Is There an Age of the Universe Problem After the Hipparcos Data?

Murat Özer
Department of Physics, College of Science, King Saud University P.O.Box 2455, Riyadh 11451, Saudi Arabia

ABSTRACT

We have reanalyzed the age of the universe problem under the assumption that the lower limit on the age of the globular clusters is 11 Gyr, as predicted by the recent Hipparcos data. We find that the globular cluster and the expansion ages in a standard $\Lambda = 0$ universe are consistent only if the present value $H_0$ of the Hubble constant is $\leq 60 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$. If $H_0 > 60 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$ some kind of modification of the standard $\Lambda = 0$ model is required. Invoking a (time-independent) cosmological term $\Lambda$ in the Einstein field equations, as has been done frequently before, we have found that due to the gravitational lensing restrictions a flat universe with the present matter density parameter $\Omega_M < 0.5$ is not problem-free. A nonflat universe with $\Omega_M \leq 1$ does not suffer from the age problem if $H_0 \leq 75 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1}$.

1. INTRODUCTION

A lower limit on the present age $t_0$ of the universe is determined by estimating the age of the oldest objects in our galaxy, the globular clusters (hereafter GC). These are stellar systems that contain about $10^5$ stars in the halo surrounding the galactic disk. The key element in estimating the age of a typical GC is the determination of its distance from us. To this end, the primary observational technique is main-sequence fitting against subdwarfs with well known parallaxes. The distance obtained this way or otherwise is used to convert the measured apparent magnitude of a GC to the absolute magnitude. The age is then estimated by applying a stellar evolution model. The estimates obtained by different astronomers agree rather well. For example, Bolte & Hogan (1995) find $15.8 \pm 2.1 \, \text{Gyr}$, Chaboyer et al. (1996) find $14.6 \pm 1.7 \, \text{Gyr}$, and Sandquist et al. (1996) find $13.5 \pm 1 \, \text{Gyr}$. These time scales are to be compared with the expansion age of the universe predicted by

\footnote{E-mail: mozer@ksu.edu.sa}
the standard model of cosmology (hereafter SM) which requires the knowledge of the present value $H_0$ of the Hubble constant. Even though the estimates of $t_0$ from GCs are based on the stellar evolution models, which are essentially the same, the situation is not the same for the $H_0$ estimates. There are a number of different techniques (see the review by Trimble 1996) which give values that differ substantially from each other. We present the most quoted estimates: $H_0 = 50 - 55 \text{km s}^{-1} \text{Mpc}^{-1}$ (Tammann & Sandage 1996) and $H_0 = 73 \pm 10 \text{km s}^{-1} \text{Mpc}^{-1}$ (Freedman, Madore, & Kennicutt 1997). In a SM flat universe $t_0$ would be 13 Gyr and 8.2 Gyr if $H_0$ were 50 km s$^{-1}$ Mpc$^{-1}$ and 80 km s$^{-1}$ Mpc$^{-1}$, respectively. Whereas in a SM open universe with $\Omega_M = 0.1$, $\Omega_M$ being the present nonrelativistic matter density parameter, the ages would be 17.6 Gyr and 11 Gyr for the same $H_0$ values as above. Thus researchers were rightfully led to think that if $H_0$ has as large a value as determined by Freedman et al. (1997) then the expansion age and the GC age of the universe are in conflict with each other.

An immediate solution to this so called age of the universe problem was suggested by including a time-independent cosmological constant $\lambda$ in the Einstein field equations (Peebles 1984; Blome & Priester 1985; Klapdor & Grotz 1986). The gravitational lensing studies, however, have shown that the cosmological constant cannot be as large as one desires to increase the expansion age to the level of GC age lest too many lensing events are predicted (Kochanek 1993, 1995; Maoz & Rix 1993). Recently, the supernova magnitude-redshift approach (Perlmutter et al. 1997) has given $\Omega_\Lambda < 0.51$ (95% confidence level) for a flat universe which is significantly lower than the gravitational upper limit $\Omega_\Lambda < 0.66$ of Kochanek (1995). Thus it had been concluded that the apparent contradiction between the GC age and the expansion age could not be reconciled in a flat universe by invoking a time-independent cosmological constant. This was the status of the age of the universe problem before Hipparcos. The purpose of this paper is to reexamine this problem in the light of the lower limit of 11 Gyr on the GC age put by the Hipparcos data (Reid 1997; Feast & Catchpole 1997; see also the news report by Schwarzschild (1997)).

