Statistics of the total number of collisions and the ordering time in a freely expanding hard-point gas

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Abstract. We consider a Jepsen gas of \(N\) hard-point particles undergoing free expansion on a line, starting from random initial positions of the particles having random initial velocities. The particles undergo binary elastic collisions upon contact and move freely in-between collisions. After a certain ordering time \(T_0\), the system reaches a ‘fan’ state where all the velocities are completely ordered from left to right in an increasing fashion and there is no further collision. We compute analytically the distributions of (i) the total number of collisions and (ii) the ordering time \(T_0\). We show that several features of these distributions are universal.

Keywords: stochastic particle dynamics (theory), stochastic processes (theory), models for evolution (theory)
1. Introduction

Exactly solvable models of interacting particle systems are important both in equilibrium and out-of-equilibrium statistical mechanics. Such solvable models, apart from their pedagogical virtues, also offer important insights and intuitions of the underlying more complex physical phenomena. Besides, they also allow us to understand better the limitations of the approximate methods one generally uses in treating many-particles systems (e.g. the Boltzmann equation).

One such very useful and instructive model was introduced a few decades ago [1,2] and is usually referred to as the Jepsen gas [3]. It consists of identical hard-point particles that undergo binary elastic collisions on a line. At these instantaneous collisions the particles exchange their velocities, while in between collisions they move freely. Due to the simplicity of the dynamics, this system admits analytical treatments for various imposed conditions, and therefore triggered quite a lot of interest in the past. The work of Jepsen [3] was followed by Lebowitz et al [4]–[6], McKean [7] and Keyes [8], who refined and extended the calculations of the stochastic properties of a ‘test’ gas particle, including its asymptotic diffusive-like behavior and a comparison with the results of the Boltzmann equation approach. There was a recent surge of interest in this model in the context of the ‘adiabatic piston problem’ [9]–[13], including the description of the stochastic dynamics of the piston-particle [14] and the analytic calculation of the energy transfer and heat flux throughout an inhomogeneously prepared, out-of-equilibrium configuration of the gas [15]. Later on, the Jarzynski theorem [16] was illustrated for the case of uniform expansion or compression of the gas [17]. The Jepsen gas also turned out to be useful in the study of spin transport processes in the nonlinear $\sigma$ model [18,19].

In the context of biological evolution, a simplified version of Eigen’s quasispecies model [20], namely, the shell model [21]–[23] corresponds to the free expansion of a Jepsen gas of $N$ particles, where initially there is one particle at each position $-k$ for $k = 1, 2, \ldots, N$ with a positive velocity $U_k$ drawn independently from a position-dependent...
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Figure 1. (a) A realization of free trajectories for $N = 4$ particles. The trajectories are labeled according to the order of their starting positions from left to right, $Y_1 < Y_2 < Y_3 < Y_4$. The slopes $\{U_1, U_2, U_3, U_4\}$ of the trajectories with respect to the ‘TIME’ axis represent velocities associated with them. (b) Actual particle trajectories. After a ordering time the system reaches a ‘fan’ state where the velocities are completely ordered from left to right in an increasing fashion and there is no further collision. The total number of binary collisions $N_c = 3$.

probability density function (pdf) $\phi_k(U_k)$. A further simplification treating the velocities as independent and identically distributed (i.i.d.) random variables with a common $k$ independent pdf (i.e. $\phi_k \equiv \phi, \forall k$) leads to the i.i.d. shell model, for which several asymptotic properties can be computed exactly [24,25]. In particular, the statistical properties of the piston-particle (the rightmost particle or the leading genotype in the biological language) exhibit universal properties. Its velocity distribution function at intermediate times ($1 \ll t \ll N^\gamma$, where $\gamma$ is related to the tail of the velocity distribution) has universal scaling behavior of only three varieties, depending exclusively on the tail of $\phi(U)$ [25]. The associated scaling functions are different from the usual extreme-value distribution of uncorrelated random variables, and this difference is due to the dynamically built, collision-generated correlations between the particles at finite times. These correlations are also responsible for the fact that the statistics of the piston collisions is not Poissonian. Indeed, both the mean and the variance of the number of collisions the piston-particle undergoes increase logarithmically with time, but with different prefactors (that are also universal).

Despite this rather rich history of the Jepsen gas, there are other interesting natural questions to which detailed answers are missing. Consider, for example, the fact that the dynamics of the freely expanding Jepsen gas provides a ‘natural’ sorting algorithm for the velocities of the particles. It leads to an asymptotic ‘fan’ state (see figure 1) in which these velocities are completely ordered from left to right in an increasing fashion. Two questions arising naturally from this ordering of the Jepsen gas are

(A) What is the total number of collisions the particles undergo?
(B) How long does it take for the gas to reach the ‘fan’ state?

