Calculable Violation of Gauge-Yukawa Universality
and Top Quark Mass in the Gauge-Higgs Unification

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Abstract

We find that the one-loop correction to the ratio of Yukawa coupling and gauge coupling in the gauge-Higgs unification, “gauge-Yukawa universality violation”, is finite and calculable in any space-time dimension. Applying this result to the ratio of top quark and W-boson masses, we show that the order one correction required to generate a viable top quark mass is indeed possible if the fermion embedding top quark belongs to the large dimensional representation of the gauge group and a vacuum expectation value of Higgs scalar field is very small comparing to the compactification scale.
Gauge-Higgs unification \([1, 2, 3]\) is one of the attractive scenarios solving the hierarchy problem without invoking supersymmetry. In this scenario, Higgs doublet in the Standard Model (SM) is identified with the extra spatial components of the higher dimensional gauge fields. One of the remarkable features is that quantum corrections to Higgs mass is insensitive to the cutoff scale of the theory and calculable regardless of the nonrenormalizability of higher dimensional gauge theory. The reason is that the Higgs mass term as a local operator is forbidden by the higher dimensional gauge invariance. Then, the finite mass term is generated radiatively and expressed by the Wilson line phase as a non-local operator. This fact has opened up a new avenue to the solution of the hierarchy problem \([4]\). Since then, much attention has been paid to the gauge-Higgs unification and many interesting works have been done from various points of view \([5]-[25]\).

The finiteness of Higgs mass has been studied and verified in various models and in various types of compactification at one-loop level \([26]-[29]\) and even at two loop level \([31]\). It is natural to ask whether any other finite physical observables exist in the gauge-Higgs unification. The naive guess is that such observables are in the gauge-Higgs sector of the theory if they ever exist. The present authors studied the structure of divergences for S and T parameters in the gauge-Higgs unification since such parameters are described by higher dimensional gauge invariant operators with respect to gauge and Higgs fields, and are expected to be finite by virtue of the higher dimensional gauge symmetry. The result is that both parameters are divergent (convergent) more than (in) five dimensions as expected from the power counting argument. However, a nontrivial prediction we have found, specific to the gauge-Higgs unification, is that some linear combination of S and T parameters is finite even in six dimensions \([32]\). One of the authors (N.M.) also has shown that the gluon fusion amplitude and the amplitude of two photon decay of Higgs boson, which are very important processes at LHC, are finite in any space-time dimension in the gauge-Higgs unification \([33]\). This is a new and only known calculable physical observable other than the Higgs mass.

Although the gauge-Higgs unification is very predictive in the gauge-Higgs sector of the standard model as mentioned above, the matter sector is too restrictive to generate a desirable flavor structure since Yukawa coupling is given by the gauge coupling, to start with. This immediately leads to the fact that fermion masses become W-boson mass and Yukawa hierarchy cannot be explained, unless some suitable mechanism is adopted. Obtaining light fermion masses is easily realized by introducing the \(Z_2\)-odd bulk mass because the zero mode wave functions for fermions with different chiralities are localized at different fixed points along the extra dimension, which naturally yields a small Yukawa.

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1. For the case of gravity-gauge-Higgs unification, see \([30]\).
2. In a toy model of QED compactified on a circle, the anomalous magnetic moment was shown to be finite in arbitrary space-time dimensions \([34]\). Recently, the cancellation mechanism of the ultraviolet (UV) divergence in the magnetic moment was further clarified in a realistic SU(3) model on an orbifold \(S^1/Z_2\) \([35]\).
coupling due to the small overlap integral of the zero mode wave functions. However, getting the top quark mass is nontrivial task since we need an enhancement factor of roughly 2, as $m_t \simeq 2m_W$. In flat space gauge-Higgs unification model, this enhancement factor can be obtained from the group theoretical factor of large dimensional representation which a bulk fermion embedding top quark belongs to [15]. In warped space case, it is known that the enhancement factor comes from the product of curvature scale and compactification radius [20].

