Deconfinement and chiral restoration within the SU(3) PNJL and EPNJL models in an external magnetic field

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The behavior of the quark condensates at zero chemical potential and finite temperature subject to an external magnetic field is studied within the three flavor Nambu-Jona-Lasinio model with Polyakov loop (PNJL) and an extension of it, the so-called entangled PNJL model (EPNJL). A comparison with recent lattice QCD data is performed. We present a possible mechanism to account for the inverse magnetic catalysis at finite temperature found in lattice calculations which so far cannot be reproduced in the context of effective chiral models. Such mechanism is based on the use of the EPNJL model with a magnetic field dependent scale parameter for the Polyakov loop potential.

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I. INTRODUCTION

Understanding the behavior of matter under extremely intense magnetic fields is important for studies involving the physics of compact objects like magnetars [1], measurements in heavy ion collisions at very high energies [2, 3] or the first phases of the universe [4]. In [5] the influence of strong magnetic fields on the QCD phase diagram covering the whole $T - \mu$ plane was investigated within one specific parametrization of the three-flavor Nambu-Jona-Lasinio model, which includes strangeness and physical quark masses in the mean field approximation. For finite chemical potentials it is found that the first order segment of the transition line becomes longer as the field strength increases and that the location of the critical end point occurs at larger temperatures for stronger fields. On the other hand, at zero chemical potential a crossover transition is obtained at temperatures that increase with an increasing magnetic field. This behavior is contrary to the recent predictions of lattice QCD calculations under strong external fields that foresee a decreasing transition temperature to a deconfined phase for an increasing external magnetic field [6]. A review of the predictions from low-energy approximations of QCD and previous lattice simulations is given in [4, 8].

Moreover, calculations of deconfinement and chiral pseudo-critical temperatures with the SU(2) Polyakov NJL (PNJL) [8] model and entangled PNJL (EPNJL) [10] influenced by magnetic fields have been discussed in [3, 11]. As in all other low-energy QCD models, these two models predict that the critical temperature for chiral symmetry restoration increases with the increase of an external magnetic field. It was also shown that within the EPNJL the split between the chiral and deconfinement transition temperatures is smaller than the split predicted by the PNJL model. In [12] the authors study the behavior of the $u$ and $d$ condensates at zero and finite temperature in an external magnetic field and confirm the magnetic catalysis phenomena observed by most of the models at zero temperature. However, for temperatures of the order of the crossover temperature the joint effect of temperature and magnetic field gives rise to a decrease of the quark condensates.

In the present work we analyze the effect of strong magnetic fields on strongly interacting matter using the SU(3) versions of the PNJL and EPNJL models. It will be shown that although these models at such do not describe the inverse catalysis effect observed in the lattice calculations of [6] at the transition temperature, some other features such as the behavior of the $u$ and $d$ condensates with an external magnetic field $B$ for zero temperature, are well reproduced. In this context it is important to recall that the parametrization of the Polyakov loop includes a scale parameter $T_0$ that was originally fixed to $T_0 = 270$ MeV in order to reproduce the pure gauge lattice results. Later on it was noted that to account for backreaction of the dynamical quarks on gauge configurations this value had to be reduced to $\sim 200$ MeV [12]. Thus, in a first step we consider $T_0 = 210$ MeV in order to get the deconfinement temperature at 170 MeV, as predicted by the lattice calculations [14]. Later on, however, we consider the possibility that the parameter $T_0$ depends on $eB$. We find that using an appropriate parametrization for $T_0(eB)$ within the EPNJL, the lattice QCD behavior of the quark condensates with temperature in an external magnetic field can be qualitatively reproduced.

