Recovering galaxy star formation and metallicity histories from spectra using VESPA

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ABSTRACT

We introduce versatile spectral analysis (VESPA): a new method which aims to recover robust star formation and metallicity histories from galactic spectra. VESPA uses the full spectral range to construct a galaxy history from synthetic models. We investigate the use of an adaptive parametrization grid to recover reliable star formation histories on a galaxy-by-galaxy basis. Our goal is robustness as opposed to high-resolution histories, and the method is designed to return high time resolution only where the data demand it. In this paper we detail the method and we present our findings when we apply VESPA to synthetic and real Sloan Digital Sky Survey (SDSS) spectroscopic data. We show that the number of parameters that can be recovered from a spectrum depends strongly on the signal-to-noise ratio, wavelength coverage and presence or absence of a young population. For a typical SDSS sample of galaxies, we can normally recover between two and five stellar populations. We find very good agreement between VESPA and our previous analysis of the SDSS sample with MOPED.

Key words: methods: data analysis – methods: statistical – galaxies: evolution – galaxies: formation – galaxies: stellar content.

1 INTRODUCTION

The spectrum of a galaxy holds vast amount of information about that galaxy’s history and evolution. Finding a way to tap directly into this source of knowledge would not only provide us with crucial information about that galaxy’s evolutionary path, but would also allow us to integrate this knowledge over a large number of galaxies and therefore derive cosmological information.

Galaxy formation and evolution are still far from being well understood. Galaxies are extremely complex objects, formed via complicated non-linear processes, and any approach (be it observational, semi-analytical or computational) inevitably relies on simplifications. If we try to analyse a galaxy’s luminous output in terms of a history parametrized by some chosen physical quantities, such a simplification is also in order. The reason is twofold: first, we are limited by our knowledge and ability to model all the physical processes which happen in a galaxy and produce the observed spectrum we are analysing; secondly, the observed spectrum is inevitably perturbed by noise, which intrinsically limits the amount of information we can recover.

Measuring and understanding the star formation history (SFH) of the Universe is therefore essential to our understanding of galaxy evolution – when, where and in what conditions did stars form throughout cosmic history? The traditional and simplest way to probe this is to measure the observed instantaneous star formation rate (SFR) in galaxies at different redshifts. This can be achieved by looking at light emitted by young stars in the ultraviolet band or its secondary effects. (e.g. Madau et al. 1996; Kennicutt 1998; Hopkins, Connolly & Szalay 2000; Bundy et al. 2006; Erb et al. 2006; Abrahams et al. 2007; Noeske et al. 2007; Verma et al. 2007).

A complementary method is to look at present-day galaxies and extract their SFH, which spans the lifetime of the galaxy. Different teams have analysed a large number of galaxies in this way, whether by using the full available spectrum (Glazebrook et al. 2003; Panter, Heavens & Jimenez 2003; Cid Fernandes et al. 2004; Heavens et al. 2004; Mathis, Charlot & Brinchmann 2006; Ocvirk et al. 2006; Cid Fernandes et al. 2007; Panter et al. 2007), or by concentrating on particular spectral features or indices (e.g. Kauffmann et al. 2003; Tremonti et al. 2004; Gallazzi et al. 2005; Barber, Meiksin & Murphy 2006), which are known to be correlated with age or metallicities (e.g. Worthey 1994; Thomas, Maraston & Bender 2003).

To do this, we rely on synthetic stellar population models to describe a galaxy in terms of its stellar components, but by modelling a galaxy in this way we are intrinsically limited by the quality of the models. There are also potential concerns with flux-calibration errors. However, using the full spectrum to recover the fossil record of a galaxy – or of an ensemble of galaxies – is an extremely powerful method, as the quality and amount of data relating to local galaxies vastly outshines that which concerns high-redshift galaxies. Splitting a galaxy into simple stellar populations (SSPs) of different ages and metallicities is a natural way of parametrizing a galaxy,

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and it allows realistic fits to real galaxies (e.g. Bruzual & Charlot 2003). Galactic archeology has become increasingly popular in the literature recently, largely due to the increase in sophistication of stellar population synthesis codes and the improvement of the stellar spectrum libraries upon which they are based, and also due to the availability of large well-calibrated spectroscopic data bases, such as the Sloan Digital Sky Survey (SDSS) (York et al. 2000; Strauss et al. 2002).

In any case, without imposing any constraints on the allowed form of the SFH, or perhaps an age–metallicity relation, the parameter space can become unsustainably large for a traditional approach. Ideally, one would like to do without such pre-constraints. Recently, different research teams have come up with widely different solutions for this problem. MOPED (Heavens, Jimenez & Lahav 2000) and STARLIGHT (Cid Fernandes et al. 2004) explore a well-chosen parameter space in order to find the best possible fit to the data. In the case of MOPED, this relies on compression of the full spectrum to a much smaller set of numbers which retains all the information about the parameters it tries to recover; STARLIGHT, on the other hand, searches for its best fit using the full spectrum with a Metropolis algorithm. STECMAP (Ocvirk et al. 2006) solves the problem using an algebraic least-squares solution and a well-chosen regularization to keep the inversions stable. All of these and other methods acknowledge the same limitation – noise in the data and in the models introduces degeneracies into the problem which can lead to unphysical results. MOPED, e.g. has produced some remarkable results concerning the average SFH of the Universe by analysing a large sample of galaxies. However, MOPED’s authors have cautioned against overinterpreting the results on a galaxy-by-galaxy basis, due to the problem mentioned above. This is directly related to the question of how finely one should parametrize a galaxy, and what the consequences of this might be.

Much of the motivation for versatile spectral analysis (VESPA) came from the realization that this problem will vary from galaxy to galaxy, and that the method of choosing a single parametrization to analyse a large number of galaxies can be improved on. VESPA is based on three main ideas, which we present here and develop further in the main text.

(i) There is only so much information one can safely recover from any given set of data, and the amount of information which can be recovered from an individual galaxy varies.

(ii) The recovered star formation fractions should be positive.

(iii) Even though the full unconstrained problem is non-linear, it is piecewise linear in well-chosen regions of parameter space.

