Monte Carlo simulations of radio pulses in atmospheric showers using ZHAireS

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Abstract

We present predictions for the radio pulses emitted by extensive air showers using ZHAireS, an AIRES-based Monte Carlo code that takes into account the full complexity of ultra-high energy cosmic-ray induced shower development in the atmosphere, and allows the calculation of the electric field in both the time and frequency domains. We do not presuppose any emission mechanism and our results are compatible with a superposition of geomagnetic and charge excess radio emission effects. We investigate the polarization of the electric field as well as the effects of the refractive index $n$ and shower geometry on the radio pulses. We show that geometry, coupled to the relativistic effects that appear when using a realistic refractive index $n > 1$, play a prominent role on the radio emission of air showers.

Key words: high energy cosmic rays and neutrinos, high energy showers, Cherenkov radio emission

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1 Introduction

In the last decades, the study of ultra high energy cosmic rays (UHECR) has been one of the most active areas in astroparticle physics [1]. The detection of extensive air showers (EAS) created by UHECR in the atmosphere has been accomplished mainly with two detection methods. The first consists on detecting the particles of the cascade reaching the ground using an array of particle detectors. The second method uses telescopes to detect the fluorescence photons emitted by the particles in the shower as they travel through the atmosphere. Only the latter method is able to directly measure the EAS longitudinal development in the atmosphere, but is subject to a very low duty
cycle (\(~ 10\%\)) since it can only be used in clear moonless nights. The surface array and fluorescence techniques are used simultaneously at the Pierre Auger cosmic ray observatory [2].

Radio emission from EAS was first observed by Jelley et al. [3] in 1964. Theoretical and experimental research in this area was very active in the sixties and early seventies. We refer the reader to a review by Allan [4] and references therein. Nevertheless, interest in this technique declined in the late seventies, mainly due to radio interference [5]. Recent developments in high speed electronics and information technology have renewed interest in the detection of radio emission in the MHz range from the particles in EAS. The radio detection technique is, in principle, sensitive to the longitudinal development of the shower, like the Fluorescence technique, but its duty cycle is much higher, since it could be used anytime except during thunderstorms.

This resurgence of the radio technique has taken form as new experiments have been developed, such as CODALEMA [6], LOPES [7,8] and AERA [9], accompanied by new calculations of the radio emission in EAS, which include analytical techniques with different levels of sophistication [10,11,12,13], Monte Carlo methods [14,15,16,17,18,19] and semi-analytical methods [20].

In this work we present ZHAireS [21,19] (ZHS+AIRES), a new simulation of radio emission in EAS that combines the full shower simulation capabilities of AIRES [22] with specific algorithms developed to calculate the electric field emitted by particles in dense media showers, implemented in the well tested ZHS code [23,24]. These algorithms are obtained from first principles, so no emission mechanism or model is presupposed. The radio emission calculations are done in parallel to the AIRES shower simulation: As each charged particle in the shower is propagated by AIRES in steps, each propagation step is taken as a single particle track and its contribution to the radio emission is calculated and added to the total electric field in both the time and frequency domains. This procedure naturally accounts for interference effects associated to the different space-time positions of the particles in the shower.

Other Monte Carlo simulations of radio emission of air showers exist, such as REAS [16,17], where the electron and positron tracks from CORSIKA [25] simulations are first histogrammed and then used to generate random \(e^\pm\) trajectories, which in turn are used for the radio emission calculations in the time domain only. An earlier version of this code, REAS2 [16], was based only on geosynchrotron emission, but was shown to be inconsistent with later simulations [17,19], which in turn are based on a pretty generic algorithm [4] that has been used for a long time to simulate pulses generated in dense media [23,24]. Another difference is that ZHAireS uses a model for the variation of the refractive index with altitude, while REAS uses a fixed refractive index equal to unity in its calculations.
This paper is organized as follows: In section 2 we give a short overview of the main radio emission mechanisms in air showers and the specific formalism used for emission calculations in ZHAireS, including a model for the variation of the refractive index with altitude. In section 3 we develop a one-dimensional toy model useful for understanding the main characteristics of the emission and stress the prominent role played by geometry. In section 4 we show results of ZHAireS simulations, discussing the influence of the distance from the antenna to the shower core, the refractive index and the shower zenith angle on the electric field pulse. We also discuss the spectrum of the radio emission. In section 5 we analyze the polarization of the electric field and discuss how it can be used to separate the contributions from various emission mechanisms. Finally, in section 6 we conclude the paper.

2 Radio Emission in Air Showers

2.1 Main emission mechanisms

In air showers, the dominant mechanism responsible for the radio emission is believed to be the deflection by the geomagnetic field of electrons and positrons in the shower [26,27]. Several approaches have been developed to model this emission. Since the direction of the Lorentz force depends on charge, it leads to a spatial separation of electrons and positrons in the shower, that can be thought of as a moving macroscopic dipole and a transverse current traveling through the atmosphere at a speed \( v \approx c \) along with the shower front, and which has been the basis for macroscopic calculations [28,11,20]. Another approach, which is adopted in this work, is to calculate the emission for each particle trajectory, i.e. a microscopic approach [23,24,21,19,17].

The geomagnetic trajectory has a quite clear signature. Its polarization is anti-parallel to the direction of the Lorentz force, i.e. in the direction of \(-\vec{\beta} \times \vec{B}\), where \(\vec{\beta}\) is the speed of the particle in \(c\) units and \(\vec{B}\) the geomagnetic field [28,29,30,31].

Another emission mechanism also thought to be important in EAS is the Askaryan effect. It was first proposed by Askaryan [34], who suggested that coherent radiation could be emitted by showers in which a charge excess develops. This mechanism, which dominates the radio emission of showers in dense media, is also known as the charge excess mechanism. For a particle following a straight track at constant speed, it can be shown [35] that:

\[
\vec{E}_{\text{rad}} \propto -e [\hat{u} \times (\hat{u} \times \vec{\beta})]
\] (1)
So the electric field emitted by particles traveling along the shower axis lies in the plane defined by the direction of the shower axis ($\vec{\beta}$) and the observation direction $\hat{u}$. Furthermore, it is perpendicular to $\hat{u}$ and its direction depends on the charge of the particle, pointing towards (away from) the shower axis for negative (positive) charges [23,24]. This means that if there is no charge excess in the shower, the net electric field is zero. However it is well known that knock-on interactions and Compton scattering incorporate electrons from molecules of the medium into the shower, leading to an excess of moving negative charges [34], which in turn are responsible for a net electric field with a radial polarization w.r.t. the shower core.

2.2 Radio emission calculation in the ZHAireS code

The algorithms used for the calculation of radio emission in ZHAireS are based on ZHS algorithms [23,24]. These were derived from first principles, namely from the Lienard-Wiechert potentials, and thus do not assume any emission mechanism. The full derivation can be seen in [23] for frequency domain calculations, and [24] for the time domain. The trajectories of shower particles are divided in tracks, which can be made arbitrarily small. Both the speed of the particle and the direction to the observer are assumed not to vary over the track. In Fig. 1 we show a schematic picture of such a track. A convenient division of trajectories in small tracks is already performed by Monte Carlo simulations of shower development in order to propagate the particles in the shower. The ZHS algorithm is then used to calculate the contribution of each track to the net emission of the shower, accounting for any interference between tracks. This approach automatically takes into account the contribution to the electric field due to the start, end, and any change in the direction and energy of each particle track. Thus any kind of deflection, scattering, creation or annihilation of a charged particle, due to any physical process used to simulate the shower is taken into account in the radio emission [19,21,37].

