Vacuum Structure of the $\mu$-Problem
Solvable Extra U(1) Models

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Abstract
Vacuum structure and related phenomenological features are investigated in $\mu$-problem solvable supersymmetric extra U(1) models. We present a framework for the analysis of their vacuum structure taking account of an abelian gauge kinetic term mixing, which can potentially modify a scalar potential and $Z^0$ gauge interactions. Applying this to data of the precise measurements at LEP, we constrain an allowed region in a space of Higgs vacuum expectation values based on consistency with potential minimum conditions. We find that such a region is confined into rather restricted one. Bounds on masses of an extra U(1) gauge boson and the lightest neutral Higgs boson are predicted. Renormalization group equations for gauge coupling constants and gaugino soft masses in an abelian gauge sector are also discussed in relation to the gauge kinetic term mixing in some detail.

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1 Introduction

Although the standard model (SM) has been confirmed in incredible accuracy through precise measurements at LEP, it is still not considered as the fundamental theory of particle physics and its various extensions have been proposed. One direction of such extensions is supersymmetrization of the SM from a viewpoint of gauge hierarchy problem. For this problem it is now considered as the most promising extension of the SM [1]. Another direction is the extension of gauge structure and it is represented by GUT models like SU(5) and SO(10). Among such extensions the simplest one is an addition of an extra U(1) factor group to the SM gauge structure. It is an interesting aspect of this extension that this kind of gauge structure often appears in the low energy effective models of perturbative superstring [2].

Even in the supersymmetrized models a theoretically unsatisfactory feature remains from the viewpoint of naturalness. This is known as a \( \mu \)-problem [3]. The supersymmetric SM has a supersymmetric Higgs mixing term \( \mu H_1 H_2 \). In order to induce the weak scale correctly, we should keep \( \mu \sim O(G_F^{-1/2}) \) by hand, where \( G_F \) is a Fermi constant. On the other hand, in the supersymmetric models a typical low energy scale is generally characterized by a supersymmetry breaking scale \( M_S \) in an observable sector. There is no reason why \( \mu \) should be such a scale since \( \mu \) parametrizes a supersymmetric term and then it seems rather natural to take it as a cut-off scale like \( M_{Pl} \).

A reasonable way to answer this question is to consider an origin of \( \mu \) scale as a result of supersymmetry breaking [4]. One of such solutions is an introduction of a singlet field \( S \) and replace \( \mu H_1 H_2 \) by a Yukawa type coupling \( \lambda SH_1 H_2 \) [5]. If \( S \) gets a vacuum expectation value (VEV) of order 1 TeV as a result of both supersymmetry breaking and radiative corrections to soft supersymmetry breaking terms [6], \( \mu \sim O(G_F^{-1/2}) \) will be dynamically realized through \( \mu = \lambda \langle S \rangle \). It is noticable that the models extended by an extra U(1) symmetry which is broken by a SM singlet field \( S \) can have this feature [7 - 12]. The existence of this U(1) can also make it free from the tadpole problem usually unavoidable in the models with gauge singlet Higgs scalars. Thus the supersymmetric SM extended with an extra U(1) symmetry can be considered as one of the simplest and most promising extensions of the supersymmetric SM. This kind of models have various interesting features and their phenomenological aspects have been studied by various authors [7 - 12].
It is worthy to note that the extra U(1) models can have another interesting feature. In principle, there can be a kinetic term mixing among abelian gauge fields because their field strength is gauge invariant. A decade ago it was suggested that such a mixing might appear in suitable unified models [13] and also in the effective theory of perturbative superstring [14, 15]. Following these works, in the supersymmetric models extended with an extra U(1), the running of gauge coupling constants [16] and also the effects on the electroweak parameters [17] due to this mixing have been studied. Recently the relation of the kinetic term mixing to the leptophobic property and the electroweak parameters has also been intensively studied in $E_6$ inspired extra U(1) models [18, 19, 20].

In supersymmetric models gauge fields are embedded in the vector superfields. This means that the similar effect appears also in other component fields contained in the vector superfields, that is, gauginos and auxiliary fields $D$. Recently, it has been shown that gauge kinetic term mixing can cause additional interesting effects in various phenomena through the neutralino sector [21, 22]. However, there still seems to remain an unstudied interesting aspect due to the modification of $D$ fields related to the vacuum structure. In this paper we investigate its effect on the scalar potential and the gauge interaction sector to examine the vacuum structure of such models. As one of its results, we can predict the bounds for masses of an extra neutral gauge boson and the lightest neutral Higgs boson.

The organization of this paper is as follows. In section 2 we review general features of physical effects of the gauge kinetic term mixing in the models with $U(1)_a \times U(1)_b$ gauge symmetry from various points of view. In section 3 we introduce the $\mu$-problem solvable extra U(1) models studied in this paper. In section 4 the discussion in section 2 are applied to formulate a framework for the study of a vacuum structure of the models in the basis of both the results of precise measurements at LEP and the potential minimization. Using numerical analysis based on this framework we constrain an allowed region in a space extended by the Higgs VEVs. We also discuss the mass bounds of the extra neutral gauge boson and the lightest neutral Higgs boson. Section 5 is devoted to the summary. In an appendix we review the derivation of the electroweak parameters in the extra U(1) models.

2 Kinetic term mixing
2.1 General feature

At first we review the general features of gauge kinetic term mixing in the case of \( U(1)_a \times U(1)_b \) model \([18]\). This is also aimed to fix various notations used in the following arguments. The Lagrangian considered here is written as

\[
L = -\frac{1}{4} F_{\mu\nu}^{a} \hat{F}^{a}_{\mu\nu} - \frac{1}{4} \hat{F}^{b}_{\mu\nu} \hat{F}^{b}_{\mu\nu} - \frac{\sin \chi}{2} \hat{F}^{a}_{\mu\nu} \hat{F}^{b}_{\mu\nu} + |\hat{D}_\mu \phi|^2 + i \bar{\psi} \gamma^\mu \hat{D}_\mu \psi. \tag{1}
\]

where \( \sin \chi \) parametrizes the gauge kinetic term mixing.

A covariant derivative \( \hat{D}_\mu \) is defined as \( \hat{D}_\mu = \partial_\mu - ig_0^a Q_a \hat{A}_\mu^a - ig_0^b Q_b \hat{A}_\mu^b \). We can change this Lagrangian into the one written in a canonically normalized basis by resolving this mixing in terms of the following transformation,

\[
\begin{pmatrix}
\hat{A}_\mu^a \\
\hat{A}_\mu^b
\end{pmatrix}
= \begin{pmatrix}
1 & -\tan \chi \\
0 & 1 / \cos \chi
\end{pmatrix}
\begin{pmatrix}
A_\mu^a \\
A_\mu^b
\end{pmatrix}. \tag{2}
\]

The resulting Lagrangian can be expressed as

\[
L = -\frac{1}{4} F_{\mu\nu} F^{a}_{\mu\nu} - \frac{1}{4} F_{\mu\nu}^{b} \hat{F}^{b}_{\mu\nu} + |D_\mu \phi|^2 + i \bar{\psi} \gamma^\mu D_\mu \psi. \tag{3}
\]

Now the covariant derivative is altered into

\[
D_\mu = \partial_\mu - ig_a Q_a A_\mu^a - i (g_{ab} Q_a + g_b Q_b) A_\mu^b, \tag{4}
\]

where

\[
g_a = g_0^a, \quad g_{ab} = -g_0^a \tan \chi, \quad g_b = \frac{g_0^b}{\cos \chi}. \tag{5}
\]

Physical phenomena should be considered by using this new Lagrangian. The mixing effects are confined into the interactions between the \( U(1)_b \) gauge field and matter fields.

In this new Lagrangian a gauge coupling constant \( g_b \) is varied from the original one and also a new off-diagonal gauge coupling \( g_{ab} \) appears.

Here we should note that Eq. (2) is not a unique choice of the basis which resolves the mixing. There is an additional freedom of the orthogonal transformation. Another useful basis \((\hat{A}_\mu^a, \hat{A}_\mu^b)\) is related to the basis \((A_\mu^a, A_\mu^b)\) by the orthogonal transformation. By fixing the definition of charges \((Q_a, Q_b)\) of \( U(1)_a \times U(1)_b \), these bases are related by the orthogonal rotation as follows,

\[
\begin{pmatrix}
A_a \\
A_b
\end{pmatrix}
= \begin{pmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
\hat{A}_a \\
\hat{A}_b
\end{pmatrix},
\]
\[
\left( \begin{array}{cc}
g_a & g_{ab} \\
0 & g_b
\end{array} \right) = \left( \begin{array}{cc}
\bar{g}_a & \bar{g}_{ab} \\
\bar{g}_{ab} & \bar{g}_b
\end{array} \right) \left( \begin{array}{cc}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{array} \right).
\] (6)

We call this new basis as the symmetric basis hereafter. Under this basis the covariant derivative is represented as

\[
D_\mu = \partial_\mu - i (\bar{g}_a Q_a + \bar{g}_{ba} Q_b) \bar{A}_\mu^a - i (\bar{g}_{ab} Q_a + \bar{g}_b Q_b) \bar{A}_\mu^b.
\]

These changes induced by resolving the mixing can bring various effects on the low energy phenomena. A typical example of such effects is a radiative correction to gauge coupling constants. For the study of the running of abelian gauge coupling constants, these effects should be taken into account. In the present model the one-loop renormalization group equations (RGEs) for these couplings generally take a \(2 \times 2\) matrix form \([16]\). If we use \(t = \ln M/M_0\) where \(M\) is a renormalization point, these can be written as

\[
\frac{d}{dt} \left( \begin{array}{cc}
\bar{g}_a & \bar{g}_{ab} \\
\bar{g}_{ba} & \bar{g}_b
\end{array} \right) = \left( \begin{array}{cc}
\bar{g}_a & \bar{g}_{ab} \\
\bar{g}_{ba} & \bar{g}_b
\end{array} \right) \left( \begin{array}{cc}
\bar{\beta}_a & \bar{\beta}_{ab} \\
\bar{\beta}_{ab} & \bar{\beta}_b
\end{array} \right),
\] (7)

in the symmetric basis. On the other hand, using the basis taken in Eq. (4), RGEs are expressed as \([18]\).

\[
\frac{d}{dt} \left( \begin{array}{cc}
g_a & g_{ab} \\
0 & g_b
\end{array} \right) = \left( \begin{array}{cc}
g_a & g_{ab} \\
0 & g_b
\end{array} \right) \left( \begin{array}{cc}
\beta_a & \beta_{ab} \\
0 & \beta_b
\end{array} \right).
\] (8)

The rotation angle \(\theta\) in Eq. (6) changes with the energy scale as

\[
\frac{d\theta}{dt} = \bar{\beta}_{ab} \cos 2\theta + \frac{1}{2}(\bar{\beta}_a - \bar{\beta}_b) \sin 2\theta.
\] (9)

Although \(\beta\)-functions in Eqs. (7) and (8) depend on the matter contents in a considering model, their general forms in the symmetric basis can easily written down \([16]\).

