COMMENTS ON INFLATIONARY COSMOLOGY

Andrei Linde
Department of Physics, Stanford University, Stanford, CA 94305

ABSTRACT

In this talk we will discuss three different issues. First of all, there exist several proposals how to solve cosmological problems by adiabatic expansion of the Universe of a specific type, without any use of inflation. We explain why these models do not solve the flatness/entropy problem. On the other hand there exist some claims that inflation also does not solve the flatness problem. We show that these claims are correct to a certain extent for old and new inflation, but are incorrect for simplest versions of chaotic inflation.

The second issue to be considered is the classification of inflationary models. At present all kinds of inflationary models are compared to each other in order to understand which of them better describes observational data. Unfortunately, this often leads to terminological misunderstandings and even to misleading conclusions. We discuss three coexisting types of classification of inflationary models and explain their relation to each other. We also briefly describe the hybrid inflation model, which combines several interesting features of different inflationary models.

Finally we show that it is possible to describe inflationary universe by a stationary (i.e. time-independent) probability distribution to find a domain of a given size containing matter with given properties. This represents a strong deviation of inflationary cosmology from the standard Big Bang paradigm.

1Based on the talks given at the 5th Canadian Conference on General Relativity, Waterloo, at the Yamada Conference on Cosmology, Tokyo, and at the Summer School on Cosmology in Aspen, 1993

2On leave from: Lebedev Physical Institute, Moscow. E-mail: linde@physics.stanford.edu
# 1 Introduction

After more than ten years of its existence, the inflationary universe scenario is gradually becoming a standard cosmological paradigm. Inflation solved many different problem of the standard cosmology, including the horizon, flatness, homogeneity and isotropy problems. For a detailed discussion of this theory see e.g. [1]. However, it would be incorrect to stop looking for other solutions of these problems. Moreover, one should remember that not every inflationary model solves these problems well enough.

Recently Freese and Levin [2] proposed a model which they called MAD. According to [2], strong variation of the gravitational constant in the very early universe may solve the horizon problem without any need for inflation and for any nonadiabatic processes such as reheating of the Universe after inflation. Their model has some interesting features, but it is not clear at all whether this model can solve the homogeneity and isotropy problem, and it appears that MAD scenario cannot solve the flatness problem.

A detailed discussion of the last issue was contained in a paper by Hu, Turner and Weinberg [3]. However, this discussion was concentrated on the model of ref. [2] and on its simplest generalizations. Meanwhile we all remember that the version of the inflationary universe scenario suggested by Guth [4] also did not work, and a year later Guth and Weinberg have almost completely proved that a consistent inflationary theory is impossible [5]. What if a year from now somebody will suggest a new or extended MAD scenario?

Unfortunately (or fortunately, depending on the point of view), there are some reasons to believe that this is impossible: all models assuming adiabatic (or almost adiabatic) expansion of the universe, including the MAD scenario [2], have a serious problem “by definition”. Indeed, the total entropy of the observable part of the Universe is greater than \( S \sim 10^{88} \). If expansion of the Universe is adiabatic, one is forced to assume that the Universe from the very beginning was huge: it contained more than \( 10^{88} \) particles. This is a manifestation of the entropy problem, which is almost equivalent to the flatness problem, but is more general [1].

The best way to understand that this problem is more general than the flatness problem is to remember the proposal to use Kaluza-Klein compactification as an alternative to inflation, see e.g. [6]. The main idea is to begin with a Universe of a very large number of dimensions, say, \( D = 100 \). Let us assume that the Universe initially was closed, and its initial size \( l_0 \) was just few times greater than the Planck length \( l_p \sim M_p^{-1} \). However, this is enough to incorporate enormous large amount of particles into such a universe, because of its huge volume. At the Planck temperature \( T \sim M_p \) such a universe contained \( \sim (l_0 T)^{99} \) particles, which is a very large number if the universe originally was few times greater than the Planck length. As a result of compactification, all these particles should fit into a Universe with only three spatial dimensions, which leads to its extremely rapid expansion.

Thus, in this scenario the Universe is initially very curved. Then, due to compactification of almost all its dimensions, the Universe becomes three-dimensionally flat without entering a
stage of vacuum energy dominance and subsequent reheating. One may argue that this solves
the flatness problem without any need of inflation. However, one should still assume that the
Universe contained at least $10^{88}$ particles from the very beginning, which is a manifestation of
the entropy problem. Until this problem is unresolved, all models which do not assume strong
violation of adiabaticity cannot compete with inflation.

But is it correct that inflation solves this problem? The answer depends on the choice of
inflationary scenario. Let us consider for example a closed universe in the new inflationary
universe scenario, which occurs when the energy density of ultrarelativistic particles becomes
smaller than the effective potential $V(0)$. One can easily show that this happens at the time

$$t_0 \sim \frac{M_p}{4} \sqrt{\frac{3}{2\pi V(0)}}.$$  (1)

Meanwhile, as it is shown in [1], a closed universe filled by the ultrarelativistic gas with a total
entropy $S$ begins collapsing after the time

$$t_c = \left(\frac{1}{90\pi^2}\right)^{1/6} S^{2/3} M_p^{-1}.\quad (2)$$

If this happens before the Universe becomes vacuum energy dominated, then the Universe recol-
lapses and no inflation occurs. Using eqs. (1) and (5) one can easily check that inflation in this
model is possible only if

$$S \gtrsim 0.3 \left(\frac{M_p^4}{V(0)}\right)^{3/4}.\quad (3)$$

For a typical potential which appears in grand unified theories $V(0) \sim 10^{-15} M_p^4$. This requires
$S > 10^{11}$. This is an improvement by at least 77 orders of magnitude as compared with any
adiabatic expansion scenario, including the MAD model [4] or the Kaluza-Klein scenario [9].
However, $10^{11}$ is still a very big number. At the Planck time the total mass of the Universe
containing $10^{11}$ particles with the Planck energy $M_p \sim 10^{19}$ GeV $\sim 10^{-5}$ g would be about $\sim 10^6$
g $\sim 10 M_{\text{Schwarzenegger}}$, which is a lot!

Fortunately, eq. (3) suggests an easy resolution of this problem. If inflation may begin at a
density not much lower than the Planck density, the entropy problem disappears. This is precisely
the case in the simplest models of chaotic inflation based on the potentials $\sim \phi^n$ or $\sim e^{\alpha \phi}$. In
such theories inflation may begin in a state with a density close to the Planck density and with
vanishing entropy. This solves both the flatness and the entropy problem [1].

---

3We use the system of units $M_{\text{Schwarzenegger}} = 10^5$ g, which differs by ten orders of magnitude from the Planck
mass $M_p = 10^{-5}$ g.
2 Classification of inflationary models and hybrid Inflation

After the discovery of anisotropy of the microwave background radiation by COBE, there were many papers trying to compare observational consequences of various inflationary models. However, some of the authors of these papers used their own definitions of new inflation, chaotic inflation, etc. This gave rise to certain purely terminological misunderstandings. For example, some authors believe that if chaotic inflation is right then extended inflation is wrong (and vice versa), that the difference between chaotic inflation and the new inflation is that the new inflation begins at small $\phi$, whereas chaotic inflation begins at large $\phi$. These and some other misconceptions appear due to the existence of several different classifications of inflationary models.

