Verification of Reachability Problems for Time Basic Petri Nets

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Abstract. Time-Basic Petri nets, is a powerful formalism for modeling real-time systems where time constraints are expressed through time functions of marking’s time description associated with transition, representing possible firing times. We introduce a technique for reachability analysis based on the building of finite contraction of the infinite state space associated with such a model. The technique constructs a finite symbolic reachability graph relying on a sort of time coverage, and overcomes the limitations of the existing available analyzers for Time-Basic nets, based in turn on a time-bounded inspection of a (possibly infinite) reachability-tree. A key feature of the technique is the introduction of the Time Anonymous concept, which allows the identification of components not influencing the evolution of a model. A running example is used throughout the paper to sketch the symbolic graph construction. The graph construction algorithm has been automated by a JAVA tool-set, described in the paper together with its main functionality and analysis capability. A use case describing a real-world example has been employed to benchmark the technique and the tool-set. The main outcome of this test are also presented in the paper.

Keywords: real-time systems, timed Petri nets, infinite-states systems, linear constraints, reachability graph, reachability problems

1 Introduction

Time-Basic (TB) Petri nets [1] belong to the category of nets in which system time constraints are expressed as numerical intervals associated to each transition, representing possible firing instants, computed since transition’s enabling time. Tokens atomically produced by the firing of a transition are thereby associated to time-stamps with values ranging over a determined set. With respect to the well-known representative of this category, i.e., Time Petri nets [7], interval bounds in TB nets are linear functions of timestamps in the enabling marking, rather than simply numerical constants. TB nets thus represent a much more expressive formal model for real-time systems. The reachability analysis of TB nets is still recognized as an open problem [15]. Available analysis techniques and tools (e.g., [13, 15]) are based on inspecting a finite portion of the potentially
infinite reachability-tree generated by a TB net. But for particular cases, only time-bounded properties can be inferred from TB net’s state-space exploration by using this kind of analyzers. The technique described in this paper tries to overcome this major limitation. It relies on a symbolic reachability graph algorithm, which is in turn based on a relative notion of time and on a symbolic state definition in which variables are used instead of numerical time-stamp values, and time dependencies are expressed by linear constraints. The core of the algorithm is a procedure verifying inclusion between symbolic states, that relies in turn on two key concepts: the erasure of absolute times and the identification of anonymous timestamps. Broadly speaking, the erasure of absolute times allow us to identify equality/inclusion relationships among states although they have a diverse displacement with respect to the initial time. The anonymous timestamp concept relies on the fact that there may exist components for which timestamp values can be ignored, as not influencing the evolution of the model. The procedure permits in many cases to build a sort of time coverage finite reachability graph. This paper represents an extended version of [4], which take a deeper look at the anonymous timestamp concept and introduces all the adopted heuristics able to find this kind of components.

The symbolic graph construction, including the search of time anonymous timestamps, has been automated by a tool-set written in Java. The output is a structure enriched with information on edges which might be exploited during property evaluation. The tool-set currently includes a module for the automatic verification of reachability properties expressed as conditions on markings. As use case we’ll use the gas burner example, that is widely used in literature as a representative of a small real system. A complete and formal description can be found in [1], and the corresponding TB net model was introduced in [6]. An excerpt will be used as running example to explain in a rather informal way the essential points of symbolic graph construction. Only some relevant new core definitions are formally given.

## 2 Time Basic Nets

Time Basic nets are Petri nets where each token is associated with a time-stamp representing the instant at which it has been created. The domain of timestamps is \( \mathbb{R}^+ \). The structure of a Time Basic net is a triplet \((P, T, F)\), where \( P \) and \( T \) are finite sets, called places and transitions, respectively, s.t. \( P \cap T = \emptyset \), and \( F \) is the flow relation, \( F \subseteq (P \times T) \cup (T \times P) \). Let \( v \in P \cup T \): \( \bullet v, v^\bullet \) denote the backward and forward adjacent sets of \( v \) according to \( F \), respectively, also called pre/post-sets of \( v \). A (time-stamp) tuple of \( t \in T \) is an association \( \text{en}: \bullet t \to \mathbb{R}^+ \). Each transition \( t \) is associated with a time function \( f_t \) which maps a tuple \( \text{en} \) of \( t \) to a (possibly empty) set of \( \mathbb{R}^+ \) values. A marking (state) is a mapping \( m: P \to \text{Bag}(\mathbb{R}^+) \), \( \text{Bag}(A) \) being the set of multiset over \( A \). A tuple \( \text{en} \) of \( t \) is said to be enabling in \( m \), in accordance to a weak semantics (as explained next), if \( \forall p \in \bullet t \text{en}(p) \in m(p) \) and \( f_t(\text{en}) \neq \emptyset \). \( f_t(\text{en}) \) represents the possible firing times for \( \text{en} \). Letting \( \text{en} \) be an enabling tuple of \( t \) in \( m \), a pair \( (\text{en}, \tau) \), \( \tau \in f_t(\text{en}) \), is
said a firing instance of \( t \) (in \( m \)). The firing of \((en, \tau)\) produces the new marking \( m'\), s.t. \( \forall p \in \, t \setminus t^{\bullet} \cdot m'(p) = m(p) - en(p) \), \( \forall p \in \, t^{\bullet} \setminus t \cdot m'(p) = m(p) + \tau \), \( \forall p \in t^{\bullet} \cap t \cdot m'(p) = m(p) - en(p) + \tau \); for all remaining places, \( m'(p) = m(p) \).

This will be as usual denoted \( m([en, \tau]) > m' \).

Hereafter a time function \( f_t \) is defined by a pair of linear functions \([lb_t, ub_t]\), denoting parametric interval bounds. \( lb_t, ub_t \) are in turn formally expressed in terms of (a non empty set of) places in \( \, t \cdot t \cdot \): \( lb_t(en), ub_t(en) \) are the numerical expressions obtained by replacing each place occurrence \( p \) with \( en(p) \). Time-functions must be monotonic, i.e., \( \forall en \cdot lb_t(en) \geq enab \equiv \max\{\{en(p)\}, p \in \, t \} \).

We will keep such assumption implicit in their formal notations.

