Flavor SU(3) in Hadronic B Decays

Amol Dighe
The Abdus Salam International Center for Theoretical Physics
34100, Trieste, Italy.

Abstract

Here we shall outline a few methods that use the flavor SU(3) symmetry in the decays of $B$ mesons to determine the angles of the unitarity triangle and to identify the decay modes which would display a significant CP violation.

1 Introduction

The decays of $B$ mesons are crucial in answering the question of whether the CKM matrix [1] can describe all the CP violation that we observe or that will be observed. This is an important question, since it probes the very basis of the charged current weak interactions and the origin of CP violation. The many decay modes available in $B$ decays provide consistency checks, which in addition probe the standard model from various angles. Thus, possibilities for the first and the surest signals of the physics beyond the standard model lie in the analyses of these decay modes.

The determination of the angles of the unitarity triangle is one of the goals of the theoretical and experimental efforts being put in this field. The search for theoretically clean and experimentally feasible decay modes is still on. The flavor SU(3) symmetry [2, 3, 4], which should approximately hold in $B$ decays, shows us some paths in this search.

\footnote{Talk given at The Workshop on B and Neutrino Physics, Mehta Research Institute, Allahabad, 1998.}
2 \( B \rightarrow PP \) amplitude polygons without time information

Let us consider the decays of a \( B \) meson into two light pseudoscalars, \( P_1 \) and \( P_2 \). Without time measurements, the only available information from a decay mode is its total decay rate, and hence the magnitude of its amplitude. In order to be able to measure the relative phases between amplitudes, we then need some theoretical relations in the form of triangles (in the complex plane) whose sides will be the amplitudes of these decay rates. The angles of triangles are determined given the lengths of their sides, and thus the relative phases between the amplitudes are known given their magnitudes. The next step is going from these relative phases to the angles of the unitarity triangle, \( \alpha, \beta \) and \( \gamma \) (also called as the CKM phases).

Two approaches have been taken to get the theoretical amplitude triangle (or quadrangle) relations. For the decays with \( B \) going into two light pseudoscalars, the total amplitude may be expressed either in the basis of six “Feynman diagrams” \( T \) (tree), \( C \) (color-suppressed tree), \( P \) (penguin), \( E \) (exchange), \( A \) (annihilation) and \( EA \) (penguin annihilation) \([5, 6]\); or in the basis of six SU(3) invariant amplitudes \([3, 7]\). Both these approaches are equivalent. (Actually, only five of these six amplitudes are independent, as shown in \([2]\).) After neglecting the annihilation-type contributions (which are expected to be suppressed by a factor of \( f_B/m_B \approx 5\% \)), numerous equivalence, triangle and quadrangle relations are obtained \([2, 5, 6, 7, 8, 9, 10, 11]\).

The decay modes need to be divided into two groups, \( \Delta S = 0 \) and \( |\Delta S| = 1 \). In a triangle, only modes from within the same group should appear, since the dominant (tree and penguin) amplitudes in these two groups contribute with different CKM strengths. Within each of these groups, all the decay modes of the type \((B^+/B^0/B_s) \rightarrow P_1P_2\) form the sides of at least one of the connected triangles, giving rise to “rigid polygons” \([10]\). Even in the presence of singlet-octet mixing in the \( \eta - \eta' \) system, it is still possible to extend the formalism and obtain amplitude triangle relations \([10]\).

The process of going from the amplitude triangles to the CKM phases is not always theoretically clean and estimations of the relative strengths of various contributions are needed in order to decide whether some of them can be neglected to simplify the relations. The hierarchy of amplitudes \([1]\) is different for \( \Delta S = 0 \) and \( |\Delta S| = 1 \) modes due to the different CKM factors.

The amplitudes for these two types of modes may be written approximately as

\[
A(\Delta S = 0) = V_{tb}^*V_{td}P + V_{ub}^*V_{ud}T = e^{-i\beta}e^{i\delta_P}P + e^{i\gamma}e^{i\delta_T}T
\]  \hspace{1cm} (1)
where primed quantities denote $|\Delta S| = 1$ decays, $\mathcal{P}', \mathcal{T}', \mathcal{P}', \mathcal{T}'$ are the magnitudes of the respective contributions, $\delta_P$, $\delta_T$ are strong phases and $\beta, \gamma, \pi$ are the weak phases from the CKM matrix elements. The corrections due to quarks other than the top quark to the penguin amplitudes have been neglected here. These corrections have been estimated in [12]. $T$ and $P$ here may contain some additional contributions due to electroweak penguins, but that does not affect the construction of triangles [10, 11].

