Chern-Simon type photon mass from fermion electric dipole moments at finite temperature in 3+1 dimensions

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ABSTRACT
We study the low energy effective field theory of fermions with electric and magnetic dipole moments at finite temperature. We find that at one loop there is an interaction term of the Chern-Simon form $\mathcal{L}_I = m_\mu A_\nu \tilde{F}^{\mu\nu}$. The four vector $m_\mu \simeq d_\mu m_i^2 \partial_\mu (\ln T)$ is interpreted as a Chern-Simon type mass of photons, which is determined by the electric (magnetic) dipole moments $d_i (\mu_i)$ of the fermions in the vacuum polarisation loop diagram. The physical consequence of such a photon mass is that, photons of opposite circular polarisations, propagating through a hot medium, have different group velocities. We estimate that the time lag between the arrival times of the left and right circularly polarised light signals from pulsars. If the light propagates through a hot plasma (where the temperature in some regions is $T \sim 100$ MeV) then the time lag between the two circularly polarised signals of frequency $\omega$ will be $\Delta t(\omega) \approx 10^{-6}/\omega$. It may be possible to observe this effect in pulsar signals which propagate through nebula at high temperatures.
Experimental consequences of fermion electric dipole moments are of interest since they signify CP (or T) violation in the underlying theory. In the low energy effective theory derived from such a CP violating fundamental theory, fermions would have an electric dipole form factor $\Gamma_{\mu e}(q) = id_i \gamma_5 q_\nu \sigma^{\mu\nu}$ which is CP odd, along with the usual magnetic dipole form factor $\Gamma_{\mu m}(q) = i\mu_i q_\nu \sigma^{\mu\nu}$. We consider the vacuum polarisation amplitude $\Pi^{\alpha\beta}(q)$ at one loop in an effective field theory with a $\Gamma_{\mu e}$ and $\Gamma_{\mu m}$ vertex. We find that at finite temperature the polarisation tensor $\Pi^{\alpha\beta}(T)$ is odd under CP and T. This vacuum polarisation amplitude corresponds to an interaction term of the form $L_I = m_\mu A_\nu \tilde{F}^{\mu\nu}$, where the four vector $m_\mu = d_i \mu_i m_i^2 \partial_\mu (\ln T)$ is determined by the electric (magnetic) dipole moments $d_i (\mu_i)$ of the fermions included in the loop for calculating the vacuum polarisation amplitude, as well as the external temperature gradient. This interaction term is similar in form to the Chern-Simon (CS) interaction [1] in 2+1 dimension $L_{CS} = \kappa \epsilon_{\mu\nu\rho} A^\mu F^{\nu\rho}$ where the CS coupling $\kappa$ is dimensionless. In 3+1 dimensions the four vector $m_\mu = d_i \mu_i m_i^2 \partial_\mu (\ln T)$ has dimensions of mass and we shall henceforth refer to it as the Chern-Simon photon mass. The 3+1 dimensional Chern-Simon interaction term $L = d_i \mu_i m_i^2 \partial_\mu (\ln T) A_\nu \tilde{F}^{\mu\nu}$ is invariant under Lorentz transformations and infinitesimal gauge transformations. The physical effect of the CS mass is that photons with opposite polarisations, propagate through a hot medium with different group velocities. The high temperature vacuum is therefore birefringent in that the refractive index for circulary polarised photons of opposite helicities are different. The magnitude of the CS mass is determined by the electric and magnetic dipole moments of fermions with mass $m_i \leq T$. The contribution of a nonzero electron edm $d_e \sim 10^{-27} ecm$ to the Chern-Simon mass is small, of the order $m_\mu \sim 10^{-17} \partial_\mu (\ln T)$. At $T \sim 100$ Mev the CS mass receives a large contribution from the muon dipole moments and is $\sim 10^{-6} \partial_\mu (\ln T)$. This number may be observable in astrophysical situations. We estimate that the time lag between the arrival times between the left and right circularly polarised light signals from pulsars. If the light propagates through a hot plasma (where the temperature in some regions is of the order $T \sim 100$ MeV) then the time lag between the two circularly polarised signals of frequency $\omega$ will be $\Delta t(\omega) \sim 10^{-6}/\omega$. 

