Research Article

Time Period Analysis of Orthotropic Skew Plate with 2-D Circular Thickness and 1-D Circular Density

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In the present study, the time period of vibration of an orthotropic parallelogram plate with 2-D (two-dimensional) circular thickness under the effect of 2-D parabolic temperature is investigated for the first time. The different edge conditions are CCCC, CCCF, CFCF, CSCF, and SFSF boundary conditions, where C, S, and F stands for clamped, simply supported, and free edges of the plate, respectively. The variation in density of plate material is considered to be 1-D (one-dimensional) circular. The Rayleigh–Ritz technique is used to solve the differential equation and evaluate the time period for the first two modes of vibration.

A convergence study of an orthotropic parallelogram, rectangle plate, and square plate for modes of frequency at various edge conditions is also carried out. The authors performed a comparative analysis of the time period and modes of frequency of orthotropic parallelogram, rectangle plate, and isotropic square plate with the available published results at various edge conditions. The main conclusion which we made from this study is that by choosing the above-mentioned plate parameters, we obtained fewer modes of frequency in comparison to other variations mentioned in the literature. Also, the study suggests that the variation in modes of frequency is less in comparison to other variations.

1. Introduction

In engineering, machines and structures vibrate, so we cannot proceed without considering vibration. Modern technology requires knowing the vibration characteristics of plates with different plate parameters. Tapered plates with uniform and nonuniform thickness under a temperature environment are generally utilized in the aeronautical field, construction industry, and submarine structures. Different analysts/researchers investigated the vibration characteristics of various plates (homogeneous or nonhomogeneous) having variable thickness with or without consideration of temperature effect. A significant work about the vibrational characteristics of plates has been reported in the literature.

The quasi-green function method (QGFM) [1] is applied to solve the free vibration of clamped orthotropic thin plates (parallelogram shape) on the Winkler foundation. In this study, a quasi-green function is established by using the fundamental solution and a principal differential equation has been solved with the help of the variable separable method. Vibration of the viscoelastic orthotropic parallelogram plate with parabolic [2] and linear thickness [3] at clamped boundary conditions was studied by using the Rayleigh–Ritz technique to determine the frequency equation. The frequency equation is derived by using the Rayleigh–Ritz technique, and two-term deflection function is used to find the modes of frequency for various values of taper constants, aspect ratios, and skew angle. Rayleigh–Ritz technique is employed to study the natural vibration of the nonhomogeneous tapered parallelogram plate with two-dimensional varying thickness [4] and one-dimensional circular variation in density parameter [5] at clamped boundary conditions under temperature field. The time period of natural transverse vibration of a nonhomogeneous skew (parallelogram) plate [6] with variable thickness and temperature field has been investigated on clamped and combination of clamped and simply supported edge conditions. The effect of sinusoidal varying thickness [7] on the
vibrations of nonhomogeneous parallelogram plates is computed at clamped edges. Here, the frequency equation is solved by using the Rayleigh–Ritz method and we analyze the vibrational behavior of frequencies of a parallelogram plate for both modes. The effect of two-dimensional circular variations in thickness [8] on a nonhomogeneous parallelogram plate under thermal effect was computed, and differential equations of motions are solved by using the Rayleigh–Ritz technique and we evaluate the vibrational frequencies at various plate parameters. A fast-converging semianalytical method [9] was developed for assessing the vibration effect on thin orthotropic skew plates. An experimental and finite element [10] was proposed to study the free vibration of isotropic and laminated composite skew plates. A method to unify the solutions for plates with different shapes [11] (circular plate, annular plate, circular sector plate, and annular sector plate) subjected to general boundary conditions was adapted to study vibration characteristics. The time period of a rectangular plate [12] with variable thickness and temperature effect was analyzed. Two-dimensional circular thickness effects on time period of the nonhomogeneous skew plate [13] for variable temperature environments are computed at various combinations of clamped, simply supported, and free edge conditions. Free vibration of an orthotropic parallelogram plate [14] with a simply supported boundary condition under the effect of biparabolic thickness variation and linear temperature distribution in both directions is carried out at simply supported edges. The effect of parabolically thick variation on vibration of a viscoelastic orthotropic parallelogram plate [15] having clamped boundary conditions on all four edges was studied by using the separation of variables method. Vibration and modes of nanocomposite plates, functionally graded sandwich plates, sector plates, sandwich panels, I-shaped graphene sheet, skew plate, cylindrical skew plates, functionally graded rectangle plates, spherical shell, and annular sector plates [16–37] have been analyzed.

In the literature till date, researchers have studied the 1–D (one-dimensional) circular tapering impact on vibrational modes of frequency for different isotropic plate structures. But in the case of orthotropic material, none of the researchers have tackled 1–D (one-dimensional) circular tapering as well as 2–D (two-dimensional) circular tapering impact on the mode of frequency. In this work, we aim to fill up this research gap. Also, none of the researchers have aimed to tackle the 2–D circular tapering impact on the mode of frequency in the case of orthotropic parallelogram plate. We will also address this issue.

In this paper, the authors studied the impact of 2–D (two-dimensional) circular tapering on the time period of vibrational modes of frequency of the nonhomogeneous orthotropic parallelogram plate on CCCC, CCCC, CFCF, CSFC, and FSFS boundary conditions. The authors also evaluated 1–D (one-dimensional) circular density and 2–D (two-dimensional) parabolic temperature impacts on the time period of vibrational modes of frequency. All the results displayed in the tabular form (refer Tables 1–3). In order to authenticate our findings, authors performed comparatively analysis of time period and modes of frequency of the orthotropic parallelogram plate (SSSS and CCCC edge condition), rectangle plate (CCCC edge condition), and square plate (CCCC, SCSC, FCFC, and FSFS edge conditions) with the available published results (refer Tables 4–10).

2. Analysis

The orthotropic parallelogram plates made with nonhomogeneous material properties with variable thickness \( l \) having skew angle \( \theta \), length \( a \), breadth \( b \), density \( \rho \), and Poisson’s ratio \( \nu \) referred to the skew coordinates \( \xi = x - y \tan \theta \) and \( \psi = y \sec \theta \) (refer Figure 1).

The thickness \( l \) of the skew plate is assumed to be circular in both dimensions (refer Figure 2), and density \( \rho \) is assumed to circular in one dimension as

\[
\begin{align*}
l &= l_0 \left[ 1 + \beta_1 \left( 1 - \sqrt{1 - \frac{\xi^2}{a^2}} \right) \right] \left[ 1 + \beta_2 \left( 1 - \sqrt{1 - \frac{\psi^2}{b^2}} \right) \right], \\
\rho &= \rho_0 \left[ 1 - m \left( 1 - \sqrt{1 - \frac{\psi^2}{b^2}} \right) \right],
\end{align*}
\]

where \( l_0 \) and \( \rho_0 \) are the thickness and density of the plate, respectively, at the origin. Also, \( \beta_1, \beta_2 (0 \leq \beta_1, \beta_2 \leq 1) \) and \( m (0 \leq m < 1) \) are taper parameters and nonhomogeneity parameter, respectively.

