Nuclear transparencies for nucleons, knocked-out under various semi-inclusive conditions

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Abstract

Using hadron dynamics we calculate nuclear transparencies for protons, knocked-out in high-$Q^2$, semi-inclusive reactions. Predicted transparencies are, roughly half a standard deviation above the NE18 data. The latter contain the effects of binned proton missing momenta and mass, and of finite detector acceptances. In order to test sensitivity we compare computed transparencies without restrictions and the same with maximal cuts for missing momenta and the electron energy loss. We find hardly any variation, enabling a meaningful comparison with data and predictions based on hadron dynamics. Should discrepancies persist in high-statistics data, the above may with greater confidence be attributed to exotic components in the description of the outgoing proton.

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The following note concerns the transparency of a nuclear medium for a proton, knocked-out in a high-$Q^2$, semi-inclusive (SI) reaction. Simplest is the electron-induced SI reaction $A(e, e'p)X$, where in addition to the scattered electron, one also measures the kinematics of a knocked-out proton. For it, the transparency $T \equiv T^{SI}$ is the ratio of the experimental yield and some reference cross section. Desiring it to be a measure for the interaction of the exiting nucleon with the remaining nuclear core, the natural choice for reference cross section is the $A(e, e'p)X$ yield where that interaction is absent, i.e. the (non-measurable) cross section in the Plane Wave Impulse Approximation (PWIA) [1]. Also in use is $Z$ times the $ep$ Mott cross section, but in that case the resulting ratio does not tend to 1 in the above limit.

To the extent that the knocked-out particle is a hadron, conventional nuclear dynamics should suffice for a description of $T$. However, for sufficiently high energies the description of the knocked-out particle is predicted to require sub-nucleonic degrees of freedom, which interact only weakly with the nuclear medium [2]. A fiducial detection of the resulting color transparency (CT) from high-$Q^2$ knock-out reactions then clearly requires

i) A measurement of $T$ under well-controlled conditions.

ii) An accurate calculation of the non-exotic component of the transparency.

Both requirements are hard to meet for SI $(e, e'p)$ processes. Data are in general not genuinely semi-inclusive, but contain contaminations, e.g. cuts in observables and detector acceptances [1,3,4]. On the theoretical side the complexity of the many-body problem stands in the way of an accurate calculation of $T$. In the absence of deconvoluted data, calculations have sometimes modeled the above problematic features, relaxing to some extent the measure of semi-inclusivity. This has led to confusion and it is not always clear what is meant by 'a standard Glauber calculation' for a SI transparency. For clarification we shall use the approach of Refs. [5,6] with modifications expounded in [7].

Having defined the transparency $T$ and the framework for computation, one may use the same to calculate related quantities, which experimental conditions may force one to study. Those become some measure for $T$, but are not identical to it. The spread in outcome of the
analysis then establishes bounds for the sensitivity of $\mathcal{T}$ for various, controlled deviations from semi-inclusivity.

Consider the SI yield when the electron transfers momentum and energy $(q,\omega)$ to the target and the knocked-out nucleon has momenta $p$. Having factored out the (off-shell) Mott cross section from the yield, the focus is on the remaining SI response per nucleon \cite{8}

$$S_{SI}(q,\omega,p) = \frac{1}{A}\sum_n |\mathcal{F}_{0n}(q)|^2 \delta(\omega + \Delta_n - e_p)$$

$$\mathcal{F}_{0n}(q) = \langle \Phi_A^{0} | \rho_{q} | \Psi_{n,p}^{+} \rangle$$

(1)

Here $\mathcal{F}_{0n}$ is the inelastic charge form factor for a transition between the ground and an excited scattering state. The latter describes asymptotically a proton with mass $M$ and total on-shell energy $e(p) = (p^2 + M^2)^{1/2}$ and a core of $A - 1$ nucleons in an excited state $n$, separated from the ground state by an energy $\Delta_n$. Exploiting the high momentum of the exiting proton one may approximately factorize those into states for the core and the knocked-out proton scattered in the field of fixed scatterers

$$\Psi_{n,p}^{+} \approx \Phi_{A}^{A-1} \psi_{p}^{+}$$

(2)

