Superinsulator-Superconductor Duality in Two Dimensions

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Abstract
For nearly a half century the dominant orthodoxy has been that the only effect of the Cooper pairing is the state with zero resistivity at finite temperatures, \textit{superconductivity}. In this work we demonstrate that by the symmetry of the Heisenberg uncertainty principle relating the amplitude and phase of the superconducting order parameter, Cooper pairing can generate the dual state with zero conductivity in the finite temperature range, \textit{superinsulation}. We show that this duality realizes in the planar Josephson junction arrays (JJA) via the duality between the Berezinskii-Kosterlitz-Thouless (BKT) transition in the vortex-antivortex plasma, resulting in phase-coherent superconductivity below the transition temperature, and the charge-BKT transition occurring in the insulating state of JJA and marking formation of the low-temperature charge-BKT state, \textit{superinsulation}. We find that in disordered superconducting films that are on the brink of superconductor-insulator transition the Coulomb forces between the charges acquire two-dimensional character, i.e. the corresponding interaction energy depends logarithmically upon charge separation, bringing the same vortex-charge-BKT transition duality, and realization of superinsulation in disordered films as the low-temperature charge-BKT state. Finally, we discuss possible applications and utilizations of superconductivity-superinsulation duality.

\textit{Keywords:} superconductivity, localization, Josephson junction arrays, nanoscale systems, disordered films, superconductor-insulator transition

1. Introduction

Hundred years ago the discovery of superconductivity, the state possessing a zero resistance at low but finite temperatures, opened a new era in physics. The discovery crowned a systematic study of electronic transport at lowest temperatures available at the time undertaken at Leiden University by Professor Kamerlingh Onnes, who strived to uncover microscopic mechanisms of conductivity in metals. It was recognized since 1905 that conductivity can be described by the Drude formula \( \sigma = n e^2 \tau / m \), where \( n \) is the charge carrier density, \( -e \) is the electron charge, \( \tau \) is the scattering time, and \( m \) is the mass of the charge carrier. A natural hypothesis was that in pure metals the scattering time will start to grow indefinitely upon approaching the absolute zero temperature, resulting in infinite conductivity. However Lord Kelvin argued that mobile charges can freeze out near the zero temperature so that the charge density \( n \) in the Drude formula goes to zero.

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and that this effect may win over the increase of $\tau$. Kamerlingh Onnes’ finding ruled out Lord Kelvin hypothesis, and, one can say – adopting extremely loose and non-rigorous interpretation of known now microscopic mechanisms of superconductivity – that superconductivity indeed complies with the $\tau$-divergence notion. It is essential for the whole concept of superconductivity that $\tau$ becomes infinite at finite temperatures. The idea about the mobile charges gradually freezing out as temperature tends to zero was found to realize in the resistivity of semiconductors (i.e. band insulators). The latter exhibit the metallic type of behavior at high temperatures but at low temperatures their charge density $n$ exponentially decreases with the decreasing temperature and wins over the gradually growing $\tau$. The next step towards what might sound as a realization of the Lord Kelvin idea, was the finding that there exist two-dimensional superconducting systems, namely, Josephson junction arrays, granular and homogeneously disordered films, which unite both types of behavior, superconducting and insulating. The idea that the material with the Cooper pairing can be transformed into an insulator traces back to Anderson [11], who considered small superconductors coupled by Josephson link, and was further discussed by Abeles [2] in the context of granular systems. The analytical proof for the existence of superconductor-insulator transition (SIT) in granular superconductors was given by Efetov [5], and the first theory of the disorder-induced superconductor-insulator transition in the 3D disordered Bose condensate was developed by Gold [4, 5]. Ever since the study of the SIT, which is an exemplary quantum phase transition, has been enjoying an intense experimental and theoretical attention and has become one of the mainstreams of contemporary condensed matter physics. In this work we will describe the further advance in understanding the SIT, the realization that at the transition point superconductivity transforms into a mirror image of superconductivity, the superinsulating state, possessing infinite resistivity at finite temperatures.

The paper is organized as follows. Next Section establishes the possibility of superinsulation from the uncertainty principle. In the Section 3 we discuss general features of two-dimensional systems enabling experimental realization of the superinsulating state; in the Section 4 we briefly remind the properties of Berezinskii-Kosterlitz-Thouless (BKT) transition and the relation between this transition and formation of the superinsulating state, presenting the phase diagram of the system in the close vicinity of superconductor-superinsulator transition; the Section 5 is devoted to realization of 2D electrostatics in disordered superconducting films that brings to the existence the charge-BKT transition; in the Section 6 we discuss a possible microscopic mechanism behind the superconducting state; in the Section 7 we present a qualitative consideration of the conductivity in the superinsulating state; Section 8 discusses possible applications and utilizations of the superconductor-superinsulator duality; and finally, we summarize the results of our work in the Section 9.

2. Superconductivity and superinsulation from the uncertainty principle

The possibility of the existence of the superinsulating state can be established from the most fundamental quantum-mechanical standpoint. Superconducting state is characterised by an order parameter, $\Psi = |\Psi| \exp(i \phi)$, where the phase, $\phi$, is well defined across the whole system. The uncertainties in the phase $\Delta \phi$ and the number of Cooper pairs in the condensate, $N = 2|\Psi|^2$, compete according to the Heisenberg uncertainty principle $\Delta \phi \Delta N \geq 1 [1, 6, 7, 8]$. This brings about the basic property of a superconductor, its ability of carrying a loss-free current. Indeed, the absence of an uncertainty in the phase, $\Delta \phi = 0$, i.e. the definite phase, implies that the number of condensate particles is undefined. It means that the particles that comprise condensate cannot scatter and dissipate energy, since every scattering event can be viewed as the measurement, and,
in principle, could be used to determine the number of particles. In two-dimensional systems, the global phase coherence leading to true superconductivity establishes itself at some finite temperature, $T_{SC}$, which is below the temperature $T_c$, where the finite modulus of the order parameter appears [9, 10]. The phase coherence then remains stable against moderate perturbations: the loss-free current maintains as long as it does not exceed the certain value $I_c$ determined by the material parameters.

Now let us assume that we have managed to tune the parameters of a 2D superconductor in such a manner that the Cooper pairs got pinned within the system. This would eliminate the uncertainty of the total charge setting $\Delta N = 0$. The Heisenberg principle requires then the finite uncertainty of the global phase, i.e. pinning the Cooper pairs results in $\Delta \phi \neq 0$. This implies that due to the Josephson relation, $d\phi/dt = (2e/\hbar)V$, where $h$ is the Planck constant, there may be a finite voltage, $V$, across the whole system while the flowing of the current is blocked. This establishes the possibility for the existence of a distinct superinsulating state [11, 12], which is dual to the superconducting one and has zero conductivity over the finite temperature range from the critical temperature $T_{SI}$ down to $T = 0$. Of course, one has to bear in mind that the term “zero conductivity” is used in the same idealized sense as the “zero resistance” of a two-dimensional superconductor. We would like to emphasize here once again the conceptual difference between the notions of the “superinsulator” and merely the “insulator,” which is that the former has zero conductivity in the finite temperatures range, while the latter would have possessed the finite, although exponentially small, linear conductivity $\sigma \propto \exp(-E_g/k_BT)$ ($E_g$ is the energy gap) at all finite temperatures, except for $T = 0$.

3. Two-dimensional superconducting systems

To gain an insight into the origin of a superinsulating state we will focus on two-dimensional tunable superconducting systems, Josephson junction arrays (JJA), the systems comprised of small superconducting islands connected by Josephson junctions. A theoretical and experimental study of large regular JJAs has been one of the main directions of the condensed matter physics for decades [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26], see [27, 28, 29] for a review. The interest was motivated by the appeal of dealing with the experimentally accessible systems whose properties can be easily controlled by tuning two major parameters quantifying the behavior of the array, the Josephson coupling energy between the two adjacent superconducting islands comprising a JJA, $E_J$, and the charging energy of a single junction, i.e. the Coulomb energy cost for transferring the Cooper pair between the neighbouring islands, $E_C = (2e^2)/(2C)$, where $C$ is the junction capacitance [1]. Moreover, JJA offers a generic model that captures essential features of two-dimensional disordered superconductors including homogeneously disordered and granular films. The remarkable feature of JJAs is that they experience a phase transition, which historically was described in terms of the zero-temperature transition between the superconducting and insulating states, superconductor-to-insulator transition (SIT), which occurs as $E_C$ compares to $E_J$. In the arrays that are already near the critical region where $E_C \approx E_J$, the SIT can be also induced by the magnetic field. The SIT studies in JJAs were paralleled by the investigations on the superconductor-insulator transition in thin granular films, which revealed the similar behavior [2, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41]. What more, even homogeneously disordered superconducting films [42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75] and layered high-$T_c$ superconductors [76, 77, 78, 79, 80, 81, 82] exhibited all the wealth of the SIT-related phenomena which were viewed as characteristic to granular superconducting systems.
This brought about the idea that in the vicinity of SIT disorder may cause the electronic phase separation (often referred to as the “self-induced granularity”) that realizes in a form the strong disorder induces an inhomogeneous spatial structure of isolated superconducting islands in thin homogeneously disordered films [47, 48, 83, 84, 85, 86, 87]. Numerical simulations of the homogeneously disordered superconducting films confirmed that indeed in the presence of sufficiently strong disorder, the system breaks up into superconducting islands separated by an insulating sea [88, 89, 91, 91]. Recent scanning tunneling spectroscopy measurements of the local density of states in TiN and InO films [92, 93] and in high-$T_c$ superconductors [94, 95] offered strong support to this hypothesis. We will discuss these issues in more detail in Section 1.4.

