Modeling the self-organization of vocabularies under phonological similarity effects

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Abstract

This work develops a computational model (by Automata Networks) of short-term memory constraints involved in the formation of linguistic conventions on artificial populations of speakers. The individuals confound phonologically similar words according to a predefined parameter. The main hypothesis of this paper is that there is a critical range of working memory capacities, in particular, a critical phonological degree of confusion, which implies drastic changes in the final consensus of the entire population. A theoretical result proves the convergence of a particular case of the model. Computer simulations describe the evolution of an energy function that measures the amount of local agreement between individuals. The main finding is the appearance of sudden changes in the energy function at critical parameters. Finally, the results are related to previous work on the absence of stages in the formation of languages.

Keywords: Phonological similarity; Working memory; Automata networks; Linguistic conventions.

Introduction

In natural language a typical linguistic interaction is not a simple sequence of individual actions, but also a form of joint activity that involves cooperation and coordination between participants (Tomasello, 2008; Fernández, 2014). What is more, inside the dialogue language users tend to converge in their choice of constructions. This mutual convergence process is called alignment, which has been extensively studied within computational studies of the formation of language through the Naming Game (Steels, 1995, 2011; Baronchelli, Felici, Caglioti, Loreto, & Steels, 2006; Loreto, Baronchelli, Mukherjee, Puglisi, & Tria, 2011). Crucially, it attempts to ask how on a population of agents only from local interactions it arises a shared word-meaning association (the simplest version of a vocabulary). This model considers a finite population of agents, where each one is endowed with a memory in which it stores an in principle unlimited number of words. At each discrete time step a pair of agents is selected: one plays the role of hearer, one plays the role of hearer. First, the speaker refers to an object by using a word. Next, the hearer tries to identify the referent. For this purpose, the hearer inspects its own memory: (1) if the word belongs to the memory of the hearer, both speaker and hearer cancel all the words in their memories, except such word; or (2) the hearer adds the word to its memory, if the word does no belong to its memory. A pairwise interaction such as in the Naming Game only is possible if the interaction is a kind of joint activity. Both agents interact in a context where the object is located and share focus on the object by means of pointing or eye-gazing. The interaction continues until both agents reach a common word associated to the object, a linguistic convention.

The adopted framework assumes that the development of linguistic conventions is founded on self-organization mechanisms arising only from local interactions between agents (Steels, 1995, 1996; Baronchelli et al., 2006). Given this self-organized nature Automata Networks (AN) (Neumann, 1966; Wolfram, 2002) provide the adequate framework to explore alignment from a computational (and mathematical) point of view. AN are extremely simple models where each vertex of a two-dimensional grid evolves following a local rule based on the states of “nearby” vertices. Despite of the simplicity of the defining rules AN exhibits astonishing rich patterns of behavior. A linguistic convention can be considered as a complex pattern.

The main question behind this paper is to understand how realistic cognitive constraints influence the emergence of language on artificial populations of individuals. The problem faces the study of computer machines which exhibit limitations and constraints of human language users. Particularly, the work stresses a novel question within computational models of self-organized processes of the formation of language: To what extent do working memory constraints influence the alignment of shared conventions on artificial populations of agents? In a more realistic scenario language users (and then the agents playing computational games of the formation of language) suffer limited working memory capacities (A. Baddeley & Hitch, 1974; A. Baddeley, 2007). Specially, phonological similarity effects suppose that individuals confound word items sharing large portions of phonological content. A classical work (A. D. Baddeley, 1966) reports experiments where subjects heard sequences of unrelated words and tried to recall in the correct order. The results suggest that memory performance was impaired for phonologically similar words (man, cad, mat, cap, can) versus dissimilar ones (pen, sup, cow, day, hot). This effect is a strong evidence for the existence of the phonological loop and, especially, of the short-term working memory system (A. Baddeley & Hitch, 1974; A. Baddeley, 2007).

