Phantom traffic jams may emerge “out of nowhere” from small fluctuations rather than being triggered by large, exceptional events. We show how phantom jams arise in a model of single lane highway traffic, which mimics human driving behavior. Surprisingly, the optimal state of highest efficiency, with the largest throughput, is a critical state with traffic jams of all sizes. We demonstrate that open systems self-organize to the most efficient state. In the model we study, this critical state is a percolation transition for the phantom traffic jams. At criticality, the individual jams have a complicated fractal structure where cars follow an intermittent stop and go pattern. We analytically derive the form of the corresponding power spectrum to be $1/f^\alpha$ with $\alpha = 1$ exactly. This theoretical prediction agrees with our numerical simulations and with observations of $1/f$ noise in real traffic.

\section{Introduction}

Our everyday experience with traffic jams is that they are annoying and worth avoiding. Intuitively, many people believe that if we could somehow get rid of jams then traffic would be more efficient with higher throughput. Here we show that this is not necessarily true. By studying a simple model of highway traffic, we find that the state with the highest throughput is a critical state with traffic jams of all sizes. If the density of cars were lower, the highway would be underutilized; on the other hand, if it were higher there would inevitably be a huge jam lowering throughput. This leaves us with the critical state as the most efficient state that can be achieved.

Finding a real traffic network operating at or near peak efficiency may seem highly unlikely. To the contrary, we find that an open network self-organizes to the critical state. The output from large jams finds the maximum allowed throughput which is barely stable to perturbations. Small perturbations in the outflow lead to traffic jams of all sizes. These phantom traffic jams may be viewed as avalanches which form a fractal in space and time. A snapshot of the jam at a certain point in time can be represented as a spatial fractal. A time series of the jam at a certain point in space will also be a fractal in time with a $1/f$ power spectrum. The space and time snapshots are related to each other since they are different cuts of the same underlying avalanche. This picture of avalanche dynamics has application to many dynamical systems in addition to traffic.

Actually about twenty years ago, it was discovered that traffic is an example of $1/f$ noise. T. Musha and H. Higuchi studied traffic on a three lane section of the

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They recorded the transit times of cars passing underneath a bridge spanning the motorway. They discovered that the power spectral density of the car current fluctuations had a $1/f$ low frequency behavior. This type of power spectrum is actually common to many granular flow systems, in addition to traffic. We find that one of the ramifications of criticality in traffic are long tails in the temporal correlation functions. We explicitly derive a $f^{-1}$ spectrum from the microscopic dynamics of a simple model for traffic. It is possible that $1/f$ noise in other granular systems can be understood from similar considerations.

In addition, we show that the fluctuations found in the 5-min measurements of traffic at capacity, by Hall and others, reflect the fact that traffic flow is intermittent and inhomogeneous with essentially two states (jammed and maximum throughput). We end our presentation highlighting applications to real traffic and a view toward economic systems.

2 The Model

The Nagel-Schreckenberg model that we study is defined on a one dimensional lattice with cars moving to the right. Cars can move with integer velocities in the interval $[0, v_{\text{max}}]$. The maximum velocity $v_{\text{max}}$ is typically set equal to 5. This velocity defines how many “car lengths” each car will move at the next time step. If a car is moving too fast, it must slow down to avoid a crash. A slow moving car will accelerate, in a sluggish way, when given an opportunity. The ability to accelerate is slower than the ability to break. Also, cars moving at maximum velocity may slow down for no reason, with probability $p_{\text{free}}$. For more details of the model, see K. Nagel’s article in the same volume.

2.1 Cruise Control Limit

We consider a limit of the model where $p_{\text{free}} \to 0$. This means that all cars which have reached maximum velocity, and have enough headway in front of them to avoid crashes, will continue to move at maximum velocity. Thus it is possible for the motion in the system to be completely deterministic.

