Brightness constraint for cooling models of young neutron stars

Hovik Grigorian†

Institut für Physik, Universität Rostock, D-18051 Rostock, Germany

Abstract

We study the systematics of neutron star cooling curves with three representative masses from the most populated interval of the estimated mass distribution for compact objects. The cooling simulations are made in the framework of the nuclear medium cooling (NMC) scenario using different combinations of possible nucleon-nucleon pairing gaps. Possible heating or enhanced cooling mechanisms in the crust are not considered. We define a constraint on the highest possible temperatures for a given age of young neutron stars and show that this limits the freedom of modeling pairing gaps and crust properties.
Studies of neutron star cooling evolution become very actual due to the presently known surface temperature and age data provided by X-ray observatories such as CHANDRA, XMM Newton and from the ROSAT catalogue [1]. These new data open a wide perspective for nuclear astrophysics for which up to now the knowledge of internal structure of the compact stars and the properties of stellar matter under extreme conditions remain central problems. Theoretical models and hypotheses about the equation of state of high density matter provide different alternatives for the explanation of the same set of observational temperature - age (TA) data points, when additional constraints are not provided. In this work we point out an existing correlation between the crust model and cooling behaviour of light neutron stars, which has a selective power in combination of TA data with the mass spectrum of neutron stars.

In our recent investigations of the cooling evolution of neutron stars (NS) we have adopted the so called nuclear medium cooling (NMC) scenario [2], which goes beyond the minimal cooling scenario [3], where in-medium modifications of cooling regulators by definition have been disregarded. Both approaches agree in the philosophy that such very effective cooling mechanism like the direct Urca process should not occur in typical NS. In these approaches the main cooling process is the modified Urca process, which in our NMC scenario also includes the in-medium softening of the pion propagator [4]. Earlier investigations within this cooling scenario [5,6] have chosen the crust model as a simplified Tsuruta law $T_{\text{sur}} = (10 T_{\text{in}})^{2/3}$. Although it is shown in Ref. [7] that the cooling evolution could be essentially affected by the inclusion of internal heating, nevertheless the latter is expected to be important for late time evolution and will not affect the results of this paper. It has been omitted in the present cooling scenario. For more recent reviews on the cooling scenarios see [8–10].

The cooling simulations presented in this work are based on a code with a number of improved inputs concerning the heat conductivity, the nucleon-nucleon pairing gaps and a new model of the neutron star crust and envelope. These models are basically taken from the recent calculations of Ref. [11], where the amount of light elements in the crust and the
influence of the magnetic field have been taken into account.

The choice of the structure of crust and envelope of the compact object becomes a central question due to the direct connection of the surface temperature with the observations.

Since simultaneous measurements of the surface temperature and either the mass or the radius of neutron stars are absent, one needs to relate the known TA data with other observational information in order to discriminate between different models.

One quantitative approach has been suggested in [12,13] and developed recently for the NMC scenario [14]. It uses the statistical method of population syntheses in combination with models for the cooling evolution and the mass distribution of isolated objects to derive a Log N - Log S dependence to be compared with observational data. This allows for an additional selection among those scenarios which have successfully passed the TA test. Nevertheless, in the routine of the Log N - Log S test the early stages of the time evolution are not included, but only the tracks of relatively old objects \( t > 3 \cdot 10^4 \) yr). This period is already in the transition from the neutrino to the photon cooling era. We will show in this work that the cooling behavior of the young objects is strongly affected by crust properties, which however do not alter the Log N - Log S distribution.

In the spirit of the Log N - Log S test we suggest an additional condition to the TA test, which requires young neutron stars \( 10^3 \leq t/\text{yr} \leq 3 \cdot 10^4 \) with low masses \( \sim 1.1M_\odot \) not to have temperatures (i.e. brightness = luminosity / \( 4 \pi \) distance\(^2\), where absorption could be neglected for hot sources since they have a hard spectrum) exceeding those of presently known objects of the same age. We will therefore call it a brightness constraint (BC).

Therefore, it is unlikely that objects with a given age are hotter than those already observed. As a consequence, cooling models which would predict such objects should be rejected.

