CHIRAL LAGRANGIANS OUT OF THERMAL EQUILIBRIUM *

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We extend chiral perturbation theory to study a meson gas out of thermal equilibrium. We assume that the system is initially in thermal equilibrium at a temperature $T_i < T_c$ and work within the Schwinger-Keldysh contour technique. To lowest order and in the chiral limit, we use a nonlinear sigma model with a time-dependent pion decay function $f_\pi(t)$, which we consistently define in terms of the axial current two-point correlator. As a first application, we analyse $f_\pi(t)$ up to one loop order and find the $T_i$ dependence of the lowest coefficients in its short-time expansion. We discuss the applicability of our results to the time evolution of the plasma formed after a heavy-ion collision and discuss the interpretation of our model in a curved space-time background.

I. INTRODUCTION

The chiral phase transition plays a fundamental role in the description of the plasma formed after a relativistic heavy-ion collision (RHIC). For $N_f$ massless quarks (the chiral limit), QCD is invariant under the chiral group $SU_L(N_f) \times SU_R(N_f)$. That symmetry is spontaneously broken at zero temperature to the vector group $SU_V(N_f)$ (isospin), the Nambu-Goldstone bosons (NGB) being the lightest mesons (pions for $N_f = 2$ plus kaons and eta for $N_f = 3$), which are the relevant low-energy degrees of freedom. The light quark masses are introduced perturbatively. This picture has proven to be very successful in describing hadron observables (like decay constants or scattering amplitudes), for low energies and small masses, through the Goldstone theorem and current algebra. On the other hand, QCD at this scale is strongly coupled to study the phase transition, which is indeed the case (see below) and allows to study the meson gas thermodynamics at low temperatures and the chiral scale $\Lambda \chi \simeq 1 \text{ GeV}$. For a review of low-energy QCD and effective lagrangians we refer to and references therein.

In thermal equilibrium at finite temperature $T$, the chiral symmetry is believed to be restored at $T_c \simeq 150-200 \text{ MeV}$ in a phase transition, which is very likely to be of second order for $N_f = 2$. It should be stressed that ChPT cannot strictly establish the existence of a phase transition since it provides an expansion in $T^2/\Lambda_{\chi}^2$ for the order parameter $\forall T$. Nonetheless, it predicts the correct behaviour of the observables as $T$ approaches $T_c$ (see below) and allows to study the meson gas thermodynamics at low temperatures. To describe the system near $T_c$, the LSM seems to be a better choice, since it undergoes a second-order phase transition in the mean field approximation, $\nu(T)$ becoming the order parameter. However, $T_c$ in the mean field LSM is independent of $\Lambda$, which suggests that the NLSM should be equally valid to study the phase transition, which is indeed the case provided one works in the large $N$ limit.

In the chiral limit and in equilibrium at temperature $T$, the only parameter of the NLSM lagrangian is a $T$-independent constant $f$ with dimensions of energy, related to the physical temperature-dependent pion decay constant as $f_\pi(T) = f(1 + O(p^2))$, where $f_\pi \simeq 93 \text{ MeV}$ at $T = 0$ ($\Lambda \chi \simeq 4\pi f_\pi$, which is the typical loop factor) is customarily measured in leptonic decays $\pi \rightarrow \nu l$. Here, $p$ denotes generically any pion momentum scale, including

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temperature and by $\mathcal{O}(p)$ we really mean $\mathcal{O}(p/\Lambda_\chi)$, as we explained before. Besides, every pion loop counts as $\mathcal{O}(p^2)$ \cite{1} and all the infinities coming from them can be absorbed in the coefficients of higher order lagrangians, which is also true for $T \neq 0$ \cite{2}. The next to leading order (NLO) one-loop corrections induce the temperature dependence of $f_\pi(T)$ and for $N_f = 2$ in the chiral limit the result is \cite{3}

$$f_\pi^2(T) = f^2 \left(1 - \frac{T^2}{6f^2}\right) \quad (1)$$

A few remarks are in order here. Firstly, at $T \neq 0$, due to the loss of Lorentz covariance in the thermal bath, one should actually consider two independent pion decay constants, $f_\pi^2$ (temporal) and $f^2$ (spatial), which in general will be complex, their real and imaginary parts being related respectively with the pion velocity and damping rate in the thermal bath \cite{4}. However, to NLO, one has $f_\pi^2 = f^2 = f_\pi(T)$ in \cite{3}. Notice that \cite{3} suggests a critical behaviour at $T_c \approx \sqrt{6}f_\pi$, since $f_\pi(T)$ behaves roughly like $v(T)$, although they are different objects \cite{6}. Despite \cite{6} being just the lowest order in the low temperature expansion (and not being too accurate to extrapolate it up to $T_c$) it predicts the right behaviour and a reasonable estimate for $T_c$. Another important comment is that one has to be careful when defining $f_\pi$ in a medium, trying to extend naively low-energy theorems to $T \neq 0$ (see section IV). Nevertheless, there is a consistent way of defining $f_\pi$ in the thermal bath from the axial-axial correlator spectral function \cite{6} as it will be explained in section IV.

The equilibrium assumption is not realistic if one is interested in the dynamics of the expanding plasma formed after a RHIC, where there are several observable nonequilibrium effects. One of them is the formation of the so called disoriented chiral condensates (DCC), regions in which the chiral field is correlated and has nonzero components in the pion direction \cite{10}. As the plasma expands, long-wavelength pion modes can develop instabilities that grow very fast as the field relaxes to the ground state. If that picture is correct, there would be observable effects, such as coherent pion emission \cite{11}. The question is whether those domains can grow fast and large enough to emit a significant number of pions. A necessary, but not sufficient, condition is that the system is out of thermal equilibrium. This problem has been extensively studied in the literature, using various non-perturbative and numerical approaches within the LSM \cite{12,13}. In all these works, thermal equilibrium at $T_0 > T_c$ is assumed for $t < 0$ and then some mechanism drives the theory off equilibrium for $t > 0$, encoded in the time dependence of the different lagrangian parameters. The simplest choice is an external quench changing the sign of the mass squared $m^2$ of the $O(4)$ field so that modes with $k^2 < m^2$ become unstable \cite{12,14}. A more realistic approach is to describe the plasma expansion in proper time and rapidity coordinates, which is analogous to study the system in a certain space-time background metric, assuming either boost \cite{15} or scale \cite{16} invariant kinematics (cylindrical and spherical expansions, respectively). During the time evolution, the pion modes propagate as if they had a negative mass squared (unstable long-wavelength modes), therefore yielding exponential growth with time of the pion propagator. The general conclusion is that the typical size of the formed DCC regions is at most 3 fm, which appears to be not big enough to contain a large observable number of pions, and the time during which the relevant plasma expansion takes place varies between 5-10 fm/c \cite{14,12}. Another important nonequilibrium effect is the photon and dilepton production \cite{17,12}. As these particles are weakly interacting, they decouple from the hot plasma carrying important information about the dynamics, so that their final spectra, dominated by meson decays, is a promising signal to analyze nonequilibrium aspects \cite{18}. A recent suggestion is that a significant number of those photons might have been produced by $\pi^0$ decay during a typical DCC nonequilibrium evolution \cite{18}.

