Effect of Doppler Shift on the Performance of Multicell Full-Duplex Massive MIMO Networks

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Abstract—This paper studies the uplink and downlink achievable rates for large-scale multiple-input–multiple-output (MIMO) systems, assuming perfect and imperfect channel state information under the effect of Doppler shift. Different from the previous relevant work, we consider a multiuser system scenario, where a full-duplex mode worked in the multicell base stations, and maximum-ratio combining maximum-ratio transmission is applied at the receiver end. Then, we derive the asymptotic uplink and downlink sum rate by utilizing the large number theorem and considering the performance comparison between the perfect and imperfect channel environment. In addition, the impact of the Doppler shift on the system performance (e.g., the uplink and downlink sum rates) is analyzed via the simulation results.

Index Terms—Doppler shift, full duplex, maximum-ratio combining maximum-ratio transmission (MRC/MRT), multicell.

I. INTRODUCTION

Due to its high-frequency spectrum and energy utilization, security, and robustness, large-scale multiple-input–multiple-output (MIMO) systems are well suited for the future broadband, digital network architectures for Internet of Things (IoT) and cloud service interconnection. It is also true that many scholars regard it as one of the key technologies of fifth-generation (5G) mobile communication and have conducted in-depth research on it. Dr. Marzetta of the Seoul Laboratories conducted a detailed analysis of the spectrum utilization and system throughput of large-scale MIMO systems [1], and proposed the main factors constraining the development of the large-scale MIMO such as pilot pollution. In [2] and [3], Jensen’s inequality and other mathematical tools were used to obtain the user reachable rate for a finite number of base station (BS) antennas, and then, used it to analyze the spectrum efficiency and throughput of the system. In detail, Ngo explained another important advantage of the massive MIMO in reducing the transmit power [4]. When the antenna size is large, the transmit power can be reduced by an order of magnitude or even more. In [4], the relation between the transmit power reduction degree and the BS antenna number ($M$) is investigated. In addition, it is pointed out that if the BS can obtain the ideal channel state information (CSI), the transmit power per user can be reduced $1/M$, and if the BS needs to estimate the channel, the transmit power per user can be reduced to $1/\sqrt{M}$.

On the other hand, the full-duplex technology can increase the spectrum efficiency of the system by nearly twice, while the premise is that it can effectively remove or suppress self-interference (SI) (e.g., the cofrequency interference) [5], [6]. Because of the extreme performance requirements in the 5G wireless networks, the use of a single item often fails to meet these demands. Thus, scholars usually combine multiple techniques to improve the system performance. In full-duplex large-scale antenna systems, the spectral efficiency of the system increases with the number of antennas. When the number of antennas is large enough, since the linear processing method effectively suppresses the SI, the spectral efficiency of the entire smart worker almost doubles that of the time-division duplex half-duplex system. Therefore, the combination of the full-duplex and large-scale antenna technologies can well satisfy the requirements of the future 5G communications, which has been studied widely in [7]–[9]. In [9], multiple pairs of users exchanged information with the aid of full-duplex relays with large-scale antennas, and proposed an optimal power allocation algorithm to minimize the total energy consumed by the system. Ngo et al. studied the linear processing method of the full-duplex one-way relay system to improve the energy efficiency [8]. Zhang studied the effect of the power scaling on the spectrum and energy efficiency in a large-scale two-way relay system [7]. Due to the effects of Doppler shifts, the channel causes channel aging. To the best of the authors’ knowledge, the effects of Doppler shifts have not been fully demonstrated in the previous work on massive MIMO. Although the influence of the Doppler shift has

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been studied in other MIMO cellular configurations, such as in multicell transmission [9], the obtained results cannot be directly applied to massive MIMO systems. Previously, we have studied the influence of the Doppler shift in a single-cell scenario on a full-duplex massive MIMO communication system. However, in contrast to the single-cell scenario, the pilot pollution caused by the reuse of pilot sequences between different cells must be considered in the multicell scenario. For the multicell downlink transmission, the authors in [10]–[13] analyzed the spectral efficiency of multicells and the impact of pilot pollution. Using pilot design [14], pilot allocation [15], and precoding [16] can effectively reduce the impact of pilot pollution. For the multicell uplink transmission, this paper proposes an optimal linear receiver by maximizing the signal-to-interference-plus-noise ratio (SINR) and deduces the achievable rates in closed forms.

The rest of this paper is organized as follows. The system model is introduced in Section II. Section III mainly analyzes the uplink and downlink rate. Section IV shows the simulation results. Section V is the summary of the full paper.

Notations: The symbols used in this paper are as follows: $(A)^T$, $(A)^H$, tr $(A)$, $\|A\|$, and $E(\bullet)$ denote the matrix transpose, conjugate transpose, matrix trace, the Euclidean norm, and expectation, respectively; $[A]_{n,n}$ denotes the $n \times n$ diagonal entry of the matrix $A$; $I_M$ represents the $M \times M$ identity matrix. $x \sim \mathcal{CN}(0, \sigma_x I)$ represents that $x$ is a circularly symmetric complex-Gaussian vector whose entry’s mean and variance are 0 and $\sigma_x$, respectively.

II. SYSTEM MODEL

This paper studies a massive MIMO system with $L$ cells in the full-duplex mode as shown in Fig. 1. Each cell consists of $K$ full-duplex single-antenna users and a full-duplex BS equipped with $N$ antennas. Transmission is considered over frequency-flat fading channels. In this paper, an additive white Gaussian noise (AWGN) is considered at each transmit antenna through a dynamic range model, in which the variance is $\kappa (n \ll 1)$ times the power of the transmit signals. Here, $\kappa$ is the dynamic range parameter. Compared with the thermal noise of the transmitter, the full-duplex transmitter noise will be propagated on the SI channel and turn into complicated. However, compared with the receiver thermal noise, the influence of the transmitter noise transmitted on the uplink/downlink channel can be ignored [17].

Here, $G_{u,jl}[0]$ is supposed as an $N \times K$ matrix that denotes the uplink channel matrix from the users in the $l$th cell to the $j$th BS at time $0$, which means the time as the all symbols are transmitted in the training phase. $G_{d,jl}[0]$ is supposed as an $N \times K$ matrix denoting the downlink channel matrix from the $j$th BS to the users in the $l$th cell. The propagation channel model in our system considers both small-scale fading caused by the multipath and the large-scale fading caused by the shadowing effect. Next, the channel vector of uplink and downlink is denoted as

$$G_{\phi,jl}[0] = H_{\phi,jl}[0]D_{\phi,jl}^{1/2}, \quad (\phi \in \{u, d\})$$

where $H_{\phi,jl}[0] \in \mathcal{CN}^{N \times K}$ represents the small-scale fading channels and its elements obey the independent and identically distribution (i.i.d.) $\mathcal{CN}(0, 1)$, and $D_{\phi,jl}$ is the large-scale fading diagonal matrix with diagonal term $[D_{\phi,jl}]_n = \beta_{\phi,jln}$, which denotes the large-scale fading between the $n$th uplink/downlink user in the $l$th cell and the $j$th BS. Let $g_{\phi,jlk}[0]$ be the $k$th column of the matrix $G_{\phi,jl}[0]$.