Our approach is based on the following assumptions made on the basis of current observational data:

- *All objects with temperatures exceeding* \( T = 4 \cdot 10^6 \) *K are potentially observable.*
The probability to find an object increases with its luminosity due to the increase of the visibility volume and reduction of the absorption of X-rays by the interstellar medium (see [15]). The absorption falls by several orders of magnitude with increase of the photon energy from 0.1 to \( \sim 10 \) keV. One can estimate the lower limit of the temperature of an object at the distance of 10 kpc or even more distant and behind clouds of the interstellar medium with maximal absorption, say for hydrogen column densities \( n_H \sim 2 \cdot 10^{22} \text{ cm}^{-2} \) (corresponding to galactic coordinates \( l=30, b=0 \)), which would be observed with instruments like ROSAT, with a lower limit for the brightness given by a few 0.01 cts/s. Using such an estimate NASA’s HEASARC tools webpage \(^1\), one obtains a limiting temperature \( T = 4 \cdot 10^6 \text{ K} \), close to the temperature of Crab pulsar \( (2.2 \cdot 10^6 \text{ K}) \). Therefore, in a good approximation the temperature of the observed object with highest temperature could be considered as a border of BC.

To specify the BC as a test we have made further assumptions:

- **The distribution of young objects is a step-like function of temperature for higher temperatures.**
- **The uncertainty of the BC border is of the same order as the error bars of the measured temperatures at the corresponding age.**

We assume that with sufficient observational data it is possible to measure the temperature distribution of the young objects. Even if the population of young objects is small \( (\bar{N} \simeq 100 \ [16]) \) one can not conclude that the probability to find an object with arbitrary high temperature is finite. The distribution is likely a step like or very stiffly falling function of temperature for higher temperatures, because the star can not have arbitrarily high temperature from the beginning of the evolution and the mass distribution of the compact objects are also a step like function for the small masses (see [14]), which are expected to

\(^1\)http://heasarc.gsfc.nasa.gov/docs/tools.html
populate the domain of slower coolers.

The statistical analysis to define the BC border is complicated by the small total number of potentially observable objects. It is assumed to be \( \approx 0.1 \bar{N} \) [17], because all known objects are colder than \( 4 \cdot 10^6 \) K [18].

Therefore we leave a corridor between the upper limits of the observed data points of higher temperatures at a given age and the BC border on the \( 2\sigma \) level of the error bar of measured temperature where it was possible and put \( \delta \log_{10}(T/K) \approx (1/\bar{N})* \) (the whole expected temperature interval of young objects)/(the probability to be observed) \( \approx 0.4 * 0.01/0.1 = 0.04 \) (for Crab), which is of the same order as the error bars of measured temperatures.

This last estimation is based on the assumption that the probability for the existence of an object is equally distributed for all available temperatures.

In order to apply this simple BC test to already published [2] scenarios we choose the strategy to follow the cooling evolution of objects with the representative values \( M = 1.1, 1.21, \) and \( 1.41M_\odot \) corresponding to the most populated bins in the mass spectrum of Fig. 1 in [14]. The basic idea behind this mass spectrum of NS is to use HIPPARCOS data on massive stars around the sun as the mass distribution of progenitors in conjunction with the calculations by [19] for a population synthesis of nearby NS.

The choice of the above representative masses for the BC test is justified by the results of [2] and [3] where it was demonstrated that all objects outside the chosen interval of the masses \( 1.1 \div 1.4 M_\odot \) are cooler than those inside this interval. Even if this systematics is not too strong, this statement is based on the set of cooling simulations [2] and is applicable to young objects and can be rephrased as follows: For a given age the brightest objects are from the most populated mass interval.

