A NOTE ON HARDY SPACES ON QUADRATIC CR MANIFOLDS

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Abstract. Given a quadratic CR manifold $\mathcal{M}$ embedded in a complex space, and a holomorphic function $f$ on a tubular neighbourhood of $\mathcal{M}$, we show that the $L^p$-norms of the restriction of $f$ to the translates of $\mathcal{M}$ is decreasing for the ordering induced by the closed convex envelope of the image of the Levi form of $\mathcal{M}$.

1. Introduction

Let $f$ be a holomorphic function on the upper half-plane $\mathbb{C}_+ = \mathbb{R} + i\mathbb{R}_+$. If $f$ belongs to the Hardy space $H^p(\mathbb{C}_+)$, that is, if $\sup_{y > 0} \|f_y\|_{L^p(\mathbb{R})}$ is finite, where $f_y: x \mapsto f(x + iy)$, then it is well known that the function $y \mapsto \|f_y\|_{L^p(\mathbb{R})}$ is decreasing on $\mathbb{R}_+$, for every $p \in [0, \infty]$. Nonetheless, if $f$ is simply holomorphic, then the lower semicontinuous function $y \mapsto \|f_y\|_{L^p(\mathbb{R})}$ need not be decreasing. Actually, the set where it is finite may be any interval in $\mathbb{R}_+$, or even a disconnected set.

Now, replace the upper half-plane $\mathbb{C}_+$ with a Siegel upper half-space $D := \{(\zeta, z) \in \mathbb{C}_n \times \mathbb{C}: \text{Im} z - |\zeta|^2 > 0\}$, and define

$$f_h: \mathbb{C}_n \times \mathbb{R} \ni (\zeta, x) \mapsto f(\zeta, x + i|\zeta|^2 + h)$$

for every $h > 0$ and for every function on $D$. This definition is motivated by the fact that

$$bD := \{(\zeta, x + i|\zeta|^2): (\zeta, x) \in \mathbb{C}_n \times \mathbb{R}\}$$

is the boundary of $D$, and the sets $bD + (0, ih)$, for $h > 0$, foliate $D$ as the sets $\mathbb{R} + iy$, for $y > 0$, foliate $\mathbb{C}_+$. If $f$ is holomorphic on $D$, then the mapping $h \mapsto \|f_h\|_{L^p(\mathbb{C}_n \times \mathbb{R})}$ is always decreasing (though not necessarily finite), in contrast to the preceding case (cf. Theorem 1). This fact is closely related with the fact that every holomorphic function defined in a neighbourhood of $bD$ automatically extends to $D$. More precisely, if one observes that $bD$ has the structure of a CR submanifold of $\mathbb{C}_n \times \mathbb{C}$, one may actually prove that every CR function (of class $C^1$) is the boundary values of a unique holomorphic function on $D$ (cf. [2, Theorem 1 of Section 15.3]).

In this note we show that an analogous property holds when $bD$ is replaced by a general quadratic, or quadric, CR submanifold of a complex space, and then discuss some examples of Šilov boundaries of (homogeneous) Siegel domains.

2. Preliminaries

We fix a complex hilbertian space $E$ of dimension $n$, a real hilbertian space $F$ of dimension $m$, and a hermitian map $\Phi: E \times E \to F_C$. Define

$$\mathcal{M} := \{ (\zeta, x + i\Phi(\zeta)): \zeta \in E, x \in F \} = \{ (\zeta, z) \in E \times F_C: \text{Im} z - \Phi(\zeta) = 0 \},$$

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where $E_C$ denotes the complexification of $E$, while $\Phi(\zeta) := \Phi(\zeta, \zeta)$ for every $\zeta \in E$. We define

$$\rho: E \times E_C \ni (\zeta, z) \mapsto \text{Im} z - \Phi(\zeta) \in F.$$ 

We endow $E \times E_C$ with the product

$$(\zeta, z)(\zeta', z') := (\zeta + \zeta', z + z' + 2i\Phi(\zeta', \zeta))$$

for every $(\zeta, z), (\zeta', z') \in E \times E_C$, so that $E \times E_C$ becomes a 2-step nilpotent Lie group, and $\mathcal{M}$ a closed subgroup of $E \times E_C$. In particular, the identity of $E \times E_C$ is $(0, 0)$ and $(\zeta, z)^{-1} = (-\zeta, -z + 2\Phi(\zeta))$ for every $(\zeta, z) \in E \times E_C$. It will be convenient to identify $\mathcal{M}$ with the 2-step nilpotent Lie group $\mathcal{N} := E \times F$, endowed with the product

