Joint Transmission Mode and Tilt Adaptation in Coordinated Small-Cell Wireless Networks

Nima Seifi, Robert W. Heath Jr., Mikael Coldrey, and Tommy Svensson

Abstract

We focus on downlink transmission in a wireless network consisting of small-sized cells, denoted as a small-cell network (SCN). We consider multi-antenna base stations (BSs) and single-antenna users, BS coordination, downlink training and channel estimation, 3D antenna patterns, distant-dependent pathloss, and fair scheduling. We investigate the impact of the variable parameter that controls the elevation angle of the BS antenna pattern, denoted as tilt, on the performance of the considered SCN when employing either a conventional uncoordinated transmission mode or a fully coordinated transmission mode. Using the results of this investigation, we propose a novel hybrid-mode transmission technique that can achieve a performance comparable to that of a fully coordinated transmission but with a significantly lower complexity and signaling requirement. The main idea is to divide the coverage area into two so-called vertical regions and jointly adapt the transmission mode and the tilt at the BSs when serving each region. A fair scheduler is used to share the time-slots between the vertical regions and among the users in each region. To make the performance analysis computationally efficient, analytical expressions for the user ergodic rates at different transmission modes are also derived.

Index Terms

Antenna tilt, ergodic rate, imperfect channel state information, base station coordination.

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I. INTRODUCTION

Increasing the area spectral efficiency of wireless networks requires a denser deployment of infrastructure and the use of more aggressive frequency reuse. A flexible approach for densifying cellular networks is to deploy self-organizing, low-power, and low-cost base stations (BSs), resulting in a so-called small-cell network (SCN) [1]. Because the number of cell edges increases, intercell interference (ICI) management has a crucial role in successful operation of SCNs.

Coordination among clusters of BSs is an efficient way to combat ICI [2]. In the most aggressive form of BS coordination, the channel state information (CSI) and the data of users are fully shared among BSs via high-speed backhaul links. These BSs then act as a single distributed multi-antenna transmitter that serves multiple users through beamforming, commonly referred to as network multiple-input multiple-output (MIMO). Although network MIMO can completely eliminates the ICI within the BSs’ coverage area, the system level performance is not necessarily better because of increased number of pilot signals required for CSI training, denoted as the training overhead [3], [4]. In frequency-division duplexing (FDD) systems, such an overhead can degrade the performance of network MIMO even below that of conventional uncoordinated transmission when the number of coordinating antennas becomes very large [5].

A less-complex approach for managing ICI is through exploiting the variable parameter that controls the vertical angle of the BS antenna pattern, commonly referred to as antenna tilt or simply tilt. By appropriately selecting the tilt, it is possible to increase the desired signal level at an intended user, while reducing ICI towards a non-intended user. In conventional cellular networks typically a fixed tilt is used at all BSs over all time-slots. This tilting strategy, denoted as cell-specific tilting, can not adapt to the particular locations of the scheduled users. Therefore, users at different locations of the cell experience different antenna gains. For example, users close to the peak of the main beam observe a high antenna gain, while those close to the side-lobes experience a low antenna gain. BS tilt can be adjusted electronically by changing the phase of the antenna excitation, a.k.a. remote electrical tilting [6]. Hence, the tilt adjustment can
potentially be done rapidly, which makes the tilt an important optimization parameter in many of the emerging technologies such as self-organizing networks [7], [8]. Thus far, the joint gains of performing both adaptive tilting and BS coordination has not been established.

In this paper, we propose a systematic transmission framework in an SCN which exploits BS coordination together with BS antenna tilting to manage the ICI experienced by users and to optimize the throughput distribution over the coverage area. The key idea consists of partitioning the coverage area into disjoint “vertical” regions and optimizing both the transmission mode and the tilt to serve the users in each vertical region. Our analytical and numerical results provide useful insights which are essential for the design of practical coordination strategies for SCNs.

A. Related Work

Due to the training overhead bottleneck of network MIMO, recent work has focused on transmission techniques that switch between conventional uncoordinated transmission and network MIMO depending on the users’ locations (see e.g. [4], [9]). Prior work, however, considers a 2D network model, ignoring the impact of vertical plane of the antenna pattern. Tilt is an important component for ICI management, yet rapid tilt adjustment has not been extensively studied in conjunction with BS coordination. In SCNs, the observed antenna gains at the users are comparable to their pathloss values owing to the reduced access distances to the BSs [10], [11]. Therefore, antenna tilt has a more noticeable impact on users’ performance in such networks.

Recent work on antenna tilt optimization has focused on a specific adaptive tilting strategy known as switched-beam tilting [6], [12]–[14]. In this method, first the coverage area is divided into vertical regions. Each BS then serves one of the vertical regions at each time-frequency resource block using one out of a finite number of fixed tilts. With this strategy, it is possible to increase the received signal power at a specific region in the desired cell, suppress the ICI at the neighboring cells, or a combination thereof. The work in [6], [12], [13] is based on system-level simulations and does not provide any design guidelines, e.g., how to determine vertical regions’ borders, how to choose the antenna tilts for different regions, or how to fairly
schedule the transmission over different regions. In [4], the authors provided a framework for forming vertical regions and applying switched-beam tilting in an isolated cell, but ICI was not considered. Furthermore, only heuristic approaches were used to determine vertical regions’ boundaries and the tilt value to be applied in each region.

One of the challenges in analyzing cellular network performance in the presence of tilt is the lack of analytical performance measures. Available techniques for the ergodic rate analysis of conventional uncoordinated MIMO systems mostly assume channel vectors with independently and identically distributed (i.i.d.) elements (see [15] and [16] and references therein). In [15], closed-form expressions for user ergodic rate under imperfect CSI were obtained in a single-cell setup without considering pathloss. Closed-form approximations for user ergodic rates with perfect CSI were derived in [16] for a multicell setup with a realistic pathloss model.

Available techniques for i.i.d. MIMO channels cannot be directly applied in the network MIMO setting. In this case each user might experience a different pathloss to each BS, and hence the elements of its aggregate channel vector to all BSs are non-identically distributed in general. In [11], [17], [18], using the results from random matrix theory, a large system approximation for the ergodic sum-rate was derived for the uplink of a network MIMO system. Closed-form approximations for the user ergodic rate in downlink network MIMO transmission were derived in [16]. A limitation of [11], [16]–[18] is that they assume perfect CSI at all BSs. In [4], a lower bound for the user ergodic rate under imperfect CSI was obtained for the special scenario in which users at fixed and symmetric set of locations in different cells are served. This symmetry causes the users to be statistically equivalent, i.e., they experience the same set of pathloss values to all the BSs, which simplifies the ergodic rate analysis [5]. In practice, however, users are usually placed in asymmetric locations, which makes the ergodic rate analysis challenging.

B. Contributions

In this paper, our aim is to exploit various system dimensions including BS antenna tilting, scheduling, and transmission mode switching to achieve a performance close to that of a fully
coordinated network, but with significantly lower complexity and signaling overhead. The main contributions of the paper are summarized as follows.

1) **We investigate the impact of tilt on the performance of an SCN with different transmission modes.** We incorporate a 3D antenna pattern at the BSs and focus on cell-specific tilting. We find the optimal tilt that maximizes some desired functions of user throughput for both conventional uncoordinated transmission and network MIMO transmission.

2) **We propose a hybrid-mode transmission technique that jointly adapts the transmission mode and the antenna tilt.** In the proposed transmission strategy BSs jointly switch their transmission modes and their applied tilts when serving different regions of the coverage area. Our simulation results show that the proposed technique outperforms the conventional uncoordinated transmission while achieving comparable performance to that of network MIMO transmission with a significantly less signaling requirement.

3) **We provide an analysis of the ergodic rate with imperfect CSI.** To make the analysis computationally efficient, we extend the results of [16] to the practical case of imperfect CSI and derive analytical expressions for user ergodic rates at different transmission modes. In particular, to simplify our derivations, we propose a novel method to approximate the non-i.i.d. network MIMO channel with an i.i.d. MIMO channel.

C. Organization

The rest of the paper is organized as follows. Section II describes the system model including antenna patterns, received signal, transmission modes, and channel estimation. Beamforming techniques together with the ergodic rate expressions are presented in Section III. A new i.i.d. approximation for the network MIMO channel is introduced in Section IV. Analytical expressions for user ergodic rate at different transmission modes are obtained in Section V. Section VI studies cell-specific tilting for different transmission modes. The proposed hybrid-mode transmission technique is presented in Section VII and conclusions are drawn in Section VIII.
D. Notation

Scalars are denoted by lower-case letters. Vectors and matrices are denoted by bold-face lower-case and upper-case letters, respectively. $\text{Tr}(\cdot), (\cdot)^{\text{H}},$ and $\log_2(\cdot)$ are trace operator, complex conjugate transpose, and 2-base logarithm, respectively. $\mathbb{E}[\cdot]$ denotes the statistical expectation. $|S|$ is the cardinality of a set $S$. The distributions of a real and a complex Gaussian random variable (RV), each with mean $0$ and variance $\sigma^2$, are respectively denoted by $\mathcal{N}(0,\sigma^2)$ and $\mathcal{CN}(0,\sigma^2)$. $\|x\|$ is the Euclidean norm of a complex vector $x$. $\text{Re}(z)$ and $\text{Im}(z)$ are the real and imaginary parts of a complex number $z$, respectively.