VESPA’s ultimate goal is to derive robust information for each galaxy individually, by adapting the number of parameters it recovers on a galaxy-by-galaxy basis and increasing the resolution in parameter space only where the data warrant it. In a nutshell, this is how VESPA works: we estimate how many parameters we can recover from a given spectrum, given its noise, shape, spectral resolution and wavelength range using an analysis given by Ocvirk et al. (2006). In that paper, singular value decomposition (SVD) is used to find a least-squares solution, and this solution is analysed in terms of its singular vectors. VESPA uses this method only as an analysis of the solution, and uses bounded-variable least squares (BVLS) (Stark & Parker 1995) to reach a non-negative solution in several regimes where linearity applies.

This paper is organized as follows. In Section 2 we present the method, in Section 3 we apply VESPA to a variety of synthetic spectra, in Section 4 we apply VESPA to a sample of galaxies from the SDSS spectroscopic data base and we compare our results to those obtained with MOPED, and finally in Section 5 we present our conclusions.

2 Method

In this section we lay down the problem to solve in detail, and explain the different steps VESPA uses to find a solution for each galaxy.

2.1 The problem

We assume a galaxy is composed of a series of SSPs of varying ages and metallicities. The unobscured rest-frame luminosity per unit wavelength of a galaxy can then be written as

\[ F_\lambda = \int_0^t \psi(t) S_\lambda(t, Z) \, dt, \]

(1)

where \( \psi(t) \) is the SFR (solar masses formed per unit of time) and \( S_\lambda(t, Z) \) is the luminosity per unit wavelength of a single stellar population of age \( t \) and metallicity \( Z \), per unit mass. The dependency of the metallicity on age is unconstrained, turning this into a non-linear problem.

In order to solve this problem, we start by discretizing in wavelength and time, by averaging these two quantities into well-chosen bins. For now we present the problem with a generalized parametrization, and discuss our choice in Section 2.3. We will use Greek indices to indicate time-bins, and roman indices to indicate wavelength bins.

The problem becomes

\[ F_j = \sum_\alpha \sum_\mu z_j \mu G(z_j \mu), \]

(2)

where \( F_j = (F_1, \ldots, F_D) \) is the luminosity of the \( j \)th wavelength bin of width \( \Delta_\lambda \), \( G(z_j \mu) \) is the \( j \)th luminosity point of a stellar population of age \( t_\mu = (t_1, \ldots, t_D) \) (spanning an age range of \( \Delta t \)) and metallicity \( Z_\mu \), and \( x_\mu = (x_1, \ldots, x_D) \) is the total mass of population \( G(Z_\mu) \) in the time-bin \( \Delta t \).

Although the full metallicity problem is non-linear, interpolating between tabulated values of \( Z \) gives a piecewise linear behaviour:

\[ G(z_j \mu) = g_\mu G(z_\mu \alpha) + (1 - g_\mu) G(Z_b \alpha), \]

(3)

and the problem then becomes

\[ F_j = \sum_\mu [g_\mu G(z_\mu \alpha) + (1 - g_\mu) G(Z_b \alpha)], \]

(4)

where \( G(z_\mu \alpha) \) and \( G(Z_b \alpha) \) are equivalent to \( G(Z) \) as above, but at fixed metallicities \( Z_\mu \) and \( Z_b \), which bind the true \( Z \). If this interpolation matches the models’ resolution in \( Z \), then we are not degrading the models in any way.

Solving the problem then requires finding the correct metallicity range. One should not underestimate the complexity this implies – trying all possible combination of consecutive values of \( Z_\mu \) and \( Z_b \) in a grid of 16 age bins would lead to a total number of calculations of the order of \( 10^9 \), which is unfeasible even in today’s fast personal workstations. We work around this problem using an iterative approach, which we describe in Section 2.3.2.

2.1.1 Dust extinction

An important component when describing the luminous output of a galaxy is dust, as different wavelengths are affected in different ways. The simplest possible approach is to use one-parameter dust model, according to which we apply a single dust screen to the
combined luminosity of all the galactic components. Equation (1) becomes
\[ F_s = f_{\text{dust}}(\tau_s) \int_0^{\tau_s} \psi(t) S_s(t, Z) \, dt, \] (5)
where we are assuming the dust extinction is the same for all stars, and characterized by the optical depth, \( \tau_s \).

However, it is also well known that very young stars are likely to be more affected by dust. In an attempt to include this in our modelling, we follow the two-parameter dust model of Charlot & Fall (2000) in which young stars are embedded in their birth cloud up to a time \( t_{\text{BC}} \), when they break free into the interstellar medium (ISM):
\[ F_s = \int_0^{t_{\text{BC}}} f_{\text{dust}}(\tau_s, t) \psi(t) S_s(t, Z) \, dt \] (6)
and
\[ f_{\text{dust}}(\tau_s, t) = \begin{cases} f_{\text{dust}}(\tau_s^{\text{ISM}}), & t \leq t_{\text{BC}} \\ f_{\text{dust}}(\tau_s^{\text{BC}}), & t > t_{\text{BC}}. \end{cases} \] (7)

where \( \tau_s^{\text{ISM}} \) is the optical depth of the ISM and \( \tau_s^{\text{BC}} \) is the optical depth of the birth cloud. Following Charlot & Fall (2000), we take \( t_{\text{BC}} = 0.03 \) Gyr.

There is a variety of choices for the form of \( f_{\text{dust}}(\tau_s) \). To model the dust in the ISM, we use the mixed slab model of Charlot & Fall (2000) for low optical depths (\( \tau_s \leq 1 \)), for which
\[ f_{\text{dust}}(\tau_s) = \frac{1}{2 \tau_s} \left[ 1 + (\tau_s - 1) \exp(-\tau_s) - \tau_s E_1(\tau_s) \right], \] (8)
where \( E_1 \) is the exponential integral and \( \tau_s \) is the optical depth of the slab. This model is known to be less accurate for high dust values, and for optical depths greater than 1 we take a uniform screening model with
\[ f_{\text{dust}}(\tau_s) = \exp(-\tau_s). \] (9)

We only use the uniform screening model to model the dust in the birth cloud and we use \( \tau_s = \tau_s(\lambda/5500 \, \text{Å})^{-0.7} \) as our extinction curve for both environments.