For every configuration of the model, one iteration consists of the following steps, which are each performed simultaneously for all vehicles (here, the quantity gap equals the number of empty sites in front of a vehicle):

- A vehicle which travels at maximum velocity $v_{\text{max}}$ and has free headway: $\text{gap} \geq v_{\text{max}}$ just maintains its velocity.
- All other vehicles are jammed. The following two rules are applied to jammed vehicles:

  - **Acceleration of free vehicles**: With probability $1/2$, a vehicle with $\text{gap} \geq v + 1$ accelerates to $v + 1$, otherwise it keeps the velocity $v$.
  - **Slowing down due to other cars of car following**: Each vehicle with $\text{gap} \leq v$ slows down to $\text{gap}$: $v \to \text{gap}$. With probability $1/2$, it overreacts and slows down even further: $v \to \text{max}[^{\text{gap}} - 1, 0]$. 

2
• Movement: Each vehicle advances \( v \) sites.

3 Phase Transition in a Closed System

Before considering the behavior of an open system, it is worthwhile to study a closed system, i.e. a ring, where the number of cars is held fixed. Our fundamental diagram, or current-density relation \( j(\rho) \), is indicated in Fig. 1. These diagrams involve steady-state, long time averages over the entire system starting from a random initial condition (see Ref. [1]). At low density, the steady state flow is laminar. All the jams present in the initial configuration have decayed away. In other words, they have been “absorbed” by the deterministic state where all cars move at maximum velocity. As a result the linear slope at low density is just \( v_{\text{max}} \).

At high density, jams present in the initial configuration are never erased from the system. These long lived jams lower the throughput and the current decreases linearly with increasing density. So the maximum throughput point separates the deterministic state with no jams at long times from the state with a finite density of jammed cars. It corresponds to a continuous phase transition where the order parameter, \( m \), is the density of jammed vehicles.

Even though the deterministic state is not dynamically accessible above maximum throughput, \( j_{\text{max}} \), it is possible to prepare special initial configurations that have no jams. In this case, the steady state will also have no jams, and the current will still be a linearly increasing function of \( \rho \) (the dotted line in Fig. 1). In this sense, the high density phase is bistable. But perturbations of the high density deterministic branch will lead to long-lived traffic jams. The jams take the system from the unstable branch to the stable branch at lower current. Below \( j_{\text{max}} \) the deterministic state is stable to perturbations. One can view the maximum throughput state as a continuous percolation transition for traffic jams. A well known example of such a transition out of an absorbing state is directed percolation. However, the critical behavior here is much simpler than directed percolation; all of the critical exponents can be obtained analytically based on a balance between cars going into a jam and cars leaving the jam. This leads to a random walk theory as described later.

4 Maximum Throughput Selection and Self-Organized Criticality

We studied the model with a very specific boundary condition which gives the highest throughput. Maximum throughput, \( j_{\text{max}} \), is selected automatically when the left boundary condition is an infinitely large jam and the right boundary is open. Traffic which emerges from the megajam operates precisely at highest efficiency. This situation is shown in Fig. 2. The horizontal axis is space and the vertical axis (down) is increasing time. The cars are shown as black dots which move to the right. The diagram allows us to follow the pattern in space and time of the traffic. Traffic jams show up as dense regions which drift to the left, against the flow of traffic. The structure on the left hand side is the front of the megajam (cars inside the megajam are not plotted). Cars emerge from the big jam in a jerky way, before
they reach a smooth outgoing pattern operating at $j_{\text{max}}$. Far away from the front of the megajam all cars eventually reach maximum velocity.

We are now in a position to show that the outflow pattern is critical. Far downstream from the megajam, a single car is perturbed slightly, by reducing its velocity from $v_{\text{max}}$ to $v_{\text{max}} - 1$. This particular car eventually accelerates to $v_{\text{max}}$. In the meantime, a following car may have come too close to the disturbed car and has to slow down. This initiates a chain reaction – the phantom traffic jam. This is shown as the structure on the right hand side of the figure, which was initiated near the top right hand corner. Eventually, the phantom jam always dissolves. Sometimes the phantom jams are large and sometimes they are small. Any downstream car not moving at $v_{\text{max}}$ is considered to be part of the phantom jam. In order to obtain statistics for the properties of the noninteracting traffic jams, the deterministic outflow is disturbed again, after the previous jam has died out. Fig. 3 shows the lifetime distribution of the jams. This is a power law, with exponent $3/2$. The absence of a characteristic scale indicates that a critical state has been reached.

