Information spreading and development of cultural centers

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(Dated: December 21, 2013)

The historical interplay between societies are governed by many factors, including in particular spreading of languages, religion and other symbolic traits. Cultural development, in turn, is coupled to emergence and maintenance of information spreading. Strong centralized cultures exist thanks to attention from their members, which faithfulness in turn relies on supply of information. Here, we discuss a culture evolution model on a planar geometry that takes into account aspects of the feedback between information spreading and its maintenance. Features of model are highlighted by comparing it to cultural spreading in ancient and medieval Europe, where it in particular suggests that long lived centers should be located in geographically remote regions.

PACS numbers: 89.65.-s, 89.70.Cf, 05.40.-a, 05.10.Gg

I. INTRODUCTION

The expansion and decline of social structures depend on information spreading in form of languages, religion, or other cultural inventions [1]. In the recent years many mathematical models have been proposed for social interactions and dynamics, trying to understand social structures. However, a main feature of most models of culture dissemination is an adaptation toward local or global consensus [2, 3], an equilibration which is also found in voter models [4], social impact theory [5], majority rules [6], the Sznajds model [7], the Deffuant model [8] and the bounded confidence models [9].

The multitude of models that emphasize consensus-dynamics contrast a reality where consensus is often broken by emergence of new cultures, languages or opinions. One driving force for heterogeneity is the need for attention, where individuals not only aim at mutual understanding, but at the same time also fight for individual attention. This attention battle is more than random fluctuations of agents [10] or rejection of other opinions [11]: The battle may, for example, involve positive feedback mechanisms as suggested by [12]. In this paper, we take the possibility for a new culture to emerge into account, in addition to the local alignment rules. Lacking a simple realistic mechanism for creation of new cultures, we here simply parametrize this “emergence” in terms of a rate \( p_{\text{new}} \) for initiation of new cultures.

Another common theme of models dealing with the spread of information is that two different pieces of information are treated on an equal footing. This is in general an incorrect assumption, as the importance of two bits of information in general is asymmetric. One sorting principle is to use the information age as a sorting criteria, reflecting the fact that the value of information typically decays with time [13]. Previous studies [14, 15] demonstrated that such a sorting principle has major consequences for the spatio-temporal dynamics of information. Importantly, newer information overriding older information has been observed in spreading of lin-
guistic features [16][17]. Furthermore, a simple model of diffusion of information from a cultural stronghold with age sorting is shown to be compatible with the observed pattern of word distribution in Japan [18].

When individuals sort information based on its age, an existing cultural center will continuously need to generate new information to maintain their sphere of influence. Accordingly, we here characterize the strength of a cultural center by the rate with which it is able to generate fashions, \( p_{\text{repeat}} \). We will take this rate as a characteristic of a cultural center, and it keeps generating new fashions until the center is eliminated by information generated from competing cultural centers.

A main feature of the famous Axelrod's model [2][3] for social alignment is conservativeness in communication, implemented by having individuals with many types of opinions and a preference for communication between individuals that share many traits. This preference makes people more open for communication towards sources where they earlier obtained information. We here parametrize such conservativeness into a single parameter, \( p \), that counts the chance that a given “site” or “agent” changes who he prefers to obtain information from.

Overall our model aim to discuss the information flow association to emerging and collapsing cultural centers, each influencing their surroundings by an ongoing generation of announcements that maintain their sphere of influence. The details of the model are presented in the next section.

II. MODEL

Our model considers many rumors / fashions / viewpoints / stories / ideas (denoted fashions in the following) competing on a two dimensional square lattice of sites which at any given moment can be occupied by one fashion only. We imagine each site as an agent, which in fact could be a whole group of people that by definition share the same taste. Each site listens to their immediate four neighboring sites with a history dependent frequency. When communicating, they accept fashions only when they are newer than the current local fashion. History, or conservativeness, is quantified in terms of a preference in listening towards the direction where the last new idea came from. This is parametrized by the probability \( p \) (\( p \leq 0.25 \)) to listen to one of the other 3 directions.

The fashions in the system come from cultural centers. Contrary to our previous model where only one cultural stronghold is placed in the system \textit{a priori} [18], we here assume that a new cultural center can emerge at any site with a small probability \( p_{\text{new}} \). Such a site is recognized as a cultural center as long as it has its own fashion and not invaded by fashions from other sites. An existing cultural center in addition broadcasts itself repeatedly by initiating a new fashion with a rate \( p_{\text{repeat}} \).

