Oscillating Quintom and the Recurrent Universe

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Current observations seem to mildly favor an evolving dark energy with the equation of state getting across -1. This form of dark energy, dubbed Quintom, is studied phenomenologically in this paper with an oscillating equation of state. We find oscillating Quintom can unify the early inflation and current acceleration of the universe, leading to the oscillations of the Hubble constant and a recurring universe. Our oscillating Quintom would not lead to a big crunch nor big rip. The scale factor keeps increasing from one period to another and leads naturally to a highly flat universe. The universe in this model recurs itself and we are only staying among one of the epochs, in which sense the coincidence problem is reconciled.

PACS numbers: 98.80.Cq

In 1998, two groups \textsuperscript{1,2} independently showed the accelerating expansion of our universe, which established the existence of dark energy where the equation of state is less than $-1/3$ today. The simplest form of dark energy is the cosmological constant. A cosmological constant which leads to current acceleration would encounter many theoretical problems, such as the fine-tuning problem and the coincidence problem\textsuperscript{3}. A light scalar field with a canonic kinetic term can only have potential incompatibility with the string theory, meanwhile quintessence\textsuperscript{4-6} which evolves with time is to some extent likely to resolve the coincidence problem. The model of phantom\textsuperscript{7} has also been put forward which leads to an equation of state $w \leq -1$. Normally quintessence with a canonic kinetic term can only have $w \geq -1$, meanwhile k-essence\textsuperscript{8} can have both $w \geq 1$ and $w < -1$. For the cosmological constant and many quintessence models the event horizon would lead to a potential incompatibility with the string theory, meanwhile for models where $w < -1$ one would get the Big Rip\textsuperscript{9} of the universe.

The accumulation of the current observational data has opened a robust window for probing the recent behavior of dark energy. Measurements from type Ia Supernova (SNe Ia), the Cosmic Microwave Background (CMB) Radiation, Large Scale Structure (LSS), weak lensing, clusters and galaxies all contain the imprints of dark energy. Specifically the recently released first year Wilkinson Microwave Anisotropy Probe (WMAP) measurement\textsuperscript{10}, the Sloan Digital Sky Survey (SDSS) measurement\textsuperscript{11} of the three-dimensional power spectrum\textsuperscript{11} and most importantly, the recent discovery of 16 SNe Ia\textsuperscript{12} with the Hubble Space Telescope during the GOODS ACS Treasury survey, together with former SNe Ia data have provided the most precise up-to-date measurements of dark energy. Many other sources have also been studied to make crosschecks and reveal new physics on the dark energy and the concordance cosmological model. Recently many authors in the literature have fitted the behavior of dark energy using various parameterizations. The best fit value of the equation of state is found to be less than $-1$ when using SNe Ia\textsuperscript{12, 13, 14, 15, 16, 17} or the X-ray mass fraction data\textsuperscript{18} with a cosmological constant still well within the central regions\textsuperscript{1}. It has also been pointed out by many authors that an evolving dark energy is indeed more favored than that with a constant equation of state. With the simplest linear parametrization of the equation of state $w$:

$$w(z) = w_0 + w'z,$$  \hspace{1cm} (1)