All the above studies showed that the Josephson junction arrays indeed offer a generic model that captures most essential features of the superconductor-insulator transition in a wide class of systems ranging from artificially manufactured Josephson junction arrays to superconducting granular systems and even the homogeneously disordered superconducting films and allows for consideration of all of them on the common ground. The properties of the films were discussed in terms of the dimensionless [measured in the quantum units $e^2/(2\pi\hbar)$] conductance $g$, which played the role of the tuning parameter for the films replacing the ratio $E_J/E_C$ characterizing JJAs. Large $g$ correspond to superconducting domain, at small $g$ films turn insulating. And at some critical value $g_c \simeq 1$ disordered films is viewed to undergo superconductor-insulator transition.

Using JJA as an exemplary system allows to understand better the microscopic mechanism of the realization of the charge-phase duality of the uncertainty principle giving rise to superinsulation. As had already been indicated in [96] the charge-vortex duality reflects the duality between the Aharonov-Bohm [97] and Aharonov-Casher [98] effects. In the Aharonov-Bohm effect, the charges moving in a field-free region surrounding a magnetic flux acquires the phase proportional to the number of flux quanta piercing the area. Accordingly, the Aharonov-Casher effect is the reverse [99]: magnetic vortices moving in a 2D system around a charge acquires a phase proportional to that charge. Now, considering an insulating side, $E_C \gg E_J$, where all the Cooper pairs are pinned at the granules by Coulomb blockade, we can conjecture, following the line of reasoning of [100, 101], that in the fluxons tunnel coherently through the Josephson links between the granules that corresponds to constructive interference of their tunnelling paths i.e. to the process having the largest quantum mechanical amplitude. Thus the phase synchronizes across the whole system remaining undefined. In other words, one can possible view the superinsulating state where the current are completely blocked as a state mediated by the coherent quantum phase slips discussed recently in the context of Josephson junction chains [102, 103] and disordered wire ring [104]. The uncertainty of the phase implies the superfluid state of fluxons at the insulating side dual to superfluid state of Cooper pair condensate, by the same token as uncertainty in the charge implies superconductivity as discussed above. Indeed the possibility of measuring the relative phase at different junctions that lifts the phase uncertainty, implies the destruction of their coherent motion, which in its turn would have meant that Cooper pairs are not blocked at their respective granules any more.

We would like to emphasize here the crucial role played by charge pinning in the formation of the superinsulating state. It has been already recognized earlier [105] that at very low temperatures the system of vortices transforms into a vortex-superfluid state in which an infinitesimal current induces an infinite voltage, i.e. the system acquires an infinite resistance and should be called a superinsulator. However the fact that in the absence of the charge pinning the charge fluctuations would destroy the superinsulating state was not appreciated in [105].
4. Superinsulation and Berezinskii-Kosterlitz-Thouless transition

As early as in 1963 Salzberg and Prager [106], in the course of their study of thermodynamics of the 2D electrolyte, derived the equation of state and noticed that due to logarithmic interaction between the ‘plus’ and ‘minus’ ions, there appears a singular temperature where the pressure of ions turns zero. They ruled that at this temperature the ion pair formation occurs. In 1970 Berezinskii [107, 108] and later Kosterlitz and Thouless [109, 110] offered more refined consideration to what had become known ever since as the binding-unbinding Berezinskii-Kosterlitz-Thouless (BKT) phase transition. The physical idea behind the transition is as follows [109].

The particle-antiparticle attraction contributes the term of $E_0 \ln(r/r_0)$ into the free energy of the system, where $E_0$ is the energy parameter characterizing the interaction and $r_0$ is the microscopic spatial cut off parameter. At the same time the entropy contribution to the free energy is $-k_B T \ln[(r/r_0)^2]$, as there are $\approx (r/r_0)^2$ ways of placing two particles within the distance $r$ apart. At low temperatures the attraction between the particles and antiparticles wins and they remain bounded. At $T_{\text{BKT}} \approx E_0/(2k_B)$ the entropy term balances the attraction, and at $T > T_{\text{BKT}}$ the particles and antiparticles get unbound. We thus see that the BKT transition is the consequence of the logarithmic interaction between the constitutive entities in two-dimensional systems (2D vortex-antivortex system, 2D charge-anticharge plasma, dislocation-antidislocation array), irrespective to their particular nature. Below we will demonstrate that it is the BKT transition that provides the mechanism by which the 2D JJA falls into either superconducting or superinsulating states at low temperatures. The choice of the particular ground state is dictated by the relation between $E_J$ and $E_C$.[2, 3, 14, 16, 17, 28, 29].

The absence of phase coherence at high temperatures and its appearance at low temperatures in a strong coupling regime, $E_J > E_C$, can be described in terms of vortex-antivortex plasma dynamics and the corresponding BKT transition. In two-dimensional superconductors and Josephson junction arrays finite temperatures $T$ generate fluctuational vortices and antivortices, the number of former and the latter remaining equal in order to conserve “flux neutrality.” At high temperatures, $T > T_{\text{BKT}}$ vortices and antivortices diffuse freely, and since each vortex (antivortex) carries the phase $2\pi$ their motion gives rise to breaking down the global phase coherence. Below the BKT transition temperature vortices and antivortices get bound into the neutral dipole pairs and cannot diffuse independently any more. As the phase gain when encircling a vortex-antivortex dipole is zero, the diffusion of the dipoles as a whole does not cause phase fluctuations on large spatial scales, and the global phase coherence sets. The crucial feature that ensures binding of vortex-antivortex pairs and setting down the phase coherence at low temperatures is the so-called vortex-antivortex confinement: the energy of vortex-antivortex interaction grows (logarithmically) with the separation between them, as long as the vortex-antivortex spacing does not exceed London penetration depth $\lambda = h^2/(2e)^2 \mu_0 E_J$. This condition reflects the restriction of the notion of 2D superconductivity: in the system with the lateral dimensions exceeding $\lambda$, unbound vortices would present at all finite temperatures (except for $T = 0$) and true superconductivity, i.e. the state which possesses the zero resistance below some finite transition temperature is absent.

Now going to the insulating side, where $E_J < E_C$, we note that the charges of Cooper pairs in the planar Josephson junction array also interact according to the logarithmic law [116]: in two dimensions the Coulomb interaction between the charges grows logarithmically with the distance, $r$, separating them, $\tilde{E}_{\text{Coulomb}} \propto \ln(r/r_0)$. Thermal fluctuations generate excessive Cooper pairs with the charge $-2e$ each and the equal number of “anti-Cooper-pairs” (i.e. local deficit of Cooper pairs) with charges $+2e$, to conserve the electro-neutrality. As long as these charge sep-
arations, $r$, does not exceed the electrostatic screening length, $\Lambda = a(C/C_0)^{1/2}$, where $C_0$ is the self-capacitance of a superconducting island (capacitance to the ground) and $a$ being the characteristic size of a single junction, the interaction energy of charges is proportional to $\ln(r/r_0)$, where in case of JJA $r_0 = a$. Therefore, planar JJAs possess a remarkable duality between the logarithmically confined magnetic vortices at the superconducting side and the logarithmically confined Cooper pairs in the insulating state. By the same ‘logarithmic token’ as above the Cooper-pairs – anti-Cooper-pairs 2D plasma undergoes the charge-BKT (CBKT) transition binding, at $T < T_{\text{CBKT}}$, Cooper pairs and “anti-pairs” into the neutral Cooper-pairs dipoles (CPD) that do not carry any charge. The charge confinement at $E_1 < E_C$ can be expressed in terms of the macroscopic Coulomb blockade that impedes the free motion of the Cooper pairs and breaks loose the global phase coherence of the condensate, according to the uncertainty principle, and, thus, gives rise to the superinsulating state of the JJA [12]. In particular, at $T = 0$ the magnetic vortex – Cooper pairs charge duality in JJA leads to a quantum phase transition between superconductivity and superinsulation at the self-dual point where $E_1 \approx E_C$ as was first noticed in [111].