In this paper, we propose an alternative mechanism to generate a viable top quark mass by taking into account one-loop corrections to Yukawa coupling in the flat space gauge-Higgs unification. Naively thinking, this seems to be clearly impossible because the loop corrections are always suppressed. However, this is not necessarily the case in the case of gauge-Higgs unification. As will be shown later, the one-loop correction effects have additional factor of Dynkin index for the representation which the matter fermion belongs to and a logarithmic factor of Higgs vacuum expectation value (VEV) other than the one-loop factor. If we consider the fermion belonging to large dimensional representation, we can have a large Dynkin index. We further note that the logarithmic factor of Higgs VEV is likely to be large since the Higgs VEV should be tiny compared to the compactification scale, typically around $O(10^{-2})$, to realize the correct pattern of electroweak symmetry breaking and obtain a Higgs mass satisfying the experimental data. Combining these effects, we can expect that the one-loop correction to Yukawa coupling becomes $O(1)$.

Quantum correction to Yukawa coupling is generally a cutoff scale dependent, especially in the nonrenormalizable higher dimensional theories, and is independent of the quantum correction to the gauge coupling. Thus, the quantum correction seems to have no definite prediction. In the gauge-Higgs unification, however, Yukawa and gauge coupling are identical, to start with, being described by the same covariant derivative, i.e. “gauge-Yukawa universality” holds. Hence, even if the universality is violated at the quantum level, the violation should be finite and calculable. It is interesting to note that a similar situation happens to MSSM, where Higgs self-coupling is provided by the gauge interaction, D-term, and the deviation of the Higgs mass from the gauge boson masses is finite, even at the quantum level [36]. According to this line of argument, we calculate here one-loop corrections to the ratio of Yukawa coupling and the gauge coupling, which will be shown to be independent of the cutoff scale of the theory, namely calculable and finite. Since Yukawa coupling is provided by a part of the gauge coupling $g\bar{\psi}A_M \Gamma^M \psi$, the renormalized Yukawa coupling is obtained by taking into account the wave function renormalization factors of extra component of the gauge field $A_y = \sqrt{Z_y} A_y^{\text{bare}}$, a fermion
\[ \psi = \sqrt{Z_{\psi}} \psi_{\text{bare}}, \]  
and the vertex correction \[ Z_{A_{\nu} \psi \bar{\psi}}. \]

\[ Y^{\text{ren}} = \frac{Z_{A_{\nu} \psi \bar{\psi}}}{Z_{\psi} \sqrt{Z_{y}}} Y = \frac{1}{\sqrt{Z_{y}}} Y \]  
(1)

where \( Y \) and \( Y^{\text{ren}} \) are the bare and renormalized Yukawa couplings and we made use of Ward identity \( Z_{A_{\nu} \psi \bar{\psi}} = Z_{\psi} \) to arrive at the final expression. On the other hand, it is well known that the renormalized gauge coupling \( g^{\text{ren}} \) is calculated from the vacuum polarization of gauge field,

\[ g^{\text{ren}} = \frac{1}{\sqrt{Z_{\mu}}} g \]  
(2)

where \( g \) is the bare gauge coupling and \( Z_{\mu} \) denotes the wave function renormalization factor for the gauge field \( A_{\mu} \), namely \( A_{\mu} = \sqrt{Z_{\mu}} A_{\mu}^{\text{bare}} \). Taking the ratio of (1) and (2) using the gauge-Yukawa universality \( Y = g \), we find

\[ \frac{Y^{\text{ren}}}{g^{\text{ren}}} = \sqrt{\frac{Z_{\mu}}{Z_{y}}} \]  
(3)

Thus, we have only to calculate the vacuum polarization diagram, defined as \( \Pi_{MN}(p^2) \), shown in Fig. 1. To be precise, we are interested in the difference between \( \Pi_{\mu\nu} \) and \( \Pi_{yy} \). Note that the nonzero KK external momenta are set to be zero since we are only interested in wave function renormalization factors of the zero modes. Each wave function renormalization factors in (3) are divergent, but the ratio is expected to be finite from the argument above and also simply from the higher dimensional Lorentz invariance.\(^3\)

In fact, the wave function renormalization factor \( Z \) for the local operator \( Z F_{MN} F^{MN} \) is universal. Thus, we can make UV-insensitive prediction for the one-loop correction to top quark mass by making use of this ratio.