In the original ZHS algorithms [23,24], the field calculation is performed in the Fraunhofer approximation. In this case the observer “sees” the radiation
arriving from all the points in the shower at the same angle, and only changes in the phase of the contribution from each point are taken into account through a simple projection. This approach works well for showers in dense media, such as ice, since the size of the shower is much smaller than the typical distance to the antennas. But in the case of air showers, the distance to the observer is usually of the order of the shower size, and thus the Fraunhofer approximation breaks down. In order to make the method valid for the closer observers in air showers, we allow the distance and direction to the observer to change from one track to the next. The direction and distance to the antenna are still taken to be constant inside a single track (see Fig. 1), i.e. we still use the Fraunhofer approximation inside each track. It can be shown [36] that this procedure reproduces the expected behavior $E \propto 1/\sqrt{R}$ of the field [38], where $R$ is the distance between the charge and the observer. Since we still use the Fraunhofer approximation inside a single track of length $L$, the following condition has to be satisfied:

$$\frac{L^2 \sin^2 \theta}{R} < \frac{\lambda}{2\pi}$$

where $\lambda$ is the wavelength of the emission. In the simulation, the vast majority of tracks satisfy this condition for frequencies up to 300 MHz, which is much higher than the frequency of the maximum of the emission, $\sim 1 - 30$ MHz.

If a particular track in the simulation does not satisfy this condition, e.g. a track passes very close to an antenna, it is further divided into several sub-tracks, and the field calculation is performed using a different $R$ and $\theta$ for each sub-track.

The positions $\vec{x}_{1,2}$, times $t_{1,2}$ and kinetic energies $E_{1,2}$ for the beginning and end points of the track are obtained directly from AIRES. This, along with the position $\vec{x}_{ant}$ of the antenna is all that is needed for the calculation of the contribution to the vector potential $\vec{A}(t, \hat{u})$ due to a given track:

$$\vec{A}(t, \hat{u}) = \frac{\mu e}{4\pi Rc} \beta_{\perp} \frac{\Theta(t - t_{1,2}^{det}) - \Theta(t - t_{2}^{det})}{1 - n\beta \cdot \hat{u}}$$

where $\vec{u} = R\hat{u}$ is the vector from the middle point of the track to the antenna, $\beta = \hat{v}/c$, $\beta_{\perp} = -[\hat{u} \times (\hat{u} \times \beta)]$ is the projection of $\beta$ onto a plane perpendicular to $\hat{u}$, $t_{1,2}^{det} = t_{1,2} + nR/c - n\beta \cdot \hat{u}$ $(t_{1,2} - t_0)$ are the retarded (detection) times for the beginning and end of the track, respectively, $t_0 = (t_1 + t_2)/2$ is the average time for the track and $\Theta(x)$ is the Heaviside step function. Note that Eq. (3) is written in the radiation gauge, since we disregard the static term of the Lienard-Wiechert potentials in its derivation [24].

1 In the formalism, as described in [24], the limit of eq. (3) for $(1 - n\beta \cdot \hat{u}) \to 0$ is used instead of eq. (3) if the denominator vanishes.
To obtain the radio signal at each antenna for the shower as a whole, the contributions of each particle track to the vector potential $\vec{A}(t)$ are added up. This net vector potential is then differentiated with respect to time to obtain the net electric field as a function of time for each antenna. In the far field, this formalism is equivalent to the one used in [17], as shown in [37].

Besides the field calculations in the time domain, ZHAireS can calculate the radio emission in the frequency domain as well. The radiation term of the electric field is $\vec{E}(t) = -\partial \vec{A}/\partial t$. Applying this to the vector potential expression (eq. 3) we obtain [24]:

$$\vec{E}(t, \hat{u}) = -\frac{\mu e}{4\pi Rc} \vec{\beta}_\perp \delta(t - t_{1}^{\text{det}}) - \delta(t - t_{2}^{\text{det}}) \frac{1}{1 - n\vec{\beta} \cdot \hat{u}}$$

(4)

In the ZHS formalism [23,24], we use the following convention for the Fourier transform:

$$\tilde{f}(\omega) = 2 \int_{-\infty}^{\infty} f(t) e^{i\omega t} dt$$

(5)

Applying this Fourier transform convention to Eq. (4) we obtain [23]:

$$\vec{E}(\omega, \hat{u}) = -\frac{\mu e}{2\pi Rc} \vec{\beta}_\perp e^{i\omega(t - t_{1}^{\text{det}})} - e^{i\omega(t - t_{2}^{\text{det}})} \frac{1}{1 - n\vec{\beta} \cdot \hat{u}}$$

(6)

which is the expression we use for the Fresnel regime frequency-domain calculations in ZHAireS. The contributions of each track to $\vec{E}(\omega)$ are added up to obtain the net spectrum at each antenna.

2.3 Variable refractive index

As stated before, in ZHAireS we use a model for the variation of the refractive index $n$ with altitude in the atmosphere. In fact there is a strong dependence of $n$ on temperature, pressure and humidity. To take the dependence with altitude into account we use an exponential model for the variation of the refractivity $\mathcal{R}$ with height $h$, motivated by the exponential decrease of density with altitude,

$$\mathcal{R}(h) = \mathcal{R}_s \exp(-K_r h)$$

(7)

where $\mathcal{R}(h) = [n(h) - 1] \times 10^6$ is the refractivity at an altitude $h$ in km and we used $\mathcal{R}_s = \mathcal{R}(h = 0) = 325$ and $K_r = 0.1218 \text{ km}^{-1}$. These values reproduce the refractivity calculated in [39] up to $h \sim 10$ km within less than $\sim 1\%$. The values in [39] take into account the humidity dependence of the refractivity,

Note that interference effects are taken into account automatically, since the contribution of each track to the vector potential is related to a specific retarded time.
which increases the refractivity at low altitudes where the effect of a variable index of refraction is most important, since the shower has a larger number of particles. At higher altitudes, the exponential model slightly overestimates the refractivity, but the effect of the variable index of refraction at these altitudes is much less important, since above 20 km the shower has barely started developing and the number of particles is small. It is also worth noting that since $R$ depends on temperature and humidity, there are large seasonal and even daily variations in the refractivity that are associated to atmospheric conditions and cannot thus be accurately described by any model\footnote{If the humidity as a function of height can be monitored using e.g. a LIDAR, then more accurate values of the refractivity for specific atmospheric conditions could be calculated by adding a humidity term to Eq. (7), as described in \cite{39}.}.

The variable atmospheric refractive index would in principle make the radio emission follow a curved path. However, in \cite{20} it was shown that the effect of the deviation from a straight path on the time structure of the pulse is negligible, and thus in ZHAireS we assume that the radio emission follows a straight path from the emission point to the antenna. However, we explicitly take into account the effect of a variable refractive index in the propagation time of the signal. For this purpose we calculate an effective refractive index $n_{\text{eff}}$ for each particle track in the ZHAireS shower simulation:

\[
n_{\text{eff}} = 1 + R_{\text{eff}} \times 10^{-6}, \quad R_{\text{eff}} = \frac{1}{R} \int_0^R R(h) \, dl
\]

where $R$ is the distance from the track to the observer, and $dl$ is an infinitesimal length along that path whose altitude $h$ varies along the path. It is important to note that while $n_{\text{eff}}$ is used for the calculation of the retarded times, $n(h)$ is used for the angular dependence of the emission, given by the denominator of Eqs. (3) and (6), i.e. the Cherenkov angle depends only on the refractive index at the emission altitude.

We expect the effect of the variable refractive index to be more important at relatively low altitudes, where $n$ is larger. Since $n$ in our model varies between $n \sim 1$ at high altitudes and $n \sim 1.000325$ close to ground, we expect the effect of the variable $n$ on the pulse characteristics discussed in section\footnote{If the humidity as a function of height can be monitored using e.g. a LIDAR, then more accurate values of the refractivity for specific atmospheric conditions could be calculated by adding a humidity term to Eq. (7), as described in \cite{39}.} (such as start-time, duration and compression in time) to be between the effects obtained for $n = 1$ and $n = 1.0003$. This can be clearly seen in Figs. 3 and 5 below.
Fig. 2. Sketch of the one-dimensional model. The shower development is represented by a 1D line neglecting the lateral structure. The shower front is assumed to travel at a speed \( c \) and reach ground at a time \( t' = 0 \). Radio emission from a height \( h \) travels along a distance \( R \) before arriving at the observer located at a distance \( r \) to the shower axis.

