The Final Theory of Physics - a Tautology?

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We culminate the idea of a final theory of physics in order to analyze its logical implications and consequences. It is argued that the rationale of a final theory is the principle of sufficient reason. This implies that a final theory of physics, presumed such a theory is possible, does not allow to incorporate substantial (non-trivial) propositions unless they are logically or mathematically deduced. Differences between physics and mathematics are discussed with emphasis on the role of physical constants. It is shown that it is logically impossible to introduce constants on the fundamental level of a final theory. The most fundamental constants emerging within a final theory are constants of motion.

It is argued that the only possibility to formulate a final theory is necessarily a tautology: A final theory of physics can only be derived from those presumptions about reality that are inherent in the idea and practice of physics itself. It is argued that a final theory is based on the notion of objectivity, but it is logically impossible that an ideal final theory supports realism.

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I. INTRODUCTION

In his book Dreams of a Final Theory Nobel laureate Steven Weinberg wrote: “It is difficult to imagine that we could ever be in possessions of final physical principles that have no explanation in terms of deeper principles.” But Weinberg (of course) did exactly this: he described the idea of a final theory in his book. Not all physicists, not even all Nobel laureates, share Weinbergs expectation that such a theory might be possible. Nevertheless we shall pick up and discuss various aspects of Weinbergs dream. Specifically we shall use the idea of the final theory to discuss the relation between physics and mathematics. Whether we believe or not that a final is possible in principle, both cases have their specific implications. Weinberg’s dream concerns a theory that is final because no deeper theory is possible. The concept of a final theory (FiT) understood in this way differs considerably from that of a grand unified theory (GUT) or a theory of everything. A GUT aims to provide a unified description of all fundamental forces of nature. However revolutionary this would be, it remains within the scope of already existing theories; the concept of a GUT does not necessarily imply finality, but completeness with respect to known experimental facts.

The conjectured finality results, as Weinberg explains, from “the beauty of simplicity and inevitability, the beauty of perfect structure, the beauty of everything fitting together, of nothing being changeable, of logical rigidity.” The keyword here is inevitability. Though the idea of a GUT implies completeness with respect to the known fundamental particles and forces, it neither implies simplicity, nor a deep or even final level of understanding, nor a high degree of explanatory power. A GUT could be ugly and incomprehensible. The form of a GUT - if we extrapolate the prominent tendencies in theoretical physics - will likely consist of a set of mathematical structures that allow to summarize the structure of the standard model of particle physics. But the very idea of a GUT does not include explanations why nature should prefer a specific mathematical structure and not some other. Or, in other words, a GUT would - in contrast to the ideal FiT - not exclude the possibility that different universes with different laws of nature might exist at least in principle.

However, we shall argue that the conjectured finality of a FiT follows a different logic than the assumed completeness of the GUT. Furthermore we shall argue that and why Weinberg’s specifications can, if at all, only be met by a theory that is based on trivial premises. The finality and inevitability that Weinberg expects from a final theory implies that it is similar to pure mathematics. This has a number of consequences.

Weinberg wrote “But (I repeat and not for the last time) I am concerned here not so much with what scientists do [...] as I am with the logical order built into nature itself”. We we shall argue that a final theory (if possible at all) will be based exactly on this: on what physicist do, what they have to do, in order to formulate a quantitative and objective description of nature.

In this essay we further elaborate the premises that we used in two preceeding publications in which

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2) One implication of the inevitability seems to be that the final theory is incompatible with the “Mathematical Universe Hypothesis” in Tegmark’s sense of enumerable possible universes: If many worlds are possible, then none of them can be regarded as inevitable.
author sketched a logical line of reasoning that allows to derive relativity, electrodynamics and the Dirac equation from (almost) trivial principles. Here we shall argue that these principles are a promising starting point of a final theory - not because we are able to deduce a GUT from it, but because they sketch a possibility to achieve the required logic of a FiT.

We shall argue that specifically the inevitability and finality of a physical theory can only be derived from a systematic analysis of the form or method of physics itself: Since the theory has to demonstrate that any physical world inevitably has the form of our universe, then it can not be purely empirical: it has to demonstrate logical necessity. And since we do (like Weinberg) not believe that it can be derived “from pure thought”, then it can only be derived as an ontology, i.e. from a description of physics itself.

In Sec. II we will try to precisely specify what we consider to be a final theory of physics. We do not know if our definition is in full agreement with Weinbergs idea of a FiT and his attitude towards such a theory. We will acuminate the idea to an extend that allows to draw conclusions. Insofar it is a thought’s experiment.

In Sec. III we discuss aspects of the relation and the differences between mathematics and physics and the consequences for a FiT. Here we put special emphasis on the difference in the methods of concept formation between mathematics and physics and what can be concluded for a final theory.

In Sec. IIIA we investigate the role played by the so-called “physical constants”, both, dimensional and dimensionless. We shall discuss where and how such constants may be introduced or derived within a final theory. The specific meaning of natural units is considered in Sec. IIIB their role within physical laws in Sec. IIIC and their importance with respect to the concept of objectivity in a final theory, in Sec. IIID. In Sec. IV we draw first conclusions and discuss their implications, mainly that the foundation of a final theory can only be formulated as a tautology. In Sec. IV A we describe the tautology, namely we provide a simple description of the premises of physics.

In Sec. IVB we sketch some consequences concerning the attitude of science towards reality. We shall argue that the formulation of a FiT first of all requires to withdraw all metaphysical presuppositions. In Secs. IVC and IVB we sketch the conclusions for the form of a final theory. In Sec. IVD we shall argue that there is an intrinsic incompatibility of rationalism with realism.

Finally we shall summarize our considerations in Sec. VIII.

II. THE SPECIFICATIONS OF A FINAL THEORY

The current idea of a GUT is that of a theory which unifies all forces of nature into a common conceptional framework. We do not intend to discuss the various candidates on the market like quantum gravity or the various string theories. It is sufficient to notice that these theories aim to be unified theories. But a final theory is more than just a unification, it is a theory which leaves no questions open. It not only allows to derive all known types of particles and interactions, but also the dimension of space-time. But even this is not sufficient to provide finality: As we shall argue, the finality implies that it even provides a formal derivation of the fundamental concepts of physics.

This specification might seem exaggerated and the idea of the possibility of a FiT hubristic, however this is not necessarily the case. The idea of a FiT only exemplifies the ratio behind the scientific project itself: Weinberg’s dream is a consequence of the well-known “principle of sufficient reason” (PSR) and it claims nothing more than that everything in nature should have a reason. Indeed it is difficult to imagine and to argue that some fact, or law, or relation in nature might have no reason at all, that it might be based on pure contingency, that the mass ratio of proton and electron is unexplainable. It would simply be unreasonable and alien to science to claim that physical relations or some law might have no reason. Because - if something in nature has no reason, why should anything have reason? Hence science has to presume reason, and that is why it is reasonable to consider that a FiT could be within reach some day - since anything that is reasonable can, by definition, be understood. Technical or financial limitations might prevent physics from performing the required experiments, or the mathematics might become so difficult, that we will never be able to solve all riddles of physics. But to question the PSR would go beyond that - to some degree it would mean to question the scientific project itself.

Gerard t’Hooft expressed his attitude towards the idea of a FiT as follows: “[...] in particular string theorists expect that the ultimate laws of physics will contain a kind of logic that is even more mysterious and alien than that of quantum mechanics. I, however, will only be content if the logic is found to be completely straightforward.” Or Leonhard Susskind (ibid.): “Not only would the theory explain why the proton and neutron are about 1,800 times heavier than the electron, but the theory would also explain itself: no other theory is possible.” If we include these requirements in the specifications of a FiT, then the theory has to carry the evidence for its truth in itself. This means that a FiT has to demonstrate that the laws governing physical reality could by no means be different than those formulated by the FiT. Then of course the FiT has to provide a formal certainty and “logical rigidity” as otherwise only known in mathematics.

