First order non-equilibrium phase transition and bistability of an electron gas

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We study the carrier concentration bistabilities that occur to a highly photo-excited electron gas. The kinetics of this non-equilibrium electron gas is given by a set of nonlinear rate equations. For low temperatures and cw photo-excitation we show that they have three steady state solutions when the photo-excitation energy is in a certain interval which depends on the electron-electron interaction. Two of them are stable and the other is unstable. We also find the hysteresis region in terms of which these bistabilities are expressed. A diffusion model is constructed which allows the coexistence of two homogeneous spatially separated phases in the non-equilibrium electron gas. The order parameter is the difference of the electron population in the bottom of the conduction band of these two steady stable states. By defining a generalized free potential we obtain the Maxwell construction that determines the order parameter. This order parameter goes to zero when we approach to the critical curve. Hence, this phase transition is a non-equilibrium first order phase transition.

Keywords: non-equilibrium phase transition; bistability; non-linear rate equations; semiconductor electron gas.

Estudiamos las biestabilidades en la concentración de portadores que le ocurren a un gas de electrones altamente fotoexcitado. La cinética de este gas de electrones fuera de equilibrio está dada por un conjunto de ecuaciones de razón no-lineales. Para bajas temperaturas y fotoexcitación en modo continuo (cw) mostramos que estas ecuaciones tienen tres soluciones de estado estacionario cuando la energía de fotoexcitación está en un cierto intervalo, el cual depende de la interacción electrón-electrón. Dos de ellas son estables y la otra es inestable. También, encontramos la región de histeresis en términos de la cual estas biestabilidades son expresadas. Construimos un modelo de difusión que permite la coexistencia de dos fases homogéneas espacialmente separadas del gas de electrones fuera de equilibrio. El parámetro de orden es la diferencia de la población electrónica en el fondo de la banda de conducción de estos dos estados estacionarios estables. Definimos un potencial libre generalizado del sistema obtenemos la construcción de Maxwell que determina entonces al parámetro de orden. El parámetro de orden va a cero cuando nos aproximamos a la curva crítica. Por eso, esta transición de fase es una transición de fase fuera de equilibrio de primer orden.

Descriptores: transición de fase fuera de equilibrio; biestabilidad; ecuaciones de razón no-lineales; gas de electrones en semiconductores.

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I. INTRODUCTION

Bistability, threshold switching transitions, spatial pattern formation, self-sustained oscillations and chaos in semiconductors are related to non-equilibrium phase transitions[1, 2]. These non-equilibrium phase transitions in semiconductors have been studied in the past decades mainly in connection with the nonlinear generation-recombination mechanism, including impact ionization[2, 3, 4]. Optical and transport properties of semiconductors strongly depend on the electron population in the bottom of the conduction band and many experimental and theoretical studies on hot photo excited electron systems have shown that this electron population depends on the excitation energy[2, 3, 5]. In this paper we present a study of the carrier concentration bistability that occur to an electron gas generated by a laser excitation and a study of a first order non-equilibrium phase transition between two homogeneous stable steady states of the excited electron gas. They both appear when the energy of the pump is varied. The order parameter is the difference between the electron populations in the bottom of the conduction band of the stable states. It depends upon the effectiveness of the electron-electron interaction. We obtain the order parameter by defining a potential which allows us to make a construction similar to the Maxwell construction for the equilibrium phase transition of a van der Waals gas[4, 5].

In following section we present the theoretical model we use to study a non-equilibrium electron gas in semiconductors. In section third we deduce the stationary state solutions to the non-linear rate equations of the model and we make the stability analysis. By considering a standard first order phase transition of the classical van der Waals gas, we analyze the non-equilibrium phase transition of the electron gas and obtain the corresponding Maxwell construction. We draw our conclusion in the final section.
II. THEORETICAL MODEL OF THE NON-EQUILIBRIUM ELECTRON GAS

We describe the non-equilibrium electron gas in semiconductors using the model equations given in Ref. 10. Here, we give a brief description of these model equations and refer the reader to the details of their derivation to Refs. 9, 10. Electrons in the bottom of the conduction band of a semiconductor play an important role in the dynamics of the whole conduction electron gas. In general, electrons with an energy in excess less than the longitudinal optical (LO) phonon energy cannot make transitions by emitting LO phonons. In the case in which the emission of LO phonons is one of the dominant mechanisms, the nonequilibrium kinetics of the electron gas in the conduction band of a semiconductor is given as follows 10.