In connection with the running of gauge coupling constants it will also be useful to comment on the relation between the charge normalization and the initial condition for RGEs study. In the usual unified models based on a simple group, there is no kinetic term mixing at the unification scale \(M_U\) among its low energy abelian factor groups. However, their kinetic term mixing can appear at the lower energy region through the multiplets splitting due to the symmetry breaking at the intermediate scale \([13]\). In this case if we assume a unified coupling constant to be \(g_U\), the abelian charges may be normalized at the unification scale as usual,

\[
g_U^2 = g_a^{02} \text{Tr} Q_a^2 = g_b^{02} \text{Tr} Q_b^2.
\] (10)
On the other hand, if we consider the models derived from superstring the kinetic term mixing can occur even at the string scale or unification scale $M_U$. In that case Eq. (10) should be modified as

$$g_U^2 = g_a^2 \text{Tr} Q_a^2 = \frac{g_a g_b}{g_a + g_b^2} \text{Tr} Q_b^2,$$

(11)

where we substitute the relations (5) into Eq. (10). This equation shows that the initial values of gauge couplings can be shifted. Usually charges are normalized as $\text{Tr} Q_a^2 = \text{Tr} Q_b^2$ and thus $g_a^0 = g_b^0$ is satisfied. However, Eq. (11) shows $g_a < g_b$ even at $M_U$ as a result of the kinetic term mixing due to some dynamics above the string scale.

In this model an additional mass mixing between $A^a_\mu$ and $A^b_\mu$ also generally appears after spontaneous symmetry breaking of $U(1)_a \times U(1)_b$ due to some VEVs of suitable Higgs scalar fields. This mixing can be resolved by the orthogonal transformation

$$\begin{pmatrix}
A^a_1 \\
A^b_1 \\
A^a_2 \\
A^b_2 
\end{pmatrix} = \begin{pmatrix}
\cos \xi & \sin \xi \\
-\sin \xi & \cos \xi 
\end{pmatrix} \begin{pmatrix}
A^a_1 \\
A^b_1 \\
A^a_2 \\
A^b_2 
\end{pmatrix}.
$$

(12)

If we write the mass matrix as

$$\begin{pmatrix}
m_a^2 & m_{ab} \\
m_{ab} & m_b^2 
\end{pmatrix},
$$

(13)

the mixing angle $\xi$ can be written as,

$$\tan 2\xi = \frac{-2m_{ab}^2}{m_b^2 - m_a^2}.
$$

(14)

### 2.2 Supersymmetric extension

We now consider a supersymmetric extension of the discussion in the previous subsection. In that case gauge fields are extended to vector superfields

$$V_{\text{WZ}}(x, \theta, \bar{\theta}) = -\theta \sigma_\mu \bar{\theta} V^\mu + i \theta \theta \bar{\theta} \bar{\lambda} - i \bar{\theta} \theta \theta \lambda + \frac{1}{2} \theta \theta \bar{\theta} \bar{\theta} D,
$$

(15)

where we use the Wess-Zumino gauge. A gauge field strength is included in the chiral superfield constructed from $V_{\text{WZ}}$ in the well known procedure,

$$W_\alpha(x, \theta) = (\bar{D}D)D_\alpha V_{\text{WZ}}$$

where

$$W_\alpha(x, \theta) = 4i\lambda_\alpha - 4\theta_\alpha D + 4i\theta^\beta \sigma_{\mu \alpha \beta} \bar{\lambda} \dot{\lambda} (\partial^\mu V^\nu - \partial^\nu V^\mu) - 4\theta \theta \sigma_{\mu \alpha \beta} \partial^\mu \lambda \dot{\lambda}.
$$

(16)

\footnote{In the definition of $\sin \chi$ in Eq. (1) the diagonal part is assumed to be canonically normalized. If there are also some corrections to them, these effects should be taken into account in the initial conditions (11) of the coupling constants at the unification scale.}
Using these superfields the supersymmetric Lagrangian can be written as

\[ \mathcal{L} = \frac{1}{32} (W^\alpha W_\alpha)_F + \left( \Phi^\dagger \exp(2g_0 Q V) \Phi \right)_D, \]

where \( \Phi = (\phi, \psi, F) \) is a chiral superfield which corresponds to a matter field. It is convenient to present a component representation of each term for the following arguments,

\[
\frac{1}{32} (W^\alpha W_\alpha)_F = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} - \frac{1}{2} i \lambda^\alpha \sigma_{\mu\nu} \partial^\mu \lambda^\gamma - \frac{1}{2} i (\partial^\mu \bar{\lambda}_\beta) \sigma^\beta_\mu \lambda_\alpha + \frac{1}{2} D^2, \]

\[
\left( \Phi^\dagger \exp(2g_0 Q V) \Phi \right)_D = |D_\mu \phi|^2 - i \left( \bar{\psi} \sigma^\beta_\mu D_\mu \psi_\alpha \right) + g_0 Q \phi^* D\phi + i \sqrt{2} g_0 Q \left( \phi^* \lambda \psi - \bar{\lambda} \bar{\psi} \phi \right) + |F|^2, \]

where \( D_\mu \) is an original covariant derivative.

If we take account of this Lagrangian, the introduction of the gauge kinetic term mixing is straightforward for \( U(1)_a \times U(1)_b \) model. Supersymmetric gauge kinetic terms are obtained by using chiral superfields \( \hat{W}^a_\alpha \) and \( \hat{W}^b_\alpha \) for \( U(1)_a \times U(1)_b \) as

\[
\frac{1}{32} (\hat{W}^{a\alpha} \hat{W}^a_\alpha)_F = \frac{1}{32} (\hat{W}^{b\alpha} \hat{W}^b_\alpha)_F + \frac{\sin \chi}{16} (\hat{W}^{a\alpha} \hat{W}^b_\alpha)_F. \]

These can be canonically diagonalized by using the supersymmetric version of the transformation (2). In the supersymmetric case this transformation affects not only the gauge field sector as Eq. (4) but also the sector of gauginos \( \hat{\lambda}_{a,b} \) and auxiliary fields \( \hat{D}_{a,b} \). These effects can be summarized as

\[
g^a_0 Q_a \hat{\lambda}^a + g^b_0 Q_b \hat{\lambda}^b = g_a Q_a \lambda^a + (g_{ab} Q_a + g_b Q_b) \lambda^b, \]

\[
g^a_0 Q_a \hat{D}^a + g^b_0 Q_b \hat{D}^b = g_a Q_a D^a + (g_{ab} Q_a + g_b Q_b) D^b, \]

where \( \lambda_{a,b} \) and \( D_{a,b} \) are canonically normalized fields. New gauge coupling constants are represented by Eq. (5). From these formulae we can extract various physical results.

Equations of motion for the auxiliary fields \( D_{a,b} \) can be easily derived as

\[ D_a = -\sum_i g_a Q^i_a |\phi_i|^2, \quad D_b = -\sum_i \left( g_{ab} Q^i_a + g_b Q^i_b \right) |\phi_i|^2. \]

The D-term contribution to the scalar potential is expressed as \( V_D = \frac{1}{2} D_a^2 + \frac{1}{2} D_b^2 \) so that the kinetic term mixing can clearly affect the vacuum structure of the model. When we introduce the Fayet-Iliopoulos D-terms \( \xi_a \hat{D}_a + \xi_b \hat{D}_b \) to the original Lagrangian, the above expressions for the auxiliary fields \( D_{a,b} \) will be modified as,

\[ D_a \rightarrow D_a - \xi_a, \quad D_b \rightarrow D_b - \left( \xi_a \frac{g_{ab}}{g^0_a} - \xi_b \frac{g_b}{g^0_b} \right). \]
If we substitute this modification into the scalar potential, the D-term contribution to the scalar potential is obtained as

$$V_D = \frac{1}{2} \tilde{D}_a^2 + \frac{1}{2} \left( \frac{g_{ab}}{g_a^0} \tilde{D}_a + \frac{g_b^0}{g_b^0} \tilde{D}_b \right)^2, \quad (24)$$

where \( \tilde{D}_{a,b} \) have the same form as the ones of no kinetic term mixing,

$$\begin{align*}
\tilde{D}_a &= \sum_i g_a^0 Q^i_a |\phi_i|^2 + \xi_a, \\
\tilde{D}_b &= \sum_i g_b^0 Q^i_b |\phi_i|^2 + \xi_b. \quad (25)
\end{align*}$$

The minimum of the D-term contribution is \( \tilde{D}_a = \tilde{D}_b = 0 \), which equals to the one in the case of \( \sin \chi = 0 \). However, the minimum of the total scalar potential including an F-term contribution is expected to be modified. Although we donot consider anomalous U(1) models here, this also may have an interesting effect on such models.

### 2.3 RGEs of abelian gauge sector

The RGEs for the abelian gauge sector are affected by the kinetic term mixing like non-supersymmetric case. Here we should add some arguments on the RGEs of gauge coupling constants and gaugino masses. In the supersymmetric case we can write down the concrete form of one-loop RGEs for gauge coupling constants in a compact form [18],

$$\begin{align*}
\frac{dg_a}{dt} &= \frac{1}{16\pi^2} g_a^3 B_{aa}, \\
\frac{dg_{ab}}{dt} &= \frac{1}{16\pi^2} \left( g_a^3 B_{aa} + 2g_{ab}g_b^2 B_{bb} + g_{ab}g_b^2 B_{ab} + 2g_a^2 g_{ab} B_{aa} + 2g_a^2 g_b B_{ab} \right), \\
\frac{dg_b}{dt} &= \frac{1}{16\pi^2} \left( g_b g_{ab}^2 B_{aa} + 2g_b^2 g_{ab} B_{ab} + g_b^3 B_{bb} \right), \quad (26)
\end{align*}$$

where we use the asymmetric basis defined by Eq. (2). The usage of this basis is convenient for the practical purpose in the unified models because U(1)\(_a\) coupling is not altered from the original one as shown in Eq. (5). The charge factor \( B_{ij} \) is defined by \( B_{ij} = \text{Tr}(Q_i Q_j) \) where \( Q_i \) and \( Q_j \) are U(1)\(_a\) or U(1)\(_b\) charges of the chiral superfields which run through the internal line. The trace should be taken for all possible chiral superfields in the loop.

As mentioned before, in the ordinary unification of the abelian factor groups into a simple group where the usual initial condition (10) is used, both of an initial value \( g_{ab} \)

2 The rotation angle \( \theta \) which relates both basis in Eq. (9) satisfies

$$\frac{d\theta}{dt} = \frac{1}{16\pi^2} (g_a g_{ab} B_{aa} + g_a g_b B_{ab}) \).
and \( B_{ab} \) vanish and nonzero \( g_{ab} \) can never appear at the low energy region if any multiplet splitting is not induced by some symmetry breakings at the intermediate scale. However, even in such models if there appear incomplete multiplets of the unification group at some intermediate region, \( B_{ab} \) becomes nonzero as its result and \( g_{ab} \) will develop nonzero value at the lower energy region \[13\]. Recently detailed analysis on the magnitude of the induced kinetic term mixing has been done in \[20\] for the \( E_6 \) inspired extra \( U(1) \) models.

