Research Article

Quantitative Analysis of the Topological Structure of Rock Pore Network

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As the most significant nonlinear reservoir, the rocks have complex structural characteristic. The pore structure of the rock is varied in shape and complex in connectivity. However, the prevailing methods for characterising the microstructure of rocks, such as the coordination number method and fractal theory, are still difficult to quantify the structural properties. In this study, based on the CT-scan method and a new complex network theory, the topological characteristics of rocks such as seepage path selection, degree of pore aggregation, pore importance, and pore module structure are analysed. The results show that the scale-free network model is more reliable in characterising the rock pore network than previously published structural models, and a small number of pores are the “key” to the seepage process. Besides, we proposed a new method to quantify the importance of rock pores and present the distribution characteristics and connectivity laws of the rock-pore network. This provides a new method to study the seepage process of the nonlinear reservoirs.

1. Introduction

The highly complex network of pore throats is the main conduit for CBM migration during the extraction [1–4]. Therefore, analysing the topological characteristics of rock pore networks, including morphology, connectivity, and seepage path selection, is one of the main approaches to unravelling the mechanism of hydrocarbon transport and to enhancing the recovery [5–8].

In recently published studies, many scholars have studied the pore network structure and seepage properties of rocks at the microscale [9–12]. Advances in CT and scanning electron microscopy (SEM) have enabled the application of the rock digital core. Using focused ion beam scanning electron microscope (FIB-SEM), scanning electron microscope (SEM), and computed tomography (CT), Leu et al. studied the pore size distribution, orientation, and mineralogical characteristics of shale and mudstone [13]. The digital core of shale with different porosity was extracted by Kelly et al. using FIB-SEM, and the coordination-number method is used to explore the network connectivity of shale pores [14]. Based on digital cores of shale, Yang et al. proposed a superposition algorithm capable of analysing both organic and inorganic pores of the matrix [15].
Furthermore, analyses related to the structural characteristics of rocks have been progressively recognized, and methods for the construction of rock pore-network models have been widely studied. Yang et al. proposed a pore network model to analyse the evolution of shale oil permeability under different organic matter contents [16]. Hajizadeh et al. propose a stochastic multidimensional numerical algorithm based on continuous two-dimensional multipoint statistical simulations coupled with multiscale conditional data using 2D images of the pore structure [17]. Civan et al. propose an improved shale gas transport model that can be used to determine the diffusion coefficient and permeability of shales [18]. Additionally, many published models have investigated the seepage properties of pore rocks which do not satisfy the Darcy’s law of permeability [19, 20]. However, the throat connecting the rock pores is irregular and unevenly distributed. Besides, current methods for characterising rock structure mainly adopt the Bieler number or macroscopic averaging perspectives. The rock pore structure is highly complex, and the conventional network models still have major difficulties in characterising pore connectivity and other topologies [21–23].

With the advent of the small-world model [24] and scale-free model [25], the theoretical framework of modern complex networks was initially formed. In the past 10 years, as a new interdisciplinary field, complex network theory has penetrated into many disciplines from life sciences to physics [26, 27]. Complex network theory reasonably characterises the structural complexity of network systems, the pore complexity, and the interaction of various complexity factors [28]. At present, studies of rock structures are mainly based on basic topological parameters such as macroscopic averaging or allotment numbers [29–32]. However, it is still difficult to characterise the connectivity of pore networks in a reasonably reliable way. Based on complex network theory, this paper presents a comprehensive analysis of the distribution patterns of rock networks such as average path length, degree distribution, the importance of pores in the seepage process, and the characteristics of pore aggregation. And the results of this study provide a rational approach to reveal the micromechanics of porous rocks.

2. Basic Structure of Rock Pore Network

In this section, the digital cores used for our structural analysis were constructed by the Blunt team at Imperial College [33]. Six different rock samples were selected, and the pore structures were extracted from the rock samples. The three-dimensional pore networks of the rock samples are shown in Figure 1 [34–36].

