Event patterns extracted from top quark-related spectra in proton-proton collisions at 8 TeV

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Abstract: We analyze the transverse momentum ($p_T$) and rapidity ($y$) spectra of top quark pairs, hadronic top quarks, and top quarks produced in proton-proton (pp) collisions at center-of-mass energy $\sqrt{s} = 8$ TeV. For $p_T$ spectra, we use the superposition of the inverse power-law suggested by the QCD (quantum chromodynamics) calculus and the Erlang distribution resulting from a multisource thermal model. For $y$ spectra, we use the two-component Gaussian function resulting from the revised Landau hydrodynamic model. The modelling results are in agreement with the experimental data measured at the detector level, in the fiducial phase-space, and in the full phase-space by the ATLAS Collaboration at the Large Hadron Collider (LHC). Based on the parameter values extracted from $p_T$ and $y$ spectra, the event patterns in three-dimensional velocity ($\beta_x-\beta_y-\beta_z$), momentum ($p_x-p_y-p_z$), and rapidity ($y_1-y_2-y$) spaces are obtained, and the probability distributions of these components are also obtained.

Keywords: top quark-related spectra, event pattern, three-dimensional space

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1 Introduction

The top quark is the heaviest particle in the standard model, and is very different from the other quarks. It is expected that the top quark may have some special characteristics and be related to new physics beyond the standard model. Therefore, it is very important to study its characteristics more. The top quark was first found by the Tevatron at the Fermi National Accelerator Laboratory [1–4], and the successful operation of the CERN Large Hadron Collider (LHC) has brought the study of the top quark into a more precise measurement period. High energy collisions are the only way to study the top quark in experiments. Generally, due to the complexity of the process in an extremely short time, theoretical physicists need to use models to analyze the properties of observables, instead of studying the interacting systems directly.

Among the kinematic observables, the transverse momentum ($p_T$) and rapidity ($y$) are always hot topics for theoretical physicists. Some phenomenological models and formulas have been proposed to fit the $p_T$ and $y$ spectra. For $p_T$ spectra, many formulas can be used, such as the standard (Fermi-Dirac, Bose-Einstein, or Boltzmann) distribution [5–8], Tsallis statistics [8–14], the inverse power-law [15–17], the Erlang distribution [18], the Schwinger mechanism [19–22], and combinations of these, while $y$ spectra can be described by the one-, two-, or three-component Gaussian function. Except for these analytic expressions, many models based on the Monte Carlo method have been used to find arithmetic solutions of the spectra of $p_T$ and $y$, and other interesting results which contain, but are not limited to, chemical and kinetic freeze-out temperatures, chemical potential, transverse flow velocity, particle ratio, and so forth.

High energy collisions are complex processes in which the production mechanisms of different types of particles are different. In particular, at the same stage of a collision process, different types of particles may undergo different ways of propagation. Although some spectra of different types of particles can be described or fitted by particular theoretical models or functions, we are interested in the differences in their production. For example, before leaving the interacting region, most light flavor particles undergo the stage of kinetic freeze-out and local thermal equilibrium. Heavy flavor particles do not undergo this stage. It is hard to learn more information from the limited spectra available in experiments. We hope to use a simple method to extract some intuitive pictures from the limited spectra, so that some differences in the production of different types of particles

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can be observed. These differences are useful in understanding the production mechanisms of different types of particles.

In order to understand the sophisticated collision process and mechanism intuitively, we can use the method of event pattern (particle scatter plot) at the last stage of particle production to obtain information about the interacting system. Using this method, we have analyzed the scatter plots of net-baryons produced in central gold-gold (Au-Au) collisions at BNL Relativistic Heavy Ion Collider (RHIC) energies in three-dimensional momentum $(p_x-p_y-p_z)$ space, three-dimensional momentum-rapidity $(p_x-p_y-y)$ space, and three-dimensional velocity $(\beta_x-\beta_y-\beta_z)$ space [23]; charged particles produced in proton-proton (pp) and lead-lead (Pb-Pb) collisions at 2.76 TeV (one of the LHC energies) in three-dimensional $p_z-p_y-p_z$ space and $\beta_x-\beta_y-\beta_z$ space [24]; as well as $Z$ bosons and quarkonium states produced in pp and Pb-Pb collisions at LHC energies in two-dimensional transverse momentum-rapidity $(p_T-y)$ space and three-dimensional $\beta_x-\beta_y-\beta_z$ space [25]. Due to the variety of produced particles, further analyses of the event patterns of other particles such as the top quarks are needed.

In this paper, we mainly use the superposition of the inverse power-law suggested by the QCD (quantum chromodynamics) calculus [15–17] and the Erlang distribution resulting from a multisource thermal model [18], and the two-component Gaussian function resulting from the revised Landau hydrodynamic model [26–29] to fit the $p_T$ and $y$ spectra of the top quark-related products produced in pp collisions at the center-of-mass energy $\sqrt{s} = 8$ TeV measured at the detector level, in the fiducial phase-space, and in the full phase-space by the ATLAS Collaboration at the LHC [30]. The related parameters can be extracted from the fitting. Based on the parameters and using the Monte Carlo method, we can obtain the event patterns in three-dimensional $\beta_x-\beta_y-\beta_z$, $p_x-p_y-p_z$, and rapidity $(y_1-y_2-y)$ spaces. The probability distributions of these components can also be obtained.

The remainder of this paper is structured as follows. A brief description of the model and method is given in Section 2. Then, the results and discussion are presented in Section 3. Finally, we summarize our main observations and conclusions in Section 4.

2 Model and method

As the heaviest particle in the standard model, the formation of the top quark is expected to be through the hard scattering process among partons (quarks and gluons) with high energy. The top quark-related $p_T$ spectra have a very wide range of distributions. This means that in some cases the spectra can in fact be divided into two parts. One part is in the relatively high $p_T$ region and mainly contributed by the real hard (the harder) scattering process, and the other part is in the relatively low $p_T$ process and mainly contributed by the not too hard (the hard) scattering process.

For the harder and hard processes, we have to choose a superposition distribution which has two components to describe the $p_T$ spectra. Of the two components, one is for the harder scattering process and the other for the hard scattering process. The relative contributions of the harder scattering process are expected to be different for different products such as top quark pairs ($t\bar{t}$ systems), hadronic top (hadronic $t$) quarks (hadronically decaying top quarks), and top ($t$) quarks (semileptonically and hadronically decaying top quarks).

