Numerical Study of Photonic-Crystal-Based Dielectric Accelerators

G. Torrisi\textsuperscript{1}, G. S. Mauro\textsuperscript{1}, L. Celona\textsuperscript{1}, D. Mascali\textsuperscript{1}, S. Gammino\textsuperscript{1}, G. Sorbello\textsuperscript{1,2}, C. De Angelis\textsuperscript{3}, A. Locatelli\textsuperscript{3}

\textsuperscript{1} INFN-LNS, Catania, Italy
\textsuperscript{2} University of Catania, Catania, Italy
\textsuperscript{3} University of Brescia, Brescia, Italy

E-mail: giuseppe.torrisi@lns.infn.it

Abstract. All-dielectric electromagnetic band gap (EBG) waveguides structures promise significant improvement of accelerating gradient of laser-driven acceleration with the potential to miniaturize the accelerator itself. In this work we study photonic crystal structures designed for acceleration of relativistic electrons. We explore the performance of the all-dielectric EBG accelerating structures thanks to full wave electromagnetic simulations of couplers and accelerating waveguides. The characteristic interaction impedance, accelerating gradient and all the key parameters that are typically used to characterize linear accelerators are evaluated and used to compare the properties of the accelerating mode field distribution in different geometries.

1. Introduction and Motivation

It is well known that conventional metallic accelerating structures suffer from electrical breakdown in the presence of strong electric fields, therefore in the recent years an alternative paradigm based on “dielectric accelerators” has been proposed to overcome this limitation [1]. The two main advantages of this approach are the larger damage threshold of dielectrics with respect to metals, and the potential reduction of cost and size of particle accelerators working at higher frequencies, in particular in the optical regime where perfect conductors do not exist. In this framework, Photonic Crystal (PhC) have opened the way to the realization of ultra-compact dielectric devices operating in different frequency regimes [2]. PhC waveguides offer the possibility of carefully engineering the dispersion properties of the guiding structures, thus have been proposed for molding phase and group velocity in a very controlled way. These features can find very interesting applications also in the field of linear accelerators. As a matter of fact, PhC waveguides seem to be the ideal choice for the realization of a dielectric accelerator: in particular, they allow hollow-core guiding of the spatially overlapped particle and light beams, the phase velocity can be tuned in order to get synchronous acceleration, the group velocity can be engineered based on the desired interaction length and higher-order modes can be efficiently suppressed. The interest in PhC guiding structures is not limited to the case of a single waveguide. As a matter of fact, PhC couplers and arrays obtained by placing several waveguides side-by-side have already been analyzed in depth. Linear accelerators based on two-dimensional (2D) PhC waveguides [3, 4], Bragg waveguides [5], woodpile waveguides [6, 7], and photonic crystal fibers [8] have already been proposed and studied in the literature.
Here we describe the design and the numerical validation of a 2D PhC planar coupler operating in the X band. The main features of the proposed all-dielectric resonant coupler is a complete conversion from a transverse electric mode to a transverse-magnetic-like accelerating mode obtained by resonant coupling between the two waveguides [9]. Although such 2D structures may not be immediately suitable for charged particle acceleration (the field is not confined in one direction), the planar geometry makes fabrication and integration simpler and, last but not least, the 2D configuration represents a simpler “testbench” that allows the evaluation of some important LINACs figure of merits in order to demonstrate the potential of the EBG accelerating structures. The accelerating waveguide of the proposed coupler is compared with other 2D synchronous accelerating structures based on Bragg reflectors [5] and high-contrast gratings (HCGs) [10]; as a comparison we include also a “conventional” 3D metallic 3-cell X band Standing Wave (SW) cavity [11].

2. 2D Photonic Crystal coupler
Photonic crystals (PhCs) are periodic arrangements of different dielectric materials exhibiting a frequency bandgap where electromagnetic waves propagation is suppressed. In this paper, for the PhC band gap computation and field evaluation we use the MIT Photonic-Bands (MPB) package [12]. The geometry has been tuned with the “supercell method” and then it has been truncated and simulated as a real coupler (including input and output regions and ports) in CST Microwave Studio [13]. It is worth to note that the two main advantages of “dielectric accelerators” - namely larger damage threshold and potential reduction of cost and size of particle accelerators - are valid at higher frequencies, in particular in the optical regime. However, in the following design we chose microwave frequency (11 GHz) and alumina as dielectric ($\varepsilon_r = 9.72$ [14, 15]), only to demonstrate physical concepts that can be scaled-up at optical frequencies to provide an all-dielectric coupler suitable for laser acceleration. The proposed directional coupler structure is based on two adjacent waveguides obtained by the optimization of two defects in a 2D PhC: in order to get an efficient energy exchange between the two waveguides, the design of the isolated guiding structures has been tuned so that the interacting modes have the same propagation constant [16]. The obtained 2D PhC is based on a triangular lattice of periodically arranged vacuum holes of radius $r = 0.3a$ and lattice constant $a = 6.62$ mm chosen to synchronize the accelerating mode with the particle beam. The complete structure consists of an alumina slab working at 11 GHz. The geometry is shown in Fig. 1: it consists of a coupler obtained by placing side by side the accelerating waveguide WG1 and the launch waveguide WG2. The two hollow waveguides are obtained by realizing two vacuum channels of width $w_1 = 1.6a = 10.59$ mm and $w_2 = 14.1$ mm respectively, with an inter-axis spacing $d = 3a + (w_1 + \delta_1)/2 + (w_2 + \delta_2)/2$ in order to accommodate three lines of holes in between. The 2D structure is assumed to be infinite in the z-direction while the electron beam and the electric field co-propagate in the x-direction.

