Model of analysis of the results of geometry scanning of turbine nozzles

E Yu Pechenina¹, V A Pechenin and M A Bolotov
Samara National Research University, 34, Moskovskoye shosse, Samara, 443086, Russia
¹E-mail: ek-ko@list.ru

Abstract. The paper presents a mathematical model to perform computer calculation for open flow area of turbine nozzles. The model is used to work with objects in *.stl format when processing data after scanning. Using the developed model, calculations of the part quality parameters will not depend on the controller qualifications, and the proposed approach will not require unique adjustment, costly repair and storage of reference patterns for all sizes of nozzle devices. The calculation was performed in three sections, which improves accuracy while reducing labor intensity and cost of measurement. Increasing the number of sections practically does not affect the labor intensity of calculations, so there are no significant restrictions on performing virtual measurements in four or more sections. The model was implemented in the mathematical package MATLAB. The model was tested with the example of STL processing of the turbine nozzle blades sector.

1. Introduction

Turbine rotors and nozzles are particularly difficult to manufacture and assemble. Quality control of assemblies is also quite a time-consuming operation [1]. Currently, aircraft engine manufacturers are equipped with optical scanners to determine deviations of manufactured parts and assemblies from nominal models.

As a result of measuring parts and assemblies on optical scanners, faceted models in *.stl format appear, to which traditional CAD modeling tools cannot be applied, and the calculation of specific geometric parameters requires additional processing of the measured data. The paper presents a model for analyzing and calculating such a key turbine assembly parameter as the nozzle passage section area (PA).

The PA of the interblade channel is defined as the area of the surface area bounded by a contour, which consists of the curves of the blade feather cut faces forming the channel, and the transition radii from the feather to the flanges, which provide the smallest width cross section in the reamer. Ultimately, the assembly is interested in the irregularity of the PA of the channels in the entire nozzle apparatus, not in the absolute values of the PA of each channel. For this reason, PAs are changed in production by hand-held indicating devices as a deviation from the reference window, with individual parameters measured, allowing the calculation to be performed using an equation:

$$F_{\text{min}} = K \cdot H \cdot (b_1 + b_2) / 2,$$  \hspace{1cm} (1)
being $K$ – empirical correction coefficient, $b_1$ and $b_2$ – width of the measured interblade channel at distances $R_1$ and $R_2$ from the motor axis respectively, $H$ – height of the interblade channel between $b_1$ and $b_2$.

Increasing the number of sections in equation (1) will increase the reliability of the calculations, but in manual measurement, it entails a significant increase in labor intensity. In the developed model, the PA calculation was performed in three sections and there are no significant restrictions on performing virtual measurements in four or more sections.

2. Model for calculating the cross-sectional area

The Stl model is a piecewise linear surface. It is characterized by the following parameters: $V_{g×3}$ – matrix of coordinates of vertices of stl-model grid, $F_{m×3}$ – matrix of combinations of three vertices (rows contain ordinal numbers of vertices) that form facets of surfaces, $N_{m×3}$ – coordinate matrix of facet normals. The calculation model is designed for a pre-segmented model, that is, one in which there are areas of facet associations responsible for certain facets of the model. The stages of the model are shown in figure 1.

The coordinate system construction involves finding the motor axis center. To do this, the cylindrical surfaces (1 and 2 on figure 2) are inscribed with replacement "cylinder" elements using the method of least squares (MLS). The axis alignment is performed then by multiplying the STL vertex matrix by the rotation matrices.

\[ \mathbf{V}_r = \mathbf{V} \cdot \mathbf{M}_{n-y} \cdot \mathbf{M}_{n-z}. \]  

being $\mathbf{M}_{n-y}$, $\mathbf{M}_{n-z}$ – the rotation matrices around the axes $OY$ and $OZ$. The rotation angles are calculated from the normal vector coordinates of cylindrical surface 2, figure 3.

Surface 1 (figure 3) shall lie "to the right" on the major axis, surface 2 "to the left", so the direction of the major axis is determined. The coordinate system center is at the midpoint $x_c$ between the centers of the cylindrical surfaces. The aligned coordinates are moved by the coordinate value of this center according to the equation:

\[ \bar{\mathbf{v}}_n = \mathbf{v}_n - \bar{x}_c, \]  

being $\bar{\mathbf{v}}_n, \mathbf{v}_n$ – coordinate vectors of the $i$-th vertex of STL before and after the displacement. As a result, a matrix of aligned and displaced STL vertex coordinates is generated $\mathbf{V}_n$ for further calculations.

