Effect of Design Parameters on the Sensitivity of a Thermal Microaccelerometer with Inertial Mass

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Abstract. An analytical method is presented that allows one to determine the influence of the design parameters of a thermal microaccelerometer with inertial mass on its output signal. In the thermal microaccelerometer, a two-dimensional domain of modeling is marked out, in which the main thermal processes take place. This domain is replaced by the equivalent rectangular one, divided into a number of regions with uniform parameters. The temperature distribution in the regions is determined by the Fourier method. The parameters characterizing the processes of heat transfer between the regions are found from the conjugation conditions. Using the proposed method, the dependencies of the output voltage on the design parameters of the microaccelerometer (the gap between the element with inertial mass and the thermally insulated structure, the thickness of the thermally insulated structure, the length of the thermally insulated structure occupied by the battery of film thermocouples, the thermal conductivity of the air in the gap) and the sensitivities of the voltage to change of these parameters are determined.

Keywords: thermal microaccelerometer, temperature distribution, Fourier method, thermally insulated structure, movable element with inertial mass, thin-film heater, battery of thin-film thermocouples

1. Introduction
Modern advances in microsystem technology have allowed a new level in the development of a number of mechanical systems and the creation of new devices. Such devices include those for measuring acceleration in the microsystem version, called microaccelerometers [1, 2]. These devices are microelectromechanical systems and, depending on the physical effects underlying their functioning, they are divided into the following types: capacitive microaccelerometers [1, 2], piezoresistive microaccelerometers [3-5], piezoelectric microaccelerometers [6], microaccelerometers based on tunneling effect [7, 8] and thermal microaccelerometers [9-21]. Of the indicated types of microaccelerometers, fundamentally new types that have no analogs in volumetric design are microaccelerometers based on the tunnel effect and thermal microaccelerometers. Their formation and development is associated with the capabilities of the microsystem technology in obtaining small air and vacuum gaps and elements with a small inertial mass.
2. Formulation of the Problem

Recently, thermal microaccelerometers have attracted considerable interest. This is due to a number of their advantages over other types of microaccelerometers. This microaccelerometers do not require complex measuring circuits, have low sensitivity to electromagnetic and thermal effects. They are characterized by small values of parasitic electromagnetic links and a weak effect of mechanical stresses arising as a result of mounting the microaccelerometer into the case or in the packageless use of it on the object under study.

The principle of operation of thermal microaccelerometers is based on the features of heat transfer processes in microsystems under the action of acceleration. Based on these features, two types of thermal microaccelerometers have been developed:

1) thermal microaccelerometers with inertial mass [9-12];
2) convective thermal microaccelerometers [13-21].

In thermal accelerometers with inertial mass, under acceleration, the size of the gap between the heated stationary element and the movable element of a certain mass changes and, accordingly, the heat flow between the elements changes. This causes the temperature of the heated element to change proportionally to the acceleration effect. In convective thermal microaccelerometers under acceleration, the spatial distribution of thermal convective flow from the heated element changes. This leads to a change in its temperature which is proportional to the magnitude of the acceleration.

For an optimal design of a thermal microaccelerometer, it is necessary to know precisely the features of the thermal processes in it, and first of all, the temperature distribution in its structure. An experimental study of these processes in a thermal microaccelerometer is a difficult task. The reasons for that are the small size and complex structure of the microaccelerometer, in which the heat generating elements are surrounded by other elements. This does not allow one to use contactless methods to study the temperature distribution in the structure of the microaccelerometer. Based on this, the most acceptable way to study thermal processes in thermal microaccelerometers is mathematical modeling. The use of mathematical modeling for these purposes was considered in a number of papers. In [14, 16, 17, 19], the mathematical modeling is used to study processes in convective thermal microaccelerometers. Analysis of processes in thermal microaccelerometers with inertial mass based on mathematical modeling is presented in [9, 10, 12].