II. System Model

We consider downlink transmission in an SCN consisting of a cluster of $B$ adjacent cells. Each cell has a BS located at a height $h_{\text{bs}}$ above the ground and is equipped with an array of $N_t$ antennas. We index all the $B$ cells in the network and their associated BSs by unique indices $b = 1, \ldots, B$. There are $K$ users uniformly distributed over the coverage area and uniquely indexed as $k = 1, \ldots, K$. Each user is at a height $h_u$ above the ground and has a single antenna. An example of an SCN consisting of three romb-shaped cells is shown in Fig. 1 where each cell has a BS with $N_t = 3$ antennas installed at one of its vertices and $K = 7$ users are uniformly distributed over the coverage area. We use this network configuration as an instructive example throughout the paper without loss of generality.

A. Antenna Radiation Pattern

For the antenna at the user, we assume a 3D isotropic unit-gain pattern. The radiation pattern of the antennas at the BS array is assumed to follow the 3D model proposed by 3GPP in [19] unless otherwise specified. According to this model, the observed antenna gain at user $k$ from
Fig. 1. An example of a 3D network consisting of 3 adjacent romb-shaped cells with schematic illustration of spherical angles.

BS $b$ is expressed in dBi as

\[
G_{k,b}^{\text{dBi}}(\beta_b) = - \min \left( \min \left( 12 \left( \frac{\phi_{k,b} - \psi_b}{\Phi_{3\text{dB}}} \right)^2, \text{SLL}_{\text{az}} \right), \min \left( 12 \left( \frac{\theta_{k,b} - \beta_b}{\Theta_{3\text{dB}}} \right)^2, \text{SLL}_{\text{el}} \right) \right) + \text{SLL}_{\text{tot}} + G_m,
\]

where $\phi_{k,b}$ denotes the horizontal angle measured between the x-axis and the line in the horizontal plane connecting user $k$ to BS $b$, and $\theta_{k,b}$ is the vertical angle measured between the horizon and the line connecting user $k$ to BS $b$. In addition, $\psi_b$ represents the fixed orientation angle of BS $b$ array boresight relative to the x-axis, while $\beta_b$ denotes the tilt of BS $b$ measured between the horizon and the line passing through the peak of the main beam. A schematic illustration of the spherical angles is shown in Fig. 1. Throughout the paper, we focus on transmission strategies in which all BSs use the same tilt, denoted as $\beta$, and hence we set $\beta_1 = \cdots = \beta_B = \beta$. We, however, highlight that the provided analysis can be readily applied to scenarios that apply different tilts at different BSs. These scenarios are beyond the scope of this paper and are left to
our future work. Moreover, \( \text{SLL}_{\text{az}} = 25 \) dB and \( \text{SLL}_{\text{el}} = 20 \) dB are the side lobe levels (SLLs) in the horizontal and vertical planes of the BS antenna pattern, respectively, while \( \text{SLL}_{\text{tot}} = 25 \) dB is used to indicate the overall SLL floor. The half-power beamwidth (HPBW) in the horizontal and vertical planes are respectively denoted as \( \phi_{3\text{dB}} = 65^\circ \) and \( \theta_{3\text{dB}} = 6^\circ \). Finally, \( G_m = 0 \) dBi denotes the peak antenna gain. Note that the model in (1) is a simplified version of the commonly used Kathrein 742215 antenna, which assumes a constant gain outside the main lobe instead of explicit side lobes [20].

### B. Received Signal Model

We focus on universal frequency reuse and a narrowband frequency-flat fading channel. The complex base-band received signal at user \( k \) can be expressed as

\[
y_k = \sum_{b=1}^{B} \sqrt{\alpha_{k,b}(\beta)} h_{k,b}^H x_b + n_k,
\]

where \( \alpha_{k,b}(\beta) \) denotes the path gain between user \( k \) and BS \( b \), \( h_{k,b} \in \mathbb{C}^{N_t \times 1} \) denotes the small-scale fading channel vector between user \( k \) and BS \( b \), \( x_b \in \mathbb{C}^{N_t \times 1} \) is the transmitted signal from BS \( b \), and \( n_k \) indicates the normalized additive white Gaussian noise (AWGN) distributed as \( \mathcal{CN}(0, 1) \). For simplicity of the analysis, we assume that the elements of \( h_{k,b}, \forall k,b \), are i.i.d. \( \mathcal{CN}(0, 1) \). This assumption corresponds to an uncorrelated Rayleigh fading across users, BSs, and antennas per BS. Moreover, we use a standard model for \( \alpha_{k,b}(\beta) \) given by [19]

\[
\alpha_{k,b}(\beta) = L_{k,b} G_{k,b}(\beta),
\]

where \( L_{k,b} \) captures the distant-dependent pathloss between user \( k \) and BS \( b \), while \( G_{k,b}(\beta) \) indicates the observed antenna gain at user \( k \) from BS \( b \) and is given in dBi scale in (1).

### C. Downlink Transmission Modes

In this work, we consider two well-known transmission modes that are adopted by the BSs when serving users:
Conventional Baseline Transmission (CBT): In this mode of transmission, each user is associated with one of the BSs to which it experiences the maximum average received power, denoted as the *home* BS. The data to each user is transmitted by its home BS, while the transmissions from other BSs are treated as ICI. This transmission strategy results in an *uncoordinated* network where adjacent cells operate independently and interfere mutually.

Network MIMO Transmission (NMT): In this transmission mode, the data and the CSI of all users are shared among the BSs through high-speed backhaul links. The BSs then act as a single distributed multi-antenna transmitter with $BN_t$ antennas to jointly serve the users in the coverage area. This transmission technique leads to a fully *coordinated* network in which the ICI is completely removed.

D. Downlink Training and Channel Estimation

We adopt a standard block-fading model [5], [21] in which channel vectors change independently from time-slot to time-slot. A time-slot corresponds to a block of length $T$ channel uses over which the small-scale fading coefficients are constant. We further focus on pilot-assisted downlink training and channel estimation in an FDD system. Each time-slot is divided into a training period of length $\tau$ channel uses and a data transmission period of length $T - \tau$ channel uses. At the beginning of each time-slot, each BS broadcasts some pilot symbols over a duration of $\tau$ channel uses to enable users to estimate their channel vectors to that BS.

In general, estimating an $M$-dimensional downlink channel vector requires at least $M$ orthogonal training sequences [22]. In CBT, each user needs to estimate its channel vector only to its home BS. Hence, to minimize the training overhead we assume that the same training sequences are re-used in each cell, thus $M \geq N_t$. In NMT, however, each user needs to estimate its channel vectors to all the BSs, thus $M \geq BN_t$, which means a larger training period compared to CBT. Details on the training and channel estimation in CBT and NMT can be found in, e.g., [22] and [5], respectively, and are not repeated here for brevity. We only mention the main results that will be used in the rest of the paper. Assuming MMSE channel estimation, the canonical
decomposition for the channel vector between user \( k \) and BS \( b, \) i.e., \( h_{k,b}, \) can be expressed as

\[
h_{k,b} = \hat{h}_{k,b} + e_{k,b}.
\] (4)

In (4), \( \hat{h}_{k,b} \sim \mathcal{CN}(0, \kappa_{k,b}^2(\beta)I) \) denotes the estimated channel vector of user \( k \) from BS \( b, \) and \( e_{k,b} \sim \mathcal{CN}(0, \sigma_{k,b}^2(\beta)I) \) is the estimation error vector such that \( \kappa_{k,b}^2(\beta) = 1 - \sigma_{k,b}^2(\beta). \) The estimation error variance is given by

\[
\sigma_{k,b}^2(\beta) = \frac{1}{1 + \frac{\alpha_{k,b}(\beta)}{\gamma^{-1} + \sum_{b' \in P_b} \alpha_{k,b'}(\beta)}},
\] (5)

where \( \gamma = MP/N_t \) and \( P_b \) denotes the set of BSs that use the same training pilots as BS \( b. \) Note that in NMT we have \( P_b = \emptyset, \forall b. \) Each user estimates either its channel vector to its home BS in CBT or its channel vectors to all BSs in NMT and feeds back the estimated channel vector(s) to its home BS. We assume genie-aided feedback links which deliver the estimated channel vector(s) to the BSs perfectly. For the case of NMT, we further assume that these vectors are shared among BSs over error- and delay-free backhaul links to enable beamformer design as explained in the next section.

### III. Multiuser MIMO Beamforming and User Ergodic Rates

With a 3D directional antenna pattern both the horizontal and vertical planes can be used for performance optimization. In this paper, we focus on a special case in which the vertical plane of the antenna pattern only controls the antenna gain via changing the tilt, while the horizontal plane is used for multiuser MIMO beamforming. In particular, for beamforming design the estimated channel vectors at the BSs are treated as if they are perfect. Moreover, we consider equal power allocation among users to simplify the analysis\(^1\). This section reviews the principles of linear multiuser MIMO zero-forcing beamforming in CBT and NMT, and presents the expressions for the user ergodic rate at each transmission mode.

