Light hadron spectrum with two flavors of $O(a)$ improved dynamical quarks: final results from JLQCD

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We present the final results of the JLQCD calculation of the light hadron spectrum and quark masses with two flavors of dynamical quarks using the plaquette gauge action and fully $O(a)$-improved Wilson quark action at $\beta = 5.2$. We observe that sea quark effects lead to a closer agreement of the strange meson and baryon masses with experiment and a reduction of quark masses by 20–30%.

1. Introduction

Precise determination of the light hadron spectrum with dynamical up, down and strange quarks is a key step for validating QCD at low energies. As a step toward this goal, we have pursued a systematic calculation in two-flavor QCD using the non-perturbatively $O(a)$-improved Wilson quark action \cite{1}. In this article, we present the final results of the hadron spectrum and quark masses from this calculation. A test of the one-loop chiral perturbation theory formula in our lattice data is discussed in a separate talk \cite{2}. Our simulations were performed at $\beta = 5.2$, where the scale is $a = 0.0887(11)$ fm, on $12^3 \times 48$, $16^3 \times 48$ and $20^3 \times 48$ lattices. We simulated five sea quark masses $K_{\text{sea}} = 0.1340, 0.1343, 0.1346, 0.1350$ and $0.1355$, which cover the range of $m_{\text{PS,sea}}/m_{\nu,\text{sea}} = 0.80 - 0.60$. For further details of the simulations, we refer the reader to the previous reports \cite{1} and a forthcoming paper \cite{3}. The main progress after the last conference is that we accumulated doubled statistics (12000 HMC trajectories) on the largest lattice.

2. Finite size effects

Finite size effects (FSE) are more pronounced in full QCD than in quenched QCD. Since our largest volume size $\simeq (1.8 \text{ fm})^3$ is not so large, it is important to check FSE in our data.

In Fig. \ref{fig1}, we plot the relative mass shift between the two larger lattices $\Delta m = (m(N_s = 16) - m(N_s = 20))/m(N_s = 20)$ for pseudo-scalar mesons and the octet baryons as a function of valence $1/K_{\text{val}}$ for each sea quark mass. We see that $\Delta m$ is consistent with zero for the whole range of valence quark mass down to the second lightest sea quark at $K_{\text{sea}} \leq 0.1350$. The size effect becomes visible and increases with reduced $1/K_{\text{val}}$ for the lightest sea quark at $K_{\text{sea}} = 0.1355$.

The wrapping of valence quarks in the spatial directions leading to FSE due to the size of hadrons is suppressed by the center $Z(3)$ symmetry in quenched QCD \cite{4}. In full QCD, $Z(3)$ symmetry is broken by the wrapping of sea quarks in the spatial directions, whose magnitude increases toward lighter sea quark. A qualitative understanding of the behavior in Fig. \ref{fig1} would be that the $Z(3)$ breaking turns on rather quickly around
our lightest sea quark.

The volume dependence of our data is well described by the power law ansatz $m(N_s) - m(N_s = \infty) \propto 1/N_s^3$. For pseudo-scalar and vector mesons, the magnitude of FSE indicated by this fit is about 3% at $K_{val} = 0.1350$, which roughly corresponds to the strange quark mass, and increases to 5% toward the chiral limit. However, since the volume dependence is expected to turn into a milder behavior $\exp[-m_{PS} N_s]$ for sufficiently large volumes, the actual size of FSE should be smaller. In particular, at strange quark mass, FSE should not exceed a few % level.

Finite size effects are more pronounced for baryon masses. For our lightest sea quark, the magnitude of FSE is about 5% at $K_{val} = 0.1350$ and increases to 10% for $K_{val} = K_{sea} = 0.1355$. This is comparable with the typical size of the quenching error. Therefore, masses of light baryons such as $m_N$ and $m_{\Delta}$ may be affected by FSE.

3. Hadron masses

In Fig. 1, we compare the valence quark mass dependence of the vector meson mass at each sea quark mass and in quenched QCD. The full QCD data have a steeper slope than the quenched QCD, and show a better agreement with the experimental points.

The increase of the slope can be explicitly seen in the $J$ parameter shown in Fig. 3. The value of $J$, which is consistent with the quenched value for heavier sea quark masses, increases as the sea quark mass decreases. The chiral limit is still significantly lower than the phenomenological value, which may be attributed to quenching of strange quarks.

Finite size effects, possibly present at a few percent level, do not alter these results since masses at larger volumes would show a steeper slope in Fig. 2.

The hadron spectrum at the physical quark mass is calculated using data on the largest lattice. We use $m_\pi$ and $m_\rho$ to fix the scale and the light quark mass $m_{ud}$. The strange quark mass $m_s$ is determined from either $K$ or $\phi$ meson mass.

Our results for the spectrum are plotted in Fig. 4. The deviation of the meson spectrum from experiment is reduced significantly by the inclusion of dynamical up and down quarks. The strange baryon masses, such as $m_{\Xi}$ and $m_{\Omega}$, also show a closer agreement with experiment in full QCD. However, sea quark effects are less clear in lighter baryons like $m_N$ and $m_{\Delta}$. This is probably due to FSE, which is enhanced for the lighter baryons.

We make a comparison of $m_{K^*}$ with $K$-input with the CP-PACS results in Fig. 5. A good agreement with their result in the continuum.
limit suggests that the scaling violation in our data is small.

4. Quark masses

Calculation of $m_{ud}$ and $m_s$ is performed with the bare quark mass defined through either the vector (VWI) or axial vector Ward identity (AWI). For the latter, we use the improved axial current with $c_A$ at one-loop. The continuum MS quark mass is obtained by the one-loop matching at scale $\mu = a^{-1}$ and evolved to $\mu = 2 \text{ GeV}$ using four-loop beta function.

It is expected that various systematic uncertainties, such as scaling violation, partially cancel in the ratio defining the AWI mass $m_q = \langle A^1 P^1 \rangle / (2 \langle P P^1 \rangle)$. This is supported by the good agreement with the CP-PACS results in the continuum limit in Fig. 5. Such a cancellation is not expected in the VVI mass. Indeed there is a sizable difference between our AWI and VVI results, and scaling violation is large in the CP-PACS results.

We therefore quote the AWI masses in Table 1 as our central values. The quark masses decrease by 20–30 % in two-flavor QCD compared to quenched QCD. Sea quark effects also reduce the difference between $m_s$ with $K$- and $\phi$-inputs. These are consistent with the observation in Ref. 6.

5. Conclusions

We have demonstrated sea quark effects at simulation points in the meson sector using the $O(a)$-improved Wilson quark action and plaquette gauge action at $a^{-1} \simeq 2 \text{ GeV}$. A closer agreement of the strange meson and baryon masses with experiment is a natural consequence of the finding. We also observed that sea quark effects lead to a significant reduction of quark masses.

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