Coulomb drag shot noise in coupled Luttinger liquids

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Coulomb drag shot noise has been studied theoretically for 1D interacting electron systems, which are realized e.g. in single-wall nanotubes. We show that under adiabatic coupling to external leads, the Coulomb drag shot noise of two coupled or crossed nanotubes contains surprising effects, in particular a complete locking of the shot noise in the tubes. In contrast to Coulomb drag of the average current, the noise locking is based on a symmetry of the underlying Hamiltonian and is not limited to asymptotically small energy scales.

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Shot noise in mesoscopic systems has started to attract much interest \textsuperscript{5} because it provides information not contained in the current-voltage characteristics alone, and can for instance reveal the fractional charge of quasiparticles in exotic electronic phases such as the fractional quantum Hall (FQH) liquid. Shot noise in FQH bars can be described by the chiral Luttinger liquid (LL) theory \textsuperscript{2}, and the predicted fractional charge $e^\ast$ smaller than the electron charge $e$ has been seen experimentally for filling factors $\nu = 1/3$ \textsuperscript{3} and $\nu = 2/5$ \textsuperscript{4}. While in Hall bars right and left moving currents are spatially separated, we address here current noise in non-chiral Luttinger liquids \textsuperscript{5}, where right and left movers interact in the same channel. Experimental realizations of this generic model for interacting electrons in 1D are for instance provided by single-wall nanotubes (SWNTs) \textsuperscript{6,7} or semiconductor quantum wires \textsuperscript{8}. For clarity, we focus on SWNTs, with very similar effects expected for other non-chiral 1D interacting metals, and restrict ourselves to the case of spinless electrons \textsuperscript{8}. According to available experiments \textsuperscript{6,7}, the standard LL interaction parameter in SWNTs is $g \approx 0.2$, indicating the presence of strong Coulomb interactions \textsuperscript{10}.

The two-terminal shot noise of a nanotube with an impurity is due to backscattered quasiparticles constructed out of the zero-modes and plasmons of the bosonized LL theory \textsuperscript{11} carrying a fractional charge $e^\ast = ge$. Naively, one might expect that what happens at the impurity should be observable in the shot noise, namely that the two-terminal shot noise in the weak impurity limit can be written as $e^\ast$ times the backscattered current \textsuperscript{4}. On the other hand, as noted previously for the average current \textsuperscript{2,12} the coupling to Fermi liquid reservoirs needs to be incorporated explicitly in the model. We demonstrate that d.c. shot noise does not probe the fractional charge but coincides with the noise in a noninteracting wire. To experimentally access the expected quasiparticle charge then requires more complicated four-terminal setups \textsuperscript{13}, which seem however difficult to implement in practice and also give only indirect evidence for $e^\ast$.

On the other hand, pronounced interaction effects in shot noise experiments arise from the Coulomb drag shot noise of two adjacent or crossed SWNTs, where we predict the remarkable effect of complete noise locking. In fact, the noise power $P_1$ in one SWNT induces a Coulomb drag shot noise $P_2$ in the other SWNT (and vice versa), where both are perfectly locked together, $P_1 = P_2$. This phenomenon would not show up in Fermi liquid systems \textsuperscript{11} and is only based on an emergent symmetry of the interacting low-energy theory. As a consequence, it can be observed at arbitrary temperature $T$ and applied voltage $U$ (within the range of validity of the LL model). Hence, the predicted complete noise locking is a basic manifestation of interaction effects with a more fundamental origin than the absolute drag effect of the average current \textsuperscript{11,13}. In contrast to the noise, the absolute drag of the current is not based on a symmetry of the Hamiltonian and is therefore restricted to asymptotically small energy scales $k_BT, eU$ and a large contact region between the tubes. Recently, current drag effects have been observed for crossed SWNTs \textsuperscript{21} and parallel semiconductor quantum wires \textsuperscript{22}. Essentially the same systems should also reveal the Coulomb drag shot noise predicted here. In future nanoscale electronic devices with 1D transmission lines the extreme noise sensitivity of these lines should play an important role in their applicability. Thus, a proper understanding of their noise properties will be essential for a successful development of future electronic components.

