Entangling successive single-photons from quantum dots

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Abstract

A method is proposed for generating and discriminating Bell states of high fidelity from consecutive single-photons generated in a semiconductor quantum dot. The use of a non-symmetric beam splitter is found to be essential and sufficient, and no nonlinear optical process such as SPDC is required at any stage. This should be of considerable importance for testing the foundations of quantum mechanics.

Keywords: entangled states, single-photons, quantum dot, quantum mechanics, LOQC

1 Introduction

Entanglement generation has become extremely important as a resource for testing the foundations of quantum theory and for quantum information processing, specially for quantum communication and quantum cryptography [1, 2, 3]. In most applications strongly correlated and entangled photon pairs are generated by spontaneous parametric downconversion (SPDC) of a laser pump beam in a nonlinear crystal [4, 5, 6, 7, 8, 9, 10, 11]. Other methods that have been used are two-photon emission from atoms [12, 13] and entangled photon pair generation in fibres [14, 15]. Of special interest in this context

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is the rapid generation of single-photons from semiconductor quantum dots in a microcavity with the feature that consecutive photons have the same wavepacket and are largely indistinguishable [16]. Here we propose a method of generating Bell states of high fidelity from a pair of such consecutively generated single-photons by making them incident on a non-symmetric and lossless beam splitter from opposite sides at the same time, and then using post-selection. For convenience and to establish the notation, the required input-output relations, originally derived by Zeilinger [17] for general lossless beam splitters, are rederived for this case. This method does not involve any nonlinear optical process like SPDC, and should be of considerable interest for linear optical computing and testing the foundations of quantum mechanics.

That entangled states can be generated from a direct product state was first shown by Yurke and Stoler [18]. Their method requires, however, the use of already entangled pairs and entanglement swapping through a Bell basis measurement. In the method proposed here, the initial states are unentangled single-photons, and non-maximal entanglement is first generated through a non-symmetric beam splitter without requiring any nonlinear process. Bell states of high fidelity are then discriminated by post-selection. The utility of non-symmetric beam splitters for linear optical quantum computation has been noted earlier also [19], though not for generating entanglement and Bell states.

2 The Method

Let a pair of consecutive collinear single-photon pulses from a quantum dot, separated by a small time interval $\Delta \tau$, arrive through a single-mode fibre and a polarization selector $PS$ at a non-polarizing 50-50 beam-splitter $NBS$, as shown in Figure 1. Let the pulses overlap and interfere on a second non-symmetric, lossless and nonpolarizing beam-splitter $BS$ from opposite sides (labelled by $a$ and $b$) after (i) reflection from two lossless mirrors $M_a$ and $M_b$ and (ii) after passing through devices $(D_a, D_b)$ which can rotate the polarizations and prepare arbitrary polarization states in each path. The overlap can be ensured by adjusting the optical path difference $\Delta \tau + \Delta t$ between the two arms, as in a Michelson interferometer. The following notation will be used. Since the Hilbert space of a single-photon is the tensor product $H_{path} \otimes H_{pol}$ of its path and polarization Hilbert spaces, we will write $|a\rangle_{in}^P \equiv |a\rangle_{in} \otimes |P\rangle$, $|b\rangle_{in}^P \equiv |b\rangle_{in} \otimes |P\rangle$ where $P$ stands for $H$ or $V$. 

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Figure 1: Schematic diagram of the experiment: Two consecutive coherent photons separated by a small time interval $\Delta \tau$ pass through a polarization selector $PS$, a non-polarizing beam splitter $NBS$ and a system of reflectors and two devices $D_a$ and $D_b$ such that two arbitrary polarization states are incident on a lossless, nonpolarizing and non-symmetric beam splitter $BS$ from opposite sides $a$ and $b$ at the same time. The reflected and transmitted states are sent through 50-50 polarizing beam splitters $PBS_a$ and $PBS_b$ and finally detected by the single-photon detectors $A_h, A_v, B_h$ and $B_v$. 
the polarization bases, and \( |a\rangle_{in} \) and \( |b\rangle_{in} \) are the incoming spatial modes corresponding to the two paths \( a = \langle x|a \rangle \) and \( b = \langle x|b \rangle \). Then the incoming state at the ports \( a \) and \( b \) of \( BS \) can be written as a tensor product state

\[
|\Psi\rangle_{in} = (\alpha |a\rangle_{in}^V + \beta |a\rangle_{in}^H) \otimes (\alpha' |b\rangle_{in}^V + \beta' |b\rangle_{in}^H) = \alpha \alpha' |a\rangle_{in}^V \otimes |b\rangle_{in}^V + \beta \beta' |a\rangle_{in}^H \otimes |b\rangle_{in}^H + \alpha' \beta |a\rangle_{in}^V \otimes |b\rangle_{in}^H + \beta' \beta |a\rangle_{in}^H \otimes |b\rangle_{in}^V,
\]