The first classification deals with the initial conditions for inflation. The old and new inflationary models were based on the assumption that the Universe from the very beginning was in a state of thermal equilibrium at an extremely high temperature, and that the inflaton field $\phi$ was in a state corresponding to the minimum of its temperature-dependent effective potential $V(\phi)$ \[4, 7\]. The main idea of the chaotic inflation scenario was to study all possible initial conditions in the Universe, including those which describe the Universe outside of the state of thermal equilibrium, and the scalar field outside of the minimum of $V(\phi)$ \[8\]. This scenario includes the possibility of inflation from the state with $\phi = 0$ in a thermal equilibrium, like in the new inflationary Universe scenario, but it contains many other possibilities as well. Therefore it can be realized in a much greater variety of models than the new inflationary Universe scenario. In fact, at present the idea of thermal beginning is almost completely abandoned, and all realistic models of inflation from the point of view of the first classification are of the chaotic inflation type \[1\].

The second classification describes various regimes which are possible during inflation: quasi-exponential inflation, power law inflation, etc. This classification is absolutely independent of the issue of initial conditions. Therefore it does not make any sense to compare, say, power law inflation and chaotic inflation, and to oppose them to each other. For example, in \[9\] it was pointed out that the chaotic inflation scenario, as distinct from the new inflationary Universe scenario, can be realized in the theories with the effective potential $e^{\alpha \phi}$ for $\alpha \ll \sqrt{16\pi}$. Meanwhile, in \[10\] it was shown that this inflation is power law. Thus, the inflationary Universe scenario in the theory $e^{\alpha \phi}$ describes chaotic power law inflation.

Finally, the third classification is related to the way inflation ends. There are two possibilities extensively discussed in the literature: slow rollover versus the first-order phase transition. The models of the first class describe slow rolling of the inflaton field $\phi$, which gradually becomes faster and faster. A particular model of this type is chaotic inflation in the theories $\phi^n$. The models of the second class should contain at least two scalar fields, $\phi$ and $\sigma$. They describe a strongly first-order phase transition with bubble production which is triggered by the slow rolling of the field $\phi$. One of the popular models of this type is the extended inflation scenario \[11\], which is a combination of the Brans-Dicke theory and the old inflationary scenario. There exist other
versions of the first-order scenario with two scalar fields, which do not require any modifications of the Einstein theory of gravity, see e.g. \[12\].

In the beginning it was assumed that the bubbles formed during the first-order phase transition could be a useful ingredient of the theory of the large scale structure formation. However, later it was realized that one should make considerable modifications of the original models in order to avoid disastrous consequences of the bubble production. According to the most recent modification \[13\], the bubble formation happens only after the end of inflation. In this case, the end of inflation occurs as in the standard slow-rollover scenario. Therefore it would be interesting to find out other possible ways in which inflation may end in the models with several different scalar fields. More generally, one may try to find out other qualitatively new inflationary regimes which may appear due to a combined evolution of several scalar fields.

Of course, one should not invent excessively complicated models without demonstrated need. However, sometimes qualitatively different inflationary regimes appear after minor modifications of the basic inflationary models, or after making their hybrids. For example, in \[14, 15\] we proposed a very simple model of two interacting scalar fields where inflation may end by a rapid rolling of the field \(\sigma\) (‘waterfall’) triggered by the slow rolling of the field \(\phi\). This regime differs both from the slow-rollover and the first-order inflation. By changing parameters of this model one can continuously interpolate between these two regimes. Therefore some hybrid models of such type may share the best features of the slow-rollover and the first-order models.

The effective potential of the hybrid inflation model of ref. \[14, 15\] is given by

\[
V(\sigma, \phi) = \frac{1}{4\lambda} (M^2 - \lambda \sigma^2)^2 + \frac{m^2}{2} \phi^2 + \frac{g^2}{2} \phi^2 \sigma^2. \tag{4}
\]

Theories of this type were considered in \[16\]–\[18\]. The main difference between the models of refs. \[16\]–\[18\] and our model is a specific choice of parameters, which allows the existence of the waterfall regime mentioned above. There is also another important difference: we will assume that the field \(\sigma\) in this model is the Higgs field, which remains as a physical degree of freedom after the Higgs effect in an underlying gauge theory with spontaneous symmetry breaking. This field acquired only positive values, which removes the possibility of domain wall formation in this theory. Usually it is rather dangerous to take the inflaton field interacting with gauge fields, since its effective coupling constant \(\lambda\) may acquire large radiative corrections \(\sim e^4\), where \(e\) is the gauge coupling constant. In our case this problem does not appear since density perturbations in our model remain small at rather large \(\lambda\), see below.

The effective mass squared of the field \(\sigma\) is equal to \(-M^2 + g^2 \phi^2\). Therefore for \(\phi > \phi_c = M/g\) the only minimum of the effective potential \(V(\sigma, \phi)\) is at \(\sigma = 0\). The curvature of the effective potential in the \(\sigma\)-direction is much greater than in the \(\phi\)-direction. Thus we expect that at the first stages of expansion of the Universe the field \(\sigma\) rolled down to \(\sigma = 0\), whereas the field \(\phi\) could remain large for a much longer time. For this reason we will consider the stage of inflation at large \(\phi\), with \(\sigma = 0\).
At the moment when the inflaton field $\phi$ becomes smaller than $\phi_c = M/g$, the phase transition with the symmetry breaking occurs. If $m^2\phi_c^2 = m^2 M^2/g^2 \ll M^4/\lambda$, the Hubble constant at the time of the phase transition is given by

$$H^2 = \frac{2\pi M^4}{3\lambda M_p^2}.$$  \hspace{1cm} (5)

Thus we will assume that $M^2 \gg \frac{\lambda m^2}{g^2}$. We will assume also that $m^2 \ll H^2$, which gives

$$M^2 \gg mM_p \sqrt{\frac{3\lambda}{2\pi}}.$$  \hspace{1cm} (6)

One can easily verify, that, under this condition, the Universe at $\phi > \phi_c$ undergoes a stage of inflation. In fact, inflation in this model occurs even if $m^2$ is somewhat greater than $H^2$. Note that inflation at its last stages is driven not by the energy density of the inflaton field $\phi$ but by the vacuum energy density $V(0,0) = \frac{M^4}{4\lambda}$, as in the new inflationary Universe scenario. This was the reason why we called this model ‘hybrid inflation’ in [14].

Let us study the behavior of the fields $\phi$ and $\sigma$ after the time $\Delta t = H^{-1} = \sqrt{\frac{3\lambda}{2\pi M^2}}$ from the moment $t_c$ when the field $\phi$ becomes equal to $\phi_c$. The equation of motion of the field $\phi$ during inflation is $3H\dot{\phi} = m^2\phi$. Therefore during the time interval $\Delta t = H^{-1}$ the field $\phi$ decreases from $\phi_c$ by $\Delta \phi = \frac{m^2\phi_c}{3H^2} = \frac{\lambda m^2 M_p^2}{2\pi g^3 M^2}$. The absolute value of the negative effective mass squared $-M^2 + g^2 \phi^2$ of the field $\sigma$ at that time becomes equal to

$$M^2(\phi) = \frac{\lambda m^2 M_p^2}{\pi M^2}.$$  \hspace{1cm} (7)

The value of $M^2(\phi)$ is much greater than $H^2$ for $M^3 \ll \lambda m M_p^2$. In this case the field $\sigma$ within the time $\Delta t \sim H^{-1}$ rolls down to its minimum at $\sigma(\phi) = M(\phi)/\sqrt{\lambda}$, rapidly oscillates near it and loses its energy due to the expansion of the Universe. However, the field cannot simply relax near this minimum, since the effective potential $V(\phi, \sigma)$ at $\sigma(\phi)$ has a nonvanishing partial derivative

$$\frac{\partial V}{\partial \phi} = m^2\phi + \frac{g^2\phi M^2(\phi)}{\lambda}.$$  \hspace{1cm} (8)

One can easily check that the motion in this direction becomes very fast and the field $\phi$ rolls to the minimum of its effective potential within the time much smaller than $H^{-1}$ if $M^3 \ll \sqrt{\lambda} g m M_p^2$. Thus, under the specified conditions inflation ends in this theory almost instantaneously, as soon as the field $\phi$ reaches its critical value $\phi_c = M/g$.

