Perturbative corrections to $\bar{B} \to X_s \gamma$ in supersymmetry at next-to-leading order

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Abstract. We give a brief overview about perturbative corrections to the inclusive decay mode $\bar{B} \to X_s \gamma$ in supersymmetric models.

1. Introduction

Perturbative QCD corrections are well-known for being the dominant contributions to the radiative inclusive penguin decay $[1–3]$. This perturbative dominance was recently reassured by a dedicated analysis $[4]$ in which non-perturbative corrections to the inclusive decay mode $\bar{B} \to X_s \gamma$ have been estimated to be well below 10%.

Within a global effort, a perturbative QCD calculation to the next-to-next-to-leading-logarithmic (NNLL) level within the Standard Model (SM) has been performed and has led to the first NNLL prediction of the $\bar{B} \to X_s \gamma$ branching fraction $[5]$. Using the photon energy cut $E_0 = 1.6$ GeV, the branching ratio reads

$$B(\bar{B} \to X_s \gamma)_{\text{NNLL}} = (3.15 \pm 0.23) \times 10^{-4}.$$  \hspace{1cm} (1)

This result is based on various highly-nontrivial perturbative calculations $[6–16]$. The combined experimental data according to the Heavy Flavor Averaging Group (HFAG) $[17]$ leads to

$$B(\bar{B} \to X_s \gamma) = (3.55 \pm 0.24 \pm 0.09) \times 10^{-4},$$  \hspace{1cm} (2)

where the first error is combined statistical and systematic, and the second is due to the extrapolation in the photon energy. Thus, the SM prediction and the experimental average are consistent at the 1.2σ level.

This is just one example among the impressive confirmation of the SM in all experiments in flavour physics during the last decade $[18–19]$, including the first generation of the $B$ factories at KEK (Belle experiment at the KEKB $e^+e^-$ collider) $[20]$ and at SLAC (BaBar experiment

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at the PEP-II $e^+e^-$ collider) [21], and the Tevatron $B$ physics programs (CDF [22] and D0 [23] experiments). Also the first results of the LHCb experiment [24] are in full agreement with the simple CKM theory of the SM.

This feature is somehow unexpected because in principle flavour changing neutral current (FCNC) processes like $B \to X_s\gamma$ offer high sensitivity to new physics (NP). Additional contributions to the decay rate, in which SM particles in the loops are replaced by new particles such as the supersymmetric charginos or gluinos are not suppressed by the loop factor $\alpha/4\pi$ relative to the SM contribution. Thus, FCNC decays provide information about the SM and its extensions via virtual effects to scales presently not accessible otherwise. This approach is complementary to the direct production of new particles at collider experiments.

2. Supersymmetric flavour problem

The experimental fact that none of the dedicated flavour experiments has observed any unambiguous sign of new physics yet, in particular no $\mathcal{O}(1)$ NP effects in any FCNC process, implies the famous flavour problem, namely why FCNC processes are suppressed. It has to be solved in any viable new physics model. The hypothesis of minimal flavour violation (MFV) [25–27], i.e. that the NP model has no flavour structures beyond the Yukawa couplings, solves the problem formally. However, new flavour structures beyond the Yukawa couplings are still compatible with the present data [28] because the flavour sector has been tested only at the 10% level in the $b \to s$ transitions.

Today supersymmetric models are often given priority in our search for NP beyond the SM. This is primarily suggested by theoretical arguments related to the well-known hierarchy problem. Supersymmetry eliminates the sensitivity for the highest scale in the theory and, thus, stabilizes the low energy theory. There are other features in supersymmetric theories which are promising like the unification of the gauge couplings and the existence of a dark matter candidate. Supersymmetry also represents the unique extension of Poincare symmetry.

The precise mechanism of the necessary supersymmetry breaking is unknown. A reasonable approach to this problem is the inclusion of the most general soft breaking term consistent with the SM gauge symmetries in the so-called unconstrained minimal supersymmetric standard model (MSSM). This leads to a proliferation of free parameters in the theory.

