Narrowing down XML Template Expansion and Schema Validation

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I. INTRODUCTION

This section gives an overview of template expansion and schema validation. At the beginning an motivational example is given in order to introduce to the topic of this thesis. The most important existing contributions are presented.

A. Motivation

In XSLT an application developer is often confronted with two basic problems related to XML documents: First, an XSLT-stylesheet needs to be instantiated. Second, a given XML document, which as one part of a more complex XML processing pipeline, needs to be validated against a known XML-schema. The automatic generation of XML-documents can be very diverse and may include numerous and even distributed applications, and so is the validation of such. For example, the instantiation of the following XSLT-stylesheet fragment:

```xml
<xsl:template match="/">
  <foos>
    <xsl:value-of select="/page"/>
    <bar/>
  </foos>
</xsl:template>
```

may be transformed into the next document by corresponding queries to the source document:

```xml
<foos>
  <bar/>
</foos>
```

In order to check validity, a RelaxNG schema may be required, for instance:

```xml
<element name="foos">
  <group>
    <data type="int"/>
    <element name="bar">
      <empty/>
    </element>
  </group>
</element>
```

Hence, apart from an XSLT-stylesheet a XML-schema is required a posteriori, which on the first view does not really look similar to the stylesheet. If only a schema could automatically be derived from a given stylesheet, both processes could be simplified. Even better, a unified view could serve this. Saying this, the total efforts saved is significant, especially when dropping document meta-information of approximately half with quite diverse document structure.

This approach is tracked by the minimalistic template-language XTL [32] used for XML-documents. Within this work instantiator and validator for XTL is implemented. Apart from that the unification in general is investigated. Especially, the following issues are discussed:

- **Formalisation of instantiation and validation.**
  How can both semantics be formalised?

- **Demonstration on examples.**
  How do prototypical implementations look like?

- **Requirements to unification.**
  Does unification disproportionately restrict either template expansion or schema validation? – How can such restrictions be overcome, if any.

- **Comparison of instantiation and validation.**
  Why does a comparison always have to focus on schema validation first?

B. Preparations

This section introduces to basics and related work as well as bordering disciplines.
1) Existing work: RelaxNG-validator.

[17] proposes the algorithm which is actually used currently in validating RelaxNG-documents. Based on Clark’s approach, for instance, Torben Kuseler [44], developed the Haskell-XML-Toolbox [58].

Transformation of regular expressions into an automaton. [63] provides practical instructions for the construction in Haskell for a string recognising determined automaton.

[9] presents the construction of so-called Glushkov-automata, which are stepwise built up by deriving transitions from previously set states.

The paper [4] widens the definitions of a partial derivation in accordance to mathematical analysis and proposes a transition calculation parametrised in comparison to [9].

Tree Automata. In [50] and [49] Makoto Murata introduces the syntax of tree automata applied to XML-documents. Essential terms, particularly tree grammars and languages as well as their properties, are provided.

[16] discusses the transformation of schema documents into regular tree expressions. The construction of tree automata is discussed in the context of database applications.

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[12] is a compendium on techniques on tree automata and its applications.

Comparisons and Field Studies. Murata, Lee and Mani [48] introduce a classification of XML-schema languages. In [46] Lee and Chu classify expressibility for selected schema languages.

Rahm and Bernstein [57] propose a classification of schema-matches.

Others, [2] and [61] both provide an survey on denotational semantics. [62] gives an introduction to Haskell. Additional material on Haskell and functional programming may be found in [55] and [56].

2) Related Work: Template-Engines are introduced in [53] and [52], where moreover [52] also provides a closer look from a Model-View-Controller perspective. Evaluation improvements are discussed in [43] on template expansion and in [8] on schema validation. [11] deals with XML typing in general, and particularly with type isomorphism in Haskell. The overall meaning of polymorphism in terms of XML is discussed in [36].

Lazy evaluation of XML-documents, refactorings of functional programs [21], [45], [64], [13] and monads/arrows [39], [37], [25], [34], [38], [66] are closely related to Haskell. Particularly, [43], [51] deals with lazy parsing of XML-documents, and [6] deals with XML-document updates.

3) Cross-bordering disciplines: Beside the just mentioned topics, this work’s topic (cross-)borders with further disciplines:

- document and schema transformation,
- parsing,
- functional programming and
- XML data binding.
This section introduces to instantiation and validation, defines basic terms with explanatory examples, followed by an introduction to the template-language XTL and some theoretical foundations. The modelling of XML-documents is described based on a tree grammar as well as the translation.

### A. Instantiation

The instantiation of XML-documents is a process, in which a template-engine generates an instance out of a template based on some source language with some instantiation data. The result is called instance and follows constraints of a target language (see fig. 1). At this moment it is agreed templates are present in XML. It is further agreed on instances have to be in well-formed XML format.

A template consists of an arbitrary number of tags and slots, which may also be nested. In general slots have to be unique among all other nodes. Slots bind instantiation data, which may differ in general. Instantiation data may be XML or form a relation. A slot may at most refer to one source. Within an instantiation step common tags are copied to the instance document, where a slot is evaluated by the template-engine. Due to its tree-structure XML-documents are processed usually top-down. So, the instantiation of templates is processed in pre-order (cmp. sect. II-D1). W.l.o.g. it is agreed, that during an instantiation step the access to already instantiated nodes is prohibited due to the transparency of the chosen model (see sect. II-D).

Slots are interfaces. Its purpose is to load data from external repositories into the instance. The loading is controlled by queries which come from the instantiation data. Queries are part of the slots. Neither its syntax nor its semantic have to follow XML or any a priori rules whatsoever. For example, a slot may contain queries in XPath or in SQL. Some source languages allow more precise queries, for instance that the returned value has to be formatted. Settings, e.g. for XTL, are going to be interpreted by Placeholder-Plugins (PHPs); see sect. II-C. This is why every template (e.g. a PHP-site) has to follow previously agreed interfaces which control the access to instantiation data.

In order to distinguish document instantiation from, for instance, object instantiation, the term template expansion is sometimes used. The term “expansion” tries to illustrate something is replaced by something bigger, however, the replacement in fact does not always have to be bigger – in fact, the opposite may actually be true. A slot is replaced by an arbitrary number of nodes. The replacement can also be interpreted as reduction rather than expansion, if the node(s) to be inserted are in total smaller than the slot. Hence, it is agreed, that both, instantiation and template expansion may be used as synonyms.

For demonstration purpose the instantiation of the following XSLT-stylesheet with XPath as embedded query language is shown:

```xml
<?xml version="1.0" encoding="UTF-8"?>
<xsl:stylesheet xmlns:xsl="http://www.w3.org/1999/XSL/Transform" version="1.0">
  <xsl:template match="/">
    <title>@title</title>
    <bibliography>
      <book author="Simon Thompson" title="Haskell - The Craft ...">
        <publications>
          <title>...</title>
          <magazin title="Informatik-Spektrum"]/>
        </publications>
      </book>
      <book author="Joshua Kerievsky" title="Refactoring to Pa ">
        <url title="XSD specification 1.0"/>
      </book>
    </bibliography>
  </xsl:template>
</xsl:stylesheet>
```

In line 6 of the stylesheet the nodes from lines 2 and 4 match. Each of these nodes is successively considered as source document itself (see lines 7-9). While processing, first, the template element node <publications/> is copied, and second the instantiation of its child nodes is continued.

```xml
<book author="Joshua Kerievsky" title="Refactoring to Pa ">
  <url title="XSD specification 1.0"/>
</book>
```

This template generates (once applied with appropriate instantiation data) for each `<book/>-node `<title>`-nodes with matching titles as text in its children nodes directly underneath `<publications/>`. Line 4 matches against the top-level element node. This means the whole document is considered. In line 6 all `<book/>-nodes starting from the top of the document are determined. Now suppose the associated source document is:

```xml
<book author="Simon Thompson" title="Haskell - The Craft ...">
  <publications>
    <title>@title</title>
    <bibliography>
      <book author="Simon Thompson" title="Haskell - The Craft ...">
        <publications>
          <title>...</title>
          <magazin title="Informatik-Spektrum"]/>
        </publications>
      </book>
      <book author="Joshua Kerievsky" title="Refactoring to Pa ">
        <url title="XSD specification 1.0"/>
      </book>
    </bibliography>
  </publications>
</book>
```

```xml
<book author="Joshua Kerievsky" title="Refactoring to Pa ">
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</book>
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  <publications>
    <title>@title</title>
    <bibliography>
      <book author="Simon Thompson" title="Haskell - The Craft ...">
        <publications>
          <title>...</title>
          <magazin title="Informatik-Spektrum"]/>
        </publications>
      </book>
      <book author="Joshua Kerievsky" title="Refactoring to Pa ">
        <url title="XSD specification 1.0"/>
      </book>
    </bibliography>
  </publications>
</book>
```

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```xml
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  <url title="XSD specification 1.0"/>
</book>
```

In order to distinguish document instantiation from, for instance, object instantiation, the term template expansion is sometimes used. The term "expansion" tries to illustrate something is replaced by something bigger, however, the replacement in fact does not always have to be bigger – in fact, the opposite may actually be true. A slot is replaced by an arbitrary number of nodes. The replacement can also be interpreted as reduction rather than expansion, if the node(s) to be inserted are in total smaller than the slot. Hence, it is agreed, that both, instantiation and template expansion may be used as synonyms.
In a second step the remaining set \( C_2 = [\text{book title="Refactoring..." ...}] \) is applied to the nested loop. The next node results:

\[
\begin{align*}
\text{<publications>}
\begin{cases}
\text{<title>Haskell...</title>}
\text{<title>Refactoring...</title>}
\text{<xsl:for-each select="//book">}
\text{<title>}
\text{<xsl:value-of select="@title"/>}
\text{</title>}
\text{</xsl:for-each>}
\end{cases}
\text{</publications>}
\end{align*}
\]

with \( C_3 = \emptyset \) as the remaining empty set. The loop condition does not hold anymore for the instantiation of loops in XSLT (cmp. [19], [67]). So, the nested loop is skipped. Since no more following nodes exist, the instantiation terminates and returns this node:

\[
\begin{align*}
\text{<publications>}
\begin{cases}
\text{<title>Haskell - The Craft ...</title>}
\text{<title>Refactoring to Patterns</title>}
\end{cases}
\text{</publications>}
\end{align*}
\]

B. Validation

Validation checks if a given XML-document is instance of a template (see fig.1). If the answer is "yes", then the given document is "valid", otherwise not. Templates generate instances, which may be described by schemas. That means template and schema de facto describe one and the same instance document. If we try to unify both processes, then both descriptions also need to be unified somehow. Otherwise, not the same template or corresponding schema may be expressed. One big problem in doing so is the fact, that both, syntax and semantic differ in both cases. The template-engine would need to be reconfigured, s.t. documents are not expanded, but schema-validated. In practise, this means validation is triggered rather than a template-engine.

The set of all valid instances of a template is described by schema. Schemas often are tagged and as such are XML-dialects. Represents of such are RelaxNG [20] and W3C's XSD [28], [60]. \( DTD \) represents a schema language for XML-documents, but itself it is not XML. Schema languages in XML can be processed the same way templates are processed (cmp. sect[III]).

XML-schemas consist of literals. Literals are childless element and text nodes as well as composed element nodes. Composed element nodes have at least one child node and any number of commando tags in arbitrary order as children. Commando-tags denote either a loop or a selection, and may be decorated with constraints.

Different to instantiation, validation does not consider instantiation data. If instantiation data was complete, an instantiation could be performed. In that case validation would be equal to checking equality of the given instance document with the instantiated document. If, however, instantiation was not complete, then conclusions would not always be right. In fact, validation could then be reduced to the problem of document reconstruction [31]. In practise, however, document reconstruction would induce too many hard restrictions, s.t. instantiation of a template would have to be invertible, for instance. That is why it is better not to make any assumption about the instantiation data instead.

Validation of instances may be considered as word problem (cmp. [16], [50]). In that case the instance documents acts as word and the template document as grammar which generates the set of all valid instance documents. In case the template contains cycles, the set of all valid instances becomes infinite, and instantiation becomes one possible derivation for a grammar, which is the template document. If the document is interpreted as term, the instance document represents a normalform in case all slots are reduced until only element and text nodes remain.

C. XTL

Until now schemas could not (satisfactorily) be represented in the language of templates. A schema formulated in a template language, if at all possible, is extraordinary difficult to read or it is too complex, or even both. That is why XSLT-templates require many helper functions and rewritings. Because of the properties an instance has to have, schemas can barely be automatically be inferred (cmp. sect[VI]). Despite that disadvantage schema languages are still being used that do not unify smoothly with templates. As already mentioned the differentiation between template and schema languages is the reason for huge increase on maintenance and heterogeneous program systems. Another disadvantage may be additional efforts in learning new schema languages.

That is why the following points are required in order to unify template and schema documents:

- keep schemas short and simple
- each element in the template shall correspond with a similar element in the schema, and vice versa.

The XML-template language XTL was defined as part of the SNOW-project [59]. One of its goals was to investigate, if tractable at all, whether documents may be reused for template expansion and schema validation. XTL in the version 1.0 currently counts 7 intrinsic tags to be explained in more detail next. The comparison between template and schema nodes is taken out in sect[VI].

\[\text{xtl:attribute}\]

The insertion of new attributes can be expressed like this:

\[
\begin{align*}
\text{<xtl:attribute name="name" select="expression"/>}
\end{align*}
\]

The tag does not have children and is under an element node [52]. Both, 'name' and 'select', have to be specified
as attributes. Attributes always have to be under an element node. In case the same attribute name has already been defined, the attribute is replaced by the new value \texttt{expression}. The definition order of attributes is arbitrary and still defines the same element node. W.l.o.g. it is agreed that attributes are in \textit{canonised} form. This means attribute assignment pairs are ordered ascending by the attribute name in lexicographic order. Attributes reference instantiation data by \texttt{select}. In case of JXPath as template-engine and PHP as template language, \texttt{expression} denotes a path expression.

\begin{verbatim}
<book id="1">
  <xtl:attribute name="author" select="/book[position()==1]/@author"/>
  <xtl:attribute name="id" select="999"/>
</book>
\end{verbatim}

For this example, sect.II-A as instantiation data and JXPath as PHP, the node \texttt{<book author="Simon Thompson" id="999"/>} is instantiated. The first attribute definition inserts a new entry to the attributes list. Since XPath maps integers on themselves \cite{11}, the new attribute’s value is "999".

\texttt{xtl:text}

The childless tag for text inclusion is:

\begin{verbatim}
<xtl:text select="expression"/>
\end{verbatim}

During instantiation, \texttt{expression} is passed to the placeholder-plugin, which will handle it further, for instance, as XPath-expression. The resulting nodes list is converted by an implicit coercion in XPath \cite{11} into a string, which finally is concatenated. The tag for XTL-text-inclusion is replaced by the concatenated text.

The expansion of two neighbouring 'xtl:text' nodes as children, is of interest, especially for validation. This interest comes from the question how to split a common string best when there are no markers that indicate boundaries. This is why the separation may become ambiguous. Hence, strategies are wanted, which allow to uniquely recognise 'xtl:text' nodes. In contrast to this 'xtl:attribute'-nodes do not have this problem.

\begin{verbatim}
<xtl:text select="/"/>
\end{verbatim}

instantiates the empty string for the example from sect.II-A because the source document is traversed in pre-order and occurrences of text nodes are accumulated and concatenated.

\texttt{xtl:include}

The inclusion of an arbitrary element node enriched by instantiation data can be achieved by the following childless tag:

\begin{verbatim}
<xtl:include select="expression"/>
\end{verbatim}

The PHP returns either one well-formed element node or none. If multiple nodes match with \texttt{expression}, then PHP chooses only the first occurrence and drops all others \cite{32}.

The attribute \texttt{expression} equals \texttt{"//url"} returns for the example from sect.II-A the element node:

\begin{verbatim}
<url title="XSD specification 1.0"/>
\end{verbatim}

\texttt{xtl:if}

Conditions in XTL have this form:

\begin{verbatim}
<xtl:if select="expression">...</xtl:if>
\end{verbatim}

If \texttt{expression} evaluates to "'true'", then the evaluation continues with its children '...'. Otherwise, the children nodes of 'xtl:if' are dropped and evaluation proceeds with the following siblings.

An example determining the second book of a bibliography, if any, looks like this:

\begin{verbatim}
<xtl:if select="/book[position()==2]">
  <xtl:include select="/book/>
</xtl:if>
\end{verbatim}

For the element node \texttt{<bibliography/>} from sect.II-A it retrieves the following node:

\begin{verbatim}
<book author="Joshua Kerievsky" title="Refactoring to Patterns"/>
\end{verbatim}

\texttt{xtl:for-each}

The tag for cycles is:

\begin{verbatim}
<xtl:for-each select="expression">...<xtl:for-each>
\end{verbatim}

The evaluation of \texttt{expression} by the PHP returns a nodes list. This list is iterated successively and each node is propagated as context for the instantiation of children nodes (cmp. sect.II-C). The instantiation of children nodes with the first element from the evaluation of 'select' returns an instantiated children list. The same counts for the ongoing instantiation. Those are linked together, until no more context exists.

The use of a context does not really restrict reachability of axes, because every node remains reachable. For example, nodes located in the upper section of a source document may by XPath \cite{11} be addressed using 'ancestor'. Contexts are using also used in XSTL \cite{19} for the sake of usability.

\texttt{xtl:macro}

Macros are defined in XTL as following:

\begin{verbatim}
<xtl:macro name="ncname">...</xtl:macro>
\end{verbatim}

Macros are \textit{symbols}, which bind arbitrary sequences of command, element and text nodes, except further macro definitions. The macro is defined by a fully qualified name \texttt{ncname} which must be unique among all macros within a template. In a template all macro definitions must be contiguous and before a sequence of non-macro definitions right underneath the top element node \cite{32}.
The macro call without children is defined as:

```
<xtl:call-macro select="ncname"/>
```

A macro call is similar to a function call without parameters. The macro call retrieves a list of element nodes. During expansion and validation the macro call is replaced by the right-hand side of element nodes from its definition. Recursive calls are permitted. Termination conditions need to be specified within `select`-expressions in XTL command tags.

Summary:

The following fragment of a XTL-schema demonstrates validation with macros and cycles.

```xml
<xtl:macro name="TDs">
  <td>
    <xtl:text select="@title"/>
  </td>
  <td>
    <xtl:text select="@author"/>
  </td>
</xtl:macro>

<table col="#FF0000">
  <th>
    <td>Title</td>
    <td>Author</td>
  </th>
  <tr col="#333300">
    <xtl:for-each select="//book">
      <xtl:if select="position() mod 2=0">
        <tr col="#333300">
          <xtl:call-macro name="TDs"/>
        </tr>
      </xtl:if>
      <xtl:if select="position() mod 2=1">
        <tr>
          <xtl:call-macro name="TDs"/>
        </tr>
      </xtl:if>
    </xtl:for-each>
    <tr>
      <td>XSD specification 1.0</td>
      <td/>
    </tr>
  </tr>
</table>
```

First, validation stores the macro ‘TDs’ defined on lines 2-7 and continues from line 10 onwards. This line matches with line 1 of the instance. The child node at lines 11-14 entirely matches with the child node of lines 2-5 from the instance. At lines 16-27 there is a non-deterministic decision to be made on whether child nodes match for a cycle. It is impossible to determine how much further a cycle needs to be unrolled in order to match the schema in the instance document at line 6 without checking following nodes. If the cycle ‘xtl:for-each’ is left too early or too late, then the `<tr/>-node from lines 28-30 may not exactly match with the expected number of nodes from the instance. According to the instance document ‘xtl:for-each’ may have no, one, two or three iterations. In order to guarantee a correct validation, it is necessary to continue on fails with alternatives, if there are any. Only after all alternatives fail, validation fails.

In the previous example the correct number of iterations, which is two, is guessed, s.t. both `<tr/>-nodes from lines 6-9 and 10-13 from the instance match consecutively with lines 7-16 from the schema. ‘select’-expressions from the conditions are ignored here. Consequently, shuffling `<tr/>-nodes in the instance document, but also a sequence of colored `<tr/>-nodes lead to a true validation.

The instantiation of the cycle is interpreted as unrolling of all books from the instantiation data. Instantiation continues with the following tags. In analogy to that, validation tests if the last node of the instance from lines 14-17 matches with `<tr/>-nodes from the schema at lines 28-31. As this is the case and no further nodes follow, the validation quits successfully.

D. Theoretic Foundations

This section introduces to theoretic foundations. First, the tree-structured data model ‘‘hedge’’ is defined, then regular tree grammars and languages. Later a short overview is given on regular automata. Examples illustrate definitions.

1) Trees: The theories introduced later in this section may be applied to tree-structured objects, like XML-schemas, XML instances and instantiation data. Especially, XML-documents can be represented as trees, since nodes of a XML-document are in a hierarchy and there are no cycles included all through ascending edges leading from leaves to the root element node. Keys as used to describe relations of a schema are not of interest regarding the syntax of a schema.

XML-documents are multi-way trees with child-rich elements as nodes, and childless elements and text nodes as leaves. Attributes of element nodes may be transformed
into element nodes with new child nodes that represent such attributes. For example, the node `<a id="1"/>` can be transformed into `<a><id>1</id><sep/></a>`, where `<sep/>` separates attribute nodes from original child nodes not representing former attributes.

Before trees and their properties are introduced, its practical meaning is recapitulated.

XML-documents can either be interpreted as unstructured, namely as text, or structured. By interpreting XML as unstructured text important information vanishes, for instance newlines or ordering. A replacement of element nodes in trees by symbols returns trees. Replacements may cause shorter nodes sequences. This is the reason why existing string-grammars are going to be extended and reused (see sect.II-D2, cmp. [50], [12]).

In structured XML-interpretations tags emphasise text regions. Tags denote meta-information and are not part of the document text. Hence, XML-document are predestined for structural interpretation, for both, instantiation with command tags as well as validation with common element and text nodes. For example, the unstructured interpretation of:

