Calibration for Axis Misalignment Angles between Photoelectric and Inertial Navigation System of Integrated Landing Navigation System

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Abstract. Aiming at the photoelectric/inertial integrated landing navigation system, this paper proposes a calibration scheme for the axis misalignment angles between photoelectric and inertial navigation system. This paper expounds the calibration scheme, analyzes the influence of various error factors on the calibration accuracy in detail, and gives the computer simulation results. The simulation results show that the calibration accuracy of pitch misalignment angle is better than 30″, the calibration accuracy of azimuth misalignment angle is better than 85”. Therefore, this scheme meets the application requirements of integrated landing navigation system.

1. Introduction
Landing of carrier-based aircraft is a relative navigation problem, which is difficult and risky. Photoelectric/inertial integrated landing navigation system is a reliable landing method. Its principle is to obtain the position and attitude relationship between carrier-based aircraft and ship from images, and calculate the position information of carrier-based aircraft and the motion information of ship by combining inertial navigation information [1-2]. Photoelectric system and inertial navigation system have their own measuring coordinate systems, and there are misalignment angles between measuring coordinate systems of different equipment. If the axis misalignment angles are not calibrated, the precision of integrated landing navigation will be seriously affected.

At present, most of the schemes for calibrating the misalignment angle between photoelectric and inertial navigation system are applied to the micro camera-IMU hybrid sensor system [3-5]. These schemes need to hold the camera-IMU system to collect the calibration target images while moving in complex motion, which is only suitable for micro-UAVs, not for large-sized aircraft.

Therefore, this paper proposes a calibration scheme for the misalignment angle between photoelectric and inertial navigation system applied to the integrated landing navigation system. This scheme can also apply to UAVs based on vision/inertial navigation.

2. Design of the calibration scheme
2.1. Definition of coordinate system
The following coordinate systems are commonly used in the scheme:

(1) Pixel coordinate system \((u, v)\)
The origin is the starting point of the upper left corner of the image. The coordinates of each image point indicate the number of columns and rows in the image matrix, in pixels.

(2) Camera coordinate system \( O_c - X_cY_cZ_c \)

The origin is the camera light center. The optical axis of the camera lens is the \( Z_c \) axis. The \( X_c \) and \( Y_c \) axes are parallel to the \( u \) and \( v \) axes respectively.

(3) World coordinate system \( O_w - X_wY_wZ_w \)

The world coordinate system is a three-dimensional coordinate system formed by vertically fixing a calibration target with feature points on a linear guide rail. The motion direction of the linear guide rail is defined as the \( Z_w \) axis, and the plane of the calibration target is defined as the \( X_wY_w \) plane.

(4) Inertial navigation coordinate system \( O_b - X_bY_bZ_b \)

The \( X_b, Y_b, Z_b \) axes points to the sensitive axes of SINS gyroscope and satisfies the right-hand rule.

(5) Geographic coordinate system \( O_n - X_nY_nZ_n \)

\( X_n \) points to the east, \( Y_n \) points to the north, \( Z_n \) points to the sky and they satisfy the right hand rule.

2.2. Design of the scheme

In this paper, a calibration scheme for the axis misalignment angle between the photoelectric system and the inertial navigation system is designed. The scheme is divided into two parts: static calibration and dynamic calibration. The block diagram of the scheme is shown in figure 2.

![Figure 1. Definition of coordinate systems.](image)

![Figure 2. Block diagram of the scheme.](image)

In the static calibration, the attitude relationship between the world coordinate system and the geographic coordinate system \( R_w^w \) is obtained by measurement. Then the attitude relationship between the camera coordinate system and the world coordinate system \( R_c^w \) is calculated by monocular vision...
pose measurement method [6], thus obtaining the attitude relationship between the camera coordinate system and the geographic coordinate system \( R^c_w = R^c_w R^w_n \).

In the dynamic calibration, the attitude relationship between the inertial navigation coordinate system and the geographic coordinate system \( \Delta R^c_r \) can be obtained after the initial alignment of the inertial navigation system. \( \Delta R \) represents the initial alignment attitude error matrix of the inertial navigation system, which needs to be estimated by motion excitation and Kalman smoothing algorithm.

\( R^c_n \) obtained by static calibration and \( R^w_n \) obtained by dynamic calibration can be used to obtain the attitude relationship between the photoelectric system and the inertial navigation system \( R^c_c = R^c_n \cdot R^w_n \).

