On the impossibility of using the longitude of the ascending node of GP-B for measuring the Lense-Thirring effect

Lorenzo Iorio
Dipartimento Interateneo di Fisica dell' Università di Bari
Via Amendola 173, 70126
Bari, Italy

Abstract

The possibility of analyzing the node $\Omega$ of the GP-B satellite in order to measure also the Lense-Thirring effect on its orbit is examined. This feature is induced by the general relativistic gravitomagnetic component of the Earth gravitational field. The GP-B mission has been launched in April 2004 and is aimed mainly to the measurement of the gravitomagnetic precession of four gyroscopes carried onboard at a claimed accuracy of 1% or better. The aliasing effect of the solid Earth and ocean components of the solar $K_1$ tidal perturbations would make the measurement of the Lense–Thirring effect on the orbit unfeasible. Indeed, the science period of the GP-B mission amounts to almost one year. During this time span the Lense-Thirring shift on the GP-B node would be 164 milliarcseconds (mas), while the tidal perturbations on its node would have a period of the order of $10^3$ years and amplitudes of the order of $10^5$ mas.

Keywords: GP-B, Lense-Thirring effect, orbit, longitude of the ascending node, orbital tidal perturbations

1 Introduction

The main scientific task of the Gravity Probe B (GP-B) mission [1], launched in April 2004, is the measurement of the general relativistic precession [2] of four gyroscopes carried onboard induced by the Earth gravitomagnetic field [3]. The claimed accuracy\(^1\) is of the order of 1% or better. The duration of the experiment is almost one year.

Measuring directly gravitomagnetism in a reliable and accurate way would be a very important test of fundamental physics.

\(^1\)See, e.g., the leaflet Gravity Probe B Launch Companion downloadable from [http://einstein.stanford.edu/index.html](http://einstein.stanford.edu/index.html)
Up to now, the only attempts to measure it in the gravitational field of the Earth have been performed by Ciufolini and coworkers [4, 5]. They have tried to measure the secular Lense-Thirring effect [6] on the whole orbits of the existing laser-ranged geodetic LAGEOS and LAGEOS II satellites by analyzing time series of the combined orbital residuals of the nodes $\Omega$ of LAGEOS and LAGEOS II and the perigee $\omega$ of LAGEOS II over time spans some years long. The adopted combination is not affected, by construction, by the first two even zonal harmonics $J_2$ and $J_4$ of the static part of the multipolar expansion of the Earth gravity potential whose induced classical precessions severely alias the genuine gravitomagnetic trends of $\Omega$ and $\omega$. The claimed accuracy is of the order of 20-30% [5]. For different, more conservative but, perhaps, more realistic estimates of the total error budget see [7, 8, 9]. However, the multi-satellite approach could turn out to be more fertile and fruitful in view of the improvements in the present and near future Earth gravity field solutions from the CHAMP [10] and, especially, GRACE [11, 12] dedicated missions. Indeed, they could allow to greatly reduce the systematic error due to the even zonal harmonics, so to discard the perigee of LAGEOS II and use the nodes only. In [9] a $J_2$-free combination which involves the nodes of LAGEOS and LAGEOS II only has been explicitly proposed: according to the recently released preliminary GRACE models, it would allow a measurement of the Lense-Thirring effect at a 15% level of accuracy (1 $\sigma$). Recently, a combination involving the nodes of the geodetic LAGEOS, LAGEOS II and Ajisai satellites and of the radar altimeter Jason-1 satellite has been put forth [13]: it would allow to reduce the error due to the geopotential by cancelling out the first three even zonal harmonics. The major problems could come from the fact that the non-gravitational perturbations on Jason–1 should be carefully dealt with. It must be pointed out that the multi-year multi-satellite $J_\ell$-free approach allows to perform, in principle, as many analyses as one wants because of the extremely long lifetimes of the satellites to be used$^2$ and of the availability of data records many years long. Moreover, since, in this case, one is interested in the precise satellites’ orbit reconstruction, such analyses can benefit from the improvements both in the observational techniques and in the modelling of the perturbing forces acting on the satellites which will become available in the future. So, the reliability and the accuracy of a measurement of the Lense-Thirring effect based on such an approach is not fixed once and for all but is increasing, at least up to certain level. The launch of another

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$^2$The lifetime of the LAGEOS satellites is of the order of $10^5$ years. LAGEOS and LAGEOS II are in orbit since 1976 and 1992, respectively.
LAGEOS–type satellite like the proposed LARES \cite{14, 15} or, more recently, OPTIS \cite{16} would further enforce the reliability and precision of such a measurement, which could be pushed to the 1\% level or, perhaps, even better.