A. Uplink Transmission

The $N \times 1$ uplink signal vector received by the $j$th full-duplex BS at time $n(n \neq 0)$ is $y_{u,jl}[n]$ 

$$y_{u,jl}[n] = \sum_{i=1}^{L} G_{u,jl}[n] x_{u,i,l}[n] + \sum_{i=1}^{L} V_{jl}[0] x_{d,i,l}[n] + \eta_{u,jl}[n]$$

where $x_{u,i,l}[n] = [x_{u,i,l}[n], \ldots, x_{u,K,l}[n]]^T$ is the uplink signal vector and $x_{d,i,l}[n]$ is an $N \times 1$ vector that denotes the downlink signal vector transmitted by the BS. We have $E(\|x_{u,i,l}[n]\|^2) = P_u$ and each BS has an average power constraint $P_d$. $V_{jl}[0]$ is supposed an $N \times N$ matrix that denotes the channel from the $l$th BS to the $j$th BS. Also, we suppose $V_{jl}[0]$ contains i.i.d. $\mathcal{CN}(0, \beta_{d,jl})$. $V_{jl}[0]$ is an SI channel at the $j$th BS. $e_{bs,j}[\eta] \in \mathcal{CN}(0, \frac{P_d}{\beta_{d,jl}}I_N)$. And the receiver noise is shown as $\eta_{u,jl}[n]$ containing i.i.d $\mathcal{CN}(0, \sigma^2, I_N)$. As we all know, if the $j$th BS knows the SI channel and downlink signal, the SI cancellation can be performed. So, (2) can be written as

$$y_{u,jl}[n] = \sum_{i=1}^{L} G_{u,jl}[n] x_{u,i,l}[n] + \sum_{i=1}^{L} V_{jl}[0] x_{d,i,l}[n] + z_{u,jl}[n]$$

where $z_{u,jl}[n] \sim \mathcal{CN}(0, (\sigma^2 + \kappa P_d\beta_{d,jl})I_N)$.

B. Downlink Transmission

In the $l$th cell, the users will receive the signals, which can be expressed as a $K \times 1$ vector $y_{d,l}[n]$, shown as

$$y_{d,l}[n] = \sum_{j=1}^{L} G_{d,jl}^{H}[n] x_{d,j}[n] + \sum_{j=1}^{L} F_{lj}[n] x_{u,j}[n] + F_{ll}[n] e_{ue,l}[n] + \eta_{d,l}[n]$$

where $F_{lj}[n]$ is a $K \times K$ matrix that denotes the user-user interference channel from $K$ uplink users in the $j$th to $K$ downlink users in the $l$th cell and the large-scale fast fading channel coefficient between the $i$th uplink user in the $j$th cell and the $k$th downlink user in the $l$th cell is $(F_{lj}[n])_{ik} = f_{ikj}[n]$, $F_{ll}[n]$ is a $K \times K$ matrix denoting the interference channels.
between $K$ full-duplex users in the $l$th cell and follows $i.i.d \sim \mathbb{C} \mathbb{N}(0, \beta_{lkjn})$. At time 0, the diagonal elements of $\mathbf{F}_l[0]$ form SI channels, where $(\mathbf{F}_l[0])_{k,k} = f_{ljk} [k], k \in K$. $\mathbf{e}_{u,l}[n]$ denotes the transmit noise at each user in the $l$th cell and follows $\mathbb{C} \mathbb{N}(0, \kappa P_u I_N)$. And the noise in the receiver is shown as $\mathbf{n}_{d,l}[n]$ with entries being $i.i.d \sim \mathbb{C} \mathbb{N}(0, \beta_{lkjn})$.

III. ANALYSIS OF ACHIEVABLE RATE

A. Perfect CSI

We first consider the case that the BS has perfect CSI. We suppose that the users begin to move at time $n$, so the autoregressive model can be applied as [18]

$$
\mathbf{G}_{\phi,jl}[n] = \alpha[n] \mathbf{G}_{\phi,jl}[0] + \mathbf{E}_{\phi,jl}[n].
$$

(5)

In this expression, $\alpha[n] = J_0(2\pi f DT_n)$ denotes a temporal correlation parameter. $J_0$ denotes the first kind Bessel function with zero order, the maximum Doppler shift is written as $f_D = v^2/c$, where $v$ denotes the relative velocity of the users, $c$ denotes the carrier frequency, and $\varepsilon$ denotes the light speed. $\mathbf{E}_{\phi,jl}[n]$ denotes the channel error vector, which is independent of $\mathbf{G}_{\phi,jl}[0]$ and $(\mathbf{E}_{\phi,jl}[n])_n = \varepsilon_{\phi,jl}[n] \sim \mathbb{C} \mathbb{N}(0, \varepsilon_{\phi,jl})$. Let $\mathbf{G}_{\phi,jl}[n] = \alpha \mathbf{G}_{\phi,jl}[0]$. Then (5) can be rewritten as

$$
\mathbf{G}_{\phi,jl}[n] = \mathbf{G}_{\phi,jl}[0] + \mathbf{E}_{\phi,jl}[n].
$$

(6)

1) Achievable Uplink Rate: The uplink will be first analyzed in this part. The $j$th BS receive signals from users in the $l$th cell, which include interference signals. Therefore, we apply maximum-ratio combining (MRC) detector for detecting the uplink signal. Thus, a $K \times 1$ signal vector can be obtained as

$$
\mathbf{r}_{u,j}[n] = \mathbf{g}^H_{u,jj}[n] \mathbf{y}_{u,j}[n]
$$

$$
= \mathbf{g}^H_{u,jj}[n] \mathbf{G}_{u,jj}[n] \mathbf{x}_{u,j}[n]

+ \sum_{l \neq j} \mathbf{g}^H_{u,jj}[n] \mathbf{G}_{u,jl}[n] \mathbf{x}_{u,l}[n]

+ \sum_{l \neq j} \mathbf{g}^H_{u,jj}[n] \mathbf{V}_{jl}[0] \mathbf{x}_{d,l}[n] + \mathbf{g}^H_{u,jj}[n] \mathbf{z}_{u,j}[n].
$$

(7)