Our basic objective in this work will be to construct an effective ChPT-based model to describe a meson gas out of thermal equilibrium, as an alternative to the LSM approach. Our only degrees of freedom will be then the NGB and we will consider the most general low-energy lagrangian compatible with the QCD symmetries. We will restrict here to $N_f = 2$ and to the chiral limit. This is the simplest approximation we can consider and it allows to build the model in terms of exact chiral symmetry. The lowest order lagrangian is then the NLSM. In principle, we will be interested in the physical regime in which the system is not far from equilibrium (we will be more specific about this point below), where an expansion in derivatives is consistent. One of the novelties of our approach is to exploit the analogy between near-equilibrium systems and ChPT.

The structure of this work is the following: in section II we introduce our non-equilibrium NLSM and establish the non-equilibrium ChPT, discussing how the different symmetries are realised. We also discuss the interpretation of the model as a curved space-time QFT, paying special attention to renormalisation. The leading order non-equilibrium pion propagator and Lehman spectral function are also analysed in that section, which also contains most of our notation and conventions. In section III, we derive the one-loop NLO correction to the propagator, which will be needed later. In section IV we provide a consistent definition of the nonequilibrium time-dependent pion decay functions (PDF) extending the equilibrium pion decay constants and we derive them up to one loop in ChPT. One of the motivations to concentrate on $f_\pi(t)$ first is that to one loop in ChPT we expect that all physical observables are obtained from the tree level ones by replacing $f(t) \rightarrow f_\pi(t)$, as it happens indeed in equilibrium. We analyse the short-time behaviour of $f_\pi(t)$ and estimate the relevant time scales involved, discussing the role of unstable modes. The issue
of axial current conservation is also analysed in section IV. Finally, in section III, we present our conclusions and discuss some open questions, as well as further applications of our approach.

II. THE NLSM AND CHPT OUT OF EQUILIBRIUM

We will assume that the system is in thermal equilibrium for \( t < 0 \) at a temperature \( T_0 < T_c \) and for \( t > 0 \) we let the lagrangian parameters be time-dependent (we are also assuming that the system is homogeneous and isotropic). The generating functional of the theory can then be formulated in the path integral formalism, by letting the time integrals run over the Schwinger-Keldysh contour \( C \) shown in Fig. 1. We will eventually let \( t_i \to -\infty \) and \( t_f \to +\infty \), although we will show below that our results are independent of \( t_i \) and \( t_f \). We remark that, even in that limit, the imaginary-time leg of the contour has to be kept, since it encodes the KMS equilibrium boundary conditions.

With the above assumptions, our low-energy model will be the NLSM with \( f \) becoming a real function \( f(t) \),

\[
S[U] = \int_C d^4x \frac{f^2(t)}{4} \text{tr} \partial_\mu U(t, x) \partial^\mu U(t, x) \tag{2}
\]

with \( \int_C d^4x \equiv \int_{C_i} dt \int d^3x, f(t < 0) = f \) and \( U(\vec{x}, t) \in SU(2) \) is the NGB field, satisfying \( U(\vec{x}, t_i + i\beta_i) = U(\vec{x}, t_i) \) with \( \beta_i = T_i^{-1} \). We will parametrise \( U \) as

\[
U(\vec{x}, t) = \frac{1}{f(t)} \left\{ \left[ f^2(t) - \pi^2 \right]^{1/2} I + i\tau_a \pi^a \right\} \tag{3}
\]

where \( \pi^2 = \pi^a \pi_a, \pi^a(\vec{x}, t) \) the pion fields satisfying \( \pi^a(t_i + i\beta_i) = \pi^a(t_i) \) \((a = 1, 2, 3)\) and \( I \) and \( \tau_a \) are the identity and Pauli matrices. In terms of the \( \pi^a \) fields, the action (2) becomes

\[
S[\pi] = \int_C d^4x \frac{1}{2} \left\{ \partial_\mu \pi^a \partial^\mu \pi^b \left[ \delta_{ab} + \frac{\pi_a \pi_b}{f^2(t) - \pi^2} \right] \right. \\
+ \frac{f(t)}{f^2(t) - \pi^2} \left\{ \frac{\pi^2}{f^2(t) - \pi^2} - 2\pi^a \dot{\pi}^a \right\} \right\} \tag{4}
\]

Note that by including \( f(t) \) in the parametrisation of the NGB field we recover the canonical kinetic term in the action. Other choices amount to a time-dependent normalisation of the pion fields, which should not have any effect on the physical observables (see section III). For instance, we could redefine \( \tilde{\pi}^a = \pi^a f(0)/f(t) \), so that for the \( \tilde{\pi}^a(\vec{x}, t) \) fields the action is the equilibrium NLSM multiplied by the time-dependent scale factor \( f^2(t)/f^2(0) \), which is directly interpreted in terms of a curved space-time background, as we will see below.

The action (3) is manifestly chiral invariant, i.e., under \( U \to LU^R \) with \( L \) and \( R \) arbitrary constant \( SU(2) \) matrices. Notice that, as we are working in the chiral limit by assumption, we have not considered a pion mass term in the action. Such term would break explicitly the chiral symmetry, being still invariant under vector isospin \( L = R \) transformations for \( m_a = m_d \). The conserved vector \( V_\mu \) and axial-vector \( A_\mu \) Noether currents for the chiral symmetry can be derived by gauging the symmetry and then taking functional derivatives with respect to the external sources \( f \). Extending that procedure to our non-equilibrium action (3) we get

\[
A_\mu^a(\vec{x}, t) = i \frac{f^2(t)}{4} \text{tr} \left[ \tau^a \left( U^\dagger \partial_\mu U - U \partial_\mu U^\dagger \right) \right]
\]

\[
V_\mu^a(\vec{x}, t) = -i \frac{f^2(t)}{4} \text{tr} \left[ \tau^a \left( U^\dagger \partial_\mu U + U \partial_\mu U^\dagger \right) \right] \tag{5}
\]

The model is also parity invariant. Lorentz covariance is lost, because we have chosen the frame in which the thermal bath is at rest for \( t < 0 \). The time translation symmetry is also broken at nonequilibrium.