The problem of Coulomb drag shot noise in two coupled SWNTs maps onto two problems of two-terminal shot noise in the presence of an impurity. Therefore, we first address the physics of that scenario before we return to the consequences on Coulomb drag shot noise after the two-terminal noise result in Eq. (6) below. We consider a SWNT of length $L$ that contains an impurity at $x = 0$ and is adiabatically coupled to external leads. Using integrability techniques the current $I$ through such a systems has recently been calculated exactly \textsuperscript{14}. The result is $I = (e^2/h)(U - V)$, where the four-terminal voltage $V$ has to be determined self-consistently for given $g$, impurity strength $\lambda$, and two-terminal voltage $U$. The two-terminal current noise, commonly measured in the
leads, $|x| > L/2$, is defined by

$$P = \int_{-\infty}^{\infty} dt \, e^{i\omega t} \langle \{ \Delta I(x,t), \Delta I(x,0) \} \rangle ,$$

(1)

where $\Delta I(x,t) = I(x,t) - \langle I \rangle$ is the current fluctuation operator. Since our calculations for $k_B T \gg eU$ reproduce the expected Johnson-Nyquist noise [1], let us focus for the moment on the zero temperature limit. As is elaborated below, in the weak impurity limit the d.c. shot noise is given by $P = 2eI_{BS}$, with the backscattered current $I_{BS} = (e^2/h)U - I = (e^2/h)V$ and, similarly, in the opposite strong backscattering limit, $P = 2eI$. This means that it is impossible to observe the fractional quasiparticle charge $e^* = ge$ in a two-terminal measurement. In a sense, the low-frequency shot noise is completely determined by the charge $e$ of a lead electron scattered by the compound LL-plus-impurity. We remark that for finite frequency $\omega$, the current noise contains more information, including spatially dependent oscillatory behaviors. Unfortunately, however, the condition $\omega L/v_F \gtrsim 1$ with the Fermi velocity $v_F$, typically requires to study frequencies above $\approx 10$ GHz in order to get significant deviations from the low frequency result. As such high frequencies are difficult to reach experimentally, we focus on the d.c. limit.

A computation of the shot noise for this problem faces conceptual difficulties, since, as mentioned above, the proper coupling to the Fermi liquid leads is essential. To obtain the current, it is sufficient to specify the mean densities of right- or left-moving electrons injected from the reservoirs, leading to radiative boundary conditions [13] that keep integrability intact [14]. To access the shot noise, however, one also needs to simultaneously specify fluctuation properties in the leads. A simple approach to accomplish this is to employ the inhomogeneous LL model [13] characterized by an $x$-dependent LL interaction parameter $g(x)$, with $g(|x| < L/2) = g$ in the SWNT and $g = 1$ in the leads. Such modelling, when combined with a careful analysis of the electrostatics under an applied voltage, is able to reproduce the Johnson-Nyquist equilibrium noise which a naive direct LL calculation does violate. Unfortunately, exact results for the shot noise at arbitrary impurity strength seem elusive for such a complicated model, but to lowest order in the impurity strength, the analytical solution is possible and outlined next.

The Hamiltonian is $H = H_0 + H_U + H_I$, where $H_U$ describes the coupling to reservoirs, $H_I$ the impurity, and the inhomogeneous LL Hamiltonian is $(\hbar = 1)$

$$H_0 = \frac{v_F}{2} \int dx \left[ \Pi^2 + \frac{1}{g^2(x)} (\partial_x \theta)^2 \right] ,$$