In this paper we provide analytical answers to these two questions.
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More precisely, we consider the free expansion of a Jepsen gas of \( N \) hard-point particles of equal mass on the infinite real line \((-\infty, \infty)\). The initial positions of the particles are drawn independently from a common pdf \( \psi(X) \) and a random velocity drawn independently from a common pdf \( \phi(U) \) is assigned to each particle. At subsequent times \((t > 0)\) each particle moves ballistically according to its assigned velocity, and upon contact between two particles they undergo elastic collision which merely interchanges their respective velocities. We assume both the set of positions and the set of velocities are continuous variables (or at least one set), so that the collisions are always binary, and there can be at most one binary collision at one instant of time. Clearly, in each collision the velocities of the colliding particles get ordered such that after the collision the particle on the right acquires the larger of the two velocities and the one on the left gets the smaller of the two velocities. Therefore, after a certain ordering time the system reaches a ‘fan’ state where the velocities of the particles are increasingly ordered from left to right. Once this ‘fan’ state is reached, evidently, there cannot be any further collision. For any given initial realization of positions and velocities, the dynamics of future evolution of the gas is completely deterministic. Therefore, both the total number of collisions (denoted by \( N_c \)) and the ordering time (denoted by \( T_o \)) are solely determined by the initial condition. Hence, \( N_c \) and \( T_o \) are random variables in the sense that they differ from one realization of initial condition to another. In this paper we analytically compute their distributions. Our main results are:

(A) The probability \( P_N(N_c) \) of having \( N_c \) collisions is completely independent of \( \psi(X) \) and \( \phi(U) \) for all \( N \geq 2 \), and for large \( N \) it approaches a Gaussian form around its mean \( \langle N_c \rangle = N(N-1)/4 \) with a variance \( \langle N_c^2 \rangle - \langle N_c \rangle^2 = N(N-1)(2N+5)/72 \).

(B) When the ordering time \( T_o \) is suitably scaled with \( N \) as \( \tau_o = T_o/\sqrt{bN^2} \)—where \( b \) is a nonuniversal scale factor which depends explicitly on \( \psi(Y) \) and \( \phi(U) \) as given by (16)—the limiting pdf of \( \tau_o \) in the scaling limit \( N \to \infty \), \( T_o \to \infty \) while keeping \( \tau_o \) fixed, becomes universal, i.e. completely independent of \( \psi(X) \) and \( \phi(U) \), and is given by the well-known Fréchet form \( f(\tau_o) = \tau_o^{-2} \exp(-1/\tau_o) \) that arises in the extreme-value statistics.

This paper is organized as follows. In section 2, we compute the statistics of the total number of collisions, and in section 3 we compute the limiting distribution of the ordering time. Section 4 contains some concluding remarks. Most of the calculational details are relegated to appendices A–C.

2. Total number of collisions

In order to express the total number of collisions \( N_c \) in terms of the initial condition, we label the particles as \( i = 1, 2, \ldots, N \) from left to right, i.e. \( i = 1 \) is the leftmost particle and \( i = N \) is the rightmost one (see figure 1). Let \( Y_i \) for each \( i = 1, 2, \ldots, N \) denote the initial position of the \( i \)th particle such that \( Y_1 < Y_2 < \cdots < Y_N \). Note that the labeled, ordered coordinates \( \{Y_i\} \)’s are no longer distributed independently according to \( \psi(Y) \); rather, their joint pdf is given by

\[
\psi_{\text{joint}}(Y_1, Y_2, \ldots, Y_N) = N! \prod_{i=1}^{N} \psi(Y_i) \prod_{j=1}^{N-1} \theta(Y_j - Y_{j+1}),
\]
where \( \theta(x) \) is the Heaviside step function. On the other hand, the initial velocities associated with the labeled particles, which we denote by \( U_i \) for \( i = 1, 2, \ldots, N \), are i.i.d. random variables drawn from the common pdf \( \phi(U) \).

Now, for a given initial configuration, the system is fully and uniquely described at all subsequent times by the set of the free trajectories \( \{Y_k + U_k t \mid k = 1, 2, \ldots, N \text{ and } t \geq 0\} \) (see figure 1(a)). Note that the \( k \)th free trajectory should not be confused with the actual trajectory of the \( k \)th particle. Indeed, each of the particles travels along such a free trajectory until it collides with another particle; in such a binary collision the particles interchange their trajectories (see figure 1(b)). In terms of the free trajectories, the total number of collisions \( N_c \) is just the total number of intersections among the \( N \) free trajectories. Two free trajectories can, of course, intersect at most once.

Consider first two extreme situations. If in the initial configuration the velocities are already in the increasing order, i.e. \( U_1 < U_2 < \cdots < U_N \) (which happens with probability \( 1/N! \)), then there cannot be any collision at subsequent times as the gas evolves. Thus, in this case, the system is always in the ‘fan’ state and \( N_c = 0 \). On the other hand, if the initial velocities are in the decreasing order, i.e. \( U_1 > U_2 > \cdots > U_N \) (which also happens with probability \( 1/N! \)), then each pair of free trajectories intersects once and there are \( \binom{N}{2} \) of them. Therefore, this configuration yields the maximum number of binary collisions, which is \( N_c = N(N - 1)/2 \). For any other realizations of the velocities, \( N_c \) lies between 0 and \( N(N - 1)/2 \). The total number of collisions \( N_c \) is a random variable, \( N_c \in [0, N(N - 1)/2] \), which differs from one realization of the initial configuration of the particle velocities to another, and whose statistical properties we address below.

Since \( Y_i < Y_j \) for \( i < j \), two free trajectories \( (Y_i + U_i t) \) and \( (Y_j + U_j t) \) intersect if and only if \( U_i > U_j \) for \( i < j \). Therefore, for a given realization of the initial velocities \( \{U_i\} \), the total number of collisions (total number of intersections among \( N \) free trajectories) can be expressed as

\[
N_c = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \theta(U_i - U_j). \tag{2}
\]

Obviously, \( N_c \) is independent of the set of initial positions \( \{Y_i\} \) of the particles and their distribution. Moreover, we will show below that \( P_N(N_c) \) is also completely independent of the velocity distribution \( \phi(U) \) and is solely determined by \( N \).

