ON THE NUCLEON DISTRIBUTION AMPLITUDE:

THE HETEROTIC SOLUTION

N. G. Stefanis and M. Bergmann

Institut für Theoretische Physik II
Ruhr-Universität Bochum
D-4630 Bochum, Germany

We present a new nucleon distribution amplitude which amalgamates features of the Chernyak-Oglobin-Zhitnitsky model with those of the Gari-Stefanis model. This “heterotic” solution provides the possibility to have asymptotically a small ratio $|G^n_M|/G^p_M \leq 0.1$, while fulfilling most of the sum-rule requirements up to the third order. Using this nucleon distribution amplitude we calculate the electromagnetic and weak nucleon form factors, the transition form factor $\gamma p \Delta^+$ and the decay widths of the charmonium levels $^3S_1$, $^3P_1$, and $^3P_2$ into $p\bar{p}$. The agreement with the available data is remarkable in all cases.

PACS numbers: 12.38.Bx, 12.38.Lg, 13.25.+m, 13.40.Fn

Internet: michaelb@photon.tp2.ruhr-uni-bochum.de
nicos@hadron.tp2.ruhr-uni-bochum.de

Bitnet: KPH509@DJUKFA11
In recent years a number of authors [1–4] have proposed various models for nucleon distribution amplitudes (NDA), based on light-cone perturbative QCD [5,6] in conjunction with QCD sum rules (SR) [1,3]. With varying degrees of conviction these models have been used in a series of analyses to calculate the electromagnetic [1,2,4,7–9] and the weak [10,11] nucleon form factors, the transition form factor $\gamma p \Delta^+$ [12–15], the cross section for proton Compton scattering at large momentum transfer $Q^2$ [16] and the exclusive $p\bar{p}$ decays of heavy quarkonia [1,8,17]. They may also be useful in determining the pion form factor at large spacelike $Q^2$ via pion electroproduction [18].

Although these models incorporate essential ingredients for a unified description of perturbative and nonperturbative aspects of the subnucleon structure, important questions still remain unanswered. One question focuses on the value of $G^n_M/G^n_p$ and the possibility that the electron-neutron differential cross section $\sigma_n$ is dominated by $G^n_E$, while $G^n_M$ is asymptotically small or equivalently that $|F^n_1| \ll |F^n_2|$ at all $Q^2$ values [19]. The Gari-Stefanis (GS) amplitude [2] was constructed to account for this behavior of the form factors and gives good agreement with the latest high-$Q^2$ SLAC data [20] at the expense that the moments (002) and (101) cannot match the SR requirements [1] of Chernyak and Zhitnitsky (CZ) in the allowed saturation range [1]. A second issue concerns whether higher terms in the Appell polynomial decomposition of the NDA [3] are significant, a point that was raised in [21]. A systematic investigation of this matter will be conducted elsewhere. In the present work we resort to second-order Appell polynomials.

In [4] Chernyak, Ogloblin, and Zhitnitsky (COZ) have recalculated the SR for the first- and second-order moments of the NDA and have derived for the first time SR for the third-order moments. Their new set of SR comprises 18 terms with restricted margins which comply with the results of the King-Sachrajda (KS) computation [3] but disagree with those obtained on the lattice for the lowest two moments [22]. They have also proposed a new NDA which satisfies all, but 6 SR, whereas the CZ amplitude and the GS amplitude violate, respectively, 13 and 14 of the new SR. The KS amplitude provides almost the same quality as the COZ amplitude with only 7 SR being broken.
In [8] the same authors pointed out that the GS model leads to a prediction for the $^{3}S_{1} \rightarrow p\bar{p}$ decay width which is about 50 times smaller than the experimental value. This, in connection with the strong violation of the SR for the moment (002) led Chernyak, Ogloblin, and Zhitnitsky to the conclusion that the GS model is unacceptable. On the other hand, they stressed [4,8] that "they have not succeeded in finding the model distribution amplitude which contains not higher than second-order Appell polynomials and gives $|F_{1}^{n}|/F_{1}^{p} < 0.4$ while fulfilling the SR."