```xml
<a>hello<b>world<c/></b></a>
```

does not allow a simple processing neither by a user nor by a program. The interpretation of its structure makes access to the documents’ content easy.

**Definition 2.1.** A hedge (after Murata [50]) is defined over a finite symbol set Σ and a finite variable set X as following:

- ε .. the empty hedge
- x .. variable with ‘x’∈ X
- a<x> .. element node ‘a’∈ Σ with hedge ‘x’
- u v .. concatenation of hedges ‘u’ and ‘v’

The two XML-element nodes `<a/>` and `<b><b/>x</b>` are representable as hedge `<a><b><b><x>x</x></b></b></a>` with the symbol set Σ={a, b} and variable set X={x}. Σ does not oblige any restriction. Element names may have any prefix and suffix. Hence, every XML-document is representable as hedge, even XTL-templates and XSLT-stylesheets.

Based on this model each navigation operator over trees can be defined (see [49], [31], [17]). For instance, the function subtree[49] distinctly determines a predecessor node for a given number-encoded path.

2) Regular Tree Grammars: As previously mentioned, string-grammars are not sufficient to describe trees. Hedges do not only grow in width by concatenation, but they also grow into depth by insertion of child nodes. Regular tree grammars are suggested as one way to resolve this issue.

**Definition 2.2.** A regular hedge-grammar (RHG, after Murata [50]) is a grammar G = (Σ, X, N, P, n_f) with:

- Σ .. finite symbol set
- X .. finite variable set
- N .. finite non-terminal set
- P .. production rules
- n_f .. final state set.

A production rule from P has either the form n → x, where n ∈ N, x ∈ X, or n → a<x>, where a ∈ Σ and x is a regular expression over Σ∪X herewith. n_f denotes a regular expression over Σ which is accepted by the grammar.

**Definition 2.3.** Regular expression over hedges.

Let r_1, r_2 be regular expressions over a finite set of non-terminals, then the following expressions are also regular:

- r_1 ∪ r_2 .. concatenation
- r_1 | r_2 .. alternative
- (r) .. parantheses
- r* .. repetition

Regular expressions are better for use in XML-schemas than relations or rigid associations, as it is demonstrated by [16]. Relations do have a fixed amount and ordering of arguments. Contrary to this, regular expressions allow short but flexible expressions.

The regularity of RHS is demonstrated also by the form of its productions. The set of valid instances for a XTL-schema:

```xml
<xtl:if select="/checked">
  <a/>
</xtl:if>
```

recognises the regular tree language

L(G) = {ε, a < ε >, a < x >, a < xx >, a < xxx >,...}

(cmp. sect.II-D2). A corresponding grammar G = {Σ, X, N, P, n1, n2} with Σ={a}, X={x}, N={n1, n2} and P: n_1 → a < n_2

n_2 → x.

RHG differs from string-grammars in variables, which can be considered as terminals, and a final state set, which describes the accepted language. Productions are similar to regular productions, whereas the right-hand side of each rule may contain further non-terminals of the form e_0 · e_1 · ... · e_r, where e_n is a non-terminal and all other e_j for ∈ [0..(n−1)] denote terminals. Terminals stand alone or to the left of a non-terminal.

Derivations for regular tree grammars work similar to string-grammars. Non-terminal symbols are derived left-to-right until the derived expression does no more contain non-terminals. The derivation of a regular tree grammar corresponds to the instantiation of a XTL-template.

The derivation of regular grammars may be non-deterministic due to multiple rules to choose from. If there are at least two derivations of a non-terminal, then an implemented automaton may not decide in general without a stack (cmp. [15], [63], [19]).

Regarding grammars the question concerning expressibility emerges, for instance, if a tree grammar is context-free or
context-sensitive. Context-free grammar do not leave open questions from a practical standpoint (see sect. III). However, context-sensitive grammars are not that easy. This is why, often, contextual information are transmitted by a different mechanism. Turing-mighty tree grammars are not further considered here.

Murata, Lee and Mani [48] classify expressibility of regular tree grammars as following:

local \(\subseteq\) single-type \(\subseteq\) ranked-competing \(\subseteq\) regular

The ordering is due to the level of non-determinism of the production rules (ambiguity). Local tree grammars are the weakest. Regular tree grammars are the most powerful. More powerful grammars totally contain weaker grammars. Moreover, more powerful grammars always contain non-empty cases which are not covered by the weaker grammars [48].

Four tree grammars may be categorised as following:

**Local:** A terminal may not occur in more than one rule.

**S.-t.:** Non-terminals of children nodes do not compete each other. This means \(\pi(e_i) \cap \pi(e_j)\) is empty for each two distinct nodes \(e_i\) and \(e_j\). \(\pi(e_j)\) determines the set of possible beginnings for some node \(e_j\).

**R.-c.:** A hedge \(r\) is uniquely decomposable. This means \(\forall U,V,W \in N : r \not\triangleright_{s} UAV\) and \(r \not\triangleright_{s} UBW\) for competing non-terminals \(A, B\) in \(r\).

**Reg.:** All regular grammars which are not ranked-competing.

The membership of a certain tree grammar is explained by the following two examples:

**Ex 1** Let the grammar \(G_1\) have the following productions:

\[
\begin{align*}
\text{Doc} & \rightarrow \text{doc}(\text{Para}_1, \text{Para}_2^*) \\
\text{Para}_1 & \rightarrow \text{para}(\text{Pdata}) \\
\text{Para}_2 & \rightarrow \text{para}(\text{Pdata}) \\
\text{Pdata} & \rightarrow \text{pdata} \varepsilon
\end{align*}
\]

\(\text{Para}_1\) and \(\text{Para}_2\) compete with each other in the first production. Hence, for a ranked-competing tree grammar no \(U, V, W\) may exist, s.t. \(r \triangleright_{s} U \text{Para}_1 V\) and \(r \triangleright_{s} U \text{Para}_2 W\), where \(r = \text{Para}_1 \text{Para}_2^*\). Since \(U\) must be different in the first derivation from the second, we just found a contradiction. Because no other decompositions exist, \(G_1\) is ranked-competing and therefore regular too.

**Ex 2** Let the grammar \(G_2\) have these productions:

\[
\begin{align*}
\text{Doc} & \rightarrow \text{doc}(\text{Para}_1^*, \text{Para}_2^*, \text{Pdata}) \\
\text{Para}_1 & \rightarrow \text{para}(\text{Pdata}) \\
\text{Para}_2 & \rightarrow \text{para}(\text{Pdata}) \\
\text{Pdata} & \rightarrow \text{pdata} \varepsilon
\end{align*}
\]

\(\text{Para}_1\) competes with \(\text{Para}_2\) in

\[
r = \text{Para}_1^* \text{Para}_2^* \text{Pdata}
\]

Hence, no \(U, V, W\) exist, s.t. \(r \triangleright_{s} U \text{Para}_1 V\) and \(r \triangleright_{s} U \text{Para}_2 W\). But, there exists the valid decomposition \(U = \varepsilon, V = V' \text{Pdata}, W = \text{Pdata}\). That is why this grammar is not ranked-competing.

3) **Regular Tree Languages:** Regular Tree Languages are formal languages that are generated by RHG, hedge-regular expressions and deterministic and non-deterministic hedge-automata (see [49]).

The transformation between the models mentioned above is very similar to those in string-based formal languages. Hedge and tree models, as well as grammar and expressions are from a computability perspective equivalent — this is shown in [49]. Trees are specialised hedges and a hedge is a tree with an empty root node.

It is worth mentioning on hedge-regular expressions, that element and text nodes can be treated nearly the same (cmp. sect. II-D1). As shown earlier, attributes may be simulated by element nodes. Examples to each of the mentioned models in this section may also be found in [49], [50], [16], [4].

4) **Finite Tree Automata:** Regular automata are being addressed in [50], [12]. Next, only such automata are characterised, which correspond to the expressibility of regular tree grammars. In order to do that, the taxonomy proposed in [48] is used, particularly the determinism and evaluation ordering is of utmost interest and is summarised in table I. Ranked-competing grammars can be recognised by bottom-up automata.

|                | deterministic | non-deterministic |
|----------------|---------------|-------------------|
| top-down       | local, single-type | regular          |
| bottom-up      | ranked-competing | regular          |

**Table I**

**Taxonomy of Tree Automata**

According to [48], grammars whose languages are recognised by non-deterministic top-down and bottom-up automata, are reducible each other to the same. Non-deterministic grammars and recognition algorithms are often simpler, however, those algorithms may be more complex and involve extensive backtracking. Bottom-up automata in contrast to top-down automata are considerably more complex and therefore by far more difficult to maintain. Especially, error messaging may become by far more difficult, since in general first all alternatives need to be checked prior to decide if a validation definitely fails. Moreover, also the “‘real’” reason would have to be tracked somehow among alternating backtraces.

Derivatives [9] is one approach, which maps regular expressions over hedges onto finite regular tree automata. During a document validation a given schema and regular expression are reduced towards an instance until both sides cannot...
be reduced any further. If both expressions are empty, then validation would succeed. Otherwise, validation fails. After each derivation step a new state is introduced. Since the incoming schema is finite and symbols are not allowed (see sect. II-C), the algorithm terminates.

Partial-Derivatives [4] is an improved approach, which calculates derivatives only when needed (see app. X). Once calculated, solutions are not determined a second time. Similar approaches on parsing cause a performance increase of 70% in parsing XML-documents [43] and 80% in instantiating approaches on parsing cause a performance increase of 70% in instantiating those [51].

The additional cost to be paid on XML-parsers is only causing approximately 10% of overhead. This algorithm has a best-case complexity of \( \theta(n) = n \) and \( O(n) = n^2 \) for the worst case, where \( n \) is the length of the given regular expression. For the reasons mentioned, this approach is of interest for practical implementations. It can be stated, that tree automata describe functions over XML-documents, particularly the template expansion and schema validation.

III. ANALYSIS

This section investigates instantiation and validation. Both functions are considered and requirements formulated for a unification on document level.

A. Current Situation

This paragraph XTL is analysed w.r.t. language features. Then instantiation and validation are investigated closer, e.g. properties of XML-template and schema languages.

1) XTL: First Considerations

Represents of template-languages are JSP, ASP, XSLT and XTL. Prolog may also be considered for instantiation of XML-documents [37].

Template languages may be distinguished by the generated target language. If both, template and target language unite, as it may be the case with XML, then it is obvious that a unification may be reached easier.

Template languages may also be distinguished by its intention. Template languages with variables and functions, for instance, are more appropriate for programming than for document processing. In any case it is worth, to separate program from document, especially when it comes to validation (cmp. sect. III-A2).

Parr [52] proposes for template languages the following minimal asset of template commands, which should also count for document processing:

1. Attribute references,
2. Conditions,
3. Recursive Template Calls,
4. Conditional Template Inclusion.

In fact, XTL, which is also a template language, already has got these features (cmp. sect. II-C). Attributes can be expressed by \( \texttt{xtl:attribute} \), conditions by \( \texttt{xtl:if} \), cycles by \( \texttt{xtl:for-each} \), text inclusions by \( \texttt{xtl:text} \) and element inclusions by \( \texttt{xtl:include} \). Applications can be expressed, but only without parameters. This is done by \( \texttt{xtl:call-macro} \).

XTL instantiates free of side-effects. This means queries to instantiation data do not alter the source nor the template. This way referential transparency is guaranteed.

If PHPs allow access to documents, then XTL indeed is also favorable for documents. Helper functions should be banned from XTL in general and moved to external sources, which may be referenced by \( \texttt{select} \).

Separation of Concerns

Czarnecki and Eisenecker [14] insist a transformation is taken out by referencing only to a minimal set of operators. This also counts for instantiations, because these are transformations too. Both suggest to restrict ourselves to loops and selections instead. In XTL this is done by the command-tags \( \texttt{xtl:for-each} \), \( \texttt{xtl:text} \), \( \texttt{xtl:include} \) and \( \texttt{xtl:attribute} \), as well as the attribute \( \texttt{select} \). Though, functions shall better be removed from instantiation and be passed to a module responsible for that particular task instead. For the same reason calculations shall also be removed from templates. This also affects arithmetic expressions, e.g. in XPath, which are not really needed for navigating instantiation data.

Furthermore, Parr [52] insists template languages clearly separate between concerns of a given instantiation problem. Fig. 2 shows a concern separation for XTL using Reenskaug's the Model-View-Controller meta-pattern. The resulting instance document shall at most be independent from the instantiation, s.t. all calculations and existing constraints fit within the chosen model. Output in View often requires formatting. In order to do that the Controller triggers the Renderer, who passes information further from Model to View. By doing so, the strict separation remains.

The separation of command language from the template language by \( \texttt{select} \) and common tags lead to an increasing similarity between instance and template. This is the foundation for the unification of instantiation and validation (see sect. VI-B).

The separation between template and command languages causes the command language has to provide conventional interfaces for text and attribute access, inclusions, conditions and loops. Since XTL does not allow direct access to instantiation data initially, it is not Turing-mighty. In order to
allow XTL to access different instantiation data, the optional attribute ‘realm’ was introduced. In XTL command tags it may be inserted by a ‘select’-attribute.

Type Safety

XTL guarantees a well-formed document during instantiation of a template, if the template is sound w.r.t. its specification [33]. Each XTL-template is well-formed XML. The only restriction is the top root node may not be a command-tags according to its specification. It must be an element node. All nodes located underneath are well-formed, because those nodes are composed of text and element nodes only. In some cases ‘select’-queries are not sound, the affected XTL-command are left empty by the XTL template engine.

PHP-functions assert type safety. Hence, instances are always type-safe here. Static types of PHP-functions make instantiation more predictable prior to running. This means command languages become exchangeable and so the source language becomes more flexible.

In XTL term-evaluator [33] and instantiation data evaluator have been introduced in order to evaluate ‘select’-expressions. The instantiation data evaluator checks types of a solution with the inferred type provided by the term-evaluator or an optional type. Instantiation data has a polymorph type, so are the types of results of queries. In order to process arbitrary instantiation data, polymorphic data needs to be transformed into non-interpretable text. This is done by the Renderer, which knows about desired output formatting.

The following table shows the types of XTL-commands, which need to be known to the Renderer before building the instance document:

| XTL Command | Type |
|-------------|------|
| xtl:attribute | String → [a] → String |
| xtl:include | String → [a] → XML |
| xtl:text | String → [a] → String |
| xtl:if | String → [a] → Bool |
| xtl:for-each | String → [a] → [a] |

The first type denotes a select-expression. The second type denotes the instantiation type, which is polymorph. The typing of PHP-functions is explained in more detail:

```
<book>
  <xtl:attribute name="title" select="..."/>
  SELECT title
    FROM books
    WHERE id='1'
</book>
```

It is assumed, a PHP-function determines for a given SQL-query a relation with exactly one result. This result is converted by the Renderer in a representable string and placed then into an instance document. The example generates a <book/>-node with attribute title="Haskell..." as relation corresponding to the example from sect.4. Though, the resulting nodes are always well-typed and well-formed for any instantiation data and PHP functions. So, every XML-document created is type-safe.

Variability

It means the exchange of the command-language. PHP has common interfaces and concrete implementations. By doing so, the internal organisation can be hidden from the user [6]. The processing of heterogeneous data is managed by previously agreed interfaces only.

Access to instantiation data is restricted by the template-engine. Access may not be granted by the template. Although, this tough restriction may not always be desired, as demonstrated by the following fragment:

```
<book>
  <xtl:include select="document('a.xml')" //title/>
</book>
```

The difficulty is the template addresses external documents during instantiation. The function ‘document’ is evaluated with the path expression following, even if the function is not XPath [11]. In order to resolve this problem, the source is either passed immediately to the instantiator, or all referenced spots are moved to a separate document – this is tractable only if there are multiple sources. In the template those entries are referenced. Moreover, the extraction of all needed sources by additional templates is in preparation.

Style

Beside the formal grammar type schema languages can also be characterised [46] by the style of the corresponding grammar.

A language has a grammar-style, if the associated schema language is described shorter by a grammar than by a corresponding regular expression [18] (see next sections). Otherwise, the language is in pattern-based style.

A schema language allows many different tags at different locations. Schemas which have a few restricting symbols only, are more often in pattern-based style than in grammar-style.

Macro definitions can easily be described in XTL in grammar-style. Right-hand sides are equal to a not necessarily right-ranked hedge (cmp. sect.14). All remaining XTL-tags may be described in pattern-based style as well as in grammar-style. In contrast we have schemas from rigid associations/relations whose elements need to be placed separately. Relations do not insist on a strict ordering. However, related entities must obey certain conventions. For example, relations need to be defined a priori, which determine uni-directional element and bi-directional attribute relations. This causes a whole graph is represented.

Regularity

Since RHG may be described by XML-schema languages [16], [30], a corresponding representation exists consisting of regular hedge-expressions. If ‘select’-expressions, which actually denote the number of repetitions, in ‘xtl:for-each’-loops are kept arbitrary, then context-sensitive and context-regular expressions become regular expressions for the price of further abstraction. Arbitrary repetitions become Kleene’s star operators (cmp. sect.11-A3). Conditions in XTL are represented either by ε, hedges, or text and element nodes as literals.
Macro definitions and macro calls require special treatment, because bodies of macro definitions may contain an arbitrary number of further macro calls anywhere within the hedge. Both extend expressibility of regular tree grammars. Therefore, macros need to be investigated separately when it comes to judgements about expressibility. After all and for sake of simplicity it still makes sense to regard XTL as a regular schema language. A more detailed investigation of the grammar class follows.

Example (a) from table II shows that XTL is not only local, because \(<a/>\) occurs in the body of the macro as well as in the hedge. This means the corresponding grammar has two productions with same right sides. Example (b) shows XTL is not only single-type. Macros ‘\(A\)’ and ‘\(B\)’ are competing – both contain ‘\(<a/>\)’ as a starting symbol, hence, \(\pi(A) \cap \pi(B)\) is not empty. Example (c) shows XTL is not only ranked-competing, because decompositions cannot always be found due to uneven macros ‘\(C\)’ and ‘\(D\)’. This is why XTL is as expressible as the class of languages generated by regular tree grammars.

Only regular tree languages are enclosed under union, intersection and complement (cmp. \([16]\)) as string-grammars are. This is particular of interest when extending XTL by command-tags composed of existing tags.

Those tags oppose non-monotone operators \([31]\), because those may change fragments of instantiation data and those do not necessarily merge together existing instantiation data. Those operators are advantageous and compact when instance shall have many details, and when the difference between instance and source documents is rather small. Rather than requesting a lot of information in order to build up the document from scratch by filling numerous slots, it may be more efficient to nearly copy the document instead. Despite non-monotone operator can reduce a template quite significantly, violated closure-properties may cause disturbance in the separation of concerns between instance and instantiation data, because validation can in general not make assumptions about the instantiation data. That is why non-monotone operators must be restricted in regular languages a priori, in order not to hinder an unification of instantiation and validation.

**Context-free Tree Languages**

In order to recognise the context-free tree language \(L(a^n b^n)\), schemas can be defined in two ways (see table III).

**TABLE II**

| Expressibility of regular schemas in XTL |
|----------------------------------------|

| Macro | Macro Call | Macro Call | Macro Call | Macro Call |
|-------|------------|------------|------------|------------|
| A     | C          | B          | D          | A          |
|       |            |            |            |            |

**TABLE III**

| Context-free schemas |
|----------------------|

| Variant 1: |
|------------|
| \(<xl:macro name="S">\) |
| \(<xl:if select="...">\) |
| \(<a/>\) |
| \(<xl:for-each select="/\A/book">\) |
| \(<xl:call-macro name="S"/>\) |
| \(<b/>\) |
| \(<xl:call-macro name="S"/>\) |

| Variant 2: |
|------------|
| \(<xl:for-each select="/\A/book">\) |
| \(<a/>\) |
| \(<xl:for-each select="/\A/book">\) |
| \(<b/>\) |

Variant 2 is universal, since \(L(a^n b^n)\) could be generated. This cannot be done with variant 1. Furthermore, \(L(a^n b^n c^n)\) and \(L(xa^y b^y c^y d^y)\) can only be recognised by variant 2. So, the difficulty is to express exactly instances that shall be recognised during validation. For instance, if ‘select’-expressions describe \(L(a^n b^n)\), then this language would be context-free. In contrast to regular lan-
language recognition the recognition of programming languages, especially those defined by a LL(k) or LR(k)-grammar is much more complex. Therefore, the recognition of context-free and context-sensitive tree languages in general might be much more complex and is doable but only with much more efforts to be spend. XTL is regular if only expressions of the command languages are not considered exactly during validation. Variant 1 describes a part of context-free schemas. However, if $L(a^nb^n)$ is to be recognised, then variant 2 would need to be chosen. But by doing so, context-free languages may not be validated exactly. This means $a/a/a/a/x/b/b/b/b$ is enclosed by $L(a^nb^n)$ and is validated by variant 2, although the input is not context-free. Hence, context-free schemas are not going to be considered further.

**Termination**

If in the body of macro definitions appear macro calls, recursion may appear. Fixpoints may appear in the template and schema. Fixpoints may be formulated with the command language and remain unreachable due to left-recursions – so instantiation and validation do not terminate. The recognition of left-recursion, however, is in general not decidable for XTL-documents due to the Halt-problem.

In contrast, it may effectively be decided whether, for instance:

