Effects of Possible $\Delta B = -\Delta Q$ Transitions in Neutral $B$ Meson Decays

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We explore the possibility that the existing data on like-sign dileptons at the $\Upsilon(4S)$ resonance consist of events arising from $B_d^0 - \bar{B}_d^0$ mixing and also from $\Delta B = -\Delta Q$ transitions. The consequences of these nonstandard transitions for certain time-asymmetries which are likely to be measured at the $B$ factories are studied.

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An interesting way in which physics beyond the Standard Model could manifest itself is through violations of the $\Delta B = \Delta Q$ rule in the decays of bottom hadrons. These violations if present in decays of neutral $B$ mesons will have obvious implications for the study of $BB$ mixing because experimentally the bottom flavour is tagged by the decay leptons. It is therefore necessary to examine the effect of possible $\Delta B = -\Delta Q$ transitions on the same-sign dilepton signal which is observed at the $\Upsilon(4S)$ resonance [1, 2, 3]. This is because not all the ‘wrong-sign’ leptons might be originating from $\Delta B = 2$ transitions, $B \leftrightarrow \bar{B}$; some may be coming from decays that do not obey the $\Delta B = \Delta Q$ rule, [4]. In terms of quarks this amounts to postulating the transition $b \rightarrow \beta + W^+$, where $\beta$ is an exotic quark carrying a charge of $(-4/3)$ units. Such exotic quarks have been envisaged in certain models; for a recent model see, e.g., Ref. [5]. These quarks could lead to the existence of exotic mesons which are doubly charged (e.g., $\beta\bar{u}$), or to a spectacular jump in the $e^+e^-$ annihilation ratio $R$ by $(16/3)$ units. Available data up to LEP energies do not indicate any evidence for such quarks. Also the present data on the semileptonic decays of neutral beons neither require nor forbid $\Delta B = -\Delta Q$ transitions which lie outside the Standard Model.

Recently Kobayashi and Sanda [6] have suggested some tests for checking $CPT$ invariance at the $B$ factories. They assumed the $\Delta B = \Delta Q$ rule of the Standard Model as being valid in semileptonic decays. On the other hand they relaxed $CPT$ invariance for the mass-matrix of the neutral beons, but not for decay amplitudes. In the following we take a complementary approach:
We assume the validity of complete CPT invariance throughout, but allow $\Delta B = -\Delta Q$ transitions. We examine how the $\Delta B = -\Delta Q$ amplitudes could generally affect the dilepton ratios and certain time-asymmetries that are measurable at the $B$ factories. Of particular interest in this context are the time-integrated asymmetries in dilepton events which arise from exclusive semileptonic decays of neutral $B$’s. We further show that the asymmetries which involve the detection of CP eigenstates in the decays of $B$ and $\bar{B}$, namely, the well-known CP-violating asymmetries [7, 8, 9] and the $y$-determining asymmetry [10] are unlikely to be affected much by the $\Delta B = -\Delta Q$ transitions.

Notation - For notational convenience in the following, we drop the superscripts and subscripts on the mesons $B_0^d$ and $\bar{B}_0^d$, and refer to them simply as $B$ and $\bar{B}$, respectively. Mixing allows the construction of eigenstates which have definite masses $m_{1,2}$ and inverse lifetimes $\Gamma_{1,2}$ in the usual way,

$$|B_1 >= p|B > + q|\bar{B} > , \quad |B_2 >= p|B > - q|\bar{B} > , \quad (|p|^2 + |q|^2 = 1) ;$$

they evolve in proper time $t$ as

$$|B_k > \rightarrow e^{(-im_k - \frac{1}{2}\Gamma_k)t} |B_k > ; \quad (k = 1, 2) .$$

We assume CPT invariance throughout, and take $|B > = CP|B >$. We define the symbols

$$g \equiv \frac{q}{p}, \quad x = \frac{m_2 - m_1}{\Gamma}, \quad y = \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \quad \Gamma = \frac{\Gamma_1 + \Gamma_2}{2} .$$