It seems, then, legitimate to ask if it would be possible to enlarge and enforce the significance of the GP-B gravitomagnetic test by measuring also the Lense–Thirring secular precession of its node. A previous treatment of this problem can be found in \cite{17}. Apparently, this possibility sounds very appealing because of the strictly polar orbital configuration retained during the science phase and of the notable accuracy reached by the most recent GRACE-based terrestrial gravity models. Indeed, the competing secular nodal precessions induced by the even zonal harmonics of the Earth gravitational field are proportional to \( \cos i \) \cite{8}, where \( i \) is the inclination of the satellite orbital plane to the Earth equator assumed as \( \{ x, y \} \) reference plane. Although GP-B is much more sensitive to the higher degree multipoles of the Earth gravity field than, e.g., the LAGEOS satellites\(^3\), the fact that \( i_{\text{GP-B}} = 90.007 \) deg together with the high accuracy of the latest solutions of the Earth gravity field from GRACE should constrain the systematic error due to the mismodelling in the even zonal harmonics of the geopotential to an acceptable level. For the combined impact of the departures from nominal polar orbital configurations and of low altitudes see \cite{18}. But, as we will show, this is not all the story.

2 The impact of the static and time-varying part of the Earth gravitational field

The secular Lense-Thirring precession of the longitude of the ascending node \( \Omega \) of a test mass freely falling in the gravitational field of a central spinning mass with proper angular momentum \( J \) is

\[
\dot{\Omega}_{\text{LT}} = \frac{2GJ}{c^2a^3(1 - e^2)^{3/2}},
\]

(1)

where \( G \) is the Newtonian gravitational constant, \( c \) is the speed of light in vacuum and \( a \) and \( e \) are the semimajor axis and the eccentricity, respectively, of the orbit of the test particle. The gravitomagnetic shift of GP-B amounts to 164 milliarcseconds per year (mas yr\(^{-1}\)) in the following. See Table 1 for the relevant parameters of the Earth-GP-B system.

\(^3\)The semimajor axis \( a \) of GP-B amounts to 7027.4 km, while \( a_{\text{LAGEOS}} = 12270 \) km. The classical nodal precessions fall off as \( R^\ell a^{-[(\ell + 3)/2]} \).
Table 1: Relevant parameters for the calculation of the solid Earth and ocean tidal perturbations for some selected tidal constituents and orbital parameters of GP-B. The degree $\ell = 2$ only tidal terms have been considered. The considered tidal constituents are the zonal ($m = 0$) lunar 18.6-year tide (055.565 in the Doodson notation) and the tesseral ($m = 1$) solar $K_1$ tide. See [19] and the references therein for the quoted numerical values of the tidal parameters. For $G, GM, R, \rho$ and $k_2'$ the IERS values have been directly adopted [20]. The proper angular momentum of Earth has been calculated as $J = I\omega$, where the value of [20] for the Earth daily angular velocity $\omega$ and of [21] for the adimensional moment of inertia $I/MR^2$ have been adopted. The orbital parameters of GP-B are those released in the leaflet *Gravity Probe B Launch Companion*.