We perform Monte-Carlo simulation of the model with parallel (synchronous) update of all agents. In the model, a fashion \( I(s,a) \) is characterized by its center \( s \) (the site at which the idea started) and its age \( a \) (how long time ago the fashion originated at the center). At any time \( t \) each site \( i \) have its current fashion \( I_i(t) = I(s_i,a_i) \) and its preferred direction \( d_i(t) \), from which this fashion was obtained.

The time step from time \( t \) to \( t + 1 \) consists of the following procedures:

(i) Emergence of new cultural centers. With a probability \( p_{\text{new}} \), a site \( i \) is randomly chosen out of all \( N \) sites in the system to become a new cultural center. It starts its own new fashion, i.e., \( I_i(t) \) is set to \( I(i,0) \).

(ii) Repeated broadcast by existing cultural centers. Each cultural center \( i \) (i.e., \( s_i = i \)) will start to spread a new fashion with probability \( p_{\text{repeat}} \), namely \( I_i(t) \) is set to be \( I(i,0) \). Putting it differently, every cultural center can re-broadcast the same fashion as a new one making it more appealing.

(iii) Spreading of ideas. For each site \( i \) in the system, the preferred site \( d_i(t) \) is chosen with probability \( 1 - 3p \) (\( p \leq 0.25 \)), or alternately one chooses one of the other neighbor sites with the probability \( p \). The age \( a_k \) of fashion \( I_k(t) = I(s_k,a_k) \) at the chosen site \( k \) is compared with the age \( a_i \) of the fashion at the site \( i \). If \( a_k < a_i \), the site \( i \) accepts the fashion from the site \( k \), namely set \( I_i(t+1) = I_k(t) = I(s_k,a_k) \) and update its preferred direction to \( d_i(t+1) = k \). Otherwise the site \( i \) keep its fashion unchanged, i.e. \( I_i(t+1) = I_i(t) = I(s_i,a_i) \) and keep its preferred direction \( d_i(t+1) = d_i(t) \). If a site \( i \) was a cultural center, the acceptance of competing idea destroys its ability as a cultural center, hence it stops repeatedly broadcasting new fashions.

(iv) Update of time. The ages of all fashions on all sites are increased by one.

We simulate the model on an \( L \times L \) square lattice under the periodic boundary conditions. We will also consider the model on a map of Europe, where the closed boundary conditions are imposed toward the sea regions. Initial condition is set so that no one has opinion nor preferred directions (all the sites weighted equally). We only investigate properties of the system after the number of cultural centers have reached the steady state value.

III. RESULTS

1. Dynamics

Dynamics of the model is depicted in Fig. [1] which presents time evolution on the European map. In the
top panel the respective cultures are presented with different colors. The middle panel shows current distance of each site from its respective cultural center, defined as the place where its current fashion was introduced. The consecutive images illustrate the dynamics of the system, with meanderings of borders, as well as emergence of new cultural centers and disappearance of others. Bottom panel presents the information pathways (river landscape), based on the preferred direction for each site $d_i(t)$. These arrows define the path to the cultural center to which every agent belongs. Intensity of points in the river landscape indicates how many times information was transmitted through every node, i.e. every time the idea is copied the number of transitions on all the nodes on the path (up to the origin) are increased by one. It shows clear river basin structure centered around respective cultural centers, much like what was obtained for the word spreading model of Ref. [18].

The role of the three parameters $p$, $p_{\text{new}}$, and $p_{\text{repeat}}$ is summarized in Fig. 2, representing behavior of $100 \times 100$ system with periodic boundary conditions. In Fig. 2, the average age, $\langle \text{age} \rangle$, (left column) and average size, $\langle \text{size} \rangle$, (right column) of cultural centers as the function of the probability of introduction of new cultures $p_{\text{new}}$ and the probability of resending new signals/rumor $p_{\text{repeat}}$. The system size is $100 \times 100$. The top row corresponds to $p = 0.25$, while the bottom presents results for $p = 0.01$.

2. Analysis of parameters

The interesting aspect of our model is the ongoing replacement of old cultural centers with new ones, a dynamics primarily governed by the emergence of new cultural centers. To quantitatively analyze this, we examine where new cultural centers tend to emerge, when there are already established cultural centers in the system. This is done by comparing the results with $200 \times 200$ system, and confirmed that the data in Fig. 2 collapses onto the data from the bigger system very well as long as the data with the same $p$, $p_{\text{repeat}}$, and $p_{\text{new}}/N$ are compared. The only exceptions are observed when the average size of fashion reaches the system size (data not shown).

