The Process $\bar{p}p \to e^-e^+$ with Polarized Initial Particles and Proton Form Factors in Time-like Region

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Abstract

The process $\bar{p}p \to e^-e^+$ is considered in the general case of polarized initial particles. A relation between the difference of the phases of the electromagnetic form factors $G_M$ and $G_E$ in the time-like region and measurable asymmetries is derived. It is shown that the moduli of the form factors can be determined from measurements of the total unpolarized cross section and of the integral asymmetry for longitudinally polarized (or transversely polarized) $\bar{p}$ and $p$. The behaviour of the proton form factors at high $q^2$ in the time-like region is also discussed. From the Phragmén-Lindelöf’s theorem it follows that the asymptotical behaviour of the form factors in the space-like and time-like regions must be the same. An analysis of experimental data in both regions based on perturbative QCD is presented.

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The charge and magnetic form factors of the nucleon, $G_E(q^2)$ and $G_M(q^2)$, are classical object of investigation. For a long time these fundamental quantities have been investigated in the region of space-like $q^2$. Starting from the seventies some informations on the electromagnetic form factors of the proton in the time-like region have been obtained. Recently rather accurate measurements of the proton form factors in the time-like region, from $q^2 = 4M^2$ up to $q^2 = 4.2 \, \text{GeV}^2$, have been done at LEAR [1,4]. Some informations on the electromagnetic form factor of the proton at high time-like $q^2$ were also obtained at Fermilab [3]. Most recent is the first experimental determination on the neutron form factor at $q^2 \approx 4.0 \, \text{GeV}^2$ [4].

There exist several QCD calculations of the electromagnetic form factors in the space-like region [5]. According to our knowledge there are no QCD-based calculations of the form factors in the time-like region. The phenomenological models which try to describe the behaviour of the form factors in both space-like and time-like regions are based on the vector meson dominance model [6–10]. However, even taking into account all known meson resonances (and one additional [10]) it is not possible to obtain a statistically acceptable description of all the existing experimental data. Let us remark that the steep decrease of the form factors near threshold in the time-like region discovered in Ref.[1] could be explained [11] by proton-antiproton interaction in the initial state.

The full understanding of nucleon electromagnetic form factors still remains a challenge for the theory. It is clear that any additional information about the form factors which could be obtained from experiment is very important.

Taking into account possible future developments of the experiments at LEAR [12] we analyze here which additional informations on the proton form factors in the time-like region can be obtained from the investigation of the process

$$\bar{p}p \rightarrow e^-e^+ \tag{1}$$

with a polarized proton target and/or a polarized antiproton beam. The possibility to polarize the antiproton beam at LEAR was considered in Ref.[13].

Let us consider the process (1) in the general case of polarized initial particles. The matrix element of the process in the one-photon approximation has the form

$$\langle f | S | i \rangle = -ie^2 \bar{\pi}(k)\gamma_\alpha u(-k') \frac{1}{q^2} \bar{\pi}(-p')\Gamma^\alpha u(p)(2\pi)^4\delta^4(k + k' - p - p'). \tag{2}$$

Here $p$ and $p'$ are the 4-momenta of the initial proton and antiproton, $k$ and $k'$ are the 4-momenta of the final electron and positron, $q = p + p'$ is the 4-momentum of the virtual photon and

$$\Gamma^\alpha = \gamma^\alpha F_1(q^2) - \frac{i}{2M} \sigma^{\alpha\beta}q_\beta F_2(q^2) \tag{3}$$

is the electromagnetic vertex of the nucleon ($M$ is the nucleon mass). The form factors $F_1(q^2)$ and $F_2(q^2)$ are connected with the magnetic $G_M(q^2)$ and charge $G_E(q^2)$ form factors.
of the nucleon by the relations

\[ G_M = F_1 + F_2, \]
\[ G_E = F_1 + \frac{q^2}{4M^2} F_2. \]

(4)