Let us now discuss how to implement a consistent ChPT out of equilibrium. The new ingredient with respect to the equilibrium case, is the temporal variation of \( f(t) \). We will then consider

\[
\frac{\dot{f}(t)}{f^2(t)} \approx O(p), \quad \frac{\ddot{f}(t)}{f^3(t)} \approx O(p^2) \tag{6}
\]

and so on, the rest of the chiral power counting being the same as in equilibrium, i.e., \( T = O(p) \), \( \partial \pi = O(p^2) \) and every pion loop is \( O(p^3) \). Therefore, in our approach we treat the deviations of the system from equilibrium perturbatively, following the ChPT guidelines. Thus, with this chiral power counting, we can expand our action (3) to the relevant order in pion fields and calculate any observable by taking into account all the Feynman diagrams that contribute to that order. The loop divergences should be such that they can be absorbed in the coefficients of higher order lagrangians, which in general will require the introduction of new time-dependent counter-terms (see below).

Notice that according to (3), we can always describe the short-time non-equilibrium regime, just by Taylor expand \( f(t) \) around \( t = 0 \). In fact, for times \( t \leq f^{-1} \), the Taylor expansion of \( f(t) \) is equivalent to a chiral expansion, since then \( \dot{f}(0)/f = O(p) \), \( \ddot{f}(0)/f^2 = O(p^2) \) and
so on. In case we are interested only in short times, the specific form of \( f(t) \) will not be important (see section IV), but we want to stress that the conditions (11) do not constrain us to work at short times, but just to remain close enough to equilibrium.

To leading order in the pion fields, the action (3) reads

\[
S_0[\pi] = - \int_C d^4x \frac{1}{2} \pi^a(\vec{x}, t) \left[ \Box + m^2(t) \right] \pi^a(\vec{x}, t) \quad (7)
\]

\[
m^2(t) = - \frac{\hat{f}(t)}{f(t)} \quad (8)
\]

where we have integrated by parts, using the boundary conditions for the pion fields. Thus, we see that the leading order non-equilibrium effect of our model can be written as a time-dependent mass term for the pions. As we commented before, time-dependent field masses are a common (and welcomed) feature of out of equilibrium models [12–16]. In our approach, that mass parametrizes the deviation of the plasma from equilibrium through the temporal variation of \( m^2(t) \). Notice that \( m^2(t) \) can be negative, so that our model accommodates unstable pion modes, whose importance we have discussed before. The effect of those modes should not be important for the short-time evolution, but for longer times we expect them to influence significantly the dynamics.

We remark that the axial current is classically conserved despite the existence of the time-dependent mass term. For instance, to leading order we have, from (3),

\[
A^a_\mu = - f(t) \partial_\mu \pi^a + \delta_{ab} \hat{f}(t) \pi^b + \mathcal{O}(\pi^3) \quad ,
\]

which satisfies \( \partial^\mu A_\mu = 0 \) using the equations of motion to the same order \( \left[ \Box + m^2(t) \right] \pi^a = 0 \). Had we included an explicit pion mass term \( m_\pi \simeq 140 \text{ MeV} \), the axial current would not have been conserved and the effective mass term in the leading order action would have been proportional to \( m_\pi^2 + m^2(t) \), so that the onset for instabilities would be rather \( m^2(t) < - m_\pi^2 \).

We will now rephrase our model in the language of curved space-time background QFT [24], which will turn out to be a very useful analogy. For that purpose, let us consider the HLSM in an arbitrary background space-time metric \( g_{\mu\nu} \),

\[
S_2[\tilde{\pi}] = \int_C d^4x \sqrt{-g} g^{\mu\nu} \left\{ \delta_{ab} \tilde{\pi}_a \partial_\nu \tilde{\pi}_b \left( \delta_{ab} \tilde{\pi}_a \tilde{\pi}_b - \frac{f^2(0)}{\pi^2} \right) \right\} + \xi S_R[\tilde{\pi}, R] \quad ,
\]

plus \( \tilde{\pi} \) independent terms, where \( g \) is the metric determinant and the term \( \xi S_R[\tilde{\pi}, R] \) accounts for the possible couplings between the pion fields and the scalar curvature \( R(x) \) (like \( R(x) \phi^2 \) for a free scalar field \( \phi \)). To compare with our nonequilibrium model, we will choose a spatially flat Robertson-Walker (RW) space-time:

\[
ds^2 = dt^2 - a^2(t)d\vec{x}^2 \quad (11)
\]

with \( a(t) \) the scale factor. For \( t < 0 \) we choose the space-time as Minkowskian, so that \( a(t < 0) = 1 \) and the system in thermal equilibrium at temperature \( T_c \). Then, if we change coordinates to the conformal time \( \eta \) defined by \( t = \int_0^\eta \frac{a(\eta')d\eta'}{f(\eta)} \), the metric becomes \( ds^2 = a^2(\eta)[d\eta^2 - d\vec{x}^2] \) and we see that the action in (10) for the minimal coupling case \( \xi = 0 \) is nothing but \( S[U] \) in (3), using the \( \tilde{\pi} \) parametrisation discussed before and identifying \( a(t) = f(t)/f(0) \). That is, our non-equilibrium model in the chiral limit is equivalent to a RW spatially flat space-time in conformal time, that starts expanding (or contracting) at \( t = 0 \) with a rate given by the scale factor \( f(t)/f(0) \) and couples minimally to the matter fields. Our model is then not only suitable as an effective theory for a RHIC environment, but also in a cosmological framework. Nonequilibrium models for RHIC in which the plasma expansion is also parametrised in a background metric have been analysed in [14–16] (see comments in section III).