Isolated pores in the rock networks do not affect the seepage process, and they are therefore removed during the construction of three-dimensional pore networks. Based on complex network theory, we obtained the basic data of the network structures for the rocks above, as shown in Table 1. Meanwhile, based on complex network theory, we analysed the structural characteristics of the rock pore networks [37]. The degree CN of a pore is defined as the number of the pore connected with other pores:

\[
\langle CN \rangle = \frac{1}{N} \sum_{i=1}^{N} K_i = \frac{2M}{N},
\]

where \(\langle CN \rangle\) is the average degree of the network, \(M\) is the number of throats, and \(N\) is the number of pores in the network.

In order to calculate the transmission efficiency of the rock network, the network diameter \(D\) and the average path length \(L\) are used, which represent the maximum value of the distance between any two pores in the rock network and the average of the distance between any two pores in the network:

\[
D = \max_{i,j} (d_{ij}),
\]

where \(d\) is the value of the shortest path length. Besides, the average path length \(L\) can be expressed as:

\[
L = \frac{2}{N(N-1)} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} d_{ij}.
\]

Theoretical experience suggests that the network diameter \(D\) and the average path length \(L\) should be proportional to network size and porosity. However, the pattern in the actual rock samples (Table 1) does not behave in line with theoretical experience. The above experience applies to the homogeneous rock network models, while in the real rock models, the network diameter and average path length could be influenced by a variety of factors such as the number of throats, pore size, and the length of throats.

We applied network density to quantify the denseness of rock pores. The pore network density \(\rho\) is the ratio of the number of throats to its upper limit:

\[
\rho = \frac{2M}{N(N-1)}.
\]

In order to analyse whether the network can be divided into different modules, we calculated the modularity \(Q\) of the pore network:

\[
Q = \sum_i \left( a_{ij} - e_{ij} \right),
\]

where \(e_{ij}\) is the proportion of the throats in the \(i\) module and the \(j\) module to the whole rock throats, \(a_{ij} = \sum e_{ij}\). And a module is a group of pores which is physically or functionally connected, working together to perform a relatively independent function.

Besides, we applied clustering coefficient to characterise the aggregation of rock pores. The clustering coefficient can be expressed as:

\[
C = \frac{E_j}{C_{CN}} = \frac{2E_j}{CN(CN-1)},
\]

where \(E_j\) is the number of the edges connected to the \(j\) module.
Figure 1: 3D pore network of different rocks (bounding box volume: 300 μm by 300 μm by 300 μm).
where \( CN \) is the degree of pore \( i \), i.e., there are \( CN \) pores connected to pore \( i \). If there are interconnected throats among these \( CN \) pores, the number of interconnected throats among the \( CN \) pores is denoted by \( E_i \).

### 3. Sandstone Pore Network Topology Analysis

In order to analyse the connectivity of the pore network, we calculated the degree distribution for different rock samples, as shown in Figure 2. The degree distributions of the rock networks fit well with the BA scale-free model. There are no eigenvalues for the rock pore networks, and the degree of the network for different samples is mainly less than 5, which can be regarded as the "degree threshold." When the degree exceeds the "degree threshold," the number of pores is drastically reduced. This inhomogeneity is probably responsible for the characteristics: a small number of pores (with large degree values) play a decisive role in the seepage process throughout the rock network. Besides, in our previous published research, we have concluded that the scale-free network model is more reliable in characterising the degree distribution of rocks than random networks and other mainstream network methods, as shown in Figure 3 [38].