The amplitude of adiabatic density perturbations produced in this theory can be estimated by standard methods [1] and is given by

$$\frac{\delta \rho}{\rho} = \frac{16\sqrt{6\pi}}{5} \frac{V^{3/2}}{M_p^3 \frac{\partial V}{\partial \phi}} = \frac{16\sqrt{6\pi} \left(\frac{M^4}{4\lambda} + \frac{m^2 \phi^2}{2}\right)^{3/2}}{5M_p^3 m^2 \phi}.$$  \hspace{1cm} (9)
In the case \( m^2 \ll H^2 \) the scalar field \( \phi \) does not change substantially during the last 60 e-foldings (i.e., during the interval \( \Delta t \sim 60H^{-1} \)). In this case the amplitude of density perturbations practically does not depend on scale, and is given by

\[
\frac{\delta \rho}{\rho} \sim \frac{2\sqrt{6\pi} g M^5}{5\lambda \sqrt{\lambda M_p^2 m^2}}. \tag{10}
\]

The definition of \( \frac{\delta \rho}{\rho} \) used in \([\text{II}]\) corresponds to COBE data for \( \frac{\delta \rho}{\rho} \sim 5 \cdot 10^{-5} \). Dividing it by \( (\text{III}) \) with an account taken of \( (\text{I}) \) gives \( M^3 \ll 5 \cdot 10^{-5} \lambda g^{-1} m M_p^2 \). This means that the ‘waterfall conditions’ \( M^3 \ll \lambda m M_p^2 \) and \( M^3 \ll \sqrt{\lambda} g M M_p^2 \) automatically follow from the conditions \( m^2 \ll H^2 \) and \( \frac{\delta \rho}{\rho} \sim 5 \cdot 10^{-5} \), unless the coupling constants \( \lambda \) and \( g \) are extremely small. Therefore the waterfall regime is realized in this model for a wide variety of values of parameters \( m, M, \lambda \) and \( g \) which lead to density perturbations \( \sim 5 \cdot 10^{-5} \).

To give a particular example, let us take \( g^2 \sim \lambda \sim 10^{-1} \), \( m \sim 10^2 \) GeV (electroweak scale). In this case all conditions mentioned above are satisfied and \( \frac{\delta \rho}{\rho} \sim 5 \cdot 10^{-5} \) for \( M \sim 1.3 \cdot 10^{11} \) GeV. In particular, we have verified, by solving equations of motion for the fields \( \phi \) and \( \sigma \) numerically, that inflation in this model ends up within the time \( \Delta t \ll H^{-1} \) after the field \( \phi \) reaches its critical value \( \phi_c = M/g \). The value of the Hubble parameter at the end of inflation is given by \( H \sim 7 \cdot 10^3 \) GeV. The smallness of the Hubble constant at the end of inflation makes it possible, in particular, to have a consistent scenario for axions in inflationary cosmology even if the axion mass is much smaller than \( 10^{-5} \) eV \([\text{III}]\). This model has some other distinctive features. For example, the spectrum of perturbations generated in this model may look as a power-law spectrum rapidly decreasing at large wavelength \( l \) \([\text{I}], [\text{I}]\).

Indeed, at the last stages of inflation (for \( \frac{M_t^4}{4\lambda} \gg \frac{m^2 \phi^2}{2} \)) the field \( \phi \) behaves as

\[
\phi = \phi_c \cdot \exp \left( \frac{- \frac{m^2}{3H} (t - t_c)}{3H} \right), \tag{11}
\]

whereas the scale factor of the Universe grows exponentially, \( a \sim e^{Ht} \). This leads to the following relation between the wavelength of perturbations \( l \) and the value of the scalar field \( \phi \) at the moment when these perturbations were generated: \( \phi \sim \phi_c \left( \frac{l}{l_c} \right)^{m^2/3H^2} \). In this case

\[
\frac{\delta \rho}{\rho} = \frac{2\sqrt{6\pi} g M^5}{5\lambda \sqrt{\lambda M_p^2 m^2}} \cdot \left( \frac{l}{l_c} \right)^{-\frac{m^2}{3H^2}}, \tag{12}
\]

which corresponds to the spectrum index \( n = 1 + \frac{2m^2}{4\lambda H^2} = 1 + \frac{\lambda m^2 M_p^2}{\pi M^4} \) \([\text{I}], [\text{I}]\). Note that this spectrum index is greater than 1, which is a very unusual feature. For the values of \( m, M, \lambda \) and \( g \) considered above, the deviation of \( n \) from 1 is vanishingly small (which is also very unusual). However, let us take, for example, \( \lambda = g = 1, M = 10^{15} \) GeV (grand unification scale), and \( m = 5 \cdot 10^{10} \) GeV. In this case the amplitude of perturbations at the end of inflation (\( \phi = \phi_c \)) is equal to \( 4 \cdot 10^{-4} \), \( n \sim 1.1 \), and the amplitude of the density perturbations drops to the desirable level \( \frac{\delta \rho}{\rho} \sim 5 \cdot 10^{-5} \).

\[\text{[This result recently was also obtained in [37].]}\]
on the galaxy scale ($l_g \sim l_c \cdot e^{50}$). One may easily obtain models with even much larger $n$, but this may be undesirable, since it may lead to formation of many small primordial black holes [20].

Note, that the decrease of $\frac{\delta \rho}{\rho}$ at large $l$ is not unlimited. At $\frac{m^2\phi^2}{2} > \frac{M^4}{4\lambda}$ the spectrum begins growing again. Thus, the spectrum has a minimum on a certain scale, corresponding to the minimum of expression (9). This complicated shape of the spectrum appears in a very natural way, without any need to design artificially bent potentials.

As we have seen, coupling constants in our model can be reasonably large, and the range of possible values of masses $m$ and $M$ is extremely wide. Thus, our model is very versatile. One should make sure, however, that the small effective mass of the scalar field $\phi$ does not acquire large radiative corrections near $\phi = \phi_c$. Hopefully this can be done in supersymmetric theories with flat directions of the effective potential.

One can suggest many interesting generalizations of our model. For example, instead of the term $\frac{m^2\phi^2}{2}$ in (1) one can use the term $\frac{\lambda\phi^4}{4}$. In this case one may have two disconnected stages of inflation. The first stage occurs at large $\phi$, as in the simplest version of chaotic inflation scenario. This stage ends at $\phi < M_p/3$, if $M^2 \ll \lambda M_p^2$. Then the field rapidly rolls down and oscillates until the amplitude of its oscillations becomes smaller than $\phi \sim \frac{M^2}{\lambda \phi M_p}$. At this moment the frequency of oscillations $\sim \sqrt{\lambda \phi} \phi$ becomes smaller that the Hubble constant, and the second stage of inflation begins. This stage of inflation ends with the waterfall at $\phi_c = M/g$. As was shown in [21], in the models with two stages of inflation with a break between them the spectrum of density perturbations may have a very reach and non-trivial structure.

3 Stationary Universe

The first models of inflation were based on the standard assumption of the Big Bang theory that the Universe was created at a single moment of time in a state with the Planck density, and that it was hot and large (much larger than the Planck scale $M_p^{-1}$) from the very beginning. The success of inflation in solving internal problems of the Big Bang theory apparently removed the last doubts concerning the Big Bang cosmology. It remained almost unnoticed that during the last ten years the inflationary theory has broken the umbilical cord connecting it with the old Big Bang theory, and acquired an independent life of its own. For the practical purposes of description of the observable part of our Universe one may still speak about the Big Bang. However, if one tries to understand the beginning of the Universe, or its end, or its global structure, then some of the notions of the Big Bang theory become inadequate.

