Influence of streamwise vortical structures on heat transfer in the far cylinder wake in a slot channel flow

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Abstract. We report on Large-eddy simulations (LES) of a flow around a cylinder in a narrow duct channel (slot) at the Reynolds number 3750 based on the bulk velocity and the diameter of the cylinder. Secondary streamwise vortical structures in the far wake are investigated appearing near the mixing layer at the midplane and near the duct channel surface. We investigate the correlation between the heat flux and friction velocity fluctuations and evaluate the propagation speed of these disturbances, which appears to be different.

1. Introduction

Flows over bluff bodies mounted between narrow rigid walls are encountered in a variety of engineering applications and environmental flows over a wide range of the Reynolds numbers. Heat exchangers, cooling systems, building sections, junctions in a wing-body, bridge piers can be mentioned amongst other various practically relevant examples of this flow. Such configurations feature a few well-known vortical structures.

In front of a bluff body the horseshoe vortex system is present due to adverse pressure gradient resulting in increased local heat transfer, acoustic noise and shear stress [1]. Behind the body quasi-periodic vortex shedding appears because Kelvin-Helmholz instability develops in separated shear layer resulting in a recirculating zone which can lead to the Kármán vortex street with large-scale vortical structures. The recirculating bubble is known to exhibit low-frequency motion featuring the fluctuation of the recirculation zone length and Reynolds stresses intensity [2-4]. In shallow wake configurations such low-frequency modulations are represented by large dominant spanwise vortical structures in near wake, which strongly affect the heat and mass transfer characteristics downstream of the cylinder [5, 6]. An interaction between Kármán vortex street and channel walls cause to appear additional streamwise vortices in the far wake region. Similar structures were earlier detected in slot jets [7,8]. In this paper we investigate a cylinder placed in a rectangular duct to qualitatively examine the effect of secondary streamwise vortical currents behind the cylinder in the far wake on heat and mass transfer from the wall.

2. Computational details

We used unstructured finite-volume computational code T-Flows [9,10] with the cell-centered collocated grid structure to solve the spatially filtered Navier–Stokes equations for incompressible fluid and a transport equation for the temperature field acting as a passive scalar within Large-eddy
The simulation framework [11,12]. The code features second order accuracy discretization in space and time. The SIMPLE algorithm is used to couple pressure and velocity fields.

The computational domain represents a rectangular duct. The total domain dimensions are $29D \times 20D \times H$ in $x \times y \times z$ (streamwise, spanwise and wall-normal direction). A circular cylinder of a diameter $D$ is placed $14D$ from the inlet boundary ($x = 0, y = 0$) and fixed between two parallel walls at $z = 0$ and $z = H$ as shown in Fig. 1. The duct channel height is $H = 0.4D$. The following boundary conditions are used for the domain boundaries: no-slip boundary conditions are set at all walls, i.e. $z = 0$ and $z = H$ as well as for side boundaries with $y = -10D$ and $y = 10D$ and the surface of the cylinder. At the inlet ($x = -14D$) we impose laminar steady parabolic velocity profile and $z$-linear temperature distribution, while the convective outflow condition is set at the outlet ($x = 15D$). The temperature on the cylinder and duct surfaces is also set to be linear varying from hot to cold as $T(z) = \Delta T(1 - z/H)$, where $\Delta T$ is temperature difference between lower and upper walls. Further all physical quantities are non-dimensionalized with the cylinder diameter $D$, bulk velocity at inflow $U_b$ and temperature difference $\Delta T$, which leads to the non-dimensional heat flux definition $q_i = -(\partial T/\partial x_i)/(Re Pr)$.

The governing equations are shown in (1-3). The Prandtl number is chosen for water at a room temperature - 6.13. The subgrid-scale velocity and temperature fluctuations are modeled using the gradient hypothesis resulting in the turbulent Reynolds $Re_t$ and Prandtl $Pr_t$ numbers which are based on the turbulent viscosity and thermal diffusivity. The turbulent viscosity is computed according to the dynamic Smagorinsky model [11,12] while $Pr_t$ is assumed to be constant (= 0.9).