Let us now consider \( \xi \neq 0 \). The lowest order \( S_R \) term we can construct has the form of an effective mass term,

\[
S_R[\tilde{\pi}, R] = R(t) f^2(0) \left[ 1 - \frac{\pi^2}{f^2(0)} \right]^{1/2} ,
\]

so that it breaks explicitly the chiral symmetry. Let us then pick up the lowest order contribution in pion fields \( S_R = - R \tilde{\pi}^2/2 \) in (12). Then recalling the expression of the scalar curvature for the spatially flat RW metric in conformal time \( R(\eta) = 6 \tilde{f}(\eta)/f^3(\eta) \), we see that the curvature term exactly cancels the \( m^2(t) \) term in (3) after rescaling \( \tilde{\pi}(x) = f(0)\pi(x)/f(t) \), provided we choose \( \xi = 1/6 \). But that value of \( \xi \) is precisely the only one making the theory scale invariant [23], so that in that case the leading order lagrangian would be scale invariant but not chiral invariant. Thus, the lagrangian chiral and conformal symmetries are not compatible in a curved background [24] or, equivalently, at nonequilibrium. Thus, for \( \xi = 0 \) we interpret the mass term \( m^2(t) \) as the minimal coupling with the background, which for \( m_\pi = 0 \) yields chiral invariance of the lagrangian, though breaking the conformal symmetry. This is the choice we will adopt here, since we want to preserve chiral symmetry.

A very important result is that all the one-loop divergences arising from (10) can be absorbed in the coefficients of the \( \mathcal{O}(p^4) \) lagrangian \( \mathcal{L}(4) \), which includes new chiral-invariant couplings of pion fields with the curvature [24]. In the chiral limit, for instance, there are two such new terms and therefore two more undetermined constants in addition to the Minkowski case ones (see [24] for details). Therefore, following this analogy, we know how to construct the higher order nonequilibrium lagrangians that eventually will absorb all the divergences we may find in our analysis. We will come back to this point below.

From the generating functional we can derive in the standard manner, the Green functions ordered in time.
along the contour $C$. The two-point function defines the pion propagator

$$G^{ab}(x, y) = -i < T_C \pi^a(x) \pi^b(y) >$$  \hspace{1cm} (13)

with $T_C$ the time-ordered product along $C$. To leading order the pion propagator is $G^{ab}_0(x, y) = \delta^{ab} G_0(x, y)$ (by isospin invariance), with $G_0(x, y)$ the solution of the differential equation

$$\{ \Box_x + m^2(x^0) \} G_0(x, y) = -\delta^{(4)}(x - y)$$  \hspace{1cm} (14)

As for the boundary conditions, firstly we have to impose KMS equilibrium conditions on the imaginary leg of the contour, that is,

$$G^\omega (\vec{x}, t_i - i\beta; y) = G^\omega (\vec{x}, t_i; y)$$  \hspace{1cm} (15)

with the advanced and retarded propagators defined as customarily along the contour $C$. In addition, the propagator must be continuous and differentiable $\forall t \in C$, so that the solution is uniquely defined [20]. Therefore, in our case, we will demand that the solution of (14) for $t \rightarrow 0^+$ matches that for $t < 0$, which is the equilibrium solution, given in momentum space by [23]:

$$G^\omega_0(q_0, \omega_q) = \rho^\omega_0(q_0, \omega_q) [\theta_C(t - t') + n_B(q_0)]$$  \hspace{1cm} (16)

where $\theta^2 = |q|^2$, $n_B(x) = [\exp(\beta x) - 1]^{-1}$ is the Bose-Einstein distribution function, $\theta_C$ is the step function along the contour $C$ and $\rho^\omega_0 = -2\pi i \text{sign}(q_0) \delta(q_0^2 - \omega_q^2)$.

We will make use of two different representations for the non-equilibrium Green functions. Notice that $G(x, x') = G(t, t', \vec{x} - \vec{x'})$ because of the lack of time translation invariance. Therefore, we will define, as customarily, the “fast” temporal variable $t - t'$ and the “slow” one $\tau \equiv (t + t')/2$, so that $F(q_0, \omega_q, \tau)$ and $F(\omega_q, t, t')$ will denote, respectively, the fast and mixed (in which only the spatial coordinates are transformed) Fourier transforms of $F(x, x')$. The fast Fourier transform depends separately on $q_0$ and $\omega_q$, because of the loss of Lorentz covariance and has the extra nonequilibrium $\tau$-dependence.

In the mixed representation, (14) becomes

$$\left[ \frac{d^2}{dt^2} + \omega_q^2 + m^2(t) \right] G_0(\omega_q, t, t') = -\delta(t - t')$$  \hspace{1cm} (17)

The general solution of (17) cannot be found analytically for $m^2(t)$ arbitrary, but it can formally be written as a Schwinger-Dyson integral equation in terms of the equilibrium solution as

$$G_0(\omega_q, t, t') = G^{\omega_0}_0(\omega_q, t - t') \right. $$ \left. + \int_C d\omega z \frac{G^{\omega_0}_0(\omega_q, t - z) G_0(\omega_q, z, t')}{z - \omega_q} \right) \hspace{1cm} (18)

with $G^{\omega_0}_0(\omega_q, t - t')$ the solution of (17) with $m^2(t) = 0$.

Another object of interest for our purposes is the Lehman spectral function, defined as [21,23]

$$\rho(x, y) = G^>(x, y) - G^<(x, y)$$  \hspace{1cm} (19)

Recall that in equilibrium and to leading order, the spectral function is $\rho_0^\omega$ in (18). Let us now discuss some useful properties of the spectral function. Firstly, by definition $G(x, y) = G(y, x)$ and hence $G^>(x, y) = G^<(y, x)$. Therefore, $\rho(x, y) = -\rho(y, x)$ and $\rho_0(\omega_q, \omega_q, \tau) = -\rho(\omega_q, \omega_q, \tau)$. The normalisation of $\rho_0$ is the same as in equilibrium, i.e., for the parametrisation [4] we have

$$\frac{1}{2\pi i} \int_{-\infty}^{+\infty} \rho_0(\omega_q, t, t') \frac{d\rho_0(\omega_q, t, t')}{dt} \bigg|_{t = t'} = -1$$  \hspace{1cm} (20)

It is not difficult to check (20), by using for instance [18] and $\rho_0^\omega(\omega_q, t, t) = 0$.

