Inverse Patch Transfer Function Method based on Steepest Gradient Descent Algorithm for Sound Field Reconstruction

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Abstract In the iPTF method with Green’s function, the classical regularization method used to solve the inverse problem can’t provide satisfactory reconstruction results around some eigen frequencies of the cavity in the iPTF method with Green’s function. In this paper, the steepest gradient descent (SGD) algorithm is introduced to solve the inverse problem in the iPTF method. In addition, to make the iPTF method with Green’s function more efficient, Contour-integration is used to split the Fourier series of Green’s function into one single-sum and one double-sum Fourier series. By applying Contour-integration, the computational efficiency of the iPTF method with Green’s function has been improved. The iPTF method with efficient Green’s function is examined in both simulations and application experiments. Experimental validations in the application are carried out in the end. The present results in experiments are also acceptable for field reconstruction.

1. Introduction

The NAH is developed by Williams et al. [1,2]. Initially, the method is based on two-dimensional (2D) Fourier transform to reconstruct sound field. This method is limited to regularly shaped sources. As target sources grow complex, the inverse boundary elements method (iBEM) was proposed to identify the irregularity shaped sources [3]. Another well-known method, the equivalent source method (ESM) [4], can also solve the problem of irregularly shaped source sound field reconstruction. Other NAH methods that combined some special algorithms have also been developed to offer more satisfactory field reconstruction results, such as statistically optimized near-field acoustical holography (SONAH) [5] and wideband holography (WBH) [6].

With the increase of the requirement of in-situ applications and the complexity of target sources, the inverse patch transfer function (iPTF) method is presented by Aucejo et al. [7]. It is first applied to the velocity field reconstruction of the target source with the L-shaped geometry [7]. Then in a reverberation environment, the normal velocities of some 3D structures such as a preliminary catalytic device, an oil pan, and an engine have also been identified by this method [8-10].

To simplify the process, the concept of Green’s function of the cavity is used in the iPTF method for the reconstruction of sparsely distributed sources[11]. In this paper, we apply this iPTF method with Green’s function of the cavity on the sound field reconstruction of continuously distributed sources, a vibration plate. It is found that the reconstruction results are unacceptable using the classical regularization method at some eigen frequencies. To get more accuracy reconstruction results, the steepest gradient descent (SGD) algorithm is introduced to solve the inverse problem of field reconstruction.
reconstruction in the present work. Meanwhile, an efficient approach is employed to compute the Green’s function of the cavity. The approach is based on Contour-integration [12,13]. In experiments, double layer pressure measurements are applied to get the normal velocity and pressure on the surfaces of the virtual cavity [11,14].

2. Theoretical background

As shown in Fig. 1, the virtual cavity \( \Omega \) surrounding the target source \( S \) is defined by the virtual surface \( \Sigma \) that has no physical meaning. The points on the virtual surface and surface \( \Sigma' \) of the target source \( S \) are respectively marked as point \( Q' \) and \( Q'' \). Some disturbing sources exist outside the cavity. The pressure at any point \( M \) in the virtual cavity satisfies Helmholtz Kirchhoff’s equation. Hence, the acoustical problem of the cavity can be described as

\[
\Delta p(M) + k^2 p(M) = 0 \quad \forall M \in \Omega, \tag{1}
\]

where \( p(M) \) is the pressure at any point in the virtual cavity. Term \( k \) is the acoustic wave number which is obtained by the angular frequency \( \omega \) and the sound speed \( c \) in the acoustic medium.

After the derivation by iPTF method, the relationship between the pressure at any point \( M \) in the measured surface and the pressure at any point on the target source surface can be impressed as

\[
\mathbf{p}_m = \mathbf{z}_{mij} \mathbf{v}_j + \mathbf{z}_{mi} \mathbf{v}_i, \tag{2}
\]

where the patch impedance matrix \( \mathbf{z} \) can be defined as

\[
\mathbf{z} = -2i\omega \rho \sum_{i=1}^{N} \left( \mathbf{G}(Q'',Q') \right) S_i. \tag{3}
\]

