4D Gauss-Bonnet Black Holes in AdS Space Surrounded by Strings Fluid Mimics Van der Waals Fluid Behavior

H. Ghaffarnejad \textsuperscript{1}, E. Yaraie \textsuperscript{a,b} and M. Farsam \textsuperscript{a,b} \textsuperscript{3}

\textsuperscript{a} Faculty of Physics, Semnan University, P.C. 35131-19111, Semnan, Iran
\textsuperscript{b} Instituut-Lorentz for Theoretical Physics, ITP, Leiden University, Niels Bohrweg 2, Leiden 2333 CA, The Netherlands

Abstract

In this paper we would like to answer the question “Can 4D Gauss-Bonnet black holes in AdS spaces surrounded by strings fluid mimics Van der Waals fluid behavior?; in context of the extended thermodynamics”. In the extended thermodynamics paradigm the pressure of the black holes can be described by cosmological constant and the conjugate variable of the pressure is the thermodynamical volume of the black holes. To examine whether there would be an analogy between 4D Gauss-Bonnet black holes in AdS spaces surrounded by strings fluid and liquid-gas system the analytical solutions of the critical points are derived and the role of strings fluid on the extended phase space of the black hole is probed. It is illuminated that in syllogism with liquid-gas phase transition in Van der Waals fluid there exists a small-large black hole phase transition in 4D Gauss-Bonnet-AdS black hole with strings fluid. Also it is discovered in the Schwartzschild limit where fluid of strings is used to rescue the small-large black holes phase transition. This means the Gauss-Bonnet black holes in AdS Space surrounded by strings fluid mimics Van der Waals fluid behavior even in the Schwartzschild limit. By comparing our results with 5D situations \cite{24}, we infer where the 4D Gauss-Bonnet black holes in AdS space surrounded by strings fluid supports the heating-cooling phase transition. This encourages us to seek answer of this question: “What story might be behind of this observation which is hidden?”

\textsuperscript{1}E-mail address: hghafarnejad@semnan.ac.ir
\textsuperscript{2}E-mail address: eyaraie@semnan.ac.ir
\textsuperscript{3}E-mail address: mhdfarsam@semnan.ac.ir
1 Introduction

Studying black hole as a fascinating predictions of Einstein’s theory of general relativity in a thermodynamic viewpoint has been considered for many years. One of main aspects which is revealed through these viewpoint is thermodynamic phase transition in AdS black holes [1]. In a new attempts in a fascinating way, the study of the AdS black hole phase transition has been generalized by the extended phase space. The extended phase space represents a phase space in which the traditional first law of black hole mechanics is corrected by an $VdP$ term so that the cosmological constant is regarded as thermodynamic pressure of the black hole with and its conjugate variable is thermodynamic volume. A large number of investigations have been done on this concept during last years [2],[3],[4],[5],[6],[7],[8],[9],[10],[11],[12],[13],[14],[15],[16],[17],[18],[19],[20],[21],[22],[23],[24],[25]. Strings fluid is a matter source that has received less attention in solving gravitational equations. The rapid expansion of the universe during inflation is thought to be related to the expansion of cosmic strings [26],[27],[28],[29],[30],[31],[32],[33],[34]. Recently, the authors in [24] studied 5-dimensional black hole in the Einstein-Gauss-Bonnet gravity in presence of cloud of strings. They demonstrated small-large phase transition can happens while there would not happens heating-cooling phase transition (Joule-Thomson effect). In the other hand in early days of 2020 year a model of Einstein-Gauss-Bonnet gravity for $(D - 4) \rightarrow 0$ limit has been introduced which does not suffers from instability of Ostrogradsky and is able to bypass the Lovelock’s theorem and so the authors [35] found a fully novel 4D static spherically symmetric black hole solution. Next, many authors investigated different subjects about this context [36],[37],[38],[39],[40],[41],[42],[43],[44],[45],[46],[47],[48],[49],[50],[51].

