ABSORBING-BOUNDARY-CONDITION METHOD FOR DRIP-LINE NUCLEI

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Absorbing-boundary-condition method and its applications to nuclear responses and breakup reactions are reported. The method facilitates calculations of the continuum states in the coordinate space of many degrees of freedom. Properties of nuclei near drip lines are discussed.

1. Absorbing boundary condition (ABC)

Advances in radioactive beams provide us with a good opportunity to study physics of weakly-bound finite-quantum systems. Since excitation spectra above the particle-emission threshold are continuum spectra, theoretical analysis requires continuum wave functions. We have recently investigated an efficient and comprehensive method of treating the continuum\textsuperscript{1,2,3,4,5,6}. This is practically identical to the one called “Absorbing Boundary Condition (ABC) method” in the chemical reaction studies\textsuperscript{7}. The method allows us to calculate the continuum wave functions with the outgoing asymptotic behavior of many-body systems in a finite spatial region where particles interact with each other.

The essential trick for the treatment of the continuum in the ABC method is to allow the infinitesimal imaginary part in the Green’s function, \( i\epsilon \), to be a function of coordinate and finite, \( i\epsilon(r) \). The \( \epsilon(r) \) should be zero in the interacting region and be positive outside the physically relevant region of space. Wave functions obtained using the ABC method are meaningful only in the interacting region. However, this is enough to solve scattering problems of particles which interact by a force with finite range.
In order to understand how the ABC method is functioning in later applications, it is useful to consider the potential scattering of a particle. The scattering wave function is given by

\[ |\psi^{(+)}_k\rangle = |k\rangle + \frac{1}{E - H + i\epsilon} V|k\rangle \equiv |k\rangle + |\psi^{(+)}_{\text{scat}}\rangle. \]  

(1)

The scattering amplitude, \( f(\Omega) \), is usually defined by its asymptotic behavior

\[ \psi^{(+)}_k(r) \to \exp(i k \cdot r) + f(\Omega) \frac{\exp(ikr)}{r}, \quad (r \to \infty), \]  

(2)

but can be written in a form

\[ f(\Omega) = -\frac{m}{2\pi\hbar^2} \langle k'|V|\psi^{(+)}_k\rangle = -\frac{m}{2\pi\hbar^2} \int dr \exp(-ik' \cdot r)V(r)\psi^{(+)}_k(r), \]  

(3)

where \( \Omega \) is the direction of \( k' \) and \( |k'| = |k| \). Equation (3) implies that the \( f(\Omega) \) can be determined by the scattering wave function, \( \psi^{(+)}_k(r) \), in the interacting region where \( V \neq 0 \). In other words, behavior of \( \psi^{(+)}_k(r) \) outside the interacting region is irrelevant to determination of the scattering properties.

Figure 1 shows calculated phase shifts for a nucleon scattered by a square-well potential:

\[ V(r) = \begin{cases} -V_0 & \text{for } r < r_0, \\ 0 & \text{for } r > r_0, \end{cases} \]  

(4)

where \( V_0 = -20 \) MeV and \( r_0 = 3 \) fm. The upper part of Fig. 1 shows the phase shifts calculated with a standard numerical procedure. Namely, we solve the radial Schrödinger equations to obtain the regular wave functions of \( s-, p-, \) and \( d\)-waves, and determine the phase shifts at a certain point of \( r > r_0 \).

On the other hand, Eqs. (1) and (3) supply another way of solving the problem. The \( |\psi^{(+)}_{\text{scat}}\rangle \) in Eq. (1) satisfies the Schrödinger equation with a source term

\[ (E - H + i\epsilon)|\psi^{(+)}_{\text{scat}}\rangle = V|k\rangle. \]  

(5)

This equation must be solved with the outgoing boundary condition. Here, the ABC plays an essential role. The infinitesimal quantity \( i\epsilon \) in Eq. (5) is now replaced by \( i\epsilon(r) \), then, the equation can be solved with the vanishing boundary condition. The conditions and limitations on \( \epsilon(r) \) are discussed in a number of works\(^1,7,8\). As is emphasized before, the obtained wave function, \( \psi^{(+)}_{\text{scat}}(r) \), is correct only in the interacting region. The scattering
amplitude, \( f(\Omega) \), is calculated using Eq. (3), and \( \arg(f(\Omega)) \) corresponds to the phase shift, which is displayed in the lower part of Fig. 1. The results are identical to those of the standard method. The agreement in results of the two methods clearly indicates that the continuum in the interacting region \( (r < r_0) \) is properly taken into account in the ABC calculation.

2. Application to nuclear breakup reactions of \(^{11}\text{Be}\)

The solution of the two-body problem in Sec. 1 was trivial. The ABC did not show any advantage over the standard techniques. However, quantum three-body scattering problems are much more difficult, and one can find usefulness of the ABC.

Let us consider a reaction of a projectile, composed of core (C) plus neutron (n), on a target nucleus (T). Denoting the projectile-target relative coordinates by \( \mathbf{R} \) and the neutron-core relative coordinates by \( \mathbf{r} \), the Hamiltonian of this three-body system is expressed as

\[
H = -\frac{\hbar^2}{2\mu} \nabla_R^2 - \frac{\hbar^2}{2m} \nabla_r^2 + V_{nC}(r) + V_{nT}(r_{nT}) + V_{CT}(\mathbf{R}_{CT})
\]  

where \( \mu \) and \( m \) are the reduced masses of projectile-target relative motion and neutron-core relative motion, respectively. \( V_{nC}, V_{nT}, V_{CT} \) are the interaction potentials of constituent particles.

