Chiralspin symmetry and its implications for QCD

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\textbf{Abstract.} In a local gauge-invariant theory with massless Dirac fermions a symmetry of the Lorentz-invariant fermion charge is larger than a symmetry of the Lagrangian as a whole. While the Dirac Lagrangian exhibits only a chiral symmetry, the fermion charge operator is invariant under a larger symmetry group, $SU(2N_F)$, that includes chiral transformations as well as $SU(2)_{CS}$ chiralspin transformations that mix the right- and left-handed components of fermions. Consequently a symmetry of the electric interaction, that is driven by the charge density, is larger than a symmetry of the magnetic interaction and of the kinetic term. This allows to separate in some situations electric and magnetic contributions. In particular, in QCD the chromo-magnetic interaction contributes only to the near-zero modes of the Dirac operator, while confining chromo-electric interaction contributes to all modes. At high temperatures, above the chiral restoration crossover, QCD exhibits approximate $SU(2)_{CS}$ and $SU(2N_F)$ symmetries that are incompatible with free deconfined quarks. Consequently elementary objects in QCD in this regime are quarks with a definite chirality bound by the chromo-electric field, without the chromo-magnetic effects. In this regime QCD can be described as a stringy fluid.

\section{Chiralspin symmetry}

The Dirac Lagrangian with $N_F$ massless flavors

\begin{equation}
L = i\bar{\psi}\gamma^\mu\partial_\mu\psi = i\bar{\psi}_L\gamma^\mu\partial_\mu\psi_L + i\bar{\psi}_R\gamma^\mu\partial_\mu\psi_R,
\end{equation}

where

\begin{equation}
\psi_R = \frac{1}{2}(1 + \gamma_5)\psi, \quad \psi_L = \frac{1}{2}(1 - \gamma_5)\psi,
\end{equation}

is chirally symmetric

\begin{equation}
SU(N_F)_L \times SU(N_F)_R \times U(1)_A \times U(1)_V.
\end{equation}

The Fermion charge, which is Lorentz-invariant,

\begin{equation}
Q = \int d^3x\bar{\psi}(x)\gamma_0\psi(x) = \int d^3x\bar{\psi}^+(x)\psi(x)
\end{equation}

is invariant with respect to any unitary transformation that can be defined in the Dirac spinor space. So far known unitary transformations were those which leave the Dirac Lagrangian invariant:

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A new unitary transformation of Dirac spinors has recently been found \[1, 2\]. It is a $SU(2)$ chiral spin (CS) transformation. The $SU(2)_{CS}$ chiral spin transformation and its generators $\Sigma^n$, $n = 1, 2, 3$, are:

$$\psi \rightarrow \psi' = \exp \left( \frac{i \varepsilon^n \Sigma^n}{2} \right) \psi,$$

$$\Sigma^n = \{\gamma_k, -i\gamma_5 \gamma_k \gamma_5\},$$

where $\gamma_k$ is any Hermitian Euclidean gamma-matrix:

$$\gamma_i \gamma_j + \gamma_j \gamma_i = 2\delta_{ij}, \quad \gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4.$$  

The $su(2)$ algebra

$$[\Sigma^a, \Sigma^b] = 2i \epsilon^{abc} \Sigma^c$$

is satisfied with any $k = 1, 2, 3, 4$.

The $U(1)_A$ chiral symmetry is a subgroup of $SU(2)_{CS}$. The free massless Dirac Lagrangian is not invariant under the $SU(2)_{CS}$ chiral spin transformation. However, it is a symmetry of the fermion charge. The fermion charge has a larger symmetry than the Dirac equation.

