CCP-Based Plant-Wide Optimization and Application to the Walking-Beam-Type Reheating Furnace

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SUMMARY In this paper, the integration of dynamic plant-wide optimization and distributed generalized predictive control (DGPC) is presented for serially connected processes. On the top layer, chance-constrained programming (CCP) is employed in the plant-wide optimization with economic and model uncertainties, in which the constraints containing stochastic parameters are guaranteed to be satisfied at a high level of probability. The deterministic equivalents are derived for linear and nonlinear individual chance constraints, and an algorithm is developed to search for the solution to the joint probability constrained problem. On the lower layer, the distributed GPC method based on neighborhood optimization with one-step delay communication is developed for on-line control of the whole system. Simulation studies for furnace temperature set-points optimization problem of the walking-beam-type reheating furnace are illustrated to verify the effectiveness and practicality of the proposed scheme.

key words: plant-wide optimization, chance-constrained programming (CCP), distributed generalized predictive control (DGPC), serially connected processes, walking-beam-type reheating furnace

1. Introduction

The walking-beam-type reheating furnace is one of the most important equipment in the steel-rolling industry, in which slabs are required to be heated to the specified temperature at the exit of the furnace for metallurgical quality and for hot rolling\[^{[1]}\]. It is important to optimize the furnace temperature and control the fuel feed flow appropriately for each zone in order to improve product quality and reduce fuel consumption. It is clear that problems in the walking-beam-type reheating furnace is a difficult problem that needs efficient control architectures and optimization algorithms to realize different production requirements, which can be considered problems of optimization and control for a class of serially connected systems\[^{[2]}\]. Many actual controlled plants such as production line, cold rolling processes, metallurgical processes, and mass control systems are composed of several serially connected dynamic subsystems, in which there is a strong incentive to control the final product values so as to minimize the variability in product quality. However, the relationship between the final product values and the desired output values of all serially connected subsystems is often nonlinear and very complex, which causes many difficulties to model-based optimization and control. In addition, there are a large number of uncertainty sources in the optimization of such systems, and these uncertainties can seriously degrade the performance of the overall optimization system. Thus it has become essential to develop efficient optimization algorithms with less computational and maintenance burden.

The above problems promote the development of plant-wide optimization (also known as real-time optimization, RTO) and control techniques. A typical RTO system is model-based optimization and implemented on top of unit-based multivariable controllers. The objective is to maintain the plant operation near an economic optimum in the face of disturbances and other external/internal changes\[^{[3]}\]. Conventional RTO strategy is based on a steady-state model of the plant and calculates set-points under input and output constraints for various plant units, which then steer their respective units to the calculated steady-state conditions. While most integrated plants have very long transient dynamics, the steady-state model based RTO formulation can be extremely limited. To overcome drawbacks existing in the steady-state RTO, many researchers proposed some dynamic RTO schemes performing at a rate same as or slower than local unit controllers to dynamically track changes in optimal operating conditions\[^{[4]}\]. Uncertainty is inherent in most process control and optimization problems, so that making robust decisions under uncertainty in the areas of real-time optimization has been paid more attention. In particular, stochastic programming handles uncertainties in the constraints or objective function using recourse functions, chance constraints and so forth, to transform the stochastic optimization problem to an equivalent deterministic optimization problem, which can be solved using a number of readily available optimization techniques\[^{[5]}\]. Chance-constrained optimization is one method of stochastic programming, in which the constraints containing stochastic parameters are guaranteed to be satisfied with a certain probability at the optimum found. Since chance-constrained optimization deals directly with uncertainty in an explicit manner, it is a good candidate technology for use in plant-wide optimization, and some robust RTO and control schemes include Refs.\[^{[6]}\]–\[^{[8]}\]. Plant-wide optimization problem with economic and model uncertainties for serially connected systems is handled in this paper by chance-constrained programming, which then can be converted into the equivalent deterministic optimization problem and solved easily.

Model predictive control (MPC) is a popular technique and has been successfully used in the control of various linear and nonlinear dynamic systems because the actual con-
control objectives and operating constraints can be represented explicitly in the optimization problem that is solved at each control instant [9], [10]. In general, MPC is implemented in a centralized scheme. Completely centralized control of such systems, on the other hand, frequently results in unacceptable control performance due to the inter-connections between the subsystems ignored [2]. Nowadays, distributed MPC where local control inputs are computed using local measurements and small-scale models of the local dynamics is also gradually developing for the control of large-scale systems, which has the advantage of being flexible to system structure, error-tolerance and less computational efforts. Some studies on distributed MPC are available in Refs. [11]–[13]. Reference [11] proposes an iterative, cooperative distributed MPC strategy in which the subsystem controllers optimize the same objective function in parallel, which guarantees performance improvement and the appropriate communication burden among subsystems. However, it is possible for each subsystem to exchange information several times during it solves the local optimization problem in the iterative algorithms. A non-iterative, non-cooperative distributed MPC scheme is presented in Ref. [12] where a neighbor-to-neighbor communication network and partial structural information are needed. However, the optimization in each local controller is to pursue the performance of local subsystem, which the performance of the non-cooperative distributed MPC frameworks is worse than that of centralized MPC. Reference [13] presents a non-iterative, cooperative distributed MPC method based on neighborhood optimization with one-step delay communication, which requires low communication requirements. The distributed GPC scheme is proposed in this paper on the basis of the work of Ref. [13] to reduce on-line computational requirement and improve control performance of the whole system.

