Experimental verification of quasi-periodic-orbit stabilization using a switched-capacitor chaotic neural network circuit

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Abstract: Pole assignment control for stabilizing a quasi-periodic orbit in a discrete-time dynamical system has been previously proposed. In this paper, the pole assignment method is applied to a switched-capacitor chaotic neural network circuit. For circuit experiments in which there are unknown circuit characteristics and parameters, and inevitable noise, the control method is modified by introducing new control input signals. As a result, the quasi-periodic orbits are successfully stabilized through pole assignment control. In order to confirm the quasi-periodicity of the obtained orbits, bifurcation diagrams and phase-plane portraits are provided. In addition, a statistical test designed for noisy experimental data, in particular, further confirms the quasi-periodicity. Through circuit experiments, the feasibility, usefulness, efficacy, and robustness of pole assignment control for quasi-periodic orbits are verified.

Key Words: quasi-periodic orbit, pole assignment control, chaotic neural network, switched-capacitor circuit

1. Introduction
One of the salient features of the FIRST Aihara Innovative Mathematical Modelling Project [1] is the

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tight collaboration and integration among the researchers of the complex dynamical system and control
theories, as illustrated in [2]. In Chapter 8 of [2], in particular, three quasi-periodic orbit stabilization
techniques, namely, the external force control, delayed feedback control, and the pole assignment
control methods, were proposed [3]. Among them, the pole assignment method is applicable and useful
for controlling quasi-periodic orbits in real dynamical systems because the information required for
control is minimal [3]. We have demonstrated the effectiveness of the pole assignment method on real
physical dynamical systems using a switched-capacitor (SC) chaotic neural network (CNN) (SC-CNN)
electrical circuit [4, 5]. There are difficulties in the circuit experiments owing to circuit non-idealities
such as mismatches, variations, drifts in the device and circuit characteristics, noise, and uncertainty
in the circuit parameter values. Therefore, necessary information for the pole assignment method
is not always available. In addition, it is difficult to guarantee that the experimentally obtained
orbits, contaminated by both dynamical and circuit noises, are certainly quasi-periodic. It is also not
practical to exactly calculate the Lyapunov exponent, a useful quasi-periodicity measure, from such
noisy experimental measurements [6].

In this paper, we further refine and extend the experiments with an SC-CNN circuit [4, 5]. For
stabilizing the quasi-periodic orbits in real circuits, based on the pole assignment method, we develop
a control technique without using the prior information for a fixed point, e.g., the derivative of a
nonlinear function, in the target system. We use asymmetric SC-CNNs with three and five neurons,
as well as a symmetric CNN with three neurons, for the experiments. To judge the quasi-periodicity
in the resulting controlled orbits, we employ the statistical test proposed in the companion paper [6],
in addition to bifurcation diagrams and phase-plane portraits. The experimental results establish
the effectiveness of the pole assignment method in real nonlinear dynamical systems with inevitable
non-idealities.

2. Stabilization techniques for unstable quasi-periodic orbits

Three methods for controlling and stabilizing unstable quasi-periodic orbits in a discrete-time non-
linear dynamical system were proposed and summarized in [3].

The first method is the external force control technique. The advantage of this method is the high
reproducibility of the unstable quasi-periodic orbit. However, this method needs information on the
target unstable quasi-periodic orbit itself, in advance.

The second stabilization method [7] is based on delayed feedback control. This technique relies
on the almost periodicity of a quasi-periodic orbit; therefore, it is difficult to stabilize the exact
quasi-periodic orbit. However, prior knowledge of the target quasi-periodic orbit is not required.

The third is the pole assignment method, based on the control of a fixed point [8]. This technique
does not require considerable system knowledge; hence, it is applicable to real physical and biological
systems. Therefore, we adopt the pole assignment method in our circuit experiments. As the control
is applied to a fixed point instead of directly to a quasi-periodic orbit, it is difficult to exactly target
a specific quasi-periodic orbit for stabilization.