The chiral spin transformations and generators can be presented in an equivalent form. With $k = 4$ they are

$$\Sigma^n = \{1 \otimes \sigma^1, 1 \otimes \sigma^2, 1 \otimes \sigma^3\}. $$

Here the Pauli matrices $\sigma^i$ act in the space of spinors

$$\left( \begin{array}{c} R \\ L \end{array} \right), $$

where $R$ and $L$ represent the upper and lower components of the right- and left-handed Dirac bispinors \[2\]. The $SU(2)_{CS}$, $k = 4$ transformation can then be rewritten as

$$\psi \rightarrow \psi' = \exp \left( \frac{i \varepsilon^n \Sigma^n}{2} \right) \psi = \exp \left( \frac{i \varepsilon^m T^m}{2} \right) \left( \begin{array}{c} R \\ L \end{array} \right). $$

A fundamental irreducible representation of $SU(2)_{CS}$ is two-dimensional and the $SU(2)_{CS}$ transformations mix the $R$ and $L$ components of fermions.

An extension of the direct $SU(2)_{CS} \times SU(N_F)$ product leads to a $SU(2N_F)$ group. This group contains the chiral symmetry $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ as a subgroup. Its transformations are given by

$$\psi \rightarrow \psi' = \exp \left( \frac{i \varepsilon^m T^m}{2} \right) \psi,$$

where $m = 1, 2, ..., (2N_F)^2 - 1$ and the set of $(2N_F)^2 - 1$ generators is

$$T^m = \{(\tau^a \otimes 1_D), (1_F \otimes \Sigma^n), (\tau^a \otimes \Sigma^n)\}.$$
with \( \tau \) being the flavor generators with the flavor index \( a \) and \( n = 1, 2, 3 \) is the \( SU(2)_{CS} \) index. \( SU(2N_F) \) is also a symmetry of the fermion charge, while not a symmetry of the Dirac equation.

The fundamental vector of \( SU(2N_F) \) at \( N_F = 2 \) is

\[
\Psi = \begin{pmatrix} u_R \\ u_L \\ d_R \\ d_L \end{pmatrix}.
\]

The \( SU(2N_F) \) transformations mix both flavor and chirality.

### 2 Symmetries of the QCD action

Interaction of quarks with the gluon field in Minkowski space-time can be splitted into temporal and spatial parts:

\[
\bar{\psi} \gamma^\mu D_\mu \psi = \bar{\psi} \gamma^0 D_0 \psi + \bar{\psi} \gamma^i D_i \psi,
\]

where \( D_\mu \) is a covariant derivative that includes interaction of the matter field \( \psi \) with the gauge field \( A_\mu \),

\[
D_\mu \psi = (\partial_\mu - ig \frac{t \cdot A_\mu}{2}) \psi.
\]

The temporal term includes an interaction of the color-octet charge density

\[
\bar{\psi}(x) \gamma^0 \frac{t}{2} \psi(x) = \psi(x)^i \frac{t}{2} \psi(x)
\]

with the chromo-electric part of the gluonic field. It is invariant under \( SU(2)_{CS} \) and \( SU(2N_F) \) \[2\]. The spatial part contains a quark kinetic term and interaction with the chromo-magnetic field. It breaks \( SU(2)_{CS} \) and \( SU(2N_F) \). We conclude that interaction of electric and magnetic components of the gauge field with fermions can be distinguished by symmetry. Such a distinction does not exist if the matter field is bosonic, because a symmetry of the Klein-Gordon Lagrangian and of charge of the \( J = 0 \) field is the same.

The quark chemical potential term \( \mu \bar{\psi}(x)^i \psi(x) \) in the Euclidean QCD action

\[
S = \int_0^\beta d\tau \int d^3 x \bar{\psi} [\gamma_\mu D_\mu + \mu \gamma_4 + m] \psi,
\]

is \( SU(2)_{CS} \) and \( SU(2N_F) \) invariant \[3\].

### 3 Observation of the chiralspin symmetry

New symmetries presented above were actually reconstructed \[1\] from lattice results on meson spectroscopy upon artificial subtraction of the near-zero modes of the Dirac operator from quark propagators \[4\]. An initial idea of these lattice experiments on low-mode truncation was to see whether hadrons survive or not an artificial restoration of chiral symmetry \[5\].