Substitution of (2) into (1) and subsequent application of closure leads to \cite{7,9} ($s = r_1 - r_1'$)

$$S_{SI}(q,y_0,p) = \frac{m^2}{q} \delta(y_0 - p_z) \int dr_1 \int dse^{is \rho_{1}(r_1, r_1')} \tilde{R}(q,r_1',s)$$

$$\tilde{R}(q,r_1',s) = \left( \prod_j \int dr_j \right) \frac{\rho_{A}(r_1', r_1; r_j)}{\rho_{1}(r_1', r_1')} \prod_{l \geq 2} \left( 1 + \gamma(q, r_1 - r_l; s) \right)$$

(3a)

(3b)

The energy loss $\omega$ in (1) has in (3a) been replaced by a scaling variable

$$y_0 \approx \bar{y}_0 \left[ 1 - \frac{1}{2A} \left( 1 + \frac{q}{y_0} \right)^{-1} \left( 1 + \frac{\omega - \langle \Delta \rangle}{M} \right) \right]$$

$$\bar{y}_0 = -q + \sqrt{2M(\omega - \langle \Delta \rangle) + (\omega - \langle \Delta \rangle)^2},$$

(4)

with $\langle \Delta \rangle$, an average nucleon separation energy for the nuclear species considered.

For any inclusive process the nuclear input required in (3a) is the nuclear density matrix $\rho_{A}$, diagonal in all coordinates except in the one of the struck particle ($'1'$). In contradistinction, the expression for $\gamma$ in (3b) which describes Final State Interactions (FSI) depends
on the type of the SI process as well as on the approximation used in the evaluation. The
induced high-energy rescattering of the knocked-out nucleon from the core is for all processes
routinely described by Glauber theory [10].

One thus finds from (1), (2) for the 'unrestricted' SI process (i.e. without binning of
data) [3]

\[ \gamma(q, r, s) = \left( 1 + \Gamma_{q}^{off}(b, z) \right) \left( 1 + \left[ \Gamma_{q}^{off}(b, z + s) \right]^{*} \right), \]

(5)

where for short-range NN interactions \( V \) one may approximately relate the elastic off-shell
profile function \( \Gamma_{q}^{off} \) in (5) to its corresponding on-shell analog \( \Gamma \). With
\( v_{q} = e(q)/q \)

\[ \begin{align*}
\Gamma_{q}^{off}(b, z) & \equiv \exp\left[-(i/v_{q}) \int_{z}^{\infty} d\zeta V(b, \zeta)\right] - 1 \\
& \approx \exp\left[-(i/v_{q}) \theta(-z) \int_{-\infty}^{\infty} d\zeta V(b, \zeta)\right] - 1 \\
& = \theta(-z) \Gamma_{q}(b) = \theta(-z) \left( \exp\left[-(i/v_{q}) \int_{-\infty}^{\infty} d\zeta V(b, \zeta)\right] - 1 \right)
\end{align*} \]

(6)

For the on-shell profile \( \Gamma \) we use the standard parametrization

\[ \Gamma_{q}(b) = e^{i\chi_{q}(b)} - 1 \approx -(\sigma_{q}^{tot}/2)(1 - i\tau_{q})A(q, b), \]

(7)

with \( \tau \), the ratio of real and imaginary parts of the forward NN elastic scattering amplitude,
and \( A(q, b) \) accounting for the range of \( \Gamma_{q}(b) \); zero-range corresponds to \( A(q, b) \rightarrow \delta^{2}(b) \).

