One-loop Quantum Corrections to the Entropy for an Extremal Reissner-Nordström Black Hole

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Abstract: The first quantum corrections to the entropy for an eternal 4-dimensional extremal Reissner-Nordström black hole is investigated at one-loop level, in the large mass limit of the black hole, making use of the conformal techniques related to the optical metric. A leading cubic horizon divergence is found and other divergences appear due to the singular nature of the optical manifold. The area law is shown to be violated.

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1 Introduction

Recently, several issues like the interpretation of the Bekenstein-Hawking classical formula for the black hole entropy, the loss of information paradox and the validity of the area law have been discussed in the literature (see, for example, the review [1]). Furthermore it has been pointed out that it should be desirable to have the usual statistical interpretation of the black hole entropy as the number of gravitational states at the horizon (see, for example, [2]).

There have also been some attempts to compute semiclassically the first quantum corrections (prefactor) to the Bekenstein-Hawking classical entropy $4\pi GM^2$ [3, 4]. With regard to this, we recall that on general grounds, the density of levels as a function of the black hole mass $M$ for $D = 4$ should read

$$\Omega(M) \simeq C(M)e^{4\pi GM^2}. \quad (1.1)$$

However, so far all, the evaluations of the prefactor $C(M)$ have been plagued by the appearance of divergences [5, 6, 7, 8, 9, 10, 11] present also in the related "entanglement or geometric entropy" [12, 13, 14]. These divergences have been related to the information loss issue [5, 9] and their presence has to be confronted with the fact that the corresponding prefactor for quantum fields and extended objects (like strings or p-branes) in ultrastatic backgrounds is computable and finite.

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In a space-time with horizons, the physical origin of these divergences can be traced back to the equivalence principle \[15, 16, 17\]. The argument goes as follows. A system in thermal equilibrium has a local Tolman temperature \(T(x) = T/\sqrt{-g_{00}}\), \(T\) being the asymptotic temperature. As a consequence, one has for the entropy \(S\) of a quantum massless field the formal quantity
\[
S = T^3 \int \frac{dV_3}{(-g_{00})^{3/2}}
\]
which is divergent if the space-time has horizons and, in turn, suggests that the use of the optical metric \(\bar{g}_{\mu\nu} = g_{\mu\nu}/|g_{00}|\), conformally related to the original one, may provide an alternative and useful framework to investigate these issues. One of the purposes of this paper is to implement this idea, which has been already proposed in Refs. \[18, 19\] and put recently forward also in Refs. \[20, 21, 11, 22\].

In Ref. \[23\], in the large black hole mass limit, we have reconsidered the case of an ordinary black hole admitting a canonical horizon, namely the case for which the quantity \(g_{00}\) has a simple zero. In this paper, we would like to investigate the case of an extremal Reissner-Nordström black hole, within the same approximation, which is reliable very near the horizon. Here the quantity \(g_{00}\) has a double zero and this corresponds to a non canonical horizon. It is known that in this case the Hawking temperature is vanishing and thus a vanishing Hawking-Bekenstein entropy is present, but the horizon area is not vanishing at all \[24\]. For this reason, it may be interesting to study the first quantum corrections to the entropy.

The content of the paper is as follows. In Sec. \[2\] we resume the conformal transformation techniques and give the relation between the physical quantities in the original manifolds and in the optical one. In Sec. \[3\] we derive, in the large mass limit, eigenfunctions and eigenvalues for a scalar field in an extremal Reissner-Nordström background and, using \(\zeta\)-function regularization, we derive the free energy and the entropy, regularizing the horizon divergences with a suitable cutoff. Finally in Sec. \[4\] we conclude with some remarks.

## 2 Conformal transformation techniques

To start with, we recall the formalism we shall use in the following in order to discuss the finite temperature effects. We may consider, as a prototype of the quantum correction, a scalar field on a 4-dimensional static space-time defined by the metric (signature \(-+++)\)
\[
ds^2 = g_{00}(x)(dx^0)^2 + g_{ij}(x)dx^i dx^j, \quad x = \{x^i\}, \quad i, j = 1, \ldots, 3.
\]

The one-loop partition function is given by (we perform the Wick rotation \(x_0 = -i\tau\), thus all differential operators one is dealing with will be elliptic)
\[
Z = \int d[\phi] \exp \left( -\frac{1}{2} \int \phi L_4 \phi d^4x \right),
\]

where \(\phi\) is a scalar density of weight \(-1/2\) and \(L_4\) is a Laplace-like operator on a 4-dimensional manifold. It has the form
\[
L_4 = -\Delta_4 + m^2 + \xi R.
\]

Here \(\Delta_4\) is the Laplace-Beltrami operator, \(m\) (the mass) and \(\xi\) arbitrary parameters and \(R\) the scalar curvature of the manifold.