The set of firing times \( f_t(en) \) can be interpreted in at least two different ways, leading to different time semantics for each transition \( t \). A first interpretation states that an enabling tuple \( en \) of \( t \) can fire at any instant \( \tau \in f_t(en) \). Transitions with one such semantics are referred to as weak. A second interpretation states that an enabling tuple must fire at an instant \( \tau \in f_t(en) \), unless it is disabled by the firing of any conflicting enabling tuple at an instant no greater than the latest firing time of \( t \). Transitions with one such semantics are referred to as strong. Thereby the enabling condition previously given must take into account also the possible presence of other strong enabling tuples [11]. Notice that the only possible semantics for Time Petri Nets [7] is strong.

In order to meet an intuitive notion of time, TB net firing sequences are restricted to the set of firing sequences whose firing times are monotonically non decreasing with respect to the firing occurrences. However, the time of a firing may be equal to the enabling time of the tuple that belongs to the firing. Intuitively this means that an effect (the firing) can occur with no delay after the cause (that enables it) is fulfilled. Therefore, it is possible to have sequences of firings where the time does not change. In practice, it is useful to restrict the attention to a subclass of TB nets, such that there exist no infinitely long firing sequences which take a finite amount of time (non Zenonicity).

Consider the excerpt from the use case, depicted in Fig. [1]. It relates to the Ignite Phase, just after the ignition transformer has been started and the gas valve has been opened. In this phase the controller must check if the flame has been lighted within a specific deadline, otherwise a recovery procedure that brings the system to Idle has to be activated. All transitions are strong, but FlameLightOff2. This permits us to express the possibility that an event occurs within a given time interval.

The flame turns on if there are Ignition and Gas (transition FlameLigthOn), but it can turn off if no gas is supplied (transition FlameLigthOff) or due to a failure, caused e.g. by wind (transition FlameLigthOff2). The time function associated with transition FlameOn (representing the system passing to burnstate after recognizing that the flame has turned on) can be interpreted as follows: FlameOn cannot fire before 0.01 time units elapse since the appearance of a token in place IGNITE_PHASE\_S (the minimum permanence time in ignitestate) and implicitly not before the timestamp in place Flame. The firing time cannot exceed the maximum between the timestamp of the token
in place $IGNITE\_PHASE\_S$ plus 0.01 time units and the time-stamp of the token in place $Flame$ plus 0.1 (i.e., the system recognizes the presence of a flame within this 0.1 units). Noticeably, this is an example of constraint that cannot be directly expressed using Time Petri Nets formalism [7].

\begin{figure}
\centering
\includegraphics[width=\textwidth]{example.png}
\caption{Running example.}
\end{figure}

3 Time coverage reachability analysis

The analysis technique presented in this paper extends the capability of the existing analyzer for TB nets [5], which uniquely permits the verification of bounded invariance and response properties, through the inspection of a time-bounded symbolic reachability tree generated from a TB net.

The new technique aims at building a finite graph instead of an infinite tree for a wide category of TB nets. A combination of three complementary ideas is exploited. First, symbolic states are compared to check subset relationships. For that purpose, using a consolidated approach, timestamp symbols no more occurring in the marking description are eliminated from the linear constraint associated to a symbolic state, independently of how it has been reached. Iden-
tifying subset relations between generated symbolic states (markings plus constraints), is necessary for recognizing cyclic paths, but it is not enough in many situations. As time progresses, periodic occurrences of equivalent conditions may be unrecognizable simply due to their different offsets with respect to system’s time zero. This observation leads us dealing with the second aspect. In the very common case a TB model contains no reference to absolute times (i.e., not as offset respect to enabling timestamps) in transition time functions, it is possible to remove any references to the “absolute zero” from symbolic states. This permits a periodic equivalent behavior to be recognized. The cost is a lossy information about state displacement along absolute time. We’ll discuss this aspects in section 6. Let us only point out that this kind of information could be recovered, if necessary, in a second step by retracing only the path(s) leading to the state of interest, or (at least partially) by combining the information on edges. The third key feature of the technique is the introduction of the time anonymous (TA) concept. This relates to the fact that in a symbolic state there may exist tokens whose timestamp values can be forgotten, as not influencing the evolution of a model. Several heuristics have been implemented, based on a mix of structural and state-dependent patterns, each characterizing one such situation. This enhances the ability of merging states, and permits facing situations where the presence of dead tokens could reintroduce a sort of symbolic absolute zero, nullifying the achievements at the previous points. Again, the cost to pay is a minor loss of information, as discussed later. There is some resemblance with the approach used in the construction of (topological) coverage graphs: the missing information is the exact timestamp of tokens instead of their exact number. TA recognition might be also exploited to introduce a topological notion of coverage for TB nets (section 9).

3.1 Basic notions

In order to understand the rationale behind the symbolic reachability graph construction technique for TB nets, we shall use once again the running example in Fig. 1. Let us only introduce a few basic notions used in the sequel, referring to [12] (where the symbolic reachability tree for TB nets is defined) for a full formalization.

Let $TS = \{T_i\}, i \geq 0$, be the set of time-stamp symbols. A symbolic state $S$ is a pair $(M, C)$, where $M : P \rightarrow Bag(TS)$, $C$ is a (satisfiable) constraint formed by linear inequalities involving $TS$ symbols occurring in $M$ (so called symbolic marking).

Unless otherwise specified, we shall refer to a normal form: if $k$ different $TS$ symbols occur in $M$, they are $T_0, \ldots, T_{k-1}$, such that $\forall i : 0 \ldots k - 2, C \Rightarrow T_i \leq T_{i+1}$.

An ordinary marking $m$ is represented by $S : (M, C)$ if and only if $m$ is obtained from $M$ by a numerical replacement $\sigma : TS \rightarrow R^+$, $\sigma$ being a solution of $C$. We say that $S$ is contained in $S'$ ($S \subseteq S'$) if and only if the corresponding represented ordinary markings are.
A mapping \(en_s : { }^*t \rightarrow TS\) is said a symbolic tuple of \(t\). The notation \((en_s, t)\) will be sometimes used. The symbolic evaluation of a time function \(f_t\), denoted \(f_t(en_s)\), is obtained by replacing each occurrence of \(p \in { }^*t\) in the formal expressions \(lb_t, ub_t\), with \(τ = en_s(p)\).