Inelastic contributions to elastic \( pp \) and \( pn \) scattering are accounted for by the imaginary
part of the eikonal phase \( \chi \) in (4) and are in the impact parameter representation related to
the 'partial' inelastic cross section \( \sigma_{q}^{inel}(b) \)

\[ \sigma_{q}^{inel}(b) \equiv 1 - e^{-2im\chi_{q}(b)} \approx A(q, b)\sigma_{q}^{inel} \]

\[ A(q, b) \approx \left( \frac{Q_{q}}{4\pi} \right)^{2} \exp\left[-(bQ_{q}/2)^{2}\right] \]

\[ \sigma_{q}^{inel} = \sigma_{q}^{tot} - \sigma_{q}^{el} \]

(8)

Using (4), (8) one shows that \( \gamma \) in Eq. (5) becomes

\[ \begin{align*}
\gamma(q, r, s) &= -A(b) \left[ \theta(-z)\langle \sigma^{inel} \rangle + \theta(z)\theta(s - z) \left( \frac{1 + i\tau}{2} \sigma^{tot} \right) \right] \\
\langle \beta\sigma \rangle &= (A - 1)^{-1} \left[ (Z - 1)\beta_{pp}\sigma_{pp} + N\beta_{pm}\sigma_{pm} \right].
\end{align*} \]

(9)
where we defined averaged (weighted) $pN$ cross sections.

Eqs. (3) describe in principle FSI to all orders in $\gamma$, but for large momentum transfers it suffices to retain terms up to second order in $\gamma$. Those involve $\rho_n, n \leq 3$$[11,12]$

$$\rho_n(r_1, r'_1; r_j) = \rho_1(r_1, r'_1) \prod_{j \geq 2}^{n-1} \rho(r_j) \zeta_n(r_1, r_j; s)$$

$$\zeta_2(r_1, r_2; s) \approx \sqrt{g(r_1 - r_2)g(r_1 - r_2 + s q)}$$

$$\zeta_n(r_1, \ldots, r_n, s) \approx \prod_{j \leq k} \zeta_2(r_j, r_k, s_z)$$

(10)

In all $\rho_n$ we factored out the non-diagonal single-particle density $\rho_1(r_1, r'_1)$ which may approximately be parametrized as [13] $(S = (r_1 + r'_1)/2; \ s = r_1 - r'_1)$

$$\rho_1(r_1, r'_1) \approx \rho(S) \int dS' \rho_1(s, S') \approx \rho(r_1) \Sigma(s)$$

$$\Sigma(s) = \int \frac{d^3p}{(2\pi)^3} \rho(p) e^{-i p s}$$

(11)

and where $\rho(r) = \rho_1(r, r)$ is the single nucleon density. Next one encounters in the expression for $\rho_2$ in ([11]) a non-diagonal pair-distribution function $\zeta_2$, approximated there by means of the diagonal pair-distribution function $g$ [12]. The non-diagonal $n$-body distribution function $\zeta_n$ in (10) is, in the independent pair approximation expressed as a product of pair analogs $\zeta_2$. Using ([11]) and (11), Eqs. (3) become

$$S^{SI}(q, y_0, p) = \frac{m}{q} \delta(y_0 - p_z) \int ds e^{i p s} \Sigma(|s|) \int dr_1 \rho(r_1) \tilde{R}(q, r_1, s_z)$$

$$= \frac{m}{q} \delta(y_0 - p_z) \int ds e^{i p s} \Sigma(|s|) \tilde{G}(q, s_z)$$

(12a)

$$\tilde{G}(q, s_z) = \int dr_1 \rho(r_1) \tilde{R}(q, r_1, s_z)$$

(12b)

$$S^{SI,PWIA}(q, y_0, p) = \frac{m}{q} \delta(y_0 - p_z) \int ds e^{i p s} \Sigma(|s|) \int dr_1 \rho(r_1) = (m/q) \delta(y_0 - p_z) n(p)$$

(12c)

Treating the retained FSI terms in the first cumulant approximation, one finds for $A \gg 1$

$$\tilde{R} = e^{\Omega} = e^{\Omega_2 + \Omega_3 + \ldots}$$

(13a)

$$\Omega_2(q, r_1; s) = (A - 1) \int dr_2 \rho(r_2) \gamma(q, r_1 - r_2, s) \zeta_2(r_1 - r_2, s)$$