The reminder of the study is organized as follows. In Sect. 2, a two-level scheme with robust dynamic plant-wide optimization and distributed GPC is presented for the serially connected processes. In Sect. 3, chance-constrained programming is employed in the plant-wide optimization under parametric uncertainty, in which the constraints containing stochastic parameters are guaranteed to be satisfied at a high level of probability. The deterministic equivalents for linear and nonlinear individual chance constraints are derived. A distributed GPC algorithm with one-step delay communication is developed in Sect. 4, in which on-line optimization of the global system is decomposed into that of several lower-order subsystems, and these subsystems can co-operate and communicate with each other in a distributed structure to achieve the objective of the whole system. An illustrative example is presented to verify the efficiency of the chance-constrained programming in Sect. 5. The simulation studies for optimization and control of the walking-beam-type reheating furnace are provided in Sect. 6 to demonstrate the practicality of the proposed dynamic plant-wide optimization and control scheme. Conclusions are drawn in Sect. 7.

2. Dynamic Plant-Wide Optimization and Control

Consider a serially connected system composed of $N$ sub-processes shown as Fig. 1, where $y_i$, $u_i$, and $z_i$ are the output vector, the control vector of the $i^{th}$ sub-process, and the final product vector, respectively. This class of serially connected systems is composed of many similar sub-processes placed after one another, in such a way that each sub-process is connected with dynamic control input coupling between its neighbor sub-processes. A two-level hierarchical architecture of the proposed dynamic plant-wide optimization and control is illustrated in Fig. 1. Dynamic plant-wide optimization performs at a rate lower than those of the local GPC units. On the top layer, the model-based plant-wide optimizer determines the optimal set-points $y_{id} = [y_{id} \cdots y_{Nid}]^T$ with parametric uncertainty, and these set-points $y_{id}$ ($i = 1, \cdots , N$) are sent to each local GPC unit at each plant-wide optimization interval. On the lower layer, a distributed GPC scheme is performed to realize the on-line control of the serially connected subsystem. For each local GPC unit, the control profile is calculated and the first move $u_i$ is implemented at each GPC sampling time. Then, the output of each sub-process $y_i$ is measured for GPC prediction update. This process is repeated until the next execution time of the plant-wide optimization is reached when the final product vector is then sampled.

3. Plant-Wide Optimization with Chance-Constrained Programming

Plant-wide optimization is model-based optimization and implemented on top of unit-based GPC controllers. The objective is to maintain the plant operation near an economic optimum in the face of disturbances and other external/internal changes. Plant/model mismatch is a crucial issue to the plant-wide optimization system. Although most plant-wide optimization systems attempt to improve model accuracy through model updating, there is a large number of uncertainty sources seriously degrade the performance of any plant-wide optimization system, including: market uncertainty, process uncertainty, measurement uncertainty and model uncertainty [4]. In this section stochastic programming is employed in the plant-wide optimization with the assumption that economic uncertainty enters the objective function.
function linearly and uncertain parameters enter the constraints nonlinearly.

3.1 Robust Plant-Wide Optimization

The dynamic plant-wide optimization of the serially connected system can be cast into the following stochastic programming resolved at each plant-wide optimization interval

$$\min_{Y_d} \mathcal{J}(t) \triangleq \mathcal{J}(Z_f(t + 1 \mid t), Y_d, \theta)$$

$$s.t. \ Z_f(t + 1 \mid t) = f(Y_d, \xi)$$

$$g(Z_f(t + 1 \mid t), Y_d) \leq 0$$

(1)

where $Y_d = [y_d^1(t) \cdots y_d^j(t + P - 1)]^T$ denotes the desired output vector with the control horizon $P$. $Z_f(t + 1 \mid t) = [z_f^1(t + 1 \mid t) \cdots z_f^j(t + P \mid t)]^T$ denotes the predicted final product state vector of horizon $P$. $\theta \in \Theta$ is the parameter vector representing the uncertain economic information, $\xi \in \Xi$ is the parameter vector representing the parametric model uncertainty; $J$ is the stochastic objective function and is often the operating profit in plant-wide optimization; $f$ is a computational mapping that establishes unique values of the predicted product quality $Z_f(t + 1 \mid t)$ for given values of $Y_d$ and $\xi$; $g$ is a set of inequality constraints and is often constrained within lower and upper bounds. The uncertain parameters $\theta$ and $\xi$ are assumed to be reasonably approximated by normal distribution $\theta \sim N(\overline{\theta}, P_{\theta})$ and $\xi \sim N(\overline{\xi}, P_{\xi})$, respectively, where $\overline{\theta}$ and $\overline{\xi}$ are the means of $\theta$ and $\xi$, $P_{\theta}$ and $P_{\xi}$ are the covariance matrices.

