Unification for the Darkly Charged Dark Matter

Ayuki Kamada,1, Masaki Yamada,2, and Tsutomo T. Yanagida3,4

1 Center for Theoretical Physics of the Universe, Institute for Basic Science (IBS), 55 Expo-ro, Yuseong-gu, Daejeon 34126, Korea
2 Tufts University, 574 Boston Avenue, Medford, MA 02155, U.S.A.
3 T. D. Lee Institute and School of Physics and Astronomy, Shanghai Jiao Tong University, 800 Dongchuan Rd, Shanghai 200240, China and
4 Kavli IPMU (WPI), UTIAS, The University of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba 277-8583, Japan

We provide a simple UV theory for a Dirac dark matter with a massless Abelian gauge boson. We introduce a single fermion transforming as the 16 representation in the SO(10)′ gauge group, which is assumed to be spontaneously broken to SU(5)′×U(1)′. The SU(5)′ gauge interaction becomes strong at an intermediate scale and then we obtain a light composite Dirac fermion with U(1)′ gauge interaction at the low-energy scale. Its thermal relic can explain the observed amount of dark matter consistently with other cosmological and astrophysical constraints. We discuss that a nonzero kinetic mixing between the U(1)′ gauge boson and the Hypercharge gauge boson is allowed and the temperature of the visible sector and the dark matter sector can be equal to each other.

Introduction.— Constructing a grand unified theory (GUT) of the Standard Model (SM) is an outstanding challenge in particle physics. The similarity of the SM gauge coupling constants and the beautiful unification of fermions in the SU(5) multiplets may support the existence of the unified theory at a very high energy scale. However, the running of the gauge coupling constants and the quark/lepton mass relation are deviated from the simplest SU(5) GUT prediction [1–5], which may imply that the GUT breaking in the visible sector is much more complicated than we expect.

In the context of cosmology, there exists dark matter, which may be a fundamental particle that barely interacts with the SM particles. Since the dark matter (DM) must be stable and neutral under the electromagnetic interaction, we consider it to be charged under a hidden U(1)′ gauge symmetry. Then one may hope that the dark sector is also unified into a GUT′ theory as in the SM sector.

In this letter, we propose a chiral SO(10)×SO(10)′ GUT as a unified model of SM and DM sectors. The first SO(10) gauge theory is a standard SO(10) GUT model, which we do not specify as it has been extensively discussed in the literature [6,11]. We focus on the second SO(10)′ gauge theory, which gives a dark sector. The fermionic matter content in SO(10)′ is a single field in the 16 representation. The SO(10)′ is assumed to be spontaneously broken to SU(5)′×U(1)′ at a very high energy scale and the SU(5)′ gauge interaction becomes strong at the energy scale of order 10^{13} GeV. Below the confinement scale, we have a light composite Dirac fermion charged under the remaining U(1)′. Therefore the DM sector results in a Dirac DM with a massless U(1)′ gauge boson, which has been discussed in Refs. [12,13]. A similar idea of the strong SU(5)′ gauge theory was used in the literature in different contexts [14,15], where they did or did not introduce the U(1)′ gauge symmetry.

As discussed in Ref. [13], a DM with a massless hidden photon is still allowed by any astrophysical observations and DM constraints even if it is the dominant component of DM. The thermal relic abundance of the Dirac fermion can explain the observed amount of DM. We find that the temperatures of SM and DM sectors can be the same with each other at a high temperature. This allows us to consider a nonzero kinetic mixing between the U(1)′ and U(1)γ gauge bosons, which presents an interesting possibility for the DM search in this model. The relic of the massless U(1)′ gauge boson affects the expansion rate of the Universe as dark radiation, which can be checked by the detailed measurements of the CMB anisotropies in the future.

Dark matter in the low-energy sector.— We first explain a low energy phenomenology in the dark sector. Let us introduce a U(1)′ gauge symmetry and a Dirac fermion η of weak-scale mass m_η with charge q. We consider the case where the U(1)′ gauge symmetry is not spontaneously broken and the gauge boson γ′ is massless until present. We denote the temperature of dark sector as T′ and that of visible sector as T. We define ξ(T′) = T′/T, which depends on the temperature. We will see that there is a viable parameter region even if ξ = 1 at a high temperature.