The cross section of process (1) in the general case of polarized initial particles is given by

\[ d\sigma = \frac{2\alpha^2}{\sqrt{q^2(q^2 - 4M^2)}} \left( k_{\alpha}k'_{\beta} - k \cdot k'g_{\alpha\beta} + k'_{\alpha}k_{\beta} \right) \frac{1}{q^4} \text{Tr} \left( \Gamma^\alpha \rho(p)\Gamma^\beta \rho(-p') \right) \delta^4(k+k'-p-p') \frac{d^3k}{k^0} \frac{d^3k'}{k'^0}. \]

(5)

Here

\[ \rho(p) = \frac{1}{2} (1 - \gamma_5 \xi) (\slashed{p} + M) \]

(6)

is the proton density matrix (\( \xi^\alpha \) is the polarization 4-vector of the proton) and

\[ \rho(-p') = -C\rho^T(p')C^{-1}, \]

(7)

where \( C \) is the charge-conjugation matrix and

\[ \rho(p') = \frac{1}{2} (1 - \gamma_5 \xi') (\slashed{p'} + M) \]

(8)

is the antiproton density matrix (\( \xi'^\alpha \) is the polarization 4-vector of the antiproton).

The polarization 4-vector of a particle in the system where its momentum is \( \vec{p} \) is connected with the polarization vector \( \vec{P} \) in its rest frame by a Lorentz boost

\[ \vec{\xi} = \vec{P} + \frac{\vec{p}(\vec{P} \cdot \vec{p})}{M(\vec{p}^0 + M)}, \]
\[ \xi^0 = \frac{\vec{\xi} \cdot \vec{p}}{M}. \]

(9)

With the help of Eqs. (4)–(9) we obtain the following expression for the cross section of
the process $\bar{p}p \rightarrow e^-e^+$ in the center of mass system \[ \left( \frac{d\sigma}{d\Omega} \right)_{\bar{p},p} = \frac{\alpha^2}{4\sqrt{q^2(q^2 - 4M^2)}} \left\{ (1 + \cos^2\vartheta) |G_M|^2 + \sin^2\vartheta \frac{4M^2}{q^2} |G_E|^2 \right. \\
+ \sin 2\vartheta \frac{2M}{\sqrt{q^2}} \text{Im}(G_M G_E^*) \left[ (\vec{P}_{\perp} \cdot \vec{n}) + (\vec{P}'_{\perp} \cdot \vec{n}) \right] \\
+ \sin 2\vartheta \frac{2M}{\sqrt{q^2}} \text{Re}(G_M G_E^*) \left[ (\vec{P}_{\perp} \cdot \vec{s}) P'_\parallel - (\vec{P}'_{\perp} \cdot \vec{s}) P_\parallel \right] \\
- \left[ (1 + \cos^2\vartheta) |G_M|^2 - \sin^2\vartheta \frac{4M^2}{q^2} |G_E|^2 \right] P_\parallel P'_\parallel \\
+ 2|G_M|^2 \sin^2\vartheta \left( \vec{P}_{\perp} \cdot \vec{s} \right) \left( \vec{P}'_{\perp} \cdot \vec{s} \right) \\
+ \sin^2\vartheta \left[ \frac{4M^2}{q^2} |G_E|^2 - |G_M|^2 \right] \vec{P}_{\perp} \cdot \vec{P}'_{\perp} \right\}. \right. \] \hfill (10)

In deriving Eq.(10) we expressed the polarization vectors of the antiproton and proton (in their rest frame) as sums of their longitudinal and transverse components

\[ \vec{P}' = \vec{P}'_\parallel + \vec{P}'_{\perp}, \]
\[ \vec{P} = \vec{P}_\parallel + \vec{P}_{\perp}, \] \hfill (11)

where $\vec{P}'_\parallel = P'_\parallel \vec{m}$ and $\vec{P}_\parallel = P_\parallel (-\vec{m})$. Here $\vec{m}$ is the unit vector in the direction of the momentum of the antiproton in the c.m.s. Further, in Eq.(11) $\vartheta$ is the angle between the momenta of the antiproton and the electron, $\vec{n} = \vec{m} \times \vec{k} / |\vec{m} \times \vec{k}|$ is the unit vector orthogonal to the reaction plane and $\vec{s} = \vec{n} \times \vec{m}$ is the unit vector in the reaction plane orthogonal to $\vec{m}$.