![Figure 1. Stages of the mathematical model.](image-url)
Figure 2. Base surfaces segmented on stl.

Figure 3. Creation of coordinate system 1, 2 – front and rear base surfaces, 3 – base axis, 4 – back surface; 5 – second major axis.

The second step is to calculate the points of intersection of the back and pressure side surfaces with the cylindrical surfaces. The axes of the cylindrical surfaces coincide with the principal axis of the coordinate system, the values of the radii are determined by measuring altitudes of the minimum section values between the surfaces of the back and pressure side. The more such measurements are performed, the more accurately it is possible to calculate the unevenness of the PA values of the inter-blade channels. In this work, the PA are calculated using three cross-sections (figure 4).

Figure 4. Calculation of the passage area for section 1 – widths of inter-blade channels at different altitudes; 2 – cross-sections of the pressure side and back surfaces; 3 – surface of the pressure side; 4 – surface of the back.
The block diagram of the algorithm for finding the point on the intersection line of the stl surface and the cylindrical surface is shown in Figure 4 (figure 5). The basic axis is OX.

1. Converting YOZ into the polar coordinate system  
2. Finding facets that intersect the radius cylinder R  
3. Finding the polar angle \( \varphi \) for a point with given coordinates \((\rho_0, x_0)\)  
4. Reconvertiong to Cartesian system

**Figure 5.** The algorithm for calculating the level line point on the surface.

The algorithm requires finding a point on the surface \( S_{stl} \) at the corresponding radius \( R \) from the coordinate system center with coordinate \( x_0 \) along the reference axis. In the first step, the coordinates of the two axes (except for the reference one) are converted to the polar coordinate system. Now, the vertices STL have the coordinates \((\rho, \varphi, x)\).

In the second step, the vertices STL have the coordinates \((\rho, \varphi, x)\) – the polar radii of the facet vertices. As a result, a significantly smaller area is selected from the surface \( S_{stl} \) to find the level line point.

The third step is to find the polar angle for a point with given coordinates \((\rho_0, x_0)\). The selection area for assigning the coordinate is \( x_0 \) chosen based on the minimum and maximum values of this coordinate for the vertices of the area \( S_{stl} \). To check if the image point belongs to the facet, three values are calculated \( d_{ver} \) (in projection on the plane \( XO\rho \)):

\[
\begin{align*}
    d_{ver1} &= (x_1 - x_0) \cdot (\rho_2 - \rho_0) - (x_2 - x_1) \cdot (\rho_1 - \rho_0), \\
    d_{ver2} &= (x_2 - x_0) \cdot (\rho_3 - \rho_0) - (x_3 - x_2) \cdot (\rho_2 - \rho_0), \\
    d_{ver3} &= (x_3 - x_0) \cdot (\rho_1 - \rho_0) - (x_1 - x_3) \cdot (\rho_3 - \rho_0),
\end{align*}
\]

being \( x_1, y_2, x_2, \rho_2, \rho_3, \rho_3, \rho_3 \) – the coordinates of the vertices \( j \) of the – facet.

If the quantities \( d_{ver} \) in equation (5) are of the same sign, the point is inside the facet.

After determining the facet to which the point belongs, the missing polar angle coordinate of the point is searched for, based on the equation of the straight facet:

\[
    \varphi_0 = z_1 + \frac{n_1(x_1 - x_0) + n_3(\rho_1 - \rho_0)}{n_2},
\]

being \( \{n_1, n_2, n_3\} \) – coordinates of the normal vector of the facet.

At the third stage of the PA calculation, the coordinates of the points to calculate the bottleneck between the sections are searched for. Consider the algorithm for calculating this bottleneck for a single section.

On the pressure side, the point \( P_{ps, P_4} \) that has the largest coordinate along the axis of rotation is selected. Through the points of the back section, the equation of the interpolating normalized spline of
degree 3 is given [2]. The coordinates of the spline point between the two set points $P_0$ and are $P_1$ determined from the equation:

$$P(u) = \begin{bmatrix} 1 - 3u^2 + 2u^3 & 3u^2 - 2u^3 & u - 2u^2 + u^3 & -u^2 + u^3 \\ P_0' & P_1' & P_0' & P_1' \end{bmatrix},$$

(7)

here the vectors $P_0', P_1'$ are vectors of derivatives at the points of $P_0$, $P_1$; $u$ – dimensionless parameter lying in the range $[0; 1]$.