The description of the cooling evolution is mainly given by the equation combining the energy balance and thermal energy transport [20]

\[
\frac{\partial}{\partial t}(Te^\phi) = -\frac{\epsilon_\nu}{c_V}e^{2\phi} + \frac{\epsilon_\lambda}{c_V r^2} \frac{\partial}{\partial r} \left( \kappa r^2 e^{\phi} + \lambda \frac{\partial}{\partial r} (Te^\phi) \right), \tag{1}
\]
where $\phi$ and $\lambda = -\frac{1}{2} \ln(1 - 2m/r)$ are metric coefficients. The heat conductivity $\kappa$, the total neutrino emissivity $\epsilon_\nu$ and the total specific heat $c_V/n$ are given as the sum of the corresponding partial contributions defined for density profiles $n(r)$ of the constituents of the matter under the conditions of the actual temperature profile $T(r,t)$. The mass of the star is the accumulated mass below the surface, $M = m(r = R)$, which together with the gravitational potential $\phi(r)$ can be determined by Oppenheimer-Volkov equations (see [6,20]), where the energy density profile $\varepsilon = \varepsilon(r)$ and the pressure profile $p = p(r)$ are defined by the condition of hydrodynamical equilibrium. The boundary condition for the solution of (1) reads $T(r = R_{in}, t) = T_{in}(t)$. The energy flux from the surface has also a contribution from photons $L_\gamma = 4\pi\sigma R^2 T_s^4$, which is governing the cooling, when the inner crust temperature $T_{in}$ falls down to $10^8$K.

For different crust models we used in our recent calculations the $T_s - T_{in}$ relations [2,21,22] shown in Fig. 1. The parameter $\eta \sim \Delta M_L/M$ is a measure for the thickness of the light element layer [23], which is related to the pressure at the bottom of the light element envelope. Thus these borders of the acceptable $T_s - T_{in}$ relations can be denoted as heavy element $\eta = 4 \cdot 10^{-16}$ (further denoted as crust model (E) corresponding to the notation in Ref. [3]) and light-element $\eta = 4 \cdot 10^{-8}$ crust models, respectively.

From the discussion we omitted the possible heating in the crust to make our focus on the general aspect of the choice of crust. In principle the heating could also be included without qualitative changes in the argumentation we use here, because it will change the cooling evolution only in the photon era, but not the evolution of young objects [24,25].

The surface temperature of NS with thick light element crust is higher during the neutrino cooling era and it shows slower cooling for young objects in contrast to the case with a heavy element rich crust (E) as shown in works [2] and [3].

For the cooling simulation done in Refs. [2,21] we follow the idea of light element decay, which is in coherence with Ref. [3]. The assumption is that during the time evolution of the envelopes chemical composition the mass fraction of the envelope consisting of light elements decays. The time dependence can be described by an exponential
\[ \Delta M_L(t) = e^{-t/\tau} \Delta M_L(0) \]  

where \( \Delta M_L(0) \) is the initial mass of light elements and \( \tau \) is the decay rate. This decay could be due to the pulsar mechanism which injects light elements into the magnetosphere or due to nuclear reactions which convert these elements into heavy ones [26,27]. This rather complicated picture we had modeled with a fit between the light and heavy element crust models corresponding to a slow decay of light elements (model (C)), whereas the heavy element crust model is denoted as fast decay (model (E)), see Fig. 1.

For the demonstration of the results given in Fig. 2 we reconsider the same cooling scenarios as in [14] using the notation given in Table 1 of that paper. We exclude only models V and II from our present discussion, since they are not in our aimed class of models. The remaining models selected are defined in Table I by the choice of the nucleon pairing gaps: (A) models I, IX - gaps from [28]; (B) models III, VI, IV, VII, VIII - gaps from [11]. For all models the \( ^3P_2 \) neutron gap is suppressed by 0.1; see Refs. [2], [14] and [21] for more details.

Each of the six models is calculated with both crust models (C) and (E). The results are displayed in the six panels of Fig. 2, grouped into two columns. The left one shows the models which successfully passed the TA test, but only for crust model (E) fulfill the BC for younger objects to not have higher temperatures than those given by the observations. In the right column all models pass the BC test, while those with crust (E) failed the TA test leaving some observed points out of explanation. Note that in Fig. 2 we have not shown cooling curves for neutron stars with masses exceeding 1.41 \( M_\odot \) which correspond to intermediate or fast coolers since we are focused here on the discussion of young and slow coolers. We want to stress, however, that all models in Fig. 2 can pass the TA test if the crust model is chosen suitably.