2.3. Static Calibration
Static calibration is divided into two parts. The first part is to measure \( R^w_n \). First, the pitch angle \( \alpha_0 \) of the \( Z_w \) axis relative to the horizontal plane and the roll angle \( \gamma_0 \) of the \( X_w \) axis relative to the horizontal plane is measured using a spirit level. Then, the longitude and latitude of multiple points along the extension line of the \( Z_w \) axis is measured by using a high precision differential GPS, and the azimuth angle \( \beta_0 \) of the \( Z_w \) axis relative to the geographic north direction is calculated by using least square straight line fitting method.

\( R^w_n \) can be obtained by the following formula:

\[
R^w_n = R_z(-\frac{\pi}{2}) R_y(\gamma_0) R_x(\alpha_0) R_z(\beta_0)
\]

(1)

The second part is to calculate \( R^c_w \) by monocular vision pose measurement method and Newton iteration method. Monocular vision pose measurement uses a photoelectric detector to complete the pose measurement based on the mathematical model of camera imaging. Newton iteration method is one of the classical methods for approximate solution of nonlinear equations. Its basic idea is to use Taylor expansion of the objective function \( f(x) \) at the iteration point \( x_k \) to find new iteration points, and to repeat this process until a point satisfying the accuracy is obtained.

According to the mathematical model of camera imaging, the objective function of Newton iteration method is selected as follows:

\[
F(X) = \begin{bmatrix}
    u \\
    v \\
    1
\end{bmatrix} - \frac{1}{P} \begin{bmatrix}
    f_x & 0 & u_0 & 0 \\
    0 & f_y & v_0 & 0 \\
    0 & 0 & 1 & 0
\end{bmatrix} \begin{bmatrix}
    R^c_w \\
    T^c_w
\end{bmatrix} \begin{bmatrix}
    X_w \\
    Y_w \\
    Z_w \\
    1
\end{bmatrix}
\]

(2)

In the above formula, the input data is the pixel coordinates of the feature points \([u, v]^T\), the world coordinates of the feature points \([X_w, Y_w, Z_w]^T\), and the camera parameters \( f_x, f_y, u_0, v_0 \). The vector \( X = [\alpha, \beta, \gamma, t_1, t_2, t_3]^T \) to be solved consists of those unknown parameters in \( R^c_c \) and \( T^c_w \).

2.4. Dynamic calibration
In the dynamic calibration, the attitude relationship between inertial navigation coordinate system and geographic coordinate system \( R^c_w \cdot \Delta R \) can be obtained after initial alignment of inertial navigation. \( \Delta R \) needs to be estimated by motion excitation and Kalman smoothing algorithm.

The block diagram of estimating \( \Delta R \) is shown in figure 3. Use high precision differential GPS system and inertial navigation system to measure vehicle motion at the same time. Compare the velocity, position of inertial navigation system and GPS system, and take the difference between them as the
measurement variables of Kalman fixed point smoothing to estimate the initial alignment error angles of the inertial navigation system.

![Block Diagram](image)

**Figure 3.** The block diagram of estimating.

Select the state variables of Kalman smoothing is

\[ X = [\Delta V_E, \Delta V_N, \Delta L, \Delta \phi, \phi_E, \phi_N, \phi_B, \epsilon_x, \epsilon_y, \epsilon_z, \epsilon_x^b, \epsilon_y^b, \epsilon_z^b, \Delta \tau] \]

(3)

In the above formula, the first 12 variables are velocity error, position error, attitude error, gyro drift and accelerometer bias of the inertial navigation system. The 13th variable is the asynchronous time between the inertial navigation system and the GPS system.

The measurement variables are the position difference and velocity difference between the inertial navigation system and the GPS system:

\[ Y = [L_1 - L_G, \lambda_1 - \lambda_G, V_{IE} - V_{GE}, V_{IN} - V_{GN}] \]

(4)

In the above formula, \( L_1, \lambda_1 \) is the latitude and longitude calculated by the inertial navigation system, \( L_G, \lambda_G \) is the latitude and longitude measured by the GPS system, \( V_{IE}, V_{IN} \) is the speed in the geographic coordinate system calculated by the inertial navigation system, and \( V_{GE}, V_{GN} \) is the speed in the geographic coordinate system measured by the GPS system.