We define a set of energy levels, each one of them representing an energy interval of width \( \Delta \varepsilon \) of the conduction band. Although not strictly necessary, for simplicity \( \Delta \varepsilon \) is set equal to \( h\omega_{LO} \), the LO phonon energy 11. We set the electron population \( \chi_i \) in these energy levels and, based on the main interaction mechanisms, the nonlinear rate equations that give the temporal behavior of these populations are obtained.

When an electron in level \( i \) emits a LO phonon it loses an amount of energy \( h\omega_{LO} \) and passes to the level \( i-1 \). If the frequency of this event is \( \nu^+ \) then we have that the rate of change of population \( \chi_i \) is \( \nu^+_0 (\chi_{i+1} - \chi_i) \). Similarly for the absorption of a LO phonon we have the rate \( \nu^- (\chi_{i-1} - \chi_i) \) where \( \nu^- \) is the frequency of this process. The electron-electron interaction gives to the rate of change of the population \( \chi_i \) at level \( i \) terms, \( ZN_{\max} \chi_{\text{tot}}(\chi_{i+1} - 2\chi_i + \chi_{i-1}) \) and \( ZN_{\max} \chi_0(\chi_i - \chi_{i-1}) \). Their form come from considering the contribution to the rate of change of the population \( \chi_i \) of the interaction between electron populations of all the energy levels, and from the use of energy conservation 11. When an electron suffers a recombination it returns to the valence band and this gives us the term \(-w\chi_i \) with \( w \) the frequency of the process.

Then, by collecting all these contributions, we have the following set of rate equations which describes the kinetics of a photo-excited electron gas in semiconductors 11,

\[
\frac{d\chi_i}{dt} = \nu^+_0 (\chi_{i+1} - \chi_i) + \nu^- (\chi_{i-1} - \chi_i) \\
+ ZN_{\max} \chi_{\text{tot}}(\chi_{i+1} - 2\chi_i + \chi_{i-1}) \\
+ ZN_{\max} \chi_0(\chi_i - \chi_{i-1}) + g_p \delta_{i,i_p} - w\chi_i , \quad (1)
\]

for \( i \neq 0 \). For \( i = 0 \), since the emission of LO phonons by an electron is not possible, we have

\[
\frac{d\chi_0}{dt} = \nu^+_0 \chi_1 - \nu^- \chi_0 + ZN_{\max} \chi_{\text{tot}}(\chi_1 - \chi_0) \\
+ ZN_{\max} \chi_0(\chi_0) + g_p \delta_{0,i_p} - w\chi_0 . \quad (2)
\]

The electron populations \( \chi_i \) have been normalized to the maximum reachable electron concentration \( N_{\max} \) and \( \chi_{\text{tot}} = \sum \chi_i \). The terms with the Kronecker delta is the generation contribution with generation rate \( g_p \). The main interaction mechanisms, the generation and recombination terms depend on the lattice temperature, carrier concentration and material parameters.

Due to the electron-electron interaction, this set of rate equations is non linear. We should say that Eqs. (1) and (2) which are the main equations of our model came from a more general theory published in Ref. 10. See, also Refs. 9, 10, 12 for other details of the model equations and other applications.

We make the following definitions \( \nu \equiv \nu^+_0 / ZN_{\max} \), \( \nu^- \equiv \nu^- / ZN_{\max} \), \( \omega \equiv w/ZN_{\max} \), \( \chi_p \equiv g_p / ZN_{\max} \), and \( \varepsilon \equiv \varepsilon / \Delta \varepsilon \). By this normalization, we have eliminated one parameter. For low temperatures 10, \( \mu \ll \nu \). Also, in steady state, \( \chi_p = \omega \), \( \chi_{\text{tot}} = 1 \) and the total carrier concentration \( N = N_{\max} \). Hence, in this case, we have only two relevant control parameters, \( \nu \) and \( \varepsilon_p \). Here \( \varepsilon_p = \varepsilon / \Delta \varepsilon \), the excitation energy in units of \( \Delta \varepsilon \).

