Leading Higgs Mass Dependent Corrections to the Standard Model Lagrangian and its Two Higgs Doublet Extension

Vidyut Jain

Max-Planck-Institut für Physik, Föhringer Ring 6, D-8000 München 40, Germany

Abstract

We present an alternative calculation for the leading Higgs mass dependent one–loop corrections to the standard model Lagrangian, using a background field technique. Cross-sections computed from our one–loop Lagrangian provide a check of and reproduce results already obtained in the existing literature using diagrammatic methods, as well as allowing an analysis of other processes involving longitudinally polarized $W^\pm$s and $Z^0$s. We concentrate on the processes $WW \rightarrow f\bar{f}$ and $f\bar{f} \rightarrow WW$. We extend our result to the case of a two Higgs doublet model, when one of the physical Higgs is much heavier than the other particles and analyze the tree+one–loop unitarity bounds on the scale at which signs of a strongly interacting sector (or the heaviest Higgs) must turn up.

1Current Address: Theoretical Physics Group, 50A-3115, L.B.L., Berkeley, CA 94720.
1. Introduction.

It has become clear that the standard model of gauge interactions has yielded a remarkably good description of nature. In spite of this, we remain completely ignorant of the symmetry breaking sector: we do not know if it is described by a simple Higgs structure as in the minimal standard model; we do not know if this sector is described by elementary scalars, or composite ones such as in technicolor scenarios; we do not know if its interactions are strong or weak.

General considerations [1] lead one to expect that something interesting will turn up at the next round of colliders (with the possible exception of the “intermediate mass” Higgs). Unitarity of partial wave amplitudes implies that something will turn up by about 1 to 2 TeV, for example a Higgs particle or signs of a strongly interacting symmetry breaking sector. In this case, even if no physical resonance is detected at, e.g., the LHC or SSC, it will still be possible to learn something about the symmetry breaking sector by studying processes involving longitudinally polarized $W$s and $Z^0$s since these are the components that arise from symmetry breaking.

In this paper we present the one–loop corrections to the standard model Lagrangian that grow as $M_H^2$ or $\ln M_H$. We include all loops containing electroweak gauge bosons, pseudoscalars, and fermions. We used a background field technique which yields a manifestly gauge invariant tree+one–loop Lagrangian, and work within the context of a nonlinear $\sigma$–model. Our calculation constitutes the evaluation of many Feynman diagrams and provides a very important independent check of various partial results already found in the existing literature. With our Lagrangian it is also possible to extract easily other one–loop corrections. Additionally, we extend our results to include the two Higgs doublet model when one of the neutral physical Higgs is much heavier than the other particles.

The current paper is the first step in finding the leading Higgs mass and top mass dependent corrections to the standard model, and its two doublet extension. For the case of the standard model, leading fermion mass dependent corrections to the tree level Lagrangian have been computed by the author in [2]. However, the physical relevance of these corrections were not discussed there. Their significance to physics at colliders will be presented in a forthcoming paper. Together with the results of this paper, they will represent the leading corrections to many interesting physical processes.

There is already an extensive amount of literature on leading Higgs mass dependent one–loop corrections, using both diagrammatic [3,4] techniques and the background field method [5,6,7]. In ref. [6], the background field method was used to find the one loop corrections to the bosonic part of the standard model Lagrangian from loops containing bosons only, and in ref. [7] this was extended to include external fermions. Our calculation gives the complete result by including fermions as both external particles and internal particles. Our additional corrections to the Lagrangian in the case of an extra Higgs doublet are also new. We have checked our results against these previous background field calculations. Furthermore, for loops containing fermions, we have checked our results against the complex diagrammatic calculations of ref. [8] for the leading corrections to
the process $f \bar{f} \to W_L W_L$.

Apart from the tree+one–loop Lagrangian, the main new results of this paper are summarized.

- For a top quark mass closer to the CDF limit [9] than the perturbative upper bound [10] we are in agreement with ref. [8] and find a 15% enhancement over the standard model tree amplitude for the process $\bar{t}t \to W^+_L W^-_L$ for $M_H \approx 2$TeV and $\sqrt{s} \approx 1$TeV. This may be modified for higher values of the top quark mass. In addition, ref. [7] gives a partial analysis of heavy pair production from $W^+_L W^-_L \to f \bar{f}$ in the standard model. With our full Lagrangian we find no significant one–loop correction to such a process.

- In the two Higgs doublet model we find corrections to processes involving not only longitudinally polarized gauge bosons, but also the $CP$–odd Higgs and the light $CP$–even Higgs. It is known that tree-level unitarity limits on the scale of new physics in a two Higgs doublet model with a very heavy Higgs can be significantly stronger than in the minimal standard model [11]. We analyze similar unitarity limits for tree+one–loop amplitudes and find that the scale at which new physics must enter to unitarize the amplitudes to be in general lower than at tree level. In fact, for fixed $M_H$, we find this scale to fall extremely rapidly with $\tan \beta$, the ratio of the vevs.

Our analysis is based on the following observation [12]. In the standard model the purely scalar sector of the Lagrangian is

$$L = \frac{1}{4} v^2 Tr \left[ \partial_\mu \Phi^\dagger \partial^\mu \Phi \right] - \frac{\lambda}{4} v^4 (Tr[\Phi^\dagger \Phi]/2 - 1)^2, \tag{1}$$

where $v$ is the Higgs vev, $\lambda$ is the self–scalar coupling, $\Phi$ parameterizes the four scalars $\sigma$ (the physical Higgs), $\pi^1, \pi^2, \pi^3$:

$$\Phi = \sigma + 2i \tau \cdot \pi, \tag{2}$$

normalized so that the scalar fields are dimensionless. Here $\tau$ are the three pauli matrices normalized so that $\tau \times \tau = i \tau$, and boldface denotes 3–vectors. We can also arrange these fields in the usual complex Higgs doublet:

$$\phi = v \begin{pmatrix} i\pi^1 + \pi^2 \\ \sigma - i\pi^3 \end{pmatrix}. \tag{3}$$

The perturbative Higgs mass, $M_H = 350 \sqrt{\lambda}$GeV, increases with $\lambda$. For $\lambda$ bigger than $\approx 10$ perturbation theory breaks down and the quantity $M_H$ no longer corresponds to the mass of any physical resonance and it becomes difficult to perform reliable calculations. However, one expects the appearance of a spectrum of new resonances above some energy.

One approach to calculating loop corrections in such a scenario [12] is to formally take the limit $\lambda \to \infty$. The potential in (1) is proportional to $(\sigma^2 + \pi^2 - 1)^2$ so that
\( \lambda \to \infty \) requires \( \sigma \to \sqrt{1 - \pi^2} \) for the energy density to remain finite. The physical Higgs is removed from the theory in this limit, and the effective Lagrangian describing the \( \pi \) fields is given by the nonlinear \( \sigma \)-model

\[
L = \frac{1}{2} v^2 g_{ij} \partial_\mu \pi^i \partial^\mu \pi^j ,
\]

with the metric on the space of scalars is given by

\[
g_{ij} = \delta_{ij} + \frac{\pi^i \pi^j}{1 - \pi^2} .
\]

The inverse is \( g^{ij} = \delta^{ij} - \pi^i \pi^j \).

To describe the standard model with large \( \lambda \) one has also to add gauge fields and fermions. In any case, because (4) describes a nonrenormalizable model, so will the corresponding effective Lagrangian for the standard model. In spite of this, loop calculations performed with it can be used to extract meaningful results. A classic example of this is the Fermi theory of weak interactions which is neither unitary nor renormalizable, yet yields a good description of four fermion interactions below the \( W \)-boson mass. Being nonrenormalizable, loop corrections in the Fermi theory contain divergent terms not in the original Lagrangian. However, since the Fermi theory can be viewed as a low energy limit of the standard model (i.e. \( M_W \to \infty \)), we may expect that loop corrections computed with the Fermi theory should have some correspondence to loop corrections computed with the renormalizable theory. Indeed, it can be shown [12] that by interpreting the regulating scale in the Fermi case as of order the \( W \)-boson mass (i.e. the scale at which the \( W \)-boson enters to dampen the otherwise divergent integrals) one can reproduce surprisingly well many of the corresponding standard model calculations. In some instances, however, equating the regulating scale with the \( W \)-boson mass may very much overestimate certain processes which in the standard model are suppressed by other means, for example the GIM mechanism. Generally speaking then, one should take the regulating scale as the scale at which new physics not described by the Fermi theory becomes important. Indeed, in the case of strangeness–changing neutral current transitions, calculations of this kind were used to predict the charm quark mass [13] even before the underlying theory was known. [Of course, there are some renormalizable divergent corrections which are identical to those found in the full underlying theory, such as the corrections which give the running of the gauge couplings.]

We will therefore work within the context of a nonrenormalizable \( \sigma \)-model and interpret the regulating (cutoff) scale \( \mu \) for the divergent integrals as the scale at which new physics enters. We emphasize two possibilities.