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2Felten & Isaacman(1986) call the models with $\lambda = 0$ ”standard models”. However, we follow the general trend in the literature and call the totality of them the ”standard model” and refer to each case by its $k$ value (see, for example, Misner, Thorne, & Wheeler 1970; Weinberg 1972.) The SM with $k = 0$ is called the Einstein-de Sitter model (Felten & Isaacman 1986).

3Reid (1997) has argued that this Freedman et al. (1997) value of $H_0$ is reduced to $H_0 = 68 \pm 9 \text{km s}^{-1} \text{Mpc}^{-1}$ because the Hipparcos data reveal a 7% increase in the distances inferred from the previous ground-based data.
2. THE AGE OF THE UNIVERSE PROBLEM

The relation between the present value $H_0$ of the Hubble constant $H = \dot{a}/a$, where $a$ is the scale factor of the universe and $\dot{a} = da/dt$, and the present age $t_0$ is given by (Al-Rawaf & Taha 1996) [4]

$$H_0t_0 = \frac{1}{(1 - \Omega_M)} \left[ 1 - \frac{\Omega_M}{(1 - \Omega_M)^{1/2}} \sinh^{-1}(\Omega_M^{-1} - 1)^{1/2} \right], \quad k = -1 \quad (1a)$$

$$= \frac{2}{3}, \quad k = 0 \quad (1b)$$

$$= \frac{1}{(\Omega_M - 1)} \left[ \frac{\Omega_M}{(\Omega_M - 1)^{1/2}} \sin^{-1}(1 - \Omega_M^{-1})^{1/2} - 1 \right], \quad k = 1. \quad (1c)$$

Here $\Omega_M$ is the present value of the nonrelativistic matter density parameter defined as the ratio of the present nonrelativistic matter density to the present critical energy density

$$\Omega_M = \frac{\rho_M}{\rho_c} = \frac{\rho_M}{3H_0^2/8\pi G}. \quad (2)$$

Expressing the Hubble constant as $H_0 = 100hkms^{-1}Mpc^{-1}$, the age in billion years is given by $t_0(Gyr) = 9.78(H_0t_0)/h$, where $(H_0t_0)$ is given in eq.(1). In Figure 1, we depict $t_0$ against $\Omega_M$ and $h$ in the SM. It is seen that $t_0$ is below the Hipparcos lower limit of 11Gyr for large values of $h$. Thus it can be stated safely that the age of the universe problem still survives if $h$ is large.

In Table 1, we display the maximum values of $h$ for which $t_0 = 11Gyr$ against $\Omega_M$. Note that the maximum $h$ values in Table 1 almost fall in the lower and upper limits of Freedman et al. (1997). Thus for each $\Omega_M$, if $h$ is greater than those given in Table 1, there is an age problem. For example, if $\Omega_M = 0.5$ and $h > 0.67$ or $\Omega_M = 1$ and $h > 0.593$ the age problem survives. Now the problem is, however, milder in the sense that before Hipparcos the age problem was thought to exist even for moderate values of $h$ whereas it now exists for large values of $h$. Emphatically, the SM has no age problem if $h < 0.593 \approx 0.6$.

