Simple Mass-estimates for Resonance(s) being 6 Top plus 6 Anti top Bound states and Combinations thereof

H.B. Nielsen *

The Niels Bohr Institute, Copenhagen, Denmark

Abstract

We have long speculated [50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 2], that 6 top + 6 anti-top quarks due to the relatively large size of the top-yukawa coupling would bind exceptionally strongly by mainly Higgs exchange. Here we present a surprisingly simple “calculation” of the mass of this speculated bound state. Even a possible resonance in scattering of two such bound states is speculated. For the “calculation” of the masses it is crucial to assume, that our since long speculated principle “Multiple Point Principle” [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18], is true. This principle says: there are several vacua all having almost zero energy density. Further we make an approximation of the Higgs Yukawa potential essentially replacing the exponential in it by a step-function. The new result means that there are now two independent calls for our bound state having the mass around 750 GeV required by our “new law of nature” the Multiple Point Principle. It should be remarked that in our picture there is no new physics in the sense of new fundamental particles, but the “Multiple Point Principle” is new in the sense of being not yet accepted. Further we get the same mass within uncertainties as earlier [2] but now from a completely different assumption, except for being from our “multiple point principle”. But the two masses are gotten from using different (speculative) vacua occurring in the pure Standard Model.

1 Introduction

We - especially C.D. Froggatt Larisa Laperashvili and myself - have long been speculating on a very strongly bound state [50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 2] of 6 top and 6 anti top quarks, being held together mainly by exchange of Higgs bosons in a picture connecting it with our (and Don Bennetts also) principle of degenerate vacua [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18]. Mostly we just claimed that we assumed/imagined, that this bound state were very strongly bound, meaning that its mass should be appreciably smaller than the collected mass of its 12 constituents $12m_t = 2076\text{GeV}$. However, C. D. Froggatt and I [20] in some appendices mainly achieved an estimate of the order of 285 GeV for the mass of this bound state. This estimate were strongly based on the assumption of the degeneracy of the present vacuum with a vacuum, in which there is a lot of copies of the bound state, the “condensate” vacuum. It is the main purpose of the present article to redo in a somewhat more direct way the calculation, made together with Colin Froggatt. As a little fun thing some corrections approximately cancel each other in such a way that we can get a rather simple
calculation at the end, and the very simple result that the mass of the bound state very crudely has the mass $4m_t$, where $m_t$ is the mass of the top quark.

The great interest of this crude estimation of the mass for our hoped for bound state from the mentioned degeneracy assumption of the “present” and the “condensate vacuum” is that recently Laperashvili, Das, and myself connected [2, 1] the mass of our bound state with the rather little difference in energy density between the present vacuum, and the vacuum corresponding to the high Higgs field minimum in the effective potential for the Higgs field. And remarkably, we found within estimation uncertainty the same mass in the two different ways of estimating the mass of the bound state. It should be stressed and understood, that we have a picture, in which we speculate there to exist three - and most importantly degenerate - vacua, and then we calculate the mass of our bound state in two different ways, namely requiring the present vacuum degenerate with the “condensate” giving - in the present article - the value $4m_t$, and with the “high Higgs field vacuum” giving about 710 GeV or 850 GeV also rather uncertain though. It is the remarkable compatibility of these two quite independent mass estimates, which is our main point! If this is not just an accident, then there must be something about both the bound state and our Multiple Point Principle about degenerate vacua!

This “Multiple Point Principle” causes restrictions between coupling constants and thus potentially serves as a candidate for a “solution” to fine tuning problems[55, 56]. This “Multiple Point Principle” (=MPP) says, indeed, that there are several vacua with extremely small energy densities. We could also say, that it means that the universe-vacuum is just at some multiple point, where several phases can coexist, much like one at the triple point for water has coexistence of ice, fluid water, and vapor for a common set of intensive variables, pressure and temperature. There may be no real good derivation or argument for our multiple point principle in spite of the fact, that we and others have published some attempts to derive this principle [5, 18, 19, 3, 4], but all such arguments would have to involve some influence of the future on the passed, or at least on the coupling constants, and that would make all such derivations of MPP (=“Multiple Point Principle”) suspicious. The reader should rather take some previous works - even prediction(s) - as well as the results of the present work as empirical evindence for this new law of nature, the “multiple point principle”.

The calculation to be presented below is indeed just a slightly renewed version of the calculation delivered in the appendices of our earlier article [20], in which it is heavily used, that there should exist a new vacuum degenerate in energy density with the present one and with an approximated structure, as if the S-particles (what we called our bound state, which so much happened to fit the by now digamma fluctuation called in its fashion F, that we shall call it F now ) were sitting as the carbon atoms in the diamond chrystal, as we shall review in section 3. Colin Froggatt and I ended in these appendices [20] with a mass 260 GeV for the bound state, but we certainly did not believe our mass prediction 260 GeV to be very accurate. For the trustworthiness of our whole story the recent work of Larisa Laperashvili, Chitta Das and myself [2] relating the mass of the speculated bound state to the degree of instability/negative energy density of the second minimum in the Standard Model Higgs field effective potential calculated without inclusion of our bound state. The point is that in order to achieve just zero energy density (as our Multiple Point Principle requires) for the vacuum represented by the second minimum the correction required is just getting right for the mass of the bound state F being in a range very close to 750 GeV. We shall return to this work in the last subsection in the conclusion 9.2.

In the following section [2] we shall review our model of there existing an exceedingly strongly bound system of 6 top + 6 anti top quarks, and of our “multiple point principle” fine tuning the coupling constants, so that for instance a condensate of the bound state can fill the vacuum...
and cause a “new vacuum” with the energy density just finetuned to be again remarkably small, of the same order as say the astronomical observation of the energy density (= cosmological constant) of the vacuum, in which we live. (This astronomically observed cosmological constant is quite negligible compared to the energy densities of any significance for high energy physics parameters such the bound state mass or the Higgs mass). A subsection 2.1 of this section 2 is assigned to our “new law of nature”, “Multiple point principle”. In the next section 3 we then model in a very crude approximation and in a non-relativistic picture the just mentioned new vacuum in our model called the “condensate vacuum” as containing a bose-condensate of the F bound states by suggesting as a very crude approximation, that this vacuum has a system/a lattice of F(750) particles interacting with their neighbors contained in the vacuum. We take the “atoms” = the F’s in this lattice to interact in analogy with the carbon atoms in a diamond chryystal. In order to compare the interactions and the binding energies we ignore the effect, that when a top quark goes around /is bound to a swarm of with the same radius bound quarks and antiquarks, it only “feels” the force from about half the number of particles in the swarm. However, we argue in section 4 that the exchange of what we call “eaten Higgses”, and which really is exchange of the longitudinal components of weak gauge bosons W and Z, happens with help of gluons also just accidentally to cancel this effect. It is very important for the success of our whole picutre of bound states and a vacuum condensate numerically, that inside the bound state F as well as in the condensate vacuum the effective Higgs-mass is appreciably lower than the Higgs mass of 125 GeV observed experimentally. Since, however, the Higgs field expectation value inside the bound state and inside the condensate vacuum is significantly lower than in the usual vacuum, the Higgs self-interaction indeed cause a smaller Higgs mass effectively in these places with many top and anti top around on the average. This diminished effective Higgs mass is discussed in section 5. In section 6 we then for simplicity make the very crude assumption of approximating the exponential factor \( \exp \left(-\frac{m_{\text{effective Higgs mass}}}{r}\right) \) in the Higgs-Yukawa-potential by a step-function, a \( \theta \left(\text{number} - m_{\text{effective Higgs mass}}\right) \) meaning, that we put the Higgs mass to zero for small distances, while we put the Higgs Yukawa potential to zero for large distances. Next in section 7 we “calculate” or rather very crudely estimate the mass of the bound state F from the requirement of the MPP assumption of the equality of the energy density of the “condensate vacuum” and the vacuum, we live in. So our new principle MPP is really cruical for our mass prediction!