For the harder scattering process, we can use the inverse power-law which results from the QCD calculus [15–17] in high energy collisions to describe the $p_T$ spectra. For the hard scattering process, we can use the Erlang distribution which results from a multisource thermal model [18]. Although the Erlang distribution is not sure to be the best choice for the hard scattering process, it is a good one to fit many data. In fact, the Erlang distribution is also used for the soft excitation process. The inverse power-law plays a significant role in the region of relatively high $p_T$, and the Erlang distribution contributes mainly in the region of relatively low $p_T$.

According to the QCD calculus [15–17], we have the inverse power-law in the form

$$f_1(p_T) = A_T \left(1 + \frac{p_T}{p_0}\right)^{-n}, \quad (1)$$

where $A$ denotes the normalization constant which makes $\int_0^\infty f_1(p_T)dp_T = 1$, and $p_0$ and $n$ are free parameters and influence the value of $A$.

According to the multisource thermal model [18], the spectra of $p_T$ for a given set of data selected in a special condition can be described by the Erlang distribution

$$f_2(p_T) = \frac{p_T^{m-1}}{(m-1)!\langle p_T^i \rangle^m} \exp \left(-\frac{p_T}{\langle p_T^i \rangle}\right), \quad (2)$$

where $\langle p_T^i \rangle$ and $m$ are free parameters. In particular, $\langle p_T^i \rangle$ denotes the average value of $p_T^i$, where $i = 1$ to $m$, and $m$ denotes the number of contribution sources, which are in fact the participant partons, which contribute the same exponential function to $p_T$. Generally, $m = 2$ or $3$ due to only two or three partons taking part in the formation of each particle.
The top quark-related $p_T$ spectrum is a superposition of the inverse power-law and the Erlang distribution. Let $k$ denote the relative contribution of the inverse power-law. Then, the relative contribution of the Erlang distribution is $1 - k$. We have the normalized distribution

$$f_0(p_T) = k f_1(p_T) + (1 - k) f_2(p_T). \quad (3)$$

In many cases, the spectra of $p_T$ in experiments are presented in terms of non-normalized distributions. To give a comparison with the experimental data, the normalized constant ($N_{p_T}$) is needed.

According to the Landau hydrodynamic model and its revisions [26–29], the $y$ spectrum is a Gaussian function [28, 29]

$$f_y(y) = \frac{1}{\sqrt{2\pi}\sigma_y} \exp \left[-\frac{(y - y_C)^2}{2\sigma_y^2}\right], \quad (4)$$

where $y_C$ denotes the mid-rapidity (peak position) and $\sigma_y$ denotes the distribution width. In the center-of-mass reference frame, $y_C = 0$ corresponds to symmetric collisions such as the pp collisions considered in the present work.

In many cases, the Gaussian function cannot describe the $y$ spectra very well, and we need at least two Gaussian functions for the $y$ spectrum. That is,

$$f_y(y) = \frac{k_B}{\sqrt{2\pi}\sigma_{y_B}} \exp \left[-\frac{(y - y_B)^2}{2\sigma_{y_B}^2}\right] + \frac{1 - k_B}{\sqrt{2\pi}\sigma_{y_F}} \exp \left[-\frac{(y - y_F)^2}{2\sigma_{y_F}^2}\right], \quad (5)$$

where $k_B$ ($1 - k_B$), $y_B$ ($y_F$), and $\sigma_{y_B}$ ($\sigma_{y_F}$) denote respectively the relative contribution ratio, peak position, and distribution width of the first (second) component, distributed in the backward (forward) rapidity region. Due to the symmetry of pp collisions, we have $k_B = 1 - k_B = 0.5$, $y_B = -y_F$, and $\sigma_{y_B} = \sigma_{y_F}$. When comparing with experimental data, the normalization constant ($N_y$) is needed.

In the present work, we use the Monte Carlo method to get discrete values which are used in the event patterns in three-dimensional $\beta_x$-$\beta_y$-$\beta_z$, $p_x$-$p_y$-$p_z$, and $y_1$-$y_2$-$y$ spaces. Let $R$, $r_i$, and $R_{1–5}$ denote random numbers distributed evenly in [0, 1]. To get the discrete values, the variable $p_T$ in Eq. (1) or in the first component in Eq. (3) obeys the following formula

$$\int_0^{p_T} f_1(p_T) dp_T < R < \int_0^{p_T + dp_T} f_1(p_T) dp_T. \quad (6)$$

The variable $p_T$ in Eq. (2) or in the second component in Eq. (3) is obtained by

$$p_T = -\langle p_T \rangle \sum_{i=1}^m \ln r_i = -\langle p_T \rangle \ln \prod_{i=1}^m r_i. \quad (7)$$

As for the variable $y$ in the first and second components in Eq. 5, we have

$$y = \sigma_y\sqrt{-2 \ln R_1 \cos(2\pi R_2)} + y_B \quad (8)$$

and

$$y = \sigma_y\sqrt{-2 \ln R_3 \cos(2\pi R_4)} + y_F \quad (9)$$

respectively.

An isotropic emission in the transverse plane results in the azimuthal angle $\varphi$ being

$$\varphi = 2\pi R_5 \quad (10)$$

which is distributed evenly in $[0, 2\pi]$. The momentum components are

$$p_x = p_T \cos \varphi, \quad (11)$$

$$p_y = p_T \sin \varphi, \quad (12)$$

and

$$p_z = \sqrt{p_T^2 + m_0^2 \sinh y}, \quad (13)$$

where $m_0$ is the peak mass in the invariant mass spectrum of the $t\bar{t}$ systems [30], or the rest mass of the top quark in the case of hadronic top quarks or top quarks.

The energy $E$ is

$$E = \sqrt{p_T^2 + m_0^2 \cosh y}. \quad (14)$$

The velocity components are

$$\beta_x = \frac{p_x}{E}, \quad (15)$$

$$\beta_y = \frac{p_y}{E}, \quad (16)$$

and

$$\beta_z = \frac{p_z}{E}. \quad (17)$$

For $y_1$ and $y_2$, we use the definitions

$$y_1 = \frac{1}{2} \ln \left(\frac{E + p_x}{E - p_x}\right) \quad (18)$$

and

$$y_2 = \frac{1}{2} \ln \left(\frac{E + p_y}{E - p_y}\right) \quad (19)$$

for the rapidities in the directions of the $ox$ and $oy$ axes respectively. We get $y$ using Eq. (8) or (9) directly.