(13b)

$$\Omega_3(q, r_1; s) = \frac{A - 1}{2} \int \int dr_2 dr_3 \rho(r_2) \rho(r_3) \gamma(q, r_1 - r_2, s) \gamma(q, r_1 - r_3, s) \zeta_3(r_1, r_2, r_3, s)$$

(13c)
We shall later on return to the ternary collision terms above and discuss first the binary collision part in (13) \[14\]

\[
\tilde{R}_2 = \tilde{R}_2^{\text{tot}} \tilde{R}_2^{\text{inel}}
\]

\[
\tilde{R}_2^{\text{tot}}(q; r_1, s_z) \approx \exp \left[ - (A - 1) \left\langle \frac{1 + i \tau_q}{2} \sigma^\text{tot}_q \right\rangle \right. \\
\int d^2b A(b) \int_0^{s_z} \! dz \rho(b_1 - b, z_1 - z) \zeta_2(b, z, s_z) \left. \right]
\]

\[
\tilde{R}_2^{\text{inel}}(q; r_1, s_z) \approx \exp \left[ - (A - 1) \left\langle \sigma^\text{inel}_q \right\rangle \right. \\
\left( \int_{-\infty}^0 \! dz \rho(b_1 - b, z_1 - z) \zeta_2(b, z, s_z) \right)
\]

(14a)

We note that the results Eq. (12)-(14) contain measurable quantities without reference to an underlying potential method. It seems then reasonable to dissociate the outcome from that model and to postulate its validity also in the high-$Q^2$ regime.

We start with Eqs. (12a), (12c) for the ‘unrestricted’ nuclear transparency, without any binning or correction for detector acceptance

\[
\mathcal{T}(q, p) \equiv \frac{S^{\text{SI}}(q, y_0, p)}{S^{\text{SI, PWIA}}(q, y_0, p)} = \frac{\int ds e^{i p s \Sigma(|s|)} \tilde{G}(q, s_z)}{\int ds e^{i p s \Sigma(|s|)}}
\]

(15a)

\[
= [n(p)]^{-1} \int ds e^{i p s \Sigma(|s|)} \tilde{G}(q, s_z) = [n(p)]^{-1} \int \frac{dp'}{2\pi} n(p'_{\perp}, s_z) G(q, p')
\]

(15b)

\( \mathcal{T} \) is from (1) expected to be a function of \( q \), the energy loss \( \omega \) or \( y_0 \), Eq. (4), and the momentum of the knocked-out proton momentum \( p \). The fact that (15) does not appear to depend on \( y_0 \) is an artifact of closure. Without its application one is lead to (15b) with non-canceling single-nucleon spectral functions instead of momentum distributions.

Next we consider conditions, intermediate between semi and total inclusive scattering and consider first cuts on the missing momentum \( p_m = p - q \). For fixed \( q \) those are the same as cuts on \( p \). We thus integrate both SI yields or responses (3) in the ratio (15a) over the momentum \( p \) of the outgoing proton, with the result

\[
\mathcal{T}^P(q, y_0) \equiv \frac{\int dp S^{\text{SI}}(q, y_0, p)}{\int dp S^{\text{SI, PWIA}}(q, y_0, p)} = \frac{\int ds_z e^{i y_0 s_z \Sigma(s_z)} \tilde{G}(q, s_z)}{\int ds_z e^{i y_0 s_z \Sigma(s_z)}}
\]

(16a)

\[
= \int \frac{dp'}{(2\pi)} F_0(y_0 - p') G(q, p')
\]

(16b)
and where we used the asymptotic limit of the NR total inclusive response \[12\]

\[
F_0(y_0) = \lim_{q \to \infty} \frac{m}{q} S^{TI}(q, y_0) = \int \frac{dP}{(2\pi)^3} n(P) \delta(y_0 - P_z)
\] (17)

After the remark following Eq. (15) one observes the reappearance of the energy loss through \(y_0\) in \(T^P\), Eq. (16). Notice that the response \(S^{SI, PWIA}\) in the ratio in (16a), when integrated over the proton momentum and energy loss is 1, which implies \(Z\sigma_{\text{Mott}}\) for the corresponding reference cross section. This does not contradict the original choice of PWIA yields as reference for the definition of \(T\): it emerges if in (15a) one consistently applies the same cuts in actual and reference yields.

