Suppression of dilepton production in hot hadronic matter

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Abstract

The pion electromagnetic form factor at finite temperature is studied using an effective chiral lagrangian that includes explicitly vector mesons. We find that in the time-like region around the rho meson resonance it decreases with increasing temperature and leads to a suppression of dilepton production from pion-pion annihilation in a hot hadronic matter. Effects on dilepton production in high energy heavy ion collisions and its relevance to the phase transition in a hot hadronic matter are discussed.

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Dileptons from relativistic heavy ion collisions have continually attracted great interest as once produced they would escape from the collision region without further interaction and are thus ideal probes of the hot dense matter formed in the initial stage of the collision [1]. In hot dense matter, chiral symmetry is expected to be partially restored and the deconfinement transition to the quark-gluon plasma is also possible if the temperature and density are sufficiently high. Dileptons can therefore provide the signatures for these new phases of hadronic matter.

It has been suggested that the masses of vector mesons would change as the hadronic matter undergoes a phase transition to the chirally symmetric phase [2]. If this is the case, one should be able to observe this effect directly through the shift of vector meson peaks in the dilepton spectra from heavy ion collisions [3]. However, based on PCAC and current algebra it has been shown that up to \( T^2 \), where \( T \) is the temperature, there is no change in vector meson masses but only a mixing between the vector and axial vector correlators [4]. This result should be satisfied by any models that include the symmetry properties of low energy hadronic physics. Indeed, results from both QCD sum-rule calculations [5,6] and effective chiral lagrangian approaches [7] are consistent with this temperature dependence, and vector meson masses obtained from these models do not change appreciably unless the temperature of the hadronic matter is very close to the critical temperature for the phase transition.

On the other hand, the yield of dileptons from hot matter may have a stronger dependence on temperature. In hot hadronic matter, the production of dileptons with invariant masses near the \( \rho \) resonance is dominated by pion-pion annihilation. According to vector meson dominance (VMD) [8], two pions in this process form a rho meson that subsequently converts into a virtual photon. The dilepton yield depends thus on the pion electromagnetic form factor,

\[
F_{\pi}(q^2) = \frac{g_{\rho\pi\pi}g_{\rho}}{m_{\rho}^2 - q^2 - im_{\rho}\Gamma_{\rho}},
\]

where \( g_{\rho} \) is the photon-\( \rho \)-meson coupling constant, \( g_{\rho\pi\pi} \) is the pion-\( \rho \)-meson coupling con-
stant, and $\Gamma_\rho$ is the neutral $\rho$ meson decay width. This form factor has been extensively used in calculating the dilepton emission rate from hadronic matter at finite temperature \cite{9,13}. In these studies, the form factor has been taken to be independent of temperature. However, recent studies using QCD sum-rules \cite{16} or QCD factorization \cite{17} have suggested that $F_\pi(q^2)$ is likely to be modified at finite temperature, and this will have effects on dilepton production in hot hadronic matter.

In the present paper, we shall study the pion electromagnetic form factor at finite temperature using an effective chiral lagrangian that includes explicitly the vector mesons and gives also the correct isospin mixing at finite temperature. Our study indicates that not only is the photon-$\rho$-meson coupling modified at finite temperature but there also exist effects due to vertex corrections and changes of pion and $\rho$ meson properties in the hot hadronic matter. Using the temperature-dependent form factor, we shall study its effect on the dilepton production rate from $\pi - \pi$ annihilation in hot matter.

Although the chiral perturbation theory \cite{18,19} has been successful in describing systematically low energy hadronic phenomena, it has not been able to account for higher energy processes related to vector mesons. In the literature, two methods have been introduced to include vector mesons and photon fields in the chiral lagrangian; the massive Yang-Mills approach \cite{20} and the hidden gauge approach \cite{21}. In the massive Yang-Mills approach, $\rho$ and $a_1$ mesons are introduced as external gauge fields of the chiral group and the photon field is introduced via VMD \cite{20}. In the hidden gauge approach, vector mesons are introduced as gauge fields of the hidden local symmetry and the photon is introduced as an external gauge field \cite{21}. These two methods have been shown to be gauge equivalent \cite{22,23} and to have identical symmetry properties at finite temperature \cite{7}.