In the upper two panels both models (IV,VI) and IX are calculated without possible \( \pi \)-condensation in contrast to the models III and I, where the condensation is possible. Therefore, the difference in these corresponding plots is only in the behavior of the configu-
ration with $M = 1.41 M_\odot$ for which $\pi$-condensation occurs. The difference between the four upper panels (III, IV) and (I, IX) comes from the difference in the nucleon-nucleon pairing gaps. Only the latter two models are microscopically justified [28]. However, without additional suppression of the neutron $3P_2$ gap, necessary to reduce the enhanced cooling by the neutron pair breaking and pair formation processes, the models would be in disagreement with the TA test (see [21]). The physical reason of this suppression could be the medium-modification of the spin-orbit interaction in neutron matter [29]. Thus the idea of slow decay of light elements in crust model (C) could be considered as a more consistent suggestion of the crust-envelope model.

Summarizing our discussion we can conclude the following. The application of the NMC scenario for the simulation of the cooling evolution of neutron stars in comparison with the existing observational data shows that

- the TA test of cooling scenarios in conjunction with the BC test can be selective for the discrimination between the crust models, when nucleon pairing is already chosen, or vice versa;

- the TA test of the cooling scenario even with the improvement of the BC test is not sufficient to make a final conclusion about the validity of neither the crust model nor the nuclear superfluidity;

- since the Log N - Log S test essentially depends on the cooling behaviour of objects older than $3 \cdot 10^4$ yr it does not interfere with the results of the BC test.

All three tests are necessary but not sufficient for the final selection. With a special choice of the crust model it is possible to change the results of the TA test while the LogN -LogS test will remain unaffected. As it is discussed in the Ref. [30] there is another accelerator of the cooling of low-mass neutron stars, the direct Urca process of neutrino emission allowed in the mantle of a neutron star near the crust-core interface, due to inhomogeneous nuclear structures. Such possibilities will make the crust model of slow decay of light elements more
preferable.

In conclusion, the present work is a contribution to the development of general testing schemes for models of compact star cooling evolution [31] and strongly interacting matter at high-densities [32] using constraints from compact star observations.

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REFERENCES

† Permanent address: Department of Physics, Yerevan State University, 375047 Yerevan, Armenia

[1] W. Voges et al., Astron. Astrophys. 349 (1999) 389.

[2] D. Blaschke, H. Grigorian and D. N. Voskresensky, Astron. Astrophys. 424 (2004) 979.

[3] D. Page, J. M. Lattimer, M. Prakash and A. W. Steiner, Astrophys. J. Supp. 155 (2004) 623.

[4] D. N. Voskresensky, Lect. Notes Phys. 578 (2001) 467.

[5] Ch. Schaab, D. Voskresensky, A.D. Sedrakian, F. Weber, and M. K. Weigel, Astron. Astrophys. 321 (1997) 591.

[6] D. Blaschke, H. Grigorian and D. N. Voskresensky, Astron. Astrophys. 368 (2001) 561.

[7] C. Schaab, A. Sedrakian, F. Weber and M. K. Weigel, Astro. Astrophys. 346 (1999) 465.

[8] D. G. Yakovlev and C. J. Pethick, Ann. Rev. Astron. Astrophys. 42 (2004) 169.

[9] D. Page, U. Geppert and F. Weber, arXiv:astro-ph/0508056 (2005).

[10] A. Sedrakian, arXiv:nucl-th/0601086 (2006).

[11] D. G. Yakovlev, O. Y. Gnedin, A. D. Kaminker, K. P. Levenfish and A. Y. Potekhin, Adv. Space Res. 33 (2004) 523.

[12] S. B. Popov, M. Colpi, M. E. Prokhorov, A. Treves and R. Turolla, Astron. Astrophys. 406 (2003) 111.

[13] S. B. Popov, R. Turolla, M. E. Prokhorov, M. Colpi and A. Treves, Astrophys. Space Sci. 299 (2005) 117.

