COUNTER EXAMPLES TO THE NONRESTRICTED REPRESENTATION THEORY

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Abstract. We shall consider nonrestricted representations of $C_l-$ type Lie algebra over an algebraically closed field of characteristic $p \geq 7$. This paper gives some counter examples to important theory relating to the representations of modular Lie algebras.

1. Introduction

We shall study the $C_l-$ type Lie algebra in section 2. In section 3, restricted and nonrestricted representations are explained. Next we exhibit irreducible modules for nonrestricted representations of modular $C_l-$ type Lie algebra in the last section 4.

This paper just gives some counter examples to [3] which states sort of the dimensions of irreducible modules over modular $C_l-$ type Lie algebras.

2. Modular $C_l-$ type Lie algebra

As is well known the symplectic Lie algebra of rank $l$, i.e., the $C_l-$ type Lie algebra over $\mathbb{C}$ has its root system $\Phi = \ldots$
\{±2\epsilon_i, ±(\epsilon_i ± \epsilon_j), i \neq j\}, where \epsilon_i, \epsilon_j are linearly independent unit vectors in \mathbb{R}^l with \ l \geq 3.

For an algebraically closed field \( F \) of prime characteristic \( p \), the \( C_l \)-type Lie algebra \( L \) over \( F \) is just the analogue over \( F \) of the \( C_l \)-type simple Lie algebra over \( \mathbb{C} \).

In other words the \( C_l \)-type Lie algebra over \( F \) is isomorphic to the Chevalley Lie algebra of the form \( \sum_{i=1}^{n} \mathbb{Z}c_i \otimes \mathbb{Z} F \), where \( n = \text{dim}_F L \) and \( x_\alpha = \) some \( c_i \) for each \( \alpha \in \Phi \), \( h_\alpha = \) some \( c_j \) with \( \alpha \) some base element of \( \Phi \) for a Chevalley basis \( \{c_i\} \) of the \( C_l \)-type Lie algebra over \( \mathbb{C} \).

3. RESTRICTED AND NONRESTRICTED REPRESENTATIONS

We assume in this section that the ground field \( F \) is an algebraically closed field of nonzero characteristic \( p \).

**Definition 3.1.** Let \((L, [p])\) be a restricted Lie algebra over \( F \) and \( \chi \in L^* \) be a linear form. If a representation \( \rho_\chi : L \rightarrow \mathfrak{gl}(V) \) of \((L, [p])\) satisfies \( \rho_\chi(x^p - x^{[p]}) = \chi(x)^p id_V \) for any \( x \in L \), then \( \rho_\chi \) is said to be a \( \chi \)-representation.

In this case we say that the representation or the corresponding module has a \( p \)-character \( \chi \). In particular if \( \chi = 0 \), then \( \rho_0 \) is called a restricted representation, whereas \( \rho_\chi \) for \( \chi \neq 0 \) is called a nonrestricted representation.
We are well aware that we have $\rho(\chi)^p - \rho(\chi[a])^{[p]} = \chi(a)^p id_V$ for some $\chi \in L^*$, for any $a \in L$ and for any irreducible representation $\rho\chi$.

4. IRREDUCIBLE NONRESTRICTED REPRESENTATIONS OF $C_l$-TYPE LIE ALGEBRA

In this section we compute the dimension of some irreducible modules of the $C_l$-type Lie algebra $L$ with a CSA $H$ over an algebraically closed field $F$ of characteristic $p \geq 7$.

**Proposition 4.1.** Let $\alpha$ be any root in the root system $\Phi$ of $L$. If $\chi(x_\alpha) \neq 0$, then $\dim_F \rho(\chi(U(L))) = p^{2m}$, where $[Q(U(L)) : Q(3)] = p^{2m} = p^{n-l}$ with 3 the center of $U(L)$ and $Q$ denotes the quotient algebra. So the irreducible module corresponding to this representation has $p^m$ as its dimension.

**Proof.** Let $\mathfrak{M}_\chi$ be the kernel of this irreducible representation, i.e., a certain (2-sided) maximal ideal of $U(L)$.

(I) Assume first that $\alpha$ is a short root; then we may put $\alpha = \epsilon_1 - \epsilon_2$ without loss of generality since all roots of a given length are conjugate under the Weyl group of the root system $\Phi$.