It is the purpose of this note to present a unique NDA which makes it possible to resolve all these problems. This novel solution turns out to be something of a hybrid between the COZ amplitude and the GS amplitude; thus the "heterotic" NDA. Specifically, we find $|G_{M}^{n}|/G_{M}^{p} \leq 0.1$ and a good agreement between the calculated proton form factor and the high-$Q^{2}$ data [20], whereas the fit to the SR has almost the same accuracy as the original COZ model. In addition, the heterotic distribution amplitude leads to predictions for the exclusive decays of the charmonium levels $^{3}S_{1}$, $^{3}P_{1}$, and $^{3}P_{2}$ into $p\bar{p}$, which are in remarkable agreement with the existing experimental data.

The theoretical basis for the description of hadronic exclusive processes within QCD is provided by the factorization property (see, e.g., [5]), meaning that all soft (nonperturbative) effects can be absorbed into quark distribution amplitudes for the hadrons in the initial and final states, while the hard-scattering subprocesses can be calculated via perturbative QCD. The leading-order definition of the NDA, modulo logarithmic corrections due to renormalization, is

$$\Phi(x, Q^{2}) = \int Q^{2} [d^{2}k_{\perp}] \psi(x, k_{\perp}^{(i)}) ,$$

where the measure is $[d^{2}k_{\perp}] = 16\pi^{3}\delta^{(2)}(\sum_{i=1}^{3} k_{\perp}^{(i)}) \prod_{i=1}^{3} \left[\frac{2k_{\perp}^{(i)}}{16\pi^{3}}\right]$ and $\psi(x, k_{\perp}^{(i)})$ is the lowest-twist Fock-space projection amplitude for finding three valence quarks inside the nucleon, each carrying a fraction $x_{i} = k_{i}^{+}/p^{+}$ (with $p^{+} = p^{0} \pm p^{3}$) of the nucleon’s longitudinal momentum $p^{+}$ and having relative transverse momentum $k_{\perp}^{(i)}$. Although $\psi(x_{i}, k_{\perp}^{(i)})$, and hence $\Phi(x_{i}, Q^{2})$ cannot be calculated within perturbative QCD, a $Q^{2}$ evolution equation

3
can be derived from QCD perturbation theory [5]. Any solution of this equation can be expressed in the form

\[ \Phi(x_i, Q^2) = \Phi_{as}(x_i) \sum_{n=0}^{\infty} B_n(\mu^2) \left( \frac{\alpha_s(Q^2)}{\alpha_s(\mu^2)} \right)^{\gamma_n} \tilde{\Phi}_n(x_i), \]

where \( \Phi_{as}(x_i) = 120x_1x_2x_3 \) and \( \{\tilde{\Phi}_n(x_i)\} \) are the eigenfunctions of the interaction kernel of the evolution equation, represented in terms of Appell polynomials (cf. Ref. [5]). The corresponding eigenvalues \( \gamma_n \) equal the anomalous dimensions of the lowest-twist three-quark operators with the appropriate baryonic quantum numbers [23].

The nonperturbative input enters Eq. (2) through the coefficients \( B_n(\mu^2) \) which represent matrix elements of appropriate three-quark operators (in the light-cone gauge \( A^+ = 0 \)) interpolating between the proton and the vacuum:

\[ < \Omega | \hat{O}^{(n_1n_2n_3)}_{\gamma}(0) | P(p) > = f_N(z \cdot p)^{n_1+n_2+n_3+1} N_{\gamma} O^{(n_1n_2n_3)}. \]

