The $O(g^6)$ coefficient in the thermodynamic potential of hot
SU(N) Gauge Theories and MQCD

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Abstract

The non-perturbative input necessary for the determination of the $O(g^6)$ part of the weak coupling expansion of the free energy density for SU(2) and SU(3) gauge theories is estimated. Although the perturbative information completing the contribution to this order is missing, we give arguments that the magnetic fluctuations are dominated by screened elementary magnetic gluons.
1 Introduction

Perturbation theory in its original formulation fails beyond $\mathcal{O}(g^5)$ in the calculation of the free energy density of non-Abelian gauge theories [1]. Very recently Braaten [2] has pointed out that the coefficients of the higher powers in the weak coupling expansion can be determined by invoking non-perturbative information from the effective theory of three-dimensional static magnetic fluctuations.

In Ref. [2] a systematic two-step separation of the perturbatively treatable fluctuations from the static magnetic sector has been proposed. In the first step the full (electric and magnetic) static sector is represented by an effective three-dimensional theory. In this theory a massive ($m_E$) adjoint scalar field stands for the screened electric fluctuations. In the second step this theory is matched onto an effective magnetic theory (MQCD) with a separation cut-off $\Lambda_M$. While the contribution of the non-static and of the static electric modes to the free energy can be safely calculated perturbatively (expansion parameters are $g(T)$ and $m_E/T$, respectively), the magnetic sector is still non-perturbative and should be investigated by numerical methods.

The high temperature free energy of the full theory can then be written in additive form:

$$f_{QCD}(T \gg T_c) = f_{NS,E}(\Lambda_M) + f_M(\Lambda_M), \quad gT \geq \Lambda_M \geq g^2T.$$

The double subscript $NS, E$ refers to the fact that the first term contains contributions to the free energy from the non-static as well as from static electric type fluctuations.

The strategy of the hierarchical calculation has been tested by Braaten and Nieto [3] by reproducing from the effective electric QCD theory the $\mathcal{O}(g^5)$ term in the free energy density of the SU(N) gauge theory, which earlier has been calculated directly in the full theory by Zhai and Kastening [4]. This is fully part of the term $f_{NS,E}$.

The necessary non-perturbative information to go beyond this stage comes from the minimal MQCD theory. The purpose of our paper is to present first results of a lattice analysis of MQCD, i.e. the 3-dimensional SU(N) gauge theory. This
does provide the non-perturbative input needed to evaluate the $O(g^6)$ part of $f_M$, denoted in the following by $f_M^{(0)}$.

This piece of information can be derived from the simplest three-dimensional gauge theory:

$$f_M^{(0)} = -\frac{T}{V} \ln \left[ \int (M)^{D} A_j^a \exp \left( -\int d^3 x L_{MQCD}^{(0)} \right) \right], \quad (1.2)$$

where

$$L_{MQCD}^{(0)} = \frac{1}{4} G_{ij}^a G^a_{ij} \quad (1.3)$$

is the only operator of dimension four contributing to leading order. The gauge coupling of MQCD with the necessary accuracy is given by $g_3^2 = g^2 T$. It is worth to note that the leading ($O(g^3)$) correction to $g_3^2$ comes from the finite wave function renormalization factor due to the integration over the static electric field.

The determination of $f_M^{(0)}$ represents a certain interest in itself. It provides information about the nature of quasi-particle excitations in the high temperature phase. Similar to the situation in the electric sector of the SU(N) gauge theories one also may expect that at very high temperature a weakly interacting gas of some (quasi)particles dominates the free energy density contribution of the magnetic sector. Yet, the nature of the "constituents" of this gas is still to be clarified.

Inspired by the reduction strategy we have recently investigated the problem of magnetic screening both in pure SU(2) and in SU(2)-Higgs models [4]. In Landau gauge we have analyzed the vector ($A_i$) two point correlations and determined the propagator mass from the corresponding Euclidean propagator. The numerical results were found to be similar to those obtained in analytical calculations from coupled gap-equations for the Higgs and the vector channels [5]. In the pure gauge theory [4] and in the SU(2) gauge-Higgs systems also heavier excitations have been identified numerically [8, 9, 10] and interpreted analytically [11]. A unified interpretation of these rich spectra is, however, missing at present. In particular one has to clarify whether the light excitations found so far only in gauge dependent correlation functions really will dominate the thermodynamics in the high temperature
ideal gas limit. Understanding the nature of (quasi)particle constituents in the high temperature limit still is one of the major challenges in non-abelian gauge theories. The single number $f_M^{(0)}$ offers an interesting input to this discussion as it is sensitive to the mass of the thermodynamically relevant lightest magnetic excitations.

