Functional mathematical definition of dependability indicators and establishment of the dependency between the integrated indicator and the unique indicators at the stage of manufacture and recovery of components that define the reliability of a machine

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Abstract. As of today, the dependability of machines in the process of their operation and repair is considered as a whole, while the norms and specifications of GOST 27.002-2015, reference and scientific/technical literature do not provide any strict mathematical definitions of dependability indicators of primary elements/components. At the same time, the parametric dependability of a machine is based on the paradigm of evaluation of dependability indicators over the whole operation period from the stage of manufacture to end of service life subject to possible repairs. Aim. Given the above, the aim of this paper is to evaluate the effect of unique dependability indicators on the integrated indicator $I_D$ (efficiency retention coefficient) at the stage of manufacture and recovery of machine components. Methods. The paper is based on the mathematical device for the identification of linear dependency between the integrated indicator and unique indicators that involves the identification of the integrated indicator when the unique indicator under consideration changes its value from the basic (first) level to the high (forth) level, while all the other unique indicators remain at the basic level, which rules out the correspondence between the unique indicator under consideration and the other indicators in the process of integrated indicator calculation. Results. Calculations show that the optimal option for increasing $I_D$ is to increase the unique indicators according to their priorities. Thus, coordinated increase at the stage of manufacture and recovery of only three unique indicators in some instances ensures 75 percent growth of the integrated indicator. Conclusions. It is suggested to classify machine components into three groups based on the value of reliability indicator of the initially installed machine component: ones that define the life until discarding ($I_{R1}>1$); ones that define machine service life ($I_{R1}=1$) and ones that define machine reliability ($I_{R1}<1$). The identification of the dependability indicators of the components in each group is based on the provisions of GOST 27.002-2015, but each group has its own unique features that must be taken into consideration in the functional mathematical definition of the dependability indicators of components in relation to the dependability of machines as a whole. For the components of the third group functional mathematical definitions were developed, dependencies and priorities between the unique indicators and increased integrated indicator were identified. Using a specific example, the economic feasibility of increasing the integrated indicator was calculated. It was established that the most promising solution would be a coordinated increase of the integrated indicator at the stages of manufacture and recovery that enables a more than a double reduction of costs, while ensuring 61 percent profitability.

Keywords: reliability, maintainability, storability, longevity, integrated dependability indicator, priorities, profitability.

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Introduction

Despite the progress in engineering, due to insufficient dependability of the machines produced, consumers have to deal with significant repair and maintenance costs that over the entire period of operation many times exceed the cost of a new machine. Thus, for trucks, the cost exceeds 6 times, and for metal-working machines, the cost exceeds 8 times. At the same time, the major part of the costs is accounted for the current repair and maintenance of the failed components [1]. Formally, a machine is a system with functionally connected elements (components), the dependability of which makes and defines the system dependability as a whole. Therefore, identifying the dependability indicators of the components relative to the machine dependability and the estimation of their contribution to this dependability will enable a differentiated approach to the calculation of unique dependability indicators in the design process and their assurance at the manufacturing and recovery stages, that is a relevant task both theoretically and practically.

Unfortunately, modern reference and scientific and technical literature in the field of machine dependability [2, 3, 4, 5, 6] does not provide any specific interrelated solutions to the task.

