The Algorithm of Local Modulus Maximum Detection Based on B-spline Wavelet

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Abstract. At the time of fault, the system signal performances singularity, and the fault state can be diagnosed effectively through effective analysis and identification of singular signals. In this paper, for the problem that the singular signal modulus maximum point detection is inaccurate and the polarity is confused due to the improper selection of wavelet basis, the algorithm of local modulus maximum detection based on b-spline wavelet is proposed. B-spline scaling function is constructed by interpolation formula, the filter coefficients of b-spline wavelet function is calculated, in the process of signal decomposition of b-spline wavelet function and the effect of endpoint effect on signal singularity is removed, the signal local modulus maximum in the wavelet decomposition coefficient is extracted to accurately determine the time when the fault occurs. The result of simulation and experimental shows that the algorithm can not only detect the location of the maximum value of the singular point modulus accurately, but also distinguish the polarity of the modulus maximum and have good anti noise characteristics.

1. Introduction

In signal system analysis, it is often encountered that the signal itself has a discontinuous point or its derivative has a discontinuous point, which is defined as a singular signal. The jump point, one of the important characteristics of the signal, is defined the singular value point of the signal, which contains very important information of the signal. Therefore, it is necessary to accurately detect and locate the location of the singularity point of the signal, which is of great significance to the detection and analysis of the mutation signal. In the aspect of singularity detection, domestic and foreign scholars have done a lot of research work. Multi-resolution singular value decomposition is proposed in literature [1], which can identify the intervals of singular points and is not accurate enough. In literature [2], a two-dimensional matrix is constructed and the signal is decomposed by singular matrix, which can accurately locate the singular point of the signal, however, it is limited by the signal data. In the process of the matrix construction of the signal, the complexity of the data window will greatly increase the time complexity of the algorithm, and it is not easy to implement. Wavelet transform with time-frequency characteristics is used to detect the point of signal singular value in literature [3-5], the accurate positioning of the singular value point modulus maximum is realized. Due to the selection of wavelet function, the polarity of the modulus maximum is chaotic. The relation between wavelet transform coefficient and wavelet transform scale is provided in literature [6-7] and the singular value points on each scale are solved by curve fitting. With the error of the curve fitting itself, the location of the singular value of the signal will be deviant. Electric field is detected by the wavelet analysis modulus maximum in literature [8]. Signal modulus maximum point is detected by wavelet analysis in...
literature [9-10]. Due to the selection of the base in the decomposition process, the singular value point can only detect the amplitude and polarity cannot be processed. Wavelet transform shows well time-frequency characteristics in the analysis of singular signals and is used widely in the analysis of singular signal. But at the same time, due to the poor anti-interference ability of the wavelet decomposition, it can cause the problem of inaccuracy and polarization of singular point modulus maximum. Therefore, the b-spline wavelet function is constructed in this paper, which is used as the basis function of wavelet transform. Based on this, a local modulus maximum detection algorithm is proposed. The result of simulation and experimental shows that the algorithm can not only accurately detect the location of singular value point modulus maximal value, but also distinguish the polarity of the modulus maximal value and have good anti-interference ability.

2. Construction of B-spline Scale Function and Calculation of the filter coefficients

From the Fourier series and Taylor's formula, a signal can be superimposed by the corresponding basic functions. To obtain the b-spline scaling function, a base must also be selected. In this paper, the basis scale function is that the zero spline function is shifted to the right by 1/2 units.

\[
N_0(u) = \begin{cases} 
1 & u \in [0,1) \\
0 & \text{others} 
\end{cases}
\] (1)

For the basic b-spline function of the zero-order, there is a formula (2):

\[
\int_{u_i}^{u_{i+1}} N_{i,0}(u) \, du = 1
\]

\[
\int_{u_i}^{u_{i+1}} N_{i,0}(u)N_{j,0}(u) = 0, i \neq j
\] (2)

\(u_i\) and \(u_{i+1}\) are non negative integers separated by one unit, \(N_{i,0}(u)\) and \(N_{j,0}(u)\) are functions on different intervals with an amplitude of 1. From equation 1 and equation 2, it can be seen that the zero-order basic b-spline function which satisfies the characteristic of orthogonality and non-negative, can be used as the scale function in wavelet analysis. With the Cox-de Boor formula in literature [11] as the equation, we can construct any b-spline scale function based on the zero-order basic b-spline scale function.

\[
N_{i,p}(u) = \begin{cases} 
\frac{u-u_i}{u_{i+p}-u_i}N_{i,p}(u) + \frac{u_{i+p+1}-u}{u_{i+p+1}-u_{i+1}}N_{i+1,p-1}(u) & \text{if } u_i \leq u \leq u_{i+1} \\
0 & \text{otherwise} 
\end{cases}
\] (3)

From the formula 3, two-order b-spline scale function is equation 4.

\[
N_{0,2}(u) = \begin{cases} 
\frac{1}{2}u^2 & u \in [0,1) \\
\frac{1}{2}u(2-u) + \frac{1}{2}(3-u)(u-1) & u \in [1,2) \\
\frac{1}{2}(3-u)(3-u) & u \in [2,3) \\
0 & \text{others} 
\end{cases}
\] (4)

The corresponding filter coefficient is the scale vector by the two-scale equation in literature [12]. The frequency domain of the two- scale equation is formula 5.