According to a (monotonic) weak time semantics, \((en_s, t)\) is said a symbolic enabling in \(S\) if \(\forall p \in { }^*t \ en_s(p) \in M(p)\) and \(C': C \land lb_t(en_s) \leq T_k \leq ub_t(en_s) \land T_{k-1} \leq T_k\) is satisfiable, i.e., there exists at least one numerical substitution (tuple) \(en\) for \(en_s\) that makes \(C\) satisfiable and \(f_t(en)\) non empty. As already said the symbolic enabling condition is a bit more complex to take into account strong enablings: an example will be provided in Sect. 3.2.

The firing of a symbolic enabling \((en_s, t)\) produces the new symbolic state \(S' : (M', C')\), where \(M'\) is obtained from \(M\) by removing \(en_s(p)\) from each place \(p \in { }^*t\), and putting the new symbol \(T_k\) in all places in \(t\), in full analogy with the ordinary firing rule. That is denoted \(M[(en_s, t)] > M'\). \(S'\) represents all the possible ordinary markings reachable from any marking represented by \(S\), by means of any firing instance corresponding to \((en_s, t)\).

### 3.2 Time-coverage graph construction

The time-coverage symbolic reachability graph generated by the running example, composed by 14 symbolic states, is presented in Fig. 2.

The adopted notation for states is: a square for symbolic states, a double square for symbolic states containing some deadlocks. Concerning edges (i.e., symbolic enablings), the format of head and tail specifies the kind of relation between source and target.

The normal case is black head and tail, e.g., from \(S_0\) to \(S_1\): considering any marking represented by \(S_0\) it is always possible to follow that edge and to reach all the markings represented by \(S_1\).

Let us consider the symbolic state \(S_8\), formally described as follows:

\[
M_8 : \text{Gas}\{T_1\}, \text{IGNITE\_PHASE}\_S\{T_0\}, \\
\text{Ignition}(TA) \land \text{NoFlame}(TA) \\
C_8 : T_1 \geq T_0 + 1.5 \land T_1 \leq T_0 + 1.8
\]

We can observe that, with respect to the original definition of symbolic state, a first extra time-stamp symbol is present, \(TA\) (time anonymous). This new symbol can occur only on the marking. Postponing an intuitive explanation of when and how symbol \(TA\) is introduced in a symbolic state representation, we can think of it as a token carrying on an unspecified time-stamp, which has been shown unessential for the computation of transition firing times.

The “candidates” for symbolic enabling in \(S_8\) are:

\[\langle (T_0), \text{GasOff2} \rangle\]
\[\langle (TA, T_1, TA), \text{FlameLightOn} \rangle.\]

\[\text{This picture has been automatically obtained by using GraphViz visualization software [14] on the output generated from the tool-set.}\]
Firing times are computed by (symbolically) evaluating transition time functions, as explained above. For \textit{GasOff2} the (only) inferred firing time is \{\(T_0 + 2\}\). Time function evaluation is slightly different for \textit{FlameLightOn}, due to the occurrence of \(TA\) in the pre-set tuple: this symbol is erased (Definition 2 in the following section) during symbolic evaluation: \(enab = \max(\{TA, T_1, TA\}) \equiv max(\{T_1\}) = T_1\). The inferred firing time in this case is \{\(T_1 + 0.5\}\).

Since both transitions have a strong semantics, there are two additional constraints specifying that the firing time of one cannot be greater than the (maximum) firing time of the other. They are \(C_{\text{GO2}}: T_0 + 2 \leq T_1 + 0.5\) and \(C_{\text{FLO}}: T_1 + 0.5 \leq T_0 + 2\), respectively.

Since both \(C8 \land C_{\text{GO2}} \land T_2 = T_0 + 2\) and \(C8 \land C_{\text{FLO}} \land T_2 = T_1 + 0.5\) are satisfiable, \((\langle T_0 \rangle, \text{GasOff2})\) and \((\langle TA, T_1, TA\rangle, \text{FlameLightOn})\) are in fact symbolic enablings in S8. It is important to note that \(C8 \Rightarrow C_{\text{GO2}} \land T_2 = T_0 + 2\), i.e., all the markings represented by S8 enable the transition \textit{GasOff2}. Instead \(C8 \not\Rightarrow C_{\text{FLO}} \land T_2 = T_1 + 0.5\), i.e., only a subset of the markings expressed by S8 enable the transition \textit{FlameLightOn}. This is highlighted in the graph by the white tail of the edge from S8 to S9.

Consider now the firing of \((\langle T_0 \rangle, \text{GasOff2})\): it only consumes tokens. In such cases the symbolic firing rule slightly differs from the original one. A second special symbol, \(TL\) (Time Last), is introduced. \(TL\) can occur only on the constraint of a symbolic state and has an intuitive meaning: it stands for the last firing
time of the TB net and it permits a correct interpretation of the model’s time semantics\(^2\). The reached symbolic state \(S_{10}\) is:

\[
M_{10} : \text{Gas}\{T_1\} \ \text{Ignition}\{TA\} \ \text{NoFlame}\{TA\} \\
C_{10} : C_8 \land T_2 = T_0 + 2 \land TL = T_2
\]

The normalization step eliminates symbols \(T_2\) (the symbolic firing time) and \(T_0\), as they occur only in \(C_{10}\), instead it leaves symbol \(TL\). That results in (after a timestamp renaming):

\[
M_{10} : \text{Gas}\{T_0\} \ \text{Ignition}\{TA\} \ \text{NoFlame}\{TA\} \\
C_{10} : TL \geq T_0 + 0.2 \land TL \leq T_0 + 0.5
\]

Another circumstance that causes the introduction of \(TL\) symbol in a symbolic state representation is when the maximum timestamp symbol \(T_k\) is replaced with \(TA\). The identification of a Time Anonymous in a given symbolic state is the next topic we treat.

The graph in Fig. 2 contains two looping paths: between states \(S_3\) and \(S_5\), and between \(S_{12}\) and \(S_{13}\) respectively. That happens because in the extrapolated sub-model (Fig. 1), no expected actions are activated after the system exits the ignition phase (e.g., closing the gas valve in the event of fail, or stopping ignition), so that an unbounded sequence of FlameLightOff;FlameLightOn is possible.