After a brief introduction and a review of the formalism, we discuss the behavior of the quark condensates in an external field within SU(3) PNJL in section III, and within SU(3) EPNJL with a magnetic field dependent Polyakov loop in section IV, and draw some conclusions in the last section.
II. MODEL AND FORMALISM

We describe (three flavor) quark matter subject to strong magnetic fields within the SU(3) PNJL model. The PNJL Lagrangian with explicit chiral symmetry breaking where the quarks couple to a (spatially constant) temporal background gauge field, represented in terms of the Polyakov loop and in the presence of an external magnetic field is given by [3]

\[
\mathcal{L} = \bar{\psi}_f (i \gamma_\mu D^\mu - m_c) \psi_f + \mathcal{L}_{\text{sym}} + \mathcal{L}_{\text{det}} + \mathcal{U} (\Phi, \bar{\Phi}; T) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},
\]

where the quark sector is described by the SU(3) version of the Nambu–Jona-Lasinio model which includes scalar-pseudoscalar and at finite temperature the spatial components that models the axial $U(1)_A$ symmetry breaking [15], with $\mathcal{L}_{\text{sym}}$ and $\mathcal{L}_{\text{det}}$ given by [15]

\[
\mathcal{L}_{\text{sym}} = \sum_{a=0}^{8} \left[ (\bar{\psi}_f \gamma_a \psi_f)^2 + (\bar{\psi}_f i \gamma_\gamma \gamma_a \psi_f)^2 \right],
\]

\[
\mathcal{L}_{\text{det}} = -K \left\{ \left| \det_f \left[ \bar{\psi}_f (1 + \gamma_\gamma) \psi_f \right] \right| + \det_f \left[ \bar{\psi}_f (1 - \gamma_\gamma) \psi_f \right] \right\}
\]

where $\psi_f = (u, d, s)^T$ represents a quark field with three flavors, $m_c = \text{diag}(m_u, m_d, m_s)$ is the corresponding (current) mass matrix, $\lambda_0 = \sqrt{2}/3I$ where $I$ is the unit matrix in the three flavor space, and $0 < \lambda_a < 8$ denote the Gell-Mann matrices. The coupling between the (electro)magnetic field $B$ and quarks, and between the effective gluon field and quarks is implemented via the covariant derivative $D^\mu = \partial^\mu - i q_f A^{\mu}_{EM} - i A^\mu$ where $\gamma_f$ represents the quark electric charge ($q_u = q_s = -q_d/2 = -e/3$). $A^{\mu}_{EM}$ and $F_{\mu\nu} = \partial_\mu A^{\nu}_{EM} - \partial_\nu A^{\mu}_{EM}$ are used to account for the external magnetic field and $A^{\mu}(x) = \text{stray}_g A^{\mu}_g(x)$ where $A^{\mu}_g$ is the SU(3) gauge field. We consider a static and constant magnetic field in the $z$ direction, $A^{\mu}_{EM} = \delta^{\mu}_z \bar{B}$. In the Polyakov gauge and at finite temperature the spatial components of the gluon field are neglected: $A^{\mu} = \delta^{\mu}_0 A^0 = -i \delta^\mu_4 A^4$. The trace of the Polyakov loop defined by $\Phi = \frac{1}{N_c^3} \langle \mathcal{P} \exp i \int_0^\beta d\tau A_4 (x, \tau) \rangle$ is the Polyakov loop which is the exact order parameter of the Z$_3$ symmetric/broken phase transition in pure gauge.

To describe the pure gauge sector an effective potential $\mathcal{U}(\Phi, \bar{\Phi}; T)$ is chosen in order to reproduce the results obtained in lattice calculations [17]:

\[
\frac{\mathcal{U}(\Phi, \bar{\Phi}; T)}{T^4} = -a(T) \Phi \bar{\Phi} + b(T) \ln \left[ 1 - 6 \Phi \bar{\Phi} + 4 (\Phi^3 + \bar{\Phi}^3) - 3 (\Phi \bar{\Phi})^2 \right],
\]

where $a(T) = a_0 + a_1 \left( \frac{T}{T_0} \right) + a_2 \left( \frac{T}{T_0} \right)^2$, $b(T) = b_3 \left( \frac{T}{T_0} \right)^3$. The standard choice of the parameters for the effective potential $\mathcal{U}$ is $a_0 = 3.51$, $a_1 = -2.47$, $a_2 = 15.2$, and $b_3 = -1.75$.