Supposing that there is an age problem, one line of attack, as in the pre Hipparcos era, is to invoke a (time-independent) cosmological constant $\lambda$ in the Einstein field equations (Peebles 1984; Blome & Priester 1985; Klapdor & Grotz 1986)

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu} = -8\pi GT_{\mu\nu}, \quad (3)$$

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[4] Equations (1a) and (1c) agree numerically with those given in Weinberg (1972) which uses a different but equivalent functional form.
where $R = R^a_a$ and $T_{\mu\nu}$ is the energy-momentum tensor. For a homogeneous and isotropic universe described by the Robertson-Walker metric

$$ds^2 = -dt^2 + a(t)^2 \left[ \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right]$$

(4)

the energy-momentum is assumed to have the perfect fluid form

$$T_{\mu\nu} = \text{diag}(\rho, p, p, p),$$

(5)

where $p$ is the pressure of the matter described by $\rho$. Equations (3) and (4) give (with $c$, the speed of light, set to 1)

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho(t) + \frac{\lambda}{3} - \frac{k}{a^2},$$

(6)

where $k = -1, 0, 1$ for a spatially open, flat and closed universe, respectively. At present, the universe is believed to be dominated by nonrelativistic massive matter rather than relativistic matter (radiation). It proves to be very useful to define the current cosmological constant density parameter

$$\Omega_\Lambda = \frac{\rho_\Lambda}{\rho_c} = \frac{\lambda/8\pi G}{\rho_c} = \frac{\lambda}{3H_0^2},$$

(7)

and the current curvature density parameter

$$\Omega_k = -\frac{\rho_k}{\rho_c} = -\frac{k}{H_0^2a_0^2},$$

(8)

where $a_0$ is the current value of the scale factor $a$ of the universe. When written in terms of the present values equation (6) gives the constraint

$$\Omega_M + \Omega_\Lambda + \Omega_k = 1$$

(9)

Equations (3) and (5) under (4) give the energy conservation equation in the matter dominated era

$$d[\rho_M(t)a^3 + \frac{\lambda}{8\pi G}a^3] + [p_M(t) - \frac{\lambda}{8\pi G}]da^3 = 0,$$

(10)

where the pressure $p_M$ of nonrelativistic matter is negligible. Thus it follows from eq.(10) that $\rho_M(t) = \rho_M a_0^3/a^3$ and the relation between $H_0$ and $t_0$ is

$$H_0t_0 = \int_0^1 y^{1/2}[\Omega_M(1 - y) + \Omega_\Lambda(y^3 - y) + y]^{-1/2}dy,$$

(11)

where $\Omega_k$ has been eliminated by using eq.(9). Now a flat universe with $\Omega_M < 1$ is rendered possible by postulating the existence of the cosmological term $\lambda$ such that $\Omega_M + \Omega_\Lambda = 1$. 
The value of \( k \) not fixed \textit{a priori}, a numerical investigation of eq.(11) reveals that it is always possible to find a set of three parameters \((\Omega_M, \Omega_\Lambda, h_{\text{max}})\) for which \( t_0 = 11\text{Gyr} \).

However, the achievement of a cosmological constant to solve the age problem and to have a flat universe with \( \Omega_M < 1 \) may be illusory. The magnitude of \( \Omega_\Lambda \) required to solve the age problem may turn out to be too large to predict plausible number of gravitational lensing events. Therefore, each such set of parameters \((\Omega_M, \Omega_\Lambda, h_{\text{max}})\) need to be confronted with the gravitational lensing statistics, which we address ourselves next.

3. THE GRAVITATIONAL LENSING STATISTICS

The integrated probability, the so-called optical depth, for lensing by a population of singular isothermal spheres of constant comoving density relative to the Einstein-de Sitter model, is

\[
P_{\text{tens}} = \frac{15}{4} \left[ 1 - \frac{1}{(1+z_s)^{1/2}} \right]^{-3} \int_0^{z_s} \frac{(1+z)^2}{E(z)} \left[ \frac{d(0,z)d(z,z_s)}{d(0,z_s)} \right]^2 dz
\]