Here \( z \) is a lightlike vector with \( z^2 = 0 \), \( N_{\gamma} \) is the proton spinor, and \( f_N \) denotes the "proton decay constant". SR calculations [1,3,4] make use of correlators between two of the \( \hat{O}^{(n_1n_2n_3)}_{\gamma} \):

\[ I^{(n_1n_2n_3,m)}(q, z) = i \int d^4 x e^{i q \cdot x} < \Omega | T(\hat{O}^{(n_1n_2n_3)}_{\gamma}(0) \hat{O}^{(m)}_{\gamma'}(x)) | \Omega > (z \cdot \gamma)(\gamma' \gamma) \]

\[ = (z \cdot q)^{n_1+n_2+n_3+m+3} I^{(n_1n_2n_3,m)}(q^2). \]

To determine the moments of the NDA [1],

\[ \Phi_{N}^{(n_1n_2n_3)} = \int_0^1 [dx] x_1^{n_1} x_2^{n_2} x_3^{n_3} \Phi_N(x_i), \]

([\( dx = dx_1dx_2dx_3 \delta(1 - x_1 - x_2 - x_3) \)]) a short-distance operator product expansion is performed at some spacelike momentum \( \mu^2 \) where quark-hadron duality is valid. By virtue of the orthogonality of the eigenfunctions \( \tilde{\Phi}_n \), the coefficients \( B_n(\mu^2) \) can be determined by inverting Eq. (5) upon imposing the SR constraints. The moments \( \Phi_{N}^{(n_1n_2n_3)} \) in terms of the coefficients \( B_n \) for \( n = 0, 1, \ldots, 5 \) have been given in [5]. Those for \( n = 6 \ldots 9 \) are calculated in [24].
Let us now outline our results. Treating the SR defined in (4) for \( n_1 + n_2 + n_3 \leq 3 \) and \( m = 1 \) within the range estimated by COZ [4], the coefficients \( B_n \) for the heterotic solution are: \( B_0 = 1, B_1 = 3.4437, B_2 = 1.5710, B_3 = 4.5937, B_4 = 29.3125, \) and \( B_5 = -0.1250 \); the value of \( B_0 \) is due to the normalization of \( \Phi_N \), i.e., \( \int_0^1 [dx] \Phi_N(x_i) = 1 \). The explicit form of \( \Phi_N^{het} \) is [4]:

\[
\Phi_N^{het}(x_i) = \Phi_{as}(x_i) \left\{ -19.773 + 32.756(x_1 - x_3) + 26.569x_2 \\
+16.625x_1x_3 - 2.916x_1^2 + 75.25x_3^2 \right\}.
\] (6)

The moments and the corresponding SR constraints are shown in Tab. 1. There is a good overall consistency, with only 7 SR being broken. Given the fact that we have taken into account only the first 6 Appell polynomials to represent \( \Phi_N \), the deviations are tolerable.

The results for the magnetic form factor \( G_M^N \) have been obtained using analytical expressions given in [4] and are plotted in Fig. 1 for the proton and Fig. 2 for the neutron. The data are from [20,25]. The form-factor evolution with \( Q^2 \) is due to the leading-order parametrization of the effective coupling constant \( \alpha_s(Q^2) \). The evolution of the coefficients \( B_n \) is a minor effect and has been neglected. Note that an average value \( \bar{\alpha}_s(Q^2) = [\alpha_s(Q^2 \times 0.427)\alpha_s(Q^2 \times 0.178)]^{1/2} \) has been used to account for the different virtualities of the involved propagators. Here and below two options are shown corresponding to two typical values of the scale parameter \( \Lambda_{QCD} \), while the proton decay constant is taken to be \( |f_N| = (5.0 \pm 0.3) \times 10^{-3} GeV^2 \), as suggested by QCD-SR [134].

