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Optimal surveillance mitigation of COVID’19 disease outbreak: Fractional order optimal control of compartment model

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ABSTRACT
In present time, the whole world is in the phase of war against the deadly pandemic COVID’19 and working on different interventions in this regard. Variety of strategies are taken into account from ground level to the state to reduce the transmission rate. For this purpose, the epidemiologists are also augmenting their contribution in structuring such models that could depict a scheme to diminish the basic reproduction number. These tactics also include the awareness campaigns initiated by the stakeholders through digital, print media and etc. Analyzing the cost and profit effectiveness of these tactics, we design an optimal control dynamical model to study the proficiency of each strategy in reducing the virulence of COVID’19. The aim is to illustrate the memory effect on the dynamics of COVID’19 with and without prevention measures through fractional calculus. Therefore, the structure of the model is in line with generalized proportional fractional derivative to assess the effects at each chronological change. Awareness about using medical mask, social distancing, frequent use of sanitizer or cleaning hand and supportive care during treatment are the strategies followed worldwide in this fight. Taking these into consideration, the optimal objective function proposed for the surveillance mitigation of COVID’19, is contemplated as the cost function. The effect analysis is supported through graphs and tabulated values. In addition, sensitivity inspection of basic reproduction number is also carried out with respect to different values of fractional index and cost function. Ultimately, social distancing and supportive care of infected are found to be significant in decreasing the basic reproduction number more rapidly.

Introduction
A deadly coronavirus that basically initiated from Wuhan city of China, all of a sudden incarcere the people all around the world. This strain of severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2) has affected more than 210 countries and territories. It has brought devastating consequences on public health as well as on social and economic activities. Governments around the world prompted surveillance on mitigating the global spread of COVID’19. Among many of these dramatic measures, majority are substantiating to be effective in reducing the virus transmission. Imposing curfew and locking down the cities in addition public awareness campaigns such as, stay-at-home, encouraging social distancing, cleanliness that include frequent washing hand, using sanitizers through digital and print media are the key measures in restraining this virus. On enforcing these policies and engaging communities in these campaigns, undoubtedly enormous social and economic cost is expected. But until an effectual vaccine or treatment becomes available, these strategies may play important roles [1–6].

Variety of research has been conducted at an extraordinary pace to analyze the COVID’19 in different perspectives [7–9]. Epidemiological dynamical systems to control the breakout of this pandemic through basic reproduction number has been obtained by various researchers [10–12]. Clinical studies to determine therapeutic solutions through the findings of the biological features of this virus [13]. Perceptions on impact of government’s preventing strategies on other environmental,
social and economic activities [14]. Decision making models to consider an effective managing prevention strategy of COVID’19 transition [15]. Machine learning models to predict the high risk and efficiently triage the patients with high accuracy [16]. Mathematical models fit out to be substantial contrivances in investigating the dynamical controls of the infectious diseases [17]. Research articles, based on optimal control models can be found in the literature to a great extent in this regard [18–24]. In the recent times of battle against COVID’19, numerous authors have added their valuable contributions in this connection. Gri-gorieva et al. formulate two SEIR-type model to investigate the cost-effective quarantine strategies and analyzed the optimal solutions numerically [25]. Analysis of interventions of COVID’19 through transmission model and observing the most effective non-pharmaceutical strategies to lessen the disease nuisance in Pakistan is found in the literature [26]. In particular, plenty of endeavors have been carried out in different context of cost-effective strategy, to control the transmission of this deadly pandemic [27,28].

In this attempt, we design mathematical model that covers two major areas, epidemiology together with dynamical optimal control. Firstly, the compartmental model is taken into account with the control variables and stability analysis are carried out. Secondly, optimizing cost functions is subjected to the compartmental model to assess the cost-effectiveness of the prevention strategies. As aforementioned, there exist significant mathematical efforts in this connection, but the novelties that invigorate the proposed assessment can be classified as:

- This study is not only susceptible, exposed, quarantine, infected and recovery compartments, but also the isolation and precautions. Thus, the model is named as SEQIMRP i.e. susceptible-expose-quarantined-infected-isolated-recovered-protected.
- The non-pharmaceutical control variables, awareness campaigns about using mask, encouraging social distancing, signifying frequent use of sanitizer and washing hands, supportive care during treatment.
- Regulatory of basic reproduction number through these campaigns.
- Incorporating fractional order derivative for dynamical scrutiny of the model with.

This significant contribution will undoubtedly add great perspicacity of COVID’19 interventions. The proposed SEQIMRP model with proportional fractional [29] signifies the broader application of the fractional definition. Its expansion elegantly converts the fractional order derivative operator into integer order that the fractional order index reallocates linearly in the equations. By virtue of this, the dynamics of COVID’19, for instance the basic reproduction number and equilibrium points can be interpreted with memory effects. Subsequently, historical values of these parameters or the compartmental functions will enable to devise defensive precautionary steps, revealed from the past experiences. In addition, the effect of memory on the optimality of awareness strategies is also illustrated through the proportional fractional derivative. The designed system provides a novel contribution in epidemiological study of epidemic and pandemic diseases. It will instruct the healthcare researchers a new mode of generating results and might be capable to investigating prior information about the risk factors or transmission rate for preparatory measures. The remaining paper contains sections of formulating the dynamical system, stability analysis of equilibrium points and optimality assessment. Furthermore, numerical discussions are also carried out to evidently establish an effective conclusion.

**Model formulation for COVID’19 optimal control**

**Susceptible-expose-quarantined-infected isolated-recovered-protected (SEQIMRP)**

Mathematical models based on disease dynamics are quite helpful in studying the functional behavior of any virus, which then helps to overcome or lessen its contaminating breakout. The destructive coronavirus converted into a pandemic within a few months and affected billions of peoples around a globe. Early laboratory research and scientific experiments to construct a drug or vaccine could not triumph. Many epidemiological models also expressed significant contributions in this connection to determine the basic reproduction number and predict the dispersion, recovery and mortality rates [11,12,30]. Here, to analyze the dynamical behavior and impact of COVID’19 pandemic, a system of differential equations is designed with respect to compartmental classes and prevention measures on the basis of following assumptions.

- Regardless of different risk rate of COVID’19 for different age-group and pre-existing disease carriers, the model assumes a homogeneous mixing of individuals in the population.
- Prevention strategies: Usage of medical mask (mm), social distancing (sd), frequently cleaning hands (ch) and supportive care (sc) during treatments are taken into account as control variables of the optimal system.
- The individuals in any compartment, following the operational prevention strategies, are assumed as will not get infected and are defined by means of protected compartment.
- Susceptible is outlined in the form of logistic growth that encompass maximum sustainability to survive in the available resources in an environment.
- Exposed are quarantined that might recover and use prevention measures later to insulate themselves from virus.
- Treatment of infected COVID’19 patients is the isolation process, which is explained in the isolation compartment. These compartments with the supportive care from staff might recover and move to recovery compartment.
- Treated individuals after recovery do not participate in transmitting the disease as they use the operational prevention strategies.
- The assessments of basic reproduction number and stability analysis are carried out in fractional calculus environment.

