Percolation on interacting, antagonistic networks

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Abstract. Recently, new results on percolation in interdependent networks have shown that the percolation transition can be first order. In this paper we show that, when considering antagonistic interactions between interacting networks, the percolation process might present a bistability of the equilibrium solution. To this end, we introduce antagonistic interactions for which the functionality, or activity, of a node in a network is incompatible with the functionality of the linked nodes in the other interacting networks. In particular, we study the percolation transition in two interacting networks with purely antagonistic interaction and differing topology.

Keywords: percolation problems (theory), random graphs, networks
1. Introduction

Over the past ten years, percolation processes, and, more generally, dynamical processes in complex networks [1, 2], have attracted great attention. In this context it has been shown that complex topologies strongly affect the dynamics occurring in networks. However, many complex systems involve interdependencies between different networks, and accounting for these interactions is crucial in economic markets, interrelated technological and infrastructure systems, social networks, disease dynamics, and human physiology. Recently, important new advances have been made in the characterization of percolation [3]–[10] and other dynamical processes [11]–[17] on interacting and interdependent networks. In these systems, one network function depends on the operational level of other networks. A failure in one network could trigger failure avalanches in the other interdependent networks, resulting in increased fragility of the interdependent system. In fact, it has been shown [3]–[7] that two interdependent networks are more fragile than a single network and that the percolation transitions in interdependent networks can be first order. These results have subsequently been extended to multiple interdependent networks [8, 9] and to networks in which only a fraction of the nodes are interdependent [10].

Here, we want to investigate the role of antagonistic interactions in the percolation transition between interacting networks. For antagonistic interactions the functionality, or activity, of a node in a network is incompatible with the functionality of the linked nodes in the other interacting networks. As happens in spin systems, where antiferromagnetic interactions can result in the frustration of the system, in interacting networks, the presence of antagonistic interactions between the nodes also introduces further complexity in the percolation problem. As a first step in investigating this complexity in this paper, we will consider two interacting networks with purely antagonistic interactions. Moreover, we assume that a node is active in a network only if it belongs to the giant component of active nodes in the network. We will show that for two Poisson networks with exclusively antagonistic interactions, the percolating configuration corresponds to the percolation of one of the two networks. Nevertheless, the solution for the model is surprising because
there is a wide region of the phase space in which there is a bistability of the percolation process: either one of the two networks might end up being percolating. Therefore, in this new percolation problem, not only might the percolation transitions be first order, but also we found that there is a real hysteresis in the system as we modify the average degrees of the two networks. Furthermore, we extend the analysis to networks with other topologies, studying the percolation transition in two antagonistic scale-free networks, and in two networks of which one is a Poisson network, while the other is a scale-free network. We characterize the rich phase diagram of the percolation transition in these networks. Interestingly, in the percolation phase diagram of these interacting networks there is a region in which both networks percolate, demonstrating a strong interplay between the percolation process and the topology of the network. Finally, these results shed new light on the complexity that the percolation process acquires when we consider percolation on interdependent, antagonistic networks.

2. Percolation on antagonistic networks

In this paper we introduce antagonistic interactions in percolation on interdependent networks. As has been done for the previously studied case of interdependent networks [3, 5, 10], we will assume that a node is active in a network only if it belongs to the giant component of active nodes in that network. The difference with respect to the case of interdependent networks is that if a node $i$ is active on one network, it cannot be active in the other one. We consider two networks of $N$ nodes. We name the networks as network A and network B with degree distributions $p^A(k)$ and $p^B(k)$ respectively. Each node $i$ is represented in both networks. In particular, each node has a set of neighbor nodes $j$ in network A, i.e. $j \in N_A(i)$, and a set of neighbor nodes $j$ in network B, i.e. $j \in N_B(i)$.

