SMC Chaos Control of a Novel Hyperchaotic Finance System Using a New Chatter Free Sliding Mode Control

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Abstract. In this paper, using a chatter free sliding mode control (SMC) strategy, SMC chaos control of a novel hyperchaotic finance system is discussed. The nonlinear uncertain chaotic systems in the presence have unknown bounded uncertainties and external disturbances. We propose a sliding mode surface with differential operator, which converts the switching term of discontinuous gain function into only one control input. The control input is smooth and time-differentiable. Based on Lyapunov stability theory and sliding mode control method, SMC stability analysis is carried out, and a theorem for designing chattering-free sliding mode control input is obtained. Finally, numerical simulation verifies the effectiveness and correctness of the control strategy.

1. Introduction

Chaos is an interesting physical phenomenon in nonlinear dynamical systems, which has been studied for decades [1-2]. Chaotic control of nonlinear dynamical systems has attracted wide attention in recent years. In the past two decades, many chaotic control methods have been proposed, such as the Ott-Grebogi-Yorke (OGY) method [3], LMI-based non-fragile control method [4], passive control method [5], and backstepping design method [6]. Therefore, chaos control and its application have become a research hotspot in the nonlinear fields.

Sliding mode control (SMC) is a variable structure control. A high-speed switch control law is used to derive the system state trajectory onto a specified user-selected surface, i.e., the so-called sliding surface, and to maintain the system state trajectory on the sliding surface for a subsequent time. SMC is a well-known robust trajectory control method, not the state of direct control system [7, 8]. In this paper, in order to solve the above problems, Lyapunov stability theory and SMC technology are combined to prove the reliability of the strategy [9, 10]. Because the control is smooth and differentiable, the chatter is weakened effectively.

In this paper, a new chatter-free sliding control strategy is applied to a novel hyperchaotic finance system [11]. The system that we first introduced is constructed in that background, where the global economic crisis shows the existence of chaos in the finance system in 2007. From the numerical examples, the hyperchaotic finance system can achieve ideal condition quickly through this method.

The structure of this paper is as follows. In Section 2, a novel hyperchaotic finance system is presented. Section 3 presents the design of chattering-free sliding mode controller. The SMC chaos
control of the new hyperchaotic financial system with uncertain parameters is studied. A numerical example is given in this section. Finally, the conclusions are given in Section 4.

2. System description

More recently, a new hyperchaotic finance system was constructed by [11]. A fractional dimensional autonomous hyperchaotic system can be described by the following differential equation:

\[
\begin{align*}
\dot{x} &= z + (y - a)x + w \\
\dot{y} &= 1 - by - x^2 \\
\dot{z} &= -x - cz \\
\dot{w} &= -dxy - kw
\end{align*}
\]

(1)

where variable \(x\) represents the interest rate in the model, variable \(y\) represents the investment demand, variable \(z\) is the price exponent and variable \(w\) is the average profit margin, \(a, b, c, d, k\) are the parameters of the system (1), and they are positive constants. When the parameters are \(a = 0.9, b = 0.2, c = 1.5, d = 0.2\) and \(k = 0.17\), the four Lyapunov exponents calculated with Wolf algorithm are 0.034432, 0.018041, 0, −1.1499. The Lyapunov fractional dimension of this hyperchaotic system is \(D_L = 3.050121\). Figure 1 shows the 3-dimensional phase portraits of hyperchaotic finance system (1). For more detailed analysis of the complex dynamics of the system, please see relative Ref. [11].

![Phase portraits of hyperchaotic finance system (1).](image)

3. SMC chaotic control of a new hyperchaotic finance system

In this part, we mainly discuss the stability of a nonlinear dynamical system. We will give some assumptions, lemma, and conditions to make sure that the system (1) will get stable.

3.1. Chatter free SMC design [9, 10]

Consider a class of controlled hyperchaotic systems described by the following nonlinear differential equations:

\[
\dot{x} = Ax + F(t, x) + d(t) + u(t)
\]

(2)
where \( x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n \) is the state vector of the system, \( F(t,x): \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^n \) denotes a nonlinear vector function. \( u(t) \in \mathbb{R}^n \) denotes the control input vector, \( d(t) \in \mathbb{R}^n \) denotes the external disturbance. \( A \in \mathbb{R}^{n \times n} \) is a coefficient matrix. The control goal is to design a chattering-free sliding mode controller under given initial conditions so that the asymptotic stability of the system (2) can be realized, i.e., \( \lim_{t \rightarrow \infty} \| x(t) \| = 0 \), where \( \| \cdot \| \) is the Euclidean norm of a vector.

There are two basic steps for SMC design. First, it needs to select a suitable switch surface. Secondly, a control law is needed to ensure the stability of the sliding surface.

