New algorithm based active method to eliminate stick-slip vibrations in drill string systems

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ABSTRACT
The drill string is always exposed to various types of vibrations among which, stick slip is one of the most important types. It is a severe state of torsional vibrations. This phenomenon can decrease the rate of penetration of drilling, wear of expensive equipment prematurely and cause catastrophic failures. In this paper, a novel adaptive sliding mode (SM) controller is proposed to eliminate stick slip in drill string systems. This proposed algorithm has a more robust capacity than existing 1st-order SM schemes in the literature regarding the robustness to parametric uncertainties, variations in weight on bit (WOB), variations in reference velocity and measurement noise. Moreover, the proposed controller does not require a priori knowledge of the upper bounds of parametric uncertainties, external disturbances and can be easily applied for any operating mode of the drill rig. A proof of stability based on the Lyapunov criterion of the system is given. Simulation results show that the proposed algorithm suppresses the stick-slip while keeping good performances compared to other SM controllers. A comparative study between the proposed controller and classic SM controllers and other adaptive SM scheme is performed in order to assess the advantages of the proposed algorithm and illustrate the overall performance improvements. The obtained results show that the proposed controller succeeded to eliminate the stick-slip phenomenon with the best performance compared to the classic SM controllers. In fact, the proposed controller presented a reduction of nearly 26% in terms of overshoot and 1.6 times better settling time values while having the smoother input signal.

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1. Introduction
During the process of drilling, the drill string always undergoes different types of vibrations at complex geological conditions (Tian et al., 2020). This results in tool failures, reduction of the drilling efficiency, minimization of the rate of penetration (ROP) which is a waste of time and money (Fan et al., 2011). To avoid this, one way is to reduce the vibrations at the drill string level. In the past few years, these vibrations were extensively investigated both theoretically and experimentally. Many techniques for the modeling and control of drill strings have been proposed. The first one is defined by the distributed parameter equations (DPE). It has high accuracy in reproducing the oscillatory behaviour to analyse the drill string vibrations (Saldivar et al., 2016). However, this model is seldom used for control purposes due to its significant complexity. The second category is the coupled PDE-ODE model. With such a model, the design of some control techniques is also significantly hard due to its complexity (Di Meglio & Aarsnes, 2015). Other category is the lumped parameter models which can describe the system dynamics in a simple way (Navarro-López & Licéaga-Castro, 2009; Shi et al., 2010; Vaziri et al., 2018). Its is the most popular model for designing controllers to eliminate the induced vibrations in the drill string system such as stick-slip (Zhang & Shi, 2019). This latter is the extreme state of torsional vibrations. Since stick-slip has a significant impact on the drilling process, several approaches have been proposed to eliminate its outcomes. Some researchers focused on so-called passive control methods such as designing sophisticated bits to limit the reactive torque that might lead to stick-slip (Pelfrene et al., 2011). Other researchers were attracted by active anti-vibration control methods which can be generally classified into linear and nonlinear controllers (Tang et al., 2019). The linear controllers are well-known control schemes based on classic structure that can be defined by a linear quadratic gaussian (LQG) (Hong et al., 2016), the proportional integral (PI) (Monteiro & Trindade, 2017; Pavković et al., 2011), proportional derivative (PD) (Lin et al., 2020), proportional...
integral derivative (PID) (Tian et al., 2020) and H-infinity ($H_{\infty}$) (Sevrarans & Kok, 1998) type controllers.

In actual operation conditions, the parameters of the drilling system are not correctly known and such systems may have a change in its weight on bit (WOB) according to need and depending on different conditions under the type of terrain. Moreover, the torque on bit is modelled by a discontinuous function to replicate the friction happening at the bit level. Consequently, the drilling system control is complicated. Then the implementation of a linear controller may not achieve the most desirable performances since they operate correctly around some specific operating ranges. A nonlinear controller is a major interest for the drill system. This motivated researchers to look at nonlinear control techniques (Kabziński, 2017; Liu, 2018; Plestan et al., 2010; Ritto, Gruzman, et al., 2009). They include fuzzy logic, adaptive, backstepping and sliding mode techniques. A fuzzy logic controller to reduce the drill string vibrations has been developed in Ritto, Gruzman, et al. (2009). Its objective is to tune the direct current motor voltage depending on the Von Moses stress that was measured near the bit. The proposed controller succeeded to reduce the vibrations with respect to the maximal stress supported by the drill string. However, the stress was measured on just one point near the bit although the drill string undergoes different vibrations in the pipes and the collars subsections. In Bu and Dykstra (2014), an indirect adaptive controller is developed by taking into account the parametric uncertainties and the nonlinear torque on the bit. Furthermore, authors in Wasilewski et al. (2019) presented a comparative study between an LQG controller and a proposed direct adaptive controller. In these research works, the simulation results proved that adaptive controllers outperform the LQG regarding the minimization of the cost functional and the closed-loop stability of the system with external disturbances. However, no experimental studies were conducted to validate these controllers’ performance. On the other hand, an inappropriate parametric initialization of several adaptive algorithms can have an effect on the controller’s convergence. In Ullah et al. (2016), a backstepping technique is studied to suppress stick-slip. To ameliorate the performance of such a controller regarding parameter uncertainties, an adaptive form was proposed in Kabziński (2017). The first order filters have been added on the bit and drive velocities thus increasing the robustness of the proposed controller compared to the noises. However, the setting of the filter’s cutoff frequency requires the prior knowledge of the noise features. Furthermore, these filters can introduce some delay on the velocity signals caused by the phase distortion phenomenon. Sliding mode control (SMC) is an advanced nonlinear control technique well known for its robustness regarding parameters variations/external perturbation satisfying matching condition. It is robust against disturbances satisfying the matching condition (Lee & Utkin, 2007). The choice of this technique is mainly due to its robustness as well as its good dynamic performance compared to other nonlinear controllers (Nobahar Sadeghi et al., 2019). For the drill string system, such a technique has been used and tested in different research works (Ghasemi & Song, 2017; Liu, 2018; Sankaranarayanan & Mahindrakar, 2009). In Sankaranarayanan and Mahindrakar (2009) authors present an SM controller (SMC) with a new switching surface proposal. However, the dry friction was neglected. In Liu (2018), the authors proposed two different 1st-order SM controllers that are compared with the conventional one. New terms were thus added to the discontinuous component of the controller to reduce the chattering effect. In Ghasemi and Song (2017), a 1st-order SMC scheme based on a cascade structure is proposed. The aim was to compensate both axial and torsional vibrations. However, the axial oscillations were not properly compensated, thus contributing to the presence of static errors. Admittedly, the new enhancements to the conventional scheme showed good chattering results, but still presents some limitations when applied in practice. One of the limitations is that it requires the prior knowledge of both uncertainties and external disturbances upper bounds. In fact, these bounds can be overestimated or underestimated. Their overestimation leads to excessive gains which can therefore cause the appearance or the amplification of chattering. On the other hand, the controller can lose its robustness when these upper bounds are underestimated. In this case, the controller gain will be smaller than the required value to counteract the perturbations.