For notational brevity we refer to a particular channel of semileptonic b-quark decay merely by the corresponding hadronic state label, and distinguish its CPT-conjugate channel by a ‘tilde’:

$$(i\ell^+) \equiv (X_i + \nu_\ell + \ell^+) , \quad (\tilde{i}\ell^-) \equiv (\bar{X}_i + \bar{\nu}_\ell + \ell^-) ;$$

thus the labels $i$ or $\tilde{i}$ are taken to include the appropriate neutrinos. The decay amplitudes obeying the $\Delta B = \Delta Q$ rule will be denoted by

$$A_i = <i\ell^+| T |B > , \quad \bar{A}_i = <\tilde{i}\ell^-| T |\bar{B} > ;$$

the corresponding amplitudes for $\Delta B = -\Delta Q$ transitions will contain complex multiplicative parameters $\rho$ as follows:

$$\rho_i A_i = <i\ell^+| T |\bar{B} > , \quad \bar{\rho}_i \bar{A}_i = <\tilde{i}\ell^-| T |B > .$$
Neglecting the final-state interactions due to electroweak forces, $CPT$ invariance leads to the relations

$$\bar{A}_i = A_i^*, \quad \bar{\rho}_i = \rho_i^*.$$  \hfill (3)

In the Standard Model, the $\rho$'s are expected to be small since they get contributions from diagrams involving at least two $W$'s; indeed in the case of kaons the ratio of the $\Delta S = -\Delta Q$ and $\Delta S = \Delta Q$ amplitudes is estimated [12] to have a magnitude of order $10^{-7}$. In what follows we shall regard the $\rho$'s as small parameters.

**Ratio of Dilepton Events** - We consider exclusive semileptonic decays of the neutral $B$ mesons into the channels $(i\ell^+) + (j\ell^+)$, where, for instance, $i$ and $j$ could stand for the states $(D^*-\nu_\ell)$ and $(D^-\nu_\ell)$. The numbers of events with same-sign dileptons from the $\Upsilon(4S)$, irrespective of the decay times, are given (apart from an overall constant) by

$$n(i\ell^+, j\ell^+) = C_{ij} \left\{(1-a)|1-r_ir_j|^2 + (1+a)|r_i - r_j|^2\right\},$$  \hfill (4)

$$n(\tilde{i}\ell^-, \tilde{j}\ell^-) = C_{ij} |g|^4 \left\{(1-a)|1-\bar{r}_i\bar{r}_j|^2 + (1+a)|\bar{r}_i - \bar{r}_j|^2\right\},$$  \hfill (5)

$$C_{ij} \equiv \left|\frac{A_iA_j^*}{\sqrt{2}g\Gamma}\right| \frac{1}{1-y^2}, \quad a = \frac{1-y^2}{1+x^2},$$  \hfill (6)

$$r_i = g\rho_i, \quad \tilde{r}_i = (\bar{\rho}_i/g).$$  \hfill (7)

Similarly, the (relative) numbers of events with opposite-sign dileptons, integrated over all times, are given by

$$n(i\ell^+, \tilde{j}\ell^-) = C_{ij} |g|^2 \left\{(1+a)|1-r_i\bar{r}_j|^2 + (1-a)|r_j - \bar{r}_i|^2\right\},$$

$$n(j\ell^+, \tilde{i}\ell^-) = C_{ij} |g|^2 \left\{(1+a)|1-r_i\bar{r}_j|^2 + (1-a)|r_j - \bar{r}_i|^2\right\},$$

where we used $|1-r_i\bar{r}_j| = |1-\bar{r}_i r_j|$.