With these definitions and assumptions and considering the finite differences as derivatives:

\[
\chi_{i+1} - \chi_i \rightarrow \frac{d\chi}{d\varepsilon}
\]

\[
\chi_{i+1} - 2\chi_i + \chi_{i-1} \rightarrow \frac{d^2\chi}{d\varepsilon^2} (\Delta\varepsilon)^2
\]

\[
\chi_i - \chi_{i-1} \rightarrow \frac{d\chi}{d\varepsilon} \Delta\varepsilon - \frac{d^2\chi}{d\varepsilon^2} (\Delta\varepsilon)^2
\]

we have that the equations (1) and (2) that give the temporal behavior of these populations are obtained.

The electron distribution function \( \chi(\varepsilon, \tau) \) is continuous and positive definite in the whole interval \( 0 \leq \varepsilon < \infty \) and has a discontinuity in its first derivative at \( \varepsilon = \varepsilon_p \). In addition, it must satisfy the conditions

\[
\chi_{\text{tot}} = \int_0^\infty d\varepsilon \chi(\varepsilon, \tau) \quad (4)
\]

\[
\chi_0 = \int_0^1 d\varepsilon \chi(\varepsilon, \tau) \quad (5)
\]

The Eq. (8) has the steady state solution 11

\[
\chi^s(\varepsilon) = \left\{ \begin{array}{ll}
Ae^{\alpha\varepsilon} + Be^{-\beta\varepsilon} ; & 0 \leq \varepsilon \leq \varepsilon_p \\
Ce^{-\beta\varepsilon} ; & \varepsilon_p \leq \varepsilon
\end{array} \right.
\]

where

\[
\alpha = \left[ \frac{(\nu + \chi^s_0)^2 + 4\omega(1 - \chi^s_0)}{2(1 - \chi^s_0)} \right]^{1/2} \equiv (\nu + \chi^s_0)
\]

\[
\beta = \frac{1}{\alpha}.
\]

The electron population at the lowest level \( \chi^s_0 \) is determined self-consistently by the equation

\[
\chi^s_0 = \frac{\omega e^{-\alpha\varepsilon_p} e^{\alpha - \beta}}{\alpha + \beta} \frac{1}{1 - \chi^s_0} \quad (7)
\]
The coefficients $A$, $B$, and $C$ are obtained using the continuity in $\chi(\epsilon)$, the discontinuity in its first derivative at $\epsilon = \epsilon_p$, and Eq. (1). In particular, we obtain that $A/B = \alpha/\beta$ (See Ref. [7] for more details).

To conclude this section we give the explicit expression for the control parameter $\nu$ that it is needed for the following discussion and we refer the reader to Refs. [4, 10, 11] for more details and the expressions of other interaction mechanisms. Then, for $\nu$ we have:

$$\nu = \frac{23/2h\omega_{LO}}{\sqrt{\pi k_B } T_c} \left(1 - \frac{E_{\infty}}{E_s}\right) (N_s + 1) S_{LO} S_{ee}^{-1} A$$

(8)

where $T_c$ is the effective electron temperature, $k_B$ is the Boltzmann constant, $E_{\infty}$ and $E_s$ are the static and optical dielectric constants, respectively, and $h$ is the Planck constant. Phonon population effects may be taken into account in $N_q$ which is the phonon population at wavenumber of magnitude $q$. The screening in the electron-LO phonon interaction is taken into account in the factor $[12]$

$$S_{LO} = \left(1 + \left(\frac{N}{N_{LO}}\right)^2\right)^{-1}$$

where

$$N_{LO} = \frac{E_{\infty} m (\hbar \omega_{LO})^3}{33/28 \pi c^2 \hbar^2 k_B T_c}$$