The key idea is to make use of the conformal transformation technique. \[25, 26, 27, 28\]. This method is useful because it permits to compute all physical quantities in an ultrastatic manifold (called the optical manifold \[29\]) and, at the end of calculations, to transform back them to a
static one, with an arbitrary $g_{00}$. The ultrastatic Euclidean metric $\bar{g}_{\mu \nu}$ is related to the static one by the conformal transformation

$$\bar{g}_{\mu \nu}(x) = e^{2\sigma(x)} g_{\mu \nu}(x), \quad (2.4)$$

with $\sigma(x) = -\frac{1}{2} \ln g_{00}$. In this manner, $\bar{g}_{00} = 1$ and $\bar{g}_{ij} = g_{ij} / g_{00}$ (Euclidean optical metric).

For the one-loop partition function it is possible to show that

$$\bar{Z} = J[g, \bar{g}] Z, \quad (2.5)$$

where $J[g, \bar{g}]$ is the Jacobian of the conformal transformation. Such a Jacobian can be explicitly computed [28], but here we shall need only its structural form. Using $\zeta$-function regularization for the determinant of the differential operator we get

$$\ln Z = \ln \bar{Z} - \ln J[g, \bar{g}] = \frac{1}{2} \zeta'(0) |\bar{\Lambda}_4 \ell^2| - \ln J[g, \bar{g}], \quad (2.6)$$

where $\ell$ is an arbitrary parameter necessary to adjust the dimensions and $\zeta'$ represents the derivative with respect to $s$ of the function $\zeta(s |\bar{\Lambda}_4 \ell^2)$ related to the operator $\bar{\Lambda}_4$, which explicitly reads

$$\bar{\Lambda}_4 = e^{-\sigma} \bar{L}_4 e^{-\sigma} = -\partial_\tau^2 - \bar{\Delta}_3 + \frac{1}{6} R + e^{-2\sigma} \left[ m^2 + (\xi - \frac{1}{6}) R \right] = -\partial_\tau^2 + \bar{\Lambda}_3. \quad (2.7)$$

The same analysis can be easily extended to the finite temperature case [27]. Since the Euclidean metric is $\tau$ independent, one obtains

$$\ln \bar{Z}_\beta = -\frac{\beta}{2} \left[ \text{PP} \zeta(-\frac{1}{2} |\bar{\Lambda}_3|) + (2 - 2 \ln 2\ell) \text{Res} \zeta(-\frac{1}{2} |\bar{\Lambda}_3|) \right]$$

$$+ \lim_{s \to 0} \frac{d}{ds} \frac{\beta}{\sqrt{4\pi t}} \sum_{n=1}^\infty \int_0^\infty t^{s-3/2} e^{-n^2 \beta^2 / 4t} \text{Tr} e^{-t\bar{\Lambda}_3} dt, \quad (2.8)$$

where PP and Res stand for the principal part and the residue of the function and one has to analytically continue before taking the limit $s \to 0$. As usual, in the definition of $\zeta$-function the subtraction of possible zero-modes of the corresponding operator is left understood. Of course, if the function $\zeta(s |\bar{\Lambda}_3)$ is finite for $s = -1/2$, the first term on the right-hand side of the latter equation is just $-\frac{\beta}{2} \zeta(-\frac{1}{2} |\bar{\Lambda}_3|)$. The latter formula is rigorously valid for a compact manifold. In the paper we shall deal with a non compact manifold, but nevertheless we shall make a formal use of this general formula, employing $\zeta$-function associated with continuum spectrum.