Where BA (Barabasi and Albert) scale-free network [25] was proposed in order to explain the generation mechanism of the power-law distribution. As a model for describing heterogeneous networks, the degree distribution function of BA scale-free network is

\[
P(CN) = \frac{2m(m + 1)}{CN(CN + 1)(CN + 2)} \propto 2m^2CN^{-3},
\]

where \( m \) is the number of new nodes connected to existing

| Rock type            | Berea sandstone | Carbonate C1 | Carbonate C2 | Sand pack F1 | Sandstone S8 | Synthetic silica A1 |
|----------------------|-----------------|--------------|--------------|--------------|---------------|---------------------|
| Porosity \( \phi \)  | 19.6%           | 23.3%        | 16.8%        | 33%          | 34%           | 42.9%               |
| Node number \( N \)  | 6057            | 3962         | 1933         | 974          | 1266          | 1620                |
| Edge number \( M \)  | 12098           | 6700         | 1932         | 2651         | 3833          | 1619                |
| Average degree \( CN \) | 3.995          | 3.382        | 1.999        | 5.444        | 6.055         | 1.999               |
| Network density \( D \) | 0.001          | 0.001        | 0.001        | 0.006        | 0.005         | 0.001               |
| Network diameter \( d \) | 33             | 31           | 137          | 16           | 17            | 48                  |
| Average path length \( L \) | 12.445        | 9.981        | 45.957       | 6.399        | 6.452         | 22.397              |
| Modularity \( Q \)   | 0.868           | 0.919        | 0.953        | 0.726        | 0.745         | 0.949               |
| Average clustering coefficient \( C \) | 0.296          | 0.408        | 0            | 0.317        | 0.425         | 0                   |

![Figure 2: Degree distribution of three-dimensional pore networks.](image-url)
nodes during network expansion, namely,

\[ P(CN) \approx CN^{-\gamma}. \]  

Among them, \( \gamma \) is power exponent. Besides, the degree distribution of ER random model satisfies [39]:

\[ P(CN) = \frac{N-1}{K} p^{CN} (1 - p)^{N-1-CN} \approx \frac{(CN)^{CN}}{CN!} e^{-CN}. \]  

And the construction algorithm of WS small world model is as follows [24]:

1. **Start Modeling with a Ring of Regular Networks.** The network contains \( N \) nodes, and each node connects to the \( K \) nodes closest to it. Where \( N >> K >> \ln (N) >> 1 \)

2. **Randomize Reconnection.** Reconnect each edge in the network randomly with probability \( p \), that is, one endpoint of the edge remains unchanged, while the other endpoint is taken as a randomly selected node in the network. It stipulates that there can be at most one edge between any two different nodes, and each node cannot have an edge connected to itself

In order to measure the importance of pores in seepage networks, eigenvector centrality is used. For two pores \( i \) and \( j \) in a rock network, \( a_{ij} = 1 \) when \( i \) and \( j \) are connected, otherwise, \( a_{ij} = 0 \). Then, the function of eigenvector centrality of pore \( i \) can be expressed as:

\[ x_i = \frac{1}{\lambda} \sum_{j=1}^{N} a_{ij} x_j, \]  

where \( x_i \) and \( x_j \) are the eigenvector centrality of pores \( i \) and \( j \), \( M(i) \) is the set of pores connected to pore \( i \), \( N \) is the total number of pores in the rock, and \( \lambda \) is the scaling constant. Denoting the adjacency matrix of the network by \( A = (a_{ij}) \), Eq. (10) can be expressed in matrix form as:

\[ AX = \lambda X, X = (x_1, x_2, \cdots, x_{(N)}). \]  

In this way, solving for eigenvector centrality can be transformed into a problem of computing eigenvalues and vectors or solving a system of equations. According to the Perron-Frobenius theorem, the vector \( X \) can only be the eigenvector corresponding to the largest eigenvalue of the adjacency matrix \( A \). Therefore, the eigenvector centrality of each pore in the rock network can be obtained from the eigenvector corresponding to the largest eigenvalue derived by the curtain iteration method.