is the threshold value for the concentration in the conduction band at which the screening becomes important $[13]$. The electron effective mass and charge are $m$ and $e$ respectively. The screening in the electron-electron is given by the factor $[10]$

$$S_{ee} = \left(1 + \frac{N}{N_{ee}}\right)^{-1}$$

and becomes important when the carrier concentration $N_{max}$ reaches a critical value

$$N_{ee} = \frac{4m E_{\infty} (k_B T_c)^2}{\pi^2 \hbar^2 c^2}$$

The frequencies associated with the collision mechanisms are in general dependent upon the energy. Therefore, $\nu$ is a function of $\epsilon$ but a smooth one. The analysis can be considerably simplified if we average it over the conduction band. The factor $A$ given by

$$A = \frac{1}{\epsilon_{\max} - 1} \int_{1}^{\epsilon_{\max}} d\epsilon \frac{1}{\sqrt{\epsilon}} \ln \left[1 + \sqrt{1 - \frac{1}{\epsilon}} \right]$$

$$- \ln \left[1 - \sqrt{1 - \frac{1}{\epsilon}} \right]$$

is this averaging process and $\epsilon_{\max}$ is the maximum energy in the conduction band that can be reached by an electron $[10]$.

The expression for $\nu$, Eq. (8), is one of the simplest approximations we can make which allows us to determine in an easy way the relative importance of the electron-electron interaction against the electron-LO phonon interaction in terms of the carrier concentration and electronic temperature.

The electron-electron interaction which gives the nonlinear character of the rate equation needs some clarification. We follow the ideas of Takenaka et al. [14] and Collet et al. [15] and use static RPA (Random Phase Approximation) to obtain $[9, 10]$

$$Z N_{max} = \frac{e^2 \sqrt{\pi m k_B T_c}}{2^2 \hbar^2 E_{\infty}} S_{ee}$$

(9)

The use of static RPA is justified for experiments which take place on longer time scales. In particular, for bulk GaAs, under low temperature cw photoexcitation which creates a not very high carrier concentrations (less than $1 \times 10^{18}$ cm$^{-3}$, where many body and occupation effects are not important and the electron-LO phonon and electron-electron dominates the dynamics of the carriers) the equations of the model are expected to be valid $[10, 17]$. Degeneracy effects are also negligible for concentrations below $1 \times 10^{18}$ cm$^{-3}$ $[17]$. As we mention before the equations of the model came from a more general theory published in Ref. [10]. These general equations take into account degeneracy effects and can be generalized to take into account quantum effects, like exchange, by using more appropriate scattering frequencies $[18]$. However, quantum effects are expected to be important in very short times and very small distances $[16, 17]$.

III. STATIONARY STATES AND STABILITY ANALYSIS

In Fig. 1 we show the low temperature dependence of $\chi_0^s$ as a function of the control parameters $\epsilon_p$ and $\nu$. Fig. 1a shows the dependence of $\chi_0^s$ upon the control parameter $\nu$ for various values of $\epsilon_p$. We have taken $\omega = 0.02$ for the recombination. When $\epsilon_p \sim 16$ and the efficiency of the electron-electron is increased ($\nu$ is diminished) we found a bifurcation, a range of values of $\nu$ where we have more than two solutions for $\chi_0^s$.

The dependence of $\chi_0^s$ upon the control parameter $\epsilon_p$ is shown in Fig. 1b for several values of $\nu$. Notice that, for example, the curve with $\nu = 0.02$, for $\epsilon_p^{(1)} = \epsilon_p^{(2)}$ has three possible values of $\chi_0^s$, that are labelled $\chi_0^{(1)}$, $\chi_0^{(2)}$, and $\chi_0^{(3)}$. We will show that two of them are stable and the other one is unstable. In addition, for $\nu_\epsilon \approx 0.2275$ we have the critical curve which separates curves which has regions of $\epsilon_p$ with three possible solutions for $\chi_0^s$ from curves with just one solution. The critical point is given
by $\varepsilon_p^c$, $\chi_0^c$, and $\nu_c$ that satisfy

\begin{align}
  \left( \frac{\partial \varepsilon_p}{\partial \chi_0} \right)_{\nu_c} &= 0 \quad (10) \\
  \left( \frac{\partial^2 \varepsilon_p}{\partial \chi_0^2} \right)_{\nu_c} &= 0 \quad (11)
\end{align}

