number of galaxies vs. $\chi - \psi$, for galaxies with $z \geq 0.3$ and maximum polarization $\geq 5\%$. 

*Galaxies with redshift $z \geq 0.3$ and maximum polarization $\geq 5\%$.***
ically at 90° in the high-redshift sample, are also evident in Figure Three and Figure Four, arguably more convincingly. However, any increased correlation is offset to some degree by the smaller number of data points. Therefore, in the remainder of this paper we will not discard those sources with maximum polarization < 5%; we will thus use precisely the same data as were analyzed in [24].

The crux of our disagreement with [24] can be found in Figure Two, the distribution of $\chi - \psi$ for sources with $z \geq 0.3$. As noted in [10], this plot constitutes vivid evidence that the polarization angle in these sources is intrinsically perpendicular to the position angle on the sky. If the claim of [24] is true, it is necessary to believe that the peak at 90° is an accident, and these data are actually drawn from a distribution which is intrinsically centered at 0°, with position- and redshift-dependent contributions of order 180°. We will argue in the next section that this is not the simplest interpretation of these data.

3 Constraints on chiral effects

Searching for a signal in the polarization data is complicated by the fact that $\chi - \psi$ is only defined modulo 180°. In testing any specific hypothesis, it is necessary to choose some reasonable procedure for resolving this ambiguity. The method chosen by the authors of [24] was the following: for any choice of direction for the vector $\vec{p}$, define an angle $\beta = \chi - \psi \pm 180°$ which is between 0° and 180° if $\cos \theta \geq 0$ and between $-180°$ and 0° if $\cos \theta < 0$, where $\theta$ (which they called $\gamma$) is the angle between $\vec{p}$ and the direction toward the source.

It was noted in [24] that this procedure necessarily introduces correlations between $\beta$ and $r \cos \theta$. It would be illegitimate, therefore, to take a statistical correlation between these two quantities as itself evidence of a signal in the data. However, if the degree of correlation were much higher than that which would be expected if there were no signal in the data, we might conclude that there was a measurable effect.

It is at this point in the analysis that we find two important flaws in the procedure followed in [24]. First, one must reliably determine the zero point for $\chi - \psi$, which would be observed in the absence of any chiral effects. In [24], the authors searched for a best fit to the data of the form $\beta = (1/2)\Lambda^{-1}s r \cos \theta + \delta$, where in the notation of Eq. 3, $\Lambda_s = p^{-1}$. They found that the favored value for the zero point was $\delta \approx 0°$. This seems to be inconsistent with the evidence of Figure Two, which exhibits a peak at 90°. The resolution is simply the fact that the definition of $\beta$, as described above, separates the data into two groups, one with $-180° < \beta < 0°$ and one with $0° < \beta < 180°$. With this procedure the favored value
for $\delta$ will always be near $0^\circ$; it arises essentially from taking the average of a group of points clustered around $90^\circ$ and another clustered around $-90^\circ$. This method of resolving the $180^\circ$ ambiguity is therefore inappropriate for data which lie naturally in the vicinity of $90^\circ$.

Nevertheless, [24] argues that the correlation found is statistically significant, as it was only very rarely reproduced in artificially generated sets of data. The procedure for generating these sets is the second important flaw that we find. Figure Two provides evidence that, regardless of the position of the source on the sky, $\chi - \psi$ is distributed approximately in a Gaussian distribution centered on $90^\circ$; a best fit to the Gaussian yields a dispersion of $\sigma = 33^\circ$. Therefore, in searching for position-dependent effects, it is appropriate to compare the actual data to data which is generated by drawing from a similar distribution. In [24], on the other hand, artificial realizations were generated completely randomly, i.e. from a flat probability distribution for $\chi - \psi$. This has a dramatic effect on the claimed significance of the result. We performed an independent analysis using two different methods of generating the artificial data sets: first by drawing from a flat distribution, and then from a Gaussian with the appropriate width. The numbers generated were values of $\chi - \psi$ for the positions and redshifts of the 71 sources in the sample with $z \geq 0.3$. In 1000 realizations of the data drawn from a flat distribution, in only 7 trials was the significance of the correlation greater than that in the actual data; this is comparable to the 6 out of 1000 reported in [24], and if reliable would be evidence of the existence of a signal. On the other hand, in 1000 realizations of the data drawn from the appropriate Gaussian distribution, the artificial data was more strongly correlated with the hypothesized test function in 911 out of 1000 trials. Even if there were no signal at all in the data, we would expect the artificial realizations to have a stronger correlation approximately 50% of the time; the fact that our trials had better correlations over 90% of the time is due to the fact that the Gaussian slightly underestimates the number of data points near $0^\circ$. This result, however, vividly demonstrates our main point: the existence of a real enhancement of $\chi - \psi$ near $90^\circ$ leads to a spuriously large correlation coefficient if one uses the procedure described in [24]. When this enhancement, which is consistent with conventional models of the sources, is taken into account, there is no sign of an additional effect such as that in Eq. 3.