Stanley Goldberg characterized the logic of mathematics as follows: “Different branches of mathematics have

3 This does not imply that everything must have an intelligible cause, though.
different rules but in all branches, since the rules are predetermined, the conclusion is actually a restatement, in a new form, of the premises. Mathematics, like all formal logic, is tautological. That is not to say that it is uninteresting or that it doesn’t contain many surprises. Hence a FrT might be possible if it is formally equivalent to mathematics. This raises the question of the differences between mathematics and physics and if and how they could be overcome. We shall come back to this in Sec. III.

The required self-evidence of a final theory (considered that such a theory is possible) does not allow to base the theory on substantial propositions, since substantial propositions are not self-evident. Using Goldbergs phrase we might say that substantial propositions are those that are not predetermined. All propositions that can be derived within the FrT must be inherent in some form in the premises of physics or they must be derivable mathematically. To substantiate the meaning of this idea, let us consider some examples of what propositions might be regarded as substantial. It is a substantial claim that space is fundamental. Though it is clear that our world appears to have this form, it is not evident why this shall be the case - why any physical world should have 3 + 1 dimensions. One might have doubts that physics will ever be able to give clear and distinct reasons why any possible physical world must have 3 + 1 dimensions, but one can not deny that the question “why 3+1 dimensions?” itself is legitimate and meaningful. Therefore a statement that the physical world has (or must have) 3+1 dimensions, is substantial and requires a derivation within a FrT. Hence, if we believe that a FrT should be possible and that it has to provide such explanations, then any physical world must have 3 + 1 dimensions.

A GUT might be based on substantial assumptions and postulates, for instance a specific dimensionality of space-time or some special general group structure or on the postulation of some type of string; A GUT could be formulated as a theory of postulates. However, Einstein remarked that “When we say that we understand a group of natural phenomena, we mean that we have found a constructive theory which embraces them.” Following Einstein, a GUT might as well be a theory of principle, but a FrT has to be anticipated as a purely constructive theory: It has to explain the 3 + 1 dimensional space-time by construction, it has to explain, why and what physical significance specific groups have, why there must be a maximal velocity of massive objects, why Maxwell’s equations have their specific form and so on. This implies that a FrT (if possible at all) also has to reformulate relativity; it has to replace all of its postulates by a reasonable construction. The same would hold for quantum mechanics. A final theory has to explain why we (have to?) use complex numbers in quantum mechanics and why the momentum should be equal to the spatial derivative of the wave function (times the unit imaginary). A FrT is a deductive theory in which all elements are deduced from unquestionable first trivial principles.

III. WHAT DISTINGUISHES PHYSICS FROM MATHEMATICS?

Since the final theory is constructive, it can not simply presume concepts like mass or velocity that originate from experience. The required inevitability implies that a final theory has to demonstrate how quantities like mass, energy, charge and field emerge within a mathematical framework that is developed on the basis of self-evident abstract principles. However there is at least one concept that has to be presumed: Time. The reason (and legitimization) is the inevitability of this concept. There is not way to describe the meaning of physics without presuming the notion of time in some way.

Goldberg described this in his book on relativity as follows: “The undefined terms for the dimensional system we will be using are "length," "time" and "mass." These are the common, undefined primitives of physics. Although I can tell you how to measure a length, or a time, or a mass by a formula, the concepts themselves are undefined. Either you know or you don’t know what I mean when I use a phrase like "time passes." The dimensions of all other quantities in physics can be expressed in terms of these fundamental quantities.” This means that the “understanding” of certain concepts can only be obtained by personal experience. If all substantial concepts of physics have to emerge within the final theory then a FrT is formally equivalent to mathematics. And then the “mapping” between mathematical form and physical meaning is reversed with respect to the usual method of physics. If substantial physical concepts like “mass” and “energy” or “spin” can not be presumed, these notions have to be identified from pure mathematical form.

Usually the arrow that describes the formation of concept in physics, points from the phenomena to the equations. In mathematics one also finds the opposite direction: Abstract concepts that have been invented on the basis of pure formal considerations and then obtain a name from conventional language like “fiber” or “pen-cil”. To establish a relationship between the mathematical form and its physical content is called interpretation. And in contrast to mathematics, the interpretation charges variables with physical meaning: the math of physics refers to things outside mathematics. It would be meaningless to ask whether names given to purely mathematical objects or functions are “correct”. Mathematical objects do not refer to anything else outside mathematics. But a FrT has to identify mathematical structures with physical objects. There is no formal way to prove the truth of this mapping. We can only verify (or falsify) its experimental predictions. But the variables that correspond to these predictions are a subset of all variables of the model: We measure mean values - even in the microscopic case. Phenomenological physical models that describe specific physical problems are to a large degree self-sufficient. It is their purpose to provide predictions. But a FrT has a different purpose, namely to provide the bridge between phenomenological
theories. It must be complete and convincing, though it is questionable, whether the required interpretations can be unique: the effectiveness of physical models is that they are applicable to a variety of phenomena.

If the phenomena (the meaning) has not been established within physics beforehand or if the FrT would describe completely unknown phenomena of nature, then the attempt of interpretation might be a serious challenge. The meaning of variables can (safely) only be obtained from already known physical concepts. That is, the path of reasoning within a FrT can not start at the phenomena and end in equations, but vice versa. The equations have to be derived on the basis of the abstract principles and the resulting variables in these equations have to be identified with physical quantities. This is a task of pattern recognition or - as Walter Smilga named it - “reverse engineering”.

This is a characteristic difference between mathematics and the classical method of physics: Mathematicians define or derive abstract objects which are invented in a purely formal way, as a kind of game, a game of thought. Then, in a second step, the forms obtain their names, which are essentially arbitrary. Physics however picks properties known from everyday experience on the basis of common language. These properties have to be defined scientifically but this should not alter their meaning, it should just render their meaning scientifically, i.e. quantitatively. The meaning of the “mass” of a material body is essentially the same in physics and in common sense, or at least it is very close.

When the physical significance of some symmetries have been understood, then physical theory might predict new particles because the specific symmetry principle suggests the particle for its completeness. But to invent a new symmetry principle without any prior experimental knowledge can be regarded as almost impossible. Hence the FrT requires for its formulation that most pieces of the final puzzle are already known.

Elementary particle physics is a special case in this respect and its methods are closer to those of mathematics: Most properties (or components) of particles are abstract and have no counterpart in the macroscopic world, like for instance (iso-) spin or parity or helicity. In this case physicists have to invent new, essentially arbitrary, names for particle substructures (“quarks”) and properties (“strangeness”, “charm”). The deeper physics dives into the micro-cosmos, the closer it seems to be to pure mathematics. Without the knowledge of the empirically developed physical concepts and notions it is hardly possible to recognize the physical meaning of the variables appearing in the supposed final theory and to find interpretations that provide a physically meaningful theory from purely formal (mathematical) relations. If a FrT is able to deduce equations from abstract principles, then the “correct” interpretation is not included for free.