By substituting (6) into (7), (7) can be rewritten as

$$
\mathbf{r}_{u,j}[n] = \mathbf{g}^H_{u,jj}[n] \mathbf{G}_{u,jj}[n] \mathbf{x}_{u,j}[n] + \mathbf{g}^H_{u,jj}[n] \mathbf{E}_{u,jj}[n] \mathbf{x}_{u,j}[n]

+ \sum_{l \neq j} \mathbf{g}^H_{u,jj}[n] \mathbf{V}_{jl}[0] \mathbf{x}_{d,l}[n] + \sum_{l \neq j} \mathbf{g}^H_{u,jj}[n]

\times \mathbf{G}_{u,jl}[n] \mathbf{x}_{u,l}[n] + \mathbf{g}^H_{u,jj}[n] \mathbf{z}_{u,j}[n].
$$

(8)

The $M \times 1$ downlink signal vectors $\mathbf{x}_{d,l}[n]$ will be transmitted by the $l$th BS by precoding the downlink messages using the maximum-ratio transmission (MRT)

$$
\mathbf{x}_{d,l}[n] = \mathbf{W}_l[n] \mathbf{s}_{d,l}[n]
$$

$$
\mathbf{W}_l[n] = \mathbf{G}^H_{d,l}[n] \left( \text{Tr} \left( \mathbf{G}^H_{d,l}[n] \mathbf{G}_{d,l}[n] \right) \right)^{-1/2}
$$

(9)

where $\text{E}(\mathbf{s}_{d,l} \mathbf{s}_{d,l}^H) = P_d \mathbf{I}_N$.

So, by substituting (9) into (8), the uplink signal from the user $k$ in the $l$th cell received by the $j$th BS is given by (10) at the bottom of this page.

Let both sides of the equal sign be divided by $\sqrt{N}$. Then

$$
I_{u,1} = \frac{\bar{g}^H_{u,jj}[n]\bar{g}_{u,jjk}[n]x_{u,jk}[n]}{\sqrt{N}}
$$

$$
I_{u,2} = \frac{\bar{g}^H_{u,jj}[n]z_{u,jk}[n]}{\sqrt{N}}
$$

$$
I_{u,3} = \frac{\sum_{l \neq j} \bar{g}^H_{u,jj}[n]V_{jl}[0]x_{d,l}[n]}{\sqrt{N} \sqrt{\text{Tr} \left( \bar{g}^H_{d,l}[n] \mathbf{G}_{d,l}[n] \right)}}
$$

$$
I_{u,4} = \frac{\sum_{l \neq j} \sum_{k} \bar{g}^H_{u,jj}[n]V_{jl}[0]s_{d,l}[n]s_{d,l}[n]}{\sqrt{N}}
$$

$$
I_{u,5} = \frac{\bar{g}^H_{u,jj}[n]z_{u,jk}[n]}{\sqrt{N}}.
$$

(11)

Lemma 1 ([20, Lemma 1]): Let us denote $n \times n$ order deterministic complex matrix by $B$. For all $n$, it has a uniformly bounded spectral radius. Let $\mathbf{x} = \frac{1}{\sqrt{N}}[x_1, \ldots, x_n]^T$ and $\mathbf{y} = \frac{1}{\sqrt{N}}[y_1, \ldots, y_n]^T$ are two complex random vectors, which are mutually independent. Their elements are i.i.d. random complex variables with unit variance and zero mean. Therefore, $\mathbf{x}^H \mathbf{B} \mathbf{x} \rightarrow \frac{1}{N} \text{Tr} (\mathbf{B})$ and $\mathbf{x}^H \mathbf{B} \mathbf{y} \rightarrow 0$ almost surely as $N \rightarrow \infty$.

In $I_{u,1}$, using Lemma 1, we can get

$$
\frac{\bar{g}^H_{u,jj}[n]\bar{g}_{u,jjk}[n]}{N} = \alpha[n] \frac{2 \bar{g}^H_{u,jj}[0]\bar{g}_{u,jjk}[0]}{N} \xrightarrow{N \rightarrow \infty} \alpha[n]^2 \beta_{u,jk}.
$$

(12)

So, we can get the power of $I_{u,1}$ as

$$
E \left[ |I_{u,1}|^2 \right] = N \mathbb{P}_u \left( \alpha[n]^2 \beta_{u,jk} \right).
$$

(13)

Continuously, the power of $I_{u,2}$ can be obtained as

$$
E \left[ |I_{u,2}|^2 \right] = \alpha[n]^2 \frac{\bar{g}^H_{u,jj}[0]\bar{g}_{u,jjk}[n]z_{u,jk}[n]}{N} \frac{\bar{g}^H_{u,jj}[0]}{N}.
$$

(14)

$$
\mathbf{r}_{u,jk}[n] = \bar{g}^H_{u,jjk}[n] \bar{g}_{u,jjk}[n] x_{u,jk}[n] + \bar{g}^H_{u,jjk}[n] z_{u,jk}[n] + \sum_{l \neq j} \bar{g}^H_{u,jjk}[n] \mathbf{g}_{u,jlk}[n] x_{u,lk}[n]
$$

$$
\frac{\sum_{l \neq j} K_{i,j,k} \bar{g}^H_{u,jjk}[n] V_{jl}[0] s_{d,l}[n]}{\sqrt{\text{Tr} \left( \bar{g}^H_{d,l}[n] \mathbf{G}_{d,l}[n] \right)}} + \bar{g}^H_{u,jjk}[n] z_{u,jk}[n].
$$

(10)
Let $\tilde{\varepsilon}_{u,j,k}[n] = \frac{\varepsilon_{u,j,k}[n]}{\|\varepsilon_{u,j,k}[n]\|}$. Then $\text{Tr}(\tilde{\varepsilon}_{u,j,k}[n] z^H_{u,j,k}[n]) = 1$.

Thus, (14) can be rewritten as

$$E \left[ |I_{u,2}|^2 \right] = \frac{N P_u \alpha [n]^2 \beta_{u,j,k}}{\sqrt{N\beta_{u,j,k}}} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{z^H_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} = 1. \quad (15)$$

Based on Lemma 1, we know

$$\frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \rightarrow \frac{1}{\sqrt{N}}. \quad (16)$$

Substituting (16) into (15), the power of $I_{u,2}$ can be obtained as

$$E \left[ |I_{u,2}|^2 \right] = \frac{P_u \alpha [n]^2}{\sqrt{N\beta_{u,j,k}}} \tilde{\varepsilon}_{u,j,k}[n] \beta_{u,j,k}. \quad (17)$$

The power of $I_{u,3}$ will be introduced as

$$E \left[ |I_{u,3}|^2 \right] = \frac{P_u \alpha [n]^4}{\sqrt{N\beta_{u,j,k}}} \sum_{(l,m) \neq (j,k)} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,m}[n] \beta_{u,j,k}. \quad (18)$$

Let $\tilde{\varepsilon}_{u,j,k}[n] = \tilde{\varepsilon}_{u,j,k}[n] \frac{\tilde{\varepsilon}_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}}$. Then $\text{Tr}(\tilde{\varepsilon}_{u,j,k}[n][\tilde{\varepsilon}_{u,l,m}[n]]) = 1$.