First we let $B_i = b_{i1} h_{\epsilon_1 - \epsilon_2} + b_{i2} h_{\epsilon_2 - \epsilon_3} + \cdots + b_{i,l-1} h_{\epsilon_l-1 - \epsilon_l} + b_{il} h_{\epsilon_l}$ for $i = 1, 2, \cdots, 2m$, where $(b_{i1}, b_{i2}, \cdots, b_{il}) \in F^l$ are chosen so that any $(l+1) - B_i$'s are linearly independent in $\mathbb{P}^l(F)$, the $B$ below becomes an $F$–linearly independent set in $U(L)$ if
necessary and \( x_\alpha B_i \neq B_i x_\alpha \) for \( \alpha = \epsilon_1 - \epsilon_2 \).

In \( U(L) / \mathcal{M}_\chi \) we claim that we have a basis \( B := \{(B_1 + A_{\epsilon_1 - \epsilon_2})^i_1 \otimes (B_2 + A_{-(\epsilon_1 - \epsilon_2)})^i_2 \otimes \cdots \otimes (B_{2l-2} + A_{-(\epsilon_1 - \epsilon_2)})^i_{2l-2} \otimes (B_{2l-1} + A_{2\epsilon_i})^i_{2l-1} \otimes (B_{2l} + A_{-2\epsilon_i})^i_{2l} \otimes (\otimes_{j=2l+1}^{2m} (B_j + A_{\alpha_j})^i_j) ; 0 \leq i_j \leq p - 1 \} \),

where we put
\[
\begin{align*}
A_{\epsilon_1 - \epsilon_2} & = x_\alpha = x_{\epsilon_1 - \epsilon_2}, \\
A_{\epsilon_2 - \epsilon_1} & = c_{-(\epsilon_1 - \epsilon_2)} + (h_{\epsilon_1 - \epsilon_2} + 1)^2 + 4 x_\alpha x_\alpha, \\
A_{\epsilon_2 \pm \epsilon_3} & = x_{\pm 2\epsilon_3} \left( c_{\epsilon_2 \pm \epsilon_3} + x_{\epsilon_2 \pm \epsilon_3} x_{-(\epsilon_2 \pm \epsilon_3)} \right) \pm x_{\epsilon_1 \pm \epsilon_3} x_{-(\epsilon_1 \pm \epsilon_3)}, \\
A_{\epsilon_1 + \epsilon_2} & = x_{\epsilon_1 - \epsilon_2} \left( c_{\epsilon_1 + \epsilon_2} + 3 x_{\epsilon_1 + \epsilon_2} x_{-\epsilon_1 - \epsilon_2} \pm 2 x_{\epsilon_1 - \epsilon_2} x_{-2\epsilon_1 - \epsilon_2} \pm 2 x_{2\epsilon_1 - \epsilon_2} x_{-2\epsilon_1 - \epsilon_2} \right), \\
A_{\epsilon_2 \pm \epsilon_k} & = x_{\epsilon_3 \pm \epsilon_k} \left( c_{\epsilon_2 \pm \epsilon_k} + x_{\epsilon_2 \pm \epsilon_k} x_{-(\epsilon_2 \pm \epsilon_k)} \right) \pm x_{\epsilon_1 \pm \epsilon_k} x_{-(\epsilon_1 \pm \epsilon_k)}, \\
A_{2\epsilon_2} & = x_{2\epsilon_3} \left( c_{2\epsilon_2} + 2 x_{2\epsilon_2} x_{-2\epsilon_2} \pm 3 x_{\epsilon_1 + \epsilon_2} x_{-\epsilon_1 - \epsilon_2} \pm 2 x_{\epsilon_1 - \epsilon_2} x_{-2\epsilon_1 - \epsilon_2} \right), \\
A_{-2\epsilon_1} & = x_{-\epsilon_3} \left( c_{-2\epsilon_1} + 2 x_{-2\epsilon_1} x_{2\epsilon_1} \pm 3 x_{-\epsilon_1 - \epsilon_2} x_{\epsilon_1 + \epsilon_2} \pm 2 x_{-\epsilon_2} x_{2\epsilon_2} \right), \\
A_{-(\epsilon_1 \pm \epsilon_3)} & = x_{-(\pm \epsilon_3)} \left( c_{-(\epsilon_1 \pm \epsilon_3)} + x_{\epsilon_2 \pm \epsilon_3} x_{-(\epsilon_2 \pm \epsilon_3)} \right) \pm x_{\epsilon_1 \pm \epsilon_3} x_{-(\epsilon_1 \pm \epsilon_3)}), \\
A_{-(\epsilon_1 \pm \epsilon_k)} & = x_{-(\epsilon_3 \pm \epsilon_k)} \left( c_{-(\epsilon_1 \pm \epsilon_k)} + x_{\epsilon_2 \pm \epsilon_k} x_{-(\epsilon_2 \pm \epsilon_k)} \right) \pm x_{\epsilon_1 \pm \epsilon_k} x_{-(\epsilon_1 \pm \epsilon_k)}), \\
A_{2\epsilon_1} & = x_{2\epsilon_1}, \\
A_{-2\epsilon_1} & = x_{-2\epsilon_1}, \\
\end{align*}
\]