The wave function may be expressed as a sum of the Coulomb wave in the incident channel and the scattered wave.

\[
\Psi^{(+)}(\mathbf{R}, \mathbf{r}) = \psi^{(+)}(\mathbf{R}) \phi_0(\mathbf{r}) + \Psi_{\text{scat}}(\mathbf{R}, \mathbf{r})
\]

where \( \phi_0(\mathbf{r}) \) is the ground state of the projectile, described as a n-C bound
The $\Psi_{\text{scat}}$ satisfies the following inhomogeneous equation in the ABC,

$$\{E + e_0 + i\epsilon_{nC}(r) + i\epsilon_{PT}(R) - H\} \Psi_{\text{scat}}(R, r) = \{V_{nT}(r_{nT}) + V_{CT}(R_{CT}) - V_C\} \psi^{(+)}(R)\phi_0(r),$$  \hspace{1cm} (8)

where $V_C$ is the Coulomb distorting potential and $e_0$ is the ground-state energy of the projectile. One should note that the right hand side $\{V_{nT} + V_{CT} - V_C\} \psi^{(+)}(R)\phi_0(r)$ is a localized function in space, being analogous to the fact that right hand side of Eq. (5) is localized. Numerical details can be found in our recent paper. We have studied a deuteron breakup reaction and compared our results with those of the continuum discretized coupled channel (CDCC) calculation. The results agree with those of CDCC. In this paper, we report the application to a breakup reaction of $^{11}\text{Be}$.

The $^{10}\text{Be}$-n potential is taken as Woods-Saxon shape whose depth is set so as to produce the $2s$ orbital binding energy. We adopt the optical potential for $^{10}\text{Be}$-$^{12}\text{C}$ and the Becchetti-Greenlees potential for $n$-$^{12}\text{C}$. The radial region up to 30 fm and 50 fm are used for $R$ and $r$, respectively. The $\epsilon_{nC}(r)$ and $\epsilon_{PT}(R)$ is non-zero in the region $20 \text{ fm} < R < 30 \text{ fm}$ and $25 \text{ fm} < r < 50 \text{ fm}$. The $n$-$^{10}\text{Be}$ relative angular momenta are included up to $l = 3$.

In Fig. 2, we show the elastic breakup cross sections of $^{11}\text{Be}$-$^{12}\text{C}$ reaction. The filled circles are the result of the ABC calculation and the open circles for the eikonal calculation. The elastic breakup cross section is substantially larger than that in the eikonal approximation at lower incident energy. The failure of the eikonal approximation is apparent at the incident energy below 50 MeV/A. There, the quantum-mechanical treatment is required for the three-body continuum.

3. Giant dipole resonances in superdeformed $^{14}\text{Be}$

In studies of giant resonances, effects of the continuum has been treated in the random-phase approximation (RPA) with Green’s function in the coordinate space. However, it is very difficult to directly apply the method to deformed nuclei because construction of the Green’s function becomes a difficult task for the multi-dimensional space. We have shown that the ABC method is very useful to treat the electronic continuum in deformed systems, such as molecules and clusters. We have also investigated the applicability of the ABC in studies of nuclear response calculations. In this section, we discuss a giant dipole resonance (GDR) in $^{14}\text{Be}$. 
We use the ABC in the time-dependent Hartree-Fock (TDHF) calculations on a three-dimensional (3D) coordinate grid. In the real-time calculations, the linear response is computed by applying the isovector-dipole field to the Hartree-Fock (HF) ground state of $^{14}\text{Be}$,

$$V_{\text{ext}}(t) = kr \left\{ \frac{1}{2} (1 - \tau_z) e - \frac{Ze}{A} \right\} \delta(t),$$

where $k$ should be small enough to validate the linear response approximation. We calculate the expectation values of the $E1$ operator as a function of time, and then Fourier transforming to get the energy response. Since all frequencies are contained in the initial perturbation, the entire energy response can be calculated with a single time evolution.

We use the Skyrme energy functional of EV8 with the SIII parameter set. For the time evolution of the TDHF state, we follow the standard prescription. The model space is a sphere whose radius is 22 fm. The $i\epsilon(r)$ is zero in a region of $r < 10$ fm, while it is non-zero at $r > 10$ fm. The TDHF single-particle wave functions are discretized on a rectangular mesh in a 3D real space. Now, we can perform the TDHF simulation together with the vanishing boundary condition at $r = 22$ fm. Time evolution is carried out up to $T = 10 \ h/\text{MeV}$.

The density distribution of the $^{14}\text{Be}$ ground state is calculated to have a
prolate superdeformed shape. Therefore, we expect the deformation splitting of the GDR peak. The $E1$ oscillator strengths of $^{14}$Be are shown in Fig. 3. Here, we use a smoothing parameter of $\Gamma = 1$ MeV. The calculation predicts the large deformation splitting of about 10 MeV. However, the width of each peak is so large that the double-peak structure is almost smeared out in the total strength (thick line). This is different from our results of stable nuclei$^{2,4}$. We have observed prominent two-peak structure in those nuclei. The significant damping width may be a peculiar nature of drip-line nuclei.

4. Summary

The idea of absorbing boundary condition is presented. The continuum is properly treated in the interacting region, although the equations can be solved with the vanishing boundary condition. Since we do not need to construct the outgoing boundary wave functions, the ABC greatly facilitates the scattering problems for cases of the continuum in many degrees of freedom. We apply the ABC to the quantum reaction studies of elastic breakup of $^{11}$Be and the linear response calculation for the isovector GDR of superdeformed $^{14}$Be.

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