Table 1. Unique dependability indicators determined the machine reliability

| Indicators | Functional definitions | Mathematical definitions |
|------------|------------------------|--------------------------|
| 1 Component dependability of the initially installed component, \( I_{R1} \) | Part of the machine life, which is realized by the component of the first installation | \( I_{R1} = \frac{T_{R1}}{L_1} \), |
| 1.2 Reliability retention coefficient of the first replacement component, \( \gamma \) | This coefficient indicates how much the reliability indicator of the first replacement component is changed, \( I_{R0} \) | \( \gamma = \frac{I_{R1}}{L_1} \cdot \frac{I_{R0}}{I_{R1}} \), where \( L_{R1} \) is the time to failure of the first replacement component |
| 1.3 Reliability indicator of the \( i \)-the replacement component, \( I_{ri} \) | Part of the machine life, which is realized by the component of the \( i \)-the replacement | \( I_{ri} = I_{R1} \cdot \gamma^{i-1} \), |
| 1.4 Number of replaced components in operation, \( N_O \) | Number of components replaced in operation for the machine life realization | \( N_O = \frac{\ln(I_{R1} + \gamma - 1) - \ln(I_{R1} \gamma)}{\ln \gamma} \), |
| 1.5 Average reliability indicator of replaced components, \( I_{R0} \) | Machine resource part, which is realized on the average by the replaced components | \( \overline{I_{R0}} = \frac{(1 - I_{R1})}{N_O} \), |
| Component maintainability | | |
| 2.1 Reliability recovery coefficient, \( I_{Rec} \) | This coefficient indicates how much the reliability indicator of recovered component is changed | \( I_{Rec} = \frac{I_{R01}}{L_{O1}} = \frac{I_{R01}}{I_{R01}} \), where \( L_{Rec} \) is the time to failure of the first recovery component, \( I_{Rec1} \) is the reliability indicator of the first recovery component |
| 2.2 Reliability indicator of the \( i \)-th recovery, \( I_{Reci} \) | Part of machine life, which is realized by the component of the \( i \)-th recovery | \( I_{Reci} = I_{R1} \cdot \gamma^{i-1} \), |
| 2.3 Component maintainability indicator, \( I_m \) | Part of machine life, which is realized by the component through \( N \)-fold recovery | | |
| 2.4 Estimated number of component recoveries, \( N_{Rec.Est} \) | Number of the component recoveries for the full machine life realization under given \( I_{Rec} \) | \( N_{Rec.Est} = \frac{\ln \left[ \frac{(1 - I_{R1}) \cdot (1 - \gamma)}{I_{Rec} \cdot I_{R1} \cdot \gamma} \right]}{\ln \gamma} \), |
| 2.5 Estimated reliability recovery coefficient, \( I_{Rec.Est} \) | Reliability recovery coefficient for the machine life realization under given number of recoveries, \( N_{Rec} \) | \( I_{Rec.Est} = \frac{(1 - I_{R1}) \cdot (1 - \gamma)}{I_{R1} \cdot \gamma \cdot (1 - \gamma^{N_{Rec}})} \), |
Functional mathematical definition of unique dependability indicators of components

According to GOST 27.002-2015, the unique dependability indicator of a component is defined by the mean time to first failure that is eliminated by current machine repair by replacing the failed component.

Considering a component as an independent object, it is worth to take the following ratio for the reliability indicator in relation to the machine:

\[ I_{R1} = \frac{I_1}{L_1} \]  

where \( I_{R1} \) is the reliability indicator of the initially installed component; \( L_1 \) is the mean time to first failure of the initially installed component; \( L_i \) is the machine mean lifetime.

From the physical point of view, the reliability indicator determines which part of the machine’s life is implemented by the initially installed component.

It is worth classifying machine components into three groups based on \( I_{R1} \): ones that define the life until discarding \( (I_{R1}>1) \); ones that define machine service life \( (I_{R1}=1) \) and ones that define machine reliability \( (I_{R1}<1) \).

Undoubtedly, the identification of the dependability indicators of the components in each group is based on the provisions of GOST 27.002-2015, but each group has its own unique features that must be taken into consideration in the functional mathematical definition of the dependability indicators of components in relation to the dependability of machines as a whole.

For the components of the third group, functional mathematical definitions of the unique dependability indicators [7] were developed by the authors. The unique dependability indicators are given in Table 1.

### Functional mathematical definition of integrated dependability indicator of component and its dependence on unique indicators

From the technical point of view, the longevity indicator, that includes all unique indicators, can be taken as the integrated indicator of reliability. However, the consumer is primarily concerned with the economic aspect of dependability, i.e. the operation costs which the consumer will bear when replacing failed components during machine life realization. According to GOST 27.002-2015, such is the efficiency retention coefficient, which is defined by the ratio...
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The value of object use efficiency indicator over certain duration of operation to the nominal value of this indicator, calculated under condition that there are no object failures during the same period of time.