As described, dust is a non-linear problem. In practice, we solve the linear problem described by equation (4) with a number of dust extinctions applied to the matrices \( G(Z) \) and choose the values of \( \tau_s^{\text{ISM}} \) and \( \tau_s^{\text{BC}} \) which result in the best fit to the data.

We initially use a binary chop search for \( \tau_s^{\text{ISM}} \in [0, 4] \) and keep \( \tau_s^{\text{BC}} \) fixed and equal to zero, which results in trying out typically around nine values of \( \tau_s^{\text{ISM}} \). If this initial solution reveals star formation at a time less than \( t_{\text{BC}} \) we repeat our search on a two-dimensional grid, and fit for \( \tau_s^{\text{ISM}} \) and \( \tau_s^{\text{BC}} \) simultaneously. There is no penalty except in CPU time to apply the two-parameter search, but we find that this procedure is robust (see Section 3.4).

2.2 The solution

In this section we describe the method used to reach a solution for a galaxy, given a set of models and a generalized parametrization. The construction of these models and choice of parameters is described in Sections 2.3 and 2.4.

We rewrite the problem described by equation (4) in a simpler way:
\[ F_j = \sum_{k=1}^{25} c_k A_k(Z_e), \] (10)
2.2.2 Noise

The inversion in equation (13) is often highly sensitive to noise, and care is needed when recovering solutions with matrix inversion methods. The fit in data space will always improve as we increase the number of parameters, but these might not all provide meaningful information. We follow an analysis given in Ocvirk et al. (2006) in order to understand how much this affects our results, and to choose a suitable age parametrization for each galaxy. This is not an exact method, and it does not guarantee that the solutions we recover have no contribution from noise. However, we found that in most cases it provides a very useful guideline (see Section 3.3, in particular Fig. 11).

We refer the reader to the above paper for a full discussion, and we reproduce here the steps used in our analysis.

We use SVD to decompose the model matrix $E$ as

$$E = U W V^T,$$

where $U$ is a $D \times 2S$ orthonormal matrix with singular data vectors $u_\kappa$ as columns, $V$ is a $2S \times 2S$ orthonormal matrix with the singular solution vectors $v_\kappa$ as columns, and $W$ is a $2S \times 2S$ diagonal matrix $W = \text{diag}(w_1, \ldots, w_{2S})$ where $w_\kappa$ are the matrix singular values in decreasing order. Replacing $E$ by this decomposition in equation (13) gives

$$c_{\text{LS}} = V W^{-1} U^T F = \sum_{\kappa=1}^{2S} \frac{\kappa}{w_\kappa} u_\kappa F v_\kappa.$$  

(16)

The solution vector is a linear combination of the solution singular values, parametrized by the dot product between the data and the corresponding data singular vector, and divided by the $\kappa$th singular value. The data vector itself is a combination of the true underlying emitted flux and noise: $F = F_{\text{true}} + \epsilon$. Equation (16) becomes

$$c_{\text{LS}} = \sum_{\kappa=1}^{2S} \frac{u_\kappa F_{\text{true}}}{w_\kappa} v_\kappa + \sum_{\kappa=1}^{2S} \frac{u_\kappa \epsilon}{w_\kappa} v_\kappa = c_{\text{true}} + c_{\epsilon},$$  

(17)

where $c_{\text{true}}$ is the solution vector to the noiseless problem and $c_{\epsilon}$ is an unavoidable added term due to the presence of noise.

It is extremely informative to compare the amplitudes of the two terms in the sum (17), and to monitor their contributions to the solution vector with varying rank. In Fig. 1 we plot $|u_\kappa F|$ and $|u_\kappa \epsilon|$ as a function of rank $\kappa$, for a synthetic spectrum with a signal-to-noise ratio (S/N) per pixel of 50 (at a resolution of 3 Å) and an exponentially decaying SFH. We observe the behaviour described and discussed in Ocvirk et al. (2006). The combinations associated with the noise terms maintain a roughly constant power across all ranks, with an average value of $(F)/(S/N)$. The data terms, however, drop significantly with rank, and we can therefore identify two ranges: a noise-dominated $\kappa$ range, in which the noise contributions match or dominate the true data contributions, and a data-dominated range, where the contributions to the solution are largely data motivated. We call the transition rank $\kappa_{\text{crit}}$. Overall, high-$\kappa$ ranks tend to dominate the solution, since the singular values $w_\kappa$ decrease with $\kappa$. This only amplifies the problem by giving greater weight to noise-dominated terms in the sum (16). Fig. 2 shows the contribution coming from each rank $\kappa$ to the final solution – the coefficient $(u_\kappa F)/w_\kappa$. We see this weight increases with rank.

Whereas this analysis gives us great insight into the problem, we do not in fact use the sum (16) to obtain $c_{\text{LS}}$, for the reasons given in Section 2.2.1.

For real data we are only able to calculate $u_\kappa F$ and estimate the noise level at $(F)/(S/N)$ and we use this information to estimate the number of non-zero parameters to recover from the data. Our aim is to have a solution which is dominated by the signal, and not by the noise. We therefore want our number of non-zero recovered parameters to be less than or equal to $\kappa_{\text{crit}}$. Estimating where this transition happens is always a noisy process. In this paper we take the conservative approach of setting $\kappa_{\text{crit}}$ to be the rank at which the perturbed singular values first cross the $(F)/(S/N)$ barrier. In the case of Fig. 1 this happens at $\kappa_{\text{crit}} = 7$.

![Figure 1. The behaviour of the singular values with matrix rank $k$. The stars are $|u_\kappa F|$ and the squares are $|u_\kappa \epsilon|$. The line is $(F)/(S/N)$, which in this case has a value of approximately 0.06.](image)

![Figure 2. The coefficients in sum (16) as a function of rank $\kappa$. We see that the highest rank modes (corresponding to the smaller singular values) tend to contribute the most to the solution.]
can explore to find a solution. This section describes this grid and the criteria used to reach a final parametrisation.