A node $i$ belongs to the percolation cluster of network A if it has at least one neighbor $j \in N_A(i)$ in the percolating cluster of network A, and has no neighbors $j \in N_B(i)$ in network B that belong to the percolating cluster of network B. Similarly, a node $i$ belongs to the percolation cluster of network B if it has at least one neighbor $j \in N_B(i)$ in the percolating cluster of network B, and has no neighbors $j \in N_A(i)$ in network A that belong to the percolating cluster of network A. The percolation steady state can be found by using a message passing algorithm [5, 18] (called by Son et al epidemic spreading). Each node $i$ sends a message to each of its neighboring nodes $j$. We call each message $y^A_{i \to j}$, if the message is sent from a node $i$ to a node $j$ in the network A (B). The message $y^A_{i \to j}$ indicates the probability that following a link $(i,j)$ in network A (B) from $j$ to $i$ we reach a node $i$ which is active in the network A (B). The probability $S^A_i$ that a node $i$ is active in network A (network B) depends on the messages $y^A_{k \to i}$ that the neighbors $k$ on network A and network B send to node $i$, i.e.

$$
S^A_i = \left[ 1 - \prod_{k \in N_A(i)} (1 - y^A_{k \to i}) \right] \prod_{k \in N_B(i)} (1 - y^B_{k \to i})
$$

$$
S^B_i = \left[ 1 - \prod_{k \in N_B(i)} (1 - y^B_{k \to i}) \right] \prod_{k \in N_A(i)} (1 - y^A_{k \to i}).
$$

(1)
Moreover the messages $y_{i \rightarrow j}^{A(B)}$ on a locally tree-like network are the fixed point solutions as $n \rightarrow \infty$ of the following iterative equations for $y_{i \rightarrow j}^{A(B),n}$:

\[
y_{i \rightarrow j}^{A,n} = \left[1 - \prod_{k \in N_A(i) \setminus j} (1 - y_{k \rightarrow i}^{A,n-1}) \right] \prod_{k \in N_B(i)} (1 - y_{k \rightarrow i}^{B,n-1})
\]

\[
y_{i \rightarrow j}^{B,n} = \left[1 - \prod_{k \in N_B(i) \setminus j} (1 - y_{k \rightarrow i}^{B,n-1}) \right] \prod_{k \in N_A(i)} (1 - y_{k \rightarrow i}^{A,n-1})
\]

(2)

In order to find the messages, usually the variables $y_{i \rightarrow j}^{A(B),n}$ are updated starting from given initial conditions until a fixed point of the iteration if found. At the fixed point the messages $y_{i \rightarrow j}^{A(B)} = \lim_{n \rightarrow \infty} y_{i \rightarrow j}^{A(B),n}$ satisfy the following relation:

\[
y_{i \rightarrow j}^{A} = \left[1 - \prod_{k \in N_A(i) \setminus j} (1 - y_{k \rightarrow i}^{A}) \right] \prod_{k \in N_B(i)} (1 - y_{k \rightarrow i}^{B})
\]

\[
y_{i \rightarrow j}^{B} = \left[1 - \prod_{k \in N_B(i) \setminus j} (1 - y_{k \rightarrow i}^{B}) \right] \prod_{k \in N_A(i)} (1 - y_{k \rightarrow i}^{A})
\]

(3)

If we average the equations (1) and (3) over an ensemble of networks with degree distributions $p^A(k)$, $p^B(k)$ we get the equations for $S^A_B = \langle S^A_i \rangle$ and $S_{A,B} = \langle y_{k \rightarrow i}^A \rangle$, where $S^A_B$ is the probability of finding a node in the percolation cluster of network A (network B), and $S_{A,B}^A$ is the probability that following a link we reach a node in the percolation cluster of network A (network B). In particular, we have

\[
S_A = [1 - G^A_0(1 - S'_A)]G^B_0(1 - S'_B)
\]

\[
S_B = [1 - G^B_0(1 - S'_B)]G^A_0(1 - S'_A)
\]

(4)

In equation (4) we have used $G^A_0(z)$ and $G^A_1(z)$ to indicate the generating functions of networks A and B defined according to the definition

\[
G_1(z) = \sum_k \frac{k p_k}{\langle k \rangle} z^{k-1} \quad G_0(z) = \sum_k p_k z^k,
\]

(5)

where we use the degree distributions $p^A(k)$ and $p^B(k)$, respectively, for network A and network B. Moreover $S_{A,B}^A$: on a locally tree-like network, satisfy the following recursive equations:

\[
S'_A = (1 - G^A_1(1 - S'_A))G^B_0(1 - S'_B) = f_A(S'_A, S'_B),
\]

\[
S'_B = (1 - G^B_1(1 - S'_B))G^A_0(1 - S'_A) = f_B(S'_A, S'_B).
\]

(6)

The solutions to the recursive equations (6) can be classified into three categories:

(i) The trivial solution in which neither of the networks is percolating: $S'_A = S'_B = 0$.