For example, already in the first version of the chaotic inflation scenario [8] there was no need to assume that the whole Universe appeared from nothing at a single moment of time associated with the Big Bang, that the Universe was hot from the very beginning and that the inflaton scalar field $\phi$ which drives inflation originally occupied the minimum of its potential
energy. Later it was found that if the Universe contains at least one inflationary domain of a size of horizon ('h-region') with a sufficiently large and homogeneous scalar field \( \phi \), then this domain will permanently produce new h-regions of a similar type. In other words, instead of a single Big Bang producing a one-bubble Universe, we are speaking now about inflationary bubbles producing new bubbles, producing new bubbles, \textit{ad infinitum}. In this sense, inflation is not a short intermediate stage of duration \( \sim 10^{-35} \) seconds, but a self-regenerating process, which occurs in some parts of the Universe even now, and which will continue without end. The most striking realization of this scenario occurs in the context of chaotic inflation \[22\], but the basic features of this scenario remain valid in old inflation \[23\], new inflation \[24, 25\] and extended inflation as well \[12\].

Thus, recent developments of inflationary theory have considerably modified our cosmological paradigm \[1\]. Now we must learn how to formulate physical questions in the new context. For example, in a homogeneous part of the Universe there is a simple relation between the density of matter and time. However, on a very large scale the Universe becomes extremely inhomogeneous. Its density, at the same ‘cosmic time’, varies anywhere from zero to the Planck density. Therefore the question about the density of the Universe at the time \(10^{10}\) years may not have any definite answer. Instead of addressing such questions we should study the distribution of probability of finding a part of the Universe with given properties, and find possible correlations between these properties.

It is extremely complicated to describe an inhomogeneous Universe and to find the corresponding probability distribution. Fortunately, there exists a particular kind of stationarity of the process of the Universe self-reproduction which makes things more regular. Due to the no-hair theorem for de Sitter space, the process of production of new inflationary domains occurs independently of any processes outside the horizon. This process depends only on the values of the fields inside each h-region of radius \( H^{-1} \). Each time a new inflationary h-region is created during the Universe expansion, the physical processes inside this region will depend only on the properties of the fields inside it, but not on the ‘cosmic time’ at which it was created.

In addition to this most profound stationarity, there may also exist some simple stationary probability distributions which may allow us to say, for example, what the probability is of finding a given field \( \phi \) at a given point. To examine this possibility one should consider the probability distribution \( P_c(\phi, t|\chi) \), which describes the probability of finding the field \( \phi \) at a given point at a time \( t \), under the condition that at the time \( t = 0 \) the field \( \phi \) at this point was equal to \( \chi \) \[22, 26\]. The same function may also describe the probability that the scalar field which at time \( t \) was equal to \( \phi \), at some earlier time \( t = 0 \) was equal to \( \chi \).

The probability distribution \( P_c \) has been studied by many authors, see e.g. \[22, 25, 27\]. Our investigation of this question has shown that in all realistic inflationary models the probability distribution \( P_c(\phi, t|\chi) \) is not stationary \[22\]. The reason is very simple. The probability distribution \( P_c \) is in fact the probability distribution per unit volume in \textit{comoving coordinates} (hence the index \( c \) in \( P_c \)), which do not change during expansion of the Universe. By considering this probability distribution we neglect the main source of the self-reproduction of inflationary domains, which is the exponential growth of their volume. Therefore, in addition to \( P_c \), we in-
troduced the probability distribution \( P_p(\phi, t|\chi) \), which describes the probability to find a given field configuration in a unit physical volume \[22\]. In the present paper (see also \[28\] for a more detailed presentation) we will show that under certain conditions the stationary probability distribution \( P_p(\phi, t|\chi) \) does exist, and a typical relaxation time during which the distribution \( P_p(\phi, t|\chi) \) approaches the stationary regime is extremely small.

First of all we should remember some details of stochastic approach to inflation. Let us consider the simplest model of chaotic inflation based on the theory of a scalar field \( \phi \) minimally coupled to gravity, with the effective potential \( V(\phi) \). If the classical field \( \phi \) is sufficiently homogeneous in some domain of the Universe, then its behavior inside this domain is governed by the equations

\[
\ddot{\phi} + 3H \dot{\phi} = -\frac{dV}{d\phi},
\]

\[
H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_p} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right).
\]

Here \( H = \dot{a}/a, a(t) \) is the scale factor of the Universe, \( k = +1, -1, \) or 0 for a closed, open or flat Universe, respectively. \( M_p \) is the Planck mass, which we will put equal to one in the rest of the paper.

Investigation of these equations has shown that for many potentials \( V(\phi) \) (e.g., in all power-law \( V(\phi) \sim \phi^n \) and exponential \( V(\phi) \sim e^{\alpha \phi} \) potentials) there exists an intermediate asymptotic regime of slow rolling of the field \( \phi \) and quasi-exponential expansion (inflation) of the Universe \[1\]. At this stage the Hubble parameter is \( H(\phi) = \sqrt{8\pi V(\phi)/3} \). In the theories \( V(\phi) \sim \phi^n \) inflation ends at \( \phi_e \sim n/12 \). In the theory with \( V(\phi) \sim e^{\alpha \phi} \) inflation ends only if we bend the potential at some point \( \phi_e \); for definiteness we will take \( \phi_e = 0 \) in this theory.

Inflation stretches all initial inhomogeneities. Therefore, if the evolution of the Universe were governed solely by classical equations of motion, we would end up with an extremely smooth Universe with no primordial fluctuations to initiate the growth of galaxies. Fortunately, new density perturbations are generated during inflation due to quantum effects. The wavelengths of all vacuum fluctuations of the scalar field \( \phi \) grow exponentially in the expanding Universe. When the wavelength of any particular fluctuation becomes greater than \( H^{-1} \), this fluctuation stops oscillating, and its amplitude freezes at some nonzero value \( \delta\phi(x) \) because of the large friction term \( 3H \dot{\phi} \) in the equation of motion of the field \( \phi \). The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field \( \delta\phi(x) \) that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the creation of more and more perturbations of the classical field with wavelengths greater than \( H^{-1} \). The average amplitude of such perturbations generated during a time interval \( H^{-1} \) (in which the Universe expands by a factor of \( e \)) is given by

\[
|\delta\phi(x)| \approx \frac{H}{2\pi}.
\]
The phases of each wave are random. Therefore, the sum of all waves at a given point fluctuates and experiences Brownian jumps in all directions in the space of fields.

The standard way to describe the stochastic behavior of the inflaton field during the slow-rolling stage is to coarse-grain it over $h$-regions and consider the effective equation of motion of the long-wavelength field [26]:

$$\frac{d\phi}{dt} = -\frac{V'(\phi)}{3H(\phi)} + \frac{H^{3/2}(\phi)}{2\pi} \xi(t).$$

(16)

Here $H = \sqrt{8\pi V/3}$, $\xi(t)$ is the effective white noise generated by quantum fluctuations, which leads to the Brownian motion of the classical field $\phi$.