For thermal accelerometers with inertial mass the following studies were performed with the help of mathematical modeling:

1) the temperature distribution in the structure of the microaccelerometer has been determined in the absence of acceleration: on the basis of numerical simulation [10]; based on analytical modeling [12];
2) the effect of the displacement of an element with inertial mass on the overheating temperature of the temperature-sensitive element of the microaccelerometer has been considered [9, 10, 12];
3) the frequency response of a thermal microaccelerometer with an inertial mass has been determined [10];
4) the magnitude of the heat fluxes in the structure of the microaccelerometer has been estimated [12].

These results do not provide complete information about the processes in thermal microaccelerometers and cannot sufficiently be the initial basis for the development of design techniques for these accelerometers.

In this connection, it is of interest to further develop methods of modeling of processes in thermal microaccelerometers and to create methods for their design on this basis. On the grounds of this, the aim of the work is to develop a mathematical model of a thermal microaccelerometer with an inertial mass and to study, on the basis of this model, the influence of design parameters of the microaccelerometer on its output signal and sensitivity.

3. Theory. Method of Modeling

The structure of the thermal microaccelerometer with inertial mass is shown in Fig. 1. The microaccelerometer is made of the three assembly units: the base of the housing 1; the frame 2 with the thermally insulated thin film structure 3, on which the thin-film heater 4 and the thin-film tempera-
Figure 1. Structure of the thermal microaccelerometer with inertial mass:
1 – base of the housing; 2 – frame for fixing thermally insulated structure; 3 – thermally isolated structure; 4 – thin-film heater; 5 – thermosensitive element (battery of thin-film thermocouples); 6 – frame for mounting the movable element; 7 – movable element with inertial mass; 8 – cantilever beam.

...ture sensitive element 5 are located; the frame 6 with the fixed mass movable element 7 fixed inside it. The thermally insulated structure is inside the frame 2 and is a two-shoulder thin-film bridge based on a film of silicon dioxide or silicon nitride. It is obtained by the method of anisotropic etching of...
silicon. A thin-film rectangular resistor is used as a heater. The thermal element consists of the two batteries of thin-film thermocouple located on both sides of the heater. The terminals of the thin-film heater and the batteries of the thermopiles are connected to the contact pads, which are located on the base of the frame. On the frame 2 with the thermally insulated structure 3 the frame 6 with the movable element with inertial mass 7 is mounted. The movable element 7 and the thin cantilever beams 8, with the help of which it is attached to the frame 6, are also formed by anisotropic silicon etching. When mounting the frame 6 to the frame 2, a certain amount of clearance is fixed between the movable element with inertial mass 7 and the thermally isolated structure 3. At the final stage of the assembly, the entire structure is mounted on the base of the housing 1.

During the operation of the thermal microaccelerometer the thermally isolated structure is heated with the help of the thin-film heater. The temperature of the structure is measured using a thin-film thermoelectric transducer. Under heating the certain heat flux flows between the thermally isolated structure and the movable element with inertial mass. The value of the flux depends on the distance between the movable element and the thermally isolated structure. Under acceleration the element with inertial mass is displaced and the given distance changes. This in turn causes a change in the magnitude of the heat flux between them and, accordingly, a change in the temperature of the thermally isolated structure.

The displacement of the element with inertial mass under acceleration is equal to

$$\xi = \frac{ml^3a}{4Ebd^5},$$

where $\xi$ is the displacement of the movable element; $m$ is the mass of the movable element; $l$, $b$, and $d$ are the length, width and thickness of the cantilever beams, respectively; $a$ is the acceleration; $E$ is Young’s modulus of the material of the cantilever beams.

In the thermal microaccelerometer under consideration, the main heat fluxes from the thermally isolated structure propagate along the $x$ and $z$ axes. The heat flux is minimal along the $y$ axis due to the use of a thermally isolated bridge structure with cavities along it limiting the heat flux in the $y$ axis direction. In Fig. 2 shows the main heat fluxes in an inertial mass thermal microaccelerometer.