The authors of Ref. [111] considered a two-dimensional Josephson junction array and showed that the zero-temperature behavior of planar JJAs in the self-dual approximation is governed by an Abelian gauge theory with the periodic mixed Chern - Simmons term describing the charge-vortex coupling. The periodicity requires the existence of (Euclidean) topological excitations, which determine the quantum phase structure of the model. Symmetry between the logarithmically interacting vortices and logarithmically interacting charges give rise to a quantum phase transition at the self-dual point. This is the transition between the superconducting phase and the phase with the confined charges, dual to superconductivity and having zero linear conductivity in the thermodynamic limit. This phase was, by analogy with the superconductors, where linear resistivity is zero, was termed “superinsulator” [111]. Therefore the quantum phase transition extensively discussed in the literature and referred to as SIT is in fact superconductor-superinsulator transition.

Turning now to the description of the phase diagram of the JJA in the vicinity of the superconductor-superinsulator transition (Fig. 1), we note first that in the 2D superconductors the symmetry of the phase-charge uncertainty relation can be recast into a picture of the vortex-Cooper pair duality (the concept of quantum superconductor-insulator transition as a transition between the state with pinned vortices and the state with pinned Cooper pairs was first introduced in the pioneering works [112, 113] as a SIT in disordered superconducting film). In this context, superconductivity is viewed as an ensemble of Cooper-pair condensate and pinned vortices. Inversely, the insulator consists of the Bose condensed vortices and of the pinned Cooper pairs. The distinctive feature of the superconductor-insulator transition is that this is the transition at which by its very definition one expects a singularity in transport properties. Thus the symmetry in the ground states is paralleled by the symmetry of the electronic transport [96,114] which can be mediated either by the tunnelling of the fluxons or by the tunnelling of the Cooper pairs. Now the both sides of the phase diagram of JJA near the superconductor-superinsulator transition (see Fig. 1) can be described from the unified viewpoint. If the linear size of the JJA is less then both characteristic screening lengths, the system would undergo both vortex- and charge-BKT transitions (VBKT and CBKT) at temperatures $T_{\text{VBKT}} \approx E_1$ and $T_{\text{CBKT}} \approx E_C$, respectively (the effects of the finite screening lengths on VBKT and CBKT transitions were discussed in [28] and [115]). The VBKT transition marks the transition between the resistive (at $T > T_{\text{VBKT}}$) and global phase coherent superconducting (at $T < T_{\text{VBKT}}$) states. Accordingly, transition at $T = T_{\text{CBKT}}$ at the insulating side separates the ‘high-temperature state,” insulator, where Cooper-pair dipoles
In disordered films $g$ is a conductance in the metallic phase, whereas in the Josephson junction arrays (JJA) it represents the ratio of coupling- and charging energies, i.e. $g = E_I/E_C$, where $E_I$ is the Josephson coupling energy of the two adjacent superconducting islands, and the charging energy $E_C$, the energy cost to transfer a Cooper pair charge $2e$ between the neighbouring islands. In the JJA the condition $g = g_c$ means $E_I = E_C$. At $g > g_c$ the finite value of the modulus of the order parameter $|\Psi|$ appears at the superconducting transition temperature $T_c$. The temperature $T_{CBKT}$ is the Berezinskii-Kosterlitz-Thouless (BKT) transition temperature in the 2D vortex-antivortex plasma. At $T_{CBKT} \approx T < T_1$ the system is in a resistive state since in this temperature range vortex-antivortex pairs are unbound and can diffuse freely breaking down the global phase coherence. At $T < T_{CBKT}$ vortex-antivortex pairs are bound, the global phase coherence establishes and the superconducting state forms. At $g < g_c$ and $T < T_1$ the system turns an insulator and in the range $T_{CBKT} < T < T_1$ it exhibits the exponentially low conductivity due to tunneling transfer of electric charges, $T_{CBKT}$ being the temperature of the charge-BKT transition. At $T < T_{CBKT}$, where the negative and positive charges are bound into dipoles, the system becomes superinsulating. The blue sector of $T > T_0$ and $T > T_1$ corresponds to a metallic state. Both low-temperature states, superinsulating and superconducting, are non-dissipative.

- (a) The sketch of the phase diagram for the superinsulator-superconductor transition in two dimensions in the close proximity to the critical point. A generic diagram is plotted in coordinates temperature ($T$) - critical parameter ($g$).
- (b) The same as (a), for the case where at the critical point all the transition temperatures turn zero.
- (c) The same as (a), for the case where the temperature of the charge-BKT transition. At $T < T_{CBKT}$, where the negative and positive charges are bound into dipoles, the system becomes superinsulating. The blue sector of $T > T_0$ and $T > T_1$ corresponds to a metallic state. Both low-temperature states, superinsulating and superconducting, are non-dissipative.

The insulating phase at $T_{CBKT} < T < T_1$ exhibits thermally activated conductivity $\sigma \propto e^{-\Delta_C/k_B T}$. The charge transfer is mediated by the tunnelling of Cooper pairs, so this state is referred to as a Cooper pair insulator. Not too far above $T_{CBKT}$, where the density of the unbound CPD is still low, $\Delta_C = E_C/\ln(L/a)$ (where $L$ is the linear size of the array, and $L \ll A$) \[11\]. That the activation energy $\Delta_C$ characterizing the insulating behavior exhibits such an unusual and peculiar dependence on the size of the system reflects the fact that the test excessive charge placed locally at any superconducting island polarizes the whole JJA \[16\]. In other words, the characteristic Coulomb energy cost associated with the excitation of an excess Cooper pair is determined by the total system capacitance $C_{tot} \approx C/\ln(L/a)$ \[12\]. Thus, since it is the whole system that participates in building the electrostatic barrier impeding the free charge propagation, we refer to this peculiar Coulomb blockade effect as to a macroscopic Coulomb blockade.

On the experimental side, the size-scaling of the activation energy of the insulator has not been demonstrated on JJAs yet and is waiting for being revealed. At the same time, the scaling of the activation energy as the logarithm of the sample size was recently observed in disordered InO.
films [68] and in TiN films [71]. We will discuss these observations in more details in the Section 1.4 below.

Upon further temperature growth, screening of an excess charge by the free unbound charges becomes more efficient and the characteristic activation energy reduces to the charging energy of a single island, $E_C$. The sketch of the phase diagram showing the possible states in the planar JJA in $g$-$T$, coordinates, where $g = E_J/E_C$ is a critical parameter, is presented in Fig. 1 and generalizes the phase diagram proposed first by Fazio and Schön [16], see also [15, 29]. We expect that the similar diagram describes the behavior of disordered superconducting films. In this case the role of the critical parameter is taken by the dimensionless conductance $g$, and the superinsulator-superconductor transition occurs at the critical conductance $g = g_c$. The metal that forms near $g_c$ in the $T \to 0$ limit has the conductance close to the quantum value $e^2/(2\pi\hbar) = (25.8\, k\Omega)^{-1}$. The corresponding distinct state has been detected via applying strong magnetic field completely destroying superconductivity in disordered films of Be, TiN, and InO, and is often referred to as “quantum metallicity” [59, 65, 54, 62, 69].

An important comment is in order. Two-dimensional systems display perfect superconductivity-superinsulation duality because of the logarithmic interaction of charges in 2D. One can ask whether this duality consideration can be extended onto three dimensional systems and whether one can expect superinsulating behavior in 3D. One can see straightforwardly that while 3D Abrikosov vortices still interact according to the logarithmic law, the charges follow the conventional 3D Coulomb law and therefore charge-anticharge confinement is absent in 3D. So absent is the 3D superinsulating state.

5. Two-dimensional universe in superconducting films

As we have already discussed, Josephson junction arrays with the junction capacitances well exceeding their respective capacitances to the ground are two-dimensional with respect to their electrostatic behavior since most of the electric force lines remain trapped within the junctions themselves. When considering behavior of homogeneously disordered superconducting films, the question can arise, to what extent they can be described by physics derived for JJAs.