We next define the ‘exclusive’ dilepton ratio $\chi_{ij}$, as the number of like-sign dilepton events relative to the total number, where all the events originate in either of the two exclusive decays having labels $i$ and $j$, or their conjugates $\tilde{i}$ and $\tilde{j}$:

$$\chi_{ij} = \frac{N_{ij}^{++} + N_{ij}^{--}}{N_{ij}^{++} + N_{ij}^{--} + N_{ij}^{+-} + N_{ij}^{-+}},$$  \hfill (8)
\[ N_{ij}^{++} \equiv n(\ell^+, \ell^+) + n(\ell^+, j^+) + n(j^+, j^+) , \]
\[ N_{ij}^{--} \equiv n(\ell^-, \ell^-) + n(\ell^-, j^-) + n(j^-, j^-) , \]
\[ N_{ij}^{+} + N_{ij}^{-} \equiv n(\ell^+, \ell^-) + n(\ell^-, j^+) + n(j^+, j^-) + n(j^+, \ell^-) . \]

In the following we treat \( \rho \) and \( \bar{\rho} \) to be small in magnitude and keep terms up to and including second order in them; for example, we shall write
\[ |1 - r_i r_j^*|^2 = |1 - (r_i r_j^* / |g|^2)|^2 \simeq |1 - 2Re (r_i r_j^*)| , \]
where in the last step we ignored the additional correction due to CP violation by setting \( |g| = 1 \) as it is multiplying the quadratically small quantity \( |r_i r_j^*| \).

Thus the modified ratio of like-sign dilepton events arising from either of the two channels \( i \) and \( j \) is
\[ \chi_{ij} = \begin{cases} 1 + 4a (Im <r_{ij}>)^2 + \frac{4a}{1 - a} & \left( \frac{|A_i A_j| |r_i - r_j|}{|A_i|^2 + |A_j|^2} \right)^2 \end{cases} \chi . \]
Here \( \chi \) is the usual dilepton ratio for inclusive channels assuming the \( \Delta B = \Delta Q \) rule [13],
\[ \chi = \frac{(1 - a)(1 + |g|^4)}{(1 - a)(1 + |g|^4) + 2(1 + a)|g|^2} ; \]
\( Im <r_{ij}> \) is the imaginary part of the weighted average of the ratios \( r_i \) and \( r_j \),
\[ <r_{ij}> = \frac{r_i |A_i|^2 + r_j |A_j|^2}{|A_i|^2 + |A_j|^2} , \]
\[ r_i = \frac{q}{p} \frac{<\ell^+ | T | B >}{<\ell^+ | T | B >} . \]
We notice that violations of the \( \Delta B = \Delta Q \) rule contribute to the ratio \( \chi_{ij} \) only quadratically. The case of a single channel say, \( i \) (together with \( \tilde{i} \)), is obtained by setting \( A_j = 0 \).

An instructive but perhaps extreme case arises if we consider \( B \leftrightarrow \bar{B} \) mixing to be altogether absent and treat the entire signal of like-sign dileptons to be solely due to the \( \Delta B = -\Delta Q \) transitions. The resulting \( \chi_{ij} \) can also be deduced from the above formulas by taking the limit of vanishing mixing parameters \( x = y = 0 \),
\[ \chi_{ij}^{NM} = 2 \left( \frac{|A_i A_j| |\rho_i - \rho_j|}{|A_i|^2 + |A_j|^2} \right)^2 ; \]
the superscript ‘NM’ denotes ‘no mixing’. Since \( |A_i|^2 \) is proportional to the partial width \( \Gamma_i \), and hence to the branching fraction \( f_i = \Gamma_i / \Gamma_{total} \), we see that
\[ |\rho_i - \rho_j| = (f_i + f_j) \sqrt{\frac{\chi_{ij}^{NM}}{2 f_i f_j}} . \]
Experimental data on dilepton events grouped in terms of exclusive channels are not available. Present data refer to the ratio of inclusive rates from the ARGUS \[1\] and CLEO \[2, 3\] groups, and we take its average value to be \(\chi_{\text{expt}} = 0.16 \pm 0.04\) . \(\text{(14)}\)