\(^1\)Although an important issue, spatial power allocation is beyond the scope of this paper and is left to our future work.
A. Beamforming and Ergodic Rates in CBT

Let \( \mathcal{K} = \{1, \ldots, K\} \) denote the set of all users in the coverage area. In addition, \( \mathcal{K}_b \subseteq \mathcal{K} \) is the set of users associated with BS \( b \) under CBT such that \( |\mathcal{K}_b| \leq N_t \), \( \bigcup_{b=1}^{B} \mathcal{K}_b = \mathcal{K} \), and \( \mathcal{K}_b \cap \mathcal{K}_{b'} = \emptyset, \forall b \neq b' \). We consider the case of \( |\mathcal{K}_b| > N_t \) when we discuss user scheduling in Section VII-B. Moreover, let \( \hat{H}_b \in \mathbb{C}^{N_t \times |\mathcal{K}_b|} \) be the channel matrix having the estimated channel vectors of the users in cell \( b \), i.e., \( \{\hat{h}_{k,b}\}_{k \in \mathcal{K}_b} \), as its columns. With only the knowledge of \( \hat{H}_b \) at BS \( b \), the unit-norm zero-forcing beamformer for user \( k \in \mathcal{K}_b \), denoted as \( w_{k,b} \in \mathbb{C}^{N_t \times 1} \), is obtained by first computing the Moore-Penrose pseudo-inverse of \( \hat{H}_b \) as \( \hat{H}_b^\dagger = \hat{H}_b (\hat{H}_b^H \hat{H}_b)^{-1} \) and then taking the normalized column of \( \hat{H}_b^\dagger \) corresponding to user \( k \). Under zero-forcing dimensionality constraint \( |\mathcal{K}_b| \leq N_t \), it holds that \( \hat{h}_{k,b}^H w_{j,b} = 0 \) for \( j \neq k \) and \( j \in \mathcal{K}_b \). The transmitted signal \( x_b \) in (7) can now be expressed as

\[
x_b = \sum_{k \in \mathcal{K}_b} w_{k,b} d_{k,b},
\]

where \( d_{k,b} \) denotes the data symbol for user \( k \). We further assume that the transmitted signal from each BS is subject to a power constraint \( P \), i.e., \( \mathbb{E} [\text{Tr}(x_b x_b^H)] = P, \forall b \). Using the canonical decomposition in (4), we can now re-write the received signal for user \( k \in \mathcal{K}_b \) in (2) as

\[
y_k = \alpha_{k,b}(\beta) \hat{h}_{k,b}^H w_{k,b} d_{k,b} + \sum_{j \in \mathcal{K}_b, j \neq k} \alpha_{k,b}(\beta) e_{k,b}^H w_{j,b} d_{j,b} + \sum_{b' \neq b} \sum_{b' \neq b} \sum_{\ell \in \mathcal{K}_{b'}} \alpha_{k,b'}(\beta) h_{k,b'}^H w_{\ell,b'} d_{\ell,b'} + n_k.
\]

The signal-to-interference-plus-noise ratio (SINR) of user \( k \in \mathcal{K}_b \) is given by

\[
\gamma_{k,\text{CBT}}(\beta) = \frac{\alpha_{k,b}(\beta) \| h_{k,b}^H w_{k,b} \|^2 \frac{P}{|\mathcal{K}_b|}}{1 + \sum_{j \neq k} \alpha_{k,b}(\beta) \| e_{k,b}^H w_{j,b} \|^2 \frac{P}{|\mathcal{K}_b|} + \sum_{b' \neq b} \sum_{\ell \in \mathcal{K}_{b'}} \alpha_{k,b'}(\beta) \| h_{k,b'}^H w_{\ell,b'} \|^2 \frac{P}{|\mathcal{K}_{b'}|}}.
\]

For our tilt optimization in Section VI (as it will be clarified later), we need to know the ergodic rate of user \( k \) at any given location in cell \( b \) given the path gain coefficients \( \{\alpha_{k,b}(\beta)\}_{b=1}^B \) at
each location. Therefore, the desired performance metric we are interested in is the conditional ergodic rate given by

\[ R_{k,CBT}(\beta) = \mathbb{E} \left[ \log_2(1 + \gamma_{k,CBT}(\beta)) \left| \{ \alpha_{k,b}(\beta) \}_{b=1}^B \right. \right], \tag{9} \]

where the expectation is taken with respect to small-scale fading realizations. In Section V, using the properties of Gamma RV, we derive an accurate analytical expression for evaluating (9).

B. Beamforming and Ergodic Rates in NMT

Define the aggregate channel vector \( h_k \) from user \( k \) to all BSs as

\[ h_k = \left[ \sqrt{\alpha_{k,1}(\beta)} h_{k,1}^\top, \ldots, \sqrt{\alpha_{k,B}(\beta)} h_{k,B}^\top \right]^\top. \tag{10} \]

We refer to \( h_k \) as the network MIMO channel vector of user \( k \) hereafter in the paper. Using the MMSE decomposition in (4), the network MIMO channel vector in (10) can be written as

\[ h_k = \hat{h}_k + e_k, \tag{11} \]

where \( \hat{h}_k \) is the estimated network MIMO channel vector given by

\[ \hat{h}_k = \left[ \sqrt{\alpha_{k,1}(\beta)} \hat{h}_{k,1}^\top, \ldots, \sqrt{\alpha_{k,B}(\beta)} \hat{h}_{k,B}^\top \right]^\top, \tag{12} \]

and \( e_k \) denotes the network MIMO estimation error vector written as

\[ e_k = \left[ \sqrt{\alpha_{k,1}(\beta)} e_{k,1}^\top, \ldots, \sqrt{\alpha_{k,B}(\beta)} e_{k,B}^\top \right]^\top. \tag{13} \]

Now let \( \hat{H} \in \mathbb{C}^{B N_t \times |K|} \) be the channel matrix having the estimated network MIMO channel vectors of all users, i.e., \( \{ \hat{h}_k \}_{k \in K} \), as its columns. Assuming the knowledge of \( \hat{H} \) at all BSs, the unit-norm beamformer \( w_k \in \mathbb{C}^{B N_t \times 1} \) is obtained by first computing \( \hat{H}^\dagger \) and then taking the normalized column of \( \hat{H}^\dagger \) corresponding to user \( k \). The aggregate transmitted signal \( x \) from all BSs can be expressed as

\[ x = \sum_{k \in K} w_k d_k, \tag{14} \]

where \( d_k \) is the data symbol for user \( k \). Here, we assume that \( x \) is subject to a sum power constraint \( BP \), i.e., \( \mathbb{E} \left[ \text{Tr}(x^\top x^H) \right] = BP, \forall b \). While a per BS power constraint is more relevant
in practice [23], [24], there is a marginal performance difference between per-base station and sum power constraint when the number of users in the coverage area is large enough [25]. Therefore, we apply the sum power constraint to simplify the ergodic rate analysis.

Remark 3.1: We notice that in CBT the beamforming vectors in cell $b$ are solely determined from $\hat{H}_b$ which contains only channel vectors with i.i.d. elements. Such beamforming vectors can point in any direction in the complex space with equal probability and are commonly referred to as isotropically distributed unit vectors [21]. This phenomenon, however, does not hold in NMT because the estimated network MIMO channel vectors contains non-i.i.d. elements, resulting in non-isotropically distributed beamforming vectors.

The received signal of user $k$ can be written as

$$y_k = h_k^H w_k d_k + \sum_{j \in K, j \neq k} e_k^H w_j d_j + n_k, \quad (15)$$

and the SINR of user $k$ is expressed as

$$\gamma_{k,NMT}(\beta) = \frac{\|h_k^H w_k\|^2_{BP}}{1 + \sum_{j \in K, j \neq k} \|e_k^H w_j\|^2_{BP}}, \quad (16)$$

Again, for tilt optimization in Section VI we need to compute the ergodic rate of user $k$ at any given location in the coverage area assuming the path gain coefficients $\{\alpha_{k,b}(\beta)\}_{b=1}^B$ are known at each location. Our desired performance metric is the conditional ergodic rate defined as

$$R_{k,NMT}(\beta) = \mathbb{E} \left[ \log_2 \left( 1 + \gamma_{k,NMT}(\beta) \right) \left| \{\alpha_{k,b}(\beta)\}_{b=1}^B \right. \right]. \quad (17)$$

Following the point in Remark 3.1 it holds that in NMT the beamformers at any time-slot depend on the particular realization of not only the small-scale fading, but also the path gains of all users. Therefore, for a given location of user $k$, the expectation in (17) should be taken with respect to all realizations of both the small-scale fading and the locations of other users. This makes the analytical evaluation of (17) very cumbersome if not impossible. To tackle this issue, we first provide a method in Section IV to approximate the network MIMO channel vector with an equivalent i.i.d. MIMO channel vector. Such an approximation will eliminate the
asymmetry due to non-i.i.d. channels, thereby facilitating the use of existing results for i.i.d. channel vectors to compute $R_{k,NMT}(\beta)$. A review of some mathematical lemmas which prove useful in the following analysis is provided in Appendix A.

IV. A NEW I.I.D. APPROXIMATION FOR NETWORK MIMO CHANNELS

In this section, we propose a new method in which each user interprets its network MIMO channel vector as an i.i.d. channel vector with an equivalent path gain and an equivalent effective degrees of freedom (DoF) per spatial dimension (to be defined later). Using (10), we first start by writing $h_k^H h_k = \sum_{b=1}^{B} \alpha_{k,b}(\beta) h_{k,b}^H h_{k,b}$. It is well-known that $h_{k,b}^H h_{k,b} = \|h_{k,b}\|^2$ is a chi-square RV with $2 N_t$ DoF scaled with $1/2$. Now, using Lemma A.3 it holds that $\alpha_{k,b}(\beta) h_{k,b}^H h_{k,b} \sim \Gamma(N_t, \alpha_{k,b}(\beta))$. Furthermore, using the independence of $h_{k,b}$ and $h_{k,b'}$, $\forall b \neq b'$, it follows that $h_k^H h_k$ is a sum of independent Gamma RVs. According to Lemma A.5 any Gamma RV with the shape parameter $\mu_{k,a}(\beta)$ and the scale parameter $\theta_{k,a}(\beta)$, which are defined as

$$\mu_{k,a}(\beta) = N_t \left( \frac{\sum_{b=1}^{B} \alpha_{k,b}(\beta))^2}{\sum_{b=1}^{B} \alpha_{k,b}^2(\beta)} \right) \quad \text{and} \quad \theta_{k,a}(\beta) = \frac{\sum_{b=1}^{B} \alpha_{k,b}^2(\beta)}{\sum_{b=1}^{B} \alpha_{k,b}(\beta)}, \quad (18)$$

has the same first and second moments as $h_k^H h_k$. It can be easily shown that $N_t \leq \mu_{k,a}(\beta) \leq BN_t$, where the upper bound becomes exact when $\alpha_{k,1}(\beta) = \alpha_{k,2}(\beta) = \cdots = \alpha_{k,B}(\beta)$, while the lower bound is attained when exactly one of the $\alpha_{k,b}(\beta)$, $\forall b$, is non-zero. Next, we present a heuristic interpretation of $\mu_{k,a}(\beta)$ and $\theta_{k,a}(\beta)$ to define an equivalent i.i.d. channel vector for each user.