The equivalent plant-wide optimization problem can be represented as

$$\min_{Y_d} \mathcal{J}(Y_d, \xi, \theta)$$

$$s.t. \ \tilde{g}(Y_d, \xi) \leq 0$$

where $\mathcal{J}(Y_d, \xi, \theta) \triangleq J(f(Y_d, \xi), Y_d, \theta)$, $\tilde{g}(Y_d, \xi) \triangleq g(f(Y_d, \xi), Y_d)$.

The above plant-wide optimization framework can be expressed in terms of the minimization of a convex combination of the expectation and the standard deviation of the objective function [14]

$$\min_{Y_d} \left\{ \alpha E[\mathcal{J}(Y_d, \xi, \theta)] + (1 - \alpha) \sqrt{\text{Var}[\mathcal{J}(Y_d, \xi, \theta)]} \right\}$$

(3)

where $\alpha \in (0, 1]$ is a user specified constant, $E[J(Y_d, \xi, \theta)]$ represents the expected value of the objective function with respect to the uncertain parameters $\xi$ and $\theta$, $\text{Var}[J(Y_d, \xi, \theta)]$ represents the spread of the distribution of objective function values about the expectation. The choice of $\alpha$ can be viewed as the emphasis on caution: as $\alpha$ assumes lower values, more conservative solutions are produced.

Some assumptions [6] are added into the plant-wide optimization in order to reduce the computational complexity of problem (3). Then the plant-wide optimization problem can reduce to

$$\min_{Y_d} \mathcal{J}(Y_d, \xi, \theta)$$

$$s.t. \ \tilde{g}(Y_d, \xi) \leq 0$$

(4)

where $\tilde{g}$ is the estimated values of model parameters $\xi$ in each plant-wide optimization interval.

It can be seen from problem (4) that the objective function is deterministic, and inequality constraints are uncertain, which reduces the complexity of problem (1). We should put emphasis on how to handle uncertain constraints $\tilde{g}(Y_d, \xi) \leq 0$ in next section.

3.2 Inequality Constraints under Uncertainty

There are four approaches to handling the constraints within stochastic programming: penalty function, constraints with conditional expectations, probability maximization, and chance-constrained programming (CCP). The practical and straightforward approach for handling the uncertainty in the constraints is the chance-constrained programming [15], in which a reliability level for the uncertain constraints is set by requiring that they must be satisfied at a high level of probability. The joint chance constrain framework is as follows

$$\Pr[\tilde{g}(Y_d, \xi) \leq 0] \geq \gamma$$

(5)

where $\gamma \in [0, 1]$ is a given reliability level. The aim is to optimize the economic objective function $\mathcal{J}(Y_d, \tilde{\xi}, \tilde{\theta})$, while satisfying all the uncertain constraints at or beyond the specified level of probability $\gamma$ simultaneously.

Such joint probability constrained (JPC) problem is very computationally expensive since it involves a semi-infinite, multivariate probability integral to evaluate the associated probability distribution function. Therefore, the individual chance constrains are more preferable

$$\Pr[\tilde{g}_j(Y_d, \xi) \leq 0] \geq \gamma_j \ (j = 1, \cdots, p)$$

(6)

where $\gamma_j$ is the $j^{th}$ inequality constraint function of $\tilde{g}$ and $\gamma_j$ is the corresponding reliability level.

It can be seen that higher values of $\gamma_j$ lead to fewer feasible solutions in individually probability constrained (IPC) problem, hence lead to optimal solutions at higher costs. Therefore, models with chance constraints give a hint to a good compromise between costs and safety.

Since chance-constrained programming deals directly with uncertainty in the constraints in an explicit and natural manner, it is a good candidate technology for use in the plant-wide optimization. After handling the uncertainty by satisfying the uncertain constraints at or beyond the specified level of probability, the next key procedure is to convert the chance-constrained programming problem into an equivalent deterministic optimization problem, which can be solved using a number of readily available optimization techniques.
3.2.1 Deterministic Equivalent for Linear Individual Chance Constraints

Consider one of the constraint functions in (6) \( g_j(Y_d, \xi) \) is linearly constrained with normally distributed stochastic parameters that can be formed as

\[
\Pr \left( \xi^T (A_j Y_d + b_j) \leq c_j \right) \geq \gamma_j
\]

and can be written as an equivalent constraint with zero mean and unit variance

\[
\Pr \left\{ \sqrt{(A_j Y_d + b_j)^T P_{\xi_j}(A_j Y_d + b_j)} \leq \frac{c_j - \xi^T (A_j Y_d + b_j)}{\sqrt{(A_j Y_d + b_j)^T P_{\xi_j}(A_j Y_d + b_j)}} \right\} \geq \gamma_j
\]

Then the above probability can be converted into

\[
\frac{c_j - \xi^T (A_j Y_d + b_j)}{\sqrt{(A_j Y_d + b_j)^T P_{\xi_j}(A_j Y_d + b_j)}} \geq F^{-1}(\gamma_j)
\]

which is equivalent to the following deterministic second-order cone constraint

\[
\xi^T (A_j Y_d + b_j) + F^{-1}(\gamma_j) \left\| P_{\xi_j}^{1/2}(A_j Y_d + b_j) \right\|_2 \leq c_j
\]

(\( j = 1, \cdots, p \))

(9)

(10)

where \( F^{-1} \) is the value of the inverse cumulative distribution function of the standard normal distribution evaluated at \( \gamma_j \).