The decay $\bar{B} \to X_s\gamma$ is sensitive to the mechanism of supersymmetry breaking because, in the limit of exact supersymmetry, the decay rate would be just zero:

$$B(\bar{B} \to X_s\gamma)_{Exact\ Susy} = 0. \tag{3}$$

This follows from an argument first given by Ferrara and Remiddi in 1974 [29]. In that work the absence of the anomalous magnetic moment in a supersymmetric abelian gauge theory was shown.

In the MSSM there are new sources of FCNC transitions. Besides the CKM-induced contributions, which are brought about by a charged Higgs or a chargino, there are generic supersymmetric contributions that arise from flavour mixing in the squark mass matrices in case they are not aligned to the ones in the quark sector. Then the gluino contribution enhanced by an extra factor $\alpha_s$ instead of $\alpha_{\text{weak}}$ significantly contributes to the decay rate.

Thus, the general structure of the MSSM does not explain the suppression of FCNC processes, which is observed in experiments; the gauge symmetry within the supersymmetric framework does not protect the observed strong suppression of the FCNC transitions. This is the crucial point of the well-known supersymmetric flavour problem.

3. Parameter bounds from the inclusive decay $\bar{B} \to X_s\gamma$

Parameter bounds on NP from flavour physics is a model-dependent issue. The present data on $\bar{B} \to X_s\gamma$ implies a very stringent bound for example on the inverse compactification radius
of the minimal universal extra dimension model (mACD) ($1/R > 600\text{GeV}$ at 95% CL) \cite{30}. The bound is much stronger than the ones derived from other measurements. Moreover, there is a bound induced by $\bar{B} \rightarrow X_s\gamma$ on the charged Higgs mass in the two Higgs-doublet model (II): $M_{H^+} > 295\text{GeV}$ at 95% CL \cite{5}. It is based on a NLL QCD calculation within this model presented in Refs. \cite{31,32}. The latter bound is not valid in general two-Higgs doublet models, especially in supersymmetric models. However, the two-Higgs-doublet model (II) is a good approximation for gauge-mediated supersymmetric models with large $\tan\beta$, where the charged Higgs contribution dominates the other supersymmetric contributions.

Simplifying assumptions about the parameters often introduce model-dependent correlations between different observables. Thus, flavour physics will also help in discriminating between the various models that will be proposed by then. In view of this, it is important to calculate the rate of the rare $B$ decays, with theoretical uncertainties as reduced as possible and general enough for generic supersymmetric models.

The rare decay $\bar{B} \rightarrow X_s\gamma$ has already carved out large regions in the space of free parameters of most of the supersymmetric models. Once more precise data from the Super $B$ factories are available, this decay will undoubtedly gain even more efficiency in selecting the viable regions of the parameter space in the various classes of models. Constraints based on nontrivial QCD calculations within various supersymmetric extensions are heavily analyzed in the literature, see for example the Refs. \cite{33–46}.

Finally, model-independent analyses in the effective field theory approach without \cite{47} and with the assumption of minimal flavour violation \cite{48} also show the strong constraining power of the $\bar{B} \rightarrow X_s\gamma$ branching fraction.

4. NLL calculations in supersymmetry

While in the SM, the rate for $\bar{B} \rightarrow X_s\gamma$ is known up to NNLL in QCD, also within supersymmetric theories higher order calculations have been pushed forward in recent years. At the LL level there are several contributions to the decay amplitude: besides the contributions solely induced by flavour mixing in the quark sector with a $W$ boson or a charged Higgs boson and a top quark in the loop, there is also a chargino contribution with an up-type squark which can be induced by the CKM matrix. If we consider also generic new sources of flavour violation induced by a disalignement of quarks and squarks, there are additional contributions from a chargino, gluino and also neutralino. The first complete analysis of the decay rate of $\bar{B} \rightarrow X_s\gamma$ has been presented in Ref. \cite{33}.

It is highly desirable to analyse these non-standard contributions with NLL precision: Besides the large uncertainties in the LL predictions, the step from the LL to the NLL precision is also necessary in order to check the validity of the perturbative approach in the model under consideration. Moreover, it was already shown in specific NP scenarios that bounds on the parameter space of non-standard models are very sensitive to NLL contributions.