For the drill string system, the knowledge of these upper bounds is not always obvious because these values depend on several factors such as operating mode, operating conditions (type of terrain, type of the rock, etc.). In fact, the external disturbances change with the operating conditions and mode. To avoid the necessity of prior knowledge of these bounds, one way is to adapt the gains of the SM controllers. Plestan et al. (2010) presented two 1st-order adaptive SM controllers with constant reaching laws. The first controller evaluates the external disturbances by means of low-pass filters. The second controller uses an algorithm containing the absolute value of the sliding surface in the gain dynamics. In Singh and Holé (2004), the authors solved the problem of design parameter selection of the second reaching law. Authors proposed a novel selection method relying on the time needed to reach the required output tracking. This new tuning technique increased the
convergence time with a similar control effort as the index number reaching law. In Rodriguez et al. (2019) and Castañeda and Gordillo (2019), similar adaptive gain algorithms based on exponent reaching laws have been investigated. Such reaching law guarantees minimal chattering compared to other reaching laws exist in the literature (Latosiński, 2017). However, with the algorithms presented in these papers, the trajectory system converges to the desired velocity with big settling time under external perturbations.

In this paper, a new switching adaptive SM controller is proposed based on an exponent reaching law. Then, our contribution consists of real time updating the controller gains. The main idea is that once the SM is reached, the adaptive gains decrease to reduce the control constraints on the system. This adaptive update is made to avoid the setting again of the controller gains even if the operating conditions are changed. Therefore, in the presence of external disturbances or some model parameter uncertainties, the controller keeps good tracking performance with minimal convergence time and energy consumption. The contribution of this paper focuses on the control methodology of the drill string system. In fact, the proposed algorithm offers automation to the adjustment of the controller gain in order to improve the performance of the controller and eliminates the stick-slip phenomenon. The auto-tuning of the controller gain is necessary with such systems working under different operating conditions. However, the considered system model to synthesize the controller is widely used in the literature. Actually, the used model offers the right compromise between the design complexity of the controller and the mathematical representation of the drill string dynamics related to the stick-slip phenomenon. The contributions of this paper can be summarized as follows: (i) novel adaptive SMC with minimum chattering effect independently from some parameter uncertainties and external disturbance upper bounds; (ii) uncertainties, the controller keeps good tracking performance with minimal convergence time and energy consumption. The contribution of this paper focuses on the control methodology of the drill string system. In fact, the proposed algorithm offers automation to the adjustment of the controller gain in order to improve the performance of the controller and eliminates the stick-slip phenomenon. The auto-tuning of the controller gain is necessary with such systems working under different operating conditions. However, the considered system model to synthesize the controller is widely used in the literature. Actually, the used model offers the right compromise between the design complexity of the controller and the mathematical representation of the drill string dynamics related to the stick-slip phenomenon. The contributions of this paper can be summarized as follows: (i) novel adaptive SMC with minimum chattering effect independently from some parameter uncertainties and external disturbance upper bounds; (ii) implement and test the proposed controller of the drill string system with different operating conditions (WOB, reference bit velocity); and (iii) comparative study between the proposed controller, adaptive and SM controllers is provided in order to evaluate the proposed algorithm performance for the drill string system.

The outline of this paper is organized as follows. Section 2 provides the lumped parameter modelling of the drill string system. Section 3 outlines the design of the proposed adaptive SM as well as a detailed demonstration of the finite-time convergence of the system using the Lyapunov direct method. The simulation results along with the evaluation of the system performances are discussed in Section 4. Finally, the conclusions are provided in Section 5.

### 2. Drill string system

Well drilling is the most adequate way to exploit oil and gas. The drilling rig is mainly composed of two parts: the higher part of the drill string includes a rotary table driven by an electrical motor which is responsible for the driving torque. This last one is transmitted to the lower part via the drill pipes. The lower part of drilling rig is called drill string and is mainly composed of drill collars and drill bit. The main role of drill collars is to provide the proper WOB. The drill-bit is a rock breaking tool. Figure 1 presents the main components of the drilling rig. The drill string’s main role is to transmit the rotation from the top drive table to the drill bit in order to elaborate the down hole drilling process.

#### 2.1. Mathematical dynamic model of the drill string system

Many modelling approaches can be used to simulate the drill string’s behaviour. Modelling the drill string as a torsional pendulum system consists of considering the drill string as a discrete structure composed of assemblies of masses, springs and dampers. Especially for torsional studies, lumped parameter modelling was widely used in the literature where their difference lies on the number of degrees of freedom (DOF) (Liu, 2018; Liu et al., 2017; Navarro-López & Cortés, 2007; Navarro-López & Licéaga-Castro, 2009; Puebla & Alvarez-Ramirez, 2008; Ritto & Ghandchi-Tehrani, 2019; Shi et al., 2010; Vaziri et al., 2018). Their accuracy, therefore, depends on the involved DOF number. For instance Liu (2018) theoretically studied an $n$-DOF model. The considered assumption here is that the model is composed of $n$ number of drill pipes with different damping and friction coefficients. However, this type of model has never been used for control purposes. In the literature review, the lumped models that are used for drill control to eliminate stick-slip are reduced to a 4-DOF or 2-DOF model. Since the focus is on eliminating the stick-slip, 4 DOF is largely sufficient. Generally, lumped parameter models with DOF higher than 4 are used to describe other coupled drill string dynamics including axial and lateral vibrations (Christoforou & Yigit, 2003; de Moraes & Savi, 2019). In fact, reducing the lumped model to 4 DOF imposes some assumption. This assumption consists in considering that all drill pipes having the same damping and friction coefficients. In Ritto and Ghandchi-Tehrani (2019) and Puebla and Alvarez-Ramirez (2008), the authors did not consider both drill pipes and drill collars dynamics. Such an assumption can more reduce the DOF of the model to 2. In Vaziri et al. (2018), authors presented experimental results of an SM controller applied to a 2 DOF
The stick-slip phenomenon was suppressed. However, the oscillations around the desired reference velocity with an overshoot equal to 30% were not eliminated. That is why multiple works have considered the 4 DOF as a model to design controller. In Navarro-López and Licéaga-Castro (2009), Shi et al. (2010), Zhang and Shi (2019) and Navarro-López and Cortés (2007), acceptable performances are obtained in eliminating the stick-slip phenomenon for the drill string systems.