So, (18) can be rewritten as

$$E \left[ |I_{u,3}|^2 \right] = \frac{N P_u \alpha [n]^4}{\sqrt{N\beta_{u,j,k}}} \beta_{u,j,k} \sum_{(l,m) \neq (j,k)} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,m}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,m}[n] \beta_{u,j,k}. \quad (19)$$

Similarly from Lemma 1, we get

$$\frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \rightarrow \frac{1}{\sqrt{N}}. \quad (20)$$

Therefore, the power of $I_{u,3}$ is

$$E \left[ |I_{u,3}|^2 \right] \rightarrow \frac{P_u \alpha [n]^4}{\sqrt{N\beta_{u,j,k}}} \beta_{u,j,k} \sum_{(l,m) \neq (j,k)} \beta_{u,l,m}. \quad (21)$$

Also, we can write the power of $I_{u,4}$ as

$$E \left[ |I_{u,4}|^2 \right] = \frac{P_d \alpha [n]^2}{\sqrt{N\beta_{u,j,k}}} \sum_{l \neq j} \sum_{k \neq j} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,k}[n] \beta_{u,j,k}. \quad (22)$$

Let $\tilde{\varepsilon}_{d,l,k}[n] = \frac{\tilde{\varepsilon}_{d,l,k}[n]}{\sqrt{N\beta_{d,l,k}}}$. Then $\tilde{\varepsilon}_{d,l,k}[n] \tilde{\varepsilon}_{d,l,k}[n] = \text{Udiag}(\epsilon_1) \text{U}$. Furthermore, we let $\text{V}_{jl}[0] = \text{V}_{jl}[0] \text{U}$. Then we can get $\text{V}_{jl}[0] (\text{diag}(\epsilon_1) \text{V}_{jl}[0] (\text{V}_{jl}[0] \text{U})^H$. Thus, the expression in (22) can be rewritten as

$$E \left[ |I_{u,4}|^2 \right] = \frac{P_d \alpha [n]^2}{\sqrt{N\beta_{u,j,k}}} \sum_{l \neq j} \sum_{k \neq j} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,k}[n] \beta_{u,j,k}. \quad (23)$$

Using the same method in (18), we can similarly let $\text{V}_{jl}[0] = \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}}$. From Lemma 1, we can easily get

$$E \left[ |I_{u,4}|^2 \right] \rightarrow \frac{P_d \alpha [n]^2}{\sqrt{N\beta_{u,j,k}}} \sum_{l \neq j} \sum_{k \neq j} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,k}[n] \beta_{u,j,k}. \quad (24)$$

So, the power of $I_{u,4}$ can be obtained as

$$E \left[ |I_{u,4}|^2 \right] \rightarrow \frac{P_d \alpha [n]^2}{\sqrt{N\beta_{u,j,k}}} \sum_{l \neq j} \sum_{k \neq j} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,k}[n] \beta_{u,j,k}. \quad (25)$$

At last, the power of $I_{u,5}$ can be shown as

$$E \left[ |I_{u,5}|^2 \right] = \frac{\alpha [n]^2}{\sqrt{N\beta_{u,j,k}}} \sum_{l \neq j} \sum_{k \neq j} \frac{\|\varepsilon_{u,j,k}[n]\|^2}{N} \frac{\tilde{\varepsilon}_{u,j,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\varepsilon_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \frac{\tilde{\varepsilon}_{u,l,k}[n]}{\sqrt{N\beta_{u,j,k}}} \varepsilon_{u,l,k}[n] \beta_{u,j,k}. \quad (26)$$

Combining (13), (17), (21), (26), and (27), the uplink rate of the user $k$ in the $l$th cell is given as in (28) at the bottom of this page.

2) Achievable Downlink Rate: Similarly, the $k$th full-duplex user in the $l$th cell can exercise the SI cancellation by taking away SI from the received signal in (2). Therefore, we can get

$$r_{d,l,k} [n] = \sum_{j=1}^{L} \frac{\tilde{\varepsilon}_{d,l,k}[n]}{\beta_{u,l,k}[n]} + \sum_{j \neq l} \sum_{k \neq j} f_{l,k} [n] x_{u,j}[n] + z_{d,l,k} [n] \quad (29)$$

in the aforementioned formula, $\text{g}_{d,l,k}[n]$ is the $k$th column of $\text{G}_{d,l}[n]$ and $z_{d,l,k} [n] \sim \text{CN}(0, (\sigma^2 + \kappa P_d \beta_{d,l,k})).$

By combining (9) and (29), the downlink signal at the $k$th full-duplex user in the $l$th cell, where $k \in K$ is written as

$$r_{d,l,k} [n] = I_{d,1} + I_{d,2} + I_{d,3} + I_{d,4} + z_{d,l,k} [n] \quad (30)$$

where $\beta_{u,j,k} = \log_2 \left( 1 + \frac{P_u \varepsilon_{u,j,k} + P_u \alpha [n]^2 \sum_{(l,m) \neq (j,k)} \beta_{u,l,m} + P_d \sum_{l \neq j} \beta_{V,l} + (\sigma^2 + \kappa P_d \beta_{d,l,k}) \right).$
where

\[ I_{d,1} = \sqrt{N} g_{d,\text{llk}}^H[n] \frac{\bar{g}_{d,\text{llk}}[n]}{\sqrt{\text{Tr}(G_{d,\text{ll}}^H[n] G_{d,\text{ll}}[n])}} s_{d,\text{lk}}[n] \]

\[ I_{d,2} = \sum_{(j,i)\neq(l,k)} \frac{g_{d,jji}[n]}{\sqrt{\text{Tr}(G_{d,jj}^H[n] G_{d,jj}[n])}} s_{d,jji}[n] \]

\[ I_{d,3} = \frac{L}{2} \sum_{j=1}^{L} \sum_{i=1}^{K} \frac{e_{d,jji}[n]}{\sqrt{\text{Tr}(G_{d,jj}^H[n] G_{d,jj}[n])}} s_{d,jji}[n] \]

\[ I_{d,4} = \sum_{(j,i)\neq(l,k)} f_{lkji}[n] x_{u,j}[n]. \]  

(31)