If the cost of a new part is taken as the nominal value of the efficiency indicator, while the operation costs for replacing the failed components when the machine realizes its life are taken as the value of the efficiency index, the integrated dependability indicator of the component is defined by the following ratio

\[ I_D = \frac{C_D}{C_D + N_{D,L} \cdot (C_D + C_0) + N_{Rec,Act} \cdot (C_{Rec} + C_0)}, \]  

(2)

where \( I_D \) is the integrated dependability indicator of the component; \( C_D \) is the cost of a new component; \( C_{Rec} \) is the cost of the recovered component; \( C_0 \) is the cost of the component replacement in operation and economic losses caused by machine downtime; \( N_{D,L} \) is the number of new components replaced in operation in addition to the recovered components; \( N_{Rec,Act} \) is the actual number of component recoveries.

Let us divide the numerator and denominator of formula (2) into \( C_D \) and introduce the term of the relative cost, determined by the following ratios

\[ \alpha = \frac{C_{Rec}}{C_D}, \quad \beta = \frac{C_0}{C_D}, \]  

(3)

where \( \alpha \) is the relative cost of the recovered component; \( \beta \) is the relative cost of the component replacement in operation.

Taking into account formula (3), sub-paragraph 4.2 and 4.3 of Table 1, after the corresponding transformations, formula (2) takes the following expression

\[ I_D = \frac{1}{1 + \frac{(1-I_{Rec})}{I_{Rec}} \cdot \left(1 + \frac{\alpha}{\beta} \cdot \frac{1}{I_{Rec} \cdot (1 + \beta) - 1}\right)}. \]  

(4)

From the economic point of view, the integrated dependability indicator determines what part of new component cost is from the total cost of operation to replace failed components.

The analysis of formula (4) shows that the product in front of the curly brackets is the relative costs in operation with no component recovery \( (I_{Rec}=0) \). The sum in the curly brackets defines the amount of change of relative costs for the component recovery, and the difference in the square brackets defines the ratio of relative costs in operation with one-time component recovery. With a positive value, the operation costs increase, with a negative value, the operation costs decrease. In this context the assessment of the feasibility of the component recovery is determined by the following inequality

\[ I_{Rec} > \frac{\alpha + \beta}{1 + \beta}. \]  

(5)

### Identification of the linear dependency between the increasing integrated dependability indicator and the unique indicators, their prioritization (by weight) and economic efficiency assessment due to increasing integrated indicator

In order to identify the linear dependency between the integrated dependability indicator and the unique indicator, a mathematical method is used [8], that involves finding the integrated indicator using formula (4) when the unique indicator under study changes its value from the basic (first) level to the upper (fourth) level, and all the others unique indicators are at the basic levels, that eliminates the relationship between the unique indicator under study and the others when calculating the integrated indicator.

| Level | Dependability indicators of the component | Component manufacturing \((I_{Rec}=0)\) |
|-------|------------------------------------------|------------------------------------------|
|       | \( I_{R1} \) | \( I_{Rec} \) | \( I_D \) | \( \gamma \) | \( I_{Rec} \) | \( I_D \) | \( \beta \) | \( I_D \) |
| Basic 1 | 0.3 | 0.18 | 0.14 | 0.8 | 0.18 | 0.14 | 0.5 | 0.14 |
| 2 | 0.4 | 0.28 | 0.24 | 0.85 | 0.2 | 0.16 | 0.4 | 0.16 |
| 3 | 0.5 | 0.39 | 0.34 | 0.9 | 0.25 | 0.19 | 0.3 | 0.18 |
| Upper 4 | 0.55 | 0.45 | 0.4 | 0.95 | 0.3 | 0.22 | 0.2 | 0.2 |