(ii) The solutions in which just one network is percolating. In this case we have either $S'_A > 0, S'_B = 0$ or $S'_B > 0, S'_A = 0$. From equations (6) we find that the solution $S'_A > 0, S'_B = 0$ emerges at a critical line of second-order phase transition, characterized

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by the condition
\[
\frac{dG^A_A(z)}{dz} \bigg|_{z=1} = \frac{\langle k(k-1) \rangle_A}{\langle k \rangle_A} = 1.
\] (7)

Similarly the solution \( S'_B > 0, S'_A = 0 \) emerges at a second-order phase transition when we have \( (\langle k(k-1) \rangle_B/\langle k \rangle_B) = 1 \). This condition is equivalent to the critical condition for percolation in single networks, as it should be, because one of the two networks is not percolating.

(iii) The solutions for which both networks are percolating. In this case we have \( S'_A > 0, S'_B > 0 \). This solution can emerge either (a) when the curves \( S'_A = f_A(S'_A, S'_B) \) and \( S'_B = f_B(S'_A, S'_B) \) cross at \( S_A = 0 \) or at \( S_B = 0 \), or (b) when the curves \( S'_A = f_A(S'_A, S'_B) \) and \( S'_B = f_B(S'_A, S'_B) \) cross at a point \( S_A \neq 0 \) and \( S_B \neq 0 \) at which they are tangent to each other. For situation (a) the critical line can be determined by imposing, for example, \( S'_A \to 0 \) in equation (6), which yields
\[
S'_B = 1 - G^B_1(1 - S'_B), \quad 1 = \frac{\langle k(k-1) \rangle_A}{\langle k \rangle_A} G^B_0(1 - S'_B).\] (8)

A similar system of equations can be found by using equations (6) and imposing \( S'_B \to 0 \). For situation (b) the critical line can be determined by imposing that the curves \( S'_A = f_A(S'_A, S'_B) \) and \( S'_B = f_B(S'_A, S'_B) \) are tangent to each other at the point where they intercept. This condition can be written as
\[
(\frac{\partial f_A}{\partial S'_A} - 1)(\frac{\partial f_B}{\partial S'_B} - 1) - \frac{\partial f_A}{\partial S'_B} \frac{\partial f_B}{\partial S'_A} = 0,
\] (9)
where \( S'_A, S'_B \) must satisfy equations (6).

2.1. The stability of the solutions

Not every solution of the recursive equations (6) is stable. Therefore here we check the stability of the fixed point solutions of equations (6) by linearizing the equations around each solution. The Jacobian matrix \( J \) of the system of equations (6) is given by
\[
J = \begin{bmatrix}
\frac{\partial f_A}{\partial S'_A} & \frac{\partial f_A}{\partial S'_B} \\
\frac{\partial f_B}{\partial S'_A} & \frac{\partial f_B}{\partial S'_B}
\end{bmatrix}.
\] (10)

The eigenvalues \( \lambda_{1,2} \) of the Jacobian can be found by solving the characteristic equation \( |J - \lambda I| = 0 \), which reads for our specific problem
\[
(\frac{\partial f_A}{\partial S'_A} - \lambda)(\frac{\partial f_B}{\partial S'_B} - \lambda) - \frac{\partial f_A}{\partial S'_B} \frac{\partial f_B}{\partial S'_A} = 0.
\] (11)

The change of stability of each solution will occur when \( \max(\lambda_1, \lambda_2) = 1 \). In the following we will discuss the stability of the solutions of type (i)–(iii).
Table 1. Stable phases in the different regions of the phase diagram of the percolation problem on two antagonistic Poisson networks (figure 1).

| Region   | Condition          |
|----------|--------------------|
| I        | $S_A' = S_B' = 0$  |
| II-A     | $S_A' > 0, S_B' = 0$ |
| II-B     | $S_B' > 0, S_A' = 0$ |
| III      | either $S_A' > 0, S_B' = 0$ or $S_B' > 0, S_A' = 0$ |

• (i) **Stability of the trivial solution** $S_A' = S_B' = 0$. The solution is stable as long as the following two conditions are satisfied:

$$
\lambda_{1,2} = \frac{\langle k(k-1) \rangle_{A/B}}{\langle k \rangle_{A/B}} < 1.
$$

Therefore the stability of this solution changes on the critical lines $\langle k(k-1) \rangle_A (k)_A = 1$ and $\langle k(k-1) \rangle_B (k)_B = 1$.