Two-dimensional steady state temperature variations on the plate are considered to be parabolic as taken in [13]

\[
\tau = \tau_0 \left( 1 - \frac{\xi^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right),
\]

where \( \tau \) and \( \tau_0 \) denotes the temperature excess above the reference temperature on the plate at any point and at the origin, respectively.

The temperature-dependent modulus of elasticity for engineering structures is taken as in [14]

\[
\begin{align*}
E_\xi (\tau) &= E_1 (1 - \gamma \tau), \\
E_\psi (\tau) &= E_2 (1 - \gamma \tau), \\
G_{\xi\psi} (\tau) &= G_0 (1 - \gamma \tau),
\end{align*}
\]

where \( E_\xi \) and \( E_\psi \) are Young’s moduli in \( \xi \) and \( \psi \) directions, respectively, \( G_{\xi\psi} \) is shear modulus, and \( \gamma \) is taken as the slope variation of moduli with temperature.

Substituting (2) in (3), we get the following expressions:

\[
\begin{align*}
E_\xi (\tau) &= E_1 \left[ 1 - \alpha \left( 1 - \frac{\xi^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right) \right], \\
E_\psi (\tau) &= E_2 \left[ 1 - \alpha \left( 1 - \frac{\xi^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right) \right], \\
G_{\xi\psi} (\tau) &= G_0 \left[ 1 - \alpha \left( 1 - \frac{\xi^2}{a^2} \right) \left( 1 - \frac{\psi^2}{b^2} \right) \right],
\end{align*}
\]

where \( \alpha = \gamma \tau_0, (0 \leq \alpha < 1) \) is called the temperature gradient.
### Table 1: Time period of the orthotropic parallelogram plate at $CCC$, $CCF$, $CFC$, $CSF$, and $SFS$ edge conditions corresponding to both tapering parameters $\beta_1$ and $\beta_2$.

| $\beta_1$ | $\beta_2$ | $\beta_2 = 0.0$ | $\beta_2 = 0.2$ | $\beta_2 = 0.4$ | $\beta_2 = 0.6$ | $\beta_2 = 0.8$ | $\beta_2 = 1.0$ |
|-----------|-----------|----------------|----------------|----------------|----------------|----------------|----------------|
| $K_1$     | $K_2$     | $K_1$          | $K_2$          | $K_1$          | $K_2$          | $K_1$          | $K_2$          |
| $\beta_1$ | $\beta_2$ | $\beta_2 = 0.0$ | $\beta_2 = 0.2$ | $\beta_2 = 0.4$ | $\beta_2 = 0.6$ | $\beta_2 = 0.8$ | $\beta_2 = 1.0$ |
| 0.0       | 0.16309   | 0.62562        | 0.15807        | 0.60548        | 0.15315        | 0.58591        | 0.14831        | 0.56693        | 0.14362        | 0.54865        | 0.13905        | 0.53102        |
| 0.2       | 0.15367   | 0.59097        | 0.14909        | 0.57262        | 0.14457        | 0.55471        | 0.14013        | 0.53734        | 0.13581        | 0.52059        | 0.13160        | 0.50445        |
| 0.4       | 0.14444   | 0.55792        | 0.14022        | 0.54114        | 0.13608        | 0.52474        | 0.13201        | 0.50884        | 0.12803        | 0.49345        | 0.12416        | 0.47862        |
| 0.6       | 0.13551   | 0.52688        | 0.13167        | 0.51155        | 0.12788        | 0.49650        | 0.12415        | 0.48192        | 0.12049        | 0.46778        | 0.11692        | 0.45412        |
| 0.8       | 0.12710   | 0.49816        | 0.12358        | 0.48403        | 0.12010        | 0.47020        | 0.11667        | 0.45676        | 0.11330        | 0.44372        | 0.11001        | 0.43109        |
| 1.0       | 0.10426   | 0.47292        | 0.11603        | 0.45864        | 0.10927        | 0.44586        | 0.10726        | 0.43345        | 0.10654        | 0.42153        | 0.10351        | 0.40966        |

### Table 2: Time period of the orthotropic parallelogram plate at $CCC$, $CCF$, $CFC$, $CSF$, and $SFS$ edge conditions corresponding to thermal gradient $\alpha$.

| $\alpha$ | $\beta_1 = \beta_2 = 0.0$ | $\beta_1 = \beta_2 = 0.2$ | $\beta_1 = \beta_2 = 0.4$ | $\beta_1 = \beta_2 = 0.6$ | $\beta_1 = \beta_2 = 0.8$ | $\beta_1 = \beta_2 = 1.0$ |
|----------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
| $K_1$    | $K_2$                     | $K_1$                     | $K_2$                     | $K_1$                     | $K_2$                     | $K_1$                     |
| 0.0       | 0.14745                   | 0.56414                   | 0.13598                   | 0.51987                   | 0.12512                   | 0.47909                   | 0.11500                   | 0.44190                   | 0.10566                   | 0.40825                   | 0.09710                   | 0.37800                   |
| 0.2       | 0.15468                   | 0.59241                   | 0.14208                   | 0.54428                   | 0.13026                   | 0.50027                   | 0.11931                   | 0.46046                   | 0.10928                   | 0.42474                   | 0.10015                   | 0.39276                   |
| 0.4       | 0.16309                   | 0.62562                   | 0.14909                   | 0.57262                   | 0.13608                   | 0.52474                   | 0.12415                   | 0.48192                   | 0.11330                   | 0.46778                   | 0.10351                   | 0.45412                   |
| 0.6       | 0.17030                   | 0.66527                   | 0.15724                   | 0.60627                   | 0.14275                   | 0.55368                   | 0.12961                   | 0.50712                   | 0.11779                   | 0.46939                   | 0.10722                   | 0.44294                   |
| 0.8       | 0.18503                   | 0.71396                   | 0.16688                   | 0.64711                   | 0.15050                   | 0.58858                   | 0.13586                   | 0.53740                   | 0.12286                   | 0.49257                   | 0.11135                   | 0.45311                   |
| 1.0       | 0.19710                   | 0.55465                   | 0.15450                   | 0.50036                   | 0.13884                   | 0.45314                   | 0.12490                   | 0.41196                   | 0.11258                   | 0.37596                   | 0.10171                   | 0.34342                   |
| 1.0       | 0.21773                   | 0.57352                   | 0.15898                   | 0.51821                   | 0.14241                   | 0.46870                   | 0.12776                   | 0.42569                   | 0.11489                   | 0.38414                   | 0.10360                   | 0.35519                   |
| 0.8       | 0.19839                   | 0.65747                   | 0.17508                   | 0.58855                   | 0.15496                   | 0.52971                   | 0.13764                   | 0.47909                   | 0.12275                   | 0.43517                   | 0.10993                   | 0.39694                   |
| 0.6       | 0.23761                   | 0.72420                   | 0.30262                   | 0.64629                   | 0.27855                   | 0.57498                   | 0.25551                   | 0.51170                   | 0.23363                   | 0.45676                   | 0.21303                   | 0.40960                   |
| 0.4       | 0.34338                   | 0.75380                   | 0.31601                   | 0.66866                   | 0.28980                   | 0.59219                   | 0.26484                   | 0.52553                   | 0.24126                   | 0.46826                   | 0.21918                   | 0.41959                   |
| 0.2       | 0.36163                   | 0.78754                   | 0.33134                   | 0.69379                   | 0.30249                   | 0.61164                   | 0.27519                   | 0.54111                   | 0.24959                   | 0.48129                   | 0.22581                   | 0.43090                   |
| 0.0       | 0.38312                   | 0.82668                   | 0.34906                   | 0.72263                   | 0.31689                   | 0.63372                   | 0.28676                   | 0.55886                   | 0.25875                   | 0.49628                   | 0.23298                   | 0.44388                   |
Table 2: Continued.