2 The 3-d magnetic free energy density

Simple analysis of the vacuum diagrams to 4 loop order in the 3-d SU(N) gauge theory with cut-off $\Lambda_M$ leads to the following dependence of $f_M^{(0)}$ on the magnetic separation scale:

$$f_M^{(0)} = T \left[ a_0 \Lambda_M^3 \ln \frac{\Lambda_M}{g_3^2} + a_1 \Lambda_M^3 + a_2 \Lambda_M^2 g_3^2 + a_3 \Lambda_M g_3^4 + \left( a_4 + a'_4 \ln \frac{\Lambda_M}{g_3^2} \right) g_3^6 \right] + \mathcal{O} \left( g_3^8 T \Lambda_M^2 \right). \quad (2.1)$$

It is worth to remark, that the appearance of the gauge coupling $g_3^2$ under the logarithms implicitly assumes the existence of an infrared scale proportional to it.

The complete free energy density does not depend on $\Lambda_M$. Therefore the coefficients of the terms diverging with $\Lambda_M$ should coincide with their perturbatively calculable values. Only in this way the $\Lambda_M$ dependence can cancel against the fully perturbatively determined $f_{NS,E}$. The finite part proportional to $a_4$, however, is fully non-perturbative, since all loop diagrams at and beyond four loops contribute to it. This is the quantity we would like to extract in a non-perturbative calculation on the lattice.

Starting from $\mathcal{O}(g^8)$, contributions to the free energy density are influenced also by the higher dimensional operators appearing in the effective magnetic theory. This implies, that the $\mathcal{O}(g^7)$ nonperturbative contribution reflects the $\mathcal{O}(g^3)$ correction of the $g_3^2 = g^2 T$ relation and is still calculable within the minimal effective theory.

The actual procedure for the determination of the coefficients in (2.1) amounts to measuring the coefficients of the weak coupling expansion of the internal energy density of the system. Since the temperature dependence appears in this theory
exclusively through $g_3^2$, one finds for the energy density

$$
\epsilon = \frac{T^2}{V} \frac{dg_3^2}{dT} \frac{d}{dg_3^2} \ln Z
\equiv -T^2 \frac{dg_3^2(T)}{dT} \epsilon_3.
\quad (2.2)
$$

The structure of the 3-dimensional energy density and its cut-off dependence is given by

$$
\epsilon_3 = -a_0\Lambda_M^3 g_3^{-2} + a_2\Lambda_M^2 + 2a_3\Lambda_M g_3^2 + (3a_4 - a_4')g_3^4 + 3a_4'g_3^4 \ln \frac{\Lambda_M}{g_3^2}.
\quad (2.3)
$$

The lattice regularization of the minimal 3-d SU(N) gauge theory is defined in the standard way:

$$
S_{LMQCD} = \beta_3 \sum_P \left( 1 - \frac{1}{2N} \text{Tr}(U_P + U_P^\dagger) \right), \quad \beta_3 = \frac{2N}{g_3^2a}.
\quad (2.4)
$$

where $U_P$ denotes the Wilson plaquette variable defined in terms of SU(N) valued variables $U_{x,i}$. The partition function is given by

$$
Z_{LMQCD} = \int \prod_{x,i} dU_{x,i} \exp(-S_{LMQCD}).
\quad (2.5)
$$

The internal energy $\epsilon_3$ and the plaquette expectation value

$$
\langle P \rangle = \left\langle 1 - \frac{1}{2N} \text{Tr}(U_P + U_P^\dagger) \right\rangle = -\frac{1}{V} \frac{d}{d\beta_3} \ln Z
\quad (2.6)
$$

can be simply related:

$$
\epsilon_3 = -3\Lambda_M^3 \frac{\beta_3}{g_3^2} \langle P \rangle.
\quad (2.7)
$$