The analogy between the planar JJA and the critically disordered thin superconducting films can be perceived from the fact that the experimentally observed in TiN films magnetic-field dependences of the activation energy, $\Delta C(B)$, and the threshold voltage $V_T(B)$ [66] are remarkably well described by formulas obtained in the framework of JJA model [11]. This hints that in the critical vicinity of the SIT a film can be viewed as an array of superconducting islands coupled by the weak links. This incipient conjecture of the early work [47] discussed further in Refs. [58, 51, 91] [66, 67, 68, 69, 47, 74, 75, 12] that near the disorder-driven SIT the electronic phase separation occurs leading to formation of the droplet-like (or island-like) texture, superconducting islands coupled via the weak links, was supported by recent scanning tunnelling spectroscopy (STS) findings [92, 93]. In general, formation of regular textures and, in particular, the spontaneous self-organization of electronic nanometer-scale structures, due to the existence of competing states is ubiquitous in nature and is found in a wealth of complex systems and physical phenomena ranging from magnetism [116, 117, 118, 119, 120], superconductivity and superfluidity to liquid crystals, see Ref. [121] for a review. The arguments that the long-range fields – for example, elastic or Coulomb – can promote phase separation were given in Ref. [122] where the island texture due to elastic interaction between the film and the substrate was found. More refined calculations for the JJA model [123] showed that in the presence of the long-range
Coulomb forces, the SIT could become a first-order transition supporting the idea of possible electronic phase separation at the SIT.

Furthermore, while at the superconducting side of the transition vortices interact logarithmically, thus guaranteeing all the BKT physics to occur, provided the size of the system remains less than the magnetic screening length, the question arises whether the charges in the same film but at the insulating side would also interact according to the logarithmic law. In simple words, in order for the film to demonstrate the 2D behavior, the electric force lines are to be trapped within the film over the appreciable distance. One can derive from the fundamentals of electrostatics that if the test charge is placed within the dielectric film of the thickness \( d \) and with the dielectric constant \( \varepsilon \), the characteristic length over which the electric field remains trapped within the film is of about \( \varepsilon d \). The quantitative description of the Coulomb interaction in a thin film was first given by Rytova [124], the logarithmic asymptotic of the solution and its implications were discussed by Chaplik and Entin [125], and a refined calculation for a film with the large dielectric constant \( \varepsilon_1 \) and \( \varepsilon_2 \) (see the left inset Fig [2]) was done by Keldysh [126]. The dependence of the electrostatic potential of the charge \(-e^*\) on the distance \( r \gg d \), is given by (see Fig. [2]):

\[
\phi(r) = -\frac{e^*}{4\pi\varepsilon_0\varepsilon d} \left[ H_0 \left( \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon} r \right) - N_0 \left( \frac{\varepsilon_1 + \varepsilon_2}{\varepsilon} r \right) \right],
\]

where \( H_0 \) and \( N_0 \) are the Struve and Neumann (or the Bessel function of the second kind) functions, respectively. This formula corresponds to the choice \( \phi(r) \to 0 \) as \( r \to \infty \). In the region \( d \ll r \ll \Lambda \), where

\[
\Lambda = (\varepsilon d)/(\varepsilon_1 + \varepsilon_2)
\]

(2)
is the electrostatic screening length, the electric force lines are trapped by the film, and the interaction energy between the charges \( e^* \) and \(-e^*\) obtained from Eq. (1) acquires the form

\[
V(\vec{r}) = \frac{(e^*)^2}{2\pi\varepsilon_0\varepsilon d} \ln \left( \frac{r}{d} \right), \quad d \ll r \ll \Lambda.
\]

(3)

The universal behavior of the electrostatic potential as a function of the reduced coordinate \( x = [(\varepsilon_1 + \varepsilon_2)/\varepsilon](r/d) \equiv r/\Lambda \) is shown in the Fig. [2] by the solid line. The dashed line shows the logarithmic approximation valid in the spatial region \( d \ll r \ll \Lambda \). The right inset displays the fit of the experimental data on the dependence of the activation energy of the InO films on the size of the sample from [68] by Eq. (3) (the dashed line) according to [11]. The intersection of the line with the x-axis gives the low-distance cutoff of 35 nm which agrees fairly well with the InO film thickness 20±25 nm quoted in [68]. The slope of the fit allows then for determination of \( \varepsilon \) and, taking where \( e^* = 2e \), yields \( \varepsilon \approx 2.3 \times 10^3 \). One can then estimate the charging energy of an ‘effective single junction’ \( E_C \approx (2e)^2/(4\pi\varepsilon_0\varepsilon d) \) and find the value for the \( T_{\text{crst}} \approx E_C/k_B \approx 0.8 \) K for InO films. Using the data for the size-dependence of the activation energy from [71] we can carry out the similar estimates for TiN films as well. The intersection of the linear fit similar to that shown in the right inset of Fig [2] one finds \( d = 3 \) nm which is fairly close to their nominal thickness of 5 nm. Then the slope of the fit gives \( \varepsilon \approx 4 \cdot 10^3 \), and accordingly, \( T_{\text{crst}} ^{\text{TiN}} \approx E_C/k_B \approx 0.06 \) K. The latter estimate is in a pretty good agreement with the experimental findings by [71, 75] that gave \( T_{\text{crst}} ^{\text{TiN}} \lesssim 60 \) mK for the TiN film. One can further estimate the electrostatic screening length \( \Lambda \). Using the parameters determined from the fit and \( \varepsilon_1 = 4 \) for SiO\(_2\) and \( \varepsilon_2 = 1 \), one finds \( \Lambda \approx 240 \mu m \). This macroscopic value of \( \Lambda \) implies that all the
The electrostatic potential (shown as the solid line) induced by the charge $-e^*$ in the film with the dielectric constant $\varepsilon$ placed in between of two media with the dielectric constants $\varepsilon_1$ and $\varepsilon_2$, $\varepsilon_1 + \varepsilon_2 \ll \varepsilon$ (see the left inset) as function of the logarithm of the reduced charge separation $x = [(\varepsilon_1 + \varepsilon_2)/\varepsilon] (r/d)$ according to Eq. (1) in the units of $e^*/(4\varepsilon_0\varepsilon d)$. The dashed line shows the logarithmic asymptote of the potential offering a perfect approximation in the region $d \ll r \ll ed/\varepsilon_1 + \varepsilon_2$. The right inset displays the fit of the data on the dependence of the activation energy of the InO films on the size of the sample from Ref. [68] by Eq. (3) (the dashed line) according to [11]. The intersection of the line with the x-axis gives the low-distance cutoff of 35 nm, which agrees fairly well with the InO film thickness 25 nm quoted in [68]. The slope of the fit allows then for determination of $\varepsilon$.

TiN samples used in experiments [12, 69, 66, 67] having 50 μm width fall well into a domain of validity of a two-dimensional electrostatics. One of the conclusions that follows from our estimates is that the condition $\varepsilon_1 + \varepsilon_2 \ll \varepsilon$ is an important requirement for the observation of the superinsulating behavior; so one has to take a special care of taking the substrate with a reasonably low $\varepsilon_1$. Another comment is that while our results allow for the fairly reliable determination of $\varepsilon$ and are remarkably self-consistent, yet the direct measurements of the film dielectric constant in the critical vicinity of the SIT are highly desirable.

An important feature of the evaluated parameters is that at the first sight the dielectric constant, especially if talking about the ‘conventional’ bulk materials, seem to come out pretty high. However, it ceases to be surprising once we recollect that the experiments were carried out in the close proximity to the SIT. Indeed, in a seminal paper of 1976, Dubrov et al. [127] presented two complimentary considerations based on the percolation theory and on the theory of the effective media showing that the static dielectric constant should become infinite at the metal-insulator transition (see also [128]). The authors were motivated by the experimental finding by Castner and collaborators [129] who revealed the onset of a divergence in the static dielectric constant of $n$-type silicon at the insulator-metal transition (this phenomenon known as polarization catastrophe was predicted as early as in 1927 [130]). Furthermore, Dubrov and collaborators investigated numerically and experimentally the model system, the square conducting network, where each bond was a parallel resistor-capacitor circuit. Breaking randomly the connections between the elements they demonstrated that near the percolation transition that the dielectric constant diverges near the percolation transition in a power-law fashion. In what follows we present the arguments, following the the considerations of Ref. [127] by which the two-phase systems that undergo the transition between the conducting and non-conducting states should exhibit the diverging dielectric constant when approaching this transition. A qualitative picture of this phenomenon is
Figure 3: A sketch of the electronic phase separation in a disordered film. (a) An insulating state, where superconducting regions (shown as white areas) are separated by the insulating spacers. (b) An insulating state on the very verge of the percolation transition: the green stripe highlights the “last” or “critical” insulating interface dividing two large superconducting clusters. The circle marks the critical (“last”) insulating “bond.” (c) The same system at the extreme proximity to the SIT on the superconducting side. As soon as the “critical” insulating bond is broken, the path connecting electrodes through the superconducting clusters shown by the dotted line appears and the film turns superconducting.