The simulations described here are based on a parameter that measures the amount of phonological confusion between words (understood as a way to describe the influence of working memory limits). The work explores the hypothesis of there being a critical range of the parameter that implies drastic changes in the language of the entire population. Therefore, the features of language (in particular, the consensus on linguistic conventions) emerge abruptly at some critical range of phonological confusion (Hauser, 1996). This hypothesis is strongly related to previous work on the absence of stages...
in the formation and evolution of human languages (see, for instance, (Ferrer-i-Canchò & Solé, 2003)).

The work is organized as follows. The second section explains basic notions on AN, the instrumentalization of phonological similarity and the rules of interaction. The third section reports simple mathematical results on the convergence of a particular case of the model. The next section describes simulation tasks based on an energy operator, over a parameter that measures the amount of similarity confusion between words. A brief discussion of the results is presented in the final section.

Model

Elements of the AN

Roughly speaking, the AN model involves the following elements:

1. A regular grid graph: vertices represent individuals; edges represent possible interactions between them (more generally, as in the Section 3, the graph can be a connected, undirected and simple network).

2. Each individual is associated to a state that eventually changes along time. This state is a way to represent the individual’s language at some time frame.

3. A set of local rules that define how the system changes. The local rule associated to one individual considers as inputs the states of the nearby individuals (the neighbors).

4. A function, the updating scheme, that indicates the order in which the individuals are updated. Two updating schemes are considered in this work: (1) the fully-asynchronous scheme, where at each time step one individual is chosen uniformly at random; and (2) the sequential scheme, defined as a permutation of the set of vertices.

In what follows some of the previous elements will be treated in greater depth.

Basic notions

The set \( P = \{1, \ldots, n\} \) represents the population of individuals. They are located on the vertices of the regular grid graph \( G = (P, E) \), where \( E \) is the set of edges. The vertex \( u \in P \) only interacts (or “talks”) with the set of adjacent vertices \( V_u = \{v \in P : (u, v) \in E\} \) (the neighborhood). Each individual considers four neighbors: up, down, left and right ones (Von Neumann neighborhood). The individual \( u \in P \) is endowed with a memory \( M_u \) in which it stores words belonging to a finite set \( W \), with \( p \) elements.

Let \( \Sigma = \{a_1, \ldots, a_k\} \) be a set of sounds. Each word of \( W \) is constructed by a combination of sounds taken from \( \Sigma \). For instance, \( a_3a_1a_6 \in W \). The length of the word \( x \) is its number of sounds. For the sake of simplicity, all the words have the same length \( L \). \( x(k) \), with \( k \leq L \), denotes the \( k \)-th sound (or position) of the word \( x \). Two related distances are defined with the purpose of measuring the amount of phonological similarity between words. The Hamming distance \( H(x,y) \) between two words \( x \) and \( y \) is the number of positions in which they differ. Consider two words \( a_4a_6a_5 \) and \( a_7a_6a_3 \), then \( H(a_4a_6a_5, a_7a_6a_3) = 2 \).

Confusion parameter

To explicitly measure the ability to distinguish between words, the confusion parameter \( \varepsilon \in [0, 1] \) is defined. Suppose that the vertex \( u \) faces two words \( x, y \). Then,

\[
\text{if } H(x,y) > \varepsilon L, \text{ } u \text{ distinguishes the words } x \text{ and } y
\]

\[
\text{else } u \text{ confounds the words } x \text{ and } y \text{ (or simply } x = y \)
\]

For instance, the individual \( u \in P \) is exposed to the words \( x = a_1a_6a_4 \) and \( y = a_1a_6a_3 \). Two values of \( \varepsilon \) are defined, 0 and 0.5. The confusion varies radically:

- (\( \varepsilon = 0 \)) \( H(x,y) = 1 > \varepsilon L = 0 \), then \( u \) distinguishes the words \( x \) and \( y \)
- (\( \varepsilon = 0.5 \)) \( H(x,y) = 1 < \varepsilon L = 2 \), then \( u \) confounds the words \( x \) and \( y \)

Local rules

In the Naming Game, the local rule associated to the individual \( u \in P \) is based on two possible actions on the memory \( M_u \) (see, for instance, (Baronchelli et al., 2006)):

- \( u \) updates its memory \( M_u \) by the addition of words; or
- \( u \) collapses its memory if \( M_u \) is updated by cancelling all its words, except one of them.