2.1 Pole assignment method for chaotic neural networks

The chaotic neural network model [9, 10] has been well-established and exhaustively studied in both
theory and application [11]. It can be presented as a simple one-dimensional discrete-time dynamical
system; therefore, it is suitable for circuit implementation [12, 13]. Moreover, we have sufficient
experience for precisely handling SC-CNN circuits, even for large-scale networks [14, 15]. Therefore,
we use the SC-CNN circuit as an experimental vehicle to verify the effectiveness of the pole assignment
method in real physical systems.

The dynamics of the \(i\)-th chaotic neuron in a network with \(N\) neurons is modeled using [9]

\[
x_i(t_{n+1}) = kx_i(t_n) - \alpha y_i(t_n) + \sum_{j=1}^{N} w_{ij} y_j(t_n) + F \cdot u_i(t_n) + \theta,
\]

\[
y_i(t_n) = f(x_i(t_n)),
\]
where \( t_n \) is the discrete-time with an integer time index, \( n \); \( x_i(t_n) \) is the internal state of neuron, \( i \), at a discrete-time, \( t_n \); \( k \) is the refractoriness damping factor; \( \alpha \) is the refractoriness scaling parameter; \( w_{ij} \) is the connection weight from neuron, \( j \), to neuron, \( i \); \( u_i(t_n) \) is the control input with a gain, \( F \), used for pole assignment; \( \theta \) is the external bias; and \( f(\cdot) \) is a monotonically increasing continuous nonlinear output function.

The pole assignment methods adopted for asymmetric [4–6] and symmetric CNNs are briefly explained below.

### 2.1.1 Pole assignment method for asymmetric chaotic neural networks

We first consider CNNs in which the constituent chaotic neurons are connected to the other neurons via asymmetric synaptic connection weights, i.e., \( w_{ij} \neq w_{ji} \) in Eq. (1). We use a network with three neurons to illustrate the pole assignment method for asymmetric CNNs: In Eq. (1), \( N = 3 \), and \( w_{ij} \) are defined by the connection matrix, \( w \), as

\[
 w = \begin{bmatrix}
 \beta & -\beta - d & -\beta + d \\
 -\beta + d & \beta & -\beta - d \\
 -\beta + d & -\beta - d & \beta 
\end{bmatrix},
\]

(3)

where \( \beta \) is the base connectivity common to all the neurons and \( d \) induces asymmetry.

If there is no control signal, i.e., \( u_i(t_n) = 0 \), the asymmetric CNN given by Eqs. (1)–(3) has a synchronous solution, where \( x_1(t_n) = x_2(t_n) = x_3(t_n) \). In this case, the system reduces to a one-dimensional chaotic neuron [9], given by,

\[
x(t_{n+1}) = k x(t_n) - \alpha y(t_n) + \theta = k x(t_n) - \alpha f(x(t_n)) + \theta.
\]

(4)

The pole assignment method focuses on the fixed point of the synchronous state of Eq. (4), which we assume to exist and define as \( x^* \). In this case, Eq. (4) leads to

\[
x^* = k x^* - \alpha f(x^*) + \theta.
\]

(5)

The system in Eqs. (1)–(3) has three eigenvalues:

\[
\lambda_1 = k - \alpha \gamma,
\]

(6)

\[
\lambda_{2,3} = k - \left( \alpha - \frac{3}{2} \beta \right) \gamma \pm \frac{i \sqrt{3}}{2} d \gamma,
\]

(7)

where, \( \gamma = f'(x^*) \). The control input, \( u_i(t_n) \) (\( i = 1, 2, 3 \)), and the gain, \( F \), in Eq. (1) for reassigning the real eigenvalue, \( \lambda_1 \), to the desired one, \( \lambda_{new} \), can be given by [4–6]

\[
u_i(t_n) = \sum_{j=1}^{3} y_j(t_n) - 3F(x^*),
\]

(8)

\[
F = \frac{\lambda_{new} - k + \alpha \gamma}{3 \gamma}.
\]

(9)

This control input renders it possible to locate \( \lambda_{new} \) such that the complex conjugate eigenvalues, \( \lambda_2 \) and \( \lambda_3 \), will become unstable through the Neimark-Sacker bifurcation before \( \lambda_{new} \). As a result, we can obtain a quasi-periodic orbit from the bifurcation of \( \lambda_2 \) and \( \lambda_3 \), instead of the periodic orbit resulting from the period-doubling bifurcation of \( \lambda_1 \), observed in the original system with \( u_i(t_n) = 0 \) [4–6].