In perturbative superstring models nonzero \( g_{ab} \) can also appear even at the Planck scale as pointed out in ref. \[14, 15\]. In that case we should use Eq. (11) as an initial condition for the RGEs study.

One-loop RGEs for \( U(1)_a \times U(1)_b \) gaugino masses take also a \( 2 \times 2 \) matrix form and can be written in the symmetric basis as,

\[
\frac{d}{dt}\begin{pmatrix} M_a & M_{ab} \\ M_{ab} & M_b \end{pmatrix} = -\begin{pmatrix} M_a & M_{ab} \\ M_{ab} & M_b \end{pmatrix} \begin{pmatrix} \gamma_m^a & \gamma_m^{ab} \\ \gamma_m^{ab} & \gamma_m^b \end{pmatrix}, \quad (27)
\]

where

\[
\begin{pmatrix} \gamma_m^a & \gamma_m^{ab} \\ \gamma_m^{ab} & \gamma_m^b \end{pmatrix} = -2 \begin{pmatrix} \bar{\beta}_a & \bar{\beta}_{ab} \\ \bar{\beta}_{ab} & \bar{\beta}_b \end{pmatrix}

= -\frac{1}{8\pi^2} \begin{pmatrix} \bar{g}_a & \bar{g}_{ba} \\ \bar{g}_{ab} & \bar{g}_b \end{pmatrix} \begin{pmatrix} B_{aa} & B_{ab} \\ B_{ab} & B_{bb} \end{pmatrix} \begin{pmatrix} \bar{g}_a & \bar{g}_{ab} \\ \bar{g}_{ba} & \bar{g}_b \end{pmatrix}. \quad (28)
\]

These RGEs show that the abelian gaugino mass mixing can appear at the low energy region as a result of the kinetic term mixing even if there is no mixing in the initial values of the soft supersymmetry breaking gaugino masses. From Eqs. (27) and (28), we cannot generally expect the unification relation which is usually predicted among the gaugino masses, if there is the kinetic term mixing.

In addition to these radiative effects the existence of an abelian off-diagonal gaugino mass may appear at the unification scale \( M_U \) in relation to the origin of soft supersymmetry breaking. In the \( N = 1 \) supergravity framework it is well known that gaugino masses are expressed at the unification scale as \[24\]

\[
M_a = \frac{1}{2} (\Re f_a)^{-1} F^j \partial_j f_a, \quad (29)
\]

where \( F^j \) is the auxiliary fields in a chiral superfield \( \Phi^j \) and its VEV induces the supersymmetry breaking. In principle, the gauge kinetic function \( f_a \) can have nonzero off-diagonal
elements $f_{ab}$ for abelian factor groups $U(1)_a \times U(1)_b$. The existence of such off-diagonal elements $f_{ab}$ was pointed out at the one-loop effect in the perturbative superstring \[\text{[15]}\]. If $f_{ab}$ has the $\Phi_j$ dependence in the case of $F_j \neq 0$, nonzero $M_{ab}$ is expected to appear at $M_U$. This means that there may be a mixing even in the initial condition of the RGEs for the abelian gaugino masses. The abelian gaugino mass mixing originated from the kinetic term mixing may be one of the interesting aspects of soft supersymmetry breakings. This point seems not to have been noted by now. Although further study of these issues seem to be worthy to clarify the detailed feature of extra $U(1)$ models, they are beyond our present scope and we will not treat them in this paper.

### 3 $\mu$-problem solvable models

There can be a lot of low energy extra $U(1)_X$ models as the extension of the MSSM.\[\text{[3]}\] Among these models we are especially interested in $\mu$-problem solvable extra $U(1)_X$ models, which satisfy the features such as, (i) the extra $U(1)_X$ symmetry should be broken by the VEV of a SM singlet scalar $S$ and (ii) the singlet chiral superfield $S$ has a coupling to the ordinary Higgs doublet chiral superfields $H_1$ and $H_2$ such as $\lambda S H_1 H_2$. In these models the ordinary $\mu$ term is forbidden in the original Lagrangian by $U(1)_X$ and the $\mu$ scale is naturally related to the mass of the extra $U(1)_X$ gauge boson. Thus they also seem to be very interesting from the phenomenological viewpoint.\[\text{[4]}\] Thus it will be worthy to prepare the framework for their analysis and to investigate detailed features of such typical models.

In this paper we confine our attention to this class of models derived from the superstring inspired $E_6$ models.\[\text{[5]}\] There are two classes of extra $U(1)_X$ models derived from superstring inspired $E_6$ models. One is a rank five model, which is called as $\eta$ model. The other ones have a rank six and there are two extra $U(1)$s in addition to the SM gauge

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3Hereafter we will use the same notation for the chiral superfields and its scalar component fields.

4There is also a possibility that the $\mu$ term is induced by a nonrenormalizable term $\lambda (S \bar{S} / M_\mu^2)^n S H_1 H_2$ in the superpotential because of some discrete symmetry.\[\text{[2]}\]. In such a case $\langle S \rangle$ should be large in order to realize the appropriate $\mu$ scale. As a result there is no low energy extra gauge symmetry which can be relevant to the present experimental front. The tadpole problem accompanying this singlet scalar $S$ may again appear. Because of these reasons we do not consider this possibility here.

5It is well known that this type of model often appears in models derived from the various construction of weak coupling superstring.\[\text{[2]}\]. The following discussion can be generalized to such models straightforwardly.
Table 1  The charge assignment of extra U(1)s which are derived from \( E_6 \). These charges are normalized as \( \sum_{i \in 27} Q_i^2 = 20 \).

| fields | \( SU(3) \times SU(2) \) | \( Y \) | \( Q_\psi \) | \( Q_\chi \) | \( Q_\eta \) | \( Q_{\xi\pm} \) |
|--------|-----------------|--------|-------------|-------------|-------------|--------------|
| \( Q \) | (3,2)           | \( \frac{1}{3} \) | \( \sqrt{\frac{5}{18}} \) | \( -\frac{1}{\sqrt{6}} \) | \( -\frac{2}{3} \) | \( \pm \frac{1}{\sqrt{6}} \) |
| \( \bar{U} \) | (3*,1)         | \( -\frac{4}{3} \) | \( \sqrt{\frac{5}{18}} \) | \( -\frac{1}{\sqrt{6}} \) | \( -\frac{2}{3} \) | \( \pm \frac{1}{\sqrt{6}} \) |
| \( D \) | (3*,1)         | \( \frac{2}{3} \) | \( \sqrt{\frac{5}{18}} \) | \( \frac{3}{\sqrt{6}} \) | \( \frac{1}{3} \) | \( \pm \frac{2}{\sqrt{6}} \) |
| \( L \) | (1,2)          | \( -1 \) | \( \sqrt{\frac{5}{18}} \) | \( \frac{3}{\sqrt{6}} \) | \( \frac{1}{3} \) | \( \pm \frac{2}{\sqrt{6}} \) |
| \( \bar{E} \) | (1,1) | \( 2 \) | \( \sqrt{\frac{5}{18}} \) | \( -\frac{1}{\sqrt{6}} \) | \( -\frac{2}{3} \) | \( \pm \frac{1}{\sqrt{6}} \) |
| \( H_1 \) | (1,2)          | \( -1 \) | \( -2\sqrt{\frac{5}{18}} \) | \( -\frac{2}{\sqrt{6}} \) | \( \frac{1}{3} \) | \( \pm \frac{3}{\sqrt{6}} \) |
| \( H_2 \) | (1,2)          | \( 1 \) | \( -2\sqrt{\frac{5}{18}} \) | \( \frac{2}{\sqrt{6}} \) | \( \frac{4}{3} \) | \( \pm \frac{2}{\sqrt{6}} \) |
| \( g \) | (3,1)          | \( -\frac{2}{3} \) | \( -2\sqrt{\frac{5}{18}} \) | \( \frac{2}{\sqrt{6}} \) | \( \frac{4}{3} \) | \( \pm \frac{2}{\sqrt{6}} \) |
| \( \bar{g} \) | (3*,1)      | \( \frac{2}{3} \) | \( -2\sqrt{\frac{5}{18}} \) | \( -\frac{2}{\sqrt{6}} \) | \( \frac{1}{3} \) | \( \pm \frac{3}{\sqrt{6}} \) |
| \( S \) | (1,1)          | \( 0 \) | \( 4\sqrt{\frac{5}{18}} \) | \( 0 \) | \( -\frac{5}{\sqrt{6}} \) | \( \pm \frac{5}{\sqrt{6}} \) |
| \( N \) | (1,1)     | \( 0 \) | \( \sqrt{\frac{5}{18}} \) | \( -\frac{5}{\sqrt{6}} \) | \( -\frac{3}{3} \) | \( 0 \) |

structure. They are known to be expressed as suitable linear combinations of two abelian groups \( U(1)_\psi \) and \( U(1)_\chi \). Their charge assignments for 27 of \( E_6 \) are summarized in Table 1. As seen from this table, there is a SM singlet \( S \) which has a coupling \( \lambda SH_1 H_2 \). The \( \eta \)-model clearly satisfies the above mentioned conditions (i) and (ii). On the other hand, in the rank six models these conditions impose rather severe constraint on the extra \( U(1)_X \) remaining at the low energy region.

In this type of rank six models a right-handed sneutrino \( N^c \) also has to get a VEV to break the gauge symmetry into the one of the SM. If we try to explain the smallness of the neutrino mass in this context, \( N^c \) should get the sufficiently large VEV. To make this possible \( N \) needs to have a massless conjugate partner \( \bar{N} \) which is a chiral superfield belonging to \( 27^* \) of \( E_6 \)\footnote{If we impose the gauge coupling unification on our models, we need to include other conjugate pairs besides \( N + \bar{N} \) from \( 27 + 27^* \) as pointed out in \[12, 20 \].}. Fortunately, it is well known that this can happen in the perturbative string models \[2\]. In such a case, as easily seen, a sector of \( \langle N, \bar{N} \rangle \) in the fields space has a D-flat direction \( \langle N \rangle = \langle \bar{N} \rangle \) and then they can get a large VEV without
breaking supersymmetry [2]. This VEV $\langle N \rangle$ can induce the large right-handed Majorana neutrino mass through the nonrenormalizable term $(NN)^n/M_{pl}^{2n-3}$ in the superpotential and then the seesaw mechanism is applicable to yield the small neutrino mass as suggested in [4, 24]. This D-flat direction may also be related to the inflation of universe and the baryogenesis as discussed in [29]. However, this introduction of $\langle N \rangle$ usually breaks the direct relation between the $\mu$ scale and the mass of the extra U(1)$_X$ gauge boson because the VEV of $N$ also generally contributes to the latter. In order to escape this situation we need to select a U(1)$_X$ by taking a suitable linear combination of U(1)$_\psi$ and U(1)$_X$ to make $N$ have zero charge of this U(1)$_X$ [4, 14, 27]. This type of model is also shown in Table 1. The difference between $\xi_{\pm}$ is the overall sign and they can be identified by the redefinition of $g_b^0$ and $\sin \chi$. In the following part we adopt the $\xi_-$ convention. Using the D-flat direction $\langle N \rangle = \langle \bar{N} \rangle$ of another extra U(1) orthogonal to this U(1)$_\xi_-$, the right-handed sneutrino gets the large VEV which breaks this extra U(1) symmetry and induces the large Majorana masses for the right-handed neutrinos as mentioned above. As a result of this symmetry breaking at the intermediate scale, only one extra U(1)$_\xi_-$ remains as the low energy symmetry.