Here $\Lambda_M \equiv a^{-1}$ is chosen to coincide with the cut-off of the lattice regularized theory.
3 Plaquette Expectation Value

The basic non-perturbative input from a lattice calculation is obtained through an evaluation of the plaquette expectation value for a SU(N) gauge theory. Its perturbative expansion for large $\beta$ has been calculated up to $\mathcal{O}(\beta^{-2})$ for arbitrary dimensions $d$ and on finite lattices \[13\].

$$\langle P \rangle \equiv \left\langle 1 - \frac{1}{2N} \text{Tr}(U_P + U_P^\dagger) \right\rangle = \sum_n c_{n,d} \beta^{-n}$$

$$= (N^2 - 1)I_d \beta^{-1} + \left(\frac{(2N^2 - 3)(N^2 - 1)}{6} I_d^2 + 4N^2(N^2 - 1)\alpha_d\right) \beta^{-2} + \mathcal{O}(\beta^{-3}),$$

where $I_d = \frac{1}{d}(1 - \frac{1}{1})$ and $\alpha_d$ is a numerical coefficient, which has a weak volume dependence. On an infinite lattice a direct evaluation gives $\alpha_4 = -0.000103$ for $d = 4$ and $\alpha_3 = -0.00095$ for $d = 3$. We note that at order $\beta^{-n}$ the dominant contribution to the expansion coefficients comes from diagrams which are proportional to $d^{-n}$. This allows to estimate also the expansion coefficient at $\mathcal{O}(\beta^{-3})$ which so far has only been evaluated in four dimensions \[14\]. These coefficients are $c_{3,4} = 0.143055$ in the case of SU(2) and 2.960467 in the case of SU(3). Multiplying with a factor $(4/3)^3$ one finds as an estimate in three dimensions

$$c_{3,3} \simeq \begin{cases} 0.34, & \text{SU(2)} \\ 7.02, & \text{SU(3)} \end{cases}.$$  

The possible appearance of a logarithmic $\beta$-dependence in the next order expansion coefficient, $c_{4,3}$, follows from combining (2.3) and (2.7). Its consequences will be discussed below.

As we are finally only interested in the finite part in $\epsilon_3$ our aim is to extract the coefficient $c_{4,3}$ in the expansion of $\langle P \rangle$. In order to do so we subtract the known part of the perturbative expansion from the numerical results for the plaquette expectation values obtained from a Monte Carlo simulation and determine the coefficients $c_{4,3}$ and $c_{3,3}$ from a fit to these differences. We have calculated plaquette expectation

\[a\]We use $\beta$ for the coupling in arbitrary dimensions.
Table 1: Plaquette expectation values and normalized differences as defined in Eq. (3.3) calculated on a $16^2 \times 64$ lattice.

| $\beta_3$ | $\langle P \rangle$ | $\Delta$ | # iterations |
|-----------|----------------------|----------|--------------|
| 6.0       | 0.1752315(78)        | 0.4593(17) | 70000        |
| 6.5       | 0.1609606(79)        | 0.4475(22) | 60000        |
| 7.0       | 0.1488726(47)        | 0.4413(16) | 50000        |
| 7.5       | 0.1384806(60)        | 0.4338(25) | 80000        |
| 8.0       | 0.1294547(50)        | 0.4276(26) | 100000       |
| 9.0       | 0.1145530(39)        | 0.4248(29) | 110000       |
| 10.0      | 0.1027277(40)        | 0.4121(40) | 90000        |
| 11.0      | 0.0931350(42)        | 0.4164(56) | 110000       |
| 12.0      | 0.0851753(35)        | 0.4060(61) | 100000       |
| 13.0      | 0.0784784(42)        | 0.4094(93) | 50000        |
| 14.0      | 0.0727477(30)        | 0.3817(83) | 50000        |

The remaining cut-off dependence results from a possible logarithmic cut-off dependence of the free energy density, i.e. the term proportional to $a'_4$ in Eq. (2.3). We thus expect $\Delta$ to depend on $\beta_3$ as follows:

$$\Delta = c_{3,3} + \left( c'_4 \ln \beta_3 + c_{4,3} \right) \frac{1}{\beta_3} + \frac{c_{5,3}}{\beta_3^2} + O(\beta_3^{-3}) .$$

