A test of the continuous spontaneous localization model involving two particles

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We describe a previously unexplored effect of the continuous spontaneous localization model whereby a correlation develops in the distributions of two nearby non-interacting particles following a period of diffusion. We propose the use of this effect as an experimental test differentiating between the continuous spontaneous localization model and standard quantum theory. The test involves building a joint probability distribution for the locations of the two particles by repeatedly releasing them from two nearby traps and subsequently measuring their positions after a brief period of time. We examine the scales of time, trap size, and particle mass necessary for observation.

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The continuous spontaneous localization (CSL) model\cite{1,2,3} is an alternative to standard quantum theory in which the standard Schrödinger dynamics are extended such that the state behaves stochastically. The model predicts that certain superposition states are unstable. In particular a macro superposition of quasi-localized position states will collapse at a rate which increases with the mass of the object. The motivation behind the CSL model is to provide a unified description of quantum dynamics encompassing both unitary evolution and state reduction. During the measurement of a micro quantum particle, the state of the particle becomes entangled with a macro state of the measuring device. Continuous spontaneous localization describes the collapse of the quantum measuring device which induces, via the entanglement, a collapse of the particle being measured.

Since the CSL model involves a modification of the Schrödinger equation, it makes predictions which are in conflict with standard quantum theory. There is therefore the possibility to experimentally test CSL. A direct way to do this is to try to observe quantum interference for objects of increasing mass. For example, in a diffraction experiment, an object is brought into a superposition of being in different locations before the different wave packets are allowed to overlap. For an object of sufficient mass, CSL predicts that the state will collapse to one of the packets before they overlap and interference will not be observed. A more detailed study shows that there should be a characteristic loss of fringe visibility with changes in the various parameters of the experiment (see Refs 1, 2, 3).

Here we consider a situation with two non-interacting particles. We will examine how the particles behave as a result of the CSL dynamics and show that there is a potentially measurable effect whereby the diffusions undergone by each particle are correlated. A key feature of the CSL model is that the localization mechanism acts on the total smeared mass density state rather than individually on each particle. This suggests that for two nearby particles, the diffusive behaviour caused by the localization mechanism will be correlated. This should have an effect on the joint spatial probability distribution. In what follows we shall solve the two-particle CSL master equation to determine the behaviour precisely.

There are two fixed parameters in the CSL model which are treated as fundamental constants. These are the localization rate \( \lambda \) and the localization length scale \( 1/\sqrt{\alpha} \). We consider an approximation in which the localization length scale is large compared to the length scale defining the spatial extent of the system. The original estimates of these parameters by Ghirardi, Rimini and Weber (GRW)\cite{8} are

\[
\lambda = 10^{-16} \text{s}^{-1}, \quad 1/\sqrt{\alpha} = 10^{-7} \text{m}.
\]

However, these values are not definitive. A recent study of the valid regions of the \( \lambda - \alpha \) parameter space consistent with experiment puts no upper bound limit on the CSL length scale\cite{8}.

We consider two non-interacting particles of the same mass. We suppose that the two particles are identical bosons although this is not crucial to our argument—the same conclusion holds if the particles are non-identical.

The CSL master equation is given by

\[
\frac{\partial \hat{\rho}_t}{\partial t} = -\frac{i}{\hbar} \left[ \hat{H}, \hat{\rho}_t \right] - \frac{\lambda}{2} \int dx \left[ \hat{M}(x), \left[ \hat{M}(x), \hat{\rho}_t \right] \right],
\]

with density operator \( \hat{\rho}_t \) and free Hamiltonian \( \hat{H} \). The smeared mass density operator is given by

\[
\hat{M}(x) = \left( \frac{\alpha}{\pi} \right)^{3/4} \frac{m}{m_0} \int dy \ e^{-\alpha (x-y)^2/2} \hat{a}^{\dagger}(y)\hat{a}(y),
\]

where \( m_0 \) is the nucleon mass (used to set the mass scale), and the field annihilation and creation operators \( \hat{a}(x) \) and \( \hat{a}^{\dagger}(x) \) satisfy

\[
[\hat{a}(x), \hat{a}^{\dagger}(y)] = \delta(x-y).
\]

The two-particle state is represented by

\[
|\psi\rangle = \int dx_1 dx_2 \psi(x_1, x_2) \frac{1}{\sqrt{2}} \hat{a}^{\dagger}(x_1)\hat{a}^{\dagger}(x_2)|0\rangle,
\]

\[
\hat{a}(x) = \int \sqrt{\frac{m_0}{\hbar\alpha}} \ e^{ikx} \ e^{-\frac{m_0}{\hbar\alpha} \left( x^2 + y^2 \right)/2} dy.
\]
where $i$ and $2$ label the two particles and $\psi$ is a symmetric wavefunction. In this representation improper position eigenstates are given by

$$|x_1, x_2\rangle = \frac{1}{\sqrt{2}} \hat{a}^\dagger(x_1) \hat{a}^\dagger(x_2)|0\rangle,$$