There is another way of quantifying our claim that a random distribution centered around

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2There are two procedures described in [24]. In the first, the correlation in the artificial data sets was calculated using the best-fit direction obtained from the real data. As the hypothesis being tested is that there is some direction of anisotropy, not that a specific direction is picked out a priori, it is more appropriate to compare correlations in each set of data (real and artificial) calculated using the best-fit direction for that set. This is equivalent to the second procedure in [24], and is the method followed here.
90° is a better fit to the data than the correlation proposed in [24]. Figure Five is a plot of \( \chi - \psi \) as a function of \( r \cos \theta \), where \( \theta \) is defined using the best-fit direction quoted in [24] and \( \chi - \psi \) is defined to be between 0° and 180°. We may think of this graph as being defined on a cylinder, where 0° is to be identified with 180°. With this in mind, we have plotted two possible relationships, a solid horizontal line at 90° and a dashed line at \((1/2)\Lambda_s^{-1}r \cos \theta + \delta\), where we have measured the parameters \( \Lambda_s \) and \( \delta \) from Figure 1(d) of [24]. If the relationship claimed in [24] is correct, the dashed line wrapping around the cylinder should be a better fit to the data than the solid horizontal line. This can be measured by calculating

\[
\chi^2 = \sum_i \left( \frac{\Delta_i}{\sigma} \right)^2 ,
\]

where we take the average error to be \( \sigma = 33° \), although the precise value is irrelevant for purposes of comparison. The quantity \( \Delta_i \), which represents the difference between the predicted and measured value of \( \chi - \psi \), is of course subject to the 180° ambiguity; however, we can resolve this ambiguity optimistically for each point, by defining \(-90° < \Delta_i < 90°\). Using this procedure, we calculate that the best fit proposed in [24] yields \( \chi^2 = 161 \), while the hypothesis of no effect yields \( \chi^2 = 69 \). Thus, the horizontal solid line in Figure Five is a much better fit than the diagonal dashed lines.

Given that there is ample evidence that the intrinsic zero point is centered on \( \chi - \psi = 90° \), we may ask how good a limit we can place on an effect such as that in Eq. (3). One approach to this problem is to define \( \chi - \psi \) to be between 0° and 180°, and to assume that the deviation from the intrinsic value is given by \( \Delta \chi = \chi - \psi - 90° \). It is then possible to do a straightforward least-squares fit to Eq. (3), with the four components of \( p_\mu \) as free parameters. Using the data at redshifts \( z \geq 0.3 \), the best-fit parameters obtained in this way are

\[
\begin{align*}
p_0 &= (0.59 \pm 0.80)H_0 \\
&= (1.25 \pm 1.71) \times 10^{-42}h_0 \text{ GeV} , \\
|\vec{p}| &= (1.13 \pm 1.40)H_0 \\
&= (2.41 \pm 2.99) \times 10^{-42}h_0 \text{ GeV} .
\end{align*}
\]

(This procedure yields separate values for each of the three spacelike components \( p_i \); since each value is consistent with no preferred direction, it is more appropriate to quote the limit on the magnitude \( |\vec{p}| \).) These values are consistent with \( p_\mu = 0 \), and similar to the limit on \( p_0 \) from [10] quoted in Eq. (3).
Figure 5: The difference between polarization and position angles as a function of $r \cos \theta$ for the best-fit direction of anisotropy proposed in [24]. Angles of 180° are to be identified with 0°; the data thus live on a cylinder. The solid line represents the predicted relationship in the absence of any signal, while the diagonal dashed line wrapping around the cylinder represents the model suggested in [24].
4 Discussion

After analyzing the data in a variety of ways, we are able to conclude with confidence that there is no evidence for a chiral effect on the propagation of photons from distant radio sources. Despite this negative result, there are still good reasons to further pursue observations such as those examined in this paper.

In Figures Six and Seven we have plotted the position of the sources in the sky, indicated by symbols related to the deviation of $\chi - \psi$ from $90^\circ$. Figure Six includes all of the galaxies, while Figure Seven is limited to the distant sources with $z \geq 0.3$. The squares represent sources with $\chi - \psi < 90^\circ$, while the $\times$'s are sources with $\chi - \psi > 90^\circ$. The size of the symbol is related linearly to the deviation from $90^\circ$, although for clarity there is an offset so that points with $\chi - \psi$ very close to $90^\circ$ still have a nonzero size. One conclusion to be drawn immediately from these graphs is that there is a need for additional data to be collected in the southern celestial hemisphere, especially at high redshifts. In the future, observations of polarization of the cosmic microwave background may be the best source of data for constraining phenomena such as these \[13, 20\].

In characterizing the limits one can place on chiral effects, for convenience we hypothesized a fixed four-vector $p_{\mu}$ which would represent a violation of Lorentz invariance. If an effect were to be found, however, it is by no means necessary that such a profound conclusion would have to be drawn. A more plausible hypothesis would be that of a very slowly-varying scalar field $\phi$ with a coupling as in \[1\]; the application of the data discussed in this paper to this possibility was examined in \[12\]. Such a field could arise as an ultralight axion, with mass of order the Hubble constant today (or less). Interestingly, such axions may appear naturally in the strongly coupled limit of heterotic string theory \[27, 28\]. Another possibility is the detection of axion-like cosmic strings; in the vicinity of such a string, the polarization angle of two light rays passing on either side will undergo rotations in opposite directions \[9\]. Although there is no obvious sign of such a signal in Figures Six and Seven, the importance of such a finding encourages us to continue the search.
Figure 6: Positions of radio sources on the sky, including all redshifts. The symbols indicate deviations from $\chi - \psi = 90^\circ$; squares are sources with $\chi - \psi < 90^\circ$, and crosses are sources with $\chi - \psi > 90^\circ$. The size of the symbol indicates the amount of deviation from 90°.
Figure 7: Same as Figure Six, including only galaxies with \( z \geq 0.3 \).
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