Noise-induced Synchronization of Crystal Oscillators

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Experimental study on noise-induced synchronization of crystal oscillators is presented. Two types of circuits were constructed: one consists of two Pierce oscillators that were isolated from each other and received a common noise input, while the other is based on a single Pierce oscillator that received a same sequence of noise signal repeatedly. Due to frequency detuning between the two Pierce oscillators, the first circuit showed no clear sign of noise-induced synchronization. The second circuit, on the other hand, generated coherent waveforms between different trials of the same noise injection. The waveform coherence was, however, broken immediately after the noise injection was terminated. Stronger modulation such as the voltage resetting was finally shown to be effective to induce phase shifts, leading to phase-synchronization of the Pierce oscillator. Our study presents a guideline for synchronizing clocks of multiple CPU systems, distributed sensor networks, and other engineering devices.

Keywords: Noise-induced synchronization, Crystal oscillator

I. INTRODUCTION

Synchronization is a ubiquitous phenomenon of coupled nonlinear oscillators found in a diverse fields of science and engineering. During the last several decades, remarkable progress has been made in both theory and experiment on synchronization of limit cycle and chaotic oscillators [1–3]. Although the most common situation presumes that the oscillators are directly coupled with each other, indirect interaction through commonly injected noise sources has been also known to synchronize uncoupled oscillators. It has been proven theoretically that a wide class of uncoupled limit-cycle oscillators can be in-phase synchronized by common weak white noise [4]. The theoretical framework has been extended to random impulses [5, 6], general, colored and non-Gaussian noise [7], and also in the presence of uncommon noise [8]. Chaotic oscillators can be also synchronized by noise inputs [9–12].

Experimental systems that show the noise-induced synchronization include neuronal systems [13–15], circuit systems [6, 16, 17] and laser systems [18, 19]. Despite intensive studies on noise-induced synchronization in a scientific framework, its application to engineering problems remains open. Natural environmental sounds have been utilized as a noise source to synchronize simulated network of distributed sensors [20]. Numerical study showed that noise can synchronize spin torque oscillators to overcome the problem of low output power in their array [21].

In the present study, we apply the method of noise-induced synchronization to crystal oscillators [22, 23]. The crystal oscillator is an electronic circuit composed of a piezoelectric resonator that determines the oscillation frequency. Because of its highly precise and stable oscillations, it has been widely used to provide clock signals to a variety of digital circuits and to stabilize frequencies of radio transmitters and receivers. The clock rate of a central processing unit (CPU) is also determined by the frequency of the crystal oscillator. Under these circumstances, synchronized operation of multiple clocks should be of significant importance in a future technology. For instance, to further accelerate the clock speed in CPUs, it is getting extremely difficult to integrate all the circuit elements into a single chip, because of the increased number of transistors, which should be located within a limited-size chip. Dividing the CPU into multiple chips is inevitable, thereby synchronizing the clocks of the divided chips should be an essential requirement. As a source of the noise to synchronize the oscillators, utilization of the internal noise, which exists inherently within circuit itself, should be advantageous in terms of an economical recycling of the oscillation energy. A variety of further applications, e.g., synchronizing the clocks of distributed sensors and CPUs, should be found. Towards establishment of a basis for such technologies, we present here an experimental study on noise-induced synchronization of the crystal oscillators. Two types of circuits were constructed: (1) Two Pierce oscillators that were isolated from each other and received a common noise input; (2) A single Pierce oscillator that repeatedly received a same noise input. As the external input signals, white Gaussian noise, Poisson spike trains, and resetting signals were applied to the circuits. Our study may provide a guideline for synchronizing clocks of multiple CPU systems, distributed sensor networks, and other engineering devices.

The present paper is organized as follows. Section II briefly introduces the theoretical framework for noise-induced synchronization of limit cycle oscillators. Section III implements two isolated Pierce oscillator circuits, to which common noise inputs were applied. Section IV applies a same noise input repeatedly to a single Pierce oscillator circuit to examine whether coherence of the output waveforms is increased by the same noise injec-
Thus, growth rate of \( \ln |\phi(t)| \) is determined by the mean Lyapunov exponent \( \Lambda = \left( \frac{d \ln |\phi(t)|}{dt} \right) \), which is statistically averaged over noise. For the Gaussian white noise, the Lyapunov exponent is calculated as \( \Lambda = -\frac{1}{2} \int_0^{2\pi} \{ Z'(\theta) \}^2 d\theta \leq 0 \), implying that \( |\phi(t)| \) shrinks on average and the oscillators get eventually synchronized with each other. This holds for arbitrary limit cycle oscillators regardless of the detailed dynamics as long as \( Z \) is differentiable \( [4] \). The theoretical framework has been extended to random impulses \( [3, 6] \), general, colored and non-Gaussian noise \( [7] \), in the presence of uncorrelated noise \( [8] \), and also to chaotic oscillators \( [12] \).

