Mental structure construction of field independent students based on initial proof ability in APOS-based learning

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Abstract. Mathematics proof plays an important role in learning the abstract algebra included group theory. The aim of this study was to describe the mental structures that might take place when FI students were learning the concept of group theory through APOS-based learning. This study was a descriptive qualitative. The participants of this study were eight (8) undergraduate students who were taking Introduction to Algebraic Structure 1, included group theory, at Universitas Negeri Semarang. Each of low and medium level of initial proof ability consisted of 3 participants, while high level of initial proof ability consisted of 2 participants. There were two instruments used to gather data: written examination in the course and a set of interview. The participants with low level of initial proof ability tended to construct the mental structure of Object for a set, did not construct the mental structure of Object for the binary operation and the axioms of group. The participants with medium level of initial proof ability constructed the mental structure of object for a set, tended to construct Object for the binary operation, the axioms of group, and tended to construct the scheme of group. The participants with high level of initial proof ability constructed mental structure of Object for a set, has not fully constructed the mental structure of Object for the binary operation and for the axioms of group yet, as well as has not fully constructed the scheme of group.

1. Introduction
The characteristic of group theory in mathematics is general and abstract. According to Piaget's cognitive development theory, undergraduate students are in formal operational stages so as to have no difficulty in studying abstract topics. However, in reality undergraduate students have difficulty in studying group theory.

The problematic of constructing or understanding proofs were faced by students at all levels. Writing proofs is crucial [1]. One of the result the preliminary study of the proof that not all mental structure on the refinement of the preliminary genetic decomposition can be constructed by participants so that there were still obstacles in the process of proving [2]. In line with this result mathematical proof was a demanding task for early undergraduate students [3]. Students of all ages face difficulties in constructing rich mathematical arguments [4].

In essence, a proof is a convincing argument that justify the truth of a mathematical statement [5]. A proof is a convincing argument expressed in the language of mathematics that a statement is true. A proof contain enough mathematical details to be convincing to the persons to whom the proof is addressed [6]. A theorem consists of some claims, each claim is justified by referencing some statements whose truth has been already established [5]. The roles of proof are to verify, explain,
systematize, discover and communicate [7]. General representations, such as mathematical symbol or quantified variables were often used in formal proofs [8].

Since every mathematical structure, object or entity can be described as a set then sets are fundamental. Logic is basic because it accepts us to know the senses of statements, to deduce facts about mathematical structures and to reveal further structures. Upon mastering the idea of function, students will prepared for advanced mathematics courses such as abstract algebra, analysis, combinatorics and topology [9]. Proof-based courses, such as abstract algebra, cover logic, sets, relations, and functions [10]. Based on these opinions, initial proof ability on group theory can be determined by some topics such as set, logic and binary operation as a function.

According to Witkin and Goodenough cognitive style is favored ways of choosing, understanding, and processing new information. The most commonly instrument to assess cognitive styles was Group Embedded Figures Test (GEFT) [11]. Cognitive style consists of two types: field independent (FI) and field dependent (FD). The FI learners used cognitive and metacognitive strategies more often than the FD counterparts [12].

The five types of mental mechanism are interiorization, coordination, reversal, encapsulation, and generalization. These five mental mechanism bring out the construction of mental structure, namely Action, Process, Object and Schema (APOS). The understanding of individual concepts depends on his/her ability to build connections among the mental structure that form it [13].

In this study, an Action was transformation of a former object(s) directed externally. An Action that occurs entirely in the mind is called a Process. The mental mechanisms interiorization or coordination or reversal constructed Processes. If a Process was understood as an entity on which Actions can be made in the mind of the individual then it becomes an Object. Objects are constructed by mental mechanism that is encapsulation. Encapsulation develops when the individual apply Actions to Processes. The mental structure Object can be de-encapsulated back to its underlying Process, if needed. A Scheme for mathematical concept is a coherent collection of mental structures and the relationship between them to give shape a frame in the mind of the individual that may be brought to the problem situation associated with the concept.