Using \( \Phi_N^{het} \) we have calculated the electromagnetic \( N - \Delta^+ \) transition form factor \( G_M^*(Q^2) \) for two [26,27] recently proposed \( \Delta^+ \) distribution amplitudes (Fig. 3) following [12]. The solid and dashed curves refer to the model of Carlson and Poor (CP) [26] for \( \Lambda_{QCD} = 100 MeV \) and \( \Lambda_{QCD} = 180 MeV \), respectively. The dotted and dash-dotted curves are their counterparts for the model of Farrar et al. (FZOZ) [27]. In all cases the CP value \( |f_\Delta| = 11.5 \times 10^{-3} GeV^2 \) has been used, which is within the spread of the FZOZ estimate. The data are compiled in [28]. The \( Q^2 \) evolution of \( G_M^*(Q^2) \) is governed by an effective coupling constant,
which we take to be the average of the two coupling constants \(\bar{\alpha}_s^{(N)}(Q^2)\) and \(\bar{\alpha}_s^{(\Delta)}(Q^2)\) with arguments weighted by the virtualities appropriate to each model: 
\[
\bar{\alpha}_s^{(CP)}(Q^2) = [\alpha_s(Q^2 \times 0.3773)\alpha_s(Q^2 \times 0.1488)]^{1/2}
\]
and 
\[
\bar{\alpha}_s^{(FZOZ)}(Q^2) = [\alpha_s(Q^2 \times 0.4643)\alpha_s(Q^2 \times 0.1015)]^{1/2}.
\]
If we take these predictions at face value, then the available data seems to favor the CP model (cf. \[29\]). On the other hand, there is as yet no definite experimental evidence whether the \(Q^4G_M^s\) curve levels off or descends rapidly to zero, as predicted by COZ-type NDA \[13\]. A priori we have no reason to favor one option over the other (and in fact the recent analysis by Stoler \[15\] of the unpublished data of the SLAC experiment E133 points to the second possibility). Further exclusive experiments to measure \(G_M^s\) at as high \(Q^2\) as possible are crucial.

The calculation for the nucleon axial form factor \(g_A(Q^2)\) according to \[10\] yields at \(Q^2 \approx 10 GeV^2\), \(Q^4g_A(Q^2) = 0.90 GeV^4\) for \(\Lambda_{QCD} = 100 MeV\), and \(Q^4g_A(Q^2) = 1.44 GeV^4\) for \(\Lambda_{QCD} = 180 MeV\). These results compare well with the value \(Q^4g_A(Q^2) \approx 1.5 GeV^4\) extrapolated from the data \[30\]. Also the ratio \(g_A(Q^2)/G_M^p(Q^2) \approx 1.19\), in the region where the calculations can be trusted, is consistent with the (extrapolated) experimental value \(g_A(Q^2)/G_M^p(Q^2) \approx 1.35\). As for the isoscalar nucleon form factor \[11\], we find at \(Q^2 \approx 10 GeV^2\), \(Q^4G_A^s(Q^2) = 0.83 GeV^4\) for \(\Lambda_{QCD} = 100 MeV\) and \(Q^4G_A^s(Q^2) = 1.34 GeV^4\) for \(\Lambda_{QCD} = 180 MeV\). Assuming isospin invariance, we combine these results with those for \(g_A\) to obtain \(G_A^{(s)}(Q^2)/G_A^{(3)} \approx 1.85\), where \(G_A^{(3)}\) is the isovector axial-vector nucleon form factor.

If a dipole form \(G_A^{(s)}(Q^2) = G_A^{(s)}(0)/(1 + Q^2/M_{AS}^2)^2\), with \(G_A^{(s)}(0) = 0.38\) from \(SU(6)\), is used to describe the \(Q^2\) dependence of \(G_A^{(s)}\) \[14\], then, in the high-\(Q^2\) region, our model yields \(M_{AS} = (1.15 - 1.22) GeV\) for \(\Lambda_{QCD} = 100 MeV\) and \(M_{AS} = (1.27 - 1.37) GeV\) for \(\Lambda_{QCD} = 180 MeV\). These results might be relevant to studies concerning the strange-quark content of the nucleon.