The IPC problem with individual chance constraints in (7) is equivalent to a deterministic optimization problem with constraints in (10), which can be cast into the second-order cone programming [16].

3.2.2 Deterministic Equivalent for Nonlinear Individual Chance Constraints

Consider one of the inequality constraints in (6), the first-order Taylor series approximation is employed for \( \tilde{g}_j(Y_d, \xi) \) at the current observation \( (Y_{d0}, \xi_0) \), and the probability constraints can be changed as

\[
\Pr \left\{ \xi \leq \tilde{h}_j^T Y_d \right\} \geq \gamma_j
\]

(11)

where

\[
\tilde{h}_j = \nabla_{Y} \tilde{g}_j \tilde{Y}_{d0} + \tilde{g}_j(Y_{d0}, \xi_0) - (\nabla_{Y} \tilde{g}_j)_{Y_{d0} = \xi_0} \tilde{Y}_{d0} - (\nabla_{\xi} \tilde{g}_j)_{Y_{d0} = \xi_0, \xi = \xi_0} \xi_j
\]

\[
\tilde{h}_j = - (\nabla_{Y} \tilde{g}_j)_{Y_{d0} = \xi_0} \tilde{Y}_{d0} + (\nabla_{\xi} \tilde{g}_j)_{Y_{d0} = \xi_0, \xi = \xi_0} \tilde{\xi}_j + \tilde{g}_j(Y_{d0}, \xi_0) - (\nabla_{Y} \tilde{g}_j)_{Y_{d0} = \xi_0} \tilde{Y}_{d0} - (\nabla_{\xi} \tilde{g}_j)_{Y_{d0} = \xi_0, \xi = \xi_0} \xi_j
\]

\( \tilde{h} \) is a bias term used to compensate for the plant/model mismatch including the effects of model linearization, and \( \beta = [\beta_1 \cdots \beta_p]^T \).

Supposing that \( \xi \) and \( \beta \) follow the multivariate normal distribution, \( \xi_j \sim N(\mu_{\xi_j}, \sigma_{\xi_j}^2) \), where the mean value \( \mu_{\xi_j} \) and variance \( \sigma_{\xi_j}^2 \) can be computed from the statistical data of \( \xi \) and \( \beta \), and the constants \( Y_{d0}, \xi_0 \). Then the above probability constraint can be converted into the following deterministic constraint

\[
\tilde{h}_j^T Y_d \geq F^{-1}(\gamma_j) \quad (j = 1, \cdots, p)
\]

(12)

The IPC problem with individual chance constrains in (6) is equivalent to a deterministic optimization problem with constraints in (12).

The individual chance constraints (11) are rearranged into the joint chance constraint

\[
\Pr \left\{ \xi \leq \tilde{h} Y_d \right\} \geq \gamma
\]

(13)

where \( \xi = [\xi_1 \cdots \xi_p]^T \) and \( \tilde{h} = [\tilde{h}_1 \cdots \tilde{h}_p]^T \).

An algorithm was proposed for solving the IPC problem with joint chance constraint (13) in Ref. [6]. A numeric stochastic optimization problem is illustrated in Sect. 5 to test the effectiveness of the algorithm to search the solution to the JPC problem.

4. Distributed GPC for Serially Connected Systems

The generalized predictive control (GPC) method has become one of the most popular MPC algorithms in industry, which is formulated as resolving an on-line open-loop optimal control problem in moving horizon style repeatedly [10]. It is claimed that GPC overcomes the following difficulties: non-minimum phase plant, open loop unstable plant, plant with variable or unknown dead-time and plants of unknown order.

Without loss of generality, suppose that each sub-process is with single input and single output. Consider the \( i^{th} \) sub-process described by the following CARIMA model

\[
A_i(z^{-1}) y_i(k) = \sum_{j=0}^{N_i} B_{ij}(z^{-1}) u_i(k-j) + \frac{T_i(z^{-1})}{\Delta} e_i(k)
\]

(14)

where \( A_i \) and \( B_{ij} \) are the denominator and the numerator polynomials of the transfer function in the backward shift operator \( z^{-1} \); \( \mathbb{N}_i = \{j\} \), called the neighborhood of the sub-process \( i \), is the set of integer \( j \) which control decision of the \( i^{th} \) sub-process has the effect on the \( i^{th} \) sub-process; \( T_i \) is the filter polynomial, \( \Delta \) is the difference operator (\( \Delta = 1 - z^{-1} \)), \( e_i(k) \) are uncorrelated zero mean white noise sequences.