The transformation of the existing concept often bring up a new mathematical concept. The genetic decomposition consists of description of Actions that student needs to perform on the existing mental Object and continues to include explanation of how the Actions are interiorized into Processes and how Processes are encapsulated into mental Object. Likely to happen that a concept consist of various distinct Actions, Processes, and Objects [13].

APOS-based learning started with composing a genetic decomposition. The genetic decomposition of group using in this study was refer to [2]. The Scheme of group consisted of a binary operation, a set and axioms. Furthermore, the learning was arranged in three stages: Activities, Classroom discussion, and Exercises. In the first stage, learners did some task on work sheet which was done outside the classroom in groups of 3-4 students. This task was intended to assist students produce mental constructions proposed in the genetic decomposition. The focal point of this task was to increase the mental mechanism. In the second stage, lecturer presented the concepts of group held in class and begun by discussing the results of worksheets. The goal of this stage was to deliver learners the moment to analyze their task on the work sheet.

In the third stage, students did exercises that were conducted outside the classroom to reinforce results at the Activities and Classroom Discussions stages. This exercise supported the continuing development of suggested mental construction in genetic decomposition. Exercises also guided learners to apply what they studied and to think about related mathematical ideas.

The goal of this study was to describe the mental structure construction of FI learners based on initial proof ability in the concept of group theory through APOS-based learning.
2. Method
The descriptive qualitative was designed for this research. We discuss our finding based on the genetic decomposition of group.

There were eight (8) undergraduate students who were taking Introduction of Algebraic Structure 1, that included group, play role as the participants of this study. Firstly, we deliver the topics set, logic and binary operation. At the end of this lecture, we conducted a test about these topics to determine students initial proof ability. This ability was divided into three level: high, medium and low. The students also took GEFT to determine the type of their cognitive style. There were eight (8) FI students. Among the eight FI students, it was obtained that each of medium and low level of initial proof ability consisted of 3 participants, while of high level of initial proof ability consisted of 2 participants. Secondly, we conducted APOS-Based Learning. After completion of the learning for the topic of group, then we held a test of proof ability. Analysis of this test was conducted to describe the mental structure of FI students based on the initial proof ability.

The data were gathered by a written test, videotape and interviews. Instrument validation was obtained through expert judgement for the test of ability to prove and experiment for the genetic decomposition of the APOS-Based Learning. The credibility of the data was done by using triangulation: test and depth interview. Miles and Huberman model was used for data analysis. The steps of analysis were data reduction, data display, and conclusion drawing/verification.

3. Result and Discussion
We discuss our finding based on the indicators of each mental structure for the concept of groups.

![Figure 1. Scheme of group](image)

The scheme of group is constructed from schemes of set, binary operation, and axioms of group as presented in the Figure 1 above. If students constructed mental structure of object for the set and binary operation and can well performed coordination mental mechanism between mental structure of Process for the set, binary operation and the axioms then students constructed the mental structure of the scheme of group. Witkin et al. stated some of the characteristics of FI students are well-organized and structured in their learning [10]. These characteristics supported students to learn group material. The following were the mental structures of the group constructed by FI students based on their initial proof ability.
3.1. FI-Low level of initial proof ability (FI-L)

An identification of inverse element has not been fully carried out by FI-L1 participant. She also could not find the identity element in the set. Based on the interview, FI-L1 participant said that e is an identity element for * on G if for each elemen a is in G there exists e is in G such that a * e = e = e * a. This showed that FI-L1 participant did not understand the concept of an identity element. While the FI-L2 and FI-L3 participants can identify elements in the set, include an identity element and inverse elements. These indicated that the FI-L participants tended to construct the mental structure of Object for a set.

The FI-L1 and FI-L2 participants did not able to show the existence of the identity element. Even, the FI-L2 failed to show the associative law. Only the FI-L3 participant succeeded in showing the existence of the inverse element. These indicated that the FI-L participants did not construct the mental stucture of Object for the binary operation.