So based on the literature review, a 4DOF model has thus been considered in our study (see Figure 2). This model includes four subsystems: a drive table, the drill-pipes, drill collars and drill-bit (Navarro-López & Licéaga-Castro, 2009).

To define such a model, some assumptions are taken into account: (i) the drill string is vertical, (ii) only torsional vibrations are considered and (iii) all drill pipes have the same inertia. The terms $c_{dt}\dot{\phi}_{dt}$ and $c_{bi}\dot{\phi}_{bi}$ present the viscous damping torques corresponding to the lubrication of the mechanical elements and the influence of drilling fluids at the bit respectively (see Figure 2). $T_{bit}$ is the nonlinear torque replicating the bit-rock interaction at the bit level. The torsional stiffness are defined by $k_{pi}$, $k_{co}$, $k_{bi}$. The torsional damping are defined by $c_{pi}$, $c_{co}$, $c_{bi}$. The equations of motion are given by Navarro-López and Licéaga-Castro (2009):

$$
\begin{align*}
J_{dt}\ddot{\phi}_{dt} + c_{dt}\dot{\phi}_{dt} + c_{pi}(\dot{\phi}_{dt} - \dot{\phi}_{pi}) + k_{pi}(\phi_{dt} - \phi_{pi}) &= u \\
J_{pi}\ddot{\phi}_{pi} + c_{pi}(\dot{\phi}_{pi} - \dot{\phi}_{dt}) + c_{co}(\phi_{pi} - \phi_{co}) + k_{pi}(\phi_{pi} - \phi_{dt}) &= 0 \\
J_{co}\ddot{\phi}_{co} + c_{co}(\dot{\phi}_{co} - \dot{\phi}_{pi}) + c_{bi}(\phi_{co} - \phi_{bi}) + k_{co}(\phi_{co} - \phi_{pi}) &= 0 \\
J_{bi}\ddot{\phi}_{bi} + c_{bi}\dot{\phi}_{bi} + c_{bi}(\dot{\phi}_{bi} - \dot{\phi}_{co}) + k_{bi}(\phi_{bi} - \phi_{co}) &= 0
\end{align*}
$$

With $u \in \mathbb{R}$ presenting the control input and $\phi_{dt}$, $\phi_{pi}$, $\phi_{co}$ and $\phi_{bi}$ are angular positions of the top drive, drill pipes, drill collars and drill bit, respectively. $J_{dt}$, $J_{pi}$, $J_{co}$,
and $J_{bi}$ are the respective moments of inertia. To put the system equations (1) into the state space form, the state vector $\mathbf{x}(t)$ is defined as Navarro-López and Licéaga-Castro (2009):

$$
\dot{\mathbf{x}}(t) = \begin{bmatrix} \dot{\psi}_{dt} \\ \dot{\psi}_{bi} \end{bmatrix} = \begin{bmatrix} \dot{\psi}_{dt} \\ \dot{\psi}_{bi} \end{bmatrix} = \begin{bmatrix} J_{dt} & J_{dt} \\ J_{dt} & J_{dt} \end{bmatrix} \begin{bmatrix} \psi_{dt} \\ \psi_{bi} \end{bmatrix} - \begin{bmatrix} -c_{dt} - c_{pi} \\ -k_{bi} \end{bmatrix} \begin{bmatrix} \psi_{dt} \\ \psi_{bi} \end{bmatrix} + \begin{bmatrix} u \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix}
$$

(2)

with $\mathbf{x}(t) \in \Re^{(7 \times 1)}$ presenting the state vector. Therefore, the system can be written as follows:

$$
\begin{align*}
\dot{x}_1 &= \frac{1}{J_{dt}} \left[ -(c_{dt} + c_{pi})x_1 - k_{pi}x_2 + c_{pi}x_3 + u \right] \\
\dot{x}_2 &= x_1 - x_3 \\
\dot{x}_3 &= \frac{1}{J_{pi}} \left[ c_{pi}x_1 + k_{pi}x_2 - (c_{pi} + c_{co})x_3 - k_{co}x_4 + c_{co}x_5 \right] \\
\dot{x}_4 &= x_3 - x_5 \\
\dot{x}_5 &= \frac{1}{J_{co}} \left[ c_{co}x_3 + k_{co}x_4 - (c_{co} + c_{bi})x_5 - k_{bi}x_6 + c_{bi}x_7 \right] \\
\dot{x}_6 &= x_5 - x_7 \\
\dot{x}_7 &= \frac{1}{J_{bi}} \left[ c_{bi}x_5 + k_{bi}x_6 - (c_{bi} + c_{bit})x_7 - T_{bit}(x(t)) \right]
\end{align*}
$$

(3)

In general form, the equations of system (3) can be written as state space form as follows:

$$
\begin{align*}
\dot{\mathbf{x}}(t) &= \mathbf{A} \mathbf{x}(t) + \mathbf{B} \mathbf{u}(t) + \mathbf{D} T_{bit}(\mathbf{x}(t)) \\
\mathbf{y}(t) &= \mathbf{C} \mathbf{x}(t)
\end{align*}
$$

