Study on Roof Safe Thickness of Rock Foundation with Karst Cave and Karst Cave- surrounding Rock Stability

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Abstract. The deformation of karst cave-surrounding rock and its roof thickness are the key indicators to evaluate the stability of karst cave foundation. According to engineering characteristics of pile foundation in karst area, Using mechanical analytical method, cave-surrounding rock elasticity theory and Griffith criterion to calculate the safe thickness of Karst cave roof. According to karst cave-surrounding rock unstable and failure conditions, such as groundwater, different pile diameter, different thickness of Karst cave roof. Through comparing and analyzing engineering cases, the results show that the mechanical analytical method to determine the safety thickness of Karst cave roof can not meet the engineering requirements, determine the safe thickness of Karst cave roof Should be based on stability of cave-surrounding rock.

1. Introduction
Karst landform is widespread in China. due attention has been paid to the influence of karst collapse on the foundation stability. The evaluation method of karst foundation stability can be classified into qualitative analysis and quantitative analysis. Qualitative analysis is the basis for quantitative analysis. The decisive method is quantitative analysis[1].

2. Quantitative analysis on the stability of karst cave foundation roof
Methods of calculating the safe thickness of the roof are as follows:

2.1. The mechanical analytic method
Calculated as cantilever beam when there is fracture in the roof span and rocks at the standoffs of both sides of the roof are firm and complete:[2]

\[ M = \frac{1}{2}pl^2 \]  \hspace{1cm} (1)

Calculated as simply supported beam when there is fracture in the Standoffs and the roof is complete:

\[ M = \frac{1}{8}pl^2 \]  \hspace{1cm} (2)

Calculated as fixed beam when strata of the Standoffs and of the roof are complete:

\[ M = \frac{1}{12}pl^2 \]  \hspace{1cm} (3)

Checking computation of shear strength:

\[ H \geq \sqrt{\frac{4Q}{\tau}} \]  \hspace{1cm} (4)
In the formula (1) (2) (3) (4), M stands for bending moment, KN·m; p for total load of the roof, kN/m; l for cave span, m; σ for the computed bending resistance of the rock mass and estimated as 1/8 of the allowable compressive strength, kPa; b for the width of the beam and slab and estimated as 1.0m; H for thickness of roof strata, m; Q for shearing force of the standoffs, Q=1/2pL; τ for computed shearing strength of the strata (in the case of limestone, estimated as 1/12 of the allowable compression strength), kPa.

2.2. stability quantitative analysis of the karst cave surrounding rock

2.2.1. Stress redistribution of circular caves elastic surrounding rock. Suppose under external force the surrounding rocks of underground caves deforms elastically and can be regarded as continuous elastic body of same nature and uniformity[3]. As shown by figure 1, there is a horizontal cave with a radius of R₀, buried in elastic rock. Suppose in this plane strain problem both the horizontal and vertical stress is main stress and put to the cave, the stress distribution formula is shown in (5):

\[
\begin{align*}
\sigma_r &= \frac{\sigma_h + \sigma_v}{2} \left[ 1 - \frac{R_0^2}{r^2} \right] + \left[ 1 + \frac{3R_0^4}{r^2} - \frac{4R_0^2}{r^2} \right] \cos 2\theta \\
\sigma_\theta &= \frac{\sigma_h + \sigma_v}{2} \left[ 1 + \frac{R_0^2}{r^2} \right] - \left[ 1 + \frac{3R_0^4}{r^2} \right] \cos 2\theta \\
\tau_{r\theta} &= -\frac{\sigma_h - \sigma_v}{2} \left[ 1 - \frac{3R_0^4}{r^2} + \frac{2R_0^2}{r^2} \right] \sin 2\theta
\end{align*}
\]

In the formula (5), σᵣ, σ₉, τᵣθ stands for radial stress, hoop stress and shear stress on the point M respectively with compressive stress as positive and tensile stress as negative; σᵥ for vertical stress and σ₃ for horizontal stress; θ for the polar angle at M with counter-clockwise direction as positive; r for vector radius and R₀ for karst cave radius. The formula of the karst cave surrounding stress (r=R₀) and the stress on the wall of the circular karst cave are as formula (6):

\[
\begin{align*}
\sigma_r &= 0 \\
\sigma_\theta &= \sigma_h(1-2\cos 2\theta) + \sigma_v(1+2\cos 2\theta) \\
\tau_{r\theta} &= 0
\end{align*}
\]