Physics and mathematics require certain notational conventions, a nomenclature, that assigns symbols to certain quantities and relations. For instance, \( a^2 + b^2 = c^2 \) is associated with the Pythagorean theorem, while \( m^2 + p^2 = E^2 \), though of identical form, reminds us of the relativistic energy formula in units where \( c = 1 \). A purely formal derivation of equations within a FrT as suggested by an analogy with pure mathematics does neither provide the correct symbols nor the correct interpretation: it only provides mathematical relationships, the form. If concepts like “mass” and “energy” are supposed to emerge from the formalism then, at some stage, the FrT must provide an equation that matches, for instance, the mentioned form of the relativistic energy formula. However the FrT can not provide a proof that the derived formula really means the relativistic energy formula. A formally derived FrT does not automatically provide physical but rather mathematical relationships. The form provides information only in the eye of the well-prepared observer. The transformation of a mathematical form into a physical meaningful law requires an identification of the terms appearing in equations with physical quantities. This process is to some degree heuristic: It is an interpretation. Therefore it is unlikely that equations that emerge within a final theory can be interpreted correctly by pure thought. The formulation of the correct principles alone is not sufficient to obtain physical meaning: Physical laws have to be known and proven meaningful in advance, before a FrT can be formulated. Modifying Goldberg’s insight, we might say that we already have to know the form and relevance of the Lorentz group in order to recognize its significance within the mathematical framework of a FrT. Therefore it is unlikely that a FrT could be formulated “from pure thought” before physics sufficiently progressed towards a fundamental level of physical reality: We can only recognize known patterns; the pure form does not generate meaning. When the abstract inevitable principles of a FrT are found, and a mathematical formalism has been developed then one still has to establish the mapping between an abstract mathematical form and known physical laws. This step in the formulation of the FrT might be summarized as “function follows form”. Though the principles of the FrT must be trivial (“self-evident”), this does not necessarily hold for the formulation of the theory. Therefore it seems unlikely that a FrT will - from beginning on - be recognized as a possible candidate for a theory of everything. This seems to be sufficiently proven by contemporary candidates like string theory: The initial ideas of theories might be simple, but the resulting
structures and mathematical forms must (by definition) be rich and complicated enough to allow for the description of a large number of particles and fields. But if they are rich and complicated enough, then they are difficult to interpret.

The basic system of “undefined” quantities, that Goldberg mentioned, is still debated, but it appears obvious that certain quantities can not be defined by other means than by a description of their respective measurement method or directly by reference to experience. This is for instance the case for the notion of time. We can not “explain” time to someone who does not know what it is because it is unique. A unique concept can not be explained by other similar concepts since it is unique. But though clocks don’t explain or define what time is, clock’s define how we measure time. And this is all that is required: Physics is neither able nor obliged to tell what things are, but physics can tell us how things behave. It could be argued that, from the complete knowledge of how things behave, we finally obtain an idea of what It could be argued that, from the complete knowledge of how things behave, we finally obtain an idea of what

properties that are to be predicted. No measurement without a reference. Hence the form of physics implies and presupposes that there is time, i.e. that the same type of quantity is allowed to appear in two forms, as a variable and as a constant reference.

Therefore we shall next discuss the role of units and of so-called “physical constants” in Sec. IIIA Inherent in the notion of time is also the concept of constancy and the specific role of constants is an important difference between physics and mathematics. Some of these constants are called “physical constants” because they do not appear in mathematics. These are first of all the constants with dimension, i.e. those that have units, like the speed of light c or the Planck’s constant h. Then there are dimensionless constants like the fine-structure constant. The latter could, at least in principle, be the result of mathematical relations. A final theory has to provide the values of these dimensionless constants - which implies that they can be derived.

A. Measurement Standards And Physical Constants

Physics used arbitrary measurement standards in its history. There is the tale about Galileo, who measured the frequency of swinging oil lamps in church using his pulse as frequency standard. The use of the own body as a basic standard is a natural and obvious thing to do, as the size of an object relative to ourselves is exactly what we naturally want to know. The number of steps on the way between point A to point B is exactly the type of practical knowledge that one may assume as the origin of measurement. And quite naturally, distances can be and likely have always been measured in units of time: A to point B might be a three day walk or a single day horse ride. The use of the own body as a reference to measure length is nearby, because the scale of our body defines the scale of our world.

It is well within proportion that the meter is roughly the size of a footstep. The unit of a foot tests this finding and shows that measurements started with scientifically arbitrary units - standards that are defined pragmatically. The first modern scientists like Galileo picked phenomenological properties like length or duration of something in motion and measured dynamical properties of swinging pendula and falling stones. Consequently the first “laws” are mere (idealized) descriptions of proportions. First scientific observations were of the sort, that the frequency of a swinging pendulum does (in first approximation) not depend on its amplitude. This continued with Keplers and Newton’s laws: Physical laws were rarely formulated as equations, but as statements of proportionality. It should be noted that laws of proportionality allow to circumvent the definition of universal units. If we say that the frequency of a swinging pendulum is inversely proportional to the square root of its length, then the experimental confirmation requires (if at all) rulers and clocks. The law itself is correct no matter what units are choosen and hence a universal system of units is not required to confirm them experimentally. Despite the fact that it is possible to formulate objectively correct laws of proportion without explicit reference to specific objects as measurement standards, the measurements that are used to find these laws, of course require rulers and clocks. But they do not need a universal gauge.

As technology evolved it became unavoidable to define systems of units: laws of proportionality do not suffice for the construction of a pendulum with a specified period. A consequence of the formulation of the laws of nature by the use of equations instead of mere laws of proportions is the appearance of conversion factors like the Avogadro constant or the gravitational constant. Often these constants are named physical constants. Wikipedia explains
the term physical constant as follows: “A fundamental physical constant is a physical quantity that is generally believed to be both universal in nature and constant in time.”\textsuperscript{14} This, however, does not reflect the understanding of the physical significance of physical constants in theoretical physics. Dirac wrote: “The information that experimentalists obtain provides us with a number of constants. These constants usually have dimensions, and then of course, they depend on what units one uses, whether centimetres or inches. Then they are not of theoretical interest.”\textsuperscript{15} Dimensional physical constants bear no physical significance. This point of view has not changed significantly until today\textsuperscript{16} - even the idea that Planck’s constant is physically significant was questioned with some convincing arguments\textsuperscript{17}.

This situation might appear paradoxical: Those physical constants that can expected to be the result of purely mathematical considerations, are supposed to be those with physical significance and a FiT would eventually have to provide their values, while those with a physical dimension, are considered to be physically meaningless. But though the numerical values of dimensionful physical constants are meaningless, as Dirac explained, their constancy is not. For instance, the numerical value of the speed of light is meaningless as it depends on the choice of units. But the fact that a maximal velocity for massive objects exists, is of course meaningful, it is even of fundamental importance. However, since a FiT must be formally derived from abstract principles, the values of dimensionful constants can not be the result of such a theory. And since their values are not significant, there is no reason why these constants should emerge from fundamental equations. In theoretical physics, the system of units is often chosen in a way that the dimensionful constants like \(c\) and \(\hbar\) can be replaced by unity, i.e. they do not appear in the equations and we can expect the same for a FiT.

For instance, it can be shown that the group velocity \(v_{gr}\) of wave-packets is given by \(v_{gr} = \nabla_k \omega(k)\). In quantum mechanics (QM) and quantum field theory (QFT), wave packets represent particles and therefore the velocity of these particles is derived from the respective dispersion relation. For massless particles this yields \(\omega(k) = |k|\) such that \(|v_{gr}| = 1\). If frequency and wave vector have the same unit, there is no theoretical need for a dimensional constant called “speed of light”. This means that a FiT constructed from formal principles can provide equations in which the so-called “physical constants” \(c\), \(\hbar\), \(\varepsilon_0\) automatically appear. In a FiT such constants have to be introduced artificially simply because these constants are artificial.

B. The Significance of Natural Units

Dimensional physical constants loose their contingent status only if natural units like the elementary charge \(e\) or the electron mass \(m_e\) can be found which allow to replace the contingent historical units with “objective” units, i.e. the corresponding constant properties of fundamental objects.\textsuperscript{5} One of the first known natural units was the elementary charge \(e\) and it is indeed remarkable that any amount of charge in nature can be expressed by an integer number. If quarks are considered, the natural unit charge is \(e/3\), but still any natural amount of charge is quantized and can be expressed as in the natural unit represented by some fundamental object that serves as reference.