Finally we consider the transparency when the cross sections in the ratio (16) are in addition integrated over the energy loss of the electron. From Eqs. (3) and (8)) one readily finds \[16\]

\[
T^{PE}(q) \approx \tilde{G}(q, 0) = \int d\mathbf{r}_1 \rho(\mathbf{r}_1) \tilde{R}(q, \mathbf{r}_1, 0)
\] (18a)

\[
= \int d\mathbf{r}_1 \rho(\mathbf{r}_1) \exp[-(A - 1)\langle \sigma_q^{\text{inel}} \rangle] \int d^2 \mathbf{b} A(\mathbf{b}) \int_0^\infty dz \rho(\mathbf{b}_1 - \mathbf{b}, z_1 - z) g(b, z)
\] (18b)

\[
\approx [(A - 1)\langle g \rangle \langle \sigma_q^{\text{inel}} \rangle]^{-1} \int d\mathbf{b} \left[1 - \exp \left(- (A - 1)\langle g \rangle \langle \sigma_q^{\text{inel}} \rangle \{t_2(b) + t_3(b) + \ldots \}\right)\right] (18c)
\]

Eq. (18c) holds only in the 0-range interaction limit and \(t_2(b) = \int_\infty^\infty \rho(b, z')\) is the standard thickness function resulting from binary FSI. \(t_3(b)\) there is due to ternary contributions, to be discussed shortly.

Although the actual derivation above leads to \(\sigma^{\text{inel}}\), one occasionally meets expressions for \(T\) with \(\sigma^{\text{inel}} \to \sigma^{\text{tot}}\) \[19\] where in particular the binary part

\[
T_2^{PP}(q) \to \int d\mathbf{r}_1 \rho(\mathbf{r}_1) \exp[-(A - 1)\langle \sigma_q^{\text{tot}} \rangle] \int_0^\infty dz \rho(\mathbf{b}_1, z_1 - z) g(0, z)
\] (19a)

\[
\approx [(A - 1)\langle g \rangle \langle \sigma_q^{\text{tot}} \rangle]^{-1} \int d^2 \mathbf{b} \left[1 - \exp \left(- (A - 1)\langle g \rangle \langle \sigma_q^{\text{tot}} \rangle \{t_2(b)\}\right)\right], (19b)
\]

is reminiscent of the transparency for an elastically scattered proton. The above has for instance been used by Benhar et al, who extended a previous description of the totally inclusive \((e, e')\) process \[20\] and reached \(T^{PP}\), Eq. (19a) \[21\]. The same holds for Frenkel, Frati and Walet \[22\] and the criticism voiced in \[5, 6\] and in particular in \[7\] pertains to it.
as well. Finally we mention work by the Rome-Perugia group, who use a spectral function beyond the mean-field approximation, but do not seem to go beyond the PWIA [23].

We now discuss Eq. (18c) in some detail. Its binary collision version (with ternary and higher order corrections $t_3 + \ldots \to 0$) resulted when Kohama et al [3] applied unnecessarily strong approximations in an evaluation of the ‘unrestricted’ transparency $\mathcal{T}$, Eq. (15).