Let $h_{k,a} \in \mathbb{C}^{BN_t \times 1}$ be a channel vector for user $k$ with elements that are i.i.d. $\mathcal{CN}(0, \theta_{k,a}(\beta))$. This can be looked at as if user $k$ is experiencing an i.i.d. channel with an equal path gain of $\theta_{k,a}(\beta)$ to all the coordinating BSs. Now, to incorporate the parameter $\mu_{k,a}(\beta)$ into the definition of $h_{k,a}$, we note that in the case where the user has an equal path gain to all the BSs, i.e., when $\theta_{k,a}(\beta) = \alpha_{k,1}(\beta) = \alpha_{k,2}(\beta) = \cdots = \alpha_{k,B}(\beta)$, it holds that $\mu_{k,a}(\beta) = BN_t$ and hence $h_{k,a}^H h_{k,a} \sim \Gamma(BN_t, \theta_{k,a}(\beta))$. This means that from the perspective of this user each spatial dimension (or antenna) contributes one unit to the shape parameter of the resulting Gamma RV. In the general case where the user experiences unequal path gains to the BSs, we have $N_t \leq \mu_{k,a}(\beta) < BN_t$. This can be looked at as if each spatial dimension in the channel offers...
\( \frac{\mu_{k,a}(\beta)}{B_{N_t}} (\frac{1}{B} \leq \frac{\mu_{k,a}(\beta)}{B_{N_t}} < 1) \) unit to the shape parameter of the resulting Gamma RV. We denote the parameter \( \frac{\mu_{k,a}(\beta)}{B_{N_t}} \) as the effective DoF per spatial dimension at a given user location. Note that the effective DoF is just a notion introduced here for simplifying the analysis in Section V and is completely different from the information theoretical DoF used in MIMO system context (see e.g. [15]). So from now on, we replace \( h_k \) with \( h_{k,a} \) in our analysis, i.e., we work with i.i.d. channel vectors, but whenever we need to consider the shape parameter for a Gamma RV in the ergodic rate analysis we consider the effective DoF per spatial dimension. This will be clarified in more details in Section V-B.

Figures 2(a) and 2(b) show the values of effective DoF per spatial dimension over the coverage area of the network in Fig. 1 when BSs use respectively a 3D isotropic unit-gain antenna pattern and the 3GPP 3D antenna pattern in Section II-A with \( \beta = 30^\circ \). We use the following values for the parameters in our simulation. The cell radius, defined as the distance from the BS to one of the vertices of the 3D rhomb-shaped cell, is set to \( D = 50 \) m. BS and user heights are chosen as \( h_{bs} = 15 \) m and \( h_u = 1.5 \) m, respectively. For the pathloss factor \( L_{k,b} \) we use a standard distance-dependent model given by \( \left( \frac{d_{k,b}}{D_0} \right)^{-\nu} \). Here, \( d_{k,b} \) denotes the distance between user \( k \) and BS \( b \), accounting for the BS and the user heights, \( D_0 \) is a reference distance which is set to 1, and \( \nu \) is the pathloss exponent which is set to 3.76 [11]. The main simulation assumptions that will also be used in the rest of the paper are summarized in Table I.

As can be seen in Fig. 2(a) an effective DoF per spatial dimension of 1 is achieved only in a small region in the center of the hexagon where the users path gains to the BSs are almost equal. In Fig. 2(b) in addition to the cell center region, three small regions close to the three non-adjacent vertices of the hexagon also achieve an effective DoF per spatial dimension of 1. This phenomenon can be attributed to the changes of the users’ path gains over the coverage area due to the employed 3GPP antenna pattern. In fact, in these three regions the antenna gains compensate for pathloss differences to the BSs, resulting in an equal path gains to all BSs. In both figures effective DoF reduces continuously as the user moves from the center of the hexagon towards the BSs, where it reaches the value of \( \frac{1}{B} = \frac{1}{3} \) in the areas near the BSs.
(a) 3D isotropic unit-gain BS antenna pattern  
(b) 3GPP 3D BS antenna pattern with $\beta = 30^\circ$

Fig. 2. Effective DoF per spatial dimension over different locations of the coverage area in the network of Fig. 1 for (a) 3D isotropic unit-gain BS antenna pattern and (b) 3GPP 3D BS antenna pattern in Section II-A with $\beta = 30^\circ$.

TABLE I

| Parameter                  | Modeling/value                     |
|----------------------------|------------------------------------|
| Cellular layout            | Network model in Fig. 1            |
| BS height, $h_{bs}$        | 15 m                               |
| User height, $h_u$         | 1.5 m                              |
| Cell radius, $D$           | 50 m                               |
| Pathloss, $L_{k,b}$        | $\left(\frac{d_k}{D_0}\right)^{-\nu}$, $D_0 = 1$, $\nu = 3.76$ |
| Peak antenna gain, $G_m$   | 0 dB                               |
| Horizontal HPBW, $\phi_{3\text{dB}}$ | 65°                                |
| Vertical HPBW, $\theta_{3\text{dB}}$ | 6°                                 |
| Horizontal SLL, SLL$_{az}$ | 25 dB                              |
| Vertical SLL, SLL$_{el}$   | 20 dB                              |
| SLL floor, SLL$_{tot}$     | 25 dB                              |

V. ERGODIC RATE ANALYSIS

In this section, we derive accurate analytical expressions for the conditional ergodic rates in (9) and (17) assuming imperfect CSI.
A. Conditional Ergodic Rate in CBT

To derive an analytical expression for (9), in the following we first determine the distributions of the desired signal term, i.e.,
\[ P_{DS}^{k,CBT}(\beta) = \alpha_{k,b}(\beta) \| \hat{h}_{k,b}^H w_{k,b} \|^2 P/|K_b|, \]
the intracell multiuser residual interference term, i.e.,
\[ P_{MRI}^{k,CBT}(\beta) = \sum_{j \in K_b, j \neq k} \alpha_{k,b}(\beta) \| e_{k,b}^H w_{j,b} \|^2 P/|K_b|, \]
and the ICI term, i.e.,
\[ P_{ICI}^{k,CBT}(\beta) = \sum_{b' = 1, b' \neq b}^{B} \sum_{\ell \in K_{b'}} \alpha_{k,b'}(\beta) \| \hat{h}_{k,b'}^H w_{\ell,b'} \|^2 P/|K_{b'}|. \]

**Desired Signal Term in CBT:** Using (4), the random part of \( P_{DS}^{k,CBT}(\beta) \) is expanded as
\begin{align*}
\| \hat{h}_{k,b}^H w_{k,b} \|^2 &= \| (\hat{h}_{k,b} + e_{k,b})^H w_{k,b} \|^2 = \| \hat{h}_{k,b}^H w_{k,b} + e_{k,b}^H w_{k,b} \|^2. \tag{19}
\end{align*}
In a classical single-cell multiuser MIMO setup, where users have identical channel statistics, the variance of \( e_{k,b} \) is usually much smaller than that of \( \hat{h}_{k,b} \) and hence the term \( e_{k,b}^H w_{k,b} \) can be safely ignored (see e.g., [15]). In a more realistic multicell scenario, where users are subject to distance-dependent pathloss as well as channel estimation impairments owing to pilot contamination [4], the variance of \( e_{k,b} \) in (5) can be comparable to that of \( \hat{h}_{k,b} \) especially at the cell edge. Hence, the term \( e_{k,b}^H w_{k,b} \) in (19) can not be ignored.

