Isospin dependence of nuclear matter symmetry energy coefficients

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Abstract

Generalized symmetry energy coefficients of asymmetric nuclear matter are obtained as the screening functions. The dependence of the isospin symmetry energy coefficient on the neutron proton (n-p) asymmetry may be determined unless by a constant (exponent) $Z$ which depend on microscopic properties. The dependence of the generalized symmetry energy coefficients with Skyrme forces on the n-p asymmetry and on the density, only from .5 up to 1.5 $\rho_0$, are investigated in the isospin and scalar channels. The use of Skyrme-type effective forces allows us to obtain analytical expressions for these parameters as well as their dependences on the neutron-proton (n-p) asymmetry, density and even temperature. Whereas the density dependence of these coefficients obtained with Skyrme forces is not necessarily realistic the dependence on the n-p asymmetry exhibit a more consistent behaviour. The isospin symmetry energy coefficient (s.e.c.) increases as the n-p asymmetry acquires higher values whereas the isoscalar s.e.c. decreases. Some consequences for the Supernovae mechanism are discussed.

PACS numbers: 21.30.-x, 21.65.+f, 26.50.+x, 26.60.+c

Key-words: Symmetry energy coefficients, nuclear density, n-p asymmetry, screening functions, Supernovae.

IF- USP - 1476/2001

To be published in Nucl. Phys A

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1 Introduction

The symmetry energy coefficients are of interest for the understanding of several aspects of nuclear structure and also reactions as for instance (neutron rich) nuclei structure, giant resonances and nuclear heavy ion collisions at intermediary and high energies \[1, 2, 3, 4, 5\]. There are also astrophysical motivations for this as, for example, in the supernovae mechanism and r-processes in nucleosynthesis \[6, 7, 8\]. However, these coefficients can depend on the medium properties as density, temperature and neutron-proton asymmetry being these dependences relevant for different phenomena. Although one usually only considers the coefficient of the isospin channel \((a_\tau)\) there are other coefficients for the spin, spin-isospin and scalar channels of the effective nucleon nucleon (NN) interaction. The former is usually defined as the energy difference between the unpolarized and polarized nuclear matter \((a_\sigma)\) while the latter to the completely symmetric unpolarized and asymmetric polarized neutron matter \((a_{\sigma\tau})\). However these two last parameters are believed to be small and will not be considered in the present article. The most well known and studied of these parameters is the isospin one \((a_\tau)\) and we will refer to it unless explicitly quoting the others. This parameter measures the restoring force of the nuclear system to a perturbation which separates protons from neutrons. It corresponds to a cost in energy which however depends on the asymmetry of the medium.

In the macroscopic mass formulae the symmetry energy coefficient (s.e.c.) (in the isospin channel) measures the difficulty of increasing the neutron-proton (n-p) asymmetry for a stable system. It contributes as a coefficient for the squared neutron-proton asymmetry:

\[
E/A = H_0(A) + a_\tau(N - Z)^2/A^2,
\]

where \(H_0\) does not depend on the asymmetry, \(Z, N\) and \(A\) are the proton, neutron and mass numbers respectively. An analogous form for the scalar channel will be considered yielding a parameter related to the nuclear matter incompressibility. Expression \([1]\) is obtained from the Fermi gas model \([2]\) as well as from microscopic calculations \([10]\) but it is also known that there are important corrections due to nucleon-nucleon interactions and correlations. Higher orders effects of the asymmetry (proportional to \((N - Z)^n\) for \(n \neq 2, n=1, 3\)) correspond to the explicit breaking of isospin symmetry among other effects \([11]\). They are expected, in principle, to be less important for the equation of state (EOS) of nuclear matter \([12, 13]\). However we show that there may occur relevant deviations for n-p asymmetries by studying the generalized s.e.c. as the nuclear matter screening functions.
For the purpose of comparison we quote one work in which the density dependence of the isospin s.e.c. worked out [14]. In this reference the relevance of $a_\tau$ for the equation of state (EOS) of dense neutron stars was studied with an effective form for the nuclear interaction. In that work the following parametrization was obtained:

$$a_\tau(\rho) = S(\rho) = (2^{\frac{2}{3}} - 1)\frac{3}{5}E^0_F(u^\frac{2}{3} - F(u)) + S_0F(u),$$

where $u = \rho/\rho_0$, $\rho_0$ being the saturation density of nuclear matter, $E^0_F$ is the Fermi energy of nuclear matter, $S_0$ is the $a_\tau$ at the saturation density (nearly 30 MeV) and $F(u)$ is a generic function which satisfies the condition $F(1) = 1$. With non relativistic microscopic approaches, $a_\tau$ is found to vary nearly linearly with the density until $\rho \simeq 2.5\rho_0$ and to saturate at higher densities. However, this behaviour may depend on the nucleon-nucleon potential [10, 15, 16].