The eigenvector centralities of the pore networks are shown in Figure 4, where the abscissa of the figure is the pore Id. For the different rock pore networks, the eigenvector centralities are as follows:

**Figure 3:** Comparison of different models with real-rock distribution characteristics (note: the degree distribution of regular model is the points of degree = 2, 3, 4, and 6).
The centrality of most pores is less than 0.6. Only a small proportion of the pores have a large centrality. The importance of a pore depends on both the number and the importance of its connected pores. The eigenvector centrality method defines the centrality of a pore as a function of the centrality of the pores it connects. Thus, as the eigenvector centrality increases, the connectivity of the pores in the pore network subsequently increases. In rock seepage network, if the well-connected pores are blocked with rock debris or sealed by the crust, it may result in a significant reduction in the overall connectivity of the rock network. We therefore speculate that pores with large eigenvector centrality have a more significant contribution to rock permeability.

In order to describe the structure conveniently, the Fruchterman-Reingold (FR) method is used to assign a uniform circular surface. The FR algorithm considers all nodes as electrons, and each node receives two forces: (1) the Coulomb force of the other nodes (repulsion) and (2) the Hooke

![Graphs showing eigenvector centrality distribution of pores.](image)

**Figure 4: Eigenvector centrality distribution of pores.**
Figure 5: Centrality of rock pores.
force of the edge-to-point (gravity). Then, with the interaction of the forces, the whole layout will eventually be called a state of equilibrium [40]. The pore centrality of the networks is shown in Figure 5, where the number is the Id of the pores, and the size of the pores expresses the centrality.

The clustering coefficients of the pores for different rock samples are shown in Figure 6. It can be concluded from Figure 6 that the increase of clustering coefficients was initially slow and then became evident. The clustering coefficients for the different networks are distributed between 0 and 0.6. And in conjunction with the conclusion of the analysis in Table 1, we concluded that the connectivity of real rocks is always less than that of the fully-coupled network with a clustering value of 1. As the distance between the pores increases, the possibility of the connection is greatly reduced.

Previously published studies have typically considered the pore network of rocks to be randomly distributed. However, as shown in Figures 2 and 3, the scale-free network model which satisfies the power-law distribution is more suitable for characterising the degree distribution. Therefore, we simultaneously calculated the modularity of different pore networks to compare the differences between the real rock and the stochastic model in the pore distribution characteristics. The modular characteristics of the different rock samples are shown in Figure 7, where pores in the same module are represented by the same colour and size.

It can be concluded from Figure 7 that the pore distributions of the different rocks have obvious modular characteristics. Besides, the modularity of different rock samples ranged from 0.8 to 1 (Table 1), which also demonstrates the modular
characteristic of the pore network. We therefore conclude that the pores of the rocks can be divided into groups based on their characteristic and function. In addition, the pore connectivity is proportional to the number of throat connecting the pores. Therefore, we predict that the pore groups with high connectivity have a tendency to the high permeability.

4. Conclusions

In this paper, we proposed a new method to analyse the distribution characteristics of rock-pore structures and the connectivity laws of the rock-pore network. The pore networks were studied by four parameters of the complex theory: degree
distribution, eigenvector centrality, clustering coefficient, and the modularity. The main conclusions are shown as follows:

(1) Compared with the random model, the BA scale-free network model is more suitable for charactering the rock-pore networks. And a small number of pores (with large degree values) play a decisive role in the seepage process throughout the rock network

(2) The importance of a pore depends on both the number and the importance of its connected pores. Accordingly, we propose a new method for quantifying the importance of rock pores. If the well-connected pores are blocked with rock debris or sealed by the crust, it may result in a significant reduction in the overall connectivity of the rock network

(3) The connectivity of real rocks is always less than that of the fully-coupled network. As the distance between the pores increases, the possibility of the connection is greatly reduced. Besides, the rock-pore networks have obvious modular characteristics. The networks can be divided into several groups according to the connectivity. Pore connectivity is proportional to the number of throat connecting the pores. We therefore predict that the pore groups with high connectivity have the high permeability

Data Availability

All data included in this study are available upon request by contact with the corresponding author. The digital cores of Blunt can be found in the website of Imperial College London: http://www.imperial.ac.uk/earth-science/research/research-groups/pore-scale-modelling/micro-ct-images-and-networks/

Conflicts of Interest

The authors declare that there is no conflict of interest regarding the publication of this paper.

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