The determination of the temperature distribution in the structure of the thermal microaccelerometer with inertial mass will be made on the basis of the analytical method proposed in [12] specifically for this type of microaccelerometers. Taking into account the features of thermal processes in the accelerometer structure, in [12] it was proposed to use a two-dimensional domain of modeling, marked out from this structure. This domain of modeling is shown in Fig. 3. Given the symmetry of the accelerometer structure in the $xz$ plane, it represents half of the accelerometer structure and is divided into the following regions:

1) regions 1 and 2, which correspond to the air gap between the thermally isolated structure and the movable element with inertial mass; region 1 is located above the section of the thermally isolated structure with the thin-film heater; region 2 – above the section of the thermally isolated structure with the battery of thin-film thermocouples;

2) regions 3 and 4, which correspond to the sections of thermally isolated structure with the thin-film heater and the battery of thin-film thermocouples, respectively;

3) regions 5 and 6, which correspond to the air gap between the thermally isolated structure and the base of the housing; region 5 is located under the section of the thermally isolated structure with the thin-film heater; region 6 – under the section of the thermally isolated structure with the battery of thin-film thermocouples.

When determining the temperature distribution by the analytical method, regions 3 and 4 must be replaced by equivalent regions with homogeneous equivalent parameters, the values of which are

1) region 3: $l_3 = b_h / 2$; $b_3 = d_{is} + d_h$; $\lambda_3 = \frac{d_{is}\lambda_{is} + d_h\lambda_h}{d_{is} + d_h}$;

2) region 4: $l_4 = (l_{is} - b_h) / 2$; $b_4 = d_{is} + d_{ic}$; $\lambda_4 = \frac{d_{is}\lambda_{is} + d_{ic}\lambda_{ic}}{d_{is} + d_{ic}}$,
Figure 2. Distribution of heat fluxes in a thermal microaccelerometer with inertial mass.

where $d_{is}$, $d_h$, and $d_{ec}$ are the thickness of the thermally insulated structure, the resistive layer of the heater and the thermocouple layers, respectively; $\lambda_{is}$, $\lambda_h$, and $\lambda_{ec}$ are the thermal conductivity of the material of the thermally insulated structure, the resistive layer of the heater and the thermocouple layers, respectively.

In region 3 the thin-film heater is located, which is a heat generating element. Taking into account that the thickness of the resistive layer of the heater is less than the thickness of the thermally isolated structure, the heat generation in the resistive layer in the real structure of region 3 can be replaced, in the equivalent structure, by heat generation at the upper boundary of region 3.

For every region of the domain of modeling of the thermal microaccelerometer, the stationary differential heat conduction equation can be represented in the following general form:

$$\frac{\partial^2 T_j}{\partial x_j^2} + \frac{\partial^2 T_j}{\partial y_j^2} = 0,$$

where $T_j$ is the temperature in the region $j$; $x_j$ and $y_j$ are the coordinates in the region $j$.

The solution of the heat equation (2) for the regions with the boundary conditions of the second kind obtained by the Fourier method can be written as follows
Figure 3. Equivalent structure of the domain of modeling for thermal microaccelerometer with inertial mass.

\[
T_j = D^{(j)}_{0,0} + 2 \sum_{k=1}^{\infty} D^{(j)}_{k,0} \cos \left( k \pi x_j / l_j \right) + 2 \sum_{m=1}^{\infty} D^{(j)}_{0,m} \cos \left( m \pi y_j / b_j \right) +
4 \sum_{k=1}^{\infty} \sum_{m=1}^{\infty} D^{(j)}_{k,m} \cos \left( k \pi x_j / l_j \right) \cos \left( m \pi y_j / b_j \right),
\]

where

\[
D^{(j)}_{k,m} = \frac{(-1)^k \delta^{(j,s)}_m - (-1)^m \delta^{(j,t)}_k + \delta^{(j,u)}_m + \delta^{(j,v)}_m}{l_j b_j \lambda_j \left[ (k \pi / l_j)^2 + (m \pi / b_j)^2 \right]};
\]

\( l_j \) and \( b_j \) is the length and width of the, respectively; \( \delta^{(j,s)}_m \), \( \delta^{(j,t)}_k \), \( \delta^{(j,u)}_m \), and \( \delta^{(j,v)}_m \) are weighting coefficients that determine the heat flux densities on the boundaries of the with the regions \( s, t, u \), and \( v \) located, respectively, right, above, left, and below the region \( j \); \( k \) and \( m \) are the summation indices by \( x \) and \( y \) coordinates, respectively.