The last issue we address in this work are the exclusive decays of the charmonium levels \(3S_1, 3P_1,\) and \(3P_2\) into \(p\bar{p}\). Such calculations have been carried out by several authors \[8,17,31,32\] within the QCD convolution framework. We here follow \[8\]. We consider
first the two $\chi_c(1P)$ states. The branching ratio for the decay of the $J^{CP} = 1^{++}$ state into $p\bar{p}$ is given by

$$BR\left(\frac{3P_1 \to p\bar{p}}{3P_1 \to all}\right) \approx \frac{0.75}{\ln(M/\Delta)} \cdot \frac{16\pi^2}{729} \cdot \frac{|f_N|}{M^2} \cdot M_1^2,$$

(7)

where $M \approx 2m_c \approx 3\text{GeV}$ and $\Delta = 0.4\text{GeV}$ (the last value from $[33]$, see also $[34]$). The nonperturbative content of Eq. (7) is due to $f_N$ and the amplitude for the process $^3P_1 \to p\bar{p}$, $M_1$, which involves $\Phi_N$. Using (6) the calculation of $M_1$ yields $M_1^{het} = 99849.6$ and as a result $BR(3P_1 \to p\bar{p}/3P_1 \to all) = 0.77 \times 10^{-2}\%$, which is in accordance with the experimental value $(0.5 - 1.0) \times 10^{-2}\%$ $[33]$.

The analogous expression to (7) for the $J^{PC} = 2^{++}$ state has the form

$$BR\left(\frac{3P_2 \to p\bar{p}}{3P_2 \to all}\right) \approx 0.85(\pi\alpha_s)^4 \cdot \frac{16}{729} \cdot \frac{|f_N|}{M^2} \cdot M_2^2,$$

(8)

which is Eq. (20) of $[8]$ with an obvious minor correction. For the heterotic NDA we find $M_2 = 515491.2$. Setting $\alpha_s(m_c) = 0.210 \pm 0.028$ (see third paper of $[33]$), we then obtain from (8) $BR(3P_2 \to p\bar{p}/3P_2 \to all) = 0.89 \times 10^{-2}\%$ in remarkable agreement with the measured value $(0.90^{+0.41}_{-0.26} \pm 0.19) \times 10^{-2}\%$ $[33]$.

The partial width of the $J^{PC} = 1^{--}$ state into $p\bar{p}$ is

$$\Gamma(3S_1 \to p\bar{p}) = (\pi\alpha_s)^6 \cdot \frac{1280}{243\pi} \cdot \frac{|f_\psi|}{M} \cdot \frac{|f_N|}{M^2} \cdot M_0^2,$$

(9)

where $f_\psi$ determines the value of the $3S_1$-state wave function at the origin. Its value can be extracted from the leptonic width $\Gamma(3S_1 \to e^+e^-) = (4.72 \pm 0.35)\text{keV}$ $[35]$ via the Van Royen-Weisskopf formula. The result is $|f_\psi| = 383\text{MeV}$ with $m_{J/\psi}$ equal to its experimental value. The heterotic solution leads to $M_0 = 13726.8$. Then, using the previous parameters, it follows that $\Gamma(3S_1 \to p\bar{p}) = 0.12\text{keV}$. From experiment $[33]$ it is known that $\Gamma(p\bar{p})/\Gamma_{tot} = 2.16 \pm 0.11 \times 10^{-3}$ with $\Gamma_{tot} = (68 \pm 10)\text{keV}$, so that $\Gamma(3S_1 \to p\bar{p}) = 0.15\text{keV}$ in excellent agreement with the model prediction. For the branching ratio we find $BR(3S_1 \to p\bar{p}/3S_1 \to all) = 1.76 \times 10^{-3}$, or $1.40 \times 10^{-3}$ if the new $[30]$ value $\Gamma_{tot} = 85.5^{+6.1}_{-5.8}\text{keV}$ is used. We emphasize that the model predictions for all considered charmonium decays are obtained with the same values of the various parameters.
By considering a number of exclusive reactions involving the NDA $\Phi_N$, we have effected that a novel solution, which we call heterotic, leads to predictions which are corroborated by experiment. This solution is consistent with the SR requirements up to the third order and allows for the possibility to analyze the form-factor data with the assumption that asymptotically $|G^a_M|/G^p_M \leq 0.1$. We have pursued our approach, in spite the objections raised by Isgur and Llewellyn Smith \[37\] and also by Radyushkin \[38\]. We nevertheless believe that our model provides a useful and predictive tool for phenomenological studies.
REFERENCES