(2)

where $\Pi(x)$ is the canonical momentum to the standard boson field $\theta(x)$ under bosonization [13]. Along the lines of Ref. [13], we obtain the boson correlator $C(x,t) = \langle \theta(x,t)\theta(0,0) \rangle$ of the clean system in closed form,

$$C(x,t) = \int_0^\infty \frac{d\omega}{4\pi} \sum_{s = \pm} \sum_{0 < k_s, \pm} \frac{g \omega e^{i\omega t}}{\kappa_s W \kappa_{s-s}} \left( \begin{array}{c} 2 e^{i\omega |x|/v_F} W^{-1-s}/g \left( 1 - e^{i\omega g |x|/v_F} \right) , \quad |x| > L/2 \\
\kappa_s e^{i\omega |x|/v_F} + \kappa_{s-s} W e^{-i\omega g |x|/v_F} , \quad |x| < L/2 
\end{array} \right) ,$$

(3)

where $\kappa_{s-s} = 1 \pm g$ and $W = \exp(i\omega L/v_F)$. Impurity backscattering at $x = 0$ then gives $H_I = \lambda \cos \sqrt{4\pi \theta}(0)$ with the impurity strength $\lambda$, while impurity forward scattering only leads to an additional phase factor of the electron field operator which does not affect the current noise and is omitted henceforth. Since $a = 1$ LL is a Fermi gas and not a Fermi liquid, to describe the coupling to the applied voltage $U$, it is essential to take into account electroneutrality in the leads. Electroneutrality implies a shift of the band bottoms strictly following the chemical potentials. Effectively, the electrostatic potentials are then $eU(x < -L/2) = \mu_L$ and $eU(x > L/2) = \mu_R$ for chemical potentials $\mu_L/R$ in the left $(L)$ and right $(R)$ reservoir, where we put $\mu_L = eU/2$ and $\mu_R = -eU/2$. In contrast to the leads, the SWNT does not remain electroneutral in presence of an applied voltage [13]. This is a consequence of the finite range of the internal Coulomb interaction screened by external gates. The band bottom in the SWNT is also shifted with a jump of size $eV$ at the impurity site. As shown in Ref. [13], the electrostatics within a voltage biased SWNT emerges naturally as a steady-state interaction effect with $U(x) = 0$ for $|x| < L/2$. The applied voltage $U$ then gives rise to the contribution

$$H_U = \frac{e}{\sqrt{\pi}} \int dx \, U(x) \partial_x \theta .$$

(4)

This formulation does not make the (incorrect) assumption of a local drop of the applied voltage $U$ at the impurity site. In reality, this potential drop is given by the four-terminal voltage $V$, and it is essential to distinguish $V$ and $U$.

To compute the shot noise to lowest order in $\lambda$, we then proceed along the lines of the Keldysh formalism [22]. Expanding in $\lambda$ and using the boson correlator $C(x,t)$ in Eq. (3) gives after straightforward yet tedious algebra for the shot noise measured in the leads

$$P = 2e^2 \lambda^2 \sin(\pi g) \cos(\pi g) \Gamma(1 - 2g) \left( \frac{eU}{\omega_c} \right)^{2g-1} ,$$

(5)

up to corrections vanishing for $L \to \infty$. The same finite $L$ corrections also appear in the backscattered current $I_{BS}$ of the inhomogenous LL system and therefore do not affect the final current-noise relation, $P = 2eI_{BS}$. This result for weak impurity backscattering shows that it is indeed the electron charge $e$ and not the fractional charge $e^* = ge$ that governs the d.c. shot noise spectrum in a
SWNT adiabatically coupled to external leads. Although the relation $P = 2eI_{\text{FS}}$ has been proposed previously [23], the derivation has been questioned, see, e.g., Ref. [1]. We believe that our careful treatment resolves this issue. In the opposite limit of a strong impurity, the contribution $H_1$ is replaced by a term $H_T$ describing electron tunneling through the “barrier” region. Then the expected relation $P = 2eI$ can be easily derived perturbatively in $H_T$, in analogy to our treatment of the weak-impurity limit.