In section 8 we also with the same picture discuss the at LHC actually first found possible resonance -of mass 1.8 TeV - seen(?) decaying into weak gauge bosons(it is very doibtful). We take this resonance to be composite from a couple of F’s as very weakly suggested from the mass of the 1.8 TeV (we shall see combining the present work with our earlier work with Laperashvili and Das [2] that an F-mass around 800 GeV is called for) possible particle being crudely twice that of F. A little problem for our interpretation of the 1.8 TeV state this way may be its relatively small width observed. The problem is dicussed a bit in the subsection 8.1.

In section 9 we review and comment our result. In the subsection 9.2 we summarize, that the value for the bound state estimated in the present article - developping the result of [20] - and the value for the bound state needed for a quite different multiple point principle requirement coincides remarkably!

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1 Our full picture has actually even more vacua, e.g. one more in pure Standard Model here called “High Higgs field vacuum”, because the Standard Model Higgs in that vacuum has a magnitude of the order of \(10^{18} \text{ GeV}\).
2 Bound State Picture and “Multiple Point Principle”

The crucial suggestion behind our bound state model of 6 tops + 6 anti tops that since Higgs exchange like any other even order tensor particle exchange delivers attraction between top and top, or top and anti top, or anti top and anti top as well, we get stronger and stronger binding between the top and anti top quarks the more of them we imagine brought together. It is because the top and anti top are the strongest binding quarks, that this type of binding becomes most relevant for the top and anti top. Now, however, the quarks are fermions and thus you cannot just unlimited clump arbitrarily many, e.g. top quarks, together. Since the top quark has a color degree of freedom taking three values, say: red, blue, and yellow, and a spin degree of freedom, that can be up and down, one can bring \(3^*2=6\) top quarks into the same orbital state, but because of fermi-statistics no more. So there can in a single orbital state be just up to 6 top + 6 anti top. Thereby a closed shell is so to speak formed (in the nuclear physics sense). In the zero Higgs mass approximation, which will be effectively valid, when the size of the bound state - the radius - multiplied by the effective Higgs mass is small, the attraction between the top-quarks or between tops and anti tops is quite analogous to that between an atomic nucleus and an electron. So we can for first orientation use the terminology from the quantum mechanical description of atomic physics. Approximating the bound state, that we suggest to be possible to form from 6 top + 6 anti top by thinking of each top or anti top going around a collected object formed from the other 11 quarks, we can talk about different “orbits” in the atomic terminology of a main quantum number \(n\) taking positive integer values and further \(l\) (the orbital angular momentum magnitude being \(\sqrt{l(l+1)}\)) and \(m\) (the angular momentum around the quantization axis). As in atomic physics the particles in the \(n=1\) orbit are bound strongest. Analogous to the helium atom having especially high excitation energies, we have because of the color factor 3 and both quark and anti quark an especially stable system being a bound state of 6 top and 6 anti top quarks.

Whether the binding of such a system of 6 top + 6 anti top now is sufficiently strong to even bind to form a resonance, with a rather small mass (as we shall need say about 750 GeV) compared to the collective mass of 6 top + 6 anti top, 12 \(m_t = 12*173 \text{ GeV} = 2076 \text{ GeV}\), is controversial,[83, 84, 85]. However, we think ourselves,[53], that making use of a long series of corrections, especially also exchange of the other three components of the Higgs than the as simple particle observed component, we can stretch the uncertainties in the calculation so far as to allow such a light bound state to be possibly formed.[53]. These other components of the Higgs are really present in the Standard model as W’s and Z longitudinal components. We call them “eaten Higgses”, but really of course it just means to include weak gauge particle Z and W exchanges.

It is important for our hope, that the bound state can indeed bind so strongly, that it gets so tightly bound, that the strong Higgs fields inside the bound state even can modify the effective mass of the Higgs significantly there. We estimated that a top-Yukawa coupling of \(g_t = 1.02 \pm 14\%\) would be just sufficient to bind an extremely light bound state of the 6 top and 6 anti top, and would match with the experimental top-Yukawa \(g_t = 0.935\). But Shuryac et al. [83, 84, 85] find, that due to the high Higgs mass, it cannot bind at all for the experimental value of \(g_t\).

2.1 MPP

The whole speculation about our bound state of 6 top + 6 anti top is a priori rather much taken out of the air by itself. However, we have all the time proposed it only connected with another
speculation, the “Multiple Point Principle”. This is, you could say, a wild guess about simplifying the fine tuning problems of the Standard Model. In order to formulate just the cosmological constant problem about, why the cosmological constant (= the vacuum energy density) compared to say Planck scale dimensional expectations is so enormously small, one needs an assumption of the form “The energy density of vacuum is extremely small!” Now you could look at the “Multiple Point Principle” as an extension of this anyway needed assumption, without really complicating it severely: “Several vacua have extremely small energy densities!”\(^2\) We almost just have put the anyway needed assumption into “plural”, or changed the “quantor” from “The physical vacuum...” to “Several vacua...”.

Now the real supporting point for this principle is, that although it is not unnecessarily complicated, it is the one, which Colin Froggatt and myself managed to use to make historically\(^9\) in 1996, long before the Higgs particle were found, to make a prediction of the Higgs mass of 135 GeV ± 10 GeV. Now our prediction using the same Multiple Point Principle would be 129.4 GeV\(^4\) but with a much smaller uncertainty, comparable to the experimental uncertainty of a few hundred MeV. So at first it then looks, that while our original prediction agreed perfectly within errors, and the Multiple Point Principle were perfectly right, it is today deviating of the order of three standard deviations from matching experiment. This formal disagreement of the theoretical prediction actually occured in spite of, that the better calculations and better top mass moved our prediction closer to the experimental value 125 GeV during the time we had predicted it. It is of course only possible that in spite of this development the agreement relative to the uncertainty could become worse, because the uncertainties in calculation and top and Higgs masses went down even faster. However, L.V. Lapershvili, C. Das and myself\(^2\) found, that the existence of the bound state F of the 6 top + 6 anti top would make a little theoretical correction to the mass of the Higgs being predicted from the multiple point, so that the agreement might indeed be improved to be perfect, if the mass of this bound state is appropriate. A mass of the bound state \(\sim 800\) would be fine for correcting the Higgs mass to be observed to agree with a perfect degeneracy of the vacua.

It should be stressed, that this successfull Higgs-mass prediction as well as Colin D. Froggatts and mine controversial argument, that the top-Yukawa-coupling \(g_t\) in order to allow for a condensate of bound states of 6 top and 6 antitop with energy density close to zero, must be close to the value \(1.02 \pm 14\%\) supports the “Multiple Point Principle” as being a principle upheld by nature. The value \(g_t = 1.02 \pm 14\%\) namely matches with the experimentally determined Higgs Yukawa coupling \(g_t = 0.935\). Really we just estimated, what the top-Yukawa coupling should be in order, that the bound state assumed to exist of 6 top + 6 anti top should have exceptionally low mass. But this should be approximately needed to have the condensating particle have mass close to zero in order for there being two degenerate vacua as required by MPP(= “Multiple Point Principle”).

This means, that even if the theoretical arguments for the MPP are not totally convincing, then there is some empirical evidence pointing in favor of this MPP. And the present article is meant to provide one more such indirect phenomenological support for MPP.

