Production Scheduling Optimization to Minimize Makespan and the Number of Machines with Mixed Integer Linear Programming

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Abstract. In production management field, production efficiency can be improved through minimizing machine downtime, optimizing production scheduling, minimizing inventory of raw materials and finished products. In this research, the production efficiency is improved through optimizing production schedule that has two goals; not only minimizing production makespan but also minimizing the number of machines used. For the purpose, a mixed integer linear programming (MILP) model was developed. The model was used to solve a problem producing 6 products with 4 production lines. In this study, product demand was first forecasted with Autoregressive Integrated Moving Average (ARIMA) method. Optimization results come up with makespan of 295.4 hours using only one production line. As addition, this research is also dealing with three real life production scenarios.

1. Introduction
Production scheduling belongs to supply chain operation type of decision that sets in weekly or daily basis [1]. Operation decision has a strong impact on overall company profitability and success. The important role of scheduling is even more this day due to the existence of globalization and the increasingly demanding customers. A company may end up into troubles if it fails to fulfill the production deadline or it faces a delay on introducing its new product to the market. On both situations, customers may shift to other brand name product to fulfill their needs. This situation will lead to a significant loss. Scheduling is also important since scheduling is also dealing with using the available limited resources, i.e. machine, man, time, and money in an efficient way[2]. Scheduling is, thus, an optimization problem that has one or more goal to be reached.

Scheduling jobs on machine(s) has many settings and objectives. The simplest scheduling is with single machine. The setting can be, for example, time dependent [3], indivisible orders[4], release date [5]. A more complex scheduling is dealing with parallel machines [2,6,7], multiple different machines, and jobshop[2,8,9]. Scheduling several different jobs to single machine requires the machine to be set up every time changing the job. Since there exist setup decisions in the system, the problem can be formulated as a mixed integer linear programming (MILP)[10]. MILP will be more efficient if it is solved by B&B algorithm [11,12,13].

[14,15]wrote time series forecasting is an important area of forecasting in which past observations of the same variable are collected and analyzed to develop describing the underlying relationship. One of the most important and widely used time series models is the autoregressive integrated moving average (ARIMA) model.

In practical setting in industry, each machine on production line has its own capacity, age, operating requirement and mode, as well as maintenance program. This variety gives different
machine capability and utilization. Production scheduling is quite challenging problem since it has many varieties of settings, big size and different target. The problem is even complicated by for instance engine failure and demand uncertainty.

In this research, production scheduling objective to be achieved is not a single target, but it has two goals, minimizing makespan and minimize the number of machines used. Issues such as replacement of product variations and variety of the number of demands are also addressed. Scheduling problem to minimize makespan and line machine utilization is, as far as the author's knowledge, new and no papers have dealt with. In this study, scheduling is combined with forecasting. The proposed method was used to solve a problem in a fast moving consumer good (FMCG) that produces soaps.

2. Materials and methods
Scheduling problems deal with a number of jobs to be processed at a number of machines[2]. In a production, a job can represent an item of a product. To perform the job, some pieces of data are required, i.e. duration time of the job processed on the machine, the time when the job is ready to be processed, the dead line of the job, and the priority factor. This later data can be ignored if all jobs have the same priority. From the machine environment angle, several scheduling settings might exist. For instance, single machine, identical parallel machine, parallel machine with different speed, flow shop, and job shop. Scheduling jobs in a single machine is the simplest scheduling environment. For this scheduling, the objective might be to minimize completion time, to minimize number of tardy jobs, to minimize makespan, etc. identical parallel machine can be considered as a number of single machines. In a job shop with some machines’ environment, each job has its own process to be completed. In a job shop, a job might or might not revisit one same machine. Figure 1 illustrates an example of scheduling 10 products on production line A (machine A is line A in single machine setting).

![Figure 1](image)

**Figure 1.** 10 different products are scheduled to be processed on production line A.

2.1. Mathematical model
The following section explain the proposed mathematical model that has two objective functions to reached. Since the model is to minimize, the goal can be achieved by minimizing the number of machines used or minimizing the makespan or minimizing both objectives at the same time.