Alternatively, desiring to simplify matters, Nikolaev et al just replaced $\mathcal{T} \to \mathcal{T}^{PE}$, obtained from cross sections integrated over missing momentum and energy-loss [15]. Those authors considered binary (‘hole’) as well as ternary (‘spectator’) contributions with the scattering operator (18a), appropriate to $\mathcal{T}^{PE}$, i.e. (5) with $s \to 0$: The integration over the energy loss eliminates the retardation in the propagation of the density disturbance. As a consequence static (diagonal) quantities replace everywhere non-diagonal ones, e.g. $\zeta_2 \to g$, etc.

Instead of $g$ Nikolaev et al prefer in [10] the use of the pair-correlation function $C = 1 - g$, thus

$$g_3(1,2,3) \equiv \zeta_3(1,2,3,0) \approx g(1,2)g(1,3)g(2,3)$$

$$\approx C(1,2)C(1,3) - C(2,3)$$

(20a)

(20b)

where for brevity we wrote $g(12) = g(r_1, r_2)$, etc. Making the additional approximations $g = -C$ and the second part of (20b) in Eqs. (13b), respectively (13c) for $s = 0$, those become

$$\Omega_2^{PE,N}(q, r_1) \approx (A - 1) \int dr_2 \rho(r_2) \gamma^{PE}(q, r_1 - r_2) C(r_1, r_2)$$

(21a)

$$\Omega_3^{PE,N}(q, r_1) \approx \frac{A - 1}{2} \int \int dr_2 dr_3 \rho(r_2) \rho(r_3) \gamma^{PE}(q, r_1 - r_2) \gamma^{PE}(q, r_1 - r_3) C(r_2, r_3)$$

(21b)

Next one shows that the product $CC$ in (20b) equals $(1/2)\left(\Omega^{PE,N}\right)^2$ and similar approximations to higher order densities produce terms of the same order in the expansion of the first cumulant of (21a). Nikolaev et al thus reach

$$R^N = \exp\left(\Omega_2^N([C]) + \Omega_3^N + \ldots\right)$$

$$\Omega_2^N[C] = \Omega_2[g]$$

(22)
In view of the replacement $g \to -C$ in the actual expression Eq. (13b) for $\Omega_2$, that approximation misses contributions to $\Omega^2$ linear in $\gamma$. Likewise, there are ternary FSI contributions quadratic in $\gamma$ which are not contained in either $1/2(\Omega^N)^2$ or in (21b).

Comparison of numerical results reflects the above shortcomings. Nikolaev et al report that the first cumulant of binary as well as ternary FSI contributions Eqs. (21) are $+ (5–6)\%$, respectively $-(2–3)\%$ of the PWIA result. However, the former fraction, computed from (13b) with $C \to -g$ is much larger, namely 15–20 \% \cite{19,7}. Even assuming (??) to be a measure for ternary FSI, those cut the binary part by only $\approx 15\%$ and not by $\approx 50\%$. Clearly, in view of the approximations involved, it seems safe to disregard ternary FSI altogether.

We now present some numerical results for the transparencies of a number of nuclear species under NE18 kinematics \cite{3,4}, i.e. for $Q^2=1.0$, 3.0, 5.0 and 6.7 GeV$^2$, $p \approx |p| q$. For the evaluation of the predictions Eqs. (12)-(18) one needs nucleon densities $\rho(r)$ and pair distribution functions $g$ (both taken to be the same for protons and neutrons) and dynamical input. Measured or interpolated values for representative $NN$ scattering parameters are given in Table I. A glance at the entries suffices for one prediction: In the region of the NE18 experiment transparencies should, for any given nuclear species decrease as function of $Q^2$ and reach a plateau beyond $Q^2 \approx 1.5$ GeV$^2$. Any significant deviation from that prediction cannot be accounted for by standard hadron physics.