At time instant \( k \), the future predictive output of the \( i^{th} \) sub-process can be expressed as

\[
y_i(k+s|k) = \sum_{j=0}^{N_i} G_{ij}^s \Delta u_j(k+s-1) + \frac{F_i^s}{T_i} y_i(k)
\]

\[ + \sum_{j=0}^{N_i} \frac{H_i^s}{T_i} \Delta u_j(k-1) \]

(15)

where \( \Delta u_i(k+s-1) \) is the increment of future manipulated variables, \( u_i(k) = u_i(k-1) + \Delta u_i(k) \), \( y_i(k+s|k) \) is the predictive output at time instant \( k+s \) based on the output at time instant \( k \). The polynomials \( F_i^s, G_{ij}^s \) and \( H_i^s \) satisfy the following Diophantine identities:

\[
\left\{ \begin{array}{l}
 T_i = E_i^s A_i \Delta + q^{-s} F_i^s \\
 E_i^s B_{ij} = G_{ij}^s T_i + q^{-s} H_{ij}^s, \quad j \in \mathbb{N}_i
\end{array} \right.
\]

(16)

As is standard, the performance index of the whole system
can be decomposed in terms of the local indexes for each subsystem
\[
\min_{\Delta u(k)} J_{GPC}(k) = \sum_{i=1}^{N} J_{GIPC}(k) = \sum_{i=1}^{N} \left\{ \sum_{j=1}^{N_i} [y_{jd} - y_i(k + s_i k)]^2 + \sum_{h=1}^{N_h} \mu(h) [\Delta u_i(k + h - 1)]^2 \right\}
\] (17)

where \(N_i, N_h\) are the prediction horizon and the control horizon, respectively, and \(\mu(h) > 0\) is the weighting sequence; \(\Delta U_i(k) = \{\Delta u_i(k) \cdots \Delta u_i(k + N_h - 1)\}^T\), \(\Delta U(k) = [\Delta U_i^T (k) \cdots \Delta U_i^T (k)]^T\). Each control variable \(u_i(k + h - 1)\), \(\Delta u_i(k + h - 1)\) and predictive output \(y_i(k + s_i k)\) are subject to the physical constraints \(u_i^{\max}, u_i^{\min}, \Delta u_i^{\max}, \Delta u_i^{\min}\), and \(y_i^{\max}, y_i^{\min}\).

The objective of the whole system is to regulate the system output to the set-point while keeping the performance index minimal and satisfying the above constraints. For the serially connected system, because of the effect of control horizon \(N_u\), the optimized vector \(\Delta U(k)\) at each sampling time is highly dimensional, the computation is intensive, which accordingly requires high performance computers or some advanced algorithms. The following distributed GPC scheme with one-step delay communication is proposed on the basis of the work of Ref. [13]. Neighborhood optimization employs a cooperative strategy so that the global optimization problem can be decomposed into a number of local optimization sub-problems.

An improved performance index for each subsystem, including cost functions within its neighborhood, can be described by
\[
\min_{\Delta U_i(k)} \tilde{J}_{GPC}(k) = \sum_{j \in \mathcal{N}_i} J_{GIPC}(k)
\] (18)
The local control decision for each subsystem can be obtained by solving the above local optimization problem if the local control decisions of its neighbors \(\Delta U_j(k)\) \((j \in \mathcal{N}_i, j \neq i)\) are available. Because there is a one-step delay in the information available from its neighbors, \(\Delta U_j(k)\) can be expressed as follows
\[
\Delta U_j(k) = \tilde{I}_j \Delta U_j(k-1) \quad (j \in \mathcal{N}_i, j \neq i)
\] (19)
where \(\Delta U_j(k - 1) = [\Delta u_j(k - 1) \cdots \Delta u_j(k + N_u - 2)]^T\), \(\tilde{I}_j = \begin{bmatrix} 0_{N_u - 1 \times 1} & I_{(N_u - 1)} \end{bmatrix} \begin{bmatrix} 0_{(N_u - 1) \times 1} \end{bmatrix} \).

Then the local control decision for each subsystem can be obtained by
\[
\Delta U_i(k) = \arg \left\{ \min_{\Delta U_i(k)} \tilde{J}_{GPC}(k) \mid \Delta U_i(k - 1) = \sum_{j \in \mathcal{N}_i, j \neq i} \Delta U_j(k) \right\}
\] (20)
At sampling time instant \(k\), the output prediction model for each subsystem can be expressed by
\[
\tilde{Y}_i(k) = G_{ii} \Delta U_i(k) + \sum_{j \in \mathcal{N}_i, j \neq i} G_{ij} \tilde{I}_j \Delta U_j(k - 1) + f_i(k)
\] (21)
where \(\hat{Y}_i(k) = [y_i(k + 1) \cdots y_i(k + N_d) [y_i^T]^T\), \(f_i(k) = [f_i(k + 1) \cdots f_i(k + N_d)]^T\), \(f_i(k + s) = \frac{f_i}{T} y_i(k) + \sum_{j \in \mathcal{N}_i} \frac{h_i}{T} \Delta u_i(k - 1), \)