Now, since \( w_{k,b}, \forall k, b \), is obtained from the normalized columns of \( \tilde{H}_{b}^\dagger \), it holds that \( \hat{h}_{k,b}^H w_{k,b} \) is a real scalar. Furthermore, from Lemma A.4 we have that \( \| \hat{h}_{k,b}^H w_{k,b} \|^2 \) is a chi-square RV with \( 2(N_t - |K_b| + 1) \) DoF scaled with \( \kappa_{k,b}^2(\beta) / 2 \). Putting the aforementioned points together it results that \( r = \hat{h}_{k,b}^H w_{k,b} \) has a generalized Rayleigh distribution with the probability density function given by [26]
\[ f_r(\rho) = \frac{2\rho^{2(N_t - |K_b| + 1) - 1}}{(\kappa_{k,b}^2(\beta))^{N_t - |K_b| + 1} (N_t - |K_b|)!} \exp\left( -\frac{\rho^2}{\kappa_{k,b}^2(\beta)} \right), \quad \rho \geq 0, \tag{20} \]
and with the moments expressed as
\[ \mathbb{E}[r^m] = (\kappa_{k,b}(\beta))^{m/2} \frac{\Gamma(N_t - |K_b| + 1 + \frac{m}{2})}{(N_t - |K_b|)!}. \tag{21} \]
In addition, using Lemma A.4 together with the independence of \( e_{k,b} \) and \( w_{k,b} \), it results that \( z = e_{k,b}^H w_{k,b} \) is a complex scalar such that both RVs \( u = \text{Re}(z) \) and \( v = \text{Im}(z) \) are independently
distributed as $N(0, \sigma^2_{k,b}(\beta)/2)$. The moments of $u$ and $v$ are given by [26]

$$
\mathbb{E}[u^m] = \mathbb{E}[v^m] = \left\{ \begin{array}{ll}
\frac{\sigma^2_{k,b}(\beta)}{2^{(m/2)}\sqrt{\pi}} & \text{for } m \text{ even} \\
0 & \text{for } m \text{ odd.}
\end{array} \right. \tag{22}
$$

Now, we re-write (19) as

$$
\|H_{k,b}^H w_{k,b}\|^2 = \|(r + u) + iv\|^2 = (r + u)^2 + v^2,
$$

where $i = \sqrt{-1}$. To obtain the distribution of $(r+u)^2+v^2$, we first note that $v^2 \sim \Gamma(1/2, \sigma^2_{k,b}(\beta))$.

Secondly, we use the independence of $r$ and $u$ and propose to model the RV $q = (r+u)^2$ as a Gamma RV with the shape parameter $\mu_q$ and the scale parameter $\theta_q$ using the Gamma second order moment matching [16, lemma 7]. To this end, we first use the moment expressions in (21) and (22) to derive $\mathbb{E}[q]$ and $\mathbb{E}[q^2]$ as

$$
\mathbb{E}[q] = \mathbb{E}[r^2] + \mathbb{E}[u^2], \quad \mathbb{E}[q^2] = \mathbb{E}[r^4] + \mathbb{E}[u^4] + 6\mathbb{E}[r^2]\mathbb{E}[u^2]. \tag{24}
$$

Then, we obtain $\mu_q$ and $\theta_q$ as

$$
\theta_q = \frac{\mathbb{E}[q^2] - \mathbb{E}[q]^2}{\mathbb{E}[q]}, \quad \mu_q = \frac{\mathbb{E}[q]}{\theta_q}. \tag{25}
$$

Finally, we note that the desired signal term in (23) is represented as the sum of two independent Gamma RVs and can be approximated by another Gamma RV using Lemma A.5.

**Intracell Multiuser Residual Interference Term in CBT:** Using the independence of $e_{k,b}$ and $w_{j,b}$, $\forall j \in \mathcal{K}_b$ and $j \neq k$, and Lemmas A.4 and A.5, it holds that $\|e_{k,b}^H w_{j,b}\|^2 \sim \Gamma(1, \sigma^2_{k,b}(\beta)/2)$.

Therefore, $P_{MRI}^{\mathcal{K}_b|\mathcal{CBT}}(\beta)$ is a sum of $|\mathcal{K}_b| - 1$ equally-weighted and independent Gamma RVs. It results from Lemmas A.1 and A.2 that $P_{MRI}^{\mathcal{K}_b|\mathcal{CBT}}(\beta) \sim \Gamma(|\mathcal{K}_b| - 1, \alpha_{k,b}(\beta)\sigma^2_{k,b}(\beta)/|\mathcal{K}_b|)$.

**ICI Term in CBT:** To compute the distribution of $P_{ICI}^{\mathcal{K}_b|\mathcal{CBT}}(\beta)$, we first re-write the ICI power as $P_{ICI}^{\mathcal{K}_b|\mathcal{CBT}}(\beta) = \sum_{b' = 1}^{B} \sum_{b \neq b'} \alpha_{k,b}(\beta)\alpha_{k,b'}(\beta)\frac{P_{\mathcal{K}'}}{|\mathcal{K}'|}$, where $\zeta_{k,b',b'} = \|h_{k,b'}^H w_{b',b'}\|^2$. Using the independence of $h_{k,b'}$ from $w_{b',b'}$, $\forall \ell' \in \mathcal{K}_b$, $\forall \ell \in \mathcal{K}_b$, and $\forall \ell' \neq b'$, it results from Lemmas A.4 and A.5 that $\zeta_{k,b',b'} \sim \Gamma(1,1)$. We note that for a given index $b'$, the RVs $\{\zeta_{k,b',b'} : \forall \ell' \in \mathcal{K}_b\}$ are in general statistically dependent as the zero-forcing beamformers applied at BS $b'$ are not necessarily orthonormal. The sets of RVs $\{\zeta_{k,b',b''} : \forall \ell' \in \mathcal{K}_{b''}\}$ and $\{\zeta_{k,b',b''} : \forall \ell'' \in \mathcal{K}_{b''}\}$, $\forall b' \neq b''$,
are, however, mutually independent. Here, to simplify the analysis we assume that the RVs \( \{ \zeta_{k,\ell,b} : \forall \ell \in K_{b'} \} \), for any given \( b' \), are statistically independent. Under this assumption, each inner summation \( \sum_{\ell \in K_{b'}} \alpha_{k,\ell,b}' \zeta_{k,\ell,b}' P_{K_{b'}} \) is distributed as \( \Gamma(|K_{b'}|, \alpha_{k,b}' P / |K_{b'}|) \), and hence the outer summation is a sum of independent Gamma RVs that can be approximated by another Gamma RV using Lemma A.5.

The conditional ergodic rate in (9), can now be expressed as

\[
R_{k,CBT}(\beta) \approx E \left[ \log_2 \left( 1 + P_{k,CBT}(\beta) + P_{MRI,k,CBT}(\beta) + P_{DS,k,CBT}(\beta) \right) \bigg| \{ \alpha_{k,b}(\beta) \}_{b=1}^B \right] - E \left[ \log_2 \left( 1 + P_{k,CBT}(\beta) + P_{MRI,k,CBT}(\beta) \right) \bigg| \{ \alpha_{k,b}(\beta) \}_{b=1}^B \right].
\] (26)

Since \( P_{k,CBT}(\beta) \), \( P_{MRI,k,CBT}(\beta) \), and \( P_{Cl,k,CBT}(\beta) \) are all mutually independent Gamma RVs, each summation inside the logarithm terms in (26) is approximated by a Gamma RV using Lemma A.5. Now, each of the expectations in (26) can be easily evaluated using Lemma A.6.

### B. Conditional Ergodic Rate in NMT

To find an analytical expression for \( R_{k,NMT}(\beta) \), we first replace the channel vector \( h_k \) of user \( k \) by its corresponding i.i.d. approximation \( h_{k,a}, \forall k \), defined in Section IV. With this replacement, the analysis is performed in a transformed network that has a “super” cell with a single distributed multi-antenna BS and users that experience the same path gain to all \( BN_i \) antennas of the BS. In such a setup, the non-i.i.d. nature of the original network MIMO channel is captured via the notion of effective DoF per spatial dimension. We apply the well-known MMSE channel estimation model on \( h_{k,a} \) to obtain a canonical decomposition as

\[
h_{k,a} = \hat{h}_{k,a} + e_{k,a}.
\] (27)

In (27), \( e_{k,a} \) is the estimation error vector with the elements that are i.i.d. \( \mathcal{CN}(0, \sigma_{k,a}^2(\beta)) \), where \( \sigma_{k,a}^2(\beta) = \theta_{k,a}(\beta) / (1 + BP\theta_{k,a}(\beta)) \) and \( \hat{h}_{k,a} \) is the estimated channel vector with the elements

\(^2\text{This is a reasonable assumption if the } |K_{b'}| \text{ users in cell } b', \forall b', \text{ are selected from a set of large number of users by employing proper user selection algorithms such as those in } [27, 28].\)
that are i.i.d. $CN(0, \kappa_{k,a}^2(\beta))$, where $\kappa_{k,a}^2(\beta) = \theta_{k,a}(\beta) - \sigma_{k,a}^2(\beta)$. Under these assumptions, the beamformer $w_{k,a}$ obtained from the nullspace of the vectors $\{\hat{h}_{i,a} : \forall i \neq k\}$ is an isotropically distributed unit-norm vector. To evaluate (17) we need the distributions of the desired signal term, i.e., $P_{k,NMT}^{DS}(\beta) = \|h_k^H w_k\|^2 BP/|K|$ and the multiuser residual interference term, i.e., $P_{k,NMT}^{MRI}(\beta) = \sum_{j \in K, j \neq k} \|e_k^H w_j\|^2 BP/|K|$. The authors in [16] have used a heuristic approach to approximate the distribution of $P_{k,NMT}^{DS}(\beta)$ with a Gamma distribution under perfect CSI. This approach, however, can not be directly extended to the case of imperfect CSI. With the proposed i.i.d. approximation, we have a systematic framework in which $h_{k,a}$, $w_{k,a}$, and $e_{k,a}$ have i.i.d. elements. In the following, we approximate the distributions of $\|h_k^H w_k\|^2$ and $\|e_k^H w_j\|^2$ using respectively the distributions of $\|h_{k,a}^H w_{k,a}\|^2$ and $\|e_{k,a}^H w_{j,a}\|^2$ together with the notion of effective DoF per spatial dimension.