\[
\frac{\partial U_k}{\partial x_k} = 0, \quad (1)
\]
\[
\frac{\partial U_i}{\partial t} + U_j \frac{\partial U_i}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_k} \left[ \left( \frac{1}{Re} + \frac{1}{Re_t} \right) \frac{\partial U_i}{\partial x_k} \right], \quad (2)
\]
\[
\frac{\partial T}{\partial t} + U_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_k} \left[ \left( \frac{1}{Re Pr} + \frac{1}{Re_t Pr_t} \right) \frac{\partial T}{\partial x_k} \right], \quad (3)
\]

A computational mesh consists of $16.6 \times 10^6$ hexahedral cells satisfying the near-wall resolution criteria for $Re = 3750$: $\Delta r^+ < 0.3$, $(RA\phi)^+ < 2.3$ and $\Delta x^+ < 2.4$, where $R = D/2$. The mesh also has constant cell size in $x$ direction starting from $x = 2.5$ and until the end of the computational domain to ensure satisfactory resolution in the far wake to investigate streamwise vortical structures.

**Figure 1.** The coordinate system and isosurface of instantaneous $Q$ - criterion for $Re = 3750$ colored with the temperature field. The plane $z = 0$ is on the bottom of the duct channel.
3. Correlation between velocity field and heat flux
Secondary coherent streamwise vortical structures in the far wake near the mixing layer shown at Fig. 1 significantly influence the near-wall heat transfer. Figure 2 demonstrates the vertical heat flux and the friction velocity in far field on the bed plane of the duct channel \((z = 0)\). Streaky longitudinal vorticies are clearly seen from \(|y| \approx 1.2\) at \(x = 4\) to \(2 < |y| < 3\) at \(x = 13\) outside of the area within dashed lines.

![Figure 2](image)

**Figure 2.** The vertical heat flux (left) and friction velocity (right) on the lower channel wall corresponding to the maximum value of the correlation coefficient. Area within dashed lines is neglected in correlation coefficient computations. Black dots correspond to the set of points selected for Fig. 3

In order to estimate how heat flux and mass transfer fluctuations caused by the presence of streamwise vortical structures are mutually connected the off-diagonal Pearson correlation coefficient \(R_{12}\), which lies in the range \([-1; 1]\), is evaluated for two vectors \(U_\tau\) and \(q_z\) within the area demonstrated on Fig. 3. Since physically these fields have different magnitude, maximum correlation value 1.0 can not be reached. The first graph can be interpreted as a direct correlation between friction velocity and heat flux fields at wide range of non-dimensional time \(t\). It features a few extrema, time instances corresponding the lowest \((t \approx 54)\) and the highest \((t \approx 136)\) \(R_{12}\) are presented at Fig. 2 and Fig. 3. Positions of friction velocity \(U_\tau\) maxima match the positions of the vertical heat flux \(q_z\) maxima outside of the dashed lines on Fig. 2, whilst this arrangement is shifted on Fig. 3. This observation can be interpreted as different propagation velocity (spatial units covered by streamwise structure per time units) of heat and mass disturbances caused by streamwise vortical structures on the wall. We estimate these velocities as 0.92 \([D/t]\) for mass transfer and 0.55 for the vertical heat transfer.
Figure 3. The vertical heat flux (left) and friction velocity (right) on the lower channel wall corresponding to the minimum value of the correlation coefficient $R_{12}$.

To further confirm this a discrete set of points shown on Fig. 2 corresponding positions of coherent longitudinal structures on the duct wall is selected to compute the correlation coefficient at Fig. 3 in the same manner as previously. Although characteristic frequency is higher for this correlation coefficient, the peak values position in time matching corresponding extrema from previous graph with a lag. Thus, we conclude that the heat transfer due to the convection of the coherent structures are not well correlated, however, these observation require further analysis.

Figure 4. Filtered off-diagonal Pearson correlation coefficient $R_{12}$ between vectors $U_\tau$ and $q_z$ for the area beyond dashed lines shown at Fig. 2 and 3 (blue line) and for the discrete set of points shown at Fig. 2-3 (red line)

Conclusion
We performed Large-eddy simulations of the wake flow in a narrow duct channel walls to study streamwise vortical structures at $Re = 3750$ with relatively high numerical resolution. Although these structures appear near midplane of the duct channel, they influence heat transfer from the duct wall. A correlation analysis is performed to demonstrate that the heat flux and friction velocity fluctuations are
mutually linked since they are caused by the same events, i.e. the vortical structures. However we observe that the correlation is not very high, thus, this issue requires further studies.

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