Both actions attempt to take into account lateral inhibition strategies (Steels, 1995, 2011) in the alignment process: the individuals add words in order to increase the chance of future strategies (Steels, 1995, 2011) in the alignment process: the individuals add words in order to increase the chance of future agreements (local consensus), and detect the words that do not cooperate with mutual understanding. Within collapses the individuals prefer the minimal word, according to the lexicographic order over the set of words. The lexicographic order \( \prec \) is a generalization of the typical alphabetical order of words on the alphabetical order of their component letters (or sounds). For example in the dictionary the word “Me” appears before “My” because the letter e comes before the letter y in the alphabet. In some sense the word “Me” is lower than the word “My”. Formally the order \( \prec \) is defined on the set \( \Sigma \). Two words \( x \) and \( y \) of length \( L \) are considered. Then, \( x \prec y \) if the first position in which they differ, say \( k \leq L \), satisfies \( x(k) < y(k) \). For instance given the set of sounds \( \Sigma = \{a,b,c,d\} \), with \( a < b < c < d \), the words \( abc, bcd \) and \( cda \) satisfy \( abc < bcd < cda \). Therefore, \( abc \) is the minimal word or, in other terms, \( \min(\{abc, bcd, cda\}) = abc \). With the previous words “Me” and “My” it is possible to write \( \min(\{“Me”, “My”\}) = “Me” \).

The preference for the minimal words can be viewed in accordance with the following scenario (Nowak & Krakauer, 1999): it is possible to think in a population of early hominids
for which leopards represents a higher risk than cows. So, the word “leopard” may be more valuable than “cow”. In the terms of this paper “leopard” could be the minimal word.

At time step \( t \) the vertex \( u \in P \) is selected according to the updating scheme (fully asynchronous or sequential). Consider a simple population example \( P = \{1,2,3,4,5\} \) (each number represents one individual). For a fully asynchronous scheme at each time step any individual of \( P \) can be selected (for instance, the individual 3). In the next step even the individual 3 can be selected again. For a sequential scheme a permutation of the set \( P \) is defined, for instance, the order 5-4-3-2-1. The individual 5 updates first, then the individual 4 updates, taking into account the effects of the changes in the first individual, and so on. At time step 6 the dynamics starts in the same previous way.

The individual \( u \in P \) is completely characterized by its state \((M_u, x_u)\), where \( M_u \) is the memory to stores words (\( M_u \) is a subset of \( W \)) and \( x_u \in M_u \) is a word that \( u \) conveys to the vertices of \( V_u \). The model induces specific communication roles: the vertex \( u \) plays the role of “hearer” (it receives the words conveyed by its neighbors); the neighbors of \( u \) play the role of “speaker” (they convey words to the vertex \( u \)). The set of all words conveyed by the speakers can be re-written as two subsets: \( N_u \) and \( B_u \). Roughly speaking \( N_u \) includes the unknown words, and \( B_u \) includes the known ones.

The state pair \((M_u, x_u)\) changes according to the following steps, which define the local rule of the automata (see Fig. 1):

**step 1** the vertex \( u \) defines two sets:

\[
N_u = \{ x_v : (v \in V_u) \land (\forall y \in M_u, H(x_v, y) > \varepsilon L) \}
\]

\[
B_u = \{ x_v : (v \in V_u) \land (\forall y \in M_u, H(x_v, y) \leq \varepsilon L) \}
\]

**step 2**

if \( N_u \neq \emptyset \), \( M_u \) adds the words of \( N_u \)

else \( M_u \) collapses in the word \( \tilde{x} \), selected at random from the set \( \{ x \in B_u : H(x, \min(B_u)) \leq \varepsilon L \} \) (that is, the new state is \((\tilde{x}), \tilde{x})\)

Step 1 comprises (1) the speaker’s behavior (the neighbors convey words to the vertex \( u \)); and (2) the definition by the hearer of the sets \( N_u \) and \( B_u \). Given a conveyed word \( x_v, v \in V_u \), the hearer \( u \) decides between: either \( x_v \) is added to \( N_u \) if for all \( y \in M_u, H(x_v, y) > \varepsilon L \); or \( x_v \) is added to \( B_u \), otherwise.

