Developing a learning design of mathematical modelling courses on understanding basic concept of mathematical modelling

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Abstract. The approach to learning mathematical modelling is an approach that focuses on mathematics learning by using the context of real world phenomena so that the use of mathematics can be used in describing a process of understanding, simplifying, and solving problems in the form of mathematical modelling in order to have deeper understanding. Therefore, undergraduate students of the mathematics education as prospective mathematics teachers in schools need to get in-depth competencies related to mathematical modelling learning. These competencies were obtained by students in mathematics modelling courses. Based on the importance of mathematical modelling courses for students of mathematics education study programs need to be made in the design of learning in the subject of mathematical modelling based on the development of questions according to the characteristics of mathematical modelling. This study aims to generate local instruction theory in mathematics modelling courses after lectures are based on designing modelling task. Therefore this research will refer to design research methods in learning by constructing the Hypothetical Learning Trajectory (HLT) which will become Local Instruction Theory in the subject of mathematical modelling. In this paper, we focussed on the first step on HLT that is the basic concept of mathematical modelling. The results of this study are forming hypotheses, developing learning designs, and testing the truth according to research design methodology.

1. Introduction
During the last two decades, the discussion about mathematics modelling has been increasingly emphasized as an educational approach in mathematics education from elementary level to higher education, we can see the proceeding of ICTMA Conference, ICMI Congress, with survey in Kaiser and in Kaiser/Blomhøj/Sriraman [1, 2]. Recently, OECD’s PISA study stimulated mathematical modelling known as Mathematical Literacy which is the ability to deal with the real world situation. Mathematical modelling is important because it can help students’ to better understand the world, support mathematics learning, develop mathematical competencies, and becomes more meaningful for students in learning mathematics [3]. However, in the most country, there is only so few modelling in everyday classroom and mostly it treated undressing the context, word problem and exercising mathematics [4]. It means that there is a gap between the educational theory and classroom practice. The main reason is that teachers have difficulty to deal with mathematical modelling. Teachers have a crucial role on basis of quality mathematics teaching. The definition of quality mathematics teacher is that a demanding orchestration of teaching the mathematical subject matter, permanent cognitive activation of the learners, and an effective classroom management [5]. In order to achieve the quality of mathematics teaching, teachers should change the new culture of task by changing mathematics teaching requires the selection of appropriate tasks and their implementation in the classroom.
The analysis of modelling task considered to the component of modelling process as a following figure [6].

![Figure 1. The modelling process.](image)

Quality teaching in mathematical modelling is not easy to accomplish. In addition, the research shows that the competency in constructing mathematical modelling problem of teacher is low [5,7]. Misunderstanding of prospective teacher in constructing the mathematical modelling problem was happened, for instance prospective teacher students still confused to select the appropriate the real world problem which has interpretation from the application problem [7]. Some of these shortcomings is not only because of a lack of practical realization of existing knowledge but also a lack of knowledge of the students’ difficulties when solving cognitively modelling task among in individual work, in pair or group work [8, 9, 10]. Therefore, it is necessary to design of mathematical modelling courses through designing task on modelling based learning.

2. Methods
The research method used in this research is educational design research. Educational Design Research which can also be called Development Research is research formed from experimental learning in class that develops the design of learning sequences and produces local theories of learning. Therefore, this learning design research aims to formulate and develop a Hypothetical Learning Trajectory (HLT) which will become a Local Instruction Theory. In one learning cycle research design has three stages, namely preliminary design, experimentation and retrospective analysis.

2.1 Preliminary design
At this initial stage, the researcher constructs the Hypothetical Learning Trajectory (HLT) which functions as a guide in designing the learning guide. HLT contains the objectives and activities of learning in mathematics modelling courses. Following this HLT has been developed in the initial stages of learning design.
2.2 Experiment
In the experiment stage, the HLT that has been designed will be tested on students. This trial will see whether the things in preliminary design are in accordance with reality or not. The things that occur during the experiment phase will be improvements or modifications to the HLT design. The focus of HLT in this stage is to learning activities, learning processes and observations.

2.3. Retrospective Analysis
At this stage, all data obtained from the experimental phase will be analyzed. The analysis process is between components in HLT before learning and after learning, followed by an analysis of possible causes in accordance with previous theories, and synthesis of improvements / modifications to the HLT design that will be used for the next cycle.

3. Result and Discussion
In this paper, we would give the result and discussion only for the first step in HLT which is the first and second activity. In the first step, the goal is to understand the basic concept of mathematical modelling. The research subject is the prospective teacher students in Mathematics Education of Teacher Training Faculty, Sriwijaya University. There were 40 prospective teacher students enrolled in mathematical modelling courses. For the analysis of the learning design, we observed the video during the teaching experiment to know how the activities could support the prospective teacher students’ in designing task. We also analyzed the written work of prospective teacher students.

3.1. Activity 1: Identifying the characteristics of mathematical modelling
There are 4 characteristic of mathematical modelling namely a) Real world problem, b) open-ended problem, c) worth it to solve it and d) mathematics as the tools to find the solutions. According to the data interview, some students still have misunderstanding to identify the first and the second characteristic of mathematical problem. The following transcript shows it.
Researcher : What do you think about real world problem?
Student : The problem in daily life.
Researcher : Do you think every problem in daily life can be mathematical modelling?
Student : Yes.
Researcher : Why?
Student : umm, because in daily life we have problem and we need mathematics to solve it.
Researcher : can you give me the example of mathematical modelling in daily life?
Student : (thinking for a while)...umm, maybe the problem about counting the money, buy or sell something.
Researcher : Do you think it is mathematical modelling
Student : Yes, because it is real world problem.

