Inexact proximal algorithms in models of Behavioral Sciences

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March, 27, 2014

The “Habitual domain” (HD) approach and the “Variational rationality” (VR) approach belong to the same strongly interdisciplinary and very dispersed area of research: human stability and change dynamics (see Soubeyran, 2009, 2010, for an extended survey), including physiological, physical, psychological and strategic aspects, in Psychology, Economics, Management Sciences, Decision theory, Game theory, Sociology, Philosophy, Artificial Intelligence, . . . .

These two approaches are complementary. They have strong similarities and strong differences. They focus attention on both similar and different stay and change problems. They use different concepts and different mathematical tools. When they use similar concepts (a lot), they often have different meaning. We can compare them with respect to the problems and topics they consider, the behavioral principles they use, the concepts they modelize, the mathematical tools they use, and their results.

1 Problems and topics

The “Habitual domain” (HD) theory. For a survey see Yu, Chen (2010).

Among other points, this approach focus attention on:

A) behavioral habitual domain principles (Yu, 1991, Yu, Chen, 2010);

B) habit formation and the dynamic of activation levels and attention efforts, following a succession of periods (Chan, Yu, 1985);

C) problem solving, modelized as a given competency set expansion (CSE) problem within a given period (see Shi, Yu, 1999). It represents the most developped part of the HD theory, which includes a lot of related papers using different mathematical tools to find the optimal expansion set (see among others, Yu, Zhang, 1990, Li, Yu, 1994, Shi, Yu 1996, Tzeng and alii, 1998, Li, Chiang, Yu, 2000, . . . .);

D) innovation dynamics (ID) consider that competency sets are dynamic and can change over time. Then, it examines a succession of competency set expansion problems which refer to decision making with change-
able spaces (DMCS) problems. Innovation dynamics examine also cover-
discover (CD) problems (Larbani, Yu, 2012). A covering problem is a com-
petency set expansion (CSE) problem. It refers to “how to transform a
given competence set CS* into a set that contains a targeted competence
set CS”. Discovering refers to the following problem. “Given a compe-
tence set, what is the best way to make use of it to solve unsolved prob-
lems or to create value? The process underlying this problem solving or
value creation involves discovering. A discovering process can be defined
as identifying how to use available tangible and intangible skills, compet-
tences, and resources to solve an unsolved problem or to produce new
ideas, concepts, products, or services that satisfy some newly-emerging
needs of people. ....Discovering contributes to reducing the charge level or
relieving the pain of some targeted people” (Larbani, Yu, 2012);

E) Decision making with changeable spaces (DMCS) games with chang-
ing minds (Yu, Larbani, 2009, Larbani, Yu, 2009, 2009b, 2011, 2012).

The “Variational rationality” (VR) approach For an extended presenta-
tion of this model, see Soubeyran (2009, 2010). This approach focus attention
on a single very general problem, the famous self regulation problem, which
includes a lot of different aspects in different disciplines, using different termi-
nologies. Among other more specific topics, it considers:

A') behavioral stability and change principles;

B') habit formation and break (HFB) problems at the individual level, as well
as routine formation and break (RFB) problems at the organizational
level;

C') exploration-exploitation (EEL) learning dynamics;

D') adaptive self regulation (SR) problems (goal setting, goal striving, goal
revision and goal pursuit) which belong to the general class of decision
making with changeable spaces, moving goals and variable preferences
(DMCSMG) problems. Such self regulation processes modelize the inter-
related dynamics of motivational (desiring and willing), cognitive (percep-
tion and knowledge acquisition) and emotional aspects of human behavior.
Bounded rational satisficing processes represent an important application;

E') variational games (VG) with self regulating agents (Attouch and alii, 2007,
Attouch and alii, 2008, Flores-Bazán, Luc, Soubeyran, 2012, Flam and alii,
2012, Cruz Neto and alii, 2013);

F') applications of self regulation problems to variational analysis. Vari-
tional analysis and most of the related algorithms: variational principles
(Luc, Soubeyran, 2013) exact and inexact alternating algorithms (Attouch
and alii, 2007, Attouch and alii, 2008, Attouch and alii, 2010), proxi-
mal algorithms with Bregman and quasi distances (Cruz Neto and alii,
2010, Moreno and alii, 2012), exact and inexact proximal algorithms on manifolds (Cruz Neto and alii, 2013), multiobjective proximal algorithms (Bento, Cruz Neto, Soubeyran, 2013), local search inexact proximal algorithms (Attouch, Soubeyran, 2010), inexact descent methods (Bento and alii, 2013), equilibrium problems (Bento and alii, 2013), and dual equilibrium problems (Moreno and alii, 2012), variational inequalities (Attouch, Soubeyran, 2006, Luc and alii, 2010), trust region methods (Villacorta and alii, 2013), Tabu search algorithms (Martinez-Legaz, Soubeyran, 2007), sequential decision making (Martinez-Legaz, Soubeyran, 2013),... use the same variational principles as VR do. These algorithms can be seen as reduced forms of the VR self regulation model.

2 Behavioral principles

**HD behavioral principles** HD theory is based on eight behavioral principles (Yu, Chen, 2010). Four hypotheses capture the basic workings of the brain. Circuit pattern hypothesis H1, Unlimited capacity hypothesis H2, Efficient restructuring hypothesis H3 and Analogy/Association hypothesis H4. Four other hypotheses summarize how our mind works. Goal setting and State evaluation hypothesis H5 (a basic function of our mind), Charge structure and attention allocation hypothesis H6 (how we allocate our attention to various events), Discharge hypothesis H7 (a least resistance principle that humans use to release their charges) and the Information internal and external inputs hypothesis H8.

**VR behavioral principles** VR approach is based on, at least, nine behavioral principles (Soubeyran, 2009, 2010):

K1) agents are bounded rational (Simon, 1955). They do not optimize, except for simple problems. As soon as a problem is complex, they satisfice within a consideration set. They consider, each step, only a limited subset of alternatives related to the behavioral chain “means and capabilities — actions — performances —— goals —— desires”. This consideration set changes from one period to the next;

K2) human activities follow a succession of temporary stays and changes;

K3) each period, the agent is satisfied, or remain unsatisfied, relative to different domains of his life;

K4) the agent problem, each step, is to choose between to temporary stay or to change (“should I stay, should I go?”);

K5) the agent balances, each step, between advantages and inconvenients to change or to stay, and more generally, motivation and resistance to change or to stay. They consider worthwhile temporary stays or changes;
K6) an agent is engaged in a “stop and go” stays and changes course pursuit, between setting, each period, the same old desired ends or (and) new ones, and finding related feasible means to reach or approach each of them;

K7) an agent is partially able to self regulate this possibly interrupted course pursuit. He can set goals, strive for them, and revise them (having reach some goals, to reset the same goals is a possibility);

K8) experience matters and partially determines the current behavioral chain. Then, almost all things, including preferences, can change. There is a course pursuit between changing preferences, actions, capabilities and beliefs. A current action is chosen on the basis of the current preference, which changes the current preference which, in turn changes the choice of the future action which in turn...;

K9) Goal pursuit stops when the agent reaches a behavioral trap, which is worthwhile to reach, starting from the initial position, and where he prefers to stay than to move again.