We shall follow the hidden gauge approach by considering the $[SU(2)_L \times SU(2)_R]_{\text{global}} \times [SU(2)_V]_{\text{local}}$ “linear” sigma model. It is constructed with two SU(2)-matrix valued variables $\xi_L(x)$ and $\xi_R(x)$, which transform as $\xi_{L,R}(x) \rightarrow h(x)\xi_{L,R}g_{L,R}^L$ under $h(x) \in [SU(2)_V]_{\text{local}}$ and $g_{L,R} \in [SU(2)_{L,R}]_{\text{global}}$. Introducing the vector meson $V_\mu$ as the gauge field of the local symmetry and the photon $B_\mu$ as an external gauge field of the global symmetry,
we have the following chirally invariant lagrangian,

\[
\mathcal{L} = f_\pi^2 \text{tr} \left[ \frac{1}{2i} (\mathcal{D}_\mu \xi_L \cdot \xi_L^\dagger - \mathcal{D}_\mu \xi_R \cdot \xi_R^\dagger) \right]^2
\]

\[
+ a f_\pi^2 \text{tr} \left[ V_\mu - \frac{1}{2i} (\mathcal{D}_\mu \xi_L \cdot \xi_L^\dagger + \mathcal{D}_\mu \xi_R \cdot \xi_R^\dagger) \right]^2 + \mathcal{L}_{\text{kin}}(V_\mu, B_\mu),
\]  

where the pion decay constant \( f_\pi = 93 \text{ MeV} \). The covariant derivative, \( \mathcal{D}_\mu \xi_{L,R} \), is given by

\[
\mathcal{D}_\mu \xi_{L,R} = \partial_\mu \xi_{L,R} + i e \xi_{L,R} B_\mu \tau_3 / 2.
\]  

In the “unitary” gauge,

\[
\xi_L^\dagger(x) = \xi_R(x) = e^{i \pi(x)/f_\pi} \equiv \xi(x),
\]  

and after rescaling \( V_\mu \rightarrow g V_\mu \), the effective lagrangian takes the form,

\[
\mathcal{L} = \frac{1}{4} \text{tr}(\partial_\mu U \partial^\mu U^\dagger) - \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \frac{1}{4} (\partial_\mu V_\nu - \partial_\nu V_\mu)^2 + \frac{1}{2} m_\rho^2 V_\mu^2
\]

\[
+ g_{\rho \pi \pi} V_\mu \cdot (\pi \times \partial_\mu \pi) - e g_B V_\mu^3 B_\mu + g_{\gamma \pi \pi} B^\mu (\pi \times \partial_\mu \pi)_3 + \mathcal{L}_{>3},
\]  

where \( U = \xi_L^\dagger \xi_R = \xi^2(x) \) and \( \mathcal{L}_{>3} \) are high order terms involving more than three fields. The parameters in eq. (5) are given as

\[
m_\rho^2 = a g^2 f_\pi^2,
\]

\[
g_\rho = a g f_\pi^2,
\]

\[
g_{\rho \pi \pi} = \frac{1}{2} a g,
\]

\[
g_{\gamma \pi \pi} = (1 - \frac{1}{2} a) e.
\]  

For \( a = 2 \), these formula are known to give automatically the universality of \( \rho \)-couplings \((g_{\rho \pi \pi} = g)\), the KSRF relations

\[
g_\rho = 2 f_\pi^2 g_{\rho \pi \pi} : \text{ KSRF (I)},
\]

\[
m_\rho^2 = 2 g_{\rho \pi \pi}^2 f_\pi^2 : \text{ KSRF (II)},
\]

and the \( \rho \) meson dominance of the pion electromagnetic form factor \((g_{\gamma \pi \pi} = 0)\).
In this effective lagrangian, the pion electromagnetic form factor in free space can be obtained at tree level from the diagram shown in Fig. 1. One sees that the vector meson dominance appears naturally so a photon converts into a vector meson which then interacts with the pion. The actual process of probing the pion is thus related more to the propagation of the vector meson than to the intrinsic size of the pion itself. The resulting pion electromagnetic form factor is exactly the same as eq. (1) assumed in VMD. The temperature effect on the pion electromagnetic form factor is obtained by taking thermal loops into consideration. We include only one-loop diagrams as the hadronic matter is rather dilute at temperatures considered here. For simplicity, we carry out the calculation in the rest frame of the photon and neglect the small finite pion mass.

First, we consider the temperature effect on the photon-vector-meson coupling as shown in Fig. 2a. This correction is related to the mixing of the vector and axial vector current correlators in hot matter. Using the relation

\[ \int_0^\infty dx \frac{x}{\pi^2 e^{x/T} - 1} = \frac{T^2}{6}, \]  

we have

\[ F_\pi^{(1)}(q, T) = -F_\pi(q^2)\frac{\epsilon}{2}, \]  

where \( q \) is the photon four momentum, \( \epsilon = T^2/6f_\pi^2 \), and \( F_\pi(q^2) \) is the pion electromagnetic form factor in free space given by eq. (1). We see that, contrary to the modification of the vector meson mass in hot matter, the pion electromagnetic form factor has a \( T^2 \) dependence correction.