4.1. NLL calculation in MFV

The MFV hypothesis is a formal model-independent solution to the NP flavour problem. It assumes that the flavour and the CP symmetry are broken as in the SM. Thus, it requires that all flavour- and CP-violating interactions be linked to the known structure of Yukawa couplings. A renormalization-group-invariant definition of MFV based on a symmetry principle is given in \cite{27}; this is mandatory for a consistent effective field theoretical analysis of NP effects. The MFV hypothesis is an important benchmark. Because any measurement which is inconsistent with the general constraints and relations induced by the MFV hypothesis \cite{48} indicates the existence of new flavour structures.

This hypothesis can also be used within the MSSM. It can be implemented by assuming that the squark and quark mass matrices can be simultaneously diagonalized (alignment). In this
Figure 1. Example diagrams for NLL gluonic corrections to the $W$ boson, charged Higgs and chargino contribution.

case there are no flavour-changing interactions induced by the gluino at the tree level.

The first NLL calculation of the inclusive decay $B \to X_s \gamma$ in the MSSM with the MFV hypothesis includes the gluon corrections to the charged Higgs and the chargino contribution [34], see Figure 1. In particular the possibility of destructive interference of the chargino and the charged Higgs contribution is studied. The analysis is done under the MFV assumption that the only source of flavour violation at the electroweak scale is that of the SM, encoded in the CKM matrix. Other flavour-changing interactions were suppressed by assuming the gluino being heavy. It is found that, in this specific supersymmetric scenario, bounds on the parameter space are rather sensitive to NLL contributions and they lead to a significant reduction of the stop-chargino mass region, where the supersymmetric contribution has a large destructive interference with the charged-Higgs boson contribution [34].

There are also further analyses within the MFV hypothesis which try to include only the potentially large contributions beyond the leading order which are enhanced by large $\tan \beta$ factors or by large logarithms of the form $\ln(M_{\text{Susy}}/M_W)$ where the masses of the supersymmetric particles are assumed to be significantly larger than the $W$-boson mass [27, 35, 36].

A practically complete MFV analysis has been presented in Ref. [42]. To LL precision this calculation includes the one-loop diagrams containing a $W$ boson and up-type quark, or a charged Higgs boson and an up-type quark, or a chargino and an up-type squark (see Figure 2). Neutralino and gluino exchange diagrams are neglected under the MFV assumption. To NLL precision the gluonic two-loop corrections to the SM and charged Higgs loops are included, also two-loop diagrams with a gluino together with a Higgs or W boson, and finally two-loop diagrams with a chargino together with a gluon or a gluino or a quartic squark coupling. As already shown in Ref. [34], the two-loop gluonic corrections to the chargino loops are not UV finite: in order to obtain a finite result one has to combine them with the chargino-gluino diagrams.

However, a MFV analysis should take into account the fact that the simultaneous diagonalization of the quark and squark mass matrices can be imposed at one scale $\mu_{\text{MFV}}$. The renormalization group evolution of the MSSM parameters then leads to a disalignment between the squark and quark mass matrices at scales different from $\mu_{\text{MFV}}$ [42]. So if the MFV condition is imposed at a scale much larger than the superparticle mass scale $M_{\text{Susy}}$, very large logarithms of $M_{\text{Susy}}/\mu_{\text{MFV}}$ occur in the Wilson coefficients. Then the soft Susy-breaking mass parameters – which are assumed to be flavour-diagonal at the scale $\mu_{\text{MFV}}$ – must be evolved down to $M_{\text{Susy}}$ with the help of the appropriate renormalization group equations (RGE), thus, generating some flavour violation in the squark mass matrices which gets absorbed in the couplings of the squark mass eigenstates with the gluinos and charginos. In Ref. [42], it is argued, that the effects of the RGE-induced flavour mixing is relatively small and, therefore, are only included to LL order,
Figure 2. Example diagrams for NLL gluino corrections to the chargino and W boson contribution.

in the one-loop diagrams with gluinos and down-type squarks and in the one-loop diagrams with charginos and up-type squarks. There is a public computer code for this MFV calculation available which includes all contributions discussed above [49].