\[
G_{ii} = \begin{bmatrix} g_{i0} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
g_{iN_u - 1} & \cdots & g_{i0} \\
\vdots & \ddots & \vdots \\
g_{iN_u - 1} & \cdots & g_{iN_u - N_u} \end{bmatrix} \begin{bmatrix} g_{i0} & \cdots & 0 \\
\vdots & \ddots & \vdots \\
g_{iN_u - 1} & \cdots & g_{i0} \\
\vdots & \ddots & \vdots \\
g_{iN_u - 1} & \cdots & g_{iN_u - N_u} \end{bmatrix}
\] (22)
The optimization problem in (20) subject to predictive output in (21) and physical inequality constraints can be cast as a quadratic programming (QP) problem
\[
\min_{\Delta U_i(k)} \tilde{J}_{GPC}(k) = \min_{\Delta U_i(k)} \frac{1}{2} \Delta U_i^T(k) Q_i \Delta U_i(k) - q_i^T(k) \Delta U_i(k)
\text{s.t.} \quad \Omega_i \Delta U_i(k) \leq d_i(k)
\] (22)
where \(Q_i = \sum_{j \in \mathcal{N}_i} G_{ij}^T G_{ji} + \Lambda_i > 0, q_i(k) = \sum_{j \in \mathcal{N}_i} G_{ji} \bar{w}_j(k)\),
\[
\bar{w}_j(k) = \omega_j(k) - \sum_{h \in \mathcal{N}_j, j \neq i} G_{hj} \Delta U_h(k - 1) - f_j(k), \quad \omega_j(k) = [y_{jd} \cdots y_{jd}]^T, \quad \Lambda_i = \text{Diag}(\mu_1(1), \cdots, \mu_j(N_u))\).
\[
M_{N_u} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\
0 & 1 & \cdots & 1 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 1 & 1 \end{bmatrix}_{N_u \times N_u}
\]
\[
\tilde{U}_i = \begin{bmatrix} u_i^{\max} - u_i(k - 1) \\
\vdots \\
u_i^{\max} - u_i(k - 1) \\
\vdots \\
u_i^{\min} - u_i(k - 1) \end{bmatrix}, \quad \Delta U_i = \begin{bmatrix} \Delta u_i^{\max} \\
\vdots \\
\Delta u_i^{\max} \\
\vdots \\
\Delta u_i^{\min} \end{bmatrix}, \quad \bar{Y}_i = [y_i^{\max} \cdots y_i^{\max}]^T, \quad Y_i = [y_i^{\min} \cdots y_i^{\min}]^T
\]
Step 3. Assignment and Implementation: Compute the instant control law \( \Delta u_i(k) = [1 \ 0 \ \cdots \ 0] \Delta U_i(k) \), and apply \( u_i(k) = u_i(k - 1) + \Delta u_i(k) \) to each subsystem;

Step 4. Receding Horizon: Move horizon to the next sampling time, that is, \( k + 1 \to k \), go to Step 1, and repeat the above steps.

5. Illustrative Example

The stochastic optimization problem to illustrate IPC and JPC programming is as follows.

\[
\begin{align*}
\min & \quad x_1 + 2x_2 + 3x_3 \\
\text{s.t.} & \quad x_1 + x_2 + x_3 \geq \xi_1 + \xi_2 + \xi_3 \\
& \quad x_2 + x_3 \geq \xi_2 + \xi_3 \\
& \quad 0 \leq x_1 \leq 2, 1 \leq x_2 \leq 3, 2 \leq x_3 \leq 4
\end{align*}
\]

where \( \xi_1, \xi_2 \) and \( \xi_3 \) follow a joint normal distribution with expectation \( \mu = [1 \ 2 \ 3]^T \), standard deviation \( \sigma = [0.1 \ 0.2 \ 0.3]^T \), and the correlation coefficient matrix \( R = \begin{bmatrix} 1 & -0.8 & -0.5 \\ -0.8 & 1 & 0.4 \\ -0.5 & 0.4 & 1 \end{bmatrix} \).

In the simulations, the specific level of probability for the satisfaction of the uncertain constraints in either IPC or JPC problems is taken as 95%. The optimal solutions derived from deterministic, IPC, and JPC problems are given in Table 1. Deterministic solution is derived from the problem in which the random variables are replaced by their expectations \( \mu \). It can be seen from Table 1 that the deterministic solution is not robust to the uncertainties. The probabilities of satisfying each individual uncertain constraint are 50%, and the probability of satisfying three uncertain constraints is only 42.27%. While the IPC and JPC approaches improve solutions robustness to provide probabilities of satisfying three constraints at high level of 92.98% and 95.01%, respectively. Accordingly the JPC approach produces the specific 95% joint constraint satisfaction probability at the expense of inflating the objective function value to 16.0794, which is a little larger than that obtained by the IPC approach and much larger than that obtained by the deterministic optimization.

| Table 1 | Optimal solutions for the illustrative example. |
|----------|-----------------------------------------------|
| Optimization variable | Deterministic solution | IPC solution | JPC solution |
| \( x_1 \) | 1 | 0.8899 | 0.9765 |
| \( x_2 \) | 2 | 2.2005 | 2.2647 |
| \( x_3 \) | 3 | 3.4935 | 3.5245 |
| \( \gamma_1 \) | 50% | 95% | 98.45% |
| \( \gamma_2 \) | 50% | 95% | 96.93% |
| \( \gamma_3 \) | 50% | 95% | 95.98% |
| \( \gamma \) | 42.27% | 92.98% | 95.01% |
| Objective function value | 14 | 15.7713 | 16.0794 |

6. Application to the Walking-Beam-Type Reheating Furnace

The walking-beam-type reheating furnace (shown in Fig. 2) consists of preheating zone, heating zone I, heating zone II and soaking zone. If furnace temperature is too high, the slabs in the furnace will be overheated. Otherwise, it cannot be heated to the desired temperature [17]. Because the slab heating process is rather complicated, the factors affecting the slab temperature at the exit of the furnace are quite a lot and the working condition is complex, the optimization the set-point of the furnace temperature for each zone has not been solved satisfactorily.