**Desired Signal Term in NMT:** Using (27), we extend the term $\|h_{k,a}^H w_{k,a}\|^2$ as

$$\|h_{k,a}^H w_{k,a}\|^2 = \|(\hat{h}_{k,a} + e_{k,a})^H w_{k,a}\|^2 (\overset{(a)}{\approx} \|\hat{h}_{k,a}^H w_{k,a}\|^2),$$

where $(a)$ follows by neglecting $e_{k,a}^H w_{k,a}$ as it is insignificant compared to $\hat{h}_{k,a}^H w_{k,a}$. This is because, contrary to CBT, ICI is non-existent in NMT and hence we have $\sigma_{k,a}^2(\beta) \ll \kappa_{k,a}^2(\beta)$ for practical values of $\theta_{k,a}(\beta)P$, $\forall k \in K$. Using Lemma A.4, $\hat{h}_{k,a}^H(\beta)w_{k,a}$ is equivalent to another vector of dimension $BN_t - |K| + 1$ with the elements that are i.i.d. $CN(0, \kappa_{k,a}^2(\beta))$, and hence $\|h_{k,a}^H w_{k,a}\|^2 \sim \Gamma(BN_t - |K| + 1, \kappa_{k,a}^2(\beta))$. We also note that the effective DoF per spatial dimension for user $k$ is equal to $\mu_{k,a}(\beta)/BN_t$. Now, we propose to approximate the distribution of $\|h_k^H w_k\|^2$ with $\Gamma((BN_t - |K| + 1) \frac{\mu_{k,a}(\beta)}{BN_t}, \kappa_{k,a}^2(\beta))$, where the shape parameter is obtained by multiplying the shape parameter of the distribution of $\|h_{k,a}^H w_{k,a}\|^2$, i.e., $BN_t - |K| + 1$, with the effective DoF per spatial dimension, i.e., $\mu_{k,a}(\beta)/BN_t$. Next, using Lemmas A.1 it follows that

$$P_{k,NMT}^{DS}(\beta) \sim \Gamma(\mu_{k,NMT}^{DS}(\beta), \theta_{k,NMT}^{DS}(\beta)), \quad \mu_{k,NMT}^{DS}(\beta) = (BN_t - |K| + 1) \frac{\mu_{k,a}(\beta)}{BN_t}, \quad \theta_{k,NMT}^{DS}(\beta) = \frac{\kappa_{k,a}^2(\beta)BP}{|K|}. \quad (29)$$

**Multiuser Residual Interference Term in NMT:** Using the independence of $e_{k,a}$ and $w_{j,a}$, $\forall j \neq k$, and Lemma A.4 it holds that $e_{k,a}^H w_{j,a}$ is equivalent to another vector of dimension 1 with
an element distributed as $\mathcal{CN}(0, \sigma^2_{k,a}(\beta))$, and hence $\|e_{k,a}^H w_{j,a}\|^2 \sim \Gamma(1, \sigma^2_{k,a}(\beta))$. Noting that the effective DoF per spatial dimension is $\mu_{k,a}(\beta)/B N_t$, similar to the case of desired signal term we approximate the distribution of $\|e_{k,a}^H w_{j,a}\|^2$ with $\Gamma(\frac{\mu_{k,a}}{B N_t}, \sigma^2_{k,a}(\beta))$. Therefore, $P_{MRI,k,NMT}(\beta)$ is approximated as a sum of independent Gamma RVs with the same scale parameter. From Lemmas A.1 and A.2 it results that $P_{MRI,k,NMT}(\beta) \sim \Gamma(\mu_{MRI,k,NMT}(\beta), \sigma_{MRI,k,NMT}(\beta))$, where

$$\mu_{MRI,k,NMT}(\beta) = \frac{|K| - 1}{B N_t} \mu_{k,a}(\beta), \quad \sigma_{MRI,k,NMT}(\beta) = \frac{\sigma^2_{k,a}(\beta) BP}{|K|}.$$  

(30)

The conditional ergodic rate in (17), is now written as

$$R_{k,NMT}(\beta) \approx \mathbb{E}\left[ \log_2(1 + P_{DS,k,NMT}(\beta) + P_{MRI,k,NMT}(\beta)) \bigg| \{\alpha_{k,b}(\beta)\}_{b=1}^B \right] - \mathbb{E}\left[ \log_2(1 + P_{MRI,k,NMT}(\beta)) \bigg| \{\alpha_{k,b}(\beta)\}_{b=1}^B \right].$$  

(31)

To derive an analytical expression for (31), we first use Lemma A.5 to approximate $P_{DS,k,NMT}(\beta) + P_{MRI,k,NMT}(\beta)$ with another Gamma RV. After that, both expectation terms on the right hand side of (31) can be easily computed using Lemma A.6.

C. Numerical Example

For our simulation purposes, we move a sample user over the line segment connecting one of the BSs to the center of the hexagon in Fig. 1. We use a 3D isotropic unit-gain pattern at the BS. The rest of the simulation parameters are the same as in Section IV. We further define the cell-edge SNR to be the SNR experienced at the edge of an isolated romb-shaped cell excluding the effect of the antenna gain. Throughout the paper, we set $N_t = 8$ and choose the BS transmit power $P$ so that the cell-edge SNR is 10 dB. For each location of the sample user, $|K| - 1$ other users are uniformly distributed over the coverage area such that there are $|K_1| = |K_2| = |K_3| = 6$ users in each cell. For CBT, the ergodic rate of the sample user at a given location is obtained by averaging the instantaneous rate over 1000 realizations of the small-scale fading for one random drop of the other $|K| - 1$ users. For NMT, in addition to averaging over small-scale fading, we also perform another averaging over 100 drops of the other $|K| - 1$ users.
Fig. 3. Validation of the conditional ergodic rate approximation for a sample user moving on the line segment which connects a sample BS to the center of the hexagon in Fig. 1.

In Fig. 3 the conditional ergodic rate of the sample user, obtained using the analytical expressions derived in this section, is compared against Monte Carlo simulations for both CBT and NMT. It can be easily seen that the match between theory and simulation is remarkably tight in CBT. In NMT, a small mismatch is observed in areas close to the BS that can be attributed to the proposed i.i.d. approximations. We also mention that the match between theory and simulation in both CBT and NMT is preserved when we change the number of users $|\mathcal{K}|$. We, however, omit these results for brevity.

VI. Cell-Specific Tilting in CBT and NMT

In this section, we investigate the performance of the SCN in Fig. 1 under cell-specific tilting. In this tilting strategy the applied tilt at the BSs is fixed at all times and does not adapt to the locations of users. Such a tilt is usually found by maximizing some desired statistical performance metric which is independent of the particular realization of the users’ locations. One popular
approach for finding such a tilt is the so-called throughput analysis\textsuperscript{\[29\]. In this approach, three different performance metrics are used, namely the \textit{average} throughput, the \textit{edge} throughput, and the \textit{peak} throughput defined respectively as the 50-percentile, the 5-percentile, and the 95-percentile of the throughput cumulative distribution function (CDF) over the cell area. Throughput distribution for any given tilt can be obtained by sampling the coverage area using a fine grid of user locations and computing the user throughput at each location by employing the analytical expressions derived in the previous section.

To this end, we let our simulation parameters follow those in Section V-C. In particular, we again assume there are $|\mathcal{K}_1| = |\mathcal{K}_2| = |\mathcal{K}_3| = 6$ users in each cell. We focus on each transmission mode (i.e., CBT or NMT) separately and save the throughput distribution for different tilts. The throughput values are further scaled with $(1 - \frac{M}{T})$, where $M = N_1$ for CBT and $M = BN_1$ for NMT, to account for the training overhead incurred. We then use these scaled throughput distributions to plot the average, edge, and peak throughput versus tilt as shown respectively in Figures 4, 5, and 6. Two different coherence block lengths of $T = 100$ and $T = 1000$ are considered to represent high- and low-mobility scenarios, respectively\textsuperscript{\[22\].}

As can be seen in Fig. 4, the maximum average throughput is attained at $\beta = 22^\circ$ for CBT, while it is reached at $\beta = 20^\circ$ for NMT. The larger tilt in CBT is required to suppress the ICI, which is non-existent in NMT. When each transmission mode is operating at its optimum tilt, we observe that NMT improves the average throughput by 25\% compared to CBT for $T = 1000$, while an improvement of only 6\% is observed for $T = 100$. The latter is due to the larger training overhead in NMT compared to CBT which suppresses the coordination gain at small values of the coherence block length.

In Fig. 4 we observe two peaks on the curves related to NMT. The first peak (on the left) occurs at a smaller tilt and corresponds to the scenario where there is an overlap among the main beams from all the BSs. In this case, users in the middle of each cell are not necessarily

\textsuperscript{3}Here, throughput refers to the conditional ergodic rates defined in Section V.
close to the peak of the main beam of any of the BSs. These users receive a significant part of their desired signal power from the neighboring BSs. The second peak (on the right) takes place at a larger tilt and relates to the case where the main beam from each BS is inside its own cell and is pointing directly to the users in the middle of the cell. Such users are close to the peak of the main beam of their home BS, and hence receive a major part of their desired signal from that BS. Interestingly, the second peak is larger than the first one, which shows the importance of tilt optimization in achieving the maximum performance gain in network MIMO.

Compared to maximum average throughput, the maximum edge throughput is attained at smaller tilts ($\beta = 21^\circ$ for CBT and $\beta = 13^\circ$ for NMT) as shown in Fig. 5. The edge throughput is mainly determined by users close to the cell edge. Therefore, to maximize the throughput of these users, the peak of the beam should be pointed more towards the edge of the cell. Moreover, we observe that even for $T = 100$ the edge throughput at optimum tilt is significantly improved (by about 130\%) in NMT compared to CBT, which shows that the coordination gain dominates the loss occurred because of training overhead. Therefore, NMT is the best transmission mode for edge throughput maximization.