| $\alpha$ | $\beta_1 = \beta_2 = 0.0$ | $\beta_1 = \beta_2 = 0.2$ | $\beta_1 = \beta_2 = 0.4$ | $\beta_1 = \beta_2 = 0.6$ | $\beta_1 = \beta_2 = 0.8$ | $\beta_1 = \beta_2 = 1.0$ |
|----------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
|          | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 0.0      | 0.34199 | 0.73689 | 0.31288 | 0.62662 | 0.28512 | 0.53401 | 0.25890 | 0.45792 | 0.23429 | 0.39622 | 0.21136 | 0.34639 |          |
| 0.2      | 0.35544 | 0.75147 | 0.32424 | 0.63649 | 0.29461 | 0.54111 | 0.26668 | 0.46351 | 0.24055 | 0.40096 | 0.21629 | 0.35073 |          |
| 0.4      | 0.37061 | 0.76699 | 0.33697 | 0.64698 | 0.30508 | 0.54871 | 0.27515 | 0.46948 | 0.24726 | 0.40621 | 0.22149 | 0.35560 |          |
| 0.6      | 0.38789 | 0.78358 | 0.35123 | 0.65817 | 0.31670 | 0.55688 | 0.28443 | 0.47611 | 0.25449 | 0.41205 | 0.22699 | 0.36110 |          |
| 0.8      | 0.40784 | 0.80142 | 0.36747 | 0.67017 | 0.32971 | 0.56374 | 0.29460 | 0.48343 | 0.26225 | 0.41865 | 0.23277 | 0.36735 |          |

Table 3: Time period of the orthotropic parallelogram plate at $CCCC$, $CCCF$, $CFCF$, $CSCF$, and $SFSF$ edge conditions corresponding to nonhomogeneity $m$.

| $m$ | $\beta_1 = \beta_2 = 0.0$ | $\beta_1 = \beta_2 = 0.2$ | $\beta_1 = \beta_2 = 0.4$ | $\beta_1 = \beta_2 = 0.6$ | $\beta_1 = \beta_2 = 0.8$ | $\beta_1 = \beta_2 = 1.0$ |
|-----|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|---------------------------|
|     | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ | $K_1$ | $K_2$ |
| 0.4 | 0.26839 | 1.4686 | 0.24490 | 1.3499 | 0.22325 | 1.2354 | 0.20355 | 1.1270 | 0.18578 | 1.0257 | 0.16983 | 0.93243 |          |
| 0.6 | 0.29229 | 1.6282 | 0.26415 | 1.4821 | 0.23879 | 1.3442 | 0.21619 | 1.2162 | 0.19612 | 1.0989 | 0.17836 | 0.99243 |          |
| 0.8 | 0.32396 | 1.8591 | 0.28876 | 1.6665 | 0.25810 | 1.4910 | 0.23149 | 1.3331 | 0.20839 | 1.1921 | 0.18832 | 1.0671 |          |

$\alpha$, $\beta$, $K$, $\alpha_1$, $\beta_1$, $K_1$, $\beta_2$, $K_2$.
Table 4: Comparison of modes of frequency of the present study (orthotropic parallelogram plate) and obtained in [14] corresponding to tapering parameter $\beta_1$ for a fixed value of aspect ratio $a/b = 1.5$.

| BC   | $\beta_1$ | $\alpha = \beta_2 = m = 0.0, \theta = 30^\circ$ | $\alpha = \beta_2 = m = 0.4, \theta = 45^\circ$ | $\alpha = \beta_2 = m = 0.8, \theta = 60^\circ$ |
|------|-----------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
|      | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ |
| 0.0  | 06.71      | 42.63        | 05.74      | 36.23        | 04.81      | 30.75        |
| 0.2  | 11.18      | 101.84       | 11.03      | 101.40       | 09.83      | 88.88        |
| SSSS | 0.4        | 12.33        | 13.78      | 129.71       | 12.77      | 119.22       |
|      | 07.50      | 52.17        | 06.59      | 45.28        | 05.72      | 38.95        |
| 0.6  | 14.73      | 139.73       | 15.20      | 145.76       | 14.28      | 135.97       |
| 0.8  | 15.98      | 154.53       | 16.64      | 162.61       | 15.79      | 153.39       |

Bold values are obtained from [14].

Table 5: Comparison of modes of frequency of the present study (orthotropic parallelogram plate) and obtained in [14] corresponding to tapering parameter $\beta_2$ for a fixed value of aspect ratio $a/b = 1.5$.

| BC   | $\beta_2$ | $\alpha = \beta_1 = m = 0.0, \theta = 30^\circ$ | $\alpha = \beta_1 = m = 0.4, \theta = 45^\circ$ | $\alpha = \beta_1 = m = 0.8, \theta = 60^\circ$ |
|------|-----------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
|      | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ |
| 0.0  | 06.71      | 42.63        | 05.74      | 38.89        | 06.50      | 43.62        | 06.43      | 45.45        |
| 0.2  | 11.18      | 101.84       | 11.06      | 107.11       | 12.29      | 115.39       | 12.08      | 115.82       |
| SSSS | 0.4        | 12.32        | 12.41      | 118.28       | 13.51      | 122.74       | 14.21      | 130.72       |
|      | 07.50      | 44.05        | 06.01      | 40.38        | 06.33      | 43.69        | 06.04      | 42.08        |
| 0.6  | 14.73      | 133.49       | 15.18      | 141.33       | 15.97      | 144.39       | 15.80      | 153.40       |
| 0.8  | 15.97      | 144.39       | 16.62      | 153.11       | 15.79      | 153.39       |

Bold values are obtained from [14].