$$(\zeta, x)(\zeta', x') := (\zeta + \zeta', x + x' + 2\text{Im} \Phi(\zeta', \zeta))$$

for every $(\zeta, x), (\zeta', x') \in \mathcal{N}$, by means of the isomorphism

$$\iota: \mathcal{N} \ni (\zeta, x) \mapsto (\zeta, x + i\Phi(\zeta)) \in E \times E_C.$$

In particular, the identity of $\mathcal{N}$ is $(0, 0)$ and $(\zeta, x)^{-1} = (-\zeta, -x)$ for every $(\zeta, x) \in \mathcal{N}$. Notice that, in this way, $\mathcal{N}$ acts holomorphically (on the left) on $E \times F_C$. Given a function $f$ on $E \times F_C$, we shall define

$$f_h: \mathcal{N} \ni (\zeta, x) \mapsto f(\zeta, x + i\Phi(\zeta) + ih) \in \mathbb{C}$$

for every $h \in F$.

Observe that the preceding groups structures show that, if we define the complex tangent space of $\mathcal{M}$ at $(\zeta, z)$ as

$$H_{(\zeta,z)}\mathcal{M} := T_{(\zeta,z)}\mathcal{M} \cap (iT_{(\zeta,z)}\mathcal{M})$$

for every $(\zeta, z) \in \mathcal{M}$, where $T_{(\zeta,z)}\mathcal{M}$ denotes the real tangent space to $\mathcal{M}$ at $(\zeta, z)$, identified with a subspace of $E \times E_C$, then

$$H_{(\zeta,z)}\mathcal{M} = dL_{(\zeta,z)}H_{(0,0)}\mathcal{M},$$

where $L_{(\zeta,z)}$ denotes the left translation by $(\zeta, z)$ (in $E \times E_C$), and $dL_{(\zeta,z)}$ its differential at $(0, 0)$. Therefore, $\dim_R H_{(\zeta,z)} = n$ for every $(\zeta, z) \in \mathcal{M}$, so that $\mathcal{M}$ is a CR submanifold of $E \times F_C$ (cf. [2] Chapter 7)], called a quadratic or quadric CR manifold (cf. [2], Section 7.3 and [10] [11]).

We observe explicitly that $\mathcal{M}$ is generic (that is, $\dim_R \mathcal{M} = \dim_R H_{(0,0)}\mathcal{M} = \dim_R E \times F_C - \dim_R M$, cf. [2], Definition 5 and Lemma 4 of Section 7.1) and that its Levi form may be canonically identified with $\Phi$ (cf. [2] Chapter 10 and [11]).

3. A Property of Hardy Spaces

We denote by $C$ the convex envelope of $\Phi(E)$.

**Theorem 1.** Let $\Omega$ be an open subset of $F$ such that $\Omega = \Omega + \overline{C}$, and set $D := \rho^{-1}(\Omega)$. Then, for every $f \in \text{Hol}(D)$, for every $p \in [0, \infty]$, for every $h \in \Omega$ and for every $h' \in \overline{C}$,

$$\|f_{h+h'}\|_{L^p(\Omega)} \leq \|f_{h}\|_{L^p(\overline{\Omega})}.$$ 

The proof is based on the 'analytic disc technique' presented in [2], Section 15.3.

Observe that the assumption that $\Omega = \Omega + \overline{C}$ is not restrictive. Indeed, if $\Omega$ is connected and $C$ has a non-empty interior $\text{Int} C$, then every function which is holomorphic on $\rho^{-1}(\Omega)$ extends (uniquely) to a holomorphic function on $\rho^{-1}(\Omega + (\text{Int} C \cup \{ 0 \}))$ by [2] Theorem 1 of Section 15.3, and $\Omega + (\text{Int} C \cup \{ 0 \}) = \Omega + \overline{C}$ since $\Omega$ is open and $\overline{C} = \text{Int} C$ by convexity. The case in which $\text{Int} C = 0$ may be treated directly using similar techniques.