The DM can annihilate into the dark photon and hence its thermal relic density is determined by the freeze-out process. The thermally-averaged annihilation cross section is given by

\[ \langle \sigma v_{\text{MD}} \rangle = \frac{\pi q^4 \alpha'^2}{m_\eta^4} \tilde{S}_{\text{ann}}(\alpha'), \]

where v_{\text{MD}} is Moller velocity and \tilde{S}_{\text{ann}} is the thermally-averaged Sommerfeld enhancement factor [17,18]. In the regime where the gauge interaction is relatively large, a bound-state formation is efficient and is relevant to determine the thermal relic abundance. Hence we have to solve the coupled Boltzmann equations for the unbound
and bound DM particles as done in Ref. [18]. For simplicity, we quote their result to plot a contour on which we can observe the neutralino amount of DM on the solid blue curve in Fig. 1, that a larger number of statistical samples of galactic scales may not be applied to this kind of models and the massless mediator is still allowed for the self-interacting DM model.

The massless dark photon remains in the thermal plasma in the dark sector and contributes to the energy density of the Universe as dark radiation. Its abundance is conveniently described by the deviation of the effective neutrino number from the SM prediction, namely

\[ \Delta N_{\text{eff}} = \frac{8}{7} \left( \frac{g^*_s(T'_d)}{g^*(T'_d)} \right)^{-4/3} \xi^4(T'_d), \]  

where \( g^*_s \) is the effective number of degrees of freedom in the dark sector and \( T'_d \) the decoupling temperature of dark sector from the SM sector. In the case where the dark sector is completely decoupled from the SM sector before the DM becomes non-relativistic and the electroweak phase transition, we should take \( g^*_s(T'_d) = 2 + 4(7/8) = 11/2 \) and \( g^*(T'_d) = 106.75 \) and obtain \( \Delta N_{\text{eff}} = 0.21 \xi^4(T'_d) \). Even if we set \( \xi(T'_d) = 1 \), the prediction is consistent with the constraint reported by the Planck data combined with the BAO observation: \( N_{\text{eff}} = 3.27 \pm 0.15 \). We can check the deviation from the SM prediction with a large significance in the near future by, e.g., the CMB-S4 experiment [24, 25]. It is also possible that the DM sector is in the thermal equilibrium with the SM sector at a high temperature and then decoupled after the DM becomes non-relativistic. This is the case when the U(1) gauge boson has a nonzero kinetic mixing with the U(1) gauge boson as we discuss later. Then we should take \( \xi(T'_d) = 1 \) and \( g^*_s(T'_d) = 2 \). As we discuss shortly, the decoupling temperature is just below the DM mass, which is of order or larger than the electronegative scale. Thus we expect \( g^*_s(T'_d) \approx 100 \), which results in \( \Delta N_{\text{eff}} \approx 0.07 \). This scenario is also consistent with the Planck data and would be checked by the CMB-S4 experiment in the future.

Dark matter from hidden \( SO(10)' \).—Now we shall provide a UV theory of the DM sector, which is similar to the SM GUT. We introduce an \( SO(10)' \) gauge group and a chiral fermion transforming as the 16 representation, assuming that the gauge group is spontaneously broken to \( SU(5)' \times U(1)' \) at the energy scale much above \( 10^{13} \) GeV and below the Planck scale. After the SSB, the fermion is decomposed into \( \psi, \chi, \text{and } N \), which transform as the 5, 10, and 1 representations in the \( SU(5)' \) gauge group, respectively. If we denote the \( U(1)' \) charge of \( N \) as \( q (= \sqrt{10}/4) \), those of \( \psi \) and \( \chi \) are \(-3q/5 \) and \( q/5 \), respectively [23]. If one starts from a generic \( SU(5)' \times U(1)' \) gauge theory instead of the \( SO(10)' \) gauge theory, the \( U(1)' \) charge \( q \) may be different from \( \sqrt{10}/4 \).
Since the SU(5)\(^{'}\) gauge interaction is asymptotically free, it becomes strong and is confined at a dynamical scale \(\Lambda_{5}^{'}\). Below the confinement scale, there is a massless baryonic state composed of three fermions like \(\eta = \psi \bar{\psi} \chi\) as the \(t\)Hooft anomaly matching condition is satisfied [27,28] (see Refs. [14–16] for other applications of this model). This can be combined with \(N\) to form a Dirac fermion. In fact, we can write down the following dimension-6 operator:

\[
\frac{c}{\Lambda_{Pl}^{2}} \psi \bar{\psi} \chi N + \text{h.c.},
\]

where \(c\) is an \(O(1)\) constant. This results in a Dirac mass term below the dynamical scale and its mass is roughly given by

\[
m_{\eta} \sim c \left(\frac{\Lambda_{5}^{'}{}^{3}}{\Lambda_{Pl}^{2}}\right).
\]

This is of order 100 GeV − 10 TeV when the dynamical scale \(\Lambda_{5}^{'}\) is of order 10\(^{13}\)−14 GeV. As a result, the low-energy sector is nothing but the DM model discussed in the previous section.