Comparisons  Circuit pattern hypothesis H1 refers to our resistance to change concept (repetitions of thoughts, ideas and actions reinforce circuits, making them difficult to abandon). Unlimited capacity hypothesis H2 supposes that “practically, every normal brain has the capacity to encode and store all thoughts, concepts and messages that one intends to”. Efficient Restructuring hypothesis H3 supposes that “the encoded thoughts, concepts and messages H1 are organized and stored systematically as data bases for efficient retrieving. Furthermore, according to the dictation of attention they are continuously restructured so that relevant ones can be efficiently retrieved to release charges”. Analogy/Association hypothesis H4 supposes that “the perception of new events, subjects or ideas can be learned primarily by analogy and/or association with what is already known. When faced with a new event, subject or idea, the brain first investigates its features and attributes in order to establish a relationship with what is already known by analogy and/or association. Once the right relationship has been established, the whole of the past knowledge (preexisting memory structure) is automatically brought to bear on the interpretation and understanding of the new event, subject or idea”. All these hypothesis are in accordance with bounded rationality (Simon, 1955) and to our resistance to change concept (see K5) where new knowledge and past knowledge are not mixed immediately. For example, as a lot of experiences on habit formation and break have shown in Psychology, many attitudes and beliefs temporary resist to change.

The goal setting and state evaluation hypothesis H5 supposes that “each one of us has a set of goal functions and for each goal function we have an ideal state or equilibrium point to reach and maintain (goal setting). We continuously monitor, consciously or subconsciously, where we are relative to the ideal state or equilibrium point (state evaluation). Goal setting and state evaluation are dynamic, interactive, and are subject to physiological forces, self-suggestion,
external information forces, current data bank (memory) and information processing capacity”. The VR approach supposes (see K6) variable ideal states (moving aspirations and desires) and an adaptive course pursuit between where we are (uncomfortable moving status quo) and where we want to be (moving aspirations, or desirable ends).

The charge structures and attention allocation hypothesis H6 supposes that “each event is related to a set of goal functions. When there is an unfavorable deviation of the perceived value from the ideal, each goal function will produce various levels of charge. The totality of the charges by all goal functions is called the charge structure and it can change dynamically. At any point in time, our attention will be paid to the event which has the most influence on our charge structure”. Discharge hypothesis H7 supposes that “to release charges, we tend to select the action which yields the lowest remaining charge (the remaining charge is the resistance to the total discharge) and this is called the least resistance principle”. The VR approach agrees with these two hypothesis H6 and H7 which are very similar to the famous “discrepancy reduction” principle in Psychology. They represent the first side of a self regulation process (see K6). The other side is a “discrepancy production” process (goal setting, goal revision; goal pursuit).

The information Input hypothesis H8 supposes that “humans have innate needs to gather external information. Unless attention is paid, external information inputs may not be processed”. The VR approach agrees with this hypothesis, which can be related to the general concept of consideration set (see K1).

3 Concepts, variables and parameters

**HD concepts.** To save space, let us list these concepts with respect to only three different HD problems:

i) Stabilization of an habitual domain problem. Following an infinite sequence of periods (a transition, in the parlance of the VR approach), the agent considers, each period \( t \), his potential domain at time \( t \), his actual domain at time \( t \), his activation probability at time \( t \), his reachable domain at time \( t \), and his attention and activation levels at time \( t \);

ii) Competency set expansion problem. Staying within a given period of time, defined as a single change in the VR approach (the case of a transition, where the agent follows an infinite sequence of periods is not examined), the agent considers his given competency set, true or perceived, his given acquired skill set, true or perceived, a given table of costs needed for acquiring a given skill directly from another given skill, his cost of acquiring a new skill, a chosen optimal expansion process, and decision traps,…;

iii) Decision making in changeable spaces (DMCS) problems. They can be represented by a time dependent list including, each time \( t \), i) a list of
changing or changeable decision elements (a subset of alternatives, a subset of criteria, an outcome measured in term of the criteria, and a preference), and ii) a list of changing or changeable decision environmental facets (information inputs, an habitual domain, a subset of other involved decision making agents, and a subset of unknowns). All the elements of this list can change or can be changed with time.

Innovation problems, like successions of competency set expansion problems and cover-discover problems, as well as DMCS game problems are DMCS very important problems which will not be examined here, because our paper considers only an agent.

VR concepts. They are all relative to the self regulation problem. More precisely, given a transition (an infinite sequence of periods), the agent follows a succession of single temporary stays or changes. Given his current experience which changes with time, he considers, each period, using his current changeable consideration set, his current changeable aspiration gap, defined as the current discrepancy between where he is, the statu quo (defined as his most recent past action and situation), and where he wants to be (which represents his current changeable aspiration level), his current changeable goals, which can help to fill a portion of the current aspiration gap and can help to approach his current aspiration level, a future chosen action to be done, repeating the old action, or doing a new action, the regeneration of old capabilities to be able to repeat the old action, the deletion of old capabilities and the acquisition of new capabilities and means to be able to do the new action, his expected new performance and payoff generated by this action, his expected costs to be able to stay and his expected costs to be able to change, his expected advantages to change, inconvenients to change, motivation to change, resistance to change. The distinction can be made between an ex ante perceived and an ex post realized concept, variable or parameter. . . . All this elements can change or can be changed over time. Then, worthwhile changes and variational traps are the keys variational rationality concepts.

The same concepts can be used to examine the functioning of variational games (for a reduced form, see inertial games, Attouch, Redont, Soubeyran, 2007, Flores-Bazán, Luc, Soubeyran, 2012). To save space we do not examine them in this paper which focus on a single agent.

Comparisons.

1) In the parlance of the VR approach, stability and change dynamics consider two dynamics: an intra period dynamic (a single temporary stay or change made of a succession of elementary stay or change operations) and an inter periods dynamic (a succession of periods, named a transition). The Chan, Yu (1985) paper on stable habitual domains examines transitions, while a competency set expansion process (Shi,Yu, 1999) refers to an inter period dynamic (a single change);
2) HD theory examines decision making with changeable spaces (DMCS) problems, while the VR approach considers decision making with changeable spaces and moving goals (DMCSMG) problems. However (DMCS) problems can choose goals as well. VR self regulation processes focus attention on i) goal setting and goal revision (hence changing aspiration levels and goals) and ii) goal striving (discrepancy or charge reduction);

3) Main HD choice variables are skills, competency sets, and activation propensities. Main VR choice variables are capabilities, actions and goals where actions refer to paths of elementary operations.

4) VR and HD payoffs are experience dependent in their most general formulations (however, the resolution of the competency set expansion problem, being mathematically so complex, seems to require the opposite. See, for example, Shi,Yu, 1996, theorem 5.1, where the expected return function \( ER \) depends only on the elements acquired from \( Tr\{Sk\ldots \} \);

5) The two concepts of competency sets and capabilities, while having some similarities, differ. For the HD theory a competency set is a subset (a collection of resources and skills). For the VR approach a capability is a path of operations including a script and a timing, and related means (physiological, physical, cognitive, motivational and emotional, tangible and intangible ingredients, downstream and upstream tools and machines used to perform these operations, following the given script); However, both approaches can consider, as alternative formulations, subsets and paths to modelize competency sets and capabilities.