In the concrete calculation, we need a few steps to generate the event patterns, i.e. the three-dimensional distributions of particle scatters.

i) We use Eq. (3) to fit the $p_T$ spectra and Eq. (5) to fit the $y$ spectra so that the values of free parameters and normalization constants can be obtained:
ii) According to the values of the free parameters obtained by the fitting in the first step, we may use Eq. (6) or (7) on the basis of \( k \) to get a discrete value of \( p_T \), and Eq. (8) or (9) with an equal probability to get a discrete value of \( y \).

iii) We use Eq. (10) to get a discrete value of \( \varphi \), and Eqs. (11)–(13) to get a set of discrete values of \((p_x, p_y, p_z)\).

iv) We use Eq. (14) to get a discrete value of \( E \), and Eqs. (15)–(17) to get a set of discrete values of \((\beta_x, \beta_y, \beta_z)\).

v) We use Eq. (18) to get a discrete value of \( y_1 \), Eq. (19) to get a discrete value of \( y_2 \), and the discrete value of \( y \) is obtained directly from Eq. (8) or (9). Then, a set of discrete values of \((y_1, y_2, y)\)

vi) Repeating steps ii) to v) 1000 times, we can obtain 1000 sets of \((p_x, p_y, p_z)\), 1000 sets of \((\beta_x, \beta_y, \beta_z)\), and 1000 sets of \((y_1, y_2, y)\).

vii) Finally, the three-dimensional distributions of particle scatters are plotted in three three-dimensional spaces.

The probability distributions of the various components are obtained by statistics.

3 Results and discussion

Before describing the comparisons with experimental data, we first introduce the meanings of “the detector level”, “the fiducial phase-space”, and “the full phase-space”. According to Ref. [30], “the event selection consists of a set of requirements based on the general event quality and on the reconstructed objects”, defined by definite conditions, “that characterize the final-state event topology”. Each requirement or condition for quantities of the considered event has to be detected, identified, and selected by various types of detectors, which is referred to as the detector level. The fiducial phase space for the measurements presented in Ref. [30] is defined “using a series of requirements applied to particle-level objects close to those used in the selection of the detector-level objects”. The full phase space for the measurements presented in Ref. [30] is defined “by the set of \( t\bar{t} \) pairs in which one top quark decays semileptonically (including \( \tau \) leptons) and the other decays hadronically”.

Although we intend to “describe” separately the spectra of top quark pairs, hadronic top quarks, and top quarks at the “detector level”, in the “fiducial” phase-space, and in the “full” phase-space, these concepts are not independent. In fact, the differences between them arise due to the decay of the top quark and also due to the finite coverage of the detector. Predicting the relationship between them is one of the great successes of perturbative QCD: the experiments only measure particle data at the detector-level, which is then extrapolated with theoretical tools to the level of top quarks and, if desired, can also be corrected to the fiducial or full phase-space.

On the short-cut process of the rest mass for the \( t\bar{t} \) systems in the cases of measuring the invariant mass spectra by the three requirements which are (i) at the detector level, (ii) in the fiducial phase-space, and (iii) in the full phase-space, we take \( m_0 = 463.2, 382.5, \) and 372.5 GeV, respectively, as in Ref. [30]. For the rest mass of (hadronic) top quark, we take \( m_0 = 172.5 \) GeV, also from Ref. [30]. It is very important to use the correct rest mass in the extraction of event patterns. In the case of giving the spectra of \( p_T \) and \( y \), the most important issue is the rest mass.

Figure 1 shows the event yields for (a)(c) \( p_T \) and (b)(d) \( y \) of (a)(b) the \( t\bar{t} \) systems and (c)(d) the hadronic top quarks produced in \( pp \) collisions at \( \sqrt{s} = 8 \) TeV, where \( y \) spectra are presented in terms of absolute values. The symbols represent the experimental data of the ATLAS Collaboration [30] measured in the combined electron and muon selections at the detector level, and the error bars are the combined statistical and systematic uncertainties, where the integral luminosity corresponds to 20.3 fb\(^{-1}\). The solid curves are our results calculated by using (a) the inverse power-law, (c) the superposition of the inverse power-law and the Erlang distribution, and (b)(d) the two-component Gaussian distribution, respectively. The dashed curve in (a) is the result calculated by using the Erlang distribution with \( n = 1 \), which is in fact the exponential distribution for the purpose of comparison. In the calculation, we use the method of least squares to determine the values of parameters. The values of free parameters \( p_0, n, k, (p_{T1}), y_F (= -y_B) \), and \( \sigma_{y_F} (= \sigma_y_B) \), normalization constants \( (N_{p_T} \) and \( N_y) \), and \( \chi^2 \) per degree of freedom \( (\chi^2/\text{dof}) \) are listed in Tables 1–3 for different sets of products and parameters, where \( k = 1 \) for the \( t\bar{t} \) systems and \( m = 3 \) for the hadronic top quarks.

One can see from Fig. 1 and Tables 1–3 that the results calculated by the hybrid model are in agreement with the experimental \( p_T \) and \( y \) spectra of both the \( t\bar{t} \) systems and hadronic top quarks produced in \( pp \) collisions at \( \sqrt{s} = 8 \) TeV measured at the detector level by the ATLAS Collaboration at the LHC. In particular, the \( t\bar{t} \) systems show only the contribution of the harder scattering process (with \( k = 1 \)), which means that \( (p_{T1}) \) and \( m \) for the \( t\bar{t} \) systems are not available. The hadronic top quarks show mainly the contribution of the hard scattering process (with a small \( k \)). This implies that more collision energy is needed to create the \( t\bar{t} \) system. As
part of the $t\bar{t}$ system, the hadronic top quark takes up part of the energy of the $t\bar{t}$ system, which results in a not too hard scattering process.

Figure 2 shows the fiducial phase-space normalized differential cross-sections for (a)(b) the $t\bar{t}$ systems and (c)(d) the hadronic top quarks, where $\sigma$ denotes the cross-section. Figure 3 shows the full phase-space normalized differential cross-sections for (a)(b) the $t\bar{t}$ systems and (c)(d) the top quarks. The corresponding parameters are also listed in Tables 1–3 for different sets of products and parameters, where $k = 1$ for the $t\bar{t}$ systems and $m = 3$ for the hadronic top quarks and top quarks, which are listed only in the captions of Tables 1 and 2, but not as a separate column in the tables.

One can also see from Figs. 2 and 3 and Tables 1–3 that the modelling results are in agreement with the experimental data of the fiducial phase-space normalization, which results in the agreement with the experimental data of the fiducial phase-space normalization, the hadronic top quarks and the (hadronic) top quarks in the full phase-space, we also have a small $k$.

As for the tendencies of the free parameters, one can see from Tables 1–3 that, for both the $t\bar{t}$ systems and the (hadronic) top quarks, $p_0$ and $n$ decrease when the experimental requirement changes from the detector level to the fiducial phase-space and then to the full phase-space. Only for the (hadronic) top quarks, $k$ slightly increases, and there is almost no change in $\langle p_{T1} \rangle$ and $m$ when the requirement changes from the detector level to the full phase-space. At the same time, both for the $t\bar{t}$ systems and the (hadronic) top quarks, both $y_F$ and $\sigma_{y_F}$ increase when the requirement changes from the detector level to the full phase-space. These tendencies may have no obvious meaning due to there being little relation among these requirements. However, because of these tendencies, we can obtain abundant structures of event patterns.