The nucleon form factors in the time-like region are complex. In the case of unpolarized initial particles the cross section depends only on the squared moduli $|G_M|^2$ and $|G_E|^2$. As it can be seen from Eq.(10), the study of process (I) with polarized initial particles could allow to obtain informations also about the phase difference $\chi = \chi_M - \chi_E$, where $\chi_M = \text{Arg} G_M$ and $\chi_E = \text{Arg} G_E$, which is an important characteristic of the form factors in the time-like region.

The value of $\sin \chi$ can be obtained from measurements of the cross section of process (I) with an unpolarized antiproton beam and a polarized proton target (or a polarized antiproton beam and an unpolarized proton target). If the target polarization is orthogonal to the beam

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1 The cross section for process (I) in the case of polarized initial particles was calculated in Ref.[14]. We present here the expression for this cross section in a different form which is more suitable for our discussion. Notice that in Eq.(10) the electron mass has been neglected.
direction, from Eq. (10) we obtain the asymmetry

\[
A(\vartheta, \varphi) = \begin{bmatrix}
\left( \frac{d\sigma}{d\Omega} \right)_{0,\vec{P}_\perp} - \left( \frac{d\sigma}{d\Omega} \right)_{0,-\vec{P}_\perp} \\
\left( \frac{d\sigma}{d\Omega} \right)_{0,\vec{P}_\perp} + \left( \frac{d\sigma}{d\Omega} \right)_{0,-\vec{P}_\perp}
\end{bmatrix} \frac{1}{|P_\perp|} \sin 2\vartheta \cos \varphi \frac{2M}{\sqrt{q^2}} |G_M| |G_E| \sin \chi
\]

\[
= \frac{1}{(1 + \cos^2 \vartheta)} |G_M|^2 + \frac{4M^2}{q^2} \sin^2 \vartheta |G_E|^2,
\]

where \( \varphi \) is the angle between \( \vec{P}_\perp \) and \( \vec{n} \). Let us notice that the asymmetry \( A(\vartheta, \varphi) \) must vanish at the threshold \( q^2 = 4M^2 \), where \( G_M = G_E \). Notice also that in the case of unpolarized proton target and polarized antiproton beam the asymmetry is the same as in the case of polarized target and unpolarized beam. This is a consequence of invariance under charge conjugation.

Information about \( \cos \chi \) can be obtained from measurements of the cross section of process (1) with a transversely polarized beam and a longitudinally polarized target (or a longitudinally polarized beam and a transversely polarized target). If the antiproton polarization vector is \( \vec{P}'_\perp \) and the proton polarization vector is \( -P_\parallel \vec{m} \) (or \( P_\parallel \vec{m} \)), for the asymmetry we have

\[
A_{||}(\vartheta', \varphi') = \begin{bmatrix}
\left( \frac{d\sigma}{d\Omega} \right)_{\vec{P}'_\perp - P_\parallel \vec{m}} - \left( \frac{d\sigma}{d\Omega} \right)_{\vec{P}'_\perp P_\parallel \vec{m}} \\
\left( \frac{d\sigma}{d\Omega} \right)_{\vec{P}'_\perp - P_\parallel \vec{m}} + \left( \frac{d\sigma}{d\Omega} \right)_{\vec{P}'_\perp P_\parallel \vec{m}}
\end{bmatrix} \frac{1}{|\vec{P}'_\perp| |P_\parallel|} \sin 2\vartheta' \sin \varphi' \frac{2M}{\sqrt{q^2}} |G_M| |G_E| \cos \chi
\]

\[
= \frac{1}{(1 + \cos^2 \vartheta')} |G_M|^2 + \frac{4M^2}{q^2} \sin^2 \vartheta' |G_E|^2,
\]

where \( \varphi' \) is the angle between \( \vec{P}'_\perp \) and \( \vec{n} \).