This Langevin equation leads to two stochastic equations for the probability distribution $P_c(\phi, t|\chi)$. The first one is called the backward Kolmogorov equation,

$$\frac{\partial P_c(\phi, t|\chi)}{\partial t} = \frac{1}{2} \frac{H^{3/2}(\chi)}{2\pi} \frac{\partial}{\partial \chi} \left( \frac{H^{3/2}(\chi)}{2\pi} \frac{\partial}{\partial \chi} P_c(\phi, t|\chi) \right) - \frac{V'(\chi)}{3H(\chi)} \frac{\partial}{\partial \chi} P_c(\phi, t|\chi).$$

(17)

In this equation one considers the value of the field $\phi$ at the time $t$ as a constant, and finds the time dependence of the probability that this value was reached during the time $t$ as a result of diffusion of the scalar field from different possible initial values $\chi \equiv \phi(0)$.

The second equation is the adjoint to the first one; it is called the forward Kolmogorov equation, or the Fokker-Planck equation,

$$\frac{\partial P_c(\phi, t|\chi)}{\partial t} = \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{H^{3/2}(\phi)}{2\pi} \frac{\partial}{\partial \phi} \left( \frac{H^{3/2}(\phi)}{2\pi} P_c(\phi, t|\chi) \right) + \frac{V'(\phi)}{3H(\phi)} P_c(\phi, t|\chi) \right),$$

(18)

This equation was derived in [26], see also [25].

One may try to find a stationary solution of equations (17), (18), assuming that $\frac{\partial P_c(\phi, t|\chi)}{\partial t} = 0$. The simplest stationary solution (subexponential factors being omitted) would be

$$P_c(\phi, t|\chi) \sim \exp \left( \frac{3}{8V(\phi)} \right) \cdot \exp \left( -\frac{3}{8V(\chi)} \right).$$

(19)

This function is extremely interesting. Indeed, the first term in (19) is equal to the square of the Hartle-Hawking wave function of the Universe [29], whereas the second one gives the square of the tunneling wave function [30]!

At first glance, this result gives a direct confirmation and a simple physical interpretation of both the Hartle-Hawking wave function of the Universe and the tunneling wave function. However, in all realistic cosmological theories, in which $V(\phi) = 0$ at its minimum, the Hartle-Hawking distribution $\exp \left( \frac{3}{8V(\phi)} \right)$ is not normalizable. The source of this difficulty can be easily understood: any stationary distribution may exist only due to compensation of the classical flow
of the field $\phi$ downwards to the minimum of $V(\phi)$ by the diffusion motion upwards. However, diffusion of the field $\phi$ discussed above exists only during inflation. Thus, there is no diffusion motion upwards from the region $\phi < \phi_e$. Therefore all solutions of equation (18) with the proper boundary conditions at $\phi = \phi_e$ (i.e. at the end of inflation) are non-stationary (decaying) [22].

The situation with the probability distribution $P_p$ is much more interesting and complicated. As was shown in [22] its behavior depends strongly on initial conditions. If the distribution $P_p$ was initially concentrated at $\phi < \phi^*$, where $\phi^*$ is some critical value of the field, then it moves towards small $\phi$ in the same way as $P_c$, i.e. it cannot become stationary. On the other hand, if the initial value of the field $\phi$ is larger than $\phi^*$, the distribution moves towards larger and larger values of the field $\phi^*$, until it reaches the field $\phi_p$, at which the effective potential of the field becomes of the order of Planck density $M_p^4$ (we will assume $M_p = 1$ hereafter), where the standard methods of quantum field theory in a curved classical space are no longer valid.

Some further steps towards the solution of this problem were made by Nambu and Sasaki [31] and Mijić [32]. Their papers contain many important results and insights. However, Mijić [32] did not have a purpose to obtain a complete expression for the stationary distribution $P_p(\phi, t | \chi)$. The corresponding expressions were obtained for various types of potentials $V(\phi)$ in [31]. Unfortunately, according to [31], the stationary distribution $P_p(\phi, t | \chi)$ is almost entirely concentrated at $\phi \gg \phi_p$, i.e. at $V(\phi) \gg 1$, where the methods used in [31] are inapplicable.

We will continue this investigation by writing the system of stochastic equations for $P_p$. These equations can be obtained from eqs. (17), (18) by adding the term $3H P_p$, which appears due to the growth of physical volume of the Universe by the factor $1 + 3H(\phi) \, dt$ during each time interval $dt$ [31]–[34], [28]:

$$\frac{\partial P_p}{\partial t} = \frac{1}{2} \frac{H^{3/2}(\chi)}{2 \pi} \frac{\partial}{\partial \chi} \left( \frac{H^{3/2}(\chi)}{2 \pi} \frac{\partial P_p}{\partial \chi} \right) - \frac{V'(\chi)}{3H(\chi)} \frac{\partial P_p}{\partial \chi} + 3H(\chi) P_p , \quad (20)$$

$$\frac{\partial P_p}{\partial t} = \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{H^{3/2}(\phi)}{2 \pi} \frac{\partial}{\partial \phi} \left( \frac{H^{3/2}(\phi)}{2 \pi} P_p \right) + \frac{V'(\phi)}{3H(\phi)} P_p \right) + 3H(\phi) P_p . \quad (21)$$

To find solutions of these equations one must specify boundary conditions. The boundary conditions at the end of inflation follow from the conservation of the probability flux [28]:

$$\frac{\partial}{\partial \phi} P_p(\phi_e) = -\frac{3}{4} \frac{V'}{V} P_p(\phi_e) , \quad \frac{\partial}{\partial \chi} \left( P_p \exp \left( \frac{3}{8V(\chi)} \right) \right) \bigg|_{\chi=\phi_e} = 0 . \quad (22)$$

The last boundary condition is especially interesting, since it indicates that at least in the vicinity of $\chi = \phi_e$ the probability distribution with respect to $\chi$ looks as a square of the tunneling wave function. However, we have found solutions of (20), (21) to be rather stable with respect to modification of these boundary conditions, whereas the conditions at the Planck boundary $\phi = \phi_p$ do play a very important role.

Investigation of these conditions poses many difficult problems. First of all, our diffusion equations are based on the semiclassical approach to quantum gravity, which breaks down at
super-Planckian densities. Secondly, the shape of the effective potential may be strongly modified
by quantum effects at $\phi \sim \phi_p$. Finally, our standard interpretation of the probability distribution
$P_c$ and $P_p$ breaks down at $V(\phi) > 1$, since at super-Planckian densities the notion of a classical
scalar field in classical space-time does not make much sense.

However, these problems by themselves suggest a possible answer. Inflation happens only in
theories with very flat effective potentials. At the Planck density nothing can protect the effective
potential from becoming steep. Hence, one may expect that inflation ceases to exist at $\phi > \phi_p$,
which leads to the boundary condition

$$P_p(\phi_p, t|\chi) = P_p(\phi, t|\chi_p) = 0,$$  \hspace{1cm} (23)

where $V(\phi_p) \equiv V(\chi_p) = O(1)$.

There is also another, much more general reason to expect that inflation kills itself as the
potential energy density approaches the Planck density $V \sim 1$. Indeed, the amplitude of fluctua-
tions of the scalar field generated during the typical time $\delta t = H^{-1}$ is given by $H/2\pi$, and
their typical wavelength at that time is $O(H^{-1})$. This means that the energy density associated
with the gradients of these perturbations is of the order of $H^4 \sim V^2$. Thus, in the domains with
$V > 1$ the gradient energy density $\sim V^2$ becomes larger than the potential energy density $V$.
This violates one of the basic assumptions necessary for inflation in domains with $V \gtrsim 1$. One
may expect also that large gradients of energy on the scale comparable to the scale of the horizon
$H^{-1}$ lead to creation of black holes rather than to the permanent self-reproduction of inflationary
$h$-regions.