The free energy is related to the canonical partition function by means of the equation

$$F_\beta = E_0 + \hat{F}_\beta = -\frac{1}{\beta} \ln Z_\beta = -\frac{1}{\beta} \left( \ln \bar{Z}_\beta - \ln J[g, \bar{g}] \right), \quad (2.9)$$

where $E_0$ is the vacuum energy while $\hat{F}_\beta$ represents the temperature dependent part (statistical sum). It should be noted that since we are considering a static space-time, the quantity $\ln J[g, \bar{g}]$ depends linearly on $\beta$ and, according to Eq. (2.3), it gives contributions only to the vacuum energy term and not to the entropy, which may be defined by

$$S_\beta = \beta^2 \partial_\beta F_\beta, \quad (2.10)$$

and for the internal energy we assume the well known thermodynamical relation

$$U_\beta = \frac{S_\beta}{\beta} + F_\beta. \quad (2.11)$$
3 Scalar fields in Reissner-Nordstr"om space-time

Let us apply this formalism to the case of a massive scalar field in the 4-dimensional Reissner-Nordstr"om background. Our aim is to compute the entropy of this field using the latter formula. We recall that the metric we are interested in reads

\[ ds^2 = -\left(1 - \frac{r_H}{r}\right)^2 (dx^0)^2 + \left(1 - \frac{r_H}{r}\right)^{-2} dr^2 + r^2 d\Omega^2, \]  

(3.1)

where we are using polar coordinates, \( r \) being the radial one and \( d\Omega^2 \) the 2-dimensional spherical unit metric. The horizon radius is \( r_H = MG = Q \), \( M \) being the mass of the black hole, \( G \) the Newton constant and \( Q \) its charge. The Hawking temperature is zero because it is proportional to the difference between the horizon radii, which coincide in the extremal case. From now on, for the sake of convenience we put \( r_H = 1 \); in this way all quantities are dimensionless; the dimensions could be easily restored at the end of calculations.

It may be convenient to redefine the Schwarzschild coordinates \((x^0, r)\) by means

\[ x^0 = x^0, \quad \rho = \frac{(r - 1)}{1 - 2(r - 1) \ln(r - 1) - (r - 1)^2}, \]  

(3.2)

where \( r \) is implicitly defined by Eq. (3.2) and has the expansion, valid near the horizon \( \rho \sim 0 \)

\[ r \sim 1 + \rho + O(\rho^2 \ln \rho). \]  

(3.3)

The optical metric \( \bar{g}_{\mu \nu} = g_{\mu \nu}/(-g_{00}) \), which is conformally related to the previous one and appears as an ultrastatic metric, reads

\[ d\bar{s}^2 = -(dx^0)^2 + \frac{1}{\rho^4} d\rho^2 + \frac{G(\rho)}{\rho^2} d\Omega^2, \]  

(3.4)

where

\[ G(\rho) = \frac{\rho^4}{1 - 2(r - 1) \ln(r - 1) - (r - 1)^2} \sim 1 + 2\rho \ln \rho + 4\rho + O(\rho^2 \ln \rho). \]  

(3.5)

Furthermore one has \( \bar{R} = O(\rho^4 \ln \rho) \). In order to perform explicit computations, we shall consider the large mass limit of the black hole and the region near the horizon. This leads to the approximated metric

\[ d\bar{s}^2 \sim -(dx^0)^2 + \frac{1}{\rho^4} d\rho^2 + \frac{1}{\rho^2} d\Omega^2. \]  

(3.6)

This can be considered as an approximation of the metric defined by Eq. (3.4) in the sense that, near the horizon \( \rho = 0 \), the geodesics are essentially the same. Eq. (3.6) defines a manifold with vanishing curvature. Then, according to Eq. (3.7), the relevant operator becomes

\[ \bar{L}_3 = -\bar{\Delta}_3 + m^2 \rho^2 = \rho^4 \partial_\rho^2 + \rho^2 (\Delta_2 + m^2), \]  

(3.7)

while the invariant measure reads

\[ d\bar{V} = \rho^{-4} d\rho dV_2, \]  

(3.8)

where \( \Delta_2 \) is the Laplace-Beltrami operator on the unitary sphere \( S^2 \) and \( dV_2 \) its invariant measure.