2.3.1 The grid

We work on a grid with a maximum resolution of 16 age bins, logarithmically spaced in look-back time from 0.02 up to 14 Gyr. The grid has three further resolution levels, where we split the age of the Universe in eight, four and finally two age bins, also logarithmically spaced in the same range.

The idea behind the multiresolution grid is to start our search with a low number of parameters (in coarser resolution), so that the entirety of the age of Universe is covered, and then increase the resolution only where the data warrant it by splitting the bin with the highest flux contribution in two, and so on. In effect, we construct one such grid for each of the tabulated metallicities, $Z_a$ and $Z_b$. We work with five metallicity values, $Z = [0.0004, 0.004, 0.008, 0.02, 0.05]$ which correspond to the metallicity resolution of the models used, where $Z$ is the fraction of the mass of the star composed of metals ($Z_\odot = 0.02$). The construction of the models for each of the time-bins is discussed in Section 2.4.

To each of the grids we can apply dust extinction as explained in Section 2.1.1.

2.3.2 The search

We go through the following steps in order to reach a solution.

(i) We begin our search with three bins: two bins of width 4 and one bin of width 8 (oldest), where here we are measuring widths in units of high-resolution bins.

(ii) We calculate a solution using equation (10) for every possible combination of consecutive boundaries $Z_a$ and $Z_b$, and we choose the one which gives the best value of reduced $\chi^2$.

(iii) We calculate the number of perturbed singular values above the noise level, as described at the end of Section 2.2.2.

(iv) We find the bin which contributes the most to the total flux and we split it into two.

(v) We find a solution in the new parametrisation, this time by trying out all possible combinations of $Z_a$ and $Z_b$ for the newly split bins only, and fixing the metallicity boundaries of the remainder bins to the boundaries obtained in the previous solution. If a bin had no stars in the previous iteration, we set $Z_d = 0.0004$ and $Z_c = 0.05$.

(vi) We return to (iii) and we proceed until we have reached the maximum resolution in all populated bins.

(vii) We look backwards in our sequence of solutions for the last instance with a number of non-zero recovered parameters equal to or less than $k_{\text{crit}}$ as calculated in (iii) and take this as our best solution.

We illustrate this sequence in Fig. 3, where we show the evolution of the search in a synthetic galaxy composed of two stellar bursts of equal SFRs – one young and one old. VESPA first splits the components which contribute the most to the total flux. In this case this is the young burst which can be seen in the first bin. Even though VESPA always resolves bins with any mass to the possible highest resolution, it then searches for the latest solution which has passed the SVD criterion explained in Section 2.2.2. In this case, this corresponds to the fifth from the top solution. VESPA chooses this solution in favour of the following ones due to the number of perturbed singular values above the solid line (right-hand panels). In this case, the solution chosen by VESPA is a better fit in parameters space (note the logarithmic scale in the y-axis – the following solution put the vast majority of the mass in the wrong bin). We observed this type of improvement in the majority of all cases studied (see Fig. 11).

2.3.3 The final solution

Our final solution comes in a parametrisation such that the total number of non-zero recovered parameters is less than or equal to the number of perturbed singular values above the estimated noise level.

The above sequence is performed for each of several combinations of $\tau_V^{\text{BC}}$, $\tau_V^{\text{ISM}}$, and we choose the attenuation which provides the best fit.

For each galaxy we recover $N$ star formation masses, with an associated metallicity, where $N$ is the total number of bins, and a maximum of two dust parameters.

2.4 The models

The backbone to our grid of models is the BC03 set of synthetic SSP models (Bruzual & Charlot 2003), with a Chabrier initial mass function (Chabrier 2003) and Padova 1994 evolutionary tracks (Alongi et al. 1993; Bressan et al. 1993; Fagotto et al. 1994a,b; Girardi et al. 1996). Although any set of stellar population models can be used, these provide a detailed spectral evolution of stellar populations over a suitable range of wavelength, ages and metallicities: $S(\lambda, t, Z)$. The models have been normalized to one solar mass at the age $t = 0$.

2.4.1 High-resolution age bins

At our highest resolution we work with 16 age bins, equally spaced in a logarithmic time-scale between now and the age of the Universe. In each bin, we assume a constant SFR

$$f_u^{\text{HR}}(\lambda, Z) = \psi \int_{\Delta t_u} S(\lambda, t, Z) \, dt$$

with $\psi = 1/\Delta t_u$.

2.4.2 Low-resolution age bins

As described in Section 2.3.1, we work on a grid of different resolution time-bins and we construct the low-resolution bins using the high-resolution bins described in Section 2.4.1. We do not assume a constant SFR in this case, as in wider bins the light from the younger components would largely dominate over the contribution from the older ones. Instead, we use a decaying SFH, such that the light contributions from all the components are comparable. Recall equation (1)

$$f_u^{\text{LR}}(\lambda, Z) = \int_{\Delta t_u} \psi(t) S(\lambda, t, Z) \, dt,$$

which we approximate to

$$f_u^{\text{LR}}(\lambda, Z) = \sum_{\alpha \in \beta} f_u^{\text{HR}}(\lambda, Z) \frac{\psi_\alpha \Delta t_u}{\sum_{\beta \in \beta} \psi_\beta \Delta t_u},$$

where low-resolution bin $\beta$ incorporates the high-resolution bins $\alpha \in \beta$, and we set

$$\psi_\alpha \Delta t_u = \frac{1}{\int_{\lambda} f_u^{\text{HR}}(\lambda, Z) \, d\lambda}.$$
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Figure 3. The evolution of the fit, as VESPA searches for a solution. Sequence should be read from top to bottom. Each line shows a stage in the sequence: the left-hand panels show the input SFH in the dashed line (red on the online version), and the recovered mass fractions on the solid line (black on the online version) for a given parametrization; the middle panel shows the input metallicities in the dashed line (red on the online version), and the recovered metallicities on the solid line (black on the online version); the right-hand panels show the absolute value of the perturbed singular values $|u \kappa F|$ (stars and solid line) and the estimated noise level $\langle F \rangle / (S/N)$. In this panel we also show the value of $\kappa_{\text{crit}}$ and the number of non-zero elements of $c_{Ls}$ in each iteration. The chosen solution is the fifth from the top, and indicated accordingly. This galaxy consists of two burst events of equal SFR—a very young and an old burst. It was modelled with a resolution of 3 Å and an S/N per pixel of 50. We see the recovery is good but not perfect—there is a 1 per cent leakage from the older population—but better than the following solutions, where this bin is split. See text in Section 2.3.2 for more details.