In this paper, we take a \((D+1)\)-dimensional \( SU(3) \) gauge theory with a triplet fermion compactified on \( S^1/Z_2 \). As will be seen later, the triplet fermion contains a doublet top quark \( t_L \) but not a singlet one \( t_R \) and a fermion belonging to large dimensional

\(^3\)Similar ratio appeared in the calculation of Higgs mass at two-loop level \([31]\).
representation accommodating $t_R$ is necessary to obtain a viable top quark mass. The
fermion of large dimensional representation also helps to get a realistic top quark mass.
The contribution by such a fermion in the large dimensional representation to top quark mass can be reduced to the contribution of a triplet fermion multiplied by an additional
group factor, namely Dynkin index. Therefore, the calculation throughout this paper is
carried out by using the triplet fermion. The $SU(3)$ symmetry is broken to $SU(2) \times U(1)$
by the orbifolding $S^1/Z_2$ and adopting a non-trivial $Z_2$ parity assignment for the members
of an irreducible representation of $SU(3)$, as stated below. The remaining gauge symmetry
$SU(2) \times U(1)$ is supposed to be broken by the VEV of the zero-mode of $A_y$, the extra
space component of the gauge field behaving as the Higgs doublet, through the Hosotani-
mechanism [3], though we do not address the question how the VEV is obtained by
minimizing the loop-induced effective potential for $A_y$ [3].

The lagrangian is simply given by
\[
\mathcal{L} = -\frac{1}{2} \text{Tr}(F_{MN} F^{MN}) + i \bar{\Psi} \gamma \cdot D \Psi
\]
where $\Gamma^M = (\gamma^\mu, i\gamma^y)$,
\[
F_{MN} = \partial_M A_N - \partial_N A_M - ig_{D+1}[A_M, A_N] (M, N = 0, 1, 2, 3, \ldots, D),
\]
\[
\bar{\Psi} = \Gamma^M (\partial_M - ig_{D+1} A_M) \left( A_M = A_M^a \frac{\lambda^a}{2} (\lambda^a : \text{Gell-Mann matrices}) \right),
\]
\[
\Psi = (\psi_1, \psi_2, \psi_3)^T.
\]
The periodic boundary conditions are imposed along $S^1$ for all fields and the non-trivial
$Z_2$ parities are assigned for each field as follows,
\[
A_\mu = \begin{pmatrix}
(+, +) & (+, +) & (-, -) \\
(+, +) & (+, +) & (-, -) \\
(-, -) & (-, -) & (+, +)
\end{pmatrix},
A_y = \begin{pmatrix}
(-, -) & (-, -) & (+, +) \\
(-, -) & (-, -) & (+, +) \\
(+, +) & (+, +) & (-, -)
\end{pmatrix},
\]
\[
\Psi = \begin{pmatrix}
\psi_{1L}(+, +) + \psi_{1R}(-, -) \\
\psi_{2L}(+, +) + \psi_{2R}(-, -) \\
\psi_{3L}(-, -) + \psi_{3R}(+, +)
\end{pmatrix},
\]
where $(+, +)$ means that $Z_2$ parities are even at the fixed points $y = 0$ and $y = \pi R$, for
instance. $y$ is the compactified space coordinate and $R$ is the compactification radius.
$\psi_{1L} \equiv \frac{1}{2} (1 - \gamma^y) \psi_1$, etc. A remarkable feature of this manipulation of “orbifolding” is that
in the gauge-Higgs sector, exactly what we need for the formation of the standard model
is obtained at low energies; one can see that $SU(3)$ is broken to $SU(2)_L \times U(1)_Y$ and the
Higgs doublet $\phi = (\phi^+, \phi^0)^t$ emerges. Namely the zero mode of the gauge-Higgs sector
takes the form,
\[
A_\mu^{(0)} = \frac{1}{2} \begin{pmatrix}
\frac{2 \sqrt{2} W_\mu}{\sqrt{3}} & \sqrt{2} W_\mu^+ & 0 \\
0 & -\frac{1}{\sqrt{3}} (\gamma_\mu + \sqrt{3} Z_\mu) & 0 \\
0 & 0 & -\frac{1}{\sqrt{3}} (\gamma_\mu - \sqrt{3} Z_\mu)
\end{pmatrix},
A_y^{(0)} = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 0 & \phi^+ \\
0 & 0 & \phi^0 \\
\phi^- & \phi^{0*} & 0
\end{pmatrix},
\]
with $W^3_\mu$, $W^\pm_\mu$, $B_\mu$ being the $SU(2)_L, U(1)_Y$ gauge fields, respectively, while in the zero-mode of the triplet fermion $t_R$ is lacking,

$$\Psi^{(0)} = \left( \begin{array}{c} t_L \\ b_L \\ b_R \end{array} \right).$$ (11)

The VEV to break $SU(2)_L \times U(1)_Y$ is written as

$$\langle A_y \rangle = \frac{v}{2} \lambda_6 \quad \langle \phi^0 \rangle = \frac{v}{\sqrt{2}}.$$ (12)