The white head of the edge from \(S_5\) to \(S_3\) means that at least one of the ordinary markings represented by \(S_3\) is not reachable by following that edge. This happens when a newly built symbolic state is recognized to be strictly contained in an existing one. What permits recognizing inclusion between states in this specific case is the usage of Time Anonymous timestamps (Definition 5). \(S_3\) is formally defined as:

\[
M_3 : \text{Gas}\{TA\} \ \text{BURN\_PHASE\_B}\{TA\} \\
\quad \text{Ignition}\{T_0\} \ \text{Flame}\{T_1\} \\
C_3 : T_1 \geq T_0 \land T_1 \leq T_0 + 0.1
\]

Without using \(TAs\), its original definition (\(S_3’\)) would be:

\[
M_3’ : \text{Gas}\{T_0\} \ \text{BURN\_PHASE\_B}\{T_1\} \\
\quad \text{Ignition}\{T_0\} \ \text{Flame}\{T_1\} \\
C_3’ : T_1 \geq T_0 \land T_1 \leq T_0 + 0.1
\]

Let us figure out what would be the model evolution from \(S_3’\), without introducing \(TA\). After the firing sequence FlameLightOff;FlameLightOn\(^3\) a state \(S_3''\) would be reached, defined in turn as:

\[
M_3'' : \text{Gas}\{T_1\} \ \text{BURN\_PHASE\_B}\{T_0\} \\
\quad \text{Ignition}\{T_1\} \ \text{Flame}\{T_1\} \\
C_3'' : T_1 \geq T_0 + 0.5 \land T_1 \leq T_0 + 0.5
\]

\(^2\) In this paper, when \(TL\) is left implicit, it coincides with the “last” generated timestamp \(T_k\).

\(^3\) We omit in this description symbolic enablings, the TB net being safe.
Since $S^3'' \not\subseteq S^3'$ and $S^3' \not\subseteq S^3''$, there is no possibility to merge them and in fact the analysis tool would produce an infinite firing sequence.

Back to $S^3$, we note it corresponds to $S^3'$ but for holding TA symbols in places $BURN\_PHASE\_B$ and $Gas$ instead of $T_1$ and $T_0$, respectively. Token $T_1$ in $BURN\_PHASE\_B$ however is not (and will never be) involved in any symbolic enabling because $BURN\_PHASE\_B$ has an empty postset (Heuristic 0 in the following section), so it is immediately marked as TA. Token $T_0$ in $Gas$ instead is in the preset of transitions $Flame\_Light\_On$ and $Flame\_Light\_Off$. As for $Flame\_Light\_On$, the tokens in place $Ignition$ and in place $Gas$ carry on the same timestamp, so either of them is enough to correctly evaluate transition's time function. As for $Flame\_Light\_Off$, the token in place $Gas$ carries on redundant information due to the simultaneous presence of $T_1$ in $Flame$, that superseded it (Heuristic 2).

$S^3''$ seems really different from $S^3$, but nearly the same heuristics permits us to replace $T_0 : BURN\_PHASE\_B$ (Ti : $p$ denotes the occurrence of a timestamp in a place) and $T_1 : Gas$ with TAs. That eliminates all the occurrences of $T_0$ from the marking. After timestamp renaming, we obtain the normal form:

\[
M^3'' : \{ TA \}_{BURN\_PHASE\_B} \{ TA \}_{Ignition} \{ T_0 \}_{Flame} \{ T_0 \}_{C3''} : true
\]

However there is still a difference with respect to $S^3$: places $Ignition$ and $Flame$ hold the same timestamp, but this boils down to a condition already represented by $S^3$ ($T_1 = T_0 \Rightarrow C3$), so $S^3''$ is recognized as a state contained in $S^3$.

Notice that the other cycle on the graph, between $S_{12}$ and $S_{13}$, is due to the adoption of a relative notion of time, i.e., it does not depend on the introduced TA concept.

An important setting of the legacy tool [13] was the time limit, a positive interval time that guaranteed the finiteness of the symbolic reachability tree of a TB net. Upon elimination of absolute time references it has been substituted by a relative time limit. This positive interval specifies the maximum admissible distance between different timestamps in a state, and allows one to deal with possibly infinite reachability graph. The tool-set checks whether a symbolic state includes any ordinary states for which the distance between $TL$ and $T_0$ (the oldest meaningful timestamp) exceeds the time limit, marking that state as not to be expanded. The rationale behind is that reaching such a user defined limit might be a symptom of the presence of unrecognized “dead tokens”, reintroducing absolute time references. If we analyzed the running example disabling TA recognition, the resulting graph would be infinite, unless a time limit is set. For example, setting this limit to 3 (time units), 25 symbolic states would be generated: 13 already included in the presented graph, the others corresponding to a partial unrolling of the loop between $S^3$ and $S^5$.

The output generated by the tool-set associates a couple of numerical values to edges of the graph, corresponding to the minimum and maximum time distances from the source node to the target node. This permits us to partially
recover time relations between nodes that were lost due to the removal of absolute times references from constraints. In the following section we’ll show how to exploit them.

4 Time Anonymous

The notion of time anonymous relies on the fact that in a symbolic state there may exist tokens whose timestamp values can be forgotten, as not influencing the evolution of a model. The adopted symbol to denote a time anonymous timestamp is $TA$, and it represents an undefined time value in the past chosen between the initial time and the time limit $TL$. The $TA$ replacement task (formally defined in the next section) allow us to build, in many cases, a finite reachability graph. In fact, the presence of “dead” tokens in a model, i.e. those tokens that cannot be consumed by firing transitions, reintroduce a sort of initial time that would prevent the discovery of equality/inclusion relationships among states.

Fig. 3: Simple TB net example generating a “dead” token.

As a simple example, let us consider the model described in Fig. 3. Transition $t_0$ is enabled in the time lapse $[T_0 + 0.2, T_0 + 0.3]$. Its firing produces two new tokens, respectively into $P_1$ and $P_2$ with a timestamp $T_1$ representing a value chosen in such a time interval. This new configuration enables $t_1$ which can fire infinitely many times, by consuming and immediately after creating a token in $P_2$, each time with a new timestamp. Although the erasure of absolute times, the presence of a “dead” token in $P_2$, creates a sort of time marker which would make the reachability graph infinite, as we can see in Fig. 4a.

After the initial state $S_0$, reachable states are all equal in terms of symbolic marking: $P_1\{T_0\}P_2\{T_1\}$ but they have different constraints:

- $C_{S_1} = 0.2 \leq T_0 \leq 1.3 \land T_0 + 0.5 \leq T_1 \leq T_0 + 0.7$
- $C_{S_2} = 0.2 \leq T_0 \leq 1.3 \land T_0 + 1.0 \leq T_1 \leq T_0 + 1.4$
- $C_{S_3} = 0.2 \leq T_0 \leq 1.3 \land T_0 + 1.5 \leq T_1 \leq T_0 + 2.1$
Fig. 4: Infinite (a) and finite (b) representations of the reachability graphs extracted from the model shown in Fig. 3.