(3.4)
| $\beta_3$ | $\langle P \rangle$     | $\Delta$       | # iterations |
|--------|-----------------|---------------|--------------|
| 12     | 0.2417305(77)   | 10.322(13)    | 10000        |
| 13     | 0.2211859(68)   | 9.942(15)     | 10000        |
| 14     | 0.2039402(64)   | 9.661(18)     | 10000        |
| 15     | 0.1892309(59)   | 9.423(20)     | 10000        |
| 16     | 0.1765317(56)   | 9.228(23)     | 10000        |
| 17     | 0.1654524(52)   | 9.074(26)     | 10000        |
| 18     | 0.1556935(48)   | 8.931(28)     | 10000        |
| 19     | 0.1470324(32)   | 8.808(22)     | 20000        |
| 20     | 0.1392884(31)   | 8.673(25)     | 20000        |
| 21     | 0.1323326(28)   | 8.618(26)     | 20100        |
| 22     | 0.1260367(27)   | 8.511(29)     | 20000        |
| 23     | 0.1203172(21)   | 8.426(26)     | 30300        |
| 24     | 0.1150972(20)   | 8.351(28)     | 30500        |
| 26     | 0.1059145(23)   | 8.242(40)     | 20000        |
| 28     | 0.0980936(21)   | 8.148(46)     | 19000        |
| 30     | 0.0913523(15)   | 8.085(40)     | 36000        |

Table 2: Plaquette expectation values and normalized differences as defined in Eq. (3.3) calculated on a $32^3$ lattice.
Figure 1: The subtracted plaquette expectation values $\Delta$ as defined in Eq. (3.3) for the SU(2) (a) and SU(3) (b) gauge theory. The solid curves show fits with $c'_4 = 0$ (case I) and the dashed line corresponds to $c'_4 \neq 0$ (case II). For the SU(3) case both fits coincide in the coupling range shown.

Our numerical data for the plaquette expectation values are limited to a $\beta$-range that varies by a factor of about 2.5. This is not large enough to be sensitive to the logarithmic term given above and at the same time allow a control over the subleading corrections proportional to $c_{5,3}$. We thus have analyzed the numerical data using two different fits. Since for the moment the 4-loop perturbative information on the coefficient of the logarithmic term as well as a direct evaluation of $c_{3,3}$ are missing we had to fit these. In order to see the sensitivity of $c_{4,3}$ to the variation of $c'_4$ we have investigated also another option, corresponding to an ideal decoupling of the magnetic fluctuations from the rest of the system: $c'_4 = 0$. It will become clear in the next section that the range of $c_4$ values obtained in this way puts already a rather stringent bound on the mass of magnetic excitations in the plasma phase.

We have fitted $\Delta$ with the ansätze

$$
\Delta = \begin{cases} 
  c_{3,3} + \frac{c_{4,3}}{\beta_3} + \frac{c_{5,3}}{\beta_3^2} & \text{case I} , \\
  c_{3,3} + \left( c'_4 \ln \beta_3 + c_{4,3} \right) \frac{1}{\beta_3} + \frac{c_{5,3}}{\beta_3^2} & \text{case II} . 
\end{cases}
$$

Results of these fits to the plaquette expectation values are summarized in Table 3. As our data for the $SU(2)$ gauge theory did not show any significant curvature in the coupling range explored by us we have fixed $c_{5,3}$ to be zero in both fits.
|       | SU(2)       | SU(3)       |
|-------|-------------|-------------|
|       | case I      | case II     | case I      | case II     |
| $c_{3,3}$ | 0.351(5)    | 0.45(5)     | 7.30(11)    | 8.11(20)    |
| $c_{4,3}$ | 0.635(37)   | 1.4(4)      | 15.2(3.6)   | 99(6)       |
| $a'_4$  | –           | -0.75(35)   | –           | -29.2(3.4)  |
| $a_{5,3}$ | –          | –           | 252(29)     | –           |

Table 3: Results of the fits using the two fitting functions defined in Eq. (3.3).

We note that the expansion coefficients for SU(2) gauge group are substantially smaller than in the case of SU(3). This is in agreement with the expected $N$-dependence for the expansion coefficients for different $SU(N)$ groups. In fact, the fitted coefficient $c_{3,3}$ agrees well with the estimate given in Eq. (3.2). This estimate does seem to favour the fit I.