with the sign chosen so that they commute with \( x_\alpha \) and with \( c_\alpha \in F \) chosen so that \( A_{\epsilon_2 - \epsilon_1} \) and parentheses are invertible. For any other root \( \beta \) we put \( A_\beta = x_\beta^2 \) or \( x_\beta^3 \) if possible. Otherwise attach to these sorts the parentheses( ) used for designating \( A_{-\beta} \) so that \( A_{\gamma} \forall \gamma \in \Phi \) may commute with \( x_\alpha \).

We shall prove that \( B \) is a basis in \( U(L) / \mathcal{M}_\chi \). By virtue of P-B-W theorem, it is not difficult to see that \( B \) is evidently a linearly independent set over \( F \) in \( U(L) \). Furthermore \( \forall \beta \in \Phi, A_\beta \notin \mathcal{M}_\chi \) (see detailed proof below).
We shall prove that a nontrivial linearly dependence equation leads to absurdity. We assume first that there is a dependence equation which is of least degree with respect to $h_{\alpha_j} \in H$ and the number of whose highest degree terms is also least.

In case it is conjugated by $x_{\alpha}$, then there arises a nontrivial dependence equation of lower degree than the given one, which contradicts our assumption. Otherwise it reduces to one of the following forms:

(i) $x_{2\epsilon_j}K + K' \in M_{\chi}$,
(ii) $x_{-2\epsilon_j}K + K' \in M_{\chi}$,
(iii) $x_{\epsilon_j+\epsilon_k}K + K' \in M_{\chi}$,
(iv) $x_{-\epsilon_j-\epsilon_k}K + K' \in M_{\chi}$,
(v) $x_{\epsilon_j-\epsilon_k}K + K' \in M_{\chi}$,

where $K, K'$ commute with $x_{\alpha}$.

For the case (i), we deduce successively $x_{\epsilon_2-\epsilon_j}x_{2\epsilon_j}K + x_{\epsilon_2-\epsilon_j}K' \in M_{\chi}$

$\Rightarrow x_{\epsilon_2+\epsilon_j}K + x_{2\epsilon_j}x_{\epsilon_2-\epsilon_j}K + x_{\epsilon_2-\epsilon_j}K' \in M_{\chi} \Rightarrow (x_{\epsilon_1+\epsilon_j} or x_{2\epsilon_1})K + x_{2\epsilon_j}(x_{\epsilon_1-\epsilon_j} or h_{\epsilon_1-\epsilon_2})K + (x_{\epsilon_1-\epsilon_j} or h_{\epsilon_1-\epsilon_2})K' \in M_{\chi}$

by $adx_{\epsilon_1-\epsilon_2}$

if $j \neq 1$ or $j = 1$ respectively, so that by successive $adx_{\alpha}$ and rearrangement we get $x_{\epsilon_1 \pm \epsilon_j}K + K'' \in M_{\chi}$ for some $K''$ commuting with $x_{\alpha}$ in view of the start equation. So (i) reduces to (iii), (iv) or (v).