3 Modelling the Condensate Vacuum

Since it is very difficult to treat bound states by the true Bethe-Salpeter\(^87\), we tend to use instead non-relativistic approximations. In spite of the fact that we consider the “condensate vacuum” a condensate of these bound states, and that they thus approximately all should be in

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\(^2\)I thank Leonard Susskind for the remark behind this argument for MPP
the same quantum single-particle-state, we tend to think that the exact boson condensate is less important than that the particles have the mutual configuration diminishing the energy density. We thus suppose that for the purpose of estimating the energy density of such a vacuum it is advisable to use rather some crystal model for this vacuum, so that the way the particles may have arranged themselves relative to each other is hopefully more realistically taken into account. Since the constituents - the quarks and anti quarks - are fermions, we imagine, that the different bound states cannot penetrate into each other more deeply than this fermi-statistics allows. The fermions in one F neighbor to an other one cannot go deeper in than to the level with the main quantum number (in the atomic physics language) \( n=2 \). The \( n=1 \) level is namely occupied by the constituents of the first F. Now there are for \( n=2 \) four orbital states, one 2s and 4 2p-states, each of which in the case of say top quarks can contain \( 2^*3 = 6 \) allowing 4 *6 =24 top quarks as constituents in the closest neighbor F’s to the given one. There are of course also similarly place for 24 anti top quarks in these neighbors. So there is in the closest layer of F’s around a given one place for \((24 +24)/12 = 4 \) neighbors. Thus the analogy with a material with tetra-valent atoms (=having configuration number 4) such as carbon is called for. Therefore diamond is a candidate for a model for this “condensate vacuum”. Colin Froggatt and I already used in an appendix in our article [20] such a diamond model. Via slightly indirect arguments we then very crudely reached a mass- prediction of 285 GeV for the bound state. In the present article the estimate is made a bit more directly, and the result comes a bit more like 690 GeV.

This analogy-choice means, that we decided to evaluate the energy density of the “condensate”, or perhaps rather a crystal of F particles sitting in a diamond shaped lattice attractted most importantly to their four nearest neighbors F’s. The attraction potential is suggestively approximately estimable by considering an F neighbor to a given one having its 12 top and anti top quarks going around the latter in the main quantum number orbit \( n=2 \). If one could use pure atomic physics and ignore the Higgs mass, it would be wellknown that the binding energy in such an \( n=2 \) orbit is just 1/4 of the binding energy in the \( n=1 \) orbit, the Rydberg. The binding energy of one top quark or of one anti top in an F is of course, if it consists of 12 quarks or anti quarks of the same sort 

\[
\frac{12m_t - m_F}{12} = \frac{12*173 \text{ GeV} - 750 \text{ GeV}}{12} = 111 \text{ GeV}
\]

for the case of top-quarks with mass \( m_t = 173 \text{ GeV} \) and an ansatz of 750 GeV for the mass of F as representing the two slightly different estimates given below for what our earlier work suggests[2], 710 GeV and 850GeV. If we ignore the screening effect, which is claimed to be just compensated for in section[3] the binding of top quark say due to a neighboring F in the diamond-like lattice is thus just 1/4 of this 111 GeV, i.e. it is 28 GeV (really 27.65).

Our calculation below now consists in observing, that such 28 GeV per constituent top or anti top means, that a full F is bound to a neighbor by 12 * 28 GeV = 331 GeV. Now there are four neighbors but if we use that, we double count by a factor 2, and so at the end the binding per F-particle in the crystal believed to be an alternative vacuum of the type suggested by MPP is 4/2 times this 331 GeV. That is to say, that the binding between neighbors in the diamond-like crystal per F runs up to 4/2 * 331 GeV = 662 GeV. If this is taking into account our very crude estimates just equal to a mass 750 GeV for the bound state. Thus in our diamond crystal model for the “condensate vacuum” we find that the Einstein(/mass) energy 750 GeV of the F in the supposed vacuum is just cancelled by the binding to their neighbors, accurately as our multiple point principle (MPP) requires!

This is a coincincence, that only happens for the mass of the resonance having the right value. But it is of course, if taken seriously, an evidence for our hypothesis the Multiple Point Principle.

Below in section[7] we shall calculate, what mass is it, that is required for this cancellation of binding energy against the Einstein energy of the F-particles thus allowing the energy density of the “condensate vacuum” to be zero.
4 An Accident Making Calculation Easy

The most important (attraktivational) interaction causing the bound states such a low mass as \( \sim 750 \text{ GeV} \) say is in our picture the exchange of Higgs particles, meaning that it is the Yukawa potential. Most simply we would make an approximation, in which the top and anti top constituents are kept together by a Higgs field centered around the “center” of the F-particle (= the bound state(which we previously called S). Then in order that this approximation should be a good one for a top or an anti top far away from the at (relatively) long distance from this “center”, we must make the strength of the Higgs Yukawa field around this “center” to be 12 or 11 times that of only one top or anti top. However, if at shorter distance we want the true strength of the Yukawa potential, we should rather only use as the strength of this the potential from this “center” being the half of that, i.e. as the one from rather 11/2 or 12/2 tops or anti tops. In other words, if we decide to work in the crude approximation of the potential being simply mathematically a Yukawa potential around a “center”, i.e. having the form

\[
V(r) = K \exp(-m_{H\text{eff}}r)/r,
\]

then the constant \( K \) should rather be proportional to 11/2 than to 11, when there are 12 constituents in the bound state F. Indeed if all the 11 other constituents than the one considered were concentrated at origo we would have

\[
\text{With no smaer out: } K = \frac{11(g_t/\sqrt{2})^2}{4\pi}, \tag{2}
\]

while, if we, as we must realistically take it, have that on the average the constituents contributing to the attraction of a given quark or anti quark are only the half of them being closer to the center than the given quark itself, then the Yukawa coupling coefficient \( K \) should rather be

\[
\text{In the average distance: } K = \frac{11/2 \times (g_t/\sqrt{2})^2}{4\pi}. \tag{3}
\]

This complicates the calculation, especially since truly the potential inside the bound state is not of the simple mathematical form \([1]\), but rather is a sum over several Yukawa potentials, one for each of the constituents.

Now, however, we want in the present article aiming at a surprisingly simple “calculation” to consider a somewhat accidental compensation of this reduction in the Higgs field strength due to the smear out of the center by the constituents away from the very center by some corrections. The corrections which Froggatt and I considered in the article \([53]\) and which could potentially help are:

- Exchange of gluons
- What we call u-channel exchange.
- Exchange of “eaten Higgses”.

These are various interactions studied by us in the article \([53]\), which for the case of the two first ones at least can in the limit of small Higgs mass be included simply by replacing the coefficient \( K \) of the Yukawa potential by a larger value.
In our work \cite{53} we in fact correct the simple (t-channel) - which is one to use, if the interacting quark is not exchanging quantum numbers under the Higgs exchange like in a u-channel scattering and the color field is screened and other exchanges ignored - Yukawa potential

$$V_{t-ch.}(r) = \frac{11/2\times(g_t/\sqrt{2})^2}{4\pi r}$$

by replacing it by a \(V_{total}\), which is given by

$$V_{total} = V_{gluon} + V_{with\ u-ch.},$$

where

$$V_{with\ u-ch.} = V_{t-ch.} + V_{u-ch.}(r) = -\frac{11/2 \times (g_t/\sqrt{2})^2}{4\pi r} + \frac{5/2 \times (g_t/\sqrt{2})^2}{4\pi r}$$

(5)

while

$$V_{gluon} = \frac{g_s^2 Tr(\lambda_n/2\lambda_n/2)\lambda_n}{4\pi Tr(I)\lambda_n r} = \frac{g_s^2}{4\pi} \frac{8/2}{3} = \frac{e_{tt}^2}{(4r)}.$$  

(6)

With \(g_s^2/(4\pi) = 0.118\) we got \(e_{tt}^2 = 1.83\).

This means that the binding energy (4) should be corrected to include the gluon exchange force by substituting

$$16g_t^2/4 \rightarrow e_{tt}^2 + 16g_t^2/4.$$  

(8)

However, even though this compared to the t-alone \(V_{t-ch}\) corresponding to \(11g_t^2/4\) is already roughly a doubling, the potential does at first to see not change its shape, and thus at first the effect of the effective charge being distributed, rather than concentrated in the center exactly, has not been corrected for. Thinking a bit more this is, however, not quite true because: The gluon exchange is actually quite absent, when we ask for the exchange force between two neighboring F-particles, because the F’s are from outside seen colorless. So when asking for the potential keeping the tops and anti tops together inside the F-bound state, the gluon force is there, but when asking for the potential between the two F’s interacting with each other, only the Higgs exchange is present. This effect gives in fact some compensation for the mistake one would do by ignoring the smearing out of the central attractor (smearing out of the analogue of the atomic nucleus in atomic physics).