(Carroll, Press & Turner 1992) where

\[
E(z)^2 = (1+z)^2(1+z\Omega_M) - z(z+2)\Omega_\Lambda
\]

and is defined by

\[
\left( \frac{\dot{a}}{a} \right)^2 = H_0^2 E(z)^2
\]
(Peebles 1993). Note that \( P_{\text{tens}} = 1 \) for the Einstein-de-Sitter model (in which \( \Omega_k = 0, \Omega_M = 1 \) and \( \Omega_\Lambda = 0 \)). \( z = (a_0/a) - 1 \) is the redshift and \( z_s \) is the redshift of the source (quasar). The angular diameter distance from redshift \( z_1 \) to redshift \( z_2 \) is

\[
d(z_1, z_2) = \frac{1}{(1+z_2) |\Omega_k|^{1/2} \text{sinn} \left[ |\Omega_k|^{1/2} \int_{z_1}^{z_2} dz \frac{d}{E(z)} \right]}
\]

where "\text{sinn}" is defined as \( \sinh \) if \( \Omega_k > 0 \), as \( \sin \) if \( \Omega_k < 0 \) and as unity if \( \Omega_k = 0 \), in which case the \( |\Omega_{k0}|^{1/2} \)'s disappear from eq.(15). To determine how much of \( P_{\text{tens}} \) is permissible, we refer to the work of the Supernova Cosmology Project (Perlmutter et al. 1997). Using the initial seven of more than 28 supernovae discovered, Perlmutter et al. (1997) have recently measured \( \Omega_M \) and \( \Omega_\Lambda \). For \( \Omega_M < 1 \), they find \( \Omega_\Lambda < 0.51 \) at the 95\% confidence level for a flat universe, and \( \Omega_\Lambda < 1.1 \) for the more general case \( \Omega_M + \Omega_\Lambda \) unconstrained.\textsuperscript{5}

In Table 2 we present \( P_{\text{tens}} \) against \( \Omega_M \) and \( \Omega_\Lambda \) for a typical source redshift of \( z_s=2 \). Table 2

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\textsuperscript{5}But of course \( \Omega_M + \Omega_\Lambda + \Omega_k = 1 \).
helps us determine the maximum allowed value of $P_{lens}$. It is seen that for $\Omega_{\Lambda} = 0.5$, which is the maximum allowed value according to Perlmutter et al. (1997), the corresponding $P_{lens}$ is 1.92. Thus we shall assume that $P_{lens}$ cannot be much larger than 2. Having determined the upper limit on $P_{lens}$, we depict in Table 3 the three parameters $\Omega_{M}$, $\Omega_{\Lambda}$ and $h_{max}$ in a flat universe and the corresponding gravitational lensing prediction for $t_0 = 11Gyr$. In preparing Table 3, we have first fixed $\Omega_{M}$ and calculated $H_0t_0$ from eq.(11) with $\Omega_{\Lambda} = 1 - \Omega_{M}$, and finally obtained the maximum value of $h$ from $h_{max} = 9.78(H_0t_0)/11$.

Discarding those set of parameters which yield $P_{lens} > 2$ or have $\Omega_{\Lambda} > 0.5$, first we confirm, from Table 3, the previous conclusions that a cosmological constant cannot solve the age problem in a flat universe with $\Omega_{M} < 0.5$ due to too many lensing predictions. Next, we see that the maximum allowed value of $h$ in a flat universe is about 0.74-0.75. This is to be compared with the pre Hipparcos lower limits for the age. For $t_0 = 13$ and $14Gyr$ the $h_{max}$ values are 0.64 and 0.60 in a flat universe, respectively.

As for a nonflat universe, one may either fix $\Omega_{M}$ and $\Omega_{\Lambda}$ first and then calculate $h_{max}$ to give $t_0 = 11Gyr$, or one may fix $\Omega_{M}$ and $h_{max}$ first and then calculate the $\Omega_{\Lambda}$ value from eq.(11) by trial and error to give again $t_0 = 11Gyr$. We have chosen the second option and constructed Figures 2 and 3, which are contour diagrams of $h_{max}$ (for $t_0 = 11Gyr$) in the ($\Omega_{M}, \Omega_{\Lambda}$) and ($\Omega_{M}, P_{lens}$) planes.