Depending on the galaxy, the final solution obtained with the sequence detailed in Section 2.3.2 can be described in terms of low-resolution age bins. In this case we should interpret the recovered mass as the total mass formed during the period implied by the width of the bin, but we cannot make any conclusions as to when in the bin the mass was formed. Similarly, the recovered metallicity for the bin should be interpreted as a mass-weighted metallicity for the total mass formed in the bin.

2.5 Errors

The quality of our fits and of our solutions is affected by the noise in the data, the noise in the models, and the parametrization we choose (which does not reflect the complete physical scenario within a galaxy). We aim to apply VESPA first to SDSS galaxies, which typically have an $S/N \approx 20$ per resolution element of 3 Å, which puts us in a regime where the main limitations come from the noise in the data.

To estimate how much noise affects our recovered solutions we take a rather empirical approach. For each recovered solution we create $n_{\text{err}}$ random noisy realizations and we apply VESPA to each of these spectra. We rebin each recovered solution in the parametrization of the solution we want to analyse and estimate the covariance matrices

$$C(x_{\lambda\beta} = \langle (x_\alpha - \bar{x}_\alpha)(x_\beta - \bar{x}_\beta) \rangle, \quad (22)$$
all plots in Sections 3 and 4 show error bars derived from $C_{\text{cov}}^{1/2}$, although it is worth keeping in mind that these are typically highly correlated.

### 2.6 Timings

A basic run of VESPA (which consists of roughly five runs down the sequence detailed in Section 2.3.2, one for each value of dust extinction) takes about 5 s. If accurate error estimations are needed per galaxy, this will add another one or two minutes to the timing, depending on how accurately one would like to estimate the covariance matrices, and depending on the number of data points. With $n_{\text{error}} = 10$, a typical SDSS galaxy takes around one minute to analyse.

### 3 Tests on Simulated Data

We tested VESPA on a variety of synthetic spectra, in order to understand its capabilities and limitations. In particular, we tried to understand the effect of three factors in the quality of our solutions: the input SFH, the noise in the data, and the wavelength coverage of the spectrum. We have also looked at the effects of dust extinction. Throughout we have modelled our galaxies in a resolution of 3 Å.

Even though we are aware that showing individual examples of VESPA’s results from synthetic spectra can be extraordinarily unrepresentative, we feel obliged to show a few for illustration purposes. We will show a typical result for most of the cases we present, but we also define some measurements of success, so that the overall performance of VESPA can be tracked as we vary any factors. We define

$$G_x = \sum_{\alpha} \left| \frac{x_\alpha - Z_\alpha}{x_\alpha} \right| \omega_\alpha, \quad (24)$$

and

$$G_Z = \sum_{\alpha} \left| \frac{Z_\alpha - Z_\alpha'}{Z_\alpha} \right| \omega_\alpha, \quad (25)$$

where $x_\alpha$ and $Z_\alpha'$ are the total mass and correspondent metallicity in bin $\alpha$ (rebinned to match the solution’s parametrization if necessary), and $\omega_\alpha$ is the flux contribution of population of age $t_\alpha$. $G_x$ and $G_Z$ are a flux-weighted average of the total absolute fractional errors in the solution, and give an indication of how well VESPA recovers the most significant parameters. A perfect solution gives $G_i = G_Z = 0$.

It is also worth noting that this statistic does not take into account the associated error with each recovered parameter – deviations from the true solution are usually expected given the estimated covariance matrices. We will also show how these factors affect the recovered total mass for a galaxy. In all cases we have renormalized the total masses such that total input mass for each galaxy is 1.

#### 3.1 Star formation histories

We present here some results for synthetic spectra with two different SFHs. All of the spectra in this section were synthesized with an $S/N$ per pixel of 50, and we initially fit the very wide wavelength range $\lambda \in [1000, 9500] \, \text{Å}$.

We choose two very different cases: first, an SFH of dual bursts, with a large random variety of burst age separations and metallicities (where we set the SFR to be 10 solar masses per Gyr in all bursts). Secondly, we chose an SFH with an exponentially decaying SFR: SFR $\propto \exp \left( -\frac{t}{\tau_w} \right)$, where $\tau_w$ is the age of the bin in look-back time in Gyr. Here we show results for $\gamma = 0.3 \, \text{Gyr}^{-1}$. Rather than being physically motivated, our choice of $\gamma$ reflects an SFH which is not too steep as to essentially mimic a single old burst, but which is also not completely dominated by recent star formation. In all cases the metallicity in each bin is randomly set. Fig. 4 shows a typical example from each type.

![Figure 4](https://example.com/figure4.png)

Figure 4. Two examples of VESPA’s analysis on synthetic galaxies. The top panels show the original spectrum in the dark line (black in the online version) and fitted spectrum in the lighter line (red in the online version). The middle panels show the input (dashed, red) and the recovered (solid, black) SFRs and the metallicity in each bin is randomly set. Note that even though many of the recovered metallicities are wrong, these tend to correspond to bins with very little star formation, and are therefore virtually unconstrained.

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Fig. 5 shows the distribution of $G_x$, $G_Z$, and total mass recovered for 50 galaxies with an S/N per pixel of 50. Solid lines correspond to dual burst and dashed lines to exponentially decaying ones. See text in Section 3.1 for details.

Fig. 6 shows the results for the same galaxies shown in Fig. 4, but results are obtained by using a smaller wavelength range. The goodness of fit in data space is still excellent, but it becomes more difficult to break certain degeneracies.