Apart from the property of the low energy extra U(1)$_X$, whether an intermediate scale can exist or not is a special feature which discriminate between the rank six $\xi_-$ model and the rank five $\eta$ model. In the following study we will concentrate our study on low energy features of two U(1)$_X$ models ($X = \eta, \xi_-$).

4 Vacuum structure of extra U(1)$_X$ models

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7 The small Majorana neutrino mass can also be directly induced through loop effects and/or non-renormalizable couplings by using this intermediate scale $\langle N \rangle$ [27, 28]. In that case some kind of discrete symmetry should play an important role. Here we should also note that all the right-handed neutrinos need not to be heavy. Light sterile neutrinos is possible in this type of models.

8 As discussed in refs. [4, 27], $Q_{\xi_-}$ can also be obtained only by changing the field assignments for $Q_X$. This insight allows us to construct new models, which can induce an interesting neutrino mass matrix [30] by using the charge assignments $Q_X$ and $Q_{\xi_-}$ for the different generations [27]. However, in this paper we shall not consider such models for simplicity.
4.1 General framework

In this section we investigate the vacuum structure of $\mu$-problem solvable $\eta$ and $\xi -$ models. The electroweak gauge structure of these models is $SU(2)_L \times U(1)_Y \times U(1)_X$ at the low energy region. The arguments in the previous section are straightforwardly applicable if we identify $A_\mu^a$ and $A_\mu^b$ with the gauge fields $B^\mu$ of $U(1)_Y$ and $X^\mu$ of $U(1)_X$, respectively. After resolving the kinetic term mixing of these abelian factor groups, the canonically normalized Lagrangian of the present models is expressed by using the component fields as,

$$
\mathcal{L} = -\frac{1}{4} W^{i\mu\nu} W_{i\mu\nu} - \frac{1}{4} B^{\mu\nu} B^\mu B^\nu - \frac{1}{4} X^{\mu\nu} X^\mu X^\nu - i \lambda^{\alpha}_{W} \sigma^{\mu}_{\alpha\beta} \partial^\mu \lambda_{W}^{\beta} - i \lambda^{\alpha}_{B} \sigma^{\mu}_{\alpha\beta} \partial^\mu \lambda_{B}^{\beta} - i \lambda^{\alpha}_{X} \sigma^{\mu}_{\alpha\beta} \partial^\mu \lambda_{X}^{\beta} \\
+ \frac{1}{2} \sum_i D_{W_i}^2 + \frac{1}{2} D_{B}^2 + \frac{1}{2} D_{X}^2 + |(\partial_\mu - G(V_\mu)) \phi|^2 + i \psi^i \sigma^{\mu}_{\alpha\beta} (\partial^\mu - G(V^\mu)) \psi^\alpha + \frac{1}{2} \phi^* G(D) \phi \\
+ i \sqrt{2} \left( \phi^* G(\lambda) \psi - \phi G(\bar{\lambda}) \bar{\psi} \right) + |F|^2 + [W(\Phi) + h.c.]_F,
$$

(30)

where $W^{i\mu\nu}, B^{\mu\nu}$ and $X^{\mu\nu}$ are the field strengths of $SU(2)_L, U(1)_Y$ and $U(1)_X$, respectively. The chiral superfields $\Phi = (\phi, \psi, F)$ should be understood to represent all necessary matter and Higgs fields, although indices for all quantum numbers are abbreviated. For the convenience we use the notation for the vector superfields

$$
G(\mathcal{F}) = \frac{i}{2} \left( g_W \tau^i F^i_W + g_Y Y F_B + (g_Y Y + g_X Q_X) F_X \right),
$$

(31)

where $\mathcal{F}$ represents the component fields $V_\mu, \lambda$ and $D$ of the vector superfield $V_{ZW}$ for the gauge group $SU(2)_L \times U(1)_Y \times U(1)_X$.

In the $\mu$-problem solvable models the gauge symmetry breaking required around the weak scale is $SU(2)_L \times U(1)_Y \times U(1)_X \rightarrow U(1)_{em}$, which is realized by the VEVs of Higgs scalar fields

$$
\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = u,
$$

(32)

where their quantum numbers for $SU(2)_L \times U(1)_Y \times U(1)_X$ are

$$
H_1 (2, -1, Q_1), \quad H_2 (2, 1, Q_2), \quad S (1, 0, Q_S).
$$

(33)

For a correct electroweak vacuum we need to impose $v_1^2 + v_2^2 = (174 \text{ GeV})^2 (\equiv v^2)$. All VEVs are assumed to be real. The vacuum of these models is described as a point in a
space of two dimensionless parameters, \( \tan \beta = v_2/v_1 \) and \( u/v \). In the rest of this section we give a general framework to constrain the allowed region in this space in terms of both electroweak precision measurements and scalar potential minimum conditions. After that we use it to analyze the vacuum structure of our models.

In order to investigate the vacuum structure in the basis of the recent experimental results we need to determine the physical states at and below the weak scale \([18]\). The mass mixing between two U(1) factor groups appears when the spontaneous symmetry breaking occurs at the neighborhood of the weak scale. In the present models the charged gauge sector is the same as that of the MSSM. In the neutral gauge sector we introduce the Weinberg angle \( \theta_W \) in a usual way,\[34\]

\[ Z_\mu = \cos \theta_W W^3_\mu - \sin \theta_W B_\mu, \quad A_\mu = \sin \theta_W W^3_\mu + \cos \theta_W B_\mu. \]

Here we use the canonically normalized basis \((Z_\mu, X_\mu)\) so that \(A_\mu\) is decoupled from the \((Z_\mu, X_\mu)\) sector. After the spontaneous symmetry breaking the mass matrix of \((Z_\mu, X_\mu)\) can be written as

\[
\begin{pmatrix}
  m_Y^2 & m_{YX}^2 \\
  m_{YX}^2 & m_X^2
\end{pmatrix},
\]

where each element is expressed as

\[
m_Y^2 = m_Z^2, \quad m_{YX}^2 = m_Z^2 s_W \tan \chi + \frac{\Delta m^2}{\cos \chi},
\]

\[
m_X^2 = m_Z^2 s_W^2 \tan^2 \chi + 2 \Delta m^2 s_W \frac{\sin \chi}{\cos^2 \chi} + \frac{M_{Z'}^2}{\cos^2 \chi}.
\]

In these expressions \(m_Z^2, \Delta m^2\) and \(M_{Z'}^2\) represent the values of corresponding components in the case of no kinetic term mixing (\(\sin \chi = 0\)). They can be written as

\[
m_Z^2 = \frac{1}{2} (g_W^2 + g_Y^2) v^2,
\]

\[
\Delta m^2 = \frac{1}{2} (g_W^2 + g_Y^2)^{1/2} g_X (Q_1 v_1^2 - Q_2 v_2^2),
\]

\[
M_{Z'}^2 = \frac{1}{2} g_X^2 \left( Q_1^2 v_1^2 + Q_2^2 v_2^2 + Q_3^2 u^2 \right).
\]

We introduce mass eigenstates which diagonalize the mass matrix (35) as follows,

\[
Z_1^\mu = \cos \xi Z^\mu + \sin \xi X^\mu,
\]

\[
Z_2^\mu = -\sin \xi Z^\mu + \cos \xi X^\mu,
\]

\[9\] In the following we use the abbreviated notation \(s_W \equiv \sin \theta_W\) and \(c_W \equiv \cos \theta_W\).
where the mixing angle $\xi$ can be given by using Eq. (13) as

$$
\tan 2\xi = \frac{-2 \cos \chi (m_Z^2 s_W \sin \chi + \Delta m^2)}{M_{Z'}^2 + 2 \Delta m^2 s_W \sin \chi + m_Z^2 s_W^2 \sin^2 \chi - m_Z^2 \cos^2 \chi}.
$$

(39)

In general, a mass eigenvalue $m_{Z_2}$ and $\xi$ are severely constrained by direct searches at Fermilab Tevatron [31] and precise measurements at LEP [32]. In the case of $\sin \chi = 0$, it is usually assumed that this constraint is satisfied because of $\Delta m^2 \ll M_{Z'}^2$. This can be realized in two ways. Simple one requires only $v_1^2, v_2^2 \ll u$ which needs no fine tuning at this stage [7, 11]. The other one requires a special situation $\Delta m^2 \sim 0$ [11, 18]. This reduces to $\tan^2 \beta \sim 1/4$ ($\eta$ model) and $3/2$ ($\xi_-$ model). Here it may be useful to remember that the radiative symmetry breaking scenario favors $\tan \beta > 1$. If $\sin \chi \neq 0$, however, there may be a new possibility to realize the small $\xi$ even if $\Delta m^2 \ll M_{Z'}^2$ is not satisfied. Such a situation can be expected to occur without any assumption for the largeness of $u$ or $\Delta m^2 \sim 0$, if the following condition is satisfied

$$
\sin \chi \sim -\frac{\Delta m^2}{m_Z^2 s_W} = -\frac{g_X Q_1 v_1^2 - Q_2 v_2^3}{v^2}.
$$

(40)

It may be useful to note that this condition is reduced to

$$
\sin \chi \sim \begin{cases} 
\frac{g_X 4 \tan^2 \beta - 1}{3g_Y \tan^2 \beta + 1} & \eta \text{ model,} \\
\frac{g_X 2 \tan^2 \beta - 3}{\sqrt{6} g_Y \tan^2 \beta + 1} & \xi_- \text{ model.}
\end{cases}
$$

(41)

This shows that there is a special $\sin \chi$ value for each $\tan \beta$ to realize $\xi = 0$. In the case of $\eta$ model $\tan \beta > 1$ can requires $\sin \chi \gtrsim 0.5g_X/g_Y$ to satisfy this condition.

In the case of $M_{Z'}^2 \gg m_Z^2$, $\Delta m^2$, the mass eigenvalues of Eq. (35) are written as

$$
m_{Z_1}^2 \simeq m_Z^2 - \frac{1}{M_{Z'}^2} (m_Z^2 s_W \sin \chi + \Delta m^2)^2
$$

(42)

$$
m_{Z_2}^2 \simeq \frac{M_{Z'}^2}{\cos^2 \chi} + \frac{1}{M_{Z'}^2} (m_Z^2 s_W \sin \chi + \Delta m^2)^2.
$$

(43)

Original states which are not canonically normalized can be related to the mass eigenstates $(\mathcal{A}^\mu, Z_{1}^\mu, Z_{2}^\mu)$ as

$$
\hat{\mathcal{A}}^\mu = \mathcal{A}^\mu - c_W \tan \chi (\sin \xi Z_{1}^\mu + \cos \xi Z_{2}^\mu),
$$

This role of $\sin \chi \neq 0$ can be played by the VEVs of new Higgs doublet scalars which have nonzero charges of U(1)$_{\chi}$ [18].