We also mention that, if $p < \infty$ and either $\Phi$ is degenerate or the polar of $\Phi(E)$ has an empty interior (that is, the closed convex envelope of $\Phi(E)$ contains a non-trivial vector subspace), then either $f_h = 0$ or $f_h \not\in L^p(\mathcal{N})$ (at least for $p \geq 1$ when $\Phi$ is non-degenerate). Cf. [6] for more details in a similar case.
Proof. For every $v = (v_j) \in \mathbb{C}^m$, consider
\[ A_v : \mathbb{C} \ni w \mapsto \left( \sum_{j=1}^m v_j w^j, i \sum_{j=1}^m \Phi(v_j) + 2i \sum_{k < j} \Phi(v_j, v_k) w^{j-k} \right) \in \mathbb{E} \times \mathcal{F}_C, \]
and observe that the following hold:

- $A_v(0) = (0, i \Psi(v))$;
- $\Psi(E^n)$ is the convex envelope of $\Phi(E)$, thanks to [12, Corollary 17.1.2];
- $\rho(A_v(w)) = 0$ for every $w \in \mathbb{T}$;
- the mapping $A : E^n \ni v \mapsto A_v \in \text{Hol}(\mathbb{C})$ is continuous (actually, polynomial).

Now, take $h \in \Omega$. By continuity, there is $\varepsilon > 0$ such that $A_v(U) + ih \subseteq D$ for every $v \in B_{E^n}(0, \varepsilon)$, where $U$ denotes the unit disc in $\mathbb{C}$, and $\overline{U}$ its closure. Then, $A_v(U) + ih' \subseteq D$ for every $v \in B_{E^n}(0, \varepsilon)$ and for every $h' \in h + \overline{D}$. For every $h'' \in \Psi(B_{E^n}(0, \varepsilon))$, denote by $\nu_{h''}$ the image of the normalized Haar measure on $\mathbb{T}$ under the mapping $\pi \circ A_v$, for some $v \in B_{E^n}(0, \varepsilon) \cap \Psi^{-1}(h')$, where $\pi : \mathbb{E} \times \mathcal{F}_C \ni (\zeta, z) \mapsto (\zeta, x) \in \mathcal{N}$. Observe that, for every $(\zeta, x) \in \mathcal{N}$ and for every $h'' \in h + \overline{D}$, the mapping
\[ \overline{U} \ni w \mapsto f((\zeta, x + i\Phi(\zeta)) \cdot [A_v(w) + (0, ih'')]) \in \mathbb{C} \]
is continuous and holomorphic on $U$, so that, by subharmonicity (cf., e.g., [13, Theorem 15.19]),
\[
|f((\zeta, x + i\Phi(\zeta)) + (h' + h''))|^{\min(1,p)} \leq \int_{\mathbb{T}} |f((\zeta, x + i\Phi(\zeta)) \cdot [A_v(w) + (0, ih'')])|^{\min(1,p)} \, dw
= \int_{\mathcal{N}} |f_{h''}(\zeta', x')(\zeta', x')|^{\min(1,p)} \, d\nu_{h''}(\zeta', x')
= |f_{h''}|^{\min(1,p)} \circ \nu_{h''},
\]
where $\nu_{h''}$ denotes the reflection of $\nu_{h''}$, while $v$ is a suitable element of $B_{E^n}(0, \varepsilon) \cap \Psi^{-1}(h')$. Since $\nu_{h''}$ is a probability measure, by Young’s inequality (cf., e.g., [4, Chapter III, § 4, No. 4]) we then infer that
\[
\|f_{h''} + h''\|_{L^p(\mathcal{N})} = \|f_{h''} + h''\|_{L^{\max(1,p)}(\mathcal{N})}^{\min(1,p)} \leq \|f_{h''}\|_{L^{\max(1,p)}(\mathcal{N})}^{\min(1,p)} = \|f_{h''}\|_{L^p(\mathcal{N})}
\]
for every $h' \in \Psi(B_{E^n}(0, \varepsilon))$ and for every $h'' \in h + \overline{D}$. Since every element of $\mathcal{C}$ may we written as a finite sum of elements of $\Psi(B_{E^n}(0, \varepsilon))$, the arbitrariness of $h''$ shows that
\[
\|f_{h''}\|_{L^p(\mathcal{N})} \leq \|f_h\|_{L^p(\mathcal{N})}
\]
for every $h' \in \mathcal{C}$, hence for every $h' \in \overline{D}$ by lower semi-continuity. The proof is complete. \qed