### 3 One-dimensional toy model

We can gain much understanding of the main features of the radio pulses in the time-domain with the aid of a very simple model. We show in this section that many of the characteristics of the pulse such as start-time, peak value of the electric field and duration in time, are mainly determined by the geometry of the system formed by the shower and the observer. In the model we assume a vertical atmospheric shower in which particles propagate along the shower axis at the speed of light \( c \), and we ignore their lateral spread. For the moment we assume a constant index of refraction \( n \). We do not make any assumption on the radio emission mechanism, and we just account for the retarded time, i.e. the radio signal is emitted at a time \( t' \) and reaches the observer located at a distance \( r \) to the shower core at a later time \( t = t' + \Delta t_p \), where \( \Delta t_p \) is given by the electromagnetic wave travel time between the emission point and the observer. The model can also be applied to understand the radio emission properties in dense media \[40\].

In Fig. 2 we show a sketch of the model. The origin of time \( t' = 0 \) is arbitrarily fixed at the time at which the shower front reaches ground. The emission time at a height \( h \) above the ground is \( t' = -h/c \), the propagation time to an antenna on the ground at a distance \( r \) from the shower core is \( nR/c \), where \( R = \sqrt{h^2 + r^2} \), and hence the arrival time of the radio emission at the antenna is:
The 1D model although simple is useful to understand at least in a qualitative way the prominent role played by the geometry in the behavior of the electric pulse signal.

3.1 Start-time of the electric field pulse

The start-time time of the electric field pulse is given by the minimum value of $t$. By doing $\partial t/\partial h = 0$, one can find the height of the shower seen first by the observer:

$$h_{\text{start}} = \frac{r}{\sqrt{n^2 - 1}}$$  \hspace{1cm} (10)

Substituting $h_{\text{start}}$ in Eq. (9), we obtain the time at which the observer sees the onset of the radio pulse:

$$t_{\text{start}} = \frac{r}{c} \sqrt{n^2 - 1}$$  \hspace{1cm} (11)

For $n > 1$, $t_{\text{start}}$ is linear in $r$ and the observer sees an increasing delay in the start time of the pulse as the distance to the shower core increases.

In Fig. 3 we plot the relation given in Eq. (9) between the observer time $t$ and the time at source $t'$ or equivalently the shower depth (see also [20]). The relation is shown at different distances $r$ to the shower core and assuming different values of $n$ including the more realistic $n$ varying with height (see section 2.3). One can see the increase of $t_{\text{start}}$ with $r$ when $n > 1$ as predicted by Eq. (11). Also, the angle $\theta$ between the observer and the shower axis (see Fig. 2) is given by $\tan \theta = r/h$. When $h = h_{\text{start}}$ as given in Eq. (10) it is straightforward to show that:

$$\tan \theta_{\text{start}} = \sqrt{n^2 - 1} = \tan \theta_C$$  \hspace{1cm} (12)

and the observer sees first a shower height $h_{\text{start}}$ at time $t_{\text{start}}$ with an angle equal to the Cherenkov angle. A similar analysis with similar results to the ones described above was developed independently and in parallel with ours by de Vries, Scholten and Werner [41], and in dense media in [42].

Due to relativistic effects associated to the speed of the shower (assumed to be $c$) being larger than the speed of the radio waves ($c/n$), at a fixed detection
Fig. 3. Relation in Eq. (9) between the observer time $t$ and the time at the source $t'$ or equivalently as depicted in the plot the depth of shower development. The relation is valid for vertical showers and is shown for observers at different distances $r$ to the shower core and for different values of the index of refraction $n$. Also shown is the longitudinal profile of a 100 PeV proton shower (from a Gaisser-Hillas function) in arbitrary units, which gives an idea of the relative number of particles at different depths.
time \( t_d \) an observer at a distance \( r \) from the shower core may see two different stages of shower development (two different heights \( h_\pm \)) simultaneously. This can be seen in Fig. 3. Solving the quadratic Eq. (9) for \( h \) with \( t_d \) constant we obtain:

\[
h_\pm = \frac{ct_d \pm nc\sqrt{t_d^2 - r^2(n^2 - 1)/c^2}}{(n^2 - 1)}
\]  

(13)

Two different real solutions exist if \( n > 1 \) and the argument of the square root is positive, which is equivalent to the condition \( t_d > t_{\text{start}} \). This apparent violation of causality is simply a relativistic effect due to the particles traveling faster that the speed of the pulse in the medium. When \( n = 1 \) the shower is seen by any antenna away from the core in a “causal” way, from beginning to end, as can be seen in Fig. 3, provided the observer is not located on the shower axis (when \( n = 1 \) and \( r = 0 \) the whole shower is seen at the same instant of time).

When \( r \) is sufficiently large or \( n = 1 \) (see Eq. (10)), \( h_{\text{start}} \) can become larger than the actual height at which the shower starts developing, namely the height \( h_0 \) where the first interaction occurs. The emission from heights above the first interaction point is due to the primary particle only and can be neglected. In that case, the observer would start to see the onset of the pulse at a time corresponding to the height \( h_0 \):

\[
t_0 = \frac{n\sqrt{h_0^2 + r^2} - h_0}{c}
\]  

(14)

The height \( h_0 \) can be considered a physical limit on \( h_{\text{start}} \), which corresponds to the depth \( X_0 \) of the first interaction. The corresponding start times \( t_0 \) can be read from Fig. 3 by truncating the curves in Fig. 3 at the minimum depth \( X_0 \). So, for any given \( n \), there is a critical distance \( r_{\text{crit}} \), at which \( h_{\text{start}} = h_0 \):

\[
r_{\text{crit}} = h_0\sqrt{n^2 - 1}
\]  

(15)

For \( n > 1 \), as we move away from the shower axis the start time first increases linearly with \( r \) (following Eq. (11)) until \( r = r_{\text{crit}} \). For larger \( r \) it follows the non-linear behavior of Eq. (14). For \( n = 1 \), the effective \( t_{\text{start}} \) is always given by \( t_0 \) in Eq. (14), regardless of the distance \( r \) to the core.

Regarding the relativistic effects described above, when \( n = 1 \), \( r_{\text{crit}} = 0 \) and the shower is seen in a causal way by any observer, as stated before. For \( n > 1 \), observers at distances greater than \( r_{\text{crit}} \) will also see the shower in a
“causal” way, starting at $h_0$, and in this model they never see the shower at the Cherenkov angle.