```xml
<xtl:macro name="M">  
<xtl:call-macro name="M"/>  
</xtl:macro>
```

contains a non-terminating cycle. But this does not work for arbitrary XTL-document prior to executing the program.

**Functions**

XTL has a very little vocabulary. Because XTL does not have parameters, formal functions cannot be defined. In XPath it is not possible to define functions with an arbitrary arity. It is also not possible to call such function from other tags. Locally defined function may not violate referential transparency of the template engine because of the strict separation between template and instantiation data. So, many functions, particularly μ-recursive functions and tail-recursives function cannot be expressed within XTL.

Tail-recursive functions can be simulated in XTL, but only under additional restrictions. For example, counters may be expressed by special 'select'-expressions. The function ‘position()’ allows to access the actual counter in XPath. The amount of loop iterations is limited in ‘xtl:for-each’ by a constant value, which is evaluated by ‘select’. Cycles simulate tail-recursive functions in XTL, but without an argument list.

Due to lack of defining and composing functions, XTL is not primitive-recursive. This holds even when while-loops are substituted by recursion with a preceding ‘xtl:if’, or tail-recursive function without arguments are simulated. Due to separation of concerns, document processing does not require sophisticated arithmetic nor logical functions.

2) Instantiation: Instantiators can be interpreted as term rewriting systems. The $\lambda$-calculus provides a mechanism for doing so (see [5]). Slots can be represented as variables, and nodes as terms. Then instantiation equals a derivation.

However, terms need to be modelled properly. For instance, it may be needed to assign each element node with a certain arity its own semantics. An evaluated node may denote a node with a qualifying functor, which encodes the number of children. Otherwise a concatenation of nodes may not be injective. Furthermore, empty nodes, attributes and hedges need to be mapped, which may require additional handling.

Values of ‘select’-expressions are bound to variables. Within terms of \(\lambda\)-abstractions this is done by applying expressions to slot-variables. The internal evaluation by PHP-functions remains invisible.

Termination is equal to reaching a normalform. Reduction is strictly monotone. This means once evaluated nodes are not reverted. The number of evaluation steps is polynomially bound, except from macros. Macros may lead to self-applications, because macro bodies may reify variables. In that case no normalform could be found, so, the template-engine would not terminate. The evaluation sequence of a hedge does not matter; since all nodes of a hedge have to be evaluated and are independent from each other. Conditions in XTL terminate when evaluated from outside in, however, they do not terminate in general in the opposite direction.

As already mentioned, slots return strings and nodes. That is why types over nodes and slots make sense. Simple element nodes may be its own type and do not require additions conventions. Simple nodes may directly be passed to instance document and do not cause any side-effects. Hence, a formal description of the instantiation benefits from denotational semantics.

The simple untyped $\lambda$-calculus is not sufficient here. The typed $\lambda$-calculus is needed for a formal description of instantiation. This is because types and constructors are helpful to the description of element and text nodes. Constructors denote parametrised types whose type variable is typed again. This means types are composed of other types. On the other side the simple $\lambda$-calculus does not provide a compact notation for function calls. Fixpoint combinators are the only possibility to mimic recursion.

In contrast to that, the denotational semantics allows us to express constructors, types and formal function in a meta-language. So, it becomes possible to compactly express pattern-matching, and to use combinators too. The construction of nodes should be done according to minimisation criteria [31].

3) Validation: In analogy to parsers validators check for a given programming language, if an incoming XML-document (program) is a valid instance of a given schema (grammar). It is more appropriate to refer to matchers [27] when validation is meant. Compilers have an invariant set of rules, but are applied to ever-changing incoming programs. When considering validators not only do the input data (instance) change, but also the set of rules (schema). Moreover, there is no translation going on, but a boolean value is calculated.
A XTL-validator checks an instance document against a regular schema. This is done node by node. In [57] a classification of schema-matchings is proposed. The XTL-validator is hence a class by its own and is schema-centric. The validation operates on hierarchic XML-nodes and therefore is structure-centric. Both, ‘xtl:for-each’ and ‘xtl:if’ are meta-operators which influence the processing of an instance document. On a lingual level they are constraints, which have nothing really in common with ‘select’-expressions. This is why the validator belongs to the class of graph-matchers.

The description of a validation can either be described textual or graphical. The relation between matching documents can be expressed by \( S \approx (s_1, s_2) \), where \( s_1 \) denotes a schema node and \( s_2 \) an instance node. Schema nodes matching with some instance node \( s_2 \) generate a set \( S_1 \), which in general is not singular. If \( S_1 \) is indeed not singular, then validation becomes valid and non-deterministic. \( S_1 \) cannot be empty, since it at least contains \( s_2 \).

The validation problem can be interpreted as typing problem \( [8] \). In [69] typing is actually proposed as static validation approach by using Haskell’s built-in type system. If a document validates, then a type can be inferred, otherwise Haskell shows up a typing mismatch. The validation takes place without an actual validation algorithm by doing so. This approach only works with dedicated constructors used for constructing in Haskell a whole XML-document.

The formal notation can base on the \( \lambda \)-calculus, but here the formalisation for validation is hard, since there seem to be no good adequacy of representation.

Context-free languages are not recognised, except the feature discussed in sect.[III-A1]. This means hedges are invalid as soon as they appear twice. The amount of opening and closing brackets is only of minor interest – this is different to programming languages. By moving a sequence \( c \) to a prefix from the language \( L\left(a^n c b^n\right) \), or by moving \( c \) to a suffix of \( a^n b^n \), a context-free schema may still be recognised. In node \( c \) may not occur, otherwise the given schema is not context-free.

In order to improve the runtime behaviour towards non-deterministic decisions determining a tree automaton requires in general regular schema whose recognition has a complexity of \( O(2^n) \). Herewith, \( n \) is the cardinality of the set of states (see [9,10], cmp. sect.[II-D4]). For a validation to be used only once these may mean too high costs. But, if many documents are going to be validated against the same schema, then situation becomes totally different, as it may be the case with database triggers.

As it was seen in sect.[7] tree automata fit only for schemas that do not change over time.

Clark [17] proposes a top-down non-deterministic algorithm for RelaxNG-schemas. Non-deterministic matchings are resolved by so-called interleavings [17]. He proposes rules for element nodes for all possible occurring cases. Inclusions denote variable symbols which may be replaced by arbitrary nodes. This causes nodes appear valid, although they do not occur at corresponding positions in the instance document. So, instead of an \( \varepsilon \)-node its successor may be taken for validation, and if validation fails then everything from there backwards need to be analysed manually, which is quite laborious. It may be more efficient though to track all possible nodes during validation and make a decision when including the next time. Unfortunately, even small documents generate such a huge search space, so it becomes not doable due to an exponential rise in complexity. The extensive search [17] should, if used at all, massively reduce invalid nodes. Obviously, there is no optimal solution for this problem. But there exist a few heuristics which may overcome the practical problem for previous domain ranges, such as the strategy ”try all valid states until a contradiction occurs”.

Antimirov [14] proposes an algorithm turning a regular expression into a non-deterministic finite automaton (NFA). Hence, XML-schemas can be considered as regular expressions, Antimirov’s approach matches for regular tree expressions also. Non-deterministic finite automata are dual to deterministic finite tree automata (cmp. [69], [48]). In contrast to [9] needed derivatives only are calculated, and those are calculated only once. In procedural and object-oriented programming languages determination can be achieved by merging non-determined states or by balancing non-determinism, e.g. by backtracking.

An alternative to validation (with XTL) is the transformation (of XTL) into another already existing schema language, like RelaxNG. Problems that need to be addressed could, but do not necessarily need to be: restriction of expressibility (of XTL) and/or coverage of the schema-language to be replaced. A schema transformation would also require further well-defined schema languages, which at least is not part of this work.

4) Properties: Instantiation as introduced in sect.[7] is a mapping whose co-domain XML is entirely covered. Instantiation can be interpreted as endomorphism, because both domain and co-domain denote the same set, namely XML. This is guaranteed by well-formedness of XTL itself. Therefore, instantiation is an enclosed operation (cmp. [54]).

If instantiation is indeed considered an operation, then associativity does not hold. Commutativity also does not hold, because instantiation of a slot-containing template document has XML as result. The result syntactically does not match in general with the origin template. Instantiation of a XML-document without slots, is idempotent for any instantiation data.

For a validation, particularly a derivation of a regular expression homomorphism holds. This follows from this equation from [44], which also holds for schemas:

\[ \text{val}(x \cdot y) = \text{val}(x) \circ \text{val}(y) \]
The operator ‘·’ concatenates two regular expressions and ‘◦’ logically ANDs two interleavings. So, some regular expression \( \text{val}(x \cdot y) \) is congruent to \( \text{val}(y) \) modulo \( \text{val}(x) \) (see sect.IV-A4). Therefore, the order does not matter, whether first regular subexpressions \( x \) and \( y \) are evaluated and concatenated second, or whether first those expressions are concatenated second and \( x \) and \( y \) are evaluated. Especially, for seeking optimal solutions this might be helpful when at least one subexpression may be dropped, for instance.

5) Arrows and Filters: An arrow is a generalised monad (see [38], [39], [58]). It encapsulates functions as parameter. An arrow in Haskell is an instance of the class \( \text{arr} \) with two functions:

\[
\text{arr} : (a \rightarrow b) \rightarrow \text{arr} a b \\
\text{>>>} : \text{arr} a b \rightarrow \text{arr} b c \rightarrow \text{arr} a c
\]

Similar to the concept of variables in programming languages, functions may also be made available in dedicated environment scopes and namespaces. This is advantageous particularly for lazy parsing and serialisation, because certain sets may be evaluated partially, allowing a higher usability so.

Instantiation may use macro definitions instead of arrows. However, during validation macro definitions shall be avoided, because apart from a macro environment further semantic fields may be required, e.g. a list of all valid element nodes — which for the sake of previous outcomes was decided not to research further in this work.

Filters denote functions having a polymorph type \( a \rightarrow \{ b \} \), where \( a \) denotes a type variable. Filters are functions with an input vector and an arbitrary output vector. They can be classified according to its behaviour, for instance, by common combinators (cmp. sect.IV-A4) and functions not in typical combinator representation. The class mentioned second are functions whose head is specified using pattern matching. If possible, implementations should make use of pattern matching — the same as semantics make. By doing so, redundant iterations of trees are avoided, and structural definitions can be reused. Both effects increase readability. The elimination of multiple iterations cost high efforts (cmp. sect.IV-A4) causing also an increase in complexity. By specifying pattern matching the gap between denotational semantics on the one side and implementations on the other side diminishes.

Both, XML-parsing and serialisation, unfortunately, violate referential transparency, but this must be, since files can neither be read from nor written to without side-effects really. Luckily, these are the only places where this is required, and there is no other place having this effect. As an alternative to arrows multi-paradigmal programming may be an option (cmp. sect.II-C). By violating the absence of side-effects read and writer operations increase the flexibility of a function in general. Input and output operations are no longer restricted to a certain location in a function, but now can be located and used anywhere. Multi-paradigmal programming shall be used, s.t. input and output operations are implemented by machine-dependent instructions, and where instantiation and validation are implemented by an abstract programming language. Here, a violation of encapsulation would increase usability. Named functions shall be strictly typed and be implemented as super-combinators.

B. Requirements

Goal of this work is the attempt of the unification of both views, instantiation and validation (see sect.II). This would have in consequence the increase of the functionality.

This requires for the sake of demonstration of usability denotational semantics to be defined for both, instantiation and validation. Algorithms based on it, an implementation and an object-oriented design with test cases are prepared.

1) Limitations: No assumptions on a concrete command language are agreed. So, no information on the syntax nor internal states nor properties are known to the instantiator. No assumptions on the structure of instantiation data are made. Only PHP-functions with previously agreed interfaces are to be considered (cmp. sect.IV-B2). Communication is achieved exclusively through these interfaces.

Validation does not interpret 'select'-expressions. Bypass- and 'realm'-attributes are not treated on instantiation. The support of 'bypass'-attributes is optional during instantiation, because this requires an interference of several expansions within one template. The effect of 'bypass' can be simulated by running several templates sequentially. 'bypass'-attributes are then redirected as command tags into the instance document or as another 'bypass'-attribute with a smaller value depending on the total amount of phases to be run.

Additional problems are made w.r.t. the detection of non-terminating loops, since this problem is in general undecidable (cmp. sect.II-C).

Encountered problems of XTL in comparison to other schema languages are to be examined and improvements shall be shown. The goal herewith is a compatible syntax extension (cmp. sect.II-D).

Another useful tool for XTL is a semi-automated schema-generator, which generates a schema from an instance document (so-called ‘validation by instance’)-approach [26]. The implementation of a schema-generator for practical use would go far beyond the goal of this work, therefore it is not considered here further (cmp. sect.II-D)

Here are some reasons why:

- **Parameter:** A schema appears useful to the user whenever it is fine-grained and recognises many regular substitutions in the instance.

But, since it is not obvious, if a hedge may be replaced by a sequence of nodes or by a cycle with conditions — there should first of all some criterion be defined regarding granularity. So, complex substitutions could reduce an instance by a line, for instance, but the obtained instance would be very hard to check by the user.
• **Ordering:** Is the ordering within a children list fixed or may it permute? The ordering of a hedge is either explicit or selective. The ordering of nodes in a hedge, also referring to following hedges, can be assigned by any attribute. Childless nodes do not necessarily need to be defined in a schema to be childless. Child-rich nodes may confirm in following nodes the exception. Such differences rest exclusively on the user and may not be considered automatically.

• **Quantity Multipliers for specifying elements occur several times, are hardly available, neither makes it really sense to specify multiplicity on each occurring hedge. Instead only those hedges should be quantified, which, for instance, occur two or more times.

• **Configuration:** All presented constraints must be configurable on the needs of a user. Rules should be assigned to membership-functions. Based on those rules, for instance, an expert system may derive optimal decisions.

1) Instatiation: Inputs are a well-formed XTL template as well as one ore more instantiation sources. Further formal and non-formal requirements not mentioned in sect.[I-A] are:

   2) The implementation is to be done in Haskell. The Haskell-Toolbox for XML-processing [58] shall be used (see sect.[V].

   3) The implementation should essentially not deviate from the denotational semantics. Data model and rules should be simple. Invalid XTL-tags should be treated as simple element nodes.

   4) Validation: A XTL-conform schema as well as any XML instance document count as validation input. The result of a validation is a ”‘yes’/’no’” answer. The requirements to a validator are as following:

   1) The implementation is to be done in Haskell. The Haskell-Toolbox for XML-processing [58] shall be used (see sect.[V].

   2) The implementation should essentially not deviate from the denotational semantics. Data model and rules should be simple. Invalid XTL-tags should be treated as simple element nodes.

   3) Validation: A XTL-conform schema as well as any XML instance document count as validation input. The result of a validation is a ”‘yes’/’no’” answer. The requirements to a validator are as following:

   4) Unification: One main goal of an unification of XTL-instantiation and validation is the lingual unification of both processes (cmp. sect.[V]). Apart from that the reuse of templates as schema shall be examined.

   A rule-based approach would be desired for a better understanding and a qualitative investigation (see sect.[V]). Here agreed data models should be used for both processes.

Helper functions should be reused as much as possible. This requires those functions to be as generic as possible. Pre-defined function should be reused (cmp. [21]).

5) Implementation in Java: The programs to be written in Haskell shall later be implemented in Java. Therefore, at least an object-oriented design as well as a translation of the denotational semantics into Java is needed (see sect.[V]). Here several questions emerge:

   1) How are polymorphism [22] and functionals implemented accordingly in Java?

   2) Can the non-deterministic top-down automata remain as is?

   3) What are appropriate class candidates? What do associations between them look like?

   4) Which roles can be abstracted and how do these roles interact?

   5) How will most generic implementations look like?

   6) What do the architectural design patterns look like?

Within this work two programs are implemented, one for instantiation and one for validation. A test suite is introduced. The well-formedness and validity of XTL-templates is guaranteed by existing XSD-schemas. Existing frameworks for XML-processing are being used where appropriate.

IV. DESIGN

In this section introduced data models and semantics for instantiation and validation are presented.

Haskell is used as programming language. Because of its functional character Haskell allows a straight transformation from denotational semantics (cmp. sect.[II]).

A. Data models

The goal of data models introduced in this section is an easy denotational semantics.

The features set of HXT is rather small. Usability is medium – so compromises must be made. This means, simplifications are useful whenever the description gets simpler. In conclusion the need arises to transform models. This takes rather small efforts when it comes to validation, since transformation back again are not needed. In contrast to this, instantiation requires both transformation directions.

Furthermore, completeness and correctness of the data model transformation need to be assured by covering both domains and co-domains. Those coverings may be used as test suite for the implementations.

\[
\text{HXT} \xrightarrow{f_1^{-1}} \text{XTL} \xrightarrow{f_2} \text{Reg} \xrightarrow{f_3} \text{NFA}
\]

Fig. 3. Data models for Instantiation and Validation

Fig. 3 shows the transformation for all the data models considered in this section. The data models HXT and XTL represent document nodes. These nodes can be transformed by the functions \( f_1 \) and its inverse \( f_1^{-1} \) into each other.
Both are the foundation for instantiation and validation. Instantiation is described by the composed mapping \( \text{HXT} \mapsto \text{XTL} \mapsto \text{HXT} \). The first transformation transfers a parsed XmlTree into a simple fine-grained XTL-representation. In conclusion the instantiated XTL document is transformed back to HXT (see sect.IV-B). In addition to that validation turns both XTL representations, for schema and instance, into the Reg-model representing regular expressions (see sect.IV-C). A transformation back to HXT is not needed.

The models HXT, XTL and Reg are concretisations. All these models are equivalent in expressibility, but each model has unique elements characteristic only for those. The HXT-model is only partially considered. All concretisations obey the following strict ordering:

\[
\text{HXT} \preceq \text{XTL} \preceq \text{Reg}
\]

The model NFA is only sketched, it is only to be an alternative proposition. Beside the disadvantages of sect.IV-A, NFAs are not really compatible by default with hedge-based data models like HXT and XTL. This is because NFAs are hard to represent as a tree, because they are graphs in general.

The completeness of a model-transformation is determined by the coverage of the domain. This is considered separately in each section.

1) HXT: The denotational semantic uses functions from the Haskell XML Toolkit [58], solely for input and output processing of XML-documents (see sect.IV). The contained data structure ‘XmlTree’ is the foundation for further processing using the toolkit. A XML-node ‘XmlTree’ and a hedge ‘XmlTrees’ are defined in Haskell as following:

```haskell
type XmlTree = NTree XNode
XmlTrees = [XmlTree] .
```

The type constructor ‘NTree’ represents multi-way trees in general and is defined in GHC as:

```haskell
type NTree a = NTree a [NTree a] .
```

Here, the node type ‘XmlTree’ explicitly denotes ‘XNode’, so child nodes may only be one of these:

- XText String
- XAttr QName
- XTag QName XmlTrees

‘QName’ denotes a qualified name. It may occur in element nodes. In HXT these are composed of the type constructor ‘Q’, a namespace prefix, a local identifier and an URI. The ordering is defined as:

```haskell
type QName = QN ns local uri
```

So, the XML-node `<a id="1"/>` is represented as XmlTree as following:

```haskell
NTree (XTag (QN "" "a"" ""))
[NTree (XAttr (QN "" "id"" "")
[NTree (XText "1") []]]]
```

The disadvantages of HXT are obvious. Even simple ‘XmlTrees’ are very long and heavily loaded with brackets. So, it would be quite hard to really experience the benefits of pattern-matching here. Since a clear and simple denotational semantic of a node is a precondition for a simple processing semantic in general, composed combinators (see sect.II) are not an equivalent.

Although the usability of the ‘XmlTree’-model in HXT is difficult at least, the amount of features in HXT is quite big (see sect.I).

Multiple functions and constructs overlay and are usable in some special cases only.

Implicit assumptions often cannot be seen by a function’s name. For example, the type constructor ‘XAttr’ and the constructor function ‘xattr’, both can be used in order to build up attribute nodes. ‘xattr’ implicitly insists attributes are specified first for children node constructions. This circumstance and unintentional constructor errors cause tree constructions quickly become hard to read and bloated. Another issue is the problem that all data types, type definitions and functions are loaded into its own environment as soon as an HXT-module is imported. This problem can be resolved by a restricted module import command, however, requires high level of awareness and can become very easy uncontrollable even with a small number of imports.

Another severe problem is the too lax syntax of an XML-node. So, many syntactic correct nodes may be generated which, however, are semantically not sound. Semantic mistakes may only be detected while serialising a document, only by throwing an exception, otherwise they will remain unnoticed. For example, attributes could accidentally be mistaken for element nodes, because they have the ‘XNode’ too. In addition, the definition of a node in HXT does not prohibit an element node is used as attribute node. The localisation of non-matching functions for that reason is a major flaw. Often errors may only be localised manually by analysing the call-stack. However, this is not sufficient. Especially, due to a lack of good tool support for Haskell despite current attempts (cmp. for instance with [29], [3]), the motivation rises even more to make data models and function as simple as possible.

2) XTL: The algebraic data type XTL is defined as:

```haskell
data XTL = XAttr String String String
| XAtt String
| XInclude String
| XMacro String [XTL]
| XCallMacro String
| XIf String [XTL]
| XForEach String [XTL]
| EIX String [(String,String)] [XTL]
| TxtX String
```

Here, ‘XAtt’ defines an attribute entry consisting of name and value. ‘XTxt’ denotes a XTL text node (see sect.II-C). The type constructors ‘XTxt’, ‘XInclude’, ‘XIf’, ‘XForEach’ have ‘select’-expression as its first String. ‘XMacro’, ‘XIf’, ‘XForEach’ denote a macro definition, condition and cycle. All of those have a ‘EI'
with [XTL] as hedge argument. ‘TxtX’ denotes an arbitrary XML text node. ‘ElX’ is a XML-node, which is composed of a name, a list of attribute entries and a hedge of child nodes.

The mapping of HXT-nodes also affects XTL-tags (see sect.II), element and text nodes (see tab.IV). Element nodes and XTL-tags differ, for instance, when control should (not) depend on data. Comments and Processing-Instruction-nodes are not considered. XTL-tags not matching with command-tags should match with rule (El) and therefore should be transformed into usual element nodes. ‘children2’ on the right-hand side of the rules (M), (If), (FE) and (El) denotes recursive continuation of the mapping onto the hedge ‘children’. qn2 denotes a qualified name as string, which is generated by qn. Because of the restriction of the HXT-model the mappings are not injective, but they are surjective, because all elements of ‘XTL’ are covered. Therefore an inverse mapping exists, which recursively applied to ‘children2’ results in ‘children’.

The node `<a id="1"><b/></a>` is represented in XTL as:

```
ElX "a" ["id","1"] [ElX "b" [] []]
```

The XTL data model is used for instantiation and validation. Despite instances do not have command tags, it is beneficial due to an unified semantics also to express instances by XTL. In addition to this, it may also be used as automated schema generator (see sect.IV-A3) or as schema parser (see sect.VI).

3) Reg: The regular data model ‘Reg’ is defined in Haskell as can be found in fig.4.