With $\mathbf{A} \in \Re^{(7 \times 7)}$, $\mathbf{B} \in \Re^{(7 \times 1)}$, $\mathbf{D} \in \Re^{(7 \times 1)}$, and $\mathbf{C} \in \Re^{(1 \times 7)}$. All the matrices are expressed as follows:

$$
\mathbf{A} = \begin{bmatrix}
-(c_{dt} + c_{pi}) & -k_{pi} & c_{pi} & 0 & J_{dt} & J_{dt} & J_{dt} \\
J_{dt} & 1 & 0 & -1 & 0 & 0 & 0 \\
c_{pi} & k_{pi} & -(c_{pi} + c_{co}) & -k_{co} & J_{pi} & J_{pi} & J_{pi} \\
J_{pi} & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & c_{co} & J_{co} & J_{co} & J_{co} & J_{co} \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-c_{co} - c_{bi} & -k_{bi} & c_{bi} & J_{co} & J_{co} & J_{co} & J_{co} \\
J_{co} & 1 & 0 & -1 & 0 & 0 & 0 \\
c_{bi} & k_{bi} & -(c_{bi} + c_{bit}) & J_{bi} & J_{bi} & J_{bi} & J_{bi}
\end{bmatrix}
$$

$$
\mathbf{B} = \begin{bmatrix} \frac{1}{J_{dt}} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T \\
\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & J_{bi} \end{bmatrix}^T \\
\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$T_{bit}$ has to properly reproduce the bit rock-interactions. The two of friction models are widely used and well known by the Stribeck and Karnopp models (Abdulgailil & Siguierdidjane, 2005; Fu et al., 2019). The major drawback of the Stribeck model is that the model cannot reproduce effectively the friction at the bit level due to the discontinuity at velocity zero. Yet, the Stribeck friction does not take into consideration the static friction. Therefore, a Karnopp friction model is used to represent the torque on bit. This last one can be described as follows:

$$
T_{bit} = \begin{cases}
T, & if \left| \psi_{bi} \right| \leq \Delta w, \left| T \right| \leq T_a \\
T_a \text{sign}(\psi_{bi}), & if \left| \psi_{bi} \right| \leq \Delta w, \left| T \right| > T_a \\
\left[ T_0 + (T_a - T_0) \\
e^{-\xi \left| \psi_{bi} - \Delta w \right|}, \right] \text{sign}(\psi_{bi}), & if \left| \psi_{bi} \right| > \Delta w
\end{cases}
$$

(5)

where $T_a = \mu_a WOB R$ presents the maximum friction torque and $T_0 = \mu_0 WOB R$ is the stick friction torque. $\Delta w$ is the threshold value between static and sliding friction, where $\xi$ presents the decline rate of friction torque. Figure 3 shows the different modes of operation for the stick-slip phenomenon. The stick region is defined by Mode 1 where $T_{bit} = T$ with $T = c_{bi}(\psi_{co} - \psi_{bi}) + k_{co}(\psi_{co} - \psi_{bi})$. This mode finishes when $T$ becomes higher than $T_a$ and terminates by entering the sliding region (Mode 2). At the moment when $\psi_{bi}$ becomes higher than $\Delta w$ which means the drill-bit enters then Mode 3. So, the sliding region is defined by two modes 2 and 3.

### 3. The proposed control strategy design

This section addresses the design of the proposed control strategy in order to ensure the regulation of the bit velocity to track the reference trajectory. The aims of this algorithm can be summarized as well: (i) reduce the chattering phenomenon caused by the classic scheme; (ii) control the drill string system taking into consideration the uncertainties/disturbances whose upper bounds are not necessarily known and (iii) offer a self-adjustment feature to the controller which facilitates its use.

Regarding the chattering problem, it should be noted that different algorithms have been proposed for its reduction. One of the methods to reduce the chattering is the state-dependent-gain method (Lee & Utkin, 2007).
Figure 3. Stick-slip modes diagram.

It is based on the design of a controller gain that varies depending on the states of the system instead of a fixed gain. One other effective solution is the higher order SM. The difference compared to the 1st-order SM is that it brings theoretically the sliding surface and its successive derivatives to zero (Taleb et al., 2014).

In contrast, this technique is only applicable for systems having relative degree greater than one which is not the case for the studied system. One other common solution to reduce the chattering effect is the boundary layer design technique (Young et al., 1999). The main idea is to replace the discontinuous sign function by a high gain saturation function (Van de Vorst et al., 1995) or a sigmoid function (Utkin et al., 1999). Such functions can smooth the controller expression.

In this paper, the sign function was replaced by the saturation function since it is the easiest method to implement and it has also been successfully applied in various works such as Navarro-López and Líceaga-Castro (2009). Apart from this solution, our proposed structure of the SM controller also allows to more smoothen the input signal of the system compared to the classical SM algorithms. In addition, the proposed controller algorithm deals with the uncertainties problem for the drill string. Such a problem has been treated in some research works.

Different methods are considered to overcome the uncertainties problem in the drill string system which can be classified into two approaches: (i) stochastic approach and (ii) deterministic approach. Several researchers investigated the uncertainties in the drill string based on the 1st approach. In Ritto, Soize, et al. (2009), uncertainties related to the bit-rock interaction model were investigated. In fact, a probabilistic model is proposed using the nonparametric probabilistic approach. In Ritto et al. (2010), the stochastic identification of the parameters of the probabilistic bit–rock interaction model is carried out using the maximum likelihood method.

Regarding the deterministic approach, adaptive techniques were extensively investigated taking into account the uncertainties. For instance, Bu and Dykstra (2014) proposed an indirect adaptive controller for the speed control of the drill string where the uncertainties were considered for all drill string parameters. In Wasilewski et al. (2019), a direct adaptive controller was proposed. The obtained result proved the robustness of this adaptive scheme regarding to the change of the friction characteristics and the presence of noise.

In this paper, a new adaptive SM controller belonging to the deterministic approach is proposed. The main idea of the proposed algorithm is to consider an interesting convergence law other than the classic controller defined by a constant reaching law. Then, a new version is proposed whose gains are dynamic according to an exponent reaching law ensuring the finite convergence time of the system. In the following subsection, all the theoretical details of the proposed controller are exposed.

3.1. Preliminaries

The sliding variable has a relative degree 1 regarding the control input. For the proposed system, the derivative of the sliding surface is given by

\[ \dot{s} = \psi(x,t) + \gamma(x,t)u \]  

where the functions \( \psi(x,t) \) and \( \gamma(x,t) \) are assumed to be as

\[ \psi(x,t) = \psi_{nom} + \Delta \psi \]
\[ \gamma(x,t) = \gamma_{nom} + \Delta \gamma \]

where \( \psi_{nom} \) and \( \gamma_{nom} \) are the nominal known functions, \( \Delta \psi \) and \( \Delta \gamma \) present the uncertainties. It is assumed that \( \psi(x,t) \) and \( \gamma(x,t) \) are bounded:

\[ |\psi(x,t)| \leq \psi_M \]
With $\psi_M, \psi_n$, and $\psi_M$ are the bounds of $\psi(x, t)$ and $\gamma(x, t)$, respectively. These bounds exist but are not known. According to Plestan et al. (2010), the sliding surface for a given system must be defined via the following set with $\rho > 0$ is the upper bound of $s$ described as