| Symbol               | Description                                      | Value          | Units                        |
|----------------------|--------------------------------------------------|----------------|------------------------------|
| $G$                  | Newtonian gravitational constant                  | $6.67259 \times 10^{-8}$ | cm$^3$g$^{-1}$s$^{-2}$      |
| $GM$                 | Earth GM                                         | $3.986004418 \times 10^{20}$ | cm$^3$s$^{-2}$            |
| $R$                  | Earth equatorial radius                          | $6378.13649 \times 10^5$ | cm                         |
| $J$                  | Earth proper angular momentum                     | $5.86 \times 10^{40}$ | g cm$^2$s$^{-1}$           |
| $\rho$               | Ocean water density                               | 1.025          | g cm$^{-3}$                 |
| $H_{1/2}(K_1)$       | Tidal height of the $K_1$ tide                   | 36.87012       | cm                          |
| $H_{3/2}(055.565)$   | Tidal height of the 18.6-year tide               | 2.792          | cm                          |
| $k_2(K_1)$           | Love number for the $K_1$ tide                   | 0.257          | -                           |
| $k_2(055.565)$       | Love number for the 18.6-year tide               | 0.315          | -                           |
| $\delta_{2,1}(K_1)$  | Lag angle for the $K_1$ tide                     | -18.36         | deg                         |
| $\delta_{2,0}(055.565)$ | Lag angle for the 18.6-year tide               | -56.29         | deg                         |
| $C_{2,1}(K_1)$       | Ocean tidal height for the $K_1$ tide            | 2.83           | cm                          |
| $\epsilon_{2,1}(K_1)$ | Ocean hydrodynamics phase shift                  | 320.6          | deg                         |
| $k_2^+$              | Load Love number                                 | -0.3075        | -                           |
| $a$                  | GP-B semimajor axis                              | $7027.4 \times 10^5$ | cm                           |
| $e$                  | GP-B eccentricity                                | 0.0014         | -                           |
| $i$                  | GP-B inclination                                 | 90.007         | deg                         |
| $n$                  | GP-B Keplerian mean motion                       | $1.0717 \times 10^{-3}$ | s$^{-1}$                   |
| $P_{\Omega}$         | GP-B nodal period                                | $3 \times 10^{10}$ (1136.746) | s (yr)                      |
| $\Omega_0$           | GP-B longitude of the ascending node             | 163.26         | deg                         |
| $\dot{\Omega}_{LT}$ | GP-B nodal Lense-Thirring shift                  | 164            | mas yr$^{-1}$               |

2.1 The role of the even zonal harmonics

In order to give a really pessimistic and conservative estimate of the systematic error due to the mismodelling even zonal harmonics of the geopotential,
we will sum the absolute values of the mismodelled classical precessions \[8\] induced by the whole range of the mismodelled \(J_\ell\) coefficients according to the variance matrix of the GGM01C GRACE-based model \[11\] up to degree \(\ell = 48\). It turns out that, at 1 \(\sigma\) level

\[
\begin{align*}
\delta \dot{\Omega}_{\text{geopot}} &= 0.8 \text{ mas yr}^{-1}, \\
\delta \dot{\Omega}_{\text{geopot}}/\dot{\Omega}_{LT} &= 5 \times 10^{-3}.
\end{align*}
\]

(2)

If a root–sum–square calculation is performed by taking the square root of the sum of the squares of the various mismodelled classical precessions a relative error of \(1 \times 10^{-3}\) is obtained.

Another source of potential bias when a single orbital element is used is represented by the secular variations of the even zonal harmonics \(\dot{J}_\ell\). In \[22\] it has been shown that an effective \(\dot{J}^\text{eff}_2 \sim \dot{J}_2 + 0.371 \dot{J}_4 + 0.079 \dot{J}_6 + 0.006 \dot{J}_8 - 0.003 \dot{J}_{10}\ldots\) can be introduced. Its magnitude is of the order of \((-2.6 \pm 0.3) \times 10^{-11} \text{ yr}^{-1}\). It turns out that its mismodelled part would induce a secular drift of the GP-B node of \(3.1 \times 10^{-3}\) mas only in one year.

2.2 The role of the solar \(K_1\) tide

As we will now show in detail, the major problems for a measurement of the nodal Lense-Thirring shift with a polar orbital geometry \[18\] comes from the classical time–varying nodal perturbations induced by the solid Earth and ocean tides.

A given tidal constituent of frequency \(f\) induces the following solid Earth tidal perturbation on the node of a satellite \[19\]

\[
\Delta \Omega_f = \frac{g}{na^2 \sqrt{1 - e^2 \sin^2 i}} \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left( \frac{R}{a} \right)^{\ell+1} A_{\ell m} \sum_{p=0}^{\ell} \sum_{q=-\infty}^{+\infty} \frac{dF_{\ell mp}(i)}{di} G_{\ell pq}(e) \frac{1}{f_p} k^{(0)}_{\ell m} H^{n}_{\ell} \sin \gamma_{\ell mpq},
\]

(3)

where \(g\) is the acceleration of gravity at the Earth equator, \(R\) is the Earth equatorial radius, \(A_{\ell m}\) is given by

\[
A_{\ell m} = \sqrt{\frac{2\ell + 1}{4\pi} \frac{\ell - m)!}{(\ell + m)!},
\]