The probability \( P_N(N_c) \) of having \( N_c \) total number of collisions can be formally expressed as

\[
P_N(N_c) = \int \cdots \int \delta \left[ N_c - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \theta(U_i - U_j) \right] \prod_{i=1}^{N} \phi(U_i) \, dU_i, \tag{3}
\]

where \( \delta[n] \), with integer \( n \), is the discrete delta function: \( \delta[0] = 1 \) and \( \delta[n] = 0 \) for any \( n \neq 0 \). Let us consider the change of variables [26]

\[
u_i = \int_{-\infty}^{U_i} \phi(U) \, dU, \quad \text{i.e.,} \quad d\nu_i = \phi(U_i) \, dU_i \quad \text{for } i = 1, \ldots, N. \tag{4}
\]

Obviously \( \nu_i \) is a monotonically increasing function of \( U_i \), and therefore \( \theta(U_i - U_j) = \theta(\nu_i - \nu_j) \). Moreover, since \( \phi(U) \) is normalized to unity, the variables \( \nu_i \) vary from 0 to 1.
and so (3) becomes
\[
P_N(N_c) = \int_0^1 dy_1 \ldots \int_0^1 dy_N \delta \left[ N_c - \sum_{i=1}^{N-1} \sum_{j=i+1}^N \theta(u_i - u_j) \right],
\]
where the velocity distribution \( \phi(U) \) simply drops out, meaning that \( P_N(N_c) \) is universal, i.e. it is the same as if the ‘new’ velocities \( \{u_i\} \) are drawn independently from an uniform distribution over \([0, 1]\).

The mean number of total collisions \( \langle N_c \rangle \) is straightforward to compute. Using the change of variables in (4), it is trivially checked that \( \langle \theta(U_i - U_j) \rangle = 1/2 \). Therefore, taking the average in (2) yields
\[
\langle N_c \rangle = \sum_{i=1}^{N-1} \sum_{j=i+1}^N \langle \theta(U_i - U_j) \rangle = \frac{N(N - 1)}{4} \sim \frac{N^2}{4} \quad \text{as} \; N \to \infty.
\]

The calculation of the variance implies several steps that are indicated in appendix A. One obtains finally
\[
\sigma^2 = \langle N_c^2 \rangle - \langle N_c \rangle^2 = \frac{N(N - 1)(2N + 5)}{72} \sim \frac{N^3}{36} \quad \text{as} \; N \to \infty.
\]

As given by (2), \( N_c \) is a sum of \( N(N - 1)/2 \) random variables that are correlated. However, each term in (2) is correlated with only \( 2(N - 2) \) other terms, since two different terms are correlated only when they have one velocity in common. Therefore, near the mean \( \langle N_c \rangle \) and within a region \( O(\sigma) \), the probability distribution \( P_N(N_c) \) for large \( N \) has a Gaussian form (see appendix B)
\[
P_N(N_c) \approx \frac{1}{\sqrt{2\pi\sigma}} \exp \left( -\frac{[N_c - \langle N_c \rangle]^2}{2\sigma^2} \right).
\]

One can check numerically that this is actually already an extremely good approximation for \( N \) as small as 25 and for practically all the values of \( N_c \), as illustrated by figure 2.

3. Ordering time

In order to compute the statistics of the ordering time \( T_o \), it is convenient to relabel the particles according to the decreasing order of the velocities (see figure 3). Let \( \{V_1, V_2, \ldots, V_N\} \) denote the decreasingly ordered set of the initial velocities, i.e.
\[
\begin{align*}
V_1 & = \max(U_1, U_2, \ldots, U_N), \\
V_2 & = \max(\{U_1, U_2, \ldots, U_N\} \setminus \{V_1\}), \\
V_3 & = \max(\{U_1, U_2, \ldots, U_N\} \setminus \{V_1, V_2\}), \\
& \quad \vdots \\
V_N & = \min(U_1, U_2, \ldots, U_N),
\end{align*}
\]
so that \( V_1 > V_2 > \cdots > V_N \). The joint pdf of these ordered velocities is given by
\[
\phi_{\text{joint}}(V_1, V_2, \ldots, V_N) = \left[ N! \prod_{i=1}^N \phi(V_i) \right] \times \left[ \prod_{i=1}^{N-1} \theta(V_i - V_{i+1}) \right].
\]
Figure 2. Probability density of the total number of collisions (scaled by the mean and the variance) for $N = 25, 50$ and $100$ (○, ▽ and +, respectively, as symbols), as compared to the Gaussian $p(x) = \exp(-x^2/2)/\sqrt{2\pi}$ (drawn with ---). We scaled the numerically obtained data by the mean $\langle N_c \rangle$ and the variance $\sigma^2$ that are given as functions of $N$ by (6) and (7), respectively. It is difficult to notice any difference between the three datasets and the Gaussian.

Figure 3. The same realization of the free trajectories shown in figure 1. However, now the trajectories are labeled according to the decreasing order of the velocities $V_1 > V_2 > V_3 > V_4$ and the initial positions $\{X_1, X_2, X_3, X_4\}$ are indexed according to the order of the velocities. Note $V_1 = U_2, V_2 = U_4, V_3 = U_1, V_4 = U_3$ and $X_1 = Y_2, X_2 = Y_4, X_3 = Y_1, X_4 = Y_3$, while comparing with figure 1.

Let $X_i$ for each $i = 1, 2, \ldots, N$ denote the initial position of the particle having the initial velocity $V_i$. Note that, when we label the particles according to the order of their velocities, their initial positions $\{X_i\}$’s are no longer ordered on the line but are i.i.d. random variables drawn from the pdf $\psi(X)$. 