The system capacitance $C$ on the insulating side is proportional to the length of the insulating interface separating two adjacent superconducting clusters, which grows near the transition as $b^{1+\psi}$, $\psi > 0$, i.e. faster than $b$. Thus the dielectric constant $\varepsilon \sim C/b$ diverges upon approach to superconductor-superinsulator transition.

illustrated in Fig. 3. The Panel (a) shows the spatial electronic structure of an insulating film in a close proximity to the percolation-like superconductor-insulator transition [131] approaching it from the insulating side. The white areas denote the superconducting regions which are separated from each other by the dark insulating areas. The Panel (b) displays the same film but “one step before” the transition: upon breaking the “last” insulating separation (the circled segment in the panel) between the superconducting clusters, the path connecting electrodes through superconducting clusters emerges and the film becomes superconducting as shown in Panel (c). The dielectric constant is proportional to the capacitance of the system, $C$, which in its turn, is proportional to the total length of the line separating the adjacent critical superconducting clusters. If we let the linear size of the superconducting cluster be $b$, this length would grow as $b^{1+\psi}$ with the exponent $\psi > 0$ (see Ref. [128]). Then the dielectric constant $\varepsilon \propto C/b \propto b^{\psi}$. As the size of the critical cluster diverges on approach the transition, so does the dielectric constant $\varepsilon$ (for more refined derivation of this divergence see [127]). Such a behavior, which is called the “dielectric catastrophe” and is very well known in semiconductor physics, has been observed experimentally (see, for example, Ref. [132], where the $\varepsilon$ divergence was observed near the metal-insulator transition). Therefore one would expect that in the critical vicinity of the superconductor-insulator transition, the dielectric constant grows large enough so that the Coulomb interaction between the charges became logarithmic over the appreciable scale comparable to the size of the system, and the 2D plasma of CPD excitations experiences the charge-BKT transition leading to formation of the low-temperature superinsulating state.

We now turn to the structure of the phase diagram of Fig. 1. At the very transition the dielectric constant $\varepsilon \rightarrow \infty$, and, accordingly, the charging energy $E_C \rightarrow 0$. Therefore, at the self-dual point, $g = g_c$, both characteristic temperatures vanish, $T_{\text{CBKT}} = T_{\text{VBKT}} = 0$. Upon decreasing $g$ and departing from the transition to the insulating side, $\varepsilon$ decreases and $T_{\text{CBKT}} \approx E_C$ grows while going deeper into the insulating side as shown in Fig. 1. Note that the activation energy $\Delta_C = E_C \ln(L/d)$ also grows upon decreasing $g$ (or increasing the resistance of the film)
as \((g_c - g)^\nu\), with the exponent \(\nu > 0\) and so does the crossover line \(T_1 \approx \Delta_C\). The width of the insulating domain in the phase diagram confined between \(T_{\text{crossover}}\) and \(T_1\) increases as well. Upon further decrease in \(g\) and the corresponding decrease in \(\varepsilon\), the electrostatic screening length \(\Lambda \propto \varepsilon d\) shrinks, and as it drops down appreciably below the sample size, the superinsulating domain at the phase diagram ceases to exist.

Having discussed the behavior of the films at the insulating side, we now review, for completeness the superconducting side. Suppression of the superconducting transition temperature, \(T_c\), by disorder in thin superconducting films can be traced back to pioneering works by Shal’nikov [133, 134], where it was noticed for the first time, that \(T_c\) decreases with the decrease of the film thickness. The next important step was made by Strongin and collaborators [42], who had found that the \(T_c\) correlates with the sheet resistance \(R = 3\pi^2\hbar/(\varepsilon^2 k_F^2 d) = 1/g\), much better than with the thickness of the film or its resistivity, and thus has to serve as a measure of disorder in the film.

Since then, numerous experimental works revealed a drastic suppression of \(T_c\), in various superconducting films with growing the sheet resistance, such as Pb [42, 133, 43, 48, 136], Al [46], Bi [42, 43, 46, 49, 51], W-Re alloys [137], MoGe [138, 139], InO [140, 145], Be [55], MoSi [141], Ta [142], NbSi [143], TiN [145, 55, 60, 66, 67, 92, 144], and PtSi [146], to name a few. The observed behavior appeared to be in a good quantitative accord with the theoretical predictions by Maekawa and Fukuyama [147], who first connected quantitatively the reduction in \(T_c\) as compared to its bulk value \(T_{c0}\) with \(R\), and with those of the subsequent work by Finkel’stein [148], who came up with the comprehensive formula for \(T_c(R)\):}

\[
\ln \left( \frac{T_c}{T_{c0}} \right) = \frac{1}{\sqrt{2r}} \ln \left( \frac{1/\gamma + r/4 - \sqrt{r/2}}{1/\gamma + r/4 + \sqrt{r/2}} \right),
\]

where \(r = R\varepsilon^2/(2\pi^2\hbar)\) and \(\gamma = \ln(\hbar/(kBT_{c0}\tau))\). The reduction of \(T_c\) with the increasing sheet resistance according to this formula is shown in Fig. 4 for several values of the product \(T_{c0}\tau\).

The physical picture behind suppressing superconductivity by disorder is that in quasi-two-dimensional systems disorder inhibits electron mobility and thus impairs dynamic screening of the Coulomb interaction. This implies that disorder enhances effects of the Coulomb repulsion between electrons which, if strong enough, breaks down Cooper pairing and destroys superconductivity. Importantly, according to [148], the degree of disorder at which the Coulomb repulsion would balance the Cooper pair coupling is not sufficient to localize normal carriers. Therefore, at the suppression point superconductors transforms into a metal. The latter can transform into an insulator upon further increase of disorder. Therefore this mechanism, which is referred to as a fermionic mechanism, results in a sequential superconductor-metal-insulator transition [148]. However, several experiments [92, 60] demonstrated that in the vicinity of the SIT the dependence \(T_c(R)\) follows perfectly well the Finkel’stein’s formula. We would like to reiterate here that in spite of the fact that Finkelstein’s theory treats superconductor-metal transition, and moreover, the films in the critical region may develop strong mesoscopic fluctuations [87], the experimentally observed \(T_c\) near the SIT where \(T_c \to 0\), is still perfectly fitted by formula (4).

At \(g = g_c\), the superfluid density of the Cooper pair condensate is zero so at the transition point \(T_{\text{vortex}} = 0\). An increase in \(g\) suppresses effects of quantum fluctuations near the transition and, correspondingly, \(T_{\text{vortex}}\) increases. Using the dirty-limit formula which relates the two-dimensional magnetic screening length to the normal-state resistance \(R_N\), Beasley, Mooij, and
Figure 4: Reduced critical temperature $T_c/T_{c0}$ vs. sheet resistance according to Eq. (4) (solid lines) for different parameters $\gamma$. The values $\gamma = 5, 6, 7$ correspond to $\tau = 10.3 \times 10^{-15}$ sec, $4.8 \times 10^{-15}$ sec, and $1.4 \times 10^{-15}$ sec, respectively, at $T_{c0} = 5$ K. The inset shows $T_{\text{VBKT}}/T_c$ vs. resistance BMO relation calculated from Eqs. (5) and (6), which is used to find $T_{\text{VBKT}}/T_{c0}$ shown by the dashed lines.

Orlando (BMO) [9] proposed the universal expression for ratio $T_{\text{VBKT}}/T_c$ (see inset to Fig. 4):

$$T_{\text{VBKT}} = T_c \left( f \left( \frac{T}{T_c} \right) \right)^{-1} = 0.561 \frac{\pi^3}{8} \left( \frac{h}{e^2} \right) \frac{1}{R_N},$$  \hspace{1cm} (5)

$$f \left( \frac{T}{T_c} \right) = \frac{\Delta(T)}{\Delta(0)} \tanh \left( \frac{\beta \Delta(T) T_c}{2 \Delta(0) T} \right),$$ \hspace{1cm} (6)

where $\Delta(T)$ is the temperature dependence of the superconducting gap and parameter $\beta = \Delta(0)/(k_B T_c)$. As it was shown in studies on InO [143] and TiN [144] the BMO formula well agrees with the experiment and correctly describes a decrease in the ratio $T_{\text{VBKT}}/T_c$ with increasing disorder, provided the proper choice of $R_N$ and $\beta$ based on the experimental data is made.