Of course, one may argue that all our considerations do not make sense at densities larger
than the Planck density. When the energy density in any $h$-region approaches the Planck density,
it may no longer be described in terms of classical space-time and should be just thrown away
from our consideration. In particular, its volume should not be considered as contributing to the
total volume of the Universe. Thus, such domains should be neglected in our definition of $P_p$. In
this case the distribution $P_p$ for the field $\phi$ will stop moving towards higher values of $\phi$ and will
approach a stationary regime when this distribution will be shifted towards $\phi \sim \phi_p$.

What we are saying is even stronger. Even if one makes an attempt to consider the domains
with $V > 1$ as a part of classical space-time, many parts of these domains drop out from the
process of inflation. This means that the total volume of inflationary $h$-regions cannot grow as
fast as $e^{3Ht}$.

We do not know which of these arguments, if any, will survive in the future theory of all
fundamental interactions. However, all these arguments point out in the same direction: For a
phenomenological description of stochastic processes in classical space-time one should impose
boundary conditions of the type of (23) which do not permit penetration of inflation deep into
the realm of super-Planckian densities. As we will show in [28], the exact form of these boundary
conditions is not very important; most of them allow the same class of solutions as the boundary
condition (23). Therefore in this paper we will use these boundary conditions, assuming for
definiteness that they are imposed at $V(\phi_p) \equiv V(\chi_p) = 1$. 

13
One may try to obtain solutions of equations (20), (21) in the form of the following series of biorthonormal system of eigenfunctions of the pair of adjoint linear operators (defined by the left hand sides of the equations below):

\[ P_p(\phi, t|\chi) = \sum_{s=1}^{\infty} e^{\lambda_st} \psi_s(\chi) \pi_s(\phi). \]  

(24)

Indeed, this gives us a solution of eq. (21) if

\[ \frac{1}{2} \frac{H^{3/2}(\chi)}{2\pi} \frac{\partial}{\partial \chi} \left( \frac{H^{3/2}(\chi)}{2\pi} \frac{\partial}{\partial \chi} \psi_s(\chi) \right) - \frac{V'(\chi)}{3H(\chi)} \frac{\partial}{\partial \chi} \psi_s(\chi) + 3H(\chi) \cdot \psi_s(\chi) = \lambda_s \psi_s(\chi). \]  

(25)

and

\[ \frac{1}{2} \frac{\partial}{\partial \phi} \left( \frac{H^{3/2}(\phi)}{2\pi} \frac{\partial}{\partial \phi} \left( \frac{H^{3/2}(\phi)}{2\pi} \pi_j(\phi) \right) \right) + \frac{\partial}{\partial \phi} \left( \frac{V'(\phi)}{3H(\phi)} \pi_j(\phi) \right) + 3H(\phi) \cdot \pi_j(\phi) = \lambda_j \pi_j(\phi). \]  

(26)

The orthonormality condition reads

\[ \int_{\phi_c}^{\phi_p} \psi_s(\chi) \pi_j(\phi) d\chi = \delta_{sj}. \]  

(27)

In our case (with regular boundary conditions) one can easily show that the spectrum of \( \lambda_j \) is discrete and bounded from above. Therefore the asymptotic solution for \( P_p(\phi, t|\chi) \) (in the limit \( t \to \infty \)) is given by

\[ P_p(\phi, t|\chi) = e^{\lambda_1t} \psi_1(\chi) \pi_1(\phi) \cdot \left( 1 + O(e^{-(\lambda_1-\lambda_2)t}) \right). \]  

(28)

Here \( \psi_1(\chi) \) is the only positive eigenfunction of eq. (25), \( \lambda_1 \) is the corresponding (real) eigenvalue, and \( \pi_1(\phi) \) is the eigenfunction of the conjugate operator (26) with the same eigenvalue \( \lambda_1 \). Note, that \( \lambda_1 \) is the highest eigenvalue, \( \text{Re}(\lambda_1-\lambda_2) > 0 \). This is the reason why the asymptotic equation (28) is valid at large \( t \). We have found [28] that in realistic theories of inflation a typical time of relaxing to the asymptotic regime, \( \Delta t \sim (\lambda_1-\lambda_2)^{-1} \), is extremely small. Typically it is only about a few thousands Planck times, i.e. about \( 10^{-40} \) sec. This means that the normalized distribution

\[ \tilde{P}_p(\phi, t|\chi) = e^{-\lambda_1t} P_p(\phi, t|\chi) \]  

(29)

rapidly converges to the time-independent normalized distribution

\[ \tilde{P}_p(\phi|\chi) \equiv \tilde{P}_p(\phi, t \to \infty|\chi) = \psi_1(\chi) \pi_1(\phi). \]  

(30)

It is this stationary distribution that we were looking for. Because the growing factor \( e^{-\lambda_1t} \) is the same for all \( \phi \) (and \( \chi \)), one can use \( \tilde{P}_p \) instead of \( P_p \) for calculation of all relative probabilities. In particular, \( \tilde{P}_p(\phi|\chi) \) gives us the fraction of the volume of the Universe occupied by the field \( \phi \), under the condition that the corresponding part of the Universe at some time in the past contained the field \( \chi \). The remaining problem is to find the functions \( \psi_1(\chi) \) and \( \pi_1(\phi) \), and to check that all assumptions about the boundary conditions which we made on the way to eq. (28) are actually satisfied.
We have solved this problem for chaotic inflation in a wide class of theories including the theories with polynomial and exponential effective potentials $V(\phi)$ and found the corresponding stationary distributions \[28\]. Here we will present some of our results for the theories $\frac{1}{4}\phi^4$ and $V_o e^{a\phi}$.

Solution of equations (25) and (26) for $\psi_1(\chi)$ and $\pi_1(\phi)$ in the theory $\frac{1}{4}\phi^4$ shows that these functions are extremely small at $\phi \sim \phi_e$ and $\chi \sim \chi_e$. They grow at large $\phi$ and $\chi$, then rapidly decrease, and vanish at $\phi = \chi = \phi_p$. With a decrease of $\lambda$ the solutions become more and more sharply peaked near the Planck boundary. (The functions $\psi_1$ and $\pi_1$ for the exponential potential have a similar behavior, but they are less sharply peaked near $\phi_p$.) A detailed discussion of these solutions will be contained in \[28\]. The eigenvalues $\lambda_1$ corresponding to different coupling constants $\lambda$ are given by the following table:

| $\lambda$ | 1   | $10^{-1}$ | $10^{-2}$ | $10^{-3}$ | $10^{-4}$ | $10^{-5}$ | $10^{-6}$ |
|------------|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| $\lambda_1$ | 2.813 | 4.418 | 5.543 | 6.405 | 7.057 | 7.538 | 7.885 |

One can find also the second eigenvalue $\lambda_2$. For example, for $\lambda = 10^{-4}$ one gets $\lambda_2 = 6.789$. This means that for $\lambda = 10^{-4}$ the time of relaxation to the stationary distribution is $\Delta t \sim (\lambda_1 - \lambda_2)^{-1} \sim 4M_p^{-1} \sim 10^{-42}$ seconds — a very short time indeed.

Note that the parameter $\lambda_1$ shows the speed of exponential expansion of the volume filled by a given field $\phi$. This speed does not depend on the field $\phi$, and has the same order of magnitude as the speed of expansion at the Planck density. Indeed, $\lambda_1$ should be compared to $3H(\phi) = 2\sqrt{6\pi}V(\phi)$, which is equal to $2\sqrt{6\pi}$ at the Planck density. It can be shown \[28\] that in the limit $\lambda \to 0$ the eigenvalue $\lambda_1$ also becomes equal to $2\sqrt{6\pi} \approx 8.681$. The meaning of this result is very simple: in the limit $\lambda \to 0$ our solution becomes completely concentrated near the Planck boundary, and $\lambda_1$ becomes equal to $3H(\phi_p)$.

