Dirichlet Solitons in Field Theories

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Abstract

I briefly describe a new class of soliton configurations in field theories. These consist of topological defects which can end when they intersect other defects of equal or higher dimensionality. Such configurations may be termed “Dirichlet topological defects”, in analogy with the D-branes of string theory. I provide a specific example - cosmic strings that terminate on domain walls - and discuss some new directions for this work, including an interesting and qualitatively different extension to supersymmetric theories.

CWRU-P2-99

The study of topological soliton solutions to classical field theories has led to many important new ideas in particle physics and cosmology. In particle physics, a common feature of the recent progress in both supersymmetry (SUSY) and string theory has been the discovery of dualities. These dualities map the calculationally difficult limit of one theory into the (hopefully) calculationally easier limit of another theory. More precisely, dualities often interchange the roles of the fundamental and solitonic degrees of freedom. For the case of Seiberg-Witten dualities in SUSY field theories, the relevant solitons are monopoles, so that the electric and magnetic degrees of freedom are interchanged. For string theories, the relevant objects are the D-branes; extended configurations on which fundamental strings can end. In both cases, the study of solitons in the theories has led to a better understanding of how particle physics works.

In cosmology, solitons, or defects, play two main roles. First, one may use cosmology to constrain candidate particle physics theories. Examples of this include the requirement that symmetry breaking schemes not allow magnetic monopoles in the early universe, and vorton constraints on theories admitting superconducting cosmic strings. Second, the dynamics, interactions, and microphysics of topological defects can provide explanations for cosmological problems. Examples of this include the use of cosmic strings and textures as

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Talk presented at Cosmo-98, Particle Physics and the Early Universe, Asilomar, CA November 15-20, 1998
seeds for the large scale structure, and the idea that topological solitons might play a role in the generation of the baryon asymmetry of the universe.

Topological defects are solitonic solutions whose stability is guaranteed by a topological conservation law. When a symmetry group $G$ is spontaneously broken to a subgroup $H$, the types of defects supported depend on the homotopy properties of the vacuum manifold, $\mathcal{M} = G/H$. In a $(d+1)$-dimensional spacetime, $p$-dimensional defects ($p < d$) exist if the homotopy group $\pi_{d-p-1}(\mathcal{M})$ is nontrivial. (For reviews see [1].) In addition to these basic defects, there are various composite solutions which combine two of the types, generally when a $(p-1)$-dimensional defect serves as the boundary of a $p$-dimensional defect. Such configurations have interesting cosmological applications, such as in the Langacker-Pi mechanism for solving the monopole problem [2].

In this brief review I will describe a new class of topological defects [6] which is complementary to those mentioned above. These consist of field configurations in which one type of topological defect can terminate when intersecting other defects of equal or higher dimensionality. Such configurations may be termed “Dirichlet topological defects”, in analogy with the D-branes of string theory. The latter are extended objects on which fundamental strings can end [7]. The models considered here are ordinary field theories, which support topological solitons which resemble these objects in fundamental string theory. Here I will only have space to provide the specific example of cosmic strings that terminate on domain walls, and to comment on future directions for investigation.

Strings arise most simply from the breakdown of $U(1)$ symmetries. Therefore consider two complex fields $\psi_i = \rho_i e^{i\theta_i}$, ($i = 1, 2$), and a single real scalar $\phi$, transforming under two $U(1)$ and one $Z_2$ symmetries in the following way

$$
\begin{align*}
Z_2 : & \{ \phi \rightarrow -\phi , \psi_1 \leftrightarrow \psi_2 \}, \\
U(1)_1 : & \psi_1 \rightarrow e^{-i\omega_1} \psi_1, \\
U(1)_2 : & \psi_2 \rightarrow e^{-i\omega_2} \psi_2.
\end{align*}
$$

The two $U(1)$’s may be taken to be either global or gauge symmetries. In the latter case, $\omega_1$ and $\omega_2$ are functions of spacetime, and there are two gauge fields $A_\mu^{(1)}$, $A_\mu^{(2)}$, with the usual transformation properties, and associated covariant derivatives.

Write a general, renormalizable potential in the convenient form

$$
V(\phi, \psi_1, \psi_2) = \lambda_\phi (\phi^2 - \bar{v}^2)^2 + \lambda_\psi \left( |\psi_1|^2 + |\psi_2|^2 - \bar{w}^2 + g(\phi^2 - \bar{v}^2) \right)^2 \\
+ h|\psi_1|^2|\psi_2|^2 - \mu \phi(|\psi_1|^2 - |\psi_2|^2),
$$

where $v = \langle |\psi| \rangle$ is the root of the cubic equation

$$
8\lambda_\phi \lambda_\psi v^3 + 6\lambda_\psi g \mu v^2 - (8\lambda_\phi \lambda_\psi \bar{v}^2 + \mu^2)v - 2\lambda_\psi (g \bar{v}^2 + \bar{w}^2) \mu = 0
$$

that reduces to $\bar{v}$ at $\mu = 0$, and $w$ is given by

$$
w = \left( \bar{w}^2 + g(\bar{v}^2 - v^2) + \frac{\mu v}{2\lambda_\psi} \right)^{1/2}.
$$

In the vacuum the real scalar $\phi$ takes the vev $\pm v$ and there may exist domain walls separating these two values. When $\langle \phi \rangle = +v$, the vacuum has $|\psi_1| = v$ and $\psi_2 = 0$, while when $\langle \phi \rangle = -v$ the vacuum has $|\psi_2| = v$ and $\psi_1 = 0$. 