- The symmetry breaking sector is strongly interacting. Then \( \mu \) is the scale of the lightest physical resonance associated with the scalar sector, generally believed to be in the 1TeV to 2TeV range [1].
The standard model with elementary scalars is not strongly interacting. It has been shown that in the gauged version of (4) that if one takes $\mu$ as the perturbative Higgs mass $M_H$ then one reproduces the leading Higgs mass dependent one–loop corrections (i.e. those that grow with $M_H^2$ or $\ln M_H$) as computed in the full standard model [5,6]. In fact, with the addition of fermions this remains true, and our results bear this out when we can compare with previous calculations.

The Lagrangian with one Higgs doublet.

The construction of the phenomenological Lagrangian describing the standard model in the limit of infinite self–scalar coupling can be found in [2]. The Lagrangian is

$$L_{SM} = -\frac{1}{4g^2} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4g'^2} G_{\mu\nu} \cdot G^{\mu\nu} + g \sqrt{2} \mu^2 \Phi + \frac{g}{2} \bar{\psi}_L \gamma^\mu \psi_L + i \bar{\psi}_R \gamma^\mu \psi_R$$

$$\bar{\psi}_L \gamma^\alpha \Phi \gamma^\beta \psi_L - \frac{1}{2} \bar{\psi}_R \gamma^\beta \Phi \gamma^\alpha \psi_R + H.c.$$

(6)

The $SU(2)_L$ field strength $F_{\mu\nu}$ and $U(1)_Y$ field strength $G_{\mu\nu}$ are given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{g}{2} \epsilon^{\mu\nu\lambda} A_\lambda$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

(7)

where $A_\mu$ are the three $SU(2)_L$ gauge fields and $B_\mu$ is the $U(1)_Y$ gauge field. The gauge covariant derivative on the scalars is

$$D_\mu \pi^i = \partial_\mu \pi^i + \frac{g}{2} \epsilon^{ijk} A_\mu \pi^j - \frac{g}{2} A_\mu \sqrt{1 - \pi^2} + \frac{g}{2} \epsilon^{i3} B_\mu \pi^3 + \frac{g}{2} \epsilon^{i2} C_\mu \pi^2.$$}

(8)

The standard model has left–handed fermion doublets transforming like $\phi$ under $SU(2)_L$, and with different $U(1)_Y$ charges. The covariant derivative on left–handed quarks and leptons is

$$D_\mu q_a^L = (\partial_\mu - iA_\mu \cdot \tau - \frac{i}{2} C_\mu \lambda_i) q_a^L,$$

$$D_\mu l_a^L = (\partial_\mu - iA_\mu \cdot \tau + \frac{i}{2} B_\mu) l_a^L.$$  

(9)

Here, and in (6), the superscripts on the fermions label generation ($a = 1 \ldots 3$). For the right–handed fermions,

$$D_\mu u_a^R = (\partial_\mu - \frac{2i}{3} B_\mu - \frac{i}{2} C_\mu \lambda_i) u_a^R,$$

$$D_\mu d_a^R = (\partial_\mu + \frac{i}{3} B_\mu - \frac{i}{2} C_\mu \lambda_i) d_a^R,$$

$$D_\mu e_a^R = (\partial_\mu + i B_\mu) e_a^R.$$  

(10)

In (6), we have not written the fermion kinetic terms explicitly; $\psi$ stands for all the quark and lepton fields. The second line of (6) gives the Yukawa couplings (normally
these are given in terms of the Higgs fields $\phi$ and $2i\tau_2\phi^*$. $\lambda_{\alpha}^{ab}$, $\lambda_{\beta}^{ab}$, and $\lambda_{\gamma}^{ab}$ are numerical matrices proportional to the usual Yukawa couplings, and for convenience we have defined $l_R^0 = (0, e_R^a)$ and $q_R^a = (u_R^a, d_R^a)$.

Notice that we have normalized the gauge fields so that the gauge coupling constants $g$, $g'$ (for $SU(2)$, $U(1)$) appear only as overall factors in (6). Additionally, we have not kept the kinetic term for the gluons as we will not calculate loop corrections due to them.

The Lagrangian with two Higgs doublets.

There is a simple extension of the standard model which is of current phenomenological interest. We add one more Higgs doublet $\chi$ to the model. Under general considerations [14] the potential is taken to be

$$V(\phi, \chi) = \lambda_1(\phi^\dagger \phi - v^2)^2 + \lambda_2(\chi^\dagger \chi - v'^2)^2 + \lambda_3((\phi^\dagger \phi - v^2)^2 + \lambda_4((\phi^\dagger \phi)(\chi^\dagger \chi) - (\phi^\dagger \chi)(\chi^\dagger \phi)) + \lambda_5[\text{Re}(\phi^\dagger \chi) - vv' \cos \xi]^2 + \lambda_6[\text{Im}(\phi^\dagger \chi) - vv' \sin \xi]^2,$$

(11)

where $v$ and $v'$ are the two vevs that break $SU(2) \times U(1)_Y$ down to $U(1)_{EM}$. The $\chi$ doublet is parameterized in terms of four real fields:

$$\chi = v' \left( \frac{i\zeta_1 + \zeta_2}{\zeta_0 - i\zeta_3} \right).$$

(12)

An important parameter that we will use later is $\tan \beta = v' / v$. The properly normalized scalars that are “eaten” by the gauge fields are no longer purely $\pi$ as in the one doublet case, but

$$w^i = v \pi^i \cos \beta + v' \zeta^i \sin \beta,$$

(13)

while the properly normalized $CP$–odd physical Higgs are

$$H^\pm = -v \pi^\pm \sin \beta + v' \zeta^\pm \cos \beta,$$

$$A^0 = -\sqrt{2}v \sin \beta \pi^0 + \sqrt{2}v' \zeta^3 \cos \beta,$$

(14)

where $\pi^\pm = \frac{1}{\sqrt{2}}(\pi^i \mp i\pi^i)$, etc.

The limit of large $\lambda_1$ corresponds to taking the (perturbative) mass of one of the $CP$–even Higgs scalars as large. In fact, in this limit the $CP$–even physical mass eigenstates are [14]

$$H^0 = \sqrt{2}v(\sigma - 1), \quad M_{H^0}^2 = 2v^2\lambda_1,$$

$$h^0 = \sqrt{2}v'(\zeta^0 - 1), \quad M_{h^0}^2 = 2v'^2(\lambda_2 + \lambda_3) + \frac{1}{4}v^2\lambda_5.$$

(15)

As with the one Higgs doublet model, when we take $\lambda_1$ to infinity, finiteness of the potential energy imposes the constraint $\phi^\dagger \phi = v^2$, or $\sigma = \sqrt{1 - \pi^2}$, and $H^0$ is removed from the theory. This is the extension of the standard model that we include here. Note, that at
tree level the masses of the physical charged Higgs states and the $CP$-odd physical Higgs state depend on $\lambda_4$ and $\lambda_6$, respectively, and are not affected by this limit.

The roles of $\phi$ and $\chi$ in the potential (11) are symmetric, even if the equation is not. Taking $\lambda_2$ to infinity instead of $\lambda_1$ reverses the role of $\phi$ and $\chi$. We can recover this case by swapping some of the parameters in (11). We do not consider the limit $\lambda_1 \to \infty$, $\lambda_2 \to \infty$ which corresponds to taking the perturbative masses of both $CP$–even Higgs to infinity. Nor do we consider the limit $\lambda_3 \to \infty$, which corresponds to taking the perturbative mass of one of the the $CP$–even Higgs eigenstates to infinity and the other mass to zero. The constraint equation in this last case does not lead to a scalar kinetic energy of the form (4).