No cataclysmic triggering event, like a traffic accident, is needed to initiate large jams. They arise from the same dynamical mechanism as small jams and are a manifestation of the criticality of the outflow regime. Our natural intuition that large events come from large disturbances is violated. It does not make any sense to look for reasons for the large jams. The large jams are fractal, with small sub-jams inside big jams ad infinitum. Between the subjams are “holes” of all sizes where cars move at maximum velocity. This represents the irritating slow and go driving pattern that we are all familiar with in congested traffic. On the diagram, it is possible to trace the individual cars and observe this intermittent pattern. This behavior gives rise to $1/f$ noise.
Figure 2: Outflow from a dense region; only the front, or interface, from the dense region is shown as the structure on the left hand side (see text). In the outflow region, a phantom jam is triggered by a small disturbance. This is the structure on the right hand side.
Figure 3: Lifetime distribution $P(t)$ for emergent jams in the outflow region; average over more than 65,000 avalanches. The dotted line has slope $3/2$. All jams larger than a numerically imposed cutoff at $t = 10^6$ were removed from the database. This imposed cutoff can be made infinite, which indicates that the outflow is precisely critical.

4.1 Random Walk Theory

It is, perhaps, surprising that the seemingly complicated structure shown in Fig.
2 is described by such a simple apparent exponent. The lifetime exponent, $3/2$,
appears to be the same as first return time exponent for a one-dimensional random
walk. In fact, one can make a self-consistent random walk theory which is valid as
long as the jams, themselves, are dense. We will return to examine the density of
jams later.

Let us consider a single jam in a large system. When the vehicle at the front
of the jam accelerates to maximum velocity it leaves the jam forever. The rate at
which vehicles leave the jam is determined by the probabilistic rules for acceleration.
Vehicles, of course, can be added to the jam at the back end. These vehicles come
in at a rate which depends on the density and velocity of cars behind the jam. If
the jam as a whole is dense, then one can ignore internal branching mechanisms
where a car may accelerate to maximum velocity (thereby changing the number $n$
of jammed cars), but not actually leave the jam without having to slow down again.
The density of a jam is defined as the number of jammed vehicles, $n$, divided by $w$,
the distance between the leftmost and rightmost jammed vehicles.

The probability distribution, $P(n, t)$, for the number of cars in the jam at time,
t, is determined by the following equation for large $n$ and $t$:

$$\frac{\partial P}{\partial t} = (r_{\text{out}} - r_{\text{in}}) \frac{\partial P}{\partial n} + \frac{r_{\text{out}} + r_{\text{in}}}{2} \frac{\partial^2 P}{\partial n^2}. \quad (1)$$
The quantities $r_{in}$ and $r_{out}$ are phenomenological parameters that depend on the density behind the jam and the rate at which cars leave a jam. They are independent of the number of cars in the jam.

When the density behind the jam is such that the rate of cars entering the jam is equal to the intrinsic rate that cars leave the jam, then the first term on the right hand side of Eq. (1) vanishes, and the jam is formally equivalent to an unbiased random walk in one dimension or the diffusion equation. The first return time of the walk corresponds to the lifetime of a phantom jam. This leads immediately to the result $P(t) \sim t^{-3/2}$ for the lifetime distribution, which agrees with the numerical observation.

This argument also shows that the outflow from a megajam is self-organized critical. This can be seen by noting that the outflow from a megajam occurs at the same rate as the outflow from a phantom jam created by a perturbation downstream from the megajam. One jam’s inflow is the other jam’s outflow, so that $r_{in} = r_{out}$. Since this is true at all scales, the branching structure is a fractal. This also shows that maximum throughput corresponds to the percolative transition for the traffic jams. Starting from random initial conditions in a closed system, the current at long times is determined by the outflow of the longest-lived jam in the system. The random walk theory enables us to determine all the critical exponents for the maximum throughput state, as explained in Table I. The agreement between our random walk theory and simulations of the model is very good, except for relations involving the spatial extent $w$ of the jam. Here internal dynamics generates logarithmic corrections, as shown in the next section. Not surprisingly, these logarithmic corrections are also responsible for $1/f$ noise.

