Bound-bound pair production in relativistic collisions

A.B. Voitkiv, B. Najjari and A. Di Piazza

Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, D-69117 Heidelberg, Germany

Electron-positron pair production is considered in the relativistic collision of a nucleus and an anti-nucleus, in which both leptons are created in bound states of the corresponding nucleus-lepton system. Compared to free and bound-free pair production this process is shown to display a qualitatively different dependency both on the impact energy and charged of the colliding particles. Interestingly, at high impact energies the cross section for this process is found to be larger than that for the analogous atomic process of non-radiative electron capture although the latter does not involve the creation of new particles.

One of the most fascinating predictions of quantum electrodynamics is the possibility of converting energy into matter. Starting with the paper by Sauter [1] electron-positron pair production from vacuum due to the presence of external electromagnetic fields has been attracting the attention of different physical communities.

Pair production has been studied theoretically in the presence of electromagnetic fields of various configurations (e. g., in the combination of Coulomb and high-energy photon fields [2], in high-energy collisions of charged particles [3], in constant and uniform fields [5], in slowly varying super-strong Coulomb fields [6], in colliding laser fields [7], in crystals [8]), and also in the presence of gravitational fields [9].

Pair production can occur with noticeable probabilities (i) if the external field is strong enough to provide an energy of the order of the electron rest energy $mc^2$ on a distance of the order of the electron Compton wavelength $\lambda_C = \hbar/mc$ [2], where $\hbar$ is the Planck’s constant, (ii) or/and if the field varies in time so rapidly that its typical frequencies multiplied by $\hbar$ are (at least) of the order of $2mc^2$.

Experimentally pair production has been explored only in the case of rapidly varying electromagnetic fields (for instance, in relativistic heavy-ion collisions, photon-laser collisions [10], in the collision of an intense laser beam with a solid target [11]).

Landau and Lifshitz [4] were the first to estimate the cross section for pair production in relativistic collisions of charged particles in which the created electron and positron freely move in space after the collision is over (see figure 1b). Such a process is termed free pair production and it was studied in much detail in a vast amount of theoretical and experimental papers (see for recent reviews e.g. [12] and also references therein).

During the last two decades another kind of pair production process occurring in relativistic nuclear collisions has attracted much attention (see e.g. [13]-[18] and references therein). In contrast to free pair production, in this process the electron is created in a bound state with one of the colliding nuclei (see figure 1c).

When the colliding nuclei possess charges of different signs, yet another pair production process becomes possible in which not only the electron but also the positron are created in a bound state (see figure 1c). Below we shall call this process bound-bound pair production. Compared to the free and bound-free cases, bound-bound pair production is expected to have a number of interesting features; in particular, it has an intrinsic non-perturbative dependence on charges of both colliding nuclei. This, as well as the fact that this process completes the picture of the basic (single-) pair production processes occurring in high-energy collisions of charged particles, makes its study of great interest. To our knowledge, bound-bound pair production has not yet been considered in the literature and it is the goal of this Letter to investigate this process.

Let us consider the collision of two nuclei with charges $Z_1$ and $Z_2$ (say $Z_1 > 0$ and $Z_2 < 0$) [19]. Impact parameter values characteristic for this process are of the order of $\lambda_C$ and, thus, are much larger than the nuclear size. Therefore, one can treat the nuclei as point-like particles. Since we consider high impact energies one can also assume that the initial velocities of these particles are not...
changed in the collision.

Our consideration of the bound-bound pair production will be based on the semi-classical approximation in which only the light particles (electron and positron) are considered using quantum theory while the heavy charges \( Z_1 \) and \( Z_2 \) are regarded as classical particles. We shall employ the rest frame of the charge \( Z_1 \) as our reference frame. We take the position of this charge as the origin and assume that in this frame the charge \( Z_2 \) moves along a straight-line classical trajectory \( \mathbf{R}(t) = \mathbf{b} + \mathbf{v}t \), where \( \mathbf{b} = (b_x, b_y, 0) \) is the impact parameter, \( \mathbf{v} = (0, 0, v) \) is the collision velocity and \( t \) is the time.

