Quantum entanglement degrees amplifier

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The quantum entangled degrees of entangled states become smaller with the transmission distance increasing, how to keep the purity of quantum entangled states is the puzzle in quantum communication. In the paper, we have designed a new type entanglement degrees amplifier by one-dimensional photonic crystal, which is similar as the relay station of classical electromagnetic communication. We find when the entangled states of two-photon and three-photon pass through photonic crystal, their entanglement degrees can be magnified, which make the entanglement states can be long range propagation and the quantum communication can be really realized.

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1. Introduction

Quantum entanglement is a unique phenomenon and its distribution over a long distance is of vital importance in quantum information. Many quantum processes require entanglement [1-3]. Quantum entanglement is the key source in current quantum information processing [4]. Most quantum communication protocols such as quantum key distribution [5], teleportation [6], quantum secret sharing [7], quantum secure direct communication [8], and quantum state sharing all need the entanglement to set up the quantum channel [9].

Currently, the most important problem for single-photon and two-photon entanglement may be the quantum repeater protocol in long distance quantum communication [10, 11]. We know the quantum entangled degree should be decreased and even approach zero, the quantum entanglement of two and three photon shall disappear. All the quantum information processes, e.g., quantum communication, quantum computation and so on can not proceed. The attenuation of quantum entangled degree is unavoidable, we can only make the quantum entangled degree magnify in the entangled states transmission process, which can be realized one-dimensional photonic crystal.

Photonic crystals (PCs) are artificial materials with periodic variations in refractive index that are designed to affect the propagation of light [12-15]. An important feature of the PCs is that there are allowed and forbidden ranges of frequencies at which light propagates in the direction of index periodicity. Due to the forbidden frequency range, known as photonic band gap (PBG) [16-18], which forbids the radiation propagation in a specific range of frequencies. The existence of PBGs will lead to many interesting phenomena. In the past ten years has been developed an intensive effort to study and micro-fabricate PBG materials in one, two or three dimensions, e.g., modification of spontaneous emission [19-22] and photon localization [23-27].

Due to various unavoidable environmental noise, the quantum entangled degrees of entangled states become smaller with the transmission distance increasing. How to keep the purity of quantum entangled states is the puzzle in quantum communication. In the paper, we have designed a new type entanglement degrees amplifier by one-dimensional photonic crystal, which is similar as the relay station of classical electromagnetic communication. We find when the entangled states of two-photon and three-photon pass through photonic crystal, their entanglement degrees can be magnified, which make the entanglement states can be long range propagation and the quantum communication can be really realized.

2. Transfer matrix and transmissivity of one-dimensional photonic crystal

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For one-dimensional conventional PCs, the calculations are performed using the transfer matrix method [18], which is the most effective technique to analyze the transmission properties of PCs. For the medium layer $i$, the transfer matrices $M_i$ is given by [18]:

$$M_i = \begin{pmatrix} \cos \delta_i & -i \sin \delta_i / \eta_i \\ -i \eta_i \sin \delta_i / \cos \delta_i & \eta_i \end{pmatrix},$$  

(1)

where $\delta_i = \frac{2\pi n_i d_i \cos \theta_i}{c}$, $c$ is speed of light in vacuum, $\theta_i$ is the ray angle inside the layer $i$ with refractive index $n_i = \sqrt{\varepsilon_i \mu_i}$, $\cos \theta_i = \sqrt{1 - (n_0^2 \sin^2 \theta_0 / n_i^2)}$, for the TE wave $\eta_i = \sqrt{\varepsilon_i / \mu_i \cdot \cos \theta_i}$, for the TM wave $\eta_i = \sqrt{\varepsilon_i / \cos \theta_i}$, in which $n_0$ is the refractive index of the environment wherein the incidence wave tends to enter the structure, and $\theta_0$ is the incident angle.

The total transfer matrix $M$ for an $N$ period structure is given by:

$$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = M_B M_A M_B M_A \cdots M_B M_A \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix} = M \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} E_{N+1} \\ H_{N+1} \end{pmatrix},$$  

(2)

where

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$  

(3)

with the total transfer matrix $M$, we can obtain the transmissivity $T$, it is

$$T = |t|^2 = \left| \frac{E_{i1}}{E_{i1}} \right|^2 = \left| \frac{2\eta_0}{A\eta_0 + B\eta_0\eta_{N+1} + C + D\eta_{N+1}} \right|^2.$$  

(4)

Where $E_{i1}$ and $E_{i1}$ are the electric field intensity of output and input, $\eta_0 = \eta_{N+1} = \sqrt{\varepsilon_0 / \mu_0 \cdot \cos \theta_0}$ for TE wave, $\eta_0 = \eta_{N+1} = \sqrt{\varepsilon_0 / \mu_0 \cdot \cos \theta_0}$ for TM wave. By the Eqs. (1) and (4), we can calculate the transmissivity of one-dimensional photonic crystal for TE and TM wave.

