Constraining $R$-parity violating couplings from $B \to PP$ decays using QCD improved factorization method

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Abstract

We investigate the role of $R$-parity violating interaction in the non-leptonic decays of $B$ mesons into two light mesons $B \to PP$. The decay amplitudes are calculated using the QCD improved factorization method. Using the combined data on $B$ decays from BaBar, Belle and CLEO, we obtain strong constraints on the various products of $R$-parity violating couplings. Many of these new constraints are stronger than the existing bounds.
1 Introduction

The Standard Model (SM) of particle physics is very successful in explaining most of the elementary particle physics phenomena at the electroweak scale. Unfortunately, despite of its eminent success, the SM suffers from certain drawbacks, the hierarchy problem being a major one. Supersymmetry (SUSY) [1] provides an elegant solution and, consequently has been extensively studied as a model beyond the SM. If such a new physics exists at the electroweak scale, then it may provide some experimental signature in the future, or even in present data. Looking for such new physics effects constitutes a major area of research in high energy physics today. There are two major ways one can see such effects. One is to observe their direct effects in the high energy collider experiments when a new particle is produced and observed through its decays. The other way, is to look for indirect evidence in the deviation from the SM prediction of low energy experimental data.

Recent results from the ongoing experiments in B-physics at CLEO, BaBar, and Belle have attracted lot of attention. Much of this attention has been devoted to the results on nonleptonic decays of B mesons, which can be used to extract information on the CKM matrix elements and CP violation. The theoretical understanding of the nonleptonic decays of B mesons is an extremely demanding challenge due to difficulties in calculating the relevant hadronic matrix elements. To have some idea of the magnitude of the matrix elements, one usually uses factorization method, factorizing the four quark operators relevant to non-leptonic B decays into the products of two currents and evaluating separately the matrix elements of the two currents. Recently the QCD improved factorization method for the hadronic B decays has been developed. This method incorporates elements of the naive factorization approach (as its leading term) and perturbative QCD corrections (as subleading contributions) allowing one to compute systematic radiative corrections to the naive factorization for the hadronic B decays [2, 3]. In our analysis we will use the formalism developed in Ref. [2]. This QCD-improved factorization method improves the analysis on several aspects, including among others the number of colors, the gluon virtuality, the renormalization scale, and the scheme dependence. The method is expected to give a good estimate of the magnitudes of the hadronic matrix elements in non-leptonic B decays, and has been used to calculate B decays in the SM [2, 1, 4, 5, 6] and models beyond [7].

The construction of most general supersymmetric extension of the standard model leads to baryon (B) and lepton (L) number violating operators in the superpotential. The simultaneous presence of both (L) and (B) number violating operators induce rapid proton decay which may contradict the strict experimental bound [8]. In order to keep the proton lifetime within the experimental limit, one needs to impose additional symmetry beyond the SM gauge symmetry to force the unwanted baryon and lepton number violating interactions to vanish. In most cases, this has been done by imposing a discrete symmetry called R-parity [3], defined as $R = (-1)^{3B + L + 2S}$,
where, $S$ is the spin of the particle. This symmetry not only forbids rapid proton decay \[^{[10]}\], it also renders stable the lightest supersymmetric particle (LSP). However, this symmetry is \textit{ad hoc} in nature. There are no strong theoretical arguments in support of this discrete symmetry. Hence, it is interesting to see the phenomenological consequences of the breaking of $R$-parity in such a way that either $B$ or $L$ number is violated both are not simultaneously violated, thus avoiding rapid proton decays. Extensive studies have been done to look for the direct as well as indirect evidence of $R$-parity violation from different processes and to put constraints on various $R$-parity violating couplings \[^{[11, 12-20]}\].

The main purpose of this paper is to constraint various $R$-parity violating couplings using the data on $B \to PP$ decay channels and a calculation based on QCD-improved factorization. Here $P$ is one of the $S(3)$ flavor octet pseudoscalars. We find that using the experimental data on the branching ratios of $B \to PP$ mode, stringent upper bounds on the products of several $L$ and $B$ violating couplings can be obtained. Many of the bounds obtained are stronger than the existing ones.

The organization of the paper is the following. In section (2) we study possible four quark operators which can induce $B \to PP$ decays with $R$-parity violating interactions. In section (3) we calculate $B \to \pi\pi, K\pi, K\bar{K}$ decay amplitudes using the QCD-improved factorization method. At last in section (4), we carry out numerical analysis, present and discuss our results and finally draw our conclusions.

### 2 New operators for $B \to PP$ with $R$-parity violation

The most general superpotential of the Minimal Supersymmetric Standard Model (MSSM) \[^{[1]}\], which describes $SU(3) \times SU(2) \times U(1)$ gauge invariant, renormalizable and supersymmetric theory, with minimal particle content, has $R$-parity violating interaction terms\[^{[2]}\]:

$$W_R = \frac{1}{2}\lambda_{[ij]k} \hat{L}_i \hat{L}_j \hat{E}_k^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_k^c + \frac{1}{2}\lambda''_{[ijk]} \hat{U}_i^c \hat{D}_j \hat{D}_k^c$$

(1)

where, $\hat{L}$ and $\hat{Q}$ are the $SU(2)$-doublet lepton and quark superfields and $\hat{E}^c, \hat{U}^c$ and $\hat{D}^c$ are the singlet superfields, while $i, j, k$ are flavor indices. In writing the above we have omitted gauge indices, which ensures that the $\lambda_{ijk}$ are antisymmetric in $i$ and $j$, and the $\lambda''$ are antisymmetric in $j$ and $k$. It is very clear that the first two terms in Equation (1) violate lepton number, while the last term violates baryon number.