It is obvious that the differential asymmetry \( A(\vartheta, \varphi) \) is maximal if the target polarization vector \( \vec{P}_\perp \) is directed along \( \vec{n} \) (i.e. \( \varphi = 0 \)). Analogously the asymmetry \( A_{||}(\vartheta, \varphi') \) is maximal if the beam polarization vector \( \vec{P}'_\perp \) is directed along \( \vec{s} \) (i.e. \( \varphi' = \pi/2 \)). In this case, from Eqs. (12) and (13) we have

\[
\tan \chi = -\frac{A(\vartheta, 0)}{A_{||}(\vartheta, \pi/2)},
\]

which is a relation between the phase difference \( \chi \) and measurable asymmetries.
Up to now we have considered differential asymmetries. It is not difficult to generalize Eq. (14) for integral asymmetries. It is obvious that the terms \((\vec{P}_\perp \cdot \vec{n})\) and \((\vec{P}_\perp' \cdot \vec{s})\) in expression (10) for the cross section vanish after integration over the total solid angle. However, we can consider the asymmetries integrated over part of the solid angle. Let us consider the process (1) with an unpolarized beam and a target with polarization \(\vec{P}_\perp\) orthogonal to the momentum of the antiproton and integrate the cross section over the angle \(\varphi\) between \(\vec{P}_\perp\) and \(\vec{n}\) from \(-\pi/2\) to \(\pi/2\) and over the angle \(\vartheta\) from 0 to \(\pi/2\). From Eq. (10), for the integral asymmetry we have

\[
A = \frac{4M}{\pi \sqrt{q^2}} |G_M||G_E| \sin \chi \cdot \frac{1}{2|G_M|^2 + \frac{4M^2}{q^2}|G_E|^2}.
\]  

(15)

Now let us consider process (1) with a longitudinally polarized proton target with polarization \(-P_\parallel \vec{m}\) (\(P_\parallel \vec{m}\)), and a transversely polarized antiproton beam with polarization \(\vec{P}_\perp\) and integrate the differential cross section over the angle \(\varphi'\) between \(\vec{P}_\perp\) and \(\vec{n}\) from 0 to \(\pi\) and over the angle \(\vartheta\) from 0 to \(\pi/2\). For the asymmetry we have

\[
A_{\perp;\parallel} = -\frac{4M}{\pi \sqrt{q^2}} |G_M||G_E| \cos \chi \cdot \frac{1}{2|G_M|^2 + \frac{4M^2}{q^2}|G_E|^2}.
\]  

(16)

From Eqs. (15) and (16) we obtain the following relation between the phase difference \(\chi\) and the integral asymmetries

\[
\tan \chi = -\frac{A}{A_{\perp;\parallel}}.
\]  

(17)

Thus measurements of the integral asymmetries \(A\) and \(A_{\perp;\parallel}\) would allow us to determine the phase difference \(\chi\) directly from experimental data. Notice that the phase difference \(\chi\) can be determined unambiguously with this method (in addition to Eq. (17) it is necessary to take into account the sign of \(A\) or \(A_{\perp;\parallel}\)). Let us stress that the knowledge of the moduli \(|G_M|\) and \(|G_E|\) is not necessary in order to obtain \(\chi\) with the help of Eq. (17).

As it can be seen from Eq. (10) if both the initial particles have longitudinal or transverse polarizations, then the integral cross section depends only on the moduli of the form factors. Let us consider first the case of longitudinal polarizations. From Eq. (10) for the asymmetry
integrated over the total solid angle we have

\[ A_{∥∥} = \frac{1}{P_{∥} P_{∥}} \left[ \frac{\sigma_{P′} \bar{m} - P_{∥} \bar{m} - \sigma_{P′} \bar{m} - P_{∥} \bar{m}}{\sigma_{P′} \bar{m} + \sigma_{P′} \bar{m} + P_{∥} \bar{m} + P_{∥} \bar{m}} \right] \]

\[ = \frac{2|G_M|^2 - \frac{4M^2}{q^2} |G_E|^2}{2|G_M|^2 + \frac{4M^2}{q^2} |G_E|^2}. \quad (18) \]