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As before, for a given initial condition, the system is fully and uniquely described at all subsequent times by the set of free trajectories \( \{X_i + V_i t \mid i = 1, 2, \ldots, N \text{ and } t \geq 0\} \), where \( V_1 > V_2 > \ldots > V_N \). The ‘fan’ state is reached when all the free trajectories become completely ordered according to the velocities, i.e. \( X_1 + V_1 t > X_2 + V_2 t > \cdots > X_N + V_N t \) at all \( t > T_o \) (see figure 3).

Let \( F_N(T) \) be the cumulative probability distribution of the ordering time, i.e. \( F_N(T) = \text{Prob}[T_o < T] \). It immediately follows that \( F_N(T) = \text{Prob}[X_1 + V_1 T > X_2 + V_2 T > \cdots > X_N + V_N T] \), given that \( V_1 > V_2 > \cdots > V_N \). This probability can be formally expressed as

\[
F_N(T) = \left\langle \prod_{i=1}^{N-1} \theta([X_i - X_{i+1}] + T[V_i - V_{i+1}]) \right\rangle,
\]

where \( \langle \cdots \rangle \) denotes the averaging over both the i.i.d. initial positions that are drawn from the common pdf \( \psi(x) \) and the initial ordered velocities, whose joint pdf is given by (9). Note that \( F_N(0) \) for any finite \( N \) is non-zero:

\[
F_N(0) = \left\langle \prod_{i=1}^{N-1} \theta(X_i - X_{i+1}) \right\rangle = \frac{1}{N!}
\]

and is universal. This represents the probability that the initial positions of the particles are also ordered according to their velocities, i.e. \( X_1 > X_2 > \cdots > X_N \), so that there cannot be any collision. The pdf of the ordering time is simply

\[
f_N(T_o) = \frac{\delta(T_o)}{N!} + \frac{dF_N(T_o)}{dT_o},
\]

where the extra weight at \( T_o = 0 \) vanishes in the limit \( N \to \infty \).

Let us introduce the variables \( x_i = X_{i+1} - X_i \) and \( v_i = V_i - V_{i+1} \) for brevity. In terms these variables we rewrite the expression (10) as

\[
F_N(T) = \left\langle \prod_{i=1}^{N-1} \left[ 1 - \theta(x_i) \theta\left(\frac{x_i}{T} - v_i\right) \right] \right\rangle,
\]

where \( v_i > 0 \) for \( i = 1, 2, \ldots, N - 1 \).

Since \( \{X_i\}'s \) are drawn independently from \( \psi(x) \), two different variables \( x_i \) and \( x_j \) with \( i \neq j \) are correlated iff \(|i - j| = 1 \). Therefore, for large \( N \), treating \( \{x_i\}'s \) as i.i.d. random variables, with a common pdf \( g(x) = \int_{-\infty}^{\infty} \psi(X) \psi(X + x) \, dX \), is a rather good approximation, which becomes exact in the limit \( N \to \infty \). On the other hand, in the variables \( v_i = V_i - V_{i+1} (i = 1, 2, \ldots, N - 1) \), the ordered velocities \( \{V_i\} \) are correlated, as can be seen from their joint pdf given by (9). Therefore, the random variables \( t_i = \theta(x_i)[x_i/v_i] \) with \( i = 1, 2, \ldots, N - 1 \), are not independent either. It turns out, however, that the limiting distribution of the ordering time \( T_o = \max(t_1, t_2, \ldots, t_{N-1}) \), when suitably scaled with \( N \), has the well-known Fréchet form (see figure 4) that arises in the extreme-value statistics of i.i.d. random variables drawn from a common parent distribution with a power-law tail. To show this, we formally expand the product in (13)
Figure 4. The pdf of the ordering time $T_o$, plotted using the scaled variable $\tau_o = T_o/[bN^2]$, where $b$ is given by (16). The points (○) were obtained by numerical simulation with $N = 1000$, averaging over $10^8$ realizations of initial configurations where particles were distributed uniformly and independently in the interval $[0,1]$, and the velocities were drawn independently from a Gaussian pdf $\phi(V) = \frac{1}{\sqrt{2\pi}} e^{-V^2/2}$. Equation (16) gives $b = 1/[12 \sqrt{\pi}]$. The solid line (——) represents the Fréchet pdf $f(\tau_o) = \tau_o^{-2} \exp(-1/\tau_o)$. The inset displays the same pdfs on a logarithmic scale.

F_N(T) = 1 - \sum_{i=1}^{N-1} \left< \theta(x_i) \theta \left( \frac{x_i}{T} - v_i \right) \right> 
+ \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \left< \theta(x_i) \theta(x_j) \theta \left( \frac{x_i}{T} - v_i \right) \theta \left( \frac{x_j}{T} - v_j \right) \right> - \cdots
+ (-1)^n \sum_{i_1=1}^{N-n} \sum_{i_2=i_1+1}^{N-n+1} \cdots \sum_{i_n=i_{n-1}+1}^{N-1} \left< \prod_{\nu=1}^{n} \theta(x_{i_\nu}) \theta \left( \frac{x_{i_\nu}}{T} - v_{i_\nu} \right) \right> 
+ \cdots
(14)