The above $R$-parity violating interaction, in general can have 27 $\lambda'$-type and 9 each of $\lambda$ and $\lambda''$-type of new couplings. Not all of them will induce $B \to PP$ decays to the lowest order. At the dimension six four-fermion interaction level only operators corresponding to the couplings $\lambda'_{ijk}$
and $\lambda_{ij}''$ will lead to hadronic $B \to PP$ decays. They are given by

$$L_{\text{eff}} = \frac{\lambda_{112}''\lambda_{13}'''}{2m_s^2}(\bar{u}_a\gamma_\mu Ru_\alpha \bar{d}_\beta \gamma_\mu Rb_\beta - \bar{u}_a\gamma_\mu Ru_\beta \bar{d}_\beta \gamma_\mu Rb_\alpha)$$

$$- \frac{\lambda_{11}''\lambda_{13}'}{2m_s^2}\bar{u}_a\gamma_\mu Lu_\beta \bar{d}_\beta \gamma_\mu Rb_\alpha$$

$$- \frac{\lambda_{11}'\lambda_{13}'}{2m_s^2}\bar{d}_a\gamma_\mu Ld_\beta \bar{d}_\beta \gamma_\mu Rb_\alpha - \frac{\lambda_{31}'\lambda_{11}'}{2m_\nu^2}\bar{d}_a\gamma_\mu Rd_\beta \bar{d}_\beta \gamma_\mu Lb_\alpha$$

$$+ \frac{\lambda_{12}''\lambda_{13}'}{2m_d^2}(\bar{u}_a\gamma_\mu Ru_\alpha \bar{s}_\beta \gamma_\mu Rb_\beta - \bar{u}_a\gamma_\mu Ru_\beta \bar{s}_\beta \gamma_\mu Rb_\alpha)$$

$$+ \frac{\lambda_{12}''\lambda_{13}'}{4m_\nu^2}(\bar{d}_a\gamma_\mu Rd_\alpha \bar{s}_\beta \gamma_\mu Rb_\beta - \bar{d}_a\gamma_\mu Rd_\beta \bar{s}_\beta \gamma_\mu Rb_\alpha)$$

$$- \frac{\lambda_{12}'\lambda_{13}'}{2m_\nu^2}\bar{u}_a\gamma_\mu Lu_\beta \bar{s}_\beta \gamma_\mu Rb_\alpha$$

$$- \frac{\lambda_{11}'\lambda_{23}'}{2m_\alpha^2}\bar{s}_a\gamma_\mu Ld_\beta \bar{d}_\beta \gamma_\mu Rb_\alpha - \frac{\lambda_{32}'\lambda_{11}'}{2m_\nu^2}\bar{s}_a\gamma_\mu Rd_\beta \bar{d}_\beta \gamma_\mu Lb_\alpha$$

$$- \frac{\lambda_{12}'\lambda_{13}'}{2m_\nu^2}\bar{d}_a\gamma_\mu Ld_\beta \bar{s}_\beta \gamma_\mu Rb_\alpha - \frac{\lambda_{31}'\lambda_{12}'}{2m_\nu^2}\bar{d}_a\gamma_\mu Rd_\beta \bar{s}_\beta \gamma_\mu Lb_\alpha$$

$$+ \frac{\lambda_{12}'\lambda_{23}'}{4m_\alpha^2}(\bar{d}_a\gamma_\alpha Rs_\alpha \bar{s}_\beta \gamma_\mu Rb_\beta - \bar{d}_a\gamma_\alpha Rs_\beta \bar{s}_\beta \gamma_\mu Rb_\alpha)$$

$$- \frac{\lambda_{22}'\lambda_{13}'}{2m_\nu^2}\bar{d}_a\gamma_\mu Ls_\beta \bar{s}_\beta \gamma_\mu Rb_\alpha - \frac{\lambda_{31}'\lambda_{22}'}{2m_\nu^2}\bar{d}_a\gamma_\alpha Rs_\beta \bar{s}_\beta \gamma_\mu Lb_\alpha$$

$$- \frac{\lambda_{22}'\lambda_{23}'}{2m_\alpha^2}\bar{s}_a\gamma_\mu Ls_\beta \bar{d}_\beta \gamma_\mu Rb_\alpha - \frac{\lambda_{32}'\lambda_{23}'}{2m_\alpha^2}\bar{s}_a\gamma_\alpha Rs_\beta \bar{d}_\beta \gamma_\mu Lb_\alpha, \quad (2)$$

where $m_f$ is the sfermion mass, $L(R) = (1 \mp \gamma_5)/2$, and $\alpha$, $\beta$ are the color indices.

There are three types of four-quark operator in the above four quark interactions,

$$O_A = (\bar{p}_\alpha q_\beta)_{V \pm A}(\bar{r}_\beta b_\alpha)_{V \mp A}.$$

$$O_B = (\bar{p}_\alpha q_\beta)_{V + A}(\bar{r}_\beta b_\alpha)_{V + A},$$

$$O_C = (\bar{p}_\alpha q_\beta)_{V + A}(\bar{r}_\alpha b_\beta)_{V + A}, \quad (3)$$

where $(\bar{p}q)_{V \pm A} = \bar{p}\gamma_\mu(1 \pm \gamma_5)q$.

The first operator above occurs in $\lambda'$ interactions and the last two are in $\lambda''$ interactions. The above operators are evaluated at the common sfermion mass scale $(m_f)$ of 100 GeV. At a scale around $\mu = m_b$, these operators will induce nonzero matrix elements causing $B \to PP$ decays. We denote $c(\mu)_{A,B,C}$ the Wilson coefficients of the operators $O_{A,B,C}$ at the scale $\mu$. Renormalization group running of these coefficients from $m_f$ to $\mu$ will modify them. For $c(\mu)_{A}$, we have

$$c(\mu)_{A} = \eta^{-8/3b_0}c(m_f)_{A}, \quad (4)$$
where $\eta = \alpha_s(m_{\tilde{f}})/\alpha_s(\mu)$, and $\beta_0 = 11 - 2f/3$ with $f$ the number of quarks with mass below $\mu$. The other two coefficients $c(m_{\tilde{f}})_B$ and $c(m_{\tilde{f}})_C$ when evolved down to the scale $\mu$ from the high scale $m_{\tilde{f}}$ will mix and are given by

$$
c(\mu)_B = \frac{1}{2} \left[ \eta^{2/\beta_0} (c(m_{\tilde{f}})_B + c(m_{\tilde{f}})_C) + \eta^{-4/\beta_0} (c(m_{\tilde{f}})_B - c(m_{\tilde{f}})_C) \right],
$$

$$
c(\mu)_C = \frac{1}{2} \left[ \eta^{2/\beta_0} (c(m_{\tilde{f}})_B + c(m_{\tilde{f}})_C) - \eta^{-4/\beta_0} (c(m_{\tilde{f}})_B - c(m_{\tilde{f}})_C) \right].
$$

To obtain the $B \to P P$ decay amplitudes induced by these operators, one needs to evaluate the related hadronic matrix elements. In the next section, we will use the QCD-improved factorization method to carry out the analysis.