$$s = \{x \in \chi, |s(x, t)| < \rho\}$$

### 3.2. Structure of the proposed controller

A sliding surface is chosen as follows (Liu, 2018):

$$s = (x_1 - x_d) + \lambda \int_0^t (x_1 - x_d) \, dt + \lambda \int_0^t (x_1 - x_\gamma) \, dt$$

where $\lambda$ is the positive constant and $x_d$ is the desired drill string velocity. Once the system is on the sliding surface where the sliding regime is reached ($s = 0$) it will be lead to the desired equilibrium point where $x_1$ will approach $x_d$ and $x_\gamma$ will approach $x_1$. The time derivative of the sliding surface can be written as

$$\dot{s} = x_1 + \lambda (x_1 - x_d) + \lambda (x_1 - x_\gamma)$$

Taking $\dot{x}_1$ from Equation (3) and replacing it in Equation (10) gives the following equation:

$$\dot{s} = 1/J_{dt}[u - (c_{dt} + c_{pi})x_1 - k_{pi}x_2 + c_{pi}x_3]
\quad + \lambda (x_1 - x_d) + \lambda (x_1 - x_\gamma)$$

Yet, the function $\psi(x, t)$ is defined as

$$\psi(x, t) = -1/J_{dt}[(c_{dt} + c_{pi})x_1 - k_{pi}x_2 + c_{pi}x_3]
\quad + \lambda (x_1 - x_d) + \lambda (x_1 - x_\gamma)$$

From Equation (1), $\gamma(x, t)$ is limited to one constant given by

$$\gamma = 1/J_{dt}$$

The equivalent control can be found from the solution of $\dot{s} = 0$ as

$$u_{eq1} = c_{pi}(x_1 - x_3) + k_{pi}x_2 + c_{dt}x_1 - J_{dt}\lambda(x_1 - x_d)
\quad - J_{dt}\lambda(x_1 - x_\gamma)$$

Thereafter, the SMC can be written as (Liu, 2018):

$$u = u_{eq1} + u_{asw1}$$

where $u_{asw1}$ is the discontinuous component of the controller. The innovation of this paper is in the design of a novel adaptive switching control. The proposed switching control $u_{asw1}$ is defined based on the adaptive exponent reaching law. The proposed adaptive SM controller is described as

$$u_{asw1} = -\hat{k}_1(t)|s|^{1/2}\text{sign}(s) - \hat{a}_1(t)s$$

With the dynamic adaptation law of the controller gain is given by

$$\ddot{k}_1(t) = \begin{cases} \mu_1 \text{sign}(|s| - \delta) & \text{if } \hat{k}_1(t) > \alpha \\ \mu_2 & \text{if } \hat{k}_1(t) \leq \alpha \end{cases}$$

With $\delta, \mu_1$ and $\mu_2$ are positive constants and $\alpha << 1$, $\alpha << 1$ are small positive constants. At $t = 0$, $\hat{k}_1(t)$ must be higher than $\alpha$. The proposed controller adapts its gains to establish minimal control effort. Once the SM is reached, the adaptive gain decreases to reduce the control constraints on the system. The 1st equation of (17) shows the gain dynamics for the convergence mode where the gain has large values. The 2nd equation shows the decreasing gain dynamics when the system reaches the SM. The block diagram of the proposed controller applied on the drill string system is shown in Figure 4.

### 3.3. The convergence proof

Using the Lyapunov direct method, the proof is determined as follows:

**Lemma**: For the controller, Equation (16) with its dynamic gain defined by Equation (17) applied on the system (3), the gain $k_1(t)$ has an upper bound defined by a positive constant $\tilde{k}_1$ with:

$$0 < \tilde{k}_1(t) \leq \tilde{k}_1, \forall t > 0$$

#### 3.3.1. Proof

The candidate Lyapunov function is as follows:

$$V(t) = \frac{1}{2}s^2 + \frac{1}{2}\tilde{k}_1(t) - \tilde{k}_1)^2$$

$$V(t) = s \dot{s} + \frac{\hat{k}_1(t) - \tilde{k}_1}{\gamma} s \mu_1 \text{sign}(|s| - \delta)$$

$$V(t) = s \dot{s} + s^\gamma \mu_1 (\hat{k}_1(t) - \tilde{k}_1) \mu_1 \text{sign}(|s| - \delta)$$

$$V(t) = s \dot{s} + s^\gamma (\hat{k}_1(t)|s|^{1/2}\text{sign}(s) - \tilde{a}_1(t)s)$
\quad + \frac{1}{\gamma} \tilde{k}_1(t) |s|^{1/2}\text{sign}(|s| - \delta)$$

$$V(t) = s \dot{s} + \frac{\hat{k}_1(t) - \tilde{k}_1}{\gamma} \mu_1 \text{sign}(|s| - \delta)$$

$$V(t) \leq |s|\psi_M - |s|\gamma\tilde{k}_1(t)|s|^{1/2} - \alpha \gamma \tilde{k}_1(t)|s|^2$$
Figure 4. The block diagram of the proposed controller.

\[
+ \frac{1}{\gamma} (\dot{k}_1(t) - \tilde{k}_1) \mu_1 \text{sign}(|s| - \delta) \tag{24}
\]

\[
V(t) = |s| \psi_M - |s| \gamma M \hat{k}_1(t) (|s|^{1/2} - \alpha |s|)
+ \frac{1}{\gamma} (\dot{k}_1(t) - \tilde{k}_1) \mu_1 \text{sign}(|s| - \delta)
+ |s| \gamma M \hat{k}_1(t) (|s|^{1/2} - \alpha |s|)
+ \frac{1}{\gamma} \mu_1 \text{sign}(|s| - \delta) \tag{25}
\]