Firstly, an integral operator is used to define the sliding surface:

\[
S_i(t) = \eta_i \int_0^t x_i(\tau) d\tau + x_i(t)
\]

where \( \eta_i \) is a positive constant \((i=1,2,\ldots,n)\). In order to obtain smooth differentiable control input and further weaken the chatter, the switching term of discontinuous symbolic function is transferred to the control input. Using the sliding mode surfaces defined above, we propose the following dynamical sliding mode surface:

\[
\sigma_i(t) = \dot{s}_i(t) + \lambda_i s_i(t)
\]

where \( \lambda_i \) presents a positive constant \((i=1,2,\ldots,n)\).

From this we can draw:

\[
\sigma_i(t) = \dot{s}_i(t) + \lambda_i \dot{s}_i(t) = \ddot{s}_i(t) + (\lambda_i + \eta_i) \dot{s}_i(t) + \dot{\lambda_i} s_i(t)
\]

\[
= A \dddot{s}_i(t) + \sum_{j=1}^n \frac{\partial F_i(t,x)}{\partial x_j} \dot{s}_j(t) + \frac{\partial F_i(t,x)}{\partial t} + \dot{d}_i(t) + \dot{u}_i(t) + (\lambda_i + \eta_i) (A \dot{x}_i + F_i(t,x) + d_i(t) + u_i(t)) + \dot{\lambda_i} s_i(t)
\]

where \( F_i \) presents the \( i \)-th row of matrix or vector \( F \).

Secondly, we need to design a sliding mode control strategy to converge the trajectory of the driving system to the sliding mode surface \( \sigma_i(t) = 0 \) \((i=1,2,\ldots,n)\).

**Assumption 1.** In equation (2), the uncertain term \( d_i(t) \) and the derivative of the uncertain term \( \dot{d}_i(t) \) are assumed to be bounded, that is, there exists a positive bounded function \( B_i(x) \) and \( \overline{B}_i(x) \) making the following inequalities hold:

\[
|d_i(t)| \leq B_i(x), \quad |\dot{d}_i(t)| \leq \overline{B}_i(x) \quad \forall x \in \mathbb{R}^n \quad (i=1,2,\ldots,n)
\]

**Assumption 2.** There exists a positive constant \( \xi_i \) satisfying the following inequalities hold:

\[
\xi_i > \overline{B}_i(x) + (\lambda_i + \eta_i) \overline{B}_i(x)
\]

where the parameters \( \eta_i \) and \( \lambda_i \) are the same as that in equation (4) \((i=1,2,\ldots,n)\).

**Barbalat lemma[12].** If \( f(t) \) is nonnegative, integrable (has a finite integral) and uniformly continuous on the interval \([\alpha, +\infty)\), then \( f(t) \) tends to 0 as \( t \to \infty \).

**Theorem 1.** For the nonlinear controlled hyperchaotic system (2), the state vector \( x(t) \) of the system will converge asymptotically to zero, if the dynamics sliding mode control law is designed as follows:

\[
\dot{u}_i = -A \dddot{s}_i + \sum_{j=1}^n \frac{\partial F_i(t,x)}{\partial x_j} \ddot{s}_j - \frac{\partial F_i(t,x)}{\partial t} - (\lambda_i + \eta_i) (A \dot{x}_i + F_i(t,x) + u_i) - \dot{\lambda_j} s_i - \varepsilon \text{sign}(\sigma_i)
\]

\((i=1,2,\ldots,n)\).

**Proof.** Construct the Lyapunov function as follows:

\[
V = \frac{1}{2} \sum_{i=1}^n \sigma_i^2
\]

By calculating the derivative of \( V(t) \) for time, we get

\[
\dot{V} = \sum_{i=1}^n \sigma_i \dot{\sigma}_i = \sum_{i=1}^n \left( \sigma_i (\dot{d}_i(t) + (\lambda_i + \eta_i) d_i(t) - \varepsilon \text{sign}(\sigma_i)) \right) = \sum_{i=1}^n \left( \sigma_i \dot{d}_i(t) + (\lambda_i + \eta_i) \sigma_i d_i(t) - \varepsilon |\sigma_i| \right)
\]

\[
\leq \sum_{i=1}^n \left( |\sigma_i| (|\overline{B}_i + (\lambda_i + \eta_i) B_i)| - \varepsilon |\sigma_i| \right) = -\sum_{i=1}^n \varepsilon (|\overline{B}_i + (\lambda_i + \eta_i) B_i| - |\sigma_i|)
\]
According to formula (6), we can get \( \dot{V} \leq 0 \).