In the present article we perform a detailed investigation of the nuclear matter s.e.c. as the screening functions of nuclear matter for the isospin and scalar channels. Preliminary results were presented in [17]. Now we extend that calculation for the case of Skyrme forces with more involved density dependence [18, 19] and make a quite complete analysis of them. Different effective Skyrme forces are considered in order to assess the possible behaviour of these functions. The use of these forces allows us to derive analytical expressions for the generalized s.e.c. One may argue that Skyrme effective interactions are parameterized purely on phenomenological grounds and therefore would not have predictive power for nuclear matter at densities different from $\rho_0$ (the saturation density) or at very large n-p asymmetries. In the present work, however, we consider different Skyrme forces fitted from diverse sources including one derived from studies with realistic interactions [20]. Therefore, a study with these different forces can supply some information on the behaviour of such phenomenological models. Furthermore, as Skyrme parameterizations are frequently used, this kind of information can be relevant in diverse situations. Besides that, a general calculation is done which yields a general expression for the isospin dependence of the screening function. The dependences of these generalized coefficients on the n-p asymmetry and on the nuclear density (in a short range: from $0.5\rho_0$ up to $1.5\rho_0$) will be studied in this article. The relevance of some results for the Supernovae (SN) mechanism is discussed. In the next section the general expression for the screening functions is derived and the particular expression for Skyrme forces is presented. The specific form of the functions of the Skyrme parameters is shown in the appendix for each of the channels. In section 3 the results are presented and discussed. In the final part the conclusions are summarized.
2 Generalized Symmetry Energy Coefficients

Generalized nuclear matter symmetry energy coefficients will be investigated in the following. It is interesting to review and to extend a qualitative argument from [21] for exploring them.

2.1 General Remarks

Let us consider a small amplitude ($\epsilon$) external perturbation which acts, through the third Pauli isospin matrix $\tau_3$, in nuclear matter separating nucleons with isospin up and down $^1$. This originates a fluctuation $\delta \rho = \rho_n - \rho_p$ of the nucleon density. The total energy of the system can be written as:

$$ H = H_0 + a_\tau \frac{\delta \rho^2}{\rho} + \epsilon \delta \rho, \quad (3) $$

where $a_\tau$ is the isospin symmetry coefficient and $H_0$ does not depend on the asymmetry. In the equilibrium the following stability conditions must hold:

$$ \frac{\delta H}{\delta \rho} = 2a_\tau \frac{\rho}{\rho} + \epsilon \delta \rho = 0. \quad (4) $$

The ratio of the amplitude of the fluctuation generated by the external perturbation to the amplitude $\epsilon$ of this perturbation yields the static polarizability ($\Pi$) (now generalizing for any channel as done in [22, 23] with $(s,t)$ for spin,isospin numbers):

$$ \frac{\delta \rho_{s,t}}{\epsilon_{s,t}} = \Pi^{s,t} = -\frac{\rho_0}{2A_{s,t}}, \quad (5) $$

Where the $A_{s,t}$ are equal to the s.e.c. In particular in the isovector channel: $A_{0,1} = a_\tau$. This expression corresponds to the static limit of the response function of symmetric nuclear matter [21].

Now let us consider asymmetric nuclear matter. In the usual parameterizations for the mass formulae, there are several effects which depend on the induced asymmetry $\delta \rho = \rho_n - \rho_p \equiv \beta$. One can thus extend expression (3) to include a more general dependence on the asymmetry. Our approach consists in considering a general parameter $A$ which is also a function of the n-p asymmetry (and therefore a function of $\beta$) instead of the s.e.c. as usually defined in mass formulae of the type of (3). If the same external perturbation is introduced in the energy density we can write:

$$ H = H_0 + A_{0,1}(\beta) \frac{\beta^2}{\rho} + \epsilon \beta. \quad (6) $$

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$^1$ This argument is valid for all the four channels -isovector, spin, spin-isospin and scalar- with suitable modification. It is enough to consider other external perturbations: $\sigma, \sigma_3 \tau_3$ and 1 for the spin, spin-isospin and scalar channels respectively.
It is worth to note that $A^{s,t}$ (in a notation of a generic channel) has rather a dependence on the n-p asymmetry $b = \rho_n/\rho_p - 1$. The exact form of this dependence is not the same that the dependence on the fluctuation $\beta$. In other words we could write, for the sake of completeness, $A = A(\beta, b)$. However these two parameters should be related to each other. In [23] two different prescriptions were discussed for $\beta$ in the calculation of the response function of asymmetric nuclear matter. The most reasonable one leads to the following relationship between the fluctuation $\beta$ and the asymmetry $b$:

$$\beta = \delta \rho_n \left( \frac{2 + b}{1 + b} \right),$$

(7)

Where $\delta \rho_n$ is the neutron density fluctuation. In the symmetric limit $\beta = 2\rho_n$ and in the opposite limit, in neutron matter, $\beta = \delta \rho_n$, as expected. Considering this, we are lead to expect that the dependence of $A$ on the asymmetry $b$ is related to its dependence on the density fluctuation $\beta$. This allows us to write shortly: $A = A(\beta)$. It is important to stress that the prescription (7) is model-dependent and different choices for it yield different results for the static screening functions as shown below. The dynamic response functions are less sensitive to this prescription, but not completely independent [23].

The above prescription (7) is based on the assumption that the density fluctuations are proportional to the respective density of neutrons and protons, i.e., $\delta \rho_n/\beta = \rho_n/\rho$, being $\rho$ the total density. In spite of being rather well suited for the isovector channel, this kind of assumption can be considered as a simplified model for the other spin/isospin channels in asymmetric nuclear matter. The derivative of $A$ with relation to $\beta$ is therefore related to $dA/db$.

If the system is stable with relation to the density fluctuations induced by the external source, the following equation holds:

$$\frac{\delta H}{\delta \beta} = 2A \frac{\beta}{\rho} + \frac{\delta A}{\delta \beta} \frac{\beta^2}{\rho} + \epsilon = 0.$$  

(8)

By substituting the variables $\Pi \equiv \beta/\epsilon$ and $B = A/\rho$ in this equation we re-write:

$$\frac{\delta B}{\delta \Pi} = -\frac{1}{\Pi^2} - \frac{2B}{\Pi}.$$  

(9)

We can face this expression as a first order differential equation whose most general solution is given by:

$$A = -\rho(\Pi - C),$$

(10)

where $C$ is a constant. From this expression we find the following algebraic equation:

$$\Pi^2 \frac{A}{\rho} + \Pi - C = 0,$$

(11)
where $C$ is a constant. The solutions are:

$$\Pi = \rho \frac{-1 \pm \sqrt{1 + 4C/A}}{2A}. \quad (12)$$

The constant $C$ is chosen in such a way that, in the limit of symmetric nuclear matter, the polarizability $\Pi$ is the one calculated in expression (5). This leads to $C = -\rho/(4A_{\text{sym}})$.

A safer and sounder way of obtaining solutions for the polarizability is to find the value of $\beta$ at which expression (8) is satisfied. We obtain a quadratic equation for $\beta$ whose solutions are given by:

$$2\beta \frac{\delta A^{0,1}}{\delta \beta} = -2A^{0,1} \pm 2A^{0,1} \sqrt{1 - \frac{\epsilon \rho}{A^{(0,1)}} \frac{\delta A^{0,1}}{\delta \beta}}. \quad (13)$$

There are two solutions for this expression and we will be concerned only with one of them (which produces sounder results) in the limit of very small amplitude $\epsilon$. In this limit for the positive sign of expression (13) the solution is trivially found to be:

$$\Pi^{s=0,t=1} = \frac{\beta}{\epsilon} = -\frac{\rho}{2A^{0,1}}. \quad (14)$$

This expression has exactly the same form of symmetric nuclear matter. The difference is that, here, $A$ does depend on $b$.