\[
\psi_L(2\omega) = H_0(\omega)\psi(\omega)
\]

\[
\psi_H(2\omega) = H_1(\omega)\psi(\omega)
\] (5)

According to the two point principle, the frequency domain relationship between adjacent scales is
formula 6.

\[ |\psi(\omega)|^2 = |\psi_L(2\omega)|^2 + |\psi_H(2\omega)|^2 \]  

(6)

By pass the equation 5 and equation 6, the same scaling relationship between high-pass filter and low-pass filter frequency domain is as follows:

\[ H^2_H(\omega) + H^2_L(\omega) = 1 \]  

(7)

The corresponding Z domain filter is as follows:

\[ H_0(z)H_0(z^{-1}) + H_1(z)H_1(z^{-1}) = 1 \]  

(8)

The Fourier transform is took for formula 4, the scale vector and the wavelet function are obtained by the filter relation. The low-pass filter of the two-order b-spline scale function in the two-scale equation is as follows:

\[ H_0^2(\omega) = \frac{\psi_L(2\omega)}{\psi(\omega)} = \frac{1}{8} + \frac{3}{8}e^{-j\omega} + \frac{3}{8}e^{-2j\omega} + \frac{1}{8}e^{-3j\omega} \]  

(9)

The low-pass filter in Z domain is as follows:

\[ H_0^2(z) = \frac{\psi_L(2z)}{\psi(z)} = \frac{1}{8} + \frac{3}{8}z^{-1} + \frac{3}{8}z^{-2} + \frac{1}{8}z^{-3} \]  

(10)

Scaling vector is as follows:

\[ h_0^2(0) = \frac{1}{8}, h_0^2(1) = \frac{3}{8}, h_0^2(2) = \frac{3}{8}, h_0^2(3) = \frac{1}{8} \]

From the two-scale equation, the high-pass filter in Z domain is as follows:

\[ H_1^2(z) = -\frac{1}{4} + \frac{3}{4}z^{-1} + \frac{3}{4}z^{-2} - \frac{1}{4}z^{-3} \]  

(11)

The high-pass filter coefficient is as follows:

\[ h_1^2(0) = -\frac{1}{4}, h_1^2(1) = \frac{3}{4}, h_1^2(2) = \frac{3}{4}, h_1^2(3) = -\frac{1}{4} \]

The two-order b-spline scale function and wavelet function can be obtained in the same ways, as shown in Fig 1.

![Scaling function and Wavelet function](image_url)

(a) scaling function  (b) wavelet function

Fig. 1 Two-order b-spline scale function and wavelet function

The above analysis shows the decomposition filter coefficients of the b-spline function at various scales, and the different frequency bands can be divided by the filter coefficients at different scales.
3. The Algorithm of Singular Signal Local Modulus Maximum Detection

In engineering applications, most of the signals are non-stationary and singular signals in literature [13]. Due to its time-frequency characteristics, wavelet analysis is often used to detect the modulus maximum of singular signals. In this paper, the three adjacent factorization coefficients in a field after the wavelet decomposition are difference, so the algorithm is that the equation is shown as follow:

\[
\begin{align*}
&W_f(s,N) > W_f(s,N-1) \\
&W_f(s,N) > W_f(s,N+1) \\
&W_f(s,N) > \beta W_f^{\text{max}}(s,N)
\end{align*}
\]  

(12)

\(W_f(s,N)\) is the stable wavelet transform coefficient, \(\beta\) is the variable constant, its range is between 0 and 1, \(s\) is number of layers, \(N\) is the number of sample points and \(N \geq 1\). If the wavelet transform coefficients satisfy equation 12, it will be defined local modulus maximum. By setting the threshold, the influence of the system noise to the singularity point of the signal is avoided, which indirectly solves the problem of the weak anti-interference ability of the wavelet transform. The algorithm flow is shown in Fig 2.

![Algorithm flow diagram](image)

Fig. 2 Algorithm flow.

4. The Example of Singular signal modulus maximum detection

4.1 Simulation Verification

In the power system, zero sequence current signal can represent the operating state in literature [14].
When single-phase ground fault occurs in the power system, the magnitude of the zero sequence current signal can distinguish the fault branch and the functional branch, the zero sequence current signal is shown in Fig 3.

![Zero sequence current signal](image)

Fig. 3 Zero sequence current signal

The fourth level wavelet decomposition coefficient is used to obtain the location and polarity of the modulus maximum point of zero sequence current signal by using local modulus maximum algorithm, as shown in Fig 4. From Fig 4, d11, d22, d33, d44 is the modulus maximum of $i_1$, $i_2$, $i_3$, $i_4$. The algorithm of local modulus maximum detection in this paper not only can accurately detect the location of the point of modulus maximum, and can determine the polarity. It can distinguish fault branch and the branch by polarity. From Fig 4, the polarity of d22 is different from the other branch, so it is clearly know that the second branch is the fault.

5. Conclusion

In this paper, b-spline scaling function based on the zero-order b-spline function through the Cox-de Boor interpolation formula and a local modulus maximum detection algorithm based on the b-spline singular signal are proposed. The simulations and experiments show that the algorithm in this paper can correctly calibrate the signal singular value maximum and polarities and it can be extremely robust for arbitrary singular signals. It has high anti-interference performance and has high practical value in practical application.
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