Table 6: Comparison of modes of frequency of the present study (orthotropic parallelogram plate) and obtained in [14] corresponding to nonhomogeneity $m$ for a fixed value of aspect ratio $a/b = 1.5$.

| BC   | $m$ | $\alpha = \beta_1 = \beta_2 = 0.0, \theta = 30^\circ$ | $\alpha = \beta_1 = \beta_2 = 0.4, \theta = 45^\circ$ | $\alpha = \beta_1 = \beta_2 = 0.8, \theta = 60^\circ$ |
|------|-----|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
|      | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_1$ | $\lambda_2$ |
| 0.0  | 06.71      | 42.63        | 06.50      | 43.62        | 06.43      | 45.45        |
| 0.2  | 11.18      | 101.84       | 12.29      | 115.39       | 12.08      | 115.82       |
| SSSS | 0.4        | 12.32        | 13.78      | 129.71       | 13.57      | 130.72       |
|      | 06.51      | 41.12        | 06.30      | 41.99        | 06.23      | 43.66        |
| 0.6  | 13.36      | 121.72       | 14.76      | 139.21       | 14.86      | 146.70       |
| 0.8  | 14.43      | 131.47       | 15.98      | 151.15       | 15.79      | 153.39       |

Bold values are obtained from [14].
Table 7: Comparison of time period of the present study (orthotropic parallelogram plate) and obtained in [15] corresponding to tapering parameter $\beta_1$ for the fixed value of aspect ratio $a/b = 1.5$.

| $\beta_1$ | $\theta = 0^\circ$ | $\theta = 45^\circ$ |
|----------|--------------------|--------------------|
|          | $K_1$              | $K_2$              | $K_1$              | $K_2$              |
| 0.0      | 0.13285            | 0.51082            | 0.16123            | 0.61588            |
|          | 0.03735            | 0.14364            | 0.02407            | 0.09130            |
| 0.2      | 0.12599            | 0.48513            | 0.15323            | 0.58578            |
|          | 0.03446            | 0.13278            | 0.02229            | 0.08463            |
| CCCC     | 0.11912            | 0.45999            | 0.14520            | 0.55628            |
| 0.4      | 0.03166            | 0.12252            | 0.02229            | 0.07832            |
|          | 0.11240            | 0.43599            | 0.13733            | 0.52804            |
| 0.6      | 0.02906            | 0.11315            | 0.01896            | 0.07254            |
|          | 0.10595            | 0.11343            | 0.01297            | 0.50137            |
| 0.8      | 0.02669            | 0.10476            | 0.01749            | 0.06734            |

Bold values are obtained from [15].

Table 8: Comparison of the time period of the orthotropic rectangle plate and obtained in [38] corresponding to nonhomogeneity $m$ for a fixed value of aspect ratio $a/b = 1.5$.

| $m$     | $\beta_1 = 0.0$ | $\beta_1 = 0.4$ |
|---------|-----------------|-----------------|
|         | $K_1$           | $K_2$           | $K_1$           | $K_2$           |
| 0.0     | 0.13285         | 0.51082         | 0.11912         | 0.45999         |
|         | 0.02978         | 0.14981         | 0.02554         | 0.12964         |
| 0.2     | 0.13511         | 0.51855         | 0.12124         | 0.46709         |
| CCCC    | 0.13733         | 0.52619         | 0.12333         | 0.47407         |
| 0.4     | 0.02982         | 0.15045         | 0.02519         | 0.12883         |
|         | 0.13952         | 0.53370         | 0.12538         | 0.48092         |
| 0.6     | 0.02983         | 0.15070         | 0.02506         | 0.12853         |
|         | 0.14168         | 0.5411          | 0.12740         | 0.48770         |
| 0.8     | 0.02984         | 0.15091         | 0.02491         | 0.12827         |

Bold values are obtained from [38].

Table 9: Comparison of the time period of the orthotropic rectangle plate and obtained in [38] corresponding to tapering parameter $\beta_1$ for aspect ratio $a/b = 1.5$.

| $\beta_1$ | $m = 0.0$ | $m = 0.4$ |
|-----------|-----------|-----------|
|           | $K_1$     | $K_2$     | $K_1$     | $K_2$     |
| 0.0       | 0.13285   | 0.51082   | 0.13733   | 0.52619   |
|           | 0.02978   | 0.14981   | 0.02982   | 0.15045   |
| 0.2       | 0.12599   | 0.48513   | 0.13035   | 0.49983   |
| CCCC      | 0.11912   | 0.45999   | 0.12333   | 0.47407   |
| 0.4       | 0.02554   | 0.12964   | 0.02519   | 0.12883   |
|           | 0.11240   | 0.43599   | 0.11645   | 0.44941   |
| 0.6       | 0.02374   | 0.12132   | 0.02328   | 0.12009   |
|           | 0.10595   | 0.41343   | 0.10984   | 0.42625   |
| 0.8       | 0.02214   | 0.11396   | 0.02160   | 0.11243   |

Bold values are obtained from [38].
The flexural rigidities $D_\zeta, D_\psi$ and torsional rigidity $D_{\zeta\psi}$ of the plate are taken as in [14]

\[
D_\zeta = \frac{E_\zeta I_\zeta^3}{12(1 - \nu_\zeta\nu_\psi)}, \quad D_\psi = \frac{E_\psi I_\psi^3}{12(1 - \nu_\zeta\nu_\psi)},
\]

\[
D_{\zeta\psi} = \frac{G_{\zeta\psi} I_{\zeta\psi}^3}{12}, \quad D_1 = \gamma_\zeta D_\zeta = \gamma_\psi D_\psi,
\]

where $\gamma_\zeta$ and $\gamma_\psi$ are Poisson’s ratios.

Using (3) and (4) in (5), we get

Table 10: Comparison of modes of frequency of the isotropic square plate and obtained in [39–50] at CCCC, SCSC, FCFC, and FSFS edge conditions.

| Boundary conditions | Ref   | $\lambda_1$ |
|---------------------|-------|--------------|
|                     | [39]  | 35.98        |
|                     | [40]  | 35.99        |
|                     | [41]  | 35.98        |
|                     | [42]  | 35.98        |
| CCCC                | [43]  | 35.99        |
|                     | [44]  | 35.99        |
|                     | [45]  | 35.99        |
|                     | [46]  | 35.99        |
|                     | Present | 33.33       |
|                     | [39]  | 28.950       |
|                     | [40]  | 28.951       |
|                     | [47]  | 28.955       |
|                     | [48]  | 29.951       |
|                     | Present | 28.911       |
| SCSC                | [39]  | 22.19        |
|                     | [40]  | 22.27        |
|                     | [49]  | 22.03        |
|                     | Present | 22.43        |
|                     | [39]  | 9.631        |
|                     | [40]  | 9.631        |
|                     | [49]  | 9.631        |
|                     | [50]  | 9.631        |
|                     | Present | 10.574       |

Figure 1: Orthotropic parallelogram plate with a skew angle $\theta$.

Figure 2: Orthotropic parallelogram plate having a two-dimensional circular thickness.
Now, introducing nondimensional variable as
\[ \zeta = \frac{E_i h_0^3}{12(1-\nu_1\nu_2)} \left[ \begin{array}{c}
\left(1 - \alpha \left(1 - \frac{c^2}{a^2}\right) \left(1 - \frac{\psi^2}{b^2}\right)\right) \\
\left(1 + \beta_1 \left(1 - \frac{c^2}{a^2}\right) \left(1 - \frac{\psi^2}{b^2}\right)\right)
\end{array} \right]^3, \]

and components of \( E_1^*, E_2^*, E^* \) and \( G_0 \) are \( E_1^* \), \( E_2^* \), sec \( \psi \), sec \( \theta \), and \( G_0 \), respectively, in \( \zeta \) and \( \psi \) directions.