It is known that the quark condensate of the vacuum, an order parameter of chiral symmetry, is connected with the density of the near-zero modes of the Euclidean Dirac operator via the Banks-Casher relation \[6\].
Figure 1. $SU(2)_L \times SU(2)_R$ classification of the $J = 1$ meson operators. Operators that are connected by the $SU(2)_L \times SU(2)_R$ and $U(1) \Lambda$ transformations are linked by the red and blue arrows, respectively. The Fig. is from Ref. [2].

$$< \bar{q}q >= -\pi \rho(0).$$

The hermitian Euclidean Dirac operator, $i\gamma_\mu D_\mu$, has in a finite volume $V$ a discrete spectrum with real eigenvalues $\lambda_n$:

$$i\gamma_\mu D_\mu \psi_n(x) = \lambda_n \psi_n(x).$$

Consequently, removing by hands the lowest lying modes of the Dirac operator from the quark propagator,

$$S = S_{Full} - \sum_{i=1}^k \frac{1}{\lambda_i} |\lambda_i\rangle \langle \lambda_i|,$$

one artificially restores the chiral symmetry. This truncation of the near-zero modes makes the theory nonlocal, but it is not a big problem. Apriori it was not clear whether hadrons would survive this truncation. If they would, then one should expect a clean exponential decay of the Euclidean correlation functions. From the slope of these functions one can extract the hadron mass.

A complete set of $J = 1$ local meson operators for $N_F = 2$ QCD is given in Fig. 1. The red arrows connect operators that transform into each other upon chiral $SU(2)_L \times SU(2)_R$. If hadrons survive the artificial chiral restoration then mesons that are connected by the red arrows should be degenerate. If in addition the $U(1) \Lambda$ symmetry is restored after low mode truncation, then there should be a degeneracy of mesons connected by the blue arrows. Consequently from the symmetry of the QCD Lagrangian one can expect degeneracy of mesons connected by the red and blue arrows. Beyond these arrows there should be no degeneracy.

It was a big surprize when a larger degeneracy than the $SU(2)_L \times SU(2)_R \times U(1) \Lambda$ symmetry of the QCD Lagrangian was found in actual lattice measurements [4, 7], see Fig. 2. This degeneracy represents the $SU(2)_{CS}$ and the $SU(2N_F)$ symmetries because it contains irreducible representations of both groups, see Fig. 3. These lattice results imply, given the symmetry classification of the QCD Lagrangian [16], that while the confining chromo-electric interaction, that is $SU(2)_{CS}$ and $SU(4)$ symmetric, contributes to all modes of the Dirac operator, the chromo-magnetic interaction, which breaks both symmetries, is
located exclusively in the near-zero modes. Consequently a truncation of the near-zero modes leads to emergence of $SU(2)_{CS}$ and $SU(4)$ in hadron spectrum. Similar results persist for $J = 2$ mesons \cite{8} and baryons \cite{9}.

The highly degenerate level seen on Fig. 2 is a $SU(2N_F)$-symmetric level of the pure electric confining interaction. The hadron spectra in nature could be viewed as a splitting of the primary level of the dynamical QCD string by means of dynamics associated with the near zero modes of the Dirac operator, i.e. dynamics of chiral symmetry breaking that in addition includes all magnetic effects in QCD.

### 4 $SU(2N_F) \times SU(2N_F)$ symmetry of confining interaction

In Fig. 2 it is well visible that the $SU(4)$ 15-plet and the $SU(4)$ singlet ($f_1$) are also degenerate. This means that a confining interaction has a larger symmetry that includes the $SU(4)$ as a subgroup. What symmetry is it \cite{10}?