As is well know, the effective potential exhibits the feature of a phase transition from color confinement ($T < T_0$, the minimum of the effective potential being at $\Phi = 0$) to color deconfinement ($T > T_0$, the minima of the effective potential occurring at $\Phi \neq 0$). $T_0$ is the critical temperature for the deconfinement phase transition in the absence of dynamical fermions and, according to lattice calculations, is 270 MeV. In order to assure that in the PNJL model the deconfinement temperature has the same value as in lattice calculations ($\sim 170$ MeV [14]), we rescale $T_0$ from 270 MeV to 210 MeV. Note that $\Phi = \Phi_0$ at zero chemical potential.

Besides the PNJL model, where $G$ denotes the coupling constant of the scalar-type four-quark interaction in the NJL sector, we consider an effective vertex depending on the Polyakov loop ($G(\Phi, \bar{\Phi})$); the EPNJL model. This effective vertex generates entanglement interactions between the Polyakov loop and the chiral condensate [10]. In our study, we use the ansatz introduced in [10]

\[
G(\Phi, \bar{\Phi}) = G \left[ 1 - \alpha_1 \Phi \bar{\Phi} - \alpha_2 (\Phi^3 + \bar{\Phi}^3) \right].
\]

For reasons of consistency we will use $T_0 = 210$ MeV also in EPNJL model.

Since the models under consideration are not renormalizable, we need to specify a regularization scheme. Here, we introduce a sharp cut-off, $\Lambda$, in 3-momentum space, only for the divergent ultra-violet integrals. The parameters of the model, $\Lambda$, the coupling constants $G$ and $K$ and the current quark masses $m_0^u$ and $m_0^d$ are determined by fitting $f_\pi$, $m_\pi$, $m_K$ and $m_{K'}$ to their empirical values. We consider $\Lambda = 602.3$ MeV, $m_u = m_d = 5.5$ MeV, $m_s = 140.7$ MeV, $G\Lambda^2 = 1.385$ and $K\Lambda^5 = 12.36$ as in [18]. The parameter set $(\alpha_1, \alpha_2)$ must satisfy the triangle region $\{ -1.50 \alpha_1 + 0.3 < \alpha_2 < -0.86 \alpha_1 + 0.32 \alpha_2, \alpha_2 > 0 \}$, and we choose $\alpha_1 = 0.25$ and $\alpha_2 = 0.10$.

The thermodynamical potential and the quark condensates ($\langle \bar{q} q_f \rangle$) for the three flavor quark sector can be evaluated following similar steps as in Refs. [19, 20]. To obtain the mean field equations we must minimize the thermodynamical potential with respect to $\langle \bar{q} q_f \rangle$, $\Phi$ and $\bar{\Phi}$ [21].

Finally, according to [12] we define the change of the light condensate due to the magnetic field as

\[
\Delta \Sigma_f (B, T) = \Sigma_f (B, T) - \Sigma_f (0, T),
\]

with

\[
\Sigma_f (B, T) = \frac{2 m_f}{m^2_T f^2} \left[ \langle \bar{q} q_f \rangle (B, T) - \langle \bar{q} q_f \rangle (0, 0) \right] + 1 \text{, (5)}
\]

where the factor $m^2_T f^2$ in the denominator contains the pion mass in the vacuum ($m_\pi = 135$ MeV) and (the chiral limit of the) pion decay constant ($f_\pi = 87.9$) MeV in PNJL model.

III. PNJL AND EPNJL MODELS

At zero temperature the chiral symmetry of QCD is explicitly broken. Consequently, at high temperature it
is expected that chiral symmetry be restored: a transition takes place separating both, the low and the high temperature regions. As a matter of fact, there is no chiral or deconfinement phase transition, merely an analytic crossover. At $eB = 0$ both, PNJL and EPNJL, models show a crossover transition: we cannot establish an unique critical temperature, only a pseudo-critical temperature depending on the observable used to define it. To identify the pseudo-critical temperatures for the chiral transition ($T^c_T = (T^u_T + T^d_T)/2$) and for the deconfinement ($T^c_u$), we use the location of the peaks for the vacuum normalized quark condensates and the Polyakov loop field $\Phi$ susceptibilities, $C_f = -m_\pi \partial \sigma_f / \partial T$ (where $\sigma_f = \langle \bar{q} q_f \rangle (B, T) / \langle \bar{q} q_f \rangle (B, 0)$) and $C_\Phi = m_\pi \partial \Phi / \partial T$ respectively. The multiplication by $m_\pi$ is only to ensure that the susceptibilities are dimensionless.