FIG. 2: (Color on-line) Phase diagram showing average age, $\langle \text{age} \rangle$, (left column) and average size, $\langle \text{size} \rangle$, (right column) of cultural centers as the function of $p_{\text{new}}$ and $p_{\text{repeat}}$. The system size is $100 \times 100$. The top row corresponds to $p = 0.25$, while the bottom presents results for $p = 0.01$.

3. Competition between cultural centers

The biggest cultural centers are observed for small $p_{\text{new}}$ and small $p_{\text{repeat}}$, see the right column of Fig. 2. Such a combination of parameters reduces the competition among cultural centers and guarantees that each culture has enough time to spread over the whole system, resulting in one dominating culture of the system size. The opposite limit of large $p_{\text{new}}$ and large $p_{\text{repeat}}$, on the other hand, means frequent emergence of new cultural centers which survives relatively well, resulting in the coexistence of many small cultural centers.

We checked the effect of the system size by comparing the results with $200 \times 200$ system, and confirmed that the data in Fig. 2 collapses onto the data from the bigger system very well as long as the data with the same $p$, $p_{\text{repeat}}$, and $p_{\text{new}}/N$ are compared. The only exceptions are observed when the average size of fashion reaches the system size (data not shown).

We re-run history multiple times with using a given snapshot of a $100 \times 100$ system as an initial configuration. For this initial configuration, we insert a new cultural center to a site at $(x, y)$ at time zero, and run the simulation to see how long the new center survives, i.e. until it is overwritten by fashions from neighbor centers. This procedure is repeated 25 times for each site to estimate the survival probability as a function of time $t$.

The top panel of Fig. 3 presents the 2D map of the survival probability after 500 time steps (left map) and 10 000 time steps (right panel) as a function of the position $(x, y)$ where new cultural center was inserted. In the plot, the positions of the cultural centers that exist in the initial configuration are marked by arrows. We see that new cultural centers are successful either when emerging very close to the existing cultural centers or when exploring remote regions.

In order to see this tendency more clearly, the middle panel presents survival probability, $p_{cc}$, as a function of
the distance to the closest cultural center, $r_{\text{min}}$. Similarly, the bottom panel of Fig. 3 shows the average size of the culture as a function of $r_{\text{min}}$. We can see that both the survival probability and average size are a non-monotonous function of the distance to the closest cultural center. The insertion points located very close to the existing cultural centers lead to maximal chances of survival ($p_{\text{cc}}$) and largest size of cultures ($\langle\text{size}\rangle$). Both $p_{\text{cc}}$ and $\langle\text{size}\rangle$ drop quickly with distance to the existing cultural center $r_{\text{min}}$ and then show a slow recovery with the distance $r_{\text{min}}$. Namely, new centers either explore a strategy of acquiring the existing network by taking over a previous center, or have to explore the weaknesses of boundary regions to build its own network of influence. The difficulty in building, rather than taking over an empire, is also reflected to the smaller size of new cultures emerging in distant regions, compared to new cultures build on depositing an existing ruler-ships. For larger times, both survival probability ($p_{\text{cc}}$) and average size ($\langle\text{size}\rangle$) decrease for all the places (Fig. 3 middle and bottom panels) since competition with existing cultural centers makes for a finite extinction rate.

4. Analysis of one dimensional model

The reported features of the present culture spreading model can be understood qualitatively by considering a simplified one dimensional model of the random walk of the boundary between two cultural centers.

Suppose that there is a cultural center $C_0$ at the site 0 and another cultural center $C_1$ at the site $R$. We consider the motion of the left most site $r$ that belongs to $C_0$. Moreover, we assume that no additional cultural center appears, i.e. $p_{\text{new}} = 0$. In this limit, the age $a_r$ of the fashion at site $r$ which belongs to $C_0$, and the age $a_{r+1}$ of the fashion at site $r' = r + 1$, which belongs to $C_1$, can be approximated as

$$a_r \approx \frac{r}{1-p} + \tau, \quad a_{r+1} \approx \frac{R-r'}{1-p} + \tau',$$

(1)

respectively. In Eq. (1), $\tau$ and $\tau'$ are independent discrete stochastic variables both having a probability distribution

$$P(\tau) = p_{\text{repeat}}(1 - p_{\text{repeat}})^\tau.$$

(2)

For the sake of simplicity, the stochasticity of the fashion propagation in the preferred direction and the time correlation of the age are ignored, see Eq. (1).

