Generalized second law of thermodynamics for FRW cosmology with logarithmic correction

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received 29 September 2010; accepted in final form 30 November 2010
published online 13 January 2011

PACS 98.80.-k – Cosmology
PACS 98.80.Jk – Mathematical and relativistic aspects of cosmology

Abstract – In the previous analyses in the literature about the generalized second law (GSL) in an accelerated expanding universe the usual relation for the entropy, i.e. \( S = \frac{A}{4} \), was used for the cosmological horizon entropy. But this entropy relation may be modified due to thermal and quantum fluctuations or corrections motivated by loop quantum gravity giving rise to \( S = \frac{A}{4} + \pi \alpha \ln(\frac{A}{4}) + \gamma \), where \( \alpha \) and \( \gamma \) are some constants whose the values are still in debate in the literature. Our aim is to study the constraints that GSL puts on these parameters. Besides, we investigate the conditions that the presence of such modified terms in the entropy puts on other physical parameters the system such as the temperature of dark energy via requiring the validity of GSL. In our study we consider a spatially flat Friedman-Robertson-Walker universe and assume that it is composed of several interacting components (including dark energy). The model is investigated in the context of thermal equilibrium and non-equilibrium situations. We show that in a (super) accelerated universe the GSL is valid whenever \( \alpha(\leq<) > 0 \) leading to a (negative) positive contribution from logarithmic correction to the entropy. In the case of super acceleration the temperature of the dark energy is obtained to be less than or equal to the Hawking temperature.

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Introduction. – After the discovery of deeper relationships between gravity and thermodynamics [1], it was realized that the notion of temperature is not restricted to black-hole horizons only. The study of quantum field theory in any spacetime with a horizon showed that all horizons have temperatures i.e. all horizons behave like a black body [2]. In this connection, Gibbons and Hawking conjectured that entropy can be associated with the cosmological horizons and other similar properties like temperature and surface gravity as well [3]. In the simplest cosmological model of de Sitter universe, the temperature goes like \( T \sim \sqrt{\Lambda} \), where \( \Lambda \) is a positive cosmological constant. This gave the motivation that the GSL can be studied for the de Sitter spacetime and probably also for other cosmological spacetimes [4]. In the literature, the GSL has been widely discussed in the framework of different gravity theories including the braneworld, Gauss-Bonnet modified gravity and \( f(R) \) gravity [5]. Also, the GSL is investigated in the presence of dark energy [6] and black hole [7]. Jacobson proved that Einstein field equation can be derived from the usual Clausius relation, \( dQ = T dS \), where \( dQ \) is the energy exchange, \( T \) is the temperature and \( dS \) the change in entropy [8]. Later on Padmanabhan showed that Einstein’s equations for a spherically symmetric spacetime can be written in the form, \( T dS = dE + P dV \), near “any” horizon [9].

The FRW universe may contain several cosmic ingredients including dark energy, dark matter and radiation. Astrophysical observations suggest that the energy density of the dark energy is the dominant component of the total cosmic energy density. In a recent study [10], it has been proven that the GSL will be valid in a cosmological scenario where the dark energy interacts with both dark matter and radiation and also that the GSL is always and generally valid, independently of the specific interaction form, of the fluids equation of state parameters and of the background (FRW) geometry. In that particular study, the authors assumed that the FRW universe is enclosed by the apparent horizon and all the interacting components were in thermal equilibrium with the apparent horizon (a trapped surface with vanishing expansion). We think

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that the latter two assumptions should be replaced with more physically motivated ones: the temperature of cosmic ingredients could be different from each other, for instance, radiation could have higher temperature than "cold" dark matter and so on; also the boundary of the FRW universe could be the apparent horizon or the future event horizon (the distance that light can travel from now till the end of time). The latter horizon is best suitable as an infrared cut-off for the universe containing the holographic dark energy [11].