## 3 $B \to P P$ decay amplitude with $R$-parity violation

The factorization approximation has been used to provide estimates for the decay amplitudes in $B$ decays. Recently it has been shown that factorization approximation in fact is supported by perturbative QCD calculations in the heavy quark limit, and the QCD-improved factorization method has been developed\[2, 4\]. This new factorization formula incorporates elements of the naive factorization approach and introduces corrections in the amplitudes.

In the heavy quark limit, the decay amplitude $B \to P_1 P_2$ due to some particular operator $O_i$ can be represented in the form \[2\]

$$
< P_1 P_2 \mid O_i \mid B > = < P_2 \mid J_2 \mid 0 > < P_1 \mid J_1 \mid B > \left[ 1 + \sum_n r_n \alpha_s^n + \mathcal{O}(\Lambda_{QCD}/m_b) \right]
$$

The above result reduces to the naive factorization if we neglect the power corrections in $\Lambda_{QCD}/m_b$ and the radiative corrections in $\alpha_s$. The radiative corrections, which are dominated by the hard gluon exchange can be computed with perturbation theory in the heavy quark limit, in terms of the convolution of the hard scattering kernel and the light cone distribution amplitudes of the mesons. Then a factorization formula for $B \to P_1 P_2$ decay can be written as \[2\]:

$$
< P_1 P_2 \mid O_i \mid B > = F_{B \to P_i}^{P_i}(0) \int_0^1 dx T_{i}^I(x) \Phi_{P_2}(x)
$$

$$
+ \int_0^1 dx dy d\xi T_{i}^{II}(\xi, x, y) \Phi_B(\xi) \Phi_{P_2}(x) \Phi_{P_1}(y)
$$

In the above formula, $\Phi_B(\xi)$ and $\Phi_{P_i}(x)$ ($i = 1, 2$) are the leading twist light cone distribution amplitudes of the $B$-meson and the light pseudoscalar mesons \[21, 22\] respectively, and the $T_{i}^{I,II}$ describes the hard scattering kernel which can be calculated in perturbative QCD \[2, 4, 5\]. The diagrams generating the hard scattering kernels $T_{i}^{I,II}$ in the SM are shown in Figure 1. Figures 1(a)-(d) depicts vertex corrections, Figures 1(e) and 1(f) penguin corrections, and Figures 1(g) and 1(h) hard spectator scattering.
Applying the QCD-improved factorization method to the operators in the previous section, we obtain the contributions from $R$-parity violating interactions. Since we are considering the leading effects, we need only evaluate Figures 1(a)-(d) for the vertex corrections and Figures 1(g) and 1(h) for the hard-spectator scatterings. The penguin types are higher order corrections. The results are listed below:

\[
A_R(B^0 \to \pi^0 \pi^0) = i f_\pi (m_B^2 - m_\pi^2) F_0^{B^+ \to \pi^0 (0)} \left[ -a'_1 \frac{\lambda''_{112} \lambda''_{132}}{8m_s^2} - a'' \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} e_i \right] \\
+ (-R_{\pi} c_A + a') \left( \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2} \right), \tag{8}
\]

\[
A_R(B^0 \to \pi^+ \pi^-) = i f_\pi (m_B^2 - m_\pi^2) F_0^{B^+ \to \pi^0 (0)} \left[ a''_2 \frac{\lambda''_{121} \lambda''_{132}}{8m_s^2} - R_{\pi} c_A \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} \right], \tag{9}
\]

\[
A_R(B^- \to \pi^- \pi^0) = i f_\pi (m_B^2 - m_\pi^2) F_0^{B^- \to \pi^0 (0)} \left[ \frac{1}{\sqrt{2}} (-R_{\pi} c_A + a') \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} e_i \right] \\
+ \frac{1}{\sqrt{2}} (-R_{\pi} c_A + a') \left( \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2} \right), \tag{10}
\]

\[
A_R(B^0 \to K^- \pi^+) = i f_K (m_B^2 - m_K^2) F_0^{B^- \to K^- (0)} \left[ -a'_1 \frac{\lambda''_{121} \lambda''_{132}}{16m_d^2} + \frac{1}{\sqrt{2}} (-R_{\pi} K R_{K} c_A + a') \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} \right] \\
+ \frac{1}{\sqrt{2}} (-R_{\pi} K R_{K} c_A + a') \left( \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2} \right), \tag{11}
\]

\[
A_R(B^- \to K^- \pi^-) = i f_K (m_B^2 - m_K^2) F_0^{B^- \to K^- (0)} \left[ a''_2 \frac{\lambda''_{121} \lambda''_{132}}{16m_d^2} + a'(\frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2}) \right] \\
- R_{K} c_A \left( \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2} \right) \tag{12}
\]

\[
A_R(B^0 \to K^0 \pi^0) = i f_\pi (m_B^2 - m_K^2) F_0^{B^+ \to K^0 (0)} \left[ \frac{1}{\sqrt{2}} a'_1 \frac{\lambda''_{121} \lambda''_{132}}{8m_d^2} + \frac{1}{\sqrt{2}} a'' \frac{\lambda''_{121} \lambda''_{132}}{8m_s^2} e_i \right] \\
+ \frac{1}{\sqrt{2}} (-R_{\pi} K c_A + R_{\pi} K R_{K} a'') \left( \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2} \right) \\
+ \frac{1}{\sqrt{2}} (-R_{\pi} K R_{K} c_A + a') \left( \frac{\lambda''_{111} \lambda''_{131}}{8m_s^2} - \frac{\lambda''_{131} \lambda''_{111}}{8m_s^2} \right), \tag{13}
\]