Hence, the fractional order epidemiological model, susceptible-expose-quarantined-infected-isolated-recovered-protected (SEQIMRP) is mathematically signified as:

\[ D^\alpha_t S(t) = r_t S(t) \left( 1 - \frac{S(t)}{K_S} \right) - (mm(t) + sd(t) + ch(t)) S(t) - \beta_t S(t) I(t) - d_t S(t) \]

\[ D^\alpha_t E(t) = \beta_t S(t) I(t) - (mm(t) + sd(t) + ch(t)) E(t) - \gamma_E(t) - d_E E(t) \]

\[ D^\alpha_t Q(t) = \gamma_E(t) E(t) - (mm(t) + sd(t) + ch(t) + sc(t)) Q(t) - \eta Q(t) - \psi_Q Q(t) - d_Q Q(t) \]

\[ D^\alpha_t I(t) = \eta Q(t) - (mm(t) + sd(t) + ch(t) + sc(t)) I(t) - \sigma_I(t) - d_I I(t) \]

\[ D^\alpha_t M(t) = \sigma_I(t) - (mm(t) + sd(t) + ch(t) + sc(t)) M(t) - \psi_M M(t) - \rho M(t) \]

\[ D^\alpha_t R(t) = \psi_Q Q(t) + \psi_M M(t) - (mm(t) + sd(t) + ch(t)) R(t) - d_R R(t) \]

With initial conditions,

\[ S(0) = O_s, \quad E(0) = O_e, \quad Q(0) = O_q, \quad I(0) = O_i, \quad M(0) = O_m, \quad R(0) = O_r, \quad P(0) = O_p. \]

(2)
Variables and parameters of the SEQIMRP.

| Compart-mental functions | Descriptions | Units | Initial values (population = in millions & time = days) | Source |
|--------------------------|--------------|-------|--------------------------------------------------------|--------|
| N(t)                     | Total population | Population / day | 113 | Estimated |
| S(t)                     | Susceptible   | Population / day | 111 | Estimated |
| E(t)                     | Exposed       | Population / day | 0   | Estimated |
| Q(t)                     | Quarantined   | Population / day | 0   | Estimated |
| I(t)                     | Infected      | Population / day | 2   | Estimated |
| M(t)                     | Infected isolated | Population / day | 0   | Estimated |
| R(t)                     | Recovered     | Population / day | 0   | Estimated |
| P(t)                     | Protected     | Population / day | 0   | Estimated |
| α                        | Order of fractional derivative | Dimensionless | 0 < α < 1 | Fitted |

Parameters

| Descriptions | Units | Value | Source |
|--------------|-------|-------|--------|
| t            |       |       |        |
| β            |       | 14.781| Fitted |
| r            |       | 1.887 × 10⁷ | Fitted |
| η            |       | 0.13266 | Fitted |
| σ            |       | 0.0714 | Fitted |
| r₅           |       | 30    | Fitted |
| k₅           |       | 100,000 | Fitted |
| ρ            |       | 0.1259 | Fitted |
| ψₐ           |       | 1.782 × 10⁵ | [35] |
| ψ₉           |       | 0.11624 | Fitted |
| ψ₉₉          |       | 0.33029 | Fitted |
| dₛ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |
| dₚ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |
| dₑ           |       | 0.0714 | Fitted |

where, O₁(0) ∈ [0, 1] for i = 1, 2, ..., 7. Table 1 further elaborates the dimensions of all the variables and parameters of system (1). Moreover, pictorial demonstration of the compartmental system, representing the flow of the diseases transmission is also given in Fig. 1. Assume N(t) is the total population density of individuals that can be structured as:

\[ N(t) = S(t) + E(t) + Q(t) + I(t) + M(t) + R(t) + P(t) \]  (3)

Moreover, \( PF_i \) articulates proportional fractional derivative of order \( α \in (0, 1) \) [29], which can be expanded as for any continuous function \( y(t) \),

\[ PF_i y(t) = \ell_0(α, t) D^\alpha_y(t) + \ell_1(α, t)y(t), 0 < α < 1 \]  (4)

where, \( \ell_0(α, t) \neq 0 \) for \( α \in (0, 1) \), with \( \lim_{α→0^+} \ell_0(α, t) = 0 \) and \( \lim_{α→1} \ell_0(α, t) = 1 \).

Fig. 1. Pictorial illustration of SEQIMRP model.
t) = 1. Additionally, ξ_1(α, t) ≠ 0 for α ∈ [0, 1), with lim \( \alpha \to 0 \) ξ_1(α, t) = 1 and lim \( \alpha \to 1 \) ξ_1(α, t) = 0. Let, \( \xi_0(\alpha, t) = \alpha \) and \( \xi(\alpha, t) = 1 - \alpha \), so Eq. (4) becomes
\[
pf D_t^\alpha y(t) = \alpha \frac{dy(t)}{dt} + (1 - \alpha)y(t)
\] (5)

Assume that all control functions (prevention steps) are constant within time, therefore, by applying expansion (5) on system (1), we get the system as:
\[
\begin{align*}
\dot{S}(t) &= \frac{1}{\alpha} \left( r_s S(t) \left( 1 - \frac{S(t)}{k_s} \right) - (mm + sd + ch)S(t) - \beta S(t) I(t) - d_s S(t) - (1 - \alpha)S(t) \right) \\
\dot{E}(t) &= \frac{1}{\alpha} \left( \beta S(t) I(t) - (mm + sd + ch + sc)E(t) - \gamma E(t) - d_e E(t) - (1 - \alpha)E(t) \right) \\
\dot{Q}(t) &= \frac{1}{\alpha} \left( \gamma E(t) - (mm + sd + ch + sc)Q(t) - \eta Q(t) - \psi_0 Q(t) - d_q Q(t) - (1 - \alpha)Q(t) \right) \\
\dot{I}(t) &= \frac{1}{\alpha} \left( \eta Q(t) - (mm + sd + ch + sc)I(t) - \sigma I(t) - d_i I(t) - (1 - \alpha)I(t) \right)
\end{align*}
\] (6)
\[
\begin{align*}
\dot{M}(t) &= \frac{1}{\alpha} \left( \alpha I(t) - (mm + sd + ch + sc)M(t) - \psi_M M(t) - \rho M(t) - (1 - \alpha)M(t) \right) \\
\dot{R}(t) &= \frac{1}{\alpha} \left( \psi_M M(t) - \psi_M M(t) - (mm + sd + ch)R(t) - d_R R(t) - (1 - \alpha)R(t) \right) \\
\dot{P}(t) &= \frac{1}{\alpha} \left( (mm + sd + ch)(S(t) + E(t) + R(t)) + (mm + sd + ch + sc)Q(t) - d_p P(t) \right)
\end{align*}
\] with the same initial conditions (2). System (6) evidently depicts the lucidity of the proportional fractional derivative, which greatly reduces the manipulation complexities of system (1).

