Analyticity of Gaussian free field percolation observables. (English) Zbl 07605989
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Summary: We prove that cluster observables of level-sets of the Gaussian free field on the hypercubic lattice $\mathbb{Z}^d$, $d \geq 3$, are analytic on the whole off-critical regime $\mathbb{R} \setminus \{ h_* \}$. This result concerns in particular the percolation density function $\theta(h)$ and the (truncated) susceptibility $\chi(h)$. As an important step towards the proof, we show the exponential decay in probability for the capacity of a finite cluster for all $h \neq h_*$, which we believe to be a result of independent interest. We also discuss the case of general transient graphs.

MSC:
82Bxx Equilibrium statistical mechanics
60Kxx Special processes
60Gxx Stochastic processes

Full Text: DOI arXiv

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