Constraints on free parameters of the simplest bilepton gauge model from the neutral kaon system mass difference

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We consider the contributions of the exotic quarks and gauge bosons to the mass difference between the short- and the long-lived neutral kaon states in SU(3)\textsubscript{C}×SU(3)\textsubscript{L}×U(1)\textsubscript{N} model. The lower bound \( M_{Z'} \sim 14 \) TeV is obtained for the extra neutral gauge boson \( Z'^0 \). Ranges for values of one of the exotic quark masses and quark mixing parameters are also presented.

The \( \Delta m = m_{K_L} - m_{K_S} \) mass difference between the long- and the short-lived kaon states was successfully used in a two generations standard model to predict the charmed quark mass \([1]\). In the following years several authors have studied the \( K^0 - \bar{K}^0 \) system in order to find constraints on parameters of new gauge theories such as gauge and scalar boson masses and mixing angles (see, for example, Refs. \([2,3]\)). The idea behind this is that, since the \( c \) quark mass was predicted with a good precision, a possible new contribution to \( \Delta m \) must be smaller than the result obtained from the two generations.

In this paper we go back to this subject in order to constrain a neutral gauge boson mass and quark mixing parameters imposed by the 3-3-1 model \([5,6]\). The 3-3-1 model is a gauge theory based on the SU(3)\textsubscript{C}×SU(3)\textsubscript{L}×U(1)\textsubscript{N} semi-simple symmetry group. It has the interesting feature that the anomaly cancelation does not happen within each generation, as in the standard model, but only when the three generations are considered together. Thus, the number of families must be multiple of the color degrees of freedom and, as a consequence, the 3-3-1 model suggests a route towards the solution of the flavor question \([3,4]\).

Let us summarize the most relevant points of the model. In the minimal particle content of Ref. \([5]\) the left-handed quark fields transform under the SU(3)\textsubscript{L} group as the triplets

\begin{align}
Q_{1L} &= \begin{pmatrix} u_1 \\ d_{1\theta} \\ J_1 \end{pmatrix} \sim (3, \frac{2}{3}), \\
Q_{2L} &= \begin{pmatrix} J_{\alpha \phi} \\ u_\alpha \\ d_{\alpha \theta} \end{pmatrix} \sim (3^*, -\frac{1}{3})
\end{align}

\((\alpha = 2, 3)\), where \(2/3\) and \(-1/3\) are the U(1)\textsubscript{N} charges. Each left-handed quark field has its right-handed counterpart transforming as a singlet of the SU(3)\textsubscript{L} group. In order to avoid anomalies one of the quark families must transform in a different way with respect to the two others. In Ref. \([5]\) the singled family is the third one, but this is not relevant here. The exotic quark \( J_1 \) carries \( 5/3 \) units of electric charge while \( J_2 \) and \( J_3 \) carry \(-4/3\) each one of them. In the gauge sector the single charged \((V^\pm)\) and the double charged \((U^{\pm\pm})\) vector bileptons \([\bar{9}]\), together with a new neutral gauge boson \( Z'^0 \) complete the particle spectrum with the charged \( W^\pm \) and the neutral \( Z^0 \) standard gauge bosons. At low energy the 3-3-1 model recovers the standard phenomenology \([\bar{10}]\).

An important property is that the bileptons can have a low energy scale. Low energy data constrain the vector bilepton masses to \( M_X > 230 \) GeV \((X \equiv V^+, U^{++})\) \([11]\). Some authors have used the running of the coupling constant to impose upper bounds on 3-3-1 gauge bosons \([12,13]\). However, this procedure involves an arbitrary normalization of \( N \) \([\bar{14}]\). Usually this is done like in the standard model, although it is not mandatory. Recently these upper bounds were reexamined and it was found that the \( M_X \) mass has no upper bound, differently from previous calculations, while \( M_X < 3.5 \) TeV \([14]\).

The \( \Delta m \) mass difference was already studied in the context of the 3-3-1 model at tree level (Fig. 1a) in order to put lower bound on the \( M_{Z'} \) mass \([15]\) and on mixing parameters \([13]\), in the last case, taking into account an upper bound on \( M_{Z'} \). Here we apply the experimental lower limits and these reexamined upper bounds on the \( M_X \) gauge boson masses in order to obtain the impositions of \( \Delta m \) upon some of the 3-3-1 free parameters. Since the bileptons couple exotic to ordinary quarks, leading to additional contributions to the box diagram for the \( K^0 - \bar{K}^0 \) transition (Fig. 1b), we combine the tree level with the box contribution.

The charged current interactions for the quarks are given in Ref. \([5]\) and we can rewrite them as

\begin{equation}
\mathcal{L} = -\frac{g}{2\sqrt{2}} \left[ \bar{U} \gamma^\mu (1 - \gamma_5) V_{\text{CKM}} DM^+ + \bar{U} \gamma^\mu (1 - \gamma_5) \xi JV^+ \right] + \text{H. c.},
\end{equation}

where

\begin{align}
U &= \begin{pmatrix} u \\ c \\ t \end{pmatrix}, \\
D &= \begin{pmatrix} d \\ s \\ b \end{pmatrix},
\end{align}

(3a)
\[ \mathcal{V}_\mu = \begin{pmatrix} V^+_{\mu} \\ U^{\mu -} \\ U^+_{\mu} \end{pmatrix}, \quad \mathcal{U}_\mu = \begin{pmatrix} U^-_{\mu} \\ V^+_{\mu} \\ V^+_{\mu} \end{pmatrix}, \]

and \( \mathcal{J} = \text{diag}(J_1, J_2, J_3) \). The \( V_{\text{CKM}} \) is the usual Cabibbo-Kobayashi-Maskawa mixing matrix and \( \zeta \) and \( \xi \) are mixing matrices containing the new unknown parameters due to the presence of the exotic quarks. Here, unlike Eqs. (a) and (b), we are working with the mass eigenstates.