3.2 Wavelength range

Wavelength range is an important factor in this sort of analysis, as different parts of the spectrum will help to break different degeneracies. Since we are primarily interested in SDSS galaxies, we have studied how well VESPA does in the more realistic wavelength range of $\lambda \in [3200, 9500]$ Å.
expected. However, a more interesting question to ask is whether this decrease in the quality of the solutions would indeed be more pronounced without the SVD stopping criterion. Fig. 11 shows a comparison between $G_i$ obtained as we have described and obtained without any stopping mechanism (so letting our search go to the highest possible resolution and taking the final solution) for 50 galaxies with an exponentially decaying SFH and an S/N of 20. The results show clearly that there is a significant advantage in using the SVD stopping criterion. Naturally, the goodness of fit in data space is consistently better as we increase the number of parameters but this improvement is illusory – the parameter recovery is worse. This is exactly the expected behaviour – we choose to sacrifice resolution in parameter space in favour of a more robust solution – even though naively one could think a lower $\chi^2$ solution would indicate a better solution. The significance of this improvement changes with the amount of noise and wavelength range of the data (and to a lesser extent with type of SFH) but we observed an improvement in all cases we have studied.

As expected, further decreasing the S/N leads to a further degradation of the recovered solutions. This is accompanied by a suitable increase in the error bars and correlation matrices, but in cases of a S/N $\approx 10$ and less it becomes very difficult to recover any meaningful information from individual spectra.

### 3.4 Dust

In this section we use simulated galaxies to study the effect of dust in our solutions. As explained in Section 2.1.1, due to the non-linear nature of the problem, we cannot include dust as one of the free parameters analysed by SVD. Instead, we fit for a maximum of two dust parameters using a brute force approach which aims to minimize $\chi^2$ in data space by trying out a series of values for $\tau_{V, ISM}^{SM}$ and $\tau_{V, ISM}^{BC}$.

For each galaxy we assign random values of $\tau_{V, ISM}^{SM} \in [0, 2]$ and $\tau_{V, ISM}^{BC} \in [1, 2]$ and we are interested in how well we recover these parameters and any possible degeneracies.

Fig. 12 shows the input and recovered values for $\tau_{V, ISM}$ for galaxies with an S/N of 50, and which were analysed using the wavelength range $\lambda \in [3200, 9500]$ Å. We show results for two different cases of SFH: 50 galaxies with an exponentially decaying SFR and 50 galaxies formed by dual bursts. We observe a good recovery of $\tau_{V, ISM}^{SM}$ in both cases, especially at low optical depths.

However, we mostly observe a poor recovery of $\tau_{V, ISM}^{BC}$, especially at high optical depths. This is unsurprisingly flagging up a certain
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Figure 9. The distribution of $G_x$, $G_z$ and total mass recovered for 50 galaxies with an exponentially decaying SFH and different S/N values. Solid lines correspond to S/N = 50 and dashed lines to S/N = 20. See text in Section 3.3 for details.

Figure 10. The recovered number of non-zero parameters as we change the noise in the data from 50 (solid line) to 20 (dashed line), in a sample of galaxies with an exponentially decaying SFR. Please note that these correspond to the total number of non-zero components in the solution vector $c_\kappa$ and not to the number of recovered stellar populations.

level of degeneracy between mass and degree of extinction, which gets worse as the optical depth increases. Essentially, it becomes difficult to distinguish between a highly obscured massive population and a less massive population surrounded by less dust. It is worth keeping in mind that young populations are affected by both dust components simultaneously, and generally, even though the recovery of the second dust parameter may not be accurate, it allows for a better estimation of the dominant dust component.

This can be tested by simulating galaxies on a two-component dust model and by analysing them using both a single component model, and a two-component model. For example, when using the more sophisticated model, we noted that the mean error on $\tau^{ISM}$ on a subsample of dual-burst galaxies (synthesized as explained in Section 3.1, but chosen to have young star formation) was reduced from 35 to 28 per cent. This simple test also revealed that we are less likely to underestimate the mass of young populations by allowing an extra dust component, but that we are also introducing an extra degeneracy, especially so in the case of faint young populations.

However, we feel that the two-parameter dust model brings more advantages than disadvantages, with the caveat being that dusty young populations can be poorly constrained. In any case, we note that each galaxy is always analysed with a one-parameter model before being potentially analysed with a two-parameter model, and both solutions are kept and always available for analysis.

Finally, our test also partly justifies our choice to first run a single dust component model and only apply a two-component model if we detect stars in the first two bins – we find that although a one-component model might underestimate the amount of young stars, it does not fail to detect them. We repeated a similar test in real data, by analysing the same sample with one- and two-parameter dust models. We found similar results, with a one-parameter model failing to yield star formation in young bins only around 1 per cent of the time (compared to the two-parameter model), and only in cases where...
where the contribution of the light from the young populations was very small (of the order of 1–2 per cent).

4 RESULTS

In this section we present some results obtained by applying VESPA to galaxies in the SDSS. Our aim is to analyse these galaxies, and to produce and publish a catalogue of robust SFHs, from which a wealth of information can then be derived. We leave this for another publication, but we present here results from a subsample of galaxies, which we used to test VESPA in a variety of ways.

4.1 Handling SDSS data

Prior to any analysis, we processed the SDSS spectroscopic data, so as to accomplish the desired spectral resolution and mask out any wanted signal.

The SDSS data files supply a mask vector, which flags any potential problems with the measured signal on a pixel-by-pixel basis. We use this mask to remove any unwanted regions and emission lines. In practical terms, we ignore any pixel for which the provided mask value is not zero.