The change of rho meson properties in hot matter gives the correction in Fig. 2b, i.e.,

\[ F_\pi^{(2)}(q, T) = -F_\pi(q^2)\frac{m_\rho^2}{q^2 - m_\rho^2}G_1(q, T), \]  

where

\[ G_1(q, T) = \frac{1}{6f_\pi^2} \int_0^\infty dx \frac{1}{\pi^2 e^{x/T} - 1} \frac{x^3}{x^2 - q^2/4}. \]  

5
In Figs. 2c-2f, we show the vertex corrections. The contribution from Fig. 2f is suppressed by the large vector meson mass and will be neglected. The other contributions can be written as

\[ F^{(3)}_\pi(q, T) = F^{(c)}_\pi(q^2)[F^{(c)}_\pi + F^{(d)}_\pi + F^{(e)}_\pi], \]  

where

\[ F^{(c)}_\pi(q, T) = \frac{m^2}{2f^2_\pi q^2}[G_2(q, T) - G_3(q, T)], \]  

\[ F^{(d)}_\pi(q, T) = -G_1(q, T), \]  

\[ F^{(e)}_\pi(q, T) = -\frac{5}{24}\epsilon. \]  

In the above, \( G_2(q, T) \) and \( G_3(q, T) \) are given by

\[ G_2 = \frac{1}{4\pi^2} \int_0^\infty dx \frac{1}{e^{x/T} - 1} \left[ 4x - \frac{m^2}{q} \ln \left( \frac{m^2 + 2qx}{m^2 - 2qx} \right) + x \ln \left( \frac{m^4}{m^4 - 4q^2x^2} \right) \right], \]  

\[ G_3 = \frac{1}{4\pi^2} \int_0^\infty dx \frac{x}{e^{x/T} - 1}(K_+ + K_-), \]  

with

\[ K_+ = \frac{1}{q\left(\frac{q^2}{2} + x\right)} \left(\frac{3}{2}q^2 + m^2 + qx\right) \left[ 2 + \frac{m^2 + qx}{qx} \ln \left( \frac{m^2}{m^2 + 2qx} \right) \right]. \]  

Contributions from thermal vector mesons are very small due to their large masses in the Boltzmann factor and are neglected in the study.

In a medium, a pion changes its properties through interactions with thermal pions and vector mesons as shown in Fig. 2g and 2h. This modification of the form factor due to the pion wave function renormalization at finite temperature is given by

\[ F^{(4)}_\pi(q, T) = F^{(3)}_\pi(q^2)Z_\pi, \]  

with

\[ Z_\pi = -\frac{\epsilon}{6} + \frac{m^2}{8f^2_\pi q} \int_0^\infty dx \frac{1}{\pi^2 e^{x/T} - 1} \ln \left( \frac{m^2 + 2qx}{m^2 - 2qx} \right). \]
In the hidden gauge approach, there is also direct photon-pion coupling at finite temperature as shown in Fig. 2i and 2j, which explicitly modifies the notion of vector meson dominance. This contribution is given by

$$F^{(5)}_{\pi}(q, T) = \frac{5}{8} \epsilon + \frac{3m_{\rho}^2}{2f_{\pi}^2}G_4(q, T),$$

(25)

where

$$G_4(q, T) = \frac{1}{4q^2} \int_0^\infty dx \frac{x}{\pi^2 e^{x/T}} \ln \left( \frac{m_{\rho}^2 + 2qx}{m_{\rho}^2 - 2qx} \right) + \ln \left( \frac{m_{\rho}^4 - 4q^2x^2}{m_{\rho}^4} \right).$$

(26)

The pion electromagnetic form factor at finite temperature is obtained by adding all contributions shown in Fig. 2. Fig. 3 shows the $q$ dependence of the pion form factor around the $\rho$ resonance for different temperatures. The form factor is seen to be reduced near the resonance as temperature increases. We obtain a reduction of the form factor by 40% at $q \sim m_{\rho}$ when $T = 180$ MeV. This result is comparable with that obtained using the QCD sum-rule approach which shows that at $q^2 \sim (1\text{GeV})^2$ the form factor at $T \sim 0.9 T_c$ is about half its value at $T = 0$ [16]. It is also consistent with that based on the perturbative QCD at high $q^2$ [17].