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Starting with some fundamental renormalisable theory with CP violation one may obtain the low energy effective theory valid below some scale $v$ (say $v \sim O(MeV)$) by integrating out all particles with masses $m_i \geq v$. The effective CP violating theory of fermions with masses $m_i \leq v$ is given by the action

$$L_{\text{eff}} = -\frac{1}{2} F^\mu\nu F_{\mu\nu} + \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i - i\frac{e}{2} d_i \gamma_5 \sigma_{\mu\nu} F^{\mu\nu} - i\frac{e}{2} \mu_i \sigma_{\mu\nu} F^{\mu\nu}) \psi_i.$$  \hspace{1cm} (1)

The last two dimension five operators in the effective action (1) arise from the loop diagrams of the fundamental theory. The electric dipole form factor arises only in theories with CP violation. Consider the one loop vacuum polarisation amplitude $\Pi^{\alpha\beta}(q)$ (fig.1) with an electric dipole vertex, $\Gamma^\mu_e = d_i q_\nu \sigma^{\mu\nu} i\gamma^5$ and a magnetic dipole vertex, $\Gamma^\alpha_m = i \mu_i q_\beta \sigma^{\alpha\beta}$. Here $d_i$ ($\mu_i$) represent the electric (magnetic) dipole moment of the $i$th fermion running in the loop. The amplitude for the process may be written as

$$\Pi^{\mu\alpha}(q) = -\sum_i (d_i \mu_i) q_\nu q_\beta \int \frac{d^4k}{(2\pi)^4} \frac{T^{\mu\nu\alpha\beta}}{(k^2 - m^2)((k + q)^2 - m^2)},$$  \hspace{1cm} (2)

where the trace

$$T^{\mu\nu\alpha\beta} = Tr[(\sigma^{\mu\nu} \gamma^5 (\gamma^\eta k_\eta + m)\sigma^{\alpha\beta}(\gamma^\delta k_\delta + \gamma^5 q_\delta + m))$$

$$+ (\sigma^{\mu\nu} (\gamma^\eta k_\eta + m)\sigma^{\alpha\beta} \gamma_5 (\gamma^\delta k_\delta + \gamma^5 q_\delta + m))].$$  \hspace{1cm} (3)

after some algebra this turns out to be

$$T^{\mu\nu\alpha\beta} = 8i\epsilon^{\mu\nu\alpha\beta} m^2.$$  \hspace{1cm} (4)

Therefore

$$\Pi^{\mu\alpha}(q) = \sum_i d_i \mu_i q_\nu q_\beta 8i\epsilon^{\mu\nu\alpha\beta} m_i^2 \Pi(q),$$  \hspace{1cm} (5)

with

$$\Pi(q) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m^2)((k + q)^2 - m^2)}.$$  \hspace{1cm} (6)

To evaluate (6) at finite temperature we follow the imaginary time prescription of Jackiw and Donald [2]. There is a technical problem in taking the zero momentum
limit of the vacuum polarization amplitude which is discussed in [3,4]. We let the frequency $k_0$ take periodic values along the imaginary axis of the contour with period $\frac{2\pi}{\beta}$ (where $\beta = 1/T$). The integral $\int dk_0$ is replaced by a sum over the discrete $k_0$:

$$\int \frac{dk_0}{2\pi} f(k_0) \rightarrow \frac{i}{\beta} \sum_{n=-\infty}^{\infty} f(k_0 = \frac{i}{\beta}(2n + 1)\pi).$$

(7)

Using this the vacuum polarization amplitude (6) at finite temperature is

$$\frac{i}{\beta} \sum_{n=-\infty}^{\infty} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^2 - m^2)((k + q)^2 - m^2)},$$

(8)

with

$$k_0 = \frac{(2\pi + 1)i\pi}{\beta}.$$

To evaluate the above integral we use the Feynman parametrization; that is

$$\frac{1}{ab} = \int_0^1 dx [ax + b(1-x)]^{-2}.$$

(9)

By using the eqn.(9) and the discrete values of $k_0$ the amplitude (eqn. 6) is given by

$$\Pi(q) = \frac{i}{\beta} \frac{\beta^4}{16\pi^4} \int \frac{d^3k}{(2\pi)^3} \int_0^1 dx \sum_{n=-\infty}^{\infty} \frac{1}{(n^2 + 2na + b)^2},$$

(10)

where

$$a = (\frac{1}{2} - \frac{i\beta q_0 x}{2\pi}),$$

(11a)

and

$$b = \frac{1}{4} - \frac{i\beta q_0 x}{2\pi} + \frac{\omega_k^2\beta^2}{4\pi^2} - \frac{(q^2 - 2k.q)\beta^2 x}{4\pi^2}. $$