In this paper we would like to investigate extended thermodynamics for 4D Gauss-Bonnet-AdS black hole in presence of strings fluid. Then we want to compare our results with the work [24] in which the authors studied extended thermodynamics of 5D Gauss-Bonnet-AdS black hole with a cloud of strings. In this work we would like to observe how extended thermodynamics of Gauss-Bonnet-AdS black hole can be affected by 4D space time. The set-up of this work is as follows:

In section 2, we review 4D Gauss-Bonnet-AdS black hole in presence of strings fluid. Then in sections 3 and 4, we investigate various aspect of the extended thermodynamics of 4D Gauss-Bonnet-AdS black hole in presence of strings fluid. To do so we study $P - V$ criticality, probing global
stability of the system which means swallowtail behavior in Gibbs energy-temperature diagram, and investigating the behavior of the system near the critical points and Joule-Thomson phenomenon.

2 4D AdS Gauss-Bonnet BH Surrounded by Strings Fluid

The action of novel $dD$ Gauss-Bonnet-AdS black hole in presence of strings fluid has be written as

$$S = \frac{1}{16\pi} \int d^d x \sqrt{g} \left( R + \frac{(d-1)(d-2)}{l^2} + \frac{\alpha}{d-4} L_{GB} - F_{\mu\nu} F^{\mu\nu} \right) + I_{NG} \tag{2.1}$$

where $\alpha$ is the second-order Lovelock coefficient (the Gauss-Bonnet coupling coefficient) which is considered to be positive, the second order Lovelock term can be read as

$$L_{GB} = R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + R^2 - 4 R_{\mu\nu} R^{\mu\nu}, \tag{2.2}$$

and the Nambu-Goto action [52] which explains the dynamics of relativistic strings is given by

$$I_{NG} = \int_{\Sigma} \rho \sqrt{-|\gamma|} d\lambda^0 d\lambda^1 \tag{2.3}$$

where $\rho$ is tension (mass per unit length or mass density) in the string and worldsheet of strings can be parameterized by local coordinates $(\lambda^0, \lambda^1)$. Induced metric and the bivector related to strings worldsheet are given respectively by

$$\gamma_{ab} = g_{\mu\nu} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b} \tag{2.4}$$

and

$$\Sigma^{\mu\nu} = \epsilon^{ab} \frac{\partial x^\mu}{\partial \lambda^a} \frac{\partial x^\nu}{\partial \lambda^b}, \tag{2.5}$$

where $\epsilon^{ab}$ is Levi-Civita tensor density. Strings fluid is described by the energy momentum tensor

$$T^{\mu\nu} = (-\gamma)^{-\frac{1}{2}} \rho \Sigma^{\mu\sigma} \Sigma^\nu_{\sigma}, \tag{2.6}$$
where \( \rho \) denotes the proper density of strings fluid. In 4-dimensional Gauss-Bonnet AdS spacetime, the electric charge-less black hole plunged into the field of a strings fluid can be expressed by the following metric.

\[
ds^2 = f(r)dt^2 - f(r)^{-1}dr^2 - r^2d\Omega^2
\]  

(2.7)

where

\[
f(r) = 1 + \frac{r^2}{2\alpha} \left( 1 - \sqrt{1 + \frac{8\alpha M}{r^3} - \frac{32}{3}P_0 \pi + \frac{4\alpha}{r^2}} \right).
\]  

(2.8)

In the above metric potential, \( M \) is the mass of black hole and in the context of the extended thermodynamics the cosmological parameter \( \Lambda \) treats as thermodynamic pressure as \( P = -\frac{\Lambda}{8\pi} \) and \( a \) is a real positive normalization factor. Applying the definitions of the Hawking temperature \( T = \frac{f'(r_+)}{4\pi} \) and the black hole entropy \( S = \int \frac{1}{T} \left( \frac{\partial M}{\partial r^+} \right) dr^+ \) in which \( r_+ \) is exterior horizon radius obtained from the horizon equation \( f(r_+) = 0 \), one can obtain \( M, T \) and \( S \) respectively as follows.

\[
M = \frac{3\alpha + 8\pi Pr^4_+ + 3r^2_+ - 3a^2r^2_+}{6r_+},
\]  

(2.9)

\[
T = \frac{8\pi Pr^4_++r^4_+-\alpha}{8\pi\alpha r_++4\pi r^2_+},
\]  