11In the $\eta$ model it is known that there is a special $\sin \chi$ value which can make U(1)$_{\eta}$ leptophobic ($\sin \chi = gx/3g_Y$) [18]. It corresponds to $\sin \chi = 1/3$ if we take $gx/g_Y = 1$ at the weak scale.
\[\hat{Z}_\mu = (\cos \xi + s_W \tan \chi \sin \xi) Z_1^\mu + (- \sin \xi + s_W \tan \chi \cos \xi) Z_2^\mu,\]
\[\hat{X}_\mu = \frac{\sin \xi}{\cos \chi} Z_1^\mu + \frac{\cos \xi}{\cos \chi} Z_2^\mu,\]  
(44)

where \(\mathcal{A}^\mu\) stands for a real photon field and \(Z_1^\mu\) is understood as \(Z_{0}^\mu\) observed at LEP. Using these mass eigenstates, the interaction terms of \(\mathcal{A}^\mu\) and \(Z_1^\mu\) with ordinary quarks and leptons can be expressed as [18],

\[\mathcal{L}_{\text{int}} = eQ_{\text{em}} \bar{\psi} \gamma_\mu \psi \mathcal{A}^\mu + \frac{g_W}{2c_W} \left[ (\bar{v}_f + \bar{v}'_f \bar{\xi}) \bar{\psi} \gamma_\mu \psi + (\bar{a}_f + \bar{a}'_f \bar{\xi}) \bar{\psi} \gamma_\mu \gamma_5 \psi \right] Z_1^\mu,\]  
(45)

\[\bar{v}_f = \left( \frac{\tau^3}{2} - 2Q_{\text{em}} s_W^2 \right) (1 + s_W \xi \tan \chi) - 2Q_{\text{em}} c_W^2 s_W \xi \tan \chi,\]
\[\bar{v}'_f = (Q^{\psi_L}_X + Q^{\psi_R}_X),\]
\[\bar{a}_f = - \frac{\tau^3}{2} (1 + s_W \xi \tan \chi),\]
\[\bar{a}'_f = (Q^{\psi_R}_X - Q^{\psi_L}_X),\]  
(46)

where we assume \(\xi\) is small enough and put \(\bar{\xi} \equiv \frac{g_X c_W}{g_W \cos \chi} \xi\). \(Q^{\psi_L}_X\) and \(Q^{\psi_R}_X\) stand for the \(U(1)_X\) charges of \(\psi_L\) and \(\psi_R\). Using these effective couplings, the partial decay width \(\Gamma_f\) and the asymmetry parameter \(A_f\) for \(Z \to \bar{f}f\) are expressed as

\[\Gamma_f = \frac{G_F m_Z^3}{6 \sqrt{2} \pi} \rho N_c \left[ (\bar{v}_f + \bar{v}'_f \bar{\xi})^2 + (\bar{a}_f + \bar{a}'_f \bar{\xi})^2 \right],\]
\[A_f = \frac{2(\bar{v}_f + \bar{v}'_f \bar{\xi})(\bar{a}_f + \bar{a}'_f \bar{\xi})}{(\bar{v}_f + \bar{v}'_f \bar{\xi})^2 + (\bar{a}_f + \bar{a}'_f \bar{\xi})^2},\]  
(47)

where \(\rho\) is a one-loop corrected ratio of the neutral current to the charged current and \(N_c\) stands for the effective color factor. The forward-backward asymmetry of \(Z \to \bar{f}f\) is given as \(A_{FB} = \frac{3}{4} A_c A_f\). Using these formulae, we can restrict the allowed region in a \((\tan \beta, u/v)\) plane by using the electroweak data.

### 4.2 Constraints from electroweak data

The deviations of various electroweak observables in the present models from the SM predictions can be strictly constrained by taking account of data obtained at LEP. Electroweak parameters corrected by the extra \(U(1)_X\) effects from the SM values are the \(\rho\) parameter and the effective Weinberg angle \(s_W^\prime\). In these parameters we only take account of the correction induced by the extra \(U(1)_X\) effect besides the radiative corrections in the SM. Other corrections yielded by the exotic matter fields are model dependent and
we assume that they are small enough \( [18] \). More concretely, we put \( \rho = 1 + \delta \rho_t + \delta \rho_M \) where \( \delta \rho_t \) is a one-loop correction due to top quark in the SM and \( \delta \rho_M \) is the extra \( U(1)_X \) effect. The detailed derivation of both deviations \( \delta \rho_M \) and \( \Delta \bar{s}_W^2 \) due to the existence of extra \( U(1)_X \) is reviewed in the appendix \([17, 18]\). We only present the results below.

\( \delta \rho_M \) appears at the tree level as a result of both of the mass and kinetic term mixing. It can be expressed as \([18]\)

\[
\delta \rho_M \simeq \frac{\xi^2}{m_Z^2 \cos^2 \chi} + 2 \bar{s}_W \xi \tan \chi,
\]

where we use a relation \( \frac{m_Z^2}{m_{Z_1}^2} \simeq \frac{M_Z^2 \xi^2}{m_Z^2 \cos^2 \chi} \) which is obtained from Eqs. (39) and (42) as far as \( \xi \ll 1 \). The second term in the right-hand side comes from the kinetic term mixing (\( \sin \chi \neq 0 \)). This deviation in the \( \rho \) parameter can also make an influence in the deviation of \( \bar{s}_W^2 \). The expression of \( \Delta \bar{s}_W^2 \) including such an effect can be found as \([18]\)

\[
\Delta \bar{s}_W^2 \simeq - \bar{s}_W^2 \xi \left( \frac{\bar{s}_W^2}{\bar{s}_W^2 - \bar{s}_W^2} \frac{M_{Z'}^2 \xi}{m_Z^2 \cos^2 \chi} - \bar{s}_W \tan \chi \right).
\]

The experimental bounds obtained at LEP for these parameters can put strong constraints on the parameters related to the extra \( U(1)_X \) sector.

Following the procedure used in Ref. \([33]\), we estimate the deviation of LEP observables \( O \) from the predictions in the present models. In the present analysis we use \( \Gamma_Z, R_\ell, \sigma_{\text{had}}, R_b, R_c, m_W/m_Z, A_{FB}^\ell, A_{FB}^c \) and \( A_{FB}^\ell \) as \( O \). These electroweak observables can be calculated by using the tree level formulae (47) and the deviation of these observables \( O \) can be approximately expanded by \( \delta \rho_M \), \( \Delta \bar{s}_W^2 \) and \( \bar{\xi} \) as

\[
\frac{\delta O}{O} = A^{(1)} \delta \rho_M + A^{(2)} \Delta \bar{s}_W^2 + B \bar{\xi}.
\]

where \( A^{(1)}, A^{(2)} \) and \( B \) are calculated by using Eqs. (46) and (47). As is easily checked, only \( B \) is different between both models. Their numerical values of the present models are given in Table 2.

Since \( \delta \rho_M \) and \( \Delta \bar{s}_W^2 \) are not independent and are related through Eqs. (48) and (49), four independent free parameters \( g_X, \sin \chi, \delta \rho_M \) and \( \bar{\xi} \) are contained in our analysis. If we take \( g_X \) and \( \sin \chi \) to be suitable values, we can carry out two parameters \( \chi^2 \)-fit between these predictions and LEP data in a \( (\delta \rho_M, \bar{\xi}) \) plane. As a result of such analyses a minimum value of \( \chi^2 \) is found to be \( \chi^2 \simeq 7.0 \) for 7 degrees of freedom at a point such as

\[
(\delta \rho_M, \bar{\xi}) \sim (-2.4 \times 10^{-4}, -4 \times 10^{-5}), \quad \eta \text{ model},
\]

\[
\sim (-2.4 \times 10^{-4}, 4 \times 10^{-5}), \quad \xi^- \text{ model} \quad (51)
\]
in the case of $\sin \chi = 0$. For this degrees of freedom the corresponding $\chi^2$ values to
goodness-of-fits of 95\% and 99\% are $\chi^2 = 14.1$ and 18.5, respectively. For the same
observables the $\chi^2$ value of the SM is 3.4. Thus at this stage there is no positive experi-
mentaltal signature for considering the low energy extra $U(1)_X$ models. These data should
be only used to put the constraint on the vacuum structure of the models so as to satisfy
the constraint on $\bar{\xi}$. In the following study our aim is to find what kind of vacua can
approximately satisfy this constraint.

By using the relations (37), (39), (48) and (49), we can project this result in the
$(\delta \rho_M, \bar{\xi})$ plane onto the ones in the various planes defined by other variables. To see a
role of the kinetic term mixing on the value of $u$, it is convenient to draw $\chi^2$-contours for
typical values of $\tan \beta$ and $g_X$ in a $(\sin \chi, u/v)$ plane. It should be reminded that the
value of $\sin \chi$ which can induces the minimum of $u$ is largely dependent on $\tan \beta$ as shown
in Eq. (41). Here we choose $\tan \beta = 1.5$ taking account of the fact that the radiative
symmetry breaking favors $\tan \beta \gtrsim 1$. These results for $\eta$ and $\xi_-$ models are presented in
Fig. 1. From this figure we can easily find that it shows very similar features to a contour
of the present upper bound of mixing angle $\xi$ given in [22] as expected. In the $\eta$ model
$\sin \chi = 1/3$ which corresponds to leptophobia in the present setting cannot make $u$ small
because $\xi$ is large for $\tan \beta = 1.5$. For this $\tan \beta$ we need $\sin \chi \sim 0.82$ to realize $\xi \sim 0$.
Here we should note that too large $\sin \chi$ value seems to be difficult to be realized [20] and
also it may contradict a perturbative picture.