Depending on these boundary conditions, KK mode expansions for the gauge fields and fermions are carried out as follows.

$$A^{(+,+)}_{\mu,y}(x, y) = \frac{1}{\sqrt{2\pi R}} \left[ A^{(0)}_{\mu,y}(x) + \sqrt{2} \sum_{n=1}^{\infty} A^{(n)}_{\mu,y}(x) \cos \left( \frac{ny}{R} \right) \right],$$ (13)

$$A^{(-,-)}_{\mu,y}(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} A^{(n)}_{\mu,y}(x) \sin \left( \frac{ny}{R} \right),$$ (14)

$$\psi^{(+,+)}_{1L,2L,3R}(x, y) = \frac{1}{\sqrt{2\pi R}} \left[ \psi^{(0)}_{1L,2L,3R}(x) + \sqrt{2} \sum_{n=1}^{\infty} \psi^{(n)}_{1L,2L,3R}(x) \cos \left( \frac{ny}{R} \right) \right],$$ (15)

$$\psi^{(-,-)}_{3L,1R,2R}(x, y) = \frac{i}{\sqrt{2\pi R}} \sum_{n=1}^{\infty} \psi^{(n)}_{3L,1R,2R}(x) \sin \left( \frac{ny}{R} \right).$$ (16)

For the calculation of one-loop corrections due to the Yukawa and gauge couplings of fermions, only the term containing fermions, $\mathcal{L}_{\text{fermion}} = i\bar{\Psi} \mathcal{D} \Psi$, in the lagrangian (4) is enough to consider. Substituting the above KK expansions for the fermion and the zero-modes for the gauge-Higgs bosons in the term and integrating over the extra space coordinate $y$, we obtain a 4D effective Lagrangian:

$$\mathcal{L}^{(4D)}_{\text{fermion}} = \sum_{n=1}^{\infty} \left\{ \left( \bar{\psi}^{(n)}_1, \bar{\psi}^{(n)}_2, \bar{\psi}^{(n)}_3 \right) \times \left( \begin{array}{ccc} i\gamma^\mu \partial_\mu - m_n & 0 & 0 \\
0 & i\gamma^\mu \partial_\mu - (m_n + m) & 0 \\
0 & 0 & i\gamma^\mu \partial_\mu - (m_n - m) \end{array} \right) \right\} \left( \begin{array}{c} \psi^{(n)}_1 \\
\bar{\psi}^{(n)}_2 \\
\bar{\psi}^{(n)}_3 \end{array} \right)$$

$$+ \frac{g_D}{2} \left( \bar{\psi}^{(n)}_1, \bar{\psi}^{(n)}_2, \bar{\psi}^{(n)}_3 \right) \left( \begin{array}{ccc} \frac{2}{\sqrt{3}} \gamma^\mu_w & W^+_\mu & -W^+_\mu \\
0 & \gamma^\mu_{W^-} & Z^\mu \\
0 & -Z^\mu & \gamma^\mu_{W^-} \end{array} \right) \gamma^\mu \left( \begin{array}{c} \psi^{(n)}_1 \\
\bar{\psi}^{(n)}_2 \\
\bar{\psi}^{(n)}_3 \end{array} \right)$$

$$+ \frac{g_D}{2} \left( \bar{\psi}^{(n)}_1, \bar{\psi}^{(n)}_2, \bar{\psi}^{(n)}_3 \right) \left( \begin{array}{ccc} 0 & \phi^+ & 0 \\
\phi^- & 0 & -i\phi^0 \\
\phi^- & i\phi^0 & 0 \end{array} \right) \left( \begin{array}{c} \psi^{(n)}_1 \\
\bar{\psi}^{(n)}_2 \\
\bar{\psi}^{(n)}_3 \end{array} \right)$$

$$+ \int \bar{\ell}_L \gamma^\mu \partial_\mu t_L \bar{b}(i\gamma^\mu \partial_\mu - m)b + \int \frac{3g_D}{6} (\bar{t}\gamma_\mu L\bar{t} + \bar{b}\gamma_\mu Lb - 2\bar{b}\gamma_\mu Rb) B^\mu$$