In the limit $\lambda_1 \to \infty$, the scalar potential becomes

$$V = \frac{1}{4} \lambda'(\chi^\dagger \chi - v'^2)^2 + \lambda_4 [(\phi^\dagger \phi)(\chi^\dagger \chi) - (\phi^\dagger \chi)(\chi^\dagger \phi)]$$

$$+ \lambda_5 \text{Re}(\phi^\dagger \chi) - vv' \cos \xi^2 + \lambda_6 \text{Im}(\phi^\dagger \chi) - vv' \sin \xi^2,$$

where $\lambda' \equiv 4\lambda_2 + 4\lambda_3$. The complete Lagrangian is the sum of three terms

$$L_{SM} = L_{SM} - V + L', \quad \text{(17)}$$

where $L'$ contains the gauge covariant kinetic term for the additional Higgs doublet and its coupling to the fermions:

$$L' = (d^\mu \chi)^i (d^\mu \bar{\chi})_i$$

$$- v'(\lambda_u^i q_u^a \Xi[1 - \tau_3] q_R^b + \lambda_d^i q_d^a \Xi[1 - \tau_3] q_R^b + \lambda_e^i q_e^a \Xi[1 - \tau_3] q_R^b + \lambda_{u}^i \bar{q}_L^a \Xi[1 - \tau_3] q_R^b + H.c.). \quad \text{(18)}$$

Here $i = 1, 2$. The covariant derivative on $\chi$ is

$$d^\mu \chi = (\partial^\mu - i \tau \cdot A^\mu - \frac{i}{2} B^\mu) \chi. \quad \text{(19)}$$

We have also defined

$$\Xi = \zeta^0 + 2i \tau \cdot \zeta. \quad \text{(20)}$$

The last line in (18) describes the Yukawa couplings of the $\chi$ fields to the fermions. To suppress tree level flavour changing neutral currents induced by scalar exchange one normally concentrates on specific models, for example case (Ia) the choice that all the $\lambda_u, \lambda_d, \lambda_e$ are zero while none of the $\lambda'_u, \lambda'_d, \lambda'_e$ are zero, case (Ib) the opposite situation, case (IIa) when only $\lambda_d, \lambda_e, \lambda'_u$ are nonzero, and case (IIb) when only $\lambda'_d, \lambda'_e, \lambda'_u$ are zero. Models IIa and IIb correspond to the Higgs coupling structure in the minimal supersymmetric standard model, but with the roles of $\phi$ and $\chi$ interchanged. Our calculations will be general, but these cases are phenomenologically interesting.

In Section 2 we give the tree+one–loop effective Lagrangian for both the one Higgs doublet case and the two Higgs doublet case. In Section 3 we discuss physical applications, and a reader who is only interested in physical results can skip Section 2.
2. One–Loop Corrections.

In this section we present the leading Higgs mass dependent one–loop corrections to the one Higgs doublet Lagrangian, eq. (6), and the two Higgs doublet Lagrangian, eq. (17). We found the one–loop corrections by using the background field expansion method together with some functional methods to evaluate one–loop determinants. In this approach, one splits the fields into background and quantum parts \( \pi = \tilde{\pi} + \hat{\pi} \), \( A_\mu = \tilde{A}_\mu + \hat{A}_\mu \), \( B_\mu = \tilde{B}_\mu + \hat{B}_\mu \), \( \psi = \tilde{\psi} + \hat{\psi} \), and \( \chi = \tilde{\chi} + \hat{\chi} \), and expand to second order in the quantum (hatted) fields. (One can also keep gluon backgrounds \( \tilde{C}_\mu^i \), but we do not). The terms linear in the quantum fields, along with appropriate source terms, are set to zero by the classical equations of motion satisfied by the background fields. One then functionally integrates over the quantum quantities in the path integral to find the one–loop effective action as a function of the background fields. One can also keep gluon backgrounds \( \tilde{C}_\mu^i \), but we do not). The terms linear in the quantum fields, along with appropriate source terms, are set to zero by the classical equations of motion satisfied by the background fields. One then functionally integrates over the quantum quantities in the path integral to find the one–loop effective action as a function of the background fields. If carefully done it allows one to keep manifest all the tree level symmetries of the theory in terms of the background fields.

However, using the fields \( \hat{\pi} \) as our quantum fields is not the best way to proceed. The Lagrangian (6) or (17) is manifestly invariant under reparameterizations of the scalar fields, i.e. under \( \pi^i \rightarrow \Pi^i(\pi) \), since \( \partial_\mu \pi^i \) transforms as a vector with the scalar metric \( g_{ij} \) transforming as a tensor. Due to the geometrical nature of the covariant derivative (8) the gauged nonlinear \( \sigma \)–model also has this property. The coordinate \( \hat{\pi}^i \) does not transform as a vector under reparameterizations so that an expansion of the action – a scalar – as a power series in the \( \hat{\pi} \) is not manifestly invariant under reparameterizations. Functionally integrating over the \( \hat{\pi} \) then leads to an effective action which is not manifestly invariant under (background field) reparameterizations. Although we cannot expect this to affect physical results, the extraction of such results is easiest done in a framework in which the reparameterization invariance is kept manifest.

Such a framework is well known [15]. Instead of \( \hat{\pi} \), a reparametrization vector \( \xi \) is chosen as the quantum field to be integrated over in the path integral. Mukhi [15] has given a very simple algorithm to generate the expansion of the action \( S[\pi] \) in terms of the three \( \xi^i \):

\[
S[\pi, \xi] = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ \int d^4x \xi(x) \cdot D^n \right] S[\pi],
\]

where \( D^n_x \) is the covariant functional derivative (with respect to \( \pi^i \)). For example, on quantities which are scalars under reparameterizations (such as the action) it is just the ordinary functional derivative, while on reparameterization vectors it is constructed with the use of the “scalar connection” which depends on the scalar metric \( g_{ij} \). One can give explicit formulas but we will not need them here.

With the background expansion complete we can use the following result [5,16,2]. For quantum bosonic fields \( \theta \) (a column vector), and four component spin-\( \frac{1}{2} \) fermions \( \hat{\psi}_A \), the Lagrangian quadratic in the quantum fields may be generally written as

\[
L = -\frac{1}{2} \theta^T Z [d_\mu d^\mu + M_\theta^2] \theta + \hat{\psi} \Delta^{-1} \hat{\psi} + \left[ (\theta^T \Sigma)_A \hat{\psi}_A + H.c. \right] + L_{G.F.}.
\]
Here \( Z \) is a matrix which gives the bosonic metric, an extension of (5); \( d_\mu \) is a covariant derivative (gauge, reparameterization, etc.); \( \Sigma \) is a matrix with mixed bosonic and fermionic indices which describes the mixing between bosons and fermions; \( M_\theta^2 \) is the mass squared matrix for the bosons; \( L_{G.F.} \) includes the terms that arise from gauge fixing (auxiliary fields and ghosts) all the local invariances; \( \Delta^{-1}_\psi = i \partial_R R + i \partial_L L - M_\psi \), where \( D_R^\mu \) is the covariant derivative on right–handed fermions (\( D_L^\mu \) is the covariant derivative on left–handed fermions), \( R \) (\( L \)) is the projection operator for right–handed (left–handed) fermions, and \( M_\psi \) is the mass matrix for the fermions which can be decomposed into right– and left–handed parts, \( M_\psi = m^R R + m^L L \).

The divergent (regulated) one–loop corrections from just the bosonic fields is [5]

\[
\delta L_\theta = -\frac{1}{2^6 \pi^2} Tr \left[ (M_\theta^4 + \frac{1}{6} J^{\mu \nu}) \ln[2 \mu_0^2/\mu^2] + 4 M_\theta^2 \mu^2 \ln 2 \right].
\]

(23)

[We have dropped total divergences, as we do throughout.] Here \( \mu \) is the regulating scale, \( \mu_0 \) is a scale characterizing the low energy theory and \( J^{\mu \nu} = [d_\mu, d_\nu] \). When \( m \) does not contain any \( \gamma \)–matrices, the divergent one–loop corrections from just the fermions is [5,16]

\[
\delta L_\psi = \frac{1}{2^4 \pi^2} Tr \left[ \left\{ (m^\dagger m)^2 - D_\mu m D^\mu m^\dagger \right\} \ln[2 \mu_0^2/\mu^2] + 4 m^\dagger m \mu^2 \ln 2 \right].
\]

(24)

The trace is over internal indices, the covariant derivative on the masses is

\[
(D_\mu m^\dagger) = D^R_\mu m^\dagger - m^\dagger D^R_\mu,
\]

\[
(D_\mu m) = D^L_\mu m - m D^L_\mu,
\]

(25)

and \( F^{R,L}_{\mu \nu} \) are the Yang–Mills field strengths \( F^{R}_{\mu \nu} = [D^R_\mu, D^R_\nu] \) and \( F^{L}_{\mu \nu} = [D^L_\mu, D^L_\nu] \). In the last equation, the derivatives act on everything to the right. For the \( \Sigma \) contributions and the gauge–fixing Lagrangian we have the corrections [2]

\[
\delta L' = \frac{1}{2^6 \pi^2} \ln[2 \mu_0^2/\mu^2] Tr \left[ 4 Z^{-1} \Sigma M_\psi^\dagger \Sigma + 2 Z^{-1} \Sigma (i \gamma^\mu d_\mu \Sigma) \right]
\]

\[
+ \frac{1}{2^5 \pi^2} Tr \left[ 2 \mu^2 \bar{M}^2 \ln 2 + \left\{ \frac{1}{2} \bar{M}^4 - \frac{1}{4} S^2 \right\} \ln[2 \mu_0^2/\mu^2] \right]
\]

\[
+ \frac{1}{2^5 \pi^2} \ln[2 \mu_0^2/\mu^2] Tr g^2 \left[ \Sigma M_\psi^\dagger \Sigma - \Sigma (i \gamma^\mu d_\mu \Sigma) \right.
\]

\[
\left. + \frac{1}{3} (\Sigma i \gamma^\nu d^\mu \Sigma)^{\alpha \beta} \eta_{\alpha \beta} + \eta_{\alpha \nu} \eta_{\beta \nu} + \eta_{\alpha \mu} \eta_{\beta \nu} \right]
\]

\[
+ \frac{2}{2^6 \pi^2} \ln[2 \mu_0^2/\mu^2] Tr \left[ \frac{1}{6} J^{A A}_{\mu \nu} J^{\mu \nu} \right].
\]