Based on Lemma 1, we can change \( I_{d,1} \) as

\[ \sum_{j,i}(37) \]

\[ I_{d,4} = \sum_{(j,i)\neq(l,k)} f_{lkji}[n] x_{u,j}[n]. \]  

(31)

The power of \( I_{d,1} \) is as follows:

\[ \text{E}[|I_{d,1}|^2] = N P_d \alpha[n]^2 \beta_{d,\text{llk}}^2 \text{Tr}(D_{d,\text{ll}})^{-1}. \]

(33)

Also, the power of \( I_{d,2} \) is written as

\[ \text{E}[|I_{d,2}|^2] = P_d \alpha[n]^2 \beta_{d,\text{llk}}^2 \frac{\sum_{(j,i)\neq(l,k)} g_{d,jji}[0]}{\text{Tr}(G_{d,jj}^H[n] G_{d,jj}[n])} g_{d,jji}[0]. \]

(34)

Let \( \tilde{g}_{d,jji}[0] = \frac{g_{d,jji}[0]}{\|g_{d,jji}[0]\|} \). Then (34) can be changed as

\[ \text{E}[|I_{d,2}|^2] = P_d \alpha[n]^2 \times \frac{N \beta_{d,\text{llk}}}{\text{Tr}(G_{d,jj}^H[n] G_{d,jj}[n])} \frac{\|g_{d,jji}[0]\|^2}{N} R_{ji} \]

(35)

where \( R_{ji} = \frac{g_{d,jji}^H[0]}{\sqrt{N \beta_{d,\text{llk}}}} \frac{\tilde{g}_{d,jji}[0]}{\sqrt{N \beta_{d,\text{llk}}}} g_{d,jji}^H[0] \).

Using the same method in (20), the power of \( I_{d,2} \) is obtained as

\[ \text{E}[|I_{d,2}|^2] = P_d \alpha[n]^2 \sum_{(j,i)\neq(l,k)} \beta_{d,jji} \frac{\beta_{d,\text{llk}}}{\text{Tr}(D_{d,\text{ll}})^{-1}}. \]

(36)

We can find that \( I_{d,3} \) is similar to \( I_{d,2} \), so the power of \( I_{d,3} \) is

\[ \text{E}[|I_{d,3}|^2] = P_d \alpha[n]^2 \sum_{j=1}^{L} \sum_{i=1}^{K} \frac{e_{d,jji}[n]}{\text{Tr}(D_{d,jj})} \beta_{d,jji}. \]

(37)

Since each user is on the move, we still utilize an autoregressive model for the channels between the users. So, \( I_{d,4} \) in (31) can be rewritten as

\[ I_{d,4} = \sum_{(j,i)\neq(l,k)} (\alpha[n] f_{lkji}[n] + \epsilon_{lkji}[n]) x_{u,j}[n]. \]

(38)

Finally, it is easy to get the power of \( I_{d,4} \) as

\[ \text{E}[|I_{d,4}|^2] = P_u \sum_{(j,i)\neq(l,k)} (\alpha[n]^2 \sigma_{lkji} + \epsilon_{lkji}). \]

(39)

Substituting (33), (36), (37), and (39) into (32), the downlink rate of the user \( k \) in the \( l \)th cell is given by (40) at the bottom of this page.

B. Imperfect CSI

In MIMO networks, to perform uplink and downlink beamforming, the BS must get the uplink and downlink channel information to perform coherent detection and precoding in the uplink and downlink, respectively. Here, the ergodic achievable rates with channel estimation error will be derived. The uplink and downlink channels are estimated by the uplink training sequences in this system, therefore, the pilot overhead is only proportional to the number of users. The uplink and downlink data transmissions begin simultaneously after the uplink training.

During the uplink training period, \( K \) mutually orthogonal pilot sequences of the length \( \tau (\tau \geq K) \) symbols are adopted to estimate the channel between each BS and its associated users within the coherent interval of \( T \). \( L \) cells will reuse the same set of pilot sequences. Due to the nonorthogonality of the reused pilot, the channel estimate will be destroyed by the pilot pollution [20]. Denote an average channel training power at each user by \( P_p \), which is dependent on the length of the pilot sequence.

To acquire an \( M \)-dimensional vector \( y_{p,jk}[0] \), the \( j \)th BS will associate the received signal from the uplink training with the pilot sequences assigned for the \( k \)th user. So, \( y_{p,jk}[0] \) can
be obtained as
\[ y_{P_r,jk}[0] = g_{\phi,jk}[0] + \sum_{l \neq j} g_{\phi,jlk}[0] + \frac{n_{jk}[n]}{\sqrt{P_p}} \phi \in \{u, d\}. \] (41)

The MMSE channel estimate of the \( k \)th user in the \( j \)th cell can be obtained as
\[ \hat{g}_{\phi,jk}[0] = \frac{P_p \beta_{\phi,jk}}{\lambda_{\phi,jk}} \left( \sum_{l=1}^{L} g_{\phi,jlk}[0] + \frac{n_{jk}[n]}{\sqrt{P_p}} \right). \] (42)

So \( \hat{g}_{\phi,jk}[0] \sim \mathcal{CN}(0, \hat{\sigma}_{\phi,jk}) \), where
\[ \hat{\sigma}_{\phi,jk} = \frac{P_p \beta_{\phi,jk}}{\lambda_{\phi,jk}} I_N \]
and
\[ \lambda_{\phi,jk} = \sigma^2 + P_p \sum_{l=1}^{L} \beta_{\phi,jlk}. \]

Since the MMSE estimator has orthogonality, the real channel can be divided into the estimated channel and channel estimation errors. Therefore, one can get
\[ g_{\phi,jk}[0] = \hat{g}_{\phi,jk}[0] + \Delta g_{\phi,jk}[0] \] (43)
where \( \Delta g_{\phi,jk}[0] \sim \mathcal{CN}(0, \hat{\sigma}_{\phi,jk}) \).

Also, the user is supposed to start moving at time \( n \) \( (n \neq 0) \), applying the autoregressive model as
\[ g_{\phi,jk}[n] = \alpha [n] g_{\phi,jk}[0] + e_{\phi,jk}[n] \] (44)
where \( e_{\phi,jk}[n] \sim \mathcal{CN}(0, \hat{\sigma}_{\phi,jk}) \) due to the time variation of the channel independent of \( g_{\phi,jk}[0] \).