Natural units refer to the properties of fundamental (stable) objects (or processes) of nature. If matter would not be composed of atoms, e.g. if matter would not be quantized in some way, such fundamental objects could not be explained within a FiT. It was a common idea in the history of physics that matter would be something that completely and continuously fills up space, pretty much as it appears to the eye. This idea is nearby, directly derived from the “empirical evidence”. Today we know that the nucleus is very small compared to the atom’s dimensions, which are defined by the electronic orbitals, so that matter is far from filling up space.

However, if matter would really be a continuous something that fills up space-time in a way that could not be derived from a more fundamental theory, then a FiT would be impossible, because this would imply that space-time is fundamental and hence unexplainable. It has been suggested that the antropic principle might serve as an explanation. But it hardly explains anything: it does not generate the kind of reason that the principle of sufficient reason recommends.

Without a description of the internal dynamics of such a material, we could by no means argue that and why a specific radius should appear. The only way to obtain a granular structure of matter would then be to postulate some kind of quantization. Otherwise there would be no means to replace the arbitrary units of mass and charge by natural units and a final theory in the sense of Weinberg would be impossible. Without quantization we could not refer to the respective properties of an electron and it seems clear, that a FiT would be unthinkable in such a world as there would be no way to get rid of the arbitrariness of dimensional units. This implies that a classical FiT (i.e. a FiT without some type of quantization) is logically impossible. And vice versa: If we speculate that the formulation of a final theory is possible, then the FiT has to contain quantization in some way. Otherwise we could not derive the objects that provide the required natural units.

\textsuperscript{5} We emphasize the importance of reference standards within the logic of physics: objectivity is based on the use of reference objects. Let us expand the notion of objects and include properties of reference processes, despite the fact that we will later conjecture that objects can - within a final theory - only be understood as processes.
C. The Form of Physical Equations

Given two physical quantities $E$ and $p$ of different dimension are supposed to be related by a physical law in the form

$$E = f(p).$$

(1)

Maybe we derived this law by a sequence of measurements $E_k$ in dependence of $p_k$. The range within which one can experimentally prepare $p$ and measure $E$ is certainly finite, such that one can always approximate $f(p)$ by a polynomial of finite degree. In the simplest case, we find that $f(p)$ is proportional to $p$ or $p^2$. If $f(p)$ is a steady function, this has to be expected for small enough ranges of $p$, as one may always write a continuous function $f(p)$ of a continuous variable $p$ as a Taylor series:

$$f(p) = \sum_{k=0}^{\infty} \frac{df^{(k)}(p)}{dp} \left|_{\tilde{p}} \right. \frac{(p-\tilde{p})^k}{k!}$$

$$= C_0 + C_1 (p-\tilde{p}) + C_2 (p-\tilde{p})^2 + \cdots$$

(2)

However, we can only add physical quantities of the same unit. Since $p$ has a dimension, then all coefficients $C_k$ have different dimensions, $C_0$ the dimension of $E$, denoted by $[E]$, $C_1$ has dimension $[E]/[p]$, $C_2$ the dimension $[E]/[p^2]$ and so on. This would lead to an infinite number of dimensional constants. However, as only two dimensionful quantities are related in Eq. 1 it is sufficient to establish two dimensionful non-zero constants $p_0$ and $E_0$ and to express $f$ in the form

$$E = E_0 f(p/p_0),$$

(3)

where now $f(x)$ can have any mathematical form since $x = p/p_0$ is dimensionless. These two dimensionful constants would fall into one, if the Taylor series had just a single term, i.e. if the true law would be

$$E = E_0 \left( \frac{p}{p_0} \right)^n,$$

(4)

where we assume $n \neq 0$ to obtain a non-trivial equation. However, in this case, we could not determine two constants $p_0$ and $E_0$ separately, but only the proportionality constants $C = E_0/p_0^n$.

This example demonstrates that the mathematical form of physical laws and the number of physical constants are indeed related. If a physical relation is continuous and can be written as a Taylor series (Eq. 2) and the required number of natural units $E_0$ and $p_0$ can be found, then one can rewrite Eq. 2 in the form

$$x = \frac{p}{p_0}$$

$$E(x) = E_0 \sum_{k=0}^{\infty} \frac{dE^{(k)}(x)}{dx} \left|_{\tilde{x}} \right. \frac{(x-\tilde{x})^k}{k!}$$

$$= E_0 \left( 1 + D_1 (x-\tilde{x}) + D_2 (x-\tilde{x})^2 + \cdots \right)$$

(5)

with an (potentially) infinite number of dimensionless constants $D_k$.

If the values of these constants would be independent such that we could not find a mathematical law that allows for their derivation, then such a law needed an infinite number of dimensionless physical constants that would have to be the result of a FrT . But since the FrT must itself be analogue to mathematics, this would be a contradiction in itself. As we specified that a FrT must derive all dimensionless physical constants mathematically, then the most fundamental laws may not contain meaningful dimensionless physical constants. The idea of a FrT implies that the dimensionless physical constants are effectively mathematical constants. Hence the Taylor series is a short polynomial or it is a transcendental function. Since the most fundamental equations of a FrT may not include dimensionless constants, these most fundamental laws have to be simple. Not because physicist prefer simplicity, but because the laws are assumed to be fundamental. Nature might turn out to be different, but if it is, then the specified FrT is logically impossible.

Let us assume the true law would be

$$f(p) = E_0 \left( \exp \left( \frac{p}{p_0} \right) - 1 \right),$$

(6)

If $p_0$ is very large compared to the experimentally realizable range of $p$, then the first experimentally verified law might be

$$f(p) = C_1 p.$$

(7)

This law would neither be absolutely right nor absolutely wrong. It would be adequate within a certain range of $p$ and increasingly inaccurate and finally useless, if the $p$ of interest is not within or not even close to the mentioned range. It is possible to determine $C_1$ by the measurements within a limited range of $p$, but not $E_0$ and $p_0$ separately. With progress of the experimental technology one might be able to extend the range of $p$ towards and beyond $p_0$, such that we would be able to find

$$f(p) = E_0 \left( \frac{p}{p_0} + \frac{1}{2} \left( \frac{p}{p_0} \right)^2 \right),$$

(8)

and maybe with further experiments (or with a deeper theoretical insight) we might conclude that the true law would have to be Eq. 8. It is obvious that the first dimensionful constant $C_1$ has, as such, no fundamental meaning. But what about $E_0$ and $p_0$?

Such constants have fundamental physical significance given that they represent natural units $E_0$ and $p_0$ or that they can be expressed by other natural constants. In the relativistic energy-momentum relation

$$E = \sqrt{m^2 c^4 + p^2 c^2}$$

(9)

these two constants are $m$ and $c$, such that with $E_0 = m c^2$ and $p_0 = m c$ one may write instead of Eq. 8

$$E/E_0 = \sqrt{1 + (p/p_0)^2}.$$
As a final theory may not contain arbitrary units but only natural units as references, these units play a fundamental role for the way, in which physical laws are and can be formulated within a FiT. The natural units however are properties of reference objects. The FiT can not derive the value of the mass of a reference object: The FiT expresses the mass of other objects in units of the reference object. The FiT can also not explain the stability of the reference object: The FiT instead uses the stability of the object as a self-sufficient fact: In an objective reality we must have stable objects.

However, the FiT may not presume dimensionless constants since the derivation of dimensionless constants was one of the requirements for a final theory. The conclusion of these considerations is that a FiT can not refer to any physical constant at the most fundamental level. Nevertheless it is supposed to provide objects with constant properties which may then serve as references and that provide natural units. This apparent paradox can (only) be solved, if the constant properties of the objects are constants of motion (COMs) as we shall explain in Sec. V.