We can calculate the rate that the position $r$ of the left most site that belongs to $C_0$ decreases (increases) by one, which happens if $a_r > a_{r+1}$ ($a_r < a_{r+1}$) when the site $r$ ($r+1$) listens to site $r+1$ ($r$) with probability $p$. The explicit form of the rates are given in the Appendix. Recall that the presented derivation is valid when $p \ll
$p_{\text{repeat}},$ where the approximation of the age in Eq. (1) is justified.

In the left panel of Fig. 4, we compare the mean first passage time (MFPT) of the boundary starting at $r = (R - 1)/2$ to reach one of the cultural centers, $T(p, p_{\text{repeat}}, R),$ which gives the typical survival time of established cultural centers separated by distance $R.$ The simplified random walk model of the boundary between two cultural centers is inserted at distance $R$ there is a cultural center at position 0, and a new cultural center is inserted at position $x \approx \pm R/2$ at time $t.$ The resulting equations are (see the Appendix),

$$\frac{\partial P(x, t)}{\partial t} = -\frac{\partial}{\partial x} J(x, t), \quad (3)$$

where

$$J(x, t) = -\frac{dU(x)}{dx} P(x, t) - \frac{\partial}{\partial x} [D(x)P(x, t)], \quad (4)$$

and

$$U(x) = |x| - \ell \left[1 - \exp \left(-\frac{|x|}{\ell} \right)\right], \quad (5)$$

$$D(x) = \frac{p}{2} \left[1 - \frac{p_{\text{repeat}}}{1 - p_{\text{repeat}}} \exp \left(-\frac{|x|}{\ell} \right)\right], \quad (6)$$

$$\ell(p, p_{\text{repeat}}) = \left[-\frac{2\ln(1 - p_{\text{repeat}})}{1 - p}\right]^{-1} \approx \frac{1 - p}{2p_{\text{repeat}}}. \quad (7)$$

The potential $U(x)$ has a minimum at $x = 0$ with harmonic behavior ($U(x) \approx x^2/(2\ell(p, p_{\text{repeat}}))$ for $|x| \ll \ell(p, p_{\text{repeat}})$) while $U(x)$ grows linearly with $|x|$ for large values of argument ($|x| \gg \ell(p, p_{\text{repeat}})$). On the other hand, the dependence of the diffusion coefficient $D(x)$ on position is rather weak and it can be considered as $D \approx p/2.$ The probability $p$ also defines the mobility, therefore the timescale of the random walk of the boundary is proportional to $1/p.$

Now we can estimate the typical size of the cultural area or the typical distance between centers $R_c.$ Suppose there is a cultural center at position 0, and a new cultural center is inserted at a distance $R.$ If new cultural center will not be inserted, the time scale when one of them will be overwritten by the other one can be estimated by the mean first passage time, $T(p, p_{\text{repeat}}, R),$ starting from the stable point $x = 0.$ During this period, however, a new cultural center can be inserted between 0 and $R$ with a probability $\approx p_{\text{new}}R/N,$ where $p_{\text{new}}/N$ is the insertion probability per site. Therefore, the insertion and the coarsening balance leads to

$$T(p, p_{\text{repeat}}, R_c)/2 = N/(p_{\text{new}}R_c). \quad (8)$$

The factor $1/2$ on left hand side of Eq. (8) comes from the fact that each centers are competing with two other centers on both sides. The comparison of the average cultural center size from the one dimensional simulation and the estimate based on Eq. (8) with the mean first passage time $T$ evaluated under continuum approximation, see Eq. (A1), are shown in the right panel of Fig. 4. The agreement is satisfactory for large $p_{\text{repeat}}$ but becomes worse for smaller $p_{\text{repeat}}.$ Consequently, the potential becomes flatter and newly inserted cultural centers have higher probability to be overwritten before the boundary reaches the central point ($x = 0$), which enhances coarsening hence increasing the average size of cultural area.