We must point out that $\nu = \nu_0^c/Z N_{\text{max}}$, then Fig. 1b can be seen as a family of curves in which the electron-electron interaction is changed, for example, by changing the electron concentration. For GaAs bulk semiconductor, at 4 K, using $N = 4.1 \times 10^{17}$ cm$^{-3}$ and $T_e = 300$ K in Eq. (8) we obtain $\nu = 0.02$. The critical curve is obtained for $N = N_c \approx 6.11 \times 10^{16}$ cm$^{-3}$.

The extrema of the curve $\varepsilon_p(\chi_0^c)$ for a given $\nu$ are $\varepsilon_p^{(1)}$ and $\varepsilon_p^{(2)}$ respectively. They are found by solving the equation $(\partial \varepsilon_p/\partial \chi_0)_{\nu_c} = 0$. They are the boundaries of what is call the hysteresis region. If we increase $\varepsilon_p$ from a value less than $\varepsilon_p^{(1)}$, we will move until the maximum value $\varepsilon_p^{(2)}$ is reached. Then an abrupt jump will take place from a $\chi_0^c$ value at the maximum $\varepsilon_p^{(2)}$ to the value marked with the arrow at the left of Fig. 1b. As we decrease $\varepsilon_p$ from a value greater than $\varepsilon_p^{(2)}$ we will move until we reach the value of $\chi_0^c$ at the minimum. Then the system will undergo an abrupt jump to the value of $\chi_0^c$ marked with the right arrow in Fig. 1b. In Fig. 2 we show this hysteresis region for two values of the recombination parameter $\omega$.

Let us consider the perturbation to the steady state, $\chi(\varepsilon, \tau) = \chi^s(\varepsilon) + \delta \chi(\varepsilon, \tau)$ produced by a time dependent excitation, $\chi_p(t) = \chi_p^s + \delta \chi_p(t)$, where $\chi^s_p$ is the constant excitation that produces the steady state. Notice that $\chi_{\text{tot}}$ and $\chi_0$ are time dependent and they are given by the relations

\begin{align}
  \chi_{\text{tot}} &= 1 + \delta \chi_{\text{tot}} \\
  \chi_0 &= \chi_0^s + \delta \chi_0
\end{align}

From Eq. (8) for $\chi_0$ and assuming that $\varepsilon_p > 1$, we have for its steady state

\begin{equation}
  \nu \chi_1^s + \chi_1^s - \chi_0^s + \chi_0^{s2} - \omega \chi_0^s = 0
\end{equation}

and the time evolution of its perturbation $\delta \chi_0$ is given by

\begin{equation}
  \frac{d}{d\tau} \delta \chi_0 = \nu \delta \chi_1 + (\delta \chi_1 - \delta \chi_0 (1 + \delta \chi_{\text{tot}}) + \delta \chi_{\text{tot}} (\chi_1 - \chi_0^s)) + 2 \chi_0 \delta \chi_0 + (\delta \chi_0)^2 - \omega \delta \chi_0
\end{equation}
which reduces to first order
\[
\frac{d}{d\tau} \delta \chi_0 = \nu \delta \chi_1 + \delta \chi_1 - \delta \chi_0 + 2\chi_0 \delta \chi_0 - \omega \delta \chi_0
\]
\[
= \left[ \frac{\delta \chi_1}{\delta \chi_0} + \frac{\delta \chi_1}{\delta \chi_0} - 1 + 2\chi_0 - \omega \right] \delta \chi_0 \tag{15}
\]

