Software for the resolution of certain kind of problems, those that rate high in the Stringent Performance Objectives adjustment factor (IFPUG scheme), can be described using a combination of game theory and autonomous systems. From this description it can be shown that some of those problems exhibit chaotic behavior, an important fact in understanding the functioning of the related software. As a relatively simple example, it is shown that chess exhibits chaotic behavior in its configuration space. This implies that static evaluators in chess programs have intrinsic limitations.

1 Introduction

IBM’s Deep Blue, a powerful chess playing machine consisting of two parallel-process tandem supercomputers programmed by a team of experts lead by team manager C. Tan [Hsu et al., 1990; Horgan, 1996; Hsu, 1990; Slate, 1984], played the world chess champion G. Kasparov several games in 1996 and 1997 with fairly even results. Actually, programmer Hsu’s estimate back in 1990 of the future machine’s playing strength was 4000 ELO points (chess’ rating system), far greater than Kasparov’s ~2800 present rating. In three minutes, which is the game’s average pondering time, the machine could calculate 20 billion moves, enough to for a 24-ply search and an up to 60-ply search in critical tactical lines. Since grandmasters can calculate just a few moves ahead, it seems very peculiar that a human could hold his own on the face of such an overwhelming opposition.

In this paper we are interested in a special kind of problem and the software written for it. It is the kind of problem whose software would score high in the
Stringent performance objectives [Abran & Robillard, 1996] adjustment factor of the International Function Point User’s Group (IFPUG). Examples are, for instance, the control of air-traffic at a busy airport, the scheduling of trains in areas with heavy traffic, and field military operations. One way of approaching this kind of problem is to treat it within the context of game theory, as a 2-player game. The first player would be the comptroller, central authority or headquarters, and the second is the system itself, that acts and reacts out of its own nature. The first player pursues to maintain control of a complicated system by choosing its moves, that is, by affecting the system in the ways available to him. He would like the system to always remain in states such that certain state variables (they could be safety, efficiency, lethality or others) are kept extremized. The performance objectives would be to extremize these state variables.

The nature of this kind of problem is such that it is necessary to see ahead what is going to happen. At least in theory, the first player must have arrived at his move only after having taken into consideration all of the possible responses of the second player. This is a common situation in game theory, and is another reason why the language of game theory is very well-suited to discuss both this kind of problem and the software developed to help deal with it. The typical program contains two fundamental sectors:

1. a ply calculator, that is able to look ahead at all possible continuations of the tree a certain number of plies ahead,

2. a static evaluator, that gives an evaluation of the resulting state of the problem at the end of each branch of plies.

Although there are many different ways of programming the ply calculator or the static evaluator, their complementary, basic functions are clear: the first is a brute force calculator of all possible states to come, and the second is an evaluator of the final resulting state of the system, intrinsically and on the basis of the state itself, without any resort to further calculation.

The different states of the problem can be seen as a function of time. If one is able to express each state using a mathematical description, the problem of the time-development of the state while maintaining certain state variables extremized can be described as an autonomous system. If the equations describing the time-development of the system are nonlinear, it is very likely that the problem is going to exhibit chaotic [Alligood et al., 1997] behavior. Therefore, the software for these problems has an intrinsic limitation on its accuracy, even if it still may be extremely useful.

As an example we will work out the case of chess, a useful one because, while nontrivial, it is not nearly as complex as some of the other problems are. Chess’ software scores high in the Stringent performance objectives adjustment factor. We will prove that chess exhibits chaotic behavior in its configuration space and that this implies its static evaluators possess intrinsic limitations: there are always going to be
states or positions that they will not be able to evaluate correctly. It is likely that this is precisely the explanation for the peculiar situation mentioned in the first paragraph: that a human being can hold his own at a chess game with a supercomputer. The ply calculator part of the program of the supercomputer would be tremendously effective, but the static evaluator would not be so faultless.

2 An abstract mathematical representation of chess

We have to describe each possible state (or position) in chess. To describe a particular state we shall use a 64 dimensional vector space, so that to each square of the board we associate a coordinate that takes a different value for each piece occupying it. A possible convention is the following:

- A value of zero for the coordinate of the dimension corresponding to a square means that there is no piece there.