\[
A_R(B^- \to K^- K^0) = A_R(B^- \to K^0 K^0) \]

\[
i f_K (m_B^2 - m_K^2) F_0^{B^- \to K^- (0)} \left[ a''_2 \frac{\lambda''_{121} \lambda''_{132}}{16m_d^2} + a'(\frac{\lambda''_{122} \lambda''_{132}}{8m_s^2} - \frac{\lambda''_{132} \lambda''_{122}}{8m_s^2}) \right] \\
- R_{K} c_A \left( \frac{\lambda''_{121} \lambda''_{132}}{8m_s^2} - \frac{\lambda''_{132} \lambda''_{121}}{8m_s^2} \right), \tag{14}
\]
\[ A_R(\bar{B}^0 \to K^-K^+) = 0, \tag{15} \]

where \( f_i, F^i_0 \) are decay constant and form factors, respectively. The parameters \( R_i \) and \( r_i \) are given by,

\[
R_x = \frac{2m^2_\pi}{\bar{m}_b(\mu)(\bar{m}_u(\mu) + \bar{m}_d(\mu))}, \tag{16}
\]

\[
R_K = \frac{2m^2_K}{\bar{m}_b(\mu)(\bar{m}_q(\mu) + \bar{m}_s(\mu))}, \tag{17}
\]

\[
r_{K\pi} = \frac{f_KF^{B\to\pi}_0(0)(m^2_B - m^2_\pi)}{f_\pi F^{B\toK}_0(0)(m^2_B - m^2_K)} \tag{18}
\]

with \( q = u \) for charged kaon and \( q = d \) for neutral kaon. The parameters \( a''_1 \) and \( a' \) are defined as

\[
a''_1 = c(\mu)_B + \frac{c(\mu)_C}{N_c} \left[ \frac{1 + C_F \alpha_s}{4\pi} V^2_\pi \right] + \frac{c(\mu)_C}{N_c} \frac{C_F \pi \alpha_s}{N_c} H_{P_2 P_1}, \tag{19}
\]

\[
a''_2 = c(\mu)_B + \frac{c(\mu)_C}{N_c} \left[ \frac{1 + C_F \alpha_s}{4\pi} V^2_\pi \right] + \frac{c(\mu)_C}{N_c} \frac{C_F \pi \alpha_s}{N_c} H_{P_2 P_1}, \tag{20}
\]

\[
a' = \frac{c(\mu)_A}{N_c} \left[ 1 - \frac{C_F \alpha_s}{4\pi} V^2_\pi \right] - \frac{c(\mu)_A}{N_c} \frac{C_F \pi \alpha_s}{N_c} H_{P_2 P_1}, \tag{21}
\]

\[
c_A = c(\mu)_A \tag{22}
\]

Here \( N_c = 3 \) is the number of colors, \( C_F = (N_c^2 - 1)/2N_c \), \( P_1 \) is the final state meson absorbing the light spectator quark from \( B \)-meson, while \( P_2 \) is another final state light meson which is composed of the quarks produced from the weak decay of \( b \) quark.

\[ V^{(i)}_P \] comes from vertex corrections \((P = \pi, K)\) (first four diagrams of Figure 1) is given by

\[
V_P = 12 \ln \frac{m_b}{\mu} - 18 + \int_0^1 dxg(x)\phi_P(x), \tag{23}
\]

\[
V'_P = 12 \ln \frac{m_b}{\mu} - 6 + \int_0^1 dxg(1 - x)\phi_P(x), \tag{24}
\]

\[
g(x) = 3\left( \frac{1 - 2x}{1 - x} \ln x - i\pi \right) + \left[ 2Li_2(x) - \ln^2 x - \frac{2\ln x}{1 - x} - (3 + 2i\pi) - (x \leftrightarrow 1 - x) \right]. \tag{25}
\]

The contributions from the hard spectator scattering as shown in the last two diagrams in Figure 2 give leading twist and chirally-enhanced twist-3 contributions to \( T_1^{(1)} \) parametrized by \( H^{(1)}_{P_2 P_1} \). The detailed expressions can be found in \[ ].

From the above, the amplitude \( A_R(\bar{B}^0 \to K^-K^+) \) is zero without annihilation contributions (shown in Figure 2). When annihilation contributions are included, this amplitude becomes nonzero and additional contributions to other decay amplitudes are generated. We list these contributions in the following:

\[
A_R^{ann}(\bar{B}^0 \to \pi^0 \pi^0) = A_R^{ann}(\bar{B}^0 \to \pi^+ \pi^-)
= i\beta f^2 \left[ b'' \frac{\lambda''_{112} \lambda''_{812}}{8m^2_s} - b'_4 \frac{\lambda'_{11} \lambda'_{13}}{8m^2_s} \right] - (b'_3 + b'_4) \left( \frac{\lambda'_{13} \lambda'_{11}}{8m^2_s} \right). \tag{26}
\]
In this section we discuss the constraints on the product of a pair of \(R\)-parity violating couplings. With \(R\)-parity violating interactions, the total \(B \to PP\) decay amplitudes contain the SM part of the decay amplitudes \(A_{SM}\) which have been obtained in Ref. [4] plus the \(R\)-parity violating amplitudes obtained in the previous section. Comparing the branching ratios obtained theoretically with known experimental data, constraints on the \(R\)-parity violating interactions can be obtained.

### 4 Results and Discussion

The parameters \(A_i\) are given by

\[
A_i^I = A_i^2 = \pi \alpha_s \left[ 18 \left( X_A - 4 + \frac{\pi^2}{3} \right) + 2 \kappa^2 X_A^2 \right],
\]

\[
A_3^I = 12 \pi \alpha_s r_\chi^2 (2X_A^2 - X_A),
\]

where \(r_\chi \approx R_\pi\), \(X_A = \int_0^1 dy/y\) parameterizes the divergent endpoint integrals, we take the value \(X_A = \ln(m_B/\Lambda_h)\) with \(\Lambda_h = 0.5\ \text{GeV}\) being the typical hadronic scale.
As we have shown in the previous section that several combinations of $R$-parity violating couplings can contribute to a particular charmless hadronic $B \rightarrow PP$ decay. In our numerical analysis, we assume that only one pair of $R$-parity violating couplings are nonzero at a time. This restriction may seem to be unnatural, however, it is an useful approach that allows one a quantitative feeling of the various experimental constraints.