3. The two-photon and three-photon polarization entangled states and quantum entanglement degree

The two-photon polarization entangled state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|HH\rangle + |VV\rangle),$$  

(5)

three-photon polarization entangled GHZ state

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|HHH\rangle + |VVV\rangle).$$  

(6)

Here the states $|H\rangle$ and $|V\rangle$ represent the horizontal (TM) and the vertical (TE) polarized single photon states.

Due to various unavoidable environmental noise, the entanglement of quantum entangled states become worse and worse with the transmission distance increasing. So, how to keep the purity of quantum entangled states is the puzzle of quantum communication. The Eqs. (5) and (6) are maximum entangled states of two-photon and three-photon, respectively. In transmission process, they should become non-maximally entangled states, they are

$$|\psi\rangle = c_1|HH\rangle + c_2|VV\rangle,$$  

(7)
$$|GHZ\rangle = c_3 |HHH\rangle + c_4 |VVV\rangle,$$

where $c_1$, $c_2$, $c_3$ and $c_4$ are real or plural coefficients, and satisfying with the normalization conditions $|c_1|^2 + |c_2|^2 = 1$ and $|c_3|^2 + |c_4|^2 = 1$. The quantum entangled degree of entangled states (7) and (8) are

$$E = - (|c_1|^2 \log_2 |c_1|^2 + |c_2|^2 \log_2 |c_2|^2).$$

With Eqs. (9) and (10), the quantum entangled degree of the two-photon and three-photon entangled states (5) and (6) are 1, i.e., maximum entanglement. For the entangled states (7) and (8), their quantum entangled degrees depend on the coefficients $c_1$, $c_2$ and $c_3$, $c_4$. When the two and three entangled photon propagate in space, the coefficients $c_1$, $c_2$ and $c_3$, $c_4$ should be changed, and the quantum entangled degree should be decreased and even approach zero, the quantum entanglement of two and three photon shall be disappeared. All the quantum information processes, e.g., quantum communication, quantum computation and so on can not proceed. The attenuation of quantum entangled degree is unavoidable, but we can magnify the quantum entangled degree in the transmission process of entangled photon, which can be achieved by one-dimensional photonic crystal.

The Eq. (4) gives the relation between the output and the input electric field intensity in one-dimensional photonic crystal, it is

$$E_{out} = t E_{in},$$

where $t$ is the transmission coefficient, $E_{out}$ and $E_{in}$ are output and input electric field intensity respectively. In Eqs. (7) and (8), when the two and three entangled photon entering one-dimensional photonic crystal, the output electric field intensity of the horizontal ($TM$) state $|H\rangle$ and the vertical ($TE$) state $|V\rangle$ are

$$E_{out}^H = t_M E_{in}^H,$$

and

$$E_{out}^E = t_E E_{in}^E,$$

where $t_M$ ($t_E$) is the transmission coefficient of horizontal (vertical) state $|H\rangle$ ($|V\rangle$), $E_{out}^H$ ($E_{out}^E$) and $E_{in}^H$ ($E_{in}^E$) are the output and input electric field intensity of horizontal (vertical) state $|H\rangle$ ($|V\rangle$) in one-dimensional photonic crystal.

The entangled states (7) and (8) are as input entangled states

$$|\psi\rangle_{in} = c_1 |HH\rangle + c_2 |VV\rangle,$$

$$|GHZ\rangle_{in} = c_3 |HHH\rangle + c_4 |VVV\rangle,$$

passing through one-dimensional photonic crystal, their output entangled states become

$$|\psi\rangle_{out} = c_1 t_M^2 |HH\rangle + c_2 t_E^2 |VV\rangle,$$

$$|GHZ\rangle_{out} = c_3 t_M^3 |HHH\rangle + c_4 t_E^3 |VVV\rangle,$$

the input entangled states and output entangled states are shown in FIG. 1, the $PC$ express the one-dimensional photonic crystal.

Eqs. (16) and (17) normalization form are

$$|\psi\rangle_{out} = d_1 |HH\rangle + d_2 |VV\rangle,$$
FIG. 1: The input entangled states $|\psi\rangle_{in}$ and $|GHZ\rangle_{in}$ pass through the photonic crystal PC become the output entangled states $|\psi\rangle_{out}$ and $|GHZ\rangle_{out}$.

$$|GHZ\rangle_{out} = d_3|HHH\rangle + d_4|VVV\rangle,$$

where normalization constants are

$$d_1 = \frac{c_1 t_M^2}{\sqrt{|c_1|^2 t_M^2 + |c_2|^2 t_E^2}}, \quad d_2 = \frac{c_2 t_E^2}{\sqrt{|c_1|^2 t_M^2 + |c_2|^2 t_E^2}},$$
$$d_3 = \frac{c_3 t_M^2}{\sqrt{|c_3|^2 t_M^2 + |c_4|^2 t_E^2}}, \quad d_4 = \frac{c_4 t_E^2}{\sqrt{|c_3|^2 t_M^2 + |c_4|^2 t_E^2}}$$