5 Avalanches and $1/f$ Noise in Traffic

A relationship between spatial fractal behavior and long-range temporal correlations can be formally established as follows: Consider Fig. 4, which is a space-time plot of a phantom traffic jam where only the jammed cars are plotted. The sites which are not occupied with cars or which have cars moving at maximum velocity are all considered to be “empty” vacuum sites. This space-time fractal, or avalanche, has the amusing property that its cuts in different directions are fractals themselves. If one makes a constant time cut, a snapshot of the jam at some instant in time is seen. The jammed cars may comprise a fractal with dimension $d_f \leq 1$. If so, then the intersection of the entire jam with a constant space cut, in the time direction perpendicular to the previous cut, will also be a fractal (see Figure). In fact, since the jams drift backwards, the fractal dimension of the time cut is exactly the same as the fractal dimension of the space cut! It turns out that a temporal sequence of points with fractal dimension $d_f \leq 1$ gives a power spectrum

$$S(f) \sim \frac{1}{f^{d_f}}. \quad (2)$$

Thus, the problem of calculating the power spectrum exponent has been reduced to the problem of calculating the spatial fractal dimension of the jammed cars at a given instant in time.
In order to proceed, one more step is needed. Consider a set of points with dimension $d_f \leq 1$ embedded on a one dimensional line. By definition, these points are separated by intervals of empty sites, or holes. The distribution of hole sizes may be a power law, i.e.

$$P_h(x) \sim x^{-\tau_h}.$$  

If so, then $d_f = \tau_h - 1$,  

as long as $\tau_h \leq 2$. In the next section, we calculate the characteristic exponent for the distribution of hole sizes and find $\tau_h = 2$ exactly. This leads immediately to the result that the power spectrum

$$S(f) \sim 1/f.$$  

Since $\tau_h = 2$, $d_f = 1$ and the jams are marginally dense. Even though the fractal dimension of jammed cars is unity, the jammed cars have zero density inside a very large jam. This is because the average size of holes inside the jam is diverging logarithmically. Thus we reach the fortuitous conclusion that the branching behavior of jams leads to complicated intermittent dynamics with a $1/f$ power spectrum, but the random walk theory for the jams, which ignores branching, is still valid up to logarithmic corrections.

5.1 A Cascade Equation for the Branching Jams

We now analyze in detail the branching behavior of jams with $v_{max} > 1$ in terms of a phenomenological cascade equation. A very large phantom jam, at a fixed point in time, consists of small dense regions of jammed cars, which we call subjams, separated by intervals, holes, where all cars move at maximum velocity. We consider the subjams to have size one.

Holes between the subjams are created at small scales by the probabilistic rules for acceleration. Each subjam can create small holes in front of it. We will ignore the details of the injection mechanism, and assume that there is a steady rate at which small holes are created in the interior of a very long lived jam. We also assume that the interior region of a long-lived jam reaches a steady state distribution of hole sizes. We do not explicitly study the distribution of hole sizes at small scales.

In order to determine the asymptotic scaling of the large holes in the interior of a long-lived jam, it is necessary to isolate the dominant mechanism in the cascade process for large hole generation. This mechanism is the dissolution of one subjam. When one subjam dissolves because the cars in it accelerate to maximum velocity, the two holes on either side of it merge to form one larger hole. Holes at any large scale are created and destroyed by this same process. This mechanism links different large scales together, and we propose that it gives the leading order contribution at large hole sizes. In the steady state, the creation and destruction of large holes must balance. This leads to a cascade equation for holes of size $x$:

$$\sum_{u=x+1}^{\infty} <h(x)h(u-x)> = \sum_{x'=1}^{x-2} <h(x')h(x-x'-1)> .$$  

8
Table 1: Critical exponents at maximum throughput. The variable $t$ is time and the variable $\Delta \rho$ is distance from the critical density.

| Physical Quantity                          | Variable | Relation                  |
|--------------------------------------------|----------|---------------------------|
| Number of Vehicles in Surviving Jams       | $n(t)$   | $n \sim t^{1/2}$          |
| Size of the Jam                            | $s(t)$   | $s \sim nt \sim t^{3/2}$  |
| Lifetime Distribution of Jams              | $P(t)$   | $P(t) \sim t^{-3/2}$      |
| Number of Jammed Vehicles in All Jams      | $\tilde{n}(t)$ | $\tilde{n} \sim t^0$     |
| Width of Surviving Jams                    | $w(t)$   | $w \sim t^{1/2} \ln(t)$  |
| Cutoff time                                | $t_{co}(\Delta \rho)$ | $t_{co} \sim (\Delta \rho)^{-2}$ |
| Probability for Infinite Jam               | $P_{\infty}(\Delta \rho)$ | $P_{\infty} \sim \Delta \rho$ |
| Distribution of Hole Sizes                 | $P_h(x)$ | $P_h(x) \sim x^{-2}$      |
| Power Spectrum                             | $S(f)$   | $S(f) \sim 1/f$           |