We now define the function \( \psi(\chi_0) \) that allows us to perform the stability analysis. From Eq. (13)
\[
\psi(\chi_0) \equiv \nu \chi_1 + \chi_1 - \chi_0 + \chi_0^2 - \omega \chi_0
\] (16)
and the steady state condition (Eq. (13)) becomes
\[
\psi(\chi_0) = 0
\] (17)
The derivative of \( \psi(\chi_0) \) with respect of \( \chi_0 \) is
\[
\frac{d\psi(\chi_0)}{d\chi_0} = \nu \frac{d\chi_1}{d\chi_0} + \frac{d\chi_1}{d\chi_0} - 1 + 2\chi_0 - \omega
\] (18)
Now, in steady state \( \chi_1 \) is function of \( \chi_0 \) and the perturbation \( \delta \chi_1 \) is written as
\[
\delta \chi_1 = \left( \frac{d\chi_1}{d\chi_0} \right)_{\chi_0=\chi_0^5} \delta \chi_0
\]
Therefore,
\[
\frac{d}{d\tau} \delta \chi_0 = \frac{d\psi}{d\chi_0} \bigg|_{\chi_0=\chi_0^5} \delta \chi_0
\] (19)
and the solution for \( \delta \chi_0 \) is
\[
\delta \chi_0(\tau) = \delta \chi_0(0) \exp \left[ \frac{d\psi}{d\chi_0} \bigg|_{\chi_0=\chi_0^5} \tau \right]
\] (20)
A steady state, given by a \( \chi_0^5 \), is stable if
\[
\frac{d\psi}{d\chi_0} \bigg|_{\chi_0=\chi_0^5} < 0
\] (21)
Eq. (7) can be written as
\[
- \frac{1}{\alpha} \ln \left( 1 - \chi_0^5 \chi_0^{\alpha} \omega \frac{\alpha + \beta}{e^\alpha - e^{-\beta}} \right) - \varepsilon_p = 0
\] (22)
In steady state Eqs. (16) and (22) are equivalent, therefore
\[
\psi(\chi_0) = - \frac{1}{\alpha} \ln \left( 1 - \chi_0 \chi_0^{\alpha} \omega \frac{\alpha + \beta}{e^\alpha - e^{-\beta}} \right) - \varepsilon_p
\] (23)
In Fig. 1b we plotted \( \varepsilon_p \) as a function of \( \chi_0^5 \) which means that we plotted the function \( \psi(\chi_0) \) and this establishes the stability of the states \( \chi_0^{(1)}, \chi_0^{(2)}, \) and \( \chi_0^{(3)} \). The middle root is an unstable steady state, while \( \chi_0^{(1)} \) and \( \chi_0^{(3)} \) are stable steady states.

IV. FIRST ORDER NONEQUILIBRIUM PHASE TRANSITION AND MAXWELL CONSTRUCTION

A thermodynamic system remains homogeneous and stable if the criteria of intrinsic stability is satisfied\[19]\
\[
\left( \frac{\partial P}{\partial V} \right)_T < 0
\] (24)
where \( P \) is the pressure of the system, \( V \) is its volume and \( T \) is its temperature. When this condition is violated a phase transition occurs\[19].

Then, from Eq. (21), we have the following correspondence :\[20]\n\[
\chi_0 \rightarrow V
\varepsilon_p \rightarrow P
\nu \rightarrow T
\]
We see that Eq. (22) turns out to be the equation of state. The homogeneous states \( \chi_0^{(1)} \) and \( \chi_0^{(3)} \) are nonequilibrium stable steady states of the system. Therefore, we call the transition between states \( \chi_0^{(1)} \) and \( \chi_0^{(3)} \), a first order out of equilibrium phase transition. The order parameter is \( \chi_0^{(3)} - \chi_0^{(1)} \) and still remains unknown.

So far, we have assumed that the populations \( \chi_0, \chi_1, \) and so on, are constants and homogeneous in space. Now, we assume that there exist spatial inhomogeneities, \( \chi_0 = \chi_0(r) \), which produce spatial gradients, and diffusion of this population. We also assume, from the structure of Eq. (13), that the spatial and temporal behavior of the populations \( \chi_i, i > 0, \) is given through \( \chi_0 \). Then, we have the following equation
\[
\frac{\partial \chi_0}{\partial \tau} = \psi(\chi_0) + \kappa \frac{\partial^2 \chi_0}{\partial z^2}
\] (25)
We suppose, for simplicity, that \( \chi_0 \) depends only on the spatial coordinate \( z \). Here, \( \kappa \) is the \( \chi_0 \) diffusion constant.