Based on the proposed scheme in the paper, functional diagram of plant-wide optimization and control for the walking-beam-type reheating furnace is shown in Fig. 3, including three functional units: plant-wide optimization with CCP, data processing and estimation using least-square method, and furnace temperature tracking control using distributed GPC. On the top layer, the plant-wide optimization calculates set-points of the furnace temperature \( T_{\text{set}} \) for each zone and these set-points are sent to the corresponding local GPC unit at each plant-wide optimization interval. On the lower layer, data processing and estimation gathers the plant and operation data (furnace temperature and fuel feed flow for each zone) and estimates the parameters of the furnace temperature model, then the distributed GPC algorithm is performed to calculate the fuel feed flow \( F_i \) for each zone, respectively, at each sampling time instant.

Fig. 2 | Structure of the walking-beam-type reheating furnace.

Fig. 3 | Functional diagram of optimization and control for the walking-beam-type reheating furnace.
predicted slab average temperatures at the exit of the furnace; a maximum limit on the difference between the predicted surface and central temperatures of the slab at the exit of the furnace. Other operation constraints include bounds on the furnace temperature for each zone. The plant-wide optimization is described as the following stochastic programming problem

\[
\begin{align*}
\min J & = \sum_{i=0}^{s_f} \alpha(s_i) T_{fd}(s_i) \\
\text{s.t.} & \quad \hat{T}(s_f) = F(T_{fd}, \bar{\xi}) \\
& \left[ T_m^s(s_f) - T_m(s_f) \right] \leq 20 \\
& T_s(s_f) - T_c(s_f) \leq 20 \\
& T_{fd}(s_f) = a s_f^2 + b s_f + c \\
& T_{fd}(s_f) \leq T_{fd}(s) \leq T_{fd}(s_f) \\
\end{align*}
\]

(24)

where \( s \) is the displacement of the slab in the furnace, \( s_f \) denotes the exit displacement of the furnace, \( s_i \) is the key position of the zone \( i \); \( \alpha(s) > 0 \) is the weighting coefficient; \( T_{fd}(s) \) denotes the desired furnace temperature distribution, \( \hat{T} \triangleq \{T_m, T_s, T_c\} \), \( T_m \) is the slab average temperature, \( T_s, T_c \) are the estimated surface and central temperatures of the slab, \( T_m^s \) is the desired slab average temperature, \( T_{fd}^\min, T_{fd}^\max \) are the minimum and the maximum limits of the furnace temperature; \( a, b, c \) are the coefficients to be optimized; \( F \) is the functional relationship between the slab temperature at the exit of the furnace \( \hat{T}(s_f) \) and the desired temperature at the key position of the zones; \( \bar{\xi} \) is the parameter vector representing the parametric model uncertainty, and \( F \) can be deduced by the following slab temperature prediction model.

\[
\begin{align*}
\frac{dT_m(t)}{dt} & = \frac{k_m}{\rho \cdot C_p} \cdot \sigma \cdot \Phi \cdot \left[T_f(x, \tau) + 273\right]^4 \left[T_i(t) + 273\right]^4 \\
T_i(t) & = T_m(t) + \frac{h}{k_c \cdot \lambda} \cdot \sigma \cdot \Phi \cdot \left[T_f(x, \tau) + 273\right]^4 \left[T_i(t) + 273\right]^4 \\
T_c(t) & = T_f(t) - \frac{h}{k_c \cdot \lambda} \cdot \sigma \cdot \Phi \cdot \left[T_f(x, \tau) + 273\right]^4 \left[T_i(t) + 273\right]^4 \\
\end{align*}
\]

(25)

Parameters in the above prediction model can be seen from Ref.\[17\]. The uncertain parameters in the slab temperature prediction model can be denoted by \( \bar{\xi} \triangleq \{h, \rho, C_p, \lambda, k_m, k_c, \lambda_c\} \), representing uncertain sizes of the heated slabs and uncertain kinds of steel.

Joint chance constraint employed into the plant-wide optimization problem in (24) for handling the uncertainty in the constraints is as follows.

\[
\Pr \left\{ \begin{align*}
& \left[ T_m^s(s_f) - T_m(s_f) \right] \leq 20 \\
& T_s(s_f) - T_c(s_f) \leq 20 \\
\end{align*} \right\} \geq \gamma 
\]

(26)

where \( \gamma \) is a given reliability level, denoting the product quality specifications.

