MASSIVE TRIADIC CHERN-SIMONS SPIN-3 THEORY

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ABSTRACT

We introduce the massive gauge invariant, second order pure spin-3 theory in three dimensions. It consists of the addition of the second order gauge invariant massless pure spin-3 action with the first order topological (generalized) Chern-Simons spin-3 term corrected with lower spin auxiliary actions which avoid lower spin ghosts propagation. This second order intermediate action completes the catalogue of massive spin-3 actions having topological structure. We also consider its spontaneous break down through the addition of the inertial spin-3 nontopological Fierz-Pauli mass term. It is shown that this non gauge invariant pure spin-3 system is the uniform generalization of linearized massive vector Chern-Simons gravity and propagates just two spin $3^\pm$ excitations having different masses.

1. Introduction

Recently an alternative curved topological gravitational theory in three dimension was found and discussed in detail\cite{1}. Although massive vector Chern-Simons gravity (VCSG) propagates one massive spin-2 excitation, like topological massive

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substantial differences between them occur. TMG is described by a third order local Lorentz and diffeomorphism invariant action whereas VCSG is a second order diffeomorphism invariant theory where local Lorentz invariance has been lost. In addition, TMG can not be broken down neither by the presence of a triadic Chern-Simons term nor by a Fierz-Pauli (FP) mass term while VCSG admits a symmetry breaking when the FP mass term is present, giving rise to a double spin-2 system having two different masses (similarly to what happens to the Maxwell-CS system when the Proca mass term is considered). It is worth recalling that the dreibein $e_{ma}$ is the most natural object to describe spin-2, its gauge (linearized) transformation law is given by $\delta \lambda h_{pa} = \partial_p \lambda_a$. Self-dual massive gravity completes the catalogue of spin-2 massive excitations in three dimensions. It is a first order theory on flat Minkowski space having no gauge invariance.

For spin-3 it is known both the topological third order theory and the self-dual first order action. The former is gauge and local Lorentz invariant while the latter does not have any local symmetry left. According to what we have learnt from the spin-2 case, we will show that a gauge invariant second-order intermediate action propagating a pure spin-3$^+$ (or spin-3$^-$) excitation indeed exists. It is the uniform generalization of VCSG having the novelty of making unavoidable the presence of auxiliary lower spin fields.

2. Triadic Chern-Simons spin-3 action

The action has the form

$$I = \frac{1}{2} \left< h_{m\bar{a}} G_{m\bar{a}} \right> - \frac{\mu}{2} \left< h_{m\bar{a}} \epsilon^{mnp} \partial_n h_{p\bar{a}} \right>$$

$$+ \mu \left< v_q \epsilon^{mnp} \partial_p h_{m\bar{a}q} \right> + \frac{\alpha}{2} \mu \left< v_p \epsilon^{mnp} \partial_m v_n \right> + \frac{\beta}{2} \mu^2 \left< v_p^2 \right>$$

$$+ \mu \left< \phi (\partial \cdot v) \right> + \frac{\gamma}{2} \left< \phi \Box \phi \right> + \frac{\delta}{2} \mu^2 \left< \phi^2 \right>,$$

wherein $h_{m\bar{a}}$ is the basic spin-3 carrier written in the triadic representation, symmetric and traceless in the barred Lorentz-like indices. The core of the action consists of the usual massless spin-3 second order action ($\sim hG$) enlarged with a typical triadic Chern-Simons first order action for spin-3 ($\sim he \partial h$) which will provide mass $\mu$. It is invariant under the natural abelian gauge transformations $\delta \lambda h_{p\bar{a}} = \partial_p \lambda_{\bar{a}}$ (similarly to massive vectorial Chern-Simons linearized gravity). The remaining terms contain lower spin contributions stemming in the auxiliary fields $v_p$, $\phi$ and a gauge invariant coupling term linking the vector with the basic spin-3 field. As is well known, in $d \geq 4$, massive higher spin fields require the presence of them to make sure the non-propagation of undesirable lower spin ghosts. The basic triadic spin-3 field $h_{m\bar{a}}$ can be decompose