(2.10)

\[
S = \pi r^2_+ + 4\pi\alpha \ln \left( \frac{r_+}{\sqrt{|a|}} \right).
\]  

(2.11)

It is worthy to note that entropy of black hole is not affected by strings fluid as 5 dimensional case [24]. In the context of the extended thermodynamics the black hole mass \( M \) is treated as Enthalpy so the first law of thermodynamics can be written as

\[
dM = TdS + VdP + Bd\alpha + Ad\alpha,
\]  

(2.12)

where

\[
V = \left( \frac{\partial M}{\partial P} \right)_{S,a,\alpha} = \frac{4}{3}\pi r^3_+,
\]  

(2.13)

is the thermodynamic volume and the conjugate quantity for the normalization factor \( a \) is

\[
B = \left( \frac{\partial M}{\partial a} \right)_{S,P,\alpha} = -\frac{1}{2}r^+_+, \]

(2.14)
and conjugate quantity for the second-order Lovelock coefficient $\alpha$ can be written as

$$A = \left( \frac{\partial M}{\partial \alpha} \right)_{S,P,a} = \frac{1}{2} r_+. \quad (2.15)$$

Comparing the above results with which ones given by the work [24] we can infer the potential $A$ depends on the black hole event horizon while in 5D case, the conjugate potential of the Gauss-Bonnet coupling parameter $\alpha$ is independent of the black hole event horizon. It must be noted we have to introduce this quantities for making the first law of thermodynamics consistent with the Smarr relation. By using the scaling argument the generalized Smarr relation for the 4 dimensional Gauss-Bonnet black hole with strings fluid reads as

$$H = 2TS + \Phi Q - 2PV + 2\alpha A + aB. \quad (2.16)$$

### 3 Thermodynamics of AdS 4D Gauss Bonnet BH Surrounded by Strings Fluid

By using the Hawking temperature it is possible to write the pressure as a function of other thermodynamic parameters so in analogy with classical thermodynamics, the equation of state of the black hole can be rewritten as

$$P = \frac{4\pi T r_+^3 + 8\pi T \alpha r_+ + \alpha + (a - 1)r_+^2}{8\pi r_+^4}. \quad (3.1)$$

Black hole equation of state is a geometric equation with respect to the event horizon so it would be needed a dimensional analysis to translate geometric equation of state to a physical one. On the other hand Van der Waals equation of state which describes phase transition between liquid and gas can be written as

$$(P + \frac{a}{v^2})(v - b) = kT, \quad (3.2)$$

where $v$ is the specific volume of the fluid, $k$ is the Boltzmann constant, the positive constants $b$ and $a$ denote the scale of the fluid particles and their relative distances. Physical temperature and pressure can be defined in Planck units by $T_{ph} = \frac{kc}{2}T$ and $P_{ph} = \frac{hc}{2}$ where $c$ is speed of light and Planck length is defined by $l_p^2 = \frac{Gh}{c^3}$. By multiplying state equation of the black
hole by $\frac{\hbar}{c}$, it can be rewritten in Planck units as follows.

$$P_{ph} = \frac{kT_{ph}}{2l_p^2r_+} + \cdots, \quad (3.3)$$

in which dots denote higher order terms of the equation (3.1). By comparing the temperature equation (3.3) with Van der Waals equation of state (3.2) for large volumes $v >> 1$ it can be concluded that specific volume can be defined as $v = \frac{1}{2}l_p^2r_+$ and by fixing $l_p = 1$ we can define specific volume by the geometric quantity namely the event horizon radius as

$$v = \frac{1}{2}r_+. \quad (3.4)$$

Substituting (3.4) into the black hole equation of state (3.5) we obtain

$$P = 2\pi T v^3 + 16\pi T \alpha v + 4\alpha + av^2 - v^2 \quad \frac{2\pi v^4}{2\pi v^4}. \quad (3.5)$$