3.2 Peak value of electric field pulse

The peak value of the radio pulse is mainly determined by three factors, namely, the distance from the shower to the observer, the number of charged particles in the shower, and also and in a very important way by geometrical effects associated to a time compression factor. The relation between the observer time and the shower height, given by Eq. (9) and shown in Fig. 3, is non linear. One can consider a given observer time interval and obtain from this relation the interval of the shower development that contributes to it. Clearly, the portion of shower development that contributes will be large when the slope of this relation is small. We can define a compression factor $f_c$ taking the derivative of Eq. (9) with respect to $h$:

$$f_c = \left| \frac{\partial t}{\partial h} \right| = \frac{1}{c} \left( 1 - \frac{nh}{\sqrt{r^2 + h^2}} \right) \quad \text{[ns m}^{-1}]$$

The absolute value accounts for the fact that the derivative of $t$ with respect to $h$ changes sign when $h = h_{\text{start}}$ (i.e. $t = t_{\text{start}}$), corresponding to a reversal in the time sequence of the shower as seen by the observer. The inverse of the compression (or Doppler) factor $f_c$ can be thought of as a measure of the compression in time [32]. Qualitatively, a small value of $f_c$ implies that the emission from a relatively large portion of the shower contributes to the pulse in a relatively small interval of observer’s time $\delta t$, increasing the pulse with respect to other cases in which the factor $f_c$ is larger. Moreover, when $h \to h_{\text{start}}$ (i.e. $t \to t_{\text{start}}$) then $f_c \to 0$ and $\delta t \to 0$. This is related to the fact that the observer sees the shower at $t = t_{\text{start}}$ with an angle equal to the Cherenkov angle, as shown in Eq. (12). In fact, since $\cos \theta = h/R$ it is straightforward to show that the factor in Eq. (16) is proportional to $|1 - n \beta \cos \theta|$. As a consequence of $f_c \to 0$, the factor $(1 - n \beta \cos \theta)$ in the denominator of Eq. (3) goes to zero, tending to enhance the peak value of the pulse, seen at the Cherenkov angle.

It is important to note that although a small $f_c$ induces a very big effect in

\[ 4 \] This singularity is not a problem for our numerical calculations for several reasons: For the singularity to actually enter the numerical calculation, $f_c$ would have to vanish (within the precision of our code) at precisely the middle of the track (see Fig. 1). This (almost) never happens, but if it does, the code uses the limit of Eq. (3) for $f_c \to 0$ [24]. Also note that just like in reality (and Monte Carlo too) there is a finite time resolution (we use $\delta t = 0.5$ ns in this work), which spreads very high (and short) vector potential peaks over the whole time bin.
the pulse in the 1D model where all particles follow the shower axis, in a real shower this effect will be less important due to the lateral and angular spread of the particles in the shower. The lateral and angular spread will change the observer angle and increase the compression factor at $h_{\text{start}}$ with respect to what the 1D model predicts ($f_c \sim 0$), since $f_c \propto -\cos \theta$. Furthermore, the lateral spread of the particles in a real shower will also change the distance to the observer, smearing the arrival time of the signal.

As stated before, the peak value of the pulse is also determined by other factors besides $f_c$. There is a non-trivial interplay between the length and height of the part of the shower seen with a small compression factor, the number of particles in that region of the shower development, and the distance $R$ to the observer. A clear illustration of this interplay can be seen in Fig. 4, where we plot $f_c$ as a function of $h$ along with a Gaisser-Hillas parameterization of shower development for a $10^{17}$ eV shower. When $n = 1.0003$ (left panel) the largest compression in time applies to the region around shower maximum for an observer at $r = 100$ m, while for an observer at $r = 50$ m the largest compression applies to a portion of the shower with fewer particles, below shower maximum.

One could think that the peak of the pulse will always drop as $r$ increases and the observer gets further away from shower axis, but the factor $f_c$ can slow down this trend (e.g. between $r = 50$ m and $r = 100$ m in Fig. 4) or even reverse it. Moreover, as $r$ increases, the width of the peak in $f_c$ becomes larger (this can be appreciated by inspection of the widths at values of $f_c = 10^{-4} \text{ ns/m}$ in Fig. 4) and this implies that a larger portion of the shower contributes quasi-simultaneously to the observed emission. On the other hand, as $r$ increases so does the distance from the emission point to the antenna, tending to decrease the signal. One can see that even in this simple 1D model the interplay of the various relevant variables is already very complicated and somewhat counter-intuitive. For observers at larger distances to the core (e.g. $r = 400$ m in the bottom panel of Fig. 3), the largest compression is achieved only at very high altitudes where the number of particles in the shower is much smaller than at shower maximum, and thus the peak value of the pulse is much smaller than that seen by an observer at $r = 50$ m (bottom panel of Fig. 3 and Fig. 4).

### 3.3 Time duration of the electric field pulse

The 1D model also allows to understand the time duration of the electric pulse. The shower arrives at ground at $t_g' = 0$ and the signal from the shower when it reaches the ground arrives at the observer at a time $t_g = nr/c$. Assuming that the emission is only due to the shower front, the time duration $\Delta t$ of the pulse, as seen by the observer, is then given by:
Fig. 4. Left Panel: Compression factor $f_c$ as defined in Eq. (16) for $n = 1.0003$ (i.e. the absolute value of the derivative of the curves labeled $n = 1.0003$ depicted in Fig. 3) as a function of altitude $h$ for observers at different distances $r$ to the shower axis. Also shown is the number of charged particles in a $10^{17}$ eV proton shower as given by a Gaisser-Hillas (GH) parameterization of the longitudinal shower development (right axis). Right Panel: Same as left, but for $n = n(h)$.

$$\Delta t = t_g - t_{\text{start}} = \frac{r}{c} \left( n - \sqrt{n^2 - 1} \right)$$

(17)

where $t_{\text{start}}$ is given by Eq. (11). This equation is only valid for $r$ greater than a few meters\(^5\) and $r < r_{\text{crit}}$ (see Eq. (15)). If $r > r_{\text{crit}}$, $t_{\text{start}}$ should be replaced by $t_0$, as defined in Eq. (14), but this does not change significantly the behavior of $\Delta t$, which stays approximately linear with $r$. So the observers see an increasingly wider pulse in time the farther they are from the shower core. Another estimate for the variation of the pulse width with distance can be found in [33].

This approach is an oversimplification as it assumes that the shower still contributes to the pulse as it reaches ground level, so it needs to be modified for very inclined showers. In addition it also neglects the delays of the particles that lag behind the shower front and also contribute to the pulse, making it wider. Still the simple expressions obtained can be quite useful to interpret the results from the full simulation, shown in the next section.

3.4 Dependence on refractive index

The 1D model also allows to study the dependence of the pulse properties on the index of refraction. For the purpose of understanding the relevance of $n$ on the pulse, we first assume $n$ to be constant with height. Using Eq. (11) it is straightforward to deduce that the start-time of the pulse increases with $n$ because the propagation time of the signal from the height of emission to the observer increases with $n$. Furthermore, the time duration of the pulse

\(^5\) For very small distances $r < L_0(n^2 - 1)/2n$, then $t_0 > t_g$ and $\Delta t = t_0 - t_{\text{start}}$. 

14
also depends on $n$. As can be seen in Eq. (17), the time duration of the pulse decreases slightly with increasing $n$.

Another dependence of the pulse properties on $n$ arises because the factor $f_c$ is also dependent on $n$. An observer at a fixed given distance $r$ from the core will see different parts of the same shower with a small $f_c$ (i.e. with a large compression in time) for different values of $n$, since $h_{\text{start}}$ (which defines the altitude at which $f_c = 0$ and the shower is seen with $\theta = \theta_C$) decreases with $n$. To illustrate this we show in Fig. 5 the factor $f_c$ as a function of $h$ for an observer at $r = 100$ m for several values of $n$. One can see that as $n$ decreases the height at which $f_c = 0$ (i.e. $h_{\text{start}}$) increases, leading to different parts of the shower being largely compressed in time. The effect on pulse height will depend on the number of particles at $h_{\text{start}}$, which in turn depends on the distance $r$ to the observer. For $r < 100$ m (not shown in Fig. 5), the highly compressed part of the shower for $n = 1.0003$ is below shower maximum. As $n$ decreases, the compressed part of the shower moves towards the maximum, tending to increase the pulse. On the other hand, for $r > 100$ m, the height $h_{\text{start}}$ is above the maximum, and a decrease in $n$ will move the compressed part of the shower further away from the maximum, tending to decrease the pulse height. For large values of $r$, $h_{\text{start}}$ is in a region with very few particles, decreasing the effect of a low $f_c$ in pulse height. Furthermore, in this part of the shower the number of particles increases only very slowly with decreasing $h$, since the density at these altitudes is low, and the effects of changes in $n$ on pulse height become less important.