Thermal transmission is carried through only from the end to the head of the furnace, and the furnace temperature model is referred from Ref.[18]. By employing the least-square identification method, the furnace temperature model for each zone can be identified. In the distributed GPC unit, the local optimization problem for each zone solved at each sampling time instant is as follows.

\[
\begin{align*}
\min_{\Delta F(k)} J_{GPC}(k) & = \sum_{i=0}^{N_i} \left[ T_{fdi} - T_f(k+s|k) \right]^2 \\
& + \sum_{h=1}^{N_h} \mu_i(h) \Delta F^2_i(k+h-1) \quad (i = 1, \ldots, 6) \\
\end{align*}
\]

(27)

where \( T_{fdi} \) is the set-point (i.e. reference trajectory) of the furnace temperature for each zone calculated from the layer of plant-wide optimization.

A six-zone walking-beam-type reheat furnace (as shown in Fig. 2) is used to demonstrate the plant-wide optimization and control technique. The loading mode of slabs is end-in side-out. The valid length, width and hearth area of the furnace is 29348 mm, 5800 mm and 161.5 m², respectively. The maximum throughput is 70 t/h. The furnace is divided into preheating zone (13398 mm), (upper, lower) heating zone I (5200 mm), (upper, lower) heating zone II (6300 mm) and (upper, lower) soaking zone (4450 mm), where preheating zone does not need to be controlled.

The uncertain parameter in the plant-wide optimization \( \bar{\xi} \) represents uncertain sizes of the heated slabs and uncertain kinds of steel. The kinds of slabs to be heated include bearing steel, stainless steel, carbon steel and spring steel, etc. The desired slab temperature at the exit of the furnace is 1030 – 1250°C. The experimental parameters of the heated slabs for simulations are shown in Ref.[17]. In the simulations, the plant-wide optimization interval is set with 1.0 hour, and the probability level in JPC optimization is taken as \( \gamma = 95\% \) for product quality constraints. Using sampling time of 2 minutes, and the parameters used in the distributed GPC scheme are set with \( N_i = 12, N_h = 4, \mu_i = 1 \).

The MATLAB based experimental results are shown in Figs. 4–6. Figure 4 shows the product quality for the illustrative furnace running for 100 plant-wide optimization intervals, where the solid and dashed lines in the top figure are the actual value and the limit of the difference between the desired and predicted slab average temperatures at the exit of the furnace, denoted by \( DT_m = T_m^s(s_f) - T_m(s_f) \); the solid and dashed lines in the bottom figure are the actual value and the limit of the difference between the predicted surface and central temperatures of the slab at the exit of the furnace, denoted by \( DT_{sc} = T_s(s_f) - T_c(s_f) \). It can be observed from Fig. 4 that almost all the constraints \( DT_m \) and \( DT_{sc} \) are within the limits (shown as dotted line) and only a few constraints violate the limits of the product quality. Taking the first experimental slab C40 (150 x 150 x 3500 mm) as an example, the slab temperature distribution along the furnace under the optimized furnace temperature is shown in Fig. 5 during one plant-wide optimization interval.

The furnace temperatures and the fuel feed flows of three zones (shown in Fig. 6 running for 4 hours) are carried out to evaluate the distributed GPC (DGPC) algorithm.
designed in this paper (shown as solid line) through performance comparisons with the centralized GPC (shown as dotted line). It can be seen that the furnace temperatures of three zones (shown as solid line) follow their set-points (optimized from the plant-wide optimization unit, shown as dash-dot line) satisfactorily. Performance comparisons, including the whole performance index and the running time for CPU between the proposed DGPC scheme and the centralized GPC, are summarized in Table 2. As can be seen, the control performance of the proposed distributed GPC scheme is worse than and close to that of the centralized GPC, and running time for CPU is less than that using centralized control method. The on-line optimization of the whole system can be decomposed into that of several small-scale subsystems in the distributed scheme. In addition, the distributed GPC algorithm with one-step delay communication exchanges information only once at each sampling time instant, which needs lower communication requirements and reduces on-line computational burden.

7. Conclusions

Focusing on the optimization and control with uncertainty for a class of serially connected processes, a two-level scheme with dynamic plant-wide optimization and distributed GPC was developed in this study. The model-based plant-wide optimization on the top layer determines set-points with constraints, and these set-points are sent to their corresponding local GPC unit on the lower layer. The stochastic optimization approach to robust plant-wide optimization was developed to deal effectively with economic and model uncertainties. CCP was employed in the plant-wide optimization in which the constraints containing stochastic parameters are guaranteed to be satisfied at a high level of probability. Algorithms were presented that treat the probabilistic constraints either independently or jointly. A distributed GPC algorithm with one-step delay communication was developed for the situation where each subsystem resolves its local optimization problem with its neighbors’ local optimal control decisions at the previous time instant available via the shared network. Finally, the good performance of the proposed robust plant-wide optimization and control was demonstrated through numerical experiments for the walking-beam-type reheating furnace. It is noted that the proposed scheme is not limited to the optimization and control for the walking-beam-type reheating furnace and can be used in a wide range of complex large-scale systems.

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