The equation for kinetic energy \( T_s \) and strain energy \( V_s \) for natural transverse vibration of a nonuniform orthotropic parallelogram is taken as in [15]

\[ T_s = \frac{1}{2} \int_a^b \int_0^d \rho \Phi^2 \cos \theta d\zeta d\psi, \]

\[ V_s = \frac{1}{2} \int_a^b \int_0^d \left[ D_1 \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 + D_\psi \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \tan^2 \theta - 2 \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \tan \theta \sec \theta + \frac{\partial^2 \Phi}{\partial \psi^2} \sec^2 \theta \right)^2 \right. \\
\left. + 2D_1 \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \tan^2 \theta + 2 \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \tan \theta \sec \theta + \frac{\partial^2 \Phi}{\partial \psi^2} \sec^2 \theta \right) + 4D_\psi \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \tan \theta \sec \theta + \frac{\partial^2 \Phi}{\partial \psi^2} \sec \theta \right)^2 \right] \cos \theta d\zeta d\psi. \]

The Rayleigh–Ritz method requires that maximum strain energy must be equal to maximum kinetic energy, i.e.,

\[ J = \delta(V_s - T_s) = 0. \]

Substituting (8) and (9) in (10), we obtained

\[ J = \frac{1}{2} \int_a^b \int_0^d \left[ D_1 \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \right)^2 + D_\psi \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \tan^2 \theta - 2 \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \tan \theta \sec \theta + \frac{\partial^2 \Phi}{\partial \psi^2} \sec^2 \theta \right)^2 \right. \\
\left. + 2D_1 \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \tan^2 \theta + 2 \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \tan \theta \sec \theta + \frac{\partial^2 \Phi}{\partial \psi^2} \sec^2 \theta \right) + 4D_\psi \left( \frac{\partial^2 \Phi}{\partial \zeta^2} \tan \theta \sec \theta + \frac{\partial^2 \Phi}{\partial \psi^2} \sec \theta \right)^2 \right] \cos \theta d\zeta d\psi. \]
Using Eqs. (1), (6) and (7), (11) becomes

\[
\begin{align*}
J &= \int_0^a \int_0^b \left[ \left\{ \cos^4 \theta + \frac{E_*^2}{E_1^2} \sin^4 \theta + 2 \frac{E_*^2}{E_1^2} \sin^2 \theta \cos^2 \theta + 4 \frac{G_0}{E_1^2} \sin^2 \theta \cos^2 \theta \right\} \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right)^2 + \frac{E_*^2}{E_1^2} \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right)^2 \right. \\
& \quad \left. - 4 \left( \frac{E_*^2}{E_1^2} \sin \theta \right) \left( \frac{\partial^2 \Phi}{\partial \zeta \partial \psi} \right) \left( \frac{\partial^2 \Phi}{\partial \psi^2} \right) \right] \, d\zeta \, d\psi \\
& = \lambda^2 \int_0^a \int_0^b \left[ (1 - m Y_1) (1 + \beta_1 Y_1) (1 + \beta_2 Y_2) \right] \Phi^2 \, d\zeta \, d\psi,
\end{align*}
\]

where \( \lambda^2 = 12 \rho_0 a^2 \cos^5 \theta/\rho_1 h_0^2 \), \( Y_1 = (1 - \sqrt{1 - \zeta^2/a^2}) \), and \( Y_2 = (1 - \sqrt{1 - \psi^2/b^2}) \).

The two-term deflection function, which satisfies all the edge conditions, can be taken as in [6]

\[
\Phi(\zeta, \psi) = \left[ \left( \frac{\zeta}{a} \right) \left( \frac{\psi}{b} \right) \left( 1 - \frac{\zeta}{a} \right) \left( 1 - \frac{\psi}{b} \right) \right]
\times \left[ \sum_{i=0}^{N} \left( \frac{\zeta}{a} \right)^i \left( \frac{\psi}{b} \right)^i \right],
\]

which is the product of two functions. Here, the first function represents the boundary conditions depending on the value of \( c, f, g, \) and \( h \), which can take different values depending upon the support edge condition. Values 0, 1, 2 are assigned for free edge, simply supported, and clamped edge, respectively. The second function represents the number of modes of frequencies and \( \Omega_i, i = 0, 1, 2, \ldots, N \), represents arbitrary constants.

In order to minimize the functional given in (12), we require the following condition:

\[
\frac{\delta J}{\partial \Omega_i} = 0, \quad i = 0, 1, 2, 3 \ldots N. \tag{14}
\]

After simplifying (14), we get a homogeneous system of equations in \( \Omega \), whose nonzero solution gives an equation of frequency as

\[
\left| P - \lambda^2 Q \right| = 0, \tag{15}
\]

where \( P = [p_{ij}]_{N+1} \) and \( Q = [q_{ij}]_{N+1} \) are the square matrix of order \( (n + 1), i = 0, 1, 2 \ldots N \) and \( j = 0, 1, 2 \ldots N \).

The following expression is used for calculating the time period:

\[
K = \frac{2 \pi}{\lambda}, \tag{16}
\]

where \( \lambda \) is a frequency obtained from (15).

### 3. Numerical Results and Discussion

For a fixed value of aspect ratio \( a/b = 1.5 \) and skew angle \( \theta = 30^\circ \), the time period \( K \) for the first two modes of vibration of an orthotropic parallelogram plate with 2–D circular thickness and 1–D circular density under 2–D parabolic temperature is computed at \( CCC \), \( CCCF \), \( CFCF \), \( CSCF \), \( SFSF \) edge conditions (refer Figure 3) corresponding to different plate parameters (tapering parameters \( \beta_1, \beta_2 \), thermal gradient \( \alpha \), and non-homogeneity \( m \)). In this study, the authors examined the impact of plate parameters on the behavior of time period of vibrational modes. During the calculation, the values of the following orthotropic material parameters are taken as in [15]:

\[
\begin{align*}
\frac{E_2^*}{E_1^*} &= 0.01, \quad \frac{E_*}{E_1} = 0.3, \quad \frac{G_0}{E_1} = 0.0333, \\
\frac{E_*}{\rho_0} &= 3.0 \times 10^5 \text{ and } \nu_0 = 0.345.
\end{align*}
\]

All the results are conferred within Tables 1–3.
Table 1 represents the time period $K$ of orthotropic parallelogram plate at $CCCC$, $CCCF$, $CFCF$, $SCSF$, and $SFSF$ edge conditions for the fixed value of thermal gradient $\alpha = 0.4$ and nonhomogeneity $m = 0.2$ corresponding to both tapering parameters $\beta_1$ and $\beta_2$. The following observations can be drawn from Table 1:

(i) Time period $K$ decreases corresponding to both increasing value of the tapering parameters $\beta_1$ and $\beta_2$ at all the edge conditions.