Figure 4: Schematic of a space-time plot of an emergent jam. The horizontal direction is space and the vertical direction is time, as in Fig. (2). Only vehicles with $v < v_{\text{max}}$, are plotted. The horizontal line is the constant time cut of the system, where a spatial fractal would be observed, and the vertical line is a constant space cut of the system, where $1/f$ noise would be observed.
Here, the angular brackets denote an ensemble average over all holes in the jam, and the quantity \( h(x)h(u-x) \) denotes a configuration where a hole of size \( x \) is adjacent to a hole of size \( (u-x) \). The right hand side of this equation represents the rate at which holes of size \( x \) are created, and the left hand side represents the rate at which holes of size \( x \) are destroyed.

Now, we make an additional ansatz; namely, for large \( x \),

\[
< h(x')h(x-x'-1) >= G(x) ,
\]  

(6)

independent of \( x' \) to leading order. That is, to leading order the probability to have two adjacent holes, whose sizes sum to \( x \) is independent of the size of either hole. \( G(x) \) then also scales the same as \( P_h(x) \), the probability to have a hole of size \( x \). Thus Eq. 5 to leading order, can be written

\[
\sum_{u=x}^{\infty} G(u) \sim xG(x) .
\]  

(7)

Differentiating leads to

\[
x \frac{\partial G(x)}{\partial x} = -2G(x) ; \quad G(x) \sim \frac{1}{x^2} .
\]  

(8)

Thus the distribution of hole sizes decays as

\[
P_h(x) \sim x^{-\tau_h} ; \quad \text{with} \quad \tau_h = 2 .
\]  

(9)

It is interesting to note that the cascade equation (5) is identical to the dominant mechanism in the exact cascade equation for forests in the one-dimensional forest fire model. The exponent \( \tau_h = 2 \) is the same as the distribution exponent for the forests, which has been obtained exactly. Since \( \tau_h = d_f+1 \), \( \tau_h < 2 \) implies that the equal time cut of the jam clusters is fractal, otherwise not. The point \( \tau_h = 2 \) is the boundary between fractal and dense behavior. At this special point, the random walk theory can still be expected to apply, although with logarithmic corrections. The borderline of fractality gives precisely \( 1/f \) behavior.

5.2 Numerical measurements

We measured the distribution of hole sizes, as shown in Fig. 5. This was accomplished by running a phantom traffic jam until the cluster reached a width of 8192, and storing the configuration of jammed cars at that time. About 60 configurations of the same size were used. The observed slope agrees well with the prediction \( \tau_h = 2 \).

The power spectrum, shown in Fig. 6, was measured in a closed system with a disturbance rate \( p_{free} = 0.00005 \) in the steady state. At low frequencies the power spectrum is consistent with the prediction \( S(f) \sim 1/f \) based on our microscopic theory for traffic.
Figure 5: Probability distribution $P_h$ for hole-sizes $x$ for $v_{max} = 2$. The dotted line has slope $-2$. The average is over 60 configurations, which all have width $w = 2^{13}$.

Figure 6: Power spectrum, $S(f)$, smoothed by averaging, for a closed system of length $L = 10^5$, with $p_{free} = 0.00005$ and $v_{max} = 5$. Dotted line has slope $-1$. 

11
6 Applications to real traffic

The following results should be general enough to be important for traffic:

- The concept of critical phase transitions is helpful for characterizing real traffic. The most efficient state for traffic that can be achieved is a critical state with jams of all sizes. Open systems will tend self-organize to this critical state. Spontaneous small fluctuations can lead to large emergent traffic jams.

- Technological advancements such as cruise-control or radar-based driving support will tend to reduce the fluctuations at maximum speed similar to our limit, thus increasing the range of validity of our results. One unintended consequence of these flow control technologies is that, if they work, they will in fact push the traffic system closer to its underlying critical point, thereby making prediction, planning, and control more difficult.