### III. NEXT TO LEADING ORDER PROPAGATOR

We will now obtain the NLO correction to the two-point Green function [13]. For that purpose, we need the action in (4) up to four-pion terms:

$$S[\pi] = S_0[\pi] + \frac{1}{2} \int_C dx \left\{ \frac{1}{f^2(t)} \left[ \partial_\pi \pi^a \partial^\mu \pi^b \pi_a \pi_b \right] + O(\pi^6) \right\}$$  \hspace{1cm} (21)

The NLO correction to the two-point function is given by the tadpole diagram a) in Fig.2. It is important to bear in mind that in principle we should also include a tree diagram coming from the $L^{(4)}$ lagrangian, which would take care of renormalisation. We shall ignore that contribution here and we will justify this approximation in section V in the context of the short-time regime.

![FIG. 2. One-loop diagrams contributing to a) The NLO pion propagator and b) The NLO axial-axial correlator](image-url)

After summing over all isospin indices in the loop, we can use the differential equation (14). However, in doing so we have to deal with the divergent quantity $\delta^{(4)}(0)$. We will use dimensional regularisation, which is a suitable scheme to deal with ChPT [4], so that $\delta^{(4)}(0) = 0$. Therefore, we can replace $\Box_z G_0(z, z) = -m^2(z) G_0(z, z)$. Let us concentrate on $t, t' \in C_1$ in Fig.2 (i.e, the $G_{11}$ component of the propagator [23]) and take $t$ and $t'$ positive, which is actually the relevant case for our purposes, since the nonequilibrium effects start...
pressed for simplicity the $d/d$ tors, the dot denotes dependent quantity, even in the chiral limit, which means that (22) is of consistency of our result. It can be checked that the equilibrium result is finite in the chiral limit, which is a good check for the $G_{\pi\pi}$ relation function and $G_{\pi\pi}$ dependence of the propagator, which is a good check of consistency of our result using the general contour $C$, which includes both imaginary-time and real-time thermal field theory.

### IV. THE NONEQUILIBRIUM PION DECAY FUNCTIONS

At $T = 0$, the pion decay constant is customarily defined through the PCAC low-energy theorem for the expectation value of the axial current between the vacuum and an asymptotic pion state and one relates that amplitude with Green functions via the LSZ formula [3]. However, in a thermal bath, the concepts of LSZ and asymptotic states are subtle and it is much more convenient to work directly with thermal Green functions. The object of interest for our purposes is the axial-axial correlator

$$A_{\mu\nu}^a(x, y) = <T_C A_{\mu}^a(x) A_{\nu}^a(y)>$$

At $T = 0$, the longitudinal part of the axial-axial spectral function is proportional, in the chiral limit, to a $\delta$-function, signaling the existence of massless NGB (see below) and the proportionality constant is $f_\pi$. However, at $T \neq 0$ that definition is more subtle: firstly, because of the loss of covariance, the tensorial structure compatible with current conservation is more general than at $T = 0$ [20] and one can define two independent and complex $f_\pi^T$ (spatial) and $f_\pi^I$ (temporal) [3]. Secondly, the pion dispersion relation is not in general a $\delta$-function as in vacuum. Nevertheless, a proper definition of $f_\pi(T)$ can still be given [3], which, to one-loop gives the result \[ i \] with $f_\pi^T = f_\pi^I = f_\pi$ real. To higher orders, $f_\pi^T \neq f_\pi^I$ and $\text{Im} f_\pi^T \neq \text{Im} f_\pi^I$ are nonzero [3].

Let us then analyze the axial-axial correlator (27) in our non-equilibrium model. As we said above, the relevant quantity, as far as $f_\pi$ is concerned, is not (27), but its spectral function $\rho_{\pi\pi}(x, y)$, defined by subtracting its advanced and retarded parts. Again, we write $\rho_{\pi\pi} = \delta^{ab} \rho_{\mu\nu}$ by isospin invariance and we concentrate on the fast Fourier transform $\rho_{\mu\nu}(q_0, \vec{q}, \tau)$. Firstly, we will write down the most general form for $\rho_{\mu\nu}$ compatible with the symmetries and analyse the restrictions imposed by the axial current conservation Ward Identity (WI).

From the definition of the spectral function, we readily realise that $\rho_{\mu\nu}(q_0, \vec{q}, \tau) = -\rho_{\nu\mu}(-q_0, -\vec{q}, \tau)$. Then, from rotational invariance,

$$\rho_{ij}(q_0, \vec{q}, \tau) = q_0 q_j \rho_L(q_0, \omega q, \tau) + \delta_{ij} \rho_d(q_0, \omega q, \tau)$$

with $\rho_L, d(q_0) = -\rho_L, d(-q_0)$ 1. On the other hand, $\rho_{j0}$ is a 3-vector, so that

$$\rho_L \text{ and } \rho_d \text{ correspond in equilibrium and in the notation of }$$

\[ 1 \rho_L \text{ to } \text{sgn}(q_0) \rho_1^0 q_0^2/q^2 \text{ and } \text{sgn}(q_0) \rho_1^0 \text{ respectively.}$$
\[ \rho_{j0}(q_0, \hat{q}, \tau) = q_j \rho_S(q_0, \omega_q, \tau) \]  

and \[ \rho_{j0}(q_0, \hat{q}, \tau) = q_j \rho_S(-q_0, \omega_q, \tau). \] Thus, \( \rho_{\mu\nu} \) is characterized, in principle by the four functions \( \rho_L, \rho_d, \rho_S \) and \( \rho_W \). However, not all of them are independent. We have seen that the axial current is classically conserved in our model and, therefore, the WI \( \partial^\mu \rho_{\mu\nu}(x, y) = \partial^\nu \rho_{\mu\nu}(x, y) = 0 \) must hold (there are no axial flavour anomalies for \( N_f = 2 \)). It can be checked, following standard path integral methods [4], that the same equilibrium WI holds out of equilibrium. Using the symmetry properties discussed before, the WI yields

\[ q^0 \rho_0 - \omega^2 \rho_S - \frac{i}{2} \rho_0 = 0 \]
\[ q^0 \rho_S - \omega^2 \rho_L + \frac{i}{2} \rho_S + \rho_d = 0 \]  

(30)

where the dot denotes \( \partial / \partial \tau \). Thus, only two components of \( \rho_{\mu\nu} \) are independent, as in equilibrium [23]. Notice that the time derivatives are not present in equilibrium.