In the following analysis, we shall assume the FRW universe to contain several cosmic fluids interacting with each other and exchanging energy densities. We also take into account the possibility of thermal non-equilibrium. We investigate the generalized second law of thermodynamics in the context of FRW cosmology by considering the logarithmic correction to the horizon entropy. The outline of this paper is as follows: In the second section, we discuss the cosmological model and calculate the entropy rate of change of cosmic ingredients; next, we construct the generalized second law with quantum-corrected entropy of the horizon. The validity of this law forces the parameters the model to satisfy some special conditions. This restricts the range of free parameters of the model introduced in the literature. In the next two subsections, we discuss GSL in thermal equilibrium and non-equilibrium settings and finally we give the conclusions.

We use units \( c = G = k_B = 1 \).

The cosmological model. – We consider the spatially flat FRW spacetime

\[
\text{d}s^2 = -\text{d}t^2 + a^2(t)(\text{d}x^2 + \text{d}y^2 + \text{d}z^2)
\]

(1)

in comoving coordinates, where \( a(t) \) is the scale factor. This universe is assumed to be composed of a \( n \)-components perfect fluid (such as dark energy, dark matter, radiation and so on): \( \rho = \sum_{i=1}^{n} \rho_i, \quad P = \sum_{i=1}^{n} P_i \)

where \( \rho \) and \( P \) are the total density of the energy and the pressure of the universe, respectively. In this spacetime the Friedman equations read

\[
H^2 = \frac{8\pi}{3}\rho,
\]

\[
\dot{H} = -4\pi(P + \rho).
\]

(2)

Here an overdot represents differentiation with respect to the time \( t \). The first law of thermodynamics implies that each of the components in the volume \( V \) satisfies [6,12,13]:

\[
dE_i = T_i \text{d}S_i - P_i \text{d}V.
\]

(3)

Here \( E_i = \rho_i V \) is the energy, \( T_i \) is the temperature and \( S_i \) is the entropy of the \( i \)-th component of the perfect fluid. From (3), we can write

\[
\dot{S}_i = \frac{1}{T_i}((P_i + \rho_i)V + V\dot{\rho}_i).
\]

(4)

The continuity equation for each element is

\[
\dot{\rho}_i + 3H(P_i + \rho_i) = Q_i,
\]

(5)

where \( Q_i \) is an interaction term which can be an arbitrary function of cosmological parameters like the Hubble parameter and energy densities [14]. This term allow the energy exchange between the components of the perfect fluid and may alleviate the coincidence problem. In our analysis, we proceed with a general \( Q_i \).

Divergenceless of the energy momentum tensor leads to \( \dot{\rho} + 3H(P + \rho) = 0 \), hence \( \sum_{i=1}^{n} Q_i = 0 \). From (4) we derive

\[
\dot{S}_i = \frac{VQ_i}{T_i} + \frac{1}{T_i}(P_i + \rho_i)(V - 3HV).
\]

(6)

To proceed further we must specify \( V \). In the literature there are different choices for \( V \) corresponding to different horizons of the universe.

In cosmological models of accelerated universe, there are horizons to which we can assign an entropy as a measure of information behind them. So to study the entropy of the universe, besides the entropy of the matter enclosed by the horizon, the horizon entropy must also be taken into account. The most natural horizon of the universe is the apparent horizon whose radius is \( R_A = H^{-1} \). The choice \( V = 4\pi t^3 \) has been adopted by many authors to study the thermodynamics of the (accelerated) universe [15].

Another cosmological horizon which conceptually more resembles to the black-hole horizon is the future event horizon, \( R_f \), defined by (in the presence of the big rip at \( t_* \), \( \infty \) must be replaced by \( t_* \))

\[
R_f(t) = a(t)\int_t^\infty \frac{dt'}{a(t')}.
\]

(7)

Due to the holographic description of dark energy proposed by [16], recently this horizon has attracted more attention to the study of the thermodynamics of the universe. Only in a de Sitter spacetime we have \( R_A = R_f \).

A detailed discussion about the relation between \( R_A \) and \( R_f \) in an accelerated universe can be found in [17].