4.2. NLL calculation in general MSSM

Beyond minimal flavour violation, the most important role is played by the non-diagonal gluino-quark-squark vertex due to the large strong coupling which comes with this vertex. As discussed above, this flavour non-diagonal vertex is induced by squark-mixing to the extent as it is misaligned with quark mixing. It represents a new flavour structure beyond the SM Yukawa couplings. To understand these new sources of flavour violation that may be present in supersymmetric models in addition to those enclosed in the CKM matrix, one has to consider the contributions to the squark mass matrices

\[ M_f^2 \equiv \begin{pmatrix} m_{f,LL}^2 + F_{f,LL} + D_{f,LL} & (m_{f,LR}^2 + F_{f,LR})^\dagger + F_{f,RL} \\ (m_{f,LR}^2 + F_{f,LR}) & m_{f,RR}^2 + F_{f,RR} + D_{f,RR} \end{pmatrix}, \tag{4} \]

where \( f \) stands for up- or down-type squarks. In the super-CKM basis, where the quark mass matrices are diagonal and the squarks are rotated in parallel to their superpartners, the \( F \) terms from the superpotential and the \( D \) terms from the gauge sector turn out to be diagonal \( 3 \times 3 \) submatrices of the \( 6 \times 6 \) mass matrices \( M_f^2 \). This is in general not true for the additional terms \( m_{f,XY}^2 \) with \( X, Y \in \{ L, R \} \), originating from the soft supersymmetric breaking potential. Because the squark-quark-gluino coupling is flavour-diagonal in the super-CKM basis, the gluino vertex in the mass eigenstate basis is non-diagonal in flavour-space due to the off-diagonal elements of the soft terms \( m_{f,LL}^2, m_{f,RR}^2, m_{f,RL}^2 \).

A complete LL analysis of the corresponding gluino contribution to the inclusive decay rate of \( \bar{B} \rightarrow X_s \gamma \) has been presented in Ref. [37]. The sensitivity of the bounds on the down squark mass matrix to radiative QCD LL corrections is systematically analysed, including the SM and the gluino contributions. In Ref. [38] the interplay between the various sources of flavour violation and the interference effects of SM, gluino, chargino, neutralino and charged Higgs boson contributions is studied. The bounds on simple combinations of elements of the soft part of the squark mass matrices are found to be, in general, one order of magnitude weaker than the bound on the single off-diagonal element, which was derived in previous work by neglecting any kind of interference effects. Some effects beyond LL precision like large \( \tan \beta \) effects are estimated in Ref. [41] in analogy to the MFV analyses of Refs. [35, 36].
Recently, the complete NLL corrections to the Wilson coefficients (at the matching scale $\mu_W$) of the various versions of magnetic and chromomagnetic operators which are induced by a squark-gluino loop have been calculated \cite{50}. In this analysis all the appearing heavy particles (which are the gluino, the squarks and the top quark) are simultaneously integrated out at the high scale. There are two classes of two-loop diagrams which have to be considered: diagrams with one gluino and a virtual gluon and diagrams with two gluinos (see Figure 3) or with one gluino and a squark-loop. The former have been presented already in Ref. \cite{51} and now confirmed, while the latter have been calculated for the first time \cite{50}.

Besides these NLL contributions due to the gluino vertex, there are of course more NLL corrections with non-minimal flavour violation; they involve electroweak (gaugino and higgsino) vertices. However, such contributions are in general suppressed compared to the ones related to the gluino. There are two types of such contributions at the NLL level: First, there are electroweak corrections to the non-minimal LL gluino contribution (in which the electroweak vertex is flavour-diagonal or MFV-like) which are naturally suppressed due to the smaller coupling constants and due to the CKM hierarchy. Second, there is also non-minimal flavour violation via squark-mixing in the electroweak vertices possible. But such contributions are already suppressed at the LL level compared to the gluino contribution due to the smaller coupling constant, apart from the chargino contributions in specific parts of the parameter space in which for example the trilinear coupling $A_{33}$ is very large. These features do not change of course when gluon- and gluino-induced NLL corrections are added to such LL contributions. Still, the leading chirally enhanced corrections can be easily calculated by inserting the effective Feynman rules of \cite{40} into the results of \cite{38}. Summing up, the complete NLL corrections induced by the gluino vertex given in Ref. \cite{50} represent the dominant contribution beyond MFV at this order in most parts of the MSSM parameter space. They are complementary to the MFV contributions at the NLL level which are given in Ref. \cite{42}. The results are presented also in public computer code \cite{50}.

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