In order to obtain first results on bound-bound pair production it suffices to employ a theoretical approach which is based on the simplest form of the transition amplitude for this process given by

\[
a_{bb}(\mathbf{b}) = -i \int_{-\infty}^{+\infty} dt \int d^3 \mathbf{r} \psi_f(\mathbf{r}, t) \tilde{W}(\mathbf{r}, t) \psi_i(\mathbf{r}, t). \tag{1}
\]

In this expression \( \psi_i \) is the state of the negative-energy electron bound in the field of the charge \( Z_2 \), \( \psi_f \) is the state of the electron bound in the field of the charge \( Z_1 \), \( \mathbf{r} = (r_x, r_y, z) \) is the lepton coordinate and \( \tilde{W} \) is the interaction. The latter can be chosen either as the interaction with the field of the charge \( Z_1 \),

\[
\tilde{W} = -\frac{Z_1}{r}, \tag{2}
\]

or with the field of the charge \( Z_2 \),

\[
\tilde{W} = -\frac{\gamma Z_2}{s} \left( 1 - \frac{v}{c} \alpha_z \right), \tag{3}
\]

where \( \gamma = 1/\sqrt{1 - v^2/c^2} \), \( s = |\mathbf{s}| \), \( \mathbf{s} = (\mathbf{r}_\perp - \mathbf{b}; \gamma(z - vt)) \) and \( \alpha_z \) is the Dirac matrix. In what follows we shall take the interaction in the simpler form (2) but one can easily show that both (2) and (3) yield identical results for the amplitude (1).

In the reference frame chosen the initial and final states read

\[
\psi_i = \sqrt{\frac{1 + \gamma}{2}} \left( 1 + \frac{\gamma}{c} \frac{1 + \gamma}{1 + \gamma} \alpha_z \right) \chi_i(s) \exp(i \varepsilon_p \gamma(t - vz/c^2))
\]

\[
\psi_f = \varphi_f(x) \exp(-i \varepsilon_e t). \tag{4}
\]

In (4) \( \chi_i \) is the initial negative-energy state, \( \varepsilon_p = mc^2 - I_p \) is the total energy of the positron where \( I_p \) is its binding energy; both these quantities are given in the rest frame of the charge \( Z_2 \). Further, \( \varphi_f \) is the bound state of the electron and \( \varepsilon_e = mc^2 - I_e \) is its total energy with \( I_e \) being the binding energy.

The amplitude (1) is written in the impact-parameter space. However, it is more convenient to calculate the cross section using the transition amplitude written in the momentum space,

\[
S_{bb}(\mathbf{q}_\perp) = \frac{1}{2\pi} \int d^2 \mathbf{b} \ a_{bb}(\mathbf{b}) \ \exp(i \mathbf{q}_\perp \cdot \mathbf{b}). \tag{5}
\]

Using Eqs. (1), (2), (4) and (5) we obtain

\[
S_{bb}(\mathbf{q}_\perp) = \frac{i Z_1}{2\pi v c} \sqrt{\frac{1 + \gamma}{2}} \int d^3 \mathbf{r} \varphi_f^*(\mathbf{r}) \frac{1}{r} \exp(iq \cdot \mathbf{r}) \times \left( 1 + \frac{\gamma}{c} \frac{1 + \gamma}{1 + \gamma} \alpha_z \right) \int d^3 \mathbf{s} \chi_i(\mathbf{s}) \exp(-i \mathbf{q}' \cdot \mathbf{s}). \tag{6}
\]

The quantities \( \mathbf{q} \) and \( \mathbf{q}' \) have the meaning of the momentum transfer as viewed in the rest frames of the charges \( Z_1 \) and \( Z_2 \), respectively, and are given by

\[
\mathbf{q} = \left( q_\perp, \frac{mc^2 - I_e + (mc^2 - I_p)/\gamma}{v} \right), \quad \mathbf{q}' = \left( q_\perp, \frac{mc^2 - I_p + (mc^2 - I_e)/\gamma}{v} \right). \tag{7}
\]

The total cross section for the bound-bound pair production reads

\[
\sigma_{bb} = \int d^2 \mathbf{q}_\perp \ | S_{bb}(\mathbf{q}_\perp) |^2. \tag{8}
\]

