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Temporal Correlation of Interference in Vehicular Networks with Shifted-Exponential Time Headways

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Abstract—We consider a one-dimensional vehicular network where the time headway (time difference between successive vehicles as they pass a point on the roadway) follows the shifted-exponential distribution. We show that neglecting the impact of shift in the deployment model, which degenerates the distribution of vehicles to a Poisson Point Process, overestimates the temporal correlation of interference at the origin. The estimation error becomes large at high traffic conditions and small time-lags.

Index Terms—Headway models, interference correlation, stochastic geometry, vehicular networks.

I. INTRODUCTION

The temporal correlation of interference in wireless networks is related to the temporal correlation of outage [1], [2], the diversity gain [3], the amount of time a node remains isolated [4], etc., thus, it is an important quantity to study. Interference becomes correlated when it originates from the same set of transmitters [5], and the link gains of interfering channels, for some of the transmitters, are correlated over sequential periods of time [6]. Mobility randomizes the link gains and naturally decreases interference correlation [1], [2].

The performance of vehicular networks has so far been studied using simplified spatial models, which cannot be considered realistic under all circumstances. Due to its analytical tractability, the Poisson Point Process (PPP) has been used to model the locations of vehicles along a roadway [1], [7]. However, it is known from transportation research that the headway distance (distance between the head of a vehicle and the head of its successor [8], or inter-vehicle distance) depends on the traffic status and it is not always exponential [9], [10].

The motivation for this letter is to study the temporal correlation of interference with mobility, considering a more realistic deployment model than the PPP. The simplest enhancement shifts the exponential distribution for the inter-vehicle distances to the right. The shift takes into account, to some extent, the interactions between successive vehicles by avoiding unrealistically small headways. We will show that the PPP overestimates the temporal correlation of interference in a high traffic scenario. This may affect in return other performance metrics, e.g., the conditional probability of success (conditioning on successful reception at the current time slot, the probability to receive successfully also in the next) which would be different than the one predicted by PPP. This may further impact the design of retransmission schemes.

II. SYSTEM MODEL

Let us assume that the headway distance consists of a constant tracking distance $c > 0$ (with probability one) and a free component following the exponential distribution with mean $\mu^{-1}$. The tracking distance models the fact that two vehicles do not come arbitrarily close to each other. In a single lane road with no overtaking, they are separated at least by the length of a vehicle plus a safety distance. Let us also assume that all vehicles travel with the same constant speed $u$ in the same direction. This might be the case in the flow of convoy or when the vehicles move with the speed limit. This deployment model degenerates to the time headway model M2 proposed by Cowan [9], i.e., the time headways follow a shifted-exponential distribution. Time-independent statistics of the interference for this model were considered in [11].

We study interference correlation at time instances $\tau$ and $(\tau + t)$, separated by the time-lag $t$. The base station is located at the origin, and the vehicles located in $[-r_0, r_0]$ are associated to it. The rest generate interference, see Fig. 1. The interferers may communicate with each other in ad hoc mode or paired with other base stations. We would like to get a preliminary insight into the impact of correlated user locations on the temporal aspects of interference at the origin; incorporating further modeling details, e.g., power control and cell association [12], has been left as a future topic to study.

The Pair Correlation Function (PCF) for a point process where the inter-point distances follow the shifted-exponential distribution has long been studied in the context of statistical mechanics for hardcore fluids/gases [13] under the name radial distribution function. The point process is stationary. For two vehicles $x, y$ with $y > x : y \in (x + kc, x + (k + 1)c)$, $k \in \mathbb{N}$ the PCF is depicted in Fig. 2 and has the following form [13]

$$\rho_k^{(2)}(y,x) = \frac{\lambda}{2\pi u c} \sum_{j=1}^{k} \frac{\mu^j (y-x-jc)^{j-1}}{\Gamma(j) e^{\mu(y-x-jc)}}, k \geq 1,$$  \hspace{1cm} (1)

where $\Gamma(j) = (j-1)!$ is the Gamma function for an integer argument, and $\lambda = \frac{\mu}{2\pi u c}$ is the intensity of vehicles [9].
The Pearson correlation coefficient at time-lag $t$ is

$$\rho^{(2)}(x, y) = \frac{\text{cov}(I^{(t)}(x), \mathbb{E}[I^{(t)}(y)]) - \mathbb{E}[I^{(t)}(x)] - \mathbb{E}[I^{(t)}(y)]}{\sqrt{\text{Var}(I^{(t)}(x)) \text{Var}(I^{(t)}(y))}}.$$ \hspace{1cm} (2)

We calculate the correlation coefficient for time-lags $t \leq 2r_0/\eta$, where the correlation is expected to be high. For this range of values, we do not get contributions to the covariance from vehicles performing handover twice. We start with the term

$$J(t) = \lambda \int g(r)(r+tu) dr,$$

where

$$J(t) = \frac{2\lambda r_0}{2\eta - 1} F_1(2\eta - 1; 2\eta, 2\eta, \frac{tu}{r_0}),$$ \hspace{1cm} (3)

where $F_1$ is the Gaussian hypergeometric function [14, pp. 556].