6) The HD definition of asymmetric costs to acquire a new skill directly from a given skill (see, for example Shi, Yu, 1996 ) is a particular and reduced form of the VR definition of costs to be able to change, defined as costs to acquire, directly and indirectly, the capability to do a new action, starting from having the capability to do again an old action. VR costs to be able to change refer to direct and indirect costs. They include direct and indirect costs to delete some old elementary capabilities, which will not be used anymore and pollute..., costs to regenerate some other old capabilities, which will be used again, and costs to acquire new elementary capabilities. HD costs to acquire a new skill from an old one are direct costs. They seem to include only acquisitions costs and they represent the infimum of costs to be able to change (see Soubeyran, 2009);

7) The VR resistance to change concept (see Soubeyran, 2009, 2010) differs from the HD resistance to change concept (Larbani,Yu, 2012). For the HD theory, the discharge hypothesis supposes that “to release charges, we tend to select the action which yields the lowest remaining charge (the remaining charge is the resistance to the total discharge) and this is called the least resistance principle” (Larbani,Yu, 2012). In the VR theory resistance to change is the disutility of inconvenients to have to change capabilities.
8) In the HD approach, motivation to change is modeled in terms of charges and discharges. The closest VR concept will be the utility-disutility of charges and discharges (as tensions). The VR motivation-resistance to change balance can be compared with the excitation-inhibition functions (Chan, Yu, 1985);

9) The definition of traps differs. The HD theory (Larbani, Yu, 2012) said that a decision maker is in a decision trap at time \( t \), if his competence set is trapped in some area and cannot expand to fully cover the targeted competence set. Then “discovering is getting out of a decision trap. The resolution of challenging problems generally involve covering and discovering or dis/covering. Discovering requires a target to cover, while the covering process requires discovering when it falls in a decision trap”. The VR approach defines a variational trap with respect to an initial situation (action, a doing, or a state, some having or being). It is a situation which is worthwhile to reach, starting from this initial situation, but, being there, not worthwhile to leave. Optima, equilibria, decision traps, habits, routines, rules and norms represent specific cases; for game situations, win-win outcomes are examples of variational traps.

10) The VR approach does not modelize the expansion of competency sets. On the contrary, Larbani, Yu (2012) give three HD toolboxes which show how to expand and enrich the actual and reachable domains and look into the depth of potential domains:

   a) the seven empowerment operators;

   b) eight methods for expanding the habitual domain M8. Learning actively, M9. Projecting from a higher position, M10. Active association, M11. Changing the relevant parameters, M12. Changing the environment, M13. Brainstorming, M14. Retreating in order to advance, M15. Praying or meditating (Larbani, Yu, 2012);

   c) nine principles of deep knowledge: M16. the deep and down principle, M17. the alternating principle, M18. the contrasting and complementing principle, M19. the revolving and cycling principle, M20. the inner connection principle, M21. the changing and transforming principle, M22. the contradiction principle, M23. the cracking and ripping principle, M24. the void principle (Larbani, Yu, 2012).

4 Rationality of a single agent.

Rationality and the HD theory In this paper we consider an isolated agent. Then, the comparisons between HD and VR game situations will be examined elsewhere. HD theory examines three different problems and proposes three different behavioral models for a single agent, who can be fully or bounded rational, depending on the model.
i) The stabilization of an habitual domain problem. This situation is modeled by a stabilization of activation propensities model, which represents a reduced form of the stabilization of an habitual domain problem (Chan, Yu, 1985). This model is a differential equation, a variant of the famous global pattern formation model (Cohen, Grossberg, 1983). The authors examine its convergence (weak and strong global stability). In this case an agent does not optimize. He is bounded rational;

ii) The competency set expansion problem. In this case the main focus is on an optimal “Problem solving” approach. More precisely an agent has a given problem E to solve. To succeed to solve this problem, he must own a collection of skills defined as the competency set Tr(E) (true or perceived), related to the full resolution of the problem. The agent starts the resolution equipped with a given competency set, the acquired skill set Sk (true or perceived) defined as the collection of skills he owns at the beginning, before starting the resolution. The problem is to find an optimal path of expansion of his competency set, from the initial position Sk to the final position Tr(E) which represents a fixed given goal. A lot of different algorithms have been used to find the optimal solution for intermediate and compound skills and asymmetric cost functions (tree expansion processes, deduction graphs, spanning trees, . . . ; see Li, Chiang, Yu, 2000). In this case the agent is fully (substantive) rational; The opposite case of a bounded rational agent which uses a satisficing process to solve a competency set expansion problem remains an open interesting problem in this area of research.

iii) Decision making and optimization in changeable spaces (DMOCS) problems. In this new setting, Larbani, Yu (2012) use some dedicated optimization methods and suggest to search for other new optimizing methods to solve them. Cover-discover problems belong to this class of new optimization problems. However, Larbani, Yu (2012) said that ”the operator Min in the models (6)-(8) should be understood in the sense of satisfaction not in the sense of absolute minimum”. They notice (Larbani, Yu, 2012, p 742) that optimization must be understood in term of reducing the charge level of the decision maker to a satisfactory or acceptable level. This is in accordance with the satisficing principle (Simon, 1955).

Bounded rationality and the VR approach VR theory proposes a unified model for human behavior, focusing on worthwhile temporary stays and changes, variational traps, and self regulation processes (goal setting, goal striving, goal revision, goal pursuit processes). It is well adapted to complex, changing and high stake decision making problems (see Kunreuther and alli, 2002), where agents cannot be fully rational. In a complex and changing world, full optimization, each step, is too costly and even not economizing, because situations (spaces of feasible means and capabilities, actions, performances, payoffs, intermediate and final goals, desires and aspirations) change each step. Then, an
optimal solution at time \( t \) can be irrelevant at time \( t+1 \). An agent tries to reach, each step, a moving satisficing level (not a fixed one). He considers, each step, worthwhile temporary stays or changes, which include, as special cases, adaptive, satisficing, local, approximate, inexact solutions (and optimal solutions as limit cases).

5 Behavioral stability issues: “how habits and routines form and break”.

Different mathematical tools for stability issues The HD theory uses, at least, three main mathematical tools to examine stability issues (convergence to a stable and desirable final situation) and innovation problems (reaching a targeted competency set, starting from a given initial one): i) the dynamics of pattern formation (Grossberg, 1973, 1978, 1980, Cohen, Grossberg, 1983) as the main tool to examine the dynamic of activation propensities (Chan, Yu, 1985), ii) mathematical programming and different graph methods to study competency set formation and innovation problems (see, among many other papers, Shi, Yu, 1996), iii) Markov chains to examine the convergence of second order DMCS games to desirable and stable issues (Yu, Larbani, 2009, Larbani, Yu, 2009, 2011, 2012). In contrast, the VR approach (Soubeyran, 2009, 2010) starts from variational rationality principles in Behavioral Sciences and offers, as immediate applications, a lot of famous mathematical principles of variational analysis (Ekeland theorem, Bronsted Lemma, and other equivalent principles, . . ; see Flores-Bazán and alii, 2012, Luc, Soubeyran, 2013). In turn, it uses a lot of well known variational algorithms (proximal algorithms, descent methods, variational inequalities, trust regions methods, equilibrium problems, . . ) to help to refine the VR approach relative to stability and innovative issues, for isolated and interacting agents (VR games).