It is also possible to extract an explicit isospin dependence of the generalized symmetry energy coefficient. From the solution of the polarizability (14) we consider the first derivative with relation to $b$:

$$\frac{d\beta}{db} = \frac{\epsilon \rho}{2A^2} \frac{dA}{db} = -\frac{\beta}{A} \frac{dA}{db}. \quad (15)$$

A complementary expression can be obtained from the relation between $b$ and $\beta$ of (7). It yields:

$$\frac{d\beta}{db} = -\frac{\beta}{2 + b}. \quad (16)$$

Equating these two last equations we obtain:

$$-A \frac{\beta}{2 + b} = -\beta \frac{dA}{db}, \quad (17)$$

From which it is possible to derive the following relation between the isospin s.e.c. and the n-p asymmetry:

$$A = A_{\text{sym}} \left( \frac{2 + b}{2} \right)^Z, \quad (18)$$

where $Z$ is a constant which depends on the channel (isovector in the present case) and eventually on other properties of the system and $A_{\text{sym}}$ is the s.e.c. of symmetric nuclear matter. In the symmetric
case $b = 0$ and hence $A = A_{sym}$. For $b = 2$ (neutron density three times larger than the proton density) we obtain $A = 2^2A_{sym}$. If we consider $Z = 1$ a considerably high value is obtained. It is therefore missing some dynamical information to fix the constant $Z$ for each channel. In the next section we find that different Skyrme forces will result different values of $Z$. We want to emphasize that the prescription (7) was the relevant information for this calculation. Any other relation between $b$ and $\beta$ will induce different asymmetry dependence of the symmetry energy coefficients.

Besides that expression (14) corresponds to the static limit of the dynamical polarizability [17]. It defines the static screening functions $A^{s,t}$ in the usual way:

$$\epsilon = C' A^{s,t} \beta,$$

where $C' = 1/\rho_0$. From this expression we see that the external field $\epsilon$ induces density fluctuations $\beta$ according to the intrinsic properties of the medium, given by the screening function $C' A^{s,t}$. These arguments are valid for any perturbation of the other channels for asymmetric nuclear matter in isospin as well as in scalar channels yielding the functions $A^{s,t}$.

We have shown that the polarizabilities of asymmetric nuclear matter yield the generalized symmetry energy coefficients. The dependence of the s.e.c. on the n-p asymmetry can be deduced unless for a constant exponent $Z$. It is however necessary to provide a prescription for the induced fluctuation $\beta$. A nearly exact expression for the dynamical polarizability of a non relativistic hot asymmetric nuclear matter for variable densities was derived with Skyrme interactions in [23]. Its static limit is exploited in the following sections for more general Skyrme forces.

### 2.2 Screening functions with Skyrme forces

In [23, 17] the dynamical polarizabilities of asymmetric nuclear matter were calculated using Skyrme forces. An alternative and independent calculation for the dynamical polarizability was done in [24]. Although the static limit has been found yielding the screening functions it has not been exploited sufficiently well. It can also be derived directly from the static Hartree-Fock equation with the same external perturbation. The static generalized symmetry energy coefficients of hot asymmetric nuclear matter, $A_{s,t}$, can be written as:

$$A_{s,t} \equiv -\frac{\rho}{2\Pi_{s,t} N} \left\{ 1 + 2 V_{0}^{s,t} N_c + 6 V_{1}^{s,t} M_p^* (\rho_c + \rho_d) + 12 M_p^* V_{1}^{s,t} V_{0}^{s,t} (N_c \rho_d - \rho_c N_d) + (V_{1}^{s,t})^2 \left[ 36 (M_p^*)^2 \rho_c \rho_d - 16 M_p^* M_c N_d \right] \right\}.$$

(19)
This expression was derived in [17] and is one of the main concerns of the present article [25]. The densities $\rho_v$, $N_v$ and $M_v$ are given by:

\[
\begin{align*}
\rho_v &= v \rho_n + (1 - v) \rho_p, \\
M_v &= v M_n + (1 - v) M_p, \\
N_v &= v N_n + (1 - v) N_p,
\end{align*}
\]

where $v$ stands for n-p asymmetry coefficients $(c, d)$ defined below (a measure of the fraction of neutron density). The above densities are defined by:

\[
(N_q, \rho_q, M_q) = \frac{2 M_0^*}{\pi^2} \int df_q(k)(k^3, k^5).
\]

In these expressions $df_q(k)$ are the differential fermion occupation numbers for neutrons ($q = n$) and protons ($q = p$). In the zero temperature limit they reduce to delta functions at the Fermi surface momentum $df_q \rightarrow -\delta(k - k_F)$. At non zero temperature one has to fix the chemical potential to calculate the Fermi occupation number and the densities. But this study of the s.e.c. dependence on the temperature would be more appropriately done with an equation of state and it is beyond the scope of this article. $V_0$ and $V_1$ are functions of the Skyrme forces parameters (calculated for Skyrme forces with more general density dependence and shown in the appendix) and $M_0^* = m_p^*/(1 + a/2)$ is an effective mass for the proton. Besides that, the four asymmetry coefficients are:

\[
\begin{align*}
a &= \frac{m_p^*}{m_n^*} - 1, \\
b &= \frac{\rho_0 n - \rho_0 p}{\rho_0 p} - 1, \\
c &= \frac{1 + b}{2 + b}, \\
d &= \frac{1}{1 + (1 + b)^2}.
\end{align*}
\]  