the quantum entangled degree of output entangled states (18) and (19) are

$$E = -(|d_1|^2 \log_2 |d_1|^2 + |d_2|^2 \log_2 |d_2|^2).$$
$$E = -(|d_3|^2 \log_2 |d_3|^2 + |d_4|^2 \log_2 |d_4|^2).$$

where

$$|d_1|^2 = \frac{|c_1|^2 T_M^2}{|c_1|^2 T_M^2 + |c_2|^2 T_E^2}, \quad |d_2|^2 = \frac{|c_2|^2 T_E^2}{|c_1|^2 T_M^2 + |c_2|^2 T_E^2},$$
$$|d_3|^2 = \frac{|c_3|^2 T_M^2}{|c_3|^2 T_M^2 + |c_4|^2 T_E^2}, \quad |d_4|^2 = \frac{|c_4|^2 T_E^2}{|c_3|^2 T_M^2 + |c_4|^2 T_E^2}.$$

From Eqs. (24) and (25) we can find the quantum entangled degree of output entangled states are related to the transmissivity $T_M$ and $T_E$ of one-dimensional photonic crystal. In one-dimensional photonic crystal, their transmissivity are $0 \leq T_M \leq 1$ and $0 \leq T_E \leq 1$. In the following calculation, we shall find the quantum entangled degree of output entangled states can be magnified by the given transmissivity $T_M$ and $T_E$ values.

4. Numerical result

In this section, we report our numerical results of the quantum entangled degrees for the two-photon and three-photon input and output entangled states. Firstly, we should calculate the transmissivity $T_M$ and $T_E$ for the TM and TE waves. The main parameters are: The center angle wavelength $\omega_0 = 1.22 \cdot 10^{15} Hz$, the medium $A$ refractive indices $n_a = 1.68$, thickness $a = 108 nm$, the medium $B$ refractive indices $n_b = 2.56$, thickness $b = 198 nm$, the structure is $(AB)^n$ and the incident angle $\theta_0 = \pi/6$. By Eq. (4), we can calculate the transmissivity of $T_M$ and $T_E$, which are shown in FIG. 2. FIG. 2 (a) and (b) are the transmissivity of $T_M$ and $T_E$, respectively. We can find their transmissivity are $0 \leq T_M \leq 1$ and $0 \leq T_E \leq 1$. We shall use the $T_M$ and $T_E$ values in calculating the quantum entangled degrees. Secondly, by Eq. (9), we calculate the quantum entangled degrees of the two-photon input entangled state (14), which is shown in FIG. 3 (a), it
FIG. 2: The one-dimensional photonic crystal transmissivity $T_M$ and $T_E$ for $TM$ and $TE$ waves.

FIG. 3: The quantum entangled degree of two-photon input and output entangled states.

FIG. 4: The quantum entangled degree of three-photon input and output entangled states.
gives the relation between coefficient $c_1$ and the entangled degrees $E$ of input entangled state. By Eqs. (22) and (24), we calculate the quantum entangled degrees of the two-photon output entangled state (16), which is shown in FIG. 3 (b), there are five output quantum entangled degrees curves. The curves 1, 2, 3, 4 and 5 are obtained by taking the transmissivity $T_E = 0.04$ and $T_M = 1.0$, $T_E = 0.12$ and $T_M = 1.0$, $T_E = 0.30$ and $T_M = 0.80$, $T_E = 0.60$ and $T_M = 0.50$, $T_E = 1.0$ and $T_M = 0.30$ respectively. The five groups different transmissivity can be easily realized by five different structure one-dimensional photonic crystals. From FIG. 3 (a), we can find the entangled degrees $E$ relaxedly increases from 0 to 1 when the coefficient $c_1$ is in the range of $0 \sim \frac{1}{\sqrt{2}}$, and the entangled degrees $E$ relaxedly decreases from 1 to 0 when the coefficient $c_1$ is in the range of $\frac{1}{\sqrt{2}} \sim 1$, i.e., the input quantum entangled degrees of two-photon tend to 1 in the vicinity of $c_1 = \frac{1}{\sqrt{2}}$, and it is relatively low in the other area of $c_1$. Then, the input entangled state of two-photon can not proceed long range propagation. In FIG. 3 (b), we can find the output quantum entangled degrees $E$, constituted by five output quantum entangled degrees curves, the quantum entangled degrees $E$ is in the range of $0.8 \sim 1$ when $c_1 = 0.02 \sim 0.99$, which has been magnified. When the output quantum entangled degrees can be magnified continuously, the output entangled state of two-photon should be achieved long range propagation. Finally, for the three-photon input and output entangled states (15) and (19), we have similarly obtained the input and output quantum entangled degrees, which are shown in FIG. 4 (a) and (b), and the results are the same as the two-photon input and output quantum entangled degrees.

5. Conclusion

In summary, we have designed a new type quantum entangled degrees amplifier by one-dimensional photonic crystal, which is similar as the relay station of classical electromagnetic communication. By calculation, we find when the entangled states of two-photon and three-photons pass through photonic crystal, their quantum entanglement degrees can be magnified, which make the entanglement states of two-photon, three-photon and multi-photon can be long range propagation and the quantum communication can be really realized.

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