| Level | Dependability indicators of the component | Component recovery \((I_{Rec}=0.3, \gamma=0.8, \beta=0.5)\) |
|-------|------------------------------------------|------------------------------------------|
|       | \( I_{R1} \) | \( I_D \) | \( I_{Rec} \) | \( I_D \) | \( \alpha \) | \( I_D \) | \( I_{Rec} \) | \( I_D \) |
| Basic 1 | 0.5 | 0.16 | 0.8 | 0.16 | 0.5 | 0.16 | 1.0 | 0.16 |
| 2 | 0.6 | 0.17 | 0.85 | 0.165 | 0.4 | 0.17 | 1.4 | 0.2 |
| 3 | 0.8 | 0.19 | 0.9 | 0.17 | 0.3 | 0.18 | 1.8 | 0.24 |
| Upper 4 | 1.0 | 0.22 | 0.95 | 0.18 | 0.2 | 0.2 | 2.2 | 0.29 |
Table 2 shows the results of the integrated dependability indicator calculation, and Figure 1 shows the graphical linear interpretation.

The weight of the unique indicator in the increasing integrated indicator is identified based on the results of the assessment of the linear dependency between the integrated indicator and the unique indicator changes and is estimated by the following ratio:

$$\eta_i = \frac{\Delta I_D}{\sum \Delta I_D},$$  

where $\eta_i$ is the weight of the $i$-th unique indicator in the increasing integrated indicator; $\Delta I_D$ is the increase of the integrated indicator from the $i$-th unique indicator defined by the difference

$$\Delta I_D = I_{Di,max} - I_{Di,basic},$$  

where $I_{Di,max}$ is the maximum value of the integrated indicator for the $i$-th unique indicator, when it is at the upper fourth level; $I_{Di,basic}$ is the value of the integrated indicator for the $i$-th unique indicator, when it is at the basic level.

From the physical point of view, the weight of the $i$-th unique indicator $\eta_i$ shows what part of the increase of the integrated indicator is attributed to the $i$-th unique indicator out of all unique indicators.

The priority of the $i$-th unique indicator is determined by ranking of the weight of all unique indicators.

The weight of the unique indicators is calculated by formulas (6) and (7), and in accordance with priorities, is presented in Table 3, while Figure 2 shows the graphical interpretation.

The analysis of calculation results (Table 2 and Table 3) shows that the optimal option for increasing $I_D$ is to change the unique indicators in accordance with their priorities. Thus, in case of simultaneous increase at the stages of manufacture and recovery, only three unique indicators ensure an increase of the integrated indicator by 75%.

The unique dependability indicators of the component are calculated from the primary factors that are formed during design, manufacture, recovery and operation of machines, the implemented of which by means of technological, process-engineering, organizational and other methods allows such indicators achieving specified values of the priority unique indicators. This requires certain expenditures, which affects the cost of new and recovered components, as well as operation costs associated with the replacement of failed components. The economic viability of costs is determined both by the resulting economic effect and profitability that defines the payback period of these costs.

### Table 3. The results of the calculation of the weight of unique dependability indicators in the increase of the integrated indicator and their prioritization

| Manufacturing | Recovery | Combined |
|---------------|----------|----------|
| $P_I$ | $I_{Di}$ | $\Delta I_{Di}$ | $\eta_i$ | $P_I$ | $I_{Di}$ | $\Delta I_{Di}$ | $\eta_i$ | $P_I$ | $I_{Di}$ | $\Delta I_{Di}$ | $\eta_i$ |
| 1 | $I_{I1}$ | 0.26 | 0.65 | 1 | $I_{I1}$ | 0.26 | 0.376 |
| 2 | $\gamma$ | 0.09 | 0.22 | 2 | $I_{I2}$ | 0.15 | 0.217 |
| 3 | $\beta$ | 0.05 | 0.13 | 3 | $\gamma$ | 0.09 | 0.13 |
| | | | | 4 | $I_{I4}$ | 0.15 | 0.52 | 4 | $\alpha$ | 0.06 | 0.086 |
| | | | | 5 | $I_{I5}$ | 0.06 | 0.2 | 5 | $\alpha$ | 0.05 | 0.072 |
| | | | | 6 | $I_{I5}$ | 0.03 | 0.11 | 6 | $\beta$ | 0.05 | 0.072 |

$$\sum \Delta I_{Di} = 0.4 \quad | \quad 1.0 \quad | \quad 0.29 \quad | \quad 1.0 \quad | \quad 0.69 \quad | \quad 1.0$$
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The operation costs for replacing failed components during the realization of machine life are defined by the product

\[ C_O = \frac{1}{I_D} - 1 \cdot C_{NC}, \]  

where \( C_O \) is the operation costs; \( I_D \) is the value of the integrated dependability indicator; \( C_{NC} \) is the cost of the new component (repair component).