For the tree diagram, the first order SU(3) corrections may be introduced through the $K/\pi$ ratio of decay constants

$$\mathcal{T}'/\mathcal{T} = |V_{us}/V_{ud}|(f_K/f_\pi) \equiv r_u(f_K/f_\pi) \equiv \tilde{r}_u .$$

Here factorization has been used for the tree contribution, which is supported by experiments [13, 14] and justified for $B \to \pi\pi$ and $\pi K$ by the high momentum with which the two color-singlet mesons separate from one another. Since factorization is questionable for penguin amplitudes, it has not been used, but it is assumed that the phase $\delta_P$ is unaffected by the SU(3) breaking. Since this phase is likely to be small [15], this assumption is not expected to introduce a significant uncertainty in the determination of the weak phases.

The $P'$ term is expected to dominate the decays of the type $|\Delta S| = 1$ and $T$ term would dominate the $\Delta S = 0$ decays [3]. Separate strategies can then be proposed for obtaining the phases from the two types of decays, which I shall illustrate with an example from each case. The details can be found in [10].

2.1 $\Delta S = 0$ :

Once the triangles have been constructed, rotate them such that the amplitudes with only penguin contributions lie along the x-axis. The (rotated) amplitude of a decay mode with nonzero tree contribution will then be

$$A_R(\Delta S = 0) = \mathcal{P} + e^{i(\beta + \gamma)} e^{i(\delta_T - \delta_P)}\mathcal{T} .$$

When the corresponding antiparticle triangle is similarly constructed and aligned, the amplitude of the corresponding CP-conjugate decay will be

$$\overline{A}_R(\Delta S = 0) = \mathcal{P} + e^{-i(\beta + \gamma)} e^{i(\delta_T - \delta_P)}\mathcal{T} .$$

If the penguin contribution is much smaller than the tree one, the angle between $A_R$ and $\overline{A}_R$ is $2(\beta + \gamma) = 2\pi - 2\alpha \equiv -2\alpha$. 

3
2.2 $|\Delta S| = 1$:

With the same reorientation as above (amplitudes with only penguin contributions along the x axis), we have

$$A_R(\Delta S = 1) = -P' + e^{i\gamma}e^{i(\delta_T - \delta_P)}T'$$

(6)

and

$$\overline{A}_R(\Delta S = -1) = -P' + e^{-i\gamma}e^{i(\delta_T - \delta_P)}T'$$

(7)

Then

$$A_R(\Delta S = 1) - \overline{A}_R(\Delta S = -1) = 2i \sin \gamma e^{i(\delta_T - \delta_P)}T'$$

(8)

Using Eq. (3) and the value of $T$ obtained from a tree-dominated $\Delta S = 0$ decay, we can obtain $\gamma$ as well as $\delta_T - \delta_P$.

3 Decays to kaons and charged pions with time information

If the measurements of time-dependent rates for $B^0(t) \to \pi^+\pi^-$ and $\overline{B}^0(t) \to \pi^+\pi^-$ are added to the measurements of the total rates for $B^0 \to \pi^-K^+$, $\overline{B}^0 \to \pi^+K^-$ and $B^\pm \to K_S\pi^\pm$, the CKM phases may be determined without having to detect $\pi^0$ as follows:

Neglecting color-suppressed electroweak penguin effects of order $|P'_{EW}/P'| = O((1/5)^2)$, we can write

$$A_{\pi\pi} \equiv A(B^0 \to \pi^+\pi^-) = T e^{i\delta_T} e^{i\gamma} + P e^{i\delta_P} e^{-i\beta},$$

$$A_{\pi K} \equiv A(B^0 \to \pi^-K^+) = \tilde{r}_u T e^{i\delta_T} e^{i\gamma} - \tilde{P}' e^{i\delta_P},$$

$$A_+ \equiv A(B^+ \to \pi^+K^0) = \tilde{P}' e^{i\delta_P}.$$  

(9)

The magnitude of the $\Delta S = 1$ penguin amplitude has been denoted by $\tilde{P}'$, to allow for SU(3) breaking. The numerous a priori unknown parameters in (9), including the two weak phases $\gamma$ and $\alpha \equiv \pi - \beta - \gamma$, can be determined from the rate measurements of the above three processes and their charge-conjugates.

