Numerical Study of Current Distribution and Thrust in Diagonal Pulsed Magnetohydrodynamic Accelerator*

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This study aims to investigate the incremental thrust in a diagonal magnetohydrodynamic (MHD) accelerator as a function of the number of segmented electrodes, the applied magnetic-flux density, and the different setting of diagonal angle. The peak incremental thrust obtained from the numerical result was agreement with that of the experimental result. For understanding the characteristics of current vector distribution in a diagonal MHD accelerator, we calculated the current vector as the function of a number of segmented electrodes, the applied magnetic-flux density and the different setting of diagonal angle. The simulations showed the current vector distribution inside the MHD channel as well as the current flow crossed into tungsten wires as a diagonal connection of the MHD channel. From these numerical simulations, the tilt angle of current vector distribution was changed by Faraday current in the MHD channel, which was controlled by the magnetic field.

Key Words: Diagonal Magnetohydrodynamic Accelerator, Incremental Thrust, Current Distribution, Magnetic-Flux Density, Diagonal Connection

1. Introduction

The research and development of electric propulsion systems have been demonstrated for station-keeping, orbit transfer, attitude control. Electric propulsion systems have a variety of the strategies for achieving very high exhaust velocities to reduce the propellant mass for a space mission.1–3) A magnetohydrodynamic (MHD) accelerator is anticipated to be one of the next-generation of long distant propulsion systems. For the further development of MHD acceleration, it is necessary to required more experimental and numerical analyses.4) The recent study concerned with the MHD acceleration is examined in research institutions including National Aeronautics and Space Administration (NASA).5–7) The MHD has the potential to achieve both high thrust and high specific impulse simultaneously using electro-magnetic Lorentz forces \((j \times B)\) attained by the interaction of electric and magnetic fields.8,9) MHD accelerator application is classified into an electric power system or an option of chemical and arc thruster. Figure 1 shows the schematic of the MHD accelerator.

In recent years, the fundamental performance of MHD accelerator has been evaluated with different electrode connection.10,11) The Faraday-type configuration has the best acceleration efficiency; whereas optimal acceleration efficiency is difficult to achieve with the Hall connection. The diagonal connection can be a cascade electrode, which improves the acceleration efficiency when operated with only one power source. The MHD acceleration force produced by Lorentz

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**Nomenclature**

\(B_z\): magnetic-flux density, \(T\)
\(C_p\): specific heat capacity, \(\text{J/(kg K)}\)
\(e\): electron charge, \(C\)
\(E\): electric field, \(\text{V/m}\)
\(E_{\text{n}}\): enthalpy density, \(\text{J/m}^3\)
\(E_x\): \(X\) component of electric field, \(\text{V/m}\)
\(E_y\): \(Y\) component of electric field, \(\text{V/m}\)
\(F\): incremental thrust, \(\text{N}\)
\(I\): current, \(\text{A}\)
\(I_x\): \(X\) component of current density, \(\text{A/m}^2\)
\(I_y\): \(Y\) component of current density, \(\text{A/m}^2\)
\(n_e\): number density of electron, \(\text{1/m}^3\)
\(P\): pressure, \(\text{Pa, J/m}^3\)
\(R\): gas constant of the working gas, \(\text{J/(K-mol)}\)
\(T\): temperature, \(\text{K}\)
\(u_0\): velocity, \(\text{m/s}\)
\(u_i\): inlet fluid velocity, \(\text{m/s}\)
\(u_o\): outlet fluid velocity, \(\text{m/s}\)
\(v_0\): collision frequency, \(\text{Hz}\)
\(V\): potential difference between the electrode, \(\text{V}\)
\(\mu\): electron mobility, \(\text{m}^2/(\text{V-s})\)
\(\theta\): electric field angle
\(\sigma\): electrical conductivity, \(\text{S/m}\)
\(\beta\): Hall parameter
\(\rho\): density, \(\text{kg/m}^3\)
\(\rho_o\): outlet gas density, \(\text{kg/m}^3\)
\(\chi_e\): electron mole fraction