**Theorem 1. (Boundedness)** Let \( \Pi \in \mathbb{R}_+^2 \) be the set of all feasible solutions of the system (6), then there exists uniformly bounded subset of \( \mathbb{R}_+^2 \) such that:
\[
\Pi = \left\{ (S, E, Q, I, M, R, P) \in \mathbb{R}_+^7 : N(t)\leq \frac{r_s}{d_q k_s} \right\}
\] (7)

**Proof:** By applying proportional fractional derivative and its expansion, as defined in the Eqs. (4)-(5), on Eq. (3), we get the expression of the form:
\[
N(t) = \frac{1}{\alpha} \left( \dot{S}(t) + \dot{E}(t) + \dot{Q}(t) + \dot{I}(t) + \dot{M}(t) + \dot{R}(t) + \dot{P}(t) - (1 - \alpha)N(t) \right)
\] (8)

On simplifying by using system (6) and suppose \( d_q \) be total proportion of deaths in all compartments i.e.
\[
d_q N(t) = d_s S(t) + d_e E(t) + d_q Q(t) + d_i I(t) + d_R R(t) + \rho M(t) + d_p P(t)
\] (9)

In addition, since \( 0 < a \leq 1 \)
\[
N(t) \leq r_s S(t) \left( 1 - \frac{S(t)}{k_s} \right) - d_q N(t)
\] (10)

where \( 0 < \frac{r_s S(t)}{k_s} \leq 1 \), so the above inequality reduces to
\[
N(t) \leq \frac{r_s S(t)}{k_s} - d_q N(t)
\] (11)

On integrating
\[
N(t) \leq e^{-d_q t} N(0) + \frac{r_s}{d_q k_s}
\] (12)

Therefore as \( t \to \infty \), we obtained the final statement of boundedness as
\[
N(t) \leq \frac{r_s}{d_q k_s}
\] (13)

**Theorem 2. (Existence and Uniqueness)** Assume the matrix of right hand side of system (6) be the real-valued function \( \Lambda(F(t)) : \mathbb{R}_+^7 \to \mathbb{R}_+^7 \), such that \( \Lambda(F(t)) \) and \( \frac{\partial \Lambda(F(t))}{\partial F(t)} \) are continuous and
\[
\| \Lambda(F(t)) \| \leq \left( \frac{X}{\alpha} - 1 \right) \| F(t) \| , \quad \forall F(t) \in \mathbb{R}_+^7 \text{ and } 0 < a \leq 1
\] (14)

Then, satisfying the initial conditions (2), there exists a unique, non-negative and bounded solution of the system (6).

**Proof:** Boundedness of system (6) can be followed from Theorem 1, now assume, the system (6) can be expressed as:
\[
F(t) = \Lambda(F(t))
\]
where,
\[
F(t) = [S(t) \quad E(t) \quad Q(t) \quad I(t) \quad M(t) \quad R(t) \quad P(t)]^T
\] (15)

and
\[
\Lambda(F(t)) = \frac{1}{\alpha} \begin{bmatrix}
    r_s (1 - \frac{S(t)}{k_s}) - (mm + sd + ch)S(t) - \beta S(t) I(t) - d_s S(t) - (1 - \alpha)S(t) \\
    \beta S(t) I(t) - (mm + sd + ch)E(t) - \gamma E(t) - d_e E(t) - (1 - \alpha)E(t) \\
    \gamma E(t) - (mm + sd + ch + sc)Q(t) - \eta Q(t) - \psi_0 Q(t) - d_q Q(t) - (1 - \alpha)Q(t) \\
    \eta Q(t) - (mm + sd + ch + sc)I(t) - \sigma I(t) - d_i I(t) - (1 - \alpha)I(t) \\
    \sigma I(t) - (mm + sd + ch + sc)M(t) - \psi_M M(t) - \rho M(t) - (1 - \alpha)M(t) \\
    \psi_M M(t) - (mm + sd + ch + sc)R(t) - d_R R(t) - (1 - \alpha)R(t) \\
    (mm + sd + ch) (S(t) + E(t) + R(t)) + (mm + sd + ch + sc) Q(t) + I(t) + M(t) + \rho M(t) + d_p P(t) \\
    -d_p P - (1 - \alpha)P(t)
\end{bmatrix}
\] (16)
\( \Lambda(F(t)) = \frac{1}{\alpha}(\Omega F(t) + S(t) \Omega F(t) + I(t) \Omega F(t) - (\alpha - 1)F(t)) \) \hspace{1cm} (17)

such that

\[
\Omega_1 = \begin{bmatrix}
  r_s - M_1 - d_3 & 0 & 0 & 0 \\
  0 & -M_1 - \gamma - d_E & 0 & 0 \\
  0 & 0 & -M_2 - \eta - \psi_o - d_0 & 0 \\
  0 & 0 & 0 & -M_2 - \sigma - d_I \\
  M_1 & M_1 & M_2 & M_2
\end{bmatrix}
\]

\( M_1 = mm + sd + ch \)

\( M_2 = mm + sd + ch + sc \)

\( \Omega = [-r_s/k_0 \ 0]_{1;3} \) and \( \Omega_2 = [-\beta \ 0 \ \beta \ 0]_{1;7}. \) Then, Eq. (17) can be rewritten as,

\[
\| \Lambda(F(t)) \| \leq \frac{1}{\alpha} \| \Omega_1 F(t) + S(t) \Omega_2 F(t) + I(t) \Omega_3 F(t) - (\alpha - 1)F(t) \|
\]

Let \( X = \| \Omega_1 \| + \| \Omega_2 \| + \| \Omega_3 \|, \) so the final statement is achieved as for \( 0 < \alpha \leq 1, \)

\[
\| \Lambda(F(t)) \| \leq \frac{X}{\alpha} \| F(t) \|
\]

where \( \nu = \left( \frac{1}{\alpha} - 1 \right). \) Next, we prove the non-negativity of the solutions by using the positivity of initial conditions (2) i.e., \( Q > 0 \) for \( i = 1, 2, \ldots, 7. \) Considering first equation of system (6), it can be deduced to:

\[
S(t) = \frac{1}{\alpha} r_s S(t) \left( 1 - \frac{S(t)}{k_s} \right) - (mm + sd + ch) S(t) - \beta S(t) F(t) - d_s S(t) - (1 - \alpha) S(t)
\]

\[ \geq - \frac{1}{\alpha} (mm + sd + ch + d_s + (1 - \alpha)) S(t) \]

On manipulating, we get

\[
S(t) \geq \frac{1}{\alpha} e^{-\frac{(mm + sd + ch + d_s + (1 - \alpha)) t}{\alpha}}
\]

(18)

Since \( 0 \leq e^{-\frac{(mm + sd + ch + d_s + (1 - \alpha)) t}{\alpha}} \leq 1 \) for \( t > 0, \) therefore Eq. (18) reduces to,

\( S(t) \geq 0 \)

Thus, proved the non-negativity of \( S(t). \) Analogously, all the remaining equations of system (6) can be proved to have non-negative solutions with the assumption of positive initial conditions.

