Effect of quark mass variation on big bang nucleosynthesis

J.C. Berengut a, *, V.V. Flambaum a, V.F. Dmitriev b

a School of Physics, University of New South Wales, Sydney 2052, Australia
b Budker Institute of Nuclear Physics, 630090, Novosibirsk-90, Russia

A R T I C L E   I N F O

Article history:
Received 14 July 2009
Received in revised form 2 December 2009
Accepted 3 December 2009
Available online 11 December 2009
Editor: W. Haxton

Abstract

We calculate the effect of variation in the light-current quark mass, m q , on standard big bang nucleosynthesis. A change in m q during the era of nucleosynthesis affects nuclear reaction rates, and hence primordial abundances, via changes in the binding energies of light nuclei. It is found that a relative variation of δ m q /m q = 0.016 ± 0.005 provides better agreement between observed primordial abundances and those predicted by theory. This is largely due to resolution of the existing discrepancies for 7 Li. However this method ignores possible changes in the position of resonances in nuclear reactions. The predicted 7 Li abundance has a strong dependence on the cross-section of the resonant reactions (d, p) 4 He and (t, n) 4 He. We show that changes in m q at the time of BBN could shift the position of these resonances away from the Gamow window and lead to an increased production of 7 Li, exacerbating the lithium problem.

© 2009 Elsevier B.V. All rights reserved.

1. Introduction

Measurements of the primordial baryon-to-photon ratio η from the cosmic microwave background from WMAP [1], coupled with precise measurements of the neutron half-life [2], have made big bang nucleosynthesis (BBN) an essentially parameter-free theory [2–4]. In this paradigm excellent agreement has been obtained between predicted and observed abundances of deuterium and 4 He (see, e.g. the Particle Data Group review [2] and references therein). However there is some disagreement for 7 Li, the only other element for which the abundance has been measured to an accuracy at which fruitful comparison with theory can be made. While the “lithium problem” has been known for some time, it has been exacerbated by recent measurements of the 4 He(α, γ) 7 Be reaction [5]. Standard BBN theory with η provided by WMAP 5 overproduces 7 Li by a factor of 2.4–4.3 (around 4–5σ) [4].

One possible solution to the lithium problem is that the physical constants of the early Universe may have been slightly different. In fact, such variations in the physical laws can be well-motivated theoretically in an expanding Universe; see [6] for a review. Ref. [7] considered variation of the deuterium binding energy B d during primordial nucleosynthesis. BBN has a high sensitivity to B d since its value determines the temperature at which deuterium can withstand photo-disintegration and hence the time at which nucleosynthesis begins. Their best-fit result Δ B d /B d = −0.019 ± 0.005 resolved then-extant discrepancies between theory and observation in both 7 Li and η (or alternatively, in 7 Li and 4 He with η fixed by WMAP).

More recently, Ref. [8] examined the response of BBN to variation of several physical parameters, including binding energies, in a linear approximation. These were coupled with calculated dependences of binding energies on m q in [9], which found that the 7 Li abundance discrepancy could be resolved by a variation in light-quark mass of δ m q /m q = 0.013 ± 0.002. Crucially, the 4 He and d abundances were found to be relatively insensitive to m q and so the existing agreement between theory and observation in these elements was maintained.

In this Letter we re-examine the dependence of light-element production on variation of the dimensionless parameter X q = m q /Λ QCD where m q is the light quark mass and Λ QCD is the pole in the running strong-coupling constant. We follow [9] and assume that Λ QCD is constant, calculating the dependence on the small parameter m q . This is not an approximation. Rather it only means that we measure all dimensions (m q , cross-sections, etc.) in units of Λ QCD . Therefore δ m q /m q should be understood as δ X q /X q . We take into account several effects that were not previously considered, most importantly the nonlinear dependence on m q and variation of resonance positions.

Note that here we will not discuss the variation of other fundamental constants (such as the fine structure constant, α) on BBN. There are two reasons for this. Firstly, BBN is known to have a particularly strong sensitivity to m q [7]. Secondly, the hypothetical unification of all interactions implies that variations of different
fundamental constants may be related [10–14]. For example, grand unification theories predict
\[
\frac{\delta X_q}{X_q} \sim 35 \frac{\delta \alpha}{\alpha}.
\]
(1)
The coefficient here is model-dependent, but large values are generic for models in which variations come from high energy scales (for a simple explanation see Ref. [9]). If these ideas are correct, the variation in \(X_q\) may be easier to detect than the variation in \(\alpha\).