Consider the Minkowski QCD Hamiltonian in Coulomb gauge \cite{11}:
\[ H_{QCD} = H_E + H_B + \int d^3 x \Psi^\dagger(x)[-i\alpha \cdot \nabla] \Psi(x) + H_T + H_C, \]  
(23)

where the transverse (magnetic) and instantaneous "Coulombic" interactions are:

\[ H_T = -g \int d^3 x \Psi^\dagger(x) \alpha \cdot A^a(x) \Psi(x), \]  
(24)

\[ H_C = \frac{g^2}{2} \int d^3 x d^3 y J^{-1} \rho^a(x) F^{ab}(x, y) J \rho^b(y), \]  
(25)

with \( J \) being Faddeev-Popov determinant, \( \rho^a(x) \) and \( \rho^b(y) \) are color-charge densities at the space points \( x \) and \( y \) and \( F^{ab}(x, y) \) is a "Coulombic" kernel.

While the kinetic and transverse parts of the Hamiltonian are only chirally symmetric, the confining "Coulombic" part (25) carries the \( SU(2N_F) \) symmetry, because the charge density operator is \( SU(2N_F) \) symmetric. However, both \( \rho^a(x) \) and \( \rho^b(y) \) are independently \( SU(2N_F) \) symmetric because the \( SU(2N_F) \) transformations at two different spatial points \( x \) and \( y \) can be completely independent, with different rotations angles. This means that confining "Coulombic" interaction is actually \( SU(2N_F) \times SU(2N_F) \)-symmetric [12].

The \( SU(4) \times SU(4) \) has an irreducible representation of \( \text{dim}=16 \) that is a direct sum of the 15-plet and of the singlet, \( 16 = 15 \oplus 1 \). Then it becomes clear why the 15-plet mesons and the singlet from Fig. 3 are degenerate in Fig. 2.

5 Topology and the near-zero modes physics

The physics of the near-zero modes is not only responsible for chiral symmetry breaking in QCD but also for the breaking of higher symmetries \( SU(2N_F) \supset SU(2)_{CS} \). The \( SU(2)_{CS} \) transformations mix the right- and left-handed components of quarks. In other words, the physics of the near-zero modes should be associated not only with breaking of chiral and \( U(1)_A \) symmetry, but also with asymmetry between the left and the right.

Below we suggest a natural microscopic mechanism that induces an asymmetry between the left- and right-handed components of quarks in the near-zero modes. This mechanism is related to the local topological (instanton) fluctuations of the global gauge configuration.

In a gauge field with a nonzero topological charge the massless fermion has exact zero modes

\[ \gamma_\mu D_\mu \psi_0(x) = 0. \]  
(26)

The zero mode is chiral, \( L \) or \( R \). According to the Atiyah-Singer theorem the difference of the number of the left- and right-handed zero modes of the Dirac operator is related to the topological charge \( Q \) of the gauge configuration,

\[ Q = n_L - n_R. \]  
(27)

With \(|Q| \geq 1\) amount of the right- and left-handed zero modes is not equal which manifestly breaks the \( SU(2)_{CS} \) symmetry. The topological configurations contain the chromo-magnetic field. What would be exact zero modes in a topological configuration with a nonzero topological charge become the near-zero modes of the Dirac operator in a global gauge configuration (with arbitrary global topological charge, zero or not zero) that contain local topological fluctuations, like in the Shuryak-Diaakonov-Petrov theory of chiral symmetry breaking in the instanton liquid [13][14]. Consequently on top of contributions from confining physics, which are manifestly \( SU(2N_F) \supset SU(2)_{CS} \) symmetric, there appear contributions from the topological fluctuations that break \( SU(2N_F) \supset SU(2)_{CS} \).
Table 1. Bilinear operators and their $SU(2)_L \times SU(2)_R$, $U(1)_A$ transformation properties. This classification assumes propagation in $z$-direction. The index $k$ denotes the components 1, 2, 4, i.e. $x, y, t$.