With \( \beta > 0 \)
\[
V(t) = |s| [\psi_M - \gamma m \hat{k}_1 (|s|^{1/2} - \alpha |s|)]
+ (\dot{k}_1(t) - \tilde{k}_1)[-|s| \gamma m \hat{k}_1(t) (|s|^{1/2} - \alpha |s|)]
+ \frac{1}{\gamma} \mu_1 \text{sign}(|s| - \delta)
+ \beta |\dot{k}_1(t) - \tilde{k}_1| + \beta |\dot{k}_1(t) - \tilde{k}_1| \tag{26}
\]

\[
V(t) = -|s| (|\psi_M + \gamma M \hat{k}_1 (|s|^{1/2} - \alpha |s|))
- \beta |\dot{k}_1(t) - \tilde{k}_1| - |\dot{k}_1(t) - \tilde{k}_1| \|s\|^{3/2}
(\gamma M \hat{k}_1(t) (1 - \alpha |s|^{1/2}))
- |\dot{k}_1(t) - \tilde{k}_1| \left( \frac{1}{\gamma} \mu_1 \text{sign}(|s| - \delta) + \beta \right) \tag{27}
\]

where \( V(t) \) is negative definite if:
\[
1 - \alpha |s|^{1/2} > 0
- \frac{1}{\gamma} \mu_1 \text{sign}(|s| - \delta) + \beta > 0 \tag{28}
\]

For the first condition, is a logic condition regarding the considered choice imposed above of the constant. For the second condition, two possible cases can be considered:

**Case 1:** if \(|s| > \delta, V(t)\) is negative definite if:
\[
\mu_1 > \gamma \beta \tag{30}
\]

**Case 2:** if \(|s| \leq \delta, V(t)\) is negative definite if:
\[
\mu_1 > -\gamma \beta \tag{31}
\]

Therefore, the following condition must be established:
\[
-\gamma \beta < \mu_1 < \gamma \beta \tag{32}
\]

Since \( \mu_1 \) is a positive constant defined in Equation (17), then the condition \( \mu_1 > -\gamma \beta \) can be replaced by \( \mu_1 > 0 \). Consequently the condition \( \mu_1 < \gamma \beta \) is enough to guarantee the stability of the system.

### 4. Results and discussion

In order to assess the advantages of the proposed control algorithm and illustrate the overall performance improvements, a comparative study is conducted between the proposed adaptive SMC, a classic sliding mode controller (CSMC), a modified sliding mode controller (SMCM) and a well-known adaptive sliding mode controller (ASMC) existing in the literature. The CSMC is designed using the well-known basic switching law (Navarro-López & Licéaga-Castro, 2009):
\[
\mu_{sw1} = -k_1 \text{sign}(s) \tag{33}
\]

It has noted that \( k_1 \) must be imposed as a big value to counteract the perturbations. The second controller
uses the following exponent reaching law (Castañeda & Gordillo, 2019):

\[ \mu_{sw2} = -k_2|s|^{1/2} \text{sign}(s) - k_3s \]  

(34)

With \( k_2 \) and \( k_3 \) being positive constants. Here, the linear term \( k_3s \) helps to reduce the chattering effect. By referring to Plestan et al. (2010), the adaptive controller used for our comparative study is given by

\[ \mu_{asw2} = -\hat{k}_4(t) \text{sign}(s) \]  

(35)

with

\[ \hat{k}_4(t) = \begin{cases} 
\mu_{11}|s|\text{sign}(|s| - \delta_2) & \text{if } \hat{k}_4(t) > \mu_{22} \\
\mu_{22} & \text{if } \hat{k}_4(t) \leq \mu_{22} 
\end{cases} \]  

(36)

With \( \hat{k}_4(0) > \mu_{22}, \mu_{11} > 0 \) and \( \mu_{22} > 0 \) being very small, and \( \delta_2 = 4\hat{k}_4(t)T_p \). Where \( T_p \) is the sampling period. Table 1 presents the different parameters of the controllers. The parameter values are chosen to have the best performance of each controller regarding a reference bit velocity \( V_{ref} \) equal to 12 rad/s and WOB equal to 90 kN. The comparison and evaluation of different controllers are done using the evaluation criteria: (i) overshoot (O%), settling time (ST), static error (SE), integral absolute error (IAE) and root mean square deviation (RMSD) described as

\[ O\% = \frac{\text{peakvalue} - \text{finalvalue}}{\text{finalvalue}} \times 100 \]  

(37)

\[ SE = |\text{referencevalue} - \text{finalvalue}| \]  

(38)

The settling time is the time required to reach and steady within a specified range of 5% of the final value. The main utility of IAE is to give an idea on the cumulative errors. IAE is described as

\[ IAE = \int |e| \, dt \]  

(39)

With \( e = V_{ref} - V_{bit} \) The RMSD criterion gives an idea on the energy consumption of the different controllers. The RMSD can be expressed as

\[ \text{RMSD} = \sqrt{\frac{1}{n} \sum (T^2)} \]  

(40)

With \( T \) presenting the control signal, \( n \) is the number of samples over time.

### 4.1. Simulation results

The simulations are implemented in the MATLAB/Simulink environment. Various tests are done to illustrate the robustness features of the different controllers. Robustness tests are defined as:

**Table 1.** Numerical values for the controllers.

| Controller | Parameters | Value |
|------------|------------|-------|
| CSMC (Navarro-López & Licéaga-Castro, 2009) | \( k_1 \) | 3 |
| | \( k_2 \) | 3 |
| | \( k_3 \) | 0.001 |
| SMCM (Castañeda & Gordillo, 2019) | \( \mu_{11} \) | 0.1 |
| | \( \mu_{22} \) | 0.01 |
| ASMC (Plestan et al., 2010) | \( \mu_{11} \) | 3 |
| | \( \mu_{22} \) | 2.001 |
| Proposed | \( \alpha \) | 0.1 |
| | \( s \) | 0.01 |

**Table 2.** Numerical values for the drill-string system parameters.

| Parameter | Symbol | Value | Units |
|-----------|--------|-------|-------|
| Inertia of the drive table | \( J_d \) | 930 | kg m² |
| Inertia of the drill bit | \( J_b \) | 471.96 | kg m² |
| Inertia of the drill pipes | \( J_p \) | 2782.25 | kg m² |
| Inertia of the drill collar | \( J_c \) | 750 | kg m² |
| Torsional damping between drive table and drill pipe | \( C_d \) | 425 | N m/s/rad |
| Torsional damping between drill pipe and BHA | \( C_{bi} \) | 50 | N m/s/rad |
| Torsional damping between BHA and drill bit | \( C_{co} \) | 190 | N m/s/rad |
| Torsional damping between BHA and drill pipe | \( C_{pi} \) | 193.61 | N m/s/rad |
| Torsional stiffness between drive table and drill pipe | \( k_{bi} \) | 698.06 | N/m/rad |
| Torsional stiffness between drill collar and BHA | \( k_{co} \) | 1080 | N/m/rad |
| Torsional stiffness between BHA and drill bit | \( k_{ci} \) | 907.48 | N/m/rad |
| Decline rate of friction torque | \( \xi \) | 0.5 | s/rad |
| Static and sliding friction threshold | \( \Delta w \) | 0.001 | rad/s |
| Radius of the drill bit | \( R \) | 0.15 | m |
| Coulomb friction coefficient | \( \mu_c \) | 0.8 | – |
| Static friction coefficient | \( \mu_s \) | 0.5 | – |

**i) Test 1:** Robustness to the variation of both reference velocity and WOB;

**ii) Test 2:** Robustness to measurement noise; and

**iv) Test 3:** Robustness to parametric uncertainties.

- **(i) Test 1:** Robustness to the variation of both reference velocity and WOB;
- **(ii) Test 2:** Robustness to measurement noise; and
- **(iv) Test 3:** Robustness to parametric uncertainties.