### 2.1.2 Pole assignment method for symmetric chaotic neural networks

This section uses the 3-neuron network again as an example to demonstrate the pole assignment method for symmetric CNNs in which \( w_{ij} = w_{ji} \), in Eq. (1). In addition, we set \( F = 1 \) in the symmetric network control, for simplicity. In a symmetric 3-neuron network, the connection weight matrix is defined as,
In this case, the eigenvalues are all real and are given by
\[
\lambda_1 = k - \alpha \gamma,
\]
\[
\lambda_{2,3} = k - \left( \alpha - \frac{3}{2} \beta \right) \gamma.
\]

Therefore, the original system does not have any inherent quasi-periodic orbit.

In order to introduce the complex conjugate eigenvalues, we design the control input, \( u_i(t_n) \), as,
\[
u_1(t_n) = \frac{d}{2} \left( y_2(t_n) - y_3(t_n) \right),
\]
\[
u_2(t_n) = \frac{d}{2} \left( y_3(t_n) - y_1(t_n) \right),
\]
\[
u_3(t_n) = \frac{d}{2} \left( y_1(t_n) - y_2(t_n) \right).
\]

As a result, the symmetric network defined by Eq. (10) is translated to the asymmetric one in Eq. (3).

To stabilize a quasi-periodic orbit in the same manner as in Section 2.1.1, the parameter, \( d \), should satisfy the following condition because we set \( F = 1 \).
\[
d > \frac{2 \sqrt{1 - (k - (\alpha - \frac{3}{2} \beta) \gamma)^2}}{\sqrt{3} \gamma}.
\]

This condition guarantees that \( \lambda_2 \) and \( \lambda_3 \) become unstable before \( \lambda_{\text{new}} \).

3. Experimental verification using SC chaotic neural network circuits

The stabilization of the quasi-periodic orbits in CNNs as per the above-mentioned pole assignment method can be easily demonstrated through numerical simulations, where all the variables and parameters are known [4–6]. However, the stabilization techniques shown in Section 2 cannot be readily applied to a real electrical circuit because of the following specific issues in circuit experiments.

1. Some details of the target dynamical system (circuit) such as its characteristics, e.g., \( f(\cdot) \) and the parameter values, are not exactly known. Therefore, information for the fixed point required for the pole assignment method is usually not fully available.

2. Fluctuations and variations such as device mismatches, offsets, drifts and thermal effects, and noise are inevitable. As a consequence, the observation and control accuracies are limited.

3. It is difficult to establish whether the controlled state is quasi-periodic based on the measurement results mainly because of 1. and 2. mentioned above.

The solutions for the problems listed in 1. and 2. will be provided in this section, through experimental setups. For item 3. above, we employ the statistical test for quasi-periodicity proposed in the companion paper [6], together with bifurcation diagrams and phase-plane portraits.

3.1 SC chaotic neural network circuit

The SC circuit implementation of the \( i \)-th neuron in a CNN [12–15], given by Eq. (1), is shown in Fig. 1 [4, 5]. In the figure, the dotted-double-rectangle marked “SC-CKT-W” is the SC circuit block for implementing the weighted sum of the outputs from the other constituent neurons, i.e.,
\[
- \sum_{j=1, j \neq i}^{N} w_{ij} y_j(t_n),
\]
where the minus sign is for circuit implementation. On the other hand, “SC-CKT-U” implements the control input, $F \cdot u_i(t_n)$. The SC-CKT-W and SC-CKT-U blocks prepared for neuron 1 (i.e., $i = 1$) are illustrated in Figs. 2 and 3, respectively, for the asymmetric network case.