Two comments are in order. Note that the merging tails of $T_{\text{VBKT}}$ and $T_c$, where they both tend to zero in the Fig. 4) is, in fact, the image of the same transition point of the phase diagram of Fig. 1 where the curves were plotted as functions of the conductance. Second, we would like to emphasize here that the behavior of the superconducting films near the SIT is fully controlled by the unique parameter, the sheet resistance.

To conclude here, we demonstrated that close to the superconductor-superinsulator transition the superconducting and insulating sides of the phase diagram are the mirror images of each other with the correspondence between the vortex- and charge-BKT transition. As we have mentioned above, this duality is paralleled by the symmetry of transport properties: the VBKT current-voltage ($I$-$V$) characteristics, $V \propto I^\alpha$, is mirrored by the CBKT $I \propto V^\alpha$ power-law behavior with the interchanging current and voltage [12]. Accordingly, the critical current of a superconducting state setting the upper bound for the loss-free currents which superconductor can still support maps onto the threshold voltage which marks the dielectric breakdown of the superinsulating state and appearance of the finite conductivity. This duality is illustrated by recent studies of the VBKT transition in TiN films [144] which revealed the classical BKT transport behavior: the power-law $V \propto I^\alpha$ current-voltage characteristics with the exponent $\alpha$ jumping from $\alpha = 1$ to $\alpha = 3$ at $T = T_{\text{VBKT}}$, in accordance with the Halperin-Nelson prediction [10], and then rapidly growing with the further decrease in temperature. The measurements of the current-voltage ($I$-$V$)
dependsences on the same material but at the insulating side of the transition \cite{71, 75}, revealed the dual $I$-$V$ characteristics which evolve upon lowering the temperature in precisely BKT-like manner, going from ohmic to power-law, $I \propto V^n$, behavior with with the exponent $n$ switching from $n = 1$ to $n = 3$ at $T = T_{\text{BKT}}$ and then rapidly growing from $n = 3$ to well above the unity with the decreasing temperature which is again an inherent feature of the BKT transition.

The dual similarity between the two states manifests itself further in that both states are the loss-free ones as the condition for the Joule loss $P = IV = 0$ holds both in superconductors and superinsulators. This and the mirror correspondence between the $I$-$V$ curves hold high potential for applications, which we will discuss below in the Section 7.

6. Microscopic mechanism of conductivity in the insulating state

The beginning of extensive studies of role of strong correlations and effects of disorder in the insulating behavior can be traced back to the classical works \cite{150, 151, 152, 153, 154}, see also \cite{155, 156}. Recent years have seen an explosive growth of investigations of the interplay between the many-body and disorder effects leading to formation of non-conventional disorder-induced insulators the properties of which are intimately connected with the localization phenomena \cite{157, 158, 159, 160, 12, 161, 162}. Yet, at present we are able to describe the tunnelling transport in the insulators only with the exponential accuracy – using euristic considerations – as an Arrhenius or Mott and/or Efros-Shklovskii behaviour. The challenge posed by experimental observations, the transition from an activated behaviour in the insulating state to the practically complete suppression of the tunnelling current in the superinsulator remains unmet.

In what follows we will discuss one of the aspects of tunnelling transport in disorder-induced insulators which may be the key to understanding the dynamic insulator-superinsulator transition. Namely, in the case of disorder-induced Cooper pair insulators the charge transfer can be viewed as a tunnelling between the localized Cooper pair sites possessing the essentially different energy levels, see Fig. 5. The tunnelling is possible then only in the presence of some energy relaxation mechanism which is able to accommodate the energy differences between the Cooper pair states at the neighbouring localize sites hosting the tunnelling process, see Fig. 5.

One can propose that an appropriate theory of transport in the insulating state should be constructed via the incorporation of the ideas of relaxation physics into a general model of transport in granular materials. As a starting point one can take a model that had already been successfully used by Efetov \cite{3} to demonstrate the very existence of the disorder-driven SIT. The customary relaxation mechanism in ‘conventional’ conductors is the energy exchange between the tunnelling charge carriers and phonons, comprising a thermal bath. In granular materials, at low
temperatures however, the role of the major relaxation processes, ensuring the tunnelling charge transfer, is taken by emission (and/or absorption) of dipole excitations. Importantly, these excitations are the same particles that mediate the charge transfer. In the Cooper-pair insulators the dipoles are thus made up of the local excess (-2e) and local deficit (+2e) in the Cooper-pairs number and form the bosonic environment [11, 12, 164, 163, 164, 165, 166]. The important features of this dipole excitations environment is that the dipoles are generated in the process of tunnelling and that the dipole environment possesses an infinite number of degrees of freedom, and as such it efficiently takes away the energy from the tunnelling particles and plays the role of the thermostat itself. Eventually, dipole excitations relax energy to the phonon bath. A theory of such a sequential, cascade relaxation in 1D granular arrays was developed in [164, 165, 166].

One of the important conclusions of this theory is that the interaction of the environment with the infinite number of degrees of freedom with disorder gives rise to broadening the levels of the tunnelling carriers. Another important result is that as soon as the spectrum of the environment excitations becomes gapped, the relaxation vanishes and the tunnelling current becomes completely suppressed. One can thus conjecture that it is the localization transition of the environmental excitation spectrum that marks the appearance of the gap in the local spectrum of environmental excitations and thus the suppression of the tunnelling charge transfer.

In a two-dimensional array the dipole excitations form the two-dimensional Coulomb plasma, such a transition, and, respectively, the suppression of the relaxation rate takes place at the temperature of the charge BKT transition, $T_{\text{BKT}} \approx E_C$, where $E_C$ is the average charging energy of a single granule. Below this temperature charges and anti-charges get bound into the neutral CPD, the glassy state forms, the gap in the local density of states of the environmental excitation spectrum appears and the tunnelling current vanishes [12, 164, 165, 166]. Although the detailed analytical calculations in two dimensions are not available at this point, the conjecture that the microscopic mechanisms of suppression of conductivity in one- and two dimensions is of the similar nature and are due to the appearance of the gap in the local density in the CPD excitation spectrum (i.e. localization in the energy space) promises to offer a route towards a quantitative microscopic mechanism behind the formation of the superinsulating state.

Recently, the transition from the insulating to superinsulating phase at low but finite temperature due to suppression of the energy relaxation was also found for the model of superconductor-insulator transition on the Bethe lattice [167, 168].

Remarkably, the employed concept of the relaxation mediated by the emission/absorption of dipole excitation explains also the simultaneous (i.e. occurring at about the same temperature) suppression of both, Cooper pairs and the normal excitations tunnelling currents. However, the detailed study of this appealing topic still remains a challenging task.

### 7. Conductivity of the Cooper-pair insulator and superinsulator: qualitative consideration

Based on the ideas of the cascade relaxation discussed in the previous section we can offer a qualitative description of the temperature evolution of conductivity at the insulating side. A crude estimate can be obtained following heuristic considerations of Ref. [12]. The tunnelling current in an array of superconducting granules can be written in the following form [169, 164, 166]

$$I \propto \exp(-E/W),$$

where $E$ is the characteristic energy barrier controlling the charge transfer between the granules, $W = \hbar/\tau_\omega$, and $\tau_\omega$ is the relaxation time, i.e. the time characterizing the rate of the energy
exchange between the tunnelling charges and the environment. One would expect that the rate of relaxation is proportional to the density of the CPD excitations, which, in its turn, can be taken proportional to the Bose distribution function (CPD excitations are bosons). The width of the local energy gap is $E_C$, therefore one can take
\[ \frac{\hbar}{\tau_w} = \frac{E_C}{\exp(E_C/T) - 1}. \] (8)

In other words, the relevant energy scale characterizing the tunnelling rate is the energy gap that enters with the weight equal to the Bose filling factor describing the probability of exciting the unbound charges. Above the charge BKT transition Eq. (8) gives $W \approx T$; this is nothing but the equipartition theorem telling us that the number of the unbound charges is merely proportional to $T / E_C$.