• (ii) **Stability of the solutions in which only one network is percolating.** For the case of $S_A' = 0$ and $S_B' > 0$, the stability condition reads

$$
\lambda_1 = \left. \frac{G_1(z)}{dz} \right|_{z=1-S_B'} < 1 \quad \lambda_2 = \frac{\langle k(k-1) \rangle_A G_0^B (1-S_B')}{(k)_A} < 1.
$$

We note here that if $\lambda_2 > \lambda_1$, we expect to observe a change in the stability of the solution on the critical line given by equation (8). A similar condition holds for the stability of the solution $S_A' > 0, S_B' = 0$.

• (iii) **Stability of the solution in which both networks are percolating** $S_A' > 0, S_B' > 0$. For characterizing the stability of the solutions of type III we have to solve equation (11) and impose that the eigenvalues $\lambda_{1,2}$ are less than 1, i.e. $\lambda_{1,2} < 1$. We observe here that for $\lambda = 1$, equation (11) reduces to equation (9). Therefore we expect to have a stability change of these solutions on the critical line given by equation (9).

### 2.2. Two Poisson networks

In order to consider a specific example of antagonistic networks we consider two antagonistic Poisson networks with average degrees $\langle k \rangle_A = z_A$ and $\langle k \rangle_B = z_B$. In the case of a Poisson network we have $G_0(z) = G_1(z) = e^{(k)(1-z)}$. Therefore we have $S_A = S_A'$ and $S_B = S_B'$. The recursive equations (6) read in this case

$$
S_A = (1 - e^{-z_A S_A}) e^{-z_B S_B} \quad S_B = (1 - e^{-z_B S_B}) e^{-z_A S_A}.
$$

In table 1 we characterize the phase diagram percolation on two antagonistic Poisson networks shown in figure 1. The critical lines are given by $z_A = 1$, $z_B = 1$ and by $z_B = \log z_A/(1 - 1/z_A)$ or $z_A = \log(z_B)/(1 - 1/z_B)$. In particular we observe a first-order phase transition along the line with $z_A > 1$ and $z_B = \log(z_A)/(1 - 1/z_A)$ and also along the line with $z_B > 1$ and $z_A = \log(z_B)/(1 - 1/z_B)$, indicated as a solid red lines in figure 1. The other lines, indicated as black dashed lines, in figure 1 are critical lines of a second-order transition.

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One should note that the solution $S'_A > 0, S'_B > 0$ in which both networks are percolating is always unstable in this case. This implies that for each realization of the percolation process, only one of the two networks is percolating.

In order to demonstrate the bistability of the percolation solution in region III of the phase diagram, we solved equations (6) recursively for $z_B = 1.5$ and variable values of $z_A$ (see figure 2). We start from the value $z_A = 4$, and we solve equations (6) recursively. We find the solutions $S'_A = S'_A(z_A = 4) > 0$, $S'_B = S'_B(z_A = 4) = 0$. Then we slightly lower $z_A$ and we again solve equations (6) recursively, starting from the initial condition $S'^0_A = S'_A(z_A = 4) + \epsilon$, $S'^0_B = S'_B(z_A = 4) + \epsilon$, and plot the result. (The small perturbation $\epsilon > 0$ is necessary in order not to end up with the trivial solution $S'_A = 0, S'_B = 0$.) Using this procedure we show that if we first lower the value of $z_A$ and then raise it again, spanning region III of the phase diagram, as shown in figure 2 panels (a) and (b), the solution presents a hysteresis loop. This means that in region III, either network A or network B might end up being percolating.

2.3. Two scale-free networks

Here, we characterize the phase diagram of two antagonistic scale-free networks with power-law exponents $\gamma_A, \gamma_B$, as shown in figure 3. The two networks have minimal connectivity $m = 1$ and varying value of the maximal degree $K$.