In Figs. 1a,b,c we entered for C, Fe and Au as function of $Q^2$ the undeconvoluted NE18 data \cite{4} and those are compared with the following predictions (none contain corrections for finite solid-angle detector acceptance).

i) Unrestricted transparencies $T$, Eq. (15), caused by binary collisions, which in turn are generated by finite, as well as by zero-range interactions.

ii) Results for $T^P$ (16), when cross sections have been integrated over missing momentum. Compared to $T$, changes are $\leq 2\%$, independent of the nuclear species or $Q^2$, and are insensitive to any reasonable choice for the average separation energy.

iii) $T^{PE}$, Eq. (18a), obtained from cross sections, in addition integrated over the electron energy loss. Relative changes now amount to (4-6)\%. We checked that in all cases only tiny
changes occur if one discards the non-negligible $\tau$ (Table I). Out of all parameters studied, only the $NN$ interaction range affects predictions.

iv) Sizable smaller transparencies $\mathcal{T}^{PP}$, Eq. (19b), result from Eq. (19a), where total cross sections replace the smaller inelastic ones.

We conclude:

1) For all targets and all $Q^2$, the most complete calculations using binary FSI produce transparencies, roughly half a standard deviation higher than the data. Lacking data with better statistics, it seems premature to ascribe the current discrepancies to color transparency, whose onset is anyhow not expected to occur for $Q^2$ as low as $\approx 1$-1.5 GeV$^2$. Ternary FSI contributions decrease the above results by only a few percent.

2) There are only modest changes in $T$ due to acceptable ranges in input parameters. Likewise $T$ is only marginally sensitive to ‘maximal’ cuts and it seems reasonable to assume the same for actual, more restricted cuts.

3) Having demonstrated the stability of calculations based on conventional hadron dynamics, firm conclusions can be drawn regarding the existence of exotic parts in $\mathcal{T}$, should discrepancies with high-statistic data persist. Presently one can only say that no conventional hadron theory can predict the rise in the undeconvoluted NE18 data towards the largest measured $Q^2$ points for all targets, or the relatively flat carbon data for the lowest $Q^2$. 
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Figure Captions.

Fig. 1a. Transparencies of C the passage of a proton, knocked out in a semi-inclusive \( A(e,e'p)X \) reaction for NE18 kinematics as function of \( Q^2 \). Data are from O’Neill \textit{et al} \textsuperscript{[4]} and differ slightly from previously published data by Makins \textit{et al} \textsuperscript{[3]}. The drawn, dotted and long dashed lines correspond to respectively \( \mathcal{T}, \mathcal{T}^P, \mathcal{T}^{PE} \), Eqs. \textsuperscript{(15)}, \textsuperscript{(16)} and \textsuperscript{(18c)}. The dot-dashed line gives the 0-range interaction result for \( \mathcal{T} \). Short dashes are for \( \mathcal{T}^{PP} \), Eq. \textsuperscript{(19a)}.

Fig. 1b. Same as Fig. 1a for Fe. Data are from O’Neill \textit{et al} \textsuperscript{[4]}.

Fig. 1c. Same as Fig. 1b for Au.

Table I. Partly interpolated \( pp \) and \( pn \) elastic scattering parameters \textsuperscript{[24]}.

| \( q \) (GeV) | \( Q^2 \) (GeV\(^2\)) | \( \sigma_{pp}^{tot} \) (mb) | \( \sigma_{pp}^{inel} \) (mb) | \( \sigma_{pn}^{tot} \) (mb) | \( \sigma_{pn}^{inel} \) (mb) | \( \tau_{pp} \) (GeV) | \( \tau_{pn} \) (GeV) | \( (Q_0)_{pp} \) (GeV) | \( (Q_0)_{pn} \) (GeV) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1.20            | 1.04           | 36.6           | 12.6          | 36             | 8              | 0.30            | -0.25          | 0.84           | 0.84           |
| 2.40            | 3.0            | 46             | 26            | 42             | 28             | -0.35           | -0.40          | 0.59           | 0.59           |
| 3.54            | 5.0            | 42.4           | 27.1          | 44.0           | 27.1           | -0.37           | -0.45          | 0.56           | 0.56           |
| 4.48            | 6.7            | 42.4           | 27.5          | 44.0           | 27.5           | -0.37           | -0.45          | 0.56           | 0.56           |
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