While strongly related to the VR approach, competency set expansion problems do not refer to stability issues but to innovative issues, while the dynamics of pattern formation (activation propensities) and second order DMCS games main focus are on stability issues. To save space, and because our inexact proximal algorithm paper chooses, for an application, habit’s and routine’s formation at the individual and organizational levels (a benchmark stability issue), we will only compare how the HD and VR approaches solve, in a different way, this very difficult problem of “habits and routines formation and break”. A comparison of the HD and VR approaches relative to DMCS problems, competency set expansion problems, innovation problems, and second order DMCS games, will be examined elsewhere. However a very preliminary step is done, later, for the comparison of second order DMCS games and VR games.

Let us compare how the HD theory, using the dynamics of pattern formation (Grossberg, 1973, 1978, 1980, 1983) and the VR approach modelizes and explain how habits and routines form and break on two grounds: i) explain convergence to a final issue, ii) explain why this final issue is desirable and stable.
Notice that the pattern formation model and the variational rationality model use, both, a balance principle. Worthwhile changes balance motivation and resistance to change. Changes in pattern allocations balance excitation and inhibition inputs and signals.

**HD theories of habit’s and routine’s formation and break**  The HD intuition is the following (Chan, Yu, 1985) “... the existence of stable HD, based on a set of hypotheses is described. Roughly, as each human being learns, his HD grows with time, but at a decreasing rate, because the probability for an arrival idea to be new with respect to HD becomes smaller as HD gets larger. Thus, unless unexpected extraordinary events arrive, HD will reach its stable state. If extraordinary events do not arrive very often, habitual ways of thinking and action will prevail most of the time. This observation is the main motivation to use ‘habitual’ as the adjective. More formally, HD theory of habit and routine formation considers the convergence of the allocations of time and effort (activities propensities) to different activities up to a final pattern (an habitual pattern of time and effort allocations), using a variant of the famous pattern formation model (Grossberg, 1973, 1978, 1980, 1983). Notice that his model of progressive pattern formation does not explain why this final allocation is desirable and stable. However, as said before, in the context of DMCS problems, win-win situations refer to desirable and stable final outcomes (Yu, Larbani, 2009, Larbani, Yu, 2009, 2011, 2012).

**VR theories of habit’s and routine’s formation and break**  The VR intuition is very different. Agents make worthwhile changes and stop to change when there is no way to be able to consider and make a new worthwhile change. The convergence can be in finite or infinite time (see Bento, Soubeyran, 2014, Flores Bazan, Luc, Soubeyran, 2012; Bento, Cruz Neto, Soares, Soubeyran, 2014). This model explains why, and under which conditions, this final issue is desirable and stable. This is the case when it is a variational trap. Moreover, the formalized VR theory of habit’s and routine’s formation fits very well (see below) the main non formalized experimental findings of the different theories of “how habits and routines form and break”, both in Psychology and Management Sciences, within a bounded rationality approach, and, to some degree, in Economics, in a perfect rationality context.

**Habit’s formation in Psychology and Economics.** In Psychology habits represent “learned sequences of acts that have become automatic responses to specific cues, and are functional in obtaining certain goals or end states”; see Verplanken, Aarts (1999). For (Duhigg, 2012), an habit is an automatized action (mental or physical experience), a more or less fixed way of thinking, willing, feeling and doing which follows an automatized three steps pattern: a given trigger which activates the action, a process (or script) that the action follows, and a reward (benefit or gain). Hence habits are learned automatic behaviors. Repetition in a similar recurrent context is a necessary condition for habits to develop. Frequency of past behavior and context sta-
bility like internal cues (moods and goals) and external cues (partners and external goals) determine habit strength. Habits represent a form of automaticity (Bargh, 1994). They are more or less conscious and intentional (wanted, i.e., the perception of contexts is more or less goal directed, triggered by goals or by other cues). They are learned in a progressive way. They can be difficult to control, hard to form, because they follow a progressive learning process, and more or less hard to break, given some weak or strong motivation and resistance to change, as the vestige of past behavior. Then, they can resist to change. Habits can be good (mentally efficient, saving on deliberation efforts). Habits can also be bad (addictions, behavioral traps, . . . ).

In Economics agents are perfectly rational and habits are defined as stocks of past experiences. A current habit is modeled as a stock of past behaviors which determines the present preference of the agent with respect to present consumption. In standard models of addictions, see (Becker, 1988), and habit formation, see (Abel, 1990, Carroll, 2000), preferences have the given current numerical representation \( U_n = U(c_n, h_n) \), where the current state \( h_n \) represents a stock of habits, \( c_n \) stands for current consumption, and \( n \) indexes time. The habit persistence hypothesis implies that instantaneous utility does not only depend on current consumption, but also on a stock of habits, \( h_n \).

**Routine’s formation in Management Sciences.** In this discipline routines are defined at the organizational level, as collective patterns of interactions. An enormous literature considers routines as organizational habits in the context of the stability and change dynamics of organizations. The excellent survey is (Becker, 2004), which lists the main points which characterize routines as: patterns of interactions, collective activities, mindlessness vs effortful accomplishments, processes (ways of doing, scripts), context dependent (embeddedness and specificity), path dependent, and triggered by related actors and external cues. Routines have several effects. They favor coordination, control, truce and stability. They also economize on cognitive resources, store knowledge, and reduce uncertainty.

**Stability issues in VR and N person second order games** Although our proximal algorithm paper considers only an agent and not a game situation, the VR approach includes the examination of variational games which follow, using reduced formulations, the VR list of nine principles given above (Attouch and alii, 2007, Attouch and alii, 2008, Flores-Bazán, Luc, Soubeyran, 2012, Flam and alii, 2012, Cruz Neto and alii, 2013). Here, the nine VR principles will not be repeated. However (to save space, this is left to the reader), to allow a possible more complete comparison of HD and VR stability issues (possible convergence to some stable final situation), let us summarize the content of N persons second order games (Yu, Larbani, 2009, Larbani, Yu, 2009, 2011, 2012). They incorporate human psychology in formulating games as people play them, using the habitual domain theory and the Markov chain theory. A Markov chain models the evolution of the states of mind of players over time as transition probabilities over them. States of mind determine the outcomes of the so called
two or \( N \)-person second-order game. The final issues are not Nash equilibria, but focal mind profiles, which are desirable to reach and globally stable solutions of the game, and win win profiles, which are focal and absorbing profiles, while Nash equilibria are not. These games can predict the average number of steps needed for a game to reach a focal or win-win mind profile where both players declare victory. Given some hypothesis, the ”possibility theorem” states that it is always possible to reach a win-win mind profile, restructuring (reframing) the data of the game, the set of players, the payoff functions, and the set of strategies of the initial game. In this reframing context, the HD information input hypothesis H8 plays a major role. In such games, players are not fully rational. They follow the rule of the HD theory. They refer to DMCS problems, where the structure of the game can be restructured (it is changeable), and information inputs help to reach a win win profile. To summarize, in a conflict situation, second order players try to reach a focal profile (which exists) and, once it is reached, they try to make it stable, as a win win profile.