SNe Ia alone favors a $w$ larger than $-1$ in the recent past and less than $-1$ today, disregard using the prior of a flat universe\textsuperscript{12, 13, 14, 15, 16} or not\textsuperscript{17}. When other "longer-armed" observations, e.g. CMB and LSS data, are also taken into account, one often takes the risk of poor parametrizations. Taking the linear parametrization as a concrete example, it gives a good approximation for the late behavior of the dark energy. However, at large redshift the amplitude of $w$ increases too much and hence CMB and LSS data would restrict $w'$ to be near zero\textsuperscript{12, 20}, spoiling the recent behavior of dark energy. Such a correlation also happens for other parametrizations. The most proper way of measuring the recent behavior of the dark energy is the principal component analysis\textsuperscript{21}. Huterer and Cooray\textsuperscript{14} based on this method and developed a simple uncorrelated and nearly model-independent band power estimates of $w$ and its density as a function of the redshift. By fitting to the recent SNe Ia data they found a marginal (2-$\sigma$) evidence for $w(z) < -1$ at $z < 0.2$. Even though SNe Ia data are still currently limited, such a method is potentially very powerful. The parametrization of the dark energy remains a paradox today, especially when confronted with observations\textsuperscript{22}. However, in any case if a time-varying dark energy model significantly reduces the $\chi^2$ value and is statistically significant, it gives strong implications for dark energy metamorphosis. In Refs.\textsuperscript{23, 24} the authors use their physical parametrizations of the dark energy and fit the models to the fully CMB, LSS and SNe Ia data

\hspace{3cm} 1 We get the similar results when using the recent data of radio galaxies\textsuperscript{12} as well as the gold samples of SNe Ia data\textsuperscript{12}.

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set, they also find that an evolving dark energy is favored indeed with \( w(z) < -1 \) today and \( w(z) > -1 \) in the recent past. However, one would have to introduce several additional parameters to give a physical interpretation for the behavior of the dark energy, resulting into a very low statistical significance of an evolving dark energy. Most of the works in the literature have not taken into account the possibly non-negligible contribution of weak lensing\(^{20, 21}\) to SNe Ia. When the flux averaging method \(^{20, 21}\) has been applied the preference of a nonzero \( w' \) is much reduced. In Ref. \(^{15}\) we have also applied the linear parametrization to the X-ray mass fraction data only and find that a negative \( w' \) is favored.

If the requirement of an evolving dark energy with \( w \) getting across \(-1\) still holds on with the accumulation of observational data, this would be a great challenge to the current cosmology. In the quintessence model, the equation of state \( w = p/\rho \) is always in the range \(-1 \leq w \leq 1 \) for \( V(Q) > 0 \). Meanwhile for the phantom which has the opposite sign of the kinetic term compared with the quintessence in the Lagrangian, one always has \( w \leq -1 \). Neither the quintessence nor the phantom alone can fulfill the transition from \( w > -1 \) to \( w < -1 \) and vice versa. Although for k-essence\(^8\) one can have both \( w > -1 \) and \( w < -1 \), it has been lately considered by Ref.\(^{20}\) that it is very difficult for k-essence to get across \(-1 \) during evolving. In Ref.\(^{17}\) we proposed a new scenario of dark energy dubbed Quintom. Quintom describes those forms of dark energy which can get \( w \) cross \(-1 \) during the time evolution.

When dark energy is not merely the cosmological constant it gives rise to many interesting consequences. There are possibly some connections between the early inflation and the current acceleration of our universe. Peebles and Vilenkin have put forward a model of quintessential inflation to unify the two epochs\(^{20}\). Dodelson, Kaplinghat and Stewart gave a simple form of oscillating quintessence which can provide a natural solution to the coincidence problem\(^{21}\). In this paper we propose a phenomenological model of oscillating Quintom and study its implications in cosmology. We will show that in our model two accelerating epochs are related and the coincidence problem can be alleviated.

The phenomenological Quintom model we consider in this paper takes the following equation of state,

\[
w(\ln a) = w_0 + w_1 \cos[A(\ln(a/a_c))],
\]

where \( a \) is the scale factor, \( w_0, w_1, A \) and \( a_c \) are parameters which we will take specific values for the detailed study below. If for a sufficiently large period the equation of state remains unchanged during the late time evolving it would fit all current data well for \( w \lesssim -1 \). Defining \( X(\ln a) = \rho_X(\ln a)/\rho_X(0) \) where the subscript \( X \) stands for Quintom and \('0' \) for today, one gets:

\[
X(\ln a) = a^{-3(1+w_0)} \exp\left\{ \frac{-3w_1}{A} \left[ \sin(A \ln \frac{a}{a_c}) + \sin(A \ln a_c) \right] \right\}.
\]