As discussed before, relativistic effects such as seeing the later parts of the shower before the earlier ones and seeing two parts of the shower simultaneously can only be observed at distances $r < r_{\text{crit}}$ from the core. Since $r_{\text{crit}}$ increases with $n$, observers further from the core will also see these effects as $n$ increases.

### 4 ZHAireS simulations

In this section we present the electric field as obtained in ZHAireS simulations of atmospheric showers. In particular in this section we will obtain the behavior of different features of the time pulse with $r$ and refractive index, but this time in realistic simulations that take into account the full complexity of atmospheric showers. We will see that the behavior follows the qualitative behavior obtained with the 1D model. At the end of this section we also show some results of the electric field calculated in the frequency domain and compare them with Fourier transforms of the time pulses.
Fig. 5. Same as Fig. 4, but for an observer at $r = 100$ m and several values of $n$, including a model for the variation of $n$ with altitude $n(h)$ (see Section 2.3).

4.1 Dependence of the electric field pulse on distance to shower axis

In Fig. 6 we show the East-West (EW) electric field component as a function of time obtained from simulations of $10^{17}$ eV vertical atmospheric showers induced by protons. We used a horizontal magnetic field $|\vec{B}| = 23 \mu T$ pointing north for this particular simulation. The electric field is shown at different distances $r$ northwards from the shower core. As predicted by the 1D model in Eqs. (11) and (17), it can be seen that the start-time of the field and the duration in time (at least the easily visible positive part of the pulse) both increase with $r$ for the observers shown in the plot.

The peak of the electric field decreases with $r$ for an observer North of the shower core, but not in a linear manner. In fact it decreases roughly as $r$ from 50 m to 100 m, but faster than $r$ from 100 m to 150 m, as can be seen in Fig. 6. As explained above, there is a non-trivial interplay between the distance from the shower to the observer, the factor $f_c$ and the number of particles in the region of the shower seen with a large compression in time, illustrated in the right panel of Fig. 4. The number of particles at $h_{\text{start}}$ increases only slightly from $r = 50$ m to $r = 100$ m, but decreases by an order of magnitude between 50 m and 150 m. Since the distance $R$ from emission point to observer increases roughly as $R \sim r$, it is twice (three times) as big at 100 m (150 m) than at 50 m. This explains why the height of the peak at 100 m is about half the height at 50 m, while it is much smaller at 150 m. We have also observed in
Fig. 6. EW component of the electric field at \( r = 50, 100, 150 \) and 400 m northwards of the shower core as obtained in ZHAireS simulations of a \( 10^{17} \) eV proton-induced vertical atmospheric shower. These simulations were performed with the exponential model for the variation with altitude of the refractive index. Note that the signal at \( r = 400 \) m is arbitrarily multiplied by a factor 20 for plotting purposes.

Our simulations that the relative height of the pulse at different distances also depends on the direction of the observer w.r.t. the shower core. In Fig. 7 we show the emission from a similar shower as in Fig. 6 but for observers East of the core. For an observer East of the shower core, the peak amplitude appears to be maximal at around \( r = 100 \) m, a result in principle compatible with \[41\].

Also, the decrease in pulse height with \( r \) seems to be slower in Fig. 7 when compared to the observer North of the core (Fig. 6). A similar dependence on antenna position can also be seen for the spectra at 1 MHz, shown in Fig. 13. This difference is, in part, due to the interference of the geomagnetic and Askaryan components of the emission, as will be discussed in section 5.

### 4.2 Dependence of the electric field pulse on the refractive index

In Fig. 8 we show the EW component of the electric field as a function of time obtained from simulations of \( 10^{17} \) eV vertical atmospheric showers induced by protons, using a magnetic field \(|\vec{B}| = 23 \mu T\) with an inclination of \(-37°\) and a declination of \(0°\). The calculations were performed with two constant refractive indices, namely \( n = 1.0 \) and \( n = 1.0003 \), as well as with a refractive index varying with altitude according to Eq. (7).
Firstly, the larger the refractive index the later the pulse starts as obtained with the 1D model above (see Eq. 11). The larger the $n$ the smaller the propagation speed, and so the signal reaches the observer at later times. For the refractive index changing with altitude, which varies between $n = 1.0$ and $n = 1.0003$, the start-time falls between the two obtained for the two constant refractive indices, as expected.

For observers close to the shower axis ($r = 100$ m in the left panel of Fig. 8), small changes in $n$ are responsible for large changes in peak height and width. This can be qualitatively understood, as described in Sections 3.2 and 3.4, in terms of the non-trivial interplay between the various geometrical factors. In Fig. 4 and 5 one can see that for $r = 100$ m the number of particles in the low $f_c$ region changes only slightly from $\sim 70 \cdot 10^6$ when $n = 1.0003$ to $\sim 60 \cdot 10^6$ when $n = n(h)$, while the length of the shower seen with $f_c < 10^{-4}$ doubles. For observers further away from the core ($r = 400$ m in the right panel of Fig. 8), the effect of the varying $n$ in pulse height is much less pronounced, since $f_c \sim 0$ only above the shower, as can be seen in the bottom panel of Fig. 3. This will make the compression factor in the shower region very similar for different values of $n$, as illustrated by the fact that the curve labeled $r = 400$ m in Fig. 4 changes only very slightly from the left panel ($n = 1.0003$) to the right panel ($n = n(h)$).
4.3 Dependence of the electric field pulse on the shower zenithal angle

The features of the electric field at a fixed distance to the shower axis are expected to depend strongly on the shower zenithal angle. The main reason for this is that the shape of the curve relating the observer time and the source depth - which largely determines the characteristics of the pulses as explained above - depends strongly on the geometry of the system formed by the observer and the shower. The electric field from a certain region of the shower depends on the angle $\theta_i$ (with which the observer sees that region) through the factor $|1 - n \beta \cos \theta_i|$ in Eq. (3), on the number of particles $N_i$ in the region, and on its distance to the observer, $R_i$. For $\theta_i$ close to the Cherenkov angle, the emission is enlarged with respect to other angles, since $|1 - n \cos \theta_i|$ is small.

The interplay between $|1 - n \cos \theta_i|$ (or equivalently the factor $f_c$) and $N_i$ is shown in Fig. 9 for a vertical (zenithal angle $\theta = 0^\circ$) and an inclined shower ($\theta = 50^\circ$). The curves in Fig. 9 were obtained for an observer at a distance $r = 400$ m (in the early part of the inclined shower) and $n = n(h)$ with a modified 1D model, similar to the one developed in Section 3, but which can handle inclined showers. Fig. 9 suggests that inclined showers produce larger signals than vertical showers for distances greater than a couple of hundred meters from the core. The main reason for this is that at larger distances to the shower axis, inclined showers are viewed with angles closer to the Cherenkov angle than vertical showers, and so the angular factor $|1 - n \beta \cos \theta_i|$ is smaller, boosting the emission and the net signal from inclined showers. A similar conclusion was also reached in [32].

Using ZHAireS, we investigated the dependence of the field on zenithal angle as a function of distance to shower axis, in a full Monte Carlo simulation of the shower development in air with a realistic model for the refractive index. We compared the signals obtained from $10^{17}$ eV proton-induced showers with
Fig. 9. Geometrical angular term $|1 - n \cos \theta|_i$ as a function of altitude $h$ for a vertical shower (dashed blue line) and a shower with zenithal angle $\theta = 50^\circ$ (solid red line). Also shown are Gaisser-Hillas parameterizations of the number of particles (linear right axis) as a function of height for a 100PeV vertical shower (thick dashed blue line) and an equivalent one with $\theta = 50^\circ$ (thick solid red line). The calculations were done numerically for $n = n(h)$ and $r = 400$ m (early part of inclined shower), using an 1D model that allows non-vertical showers, similar to the one described for vertical showers.

