Batalin–Vilkovisky gauge–fixing of a chiral two–form in six dimensions

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Abstract

We perform the gauge–fixing of the theory of a chiral two–form boson in six dimensions starting from the action given by Pasti, Sorokin and Tonin. We use the Batalin–Vilkovisky formalism, introducing anti-fields and writing down an extended action satisfying the classical master equation. Then we gauge–fix the three local symmetries of the extended action in two different ways.

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1 Introduction

Chiral bosons are antisymmetric tensors of rank $2n$ in $4n + 2$ dimensions with (anti)self–dual field strengths. Their description in terms of a manifestly Lorentz-covariant action has been a longstanding problem. The core of this problem is the first order self–duality condition. The absence of a Lorentz covariant action makes an analysis of quantum properties of these theories more cumbersome. In 6 dimensions, the chiral boson is a (anti)self–dual two–tensor. It appears in the worldvolume action of the $M5$–brane [1] or in 6–dimensional chiral supergravity [2].

Some proposals for Lorentz invariant Lagrangians involved squaring the second class self–duality constraint [3] or introducing an infinite tower of auxiliary fields [4]. Their quantum properties were studied in [4, 5]. The problem with the infinite tower of auxiliary fields of [4] is to find a consistent truncation. This approach was generalised to higher order forms in [6].

By giving up explicit Lorentz invariance, it became possible to construct a new class of actions [7, 8]. In [8], 6–dimensional actions were constructed with manifest 5-dimensional Lorentz symmetry. The proof of 6–dimensional Lorentz invariance needed a non-trivial check.

By introducing one auxiliary scalar field, Lorentz covariant actions for chiral $p$–forms were constructed in [9, 10, 11, 12]. This auxiliary scalar field appears non-polynomially in the action. The action is invariant under two new gauge symmetries that depend on this scalar. One of the gauge symmetries makes it possible to gauge away this new auxiliary field. The relation to the approach with an infinite number of Lagrange multipliers [3] and the formulation with manifest 5–dimensional Lorentz invariance [9] is explained in [9]. Later, also $\kappa$-symmetric, Lorentz invariant world–volume actions were constructed for the 5–brane of $M$–theory [10, 11, 13].

In this paper, we study the free self–dual two–tensor in 6 dimensions in the Batalin–Vilkovisky (BV) formalism. This free action is superconformally invariant and it is the gauge–fixed action of the $\kappa$–symmetric $M5$–brane worldvolume action, restricted to quadratic terms, as proven in [13]. Up to now, a description for interacting tensor multiplets is lacking, although some attempts have been made [14].

The BV–formalism [15] is suited to study systems with different types of gauge symmetry structures, e.g. reducible gauge algebras. The classical BV–formalism leads to a gauge–fixed action. The quantum BV–formalism enables the calculation of anomalies. Here, we restrict ourselves to the classical analysis of the action of the chiral 2–form in 6 dimensions.

We start from the Pasti-Sorokin-Tonin (PST) action [9]. We build an extended action for the chiral tensor that satisfies the classical master equation. For this,
we have to introduce ghosts for the gauge symmetries and ghosts for ghosts for the reducible symmetries.

Using different canonical transformations, we find gauge–fixed actions corresponding to two gauge choices. The first action is a new, fully covariant one, while another gauge choice gives rise to the gauge–fixing of \[3\].

The paper is organised as follows. In section 2, a short review of the classical part of the BV–formalism is given. In section 3, the extended action is built for the self–dual 2–tensor using the BV–formalism. Two possible gauge–fixings of this extended action are given in section 4.