All distances between the set points of the spline and the point $P_{ps, PA}$ are calculated. Two spline segments in the vicinity of the set point closest to $P_{ps, PA}$. Calculation of the back point coordinates $P_{bp, PA}$, located at the section end of the smallest width, was carried out by the method of golden section [3]. The search criterion was to minimize the distance between the points $P_{ps, PA}$ and $P_{bp, PA}$.

The error of the algorithm $\varepsilon$ was assumed to be $10^{-4}$.

The final stage of calculating the PA is based on calculating the sum of the areas of the quadrangles formed by adjacent sections of the smallest width, the ends of which are connected by segments. Equation for calculating the area of the passage section with a set of section points $P_{ps, PA,i}$, $P_{bp, PA,i}$ looks like:

$$S_{PA} = \sum_{i=1}^{m-1} d_1 \cdot d_2 / 2 \cdot \sin \beta,$$

(8)

being $d_1$ and $d_2$ – are the diagonals of each quadrilateral, distances $|P_{ps, PA,i}, P_{bp, PA,i+1}|$ and $|P_{ps, PA,i+1}, P_{bp, PA,i}|$; $\beta$ – acute angle between the diagonals of the $i$-th quadrilateral; $m$ – number of sections.

3. Segmentation of the scanned surface edges

Each point of a regular surface in three-dimensional Euclidean space is characterized by two extreme values of curvatures $k_{min}$ and $k_{max}$, called principal [2]. For a linear STL-surface, the calculation of the principal curvatures based on the derivatives is a nontrivial problem; various approaches have been developed to calculate them correctly [4,5,6], in particular, the results from [4] were used in the study.

Based on the principal curvature tensor, the surfaces are segmented (divided into separate regions), in the current study, the algorithm was used from [7].

After model segmentation, recognition of the target faces (base, back and pressure side surfaces) was performed. Recognition can be performed manually, but this will increase the labor intensity of the calculations, so this stage has been automated by using computer vision techniques [8], in particular convolutional neural networks [9]. To train a neural network, a training sample is needed to prepare, and to assess the quality – a test sample. Various STL geometric parameters can be used as input data [10]. Taking into account the segmentation already performed, the task is simplified, and the projections of the model onto six planes (front, back, left, right, top and bottom view) parallel to the coordinate planes are used as input data to the neural networks. STL object is placed as if in a cube the faces of which are parallel to the coordinate planes [11]. The Roberts algorithm was used to prepare the projections.
The output is also six projections of the parts, with the difference that the faces on them are marked. The target facets for recognition are the facets on which the assembly is mated and controlled. Accordingly, these facets are distinguished by different colors, the other facets of the body have a single and different color.

In order to perform recognition, we used U-net neural network [12], the output of which is a picture in "shades of gray". A view of one of the stl projections, the same projection marked for network training, and the output of the neural network is shown in figure 6.

![Figure 6. a) original stl projection; b) marked projection; c) recognition using U-net.](image)

Segmentation and facet recognition is a necessary step to automate calculations of the PA parameter using the developed model.

4. The results of the experimental study

The developed model was implemented in the mathematical package MATLAB. For testing, a sector model of three nozzle blades was used. In the NX package, sections and PA were measured. The solid model was converted to STL with 411524 facets (figure 7). In order to perform the calculations, the STL surface was segmented into individual facets. The training sample for U-net was 1000 artificially generated cases, the test sample was 50 cases. Using the recognition results, the facets required for the calculations were selected (figure 8). The number of back facets was 24266, the pressure side – 28336.

![Figure 7. STL sector model.](image)

![Figure 8. Segmented facets.](image)
Measurement radii $R_1$, $R_2$ and $R_3$ are 205.5; 232.5 and 257.5 mm, respectively. The calculated area on the model was 701.234 mm$^2$. The boundaries of the segmented facets are not perfectly flat, since they represent many adjacent triangles in space. Therefore, it is important to select the necessary offset from the boundaries of the measurement sections, while at the same time, that it was not too big. Table 1 shows the calculation results of the cross sections for the smallest width and PA when changing the offsets from the pressure side boundary.