$\theta = 0^\circ$ and $\theta = 50^\circ$. We found that the results obtained with ZHAireS are in qualitative agreement with our 1D model and with the calculations in [32]. In Fig. 10 we compare the East-West components of the electric field for the vertical and inclined showers for an antenna at $r = 100$ m (left) and $r = 800$ m (right) located in the North-East of the shower core. Clearly at large distances to the core the peak value of the electric field in the inclined shower is much larger than that of the vertical shower, a factor $\sim 5$ in this particular simulation. This trend of an increased non-vertical shower signal at large distances from the core is present in all directions with respect to the shower core. Note also the shift in the start-time of the pulse in the inclined shower w.r.t. the vertical one, due to the different geometry of both showers.

We have also found that the signal in non-vertical showers at large distances from the core is very sensitive to the decrease of the refractive index with altitude. In Fig. 11 we plot the electric field for the same showers as in Fig. 10 but instead of the model of variable refractive index with altitude, the fields shown were calculated using a constant refractive index $n = 1.000325$. It can be clearly seen in the left panel of Fig. 11 that for the antenna at $r = 100$ m North-East from the core, the use of a constant $n$ decreased both the vertical
Fig. 10. Comparison between the electric field as a function of time as obtained in ZHAireS simulations of $10^{17}$ eV proton-induced showers with zenithal angle $\theta = 0^\circ$ (blue solid) and $\theta = 50^\circ$ (red dashed), for antennas at $r = 100$ m (left) and $r = 800$ m (right) NE of the shower core on the ground. The inclined shower comes from the SE. The simulations were done using a variable refractive index model for the atmosphere.

Fig. 11. Same as Fig. 10 but using a constant refractive index $n = 1.0003$. and non-vertical EW field component w.r.t. the variable $n$, while at $r = 800$ m (right panel), the signal in the vertical shower stayed practically unchanged and the peak in the non-vertical shower more than doubled in value. Again this behavior can be traced back to the geometrical dependence of the interplay between the various key elements in the calculation of the electric field.

4.4 Radio pulses in the frequency domain

In Fig. 12 we compare the results of ZHAireS frequency domain calculations with Fast Fourier Transforms (FFT) of the calculated time domain signals for the same shower. The convention used for the normalization of the Fourier transform is in Eq. (5). One can see that the agreement between the spectra and the FFT is very good up to high frequencies, at which the width of the positive peak in the time domain becomes important, since it is the smallest
large scale structure of the time pulse. More quantitatively, above $\nu \sim 1/\Delta T$, where $\Delta T$ is the characteristic width of the peak of the pulse in time, the FFT starts fluctuating. For 0 m (400 m), the peak width is around 5 ns (50 ns), leading to large fluctuations starting at around 200 MHz (20 MHz). This high frequency part of the spectrum is largely incoherent and sensitive to shower to shower fluctuations and to the thinning level used in the simulation. These fluctuations are partly physical (incoherence level, as will be discussed below) and partly unphysical (bin size effects, thinning, FFT numerical error, etc). Also, in the direct frequency domain calculations they are slightly smaller than in the Fourier-transform of the time pulse. This means that to study the spectrum at high frequencies it is best to calculate shower emission directly in the frequency domain, reducing the unphysical contributions to the fluctuations.

The spectrum of the radio emission depends on the distance $r$ from the core on the ground to the antenna. As can be seen in figure 12, the frequency at which the Fourier components reach a maximum decreases from $\sim 30$ MHz at $r = 0$ m to $\sim 3$ MHz at 400 m. This cutoff frequency is related to the time duration of the pulse for a specific observer and serves as a boundary

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6 Similar fluctuations can also be seen in [17].
7 Note that due to computing time issues, the frequency bin used in the frequency domain simulations shown in Fig. 12 is much wider than the frequency bin used in the FFT, making the fluctuations appear much smaller than they really are.
between the fully coherent and incoherent parts of the spectrum. Above this frequency, the emission is subject to shower to shower fluctuations. At even higher frequencies, effects due to thinning in the simulation become important, since the spectrum becomes sensitive to the fine details of the shower, even down to single particles for the highest frequencies (not shown). A 2D model that explains this behavior in the Fraunhofer regime, but which is still relevant, can be found in [43].

The fact that the cutoff frequency of the emission decreases with distance to the core has implications on the measured lateral distribution of the signal. Depending on the measured frequency range, the Fourier components of the signal will drop at different rates as the distance increases. In Fig. 13 we show the average total field as a function of distance to the core at 1 MHz (left) and at 60 MHz (right), along with their RMS, calculated from simulations of 10 proton showers of 100 PeV. At 60 MHz, i.e. in the incoherent part of the spectrum, the field decreases much faster with increasing distance to the core compared to the coherent part of the spectrum at 1 MHz. This behavior can be relevant for the design of radio detector arrays. An explanation of the reason why the largest (smallest) pulses are received at antennas located in the East (West) of the shower is given in the next section.

At the fully coherent 1 MHz frequency (Fig. 13 left), one can also see that a maximum appears at around \( r \sim 100 \) m. A similar maximum was reported in [41], but for full bandwidth pulse height. Furthermore, Fig. 13 suggests that this maximum is dependant not only on frequency, but also on observer direction w.r.t. the shower core, disappearing for antennas to the west. Our full bandwidth pulses (Figs. 6 and 7) show a similar dependence of the maximum of the emission with observer direction. This suggests that the mechanism responsible for this maximum is weaker than the interference effect between the geomagnetic and Askaryan components of the emission, which is discussed in the next section.

5 Polarization properties of the signal and asymmetries

The polarization of the electric field depends on the relative importance of the two main mechanisms thought to be responsible for the radio emission [20]. Cherenkov radio emission due to the excess of electrons in the shower (Askaryan effect) has a characteristic axial polarization in relation to the shower axis. On the other hand, the polarization of the emission due to geomagnetic follows \(-\vec{v} \times \vec{B}\), and depends on the direction \(\vec{v}\) of the charged particles and the geomagnetic field \(\vec{B}\). In contrast to the charge excess mechanism, the geomagnetic emission of electrons and positrons add up since they have opposite charges but their trajectories curve in opposite directions (see Section 2.1). In the top
Fig. 13. Average electric field vs distance to core for 10 vertical showers of 100 PeV simulated with ZHAireS at 1 MHz (left) and 60 MHz (right). The observers are placed at positions N, S, E and W of the shower core. Note that for observers N and S the fields are similar and lie on top of each other in the figure. Also shown as points are the RMS of the electric field.

In the panel of Fig. 14 we show schematically the projection of the polarization on the ground induced by each of the emission mechanisms for a vertical shower [17]. A horizontal magnetic field pointing north (N) is assumed.

The interplay between the different polarizations makes the projection of the net field on ground asymmetric with respect to the shower core [15]. This can be easily understood from the sketch at the top of Fig. 14. Since the Askaryan and geomagnetic polarizations point in the same direction for observers East of the core, and in opposite directions for observers to the west, we expect the EW component of the electric field ($E_{EW}$) to be larger in an antenna eastwards of the shower core than in an antenna West of the core, at the same distance. We also expect $E_{EW}$ to be of the same order northwards and southwards from the shower core. Another prediction is that the NS component of the electric field ($E_{NS}$) should be close to zero for observers along the East-West direction and largest for observers along the North-South one [17]. As discussed in [15], as the distance $r$ from the antenna to the shower axis increases, the relative contribution to the electric field of the geomagnetic mechanism decreases and as a consequence the Askaryan mechanism accounts for a larger fraction of the emission. The pattern of the electric field at ground level is then expected to recover a symmetric behavior with respect to the shower core for very large distances, when the Askaryan contribution dominates.