Based on the parameter values obtained from Figs. 1–3 and listed in Tables 1–3, Monte Carlo calculation can be performed and the values of a series of kinematical quantities can be obtained. Thus, we can get different kinds of diagrammatic sketches at the last stage of particle production in the interacting system formed in $pp$ collisions. Figures 4–6 give the event patterns which are displayed by the particle scatter plots in the three-dimensional $\beta_x-\beta_y-\beta_z$, $p_x-p_y-p_z$, and $y_1-y_2-y$ spaces, respectively. In these figures, panels (a)–(f) correspond to the results for the $t\bar{t}$ systems at the detector level, the hadronic top quarks at the detector level, the $t\bar{t}$ systems in the fiducial phase-space, the hadronic top quarks in the fiducial phase-space, the $t\bar{t}$ systems in the full phase-space, and the top quarks in the full phase-space, respectively. The total number of particles for each panel is 1000. The blue and red globules represent the contributions of the inverse power-law and Erlang distribution respectively. The values of root-mean-squares $\sqrt{\beta_z^2}$ for $\beta_x$, $\sqrt{\beta_y^2}$ for $\beta_y$, and $\sqrt{\beta_z^2}$ for $\beta_z$, as well as the maximum $|\beta_z|$, $|\beta_y|$, and $|\beta_x|$ (i.e., $|\beta_z|_{\text{max}}$, $|\beta_y|_{\text{max}}$, and $|\beta_x|_{\text{max}}$) are listed in Table 4. The values of root-mean-squares $\sqrt{p_x^2}$ for $p_x$, $\sqrt{p_y^2}$ for $p_y$, and $\sqrt{p_z^2}$ for $p_z$, as well as the maximum $|p_x|$, $|p_y|$, and $|p_z|$ (i.e., $|p_x|_{\text{max}}$, $|p_y|_{\text{max}}$, and $|p_z|_{\text{max}}$) are listed in Table 5. The values of root-mean-squares $\sqrt{y_1^2}$ for $y_1$, $\sqrt{y_2^2}$ for $y_2$, and $\sqrt{y^2}$ for $y$, as well as the maximum $|y_1|$, $|y_2|$, and $|y|$ (i.e., $|y_1|_{\text{max}}$, $|y_2|_{\text{max}}$, and $|y|_{\text{max}}$) are listed in Table 6.

From Figs. 4–6 and Tables 4–6, one can see that the event patterns in the three-dimensional $\beta_x-\beta_y-\beta_z$ space for the $t\bar{t}$ systems in the three requirements are rough cylinders with $\sqrt{p_x^2} \approx \sqrt{\beta_y^2} \ll \sqrt{\beta_z^2}$ and $|\beta_z|_{\text{max}} \approx |\beta_y|_{\text{max}} < |\beta_x|_{\text{max}}$, though few differences among the three requirements are observed. The event patterns for the (hadronic) top quarks in the three requirements are rough ellipsoids, with similar relations among these quantities and few differences between the three requirements. An obvious difference between the event patterns for the $t\bar{t}$ systems and the (hadronic) top quarks is observed due to their different production processes. Meanwhile, both the root-mean-squares and the maxima for the $t\bar{t}$ systems are less than those for the (hadronic) top quarks, and the differences in relative sizes between transverse and longitudinal quantities for the $t\bar{t}$ systems are larger than those for the (hadronic) top quarks.

The event patterns in the three-dimensional $p_x-p_y-p_z$ space for the $t\bar{t}$ systems in the three requirements are relatively thin and very rough ellipsoids with $\sqrt{p_z^2} \approx \sqrt{p_y^2} < \sqrt{p_x^2}$ and $|p_y|_{\text{max}} \approx |p_y|_{\text{max}} < |p_z|_{\text{max}}$, though few differences among the three requirements are observed. The event patterns for the (hadronic) top quarks in the three requirements are relatively fat and very rough ellipsoids with similar relations among these quantities and few differences between the three requirements. An obvious difference between the event patterns for the $t\bar{t}$ systems and the (hadronic) top quarks is observed. Meanwhile, the transverse quantities for the $t\bar{t}$ systems are less than those for the (hadronic) top quarks, while the opposite is true for the longitudinal quantities. The differences in relative sizes between
Fig. 1. (a)(c) Transverse momentum and (b)(d) rapidity spectra of (a)(b) the $t\bar{t}$ systems and (c)(d) the hadronic top quarks produced in $pp$ collisions at $\sqrt{s} = 8$ TeV, where the rapidity spectra are presented in terms of absolute values. The symbols represent the experimental data of the ATLAS Collaboration [30] measured in the combined electron and muon selections at the detector level. The solid curves are our results calculated by using the (a) inverse power-law, (c) superposition of inverse power-law and Erlang distribution, and (b)(d) two-component Gaussian distribution, respectively. The dashed curve in (a) is the result calculated by using the Erlang distribution with $m = 1$, which is in fact the exponential distribution.

Table 1. Values of free parameters ($p_0$ and $n$), normalization constant ($N_{p_T}$), and $\chi^2$/dof corresponding to the curves in Figures 1(a), 2(a), and 3(a), where $k = 1$, which means that $\langle p_T \rangle$ and $m$ are not available.

| Figure     | Type | $p_0$ (GeV/c) | $n$      | $N_{p_T}$         | $\chi^2$/dof |
|------------|------|--------------|----------|-------------------|--------------|
| Figure 1(a)| $t\bar{t}$ | 162.86 ± 4.20 | 7.70 ± 0.40 | (1.93 ± 0.05) x 10^5 | 2.88         |
| Figure 2(a)| $t\bar{t}$ | 52.73 ± 2.60  | 4.34 ± 0.20 | 1.00              | 4.26         |
| Figure 3(a)| $t\bar{t}$ | 45.83 ± 2.50  | 4.30 ± 0.20 | 1.00              | 1.43         |
Fig. 2. (a)(c) Transverse momentum and (b)(d) rapidity normalized differential cross-sections of (a)(b) the $t\bar{t}$ systems and (c)(d) the hadronic top quarks produced in $pp$ collisions at $\sqrt{s} = 8$ TeV. The symbols represent the experimental data of the ATLAS Collaboration [30] measured in the combined electron and muon selections at the fiducial phase-space level. The solid curves are our results calculated by using the (a) inverse power-law, (c) superposition of inverse power-law and Erlang distribution, and (b)(d) two-component Gaussian distribution, respectively. The dashed curve in (a) is the result calculated by using the Erlang distribution with $m = 1$, which is in fact the exponential distribution.