In Fig. 1, the nonlinear output function of Eq. (2) is realized by the circuit shown in Fig. 4. Note that this circuit realizes $-f(\cdot)$ rather than $f(\cdot)$. The circuit parameters in Fig. 4 are $R_1 = 22 \, \text{k}\Omega$, $R_2 = 1 \, \Omega$, $R_3 = 11 \, \text{k}\Omega$, $R_4 = 83 \, \text{k}\Omega$, and $R_5 = 22 \, \text{k}\Omega$.

On the printed circuit board (PCB) shown in Fig. 5, the five chaotic neuron circuits depicted in Fig. 1 were mounted along with auxiliary circuits. On the PCB, various CNNs can be easily constructed with different neuron numbers, and different connection patterns with different sets of weights. In the following sections, we present the experimental results obtained using the PCB shown in Fig. 5. Through the circuit experiments, we deal with the specific practical issues listed in the
beginning of Section 3.

3.2 Experimental results for asymmetric chaotic neural network circuits

In this section, the experimental results for asymmetric CNNs with three and five neurons, respectively, are demonstrated.

3.2.1 3-neuron asymmetric chaotic neural network

We constructed the asymmetric 3-neuron CNN defined by Eq. (3) using the circuit shown in Fig. 1, on the PCB in Fig. 5.

As mentioned in Section 2.1.1, we can stabilize a quasi-periodic orbit by applying the control signal represented by Eqs. (8) and (9). However, $\gamma$ in Eq. (9), is not available for a real circuit; even the exact shape of $f(\cdot)$ is unknown. Hence, we cannot use these equations for control in real situations. Thus, we modify them to avoid the use of $\gamma$ and $f(\cdot)$, as follows:

First, we introduce a modified control input,

$$u_i(t_n) = \sum_{j=1}^{3} y_j(t_n) - 3Q,$$

where, $0 \leq Q \leq 1$ is a constant parameter. With this $u_i(t_n)$, the fixed point of the synchronized network (Eq. (4)) is now given by the reduced equation,

$$x^* = kx^* - \alpha f(x^*) + 3Ff(x^*) + A,$$
where we define $A \triangleq \theta - 3FQ$.

In order to demonstrate the feasibility of this modification, we derive the eigenvalues of the modified network as,

$$
\lambda_1 = k - \alpha \gamma + 3F\gamma,
\lambda_{2,3} = k - \left(\alpha - \frac{3}{2} \beta\right) \gamma \pm \frac{i}{2} \sqrt{3} d\gamma.
$$

(20) (21)

It can be observed that $\lambda_{2,3}$ are not affected by the modified control input of Eq. (18); in contrast, we can control $\lambda_1$ through $F \cdot u_i(t_n)$. Therefore, we can use the pole assignment method with Eq. (18) as the control input.

Next, we determine the control gain, $F$, in Eq. (9), when $u_i(t_n)$ is given by Eq. (18). To preserve the original dynamics as much as possible, even with $u_i(t_n)$, the value of $F$ should be as small as possible. Therefore, we derive the lower bound for $F$.

Assuming that the monotonically increasing continuous nonlinear function, $f(\cdot)$, is symmetric around the origin, as in a sigmoidal function, the largest value of $f'(x)$ would be obtained at the origin, i.e., $f'(0)$. This results in

$$
0 \leq \gamma \leq f'(0).
$$

(22)

For quasi-periodic orbit stabilization through the pole assignment method, we should guarantee that the real eigenvalue, $\lambda_1$, resides in the unit circle, even when $\gamma = f'(0)$. Therefore, from Eq. (20),

$$
k - \alpha f'(0) + 3F f'(0) > -1.
$$

(23)

From this equation, the lower bound for $F$ can be obtained as

$$
F > \frac{\alpha f'(0) - k - 1}{3f'(0)}.
$$

(24)

As a result, we can use Eq. (24) to set the value of $F$ as small as possible.