The amplitudes for the corresponding charge-conjugate decay processes are simply obtained by changing the signs of the weak phases $\gamma$ and $\beta$. We denote the charge-conjugate amplitudes corresponding to (9) by $\overline{A}_{\pi\pi}, \overline{A}_{\pi K}, A_-,
respectively. A state initially tagged as a $B^0$ or $\bar{B}^0$ will be called $B^0(t)$ or $\bar{B}^0(t)$. The time-dependent decay rates of these states to $\pi^+\pi^-$ are given by

$$
\Gamma(B^0(t) \rightarrow \pi^+\pi^-) = e^{-\Gamma t} \left[ |A_{\pi\pi}|^2 \cos^2\left(\frac{\Delta m}{2} t\right) + |\overline{A}_{\pi\pi}|^2 \sin^2\left(\frac{\Delta m}{2} t\right) 
+ \text{Im}(e^{2i\beta A_{\pi\pi} \overline{A}_{\pi\pi}} \sin(\Delta m t)) \right],
$$

$$
\Gamma(\bar{B}^0(t) \rightarrow \pi^+\pi^-) = e^{-\Gamma t} \left[ |A_{\pi\pi}|^2 \sin^2\left(\frac{\Delta m}{2} t\right) + |\overline{A}_{\pi\pi}|^2 \cos^2\left(\frac{\Delta m}{2} t\right) 
- \text{Im}(e^{2i\beta A_{\pi\pi} \overline{A}_{\pi\pi}} \sin(\Delta m t)) \right].
$$

Measurements of these decay rates determine the quantities $|A_{\pi\pi}|^2$, $|\overline{A}_{\pi\pi}|^2$ and $\text{Im}(e^{2i\beta A_{\pi\pi} \overline{A}_{\pi\pi}})$. It is convenient to define sums and differences of the first two quantities, and we find

$$
A \equiv \frac{1}{2}(|A_{\pi\pi}|^2 + |\overline{A}_{\pi\pi}|^2) = T^2 + P^2 - 2TP \cos \delta \cos \alpha,
$$

$$
B \equiv \frac{1}{2}(|A_{\pi\pi}|^2 - |\overline{A}_{\pi\pi}|^2) = -2TP \sin \delta \sin \alpha,
$$

$$
C \equiv \text{Im}(e^{2i\beta A_{\pi\pi} \overline{A}_{\pi\pi}}) = -T^2 \sin 2\alpha + 2TP \cos \delta \sin \alpha,
$$

where we use $\beta + \gamma = \pi - \alpha$ and where we define $\delta \equiv \delta_T - \delta_P$.

The rates of the self-tagging modes $\pi^-K^+$, $\pi^+K^-$ and $\pi^+K^0$ or $\pi^-\bar{K}^0$ determine $|A_{\pi K}|^2$, $|\overline{A}_{\pi K}|^2$ and $|A_\pm|^2$, respectively. Again, we can take sums and differences of the first two, and find

$$
D \equiv \frac{1}{2}(|A_{\pi K}|^2 + |\overline{A}_{\pi K}|^2) = (\tilde{r}_u T)^2 + \tilde{P}'^2 - 2\tilde{r}_u T \tilde{P}' \cos \delta \cos \gamma,
$$

$$
E \equiv \frac{1}{2}(|A_{\pi K}|^2 - |\overline{A}_{\pi K}|^2) = 2\tilde{r}_u T \tilde{P}' \sin \delta \sin \gamma,
$$

$$
F \equiv |A_+|^2 = |A_-|^2 = \tilde{P}'^2.
$$

The rates for $B^+ \rightarrow \pi^+K^0$ and $B^- \rightarrow \pi^-\bar{K}^0$ are expected to be equal, since only penguin amplitudes are expected to contribute to these processes. Measurement of the six quantities $A - F$ suffices to determine all six parameters $\alpha$, $\gamma$, $T$, $P$, $\tilde{P}'$, $\delta$ up to discrete ambiguities. The accuracy to which they can be determined is estimated in [17] and the discrete ambiguities are studied in [18]. The CKM parameter $r_t \equiv |V_{ts}/V_{td}|$, which is still largely unknown, is obtained from the unitarity triangle in terms of $\alpha$ and $\gamma$ as $r_u r_t = \sin \alpha / \sin \gamma$. 

5
B and E are proportional to \( \sin \delta \), and thus would vanish in the absence of a strong phase difference. In that case, the number of equations becomes less than the number of unknowns and one would have to assume a relation between \( \tilde{P}' \) and \( P \) or some other constraint in order to obtain a solution.