(26)

In the first line, the trace is over all bosonic indices, whilst for the remainder the trace is only over gauge indices. \( \bar{M}^2 \) is the (background) mass squared term for the gauge fields, and \( S \) describes the mixing between the gauge fields and the other scalar fields.
precisely if $\theta^T = (\hat{A}_i^\mu, \hat{\phi}^j)$ then the Lagrangian contains the terms $L \ni \frac{1}{2} \hat{A}_i^\mu \eta_{\mu\nu} (\hat{M}^2)^{ij} \hat{A}_j^\nu + \hat{\phi}^j (S_\mu)^j_i \hat{A}_i^\mu$. Furthermore $d_\mu \Sigma$ is the covariant derivative acting on $\Sigma$, i.e. that it transforms in the same way as $\Sigma$ under some (background) transformations. Finally, $g^2$ in (26) is a diagonal matrix in gauge space with entries $(g^2, g^2, g^2)$.

With these rather general results it is straightforward to complete the background field expansion and computing the explicit corrections given by equations (23), (24), and (26). For the Lagrangian (6) the details of the calculation are given in [2]. In what follows we dropped the tildes denoting background fields.

**One Higgs Doublet Model.**

The complete leading tree+one–loop Lagrangian from (6) is:

$$L_{SM}(\mu^2) = -\frac{1}{4g^2} Z_g F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{4g'^2} Z_{g'} G_{\mu\nu} G^{\mu\nu} + \frac{1}{2} \psi^2 Z_v g_{ij} D_\mu \pi^i D_\nu \pi^j$$

$$+ i \psi \Sigma \Sigma' + i \psi R \Sigma' \Sigma R - v [\psi_L \Phi \Sigma' \Lambda \psi_R + H.c.]$$

$$+ \frac{1}{3} \frac{\mu^2}{\mu_0^2} g_{ij} D_\mu \pi^i D_\mu \pi^j + \frac{3}{4} \psi \Sigma' \Sigma R + [\psi_L \Phi \Sigma' \Lambda \psi_R + H.c.]$$

$$- \frac{1}{6} F^{\mu\nu} G^{\mu\nu} + \frac{1}{6} [K_{\mu\nu}^{ij} \pi^k + 2 R_{\mu\nu}^{ij} + 2 K_{\mu\nu}^{ij} \pi^k - \frac{1}{2} (\pi \cdot D_\mu \pi) [F^{\mu\nu} \cdot (\pi \times D_\mu \pi) + G^{\mu\nu} (\pi \times D_\mu \pi)^3] \right\}. \) (27)

Here

$$Z_g = 1 - \frac{13 g^2}{64 \pi^2} \ln[\mu^2/\mu_0^2],$$

$$Z_{g'} = 1 + \frac{81 g'^2}{192 \pi^2} \ln[\mu^2/\mu_0^2],$$

$$Z_v = 1 - \frac{\eta^2}{v^2} + \frac{2 \Lambda^2}{8 \pi^2} \ln[\mu^2/\mu_0^2],$$

$$Z_L = 1 - \frac{1}{16 \pi^2} \ln[\mu_0^2/\mu^2] [tr \Lambda \Lambda^T - \frac{1}{2} \Lambda \Lambda^T],$$

$$Z_R = 1 - \frac{3 \Lambda^2}{16 \pi^2} \ln[\mu_0^2/\mu^2],$$

$$Z_A = 1 - \frac{3 \eta^2}{32 \pi^2} + \frac{1}{8 \pi^2} [tr \Lambda \Lambda^T - \frac{1}{2} \Lambda \Lambda^T] + \frac{3 \ln[\mu_0^2/\mu^2]}{64 \pi^2} g^2 Y_L \Lambda Y_R \Lambda^{-1}, \) (28)
are renormalization factors. For compactness of notation, the terms involving the fermions \( \psi \) are implicitly summed over leptons and quarks, and \( \Lambda \) embodies all the Yukawa couplings in a large matrix:

\[
\Lambda = \begin{pmatrix}
\lambda^a_{u} \left[ \frac{1}{2} + \tau_3 \right] + \lambda^a_{d} \left[ \frac{1}{2} - \tau_3 \right] & 0 \\
0 & \lambda^b_{e} \left[ \frac{1}{2} - \tau_3 \right]
\end{pmatrix}, \quad \psi = \begin{pmatrix}
q^a_m \\
\nu_m
\end{pmatrix},
\]

(29)

where \( q^1, q^2, q^3 \) are the three generation of colour triplet quarks and \( l^1, l^2, l^3 \) are the three generations of colour singlet leptons. The lower-case “tr” is a trace only over the pauli matrices in (29), and \( Tr \Lambda \Lambda = 3 \lambda_u^{iab} \lambda_u^{abc} + 3 \lambda_d^{iab} \lambda_d^{abc} + \lambda_e^{iab} \lambda_e^{abc} \), with a sum over \( ab \). The factors of 3 are due to colour. \( Y_L \) and \( Y_R \) are also large matrices here. They are diagonal and their entries correspond to the hypercharge assignments of the different fermions. To be exact, from the covariant derivatives (9) and (10), we have \( Y_L(l^p_k) = -1, Y_L(q^p_k) = 1/3, Y_R(l^p_k) = -2, \) and \( Y_R(q^p_k) = \text{diag}(4/3, -2/3) \). With this understanding, \( \Phi \) in (27) is also a large matrix in generation space. It is block diagonal, with each entry corresponding to eq. (2).

The other undefined quantities in the above equation are

\[
S^\mu_{ji} = g_{ij} D^\mu \pi^j \times \begin{cases}
\epsilon^i_{3d} + \delta^i_{3j} \sigma_i + \pi^i g_{lm}(e^m_{3q} \pi^q + \delta^m_3 \sigma), & I = 0, \\
\epsilon^i_{pd} - \delta^i_p \sigma_i + \pi^i g_{lm}(e^m_{pq} \pi^q - \delta^m_p \sigma), & I = p,
\end{cases}
\]

\[
R^i_{\mu \nu j} = g_{ij} D^\mu \pi^i D^\nu \mathbf{\pi}^j - (\mu \leftrightarrow \nu),
\]

\[
I^i_{\mu \nu j} = \frac{1}{2} \epsilon_{mji} F^m_{\mu \nu} + \frac{1}{2} \epsilon_{3ij} G_{\mu \nu},
\]

\[
K^i_{\mu \nu jk} = \pi^i g_{ij} I^l_{\mu \nu k} + O(\pi^2).
\]

(30)

Also, \( b^j_{\mu} = \frac{1}{2} A^j_{\mu} \) for \( j = 1, 2 \), and \( b^3_{\mu} = \frac{1}{2} A^3_{\mu} - \frac{1}{2} B^3_{\mu} \), and the sum over \( p \) in the \( S^2 \) term of eq. (27) is over \( SU(2)_L \) gauge indices. The complete form of the matrix \( K_{\mu \nu} \) can be extracted from [2], but the higher order terms are suppressed by inverse powers of \( v \) and are not phenomenologically interesting.

We have introduced \( \rho \) and \( \eta \) to parameterize the dependence of the exact answer on the scheme chosen to regularize the divergent integrals. For the double subtraction scheme that was used [5,17], \( \eta = 2 \ln 2 \) and \( \rho = \frac{1}{2} \). In our methodology, these may also be taken to parametrize the uncertainty about the detailed way in which the underlying theory enters to dampen the otherwise divergent integrals. In this case we should properly use different parameters for the different terms in our one–loop corrections. Of course, some of the \( \ln \mu^2 \) terms should not be taken as finite in the sense we have advocated – these are the renormalizable terms that appear regardless of whether we work with a linear or nonlinear sigma model. The corrections to the gauge kinetic terms, for example, determine the \( \beta \)–functions for the running of the gauge coupling constants. We agree with previous results [6,18]. Similarly, the \( \ln \mu^2 \) corrections to the Yukawa couplings determine their \( \beta \)–functions. For the nonrenormalizable terms we identify the cutoff \( \mu \) with the physical mass of the Higgs (or the heaviest \( CP \)-even Higgs in the two Higgs doublet model). The characteristic scale \( \mu_0 \) is more problematic. Since these corrections arise from diagrams containing the pseudo–goldstone bosons the most natural choice is the
W–boson mass. However, one has to be more careful. To obtain the correct identification means evaluating terms subleading to those found here since we were only interested in the ultraviolet divergent terms. This is the reason for the artificial infrared divergence for \( \mu_0 \ll M_H \) which should only happen for special values of the external momentum. By examining the infrared divergences more carefully and resuming the derivative expansion, Cheyette and Gaillard \[6\] have shown that in this limit the leading kinematic factor for scattering processes involving four external scalars is given by the replacement \( \mu_0^2 \rightarrow \partial^2 \).