For a numerical discussion we fix the model parameters and consider a specific \( w \) given below

\[
w(\ln a) = -1 - 1.5 \cos(0.032 \ln a - \frac{4\pi}{9}).
\]

In this model the \( w \) at present time is \( w = -1.26 \) (we have checked that if tuning \( a_c \) to get \( w \) more closer to \(-1 \) the picture on the evolution of the universe will not be changed from our current choice of the parameters), which is well within the limit of current observations. In Fig.1 we show the evolution of the universe filled with only the matter of the oscillating Quintom. As we’ve fixed \( w_0 = -1 \), one can see from Eq.\(^8\) that the energy density, hence the Hubble parameter \( H \) simply oscillates with a long period. Since the time is a simple integration of \( d\ln a/H \) and \( X(\ln a) \) evolves periodically with \( \ln a \), time increases equally for every span of \( \ln a \). One can also easily find from Fig.1 that as \( X \) is an integration of \( w(\ln a) \) they differ in the phase of oscillation. Since the Hubble parameter increases gradually during phantom-dominated phase of dark energy, this will make the universe evolve into a regime with high energy again\(^2\), which may be regarded as the recurrence of an early phantom inflation\(^{31}\) of our universe, (see \(^{32}\) for a earlier study) where the initial perturbation in the horizon will exit the horizon, and reenter the horizon after the transition to a late-time expanding phase with the ordinary matter contents. Thus our model unifies the inflation and dark energy into a single process, in which the universe evolves periodically. While we live in one period, the present acceleration with \( w < -1 \) is just a start of the phantom inflation for the next period of universe. During this process, all ordinary forms of matter and radiation are diluted by inflation. But in the next period new structures will be formed again by the fluctuations generated during the present acceleration, i.e. phantom inflation\(^{31}\) (see \(^{33, 34}\) for a detailed investigation of primordial perturbations from phantom phase). The universe can also be reheated with help of the reheating mechanisms such as instant preheating\(^{35}\) and curvaton reheating\(^{36}\).

We show now that our model works the same as described above when including matter and radiation. Assuming \( \Omega_m = 0.3 \) and \( \Omega_X = 0.7 \) today with the Hubble parameter \( h = 0.7 \) (this determines the abundance of radiation) and a negligible curvature, we show in Fig.2 the evolution of the universe for 1-period. Here we assume that the radiation starts to dominate our universe promptly at the moment when \( \ln(a/a_0) = -60 \) ( \( a_0 \) is set to be unity today ). From Fig.2 one can see that firstly the universe is just like today with \( \Omega_X = 0.7 \) and \( w = -1.26 \), with time \( w \) evolves downward, \( \rho_X \) increases and \( \rho_m \) decreases, then the universe begins inflating. When \( \ln a \sim -150 \), \( w \) starts to increase. And
at $\ln a \approx -90$ we have $w \sim -\frac{1}{3}$, inflation stops and the energy density continues dropping. At $\ln a = -60$ the universe enters radiation domination epoch. $\rho_X$ starts to increase recently and begins to dominate the universe with $\Omega_X = 0.7$ today. The universe starts the inflation since today and is recurring into the next period.

We should point out that for our phenomenological parametrization in Eq. (2) and Eq. (4) to realize our recurrent picture of the universe, the value of $w_0$ has to be extremely close to -1, otherwise the next and previous "cycles" would be different from the current one and we would also be encountered with too high or low inflation energy scales, as can be clearly seen in Eq. (3). While for the other several parameters less fine tunings are needed: the period is determined by $A$ and the variation of the energy density is by $w_1$ and $A$, and finally, the value of $a_c$ somewhat determines the present value of $w$. To realize the recurrence picture one requires that inflationary epoch is satisfactory and hence we have $2\pi/A \sim 100$, on the other hand we need $H_{inf}/M_{Pl} < 10^{-5}$ to satisfy the primordial gravitational wave constraints, these will put considerable constraints on $w_1$. For our dynamical model of dark energy as the period of oscillation is required to be quite large, the crossing of the cosmological boundary appears at some high redshifts where dark energy is typically negligible compared with the background matter components, where such a crossing behavior is hard to be detected by the observations. Nevertheless such a crossing is crucial to realize our picture of oscillating quintom. The dynamics of dark energy is not well imprinted on the redshift range covered by current SNe Ia surveys, where a running of the equation of state $dw/dz$