Let \( \hat{g}_{\phi,jk}[n] = \alpha [n] \hat{g}_{\phi,jk}[0] + e_{\phi,jk}[n] = \alpha [n] \Delta g_{\phi,jk}[0] + e_{\phi,jk}[n] \), where \( (E_{\phi,jl}[n])_n = \epsilon_{\phi,jl}[n] \sim \mathcal{CN}(0, \hat{\epsilon}_{\phi,jl}) \). Then (44) can be rewritten as
\[ g_{\phi,jk}[n] = \alpha [n] (\hat{g}_{\phi,jk}[0] + \Delta g_{\phi,jk}[0]) + e_{\phi,jk}[n] \] (45)
where \( \epsilon_{\phi,jk}[n] \sim \mathcal{CN}(0, \hat{\epsilon}_{\phi,jk}) \).

1) Achievable Uplink Rate: By substituting (5) into (7), then can be rewritten as
\[ r_{u,j}[n] = \hat{G}_{u,j}^{H}[n] \hat{G}_{u,j}[n] x_{u,j}[n] + \sum_{l \neq j} \hat{G}_{u,j}^{H}[n] \hat{G}_{u,jl}[n] x_{u,l}[n] + \sum_{l=1}^{L} \hat{G}_{u,j}^{H}[n] E_{u,jl}[n] x_{u,l}[n] \times \sum_{l \neq j} \hat{G}_{u,j}^{H}[n] v_{jl}[0] x_{d,l}[n] + \hat{G}_{u,j}^{H}[n] z_{u,j}[n]. \] (46)

Substituting (9) into (46), the uplink signal from the user \( k \) in the \( \ell \)th cell received by the \( j \)th BS is given by (47) at the bottom of this page.

Let both sides of the equal sign be divided by \( \sqrt{N} \). Then
\[ I_{u,1} = \sqrt{N} \hat{G}_{u,jk}^{H}[n] \hat{g}_{u,jk}[n] x_{u,jk}[n], \]
and based on Lemma 1, we have
\[ \frac{\hat{G}_{u,jk}^{H}[n] \hat{g}_{u,jk}[n] \hat{g}_{u,jk}[n]}{N} = \alpha [n]^2 \hat{G}_{u,jk}^{H}[n] \hat{g}_{u,jk}[n] \frac{\hat{g}_{u,jk}[n]}{N} \rightarrow \alpha [n]^2 \hat{\beta}_{u,jk}. \] (48)

Thus, the power of \( I_{u,1} \) is
\[ E \left[ I_{u,1}^{2} \right] = N P_u (\alpha [n]^2 \hat{\beta}_{u,jk})^2. \] (49)

Let \( I_{u,2} = \sum_{l(l,m) \neq (j,k)} \hat{G}_{u,jl}^{H}[n] \hat{g}_{u,jl}[n] x_{u,l}[n] \sqrt{N} \). Then the power of \( I_{u,2} \) can be written as
\[ E \left[ I_{u,2}^{2} \right] = P_u \alpha [n]^4 \times \sum_{l(m) \neq (j,k)} \hat{G}_{u,jl}^{H}[n] \hat{g}_{u,jl}[n] \hat{g}_{u,jl}[n] \hat{G}_{u,jl}[n] \frac{\hat{G}_{u,jl}[n]}{N} \rightarrow \hat{\beta}_{u,jl}. \] (50)

We can make \( \hat{G}_{u,jl}[n] = \hat{G}_{u,jl}[n] \frac{\hat{G}_{u,jl}[n]}{N} \). so \( \text{Tr}(\hat{g}_{u,jl}[n]) = 1 \) and (50) can be rewritten as
\[ E \left[ I_{u,2}^{2} \right] = N P_u \alpha [n]^4 \hat{\beta}_{u,jl} \times \sum_{l(m) \neq (j,k)} \frac{\hat{G}_{u,jl}^{H}[n] \hat{G}_{u,jl}[n] \hat{G}_{u,jl}^{H}[n] \hat{G}_{u,jl}[n]}{N} \rightarrow \hat{\beta}_{u,jl}. \] (51)

Applying Lemma 1, we have
\[ \frac{\hat{G}_{u,jk}^{H}[n] \hat{g}_{u,jk}[n] \hat{g}_{u,jk}[n] \frac{\hat{g}_{u,jk}[n]}{N}}{N \hat{\beta}_{u,jk}} \rightarrow 1 \frac{1}{N}. \] (52)
\[ \frac{||\hat{G}_{u,jl}[n]||^2}{N} \rightarrow \hat{\beta}_{u,jl}. \] (53)

Therefore, the power of \( I_{u,2} \) is
\[ E \left[ I_{u,2}^{2} \right] = P_u \alpha [n]^4 \hat{\beta}_{u,jl} \sum_{l(m) \neq (j,k)} \hat{\beta}_{u,jl}. \] (54)

Let \( I_{u,3} = \sum_{l=1}^{L} \sum_{k=1}^{K} \hat{G}_{u,jk}^{H}[n] \epsilon_{u,jl}[n] x_{u,jl}[n] \sqrt{N} \). The formula of \( I_{u,3} \) is analogous to \( I_{u,2} \).

\[ r_{u,jk}[n] = \hat{G}_{u,jk}^{H}[n] \hat{g}_{u,jk}[n] x_{u,jk}[n] + \sum_{l \neq j} \hat{G}_{u,jk}^{H}[n] \hat{G}_{u,jl}[n] x_{u,l}[n] + \sum_{l=1}^{L} \hat{G}_{u,jk}^{H}[n] \epsilon_{u,jl}[n] x_{u,jl}[n] \times \sum_{l \neq j} \hat{G}_{u,jk}^{H}[n] v_{jl}[0] x_{d,l}[n] + \hat{G}_{u,jk}^{H}[n] z_{u,jk}[n]. \] (47)
Thus, the power of $I_{u,3}$ can be obtained as

$$E \left[ |I_{u,3}|^2 \right] = P_u \alpha [n]^2 \hat{\beta}_{u,jjk} \sum_{l=1}^{L} \sum_{i=1}^{K} \varepsilon_{u,jli}. \tag{55}$$

Let $I_{u,4} = \sum_{l \neq j}^{L} \sum_{i=1}^{K} \frac{\tilde{g}_{d,jjk}^H [0] \tilde{V}_{j,l,1} [0] \tilde{V}_{j,l,1}^H [0] \tilde{g}_{u,jjk} [0]}{N}$. The power of $I_{u,4}$ is

$$E \left[ |I_{u,4}|^2 \right] = \frac{P_d \alpha [n]^2}{\text{Tr} \left( \tilde{G}_{d,dl}^H [0] \tilde{G}_{d,dl} [0] \right)} \times \frac{L}{K} \sum_{l \neq j}^{L} \sum_{i=1}^{K} \frac{\tilde{g}_{d,jjk}^H [0] \tilde{V}_{j,l,1} [0] \tilde{V}_{j,l,1}^H [0] \tilde{g}_{u,jjk} [0]}{N}. \tag{56}$$