It is more obvious that the third type of correction - “the eaten Higgs exchange” - is \textit{not of the same shape} and thus can make a difference. The point is that, when an “eaten Higgs”, meaning really exchange of a longitudinal W or \(Z^0\), is exchanged, the top will go into a different state, such as a left b-quark state instead. Such a changed state does not bind equally strongly as the then missing top or anti top would have done. This means that the modified top or anti top being the left bottom quark or left anti bottom should rather quickly be brought back to become a top or anti top, if a large energy increase shall be avoided, and we of course think of the ground state. But some amplitude for there being a component of the modified top or anti top is calculable in second order perturbation theory. We may think of this interaction expressed just in terms of top and anti top effectively taking into account the second order perturbation, wherein the “eaten Higgs” has been exchanged \textit{twice}. Such a double eaten Higgs exchange effectively represents, rather than the long range Yukawa type interaction, a shorter range interaction having rather the form \(\propto 1/r^2\) than the Yukawa going rather \(\propto 1/r\). So the eaten Higgs correction will change the effective shape of the potential. It namely contributes mainly only at short distances \(r\).

At formally very small distances the three eaten Higgses couple very similarly to the un eaten one, so one should think it would be like \(g_t^2\) were increased by a factor 4. But as the distance
gets larger the eaten Higgs attraction will completely disappear. But taking it more in conformity with the speculations in our work\cite{[53]} it is the combined effect of two Higgses (eaten or not) which goes by a Feynman diagram with four $g_t$ factors, that goes up by the factor 4 being the number of eaten plus uneaten Higgses compared to the only one uneaten Higgs.

If we take this correction of the coupling square $g_t^2$ to mean that it gets increased by a factor 2 in the tight region, while there is approximately no change due to the eaten Higgses for large distance $r$, then we could say: The $g_t^2$ effectively to be used for the main quantum number $n=1$ should be increased, at most though up to being doubled, while the force in the main quantum number $n=2$ orbit should be essentially unchanged by the eaten Higgses. This would potentially compensate the effect of the only $11/2$ or $12/2$ instead of $11$ or $12$ due to the screening, that half the attracting quarks or anti quarks are outside the quark, say, to be attracted and thus do not provide any attraction. However, as just said the factor 2 increase of the effective $g_t^2$ due to eaten Higgses were rather an upper bound than the most honest estimate. But now it is then very good for obtaining an approximate compensation of making the error of ignoring the smear out of the cloud of the attracting particles, that also the gluon exchange gives a contribution to the attraction of the tops and anti tops being bound to form the F, while NOT contributing to the attraction between neighboring F’s.

Now what we are really going to use our “accidental” cancellation of the smear out of the consisuent now is to obtain the ratio of the binding energy of a constituent in the $n=2$ orbit to be just $1/4$ of that in the $n=1$. Of course as long as the effeective Higgs mass would be positive, there would always be an error in this use because the $n=2$ binding energy will be numerically suppressed more the effective Higgs mass than the $n=1$ orbit. The rudiment of the Higgs mass non-zero effect will also contribute to counteract to the ignored effect of the smearing out of the constituents.

So as a crude, more or less accidental, cancellation we shall simplify our calculation by taking it, that the eaten Higgs correction helped by a couple of smaller effects drops out against the effect of half the quark or antiquark sources being outside for the $n=1$ orbit. This then means that we can formally allow ourselves to calculate as if there were no sources-outside-effect and no eaten Higgs effect, and even as if the gluon attraction had same coupling in both $n=1$ and $n=2$, being totally absorbable in the Higgs exchange. So we could go on very simply both ignoring gluons - absorbed into Higgs - and the smearing out of the attracting cloud, being compensated for by other effects.

This would mean, that we could calculate as if: Each quark were bound by a completely central point Yukawa potential and the very same strength could be used then for all the orbits $n=1, n=2, and so on.$

That is to say we could claim, that after this cancellation - a bit accidentally - the binding energy of say a quark in the zero Higgs mass limit would be just bound by a factor 1/4 in the $n=2$ orbit compared to its binding in an $n=1$ orbit.

5 Higgs-field and Higgs Effective Mass in the Different Vacua

The zero Higgs mass approximation is, however, not so obvious, and if we cannot use it, then of course the binding in the $n=2$ orbit might not even be 1/4 of that in the the $n=1$ orbit as we claimed after our assumed cancellation, see section $[4]$.

We already mentioned, that we here would make another rather drastic approximation:

We would replace the exponential factor in the Yukawa potential by a theta-function like function. That is to say we would in the different situations, meaning respectively
• inside the condensate of F’s, and
• in the ordinary/physical vacuum, in which we live.

use different effective Higgs masses and thus different theta-like functions for the exponential factor in the Yukawa-potential

6 Crude But Easy Treatment of the Yukawa Potential Due to Higgs Boson

Since the average Higgs field in the condensate vacuum is clearly smaller than in the ordinary or physical one, the effective Higgs mass will due to self-interaction easily be seen to be also smaller in the condensate than in the ordinary vacuum. This is seen from the following:

• Since say the top quark gets its mass from the interaction with the vacuum Higgs field, it is obvious that a diminishing numerically of the Higgs field just around a top quark would lower the energy/mass. This possible adjustment to lower the energy is in fact what brings about the Yukawa potential around say a top quark. To minimize the energy by adjusting the Higgs field around a top it actually pays energetically to let the Higgs field diverge infinitesimally near to the top, but in the surrounding region the Higgs field further and further away go back to its usual vacuum expectation value. This is how the Yukawa potential comes about.

• If the Higgs field in some region remains very close to some value \( \phi_0 \) say, then small deviations in the Higgs field \( \phi_H \) from this \( \phi_0 \) will behave as if the Higgs mass squared were

\[
\frac{\partial^2 V_{\text{eff}}(\phi_H)}{\partial \phi_H^2}|_{\phi_H=\phi_0}.
\]

Now the Higgs effective potential actually has such a form, that when the value is numerically lowered the second derivative also becomes lower, end even at some point becomes zero and then negative. It is wellknown that for zero Higgs field the effective Higgs mass square in our sense here is the tachyonic Higgs mass square, i.e. negative.

That of course in turn means that the zero Higgs mass approximation gets better in the condensate than in the ordinary vacuum. It is therefore possible and we shall assume that - hopefully after a numerical estimate, that it happens to be so approximately - while for the condensate vacuum the zero Higgs mass can be used including the \( n=2 \) orbit, i.e. for orbit \( n=1 \) and \( n=2 \), in the ordinary vacuum only the \( n=1 \) orbit allows the zero Higgs approximation, for the higher ones we let the Yukawa potential completely be approximated by zero(this corresponds to infinite Higgs mass relative to the inverse radius for these higher than or equal to \( n=3 \) orbits).

The here suggested treatment of the Yukawa potential is not exactly to take it as a step function, but it is approximately so.

To summarise the rule suggested - and to be confirmed by some estimations - :

• Condensate vacuum In the condensate vacuum we take the Higgs mass zero for orbits \( n=1 \), and 2, and huge, say infinite, for \( n=3,4,... \).

• Present vacuum while in the vacuum, we live in, we instead take Higgs mass zero (only) for \( n=1 \), while infinite or huge for \( n=2,3,4,... \).
7 Calculation of the Mass of the F Bound State

The basic requirement used in the present “calculation” of the mass of bound state \( S = F \) of 6 tops + 6 anti tops is, that in a presumably too naive nonrelativistic thinking the energy density of the condensate vacuum shall be zero. Of course this zero should be understood as being compared to the background being identified with the vacuum, in which we live. Taking the zero to be relative to energy density in the usual vacuum, means that we only have to include those energy carrying ingredients, which are not also present in the usual vacuum, in which we live. Since the “condensate vacuum” is characterized by its extra \( F \)-particles, it is the energy density resulting from these particles and their mutual interactions, that should be added up and required by MPP to be zero.