In Fig. 3 we show the evolution of the universe with multi periods. For each period we assume that the radiation starts domination around the same time. As radiation and Quintom domination epoch contribute little to time compared with when matter is non-negligible, one can see from Fig. 3 that time only increases equally around every short epoch of matter domination.
would be typically of order 0.01, this is different from the model of Quintom investigated earlier where a significant dynamical dark energy behavior displays in a very low and short redshift range. For the specified example in Eq. 4, the running of $w$ is $\sim 0.02$, which is difficult to be excluded by future observations like SNAP [31] or Dark Energy Survey (DES) [43]. On the other hand, if a significant dynamical behavior of dark energy were verified by future observations, this would be a strong argument to rule out our model. Again we should point out that Eq. 4 is one of the typical examples where the current equation of state is quite adjustable which does not affect our picture. To satisfy the current observations we need the equation of state close to $-1$ today, which in turn requires in Eq. 2 the term $w_1 \cos[\pm a c]$ to be close to zero.

During the phantom-dominated phase, the energy density increases as well as the Hubble parameter. If there is not a transition, the exit to observable universe is hardly possible, which is also a problem puzzling phantom inflation [3]. But in our model it is avoided by the oscillation of $w$. In some sense, our model is similar to the Pre Big Bang scenario [42] (see [43] for a review) and the recently proposed cyclic model [44] where the primordial perturbations are seeded by the contraction phase with a close to zero.

The exit may be implemented by using growing wormholes and ringholes [45], which can be regarded as a connection between the phantom phase and late-time expanding phase, see [46] for a further discussion.

Acknowledgements: We thank the anonymous referee for helpful comments and suggestions. This work is supported in part by National Natural Science Foundation of China under Grant Nos. 10405029, 90303004 and 19925523 and by Ministry of Science and Technology of China under Grant No. NKBRSF G19990754.