This asymmetry depends only on $|G_M|^2$ and $|G_E|^2$. The value of $|G_M|^2$ and $|G_E|^2$ can be determined from the measurement of the $\cos^2 \vartheta$ dependence of the differential cross section of process (1) in the case of unpolarized initial particles. However, this method requires a rather high statistical accuracy of the data. Here we discuss an alternative method for the determination of $|G_M|^2$ and $|G_E|^2$. If the asymmetry $A_{∥∥}$ is measured, these quantities can be determined directly from the value of $A_{∥∥}$ and the value of the total cross section of process (1) with unpolarized initial particles

\[ \sigma_0 = \bar{\sigma} \left[ 2|G_M|^2 + \frac{4M^2}{q^2} |G_E|^2 \right], \quad (19) \]

where

\[ \bar{\sigma} = \frac{2\pi \alpha^2}{3\sqrt{q^2 (q^2 - 4M^2)}}. \quad (20) \]

Indeed from Eqs.(18) and (19) we have

\[ |G_M|^2 = \frac{1}{4} \left( 1 - A_{∥∥} \right) \frac{\sigma_0}{\bar{\sigma}}, \]

\[ |G_E|^2 = \frac{q^2}{8M^2} \left( 1 + A_{∥∥} \right) \frac{\sigma_0}{\bar{\sigma}}. \quad (21) \]

As can be seen from Eq.(19), the cross section of process (1) depends only on $|G_M|^2$ and $|G_E|^2$ also if both beam and target have transverse polarizations. In this case, assuming that the polarizations of the proton and antiproton are parallel (antiparallel), for the integral asymmetry we have

\[ A_{⊥⊥} = \frac{4M^2}{q^2} \frac{|G_E|^2}{2|G_M|^2 + \frac{4M^2}{q^2} |G_E|^2}. \quad (22) \]

With the help of Eqs.(19) and (22) we obtain

\[ |G_M|^2 = \frac{1}{2} \left( 1 - A_{⊥⊥} \right) \frac{\sigma_0}{\bar{\sigma}}, \]

\[ |G_E|^2 = \frac{q^2}{4M^2} A_{⊥⊥} \frac{\sigma_0}{\bar{\sigma}}. \quad (23) \]
So an investigation of the process $\bar{p}p \to e^-e^+$ in the case of longitudinally polarized or transversely polarized beam and target will allow us to determine the moduli of the charge and magnetic proton form factors from measurements of the integral cross section.

The experiments discussed here without any doubt are very difficult. However, taking into account the importance of the informations which could be obtained from the investigation of polarization effects in process (1), we think that it is appropriate to discuss the possibility of their measurement.

In conclusion we would like to make the following remarks

1. As it can be seen from Eq.(10), in the cross section of process (1) with unpolarized particles $(\frac{d\sigma}{d\Omega})_{0;0}$ the value of $|G_E|^2$ is multiplied by $4M^2/q^2$. This means that at high $q^2$ the main contribution to $(\frac{d\sigma}{d\Omega})_{0;0}$ comes from $|G_M|^2$. On the other hand, from Eq.(16) it follows that at high $q^2$

$$A_{\perp;\parallel} \simeq -\frac{2M}{\pi \sqrt{q^2}} \frac{|G_E|}{|G_M|} \cos \chi .$$

So measurements of the asymmetry $A_{\perp;\parallel}$ allow to determine the ratio $|G_E|/|G_M|$ at high $q^2$ (if the phase $\chi$ is known from measurements of the asymmetries $A$ and $A_{\perp;\parallel}$).

2. In order to illustrate the possible behaviour of the considered asymmetries, we calculate them using the parametrizations of the form factors proposed in Ref.[8] and Ref.[10]. These parametrizations are based on the vector meson dominance model. The parameters were determined in Ref.[8] and Ref.[10] from a fit of existing experimental data. The results of our calculations of the asymmetries $A$ and $A_{\perp;\parallel}$ are presented in Fig.1 and Fig.2, respectively. It can be seen from Fig.1 and Fig.2 that different models predict quite different behaviours of the asymmetries.