2 Review of the Batalin–Vilkovisky formalism

In this section we give a brief account of the classical BV–procedure. More elaborate reviews can be found in \[15\]. We will emphasize the case where the gauge symmetries of the classical action have zero modes, so that a second generation of ghosts will have to be introduced. A ghost number \(g\) is assigned to the \(g\)th generation of ghosts, where the classical fields have \(g = 0\). For every field, including the ghosts, an antifield with opposite statistics is introduced. We denote the physical and ghost fields by \(\Phi^A\) and their antifield counterparts by \(\Phi^*_A\). The antifields are assigned a ghost number such that

\[ g(\Phi^A) + g(\Phi^*_A) = -1. \]

We also define an antifield number, which is zero for fields and equal to \(-g\) for antifields. For functionals \(F(\Phi, \Phi^*)\), \(G(\Phi, \Phi^*)\), we introduce the antibracket

\[ (F, G) = F \frac{\delta}{\delta \Phi^A} \frac{\delta}{\delta \Phi^*_A} G - F \frac{\delta}{\delta \Phi^*_A} \frac{\delta}{\delta \Phi^A} G, \]

where summation over \(A\) is understood. Sometimes we will add to the antibracket an index between brackets: \((F, G)_n\) means that in \([2]\), only the terms with antifield number \(n\) are to be included.

The minimal extended action is defined by adding to the classical action a part containing the antifields, multiplied by the BRST-transformations. Becchi, Rouet, Stora and Tuytin replaced the parameters of the gauge transformations into ghost fields:

\[ S_{\text{min}} = S_d + S^1 \]
\[ = S_d + \Phi^*_A R^A_B c^B, \]

where \(\delta \Phi^A = R^A_B (\phi^C) \varepsilon^B\) is the infinitesimal gauge transformation with parameter \(\varepsilon^B\). The extra term has antifield number 1.
If the gauge transformations of $S_{cl}$ have zero modes, extra terms have to be introduced which have an analogous form: an antifield which will be the antighost of the relevant gauge transformation, multiplied by the zero mode transformation, where the parameter has been replaced by a ghost for ghost. This yields a term of antifield number 2. An additional term of the same antifield number is introduced for the commutators of the gauge symmetries, of the form

$$\int dx \frac{1}{2} (-)^B c_A^* T_{BC}^A c^B c^C, \quad (4)$$

where the $T_{BC}^A$ are the structure functions associated to the commutators of the gauge symmetries and $(-)^B$ is the fermion number of ghost $c^B$. Other terms may have to be added to the action until it satisfies the properness condition. This essentially states that for every gauge symmetry a ghost has been introduced, and for every zero mode a ghost for ghost. This can be checked by counting the number of zero modes of the Hessian

$$S_{AB} = \frac{\delta}{\delta \Phi_A} S \frac{\delta}{\delta \Phi_B}, \quad (5)$$

at the stationary surface. This is the surface where the classical fields satisfy the field equations and the ghosts and antifields are set to zero. The result should be half the number of fields and antifields.

More terms of higher antifield number, $S^3, \ldots, S^n$, are added until the resulting action, called the extended action, satisfies the classical master equation, which is

$$(S_{ext}, S_{ext}) = 0. \quad (6)$$

The truncation of the extended action to the antifield–independent part should be the classical action.

Canonical transformations from the basis $\{\Phi^A, \Phi_A^*\}$ to another set $\{\bar{\Phi}^A, \bar{\Phi}_A^*\}$ are defined to leave the antibrackets invariant. Often they can be determined by a fermionic generating function of ghost number $-1$, $F(\Phi, \bar{\Phi}^*)$, such that

$$\bar{\Phi}^A = \frac{\delta F(\Phi, \bar{\Phi}^*)}{\delta \Phi_A^*}, \quad \Phi_A^* = \frac{\delta F(\Phi, \bar{\Phi}^*)}{\delta \bar{\Phi}^A}. \quad (7)$$

The generating function is of the form

$$F(\Phi, \bar{\Phi}^*) = \Phi^A \bar{\Phi}_A^* + \Psi(\Phi). \quad (8)$$

The function $\Psi$, also a fermion of ghost number $-1$, is called the gauge fermion.
Gauge-fixing is performed by canonical transformations on the set of fields and antifields, such that the antifield-independent part of the extended action has a Hessian that is invertible on the stationary surface. In that case, well-defined propagators can be calculated. To do the gauge-fixing, it will be necessary to introduce auxiliary fields and their antifields \( b \) and \( b^* \). They should not have any influence on the master equation, and the physical content of the action should not be changed. For example, one gives the field \( b \) a ghost number \(-1\), so that its antifield has ghost number zero, and one adds to the action the term \( S_{nm} = (b^*)^2 \). Fields with negative ghost number are called non-minimal.