(20)

(The coefficient $b$ is related to a frequently used asymmetry coefficient: $\alpha = (2 \rho_0 n - \rho_0 p)/\rho_0$, by the expression: $b = 2\alpha/(1 - \alpha)$).

In the scalar channel the screening function will be referred to as the dipolar incompressibility, $K_D$, and is related to the usual nuclear matter incompressibility, $K_\infty$ by the following expression in terms of the SLyb Skyrme force parameters [20]:

\[
K_D = K_\infty + \frac{4}{5} T_F - 2V_1 k_F^2 \rho_0 + \frac{3}{4} t_3 \rho_0^{a+1}.
\]  

(21)

This relationship is slightly different for the SkSC interactions [19]. In this reference the authors fitted neutron star properties with parameterizations that yielded different values for the n-p symmetry energy at saturation density: SkSC4 (27MeV), SkSC6 (30MeV) and SkSC10 (32MeV). At different densities and n-p asymmetries (and also temperatures) the static screening functions have different values. They are studied in the next section.
3 Results and discussion

In this section we show some figures which exhibit the isospin and density dependences of the nuclear matter screening functions of expression (19) for the isospin and scalar channels. For this we use the functions $V_0$ and $V_1$ (written in the appendix) calculated for Skyrme functions with different density dependences. The parameters of one of them (SLyb) were fitted from results of neutron matter properties obtained from microscopic calculations in [20]. Other forces (SkSC4, SkSC6 and SkSC10), which have slightly different density dependences, had their parameters fixed by adjusting a large amount of nuclear masses yielding the same results of the Extended Thomas Fermi method and shell corrections calculated by the Strutinsky-integral method [18, 19]. One of the strong characteristic of the forces SkSC is the value of the effective mass which is kept to be equal to the free nucleon mass. This may be interpreted as a result of the coupling between particle modes and surface vibration modes.

In figure 1 the generalized isospin symmetry energy coefficient $A_{0,1}$ is shown as a function of the ratio of the density to the saturation density for Skyrme interactions SLyb, SkSC4, SkSC6 and SkSC10. Since Skyrme forces are not necessarily expected to describe physics at high densities we decided to investigate the dependence of the screening functions up to $1.5 \rho_0$. Most of the isovector screening functions of figure 1 does not vary much. Most of them have the tendency to keep nearly constant in a value close to that at the saturation density and decrease at higher densities. For the SLyb force two curves are shown. $A_{0,1}$ corresponds to the complete s.e.c. expression (19) whereas $a_\tau$ neglects the $V_1$ term present in expression (A.1). They exhibit nearly the same behaviour. The SkSC10 force, however, yields a different behaviour which is more compatible with that of the parameterizations of expression (2), with $F(u) = u$ from [14] (plotted with circles). There are no means (not yet) of choosing the most realistic (and reliable) of the forces in this channel for varying densities. The force SkSC10 reproduces rather the behaviour of relativistic parameterizations. In figure 2 the generalized isospin s.e.c. $A_{0,1}$ is shown as a function of the n-p asymmetry $b$ for the different forces at the saturation density $\rho_0$. There is a trend which is absolutely dominant for all the interactions. It is the increase of $A_{0,1}$ with $b$. One big difference between these curves for each of the forces is given by the slope which can be larger (eg. for SkSC6) or smaller (for SLyb). It seems to us that if the slope were so large as that obtained from SkSC6 it would have already manifested in studies of neutron matter as well as in heavy nuclei observables (for the lead nucleus $b = 0.54$). Further studies of neutron stars, supernovae and
heavy ions collisions may lead to a more precise determination of this slope. In section 2.1 a general expression for the isospin dependence of the isospin s.e.c. has been derived making use of relation (7). However it was not possible to fit any of the results obtained using the above Skyrme forces with expression (19), for any $Z$. The form of the curves (slopes) obtained with the used Skyrme forces, in figure 1, indicates that $Z$ would be smaller than one whereas the numerical values of $A_{0,1}$ (with the same Skyrme interactions) suggest that $Z \geq 1$. This indicates that the calculation done in section 2 is not completely compatible with the Skyrme forces which have been used in this paper. This could eventually be solved by means of microscopical calculations (e.g. using Monte Carlo techniques as done in [29]) and could, hopefully, also be checked by new parameterizations of mass formulae.