6 Variational rationality, changeable payoffs and decision sets, and the inexact proximal algorithm

The Variational rationality approach considers the rationality of agents, when many things change or are changeable, and other things must temporary stay. Let us summarize our finding. First, the inexact proximal algorithm (with relative resistance to change) is a reduced form of the variational rationality model. Second, it is not a repeated optimizing problem in changeable spaces as an exact proximal algorithm can be. In this section, we will show that it represents a worthwhile to stay and change dynamic. Then, it is an adaptive and satisfying course pursue problem, with changeable payoffs, goals, and decision spaces.

Exact proximal algorithms as repeated optimization problems with variable payoffs and changeable spaces Let us consider, first, exact proximal algorithms which are benchmark cases of their inexact versions. They are not satisfying models of human behaviors. However, they can be considered as repeated optimization problems with variable payoffs and changeable decision sets (as worthwhile to change sets, see the Lemma). The fact that they consider variable payoffs has been shown above.

Let \( X = \mathbb{R}^n \) be an action space, \( f : X \to \mathbb{R} \cup \{+\infty\} \) a proper, lower semi-continuous function bounded from below and consider the following problems.

- The fixed payoff and decision set optimization problem (1):

\[
\text{PROBLEM 1 : } \inf \{ f(y) : y \in X \}.
\]

This problem is the substantive (global) rationality minimization problem.
• The fixed payoff and fixed decision set exact proximal algorithm (2)

\[ \text{PROBLEM 2 : } \inf \left\{ f(y) + \lambda_k \Gamma [q(x^k, y)] , \ y \in X \right\}, \]

where \( x^k \in X \) and \( \lambda_k \in \mathbb{R}_{++} \) are given for each \( k \in \mathbb{N} \), \( q : X \times X \to \mathbb{R}_+ \) is a quasi-distance and \( \Gamma : \mathbb{R} \to \mathbb{R} \) represent the relative resistance to change. This problem is the exact version of our inexact proximal problem with relative resistance to change. It is a repeated optimization problem.

• The variable payoffs and fixed decision set problem (2’):

\[ \text{PROBLEM 2’ : } \inf \left\{ f_{E^k}(y) + \eta_k \Gamma [q(x^k, y)] , \ y \in X \right\}. \]

where the variable payoff function is \( f_{E^k}(y) = f_{E^k}(y) = v(E^k) f(y) \). Equality \( \lambda_k = \eta_k / v(E^k) \) shows that this problem is equivalent to Problem 2. It represents a variable and experience dependent payoff with fixed decision set problem. It allows to define a course pursuit problem with variable preferences.

• Variable payoffs and variable decision set problems.

\[ \text{PROBLEM 3 : } \inf \left\{ f(y) + \lambda_k \Gamma [q(x^k, y)] , \ y \in W_{\lambda_k}(x^k) \right\}, \]

where \( W_{\lambda_k}(x^k) = \{ y \in X : f(x^k) - f(y) \geq \lambda_k \Gamma [q(x^k, y)] \} \), \( k = 0,1, \ldots \).

Lemma 1 PROBLEM 1 and PROBLEM 2 are equivalent, i.e.,

\[ \arg \min_{y \in X} \left\{ f(y) + \lambda_k \Gamma [q(x^k, y)] \right\} = \arg \min_{y \in W_{\lambda_k}(x^k)} \left\{ f(y) + \lambda_k \Gamma [q(x^k, y)] \right\}. \]

Proof. Take \( x^k_3 \in \arg \min_{y \in X} \left\{ f(y) + \lambda_k \Gamma [q(x^k, y)] \right\} \). Taking into account that \( \Gamma [q(x^k, x^k)] = 0 \), from the definition of \( W_{\lambda_k}(x^k) \), it follows immediately that \( x^k_3 \in W_{\lambda_k}(x^k) \). Now, take

\[ x^k_3 \in \arg \min \left\{ f(y) + \lambda_k \Gamma [q(x^k, y)] , y \in W_{\lambda_k}(x^k) \right\}. \]

It is easy to see that:

\[ f(x^k_3) + \lambda_k \Gamma [q(x^k, x^k_3)] \leq f(x^k_2) + \lambda_k \Gamma [q(x^k, x^k_2)]. \]

On the other hand, since \( W_{\lambda_k}(x^k) \subset X \) and \( x^k_3 \in W_{\lambda_k}(x^k) \), definition of \( x^k_2 \) implies:

\[ f(x^k_3) + \lambda_k \Gamma [q(x^k, x^k_3)] \geq f(x^k_3) + \lambda_k \Gamma [q(x^k, x^k_3)], \]

and the result follows. ■

Remark 2 This lemma show that our inexact proximal algorithm is a changeable payoff and decision set process which belongs to the class of “Decision Making and Satisficing (not necessarily Optimizing) Problems in Changeable Spaces”, where the changeable payoff is \( P_{\lambda_k}(x^k, y) = f(y) + \lambda_k \Gamma [q(x^k, y)] \), and the changeable decision set is the current worthwhile to change set \( W_{\lambda_k}(x^k) \).
Inexact proximal algorithms as adaptive satisficing dynamics with variable payoffs and changeable spaces

If the agent follows an inexact proximal algorithm, he will choose to perform, each step $k$, a worthwhile change in the set $W_{e_k,\xi_k}(x^k)$ where $\xi_k = \lambda_k \mu_k > 0$.

$$f(x^k) - f(y) \geq \lambda_k \mu_k \Gamma[q(x^k, y)], \quad 0 < \mu_k \leq 1.$$  

This inexact proximal algorithm (with relative resistance to change) introduces a lot of simplifications, as a reduced form of the variational rationality model. Among others, it identifies actions $x \in X$ to activities $x \in \langle x, [x] \rangle$ where $[x] \in [X]$ refers to a given capability to do an action $x$. It supposes a strictly increasing and invertible pleasure (utility) function $U_c[A_x] = A_x$, and defines a relative resistance to change function $\Gamma(I_x) = U^{-1}[D_e[I_x]]$ where pain, i.e., the disutility of inconvenient to change is $D_e[I_x] = I_x$. It considers separable experience dependent unsatisfied need functions $f_{E^k}(y) = v(E^k)f(y)$ or separable payoff (profit) functions $g_{E^k}(y) = v(E^k)g(y)$, where $E^k = \langle x^1, x^2, ..., x^k \rangle$ is the history of past actions, and the influence of experience is modeled via the coefficient $v(E^k) > 0$. It considers infimum costs to be able to change $C(x, y) = \inf \{ C([x], [y], \omega_{[x], [y]}), \omega_{[x], [y]} \in \Omega([x], [y]) \}$, among all finite costs to be able to change $C([x], [y], \omega_{[x], [y]}) < +\infty$, following a path of change, $\omega_{[x], [y]} \in \Omega([x], [y])$, from a given capability $[x]$ to do an action $x$ to a given capability $[y]$ to do a new action $y$. This inexact proximal algorithm allows to define worthwhile changes $x^k \sim y$ as $g_{E^k}(y) - g_{E^k}(x^k) \geq \eta_k \Gamma[q(x^k, y)]$, where $\eta_k > 0$ is an adaptive worthwhile to change satisficing ratio, which can be changed from period $k$ to period $k+1$. If we note $\lambda_k = \eta_k/v(E^k)$, then, a worthwhile change $x^k \sim y$ is defined as $g(y) - g(x^k) \geq \lambda_k \Gamma[q(x^k, y)]$. In term of separable experience dependent unsatisfied need functions $f_{E^k}(y) = v(E^k)f(y)$, a worthwhile change $x^k \sim y \in W_{e_{k+1},\xi_{k+1}}(x^k)$ is such that $f(x^k) - f(y) \geq \lambda_k \Gamma[q(x^k, y)]$.