However, the signal from the inclusive semileptonic decay (total branching fraction \(\simeq 10\%\)) seems to be arising only from a few channels;

\[
f(B \rightarrow D^- \ell^+ \nu_\ell) = (1.8 \pm 0.5)\% \quad \text{(Ref. \[14\])}
\]

\[
f(B \rightarrow D^+ \ell^+ \nu_\ell) = (5.2 \pm 0.8)\% \quad \text{(Ref. \[15\])}
\]

Therefore in order to get an estimate of the \(\rho\)'s we may assume that the contribution to \(\chi_{\text{expt}}\) is almost entirely due to the above two decay modes. This allows us to identify them \[16\] with labels \(i\) and \(j\) in Eq. \(\text{(13)}\), and obtain

\[
|\rho_i - \rho_j| = 0.65 \pm 0.10 . \quad \text{(15)}
\]

In other words, if we view the like-sign dilepton signal from \(\Upsilon(4S)\) as being purely due to an admixture of the \(\Delta B = -\Delta Q\) decay amplitudes, we would require their relative magnitudes to be at least \(0.33 \pm 0.05\) [i.e., half the number appearing in Eq.\(\text{(13)}\)]. In comparison, the amplitude ratio \(\rho(K)\) corresponding to \(\Delta S = -\Delta Q\) decays in \(K_0^0\) is known \[14\] to be very small, \(\rho(K) = (0.6 \pm 1.8\%) - i(0.3 \pm 2.6\%)\) .

An interesting ratio that can be constructed out of same-sign dilepton events which emerge from two channels \(i\) and \(j\), is the asymmetry

\[
\alpha_{ij} \equiv \{(N_{ij}^{++} - N_{ij}^{--}) / (N_{ij}^{++} + N_{ij}^{--})\} . \quad \text{(16)}
\]

Keeping terms up to the bilinear ones in \(r_i\) and \(r_j\), we see that

\[
\alpha_{ij} = \frac{1 - |g|^4}{1 + |g|^4} \left\{ 1 + \frac{4}{1 + |g|^4} \left[ \text{Re} \left( <r>_{ij} \right)^2 - \frac{|g|^2}{2} \frac{1 + a}{1 - a} \chi_{ij}^{NM} \right] \right\} . \quad \text{(17)}
\]

The terms in the square brackets are indeed the correction terms in expressing the denominator in Eq. \(\text{(16)}\). Thus a nonzero value of \(\alpha_{ij}\) would require not only mixing \((a \neq 1)\) but also \(CP\) violation of the mass-matrix \((|g| \neq 1)\). The well-known inclusive version of \(\alpha_{ij}\) given by Okun \textit{et al} \[13\]...
(which assumes the $\Delta B = \Delta Q$ rule) also has these two requirements. If $\Delta B = -\Delta Q$ transitions were the only source of the like-sign dilepton events, the asymmetry $\alpha_{ij}$ would vanish but not the ratio $\chi_{ij}$. Available data on the $CP$-violating dilepton asymmetry however refer to the inclusive semileptonic channels and the present experimental value \[ \alpha = \frac{(1 - |g|^4)}{(1 + |g|^4)} = (3.1 \pm 9.6 \pm 3.2)\% \]

*Time-Asymmetries* - Consider the time-asymmetry among the events with same-sign dileptons say, $\ell^+\ell^+$, which result from decays of neutral $B$'s via either of the two channels, labelled $i$ and $j$ :

$$A_{\ell^+\ell^+}(ij) = \frac{[\nu(ij) - \nu(ji)]}{[\nu(ij) + \nu(ji)]}.$$  \hspace{1cm}(18)

The symbol $\nu(ij)$ stands for the number of dilepton events in which the $\ell^+$ associated with channel $i$ occurs earlier than the one associated with channel $j$; this is ensured simply by integrating the rates with respect to the variable $\tau = (t_j - t_i)$ over the range $0$ to $\infty$. We see that

$$A_{\ell^+\ell^+}(ij) = \frac{-2}{1-a} \left[ ax \text{ Im} (r_i - r_j) + y \text{ Re} (r_i - r_j) \right];$$ \hspace{1cm}(19)

similarly we also have

$$A_{\ell^-\ell^-}(ij) = \frac{\nu(ij) - \nu(ji)}{\nu(ij) + \nu(ji)} = \frac{2}{1-a} \left[ ax \text{ Im} (r_i - r_j) - y \text{ Re} (r_i - r_j) \right].$$ \hspace{1cm}(20)

Obviously these asymmetries vanish in the case of a single channel (namely $i = j$) because the initial state $\Upsilon(4S)$ is antisymmetric under the interchange of the two beons, while the final state is symmetric (when $i = j$).