In the two-waveguide coupler structure, for the proper choice of the length of the coupler $L_B = \frac{\pi}{\Delta \beta} = 31.4a$, the power can be totally transferred. Synchronization of the accelerating and a transverse mode can be achieved by varying $w_{1,2}$ and the pad width $\delta_{1,2}$ [17, 16].

We show in Fig. 2 the full-wave CST simulated electric field: it can be observed that the TE mode is totally transferred to the accelerating waveguide after a beating period. The field pattern shown in the coupler in Fig. 2 is given by the linear superposition of the two coupler supermodes that are, in first approximation, the even/odd combination of the accelerating and the transverse mode of the isolated WG1 and WG2 waveguides. The field in the narrower waveguide WG1 has a strong longitudinal $E_x$ component and and it is denoted as “the accelerating mode”; the field in the larger waveguide WG2 has a strong transverse $E_y$ component similar to the classic transverse mode $TE_{10}$ and it is denoted as “telecom mode”. The waveguide WG1 allows propagation of a transverse-magnetic (TM) like mode for acceleration; however cannot be easily coupled by end fire butt-coupling with a standard telecom laser Gaussian mode or the TEM mode of a parallel
Figure 1. Geometry of a 2D PhC coupler consisting of vacuum holes in alumina. TE wave is launched from the side waveguide WG2 and coupled to the TM mode on waveguide WG1. Here $r = 0.3a$ and $a = 6.6$ mm.

Figure 2. CST MWS simulated electric field intensity distribution inside the 2D PhC coupler: the transverse electric field of WG2 is excited by a Transverse Electromagnetic (TEM) mode obtained by imposing Perfect Electric Conductor (PEC) and Perfect Magnetic Conductor (PMC) boundary condition.

plate metallic waveguide since its overlap integral [18] with such mode would be zero.

The dependence of coupling length and maximum transfer power on the distance of two parallel waveguides are discussed in [16]: the results show that the longitudinal power flux in the accelerating waveguide concentrates mainly on the edge of the defect which confirms that the accelerating mode can not be easily excited by end coupling. Of course the use of a side-coupling structure is required also by the necessity to leave a clearance for the electron beam path. By controlling the padding thickness $\delta_1/2$, the proposed structure can easily provide an accelerating mode with any required phase velocity. In our case, the latter is chosen equal to the electron velocity to accelerate relativistic electrons. Our asymmetric coupler configuration as compared to [19] does not require two drive guides excited 180° out-of-phase.

3. Accelerating Parameters
In this section we evaluate the characteristic parameters for different configurations of accelerating structures. To accelerate particles efficiently, electromagnetic waves must be guided or confined into the region in which the particles travel. To accelerate particles over significant distances, the wave phase velocity (in the direction of particle travel) must be synchronized with the particle velocity, and the waves must have an electric field component in the direction of acceleration. Fig. 3 shows that the latter condition is fulfilled by the 2D PhC coupler. The computed transit time factor [20] is $T = 0.99$. For dielectric structure it can defined the
characteristic interaction impedance [1], as \( Z_c = \left( \frac{E_{\text{acc}} \lambda}{P} \right)^2 \) (\( \Omega \)) which is a measure of the effective accelerating field acting on the electrons, \( E_{\text{acc}} \), for a given total electromagnetic power, \( P \), flowing into the system [21]. When considering a 2D slab structure with an indefinite height \( h \) (thickness in \( z \) direction in our case) it is also possible to define a characteristic interaction impedance times the unit height \( Z'_c = \left( \frac{E_{\text{acc}} \lambda}{P'} \right) (\Omega \text{m}) \) where \( P' \) (W/m) is the total power per unit height of the structure [5, see Eq. (37) and (38)]. If we fix the height \( h \) of the structure, also the power flowing in its section is fixed equal to \( P = P' h \) (W), and the characteristic interaction impedance times the unit height, \( Z'_c \), of the 2D structure can be put in relation with the characteristic interaction impedance of the corresponding 3D structure of height \( h \): \( Z'_c = Z_c h \) (\( \Omega \text{m} \)). The field component of a speed-of-light accelerating mode in the vacuum region have a simple expression [22, Eq. (2)] in that in reference system of Fig. 3 reads (\( \beta = \omega/c \)):