As a result of \( \dot{V} \leq 0 \), we can obtain that of \( \sigma_i \) on time \( t \) is integrable and uniformly continuous on \([0, +\infty)\). According the Barbalat lemma, we get \( \sigma_i \to 0 \) as \( t \to \infty \). So the sliding mode surface is globally asymptotically stable at its equilibrium point \( \sigma_i = 0 \) \((i=1,2,\ldots,n)\). Therefore, equation (4) is equivalent to \( \dot{s}_i = -\lambda_i s_i \), which means that \( s_i \to 0 \) as \( t \to \infty \). So we get

\[
x_i(t) = -k_i \int_0^t x_i(\tau) d\tau, \quad (i=1,2,\ldots,n)
\]

When \( s_i \to 0 \), by differentiability of \( x_i(t) \)

\[
\dot{x}_i(t) = -k_i x_i(t), \quad (i=1,2,\ldots,n)
\]

Therefore, the state vector \( x(t) \) of the system (2) will converge asymptotically to zero. The proof is complete.

3.2. SMC control of hyperchaotic finance system

The system is described by the following hyperchaotic finance system [11]:

\[
\dot{x} = Ax + F(t, x)
\]

where

\[
x^T = (x_1, x_2, x_3, x_4), \quad A = \begin{bmatrix}
a & 0 & 1 & 1 \\
0 & -b & 0 & 0 \\
-1 & 0 & -c & 0 \\
0 & 0 & 0 & -k
\end{bmatrix}, \quad F(t, x) = \begin{bmatrix}
x_2x_3 \\
1-x_1^2 \\
0 \\
-d_3x_1x_3
\end{bmatrix}.
\]

Considering the characteristics of hyperchaotic finance system, so as state variable \( x_3 \) is stabilized to zero, then the others state variables will automatically converge to zero. Therefore, we must design the control input \( u_3 \), and then it will be placed on the right side of the third equation in equation (7) so that all states can be stabilized to its origin.

Consider adding an uncertain disturbance \( d(t) \) to the right of the system (8). Uncertain hyperchaotic financial systems can be written as follows

\[
\dot{x} = Ax + F(t, x) + d(t) + u
\]

where \( d(t) \) and \( u \) be represented as \([0,0,d_3(t),0]^T\) and \([0,0,u_3,0]^T\), representing uncertainties and control inputs, respectively. In addition, it is assumed that \( d_3(t) \) satisfies conditions required to ensure that the system defined in has a unique solution in the interval \([t_0, +\infty), t_0 > 0\), for any given initial condition.

In this example, let uncertainty \( d_3(t) \) be \( \sin(t) \), according to the above assumptions, the control parameters could be designed as \( k_3 = \lambda_3 = 2 \), \( B_3(x) = B_3(x) = 2 \) and \( \epsilon = 10 \).

According to (3), the sliding mode surface \( s_3(t) \) is designed as follows

\[
s_3 = 2 \int_0^t x_3(\tau) d\tau + x_3(t)
\]

thus

\[
\sigma_3 = \dot{s}_3 + 2s_3
\]

According to formula (7), the dynamics sliding mode control law is designed as follows:

\[
\dot{u}_3 = -\dot{s}_3 - c_3s_3 - 4(-s_3 - c_3s_3 + u_3) - 4s_3 - 10\text{sign}(\sigma_3)
\]

Then system (9) satisfied all conditions in Theorem 1. Thus, the sliding surface of the uncertain hyperchaotic financial system (9) is globally asymptotically stable at its equilibrium point.

To verify the effectiveness of a chatter free SMC method, in the numerical simulations, the initial values of the hyperchaotic system chosen as \([x_1(0), x_2(0), x_3(0), x_4(0)] = [-1, -2, -3, -4]\), the step size
\( \tau = 0.001 \). Figure 2 shows the sliding mode surface \( s(t) \). Figure 3 shows the dynamic sliding mode surface \( \sigma(t) \). Figure 4 shows the response of control input \( u(t) \) with time \( t \).

![Figure 2](image1)

**Figure 2.** Simulation results of the sliding mode surface \( s(t) \).

![Figure 3](image2)

**Figure 3.** Simulation results of the dynamic sliding mode surface \( \sigma(t) \).

![Figure 4](image3)

**Figure 4.** Simulation results of the response of control input \( u(t) \) with time \( t \).

### 4. Conclusions

Aiming at the control problem of hyperchaotic finance system, a new chattering-free sliding mode control strategy is proposed. Based on the Lyapunov stability theory and sliding mode control method, a sliding mode controller is designed from state vector to a desired point in the state space. The simulation results have confirmed the effectiveness of the proposed scheme to control the hyperchaotic finance system. The control is smooth. It shows that the designed controller has lower cost and complexity.

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