3. Since $G_E(4M^2) = G_M(4M^2)$, it is clear that the asymmetry $A$ vanishes at the threshold. It is possible to show that the asymmetry $A$ goes to zero at $q^2 \to \infty$. In fact the electromagnetic form factors $G_{E,M}(q^2)$ are limiting values of the functions $G_{E,M}(z)$

$$G_{E,M}(q^2) = \lim_{\epsilon \to 0^+} G_{E,M}(q^2 + i\epsilon)$$

which are analytical in the upper half of the complex $z$ plane and increase at infinity not faster than a power of $z$. We can apply [16] to the form factors the Phragmén-Lindelöf’s theorem [17] which was used [18,19] to proof the Pomeranchuk theorem and its generalizations [16]. From this theorem it follows that the form factors have the same asymptotical behaviour in the space-like and time-like regions. In the space-like

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2 Let us notice that the proton polarization in the process $e^-e^+ \to \bar{p}p$ was discussed in Ref.[15].
region the form factors are real. This means that the form factors are real in the time-like region at asymptotically high $q^2$ and from Eq.(12) it follows that $A \to 0$ at $q^2 \to \infty$.

4. Let us discuss in some more detail the asymptotical behaviour of the electromagnetic form factors of the nucleon in the time-like region. In accordance with the quark counting rule [20,21], which is based on the scaling hypothesis and dimensional arguments, at high $|q^2|$ the form factors of the nucleon (system of three quarks) behave as

$$G_M(q^2) \sim F_1(q^2) \sim \frac{1}{q^4}.$$  \tag{26}

The quark-gluon interaction leads to violation of scaling and additional logarithmic $q^2$ dependence of the form factors. Let us write in the space-like region ($q^2 < 0$)

$$G_M(q^2) \sim \frac{C_s}{q^4} \Phi(q^2),$$  \tag{27}

where $\mu_p = 2.79$ is the proton magnetic moment in nuclear magnetons. From the leading order perturbative QCD it follows that $\Phi(q^2)$ is given by [22]

$$\Phi(q^2) = \alpha_s^2(-q^2) \left[ \ln \left( -\frac{q^2}{\Lambda^2} \right) \right]^{-4/3\beta},$$  \tag{28}

where

$$\alpha_s(-q^2) = \frac{4\pi}{\beta \ln \left( -\frac{q^2}{\Lambda^2} \right)}, \quad \beta = 11 - \frac{2}{3} n_f.$$  \tag{29}

$n_f$ is the number of flavours and $\Lambda$ is the QCD scale parameter. The value of the constant $C_s$ is determined by the wave function of the nucleon [22–24]. The Phragmén-Lindelöf’s theorem implies that in the time-like region $(q^2 > 0)$ the form factors have the following asymptotical behaviour

$$G_M(q^2) \sim \frac{C_t}{q^4} \Phi(q^2)$$  \tag{30}

with

$$C_t = C_s.$$  \tag{31}

Let us compare the behaviour of the form factors at large space-like and time-like momentum transfer. In a recent SLAC experiment [25] the elastic electron-proton cross section was measured in a wide range of momentum transfer, from $-q^2 = 2.9 \text{GeV}^2$ to $-q^2 = 31.3 \text{GeV}^2$. From these measurements the values of the form factor $G_M(q^2)$ at high $-q^2$ can be extracted (the contribution to the cross section of the form factor $G_E(q^2)$ at high $-q^2$ is small and can be neglected). In Fig.3 the experimental points for
$|G_M|q^4/\mu_p$ at $-q^2 > 10 \text{GeV}^2$ are plotted together with curves obtained from fits of the data using Eq. (27). With $\Lambda = 100 \text{MeV}$ we obtain $C_s = 12.3 \pm 0.2 \text{GeV}^4$ ($\chi^2/\text{NDF} = 4.6/5$) and with $\Lambda = 200 \text{MeV}$ we get $C_s = 7.8 \pm 0.1 \text{GeV}^4$ ($\chi^2/\text{NDF} = 10.2/5$). Let us notice that the quality of the fit depends rather strongly on the value of $\Lambda$ (smaller values of $\Lambda$ are preferable).