In the two dimensional case, the size becomes proportional to $R_c^2,$ but other parameter dependence are expected to be qualitatively the same. The mean first passage time $T$ is proportional to $1/p,$ see Eq. (A1), which is the timescale of the dynamics, while rapidly growing function of both $p_{\text{repeat}}$ and $R.$ Therefore, the average size $R_c^2$ estimated with (8) are expected to decrease with $p,$ $p_{\text{repeat}},$ and $p_{\text{new}},$ what is consistent with Fig. 2.

5. Replaying history of Europe

![FIG. 5: (Color on-line) The frequency histogram, presenting the fraction of time for which each point constituted a cultural center. Parameters: $p_{\text{new}} = 0.004$ and $p_{\text{repeat}} = 0.015, p = 0.01.$ Simulation time $T = 10^5$ Monte Carlo steps.](image-url)
20 cultural centers coexists (with $p = 0.01$, $p_{\text{new}} = 0.004$ and $p_{\text{repeat}} = 0.015$; as also used in Fig. 1). Assuming that the fastest time it takes a rumor to cross Europe is about 10 years, the corresponding number of updates on our $200 \times 200$ will be $\sim 200$. In this perspective the subsequent snapshots in Fig. 1 correspond to 50 years, a timescale where changes on the European map indeed occurred throughout the last millennium. Fig. 6 shows a fraction of time for which respective sites was a cultural center, illustrating that centers tend to be more stable or more often established at the tips of peninsulas or other remote regions of the continents. That is, the chance to be invaded by competing fashions and cultures diminishes in these remote edge regions.

Additionally, we have checked robustness of the observed patterns in Fig. 5. More precisely, we have constructed frequency histograms for size dependent broadcast probability $p_{\text{repeat}}$ ($p_{\text{repeat}} \sim \text{size/system size}$). This modification is based on a picture that new fashions come more frequently when the size of the culture is bigger, which in consequence makes a bigger culture more stable and convincing. The presence of such a positive feedback weakened the contrast between peripheries and internal regions. Nevertheless, distant points remained harder to be invaded than internal points (data not shown). We also studied the effect of short-cuts that connect two randomly chosen remote sites, having the possibility to building main roads between cities in mind. Presence of short-cuts further reduced the contrast between remote and central points (data not shown).

It should also be pointed out that the studied model does not account for geographical constraints like rivers, mountain chains, climate and population distribution, which are crucial for the spread of fashions and cultures in real life situations. It is assumed that transmission of fashions are purely local and in particular that fashions does not travel overseas. As a consequence, the European simulation is more an illustration of the basic principle of the model than a valid simulation of available information highways on an ancient European landscape.

The incorporation of mentioned constraints can significantly change the properties of the model. Rivers and roads constituted information paths in pre-telegraph Europe while mountain chains provide natural communication barriers. Contrary to geographical landscape, the role of varying population density is more complex and less apparent. On the one hand, it is natural to imagine that large population density leads to larger creativity to start a new culture, and the number of people sharing the same culture also affects the ability of the culture to convince other people. On the other hand, it is likely that there is a positive feedback from cultural center to the local population, i.e., larger population density appears in places which are close to existing cultural centers. It would be an interesting future project to incorporate such an effect in the present model.

IV. SUMMARY AND DISCUSSIONS

We have explored a simple model for emergence and decline of cultural strongholds, parametrized with rigidity in local social network $p$, probability of emergence $p_{\text{new}}$ and the probability at which an existing cultural center broadcast fashions $p_{\text{repeat}}$.

The overall assumption of the model was the postulate that individuals always accept the newest (locally) available viewpoint as their own, but obviously cannot adopt a viewpoint that is not available in their social surroundings. Social surroundings were restricted to their 4 nearest neighbors on a 2D square lattice, and further biased with their preference for listening in the direction where they last obtained a useful information. As probability to listen in other competing directions decreases ($p \rightarrow 0$) the cultural map freezes into many small regions.

The acceptance of a fashion according to its age only is an important feature of our model. In some models of opinion dynamics, on the other hand, a set of rules, which on average makes an agent accept a fashion shared by majority, have been adopted (e.g., voter models 1 or majority rules models 2). Incorporation of such rules to the present model should make it more difficult for a new culture center to emerge and grow, though quantitative effect depends on the exact rules.