The critical lines of the phase diagram depend on the value of the maximal degree $K$ of the networks. The critical lines of the phase diagram are dependent on the value of the cutoff $K$ of the scale-free degree distribution and therefore for finite values of $K$ we observe an effective phase diagram converging in the $K \to \infty$ limit to the phase diagram of an infinite network. In the infinite network limit the recursive equations (6) can be
Panels (a) and (b) show the hysteresis loop for the percolation problem on two antagonistic Poisson networks with $z_B = 1.5$. Panels (c) and (d) show the hysteresis loop for the percolation problem on two antagonistic networks of differing topology: a Poisson network of average degree $z_A = 1.8$ and a scale-free network with power-law exponent $\gamma_B$, minimal degree $m = 1$ and maximal degree $K = 100$. The hysteresis loop is obtained using the method explained in the main text. The value of the parameter $\epsilon$ used in this figure is $\epsilon = 10^{-3}$.

written as

$$S'_A = \left(1 - \frac{\text{Li}_{\gamma_A-1}(1 - S_A')}{(1 - S_A')\zeta(\gamma_A - 1)}\right) \frac{\text{Li}_{\gamma_B}(1 - S_B')}{\zeta(\gamma_B)}$$

$$S'_B = \left(1 - \frac{\text{Li}_{\gamma_B-1}(1 - S_B')}{(1 - S_B')\zeta(\gamma_B - 1)}\right) \frac{\text{Li}_{\gamma_A}(1 - S_A')}{\zeta(\gamma_A)}$$

where $\zeta(s)$ is the Riemann zeta function and $\text{Li}_n(z)$ is the polylogarithm function. Solving these equations, and studying their stability as described in the previous paragraphs, we can draw the phase diagram of the model. The phase diagram is rich, showing a region (region III) in the figure where both networks are percolating, demonstrating an interesting interplay between the percolation and the topology of the network. The hub nodes of a network are the nodes which are more likely to be active in that network. Therefore hub nodes act as kinds of ‘pinning centers’ for the percolating cluster. Since the two antagonistic networks in our model have uncorrelated degrees, a node that is a hub in a network is unlikely to also be a hub in the other network, offering the chance of
Figure 3. The phase diagram of the percolation process in two antagonistic scale-free networks with power-law exponents $\gamma_A, \gamma_B$. The minimal degree of the two networks is $m = 1$ and the maximal degree, $K$. Panel (a) show the effective phase diagram with $K = 100$; panel (b) shows the phase diagram in the limit of an infinite network, $K = \infty$.

Table 2. Stable phases in the different regions of the phase diagram of the percolation on two antagonistic scale-free networks (figure 3).

| Region   | $S'_A = S'_B = 0$ | $S'_A > 0, S'_B = 0$ | $S'_B > 0, S'_A = 0$ | $S'_A > 0, S'_B > 0$ |
|----------|-------------------|----------------------|----------------------|----------------------|

having two percolating clusters in the two antagonist networks. This observation offers a qualitative understanding of why we can observe in two antagonist uncorrelated networks the coexistence of two percolating clusters, while in the case of two Poisson networks where the degrees of the nodes are more homogeneous, this phase is not observed. The importance of the hub nodes in pinning the percolation cluster on one network can also help us understand qualitatively the strong effects that a finite upper cutoff $K$ in the degree has in the phase diagram. A description of the stable phases in the different regions of the phase diagram is provided by table 2. In this case all the transitions are second order.

2.4. A Poisson network and a scale-free network

Finally we consider the case of a Poisson network (network A) with average connectivity $\langle k \rangle_A = z_A$, and a second network (network B) with a scale-free degree distribution and a power-law exponent of the degree distribution $\gamma_B$. The scale-free network has minimal connectivity $m = 1$ and maximal degree given by $K$. In figure 4 we show the phase diagram of the model in the plane $(\gamma_B, z_A)$. The critical lines of the phase diagram are dependent on the value of the cutoff $K$ of the scale–degree distribution and therefore for finite value
of $K$ we observe an effective phase diagram converging in the $K \to \infty$ limit to the phase diagram of an infinite network. In the infinite network limit, the recursive equations (6) can be written as

\begin{align}
S'_A &= (1 - e^{-z_A S_A}) \frac{\text{Li}_{\gamma_B}(1 - S'_B)}{\zeta(\gamma_B)} \\
S'_B &= \left(1 - \frac{\text{Li}_{\gamma_B-1}(1 - S'_B)}{(1 - S'_B)\zeta(\gamma_B - 1)}\right) e^{-z_A S_A}
\end{align}