For the numerical computations we use the averaged value of the experimental input as shown in Table 1. We average the data from BaBar, Belle, CLEO B factories as their results are uncorrelated. If the experimental branching ratio has only upper limits, we select the most stringent one. It is clear from the Table 1 that for $B \rightarrow \pi^-\pi^0, B \rightarrow \pi^0\pi^0$ and $B \rightarrow KK$ modes have only the 90% C.L. upper limit while others have results at 95% C.L.

In our numerical calculation we use $m_b(m_b) = 4.2$ GeV for the b quark mass. For the other lighter quark masses we take their central values $m_c(m_c) = 1.3 \pm 0.2$ GeV, $m_s(2$ GeV) = 110.0 ± 0.25 MeV, $(m_u + m_d)(2$ GeV) = 9.1 ± 2.1 MeV. For the decay constants and form factors, we use $f_\pi = 0.131$ GeV, $f_K = 0.160$ GeV, $f_B = 0.180$ GeV, $F_{B \rightarrow \pi} = 0.28$, $r_{\pi K} \simeq \frac{F_{B - K}}{F_{B + K}}f_K = 0.9$. For the KM matrix elements, we fix $\lambda = 0.2196$, and take the central values of $|V_{cb}| = 0.0402 \pm 0.0019$ $|V_{cb}| = 0.090 \pm 0.025$, but allow the CP violating phase $\gamma$ to vary from 0 to $2\pi$. In the SM, the CP violating phase $\gamma$ is well constrained. However, with $R$-parity violation this constraint may be relaxed. To accommodate this we vary $\gamma$ from 0 to $2\pi$ to obtain conservative limits on $R$-parity violating couplings.

We also need to know various light cone distribution amplitudes. The leading-twist light cone distribution amplitudes of the light pseudoscalar mesons can be expanded in Gegenbauer polynomials. We truncate this expansion at $n = 2$.

$$\phi_P(x, \mu) = 6x(1 - x) \left[ 1 + \alpha_1^P(\mu)C_1^{(3/2)}(2x - 1) + \alpha_2^P(\mu)C_2^{(3/2)}(2x - 1) \right], \quad (35)$$

where $C_1^{(3/2)}(u) = 3u$ and $C_2^{(3/2)}(u) = \frac{3}{2}(5u^2 - 1)$. The distribution amplitude parameters $\alpha_{1,2}^P$ for $P = K, \pi$ are: $\alpha_1^K = 0.3$, $\alpha_2^K = 0.1$, $\alpha_1^\pi = 0$ and $\alpha_2^\pi = 0.1$.

The hard spectator contributions to the coefficients $a_{1,2}'$, and $a'$ are parametrized in terms of a single (complex) quantity $H'_{P_2P_1}$, which suffers from large theoretical uncertainties related to the regularization of the divergent endpoint integral. Following we use $H_{\pi K} = H_{KK} = 0.99$ at the scale $\mu = m_b$ and

$$H'_{P_2P_1} = H_{P_2P_1}, H_{\pi K} = H_{KK} = r_{\pi K}H_{\pi K}. \quad (36)$$

Here we should like to discuss the numerical changes of the coefficients $a_{1,2}''$, and $a'$ due to our use of QCD improved factorization calculations instead of the naive factorization approximation. For example, in the $B \rightarrow \pi\pi$ decay mode using the naive factorization scheme the coefficients $a''_1$, and $a'$ have values 0.939455 and 0.661932 respectively. Using the QCD improved factorization method we get $1.0164 + 0.105414i$ and $0.581234 + 0.148545i$ for $a''_1$ and $a'$ respectively. For the
coefficient $a^\nu_1$ we get an enhancement of the order 10% in the real part and a new imaginary contribution, while for the coefficient $a^\prime$ we have a negative contribution of the order 12% in the real part but a new imaginary contribution as before. The QCD improved factorization gets contributions from vertex corrections, hard scattering and from weak annihilation. The contribution from annihilation diagrams is small compared with the current-current contributions. It is less than 5% in decays involving $a^\nu_{1,2}$ while for decays only involving $a^\prime$ the annihilation contributions can be as large as 20%.

The results are shown in Tables 3 and 4. In the Table 3 we display limits on different pair of $\lambda^\nu$-type couplings at 95% C.L. For the published experimental data with 90%C.L. it is hard to get the upper limit corresponding to 95%C.L., so we denote those bounds by (*) in the Table 3. In Table 4 we show the bounds for the product of $\lambda^\nu$-type of couplings. The bounds are obtained assuming 100 GeV common sfermion masses, so for other values of the sfermino mass, the bounds on the couplings can be easily obtained by scaling them by $m_f^2/(100\text{GeV})^2$.

We note that for several decays R-parity violation contributes only through the annihilation contribution. There are possible large uncertainties in the evaluation of the weak annihilation contribution to charmless B decays due to the fact that they are power suppressed in the heavy quark limit and the weak annihilation effects exhibit endpoint singularities. We therefore denote the modes which only receive annihilation contributions with (†) to remind that there may be large uncertainties there.