Indexes:
- \( i \) = product \( i = 1, \ldots, N \) (\( N \) = number of products made)
- \( j \) = production line \( j = 1, \ldots, M \) (\( M \) = the number of machines used)

The decision variables:
- \( M_{ij} \) = the start time of product \( i \) on line \( j \)
- \( S_{ij} \) = time finished product \( i \) on line \( j \)
- \( d_{ij} \) = the remainder of the production of \( I \) on line \( j \)
- \( C \) = constant
- \( G \) = constant
- \( Q_i \) = the number of products \( i \) requested
\( V_j \) = the speed of the production line \( j \)
\( \frac{Q_i}{V_j} \) = time of production order \( i \)
\( MS_j \) = makespan

Min \( Z = C \sum_{j}^{M} z_j + MS_j \) .............. (1)

St:

\[ \sum_{i}^{N} x_{ij} \leq 1 \quad \forall j \] .................. (2)

\[ \sum_{j}^{M} x_{ij} = 1 \quad \forall i \] .................. (3)

\[ S_{ij} = M_{ij} + \sum_{i}^{N} \frac{Q_i}{V_j} x_{ij} + 0,5(x_{ij}) \] .............. (4)

\[ M_{i} \geq M_{ij} \quad \forall i \quad \forall j \] .................. (5)

\[ S_{i} \geq S_{ij} \quad \forall i \quad \forall j \] .................. (6)

\[ M_{ij} \leq S_{i+1j} \quad \forall i; \forall j \] .................. (7)

\[ M_{i+1j} \leq S_{ij} \quad \forall i; \forall j \] .................. (8)

\[ G(1-y_{i+1j}) + M_{ij} - S_{i+1j} \geq 0 \forall i; \forall j \] .... (9)

\[ M_{ij} = 0 \forall i; \; i = 1; \quad \forall j \] ..............(11)

\[ 550 - S_{ij} = d_{ij} \forall i; \; \forall j \] .............. (12)

\[ 550 \leq S_{ij} + z_{ij} \forall i; \; \forall j \] .............. (13)

\[ MS_{j} \geq S_{ij} \forall i; \; \forall j \] .............. (14)

\[ MSS \geq MS_{j} \forall j \] .................. (15)

\[ x_{ij} \in \{0,1\} \forall i, j \] .................. (16)

\[ z_{ij} \in \{0,1\} \forall j \] .................. (17)

\[ y_{ijk} \in \{0,1\} \forall i, j, k \] .................. (18)

Equation (1) is the objective function of the model, i.e. minimize the makespan and the number of production lines used. Equations 2 to 18 are the model constraint. Equation (2) is line constraint that says on one-line \( j \) at most only one product is done. Equation (3) is product constraint that says a product \( i \) can only be worked on one line \( j \). Equation (4) shows the constraints of the time needed to work on the product \( i \), in line \( j \). Equation (5) and equation (6) show the relationship \( M_{ij} \) and \( S_{ij} \) will be zero if the work was not done on the machine that there is a new variable that is \( M_i \) and \( S_i \) are positive that the product \( i \) worked on one machine \( j \). Equations (7) and (8) are a constraint options, if there are two products \( i \) and \( i + 1 \) can be done at the same time instance, then the constraints will sequence \( i \) after \( i + 1 \) or \( i + 1 \) after \( i \). Equations (9) and (10) are the linear form of equations (7) and (8). Equation (11) shows a beginning process of time constraint. Equation (12) is the rest of the time constraint. Equation (13) is the line usage constraints. Equation (14) shows makespan line \( j \) as the longest time of all time completion of work on the line. Equation (15) the makespan of the line is the longest time of
all existing job completion time. Equations (16, 17, 18) are binary constraints. These constraints will give an option whether to utilize or not to utilize the machine.

2.2. Model verification and validation

To be able to use the proposed mathematical model to solve real problems, the model is translated to a computer program with Lingo software version 11.0. To check and to make sure that the computer coding is correct, the computer code is verified and validated. The program is verified if it can be run without error and give an output result. While to validate the program, a small scale problem is prepared and solved manually and using the program. If the results with manual calculation are the same with the results solved by the program computer, the program is validated.