Table 2. Values of free parameters ($p_0$, $n$, $k$, $\langle p_{T,i} \rangle$, and $m$), normalization constant ($N_{p_T}$), and $\chi^2$/dof corresponding to the curves in Figures 1(c), 2(c), and 3(c), where the values of $m$ in the Erlang distribution are invariably taken to be 3 and are not listed in a separate column.

| Figure | Type             | $p_0$ (GeV/c) | $n$     | $k$     | $\langle p_{T,i} \rangle$ (GeV/c) | $N_{p_T}$ | $\chi^2$/dof |
|--------|------------------|---------------|---------|---------|-----------------------------------|-----------|--------------|
| Figure 1(c) hadronic | $t$  | 225.00 ± 22.50 | 6.50 ± 1.00 | 0.10 ± 0.02 | 39.70 ± 0.50 | (1.99 ± 0.05) × 10^7 | 11.34 |
| Figure 2(c) hadronic | $t$  | 204.00 ± 20.40 | 6.05 ± 1.00 | 0.13 ± 0.02 | 40.50 ± 0.05 | 1.00 | 14.33 |
| Figure 3(c) $t$  | 165.00 ± 16.50 | 5.60 ± 1.00 | 0.14 ± 0.02 | 39.20 ± 0.05 | 1.00 | 3.76 |
Fig. 3. (a)(c) Transverse momentum and (b)(d) rapidity normalized differential cross-sections of (a)(b) the $t\bar{t}$ systems and (c)(d) the top quarks produced in $pp$ collisions at $\sqrt{s} = 8$ TeV. The symbols represent the experimental data of the ATLAS Collaboration [30] measured in the combined electron and muon selections at the full phase-space level. The solid curves are our results calculated by using the (a) inverse power-law, (c) superposition of inverse power-law and Erlang distribution, and (b)(d) two-component Gaussian distribution, respectively. The dashed curve in (a) is the result calculated using the Erlang distribution with $m = 1$, which is in fact the exponential distribution.

Table 3. Values of free parameter [$y_F (= -y_B)$ and $\sigma_{y_F} (= \sigma_{y_B})$], normalization constant ($N_\eta$), and $\chi^2$/dof corresponding to the curves in Figures 1(b), 1(d), 2(b), 2(d), 3(b), and 3(d).

| Figure | Type | $y_F (= -y_B)$ | $\sigma_{y_F} (= \sigma_{y_B})$ | $N_\eta$ | $\chi^2$/dof |
|--------|------|----------------|-------------------------------|---------|--------------|
| Figure 1(b) | $tt$ | $0.36 \pm 0.02$ | $0.53 \pm 0.02$ | $(196.00 \pm 2.00) \times 10^3$ | 2.27 |
| Figure 1(d) | hadronic $t$ | $0.53 \pm 0.02$ | $0.68 \pm 0.02$ | $(199.00 \pm 2.00) \times 10^3$ | 2.21 |
| Figure 2(b) | $tt$ | $0.39 \pm 0.02$ | $0.55 \pm 0.02$ | $1.00$ | 7.88 |
| Figure 2(d) | hadronic $t$ | $0.57 \pm 0.02$ | $0.69 \pm 0.02$ | $1.01 \pm 0.01$ | 13.46 |
| Figure 3(b) | $tt$ | $0.52 \pm 0.02$ | $0.79 \pm 0.03$ | $1.04 \pm 0.01$ | 4.64 |
| Figure 3(d) | $t$ | $0.58 \pm 0.02$ | $0.98 \pm 0.03$ | $1.04 \pm 0.01$ | 6.55 |
transverse and longitudinal quantities for the $t\bar{t}$ systems are larger than those for the (hadronic) top quarks. The maximum quantities do not show an obvious tendency for the $t\bar{t}$ systems and the (hadronic) top quarks.

The event patterns in the three-dimensional $y_1$-$y_2$-$y$ space for the $t\bar{t}$ systems in the three requirements are very rough ellipsoids with $\sqrt{y_1^\text{max}} \approx \sqrt{y_2^\text{max}} \ll \sqrt{y^\text{max}}$ and $|y_1|_{\text{max}} \approx |y_2|_{\text{max}} \ll |y|_{\text{max}}$; though few differences among the three requirements are observed. The event patterns for the (hadronic) top quarks in the three requirements are very rough rhomboids with similar relations among these quantities and few differences between the three requirements. An obvious difference between the event patterns for the $t\bar{t}$ systems and the (hadronic) top quarks is observed. Meanwhile, both the root-mean-squares and the maxima for the $t\bar{t}$ systems are obviously less than those for the (hadronic) top quarks. The differences in relative sizes between transverse and longitudinal quantities for the $t\bar{t}$ systems are larger than those for the (hadronic) top quarks.

According to these scatter plots (Figs. 4–6), we can obtain the probability distributions of the considered quantities. Using higher statistics, Figs. 7–9 present the probability distributions of $\beta_i$ ($i = x$, $y$, and $z$), $p_i$ ($i = x$, $y$, and $z$), and $y_i$ ($i = 1$ and 2), respectively, where $N$ denotes the number of particles. Different panels correspond to different requirements and different curves correspond to different quantities shown in the panels. One can see that the distributions of $x$ and $y$ components are almost the same, if not equal to each other at the pixel level, due to the assumption of isotropic emission in the transverse plane. All distributions of $x$, $y$, and $z$ components are symmetric at zero.

On the velocity components, $\beta_x$ and $\beta_y$ are distributed only in a small region near zero. The $t\bar{t}$ systems have a narrower region and a higher peak than the (hadronic) top quarks. For the requirements from the detector level to the fiducial phase-space then to the full phase-space, the peak value increases obviously. $\beta_z$ is distributed almost uniformly in a wide region for different particles and requirements. In particular, the distributions of $|\beta_z|$ of the (hadronic) top quarks increase slightly with the increase of $|\beta_z|$, while the $t\bar{t}$ systems show an opposite or static tendency. Moreover, the tendency in the full phase-space is more obvious than in the fiducial phase-space and at the detector level. The main differences appear near $|\beta_z|_{\text{max}}$ and are caused by different $m_0$.

On the momentum components, all the distributions of $p_x$, $p_y$, and $p_z$ for different particles and requirements have a peak at zero, though the distribution of $p_z$ has a wider range and a lower peak than those of $p_x$ and $p_y$. The $t\bar{t}$ system has an increasingly high peak in the $p_x$ and $p_y$ distributions at the detector level, in the fiducial phase-space, and in the full phase-space, while the other three types of distributions have similar shapes and do not show an obvious tendency to increasing peak height.