At first glance, independence of the speed of expansion of volume $e^{\lambda_1 t}$ on the value of the field $\phi$ may seem counterintuitive. The meaning of this result is that the domain filled with the field $\phi$ gives the largest contribution to the growing volume of the Universe if it first diffuses towards the Planckian densities, spends there as long time as possible expanding with nearly Planckian speed, and then diffuses back to its original value $\phi$.

But what about the field $\phi$ which is already at the Planck boundary? Why do the corresponding domains not grow exactly with the Planckian Hubble constant $H(\phi_p) = 2\sqrt{6\pi}/3$? It happens partially due to diffusion and slow rolling of the field towards smaller $\phi$. However, the leading effect is the destructive diffusion towards the space-time foam with $\phi > \phi_p$. One may visualize this process by painting white all domains with $V(\phi) < 1$, and by painting black domains filled by space-time foam with $V(\phi) > 1$. Then each time $H^{-1}(\phi_p)$ the volume of white domains with $\phi \sim \phi_p$ grows approximately $e^3$ times, but some ‘black holes’ appear in these domains, and, as a result, the total volume of white domains increases only $e^{3\lambda_1/2\sqrt{6\pi}}$ times. This suggests (by anal-
ogy with \cite{35} calling the factor $d_f = 3\lambda_1/2\sqrt{6\pi}$ ‘the fractal dimension of classical space-time’, or ‘the fractal dimension of the inflationary Universe’. (Note that $d_f < 3$ for $\lambda \neq 0$; for example, $d_f = 2.6$ for $\lambda = 10^{-5}$.) However, one should keep in mind that the fractal structure of the inflationary Universe in the chaotic inflation scenario in general is more complicated than in the new or old inflation and cannot be completely specified just by one fractal dimension \cite{28}.

The distribution $\tilde{P}_p(\phi|\chi) = \psi_1(\chi) \pi_1(\phi)$ which we have obtained does not depend on time $t$. However, in general relativity one may use many different time parametrizations, and the same physics can be described differently in different ‘times’. One of the most natural choices of time in the context of stochastic approach to inflation is the time $\tau = \ln \frac{a(x,t)}{a(x,0)} = \int H(\phi(x,t), t) \, dt$ \cite{26, 27}. Here $a(x,t)$ is a local value of the scale factor in the inflationary Universe. By using this time variable, we were able to obtain not only numerical solutions to the stochastic equations, but also simple asymptotic expressions describing these solutions. For example, for the theory $\frac{1}{2} \dot{\phi}^4$ both the eigenvalue $\lambda_1$ and the ‘fractal dimension’ $d_f$ (which in this case refers both to the Planck boundary at $\phi_p$ and to the end of inflation at $\phi_e$) are given by $d_f = \lambda_1 \sim 3 - 1.1 \sqrt{\lambda}$, and the stationary distribution is

$$\tilde{P}_p(\phi, \tau|\chi) \sim \exp\left(-\frac{3}{8V(\chi)}\left(\frac{1}{V(\chi)} + 0.4 - \frac{1}{1.4}\right) \cdot \phi \exp\left(-\pi (3 - \lambda_1)\phi^2\right)\right) \sim \exp\left(-\frac{3}{2\lambda_4}\right)\left(\frac{4}{\lambda_4 + 1.6} - \frac{1}{1.4}\right) \cdot \phi \exp\left(-3.5\sqrt{\lambda}\phi^2\right).$$

(31)

Note that the first factor coincides with the square of the tunneling wave function \cite{30}. This expression is valid in the whole interval from $\phi_e$ to $\phi_p$ and it correctly describes asymptotic behavior of $\tilde{P}_p(\phi, \tau|\chi)$ both at $\chi \sim \chi_e$ and at $\chi \sim \chi_p$.

A similar investigation can be carried out for the theory $V(\phi) = V_o \, e^{\alpha\phi}$. The corresponding solution is

$$\tilde{P}_p(\phi, \tau|\chi) \sim \exp\left(-\frac{3}{8V(\chi)}\right)\left(\frac{1}{V(\chi)} - 1\right) \cdot \left(\frac{1}{V(\phi)} - 1\right) V^{-1/2}(\phi).$$

(32)

This expression gives a rather good approximation for $\tilde{P}_p(\phi, \tau|\chi)$ for all $\phi$ and $\chi$.

The main result of our work is that under certain conditions the properties of our Universe can be described by a time-independent probability distribution, which we have found for theories with polynomial and exponential effective potentials. A lot of work still has to be done to verify this conclusion, see \cite{28}. However, once this result is taken seriously, one should consider its interpretation and rather unusual implications.

When making cosmological observations, we study our part of the Universe and find that in this part inflation ended about $t_e \sim 10^{10}$ years ago. The standard assumption of the first models of inflation was that the total duration of the inflationary stage was $\Delta t \sim 10^{-35}$ seconds. Thus one could come to an obvious conclusion that our part of the Universe was created in the Big Bang, at the time $t_e + \Delta t \sim 10^{10}$ years ago. However, in our scenario the answer is quite different.
Let us consider an inflationary domain which gave rise to the process of self-reproduction of new inflationary domains. For illustrative purposes, one can visualize self-reproduction of inflationary domains as a branching process, which gives a qualitatively correct description of the actual physical process we consider. During this process, the first inflationary domain of initial radius $\sim H^{-1}(\phi)$ within the time $H^{-1}(\phi)$ splits into $e^3 \sim 20$ independent inflationary domains of similar size. Each of them contains a slightly different field $\phi$, modified both by classical motion down to the minimum of $V(\phi)$ and by long-wavelength quantum fluctuations of amplitude $\sim H/2\pi$. After the next time step $H^{-1}(\phi)$, which will be slightly different for each of these domains, they split again, etc. The whole process now looks like a branching tree growing from the first (root) domain. The radius of each branch is given by $H^{-1}$; the total volume of all domains at any given time $t$ corresponds to the ‘cross-section’ of all branches of the tree at that time, and is proportional to the number of branches. This volume rapidly grows, but when calculating it, one should take into account that those branches, in which the field becomes larger than $\phi_p$, die and fall down from the tree, and each branch in which the field becomes smaller than $\phi_e$, ends on an apple (a part of the Universe where inflation ended and life became possible).

One of our results is that even after we discard at each given moment the dead branches and the branches ended with apples, the total volume of live (inflationary) domains will continue growing exponentially, as $e^{\lambda_1 t}$. What is even more interesting, we have found that very soon the portion of branches with given properties (with given values of scalar fields, etc.) becomes time-independent. Thus, by observing any finite part of a tree at any given time $t$ one cannot tell how old the tree is.

To give a most dramatic representation of our conclusions, let us see where most of the apples grow. This can be done simply by integrating $e^{\lambda_1 t}$ from $t = 0$ to $t = T$ and taking the limit as $T \to \infty$. The result obviously diverges at large $T$ as $\lambda_1^{-1} e^{\lambda_1 T}$, which means that most apples grow at an indefinitely large distance from the root. In other words, if we ask what is the total duration of inflation which produced a typical apple, the answer is that it is indefinitely large.

This conclusion may seem very strange. Indeed, if one takes a typical point in the root domain, one can show that inflation at this point ends within a finite time $\Delta t \sim 10^{-35}$ seconds. This is a correct (though model-dependent) result which can be confirmed by stochastic methods, using the distribution $P_c(\phi, \Delta t | \chi)$ [22]. How could it happen that the duration of inflation was any longer than $10^{-35}$ seconds?