Step 2 summarizes the behavior of the hearer in order to align itself with the speakers. In the case that \( N_u \neq \emptyset \), the hearer simply adds to its memory the words of \( N_u \). Otherwise (\( N_u = \emptyset \)), the hearer collapses its memory in the word \( \tilde{x} \). The word \( \tilde{x} \) is selected uniformly at random from \( \{ x \in B_u : H(\cdot, \min(B_u)) \leq \varepsilon L \} \). Thus, the preferred word \( \min(B_u) \) can be confused by “similar” ones (with \( H(\cdot, \min(B_u)) \leq \varepsilon L \)).

**Dynamics of the automata**

The dynamics of the automata model evolves as follows. As initial configuration each individual receives uniformly at random a word constructed by a random combination of \( L \) sounds from the set of symbols \( \Sigma \). Thus, at \( t = 0 \) each individual is associated to a state of the form \((\{x\}, x)\), with \( x \in W \). Each discrete time step \( t \geq 0 \) supposes that one individual, \( u \in P \), is selected uniformly at random (the fully-asynchronous scheme) or according to a permutation of the set of vertices (the sequential scheme). The individual receives the words conveyed by its neighbors. Then, it plays the role of hearer, and the neighbors play the role of speaker. Regarding to the conveyed words the individual follows the two previously defined steps in order to decide possible changes in its own language state:

**step 1** the individual defines the sets \( N_u \) (unknown words) and \( B_u \) (known words).

**step 2** the individual either adds words if \( N \) is non empty or collapses its memory in the minimal word of \( B_u \), if \( N_u \) is empty.

Both steps involve the possibility of confusion between similar words.

**A brief note on convergence at \( \varepsilon = 0 \)**

An interesting problem related to the formation of consensus on linguistic conventions is to propose mathematical convergence proofs. Given the mathematical framework of this paper the problem becomes to count (in the worst case) the number of simulation steps needed to stop the dynamics, that is, to prove that after a finite number of time steps the population reaches a shared linguistic convention. Despite that other works have solved similar tasks (see (DeVylder & Tuyls, 2006)), the novelty of the rest of this section is to develop a convergence proof based on the worst-case complexity, which measures the amount of resources (running time) needed by the dynamics if it is considered as an algorithm (Cormen, Leiserson, Rivest, & Stein, 2001). The case of running time (number of steps until the entire population reaches a global consensus language) indicates the largest dynamics performed by the automata given the size \( n \) of the population (denoted \( O(f(n)) \), where \( f(n) \) is a function of \( n \)). For instance \( O(n^2) \) means that in the worst-case the running time has a growth rate scaling as \( n^2 \).

A fixed point is a configuration which is invariant under the application of local rules. A fixed point can be interpreted as a final consensus configuration, where all individuals agree about some linguistic convention.

This section considers \( \varepsilon = 0 \) and individuals located on a general undirected and connected network (not necessarily a regular grid). It is straightforward to notice that at \( \varepsilon = 0 \) the individuals distinguish any pair of words. Indeed, at \( \varepsilon = 0 \) (\( \forall y \in M_u, H(x_v, y) > \varepsilon L \)) is equivalent to \((x_v \notin M_u) \). Thus, the two steps of the rule take a simpler form:

**step 1** the vertex \( u \) defines two sets:

\[
N_u = \{ x_v : (v \in V_u) \land (x_v \notin M_u) \}
\]
The automata model with the set of $p$ words $W$. Then, Theorem 1 minimum word. til it reaches a fixed point where all individuals convey this collapsed at least once. As the theorem shows in detail, the steps (there are $p$ words. Then, in at most $p − 1$ steps (there are $p$ words). Since the population has size $n$, after $n(p − 1)$ steps the individuals must collapse their memories. Then, at step $t^* = n(p − 1)$ all individuals have been collapsed at least once. As the theorem shows in detail, the minimum conveyed word at $t^*$ propagates in the system until it reaches a fixed point where all individuals convey this minimum word.