In Fig. 1 we plot the quark condensates and the Polyakov field $\Phi$, together with the respective susceptibilities as functions of $T$ for several magnetic field strengths in PNJL model.

One interesting aspect is the fact that pseudo-critical temperatures for $u$ and $d$ quark transitions become different as $eB$ increases (see Table I). Due to its larger electric charge, the $u$ quark has an effective mass that becomes larger (which is manifested in the behavior of the respective condensate, top panel of Fig. 1) so the partial restoration of chiral symmetry in the $u$ sector is delayed and the respective transition occurs at higher temperature than the transition in the $d$ sector. It is also observed that as the magnetic field becomes stronger, the separation between the temperatures $T^c_T$ and $T^c_u$ increases as seen from Fig. 1 bottom panel, and also in Table I. However, the magnetic field has a smaller impact on the location of the deconfinement crossover as already noticed in [11] for the SU(2) sector: $T^c_u$ has just a weak increase (see Fig. 1 bottom panel and also Table I). Moreover, the Polyakov loop susceptibilities become narrower with an increasing magnetic field and eventually for sufficient strong magnetic fields a first order phase transition takes place.

It also can be seen that the condensate susceptibilities display small peaks around the peak of $C_\Phi$. As pointed out in [21], the latter effect is related to the fastening of the phase transition induced by the Polyakov loop and, obviously, does not signal a phase transition since the variation of the order parameter around this temperature is small. These peaks are visible for all range of values of $eB$ studied.

Next, we focus our study on the quark condensates as functions of $eB$ at $T = 0$, having in mind the comparison of the PNJL model with lattice results for the quark condensates in external magnetic field [12]. Note that for $T = 0$ three models NJL, PNJL and EPNJL coincide. In Fig. 2 top panel, we compare the PNJL model results for the change of the renormalized condensate $\Delta (\Sigma_u + \Sigma_d)/2$ with lattice results extracted from [12]. We observe that our results work quantitatively well and even at $eB = 1$ GeV$^2$ there is a discrepancy of the order of $\sim 15\%$ much smaller than the prediction of chiral perturbation theory ($\chi$PT) and SU(2) PNJL model (see Ref. [12]). In Table II we present the continuum extrapolated lattice results for the light condensates at zero temperature, as functions of $eB$ [12], together with the results obtained for the PNJL model. The average of the light condensates (“+/2”) is in very good agreement with lattice results, specially at low magnetic fields. Even for $eB = 1$ GeV$^2$ the average of the light condensates do not differ more than $\sim 10\%$.

In Fig. 2 bottom panel, the average of $u$ and $d$ conden-
sates is plotted as a function of the magnetic field intensity for several temperatures in PNJL model. For $T < T^\chi(eB = 0)$ the condensates average increase with $eB$ being its value greater the higher the temperature, due to the magnetic catalysis effect. When $T > T^\chi(eB = 0)$ we are in the region where the partial restoration of chiral symmetry already took place. In this region there are two competitive effects: the partial restoration of chiral symmetry and the magnetic catalysis. The former effect prevails at lower values of $eB$, making the condensates average approximately zero. The latter effect becomes dominant as the magnetic field increases and the average condensate becomes nonzero. Let’s take as an example the case $T = 300$ MeV: since $T = 300$ MeV is larger than $T^\chi(eB = 0)$, the average condensate is approximately zero for small values of $eB$ and starts to increase around $eB = 0.8$ GeV$^2$, a magnetic field strong enough to prevent the partial restoration of chiral symmetry that would have occurred at zero magnetic field.

At this point it is important to notice that the results within EPNJL model are qualitatively similar with the results of PNJL model. However, it is important to comment some new features of EPNJL in the presence of an external magnetic field. From Table I it is seen that the coincidence existing between the deconfinement and chiral transition temperatures at $eB = 0$ is destroyed in the presence of an external magnetic field. When compared with PNJL, the effect of entanglement present in the EPNJL is seen on the larger (smaller) increase of $T^\chi$ ($T^\delta$).