**Table 1. Errors in changing the offsets from the surface boundaries.**

| Offset, % | Reference, mm | Calculated values, mm | Offset, mm |
|-----------|---------------|-----------------------|------------|
|           | 1 Flow area   | 2 Flow area           | 1 Flow area | 2 Flow area |
| 0         | 11.95         | 13.52                 | 0.11       | 0.17       |
| 5         | 12.06         | 13.69                 | 0.08       | 0.07       |
| 8         | 12.09         | 13.66                 | 0.14       | 0.13       |
| 15        |               |                       |            |            |

Optimal in terms of accuracy was the use of 5% offset from the boundaries of the surfaces. In addition, the calculation accuracy is also affected by the number of section points. Table 2 shows the calculation results of the sections for the smallest width and PA when changing the number of points on the splines (4).

**Table 2. Errors in changing the number of spline points**

| Number of points | Reference, mm | Calculated values, mm | Offset, mm |
|------------------|---------------|-----------------------|------------|
|                  | 1 Flow area   | 2 Flow area           | 1 Flow area | 2 Flow area |
| 10               | 11.95         | 13.52                 | 0.09       | 0.07       |
| 20               | 12.04         | 13.59                 | 0.08       | 0.07       |
| 30               | 12.03         | 13.59                 | 0.08       | 0.07       |
| 50               | 12.03         | 13.59                 | 0.08       | 0.07       |
|                  |               |                       |            |            |
| Number of points | 1 Flow area   | 2 Flow area           | 1 Flow area | 2 Flow area |
| 10               |               |                       | 0.17       | 3.54       |
| 20               | 14.967        | 701.234               | 0.06       | 2.48       |
| 30               | 15.03         | 703.72                | 0.06       | 2.44       |
| 50               | 15.03         | 703.70                | 0.06       | 2.47       |

The deviation of the sections when using 30 points did not exceed 0.09 mm, the deviation of PA – 2.44 mm. Taking into account the measurement errors of scanners 0.05-0.1 mm and relatively low discreteness of the partition of the considered STL, the obtained calculation errors are acceptable.

The instrumental error in existing measurement technologies using indicators is from 1 mm$^2$, so that the computer calculation is comparable in accuracy with manual measurement, and in terms of performance is significantly higher.

**5. Conclusion**

The results allow us to automate and improve the accuracy of calculating the quality parameters of individual parts, as well as significantly reduce the labor intensity of control operations after assembling.
The considered model is one of the stages of nozzle assembly using the "virtual assembly" approach. The measured geometry can be used to perform a virtual assembly [13] that is to create a set of adequate digital mathematical models of the assembled product, taking into account the most essential properties and processes of this particular instance of any real product. Based on such a virtual product, assembly geometric and physical parameters can be determined prior to real assembly and testing. Thus, virtual assembly technology makes it possible to reduce the labor intensity of the process and to choose the optimal assembly option that provides the required quality indicators.

Acknowledgments
This paper was financially supported by the Ministry of Science and Higher Education of the Russian Federation as part of the Russian Federation Presidential Scholarship (number SP-262.2019.5). Experimental studies were performed on the equipment of the common use center of CAM-technologies (RFMEFI59314X0003).

References
[1] Zakharov O V, Balaev A F, Kochetkov A V 2017 Procedia Engineering 206 1458-63
[2] Rogers D, Adams J 1990 Mathematical Elements for Computer Graphics (London: McGrawHill) p 604
[3] Powell M J D 1978 ed. G.A. Watson, Lecture Notes in Mathematics (Berlin: Springer Verlag) 144-57
[4] Cohen-Steiner D and Morvan JM 2003 In Proceedings of the nineteenth annual symposium on Computational geometry 8 312-21
[5] Meyer M, Desbrun M, Schröder P and Barr 2003 In Visualization and mathematics III Berlin Heidelberg 35-57
[6] Taubin G 1995 In Proceedings of IEEE International Conference on Computer Vision. 902-07
[7] Lavoué, G Dupont F and Baskurt A 2005 Comput. aided des. 37 975-87
[8] Zakani F R Arhid K Bouksim M Aboulfatah M and Gadi T 2018 Computer Optics 42 312-19
[9] Rosebrock A Deep Learning for Computer Vision with Python 2017 (PyImageSearch.com) p. 332
[10] Qi C R Su H Mo K and Guibas L J 2017 In Proceedings of the IEEE conference on computer vision and pattern recognition 652-60
[11] Bolotov M A, Pechenin V A, Ruzanov N V, Kolchina E J 2019 CEUR Workshop Proceedings 2391 342-49
[12] Ronneberger O Fischer P and Brox T 2015 October In International Conference on Medical image computing and computer-assisted intervention 234-41
[13] Rezchikov A F, Kochetkov A V and Zakharov O V. 2017 MATEC Web Conf. 129 01054