Theoretically, the stress redistribution is considered within the radius range six times the karst cave radius (r=6R₀) and surrounding rocks outside the range is considered under little influence of the stress distribution. This paper analyzes the stress 5R₀ away from the cave center. The vertical and horizontal stresses of the karst cave (r=5R₀) are calculated according to the following formulas(7):
\[
\begin{aligned}
\sigma_r &= a_A P_0 + \sigma_A \\
\sigma_v &= \beta(a_B P_0 + \sigma_B)
\end{aligned}
\]  

(7)

In the formula (7), \(a_A\) and \(a_B\) for the additional stress coefficient at A and B stand respectively; \(P_0\) for the additional stress of the substrate; \(\sigma_A\) and \(\sigma_B\) for the self-weight stress from the soils and rocks at A and B respectively; \(\beta\)—lateral pressure coefficient of the rock mass, \(\beta=\mu/1-\mu\) (\(\mu\) limestone regarded as 0.2)[4].

2.2.2. Stress redistribution of other shapes caves. The underground caves of karst terrain are usually not of regular circular structure. Suppose the karst cave is of oval structure, the horizontal and vertical diameters are \(b\) and \(a\) respectively, the horizontal and vertical stresses of the karst cave surrounding rocks are \(\sigma_h\) and \(\sigma_v\) respectively, the stress component of the karst cave chambers can be calculated by the following formula (8):

\[
\sigma_\theta = \frac{\sigma_r [m(m+2)\cos^2 \theta - \sin^2 \theta] + \sigma_\phi [(2m+1)\sin^2 \theta - m^2 \cos^2 \theta]}{\sin^2 \theta + m^2 \cos^2 \theta}
\]

\[
\sigma_r = \tau_{r\theta} = 0
\]

(8)

In the formula (8), \(m\) for the ratio of the ellipse axes, \(m=a/b\); \(\theta\) for the eccentric stands angel of the calculation position around the karst cave.

2.2.3. Judge the karst cave surrounding rocks stability. When the the karst cave surrounding rocks at the position \(r=5R_0\) do not wreak damage under the redistribution of stress, the karst cave is stable and the expression of judging its stability in accordance with the Griffith Criterion is the following formulas (9):

\[
\begin{aligned}
\left(\sigma_1 - \sigma_3\right)^2 &= 8\sigma_t, \quad \sigma_1 + 3\sigma_3 \geq 0 \\
\sigma_1 + \sigma_3 &= -\sigma_t, \quad \sigma_1 + 3\sigma_3 < 0
\end{aligned}
\]

(9)

In the formula (9), \(\sigma_1\) and \(\sigma_3\) for the maximal and minimal stress respectively with the compression as positive; \(\sigma_t\) for uniaxial tensile strength of the rock with a negative sign. In terms of the stress around the cave, the tangential stress \(\sigma_\theta\) and the radial stress \(\sigma_r\) are the maximal and the minimal stress respectively, shearing stress \(\tau_{r\theta} = 0\). Hence, \(\sigma_\theta=\sigma_1\) and \(\sigma_r=\sigma_3\) are Principal stress, namely \(\sigma_\theta = \sigma_1 = 0\), which is substituted in the criterion (8). Judge the stability with the first formula if \(\sigma_1 + \sigma_3 \geq 0\) and with the second formula if \(\sigma_1 + \sigma_3 < 0\).

3. Engineering examples

3.1. Project Overview

There is a building for both commercial and residential purposes in the area located near the middle and lower reaches of Yangtze River, which has karst development. After survey, it is found that there is a cave of irregular round form under the 3# foundation. The institute of design suggests that 1500mm pile bored piles be used, the design value of single pile be 6300kN, the macro and minor axis be 0.7m (the height of the cave) and 0.6m (the cave span) respectively, the pile foundation be 0.5m under the fossilized limestone surface, as shown in picture1. Take 31000kpa as the mono-axial compression strength value and 1950kpa as the mono-axial tensile strength value.
Table 1. Parameter of the strata of and above the cave roof

| The number | Soil lithology             | Thickness (m) | Volume weight (KN/m³) |
|-----------|---------------------------|---------------|-----------------------|
| 1         | Miscellaneous fill        | 4             | 18                    |
| 2         | Silty clay                | 3             | 17.64                 |
| 3         | Silt                      | 1.5           | 17.2                  |
| 4         | Silty clay                | 1.5           | 17.64                 |
| 5         | Slightly weathered carbonate | 2.5         | 26                    |
|           | Total thickness of the strata | 12.5        |                       |
|           | Weighted average volume weight | 20           |                       |