[1] V. L. Chernyak and I. R. Zhitnitsky, Nucl.Phys. B246, 52 (1984)

[2] M. Gari and N. G. Stefanis, Phys. Lett. B175, 462 (1986); Phys. Rev. D35, 1074 (1987)

[3] I. D. King and C. T. Sachrajda, Nucl.Phys. B279, 785 (1987)

[4] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, Z. Phys. C42, 569 (1989)

[5] G. P. Lepage and S. J. Brodsky, Phys. Rev. D22, 2157 (1980)

[6] V. A. Avdeenko, S. E. Korenblit, and V. L. Chernyak, Yad. Fiz. 33, 481 (1981) [Sov. J. Nucl. Phys. 33, 252 (1981)]

[7] C.-R. Ji, A. F. Sill, and R. M. Lombard-Nelsen, Phys. Rev. D36, 165 (1987)

[8] V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, Z. Phys. C42, 583 (1989)

[9] N. G. Stefanis, Phys. Rev. D40, 2305 (1989); ibid. D44, 1616 (E) (1991)

[10] C. E. Carlson and J. L. Poor, Phys. Rev. D34, 1478 (1986)

[11] C. E. Carlson and J. L. Poor, Phys. Rev. D36, 2169 (1987)

[12] C. E. Carlson, Phys. Rev. D34, 2704 (1986)

[13] C. E. Carlson, M. Gari, and N. G. Stefanis, Phys. Rev. Lett. 58, 1308 (1987)

[14] G. A. Warren and C. E. Carlson, Phys. Rev. D42, 3020 (1990)

[15] P. Stoler, Phys. Rev. Lett. 66, 1003 (1991); Phys. Rev. D44, 73 (1991)

[16] A. S. Kronfeld and B. Nižić, Phys. Rev. D44, 3445 (1991)

[17] A. Andrikopoulou, Z. Phys. C22, 63 (1984)

[18] C. E. Carlson and J. Milana, Phys. Rev. Lett. 65, 1717 (1990)

[19] J. G. Körner and M. Kuroda, Phys. Rev. D16, 2165 (1977); R. G. Arnold, C. E. Carlson,
and F. Gross, Phys. Rev. C21, 1426 (1980); M. Gari and W. Krümpelmann, Z. Phys. A322, 689 (1985); Phys. Lett. B173, 10 (1986)

[20] R. G. Arnold et al., Phys. Rev. Lett. 57, 174 (1986), and earlier references cited therein

[21] A. Schäfer, Phys. Lett. B217, 545 (1989); J. Hansper, R. Eckardt, and M. F. Gari, Z. Phys. A341, 339 (1992)

[22] D. G. Richards, C. T. Sachrajda, and C. J. Scott, Nucl.Phys. B286, 683 (1987); G. Martinelli and C. T. Sachrajda, Phys. Lett. B217, 319 (1989)

[23] M. Peskin, Phys. Lett. B88, 128 (1979)

[24] M. Bergmann and N. G. Stefanis, in preparation

[25] S. Platchkov et al., Nucl. Phys. A510, 740 (1990); P. E. Bosted et al., Phys. Rev. Lett. 68, 3841 (1992), and references therein

[26] C. E. Carlson and J. L. Poor, Phys. Rev. D38, 2758 (1988)