$$h_{m\bar{a}} = \epsilon_{mca} h_{\bar{c}b \bar{a}} + \epsilon_{mbc} h_{\bar{c}a \bar{b}} + \frac{3}{10} \left[ \eta_{pa} h_b + \eta_{pb} h_a - \frac{2}{3} \eta_{ab} h_p \right].$$
The 15 independent components of $h_{\bar{m}\bar{a}b}$, are represented by the 7 components of $h_{\bar{m}\bar{a}b}$ plus the 5 needed to describe $h_{\bar{b}e}$ plus the last 3 which determine $h_p = h_{\bar{m}\bar{m}\bar{p}}$, the unique non-vanishing trace of $h_{\bar{m}\bar{a}b}$.

Independent variations of $h_{\bar{m}\bar{a}b}$, $v_p$, $\phi$ lead to the field equations

$$E_{m\bar{a}b} \equiv G_{m\bar{a}b} - \mu \epsilon^{mpn} \partial_n h_{p\bar{a}b} + \frac{\mu}{2} [\epsilon^{ma} \partial_n v_b + \epsilon^{mb} \partial_n v_a - \frac{2}{3} \eta_{ab} \epsilon^{pmn} \partial_m v_n] = 0,$$

(3)

$$F^p = \alpha \mu \epsilon^{pmn} \partial_m v_n + \beta \mu^2 v_p + \mu \epsilon^{pmn} \partial_p h_{m\bar{m}\bar{p}} - \mu \partial_p \phi = 0,$$

(4)

and

$$G \equiv \mu (\partial Hv) + \gamma \Box \phi + \delta \mu^2 \phi = 0.$$  

(5)

In order to see their dynamical content we perform a covariant analysis using the harmonic gauge defined in this case by

$$\partial_m (h_{m\bar{a}b} + h_{a\bar{m}b} + h_{b\bar{a}m} - (\partial_a h_b + \partial_b h_a) = 0.$$  

(6)

Taking into account Eq. 2, this gauge fixing condition implies that

$$\partial_m h_{m\bar{a}b} = \frac{1}{3} [\partial_a h_b + \partial_b h_a - \frac{2}{3} \eta_{ab} \partial_m h_m],$$

(7)

and the Eq 3 becomes

$$\Box [h_{m\bar{a}b} + h_{a\bar{m}b} + h_{b\bar{a}m} - \eta_{ma} h_b - \eta_{mb} h_a] - \mu \epsilon^{mpn} \partial_n h_{p\bar{a}b}$$

$$+ \frac{\mu}{2} [\epsilon^{ma} \partial_n v_b + \epsilon^{mb} \partial_n v_a - \frac{2}{3} \eta_{ab} \epsilon^{pmn} \partial_m v_n] = 0.$$  

(8)

It is easy to check than none of the spin-2 variables have any dynamical behaviour. They do not propagate. Now, let us go to the spin-1 sector. The variables are $\rho_m h^T_{mn}, h^T_p (\rho_p \equiv \frac{\partial_k h_k}{\rho}, \rho \equiv (\Box)^2)$. The spin-1 dynamical behaviour is determined by the equations $E_p \equiv E_{m\bar{m}\bar{p}} = 0$, $\epsilon^{mac} \partial_c E_{m\bar{a}b} = 0$ and $F^p = 0$. In order not to have any spin-1 excitation alive we must choose

$$\alpha = -\frac{27}{6}, \quad \beta = -\frac{16}{3},$$

(9)

in order to make the spin-1 inverse propagator a non-vanishing number.