That is to say we shall assume, that the energy in the condensate vacuum per \( F \)-particle, i.e. really \( \rho_F^{-1} \rho_{\text{energy}} \), is zero. Here we used the notation that \( \rho_F \) is the density inside the condensate of the particles \( F \), while the energy density of the condensate, counted relative to the vacuum, we live in, and which have no \( F \)'s in first thinking (of course there are some vacuum fluctuations, but that is not counted into neither \( \rho_F \) nor \( \rho_{\text{energy}} \), if the same is present in the vacuum, we live in.)

This MPP-requirement is written

\[
0 = m_F - \text{“binding per } F\text{”} \tag{10}
\]
\[
= m_F - \frac{\#\text{neighbors}}{2} \ast \text{“binding to neighbor } F\text{”} \tag{11}
\]
\[
\approx m_S - \frac{4}{2} \ast \text{“binding of } F \text{ in } n=2 \text{ arround another } F\text{”} \tag{12}
\]
\[
\approx m_F - \frac{4}{2} \ast \frac{1/2^2}{1/1^2} \tag{13}
\]
\[
= m_F - \frac{1}{2} \ast \text{“binding of } F\text{”} \tag{14}
\]
\[
= m_F - \frac{1}{2} \ast (12m_t - m_F) \tag{15}
\]
\[
= \frac{3}{2}m_F - 6m_t \tag{16}
\]

We shall indeed follow an appendix of our earlier work[20] and assume, that the structure of the condensate can be approximated as being a diamond lattice structure, so that there are just \( \#\text{neighbors} = 4 \), i.e. other \( F \)-particles surrounding each one of them in the lattice. When we count all the binding energy per \( F \) present in the condensate “binding per \( F\)” as being the number of neighbors \( \#\text{neighbors} \) times the binding of one \( F \) to its neighbor “binding to neighbor \( F\)”, we \textit{double count}, because we count the same binding from both the one \( F \) and from the other one it binds. Therefore we must have the denominator 2 seen in the formula (16).

We made then the approximation, that we can effectively consider it, that the neighboring top quarks and anti topquarks contained in an \( F \) neighboring to another one are \textit{in effect in the } \( n=2 \) \textit{orbit} of the latter. Thus we can take the binding energy of a neighboring \( F \) to a given one “binding to neighbor \( F\)’’ to be as, if the top and anti tops were in an \( n=2 \) orbit or some superposition thereof. Thus the binding of the neighbors occur with binding energy “binding of \( F \) in \( n=2 \) arround another \( F\)”.

As long as we can take the effective Higgs mass for the two lowest orbits \( n = 1 \) and 2 to be zero, we can count, that the binding energy, for top say, in the orbit \( n=2 \) is just one quarter of that in the \( n=1 \) orbit, provided we can use the same potential of the form \( \propto 1/r \). But now
that were, what our above discussion “accidental cancellation” in section 4 should ensure, and so even for an F-particle, which consists of tops and anti tops the ratio of the binding energies should be $1/2^2 = 1/4$.

From the last step in (16) we easily derive of course

$$m_F = \frac{2}{3} \times 6m_t = 4m_t = 173GeV \times 4 = 692GeV$$

agreeing well with 710GeV or 850GeV! (17)

8 Calculation of the Mass of the into Gauge Bosons Decaying Observed (?) 1.8 TeV-Resonance

In general our model means that the Higgs coupling of our bound state, F or S, is very strong, and several such bound states interact - actually attract - each other. Thus we expect that there should exist further resonances formed from two or more bound states. After all we even have used the picture that there is new phase a new vacuum, which might be considered a huge bound state of an infinite number of our bound state of 6 top and 6 anti top, F. The most significant of such further bound states of our bound states would presumably be a resonance of just two of them.

It happens that the resonance doubtfully seen in the decay into Z’s or W’s with mass 1.8 TeV could be identified with a resonance consisting essentially of two F-particles, which in turn are the bound states of the 6 tops and 6 anti tops each.

Already the fact that the mass 1.8 TeV is very crudely just that of two F-particles(with mass as our bound state calculation based on MPP suggests), $2 \times 750 \text{GeV} = 1500 \text{GeV} = 1.5 \text{TeV}$, suggests such a thinking. But now we shall estimate the mass of the 1.8 TeV resonance by the following procedure:

If we could calculate as above that the attraction in the n=1 and n=2 orbits were as if the Higgs mass were zero, we could have one F bound to another one by having all the tops and anti tops of the one going into the n=2 orbit of the other one. Then the binding would according to our rule used above be just 1/4 of the binding of one F-particle $12m_t - m_F$. Now, however, because of the effective Higgs mass being smaller in the situation, wherein we have only two F-particles rather than a full condensate, as we thought about above, we should take it that the attraction in the n=2 disappears, when going from the condensate to the system - the 1.8 particle we hope - of only two F’s. This change we then treat as a perturbative correction to the first mass for the two F-bound state of $2m_F - \frac{1}{4} \times (12m_t - m_F)$ due to the change of the potential energy between the two F-particles disappearing.

Now in potentials of the $1/r$ form, which we use as our approximation here, the virial theorem allows us to use, that the pototential energy is negative and just twice the binding energy. The energy or mass of the two-F-resonance will thus rather be increased compared to the mass of two F-particles than decreased by the amount of $\frac{1}{4} \times (12m_t - m_F)$.

That is to say the mass of the resonance - which we want to identify with the 1.8 TeV finding - is given as

$$m_{2F\text{Resonance}} = 2m_F + \frac{1}{4} \times (12m_t - m_F) = 1.75m_F + 3m_t = 1313GeV + 519GeV = 1832GeV = 1.83TeV,$$

which agrees - accidentally? - wonderfully with the number 1.8 TeV from the experiment!
8.1 Excuse for Narrow Width

When you think about our huge Higgs mass approximation for the case of binding of just two F-particles, you see that formally there is no interaction at all between the two! This would not allow a narrow resonance. Therefore we need some helping story to excuse that the width after all gets so small, that we experimentally shall conceive of the resonance with mass 1.8 TeV as a resonance at all.

The excuse suggested is, that during the interaction assumed to be mainly given by Higgs exchange in our calculation the two 6tops + 6 anti tops boundstates F are \textit{partly annihilated} so as to be in reality not really two true F-particles, but rather some similar structures with a bit smaller numbers of constituents. We could still hope that although such a partial annihilations, that could go back and forth, while the two F’s move around, could help to decrease the decay rate and making the width of the 1.8 TeV-resonance smaller, it would not severely modify our crude estimate above.

The narower we can speculate the 1.8 TeV resonance to be w.r.t. decaying into two F’s the bigger we can hope for the partial width into other channels than the channel into two F’s. Since it is so far only seen in vector boson channels, it is needed to be speculated, that the two F’s channel does not take away all the 1.8 TeV particles.

9 Conclusion

9.1 The New “Calculation”

We have presented an overly simple “calculation” leading to there being due to the strong top-Yukawa coupling and under the assumtion of a finetuning ensuring a with the normal vacuum degenerate one with a condensate of bound states S=F of six top + six anti top, two “resonances” with masses respectively

\[ m_F = \frac{12m_t}{3} = 4m_t = 692\text{GeV} \]  \hspace{1cm} (19)

\[ m_{1.8} = 2m_F + \text{“kinetic energy of } n=2 \text{ orbit”} = 2m_F + \frac{12m_t}{6} = 2m_F + 2m_t = 1846\text{GeV} \]  \hspace{1cm} (20)

These results were obtained in the philosophy, that the coupling constants - especially say the top-Yukawa-coupling \( g_t \) - are by the new principle “multiple point principle” adjusted/finetuned to make the energy density of a condensate of F-particles - which we earlier caled S-particles - (the bound states of 6 top + 6 anti top quarks) have just the same energy density as the usual vacuum (in which we live). That were to say, that the interaction between the F-particles in the condensate should be just so strong an attraction between them, that the total energy (density) of the condensate just becomes zero (relative to the normal vacuum). That is to say the binding between the neighboring F-states just equals the mass-energies of these F-states. If one therefore considers our good agreement of the masses as an evidence for the truth of the assumptions having been used, then there is a significant evidence for our long speculated “Multiple Point Principle”!