In real situations, we cannot directly use Eq. (24) because we do not know the value of $f'(0)$. However, the value of $f'(0)$ (not the entire shape of $f(\cdot)$) can be obtained by direct measurement or it can be estimated through experimental results such as return maps. In the following circuit experiments, we use the value, $f'(0) = 7.92 \, \text{V/V}$, obtained by direct measurement on the circuit in Fig. 4. In addition, we illustrate the experimental results with $k = 0.7$, $\alpha = 1$, $\beta = 0.66$, and $d = 0.15$. In this case, $F > 0.262$. Therefore, we use $F = 0.27$ in the experiments.

We can determine the capacitor values in Figs. 1, 2, and 3 as,

$$
\frac{C_K}{C_0} = 1 - k,
\frac{C_{ii}}{C_0} = \alpha - \beta, \text{ for } i = 1, 2, 3,
\frac{C_{13}}{C_0} = \frac{C_{31}}{C_0} = \frac{C_{21}}{C_0} = \frac{C_{32}}{C_0} = \frac{\beta + d}{2},
\frac{C_{12}}{C_0} = \frac{C_{23}}{C_0} = \frac{C_{31}}{C_0} = \frac{\beta - d}{2},
\frac{C_U}{C_0} = F.
$$

(25) (26) (27) (28) (29)

The capacitor values used in the experiments are summarized in the left column of Table I.

Figure 6 displays the bifurcation diagram of $x_1(t_n)$, when $A$ is varied as the bifurcation parameter. From the figure, we can presume that a fixed point bifurcates to a quasi-periodic orbit through the Neimark-Sacker bifurcation at $A \approx -0.3 \, \text{V}$. It should be noted that this bifurcation diagram differs from that of an ordinary (i.e., without control) chaotic neuron [5, 9], where a period-doubling route to chaos can be typically observed.
Table I. Capacitor values for the circuit experiments.

| Asymmetric 3-Neuron CNN | Asymmetric 5-Neuron CNN | Symmetric 3-Neuron CNN |
|-------------------------|-------------------------|------------------------|
| $C_0$                   | 470 pF                  | $C_0$                  | 470 pF                  |
| $C_K$                   | 141 pF                  | $C_K$                  | 141 pF                  |
| $C_A$                   | 470 pF                  | $C_A$                  | 470 pF                  |
| $C_U$                   | 127 pF                  | $C_U$                  | 75 pF                   |
| $C_{11}, C_{22}, C_{33}$| 157 pF                  | $C_{11}, C_{22}, C_{33}, C_{44}, C_{55}$| 157 pF |
| $C_{12}, C_{23}, C_{31}$| 121 pF                  | $C_{12}, C_{23}, C_{34}, C_{45}, C_{51}$| 37 pF |
| $C_{13}, C_{21}, C_{32}$| 192 pF                  | $C_{13}, C_{24}, C_{35}, C_{41}, C_{52}$| 84 pF |
| -                       | -                       | $C_{14}, C_{25}, C_{31}, C_{42}, C_{53}$| 72 pF |
| -                       | -                       | $C_{15}, C_{21}, C_{32}, C_{43}, C_{54}$| 119 pF |

![Fig. 6](image6.png)

Fig. 6. Bifurcation diagram of $x_1(t_n)$, experimentally obtained from an asymmetric 3-neuron CNN circuit under pole assignment control, when $A$ is swept as the bifurcation parameter.

![Fig. 7](image7.png)

Fig. 7. Phase-plane portraits of $x_1(t_n)$ and $x_2(t_n)$ in the 3-neuron asymmetric CNN circuit for (a) $A = -0.2$ V, (b) $A = 0$ V, and (c) $A = 0.2$ V.