It follows from Eqs. (6)-(7) that at asymptotically high collision energies the only dependence of the amplitude \( S_{bb} \) on the collision energy is given by the factor \( 1/\sqrt{\gamma} \). Besides, taking into account the form of the bound states one can show that this amplitude is roughly proportional to \( Z_1^{5/2} Z_2^{5/2} \). Correspondingly, we obtain that the asymptotic form of the cross section for the bound-bound pair production is given by

\[
\sigma_{bb} \sim \frac{Z_1^5 Z_2^5}{\gamma}. \tag{9}
\]

This dependence is significantly different from the corresponding ones in the case of free and bound-free pair productions which read \( \sigma_f \sim Z_1^2 Z_2^2 \log^2(\gamma) \) and \( \sigma_{ff} \sim Z_1^3 Z_2^3 \log(\gamma) \), respectively (see e.g. [12, 17]). Note that in \( \sigma_{bb} \) \( Z_1 \) is the charge of the nucleus carrying away the created electron.

In figure 2 we show the cross section for the reaction \( p^- + U^{92+} \rightarrow \pi^-(1s) + U^{91+}(1s) \) (solid curve). The dependence of the cross section on the impact energy is not monotonomous. At the relatively low collision energies the cross section increases with the energy reaching a maximum at about 5-7 GeV/u. With a further energy increase the cross section starts to decrease with an increasing slope and reaches its asymptotic energy dependence \( \sim 1/\gamma \) already within the energy interval displayed in the figure.

The cross section for the bound-bound pair production can be compared with that for the bound-free pair creation. We have calculated the cross section for the latter
process (shown in figure 2 by dash curve) treating it as a transition between the negative- and positive-energy Coulomb states centered on the antiproton which is induced by the interaction with the field of the charge $Z_1$ taken into account in the lowest order perturbation theory.

At relatively low impact energies both cross sections increase and are rather close in magnitude to each other. However, at larger impact energies the cross sections start to demonstrate qualitatively different behaviours and the difference in the magnitude between them increases very rapidly.

Such an interrelation between these cross sections can be understood by noting the following. At very low collision energies the spectrum of the electromagnetic field generated by the colliding particles does not have enough high-frequency components necessary to create an electron-positron pair. As a result, the cross sections for both pair production processes are very small. An increase in the impact energy leads to an increase of the high-frequency component of the field and both cross sections grow rather rapidly. However, when the impact energy increases further the conditions for the bound-bound pair production begin to deteriorate. Indeed, the electron and positron are created on different nuclei and, therefore, the difference between their momenta increases with the impact energy. This reduces the overlap between the states $\psi_i$ and $\psi_f$ making bound-bound pair production more difficult to occur.

This, of course, does not occur in bound-free pair production since both the leptons are created on/around the same nucleus. In this case when the impact energy grows the range of the impact parameters efficiently contributing to the process grows as well ($\sim \gamma$) leading to the logarithmic increase in the bound-free pair production cross section.

Bound-bound pair production can be viewed as a collision-induced transition between states of the electron with negative and positive total energies bound in the field of the charge $Z_2 < 0$ and charge $Z_1 > 0$, respectively. This is reminiscent of the atomic collision process of non-radiative electron capture (for a review see e.g. [17]–[18]) in which an electron initially bound in the atom undergoes a transition into a bound state in the ion: $(Z_a + e^-) \rightarrow Z_a + (Z_i + e^-)$, where $Z_a$ and $Z_i$ are the charges of the atomic and ionic nuclei.

Indeed, within the simplest description of non-radiative capture its amplitude is given by Eq. (1) in which $\psi_i$ and $\psi_f$ are now the states of the electron bound in the atom and ion, respectively, and, therefore, it is of interest to compare the cross sections for these two processes.

Such a comparison is presented in figure 2 where dot curve shows twice the cross section for the reaction $H(1s) + U^{92+} \rightarrow p^+ + U^{91+}(1s)$ calculated using the simplest description mentioned above. At relatively low and intermediate collision energies, where the electron capture is much more probable than the bound-bound pair production, the two cross sections show a qualitatively different behaviour. However, at higher impact energies the cross sections approach each other, cross and, when the energy increases further, demonstrate exactly the same energy dependence with the bound-bound pair production cross section being a factor of 2 larger.