The topic of our paper is not exact proximal algorithms, but inexact ones.

Inexact proximal algorithms examined in this paper represent adaptive satisficing dynamics (dealing with changeable satisficing levels), variable and experience dependent preferences and changeable decision sets, which belong to the class of decision making with changeable spaces and changeable goals problems, noted DMCSCG problems. Let us show this important point, which helps the comparison with the Habitus domain theory and DMCS decision making problems with changeable spaces (Larbanì, Yu, 2012). Our VR point of view is the following. An inexact proximal algorithm is a specific instance of a VR worthwhile to stay and change dynamics $x^{k+1} \in W_{e_{k+1},\xi_{k+1}}(x^k), k \in N$. This dynamic is both satisficing and considers changeable goals and decision sets.

i) This dynamic is satisficing because the worthwhile to change condition generalizes the Simon (1955) definition in a dynamical context, balancing satisfactions to change with sacrifices to change. The moving goal is, each period, the chosen worthwhile to change satisficing level $\xi_{k+1} > 0$.

ii) This dynamic is adaptive, and considers changeable decision sets $W_{e_{k+1},\xi_{k+1}}(x^k)$. Each period, the agent can choses how much changes must be worthwhile (the size of $\xi_{k+1}$) to accept to change. Then, the worthwhile to change set is chosen, each period.
Local inexact proximal algorithms  For each $k \in \mathbb{N}$ fixed, let $X^k \subset X$, $x^k \in X$, $\lambda_k \in \mathbb{R}_+$ and consider the following problem:

PROBLEM 4 : $\inf \{ f(y) + \lambda_k \Gamma [q(x^k, y)] , \ y \in X^k \}$.

This a variable preference proximal problem with a changeable feasibility space $X^k \subset X$. In Attouch, Soubeyran (2010) it is a changing ball $X^k = B_r(x^k)$ of constant radius $r > 0$. We can define the indicator function $I_{X^k}$ and consider the changeable payoffs and decision spaces proximal problem,

$$\inf \{ f(y) + I_{X^k}(y) + \lambda_k \Gamma [q(x^k, y)] , \ y \in X \}.$$

Bento and alii (2013) have also examined an exact local search multiobjective proximal problem, where $X^k$ is a lower countour set of the multi-objective function.

Now, let us consider “fixed payoff and variable decision set problems”, namely, PROBLEM 5 : $\inf \{ f(y), y \in W_{\lambda_k}(x^k) \}$.

Let us assume that $\{ x^k_2 \}$ is a generated sequence from the iterative process

$$x^k_2 \in \text{argmin} \{ f(y), y \in W_{\lambda_k}(x^k) \}.$$

Since $x^k_2 \in W_{\lambda_k}(x^k)$, by definition, the agent follows, each step, a worthwhile change.

**Lemma 3** Let $\{ x^k_2 \}$ be a generated sequence from the iterative process

$$x^k_2 \in \text{argmin}_{y \in X} \{ f(y) + \lambda_k \Gamma [q(x^k, y)] \},$$

such that $\{ f(x^k_2) \}$ converges to $\underline{f} := \inf \{ f(y), y \in X \}$. Then the sequence $\{ x^k_2 \}$ is a minimizing sequence for PROBLEM 1.

**Proof.** Take $k \in \mathbb{N}$ arbitrary. Note that $x^k_2 \in W_{\lambda_k}(x^k)$ (see proof of Lemma 2). It is easy to see that $f(x^k_2) \leq f(x^k_1)$, i.e., each solution of PROBLEM 2 gives an estimation-majoration $f(x^k_2)$ to the minimal payoff $f(x^k_1)$ of PROBLEM 5. Now, from the definition of $\underline{f}$, we have

$$f \leq f(x^k_1) \leq f(x^k_2).$$

Therefore, the desired result follows from $\square$ together the fact of $\{ x^k_2 \}$ be a minimizing sequence for PROBLEM 1.

**Mathematical stability issues**

1) **Stability of variational traps.** Let $\lambda_k = \eta_k / v(E^k)$ be a stage $k$ proximal ratio, where $\eta_k > 0$ is a stage $k$ worthwhile to change ratio, and $v(E^k) > 0$ is the experience rate of influence at stage $k$. Let $\lambda_* = \eta_*/v(E^*)$
be the limit proximal ratio. It is easy to show that if $x^* \in X$ is a variational trap, given the proximal ratio $\lambda_*$, then, $x^* \in X$ is also a variational trap, for any higher proximal ratio $\lambda > \lambda_*$. This comes from the implication: $W_{\lambda_*}(x^*) = \{x^*\}$ implies $W_{\lambda}(x^*) \subset W_{\lambda_*}(x^*)$ for $\lambda > \lambda_*$. Then, $W_{\lambda}(x^*) = W_{\lambda_*}(x^*) = \{x^*\}$ for $\lambda > \lambda_*$. A higher proximal ratio $\lambda = \eta/v(E) > \lambda_* = \eta_*/v(E^*)$ means a higher worthwhile to change ratio $\eta > \eta_*$ or (and) a lower experience rate of influence $v(E) < v(E^*)$. It can also mean higher costs to be able to change parameters and, each period, a longer length of the exploitation phase, or a shorter length of the exploration phase (Soubeyran, 2009, 2010). For example, stability issues with respect to adaptive parameters like $\lambda_k \geq \lambda_\infty > 0$, where $\lambda_k$ represents a changing preference, resistance to change, costs to be able to change, and length of the exploitation period relative to the exploration period, will be examined elsewhere.

2) **Stability of worthwhile to change trajectories.** For an exact quadratic proximal problem,

$$
\inf \{ f(y) + \lambda \Gamma[q(x, y)], y \in X \}, \quad \Gamma[q(x, y)] = \|y - x\|^2,
$$

this is a well known result. The proximal mapping

$$
X \ni y \mapsto \varphi_\lambda(x) := \arg \min \{ f(y) + \lambda \Gamma[q(x, y)], y \in X \},
$$

is well defined (recall that $f$ was assumed be bounded from below) and, if $f$ is convex, it is non expansive, i.e.,

$$
|\varphi_\lambda(x') - \varphi_\lambda(x)| \leq \|x' - x\|, \quad x, x' \in X,
$$

see, for example, (Attouch, Soubeyran, 2010). For an inexact proximal algorithm with a non quadratic regularization term and a non convex $f$, this is an open difficult question.