It is to note that the tuning of the controllers was done once for all different tests. The chosen sampling frequency is \( f_s = 10^3 \) Hz. The drill string parameters used in the simulations are presented in the Table 2. The chosen parameters mentioned in Table 2 correspond to a real drill string reported in Navarro-López and Licéaga-Castro (2009) with a pipeline consisting of 130 drill pipes of 5 inches outer diameter and 9 meters of length and a drill bit of 1.5 metres of length.
4.1.1. Robustness to the variation of reference velocity and WOB

The 1st tests are conducted to illustrate the robustness regarding the variation of the reference velocity $V_{\text{ref}}$ and WOB. Figure 5 illustrates the reference velocity and WOB trajectories for the upcoming simulations.

Figure 6 presents bit velocities curves of the different controllers. It is shown that for a reference velocity equal to 12 rad/s and WOB equal to 90 kN, the proposed controller had minimal overshoot and settling time values of 45% and 33 s respectively (see zoomed range). At $t = 100$ s, the WOB changes to 70 kN. The decrease of the WOB induced a slightly increase in the bit velocities and a decrease in the input signals for an interval time of 25 s then the system reached the steady state again. However, the bit velocity curve of the proposed controller for $t = 100$ s had a minimal settling time compared to the other controllers.

At $t = 150$ s, the reference velocity now increases to 13 rad/s. The small change of reference velocity (1 rad/s) gives that all controllers had similar overshoot values. However, the proposed controller had a slightly better settling time compared to the other controllers.

4.1.2. Robustness to measurement noise

In practice, the measurement noise can generate problems for the controlled system. This problem is more important specifically for controllers based on the SM technique since they already suffer from the chattering problem. Various sources of noise exist in practice such as thermal noise and shot noise. Whatever the source of the noise, it is characterized by a random behaviour which is generally added to the useful signal in an additive way. These random signals can be stationary such as white noise. This last one is a stochastic process which has the same power spectral density at all frequencies. It can therefore be a representative of measurement noise (Monnerie et al., 2004). In Figures 8 and 9, the white noise amplitude $10^{-4}$ db was chosen in order to approximate the true measurement noise of the bit velocity given by a real sensor. In other words, to have the velocity curve close to that which can be given by a speed sensor. But it is true that this remains relative to the quality of the measurement sensor.

In order to have a quantitative idea, the RMSD is calculated. Table 3 gives RMSD values of each controller.

The smallest energy consumption belongs to the proposed controller. This proves that the least impact of the noise belongs to the proposed controller. The proposed adaptive gain dynamics proved to be better than the ASMC mentioned in Plestan et al. (2010).
4.1.3. Robustness to parametric uncertainties

Regarding the type of parametric uncertainties, the choice of the sliding surface imposed the equivalent control scheme. This latter furthermore imposed the choice of the parameters under uncertainties. Yet, the equivalent control given by Equation (14) is replaced by the following equation:

$$u_{eq2} = \Delta_1 c_{pl}(x_1 - x_3) + \Delta_2 k_{pl}x_2 + \Delta_3 c_{dl}x_1 - J_{dt}\lambda(x_1 - x_d) - J_{dt}\lambda(x_1 - x_f) \quad (41)$$

The amplitude of parametric uncertainties was chosen in order to assess the performance of different controllers.
in all possible cases. As shown in Equation (41), parametric uncertainties are imposed on the drill pipes stiffness and damping and the drive table damping. To evaluate the robustness of each controller regarding these parametric uncertainties, different variations of uncertainties are taken into account (see Table 4). Figure 10 shows that with small parametric uncertainties, the CSMC kept acceptable performance compared to the nominal case. However, the controller lost its robustness for medium parametric uncertainties and the trajectory of the system diverged completely for big parametric uncertainties.

Regardless of the values of parametric uncertainties, Figure 11 shows that the response of the system diverges. It is noteworthy to mention that in both cases 2 and 3, the...
Table 3. RMSE values of different controllers under measurement noise.

| Controller | RMSD      |
|------------|-----------|
| CSMC       | $1.3097 \times 10^4$ |
| SMCm       | $1.3032 \times 10^4$ |
| ASMC       | $1.3046 \times 10^4$ |
| Proposed controller | $1.3016 \times 10^4$ |

Table 4. Different cases of parametric uncertainties.

| Parametric uncertainties | $\Delta_1$ (%) | $\Delta_2$ (%) | $\Delta_3$ (%) |
|--------------------------|-----------------|-----------------|-----------------|
| Nominal Case             | 0               | 0               | 0               |
| Case 1 (small parametric uncertainties) | 20              | 20              | 20              |
| Case 2 (medium parametric uncertainties) | 50              | 50              | 50              |
| Case 3 (large parametric uncertainties) | 80              | 80              | 80              |

Figure 9. Input signals of all controllers under measurement noise: (a) full range and (b) zoomed range.
Figure 10. System responses with the CSMC for different cases of parametric uncertainties.

Figure 11. System responses with the SMCm for different cases of parametric uncertainties.

For all cases, the ASMC converged to the reference velocity in a reasonable time as can be seen from Figure 12. However, for large parametric uncertainties (case 3), the settling time increased significantly of nearly 20%.

For the different cases of parameter variations, the proposed controller kept its good performance, see Figure 13. For instance, the convergence time was not
affected in all cases. In Table 4, a closer look at overshoot, settling time and static error values of the different control strategies along with a comparative overview is presented regarding the robustness to parametric uncertainties and measurement noise for a WOB equal to 90 kN and $V_{\text{ref}}$ equal to 12 rad/s.