- The fact that traffic jams at the border of fractal behavior means that, from a single “snapshot” of a traffic system, one will not be able to judge which traffic jams come from the same ‘reason’. Concepts like queues or single waves do not make sense when traffic is close to criticality. Phantom traffic jams emerge spontaneously from the dynamics of branching jam waves, and give rise to $1/f$ noise.

- The regime near maximum throughput corresponds to large “holes” operating practically at $j_{\text{max}}$ and critical density, plus a network of branched subjams. The fluctuations found in the 5-minute-measurements of traffic at capacity therefore reflect the fact that traffic flow is intermittent and inhomogeneous with essentially two states (jammed and maximum throughput). The result of each 5-minute-measurement depends on how many jam-branches are measured during this period.

7 Traffic and Economics

Traffic jams have negative economic impact. One may note that in 1990 (1980), 14.8% (16.4%) of the U.S. GNP was absorbed by passenger and freight transportation costs. The conventional view is that one should try to get rid of traffic jams in order to increase efficiency and productivity. However, we find that the critical state, with traffic jams of all sizes, is the most efficient state that can actually be achieved. A carefully prepared state where all cars move at maximum velocity would have higher throughput, but it would be dreadfully unstable. The very efficient state would catastrophically collapse from any small fluctuation. A similar situation occurs in the familiar sand pile models of SOC. One can prepare a sand pile with a supercritical slope, but that state is unstable to small perturbations. Disturbing a supercritical pile will cause a collapse of the entire system in one gigantic avalanche.

But there is perhaps even a deeper relationship between traffic and economics (see also Refs. [15,16]). In an economy, humans interact by exchanging goods and services. In the real world, each agent has limited choices, and a limited capability
to monitor his changing environment. This is referred to as bounded rationality. The situation of a car driver in traffic can be viewed as a simple example of an agent trying to better his condition in an economy. Each driver’s maximum speed is limited by the other cars on the road and posted speed limits. His distance to the car in front of him is limited by his ability to stop and his need for safety in view of the unpredictability of other drivers. He is also exposed to random shocks from the road or from his car. He may be absent minded. If traffic is a paradigm for economics in general, then perhaps we have found a new economic principle: the most efficient state that can be achieved for an economy is a critical state with fluctuations of all sizes.

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References

1. K. Nagel and M. Paczuski, Phys. Rev. E 51, 2909 (1995).
2. S. Maslov, M. Paczuski, and P. Bak, Phys. Rev. Lett. 73, 2162 (1994).
3. M. Paczuski, S. Maslov, and P. Bak, Phys. Rev. E (in press).
4. T. Musha and H. Higuchi, Jap. J. Appl. Phys. 15, 1271 (1976); ibid 17, 811 (1978).
5. H. Herrmann’s article in the same volume.
6. F.L. Hall, B.L. Allen, and M.A. Gunter, Trans. Res. A 20, 197 (1986); see Hall’s article in the same volume.
7. K. Nagel and M. Schreckenberg, J. Phys. I (France) 2, 2221 (1992).
8. K. Nagel, Int. J. Mod. Phys. C 5, 567 (1994).
9. W. Feller, An Introduction to Probability Theory and Its Applications, Vol. 1 (Wiley, New York 1968).
10. Note that for \( v_{\text{max}} = 1 \) the jams do not branch, and \( 1/f \) noise is not observed.
11. M. Paczuski and P. Bak, Phys. Rev. E 48, 3214 (1993).
12. B. Drossel, S. Clar, and F. Schwabl, Phys. Rev. Lett. 71, 3739 (1993).
13. Eno Foundation for Transportation, Transportation in America, published by Transportation Policy Associates (1992); Contained in National Transportation Statistics, D.O.T. Report DOT-VNTFC-RSPA-92-1 (1992).
14. P. Bak, C. Tang, and K. Wiesenfeld, Phys. Rev. A 38, 368 (1988).
15. K. Nagel and S. Rasmussen, in Proceedings of Alife 4, edited by R. Brooks and P. Maes (MIT Press, Cambridge 1994).
16. P. Bak, Self-Organized Criticality: Why Nature is Complex, (Springer, New York, 1996).