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In recent years, the fundamental performance of MHD accelerator has been evaluated with different electrode connection.10,11) The Faraday-type configuration has the best acceleration efficiency; whereas optimal acceleration efficiency is difficult to achieve with the Hall connection. The diagonal connection can be a cascade electrode, which improves the acceleration efficiency when operated with only one power source. The MHD acceleration force produced by Lorentz
force, which applied with magnetic field in the perpendicular direction to gas flow so it accelerated the working gas.\(^9,10,12\) Over the past several years, the design and development of diagonalized MHD accelerator have demonstrated with three-dimensional simulation. These studies reported that the diagonal angle of \(45^\circ\) should be utilized for the electrode pairs.\(^11\) The performance of diagonal-type MHD accelerator with an air plasma as a working gas and a one-dimensional numerical simulation with the thermal-equilibrium condition was relatively performed to the basic MHD accelerator channel. The results confirmed that the best performance was obtained by setting a constant diagonal angle.\(^13,14\)

In previous studies, the proof of the principal experiment of the MHD accelerator was demonstrated by using a pulsed-power discharge and a model rocket engine. The results indicated that the thrust efficiency of the pulsed-MHD accelerator improved when the cross-section of electrode decreased.\(^15,16\)

To improve MHD acceleration performance, diagonal connection of a MHD accelerator was performed. The objective of this study is to investigate the incremental thrust and explain the effect of current vector flow in the MHD channel. This study consisted of different number of segmented electrodes, various applied magnetic-flux densities and the different diagonal angle settings. To verify the numerical results, they were compared with experimental results.

2. Experiment Setup

The current study investigates the incremental thrust in a pulse diagonal MHD acceleration, so that the schematic diagram of an experimental setup and apparatus are shown in Fig. 2.\(^14,15\) The experimental apparatus consists of a pulse forming network, an MHD acceleration channel, a model rocket engine, and electromagnet. The MHD accelerator has a diagonal electrode connection with a channel length of 150 mm, channel cross-section of \(20 \times 20\) mm. Tungsten wires are set along the MHD channel as segmented electrodes, which have a cross-section of \(0.5 \times 20\) mm. To obtain the acceleration performance via Lorentz force and thermal pressure force. Current is generated through a Pulse Forming Network (PFN) and measured using a current transformer (CT) as shown in Fig. 3. The high-voltage power supply provides 5 kV for discharging the capacitor of PFN and the discharge current with pulse width is generated for 120 \(\mu\)s with a peak of 1.1 kA as shown in Fig. 4. The combustion gas of a model rocket engine was produced as a working fluid for the experimental setup.\(^17\) Figure 5 shows the magnetic-flux density in the MHD channel was measured by magnet gauss meter and it was uniformly applied by using an electromagnet. Model of the electrical conductivity in working fluid was estimated using Ref. 10) as shown in Fig. 6.\(^10,18\). The incremental thrust was measured using pressure sensor which connected to the model rocket engine in the diagonal type MHD accelerator.

2.1. Number of segmented electrodes

Figure 7 shows the schematics of the diagonal MHD accelerators as a function of a number of segmented electrodes

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Fig. 1. Schematic of MHD accelerator.

Fig. 2. Experimental setup.

Fig. 3. Pulse forming network (PFN).

Fig. 4. Current pulsed width of 120\(\mu\)s.

Fig. 5. Measurement of magnetic-flux density in MHD channel.
with one, three, five, and seven pairs. Applied magnetic-flux density is 0.3 T with a diagonal angle of 36°.

2.2. Applied magnetic-flux density

Figure 8 shows schematic of a diagonal MHD accelerator channel as a function of various applied magnetic-flux density varying between 0–0.7 T. The setting of diagonal angle 36° with seven electrode pairs.

2.3. Setting of different diagonal angle

In case of a diagonal angle, Hall and Faraday’s currents are expressed by,

\[ j_x = \frac{(1 - \beta \tan \theta) j_1 + \sigma u B_z \tan \theta}{1 + \tan^2 \theta} \]  
\[ j_y = \frac{(1 + \beta \tan \theta) j_1 - \sigma u B_z}{1 + \tan^2 \theta} \]  

To repress Hall current, the optimum angle of the diagonal MHD accelerator is neutralized using magnetic field which consisted velocity and electrical conductivity. The neutralized Hall current is determined as follow,

\[ \theta = \tan^{-1}\left( \frac{j_1}{\beta j_1 - \sigma u B} \right) \]

Using Eqs. (1)–(3) and experimental parameters, the diagonal angle was set as 30°, 45°, 56° and 70°. Figure 9 shows the schematics of a diagonal MHD accelerator channels with three electrode pairs and by varying the electrode width, different diagonal angle settings of 30°, 45°, 56° and 70° obtained. Applied magnetic-flux density is 0.3 T.