7 Habit’s and Routine’s Formation: Inexact Proximal Processes with Weak Resistance to Change

In this section (see the Arxiv version of our paper, Bento, Soubeyran, 2013), we detail how our inexact proximal algorithm modelizes habit/routine formation and break, using the VR resistance to change modelization.

Comparing worthwhile changes and stays processes, inexact proximal algorithms and habituation-routinization processes. As an application we will consider habit/routine formation as an inexact proximal algorithm in the context of weak resistance to change. Because of its strongly interdisciplinary aspect (Mathematics, Psychology, Economics, Management), to be carefully justified, this application needs several steps. This, because we need to compare
three different processes. First, a general worthwhile change and stay process. Then, as two specific instances, an inexact proximal algorithm and an habituation/routinization process. The comparison must consider three aspects; a dynamical system (which one?), which converges (how?), to an end point (which one?).

- **Step 1.** At the mathematical level, an exact proximal algorithm represents a dynamical system, which, each step, minimizes a proximal payoff \( f + \lambda_k [q(x^k, \cdot)]^2 \) over the whole space \( X \). The perturbation term is \( \lambda_k [q(x^k, y)]^2, \lambda_k > 0, y \in X \), while the payoff function is \( f \). An inexact proximal algorithm represents a dynamical process which, each step, uses a descent condition and a temporary stopping rule. In both cases, the problem is to give conditions under which this process converges to a limit point. Then, an exact or inexact proximal algorithm considers three points:
  
i) a dynamical process. It represents the proximal sub-problem which can be the minimization of the current proximal payoff (for an exact proximal algorithm), or a descent condition, i.e., a sufficient decrease of the proximal payoff (in the inexact case):
  
ii) the existence of some end points, which can be, or not, a critical point, a local, or global minimum of \( f \):

iii) and the convergence of the process towards an end point. This convergence can be linear, quadratic... .

- **Step 2.** At the behavioral level, in the context of his “Variational rationality approach”, (Soubeyran, 2009, 2010) has examined “worthwhile change and stay” processes. Such dynamical processes refer to successions of temporary worthwhile changes and worthwhile stays. The end points are variational traps. The convergence of the process materializes in small steps, whose length goes to zero. Let us remind that, in this behavioral context, end points represent traps (reachable, i.e., more or less easy to reach, but difficult to leave). Worthwhile changes balance, each step, motivation and resistance to change forces. The motivation force is the utility \( U \) of the advantages to change function \( A(x, y) = f(x) - f(y) \) where \( f \) represents an unsatisfied need to be minimized. The resistance to change force represents the disutility \( D \) of the inconveniences to become able to change \( I(x, y) = q(x, y) \), where \( q(x, y) \) is a quasi distance. In the context of this paper, the variational approach considers the relative resistance to change or aversion to change function \( \Gamma[q(x, y)] = U^{-1} [D[q(x, y)]] \). This shows that the perturbation term of an exact or inexact proximal algorithm is a specific instance of a relative resistance to change function \( \Gamma[q(x, y)] \). (Moreno et al. 2011) have considered the specific quadratic case of weak resistance to change, where \( \Gamma[q(x, y)] = q(x, y)^2 \). Then, at the behavioral level, two main concepts, among others, drive a “worthwhile changes and stays” process: i) the unsatisfied need function \( f \) (which
materializes the motivation to change) and, ii) inertia (the relative resistance to change function $\Gamma[q(x, y)]$).

- **Step.3.** Habit formation/routinization processes. A very short survey has been given in the previous section Behavioral stability issues: “how habits and routines form and break”. These processes consider, i) a repetitive process where an action is repeated again and again in the same recurrent context, ii) a final stage where this action becomes a permanent habit, iii) the convergent process which describes a slow learning habituation process where this action, being repeated again and again in the same recurrent context becomes gradually automatized.

- **Step.4.** Then, it becomes clear that habituation/routinization processes are specific instances of worthwhile change and stay processes. Unsatisfied needs and inertia play a major role.

- **Step.5.** In our paper we have shown how both exact and inexact proximal algorithms are specific instances of “worthwhile change and stay” processes, because: i) minimization of the current proximal payoff, descent conditions and current stopping rules are special cases of worthwhile changes and marginal worthwhile stays, ii) critical points, local and global minimum are specific representations of variational traps, iii) convergence of the proximal algorithm shows how proximal worthwhile changes converge, depending of the shape of the payoff function (which can be convex, lower semicontinuous, or which can satisfy a Kurdyka-Lojasiewicz inequality, . . .) and of the shape of the perturbation term (linear, convex,. . . with respect to distance or quasi distance).

**Why resistance to change matters much.** The benchmark case of lower semicontinuous unsatisfied need functions $f$ and strong resistance to change functions $\Gamma[q(x, y)]$ (where costs to be able to change are higher than a quasi distance) have been examined by (Soubeyran, 2009, 2010) who considered worthwhile change and stay processes. It has been shown that when a worthwhile change and stay process converges to a variational trap, this variational formulation offers a model of habit/routine formation which modelizes a permanent habit/routine as the end point of a convergent path of worthwhile change and temporary habits, where, for a moment, there is no way to do any other worthwhile change, except repetitions.

The opposite case of weak resistance to change was left open. In the context of an exact proximal algorithm, Moreno et al.(2011) have examined a specific case of weak resistance to change, namely the quadratic case $\Gamma[q(x, y)] = q(x, y)^2$. However, exact proximal algorithms represent a very specific case of worthwhile changes, where, each step, the descent condition is optimal. This means that, each step, the process minimizes the proximal payoff $f + \lambda_k \Gamma[q(x^k, \cdot)]$ on the whole space $X$. Then, such optimizing worthwhile changes are not step by step economizing behaviors because they require to explore, each step, the whole state space, again and again. The present paper considers the
generalized weak resistance to change case in the context of an inexact proximal algorithm instead of an exact one. In both papers the unsatisfied need function $f$ satisfies a Kurdyka-Lojasiewicz inequality. How the strength of resistance to change impacts the speed of habit’s/routine’s formation is, as an application, the topic of the related paper Bento, Soubeyran (2014).

To summarize, we have compared an inexact generalized proximal algorithm with a worthwhile change and stay process with respect to three aspects: A) as a dynamical system, B) with an end point, C) which converges to that end point. To end this paper, it remains to compare an inexact generalized proximal algorithm with an habituation and routinization process, as it is described in Psychology and Management Sciences, using the same three criteria.

**Inexact generalized proximal algorithm and habituation/routinization process are dynamical systems.**

- **Habituation/routinization process.** They represents the repetition of an action of a given kind (some activity related to a given goal), in order to satisfy a recurrent unsatisfied need in a stable context. The repetition concerns the action and what becomes more and more the same is “the way of doing it” (the script). This repetition follows a succession of worthwhile changes and stays. An inexact proximal algorithm represents a step by step processes, a succession of moves in order to “decrease enough” some proximal payoff function. Usually, both dynamical processes are unable to reach the goal in one step. Each step, the level of satisfaction of the recurrent need increases, but some unsatisfaction remains.