```
data Reg = MacroR String  
  | AttrR String String  
  | TextR String  
  | IncludeR String  
  | ElXR String [String,String] Reg  
  | TxtR String  
  | Epsilon  
  | Or Reg Reg  
  | Then Reg Reg  
  | Star Reg 
```

![Fig. 4. Data model Reg](image)

‘Reg’ follows the model presented in sect.IV-A2 ‘AttrR’, ‘TextR’, ‘IncludeR’, ‘ElXR’ and ‘TxtR’ represent literals. Although element nodes are recursive, each can still be considered as literal, especially when they are empty hedges or being processed. The type constructor ‘AttrR’ has the same structure as ‘XAtt’, ‘TextR’ the same as ‘XTxt’ and ‘IncludeR’ as ‘XInclude’. ‘Epsilon’ denotes the empty word, ‘Or’ denotes selection, ‘Then’ denotes concatenation and ‘Star’ denotes arbitrary repetition (cmp. [B3]).

In contrast to sect.IV-A2 regular expressions may not contain arbitrary macro calls. Otherwise, this could be considered as a non-right congruent derivation. This would be a context-free derivation. Regular expressions still can contain macro calls and can be unrolled initiated by a caller. In the following only those macros shall be considered, whose derivation terminates. This means whose expression is regular. In substitution of regular expressions by other regular expressions is according to the definition of tree grammars regular again (see sect.II-D2).

The node `<a id="1">` looks as ‘Reg’:

```
<a id="1">
  <xtl:attribute name="title" select="/AAA" />
</a>
```
The introduced regular expressions can be interpreted as OBDDs. OBDDs are graphical representations of terms. Terms consist of variables and terms again, constants and binary functors. The graphical notation proposed in fig. 5 is extended by the unary functors ‘Star’ and ‘ε’, completely covering all what is needed for OBDDs together with ‘Then’, ‘Or’ and ‘@’. Element nodes have a non-empty empty. Element nodes are already tree-structured and therefore can also be interpreted as nodes in OBDDs. Childless element nodes are leaves in a tree. This interpretation is isomorphic to boolean terms, where ‘∧’ and ‘∨’ represent the binary functors, and boolean variables represent leaves.

Fig. 5. Graphical Representation of Reg

The consideration of OBDDs has mainly two advantages here. First, the difference between nodes and hedge vanishes. A node representing a hedge with exactly one child is represented the same as a hedge with multiple children by Then. Second, the set of alternatives is represented the same way (cmp. [63], [17]). One advantage of a binary tree over a multi-way tree is a simpler specification of nodes.

Furthermore, the graphical notation presented makes functions safer, because less exception and cases need to be distinguished and in conclusion there are less places to commit an error by the developer here. There is either a ‘Epsilon’ or a ‘Then’, where it is agreed ‘Then’ may not have a ‘Epsilon’ as its left child. Same as lists, the OBDD-notation allows lazy evaluation, so infinite OBDDs are a meaningful completion to the test cases from sect.V.

A set of alternatives \{a_0, ..., a_n\} can within a ‘Then’ be arranged on different ways (see fig. 6). For instance, the ‘XTL’-node:

\[
\begin{align*}
\text{ElX} \ "a" & \ (\{(\text{id}, "1")\}) \ \text{[ElX} \ "b" \ [\ ] [\ ]]) \\
\text{is transferred into the ‘Reg’-node:} \\
\text{ElR} \ "a" & \ (\{(\text{id}, "1")\}) \ \text{Then (ElR} \ "b" \ [\ ] [\ ])
\end{align*}
\]

The validation can be simplified by normalisation of regular expressions. Arbitrary orderings of ‘Then’ and ‘Or’ are disallowed, which significantly reduces the amount of rules to be considered. It is agreed, that a regular expression is in normalform, if:

- No two text nodes are neighbours. Text inclusions are exempted.
- Strings can be separated only with difficulties. This is because it is hard to decide with truncation of a string is correct. The intention of two ‘xtl:text’ could be the merge into one given string. That is why a truncation may not make sense.
- ‘Then’ and ‘Or’ are right-associative.
This means, on both, on the left and on the right there is a regular expression and the left expression does not match with the parent node.

- `XAtt` directly underneath a `Then` as first contiguous sequence in the corresponding hedge `ElR` (cmp. [15]).

By the use of OBDDs qualitative properties may be probed (see sect[17] cmp. [13]), because less base cases require consideration.

The mapping XTL \(\mapsto\) Reg is total. Each `XTL`-element is injectively assigned to an element from `Reg`. If `select`-expressions are ignored (see sect[18] and arbitrary non-fixed repetitions can be introduced by using Kleene’s star operator. By doing this the mapping is from now on than no more invertible, s.t. the origin is reproducible exactly. `XMacro` can be mapped onto empty, since it is just added to a macro environment and no real regular expression is associated with it.

### Table V

| XTL Mapping | REG Mapping |
|-------------|-------------|
| XIf \(\_\_\_\_\_\) | Or Epsilon 12 |
| XForEach \(\_\_\_\_\_\) | Star 12 |
| XAtt name value \(\_\_\_\_\) | AttrR name value |
| XTxt select \(\_\_\_\_\) | TextR select |
| XInclude select \(\_\_\_\_\) | IncludeR select |
| ELX name attrs l \(\_\_\_\_\) | ELR name attrs 12 |
| XCallMacro mname \(\_\_\_\_\) | MacroR mname |

The exact mapping is in table[17]. `\_\_\_\_` denotes hedges which are recursively generated by the hedge `\_\_\_\_`. The inverse mapping from `Reg` onto `XTL` is not possible, not even by introducing `”don’t-care”` variables `\_\_\_\_` or referring to implicit macro environments. However, validation does not really insist on it.

4) Non-deterministic Finite Automaton: As already mentioned in sect[17] regular tree automata just perfectly fit when it comes to recognising tree-structured regular input data, for instance, as XML-documents are (cmp. [50]). The construction of a Non-deterministic Finite Automaton (NFA) over hedges is similar to that over strings, it follows the so-called `”toolbox”`-principle. This principle states all nodes are applied successively nodes, for instance during concatenation. After the end-points of one component are connected with each other, so the resulting automaton grows (cmp. [53]).

The partial-derivatives algorithm [9] derives only symbols lying in \(\pi(t)\), where \(\pi\) denotes the set of all valid beginnings, and \(t\) denotes an arbitrary yet to be determined regular expression. Due to the homomorphism for regular expressions (see sect[19]), the calculations can be placed within the remainder field modulo a literal. In the congruency \(a \equiv b \mod (c)\), \(a\) stands for an initial regular expression, \(b\) stands for the abstracted congruency reduced by \(c\), and \(c\) is the remainder partition or with other words another possible beginning of \(a\), so \(c \in \pi(a)\).

The construction of the corresponding NFA always depends on the current derivation. All calculated derivations are put into a hashing table. Every time an expression is to be derived, it is first checked whether this derivation already is in the hashing table. For the original schema \(x^* \cdot (x + y)^*\) either \(x\) (4.1) or \(y\) (4.3) can be derived. Due to the star-operator (4.1) can either have the same expression again or a \(x\), because \(x^*\) would have been removed from \(x^* \cdot (x + y)^*\) and from the second subexpression \((x + y)^*\ x\) would follow after \(x\), which, however, would follow another \((x + y)^*\) (4.2). The congruencies (4.4)-(4.6) can be obtained after further derivations. Until (4.6) all right sides are determined. So, the corresponding NFA has no new transitions to be added (cmp. app[19]).

\[
x^* \cdot (x + y)^* \equiv x^* \cdot (x + y)^* \mod (x) \quad (4.1)
\]
\[
x^* \cdot (x + y)^* \equiv x \cdot (x + y)^* \mod (x) \quad (4.2)
\]
\[
x^* \cdot (x + y)^* \equiv (x + y)^* \mod (y) \quad (4.3)
\]
\[
x \cdot (x + y)^* \equiv (x + y)^* \mod (x) \quad (4.4)
\]
\[
(x + y)^* \equiv x \cdot (x + y)^* \mod (x) \quad (4.5)
\]
\[
(x + y)^* \equiv (x + y)^* \mod (y) \quad (4.6)
\]

Such an approach is appropriate for complex schemas, which may be reused (cmp. sect[17]). The state-based approach is also appropriate for error location. However, multiple transitions leaving a terminal and \(\varepsilon\)-transitions may make recognition not determined. Determination of the NFA (see [23]) comes for the cost of Rabin-Scott’s powerset construction. A validation would be beneficial only if the automaton was determined. The additional cost pays off if multiple instances are validated with the same DFA.

On the one side there is the direct approach as described in sect[17]. On the other side there is the graph-based approach. The validation problem can be interpreted as path problem in a graph.

### B. Instantiation

Before presenting the denotational semantics for instantiation and validator a brief discussion on semantics should sum up pros and cons.

1) Semantics Form: As already described in sect[17] the untyped \(\lambda\)-calculus and attribute grammar, both are not really appropriate for the description of the instantiation semantics. A logical model seems reasonable at the first glance, since a matcher-algorithm would be needed to be designed (cmp. sect[17]). A first implementation may even allow back-tracking until an optimisation could be found.

The structure to be matched against can certainly be represented as term expression (cmp. [31]). Both, schema and instance, and helper functions can be transformed into Horn-clauses. Functions can only poorly be expressed by relations, since logical programming mainly interprets terms and relations. Functions, however, have "only" static mappings. Apart from that future implementations in an object-oriented programming language should guarantee referential transparency and the evaluation ought to proceed forward. Concepts like backtracking and cuts in a logical
programming language, however, disallow this – just to mention some on the example of Prolog, for instance.

For this reasons the semantics of instantiation and validation should be applied to the functional paradigm (cmp. [2], [61]).

The semantics shall be as easy as possible by using type constructors, so a comprehending implementation in Haskell matches the denotational semantics. Node specifications using type constructors describe interfaces and attributes of classes (cmp. sect.IV-D).

2) Semantic: The complete semantics for instantiation and validation are enclosed in app.IX. Instantiation turns a template in an instance. The instance document can be represented as XTL-term (cmp. sect.IV-A2). The denotational semantics is described in Haskell. Next some helper functions are going to be described in the denotational semantic:

The functions:

\[
\begin{align*}
\text{filter} & \colon (a \to \text{Bool}) \to [a] \to [a] \\
\text{concatMap} & \colon (a \to [b]) \to [a] \to [b]
\end{align*}
\]

are defined in the GHC-package GHC.List. The function ‘filter’ filters any given list using a predicate (simply means a function returning a boolean value here). So, for instance, filter (odd) [1..10] returns as result the list [1, 3, 5, 7, 9]. The function ‘concatMap’ applies for any given list for each element some given mapping, where for each list element a list type is returned as element of the co-domain. Later all created sub-lists are concatenated with each other. concatMap (\(x\to[\{x\}]) \ [1..10]\ returns "aaaaaaaaaaa".

The function ‘qSort’ of type ‘Ord a \to [a] \to [a]’ is a generalised (so-called lifted) function sorting a list of any comparable type. ‘qSort’ is used for canonicalisation.

The functions \(f_1, f_2, f_3, f_4\) denote the agreed PHP-functions (cmp. sect.III). These functions describe in the given order access to external texts, the filtering mode if there are multiple solutions, the checking on satisfiability and the return of an element node. The corresponding types can be found in tab[IV]. The type ‘a’ denotes arbitrary instantiation elements, for instance, element nodes in case of XML-documents. Be aware of ‘XInclude’. It always produces a ‘XmlTree’ to the instance document. The result of function \(f_0\) is ‘[a]’. This means the polymorphic results are being passed through as context to the loop body.

The helper function getMacro of type ‘[XTL] \to [XTL]’ from (A3) returns for a macro environment \(\mu\) the corresponding macro body. It is further assumed, a matching macro is defined prior to calling it. Otherwise, an according error message shall be dumped. The name of the wanted macro definition is transferred by the surrounding rule by name \(\text{m}_1\).

The abbreviation (S) stands for the starting rule of instantiation, (E) stands for elimination of XTL-attributes, which are located directly under the top-level element node. Rules (I1)-(I3) initiate instantiation by filtering all macro-definitions first and passing those to the instantiation second. Rules (A1)-(A7) are the core instantiation rules for XTL-tags and non-XTL-tags.

The helper function \(\varepsilon^\text{MA}\) of type ‘XTL\to\text{Bool}’ checks for a given ‘XTL-node if it is in fact a XTL-attribute or not. In analogy, \(\varepsilon^\text{MM}\) checks for macro-definitions. The semantic fields include macro-environment and context (see app.IX). Here, a macro environment \(\mu\) is built up in (I3). \(\mu\) consists of the mapping ‘‘-Macro-Name\times XTLL-Hedge’’. The hedge represents a list. The macro environment \(\mu\) is an invariant part in \(\varepsilon\) (see app.IX). Tab.VII demonstrates the evaluation of the macro environment (cmp. sect.II-C) until the selected row.

Context has multiple purposes, one of which is simplification. For instance, the simplification of XPath-expressions (see sect.II–III). It returns a node or a value of arbitrary type and is used while calling PHP-functions as instantiation data source.

The mappings use the domains from tab.VIII. ‘XTL’ describes the domains in more detail in sect.II-C. ‘[XTL]’ denotes a hedge of type ‘XTL’. The hedge type is a list in terms of denotational semantic. It is worth noting the type is monadic.

The considered denotational semantic variables are used quite often. Anonymous and ‘‘-don’t-car’’ variables are marked as ‘\_’. These variables often stand in constructor specification and cannot be addressed in a rule’s body. Apart from that there are underlined variables written in italics – these specify XTL-nodes and fragments of it. These can also be memoised. For example, XTL-attributes in (E) can be interrupted and escaped prior to reaching (E)’s end. The same counts for non-XTL-attributes in nodes too. Non-underlined
variables with a Greek letter \( \pi \) and \( \mu \) denote each PHP-tuple \((f_0^1, f_0^2, f_0^3, f_0^4)\) and also macro environment. Functions are written in italics and each has an upper index, which denotes the arity of that function. The only exceptions are doubly-indexed PHP-function from \( f_0^3 \) to \( f_0^4 \). The arities of those functions is slightly different and is fully listed in tab[V]

Rules of the denotational semantic

The rule

\[(S)\] \( \mathcal{E}^\text{Start} \circ \xi_\pi := \mathcal{E} \circ \xi \circ \pi \) for \( \xi \).

\( \mathcal{E} \) denotes the template, \( \xi \) denotes the context, and \( \pi \) denotes the placeholder-function 4-tuple. The given template is a non-empty XTL-node. The context is the source document at the beginning. Because \( \xi \) may also have XTL-attribute in the hedge, \( \xi \) is reduced first of all with \( \mathcal{E} \).

\( \mathcal{E} \) requires one 'XTL' only:

\[(E)\] \( \mathcal{E} \circ \xi \text{XAtt} \) a \( \mathcal{E} \circ \xi \text{XAtt} \) a.

\( \mathcal{E} \) and \( \pi \) denote the template, \( \xi \) denotes the context, and \( \pi \) denotes the placeholder-function 4-tuple. The given template is a non-empty XTL-node. The context is the source document at the beginning. Because \( \xi \) may also have XTL-attribute in the hedge, \( \xi \) is reduced first of all with \( \mathcal{E} \).

This means the hedge of type \([\text{XTL}]\) establishes, after a XTL-node, instantiation data of type 'a', and a macro environment are passed. The macro environment consists of a list of a list of tuples herewith, where each tuple has the mapping "'Macro-Name'\to'Macro-Body'". The macro body is a hedge of type \('[\text{XTL}]' \) and may contain all XTL-tags except 'XMacro'.

Instantiation is described by rules (A1) until (A7). Notably, the tag 'XMacro' is missing. This is because the extraction of \( \mu \) happens before. Furthermore, all 7 rules are complete, w.r.t. XTL-tags are not imposed any further restrictions. The rules are not prioritised. Element nodes which are non XTL-tags are handled as common element nodes in the semantic (cmp. sect[IV-A2]).

The handling of conditions is in (A1).

\[(A1)\] \( \mathcal{E} \circ \text{XIf} \) \( \xi \) \( \mathcal{E} \circ \text{XIf} \) \( \xi \) \( \mathcal{E} \circ \text{XIf} \) \( \xi \) \( \mathcal{E} \circ \text{XIf} \) \( \xi \).

In order to do that the string \( \xi \) is passed to the PHP-function \( f_0^3 \) together with \( \xi \), which is the source of instantiation data. If \( f_0^3 \) succeeds (cmp. sect[II]) it returns 'True'. 'False' otherwise. The Haskell syntax implies that on success instantiation continues with child node \( \xi \). In case of error it returns an empty list. However, empty list is neutral w.r.t. list concatenation, and this why no special handling is required on the caller's side.

The handling of loops is done in (A2).

\[(A2)\] \( \mathcal{E} \circ \text{XForEach} \) \( \xi \) \( \mathcal{E} \circ \text{XForEach} \) \( \xi \) \( \mathcal{E} \circ \text{XForEach} \) \( \xi \) \( \mathcal{E} \circ \text{XForEach} \) \( \xi \).

The variable \( \xi \) binds all nodes of type 'a', which establish during the evaluation of the 'select'-expression \( \xi \), the source of instantiation data \( \xi \), and the PHP-function \( f_0^3 \). In case \( \xi \) equals an empty list, then function \( \mathcal{E} \) results in the empty set. This is because Haskell has a non-stric evaluation order. Otherwise, first, instantiation continues for each child node \( \xi \) of hedge \( \xi \), and second, all determined hedges are then concatenated together into eventually one resulting hedge. It is worth noting, the list \( \xi \) is successively propagated through to \( \mathcal{E} \) using the bound variable \( \xi \). Each element of \( \xi \) of type 'a' is passed over to each child node \( \xi \) for hedge \( \xi \) as new instantiation data \( \xi \).

The handling of macro calls is done in (A3).

\[(A3)\] \( \mathcal{E} \circ \text{XCallMacro} \) \( \xi \) \( \mathcal{E} \circ \text{XCallMacro} \) \( \xi \) \( \mathcal{E} \circ \text{XCallMacro} \) \( \xi \) \( \mathcal{E} \circ \text{XCallMacro} \) \( \xi \).

The function 'getMacro' was mentioned initially. It determines for a given macro name \( \xi \) and a macro environment \( \xi \) the corresponding macro body \( \xi \), which is a hedge. The instantiation proceeds for each node sequentially from the
hedge with the same initial context \( s \), PHP-tuple \( \pi \) and the same macro environment \( \mu \). The resulting hedge as list is
concatenated, so the returned value of this rule has type `XTL`.

The inclusion of text follows rule (A4).

(A4) \( \mathcal{E}^a\{\text{XTxt } \xi\}([s, \pi, (\tilde{t}_0, \_ \_ \_)]) := [\text{XTxt } \tilde{t}_0 \_ \_ \_] \)

Access is granted by the function \( \tilde{t}_0 \) for each ‘select’-expressions \( \xi \) and instantiation data \( s \). The result of the
instantiation is a singular list of text nodes, which is determined by the evaluation. Access to XML-nodes of instantiation data is granted by rule (A5), which is in analogy to (A4) except that function \( \tilde{t}_0 \) returns a XML-node.

(A5) \( \mathcal{E}^a\{\text{XInclude } \xi\}([s, \mu, (_, \_ \_ \_ \_ \_ \_)]) := [\tilde{t}_0 \_ \_ \_] \)

Instantiation of common nodes is done by (A6)-(A7).

(A6) \( \mathcal{E}^a\{\text{ElX } n \_ \_ \_ \_ \_ \_ \_ \_ \_ \_}]([s, \mu, \pi]) := 
\{\text{ElX } n \_ \_ \_ \_ \_ \_ \_ \_ \_ \_}] (\text{concatMap} (\lambda \text{child}. \mathcal{E}^a\{\text{ElX } \text{child}\}[s, \mu, \pi])) \}

(A7) \( \mathcal{E}^a\{\text{TtxtX } \xi\}([\_ \_ \_ \_ \_ \_ \_ \_]) := [\text{TtxtX } \xi] \)

Element nodes are taken as is. Instantiation proceeds with children nodes with the same instantiation data, macro environment and PHP-tuple. Text nodes are just copied unconditionally.

Example

Let the following document be given

\[
<\text{books}>
  <\text{xt:for-each select="//book">}
  <\text{title}>
    <\text{xt:text select="@title"/>}
  </\text{title}>
  </\text{xt:for-each}>
</\text{books}>
\]

where \( \xi \) is the bibliography document wanted from sect.[II-A]

So, the following then holds:

\[
\begin{align*}
\pi &= \text{ElX } \text{"books" } [] \text{ [XForEach } "//book" \text{ [ElX } \text{"title" } [] \text{ [Ttxt } \"@title\"]]} \\
\phi &= \text{ElX } \text{"books" } [] \text{ [XForEach } "//book" \text{ [ElX } \text{"title" } [] \text{ [Ttxt } \"@title\"]]}
\end{align*}
\]

where

\[
\begin{align*}
\text{sel1} &= \text{ElX } \text{"book" } [[\text{"author"}, \text{"Simon ..."}], \text{["title"}, \text{"Has..."}]] [] \\
\text{sel2} &= \text{ElX } \text{"book" } [[\text{"author"}, \text{"Joshua ..."}], \text{["title"}, \text{"Re..."}]] []
\end{align*}
\]

denote element nodes from \( \phi \). These are reused as following:
The derivation during instantiation starts with \( \mathcal{E}\{\text{Start}[\] and is interrupted on first occurrence of ‘...’ by \( \mathcal{E}\{\). The remaining segments are composed in analogy to that, and are also disrupted by minor calculations.
C. Validation

Schema and instance are represented as regular expressions on validation (see sect.IV-A3). Instances, however, do not have alternatives, no star-operator, no XTL attributes and no text nodes – in contrast to schemas. Instances can be described neatly by ε and "Then". In the normalization form the last node of a recursive node in an OBDD ε is used. "Then" is recommended for node concatenation to a hedge.

This means, a matcher (see sect.II) must recognize the cases from fig.7. The left side denotes the set of instance nodes, and the right side denotes the set of schema nodes. Obviously, all 32 cases of the bipartite graph need to be handled, because instances a priori do not contain XTL-tags (cmp. [32]).

XTL is well-formed and safe according to sect.II. Except bypasses and node inclusion, there is no other way to generate element nodes. A matching of hedges with node inclusion is no longer considered here, because the essential question researched here is w.l.o.g. if two generated languages are the same or not – and for that node inclusion does not matter in practice. In order to address this issue, equality solver are needed, which would determine solutions just from the relations. Here, a solution does not necessarily have to exist (see [23]). So, it is guaranteed element nodes of the instance indeed match only with tags that are not XTL Commands – meaning only element nodes.

1) Semantic: The validation bases on regular expressions 'Reg', which are mapped onto boolean values Μ. Interpretations of Μ map onto {True, False}. All XML-instances and XTL-schemas are definable over 'Reg' (see sect.IV-A3). Macros consist of a head, which is determined by a macro name, and a macro body, which is a hedge of element nodes. The hedge is represented by a top-level 'Then' and which is a OBDD. The information about which macro name is assigned which macro body must be available on validation as context information μ. The macro environment μ has the typing String→Reg.

The following semantic-rules variables will be used differently, namely for the specification of instance and schema nodes, and or strings. This allows a shorter rule representation than without and by memoisation it avoids redundant calculations.

The validation is done by:

\[
\mathcal{E}^\alpha_1 \in \mathbb{S} : \text{Reg} \to \text{Reg} \to \text{Bool}
\]

where \(\mathbb{S}\) denotes the instance and \(\mathbb{S}\) the schema document.

Used helper functions are listed in tab.X. The function \(\text{qSort}\) is a lifted function of type \([a] \to [a]\) and is used, particularly, for canonisation. The functions frontSplits and \(\text{splits}\) divides non-deterministically a regular expression into two disjoint parts. All parts of a 'Reg' are calculated lazily. In contrast to \(\text{splits}\), frontSplits calculates a partition less, namely it skips the trivial partition (Epsilon,Reg). The partition is considered here as inversion of the 'Then'-
concatenation. It works in analogy to the non-deterministic partition of strings in \texttt{frontSplitText} and \texttt{splitText}.

For instance, \texttt{splitText} "ab" returns the list \(["a", "b"]\), \(["a", "b"]\), \(["a", "b"]\), but \texttt{frontSplitText} "ab" only returns the two last tuples.

The function \texttt{extractAttributes} extracts from an element node given by a 'Reg' all attributes 'AttrR' and adds to pre-existing ones into the element node. The function \texttt{getMacro} scans \(\mu\) and find for a given macro name the matching body which is a 'Reg'. It is assumed during validation all macros are defined prior to running it.

The validation of a hedge is successful, if all children of the schema successively match – however, not necessarily with the same position index. This is implied by the logical operator '∧'. Alternatives are evaluated by '∨'.

**The rules of the denotational semantic**

The rules of the denotational semantic obey fig\[
\text{Fig. 7. Interleaving of cases}
\]
\] In order to illustrate the topic for the sake of a better understanding and didactics validation rules are not going to be explained just sequentially.

First of all, rules do not follow any order a priori here. However, it is still agreed upon prioritisation of listed rules. This shall mean the precedence of interleaving rules raises with the rule number. The following rules can so be described shorter: Moreover, it is agreed upon in the matching relation \(\cong\) (see sect.\[ III-A\]) first argument in is the instance \(I\) to the left, and second argument the schema \(S\) to the right. Hence, \(\cong\) as denotational semantic can be divided into four partitions according to the 'Reg'-structure of the instance.

(\textit{E1}) \(\mathcal{E}([\text{Epsilon}, \text{TextR}])\mu := True\)
(\textit{E2}) \(\mathcal{E}([\text{Epsilon}, \text{TextR}])\mu := False\)
(\textit{E3}) \(\mathcal{E}([\text{Epsilon}, \text{Epsilon}])\mu := True\)
(\textit{E4}) \(\mathcal{E}([\text{Epsilon}, \text{ElR}])\mu := False\)
(\textit{E5}) \(\mathcal{E}([\text{Epsilon}, \text{Star}])\mu := True\)
(\textit{E6}) \(\mathcal{E}([\text{Epsilon}, \text{TextR}])\mu := True\)
(\textit{E7}) \(\mathcal{E}([\text{Epsilon}, \text{Then}])\mu := True\)

In the first paragraph empty instance nodes match with empty text nodes (E1) and (E6), and empty schema nodes (E3) matches with star-operator (E5) Exemptions apply to element nodes (E4) and non-empty strings (E2). The rules (E2) and (E1) overlap, because (E1) is a special case of (E2). However, the relatively simple rule (E2) is preferred over an explicit representation. That is why empty text nodes shall first match with (E1). (E7) considers the case a schema has a concatenation, whose left 'Then'-branch is not \(\varepsilon\) – but this node could still be derived to \(\varepsilon\). That is why both branches of the schema-node must be derivable to \(\varepsilon\).

The second passage treats concatenations of instance nodes. This means hedges. Since the normalform of OBDDs in instances excludes two consecutive \(\varepsilon\) as well as text nodes (see sect.\[ IV-A\]), and since the left branch of a 'Then' may not be empty, it can be inferred an instance-hedge is not derivable to \(\varepsilon\) (Then1).

(Then 1) \(\mathcal{E}([\text{Then}])\mu := False\)
Moreover, the normalform implies that a hedge either entirely contains a string in the left branch and the right branch validates against \(\varepsilon\), or no validation at all is valid here (Then2). The derivation of the right branch is needed, because the left branch does not have to be syntactically identical – it could also be the result of a hedge evaluation that requires attention.

(Then 2) \(\mathcal{E}([\text{Then}])\mu := True\)
This is different with:

(Then 8) \(\mathcal{E}([\text{Then}])\mu := True\)
(Then 9) \(\mathcal{E}([\text{Then}])\mu := False\)

Because an instantiation of a 'txt1:text' insists a text node in the instance document follows, there is no other validation (Then9).

The validation of a hedge with the element node at the beginning (Then3) can only be successful against an element node in the schema, if both element nodes are homomorphic w.r.t. validation and all remaining nodes of the hedge validate against \(\varepsilon\). All other cases lead to an unsound instance (Then4).

(Then 3) \(\mathcal{E}([\text{Then}])\mu := True\)
(Then 4) \(\mathcal{E}([\text{Then}])\mu := False\)

Rule (Then5) attempts to non-deterministically match a repeating sequence from a hedge. Here the given sequence
is divided, s.t. the first part with hedge from the cycle, and the right part matches with the whole cycle which may also be empty. Only if in the given hedge there is a repetition, then the right 'Then'-branch may match with the remaining hedge. One corner cases occurs when the left 'Then'-branch contains the whole cycle, so the remaining hedge ε successfully matches with Star ε.

(Then 5) $E \text{TxtR \ 'text', Then } r_1 \ r_2\ \mu := \forall [True]$

$(s_1, s_2) \leftarrow \text{frontSplits}(r_1, r_2) \land E[s_1, s_2] \land E[s_2, Star \ s_2] \mu$

(Then 6) $E \text{Then } r_1 \ \text{Epsilon}, \text{Then } s_1 \ s_2 \ \mu := \forall [True]$

$(s_1, s_2) \leftarrow \text{frontSplits}(r_1, r_2) \land E[s_1, s_2] \mu \land E[s_2, Star \ s_2] \mu$

(Then 7) $E \text{Then } r_1 \ r_2, \text{Then } s_1 \ s_2 \ \mu := \forall [True]$

$(t_1, t_2) \leftarrow \text{splitText}'(t_1, t_2) \land E[t_1, t_2] \land E[s_1, s_2] \mu$

The third passage considers text nodes. Empty text nodes are derivable to ε (#2), non-empty texts, however, cannot be derived to ε (#3). Star-operators are also derivable to ε, so empty text nodes are derivable to arbitrary star-operators (#4).

(Then 8) $E \text{TxtR ' ', Epsilon} \mu := \forall [True]$

(Then 9) $E \text{TxtR ' ', Star} \mu := \forall [True]$

It still is possible (#6), that 'xtl: 'text' in the instance can generate arbitrary text as output.

(Then 6) $E \text{TxtR 'text', TextR ' ' } \mu := \forall [True]$

The comparison of two text nodes (#7) is trivial, the same as a validation of a text node against an arbitrary element node is (#8).

(Then 7) $E \text{TxtR 'text1', TxtR 'text2'} \mu := text1 == text2$

(Then 8) $E \text{TxtR 'text', ElR ' ' ' } \mu := \forall [True]$

The validation of a string against a hedge is similar to a validation of element nodes against a hedge (#5). A repeating pattern is searched. If no non-deterministically obtained partitions matches, so the considered string may only derive to ε.

(Then 5) $E \text{TxtR 'text', Star ' ' } \mu :=$

if (h == True) then True else $E \text{TxtR 'text', Epsilon} \mu$

for h := $\forall [True]$

$(s_1, s_2) \leftarrow \text{frontSplitText}'(text)$

$E[s_1, s_2] \mu \land E[s_2, Star s_2] \mu$

The validation against a hedge is only valid, if a valid, possibly empty, partition of a hedge exists, so both texts concatenated equals the wanted text.

(#1) $E \text{TxtR 'text', Then } r_1 \ r_2\ \mu := \forall [True]$

$(s_1, s_2) \leftarrow \text{splitText}'(text) \land E[s_1, s_1, r_1] \mu \land E[s_2, s_2, r_2] \mu$

The fourth passage considers element nodes as instance. Element nodes are in contrast to text nodes atomic. This means an element cannot be part of another element node at the same level. This atomicity leads in (ElR8), (ElR9) and (ElR10) to schema-tags are excluded from the very beginning.

(ElR8) $E \text{ElR 'name', TextR ' ' } \mu := \forall [True]$

(ElR9) $E \text{ElR 'name', Epsilon} \mu := \forall [True]$

(ElR10) $E \text{ElR 'name', TextR ' ' } \mu := \forall [True]$

Case (ElR7) states an element node validates against a star-operator only, if the subexpression underneath validates against the instance node. So, the loop body's beginning and ending, both are nodes derivable to ε, and in between there is only the element node.

(ElR7) $E \text{ElR 'name', AttrR 'attribute', Star ' ' } \mu := E \text{ElR 'name', AttrR 'attribute', ' ' } \mu$

Rule (ElR1) may only look trivial at the first glance. It validates an element node against another. It is important to notice, however, besides equality of the element name there is also the set of existing schema nodes that need to match after canonisation. In case of schema-nodes 'AttrR' also need to be considered. In any case validation has to continue with the reduced schema hedge r_3.

(ElR1) $E \text{L, R } \mu :=$

$L = \text{ElR 'name1', AttrR 'attribute1', R1}$

$R = \text{ElR 'name2', AttrR 'attribute2', R2}$

$(\text{name1} = \text{name2}) \land (\text{qSort 'attribute1' = 'attribute2'})$

$\forall [True]$

$E[r_1, r_2] \mu$

for (ElR ' ' 'attribute3', r_3) = extractAttributes (ElR 'attribute3', ' ' 'attribute3')

Validation against Then cannot just proceed. In fact, it needs to answer first the question which element ought to be validated really first. So, π(Δ) would need to be determined, which is not, at least not initially (see sect. III). So the left branch of 'Then' needs to be checked. In dependency of that the cases (ElR2), (ElR3), (ElR4) and (ElR6) result for all remaining schema nodes.

(ElR6) $E \text{ElR ' ' 'attribute', Then ' ' ' } \mu := \forall [True]$

The cases (ElR2), (ElR3) and (ElR4) are obvious and do not require further explanations.

(ElR2) $E \text{L, R } \mu :=$
\[ L = \text{EIR } \text{name1 } \text{atts1 } r_1, \]
\[ R = \text{Then } (\text{EIR } \text{name2 } \text{atts2 } r_2) \implies \]
\[ \varepsilon \ll L, \text{EIR } \text{name2 } \text{atts2 } r_2 \ll \mu \]
\[ \land \varepsilon \ll \text{Epsilon}, \varepsilon \ll \mu \]

(EIR3) \[ \varepsilon \ll L, R \ll \mu := \]
\[ L = \text{EIR } \text{name1 } \text{atts1 } r_1, \]
\[ R = \text{Then } (\text{Or } s_1 s_2) \implies \]
\[ (\varepsilon \ll \text{EIR } \text{name1 } \text{atts1 } r_1, \text{Or } s_1 s_2) \ll \mu \]
\[ \land \varepsilon \ll \text{Epsilon}, \varepsilon \ll \mu \]
\[ \lor (\varepsilon \ll \text{Epsilon}, \text{Or } s_1 s_2) \ll \mu \]
\[ \land \varepsilon \ll \text{EIR } \text{name1 } \text{atts1 } r_1, \varepsilon \ll \mu \]

In case of (EIR5) it is checked, if the element node from the instance is in the macro body. – If so, the following hedge \( \varepsilon \) has to be derivable to \( \varepsilon \). Alternatively, the element node is in \( \varepsilon \). Then the macro body must be derivable to \( \varepsilon \).

(EIR5) \[ \varepsilon \ll L, R \ll \mu := \]
\[ L = \text{EIR } \text{name1 } \text{atts1 } r_1, \]
\[ R = \text{Then } (\text{MacroR } m) \implies \]
\[ (\varepsilon \ll \text{EIR } \text{name1 } \text{atts1 } r_1, \text{MacroR } m) \ll \mu \]
\[ \land \varepsilon \ll \text{Epsilon}, \text{MacroR } m \ll \mu \]
\[ \lor \varepsilon \ll \text{EIR } \text{name1 } \text{atts1 } r_1, \text{MacroR } m \ll \mu \]

Both rules (\( \Phi \)) and (\( \Omega \)) are universal w.r.t. instance nodes. A macro call in the schema has – independent from the actual instance node – an unfolding or substitution by the macro body in consequence, which is represented by exactly one \( \text{OBDD-expression} \).

(\( \Phi \)) \[ \varepsilon \ll \text{inst}, \text{MacroR } m \ll \mu := \]
\[ \varepsilon \ll \text{inst}, \text{word} \ll \mu \]
\[ \text{for word} = \text{getMacro } m \ll \mu \]

The same is with ‘Or’ in the schema – independent from the concrete instance a matching case \( r_1 \) or case \( r_2 \) is validated.

(\( \Omega \)) \[ \varepsilon \ll \text{inst}, \text{Or } r_1 r_2 \ll \mu := \]
\[ \varepsilon \ll \text{inst}, r_1 \ll \mu \lor \varepsilon \ll \text{inst}, r_2 \ll \mu \]

Example

The following example shows the validation of a simple document. Let the schema \( \varepsilon \) be given from fig.\( 8(a) \) and a corresponding instance document \( \iota \) from fig.\( 8(b) \).

The textual notations are:

\[ \varepsilon = \text{EIR } \text{"book" } [ ] \text{Then } (\text{EIR } \text{"title" } \text{Then } (\text{TxtR } \text{"Haskell") Epsilon)} \text{Then } (\text{Star } (\text{EIR } \text{"checked" } [ ] \text{Epsilon})) \text{Then } (\text{EIR } \text{"author" } [ ] \text{Then } (\text{TxtR } \text{"Simon...") Epsilon})) \text{Then } (\text{EIR } \text{"author" } [ ] \text{Then } (\text{TxtR } \text{"Simon...") Epsilon}) \text{Then } (\text{EIR } \text{"checked" } [ ] \text{Epsilon}) \text{Epsilon} \]

\[ \iota = \text{EIR } \text{"book" } [ ] \text{Then } (\text{EIR } \text{"title" } \text{Then } (\text{TxtR } \text{"Haskell") Epsilon)} \text{Then } (\text{Star } (\text{EIR } \text{"author" } [ ] \text{Then } (\text{TxtR } \text{"Simon...") Epsilon})) \text{Then } (\text{EIR } \text{"author" } [ ] \text{Then } (\text{TxtR } \text{"Simon...") Epsilon}) \text{Then } (\text{EIR } \text{"checked" } [ ] \text{Epsilon}) \text{Epsilon} \]

\[ a_1 = \text{Then } (\text{EIR } \text{"author" } [ ] \text{Then } (\text{TxtR } \text{"Simon...") Epsilon}) \text{Epsilon} \]
\[ a_2 = \text{EIR } \text{"author" } [ ] \text{Then } (\text{TxtR } \text{"Simon...") Epsilon} \]
\[ a_3 = \text{EIR } \text{"title" } [ ] \text{Then } (\text{TxtR } \text{"Haskell") Epsilon} \]

The derivation looks as following:
$\mathcal{E}[\text{l, s}]$

$\mathcal{E}[\text{l, E1R "book" [ ] ...}]
\quad L=E1R "book" [ ] ...
\quad ("book"=="book") \land \mathcal{E}[Then (E1R "title" [ ]..., r3)]$

for $(E1R \_ \_ \_atts3 \_ r3) = extractAttributes s$

$\mathcal{E}[Then (E1R "title" [ ]..., r3)]$

$\mathcal{E}[L=Then (E1R "title" [ ]..., Then (r3))]
\quad for (E1R \_ \_atts3 \_ r3) = s$

$\mathcal{E}[L=Then (E1R "title" [ ]..., Then (r3))]
\quad for (E1R \_ \_atts3 \_ r3) = s$

$\mathcal{E}[L=Then (E1R "author" [ ]...]
\quad R=Then (Star ...)
\quad (Then6)

$\mathcal{E}[L, R]$

$L=\text{E1R "title" [ ]...},$

$R=\text{E1R "author" [ ]...},$

$\land \mathcal{E}[L2, Then (Star ...),
\quad L2=Then (E1R "author" [ ]...]

$\mathcal{E}[Then (E1R "author" [ ]...],
\quad R=Then (Star ...)]$

$(t1,t2) \leftarrow \text{splits (Then L R)}$

$L=(\text{E1R "title" ...)},
\quad R=\text{Then (E1R "author" ...),}
\quad \land \mathcal{E}[t1, \text{E1R "title" ...}]
\quad \land \mathcal{E}[t2, \text{Then (Star ...)}]

= \land \land \land \lor$

$(t1,t2) \leftarrow \text{splits (Then L Then ...)}$

$L=(\text{E1R "author" ...}),$

$R=\text{Then (Star ...),}
\quad \land \mathcal{E}[L2, \text{Then R Epsilon}]
\quad R=\text{(E1R "checked" [ ] Epsilon)}$

$\lor \lor \lor \land$

$\mathcal{E}[S=\text{Star (E1R "author" [ ] R),
\quad R=Then (TextR ".") Epsilon,}
\quad \land \mathcal{E}[L2, R2],
\quad L2=\text{Then (E1R "author" Epsilon),}
\quad R2=\text{Then (E1R "author" Epsilon)}
\quad \land \mathcal{E}[\text{E1R "checked" [ ] Epsilon,}
\quad R2=\text{Then (E1R "checked" [ ] Epsilon)}$

$\land \land \land \lor$

$(t1,t2) \leftarrow \text{splitsText "Simon..."}$

$L=\text{Then (TextR "Haskell") Eps, r3}$

$\text{for (E1R \_ \_atts3 \_ r3) = extractAttributes (E1R "title" [ ] Then (TextR ".") Epsilon)}$

$\lor \lor \lor \land$

$\mathcal{E}[\text{E1R "author" [ ] R}]
\quad R=\text{Then (TextR ".") Epsilon}$

$\land \land \land \lor$

$(t1,t2) \leftarrow \text{splitsText "Haskell..."}$

$L=\text{Then (TextR "Haskell") Eps, r3}$

$\text{for (E1R \_ \_atts3 \_ r3) = EIR "title" [ ] R}$

$\lor \lor \lor \land$

$\mathcal{E}[\text{E1R "author" [ ] R}]
\quad R=\text{Then (TextR "Haskell") Epsilon}$

$\land \land \land \lor$

$(t1,t2) \leftarrow \text{splitsText "author..."}$

$L=\text{Then (TextR "author") Eps, r3}$

$\text{for (E1R \_ \_atts3 \_ r3) = extractAttributes (E1R "author" [ ] Then (TextR ".") Epsilon)}$

$\lor \lor \lor \land$

$\mathcal{E}[\text{E1R "author" [ ] R}]
\quad R=\text{Then (TextR ".") Epsilon}$

$\land \land \land \lor$

$(t1,t2) \leftarrow \text{splitsText "author..."}$

$L=\text{Then (TextR "author") Eps, r3}$

$\text{for (E1R \_ \_atts3 \_ r3) = extractAttributes (E1R "author" [ ] Then (TextR ".") Epsilon)}$
\[ E[l \in R \text{ "checked" } \{\} \text{ Eps}, R] \]
\[ \text{Then } (E[l \in R \text{ "checked" } \{\} \text{ Eps}) \text{ Eps} = E[L, L] \]
\[ \text{L} = E[l \in R \text{ "checked" } \{\} \text{ Epsilon} \]
\[ \Rightarrow \text{Eps}_1 \text{ Epsilon} \]
\[ (\text{"checked"}{=}\text{"checked"}) \]
\[ \text{Eps}_2 \text{ Epsilon} \]
\[ \text{for } (E[l \in \text{atts}_3 \text{ r}_3]) = \text{extractAttributes}(E[l \in R \text{ "checked" } \{\} \text{ Eps}) \]
\[ (qSort[\{}{=}\text{atts}_3) \]
\[ \text{Eps}_3 \text{ Epsilon} \]
\[ \text{for } (E[l \in \text{atts}_3 \text{ r}_3]) = \text{Eps}_4 \text{ Epsilon} \]
\[ \text{True} \]

In the derivation below the equality sign the applied validation rules are provided. Rules labelled with ‘(nd)’ indicate non-deterministic selection is made. When searching a solution, the program requires numerous executions which have to be refined each time. The rule ‘(nd)’ therefore is only of didactic help. The continuations ‘...’ are shortened regular expressions. Those complete preceding rules.

Although the selected instance document is rather small, the derivation shows that even a few non-deterministic cases can lead to extensive search.

V. IMPLEMENTATION

A. Overview

Haskell is recommended for implementing denotational semantics, because of the lack of side-effects and its functional paradigm. Haskell’s features are very close to the syntax and semantic of denotational semantics. It includes, for instance, higher-order functions, a strict static typing, data encapsulation, lazy evaluation and generic polymorphism \[63\], \[62\]. Some functional programming languages, like LISP or Miranda, however, do not have at least one of the mentioned advantages. This is the reason why Haskell is chosen.

The Haskell XML-Toolbox (HXT) \[58\] provides a huge library of functions processing XML. In version 7.0 HXT contains over 114 non-empty sub-packages. This includes a XML data model, XML parser and serialiser, a validator for RelaxNG and DTD schemas and processors for XPath and XSLT. Apart from that HXT has numerous navigation function and constructors. Parsing is done by the function ‘readDocument’. Serialisation is done by ‘writeDocument’. Both functions use stateful arrows (cmp. sect.III), which, however, guarantee referential transparency to the outside. Both functions obey a strict sequential evaluation ordering. Filters may be used as a substitution to arrows. Filters can be used without the binary sequential operator ‘>>>’ (comparable to ‘;’ in C or Pascal). This makes lazy evaluation possible and not needed calculations may be dropped. HXT is a toolbox. HXT does not come as a framework, because control always sticks to the application programmer and never changes. Just a few filters allow a semi-automatic processing of XML-documents by using user-defined helper functions.

Localisation of files works with URIs-addressing. Thus XML sources are independently addressable from the underlying system. Unfortunately, the recent HTTP-module does not support relative addressing to full extent.

In contrast, HaXML \[68\] includes 25 non-empty subpackages. HaXML is reduced to essential XML-operations, a pretty-printer and HTML-processing. It uses filters as combinators. Arrows are currently not foreseen. Despite that the integrated HaXML-parser is based on memoisation. So, needed previously calculated subexpressions are calculated only once.

The selection for the right toolkit wins HXT due to its huge amount of supported features.

B. Architecture

This section introduces to the main functions and modules written in Haskell. The implementation of instantiator and validator are described briefly. Apart from that tests are shortly demonstrated in order to assure valid implementations. Introduced models are checked visually and briefly.
1) Function Dependency Graph: The function dependency graph for each, instantiator and validator, is illustrated in Fig. 9. Essentially, the implementation consists of three modules: Main, Instantiator and Validator. Helper functions as ‘getXPathSubTrees’ are skipped.

Fig. 9. Function Dependency Graph for Instantiator and Validator.

Functions for instantiation and validation are located in module Main. Here the functions that appear filled are used by both programs. In contrast to function ‘readDocument’, ‘getXmlDocument’ reads XML-documents by avoiding arrows, so further XML-documents in ‘main’ can be processed.

The function ‘instantiateXTL’ performs an instantiation using a template and a source for instantiation data. ‘validateDocument’ performs a validation of an instance document against a XTL-schema. Both functions are implemented as arrows. These are in ‘main’ together with input and output function in a ‘do’-environment. Because ‘instantiateDocument’ and ‘validateDocument’ require a second document, the effective function are implemented as partially defined arrows.

‘instantiateDocument’ transforms multiple ‘XmlTree’ into the data model ‘XTL’.

This corresponds to $E^{Start} [[]]$ in terms of denotational semantics. An instantiation is triggered by ‘instantiateXTL’. In a pre-calculation step within the top-level node of the template XTL-attributes are united with preexisting attributes from element nodes. This corresponds to $E^{[]}$ $[]$. Then instantiation resumes recursively. For an element node this is described by the function ‘instantiate’ (namely $E^{[]} []$) and ‘instantiate2’ ($E^{[]}$ $[]$) describes it for hedges. ‘matchXTLMacro’ distinguishes on the top-level for an element node, if it is a macro or not.

Validation is triggered by ‘validate’ as soon as a given ‘XTL’ is transferred to the regular data model ‘Reg’. Validation equals $E^{Start} [[]]$ in terms of denotational semantics. A call to ‘extractMacros’ filters at the beginning all macros. This corresponds to $E^{[]}$ $[]$. Validation continues recursively with ‘matches’ with the matching of regular expressions for both, instance and schema. The function ‘matches’ corresponds to $E^{[]} []$. Macro calls are implemented by ‘getMacro’. The non-deterministic splitting of interleavings is done by ‘frontSplitText’, ‘splitText’, ‘splits’ and ‘frontSplits’ (cmp. sect.III-A3) and depends on the node type for each element node.

Both implementations of both processes strictly follow the denotational semantics from the appendix. So, on validation match-rule (E1) is noted in Haskell as following (see sect.IV):

$$\text{matches Epsilon (TxtR "") macros = True.}$$

Same holds for instantiation (cmp. app.IX).

Not well-formed documents cannot be instantiated. Since ‘readDocument’ parses lazily and traverses XML-documents in pre-order, errors are issued with a position. Element nodes, which do not obey conventions from sect.II-C are interpreted as usual element nodes (cmp. app.IX).

As presented in sect.IV-B2 instantiation data shall be passed in a context in ‘xtl:for-each’. Binding instantiation data implicitly to PHP-function is disadvantageous. Children of ‘xtl:for-each’ access on portions of instantiation data.

Errors are issued locally. This does not mean, however, a thrown error is the reason for a failure during the validation (cmp. sect.IV). In order to meet the requirement of error localisation, the pre-order processed schema is serialised pre-order – this allows a faster localisation by the user. An error stack would simplify bug tracking on the one side. On the other side, it would, however, rapidly increase execution time. The serialised document contains the last valid node, because validation operates lazy.
2) Checking Validity: The proof for a correct model transformation was done in sect. IV-A. The proof for completeness of the data models is already done in sect. IV. Here, all XML-document composed of text and element nodes can be expressed by ‘XTL’ and ‘Reg’. Properties of instantiation and validation are discussed in sect. III.

Correctness of the implementations is guaranteed by a precise translation of the denotational semantic and the referential transparency of Haskell (cmp. sect. IV-B, IV-C). Implementation validity is assured by HUnit-tests [35]. During instantiation XPath is prototypically used as command language. Tests simply cover the rules of the denotational semantics and also in combination with other rules. Tests check for valid and invalid input. The validator has currently 818 tests, where the instantiator has 62 tests. The big discrepancy in exponential rise of complexity is due to the additional transitions by the validator. The instantiator is determined and much simpler in comparison to the validator.

A documentation for the Haskell-functions is given by a Haddock-helpfile [37].

As suggested in sect. IV test cases should be enriched by infinite documents. Infinite OBDDs can be passed to ‘validate’-functions. The termination behaviour may practically be restricted, since in case of a left-recursion it is for sure after a finite number of steps a result eventually is available, or a non-termination ‘⊥’ is the ‘result’. The following example shows infinite data structures as an extension to existing test cases:

\[
\text{genTree, genTree2, genTree3 :: Tree String}
\]

\[
\text{genTree = Node "a" (Node "b" genTree Eps) (Node "c" genTree Eps)}
\]

\[
\text{genTree2 = Node "a" (Node "b" Eps genTree2) (Node "c" genTree2 Eps)}
\]

\[
\text{genTree3 = Node "a" Eps (Node "b" genTree3 Eps)}
\]

test Eps _ = True

test (Node _ l r) c =
\begin{cases}
\text{True} & \text{if (c:=10)} \\
\text{else or } [(\text{test l (c+1)}), (\text{test r (c+1)})]
\end{cases}

For the sake of a clear explanation a binary tree ‘Tree’ is used here. This tree has a similar structure as regular expressions ‘Then’ and ‘Or’ in ‘Reg’ (cmp. sect. IV-A).

The underlying data model Tree is defined as:

\[
\text{data Tree a = Node a (Tree a) (Tree a) | Eps.}
\]

The test function ‘test’ has the type ‘Tree a -> Integer -> Bool’. The starting value ‘c:=0’ iterates a polymorphic infinite ‘Tree’, until a node of depth 10 is visited for the first time. The equivalence partition of positive test cases encloses all (Tree Integer, c), where ‘c’ is an integer less equals 10. For this partition the function terminates with the result ‘True’. For c>10 it results in ‘⊥’, because the data structure is infinite and the base case ‘"test Eps _ = True"’ is unreachable. This example demonstrates the meaning of infinite OBDDs as test case. If the test function was passed a negative finite input, then it would always return a result not equal to ‘⊥’. For an invalid input the test function returns ‘⊥’. Match-rules of the validator that return a result for ‘⊥’ and ‘⊥’ otherwise, have a similar behaviour. That is why test functions should consider selected infinite test data or data structures as ‘genTree’, ‘genTree2’ and ‘genTree3’.

C. Optimisations

Recommendations for improvement on two examples in Haskell are shown.

1) Multiple Iterations: Whilst instantiation hedges are iterated several times in order to filter nodes with a certain property. The rules (E) and (I3) are considered as patterns from app.IX:

\[
\text{let attDefs = filter2}]\rightarrow\text{child} \in \text{MA}\text{child}[\text{child}] \in \text{E}\text{MA}\text{child} [\text{child}].\text{c}
\]

\[
\text{nodes = filter} \rightarrow \text{child} \not\in \text{E}\text{MA}\text{child} [\text{child}] \text{c}.
\]

It would be more efficient to comprehend both resulting variables, which would be a tuple, and to accumulate then one of both tuple-variants. The following fragment shows a corresponding improvement towards reducing number of iterations:

\[
\text{let (attDefs, nodes) =}
\]

\[
\text{foldl}(λ(m, n) ∈ \text{E}\text{MA}\text{child} then (m ++ |c|, n)
\]

\[
\text{else (m, n ++ [c])}) (|{}|, |{}|) \epsilon.
\]

By bundling child nodes are processed once. Lazy evaluation still holds, but in every case ‘let’ is evaluated before the function body. The performance bargain which looks reasonable at the beginning, because the new variant has an alternative with a condition and alternative. The supposed advantage in performance is bought by a rather complex program, since the new variant has a condition with alternative. The rows, which initially used to be rather short, still hardly differ. The gained improvement has an unsymmetrical behaviour.

The separation between program and optimisation rules can be achieved by GHC by introducing an additional comment within a Haskell-program. This comment is checked while interpretation and matching optimisation rules are applied to defined functions.

In relation to sect. V-D this simplification has a minor meaning, because object-oriented modelling interprets operations over aggregated child nodes certainly different.

2) Lazy Evaluation: Left-associative folds have a performance advantage over right-associative folds because of the lazy evaluation. Because of the homorphism-condition holding during validation (see sect. III-A) regular subexpression may be evaluated in arbitrary ordering. Subexpressions, however, may be skipped. So, list comprehensions, which are left-associative, lead to ‘frontSplits’ calculates all partitions ascending by the length of a regular expression. This skips partitions quicker.

Hash-tables should be used for access to attribute entries during validation, because attributes are accessed quite often
and those often not canonicalised in applications. The algorithm from app[1X] uses a lazy evaluation strategy, which can be speed up further using hash-tables. The table size should grow in proportion to the length of the derived regular expression (see sect[IV]).

D. Implementation in Java

Before the Haskell-implementation is translated into Java, several main caveats need be considered. Xerces [72] may be used for input and output of XML-documents. JXPath [40] may be used for XPath-queries.

Data models 'XTL' and 'Reg' which are given as algebraic data types, must first be modelled as parametrised classes. So, for 'Reg' an abstract class 'RegEx' is defined. 'RegEx' has subclasses, for instance ‘Then’ and ‘Or’.

Haskells generic polymorphism is restricted further by ad-hoc polymorphism [22] and is used, for instance, by sorting and when processing instantiation data. Java 5.0 provides the possibility to replace generic polymorphism by templates which are determined on runtime.

Unfortunately, lazy evaluation cannot directly be simulated in Java. By explicit checks the evaluation ordering needs to be influenced, s.t. many cases are eliminated. Alternatively, the most-likely cases need to be checked. For the encapsulation of lazy methods the STRATEGY-pattern is promising.

Higher-order function need to be mimicked by polymorphic classes. Here, HOF are implemented as concrete classes, which calls polymorphic class methods. Partial functions can be mimicked in Java without sending messages. Partial arguments can be determined by queries to ‘get’-methods on the called object. Static typing of PHP-function can be described by interfaces.

Matching rules of the validation algorithm can either be adapted as is. In this case the validator operates recursively descendant and consumes many resources. The alternative is to consider a pushdown-automaton. However, the overall functional paradigm will fiercely change and so will the denotational semantics. In this case it may be better to refer to an operational semantics instead. However, this is not aim of this work (see sect[III-A]).

1) Proposition of Class Diagram: Fig[V-D1] shows a possible design for validation. The class Validator represents the caller, which is initiating validation of an instance document against a given schema document. The abstract class RegEx represents regular expressions. It currently has eight subclasses. Subclasses containing one or two RegEx allow variable child nodes. The abstract methods validate and getIdentifier are implemented in the subclasses. The method getIdentifier returns the identifier of a subclass. This method actually breaks encapsulation. However, it may be tolerated only, if all regular expressions are fully covered. By object-centric identifiers every validate-method can be implemented by switch-constructs. Implementation follows the rules of the corresponding denotational semantics closely herewith.

Since the methods validateOr and validateMacro are universal (see sect[IV-C]), those can be implemented by RegEx. The class MacroEntry represents an entry of the macro environment μ (see sect[II-C]).

In this class diagram several design and architectural pattern are hidden (after Fowler [27] and Kernievsky [22]). Initially, regular expressions are constructed by the counted subclasses by using the BUILDER-pattern. Here Validator acts as Director and RegEx as AbstractBuilder. The common method is create. The DECORATOR-pattern allows some RegEx-classes the reuse of previously implemented code. The classes TextR, Epsilon and TextR act as ConcreteComponent, RegEx as Component and ElR, Star, MacroR, Or, Then as ConcreteDecorator.

The INTERPRETER-pattern describes a regular language. The client Validator triggers validation by calling method validate. The IMPLICIT-LANGUAGE-pattern consists of terminals and non-terminals, which generate the language. Subclasses not containing composition act as Terminals. Classes, which have a simple composition RegEx act as Non-Terminal. Both symbols are interpretable expressions.

On validation each node type is matched with an instance node. Invalid combinators are skipped and validated against ‘False’. The classes do not implement validate for every RegEx-subclass. That is also because the structure of regular expression for schemas does not extend in a meaningful way. Hence, each class type during evaluation returns identifiers or type information. This information can be obtained by the function getIdentifier. That is why a separation between data model and tree traversal does not really make sense. Hence, an implementation by the VISITOR-pattern is not appropriate here.

Besides, the universal DESCRIPTOR-pattern occurs whose INSTANCE is the class TextSplitting and whose DESCRIPTION is TextR. In analogy, ElR and AttrEntry also matches the DESCRIPTOR-pattern. Apart from this the TEMPLATE-METHOD-pattern can be found. The abstract class here is RegEx, where validateMacro and validateOr act as Template. The subclasses listing the same methods are Hooks, which are referring to templates.

In a next step the data model ‘XTL’ and polymorphic instantiation data as well as the instantiator can be modelled. The proposed test suite can be used in addition to the module test.
VI. Comparison

In this section dedicated schema languages are compared according to previously selected criteria. Here, language features are more of importance than implementations. The following questions will be taken into consideration:

1) What are positive factors towards unification?
   Here, semantics, syntax and algorithms for both, instantiation and validation shall be considered. The criteria weights shall be considered thoroughly. Criteria in favor and against an unification shall be clarified.

2) Which lingual features are in favor and which ones are against unification?
   Is it possible to adapt and adequately represent features from other schema languages? A metric shall be provided, if any possible. In case a comparison with other XML schema languages shows up weakness, investigate if the weakness may be resolved.

3) Which consequences do unification have in practice?
   What is the practical benefit of unification?

Furthermore, differences and mutual grounds of features towards an unification from previous sections shall thoroughly be analysed.

A. Considered Languages

The considered languages are illustrated in fig. 11. It is XTL which may be considered as one fraction, and rather popular schema languages like XSD, RelaxNG and DTD. Since DTD may not be described within XML, DTD will only be referred to at some positions only where appropriate for comparison purposes only.

XSLT is considered for the sake of comparison between template and schema languages.

B. Criteria

The criteria are applied to XTL as well as to selected schema languages. Criteria are supposed to be as common as possible and are focused towards an unification. In conclusion, comparisons are mostly qualitatively.

An unification of template expansion and schema validation is achieved by involved documents and by the referred schema languages (cmp. sect. III). Orthogonality and distributivity are properties which have a minor meaning here. Both properties may already be covered by modularity.
**Similarity**

Similarity refers to template and instance documents on the one side, and to template and schema documents on the other side. If template and schema document are the same, then unification is achieved. Otherwise, there are commands in the template language which do not correspond to commands in the schema language or vice versa. This is distinguishing instantiation from validation.

Moreover, similarity means an adequate representation of a template document w.r.t. an instance document. If both are too different then similarity is too little. This means template markups can be transferred into instance markups by only a big amount of changes.

So, if template and schema document, or template and instance document were only close enough, then a general unification is achievable.

**Expressibility**

Expressibility asks if a template language may be expressed by features from the schema language and vice versa.

Even so regularity properties and type safety are researched for an unification. An essential question to find here is whether functions and symbols can be validated as expression.

Even the representation of rules themselves may influence the unification. So, for instance, one question emerges if rule representation may improve unification, if rules are based purely on filters and/or pattern-matching. It is generally known from observations, that pattern-matching often is better for more compact notation, if possible at all, than filters for instance.

1) Classification: The taxonomies shown in tab.X, XI may be derived from sect.I. The three views are mostly related to schema languages.

## TABLE X

| Syntax                  | Semantic               | Complexity              |
|-------------------------|------------------------|-------------------------|
| Schema-Style Ordering   | Typing                 | Time-/ Memory usage     |
| Syntactic Sugar         | Functions              | Automa Class            |
| Pattern-Matching        | Constraints            | Evaluation Ordering     |
| Regularity              | Error Handling         |                         |
| Symbols                 |                         |                         |
| Control Flow            |                         |                         |

**TABLE XI**

| Criteria                  | Objective | Subjective |
|---------------------------|-----------|------------|
| Openness                  |           |            |
| Extensibility             |           |            |
| Variability               |           |            |
| Rules set                 | Command Tags |
| Constraints               | Modules/NAMESpaces |
| Command Language          | Instantiation Data |

**TABLE XI**

| Similarity                | Objective | Subjective |
|---------------------------|-----------|------------|
| Expressibility             |           |            |
| Automata Class            |           |            |
| Control Flow              |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |
| Expressibility             |           |            |
| Symbols                   |           |            |
| Complexity                |           |            |

2) Assessment: In this section comparisons base on tab.X. Taxonomies (a) and (b) of tab.X are compared with the first table in addition. The criteria from tab.X are loosely coupled. Constraints and typing are of overall practical meaning. “Error handling” is a secondary requirement to usability. Functions are of utmost importance to the expressibility of template languages.

Syntax summarises the most important criteria in comparison to semantic and complexity. “Ordering” and “Symbols” may be considered as syntactic sugar. Symbols replace well-defined nodes and hedges. Therefore, they improve reuse. Control flow in the validation process is of key importance. From the application’s perspective the schema-style is important. If a schema language is pattern-styled, then even complex schemas with a difficult syntax description may easily be expressed. This relates to the regularity of schema languages.

W.r.t. to complexity, time and memory consumption play a key role. However, because of heterogeneous program systems this criterion if often totally underestimated. Because of the prototypical Haskell-implementation, a dynamic profile does not make sense. Both, evaluation order as well as eliminated
bottle-necks directly affect the efficiency of validators.

In addition to openness a structured rules may improve the comparison towards unification. Excluding constraints may cause contradiction towards unification and make the definition of new predicates error-prone and bloated. That is why an investigation of including and excluding specifications shall be done. Regarding modularisation of template and schema the following points seem most promising: integration into namespaces, node inclusion and exchangeable command languages. The taxonomies from tab. X (a) are already covering the criteria from tab. X (b).

C. Semantic

This section introduces features of the considered schema languages and one template language. Typing is here of special interest in schema and command languages as well as the representation of functions, the use of constraint and error handling.

1) Typing: Each template node must be well-typed. This is a precondition on instantiation and validation. On the one hand there are element and text nodes. On the other hand there are command tags. Depending on the concrete command those tags also return either an element node, a text node, or may return a hedge containing both as instance (cmp. 31). If this condition is fulfilled, then implicitly well-formedness of the instance is guaranteed. Access to attributes can be replaced as access to element nodes as was discussed earlier and therefore does not require further investigation here. Depending on a given schema language, typing may also include referential integrity.

Instantiation uses PHP-functions, where validation does not. During validation the type of the associated command-tag is considered, so descriptions are reused in an unified approach.

Unification does not really matter about command tags. However, the less possibilities exist to instantiate a template node the faster validation can be performed (see sect. VI-F).

In XSLT type safety is guaranteed in XPath-expressions for command tags. The result is either a hedge, an element node or a boolean value (see 11, 67). Numbers and dates are included in hedges whose children nodes have exactly one node and a list of <book/>-nodes as a hedge with a certain amount of element nodes. Similarly to XTL singular types are used for inclusion of and boolean types for controlling the instantiation. The consideration of the typing of each command tag is essential to validation, because for each command tag a decision needs to be made with how many instance nodes does it correspond with. In other words this means, the type of the inferred template node is compatible with the type of the instance node.

This raises the question, whether a command retrieving text could not accidentally return a singular node or a hedge. The interpretation of text as element nodes, however, would insist tag-brackets are generated at least in the output. But this is not possible in the default configuration of XTL and XSLT, because special characters are treated as XML-entities. Type safety on text output can be influenced in XSLT by the attribute ‘disable-output-escaping’. Even well-formedness is not guaranteed in XSLT, because an upper-most root node does not need to exist. Not only because of this, XSLT is not appropriate for an unification of instantiation and validation. Despite this, XTL does guarantee type safety.

Guaranteeing referential integrity is important, especially in XML databases. In XTL most existing command tags can be adapted without prior configuration. However, unique values and foreign keys must be supported by the command language. XSD has techniques to assure referential integrity. DTD and RelaxNG do not support keys. DTD supports only very basic unique elements. However, there is no modularisation. Therefore, no separation and no distinction by element names are possible.

Key information and uniqueness do not really hinder unification. However, they are only relevant to validation, but not to instantiation. Another approach to separate a schema is to hoist key relations. In order to do that schema nodes need to be referenced. This is done by XPath-expressions — in analogy to XSD. By doing so the key relation remains free of redundant nodes, but it may also invalidate schema relations if path expressions become invalid.

From the standpoint of common rules (see sect. VI-F2), the support for referential integrity is a violation of the unification of instantiation and validation. This is because instantiated primary runs for a given document only once. However, validation runs the first time in order to localise identifiers and unique elements and attributes. The first run is then still needed in order to check uniqueness and key relations.

2) Functions: Functions are an important acceptance criterion for template languages. In JSP and ASP small calculations can be performed by functions. Local and remote function call may be triggered. Template languages like XSLT define functions with arity greater equals zero by named templates (cmp. 31). The most popular schema languages do not have corresponding mechanisms. Functions require among its values there is a possibility to check each value has a correspondence to at least one instantiation data field. If this is the case, then this value is valid according to the function. Otherwise, the function value is not valid according to a schema node. In other words, functions must be invertible and instantiation of a template must be an isomorphic mapping 31.

Since in practice this tough restriction is too hard, in general, functions may not be validated in general. This means generalised functions prevent the unification. Restrictions must be imposed.

Functions may be not defined in XTL, and this would not even be meaningful for the reason just mentioned. Referential transparency disallows it. Only macros may be stored locally
to the global symbol environment. That is why only functions within the command language may be used.

3) Constraints: Constraints may be required in both processes. During validation constraints restrain nodes and text. So, specifications may be including or excluding. XTL, RelaxNG and DTD belong to schema languages with including specifications. XSD is a hybrid. XSD may allow contradicting specifications for which it is impossible to define a valid document. For instance:

```xml
<xsd:element name="top">
  <xsd:complexType>
    <xsd:sequence>
      <xsd:element name="second" type="BBB2" minOccurs="0"/>
    </xsd:sequence>
  </xsd:complexType>
</xsd:element>

<xsd:complexType name="AAA">
  <xsd:sequence>
    <xsd:element name="x" type="xsd:string" minOccurs="3" maxOccurs="3"/>
  </xsd:sequence>
</xsd:complexType>
</xsd:complexType>

<second/> is not a valid instance, because ‘BBB2’ requires an empty content-model and requires an element node as child, so "<second><x>1</x></second>" is also invalid.

An example for constraints is the explicit null. In order to specify an element node AAA may not have child nodes within DTD, children shall be specified EMPTY. The full element node corresponding will be <!ELEMENT AAA EMPTY>. In RelaxNG <empty/> has the same effect. In XSD the same is achieved by an empty sequence.

Constraints on elements are helpful to restrict. However, partially specified names can be inconvenient if names do no more differ (enough). In contrast to that the restriction of child nodes is important. So, for example, a schema containing <a/> with a text node, followed by <b/> is specified in XTL as <a><xtl:for-each select="'/a'"/>b</a>. Similar looks the specification for RelaxNG and DTD. The XSD is relatively long (see fig.12). Here, <a>#$b</a> is not exactly expressed, where ‘#$’ denotes an arbitrary text node. The weakness of content-models in XSD is its imprecise position for text and element nodes. So, <a><b/>#$</a> and <a>$<b/>#$</a> are accepted, even so this was not originally intended.

That is why including schemas are easier to check than excluding, because validation has only to check whether both, schema and instance nodes fulfill some predicates. In excluding schemas for an instance node all listed specifications need to be checked whether they do not fulfill. Excluding schemas make unification more difficult, because a fixed amount of excluding predicates needs to be considered.

On unification attribute-constraints may not specify attribute values any closer. For this reason attribute names must be given in a complete form – the same as it is with element nodes. In contrast to elements, the exclusion of attributes in schema languages is weaker and has to obey only weaker restrictions. So, in RelaxNG within a ‘except’-node ‘nsName’ may exclude certain namespaces. In XSD only a certain namespace may either be included or excluded. On the contrary, arbitrary attributes without namespaces can be added by ‘anyAttribute’. This reminds the mixed content-model, but is still different. In XTL Tag ‘attribute’ allows to arbitrarily specify attributes. From this perspective XTL offers the best usability.

Fig. 12. XSD-Schema for a mixed Content-Model

Multiplicities are another possibility to restrict hedges. In XSD any non-negative multiplicities are expressible by attributes. RelaxNG only allows the multiplicities ‘0...*’ and ‘1...*’.

Multiplicities cannot by expressed dynamically, but only by predetermined values and variables. This ensures, that for instance in XSD regular expressions can be composed (see sect.[VI-E]) and cycles do not depend on variables. In XTL there is currently no option to repeat element nodes any time. A fixed number of repetitions is allowed however. That is why <a>/<a/> cannot be described by L(a3). Multiplicities in XTL are recommended for the same. Kleene’s star operator ‘unbound’ may remain unnoticed on definition, because it is already covered by ‘xtl:for-each’. Since multiplicities remain the properties on regularity untouched, unification gets another valid construct to be used.

4) Error Handling: Error handling heavily depends on the concrete automaton (see sect.[VI-E]). The handling on errors in XTL differs from the behaviour in XSD and RelaxNG. This is mainly due to the prototypical stage of XTL. XSD and RelaxNG issue an error message containing the error position. The
schema is well-formed and suffices the syntactic requirements of the schema language. Although RelaxNG may also be simulated by a non-deterministic automaton (see sect. VI-E), the output of all previous locally occurred errors is avoided by the implementation of an error stack. This is a big help to schema developers, but requires further resources. In XTL there shall be an error stack too.

D. Syntax

In this section syntactic features of schema languages are considered. The more features a schema language has the more powerful it is – this assumption is false in general. Beside unification criteria learnability is also being considered. The goal of this section is to find answers to the following questions:

1) Do lingual features improve unification?
   What can be done, if possible at all, in order to increase unification?

2) Can lingual features of other languages be expressed within XTL?
   Apart from the question if it is possible in general, the question of complexity arises second and of type safety third (see sect. VI-C1). Does the other way round also function?

3) How important are those lingual features in detail?

4) Is XTL minimal w.r.t. lingual feature amount? Are some template and schema features missing?

XTL combines the advantages of XML-schema languages, especially the huge amount of available tools and APIs. One advantage, however, is that still missing in RelaxNG and XTL: it is the fixed association between instance document and schema. It should explicitly be placed in a XML processing instruction or in the top element node, so the relation does not get lost, especially when several schemas are in use. The information regarding embedded command languages should also be explicit. This can be achieved by the ‘realm’-attribute.

1) Symbols: In schemas it is often useful to summarise hedges and reuse them at different locations. The insertion of a hedge is comparable to the use of constant symbols, because the hedge is assigned only once and remains invariant afterwards. The invariance effects template and schema nodes. That is why invariant hedges are sometimes called ‘symbols’ which can be expressed by constants. Symbols are visible everywhere within a given schema for all considered schema languages. Some template languages like XSLT version 2.0, but also Prolog (cmp. sect. VI-A) offer variable agreements, which may be used for memoisation and as placeholder. These variables are not considered as symbols, because they are dynamic and therefore violate referential transparency.

Symbols are so-called Substitution groups in XSD, Definitions in RelaxNG, and Macros in XTL. Symbols in XSD may restrict or extend and always refer to element nodes. Symbols introduced in RelaxNG behave isomorphic to ‘xtl:call-macro’ and ‘xtl:macro’. In contrast to XSD in XTL and RelaxNG text and element nodes may be appended before and after a macro call, where the ordering does not change (see sect. VI-D3). Symbols do not violate referential transparency due to determinism, so these are well-defined before an instantiation and validation. That is why they harmonise both processes, instantiation and validation. Variables with mutable values on the other side hinder unification, because these immediately influence validation (see sect. VI-C2).

The next coarse-grained structure would be namespaces. In XTL namespaces are present, but because of its prototypical characteristics, they are not supported by XTL-implementations. In other schema languages namespaces can either be within the top root node or in RelaxNG or be local in RelaxNG in all children nodes. Apart from that namespaces in RelaxNG and XTL can also be applied to attributes. The attribute ‘ns’ in element nodes does just that. Namespaces are modules too. By doing so, namespaces contribute to an ongoing unification of template language and schema languages.

XSD has numerous simple and complex data types. RelaxNG in contrast has only 2, and vocabularies of other schema languages are imported. XTL has only the data types from the previous section.

2) Control Flow: The control flow is determined by XML tags in schemas. The flow controls validation. Upon closer examination ‘xtl:if’ and ‘xtl:for-each’ can be interpreted as regular expressions. Simple conditions are represented by ‘’’; nested conditions with optional alternatives, and by a selection of ‘’‘’, where ‘’’’ denote hedges of corresponding consequences.

‘xtl:if’, ‘xtl:for-each’ and ‘xtl:call-macro’ determine the control flow in XTL. Macro calls correspond to hedge substitutions (cmp. sect. VI-C3). Every ‘xtl:if’ can be substituted by a matching ‘xtl:for-each’, but not vice versa. Hence, ‘xtl:for-each’ is an universal element. In XSD conditions may also be expressed by multiplicities with lower bound ‘0’ and upper bound ‘1’. Multiple selections and nested conditions can be expressed by ‘xtl:if-conditions. These are the same from the standpoint of validation, if the proposed right-associated OBDD are chosen (cmp. sect. VI). So, a simple and a nested condition as well as multiple selection are preferred for unification. When it comes to instantiation refined conditions are beneficial, since unconsidered cases could be dropped and the overall instantiation increased.

3) Ordering: The ordering is of utmost importance to validation. Without an order the increase of complexity for any validation algorithm could be tremendous (cmp. sect. VI-E). This is because an unspecified ordering requires to test for all possible permutations of all possible hedges. The amount of permutations is \( n! \), where \( n \) is the number of children for a given hedge.

In XSD the ordering can optionally be specified. By doing so readability suffers and redundancy increases. A specification would be more efficient in terms of adequacy if only the
provided ordering was allowed. The concrete ordering should always be taken out by expressions of a given schema language and should be done explicitly. Often certain permutations are not desired and a systematic exclusion of all unwanted would critically increase complexity too. For example, in XSD there exist different representations of "\(\langle a/\rangle b\)". This hedge may be represented by the unordered regular expression \((a|b)\). This expression is interpreted under a certain ordering mode and is expressed by multiplicities. So,

\[
\text{permutate}
\]

\[
\text{expression (a}| \text{b)}
\]

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\[
\text{permutate}
\]

\[
\text{expression (a}| \text{b)}
\]

This representation of \(\text{permutate}\) is obviously syntactic sugar and is believed to be safe due to the closedness of the operation. However, permutation would need to have a meaningful counterpart on instantiation when it comes towards unification. If instantiated hedges ought to be permuted, then it can be stated, that permutation increases unification w.r.t. instantiation and validation. A \('\text{permutate}'\)-tag shall be introduced to XTL for explicitly specified permutation.

Because of the described dramatic drop in performance on validation in general, unknown hedges and suffixes of known nodes are not considered currently in XTL and therefore are not implemented yet. Unknown prefixes can be expressed by \('\text{include}'\)-tags and are guarded by \('\text{xtl:for-each}'\). The only severe disadvantage is extensive search which takes place right at the beginning of the unknown prefix. It is a practical advantage to place known nodes to the start of a hedge and as close as possible to the beginning of the document.

4) Patterns: XSD is the most pattern-styled language among all considered schema languages (cmp. [48]). In contrary, XTL, RelaxNG and DTD have a grammar-styled representation which is too less pattern-styled (see [52], [17]).

Beside the style of a schema language, syntactic notation, e.g. by using regular operator, have a big influence on usability. In RelaxNG the tag \('\text{oneOrMore}'\) is used as a replacement for the plus-operator. Operators of the command language do not obey regularity criteria in XSD, because restrictions on symbols (see sect[VI-D1]) violate those. Expressions of the command language do not have any significance, because they are ignored on validation.

5) Usability: Usability is worsened in XSD by redundant \('\text{complexType}'\)-definitions. Both, XSD and RelaxNG lack of adequate representations of element nodes. So, \(<\text{xsd:element name="..."}>\) or \(<\text{element}> <\text{name .../}>\ldots \) must be used as node constructors. Node definitions in DTD are not adequate. However, this cannot be resolved by any other means because of its non-XML notation. In contrast to this, XTL is able to take schema nodes exactly as are. Specifications taking nodes exactly as are simplify unification, because template, schema and instance nodes all are congruent.

6) Syntactic Sugar: Syntactic Sugar in the scope of this work means command tags which can be removed from a given language, s.t. expressibility neither of schema language nor of template language diminishes. The more features are shared among schema and template languages, the more unification increases, because in fact expressibility increases relatively.

Idioms and syntactic sugar have the following characteristics:

- **Openness:** Idioms must be expressible just by some core functions. Herewith, neither referential transparency may be violated, nor additional assumptions about instantiation data are allowed.
- **Extension:** Idioms may not extend the expressibility of a schema language. This means, for example, regular schema languages may, naturally, recognise ranked-competing fragments also as a result of a weaker tag. But, ranked-competing schema languages may not introduce tags, which would enable regular schema language recognition.

Openness causes both processes, validation and instantiation, are proceeded by another step turning its product, the document, into a form, which is free of sugared tags. Simple forms lead validators and schema simplifiers (see sect[VI-F2]) to heavily reduced rules sets. For instance, in RelaxNG this is called \('\text{simple syntax}'\) [20].

An extension of a ranked-competing schema language to a lower leads to most rules of a validator must be dropped, since an increase in non-determinism leads immediately to further matching cases. Here, a validator is assumed to be fully described by a rules set. The inversion makes a conflict visible now, since non-deterministic matching rules need to be excluded by the syntax.

It is compulsory syntactic sugar has a unique semantics in both, the template language and the schema language. If an idiom has only in one of both languages a non-empty semantic, then exclusions may only be hard to formulate and are less plausible.

E. Complexity

A closer description of complexity is given in sect[IV]. The validation algorithm is not bound by polynomial complexity w.r.t. schema length (see [11]) herewith. The complexity
is caused by rules splitting string and ‘Then’-nodes non-deterministically. Hereby the potential search space of validation is drastically expanded. But this approach ends with a higher expressibility than XSD or DTD. The memory consumption is directed by runtime behaviour. When an alternative occurs the last valid state before entering the recursion has to be stored. The XTL validation requires only the memory needed according to the maximal recursion depth. Solutions once dropped are not considered a second time. Only open solution not yet considered must be stored on lazy evaluation.

The separate handling of attributes does not cause a significant raise in complexity.

The infinite OBDDs proposed in sect[7] have non-polynomial complexity on lazy evaluation, but only if fixpoints exist in the schema. From the practical perspective, infinite instance documents are disallowed, because validation only considers enclosed documents. Instantiation can accept infinite OBDDs as input. Because in contrast to validation, instantiation does not require the entire document is present. Consecutive tags in a hedge may be instantiated independently (see sect[III-A2]). Exceptions are macros having infinite bodies but whose overall structure is well-defined. In case of finite templates without left-recursion instantiation always terminates with a well-formed resulting instance document. The runtime complexity of instantiation is bound by a polynomial which degree is the maximum number of nested loops. However, this does not hold in general for macro calls. Macros lead to a non-polynomial runtime behaviour, which may be compensated by lazy evaluation.

Depending on a tree language (see sect[III]), a schema language automata may recognise different levels of granularity. DTD is expressed by a single-typed grammar, where XSD is expressed by a ranked-competing grammar. The latter allows more freedom [48]. RelaxNG and XTL are generated by regular grammars and allow maximal expressibility. So, XTL allows to define arbitrary (but at most regular) sequences of nodes.

On the other side there is syntax to be considered. XTL has a minimalist syntax, since no element is sugared (see sect[IV]). RelaxNG has a relatively tiny amount of features, for instance, in comparison to XSD. However, this does not affect expressibility. XSD has a poor expressibility, but lots of sugar. This quickly leads to hard to read documents. By missing macros or definitions schema validation always terminates in XSD. Only for XTL and RelaxNG closedness properties hold regarding intersection, union and complement. That is why those languages are perfect for extensions. The same expressibility class practically means, transformations into each other are possible without any severe hinder. There may also be transformations from XSD and DTD to XTL and RelaxNG. Beware the other direction is not possible in general.

Schemas in DTD are recognised by top-down deterministic validators. RelaxNG and XTL are recognised by top-down non-deterministic validators. XSD is generated by a single-typed tree grammar [48]. That is why XSD is recognised by a top-down deterministic validator.

F. Unification

This section unification is considered in general towards documents and rules sets.

1) Documents: In order to unify templates and schemas, the following restrictions are recommended:

1) Type Safety. Both, template and instance document must be XML. Existing command-tags need to have a semantic, which does not depend on surrounding tags and is still distinct among other command tags. In conclusion this means, validation should have rather a little possibilities only to validate against an instance node.

2) Abstraction from the Command Language. Expressions to be calculated should be enclosed in the document structure. This means, functions must be eliminated and command languages ignored during validation. Instantiation data must be hidden during validation. Instantiation data may not be in the schema, so access is granted by attribute entries in command tags.

3) Self-Similarity. The bigger similarities between schema and instance document are, the easier validation is to describe and the bigger the intersection of shared language features is.

2) Rules Set: The essential difference between instantiators and validators is the underlying algorithms. A validator matches nodes, where an instantiator substitutes nodes (see sect[57]). The instantiator may process nodes in parallel, where a validator processes an instance document sequentially.

Instantiators and validators are based on rules sets. Denotational semantics for instantiation and validation may be expressed by rules of the form: $p \rightarrow a_0, ..., a_n \ q$. Here, ‘p’ denotes a premise. The premise of instantiation contains a template node, where the premise of validation contains an instance node. ‘a_i’ represent constraints. ‘q’ denotes the result of an instantiation or validation. The evaluation ordering of instantiation and validation is controlled by constraints (cmp. [22]). Rules sets do not contain instantiation data. However, this is actually a problem, because validation iterates the instance document in parallel to the schema document. Instantiation does not allow an even procedure. First, this is because, instantiation data may be arbitrary heterogeneous structures, which cannot be compared with the template-document in general. Second, any assumptions about the internal structure of instantiation data is strictly prohibited as it was shown in sect[III].