where \((\alpha, \beta, \alpha', \beta')\) are tunable complex amplitudes with \(|\alpha|^2 + |\beta|^2 = 1, |\alpha'|^2 + |\beta'|^2 = 1\). In quantum theory the incoming states are converted to the outgoing states by a unitary operator. Following Zeilinger [17] (but changing his notation to suit our purpose), we let the incident states be piloted to the outgoing states by the general matrix beam splitter relation

\[
(a \ b)_{out}^P = \begin{pmatrix} r_p^a & t_p^b \\ t_p^a & r_p^b \end{pmatrix} (a \ b)_{in}^P \equiv B_p (a \ b)_{in}^P
\]

where \( r_p^a, r_p^b, t_p^a, t_p^b \) are the reflection and transmission coefficients of the polarization state \( P \) on the two sides of the beam splitter. The underlying dynamics is defined by \( B_p = \exp\{i\epsilon H_p\} \) with \( H_p^\dagger = H_p \) as the Hamiltonian. Inverting this, we get

\[
B_p^{-1} = \Delta_p^{-1} \begin{pmatrix} r_p^b & -t_p^b \\ -t_p^a & r_p^a \end{pmatrix} = \begin{pmatrix} r_p^{a*} & t_p^{a*} \\ t_p^b & r_p^{b*} \end{pmatrix}
\]

with \( \Delta_p = r_p^a r_p^b - t_p^a t_p^b = e^{i\chi} \) because the determinant of a unitary matrix is of modulus unity. Since \( \chi \) is an overall phase, we can set \( \chi = 0 \) so that

\[
\Delta_p = 1.
\]

Unitarity of \( B_p \) implies

\[
r_p^b = r_p^{a*}, \quad t_p^a = -t_p^{b*}.
\]

One therefore has, as first shown by Zeilinger,

\[
|r_p^a| = |r_p^b|, \quad |t_p^a| = |t_p^b|, \quad \delta_{t_p^a} - \delta_{r_p^a} + \delta_{t_p^b} - \delta_{r_p^b} = \pi,
\]

where \( \delta \)'s are phases. Apart from this relation, any phase relation between the transmitted and reflected states is allowed. Accordingly, we will set all
the reflection amplitudes to be real and $t_{ap}^{a,b} = -i|t_{ap}^{a,b}|$. Henceforth, therefore, to simplify the notation, we will omit the superscripts $a$ and $b$ and also write $r_p, t_p$ etc instead of $|r_p|, |t_p|$. Thus,

$$B_p^{-1} = \Delta_p^{-1} \begin{pmatrix} r_p & it_p \\ it_p & r_p \end{pmatrix}$$

with $\Delta_p = r_p^2 + t_p^2 = 1$. For a 50-50 beam splitter, $r_p = t_p = \frac{1}{\sqrt{2}}$ and

$$\delta_{r_p} - \delta_{t_p} = \pi/2, \quad (6)$$

which is widely used in the literature. *Violation of this set of relations by a non-symmetric beam splitter will turn out to be of crucial importance for our purpose.*

We have, therefore, the following relations for a non-symmetric beam splitter:

$$|a\rangle_p^p = r_p|a\rangle_p^{out} + it_p|b\rangle_p^{out}, \quad (7)$$

$$|b\rangle_p^p = it_p|a\rangle_p^{out} + r_p|b\rangle_p^{out}. \quad (8)$$

Finally, omitting the subscript $out$ from all kets and the symbol $\otimes$ between the kets, we get

$$|\Psi\rangle_{out} = \left[ \alpha^\prime (r_v^2 - t_v^2)|a\rangle^V|b\rangle^V + \beta^\prime (r_h^2 - t_h^2)|a\rangle^H|b\rangle^H \right]$$

$$+ \left[ \left( \alpha^\prime \beta^\prime (\alpha^\prime \beta^\prime -\alpha^\prime \beta^\prime r_h t_v - \beta^\prime \alpha^\prime r_h t_v)\right)|a\rangle^V|b\rangle^H \right]$$

$$+ \left[ \left( \alpha^\prime \beta^\prime (\alpha^\prime \beta^\prime -\alpha^\prime \beta^\prime r_h t_v - \beta^\prime \alpha^\prime r_h t_v)\right)|a\rangle^H|b\rangle^V \right]$$

$$+ i\alpha^\prime \beta^\prime r_h t_v (|a\rangle^V|a\rangle^V + |b\rangle^V|b\rangle^V)$$

$$+ i\beta^\prime \alpha^\prime r_h t_v (|a\rangle^H|a\rangle^H + |b\rangle^H|b\rangle^H). \quad (9)$$