We continue with the contributions to the covariance due to pairs. Let us assume that at time $\tau$, one vehicle is located at the infinitesimal interval $d\tau$, centered at $x$, and the other at the infinitesimal interval $dy$, centered at $y$. Since the speed $u$ is constant, the displacement within $t$ is deterministic. After writing $I(t)$ as an integral and using the PCF in (1), we get

$$I(t) = \int g(x) g(y + tu) \rho^{(2)}(x, y) dy dx,$$

$$= \lambda \int_0^\infty \left( \int_0^\infty \left( \int_0^\infty (y-x-ka)^k \frac{e^{-y\mu}}{\mu^{y-x-ka}} dy \right) dx \right) \left( \int_0^\infty \frac{e^{-x\mu}}{\mu^{x-y-ka}} dx \right)$$

$$= \lambda \int_0^\infty \left( \int_0^\infty \left( \int_0^\infty (y-x-ka)^k \frac{e^{-y\mu}}{\mu^{y-x-ka}} dy \right) dx \right) \left( \int_0^\infty \frac{e^{-x\mu}}{\mu^{x-y-ka}} dx \right)$$

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$$= \lambda \int_0^\infty \left( \int_0^\infty \left( \int_0^\infty (y-x-ka)^k \frac{e^{-y\mu}}{\mu^{y-x-ka}} dy \right) dx \right) \left( \int_0^\infty \frac{e^{-x\mu}}{\mu^{x-y-ka}} dx \right)$$

Equation (4) does not provide much insight about the impact of traffic parameters, $\lambda, c, \mu$ on the covariance. To get that, we will assume a small tracking distance $c$ as compared to the mean inter-vehicle distance $\lambda^{-1}$. 

![Fig. 2. Normalized PCF $\rho^{(2)}(x, y)$ with respect to the normalized distance $|y-x|^a$].

![Fig. 3. I_1^t + I_2^t = \lambda \int g(r)(r+tu) dr + \lambda \int g(r)(r+tu) dr \equiv 2\lambda^2 r_0^2 F_1(2\eta - 1; 2\eta, 2\eta, \frac{tu}{r_0})].
IV. APPROXIMATION FOR THE COVARIANCE

For $\lambda c \ll 1$, we may approximate the PCF with the PCF of PPP for distances separation larger than $2c$, without introducing much error, $g^{(2)}(y, x) \approx \lambda^2 |y - x| > 2c$. For small $\lambda c$, it is the random part dominating the deployment. Having a vehicle at $x$ imposes little constraint on the probability of finding a vehicle at $y$, given that $x$ and $y$ are far apart. This justifies why for small $\lambda c$, the PCF converges at few multiples of $c$ to the PCF of PPP, see Fig. 2. According to (4), the pairs separated by more than $2c$ give a contribution to the term $I(t)$ which can be written as a sum of four terms $I_{2c}(t) = \sum_{j=1}^4 I_j$.

$$I_1 = \lambda^2 \int_{r_0}^{\infty} \int_{x+2c}^{x-2c} g(x) g(y+tu) \, dy \, dx,$$

For the term $I_2$, we have to separate between the vehicles at the left- and the right-hand side of the cell at $(\tau + t)$.

$$I_2 = \lambda^2 \int_{r_0}^{\infty} \int_{x-2c}^{x+2c} g(x) g(y+tu) \, dy \, dx = \lambda^2 \int_{r_0}^{\infty} \int_{x-2c}^{x+2c} g(x) g(y+tu)dy + \int_{x+2c}^{x-2c} g(x) g(y+tu)dy \, dx \quad \text{(a)},$$

$$= \lambda^2 \int_{r_0}^{\infty} \int_{x-2c}^{x+2c} g(x) g(y)dy + \int_{x+2c}^{x-2c} g(x) g(y)dy \, dx = \lambda^2 \int_{r_0}^{\infty} \int_{x-2c}^{x+2c} g(x) g(y)dy \, dx = \frac{1}{2} \mathbb{E}[\mathcal{I}]^2 - \lambda^2 \int_{x-2c}^{x+2c} g(x) g(y)dy \, dx,$$

where (a) follows from $z = y + tu$ in both integrals and the assumption that $t > \frac{2\nu}{\lambda c} \approx t_1$ in the second.

After changing the variable $z = y + tu$ also in $I_1$, and using the above form for $I_2$, the sum $(I_1 + I_2)$ can be read as

$$I_1 + I_2 = \frac{1}{2} \mathbb{E}[\mathcal{I}]^2 - \lambda^2 \int_{x-2c}^{x+2c} g(x) g(y)dy \, dx, \quad t \geq t_1.$$
The temporal correlation coefficient of interference for a PPP is independent of the intensity of vehicles and tracking distance on the correlation coefficient. The performance assessment of a receiver at the origin, e.g., temporal outage probability, local delay, etc., and the mechanisms to cope with the high correlation predicted by the PPP model may need to be revisited. Future topics of study may include more realistic headway models and may also incorporate more accurate models for the uplink.

VI. CONCLUSIONS

The PPP allows unrealistically small headways. Avoiding this by shifting the exponential distribution for the inter-vehicle distances to the right, reduces the temporal correlation of interference. For fixed tracking distance c, the interference might not be that correlated, particularly in high traffic conditions, large λ. The performance assessment of a receiver at the origin, e.g., temporal outage probability, local delay, etc., and the mechanisms to cope with the high correlation predicted by the PPP model may need to be revisited. Future topics of study may include more realistic headway models and may also incorporate more accurate models for the uplink.

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