[1] A. G. Riess et al., Astron. J. 116, 1009 (1998).
[2] S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[3] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[4] B. Ratra and P. J. E. Peebles, Phys. Rev. D 37, 3406 (1988); P. J. E. Peebles and B. Ratra, Astrophys. J. 325, L17 (1988).
[5] R. D. Peccei, J. Sola and C. Wetterich, Phys. Lett. B 195, 183 (1987); C. Wetterich, Nucl. Phys. B 302, 668 (1988); C. Wetterich, Astron. Astrophys. 301, 321 (1995).
[6] P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D 59, 123504 (1999).
[7] R. R. Caldwell, Phys. Lett. B 545, 23 (2002).
[8] C. Armendariz-Picon, V. Mukhanov and P. J. Steinhardt, Phys. Rev. Lett. 85, 4438 (2000); Phys. Rev. D 63, 103510 (2001); T. Chiba, T. Okabe and M. Yamaguchi, Phys. Rev. D 62 (2000) 023511.
[9] R. R. Caldwell, M. Kamionkowski and N. N. Weinberg, Phys. Rev. Lett. 91, 071301 (2003).
[10] D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003).
[11] M. Tegmark et al., Phys. Rev. D69, 103501 (2004).
[12] A. G. Riess et al. [Supernova Search Team Collaboration], Astrophys. J. 607, 665 (2004).
[13] U. Alam, V. Sahni, and A. A. Starobinsky, JCAP 06, 08 (2004).
[14] D. Huterer and A. Cooray, Phys. Rev. D 71, 023506 (2005).
[15] B. Feng, X. Wang and X. Zhang, Phys. Lett. B 607, 35 (2005).
[16] T. R. Choudhury and T. Padmanabhan, Astron. Astrophys. 429, 807 (2005).
[17] D. A. Dieus and W. W. Repko, Phys. Rev. D 70, 083527 (2004).
[18] S. W. Allen, R. W. Schmidt, H. Ebeling, A. C. Fabian, and L. van Speybroeck, Mon. Not. Roy. Astron. Soc. 353, 457 (2004).
[19] R. A. Daly and S. G. Djorgovski, Astrophys. J. 612, 652 (2004).
[20] Y. Wang and M. Tegmark, Phys. Rev. Lett. 92, 241302 (2004).
[21] D. Huterer and G. Starkman, Phys. Rev. Lett. 90, 031301 (2003).
[22] E. V. Linder, Phys. Rev. D 70, 061302 (2004).
[23] P. S. Corasaniti, M. Kunz, D. Parkinson, E. J. Copeland and B. A. Bassett, Phys. Rev. D 70, 083006 (2004).
[24] S. Hannestad and E. Mortsell, JCAP 0409, 001 (2004).
[25] Y. Wang, JCAP 0503, 005 (2005).
[26] Y. Wang and P. Mukherjee, Astrophys. J. 606, 654 (2004).
[27] Y. Wang, Astrophys. J. 536, 531 (2000).
[28] A. Vikman, Phys. Rev. D 71, 023515 (2005).
[29] P. J. E. Peebles and A. Vilenkin, Phys. Rev. D59, 063505 (1999).
[30] S. Dodelson, M. Kaplinghat and E. Stewart, Phys. Rev. Lett. 85, 5276 (2000).
[31] Y. S. Piao and Y. Z. Zhang, Phys. Rev. D 70, 063513 (2004).
[32] Y. S. Piao and E Zhou, Phys. Rev. D68, 083515 (2003).
[33] Y. S. Piao and Y. Z. Zhang, Phys. Rev. D 70, 043516 (2004).
[34] Y. S. Piao, Phys. Lett. B 606, 245 (2005).
[35] G. Felder, L. Kofman and A. Linde, Phys. Rev. D60, 103505 (1999).
[36] B. Feng and M. Z. Li, Phys. Lett. B564, 169 (2003).
[37] P. F. Gonzalez-Diaz, Phys. Rev. D68, 084016 (2003).
[38] P. F. Gonzalez-Diaz, J. A. Jimenez-Madrid, hep-th/0406261
[39] e.g. G. Aldering et al. [SNAP Collaboration], arXiv:astro-ph/0405232
[40] Available at http://snap.lbl.gov/.
[41] A. Crotts et al., arXiv:astro-ph/0507043
[42] Available at http://jedi.nhn.ou.edu/.
[43] T. Abbott et al. [Dark Energy Survey Collaboration], arXiv:astro-ph/0510346
[44] M. Gasperini and G. Veneziano, Astropart. Phys. 1 (1993) 317.
[45] G. Veneziano, hep-th/0002094 J. E. Lidsey, D. Wands and E. J. Copeland, Phys. Rept. 337 (2000) 343; M. Gasperini and G. Veneziano, Phys. Rept. 373 (2003) 1.
[46] P. J. Steinhardt and N. Turok, Science 296, (2002) 1436; Phys. Rev. D65 126003 (2002); New Astron. Rev. 49, 43 (2005).
[47] For some other relevant studies see e.g. J.D. Barrow, Class. Quant. Grav. 21, L79 (2004); I. Brevik, S. Nojiri, S. D. Odintsov and L. Vanzo, Phys.Rev. D70, 043520 (2004).
[48] B. Feng, M. Li, Y. S. Piao and X. Zhang, in preparation.