To investigate these expectations we have obtained the $E_{EW}$ and $E_{NS}$ components of the field, at a frequency of 60 MHz, from ZHAireS simulations of vertical proton showers with energy $10^{17}$ eV. These components are shown in Fig. 14. In the left panel we show $E_{NS}$ as a function of the position on the ground, while in the right panel we show the corresponding $E_{EW}$. One can see the expected EW asymmetry in the $E_{EW}$ component, with larger fields to the east of the core, while it is approximately the same North and South of the shower core. The $E_{NS}$ component (left panel) is mainly due to the
Fig. 14. (Color online) Top: Sketch of the projection on the ground of the electric field induced by the Askaryan and geomagnetic emission mechanisms in a vertical shower $\theta = 0^\circ$ (see also [17]). Bottom: Left panel: North-South component of the electric field $E_{NS}$ as a function of the position around the shower core as obtained in ZHAireS simulations of vertical showers induced by protons of energy $10^{17}$ eV. Right panel: East-West component $E_{EW}$ of the electric field obtained in the same simulations. The color scale indicates the magnitude of the components of the field in ($V/m/\text{MHz}$), note the different scale in the left and right panels.

Askaryan mechanism, and hence it is largest along the NS direction, while it gradually decreases as the observer moves to the EW direction because neither the Askaryan nor the geomagnetic mechanism induce a significant $E_{NS}$ component along observers in the EW direction (see sketch in Fig. 14).

Comparing $E_{NS}$ and $E_{EW}$ for observers along the NS axis, one can clearly see that at distances relatively close to the shower core ($r \lesssim 150$ m), the NS component of the field is a factor $\sim 4$ smaller than the EW component, because the geomagnetic mechanism dominates the emission close to the core, while this factor tends to diminish at larger distances (see also Fig. 13).
5.1 $B = 0$ vs $B \neq 0$: Separation of the geomagnetic and Askaryan components.

In order to disentangle the geomagnetic and Askaryan components of the radio emission, we also simulated showers turning off the geomagnetic field. As discussed in section 5, there is an EW asymmetry in the signal strength due to an interference effect between the polarizations of the different emission mechanisms, making the signal larger to the East. In fig. 15 we compare the EW components (positive to the East) at 100 m (top) and 400 m (bottom) West (left) and East (right) of the core, obtained with simulations with and without the geomagnetic field. The simulation with the magnetic field on shows the net field due to both emission mechanisms, while the simulations without the magnetic field has only the Askaryan component. One can see that, as expected, the polarization of the Askaryan component changes direction between antennas East and West of the core (dashed red lines of fig. 15), and this causes the difference in the peak height of the net field (solid blue lines). Also, the height of the pure Askaryan peak (dashed red) is roughly half the difference between the E and W peaks of the net field (solid blue), as expected (see also the sketch in Fig. 14) . At larger distances (e.g. at 400 m, at the bottom of fig 15), the ratio between the Askaryan and geomagnetic components gets higher. At larger distances, the Askaryan mechanism starts to dominate the emission.

To further investigate the polarization of the emission we use a parameter $R_p$, as defined in [44], which is sensitive to the polarization of the electric field. In the case of a horizontal (parallel to ground) magnetic field pointing north, the polarization vector of the geomagnetic contribution to the electric field points East, and $R_p$ is given by:

$$R_p = \frac{\sum t (E_{EW} \cdot E_{NS})}{\sum t (E_{EW}^2 + E_{NS}^2)} ,$$

where the sum runs over all bins in time having a non-zero electric field at the antenna.

From the sketch in the top of Fig. 14 we expect $R_p$ to vary as a function of the azimuthal angle $\phi$ on the ground, defined so that $\phi = 0$ for an antenna in the East direction and $\phi = 90^\circ$ for an antenna at the North. $R_p = 0$ when either $E_{EW}$ or $E_{NS}$ equal zero. We then expect that if the polarization were only due to the geomagnetic contribution (the unphysical case of no charge excess in the shower), the NS polarization would always be zero and so would $R_p$. If at a certain distance $r$ to the shower core the Askaryan and geomagnetic contributions are important, then $R_p \sim 0$ at $\phi = 0^\circ$ and $\phi = 180^\circ$, because $E_{NS} \sim 0$ along those two directions [44]. However, in contrast to what was
expected in [44], if the geomagnetic component is absent then we clearly expect $R_p \sim 0$ at $\phi = 0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$, because either $E_{EW} \sim 0$ or $E_{NS} \sim 0$ along those directions. This means that in the absence of a magnetic field, $R_p$ should exhibit a $180^\circ$ periodicity in azimuthal angle $\phi$. We checked this in our ZHAireS simulations by switching off the magnetic field. The result is plotted in Fig. 16 where one can see that $R_p$ exhibits a periodicity of $180^\circ$ in $\phi$, regardless of the distance to the observer.

In contrast, in Fig. 17 we show $R_p$ as a function of $\phi$ as obtained in ZHAireS simulations of 10 vertical proton showers of energy $10^{17}$ eV, but in this case with the geomagnetic field on. One can see that for distances closer to the core ($r < 300$ m), the $R_p$ parameter reaches zero only at $\phi = 0^\circ$ and $\phi = 180^\circ$, as expected when both the geomagnetic and Askaryan components are important, while at larger distances from the core, the period of $R_p$ vs $\phi$ changes from $360^\circ$ to $\sim 180^\circ$. This further confirms that at large distances to the core ($r \gtrsim 600$ m), the Askaryan mechanism dominates the full bandwidth emission. A similar behavior was reported in [44] for REAS3 simulations convoluted with the detector response, but this periodicity change in $R_p$ was interpreted as a signature of a dipole polarized field instead of a dominant charge excess contribution. In the same paper [44], MGMR simulations did not exhibit this behavior.
Fig. 16. ZHAireS code predictions for the polarization sensitive parameter $R_p$ (as defined in Eq. (18)), as a function of the azimuthal angle $\phi$ (see top of Fig. 14) for 10 vertical proton showers of energy $10^{17}$ eV. The magnetic field was switched off in the simulations. The $R_p$ parameter is plotted for antennas close to the shower core (blue circles: $r < 300$ m), and farther from it (red stars: $r > 600$ m).

If we assume a perfect shower symmetry in $\phi$, the modulus $E_{Ch}^e$ of the electric field due to Askaryan mechanism would be the same for antennas at the same distance from the core, and the EW and NS components could be written as:

$$E_{EW}^{Ch} = -E_{NS}^{Ch} = -E_{Ch}^e \sin \phi$$  \hspace{1cm} (19)

If we further assume no time dependence of the polarization, and take the numerator of Eq. (18):

$$R_p^{Ch} \propto (E_{Ch}^e)^2 \sin 2\phi$$  \hspace{1cm} (20)

which leads to $R_p^{Ch}$ with a period of 180° in $\phi$, as shown in Fig. 16.

The field with both the Askaryan $E_{Ch}^e$ and geomagnetic $E_{geo}^e$ contributions can be written as (see top of Fig. 14):

$$E_{EW} = -(E_{geo}^e + E_{Ch}^e \cos \phi), \quad E_{NS} = -E_{Ch}^e \sin \phi$$  \hspace{1cm} (21)

leading to:

$$R_p \propto E_{geo}^e E_{Ch}^e \sin \phi + (E_{Ch}^e)^2 \sin 2\phi / 2$$  \hspace{1cm} (22)
Fig. 17. ZHAireS code predictions for the polarization sensitive parameter $R_p$ (as defined in Eq. (18)), as a function of the azimuthal angle $\phi$ for 10 vertical proton showers of energy $10^{17}$ eV, with a horizontal magnetic field pointing north. The $R_p$ parameter is plotted for antennas close to the shower core (blue circles: $r < 300$ m) and farther from it (red stars: $r > 600$ m).