The BC03 synthetic models produce outputs at a resolution of 3 Å, which we convolve with a Gaussian velocity dispersion curve with a stellar velocity $\sigma_V = 170$ km s$^{-1}$, this being a typical value for SDSS galaxies. We take the models’ tabulated wavelength values as a fixed grid and rebin the SDSS data into this grid, using an inverse-variance weighted average. We compute the new error vector accordingly. Note that the number of SDSS data points averaged into any new bin is not constant, and that the rebinning process is done after we have masked out any unwanted pixels. Additionally to the lines yielded by the mask vector, we mask out the following emission line regions in every spectrum’s rest-frame wavelength range: [5885–5900, 6702–6732, 6716–6746, 6548–6578, 6535–6565, 6569–6599, 4944–4974, 4992–5022, 4846–4876, 4325–4355, 4087–4117, 3711–3741, 7800–11000] Å.

These rebinned data and noise vectors are essentially the ones we use in our analysis. However, since the linear algebra assumes white noise, we pre-whiten the data and construct a new flux vector $F_j' = F_j/\sigma_j$, which has unit variance, $\sigma_j' = 1$, $\forall j$, and a new model matrix $A_j' = A_j/\sigma_j$.

4.2 Duplicate galaxies

There are a number of galaxies in the SDSS data base which have been observed more than once, for a variety of reasons. This provides an opportunity to check how variations in observation-dependent corrections affect the results obtained by VESPA.

We have used a subset of the sample of duplicate objects in Brinchmann et al. (2004)$^1$ to create two sets of observations for 2000 galaxies, which we named list A and list B. We are interested in seeing how the errors we estimate for our results compare to errors introduced by intrinsic variations caused by changing the observation conditions (such as quality of the spectra, placement of the fibre, sky subtraction or spectrophotometric calibrations).

Fig. 13 shows the average star formation fraction as a function of look-back time for both sets of observation. The error bars showed are errors bars on the mean for each age bin. We show only the errors from list A to avoid cluttering – the errors from list B are of similar amplitude.

$^1$ Available at http://www.mpa-garching.mpg.de/SDSS/.
significantly differences in their continuum, but after further investigation it remains unclear what motivates such a difference. The simplest explanation is that the spectrophotometric calibration differs significantly between both observations, and that might have been the reason the plate or object was re-observed. Whatever the reason however, the clear conclusion is that stellar mass estimates are highly sensitive to changes in the spectrum continuum, and the errors we estimate from the covariance matrix alone might be too small.

We did not find any signs of a systematic bias in any of the analysis we carried out.

### 4.3 Real fits

In this section we discuss the quality of the fits to SDSS galaxies obtained with VESPA.

As explained in Section 2, VESPA finds the best-fitting solution in a $\chi^2$ sense for a given parametrization, which is self-regulated in order to not allow an excessive number of fitting parameters. We have shown that this self-regularization gives a better solution in parameter space (Fig. 11), despite often not allowing the parametrization which would yield the best fit in data space (Fig. 3). However, our aim is still to find a solution which gives a good fit to the real spectrum. Fig. 15 shows the one-point distribution of reduced values of reduced $\chi^2$ for one plate of galaxies. This distribution peaks at around $\chi^2_{\text{reduced}} = 1.3$, and Fig. 16 shows a fit to one of the galaxies with a typical value of goodness of fit.

It is worth noting that the majority of the fits which are most pleasing to the eye, correspond to the ones with a high S/N and high value of reduced $\chi^2$. One would expect the best fits to come from the galaxies with the best signal. However, we believe the fact that they do not is not a limitation of the method, but a limitation of the modelling. There are a number of reasons why VESPA would be unable to produce very good fits to the SDSS data. One is the adoption of a single velocity dispersion (170 km s$^{-1}$) which could easily be improved upon at the expense of CPU time. However, the dominant reason is likely to be lack of accuracy in stellar and dust modelling – whereas BC03 models can and do reproduce a lot of the observed features, it is also well known that this success is limited as there are certain spectral features not yet accurately modelled, or even modelled at all. There are similar deficiencies in dust models and dust extinction curves. The effect of the choice of modelling should not be overlooked, and we refer the reader to a discussion in Section 4.5 of Panter et al. (2007), where these issues are discussed.

### 4.4 VESPA and MOPED

In this section we take the opportunity to compare the results from VESPA and MOPED, obtained from the same sample of galaxies. The VESPA solutions used here are obtained with a one-parameter dust model, to allow a more fair comparison between the two methods. Both methods make similar assumptions regarding stellar models, but MOPED uses a Large Magellanic Cloud (Gordon et al. 2003) dust extinction curve, and single screen modelling for all optical depths.

Our sample consists of two plates from the SDSS Data Release 3 (DR3) (Abazajian et al. 2005) (plates 0288 and 0444), from which we analyse a total of 821 galaxies. We are mainly interested in comparing the results in a global sense. MOPED in its standard configuration attempts to recover 23 parameters (11 star formation fractions, 11 metallicities and one dust parameter), so we might expect considerable degeneracies. Indeed, in the past the authors of MOPED have cautioned against using it to interpret individual galaxy spectra too precisely. We have observed degeneracies between adjacent bins in MOPED, but on the other hand a typical MOPED solution has many star formation fractions which are essentially zero, so the number of significant contributions is always much less than 23.

Fig. 17 shows the recovered average SFH for the 821 galaxies using both methods. In the case of VESPA, solutions parametrized by low-resolution bins had to be reparametrized in high-resolution bins, so that a common grid across all galaxies could be used. This was done using the weights given by (21). The lines show a remarkably good agreement between the two methods. Having recovered an SFH for each galaxy, one can then estimate the stellar mass of a galaxy. We calculated this quantity for all galaxies using the solutions from both methods, and with similar assumptions regarding

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**Figure 14.** Total stellar mass recovered for two sets of observations of 500 galaxies in the main galaxy sample. The error bars are calculated from $C(x)$.

**Figure 15.** The distribution of reduced values of $\chi^2$ for a sample of 360 galaxies analysed by VESPA.
cosmological parameters and fibre-size corrections. Explicitly, we have done the following.

(i) We converted from flux to luminosity assuming the set of cosmological parameters given by Spergel et al. (2003).

(ii) We recovered the initial mass in each age bin using each method.

(iii) We calculated the remaining present-day mass for each population after recycling processes. This information is supplied by the synthetic stellar models, as a function of age and metallicity.