For generality, in our study we consider both the choices: \( R_A \) and \( R_f \).

Using

\[
\dot{R_f} = H R_f - 1,
\]

(8)

eq. (4) becomes

\[
\dot{S}_i = \frac{VQ_i}{T_i} - 4\pi R_f^2 \frac{P_i + \rho_i}{T_i}.
\]

(9)

Similarly, if we take the horizon as the apparent horizon: \( R_A = H^{-1} \), we obtain

\[
\dot{S}_i = \frac{4\pi}{3H^3} \frac{Q_i}{T_i} - 4\pi \left( \frac{H}{H_*} + \frac{1}{H^2} \right) \frac{P_i + \rho_i}{T_i}.
\]

(10)

Hence even in the thermal equilibrium: \( T_i = T \) (where \( T \) is the temperature of the horizon) or in the absence of
interactions: $Q_i = 0$, the total entropy of the components within the volume $V$ is not a constant. In the thermal equilibrium we have $S_{th} = \frac{R^2}{T} H / T$ and $S_{in} = \frac{1}{T} \left( \frac{H}{T} + 1 \right) \frac{H}{T}$, corresponding to (9) and (10), respectively. Note that we have $S_{in} < 0$ for an accelerated expanding universe in the quintessence phase (i.e. for $\dot{H} + H^2 > 0$ and $\dot{H} < 0$).

**Generalized second law of thermodynamics with logarithmic correction.** – To study the evolution of the entropy of the Universe, we take into account the contribution of the entropy associated to the horizon. Note that the expression of entropy is a quantity that is derived from the theory of gravity under consideration. In Einstein’s gravity, the entropy of the horizon (both for black holes and the FRW universe) is proportional to the area of the horizon, $S \propto A$. When the gravity theory is modified by adding extra curvature terms in the action functional, it ultimately modifies the entropy-area relation, for instance, in $f(R)$ gravity, the entropy-area relation is, $S \propto f'(R) A$ [18]. In the context of loop quantum gravity, the entropy-area relation can be expanded in an infinite series expression. The leading-order term in this expression is the logarithmic correction term to the entropy-area relation. Mathematically, we have [19]

$$S_h = \frac{A}{4\hbar} + \tilde{\alpha} \ln \frac{A}{4\hbar} - \tilde{\alpha}_1 \frac{A}{A} + \tilde{\alpha}_2 \frac{16\hbar^2}{A^2} - \cdots$$

$$= S_0 + \tilde{\alpha} \ln S_0 - \sum_{i=1}^{\infty} \tilde{\alpha}_i \frac{S_i}{S_0}.$$

Here $S_0$ is the classical entropy of the black hole while the second- and higher-order terms are non-logarithmic terms and are called quantum corrections. Here $\tilde{\alpha}_i$ are finite constants and $A$ is the area of the horizon. It is obvious from the last expression that higher-order terms are negligible due to the smallness of $\hbar$ and only the first-order correction term is relevant for the analysis. (From here onwards we shall fix $\hbar = 1$.) The issue of the value of $\tilde{\alpha}_i$’s is highly debated. There are different interpretations found in the literature. The prefactor of the logarithmic term, $\tilde{\alpha}$, for example, is given to be $-3/2$ in [20], and $-1/2$ in [21]. Similarly, some authors [22] take it to be a positive integer, while others find it even to be zero [23].

Note that besides the framework of loop quantum gravity, the same result for the corrected entropy can be derived by considering the effects of quantum [24], and thermal fluctuations around equilibrium [25], charge and mass fluctuations [26].