For the distributions of $y_1$ and $y_2$, one can see an increasingly high peak for the $t\bar{t}$ system going from the detector level to the fiducial phase-space then to the full phase-space. For the hadronic top quarks, the distributions of $y_1$ and $y_2$ at the detector level and in the fiducial phase-space are almost the same, and they are different in shape and slope around the peak region from the distributions for the top quarks.

It should be noted that, in the above discussions, we have produced fits to the ATLAS 8 TeV data on $p_T$ and $y$ of the top quarks and the $t\bar{t}$ pairs. The fits for these one-dimensional kinematic distributions are then used in such a way that fully differential distributions for the same final states are “predicted”. It seems that, in general, it is impossible to achieve this because one cannot reconstruct a generic multidimensional distribution from its one-dimensional (marginal) projections due to the correlations between variables not being known. In fact, our procedure of reconstruction works well due to the correlations between variables being considered through Eqs. (11) and (14).

For the correlations between $x$- and $y$-components, we have used an isotropic assumption in the transverse plane. This results in Eqs. (10)–(12) for the azimuthal angle, $p_x$, and $p_y$, respectively. We would like to point out that the effect of elliptic flow for the top quarks and the $t\bar{t}$ pairs are neglected due to this effect appearing.

| Figure | Type | $\sqrt{y_1^\text{max}}$ | $\sqrt{y_2^\text{max}}$ | $\sqrt{y^\text{max}}$ | $|\beta_1|_{\text{max}}$ | $|\beta_2|_{\text{max}}$ | $|\beta_3|_{\text{max}}$ |
|--------|------|-----------------|-----------------|-----------------|----------------|----------------|----------------|
| Figure 4(a) | $t\bar{t}$ | $0.11 \pm 0.01$ | $0.11 \pm 0.01$ | $0.50 \pm 0.01$ | $0.58$ | $0.56$ | $0.96$ |
| Figure 4(b) | hadronic $t$ | $0.31 \pm 0.01$ | $0.32 \pm 0.01$ | $0.59 \pm 0.01$ | $0.79$ | $0.82$ | $0.99$ |
| Figure 4(c) | $t\bar{t}$ | $0.13 \pm 0.01$ | $0.14 \pm 0.01$ | $0.52 \pm 0.01$ | $0.76$ | $0.67$ | $0.97$ |
| Figure 4(d) | hadronic $t$ | $0.31 \pm 0.01$ | $0.32 \pm 0.01$ | $0.61 \pm 0.01$ | $0.81$ | $0.87$ | $0.99$ |
| Figure 4(e) | $t\bar{t}$ | $0.10 \pm 0.01$ | $0.10 \pm 0.01$ | $0.62 \pm 0.01$ | $0.52$ | $0.46$ | $0.99$ |
| Figure 4(f) | $t$ | $0.28 \pm 0.01$ | $0.29 \pm 0.01$ | $0.67 \pm 0.01$ | $0.77$ | $0.84$ | $1.00$ |
Fig. 4. Event patterns (particle scatter plots) in three-dimensional $\beta_x$-$\beta_y$-$\beta_z$ space in $pp$ collisions at $\sqrt{s} = 8$ TeV (a)(b) at the detector level, (c)(d) in the fiducial phase-space, and (e)(f) in the full phase-space, for (a)(c)(e) the $t\bar{t}$ systems, (b)(d) the hadronic top quarks, and (f) the top quarks. The number of particles for each panel is 1000. The blue and red globules represent the results corresponding to the inverse power-law function and the Erlang distribution for $p_T$, respectively.
Fig. 5. Event patterns (particle scatter plots) in three-dimensional $p_x$-$p_T$-$p_z$ space in $pp$ collisions at $\sqrt{s} = 8$ TeV (a)(b) at the detector level, (c)(d) in the fiducial phase-space, and (e)(f) in the full phase-space, for (a)(c)(e) the $t\bar{t}$ systems, (b)(d) the hadronic top quarks, and (f) the top quarks. The number of particles for each panel is 1000. The blue and red globules represent the results corresponding to the inverse power-law function and the Erlang distribution for $p_T$, respectively.
Fig. 6. Event patterns (particle scatter plots) in three-dimensional $y_1$-$y_2$-$y$ space in $pp$ collisions at $\sqrt{s} = 8$ TeV (a)(b) at the detector level, (c)(d) in the fiducial phase-space, and (e)(f) in the full phase-space, for (a)(c)(e) the $t\bar{t}$ systems, (b)(d) the hadronic top quarks, and (f) the top quarks. The number of particles for each panel is 1000. The blue and red globules represent the results corresponding to the inverse power-law function and the Erlang distribution for $p_T$, respectively.
Fig. 7. Distributions of velocity components $\beta_x$, $\beta_y$, and $\beta_z$ in $pp$ collisions at $\sqrt{s} = 8$ TeV (a)(b) at the detector level, (c)(d) in the fiducial phase-space, and (e)(f) in the full phase-space, for (a)(c)(e) the $t\bar{t}$ systems, (b)(d) the hadronic top quarks, and (f) the top quarks. The dotted, dashed, and solid curves correspond to the distributions of $\beta_x$, $\beta_y$, and $\beta_z$, respectively, where the distributions of $\beta_x$ and $\beta_y$ are nearly the same.

Table 5. Values of the root-mean-squares $\sqrt{p_x^2}$ for $p_x$, $\sqrt{p_y^2}$ for $p_y$, and $\sqrt{p_z^2}$ for $p_z$, as well as the maximum $|p_x|$, $|p_y|$, and $|p_z|$ ($|p_x|_{\text{max}}$, $|p_y|_{\text{max}}$, and $|p_z|_{\text{max}}$) corresponding to the scatter plots for different types of products, where the corresponding scatter plots are presented in Fig. 5. Both the root-mean-squares and maximum momentum components are in units of GeV/$c$.