$$+ \frac{g_D}{\sqrt{2}} (\bar{t}\gamma_\mu LbW^+ + \bar{b}\gamma_\mu L W^-) + \frac{g_D}{2} (\bar{t}\gamma_\mu L - \bar{b}\gamma_\mu Lb) W^\mu_3$$ (17)
where \( m_n = \frac{n}{R} \), \( g_D = \frac{g_D'}{\sqrt{2} \pi R} \) is the \( D \)-dimensional gauge coupling and \( m = \frac{g_D}{2} (= m_W) \) is the bottom quark mass \( m_b \). In deriving the 4D effective Lagrangian (17), a chiral rotation

\[
\psi_{1,2,3} \rightarrow e^{-i \frac{\pi}{4} \gamma^y} \psi_{1,2,3}
\]

has been made in order to get rid of \( i \gamma^y \). We easily see that the non-zero KK modes in the mass eigenstates \( \tilde{\psi}_2^{(n)} \), \( \tilde{\psi}_3^{(n)} \) after the electroweak symmetry breaking are obtained as,

\[
\begin{pmatrix}
\tilde{\psi}_1^{(n)} \\
\tilde{\psi}_2^{(n)} \\
\tilde{\psi}_3^{(n)}
\end{pmatrix} = O \begin{pmatrix}
\psi_1^{(n)} \\
\psi_2^{(n)} \\
\psi_3^{(n)}
\end{pmatrix}, \quad O = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}.
\]

The relevant Feynman rules for our calculation can be readily read off from this lagrangian.

First, we compute \( \mu \nu \) components (i.e. \( D \)-dimensional components) of the vacuum polarization tensor \( \Pi_{MN}(p^2) \) where a zero mode and nonzero KK modes of triple fermions are running in the loop. As an example, we consider the polarization tensor \( \Pi_{\mu\nu} \) of photon \( \gamma_\mu \). The result is

\[
\Pi_{\mu\nu}(p^2) = \frac{2^{[D/2]} g_D^2}{2(4\pi)^{D/2}} (p_\mu p_\nu - p^2 g_{\mu\nu}) \int_0^\infty dt t^{1-D/2} \sum_{n=-\infty}^{\infty} R \sqrt{\frac{\pi}{t}} e^{-\left(\pi R m\right)^2 t - 2 \pi i a n} \tag{20}
\]

where a dimensionless constant \( a \) is defined as \( a \equiv m_W R \). In the above calculation, Poisson resummation formulae are applied.

\[
\sum_{n=-\infty}^{\infty} e^{-\left(\frac{\pi i a n}{R}\right)^2 t} = \sum_{m=-\infty}^{\infty} R \sqrt{\frac{\pi}{t}} e^{-\left(\pi R m\right)^2 t - 2 \pi i a m}, \tag{21}
\]

\[
\sum_{n=-\infty}^{\infty} \left(\frac{n + a}{R}\right)^2 e^{-\left(\frac{\pi i a n}{R}\right)^2 t} = \sum_{m=-\infty}^{\infty} R \left(\frac{1}{2} \sqrt{\frac{\pi}{t^3}} - \sqrt{\frac{\pi}{t^5}} \left(\pi R m\right)^2 \right) e^{-\left(\pi R m\right)^2 t - 2 \pi i a m}. \tag{22}
\]

Note that only the relevant terms of order \( O(p^2) \) for the wave function renormalization factor are extracted in (20). The divergence appears only in the zero winding mode \( (n = 0 \text{ mode after Poisson resummation}) \), we thus obtain the divergent and finite part of the wave function renormalization factor as

\[
\Pi_{\mu\nu}^\text{div}(p^2) = \frac{2^{[D/2]} g_D^2}{2(4\pi)^{D/2}} R \sqrt{\pi}(p_\mu p_\nu - p^2 g_{\mu\nu}) \int_0^\infty dt t^{1-D/2} \tag{23}
\]

\[
\Pi_{\mu\nu}^\text{finite}(p^2) = \frac{2^{[D/2]} g_D^2}{2(4\pi)^{D/2}} (p_\mu p_\nu - p^2 g_{\mu\nu}) \int_0^\infty dt t^{1-D/2} \sum_{n=1}^{\infty} R \sqrt{\frac{\pi}{t}} e^{-\left(\pi R m\right)^2 t - 2 \cos(2\pi n a)} \tag{24}
\]

\[
= \frac{2^{[D/2]} g_D^2}{2(4\pi)^{D/2}} 2 R \sqrt{\pi} \Gamma \left(\frac{D - 3}{2}\right) (p_\mu p_\nu - p^2 g_{\mu\nu}) \sum_{n=1}^{\infty} \frac{\cos(2\pi n a)}{(\pi R m)^{D-3}}.
\]