The crystal circuit system generates limit cycle oscillations, which fall within the above theoretical framework. In the following Sections, we experimentally apply common noise inputs to the crystal oscillator system.

\section{III. TWO UNCOUPLED PIERCE CIRCUITS}

As the piezoelectric crystal oscillator circuits, two Pierce oscillators \( [24] \), each of which was composed of a single digital inverter (Toshiba 74HCU04AP), one resistor, two capacitors, and one quartz crystal (32.768 kHz, SII VT-200-F), were built as shown in Fig. I (a). The inverters (INV\(1 \) and INV\(3 \)) amplified oscillations of the crystals (XTAL\(_1 \) and XTAL\(_2 \)) with feedback resistors \( (R_3 \) and \( R_4 \)), while the capacitors \( (C_1, C_2, C_4, \) and \( C_5 \)) adjusted their oscillation frequencies. The output signals, denoted as \( V_1 \) and \( V_2 \), respectively, were generated through the buffers (INV\(2 \) and INV\(4 \)). To isolate the two Pierce oscillators from each other, Schottky diodes \( (D_1 \) and \( D_2 \)) that rectified their inputs were inserted. The coupling capacitors \( (C_3 \) and \( C_6 \)) passed the alternating input signals, while the resistors \( (R_3 \) and \( R_6 \)) discharged them. Physical parameters of the electric components used in the present experiment are summarized in Table I. To inject a common noise input, a function generator (Keysight 33500B), which generates various noise signals including white Gaussian noise and Poisson spike trains, was utilized. The output voltages \( (V_1, V_2) \) were simultaneously observed by an oscilloscope (Keysight InfiniVision DSOX2014A) and recorded into a data logger (Keyence NR-600 and NR-HA08) with a sampling frequency of \( f_s = 500 \) kHz and with a data point of \( N = 2 \times 10^6 \).

To quantify the level of synchronization between the two crystal oscillators, correlation coefficient between the two outputs, \( V_1 \) and \( V_2 \), was computed. To study dependence of the synchronization on the noise strength, peak-to-peak voltage of the input noise was increased from 0 to 10 V. Fig. I (b) shows the results of applying Gaussian white noise. For each setting of the noise strength, 10 measurements were repeatedly made with different realizations of the noise signals and their average was drawn with the standard deviation indicated by the error-bar. The correlation coefficient remains close to zero even if the noise strength is increased to the maximum level of 10 V. Fig. I (c), on the other hand, shows the results of applying Poisson spike trains with an average spike frequency of 32.768 kHz, which is close to the natural frequency of the crystal oscillator. Again, the correlation coefficient stays close to the zero line up to the maximum spike amplitude of 9 V. The results were the same for different setting of the spike frequency, which was varied as \( ?? \), with the peak-to-peak voltage set to 9 V (Fig. I (d)).

To visualize the inter–relationship between the two Pierce circuits, Lissajous plot of \( V_1 \) vs. \( V_2 \) was drawn for the Poisson spike input (amplitude: 9 V, average frequency: 32.768 kHz) in Fig. I (e). The phase plots, which filled the entire square region, showed no sign of coherence. This indicates that the noise-induced synchronization was not observed for the present two uncoupled crystal oscillator circuits. As shown in \( [12] \), noise-induced synchronization is, in a strict sense, not achievable in the presence of frequency detuning between the
two limit cycle oscillators. To observe synchronization, frequency mismatch, which inevitably exists between the two crystal oscillators, should be largely reduced.

**FIG. 1.** (a) Schematic diagram of two uncoupled Pierce oscillator circuits. Each circuit consists of inverting operational amplifier circuit (INV1 or INV2), feedback resistor (R1 or R4), crystal (XTAL1 or XTAL2), and capacitors (C1, C2 or C4, C5). The function generator (Keysight 33500B) injects external signals (Gaussian white noise or Poisson spike trains) to each oscillator via diode (D1 or D2), coupling capacitor (C1 or C4), and ground resistance (R2 or R4). (b) Dependence of the correlation coefficient between V1 and V2 on the peak-to-peak voltage of Gaussian white noise input. Averaged value over 10 different measurements is plotted with the standard deviation as the error-bar. (c) Dependence of the correlation coefficient on the amplitude of Poisson spike trains. The mean spike frequency was approximately 32.768 kHz, which is close to the natural frequency of crystals. (d) Dependence of the correlation coefficient on frequencies of the Poisson spike inputs (amplitude: 9 V). (e) Lissajous plot of V1 vs. V2 in the case of Poisson spike inputs (frequency: 32.768 kHz, amplitude: 9 V). Poisson spike noise.