In order to examine the vacuum structure we should draw the $\chi^2$-contours in the
$(\tan \beta, u/v)$ plane since the vacuum is parameterized by two dimensionless variables
$\tan \beta$ and $u/v$. In this analysis relevant free parameters in the model are $g_X$ and $\sin \chi$

| $O$ | $\Gamma_Z$ | $R_\ell$ | $\sigma_{\text{had}}$ | $R_p$ | $R_c$ | $m_W/m_Z$ | $A_{FB}^b$ | $A_{FB}^c$ | $A_{FB}^\ell$ |
|-----|-----------|---------|----------------------|------|------|-----------|---------|---------|---------|
| $A^{(1)}$ | 0.99 | 0 | 0 | 0 | 0.38 | 0 | 0 | 0 |
| $A^{(2)}$ | -1.06 | -0.84 | 0.097 | 0.18 | -0.35 | -1.00 | -54.15 | -58.66 | -106.93 |
| $B(\eta)$ | 0.52 | -0.82 | 0.20 | 2.56 | -4.86 | 0 | -26.61 | -25.41 | -54.78 |
| $B(\xi_-)$ | 0.33 | -4.31 | 5.03 | 1.86 | -3.54 | 0 | 7.21 | 9.71 | 16.98 |

Table 2
Fig. 1 $\chi^2$-contours in the $(\sin \chi, \ u/v)$ plane for (A) $\eta$ and (B) $\xi_-$ models with $\tan \beta = 1.5$ and $g_X/g_Y = 1$. They are shown by a solid line for $\chi^2 = 18.5$ and a dotted line for $\chi^2 = 14.1$, respectively. Dash-dotted lines represent the $m_{Z^2}$ contours such as a: 400 GeV, b: 800 GeV, c: 1200 GeV and d: 1600 GeV, respectively.

 alone. We vary these parameters in the following regions,

$$0 \leq \sin \chi \leq 0.3, \quad 0.8 \leq g_X/g_Y \leq 1.2.$$  \hspace{1cm} (52)

The study of RGEs in Eq. (26) for various $E_6$ inspired extra U(1) models [20] seems to verify this assumption on $\sin \chi$ and $g_X/g_Y$ at the weak scale. The reason to take $\sin \chi$ positive has already discussed in the previous subsection. In Fig. 2 we present the $\chi^2$-contours in this plane for both models with typical values of these parameters. In the same plane we also plot contours of the extra neutral gauge boson mass $m_{Z^2}$ as a reference. We can observe that $u$ can be small and result in a rather small $m_{Z^2}$ value for a suitable region of $\tan \beta$. This $\tan \beta$ region is somehow different between two models.

A gross shape of the $\chi^2$-contours shown in Fig.2 is general and they donot show so strong $g_X$ and $\sin \chi$ dependence. However, the value of $\tan \beta$ allowed at the small $u/v$ region somehow changes depending on the $\sin \chi$ value as expected from Eq. (41). If we note a $\chi^2 = 18.5$ contour, we find that the region of $u/v \leq 10$ can be realized at very restricted $\tan \beta$ values as follows,

$$\eta \ model \ \left\{ \begin{array}{l} 0 < \tan \beta \lesssim 1.1 \quad (\sin \chi = 0), \\ 0.4 \lesssim \tan \beta \lesssim 1.2 \quad (\sin \chi = 0.3), \end{array} \right.$$
Fig. 2 $\chi^2$-contours in the $(\tan \beta, u/v)$ plane for (A) $\eta$ model with $\sin \chi = 0.3$ and (B) $\xi_-$ model with $\sin \chi = 0.2$. In both cases $g_X/g_Y = 1$ is assumed. Contours are shown by a solid line for $\chi^2 = 18.5$ and a dotted line for $\chi^2 = 14.1$. Dashed-Dotted lines represent the $m_{Z_2}$ contours a: 400 GeV, b: 800 GeV, c: 1200 GeV and d: 1600 GeV, respectively.

\[ \xi_- \text{ model } \begin{cases} 0.9 < \tan \beta \lesssim 1.9 & (\sin \chi = 0), \\ 1.2 < \tan \beta \lesssim 2.6 & (\sin \chi = 0.2). \end{cases} \tag{53} \]

In both models the somehow larger $\sin \chi(\geq 0)$ value is necessary to realize $\tan \beta$ in a favorable region like $\tan \beta \gtrsim 1$ as far as we require that rather small $u/v$ value is allowable. Generally, in the $\xi_-$ model the naturally small $\sin \chi(\geq 0)$ values can make $u/v$ rather small for the $\tan \beta \gtrsim 1$ region. Thus the $\xi_-$ model may be more promising than the $\eta$ model from this viewpoint.

### 4.3 Minimization of scalar potential

Next we study the vacuum structure through minimizing the scalar potential \[7, 11, 12\]. We again take account of the influence on the scalar potential caused by the kinetic term mixing among abelian gauge fields. It should be reminded that the abelian auxiliary fields $D_{Y,X}$ are changed as shown in Eq. (21). For the detailed investigation of the scalar potential in the present models the superpotential and the soft supersymmetry breaking terms should be fixed definitely. In this paper we assume the following superpotential,

\[ W = h_U Q H_2 \bar{U} + h_D Q H_1 \bar{D} + h_E L H_1 \bar{E} + h_N L H_2 \bar{N} + \lambda S H_1 H_2 + k S g g. \tag{54} \]
Here we explicitly write the minimal part which is necessary for this model to be realistic. Generation indices are abbreviated. Other terms including the exotic fields are omitted. As soft supersymmetry breaking terms we consider the following ones.

\[ \mathcal{L}_{\text{soft}} = - \sum_k m_k^2 |\phi_k|^2 - m_1^2 |H_1|^2 - m_2^2 |H_2|^2 - m_S^2 |S|^2 \]

+ \[ A_U h_U QH_2 \bar{U} + A_D h_D QH_1 \bar{D} + A_E h_E LH_1 \bar{E} + A_N h_N LH_2 \bar{N} \]

+ \[ A_\lambda \lambda_1 H_1 H_2 + A_k k \bar{g} g \]

+ \[ \frac{1}{2} \left( M_W \sum_i \lambda_W \lambda_W_i + M_Y \lambda_Y \lambda_Y + M_X \lambda_X \lambda_X \right) + h.c. \] (55)

where \( \phi_k \) stands for the superpartners of ordinary quarks and leptons. For simplicity, \( A \)-parameters and Yukawa couplings are assumed to be real.

The changes induced in the scalar potential by the extra \( U(1)_X \) is expected to be reflected in the vacuum structure. In order to investigate this aspect we write down the vacuum energy by using the VEVs of \( H_1 \), \( H_2 \) and \( S \). Under the assumption (32) for these VEVs, we can write it as follows,

\[ V = \frac{1}{8} \left( g_W^2 + g_Y^2 \right) \left( v_1^2 - v_2^2 \right)^2 \]

+ \[ \frac{1}{8} \left( g_Y \tan \chi (v_1^2 - v_2^2) + \frac{g_X}{\cos \chi} \left( Q_1 (v_1^2 - u^2) + Q_2 (v_2^2 - u^2) \right) \right)^2 \]

+ \[ \lambda^2 v_1^2 v_2^2 + \lambda^2 u^2 v_1^2 + \lambda^2 u^2 v_2^2 + m_1^2 v_1^2 + m_2^2 v_2^2 + m_S^2 u^2 - 2A \lambda w^2 (v_1 v_2) \] (56)

Here we use the relation \( Q_1 + Q_2 + Q_S = 0 \) for the extra \( U(1)_X \) charge, which comes from the \( \lambda_1 H_1 H_2 \) coupling in the superpotential \( W \). In the second line we can see the effect of the kinetic term mixing. A linear term of \( u \) appears in this potential only in a form \( Au \) so that a sign of \( A \) is relevant to the one of \( u \).

Now we proceed an analysis of the feature of the minimum of this potential. The analytic study is difficult and then the potential minimum must be numerically studied by solving the minimum conditions,

\[ \frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial u} = 0. \] (57)

\textsuperscript{12}We also drop the usual R-parity violating terms to guarantee the proton stability. This may be justified due to some discrete symmetry. The last term is necessary for the radiative symmetry breaking of \( U(1)_X \) as discussed in \textsuperscript{[1]} [12]. In the \( \eta \) model \( h_N = 0 \) should be assumed.

\textsuperscript{13}We donot consider the abelian gaugino mass mixing term \( M_X \lambda_Y \lambda_X \), for simplicity.
These conditions can be translated into the equations for Higgs masses,

\[
\left( \frac{m_1^2}{v} \right)^2 = \frac{g_Y^2 + g_Y^2}{4} \left( \sin^2 \beta - \cos^2 \beta \right) - \frac{\zeta_1}{4} \left( \zeta_1 \cos^2 \beta + \zeta_2 \sin^2 \beta + \zeta_3 \left( \frac{u}{v} \right)^2 \right) \\
- \lambda^2 \left( \sin^2 \beta + \left( \frac{u}{v} \right)^2 \right) + \lambda \frac{A}{v} \frac{u}{v} \tan \beta,
\]

(58)

\[
\left( \frac{m_2^2}{v} \right)^2 = -\frac{g_Y^2 + g_Y^2}{4} \left( \sin^2 \beta - \cos^2 \beta \right) - \frac{\zeta_2}{4} \left( \zeta_1 \cos^2 \beta + \zeta_2 \sin^2 \beta + \zeta_3 \left( \frac{u}{v} \right)^2 \right) \\
- \lambda^2 \left( \cos^2 \beta + \left( \frac{u}{v} \right)^2 \right) + \lambda \frac{A}{v} \frac{u}{v} \cot \beta,
\]

(59)

\[
\left( \frac{m_S^2}{v} \right)^2 = -\frac{\zeta_3}{4} \left( \zeta_1 \cos^2 \beta + \zeta_2 \sin^2 \beta + \zeta_3 \left( \frac{u}{v} \right)^2 \right) - \lambda^2 + \lambda \frac{A}{v} \frac{u}{v} \cos \beta \sin \beta,
\]

(60)

where \( v_1^2 + v_2^2 = v^2 \) should be satisfied. We define \( \zeta_1, \zeta_2 \) and \( \zeta_3 \) as

\[
\zeta_1 = g_Y \tan \chi + \frac{g_X Q_1}{\cos \chi}, \quad \zeta_2 = -g_Y \tan \chi + \frac{g_X Q_2}{\cos \chi}, \quad \zeta_3 = \frac{g_X Q_S}{\cos \chi}. \]

(61)

A solution of Eqs. (58)-(60) is realized as a triple crossing point of the contours of \( m_1^2 \), \( m_2^2 \) and \( m_S^2 \) in the \((\tan \beta, u/v)\) plane for a suitable set of parameters \( g_X, \sin \chi, A \) and \( \lambda \). It is not an easy task to solve the coupled RGEs for all physical parameters and find solutions of Eqs. (58)-(60) varying the initial conditions for all parameters at the unification scale. In the present analysis we do not practice this procedure but adopt more convenient method.

To know the existence of such solutions we should search the parameters region for which the contour bands of \( m_1^2 \), \( m_2^2 \) and \( m_S^2 \) simultaneously cross each other within a suitable width like \( |m^2| \lesssim (1 \text{ TeV})^2 \). The absolute values of these squared masses are naturally considered to be near the weak scale from a viewpoint of the radiative symmetry breaking induced by the quantum corrections to the soft supersymmetry breaking parameters. These parameters are usually considered to be from a few hundred GeV to 1 TeV at the unification scale and run towards the low energy region mainly under the control of the contribution from large Yukawa coupling constants. If we take this viewpoint, we can roughly know the consistent parameter region with the radiative symmetry breaking scenario. We follow this simplified method. The complete RGEs study done in [7, 11, 12] seems to support the result of this method.