The eigenfunctions of the operator \(-\Delta_2 + m^2 \) are the spherical harmonics \( Y_{lm}(\vartheta, \varphi) \) and the eigenvalues \( \lambda_l^2 = l(l + 1) + m^2 \). Let \( \Psi_{\lambda l m}(\vartheta, \varphi) \) be the eigenfunctions of \( L_3 \)
with eigenvalues $\lambda^2$. The differential equation which determines the related continuum spectrum turns out to be
\[
\left[ \rho^4 \partial_\rho^2 - \rho^2 \lambda^2 + \lambda^2 \right] \phi_\lambda(\rho) = 0.
\] (3.9)

The only solutions with the correct decay properties at infinity are the Bessel functions and we have
\[
\phi_\lambda(\rho) = \sqrt{\rho} J_{\nu}(\rho^{-1} \lambda),
\] (3.10)
where $\nu_1^2 = (l + \frac{1}{2})^2 + m^2$ has been set. For any suitable function $f(\bar{L}_3)$, we may write
\[
< x | f(\bar{L}_3) | x > = \int_0^\infty f(\lambda^2) \sum_{lm} \mu_l(\lambda) Y_l^m(\vartheta, \varphi) \phi_\lambda^*(\rho) Y_l^m(\vartheta, \varphi) \phi_\lambda(\rho) d\lambda,
\] (3.11)
where $\mu_l(\lambda)$ is the spectral measure (density) associated with the continuum spectrum. It may be defined by means of equation
\[
(\phi_\lambda, \phi_{\lambda'}) = \frac{\delta(\lambda - \lambda')}{\mu_l(\lambda)}.
\] (3.12)

Using the asymptotic behaviour of the Bessel functions at infinity [30] one can show that
\[
\mu(\lambda) \equiv \mu_l(\lambda) = \lambda.
\] (3.13)

This is the spectral measure associated with the Hankel inversion formula [31]. As a consequence, the heat kernel of $\bar{L}_3$ may be written as
\[
K_t(x|\bar{L}_3) = \int_0^\infty d\lambda e^{-\frac{t}{\lambda^2}} \sum_{l=0}^\infty \frac{(2l + 1)}{4\pi} \rho J_{\nu_1}^2(\rho^{-1} \lambda).
\] (3.14)

To go on, we use a method based on the Mellin-Barnes representation. In fact, we have [32]
\[
\sum_{l=0}^\infty \frac{(2l + 1)}{4\pi} \rho J_{\nu_1}^2(\rho^{-1} \lambda) = \frac{(4\pi)^{-3/2}}{\pi i} \int_{\text{Re} s = 3/2 + c} \frac{\Gamma(s - \frac{1}{2})}{\Gamma(s)} \rho^{3-2s} \lambda^{2s-2} I(s) ds,
\] (3.15)
where $c$ is a positive number smaller than $m^2$ or than 1, according to whether one is considering the massive or the massless cases respectively. We have introduced the function
\[
I(s) = \sum_{l=0}^\infty (2l + 1) \frac{\Gamma(\nu_1 - s + 1)}{\Gamma(\nu_1 + s)},
\] (3.16)
which is certainly convergent for $\text{Re} s = 3/2 + c$, since for large $l$ one has [32]
\[
\frac{\Gamma(\nu_1 - s + 1)}{\Gamma(\nu_1 + s)} \sim \nu_1^{1-2s} \sum_{j=0}^\infty G_j(s) \nu_1^{-j} \equiv \nu_1^{1-2s} Q(s, \nu_1),
\] (3.17)

$G_j(s)$ being computable polynomials in $s$. For example we have
\[
G_0(s) = 1, \quad G_1(s) = 0, \quad G_2(s) = \frac{s(s - 1)(s - 1/2)}{3}.
\] (3.18)

Eq. (3.17) is useful in order to obtain the analytical continuation of $I(s)$ for $\text{Re} s < 3/2$. In the massive case, the result is
\[
I(s) = \sum_{j=0}^\infty G_j(s) f\left(s + \frac{i-1}{2}\right) + \text{analytical part},
\] (3.19)
where

\[ f(z) = \sum_{l=0}^{\infty} (2l + 1) \nu_l^{-2z} , \tag{3.20} \]

while, in the massless case

\[ I(s) = \frac{\Gamma(\frac{3}{2} - s)}{\Gamma(\frac{1}{2} + s)} + 2 \sum_{j=0}^{\infty} G_j(s) \zeta_H(2s + j - 2; \frac{1}{2}) + \text{analytical part} , \tag{3.21} \]