Our result (27) agrees with those of Refs. \[6\] and \[7\] when it was possible to compare, except for the \((D_\mu \pi^i)^2 [\bar{\psi} \Phi \Lambda \psi + H.c.]\) term on the sixth line for which we find the opposite sign to the corresponding term in eq (4.23) of \[7\]. In ref. \[19\] a similar computation was performed for the terms involving both fermions and bosons that come from mixed fermionic and bosonic loops (our \( \Sigma \) terms). The corrections are given in eq. (3.10a,b,e,f,h-j) of that paper. We find complete agreement except for the only factor these authors did not explicitly compute, eq. (3.10), for which we find an answer three times as big. However this term, which corrects the kinetic term for the right–handed fermions (i.e. \( Z_R \)) does not seem to contain much interesting physics. One further term bears comment: the \( S^2 \) term on the third line of (27). This term gives the scalar loop contribution to the \( \rho \) parameter, already found in a slightly different form for the nonlinear \( \sigma \)–model by Cheyette \[6\], and it can be shown to agree with the standard result \[20\].

The first three lines of (27) contain terms appearing in the tree Lagrangian, eq. (6), and may be renormalized by the addition of suitable counterterms. For example, Costa and Liebrand \[7\] consider some of the resulting wave–function renormalizations, and so on. All the quadratically divergent corrections are renormalizable so that physically measurable effects are insensitive to the exact value of this cutoff, in agreement with M. Veltman’s screening theorem \[21\].

We would like to stress that our background field calculation gives corrections manifestly invariant under \( SU(2)_L \times U(1)_Y \) gauge transformations, as well as reparameterizations of the scalar fields. We could fix to the unitary gauge and remove all dependence on the pseudo–scalars, and in this form the corrections to physical scattering processes may be more transparent. For example, the first term on the fourth line of (27) is proportional to the square of the tree level scalar kinetic term, so that in the unitary gauge this term contains four gauge bosons and describes corrections to such processes as \( W_L W_L \rightarrow W_L W_L \). However, for the energy range we shall be interested in (next section) it is more convenient to think of the \( \pi \) as the longitudinal components of the massive gauge fields and use the equivalence theorem \[22\] to extract the relevant cross–sections.

In section three we write out some of the terms in (27) for the top/bottom fermion doublet. This gives an indication of how to extract corrections for other processes.

**Two Higgs Doublet Model.**

For the two Higgs doublet model given by eq. (17), it is straightforward to extend the
previous results. We find in addition to the results of (27) the following corrections:

\[
\delta L = -\frac{\mu^2}{16\pi^2} \ln \frac{\mu^2}{\rho^2} [M_2 + M_4]
\]

\[
-\ln \frac{\mu^2}{\rho^2} \{ V_4 - \frac{1}{12} F_{\mu\nu} \cdot F^{\mu\nu} - \frac{1}{12} G_{\mu\nu} G^{\mu\nu} - \frac{1}{6} F_{\mu\nu}^3 G^{\mu\nu} - \frac{3}{2} S \}
\]

\[
-\ln \frac{\mu^2}{\rho^2} \{ V_8 + 2u [8\tilde{V} + 3V' - 8\lambda'_4(\chi^\dagger\phi)(\phi^\dagger\chi) + 4v^2\lambda'_4(\chi^\dagger\chi)]
\]

\[
+ 2g_{ij}D_{\mu}^jD_{\nu}^i \frac{1}{2} \left[ -2V' - 6\tilde{V} + 6\lambda'_4(\chi^\dagger\phi)(\phi^\dagger\chi) - 4v^2\lambda'_4(\chi^\dagger\chi) \right]
\]

\[
+ 2D_{\mu}^jD_{\nu}^i \frac{1}{2} \left[ \lambda'_4 [(\chi^\dagger\phi)](\phi^\dagger\chi) + H.c.] - \frac{1}{2}(\lambda_5 - \lambda_6) [(\phi^\dagger\chi)](\phi^\dagger\chi) + H.c.] \right). \]

(31)

Here, \(M_2, M_4, V_4, V_8\) are corrections to the potential terms. \(M_2\) comes from loops involving the \(\chi\) and gauge fields only; \(M_4\) and \(V_8\) from loops containing the \(\pi\) fields only. We have:

\[
M_2 = + (3\lambda' + 6g^2 + 2g^2 - 4\lambda'_4)\chi^\dagger\chi + (2\lambda_4 + \lambda_5 + \lambda_6)\phi^\dagger\phi
\]

\[
M_4 = + 8\lambda'_4 v^2(\chi^\dagger\phi)(\phi^\dagger\chi) + 6v'/v \left[ \lambda_5 \cos \xi \text{Re}(\phi^\dagger\chi) + \lambda_6 \sin \xi \text{Im}(\phi^\dagger\chi) \right]
\]

\[- 2(\lambda_5 - \lambda_6)v^2 \left[ (\phi^\dagger\chi)^2 + H.c.] \right).
\]

\[V_4 = \left[ \frac{5}{2} \lambda'^2 + 3g^4 + 2g^2g'^2 + g'^4 \right] (\chi^\dagger\chi)^2
\]

\[+ \left[ -3\lambda'^2v^2 + (6g^4 + 2g'^4 - 4g^2g'^2)v^2 + 4\lambda_4^2v^2 + 2\lambda'(5\lambda_4 + \lambda_5)v^2 \right] (\chi^\dagger\chi)
\]

\[+ \left[ -2\lambda'_4 + 8g^2g'^2 \right] (\chi^\dagger\phi)(\phi^\dagger\chi)
\]

\[v^4V_8 = 20\lambda'^2(\chi^\dagger\phi)^2(\phi^\dagger\chi)^2 - 16\lambda'_4 v^2(\chi^\dagger\phi)(\phi^\dagger\chi)(\chi^\dagger\chi)
\]

\[- \lambda'_4(\chi^\dagger\phi)(\phi^\dagger\chi) \left[ 16V' + 40\tilde{V} \right] + v^2\lambda'_4(\chi^\dagger\chi) \left[ 8V' + 16\tilde{V} \right]
\]

\[+ 3V'^2 + 20\tilde{V}^2
\]

\[- \lambda'_4(\lambda_5 - \lambda_6) \left[ (\chi^\dagger\phi)(\phi^\dagger\chi) + H.c.] \left[ (\phi^\dagger\chi)(\phi^\dagger\chi) + H.c.] \right) g^{ij}g^{ij}
\]

\[+ \lambda'^2 \left( (\chi^\dagger\phi)(\phi^\dagger\chi) + H.c.] \right)^2
\]

\[u = g_m D_{\mu}^\pi \pi^\dagger b_{\mu}^m \sigma + \frac{1}{v} \left[ \tilde{\psi}_L \Phi A \psi_R + H.c.] \right)
\]

\[S = \text{ k.e. terms for scalars,}
\]

\[\mathcal{T} = \frac{i}{v}[\tilde{\psi}_L Z_L^\dagger \tilde{\psi}_L + i\tilde{\psi}_R Z_R^\dagger \tilde{\psi}_R - v'[\tilde{\psi}_L \Xi Z' \Lambda' \psi_R + H.c.].
\]

(32)

We have defined \(\lambda'_4 = \lambda_4 - \frac{1}{2}\lambda_5 - \frac{1}{2}\lambda_6\). We also have:

\[V' = -2\lambda_5 v v' \cos \xi \text{Re}(\phi^\dagger\chi) - 2\lambda_6 v v' \sin \xi \text{Im}(\phi^\dagger\chi),
\]

\[\tilde{V} = \frac{1}{4}(\lambda_5 - \lambda_6) \left[ (\phi^\dagger\chi)^2 - (\chi^\dagger\chi)^2 \right],
\]

(33)

\(\Lambda'\) embodies all the Yukawa couplings of (18) in a large matrix:

\[
\Lambda' = \left( \begin{array}{ccc} 
\lambda^{ab}_{\tau} \frac{1}{2} + \tau_3 & \lambda^{ab}_\tau \frac{1}{2} & 0 \\
\lambda^{ab}_\tau \frac{1}{2} & \lambda^{ab}_\tau \frac{1}{2} - \tau_3 & 0 \\
0 & 0 & \lambda^{ab}_c \frac{1}{2} - \tau_3
\end{array} \right).
\]

(34)
The first two lines of (31) are renormalizable, it is only the following lines which contain new terms. \( S \) contains the leading Higgs mass dependent correction to the kinetic terms of the \( \chi \). \( T \) are corrections from loops involving fermions and \( \chi \) fields, and they modify the kinetic terms and Yukawa couplings of the fermions. The quantities \( Z'_L, Z'_R, Z'_A \) which give the precise one–loop shifts to the fermion kinetic and mass terms are as in the nonmassive two Higgs doublet models and can be easily deduced from the results of [2]. We do not give them here, as they are not important to the physical processes we will be interested in. (The shifts to the scalar kinetic energy will give additional contributions to the \( \rho \) parameter, but this is already well known [14].) In addition, in contrast to (27), we have normalized the gauge fields so that there are no overall factors of gauge couplings outside the gauge kinetic terms.