and the two-particle density matrix is represented in coordinate space as

$$\rho_t(x_1, y_1, x_2, y_2) = \langle x_1, x_2 | \rho_t | y_1, y_2 \rangle.$$  

The coordinate space representation of Eq. (2) for the two-particle state in the limit of large localization length is found to be

$$\frac{\partial}{\partial t} \rho_t = \frac{i\hbar}{2m} \left( \frac{\partial^2}{\partial x_1^2} - \frac{\partial^2}{\partial y_1^2} \right) \rho_t + \frac{i\hbar}{2m} \left( \frac{\partial^2}{\partial x_2^2} - \frac{\partial^2}{\partial y_2^2} \right) \rho_t - \frac{D}{\hbar^2} \left[ (x_1 - y_1)^2 + (x_2 - y_2)^2 - (x_1 - x_2)^2 - (y_1 - y_2)^2 \right] \rho_t,$$

where

$$D = \frac{\hbar^2 \lambda}{4} \left( \frac{m}{m_0} \right)^2.$$  

Here, the limit of large localization length specifically means that $1/\sqrt{\alpha} \gg |x_i - y_i|$ for $i = 1, 2$, and $1/\sqrt{\alpha} \gg |x_1 - x_2|$. From here on we treat Eq. (8) in one dimension, effectively having traced out the other two.

We now present a solution of Eq. (8) (further details of the techniques used can be found in an accompanying article [10]). The solution is represented in terms of a density matrix propagator $J$ (see Refs [11, 12]) as

$$\rho_t(x_1, y_1, x_2, y_2) = \int dx_1' dy_1 dx_2 dy_2 J(x_1, y_1, x_2, y_2, t|x_1', y_1', x_2', y_2', 0) \times \rho_0(x_1', y_1', x_2', y_2').$$  

We find that for Eq. (8), $J$ is given by

$$J(x_1, y_1, x_2, y_2, t|x_1', y_1', x_2', y_2', 0) = \left( \frac{m}{2\pi\hbar t} \right)^2 e^{-(im/2\hbar t) \sum [(x_i - x_i')^2 - (y_i - y_i')^2]} \times e^{-(D t/3\hbar)^2 \sum [(x_i - x_i')^2 + (x_i - y_i')(x_i' - y_i') + (x_i' - y_i')^2]} \times e^{(D t/3\hbar)^2 [(x_i - x_i')^2 + (x_i - y_i')(x_i' - y_i') + (x_i' - y_i')^2]} \times e^{(D t/3\hbar)^2 [(y_i - y_i')^2 + (y_i - y_2')(y_i' - y_2') + (y_i' - y_2')^2]},$$  

where $i, j = 1, 2$.

We consider an initial wave function of the form

$$\psi(x_1, x_2) = \frac{1}{\sqrt{4\pi \sigma^2}} e^{-(x_1 - \mu)^2/4\sigma^2} e^{-(x_2 + \mu)^2/4\sigma^2} + \frac{1}{\sqrt{4\pi \sigma^2}} e^{-(x_1 + \mu)^2/4\sigma^2} e^{-(x_2 - \mu)^2/4\sigma^2}.$$  

This represents the particles residing in adjacent harmonic traps of width $\sigma$ and spaced by a distance $2\mu \gg \sigma$. The initial density matrix is

$$\rho_0(x_1, y_1, x_2, y_2) = \psi(x_1, x_2) \psi^*(y_1, y_2).$$  

We suppose that the particles are simultaneously released from the traps and allowed to diffuse freely in the direction parallel to the line connecting the two traps. However, they remain sufficiently separated that there is negligible conventional interaction between them. Using Eqs (10) and (11) we have calculated the diagonal part of the density matrix

$$P_t(x_1, x_2) = \rho_t(x_1, x_1, x_2, x_2),$$  

following a time period $t$ after their release. This represents the joint probability distribution for the subsequently measured positions of the two particles. The result is simplest when expressed in terms of the variables

$$X = \frac{x_1 + x_2}{2} \quad \text{and} \quad \xi = x_1 - x_2.$$  

where we find

$$P_t(X, \xi) = \frac{1}{4\pi \sigma^2 \sqrt{L}} e^{-X^2/2\sigma_X^2} e^{-(\xi/2 - \mu)^2/2\sigma_{\xi/2}^2} + \frac{1}{4\pi \sigma^2 \sqrt{L}} e^{-X^2/2\sigma_X^2} e^{-(\xi/2 + \mu)^2/2\sigma_{\xi/2}^2},$$  

with

$$L = \frac{D\hbar^2 \lambda^5}{3m^4 \sigma^6} + \frac{\hbar^4 t^4}{16m^4 \sigma^8} + \frac{4Dt^3}{3m^2 \sigma^2} + \frac{\hbar^2 t^2}{2m^2 \sigma^4} + 1,$$  

and

$$\sigma_X^2 = \frac{1}{2} \sigma^2 L \left( \frac{\hbar^2 t^2}{4m^2 \sigma^4} + 1 \right)^{-1},$$  

$$\sigma_{\xi/2}^2 = \frac{1}{2} \sigma^2 L \left( \frac{4Dt^3}{3m^2 \sigma^2} + \frac{\hbar^2 t^2}{4m^2 \sigma^4} + 1 \right)^{-1}. $$  

The distribution consists of two peaks, one about $X = 0, \xi/2 = \mu$ and another about $X = 0, \xi/2 = -\mu$. Note that we have ignored a possible interference term between the two peaks in Eq. (11) since we assume that they do not disperse enough to overlap. The feature that we are interested in is the shape of these peaks and in particular their rate of dispersion in the two directions $X$ and $\xi/2$. The $X$ and $\xi$ spreads of each of the peaks are defined by $\sigma_X$ and $\sigma_{\xi/2}$ respectively. We see that the spread after time $t$ in the distribution of $X$ is greater than the spread in $\xi/2$. The difference is due to the presence of the CSL diffusion parameter $D$ and is therefore an effect of CSL.