We next define the time-asymmetry for opposite-sign dilepton events arising from any of the two exclusive channels $i$ and $j$ :

$$A_{\ell^+\ell^-}(ij) = \frac{\nu(ij) + \nu(ji)}{\nu(ij) + \nu(ji)} - \frac{\nu(ij) + \nu(ji)}{\nu(ij) + \nu(ji)} = \frac{4ax}{1+a} \text{ Im} <r>_{ij};$$ \hspace{1cm}(21)

Keeping terms upto the linear ones in $r_i$ and $r_j$, we obtain this $CP$-violating asymmetry to be

$$A_{\ell^+\ell^-}(ij) = \frac{4ax}{1+a} \text{ Im} <r>_{ij};$$ \hspace{1cm}(22)

where $<r>_{ij}$ is defined in Eq. (11), and the factor multiplying it is $\simeq 1.1$ (for the typical values $x \simeq 0.67$ and $y \simeq 0$). The case of a single channel was reported earlier by one of us \[17\], and
generalization to more than two channels is straightforward. Notice that this asymmetry, unlike the ones in Eqs. (19,20), does not vanish even if all the \( r_i \)'s are equal. We emphasize that a nonzero value of this asymmetry would establish the presence of \( \Delta B = -\Delta Q \) transitions in decays of neutral \( B \) mesons, [18].

A related asymmetry is the one with respect to ‘channels’, without regard to the leptonic charge; in other words we look at the difference in rates when the decay channels \( j \) and \( \tilde{j} \), follow (or precede) the decay channels \( \tilde{i} \) and \( i \):

\[
A_{\ell^+\ell^-} = \frac{\nu(\tilde{i}\ell^- + \tilde{i}\ell^+)}{\nu(\tilde{i}\ell^+ + \tilde{i}\ell^-)} - \frac{\nu(\tilde{j}i + \tilde{j}i)}{\nu(\tilde{j}i + \tilde{j}i)} = -\frac{2y}{1 + a} \text{Re} (r_i - r_j).
\]

Although this signal is \( CP \)-conserving, in contrast to Eq. (22), it is unfortunately suppressed by the factor \( y \) which could be quite small for the \( B \) mesons.

*Decays to \( CP \) Eigenstates*- It is now well recognized that at the forthcoming \( B \) factories the main thrust of the experimental effort would be towards measuring the \( CP \)-violating asymmetries. Of special interest are the time-dependent rate asymmetries between \( B \) and \( \bar{B} \) decays to specific \( CP \) eigenstates \( f \) \((= J/\psi K_S, \pi^+\pi^-, ... )\), as they would enable us to test the \( CP \)-violating mechanism of the Standard Model; for a recent review see, e.g., [11]. How do these \( CP \)-violating signals get modified when some of the tag-leptons arise from \( \Delta B = -\Delta Q \) transitions? This is the item we discuss next.