2. Variation of binding energies

The energy released in each reaction, \(Q\), is determined by the masses of the reactants and products, which in turn are determined by the nuclear binding energies. As noted in [8], the \(Q\)-values affect the forward (exothermic) reaction rates via space and radiative emission factors. For radiative capture reactions at low energy \(E\) the \(Q\)-dependence is
\[
\sigma(E) \propto E_0^3 \sim (Q + E)^3.
\]
(2)
For low-energy reactions with two nucleons in the exit channel the dependence is proportional to the outgoing channel velocity, \(v \sim (Q + E)^{1/2}\). When the outgoing particles are charged, the Gamow factor of the exit channel can also contribute:
\[
\sigma(E) \sim (Q + E)^{1/2} e^{-\sqrt{E_0/(Q+E)}}.
\]
(3)
The Gamow factor appears because of the Coulomb barrier to the reaction; \(E_0 = 2\pi^2 Z_1^2 Z_2^2 \alpha^2 \mu c^2\) where \(\alpha\) is the fine-structure constant, \(Z_1\) and \(Z_2\) are the charge numbers of the products, and \(\mu\) is the reduced mass of the products. At BBN temperatures we can usually assume that \(E \ll E_0\). Expanding in \(Q\),
\[
\sigma = \sigma_0 \left[ 1 + \frac{1}{2} \left(1 + \frac{E_0}{Q}\right) \frac{\delta Q}{Q} + \cdots \right]
\]
(4)
and we see that the Gamow term in (3) is generally small (it was neglected in [8]). However it can be important for some reactions, for example in \(^8\text{Be}(n, \gamma)\text{Li}\), \(E_0/Q = 2.17\), i.e. it triples the effect of \(\delta Q\) on the reaction rate.

The reverse reaction rates are simply related to the forward rates via statistical factors. From detailed balance one finds
\[
\frac{\langle \sigma v \rangle_{\text{rev}}}{\langle \sigma v \rangle_{\text{fwd}}} \sim e^{-Q/T}
\]
(5)
and we see that the reverse reactions also provide sensitivity to \(Q\).

An exception to the rule (2) is found in the reaction \(p(n, \gamma')d\), an important reaction because \(d\) is a precursor to all further nucleosynthesis. This reaction is sensitive not only to \(Q\) but also to the position of the virtual level with energy \(E_\nu = 0.07\) MeV. The sensitivity of this reaction to \(Q\) was calculated in [7]
\[
\langle \sigma v \rangle \sim \left[ 1 + \left(\frac{5}{2} + \frac{Q}{E_\nu}\right) \frac{\delta Q}{Q} \right].
\]
(6)
Note that [8,9] did not take variation of the virtual level into account. In Table 1 we show the linear dependence of abundances on the deuterium binding energy with different theories of variation. It shows the effect of variation of the virtual level, as well as the effect of including \(B_q\) variation on other \(Q\)-values and reaction rates.

We denote the sensitivity of nuclear binding energies to the light-current quark mass \(m_q\) by

| Method | \(d\) | \(^3\text{He}\) | \(^4\text{He}\) | \(^4\text{Li}\) | \(^7\text{Li}\) |
|--------|------|--------|--------|--------|--------|
| 1      | 4.04 | -1.75  | 0.68   | -3.17  | 10.59  |
| 2      | 2.91 | -2.08  | 0.67   | -6.58  | 9.41   |
| 3      | -5.12| -1.29  | 0.70   | -4.23  | 17.99  |
| 4      | -4.00| -1.62  | 0.69   | -7.64  | 16.81  |

\[ K = \frac{\delta \Lambda_{\text{nuc}}/m_q}{m_q} \] (7)

Values of \(K\) for several light nuclei were presented in Refs. [9,15]. We use the “best values” from these papers, given by the AV18 + UIX nuclear Hamiltonians, with hadron mass variations calculated in terms of the \(m_q\) using the Dyson–Schwinger equation calculation of [16]. From these one calculates the \(m_q\)-dependence of the \(Q\) values, and therefore the reaction rates, and therefore the primordial abundances of light elements in BBN.