Let us contrast this with the case of standard quantum mechanics. By setting $D = 0$ in Eqs (18) and (19) we find

$$\sigma_X = \sigma_{\xi/2} = \frac{1}{2} \sigma^2 \left( \frac{\hbar^2 t^2}{4m^2 \sigma^4} + 1 \right).$$  


This reflects the fact that the two particles are behaving independently and their distributions are uncorrelated.

The physical reason for the difference between $\sigma_X$ and $\sigma_{\xi/2}$ when $D \neq 0$ is that the diffusive shifts in position of the two particles caused by the localization mechanism will be positively correlated if the particles are closer together than the localization length scale. This tends to prevent $\xi$ from spreading even though the system as a whole diffuses.

Our result is clearly dependent on the fact that the localization length scale $1/\sqrt{\alpha} > 2\mu$, but in this limit the spreads in $X$ and $\xi/2$ do not depend on the separation $2\mu$. From an experimental point of view this implies that $\mu$ does not need to be accurately controlled provided that it can be accurately measured (to set the centre for the $\xi/2$ distribution). For $1/\sqrt{\alpha} < \mu$ the particles would behave independently under CSL. A null observation could therefore be used to put a constraint on $\alpha$.

![Diagram](attachment:image.png)

**FIG. 1**: For a range of different trap sizes $\sigma$, we show the mass of each of the two particles in nucleon masses, $n$, versus the time in seconds following their release from the traps after which $\sigma_X$ is 10% greater than $\sigma_{\xi/2}$.

Figure 1 shows the point at which $\sigma_X$ is 10% greater than $\sigma_{\xi/2}$ in order to give an idea of the scales of time, trap size, and particle mass necessary to observe this effect. We have defined

$$m = nm_0,$$

and we assume that the combination $\lambda \alpha$ takes the GRW value of $10^{-2} m^{-2} s^{-1}$.

For example, a trap of size 10nm with particles of mass $10^6 m_0$ requires dispersion for 10s in order to see the effect at 10%. An observation would require measuring both particle positions and repeating the experiment many times in order to build up a joint probability distribution function. Note that for a given value of $\sigma$ there is a lower bound in time at which the effect is seen at 10%, irrespective of the mass of the particles. The final observed values of $\sigma_X$ and $\sigma_{\xi/2}$ will be of order or greater than $\sigma$.

If the particles are initially in the ground state of the same trap there is a similar effect [10]. However, observation in this case would rely on a precise account of the particle interactions which here we can ignore.

In summary, we have demonstrated an effect of the CSL model in which the diffusive behaviour of two sufficiently nearby particles is correlated. This means that if the two particles are simultaneously released from known localized states and subsequently measured in position space, there will be a correlation in their measured positions; if one particle is found to have randomly moved in one direction, the other particle is more likely to have moved in the same direction. We propose attempting to observe this effect as a test of CSL against standard quantum theory. The test does not involve having to create a macro superposition state and the fact that it involves a comparison between two similar observations, $\sigma_X$ and $\sigma_{\xi/2}$, should remove sources of systematic error.

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[1] P. Pearle, Phys. Rev. A39, 2277 (1989).
[2] G.C. Ghirardi, P. Pearle, & A. Rimini, Phys. Rev. A42, 78 (1990).
[3] A. Bassi & G.C. Ghirardi, Phys. Rept. 379, 257 (2003).
[4] A. Bassi, K. Lochan, S. Satin, T. P. Singh, & H. Ulbricht, Rev. Mod. Phys. 85, 471 (2013).
[5] S. Nimmrichter, K. Hornberger, P. Haslinger, & M. Arndt, Phys. Rev. A83, 043621 (2011).
[6] W. Marshall, C. Simon, R. Penrose, D. Bouwmeester, Phys. Rev. Lett. 91, 130401 (2003).
[7] T. Kimpenberg & K. Vahala, Science 321, 1172, 2008
[8] G.C. Ghirardi, A. Rimini, & T. Weber, Phys. Rev. D34, 470 (1986).
[9] W. Feldmann & R. Tumulka, J. Phys. A: Math. Theor. 45, 065004 (2012).
[10] D. J. Bedingham, *Effects of the continuous spontaneous localization model in the regime of large localization length scale*.
[11] A. O. Caldeira & A. J. Leggett, Physica A121, 587 (1983).
[12] C. Anastopoulos & J. J. Halliwell, Phys. Rev. D51, 6870 (1995).