Based on the transcript above, the researcher asks about the meaning of real world problem, we can see that student attempt to explain the meaning of the real is that every daily life problems. However, student did not the problem in detail whether it is mathematical modelling or not because we cannot say that every daily life problem is mathematical modelling. As we know that in mathematics problem, we have word problem, application problem and modelling problem. In modelling problem, we need the interpretation to solve the problem [12,13]. And, if we see the example of mathematical modelling problem in the transcript above, it is not the correct example of the modelling problem. There is no interpretation on it. Student give the example about counting the money in buying or selling and it is just the word problem. It seems that student cannot distinguish the difference between word problem and modelling problem.

When designing modelling task, the problem are often related to mathematical application problems and solving application problems which are integrated in problem solving. The modelling problem is linked to the mathematical applications problems to reality or to other subjects, and mathematical interpretation of the term modelling. This connection can be examined as analyzing, assimilating, interpreting and validating problem [14, 15].

3.2. Activity 2: Solving the mathematical modelling problem
The problem solver must understand the problem situation to be simplified, structured and made more precise for constructing the real model situation. The transformation from the real model into a mathematical model consists of certain equation [16, 17]. Solving the equation generates mathematical results that are interpreted in the real world as real results. Therefore, the other mathematical competencies (reading, communicating, applying the strategies, and working mathematically) are helpful for analyses of modelling tasks. The seven-step in modelling cycle refers to construct, simplify, structure, mathematize, work mathematically, interpret, validate and expose.

In the second activity, the prospective teacher students solved the modelling problem. It has been solved by working in group consist of 3 to 4 prospective teacher students. The goal of this activity is to know the process of solving modelling tasks in detail and referring to the different phases of modelling process. The following tasks in this paper are taken from own projects which is already valid as a modelling task [8]. Therefore, the task can be used to analyse how modelling route in deal with cognitively demanding modelling problem.
Figure 3. Example of modelling problem.

The figure shows the modelling problem about transportation. Students were asked to find the best price between grab-bike or go-jeck. Students should compare the data before the judge which one is the best price. Students followed the 4 steps in finding the solution, namely 1) Identifying the problem, 2) Making assumption and defining the variables, 3) Making the formula and solving it, and 4) Reporting the result. Here is the students' answer.

Figure 4. Identifying the problem.

From Figure 4, we can see that student identify the problem by writing what is the problem and make their statement from the problem by using their own experience in daily life to more understand the situation problem. In their work, it shows that “the best price” of the task means the cheapest fee related to the minimum fee, distance, and time. This statement generates based on a practical understanding on which the real model will be constructed at the beginning of the modelling cycle [20].
Figure 5. Making assumption and defining the variables.

In the figure 5, the assumption in the problem is that there is no difference on the interesting figure of the modelling problem, the distance from the modelling problem is same, and grab or gojek (the option of the problem) is available. Then, students choose the important object in the modelling problem which is distance, time and fee. This important object becomes the variable of the modelling problem [18,19]. Lastly, the identify the relation among the variable to generate the formula. The following figure shows the students’ work on it by making the formula of the fee considering the distance as the variable and the busy time.

Figure 6. Making the formula and solving it.

The piecewise function is constructed by students working. The piecewise function is defined by two and three function and each function applying to a particular interval of the domain from the main function. The interval related to the distance (s), because the price of the grab or gojek is influenced by the the distance. We can see that if s is less than 2, then the minimum rate and initial rate will be applied. And, if s is greater than 2, the rate will be different from the previous function. In addition, the normal time and bust time is also considered in this piecewise function. This formula leads student to solve the modelling problem by calculating for each function (situation modelling problem) from the difference interval (refers to different travel distance). The following table figures the detail function.
Overall, from the four figures above, some students show their skill to solve the mathematical modelling problem. In the figure 4, student identifies the problem based on the information from the problem and the property of the problem in daily life. In this process, student attempts using their prior knowledge to write clearly the problem from their point of view. In the figure 5, student makes assumption based on their prior knowledge and making the independent and dependent variables from the problem. For the figure 6, student creates the formula which is the steps function to describe the situational problem into the mathematics expression. And the last figure, student makes the tables to show the detail solution for the every single case in the problem. Then, in the end the process, students conclude the solution from the table solution.

4. Conclusion
From the HLT, in this paper we focus to the first step that is understand the basic concept of the mathematical modelling. This step consists of two activities namely identifying the characteristics of mathematical modelling and solving the mathematical modelling problem. In the first activity, there was still misunderstanding about the characteristic of mathematical modelling that are real world problem and open ended problem. For the second activity, students show the appropriate process in mathematical modelling. There are four process namely 1) Identifying the problem, 2) Making assumption and defining the variables, 3) Making the formula and solving it, and 4) Reporting the result.
5. Acknowledgment
This project is founded by Sains, Teknologi dan Seni (SATEKS) 2019 Award Universitas Sriwijaya.

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