In Fig. 1 we present our predicted values of \(^4\text{He}\), \(d\), and \(^7\text{Li}\) with different values of light quark mass. Details of the calculations and explanation of observational abundances are presented in the appendices. Comparing the observed and predicted abundances from the figures we obtain for \(^4\text{He}\), \(d\), and \(^7\text{Li}\) respectively, \(\delta m_q/m_q = -0.002 \pm 0.037, 0.012 \pm 0.011,\) and \(0.018 \pm 0.006\). These concordance regimes are simply obtained by adding in quadrature the observational and theoretical errors presented in the appendices. The uncertainties in our theoretical predictions, represented by the ranges of the dashed lines in Fig. 1, include uncertainties in the WMAP data and reaction rates, as calculated in [4]. The three data sets are therefore consistent, with weighted mean

\[ \delta m_q/m_q = 0.016 \pm 0.005. \] (8)

It is seen that the \(^4\text{He}\) abundance has a low sensitivity to \(m_q\); furthermore we show in Fig. 1 the conservative observational error bounds provided by [17]. Therefore, it is worth pointing out that the more tightly constrained abundance, \(Y_p = 0.2477 \pm 0.0029\) [18], is also consistent with the variation (8). It is clear from Fig. 1 that taking into account the nonlinear dependence of BBN abundances on \(m_q\) is important, particularly for \(^7\text{Li}\). In fact, if we assume a linear response, as was done in [8,9], we instead obtain \(\delta m_q/m_q = 0.014 \pm 0.002\).

As noted in [9], to take into account uncertainties in the theoretically derived quantities \(K\) (Eq. (7)) the final result (8) should be interpreted as \(\delta m_q/m_q = k \cdot (0.016 \pm 0.005)\) where \(k \sim 1\) and the accuracy in \(k\) is approximately a factor of two.

3. Resonances

Of the most important reactions in BBN, the mirror reactions

\(^3\text{He}(d, p)^4\text{He}\) (reaction 1),
\(t(d, n)^4\text{He}\) (reaction 2)

are the only reactions where the cross-section is dominated by a fairly narrow resonance. Therefore, one can hope for sensitivity of primordial abundances to the position of these resonances. (Note that the reaction \(^7\text{Be}(n, \gamma)\text{Li}\) is also dominated by a near-threshold resonance, however in this case the resonance is a rather broad and hence strong sensitivity can hardly be expected.)
Consider modification of the resonance positions, $E_r \rightarrow E_r + \delta E_r$, due to a variation of the fundamental constant $m_q$. Reaction 1 will be affected in the following way. The resonance is an excited state of $^5$Li; that is, a compound nucleus with three protons and two neutrons: we call this state $^5$Li*. Similarly there is a state $^5$He* for reaction 2. Then

$$E_r^{(1)} = E_{51}^{Li} - E_{3He} - E_d,$$

$$E_r^{(2)} = E_{5He}^* - E_t - E_d$$

and so $E_{51}^{Li} = -9.76$ MeV and $E_{3He}^* = -10.66$ MeV. The change in the resonance position due to a variation in $m_q$ is therefore

$$\delta E_r^{(1)} = \delta E_{51}^{Li} - \delta E_{3He} - \delta E_d = (K_{51}^{Li}E_{51}^{Li} - K_{3He}^{Li}E_{3He} - K_dE_d) \frac{\delta m_q}{m_q}$$

with the $K$ defined by (7).

One could also consider the effect of variation of fundamental constants on the total resonance widths, $\Gamma$. Variation of the width of the exit channel is already taken into account using Eq. (3). Variation of other widths is not as large as the relative variation of the shallow level $E_r$, which is enhanced due to the approximate cancellation of the large potential and kinetic energy contributions. Therefore the final BBN abundances have no great sensitivity to these parameters and we have not included them in our analysis.