Only the first two terms in square braces represent entangled states, the rest being bunched photon terms representing both photons on the same side of the beam splitter. The state $(9)$ can be written as

$$|\Psi\rangle_{out} = c_\Phi^+|\Phi^+\rangle + c_\Phi^-|\Phi^-\rangle + c_\Psi^+|\Psi^+\rangle + c_\Psi^-|\Psi^-\rangle + \cdots \quad (10)$$
where, juxtaposing the notations for the path and polarization appropriately,

\[
|\Phi^+\rangle = \frac{1}{\sqrt{2}}[(a^H|b\rangle + |a\rangle|V\rangle)] = \frac{1}{\sqrt{2}}[|H\rangle^a|H\rangle^b + |V\rangle^a|V\rangle^b], \quad (11)
\]

\[
|\Phi^-\rangle = \frac{1}{\sqrt{2}}[(a^H|b\rangle - |a\rangle|V\rangle)] = \frac{1}{\sqrt{2}}[|H\rangle^a|H\rangle^b - |V\rangle^a|V\rangle^b], \quad (12)
\]

\[
|\Psi^+\rangle = \frac{1}{\sqrt{2}}[(a|V\rangle|b\rangle + |a\rangle^H|b\rangle^H)] = \frac{1}{\sqrt{2}}[|V\rangle^a|H\rangle^b + |H\rangle^a|V\rangle^b], \quad (13)
\]

\[
|\Psi^-\rangle = \frac{1}{\sqrt{2}}[(a|V\rangle|b\rangle - |a\rangle^H|b\rangle^H)] = \frac{1}{\sqrt{2}}[|V\rangle^a|H\rangle^b - |H\rangle^a|V\rangle^b] \quad (14)
\]

are the Bell states, the dots represent the bunched photon states, and

\[
c_{\Phi^+} = \frac{1}{\sqrt{2}}[\alpha\alpha'(r_v^2 - t_v^2) + \beta\beta'(r_h^2 - t_h^2)], \quad (15)
\]

\[
c_{\Phi^-} = \frac{1}{\sqrt{2}}[\beta\beta'(r_v^2 - t_v^2) - \alpha\alpha'(r_h^2 - t_h^2)], \quad (16)
\]

\[
c_{\Psi^+} = \frac{(\alpha\beta' + \beta\alpha')}{\sqrt{2}}[r_v r_h - t_v t_h], \quad (17)
\]

\[
c_{\Psi^-} = \frac{(\alpha\beta' - \beta\alpha')}{\sqrt{2}}[r_v r_h + t_v t_h]. \quad (18)
\]

Note that the four Bell states occur with different coefficients, and hence entanglement is not entirely cancelled. Note also that even with a fixed initial polarization state, i.e. \(\alpha = \alpha' = 1, \beta = \beta' = 0\) or \(\beta = \beta' = 1, \alpha = \alpha' = 0\), a state like \(|V\rangle^a|V\rangle^b\) or \(|H\rangle^a|H\rangle^b\) is produced and the photon bunching rule [20, 21] does not hold. This is because the special relationship responsible for photon bunching does not hold for a non-symmetric beam splitter. This is easily checked by using the conditions for a symmetric 50-50 beam splitter, namely \(r_v = t_v\) and \(r_h = t_h\) in Eqn. (9) and verifying that these terms do indeed disappear. On the other hand, even with a symmetric 50-50 beam splitter, the state \(|\Psi^-\rangle\) can still be produced unless \(\alpha\beta' = \beta\alpha'\). The use of a non-symmetric beam splitter together with the use of incoming states of arbitrary polarization that can be manipulated turns out to be critical.

We will now consider two alternative arrangements.

1. Set \(\alpha = \beta = \alpha' = \beta' = 1/\sqrt{2}\) so that \(\alpha\alpha' = \beta\beta' = 1/2\) and \(\alpha\beta' = \beta\alpha' = 1/2\). Also, choose a beam splitter and the angles of incidence such that \(r_v^2 - t_v^2 = r_h^2 - t_h^2 \neq 0\). Then, \(c_{\Psi^-} = c_{\Phi^-} = 0\) but \(c_{\Phi^+} \neq 0, c_{\Psi^+} \neq 0\).
2. Set $\alpha = \beta = \alpha' = -\beta' = 1/\sqrt{2}$ so that $\alpha\alpha' = -\beta\beta' = 1/2$ and $\alpha\beta' = -\beta\alpha' = -1/2$ and also choose $r_v^2 - t_v^2 = r_h^2 - t_h^2 \neq 0$ as before.

Then, $c_{\Phi^+} = c_{\psi^+} = 0$ but $c_{\Phi^-} \neq 0$, $c_{\psi^-} \neq 0$.