(a) Reachability graph without TA replacement.

(b) Reachability graph with TA replacement.

and so forth, departing $T_1$ from $T_0$ further and further. Anyway, it is worth noting that $T_0$ does not influence the evolution of the model, thus we can forget about this value replacing it with an anonymous timestamp $TA$. The $TA$ replacement cause the erasure of $T_0$ from constraints enabling the identification of equality relationships among states. In fact, a $TA$ timestamp does not have any relationships with other symbolic values because it represents any time value in the past. Therefore, all the states after the initial one, would have the same constraint: $C_{S_1} = TRUE$. The finite reachability graph, resulting from the analysis of Fig. 3 using $TA$ replacements, is shown in Fig. 4b.

We identified three different typologies of tokens disclosing a negligible symbolic time:

- The first category is composed of “dead” tokens. A token $t_k$ is dead if belongs to a place with an empty postset. Therefore such a token will be never consumed by firing transitions. It is possible to statically identify places that may contain dead tokens.

- The second category contains all tokens $t_k$ such that $t_k$ belongs to a place $p$ with a non empty postset, and $t_k$ cannot be consumed by firing transitions. I.e. foreach $t \in p^*$, any symbolic tuple $(en_s, t)$, such that $en_s(p) = t_k$ is not an symbolic enabling. It is not possible to statically evaluate places containing such a tokens.

- This latter category regards all tokens $t_k$ such that $t_k$ can be consumed by a firing transition, but its firing time is not evaluated in terms of the timestamp associated with $t_k$. As the previous category, we must search for such a tokens dynamically, during the graph construction.

It is worth noting that, a symbolic enabling $(en_s, t)$ such that $lb_t(en_s) = TA$ makes the lower bound $lb_t(en_s)$ equals to $TL$, in fact a $TA$ lower bound means that $TL$ exceeds the minimum enabling time. Anyway, in case the preset of a transition $t$ contains only “$TA$ tokens”, $t$ cannot fire because both the
lower bound and the upper bound of $t_f$ would be any time value in the past, thus we cannot determine whether it represents an empty set. The reason of a TA replacement of all tokens belonging to $\bullet t$ could be that foreach symbolic tuple $(en_s, t), TL > ub_t(en_s)$. Thus, if such a tokens does not contribute to the evaluation of possible firing times of other transitions, we can forget about all their symbolic times.

The next section introduces a formal definition of a “TA replacement” and all the adopted heuristics in order to find time anonymous timestamps during the graph building.

5 Formal Definitions

Let us formalize some core concepts previously outlined, focusing in particular on TA and coverage. For the sake of readability, definitions involving transitions refer to the weak semantics.

**Definition 1 (symbolic state).** A symbolic state $S$ is a pair $\langle M, C \rangle$, where $M$ is a function $P \rightarrow Bag(TS \cup \{TA\})$, and $C$ is a (satisfiable) linear constraint defined on $TS_M \cup \{TL\}$, $TS_M \subset TS$ being the finite set of symbols $T_i$ occurring on $M$, such that $\forall T_i \in TS_M, C \Rightarrow TL \geq T_i$.

**Definition 2 (well-defined erasure).** Let $g_t$ be the formal expression of a linear function. The erasure of a set of symbols $E \subset \bullet t$ from $g_t$, denoted $g_{t|\sim E}$, is well-defined if it doesn't violate the arity of any operators occurring in $g_t$.

Consider for instance $t$, s.t. $\bullet t = \{p_1, p_2\}$, and $f_t : [\max\{p1, p2\}], p2 + 0.5]$, where, $\max : 2^{R^+} \setminus \emptyset \rightarrow R^+, + : R^+, R^+ \rightarrow R^+$. Then, the erasure $f_{t|\sim \{p_1\}}$ is well-defined and results in $[p2, p2 + 0.5]$, instead $f_{t|\sim \{p_2\}}$ is not well-defined.

A symbolic instance of $t$ is a mapping $en_s : \bullet t \rightarrow TS \cup \{TA\}$.

Let $en_s^{-1}(\tau) = \{p\}$, $en(p) = \tau$.

**Definition 3 (symbolic enabling).** $(en_s, t)$ is said a symbolic enabling in $S = \langle M, C \rangle$ if and only if:

- $\forall p \in \bullet t, en_s(p) \in M(p)$
- $f_{t|\sim en_s^{-1}(TA)}$ is well-defined
- $C \land lb_{t|\sim en_s^{-1}(TA)}(en_s) \leq ub_{t|\sim en_s^{-1}(TA)}(en_s)$ is satisfiable

Let $C \setminus X$ denotes the constraint obtained by eliminating variable $X$ from $C$, in such a way that the solutions of $C \setminus X$ are “projections” of the solutions of $C$.

**Definition 4 (symbolic firing).** Let $(en_s, t)$ be a symbolic enabling in $S = \langle M, C \rangle$, $k = |TS_M|$. The firing of $(en_s, t)$ produces the new symbolic state $S' : \langle M', C' \rangle$, where

- $\forall p \in \bullet \setminus \bullet t, M'(p) = M(p) - en_s(p)$
- $\forall p \in \bullet \setminus \bullet t, M'(p) = M(p) + T_k$
\[
\forall p \in T \cap t, M'(p) = M(p) - \text{en}_a(p) + T_k \\
\text{for all remaining places, } M'(p) = M(p) \\
C' = C - TL \land \text{lb}_{t[\text{en}_a^{-1}(TA)]}(\text{en}_a) \leq T_k \land T_k \leq \text{ub}_{t[\text{en}_a^{-1}(TA)]}(\text{en}_a) \land T_k \geq T_{k-1} \land TL = T_k
\]

C' may contain some symbols \( T_i \) that have been withdrawn from \( M' \). After eliminating redundant variables, and (possibly) renaming left symbols, the reached state meets definition \( \square \) and is in normal form.