| Figure | Type | $\sqrt{p_x^2}$ | $\sqrt{p_y^2}$ | $\sqrt{p_z^2}$ | $|p_x|_{\text{max}}$ | $|p_y|_{\text{max}}$ | $|p_z|_{\text{max}}$ |
|--------|------|---------------|---------------|---------------|----------------|----------------|----------------|
| Figure 5(a) | $t\bar{t}$ | 63.2 ± 2.8 | 64.9 ± 2.8 | 534.4 ± 11.0 | 427.4 | 469.0 | 1545.0 |
| Figure 5(b) | hadronic $t$ | 97.0 ± 2.7 | 98.8 ± 2.5 | 379.1 ± 10.8 | 433.6 | 384.8 | 1347.2 |
| Figure 5(c) | $t\bar{t}$ | 75.4 ± 4.7 | 74.9 ± 4.4 | 338.2 ± 10.3 | 589.3 | 632.2 | 1399.1 |
| Figure 5(d) | hadronic $t$ | 100.9 ± 3.1 | 102.8 ± 3.0 | 304.9 ± 13.4 | 498.6 | 560.5 | 1999.9 |
| Figure 5(e) | $t\bar{t}$ | 52.4 ± 2.1 | 53.2 ± 2.0 | 574.6 ± 23.7 | 273.5 | 255.6 | 3189.8 |
| Figure 5(f) | $t$ | 95.0 ± 2.6 | 96.8 ± 2.5 | 513.9 ± 28.8 | 393.8 | 423.9 | 3079.7 |
Fig. 8. Distributions of momentum components $p_x$, $p_y$, and $p_z$ in $pp$ collisions at $\sqrt{s} = 8$ TeV (a)(b) at the detector level, (c)(d) in the fiducial phase-space, and (e)(f) in the full phase-space, for (a)(c)(e) the $t\bar{t}$ systems, (b)(d) the hadronic top quarks, and (f) the top quarks. The dotted, dashed, and solid curves correspond to the distributions of $p_x$, $p_y$, and $p_z$, respectively, where the distributions of $p_x$ and $p_y$ are nearly the same.

Table 6. Values of the root-mean-squares $\sqrt{y_1^2}$ for $y_1$, $\sqrt{y_2^2}$ for $y_2$, and $\sqrt{y^2}$ for $y$, as well as the maximum $|y_1|$, $|y_2|$, and $|y|$ ($|y_1|_{\text{max}}$, $|y_2|_{\text{max}}$, and $|y|_{\text{max}}$) corresponding to the scatter plots for different types of products, where the corresponding scatter plots are presented in Fig. 6.

| Figure | Type     | $\sqrt{y_1^2}$ | $\sqrt{y_2^2}$ | $\sqrt{y^2}$ | $|y_1|_{\text{max}}$ | $|y_2|_{\text{max}}$ | $|y|_{\text{max}}$ |
|--------|----------|----------------|----------------|--------------|-------------------|-------------------|-------------------|
| Figure 6(a) | $t\bar{t}$ | $0.11 \pm 0.01$ | $0.11 \pm 0.01$ | $0.65 \pm 0.01$ | $0.67$ | $0.63$ | $1.92$ |
| Figure 6(b) | hadronic $t$ | $0.34 \pm 0.01$ | $0.36 \pm 0.01$ | $0.87 \pm 0.02$ | $1.08$ | $1.16$ | $2.53$ |
| Figure 6(c) | $t\bar{t}$ | $0.14 \pm 0.01$ | $0.15 \pm 0.01$ | $0.68 \pm 0.01$ | $1.01$ | $0.81$ | $2.01$ |
| Figure 6(d) | hadronic $t$ | $0.34 \pm 0.01$ | $0.36 \pm 0.01$ | $0.90 \pm 0.02$ | $1.12$ | $1.34$ | $2.60$ |
| Figure 6(e) | $t\bar{t}$ | $0.10 \pm 0.01$ | $0.11 \pm 0.01$ | $0.95 \pm 0.02$ | $0.58$ | $0.49$ | $2.84$ |
| Figure 6(f) | $t$ | $0.30 \pm 0.01$ | $0.33 \pm 0.01$ | $1.15 \pm 0.03$ | $1.02$ | $1.22$ | $3.46$ |
Fig. 9. Distributions of \( y_1 \) and \( y_2 \) in pp collisions at \( \sqrt{s} = 8 \) TeV (a)(b) at the detector level, (c)(d) in the fiducial phase-space, and (e)(f) in the full phase-space, for (a)(c)(e) the \( t\bar{t} \) systems, (b)(d) the hadronic top quarks, and (f) the top quarks. The dotted and dashed curves correspond to the distributions of \( y_1 \) and \( y_2 \) respectively, where the two distributions are nearly the same.
mainly in the soft process. In the case of considering the elliptic flow for the soft process, we are expected to study the fine-structure of event patterns [31], which is beyond the focus of the present work, though this effect is small and can be neglected. In any case, conservation of energy and momentum is satisfied in the calculation.

For comparisons with our recent works [23–25], as an example, we can see the similarities and differences in the three-dimensional $\beta_x-\beta_y-\beta_z$ space. The scatter plots of $t\bar{t}$ systems and (hadronic) top quarks are similar to those of $Z$ bosons and quarkonium states [25] due to them being heavy particles. In fact, the scatter plots of heavy particles show that the root-mean-square velocities form a rough cylinder or ellipsoid surface and the maximum velocities form a fat cylinder or ellipsoid surface, due to their production being at the initial stage of collisions. The scatter plots of charged particles show that the root-mean-square velocities form an ellipsoid surface and the maximum velocities form a spherical surface [23, 24], due to their production being mostly at the intermediate stage of collisions and suffering particularly the processes of thermalization and expansion of the interacting system.

As for comparisons with other modelling or theoretical works, although thousands of papers on top quark-related subjects have been published since (at least) the 1980s, and the number of top quark-related publications from the LHC is also in the hundreds (for example, see Refs. [32–36]), few of them are directly related to the event patterns or particle scatter plots. In fact, we cannot give a direct comparison with other works due to the available results not being obtained. In addition, although one might just use some event generators such as Pythia, JETSET, and HERWIG instead [37–41], they are not just fitted to transverse momentum and rapidity spectra and require information about the underlying event, pileup, and so on. The present work provides a simple and alternative method to structure event patterns displayed by the scatter plots of different particles. Using this alternative method, one can obtain some direct and iconic pictures for production of different particles.

Although we have used the hybrid model to fit the experimental $p_T$ and $y$ spectra to extract the parameter values and to restructure the event patterns, the event patterns we have obtained are model-independent. In particular, similar or related experimental spectra are also described or predicted by other perturbative QCD calculations such as the Next-to-Leading Order (NLO) in QCD [42–44], Next-to-Next-to-Leading Order (NNLO) in QCD [45–50], Next-to-Next-to-Leading Logarithms (NNLL) [51–53], etc. The event patterns are independent of these theories. In any case, the event patterns are only dependent on the discrete values of experimental probability density distributions of $p_T$ and $y$. What we fitted in the above by using the hybrid model is only parameterizations for the $p_T$ and $y$ spectra. These parameterizations smooth only the experimental probability density distributions and help us to restructure the event patterns.