At \( T = 0 \) one has \( \rho_L = 2\pi f^2_\pi \text{sgn}(q^0) \delta(q^2) \), which defines \( f_\pi \) and states that in the chiral limit there are NGB with the same quantum numbers as the axial current, according to Goldstone theorem. That is not the case in equilibrium at \( T \neq 0 \) [20], where to define properly \( f_\pi(T) \) requires taking the \( \omega_\pi \to 0^+ \) limit, in which Goldstone Theorem requires that a zero energy excitation still exists [3]. Extending the above ideas to non-equilibrium, we will define the spatial PDF

\[ \left[ f^2_\pi(t) \right] = \frac{1}{2\pi} \lim_{\omega_\pi \to 0^+} \int_{-\infty}^{\infty} dq_0 \rho_L(q_0, \omega, t) \]
\[ = \lim_{\omega_\pi \to 0^+} i \frac{d}{dt} \rho_L(\omega, t, t') \bigg|_{t=t'} \]  

(31)

where in the second line we have written \( \rho_L \) in the mixed representation. In equilibrium, the above gives \( f_\pi \) independent of \( \omega \) and then there is no need of taking the \( \omega_\pi \to 0^+ \) limit [3], meaning that the pion distribution function still behaves as a \( \delta \)-function to this order of approximation. It is not obvious in the least that the above has to hold out of equilibrium to one-loop, as it indeed is (see below).

Let us check the consistency of the definition [33]. To leading order, from (31), we have

\[ \rho_L^{\text{LO}}(\omega_q, t, t') = i f(t)f(t') \rho_0(\omega_q, t, t') \]  

(32)

with \( \rho_0 \) the spectral density [13] to leading order. Then, using (20), we readily find \( f^2_\pi(t)_\text{LO} = f^2(t) \), i.e, the PDF coincides with \( f(t) \) to leading order, as it should be. Following the same ideas, we will introduce

\[ f_\pi(t) f_\pi(t) = \frac{1}{2\pi} \lim_{\omega_\pi \to 0^+} \int_{-\infty}^{\infty} dq_0 \rho_S(q_0, \omega, t) \]  

(33)

\[ f_\pi(t) g_\pi(t) = -i \frac{1}{2\pi} \lim_{\omega_\pi \to 0^+} \int_{-\infty}^{\infty} dq_0 \rho_S(q_0, \omega, t) \]  

(34)

The function \( f_\pi^2(t) \) is the nonequilibrium extension of the spatial pion decay constant, whereas \( g_\pi(t) \) vanishes in equilibrium. However, according to our previous discussion on the WI, the three PDF we have introduced are not independent. In fact, from their above definitions and integrating in \( q_0 \) in (33), we find

\[ f_\pi^2(t) g_\pi(t) = \frac{1}{2} \frac{d}{dt} \left[ f_\pi^2(t) f_\pi^2(t) \right] \]  

(35)

and then there are only two independent PDF, also as in equilibrium [3].

As we did above with \( f_\pi^2(t) \), we can check now the above expressions to leading order. From [33] we find \( f_\pi^2(t)_\text{LO} = f_\pi(t)_\text{LO} \) and \( g_\pi(t)_\text{LO} = f(t)_\text{LO} \), so that our definitions are consistent to LO.

To NLO, we need the axial current to \( \mathcal{O}(\pi^3) \). From [33] we have

\[ A_\mu^a(x, t) = -f(t) \partial_\mu \pi^a + 2\pi f(t) \rho_{00} \pi^a - \frac{1}{2f(t)} \left( \pi^a \partial_\mu \pi^2 - \pi^2 \partial_\mu \pi^a - \delta_{\mu0} \pi_0 \right)^2 + \mathcal{O}(\pi^5) \]  

(36)

Therefore, according to the chiral power counting explained in previous sections, we have two types of contributions to the axial-axial correlator NLO corrections. The first one comes from the NLO correction to the pion propagator, which we have evaluated in section [11] in the product of the \( \mathcal{O}(\pi) \) terms above, whereas the second is the product of the \( \mathcal{O}(\pi) \) with the \( \mathcal{O}(\pi^3) \). These two contributions are represented in Fig.1 by the diagrams a) and b) respectively. As we did in the evaluation of the NLO propagator, we are not considering the \( \mathcal{L}^{(4)} \) contributions (see below). After evaluating the loops, we obtain \( \rho_{\mu\nu} \) to NLO. We give here the result for the spatial-spatial component:

\[ \rho_L^{NLO}(\omega_q, t, t') = \rho_L^L(\omega_q, t, t') + i f(t) f(t') \rho_{00}(\omega_q, t, t') \]
\[ + \frac{1}{2} \rho_0(\omega_q, t, t') \left[ f(t) f(t') G_0(t) - \frac{f(t)}{f(t')} G_0(t) \right] \]  

(37)

with \( \rho_L^L \) in [33] and \( \rho_{00} = G_{NLO}^< - G_{NLO}^> - \rho_0 \). The \( \rho_{00} \) contribution in the above equation comes from the diagram a) in Fig.2 and the rest from diagram b). From our definitions (31) and (32) we obtain

\[ f_\pi^2(t)_\text{NLO} = f_\pi^2(t)_\text{NLO} = f_\pi^2(t) \]
\[ - i \left[ G_0(t) - f_\pi^2(t) H(t) \right] \]  

(38)

\[ f_\pi(t) g_\pi(t)_\text{NLO} = f(t) f(t) \]
\[ - i \left[ G_0(t) + G_0(t) \frac{f(t)}{f(t)} + f(t) f(t) H(t) \right] \]  

(39)

with
where we have made use of (23), taking, without loss of generality, both \( t, t' \in C_1 \) and positive (we know the equilibrium answer for the negative real axis). We realise that \( \Delta_1(t, \omega) \) in (23) does not contribute to \( \mathcal{H}(t) \) and that the whole NLO result for the PDF is independent of \( \omega \), so that there is no need of taking the \( \omega \to 0^+ \) limit, which together with \( f_z^2(t) = f_z^1(t) \) in (23) and according to our previous discussion, we may interpret as the massless parameter (this to order) of undamped NGB in the nonequilibrium plasma, propagating at the speed of light (see [3]).