To see the violation of gauge-Yukawa universality, we next calculate the \( yy \) component of the vacuum polarization tensor. As a matter of fact, the \( A_y \) partner of the photon, say \( \gamma_y \)

\footnote{Top Yukawa coupling is not generated in the case of triplet fermion. We will later consider the fermion in the large dimensional representation inducing top Yukawa coupling, such as 15.}
does not have a zero mode. We, however, expect that at least the UV-divergence due to the quantum correction to an SU(3) invariant local operator is common, irrespectively of the choice of the gauge generator. So, $\gamma_y$ is expected to mimic, say $\phi^0_0$. The result reads as

$$\Pi_{yy}(p^2) = \frac{2[D/2]}{2(4\pi)^{D/2}p^2} \sum_{n=-\infty}^{\infty} \int_0^\infty dt \frac{1}{t^2 (1 + \frac{(\pi Rn)^2}{t})} e^{-\frac{(\pi Rn)^2}{t} - 2\pi in a}. \quad (25)$$

The divergent and finite part are found,

$$\Pi_{yy}^{\text{div}}(p^2) = \frac{2[D/2]}{2(4\pi)^{D/2}p^2} \int_0^\infty dt t^{(1-D)/2} R \sqrt{\pi}, \quad (26)$$

$$\Pi_{yy}^{\text{finite}}(p^2) = \frac{2[D/2]}{2(4\pi)^{D/2}2p^2} \sum_{n=1}^{\infty} \int_0^\infty dt t^{(1-D)/2} R \sqrt{\pi} \left(1 + \frac{(\pi Rn)^2}{t}\right) e^{-\frac{(\pi Rn)^2}{t}} \cos(2\pi na)$$

$$= \frac{2[D/2]}{2(4\pi)^{D/2}2R^2} R \sqrt{\pi} \Gamma \left(\frac{D-3}{2}\right) p^2 \left(\frac{D-1}{2}\right) \sum_{n=1}^{\infty} \frac{\cos(2\pi na)}{(\pi Rn)^{D-3}}. \quad (27)$$

We can see that the divergence coefficients of $p^2 g_{\mu\nu} - p_\mu p_\nu$ components in $\Pi_{\mu\nu}$ and $p^2$ components in $\Pi_{yy}$ agree as it should be from the $D + 1$ dimensional Lorentz invariance. That means that the ratio of the wave function renormalization factors $Z_\mu/Z_y$ are finite, namely the explicit finite part expression is found,

$$\sqrt{\frac{Z_\mu}{Z_y}} = 1 + \frac{2[D/2]}{2(4\pi)^{D/2}} R \sqrt{\pi} \Gamma \left(\frac{D-1}{2}\right) \sum_{n=1}^{\infty} \frac{\cos(2\pi na)}{(\pi Rn)^{D-3}}$$

$$\rightarrow 1 + \frac{g_4^2}{16\pi^2} \sum_{n=1}^{\infty} \frac{\cos(2\pi na)}{n} (D \rightarrow 4)$$

$$= 1 - \frac{g_4^2}{16\pi^2} \log(2 \sin(\pi a)) \quad (28)$$

where $\Gamma(3/2) = \sqrt{\pi}/2$. In the second line, we have taken the limit corresponding to the five dimensional case $D \rightarrow 4$. The mode sum can be carried out exactly in the last line. In this way, we have shown that the gauge-Yukawa universality violation at one-loop is finite and calculable regardless of the non-renormalizability of the model.

Next, we apply this calculable violation of gauge-Yukawa universality to generate a viable top quark mass. First we note that the ratio between top quark mass and W-boson mass can be obtained by multiplying the Higgs VEV to both the numerator and the denominator of the ratio $m_t$. Second, let us also note that the violation of the universality in $g_4^2/16\pi^2$ rapidly increases for small Higgs VEV $a$, as is shown in Fig. 2. Though we leave $a$ as a free parameter in this analysis, since it is highly dependent on the detail of the matter content, small $a$ is needed anyway to realize the electroweak symmetry breaking $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$ and sufficiently large Higgs mass. Because of the one-loop factor $g_4^2/(16\pi^2)$ in $g_4^2/16\pi^2$, we further need additional enhancement factor. In this paper, we consider the case that such an enhancement factor comes from group theoretical factor, i.e. a second rank Dynkin index $C_2(R)$ of representation $R$ defined as
Figure 2: Higgs VEV dependence of the mode sum. The horizontal axis is Higgs VEV \( a \) and the vertical axis denotes the mode sum \( \sum_{n=1}^{\infty} \frac{\cos(2\pi na)}{n} = -\log(2\sin(\pi a)) \).