As for the SM sector, we consider also an SO(10) GUT, motivated by the thermal leptogenesis [29] (see, e.g., Refs. [30–33] for recent reviews) and seesaw mechanism [34–37]. Here, we introduce a right-handed neutrino with mass of order or larger than 10\(^9\) GeV in the SM sector. Then, we expect an SO(10) \(\times\) SO(10)\(^{'}\) gauge theory to be a unified model of the SM and DM sectors. The similarity of the SM and DM sectors may be because a fermion in the 16 representation is the minimal particle content for the anomaly-free chiral SO(10) gauge theory.

An example of renormalization group running of gauge coupling constants is shown in Fig. 2 where we note that there are three flavors for quarks and leptons while there is only one “flavor” in the dark sector. Although an explicit construction of the GUT model in the SM sector is beyond the scope of this paper, we present a gauge coupling unification in a simple GUT model proposed in [38]. They introduced adjoint fermions for SU(3)\(_c\) and SU(2)\(_L\) at an intermediate scale and at the TeV scale, respectively. Although the SU(2)\(_L\) adjoint fermion is stable, we assume that it is a subdominant component of DM or there is another field that makes it unstable. Noting that this is just one example of GUT in the Standard Model sector, we plot the gauge coupling unification in the simplest case in the figure. We do not introduce such adjoint fermions in the dark sector or we assume that they are heavier than the dynamical scale if present.

We are interested in the case where \(q = \sqrt{10}/4\) and the SU(5)\(^{'}\) gauge coupling \(\alpha_{5}^{'}\) becomes strong at \(\Lambda_{5}^{'} \sim 10^{13}\) GeV. Starting from \(\alpha_{5}^{'} \simeq 4.2 \times 10^{-2}\) and \(2.5 \times 10^{-2}\) at the electroweak scale, we find that the SU(5)\(^{'}\)\(\times U(1)\) gauge group can be unified at the energy scale of \(M_{\text{GUT}}^{'} = 10^{16}\) GeV and the Planck scale, respectively. These gauge coupling constants are shown as the upper and lower dashed lines in Fig. 4. It shows that the DM mass should be about 1.1 TeV and 600 GeV, respectively, to explain the observed amount of DM if \(\xi(T_{\Delta}^{'}{\Lambda}_{Pl}^{2}) = 1\).

We note that the gauge coupling constants in the dark sector does not need to be unified at the same scale as the GUT scale in the SM but can be unified at the energy scale between the dynamical scale \(\Lambda_{5}^{'}\) (\(\sim 10^{13}\) GeV) and the Planck scale. Thus the U(1)\(^{'}\) gauge coupling constant can be as large as \(q^{2} \alpha_{5}^{'} \sim 0.2\) at the electroweak scale. However, we expect that the gauge coupling constant at the unification scale is of the same order with that of the SM gauge coupling constants and hence \(M_{\text{GUT}}^{'} = O(10^{16\text{-}18})\) GeV. In this case, \(\alpha_{5}^{'}\) must be within the region between the dashed lines in Fig. 4 namely,

\[
\alpha_{5}^{'} = (2.5-4.2) \times 10^{-2}, \quad m_{\eta} = 0.6 - 1.1 \text{ TeV}.
\]

This is the prediction of the chiral SO(10)\(^{'}\) gauge theory in the DM sector.