**Corollary 2.** Assume that $C$ has a non-empty interior $\Omega$, and set $D := \rho^{-1}(|\Omega|)$. Then, for every $p \in [0, \infty]$ and $f \in \text{Hol}(D)$,
\[
\sup_{h \in \Omega} \|f_h\|_{L^p(\mathcal{N})} = \lim_{h \to 0, h \in \Omega} \inf \|f_h\|_{L^p(\mathcal{N})}.
\]

In particular, if we define the Hardy space $H^p(D)$ as the set of $f \in \text{Hol}(D)$ such that $\sup_{h \in \Omega} \|f_h\|_{L^p(\mathcal{N})}$ is finite, the preceding result states that $H^p(D)$ may be equivalently defined as the set of $f \in \text{Hol}(D)$ such that $\liminf_{h \to 0, h \in \Omega} \|f_h\|_{L^p(\mathcal{N})}$ is finite. This result should be compared with [3], where the boundary values of the elements of $H^p(D)$ are characterized as the CR elements of $L^p(\mathcal{N})$, for $p \in [1, \infty]$. In particular, Corollary 2 could be deduced from the results of [3], when $p \in [1, \infty]$, though at the expense of some further technicalities.

This result extends [7, Corollary 1.43].
4. Examples

We shall now present some examples of homogeneous Siegel domains $D = \rho^{-1}(\Omega)$ for which $\overline{\Omega}$ is the closed convex envelope of $\Phi(E)$, so that Corollary 2 applies.

We recall that $D$ is said to be a Siegel domain if $\Omega$ is an open convex cone not containing affine lines, $\Phi$ is non-degenerate, and $\Phi(E) \subseteq \overline{\Omega}$. In addition, $D$ is said to be homogeneous if the group of its biholomorphisms acts transitively on $D$. It is known (cf., e.g., [3] Proposition 1) that $D$ is homogeneous if and only if there is a triangular Lie subgroup $T_+$ of $G\ell(F)$ which acts simply transitively on $\Omega$, and for every $t \in T_+$ there is $g \in G\ell(E)$ such that $t\Phi = \Phi(g \times g)$.

If $T_+$ is another Lie subgroup of $G\ell(F)$ with the same properties as $T_+$, then $T_+$ and $T_+'$ are conjugated by an automorphism of $F$ preserving $\Omega$. Thanks to this fact, we may use the results of [7] even if a different $T_+$ is chosen. In particular, there is a surjective (open and) continuous homomorphism of Lie groups

$$\Delta: T_+ \to (\mathbb{R}^*_+)^r$$

for some $r \in \mathbb{N}$, called the rank of $\Omega$, so that

$$\Delta^s = \Delta^s_{11} \cdots \Delta^s_{rr},$$

$s \in \mathbb{C}^r$, are the characters of $T_+$. Once a base point $e_D \in \Omega$ has been fixed, $\Delta^s$ induces a function $\Delta^s_{t}$ on $\Omega$, setting $\Delta^s_{t}(t(e_D)) = \Delta^s(t)$ for every $t \in T_+$.