- For the White pieces the convention would be: a value of 1 for the coordinate means the piece is a pawn, of 2 means it is a pawn without the right to the en passant move, of 3 that it is a knight, of 4 a bishop, of 5 a rook, of 6 a queen, of 7 a king, and of 8 a king without the right to castle.

- The values for the Black pieces would be the same but negative.

Let us represent the 64-component vector by the symbol $x$. A vector filled with the appropriate numbers can then be used to represent a particular state of the game. We shall call the 64-dimensional space consisting of all the coordinates the configuration space $C$ of the game. The succeeding moves of a pure strategy can be plotted in this space, resulting in a sequence of points forming a path.

Now we construct a function $f : C \to C$ that gives, for any arbitrary initial state of a chess game, the control strategy to be followed by both players. The existence of this function is assured by the Zermelo-von Neumann theorem [von Neumann & Morgenstern, 1944] that asserts that a finite 2-person zero-sum game of perfect information is strictly determined, or, in other words, that a pure strategy exists for it. For a given initial chess state this means that either

- White has a pure strategy that wins,

- Black has a pure strategy that wins,

- both are in possession of pure strategies that lead to a forced draw.

Consider a certain given initial state of the game where White has a pure strategy leading to a win. (The two other cases, where Black has the win or both have drawing strategies can be dealt with similarly and we will not treat them explicitly.) Let the initial chess state be given by the 64-component vector $x_0$, where we are assuming
that White is winning. The states following the initial one will be denoted by \( x_n \),
where the index is the number of plies that have been played from the initial position.
Thus \( x_1 \) is the position resulting from White’s first move, \( x_2 \) is the position resulting
from Black’s first move, \( x_3 \) is the position resulting from White’s second move, and so on. Since White has a winning pure strategy, it is obvious that, given a certain
state \( x_n \), \( n \) even, there must exist a vector function \( f \) so that, if \( x_{n+1} \) is the next state
resulting from White’s winning strategy, then \( f(x_n) = x_{n+1} \). On the other hand, if
\( n \) is odd, so that it is Black’s turn, then we define \( f \) to be that strategy for Black
that makes the game last the longest before the checkmate. Again, the pure strategy
that is available to Black according to the Zermelo-von Neumann theorem allows us
to define a function \( f(x_n) = x_{n+1} \). The function \( f \) is thus now defined for states with
\( n \) both even and odd.

The function \( f \) allows us to define another function \( g : \mathbb{C} \rightarrow \mathbb{C} \), the control strategy
vector function [Abramson, 1989], defined by \( g(x_n) = f(x_n) - x_n \). With it we can
express the numerical difference between the vectors corresponding to two consecutive
moves as follows:

\[
g(x_n) = x_{n+1} - x_n.
\] (1)

Given any initial state \( x_0 \), this function gives us an explicit control strategy for the
game from that point on.

### 3 Chaos in configuration space

A set \( N \) of simultaneous differential equations,

\[
g(x) = \frac{dx}{dt},
\] (2)

where \( t \) is the (independent) time variable, \( x \in \mathbb{R}^N \) and the \( g \) are known \( g : \mathbb{R}^N \rightarrow \mathbb{R}^N \)
vector functions, is called an autonomous system [Alligood et al., 1997]. The time
\( t \) takes values in the interval \( 0 \leq t \leq T \). Let us discretize this variable, as is often
done for computational purposes [Parker & Chua, 1989]. We assume it takes only
 discrete values \( t = 0, \Delta t, 2\Delta t, \ldots , T \). After an appropriate scaling of the system one
can take the time steps to be precisely \( \Delta t = 1 \). Let the initial condition of the system
be \( x(0) \equiv x_0 \), and let us define \( x(1) \equiv x_1 \), \( x(2) \equiv x_2 \), and so on. By taking \( N = 64 \)
one can then rewrite (2) in a form that is identical to (1).