(16)

where $\zeta(s)$ is the Riemann zeta function and $\text{Li}_n(z)$ is the polylogarithm function. Solving these equations and studying their stability as described in the previous paragraphs, we were able to draw the phase diagram of the model. The phase diagram includes two regions (region III and region V) with bistability of the solutions and two regions (region III and region IV) in which the solution in which both networks are percolating is stable. We have indicated with red solid lines the lines where a first-order phase transition can be observed in correspondence with a change of the stability of the solutions and we have indicated with black dashed lines the lines of a second-order phase transition. Also in this case, the importance of hubs as centers for the pinning of a percolating phase is apparent. In fact we observe strong effects of a finite cutoff $K$ in the degrees of the nodes, leading to the disappearance of region V when the cutoff $K$ goes to infinity. The fact that region

Figure 4. Phase diagram of the percolation process on a Poisson network with average degree $\langle k \rangle_A = z_A$ interacting with a scale-free network of power-law exponent $\gamma_B$ and minimal degree $m = 1$. Panel (a) shows the effective phase diagram for maximal degree $K = 100$; panel (b) shows the phase diagram in the limit of an infinite network, i.e. for $K = \infty$. 

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Table 3. Stable phases in the phase diagram for the percolation on two antagonist networks: a Poisson network (network A) and a scale-free network (network B); see figure 4.

| Region   | Condition                  |
|----------|----------------------------|
| Region I | $S_A' = S_B' = 0$          |
| Region II-A | $S_A' > 0, S_B' = 0$    |
| Region II-B | $S_B' > 0, S_A' = 0$    |
| Region III | $S_A' > 0, S_B' > 0$     |
| Region IV | either $S_B' > 0, S_A' = 0$ or $S_A' > 0, S_B' > 0$ |
| Region V  | either $S_A' > 0, S_B' = 0$ or $S_B' > 0, S_A' = 0$ |

V is only observed in the finite size network can be explained by the effect that highly connected hubs have in the stabilization of a percolation phase in network B. Region V in fact contains a phase in which network B is not percolating; this phase is allowed only if the hubs are below a certain connectivity. Therefore this phase disappears in the limit of an infinite network. In table 3 we describe the percolation stable solutions in the different regions of the phase diagram shown in figure 4.

In order to demonstrate the bistability of the percolation problem, we solved equations (6) recursively for $z_B = 1.8$ (see figure 2). We start from the value $\gamma_B = 3$, and we solve the equations (6) using the same method as was explained for the two antagonistic Poisson networks. Using this procedure we show in figure 2 panels (c) and (d) that the solution presents a second-order phase transition to a phase in which both networks are percolating and also a hysteresis loop in correspondence with region IV. This demonstrates the bistability of the solutions in region IV and the existence of a phase in which both networks percolate in region III.

3. Conclusions

In conclusion, we have investigated how much antagonistic interactions modify the phase diagram of the percolation transition. The percolation process on two antagonist networks shows important new physics of the percolation problem. In fact, the percolation process in this case shows a bistability of the solutions. This implies that the steady state of the system is not unique. In particular, we have demonstrated the bistability of the percolation solution for the percolation problem on two antagonist Poisson networks, or two antagonist networks with differing topology: a Poisson network and a scale-free network. Moreover, in the percolation transition between two scale-free antagonist networks and in the percolation transition between two antagonist networks with a Poisson network and a scale-free network, we found a region in the phase diagram in which both networks are percolating, despite the presence of antagonistic interactions. We believe that this paper opens up new perspectives in the percolation problem on interdependent networks, which might include both interdependences and antagonistic interactions eventually combined in a Boolean rule. In an increasingly interconnected world, understanding how much these different kinds of interactions affect percolation transitions is becoming key to answering fundamental questions about the robustness of interdependent networks.

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