**TABLE I. Parameters of Two Pierce Circuits.**

| Circuit elements of Pierce oscillators | | |
| --- | --- | --- |
| XTAL1 and XTAL2 | 32.768 KHz | |
| INV1, INV2, INV3 and INV4 | 74HC04AP | |
| R1 and R4 | 1 MΩ | |
| C2, C3, C5 and C6 | 22 pF | |
| Unidirectional input paths | | |
| D1 and D2 | BAT43 | |
| R3 and R4 | 10 kΩ | |
| C1 and C4 | 20 pF | |

**IV. SINGLE PIERCE CIRCUIT WITH REPEATED NOISE INJECTION**

In the previous Section, we have seen that the noise-induced synchronization was not observed for the two uncoupled crystal circuit systems. The frequency mismatch, which inherently exists between the two circuits, is considered as one of the primary causes that prevented the synchrony. In order to exclude such primary cause, here, we utilize a single Pierce oscillator circuit. By repeatedly injecting a same noise sequence to the circuit, we may measure the coherence of the circuit outputs between different trials. By considering that each trial output is generated from different circuit having exactly the same oscillator properties, noise-induced synchronization can be examined between identical oscillators that have no frequency mismatch, as demonstrated in [13].

To carry out the experiment described above, we implemented a single Pierce oscillator circuit, which has the same parameter setting as in the previous Section (see Fig. 2(a) and Table I). The function generator was composed of an embedded computer (Raspberry Pi 2 Model B) and a D/A converter (Sunhayato MM-5102), which is capable of generating a same sequence of white Gaussian noise or Poisson spike trains as an audio signal with a sampling rate of 192 kHz.

For each trial, output from the circuit and the noise signal were recorded simultaneously. To align the time sequences of different trials, the time shift that maximized the correlation coefficient between the noise signals was sought. By this alignment, all trials give rise to the same timing of noise inputs and thus the circuit outputs can be regarded as those receiving the same noise input simultaneously. Then, to quantify the level of synchrony, correlation coefficient between the circuit outputs of different trials was computed.

Fig. 2(b) shows the results of applying Gaussian white noise. Peak-to-peak voltage of the noise signals was increased from 1.5 to 3 V. For each noise intensity, 10 trials with the same noise signal were repeatedly made. For 45 pairs of all possible combinations, the correlation coefficients between the circuit outputs were computed. Their average was drawn by the solid line, while the standard deviation was indicated by the error-bar. We see that the output coherence increases slightly as the noise intensity
is increased. To compare with the case that no noise was injected, the correlation coefficient, which was computed in a similar manner between the circuit outputs before the noise was injected, was plotted by the dotted line. The significant difference (*: t-test, p < 0.05) was detected for the noise intensity of 3 V, indicating that the observed increase in the level of output coherence was due to the common Gaussian white noise.

Fig. 2(c), on the other hand, shows the results of applying Poisson spike trains (mean frequency: 20 kHz). Again, the correlation coefficient between the circuit outputs increased clearly as the noise intensity was increased. For the noise intensity of 2.5 V and 3 V, significant difference (**: t-test, p < 0.001) from the control signals was detected, indicating that the oscillator outputs were highly correlated with each other due to the common spike inputs.

To see the post effect of the noise-induced coherence, the circuit outputs were further examined after the noise injection was terminated. For Poisson spike trains (mean frequency: 20 kHz, spike voltage: 3 V), the output signals (V1) were drawn simultaneously for 10 different trials. Fig. 3(a) and (b) correspond to the plots during and after the noise injection, respectively. Strongly correlated outputs discernible during the noise injection quickly disappeared after the noise injection. To quantitatively see the change in the correlation, Fig. 3(c) displays distribution of the 45 correlation coefficients computed for all pairs of the 10 trials during (stripe patterns) and after (gray patterns) the noise injection. The high correlation, which existed during the noise injection, disappeared after the noise injection was stopped. According to the Kolmogorov-Smirnov test, their difference was significant (p = 0.0069).