For the purpose of validation, a small scale scheduling problem is shown on tables 1 and 2. Table 1 is listed 4 products with their respective demand. Table 2 tells about 2 production lines with their capacities.

**Table 1.** Small scale problem demand data.

| Product | Demand (box) |
|---------|--------------|
| i1      | 8,000        |
| i2      | 9,000        |
| i3      | 3,500        |
| i4      | 8,000        |

**Table 2.** Machine capacity data.

| Line | Capacity V (box/hour) |
|------|-----------------------|
| j1   | 100                   |
| j2   | 600                   |

Table 3 shows the processing time of each product if it is produced at the available line. The processing time is calculated by dividing the demand with machine production capacity \( Q/V_j \).

**Table 3.** The processing time required for product.

| Line | Product | Process Time (hour) |
|------|---------|---------------------|
| j1   | i1      | 80.0                |
|      | i2      | 90.0                |
|      | i3      | 35.0                |
|      | i4      | 80.0                |
|      | i1      | 13.3                |
| j2   | i2      | 15.0                |
|      | i3      | 5.8                 |
|      | i4      | 13.3                |

Table 4 shows that for the lines j1, the smallest processing time required is 35 hours for product i3, while on the line j2 the maximum processing time is 15 hours for product i2. Since the processing time of line j1 is bigger than j2 processing time, j1 is not selected. Table 4 shows the finish time of all products in line j2. The completion time is calculated using Equation (4).
Table 4. Manual calculation result.

| Line | Product | Start Time (hour) | Processing Time (hour) | Completion Time (hour) |
|------|---------|-------------------|------------------------|------------------------|
| J2   | i1      | 0.0               | 13.3                   | 13.8                   |
|      | i2      | 13.8              | 15.0                   | 29.3                   |
|      | i3      | 29.3              | 5.8                    | 35.7                   |
|      | i4      | 35.7              | 13.3                   | 49.5                   |

Validation with Lingo version 11.0 was performed on a computer with Intel (R) Core (TM) i5-3210M CPU @ 2.5 GHz, 4 GB of RAM. Computer operating system is Windows 7. Table 5 tabulates the results. Furthermore, if the results on Table 6 is plotted on a Gantt chart, the chart is displayed in Figure 2.

Table 5. Lingo optimization results.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| J2   | i1      | 0        | 13.3     | 13.8     |
|      | i3      | 13.8     | 5.8      | 20.2     |
|      | i2      | 20.2     | 15       | 35.7     |
|      | i4      | 35.7     | 13.3     | 49.5     |

Description: M: starting time, T: processing duration time, S: completion time

Since the makespan result of the optimization shown in table4 and the makespan result of the manual calculations shown on Table5 is the same, i.e. 49.5 hours, it is concluded that s the proposed computer algorithm is validated. The algorithm is ready to be used to solve real case problems.

| Line | Monday | Tuesday | Wednesday |
|------|--------|---------|-----------|
| j2   | i1     | i3      | i2        | i4        |

Figure 2. Gantt chart of example problem optimization result.

Gantt chart on figure 2 is the real schedule plotted on the calendar time. The chart tells that one complete production cycle is taking 2.5 working days. It will be 2 changeovers on the first working day and one changeover on the second day.

3. Results and discussion
The model developed was used to determine production schedule of an FMCG company that produces soaps. 6 products will be made on 4 production lines each of which has a single machine. Each line has different speed (different capacity). Table 6 describes the production line capacity of each line.

Table 6. Machine capacity.

| Line | box/shift | box/hour |
|------|-----------|----------|
| A    | 1,500     | 214      |
| B    | 800       | 114      |
| C    | 1,200     | 171      |
| D    | 2,500     | 357      |
Assumptions used:
- Scheduling is conducted for a product with 6 different colors that have a certain demand quantity (in number of boxes).
- When a machine is producing different color, there is a changeover time of 0.5 hours for the 24-hour product replacement.
- If a given demand meets/exceeds the capacity of the machine, the line can be run all. However, if the demand is granted under a certain line, then it will be a line(s) that can be turned off.
- The production of certain lines in accordance with the capacity of these lines (assuming no product damage in lines).
- Working time for 1 month is 25 days, and 1 day = 22 hours. So that the total working hours within 1 month is 22 x 25 = 550 working hours.