(4)

the quantities \(F_{\ell mp}(i)\) and \(G_{\ell pq}(e)\) are the so called inclination and eccentricity functions \[23\], \(k^{(0)}_{\ell m}\) and \(H^{n}_{\ell}\) are the Love numbers and the solid Earth

\[\text{For the LAGEOS satellites a calculation up to } \ell = 20 \text{ is well adequate.}\]
tidal heights, respectively, $f_p$ is the frequency of the orbital perturbation given by
\begin{equation}
  f_p = (\ell - 2p)\omega + (\ell - 2p + q)\dot{M} + m(\dot{\Omega} - \dot{\theta}) + \sigma,
\end{equation}
with
\begin{equation}
  \sigma = j_1 \dot{\theta} + (j_2 - j_1)\dot{s} + j_3 \dot{h} + j_4 \dot{p} + j_5 \dot{N'} + j_6 \dot{p}_s,
\end{equation}
while the phase of the sinusoidal function is
\begin{equation}
  \gamma_{\ell mpqf} = (\ell - 2p)\omega + (\ell - 2p + q)\dot{M} + m(\Omega - \theta) + \sigma t - \delta_{\ell mf}.
\end{equation}
The angular variable $\mathcal{M}$ is the satellite’s mean anomaly, $s, h, p, N', p_s$ are the luni-solar mean longitudes, $\theta$ is the Greenwich sidereal time and the $j_k$, $k = 1, ..6$ are small integers which can assume negative, positive or null values. They are arranged in the so called Doodson number
\begin{equation}
  j_1(j_2 + 5)(j_3 + 5)(j_4 + 5)(j_5 + 5)(j_6 + 5)
\end{equation}
by means of which each tidal constituent $f$ is named. In it the integer $j_1$ classifies the tides in long period or zonal ($j_1 = 0$), diurnal or tesseral ($j_1 = 1$) and semidiurnal or sectorial ($j_1 = 2$). Finally, $\delta_{\ell mf}$ is the phase lag angle of the response of the solid Earth with respect to the considered tidal constituent.

The ocean tidal perturbation on the node of an Earth satellite induced by a tidal constituent of frequency $f$ is
\begin{equation}
  \Delta \Omega_f = \frac{1}{na^2\sqrt{1 - e^2}} \sin i \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} A_{\ell mf}^\pm \times \
  \sum_{p=0}^{\ell} \sum_{q=-\infty}^{+\infty} \frac{dF_{\ell mp}^f}{di} G_{\ell pq} \int_{-\infty}^{+\infty} \left[ \sin \gamma_{\ell mpqf}^\pm - \cos \gamma_{\ell mpqf}^\pm \right]^{\ell-m \text{ even}}_{\ell-m \text{ odd}},
\end{equation}
where $\pm$ indicates the prograde and retrograde components,
\begin{equation}
  A_{\ell mf}^\pm = 4\pi G\rho \left( \frac{1 + k'_\ell}{2\ell + 1} \right) C_{\ell mf}^\pm,
\end{equation}
$\rho$ is the ocean water density, $k'_\ell$ is the Load Love number, $C_{\ell mf}^\pm$ are the ocean tidal heights,
\begin{equation}
  \gamma_{\ell mpqf}^\pm = (\ell - 2p)\omega + (\ell - 2p + q)\mathcal{M} + m(\Omega - \theta) \pm (\sigma t - \varepsilon_{\ell mf}^\pm),
\end{equation}
and $\varepsilon_{\ell mf}^\pm$ is the phase shift due to hydrodynamics of the oceans.
We will deal with the long-period components averaged over one orbital revolution \((\ell - 2p + q = 0)\) because we have to compare their effects with the secular Lense-Thirring effect. In particular, we will focus our attention on the degree \(\ell = 2\) terms. For \(\ell = 2\), \(p\) runs from 0 to 2, and so, in virtue of the condition \(\ell - 2p + q = 0\), \(q\) assumes the values \(-2, 0, 2\). Since the only non vanishing eccentricity function of degree \(\ell = 2\) is that for \(p = 1, q = 0\), it follows that the condition \(\ell - 2p = 0\) is also fulfilled. The frequencies of the perturbations are, in this case

\[ f_p = \dot{\Gamma}_f + m\dot{\Omega}, \tag{12} \]

with

\[ \dot{\Gamma}_f = (j_2 - m)s + j_3h + j_4p + j_5\dot{N} + j_6\dot{s}. \tag{13} \]