For each gauge invariance one introduces such a non-minimal field \( b^i \), and gauge-fixing functions \( F_i(\Phi_{cl}) \), where the \( \Phi_{cl} \) are the classical fields. The gauge fermion is then taken to be

\[
\Psi = b^i F_i(\Phi_{cl}).
\]

One also has to take care of the zero modes of the gauge transformations. Therefore, one has to introduce new auxiliary fields, but since they will have to be fermionic, it will be impossible to add a term \((b^*)^2\) to the extended action. We will deal with this problem as it arises.

This concludes our summary of the BV–formalism. For justification of the statements made here, and for further details, one should consult one of the references [15].

3 The extended action for a chiral 2–form

In this section, we first give the notations of the six-dimensional PST–action, and then build the extended action for the chiral 2–form introducing ghosts and ghosts for ghosts associated to the gauge symmetries and their zero modes. Since we are interested in studying the gauge symmetries of the chiral tensor, we will restrict the action to the terms with the chiral two–tensor and not consider the full supersymmetric model.

3.1 The classical action and its symmetries

The classical action for the self–dual tensor multiplet in 6 flat dimensions in the PST–formalism [9] is

\[
S_{cl} = \int d^6x - \frac{1}{2} H_{ab}^- H^{*ab}.
\]

The action contains a tensor \( B_{ab} \) and an auxiliary field \( a \). This is the action for one self–dual tensor. The description for interacting tensors, the non–abelian generalisation of the free model [10], is not known. To realise a self–dual tensor in
a supersymmetric context, chiral supersymmetry in 6 dimensions is needed. The action of the self–dual tensor multiplet with rigid superconformal symmetry is presented in [13]. We will confine ourselves to the study of the gauge symmetries of the bosonic model (10). The following notations are used:

\[ u_a = \partial_a a, \quad u^2 = u^a u_a, \]
\[ H = dB, \quad H_{abc} = 3\partial_{[a} B_{bc]}, \]
\[ H_{ab} = \frac{u^c}{\sqrt{u^2}} H_{abc}, \]
\[ H^*_{ab} = \frac{u^c}{\sqrt{u^2}} H^*_{abc}, \quad \text{where} \quad H^*_{abc} = \frac{1}{6} \epsilon_{abcdef} H_{def}, \]
\[ H^\pm_{ab} = \frac{u^c}{\sqrt{u^2}} H^\pm_{abc}, \quad \text{where} \quad H^\pm_{abc} = \frac{1}{2}(H_{abc} \pm H^*_{abc}). \] (11)

This action has the following three gauge symmetries:

\[ \delta_I B_{ab} = 2\partial_{[a} \Lambda_{b]}, \quad \delta_I a = 0, \]
\[ \delta_{II} B_{ab} = 2H^*_{ab} \frac{\phi}{\sqrt{u^2}}, \quad \delta_{II} a = \phi, \]
\[ \delta_{III} B_{ab} = u_{[a} \psi_{b]}, \quad \delta_{III} a = 0. \] (12)

The first symmetry is the usual gauge symmetry for a \( p \)-form. The second and the third symmetries are ‘new’ symmetries, enabled by the introduction of the scalar field \( a \). The first and the third symmetries are reducible. They have 3 zero modes:

\( (a) \quad \Lambda_a = \partial_a \Lambda, \)
\( (b) \quad \psi_a = u_a \psi, \)
\( (c) \quad \Lambda_a = u_a \Lambda', \quad \psi_a = 2\partial_a \Lambda'. \) (13)

### 3.2 The extended action of the chiral 2–form

Using the existence of the gauge symmetries and their reducibility conditions, we will build the extended action of the chiral 2–form. The first step to achieve an extended action is to introduce antifields \( B^*_{ab} \) and \( a^* \) for the classical fields, and ghosts \( c_a, c \) and \( c_b \) associated respectively to symmetries I, II and III. The ghost number and statistics of the different fields and antifields can be found in table [4].