The reduction of the pion electromagnetic form factor at finite temperature is mainly due to the modification of the photon-vector meson coupling, the increase of the $\rho$-meson width, and the vertex corrections. These are probably related to chiral symmetry restoration and the deconfinement phase transition in hot hadronic matter. The photon-$\rho$-meson coupling is modified due to the isospin mixing at finite temperature which has been regarded as a possible signature for the partial restoration of chiral symmetry in hot matter [24]. The resonance width has also been expected to increase in hot hadronic matter as the system undergoes chiral symmetry restoration and the deconfinement phase transition [2]. Since

$$\Gamma(\rho \to 2\pi) \sim \frac{g_{\rho\pi\pi}^2 p_{\pi}^3}{m_{\rho}^2} \sim \frac{p_{\pi}^3}{f_{\pi}^2},$$

(27)

the $\rho$-meson width will increase as $f_{\pi} \to 0$ when the temperature is close to the critical value. The vertex corrections, which lead to a reduction of the rho-pion coupling constant at finite
temperature, may be related to the recent suggestion that the pion-vector meson coupling constant vanishes when chiral symmetry is restored in the vector limit [25]. The possible relation between the suppression of the form factor and phase transition in hot hadronic matter also has been suggested via QCD sum rules [16] and the QCD factorization formula [17] to lead to similar suppressions in the pion electromagnetic form factor.

We have calculated the dilepton production rate from pion-pion annihilation in hot hadronic matter. The production rate of dileptons with vanishing three momentum in hot hadronic matter is then given by [26]

\[ \frac{d^4 R}{d^4 q dM} \bigg|_{q=0} = \frac{\alpha^2}{3(2\pi)^4} \frac{|F_\pi(M, T)|^2}{(e^{\omega/T} - 1)^2} \sum_k k^4 \left( \frac{d\omega}{dk} \right)^{-1} , \]

where \( M \) is the dilepton invariant mass. The momentum and energy of the pion are denoted by \( k \) and \( \omega \), respectively, and are related by its dispersion relation in the medium. The last factor takes into account this effect. The sum over \( k \)'s is restricted by \( \omega(k) = M/2 \). However, the modification of the pion dispersion relation at finite temperature is small [27] and will be neglected.

The dilepton production rate is shown in Fig. 4 for \( T = 180 \). The result obtained with the modified pion form factor (solid line) is compared with that calculated using the form factor in free space (dotted line). Since the production rate is proportional to the square of the form factor, we obtain a larger reduction with temperature in the dilepton production rate than in the form factor above. Near the \( \rho \) meson resonance we have \( dR[F_\pi(M, T)] \sim (3/5)^2 dR[F_\pi(M, 0)] \) at \( T = 180 \) MeV, and the dilepton production rate is reduced by almost a factor of three.

Recent experiments at CERN have reported the “unaccounted” excess of dileptons in central S-W nuclear collisions, compared to pA and peripheral collisions [28]. The data might be described by assuming, as for photon production [29], that a thermalized quark-gluon plasma is formed in the collision, which then cools due to expansion and hadronizes before undergoing a freeze-out. In this model, the most important contribution to dilepton production comes from the hadronic component of the mixed phase at \( T_c = 160 \sim 180 \)
MeV $^{29,30}$. Our results, however, imply a suppression in the production rate due to the temperature dependence of the form factor. This may suggest that other mechanisms are contributing to photon and dilepton production from hot hadronic matter. One possibility is the slow expansion model $^{30}$ in which a mixed phase expands very slowly and produces more dileptons to compensate for the suppression due to the modification of the form factor $^{31}$.

Also, it is of interest to extend present calculations to the $SU(3)$ limit and to study the temperature dependence of the form factor needed in $K^-K$ annihilation. This will be relevant to the double phi meson peak in the dilepton spectrum, which has recently been suggested as a possible signal for the phase transition in hot matter $^{32}$. The second phi peak in the dilepton spectrum is from the decay of phi mesons in the mixed phase, which have reduced masses as a result of partial restoration of chiral symmetry.

In summary, we have studied the pion electromagnetic form factor in hot hadronic matter using an effective lagrangian with vector mesons. We find that there is a reduction in the magnitude of the form factor, which could be understood in terms of the partial restoration of chiral symmetry and the deconfinement transition in hot hadronic matter. The reduction in the electromagnetic form factor leads to a suppression of dilepton production from pion-pion annihilation in hot matter. This effect needs to be included in future studies of dilepton production from heavy ion collisions.

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Figure Captions

**Fig. 1:** The pion electromagnetic form factor at tree level.

**Fig. 2:** One-loop corrections to the pion electromagnetic form factor. Solid, wavy, and dotted lines denote, respectively, the vector meson, the photon and the pion.

**Fig. 3:** The pion electromagnetic form factor at finite temperature.

**Fig. 4:** The thermal dilepton production rate from pion-pion annihilation at $T = 180$ MeV. Solid and dotted lines are results obtained with modified and free form factors, respectively.
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