For estimating the number of jobs (the number of products) for each product, a forecasting with ARIMA method is conducted. The forecast uses 27-months historical selling data for the six products. Table 7 shows the forecasting results including their error.

### Table 7. Forecasting results with ARIMA.

| Product | Arima Model (p,d,q) | MAPE (%) | Demand Forecasting (box) |
|---------|---------------------|----------|--------------------------|
| 1       | 2,0,0               | 8.534    | 29,192                   |
| 2       | 2,0,0               | 8.92     | 23,209                   |
| 3       | 2,0,0               | 10.26    | 16,182                   |
| 4       | 2,0,0               | 9.369    | 9,470                    |
| 5       | 2,0,0               | 14.96    | 14,794                   |
| 6       | 2,0,0               | 12.735   | 11,530                   |

### 3.1. Existing scheduling
Currently the production scheduling at the company is developed subjectively by the employees who have experience in making the schedule. The schedule is based on achieving the minimum possible machine changeover. Using the number of products obtained from ARIMA forecasting shown in table 8, the existing company scheduling result is shown in table 8.

### Table 8. Scheduling results using subjective method.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| A    | 1       | 0.0      | 136.4    | 136.9    |
| B    | 6       | 0.0      | 101.1    | 101.6    |
|      | 4       | 101.6    | 83.1     | 185.2    |
| C    | 3       | 0.0      | 94.6     | 95.1     |
| D    | 5       | 95.1     | 86.5     | 182.1    |

Table 8 shows that the makespan of the schedule is 185.2 hours. The table also shows that all the four production lines are utilized. Lines A and D are totally used to make products 1 and 2.
respectively. Line B is used to make products 6 and 4. Products 3 and 5 are processed at line C. Scheduling on Table 9 presented with Gantt chart is shown on figure 3.

![Gantt chart for scheduling product of table 8.](image)

Table 9. Scheduling result using optimization to minimize makespan.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| A    | 2       | 0        | 108.5    | 109      |
| B    | 5       | 0        | 129.8    | 130.3    |
| C    | 6       | 0        | 67.4     | 67.9     |
|      | 4       | 67.9     | 55.4     | 123.8    |
| D    | 1       | 0        | 81.8     | 82.3     |
|      | 3       | 82.3     | 45.3     | 128.1    |

Table 10 shows that using optimization approach, the makespan is 130.3 hours. This value is almost 35 hours less than the makespan of subjective scheduling method. This shorter makespan is achieved through utilizing all the four lines like the results of subjective method but with different arrangement. Lines A and B are used dedicatedly to make products 2 and 5, respectively. While lines C and D are used to make products 6 and 4, and 1 and 3, respectively. Scheduling results on table 10 presented with Gantt chart is displayed on figure 4.

![Gantt chart of the optimization of Table 10](image)

Gantt chart on figure 4 informs that the demand on table 8 can be made within 6 working days (Monday to Monday). This production cycle is 2 days shorter than subjective scheduling. Scheduling
with objective to minimize makespan, fortunately has also 2 times changeover; one changeover at line C and the other is at line D. Another benefit of this approach is a high machine utilization. Lines B and D have 100% utilization. The lowest machine utilization is Line A which is 83.3%.