In the following we only take into account the contribution of the logarithmic correction such that [26]

$$S_h = \frac{A}{4} + \pi \alpha \ln \left( \frac{A}{4} \right) + \gamma,$$

where $\pi \alpha \sim O(1)$ and $\gamma$ are constants. This logarithmic term also appears in a model of entropic cosmology which unifies the inflation and late time acceleration [27]. It is very interesting if one can determine the coefficient $\alpha$ in front of log correction term by observational constraints. In this connection, the same study showed that the coefficient might be extremely large, of the order of $10^{16}$, due to the current cosmological constraints which inevitably brought in a fine-tuning problem to entropy corrected models [27]. Hence the logarithmic correction might play a great role in cosmology. From (10) and (11), the time evolution of the total entropy (or the generalized entropy) defined by $\dot{S} = \dot{S}_h + \dot{S}_{in}$, for $R = R_A$ is

$$\dot{S} = 4\pi \sum_{i=1}^{\infty} \frac{Q_i}{T_i} - 4\pi \left( \frac{H}{H^4} + \frac{1}{H^2} \right) \sum_{i=1}^{\infty} \frac{P_i + \rho_i}{T_i}$$

$$-2\pi \frac{\dot{H}}{H^3} (1 + \alpha H^2),$$

(12)

while for $R = R_f$, the total entropy is obtained as

$$\dot{S} = 4\pi \sum_{i=1}^{\infty} \frac{Q_i}{T_i} - 4\pi \sum_{i=1}^{\infty} \frac{P_i + \rho_i}{T_i}$$

$$+ 2\pi (H R_f - 1) \left( \frac{R_f + \alpha R_f}{R_f} \right).$$

(13)

The GSL requires $\dot{S} \geq 0$, i.e. the sum of the entropies of the perfect fluids inside the horizon and the entropy attributed to the horizon is a non-decreasing function of the comoving time. In the following we discuss the validity of this law specially in the presence of dark energy.

**GSL in thermal equilibrium.** In thermal equilibrium, i.e. $\forall i: T_i = T$, eq. (12) becomes

$$\dot{S} = \frac{\dot{H}}{T} \left( \frac{\dot{H}}{H^4} + \frac{1}{H^2} \right) - 2\pi \frac{\dot{H}}{H^3} (1 + \alpha H^2).$$

(14)

When the horizon is the apparent horizon, we take the temperature as the Hawking temperature $T = \frac{H}{\pi}$ and the GSL for (12) reduces to

$$\dot{S} = 2\pi \frac{\dot{H}}{H^3} \left( \frac{\dot{H}^2}{H^2} - \alpha H^2 \right) \geq 0.$$  

(15)

Hence the GSL is always satisfied in the absence of the correction term. Note that in a de Sitter spacetime, i.e. $\dot{H} = 0$, the expansion is isentropic: $\dot{S} = 0$.

Equation (15) puts some constraint on $\alpha$. As an illustration consider the case of small perturbations around the de Sitter space (quasi-de-Sitter spacetime):

$$H = H_0 + H_0 \epsilon t + O(\epsilon^2), \quad \epsilon := -\frac{\dot{H}}{H^2}, \quad |\epsilon| \ll 1, \quad \dot{\epsilon} = O(\epsilon^2).$$

(16)

When $\dot{H} > 0$, i.e. in a super accelerated universe, the GSL is satisfied only when $\alpha < \frac{\pi^2}{12}$. In units where $G \neq 1$ this can be rewritten as $8\pi\alpha < \frac{\pi^2}{12}$, where $m_P^2 = \frac{8\pi}{G}$, and $m_P$ is the Planck mass. Note that, as $\epsilon \ll 1$, the left-hand side
of this inequality may be a number of order unity. \( \alpha < 0 \) is a sufficient (although not necessary) condition to satisfy the GSL in this case for all small positive values of \( \epsilon \). In the same way when \( \dot{H} < 0 \), e.g. in the dark-energy-dominated era and when the universe is in the quintessence phase, the GSL is satisfied for \( \alpha > \frac{\pi^2}{27} \) or \( 8\pi \alpha > \epsilon \frac{n^2}{m} \). Here, \( \alpha > 0 \) is a sufficient condition for the validity of the GSL. Note that these inequalities do not determine the precise value of \( \alpha \), and corresponding to the model proposed (e.g. for inflation or late time accelerated expansion of the universe where we can consider a quasi-de-Sitter spacetime), only give us the upper or lower bound of \( \alpha \). Note that as our results are general and independent of the explicit form of dark energy or inflaton, we have not determined numerically these bounds which are model dependent.