To confirm the quasi-periodicity, we plot the phase-plane diagram of $x_1(t_n)$ and $x_2(t_n)$, as shown in Fig. 7(a) for $A = -0.2$ V, together with those for $A = 0$ V and $A = 0.2$ V in Figs. 7(b) and (c), respectively. As shown in Fig. 7, invariant closed-loop curves on the phase-plane were observed. This strongly suggests that the observed attractors are quasi-periodic.

To further establish the quasi-periodicity of the obtained attractors in Fig. 7, the Lyapunov exponent can be a valuable and valid measure. However, it is difficult to calculate the exact value of the Lyapunov exponent from measured data, particularly from noisy data as in Fig. 7 [6]. Therefore, we use a statistical test for the quasi-periodicity in noisy data, proposed in the companion paper [6],
instead of the Lyapunov exponent. The details of the test are available in [6] included in this volume. In summary, the test uses the equivalence between a quasi-periodic system in the presence of dynamical noise and a random walk model. Since there are general tests for the random walk, we can statistically evaluate quasi-periodicity of a time series based on these tests. In our test method, we adopted the augmented Dicky-Fuller test. The test provides the p-value for a given noisy time series for determining the quasi-periodicity. If the resulting p-value is greater than 0.05, the given measured data is quasi-periodic [6]. As per [6], we calculated the p-values for the attractors shown in Fig. 7. The left column in Table II summarizes the results. As shown in the table, the p-values obtained from the attractors in Fig. 7 are all greater than 0.05. Thus, we conclude that these attractors are quasi-periodic orbits.

Therefore, the stabilization of the quasi-periodic orbits using the modified pole assignment method described above was successful.

### 3.2.2 5-neuron asymmetric chaotic neural network

In this section, we experimentally demonstrate the versatility of the quasi-periodic orbit stabilization technique based on the pole assignment method by extending the above-mentioned 3-neuron network to an asymmetric CNN with five neurons [5].

We consider the 5-neuron CNN given by the weight matrix,

\[
W = \begin{bmatrix}
\beta & \beta & \beta & \beta & \beta \\
-\beta + d_1 + d_2 & -\beta + d_1 + d_2 & -\beta + d_1 + d_2 & -\beta + d_1 + d_2 & -\beta + d_1 + d_2 \\
-\beta - d_1 - d_2 & -\beta - d_1 - d_2 & -\beta - d_1 - d_2 & -\beta - d_1 - d_2 & -\beta - d_1 - d_2 \\
-\beta + d_1 + d_2 & -\beta + d_1 + d_2 & -\beta + d_1 + d_2 & -\beta + d_1 + d_2 & -\beta + d_1 + d_2 \\
-\beta - d_1 - d_2 & -\beta - d_1 - d_2 & -\beta - d_1 - d_2 & -\beta - d_1 - d_2 & -\beta - d_1 - d_2 \\
\end{bmatrix},
\]

where \( \beta \) is the base connectivity common to all the neurons, and \( d_1 \) and \( d_2 \) induce asymmetry.

In this case, the control input is derived as

\[
u_i(t_n) = \sum_{j=1}^{5} y_j(t_n) - 5Q,
\]

where \( 0 \leq Q \leq 1 \) is a constant parameter. With this \( u_i(t_n) \), the fixed point of the synchronized network is given by the reduced equation,

\[
x^* = kx^* - \alpha f(x^*) + 5F f(x^*) + A,
\]

where we define \( A \equiv \theta - 5FQ \).

As a result, the eigenvalues of the network are

\[
\lambda_1 = k - \alpha \gamma + 5F \gamma,
\]

\[
\lambda_{2,3} = k - \left( \alpha - \frac{5}{4} \beta \right) \gamma \pm i \sqrt{\frac{5 \left( d_1^2 + d_2^2 \right) \gamma^2 + 2 \sqrt{5} \left( d_1^2 + d_1 d_2 - d_2^2 \right) \gamma^2}{4}},
\]

\[
\lambda_{4,5} = k - \left( \alpha - \frac{5}{4} \beta \right) \gamma \pm i \sqrt{\frac{5 \left( d_1^2 + d_2^2 \right) \gamma^2 - 2 \sqrt{5} \left( d_1^2 + d_1 d_2 - d_2^2 \right) \gamma^2}{4}}.
\]

From these equations, we can control \( \lambda_1 \) through \( F \cdot u_i(t_n) \) without affecting \( \lambda_{2,3,4,5} \); hence, the pole assignment method can be used on \( \lambda_1 \).