\[ Tr(T^a(R)T^b(R)) = C_2(R)\delta^{ab} \] when the fermions embedding a top quark belonging to large dimensional representation. In this case, the result (28) is modified only by multiplying a Dynkin index,

\[ \frac{m_t}{m_W} = \sqrt{\frac{Z_\mu}{Z_y}} = 1 + \frac{g_4}{8\pi^2}C_2(R) \sum_{n=1}^{\infty} \frac{\cos(2\pi na)}{n} (D \to 4) = 1 - \frac{g_4^2}{8\pi^2}C_2(R)\log(2\sin(\pi a)) \] (29)

where we restricted to the case of five dimensional space-time.

For instance, if we consider a fermion belonging to the representation with rank 4 discussed in [15] to reproduce top quark mass, their Dynkin indices are given as

\[
\begin{array}{c|ccc}
R/C_2(R) & 15 & 24 & 27 \\
17.5 & 25 & 27 \\
\end{array}
\] (30)

where the normalization is taken to be \( C_2(\Box) = 1/2 \) for the fundamental representation.

Now, the corresponding one-loop corrections are displayed for each representation in Fig. 3. To obtain \( O(1) \) correction by compensating a factor \( g_4^2/(16\pi^2) \), we found the upper bound on \( a \) as \( a < 0.002 \) for 15, \( a < 0.005 \) for 24 and \( a < 0.01 \) for 27. In other words, these constraints can be translated into those for the compactification scale through \( m_W = a/R, R^{-1} > 40 \) TeV for 15, \( R^{-1} > 16 \) TeV for 24 and \( R^{-1} > 8 \) TeV for 27.

The difference of the approaches between [15] and ours is that the standard model fermions are not needed to be localized at the branes in our case. All of the standard model fermions may be embedded in the bulk fields and their Yukawa coupling can be uniquely generated by the bulk gauge coupling. Furthermore, we do not need extra massive bulk fermions. This feature makes the model building (in particular the flavor sector) in the gauge-Higgs unification greatly simplified.

Next, let us consider whether the fermion with twisted boundary condition along the extra dimension can improve the above result. The contribution of the fermion with
The mode sum is shown in Fig. 4. We immediately see that the mode sum of twisted fermion is negative in the range $0 < a < 1$ and its contribution does not help to enhance the Yukawa coupling.

In summary, we have discussed the violation of “gauge-Yukawa universality” in the gauge-Higgs unification. Although the Yukawa coupling is given by the gauge coupling in the gauge-Higgs unification at the classical level, such universality is violated by quantum corrections to each coupling. We have shown that the violation of gauge-Yukawa universality is finite and calculable using in the $SU(3)$ gauge-Higgs unification model with arbitrary space-time dimension compactified on an orbifold $S^1/Z_2$. The point is that the gauge-Yukawa universality violation is parameterized by the ratio of Yukawa coupling and the gauge coupling and the ratio is further expressed by the ratio of the wave function...
renormalization factor for the gauge field and the extra component of the gauge field. The ratio is clearly understood to be finite from the higher dimensional Lorentz invariance.

As an interesting application, we have proposed a mechanism to generate a viable top quark mass in flat space gauge-Higgs unification, alternative to the mechanism in \[15\]. By multiplying the Higgs VEV, the violation of gauge-Yukawa universality can lead the quantum correction to the top quark mass. We have shown that the order one correction to the top Yukawa coupling is possible if the fermion belongs to the large dimensional representation and the Higgs VEV is very small compared with the compactification radius, i.e. \(a = M_W R \ll 1\). As a result, we have obtained the constraints for the compactification scale in the case where fermions belong to the rank 4 representation (15, 24, and 27 representations of \(SU(3)\)) discussed in the literature \[15\]. One of the advantages of our approach is that the standard model fermion is not needed to be localized on the branes and no extra massive bulk fermions are required. This makes the model building of the gauge-Higgs unification (in particular the flavor physics) greatly simplified.

We hope that this approach shed some new insights on the flavor physics of the gauge-Higgs unification.

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