For the future event horizon, and in the absence of a well-defined temperature, we assume that \( T \) is proportional to the Hawking temperature \[ T = \frac{bH}{2\pi}. \tag{17} \]

The GSL requires

\[ \dot{S} = 2\pi \frac{\dot{H}}{bH} R_{f}^{2} + 2\pi(HR_{f} - 1) \left( R_{f} + \frac{\alpha}{R_{f}} \right) \geq 0. \tag{18} \]

This indicates that

\[ \frac{d}{dt} \ln \left( R_{f} H^{\frac{1}{2}} e^{\frac{\alpha}{2\pi T}} \right) \geq 0. \tag{19} \]

If the expansion is isotropic, then \( H = \lambda R_{f}^{-b} e^{\frac{\alpha}{2\pi T}} \), where \( \lambda \) is a positive constant. By using (8) we conclude that the future event horizon satisfies

\[ \dot{R}_{f} = \lambda R_{f}^{-b} e^{\frac{\alpha}{2\pi T}} - 1. \tag{20} \]

If the universe remains in the phantom phase, then \( \dot{R}_{f} < 0 \) \[6,12\]. This implies \( \lambda < R_{f}^{b-1} e^{\frac{\alpha}{2\pi T}} \). But near the big rip we have \( \lim_{t \to t_{s}} R_{f} = 0 \), hence the positivity of \( \lambda \) requires \( \alpha < 0 \). In this case we do not need to have \( b > 1 \), which is true only when \( \alpha = 0 \).

To see more explicitly how the validity of GSL depends on the value of \( \alpha \), let us consider a model, discussed in the literature (see \[6\] and references therein), corresponding to a super accelerated universe with a big rip at the finite time \( t_{s} \), near which \( H \) becomes very large. This phantom-dominated universe of polelike type is described by the scale factor

\[ a(t) = a_{0}(t_{s} - t)^{-n}. \tag{21} \]

where \( a_{0} \) and \( n \) are positive constants. The Hubble parameter and the future event horizon are given by \( H = \frac{n}{t_{s} - t} \) and \( R_{f} = \frac{t_{s} - t}{n + 1} \), respectively. Hence (18) reduces to

\[ (b - 1)(t_{s} - t)^{2} + b\alpha(n + 1)^{2} \leq 0. \tag{22} \]

For \( \alpha = 0 \), the GSL is satisfied provided that \( b \leq 1 \) (the temperature is less than the Hawking temperature). In the presence of the correction term, for \( b > 1 \) (i.e. the temperature is greater than the Hawking temperature), the GSL is satisfied when

\[ (t_{s} - t)^{2} \leq \frac{b\alpha(n + 1)^{2}}{1 - b}. \tag{23} \]

In this case we must have \( \alpha < 0 \). For temperatures less than the Hawking temperature, the GSL is valid when

\[ (t_{s} - t)^{2} \geq \frac{b\alpha(n + 1)^{2}}{1 - b}, \tag{24} \]

which is true for all \( t \), provided that \( \alpha < 0 \).

**GSL with thermal non-equilibrium.** If we leave the thermal-equilibrium condition, the problem of investigating the validity of the GSL becomes more complicated. However to get an insight about what happens in this case, let us consider a simple model. Assume that the universe is dominated by two subsets with different temperatures: \( \rho = \rho_{1} + \rho_{2} \), such that each of them satisfies the continuity equation: \( \dot{\rho}_{j} + 3H(P_{j} + \rho_{j}) = 0 \), \( j = 1,2 \) and evolves with its own temperature. Note that in this model \( \rho_{1} \) and \( \rho_{2} \) consist of ingredients in thermal equilibrium (in each subset), which even if do not interact with the elements of the other subset, can interact with each other. Hence in this situation using (12), (13) one can show that GSL requires