We show in appendix C that $T$ scales as $N^2$ when both $T$ and $N$ are large. Moreover, as detailed in appendix C, considering first $n \ll N$ and then taking the scaling limit $T \to \infty$ and $N \to \infty$, but keeping $T/N^2$ fixed, the $n$th term (counting from the ‘zero’-th term, which equals unity) in (14) becomes

$$
\lim_{N \to \infty} \sum_{i_1=1}^{N-n} \sum_{i_2=i_1+1}^{N-n+1} \cdots \sum_{i_n=i_{n-1}+1}^{N-1} \left< \prod_{\nu=1}^{n} \theta(x_{i_\nu}) \theta \left( \frac{x_{i_\nu}}{N^2 T} - v_{i_\nu} \right) \right> = \frac{1}{n!} \left( b \right)^n,
(15)
$$

where

$$
b = \left[ \int_0^\infty x g(x) \, dx \right] \times \left[ \int_{-\infty}^{\infty} \phi^2(V) \, dV \right],
(16)$$

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with, recall,
\[ g(x) = \int_{-\infty}^{\infty} \psi(X) \psi(X + x) \, dX. \]  
(17)

Thus, (14) yields
\[ \lim_{N \to \infty} F_N(b \, N^2 \, \tau) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \frac{1}{\tau} \right)^n = \exp \left( -\frac{1}{\tau} \right). \]  
(18)

The limiting pdf of the scaled ordering time \( \tau_o = T_o / [b \, N^2] \) — where the scale factor \( b \) is nonuniversal and is given explicitly by (16)—is therefore given by the universal function
\[ f(\tau_o) = \tau_o^{-2} \exp(-1/\tau_o). \]  

Figure 4 compares the results of the numerical simulation of the gas with this limiting pdf.

The Fréchet cumulative distribution (18) usually emerges as the cumulative distribution of the maximum of a set of \( N^2/2 \) i.i.d. random variables \( \{\tau_i\} \) each drawn from the common pdf having a power-law tail \( p(\tau) \sim 2b/\tau^2 \). In the context of the Jepsen gas, let us consider two free trajectories \((X_i + V_i t)\) and \((X_j + V_j t)\) chosen at random. The ordering time \( \tau_{ij} \) between these two free trajectories has the pdf
\[ p(\tau_{ij}) = \left\langle \delta \left( \tau_{ij} - \theta \left( \frac{X_j - X_i}{V_i - V_j} \right) \left[ \frac{X_j - X_i}{V_i - V_j} \right] \right) \right\rangle \sim \frac{2b}{\tau_{ij}^2} \quad \text{for large } \tau_{ij}, \]  
(19)

where \( b \) is given by (16). The ordering time \( T_o \) of the full system is clearly the maximum of the ordering times \( \{\tau_{ij}\} \) between each of the \( N(N - 1)/2 \) (\( \approx N^2/2 \) for large \( N \)) pairs of free trajectories. Therefore, it turns out simply that, in the \( N \to \infty \), the ordering times between different pairs of free trajectories become uncorrelated, and can be treated as i.i.d. random variables drawn from a common pdf having power-law tail \( p(\tau) \sim 2b/\tau^2 \).

4. Concluding remarks

In this paper we have computed the statistics of the total number of collisions of a freely expanding gas of \( N \) hard-point particles and found that the mean is \( O(N^2) \). On the other hand, the variance is \( O(N^3) \), due to the correlations between different particle collisions, without which the variance would also have been \( O(N^2) \). However, despite these correlations, the probability distribution of the total number of collisions near the mean is very well approximated by a Gaussian form.

In the context of biological evolution of quasispecies, the evolution time \( T_e \)—which is defined as the time at which the rightmost particle undergoes the last collision, i.e. the free trajectory with the largest slope (with respect to the time axis) becomes the rightmost trajectory—was studied recently \([21, 27]\). It was estimated that its pdf has a power-law tail \( p(T_e) \sim T_e^{-2} \). The ordering time studied here represents obviously the upper bound to this evolution time, \( T_e \leq T_o \). We have found that the limiting distribution of the ordering time, when suitably scaled with \( N \), becomes universal and is given by the Fréchet form. This Fréchet form usually appears as the limiting distribution of the maximum of a set of i.i.d. random variables drawn from a common distribution with a power-law tail. Our result here provides another mechanism of generating Fréchet form as a limiting distribution.

Finally, while we have studied the statistics of the total number of collisions, it would be interesting to study the number of collisions of the particles \( N_c(t) \) up to time \( t \). Our
results correspond to the limit $t \to \infty$. In this context we point out that recently the collision statistics of a tagged particle in a $d$-dimensional hard-sphere gas at equilibrium was investigated using the Boltzmann equation approach [28] and was found to be non-Poissonian. It would be interesting to verify this conclusion for a tagged particle in the 1d Jepsen gas for which an exact result may be possible to obtain.

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Appendix A. Variance of the total number of collisions

Subtracting the mean given by (6) from (2), then taking the square and the average gives

$$\sigma^2 = \langle (N_c - \langle N_c \rangle)^2 \rangle = \frac{1}{4} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{k=1}^{N-1} \sum_{l=k+1}^{N} \langle s_{ij} s_{kl} \rangle,$$

(A.1)

where

$$s_{ij} = 2 \theta(U_i - U_j) - 1 = \pm 1.$$  

(A.2)

Note that $\langle s_{ij} \rangle = 0$, $\langle s_{ij}^2 \rangle = 1$ and $\langle s_{ij} s_{kl} \rangle = 4 \theta(U_i - U_j) \theta(U_k - U_l) - 1$.

The total number of terms in the summation in (A.1) is $[N(N - 1)/2]^2$. These can be grouped according to the correlation functions $\langle s_{ij} s_{kl} \rangle$'s that are of similar kind. This can be conveniently represented by diagrams as shown in figure A.1. Then using the change of variables (4), it is easy to compute the correlations $\langle \theta(U_i - U_j) \theta(U_k - U_l) \rangle$ and hence $\langle s_{ij} s_{kl} \rangle$, corresponding to each of diagrams (a)–(f) in figure A.1.