(11b)

Factorizing $(n^2 + 2na + b)$ by partial fraction and using the formula

$$\sum_{n=-\infty}^{\infty} (n - x)^{-2} = -\pi^2 cosech^2(i\pi x),$$

(12)

the mode sum in the limit $q \to 0$ is

$$\sum_{n=-\infty}^{\infty} (n^2 + 2na + b)^{-2} = -\frac{2\pi^4}{\beta^2\omega_k^2} \left[sech^2\frac{\beta\omega}{2} + \frac{2}{\beta\omega} \tanh\frac{\beta\omega}{2}\right],$$

(13)
with $\omega_k = (\vec{k}^2 + m^2)^{1/2}$. Then the vacuum polarisation amplitude, using eqn.(13) yields

$$\Pi(T) = -2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_k^2} \{ \tanh \frac{\beta \omega}{2} + \frac{\beta \omega}{2} \text{sech}^2 \frac{\beta \omega}{2} \}. \quad (14)$$

The interaction term corresponding to the vacuum polarisation amplitude eqn.(14) is obtained by supplying the external legs for photons to the polarisation tensor (5)

$$L_I = \epsilon_\mu(q)\epsilon_\alpha(q) \Pi^{\mu\alpha}(q)$$

$$= \sum_i d_i \mu_i m_i^2 \Pi(T) \epsilon^{\mu\nu\alpha\beta} F_{\mu\nu} F_{\alpha\beta}$$

$$= \sum_i d_i \mu_i m_i^2 \Pi(T) F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (15)$$

with $\Pi(T)$ given by eqn.(14). Integrating eqn.(15) by parts and using Bianchi identity $\partial_\mu \tilde{F}^{\mu\nu} = 0$, we have

$$L_I = -\sum_i d_i \mu_i m_i^2 (\partial_\mu \Pi(T)) A_\nu \tilde{F}^{\mu\nu}. \quad (16)$$

The gradient $\partial_\mu \Pi(T)$ may be simplified as

$$\partial_\mu \Pi(T) = \partial_\mu T \left( -\frac{1}{T^2} \right) \frac{\partial}{\partial \beta} \Pi(T)$$

$$= (\partial_\mu T) \frac{1}{T} \int_{-\frac{m^2}{\beta^2}}^{\infty} \frac{dx}{x^2} \left[ x^2 - \frac{\beta^2 m^2}{4} \right]^{1/2} \text{sech}^2 x (1 - x \tanh(x))$$

$$= I(m\beta) \partial_\mu(\ln T), \quad (17)$$

where the definite integral has the value $I(m\beta) \sim 0.4$, for $m \sim T$. The Chern-Simon interaction can be written as

$$L_{CS} = -\frac{1}{2} \sum_i I(\beta m_i) d_i \mu_i m_i^2 (\partial_\mu \ln T) A_\nu \tilde{F}^{\mu\nu} = m_\mu A_\nu \tilde{F}^{\mu\nu}, \quad (18)$$

where

$$m_\mu = \sum_i I(\beta m_i) d_i \mu_i m_i^2 (\partial_\mu \ln T), \quad (19)$$
is the four vector Chern-Simon photon mass which is determined by the electric, magnetic dipole moments of fermions with masses $m_i \leq T$.

Under gauge transformation $\delta A_\mu = \partial_\mu \Lambda$ the CS interaction varies as

$$\delta \mathcal{L}_{CS} = \frac{C}{4} \Lambda \tilde{F}^{\beta \alpha} (\partial_\alpha \partial_\beta (\ln T)) - \partial_\beta \partial_\alpha (\ln T) = 0,$$

(20)

with $C = \mu_i d_i m_i^2$. Therefore unlike the Proca mass the CS mass term does not violate gauge invariance. Also since $\partial_\mu \Pi(T)$ transforms like a covariant vector under Lorentz transformations. The expression in eqn. (19) is therefore Lorentz invariant. If the four vector $m_\mu$ were a fixed vector instead of being the gradient of a scalar then the interaction term would have violated Lorentz invariance. Carroll, Field and Jackiw (CFJ) [5,6] have studied the phenomenological consequences of such a Lorentz violating term $p_\mu A_\nu \tilde{F}^{\mu \nu}$ (where $p_\mu$ is a constant vector) and put an upper bound on the coefficient $|p_\mu|$. We will see that the magnitude of the CS mass $m_\mu$ turns out to be well below the upper bound established by CFJ.