The state equation and measurement equation are:

\[
\begin{align*}
\dot{X}(t) &= F(t)X(t) + G(t)W(t) \\
Y(t) &= H(t)X(t) + V(t)
\end{align*}
\]

(5)

Kalman smoothing is divided into two parts: filtering and smoothing [7-9]. The filtering equation is:

\[
\begin{align*}
\hat{X}_{k|k-1} &= \Phi_{k|k-1} \hat{X}_{k-1} \\
P_{k|k-1} &= \Phi_{k|k-1} P_{k-1} \Phi_{k|k-1}^T + G_k Q_k G_k^T + R_k \\
K_k &= P_{k|k-1} H_k^T (H_k P_{k|k-1} H_k^T + R_k)^{-1} \\
\hat{X}_k &= \hat{X}_{k|k-1} + K_k (Y_k - H_k \hat{X}_{k|k-1}) \\
P_k &= (I - K_k H_k) P_{k|k-1}
\end{align*}
\]

(6)

The fixed point smoothing equation is:

\[
\begin{align*}
\hat{X}_{j+k} &= \hat{X}_{j+k-1} + K_k^u (Z_k - H_k \hat{X}_{k|k-1}) \\
K_k^u &= P_{k|k-1}^u H_k^T (H_k P_{k|k-1}^u H_k^T + R_k)^{-1} \\
P_{k+1|k} &= P_{k+1|k}^u (I - K_k H_k)^T F_k^T
\end{align*}
\]

(7)

**3. Static calibration simulation and experiment**

**3.1. Simulation**

The accuracy of monocular vision positioning algorithm to solve \( R^u \) is affected by many factors and needs to be analyzed by computer simulation. The simulation principle is shown in Figure 4. After adding the error to the real value, it is input into Newton iterative program to obtain the solution value of attitude angles. The solution error is obtained by subtracting the solution value from the real value.
The mathematical model of camera imaging Newton iteration method error error error, , , ( , , ) w w w X Y Z 00, xy f f u v, , ++ - 1 2 3, , , , , t t t    ++ Solution error,, , , ,   

Figure 4. Static calibration simulation block diagram.

The simulation conditions are as follows: set the X_w Y_w plane to have 4 rows and 4 columns with a total of feature 16 points evenly distributed at an interval of 9cm, collect images at Z_w1 = 0.01m and Z_w2 = 1.5m respectively, and obtain 32 feature points. The parameters of the camera are f_x = f_y = 1196, u_0 = 160, v_0 = 128, and the image size is 320×256. The relative pose parameters are: \( \alpha = 1^\circ, \beta = 1^\circ, \gamma = 1^\circ \), \( T = [-0.135m, -0.134m, 1.5m]^T \).

The error sources of the monocular vision pose measurement are mainly as follows: the coordinate error of feature points in the pixel coordinate system, the coordinate error of feature points in the world coordinate system, and the parameter error of camera. In the simulation, the above errors are added separately. 100 random experiments are carried out at each random error level, and the mean of 100 random experiments is taken as the result of each error level.

1. Adding random errors to the pixel coordinates of each feature point, the range of random errors increases from \([-0.2, 0.2]\) pixel to \([-2, 2]\) pixel, with an increase of 0.1 pixels. The simulation results are shown in figure 5.

2. Adding random errors to the world coordinates of each feature point, the range of random errors increases from \([-0.2, 0.2]\) mm to \([-2, 2]\) mm with an increase of 0.1 mm. The simulation results are shown in figure 6.

3. Adding constant error to the camera parameter f_x and f_y, the range of constant error increases from -10 to 10 with an increase of 1. The simulation results are shown in figure 7.

From the figure above, it can be seen that with the increase of the error of pixel coordinates and the error of world coordinates, the errors of attitude angles also increase. These errors have influence on \( \alpha \) and \( \beta \), and have greater influence on \( \gamma \). The error of camera parameter have influence on \( \alpha \) and \( \beta \), but have little influence on \( \gamma \) angles. With the increase of camera parameter error, the errors of attitude angles also increase.
The level of each error in the experiment is shown in Table 1. These errors are added to the simulation at the same time to observe the attitude angle error. The results are shown in Table 2.

### Table 1. Error level in the experiment.

| Error level                                      | Level   |
|--------------------------------------------------|---------|
| random errors to the pixel coordinates           | ±0.5pixel |
| random errors to the world coordinates           | (±0.3mm,±0.3mm,±0.1mm) |
| constant error to the parameter of camera         | ±3      |

### Table 2. Attitude angles error.

| Angle       | Error(″) |
|-------------|----------|
| Pitch angle | 21.4516  |
| Azimuth angle | 22.5043  |
| Roll angle  | 173.657  |

#### 3.2. Experiment

In the static calibration experiment, the measurement results of $\alpha_0$ and $\gamma_0$ are shown in tables 3 and 4 respectively. It can be seen that the measurement accuracy of $\alpha_0$ and $\gamma_0$ are 21.98″ and 26.94″ respectively.