In Table II we also list the values for the difference between $u$ and $d$ quark condensates (“−”) in comparison with lattice calculations. Once again the results are in good agreement namely to lower values of $eB$. A larger difference between PNJL and lattice calculations occurs for larger values of $eB$ with the lattice predicting a larger difference between both condensates. This means that an effect just due to the electric charge quark difference is stronger in the lattice calculations.

In Fig. 3 the average (upper panel) and the difference (lower panel) between light quark condensate is plotted as a function of $T/T^\chi(eB)$ for several values of $eB$ for PNJL (dashed lines), EPNJL (full lines) and lattice data [12]. The temperature normalization was done in order to remove the inverse magnetic catalysis effect in the lattice results.

The comparison plotted in Fig. 3 upper panel, for the average of the light condensates shows that in general PNJL and EPNJL have the same behavior as the lattice results except for a too fast drop above the respective transition temperatures.

The effect of a stronger magnetic catalysis for the $u$ quark, due to its larger electric charge, present in both models at finite temperatures is clear in the plot of the difference between the condensates: the larger the magnetic field the larger the difference between $u$ and $d$ condensates, and the respective chiral transition temperatures (see Table I). This feature is particularly strong close to the transition temperature, where the curves for the stronger fields have a larger bump. After the transition temperature $T^\chi$, the masses of the quarks are smaller, due to the partial restoration of chiral symmetry, prevailing this effect over the magnetic catalysis, being the $u$ and $d$ quark condensate difference smaller.

| $T$ | $eB = 0$ | $eB = 0.2$ GeV$^2$ | $eB = 0.4$ GeV$^2$ |
|-----|----------|------------------|------------------|
|     | +/2      | −                | +/2              |
| (E)PNJL | 1     | 0.11             | 1.32             |
| Latt. [12] | 1     | 0.09             | 0.28             |

| $T = 0$ | $eB = 0.6$ GeV$^2$ | $eB = 0.8$ GeV$^2$ | $eB = 1.0$ GeV$^2$ |
|---------|------------------|------------------|------------------|
|         | +/2              | −                | +/2              |
| (E)PNJL | 1.55             | 1.79             | 2.02             |
| Latt. [12] | 1.63(3)         | 1.90(3)          | 2.16(3)          |

TABLE II. Results obtained for PNJL (EPNJL) model together with the continuum extrapolated lattice results for the light condensates at $T = 0$ [12]. Columns labeled “+/−” contain the average of the light condensates, while those with “−” contain the difference.

FIG. 2. Top panel: comparison of the PNJL model results to the continuum limit of the change of the renormalized condensate [12] at $T = 0$. At this temperature NJL, PNJL and EPNJL coincide. Bottom panel: the change of the renormalized condensate as a function of $eB$ for several temperatures close to the transition temperature within PNJL model.
which the deconfinement phase transition becomes of first order. On the other hand, introducing an effective four-quark vertex that depends on the Polyakov loop as done in the EPNJL, the chiral condensates and the Polyakov loop become entangled. Thus, the chiral transition temperatures are pulled down to temperatures close to the deconfinement transition temperature. This model, however, still predicts a first order transition for $T_0$ too small at moderate magnetic fields. Hence, to test whether the use of a magnetic field dependent $T_0(B)$ can lead to inverse magnetic catalysis while avoiding too small values of $T_0$, we take for $B=0$ $T_0=270$ MeV as obtained in pure gauge, which gives $T_\chi=215$ MeV, 56 MeV higher than the prediction of lattice QCD data in [6]. For the finite magnetic field we take for $T_\chi$ the data from [8] shifted 56 MeV and obtain the function

$$T_0(eB) = 270 + \xi(eB)^2 + \eta(eB)^4, \quad eB < 0.75 \text{GeV}^2 \quad (6)$$

that reproduces the shifted $T_\chi(eB)$, which can be read off Table III for $\xi = -282.541$ and $\eta = 38.123$. For magnetic fields above 0.75 GeV$^2$, $T_0$ becomes too small and the transition becomes of first order, therefore this is the limit of our approach.