Diagram (a): $\langle s_{ij} s_{kl} \rangle \equiv \langle s_{ij}^2 \rangle = 1$.

Diagram (b): $\langle s_{ij} s_{kl} \rangle \equiv \langle s_{ij} s_{il} \rangle = 1/3$.

Diagram (c): Same as for diagram (b).

Diagram (d): $\langle s_{ij} s_{kl} \rangle = \langle s_{ij} s_{ki} \rangle = -1/3$.

Diagram (e): $\langle s_{ij} s_{kl} \rangle = \langle s_{ij} s_{ki} \rangle = 1/3$.

Diagram (f): $\langle s_{ij} s_{kl} \rangle = \langle s_{ij} s_{ki} \rangle = -1/3$.

The number of such terms in (A.1) is just the number of ways of choosing three indices $\{k, i, l\}$ out of $N$, which is

$$\binom{N}{3} = \frac{N(N-1)(N-2)}{6}.$$
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**Figure A.1.** Diagrammatic representations of the different types of terms appearing in (A.1) for the computation of $\langle N_c^2 \rangle$. In all the diagrams $j > i$ and $l > k$. The solid lines connect two indices having equal values. One has: (a) $i = k$ and $j = l$; (b) $i = k$ and $j \neq l$; (c) $i \neq k$ and $j = l$; (d) $i = l$; (e) $j = k$; (f) $i \neq k$ and $j \neq l$.

**Diagram (e):** Same as for diagram (d).

**Diagram (f):** $\langle s_{ij}s_{kl} \rangle = \langle s_{ij}\rangle \langle s_{kl}\rangle = 0$. These terms do not contribute to the sum in (A.1).

Therefore, finally (A.1) yields

$$\sigma^2 = \frac{1}{4} \left[ 1 \cdot \binom{N}{2} + 2 \cdot \frac{1}{3} \cdot \binom{N}{3} \right] = \frac{N(N-1)(2N+5)}{72}.$$  \hspace{1cm} (A.3)

**Appendix B. Probability distribution of the total number of collisions**

Let us consider the deviation of the total number of collisions given by (2) from its mean given by (6):

$$M = N_c - \langle N_c \rangle = \frac{1}{2} \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \text{sgn}(U_i - U_j) = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} S_{i,j},$$  \hspace{1cm} (B.1)

where $\text{sgn}(x) = 2\theta(x) - 1$, and $S_{i,j} \equiv (1/2) \text{sgn}(U_i - U_j)$. Obviously $\langle M \rangle = 0$ and we have already calculated the second moment in appendix A, which is

$$\langle M^2 \rangle \equiv \sigma^2 = \frac{N(N-1)(2N+5)}{72} \approx \frac{N^3}{36} \quad \text{for large } N.$$  \hspace{1cm} (B.2)

Assuming the pdf of the velocities to be symmetric, i.e. $\phi(-U) = \phi(U)$, and noting that $\text{sgn}(-x) = -\text{sgn}(x)$, it can be easily shown that the probability distribution of $M$ is symmetric about zero. On the other hand, we have shown in section 2 that the probability distribution of $N_c$, and hence that of $M$, is independent of the velocity distribution. Therefore, the distribution of $M$ must be symmetric for any velocity distribution. This implies the vanishing of all the odd moments, i.e.

$$\langle M^{2n+1} \rangle = 0, \quad \text{for } n = 0, 1, 2, \ldots$$  \hspace{1cm} (B.3)

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By the same argument, one can also show that the average over any product of odd number of $S_{(\ldots)}$’s is zero. $\langle M^{2n+1} \rangle$ is just a sum of such terms.

The even moments are given by

$$\langle M^{2n} \rangle = \sum_{i_1=1}^{N-1} \sum_{j_1=i_1+1}^{N} \cdots \sum_{i_{2n}=1}^{N-1} \sum_{j_{2n}=i_{2n}+1}^{N} \prod_{\nu=1}^{2n} S_{i_{\nu},j_{\nu}},$$  \hspace{1cm} (B.4)

for $n = 1, 2, 3, \ldots$. We first break the average as the product of averages over $n$ different factors of a pair of $S_{(\ldots)}$’s, and then sum over the indices independently under each average of factors, each of which gives the second moment $\langle M^2 \rangle \equiv \sigma^2$ making it $\sim \sigma^{2n}$. This is $O(N^{3n})$. The corrections to this come from the terms where the sets of indices between different factors are not independent. If there is one common index between two different factors, we will have to take the average of them together. It will reduces one summation and therefore is $O(N^{3n-1})$. The number of ways of grouping $2n$ objects into $n$ pairs is clearly $(2n)!/n!2^n$. Therefore

$$\langle M^{2n} \rangle = \frac{(2n)!}{n!2^n} \sigma^{2n} + O(N^{3n-1}), \quad \text{for } n = 2, 3, \ldots \hspace{1cm} (B.5)$$

Let us now consider the scaled variable $z = \lim_{N \to \infty} \frac{M}{\sigma}$. (B.6)

Since $\sigma^{2n}$ is $O(N^{3n})$, we have

$$\langle z^{2n} \rangle = \frac{(2n)!}{n!2^n} \quad \text{and} \quad \langle z^{2n+1} \rangle = 0, \quad \text{for } n = 0, 1, 2, 3, \ldots. \hspace{1cm} (B.7)$$

The characteristic function of the probability density of $z$ is then

$$\langle e^{i\lambda z} \rangle = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \lambda^{2n} \langle z^{2n} \rangle$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \left( \frac{\lambda^2}{2} \right)^n = \exp \left( -\frac{\lambda^2}{2} \right). \hspace{1cm} (B.8)$$

The inversion of this Fourier transform gives the pdf of $z$ as

$$p(z) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{z^2}{2} \right), \hspace{1cm} (B.9)$$

which says that the scaled variable $[N_c - \langle N_c \rangle]/\sigma$ has a Gaussian distribution.