Changes to the cross-section of reaction 1 affects the primordial abundances of $^4$He and $^7$Be, while changes in reaction 2 affect abundances of $^t$ and $^3$Li. Since $t$ and $^3$He are not well constrained observationally, we choose to focus on $^3$Li. In Fig. 2 we present $^3$Li abundance against variation of light quark mass $\delta m_q/m_q$ at $\eta = 6.23 \times 10^{-10}$, the WMAP5 value. For such a value of $\eta$, the majority of $^3$Li is created as $^7$Be (which $\beta$-captures to $^7$Li) via the reaction $^3$He($^4$He,$\gamma$)$^7$Be.

We need to find $K_{51}^{Li*}$ (and similarly $K_{3He}^{He*}$). One assumption is that the mass-energy of the resonance varies with the mass-energy in the incoming channel [8]; in this case the resonance does not shift. This assumption corresponds to $K_{51}^{Li} = -1.54$ and $K_{3He} = -1.44$. It corresponds to the solid line in Fig. 2.

A more reasonable guess is to assume that the variation of the resonant state $^5$Li* will be approximately the same as that of the ground state $^3$Li. This can be seen by considering the resonance and the ground state configurations as residing in the same potential. The sensitivity of the ground state $^3$He to $m_q$ has been calculated $K_{3He} = -1.24$ [15]; $K_{51}^{Li}$ was not calculated explicitly, but its value will be very close to that of $^3$He. Our assumption of equal variation of the ground and excited state then gives

$$K_{51}^{Li} = -3.35,$$

$$K_{3He} = -3.19.$$  

This assumption corresponds to the dashed line in Fig. 2.

The equal-variation assumption in the previous paragraph represents an upper limit on the relationship between the ground and excited state. In reality the potential-dependence of the states may be different, in which case the shift of the $^3$Li (or $^3$He) resonance may be smaller than the shift of the ground state. On the other hand a minimum value of $K$ for the resonance states is that of the ground state, $K_{51}^{Li} = K_{3He} = -1.24$. A reasonable, conservative, estimate is to take the average of these extremal values: $K_{51}^{Li} = -2.29$ and $K_{3He} = -2.21$; this is the dot-dashed line in Fig. 2. Ultimately however, we require a nuclear calculation of sensitivity, of the kind presented in Refs. [9,15].

The effect of $\delta E_r^{(1)}$ on BBN can be understood in the following way. When the cross-section is convolved with a Maxwellian

\[ \sigma(E) = \frac{P(E)}{E} \frac{1}{(E - E_r)^2 + \Gamma_r^2/4} \]

where $E_g$ is the Gamow energy of the reactants, $E_r$ and $\Gamma_r$ are resonance parameters, and $P(E)$ is a polynomial chosen to fit the measured reaction cross-section. In this work we use the cross-section fits of Ref. [19], which give $E_r^{(1)} = 0.183$ MeV, $\Gamma_r^{(1)} = 0.256$ MeV and $E_r^{(2)} = 0.0482$ MeV, $\Gamma_r^{(2)} = 0.0806$ MeV for reactions 1 and 2, respectively.
The effect of $\delta E^{(2)}$ is very similar: it reduces the amount of $t$ destroyed in reaction 2, leaving more tritium to react with $^4\text{He}$ to produce $^7\text{Be}$. On the other hand the effect of this reaction on $^7\text{Li}$ and $^4\text{He}$ abundances is minimal.

The effect of $\delta E^{(2)}$ is very similar: it reduces the amount of $t$ destroyed in reaction 2, leaving more tritium to react with $^4\text{He}$ to produce $^7\text{Li}$ directly. Despite this production channel being suppressed at high $\eta$, the effect of $\delta E^{(2)}$ is still important for $^7\text{Li}$ production because the relative effect of the variation is larger: $\delta E^{(2)}/E^{(2)} > \delta E^{(1)}/E^{(1)}$. The trends seen in Fig. 2 are the same even at low $\eta$ since both reaction pathways behave in much the same way to variation in $m_q$.

From Fig. 2 we see that taking shifts in the resonance positions into account can destroy the agreement between theory and observation previously obtained by varying $m_q$. In the case where the shifts in the ground and resonant states vary by the same amount (dashed line), the $^7\text{Li}$ discrepancy actually gets worse with variation in light quark mass. On the other hand the milder “averaged $K$” response (dot-dashed line) still significantly challenges the conclusions of Section 2. It is not appropriate to directly compare primordial $^3\text{He}$ abundances with observations because of the complexity of the stellar evolution of this isotope [20], however we note that primordial $^3\text{He}$ production could also be greatly increased by movement of these resonances (Fig. 3).