To complete the estimate we have to evaluate the characteristic energy $E$. In a 2D JJA or in a 2D disordered film in the vicinity of the SIT, where the electrostatic screening length $\Lambda$ is large and exceeds the linear sample size $L$, the characteristic energy $E = E_C \ln(L/a)$, where $a$ is the size of a single Josephson junction. One realizes that well above $T_{\text{CBKT}}$, the logarithmic charge interaction is screened so that $\Lambda \approx a$ and $E$ is reduced to $E_C$. In this temperature region the system exhibits the "bad metal" behavior. However, if one is not too far from the charge-BKT transition, $T \gtrsim T_{\text{CBKT}}$, one still has $\Lambda > L$ and conductivity acquires Arrhenius thermally activated form with the activation energy that scales as $\ln(L/a)$:
\[ \sigma \propto \exp[-E_C \ln(L/a)/T], \quad T \gtrsim T_{\text{CBKT}}. \] (9)

Notably, Eq. (9) looks like a formula for thermally activated conductivity. Yet, one has to remember that the physical mechanism behind the considered charge transfer is quantum mechanical tunnelling process which can take place only if the mechanisms for the energy relaxation are switched on.

Turning now to very low temperatures, $T \ll T_{\text{CBKT}} \approx E_C$, one sees that from Eq. (9) follows that the characteristic energy $W = E_C \exp(-E_C/T)$, i.e. the relaxation rate becomes exponentially low. The unbound charges that have to mediate the energy relaxation from the tunnelling carriers are in an exponentially short supply, and the estimate for conductivity yields:
\[ \sigma \sim \exp[-E_C \ln(L/a) \exp(E_C/T)], \quad T \ll T_{\text{CBKT}}, \] (10)
in accordance with the earlier estimate [11]. One has to bear in mind though that the latter formula has indeed a character of a very crude estimate showing that conductivity at $T < T_{\text{CBKT}}$ is practically zero since the spectrum of the environment excitations acquires the gap $\approx E_C$. In reality, this "zero-conductivity" regime will be shunted at very low temperatures by phonons and quantum fluctuations (the discussion of which we leave for the forthcoming publication) which will yield the finite conductivity.

Let us note that interpolation formula (8) becomes very inaccurate in the close vicinity of the transition, since it does not take into account the fact that the typical distance between the free unbound charges (called the correlation length, $\xi_{\text{CBKT}}$) diverges upon approach the transition as [13, 28]:
\[ \xi_{\text{CBKT}} \approx (1/a^2) \exp\left(\frac{b}{(T/T_{\text{CBKT}}) - 1}\right), \] (11)
where $b$ is a constant of the order of unity. This implies that the density of the environment excitations starts to drop exponentially $\propto \xi_{\text{CBKT}}^{-2}$ and that one has to observe an appreciably increasing
Figure 6: Plots of the logarithm of the sheet resistance \( R \) versus \( 1/T \) taken at two values of the magnetic field, \( B = 0 \) T and \( B = 0.3 \) T (data from [69]) (red filled and blue open symbols, respectfully). The solid lines represent the fit given by Eq. (12) with parameters \( T_{\text{CBKT}} = 0.062 \) mK, \( b = 1 \), at \( B = 0 \) T, and \( T_{\text{CBKT}} = 0.175 \) mK, \( b = 0.5 \) at \( B = 0.3 \) T. The pre-exponential factors are the same for both curves and are taken as \( R_0 = 8 \) kΩ and \( A = 1 \). The dashed straight line corresponds \( T_I = 0.63 \) K. Equation (12) is valid till the square root in the exponent exceeds unity. In the presented data this condition is satisfied at \( T \leq 0.31 \) K for \( B = 0 \) and at \( T \leq 0.35 \) K for \( B = 0.3 \) T. The overlap between the ranges of applicability of Eq. (12) and activated behaviour means that indeed the screening length remains large enough down to \( T \approx 1 \) K. Note that \( T_{\text{CBKT}} = 0.062 \) mK, at zero field well coincides with the above estimate (60 mK) for a similar sample derived from the data on the size-dependent activation energy.

and deviating from its Arrhenius form resistance as a precursor of the charge-BKT when approaching this transition from the above. This should be seen as an upturn in the \( \ln R \) vs. \( 1/T \) curves and one can attribute the hyperactivated behaviour found [69] in the 5 nm thin disordered titanium nitride (TiN) film, which was, by its degree of disorder, in the extreme proximity to the disorder-driven SIT, approaching it from the insulating side. To test it we took \( \ln R(T) \) vs. \( 1/T \) plots from [69] for zero field and \( B = 0.3 \) T and fitted it with

\[
R = R_0 \exp \left( A \exp \sqrt{\frac{b}{T/T_{\text{CBKT}}} - 1} \right),
\]

(12)

following from the fact that at \( T \approx T_{\text{CBKT}} \) should be \( W \propto \xi_{\text{CBKT}}^{-2} \). One sees an excellent agreement of this formula (solid curves) with the experimental data. The dashed line indicates the thermally activated behaviour, and one sees that the upturn smoothly transforms into the Arrhenius regime.

As early as in '90-s, an upturn in the \( \log R(T) \) vs. \( 1/T \) dependence indicating the “stronger than activation” behavior was observed in JJAs [22, 21, 25]. They also viewed it as a precursor of the charge-BKT behaviour and fitted it with \( R \propto \exp \left( A \exp \sqrt{b/(T/T_{\text{CBKT}})} - 1 \right) \) analogously to the procedure adopted for description of the vortex-BKT data [28]. We have found that such a procedure also gives a reasonable fit to the hyperactivated behaviour data of Ref. [69], with
slightly different fitting parameters. This is not surprising since we work in the temperature range which does not include the extreme proximity to $T_{CBKT}$, i.e. in the range where $\xi_{CBKT}$ is not excessively large.

8. Applications

The existence of the superinsulating state opens a route for a new class of devices for cryogenic electronics, utilizing the duality between the superinsulation (SI) and superconductivity (SC). The left panel of Fig. 7 sketches the dual-diode current-voltage ($I-V$) characteristics of a film or Josephson junction array corresponding to superconducting and superinsulating states respectively. Application of a moderate voltage $V$ below the threshold voltage $V_T$ realizes the 'logical unit' operational mode of the device. When in a superconducting state ('on' regime), the device carries a loss-free supercurrent, in the superinsulating state ('off' regime) the current is blocked since $V < V_T$. Building the superswitch into an electric circuit, one designs a binary logical unit, where (0) corresponds to, say, the SI zero-current and (1) corresponds to the current-carrying SC state. Note that in both, 'on' and 'off' regimes the superswitch does not lose any power, remaining non-dissipative in both operating states.

Superinsulators can serve as a working body of a detector or bolometer (see Fig. 7). To realize this mode one applies the bias $V$ less but close to $V_T$ and takes $T \lesssim T_{SI}$ as a working temperature. Heating by an irradiation shifts $V_T$ below the voltage of the working point and the current jump over several orders of magnitude occurs. The choice of $T \lesssim T_{SI}$ as a working point ensures that the current jump is controlled by the heating instability solely, eliminating effects of disorder and imperfections, which would randomly shifted the voltage at which the switching to the 'hot' branch takes place if the working point were chosen deep in the superinsulating state. The important feature of the prospective superinsulator-based electronic devices is that one can expect their sensitivity to be very high since the current in the closed mode is completely blocked.

The switching between the regimes, i.e. between the superconducting and superinsulating states, can be implemented with the aid of the magnetic field which modulates the Josephson coupling $E_J$, changing thus the actual $E_J/E_C$ ratio. The Fig. 7(b) displays the energy-magnetic field phase diagram for a particular case of the square JJA array, the $E_J(B)$ dependence for which was calculated in Ref. [170] in the nearest neighbors approximation.

From the point of view of applications the most appealing device would be the one that could be operated by means of the electric field. The technologically attractive possibility for implementation of such a device is to construct a planar Josephson junction array in which the electric field tunes the ratio $E_J/E_C$, and, therefore, switches the device between the SI and SC states. A practical design of such a superswitch which is realized by utilizing the field effect transistor principle, a superconductor-superinsulator field effect transistor (SSFET) is shown in Fig. 8. In this device in the absence of voltage at the gate the superconducting islands are decoupled due to high tunnelling resistance, $R_T$, of the dielectric separating them, and the array is in a superinsulating state. Applying the sufficient gate voltage the insulating channels can be turned conducting (or, at least, their $R_T$ becomes reduced). This will give rise to increase of the ratio $E_J/E_C$ and will drive the array in a superconducting state. Thus, the proposed switching technique is built on changing the conductivity of the medium confined between the adjacent superconducting islands by tuning the gate voltage.