It is clear in Table 5 that the proposed controller succeeded to eliminate the stick-slip phenomenon with the least settling time and overshoot values compared to the other controllers. The proposed algorithm had the least IAE value which means that the nearest response to the reference velocity belongs to the proposed controller. Thus, it had great performance in eliminating torsional vibrations occurring in the drill string with great adaptation to parametric uncertainties and operating conditions.

### 4.2. Discussion

In this subsection, the discussions are elaborated to give a more global look on overshoot and settling time values of the CSMC, SMCm, ASMC and the proposed controller. To enrich the discussions of the obtained results, different tests were carried out based on different criteria (see Equations (38)–(39)) that are evaluated via boxplots. All controllers were tested under 10 different values of reference velocity and WOB. Reference velocities values are between 7 and 16 rad/s considering a step of 1 rad/s. WOB values are between 70 and 160 kN with a step of 10 kN. For the 10 different simulations, a change in the reference velocity and WOB at $t = 0$ is done to investigate different initial conditions for the drill string system. Overshoot values for different $V_{\text{ref}}$ are presented in Figure 14. The least overshoot values belong to the proposed controller with variation between 44% and 52%. The biggest overshoot values are recorded for the SMCm reaching nearly 84%. The SMC and the ASMC have relatively similar overshoot intervals. However, the ASMC has a lower overshoot value.

Figure 15 shows that the least settling time values belong to the proposed controller with a minimal value of 32 s. The biggest settling time values are of the SMCm with an increase of 1.3 times compared to the proposed controller. It is also noticed that the CSMC and the ASMC have similar settling time variations. This figure shows also that the proposed controller’s significantly small settling time almost did not change which proves that the proposed algorithm guarantees the fast convergence independent of $V_{\text{ref}}$.

The evaluation of overshoot and settling time values for 10 different WOB values between 70 and 160 kN are illustrated in Figures 16 and 17. It is clear in Figure 17 that the SMCm has both the biggest overshoot values and the largest variations. And, the ASMC presents a minimal variation of overshoot values for different WOB values. Yet, the proposed controller has the least overshoot values compared to all the other controllers. This proves that the proposed control strategy is the most adequate for controlling the drill string system under different operating conditions.
**Figure 13.** System responses with the proposed controller for different cases of parametric uncertainties.

**Figure 14.** Overshoot values for ten values of reference velocity.

**Table 5.** Recapitulative table of results for WOB = 90 kN and $V_{ref} = 12$ rad/s.

| Controller | Overshoot | Static error | Settling time | IAE | RPU | RMN | EC |
|------------|-----------|--------------|---------------|-----|-----|-----|----|
| $u_{\text{sw1}}$ | 58% | 0.003 rad/s | 38 s | 114.8785 | + | + | + |
| $u_{\text{sw2}}$ | 60% | 0.006 rad/s | 45 s | 146.9782 | - | + | ++ |
| $u_{\text{sw2}}$ | 56% | 0.002 rad/s | 36 s | 164.3164 | ++ | ++ | ++ |
| Proposed | 45% | 0.001 rad/s | 33 s | 106.9883 | +++ | +++ | +++ |
It can be seen from Figure 17 that the SMCm had the biggest settling time regarding different WOB values with a minimal value of 42 s. Yet, the proposed controller had the smallest settling time values which guarantees the fast convergence of the proposed scheme regarding different WOB values.

As shown in the Figures 14–17, both criteria, the settling time and overshoot, remained almost unchanged with the proposed algorithm against $V_{ref}$ variations. Therefore, the proposed controller keeps the best and the same performance compared to others for such variation. On the other hand, these two criteria vary slightly with the WOB variation but their variation intervals remain small. Moreover, the bounds of these intervals are also the lowest compared to the other algorithms. This means that the proposed algorithm can be implemented with...
different drilling rigs and for various operating conditions without worrying about its parametric adjustment.

5. Conclusion

A new adaptive sliding mode controller has been proposed for the bit velocity regulation of a drill string system. It was developed to suppress the stick-slip phenomenon taking into consideration the external perturbations and some parametric uncertainties. A comparative study with well-known controllers in the literature is carried out with different operating conditions. The obtained results highlight the performance of the proposed algorithm. This last one proved to have better performance compared to both classical SM and adaptive SM control strategies in terms of precision, settling time and overshoot in static and in dynamic performances. The proposed algorithm presents also an appreciable improvement that has been noticed regarding the robustness to the some parametric uncertainties and the measurement noise. The proposed controller also enhances the smoothing of the input signal of the system and reduces the chattering effect. Therefore, these improvements will have an impact on the life time of the drill string system. All these results give the proposed controller the feature that it can be implemented with different drilling rigs and under different operating conditions. Future research work will include incorporating fuzzy tuning techniques to the proposed controller and testing the proposed controller on a drill-string test bench. A full-size test rig to validate the proposed controller and prospective tuned controllers is currently under fabrication (Tucker et al., 2020).

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Appendices

Appendix 1. Definitions of controllers parameters

| Parameter | Symbol |
|-----------|--------|
| Sliding surface | \( s \) |
| Uncertain functions belonging to the set | \( \mathcal{H}, \psi, \Gamma \) |
| Control input | \( u \) |
| Positive constant | \( \lambda \) |
| The time derivative of the sliding surface | \( \dot{s} \) |
| The candidate Lyapunov function | \( V \) |
| Positive constants | \( \delta, u_1, u_2, \alpha \) |
| Adaptive gain upper bound | \( k_1, k_2, k_3, k_4 \) |
| Bit velocity | \( V_{\text{bit}} \) |
| Required reference velocity | \( V_{\text{ref}} \) |
| Positive constants | \( \Delta_2, u_1, u_2, u_3, u_4, u_5 \) |

Appendix 2. Abbreviations

| Acronym | Definition |
|---------|------------|
| ASMC | Adaptive Sliding Mode Controller |
| CPDE | Coupled Partial Differential Equations |
| CSMC | Classic Sliding Mode Controller |
| DOF | Degrees Of Freedom |
| LQG | Linear-Quadratic-Gaussian |
| ODEs | Ordinary Differential Equations |
| PD | Proportional-Derivative |
| PI | Proportional-Integral |
| PID | Proportional-Integral-Derivative |
| ROP | Rate Of Penetration |
| SM | Sliding Mode |
| SMC | Sliding Mode Controller |
| SMCM | Modified Sliding Mode Controller |
| WOB | Weight On Bit |