To study phase transition itself and other thermodynamic phenomena related to the phase transition of the black hole system, we must be choose a critical hyper-surface at the parameter space and study critical behavior of the system at neighborhood of a critical point on the hyper-surface while their other parameters are kept fixed. The behavior of the thermodynamic system below and above of the critical points of the phase space are different. It demonstrates a phase transition between gas-liquid phase transition in typical thermodynamic systems which in context of black hole mechanics it translates to the small-to-large black hole phase transition. In fact this latter phase transition is happened by connecting an unstable medium size black hole. By using the thermodynamic equation of state given by (3.5) one can obtain location of the critical points by solving the following equations.

$$\left|\frac{\partial P}{\partial v}\right|_{T=T_c} = 0, \quad (3.6)$$

$$\left|\frac{\partial^2 P}{\partial v^2}\right|_{T=T_c} = 0. \quad (3.7)$$

The above equations for the 4D Gauss-Bonnet black holes in AdS space surrounded by strings fluid read to the following critical points.

$$v_c = 2\sqrt{\frac{(3a - 6 - \zeta)\alpha}{a - 1}}, \quad (3.8)$$
\[ T_c = \frac{(3\alpha - \zeta - 4)(a - 1)^2}{2\sqrt{(a - 1)(3a - 6 - \zeta)\alpha\pi(9a - 12 - \zeta)}}, \quad (3.9) \]

\[ P_c = -\frac{(a - 1)^2(4\zeta a - 12a^2 - 7\zeta + 55a - 52)}{8\alpha\pi(-3a + 6 + \zeta)^2(-9a + 12 + \zeta)}, \quad (3.10) \]

where \( \zeta = \sqrt{9a^2 - 48a + 48} \), and subscript \( c \) indicates the word "critical point". It is worthy to note that by comparing our result with 5 dimensional case [24], the effect of strings fluid can be seen in all critical points and the universal Van der Waals ratio depends on string fluid agent. The isothermal diagram for \( P - v \) has been portrayed for different values of the Hawking temperature with \( \alpha = 0.001 \) and \( a = 0.1 \) in figure 1. The blue line shows the ideal gas phase transition happens in monotonically decreasing behavior when the Hawking temperature is more than the critical value. When the Hawking temperature is lower than critical value represented by the red line, the system experience small-large black hole phase transition which is similar to Van der Waals liquid-gas phase transition. It is viewed in \( T < T_c \) case there would be a black hole phase transition of small-medium-large scale which mimics Van der Waals liquid-gas phase transition. In contrast to the large and the small black hole which are stable, medium black hole is quite unstable. Gibbs free energy is a easy-to-use gadget to investigate global stability of a thermodynamic system. Gibbs free energy in the extended thermodynamics formalism can be written as \( G = M - TS \). Any thermodynamic system always exist in a phase which has minimum value of free energy among other possible values. A phase transition happens when two branches of these minimum free energies cross each other, however if this two branches have the same value of free energy then this two phases are called coexist. The 4D Gauss-Bonnet black holes in AdS space surrounded by strings fluid has a following form for the Gibbs free energy.

\[ G = -\frac{1}{36}\left(\frac{4P\pi r_+^6 + 144P\pi \alpha r_+^4 + 4ar_+^3 - 3r_+^4 + 18\alpha r_+^2 - 72\alpha^2}{r_+^2 + 4\alpha}\right). \quad (3.11) \]

By studying behavior of the Gibbs free energy in figures 5,6, and 7, we can infer at pressures lower than the critical pressure, the free energy shows specific swallowtail behavior which confirms the AdS 4D Gauss-Bonnet black holes surrounded by strings fluid, can mimics Van der Waals liquid-gas phase transition in the extended phase space. This swallowtail behavior indicates that the black hole undergoes the transition from the small-large phases. It
is very important where when $\alpha \to 0$ and $a \to 0$, the branch of the diagram with small black hole phase aligns with the temperature axis which means the small black hole evaporates completely and reaches to AdS space. In other words the small-large black hole phase transition attenuates to other phase transition called as phase transition of a black hole to global Anti de Sitter space. A very interesting point is this: in the presence of strings fluid, the 4D Gauss-Bonnet black holes in AdS space surrounded by strings fluid in Schwarzschild limit would experience the small-large black hole phase transition. It is worthy to note that in Schwarzschild black hole small-large black hole phase transition does not happen and the effect of strings fluid can recover small-large black hole phase transition.