The nonrenormalizable terms contain corrections with multiple scalar fields, as well as couplings between fermions and scalars. Our interpretation of the scale \( \mu \) will be the physical mass of the \( CP \)–even Higgs we eliminated from the theory, i.e. \( H^0 \) in (15). To find the corrections to processes involving physical particles one has to rewrite the sum of (27) and (31) by use of the physical scalars of (14), and then to eliminate the “eaten” scalars of (13) by going to the unitary gauge (or to use the equivalence theorem for these in the appropriate energy range).

3. Physical Results.

Amplitudes computed from our effective Lagrangians should give the tree + leading Higgs mass dependent one–loop results. These are the leading corrections for large enough Higgs mass, and for illustrative purposes we consider some physical processes when this is the case.

One Higgs Doublet Model.

For the strongly interacting standard model, one–loop corrections to the Lagrangian were already computed for pure boson loops in [6,7]. In [6] these corrections were used to study the consequences for longitudinally polarized gauge boson rescatterings. We focus here on corrections due to loops with internal fermions and bosons, not computed for the standard model before. These have two or more external fermions, and any number of external pseudo–goldstone scalars and/or \( SU(3)_c \times SU(2)_L \times U(1)_Y \) Yang–Mills gauge fields. We kept only the leading Higgs mass dependent \( (M_H^2 \text{ or } \ln M_H) \) corrections. These corrections contain only two external fermions, and at most one external gauge boson. They may contain any number of external scalars since we have taken the nonlinear limit in which even the tree level Yukawa couplings contain fermion interactions with an arbitrary number of scalars. We expect that loops containing more than two external fermions or more than one external gauge boson yield much smaller corrections for large enough Higgs mass.

These mixed fermion/boson loop corrections depend on the Yukawa couplings, or equivalently the fermion masses, so we expect them to be largest for the top quark whose mass is at least 89 GeV [9,10] and is perturbatively constrained by the one–loop \( \rho \)–
parameter to be within the approximate range 125 GeV to 195 GeV in the standard model with a Higgs mass between 0.5 TeV and 1 TeV [10]. (Both the upper and lower limits are lowered by as much as 40 GeV for a Higgs mass near 100 GeV.) Taking our result of (27), ignoring $K M$-matrix mixing angles and writing it out in full for the top/bottom doublet, we have the following one loop correction to the tree-level Lagrangian:

$$\delta L \ni \frac{i \ln[M_H / M_W]}{32 \pi^2 v^4} \left\{ (m_t^2 - m_b^2) \bar{\psi}_L (2 \tau_3, \Phi) \Phi^\dagger \bar{\psi}_L (\Phi \Phi) \Phi^\dagger \psi_L \\
- (m_t^2 - m_b^2) \bar{\psi}_L (\Phi \Phi) (2 \tau_3) \Phi^\dagger \psi_L + \frac{(m_t^2 + m_b^2)^2}{2} \bar{\psi}_R (\phi^\dagger \Phi \Phi) \psi_R \\
+ \frac{(m_t^2 - m_b^2)}{2} \bar{\psi}_R (2 \tau_3, (\Phi^\dagger \Phi \Phi)) \psi_R + \frac{(m_t - m_b)^2}{2} \bar{\psi}_R (2 \tau_3) (\Phi^\dagger \Phi \Phi) (2 \tau_3) \psi_R \right\}. \tag{35}$$

In this equations $M_H$ is meant to be taken as the mass of the physical Higgs, $v$ is the Higgs vev, and we have taken the $W$-boson mass for $\mu_0$. In addition, we have returned to dimensionful scalar fields which are given by

$$\Phi = \begin{pmatrix} \sqrt{v^2 - \pi^2} + i \pi^3 \\ i \sqrt{2} \pi^- \\ i \sqrt{2} \pi^+ \\ \sqrt{v^2 - \pi^{-2} - i \pi^3} \end{pmatrix}, \tag{36}$$

where $\pi^\pm = \frac{1}{\sqrt{2}} (\pi^1 \mp i \pi^2)$. In our unconventional normalization for the pauli matrices, $2 \tau_3 =$ diag$(1, -1)$. The fermions are doublets

$$\psi = \begin{pmatrix} t \\ b \end{pmatrix}. \tag{37}$$

We are interested in processes involving longitudinally polarized electroweak gauge bosons. Here, we can make use of the famous equivalence theorem [22] which states that physical amplitudes involving the longitudinal bosons are related to unphysical amplitudes involving the pseudo–goldstone bosons via

$$A \left[ W_L^\pm(p), \ldots, Z_L^\pm(k), \ldots \right] = A \left[ \pi^\pm(p), \ldots, \pi^3(k), \ldots \right] + O \left( \frac{M_W}{E} \right). \tag{38}$$

On the RHS the matrix element is to be evaluated in a gauge in which the pseudo–goldstone scalars still appear in the Lagrangian, and $E$ is the energy of the process. This holds for any number of external states, including those other than the longitudinal bosons. More importantly it holds to all orders in the Higgs self-coupling $\lambda$, which is important in the case of a strong $\lambda$. Although we could take our tree+one-loop Lagrangian, fixed to the unitary gauge, and calculate the actual gauge boson scattering amplitudes this turns out to be more work than finding the corresponding amplitudes involving the $\pi$. Using this procedure, our calculations should faithfully reproduce the full answers in the energy range

$$M_W \ll E \ll M_H. \tag{39}$$
One can now see that equation (35) contains some one-loop corrections to many processes, e.g. $\bar{t} t \rightarrow W^+_L W^-_L$, or $\bar{t} b \rightarrow W^+_L Z^0_L$. Since $\Phi$ contains any number of $\pi$, the final states may contain any number of longitudinal bosons (however each additional boson suppresses the amplitude by $v$).

There are other terms in (27) that contribute to such processes. Consider $W^+_L W^-_L$ which can proceed through an intermediate photon or $Z^0$, as well as bottom exchange. In the linear $\sigma$-model (i.e. keeping the Higgs) it also proceeds through an intermediate Higgs. In our case the corresponding diagram is an elementary four point vertex (the Higgs propagator is “shrunk” to a point). We need to examine all the one-loop corrections to these. In our analysis here we ignore the finite fermion loops. One-loop corrections to, e.g., the $Z^0 \pi^+ \pi^-$ coupling are very small (as a case in point, consider the term that gives the Higgs correction to the $\rho$ parameter). In fact, we find the leading correction apart from (35) is a correction to the elementary four point vertex (the “Higgs exchange”) as given on the sixth line of (27). Writing it out for the top/bottom doublet with the same normalization as (35) we find

$$\delta L \ni - \frac{\ln[M_H/M_W]}{4\pi^2v^4} \left\{ m_t \bar{t} t + m_b \bar{b} b \right\} D_\mu \pi^+ D^\mu \pi^- . \quad (40)$$

This correction is due to loops involving the pseudo-goldstone scalars only, with two external scalars and an external fermion bilinear $\bar{\psi} \psi$. The relevant tree level terms that we must compare with are found from (6) to be

$$L_{\text{tree}} = \frac{m_t}{v^2} \pi^+ \pi^- \bar{t} t + \frac{m_b}{v^2} \pi^+ \pi^- \bar{b} b$$

$$+ i\sqrt{2m_t} \left( \pi^+ \bar{t} R b_L - \pi^- b_L \bar{R} t_R \right) + i\sqrt{2m_b} \left( \pi^- \bar{b} R t_L - \pi^+ \bar{R} b_L \right)$$

$$+ \frac{g}{2} \bar{t} L A^3 t_L + \frac{g'}{6} \bar{b} L B t_L + \frac{2g'}{3} \bar{t} R b_L + \frac{g}{2} \bar{b} L A^3 b_L + \frac{g'}{6} \bar{b} L B b_L - \frac{g'}{3} \bar{b} R B b_R$$

$$+ i \left( g A_{\mu}^3 + g' B_{\mu} \right) \left\{ \pi^- D^\mu \pi^+ - \pi^+ D^\mu \pi^- \right\} . \quad (41)$$

We have reinserted the gauge coupling constants $g$ and $g'$ for the $SU(2)_L$ gauge fields $A_{\mu}$ and hypercharge gauge field $B_{\mu}$, respectively, as compared to (6).