**Theorem 1** Consider a population of $n$ individuals playing the automata model with the set of $p$ words $W$. Then, (I) for the sequential scheme, the system converges to fixed points in at most $O(n^2 p)$ steps; (II) for the fully-asynchronous scheme, the system converges to a fixed point in expected time $O(n^2 p \log(n))$.

**Proof 1** (I) Initially there are $p$ words. Then, in at most $p − 1$ updates a vertex has collapsed for the first time (in the worst case the vertex must add every possible word, one at a time). This implies that in $n(p − 1)$ steps ($p − 1$ updates of each vertex) all vertices have collapsed at least one time. Let $m$ be the minimum conveyed word at step $t^* = n(p − 1)$, and let $u$ be a vertex such that $x_u = m$ (in more precise terms, $m = \min\{x_u \mid u \in P\}$).

Since $u$ has collapsed at least one time, the updating scheme is sequential, and $m$ is the minimum word, then $u$ must have another neighbor $v \in V_u$ conveying $m$. In consequence, after $t^*$ both vertices $u$ and $v$ will remain conveying $m$, and at each time a neighbor of any of these two vertices will necessarily collapse in the word $m$. The graph has a diameter of $O(n)$ and each $np$ steps it occurs a collapse. Therefore, in at most $O(n^2 p)$ steps the system converges to a fixed point where all vertices convey the same word $m$.

(II) In the fully-asynchronous scheme, at each time step a single vertex is picked independently and uniformly at random. The expected convergence time grows by a factor of $O(\log(n))$ with respect to the sequential scheme. Notice that in the proof of the part (I) it is shown that after updating $[O(np)$ times] [each vertex], a fixed point is reached. From the well known coupon collector’s problem (see, for instance, (Grimmett & Stirzaker, 2001)), the expected number of steps required to update [at least once] [every vertex in the graph (or pick every coupon)] is $O(n \log(n))$. Since in a fully asynchronous updating scheme each step is independent to the others, the result follows.
Simulations

To describe the amount of agreement between the languages of the individuals, an energy operator is defined. This energy-based approach arises from a “physicist” interpretation (for a related approach, see (Regnault, Schabanel, & Thierry, 2009)): the energy measures the amount of local instability of the system. At each neighborhood \( V_u \) the function \( \sum_{v \in V_u} H(x_u, x_v) \) is defined, which measures the Hamming distance between the word \( x_u \) and the words conveyed by the neighbors of the individual \( u \). This function is bounded by 0 (in case of agreement, that is, the individual \( u \) and its neighbors convey the same word) and \( 4L \) (disagreement, which means that the individual \( u \) and its neighbors convey radically different words). Summing over all individuals defines the total energy function at some time step \( t \):

\[
E(t) = \frac{1}{4Ln} \sum_{u \in P} \sum_{v \in V_u} H(x_u, x_v) \tag{1}
\]

The function \( E(t) \) is bounded. Indeed, \( 0 \leq E(t) \leq 1 \). Thus, the dynamics of the AN model can be understood as the trajectory between initial configurations associated to large amounts of instability (with \( E(0) \sim 1 \)) and final consensus configurations where \( E(t) \sim 0 \), or equivalently, all individuals convey similar - in the sense defined by the Hamming distance- words.

The analysis focuses on two-dimensional periodic lattices of size \( n = 128^2 \) with Von Neumann neighborhood. The simulations describe \( 500n \) steps of the energy function \( E(t) \). At each time step one vertex (playing the role of hearer) is selected uniformly at random (fully-asynchronous scheme). The plots show average values over 20 initial conditions. An initial condition is defined as follows: each individual receives uniformly at random a word constructed by a random combination of \( L \) sounds from the set of symbols \( \Sigma \), \( |\Sigma| = 10 \). Word-length \( L \) varies from \( \{2, 4, 8, 16, 32, 64\} \). \( \epsilon \) is varied from 0 to 1 with an increment of 10%.