Let \( R(S) \) be the set of symbolic states reachable from \( S \)

**Definition 5 (valid TA-replacement).** Given a state \( S \), a timestamp occurrence \( T_i : p \) is replaceable with \( TA : p \) if and only if for each \( S' = (M',C') \in R(S) \) in which token \( T_i : p \) is left (modulo timestamp renaming), for each symbolic enabling \( (\text{en}_a,t) \) in \( S' \) s.t. \( \text{en}_a(p) = T_i \), \( f_{t[\{-p\}]} \) is a well-defined erasure and

\[
C' \land \max(\{TL,\text{lb}_t(\text{en}_a)\}) \leq \text{ub}_t(\text{en}_a) \Leftrightarrow C' \land \max(\{TL,\text{lb}_{t[\{-p\}]}(\text{en}_a)\}) \leq \text{ub}_{t[\{-p\]}(\text{en}_a)
\]

The new semantics of a symbolic state is provided by the following coverage notion.

**Definition 6 (symbolic state coverage).** Let \( S = (M,C) \) be a symbolic state. An ordinary marking \( m \) is covered by \( S \) if and only if it corresponds to a numerical substitution \( \sigma \) of symbols occurring in \( M \), s.t. \( \sigma \) satisfies \( C \) and for each ordinary enabling \( \text{en} \) of \( t \) in \( m \), for each symbolic tuple \( (\text{en}_a,t) \) in \( S \) s.t. \( \text{en} \) is a numerical substitution of \( \text{en}_a \),

- \( \text{lb}_{t[\text{en}_a^{-1}(TA)]}, \text{ub}_{t[\text{en}_a^{-1}(TA)]]} \) are well defined
- \( \text{lb}_{t[\text{en}_a^{-1}(TA)]}(\text{en}) = \text{lb}_t(\text{en}) \land \text{ub}_{t[\text{en}_a^{-1}(TA)]}(\text{en}) = \text{ub}_t(\text{en}) \)

The next lemma sets the relationship between ordinary and symbolic instances (state transitions).

**Lemma 1.** Let \( m \) be covered by \( S \). If \( m[(en,\tau)] > m' \), then there exists a symbolic enabling \( \text{en}_a \), s.t. \( \text{en} \) is a numerical substitution of \( \text{en}_a \), \( S[(\text{en}_a,t)] > S' \) and \( m' \) is covered by \( S' \)

Let us finally report all the heuristics implemented by the tool to identify the TA replacements commented in the previous sections.

Formally, a valid replacement of a timestamp occurrence \( T_i : p \) with \( TA : p \), in \( S = (M,C) \), according to definition \( \square \) takes place whenever at least one of the following heuristic, is verified foreach \( t \in t^* \). Note that if \( p^* = \emptyset \) (Heuristic 0), this condition is trivially true.

**Heuristic 1** \( \forall p' \in t^*, M(p') \neq \emptyset \)

- \( f_i \) is in the form \( \lbrack \text{enab} + c,\text{enab} + c' \rbrack \)
- \( \exists p' \in t^*(\forall T_j \in M(p') C \Rightarrow T_j \geq T_i) \)

All places belonging to \( t^* \) are marked, \( f_i \) is in the form \( \lbrack \text{enab} + c,\text{enab} + c' \rbrack \), but there exist another place containing only newer tokens. Thus tokens belonging to \( p \) won’t be used to compute the enabling time.
Heuristic 2 \( \forall p' \in \mathbf{t}, \ M(p') \neq \emptyset \)
\( \land f_i \) does not contain \( p \)
\( \land f_i \) does not contain \( \text{enab} \)

All places belonging to \( \mathbf{t} \) are marked, but \( p \) will not be used to compute possible firing times of \( f \) because \( f_i \) does not contain either the variable \( p \) or \( \text{enab} \).

Heuristic 3 \( \forall p' \in \mathbf{t}, \ M(p') \neq \emptyset \)
\( \land f_i \) is in the form \([ \text{max}(...) + c, \text{max}(...) + c' \])
\( \land \forall (en_s, t) \) symbolic enabling, \( lb_{k[\neg\{p\}]}(en_s) = lb_i(en_s) \land ub_{k[\neg\{p\}]}(en_s) = ub_i(en_s) \)

All places belonging to \( \mathbf{t} \) are marked, \( f_i \) is in the form \([ \text{max}(...) + c, \text{max}(...) + c' \]) but foreach enabling tuple \( en_s, f_i(en_s) \) equals \( f_i[\neg\{p\}](en_s) \) (well defined erasure). Thus neither \( lb_i(en_s) \) nor \( ub_i(en_s) \) refers to \( T_i \).

Heuristic 4 \( \forall p' \in \mathbf{t}, \ M(p') \neq \emptyset \)
\( \land \forall (en_s, t) \) symbolic enabling, \( C \Rightarrow (TL > ub_i(en_s) \land TL > lb_i(en_s)) \)

All places belonging to \( \mathbf{t} \) are marked, \( t \) is not enabled \( (TL > ub_i(en_s)) \) and tokens in \( p \) won't be used to compute the lower bound of \( f_i \) even if \( t \) would be re-enabled by other tokens \( (TL \geq lb_i(en_s)) \).

Heuristic 5 \( \forall p' \in \mathbf{t}, \ M(p') \neq \emptyset \)
\( \land \forall (en_s, t) \) symbolic enabling, \( C \Rightarrow (lb_i(en_s) > ub_i(en_s) \land TL \geq lb_i(en_s) \lor lb_{k[\neg\{p\}]}(en_s) = lb_i(en_s)) \)

All places belonging to \( \mathbf{t} \) are marked, \( t \) is not enabled \( (lb_i(en_s) > ub_i(en_s)) \) and tokens in \( p \) won't be used to compute the lower bound of \( f_i \) even if \( t \) would be re-enabled by other tokens, in fact \( TL \geq lb_i(en_s) \) or \( p \) does not contribute to the evaluation of \( lb_i(en_s) \).

Heuristic 6 \( \exists p' \in \mathbf{t}, \ M(p') = \emptyset \)
\( \land f_i \) does not contain \( p \)

\( t \) is disabled in \( S \) and \( p \) does not contribute to the evaluation of \( f_i \) foreach possible future symbolic enabling.

Heuristic 7 \( \exists p' \in \mathbf{t}, \ M(p') = \emptyset \)
\( \land lb_i \) contains \( p \)
\( \land ub_i \) does not contain \( p \)
\( \land \forall en \ s.t. \ (en, t) \) future symbolic enabling, \( C \Rightarrow TL \geq lb_i(en) \)

\( t \) is disabled in \( S \), \( ub_i \) does not contain the variable \( p \), and foreach possible future symbolic enabling \( (en, t) \), the lower bound \( lb_i(en) \) will be greater or equal to \( TL \).