Similarly as in (i) and by adjoint operations, (ii) reduces to (iii), (iv) or (v). Also (iii), (iv) reduces to the form (v) putting $\epsilon_j = -(-\epsilon_j), \epsilon_k = -(-\epsilon_k)$. 

Hence we have only to consider the case (v). We consider
\[ x_{\epsilon_k-\epsilon_2} x_{\epsilon_j-\epsilon_k} K + x_{\epsilon_k-\epsilon_2} K' \in \mathcal{M}_\chi, \]
so that \( (x_{\epsilon_j-\epsilon_2} + x_{\epsilon_j-\epsilon_k} x_{\epsilon_k-\epsilon_2}) K + x_{\epsilon_k-\epsilon_2} K' \)
\[ \in \mathcal{M}_\chi \] for \( j, k \neq 1, 2. \) We thus have \( x_{\epsilon_j-\epsilon_2} K + (x_{\epsilon_j-\epsilon_k} x_{\epsilon_k-\epsilon_2} K + x_{\epsilon_k-\epsilon_2} K') \in \mathcal{M}_\chi, \)
so that we may put this last ( ) as another \( K' \) alike as in the equation (v).

Hence we need to show that \( x_{\epsilon_j-\epsilon_2} K + K' \in \mathcal{M}_\chi \) leads to absurdity. We consider \( x_{\epsilon_2-\epsilon_j} x_{\epsilon_j-\epsilon_2} K + x_{\epsilon_2-\epsilon_j} K' \in \mathcal{M}_\chi \Rightarrow \)
\( (h_{\epsilon_2-\epsilon_j} + x_{\epsilon_j-\epsilon_2} x_{\epsilon_2-\epsilon_j}) K + x_{\epsilon_2-\epsilon_j} K' \in \mathcal{M}_\chi \Rightarrow (x_{\epsilon_1-\epsilon_2} \pm x_{\epsilon_j-\epsilon_2} x_{\epsilon_1-\epsilon_j}) K + x_{\epsilon_1-\epsilon_j} K' \in \mathcal{M}_\chi \) by \( \text{ad} x_{\epsilon_1-\epsilon_2} \) \Rightarrow either \( x_{\epsilon_1-\epsilon_2} K \in \mathcal{M}_\chi \) or ( \( x_{\epsilon_1-\epsilon_2} + x_{\epsilon_j-\epsilon_2} x_{\epsilon_1-\epsilon_j} K + x_{\epsilon_1-\epsilon_j} K' \in \mathcal{M}_\chi \)
depending on \( [x_{\epsilon_j-\epsilon_2}, x_{\epsilon_1-\epsilon_j}] = +x_{\epsilon_1-\epsilon_2} \) or \(-x_{\epsilon_1-\epsilon_2}. \) The former case leads to \( K \in \mathcal{M}_\chi, \)
a contradiction.

For the latter case we consider
\[ x_{\epsilon_1-\epsilon_2} K + (x_{\epsilon_j-\epsilon_2} x_{\epsilon_1-\epsilon_j} K + x_{\epsilon_1-\epsilon_j} K') \in \mathcal{M}_\chi. \]

So we may put
\[ (*) x_{\epsilon_1-\epsilon_2} K + K'' \in \mathcal{M}_\chi, \]
where \( K'' = x_{\epsilon_j-\epsilon_2} x_{\epsilon_1-\epsilon_j} K + x_{\epsilon_1-\epsilon_j} K'. \)
Thus \( x_{\epsilon_2-\epsilon_1} x_{\epsilon_1-\epsilon_2} K + x_{\epsilon_2-\epsilon_1} K'' \in \mathcal{M}_\chi. \) From \( w_{\epsilon_1-\epsilon_2} := (h_{\epsilon_1-\epsilon_2} + 1)^2 + 4 x_{\epsilon_2-\epsilon_1} x_{\epsilon_1-\epsilon_2} \in \text{the center of } U(\mathfrak{sl}_2(F)), \) we get \( 4^{-1} \{ w_{\epsilon_1-\epsilon_2} - (h + 1)^2 \} K + x_{\epsilon_2-\epsilon_1} K'' \equiv 0 \) modulo \( \mathcal{M}_\chi. \)
If \( x_{\epsilon_2-\epsilon_1} \equiv c \) which is a constant, then
\[ (**) 4^{-1} x_{\epsilon_2-\epsilon_1} \{ w_{\epsilon_1-\epsilon_2} - (h_{\epsilon_1-\epsilon_2} + 1)^2 \} K + c K'' \equiv 0 \]
is obtained. From \( (*), (**) \), we have \( 4^{-1} x_{\epsilon_2-\epsilon_1} \{ w_{\epsilon_1-\epsilon_2} - (h_{\epsilon_1-\epsilon_2} + 1)^2 \} K - c x_{\epsilon_1-\epsilon_2} K \)
≡ 0. Multiplying \( x_{\epsilon_1 - \epsilon_2}^{p-1} \) to this equation, we obtain