**Optimal control problem**

Furthermore, the dynamical model (6) of COVID’19 would be incomplete if the assumption of optimal control of infection and intervention cost is not incorporated. Therefore, we formulate optimal control problem by means of the cost function type of quadratic function as:

\[
J(Y_i, U_k) = \int_0^\infty \left( \sum_{i=1}^7 w_i Y_i^2 + \varphi_i mm^2 + \varphi_i s d^2 + \varphi_i ch^2 + \varphi_i \alpha c^2 \right) dt
\]

where, \( Y_i \geq 0 \) for \( i = 1, 2, \ldots, 7 \) are replace by \( S, E, Q, I, M, R, P, \)

respectively. Moreover, here \( w_i, \) for \( i = 1, 2, \ldots, 7, \) are the weights of human population cost, whereas \( \varphi_k, \) for \( K = 1, 2, 3, 4, \) are the weights of undertakent intervention cost for COVID’19. At this juncture, intervention cost comes from government campaigns of using mask, social distancing and frequently washing hand. In addition, the hospitalization cost for drugs, ventilators and trained medical staffs for supportive care of the COVID’19 infected individuals also become higher with the increase in number of patients. Therefore, if greater cost is implemented of campaigns of enforcing the people on usage of mask, social distancing and frequently washing hand will reduce the COVID’19 transmission, which on the other hand it reduces the supportive care cost. Thus, we assume \( \varphi_k > 0 \) for \( K = 1, 2, 3, 4. \) Analogously, the objective of the present scenario is to control the spread out of COVID’19, which ultimately leads to minimize the infected individuals, therefore we consider \( w_i \) to be found remaining equal to zero.

**Basic reproduction number \( R_0 \)**

In this sequel, we utilize the next generation method, to structure the \( R_0 \) for the governing model (6). For this purpose, a sub-model of the SEQIMRP is considered that includes the four infected classes i.e. exposed, quarantine, infected and isolated individuals. Therefore, the equation:

\[
d\vec{X}/dt = F(\vec{X}) - V(\vec{X})
\]

will have \( \vec{X} \) as a vector of the \( E(t), Q(t), I(t), \) and \( M(t), \) which is outlined as,

\( \vec{X} = [E \ Q \ I \ M]^T \)

with, \( F(\vec{X}) \) expressed as,

\[
F(\vec{X}) = [\beta S(t)/\alpha \ 0 \ 0 \ 0]^T
\]

On the other hand, \( V(\vec{X}), \) can be further split down as,

\[
V(\vec{X}) = \begin{bmatrix}
  (mm + sd + ch + \gamma + d_s + (1 - \alpha)) E(t)/\alpha \\
  (mm + sd + ch + \gamma + \varphi_o + d_0 + (1 - \alpha)) Q(t)/\alpha \\
  (mm + sd + ch + \gamma + \varphi_o + d_0 + (1 - \alpha)) I(t)/\alpha \\
  (mm + sd + ch + \gamma + \varphi_o + d_0 + (1 - \alpha)) M(t)/\alpha \\
  0 \\
  -\eta E(t)/\alpha \\
  -\eta Q(t)/\alpha \\
  -\sigma I(t)/\alpha
\end{bmatrix}
\]

Taking Jacobian matrix of Eq. (20) at disease free equilibrium point, \( \Pi_1( - k_s (1 + d_s - r_s + mm + sd + ch - \alpha)/rs, 0, 0, 0, 0, 0, 0, 0 ) \), we get,

\[
J = \begin{bmatrix}
  0 & H_{13} & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(21)
\[\Pi_1(-k_s(1 + d_s - r_s + mm + sd + ch -\alpha)/r_s, 0, 0, 0, 0, 0) \in \mathbb{R}_+^7\]

For \(R_0 > 1 + d_s + mm + sd + ch -\alpha\), it is locally asymptotically stable if \(R_0 > 1\) and unstable when \(R_0 > 1\), for \(0 < \alpha < 1\).

**Proof:** On manipulating Jacobian at \(\Pi_1(-k_s(1 + d_s - r_s + mm + sd + ch -\alpha)/r_s, 0, 0, 0, 0, 0) \in \mathbb{R}_+^7\), the negative eigenvalues i.e. \(\lambda_i \in \mathbb{R}_-^7\) for \(i = 1, 2, 3, 4\), are attained as:

\[
\lambda_1 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a, \\
\lambda_2 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a, \\
\lambda_3 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a, \\
\lambda_4 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a.
\]

With the equation,

\[
P(\lambda) = \lambda^3 + b_1 \lambda^2 + b_1 \lambda + b_2(1 - R_0) = 0
\]

where

\[
b_1 = \frac{1}{\mu} (3 + d_s + d_s + d_s + 3mm + 3sd + 3ch - 3a +\gamma +\eta +\sigma +\psi_0),
\]

Dynamical analysis

In this section, on the strength of proportional fractional derivative, dynamical analysis of equilibrium points and optimality conditions are discussed in fractional environment as follows:

**Systematic stability analysis**

**Theorem 3.** *(Trivial Equilibrium Points)* The trivial equilibrium solution, \(\Pi_0(0, 0, 0, 0, 0, 0) \in \mathbb{R}_+^7\), of system (6), is asymptotically unstable, for \(0 < \alpha < 1\).

**Proof:** It can be easily proved by eigenvalues of \(J\) at \(\Pi_0(0, 0, 0, 0, 0, 0) \in \mathbb{R}_+^7\), for all \(0 < \alpha < 1\),

\[
\lambda_1 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a, \\
\lambda_2 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a, \\
\lambda_3 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a, \\
\lambda_4 = \frac{1}{\mu} (-d_m - d_s - r_s + mm + sd + ch - a)/a.
\]

Since \(R_0 > 1 + d_s + mm + sd + ch - a\), it is clear that \(\lambda_3 > 0\), for \(0 < \alpha < 1\). Thus, \(\Pi_0 \in \mathbb{R}_+^7\) is unstable.

**Theorem 4.** *(Disease Free Equilibrium Point)* The disease-free equilibrium of the system (6)

\[
\text{where}
\]

\[
Z = (1 + d_m + mm + sd + ch - a +\gamma +\eta +\sigma +\psi_0)/(1 + d_s + mm + sd + ch - a +\sigma +\psi_0) + (1 + d_m + mm + sd + ch - a +\sigma +\psi_0)\]

On applying Routh-Hurwitz criteria [31–34] i.e. if \(b_2 > 0\), \(b_0(1 - R_0) > 0\) and \(b_1b_2 > b_0(1 - R_0)\), then polynomial (23) is greater than zero and thus all the real part of the eigenvalues must be negative. It can be evidently seen that \(b_1 > 0\) for \(i = 0.1.2\), now the thing which left to prove is \((1 - R_0) > 0\). Hence, \(\Pi_1 \in \mathbb{R}_+^7\) is locally asymptotically stable if \(R_0 < 1\) and if \(R_0 > 1\), \((1 - R_0) < 0\) implies \(P(\lambda) < 0\) that is Eq. (23) must have a nonnegative real part, thus \(\Pi_1 \in \mathbb{R}_+^7\) becomes unstable.

**Theorem 5.** *(Endemic Equilibrium Point)* The endemic equilibrium \(\Pi_2(\bar{S}, \bar{E}, \bar{Q}, \bar{I}, \bar{M}, \bar{R}, \bar{P}) \in \mathbb{R}_+^7\) is locally asymptotically stable if and only if, \(R_0 > 1\), for \(0 < \alpha < 1\).