Another issue which can be taken from figures 1 and 2 is the isospin dependence of the s.e.c. at diverse densities. The effects of changing the asymmetry and the density sum up (with relation to the symmetric nuclear matter at saturation density) reproducing behaviours already present in figures 1 and 2 for given $\rho$ and $b$. This means that the same results of figure 1 (density dependence) will be present for nuclear matter with an asymmetry $b$ (shown in 2). The difference is that: the more asymmetric is the nuclear matter more repulsive is the isovector interaction (and hence higher the $A_{0,1}$).

In [19] the EOS of a collapsing star is studied and the results support forces for which $a_\tau$ is considerably greater than 27MeV to keep neutron matter stable. However these higher values spoil the fit of nuclear masses within the frame of the extended Thomas-Fermi plus Strutinsky integral mass formula [8]. Our results make these discrepancies be compatible in a certain sense. We have found that for larger values of the n-p asymmetry $b$ the isospin s.e.c. increases considerably corresponding to the case of neutron stars medium. Therefore for the same Skyrme force several values of $a_\tau$, or rather $A_{b,\rho} = A_{0,1}(b,\rho)$, can be considered. For the description of (not very asymmetric) nuclei the lower values can be expected to yield better results.

For the generalized scalar dipolar s.e.c., figures 3 and 4 show that, roughly speaking, all the interactions are attractive for lower densities (figure 3) and higher n-p asymmetries (figure 4). This corresponds to instabilities which are related to those found, for example, in [26]. For neutron stars it may seem that all these forces (except SLy6) lead to instabilities according to figure 4 (for fixed density $\rho_0$). However higher densities may stabilize the system (while lower densities lead to still more unstable systems). As discussed for the isospin case the effects of varying the nuclear density and the asymmetry can be considered simultaneously, i.e., to obtain the n-p asymmetry dependence of the
scalar s.e.c. at a density $\rho \neq \rho_0$ the behaviours of figures 3 and 4 can be analysed together. Therefore, for higher densities (making the $A_{0,0}$ to increase) the system may experience higher asymmetries ($b$) without the instabilities present for forces SkSC4 and SkSC6. We do not exhibit, however, a quantitative estimate of the instabilities points in the variables $b$ and $\rho$ due to the diverse behaviour of Skyrme forces. Moreover these points may be out of the validity of the application of these interactions. For the moment there is no way of verifying this. The interaction in this channel becomes more and more repulsive for higher densities. The increase of the n-p asymmetry on the other hand, make the interaction continuously more attractive for most of the forces as shown in figure 4. However, the slopes for interactions SkSC seem to be too negative. In figure 4 it is also shown the parameterization for n-p asymmetry dependence of the incompressibility of nuclear matter ($K_\infty(b)$) of reference [27] -from the equation of state- (not the incompressibility $K_\infty$ itself, but its asymmetry dependence- for the case of SLyb). This parameterization (using the n-p asymmetry parameter of our work) is given by:

$$K(b) = K(b = 0)(1 - a'(b/(b + 2))^2)$$  \hspace{1cm} (22)

where $a'$ (in [27]) is of the order of 1.28 up to 1.99 for several Skyrme interactions (zero temperature) and $K(b)$ stands for $K_\infty$ or $A_{0,0} = K_D$. It is seen in figure 4 that this case (from [27]) has nearly the same behaviour (dependence on the n-p asymmetry $b$ with $\rho = \rho_0$) as our calculation for force SLyb, a remarkable feature. In another work [19] it was found the same kind of dependence on the n-p asymmetry for the incompressibility of nuclear matter but with other numerical coefficients:

$$K(b) = K_v + K_s(b/(b + 2))^2$$  \hspace{1cm} (23)

These parameters assume the following values for the forces used in that article:

$$\begin{align*}
SkSC4(K_v = 234.7\text{MeV}, & \quad K_s = -334.9\text{MeV}); & SkSC10(K_v = 235.4\text{MeV}, & \quad K_s = -203.5\text{MeV}); \\
SkSC6(K_v = 235.8\text{MeV}, & \quad K_s = -136.6\text{MeV}).
\end{align*}$$  \hspace{1cm} (24)

These values are (a little smaller) of the same order of those obtained in the figure 4 and expression (22). It can expected that this dipolar incompressibility is also of relevance for the study of the Isoscalar Dipole Giant Resonance [28].