(ii) Under $CSCF$ edge condition, the time periods $K$ of vibrational modes are higher and less on $CCCC$ edge condition. For different edge conditions, the time period $K$ of vibrational modes in increasing order corresponding to tapering parameters $\beta_1$ and $\beta_2$, is $CCCC < CCCF < SFSF < CFCF < CSCF$.

(iii) The rate of decrement in the time period $K$ corresponding to the tapering parameter $\beta_1$ is higher at $CCCC$, $CFCF$, and $SFSF$ in comparison to the rate of decrement in the time period $K$ corresponding to the tapering parameter $\beta_2$, while the rate of decrement corresponding to the tapering parameter $\beta_2$ is higher at $CFCF$ and $SCSF$ corresponding to tapering parameter $\beta_2$ in comparison to the rate of decrement in the time period $K$ corresponding to the tapering parameter $\beta_1$.

(iv) For different edge conditions, the rate of decrement in time period $K$ of vibrational modes in ascending order corresponding to tapering parameter $\beta_1$ is $CSCF < CFCF < CFCF < CCCF$, while for different edge conditions, the rate of decrement in time period $K$ of vibrational modes in ascending order corresponding to tapering parameter $\beta_2$ is $SFSF < CCCF < CFCF < CSCF$, $CCCC$. But the rate of decrement in time period $K$ of vibrational modes in ascending order corresponding to tapering parameters $\beta_1$ and $\beta_2$ is $CSCF < CFCF < SFSF < SCSF < CSCF < CCCF$.

Table 2 displays the time period $K$ of the orthotropic parallelogram plate having variation in tapering parameters $\beta_1$ and $\beta_2$ from 0.0 to 0.8 and for fixed value of nonhomogeneity $m = 0.2$ corresponding to thermal gradient $\alpha$ at $CCCC$, $CCCF$, $CFCF$, $SCSF$, and $SFSF$ edge conditions. The subsequent observations can be drawn from Table 2:

(i) As the value of thermal gradient $\alpha$ increases, the time period $K$ of vibrational modes also increases but the time period $K$ of vibrational modes decreases with the increasing value of tapering parameters $\beta_1$ and $\beta_2$ at all edge conditions.

(ii) Like in Table 1, the time period $K$ of vibrational modes is higher on $CSCF$ edge condition and less on $CCCC$ edge condition. For different edge conditions, the time period $K$ of vibrational modes in increasing order corresponding to thermal gradients $\alpha$ is $CCCC < CCCF < SFSF < CFCF < CSCF$.

(iii) At $CCCC$ edge condition, the rate of increment in time period $K$ is higher corresponding to thermal gradient $\alpha$ in comparison to the rate of decrement corresponding to tapering parameters $\beta_1$ and $\beta_2$, while on the rest of the edge conditions, i.e., $CCCC$, $CFCF$, $SCSF$, and $SFSF$, the rate of increment in time period $K$ is smaller corresponding to thermal gradient $\alpha$ in comparison to the rate of decrement corresponding to tapering parameters $\beta_1$ and $\beta_2$.

(iv) The rate of increment in time period $K$ of vibrational modes is higher on $CCCC$ edge condition and less in $CCCF$ edge condition corresponding to thermal gradient $\alpha$, while the rate of decrement in time period $K$ of vibrational modes is higher on $CCCC$ edge condition and less in $CCCF$ edge condition corresponding to tapering parameters $\beta_1$ and $\beta_2$. For different edge conditions, the rate of increment in time period $K$ of vibrational modes in ascending order corresponding to thermal gradient $\alpha$ is $CCCC < CCCF < SFSF < CFCF < CSCF$. But the rate of decrement in time period $K$ of vibrational modes in ascending order corresponding to tapering parameters $\beta_1$ and $\beta_2$ is $CSCF < CFCF < SFSF < SCSF < CSCF < CCCF$.

Table 3 provides the time period $K$ of the orthotropic parallelogram plate at $CCCC$, $CCCF$, $CFCF$, $SCSF$, and $SFSF$ edge conditions corresponding to nonhomogeneity $m$ for a fixed value of thermal gradient $\alpha = 0.4$ and variable values of tapering parameters $\beta_1$ and $\beta_2$, from 0.0 to 0.8. From Table 3, the following facts can be interpreted:

(i) As the value of nonhomogeneity $m$ increases, the time period $K$ of vibrational modes also increases, but time period $K$ of vibrational modes decreases with the increasing value of tapering parameters $\beta_1$ and $\beta_2$, $\beta_1$, $\beta_2$ at all edge conditions.

(ii) Like in Tables 1 and 2, here also the time period $K$ of vibrational modes is higher on $CSCF$ edge condition and less on $CCCC$ edge condition. For different edge conditions, the time period $K$ of vibrational modes in increasing order corresponding to nonhomogeneity $m$ is $CCCC < CCCF < SFSF < CFCF < CSCF$. 

Figure 3: Orthotropic parallelogram plate with different edge conditions.
(iii) At all edge conditions, i.e., CCCC, CCCF, CF CF, CSCF, and SFSF, the rate of increment in time period $K$ is less corresponding to nonhomogeneity $m$ in comparison to the rate of decrement corresponding to tapering parameters $\beta_1$ and $\beta_2$.

(iv) The rate of increment in time period $K$ of vibrational modes is higher on CCCC edge condition and less in CSCF edge condition corresponding to nonhomogeneity $m$, while the rate of decrement in time period $K$ of vibrational modes is higher on CCF edge condition and less in CCC edge condition corresponding to tapering parameters $\beta_1$ and $\beta_1, \beta_2$. For different edge conditions, the rate of change in time period $K$ of vibrational modes in ascending order corresponding to nonhomogeneity $m$ is CSCF < CF CF < CCF < SFSF < CCCC. But the rate of decrement in time period $K$ of vibrational modes corresponding to tapering parameters $\beta_1$ and $\beta_1, \beta_2$ is CCCC < CF CF < SFSF < CSCF < CCF.

4. Convergence of Results

In this section, the authors report a convergence study on modes of frequency $\lambda$ for the following:

(i) Orthotropic parallelogram plate (by taking $\theta = 30^\circ, a/b = 1.5$ in the present study) at CCCC, CCCF, and SFSF edge conditions (refer Table 11)

(ii) Orthotropic rectangle plate (by taking $\theta = 0^\circ, a/b = 1.5$ in the present study) at CCCC, CCCF, CF CF, and SFSF edge conditions (refer Table 12)

(iii) Orthotropic square plate (by taking $\theta = 0^\circ, a/b = 1.0$ in the present study) at CCCC, CCCF, CF CF, SFSF, and CSCF edge conditions (refer Table 13)

When the order of approximation increased for all values of plate parameters in the ranges specified, i.e., $\beta_1 = \beta_2 = m = \alpha = 0, E_1^F/E_1^I = 0.01, E_2^F/E_1^I = 0.3, G_{12}/E_1^I = 0.0333$, and $v_0 = 0.345$. All the results are presented in tabular form.