The weighting coefficients \( \delta^{(j,s)}_m \), \( \delta^{(j,t)}_k \), \( \delta^{(j,u)}_m \), and \( \delta^{(j,v)}_m \) characterize the heat flux densities on the boundaries of the regions. Their values are unknown and can be determined from the boundary conditions of the first kind (the boundaries of regions with the environment), the conjugation boundary
conditions for temperature on the boundaries between the regions and the condition of equality of heat flux densities on the heat generating boundary (the boundary between regions 1 and 3). The given conditions are discussed in detail in [12, 22]. The use of these conditions gives a generalized linear system of equations for the weighting coefficients, whose solution allows us to determine their values [22]. The found values of the weighting coefficients make it possible to find the temperature distribution in the regions of the domain of modeling of the thermal microaccelerometer in accordance with expressions (3) and (4).

Using the found temperature distribution in region 4, the output signal of the thermal accelerometer with inertial mass is determined

\[ U = 2N(\alpha_p - \alpha_n)(T_i|_{x_i|m_{in}} - T_0), \]

where \( U \) is the output voltage of the thermocouple batteries; \( N \) is the number of thermocouples in the right or left battery thermocouples; \( \alpha_p \) and \( \alpha_n \) is the Seebeck coefficient of the material of the positive and negative thermocouple branches, respectively; \( T_4 \) is the temperature in the region 4; \( l_{bg} \) is the distance from the beginning of the region 4 to the line of the location of the hot junctions of thermocouples.

4. Results of Modeling

The proposed modeling method was used to analytically study the influence of the design parameters on the output signal of the thermal microaccelerometer and to determine the sensitivity of the output signal to changes of these parameters. The following values of the design parameters of the thermal microaccelerometer were used in the simulation. As the material of thermally isolated structure, silicon dioxide with the thermal conductivity coefficient of 1.1 W/(m·K) was chosen. The dimensions of the thermally isolated structure varied within the following limits: length – 60 ... 800 μm; thickness – 0.5 ... 3.0 μm. The width of the thermally isolated structure was considered constant and was 250 μm. The distance from the thermally insulated structure to the base of the housing was equal to the thickness of the silicon wafer and was 200 μm. The branches of the thin-film thermocouples were made of p- and n-type polysilicon and had a thickness of 0.3 μm. The Seebeck coefficients of polysilicon were: p-type – +200 μV/K; n-type – −200 μV/K. The branches of film thermocouples had the same width of 5 μm. The distance between the branches was equal to their half width. The number of thermocouples in each battery was 15. The resistive layer of the thin-film heater was also made of polysilicon and had 0.3 μm thick. The thermal conductivity coefficient of polysilicon was 22 W/(m·K). The parameters of the thin-film heater were equal to: the resistance – 100 Ohm; the temperature coefficient of resistance – 0.83·10^{-3} 1/K; resistive layer width – 20 μm; form factor – 12; current through the heater – 2.5 mA. The remaining parameters had the following values: the ambient temperature – 300 K; the thermal conductivity coefficient of air – 0.026 W/(m·K).

For the structure of the thermal microaccelerometer under consideration, the following dependencies of its output voltage and the sensitivity of the output voltage to changes of its design parameters were determined:

1) the dependencies on the distance between the element with inertial mass and the thermally isolated structure (Fig. 4);
2) the dependencies on the thickness of the thermally isolated structure (Fig. 5);
3) the dependencies on the length of the section of the thermally isolated structure occupied by the battery of the thin-film thermocouples (Fig. 6);
4) the dependencies on the thermal conductivity of the air in the gap (Fig. 7).