The gauge symmetries yield the following contribution to the extended action at
antifield number 1 as given by (3):

$$S_1 = \int d^6x \left( \Phi^{ab} (2 \partial_a c_b + 2 H^{-}_{ab} c_{\sqrt{u^2}} + u a c'_b) + a^* c \right).$$

(14)

A contribution at antifield number 2 comes from the ghosts for ghosts $d_1$, $d_2$ and $d_3$, associated to the 3 zero modes of the gauge symmetries (13). This gives

$$S^2_1 = \int d^6x (c^*_a (\partial^a d_1 + u^a d_3) + c^*_a (u^a d_2 + 2 \partial a) d_3),$$

(15)

where $c^*_a$ and $c'^*_a$ are the antifields associated to the ghosts $c_a$ and $c'_a$. In $S^2$, we also have to include a term related to the commutators of the symmetries as indicated in (4). These commutators are

$$[\delta_{\Pi} (\phi_2), \delta_{\Pi} (\phi_1)] = \delta_{\Pi}(4 \frac{H^{-}_{ab}}{(u^2)^{3/2}} (\phi_1 \partial^b \phi_2 - \phi_2 \partial^b \phi_1)),$$

(16)

$$[\delta_{\Pi} (\phi), \delta_{\Pi} (\psi_a)] = \delta_{\Pi}( (\frac{1}{2} \psi a \phi) + \delta_{\Pi}(2 \partial [a \psi c] \cdot u^a \phi u^2).$$

(17)

We get the additional term in the action

$$S^2_2 = \int d^6x \left( \frac{1}{2} c^*_a c'^* a^* c - c^*_a \partial^a c'^* b \cdot \frac{u^b}{u^2} c + 4 c^*_a \frac{H^{-}_{ab}}{(u^2)^{3/2}} c \partial b c \right).$$

(18)

It is straightforward to check that the action

$$S = S_{cl} + S^1 + S^2,$$

(19)

with $S^2 = S^2_1 + S^2_2$, satisfies the properness condition, i.e. that the Hessian

$$S_{AB} = \frac{\delta}{\delta \Phi A} S\frac{\delta}{\delta \Phi^*_B}$$

(20)

at the stationary surface has a number of zero modes that is exactly half its dimension. To check it, it is convenient to choose a point on the surface where $H^{-}_{abc} = 0$. The number of zero modes of the Hessian cannot vary along the stationary surface.
As far as antibrackets are concerned, one has
\[(S_{cl}, S^1) = 0.\] (21)

This is simply a consequence of the gauge invariance of the classical action. However, the classical master equation is not yet satisfied; in particular, one has the antibracket
\[
2(S^1, S^2)(2) + (S^2, S^2)(2) = 8c_{1*}' \frac{H_{bde}}{(u^2)^3} u^a u^d \partial^e c \cdot c \partial^b c + c_{1*}' \left(-2 \frac{u^a u^b}{u^2} \partial_b d_2 \cdot c + 2 \partial^a (d_2 c) \right)
+ c_{1*}' (2 \partial^a (d_3 c) - u^a d_2 c) + 2c_{2*}' \frac{u^a u^b}{(u^2)^2} \partial_c c_{ij} \cdot c \partial^c c. \] (22)