The first steps on the realization of the electrostatically-driven SIT have already been undertaken. The system of the superconducting islands placed on the graphene layer was used in the recent work [171] in order to tune the $E_J/E_C$ ratio as proposed above and the pronounced SIT
Figure 7: Switching between superconducting and superinsulating states. (a) Exemplary dual threshold current-voltage ($I$-$V$) characteristics in the superconducting and superinsulating states of a 5 nm thin TiN film in the double-log coordinates. The current and voltage are measured in units of the critical current, $I_c$, and the threshold voltage, $V_T$, respectively. The lower inset illustrates switching between the ‘off’ (superinsulating) and ‘on’ (superconducting) states in the logic unit operation mode. The upper inset illustrates the sensor (bolometer) mode utilizing the threshold character of the $I$-$V$ characteristics in the superinsulating state: when some pre-threshold voltage is applied to the film it remains superinsulating state with the zero current. As soon as the film is heated by some irradiation, the threshold voltage decreases below the applied value and the current jump over the six orders of magnitude occurs. An advantage of using a superinsulator as the working body for a sensor over the superconductor-based sensor is that due to huge resistance the current in the superinsulating state is extremely low, and, therefore, superinsulator is less sensitive to background noise. (b) A sketch of the magnetic field controlled switching between two non-dissipative states in a square Josephson junction array (JJA). The line separating the superinsulating and superconducting domains depicts the dependence of Josephson coupling energy, $E_J$ (measured in the units of the Josephson coupling energy, $E_{J0}$, in the zero magnetic field), on the applied magnetic field, $B$. The arrow shows the evolution of the system, with the ratio $E_C/E_{J0}$ taken slightly less than unity, where $E_C$ is the charging energy of a single junction. If $E_C/E_{J0} < 1$, the system is a superconductor, and if $E_C/E_{J0} > 1$, the system is superinsulator in the zero magnetic field. Upon increasing field, the system evolves along the arrow and, starting from the superconducting state transits into the superinsulator as soon as it crosses the line $E_J(B)$, i.e. as soon as $E_J(B)$ becomes less than the charging energy $E_C$.

with the resistance as high as $10^7 \Omega$ at 40 mK was observed in the insulating state. The change of the resistance as a function of the gate voltage was about seven orders of magnitude.

The story, however, would have been incomplete, without mentioning the approach based on the electric field-induced change of the properties of the superconducting films themselves [172]. Such an approach, a direct varying of the charge carrier density in the superconducting film by applying a gate voltage in a field effect transistor configuration was taken in many works [173, 174, 175, 176, 177, 178, 180, 82]. At present two kinds of materials are used as gate insulators: SrTiO$_3$ (STO) crystal and/or various electrolytes (or ionic liquids). We would like to note that STO is an insulator with the very high dielectric constant $\varepsilon \sim 20000$. Such a dielectric sandwiching or covering the superconducting film is detrimental to formation of the superinsulating state since it catastrophically reduces the electrostatic screening length, see Eq. (2). Indeed, in none of the quoted works the sheet resistance in the non-superconducting state does not exceed 40 K$\Omega$, thus the very possibility of the transition into an insulating state remains to be investigated. Turning to the electrolytes as gate insulators, their disadvantage, from the viewpoint of the practical applications, is that the operating mode requires the thermocycling up to room temperatures for changing the gate voltage and influencing the properties of superconducting

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9. Results

Our results can be summarized as follows:

1. On the fundamental level, the phenomenon of superinsulation rests on the duality between superconducting vortices and charges in two-dimensional systems experiencing the superconductor-insulator transition. At the superconducting side logarithmic interaction between vortices (i.e. vortex confinement) leads to superconductivity, the state with the zero linear resistance below the vortex-BKT transition where global phase coherence establishes. Correspondingly, the logarithmic interaction between charges (charge confinement) at the insulating side leads to a superinsulating state with the zero linear conductivity. This duality follows from the Heisenberg uncertainty principle: at the superconducting side the phase uncertainty is zero implying that charges that comprise Cooper pair condensate move without scattering. At the superinsulating side all the charges are pinned below the charge-BKT transition implying, according to the Aharonov-Casher effect, the coherent quantum tunnelling of all the fluxons (the synchronized quantum slips of globally uncertain phase) i.e. that fluxons are in a superfluid state.

2. The condition for a charge confinement i.e. logarithmic growth of the interaction between the charges as a function of the distance between them in the insulating state of a planar Josephson junction array is realized if the electrostatic screening length exceeds the linear size of the array. This requires that a single junction capacitance well exceeds the capacitance to the ground. This logarithmic interaction leads to the logarithmic scaling of the activation energy of the Cooper pair insulator in Josephson junctions array with the linear size of the array. We refer to this phenomenon of emergence of the large energy scale controlling the activated behaviour and growing with the size of the system as to a macroscopic Coulomb blockade.
3. Disordered thin films in the critical vicinity of the superconductor-superinsulator transition have huge, diverging on approach to the transition to the superconducting state, dielectric constant. This leads to the two-dimensional logarithmic Coulomb interaction between the unscreened charges in the film, which, in its turn results in the charge Berezinskii-Kosterlitz-Thouless transition. The logarithmic interaction holds as long as the distance separating charges does not exceed the electrostatic screening length. Accordingly the CBKT transition is most pronounced in the systems whose linear size does not exceed this length.

4. One can expect that in the critical vicinity of the superconductor-superinsulator transition where long-range Coulomb interactions become relevant, electronic phase separation can occur and the texture of weakly coupled superconducting islands may form. This indicates that near the transition the transport behaviour of disordered films is analogous to that of planar Josephson junctions arrays.

5. Logarithmic interaction between charges in the Cooper-pair insulating phase of the two-dimensional Josephson junction array is dual to the logarithmic interactions between the vortices at the superconducting side. This implies the dual phase diagram in the vicinity of the superconductor-superinsulator transition: Transformation of the resistive state into superconductivity possessing the global phase coherence occurring at $T_{\text{CBKT}}$ is mirrored by the transition from the insulator to superinsulator at the charge BKT transition temperature $T_{\text{CBKT}}$. We identify a superinsulator as a low-temperature charge-BKT phase, possessing infinite resistivity in the finite temperature range in the same sense as the low-temperature vortex BKT phase is a superconductor having infinite conductivity in a finite temperature range as well.

6. In the Cooper-pair insulator the current occurs via tunnelling of the Cooper pairs across the Josephson links. To ensure tunnelling transport in the random system, the energy relaxation mechanism providing the energy exchange between the charge carriers and some bosonic environment is required. At low temperatures tunnelling in Josephson junction array is mediated by the self-generated bosonic environment of the Cooper-pair dipole excitations, comprised of the same Cooper pairs that tunnel and mediate the charge transfer. Importantly, being a system with the infinite number degrees of freedom, the dipole environment plays itself the role of the thermostat. Opening the gap in the local dipole excitations spectrum suppresses energy relaxation and impedes the tunnelling current. In the two-dimensional Josephson junction array this gap appears below $T_{\text{CBKT}}$. We expect that the gap in the local dipole spectrum is associated with formation of a low-temperature glassy state. This constitutes the microscopic mechanism of superinsulation.

10. Conclusions

In our work we have shown that the existence of the superinsulating state is a fundamental phenomenon intimately connected with the very existence of superconductivity and that it follows from the symmetry of the uncertainty principle setting a competition between the conjugated quantities, the charge and the phase of the wave function of the Cooper pair condensate. This symmetry can be expressed in terms of the charge-vortex duality in two-dimensional superconducting systems and manifests itself via the reversal between the Aharonov-Bohm and the Aharonov-Casher effects. We have described the conditions for emerging of superinsulation in real experimentally accessible two-dimensional systems, lateral Josephson junction arrays and...
highly disordered thin superconducting films and demonstrated that the critical component of the phenomenon is the high dielectric constant of the insulating phase of the system in question which develops in the close vicinity of superconductor-insulator transition. The latter ensures the logarithmic interaction between the charges and brings with it the effect of the macroscopic Coulomb blockade, which, in its turn manifests as a size-dependent characteristic energy that controls activation processes in the insulating phase. We examined these concepts testing them on the original experimental findings and demonstrated that they work perfectly well offering self consistent physical picture describing collective behaviour of two-dimensional superconducting systems in the vicinity of superconductor-insulator transition. We constructed and analysed the phase diagram for a planar Josephson junction array and/or disordered superconducting film in the vicinity of the superconductor insulator transition and identified superinsulating state as a low temperature charge-BKT phase. And, finally, we discussed the perspectives for practical applications of the superinsulating systems as a working body for new generation of electronic devices utilizing the duality between superinsulation and superconductivity.

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