\( \zeta_H(z; q) \) representing the Riemann-Hurwitz zeta-function, which has only a simple pole at \( z = 1 \), with residue equal to 1. In the same way, the function \( f(z) \) admits an analytical continuation in the whole complex \( z \) plane with a simple pole at \( z = 1 \) and residue equal to 1. This means that in both the cases, for \( \Re s < 3/2 + c \), \( I(s) \) is a meromorphic function with possible simple poles at \( s = \frac{3-j}{2} \) (\( j = 0, 1, ... \)). However we observe that only \( s = 3/2 \) is a true pole, with residue equal to 1, since the residues \( G_j(\frac{3-j}{2}) \) \( (j \geq 1) \) in all other possible poles are equal to 0. This can be easily seen by observing that for integer or half-integer \( s = \pm \frac{N}{2} \) \((N \geq 0) \) we have

\[ Q(\pm \frac{N}{2}, \nu_l) = \prod_{0 \leq k = \text{odd/even}}^{N} \left( 1 - \frac{k^2}{4
u_l^2} \right) , \tag{3.22} \]

\[ Q(\frac{N}{2}, \nu_l) = \prod_{0 \leq k = \text{odd/even}}^{N-2} \left( 1 - \frac{k^2}{4
u_l^2} \right)^{-1} , \tag{3.23} \]

From equations above, we see that \( Q(\pm \frac{N}{2}, \nu_l) \) is an even function in \( \nu_l^{-1} \) and this means that the corresponding \( G_j \) are vanishing if \( j \) is odd. Moreover, from Eq. (3.22) we see that \( Q(\pm \frac{N}{2}, \nu_l) \) is a polynomial of degree \( N + 1 \) or \( N \) according to whether \( N \) is odd or even. Then \( G_j(\frac{3-j}{2}) \equiv 0 \) if \( 3 - j \leq 0 \). The coefficients for \( s = 0, \frac{3}{2} \) and 1 can trivially computed using Eq. (3.23). They all are vanishing but \( G_0 \), which is equal to 1 for any \( s \). As a result, \( I(s) \) is analytic for \( \Re s < 3/2 \).

The heat kernel reads

\[ K_t(x|L_3) = \frac{(4\pi)^{-3/2}}{2\pi i} \int_{\Re s = 3/2 + c} \Gamma(s - \frac{1}{2}) \rho^{3-2s} t^{-s} I(s) ds . \tag{3.24} \]

Since we are mainly interested in global quantities, like the partition function, we integrate Eq. (3.24) over \( \hat{M}^3 \), paying attention to the fact that the integration over \( \rho \) formally gives rise to horizon divergences. In order to regularize such divergences, we introduce a horizon cutoff \( \varepsilon > 0 \). At the end of the computation, we shall take the limit \( \varepsilon \to 0 \). In this way we have

\[ \text{Tr} e^{-t L_3(\varepsilon)} = \frac{A(4\pi)^{-3/2}}{4\pi i} \int_{\Re s = 3/2 + c} \frac{\Gamma(s - \frac{1}{2})}{s} \varepsilon^{-2s} t^{-s} I(s) ds , \tag{3.25} \]

where the horizon area \( A \) is equal to \( 4\pi r_H^2 \) and the integration in \( \rho \) has been performed.

Now we can shift the vertical contour to the left hand side in the complex plane. There are simple poles at all the half-integers \( s \leq 3/2 \) and also in \( s = 0 \). Since we have to take the limit \( \varepsilon \to 0 \), only the non negative poles will give non vanishing (divergent) contributions to the integral. As a result

\[ \text{Tr} e^{-t L_3(\varepsilon)} = \frac{A}{(4\pi t)^{3/2}} \left[ \frac{1}{3\varepsilon^3} + \frac{I(\frac{1}{2})}{\varepsilon} t - \sqrt{\pi} I(0) t^{3/2} \right] . \tag{3.26} \]