The equation above predicts that when the Askaryan and geomagnetic mechanisms compete, $R_p$ is proportional to a rather complicated function of $\phi$ whose shape depends on the relative values of $E_{\text{Ch}}$ and $E_{\text{geo}}$.

In Fig. 18 we show the numerator of $R_p$, given by the product $E_{EW} \cdot E_{NS}$, as a function of $\phi$ obtained in ZHAireS simulations of a vertical proton shower of $10^{17}$ eV, for antennas at $r = 200$ m (left) and $r = 1200$ m (right) from the shower core. We then fitted Eq. (22) to these simulations with only $E_{\text{Ch}}$ and $E_{\text{geo}}$ as free parameters. The fit based on the simple model above reproduces remarkably well the behavior with $\phi$ obtained in the simulations, with a ratio $E_{\text{geo}}/E_{\text{Ch}} \sim 1.74$ at $r = 200$ m, and $E_{\text{geo}}/E_{\text{Ch}} \sim 0.51$ at $r = 1200$ m, decreasing with $r$ as the Askaryan component is expected to dominate at large distances to the core. It is interesting to note that in the ZHAireS simulation for antennas south of the core, the ratio between the peaks of $E_{EW}$ (expected to be purely geomagnetic) and $E_{NS}$ (expected to be purely Askaryan) decreases from 2.32 at $r = 200$ m to 0.57 at $r = 1200$ m, for this same shower. It is also interesting to see in Fig. 18 how the parameter $R_p$ gradually changes from having a 360° periodicity in $\phi$ to a 180° periodicity as the distance to the shower axis increases. We believe that similar, but more refined analysis methods could be derived to separate the contributions of the different emission...
Fig. 18. Points: The numerator of $R_p$ in Eq. (18) as a function of $\phi$ for a vertical proton shower of $10^{17}$ eV simulated with ZHAireS, for antennas at $r = 200$ m (left) and $r = 1200$ m (right) from the core. Solid lines: Fit of Eq. (22) (obtained using a simple model of the polarization - see text for more details) to the simulated electric fields.

Fig. 19. Sketch of the electric field in non-vertical showers and its projection onto the ground plane.

mechanisms to the electric field.

5.2 Polarization of the electric field in non-vertical showers

In the case of non-vertical showers, the projection on the ground of the polarization of both, the geomagnetic and Askaryan components will depend on the azimuthal angle $\phi$ of the antenna position on the ground in relation to the core, as can be seen in schematic form in Fig. 19. Furthermore, the direction of the shower axis with respect to the geomagnetic field will change the direction of the Lorentz force acting on the charged particles. In vertical showers the force is always almost parallel to the ground, but in non-vertical showers the Lorentz force will only be horizontal if the plane defined by $\vec{B}$ and the shower axis is perpendicular to the ground (e.g. Fig. 20 top left). In most geometries there will be a vertical component to the Lorentz force (e.g. Fig.
which shows the results of full ZHAireS simulations of the N-S (top), E-W (middle) and Z (bottom) components of the electric field at 60 MHz for 100 PeV proton shower with $\theta = 45^\circ$ coming from the north (left) and the west (right), corresponding to the geometries shown schematically on the top left and top right of Fig. 20 respectively. Note that different scales were used for the field in the different panels. A horizontal magnetic field pointing north was used in the simulations. For the shower coming from the north (left), one can see that the main asymmetry is in the E-W component (left-middle), similar to the vertical shower case, while the N-S and Z components are smaller and very similar to each other, because the dominant geomagnetic contribution is horizontal. On the other hand, in the case of the shower coming from the West (right), there is a large asymmetry to the East on both, the E-W and Z components, since in this particular geometry the dominant geomagnetic contribution makes an angle of $\sim 45^\circ$ with the horizontal, and thus should have very similar E-W and Z components.

This dependence of the polarization on the azimuthal angle of the shower direction is relevant for studies trying to disentangle the contributions of the emission mechanisms using polarization. Since the inclination of the polarization vector of the dominant geomagnetic emission with respect to the horizontal plane changes with the shower azimuthal angle, it may be important to also measure the vertical component of the net electric field, since for showers with $\theta > 45^\circ$, the vertical component of the field can be even larger than the horizontal ones.

6 Conclusions

In this work we present predictions for the radio pulse emitted by extensive air showers. Our results are obtained using the ZHAireS Monte Carlo, an AIRES-based code that takes into account the full complexity of ultra-high energy cosmic-ray induced shower development in the atmosphere, and allows the calculation of the electric field in both the time and frequency domains based on the algorithms developed in references [23][24]. Although our approach does not presuppose any a priori emission mechanism, our results confirm that the emission at radio frequencies can be understood as the superposition of radiation from the charge separation induced by the magnetic field of the Earth (geomagnetic effect), and that coming from the net excess negative charge evolving as the shower develops in the atmosphere (Askaryan effect).

We have pointed out the relevance of the refractive index in the time structure and intensity of the radio pulses, especially at short distances to the shower.
Fig. 20. Results of full ZHAireS simulations of the N-S (top), E-W (middle) and Z (bottom) components of the electric field at 60MHz for 100PeV proton shower with \( \theta = 45^\circ \) coming from the north (left) and the west (right). See text for more details.
axis. Another interesting work [41], developed independently and in parallel with ours, deals with similar issues. The refractive index determines the angular distribution of the radiation at the emission point, as well as its propagation through the atmosphere and non-trivial relativistic effects arise due to the refractive index $n > 1$. We have developed a simple 1-dimensional model to address the characteristics of the radio pulse, which qualitatively allows us to interpret the pulse height, width and its dependence on the refractive index and the distance from antenna to shower core [41]. The intensity of the radio pulses is typically highest for those observers that see a region of the shower containing a large number of particles (such as shower maximum) with an angle close to the Cherenkov angle. There is a non-trivial interplay between the distance from the emission point in the shower to the observation point, the angle between the particle direction and the observer, and the number of particles in the region of the emission, whose effects can only be accurately determined with Monte Carlo simulations such as those developed in this work. These key elements determine for instance that far from the shower axis inclined showers typically induce a more intense radiation than vertical ones [32] as we show with our ZHAireS simulations.

We have shown that the frequency at which the emission spectrum is maximum, which is in the range $\sim 3-30$ MHz for $0 < r < 400$ m, decreases as the distance $r$ from the antenna to the shower core increases (for $r > 100$ m see also [11,17,46]). So most experiments, which typically are only sensitive to frequencies above 30 MHz, in fact only measure the incoherent part of the spectrum, even very close to the shower core. We also observed that the signal at higher frequencies decreases more rapidly with $r$ than at the fully coherent 1 MHz range, where we observed a maximum in the emission at around $r \sim 100$ m, similar to the one reported in [41]. Our results also suggest that this maximum is dependant on frequency and on observer direction w.r.t. the shower core. We have also explored the polarization properties of the radiation, confirming the expectation that the radiation is mainly polarized in the opposite direction to the Lorentz force induced by the magnetic field of the Earth [28,29,30,31]. Far from the shower core, our simulations show that the polarization is compatible with the presence of a significant amount of radiation due to the Askaryan effect, as in [17]. We have also shown, using a very simple model, that as the Askaryan component of the emission becomes dominant over the geomagnetic one at larger distances to the core, the parameter $R_p$ changes its periodicity on the azimuthal angle of the observer. When both, the geomagnetic and Askaryan components are significant, the periodicity is 360°, but when the emission is dominated by the Askaryan effect, a 180° periodicity can be clearly appreciated. In inclined showers we have stressed the role played by the relative orientation of the shower axis and the magnetic field on the polarization. A significant vertical component of the electric field arises in inclined showers which calls for detection instruments able to observe it.
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