Since this is not 0, we have to add terms at antifield number 3. The choice that works is
\[
S^3 = -d_3^* d_3 c
+ d_2^* \left(-4 \frac{H_{ab}}{(u^2)^{5/2}} \partial^c c \cdot c \partial^a c + 2 \frac{u^a}{(u^2)^2} \partial_{[a} c_{ij]} \cdot c \partial^b c + \frac{u^a}{u^2} \partial_a d_2 \cdot c \right)
+ d_3^* d_2 c. \] (23)

Then
\[
2(S^2, S^3)(2) - 2(S^1, S^3)(2) + (S^2, S^2)(2) = 0,
2(S^1, S^3)(3) + 2(S^2, S^3)(3) + (S^3, S^3)(3) = 0. \] (24)

We conclude that the sum of (10), (14), (15), (18) and (23) is a good extended action. This action satisfies the classical master equation, the properness condition, and the classical limit (deleting all terms with non-zero ghost number in the action) gives the classical action (10). So, all the conditions to have a good extended action are fulfilled.

4 Gauge–fixings of the extended action

In this section, we present three different gauge–fixings. First, there is the gauge–fixing that gives rise to the explicit self–duality condition of the 2–form [9]. Then a covariant gauge–fixing of the extended action is given. By allowing the gradient of the auxiliary scalar to point in an arbitrary direction, a non–covariant gauge–fixing, as used in [8, 10], is found.
4.1 The self–duality of the two–form

The field equation of the two–form $B_{ab}$ is

$$\epsilon^{lmnpqr} \partial_n \left( \frac{1}{u^2} u_p H_{qrs} u^s \right) = 0. \quad (25)$$

The most general solution of (25) is [9]

$$H_{lmn} u^n = u^2 \partial[l \Phi_m] + u^n (\partial_n \Phi[l] u_m) + u[m] (\partial_m \Phi_n) u^n, \quad (26)$$

This is a gauge transformation III of $H_{lm}$. By using a Schouten identity, it can be proven that $H_{lm} = 0$ is equivalent to $H_{lmn} = 0$. This means that it is possible to find the self–duality of $B_{ab}$ by picking a gauge choice for the third gauge transformation. So, the formulation with an auxiliary field gives rise to the explicit self–duality equation.

4.2 A covariant gauge–fixing

In this section we will present a covariant gauge–fixing of the extended action using gauge fermions in the BV–formalism. We start by gauge–fixing the three gauge symmetries (12) of the classical action. We will use the gauges

$$\partial_a B^{ab} = 0, \quad (27)$$
$$u^2 = 1, \quad (28)$$
$$u_a B^{ab} = 0. \quad (29)$$

(28) is a Lorentz invariant gauge–fixing for symmetry II. The gauge (29) fixes symmetry III and is the analogue of the Lorentz gauge; using a transformation III, one can remove the component of $B_{ab}$ that is parallel to the vector $u^a$.

To facilitate the gauge–fixing of symmetry I, we first rewrite the classical action as

$$S_{cl} = \int d^d x \left( -\frac{1}{24} H^{abc} H_{abc} + \frac{1}{2} H^{-ab} H^a_{ab} \right). \quad (30)$$

For each of these gauge–fixings, one introduces a new non–minimal set of fermionic fields and their antifields and adds to the action a term quadratic in the antifields. In table 2, they are denoted by $b_a$.

The gauge–fixing is done by introducing a gauge fermion for each symmetry:

$$\Psi_1 = b_a \partial_b B^{ab} \quad (31)$$
$$\Psi_2 = b'_a u_b B^{ab} \quad (32)$$
$$\Psi_3 = b(u^2 - 1), \quad (33)$$
Table 2: The fields for the gauge–fixing of theories with a reducible gauge algebra, with (ghost number, ghost number of antifield) and a schematic indication of non–degeneracy conditions of the gauge fixing and the connected antifields in the non–minimal extended action.

and adding to the action the non–minimal terms

\[ S_{nm1} = -\frac{1}{4}b^*_a b^{*a} - \frac{1}{2}b^{*a}_b b^{*a}_b + b^* b^2. \] (34)

The gauge symmetries of the classical action also had three zero modes. Their gauge–fixing is done in general by introducing two extra sets of bosonic fields and their antifields for each zero mode. In the table 2 they are denoted by \( b'^a_1 \) and \( b_1 \). The arrow between them is used to indicate that one adds a non–minimal term to the action which is a product of their antifields.