From the Table 3 and Table 4 we choose the pairs of R-parity violating couplings with most stringent bounds and we display them in Table 5 with the existing limits on such product of couplings. Among the existing limits, there are several cases, where there is no direct limit on the products of the couplings, in that case we take the products of their individual bounds from Ref.[11]. As it can be seen in [11] that the bounds on the first two generation individual couplings are much stronger than the third generation. For this reason, in most of the cases, the product of either or both first two generation individual couplings are much stronger than our prediction. Furthermore, we find that bounds on the pair of couplings $\lambda^\nu_{12}\lambda^\nu_{113}$, and $\lambda^\nu_{112}\lambda^\nu_{123}$ obtained from $n - \bar{n}$ oscillation and double nucleon decays, $\lambda^\nu_{31}\lambda^\nu_{122}$, $\lambda^\nu_{232}\lambda^\nu_{211}$ and $\lambda^\nu_{332}\lambda^\nu_{311}$, obtained from $\Delta m_K$ are stronger than ours. It is, however, interesting to note that, more than half of the bounds obtained in this paper are better than the existing ones. In addition to that, the bounds on $\lambda^\nu_{212}\lambda^\nu_{213}$, $\lambda^\nu_{312}\lambda^\nu_{313}$, $\lambda^\nu_{212}\lambda^\nu_{223}$, and $\lambda^\nu_{312}\lambda^\nu_{323}$ couplings are completely new, in a sense that there were no previous bounds on these pairs of couplings from any experimental data, they were from perturbative unitarity. In two of these cases ($\lambda^\nu_{212}\lambda^\nu_{213}$ and $\lambda^\nu_{312}\lambda^\nu_{313}$) we have improved the existing bound by over two orders of magnitude.

Before we go conclude we would like mention the possible theoretical uncertainties in the bounds arising from several input parameters. As described before, we have obtained all of our bounds by using the central values for the relevant parameters, the CKM matrix elements, the
quark masses, the form factors. We have also parametrized the hard spectator contribution by some default number. Some of these parameters may change up to 20%, and the uncertainty in the hard spectator contribution may be even larger. Even allowing for this, we do not expect our result to change more than factor of two.

In this paper we have studied $B \to \pi\pi$, $B \to \pi K$ and $B \to K\bar{K}$ decays based on the QCD improved factorization approach in the presence of $R$-parity violating couplings. Comparing our calculated branching ratios $B \to PP$ with the experimental data, we have obtained bounds on the products of two $R$-parity violating couplings. We found that most of the bounds we obtained on combinations of the $\lambda'$ and $\lambda''$ type of couplings are stronger than the existing limits. Rare hadronic B decays can provide important information about $R$-parity violating interactions.

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et al. (Belle Collaboration), *Phys. Rev. Lett.* **87**, 101801 (2001); B. Aubert, et al. (BABAR 
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### Table 1: Experimental results for the CP-averaged $B \to \pi\pi$, $B \to \pi K$ and $B \to KK$ branching ratios in units of $10^{-6}$.

| Decay Mode         | CLEO        | Belle          | BaBar          | Average      |
|--------------------|-------------|----------------|----------------|--------------|
| $B^0 \to \pi^+\pi^-$ | $4.3^{+1.6}_{-1.4} \pm 0.5$ | $5.6^{+2.3}_{-2.0} \pm 0.4$ | $4.1 \pm 1.0 \pm 0.7$ | $4.4 \pm 0.9$ |
| $B^- \to \pi^-\pi^0$ | $< 12.7$ (90% C.L.) | $< 13.4$ (90% C.L.) | $< 9.6$ (90% C.L.) | $< 9.6$ (90% C.L.) |
| $B^0 \to \pi^0\pi^0$ | $< 5.7$ (90% C.L.) | -              | -              | $< 5.7$ (90% C.L.) |
| $B^0 \to K^+\pi^-$ | $17.2^{+2.3}_{-2.4} \pm 1.2$ | $19.3^{+3.4+1.5}_{-3.2-0.6}$ | $16.7 \pm 1.6 \pm 1.3$ | $17.3 \pm 1.5$ |
| $B^- \to K^-\pi^0$ | $11.6^{+3.9+1.4}_{-2.7-1.3}$ | $16.3^{+3.5+1.6}_{-3.3-1.4}$ | $10.8^{+2.1}_{-1.9} \pm 1.0$ | $12.1 \pm 1.7$ |
| $B^- \to K^0\pi^-$ | $18.2^{+4.6}_{-4.6} \pm 1.6$ | $13.7^{+5.7+1.9}_{-4.8-1.8}$ | $18.2^{+3.3}_{-3.0} \pm 2.0$ | $17.3 \pm 2.7$ |
| $B^0 \to K^0\pi^0$ | $14.6^{+5.9+2.4}_{-5.1-3.3}$ | $16.0^{+7.2+2.5}_{-5.9-2.7}$ | $8.2^{+2.1}_{-2.7} \pm 1.2$ | $10.4 \pm 2.7$ |
| $B^0 \to K^+K^-$ | $< 1.9$ (90% C.L.) | $< 2.7$ (90% C.L.) | $< 2.5$ (90% C.L.) | $< 1.9$ (90% C.L.) |
| $B^- \to K^-K^0$ | $< 5.1$ (90% C.L.) | $< 5.0$ (90% C.L.) | $< 2.4$ (90% C.L.) | $< 2.4$ (90% C.L.) |
| $B^0 \to K^0K^0$ | $< 17.0$ (90% C.L.) | -              | $< 7.3$ (90% C.L.) | $< 7.3$ (90% C.L.) |
Table 2: Wilson coefficients, and related parameters due to $R$-parity violating interactions in different decays, $\mu = m_b$, assuming 100 GeV sfermion mass. The first final state meson is made of spectator quark, while the second final state meson is formed of quark pairs originating from $b$ quark weak decay vertex. In the above $r_A = f_B f_\pi / (m_B^2 F_0^{B \to \pi}(0))$.