In addition, when we substitute (41) in (38) and (39), we see that \( g_\pi(t) \) to NLO satisfies the WI relation (35), which is a consistency check. We finally obtain

\[
[f_z^2(t)]^2_{NLO} = [f_z^1(t)]^2_{NLO} = f^2(t) - 2i G_0(t) \tag{41}
\]

This is the one-loop relationship between the PDF and \( f(t) \), extending the equilibrium result (1), which we recover (for the general contour \( C \)) simply by replacing (25) in (11). We want to stress that the PDF are observable and therefore (11) should be independent of the parametrisation chosen for the pion fields. We have checked it explicitly by calculating with the \( \bar{\pi} \) fields defined before. Both the propagator and the axial-axial spectral functions change, but (11) remains the same with the same definitions (31) and (33) and the same \( G_0(t) \). In turn, let us mention that by extending naively the low \( T \) results in the LSM (see [5]) to nonequilibrium we would have \( v^2(t) = f_z^2(t) - i G_0(t) \) with \( v(t) = \langle \sigma \rangle(t) \). We will make use of this assumption below to compare our results with [13].

Notice also that it is licit to replace \( f(t) \) by \( f_z(t) \) for \( G_0(t) \) in (11), to the order we are considering here. Therefore, using (41) allows to express all physical nonequilibrium observables (like decay rates, masses, etc) obtainable from our model to one loop in ChPT, in terms of the physical \( f_z(t) \), which could be determined, for instance, by analysing nonequilibrium lepton decays \( \pi \to lvq \). A very important aspect in this program is renormalisation. As we said before, \( G_0(t) \) contains in general time-dependent infinities. However, as we are going to see, in certain regimes (for instance, at short times) it is still possible to find a finite answer for \( G_0(t) \) in dimensional regularisation (as in equilibrium in the chiral limit) so that the counterterms coming from \( \mathcal{L}^{(4)} \) are not needed. There still could be finite contributions coming from \( \mathcal{L}^{(4)} \) but those are typically of \( \mathcal{O}(10^{-3}) \) with respect to those coming from the loops [3,24], so that our approximation of neglecting \( \mathcal{L}^{(4)} \) would be justified in that case, which is the one we will consider in the remaining of this work. Explicit knowledge of \( f(t) \) would require to solve the hydrodynamic equations for the plasma, or, equivalently, the Einstein equations for the metric. Alternatively, one can follow the approach of treating \( f(t) \) as external, choosing for instance different parametrisations compatible with (1). This is left for future investigation.

For short times, the particular form of \( f(t) \) is not important and we can parametrise the nonequilibrium dynamics in terms of a few parameters, which are the values of \( f(t) \) and its derivatives at \( t = 0 \). This is equivalent to describe the expansion of the Universe in terms of the Hubble parameter, the deceleration parameter and so on. As we discussed in section 4, this approach is justified for times \( t < t_{\text{max}} \) with \( t_{\text{max}} \simeq 1/f_s(0) \simeq 2 \text{ fm/c} \), which are indeed not so short, regarding the typical time scales involved in a RHIC (see our comments below and in section 1). Let us then start by considering the differential equation (17) with KMS conditions at \( t_s \). In addition, as we have discussed in section 4, we shall impose that \( G_0(\omega_q, t, t') \) be continuous and differentiable in \( t = 0 \) and \( t' = 0 \). The solution of (17) can be written in terms of two arbitrary solutions \( g_{\pi}(t) \) of the homogeneous equation, satisfying the Wronskian condition \( g_1 g_2 - g_2 g_1 = 1 \). In [23], the general solution of (17) satisfying KMS conditions was constructed in terms of the \( g_i(t) \) functions, for \( t \) and \( t' \) in the contour \( C \) (as we have said before, it is enough for us to take \( t, t' \in C_1 \)). We remark that the solution of the homogeneous equation is only known explicitly for a some special choices of \( m^2(t) \) [24,25].

Therefore, we expand both \( f(t) \) and the solution of the homogeneous equation near \( t = 0 \). Both the differential equation and the Wronskian condition reduce then to algebraic relations between the first coefficients in the expansion. With the boundary, continuity and differentiability conditions discussed above, the solution is uniquely defined and we find to lowest order

\[
G_0(\omega_q, t, t') = -\frac{i}{2\omega_q} \coth \left( \frac{\beta \omega_q}{2} \right) \left[ 1 - m^2 t^2 + \mathcal{O}(m^4 t^4) \right]
\]

with \( m^2 \equiv m^2(0) = -\langle f(0) \rangle \). For \( m^2 < 0 \) we see the onset of unstable modes, making the pion correlation function grow with time. We also observe that for short times, the time dependence of the mixed propagator factorises. Therefore, the momentum dependence is the same as in equilibrium and then we can integrate it in dimensional regularisation, using (25), to get a finite answer. As we said before, in this regime we are able to avoid renormalisation. Then, from (41) and (2) we get

\[
f_z^2(t) = f^2 \left[ a(T_i) + 2H t - c(T_i) t^2 + \mathcal{O}(m^3 t^3) \right]
\]

where \( f = f(0) \), \( H = f(0)/f(0) \) (\( H \) and \( m \) are \( \mathcal{O}(p) \) and play the role of the Hubble and deceleration parameters in the expansion, respectively) and

\[
a(T_i) = 1 - \frac{T_i^2}{T_c^2}
\]

\[\text{Notice that } G_0(t) \text{ in (41) depends implicitly on } f(t), \text{ through (1).}\]
where $T_c \equiv \sqrt{6} f_\pi \simeq 228$ MeV. Therefore, (44) provides the nonequilibrium relationship between the LO Taylor coefficients, independent of $T_e$, appearing in the lagrangian and the physical $T_c$-dependent NLO ones, extending the equilibrium formula (1) when $f$ is the only lagrangian parameter. The coefficients of order zero and two in the Taylor expansion of $f_\pi(t)$ are corrected by the same one-loop $T_c$-dependent factor, whereas the first order coefficient $H$ does not acquire NLO corrections. We would like to remark that the corrections coming from $\mathcal{L}^{(4)}$ that we have neglected are, in addition, independent of $T_c$, since they only contribute at tree level to NLO.