Heuristic 8 \( \exists p' \in \mathbf{t}, \ M(p') = \emptyset \)
\( \land f_i \) is in the form \([ \text{max}(...) + c, \text{max}(...) + c' \])
\( \land \forall en \ s.t. \ (en, t) \) future symbolic enabling,
\( lb_{k[\neg\{p\}]}(en) = lb_i(en) \land ub_{k[\neg\{p\}]}(en) = ub_i(en) \)
Heuristic 9  \( \exists p' \in t : M(p') = \emptyset \)
\( \land \forall en \text{ s.t. } (en,t) \) future symbolic enabling,
\( C \Rightarrow (TL > ub_t(en) \land TL \geq lb_t(en)) \)

Heuristic 10  \( \exists p' \in t : M(p') = \emptyset \)
\( \land \forall en \text{ s.t. } (en,t) \) future symbolic enabling,
\( C \Rightarrow (lb_t(en) > ub_t(en) \land TL \geq lb_t(en)) \)

Heuristics 8, 9, 10 are respectively conceptually similar to 3, 4, 5 except they refer to future symbolic enablings, being \( t \) disabled within S.

Heuristic 11 Given a place \( p' \) and a symbolic tuple \( en_s \), let \( \phi_{TA}(en_s,p') \) be a new symbolic tuple such that:
\[
\phi_{TA}(en_s,p')(p) = \begin{cases} 
    en_s(p) & \text{if } p = p' \\
    TA & \text{otherwise}
\end{cases}
\]
\( \forall (en_s,t) \) symbolic enabling,
\( C \Rightarrow (TL > ub_t(\phi_{TA}(en_s,p)) \land TL \geq lb_t(\phi_{TA}(en_s,p))) \)

This heuristic assesses whether the symbolic time \( T_i \) influences the evaluation \( f_t(en_s) \). To this end, we consider \( T_i \) as the last produced token by replacing each timestamp of \( en_s \), except \( T_i \), with a TA. If \( T_i \) does not contribute to evaluate \( f_t(en_s) \), even if this condition holds, we can replace it with a TA timestamp.

6 Property Evaluation

The symbolic (time coverage) reachability graph contains several exploitable information.

The tool recognizes deadlocks even if they are topologically hidden by the presence of outgoing edges. In fact if all the outgoing edges have a white tail, it is still possible that a proper subset of the corresponding symbolic state is composed by deadlock marking. In the running example however no deadlock marking is reachable.

Disregarding time specification (i.e., considering only the number of tokens distributed over places), the graph nodes exactly identify all the reachable (topological) markings: if a marking matches a symbolic node then there exists at least one path from the initial state to such a marking, conversely if a marking matches no symbolic nodes, it is not reachable. It is thereby possible to verify P-invariants from a specified marking. In case of finite graph, it is possible to answer questions about maximum (minimum) number of tokens in some (combinations) of places.

In general, due to TA introduction, the set of ordinary markings covered (Definition 6) by the states of the symbolic graph built from a TB net is a superset of the reachable ordinary markings of the TB net. Given a symbolic state \( S = (M,C) \), each numerical substitution of \( \{T_i\} \) symbols occurring in \( M \)
and satisfying $C$ corresponds to the projection of reachable ordinary states. If we are interested in checking timing relations between token’s timestamps on the states of the graph we can get three different answers upon graph inspection: a positive one (e.g., there exists a node that satisfies the condition), a negative one (e.g., no nodes satisfy the condition), or a possibly positive. For example, if we are looking for a state where a token in place $\text{Flame}$ carries on a timestamp greater than the one in place $\text{IGNITION\_PHASE\_S}$, state $S9$ provides us with a positive answer. Instead, if we are checking whether places $\text{Gas}$ and $\text{Ignition}$ can ever hold the same timestamp the answer is may be (the presence of $\text{TA}$ in either places covers that condition).

As for timing relations between token’s timestamps in different markings, or between firing times in a transition firing sequence, the symbolic graph permits identifying critical paths by combining the information on edges. In particular, conservative bounds can be established. In the case they are not enough to exclude incorrect timing behaviors, it is possible to carry out a more accurate analysis by rebuilding a portion of the graph, retracing some critical paths and reintroducing absolute time references. For example, looking at the time information on edges, it is possible to establish that state $S10$ is not reachable from $S0$ in less than 1.7 time units. We cannot directly infer that $S10$ is reachable in exactly 1.7 time units.

![Fig. 5: Critical case for path feasibility.](image)

Concerning feasibility of firing sequences (Lemma 1), the symbolic graph expresses all the possibilities (an ordinary firing sequence is matched by any firing sequence on the graph). A possible critical situation is a white-arrow edge (meaning that we reach only a subset of the target state) is followed by a white-tail edge as shown in Fig. 5 (meaning that the transition is enabled only in a subset of the ordinary states represented by the node). In this case there is still the possibility that this path actually is not feasible. Also such critical paths could be retraced. Let us stress (back to the reachability problem) that by construction, for every node on the graph there exists a path from the initial state to such a node formed exclusively by black-arrow edges.

The available tool’s evaluation component is still very simple, its integration with some existing model checking engines is currently under investigation. However it already permits examining the input graph looking for interesting properties on topological definition of markings:
existence of a state with a marking satisfying a constraint (i.e., a boolean combination of condition on the number of tokens in places)

maximum (minimum) value of an expression involving the number of tokens in places (possibly restricting the evaluation to markings satisfying a given constraint)

7 Tool Architecture

The analysis technique described in this paper has been implemented as a command line tool written in Java. The tool architecture depicted in Fig. 6 presents the various components that communicate by means of files. The \textit{tgraphgen} module receives as input a Time Basic Petri net (either in the legacy file format used by the Cabernet tool, or in a PNML format generated, for example, by a customized version of PIPE2 open source tool [10]). It generates as outputs the graph in binary format (used by the property verification module \textit{tgrapheval}), and in an annotated DOT text format (used by the GraphViz tool). The tool is also integrated as an analysis module in the customized PIPE2 open source tool. That will permit accessing all the functions by means of menu, and exploiting in an integrated environment consolidated structural analysis algorithms for the verification of the untimed part of TB nets (e.g., P/T nets invariant analysis). Both the command line tool and the customized version of PIPE2 are available for download at \url{http://camilli.di.unimi.it/graphgen}, together with a brief user guide and some running examples.