| Name                  | Dirac structure | Abbreviation | Transformation |
|-----------------------|-----------------|--------------|---------------|
| Pseudoscalar          | $\gamma_5$     | PS           | $U(1)_A$      |
| Scalar                | $\mathbb{1}$   | S            |               |
| Axial-vector          | $\gamma_k\gamma_5$ | A          | $SU(2)_A$    |
| Vector                | $\gamma_k$     | V            |               |
| Tensor-vector         | $\gamma_k\gamma_3$ | T          | $U(1)_A$      |
| Axial-tensor-vector   | $\gamma_k\gamma_3\gamma_5$ | X          |               |

6 Observation of $SU(2)_{CS}$ and $SU(4)$ symmetries at high temperatures and their implication

So far we have discussed $SU(2)_{CS}$ and $SU(4)$ symmetries in hadrons that emerge upon artificial truncation of the near-zero Dirac modes in $T = 0$ calculations. The near-zero modes of the Dirac operator are naturally suppressed at high temperature above the chiral restoration crossover. Then, one can expect emergence of $SU(2)_{CS}$ and $SU(4)$ symmetries at high $T$ without any truncation [15].

Given this expectation $z$-direction correlators

$$C_{\Gamma}(n_z) = \sum_{n_x, n_y, n_t} <O_\Gamma(n_x, n_y, n_z, n_t)O_\Gamma(0, 0)^\dagger>$$

of all possible $J = 0$ and $J = 1$ local isovector operators $O_\Gamma(x) = \bar{q}(x)\Gamma \frac{T}{2} q(x)$ have been calculated on the lattice at temperatures up to 380 MeV in $N_F = 2$ QCD with the chirally symmetric domain wall fermions [16]. The operators and their $SU(2)_L \times SU(2)_R$ and $U(1)_A$ transformation properties are presented in Table 1.

Figure 4 shows the correlators for all operators from Table 1. The argument $n_z$ is proportional to the dimensionless product $zT$. We observe three distinct multiplets:

- $E_1 : PS \leftrightarrow S$ (29)
- $E_2 : V_x \leftrightarrow T_t \leftrightarrow X_t \leftrightarrow A_x$ (30)
- $E_3 : V_t \leftrightarrow T_x \leftrightarrow X_x \leftrightarrow A_t.$ (31)

$E_1$ is the pseudoscalar-scalar multiplet connected by the $U(1)_A$ symmetry. The $E_2$ and $E_3$ multiplets contain however some operators that are not connected by $SU(2)_L \times SU(2)_R$ or $U(1)_A$ transformations. The symmetries responsible for emergence of the $E_2$ and $E_3$ multiplets are $SU(2)_{CS}$ and $SU(4)$.

The $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries are exactly or almost exactly restored at temperatures above 220 MeV. At the same time the $SU(2)_{CS}$ and $SU(4)$ symmetries are only approximate. The correlators from the $E_1$ and $E_2$ multiplets at the highest available temperature 380 MeV are shown in Fig. 5. A remaining $SU(2)_{CS}$ and $SU(4)$ breaking is at the level of 5%. We also show there correlators calculated with the noninteracting quarks (abbreviated as "free"). In the latter case only $U(1)_A$ and $SU(2)_L \times SU(2)_R$ symmetries are seen in the correlators.

Scalar (S) and pseudoscalar (PS) systems are bound state systems because the slopes of the PS and S correlators are substantially smaller than for the free quark-antiquark pair. In the free quark case the minimal slope is determined by twice of the lowest Matsubara frequency (due to the antiperiodic boundary conditions for quarks in time direction). If the quark-antiquark system is bound and of
the bosonic nature, the bosonic periodic boundary conditions do allow the slope to be smaller. For the $J = 1$ correlators the difference of slopes of dressed and free correlators is smaller but is still visible. The observed approximate $SU(2)_{CS}$ and $SU(4)$ symmetries in $J = 1$ correlators rule out asymptotically free deconfined quarks because free quarks do not have these symmetries. Correlators with such symmetries cannot be obtained in the weak coupling regime, because perturbation theory relies on a free Dirac equation that is not $SU(2)_{CS}$- and $SU(4)$-symmetric.