3.2. checking computation of cave roof’s thickness

Given the survey that the karst cave roof is complete, but there is weathering fracture development, according to the above geological condition, calculate the safe thickness of the karst cave roof:

1) considered as simple-supported beam, the safe thickness of the karst cave roof is:

\[
H \geq \sqrt{\frac{6 \times \left(26 \times 2 + 10500 \times 0.6^2\right)}{1 \times 8 \times 31000}} \geq 0.86m
\]

2) considered as fixed beam, the safe thickness of the cave roof is:

\[
H \geq \sqrt{\frac{6 \times \left(1 \times 8 \times 6300 \times 0.6 + \frac{121}{12} \times (26 \times 2) \times 0.6^2\right)}{1 \times 0.1 \times 31000}} \geq 0.96m
\]

3) checked as shear strength, the thickness should meet the following calculations:

\[
H \geq \sqrt{\frac{4Q}{\tau}} = \sqrt{\frac{4 \times \frac{1}{2} \times (10500 + 26 \times 2) \times 0.6}{1 \times 31000}} \geq 2.21m
\]

Take 1.2 as safety coefficient, the safety thickness should be 2.65m. According to the preliminary design, the pile foundation is 0.5m embedded in the limestone surface, and the thickness of the thinnest part is 2.0m, which doesn’t meet the safety requirement. Thus, improvement is needed in the pile design. Through analyzing related factors influencing the cave stability.

3.3. Karst cave surrounding rock stress calculation and stability judgment

3.3.1. Calculate Karst cave surrounding rock stress. (1) Calculate \( \sigma_0 \) of surrounding rocks \((r=5R_0)\)

Calculate the vertical and horizontal stress according to formula(7), the results are that \( p_0 = 3354KN/m^2; \alpha_A = 0.756, \sigma_0A = 229kPa; \sigma_0B = 275kPa; \) take 0.25 as \( \beta \) value, the results are as followed:

\[
\sigma_v = 2765 kpa \quad \sigma_0 = 123kpa
\]

Calculate the hoop stress \( \sigma_0 \) at A point as in Fig.2, namely the maximal stress of the karst cave roof \((\theta = 90^\circ \text{or} 270^\circ)\). Apply formula(8): \( \sigma_{0A} = -2355kpa \)

Likewise, calculate the hoop stress \( \sigma_{0B} \) at B point, namely the maximal stress on both sides of the karst cave \((\theta = 0^\circ \text{or} 180^\circ)\), \( \sigma_{0B} = 7395kpa \)

3.3.2. Judge stability of the karst cave surrounding rock. Use formula(9) to judge the stress value at A and B

at A point \((\theta = 90^\circ \text{or} 270^\circ)\), \( \sigma_{0A} = -2355kpa, \sigma_{0A} = 0kpa; \) on the roof appears large tensile stress.

\[
|\sigma A| = 2355kpa > 1950kpa \quad \text{(unstable)}
\]

at B point \((\theta = 0^\circ \text{or} 180^\circ)\), \( \sigma_{0B} = 7395kpa, \sigma_{0B} = 0kpa; \)

\[
\frac{- (\sigma_1 - \sigma_3)^2}{8(\sigma_1 + \sigma_3)} = 924kpa < 1950kpa \quad \text{(stable)}
\]
The results show that tensile failure appears on the karst cave roof.

3.3.3. Influence of Groundwater to the stability of the karst cave. The influence is insignificant for the karst cave if the water level rises. If the water level declines below the cave, the cave would be under inhaling vacuum corrosion and negative pressure would appear with a value of $10^5$pa, which equals large-area additional load of 100kpa on the ground surface.

$$\sigma_v=2865\text{kpa}, \sigma_h=148\text{kpa};$$

Apply formula (10) to calculate $\sigma_{\theta A}$ at A point and $\sigma_{\theta B}$ at B point as shown in picture.2 as followed:

$$\sigma_{\theta A}=-2372\text{kpa}; \sigma_{\theta B}=7642\text{kpa};$$

The results show that $\sigma_{\theta}$ at the karst cave floor and at both sides increase, the tensile stress at A point changes from 2355kpa to 2372kpa, $\sigma_{\theta B}$ at B point changes from 7395kpa to 7642kpa. The degree of change is slight and the inhaling corrosion of vacuum produced by the dynamic change of the groundwater level has little influence on the stability of the karst cave surrounding rocks.