From Tables 11 and 12, the authors conclude that the first two modes of frequency $\lambda$ of the orthotropic parallelogram plate at CCCC, CCCF, and SFSF edge conditions and orthotropic rectangle plate at CCCC, CCCF, CF CF, and SFSF edge conditions converge up to four decimal places in the fifth approximation, while Table 13 shows that the first two modes of frequency $\lambda$ of orthotropic square plate at CCCC, CCCF, CF CF, SFSF, and CSCF edge conditions converge up to four decimal places in the sixth approximation.

5. Result Comparison

A comparative analysis of modes of frequency $\lambda$ and time period $K$ obtained in the present study was done with the following available published results:

(i) Modes of frequency $\lambda$ of the orthotropic parallelogram plate obtained in [14] at SSSS edge condition corresponding to tapering parameters $\beta_1$ and $\beta_2$ and nonhomogeneity $m$ (refer Tables 4–6)

(ii) Time period $K$ of the orthotropic parallelogram plate obtained in [15] at CCCC edge condition corresponding to tapering parameter $\beta_1$ (refer Table 7)

(iii) Time period $K$ of the orthotropic rectangular plate obtained in [38] at CCCC edge condition corresponding to nonhomogeneity $m$ and tapering parameter $\beta_1$ (refer Tables 8 and 9)

(iv) Modes of frequency $\lambda$ of isotropic square plate obtained in [39–50] at CCCC, SCSC, FCFC, and FSFS edge conditions (refer Table 10)

Tables 4 and 5 present the comparison of modes of frequency $\lambda$ obtained in the present study (orthotropic parallelogram plate) and obtained in [14] corresponding to tapering parameters $\beta_1$ and $\beta_2$, respectively, for a fixed value of aspect ratio $a/b = 1.5$ and for the varying values of tapering parameters $\beta_2$ and $\beta_1$, nonhomogeneity $m$, thermal gradient $\alpha$, and skew angle $\theta$, i.e., $\beta_2 = m = \alpha = 0.0, 0.4, 0.8, \theta = 30^\circ, 45^\circ, 60^\circ$ (refer Table 4) and $\beta_1 = m = \alpha = 0.0, 0.4, 0.8, \theta = 30^\circ, 45^\circ, 60^\circ$ (refer Table 5) at SSSS edge condition. From Tables 4 and 5, the authors conclude that modes of frequency $\lambda$ obtained in the present study (orthotropic parallelogram plate) is less in comparison to modes of frequency $\lambda$ obtained in [14] with the increasing value of the tapering parameters $\beta_1$ as well as $\beta_2$ at SSSS edge condition. The comparison of modes of frequency $\lambda$ obtained in the present study (orthotropic parallelogram plate) and obtained in [14] corresponding to nonhomogeneity $m$ for a fixed value of aspect ratio $a/b = 1.5$ and for the varying values of tapering parameters $\beta_1$ and $\beta_2$, thermal gradient $\alpha$, and skew angle $\theta$, i.e., $\beta_1 = \beta_2 = m = \alpha = 0.0, 0.4, 0.8, \theta = 30^\circ, 45^\circ, 60^\circ$ at SSSS edge condition, are displayed in Table 6. From Table 6, one can conclude that modes of frequency $\lambda$ obtained in the present study (orthotropic parallelogram plate) is less in comparison to modes of frequency $\lambda$ obtained in [14] with the increasing value of nonhomogeneity $m$ at SSSS edge condition.

Table 7 shows the comparison of the time period $K$ of the present study (orthotropic parallelogram plate) and obtained in [15] corresponding to tapering parameter $\beta_1$ for two different sets of values of skew angle $\theta$, i.e., $\theta = 0^\circ, 45^\circ$, at CCCC edge condition for a fixed value of aspect ratio $a/b = 1.5$. Here, the authors exclude nonhomogeneity $m$, thermal gradient $\alpha$, and tapering parameter $\beta_2$ because these plate parameters are not considered in [15]. Table 7 enlightens the fact that the time period $K$ obtained in the present study is higher in comparison to the time period obtained in [15] for increasing value of tapering parameter $\beta_1$. But the rate of decrement in the time period $K$ obtained in the present study is less in comparison to the rate of decrement obtained in [15].

For comparison of time period $K$ of the orthotropic rectangle plate (by considering skew angle $\theta = 0^\circ$ in the present study) and obtained in [38], the authors exclude thermal gradient $\alpha$ and tapering parameter $\beta_2$ because both plates parameters are not considered in [38] (refer Tables 8 and 9). Table 8 incorporates the comparison of time period $K$ of the orthotropic rectangle plate and obtained in [38]
Table 11: Convergence study of modes of frequency of the orthotropic parallelogram plate at CCCC, CCCF, and SFSF edge conditions.

| N  | \(\lambda_1\) | CCCC | \(\lambda_2\) | CCCF | \(\lambda_1\) | CCCC | \(\lambda_2\) | CCCF | \(\lambda_1\) | CCCC | \(\lambda_2\) | SFSF |
|----|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|
| 2  | 12.2997        | 46.5454 | 12.5111       | 38.3289 | 04.4441 | 23.9076 |                      |      |                      |      |                      |      |
| 3  | 12.2997        | 46.5454 | 12.5111       | 38.3289 | 04.7723 | 18.7564 |                      |      |                      |      |                      |      |
| 4  | 12.2997        | 46.5454 | 12.5111       | 38.3289 | 08.1298 | 19.6771 |                      |      |                      |      |                      |      |
| 5  | 12.2997        | 46.5454 | 12.5111       | 38.3289 | 09.8217 | 21.2732 |                      |      |                      |      |                      |      |

Table 12: Convergence study of modes of frequency of the orthotropic rectangle plate at CCCC, CCCF, CFCF, and SFSF edge conditions.

| N  | \(\lambda_1\) | CCCC | \(\lambda_2\) | CCCF | \(\lambda_1\) | CCCC | \(\lambda_2\) | CFCF | \(\lambda_1\) | CCCC | \(\lambda_2\) | SFSF |
|----|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|
| 2  | 12.3061        | 43.3389 | 11.4976       | 37.1324 | 04.8196 | 26.2159 |                      |      |                      |      |                      |      |
| 3  | 11.3061        | 42.7072 | 11.4948       | 35.5899 | 04.6538 | 24.0062 |                      |      |                      |      |                      |      |
| 4  | 11.3061        | 42.7061 | 11.4936       | 35.5099 | 04.4441 | 23.9076 |                      |      |                      |      |                      |      |
| 5  | 11.3061        | 42.7061 | 11.4936       | 35.5099 | 04.2815 | 23.7995 |                      |      |                      |      |                      |      |

Table 13: Convergence study of modes of frequency of the orthotropic square plate under CCCC, CCCF, CFCF, SFSF, and CSCF edge conditions.