It is seen that for each contour there is a minimum value of $\Omega_{M}$ before which the age is greater or equal to $11Gyr$ for $\Omega_{\Lambda} = 0$. In drawing Figures 2 and 3, we have assumed that the maximum allowed value of $\Omega_{\Lambda}$ is about 1.1, in accordance with the findings of Perlmutter et al. (1997). The age problem is seen to survive for $\Omega_{M} \geq 0.3$ only if $h$ is as large as 0.8 for which lensing predictions are larger than 2. There is no age problem in a non-flat universe provided $h \leq 0.75$ for all $\Omega_{M} \leq 1$.

4. CONCLUSIONS

That the Hipparcos data (Reid 1997; Feast & Catchpole 1997) imply that GCs may be as young as $11Gyr$ has raised the hopes to reconcile the age of GCs and the expansion age of the universe. We have studied this matter in this work. As is well known, and as born out by our results, the realization of this hope depends solely on what the value of $H_0$ is. If $H_0$ is as large as the upper limit of the Freedman et al. (1997) value, the age of the universe problem continues to exist in the SM. The problem, however, is now milder than it was before Hipparcos. Previously, it was thought to exist even for moderate values of $h$, whereas it seems to exist for large values of $h$ now. If, however, $H_0$ is as low as favored
by Tammann & Sandage (1996) then the GC and the expansion ages of the universe are consistent with each other in the SM.

Assuming that $H_0$ is high and hence modifying the SM by invoking a (time-independent) cosmological term in the Einstein field equations, as has been done before (Peebles 1984; Blome & Priester 1985; Klapdor & Grotz 1986), we have confirmed the conclusion of previous workers that due to lensing restrictions the age problem still survives in a flat universe for $\Omega_M < 0.5$, and at the same time concluded that $h$ cannot be larger than about 0.75. As for a nonflat universe, we have shown that the age problem does not exist for all $\Omega_M \leq 1$ provided $h \leq 0.75$.

The above mentioned hope is realized in the SM only if $h \leq 0.6$ (see Table 1). Otherwise, some kind of modification of the SM is called for. One such, and the most-studied, attempt is the inclusion of a cosmological term in the field equations. With such a term, the age problem has a better standing in a nonflat (open or closed) universe with $\Omega_M \leq 1$. It should be noted, in the light of recent works, that such a cosmological term need not be a pure time-independent constant. Scalar fields, cosmic strings or some kind of stable textures with an energy density varying as $a^{-2}$ lead to viable cosmological models that stand as alternatives to the SM (Kamionkowski & Toumbas 1996; Spergel & Pen 1997; Özer 1999).

REFERENCES

Al-Rawaf, A. S., & Taha, M. O. 1996, GRG, 28, 935
Blome, H.J., & Priester, W. 1985, Ap&SS, 117, 327
Bolte, M., & Hogan, C. J. 1995, Nature, 376, 399
Carroll, S. M., Press, W. H., & Turner, E. L. 1992, Annu.Rev. Astron. Astrophys., 30, 499
Chaboyer, B., Demarque, P., & Sarajedini, A. 1996, ApJ, 459, 558
Feast, M. W., & Catchpole, R. M. 1997, MNRAS, 286, L1
Felten, J. E., & Isaacman, R. 1986, Rev. Mod. Phys., 58, 689
Freedman, W. L., Madore, B. F., & Kennicut, R. C. 1997, in The Extragalactic Distance Scale, edited by M. Donahue and M. Livio (Cambridge University Press, Cambridge)
Kamionkowski M. & Toumbas N. 1996, Phys. Rev. Lett., 77, 587.
Klapdor, H. P., & Grotz, K. 1986, ApJ, 301, L39
Kochanek, C. S. 1993, ApJ, 419, 12
Kochanek, C. S. 1995, ApJ, 453, 545
Maoz, D., & Rix, H.-W. 1993, ApJ, 416, 425
Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1970, Gravitation (W. H. Freeman and
  Company, San Fransisco)
Özer, M. 1999, ApJ, 520, 45
Peebles, P. J. E. 1984, ApJ, 284, 439
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton University Press,
  Princeton)
Perlmutter, S., et al. 1997, ApJ, 483, 565
Reid, I. N. 1997, AJ, 114, 161
Sandquist, E. L., Bolte, M., Stetson, P. B., & Hesser, J. E. 1996, ApJ, 470, 910
Schwarzschild, B. 1997, Phys. Today, September, 19
Spergel, D. & Pen U. 1997, ApJ, 491, L67
Tammann, G. A., & Sandage, A. 1996, IAU Symposium 168, p.163
Trimble, V. 1996, Nucl. Phys. B (Proc. Suppl.), 51B, 5
Weinberg, S. 1972, Gravitation and Cosmology (John Wiley & Sons, New York)