5. Results Discussion

The dependence of the output voltage of the thermal microaccelerometer on the distance between the movable element with inertial mass and the thermally isolated structure is shown in Fig. 4. As can be seen from this dependence, the output voltage of the microaccelerometer varies nonlinearly with increasing the distance between the movable element and the structure. Taking into account that the
displacement of the movable element is directly proportional to acceleration, this feature lead to a non-linear dependence of the output voltage of the microaccelerometer on the applied acceleration. This is confirmed by the dependence of the sensitivity of the output voltage to the change in the distance between the movable element with inertial mass and thermally isolated structure on this distance (Fig. 4). The presented data show that with increasing the distance between the movable element and the thermally isolated structure, the sensitivity of the output voltage to a change in this distance decreases by 10 times.

Fig. 5 shows the dependence of the output voltage of the thermal microaccelerometer on the thickness of the thermally isolated structure and the sensitivity of the output signal to change of the this thickness. The data show, that with increasing the thickness of the thermally isolated structure by 6 times from 0.5 μm to 3 μm, the output voltage decreases almost linearly by only 15%. This indicates the fact that for the selected values of other design parameters, the influence of the thickness of the thermally isolated structure on the output voltage of the microaccelerometer does not matter much. This conclusion is confirmed by the dependence of the sensitivity of the output signal to change of the thickness of this structure. The sensitivity is greatest when the thickness of the thermally isolated structure is small and is −8 mV / μm. As the thickness increases to 3 μm, it decreases, reaching about −5.5 mV/μm.

The dependences of the output voltage of the thermal microaccelerometer on the length of the section of the thermally insulated structure occupied by the battery of thermocouples and the sensitivity of the output voltage to change of the length of this section are shown in Fig. 6. Analysis of these dependences indicates that with increasing the length of the section of the thermally isolated structure occupied by the battery of thermocouples, a sharp increase in the output signal of the thermocouples is observed, about 3 times. The further increase of the length of this section does not lead to increasing the output voltage of the microaccelerometer. Its value becomes almost constant. This is also confirmed by the dependence of the sensitivity of the output voltage to change of the length of the given section of the thermally isolated structure. The sensitivity decreases to zero.

Fig. 7 shows the dependence of the output voltage of the thermal microaccelerometer on the thermal conductivity of the air in the gap between the movable element with inertial mass and the thermally isolated structure. With increasing the thermal conductivity of air in the gap by 10 times (from 0.02 W/(m·K) to 0.2 W/(m·K)), the output voltage of the microaccelerometer decreases almost 4 times. For large values of this thermal conductivity, the sensitivity of the output signal of the microaccelerometer to change its value has negative value and with increasing the thermal conductivity is significantly reduced in absolute value.

6. Conclusion
The paper proposes a method of analytical modeling, which allows one to determine the influence of design parameters on the characteristics of a thermal microaccelerometer with inertial mass: its output voltage and sensitivity of the output voltage to changes in design parameters. As a result of the simulation, it was found that on the output voltage of the thermal microaccelerometer with an inertial mass the following design parameters have the greatest effect:

1) size of the gap between the movable element and the thermally isolated structure at small values of the gap size;
2) length of the sections on which the batteries of thin-film thermocouples are located with small sizes of these sections;
3) thermal conductivity coefficient of the gas medium in the gap between the movable element with inertial mass and the thermally isolated structure at small values of this thermal conductivity coefficient.

The proposed method can be used to study the dependence of the output signal of a thermal microaccelerometer on the magnitude of the acceleration and on the values of the design parameters of the microaccelerometer. Based on this method, CAD elements of design systems for thermal microaccelerometers with inertial mass can be developed.
**Figure 4.** Output voltage of the thermal microaccelerometer and the sensitivity of the output voltage to the change of the distance between the element with inertial mass and the thermally isolated structure as functions of this distance.

**Figure 5.** Output voltage of the thermal microaccelerometer and the sensitivity of the output voltage to the change of the thickness of the thermally isolated structure (TIS) as functions of the thickness of this structure.

**Figure 6.** Output voltage of the thermal microaccelerometer and the sensitivity of the output voltage to the change of the length of section of the thermally isolated structure (TIS) occupied by the thermocouples as functions of the length of this section.

**Figure 7.** Output voltage of the thermal microaccelerometer and the sensitivity of the output voltage to the change of the thermal conductivity of air in the gap between the element with inertial mass and the thermally isolated structure as functions of the value of this thermal conductivity.
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