| Couplings | Bound | Process |
|-----------|-------|---------|
| $| \lambda''_{112} \lambda''_{132} |$ | $3.69 \times 10^{-3}$ | $B^0 \to \pi^+ \pi^-$ |
| | $4.59 \times 10^{-3}$ | $B^0 \to \pi^+ \pi^-$ |
| | $3.30 \times 10^{-3}$ | $B^0 \to K^- K^0$ |
| | $4.46 \times 10^{-3}$ | $B^0 \to K^- K^0$ |
| | $3.46 \times 10^{-3}$ | $B^0 \to K^- K^0$ |
| | $5.03 \times 10^{-1}$ | $\bar{B}^0 \to K^- \pi^+$ |
| | $1.58 \times 10^{-2}$ | $B^- \to K^- \pi^0$ |
| | $1.03 \times 10^{-2}$ | $B^- \to K^- \pi^0$ |
| | $6.80 \times 10^{-3}$ | $\bar{B}^0 \to K^- \pi^0$ |
| | $7.45 \times 10^{-11}$ | $\bar{B}^0 \to K^- \pi^+$ |
| | $1.58 \times 10^{-2}$ | $B^- \to K^- \pi^0$ |
| | $1.03 \times 10^{-2}$ | $B^- \to K^- \pi^0$ |
| | $7.55 \times 10^{-11}$ | $\bar{B}^0 \to K^- \pi^0$ |
| $| \lambda''_{112} \lambda''_{113} |$ (i $\neq$ 1) | $3.19 \times 10^{-3}$ | $B^+ \to K^- K^0$ |
| | $4.46 \times 10^{-3}$ | $\bar{B}^0 \to K^0 \bar{K}^0$ |
| | $1.64 \times 10^{-1}$ | $\bar{B}^0 \to K^+ K^-$ |

Table 3: 95% C.L. limits on the products of $\lambda''$-type $R$-parity couplings assuming 100 GeV common sfermion mass. The limits correspond to the label (+) are at 90% C.L., while the limits denoted by (†) are obtained from decay modes with only annihilation contributions.
Table 4: 95% C.L. limits on the products of $\lambda'$ $R$-parity violating couplings for a common sfermion mass $m_f = 100$ GeV. The limits correspond to the label (*) are at 90% C.L., while the limits denoted by (†) are from decay modes with only annihilation contributions.

