Griette, Quentin
Singular measure traveling waves in an epidemiological model with continuous phenotypes.
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The work considers the reaction-diffusion equation of the Fisher-Kolmogorov type by with inclusion of
the non-local reaction terms and a mutation kernel acting on the phenotypic space. The main result
consists of proving the existence of a measure traveling wave that is achieved by applying the principal
eigenvalue approach. The latter gives also the value of the expected spreading speed.

Reviewer: Eugene Postnikov (Kursk)

MSC:
35R09 Integro-partial differential equations
35K57 Reaction-diffusion equations
35C07 Traveling wave solutions
35D30 Weak solutions to PDEs
35Q92 PDEs in connection with biology, chemistry and other natural sciences
92D30 Epidemiology
92D15 Problems related to evolution

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