The economic effect of increasing the integrated dependability indicator from \( I_{D1} \) to \( I_{D2} \) is defined by the difference

\[ C_{EE} = C_{D2} - C_{D1}, \]  

where \( C_{EE} \) is the economic effect of increasing the integrated dependability indicator.

The profitability of increasing the integrated dependability indicator is defined by the ratio

\[ R = \frac{C_{EE}}{C_O} \cdot 100\%, \]  

where \( R \) is the profitability of costs in %.

Let us consider, as an example, an assessment of the economic feasibility of increasing the integrated dependability indicator of a component. The initial values of the indicators are as follows: \( I_{D1} = 0.17 \); \( C_{NC} = 4 \) thousand rubles; \( C_{Rec} = 2 \) thousand rubles; \( C_{Rec} = 1 \) thousand rubles; \( I_{Rec} = 0.3 \); \( I_m = 0.5 \); \( I_s = 0.85 \). The required values of the indicators are as follows: \( I_{D2} = 0.4 \); \( C_{Rec} = 5 \) thousand rubles; \( I_{Rec} = 0.4 \); \( I_m = 1 \); \( I_s = 0.9 \); \( I_{Rec} = 1.5 \).

The results of the calculations are given in Table 4.

**Conclusion**

1. It is proposed to classify machine components into three groups based on the value of reliability indicator of the initially installed machine component: ones that define the life until discarding; ones that define machine service life; and ones that define machine reliability.

2. For the components of the third group functional mathematical definitions were developed, dependencies and priorities between the unique indicators and increased integrated indicator were identified.

**Table 4. The results of the calculations of the dependability indicators and economic efficiency of increasing \( I_D \) from 0.17 to 0.41.**

| Dependability indicators of component | \( C_O \) | \( C_{EE} \) | \( R \) |
|--------------------------------------|---------|---------|------|
| \( \alpha \) | \( \beta \) | \( \gamma \) | \( I_{Rec} \) | \( I_m \) | \( I_s \) | \( I_{Rec} \) | Thousand rubles | Thousand rubles | % |
| Component manufacturing stage       |         |         |       |       |       |       |         |         |    |
| -                                   | 0.25    | 0.8     | 0.3   | -     | -     | -     | 0.17    | 19.5    | -   |
| -                                   | 0.2     | 0.8     | 0.4   | -     | -     | -     | 0.28    | 12.8    | 6.7  |
| Component recovery stage            |         |         |       |       |       |       |         |         |    |
| 0.5                                 | 0.25    | 0.8     | 0.3   | 0.5   | 0.85  | 1.0   | 0.2     | 16.8    | 2.7  |
| 0.65                                | 0.25    | 0.8     | 0.3   | 1.0   | 0.9   | 1.5   | 0.26    | 14.2    | 5.3  |
| Manufacturing and recovery          |         |         |       |       |       |       |         |         |    |
| 0.65                                | 0.2     | 0.8     | 0.4   | 1.0   | 0.9   | 1.5   | 0.41    | 8.4     | 11.1 |

Figure 2. Graphical dependence between \( I_D \) increase and priorities of the unique dependability indicators of the component

The operation costs for replacing failed components during the realization of machine life are defined by the product

\[ C_O = \frac{1}{I_D} - 1 \cdot C_{NC}, \]  

where \( C_O \) is the operation costs; \( I_D \) is the value of the integrated dependability indicator; \( C_{NC} \) is the cost of the new component (repair component).
3. Using a specific example, the economic feasibility of increasing the integrated indicator was calculated. It was established that the most promising solution would be a co-ordinated increase of the integrated indicator at the stages of manufacture and recovery that enables a more than a double reduction of costs, while ensuring 61 percent profitability.

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