\[ \dot{S} = \left( \frac{\dot{H}}{H^{2}} + \frac{1}{H^{2}} \right) \left( -4\pi\rho_{1} + P_{2} \right) \left( \frac{1}{T_{1}} - \frac{1}{T_{1}} + \frac{2\pi H}{T_{1}} \right) \notag \]

\[ + 2\pi \left( HR_{f} - 1 \right) \left( R_{f} + \frac{\alpha}{R_{f}} \right) \geq 0, \tag{25} \]

and

\[ \dot{S} = -4\pi R_{f}^{2} \left( P_{2} + \rho_{2} \right) \left( \frac{1}{T_{2}} - \frac{1}{T_{1}} \right) + \frac{R_{f}^{2} \dot{H}}{T_{1}} \notag \]

\[ + 2\pi \left( HR_{f} - 1 \right) \left( R_{f} + \frac{\alpha}{R_{f}} \right) \geq 0, \tag{26} \]

respectively. These inequalities written in this special form allow us to study the validity of the GSL by knowing the EoS parameter of only one of the sectors. If we take the first sector as the dark-energy perfect fluid whose temperature is assumed to be \( T_{1} = \frac{bH}{2\pi} \) and the other sector as a barotropic perfect fluid, \( P_{2} = w_{2}\rho_{2} \), eq. (25) leads to

\[ \dot{S} = \left( -4\pi(w_{2} + 1)\rho_{2}a^{-3(w_{2} + 1)} \left( \frac{1}{T_{2}} - \frac{2\pi}{bH} \right) + \frac{2\pi H}{bH} \right) \notag \]

\[ \times \left( \frac{\dot{H}}{H^{2}} + \frac{1}{H^{2}} \right) - 2\pi \frac{\dot{H}}{H^{3}} \left( 1 + \alpha H^{2} \right) \geq 0, \tag{27} \]

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where $\rho_{02} = \rho_2 (a = 1)$ is a constant and we have used the continuity equation to write $\rho_2 = \rho_{02} a^{-3(w_2 + 1)}$. In the same way (26) casts to

$$\dot{S} = -4\pi(1 + w_2)\rho_{02} R_j^2 a^{-3(w_2 + 1)} \left( \frac{1}{T_2} - \frac{2\pi}{bH} \right) + 2\pi R_j \left( \frac{\dot{R}_j}{bH} \right) + 2\pi\dot{H} \frac{R_j^2}{bH} \left( R_f + \frac{\alpha}{R_f} \right) \geq 0. \quad (28)$$

The non-equilibrium situation changes drastically the previous necessary conditions for validity of GSL. For example, as is obvious from (25) or (27), even in a de Sitter spacetime the expansion is no more isentropic and the GSL is valid provided that

$$(\rho_2 + P_2) \left( \frac{1}{T_2} - \frac{1}{T_1} \right) = (\rho_1 + P_1) \left( \frac{1}{T_1} - \frac{1}{T_2} \right) \leq 0. \quad (29)$$

Hence the GSL implies that the temperature of a sector which is in the quintessence (phantom) phase is greater (less) than the other component.

For a known $T_2$ (in terms of the scale factor), the inequalities (27) and (28) determine the time derivative of the total entropy in terms of the scale factor, its time derivatives, and its time integral. So in principle they specify the necessary conditions for the GSL to be true. As a possible choice one can take the second sector as an ordinary matter like radiation, whose temperature may be obtained as $T_2 = T_{02} a^{-3w_2}$, where $T_{02} = T(a = 1)$. This follows from the equation

$$\dot{T}_i = w_i \frac{Q_i - 3H(P_i + \rho_i)}{P_i + \rho_i}. \quad (30)$$

For non-interacting radiation this gives the well-known result: $T_2 = T_{02} a^{-3}$. Another choice is to take the temperature of the second sector like the first one as $T_2 = \frac{b_2}{2\pi a}$ where $b_2 \neq b$ is a positive constant.