The peak throughput is usually attained by users close to the BS. So we expect the maximizing tilt for peak throughput to be larger than that for average throughput. This is verified in Fig. 6 where we see that the optimum tilt for both CBT and NMT is $36^\circ$. For $T = 1000$, the performance of CBT is almost the same as that of NMT for a tilt greater than $22^\circ$. For $T = 100$, CBT outperforms NMT for a tilt greater than $20^\circ$. This shows that for users close to the BS the loss owing to training overhead in NMT dominates the gain brought by coordination.

Notice that Figures 4, 5, and 6 have been plotted assuming 6 users per cell. This choice of number of users is motivated by the results in [15] indicating that the number of users in each cell for multiuser MIMO transmission should be less than $N_t$ in the presence of imperfect CSI. Our results from separate simulations, however, show that the number of users has a negligible effect on the optimum tilts for different performance metrics. These results are omitted here due to space limitations.
Fig. 4. Average throughput of CBT and NMT vs. tilt for $|\mathcal{K}_1| = |\mathcal{K}_2| = |\mathcal{K}_3| = 6$ and $T = 100, 1000$.

Fig. 5. Edge throughput of CBT and NMT vs. tilt for $|\mathcal{K}_1| = |\mathcal{K}_2| = |\mathcal{K}_3| = 6$ and $T = 100, 1000$. 
One important highlight here is that with cell-specific tilting, it is not possible to optimize all the performance metrics, i.e., average, edge, and peak throughput, at the same time as each of these metrics is maximized at a different tilt. One low-complexity solution, which has attracted a lot of attention recently [6], [13], [14], is the switched-beam tilting in which at each time-slot one out of a set of finite tilts is applied at the BSs depending on the locations of the users. In the next section, we exploit the idea of switched-beam tilting and propose a novel transmission strategy that is capable of achieving a tradeoff in maximizing all performance metrics simultaneously.

VII. PROPOSED HYBRID-MODE TRANSMISSION

In the previous section, NMT was shown to be the best transmission strategy for edge throughput maximization. Furthermore, the peak throughput performance of CBT was shown to be at least as good as NMT for tilts greater than some threshold (22° for the considered scenario). For average throughput, we observed that NMT has moderate superiority over CBT mainly at low-mobility scenarios.
The aforementioned argument eventually leads to the following hypothesis: a transmission strategy that would serve the users in different regions of the cell, namely, the cell-interior region or the cell-edge region, with an appropriate transmission mode, i.e., NMT or CBT, and a corresponding appropriate tilt could potentially achieve a tradeoff in maximizing all the performance metrics simultaneously. Hence, we propose a hybrid-mode transmission (HMT) technique with the following components:

1) A division of the coverage area in Fig. 1 into two so-called *vertical regions* as follows: i) a cell-interior region consisting of three disjoint vertical regions each associated with one of the BSs. Each of these vertical regions is obtained as the intersection of the coverage area with a circle of radius $D_{int}$ centered at the corresponding BS; ii) a cell-edge region shared among all BSs. This is illustrated in Fig. 7.

2) A transmission technique that at each time-slot serves either the cell-interior region using CBT or the cell-edge region using NMT. A switched-beam tilting strategy is also employed which applies at all BSs a fixed tilt $\beta_{CBT}$ when serving the cell-interior region (see Fig. 7(a)), or a fixed tilt $\beta_{NMT}$ when serving the cell-edge region (see Fig. 7(b)). We emphasize that only one of these transmission modes can be active at each time-slot.4

3) A scheduler to share the available time-slots between the cell-interior region and the cell-edge region and to further serve the users in each region.

Notice that in the proposed HMT, the parameters $D_{int}$, $\beta_{CBT}$, $\beta_{NMT}$, and the fraction of time-slots that the scheduler should allocate to each vertical region are unknown and need to be determined. Next, we present the methods used to determine these parameters.

A. *Determining $D_{int}$, $\beta_{CBT}$, and $\beta_{NMT}$*

To determine $D_{int}$, $\beta_{CBT}$, and $\beta_{NMT}$, we focus on average throughput maximization. Note that in HMT the cell-interior users are served by CBT and the cell-edge users are served by

4In systems with multiple frequency subbands such as LTE, it is possible to interlace the two transmission modes over subbands at each time-slot so that the users in each vertical region can be served at all time-slots.
NMT. Therefore, from the discussion in Section VI, we expect the edge and peak throughput performance to be at a satisfactory level for any choice of $\beta_{\text{CBT}}$ and $\beta_{\text{NMT}}$ that would maximize the average throughput.

The average throughput for a given $D_{\text{int}}$, denoted as $\bar{R}(D_{\text{int}})$, is determined by the users’ throughput both in the cell-interior region and in the cell-edge region. Although the user throughput in both regions can be obtained using the analytical expressions in (26) and (31), it is very difficult to draw any insight about how $\bar{R}(D_{\text{int}})$ changes with $D_{\text{int}}$. In the following we provide a heuristic discussion about the relationship between $\bar{R}(D_{\text{int}})$ and $D_{\text{int}}$.

On one hand, if $D_{\text{int}}$ becomes too small, most of the users in the interior part of the cell are served using NMT. Since $\beta_{\text{NMT}}$ is set so that the peak of the beam is pointing more towards the cell-edge, many of these users are close to the side-lobe of the antenna beam. Such users could potentially achieve a higher throughput if they were served by CBT with $\beta_{\text{CBT}} > \beta_{\text{NMT}}$. In that case, they would be both closer to the peak of the beam of their home BS and very well protected against ICI (see Fig. 7(a)). On the other hand, if $D_{\text{int}}$ becomes too large, most of the users in the vicinity of the cell edge are served by CBT. Such users will experience a low
throughput as they are both close to the side-lobe of the antenna beam of the their home BS when the tilt is equal to $\beta_{\text{CBT}}$ and subject to a large ICI. As a result, the optimum $D_{\text{int}}$ which maximizes $\bar{R}(D_{\text{int}})$ is expected to be somewhere in the middle of the cell. To determine the optimum $D_{\text{int}}$, $\beta_{\text{CBT}}$, and $\beta_{\text{NMT}}$, we simulate $\bar{R}(D_{\text{int}})$ for $D_{\text{int}} \in [0.15D, 0.95D]$ such that for any given $D_{\text{int}}$ we exhaustively search for $\beta_{\text{CBT}}$ and $\beta_{\text{NMT}}$ that maximizes $\bar{R}(D_{\text{int}})$. Our simulation setup is the same as in Section VI. In Fig. 8 the average throughput $\bar{R}(D_{\text{int}})$ is plotted versus the normalized cell-interior region radius, i.e., $D_{\text{int}}/D$. As can be seen in the figure, the maximum average throughput is achieved at $D_{\text{int}} = 0.65D$ and by using a corresponding $\beta_{\text{CBT}} = 26^\circ$ and $\beta_{\text{NMT}} = 19^\circ$ for both low- and high-mobility scenarios.

B. Fair Scheduling

So far we have only considered the users that are served at each time-slot, denoted as the active users. The number of such users in the network is limited by the available resources that can be used for transmission. For example, under zero-forcing beamforming, the BSs in NMT
can serve at most $BN_t$ users. In practice the number of available users in the cell-edge region might be larger than $BN_t$. Hence, a scheduler is employed to select a subset of the available users at each time-slot. Let $\mathcal{K}_{\text{CBT}}$ and $\mathcal{K}_{\text{NMT}}$ denote the set of all users in the cell-interior region and cell-edge region, respectively. We focus on one drop of $|\mathcal{K}_{\text{CBT}}| + |\mathcal{K}_{\text{NMT}}|$ users over the coverage area and assume that the path gain coefficients of all users are known. Under user scheduling, the throughput of user $k$ is defined as

$$R_{k}^{\text{sch}} = \begin{cases} \nu_{\text{CBT}} R_{k,\text{CBT}}^{\text{sch}}(\beta_{\text{CBT}}) & \text{if } k \in \mathcal{K}_{\text{CBT}} \\ \nu_{\text{NMT}} R_{k,\text{NMT}}^{\text{sch}}(\beta_{\text{NMT}}) & \text{if } k \in \mathcal{K}_{\text{NMT}}, \end{cases}$$

(32)

where $\nu_{\text{CBT}}$ ($0 \leq \nu_{\text{CBT}} \leq 1$) denotes the cell-interior region activity factor, i.e., the fraction of the total time-slots in which the cell-interior region is active. Similarly, $\nu_{\text{NMT}}$ ($0 \leq \nu_{\text{NMT}} \leq 1$) is the cell-edge region activity factor such that $\nu_{\text{CBT}} + \nu_{\text{NMT}} = 1$. Moreover, $R_{k,\text{CBT}}^{\text{sch}}(\beta_{\text{CBT}})$ and $R_{k,\text{NMT}}^{\text{sch}}(\beta_{\text{NMT}})$ indicate the user per region throughput for user $k$ in the cell-interior region and cell-edge region, respectively. $R_{k,\text{CBT}}^{\text{sch}}(\beta_{\text{CBT}})$ ($R_{k,\text{NMT}}^{\text{sch}}(\beta_{\text{NMT}})$) is obtained by averaging the instantaneous rate over all the time-slots in which the cell-interior region (cell-edge region) is active. Note that user $k \in \mathcal{K}_{\text{CBT}}$ ($k \in \mathcal{K}_{\text{NMT}}$) might not necessarily be served at each time-slot in which the cell-interior region (cell-center region) is active, in which case its instantaneous rate is zero. Therefore, $R_{k,\text{CBT}}^{\text{sch}}(\beta_{\text{CBT}})$ and $R_{k,\text{NMT}}^{\text{sch}}(\beta_{\text{NMT}})$ are in general different from the conditional ergodic rates defined in Section V. To determine $\nu_{\text{CBT}}$ and $\nu_{\text{NMT}}$, the scheduler has to solve the following convex optimization problem:

$$\text{maximize } g(\mathbf{R}^{\text{sch}})$$

subject to

$$R_k^{\text{sch}} \leq \begin{cases} \nu_{\text{CBT}} R_{k,\text{CBT}}^{\text{sch}}(\beta_{\text{CBT}}) & \text{if } k \in \mathcal{K}_{\text{CBT}} \\ \nu_{\text{NMT}} R_{k,\text{NMT}}^{\text{sch}}(\beta_{\text{NMT}}) & \text{if } k \in \mathcal{K}_{\text{NMT}}, \end{cases} ,$$

$$\nu_{\text{CBT}} + \nu_{\text{NMT}} = 1, \nu_{\text{CBT}}, \nu_{\text{NMT}} \geq 0.$$  