[27] G. R. Farrar et al., Nucl. Phys. B311, 585 (1988/89)

[28] W. Bartel et al., Phys. Lett. B28, 148 (1968); S. Galster et al., Phys. Rev. D5, 519 (1972); J. C. Alder et al., Nucl. Phys. B46, 573 (1972); S. Stein et al., Phys. Rev. D12, 1884 (1975); R. Siddle et al., Nucl. Phys. B35, 93 (1975)

[29] N. G. Stefanis and M. Bergmann, Preprint RUB-TPII-22/92 (1992)

[30] N. J. Baker et al., Phys. Rev. D23, 2499 (1981); K. L. Miller et al., Phys. Rev. D26, 537 (1982); T. Kitagaki et al., Phys. Rev. D28, 436 (1983)

[31] S. J. Brodsky and G. P. Lepage, Phys. Rev. D24, 2848 (1981)

[32] P. H. Damgaard, K. Tsokos, and E. L. Berger, Nucl. Phys. B259, 285 (1985)

[33] R. Barbieri, R. Gatto, and R. Kögerler, Phys. Lett. B60, 183 (1976); R. Barbieri, R. Gatto, and E. Remiddi, Phys. Lett. B61, 465 (1976); ibid. B106, 497 (1981)
[34] V. A. Novikov et al., Phys. Rep. C41, 1 (1978)

[35] Particle Data Group, J. J. Hernández et al., Phys. Lett. B239, 1 (1990), and references therein

[36] S. Y. Hsueh and S. Palestini, Phys. Rev. D45, R2181 (1992)

[37] N. Isgur and C. H. Llewellyn Smith, Phys. Rev. Lett. 52, 1080 (1984); Nucl. Phys. B317, 526 (1989)

[38] A. V. Radyushkin, Nucl. Phys. A527, 153c (1991); ibid. A532, 141c (1991); A. P. Bakulev and A. V. Radyushkin, Phys. Lett. B271, 223 (1991)
FIGURES

FIG. 1. The proton magnetic form factor calculated with the heterotic distribution amplitude in comparison with the data.

FIG. 2. The neutron magnetic form factor calculated with the heterotic distribution amplitude in comparison with the data.

FIG. 3. Comparison with available data of the transition form factor $\gamma p \Delta^+$ calculated with the heterotic nucleon distribution amplitude and two different models for the $\Delta^+$ resonance, as explained in the text.
TABLE I. Moments $n_1 + n_2 + n_3 \leq 3$ of the heterotic nucleon distribution amplitude $\Phi_N$ in comparison with the sum-rule constraints

| Moments $(n_1 n_2 n_3)$ | Sum rules | $\Phi^{(n_1 n_2 n_3)}_{N/het}$ |
|-------------------------|-----------|-------------------------------|
| (000)                   | 1         | 1                             |
| (100)                   | 0.54—0.62 | 0.572                         |
| (010)                   | 0.18—0.20 | 0.184                         |
| (001)                   | 0.20—0.25 | 0.244                         |
| (200)                   | 0.32—0.42 | 0.338                         |
| (020)                   | 0.065—0.088 | 0.066                        |
| (002)                   | 0.09—0.12 | 0.170                         |
| (110)                   | 0.08—0.10 | 0.139                         |
| (101)                   | 0.09—0.11 | 0.096                         |
| (011)                   | −0.03—0.03 | −0.021                       |
| (300)                   | 0.21—0.25 | 0.21                          |
| (030)                   | 0.028—0.04 | 0.039                        |
| (003)                   | 0.048—0.056 | 0.139                      |
| (210)                   | 0.041—0.049 | 0.079                     |
| (201)                   | 0.044—0.055 | 0.049                     |
| (120)                   | 0.027—0.037 | 0.050                     |
| (102)                   | 0.037—0.043 | 0.037                     |
| (021)                   | −0.004—0.007 | −0.023                    |
| (012)                   | −0.005—0.008 | −0.007                    |