**Proof:** The Jacobian at \(\Pi_2(\bar{S}, \bar{E}, \bar{Q}, \bar{I}, \bar{M}, \bar{R}, \bar{P}) \in \mathbb{R}_+^7\), generates the negative real eigenvalues,

\[
\lambda_1 = \frac{1}{\mu} (-d_m + mm + sd + ch - a)/a, \\
\lambda_2 = \frac{1}{\mu} (-d_m + mm + sd + ch - a)/a, \\
\lambda_3 = \frac{1}{\mu} (-d_m + mm + sd + ch - a)/a.
\]

With the polynomial equation,
\[ D(\lambda) = A^2 + K_3 \lambda^3 + K_2 \lambda^2 + K_1 \lambda + K_0 = 0 \]

where,
\[ K_1 = -\frac{(1 + d_5 - r_5 + mm + sd + ch - \alpha) \omega}{\alpha R_0} + b_2 \]
\[ K_2 = (K_3 - b_1) b_2 + \frac{B}{\alpha^2} \]
\[ K_3 = \left( \frac{B(K_3 - b_1)}{\alpha^2} \right) \]
\[ K_0 = \frac{(1 + d_3 - r_3 + mm + sd + ch - \alpha) \left( k_5(1 + d_5 - r_5 + mm + sd + ch - \alpha) \right) \psi \eta}{\alpha R_0} + A \]

where,
\[ A = 1 + d_0 + 3mm + 2d_2 d_4 + 3mm^2 + d_2 d_4 mm + mm^3 + 3sd + 2d_4 sd + 6mm sd + 2d_2 d_4 mm + 3mm^2 + 3sd^2 + 3d_2 d_4 sd + 3mm^2 + sd + 3ch + 2d_4 ch + 6mm ch + 3mm^2 ch + 6sd ch + 2d_2 sd ch + 6mm sd ch + 3sd^2 + 3mm ch^2 + 3mm ch^2 + 3sd^2 + ch^2 + 2sc + d_4 sc + 4mm sc + d_4 mm sc + 2mm sc^2 + 4sd sc + d_4 sd sc + 4mm sd sc + 2sd^2 sc + 4ch sc + d_4 sc + 4mm sc + 4sd sc + 24ch sc + sc^2 + mm sc^2 + sd sc^2 + ch sc^2 + 3 \alpha \left( 1 + d_5 + 6mm + 2d_2 d_4 + 6mm + 2sd + 2d_4 sd + 6mm + 2sd + 2ch \right) + \eta \left( \left( 1 + d_5 + 6mm + 2d_2 d_4 + 6mm + 2sd + 2d_4 sd + 6mm + 2sd + 2ch \right) \right) + \psi \left( \left( 1 + d_5 + 2mm + mm + mm + mm + 2sd + d_4 sd + 2mm + sd + 2ch \right) + \sigma \left( 1 + d_5 + 2mm + mm + mm + 2sd + d_4 sd + 2mm + sd + 2ch \right) \right) + ch sc + ch sc - 2a - \eta - \alpha \eta - \eta + \sigma \left( 1 + mm + sd + ch - \alpha + \gamma \right) \left( 1 + mm + sd + ch - \alpha + \sigma \right) \psi \left( 1 + mm + sd + ch - \alpha + \gamma \right) + d_4 \left( 1 + mm + sd + ch - \alpha + \gamma \right) \left( 1 + d_5 + mm + mm + mm + mm + mm + sd + ch - \alpha + \eta + \psi \right) \]

and
\[ B = 3 + 2d_0 + 6mm + 2d_2 d_4 + 3mm^2 + 6sd + 2d_4 sd + 6mm sd + 3sd^2 + 6ch + 2d_4 ch + 6mm ch + 6sd ch + 3ch^2 + 4sc + d_4 sc + 4mm sc + 4sd sc + 4sc + 2ch + sc^2 - 6a - 2d_4 ch - 4mm - 6sd - 6ch - 4sc \left( 3 \alpha^2 + 2r + d_4 r + 2mm + 2sd + 2ch + 2sc - 2a + \gamma + \psi \right) + d_4 \left( 2 + d_0 + 2mm + 2sd + 2ch + sc - 2a + \gamma + \psi \right) + (2 + d_0 + 2mm + 2sd + 2ch + sc - 2a + \gamma + \psi) \]

The factor \( \frac{\psi \left( 1 + d_5 - r_5 + mm + sd + ch - \alpha \right) \psi}{\alpha R_0} > 0 \), which implies that \( K_0 \) becomes positive if and only if \( R_0 > 1 \). Thus with reference to Lemma 5.1 of [20], the positive constant of the polynomial \( D(\lambda) \) implies \( \Pi(t) \left( S, \tilde{E}, \tilde{Q}, \tilde{I}, \tilde{M}, \tilde{R}, \tilde{P} \right) \in \mathcal{N}_r^+ \) is locally asymptotically stable if \( R_0 > 1 \).

**Characterization of optimal control**

It is evidently clear from Theorem 1 that there exist a unique solution of system (6). Now to optimize the solution, we define the Lagrangian by
\[ L(Y) = \sum_{i=1}^{7} w_i Y_i^2 + \varphi_1 mm^2 + \varphi_2 sd^2 + \varphi_3 ch^2 + \varphi_4 sc^2 \]

In addition, describing the Hamiltonian \( H \) as the inner product of the right hand side of the state system (6) and the adjoint variables \( \Omega = (a_0, a_2, a_4, a_6, a_8, a_0, a_9) \), we get
\[ H(S, E, Q, I, M, R, P, \Omega, t) = L(Y) + a_0(t) \hat{S}(t) + a_2(t) \hat{E}(t) + a_4(t) \hat{Q}(t) + a_6(t) \hat{I}(t) + a_8(t) R(t) + a_0(t) \hat{P}(t) \]

where \( \Omega \) is determined. Now, utilizing the Pontryagin’s maximum principle for the Hamiltonian \( H \), following theorem is obtained to determine the adjoint variables.

**Theorem 6.** (Existence of adjoint variables) For the controlling functions \( mm, sd, ch \) and \( sc \) together with the solution \( (S(t), E(t), I(t), Q(t), M(t), R(t), P(t)) \) of the corresponding system (6), there exists adjoint variables \( \Omega = (a_0, a_2, a_4, a_6, a_8, a_0, a_9) \) that satisfy
\[ \frac{da_0(t)}{dt} = -\left( 1 - d_0 - mm - sd - ch + \alpha - \gamma \right) a_0(t) \left( \gamma a_0(t) \right) - \frac{(mm + sd + ch + \alpha - \gamma) a_0(t)}{\alpha} \]
\[ \frac{d\omega_1(t)}{dt} = \left( -1 - d_k - r_3S(t) \frac{k_1}{k_1} - r_3 \left( 1 - S(t) \right) - mm - sd - ch + \alpha - I^*(t) \beta \right) \omega_1(t) \]

\[ \frac{1}{\alpha} \left( I^*(t) \beta \omega_2(t) \right) - \frac{(mm + sd + ch) \omega_2(t)}{\alpha} \]

\[ mm^* = \max \left( \min \left( \frac{B_0}{2\alpha q_1}, mm^{\text{max}} \right), 0 \right), \]

\[ sd^* = \max \left( \min \left( \frac{B_0}{2\alpha q_2}, sd^{\text{max}} \right), 0 \right), \]