In Fig. 4 the change of the renormalized condensate are plotted as a function of the magnetic field for $eB < 0.75 \text{ GeV}^2$ and several temperatures close to $T_\chi(eB = 0)$. The main conclusions are: a) the qualitative behavior shown in Fig 2 of Ref. [12] is reproduced; b) the $T = 0$ curve has the highest $\Delta(S_u + S_d)/2$, contrary to the results of Fig. 2 bottom panel, for PNJL with fixed $T_0$; c) for $200 < T < 215$ MeV the strong interplay between the partial restoration of chiral symmetry, that for stronger magnetic fields occurs at smaller temperatures (see Table III), gives rise to curves that for small $eB$ values increase (magnetic catalysis) and as soon as the partial restoration of chiral symmetry becomes dominant the curve starts to decrease; d) For $T > 215$ MeV, the curves are negative because $\Delta\Sigma(B,T)$ includes the subtraction of the condensate $\Sigma(0,T)$, when partial restoration of chiral symmetry has still not occurred.

V. CONCLUSIONS

In the present work the behavior of the quark condensates at zero chemical potential and finite temperature under the influence of an external magnetic field are studied within three flavor PNJL and EPNJL. The results are compared with the lattice QCD data discussed in [6,12].

| $eB$ [GeV$^2$] | 0 0.2 0.4 0.6 0.75 |
|---------------|---------------------|
| $T^\chi_0$ [MeV] | 215 214 211 206 200 |
| $T^\chi_5$ [MeV] | 214 211 200 198 200 |

TABLE III. Pseudo-critical temperatures for chiral transition and for the deconfinement in EPNJL model with $T_0(eB)$. 

FIG. 3. Average of light quark condensate, together with respective lattice results taken from [12], (top panel) and light quark condensate difference (bottom panel) as a function of $T$ for several values of $eB$. 

IV. INVERSE MAGNETIC CATALYSIS AT FINITE $T$

In the last section we have seen that SU(3) (E)PNJL reproduces quite well the lattice QCD quark condensate behavior in an external magnetic field except for the inverse magnetic catalysis effect predicted by lattice QCD calculations at temperatures of the order of the transition temperature and high magnetic fields. In the EPNJL the deconfinement is described by the Polyakov loop which couples weakly to the magnetic field as referred above. It should be noted that originally the parametrization of the Polyakov loop potential was built in order to reproduce the pure gluonic lattice data. Later, it was realized that the inclusion of dynamical quarks leads to a decrease of the scale parameter $T_0$. Since strong magnetic fields have certainly an effect on those dynamical quarks, one could expect that their presence could affect the value of $T_0$. Unfortunately, so far there is no available calculation of the effect of the magnetic fields on the Polyakov potential. In this situation one possible approach is to choose a magnetic field dependent $T_0(eB)$ so that the correct transition temperatures are reproduced. Within the PNJL this is not possible because the chiral transition temperatures increase strongly with the external magnetic field. In order to bring these temperatures down it would be necessary to use very small values of $T_0$, for which the deconfinement phase transition becomes of first order.

| $\Sigma = 0$ | 0.2 0.4 0.6 0.75 |
| $\Sigma = 2$ | 0.2 0.4 0.6 0.75 |

FIG. 4. Change of the renormalized condensate for $eB < 0.6 \text{ GeV}^2$. 

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Previously the two-flavor version of both models had been studied to use the QCD phase diagram in the presence of an external magnetic field. The chiral and deconfinement transition temperatures increase in the presence of an external magnetic field, although the deconfinement transition temperature de- creases with the increase of the magnetic field strength, reproducing lattice calculations.

**FIG. 4.** The change of the renormalized condensate as a function of $eB$ for several temperatures in the PNJL model with $T_0(eB)$ defined in Eq. (9).

$T = 0$ the quantitative behavior of SU(3) PNJL and EPNJL is closer to the lattice results. Another aspect that should be referred if the effect of the magnetic field on the EPNJL deconfinement and chiral transition temperatures: the existing coincidence at $eB = 0$ is destroyed by the magnetic field.

Once the EPNJL model is used to compute the above mentioned quantities, a magnetic field dependent parametrization of the Polyakov loop can invert the magnetic catalysis and the pseudo-critical temperature decreases with the increase of the magnetic field strength.

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