\[
(\ast\ast\ast)4^{-1}x_{\epsilon_1 - \epsilon_2}^{p-1}x_{\epsilon_2 - \epsilon_1}^{p-1}\{w_{\epsilon_1 - \epsilon_2} - (h_{\epsilon_1 - \epsilon_2} + 1)^2\}K - cx_{\epsilon_1 - \epsilon_2}^{p-1}K \equiv 0.
\]

By making use of \( w_{\epsilon_1 - \epsilon_2} \), we may have from \((\ast\ast\ast)\) an equation of the form

(a polynomial of degree \( \geq 1 \) with respect to \( h_{\epsilon_1 - \epsilon_2} \))

\[
K - cx_{\epsilon_1 - \epsilon_2}^{p-1}K \equiv 0.
\]

Finally if we use conjugation and subtraction, then we are led to a contradiction \( K \in \mathcal{M}_\chi \).

(II) Assume next that \( \alpha \) is a long root; then we may put \( \alpha = 2\epsilon_1 \) because all roots of the same length are conjugate under the Weyl group of \( \Phi \). Similarly as in (I), we let \( B_i := \) the same as in (I) except that this time \( \alpha = 2\epsilon_1 \) instead of \( \epsilon_1 - \epsilon_2 \).

We claim that we have a basis \( B := \{(B_1 + A_{2\epsilon_1})^{i_1} \otimes (B_2 + A_{-2\epsilon_1})^{i_2} \otimes (B_3 + A_{\epsilon_1 - \epsilon_2})^{i_3} \otimes (B_4 + A_{-(\epsilon_1 - \epsilon_2)})^{i_4} \otimes \cdots \otimes (B_{2l} + A_{-(\epsilon_1 - \epsilon_l)})^{i_{2l}} \otimes (B_{2l+1} + A_{2\epsilon_1})^{i_{2l+1}} \otimes (B_{2l+2} + A_{-2\epsilon_1})^{i_{2l+2}} \otimes (\otimes_{j=2l+3}(B_j + A_{\alpha_j})^{i_j}); 0 \leq i_j \leq p - 1\}, \)

where we put

\[
\begin{align*}
A_{2\epsilon_1} &= x_{2\epsilon_1}, \\
A_{-2\epsilon_1} &= c_{-2\epsilon_1} + (h_{\epsilon_1} + 1)^2 + 4x_{-2\epsilon_1}x_{2\epsilon_1}, \\
A_{-\epsilon_1 \pm \epsilon_2} &= x_{-\epsilon_1 \pm \epsilon_2}(c_{-\epsilon_1 \pm \epsilon_2} \pm x_{-\epsilon_1 \pm \epsilon_2}x_{\epsilon_1 \pm \epsilon_2} \pm x_{\epsilon_1 \pm \epsilon_2}x_{-\epsilon_1 \pm \epsilon_2}), \\
A_{-\epsilon_1 \pm \epsilon_j} &= x_{-\epsilon_2 \pm \epsilon_j}(c_{-\epsilon_1 \pm \epsilon_j} + x_{\pm \epsilon_j}x_{\epsilon_1 \pm \epsilon_j} \pm x_{\epsilon_1 \pm \epsilon_j}x_{-\epsilon_1 \pm \epsilon_j}) \text{, and for any other root } \beta \text{ we put } A_\beta = x_\beta^2 \text{ or } x_\beta^3 \text{ if possible. Otherwise attach to these the parentheses ( ) used for designating } A_{-\beta}. 
\end{align*}
\]

Likewise as in case (I), we shall prove that \( B \) is a basis in \( U(L)/\mathcal{M}_\chi \). By virtue of P-B-W theorem, it is not difficult
to see that $B$ is evidently a linearly independent set over $F$ in $U(L)$. Moreover $\forall \beta \in \Phi$, $A_\beta \not\in \mathfrak{M}_\chi$(see detailed proof below).