Let $\tilde{g}_{d,dl} [0] = \frac{\tilde{g}_{d,dl} [0]}{\sqrt{\text{Tr} \left( \tilde{G}_{d,dl}^H [0] \tilde{G}_{d,dl} [0] \right)}}$. Then $\tilde{g}_{d,dl} [0] \tilde{g}_{d,dl}^H [0] = \text{Udiag}(\epsilon_1) \text{U}$. Furthermore, we let $\tilde{V}_{j,l,1} [0] = \tilde{V}_{j,l,1} [0] \text{U}$. Then we can get $\tilde{V}_{j,l,1} [0] \text{diag}(\epsilon_1) \tilde{V}_{j,l,1}^H [0] = \tilde{V}_{j,l,1} [0] (\tilde{V}_{j,l,1} [0]^H)$. Thus, the expression in (56) can be rewritten as

$$E \left[ |I_{u,4}|^2 \right] = \frac{P_d \alpha [n]^2}{\text{Tr} \left( \tilde{G}_{d,dl}^H [0] \tilde{G}_{d,dl} [0] \right)} \times \frac{L}{K} \sum_{l \neq j}^{L} \sum_{i=1}^{K} \frac{\tilde{g}_{d,jjk}^H [0] \tilde{V}_{j,l,1} [0] \tilde{V}_{j,l,1}^H [0] \tilde{g}_{u,jjk} [0]}{N}. \tag{57}$$

Similarly, let $\tilde{V}_{j,l,1} [0] = \frac{\tilde{V}_{j,l,1} [0]}{\|\tilde{V}_{j,l,1} [0]\|}$. Then based on

$$\tilde{g}_{d,jjk}^H [0] \tilde{V}_{j,l,1} [0] (\tilde{V}_{j,l,1}^T [0]) \frac{\tilde{g}_{u,jjk} [0]}{\sqrt{N} \hat{\beta}_{u,jjk}} \xrightarrow{N \rightarrow \infty} \frac{1}{N} \tag{58}$$

and

$$\frac{\tilde{g}_{d,dl}^H [0] \tilde{G}_{d,dl} [0]}{N} \xrightarrow{N \rightarrow \infty} \tilde{D}_{d,dl} \tag{59}$$

can we get the power of $I_{u,4}$ as

$$E \left[ |I_{u,4}|^2 \right] \xrightarrow{N \rightarrow \infty} P_d \alpha [n]^2 \hat{\beta}_{u,jjk} \sum_{l \neq j}^{L} \beta_{b,jjl}. \tag{60}$$

Finally, we can easily know

$$E \left[ |I_5|^2 \right] = \alpha [n]^2 \frac{\tilde{g}_{u,jjk}^H [0] z_{u,jjk} [n] \tilde{g}_{u,jjk} [0]}{N} \xrightarrow{N \rightarrow \infty} \alpha [n]^2 \left( \sigma^2 + \kappa P_d \beta_{b,jjl} \right) \tilde{g}_{u,jjk}. \tag{61}$$

Considering (49), (54), (55), (60), and (61), the uplink rate of the user $k$ in the $l$th cell is given by (62) at the bottom of this page.

$$R_{u,k} \rightarrow \log \left( 1 + \frac{N P_u \alpha [n]^2 \hat{\beta}_{u,jjk}}{P_u \alpha [n]^2 \sum_{l \neq j}^{L} \beta_{u,jlm} + P_u \sum_{l=1}^{L} \sum_{i=1}^{K} \varepsilon_{u,jli} + P_d \sum_{l \neq j}^{L} \beta_{b,jjl} + (\sigma^2 + \kappa P_d \beta_{b,jjl})} \right) \tag{62}$$

2) **Achievable Downlink Rate:** The signal received by the $k$th full-duplex user in the $l$th cell, where $k \in K$ is written as

$$r_{d,ilk} [n] = I_{d,1} + I_{d,2} + I_{d,3} + I_{d,4} + z_{d,ilk} [n] \tag{63}$$

where

$$I_{d,1} = \sqrt{N} \tilde{g}_{d,ilk}^H [n] \frac{\tilde{g}_{d,ilk} [n]}{\sqrt{\text{Tr} \left( \tilde{G}_{d,ilk}^H [n] \tilde{G}_{d,ilk} [n] \right)}} s_{d,ilk} [n]$$

$$I_{d,2} = \sum_{(j,s) \neq (l,k)} \tilde{g}_{d,jsj}^H [n] \frac{\tilde{g}_{d,jsj} [n]}{\sqrt{\text{Tr} \left( \tilde{G}_{d,jsj}^H [n] \tilde{G}_{d,jsj} [n] \right)}} s_{d,jsj} [n]$$

$$I_{d,3} = \sum_{(j,s) \neq (l,k)} \tilde{g}_{d,jsj}^H [n] \frac{\tilde{g}_{d,jsj} [n]}{\sqrt{\text{Tr} \left( \tilde{G}_{d,jsj}^H [n] \tilde{G}_{d,jsj} [n] \right)}} s_{d,jsj} [n]$$

$$I_{d,4} = \sum f_{lkji} [n] x_{u,jli} [n]. \tag{64}$$

Applying Lemma 1, we have

$$\tilde{g}_{d,ilk}^H [n] \frac{\tilde{g}_{d,ilk} [n]}{\sqrt{\text{Tr} \left( \tilde{G}_{d,ilk}^H [n] \tilde{G}_{d,ilk} [n] \right)}} \xrightarrow{N \rightarrow \infty} \alpha [n] \frac{\tilde{g}_{d,ilk}^H [n] \tilde{g}_{d,ilk} [n]}{N} \tilde{D}_{d,ilk} \left( \tilde{D}_{d,ilk} \right)^{-1/2} \tag{65}$$

so the power of $I_{d,1}$ is

$$E \left[ |I_{d,1}|^2 \right] = P_d \alpha \alpha [n]^2 \tilde{\beta}_{d,ilk} \tilde{D}_{d,ilk} \left( \tilde{D}_{d,ilk} \right)^{-1/2} \tag{66}$$

Next, the power of $I_{d,2}$ can be written as

$$E \left[ |I_{d,2}|^2 \right] = P_d \alpha \alpha [n]^2 \sum_{(j,s) \neq (l,k)} \tilde{g}_{d,jsj}^H [n] \frac{\tilde{g}_{d,jsj} [n]}{\sqrt{\text{Tr} \left( \tilde{G}_{d,jsj}^H [n] \tilde{G}_{d,jsj} [n] \right)}} s_{d,jsj} [n] \tag{67}$$