On Fig. 6 we show a ratio of the correlators from the multiplet $E_2$ at different temperatures. We also show a ratio calculated with the free noninteracting quarks. For exact $SU(2)_{CS}$ symmetry the ratio should be 1. At a temperature just above the chiral crossover the ratio is essentially larger than 1. This can happen only if a contribution from the chromo-magnetic interaction is still large. Increasing the temperature, the role of the chromo-magnetic interaction is diminishing and the interaction between quarks is almost entirely chromo-electric. One concludes that elementary objects at $T \sim 2T_c$ are not free deconfined quarks, but rather quarks with a definite chirality bound by the chromo-electric field, something like a string. This conclusion remains also true in matter with finite chemical potential.

How should such a state of matter be called? It is not a plasma, because according to the standard definition plasma is a system of free charges with Debye screening of electric field. From our results it follows that there are no free deconfined quarks and in addition it is a chromo-magnetic, but not a chromo-electric field, that is screened. So conditionally one could call this matter a stringy fluid, see Fig. 7. However, the elementary objects are not usual hadrons. This can be clearly deduced from the correlators on Fig. 4. E.g. at zero temperature both $V_t$ and $V_x$ operators couple to one and the same $\rho$-meson. Above the cross-over we observe that properties of objects that are created by $V_t$ and $V_x$ operators are very different, because these correlators are very different. This means that usual $\rho$-mesons get splitted into two independent objects with not yet known properties. This could explain observed fluctuations of conserved charges and could perhaps be experimentally detected via dileptons.
Figure 5. $E_1$ and $E_2$ multiplets (29, 30) for interacting (dressed) and non-interacting (free) calculations at $T=380$ MeV. The Fig. is from Ref. [16].

Figure 6. Ratios of normalized correlators, that are related by $U(1)_A$ and $SU(2)_{CS}$ symmetries. The Fig. is from Ref. [16].

References

[1] L. Y. Glozman, Eur. Phys. J. A 51 27 (2015).
[2] L. Y. Glozman and M. Pak, Phys. Rev. D 92 016001 (2015).
[3] L. Y. Glozman, Eur. Phys. J. A 54, no. 7, 117 (2018).
[4] M. Denissenya, L. Y. Glozman and C. B. Lang, Phys. Rev. D 89 077502 (2014).
[5] C. B. Lang and M. Schrock, Phys. Rev. D 84, 087704 (2011).
[6] T. Banks and A. Casher, Nucl. Phys. B 169 103 (1980).
[7] M. Denissenya, L. Y. Glozman and C. B. Lang, Phys. Rev. D 91 034505 (2015).
[8] M. Denissenya, L. Y. Glozman and M. Pak, Phys. Rev. D 91 114512 (2015).
[9] M. Denissenya, L. Y. Glozman and M. Pak, Phys. Rev. D 92 074508 (2015).
Figure 7. A schematic phase diagram

[10] T. D. Cohen, Phys. Rev. D 93, no. 3, 034508 (2016).
[11] N. H. Christ and T. D. Lee, Phys. Rev. D 22, 939 (1980) [Phys. Scripta 23, 970 (1981)].
[12] L. Y. Glozman, [arXiv:1511.05857 [hep-ph]].
[13] E. V. Shuryak, Nucl. Phys. B 203, 93 (1982).
[14] D. Diakonov and V. Y. Petrov, Nucl. Phys. B 272, 457 (1986).
[15] L. Ya. Glozman, Proceedings of the "Critical point and onset of deconfinement" conference, Wroclaw, Poland, May 30 - June 4, 2016, Acta Physica Polonica B, Proceedings Supplement, vol. 10, N. 3, 583 (2017) [arXiv:1610.00275]
[16] C. Rohrhofer, Y. Aoki, G. Cossu, H. Fukaya, L. Y. Glozman, S. Hashimoto, C. B. Lang and S. Prelovsek, Phys. Rev. D 96, 094501 (2017).