**Table II.** Resulting p-values from the quasi-periodicity tests [6] on experimental data.

| Asymmetric 3-Neuron CNN | Asymmetric 5-Neuron CNN |
|--------------------------|-------------------------|
| Fig. 7(a)                | 0.16                    | Fig. 9(a)                | 0.9563 |
| Fig. 7(b)                | 0.646                   | Fig. 9(b)                | 0.2757 |
| Fig. 7(c)                | 0.9679                  | Fig. 9(c)                | 0.6591 |
From Eq. (33), we can obtain the lower bound for $F$, assuming again that $f(\cdot)$ has its largest derivative at the origin, $f'(0)$, given by

$$F > \alpha f'(0) - k - \frac{1}{5f'(0)}. \quad (36)$$

According to Eq. (36), we select a small value of $F$ that satisfies this condition. If $f'(0) = 7.92 \, \text{V/V}$, $k = 0.7$, $\alpha = 1$, $\beta = 0.66$, $d_1 = 0.15$, and $d_2 = 0.2$, $F > 0.157$. Therefore, in the following experiments we use $F = 0.16$.

The capacitor values in Figs. 1, 2, and 3 can be determined as,

$$\frac{C_K}{C_0} = 1 - k, \quad (37)$$

$$\frac{C_i}{C_0} = \alpha - \beta, \quad \text{for } i = 1, 2, 3, 4, 5, \quad (38)$$

$$\frac{C_{12}}{C_0} = \frac{C_{23}}{C_0} = \frac{C_{34}}{C_0} = \frac{C_{45}}{C_0} = \frac{C_{51}}{C_0} = \frac{\beta - d_1 - d_2}{4}, \quad (39)$$

$$\frac{C_{13}}{C_0} = \frac{C_{24}}{C_0} = \frac{C_{35}}{C_0} = \frac{C_{41}}{C_0} = \frac{C_{52}}{C_0} = \frac{\beta - d_1 + d_2}{4}, \quad (40)$$

$$\frac{C_{14}}{C_0} = \frac{C_{25}}{C_0} = \frac{C_{31}}{C_0} = \frac{C_{42}}{C_0} = \frac{C_{53}}{C_0} = \frac{\beta + d_1 - d_2}{4}, \quad (41)$$

$$\frac{C_{15}}{C_0} = \frac{C_{21}}{C_0} = \frac{C_{32}}{C_0} = \frac{C_{43}}{C_0} = \frac{C_{54}}{C_0} = \frac{\beta + d_1 + d_2}{4}, \quad (42)$$

$$\frac{C_U}{C_0} = F. \quad (43)$$

The values of the capacitors used in the experiments are summarized in the middle column of Table I.

The bifurcation diagram of $x_1(t_n)$ is shown in Fig. 8. From the figure, it appears that a fixed point bifurcates to a quasi-periodic orbit at $A \approx -0.23 \, \text{V}$. The phase-plane portraits of $x_1(t_n)$ and $x_2(t_n)$ are shown in Fig. 9 for $A = -0.2$, 0, and 0.2 V. The noisy closed-loop curves in Fig. 9 are strong signs of quasi-periodic orbits. Therefore, we applied the statistical test for quasi-periodicity [6] to the measured data. The results are shown in the right column of Table II. All the $p$-values in the table are greater than 0.05; hence, it is highly probable that the orbits stabilized by the pole assignment method are quasi-periodic.