Since the science phase of GP-B will be almost one year long, it is of the utmost importance to see if there are some tidal perturbations, with relevant amplitudes, which could resemble as superimposed linear trends on the gravitomagnetic signal during such a time span. The most powerful tidal constituents in affecting satellite orbits are the zonal \((m = 0)\) lunar 18.6-year tide, with Doodson number \((055.565)\), and the tesseral \(m = 1\) solar \(K_1\) tide with Doodson number \((165.555)\). For such tidal lines the inclination factor of the perturbing amplitudes are

\[ \frac{1}{\sin i} \frac{dF_{201}}{dt} = \frac{3}{2} \cos i, \tag{14} \]

\[ \frac{1}{\sin i} \frac{dF_{211}}{dt} = -\frac{3}{2} \left( \frac{\cos^2 i}{\sin i} - \sin i \right), \tag{15} \]

respectively. This implies that a nearly polar orbital geometry will be affected mainly by the \(K_1\) tide than by the 18.6-year tide. This is an important feature because the period of the nodal perturbation induced by \((055.565)\) depends only on the luni-solar variables and amounts to 18.6 years. The situation is quite different for the \(K_1\) tide. Indeed, from eqs.\((12)\)-\((13)\) it follows that its nodal perturbation has the same period of the node of the satellite. For GP-B it amounts to 1136.746 years. From Table \(4\) and by assuming a time span of one year eq.\((8)\) yields a nominal shift of \(\Delta \Omega(K_1)^{\text{solid}}_{\ell=2} = 2.01264 \times 10^5\) mas. By assuming a 0.5% uncertainty in the Love number \([19]\) one gets \(\delta[\Delta \Omega(K_1)^{\text{solid}}_{\ell=2}] = 1000.6\) mas. Eq.\((9)\) yields a shift of \(\Delta \Omega(K_1)^{\text{ocean}}_{\ell=2} = 4.94785 \times 10^5\) mas after one year. By assuming an uncertainty of 3.8% in the ocean tidal height \([24]\), one gets \(\delta[\Delta \Omega(K_1)^{\text{ocean}}_{\ell=2}] = 1.8801 \times 10^4\) mas. The 18.6-year tide would not pose
particular problems. Indeed, the nominal amplitude of its nodal perturbation would amount to 2.7 mas only; the error in the Love number at that frequency is estimated to be [19] 1.5%, so that $\delta[\Delta \Omega(055.565)] = 0.04$ mas. These results clearly show that the bias induced by the $K_1$ tide on the node of GP-B would not allow to use it in order to measure the Lense-Thirring effect. Such results lead to the same conclusions of [17].

2.3 A multi-satellite approach

At this point one could ask if it would be possible to include the node of GP-B in some multi-satellite $J_\ell$-free combination. The answer, also in this case, is negative because of the polar geometry of its orbit. As shown in [18], the coefficients with which the residuals of the orbital elements of a polar satellite would enter some combinations tends to diverge for $i \sim 90$ deg. In the case of GP-B one could consider, e.g., a $\delta \Omega_{\text{LAGEOS}} + c_1 \delta \Omega_{\text{GP-B}}$ combination. The coefficient $c_1$ of GP-B would be equal to $5 \cdot 398$ so that the impact of the $K_1$ tide would be further enhanced.

3 Conclusions

In this paper we have investigated the possibility of measuring the gravitomagnetic Lense-Thirring effect on the orbit of a test particle by analyzing also the nodal rate of the GP-B satellite in the gravitational field of the Earth. It has been launched in April 2004 and its main task is the measurement of the gravitomagnetic precession of four gyroscopes carried onboard at a claimed accuracy of 1%. The main problems come from the fact that the orbital perturbations induced by the solid Earth and ocean components of the solar $K_1$ tide on the GP-B node would resemble aliasing linear trends superimposed on the genuine relativistic linear signal. Indeed, the observational time span would be of almost one year while the period of such perturbations is of the order of $10^3$ years. Moreover, their amplitudes are three orders of magnitude larger than the Lense-Thirring effect. This drawback would be further enhanced by including the GP-B node in some multi-satellite $J_\ell$-free combinations because the coefficient with which it would enter them would be very large due to the polar geometry of the satellite.

\[ ^5 \text{The systematic error due to the mismodelling in the even zonal harmonics would amount to 50-100\%, according to GGM01C model.} \]
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