The Bell states can now be discriminated in the coincidence basis as follows. Let the outgoing modes from $BS$ pass through two 50-50 polarizing beam splitters $PBS_a$ and $PBS_b$ which separate out the horizontal and vertical polarization states (Fig. 1). Let $A_h, A_v, B_h, B_v$ be single-photon detectors. Then, in case 1, coincidences at $A_hB_v$ and $A_vB_h$ signal $|\Psi^+\rangle$ and coincidences at $A_hB_h$ and $A_vB_v$ signal $|\Phi^+\rangle$. In case 2, coincidences at $A_hB_v$ and $A_vB_h$ signal $|\Psi^-\rangle$ and coincidences at $A_hB_h$ and $A_vB_v$ signal $|\Phi^-\rangle$. The bunched photon states produce double counts and coincidences at $A_vA_h$ and $B_vB_h$ and can be ignored.

Finally, consider the fidelity of the target states $|\Phi^+\rangle$ and $|\Psi^+\rangle$ in case 1 as the parameters $\alpha, \beta, \alpha', \beta'$ are varied around $1/\sqrt{2}$. Using the reparametrizations $\alpha = \alpha' = \frac{1}{\sqrt{2}} = \cos(\pi/4), \beta = \beta' = \frac{1}{\sqrt{2}} = \sin(\pi/4)$, we can rewrite (9) as

$$|\Psi\rangle = c_{\Phi^+} |\Phi^+\rangle + c_{\psi^+} |\Psi^+\rangle + \cdots$$

$$= \left( \frac{r_v^2 - t_v^2}{\sqrt{2}} \right) |\Phi^+\rangle + \left( \frac{r_vr_h - t_vt_h}{\sqrt{2}} \right) |\Psi^+\rangle + \cdots \quad (19)$$

Varying the parameters, we get

$$|\Psi\rangle' = (r_v^2 - t_v^2).$$

$$\left[ \cos(\frac{\pi}{4} + \epsilon) \cos(\frac{\pi}{4} + \epsilon') |V^a\rangle |V^b\rangle + \sin(\frac{\pi}{4} + \epsilon) \sin(\frac{\pi}{4} + \epsilon') |H^a\rangle |H^b\rangle \right]$$

$$+ \left[ \left( \cos(\frac{\pi}{4} + \epsilon) \sin(\frac{\pi}{4} + \epsilon') r_v r_h - \sin(\frac{\pi}{4} + \epsilon) \cos(\frac{\pi}{4} + \epsilon') t_v t_h \right) |V^a\rangle |H^b\rangle \right]$$

$$- \left[ \left( \cos(\frac{\pi}{4} + \epsilon) \sin(\frac{\pi}{4} + \epsilon') t_v t_h - \sin(\frac{\pi}{4} + \epsilon) \cos(\frac{\pi}{4} + \epsilon') r_v r_h \right) |H^a\rangle |V^b\rangle \right]$$

$$+ \cdots \quad (20)$$

Therefore,

$$\frac{\langle \Phi^+|\Psi\rangle'}{\langle \Phi^+|\Phi^+\rangle} = \cos(\epsilon - \epsilon') \quad (21)$$

and

$$\frac{\langle \Psi^+|\Psi\rangle'}{\langle \Psi^+|\Psi^+\rangle} = \cos(\epsilon + \epsilon'). \quad (22)$$
Similarly, for case 2 in which $|\Phi^−\rangle$ and $|\Psi^−\rangle$ are produced, we have

$$\frac{\langle\Phi^−|\Psi\rangle'}{\langle\Phi^−|\Psi\rangle} = \cos(\epsilon + \epsilon')$$

(23)

and

$$\frac{\langle\Psi^−|\Psi\rangle'}{\langle\Psi^−|\Psi\rangle} = \cos(\epsilon - \epsilon').$$

(24)

Hence, the fidelity of these states is $\cos^2(\epsilon \pm \epsilon')$.

This completes the theoretical demonstration.

3 Conclusion

The new result is that arbitrary superpositions of the Bell states can be generated from consecutive and indistinguishable single-photon pulses whose polarization states can be manipulated by making them overlap on a nonpolarizing and non-symmetric beam splitter $BS$ from opposite sides (Fig. 1). This is impossible with symmetric beam splitters. By using a beam splitter $BS$ with the right reflection and transmission coefficients and by also tuning the amplitudes of the $H$ and $V$ components of the incident photons, a superposition of two Bell states with different amplitudes can be produced for each arrangement. These two Bell states of high fidelity can be discriminated in the coincidence basis by post-selection. This method therefore has the potential for use as a versatile resource for quantum information processing without the need for nonlinear processes such as SPDC at any stage, and also for testing the foundations of quantum mechanics.

4 Acknowledgement

I thank the National Academy of Sciences, India for the award of a Senior Scientist Platinum Jubilee Fellowship which allowed this work to be undertaken. I also thank Anirban Mukherjee for help with the fidelity calculation.

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