The answer is related to the choice between $P_c$ and $P_p$, or between roots and fruits. Typical points in the root domain drop out from the process of inflation within $10^{-35}$ seconds. The number of those points which drop out from inflation at a much later stage is exponentially suppressed, but they produce the main part of the total volume of the Universe. Note that the length of each particular branch continued back in time may well be finite [30]. However, there is no upper limit to the length of each branch, and, as we have seen, the longest branches produce almost all parts of the Universe with properties similar to the properties of the part where we live now. Since by local observations we can tell nothing about our distance in time from the root domain, our probabilistic arguments suggest that the root domain is, perhaps, indefinitely far away from us. Moreover, nothing in our part of the Universe depends on the distance from the root domain.
and, consequently, on the distance from the Big Bang.

Thus, inflation solves many problems of the Big Bang theory. However, after making all kinds of improvements of this theory, we are now winding up with a model of a stationary Universe, in which the notion of the Big Bang loses its dominant position, being removed to the indefinite past. One should note, that for all practical purposes the end of inflation in our part of the Universe can be considered as a Big Bang (or, more precisely, as a Pretty Big Bang), after which our part of the Universe became hot and looked like an expanding ball of fire. However, the global structure of the inflationary universe cannot be described by this simple picture.

From this point of view, inflationary theory cannot be considered as a part of the Big Bang theory or as its improved version. On the contrary, the Big Bang theory can be embedded into inflationary cosmology, and after this embedding the Big Bang cosmology provides an excellent description of the local structure of the inflationary Universe.

References

[1] A.D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).

[2] J. Levin and K. Freese, Michigan Univ. preprint, MIT-AT-92-01, astro-ph@babbage.sissa.it - 9211011; J. Levin and K. Freese, Michigan Univ. preprint, MIT-AT-92-02, astro-ph@babbage.sissa.it - 9211008.

[3] Y. Hu, M. Turner and E.J. Weinberg, Columbia University preprint CU-TP-581, astro-ph@babbage.sissa.it - 9302002.

[4] A.H. Guth, *Phys. Rev.* D23, 347 (1981).

[5] A. H. Guth and E. J. Weinberg, *Nucl. Phys.* B212, 321 (1983).

[6] R. Abbott, S. Barr and S. Ellis, *Phys. Rev.* D30, 720 (1984); *ibid* D31, 673 (1985).

[7] A.D. Linde, *Phys. Lett.* 108B, 389 (1982); A. Albrecht and P.J. Steinhardt, *Phys. Rev. Lett.* 48, 1220 (1982).

[8] A.D. Linde, *Phys. Lett.* 129B, 177 (1983).

[9] A.D. Linde, *Phys. Lett.* 162B, 281 (1985).

[10] F. Lucchin and S. Matarrese, *Phys. Rev.* D32, 1316 (1985); J. Halliwell, *Phys. Lett.* B 185, 341 (1987); A.B. Burd and J.D. Barrow, *Nucl. Phys.* B308, 929 (1988); A.R. Liddle, *Phys. Lett.* B220, 502 (1989).
[11] D. La and P.J. Steinhardt, *Phys. Rev.*. Lett. **62**, 376 (1989).

[12] A.D. Linde, *Phys. Lett.* **B249**, 18 (1990).

[13] R. Crittenden and P.J. Steinhardt, *Phys. Lett.* **B293**, 32 (1992).

[14] A.D. Linde, *Phys. Lett.* **B259**, 38 (1991).

[15] A.D. Linde, *Hybrid Inflation*, Stanford University preprint SU-ITP-93-17, astro-ph/9307002, submitted to *Phys. Rev.*

[16] L.A. Kofman and A.D. Linde, *Nucl. Phys.* **B282**, 555 (1987).

[17] L.A. Kofman, D.Yu. Pogosyan, *Phys. Lett.* **B214**, 508 (1988); D. S. Salopek, J.R. Bond and J.M. Bardeen, *Phys. Rev.* **D40**, 1753 (1989); L.A. Kofman, *Physica Scripta* **T36**, 108 (1991).

[18] H. Hodges, G. Blumenthal, L. Kofman and J. Primack, *Nucl. Phys.* **B335**, 197 (1990).

[19] A.R. Liddle and D.H. Lyth, Sussex University preprint AST 92/8-2.

[20] A.G. Polnarev and M.Yu. Khlopov, *Sov. Phys. Usp.* **28**, 213 (1985).

[21] V.F. Mukhanov and M.I. Zelnikov, *Phys. Lett.* **B263**, 169 (1991); D. Polarski and A.A. Starobinsky, *Nucl. Phys.* **B385**, 623 (1992).

[22] A.D. Linde, *Phys. Lett.* **175B**, 395 (1986); A.S. Goncharov, A.D. Linde and V.F. Mukhanov, *Int. J. Mod. Phys.* **A2**, 561 (1987).

[23] A.H. Guth, *Phys. Rev.* **D23**, 347 (1981); J.R. Gott, *Nature* **295**, 304 (1982); K. Sato, H. Kodama, M. Sasaki and K. Maeda, *Phys. Lett.* **B108**, 35 (1982).

[24] P.J. Steinhardt, in: *The Very Early Universe*, G.W. Gibbons, S.W. Hawking, S. Siklos, eds., Cambridge U.P. Cambridge, England (1982), p. 251; A.D. Linde, *Nonsingular Regenerating Inflationary Universe*, Cambridge University preprint (1982).

[25] A. Vilenkin, *Phys. Rev.* **D27**, 2848 (1983).

[26] A.A. Starobinsky, in: *Fundamental Interactions* (MGPI Press, Moscow, 1984), p. 55; A.S. Goncharov and A.D. Linde, Sov. J. Part. Nucl. **17**, 369 (1986); A.A. Starobinsky, in: *Current Topics in Field Theory, Quantum Gravity and Strings*, Lecture Notes in Physics, eds. H.J. de Vega and N. Sanchez (Springer, Heidelberg 1986) **206**, p. 107.

[27] D.S. Salopek and J.R. Bond, *Phys. Rev.* **D42**, 3936 (1990); *ibid* **D43**, 1005 (1991).

[28] A.D. Linde and A. Mezhlumian, *Phys. Lett.* **B307**, 25 (1993); A.D. Linde, D.A. Linde, A. Mezhlumian, *From the Big Bang Theory to the Theory of a Stationary Universe*, Stanford University preprint, SU-ITP-93-13, gr-qc@xxx.lanl.gov - 9306035, submitted to *Phys. Rev.*; A.D. Linde and A. Mezhlumian, in preparation.

[29] J.B. Hartle and S.W. Hawking, *Phys. Rev.* **D28**, 2960 (1983).
[30] A.D. Linde, JETP 60, 211 (1984); Lett. Nuovo Cim. 39, 401 (1984); Ya.B. Zeldovich and A.A. Starobinsky, Sov. Astron. Lett. 10, 135 (1984); V.A. Rubakov, Phys. Lett. 148B, 280 (1984); A. Vilenkin, Phys. Rev. D30, 549 (1984).

[31] Y. Nambu and M. Sasaki, Phys. Lett. B219, 240 (1989); Y. Nambu, Prog. Theor. Phys. 81 (1989) 1037.

[32] M. Mijić, Phys. Rev. D42, 2469 (1990); Int. J. Mod. Phys. A6, 2685 (1991).

[33] Ya.B. Zeldovich and A.D. Linde, unpublished (1986).

[34] A. Mezhlumian and S.A. Molchanov, J. Stat. Phys. 71, 799 (1993).

[35] M. Aryal and A. Vilenkin, Phys. Lett. B199, 351 (1987).

[36] A. Vilenkin, Phys. Rev. D46, 2355 (1992).

[37] S. Mollerach, S. Matarrese and F. Lucchin, Blue Perturbation Spectra From Inflation, Padova University preprint, submitted to ApJ.