| N  | \(\lambda_1\) | CCCC | \(\lambda_2\) | CCCF | \(\lambda_1\) | CCCC | \(\lambda_2\) | CFCF | \(\lambda_1\) | CCCC | \(\lambda_2\) | SFSF | \(\lambda_1\) | CSCF |
|----|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|----------------|------|
| 2  | 24.7880        | 98.0261 | 25.0290       | 75.9606 | 22.4259 | 44.6728 | 10.4990       | 36.4318 | 10.1751       | 49.7153 | 8.4351       | 35.7986 |                      |      |
| 3  | 24.7880        | 95.3261 | 25.0287       | 75.1361 | 21.4911 | 41.3478 | 10.3099       | 49.7645 | 14.7061       | 36.6312 |                      |      |                      |      |
| 4  | 24.7880        | 95.3261 | 25.0287       | 75.1361 | 22.4259 | 44.6728 | 10.4990       | 36.4318 | 10.1751       | 49.7153 | 8.4351       | 35.7986 |                      |      |
| 5  | 24.7880        | 95.1864 | 25.0279       | 75.0792 | 14.0662 | 36.4318 | 10.1751       | 49.7153 | 8.4351       | 35.7986 |                      |      |                      |      |

Comparison of time period \(K\) of the orthotropic rectangle plate and obtained in [39] corresponding to tapering parameter \(\beta_1\), i.e., \(\beta_1 = 0.0, 0.4\), at CCCC edge condition for a fixed value of aspect ratio \(ab = 1.5\). In Table 8, it has been seen that time period \(K\) as well as the rate of increment in time period \(K\) obtained in orthotropic rectangle plate is higher as compared to time period \(K\) as well as the rate of increment in time period \(K\) obtained in [38] with the increase in value of non-homogeneity \(m\).

Comparison of time period \(K\) of the orthotropic parallelogram plate and obtained in [38] corresponding to tapering parameter \(\beta_1\) for two different sets of values of nonhomogeneity \(m\), i.e., \(m = 0.0, 0.4\), at CCCC edge condition for fixed value of aspect ratio \(ab = 1.5\) is incorporated in Table 9. Here, the authors observed that the behavior of time period \(K\) obtained for the orthotropic rectangle plate (refer Table 9) is the same as the behavior of the time period obtained for the orthotropic parallelogram plate (refer Table 7), i.e., the time period \(K\) obtained in the present study is higher in comparison to the time period obtained in [38] for increasing value of tapering parameter \(\beta_1\). But the rate of decrement in the time period \(K\) obtained in the present study is less in comparison to the rate of decrement obtained in [38].

For comparison of the mode of frequency \(\lambda\) of an isotropic square plate (by considering skew angle \(\theta = 0^\circ\) and aspect ratio \(ab = 1\) in the present study) with the mode of frequency \(\lambda\) obtained in [39–50], the authors take the value of the plate parameters as \(E_2/E_1 = 1\), \(\alpha = \beta_1 = \beta_2 = m = 0.0\) (refer Table 10).

Table 10 incorporates the comparison of the mode of frequency \(\lambda\) (first mode) obtained for the square plate and obtained in [39–50] at CCCC, SCSC, FCFC, and FSFS edge conditions. From Table 10, the following facts can be drawn:

(i) The mode of frequency \(\lambda\) (first mode) obtained for the square plate is less in comparison to the mode of frequency \(\lambda\) (first mode) obtained in [39–46] at CCCC and [39, 40, 47, 48] SCSC edge condition, respectively.

(ii) The mode of frequency \(\lambda\) (first mode) obtained for the square plate is slightly higher in comparison to the mode of frequency \(\lambda\) (first mode) obtained in [39, 40, 49] at FCFC and [39, 40, 49, 50] FSFS edge condition, respectively.

6. Conclusions

The present study shows the effect of 2–D tapering parameters \(\beta_1, \beta_2\), 2–D thermal gradient \(\alpha\), and 1–D non-homogeneity \(m\) on the time period \(K\) of the mode of vibration of an orthotropic parallelogram plate at CCCC, CCCF, CFCF, CSCF, and FSFS edge conditions. From the above discussion and results comparison, the authors would like to conclude the following facts:

(i) In the case of two-dimensional circular variation in thickness, the modes of frequency \(\lambda\) obtained for orthotropic parallelogram plate (present study) are...
less in comparison to modes of frequency $\lambda$ obtained in [14] in the case of two-dimensional linear variation in thickness (refer Tables 4 and 5) at $\text{SSSS}$ edge condition.

(ii) In the case of one-dimensional circular variation in density, the modes of frequency $\lambda$ obtained for orthotropic parallelogram plates (present study) are less in comparison to modes of frequency $\lambda$ obtained in [14] in the case of one-dimensional linear variation in density (refer Table 6) at $\text{SSSS}$ edge condition.

(iii) In the case of one-dimensional circular variation in thickness, the time period $K$ obtained for the orthotropic parallelogram plate (present study) is higher in comparison to time period $K$ obtained in [15] in the case of one-dimensional parabolic variation in thickness (refer Table 7) at $\text{CCCC}$ edge condition.

(iv) In the case of one-dimensional circular variation in density, the time period $K$ obtained for the orthotropic rectangle plate (present study) is higher in comparison to the time period $K$ obtained in [38] in the case of one-dimensional linear variation in density (refer Table 8) at $\text{CCCC}$ edge condition.

(v) In the case of one-dimensional circular variation in thickness, the time period $K$ obtained for the orthotropic rectangle plate (present study) is higher in comparison to the time period $K$ obtained in [38] in the case of one-dimensional parabolic variation in thickness (refer Table 9) at $\text{CCCC}$ edge condition.

(vi) At $\text{CCCC}$, $\text{CFCF}$, and $\text{SFSF}$ edge conditions, the tapering parameter $\beta_1$ dominates the rate of change in time period $K$ more than the tapering parameter $\beta_2$, while at $\text{CCCC}$ and $\text{CFCF}$, the tapering parameter $\beta_2$ dominates the rate of change in time period $K$ more than the tapering parameter $\beta_1$ (refer Table 1).

(vii) At $\text{CCCC}$ edge condition, the thermal gradient $\alpha$ dominates the rate of change in time period $K$ more than the tapering parameters $\beta_1, \beta_2$, while at $\text{CCCC}$, $\text{CFCF}$, $\text{CFCF}$, and $\text{SFSF}$, the tapering parameters $\beta_1, \beta_2$ dominate the rate of change in time period $K$ more than the thermal gradient $\alpha$ (refer Table 2).

(viii) At all edge conditions, i.e., $\text{CCCC}$, $\text{CCCC}$, $\text{CFCF}$, $\text{CSCF}$, and $\text{SFSF}$, the tapering parameters $\beta_1, \beta_2$ dominate the time period $K$ (rate of decrement) more than the nonhomogeneity $m$ (refer Table 3).

(ix) The time period $K$ of modes of frequency decreases with the increasing value of tapering parameters $\beta_1, \beta_2$ (refer Table 1), while the time period $K$ of modes of frequency increases with the increasing value of thermal gradient $\alpha$ and nonhomogeneity $m$ (refer Tables 2 and 3).

From the abovementioned conclusions, we can say modes of frequency $\lambda$, time period $K$, and variation in time period $K$ can be minimized and controlled by choosing the circular variation in the plate parameters.

**Data Availability**

The research data used to support the findings of this study are currently under embargo, while the research findings are commercialized. Requests for data, 6 months after the publication of this article, will be considered by the corresponding authors.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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