\[ f(x) = \frac{1}{1-x}\left(1 + \frac{x^2 \ln x}{1-x}\right). \]

In Eq. (4) \( a_{q_i}^X = (m_{q_i}/M_X)^2 \) is the square ratio of the \( m_{q_i} \) quark and the \( M_X \) gauge boson masses. We have defined \( \mathcal{O}_{sd} = [\bar{s}_x \gamma_\mu (1-\gamma_5) u_d]^2 \). The sum in Eq. (4) runs over all quarks coupled by the \( X \) gauge boson. The \( \Delta m_X \) mass difference due to the \( X \) gauge boson contribution is given by

\[ \Delta m_X = -2 \text{Re} \langle \overline{K^0} | \mathcal{L}_X | K^0 \rangle \]

with \( \mathcal{L}_X \equiv A_X \). Taking into account the relation

\[ \langle \overline{K^0} | \mathcal{O}_{sd} | K^0 \rangle = \frac{4}{3} f_K^2 m_K, \]

where \( m_K = 498 \text{ MeV} \) and \( f_K = 161 \text{ MeV} \) are the kaon meson mass and its decay constant, respectively, the total mass difference is

\[ \Delta m = \frac{8 \eta m_K}{3} \left( \frac{f_K M_X}{s_W M_X} \right)^2 \sum_{i,j} \Gamma_i \Gamma_j \frac{f(a_{q_i}^X) - f(a_{q_j}^X)}{a_{q_i}^X - a_{q_j}^X}, \]

where \( s_W = \sin \theta_W \), \( \theta_W \) is the Weinberg weak mixing angle and we have included the leading order QCD correction factor \( \eta = 0.55 \).

Despite the uncertainties undergone by the evaluation of the effective Lagrangian \( \mathcal{L}_X \), the application of this method to well known processes tells us that the procedure is reliable. In order to get physical results from Eq. (8) we perform some hypotheses on the free parameters. Firstly we notice that the standard model does not give the whole \( \Delta m \) mass difference. Considering only two quark generations we find \( \Delta m_{q_2} = 2 \times 10^{-15} \text{ GeV} \), while the experimental value is \( \Delta m_{q_{exp}} = 3.491 \times 10^{-15} \text{ GeV} \). The top quark contribution is negligible because of the smallness of the corresponding mixing parameters. Thus, if the 3-3-1 model is realized in Nature, we can expect that its pure contribution to \( \Delta m \) is about \( 10^{-15} \text{ GeV} \).

Let us examine individual 3-3-1 contributions under the conservative hypothesis in which each one of them gives the total assumed 3-3-1 counterpart to \( \Delta m \), i.e., \( 10^{-15} \text{ GeV} \). We begin analyzing the contribution of the
$U$ bilepton and the $J_1$ quark. By convenience we assume for the $\xi$ mixing matrix the same parametrization advocated by the Particle Data Group for $V_{C,K,M}$ [13]. In this case, since we choose all the mixing angles in the first quadrant, the parameter $\xi_{1d}^*\xi_{1s}$ is positive. We do not consider here CP violating phases in the mixing matrices. In Fig. 2 we plot this mixing parameter as function of $M_{\xi}$, according Eq. (8), where we are using the bounds $M_{\xi} > 230$ GeV [11] and $M_{\xi} < 3500$ GeV [13]. Hence, we can see that $0.014 \lesssim \xi_{1d}^*\xi_{1s} \lesssim 0.55$. Physically this range for the mixing angles implies $m_{J_1} \lesssim M_{\xi}$. In order to apply the lower limit on the mixing parameter for obtaining a lower bound for $M_{\xi}$, we use the result of Ref. [12] for the $Z'$ contribution to $\Delta m$, but taking into account that we are assuming that the maximal contribution to $\Delta m$ is $\sim 10^{-15}$ GeV (not the whole experimental value). We introduce also the leading order QCD correction factor $\eta = 0.55$. This leads to $M_{\xi} \sim 1.03 \times 10^3 \left[ \text{Re} \left( V_{11}^D V_{12}^D \right) \right]^{1/2}$ TeV, where $V_{11}^D$, the mixing matrix relating the symmetry left-handed quark states carrying $-1/3$ units of electric charge ($D_1^L$) with the physical ones ($D_L$), is defined by $D_L = V_{11}^D D_1^L$. Since only the $J_1$ quark carries $5/3$ units of electric charge it no mix. Therefore, the mixing parameter $V_{11}^D V_{12}^D$ is the same as $\xi_{1s}^*\xi_{1d}$, whose lower bound provides $M_{\xi} \sim 14.4$ TeV.

A more rigorous analysis, considering the contributions of diagrams exchanging the single charged $V$ bilepton, $J_2$ and $J_3$ exotic quarks, is complicated since these two quarks can mix and the sign of the mixing parameters is not defined according to our parametrization of the mixing matrices and the choice of the mixing angles. However it is not expected an appreciable difference in the results.

$\Delta m$ mass difference on free parameters of the 3-3-1 model. However, differently from previous calculations [13, 14], here we are taking into account the contribution of the box diagram exchanging the double charged $U^{--}$ bilepton. Our results differ from the previous ones because we have combined the analysis for the tree level and the box diagram. Therefore, if the 3-3-1 Higgs contribution to $\Delta m$ is not important we can assume the value we have estimated for $M_{\xi}$ as an approximate lower bound (i.e., $M_{\xi} \gtrsim 14$ TeV). We stress that this bound does not depend on the aforementioned arbitrary normalization of $N$. The crucial parameter for this value is the experimental lower bound on the mass of the $U^{--}$ bilepton gauge boson.

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