Unfortunately this is not the last step in order to get a pure spin-3 propagation. There are still three scalar ghosts($\partial_m h_{p\bar{b}}, \partial_p h_p, \partial_p v_p$) that might propagate. This is the reason why we have to introduce the auxiliary scalar field $\phi$. The key equations for the scalar sector are $\partial_p E_p = 0, \epsilon^{mac} \partial_c E_{m\bar{a}b} = 0, \partial_p F^p = 0$ and $G = 0$. One finds that

$$\gamma = 0, \quad \delta = -\frac{1}{6},$$

(10)
entails the non-propagation of the whole scalar sector. Consequently, the dynamics is contained in the symmetric, traceless, transverse part of the triadic spin-3 field: $h_{m\bar{n}\bar{p}}^T$. Its two independent components can be split into parity sensitive parts according to

$$h_{m\bar{n}\bar{p}}^T = h_{m\bar{n}\bar{p}}^{T+} + h_{m\bar{n}\bar{p}}^{T-},$$

with

$$h_{m\bar{n}\bar{p}}^{T\pm} \equiv \frac{1}{2} h_{m\bar{n}\bar{p}}^T \pm \frac{1}{6} (\epsilon_{rsn}\rho_r h_{s\bar{n}\bar{p}}^T + \epsilon_{rsn}\rho_r h_{s\bar{n}\bar{p}}^T + \epsilon_{rsp}\rho_r h_{s\bar{m}\bar{n}}^T).$$

Analyzing the field equation (3), we find that $h_{m\bar{n}\bar{p}}^{T-} = 0$ for $\mu > 0$. An iterative process yields

$$\Box - \frac{\mu^2}{9} h_{m\bar{n}\bar{p}}^{T+} = 0,$$

i.e. our action describes a single pure massive spin-3$^+$ excitation.

### 3. Spontaneous break-down of translational gauge invariance

We want to analyse the possibility of breaking down the (translational) local gauge invariance. We introduce a Fierz-Pauli spin-3 mass term $\sim m^2 hh$ and consider the following action

$$I = \frac{1}{2} < h_{m\bar{a}\bar{b}} G_{m\bar{a}\bar{b}} > + \frac{\mu}{2} < h_{m\bar{a}\bar{b}} \epsilon^{mnp} \partial_n h_{p\bar{a}\bar{b}} > - \frac{1}{6} m^2 < \epsilon^{mnn} \epsilon^{abc} \eta^p h_{m\bar{a}\bar{b}} h_{n\bar{c}\bar{d}} > + \mu^2 < h_p v_p > + \frac{\mu}{2} \alpha < v_p \epsilon^{pnm} \partial_n v_m > + \frac{\mu^2}{2} \beta < v_p^2 > + \mu < \phi \partial_p v_p > + \frac{\mu^2}{2} \gamma < v_p > + \frac{1}{2} \gamma < \phi \Box \phi > .$$

where we have taken for simplicity an algebraic (non-gauge invariant) coupling term $\sim \mu^2 h.v$ instead of the (gauge invariant) differential one we introduced in the initial action.

It is straightforward to prove, performing a covariant analysis that

$$\alpha = -18 \left( \frac{\mu}{m} \right)^4 = \left( \frac{\mu}{m} \right)^2 \beta, \quad \gamma = 0, \quad \delta = \frac{m^4}{24 (\mu^4 + \mu^2 m^2)}$$

induce the vanishing of all lower spin and our system only propagates two spin-3 excitations represented by the traceless, transverse, parity sensitive parts of $h_{m\bar{a}\bar{b}}^T$ defined in Eq. 11. Their evolution equations are found to be

$$\Box h_{m\bar{a}\bar{b}}^{T+} + \frac{1}{3} \mu (\Box) \frac{1}{2} h_{m\bar{a}\bar{b}}^{T+} - \frac{1}{9} m^2 h_{m\bar{a}\bar{b}}^{T+} = 0,$$

$$\Box h_{m\bar{a}\bar{b}}^{T-} - \frac{1}{3} \mu (\Box) \frac{1}{2} h_{m\bar{a}\bar{b}}^{T-} - \frac{1}{9} m^2 h_{m\bar{a}\bar{b}}^{T-} = 0,$$
giving rise to two different masses

\[ m_{\pm} = \frac{\mu}{6} \left[ (1 + 18 \frac{m^2}{\mu^2})^{\frac{1}{2}} \mp 1 \right]. \] (17)

corresponding to the two spin-3\(^\pm\) excitations that action (14) propagates.

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