Before giving conclusions, we would like to emphasize briefly the significance of the present work. In our opinion, the present work supports the methodology which restructures the event patterns or particle scatter plots from both $p_T$ and $y$ spectra. This method can be used in the studies of other particles [23–25, 31] as discussed above, which allows us to give comparisons of the production of different types of particles. Indeed, from the three-dimensional distribution, we have obviously observed some differences for different type of particles. These differences are useful in the understanding of particle production and event reconstruction.

4 Conclusions

We summarize here our main observations and conclusions.

(a) We have used the hybrid model to fit the top quark-related spectra of $p_T$ and $y$, which include the spectra of $t\bar{t}$ systems, hadronic top quarks, and top quarks produced in $pp$ collisions at $\sqrt{s} = 8$ TeV measured by the ATLAS Collaboration at the LHC. The hybrid model uses the superposition of the inverse power-law and the Erlang distribution for the description of $p_T$ spectra and the two-component Gaussian function for the description of $y$ spectra. The inverse power-law, the Erlang distribution, and the two-component Gaussian function are derived from the QCD calculus, the multisource thermal model, and the Landau hydrodynamic model, respectively. We have used the inverse power-law and the Erlang distribution to fit the harder and hard scattering processes respectively.

(b) The modelling results are in agreement with the experimental data of the $t\bar{t}$ systems and the hadronic top quarks measured at the detector level, the fiducial phase-space normalized differential cross-sections for the $t\bar{t}$ systems and the hadronic top quarks, and the full phase-space normalized differential cross-sections for the $t\bar{t}$ systems and the top quarks. The $t\bar{t}$ systems show only the contribution of the harder scattering process (with $k = 1$). The (hadronic) top quarks show mainly the contribution of the hard scattering process (with a small $k$). This implies that more collision energy is needed to create the $t\bar{t}$ system. As a part of the $t\bar{t}$ system, the (hadronic) top quark takes up part of the energy of the
the $t\bar{t}$ system, which results in a not too hard scattering process.

(c) When the experimental requirement changes from the detector level to the fiducial phase-space and then to the full phase-space, for both the $t\bar{t}$ systems and the (hadronic) top quarks, $p_\perp$ and $n$ decrease, and $y_F$ and $\sigma_{y_F}$ increase. Only for the (hadronic) top quarks, $k$ slightly increases, while there is almost no change in $\langle p_{T1}\rangle$ and $m$. Although these tendencies of the parameters may have no obvious meaning, due to there being little relation among these experimental requirements, these parameters can be used in the extraction of discrete values of some kinematic quantities. In fact, based on these parameters, we have obtained some discrete values of the velocity, momentum, and rapidity components. Based on these discrete values, the event patterns in some three-dimensional spaces are obtained.

(d) The event patterns in the three-dimensional $\beta_x^{-1}-\beta_y^{-1}-\beta_z^{-1}$ space for the $t\bar{t}$ systems in the three requirements are rough cylinders with $\sqrt{\beta_x^2} \approx \sqrt{\beta_y^2} \ll \sqrt{\beta_z^2}$ and $|\beta_x|_{\text{max}} \approx |\beta_y|_{\text{max}} < |\beta_z|_{\text{max}}$. The event patterns for the (hadronic) top quarks in the three requirements are rough ellipsoids with similar relations among these quantities. Both the root-mean-squares and the maxima for the $t\bar{t}$ systems are less than those for the (hadronic) top quarks, and the differences in relative sizes between transverse and longitudinal quantities for the $t\bar{t}$ systems are larger than those for the (hadronic) top quarks.

(e) The event patterns in the three-dimensional $p_x-p_y-p_z$ space for the $t\bar{t}$ systems in the three requirements are relatively thin and very rough ellipsoids with $\sqrt{p_x^2} \approx \sqrt{p_y^2} \ll \sqrt{p_z^2}$ and $|p_x|_{\text{max}} \approx |p_y|_{\text{max}} < |p_z|_{\text{max}}$. The event patterns for the (hadronic) top quarks in the three requirements are relatively fat and very rough ellipsoids with the similar relations among these quantities. The transverse quantities for the $t\bar{t}$ systems are less than those for the (hadronic) top quarks, and the situations of longitudinal quantities are opposite. The differences in relative sizes between transverse and longitudinal quantities for the $t\bar{t}$ systems are larger than those for the (hadronic) top quarks. The maximum quantities do not show an obvious tendency for the $t\bar{t}$ systems and the (hadronic) top quarks.

(f) The event patterns in the three-dimensional $y_1-y_2-y$ space for the $t\bar{t}$ systems in the three requirements are very rough ellipsoids with $\sqrt{y_1^2} \approx \sqrt{y_2^2} \ll \sqrt{y^2}$ and $|y_1|_{\text{max}} \approx |y_2|_{\text{max}} < |y|_{\text{max}}$. The event patterns for the (hadronic) top quarks in the three requirements are very rough rhomboids with similar relations among these quantities. Both the root-mean-squares and the maxima for the $t\bar{t}$ systems are obviously less than those for the (hadronic) top quarks. The differences in relative sizes between transverse and longitudinal quantities for the $t\bar{t}$ systems are larger than those for the (hadronic) top quarks.

(g) According to these scatter plots, we have obtained the probability distributions of the considered quantities such as $\beta_z$, $p_t$, and $y_F$. The distributions of $x$ and $y$ components are almost the same, if not equal to each other at the pixel level, due to the assumption of isotropic emission in the transverse plane. $\beta_x$ and $\beta_y$ are distributed only in a small region near zero. The $t\bar{t}$ systems have a narrower region and a higher peak than the (hadronic) top quarks. $\beta_z$ is distributed almost uniformly in a wide region for different particles and requirements. In particular, the distributions of $|\beta_z|$ of the (hadronic) top quarks increase slightly with the increase of $|\beta_z|$, while the $t\bar{t}$ systems show an opposite or static tendency.

(h) All the distributions of $p_x$, $p_y$, and $p_z$ for different particles and requirements have a peak at zero, though the distribution of $p_z$ has a wider range and a lower peak than those of $p_x$ and $p_y$. The $t\bar{t}$ systems have a higher peak in the $p_x$ and $p_y$ distributions, while the other three types of distributions have similar shapes and do not obviously show such a high peak. In the distributions of $y_1$ and $y_2$, a higher peak for the $t\bar{t}$ systems than for the (hadronic) top quarks is observed. For the hadronic top quarks, the distributions of $y_1$ and $y_2$ at the detector level and in the fiducial phase-space are almost the same, and they are different in shape and slope around the peak region from the distributions for the top quarks.

Conflict of Interests
The authors declare that there is no conflict of interests regarding the publication of this paper.

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