Propagation of electromagnetic waves is governed by Maxwells Lagrangian augmented by the Chern-Simon interaction. Thus the Lagrangian is written as

$$\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{2} m_\mu A_\nu \tilde{F}^{\mu \nu}.$$

(21)

Then the equations of motion from eqn. (21)

$$\partial_\mu F^{\mu \nu} + m_\mu \tilde{F}^{\mu \nu} = 0,$$

(22)

gives the inhomogenous Maxwells equations

$$\nabla . \vec{E} + \vec{m} . \vec{B} = 0,$$

(23a)

$$-\partial_0 \vec{E} + \nabla \times \vec{B} + m_0 \vec{B} + \nabla \times \partial_0 \vec{E} = 0.$$

(23b)

The homogenous Maxwells equations follow from the Bianchi identity

$$\partial_\mu \tilde{F}^{\mu \nu} = 0,$$

(24)

which in terms of $\vec{E}$ and $\vec{B}$ reads

$$\nabla . \vec{B} = 0,$$

(25a)
and
\[ \partial_0 \vec{B} + \nabla \times \vec{E} = 0. \] (25b)

Eqns. (23) and (25) can be combined to yield the wave equation
\[ \partial_0^2 \vec{E} - \nabla^2 \vec{E} + \nabla (\nabla \cdot \vec{E}) + m_0 \nabla \times \vec{E} + \vec{m} \times \partial_0 \vec{E} = 0. \] (26)

Consider a wave propagating in the direction of the temperature gradient \( \nabla T \). Choosing the \( z \)-axis along the direction of wave propagation we find that for the circularly polarised combinations
\[ E_\pm(z, t) = (E_x \pm iE_y) \exp(ikt - i\omega t), \] (27)

the wave equation (26) decouples as
\[ (\omega^2 - k^2 - \omega_p^2)E_\pm) \pm \omega m_z E_\pm = 0. \] (28)

Where the plasma frequency \( \omega_p = Ne/m_e \) is determined primarily by the electron number density in the hot medium. The effect of the CS term is to introduce a difference in the phase and the group velocities of the two circularly polarised modes. This is reflection of the P and T violation caused by the CS interaction. From the dispersion relation (28) we obtain the group velocities of the two polarisation modes
\[ v_\pm = \frac{d\omega_\pm}{dk} = (1 + \frac{\omega_p^2}{(\omega^2 - \omega_p^2)} \mp \frac{m_z \omega}{(\omega^2 - \omega_p^2)})^{-1/2}. \] (29)

The group velocities of the two circularly polarised modes of the same wavelength are different. The group velocity of either polarisation never exceeds the velocity of light in the zero temperature vacuum since at finite temperature the plasma frequency is always larger than the Chern-Simon photon mass.

Consider the propagation of a pulsar signal through the hot nebula surrounding it. As the light pulse propagates through the hot plasma the (-) helicity modes with the smaller group velocity will fall behind the (+) helicity modes. The time lag between the two modes after propagating through a distance \( D \) of the nebula be computed from the arrival times \( t_\pm \) given by
\[ t_\pm (\omega) = \int_{z_i}^{z_i + D} dz \left( 1 + \frac{\omega_p^2}{(\omega^2 - \omega_p^2)} \mp \frac{d_i \mu_i m_i^2}{(\omega^2 - \omega_p^2)} (\partial_\mu \ln T) \omega \right)^{1/2}. \] (32)
The magnitude of the time lag $\Delta t = (t_+ - t_-)$ depends upon the dipole moments of the fermions, with $m_i \leq T$ is given by

$$\Delta t(\omega) \simeq \frac{d_i \mu_i \Delta T / T}{(\omega^2 - \omega_p^2)^{1/2}}. \quad (33)$$

At $T \sim Mev$ only electrons $d_1 \sim 10^{-27} e\, cm$ contribute. $\Delta t$ in this case turns out to be extremely small $\Delta t(\omega) \sim 10^{-17}/\omega$. If there are regions in the nebula where the temperature could be as high as $T \sim 100 Mev$ then muons with electric dipole moment $d_\mu \sim 10^{-19} e\, cm$ [10] will contribute substantially. The time lag in this case turns out to be $\Delta t(\omega) \sim 10^{-6}/\omega$. There remains the possibility that this polarisation dependent time lag in the signals from pulsars may be observed in practice.

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