### Table 3. The measurement results of $\alpha_0$.

|         | 1       | 2       | 3       | 4       | 5       | Standard deviation |
|---------|---------|---------|---------|---------|---------|-------------------|
| $\alpha_0$ | 0.0630° | 0.0648° | 0.0616° | 0.0647° | 0.0637° | 4.28″             |

### Table 4. The measurement results of $\gamma_0$.

|         | 1       | 2       | 3       | 4       | 5       | Standard deviation |
|---------|---------|---------|---------|---------|---------|-------------------|
| $\gamma_0$ | 0.48°   | 0.50°   | 0.49°   | 0.48°   | 0.49°   | 26.94″            |

The fitted straight line of the $Z_w$ axis is shown in figure 3, and the residual error is shown in figure 4. It can be seen that the residual error is less than 2cm, so the measurement accuracy of $\beta_0$ is better than 38″.

Six experiments with different attitude angles were carried out in the monocular vision pose measurement experiment. The calculation errors of angles $\alpha$, $\beta$, and $\gamma$ are shown in table 5. It can be seen that the measurement accuracy of $\alpha$ is 21.98″, the measurement accuracy of $\beta$ is 18.61″, and the measurement accuracy of $\gamma$ is 112.82″, which can verify the simulation results.
Table 5. The calculation errors of angles.

| Experimental serial number | Error of $\alpha$ (″) | Error of $\beta$ (″) | Error of $\gamma$ (″) |
|----------------------------|------------------------|----------------------|-----------------------|
| 1                          | -28.8000               | 7.2000               | 101.5200              |
| 2                          | -20.3184               | -39.5672             | -17.1847              |
| 3                          | -20.7300               | -6.0447              | 65.2073               |
| 4                          | 5.2165                 | 6.7048               | 10.9152               |
| 5                          | 34.6444                | -1.0374              | 209.0185              |
| 6                          | 0.9130                 | 19.4442              | -133.0865             |
| Standard deviation         | 21.9859                | 18.6102              | 112.8224              |

4. Dynamic calibration simulation
The block diagram of dynamic calibration simulation is shown in figure 10. The trace generator is used to simulate the vehicle motion to generate ideal navigation information. Ideal navigation information adds GPS noise to simulate actual GPS information. Ideal navigation information adds inertial instrument error to simulate gyro and accelerometer information, which is input into SINS calculation to solve the inertial navigation information. Inertial navigation information and GPS information are input into Kalman smoother to estimate the initial alignment error angles of inertial navigation.

The simulation conditions are as follows: initial alignment error angles $\Delta \phi_k = -30''$, $\Delta \phi_n = 30''$, $\Delta \phi_v = 360''$. The gyro constant drift is $2^\circ$/h and the gyro random drift is $0.3^\circ$/h. The constant bias of accelerometer is $100 \mu g$, and the random bias of accelerometer is $50 \mu g$. The constant error of GPS position measurement is 0.1m, the mean square error of random noise of GPS position measurement is 0.1m. The constant error of GPS velocity measurement is 0.03m/s, and the mean square error of random noise of GPS velocity measurement is 0.03 m/s.

The motion path of the vehicle is a rectangular motion path: uniform acceleration linear motion 10s, initial speed 0m/s, and final speed 15m/s; Uniform linear motion for 20s; Turn horizontally for 5s and rotate clockwise for 90 degrees; Uniform linear motion 120s; Turn horizontally for 5s and rotate clockwise for 90 degrees; Uniform linear motion 120s; Turn horizontally for 5s and rotate clockwise for 90 degrees; Uniform linear motion 120s; Turn horizontally for 5s and rotate clockwise for 90 degrees; Uniform linear motion 300s.

The estimated errors of 20 simulations is shown in figure 11. The standard deviation of the estimated error of the pitch error angle $\phi_k$ is $1.72''$ and that of the roll error angle $\phi_n$ is $1.59''$ and that of the azimuth error angle $\phi_v$ is $18.19''$.  

Figure 10. block diagram of dynamic calibration simulation.
Figure 11. Estimated errors of 20 simulations.

5. Conclusion
In this paper, a calibration scheme of the axis misalignment angle between the photoelectric system and the inertial navigation system based on the integrated landing navigation system is proposed. And the influence of various error factors on the calibration accuracy is analyzed in detail. The simulation results show that the calibration accuracy of pitch misalignment angle is better than 30″, the calibration accuracy of azimuth misalignment angle is better than 80″ and the calibration accuracy of roll misalignment angle is better than 200″. In the integrated landing navigation system, the calibration accuracy requirements for pitch, azimuth and roll misalignment angles are 1′, 2′, 0.1°, so the calibration scheme can meet the requirements of the integrated landing navigation system.

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