From the tree amplitudes of (35), (40), and (41) one can extract the exact answer for the tree + one-loop amplitudes found from the tree Lagrangian (up to subleading terms we have neglected). Let us consider two cases within the context of the standard model.

**The process** $\bar{t} t \rightarrow W^+_L W^-_L$. Neglecting $m_b$ in comparison to $m_t$ we find the relevant one-loop corrections are

$$\delta L \ni \frac{i}{16\pi^2v^4} \ln[M_H/M_W] \left\{ \bar{t} L \gamma^\mu t_L \left( \pi^- D_\mu \pi^+ + \pi^+ D_\mu \pi^- \right) \right.$$

$$+ i\bar{t} R \gamma^\mu t_R \left( \pi^+ D_\mu \pi^- - \pi^- D_\mu \pi^+ \right) + 2\pi^+ \pi^- i\bar{t} L B t_L \right\}$$

$$+ \sqrt{2} m_t \frac{i}{16\pi^2v^3} \ln[M_H/M_W] \left\{ \bar{b} L \gamma^\mu t_L D_\mu \pi^- + \pi^- \bar{b} L B t_L - \pi^+ \bar{t} L B t_L \right\}$$

15
\begin{equation}
-m_t \ln \left[ \frac{M_H/M_W}{4\pi^2 v^4} \right] D_{\mu} \pi^+ D^\mu \pi^- \bar{t} t.
\end{equation}

These corrections are subdued by powers of \( v \) compared to the tree Lagrangian of (41) so we expect them to be largest for large top mass and energy. For top masses closer to the CDF limit than the perturbative upper bound we may expect that at high enough energies the dominant correction is from the last term in (42). In fact, H. Veltman [8] has shown this to be the case by explicit computation of all the leading amplitudes. Ref. [8] also shows that the dominant tree contribution in this case is what corresponds to our four point vertex, the first term in (38). To find the ratio of tree+one–loop to tree amplitude we use \( \partial_{\mu} \pi^+ \partial_{\mu} \pi^- \to -(p_+ \cdot p_-) \pi^+ \pi^- \to -\frac{1}{2} \hat{s} \pi^+ \pi^- \), where \( p_\pm \) is the four–momentum of the outgoing \( \pi^\pm \) and \( \hat{s} = (p_+ + p_-)^2 \). In addition, following Cheyette and Gaillard [6] we will use \( \ln \left[ \frac{M_H^2}{M_W^2} \right] \to \ln \left[ \frac{M_H^2}{(-\hat{s})} \right] \) for the correct kinematic factor for our scattering process. Then we immediately find

\begin{equation}
\frac{A_{\text{tree+one–loop}}(\bar{t} t \to \pi^+ \pi^-)}{A_{\text{tree}}(\bar{t} t \to \pi^+ \pi^-)} \approx 1 + \frac{1}{16\pi^2 v^2} \hat{s} \ln \left[ \frac{M_H^2}{(-\hat{s})} \right],
\end{equation}

in agreement with ref. [8]. For example, this can lead to about a 15% enhancement over the tree amplitude when \( M_H \approx 2 \text{TeV} \) and \( \sqrt{\hat{s}} \approx 1 \text{TeV} \). By the equivalence theorem this is also an approximation to the correction \( \bar{t} t \to W^+ L W^- L \) so we may expect a 30% enhancement in the elementary cross–section with our values. Our result provides an important and independent check of the result of ref. [8].

For \( m_t \) closer to \( v \) the other tree diagrams from (42) must be more carefully considered.

**The process** \( W^+_L W^-_L \to \bar{\Psi} \Psi \) for heavy fermion pair production. Since mass splittings between a fermion doublet contribute to the one–loop \( \rho \)–parameter we consider an almost degenerate fermion doublet \( (\Psi, \Xi) \) with average mass \( M \). In this case the leading corrections similar to (35) and (40) for this process are:

\begin{align*}
\delta L \equiv &-iM^2 \ln \left[ \frac{M_H/M_W}{8\pi^2 v^4} \right] (\pi^- D_{\mu} \pi^+ - \pi^+ D_{\mu} \pi^-) \bar{\Psi} \gamma^\mu \Psi \\
&-\sqrt{2}M^2 \ln \left[ \frac{M_H/M_W}{16\pi^2 v^3} \right] \left[ \bar{\Psi} (\not{D} \pi^+) \Xi + \text{H.c.} \right] - M \ln \left[ \frac{M_H/M_W}{4\pi^2 v^4} \right] D_{\mu} \pi^+ D^\mu \pi^- \bar{\Psi} \Psi.
\end{align*}

\begin{equation}
(44)
\end{equation}

We must compare this with the corresponding tree contributions, a minor modification of (41), and also production via quark–antiquark and gluon fusion. This process was examined by Costa and Liebrand [7] who only considered corrections from the last term in (44). For \( M^2 \sim \hat{s} \) the other terms are also important and a more complete analysis is necessary. For heavy quark production, the QCD background at the LHC or SSC will be enormous [23] so our extra terms are do not alter the conclusion of [7] that one–loop corrections give only an infinitesimal contribution to the total rate. For equally heavy leptons, the QCD background is much smaller so these corrections may be observable (if they are detectable at all, since they will decay into a sea of \( W^+ W^- \)).
Two Higgs Doublet Model.

Our corrections (31) in the case of a strongly interacting two Higgs doublet model are entirely new. Of course, there are many more parameters than in the minimal standard model, and we have only studied the case when one of the CP-even physical scalars is very massive. The leading fermion mass dependent corrections, as mentioned previously, will be presented elsewhere. However, we are able to extract what should be the leading corrections at high enough energy. This would be physically relevant if some evidence for at least an extra Higgs doublet is found (say CP-odd scalars like $H^\pm$) but the scalar sector is strongly interacting.

It is well known [1] that tree level amplitudes in the minimal standard model with very large Higgs self-coupling violate unitarity at $\sqrt{s_{c}}=1.7$ TeV. The tree level unitarity bound can be simply found by considering the nonlinear $\sigma$-model given by (4). This Lagrangian gives, using the equivalence theorem, the leading two body to two body scattering amplitudes for processes involving longitudinally polarized $W$s and $Z$s. In particular, the $J=0$ partial wave amplitude $a_0$ for the process $W_L^+ W_L^- \rightarrow Z_L Z_L$ grows with energy to lowest order as

$$|a_0(W_L W_L \rightarrow Z_L Z_L)| \approx \frac{s}{16\pi v^2} = \frac{s}{(1.7\text{TeV})^2}$$

(45)

for $\sqrt{s}$ much less than the perturbative Higgs mass $M_H = 350\sqrt{s}/\text{GeV}$ (see eq. (1)). Demanding $|a_0| < 1$ gives the approximate bound on $\sqrt{s}$ for the scale at which new physics, for example the physical Higgs, should appear to make the full theory unitary.

One loop corrections to such unitarity bounds (for the one Higgs doublet case) have been considered in [4]. One finds that low energy unitarity bounds can be made significantly stricter. While one should be careful in believing perturbative results in the strongly interacting case, it is important to note that such one-loop calculations support the belief [1] that a Higgs or new physics should appear at or before SSC energies.

Since there is no a priori reason to believe in the minimal standard model it is important to investigate other models to understand what we may be able to detect at the next generation of colliders. There are two useful limits: $\tan \beta = v'/v$ small and $\tan \beta$ large. These limits correspond to the $\pi$ fields of the strongly interacting scalar doublet being purely the eaten bosons (the longitudinal gauge bosons) or being purely the CP-odd physical Higgs, respectively. The relevant four pion tree level vertex is in both cases given by (4). For small $\tan \beta$ the vev $v$ is fixed from experiment by $1/v^2 = \sqrt{2}G_F$ and the the process $w^+w^- \rightarrow zz$ gives the same tree level unitarity bound as from (45), namely $\sqrt{s_c} < 1.7\text{TeV}$. For large $\tan \beta$ the process $H^+H^- \rightarrow A^0A^0$ gives the same partial wave amplitude as (45), but now $1/(v^2 + v'^2) = \sqrt{2}G_F$ fixes $v^2 \approx v'^2/(\tan \beta)^2$ and we get the bound $\sqrt{s_c} < 1.7\text{TeV}/\tan \beta$. For $\tan \beta = 2$ this gives the scale of unitarity violation as just $875$ GeV.

A more careful tree level analysis of unitarity in the two Higgs doublet model has been
carried out in [11]. One finds that
\[ \sqrt{s_c} = \frac{1.7TeV}{\sqrt{1 + (\tan \beta)^2}}, \] (46)
for the heavy Higgs limit we assumed (see eq. (15)). This result comes from considering all the possible two body to two body scatterings with neutral initial and final states, and finding the amplitude \( a_0 \) with the largest absolute value. Therefore, the energy at which new phenomena may appear in the strongly interacting two Higgs doublet model may be significantly lower than in the standard model.