(33)

In (33), $g(\cdot)$ is a concave and componentwise non-decreasing utility function with a suitable notion of fairness [30] and $\mathbf{R}^{\text{sch}}$ denotes the vector of throughputs of all users in the coverage area.
area. Here, we focus on the popular choice of proportional fair scheduling \[30\] whose utility function is given by

$$g(R^{\text{sch}}) = \sum_{k \in K_{\text{CBT}}} \log(R_{k}^{\text{sch}}) + \sum_{k \in K_{\text{NMT}}} \log(R_{k}^{\text{sch}}).$$ (34)

Solving (33) using the utility function in (34), we obtain the activity factors of the cell-interior region and the cell-edge region as

$$\nu_{\text{CBT}} = \frac{|K_{\text{CBT}}|}{|K_{\text{CBT}}| + |K_{\text{NMT}}|}, \quad \nu_{\text{NMT}} = \frac{|K_{\text{NMT}}|}{|K_{\text{CBT}}| + |K_{\text{NMT}}|}. \quad (35)$$

Notice that for proportional fair scheduling the values of $\nu_{\text{CBT}}$ and $\nu_{\text{NMT}}$ are independent of \{$R_{k,\text{CBT}}^{\text{sch}}(\beta_{\text{CBT}})$\}$_{k \in K_{\text{CBT}}}$ and \{$R_{k,\text{NMT}}^{\text{sch}}(\beta_{\text{NMT}})$\}$_{k \in K_{\text{NMT}}}$, which are usually difficult to compute analytically. Calculating the activity factors of vertical regions for other utility functions is beyond the scope of this paper and is left to our future work.

C. Numerical Results

In this section, the performance of the proposed HMT technique in Section \[\text{VII}\] is evaluated via Monte Carlo simulations. Our simulation parameters follows those in Section \[\text{VI}\]. We use a drop-based simulation, where at each drop 20 users are randomly placed in each romb-shaped cell. The users are associated with the cell-interior region or the cell-edge region based on their locations in the cell. At each time-slot, we use the standard proportional fair user selection together with the multiuser MIMO zero-forcing in Section \[\text{III}\] \[3\]. For each drop of users, we simulate a sufficient number of small-scale fading realizations such that all users achieve their limiting throughputs. We then stack the users’ throughputs over all drops to obtain the throughput distribution over the coverage area. We compare the performance of three different transmission strategies as follows. 1) CBT with $\beta = 21^\circ, 22^\circ, 36^\circ$; 2) NMT with $\beta = 13^\circ, 20^\circ, 36^\circ$; and 3) HMT with $\beta_{\text{CBT}} = 26^\circ$ and $\beta_{\text{NMT}} = 19^\circ$. By design these tilts are obtained using the throughput analysis in Sections \[\text{VI}\] and \[\text{VII-A}\] that assume no particular user scheduling algorithm.$^5$

$^5$The choice of the three tilts for CBT and NMT corresponds respectively to the edge, average, and peak throughput maximizing tilts obtained in Section \[\text{VI}\]
Figure 9 compares the edge, average, and peak throughput of CBT (Fig. 9(a)) and NMT (Fig. 9(b)) with those of the proposed HMT assuming no training overhead. As can be seen, for the edge and average throughput maximizing tilts, the proposed HMT completely outperforms CBT, while shows a comparable performance to NMT at all the considered performance metrics. For peak throughput maximizing tilt, HMT underperforms both CBT and NMT in peak throughput, but shows a significant performance gain in edge and average throughput. Moreover, for both CBT and NMT, setting the tilt to maximize one performance metric (e.g., the peak throughput) results in loss in the other two performance metrics. The proposed HMT, however, achieves a tradeoff in simultaneously maximizing all the performance metrics. We further highlight that in the proposed HMT a fraction $A_{\text{CBT}}/A_{\text{cov}}$ of the coverage area is served using CBT, where $A_{\text{CBT}}$ is the area of the cell-interior region and $A_{\text{cov}}$ denotes the area of the whole network. In the considered system setup, $A_{\text{CBT}} = \pi(0.65D)^2$ and $A_{\text{cov}} = 3\sqrt{3}D^2/2$, resulting in $\approx 50\%$ of the coverage area to be served using CBT. Equivalently, this means that the proposed HMT requires about 50% less signaling and data sharing overhead compared to NMT, while achieving a comparable performance.
VIII. CONCLUDING REMARKS

In this paper, we investigated downlink transmission in a network with small-sized cells that exploits BS coordination and BS antenna tilting for interference management as well as throughput optimization. Cell-specific tilting was studied for two well-known transmission modes, namely, conventional uncoordinated transmission and fully coordinated transmission, taking into account the impact of training overhead. Using the conclusions from this study, a novel hybrid-mode transmission technique was proposed that adapts both the transmission mode and the applied tilt at the BSs when serving users in different regions of the coverage area. Analytical expressions for the user ergodic rates with imperfect CSI were derived for both conventional uncoordinated and fully coordinated transmission modes to facilitate systematic and computationally efficient performance analysis. Numerical results showed the superiority of the proposed transmission technique over the conventional uncoordinated transmission. The proposed hybrid-mode transmission also seems to provide a superior performance-complexity tradeoff compared to fully coordinated transmission.

APPENDIX A

MATHEMATICAL LEMMAS

In this appendix, we provide some well-known lemmas which will prove useful in the analyses in Sections IV and V.

Lemma A.1: If $Y$ is a Gamma RV with shape parameter $\mu$ and scale parameter $\theta$, i.e., $Y \sim \Gamma(\mu, \theta)$, and $b$ is a positive constant, then $bY \sim \Gamma(\mu, b\theta)$.

Lemma A.2: If $Y_i \sim \Gamma(\mu_i, \theta)$ for $i = 1, \ldots, N$, then $\sum_{i=1}^{N} Y_i \sim \Gamma\left(\sum_{i=1}^{N} \mu_i, \theta\right)$.

Lemma A.3: If $Z$ is a chi-square RV with $2r$ DoF, denoted as $Z \sim \chi^2_{2r}$, and $a$ is a positive constant, then $aZ \sim \Gamma(r, 2a)$.

Lemma A.4 (Muirhead [31]): The projection of an $M$-dimensional vector with i.i.d. $CN(0, \sigma^2)$ elements onto a subspace of dimension $s$, for $s \leq M$, is another vector of dimension $s$ with i.i.d. $CN(0, \sigma^2)$ elements.
Lemma A.5: Assume \( \{Y_i\} \) are independent Gamma RVs with parameters \( \mu_i \) and \( \theta_i \). The RV 
\( W \sim \Gamma(\mu, \theta) \) has the same first and second order moments as the RV 
\( Y = \sum_i X_i \), where 
\[
\mu = \frac{\left(\sum_i \mu_i \theta_i\right)^2}{\sum_i \mu_i \theta_i^2} \quad \text{and} \quad \theta = \frac{\sum_i \mu_i \theta_i^2}{\sum_i \mu_i \theta_i} \tag{36}
\]

Lemma A.6: Let \( X \sim \Gamma(\mu, \theta) \), then \( \mathbb{E}_X[\log_2(1 + X)] \) is computed as 
\[
\mathbb{E}_X[\log_2(1 + X)] = \frac{1}{\Gamma(\mu) \ln 2} G_{3,2}^{1,3}\left[ \begin{array}{c|c}
1 - \mu, 1, 1 \\
1, 0 
\end{array} \right] 
\]
where \( G_{p,q}^{m,n}\left[ \begin{array}{c|c}
x \end{array} \right] \) denotes the Meijer G-function [32, Eq. (9.301)].

Proof: 
\[
\mathbb{E}_X[\log_2(1 + X)] = ^{(a)} \frac{1}{\theta^\mu \Gamma(\mu) \ln 2} \int_0^\infty G_{2,2}^{1,2}\left[ \begin{array}{c|c}
x \end{array} \right]^{1,1,1} \left(1 - e^{-x/\theta}\right) dx
\]
\[
= ^{(b)} \frac{1}{\Gamma(\mu) \ln 2} G_{3,2}^{1,3}\left[ \begin{array}{c|c}
1 - \mu, 1, 1 \\
1, 0 
\end{array} \right].
\]

Here, (a) follows by expressing the logarithmic term \( \ln(1+x) \) via a Meijer’s G-function according to [33, Eq. (8.4.6.5)] and (b) results by evaluating the integral expression in (a) using the integral identity for Meijer’s G-functions from [32, Eq. (7.813.1)] and some algebraic simplifications.

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