It is also worth mentioning the relation between the present model and Axelrod model 2, which is a widely accepted model of formation of cultural area both by social scientists and physicists 3. In the Axelrod model, each agents has a set of opinions as a vector and interact with the neighboring agents according to the overlap of opinions: It is more likely to interact when the opinions are close, and when they interact the agent copies one of the different opinion from the neighbor to its opinion set. In this dynamics, the conservativeness is taken into account as tendency to talk to the agents that has close opinion, and the cultural area is formed as the agents align their opinions with their neighbors. In a sense, a culture spontaneously appears via interactions between agents in this model. The model can show coexistence of multiple cultural areas but it turned out that coexistence is unstable against spontaneous flipping of the opinions 19, and several modifications of the model has been and is studied to realize stable coexistence of cultures 8.

On the contrary, in our model cultural area is defined as the area that shares the same information source. The random appearance of the new cultural center, which can be viewed as a spontaneous change of the opinion set in Axelrod model, is actually the important feature to keep the multiple cultural centers against one culture taking over the whole system. The key feature of our model to make this possible is the importance of newer information, which give some chance for newcomer to win against existing cultural centers. It can be interesting to add a similar feature to the Axelrod model, i.e., give some rate to renew opinions and value newer information more to verify whether multiple cultures can coexist in that case.
Acknowledgments

The authors acknowledge the Danish National Research Foundation for financial support. Computer simulations have been performed at the Academic Computer Center, Cyfronet AGH (Kraków, Poland) and CMOL Niels Bohr Institute.

Appendix A: 1D random walk model of the boundary between two cultural centers

Here we derive the simplified one dimensional model of the random walk of the boundary between two cultural centers. As summarized in subsection III 4, two cultural centers $C_0$ and $C_1$ are located at the site 0 and at the site $R$, respectively. We analyze the motion of the left most site $r$ that belongs to $C_0$ (the site $r+1$ belongs to $C_1$).

The rate the position $r$ decreases (increases) by one, $W_{r\rightarrow r-1}$ ($W_{r\rightarrow r+1}$), is given by the probability that $a_r > a_{r+1}$ ($a_r > a_{r+1}$) when the site $r$ ($r+1$) listens to the preferred direction $r+1$ ($r$) at a given time step. From [1] and [2] we get (note that $0 < r < R$)

$$W_{r\rightarrow r-1} = \begin{cases} \frac{(1-\ repeat\)}{2\ repeat} & \text{for } r \leq \frac{R-p}{2}, \\ \frac{1-(1-\ repeat)}{2\ repeat} & \text{for } r > \frac{R-p}{2}, \end{cases}$$

and

$$W_{r\rightarrow r+1} = \begin{cases} \frac{1-(1-\ repeat)}{2\ repeat} & \text{for } r \leq \frac{R-p}{2}, \\ \frac{(1-\ repeat)}{2\ repeat} & \text{for } r > \frac{R-p}{2}. \end{cases}$$

Using above transition rates one can write the master equation for the probability density $P_{r,t}$ that the boundary is at site $r$ at time $t$:

$$P_{r,t+1} - P_{r,t} = W_{r-1\rightarrow r} P_{r-1,t} - W_{r\rightarrow r+1} P_{r,t} + W_{r+1\rightarrow r} P_{r+1,t} - W_{r\rightarrow r-1} P_{r,t}.$$ 

Assuming that the time step and the lattice spacing are small, we obtain the following Fokker-Planck equation [20]

$$\frac{\partial P(r,t)}{\partial t} = \frac{\partial}{\partial r} \left[ (W(r \rightarrow r-1) - W(r \rightarrow r+1))P(r,t) \right] + \frac{1}{2} \frac{\partial^2}{\partial r^2} \left[ (W(r \rightarrow r-1) + W(r \rightarrow r+1))P(r,t) \right].$$

Substituting $r$ with $x = r - (R-1)/2$ one gets Eqs. [3] to [7].

The mean first passage time $T$ starting from $x = 0$ to reach $x = \pm R/2$ is given by the closed formula [20]

$$T(p, \ repeat, R) = -\int_{-R/2}^{R/2} ds \left\{ e^{\Phi(s)} \int_0^s e^{-\Phi(y)} D(y) dy \right\},$$

where

$$\Phi(x) = \int_0^x \frac{pU(y)}{D(y)} dy.$$ 

Approximating $D(x)$ with $p/2$, see Eq. [6], one gets

$$T(p, \ repeat, R) \approx \frac{2}{p} \int_{-R/2}^{R/2} ds \left\{ e^{2U(s)} \int_0^s e^{-2U(y)} dy \right\},$$

from which Eq. [8] can be evaluated using Wolfram Mathematica.

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