And \( \mathbf{z}_{mij} \) is the patch impedance matrix between the patches on the source surface and that on the virtual surfaces. Matrix \( \mathbf{z}_{mi} \) establishes the relation between the \( m \)th patch and \( i \)th patch on virtual surfaces. \( S_i \) denotes the area of each patch. After inverting Eq. (2), we can get the formulation of the iPTF method as follows

\[
\mathbf{v}_j = \mathbf{z}_{mij}^{-1} (\mathbf{p}_m - \mathbf{z}_{mi} \mathbf{v}_i). \tag{4}
\]

Also, after achieving the velocity distribution on the target sources, the pressure \( \mathbf{p}_k \) at any point in the field can be obtained as

\[
\mathbf{p}_k = \mathbf{z}_{kJ} \mathbf{v}_J + \mathbf{z}_{ki} \mathbf{v}_i, \tag{5}
\]

by substituting \( \mathbf{v}_j \) in Eq. (4) into Eq. (5). In Eq. (5), \( \mathbf{z}_{kJ} \) is the patch transfer matrix between the points in the cavity and that on the source surface and \( \mathbf{z}_{ki} \) represents the patch transfer matrix between the points in the cavity and that on the virtual surfaces.

To improve its computational efficiency, the Contour-integration [12, 13] is introduced in the present work. The partial infinite sum in cavity Green’s function can directly be carried out by the contour integration [16,17]. The function \( G \) can be reduced as

\[
G = \frac{1}{V} \left( \sum_{n=0}^{\infty} e_n \cos k_x x_0 \sum_{n=0}^{\infty} e_n \cos k_y y_0 \frac{e_n \cos k_x x_0 y_0}{k_x^2 + k_y^2 - k^2} + 2 \sum_{n=0}^{\infty} e_n \cos k_x x_0 \sum_{n=0}^{\infty} e_n \cos k_y y_0 \cos k_y y_0 S(n) \right), \tag{6}
\]
where $S$ is defined as

$$S(n) = \frac{1}{2} \left[ \frac{1}{\pi} \right] \left[ \frac{\cosh \alpha_n (\pi - z^0) + \cosh \alpha_n (\pi - z^0)}{\sinh \alpha_n \pi} \right] \left[ \frac{4}{\alpha_n^2} \right] = \frac{1}{2} \left[ \frac{1}{\pi} \right] \left[ \frac{\pi}{2} \alpha_n (N_{n1}^+ + N_{n2}^- - \frac{1}{\alpha_n^2}) \right],$$

(7)

where term $N_{n1}^\pm$ is defined as

$$N_{n1}^\pm = \cosh \alpha_n (\pi - z^0),$$

$$N_{n2}^\pm = \frac{e^{\alpha_n (\pi - z^0)} + e^{\alpha_n (\pi - z^0)}}{e^{\alpha_n z} - e^{-\alpha_n z}} = \frac{1}{e^{\alpha_n z} - e^{-\alpha_n z}} + \frac{1}{e^{\alpha_n (2z - z^0)} - e^{-\alpha_n (2z - z^0)}}.$$  

(8)

The SGD algorithm is introduced to solve the inverse problem in the iPTF method with Green’s function of a virtual cavity. The algorithm can offer satisfactory reconstruction results by achieving a solution $V_j$ that minimizes the squared residual function

$$F(v_j) = \frac{1}{2} \| r(v_j) \|^2 = \frac{1}{2} \| p - z_{n1} v_j \|^2.$$  

(9)

In the algorithm, an iteration process is utilized in the steepest descent direction to obtain the solution $V_j$. The steepest descent direction is defined as the negative of the gradient vector of $F$ which is expressed as

$$\nabla F(v_j) = -z_{n1}^j \left( p_i - z_{n1} v_j^i \right).$$  

(10)

Then the iterative formula is given as

$$v_j^{i+1} = v_j^i + s_j \left( -\nabla F(v_j^i) \right),$$  

(11)

where $s_j$ is the moving step length in the iteration process. It can be deduced by minimizing the function $F$ as

$$F(v_j^{i+1}) = \frac{1}{2} \left[ \| r^2 - 2s_j (\nabla F(v_j^i))^j \nabla F(v_j^i) + s_j^2 (z_{n1}^j \nabla F(v_j^i))^j (z_{n1}^j \nabla F(v_j^i)) \right].$$  

(12)

Thus $s_j$ can be expressed as

$$s_j = \left( \frac{\left( \nabla F(v_j^i) \right)^j \nabla F(v_j^i)}{(z_{n1}^j \nabla F(v_j^i))^j (z_{n1}^j \nabla F(v_j^i))} \right).$$  

(13)

After iterations, the exact velocities distribution solution $V_j$ can be obtained. The main vibrating positions of sources can be detected by the identification map.