Only the existence of fundamental objects allows to refer to fundamental quantities. This in turn allows to express physical quantities by real (or even by natural) numbers. The final theory became thinkable only by the discovery of the sufficient number of physical constants, e.g. by the invention of relativity and quantum mechanics. Without the natural objective constants which may then serve as references and that provide natural units. This apparent paradox can (only) be solved, if the constant properties of the objects are constants of motion (COMs) as we shall explain in Sec. V.

Dimensionless constants like the fine-structure constant $\alpha \approx 1/137$ or the mass ratio of electron and proton $m_e/m_p \approx 1/1836$ represent proportions between the properties of fundamental objects or processes. A FiT should provide the mathematical arguments that allow to derive these proportions and in case of success these constants are effectively mathematical constants like $\pi$, which represents an extremal proportion between circumference and diameter.

D. Reference Objects and Constants of Motion

On the most fundamental level, a FiT may neither refer to dimensional constants nor to dimensionless constants, because the former are arbitrary and the latter have to be derived within the FiT. But on the other hand objects with constant properties are required as references in order to allow for measurements. This apparent contradiction can be resolved if we draw the following conclusions: Firstly, all quantities that belong to the fundamental level, must be variables in the sense that they have to vary at all times. Secondly, since constants are required to enable for measurements, then the most fundamental level obviously does not allow for a direct measurement. However, there must be a one or several functions of the fundamental variables that generate constants of motion and therefore the reference constants must be constants of motion. If all fundamental variables continuously vary, the constants of motion must at least be of second order in terms of the fundamental variables.

Constants of motion appear in all dynamical laws of physics. The best known constants of motion are energy, momentum and angular momentum and a FiT has to provide a mechanism from which these COMs emerge. From Noether’s theorems we know that constants of motion and invariance properties are closely connected. As described in Ref.20,21 the suggested trivial principles allow to derive Hamilton’s equations of motion (EQOM). But the fundamental variables are not “coordinates of mass points”, but purely abstract variables that can not be directly measured. This is a logical implication of the idea of a FiT and if we interpret fundamental variables as (components of) quantum mechanics wave functions, then we found a reason why the wave function itself can not be directly measured: Quantum mechanics has the task to analyze the behavior of ensembles solely on the basis of their observables. But the fundamental objects, represented by the wave function, can not be directly measured: There is no direct unit for the wave function, because the (components of the) wave function vary at all times.

A consequence of this argument is that truly fundamental quantities can never be subject of a direct measurement. This is the trivial and evident reason, why we can not measure (components of) $\psi$ and claim that “the first component of the wave function at position $x$ is five puminis”6 while there is no (principle) problem to obtain the value of a magnetic field $B(x)$ at some position in units of Gauss or Tesla. Then, however, it follows that a magnetic field value can not be regarded as a fundamental quantity: no directly measureable physical quantity can be regarded as fundamental.

IV. WHAT COULD PROVIDE THE TRIVIAL EVIDENCE OF A FINAL THEORY?

If we apply Goldbergs characterization of mathematics to physics, then a FiT has to be a tautology. As mentioned in the introduction, we suspect that a FiT might be formulated on the basis of what we do when we perform physical experiments, physical measurements, i.e. what we have to assume unless our experiments and measurements are meaningless. Here we might also find the

6 A pumi is a fictitious unit for wave functions.
final answer to the question in what respect physics differs from pure mathematics.

Therefore the first step towards a FiT would be to provide an inventory of the primary principles that are trivially true in any world that allows for the formulation of physical laws and to remove all presumptions that can not be considered to be trivially fulfilled. Since substantial presumptions can not be prerequisites for the formulation of a FiT, they should be critically reviewed. Essentially this requires a description of the form of physics.

A. The Form of Physics: What Physicists Do

Mathematics is itself not a model of anything but a logical framework, and also a FiT can not in its origin be a model of anything. Hence it can not be directly derived from the phenomena. If a FiT is possible, then the description of the physical aspects of reality is based on a set of formal principles and we will show that the form of physics provides trivial principles that can be used as a starting point.

Physics is a scientific discipline because it is based on a well-defined method to test theories. The method of physics is “objective” - but not because it describes “real” objects or some kind of “objective reality”. It is objective because it is based on the comparison of objects with objects. Measurements are comparisons - physicists compare the size, weight, velocity, energy and other quantitative properties with the size, weight, velocity and energy of reference objects: the prototype meter, the kilogram and the velocity of light are - or have been - used as reference. The branch of physics that provides the required references, is called metrology. The definition of units and the production of reference artifacts might appear to be an inevitable but merely formal act. However, it is exactly of the kind of inevitability which is required for the formulation of a FiT. The inevitable aspects of physics are those which in summary are the form or method of physics. Since physical theories are tested by measurements, the things that physicists do when they perform measurements, is an essential part of the form of physics.

Einstein wrote about his theory of special relativity that “It is striking that the theory (except for the four-dimensional space) introduces two kinds of things, i.e. (1) measuring rods and clocks, (2) all other things, e.g., the electromagnetic field, the material point, etc. This, in a certain sense, is inconsistent; strictly speaking, measuring rods and clocks should emerge as solutions of the basic equations [...] not, as it were, as theoretically self-sufficient entities. This confession should be surprising and disconcerting to physicists and philosophers of science, as it suggests that there might be missing piece, a systematic incompleteness in the logic of objectivity of the natural sciences. However (to the knowledge of the author), the fundamental role that metrology plays with respect to the method of physics is rarely mentioned, discussed or even noticed, despite the fact that the use of objects as reference standards is undeniable the fundamental of objectivity in physics. A final theory, if considered possible at all, has to incorporate the trivial and fundamental role played by measurement standards.

In the mentioned quote, Einstein describes what he did when he formulated special relativity, but it is just an example of what physicists always do, namely to introduce references combined with certain invariance assumptions. As they seem blatantly trivial, these assumptions are rarely spelled out explicitly; but to define rods and clocks (e.g. some objects) as references, implies a number of presumptions about the relevant properties of these objects, namely that they are invariant. First of all they are assumed to be constant in time, but the meter rod is also assumed to be invariant with respect to its orientation in space. Certainly we won’t suggest to doubt these invariance principles, but we should be aware of the implications of this method. Because the presumed invariance principle leads, if violated, to the introduction of fields: If the spin of an electron has no preferred direction, then this is phenomenologically identical to the absence of a magnetic field. And vice versa: The violation of the isotropy of space is phenomenologically identical with the presence of a magnetic field. Contrary to what one might expect, but the presumed isotropy of space does not require an experimental proof: Where we find space to be anisotropic, we (have to) presume a reason, namely a (gauge) field. Newton’s axiom of inertia obeys the same logic: According to this axiom a body moves with constant velocity on a straight line unless a force is applied. Not only that, at the time of Newton, it was technically impossible to provide experimental evidence for the presumed straight motion, it is logically impossible to prove Newton’s premise as it is itself the definition of (the absence of) a force. Therefore it is introduced as an axiom. The underlying principle of this method is the principle of (sufficient) reason. Science has to provide reasons for distinctions, for asymmetries, for differences. And if such reasons are not at hand, then we (have to) assume symmetry, homogeneity, equivalence. This is not a principle of nature, it is a principle of thought.