Up to modify $\Delta$, we may then assume that the functions $\Delta^s_{t}$ are bounded on the bounded subsets of $\Omega$ if and only if $\text{Res} \in \mathbb{R}^*_+$ (cf. [7] Lemma 2.34]). In particular, there is $b \in \mathbb{R}^*_+$ such that $\Delta^{-b}(t) = |\det g|^2$ for every $t \in T_+$ and for every $g \in G\ell(E)$ such that $t\Phi = \Phi(g \times g)$ (cf. [7] Lemma 2.9]), and one may prove that $b \in (\mathbb{R}^*_+)^r$ if and only if $\Phi(E)$ generates $F$ as a vector space, in which case $\Omega$ is the interior of the convex envelope of $\Phi(E)$ (cf. [7] Proposition 2.57 and its proof, and Corollary 2.58]). Therefore, we are interested in finding examples of homogeneous Siegel domains for which $b \in (\mathbb{R}^*_+)^r$.

Notice, in addition, that if $b \not\in (\mathbb{R}^*_+)^r$, then $\Phi(E)$ is contained in a hyperplane, so that the interior of its convex envelope is empty.

The Siegel domain $D$ is said to be symmetric if it is homogeneous and admits an involutive biholomorphism with a unique fixed point (equivalently, if for every $(z, \bar{z}) \in D$ there is an involutive biholomorphism of $D$ for which $(z, \bar{z})$ is an isolated (or the unique) fixed point). The domain $D$ is said to be irreducible if it is not biholomorphic to the product of two non-trivial Siegel domains.

It is well known that every symmetric Siegel domain is biholomorphic to a product of irreducible ones, and that the irreducible symmetric Siegel domains can be classified in four infinite families plus two exceptional domains (cf., e.g., [11 §§ 1, 2]). In particular, for an irreducible symmetric Siegel domain, either $b = 0$ (that is, $E = \{ 0 \}$, in which case $D$ is 'of tube type'), or $b \in (\mathbb{R}^*_+)^r$ (cf., e.g., [11 Example 2.11]). Hence, when $D$ is a symmetric Siegel domain, $\overline{\Omega}$ is the closed convex envelope of $\Phi(E)$ if and only if none of the irreducible components of $D$ is of tube type. Note that these domains can be also characterized as those which do not admit any non-constant rational inner functions, thanks to [8].

We now present some examples of (homogeneous) Siegel domains.

Example 3. Let $K$ be either $\mathbb{C}$ or the division ring of the quaternions. In addition, fix $r, k, p \in \mathbb{N}$ with $p \leq r$, and define

- $E$ as the space of $k \times r$ matrices over $K$ whose $j$-th columns have zero entries for $j = p + 1, \ldots, r$;
- $F$ as the space of self-adjoint $r \times r$ matrices over $K$;
- $\Omega$ as the cone of non-degenerate positive self-adjoint $r \times r$ matrices over $K$;
- $\Phi: E \times E \ni (\zeta, \zeta') \mapsto \frac{1}{2}(\zeta^* \zeta + \zeta' \bar{\zeta'}) + i(\zeta^* i\zeta' - \zeta'^* i\zeta) \in F$;
- $T_+$ as the group of upper triangular $r \times r$-matrices over $K$ with strictly positive diagonal entries, acting on $\Omega$ (and $F$) by the formula $t \cdot h := th t^*$;
- $\Delta: T_+ \ni t \mapsto (t_{1,1}, \ldots, t_{r,r}) \in (\mathbb{R}^*_+)^r$. 

\[ \to \]
Then, $\Omega$ is an irreducible symmetric cone of rank $r$ on which $T_+$ acts simply transitively by $[7]$ Example 2.6. In addition, $\Phi$ is well defined, since $\zeta^*\zeta + \zeta'^*\zeta' - \zeta^*\zeta' \in F$ for every $\zeta, \zeta' \in E$, and clearly $\Phi(\zeta) \in \mathcal{F}$ and
\[ t \cdot \Phi(\zeta) = t \cdot (\zeta^* \zeta) = (\zeta'^* \zeta') = \Phi(\zeta'^* \zeta') \]
for every $t \in T_+$ and for every $\zeta \in E$ (with $\zeta'^* \zeta \in E$), so that $D$ is homogeneous. Then, $b = (b_j)$, with $b_j = -k \text{dim}_c K$ for $j = 1, \ldots, p$ and $b_j = 0$ for $j = p + 1, \ldots, r$. Consequently, $\mathcal{F}$ is the closed convex envelope of $\Phi(E)$ if and only if $p = r$ and $k > 0$.