\[ ch^* = \max \left( \min \left( \frac{B_0}{2\alpha p_2}, ch^{\text{max}} \right), 0 \right), \]

\[ sc^* = \max \left( \min \left( \frac{Q(t)\omega_3(t) + I(t)\omega_4(t) + M(t)\omega_5(t) - \omega_1(t)(I'(t) - M'(t) - Q'(t))}{2\alpha q_4}, 0 \right) \right). \]

\[ \frac{d\omega_3(t)}{dt} = \frac{-dH}{dS} = \frac{-d_4 = mm - sd - ch - sc + \alpha - \eta - \Psi(t)}{\alpha} \]

\[ \frac{(mm + sd + ch + sc) \omega_2(t)}{\alpha} - \frac{\omega_4(t)}{\alpha} \]

\[ B_0 = S'(t)\omega_1(t) + E'(t)\omega_2(t) + Q'(t)\omega_3(t) + I'(t)\omega_4(t) + M'(t)\omega_5(t) \]

\[ + R'(t)\omega_6(t) - \omega_1(t)(E'(t) + I'(t) + M'(t) + Q'(t) + R'(t) + S'(t)) \]

**Proof:** By using Pontryagin’s maximum principle in state, the adjoint equations with transversality conditions is stated as:

\[ \frac{d\omega_4(t)}{dt} = \frac{-dH}{dE} = \frac{-d_5 = \omega_1(t)}{\alpha} \]

\[ \frac{(mm + sd + ch + sc) \omega_2(t)}{\alpha} - \frac{\omega_5(t)}{\alpha} \]

\[ \frac{d\omega_5(t)}{dt} = \frac{d_6 = mm - sd - ch - sc + \alpha - \rho - \Psi(t)}{\alpha} \]

\[ \frac{(mm + sd + ch + sc) \omega_2(t)}{\alpha} - \frac{\omega_6(t)}{\alpha} \]

\[ \frac{d\omega_6(t)}{dt} = \frac{-d_7 = mm - sd - ch + \alpha}{\alpha} \]

\[ \frac{(mm + sd + ch + sc) \omega_2(t)}{\alpha} - \frac{\omega_7(t)}{\alpha} \]

\[ \frac{d\omega_7(t)}{dt} = \frac{-d_8 = mm - sd + ch}{\alpha} \]

\[ \frac{(mm + sd + ch + sc) \omega_2(t)}{\alpha} - \frac{\omega_8(t)}{\alpha} \]

(27)
\[
\frac{d\phi}{dt} = -2\Gamma(t)w_4 + \frac{S(t)\phi_0}{\alpha} - \frac{S(t)\phi_0}{\alpha} \frac{(mm + sd + ch + sc)\phi(t)}{\alpha} - \frac{(mm + \phi(t)(mm + sd + ch + sc)\phi(t))}{\alpha} - \frac{d(1 - \delta - mm' - sd - ch - \phi(t)) + \sigma\phi(t)}{\alpha} - \frac{\alpha\phi(t)}{\alpha}
\]

\[
\frac{d\phi}{dt} = -\frac{\partial H}{\partial \phi} = \left( \frac{-1 - \delta - mm' - sd - ch - \phi(t)}{\alpha} \right) - \frac{\phi_{\phi}(t)}{\alpha} \left( \frac{(mm + sd + ch + sc)\phi(t)}{\alpha} \right) - \frac{\alpha\phi(t)}{\alpha}
\]

\[
\frac{d\phi}{dt} = \frac{-dH}{\partial \phi} = \left( \frac{-1 - \delta - mm' - sd - ch + \phi(t)}{\alpha} \right) - \frac{(mm + sd + ch + sc)\phi(t)}{\alpha}
\]

\[
\frac{d\phi}{dt} = \frac{-dH}{\partial \phi} = \left( \frac{-1 - \delta - mm' - sd - ch + \phi(t)}{\alpha} \right) - \frac{(mm + sd + ch + sc)\phi(t)}{\alpha}
\]

with transversality \(\phi_0(T) = 0, i = 1, 2, \ldots, 7\) where \(T = t_{\text{final}}\). By using optimality condition, we deduce the optimal control pairs as:

\[
\frac{dH}{\partial mm} = 0 \Rightarrow mm' = \frac{B_0}{2\alpha\phi_1}
\]

\[
\frac{dH}{\partial sd} = 0 \Rightarrow sd' = \frac{B_0}{2\alpha\phi_2}
\]

\[
\frac{dH}{\partial ch} = 0 \Rightarrow ch' = \frac{B_0}{2\alpha\phi_3}
\]

\[
\frac{dH}{\partial sc} = 0 \Rightarrow sc' = \frac{Q'(t)\phi_0(t) + \Gamma(t)\phi_0(t) + M'(t)\phi_0(t) - \phi_0(t)(\Gamma(t) + M'(t) + Q'(t))}{2\alpha\phi_4}
\]

Further, taking into account the property of the control space, we achieve,

\[
mm'(t) = \begin{cases} 0 \text{ if } X_1 < 0 \\ X_1 \text{ if } 0 \leq X_1 < \text{max} \text{ mm} \\ X_1 \text{ if } X_1 \geq \text{max} \text{ mm} \end{cases}
\]

\[
sd'(t) = \begin{cases} 0 \text{ if } X_2 < 0 \\ X_2 \text{ if } 0 \leq X_2 < \text{max} \text{ sd} \\ X_2 \text{ if } X_2 \geq \text{max} \text{ sd} \end{cases}
\]

\[
ch'(t) = \begin{cases} 0 \text{ if } X_3 < 0 \\ X_3 \text{ if } 0 \leq X_3 < \text{max} \text{ ch} \\ X_3 \text{ if } X_3 \geq \text{max} \text{ ch} \end{cases}
\]

\[
sc'(t) = \begin{cases} 0 \text{ if } X_4 < 0 \\ X_4 \text{ if } 0 \leq X_4 < \text{max} \text{ sc} \\ X_4 \text{ if } X_4 \geq \text{max} \text{ sc} \end{cases}
\]

where,

\[
\dot{P}(t) = \frac{1}{\alpha} \left( \frac{(mm'(t) + sd'(t) + ch'(t) + \psi(t)(mm'(t) + sd'(t) + ch'(t) + \psi(t))(I'(t) + \psi(t))}{(mm'(t) + sd'(t) + ch'(t) + \psi(t))(I'(t) + \psi(t))} \right)
\]

and

\[
\frac{d\phi_0(t)}{dt} = \left( \frac{-1 - \delta - \frac{S_0(t)}{\kappa_1} + \psi(t) \left( \frac{S_0(t)}{\kappa_1} \right) - (mm'(t) + sd'(t) + ch'(t) + \psi(t))(I'(t) - \Gamma'(t))}{\alpha} \right) \frac{\alpha}{\alpha}
\]

\[
\frac{d\phi_0(t)}{dt} = \left( \frac{-1 - \delta - \frac{S_0(t)}{\kappa_1} + \psi(t) \left( \frac{S_0(t)}{\kappa_1} \right) - (mm'(t) + sd'(t) + ch'(t) + \psi(t))(I'(t) - \Gamma'(t))}{\alpha} \right) \frac{\alpha}{\alpha}
\]

\[
X_1 = \frac{B_0}{2\alpha\phi_1}
\]

\[
X_2 = \frac{B_0}{2\alpha\phi_2}
\]

\[
X_3 = \frac{B_0}{2\alpha\phi_3}
\]

\[
X_4 = \frac{Q'(t)\phi_0(t) + \Gamma'(t)\phi_0(t) + M'(t)\phi_0(t) - \phi_0(t)(\Gamma'(t) + M'(t) + Q'(t))}{2\alpha\phi_4}
\]