4 Further thermodynamic discussions

4.1 Critical exponents

A way to have a better understanding of the behavior of a thermodynamical system near the critical points is study of its critical exponents. This would be defined in temperatures lower than the critical temperature where the system behaves as a Van der Waals thermodynamic system \cite{53,54}. Regarding the critical values which are obtained in previous section one can reach to a dimensionless form of the state equation of the black hole as follows:

$$p = \left( \frac{T_c}{v_c P_c} \right) \frac{\tau}{\nu} + \left( \frac{8 \alpha T_c}{v_c^3 P_c} \right) \frac{\tau}{\nu^3} + \left( \frac{2 \alpha}{\pi v_c^4 P_c} \right) \frac{1}{\nu^4} + \left( \frac{a - 1}{2 \pi P_c v_c^2} \right) \frac{1}{\nu^2},$$  \hspace{1cm} (4.1)

in which $\nu = v_c, \; p = P_c \; P$ and $\tau = T_c \; T$. By expanding the parameters of temperature and volume around the critical point in an isobaric process like:

$$\tau = 1 + t, \; \nu = 1 + \omega,$$  \hspace{1cm} (4.2)

the equation of state can be approximated as

$$p = 1 + At + B\omega + Ct\omega + \cdots$$ \hspace{1cm} (4.3)

where all coefficients $A, B, C, \cdots$ are some regular functions versus the $a$ parameter and here for our aims their exact forms are not important. In this regime one can see the specific heat at constant volume behaves like
$C_v \sim |t|^{-\alpha}$, the isothermal behavior of the order parameter like $\eta \sim |t|^{\beta}$, the isothermal compressibility changes as $\kappa \sim |t|^{-\gamma}$ and also the behavior of pressure in an isothermal process at $T = T_c$ is $P - P_c \sim |v - v_c|^{\delta}$. Studying all these critical exponents called as $(\alpha, \beta, \gamma, \delta)$ would be so helpful near the critical points.

It is easy to see the entropy of our thermodynamic system is independent from the temperature and from $C_v = T \left( \frac{\partial S}{\partial T} \right)_v$, one can infer the specific volume is related to derivative of the entropy versus the Hawking temperature so that $C_v = 0$ when $\alpha = 0$. On the other side, in an isothermal process behavior of the order parameter depends to change of the specific volume for which we have two equations: constant pressure equation $p_l = p_s$ and the Maxwell’s equal area $\int_{\omega_l}^{\omega_s} \omega \frac{dp}{d\omega} d\omega = 0$. Regarding the dimensionless equation of state one can solve these two equations and obtain $\omega = -\omega s \sim \sqrt{-t}$ or $\eta \sim \sqrt{-t}$. So we have another exponent as $\beta = \frac{1}{2}$. For the isothermal compressibility, one can obtain $\kappa_T = -\frac{1}{v} \left( \frac{\partial \omega}{\partial \omega} \right) \left( \frac{\partial \omega}{\partial P} \right)_T \sim \frac{1}{t}$ which leads to $\gamma = 1$. The last exponent which we would be derived from the behavior of the critical isothermal of the pressure at critical temperature corresponds to $t = 0$ in the equation (4.3) or $p - 1 = B \omega$ yielding $\delta = 1$.