Nonlinear autonomous systems in several dimensions are always chaotic, as expe-
rience shows. Is the control strategy function nonlinear? A moment’s consideration
of the rules of the game tell us that the control function has to be nonlinear and that,
therefore, the system described by (1) has to be chaotic.

For some kinds of chess moves the difference \( x_{n+1} - x_n \) has a relatively large value
that would correspond to a jerky motion of the system, and the question can be raised
if such a motion could really occur in a an autonomous system. But the important
thing to realize is that if even typical autonomous nonlinear systems (that possess a smooth function $g(x)$) do show chaotic behavior, then certainly the system that represents chess (with a jerky control strategy function) should also show it.

The chaotic nature of the paths in configuration space has several immediate implications, but certainly one of the most interesting is the following:

**Proposition 1** It is not possible to program a static evaluator for chess that works satisfactory on all positions.

**Proof.** The point of the proposition is that a program with a good static evaluator is always going to have shortcomings: it will always evaluate incorrectly at least some positions. If one programs another static evaluator that evaluates correctly these positions, one will notice soon that there are others that the new program still cannot evaluate correctly. In last analysis the perfect evaluator for chess would have to be an extremely long program, and for more complex systems of this kind, an infinite one. To see it is not possible to program a static evaluator for chess that works correctly on all positions, notice that it would have to evaluate on the basis of the state itself without recourse to the tree. The evaluation of the state has to be done using heuristics, that is, using rules that say how good a state is on the basis of the positions of the pieces and not calculating the tree. But this is not possible if chess is chaotic because then we know the smallest difference between two states leads to completely diverging paths in configuration space, that is, to wholly differing states a few plies later. Therefore the heuristic rules of the static evaluator have to take into account the smallest differences between states, and the evaluators have to be long or infinite routines. Static evaluators, on the other hand, should be short programs, since they have to evaluate the states at the end of each branch of the tree.

Another interesting point is that chaos exacerbates the horizon effect [Berliner, 1973]. This is the problem that occurs in game programming when the computer quits the search in the middle of a critical tactical situation and thus it is likely that the heuristics return an incorrect evaluation [Shannon, 1950]. In a sense, what the proposition is saying is that practically all states are critical, and that the horizon effect is happening all the time and not only for some supposedly very special positions.

## 4 Comments

We have seen that it is likely that the pure strategy paths of chess in configuration space follow chaotic paths. This implies that practical static evaluators must always evaluate incorrectly some of the states. As a result the horizon problem is exacerbated.

The reason why a machine such as Deep Blue is not far, far stronger than a human has to do again with the problem of programming a static evaluator. Even though the machine searches many more plies than the human does, at the end of each branch it has to use a static evaluator that is bound to incorrectly evaluate some states. This adds an element of chance to the calculation. The fact that Deep Blue at present has
a playing strength similar to the best human players tells us that the human mind has a far better static evaluator than Deep Blue (assuming one can apply these terms to the human mind). If chess were not chaotic the overwhelming advantage in ply calculation that the machine has would allow it to play much better than any human could.

In practice, of course, as long as computers keep getting faster and having more memory available, it is always possible to keep improving the static evaluators. If computers can be programmed to learn from their experience they could improve their static evaluators themselves. This was the idea of the program for another game, link-five [Zhou, 1993].

Now, in a general vein, it should be clear why programs that would score high in the IFPUG’s Stringent performance objectives adjustment factor would tend to exhibit chaotic behavior in their configuration spaces. The software of this type of program has to foresee what is going to happen while extremizing certain state variables, as we mentioned before. This kind of problem is equivalent to an autonomous system of differential equations that exhibits chaos, so that the control strategy vector function $g$ of the system is extremely sensitive to the smallest differences in a state $x$. Thus any static evaluator that one programs (that has to be heuristic in nature) is going to be severely limited.

Nevertheless, the consideration we made two paragraphs ago for chess is also true for this kind of problem in general: as long as computers get faster and have more memory the programs can be prepared to deal with more and more situations. Rules of thumb that humans have learned from experience can be added to the evaluators. Alternatively, the programs can be written so that they learn from their experience. But they are always going to be very long programs.

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