**Kinetic mixing.**—Finally, we comment on the kinetic mixing between the U(1)\(_Y\) and U(1)\(^{'}\) gauge bosons. For this purpose, we need to specify how to break the gauge groups at the GUT scale. We first note that a scalar field transforming as the 45 representation in SO(10) is decomposed into scalar fields in the \(1 + 10 + 10 + 24\) representations under an SU(5) (\(\subset SO(10)\)) gauge group. The singlet 1 can be used to break SO(10) to SU(5) \(\times U(1)\). We assume that SO(10) and SO(10)\(^{'}\) are spontaneously broken to SU(5) \(\times U(1)_{B-L}\) and SU(5)\(^{'}\) \(\times U(1)\) by nonzero VEVs of 45\(_{H}\) and 45\(^{'}\)\(_{H}\), respectively. The remaining SU(5) in the visible sector is also assumed to be spontaneously broken to the Standard Model gauge
The effective electromagnetic charge of DM is given by
\[ q_{\text{EM}} = q_{\text{EM}}' \cos \theta_W / \epsilon_{\text{EM}} \]
where the atmosphere \( \xi \). The measurement of CMB temperature anisotropies also constrain the millicharged DM for a larger charge region \([42, 43]\). In combination, there is an allowed range such as
\[ 10^{-6} \left( -\frac{m_\eta}{10^3 \text{ GeV}} \right) \lesssim \epsilon \lesssim 3 \times 10^{-5} \left( -\frac{m_\eta}{10^3 \text{ GeV}} \right)^{1/2} \]
distinguish our model from the self-interacting DM model with a velocity-independent cross section, like the ones studied in Refs. [22, 77–79]. It is also worth to investigate if the self-interacting DM with a massless vector mediator solves the small-scale issues for the cosmological structure formation [22, 77–79].

Acknowledgments.– A. K. was supported by Institute for Basic Science under the project code, IBS-R018-D1. A. K. would like to acknowledge the Mainz Institute for Theoretical Physics (MITP) of the Cluster of Excellence PRISMA+ (Project ID 39083149) for enabling A. K. to complete a significant portion of this work. T. T. Y. was supported in part by the China Grant for Talent Scientific Start-Up Project and the JSPS Grant-in-Aid for Scientific Research No. 16H02176, No. 17H02878, and No. 19H05810 and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. T. T. Y. thanks to Hamamatsu Photonics.

[1] J. R. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B249, 441 (1990).
[2] J. R. Ellis, S. Kelley, and D. V. Nanopoulos, Phys. Lett. B260, 131 (1991).
[3] U. Amaldi, W. de Boer, and H. Furstenau, Phys. Lett. B260, 447 (1991).
[4] C. Giunti, C. W. Kim, and U. W. Lee, Mod. Phys. Lett. A6, 1745 (1991).
[5] P. Langacker and M.-x. Luo, Phys. Rev. D46, 2261 (1993).
[6] Y. Mambrini, N. Nagata, K. A. Olive, J. Quevillon, and B. Zaldiver, Phys. Rev. Lett. 110, 241306 (2013) [arXiv:1302.4438 [hep-ph]].
[7] T. Yanagida, Proceedings: Workshop on the Unified Theories and the Baryon Number in the Universe: Tsukuba, Japan, February 13-14, 1979, NATO Sci. Ser. B59, 135 (1980).
[8] S. Dimopoulos, S. Raby, and L. Susskind, Nucl. Phys. B173, 208 (1980).
[9] M. Fukugita and T. Yanagida, Phys. Lett. B174, 45 (1986).
[10] Y. Mambrini, K. A. Olive, J. Quevillon, and B. Zaldiver, Phys. Rev. Lett. 110, 241306 (2013) [arXiv:1302.4438 [hep-ph]].
[11] S. Bertolini, L. Di Luzio, and M. Malinsky, Phys. Rev. D81, 035015 (2010) [arXiv:0912.1796 [hep-ph]].
[12] D. Chang, R. N. Mohapatra, J. Gipson, R. E. Marshak, and M. K. Parida, Phys. Rev. D31, 1718 (1985).
[13] P. Agrawal, F.-Y. Cyr-Racine, L. Randall, and A. Strumia, Nucl. Phys. B685, 362 (2004) [Erratum: Nucl. Phys. B793, 362 (2008)] [arXiv:hep-ph/0205349 [hep-ph]].
[14] A. K. was supported by Institute for Basic Science under the project code, IBS-R018-D1. A. K. would like to acknowledge the Mainz Institute for Theoretical Physics (MITP) of the Cluster of Excellence PRISMA+ (Project ID 39083149) for enabling A. K. to complete a significant portion of this work. T. T. Y. was supported in part by the China Grant for Talent Scientific Start-Up Project and the JSPS Grant-in-Aid for Scientific Research No. 16H02176, No. 17H02878, and No. 19H05810 and by World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. T. T. Y. thanks to Hamamatsu Photonics.