### 4.2 Joule-Thomson Effect

In the context of the extended thermodynamics the rich physics of the Anti de Sitter black holes can be disclosed by studying the Joule-Thomson expansion [18],[19]. In the Joule-Thomson effect a gas moves from a high pressure region to a low-pressure region. In this context the behavior of isenthalpic curves in $T - P$ plane illustrates some interesting points: Inversion curve divides isenthalpic curves into two branches where the intersection point is called inversion point. This special point identifies the cooling-to-heating phase transition. This special point identifies the phase transition of cooling-to-heating. In the cooling process the branch of the isenthalpic curve possess positive slope in contrast to the heating process has negative slope. Actually Joule-Thomson expansion explains the variation of a gas or liquid temperature through a process in which the temperature of the system changes with respect to pressure in a constant Enthalpy and it could be seen a transition between its heating and cooling phases. To be more precise when the Joule-Thomson coefficient $\mu_{JT} = \left( \frac{\partial T}{\partial P} \right)_H$ changes its sign from negative to positive then there would be an expansion from the heating-to-cooling phase, respec-
tively. To investigate whether this behavior would be seen in our model we can just fix parameters of the model and study $T - P$ behavior. In Joule-Thomson expansion, one can see isenthalpic behavior of a gas in which the first temperature of gas increases by reaching to a maximum value and then decreases to lower temperatures. Change of this behavior is happened at the point of the inversion temperature curve which is obtained from $\mu_{JT} = 0$. From $T - P$ plots given in the figures 5, 6 and 7, one can see evolution of temperature with respect to variations of any black hole thermodynamics parameters such as $M$, $a$ and $\alpha$. These diagrams show the inversion point happens at higher temperature with higher pressure for massive black holes with larger mass. On the other side, for fixed mass, the increasing of $a$ or $\alpha$ leads to decreasing of inversion point. By comparing our results with 5 dimensional case [24] one can infer in contrast to 5 dimensional case, $4D$ Gauss-Bonnet black holes in AdS spaces surrounded by strings fluid supports the heating-to-cooling phase transition.

5 Conclusion

In this paper we used the extended thermodynamic formalism to give an suitable answer to the question ”Can $4D$ Gauss-Bonnet black holes in AdS space surrounded by strings fluid mimics Van der Waals fluid behavior ?”. In general, we should say in an extended thermodynamic paradigm, the pressure of the black holes can be described by cosmological constant so that its conjugate variable plays the role of thermodynamic volume of the black holes. To examine the analogy between AdS $4D$ Gauss-Bonnet black holes surrounded by strings fluid with liquid-gas system we derived analytical soluions of the critical points. We have demonstrated in syllogism with liquid-gas phase transition in Van der Waals fluid, there would be small-large black hole phase transition in AdS $4D$ Gauss-Bonnet black hole surrounded with strings fluid. At last we obtained, the role of strings fluid is rescuable in the Schwarzschild limit, for the small-to-large black hole phase transition, that it means Gauss-Bonnet black holes in AdS Spaces surrounded by strings fluid mimics Van der Waals fluid behavior. By comparing our results with 5 dimensional case [24] it is divulged in contrast to 5 dimensional case, $4D$ Gauss-Bonnet black holes in AdS spaces surrounded by strings fluid supports the heating-to-cooling phase transition.
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Figure 1: $T - P$ diagram when $a = 1$ and $\alpha = 0.1$ are fixed and $M = 1, 1.5, 2$ for red, green and blue lines, respectively.
Figure 2: $T - P$ diagram when $a = 1$ and $M = 1$ are fixed and $\alpha = 0.3, 0.2, 0.1$ for red, green and blue lines, respectively.

Figure 3: $T - P$ diagram when $M = 1$ and $\alpha = 0.1$ are fixed and $a = 3, 2, 1$ for red, green and blue lines, respectively.
Figure 4: $P$ is plotted vs $v$ for $a = 0.1$, $\alpha = 0.001$. Lines denote to: $T = \frac{2.2399}{\pi}$ for green, $T_c = \frac{2.8399}{\pi}$ for blue and $T = \frac{1.8399}{\pi}$ for red.

Figure 5: $G$ is plotted vs $T$ for $a = 0.9$ and $\alpha = 0.0000001$. Lines denote to: $P = \frac{60.69}{\pi}$ for green, $P_c = \frac{88.69}{\pi}$ for red and $P = \frac{70.69}{\pi}$ for blue.
Figure 6: $G$ vs $T$ for $a = 0.1$, $\alpha = 0.00001$ illustrates Hawking-Page phase transition in $\alpha \to 0$ limit can be vanished by strings fluid effect. Red, blue and green lines denote to $P_c$, $\frac{P_c}{2}$ and $\frac{P_c}{4}$ respectively.

Figure 7: $G$ is plotted vs $T$ for $\alpha = 0.1$ and different values of the string fluid factor: Red, orange, green, light blue, purple and blue lines denote to $a = 0$, $a = 0.1$, $a = 0.2$, $a = 0.5$, $a = 0.6$ and $a = 0.7$ respectively.