Following (28), for $T_2 = T_{02} a^{-3w_2}$, the GSL is true whenever

$$A'(t_a - t)^{3(n + 1)} + B(t_a - t)^{3n(w_2 + 1)} + C(t_a - t)^2 \geq \alpha, \quad (31)$$

where

$$A' = -\frac{2(1 + w_2)\rho_{02}}{T_{02}(n + 1)^2a_0^3}, \quad B = \frac{4\pi(1 + w_2)\rho_{02}}{bn(n + 1)^2a_0^3},$$

$$C = \frac{1 - b}{b(n + 1)^2}. \quad (32)$$

Note that as $A'$ and $B$ have different signs, even in the case $\alpha < 0$ and $b < 1$, the GSL may not be satisfied for some times. Now let us examine the other choice, $T_2 = \frac{b_2}{2\pi a}$. In this case GSL is valid when

$$F(t_a - t)^{3n(w_2 + 1)} + C(t_a - t)^2 \geq \alpha, \quad (33)$$

where

$$F = -\frac{4\pi n(1 + w_2)\rho_{02}}{(n + 1)^2a_0^3} \left( \frac{1}{b_2} - \frac{1}{b} \right).$$

If the dark-energy temperature is greater than the Hawking temperature, $b > 1$, then $C < 0$ and for $\alpha > 0$, GSL is satisfied for $F > 0$. If the dark-energy temperature is less than the Hawking temperature $b < 1$, the GSL is satisfied always provided that $\alpha < 0$ and $F > 0$. Note that $F > 0$ leads to $b_2 \geq b$ for quintessence (phantom) like $w_2$.

At the end we would like to note that if one considers an interaction between the components which have different temperatures, then the first terms in (12) and (13) must be taken into account too, and the problem becomes very complicated. This is due to the fact that for a general interaction obtaining an expression for the energy densities $\rho_i$ or for the temperature in terms of the scale factor is not straightforward.

**Conclusion.** – In this paper, we considered the FRW cosmological spacetime (see eq. (1)) composed of interacting components and discussed the generalized second law (GSL) of thermodynamics by considering the logarithmic correction to the horizon entropy:

$$S = \frac{A}{4} + \pi \alpha \ln \left( \frac{A}{4} \right) + \gamma.$$

This correction is specified by two constants $\alpha$ and $\gamma$ whose values are in debate in the literature (see the discussion at the beginning of the third section and after eq. (11)). For the sake of generality we performed our analysis by focusing on both the apparent and future event horizons and re-expressed the GSL in terms of the parameters of the model (see eqs. (12) and (13)). We showed that, in thermal equilibrium, although in the absence of corrections, GSL is in general valid for the apparent horizon, but in the presence of corrections this is not true (see eq. (15)). In a quasi-de-Sitter spacetime which is a model corresponding to the expansion of the universe in the early universe or at late time (see eq. (16)), it was shown that that negativity (positivity) of $\alpha$ in a (super) accelerated universe is a sufficient condition for the validity of the GSL leading to a negative (positive) contribution from correction to the entropy. The constraints coming from the GSL which are stated as inequalities (depending on the model proposed for the evolution of the universe), only enabled us to determine the upper or lower bounds of $\alpha$.

We performed a similar analysis for the future via some examples concerning the pole-like-type expansion and big rip (see eq. (21)) where the Hubble parameter may become very large with respect to the present time. We showed that in order that the GSL be satisfied the temperature of dark energy must be less than the Hawking temperature, and $\alpha$ must be negative: therefore the correction decreases the horizon entropy.

A brief analysis was also done for a universe dominated by two non-interacting components with different temperatures (although each components may contain interacting ingredients in thermal equilibrium) (see eqs. (25) and (26)). In the case of the future event horizon we verified that the GSL always holds provided that $\alpha < 0$, so as before the horizon entropy is decreased by the logarithmic correction.
Note that, as the GSL deals with the time derivative of the total entropy, our analysis does not give any information about $\gamma$.

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We would like to thank J. Bekenstein for useful comments on this paper.

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