It is one of the revolutionary but also one of the most questionable aspects of relativity, that it does not simply continue to presume the invariance of rods and clocks, but describes precise conditions of invariance and covariance and the corresponding transformation properties. And in doing so, it apparently breaks with a rule of thought: Rulers are, as we explained above, by definition constant. However, this holds only, if the we assume that space-time is a fundamental notion, an assumption that is, as we argued above, incompatible with the logic of a FiT: it would deny the possibility to argue why space should be three-dimensional. Whence there is no contradiction between relativity and the rules of thought, if a FiT is possible. The FiT transforms the contradiction into an obligation: The FiT has to formulate a criterion that allows to characterize dimensionalities and to show how spatial notions emerge.
Prior to the invention of relativity, physicists simply presumed that clocks and rods are the same for all observers under all conditions. The theory of special relativity is the first theory that questioned this naive presumption. Now, after the experimental confirmation of relativity, we believe to know that (perfect) rods are invariant in proper time and under rotations, but not under Lorentz boosts. As rods are rarely used to measure a length when they move with relativistic speeds, this effect is irrelevant in all practical situations. It is difficult to imagine a situation in which the “length contraction” could have been discovered by the direct use of rods for length measurements and accordingly it is difficult to confirm this “effect” by direct measurements. It is a consequence of a formalism that has been confirmed by a variety of other methods and predictions. Despite this, the theory of special relativity has achieved the status of an almost unquestionable corner-stone of physics. Namely in accelerator and particle physics, it has reached the same status as daily applied engineering knowledge as electrodynamics and is usually regarded as part of classical physics. To the general public, relativity is usually presented as a theory of space-time and rarely regarded as an extension of metrology. We think that this might be a mistake: Seen by light, Einstein’s work is full of hints that relativity is intimately connected to problems of measurement standards and relative calibration (clock synchronization).

Our considerations result in the insight that it is not the constancy of the length of a specific (inertial) ruler that we have to presume, but we have to presume that regardless of the technical problems to provide a really constant ruler, it is in principle possible. This is part of the form of physics: Despite all practical problems, we presume that there is no general physical law that prevents us in principle from manufacturing a constant ruler or a perfect kilogram. As a final theory has to be free of logical circles, the final theory can not presume stability of the natural units and rulers and provide at the same time an explanation for this stability.

A FvT can not be based on the assumption that a specific property of a specific object is constant in time, but it may be based on the more abstract assumption that there are constants. It has to incorporate the basic assumption of metrology, namely that nature provides objects with constant properties that can be used as references. It is then logically impossible to prove this assumption within the theory. Any attempt to do so provides merely the proof of internal consistency: It would just show that the theory allows to reproduce the assumptions it is based upon.

B. Existence, Appearance and Philosophy

But how at all can it happen that preconceptions based on empirical evidence fail, that matter is almost empty space, that Newton’s concept of absolute time and space turned out to be wrong? Our preconceptions about space and time are taken from “reality” as it appears to the senses and as it is processed by our mind. But is it not science (and specifically physics) that provides evidence and confirmation for the existence of an objective reality? Steven Weinberg dedicated a complete chapter of his book to argue “against philosophy”. Nevertheless he admits on the second page of this chapter: “Physicists do of course carry around with them a working philosophy. For most of us, it is a rough-and-ready realism, a belief in the objective reality of the ingredients of our scientific theories.”

It is irritating that Weinberg argues against philosophy, as his book appears to be more about philosophy than about anything else. But as he explains, he argues not against philosophy as such but against dogmatism. He writes: “Although naive mechanism seems safely dead, physics continues to be troubled by other metaphysical presuppositions, particularly those having to do with space and time.” and on the same page he writes: “Even now, almost a century after the advent of special relativity, some physicists still think that there are things that can be said about space and time on the basis of pure thought.” One might be tempted to remark that the idea of a final theory, as Weinberg (pro-) poses it, may as well suit the purpose of arguing in favour of a theory derived from pure thought: The inevitability, logical necessity and coherence that he expects from a final theory, are properties that can otherwise only be found in mathematics, that is: in a branch of science that is based on pure thought. Again we arrive at an irritating dichotomy. On the one hand, physics defines itself as an experimental science, but on the other hand it is the final objective of (theoretical) physics to find a final theory which must be similar to pure mathematics, in various aspects. However, the keyword in the above quote is “metaphysical presupposition” and in this respect the author fully agrees with Weinberg. A theory should not (and a final theory may not) contain metaphysical presuppositions. This is a key requirement that we specified for a final theory: It may not contain any substantial distinctions without sufficient reason. Specifically we deny the possibility that a FvT could be based on metaphysical assumptions. Within the context of a FvT we have to refuse any assumption that is not based on reason, any distinction that is not inevitable. And in consequence this means that we refuse any substantial claim.

It appears that Weinberg is aware of the fact that the “rough-and-ready realism”, understood as a habit or a tendency to be philosophically attached to the appearance of reality, insofar as it is similar to “naive mechanisms”, is disproven by experimental evidence.

In his allegory of the cave, the Greek philosopher Plato addressed the question, whether appearance generates a faithful image of the true form of reality: “Plato has Socrates describe a gathering of people who have lived chained to the wall of a cave all of their lives, facing
a blank wall. The people watch shadows projected on the wall from things passing in front of a fire behind them, and they begin to give names to these shadows. The shadows are as close as the prisoners get to viewing reality. In Plato’s allegory the perception of reality does not show something unreal or an illusion, but something that is a projection or a derivation from the true world. Thus our preconceptions fail, because they are based on notions derived from the appearance of the world, namely (as Weinberg emphasizes) on wrong preconceptions about space and time. Our preconceptions fail because they are based on “empirical evidence”, but not on the kind of empirical evidence of a physics experiment, but on the kind of naive “empirical evidence” of common sense.

Weinberg wrote: “It is just that a knowledge of philosophy does not seem to be of use to physicists”. Here we clearly disagree with Weinberg. First of all, we believe that some knowledge of philosophy is useful to any human being, but more than that, we doubt that the usefulness of philosophy can be derived from the possession of some kind of knowledge. Same as physics, philosophy is mainly a proficiency. Because a final theory may not be based on metaphysical assumptions, the proficiency to recognize assumptions as metaphysical is required. Regardless on how we arrive at a FiT - only if the theory can be (re-) constructed on the basis of pure thought, it can be said to have met his specifications. And philosophy (even more than mathematics) is the domain of pure thought.

Besides the major concepts of physics also the specific (apparently relativistic) form of space and time including Lorentz transformations and the dimensionality of spacetime must be the result of a final theory. In order to obtain these notions as results from a final theory, the theory has to be based on something else, e.g. not on the presumption of a self-sufficient space-time model. It has to include a part that describes the emergence (or appearance) of spatial notions.

C. The Form of Dynamical Laws

The example of a physical law as described in Sec. 11C concerned some general relation between two physical quantities like (for instance) energy and momentum. Such relations allow to reduce the number of free variables by uncovering a fixed relation between quantities (variables). If we discuss whether a specific process (or nature as a whole) is deterministic (i.e. if it could be predicted by an algorithm), then we refer to a different kind of physical law, namely to a dynamical law, to so-called equations of motion (EQOM). EQOMs refer to a number of dynamical variables that can be expressed by the components of a state vector \( \psi \) and have the archetypical form

\[
\dot{\psi} = f(\psi),
\]

- where the dot indicates the time derivative - such that, if the state of a system is represented by \( \psi \) at a certain point in time, all future states of the system can be computed by integrating Eq. (11). We do not consider partial differential equations here, for two reasons: Firstly, we said that a FiT is allowed to be based on the notion of time, but not on space - and secondly, we expect that the introduction of spatial notions can be delayed to a later stage, as it is always possible to use a Fourier transformation to introduce spatial notions.