\[
E_x = E_0 \exp(-j\beta x); \quad E_y = j\beta y E_0 \exp(-j\beta x)
\]  

from (1) an upper bound for the \( Z'_c \) can be calculated [5, Eq. (38)] and in our notation the formula for the maximum characteristic interaction impedance times the unit height reads \( Z'_{c\text{(max)}} = \frac{3\eta_0 \lambda^4}{4\pi^2 (w_1/2)^2} \) (\( \Omega \text{m} \)). For 2D structures it is a common practice to fix \( h = \lambda \) and to plot/consider \( Z = Z'/\lambda \). In this case the upper bound is for the interaction impedance reads

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**Figure 3.** Electric field components along the axis of the accelerating waveguide WG1 for ten repetitions of the elementary cell: the accelerating field \( E_x(x) \) is almost constant on the channel axis. The transverse field \( E_y(x) \) is negligible on the waveguide axis. Both fields are synchronous with a speed of light electron beam.

**Figure 4.** Electric fields components \( E_x \) and \( E_y \), and refractive index \( \epsilon_r \) at \( x = 0 \) (top graph). The \( x = 0 \) cut line is shown with a dashed red line on the 2D map of the refractive index (bottom map). The field component in the vacuum channel are compatible with a speed-of-light accelerating mode (1).
Table 1. Accelerating structures figures of merit comparison. The accelerating gradient is calculated as $E_0 = \frac{\sqrt{PZ_c/\lambda}}{\lambda}$ with an input power $P = 1$ W

| Parameter          | this work | [5] | [10] | [11] |
|--------------------|-----------|-----|------|------|
| materials          | alumina   | silicon | silicon | copper |
| $\epsilon_r^{(\text{max})}$ | | 9.7 | 4 | 3.48 | – |
| $D/\lambda$       | | 0.19 | 0.3 | 0.5 | – |
| $Z_c = \frac{Z_c^{(\text{max})}}{\lambda}$ | | 1.78 | 147 | 162 | 1.19 $\cdot 10^6$ |
| $E_0$ (MV/m)       | | 4.9 $\cdot 10^{-5}$ | – | 8.4 | 4.0 $\cdot 10^{-2}$ |

For the waveguide WG1 we have: $Z_c^{(\text{max})} = 3914$ (Ω) and $Z_c^{(\text{max})} = 106.8$ (Ωm) corresponding to a field totally confined in the vacuum core of half width $D_1 = w_1/2$ and zero in the cladding. In Table 1 we list: channel size $D/\lambda$ (in our case $D = w_1/2$), characteristic interaction impedance, $Z_c$, and accelerating gradient, $E_0$, for a nominal input power of 1 W for 1) the 2D PhC accelerating waveguide of the coupler presented in this paper; 2) the 2D optical Bragg accelerator [5]; 3) the HCG based accelerator [10]; 4) a “conventional” metallic 3-cell X band Standing Wave cavity [11]. It is worth to note that we are comparing a coupler with accelerating structures; for the latter both an high dielectric constant and a small channel lead to a desiderable increase of the interaction impedance value by confining the field in the vacuum channel but at the same time make less practical both the resonant side coupling and the coupler itself fabrication tolerances. As it is apparent from Fig. 4 (and also from Fig 2) the accelerating mode, supported by WG1, is a surface mode: transverse field $E_y$ and longitudinal Poynting vector $P_x$ are confined at the PhC - vacuum interface. The longitudinal field $E_x$ is rather intense and uniform along the vacuum channel where light and particles travel in a synchronous fashion in agreement with (1). Since these 2D structures do not include an efficient method for a vertical confinement of the electromagnetic field they are suitable for the acceleration of a ribbon-like electron beam; a further step would be to consider 3D structures such as the “woodpile” photonic crystal proposed in [23, 24].

4. Conclusion
We have found accelerating mode for a 2D Photonic Band Gap structure. We examined parameters of the accelerating modes. We have demonstrated that our 2D structure supports a complete conversion of the input power in a TM$_{01}$-like uniform accelerating mode with a longitudinal field component synchronous with relativistic electrons. The on axis interaction impedance could be increased by a further optimization and increasing the refraction indices contrast. Finally, to include a proper vertical confinement of the electromagnetic field, the concepts discussed here must be explored in a 3D case such as the woodpile structure proposed in [24].

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