Ultimately, the control pair and state variables are found by using the following composed systems:

\[
S(t) = \frac{1}{\alpha} \left( I(t) - \frac{(mm'(t) + sd'(t) + ch'(t) + \psi(t))(I(t) - \psi(t))}{\alpha} \right)
\]

\[
E(t) = \frac{1}{\alpha} \left( \psi(t)(mm'(t) + sd'(t) + ch'(t) + \psi(t)) + \psi(t)(I'(t) - \psi(t)) \right)
\]

\[
Q(t) = \frac{1}{\alpha} \left( \psi(t)(mm'(t) + sd'(t) + ch'(t) + \psi(t)) + \psi(t)(I'(t) - \psi(t)) \right)
\]

\[
I(t) = \frac{1}{\alpha} \left( \psi(t)(mm'(t) + sd'(t) + ch'(t) + \psi(t)) + \psi(t)(I'(t) - \psi(t)) \right)
\]

\[
M(t) = \frac{1}{\alpha} \left( \psi(t)(mm'(t) + sd'(t) + ch'(t) + \psi(t)) + \psi(t)(I'(t) - \psi(t)) \right)
\]

\[
R(t) = \frac{1}{\alpha} \left( \psi(t)(mm'(t) + sd'(t) + ch'(t) + \psi(t)) + \psi(t)(I'(t) - \psi(t)) \right)
\]
The sensitivity analysis of $R_0$ by means of control variables are described in Table 2 and Figs. 2-7 for the parameters mentioned in Table 1 and at different values of $\alpha$. These control variables define the strategic campaigns utilized to prevent the deadly transmission of the COVID-19. It can be clearly seen from the Figs. 2-7 that at each value of $\alpha$, the influential strength of each campaign together minimizes the significance of $R_0$. The generation of colorized output in these figures, ranging from light to dark, indicates the gradual decrease in $R_0$ from largest to lowest value. The obtained value of $R_0$ without any awareness campaign is greater than 1, which gradually reduces to less than 1 on increasing awareness campaigns that can be seen from the Table 2 and Figs. 2-7. Furthermore, the lines of $R_0$ on the Fig. 2, which are attained by fixing $ch = 0.1$ and $sc = 0.1$ and varying $mm$ and $sd$ represent decrease in value starting from 1.4 to 0.4. In the same way, Figs. 3 and 5 plotted for $mm = 0.1$, $sc = 0.1$ and $mm = 0.1$, $sd = 0.1$, respectively which demonstrate the same pattern of decrease in $R_0$. On the other hand, Fig. 4 exhibit the decrease in $R_0$ starting from 1.8 to 0.4, for $mm = 0.1$, $sd = 0.1$. Similarly, same sketches are found in Figs. 6 and 7, which are produced by fixing $mm = 0.1$, $ch = 0.1$ and $sd = 0.1$, $ch = 0.1$, accordingly. Besides, Table 2 explains the sensitivity of $R_0$ with some different values of intervention strategies, which elucidates that for $mm = 0.3$, $sd = 0.7$, $ch = 0.5$, $sc = 0.9$, the value of $R_0$ decreases more rapidly than

### Numerical simulation and deliberation

In this segment, numerical investigations of the aforementioned system are carried out by considering some numerical values of the parameters, as shown in Table 1. The graphical predisposition analysis of $R_0$ with respect to the strategies are also added in the discussion. Moreover, the simulations of all compartmental class, with prevention and without prevention campaign cases are plotted and tabulated by using Mathematica 11.0.

![Fig. 2. Sensitivity inspection of $R_0$ with respect to mm and sd for $ch = 0.1$, $sc = 0.1$, at $\alpha = 0.95$.](image-url)
the other combinations, at each value of $\alpha$. In addition, the last column of Table 2 also elaborates the minimum cost function $J$ against each mitigation strategy for weights $w_4 = 200, \phi_1 = 100, \phi_2 = 20, \phi_3 = 150$ and $\phi_4 = 300$. Evidently, the optimal values of $J$ for the cost efforts of surveillance mitigations, $mm = 0.1, sd = 0.7, ch = 0.5, sc = 0.9$, which greatly reduce $R_0$ are 9068.92, 9090.57 and 9098.23 for $\alpha = 0.8, 0.95, 1$, respectively and $t \in [0, 30]$. According to these values, increasing the awareness about social distancing and supportive care of the infected individuals will significantly affect the transmission of COVID’19 with optimal cost efforts, comparative to other combinations of mitigations.

Moreover, solving SEQIMRP system different plots are attained that define the stability of $\Pi_1$ and $\Pi_2$. In the current scenario, evaluations of these equilibrium points are produced on the basis of the prevention campaigns. Commencing from Table 3, the values are generated for $mm = 0, sd = 0, ch = 0$ and $sc = 0$, at $\alpha \in [0.1]$ and $t \in [0, 30]$. Manifestly, it can be seen when no prevention measures are taken $R_0$ increases gradually and endemic state of the pandemic becomes stable. Additionally, Figs. 8-14 also plotted for $mm = 0, sd = 0, ch = 0$ and $sc = 0$, at $\alpha = 0.8, 0.95, 1$ and $t \in [0, 30]$ represent the stability of the deadly endemic state of COVID’19 for the current rate of transmission.
recovery and mortality. Since, no prevention measures are taken at initial spread stage of COVID’19, therefore the curve of protected population yields a constant straight line on zero. This further elaborates the circumstances where everyone is at high risk of being infected that the population yields a constant straight line on zero. This further elaborates the recovery and mortality. Since, no prevention measures are taken at initial spread stage of COVID’19, therefore the curve of protected population yields a constant straight line on zero. This further elaborates the circumstances where everyone is at high risk of being infected that the population yields a constant straight line on zero. This further elaborates the circumstances where everyone is at high risk of being infected that the pandemic situation becomes worst. Contrarily Table 4 depicts the values, which are generated for \( mm = 0.2, \ sd = 0.3, \ ch = 0.35 \) and \( sc = 0.65 \), at \( \alpha \in [0, 1] \) and \( t \in [0, 30] \). Evidently from Table 4, when prevention measures are taken into account to some extent, we attain the disease-free state of the dynamics at each value of \( \alpha \). In addition, it also shows the value of \( R_0 \) to be less than one, which is proved in Theorem 4. Figs. 15-21 graphically demonstrate the stability of \( \Pi_1 \) for \( mm = 0.2, \ sd = 0.3, \ ch = 0.35 \) and \( sc = 0.65 \), at \( \alpha = 0.8, \ 0.95, \ 1 \) and \( t \in [0, 30] \). Contrary to endemic, in disease free case the infected cells become zero whereas susceptible and protected individuals remain at a population level other than zero. Running awareness campaigns about using mask, social distancing, hand wash and also invigorating supportive care of the patients will decrease the basic reproduction number and eventually the deadly spread of COVID’19.