Now we turn to the more interesting problem of Coulomb drag shot noise, which has immediate application potential for SWNTs. The setup under consideration involves two SWNTs that are separately contacted with applied voltages $U_{1,2}$, but remain in contact over some region of length $L_c$. Again we assume good (adiabatic) contact with the leads, which is possible using present-day technology [12]. The contact between the two SWNTs may be achieved either by arranging the SWNTs parallel to each other as illustrated schematically in Fig. 1, implying $L_c \gg a$ with the lattice spacing $a$, or by crossing them with $L_c \approx a$. We note that crossed SWNTs in good contact to separate lead electrodes have already been realized experimentally [20], and a measurement of noise properties can be anticipated without major difficulties. The question then arises: What is the shot noise transferred via the Coulomb interaction coupling the two SWNTs ("Coulomb drag shot noise")? As we discuss below, the combined presence of intra- and inter-tube interactions gives rise to interesting shot noise phenomena. We focus on the case of short-to-intermediate contact length $L_c$. Therefore our treatment directly applies to crossed SWNTs, but also gives a correct qualitative picture for parallel SWNTs. This can be understood from a renormalization group calculation [13], showing that the low-energy properties of an extended contact are captured by an effective pointlike contact (with renormalized coupling strength) for not exceedingly large $L_c$. In practice, a contact with $L_c \lesssim 20 a$ can still be modelled by such a pointlike contact to high accuracy.

Taking adiabatic contacts to the leads, the two uncoupled clean SWNTs ($\alpha = 1, 2$) are described by the above Hamiltonian, $H_\alpha = H_0 + H_{U_\alpha}$, where we take the same $g$ parameter for both SWNTs. Since the band curvature is extremely small [12], both Fermi velocities are identical, $v_{F1} = v_{F2} = v_F$, and independent of the Fermi momentum $k_F = |\hat{E}_F|/v_F$. The Fermi level can in practice be tuned separately for both SWNTs by changing the mean chemical potential in the attached leads, and therefore in general $k_{F1} \neq k_{F2}$. We assume that the inter-tube coupling is sufficiently weak such that neither electron backscattering within each SWNT alone nor electron tunneling between the SWNTs is significant. In any case, the strong interactions present in SWNTs ($g \approx 0.2$) highly suppress tunneling into or between them, and recent experiments [24] are indeed consistent with very small single-electron tunneling probabilities. However, it is crucial to retain the electrostatic coupling between the SWNTs. Modelling this coupling by a local interaction in the contact region $-L_c/2 < x < L_c/2$ as indicated in Fig. 1, we obtain the contribution [2]

$$H' = \int dx \zeta(x) \left\{ V_{\text{FS}} \prod_\alpha \partial_x \theta_\alpha + V_{\text{BS}} \prod_\alpha \sin[2k_{F\alpha}x + 4\pi \theta_\alpha(x)] \right\},$$

where $\zeta(x) = 1$ in the contact region and zero otherwise, and $V_{\text{FS}}$ ($V_{\text{BS}}$) is the forward (backward) scattering inter-tube interaction strength. The $V_{\text{FS}}$ term has no effect on noise nor current for a local coupling (small $L_c$), and gives only an insignificant renormalization of LL parameters for large $L_c$ that will be ignored here.

Progress can then be made by switching to new boson fields $\theta_\pm = (\theta_1 \pm \theta_2)/\sqrt{2}$, which completely decouple the full Hamiltonian $H = H_1 + H_2 + H' = H_+ + H_-$ in the $\pm$ channels,

$$H_\pm = \frac{v_F}{2} \int dx \left[ \Pi_\pm^2 + \frac{1}{g^2(x)}(\partial_x \theta_\pm)^2 \right] + \frac{e}{\sqrt{\pi}} \int dx \left[ U_\pm(x) \partial_x \theta_\pm \mp \frac{V_{\text{BS}}}{2} \int dx \zeta(x) \cos[2(k_{F1} \pm k_{F2})x + \sqrt{8}\pi \theta_\pm(x)].$$

Now we make use of the small-$L_c$ assumption which allows to formulate the last line of Eq. (7) as an effectively point-like coupling $+\lambda_\pm \cos[\sqrt{8}\pi \theta_\pm(0)]$. The effective impurity strength in the two channels is then

$$\lambda_\pm = \frac{V_{\text{BS}}}{2} \int dx \zeta(x) \cos[2(k_{F1} \pm k_{F2})x].$$

