Hydrodynamics of the developed pulsating laminar flow in a flat channel in the quasi-stationary region

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Abstract. Abstract. The hydrodynamic characteristics of the developed pulsating laminar flow in a flat channel — the longitudinal component of the velocity vector, the dimensionless pressure gradient, and the shear stress on the wall — were investigated. The calculations were carried out in a quasistationary region, i.e., under relatively low frequencies of oscillations of the cross-section averaged velocity. To obtain data in a quasistationary region, we analysed and used the results of the calculations of a stationary developed flow. The calculations of the system of the stationary motion equations and continuity equations were carried out by the finite difference method. The calculation results turned out to be consistent with the data obtained earlier by other authors. It has been found that at high amplitude of oscillations of velocity averaged over the cross section and exceeding one, coefficients of hydraulic resistance and friction resistance near the channel entrance averaged over the period are higher than these values for a steady flow.

1. Introduction
From the middle of the last century, the question of the influence of flow rate pulsations on the heat transfer has remained open; this problem was studied both experimentally and theoretically by calculation. The contradictory nature of this influence has been explained only relatively recently [1]. It turns out that the Nusselt number $\bar{\text{Nu}}$, averaged over the oscillation period, only for small relative amplitudes of the flow rate oscillations $A < 1$ does not practically change compared with its $\text{Nu}_s$ value in a stationary flow and may even decrease. Near the entrance to the heated section of the channel, there is a slight, up to several percent, maximum ratio $\frac{\bar{\text{Nu}}}{\text{Nu}_s}$. However, this maximum increases markedly with an increase in the amplitude of oscillations $A > 1$.

Obtaining correct results for the numerical solution of the heat exchange problem with a pulsating flow with $A > 1$ is possible only with special formulation of the boundary conditions. Before the heated area it is necessary to place an adiabatic (unheated) area. The length of this area should be several heat wave lengths, and then its presence will not affect the calculation results in the heated area. The heat wavelength is inversely proportional to the oscillation frequency, therefore, in the quasistationary region of low-frequency oscillations, performing calculations with a pre-included adiabatic section is not possible. In this case, you can use the technique applied in [2] – to attract data for a stationary flow.
The calculations in [1], [2] were performed for a developed flow. At the initial hydrodynamic section with a pulsating flow, data on hydrodynamics and heat transfer are absent. Since heat exchange is largely determined by the hydrodynamics of the flow, first of all it is advisable to consider the problem of a pulsating flow in the initial hydrodynamic section.

In [3], the length of the section of hydrodynamic stabilization with a pulsating flow in a quasistationary region was investigated. The dependences for the stationary flow are used in the literature. In [3], the dependence for the length of the initial hydrodynamic section on the Reynolds number was obtained for the condition when a uniform velocity profile is specified at the input and the no-slip on the wall condition is satisfied. In [4], a similar dependence was proposed for a different formulation of boundary conditions on the wall. At the entrance to the channel, a uniform velocity profile was set, but the boundary condition of no-slip was set only at a certain distance from the entrance. It is believed that in the area adjacent to the entrance, the wall is the streamline. In the latter case, there is a certain analogy between the formulation of hydrodynamic and thermal problems, when there is a pre-included adiabatic segment in front of the heated section. In [5], it was established that the effect of the two specified problem statements on the length of the initial hydrodynamic section is manifested at low Reynolds numbers and does not exceed a few percent. Therefore, in this work, a uniform longitudinal velocity profile is specified at the channel input.

2. Problem statement and numerical method

The system of equations of motion and continuity for a stationary flow in a flat channel was solved in the approximation of a narrow channel. We present the system in a dimensionless form:

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = P_0 + 4 \frac{\partial^2 U}{\partial Y^2},
\]

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0.
\]

Here \( X = x / (h \text{Re}) \), \( x \) is the longitudinal coordinate, \( h \) is the width of the channel, \( \text{Re} = \frac{\bar{u} h}{\nu} \) is the Reynolds number, \( \bar{u} \) is the velocity averaged over time and cross section; \( Y = y / (h / 2) \) is the transverse coordinate; \( U = u / \bar{u} \), \( u \) is the longitudinal component of the velocity vector, \( V = 2 \text{Re} \frac{v}{\bar{u}} \), \( v \) is the transverse component of velocity, \( P_0 = \frac{h^2 dp}{\mu dx} \) is the Poiseuille number (dimensionless pressure gradient).