It should be noted the appearance of a horizon divergence more severe than the one which one has in the non extremal case.
The corresponding partition function may be evaluated making use of Eq. (2.8). We have

\[
\ln Z_\beta = -\beta F_\beta = \beta A \left[ j_\epsilon - \frac{1}{2} \zeta (-\frac{1}{2}|L_\beta) \right] + \beta A \left[ \frac{8\pi^4}{135\epsilon^3 \beta^4} + \frac{2\pi^2 I(\frac{1}{2})}{3\epsilon^2 \beta^2} + \frac{2\pi I(0)}{\beta} \ln \beta \right], \quad (3.27)
\]

where we have written the Jacobian contribution due to the conformal transformation in the form \( A \beta j_\epsilon \). The horizon divergences which appear in Eq. (3.27) are also contained in the Jacobian factor, while ultraviolet divergences are present in the vacuum energy part. As is well known, one needs a renormalization in order to remove the vacuum divergences. We recall that these divergences, as well as the Jacobian conformal factor, being linear in \( \beta \), do not make contributions to the entropy. As a consequence, the first quantum corrections at temperature \( T = 1/\beta \) to entropy read

\[
S_\beta = A \left[ \frac{2\pi^2}{135\epsilon^3 \beta^3} + \frac{I(\frac{1}{2})}{12\epsilon^2 \beta} + \frac{I(0)}{8\pi} (\ln \beta - 1) \right]. \quad (3.28)
\]

The functions \( I(1/2) \) and \( I(0) \) can be evaluated in the sense of analytical continuation. In the massive case we have

\[
I(\frac{1}{2}) = f(0) = 2\zeta_H (-1|\frac{1}{2}) = \frac{1}{12}, \quad (3.29)
\]

while for the massless one, \( I(\frac{1}{2}) \) is again given by Eq. (3.29), but \( I(0) \) vanishes.

Then we have the final result (restating \( r_H \) by dimensional arguments)

\[
S_\beta = A \left[ \frac{2\pi^2 r_H^4}{135\epsilon^3 \beta^3} + \frac{1}{144\epsilon^2 \beta} + \frac{f(-\frac{1}{2})}{8\pi} \left( \ln \frac{\beta}{r_H} - 1 \right) \right], \quad (3.31)
\]

valid for massive and massless fields respectively. The leading term of above equations is in agreement with Refs. [33, 22].

\[ \text{valid for massive and massless fields respectively. The leading term of above equations is in agreement with Refs. [33, 22].} \]

4 Concluding remarks

In this paper, making use of conformal transformation techniques, we have investigated in some detail, the first quantum corrections to an extremal Reissner-Nordström black hole. Our approach does not make use of brick-wall boundary conditions, but the horizon cut-off is introduced in the computation of the heat-kernel trace. Although the spectrum of the relevant operator is continuous, the computation of the statistical sum can be done, in the large mass limit of the black hole and the leading divergences of the entropy are in agreement with the ones obtained in Refs. [33] (brick-wall method) and [22], in which the optical manifold method is implemented in an alternative way. However, we would like to stress a technical point. In the case of canonical horizons, the optical spatial section turns out to be smooth, for example in the Rindler space-time, it coincides with the hyperbolic space. In the case of non canonical horizons, the
optical spatial section has conical singularities and this fact leads to the appearance of a linear divergence besides the leading cubic one.

As we have already recalled, in this extremal case the Hawking temperature vanishes, but it has been recently observed that the corresponding Euclidean solution may be identified with an arbitrary $\beta$ [24, 34]. As a result, in the extreme case, one has a quantum divergent entropy which does not satisfy the area law. Since our techniques can be applied to every space-time with non canonical horizons, we may conclude that this area law violation holds also in these cases. We note, however, that the area law violation is also present in the extremal dilatonic black hole [3], where the linear divergence is vanishing in the extremal case.

Finally few words about these horizons divergences. In the case of canonical horizons, it has been recently proposed to cure the linear and logarithmic divergences by standard one-loop renormalization of the Newton constant \[ \beta \] within the formalism of the quantum gravity effective action (for a recent review on this issue see, for example, [37]). However, apart the criticism of Ref. [38], it is not clear to us whether the same procedure may be applied to the case of space-times with non canonical horizons, since here, as we have shown, the leading horizon divergence is cubic.

On the other hand, in the non extremal case, one may choose the horizon cutoff of the order of the Planck length ($\epsilon^2 = G$), in order to reproduce the area law [5, 20]. In the extremal case, this seems problematic, since there is no natural value for $\beta$. Alternatively, for this problem, one may try to make use of string theory, which may be considered a perturbatively ultraviolet finite model for quantum gravity (see for example Refs. [1, 24, 31, 11, 12]).

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