It must, however, be admitted that the present very simplified “calculation” were based on a very crude treatment of the Yukawa potential representing the Higgs exchange between the F-particles, which is the (main) interaction between these F-particles in our picture. In fact we approximated Higgs exchange by letting the Higgs be effectively massless, when the top-antitop quarks are in relative orbits with atomic main quantum number \( n=1 \), while we let the Higgs potential be totally cut away for \( n=3, 4, \ldots \). For the main quantum number \( n=2 \) we made
the more complicated assumption of letting it be either as for a zero mass Higgs or cut down
to zero according to the surroundings, which influence the average Higgs field. Indeed we took
for n=2 the Higgs exchange force to be like for massless Higgs inside the condensate vacuum,
while we put the exchange potential to be zero, when applied in the two-F-system identified with
resonance hoped for with the mass 1.8 TeV. Although the resonance previously experimentally
suggested as excess seen decaying into weak gauge bosons does not seem much supported, it
would for our system be very fine with such an 1.8 TeV resonance. In general some related
resonances to our main bound state with mass in the 750 GeV range are in our scheme not
unexpected, since we after all have a scheme with - a new type(i.e. not just QCD.) of - strong
interactions. The mass region at about 1.8 TeV is the first suggested such further resonance to
be expected.

Thus more severe calculations are to be performed to truly settle, if our calculations are
right. It should in this connection be stressed, that since our model is in principle only Standard
Model extended with our Multiple Point Principle used to restrict the coupling constants, one
should in principle be able to calculate whatever one wants. With a relatively strong coupling
$g_t$ being the very basis for the whole story there is though of course the complication of not
having in principle the basis for perturbation theory.

9.2 Main Coincidence! Earlier Bound State Mass Fit from MPP

It should be stressed that the main point and result of our estimate that our mass estimate
coincides with an earlier result obtained also using MPP but using a different vacuum the “high
Higgs field vacuum” - we could call it:-

A recent work by Larisa Laperashvili, Chitta Das and myself[2], in which we have the bound
state, F-resonance, give a little correction to the mass of the Higgs, that should be measured
relative to the one associated with the running self coupling at the weak scale, improves the
agreement with experiment of requirement of Multiple Point Principle for yet a vacuum. Indeed
we have in our picture a third vacuum (in addition to the usual one and the condensate vacuum
with its F’s in it), namely one with a very high Higgs field expectation value. According to
Standard Model caculations witout our bound state the high Higgs field vacuum has a slightly
negative energy density (compared to the two other vacua). However, we find a little correction
depending among other quantities on the mass of the bound state. We found that this
bound state mass put to 750 GeV would fit wonderfully and consider, that this fact strongly
supports the truth of MPP. In fact it turns out that the mass 750 GeV for the bound state
F is perfect for the correction of ours just to bring the energy density of the vacuum with the
high Higgs field expectation value from its otherwise slightly negative value to zero. This means
that our Multiple Point Principle using a quite third vacuum, namely one with a high Higgs field
expectation value of the order of $10^{18}$ GeV, leads to a need for a particle - our bound state
indeed - with a mass about 750 GeV also. This means that now , when the F(750) fluctuation
digamma once so fashionable turned out being only a statistical fluctuation, then we would
nevertheless from Multiple Point Principle get two different and essentially independent reasons
for our bound state to have the mass near the value 750 GeV. That is to say we would then
claim, well our bound state should be with a mass close to 750 GeV. We have already two
calculations of this mass in different ways, although both originating from the same principle
MPP, but involving quite different data to fit.

In fact Larisa Laperashvili et al. [2] uses a correction due to the diagram
in order to correct the running self coupling of the Higgs $\lambda_{\text{run}}(10^{18}\text{GeV})$ corresponding to the observed Higgs mass 125 GeV to go from the value obtained by DeGrassi et al. [44] of

$$\lambda(\phi^\text{"high field"}) = -0.01 \pm 0.002.$$  \hfill (21)

at the high field scale $\phi^\text{"high field"}$ to the value very accurately zero requied by Multiple Point Principle(=MPP). Since bound state F is an extended object we must include a formfactor, when using it.

Defining a quantity $b$ denoting the radius of the bound state measured with top quark Compton wave length $1/m_t$ as unit by:

$$\langle \vec{r}^2 \rangle = 3r_0^2,$$  \hfill (22)

$$r_0 = \frac{b}{m_t},$$  \hfill (23)

we obtain a theoretical estimate

$$b = \sqrt{\frac{\langle \vec{r}^2 \rangle}{3}} m_t \approx 2.34,$$  \hfill (24)

crudely confirmed by a slightly different estimate.

The dominant diagram/correction - the first and quadratic of the diagrams on the figure just above - is

$$\lambda_S \approx \frac{1}{\pi^2} \left( \frac{6g_t}{b} \frac{m_t}{m_S} \right)^4$$

where we have the estimated or measured values

$$g_t = 0.935; \ m_t = 173\text{GeV}; \ b \approx 2.34\text{or}2.43$$

Using the after all rather small deviation from perfect MPP

$$\lambda_{\text{high field}} = -0.01 \pm 0.002$$

and requiring it to be cancelled by the correction from the bound state we get the requirement

$$\lambda_S = \frac{1}{\pi^2} \left( \frac{6g_t}{b} \frac{m_t}{m_F} \right)^4 \ast (\sim 2) \approx 0.01 \pm 0.002,$$  \hfill (25)
where \( g_t = .935, \ m_t = 173 GeV, \ b \approx 2.43 \) and the factor “(\( \sim 2 \))” were taken in to approximate some neglected diagrams, as the next on the figure. If a nearer study should show that the next diagrams add up to roughly as much as the first one should include the factor \( \sim 2 \) to take into account the neglected Feynman diagrams correcting the Higgs self coupling.

The solution w.r.t. the mass of the bound state \( m_F(750) \) gives

\[
m_F \approx \frac{6g_t m_t}{b} \left( \frac{\sim 2}{\pi^2 \cdot 0.01 \pm 0.002} \right)^{1/4}
\]

\[
\approx 2.31 \cdot 173 GeV \cdot 2.1 = 4.9 \cdot 173 GeV = 850 GeV \pm 20%
\]

or without the \( \sim 2 \):
\[
m_F = 2.31 \cdot 173 GeV \cdot 1.8 = 4.1 \cdot 173 GeV = 710 GeV \pm 20%
\]

The “without the \( \sim 2 \)” means what one shall do if the first diagram indeed dominates strongly.

In this way we got even two calculations for the bound state mass - using in addition crude estimation -

\[
m_F(\text{from “high field vacuum”}) \approx 850 GeV \pm 30\% \text{with } \sim 2 \tag{26}
\]

\[
m_F(\text{from “high field vacuum”}) \approx 710 GeV \pm 30\% \text{ without } \sim 2 \tag{27}
\]

\[
m_F(\text{“condensate vac.”}) \approx 692 GeV \pm 40\% \tag{28}
\]

The agreement of the value “692 GeV” with the estimate(s) from the completely different vacuum with the high Higgs field “850 GeV” or “710 GeV” is encouraging and a support of our “Multiple Point Principle”!

### 9.3 No Genuine New Physics

If our numbers are taken as so convincing that our picture should be taken seriously then we would have the consequences:

- We must take our “Multiple Point Principle” as a true new physical law, even the mechanism behind it may not be clear.
- We must accept that the Standard Model except for “smaller” deviations, that are too small to significantly modify the running of the Higgs self coupling, is valid all the way to about \( 10^{18} \) GeV. Some see-saw neutrinos may be acceptable, as long as they do not couple too strongly to the Higgs to influence the running of its self coupling. Otherwise it would be very accidentally that a pure Standard model calculation would give so consistent results.

So there would not be much place for new physics except for the various resonances formed from the bound states, because we now have a new regime of strong interactions that can only be treated by non-perturbative methods. (The suggestion for the 1.8 TeV resonance is an example for how there can be more particles in such a new strong interaction regime)

- There should be seen sooner or later bound state of the 6 top + 6 anti top with a mass not far from 800 GeV.