Notice that $D$ is irreducible since $\Omega$ is irreducible (cf. [8] Corollary 4.8), and that $D$ is symmetric if $kp = 0$ or if $p = r$ and $K = C$ (cf. [7] Examples 2.14 and 2.15). If $kp(r - p) > 0$, or if $K \neq C, r \geq 3$, and $k \geq 2$, then $D$ cannot be symmetric.

**Example 4.** Take $k, p, q \in \mathbb{N}, p \leq 2$. Define:

- $E$ as the space of formal $k \times 2$ matrices whose entries of the first column belong to $\mathbb{C}$ (and are 0 if $p = 0$), and whose entries of the second column belong to $\mathbb{C}^q$ (and are 0 if $p \leq 1$);
- $F$ as the space of formally self-adjoint $2 \times 2$ matrices whose diagonal entries belong to $\mathbb{R}$, and whose non-diagonal entries belong to $\mathbb{C}^q$;
- $\Omega$ as the cone of $(\frac{a}{b} c) \in F$ with $a, c > 0, b \in \mathbb{C}^q$, and $ac - |b|^2 > 0$;
- $\Phi$ so that
\[ \phi \left( \begin{array}{cc} a_1 & b_1 \\ \vdots & \vdots \\ a_k & b_k \end{array} \right) = \left( \sum_j |a_j|^2 \sum_j \overline{a_j}b_j \right) \left( \sum_j |b_j|^2 \right) \]
for every $\begin{pmatrix} a_1 & b_1 \\ \vdots & \vdots \\ a_k & b_k \end{pmatrix} \in E$;
- $T_+$ as the group of formal $2 \times 2$ upper triangular matrices with diagonal entries in $\mathbb{R}_+^q$ and non-diagonal entries in $\mathbb{C}^q$, with the action
\[ t \cdot \left( \begin{array}{cc} a & b \\ 0 & c \end{array} \right) = \left( \begin{array}{cc} a' & b' \\ c' & c' \end{array} \right) := \left( \begin{array}{cc} a' a^2 + c'|b|^2 + 2a \text{Re}(b, b') & ab^* + cc' \bar{b} \\ ac\bar{b}' + cc'^* \bar{b} & c'^2 \bar{b}' \end{array} \right); \]
- $\Delta$: $T_+ \ni t \mapsto (t_{1,1}, t_{2,2})$.

Then, $\Omega$ is an irreducible symmetric cone of rank 2 on which $T_+$ acts simply transitively (cf. [7] Example 2.7). In addition, $\Phi(\zeta) \in \mathcal{F}$ for every $\zeta \in E$, and
\[ t \cdot \Phi(\zeta) = \Phi(\zeta'^* \zeta') \]
for every $t \in T_+$ and $\zeta \in E$ (with $\zeta'^* \zeta \in E$), provided that $p \leq 1$. Then, $D$ is an irreducible Siegel domain, and it is homogeneous if $p \leq 1$ (it is symmetric if $p = 0$). In addition, $b = \mathbf{0}$ if $p = 0$, while $b = (k, 0)$ if $p = 1$. Further, if $p = 2$, then $\Phi(E)$ contains the boundary of $\Omega$, since $\left( \frac{a}{b} c \right) = \Phi \left( \begin{array}{ccc} a^{1/2} & a^{-1/2}b & 0 \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \end{array} \right)$, for every $a > 0$, for every $c > 0$ and for every $b \in \mathbb{C}^q$ such that $|b|^2 = ac$ (the case $a = 0, b = 0, c > 0$ is treated similarly). Then, $\mathcal{F}$ is the closed convex envelope of $\Phi(E)$ if and only if $p = 2$.

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1A cone is said to be homogeneous if the group of its linear automorphisms acts transitively on it. It is said to be symmetric if, in addition, it is self-dual for some scalar product. A convex cone is said to be irreducible if it is not isomorphic to a product of non-trivial convex cones.

2Formally, $\left( \frac{a}{b} c \right) \cdot \left( \frac{a'}{b'} c' \right) = \left( \frac{a}{b} c \right) \left( \frac{a'}{b'} c' \right) \left( \frac{a}{b} c \right)^*$. 
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