The boundary conditions for the equations are as follows. A uniform velocity profile is set at the input \( X = 0 \): \( U = 1, V = 0 \). The no-slip and impermeability conditions are set on the wall \( Y = 0 \): \( U = V = 0 \). The symmetry condition is fulfilled on the channel axis \( Y = 1 \): \( \partial U / \partial Y = 0 \).

Initially, calculations were carried out for a stationary flow. The system of equations (1) – (2) was solved by the finite difference method. The dimensionless pressure gradient was found by the splitting method with using the integral of the continuity equation \( \int_0^1 UdY = 1 \). The motion equation balance was under control, and imbalance did not exceed a few percent.

The computationally obtained change of the velocity profile along the length coincides with the data obtained earlier in the initial hydrodynamic part of the flat channel, for example, in [6]. As a result of the calculation, the following dependences have been obtained for the stationary flow \( U = U_s(X, Y), P_0 = P_{0s}(X) \). To find the dimensionless shear stress on the wall, the computationally obtained velocity profile was used. By analogy with the Poiseuille number for
pressure, we introduce the Poiseuille number for the shear stress on the wall \( \tau = \frac{-2\tau_c h}{\mu <u> \mu} \) is the shear stress on the wall).

When obtaining the characteristics of a quasistationary flow from the data for a stationary flow, the concept of quasistationarity is used, in which the flow characteristics at each moment of time correspond to the value of the Reynolds number at that moment. For a pulsating flow, the Reynolds number and the velocity average over the cross section vary in time according to the harmonic law
\[
< U > = 1 + A \cos(\omega t),
\]
where \( A \) is the amplitude of oscillations, \( \omega \) is the circular frequency, and \( t \) is the time.

The velocity value and pressure gradient value at each time point are determined by the Reynolds number at that moment. Then the correct equations are as follows:
\[
\frac{u}{<u>(\omega)} = U, \left[ X < U > (\omega), Y \right],
\]
\[
U = \frac{u}{<u>(\omega)} = U, \left[ X < U > (\omega), Y \right] < U > (\omega),
\]
\[
\tau_0 = \tau_0, \left[ X < U > (\omega) \right]< U > (\omega),
\]
\[
\tau_0 = \tau_0, \left[ X < U > (\omega) \right]< U > (\omega).
\]

3. Results of calculations
The results of the calculations of the above functions depend on the amplitude of oscillations of velocity averaged over the cross section, which varied in the range of \( A = 0.5 \div 5 \). For the amplitude of oscillations exceeding unity, in a certain interval of the oscillation period there is a return flow of fluid moving in the negative direction of the axis \( X \). This flow is believed to be the developed one. An estimate of the channel length necessary to stabilize the flow is given in [3].

\[
\text{Figure 1. Velocity profiles diagrams. } a - A = 0.5, \ b - 5. \ I - X = 0; \ X = 0.00145 \ (a), \ X = 0.00526 \ (b); 2 - U (\omega), \ 3 - U, \ 4 - U; \ X = 0.075 \ (a), \ X = 0.3 \ (b); 5 - U (\omega); I - \omega = 0, \ II - \omega = \pi / 2, \ III - \omega = \pi.
\]
Fig 1 shows the change in time period for the longitudinal velocity profile for two cross sections: near the input and in the development section. Calculations are presented for two amplitudes of oscillations of velocity averaged over a cross section – relatively small and much higher than one. For both amplitudes far from the entrance outside the development section the oscillations are harmonic and correspond to the oscillations of velocity averaged over the cross section. The velocity profile averaged over the oscillation period \( \bar{U}(Y) \) coincides with the velocity profile under the stationary flow \( U_s(Y) = 1.5Y(2-Y) \).