But even if it thus looks a bit sad w.r.t. much new physics, one should not forget that having our Multiple Point Principle established would be a strong element of new physics, perhaps then of an a bit unexpected type.
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Astri Kleppe even presented our “Mutiple Point Principle” for the workshop.

References

[1] C. D. Froggatt and H. B. Nielsen, “Production and Decay of 750 Gev state of 6 top and 6 antitop quarks,” [arXiv:1605.03909 [hep-ph]].

[2] L. V. Laperashvili, H. B. Nielsen and C. R. Das, “New results at LHC confirming the vacuum stability and Multiple Point Principle,” Int. J. Mod. Phys. A 31 (2016) no.08, 1650029 doi:10.1142/S0217751X16500299 [arXiv:1601.03231 [hep-ph]].

[3] Y. Hamada, H. Kawai and K. Kawana, Evidence of the Big Fix, Int. J. Mod. Phys. A 29, no. 17, 1450099 (2014) [arXiv:1405.1310 [hep-ph]].

[4] Y. Hamada, H. Kawai and K. Kawana, Weak Scale From the Maximum Entropy Principle, [arXiv:1409.6508 [hep-ph]]

[5] D.L. Bennett and H.B. Nielsen, Int. J. Mod. Phys. A9, 5155 (1994).

[6] D.L. Bennett and H.B. Nielsen, Int. J. Mod. Phys. A14, 3313 (1999).

[7] D.L. Bennett, C.D. Froggatt and H.B. Nielsen, in Proc. 27th Int. Conf. on High Energy Physics, Glasgow, Scotland, 1994, eds. P. Bussey and I. Knowles (IOP Publishing Ltd., 1995), p. 557.

[8] D.L. Bennett, C.D. Froggatt and H.B. Nielsen, in Perspectives in Particle Physics 94, eds. D. Klabu.car, I. Picek and D. Tadi.c (World Scientific, Singapore, 1995), p. 255.

[9] C.D. Froggatt and H.B. Nielsen, Phys. Lett. B368, 96 (1996).

[10] L.V. Laperashvili, Yad. Fiz. 57, 501 (1994) [Phys. Atom. Nucl. 57, 471 (1994)].

[11] C.D. Froggatt, L.V. Laperashvili, R.B. Nevzorov and H.B. Nielsen, Phys. Atom. Nucl. 67, 582 (2004) [Yad. Fiz. 67, 601 (2004)], [arXiv:hep-ph/0310127].

[12] C.D. Froggatt, L.V. Laperashvili and H.B. Nielsen, Phys. Atom. Nucl. 69, 67 (2006), [hep-ph/0407102]

[13] D.L. Bennett, L.V. Laperashvili and H.B. Nielsen, Relation between fine structure constants at the Planck scale from multiple point principle, in Proc. 9th Workshop: What Comes Beyond the Standard Models, Bled, Slovenia, eds. M. Breskvar et al. (DMFA, Zaloznistvo, Ljubljana, 2006), p. 10, [arXiv:hep-ph/0612250].
[14] D.L. Bennett, L.V. Laperashvili and H.B. Nielsen, *Finestructure constants at the Planck scale from multiple point principle*, in Proc. 10th Workshop on What Comes Beyond the Standard Model, Bled, Slovenia, 1727 Jul 2007, arXiv:0711.4681.

[15] C.D. Froggatt, R.B. Nevzorov and H.B. Nielsen, *Smallness of the cosmological constant and the multiple point principle*, J. Phys. Conf. Ser. 110, 072012 (2008), arXiv:0708.2907.

[16] C.D. Froggatt, R.B. Nevzorov, H.B. Nielsen, A.W. Thomas, Phys. Lett. B737, 167 (2014), arXiv:1403.1001.

[17] C.D. Froggatt, R.B. Nevzorov, H.B. Nielsen, A.W. Thomas, *On the smallness of the cosmological constant in SUGRA models with Planck scale SUSY breaking and degenerate vacua*, 2015 European Physical Society Conference on High Energy Physics (EPS-HEP 2015), 22-29 Jul, 2015, Vienna, Austria; arXiv:1510.05379.

[18] H. B. Nielsen and M. Ninomiya, “Degenerate vacua from unification of second law of thermodynamics with other laws,” Bled Workshops Phys. 12 (2011) no.2, 199 hep-th/0701018.

[19] Stillits, Cand. Scient. thesis at the Niels Bohr Institute.

[20] C. D. Froggatt and H. B. Nielsen, “Tunguska Dark Matter Ball,” Int. J. Mod. Phys. A 30 (2015) no.13, 1550066 doi:10.1142/S0217751X15500669 [arXiv:1403.7177 [hep-ph]].

[21] C.D. Froggatt and H.B. Nielsen, *Origin of Symmetries*, World Scientific, Singapore, 1991.

[22] C.R. Das and L.V. Laperashvili, Int. J. Mod. Phys. A20, 5911 (2005).

[23] N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B158, 295 (1979).

[24] P.Q. Hung, Phys. Rev. Lett. 42, 873 (1979).

[25] R.A. Flores, M. Sher, Phys. Rev. D27, 1679 (1983).

[26] M. Lindner, Z. Phys. 31, 295 (1986).

[27] D.L. Bennett, H.B. Nielsen and I. Picek, Phys. Lett. B208, 275 (1988).

[28] M. Sher, Phys. Rept. 179, 273 (1989).

[29] M. Lindner, M. Sher, H.W. Zaglauer, Phys. Lett. B228, 139 (1989).

[30] P.B. Arnold, Phys. Rev. D40, 613 (1989).

[31] G. Anderson, Phys. Lett. B243, 265 (1990).

[32] P. Arnold and S. Vokos, Phys. Rev. D44, 3620 (1991).

[33] C. Ford, D.R.T. Jones, P.W. Stephenson, M.B. Einhorn, Nucl. Phys. B395, 17 (1993).

[34] M. Sher, Phys. Lett. B317, 159 (1993).

[35] G. Altarelli and G. Isidori, Phys. Lett. B337, 141 (1994).

[36] J.A. Casas, J.R. Espinosa, M. Quiros, Phys. Lett. B342, 171 (1995).

[37] J.R. Espinosa, M. Quiros, Phys. Lett. B353, 257 (1995).
[38] J.A. Casas, J.R. Espinosa, M. Quiros, Phys. Lett. B382, 374 (1996).

[39] B. Schrempp and M. Wimmer, Prog. Part. Nucl. Phys. 37, 1 (1996).

[40] C.D. Froggatt, H.B. Nielsen, and Y. Takanishi, *Standard model Higgs boson mass from borderline metastability of the vacuum*, Phys. Rev. D64, 113014 (2001).

[41] V. Brancina and E. Messina, Phys. Rev. Lett. 111, 241801 (2013), arXiv:1307.5193.

[42] V. Brancina, E. Messina and M. Sher, Phys. Rev. D91, 013003 (2015), arXiv:1408.5302.

[43] V. Brancina, E. Messina and A. Platania, JHEP 1409, 182 (2014), arXiv:1407.4112.

[44] G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori and A. Strumia, JHEP 1208, 098 (2012); arXiv:1205.6497.

[45] D. Buttazzo, G. Degrassi, P.P. Giardino, G.F. Giudice, F. Salab, A. Salvio, A. Strumia, JHEP 1312, 089 (2013); arXiv:1307.3536.

[46] G. Isidori, G. Ridolfi, A. Strumia, Nucl. Phys. B609, 387 (2001).

[47] J.R. Espinosa, G.F. Giudice and A. Riotto, JCAP 0805 (2008) 002.

[48] J. Ellis, J.R. Espinosa, G.F. Giudice, A. Hoecker and A. Riotto, Phys. Lett. B679, 369 (2009).

[49] J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Riotto, A. Strumia, Phys. Lett. B709, 222 (2012).