This problem may be overcome if the SU(3)-related decays \( B^0_s \rightarrow \pi^+K^- \), \( B^0_s(t) \rightarrow K^+K^- \) and \( B^0 \rightarrow K^0\bar{K}^0 \) are included \[19\]. One can then also get rid of some of the discrete ambiguities and \( \gamma \) can be obtained cleanly with no penguin contamination. But a large number of quantities (12 decay rates) need to be measured (some of them involving \( B_s \)), so it will be difficult to implement this method in the near future.

### 4 Angular distributions of \( B \) decays to two vector mesons

The decay modes of \( B \) into two vector mesons, each of which decay into two particles each, are very promising, mainly because of the larger number of observables at one’s disposal through the angular distributions \[20, 21\] of the decays. The time-dependent angular distributions contain information about the lifetimes, mass differences, strong and weak phases, form factors, and CP violating quantities.

Let me illustrate with the example of \( B_s \rightarrow J/\psi \phi \). With the angles \( \theta, \phi, \psi \) defined as in \[22\], the angular distribution is

\[
\frac{d^3\Gamma(B_s(t) \rightarrow J/\psi(\rightarrow l^+l^-)\phi(\rightarrow K^+K^-)]}{d\cos \theta \, d\varphi \, d\cos \psi} \propto \frac{9}{32\pi} \left[ 2|A_0(t)|^2 \cos^2 \psi(1-\sin^2 \theta \cos^2 \varphi) \\
+ \sin^2 \psi \{ |A_{\parallel}(t)|^2 (1-\sin^2 \theta \sin^2 \varphi) + |A_{\perp}(t)|^2 \sin^2 \theta - \text{Im} \, (A_0^*(t)A_{\perp}(t)) \sin 2\theta \sin \varphi \} \\
+ \frac{1}{\sqrt{2}} \sin 2\psi \{ \text{Re} \, (A_0^*(t)A_{\parallel}(t)) \sin^2 \theta \sin 2\varphi + \text{Im} \, (A_0^*(t)A_{\perp}(t)) \sin 2\theta \cos \varphi \} \right].
\]

(13)

The time evolutions of the coefficients of the angular terms is given in Table I. Here \( \Gamma_L \) and \( \Gamma_H \) are the widths of the light and heavy \( B_s \) mass eigenstates, \( B_s^L \) and \( B_s^H \) respectively, and \( \Delta m \) is the mass difference between them. \( \bar{\Gamma} \) is the average of \( \Gamma_L \) and \( \Gamma_H \). Here \( \delta_1 \equiv \text{Arg}(A_0^*(0)A_{\perp}(0)) \) and \( \delta_2 \equiv \text{Arg}(A_0^*(0)A_{\parallel}(0)) \) are the strong phases, and \( \Delta \phi \approx 2\lambda^2\eta \) is related to an angle of a (squashed) unitarity triangle \[23\], which is very small in the standard model \[ \approx (0.03) \].
Observables & Time evolutions
\[ |A_0(t)|^2 \rightarrow |A_0(0)|^2 [e^{-\Gamma_L t} - e^{-\Gamma_L t} \sin(\Delta m t) \delta \phi] \]
\[ |A_\parallel(t)|^2 \rightarrow |A_\parallel(0)|^2 [e^{-\Gamma_L t} - e^{-\Gamma_L t} \sin(\Delta m t) \delta \phi] \]
\[ |A_\perp(t)|^2 \rightarrow |A_\perp(0)|^2 [e^{-\Gamma_H t} + e^{-\Gamma_L t} \sin(\Delta m t) \delta \phi] \]
\[ \text{Re}(A_0^*(t) A_\parallel(t)) \rightarrow |A_0(0)||A_\parallel(0)| \cos(\delta_2 - \delta_1) [e^{-\Gamma_L t} - e^{-\Gamma_L t} \sin(\Delta m t) \delta \phi] \]
\[ \text{Im}(A_\parallel^*(t) A_\perp(t)) \rightarrow |A_\parallel(0)||A_\perp(0)| \left[ e^{-\Gamma_H t} \sin(\delta_1 - \Delta m t) + \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_1) \delta \phi \right] \]
\[ \text{Im}(A_0^*(t) A_\perp(t)) \rightarrow |A_0(0)||A_\perp(0)| \left[ e^{-\Gamma_H t} \sin(\delta_2 - \Delta m t) + \frac{1}{2} (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \cos(\delta_2) \delta \phi \right] \]

| Table 1: Time evolution of the decay \( B_s \rightarrow J/\psi \rightarrow l^+l^- \phi \rightarrow K^+K^- \) of an initially (i.e. at \( t = 0 \)) pure \( B_s \) meson. |

The time-dependent observables in all these decays provide information about the corresponding values of \( \Gamma_L, \Gamma_H \) and \( \Delta m \). If we integrate over the angles \( \varphi \) and \( \psi \), the angular distribution in the remaining “transversity” angle \( \theta \) can help in separating out the CP even and odd components and in determining their lifetimes separately [21].