We numerically examine the possible vacuum structure by drawing \( m_1^2, m_2^2 \) and \( m_S^2 \) contours in the \((\tan \beta, u/v)\) plane for the various values of parameters \( g_X, \sin \chi, g_Y, \lambda \) and \( A \). We vary these in the region (52) and

\[
0.1 \leq \lambda \leq 0.9, \quad |A/v| \leq 10.
\]

(62)
Fig.3 Contours of $m_1^2$, $m_2^2$ and $m_S^2$ in the $(\tan \beta, u/v)$ plane for (A) $\eta$ model with $\sin \chi = 0.3$ and (B) $\xi -$ model with $\sin \chi = 0.2$. In both cases $g_X/g_Y = 1$, $\lambda = 0.5$ and $A/v = 2$ are assumed. Contours of $m_1^2$, $m_2^2$ and $m_S^2$ are drawn by dash-dotted, dotted and solid lines, respectively. The value associated with contours are $(m_{1,2,S}/v)^2 = -30, -25, -20, \cdots, 20, 30$. In particular, contours for $(m_{1,2,S}/v)^2 = -30$ are marked by x at their neighborhood.

Since the parameter $A$ is varied in the above region including a negative $A$, we only need to search the positive $u/v$ region. As such an example, in Fig. 3 we draw those contours for a typical set of these parameters.

Here we summarize the general features found from this analysis. The $\sin \chi$ and $g_X$ dependence of these contours is non-negligible but weak. As expected from the largeness of $u/v$, $m_S^2$ is almost dominated by the first term in Eq. (60) and then it is not affected so large by parameters $\lambda$ and $A$ in our interesting $(\tan \beta, u/v)$ region. The contours of $m_S^2$ are almost equivalent to the constant $u/v$ lines satisfying $u/v \lesssim 10$. On the other hand, parameters $\lambda$ and $A$ play crucial role to determine the contours $m_1^2$ and $m_2^2$. This feature is mainly related to the behavior of the last terms in Eqs. (58) and (59). The behavior of contours $m_1^2$ and $m_2^2$ in our considering $(\tan \beta, u/v)$ regime may be explained as follows. For the small $\lambda$ and $|A/v|$ values, the $m_1^2$ and $m_2^2$ contours are also almost constant $u/v$ lines since the first two terms in Eqs. (58) and (59) dominate their values. As its result there appears an allowed wide $\tan \beta$ region where three contours satisfying $|(m/v)^2| < 30$ can cross at a point. If we take $|A/v|$ larger, the overlapping region of three
contours shrinks around \( \tan \beta \sim 1 \) and also its upper bound on \( u/v \) becomes smaller. This is because the contours of \( m_1^2 \) and \( m_2^2 \) have a shape which is determined by the last terms of Eqs. (58) and (59). If we make \( \lambda \) larger, their shape does not change largely. Although a wide \( \tan \beta \) region gives a solution and no favorite \( \tan \beta \) region appears, the upper bound on \( u/v \) becomes smaller. When both of \( \lambda \) and \( |A/v| \) are taken larger, the contours of \( m_1^2 \) and \( m_2^2 \) cross at the region where \( u/v \) has too large or small values like \( u/v \gtrsim 10 \) or \( u/v \lesssim 1 \). In that case we cannot find a solution in a suitable \((m_S/v)^2\) region. Some of these features can be found in Fig. 3. Similar features are reported in the RGEs study [12].

These results also may give us a hint for the RGEs study. As found in this argument, in such a study one of the important problems is clearly to find what kind of input parameters can realize the negative smaller \( m_3^2 \) at the weak scale to obtain such a solution as \( u/v \gtrsim 2 \). In that case we may need non-universal soft supersymmetry breaking scalar masses at least in the Higgs sector [4, 12].

Now by combining this result with the previously discussed \( \chi^2 \)-fits using the precise measurements, we can restrict the allowed region in the \((\tan \beta, u/v)\) space of the present models. In that region the model can simultaneously satisfy both of the condition for the radiative symmetry breaking and the requirement to fulfill the constraint from the data of electroweak precise measurements. As an example, if we use Figs. 2 and 3 for this purpose, we can find such solutions at the following values,

\[
\begin{align*}
\tan \beta &\sim 0.8, \quad 2.5 \lesssim u/v \lesssim 8 \quad (\eta \text{ model with } \sin \chi = 0.3), \\
\tan \beta &\sim 1.5, \quad 2 \lesssim u/v \lesssim 8 \quad (\xi^- \text{ model with } \sin \chi = 0.2),
\end{align*}
\]

(63)

where the parameters are taken as \( g_X/g_Y = 1, \lambda = 0.5 \) and \( A/v = 2 \). If we deviate \( \tan \beta \) from the above one within the region (53), the lower bound of \( u/v \) suddenly becomes larger. These are general features.

We comment in some detail on the parameters dependence of these solutions. The drastic effect of \( \sin \chi \neq 0 \) on this vacuum structure can not be found but we should note that it has an important role to determine \( \tan \beta \) which makes small \( u \) values allowable. Generally the consistent solutions tends to be found for not so large values of \( \lambda \) and \( |A/v| \). This tendency can be understood from the previously mentioned features of the parameter dependence of Fig. 2 and Fig. 3. If we take account of the well known result \( \tan \beta \gtrsim 1 \) in
the study of the radiative symmetry breaking due to large Yukawa couplings, the above results shows the $\eta$ model seems not to be realized as a consistent model in our framework assuming $| (m_{1,2}/v)^2 | \leq 30$. The $\xi_-$ model has a good nature also in this aspect.

4.4 Masses of an extra $Z$ and a neutral Higgs scalar

It is very interesting that in this study the vacuum solutions can be found only at the stringently restricted region in the $(\tan \beta, u/v)$ plane. This makes us possible to find the bounds on $m_{Z_2}$ and the lightest neutral Higgs scalar mass $m_{h^0}$. Here we estimate these mass bounds in the basis of the study in the previous subsections.

The neutral Higgs scalars mass matrix can be written in the basis of $(H_1^0, H_2^0, S)$ by using the minimization conditions (58)-(60) as,

$$
\frac{m^2_{h^0}}{g^2} = \begin{pmatrix}
\cos^2 \beta (\tilde{g}^2 + \zeta_1^2) + 2\lambda \tilde{A} \tan \beta & \frac{1}{2} \sin 2\beta (-\tilde{g}^2 + X_{12}) - 2\lambda \tilde{A} \tilde{u} & \cos \beta (\tilde{u} X_{13} - 2\lambda \tilde{A} \tan \beta) \\
\frac{1}{2} \sin 2\beta (-\tilde{g}^2 + X_{12}) - 2\lambda \tilde{A} \tilde{u} & \sin^2 \beta (\tilde{g}^2 + \zeta_2^2) + 2\lambda \tilde{A} \cot \beta & \sin \beta (\tilde{u} X_{23} - 2\lambda \tilde{A} \cot \beta) \\
\cos \beta (\tilde{u} X_{13} - 2\lambda \tilde{A} \tan \beta) & \sin \beta (\tilde{u} X_{23} - 2\lambda \tilde{A} \cot \beta) & \zeta_3^2 \tilde{u}^2 + \frac{4}{\tilde{u}} \lambda \sin 2\beta
\end{pmatrix},
$$

where $\tilde{g}^2 = g_W^2 + g_X^2$ and $X_{ij} = \zeta_i \zeta_j + 4\lambda^2$. In this expression we also use abbreviations $\tilde{A} = A/v$ and $\tilde{u} = u/v$. In order to estimate $m_{h^0}$ we numerically diagonalize this matrix and plot the contours of the smallest mass eigenvalue in the $(\tan \beta, u/v)$ plane. Since in the present models the value of $u/v$ is severely restricted by the bounds on $m_{Z_2}$ and $\xi$, it cannot be so small that the lightest neutral Higgs scalar cannot be generally dominated by a scalar partner of $S$ which has no electroweak interactions. This situation is completely different from the case of NMSSM where the lightest neutral Higgs scalar can be dominated by the scalar component of $S$ in the case of the small $u/v$ [34]. Thus the MSSM bound on the lightest neutral Higgs scalar mass might be almost applicable and we could impose $m_{h^0} \gtrsim 62.5$ GeV [33]. This can give an additional constraint on the parameters in the present models.

To make our discussion definite we consider the $\xi_-$ model. We know from the discussions up to now that $\tan \beta \gtrsim 1$ and $2 \lesssim u/v \lesssim 8$ are required. This makes us possible to examine the allowed parameters region by checking whether this condition on $m_{h^0}$ is satisfied or not in the constrained $(\tan \beta, u/v)$ plane. Although $m_{h^0}$ is found to have only a negligible dependence on $\sin \chi$ and $g_X$, it is crucially dependent on $\lambda$ and $A$. In fact, for $A/v \lesssim 0.1$, $m_{h^0} \gtrsim 62.5$ GeV cannot be satisfied in the favorable part of the $(\tan \beta, u/v)$
Fig. 4 Contours of the lightest neutral Higgs scalar mass $m_{h^0}$ in the $(\tan \beta, u/v)$ plane for $\xi_-$ model with $\sin \chi = 0.2$, $g_X/g_Y = 1$, $\lambda = 0.5$ and $A/v = 2$.

plane for any $\lambda$ in the region (62). For small $A/v$, the larger $\lambda$ has a tendency to make $m_{h^0}^2 < 0$ there and then the vacuum unstable. We can estimate the allowed perturbative region of $\lambda$ for the typical values of $A/v$, for which $m_{h^0}$ can satisfy the experimental lower bound in a certain point of the above mentioned constrained $(\tan \beta, u/v)$ plane. Such an example is

\begin{align}
0.2 \lesssim \lambda \lesssim 0.5 \quad &\text{for } A/v = 1, \\
0.4 \lesssim \lambda \lesssim 0.9 \quad &\text{for } A/v = 5.
\end{align}

(64)

For the larger $A/v$, the larger $\lambda$ is favored and the larger $A/v$ tends to make $m_{h^0}$ larger.

To find the absolute value of $m_{h^0}$ consistent with other conditions, we take the same parameters as those used to draw Fig. 2 and Fig. 3. In this case contours of the lightest neutral Higgs scalar mass are shown in Fig. 4. One of interesting features of this figure may be a behavior of the contours for $\tan \beta$. As mentioned before, the lightest Higgs scalar in this model is mostly composed in the same way as the one of the MSSM whose mass increases with increasing $\tan \beta$. On the other hand, Fig. 4 shows a completely different behavior from that.\footnote{The author would like to thank the referee for pointing this out.} In the present Higgs scalar mass matrix there is a D-term contribution of the extra $U(1)_X$ even in a $2 \times 2$ submatrix for $(H_1^0, H_2^0)$. This contribution seems to make us possible to understand the feature in Fig. 4. In order to see this briefly,
we may extract a $2 \times 2$ submatrix for $(H_0^1, H_0^2)$ and carry out orthogonal transformation with an angle $\beta$. Then we can find the lightest Higgs mass upper bound as

$$m_{h^0}^2 \leq m_Z^2 \left[ \cos^4 \beta \left( 1 + \frac{\zeta_1^2}{g^2} \right) + \sin^4 \beta \left( 1 + \frac{\zeta_2^2}{g^2} \right) + 2 \sin^2 \beta \cos^2 \beta \left( -1 + \frac{X_{12}}{g^2} \right) \right],$$

where $\zeta_1$, $\zeta_2$ and $X_{12}$ represent the extra $U(1)_X$ effect. Although this upper bound increases with increasing $\tan \beta$ in the case of $\zeta_1 = \zeta_2 = 0$, we can find that this can decrease with increasing $\tan \beta$ in certain extra $U(1)_X$ models like the present case. The extra $U(1)_X$ model may give an example of the lightest Higgs scalar with the similar composition to the MSSM one but with the different mass behavior for $\tan \beta$ from that.