Fig. 6: Reference architecture.
8 Use Case and Comparison with other tools

In order to make a comparison with the available analysis techniques and tools for TB nets, we consider now the complete gas burner example analyzed in [6], also reported in Fig. 5 for completeness.

The main critical parameter of the system was identified in the concentration value of unburned gas. With the old analyzers it was only possible to do an approximate analysis, by verifying the safety requirement within a fixed time threshold [6], or by empirically guiding the construction of a portion of the reachability tree looking for a state invalidating the property [8]. These techniques were only able to verify the unsatisfiability of the time bounded safety property by ending the construction of the tree after reaching a state with a concentration exceeding a critical value (i.e., according the specification, one second of unburned gas). A significant improvement is that our technique computes the graph representing the complete behavior of the system, and thus for example permits calculating the actual concentration upper bound.

![Fig. 7: State creation advancement.](image)

Table 1 reports the outcomes of the analysis on the use case. In particular the considered parameter has been measured with three versions of the net. They differ in the time granularity used for the unburned gas process, i.e., the time function of the transition \( \text{Inc}_\text{Conc} \). The first thing to note is however that the analysis result is coherent in the various situations, identifying the maximum amount of unburned gas as corresponding to a leaking period of two seconds.
The test has been performed on a Toshiba Notebook with 2.4Ghz Intel Core 2 Duo processor and 4GB of memory. The operating system is Ubuntu 10.10 and the Java Virtual Machine is OpenJDK IcedTea6 1.9.5.

On the table we report also the number of states of the final reduced graph against the overall number of states generated by the algorithm, and the execution times.

In Fig. 7 some profiling data – relating the 0.1 time granularity version of the model – are presented. On the x axis there is the execution time expressed in minutes, on the y axis there are the number of built nodes, of reduced (final) nodes, and of nodes ready to be processed, respectively. This picture is important for two reasons: first it shows that the performance degradation of state construction process is very small (the number of states created is pretty much constant in time after an initial burst); second, it supports the idea that a parallel (distributed) version of the graph builder, introduced in [2,3,9] should substantially improve the performances (the front of expansion remaining consistently wide).

Table 1: Use case analysis results.

| Inc-Conc gran | max(Conc) | # [final/built] states | exec. time |
|---------------|-----------|------------------------|------------|
| 0.5           | 4         | 865/1217               | $\approx 75$ secs |
| 0.25          | 8         | 2233/2983              | $\approx 400$ secs |
| 0.1           | 20        | 14563/23635            | $\approx 7.5$ hrs |

9 Conclusion and future works

The analysis technique presented in this paper overcomes the existing available analysis technique for Time Basic Nets (a very expressive timed version of Petri nets) because it permits the building of a sort of (symbolic) time-coverage reachability graph keeping interesting timing properties of the nets. In particular the introduction of the concept of time anonymous timestamps, allows for a major factorization of symbolic states. An extension of the technique that further exploits the time anonymous concept in order to deal with topologically unbounded nets (by means of a coverage of TA tokens, i.e., a sort of $\omega_{TA}$) is under definition.

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Initial marking: \( IDLE\_PHASE(T_0), IDLE\_PHASE\_bis(T_0), No\_Ignition(T_0), No\_HeatReq(T_0), No\_Gas(T_0), No\_Flame(T_0), NO\_FLAME\_bis(T_0) \)

Initial constraint: \( 0 \leq T_0 \leq 10 \)

Time-Functions:

- **HrOn**
  \[ IDLE\_PHASE + 0.01, max(\{IDLE\_PHASE + 0.01, HeatReq + 0.1\}) \]

- **HrOff**
  \[ BURN\_PHASE\_S + 0.01, max(\{BURN\_PHASE\_S + 0.01, NoHeatReq + 0.1\}) \]

- **IgnOn**
  \[ max(\{PURGE\_PHASE + 0.01, IDLE\_PHASE\_bis + 30\}), max(\{PURGE\_PHASE + 0.01, IDLE\_PHASE\_bis + 30\}) \]

- **CloseValve**
  \[ ValActCloseReq + 0.2, ValActOpenReq + 0.2 \]

- **OpenValve**
  \[ ValActOpenReq + 0.2, ValActOpenReq + 0.2 \]

- **FlameOff**
  \[ STOP\_PHASE\_F + 0.01, max(\{STOP\_PHASE\_F + 0.01, NoFlame + 0.1\}) \]

- **FlameOff2**
  \[ BURN\_PHASE\_S + 0.01, max(\{BURN\_PHASE\_S + 0.01, NoFlame + 0.1\}) \]

- **FlameOn**
  \[ IGNITE\_PHASE\_S + 0.01, max(\{IGNITE\_PHASE\_S + 0.01, NoFlame + 0.1\}) \]

- **IgnLightOn**
  \[ IgnActOnReq + 0.2, IgnActOnReq + 0.2 \]

- **IgnLightOff**
  \[ IgnActOffReq + 0.2, IgnActOffReq + 0.2 \]

- **FlameLightOn**
  \[ max(\{Gas, Ignition\}) + 0.5, max(\{Gas, Ignition\}) + 0.5 \]

- **FlameLightOff**
  \[ enab, NoGas + 0.1 \]

- **FlameLightOff2**
  \[ enab + 0.01, enab + 0.1 \]

- **GasOn**
  \[ enab + 0.01, enab + 0.1 \]

- **GasOff**
  \[ enab + 0.01, enab + 0.1 \]

- **GasOff2**
  \[ enab + 2, enab + 2 \]

- **GasOff3**
  \[ enab + 0.01, enab + 0.1 \]

- **IgnOff**
  \[ enab + 0.01, enab + 0.1 \]

- **IgnOff2**
  \[ enab + 0.01, enab + 0.1 \]

- **SwitchHROn**
  \[ enab, enab + 10 \]

- **switchHROff**
  \[ enab + 120, enab + 120 \]

- **Inc_Conc**
  \[ enab + 0.1, enab + 0.1 \]

- **Dec_Conc**
  \[ enab + 30, enab + 30 \]

---

Fig. 8: Use case net: gas burner.