Consequences concerning the supernovae mechanism can be discussed now. The higher the n-p symmetry energy coefficient the smaller is the deleptonization (electron capture) in the quasi-static phase of the supernovae mechanism yielding a larger final proton fraction and faster cooling (via
neutrino emission). As we have found that $A_{0,1}$ increases with the n-p asymmetry it can be expected that as deleptonization proceeds it becomes more and more difficult to convert protons into neutrons. This also means that the final (neutronized) star would be less asymmetric than expected with smaller (fixed) values for $a_\tau$. This picture yields a stronger shock wave since the energy loss inside the star due to deleptonization is smaller. Therefore the increase of isospin symmetry energy coefficient (which occurs for higher n-p asymmetries) helps a successful explosion of the (contracting) star (keeping the density fixed). This conclusion has the essential feature compatible with the analysis done for the SN 1987a event [6, 29, 30].

4 Conclusions

Summarizing, generalized symmetry energy coefficients of nuclear matter were investigated. Firstly we have shown that the polarizabilities of asymmetric nuclear matter yield the generalized symmetry energy coefficients. The dependence of the isospin s.e.c. on the n-p asymmetry, by means of the relation (7) was found to be:

$$A = A_{sym} \left( \frac{2 + b}{2} \right)^Z,$$

where $Z$ is a constant. To obtain such expression it was necessary to provide a prescription for the dependence of the induced fluctuation $\beta$ on the n-p asymmetry $b$ (expression (7)) [23] which could be chosen differently. The form of the curves obtained from the used Skyrme forces in figure 1 indicate that $Z$ would be smaller than one whereas the numerical values of $A_{0,1}$ suggest that $Z \geq 1$. This constant seems to contain relevant dynamical information and it could be determined by the study of the symmetry energy coefficient microscopically.

Secondly a study of the generalized symmetry energy coefficients (isovector and scalar) was done with Skyrme forces. However these results are not adjusted by the function obtained (the expression (18)) The use of Skyrme-type interactions allowed to obtain analytical expressions for the s.e.c. Their density and n-p asymmetry dependences were analyzed for different Skyrme forces which may yield very different behaviours. In some cases it is possible to discard unphysical results but there are results which cannot be chosen due to non existing experimental knowledge. Nevertheless neutron stars and eventually very asymmetric nuclei properties as well as heavy ion collisions can provide valuable information. Although one should not believe that only one Skyrme force parameterization could account for the description of all nuclear observables at different n-p asymmetries (as well as
densities and temperatures) it is acceptable the idea that several parameterizations could hopefully
describe different ranges of the dependence of nuclear observables (as the s.e.c.) with these three
variables. Nevertheless it is tempting to conclude that the force SLyb, which was adjusted to reproduce
general properties of asymmetric nuclear matter, yielded, in general, more reasonable behaviour for
the symmetry energy coefficients for varying n-p asymmetries. The dependence of the isospin s.e.c.
on the n-p asymmetry indicates that the deleptonization process in supernovae is suppressed as the
neutronization occurs. This would make final proton fraction bigger with less energy loss helping the
SN explosion (and also less asymmetric neutron stars matter) in agreement with expectations. A
quantitative estimate of this effect however will only be possible after a more precise determination of
the dependence of the s.e.c. on \( b \).

Acknowledgements

This work was supported by FAPESP, Brazil.

Appendix: Functions \( V_{i,t}^{s,t} \)

We write the parameters \( V_0 \) and \( V_1 \) from the linearization of the Hartree Fock equation (for nuclear
matter) for the Skyrme forces SkSC used by [19]. They contain more involved density dependence:

\[
V_{0}^{s,t} = V_{0}^{s,t} + V_{2}^{s,t} - \frac{2 V_{1}^{s,t} s_{p} \rho_0}{1 - 4 V_{1}^{s,t} m_p \rho_0},
\]

where only the \( V_0 \) and \( V_2 \) were modified.