For the gauge–fixing of the three reducible gauge symmetries, one introduces the bosons \( k, m, p \) and \( l, n, q \). The following gauge fermions can be used:

\[
\begin{align*}
\Psi_4 &= k \partial_b c^b + l \partial_a b^a \\
\Psi_5 &= m u^a c^a + n u_a b^a \\
\Psi_6 &= p (u_a c^a - 2 \partial_a c^a) + q (u_a b^a - 2 \partial_a b^a).
\end{align*}
\] (35) (36) (37)

To the action the following non–minimal terms are added:

\[ S_{nm2} = k^* l^* + \frac{1}{2} m^* n^* + p^* q^*. \] (38)

The antifield–independent part of the action becomes

\[
S = \int d^6 x \left[ \frac{1}{8} B_{ab} \square B_{ab} - \frac{1}{2} H^{-ab} H_{ab} - \frac{1}{4} q^2 u^2 - 2q u^a \partial_a l \\
- l \square l + (u^2 - 1)^2 + q \square q + n u^a \partial_a q - \frac{1}{4} n^2 u^2 - \frac{1}{4} u_b B^{ab} u^c B_{ac} \\
-b^{*a} \partial_a u_b . c^b - b^{*a} u^a \partial_a c_b - \frac{1}{2} b^{*b}_b u^2 c'_b - b^{*a} c_a \\
+ 2 \partial^a b^b \cdot H_{ab} \frac{c}{\sqrt{u^2}} - \frac{1}{2} c'_b u^a \partial_a b^b + \frac{5}{2} u_a b^a \partial_b c'_b + \frac{1}{2} b^{*a} \partial u_a \cdot c^b
\right]
\]
\begin{align*}
  &u_a c^a u_b b^b + 4 \partial_c c^a \cdot \partial_b b^b + \left( q b^a + 2 b u^a + n b^a \right) \partial_a c \\
  &+ p u^a \partial_a d_1 + k \Box d_1 + p u^2 d_3 + d_3 u^a \partial_a k + \frac{1}{2} p u_a c^a c + \frac{1}{2} k \partial^a (c^a c) \\
  &- 2 d_2 u^a \partial_a p + m u^2 d_2 + 4 p \Box d_3 + 2 m u^a \partial_a d_3 + 2 \partial_a p \cdot \partial^b c^b \cdot \frac{u_b}{u^2} \\
  &- 8 \partial_a p \cdot \frac{H^{-ab}}{(u^2)^{3/2}} c \partial_b c + (pc^a + mc^a) \partial_a c \bigg] .
\end{align*}

(39)

This is a covariant gauge–fixed action for the self–dual 2–form in 6 dimensions. In principle this can be used to derive gravitational anomalies as in \([16, 17]\), but the presence of the auxiliary scalar in the propagators makes an analysis of anomalies using this action very hard. The non-covariant gauge–fixing of the next section can be used for this.

### 4.3 A non–covariant gauge–fixing

The non–covariant gauge–fixing of the second symmetry in this section corresponds to \( a = x^5 \) in \([8]\). In \([16]\), the second gauge symmetry was fixed by imposing \( a = n_a x^a \) where \( n_a \) is a unit vector such that \( u^2 = 1 \). Using the Faddeev–Popov approach, it was proven that this gauge–fixing gives rise to the propagators postulated and used in \([17]\) to calculate the gravitational anomalies of a chiral 2–form in 6 dimensions.