**Appendix C. Evaluation of the terms in the series expansion of $F_N(T)$**

**First term:**

Let us compute the average $\langle \theta(x_i) \theta(x_i/T-v_i) \rangle$, in the first term (counting from the ‘zero’-th term, which equals unity) in (14). The pdf of $v_i$ is given by

$$\rho_i(v) = \theta(v) \frac{N!}{(N-i-1)!(i-1)!} \times \int_{-\infty}^{\infty} dV \phi(V) [1 - \Phi(V)]^{N-i-1} \phi(V+v) \Phi(V+v)^{i-1}, \hspace{1cm} (C.1)$$

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in which
\[ \Phi(w) = \int_w^\infty \phi(V) \, dV; \quad \text{i.e.,} \quad d\Phi = -\phi(V) \, dV. \] (C.2)

The integrand in (C.1) merely specifies that \((N - i - 1)\) velocities are smaller than \(V\), \((i - 1)\) velocities are larger than \((V + v)\), and two velocities are \(V\) and \((V + v)\), respectively, such that their difference is \(v\). The combinatorial prefactor gives the number of such arrangements and we finally integrate the integrand over all possible values of \(V\). It is checked that this pdf is normalized, \(\int_0^\infty \rho_i(v) \, dv = 1\). Using (C.1) we first take the average over \(v_i\), which for all \(i = 1, 2, \ldots, N - 1\) gives
\[ \theta(x_i) \left< \theta \left( \frac{x_i}{T} - v_i \right) \right>_{v_i} = \frac{\theta(x_i) \, N!}{(N - i - 1)! \, (i - 1)!} \int_{-\infty}^\infty dV \, \phi(V) \left[ 1 - \Phi(V) \right]^{N-i-1} \]
\[ \times \frac{1}{i} \left[ \Phi(V) \right]^i - \left[ \Phi(V + x_i/T)i \right]. \] (C.3)

When \(T \gg N\)
\[ \frac{1}{i} \left[ \Phi(V) \right]^i - \left[ \Phi(V + x_i/T)i \right] = \frac{x_i}{T} \left[ \Phi(V) \right]^{i-1} \phi(V) + O \left( \frac{i x_i^2}{T^2} \right). \] (C.4)

Note that the pdf of \(x_i\) as given by (17) is independent of the index \(i\). Substituting (C.4) in (C.3), then taking the average over \(x_i\), and finally summing over \(i\) yields
\[ \sum_{i=1}^{N-1} \left< \theta(x_i) \theta \left( \frac{x_i}{T} - v_i \right) \right> = b \frac{N(N-1)}{T} + O \left( \frac{N^3}{T^2} \right), \] (C.5)
with \(b\) given by (16).

Equation (C.5) suggests that \(T\) scales as \(N^2\), which indicates that the natural scaling limit corresponds to \(T \to \infty\) and \(N \to \infty\), while keeping \(T/N^2\) fixed. In this limit the term (C.5) becomes
\[ \lim_{N \to \infty} \sum_{i=1}^{N-1} \left< \theta(x_i) \theta \left( \frac{x_i}{N^2 \tau'} - v_i \right) \right> = \frac{b}{\tau'}. \] (C.6)

Second term:
Let us now consider the second term in (14) involving summation over \(i\) and \(j\). We substitute \(T = N^2 \tau'\), and rewrite it as
\[ \sum_{i=1}^{N-1} \sum_{j=i+2}^{N-1} \left< \theta(x_i) \theta(x_j) \theta \left( \frac{x_i}{N^2 \tau'} - v_i \right) \theta \left( \frac{x_j}{N^2 \tau'} - v_j \right) \right> \] (C.7)
\[ + \sum_{i=1}^{N-2} \left< \theta(x_i) \theta(x_{i+1}) \theta \left( \frac{x_i}{N^2 \tau'} - v_i \right) \theta \left( \frac{x_{i+1}}{N^2 \tau'} - v_{i+1} \right) \right>. \] (C.8)

Now in the first term (C.7), the variables \(x_i\) and \(x_j\) are uncorrelated, and each of them has the same pdf (17). Following the similar reasoning that we used to write down the pdf of the single variable \(V_i\) in (C.1), we can also write down the joint pdf of \(v_i\) and \(v_j\) for
such that (C.12) contains only the terms with $\rho$ can be written as

$$
\rho_{i,j}(v_i, v_j) = \theta(v_i) \theta(v_j) \frac{N!}{(N - j - 1)!(j - i - 2)!(i - 1)!} \prod_{\nu=1}^{N} \Phi(\nu)
$$

&

$$
\times \int_{-\infty}^{\infty} dV \phi(V) \int_{-\infty}^{\infty} dV' \phi(V') \theta(V - V') \left[ 1 - \Phi(V') \right]^{N-j-1}
$$

&

$$
\cdot \theta(V - V' - v_j) \phi(V + v_i) \phi(V' + v_j)
$$

&

$$
\cdot \left[ \Phi(V' + v_j) - \Phi(V) \right]^{j-i-2} \left[ \Phi(V + v_i) \right]^{i-1},
$$