![Bifurcation diagram of $x_1(t_n)$](image)

**Fig. 8.** Bifurcation diagram of $x_1(t_n)$, experimentally obtained from an asymmetric 5-neuron CNN circuit under pole assignment control, when $A$ is swept as a bifurcation parameter.
Phase-plane portraits of $x_1(t_n)$ and $x_2(t_n)$ in the 5-neuron asymmetric CNN circuit for (a) $A = -0.2 \text{ V}$, (b) $A = 0 \text{ V}$, and (c) $A = 0.2 \text{ V}$.

3.3 Experimental results for a symmetric chaotic neural network circuit

In this section, the experimental results for the symmetric CNN with three neurons, discussed in Section 2.1.2 and represented by Eq. (10), are illustrated.

In this case, the control signals, $u_i(t_n)$ for $i = 1, 2, 3$, are given by Eqs. (13) to (15), respectively. These differential inputs can be easily generated using the SC circuit shown in Fig. 10, illustrated for $i = 1$ as an example. Therefore, we use the circuit in Fig. 10 as the SC-CKT-U in Fig. 1, instead of the one in Fig. 3, in the following experiments. With this SC-CKT-U, the capacitor-ratios in the symmetric 3-neuron CNN circuit can be determined as,

$$\frac{C_K}{C_0} = 1 - k,$$

$$\frac{C_{ii}}{C_0} = \alpha - \beta,$$ for $i = 1, 2, 3$,

$$\frac{C_{ij}}{C_0} = \frac{\beta}{2}$$ for $i = 1, 2, 3$, $j = 1, 2, 3$ and $i \neq j$,  

$$\frac{C_U}{C_0} = \frac{d}{2}.$$  

In addition, we can obtain the lower bound for $d$ from Eq. (16). In our case, for $k = 0.7$, $\alpha = 1$, $\beta = 0.66$, and $f'(0) = 7.92 \text{ V/V}$, $d > 0.825$. Therefore, we set $d = 1$ for the experiments. The capacitor values for the symmetric 3-neuron CNN circuit used in the experiments are listed in the right column of Table I.

Figure 11 shows the bifurcation diagram of $x_1(t_n)$. It is difficult to find a quasi-periodic orbit from the figure. Therefore, using the phase-plane portraits of $x_1(t_n)$ and $x_2(t_n)$ by changing $A$, we tried to determine the quasi-periodic orbits. As a result, some quasi-periodic orbit candidates were found. One of them is shown in Fig. 12 at $A = -3 \text{ V}$. The noisy closed-loop curve in the figure suggests that the attractor is quasi-periodic. We employed the quasi-periodicity test [6] for Fig. 12. As a result, a $p$-value of 0.2156 was obtained. This supports our conclusion that the stabilized orbit in Fig. 12 is quasi-periodic.

![SC circuit block for the control input, $u_i(t_n)$, in the symmetric networks. The circuit for $i = 1$ (Eq. (13)) is shown as an example.](image)
4. Conclusions

The experimental results of the pole assignment control technique for stabilizing quasi-periodic orbits have been presented. For the experiments, we built an SC-CNN circuit board as a test bed. With this circuit board, we constructed 3- and 5-neuron asymmetric CNNs and a 3-neuron symmetric CNN. Furthermore, we modified the control signal for real circuits considering non-ideal circuit characteristics such as unknown output functions. In the experiments, we first demonstrated bifurcation diagrams, where bifurcation from a fixed point to a quasi-periodic orbit was suggested. We then presented the phase-plane portraits, showing the closed-loop attractors as strong evidence of the controlled orbit quasi-periodicity. Finally, we used the statistical test [6] for quasi-periodicity, applicable and effective for noisy circuit experimental data, to further confirm the quasi-periodicity. Through the experiments, we successfully verified the feasibility and efficacy of pole assignment control on a quasi-periodic orbit in a real noisy circuit without exact knowledge of the circuit characteristics and parameter values. This confirms the robustness of pole assignment control.

In future, we intend to apply the control method to other classes of circuits such as coupled map lattices [16], and to further extend the control method to systems other than electrical circuits, for example, biological systems such as biological neuron cultures.

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