In classical physics and namely in classical statistical mechanics, the state vector contains the coordinates and momenta (Hamiltonian mechanics) or the coordinates and velocities (in Lagrangian mechanics), respectively. In the former case, the EQOMs are derived from a conservation law (usually the conservation of energy) and in the latter case from a variational principle (the so-called principle of least action). The EQOM depend on a single variable \( t \) which can be interpreted as time, the primary “quantity” that we accept as being undefinable. Time is the (first) variable for which the question “why time” makes no sense within a final physical theory: Already the concept of physical variable and of measurement implies that we refer to change, variation and constancy, i.e. to those notions that are associated with time. And we shall use the required distinction of variables and constants to discuss the possible forms of Eq. (11).

V. A Survey of (Trivial) Principles of Physics

Our considerations allow to give a first, possibly incomplete, inventory of formal first principles. The most important being the principle of sufficient reason. Sir Hamilton summarized this principle in the short formula: “I infer nothing without a ground or reason.”. The idea of a FiT itself is based on this principle as we explained in Sec. II.

The principle of reason (PSR) is the first principle that trivially has to be respected in a FiT. The believe that things are comprehensible is a precondition for science in general, of course. But the principle includes more than that. From the specifications of Weinberg, t’Hooft and Susskind it follows that if a FiT is supposed to be inevitable, evidently true, complete and final, then a FiT may not contain any arbitrary distinction or selection, no concept should be introduced without a clear and evident reason. The final theory does not have to pro-
vide causal explanations for individual phenomena, but it should provide sufficient reasons for the general form of the phenomena. Hence a FrT can not (and does not have to) tell what time is, since we can not even formulate the idea of what physics is without presuming that the concept of time is already known.

Secondly, as we explained before, the presumed concepts of time and measurement inevitably lead to the trivial confession that physical models contain variables and constants. As we argued, for logical reasons, constants may not appear at the fundamental level, neither dimensional nor dimensionless. Despite this, the idea of a measurement requires the appearance of true constants for reference. We might call this requirement “the principle of objectivity” (POO). It is the basis of the relation between a physical theory and “objective reality”: Physical equations refer to relations between properties of objects and though this is rarely within the focus of attention, these objects have to be physically made, if the equations are to be confirmed. The production of artifacts with constant properties must after all be possible in principle. If this is not the case, then we can not directly measure this property. Even worse, strictly speaking it could be questioned if such a property is objectively there.

The dichotomy that we may not directly introduce constants while measurements are nevertheless based on the availability of such constants, can be solved: dimensional constants are logically connected to the laws of motion - they appear in the form of constants of motion (COMs). Within a FrT constants of motion are the only possible method to introduce dimensional constants. Either we derive constants of motion from a law of motion, or vice versa, but in the former case we have to presume substantial laws, while in the latter case we just presume the trivial fact that constants must exist. The latter presumption is sufficient trivial to serve as a first principle, but we have a prize to pay: if the most fundamental constants are constants of motion that come along with some law of motion of some set of fundamental variables $\psi = (\psi_1, \psi_2, \ldots, \psi_k)$, then a reference for a measurement of the components of $\psi$ is logically impossible. The fundamental level is a level of pure variables, while the “level” of constants of motion is at least second order in $\psi$. As we have shown in Ref.\textsuperscript{20,21}, the constants of motion are the even moments of $\psi$. Since COMs are required to obtain observables, then, within a FrT, the fundamental level of reality can not be observed: The final theory must distinguish between existence and appearance.

In Ref.\textsuperscript{20} we used an “ontology of time” as the basis for the same reasoning. But as we have shown in this essay, the suggested principle of variation (POV) can be derived from the idea of a FrT, hence finally from the principle of sufficient reason. A genuine ontology is not required, as we derived the law, that all variables on some fundamental level have to vary, from distinct formal considerations.

Besides the distinction of variables from constants, we have to distinguish quantity from structure. It is trivially true that physical models contain variables that represent quantities. But structures have to be derived. A quantity can be represented by a number and a unit. A structure is given by an algebra. The complex plane for instance is represented by an algebra: The unit imaginary $i$ can not be represented by a number and therefore we (have to) introduce a symbol. Something similar holds for the number $\pi$, namely the fact that we can not write it down explicitly, suggests the use of a symbol. But unlike $\pi$, which can by no means be completely written down, we can represent the unit imaginary by a structure that fully characterizes the algebra of the complex numbers, namely that of $2 \times 2$-matrices. The introduction of a new symbol like $i$ is equivalent to the introduction of an algebraic structure. It is therefore reasonable to ask, if the unit imaginary is essential for quantum mechanics or not - and if it is, then why? The use of complex numbers in quantum mechanics can be questioned and hence it is a substantial choice. Since a FrT may not presume what it is supposed to derive, it is not legitimate to simply presume a prominent role for complex numbers in a FrT; the FrT has to provide a reason.

This preliminary survey does not claim to be complete. There are likely more trivial principles that belong to the form of physics which we did not even mention. One of the principles that we did not discuss is implied in the possibility of a quantitative description: If there are observable quantitative properties of objects, then these properties must be additive: The length is of two rods is twice the length of a single rod. And there are certainly more principles of this kind. We summarized these principles in the formula “a world that allows for a physical description”, which is both, precise enough for the purpose of this paper and vague enough to include those principles that we did not discuss explicitly.

Let us finally emphasize that the describes principles imply a strong selection: Whatever might be “out there”, we can only measure those things that obey a law of motion which provides the necessary constants of motion as reference standards. Everything else is noise. Scientists should in general avoid to make claims about unmeasurable things. However, as we argued, with respect to a final theory, we can not avoid to make at least one claim about the wave-function, namely that its components are not directly measurable. This is not unreasonable, but a logical necessity.

VI. OBJECTIVITY AND CLASSICAL REALISM

As explained in the previous sections, it appears that, given a final theory would be possible or even the only logical consequence of the program of some specific rationalism, then this seriously questions that objectivity and realism are compatible with each other; if the final theory would be found and could be regarded as objectively
and evidently (trivially) correct, then physical appearance and existence must be distinguished and classical realism must be regarded as disproven. Or, if we choose to believe that classical realism must be correct, then a final theory is impossible and there must be something unreasonable, i.e. some kind of significant but unreasonable arbitrariness in the world, something outside the PSR, something that a FiT could not derive or explain. Hence it appears that there is an intrinsic contradiction between realism and rationalism. The rationalist would expect that a FiT is possible - but then a “realistic” worldview is logically impossible.

We defined the notion of objectivity of experimental physics and we believe it is as clear as it is practically and theoretically relevant, though it refers to the notion of object, i.e. to something that is given only through experience. Realism, the complementary notion of objectivity, on the other hand, is more difficult to specify, also, because is appears in a number of variants. Often realism and objectivity are used in combination as objective reality, some sort of common sense agreement about what is meant by “the world”. Instead of making attempts to give a theoretical definition of “realism”, let us instead refer to a number of statements, that a strong realist would likely support:

- The world is in a fundamental way as we perceive it, i.e. composed of objects located in space and time. Objects are made of matter.
- Every physical effect has an exclusively physical cause: The physical world is closed. This includes:
  - The only possible way to influence the physical world is by physical action (i.e. through actions of the body).
  - All valid information about the world originates from information that has been received physically, i.e. with finite velocity $v \leq c$ via physical signals through our senses.

It is noteworthy, that while the principle of objectivity defines (and therefore limits) the scope of empirical sciences, especially that of physics, realism defines (and therefore limits) reality, most often to the notions of space, time, matter and causality, or by suggesting limits of possible information transfer. The former leaves the question open if the objectively real is a subset of the real, while the latter reduces the world to the objective by definition. Strange enough that these notions are rarely distinguished by physicists. We do not intend to discuss this much detail, but we would like to emphasize that the principle of objectivity defines a fundamental rule for quantitative empirical science that can not be questioned on scientific grounds, while realism tries to establish a specific ideology that is based to a large degree on unproveable metaphysical claims. One might summarize this as follows: it is logically impossible to question that physical laws are objectively true, unless one questions that a kilogramm, a meter rod or a clock can be used as measurement standard. But classical realism reverses this logic and claims that only the “objective” aspects of the world “really” exist. We think that this type of restrictive realism is not only not required, but it builds a barrier between natural and social sciences that is not helpful. Besides that, the metaphysical assumptions of realism are not required to obtain objective knowledge of nature.