3. Numerical tests of the iPTF method with Green’s function and its efficient form

In numerical tests, the identification process is performed on a baffled plate excited by a harmonic point force as Fig. 2. The plate is 0.35m long, 0.30m width, and 0.0015m thick. A 0.35x0.30x0.05 m^3 virtual cavity is defined as shown in Fig. 2(a). A unit point force is added at point (0.1, 0.1, 0) m on the plate in Fig. 2(b). Fig. 2(c) shows the FE mesh of the virtual cavity for solving the direct problems in simulations. The field points, as measured points on the patches of the virtual cavity, are illustrated as Fig. 2(d). The radiated field obtained in simulations is provided by utilizing a finite element method in an infinite domain in software ACTRAN.
In this part, the velocity field of vibration plate at 380Hz is reconstructed as shown in Fig. 3. It can be seen that the identified results of the velocity field computed by the three methods all match the reference results well at this frequency. At 720Hz shown in Fig. 4, results with the SGD algorithm are almost consistent with reference results. However, the reconstruction results in Tikhonov regularization method miss some important information on the source surface, which is unacceptable.

Fig. 3 Velocity field (a) computed on the surface of the plate, (b) identified results by the SGD algorithm, (c) identified results by the SGDE algorithm and (d) identified results by Tikhonov regularization; Sound pressure field (e) computed on the surface of plate (reference), (f) identified results by the SGD algorithm, (g) identified results by the SGDE algorithm and (h) identified results by Tikhonov regularization at 380Hz.

Fig. 4 Velocity field (a) computed on the surface of the plate (reference), (b) identified results by the SGD algorithm, and (c) identified results by Tikhonov regularization; Sound pressure field (e) computed on the surface of plate (reference), (f) identified results by the SGD algorithm, (g) identified results by the SGDE algorithm and (h) identified results by Tikhonov regularization at 720Hz.
4. Real Experiment on a vibration plate

4.1 Experimental setup
An application of the proposed methods has been performed as an experimental validation to illustrate the numerical simulation as shown in Fig. 2. A rectangular plate made of steel is fixed on a wood frame by four G-clamps. This plate, with 0.35m long, 0.30m width, and 0.002m thick, is excited by an electrodynamic shaker that fed with cosine signal in an ordinary room where sound reflection and random disturbing sources both exist. Instead of using a P-U probe to measure the velocities and pressure on the holography, the double layer pressure measurement [11,18] is employed.

4.2 Experimental validation
Compared with the measured reference pressure field, the reconstructed pressure field results by the SGD and the SGDE algorithm are satisfactory and acceptable. The positions of the area of maximum and minimum sound pressure are identified accurately and the amplitude of the maximum of sound pressure approaches that of the reference measurement results. Because the reconstructed pressure field is deducted by the identified velocity field on the plate surface, the identified velocity field by two methods in Fig. 6(b) and (d) are also accurate.

Fig. 5 Experimental situ

Fig. 6 Experimental comparison between reference and reconstructed field at 380 Hz at 0.035m from the plate surface: (a) reference pressure field; (b) the reconstructed pressure field by the SGD algorithm; (d) the reconstructed pressure field by the SGDE algorithm; and real part of the identified velocity field at the plate by (c) the SGD algorithm and (e) the SGDE algorithm

5. Conclusions
In this paper, the iPTF method with Green’s function with normal modal expansions is used to reconstruct the sound field of the vibration plate. In the processing of solving the inverse problem,
Tikhonov regularization fails at some frequencies around the characteristic frequency as shown in simulations. To solve this problem, the SGD algorithm is introduced. Both the simulations and experiments indicate that the algorithm is proper for solving the inverse problems of sound field reconstruction in the iPTF method with Green’s function with normal modal expansions.

To make the iPTF method with Green’s function with normal modal expansions more efficient, the form of Green’s function is modified by Contour-integration that makes the computation of Green’s function more efficient. The results are accurate and the robust is better than the original form. In the experiments, the reconstruction results obtained by the efficient form are also acceptable.

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