Notably, these couplings could be tuned by either (i) changing the distance between the two SWNTs which modifies $V_{\text{BS}}$, (ii) changing the contact length $L_c$, or (iii) changing the Fermi momenta $k_{F1,2}$ relative to each other. Such a tunable impurity strength gives an exceptional flexibility in transport measurements. Furthermore, the electrostatic potentials $U_\pm(x)$ are of the same form as before, but with effective applied voltages $U_\pm = (U_1 \pm U_2)/\sqrt{2}$. We are then left with two independent effective single-impurity problems in the $\pm$ channels. Due to the doubled argument in front of $\theta_\pm(0)$ in the impurity cosine, these models live at the doubled interaction parameter, $g \rightarrow 2g$.

Applying this transformation shows that the shot noise $P_\alpha$ in each physical SWNT ($\alpha = 1, 2$) is identical. Formally, this follows from the vanishing of cross-correlations between the $\theta_+$ and $\theta_-$ fields. In terms of the shot noise $P_\alpha$ in the $\pm$ channel, which for small $V_{\text{BS}}$ is given by Eq. (8) with $g \rightarrow 2g$, $\lambda \rightarrow \lambda_\pm$, $U \rightarrow U_\pm$. 


\[ P_1 = P_2 = (P_+ + P_-)/2 \, . \] (9)

The predicted complete locking of shot noise, \( P_1 = P_2 \), is solely based on the decoupling \( H = H_+ + H_- \), and therefore survives thermal fluctuations and finite applied voltages. As the decoupling, Eq. (1), is valid for any contact length \( L_c \), the noise locking is also observable in parallel SWNTs with an extended contact region. Of course, the noise locking is affected by inter-tube tuneling processes, but as these are highly suppressed for strong intra-tube interactions (small \( g \)), Eq. (1) should be valid to high accuracy. Note that noise locking happens already for a very short contact, \( L_c \approx a \), while the transconductance and hence the absolute drag effect would vanish in that limit (17).

To demonstrate the consequences of changing \( \lambda_\pm \) relative to each other, we finally point out an interesting feature for \( \lambda_+ \ll \lambda_- \). This situation applies to the case of parallel SWNTs characterized by intermediate-to-long \( L_c \), where \( \lambda_- \) is suppressed by the oscillating integrand in Eq. (1) and can be treated perturbatively, while \( \lambda_+ \) is quite large and can even reach the strong-impurity limit. Exploiting duality relations (1), the shot noise \( P = P_1 = P_2 \) is found from Eq. (1) as \( P = e(I_{BS}^g + L_-) \), where \( I_{BS}^g \sim \lambda_+^2 |U_1 + U_2|^{g-1} \) and \( L_- \sim \lambda_-^{-1/g} |U_1 - U_2|^{1/g-1} \). Apparently, for \( U_1 = U_2 \), shot noise is only determined by the + channel and does not depend on \( \lambda_- \) at all. For \( U_1 \neq U_2 \), however, one can reach the strong-impurity limit in the – channel. Using Eq. (1) we then predict a \( g \)-dependent power-law decrease of \( P \) as a function of the inter-tube coupling, \( P \sim V_{BS}^{-1/g} \). As a consequence, there should be a maximum in the noise as a function of \( V_{BS} \) at some (unknown) intermediate value.

In summary, we have shown that the Coulomb drag shot noise of two coupled SWNTs reveals unambiguous evidences for inter- and intra-tube interaction effects. In particular, we have predicted a complete noise locking in each physical tube due to an emergent symmetry of the Hamiltonian. The system under consideration may be realized experimentally by crossing two SWNTs with a remarkable tunability of the effective inter-tube coupling strength.

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FIG. 1. Two coupled SWNTs with a local contact region of length $L_c$. The SWNTs are adiabatically connected to separate reservoirs.