These results suggest that, in our experiment of repeating the same noise injection to the single Pierce circuit, the noise input increased coherence of the oscillator outputs by modulating the circuit system. This noise perturbation was, however, not strong enough to induce phase shift of the crystal oscillators, since the coherence was lost instantly after the noise injection was stopped. To induce phase synchronization of limit cycle oscillators, the phase shift, which lasts permanently after the perturbation, is required [1-3]. In this sense, the present experimental framework is not sufficient to realize noise-induced synchronization of the crystal oscillators.
V. VOLTAGE ResetTING AS THE STRONGEST PERTURBATION

Our experiment in the previous Section indicated that the repeated injection of the same noise to the single Pierce circuit was not strong enough to induce the real phase shift, which should last permanently after the noise input was terminated. As much stronger stimulus to induce such phase shift, we introduced a voltage resetting of the Pierce oscillator circuit through the nMOS transistor \((Q_1)\) as shown in Fig. 4(a). The function generator, used in our previous experiment as Fig. 2(a), transmitted a resetting signal of high voltage to the transistor \(Q_1\). By this signal, the output voltage was reset from INV1 to 0 V, during the time when the transistor \(Q_1\) was turned on.

In our first experiment, the resetting time, during which the output voltage was reset to 0 V, was set to 2 ms. In each trial, a single resetting signal was injected to the Pierce circuit. Then, the resetting signal and the output signal were recorded simultaneously. This measurements were repeated for 10 times. As in our previous experiment, to align the time sequences of different trials, the time shift that maximized the correlation coefficient between the resetting signals was sought. By this alignment, all trials give rise to the same timing of resetting signals and thus the circuit outputs can be regarded as those simultaneously reset. Then, to quantify the level of synchrony, correlation coefficient between the circuit outputs of different trials was computed.

Fig. 4 (c) and (d) show simultaneous plots of the output signals \((V_1)\) with 10 different trials. No clear difference is observed between the plots before (c) and after (d) the input of resetting signal. Fig. 4(e) and (f), on the other hand, show the case that the resetting time was set to 0.8 s. The phase shifts were induced clearly after the resetting in (f), showing a sign of synchronization among the outputs of different trials. These phase shifts were not instantaneous but they lasted for a very long time after the voltage reset.

To quantify the level of synchrony, correlation coefficient between the circuit outputs of different trials was computed for all pairs of different trials. Fig. 4(b) shows dependence of the averaged correlation coefficient on the resetting time duration. The dashed and solid lines represent the cases before and after the reset signals, respectively (**: \(p < 0.001\), t-test). The error-bars represent standard deviation over all pairs of 10 trials (for resetting time of 2 ms, 50 trials were made). (c-f) Simultaneous plot of the output signals \(V_1\) for 10 different trials before (c,e) and after (d,f) the voltage resetting. The resetting time was set to 2 ms (c,d) and 0.4 s (e,f). The waveforms were sorted so that the reset timings become the same.

VI. DISCUSSIONS AND CONCLUSIONS

The present study focused on the noise-induced synchronization of crystal oscillators. Two uncoupled Pierce circuits receiving a common noise input and a single Pierce circuit repeatedly forced by a same noise input were implemented in our hardware. Regardless of noise amplitude and noise types, synchronization was not achieved between the two uncoupled crystal oscillators. Then, to exclude the frequency mismatch of the oscillators as a primary cause of avoiding the synchrony, a same noise input was injected repeatedly to a single Pierce circuit. Although the coherence of the circuit outputs was increased during the noise injection, the output coherence disappeared immediately after the noise injection, implying that the noise was not strong enough to induce the real phase shifts. As the strongest perturbation, the voltage resetting was finally examined. As the resetting time was increased, a clear phase shift was induced, giving rise to synchronized outputs of the circuit oscillator.
Taken together, our study indicated that the crystal oscillator was robust against eternal perturbations such as noise injections in the sense that its phase was not easily shifted. Much stronger perturbation such as the voltage resetting was needed to induce the phase shifts, leading to synchrony.

Recently, noise-induced synchronization has been applied to several engineering problems. For instance, simulation study examined environmental noise as a possible source for synchronizing wireless sensor networks [20, 25]. Noise-induced synchronization has been also utilized in a simulated array of spin torque oscillators to overcome their low output power [21]. Along the line of these works, synchronization of the crystal oscillators would provide a rich technological basis for unifying multiple CPUs, sensor networks, and other distributed clock devices. Towards such applications, our study provides a guideline for realizing noise-induced synchronization of the crystal oscillators, where design of a very strong signal is needed to achieve their synchrony. In our present experiment, a single resetting pulse has been applied to the crystal oscillator. The important future study would be to apply multiple resetting pulses [26, 27] with a short resetting time to induce more efficient phase shifts, leading to noise-induced synchronization of crystal oscillators.

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