Consider the events wherein the tag-lepton is emitted in an exclusive semileptonic channel \( (\tilde{i}\ell^-) \) or \( (i\ell^+) \). We express the asymmetry in terms of the rates summed over the channel index \( i \) as follows:

\[
A_f = \frac{\sum_i \left[ \nu(\tilde{i}\ell^- f) - \nu(i\ell^+ f) \right]}{\sum_i \left[ \nu(\tilde{i}\ell^- f) + \nu(i\ell^+ f) \right]},
\]

where \( \nu(i\ell^+ f) \) represents the number of events arising from \( \Upsilon(4S) \) in which the \( B \) decay to \( f \) occurs at any time later than the related semileptonic \( B \) decay into \( (i\ell^+) \). In the Standard Model the formula for this asymmetry is

\[
A_f^{(SM)} = \frac{\alpha}{\Omega} \frac{-2x \text{Im} (u_f) + 1 - |u_f|^2}{1 + |u_f|^2 + 2y \text{Re} (u_f)} - \frac{\Omega}{\alpha} \frac{1 + |u_f|^2 + 2y \text{Re} (u_f)}{1 + |u_f|^2} \Omega \frac{1 + |g|^2}{1 + |g|^2};
\]  

\[
u_f \equiv g \frac{<f|T\bar{B}>}{<f|T|B>}, \quad \Omega \equiv <B_2|B_1> \quad = \frac{1 - |g|^2}{1 + |g|^2}.
\]
In the limit of $CP$ conservation we have $\Omega = 0$ and $u_f = \xi_f$, where $\xi_f$ is the $CP$ eigenvalue, $CP|f > = \xi_f |f > = \pm |f >$; thus we see that $A_f^{(SM)} = 0$ in that limit.

Going beyond the Standard Model and retaining the $\Delta B = -\Delta Q$ amplitudes up to first order, i.e., by keeping terms up to linear ones in the parameter $r_i$, we obtain

$$A_f = \left[ 1 + \frac{\Re(u_f) + y}{1 + y \Re(u_f)} 2 \Re <r> \right] A_f^{(SM)},$$

(27)

where the correction depends on the weighted average of the ratio of the nonstandard to standard amplitudes,

$$<r> = \frac{\Sigma_i (r_i |A_i|^2)}{\Sigma_i |A_i|^2}.$$  

(28)

Since $A_f^{(SM)}$ is already of first order in $CP$ violation, and since $<r>$ is expected to be small, Eq. (27) shows that the $CP$-violating asymmetry $A_f$ will be hardly affected by the presence of $\Delta B = -\Delta Q$ transitions.

Finally we comment on the interesting asymmetry that provides a direct measure of the parameter $y$. This needs the same data-base as the $CP$-violating $A_f$ with the difference that the lepton here serves to fix the ‘zero’ of the time of decay into the $CP$ eigenchannel $f$. The time-integrated rate for the emission of a lepton of either charge to emerge earlier than the channel $f$ is

$$\nu(\ell, f) \equiv \sum_i \left[ \nu(i\ell^+, f) + \nu(i\ell^-, f) \right].$$

(29)

In a similar way let $\nu(f, \ell)$ determine the rate wherein the semileptonic decays occur after the decay to $f$; it is obtained by integrating with respect to the relative time $\tau = (t_f - t_\ell)$ over negative values. With the help of these quantities we formulate the asymmetry that determines $y$:

$$A_y = \frac{\nu(\ell, f) - \nu(f, \ell)}{\nu(\ell, f) + \nu(f, \ell)} = \left[ \frac{2 \Re(u_f)}{1 + |u_f|^2} \right] \left\{ 1 - 4(\xi_f - \Re u_f) \Re <r> \right\} y.$$  

(30)

Thus the value obtainable for $y$ will be hardly modified by the inclusion of $\Delta B = -\Delta Q$ contributions since the parameter $<r>$ of Eq. (28) occurs in multiplication with a $CP$-violating effect in the decay $B \to f$.

In summary, signals for physics beyond the Standard Model could appear at $B$ factories as nonvanishing time-asymmetries for dilepton events; Eqs. (18-23). Of particular significance is the asymmetry $A_{\ell^+\ell^-}$ of Eq. (22) which is $CP$-violating and which does not vanish in the limit of
$y = 0$. On the other hand $\Delta B = -\Delta Q$ contributions, enter only bilinearly in the time-integrated dilepton ratios $\chi_{ij}$ and $\alpha_{ij}$, and hardly affect the interesting asymmetries which depend on beon decays to $CP$ eigenstates $f$.

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