Near the input, the oscillations are not harmonic, but at small values of \( A \) they are almost imperceptible. Under \( A < 1 \) the value of the velocity averaged over the period of oscillations near the wall slightly differs from the values in the case of stationary flow, and under \( A > 1 \) this difference becomes more noticeable. In this case, at small distances from the entrance, the developed flow exists only in the initial phase of oscillations.

Under \( A > 1 \) near the wall, the amplitude of oscillations in the maximum \( (U - \sqrt{1+A\cos \omega t}) \) is lower than that in the minimum \( (U-1+A\cos \omega t) \). This is due to the fact that in a certain part of the period the section averaged velocity decreases to zero and changes its direction; the flow is the developed one.

Fig. 2 shows the Poiseuille number for a developed flow (near the entrance to the channel). These oscillations are not harmonic, especially when the amplitudes of oscillations exceed unity. This fact is explained as follows. For a stationary developed flow, the dependence of friction resistance on the longitudinal coordinate \( x \) is similar to this dependence for a flow past a flat surface.

![Figure 2](image.png)

**Figure 2.** The dimensionless share stress changes on the wall over time period. \( a = A = 0.5, \) \( X = 0.00145, b = 5, X = 0.00526. I - \frac{Po_t}{Po_r}, 2 - \frac{Po_t}{Po_{\infty}}, 3 - \frac{Po_{rs}}{Po_{\infty}}, 4 - \frac{<U>}{\sqrt{x/\mu_{\infty}}} \).

In the latter case the friction resistance coefficient \( C_f = \frac{\tau_c}{\rho u_{\infty}^2/2} \sim 1/\sqrt{x/\mu_{\infty}} \) and shear stress on the wall \( \tau_c \sim \frac{u_{\infty}}{\sqrt{x/\mu_{\infty}}} \). Then for the developed flow near the input to the channel \( \tau_c \sim \frac{<U>}{\sqrt{x/\mu_{\infty}}} \).
and \( P_{o_{\tau}} \sim \frac{\tau_{\tau}}{\langle u \rangle} \sim \frac{\langle u \rangle > (1 + A \cos \omega \omega)}{\langle u \rangle > \sqrt{X (1 + A \cos \omega \omega)}} = \sqrt{1 + A \cos \omega \omega} \). Maximum amplitude of this function (under \( \omega \omega = 0 \)) is approximately equal to \( \sqrt{1 + A - 1} \) and is less than the amplitude of harmonic oscillations \( A \). Fig. 2 shows, that under \( A < 1 \) the Poiseuille number value averaged over the oscillation period \( \overline{P_{o_{\tau}}} \) coincides with its value at a stationary value \( P_{o_s} \) (lines 2 and 3 are slightly different from each other). At \( A > 1 \) \( \overline{P_{o_{\tau}}} > P_{o_s} \). Both for the stationary and for quasistationary developed flows far from the input values \( P_{o_{s_{\tau}}} = P_{o_{s_{m}}} = 12 \).

Fig. 3 shows the distribution of the Poiseuille number along the length of the channel. The maximum value of \( \overline{P_{o_{\tau}}} (x) \) is achieved in the phase of the maximum velocity averaged over the cross section (at \( \omega \omega = 0 \)), and the minimum is reached in the phase of the minimum one \( \langle U \rangle \) (at \( \omega \omega = \pi \)).

The Poiseuille number \( \overline{P_{o_{\tau}}} \) averaged over the oscillation period at \( A < 1 \) practically does not differ from its value with a stationary flow \( P_{o_{s_{\tau}}} \). When \( A > 1 \) \( \overline{P_{o_{\tau}}} \) is noticeably higher than \( P_{o_{s_{\tau}}} \): about two times for \( A = 5 \). The Poiseuille number in the phase \( \omega \omega = \pi / 2 \) is near to this number at the stationary flow (lines 2 and 5 are slightly different).