The width difference \( \Delta \Gamma \equiv \Gamma_H - \Gamma_L \) may be sizeable [24]. Because of this difference, the interference effects between the CP-even and CP-odd final-state configurations give rise to a term in the time evolution of the untagged rate, which is proportional to \( (e^{-\Gamma_H t} - e^{-\Gamma_L t}) \delta \phi \) [25]. Thus, with the angular distribution, CP violating effects may be observable even without tagging the flavor of the initial \( B \). This feature may be important, because it provides an alternative to previous investigations, which have shown how to extract \( \sin \phi_{\text{CKM}} \) from tagged, time-dependent analyses [26].

The observables of the angular distributions can be determined from experimental data by an angular-moment analysis [22, 27, 28] in which the data are weighted by judiciously chosen weighting functions in order to arrive directly at the observables. In [22], a method applicable to all kinds of angular distributions is indicated, where the weighting functions can be determined without any \textit{a priori} knowledge of the values of the coefficients. This method is almost as efficient as the likelihood-fit method for a small number of parameters and is expected to stay robust even with a large number of parameters where the maximum likelihood fit method may be unreliable [29].

Using appropriate weighting functions for the angular distributions of the decay products in the transitions \( B_s \rightarrow J/\psi \phi \) and/or \( B_s \rightarrow D_s^{*+}D_s^{-} \), one can extract \( (\Gamma_H, \Gamma_L, \Delta m)_{B_s} \). The observables of the angular distri-
butions of $B_s \rightarrow J/\psi \phi$, $D_s^+ D_s^-$ can be related to those of the decays $B \rightarrow J/\psi K^*, D_s^+ D^*$ by using the $SU(3)$ flavor symmetry, where $B$ stands for $B_d$ or $B^+$. Determination of the time-dependent angular distributions will also be useful in testing form factor models \cite{30} and furthermore in determining the extent to which factorization or the $SU(3)$ flavor symmetry hold in these decays. The full angular distributions for all these transitions are given explicitly, and the corresponding weighting functions are specified in \cite{22}.

Whereas $B \rightarrow J/\psi K^*$ angular distributions (or the “gold-plated” mode $B \rightarrow J/\psi K_S$) give the value of $\sin(2\beta)$, a discrete ambiguity still remains in the determination of $\beta$. Using $B_s \rightarrow J/\psi \phi$ angular distributions in addition, and using the $SU(3)$ flavor symmetry (only weakly), this ambiguity can be resolved \cite{31}.

5 Model-independent estimations of $B \rightarrow PP$ and $B \rightarrow VP$ amplitudes

Model-dependent rate calculations of $B \rightarrow PP$ and $B \rightarrow VP$ amplitudes have been made in \cite{4,32,33,34}. While flavor $SU(3)$ by itself cannot predict the rate of a process, when combined with the experimental observations of some decay rates, reliable predictions can be made about the others. This helps in identifying the decay channels which would be expected to display a significant CP violation.

5.1 $B \rightarrow PP$

The measured branching ratios (in units of $10^{-5}$) \cite{35} $B(B^0 \rightarrow K^+\pi^-) = 1.4 \pm 0.3 \pm 0.1$ and $B(B^+ \rightarrow K^0\pi^+) = 1.4 \pm 0.5 \pm 0.2$, when compared with \cite{36} $B(B^+ \rightarrow K^+\eta') = 7.4^{+9.8}_{-1.3} \pm 0.9$ may imply a significant contribution from the flavor $SU(3)$ singlet component of the $\eta'$ \cite{37}. While one possibility for this contribution \cite{35,36,37} is an intrinsic $c\bar{c}$ component in the $\eta'$, more conventional mechanisms \cite{10,11} (e.g., involving gluons) also seem adequate to explain the observed rate. Parametrizing this contribution as $S$ or $S'$ (by using the same Feynman diagram basis mentioned in Sec. 2), and adding the information from the branching ratio of $B \rightarrow \pi\pi$, one can estimate the magnitudes of $T, T', \mathcal{P}, \mathcal{P}', S$ and $S'$. Since all the $B \rightarrow PP$ decay processes are dominated by one or more of these contributions, the decay rate of any process can thus be estimated. This has been explicitly done in \cite{37}. (The
relative phases between these amplitudes are still unknown, so only a range for the branching ratio of a decay mode can be given.)