### 3.3. Scheduling with objective function minimizing makespan and total line used

Table 10 shows the optimization results with objective function to minimize makespan and the number of lines being used.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| 1    | 0       | 81.8     | 82.3     |          |
| 3    | 82.3    | 45.3     | 128.1    |          |
| 6    | 128.1   | 32.3     | 160.9    |          |
| 2    | 160.9   | 65       | 226.4    |          |
| 5    | 226.4   | 41.4     | 268.3    |          |
| 4    | 268.3   | 26.5     | 295.4    |          |

With the objective function minimizing the number of lines as addition to minimizing makespan, the result on Table 11 shows that the schedule is only using one line, i.e. line D. the schedule is possible since line D has the highest capacity (see Table 6). For this schedule, the makespan is 295.4 hours. When production is only using one machine from some m available parallel machines, the problem become scheduling with single machine problem. Figure 5 shows the Gantt chart for scheduling result of Table11. The figure shows that the production cycle is 3 weeks. During that period, the number of changeovers is 5.

| Line | Mon-Thurs | Fri | Sun | Mon | Tues-Wed | Thur-Fri | Sun | Mon | Tues-Wed | Thurs-Fri |
|------|-----------|-----|-----|-----|----------|----------|-----|-----|----------|-----------|
| D    | 1         | 3   | 3   | 6   | 2        | 2        | 5   | 4   | 3        |

**Figure 5.** Gantt chart optimization results of Table 11

### 3.4. Cost Perspective Analysis

When dealing with 2 different parameters that giving 2 different results, it is not easy to determine which condition is better. For example is comparing results of optimization methods with one and two objective functions. In this situation, comparison in term of costs can be used. Table 11 shows data used for conducting cost analysis.

| Line | Line Capacity (Box/hour) | Electricity Power (kW) | Man Power (man) |
|------|--------------------------|------------------------|-----------------|
| A    | 214                      | 1,000                  | 12              |
| B    | 114                      | 723                    | 10              |
| C    | 171                      | 1,234                  | 12              |
| D    | 357                      | 1,364                  | 20              |

Assuming that the 0.5 hour changeover is comparable to produce 2,500 pcs of product, and if a product price is 1,000 IDR, the changeover’s worth is equal to 2,500,000 IDR. Based on the minimum government wage regulation, wage per hour for the worker is calculated as 6,090.9 IDR. Electricity
rate is 1,150 IDR per kWh. In this case, the total cost is the operating cost which is the summation of electrical cost to run the lines, wage to pay the workers running the machine and changeover equivalent. Furthermore, the electrical cost is a multiplication of electricity rate (in IDR/kWh), electricity power to run the machine (in kW) and the duration to run the line (in hours). While the wage is a multiplication of the number of workers to operate the line (man), wage rate per hour (IDR/man) and the working duration (hour). Using working duration listed on tables 9, 10, and 11, table 12 displays the summary of the calculation.

Table 12. Summary of the total cost.

| Scheduling Model     | Cost (IDR) | % saving |
|----------------------|------------|----------|
|                      | Electrical | Wage     | Changeover | Total      |
| Existing (subjective)| 672,580,950| 42,575,454| 5,000,000  | 720,156,405| -          |
| Single objective     | 610,310,175| 40,556,927| 5,000,000  | 655,867,102| 8.9        |
| Two objectives       | 463,364,440| 55,505,236| 15,000,000 | 533,869,676| 25.87      |

Table 12 clearly shows that the scheduling model with two objective functions (minimize makespan&the number of lines used) will give the least total cost for producing the same product quantity compare to the subjective and single objective scheduling model (minimize makespan only). The model can reduce 25.87% of cost compare to the manual existing one even though the makespan of this model is longer than manual scheduling and single objective scheduling model.

3.5. Sensitivity Analysis

Three scenarios are provided to check the proposed model applicability to provide optimal results. The first is demand increases triple or more scenario, changing production time scenario, and unused production line scenario. The third scenario is for example a situation due to predicted line maintenance activity that should be done without disrupting production scheduling engine.

3.5.1. Scenario 1: Surging demand

This scenario is a situation where additional demand occurs threefold or more than the forecasting value. This situation is not an impossible situation. In real life, demand can increase triple or more when special events occur. For example, during religious holiday or at the end of year. To cope such a situation, companies usually carry safety inventory for a given period[1]. However, if the sudden demand is beyond the safety inventory, company can implement overtime strategy or sub-contracting strategy. In this scenario, say the demand goes up like shown at Table 13. The demand goes up 4 times than the forecast. For this kind of demand, will the existing production capacity be able to handle? If the answer is yes, what is the production arrangement and the schedule?