(iv) We summed this across all bins to calculate the total stellar present-day mass in the fibre aperture, \( M \).

(v) We corrected for the aperture size by scaling up the mass to \( M_{\text{stellar}} \) using the Petrosian and fibre magnitudes in the \( \lambda \) band, \( M_p(\lambda) \) and \( M_f(\lambda) \), with \( M_{\text{stellar}} = M \times 10^{0.4[M_p(\lambda) - M_f(\lambda)]} \).

Fig. 18 shows the recovered galaxy masses as recovered from MOPED and from VESPA. We see considerable agreement between VESPA and MOPED. Over 75 per cent of galaxies have \( 0.5 \leq M_{\text{VESPA}} / M_{\text{MOPED}} \leq 1.5 \). There is a tail of around 10 per cent of galaxies where VESPA recovers two to four times the mass recovered by MOPED. The main reason for this difference is in the dust model used – we find a correlation between dust extinction and the ratio of the two mass estimates. This again reflects the fact that
total stellar mass estimates are highly sensitive to changes in the spectrum continuum (see also Section 4.2).

Our subsample of 821 includes galaxies with a wide range of S/N values, SFHs and even wavelength range (mainly due to each galaxy having different masks applied to it, according to the quality of the spectroscopic data). Fig. 19 shows the number of recovered non-zero parameters in the sample, using VESPA. As an average, it falls below the synthetic examples studied in Section 3. This is not surprising, though, as each galaxy will be have an unique and somewhat random combination of characteristics which will lead to a different number of parameters being recovered. The total combination of these sets of characteristics would be impossible to investigate using the empirical method described in Section 3, and here lies the advantage of VESPA of dynamically adapting to each individual case. Also important to note is the fact that the wavelength coverage is normally not continuous in an SDSS galaxy, due to masked regions. This was not modelled in Section 3, and is likely to further reduce the number of recovered parameters in any given case.

Perhaps more useful is to translate this number into a number of recovered significant stellar populations for each galaxy. We define a significant component as a stellar population which contributes 5 per cent or more to the total flux. Fig. 20 shows the distribution of the number of significant components for our subsample of galaxies, as recovered by MOPED and VESPA. It is interesting to note that both methods recover on average a similar amount of components, even though MOPED has no explicit self-regularization mechanism, as VESPA clearly does.

5 CONCLUSIONS

We have developed a new method to recover star formation and metallicity histories from integrated galactic spectra – VESPA. Motivated by the current limitations of other methods which aim to do the same, our goal was to develop an algorithm which is robust on a galaxy-by-galaxy basis. VESPA works with a dynamic parametrization of the SFH, and is able to adapt the number of parameters it attempts to recover from a given galaxy according to its spectrum. In this paper we tested VESPA against a series of idealized synthetic situations, and against SDSS data by comparing our results with those obtained with the well-established code, MOPED.

Using synthetic data we found the quality and resolution of the recovered solutions varied with factors such as type of SFH, noise in the data and wavelength coverage. In the vast majority of cases, and within the estimated errors and bin correlations, we observed a reliable reproduction of the input parameters. As the S/N decreases, it becomes increasingly difficult to recover robust solutions. Whereas our method cannot guarantee a perfect solution, we have shown that the self-regularization we imposed helped to obtain a cleaner solution in an overwhelming majority of the cases studied.

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**Figure 18.** Galaxy stellar mass (in units of solar masses) as recovered by VESPA and MOPED for a subsample of 821 SDSS galaxies. The small percentage of galaxies with significantly larger VESPA masses have large extinction. The difference is accounted for by the fact that MOPED and VESPA use different dust models.

**Figure 19.** Number of non-zero parameters in solutions recovered from 821 SDSS galaxies with VESPA. Please note that these correspond to the total number of non-zero components in the solution vector $c$, and not to the number of recovered stellar populations. For information about the number of recovered populations see Fig. 14.

**Figure 20.** The distribution of the total number of recovered stellar populations which contribute 5 per cent or more to the total flux of the galaxy, as recovered from MOPED (dashed line) and VESPA (solid line).
On the real data analysis, we have studied possible effects from systematics using duplicate observations of the same set of galaxies, and have also compared the VESPA’s results to those of MOPED obtained using the same data sample. We found that in the majority of cases our results are robust to possible systematics effects, but that in certain cases and particularly when calculating stellar masses, VESPA might underestimate the mass errors. However, we found no systematic bias in any of our tests. We have also shown that VESPA’s results are in good agreement with those of MOPED for the same sample of galaxies. VESPA and MOPED are two fundamentally different approaches to the same problem, and we found good agreement both in a global sense by looking at the average SFH of the sample, and in an individual basis by looking at the recovered stellar masses of each galaxy. VESPA typically recovered between two and five stellar populations from the SDSS sample.

VESPA’s ability to adapt dynamically to each galaxy and to extract only as much information as the data warrant is a completely new way to tackle the problem of extracting information from galactic spectra. Our claim is that, for the most part, VESPA’s results are robust for any given galaxy, but our claim comes with two words of caution. The first one concerns very noisy galaxies – in extreme cases ($S/N \approx 10$ or less, at a resolution of 3 Å), it becomes very difficult to extract any meaningful information from the data. This uncertainty is evident in the large error bars and bin correlations, and our solutions can be essentially unconstrained even at low resolutions. We are therefore limited when it comes to analysing individual high-noise galaxies, which is the case of many SDSS objects. Our second word of caution concerns the stellar models used to analyse real galaxies – any method can only do as well as the models it bases itself upon. We are limited in our knowledge and ability to reproduce realistic synthetic models of stellar populations, and this is inevitably reflected in the solutions we obtain by using them. On the plus side, VESPA works with any set of synthetic models and can take advantage of improved versions as they are developed.

VESPA is fast enough to use on large spectroscopic samples (a typical SDSS galaxy takes 1 min on an average workstation), and we are in the process of analysing SDSS DR3, which consists of roughly half a million galaxies. Our aim is to publish and exploit a catalogue of robust SFHs, which we hope will be a valuable resource to help constrain models of galaxy formation and evolution.

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