3. Numerical Procedure

3.1. Governing equations of current distribution in a diagonal MHD channel

In order to evaluate the current distribution in the MHD channel, the current stream function \( \psi \) is defined by,

\[ j_x = \frac{\partial \psi}{\partial y}, \quad j_y = \frac{\partial \psi}{\partial x} \]  

Diagonal electrode connection of the MHD accelerator is considered and discussed under the condition where the magnetic field is uniform and parallel to z-axis. In the diagonal MHD accelerator devices, the electric field is forced to be perpendicular to the diagonal angle due to wiring of a pair of short electrodes. Angle between two direction electric field can be obtained by,

\[ \theta = \arctan \frac{E_y}{E_x} \]

The current densities are determined using the generalized...
Ohm’s law as follows:

\[ j_x = \frac{\sigma}{1 + \beta^2} \left( E_x - \beta (E_y - u_x B_z) \right) \]  
\[ j_y = \frac{\sigma}{1 + \beta^2} \left( \beta E_x + (E_y - u_x B_z) \right) \]  

The following simplified Maxwell Eqs. (8) are introduced with the set of MHD Eqs. (6) and (7):

\[ \nabla \times E = 0, \quad \nabla \cdot j = 0 \]  

Equations (6)–(8) were solved using a Gauss–Saidel iteration under the boundary condition. In addition, to evaluate the behaviors of diagonal electrode connection of an MHD accelerator, the electrical mobility, electrical conductivity, and Hall parameter are required. The electron mobility is determined using the following expression:

\[ \mu = \frac{e}{m_e v_e} \]  

The Hall parameter can be obtained using the following expression with the electron mobility \( \mu \):

\[ \beta = \frac{e B}{m_e v_e} = B \mu \]  

The electron number density \( n_e \), electron collision frequency \( v_e \), and electrical conductivity \( \sigma \) are given by the combustion plasma of the model rocket engine under the thermal-equilibrium condition. Using Chemical Equilibrium with Application (CEA) developed at NASA, the temperature values of 1500–20000 K with pressure values of 0.2–3.0 MPa was estimated. The electron number density is obtained using the following equation.

\[ n_e = 7.3395 \times 10^{27} \frac{P}{T} \]  

The electron collision frequency \( v_e \) is given by

\[ v_e = \frac{e^2 n_e}{m_e \sigma} \]  

The initial condition of working fluid was assumed to be in steady state and spread through out the channel as shown in Table 1.10,19,20)

The boundary condition of the electric field on the electrode is defined as:

\[ E_x = 0 \]  

The boundary condition of current density on the insulating wall is defined as:

\[ j_y = 0 \]  

The boundary conditions on the inlet and outlet channels are defined as:

\[ j_x = 0 \]  

According to the estimation, time steps are used for calculating the flow properties with time variation. The total of mesh number is set of 270000 meshes and the solver is run for 500\( \mu \)s with a time step of 1\( \mu \)s starting at time 0.

### 3.2 Governing equations of the MHD accelerator

MHD governing equations comprise continuity, momentum and energy equations, including MHD effect as follows:
\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{u} &= 0 \quad (16) \\
\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} &= -\frac{1}{\rho} \frac{\partial \mathbf{p}}{\partial x} + \mathbf{j}_y B_z \quad (17) \\
\frac{\partial E_n}{\partial t} + u_x \frac{\partial (E_n + P)}{\partial x} &= \frac{j_x^2 + j_y^2}{\sigma} - \left( E_n + P \right) \frac{\partial u_x}{\partial x} \quad (18) \\
E_n &= \rho \left( C_p T + \frac{1}{2} \alpha^2 \right) \quad (19) \\
P &= \rho RT \quad (20)
\end{align*}
\]

The resulting thrust has two contributions. One is due to the mass flux and flow acceleration. The second is from pressure differences between the engine exit and atmosphere. Therefore, the incremental thrust \( \Delta F \) generated at the outlet of the channel which is investigated using

\[\Delta F = (\rho_i u_i^2 - \rho_o u_o^2)A + (P_o - P_i)A \quad (21)\]

### 4. Results and Discussion

#### 4.1. Number of segmented electrodes

The comparison between numerical and experimental results as a function of a number of segmented electrodes is shown in Fig. 10. This study applied magnetic-flux density of 0.3 T with diagonal angle of 36°. The numerical results were quantitatively close to the experimental results. These results indicate that increasing of number of segmented electrode will increase the incremental thrust, because of Lorentz force is increased when increasing the number of current flow in MHD channel.