We will proceed now to discuss some physical effects related to the behaviour of $f_\pi(t)$. For that purpose, and based upon (1), let us define a time-dependent effective temperature $T(t)$ as

$$f_\pi(T) = f^2 \left[ 1 - \frac{T^2(t)}{T_c^2} \right],$$

i.e, we parametrise the nonequilibrium effects of $f_\pi(t)$ in $T(t)$. Let us define in addition the critical time $t_c$, as $f_\pi(t_c) = 0, (T(t_c) = T_c)$ and the freezing time $T(t_f) = 0, f_\pi(t_f) = f$. These are the relevant time scales when the system is heating or cooling down, respectively. The sign of $H$ determines whether the plasma is initially heated ($H < 0$) or cooled down ($H > 0$). However, it is the sign of $c(T_i)$ in (14) what determines the behaviour at longer times. Thus, we will follow the short-time evolution of $f_\pi(t)$ until it reaches either $t_c$ or $t_f$, imposing that $0 < T(t) < T_c$. We are following a similar approach as that of equilibrium when one extrapolates (1) until $T_c$.

In order to estimate the above time scales, let us take $H^2 \simeq |m^2|$ and retain only the leading order in $x = T^2(t)/T^2_c$, consistently with the chiral expansion. Then, we have the following cases:

1. $H > 0$: Cooling down until $\tilde{t}_f \simeq x/2$.

2. $H < 0, m^2 > 0$: Heating until $\tilde{t}_c \simeq (1 - 3x/4)/2$.

3. $H < 0, m^2 < 0$: Heating until $\tilde{t}_m \simeq (1 + x/2)/2$ and then cooling down until $\tilde{t}_f \simeq 1 + x$, passing through the equilibrium time $\tilde{t}_{eq} = 2\tilde{t}_m$ such that $f_\pi(t_{eq}) = f_\pi(0)$.

4. $H = 0, m^2 > 0$: Heating until $\tilde{t}_m^2 m^2 = 1$, independent of $T_c$.

5. $H = 0, m^2 < 0$: Cooling until $\tilde{t}_m^2 m^2 \simeq -x(1 + x)$

where $\tilde{t} = t|H|$. We observe that the effect of the unstable modes ($m^2 < 0$) is always to cool down the system. The freezing time $t_f$ for $H < 0$ is much longer (according to our power counting) than that for $H > 0$, which is $\mathcal{O}(x/H)$. Indeed, the $\mathcal{O}(1/H)$ time scales are much longer than those to which our short-time approximation remains valid and they have to be understood as estimates, similarly to the equilibrium case, where one estimates the critical temperature, even though the low $T$ approach breaks down for $T \simeq T_c$. Taking typical values $T_i \simeq |m| \simeq 100$ MeV ($x \simeq 0.16$) we get $t_f \simeq 0.2$ fm/c for cases 1 and 5, $t_c \simeq 2$ fm/c for cases 2 and 4 and $t_f \simeq 2.3$ fm/c for case 3.

On the other hand, evaluating the above time scales for initial temperatures closer to $T_c$, by expanding now for small $a(T_i)$ in (14), we get $t_f \simeq \sqrt{2} - 1$ for case 1, $\tilde{t}_c \simeq a(T_i)/2$ for case 2, $\tilde{t}_m \simeq 1$, $\tilde{t}_f \simeq 1 + \sqrt{2}$ for case 3 and $t_f^2 m^2 = -x/a(T_i)$ for case 5. We can compare with $\nu(t)$ in (19) (see our comment above), using their initial values $T_i \simeq 200$ MeV and $|H| \simeq 400$ MeV (which are clearly too high for our low-energy approach). We see that our estimates for the time evolution duration are of the same order as those in (19), although ours are somewhat lower. An important remark is that in typical LSM simulations like (14), $\nu(t)$ reaches a stationary value, about which it oscillates, the relevant time scale being the time it takes the system to reach such value. It is clear that we cannot predict that type of behaviour only within our short-time approach, quadratic in time, but only estimate the time scales involved $\tilde{t}_i$ i.e, we cannot see the system thermalise in that sense.

V. CONCLUSIONS AND OUTLOOK

We have extended chiral lagrangians and ChPT out of thermal equilibrium. The chiral power counting requires all time derivatives to be $\mathcal{O}(p)$ and to lowest order our model is a NLSM with $f \to f(t)$. This model accommodates unstable pion modes and corresponds to a spatially flat RW metric in conformal time with $a(t) = f(t)/f(0)$ and minimal coupling. For a non-minimal coupling, the chiral symmetry would be broken and conformal invariance restored for $\xi = 1/6$.

We have applied our model to study the time dependent PDF, which extend the equilibrium pion decay constants. We find that in general there are two independent PDF, as in equilibrium. To one loop in ChPT such functions coincide and we have given their expression in terms of the equal-time correlation function, analysing the lowest order short-time coefficients of $f_\pi(t)$. We have studied the relevant time scales and their dependence with $T_i$, paying special attention to the role of the unstable modes in cooling down the plasma. In this work we have treated nonequilibrium renormalisation by remaining within the short-time expansion, where the equal-time correlator turns out to be free from UV divergences in the chiral limit in dimensional regularisation and hence it is reasonable to neglect the contributions coming from

\[ ^3 \text{Similarly as to why ChPT cannot see the phase transition (see comments in section 6).} \]
\( \mathcal{L}^{(4)} \) in our analysis. Nonetheless, we have seen that, thanks to the analogy with curved space-time, we have a well-defined procedure to construct \( \mathcal{L}^{(4)} \) so as to absorb all the loop divergences, which in general will be time-dependent, and find a finite answer for the observables.

Among the aspects we would like to study further is to include the nonequilibrium \( \mathcal{L}^{(4)} \) tree contributions, so that we can renormalise \( f_\pi(t) \) and analyse the long time behaviour properly. An interesting remark is that for long times with unstable modes of negative mass squared \( m^2 \), only the \( k^2 < |m^2| \) modes are important, so that there is an effective ultraviolet cutoff and we can forget about UV divergences in that case as well. Another point we leave for future investigation is the behaviour of the two-point correlation function at different space points for different choices of \( f(t) \), which would allow us to investigate the formation of regions of unstable vacua (DCC) in our model. Other applications and extensions of our approach, to be explored in the future, include photon production in the pion sector (by gauging the theory and including \( \pi^0 \) decay photon production through the anomalous Wess-Zumino-Witten term), the quark condensate time dependence (by considering the mass explicit symmetry-breaking terms we have neglected here) and the \( N_f = 3 \) case. Finally, we hope that by implementing resummation methods like large \( N \) in our model, we may be able to study the long-time thermalisation process.

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