While the coefficient $c_{4,3}$ only changes by a factor 2 in the case of $SU(2)$ for both fits, the variation is about twice as large in the case of $SU(3)$. However, it will become clear from the discussion in the following section that already our present estimates for $c_{4,3}$ are rather restrictive for the effective mass of magnetic excitations in the plasma phase, since this latter is only sensitive to its cubic root.

4 The $O(g^6)$ coefficient and its interpretation

Combining (2.7), (3.1), (3.3) and (3.4) for case II we relate the lattice and continuum coefficients in the following way:

\[
\begin{align*}
    a'_4 &= -\frac{1}{(2N)^3} c'_4, \\
    a_4 &= -\frac{1}{(2N)^3} \left( c_{4,3} + c'_4 (\log(2N) + \frac{1}{3}) \right).
\end{align*}
\]  

(4.1)

Case I corresponds to setting $c'_4 = 0$. The values of $c_{4,3}$ and $c'_4$ provide the input from the magnetic sector for the determination of the $O(g^6, g^6 \log g)$ terms in the weak coupling series of the free energy density.
The numerical values of the previous section lead to:

\[ a_4^{(I)} = \begin{cases} 
-(0.010 \pm 0.001), & \text{SU(2)} \\
-(0.07 \pm 0.02), & \text{SU(3)} 
\end{cases} \]  

(4.2)

and

\[ a_4^{(II)} = \begin{cases} 
-(0.002 \pm 0.016), & \text{SU(2)} \\
-(0.17 \pm 0.06), & \text{SU(3)} 
\end{cases} \]  

(4.3)

For the SU(2) group the estimated value for \( a_4 \) in case II is compatible with zero within errors. Clearly, this means that the accuracy of our simulation in the SU(2) case is yet insufficient to become sensitive to logarithmic corrections. We note, however, that in the SU(2) as well as in the SU(3) case the inclusion of a possible logarithmic term in the fit at most increases the value of \( a_4 \) by a factor of about 2.5.

Let us try to get some feeling for the magnitude of \( a_4 \) and its interpretation in terms of physical excitations. The 3-d theory represents the magnetic fluctuations of the high-T (3+1)-dimensional gauge theory. It is natural to expect that the \( O(g^6) \) contribution to the free energy can be viewed also as resulting from a weakly interacting (almost ideal) gas of some pseudo-particles of mass \( m_M \sim g^2 T \) and degeneracy \( N_D \). This is quite analogous to the electric sector, where the \( O(g^3) \) contribution to the free energy density is just the contribution of a massive free gas of pseudo-particles with mass \( m_E \sim gT \). The well-known 3-d, 1-loop vacuum energy has the cut-off independent part:

\[ \epsilon_3 = -\frac{3N_D m_M^2}{12\pi} \frac{dm_M}{dg_3^2} = -\frac{N_D m_M^3}{4\pi g_3^2}. \]  

(4.4)

In the last equality the proportionality of \( m_M \) to \( g_3^2 \) has been exploited.

The picture of an ideal gas of pseudo-particles cannot account for the logarithmic piece. If present at all this term would arise from higher order interactions of the effective degrees of freedom. The natural combination appearing in the argument of the logarithm is the ratio of the ultraviolet scale (the lattice constant) and a dynamically generated infrared scale \( m_G^{-1} \). This scale is proportional to \( g_3^2 \): \( m_G = \)}
This fixes the separation of the terms proportional to $g^6$ and to $g^6 \log g$, necessary for the quantitative investigation of the non-logarithmic piece:

$$\alpha_4^{\text{nonpert}} = -\frac{1}{(2N)^3} \left[ c_4 + c_4' \left( \frac{1}{3} + \ln(2Nc_G) \right) \right]. \quad (4.5)$$

The comparison of (4.4) to (2.7) via (3.1) leads to

$$m_M = \left( \frac{-12\pi \alpha_4^{\text{nonpert}}}{N_D} \right)^{1/3} g_3^2. \quad (4.6)$$