Let $\tilde{g}_{d,jsj} [n] = \frac{\tilde{g}_{d,jsj} [n]}{\|\tilde{g}_{d,jsj} [n]\|}$. Then (67) can be rewritten as

$$E \left[ |I_{d,2}|^2 \right] = \frac{P_d \alpha \alpha [n]^2}{\text{Tr} \left( \tilde{G}_{d,jsj}^H [n] \tilde{G}_{d,jsj} [n] \right)} \frac{\|\tilde{g}_{d,jsj} [n]\|^2}{\sqrt{N} \tilde{D}_{d,jsj}} R_{jsj} \tag{68}$$

where

$$R_{jsj} = \frac{\tilde{g}_{d,jsj}^H [n] \tilde{g}_{d,jsj} [n]}{\sqrt{N} \tilde{D}_{d,jsj}} \tag{62}$$
So, using Lemma 1, the power of $I_{d,2}$ is

$$E\left[|I_{d,2}|^2\right] = P_d\alpha[n]^2 \sum_{(j,i)\neq(l,k)} \frac{\hat{\beta}_{d,ijkl}}{\text{Tr}(\hat{D}_{d,ll})} \hat{\beta}_{d,ji}. \quad (69)$$

Also, the power of $I_{d,3}$ is

$$E\left[|I_{d,3}|^2\right] = P_d\alpha[n]^2 \sum_{j=1}^{L} \sum_{i=1}^{K} \frac{\hat{\beta}_{d,ijkl}}{\text{Tr}(\hat{D}_{d,jj})} \hat{\beta}_{d,ji}. \quad (70)$$

Since each user is on the move, we also use an autoregressive model for the channels between the users. Thus, we can turn $I_{d,4}$ in (64) into

$$I_{d,4} = \sum_{(j,i)\neq(l,k)} (\alpha[n] f_{ikji}[0] + \epsilon_{ikji}[n]) x_{u,ji}[n]. \quad (71)$$

Finally, it is easy to get the power of $I_{d,5}$ as

$$E\left[|I_{d,4}|^2\right] = P_u \sum_{(j,i)\neq(l,k)} \left(\alpha[n]^2 \sigma_{ikji} + \epsilon_{ikji}\right). \quad (72)$$

Therefore, the downlink rate of the user $k$ in the $l$th cell is obtained as (73), shown at the bottom of this page.

### IV. SIMULATION RESULTS

We consider seven cells, where the radius of each cell is 1000 m, and all the users ($K = 4$) are uniformly distributed within the cell. Each cell has a full-duplex BS. Assuming that each cell has a guard range of $r_0 = 100$ m that means the distance between the nearest user and the BS. The large-scale fading can be modeled as $\beta_k = z_k / (r_k/r_0)^\eta$. In this expression, $z_k$ is a log-normal random variable with standard deviation $\sigma$ representing the shadow fading effect and the distance between the $k$th user and the BS is denoted by $r_k$ ($100 \leq r_k \leq 1000$). $\eta$ is the path loss in the system [20]. In simulation, we use $\sigma = 8$ dB and $\eta = 3.8$. In Doppler frequency shift factor, we use the carrier frequency $f_c = 2.5$ GHz, the channel sampling interval $T_s = 5$ ms, the dynamic range parameter $\kappa = 0.013$ and an average power constraint $P_d = 10$ dB. As the user rate $v = 3$ km/h, we can get $\eta = 0.9881$; as the user rate $v = 250$ km/h, also we can get $\eta = 0.0204$. So, it is easy to see that the Doppler shift is getting worse with the increase of users velocity. Figs. 2 shows uplink rate versus the transmit power of the user. From the curve, we can see that the rate increases with the increasing transmit power of the user. And when the transmission power is fixed, increasing the number of antennas can increase the rate.

Therefore, we can add the number of antennas to reduce the transmit power of the user. Figs. 3 and 4 present the uplink and downlink sum rate versus the normalized Doppler shift with perfect CSI. From Fig. 3, we can easily find that uplink sum rate will deduce as the normalized Doppler shift increases, especially in the second peak. Also, when the normalized Doppler shift increases (i.e., user speed increases), deploying more antennas can make up for the reduced rate. Besides, the uplink can resist the decline better than the uplink. Similarly, Fig. 5 shows the sum rate versus the normalized Doppler shift with perfect CSI. And the same conclusion can be drawn.

$$R_{d,k} = \log \left(1 + \frac{NP_d\alpha[n]^2 \beta_{d,ijkl}^2 \text{Tr}(\hat{D}_{d,ll})^{-1}}{P_d\alpha[n]^2 \sum_{(j,i)\neq(l,k)} \frac{\beta_{d,ijkl}}{\text{Tr}(\hat{D}_{d,jj})} \hat{\beta}_{d,ji} + P_d\alpha[n]^2 \sum_{(j,i)\neq(l,k)} \frac{\beta_{d,ijkl}}{\text{Tr}(\hat{D}_{d,jj})} \hat{\beta}_{d,ji}} + P_u \sum_{(j,i)\neq(l,k)} \left(\alpha[n]^2 \sigma_{ikji} + \epsilon_{ikji}\right) + \left(\sigma^2 + \kappa P_u \hat{\beta}_{ijkl}\right)\right) \quad (73)$$
Figs. 6 and 7 present the uplink and downlink sum rate versus the normalized Doppler shift with imperfect CSI. As is shown in the figure, uplink sum rate will deduce with the normalized Doppler shift increasing, especially in the second peak. Also, when the normalized Doppler shift increases (i.e., user speed increases), deploying more antennas can make up for the reduced rate. Besides, the uplink can resist the decline better than the uplink. Fig. 8 shows the sum rate versus the normalized Doppler shift with imperfect CSI. And the same conclusion can be drawn. Fig. 9 shows the sum rate changing with the uplink transmit power.
power. Obviously, as the transmit power gets larger, the rate increases. And, increasing the number of antennas can further increase the rate. Fig. 10 provides a contrast for the uplink sum rate under perfect and imperfect channel. From the picture, we can easily find the influence of the imperfection of the channel. Moreover, the bigger the value of time $n_t$, the more serious the effect of the Doppler shift on the performance of the system. As is shown in Fig. 5, the rate of the imperfect channel is nearly half that of perfect channels under the same conditions.

V. CONCLUSION

This paper mainly discusses the performance of the full-duplex system with effects of Doppler shift. Both perfect and imperfect CSI are, respectively, considered. We use an autoregressive model, to model the effects of the Doppler shift. In this paper, we assume a scenes with $L$ cells. Then, we apply MRC/MRT to optimize the system performance. Through simulation, we find that when the normalized Doppler shift increases (i.e., user speed increases), deploying more antennas can make up for the reduced rate and as the transmit power gets larger, the rate increases. Also, increasing the number of antennas can further increase the rate.

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