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Table 1. Maximum values of $h$ in the SM for which $t_0 = 11\, Gyr^a$

| $\Omega_M$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1.0 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $h_{\text{max}}$ | 0.799 | 0.753 | 0.719 | 0.692 | 0.67 | 0.651 | 0.634 | 0.619 | 0.605 | 0.593 |

Note that if $h = h_{\text{max}}$ then $t_0 = 11Gyr$, if $h > h_{\text{max}}$ then $t_0 < 11Gyr$ and if $h < h_{\text{max}}$ then $t_0 > 11Gyr$.

Table 2. Normalized optical depths.

| $\Omega_M$ | $\Omega_\Lambda$ | $P_{\text{lens}}$ |
|------------|-----------------|-----------------|
| 0          | 1.0             | 13.25           |
| 0.1        | 0.9             | 5.98            |
| 0.2        | 0.8             | 3.94            |
| 0.3        | 0.7             | 2.93            |
| 0.4        | 0.6             | 2.33            |
| 0.5        | 0.5             | 1.92            |
| 0.6        | 0.4             | 1.63            |
| 0.7        | 0.3             | 1.42            |
| 0.8        | 0.2             | 1.25            |
| 0.9        | 0.1             | 1.11            |
| 1.0        | 0.0             | 1.00            |
| 1.0        | 1.1             | 1.61            |
| 0.8        | 1.1             | 1.99            |
| 0.6        | 1.1             | 2.57            |
| 0.4        | 1.1             | 3.61            |
| 0.2        | 1.1             | 6.05            |
Table 3. Maximum values of $h$ in a flat universe for which $t_0 = 11Gyr$.

| $\Omega_M$ | $\Omega_\Lambda$ | $h_{\text{max}}$ | $P_{\text{lens}}^a$ |
|------------|------------------|-------------------|---------------------|
| 0.1        | 0.9              | 1.14              | 5.98                |
| 0.2        | 0.8              | 0.96              | 3.94                |
| 0.3        | 0.7              | 0.86              | 2.93                |
| 0.4        | 0.6              | 0.79              | 2.33                |
| 0.45       | 0.55             | 0.76              | 2.11                |
| 0.5        | 0.5              | 0.74              | 1.92                |
| 0.6        | 0.4              | 0.70              | 1.63                |
| 0.7        | 0.3              | 0.67              | 1.42                |
| 0.8        | 0.2              | 0.64              | 1.25                |
| 0.9        | 0.1              | 0.61              | 1.11                |
| 1.0        | 0                | 0.59              | 1.00                |

$^a$Recall that $P_{\text{lens}}$ is independent of $h$ (see equations (12)-(15)).
Fig. 1.— The age of the universe in the SM for $k = -1$ (solid lines), $k = 0$ (dots) and $k = 1$ (dashed lines) versus the present value of the matter density parameter $\Omega_M$. 
Fig. 2.— Contours of $h_{\text{max}}$ for which $t_0 = 11\text{Gyr}$ in the $(\Omega_M, \Omega_\Lambda)$ plane.
Fig. 3.— Contours of $h_{\text{max}}$ for which $t_0 = 11 \text{Gyr}$ in the $(\Omega_M, P_{\text{lens}})$ plane.