At least two simple cases can be put forward for the pseudo-particle excitations resulting from the high-$T$ magnetic modes. With the ansatz given by Eq. (4.6) they correspond to different choices for the degeneracy factor $N_D$. The extreme cases clearly are to set $N_D$ equal to the number of gluonic degrees of freedom or to unity. The first case may be considered as being the analogous interpretation to the electric sector, while the second assumes that there do exist only color singlet magnetic modes above $T_c$:

i) Adjoint SU(N) multiplet of screened gluons

Our previous determination of the magnetic screening mass of the gluons has been performed in Landau gauge \[5\]. For reasons of comparability we continue to use this gauge. Then the magnetic mass and the infrared regularization mass are naturally identified. The number of degrees of freedom in this gauge is then given by $N_D = 2(N^2 - 1)$. The solution $c_G$ of the equation (4.6) leads to the following mass estimates:

$$m_{\text{gluon}}^{(I)} = \begin{cases} 
(0.397 \pm 0.008) g_3^2, & \text{SU(2)} \\
(0.55 \pm 0.04) g_3^2, & \text{SU(3)}
\end{cases} \quad (4.7)$$

and

$$m_{\text{gluon}}^{(II)} = (0.78 \pm 0.29) g_3^2, \quad \text{SU(3)}. \quad (4.8)$$

This later value is still compatible within error bars with case I. Within the errors of the simulation the SU(2) result is perfectly compatible with the gluon mass calculations presented in Ref. \[5\]. We also note that the ratio of the SU(2) and SU(3) mass
values for case I is very close to $2/3$, which is expected from the 1-loop gap equation approach of [3], since the 1-loop diagrams contribute a quantity proportional to $N$ to the self energy.
ii) Scalar, SU(N) singlet glueballs ($N_D=1$)

One arrives at the following prediction for the glueball mass:

\[
m_{\text{singlet}}^{(I)} = \begin{cases} 
0.721 \pm 0.014 & \text{SU}(2), \\
1.38 \pm 0.11 & \text{SU}(3),
\end{cases}
\]  

and

\[
m_{\text{singlet}}^{(II)} = (2.08 \pm 0.67) g_3, \quad SU(3).
\]  

For the determination of $a_4$ in case II one requires as additional input the infrared regularization scale. For the SU(3) case we used the measured magnetic screening mass, determined for SU(2) \[^5\], and scaled it up by a factor 3/2, that is we used $c_G \sim 0.6$ in Eq. (4.5).

These values can be partly compared with existing numerical results on 3-d glueballs. For the SU(2) theory the lowest glueball mass \[^1\] calculated in numerical simulations $m_{\text{glueball}} = 6.34(6) g_3^2$ is clearly much larger than our estimate. Additional (higher) glueball states would make this discrepancy even more dramatic. Also the estimate of zero temperature glueball masses in four dimensions and estimates of finite temperature glueball screening masses in the (3+1)-d SU(3) gauge theory lead to larger values \[^2\], though the value found in case II within errors is on the edge of being compatible with the MC estimate.

The above comparisons seem to present a rather strong evidence against an interpretation of the thermodynamics of magnetic fluctuations in terms of singlet excitations with a mass similar to a typical zero temperature glueball mass. This analysis suggests that the relevant thermodynamic degrees of freedom at high temperatures in the magnetic sector of the non-Abelian plasma are screened elementary gluons.

5 Conclusions

We have performed a first analysis of the non-perturbative contribution to the $O(g^6)$ coefficient of the free energy of a finite temperature SU(N) gauge theory. We stress
that the analysis we have presented here is not complete. At the present stage we tried to investigate whether a lattice calculation can provide the necessary non-perturbative information that is needed to determine the complete $O(g^6)$ term in the free energy of an $SU(N)$ gauge theory. Our calculation shows that this is indeed feasible. Combined with a rather straightforward calculation of the expansion coefficient $c_{3,3}$ within lattice perturbation theory and an analytic determination of $c_4'$ a much better determination of $a_4$ can be achieved already on the basis of the numerical results presented here. The quality of the numerical input may also be further improved by exploring improved actions which reduce the size of the strongly cut-off dependent parts of observables like the expectation value of the Euclidean action. It is very encouraging, however, that already at the present stage we can distinguish between qualitatively different ideas on the nature of the high temperature magnetic excitations.

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