An habituation process is driven by two balancing forces: a motivation to change function $M[A(x, y)]$ (an habit/routine must serve us), and a resistance to change function $D[I(x, y)]$, (habits/routines are hard to form and hard to break because learning and unlearning are costly). An inexact proximal algorithm is driven by the two terms $f(y)$ and $\Gamma[q(x, y)]$ of its proximal payoff $f(y) + \lambda_k \Gamma[q(x, y)]$ where $q(x, y) = C(x, y)$. This balance describes the goal-habit interface.

The rationality of an habituation process is to improve by repetition the way of doing a similar action in the same context. The agent improves with costs to change. He satisfices, doing worthwhile changes, without exploring too much each step (local consideration and exploration; see Soubeyran, 2009, 2010 for this important aspect). An inexact proximal algorithm follows, each step, some descent condition and marginal stopping rule, without optimizing each step.

The influence of the past differs from one process to the other. For an habituation process the impact of the past can be very important (the past sequence matters much). For an inexact proximal algorithm it is as if only the last action matters. The influence of the past is minimal (it is as if the agent has a short memory). The influence of the future seems identical in both cases: myopia seems to be the rule. Only the
next future action matters. Agent’s behavior driven by habits/routines are not forward looking. For more forward looking worthwhile to change behavior; see Soubeyran, (2009, 2010).

Convergence: see the last paragraph (“why resistance to change matters much”)

End points. Our inexact proximal algorithm converges to a critical point, which is not an end point, unless it can be shown that a critical point is a variational trap (as this is done in our paper). A variational trap is worthwhile to reach and not worthwhile to leave. An habituation process ends in a permanent habit/routine which is hard to form and hard to break. It represents the vestige of a past repeated behavior.

8 Making the assumptions of the proximal algorithms clear in behavioral terms

We have to show how, in Behavioral Sciences, our three proximal algorithms modelize, at least in a reduced form, how habits form and break.

A first step has been given before, in Section 7. This have been done in five steps. We have shown that inexact proximal algorithms and habitual processes are dynamical systems, we have compared their end points, critical points or variational traps, we have examined how the strength of resistance to change influences their abilities to converge, and we have linked resistance to change to loss aversion, a famous behavioral concept.

A second step is to detail the behavioral content of all the hypothesis which drive our three proximal algorithms. There are general behavioral hypothesis which are common to the three proximal algorithms, and specific hypothesis relative to each of them. More explicitly, the three algorithms suppose that costs to be able to change are quasi distances. They consider, each step, worthwhile and marginally worthwhile changes (descent conditions), and suppose weak resistance to change, as well as a marginal stopping rule. The first algorithm is targeted to converge to a critical point. The second and third algorithms are targeted to converge to a variational trap.

General behavioral hypothesis

- H.1. Costs to be able to change $C$ are modelized as quasi-distances. For an agent, costs to be able to change $C(x, y) = q(x, y)$ refer to the infimum of the costs to be able to change his capabilities, from having the capability to do an action $x$, to the capability to do an action $y$. Then $C(x, x) = 0$ means that if the agent is able to do an action $x$, he is able to do this action at no cost. The condition $C(x, y) = 0 \iff y = x$ means that if the agent is able to move at no cost, then, he can only repeat the same action, if he is able to do it. The triangle inequality $C(x, z) \leq C(x, y) + C(y, z)$ for all $x, y, z \in X$ means that, for an agent, the infimum cost to change
his capabilities from the initial capability to do an action \( x \) to the final capability to do an action \( z \) is lower than the infimum cost to change his capabilities from the initial capability to do action \( x \) to the intermediate capability to do an action \( y \) and the infimum cost to change his capabilities from the intermediate capability to do action \( y \) to the final capability to do action \( z \), because the way to change successively from an initial capability to an intermediate capability and from this intermediate capability to a final capability is an indirect way to change from the initial to the final capability.

- H.2. Unsatisfied needs \( f : \mathbb{R}^n \rightarrow \mathbb{R} \cup \{+\infty\} \) is a proper lower semicontinuous function. This is a regularity assumption which supposes no free lunch. The agent cannot reduce his unsatisfied need in a given small amount without changing enough his action (no jump downward are allowed).

- H.3. Advantages to change \( A(x, y) = f(x) - f(y) \) refer to separable advantages to change functions, linked, when positive, to a decrease in unsatisfied needs.

- H.4. The ratio \( \xi_k > 0 \) modelizes how much a change can be worthwhile (the adaptive satisficing case). This ratio can change from period to period. This means that the agent can adapt with delay or not, each step, his degree of satisficing. In this paper he adapts with one period delay (writing \( \xi_k > 0 \) instead of \( \xi_{k+1} > 0 \), to fit with a proximal formulation).

- H.5. The utility function \( U[\cdot] \) is invertible with \( U[0] = 0 \). This is the case for a strictly increasing utility function, relative to advantages to change (the usual case).

- Condition (4) supposes that \( \Gamma(C) = U^{-1} [D[C]] \) is twice differentiable with respect to \( C \). It is a regularity condition, relative to the relative resistance to change function.

- Condition (7) means that the relative resistance to change function is “flat enough in the small” (in the ”weak enough resistance to change” case). It supposes that the marginal relative resistance to change must be “lower enough” with respect to the mean relative resistance to change, at least for “low enough” costs to be able to change.

- Assumption 3.1 supposes that costs to be able to change are high (low) enough iff the old and new actions are different (similar) enough.

**Hypothesis relative to Algorithm 3.1** Algorithm 3.1 supposes three conditions (20), (21), (22). Condition (20) imposes worthwhile changes (a sufficient descent assumption) along the process. Condition (21) defines subgradients for the unsatisfied needs and costs to be able to change functions. Condition (22) refers, each step, to a stopping rule, where the norm of the marginal decrease of
the unsatisfied need is lower than the norm of the marginal relative resistance to change.

The Kurdyka-Lojasiewicz inequality (Definition 3.2) refers to a curvature property of the unsatisfied need function, near a critical point. The unsatisfied need must be lower than some increasing function of the marginal unsatisfied need, close to a critical point.

Assumption 3.2 supposes that the marginal relative resistance to change function is lower than a power function, near the origin. It is a curvature property.

**Hypothesis relative to the algorithm (5.2)** This algorithm (52) supposes approximate (almost) worthwhile changes, including, each step, an error term $\varepsilon_k > 0$.

Condition (53) supposes that the worthwhile to change satisficing ratio $\lambda_k > 0$ converge along the process.

**Hypothesis relative to Algorithm 4.1** This algorithm supposes three conditions (54), (55), (56). Condition (54) is a modified worthwhile to change assumption. Condition (55) defines subgradients for the unsatisfied needs and costs to be able to change functions. Condition (56) is the stopping rule condition (22). This algorithm adopts all the hypothesis of Algorithm 3.1

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