Finally, if we combine this with Figs. 2 and 3, the bounds of $m_{Z_2}$ and $m_{h^0}$ can be estimated as

$$400 \text{ GeV} \lesssim m_{Z_2} \lesssim 1500 \text{ GeV}, \quad 62.5 \text{ GeV} \lesssim m_{h^0} \lesssim 95 \text{ GV}. \quad (65)$$

In the case of $m_{Z_2}$ the lower and upper bounds are mainly constrained by the values of $\sin \chi$ and $m_{1,2,S}^2$, respectively. On the other hand, we should note again that $\lambda$ and $A$ are the most important to determine the bound of $m_{h^0}$. Although we assume that the soft supersymmetry breaking scalar masses $m_1^2$, $m_2^2$ and $m_3^2$ should be in the reasonable range $|m^2| < (1 \text{ TeV})^2$ from the viewpoint of naturalness consideration, it seems to be difficult to change this condition largely so that we expect our estimation method and then the obtained results are not so bad as far as $\lambda$ and $A$ are fixed in a suitable range.

5 Summary

The weak scale extra $U(1)_X$ models are very promising candidates of a solution for the $\mu$-problem. They are characterized by the existence of an additional neutral gauge boson around a several hundred GeV or a TeV. Thus they are also interesting from a viewpoint of experiments at the high energy front. As such a concrete example, at least we have two simple models, $\eta$ model and $\xi_-$ model which can be induced from the perturbative superstring inspired $E_6$ model. For these models it is an important issue to check their consistency as realistic ones.

In this paper we payed our attention on their vacuum structure parametrized by the VEVs of three Higgs scalars to study it. We investigated them based on the constraints
from the precise measurements of electroweak observables and the radiative symmetry breaking at the weak scale. We searched the allowed region in the \((\tan \beta, u/v)\) plane. When we proceeded this study, we took account of the effects of the gauge kinetic term mixing of abelian factor groups, which might be a special feature of the extra \(U(1)\) models. In general they can potentially bring some effects not only on the usual gauge interaction sector but also on the D-term scalar potential. The latter effect has not been payed any attention before.

Based on the numerical study we had the following results. First, the consistent region in the \((\tan \beta, u/v)\) plane is strongly restricted but we could find that there was the interesting allowed region for suitable values of parameters \(\lambda\) and \(A\). This means that the weak scale extra \(U(1)_X\) models could be realistic as the \(\mu\)-problem solvable electroweak model at least from the viewpoint of consistency of the precise measurements and the radiative symmetry breaking. Using our result for the allowed region in the \((\tan \beta, u/v)\) plane, we could estimate the mass bounds on an extra neutral gauge boson and the lightest neutral Higgs scalar. It is noticable that we can bring the upper bound on the lightest neutral Higgs scalar mass. Since we did not solve the RGEs of physical parameters like Yukawa coupling constants, soft supersymmetry breaking parameters and so on explicitly in this analysis, these bounds could be somehow rough ones. However, it is an interesting feature of this type of analysis to put the upper bound for them. To obtain more fine bounds, we need to analyze it by solving RGEs explicitly.

Second, the effect on the scalar potential itself coming from the kinetic term mixing was found not to be so large. However, its effect could not be negligible as it could rather largely affect the gauge interaction sector. Since the allowed region in the \((\tan \beta, u/v)\) plane is influenced by both sectors, we could see non-negligible effects on the values of \(\tan \beta\) and \(u\) due to the kinetic term mixing. We need further quantitative study for the origin of this kinetic term mixing in the present type of models as done in [15, 20].

The \(\mu\)-problem solvable models, in particular, which have an intermediate scale like our \(\xi_-\) model, show very interesting phenomenological features such as an explanation of small neutrino masses. Although we only discuss the \(\xi_-\) model as an example of this kind of models, there can be other models and it seems to be worthy to construct such models by various methods, especially, as the low energy effective models of superstring. If we consider these models seriously, we also need to study the problems related to the exotic
fields included in the models, like proton stability and flavor changing rare processes [27]. Such a study may enlarge our scope for the unification scenario different from the usual grand unification schemes.

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Appendix

In this appendix we derive the formulae for the deviation of the electroweak parameters \((\rho, \bar{s}_W^2)\) from the SM prediction due to the existence of an extra \(U(1)_X\) gauge field based on Refs. [17], [18]. They are defined in the SM through the neutral current effective interaction Lagrangian as follows,

\[
\mathcal{L}_{\text{int}} = \frac{e}{2s_W c_W} \rho_{\text{SM}}^{1/2} \left( J_3^\mu - 2 \bar{s}_W^2 J_{\text{em}}^\mu \right) Z_\mu, \tag{66}
\]

where \(\rho_{\text{SM}} \simeq 1 + \delta \rho\). The corresponding effective interaction Lagrangian of \(Z_1^\mu\) in the extra \(U(1)_X\) model is defined as

\[
\mathcal{L}_{\text{int}} = \frac{e}{2s_Z c_Z} \rho^{1/2} \left( J_3^\mu - 2 \bar{s}_Z^2 J_{\text{em}}^\mu \right) Z_{1\mu}. \tag{67}
\]

We assume that the radiative correction dominantly comes from the SM sector and others can be neglected. Thus the one-loop corrected on-shell Weinberg angle \(\bar{s}_W^2\) in the SM and the corresponding Weinberg angle \(\bar{s}_Z^2\) in the present models are defined in terms of the observed values \(\alpha\) and \(G_F\) as

\[
\bar{s}_W^2 \bar{c}_W^2 = \frac{\pi \alpha(m_Z^2)}{\sqrt{2} G_F m_Z^2 \rho_{\text{SM}}}, \quad \bar{s}_Z^2 \bar{c}_Z^2 = \frac{\pi \alpha(m_Z^2)}{\sqrt{2} G_F m_Z^2 \rho_{Z1} \rho_{\text{SM}}}, \tag{68}
\]

where the relation between \(m_{Z1}^2\) and \(m_Z^2\) is given by Eq. (42). From Eq. (68) we can easily obtain

\[
\bar{s}_Z^2 \simeq \bar{s}_W^2 + \frac{\bar{s}_W^2 c_W^2}{c_W^2 - \bar{s}_W^2} \left( \frac{m_Z^2}{m_{Z1}^2} - 1 \right). \tag{69}
\]

In the present model vector interaction parts in the effective Lagrangian for \(Z_{1\mu}\) are summarized by using Eqs. (44) and (45) as follows,

\[
\mathcal{L}_{\text{int}} = \frac{e}{2s_W c_W} \rho_{\text{SM}}^{1/2} (1 + s_W \xi \tan \chi) \left( J_3^\mu - 2 \bar{s}_Z^2 J_{\text{em}}^\mu \right) Z_{1\mu}
- e c_W \xi \tan \chi J_{\text{em}}^\mu Z_{1\mu} + \frac{g_X \xi}{2 \cos \chi} J_{\mu}^\text{em} Z_{1\mu}. \tag{70}
\]

The last term is taken into account as a \(\bar{v}_f^\prime\) term in Eq. (45) and then it is irrelevant to the present calculation. The first two terms in the right-hand side can be rearranged into the following form,

\[
\mathcal{L}_{\text{int}} \simeq \frac{e}{2s_W c_W} \rho_{\text{SM}}^{1/2} (1 + s_W \xi \tan \chi) \left( J_3^\mu - 2 \bar{s}_Z^2 J_{\text{em}}^\mu \right) Z_{1\mu}. \tag{71}
\]
Since the $Z_\mu$-$X_\mu$ mixing introduces a new interaction $-ec_W \xi \tan \chi J^\mu_{em} Z_{1\mu}$ for $Z_{1\mu}$ at tree level in comparison with the SM case, $\bar{s}_Z^2$ deviates from $s_Z^2$ as

$$\bar{s}_Z^2 \simeq s_Z^2 + s_W c_W^2 \xi \tan \chi.$$  \hfill (72)

We can also rewrite the right-hand side of Eq. (71) as

$$\frac{e}{2s_Zc_Z} \rho_{SM}^{1/2} \left(1 + s_W \xi \tan \chi \right) \left[1 + \frac{1}{2} \left( \frac{m_Z^2}{m_{Z_1}^2} - 1 \right) \right] \left(J^\mu_3 - 2s_Z^2 J^\mu_{em} \right) Z_{1\mu}.$$  \hfill (73)

From this expression we can extract the expression of $\rho$ parameter in this model,

$$\rho = \rho_{SM} (1 + s_W \xi \tan \chi)^2 \left[1 + \frac{1}{2} \left( \frac{m_Z^2}{m_{Z_1}^2} - 1 \right) \right]^2.$$  \hfill (74)

Thus up to the first order of the small quantities the expression of $\delta \rho_M$ can be read off as,

$$\delta \rho_M \simeq 2s_W \xi \tan \chi + \left( \frac{m_Z^2}{m_{Z_1}^2} - 1 \right).$$  \hfill (75)

From Eqs. (69) and (72) the deviation of $\bar{s}_W^2$ can be derived as

$$\Delta \bar{s}_W^2 \equiv \bar{s}_Z^2 - \bar{s}_W^2 \\ \simeq -s_W c_W \left( \frac{m_Z^2}{m_{Z_1}^2} - 1 \right) + s_W c^2_W \xi \tan \chi.$$  \hfill (76)

If we remind the definition of famous $S$ and $T$ parameters \[37\]

$$\rho = 1 + \alpha T, \quad \Delta \bar{s}_W^2 = \frac{\alpha}{c_W^2 - \bar{s}_W^2} \left(-c_W^2 \bar{s}_W T + \frac{1}{4} S \right),$$  \hfill (77)

we can obtain the contributions from the abelian gauge kinetic term mixing and also the mass mixing to these parameters as follows,

$$\alpha T_M \simeq 2s_W \xi \tan \chi + \left( \frac{m_Z^2}{m_{Z_1}^2} - 1 \right),$$  \hfill (78)

$$\alpha S_M \simeq 4c_W^2 s_W \xi \tan \chi.$$  \hfill (79)

Since the difference between $\bar{s}_W$ and $s_W$ is a higher order effect in Eqs. (75)-(79), we can replace $s_W$ with $\bar{s}_W$ in those formulae for the practical calculation.
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