Table 13. Scenario 1 surging demand.

| Product | Demand (Box) | Demand (Box) |
|---------|--------------|--------------|
|         | Forecasting  | Scenario 1   |
| 1       | 29,192       | 89,564       |
| 2       | 23,209       | 73,826       |
| 3       | 16,182       | 71,975       |
| 4       | 9,470        | 51,554       |
| 5       | 14,794       | 58,290       |
| 6       | 11,530       | 68,740       |
Table 14 is the optimization result for the scenario displayed on Table 13. The result show that the current production lines are able to handle a demand spike of 4 folds if the machine is scheduled properly. Table 14 clearly shows that to handle the high jump of the demand, all the four lanes are utilized with the makespan 540.1 hours. This makespan makes the production lines almost in 100% utilization. The makespan is approaching the total machine available time, i.e. 550 hours. For this scenario, Lines A, B, and C are dedicated to produce products 2, 5, and 1, respectively. While the rest of products, i.e. products 3, 4, and 6 are scheduled at Line D with the production sequence as product 6, product 3, and product 4.

Table 14. Scheduling result for Scenario 1.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| A    | 2       | 0        | 345      | 345.5    |
| B    | 5       | 0        | 511.3    | 511.8    |
| C    | 1       | 0        | 523.8    | 524.3    |
|      | 6       | 0        | 192.5    | 193      |
| D    | 3       | 193      | 201.6    | 395.2    |
|      | 4       | 395.2    | 144.4    | 540.1    |

3.5.2. Scenario 2: Changing production time

This scenario happened when the total available time for the lines to be operated is only 295.4 hours instead of 505 hours. This scenario happens for instance if national holidays are observed at one particular month. The holiday will reduce the regular production time. The scenario may frequently happen in Indonesia so it should be anticipated.

Table 15 shows the optimization scheduling results for this scenario. The results indicate to use the two largest capacity machines, i.e. lines D and A. Line D is used for producing products 3 and 2, while line A is for products 1, 5, 6, and 4. The makespan for this scheduling scheme is 185.1 hours. The two lines almost utilized at the same utilization rate.

Table 15. Scheduling result for Scenario 2.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| D    | 3       | 0        | 75.6     | 76.1     |
|      | 2       | 76.1     | 108.5    | 185.1    |
|      | 1       | 0        | 81.8     | 82.3     |
|      | 5       | 82.3     | 41.4     | 124.2    |
|      | 6       | 124.2    | 32.3     | 157      |
|      | 4       | 157      | 26.5     | 184      |

3.5.3. Scenario 3: Production line is not being used
Scenario 3 is a situation where the number of products needed is normal but a line with the largest capacity is broken or turned off for maintenance. If this scenario happens, what is the optimum scheduling to be used? In this research, the scenario is only checked for the largest capacity machine since this scenario will make sure the production will be fulfilled if a lesser capacity machine was broken.

Table 16 shows the optimization result for scenario 3. The production can be scheduled with the second largest capacity machine, Line A. The makespan for this scenario is 490.7 hours and the production sequence is product 1, 3, 2, 5, 4, and 6.

### Table 16. Scheduling result for Scenario 3.

| Line | Product | M (hour) | T (hour) | S (hour) |
|------|---------|----------|----------|----------|
| A    | 1       | 0        | 136.4    | 136.9    |
|      | 3       | 136.9    | 75.6     | 213      |
|      | 2       | 213      | 108.5    | 322      |
|      | 5       | 322      | 69.1     | 391.6    |
|      | 4       | 391.6    | 44.3     | 436.4    |
|      | 6       | 436.4    | 53.9     | 490.7    |

### 4. Conclusion

The proposed scheduling mathematical model with an objective function to minimize makespan and the number of lines can produce a better makespan and better machine utilization compare to a single optimization model and subjective approach. From cost point of view, the model provides a smaller fee than a model with just to minimize makespan. The model can be used in various settings with a single machine scheduling and/or parallel machine scheduling.

In the case study, the model with two objective functions may obtain makespan of 295.4 hours with a single line machine. It can reduce 25.87% of cost compare to the manual existing schedule.

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