In the SkSC forces the density dependence implies that this part of the interaction between two
protons depends only on the proton density being more reasonable. It can be written in the following
form:

\[
\frac{t_3}{6} (1 + x_3 P_\sigma) (\rho_{qi} + \rho_{qj})^\alpha \delta(r_{i,j}) + a_2 \rho^\alpha,
\]

where \( P_\sigma \) is the two-body spin exchange operator, \( q \) indicates proton or neutron and \( a_1, a_2 \) are taken
to be 0, 1 or 1, 0 for Skyrme interactions SkSC and SLyb respectively.
In the isovector channel one obtains:

\[ V_{0.1}^0 = \left( -\frac{t_0}{2} \left( x_0 + \frac{1}{2} \right) - \frac{t_3}{12} \left[ a_2 \left( x_3 + \frac{1}{2} \right) + a_1 \left( 1 + \frac{x_3}{2} \right) - \frac{1}{4} (1 - x_3) (\alpha + 2)(\alpha + 1) \right] \rho_0^0 \right) (1 + bc), \]

\[ V_{1.1}^0 = \frac{1}{16} (t_2(1 + 2x_2) - t_1(1 + 2x_1)), \]

\[ V_{2.1}^0 = t_3 \left[ a_2 \left( \frac{1}{2} + x_3 \right) \alpha \rho_0^{\alpha - 1}(c \rho_n + (c - 1) \rho_p) + \right. \]

\[ + a_1 \left( 1 + \frac{x_3}{2} \right) \alpha \rho^{\alpha - 1}(c \rho_n + \rho_p(c - 1)) + 2(1 - x_3)(\alpha + 2)(\alpha + 1)(c \rho_n^\alpha + \rho_p^\alpha(c - 1))/16 \right] /12, \]

where \( \rho_n, \rho_p \) and \( \rho_0 \) are the proton, neutron and total saturation densities of asymmetric nuclear matter. For the scalar channel:

\[ V_{0.0}^0 = \left( t_0/4 + (\alpha + 1) (\alpha + 2) t_3 \rho_0^\alpha \right) \left[ a_1 \left( 1 + \frac{x_3}{2} \right) \left( \frac{1 + b}{2 + b} \right) \frac{1}{16} + a_2 \left( 1 + \frac{x_3}{2} \right) \frac{1}{12} \right] (1 + bc), \]

\[ V_{1.0}^0 = t_3 \alpha (c \rho_n + (c - 1) \rho_p), \]

\[ V_{2.0}^0 = \frac{t_3}{12} \left[ (x_3 + .5)(c \rho_n + (c - 1) \rho_p \rho_0^{\alpha - 1}) + \right. \]

\[ + a_1 \alpha (1 + 2)(1 - x_3) \left( \frac{(2 \rho)^\alpha}{(2 + b)^{\alpha + 2}} + 2 \frac{(1 + b)^2 \rho^\alpha}{(2 + b)^{\alpha + 2}} \right) /2 - a_2 (1 + (1 + b)^2 \rho^\alpha)/(2 + b)^2 \right] . \]

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Figure caption

Figure 1 Isovector screening function \( A_{0,1} = \rho/(2\Pi_{R}^{0,1}) \) of symmetric nuclear matter as a function of the ratio of density to density at saturation \( u = \rho/\rho_0 \) for interactions SLyb (solid), SkSC4 (dotted), SkSC6 (dashed), SkSC10 (long dashed). The simplified expression \( a_{\tau} \), i.e. without terms of order of \( V_{1}^{2} \), for the force SLyb (thin dotted-dashed). Circles (P.A.L. 1) for the expression of \( a_{\tau} \) as a function of \( u \) with three different functions \( F(u) \) from reference [14].

Figure 2 Isovector screening function \( A_{0,1} = \rho/(2\Pi_{R}^{0,1}) \) as a function of the asymmetry coefficient \( b \) (at \( \rho_0 \)): dotted line for SkSC4, dashed lines for SkSC6, solid line for SLyb and long-dashed lines for SkSC10.

Figure 3 Scalar (dipole) screening function \( A_{0,0} = \rho/(2\Pi_{R}^{0,0}) \) as a function of the ratio of density to density at saturation \( u = \rho/\rho_0 \) for the different interactions with the conventions of figure 1.

Figure 4 Scalar (dipole) screening function \( A_{0,0} = \rho/(2\Pi_{R}^{0,0}) \) as a function of the asymmetry coefficient \( b \), with \( \rho_0 \), with the different forces (the conventions of figure 2) and also the n-p asymmetry dependence of the compressibility \( K_{\infty} \) of nuclear matter of reference [27] using \( K_D(b = 0) \) from the SLy force (star).
Figure 3

Figure 4