3.4. Improved methods of pile foundation

3.4.1. Influence of pile diameter to the surrounding rock. When the load doesn’t change while the pile diameter changes, the corresponding change of $\alpha_A, \alpha_B, \sigma_v, \sigma_h, \sigma_{\theta A}$ and $\sigma_{\theta B}$ are shown in Table 2 (the influence of groundwater considered):

| Pile diameter (m) | $\alpha_A$ | $\alpha_B$ | $\sigma_v$ (kpa) | $\sigma_h$ (kpa) | $\sigma_{\theta A}$ (kpa) | $\sigma_{\theta B}$ (kpa) |
|-------------------|------------|------------|-----------------|-----------------|-------------------------|-------------------------|
| 1.3               | 0.729      | 0.043      | 2774            | 130             | -2341                   | 7412                    |
| 1.2               | 0.701      | 0.037      | 2680            | 125             | -2264                   | 7162                    |
| 1.0               | 0.646      | 0.028      | 2496            | 117             | -2006                   | 6397                    |

of the surrounding rocks declines to different degrees, yet the cave is still unstable. When the pile diameter is 1.0m, the tensile stress of the roof is 2006kpa >1950kpa, which means that the size of pile bottom has great influence on the cave stability. Pile diameter out of a certain range would surpass the standard bearing capacity value of the limestone pile tip.

3.4.2. Influence of roof thickness to the karst cave surrounding rock. When the load and the pile diameter do not change while thickness of the roof changes, the corresponding change of $\alpha_A, \alpha_B, \sigma_v, \sigma_h, \sigma_{\theta A}$ and $\sigma_{\theta B}$ are shown in Table 2. (the influence of groundwater considered):

| Roof thickness (m) | $\alpha_A$ | $\alpha_B$ | $\sigma_v$ (kpa) | $\sigma_h$ (kpa) | $\sigma_{\theta A}$ (kpa) | $\sigma_{\theta B}$ (kpa) |
|-------------------|------------|------------|-----------------|-----------------|-------------------------|-------------------------|
| 2.3               | 0.646      | 0.060      | 2503            | 146             | -2017                   | 6659                    |
| 2.4               | 0.55       | 0.058      | 2184            | 145             | -1701                   | 5793                    |
| 2.5               | 0.43       | 0.055      | 1784            | 143             | -1308                   | 4707                    |

The results show that when roof thickness is 2.3, there is a large tensile stress, $|\sigma_{\theta A}|=2017\text{kpa}>1950\text{kpa}$, the cave is unstable. Apply formula (9) to judge the stability of the karst cave surrounding rocks when the roof thickness is 2.40m:

1. at A ($\theta=90^o$ or $270^o$), $\sigma_{\theta A}=-1701\text{kpa}$; $\sigma_{\theta B}=0\text{kpa}$. $|\sigma_{\theta A}|=1701\text{kpa}<1950\text{kpa}$  (stable)
2. at B ($\theta=0^o$ or $180^o$), $\sigma_{\theta A}=5793\text{kpa}$, $\sigma_{\theta B}=0\text{kpa}$.
$$\frac{|\sigma_{\theta A} - \sigma_{\theta B}|}{8(\sigma_f + \sigma_t)} = 724\text{kpa}<1950\text{kpa}$$

(stable)

The results show that after the change of roof thickness, the surrounding rocks are in a state of stability. When the roof thickness is 2.4m, the top and floor and both sides of the karst cave are not damaged. When the safety factor is 1.2, the roof thickness needed is 2.9m. Thus, the safety roof thickness of 2.65m calculated by mechanical analysis cannot guarantee cave stability and the foundation must be improved.
4. Conclusions
(1) Apply elastic theory of surrounding rocks to judge the foundation stability when the supporting layer of the foundation is strata of karst cave. The safety thickness of the karst cave roof chosen in this way can guarantee both the roof safety and cave stability.
(2) The case study shows that within the standard value of the foundation supporting layer, the smaller the foundation bottom, the more stable the cave is. Also, an effective method of ensuring the foundation stability is to change the safety thickness of roof.
(3) A conclusion drawn from the calculations above is that the inhaling corrosion of vacuum caused by groundwater is not a key factor[5].
(4) According to Regulation 6.5.3 in GB50007-2002 Code for Design of Building Foundation, the influence of vertical grike with a width less than 1m and of the section near sinkholes on the foundation stability can be ignored. This paper suggests foundation stability be re-judged in those situations.

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