An important benefit of a rule-based semantic is the openness towards compilers and interpreters with a rigid structure (cmp. [12]) – as it is the case of denotational semantics in sect[IV] and of implementation in sect[V]. So, new elements are comfortably inserted into or removed from existing rules. So, $a_0 + n$ new cases are inserted to the rules set of a validator in case of insertion of a new tag, where the bipartite graph consists of at most $m \times n$ edges (cmp. fig[7]). This means, insertion of a new element has a
linear complexity increase in conclusion (even the constant factor \(\frac{1}{2}\)). Despite this are bottom-up parsers, which often require a change in a quite considerable amount of rules. It is worth mentioning, that \(m \times n\) cases could have further distinction, for instance, for the concatenation of (possibly empty) command tags (cmp. sect. 4.2). In contrast to this, an instantiator requires the insertion of just one rule into the denotational semantics, since there are no non-deterministic cases and a new command tag has the same behaviour as other tags.

The reuse of short and self-explanatory templates is simplified. In order to effectively reuse documents of a schema language, simplifications with the document structure are recommended. That is why rule-based simplifiers shall be developed, where weak regular schema languages are advantageous. For example, the schema \(a^* (ba^*)^*\) shall be restructured to \((a|b)^*\) [23].

From a lingual perspective commands hinder unification, which either support instantiation or validation. But their implementation causes the assumption the other function does not make assumptions of the origin. On instantiation, e.g. XSLT-stylesheets are XSLT-templates, and regular text patterns on validation.

**VII. Summary**

This work examined how much template instantiation can narrow schema validation for XML-documents in an unification attempt. First, instantiation and validation were formalised. Properties towards their practical meaning were probed, an implementation was developed. Requirements for an unification were elaborated and based on these results a comparison was done.

The semantics were formulated in different ways. On the one side denotational semantics specified the programs’ behaviour. On the other side rules demonstrated introduced data models used and transformed. The tree automaton model was used for evaluation. Optimisation techniques were discussed. The formalisation made it clearer instantiation is adequately represented as a term-rewriting system and validation as graph-matcheer, also a rule and term-based system.

Both semantics showed, that the rules set for both, instantiation and validation, cannot entirely be unified. However, reuse of simplified code simplifies unification.

The implementation allowed an unification of both processes on document-level. The validity of all implementations is guaranteed by a comprehensive test suite. A stack for error should be integrated in future with a Java-implementation. The regular data model was prototypically introduced in Java.

Analysis showed, XTL has regular grammar properties, except macros, which extend expressibility but also violate certain closedness-properties. The extension requires further research towards practical means. Moreover, it was shown termination does not hold and should not hold in general. An explanation was given, why filters and arrows are not best, especially when XTL is going to be variable and extensive. Recommendations for improvement were given.

Instantiation showed, XTL is not as universally applicable as, for instance, XSLT. For instance, there is no possibility to define arbitrary functions in a schema, which could be used later. It was found out, the expressibility of XTL directly depends on the command language. Instantiators work deterministic, where validators do not by default. Here, parallel validation should be considered further.

It was found out, it is advantageous to restrict unification. If a schema language assigns a type each slot, then validation overall may be simplified quite considerable. Regular properties implies syntactic sugar can be defined without any change on the schema language’s expressibility. In order to obtain most flexibility command languages require adaptations. The introduced rules for instantiation and validation have in consequence, that changes can be done easily. An effective possibility to beat non-determinism is the construction of automata as described and the restriction of a schema language’s expressibility. It was noted, however, that there can be drawbacks here. First, the Rabin-Scott’s powerset construction may cost too high efforts for XML. Second, a restriction to single-typed or ranked-competitive is not really a solution, because closedness-properties are violated.

The comparison showed, the set of potentially all generated instance documents has a huge impact on the unification – much more than the expressibility of encapsulated queries, for example. Comparison criteria were introduced regarding syntax and semantics. Comparisons were taken out and marked accordingly. Despite its huge syntax definitions XSD was found weaker than XTL or RelaxNG. XTL as template language is quite universal. Because of its universality it is possible, for instance, to define keys for referential integrity. Variable orderings of foreseen attributes shall be considered as syntactic sugar. It is recommended to define a rule-based simplifier for XTL-schemas, because it is estimated simple schemas and tools will raise the acceptance of XTL.
VIII. Glossary

Abstraction.
denotes an anonymous function in the \( \lambda \)-calculus.

Ad-hoc polymorphism.
similar to \( \triangleright \) generic polymorphism, abstracts from and restricts a data type, e.g. by subclasing.

Active Server Pages.
a certain \( \triangleright \) template-language.

Ambiguity.
non-determinism in grammars caused by overlapping rules.

Application.
denotes in terms of \( \lambda \)-calculus the application of a given term to an \( \triangleright \) abstraction. Is equal to \( \beta \)-reduction. Applications with term and abstraction being the same are self-applicative.

Arrow.
generalised \( \triangleright \) monad having functions as input.

Arity.
The amount of parameters a function has. A function with arity zero is a constant.

Command Language.
A language embedded in some \( \triangleright \) template-language for accessing \( \triangleright \) instantiation data by placeholder-plugins. XPath is the command language for \( \triangleright \) XSLT, where JXPath is a reference-implementation.

Ranked-Competing.
\( \triangleright \) Regular tree grammar whose \( \triangleright \) hedges are uniquely decomposable.

Bypass Attribute.
a \( \triangleright \) XTL-attribute for the stepwise instantiation of a \( \triangleright \) template.

Call-by-need evaluation.
a special case of \( \triangleright \) lazy evaluation on which intermediate results are not calculated twice.

Constraints.
denotes general restrictions. Constraints during \( \triangleright \) instantiation and \( \triangleright \) validation select and specify nodes with \( \triangleright \) command tags. In \( \triangleright \) XTL constraints are specified by "select" expressions. Constraints may also be used to specify valid and invalid variable and function domains. In the context of databases constraints guarantee referential integrity.

Content-Model.
describes a \( \triangleright \) hedge in \( \triangleright \) XSD.

Definitions.
denote dedicated \( \triangleright \) symbol nodes in \( \triangleright \) RelaxNG.

Denotational Semantics.
also known as functional semantics, denotes the functional behaviour for a given program. It uses an universal language for its syntax, e.g. set operators or an abstract programming language, and defines a relation between syntactical constructs and their meaning. Denotational semantics abstract from a certain machine platform and focus on the calculation of output for a given input.

Derivatives.
denote mappings from regular expressions to regular automata as proposed by Brzozowski and Glushkov.

Document reconstruction.
reconstructs unknown \( \triangleright \) instantiation data. This is in contrast to \( \triangleright \) validation where instantiation data is known. Document reconstruction can be considered as inverse operation to \( \triangleright \) instantiation.

Document Object Model.
is a data model for a XML document.

Regular (Tree Grammar).
the most powerful of all considered \( \triangleright \) regular tree grammars, which has no restriction.

Endomorphism.
mapping whose domain and codomain both denote the same set.

Exhaustive Search.
searches for a sound non-deterministic \( \triangleright \) validation.

Lazy Evaluation.
evaluation ordering which includes only those steps really necessary for obtaining the final result. Within the \( \lambda \)-calculus it corresponds to the outermost term reduction.

Filter.
functions having a polymorphic \( \triangleright \) type \( a \rightarrow \{ a \} \).

Fixpoint.
in geometry denotes a point which is fix for a given mapping. Similar to that, a fixpoint in templates and schemas is a synonym for conditions that in loops do no further change. In \( \lambda \)-calculi, a fixpoint in templates and schemas is a condition with an invariant (state) in recursions. \( \triangleright \) left-recursions lead to unreachable fixpoints during \( \triangleright \) instantiation and \( \triangleright \) validation.

Functional.
\( \triangleright \) higher-order functions.

Higher-order Functions.
functions which consume functions as input and output. Known list-\( \triangleright \)functionals include fold-left \( \langle \text{foldl}, \langle \text{map} \rangle \) and filter \( \langle \text{filter} \rangle \).

Generic Polymorphism.
abstraction of a certain data-type without further restrictions, see \( \triangleright \) ad-hoc polymorphism.

Glushkov-Automaton.
finite determined automaton which recognises regular expressions.

Grammar Style.
schema language which syntax can be expressed good by a grammar — in contrast to \( \triangleright \) pattern-like schema languages.

Graph-Matcher.
program which checks if two graphs are identical.

HaXML.
Haskell-API for processing XML documents (also see \( \triangleright \) HXT).

Hedge.
synonymous for a list of children nodes in a XML document.

Homomorphism.
mapping for which the following equation holds: the product of the codomains equals the codomain of its products. Homomorphism holds, for instance, for the addition in a residue class ring.
HXT.
Haskell-API for processing XML documents (also see ↘ HaXML).

Instance.
result of the ↗ instantiation.

Instantiation.
process turning a ↗ template with ↗ instantiation data into a ↗ instance.

Instantiation data evaluator.
part of the ↗ placeholder-plugin, which on request by the ↗ template-engine issues ↗ instantiation data.

Instantiation Data.
denotes data sources, which are bound during an ↗ instantiation to ↗ slots.

Interleaving (Matching Rule).
denotes in terms of ↗ RelaxNG a non-deterministic matching rule.

Interpretation.
generally denotes the codomain of a function. ↗ Instances may be considered as interpretation of a ↗ template.

Java Server Pages.
a ↗ template-language.

Canonisation.
recursively sorts all attributes within element nodes ascending by name.

Kleene’s star operator.
denotes the star operator within regular expressions.

Combinator.
denotes an ↗ abstraction in the λ-calculus where no variables are free.

Commando-Tag.
denotes all tags in ↗ template-languages, which control the instantiation.

Competing.
Two non-terminals compete with each other, if their sets of possible beginnings share at least one terminal.

Left-recursion.
causes the instantiation of a macro does not terminate.

Literal.
denotes text nodes and childless element nodes.

Local Tree Grammar.
is the weakest of all considered ↗ regular tree grammars, in which a terminal does not appear in any other rule.

Macro.
denotes in ↗ XTL a ↗ symbol.

Tag.
is for the distinction of text.

Matcher.

↗ Graph-Matcher.
Memoisation.
denotes the ↗ call-by-need evaluation in programming languages.

Model-View-Controller.
ar littoral principle for the separation of concerns between model, view and control.

Monad.
denotes in Haskell an algebraic data type. Lists with ↗ as neutral element represents a monad w.r.t. the associative operation ↗.

Pattern Style.
schema language whose syntax can be described by a regular expression – rather than ↗ in a grammar style.

Non-deterministic Finite Automaton.
an automaton, which recognises regular expressions and has a finite number of states.

Non-monotone and monotone operators.
Non-monotone operators alter the structure of passed (fragments of) documents. Monotone operators keep passed documents as is or extend those.

Non-strict Functions.
functions, which accept indefinite data structures as argument and do terminate after a finite leap of time.

OBDD.
on-ordered binary decision tree.

Partial Derivatives Algorithm.
Successive construction of a NFA from a regular expression.
Partial derivatives for a certain terminal symbol are evaluated ↗ in call-by-need mode.

Permutation.
↗ commando-tag that swaps nodes.

Path Problem.
Theoretic questions regarding paths in a given graph.

Placeholder-Plugin.
a module evaluating certain requests formulated in a ↗ command language.

Precedence (of an operator).
synonymous for inverse operator priority. Operators with the lowest precedence have the highest priority over other operators.

Processing-Instruction.
XML-nodes containing a non-functional, but descriptive annotation to a XML node.

Source Language.
Formal Language ↗ instantiation data has to obey.

Redex.
denotes in λ-calculus terms, which may be reduced.

Referential Transparency.
A property that holds, when the evaluation of a function or an expression does not cause any side effects.

Regular Tree Automaton.

A tree automaton, that accepts regular ↗ tree languages.

Regular Tree Grammar.
A regular formal grammar whose terminal symbols extend to trees.

Regular Tree Language.
A formal language, which is generated by a ↗ regular tree.

RelaxNG.
XML-schema language, also see ↗ XSD.

Schema.
specifies a XML-dialect, for checking validity of a XML document.
Transformation. Transforms a XML-document obeying one XML-schema to another XML-document possibly obeying another XML-schema.

Schematron. a XML-schema language.

Safe Commands. special \textit{commando-tags}, which when added to a certain schema language, do not violate the closure property under union, intersection, minus. See also \textit{Syntactic Sugar}.

SINGLE-TYPE GRAMMAR. \textit{Regular tree grammar} whose non-terminals inside of \textit{hedges} do not compete with each other.

Slot. part of a \textit{template}, which is substituted/filled during \textit{instantiation}.

Slot-Markup Language. \textit{template-language}.

Structured Query Language. the most popular standardised database query language.

String-grammar. grammar whose derivations always generate words, also see \textit{word problem}.

Stylesheet. \textit{XSLT-\textit{template-document}}. Substitution group.
denotes a \textit{symbol} in \textit{XSD}.

Super-combinator. A specialised \textit{combinator}, with all containing \textit{abstractions} being super-combinator again. Combinators in imperative/procedural languages lead to modular programs. The introduction of new bindings, for example, turns $\lambda w.(\lambda x.zw)$ into $\lambda w.((\lambda xy.xy)w)$.

Symbol. synonymous for a symbol binding to a \textit{hedge}.

Syntactic Sugar. Constructs or idioms of a programming language, which improve the usability, but which can be replaced by more complicated constructs from the same language. Sugar does not extend functionality, it may only increase expressibility. Idioms worsening the expressibility of a language are called Syntactic Salt.

Template. a XML-document containing \textit{slots}. It is used for \textit{instantiation} and obeys the rules of a \textit{template-language}.

Template-Engine. Software, which instantiates \textit{templates}.

\textit{Template-Expansion}.

\textit{Template-Language}.

is described by \textit{commando-tags}, text nodes, element nodes.

Term-Evaluator. Part of \textit{placeholder-plugins} providing the \textit{template-engine} with formatted instantiation data.

Tracing. Tracking down errors by using a stack.
IX. APPENDIX A: Denotational Semantics

Instantiation

Source: https://rhaber123.github.io/web-page/

(S) $\mathcal{E}^\text{Start}(\mathbf{x})^{\#} = \mathcal{E}(\mathbf{x}^{2})^{\#}$ for $\mathbf{x}^{2} = \mathcal{E}''(\mathbf{x})$

(E) $\mathcal{E}'(\text{EX} \circ \text{EX})^{\#} = [\text{let attDefs = filter} \; \lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} ;
\text{nodes} = \text{filter} \; \lambda \text{child} \; \text{not} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \text{in} \; \text{EX} \circ \text{EX} \circ (\text{qSort} \; \theta^{\#} + \text{attDefs}) \; \text{nodes}]^{\#}$

(11) $\mathcal{E}''(\text{TxtX} \circ \text{EX})^{\#} := \text{XTxt} \circ \text{EX}$

(12) $\mathcal{E}''(\text{XAtt} \circ \text{EX})^{\#} := \text{XAtt} \circ \text{EX}$

(13) $\mathcal{E}''(\text{EX} \circ \text{EX})^{\#} = [\text{let mdefs = filter} \; \lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} ;
\text{nodes} = \text{filter} \; \lambda \text{child} \; \text{not} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \text{in} \; \text{EX} \circ \text{EX} \circ (\text{concatMap} \; \lambda \text{node} \; \text{EMA} \; \text{node}^{\#} (\varsigma, \text{mdefs}, \pi)) \; \text{nodes}]^{\#}$

(A1) $\mathcal{E}''(\text{XIf} \circ \text{EX})^{\#} :=
\begin{cases}
\text{concatMap} \; \lambda \text{x2c} \in \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \circ \text{gc} \\
\text{false}
\end{cases}
$

(A2) $\mathcal{E}''(\text{XForEach} \circ \text{EX})^{\#} :=
\begin{cases}
\text{concatMap} \; \lambda \text{x2c} \in \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \circ \text{gc} \\
\text{false}
\end{cases}
$

(A3) $\mathcal{E}''(\text{XCallMacro} \circ \text{EX})^{\#} :=
\begin{cases}
\text{getMacro} \; \lambda c \in \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \mu_{2} \\
\text{false}
\end{cases}
$

(A4) $\mathcal{E}''(\text{TXT} \circ \text{EX})^{\#} :=
\begin{cases}
\text{getMacro} \; \lambda \text{x2c} \in \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \circ \text{xs} \\
\text{false}
\end{cases}
$

(A5) $\mathcal{E}''(\text{XInclude} \circ \text{EX})^{\#} :=
\begin{cases}
\text{getMacro} \; \lambda \text{x2c} \in \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \circ \text{xs} \\
\text{false}
\end{cases}
$

(A6) $\mathcal{E}''(\text{EX} \circ \text{EX})^{\#} :=
\begin{cases}
\text{concatMap} \; \lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \circ \text{xs} \\
\text{false}
\end{cases}
$

(A7) $\mathcal{E}''(\text{TXT} \circ \text{EX})^{\#} :=
\begin{cases}
\text{getMacro} \; \lambda \text{x2c} \in \text{EX} \; \varsigma \; \varsigma \left(\lambda \text{child} \; \text{EMA} \; \text{child}^{\#})_{c}^{\#} \right) \circ \text{xs} \\
\text{false}
\end{cases}
$

Validation

(S) $\mathcal{E}^\text{Start}(\mathbf{x})^{\#} := \mathcal{E}(\mathbf{x}^{2})^{\#}$ for $\mathbf{x}^{2} = \mathcal{E}''(\mathbf{x})$

(E1) $\mathcal{E}''(\text{Epsilon, TxtR ""}^{\#}) := \text{True}$

(E2) $\mathcal{E}''(\text{Epsilon, TxtR ""}^{\#}) := \text{False}$

(E3) $\mathcal{E}''(\text{Epsilon, Epsilon}^{\#}) := \text{True}$

(E4) $\mathcal{E}''(\text{Epsilon, Epsilon}^{\#}) := \text{True}$

(E5) $\mathcal{E}''(\text{Epsilon, Star}^{\#}) := \text{True}$

(E6) $\mathcal{E}''(\text{Epsilon, TextR}^{\#}) := \text{True}$

(E7) $\mathcal{E}''(\text{Epsilon, Then}^{\#}) :=
\begin{cases}
\mathcal{E}''(\text{Epsilon, r1}^{\#} \land \mathcal{E}''(\text{Epsilon, r2}^{\#})
\end{cases}$

(Then1) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then2) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then3) $\mathcal{E}''(\text{Then}^{\#}) :=
\begin{cases}
\text{Epsilon}^{\#}
\end{cases}$

(Then4) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then5) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then6) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then7) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then8) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Then9) $\mathcal{E}''(\text{Then}^{\#}) := \text{Epsilon}^{\#}$

(Φ) $\mathcal{E}''(\text{inst, MacroR mname}^{\#}) :=
\begin{cases}
\text{Epsilon}^{\#}
\end{cases}$

(Ω) $\mathcal{E}''(\text{inst, r1 r2}^{\#}) :=
\begin{cases}
\text{Epsilon}^{\#}
\end{cases}$
X. APPENDIX B: PARTIAL-DERIVATIVES ALGORITHM

Given: Regular expression $t = x^* \cdot (x \cdot x + y)^*$

To be found: NFA with $L(t)$?

Step 1: Determine linear form

$1f(t) = \text{lf}(x^*) \circ r \cup \text{lf}(r) = (\text{lf}(x) \circ x) \circ r \cup \text{lf}(r)$

$2\text{lf}(y) = \text{lf}(x \cdot x) \circ y \cup \text{lf}(y)$

$3\text{lf}(y) = (\text{lf}(x \cdot x) \cup \text{lf}(y)) \circ r = ((\text{lf}(x) \circ x) \cup \text{lf}(y)) \circ r$

Step 2: Apply the Partial-Derivatives algorithm:

All linear forms are determined now for the second component.

$< PD_0, \Delta_0, \tau_0 > := \emptyset, \{ t \}, \emptyset$

$PD_1 := PD_0 \cup \Delta_0 = \{ t \}$

$\Delta_1 := \cup_{p \in \Delta_0} \{ q \mid < x, q > \in lP(p) \land q \notin PD_1 \} = \{ r, r \}$

$\tau_1 := \tau_0 \cup \{ < p, x, q > \mid p \in \Delta_0 \land < x, q > \in lP(p) \} = \{ < t, x, t >, < t, x, r >, < t, y, r > \}$

$< PD_1, \Delta_1, \tau_1 > := \{ t \}, \{ x \cdot r, r \}, \{ < t, x, t >, < t, x, r >, < t, y, r > \}$

$PD_2 := \{ t, x \cdot r, r \}$

$\Delta_2 := \emptyset$

$\tau_2 := \{ < t, x, t >, < t, x, r >, < t, y, r >, < x \cdot r, r >, < r, x \cdot r >, < r, y, r > \}$
<PD₂, Δ₂, τ₂ > := < {t, x · r}, ∅, τ₂ >
PD₃ = {t, x · r, r}
Δ₃ = ∅
τ₃ := τ₂ → Halt!

Final states:
F ⊆ PD₃ := \{ f | f ∈ PD₃ ∧ f ∈ τ₁ \} = \{ r \}

Remaining states:
PD₃ \ F = \{ t, x · r \}

Step 3: Building NFA:

τ₂Reg₀ .. terms that do not contain ε
τ₂Reg₁ .. terms that do contain ε

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< PD₂, Δ₂, τ₂ > := < {t, x · r}, ∅, τ₂ >
PD₃ = {t, x · r, r}
Δ₃ = ∅
τ₃ := τ₂ → Halt!
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