The FI-L Participants did not able to show the existence of the identity element and the existence of the inverse elements. The FI-L2 participant did not able to show the associative law. This indicated that FI-L2 did not construct the mental structure of Object for the axioms of group.

Based on the conditions above, the mental mechanism of coordination between the set, binary operation, and group axioms cannot be well done. The FI-L participants were not able to verify that a set was a group under a binary operation. This led to the mental structure of the scheme of group has not been constructed. There are two separate processes and there are no hints of coordination between the two [14]. According to Miyakawa, the mathematical symbols support students to take proof ideas [15]. In contrast with Miyakawa, other finding of this study was the FI-L participants tended not to be able to use mathematical symbols and language correctly. This was in accordance with Canadas et all that students experienced the particular obstacles when asked to give meaning to symbolic statements [16]. In addition, related to the notation (symbol), Isnarto reported that students were not able to use notations, symbols or mathematical terms [17].

3.2. FI-Medium level of initial proof ability (FI-M)

The FI-M participants succeeded in identifying element in the set. The concept of set has been constructed as an Object by the FI-M participants.

The FI-M1 participant had difficulties to show that the binary operation satisfied the properties of associative, existence of an identity element, and existence of an inverse element. While the FI-M2 participant succeeded in showing the existence of an identity element, and the existence of an inverse element, but failed to show the associative law. The FI-M3 participant showed well the associative law, the existence of an identity element, and the existence of an inverse element. These indicated that the binary operation tended to be constructed as an Object.

The way to show the properties of associative, existence of an identity element, and existence of an inverse element lead to the conclusion that the axioms of group tended to be constructed by FI-M participants as an Object. Moreover, the mental mechanism of coordination between set, binary operation, and axioms tended to be well done. The FI-M participants tended to construct the scheme of group. However, the FI-M participants were able to propose an example of a group but they were not able to prove it as a group. This was consistent with the characteristics of participants with low level of initial proof ability. Syntactic skills were needed by students at the basic level, namely the competence to annihilate and logically manipulate definitions, to construct proof [18]. It appeared that this skill has not been owned by FI-M participants, namely de-encapsulating objects into processes so that they can be coordinated with other processes. Additionally, the mathematical symbols and languages tended to not be used correctly. Regarding the use of language and symbols, this result in line with the finding of Viviane et all that most learners were challenged for how to use notations correctly [19].
3.3. FI-High level of initial proof ability (FI-H)
The FI-H participants can identify elements in the set well and can de-encapsulate the set in order that it can be coordinated with other processes. In this case, the FI-H participants constructed mental structure of Object for set.

The FI-H1 participant constructed the mental structure Object for the binary operation. While the FI-H2 participant did not construct the mental structure of Object for the binary operation. This was indicated by the FI-H2 participant was not able to investigate the axioms of associative, the existence of an identity element, and the existence of an inverse element of the binary operation using the correct steps. These showed that the FI-H participants has not fully constructed the mental structure of Object for the binary operation yet. The conditions above also led to the mental structure of Object for the axioms of group has not been fully constructed. These were in accordance with [20] that students still experience difficulties with proving because the lack of knowledge of mathematical theorems, definitions and concepts needed for production of proof.

On the other hand, the FI-H participants has not fully coordinated set, binary operation, and axioms. It meant that the scheme of group has not been fully constructed. The mathematical symbols and languages have not been fully used correctly. This was in conjunction with Isnarto [17].

4. Conclusion
Conclusions of this study were the FI-L participants tended to construct the mental structure of Object for a set, did not construct the mental structure Object for the binary operation and the axioms of group. In addition, the FI-L participants tended not to be able to use mathematical symbols and language correctly.

The FI-M participants constructed the mental structure Object for a set, tended to construct Object for the binary operation, the axioms of group, and tended to construct the scheme of group. The FI-M participants tended not to be able to use mathematical symbols and language correctly.

The FI-H participants constructed mental structure of Object for a set, has not fully constructed the mental structure of Object for the binary operation and for the axioms of group yet, as well as has not fully constructed the scheme of group. The mathematical symbols and languages have not been fully used correctly.

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