The second gauge symmetry can be fixed by a canonical transformation that cannot be generated by a gauge fermion:

\begin{align*}
  a &\rightarrow -b^* + n_a x^a \\
  a^* &\rightarrow b .
\end{align*}

(40)

This means that

\begin{align*}
  u_a c^a &\rightarrow -\partial_b b^* + n_c \\
  u^2 &\rightarrow 1 - 2n^a \partial_a b^* + (\partial b^*)^2 .
\end{align*}

(41)

Using the gauge fermions

\begin{align*}
  \psi_1 &= b_a \partial_b B^{ab} \\
  \psi_3 &= b'_a n_b B^{ab} \\
  \psi_4 &= k \partial_c b^c + l \partial_b b^b \\
  \psi_5 &= mn_a c^{a^*} + mn_a b^{a^*} \\
  \psi_6 &= p (n_a c^a + 2 \partial_a c^{a^*}) + q (n_a b^a + 2 \partial_a b^{a^*}) ,
\end{align*}

(42-46)
for the other symmetries, and the corresponding non-minimal terms in the action

\[ S_{nm} = -\frac{1}{4} b^*a b^*a + \frac{1}{2} b^*a b^*a - k^*l^* + \frac{1}{2} m^*n^* - \frac{1}{2} p^*q^* , \]

the other symmetries of the action are gauge-fixed. The antifield-independent part of the action is:

\[ S = \int d^6x \left[ \frac{1}{2} B_{ab} \nabla B_{ab} - \frac{1}{2} H_{abc} H_{abcd} u^d - b^a \nabla c_a - b^a n^b \partial_b c_a \right. \\
+ 2b^a \partial^b H_{abc} \cdot n^c + \frac{3}{2} n^a b_a \partial^a c' - \frac{1}{2} b^a n^b \partial_b c'_a - \frac{1}{2} b^a c'_a + bc \\
+ k \nabla d_1 + k n^a \partial_a d_3 + \frac{1}{2} k \partial^a (c'_a c) + md_2 + 2m n^a \partial_a d_3 + \frac{1}{2} q n^a \partial_a l + \frac{1}{4} l \nabla l \\
+ \frac{1}{2} n^b B_{ab} n^c B_{ac} + \frac{1}{2} n^2 - 2n^a \partial^a q - 2q \nabla q + \frac{1}{2} n^a c_a n^b b_b - 2C_{ab} c^a \cdot \partial b^b \\
+ p n^a \partial_a d_1 + pd_3 + \frac{1}{2} p n^a c'_a c - 2d_2 n^a \partial_a p + 4p \nabla d_3 + 2\partial^a p \cdot \partial_b c'_a \cdot n^b c \\
\left. - 8\partial^a p \cdot H_{abc} n^c c \partial^b c \right] . \tag{47} \]

5 Conclusions

In this article, we were able to find the full extended action of the chiral two–form in six dimensions using an auxiliary field $a$. For this, we had to introduce ghosts for the three gauge symmetries of the classical action and ghosts for ghosts for the three zero modes and add corresponding terms to the classical action. We showed that the expansion in antifield number of the action satisfying the classical master equation terminates at antifield number 3. Since this action satisfies the properness condition and gives the classical action in the classical limit, it is a good extended action.

We treated two different gauge–fixings, a non-covariant one as well as a covariant one. We note in passing that the non-covariant gauge fixing allows for an easy counting of the on–shell degrees of freedom: starting with 16 (15 for the tensor and 1 for the auxiliary scalar), one ends up with 3 degrees of freedom after successive gauge–fixings of the three gauge symmetries.

The two gauge–fixing schemes were implemented within the BV–formalism. The first one resulted in a fully Lorentz covariant gauge–fixed action. These gauge–fixed actions lead to different propagators that can be used in calculations of gravitational anomalies. From the form of the actions we derived, it is clear that our second gauge–fixing, also used in $[16]$, is easiest because of the absence of the auxiliary field $a$ in the propagators. However, in the BV–formalism, a cohomology can be defined $[15]$, such that anomalies derived for different gauge–fixings (like the covariant one) are in the same cohomology class.
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