Given the worst-case complexity approach to theoretical aspects of the convergence of the model, it is important to notice that on low-dimensional lattices it seems hard that one individual adds \( O(np) \) words after it collapses, because lattices are low-connected. This intuition suggests that the running times on lattices will be lower than theoretical bounds \( \Theta(n^2p \log(n)) \) for the fully-asynchronous scheme).

Results

There are several remarkable aspects, as shown in Fig. 2 and Fig. 3. First of all, \( E(t = 500n) \) versus \( \epsilon \) exhibits at \( \epsilon = 1 \) a maximum which is close to 0.5 for all values of \( L \) (Fig. 2). Secondly, to the extent \( L \) grows, \( E(t = 500n) \) versus \( \epsilon \) evolves more “smoothly”: \( L \leq 8 \) supposes “ladder” steps which mean that different values of the confusion parameter \( \epsilon \) lead to the same energy. Finally, focusing the description on \( L = 32 \) (Fig. 3), it is interesting to notice that at \( \epsilon = 0.7 \) the average value of \( E(t) \) stops approximately after \( t = 200n \) steps, that is, only a few set of runs do not converge to the global minimum of the \( E(t) = 0 \). This strongly suggests the appearance of a critical parameter \( \epsilon^* \sim 0.7 \) which clearly defines two phases in the evolution of \( E(t = 500n) \) versus \( \epsilon \): (1) \( \epsilon < \epsilon^* \) implies the convergence to the global minimum \( E(t) = 0 \), where all individuals convey the same word; and (2) for \( \epsilon \geq \epsilon^* \) the dynamics changes drastically until it reaches local minima of the energy function \( (E(t) \to 0.5) \).

Figure 2: \( E(t = 500n) \) versus \( \epsilon \). On a two dimensional grid of size \( n = 128^2 \), the figure shows the final value of the energy function versus the parameter \( \epsilon \), after \( 500n \) steps or until they reach the global minimum \( E(t) = 0 \). The plots show averages over 20 initial conditions: for an initial condition, each vertex receives a word constructed by a random combination of \( L \) sounds from the set of symbols \( \Sigma \), \( |\Sigma| = 10 \). Word-length varies from \( \{2, 4, 8\} \) (top); and \( \{16, 32, 64\} \) (bottom). \( \epsilon \) is varied from 0 to 1 with an increment of 10%.

Conclusion

This work summarizes an AN approach to the formation of linguistic conventions under phonological similarity (and, in general, working memory) constraints. The paper presents the evolution of an energy functional, defined as a word “confusion” average, during the alignment game. Two aspects are remarkable. On the one hand, the appearance of drastic transitions can be related to previous works that focus on the absence of stages in the formation and evolution of human languages (see, for instance, (Ferrer-i-Cancho & Solé, 2003)). On the other hand, the proposed model becomes an
alternative (mathematical) framework for agent-based studies on language formation.

As a first approach to the convergence of the formation of linguistic conventions, this paper presents within an AN account simple results of the number of steps needed to reach fixed points. As the main tool, the proofs are based in to describe the worst case of convergence.

Many extensions of the proposed model should be studied with the purpose to describe the role of cognitive constraints (and, in particular, the role of (working) memory limits) on the formation of linguistic conventions. First, within a mathematical point of view it seems interesting to explore convergence bounds on regular lattices. Also, a comparison between theoretical and numerical convergence times should be carried out. Second, AN allow to study new aspects of the formation of linguistic conventions. Indeed, the model can be extended to other updating schemes, for example, the synchronous one, where at each time step all individuals are updated. Third, the results should be compared with larger word lengths (L) and several number of symbols (Σ). Finally, future work should involve more realistic ways to measure the amount of phonological confusion and its effects on the formation of conventions.

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