Since we have explicitly calculated the relevant leading terms we can easily determine one loop corrections to these unitarity bounds. We consider only \( w^+w^- \rightarrow zz \) for small \( \tan \beta \) and \( H^+H^- \rightarrow A^0A^0 \) for large \( \tan \beta \). Since at tree level these channels reproduce the exact result for a wide range of \( \tan \beta \), they will give a good indication of how one loop corrections change such unitarity bounds.

The relevant tree+one–loop Lagrangian for large \( \lambda_1 \) is given by (27) and (31). We will probe the region \( M_2^2W \ll s \ll 2\lambda_1v^2 \) so we need only consider the derivative terms. Keeping the necessary terms to lowest order in the scalar fields and ignoring renormalizable corrections, we find that the relevant terms are
\[ L \ni \frac{1}{2} v^2 g_{ij} D_\mu \pi^i D^\mu \pi^j + \frac{\ln[M_H^2/\rho \partial^2]}{64\pi^2} \left[ \frac{2}{3}(\delta_{ij} D_\mu \pi^i D^\mu \pi^j)^2 + \frac{4}{3}(\delta_{ij} D_\mu \pi^i D_\nu \pi^j)^2 \right] + 2\delta_{ij} D_\mu \pi^i D^\mu \pi^j \left( -2V' - 6\tilde{V} + 6\lambda_4'(\chi^\dagger \phi)(\phi^\dagger \chi) - 4v^2 \lambda_5'(\chi^\dagger \chi) \right) + \frac{2}{3} D_\mu \pi^i D^\mu \pi^j \left( \lambda_5'[\chi^\dagger \phi_i](\phi^\dagger_j \chi) + H.c. \right) - \frac{\lambda_5 - \lambda_6}{2} \left[ (\phi^\dagger_i \chi)(\phi^\dagger_j \chi) + H.c. \right] \right]. \] (47)

We have inserted the correct factor in the logarithm \([6]\) for the scattering process and substituted the perturbative Higgs mass (of \( H^0 \)) for \( \mu \); \( \rho \) is undetermined in our approach and since we do not have the next to leading one loop corrections the best we can do is assume it is of \( O(1) \). In what follows we drop it. In spite of this uncertainty the above prescription should give a good indication of the amount by which the one loop corrections modify the tree level result.

**Small** \( \tan \beta \). Writing out the leading terms in (47) that are important for \( w^+w^- \rightarrow zz \), we get
\[ L \ni \frac{1}{v^2} \left[ w^+z(D_\mu w^-)(D^\mu z) + H.c. \right] + \frac{\ln[M_H^2/\partial^2]}{64\pi^2} \left[ \frac{8}{3} v^4 (D_\mu w^+)(D^\mu w^-)(D_\nu z)(D^\nu z) \right] + \frac{\ln[M_H^2/\partial^2]}{64\pi^2} \left[ \frac{16}{3} v^4 (D_\mu w^+)(D_\nu w^-)(D^\mu z)(D^\nu z) \right]. \] (48)
Note that the one–loop corrections in the last two lines of (47) do not contribute in this limit.

Eq. (48) tells us that the leading corrections are as in the standard model for this process [6], and one–loop unitarity bounds have been well studied [4,24]. Ignoring external masses, the amplitude for \( w^+w^- \rightarrow zz \) is

\[
A(\hat{s}, \hat{t}, \hat{u}) = \frac{1}{v^2} \left[ s + \frac{1}{16\pi^2v^2} \left( \frac{s^2}{2} \ln[M_H^2/(-\hat{s})] + \frac{2\hat{t}^2 + \hat{s}\hat{t}}{6} \ln[M_H^2/(-\hat{t})] + \frac{2\hat{u}^2 + \hat{u}\hat{s}}{6} \ln[M_H^2/(-\hat{u})] \right) \right].
\] (49)

The \( J = 0 \) partial wave is easily found (see for example ref. [1]) by using \( \hat{s} + \hat{t} + \hat{u} = 0 \) and integrating over \( \hat{t} \). We find for the real part

\[
a_0(\hat{s}) = a^0_{\hat{s}} \left[ 1 + \frac{a^0_{\hat{s}}}{\pi} \left( \frac{73}{36} \ln[M_H^2/s] - \frac{1}{108} \right) \right],
\] (50)

where \( a^0_{\hat{s}} \) is the tree result of (45), which does not depend on the parameter \( M_H \). One finds, for example, that unitarity is saturated at \( \sqrt{s_c} = 1740 \text{ GeV} \) when \( M_H = 1750 \text{ GeV} \).

**Large** \( \tan \beta \). This case is much more interesting. It is easy to check that the relevant Lagrangian for \( H^+H^- \rightarrow A^0A^0 \) is similar to (48). We have

\[
L \ni \frac{(\tan \beta)^2}{v^2} \left[ H^+A^0(D_\mu H^-)(D^\mu A^0) + H.c. \right] + \frac{\ln[M_H^2/\partial^2]}{64\pi^2} \left( \frac{8(\tan \beta)^4}{3v^4} (D_\mu H^+)(D^\mu H^-)(D_\nu A^0)(D^\nu A^0) \right. \\
\left. + \frac{16(\tan \beta)^4}{3v^4} (D_\mu H^+)(D_\nu H^-)(D^\mu A^0)(D^\nu A^0) \right].
\] (51)

The partial wave amplitude for our process is then again given by (50), with

\[
a^0_{\hat{s}}(\hat{s}) = \frac{(\tan \beta)^2\hat{s}}{16\pi v^2} = \frac{(\tan \beta)^2\hat{s}}{(1.7T\text{eV})^2}.
\] (52)

The perturbative tree level heavy Higgs mass grows with \( \sqrt{\lambda_1}/\tan \beta \) in this limit. To study the large mass limit we may study either the limit \( \lambda_1 \) large and fixed or \( M_H \) large and fixed. In both cases large means \( M_H \) is much larger than \( M_W, M_{\text{top}} \) and \( \sqrt{s_c} \). To get an idea of \( \sqrt{s_c} \) in this case we computed the scale at which (50) is 1 for two cases: (i) \( M_H = 1750 \text{ GeV} \) and (ii) \( M_H = 1750 \text{ GeV}/\tan \beta \). For the last case, the tree+one–loop unitarity bound is approximately the same as the tree level bound, 1750 GeV/(\tan \beta) for the values of \( \tan \beta \) we looked at (2 < \( \tan \beta < 10 \)). In fact the bound is a little higher, since at \( M_H^2 = s = 1750\text{GeV}/(\tan \beta) \) the logarithm in (50) vanishes and the remaining one–loop term helps to unitarize the theory. However, since we have dropped subleading one–loop corrections, as well as ignoring the imaginary part of the partial wave, all we can say in this case is that there is no significant shift from the tree level unitarity result.
For case (i) with fixed $M_H$ the unitarity bound drops faster at large tan $\beta$ with tan $\beta$ than the tree level result. For example, at tan $\beta = 2$ we get $\sqrt{s_c} = 666$ GeV and at tan $\beta = 10$ we get $\sqrt{s_c} = 110$ GeV. This is to be contrasted with the tree level results of $\sqrt{s_c} = 875$ GeV at tan $\beta = 2$ and $\sqrt{s_c} = 175$ GeV at tan $\beta = 10$ in the large tan $\beta$ limit. In our figure we plot the tree+one loop tan $\beta = 0$ unitarity bound as well as the large tan $\beta$ limit results for $2 < \tan \beta < 10$. For extremely high values of tan $\beta$ our results are not to be trusted since $\sqrt{s_c}$ is driven near $M_W$, $M_{top}$ and our approximations are not good. For tan $\beta$ in the approximate range 0.5 to 2, one needs a more careful analysis starting with equation (47). In this case it is expected that the last two terms in (47) will be important, since they contribute for arbitrary tan $\beta$ to any of the neutral channels.

In summary, our results indicate that in the case of a two Higgs doublet model either signs of the possible strong interactions of the scalars or the scalars themselves may turn up at relatively low energies. At tan $\beta = 4$, this scale can be as low as 300 GeV. In addition, if one carries out a more detailed analysis keeping the subleading terms and the imaginary parts (as well as perhaps higher order corrections and top mass dependent one–loop corrections) it would be possible to gain information about $M_H$ as a function of tan $\beta$ by studying cross-sections at future experiments. For example, the scale of unitarity violation given by (50) depends on $M_H$: for tan $\beta = 3$ we have $\sqrt{s_c} = 500, 415, 380$ GeV at $M_H = 750, 2000, 4000$ GeV, respectively. This simple example demonstrates the interesting interplay between unitarity, tan $\beta$ and the Higgs masses in the two Higgs doublet model. One further analysis which would be useful is to take all the perturbative physical scalar masses to infinity, and then study unitarity violations. We leave these as future projects.

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