We shall prove that a nontrivial linearly dependence equation leads to absurdity. We assume first that there is a dependence equation which is of least degree with respect to $h_{\alpha_j} \in H$ and the number of whose highest degree terms is also least. If it is conjugated by $x_\alpha$, then there arises a nontrivial dependence equation of least degree than the given one, which contravenes our assumption.

Otherwise it reduces to one of the following forms:

(i) $x_{2\epsilon_j}K + K' \in \mathfrak{M}_\chi$,
(ii) $x_{-2\epsilon_j}K + K' \in \mathfrak{M}_\chi$,
(iii) $x_{\epsilon_j+\epsilon_k}K + K' \in \mathfrak{M}_\chi$,
(iv) $x_{-\epsilon_j-\epsilon_k}K + K' \in \mathfrak{M}_\chi$,
(v) $x_{\epsilon_j-\epsilon_k}K + K' \in \mathfrak{M}_\chi$,

where $K$ and $K'$ commute with $x_\alpha = x_{2\epsilon_1}$.

For the case (i), we consider a particular case $j=1$ first; if we assume $x_{2\epsilon_1}K + K' \in \mathfrak{M}_\chi$, then we are led to a contradiction according to the similar argument (*) as in (I). So we assume $x_{2\epsilon_j}K + K' \in \mathfrak{M}_\chi$ with $j \geq 2$. Now we have $x_{2\epsilon_j}K + K' \in \mathfrak{M}_\chi \Rightarrow x_{-\epsilon_1-\epsilon_j}x_{2\epsilon_j}K + x_{-\epsilon_1-\epsilon_j}K' \in \mathfrak{M}_\chi \Rightarrow x_{-\epsilon_1+\epsilon_j}K + x_{2\epsilon_j}x_{-\epsilon_1-\epsilon_j}K + x_{-\epsilon_1-\epsilon_j}K' \in \mathfrak{M}_\chi \Rightarrow$ by $adx_{2\epsilon_1}, x_{\epsilon_1+\epsilon_j}K + x_{2\epsilon_j}x_{\epsilon_1-\epsilon_j}K + x_{\epsilon_1-\epsilon_j}K' \in \mathfrak{M}_\chi$ is obtained. Hence (i) reduces to (iii).

Similarly (ii) reduces to (iii) or (iv) or (v). So we have only to consider (iii), (iv), (v). However (iii), (iv), (v) reduce to $x_{2\epsilon_1}K + K'' \in \mathfrak{M}_\chi$ after all considering the situation as in (I).
Similarly following the argument as in (I), we are led to a contradiction $K \in \mathcal{M}_\chi$.

\[ \square \]

**Corollary 4.2.** The Weyl module $W_\chi(L) = \text{the Verma module } V_\chi(L)$, where $\chi$ is a nonzero character of $L$ over an algebraically closed field $F$ of characteristic $p \geq 7$. In other words the dimension of any irreducible $L$-module over $F$ is $p^m$ if the irreducible module is associated with a nonzero character $\chi \neq 0$.

**Proof.** It is obvious by the proposition(4.1).

\[ \square \]

So we might as well extend the argument in the proof of the proposition(4.1) to that of other type simple Lie algebras.

**Remark 4.3.** James E. Humphreys indicated in [3] that there is an $\text{sp}_4(F)$—irreducible module of dimension $p^3$ in the above situation which has nearly nothing to do with characteristic $p$.

However according to the argument similar as our argument in the above propositions, the dimension of $\text{sp}_4(F)$—irreducible module is equal to $p^4$ in our situation because $l = 2, m = 4$, and $n = 10$. Our computation so far makes us to conjecture results alike for other classical modular simple Lie algebras.

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