The cross section of the process $\bar{p}p \rightarrow e^-e^+$ at high $q^2$ ($q^2 = 8.9, 12.4, 13.0 \text{GeV}^2$) was measured recently in a Fermilab experiment [3]. In Fig. 3 we have plotted the values of $|G_M|q^4/\mu_p$ obtained from this experiment. There are also shown two fits of these data made using Eq. (30) with $\Lambda = 100 \text{MeV}$ ($C_t = 30.9_{-1.8}^{+4.1} \text{GeV}^4$, $\chi^2/\text{NDF} = 0.29/1$) and $\Lambda = 200 \text{MeV}$ ($C_t = 21.2_{-3.3}^{+2.8} \text{GeV}^4$, $\chi^2/\text{NDF} = 0.29/1$). We have taken into account the imaginary part of the form factor in the time like region that arises due to $\ln \left(-\frac{q^2}{\Lambda^2}\right) = \ln \left(\frac{q^2}{\Lambda^2}\right) + i\pi$. Thus the experimental data in the space-like as well as in the time-like regions of $q^2$ are described by expressions (27) and (30), respectively. The accuracy of the data in the time-like region is much worse than that in the space-like region. As a consequence, the corresponding accuracy of the determination of the constant $C_t$ in the time-like region is much worse than that of the constant $C_s$ in the space-like region. However, the average values of $C_t$ and $C_s$ are so different that, even with such a low accuracy in the determination of $C_t$, we can conclude that these constants are different ($C_t$ is more than $3\sigma$ higher than $C_s$). From our point of view this difference means that the range of high $q^2$ values investigated in present experiments is not asymptotic and the Phragmén-Lindelöf’s theorem does not apply in this range. Nonperturbative effects [26], nonleading log corrections and other effects could be important in this region.

Further investigations of the processes $ep \rightarrow ep$ and $\bar{p}p \rightarrow e^-e^+$ and comparisons of the behaviour of the form factors in the regions of space-like and time-like high $q^2$ seems to us very important from the point of view of the possibility to test the perturbative QCD prediction of the asymptotical behaviour of the proton form factor in a way which does not dependent on the choice of the nucleon wave function.

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3 We are grateful to Dr. S. Brodsky for calling our attention to the possible numerical importance of the imaginary part of the form factor in the range of values of $q^2$ presently available. Notice that at $q^2 \simeq 10 \text{GeV}^2$ the relative contribution to $|G_M|$ of the imaginary part of $G_M$ is about 20%.
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Figure Captions

Fig.1 Integral asymmetry $A$ for the case of unpolarized antiproton beam and polarized proton target as a function of $q^2$ in different models: BHTZ (Ref.[8]), BDDS21 (Ref.[10], Table 2, Column 1), BDDS31 (Ref.[10], Table 3, Column 1), BDDS32 (Ref.[10], Table 3, Column 2).

Fig.2 Integral asymmetry $A_{\perp\parallel}$ for the case of transversely polarized antiproton beam and longitudinally polarized proton target as a function of $q^2$ in different models: BHTZ (Ref.[8]), BDDS21 (Ref.[10], Table 2, Column 1), BDDS31 (Ref.[10], Table 3, Column 1), BDDS32 (Ref.[10], Table 3, Column 2).

Fig.3 $|G_M|q^4/\mu_p$ for $|q^2| > 10$ GeV$^2$ in the space-like and time-like regions. The experimental points are taken from Ref.[25] and Ref.[3]. The curves are obtained from fits of the data using Eq.(27) with $\Lambda = 100$ MeV and $\Lambda = 200$ MeV.
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Figure 1
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Figure 2
Figure 3