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In this model, therefore, the unbroken symmetry group in the true vacuum is U(1), and the vacuum manifold is $\mathcal{M} = [U(1) \times U(1) \times \mathbb{Z}_2]/U(1) = S^1 \times \mathbb{Z}_2$, admitting walls and strings. When $\langle \phi \rangle = +v$, the complex field $\psi_1$ can form cosmic strings with winding number $n$, around which $\theta_1$ will change by $2\pi n$. Such a string ends if it intersects a D-wall, since $\langle \psi_1 \rangle = 0$ on the other side. Analogous statements hold for the $\psi_2$ field when $\langle \phi \rangle = -v$.

In the core of a string the corresponding U(1) symmetry is restored. In the gauge case, therefore, the gauge bosons associated with, for example, $U(1)_1$ are massless both in the core of a $\psi_1$-string on the $\langle \phi \rangle = v$ side of the D-wall, and anywhere on the $\langle \phi \rangle = -v$ side of the D-wall. As usual, outside the $\psi_1$-string the gauge field is pure gauge, such that it cancels the gradient energy of the scalars by enforcing the vanishing of the covariant derivative. The gauge field is thus given by $A^{(1)}_{\mu} = -\partial_{\mu} \theta_1$. Consequently, there is magnetic flux through the string (which I take to have winding number $n$), given by $\Phi^{(1)} = -n\pi$. This flux flows through the string until it hits the wall; on the other side of the wall the symmetry is unbroken everywhere, and the magnetic field describes a monopole configuration emanating from the point where the string intersects the wall.

Configurations of this type, with strings ending on walls, have recently been discussed in the context of supersymmetric QCD [8]. There, the string consists of non-Abelian flux, and the wall separates different chiral vacua, with shifted values of the QCD $\theta$-parameter. The intersections of strings and domain walls can be thought of as quarks. The structures of these QCD configurations and the scalar field models discussed here are obviously very similar, and the relationship between them deserves further investigation. (One difference is that the flux in the strings considered in [8] does not propagate freely on the other side of the wall, as the symmetry is still broken there; rather, it is confined to the wall itself. It should not be difficult to extend models of the type considered in this paper to include such situations.)

I have described a class of topological defects in classical field theories in (3+1) dimensions, consisting of Dirichlet defects on which fundamental defects of lower dimension can terminate. While the search for models supporting these configurations is inspired by the appearance of D-branes in string theory, there are important differences between the two sets of objects. In all of the theories considered, the basic degrees of freedom are scalar fields and gauge fields, out of which all of the higher-dimensional objects are constructed. Gravity and supersymmetry are not included (although there are no obstacles to the appropriate generalizations [9]). Furthermore, the specific dependence of D-brane energy on the string coupling constant is not a feature of our models, and the Ramond-Ramond gauge fields to which D-branes couple are absent. Nevertheless, it may be interesting to compare the dynamical behavior of Dirichlet defects to that of D-branes in string theory, and search for models in which the similarities between the two systems are even stronger.

One obvious direction in which to generalize the models considered here is to consider $q$-dimensional defects ending on $p$-dimensional D-defects in $d$ spatial dimensions. (There are a variety of such objects in string theory and M-theory, with configurations governed by charge conservation.) A number of interesting issues arise in this case, especially for gauge symmetries. For example, to make topological defects of dimension $q$ in $d$ spatial dimensions requires that $\pi_{d-q-1}(\mathcal{M})$ be nontrivial, for example by breaking $SO(d-p)$ to $SO(d-p-1)$ (for which $\mathcal{M} = S^{d-p-1}$). In such a model, the unbroken symmetry group $SO(d-p-1)$ is non-Abelian for $p \leq d - 4$ and the low-energy gauge theory is expected to be strongly
coupled, and the resulting defects to be confined.

Finally, as with any species of topological defect, it is also natural to ask what the cosmological consequences of the formation of these objects in the early universe might be.

ACKNOWLEDGMENTS

I would like to thanks my collaborators, Sean Carroll and Simeon Hellerman for a lot of fun working on these topics. Thanks also to the organizers, for a great meeting. This work was supported by the Department of Energy (D.O.E.).
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