Conclusion

The declaration of PHEIC by the WHO about the COVID’19 outbreak, agitate the scientific community and the healthcare professionals of the countries. After the failure of several experiments on the inoculations, the only operational plan of action to decelerate the spread of COVID’19 is to adopt non-pharmaceutical restrictions. For this purpose, different unprecedented measures are taken into account such as lockdown, closure of institutions and initiating different awareness campaigns. Here, we discussed the cost and public effectiveness of the awareness campaigns taken into consideration by the stakeholders. These maneuvers include the strict imposition of using medical mask in public places, social distancing of 6 feet, frequent use of hand wash and sanitizers, training medical staffs and officers for extraordinary supportive care of COVID’19 patients in hospitals. The optimal control function was designed with the epidemic dynamical system SEQIMRP to mutually

| \( \alpha \) | \( R_0 \) | \( S(t) \) | \( E(t) \) | \( Q(t) \) | \( I(t) \) | \( M(t) \) | \( R(t) \) | \( P(t) \) |
|---|---|---|---|---|---|---|---|---|
| 0.4 | 1.07658 | 90564.3 | 130,860 | 0.0166747 | 0.14077 | 0.0108021 | 0.00775503 | 0 |
| 0.5 | 1.2482 | 78379.6 | 341,367 | 0.0437936 | 0.394839 | 0.0339465 | 0.0267258 | 0 |
| 0.6 | 1.46208 | 67141.9 | 503,966 | 0.0650954 | 0.629689 | 0.304305 | 0.0546976 | 0 |
| 0.7 | 1.73309 | 56834.8 | 623,165 | 0.0810462 | 0.8465 | 0.615493 | 0.0584976 | 0 |
| 0.8 | 2.08323 | 47,442 | 703,299 | 0.0921023 | 1.04305 | 0.140393 | 0.18117 | 0 |
| 0.9 | 2.54613 | 38947.9 | 748,529 | 0.0987099 | 1.22222 | 0.202724 | 0.373485 | 0 |
| 1 | 3.17529 | 31335.7 | 762,847 | 0.101305 | 1.38348 | 0.298912 | 1.00458 | 0 |
Table 4
Basic reproduction number $R_0$ and disease free equilibrium points $\Pi_1$, for parameters describe in Table 1, for $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at different values of $\alpha$ and $t \in [0, 30]$.

| $\alpha$ | $R_0$ | $S(t)$ | $E(t)$ | $Q(t)$ | $I(t)$ | $M(t)$ | $R(t)$ | $P(t)$ |
|----------|-------|--------|--------|--------|--------|--------|--------|--------|
| 0.4      | 0.297661 | 94566.7 | 0      | 0      | 0      | 0      | 97904.3 |
| 0.5      | 0.324551 | 94,900  | 0      | 0      | 0      | 0      | 111,349 |
| 0.6      | 0.354992 | 95233.3 | 0      | 0      | 0      | 0      | 128,931 |
| 0.7      | 0.389628 | 95566.7 | 0      | 0      | 0      | 0      | 152,907 |
| 0.8      | 0.429253 | 95,900  | 0      | 0      | 0      | 0      | 187,538 |
| 0.9      | 0.474857 | 96233.3 | 0      | 0      | 0      | 0      | 241,958 |
| 1        | 0.527691 | 96566.7 | 0      | 0      | 0      | 0      | 339,915 |

Fig. 11. Dynamics of $I(t) \in \Pi_2$ of SEQIMRP, for parameters described in Table 1 for $mm = 0$, $sd = 0$, $ch = 0$ and $sc = 0$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

Fig. 12. Dynamics of $M(t) \in \Pi_2$ of SEQIMRP, for parameters described in Table 1 and $mm = 0$, $sd = 0$, $ch = 0$ and $sc = 0$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

Fig. 13. Dynamics of $R(t) \in \Pi_2$ of SEQIMRP, for parameters described in Table 1 and $mm = 0$, $sd = 0$, $ch = 0$ and $sc = 0$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

Fig. 14. Dynamics of $P(t) \in \Pi_2$ of SEQIMRP, for parameters described in Table 1 and $mm = 0$, $sd = 0$, $ch = 0$ and $sc = 0$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

Fig. 15. Dynamics of $S(t) \in \Pi_1$ of SEQIMRP, for parameters described in Table 1 and $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

Fig. 16. Dynamics of $E(t) \in \Pi_1$ of SEQIMRP, for parameters described in Table 1 and $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

Fig. 17. Dynamics of $Q(t) \in \Pi_1$ of SEQIMRP, for parameters described in Table 1 and $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at $\alpha = 0.8$, 0.95, 1and $t \in [0, 30]$.

study its dynamical stability and the feasibility of the prevention tactics. The system was formulated with the proportional fractional derivative, in order to analyze the basic reproduction number at each chronological change. Ultimately, through the aforementioned analytical and numerical illustrations, the following propitious facts can be extracted:

- The strategies of using medical mask, social distancing, frequently sanitizing hands and supportive care of COVID ‘19 for speedy...
Fig. 18. Dynamics of $I(t) \in \Pi_1$ of SEQIMRP, for parameters described in Table 1 and $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at $\alpha = 0.8, 0.95$, $1$ and $t \in [0, 30]$.

Fig. 19. Dynamics of $M(t) \in \Pi_1$ of SEQIMRP, for parameters described in Table 1 and $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at $\alpha = 0.8, 0.95$, $1$ and $t \in [0, 30]$.

Fig. 20. Dynamics of $R(t) \in \Pi_1$ of SEQIMRP, for parameters described in Table 1 and $mm = 0.2$, $sd = 0.3$, $ch = 0.35$ and $sc = 0.65$, at $\alpha = 0.8, 0.95$, $1$ and $t \in [0, 30]$.

recovery are significant attempts to win this battle against this pandemic.

- The awareness and necessitating of these lines of attacks may change the state of pandemic into a stable disease-free environment.
- These can greatly lesser the basic reproduction number from $R_0 > 1$ to $R_0 < 1$.
- The optimal surveillance mitigation with respect to cost effectiveness, social distancing and supportive care may reduce the diffusion of COVID’19 more hastily.
- Illustrations at different fractional derivative index show systematic reading in the susceptible, expose, quarantined, infected, isolated, recovered and protected population.
- Without precautions, as the fractional derivative approaches the whole change, the readings represent step by step increase in susceptible, expose, quarantined, infected, isolated and recovered population.
- Following precautions, as the fractional derivative approaches the whole change, the number individuals in protection increases gradually, while expose, quarantined, infected and recovered remain zero.
- Competency in prior recognition of the track of COVID’19 transmission risk through the proportional fractional derivative model.
- Proficiently trace the basic reproduction number and take preparatory measures before becoming a deadly pandemic.

In the current phase, understanding the epidemiological characteristics is a serious bone of contention question for researchers and health professionals. The successful investigations may significantly help out the stakeholders in making effective standard operational procedures of interventions. The designed model SEQIMRP will categorically aid a great contribution in dynamically scrutinizing and exhibiting the optimal strategy to control the deadly escalation of COVID’19.

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