We have shown in Section 2 that a variation in the light quark mass during the era of big-bang nucleosynthesis of $\delta m_q/m_q = 0.016 \pm 0.005$ provides better agreement between theory and the observed primordial abundances. This is largely because it resolves the existing disagreement in $^7\text{Li}$ abundances [4].

However this conclusion is threatened when movement of the resonance positions in either direction will reduce the cross-section for this reaction at the relevant temperatures. In turn this reduces the amount of $^3\text{He}$ that is destroyed via reaction 1, leaving more to re-

**Fig. 3.** Calculated $^3\text{He}$ abundance vs. relative change in light quark mass $m_q/\Lambda_{\text{QCD}}$. Solid line: no shifts in resonance positions included; dashed line: resonance shifts according to assumption that resonant state varies as much as the ground state (Eqs. (14) and (15)); dot-dashed line: an averaged value of resonance-position sensitivity used.

**4. Conclusion**

This work is supported by the Australian Research Council. J.C.B. thanks Daniel Wolf Savin for hospitality during the early stages of this work.

**Appendix A. Observational abundances**

Our observational abundances largely follow the recommendations of the Particle Data Group review [2]. The deuterium abundances are derived from several studies of isotope-shifted $\text{Ly}-\alpha$ spectra in quasar absorption systems. Combined these give

$$
\frac{d}{H} = (2.84 \pm 0.26) \times 10^{-5}
$$

where the errors have been increased to account for the scatter between different systems.

$^4\text{He}$ is observed in H II regions of low-metallicity dwarf galaxies. A very conservative estimate of observed $^4\text{He}$ abundance comes from [17]

$$
Y_p = 0.249 \pm 0.009.
$$

The error here is significantly larger than other extrapolations to zero metallicity, e.g. [21,18].
Primordial $^7\text{Li}$ abundance is determined from metal-poor Pop II stars in our galaxy. Lithium abundance does not vary over many orders of magnitude of metallicity in such stars; this is the Spite plateau [22]. Recent studies give abundances of $(1.1-1.2 \pm 0.1) \times 10^{-10}$ [23], $(1.26 \pm 0.26) \times 10^{-10}$ [24], and $(1.1-1.5) \times 10^{-10}$ [25]. Significantly higher results were obtained with different methods of obtaining effective temperature of the stars [26] since the derived lithium abundance is very sensitive to temperature. However no evidence for high temperatures was found in the studies of Cyburt, Fields, Ka. Olive [4] which use similar methods of calculation. The important results are those of the globular cluster NGC 6397 which give values of $(25,23)$. For a more detailed review and discussion of systematics, see, e.g. [2,23,4].

In this Letter we use the conservative range of

$^7\text{Li}/H = (1.5 \pm 0.5) \times 10^{-10}$,

which was also adopted in [8], although we note that some of the studies listed above give ranges as high as $2.5 \times 10^{-10}$.

Appendix B. Computer code, reaction rates, and theoretical uncertainties

In this work we calculate BBN abundances using a modified Kawano code [31], with updated reactions from the NACRE Collaboration [32]. For the most important reactions we use the cross-section fits in [19]. The exceptions are $p(n,\gamma)d$ where we use the calculations of [33], and the recently measured reaction $^3\text{He}(^4\text{He},\gamma)^7\text{Be}$ for which we use the fits provided in [5]. Our results are

$$Y_p = 0.2486,$$
$$d/H = 2.53 \times 10^{-5},$$
$$^7\text{Li}/H = 5.05 \times 10^{-10}.$$  

We have not calculated errors in the theoretical prediction; instead we simply take the relative errors from the recent calculations of Cyburt, Fields and Olive [4] which use a very similar reaction network and the latest physical data. Their results are $Y_p = 0.2486 \pm 0.0002$, $d/H = (2.49 \pm 0.17) \times 10^{-5}$, and $^7\text{Li}/H = (5.24^{+0.71}_{-0.62}) \times 10^{-10}$, which compare well with the results from our code.