[14] S. Popov, H. Grigorian, R. Turolla and D. Blaschke, Astron. Astrophys. 448 (2006)
[15] J. Wilms, A. Allen and R. McCray, Astrophys. J. **542**, (2000) 914.

[16] S. B. Popov, R. Turolla and A. Possenti, Mon. Not. Roy. Astron. Soc. **Lett.** (2006) 34.

[17] C. A. Faucher-Giguere and V. M. Kaspi, arXiv:astro-ph/0512585.

[18] D. L. Kaplan, D. A. Frail, B. M. Gaensler, E. V. Gotthelf, S. R. Kulkarni, P. O. Slane and A. Nechita, Astrophys. J. Suppl. **153** (2004) 269.

D. L. Kaplan, B. M. Gaensler, S. R. Kulkarni and P. O. Slane, arXiv:astro-ph/0602312.

[19] S.E. Woosley, A. Heger and T. A. Weaver, Rev. Mod. Phys. **74** (2002) 1015.

[20] F. Weber. *Pulsars as Astrophysical Laboratories for Nuclear and Particle Physics*, (IOP Publishing, Bristol, 1999).

[21] H. Grigorian and D. N. Voskresensky, Astron. Astrophys. **444** (2005) 913.

[22] H. Grigorian, AIP Conf. Proc. **775** (2005) 182.

[23] D. G. Yakovlev, K. P. Levenfish, A. Y. Potekhin, O. Y. Gnedin and G. Chabrier, Astron. Astrophys. **417** (2004) 169.

[24] K. Van Riper, B. Link and R. Epstein, arXiv:astro-ph/9404060 (1994).

[25] S. Tsuruta, IAU Symposium 218: *Young Neutron Stars and Their Environment*, (2004) L18.

[26] P. Chang and L. Bildsten, Astrophys. J. **585** (2003) 464.

[27] P. Chang and L. Bildsten, Astrophys. J. **605** (2004) 830.

[28] T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. **112** (2004) 37.

[29] A. Schwenk and B. Friman, Phys. Rev. Lett. **92** (2004) 082501.

[30] M. E. Gusakov, D. G. Yakovlev, P. Haensel and O. Y. Gnedin, Astron. Astrophys. **421**
(2004) 1143.

[31] S. Popov, H. Grigorian and D. Blaschke, arXiv:nucl-th/0512098.

[32] T. Klähn et al., arXiv:nucl-th/0602038.
TABLE I. Classification of the models by the possible transition to $\pi$-condensate and the choice of proton ($p^{-1}S_0$) and neutron ($n^{-1}S_0$, $n^{-3}P_2$) pairing gaps with corresponding suppression factors.

| Class of model | Models with $\pi$-condensate | Models without $\pi$-condensate | Gaps |
|----------------|-------------------------------|----------------------------------|------|
| A              | I                             | IX                               | [28]; $n^{-3}P_2\ast 0.1$ |
| B              | III                           | IV & VI$^2$                      | [11]; $n^{-3}P_2\ast 0.1$ |
| B’             | -                             | VII                              | [11]; $p^{-1}S_0\ast 0.5$; $n^{-3}P_2\ast 0.1$ |
| B”             | VIII                          | -                                | [11]; $p^{-1}S_0\ast 0.2$; $n^{-1}S_0\ast 0.5$; $n^{-3}P_2\ast 0.1$ |

$^2$The model VI is the same as model IV, which was already calculated with crust (E) in Ref. [2]
FIG. 1. (Color online) The relation between the inner crust temperature and the surface temperature for different models. Dash-dotted curves indicate boundaries of the uncertainty band. Notations of lines are determined in the legend. For more details see [2,3] and [23].
FIG. 2. (Color online) Cooling evolution for representative NS from the most populated mass bins $M = 1.1, 1.21, 1.41M_{\odot}$ according to the NMC scenario. The classification of models is taken from Ref. [14]. The cooling curves for slow decay (C) are shown with thick lines and those for fast decay (E) with thin lines. The data points correspond to Fig. 1 of Ref. [3]. According to the brightness constraint applied for young objects the shaded regions should not be populated and cooling scenarios entering there shall be rejected.