The modeling calculation of the current vector distribution can be evaluated by assuming constant physical parameters for the gas velocity, magnetic-flux density, and electrical conductivity, which obtained using the generalized Ohm’s law and Maxwell’s equations (Eqs. (6)–(8)).

The characteristics of current vector distribution as a function of the number of segmented electrodes with one, three, five, and seven pairs are shown in Fig. 11. Numerical results indicate that the current vector passed between electrode pairs inside the channel. The current vector distribution was inclined by applying a magnetic field of 0.3 T.

#### 4.2. Applied magnetic-flux density

In the second case, the dependence of incremental thrust on the magnetic-flux density with setting diagonal angle of 36° and seven electrode pairs are shown in Fig. 12. Both numerical and experimental results indicate that incremental thrust increased from 0 T to 0.5 T. However, it decreased at the magnetic-flux density of 0.7 T. The maximum incremental thrust was obtained at the optimum magnetic-flux density of 0.5 T. The Lorentz force \( (\mathbf{j} \times \mathbf{B}) \) will be increased when increasing the magnetic-flux density, which is directly increase the incremental thrust. However, increasing the magnetic-flux density over 0.5 T will decrease both Lorentz force and incremental thrust by increasing Hall current.

The current vector distributions were examined with a configuration of seven electrode pairs under variable magnetic-flux densities, as shown in Fig. 13. The applied magnetic-flux densities were varied between 0–0.7 T at a diagonal angle of 36°. It was found that current vector occurred on each electrode surface inclined other than 0.5 T magnetic-flux density. From this result, it was found that the maximum Faraday current was obtained when applying the magnetic flux density of 0.5 T, which achieved the maximum incre-
mental thrust. The higher magnetic-flux density at 0.7 T made the current vector in the direction of the channel inlet. Therefore, to maximize the Faraday current, it is necessary to set the current vector to the optimum tilt angle by changing magnetic-flux density.

4.3. Settings of different diagonal angle

Finally, incremental thrust was examined as a function of setting different diagonal angle for the applied magnetic-flux density varies between 0–0.7 T by setting diagonal angle of 36° with seven electrode pairs. The results revealed that the best incremental thrust of a diagonal MHD accelerator was obtained when setting a diagonal angle at 56° for the uniform magnetic-flux density of 0.3 T as shown in Fig. 14.

Current vector distribution is plotted in Fig. 15. The setting of diagonal angle was changed by increasing distance between electrodes. The results found that the current vector flowing inside the MHD channel was affected by increasing...
of electrode distance. Current vector distributions smoothly flowed from anode to cathode up to setting diagonal angle of 56°. When setting diagonal angle at 70°, the current vector observed to be discreted. This may be because of longer distance between electrodes comparing to other diagonal angles. Therefore, the diagonal angle affect to working fluid and current vector distribution flow.

5. Conclusions

A detailed analysis of the improvement of MHD acceleration performance was numerically clarified using numerical simulations. In addition, effect of current vector distribution behavior in a diagonal MHD accelerator channel was calculated. The present analysis explained the dependency of incremental thrust on the number of electrode pairs, magnetic-flux density, and diagonal angle of the MHD accelerator. The following conclusions were drawn from the study.

1) Evaluation of MHD acceleration performance was numerically performed. The thrust was increased when increasing number of segmented electrode.

2) The optimum incremental thrust was applied based on a magnetic-flux density of 0.5 T and a diagonal angle of 36°.

3) The total current vector of the electrodes was changed by changing the magnetic-flux density.

4) The total current vector from electrode was perpendicular to the working fluid when obtained the maximum incremental thrust.

5) The best diagonal MHD accelerator was obtained by setting the diagonal at an angle of 56°.

From these results, it was concluded that the present numerical model and numerical simulation are effective for designing a diagonal MHD accelerator and for estimating acceleration performance. An important factor to improve the thrust efficiency of diagonal MHD accelerator, pulse repetitive operation will be studied in future.

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