![Figure 3](image)

**Figure 3.** The dimensionless shear stress changes on the wall along the channel length. \( a - A = 0.5, \quad b - 5, \quad 1 - \omega \omega = 0, \quad 2 - \omega \omega = \pi / 2, \quad 3 - \omega \omega = \pi, \quad 4 - \overline{P_{o_{\tau}}} / P_{o_{s}}, \quad 5 - P_{o_{s_{\tau}}} / P_{o_{s}}. \)

Calculations have shown that changes in the Poiseuille number for pressure and shear stress are qualitatively similar, but for a developed flow \( P_{o} > P_{o_{\tau}} \) is due to the contribution into the pressure gradient of the convective terms of the motion equation. Under \( A = 5 \) near channel input the ratio \( \overline{P_{o}} / P_{o_{s}} = 3.5 \).

**4. Conclusion**

Calculations of the hydrodynamics process with a pulsating laminar developed flow have shown that the results of calculations of the hydrodynamic characteristics of the flow are significantly influenced by the amplitude of oscillations of velocity averaged over the cross section. Fluctuations of hydrodynamic quantities at a quasi-stationary developed flow are not harmonic. The amplitudes of
these oscillations are less than the amplitude of oscillations of velocity averaged over cross-section \( A \). However, this difference appears only at high values of \( A \).

Under \( A < 1 \) all the hydrodynamic characteristics averaged in time period — the longitudinal velocity, shear stress on the wall, the pressure gradient — slightly differ from their values at a stationary value, which is also observed for a developed flow.

Under \( A > 1 \) near the channel input there is a significant increase in the values of these characteristics compared to those in a stationary flow. For example, for \( A = 5 \) the Poiseuille number ratio for pressure gradient and shear stress to these values at stationary flow \( \frac{\overline{\text{Po}}}{\overline{\text{Po}}_s} \approx 3.5 \) and \( \frac{\overline{\tau}}{\overline{\tau}_s} \approx 2.4 \). This increase is due to an increased length of the input development section in the initial phase of changes in velocity averaged over the cross section.

Since, for both stationary and quasi-stationary flows, there is the Reynolds analogy, we should expect that the period averaged Nusselt number \( \overline{\text{Nu}} \) for the developed flow will also exceed its value for a stationary flow \( \text{Nu}_s \). This conclusion is important for practice. In many cases, in order to increase the transferred thermal power, additional power is required to pump the coolant. It should be noted that the calculations performed in [2] at the initial thermal section of the developed flow showed that the ratio \( \overline{\text{Nu}} / \text{Nu}_s \) near the input to the flat channel does not exceed 1.15 (for \( A = 5 \)).

In the future, it is supposed to carry out calculations of heat transfer in a developed pulsating flow on the basis of the same approach that was used in this work, i.e. using data for the stationary flow.

References
[1] Valueva E P, Purdin M S 2018 Hydrodynamics and heat transfer for large amplitude pulsating laminar flow in channels Thermophysics and Aeromechanics vol 25 N 5 p 705
[2] Valueva E P, Purdin M S 2016 Heat exchange at laminar flow in rectangular channels Thermophysics and Aeromechanics vol 23 N 6 p 857
[3] Valueva E P, Zyukin V S 2019 Influence of axial thermal conductivity of the wall on the temperature regime and efficiency of heat exchangers with parallel movement of heat carriers Proceedings of the School-seminar under the guidance of acad. A. I. Leontiev «Problems of gas dynamics and heat and mass transfer in power plants» ISBN 978-5-6042605-6-2 pp 66-69
[4] Durst F, Ray S, Ünsal B, Bayoumi O A 2005 The development lengths of laminar pipe and channel flows ASME J. Fluids Eng. vol 127 pp 1154-1160
[5] Joshi Y, Vinoth B R 2018 Entry lengths of laminar pipe and channel flows J. Fluids Eng. vol 140 N 6 pp 061203-061203-8
[6] Bodoia J R, Osterle J F 1961 Finite difference analysis of plane Poiseuille and Coutte flow developments Appl. Sci. Res. Vol 10 p 265