The potential for CP-violating rate asymmetries to be exhibited in decays of $B$ mesons to pairs of charmless mesons has been noted in [42]. To observe direct CP violation in a decay mode, we need at least two significant contributions to that mode, which have different strong as well as weak phases. As a rule of thumb, one must at least be able to observe the square of the lesser of the two interfering amplitudes at the $n\sigma$ level in order to observe an asymmetry at this level [43]. With the above estimation of the contributions of different amplitudes, it is seen that this sensitivity threshold is passed for decays of the form $B^+ \to \pi^+\eta$ and $B^+ \to \pi^+\eta'$ when branching ratios of order $10^{-6}$ become detectable in experiments sensitive to both charged and neutral final-state particles. These two modes thus emerge as promising ones for observing direct CP violation [37].

5.2 $B \to VP$

The observation of the decays $B^+ \to \omega\pi^+$ and $\omega K^+$ at branching ratio levels of about $10^{-5}$ by the CLEO Collaboration [44] can be used, with the help of flavor SU(3), to anticipate the observability of other charmless $B \to VP$ decays in the near future [13].

Now we need to have twice the number of amplitudes for our basis. The amplitudes will depend upon whether the spectator quark (the quark other than the $b$ in the decaying $B$ meson) ends up in the final state scalar or vector meson. The amplitudes contributing to a significant extent will then be denoted by $t_P, t_V, c_P, c_V, p_P, p_V, s_P, s_V$ (and their primed counterparts for $|\Delta S| = 1$ decays) where the subscript denotes where the spectator quark goes.

One can then study the hierarchy of these amplitudes, on similar lines to the hierarchy in $B \to PP$ mentioned in Sec. 2. The fact that the $B^+ \to \omega\pi^+$ and $B^+ \to \omega K^+$ branching ratios are comparable to one another and each of order $10^{-5}$ indicates that the dominant contribution to $\omega\pi^+$ is most likely $t_V$, while the dominant contribution to $\omega K^+$ is most likely $p_V'$. An appreciable value for the amplitude $p_V'$, somewhat of a surprise on the basis of conventional models [1, 33, 34], implies that the decays $B \to \rho K$ should be observable at branching ratio levels in excess of $10^{-5}$. The smallness of the ratio $B(B^+ \to \phi K^+)/B(B^+ \to \eta K^+)$ indicates that $|p'_P| < |p'_V|$. The amplitude $p'_P$ should dominate not only $B \to \phi K$ but also $B \to K^*\pi$ decays. Evidence for any of these would then tell us the magnitude of $p'_P$. The relative phase of $p'_P$ and $p'_V$ is probed by $B \to K^*(\eta, \eta')$ decays.
The amplitude $s'_V$, coupling to the flavor SU(3) singlet component of the $\eta$ and $\eta'$, can be as large as or even larger than $p'_V$. Several tests can be suggested for non-zero singlet amplitudes, including a number of triangle and rate relations. A program for determining the magnitude and phase of $s'_V$ has also been outlined in [45].

Once the dominant $t_V$, $p'_V$, and $s'_V$ amplitudes have been determined, one can use flavor SU(3) to predict the amplitudes $t'_V$, $p_V$, and $s_V$. It is then a simple matter (along the lines indicated in Sec. [5]) to determine which processes have the potential for exhibiting noticeable interferences between two or more amplitudes, and thereby displaying CP-violating asymmetries.

### 6 SU(3) Breaking

Flavor SU(3) is not an exact symmetry and while making predictions on its basis, the errors due to the SU(3) breaking effects need to be estimated and the corrections need to be taken into account. There is no theoretically clean way of calculating these corrections, however a parametrization has been proposed in [46] in the basis of Feynman diagrams, where the implications of the SU(3) breaking effects for the extraction of weak phases are also examined.

Within the framework of generalized factorization, the SU(3) breaking can be looked upon as arising from the following sources:

- different decay constants of the final state particles, $f_P$ and $f_V$
- different form factors
- different masses of the final state particles.

A study of the SU(3) breaking corrections using generalized factorization is currently in progress [47].

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