[50] C.D. Froggatt and H.B. Nielsen, *Trying to understand the Standard Model parameters*. Invited talk by H.B. Nielsen at the "XXXI ITEP Winter School of Physics", Moscow, Russia, 18-26 February, 2003; Surveys High Energy Phys. 18, 55-75 (2003); hep-ph/0308144.

[51] C.D. Froggatt, H.B. Nielsen and L.V. Laperashvili, *Hierarchy-problem and a bound state of 6 t and 6 anti-t*, in: Proceedings of Coral Gables Conference on Launching of Belle Epoque in High-Energy Physics and Cosmology (CG 2003), Ft. Lauderdale, Florida, 17-21 December, 2003.

[52] C.D. Froggatt, H.B. Nielsen and L.V. Laperashvili, Int. J. Mod. Phys. A 20, 1268 (2005); hep-ph/0406110.

[53] C.D. Froggatt and H.B. Nielsen, Phys. Rev. D 80, 034033 (2009); arXiv:0811.2089.

[54] C.D. Froggatt, H.B. Nielsen, *Hierarchy Problem and a New Bound State*, in Proc. to the Euroconference on Symmetries Beyond the Standard Model, p.73, Slovenia, Portoroz, 2003 (DMFA, Zalojnstvo, 2003); ArXiv: hep-ph/0312218.

[55] C.D. Froggatt, *The Hierarchy problem and an exotic bound state*, in: Proceedings of 10th International Symposium on Particles, Strings and Cosmology, (PASCOS 04), Boston, Massachusetts, 16-22 Aug, 2004. Published in: “Boston 2004, Particles, strings and cosmology”, pp.325-334; hep-ph/0412337.

[56] C.D. Froggatt, L.V. Laperashvili and H.B. Nielsen, *A New bound state 6t + 6 anti-t and the fundamental-weak scale hierarchy in the Standard Model*, in: Proceedings of 13th International Seminar on High-Energy Physics: QUARKS-2004, Pushkinskie Gory, Russia, 24-30 May, 2004; hep-ph/0410243.
[57] C.D. Froggatt, L.V. Laperashvili and H.B. Nielsen, Phys. Atom. Nucl. 69, 67 (2006) [Yad. Fiz. 69, 3 (2006)]; hep-ph/0407102.

[58] C.D. Froggatt, L.V. Laperashvili, R.B. Nevzorov and H.B. Nielsen, The Production of $6t + 6\bar{t}$ bound state at colliders. A talk given by H.B. Nielsen at CERN, 2008, preprint CERN-PH-TH/2008-051.

[59] C.D. Froggatt, L.V. Laperashvili, R.B. Nevzorov, H.B. Nielsen and C.R. Das, New Bound States of Top-anti-Top Quarks and T-balls Production at Colliders (Tevatron, LHC, etc.), Report CHEP-PKU-1-04-2008, CERN (2008); arXiv:0804.4506.

[60] C.D. Froggatt and H.B. Nielsen, in: Proceedings to the 34th International Conference on High Energy Physics (ICHEP 2008), 30 Jul - 5 Aug 2008, Philadelphia, Pennsylvania; arXiv:0810.0475.

[61] C.R. Das, C.D. Froggatt, L.V. Laperashvili and H.B. Nielsen, Int. J. Mod. Phys. A 26, 2503 (2011); arXiv:0812.0828.

[62] C.D. Froggatt, C.R. Das, L.V. Laperashvili and H.B. Nielsen, New indications of the existence of the $6$ top-anti-top quark bound states in the LHC experiments, a talk by L.V. Laperashvili at the Conference ”Quarkonium-2012”, Moscow, Russia, MEPhl, November, 12 - 16, 2012, arXiv:1212.2168 [hep-ph]; Yad. Fiz., 76, 172 (2013).

[63] C.D. Froggatt, C.R. Das, L.V. Laperashvili, H.B. Nielsen, Int. J. Mod. Phys. A 30, 1550132 (2015); arXiv:1501.00139.

[64] LHC seminar ”ATLAS and CMS physics results from Run 2”, talks by Jim Olsen and Marumi Kado, CERN, 15 Dec. 2015. http://indico.cern.ch/event/442432/.

[65] ATLAS Collaboration, ATLAS-CONF-2015-081, ”Search for resonances decaying to photon pairs in 3.2 fb$^{-1}$ of pp collisions at $\sqrt{s} = 13$ TeV with the ATLAS detector”.

[66] CMS Collaboration, CMS PAS EXO-15-004, ”Search for new physics in high mass diphoton events in proton-proton collisions at 13TeV”; http://indico.cern.ch/event/442432/.

[67] ATLAS Collaboration (G. Aad et al.), Phys. Rev. Lett. 114, 081802 (2015).

[68] Particle Data Group Collaboration (K.A. Olive (Minnesota U.) et al.), 2014, 1676 pp. Chin. Phys. C 38, 090001 (2014).

[69] ATLAS, CMS, D0 Collaborations, arXiv:1403.4427.

[70] CERN CMS, CMS Collaboration, CMS Collaboration CMS-PAS-EXO-14-010, “Search for massive WH resonances decaying to $b\bar{b}$ final state in the boosted regime at $s=8$ TeV”.

[71] F.L. Bezrukov, M. Shaposhnikov, Phys. Lett. B 659, 703 (2008).

[72] F.L. Bezrukov, A. Magnin, M. Shaposhnikov, Phys. Lett. B 675, 88 (2009).

[73] F.L. Bezrukov and D. Gorbunov, JHEP 1307, 140 (2013), arXiv:1303.4395.

[74] F.L. Bezrukov and M. Shaposhnikov, J. Exp. Theor. Phys. 120, 335 (2015) [Zh. Eksp. Teor. Fiz. 147, 389 (2015)], arXiv:1411.1923.
[75] F. Bezrukov, J. Rubio and M. Shaposhnikov, *Living beyond the edge: Higgs inflation and vacuum metastability*, Phys. Rev. D**92**, 083512 (2015), [arXiv:1412.3811](http://arxiv.org/abs/1412.3811).

[76] L.N. Mihaila, J. Salomon and M. Steinhauser, Phys. Rev. Lett. **108** (2012) 151602.

[77] K. Chetyrkin and M. Zoller, JHEP **06**, 033 (2012).

[78] F. Bezrukov, M.Yu. Kalmykov, B.A. Kniehl, M. Shaposhnikov, JHEP **1210**, 140 (2012).

[79] C.D. Froggatt, C.R. Das, L.V. Laperashvili, H.B. Nielsen, Int. J. Mod. Phys. A**30**, 21, 1550132 (2015); [arXiv:1501.00139](http://arxiv.org/abs/1501.00139).

[80] A.L. Macpherson and B.A. Campbell, Phys.Lett. B**306**, 379 (1993); ArXiv: [hep-ph/9302278](http://arxiv.org/abs/hep-ph/9302278).

[81] A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn and V.F. Weisskopf, Phys.Rev. D**9**, 3471 (1974).

[82] W.A. Bardeen, M.S. Chanowitz, S.D. Drell, M. Weinstein and T.-M. Yan, Phys.Rev. D**11**, 1094 (1975).

[83] M.Yu. Kuchiev, V.V. Flambaum, E. Shuryak, Phys. Rev. D**78**, 077502 (2008); [arXiv:0808.3632](http://arxiv.org/abs/0808.3632).

[84] M.Yu. Kuchiev, V.V. Flambaum, E. Shuryak, Phys. Lett. B**693**, 485 (2010); [arXiv:0811.1387](http://arxiv.org/abs/0811.1387).

[85] M.Yu. Kuchiev, Phys. Rev. D**82**, 127701 (2010); [arXiv:1009.2012](http://arxiv.org/abs/1009.2012).

[86] Jean-Marc Richard, *About the stability of the dodecatoplet*, Few Body Syst. **45**, 65 (2009); [arXiv:0811.2711](http://arxiv.org/abs/0811.2711).

[87] H. Bethe, E. Salpeter (1951). ”A Relativistic Equation for Bound-State Problems”. Physical Review 84 (6): 1232. Bibcode:1951PhRv...84.1232S. doi:10.1103/PhysRev.84.1232.