Let us introduce the “potential” \( \Phi(\chi_0) \) with the definition\[4,8]\n\[
\psi(\chi_0) = \frac{\partial}{\partial \chi_0} \Phi(\chi_0)
\] (26)
Then, the steady state satisfies the equation
\[
\kappa \frac{\partial^2 \chi_0}{\partial z^2} = - \frac{\partial}{\partial \chi_0} \Phi(\chi_0)
\] (27)
We have seen from Fig. 1b that, when \( \nu = 0.02 \) and \( \varepsilon_p \) is in the energy interval, \( \varepsilon_p^{(1)} < \varepsilon_p < \varepsilon_p^{(2)} \), the system has two homogeneous stable steady states. Let us find a solution \( \chi_0(z) \) such that, \( \chi_0(+\infty) = \chi_0^{(1)} \) and \( \chi_0(-\infty) = \chi_0^{(3)} \). In such a case two steady states coexist. Obviously, the “potential” \( \Phi(\chi_0) \) has two maxima in \( \chi_0^{(1)} \) and \( \chi_0^{(3)} \).
Coexistence occurs for a value of \( \varepsilon_p \) such that the two maxima are indistinguishable for the system,

\[
\Phi(\chi_0^{(1)}) = \Phi(\chi_0^{(3)})
\]  

(28)

Then

\[
0 = \Phi(\chi_0^{(3)}) - \Phi(\chi_0^{(1)}) = \int_{\chi_0^{(1)}}^{\chi_0^{(3)}} d\chi_0 \psi(\chi_0)
\]

\[
= \int_{\chi_0^{(1)}}^{\chi_0^{(3)}} d\chi_0 \left( \nu \chi_1 + \chi_1 - \chi_0 + \chi_0^2 - \omega \chi_0 \right)
\]  

(29)

The last equation is the Maxwell construction for the vapor pressure in the van der Waals gas from which the order parameter \( \chi_0^{(3)} - \chi_0^{(1)} \) can be calculated. We also notice that \(-\Phi\) corresponds to the Helmholtz free potential. Moreover, the condition that \( \Phi \) is at a maximum in a stable steady state corresponds to that the generalized free potential \( H = -\Phi \) is at a minimum. This is consistent with the condition that an equilibrium thermodynamic system is in a state of minimum Helmholtz free energy \(^{[19]}\).

\[\text{V. CONCLUSIONS}\]

We have found carrier concentration bistabilities in a low temperature nonequilibrium electron gas and we study them in terms of the hysteresis region. The diffusion model consider here allows the coexistence of two phases of the open far from equilibrium gas. The phases are distinguished by the two values of carrier concentration at the bottom of the conduction band, namely, \( \chi_0^{(1)} \) and \( \chi_0^{(3)} \). These two corresponding homogeneous stable steady states are obtained using the Maxwell construction. We introduced the function \( \psi \) and the stability condition for these states was given by Eq. (21). A generalized free potential was defined by Eq. (26) from which we obtained Eq. (29) that corresponds to the vapor pressure Maxwell construction of a van der Waals gas. The difference of the electron population in the bottom of the conduction band is the order parameter and is then calculated and goes to zero when \( \nu \) approaches the critical value \( \nu_c \approx 0.2275 \). This is the reason we call this phase transition a first order nonequilibrium phase transition. For bulk GaAs at 4 K the critical curve is obtained using \( N = N_c \approx 6.11 \times 10^{16} \text{ cm}^{-3} \) and \( T_c = 300 \text{ K} \) in Eq. (8). We have to point out that the electron-electron interaction, which gives the nonlinear character of the rate equations, is the necessary main ingredient for the existence of the phase transition.

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