Especially the closure claim of classical realism can be considered problematic, especially in the context of the questions of free will and the nature of consciousness.

VII. SOME FORMAL CONSIDERATIONS

While we measure the weight of an object, by comparing it to reference weights, we do not bother too much about the number system used. This might be deprecated by pure mathematicians, but for most experimental physicist (and likely for most engineers), there are only two types of numbers required: integers and reals. The question, whether it is logically necessary to use the reals instead of the rational numbers, can be pragmatically answered insofar as there is no immediate reason to restrict the number system to the rationals, despite the fact that we can never obtain a non-rational measurement result in measurements of limited precision. Though we can not measure $\pi$ with infinite precision, we are still able to find ways to measure $\pi$ to any required precision by the use of a sufficiently large circles. Insofar the experimental determination of $\pi$ via measurement is in principle not different from its computation: No one can compute (or write down or store on a computer disc) the number $\pi$ with infinite precision; but it can be done with any required precision: there is no logical limit to the precision with which we can either compute or measure $\pi$.

Sometimes it is argued that the reals are - besides the complex numbers and the quaternions - just the simplest field over which one can define division rings and that this fact suggests that the most general mathematical objects (i.e. the quaternions) should be used on the fundamental level of a final physical theory. Despite the fact, that quaternions might play a prominent role in some areas of physics, we reject the general argument. Both the complex numbers as well as the quaternions, can be represented by real valued structures, namely the complex numbers by real $2 \times 2$ and the quaternions by real $4 \times 4$ matrices. Therefore they are not more fundamental than real matrices, but less, as they refer to specific matrix algebras. Instead of presuming a fundamental role for a specific algebraic structure, a FiT should derive and explain this structure and prove its fundamentality. The intransparent mixture of quantity and structure however may lead to confusion about the meaning and role of complex numbers in quantum mechanics and physics in general. This is indicated by the discussion about the meaning of complex numbers in QM that is ongoing for
decades.\textsuperscript{22–34} We do not recognize the progress in mixing quantity and structure on the fundamental level. Rather, if complex numbers are a preferable, this should be a result of the algebraic properties of the FrT\textsuperscript{.} This fact is reflected by the architecture of number systems in computers, computers which are able to perform simulations of quantum systems, i.e., integers and reals. A computer is usually understood as purely based on binary digits. Though this is trivially true for contemporary digital computers, the existence of analogue computers and the fact that the personal computer had originally two processing units, the central processing unit (CPU) and the floating point unit (FPU), can be regarded as an argument against the fundamentality of the bit: Even, if the memory of modern computers is by design based on bits, it is undeniable that we can not use it in a general scientifically meaningful way without introducing a representation of floating point numbers. This representation is based on bits for purely practical reasons. Though any given coding scheme of floating point numbers allows only for a limited precision, we are able to construct coding schemes for any required precision.

A. Structure Preservation

\textit{Any finite} physical system can be described by a (possibly infinite) set of varying real fundamental quantities as a function of time $\psi(t)$. But all observable (measurable) properties are of even (at least second) order in these fundamental variables. The variables themselves can not be directly measured due to the lack of constant references. References for measurements are constants of motion and they are at least of second order in $\psi$. Once we presume some (arbitrary) constant of motion $\mathcal{H}(\psi)$, then further constants of motion emerge in the description of this fundamental “objects”\textsuperscript{20,21}, namely the traces of powers of the autocorrelation matrix (matrix of second moments), multiplied by the symplectic unit matrix. When the general constant of motion $\mathcal{H}(\psi)$ can be written as a (multivariate) Taylor series, then the leading order terms automatically generate structure preservation. The presumed constancy of $\mathcal{H}(\psi)$ generates a self-identity of “objects” via the identity of motional patterns. No other identity is required and possible. As we argued in Ref.\textsuperscript{20,21}, the most general framework for structure preserving motion is symplectic motion and it is a logical consequence of the constancy of $\mathcal{H}$. Furthermore it has been shown that the most simple, fundamental, and non-trivial algebra that describes the general properties of symplectic motion, is given by the real Dirac (Clifford-) algebra $\text{Cl}(3,1)$ which implies that the fundamental set of observables suggests the pattern of Minkowski space-time. There are 10 symplectic generators within this Lie-algebra that require an interpretation: a $3+1$ dimensional vector and a six-component bi-vector. The transformation properties of these components suggest the interpretation in terms of energy, momentum, electric- and magnetic field\textsuperscript{20,21} and the equations of motion of the autocorrelation matrix allow for a derivation of the Lorentz force. However, the primary and fundamental variables are not space-time coordinates any more. Instead the fundamental variables are the (abstract) components of $\psi$, which can best be described as points in an abstract phase space. These ideas thus reverse the role between “real” space and phase space: The $3+1$ dimensional “energy-momentum space” is an effective space that emerges on the basis of a fundamental 4-dimensional phase space described in proper time and the concept of space-time emerges through Fourier transformation. Effectively this Fourier transformation is similar to the characteristic function of probability theory, but its argument is not the probability density function (PDF) $\rho$, but the product of $\psi$ and $\sqrt{\pi}$. The 6-dimensional phase space of a “mass point” emerges from the equations of motion of the second moments and is symplectic only in a low-energy and low-field approximation.

Hence it seems possible to derive the concepts of mass and spin (the two invariants of motion of a 4-spinor), of energy-momentum 4-vector, of the electromagnetic field and the transformation properties of the Lorentz group (rotations and boosts) on the basis of trivial principles. This is a first “proof of principle” that a FrT might indeed be possible - as a tautological theory.

VIII. SUMMARY

In this paper we picked up the idea of a final theory of physics. First we formulated the requirements of a final theory. These seem to be extreme in view of the severe difficulties of modern physics to unify gravity and quantum mechanics. But regardless of the current status of theoretical physics, the idea of a final theory has its roots in the principle of sufficient reason (PSR) and hence essentially in rationalism. Specifically the idea that a final theory would be inevitable and that it would carry the evidence for its truth in itself seems to suggest that - following Goldberg’s characterization of mathematics - a final theory of physics must be a tautology. This implies that all statements of the theory are predetermined in its premises such that it is possible to derive the content of physics logically from the premises, but also to elaborate to a higher degree of precision what a final theory is able to tell us about the world, and even more, what it is not able to tell. The absence of metaphysical presuppositions is a logical necessity of the finality of a FrT - but it does not imply the absence of “metaphysical” conclusions. The final theory as we sketched it, might tell a lot about reality, but likely not what some of us expected to
find. It will not tell us, why there is something instead of nothing. It will not even tell us, what type of substance matter “really” is. The tautological FiT - and we claim that no other FiT is possible - is, well, tautological.

Besides the PSR, which provides the rationale for a final theory, and is therefore obligatory, we discussed some premises for a FiT that may in summary be called the *form of physics*. We presented a first primitive analysis of the conjectured possibility of a FiT. We identified three types of physical constants, namely dimensionless constants, dimensional conversion factors and natural units. We explained why the only possible dimensional physical constant within a FiT is the natural unit and that it can only emerge from the theory as a constant of motion. We argued that this result logically implies that the fundamental level of a FiT is not directly measurable - not because the fundamental level is less “real”, but because there is an unavoidable lack of reference.

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