| Couplings | Bound | Process |
|-----------|-------|---------|
| | $|\lambda'_{111}\lambda'_{113}|$ | $1.49 \times 10^{-3}$ | $\bar{B}^0 \rightarrow \pi^0\pi^0$ |
| | | $1.86 \times 10^{-3}$ | $\bar{B}^0 \rightarrow \pi^+\pi^-$ |
| | | $1.13 \times 10^{-2}$ | $\bar{B}^0 \rightarrow K^-K^0$ |
| | | $3.68 \times 10^{-2}$ | $B^- \rightarrow K^-K^+$ |
| | | $1.39 \times 10^{-2}$ | $\bar{B}^0 \rightarrow K^0K^0$ |
| | $|\lambda'_{131}\lambda'_{111}|$ | $1.95 \times 10^{-3}$ | $\bar{B}^0 \rightarrow \pi^0\pi^0$ |
| | | $2.91 \times 10^{-3}$ | $\bar{B}^0 \rightarrow \pi^+\pi^-$ |
| | | $3.49 \times 10^{-3}$ | $B^- \rightarrow \pi^-\pi^0$ |
| | $|\lambda'_{112}\lambda'_{113}|$ | $3.16 \times 10^{-2}$ | $B^- \rightarrow K^-\pi^+$ |
| | | $2.78 \times 10^{-3}$ | $B^- \rightarrow K^-\pi^0$ |
| | | $2.66 \times 10^{-3}$ | $B^- \rightarrow K^0\pi^-$ |
| | | $1.71 \times 10^{-3}$ | $\bar{B}^0 \rightarrow K^0\pi^0$ |
| | $|\lambda'_{131}\lambda'_{121}|$ | $4.19 \times 10^{-2}$ | $\bar{B}^0 \rightarrow K^-\pi^+$ |
| | | $1.33 \times 10^{-3}$ | $B^- \rightarrow K^-\pi^0$ |
| | | $2.13 \times 10^{-3}$ | $B^- \rightarrow K^0\pi^-$ |
| | | $1.99 \times 10^{-3}$ | $\bar{B}^0 \rightarrow K^0\pi^0$ |
| | $|\lambda'_{111}\lambda'_{123}|$, $|\lambda'_{132}\lambda'_{111}|$ | $2.64 \times 10^{-1}$ | $\bar{B}^0 \rightarrow K^-\pi^+$ |
| | | $3.28 \times 10^{-2}$ | $B^- \rightarrow K^-\pi^0$ |
| | | $8.30 \times 10^{-3}$ | $B^- \rightarrow K^0\pi^-$ |
| | | $2.18 \times 10^{-3}$ | $\bar{B}^0 \rightarrow K^0\pi^0$ |
| | $|\lambda'_{122}\lambda'_{113}|$, $|\lambda'_{131}\lambda'_{122}|$ | $2.88 \times 10^{-3}$ | $B^- \rightarrow K^-\pi^0$ |
| | | $5.26 \times 10^{-3}$ | $\bar{B}^0 \rightarrow K^0\pi^0$ |
| | | $5.83 \times 10^{-3}$ | $\bar{B}^0 \rightarrow K^-K^+$ |
| | $|\lambda'_{121}\lambda'_{123}|$, $|\lambda'_{132}\lambda'_{112}|$ | $6.70 \times 10^{-4}$ | $B^- \rightarrow K^-\pi^0$ |
| | | $9.49 \times 10^{-4}$ | $\bar{B}^0 \rightarrow K^0\pi^0$ |
| | | $3.68 \times 10^{-2}$ | $\bar{B}^0 \rightarrow K^-K^+$ |
| Product of couplings | our limits | previous limits | process of others constraints |
|----------------------|------------|----------------|-------------------------------|
| \( \lambda_{112} \lambda_{113} \) | \([6.80 \times 10^{-3}]\) | \(2 \times 10^{-8}\) | \(n \rightarrow \bar{n}\) and double nucleon decay \([19]\) |
| \( \lambda_{212} \lambda_{213} \) | \(1.03 \times 10^{-2}\) | 1.5 | perturbativity bound \([19]\) |
| \( \lambda_{212} \lambda_{313} \) | \(1.03 \times 10^{-2}\) | 1.5 | perturbativity bound \([19]\) |
| \( \lambda_{112} \lambda_{123} \) | \([3.30 \times 10^{-3}]\) | \(2 \times 10^{-8}\) | double nucleon decay \([19]\) |
| \( \lambda_{212} \lambda_{223} \) | \(3.19 \times 10^{-3}\) | 1.5 | perturbativity bound \([19]\) |
| \( \lambda_{312} \lambda_{323} \) | \(3.19 \times 10^{-3}\) | 1.5 | perturbativity bound \([19]\) |
| \( \lambda_{111} \lambda_{113} \) | \([1.49 \times 10^{-3}]\) | \(1.1 \times 10^{-5}\) | \[20\] |
| \( \lambda_{211} \lambda_{213} \) | \(1.49 \times 10^{-3}\) | \(3.6 \times 10^{-3}\) | \(\Delta m_B\) \([20]\) |
| \( \lambda_{111} \lambda_{123} \) | \([2.18 \times 10^{-3}]\) | \(2.2 \times 10^{-5}\) | \[20\] |
| \( \lambda_{211} \lambda_{223} \) | \(2.18 \times 10^{-3}\) | \(1.2 \times 10^{-2}\) | \[20\] |
| \( \lambda_{311} \lambda_{323} \) | \(2.18 \times 10^{-3}\) | \(1.6 \times 10^{-2}\) | \(\Delta m_B\) \([20]\) |
| \( \lambda_{111} \lambda_{131} \) | \([1.95 \times 10^{-3}]\) | \(1.0 \times 10^{-5}\) | \[20\] |
| \( \lambda_{211} \lambda_{231} \) | \(1.95 \times 10^{-3}\) | \(1.1 \times 10^{-2}\) | \[20\] |
| \( \lambda_{311} \lambda_{331} \) | \(1.95 \times 10^{-3}\) | \(5.0 \times 10^{-2}\) | \[20\] |
| \( \lambda_{111} \lambda_{132} \) | \([2.18 \times 10^{-3}]\) | \(1.4 \times 10^{-4}\) | \[20\] |
| \( \lambda_{211} \lambda_{232} \) | \([2.18 \times 10^{-3}]\) | \(4.7 \times 10^{-4}\) | \(\Delta m_K\) \([20]\) |
| \( \lambda_{311} \lambda_{332} \) | \([2.18 \times 10^{-3}]\) | \(4.7 \times 10^{-4}\) | \(\Delta m_K\) \([20]\) |
| \( \lambda_{121} \lambda_{113} \) | \([1.71 \times 10^{-3}]\) | \(4.4 \times 10^{-4}\) | \[20\] |
| \( \lambda_{212} \lambda_{213} \) | \(1.71 \times 10^{-3}\) | \(3.5 \times 10^{-3}\) | \[20\] |
| \( \lambda_{312} \lambda_{313} \) | \(1.71 \times 10^{-3}\) | \(1.2 \times 10^{-2}\) | \[20\] |
| \( \lambda_{112} \lambda_{132} \) | \(6.70 \times 10^{-4}\) | \(5.9 \times 10^{-3}\) | \[20\] |
| \( \lambda_{212} \lambda_{232} \) | \(6.70 \times 10^{-4}\) | \(3.3 \times 10^{-2}\) | \[20\] |
| \( \lambda_{312} \lambda_{332} \) | \(6.70 \times 10^{-4}\) | \(5.0 \times 10^{-2}\) | \[20\] |
| \( \lambda_{113} \lambda_{122} \) | \([2.88 \times 10^{-3}]\) | \(9.0 \times 10^{-4}\) | \[20\] |
| \( \lambda_{213} \lambda_{222} \) | \([2.88 \times 10^{-3}]\) | \(1.2 \times 10^{-2}\) | \[20\] |
| \( \lambda_{313} \lambda_{322} \) | \(2.88 \times 10^{-3}\) | \(5.7 \times 10^{-2}\) | \[20\] |
| \( \lambda_{121} \lambda_{123} \) | \([6.70 \times 10^{-4}]\) | \(1.4 \times 10^{-3}\) | \(\Delta m_B\) \([20]\) |
| \( \lambda_{221} \lambda_{223} \) | \([6.70 \times 10^{-4}]\) | \(1.4 \times 10^{-3}\) | \(\Delta m_B\) \([20]\) |
| \( \lambda_{321} \lambda_{323} \) | \([6.70 \times 10^{-4}]\) | \(1.4 \times 10^{-3}\) | \(\Delta m_B\) \([20]\) |
| \( \lambda_{121} \lambda_{131} \) | \([1.33 \times 10^{-3}]\) | \(8.2 \times 10^{-4}\) | \[20\] |
| \( \lambda_{221} \lambda_{231} \) | \(1.33 \times 10^{-3}\) | \(3.2 \times 10^{-2}\) | \[20\] |
| \( \lambda_{321} \lambda_{331} \) | \(1.33 \times 10^{-3}\) | \(2.3 \times 10^{-1}\) | \[20\] |
| \( \lambda_{122} \lambda_{131} \) | \([2.88 \times 10^{-3}]\) | \(1.0 \times 10^{-4}\) | \(\Delta m_K\) \([20]\) |
| \( \lambda_{222} \lambda_{231} \) | \([2.88 \times 10^{-3}]\) | \(1.0 \times 10^{-4}\) | \(\Delta m_K\) \([20]\) |
| \( \lambda_{322} \lambda_{331} \) | \([2.88 \times 10^{-3}]\) | \(1.0 \times 10^{-4}\) | \(\Delta m_K\) \([20]\) |

Table 5: Comparison of constraints obtained in this paper with other existing constraints. 95\% C.L. limits on the products of different \(R\)-parity couplings for sfermion masses \(m_f = 100\) GeV. The limits correspond to the label (\(\ast\)) are at 90\% C.L.. The limits shown in the square bracket are weaker than the existing bounds. The bounds corresponding to the processes not shown in the Table are coming from the product of individual couplings \([19]\).
Figure 1: Order of $\alpha_s$ corrections to the hard-scattering kernels $T^I$ and $T^{II}$. The quark lines directed upwards represent the ejected quark pairs from weak decays of $b$-quark.

Figure 2: Annihilation diagrams