The factor of 2 is of statistical origin reflecting the difference between the averaging over the projections of the angular momentum in the initial state in the case of electron capture and the corresponding summation in the case of pair production. Apart from it the cross sections are identical. This circumstance can be understood by taking into account the symmetry between these two processes and observing that at $\gamma \gg 1$ the absolute values of the momentum transfers in both processes become essentially the same. Thus, at asymptotically high impact energies it would be easier to capture electron and positron from vacuum into the corresponding bound states in the collision with an antiproton than to pick up the already existing electron from the atomic hydrogen.

In order to illustrate the very strong dependence of the bound-bound pair production on the charges of the colliding particles in figure 3 we present the cross section for the (hypothetical) reaction $\overline{p}^{92-} + U^{92+} \rightarrow \overline{p}^{91-}(1s) + U^{91+}(1s)$. It is seen that compared to the case shown in figure 2 the magnitude of the cross section has increased roughly by 10 orders with the dependence on the collision energy remaining basically the same. In figure 3 the bound-bound cross section can also be compared with those for the bound-free pair production in $U^{92+} + U^{92+}$ collisions and for the electron capture in collisions between $U^{90+}(1s^2)$ and $U^{92+}$.

Before we proceed to conclusions, two brief remarks...
atomic processes appear: excitation, ionization and electron capture. The relativistic theory adds up the negative energy states into consideration. This results in the existence of pair production and the corresponding expansion of the group of the basic atomic collision processes to the six. Thus, the bound-bound pair production not only fills in the ‘vacancy’ in the set of the (single-) pair production processes but can also be viewed as completing the whole picture of the basic (single-lepton) atomic processes possible in ion-atom (ion-ion) collisions.

The cross section for bound-bound pair production is very small. Therefore, the detection of this process in laboratory seems to be feasible only provided high-luminosity beams of heavy nuclei and antiprotons (or muons) are available. The possibility to detect bound-bound pair production using the future facilities at GSI (Darmstadt, Germany) is currently under discussion.

A.D.P. thanks C.H. Keitel for helpful discussions.

[1] F. Sauter, Z. Phys. 69, 742 (1931).
[2] The so called critical field whose strength is given by $E_{cr} = m^2 e^4 / 4\pi \epsilon_0 = 1.3 \times 10^{16}$ V/cm where $e$ is the absolute value of the electron charge.
[3] H. A. Bethe and W. Heitler, Proc. Roy. Soc. A 146, 83 (1934).
[4] L.D. Landau and E.M. Lifshitz, Phys. Z. Sowjet. 6 244 (1934)
[5] W. Heisenberg and H. Euler, Z. Phys. 98, 714 (1936).
[6] W. Greiner, B. Müller, and J. Rafelski, Quantum Electrodynamics of Strong Fields, (Springer, Berlin, 1985).
[7] E. Brézin and C. Itzykson, Phys. Rev. D 2, 1191 (1970).
[8] V. N. Baier and V. M. Katkov, Phys. Rep. 409, 261 (2005).
[9] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, (Cambridge University Press, Cambridge, 1984)
[10] D. L. Burke, et al., Phys. Rev. Lett. 79, 1626 (1997).
[11] H. Chen, et al., Phys. Rev. Lett. 102, 105001 (2009).
[12] C.A. Bertulani and G. Baur, Phys. Rep. 163, 299 (1988); G. Baur, K. Hencken and D. Trautmann, J. Phys. G 24, 1637 (1998); G. Baur, K. Hencken, D. Trautmann, Phys.Rep. 453 1 (2007).
[13] R. Anholt and H. Gould, Adv. At. and Mol. Phys. 22 315 (1986).
[14] U. Becker, N. Grün and W. Scheid, J.Phys. B 20 2075 (1987); U. Becker, J.Phys. B 20 6563 (1987).
[15] A. Belkacem, et al, Phys.Rev.Lett. 71 1514 (1993); Phys.Rev.Lett. 73 2432 (1994); Phys.Rev. A 58, 1253 (1998).
[16] H.F. Krause et al, Phys. Rev. Lett. 80, 1190 (1998).
[17] J. Eichler and W. Meyerhof, Relativistic Atomic Collisions, (Academic Press, San Diego, 1995).
[18] D.S.F. Crothers, Relativistic Heavy-Particle Collision Theory (Kluwer Academic/Plenum Publishers, London, 2000).
[19] From now on atomic units are used unless otherwise indicated.