(C.9)

where $\Phi(w)$ is given by (C.2). It is checked that the joint pdf is normalized to unity, i.e.

$$
\int_{0}^{\infty} dv_i \int_{0}^{\infty} dv_j \rho_{i,j}(v_i, v_j) = 1.
$$

Now, using (C.9) we first compute the average over $v_i$ and $v_j$ in (C.7), then we expand the result in Taylor series (assuming large $N$), and then average over $x_i$ and $x_j$. Finally, we sum over $i$ and $j$ and take the limit $N \to \infty$. Noting that

$$
\int_{-\infty}^{\infty} dV \phi^2(V) \int_{-\infty}^{\infty} dV' \phi^2(V') \theta(V - V') = \frac{1}{21} \left[ \int_{-\infty}^{\infty} \phi^2(V) dV \right]^2,
$$

we find

$$
\lim_{N \to \infty} \sum_{i=1}^{N-3} \sum_{j=i+2}^{N-1} \left\langle \theta(x_i) \theta(x_j) \theta \left( \frac{x_i}{N^2 \tau} - v_i \right) \theta \left( \frac{x_j}{N^2 \tau} - v_j \right) \right\rangle = \frac{1}{21} \left( \frac{b}{\tau} \right)^2,
$$

where $b$ is given by (16). The term (C.8) goes to zero in the limit $N \to \infty$.

The $n$th term:

Following exactly the same steps, we can evaluate the $n$th term of the expansion of $F_N(T)$ in the expression (14). We first assume that $n \ll N$, and later take the limit $N \to \infty$. We again write the sum as

$$
\sum_{i_1=1}^{N-2n+1} \sum_{i_2=i_1+2}^{N-2n+3} \cdots \sum_{i_n=i_{n-1}+2}^{N-1} \left\langle \prod_{\nu=1}^{n} \left[ \theta(x_{i_{\nu}}) \theta \left( \frac{x_{i_{\nu}}}{N^2 \tau} - v_{i_{\nu}} \right) \right] \right\rangle
$$

&

$$
+ \text{[remaining terms]},
$$

(C.13)

such that (C.12) contains only the terms with $i_k \geq i_{k-1} + 2$ for all $k = 2, \ldots, n$. As before, the variables $\{x_{i_{\nu}}, \nu = 1, 2, \ldots, n\}$ in (C.12) are uncorrelated, and the joint pdf of $\{v_{i_{\nu}}\}$ can be written as

$$
\rho_{i_1,i_2,\ldots,i_n}(v_{i_1}, v_{i_2}, \ldots, v_{i_n}) = \left[ \prod_{\nu=1}^{n} \theta(v_{i_{\nu}}) \right] C_N(i_1, i_2, \ldots, i_n)
$$

$$
\times \int_{-\infty}^{\infty} dV_{(1)} \phi(V_{(1)}) \int_{-\infty}^{\infty} dV_{(2)} \phi(V_{(2)}) \cdots \int_{-\infty}^{\infty} dV_{(n)} \phi(V_{(n)})
$$

$$
\cdot \left[ \prod_{k=1}^{n-1} \theta \left( V_{(k)} - V_{(k+1)} \right) \right] \cdot \left[ 1 - \Phi \left( V_{(n)} \right) \right]^{N-i_{n}-1}
$$

$$
\cdot \left[ \prod_{k=2}^{n} \theta \left( V_{(k-1)} - V_{(k)} - v_{i_{k}} \right) \right] \cdot \left[ \prod_{k=1}^{n} \phi(V_{(k)} + v_{i_{k}}) \right]
$$

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\[ 
\frac{1}{N!} \int_0^\infty x g(x) \, dx 
\]

(C.17)

\[ 
\times \int_{-\infty}^\infty \cdots \int_{-\infty}^\infty \left[ \prod_{k=1}^n dV_k \phi^2(V_k) \right] \cdot \left[ \prod_{k=1}^{n-1} \theta(V_k - V_{k+1}) \right]. 
\]

(C.18)

Now, the term inside the first square brackets of the integrand in (C.18) remains unchanged under permutations of the variables \( \{V_k\} \). While the term inside the second square brackets of the integrand changes under permutations, summing over all possible permutations yields unity. However, note that \( \{V_k\} \)’s are just dummy variables in the multiple integral (C.18), so it must remain unchanged under any of the \( n! \) permutations of these variables. Therefore, the integral (C.18) equals

\[ 
\frac{1}{n!} \int_{-\infty}^\infty \cdots \int_{-\infty}^\infty \left[ \prod_{k=1}^n dV_k \phi^2(V_k) \right] = \frac{1}{n!} \int_{-\infty}^\infty \phi^2(V) \, dV
\]

(C.19)

Therefore, in the limit \( N \to \infty \) the expression (C.12) becomes (see (C.17) and (C.18))

\[ 
\lim_{N \to \infty} \sum_{i_1=1}^{N-2n+1} \sum_{i_2=i_1+2}^{N-2n+3} \cdots \sum_{i_n=i_{n-1}+2}^{N-1} \left\langle \prod_{\nu=1}^n \theta(x_{i_\nu}) \theta \left( \frac{x_{i_\nu}}{N^2 \tau^j} - v_{i_\nu} \right) \right\rangle = \frac{1}{n!} \left( \frac{b}{\tau^j} \right)^n,
\]

(C.20)

where \( b \) is given by (16). The remaining term (C.13) goes to zero in the limit \( N \to \infty \).

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