Understanding the mechanical response of glass and carbon fibres: stress-strain analysis and modulus determination

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Abstract. Accurate characterization of fibres is crucial for the understanding the properties and behaviour of fibre-reinforced composite materials. Fibre properties are key parameters for composite design, modelling and analysis. In this study, characterization of mechanical properties of glass and carbon fibres has been performed using a semi-automated single-fibre testing machine. Based on a sample set of 150 glass and carbon fibers fibres, engineering and true stress-strain curves are analyzed. Different modulus determination methods are discussed based on true stress-strain and tangent modulus-strain relationships. For glass fibres, the true stress-strain based tangent modulus is found to be independent of applied strain, whereas for carbon fibres, a tendency of tangent modulus to increase with applied strain is observed. The modulus of glass fibres is found to be independent of fibre diameter, whereas carbon fibres with smaller diameter show higher modulus compared with carbon fibres with larger diameters.

1. Introduction
Fibre-reinforced composites are seeing a rapid growth in their development and application in wind energy, aerospace and automotive industries. It is due to their excellent mechanical properties combined with low density. The use of fibres as reinforcement with high levels of specific modulus and specific strength impacts desirable mechanical performance to composites. Reliable characterization of the reinforcing fibres is critically important for reliable predictions of the mechanical behaviour of composites, as they are the principal load-bearing members.

Different methods are used to characterize the mechanical properties of fibres like single-fibre tensile test and fibre bundle test. The single-fibre tensile test is the most commonly used method [1]. Individual fibres are tested one by one by subjecting them to an increasing tensile load until failure. The traditional card frame method is a labour intensive and time-consuming process. Due to the small diameter of fibres, measurement of their cross-sectional area becomes challenging. In addition, a careful processing and handling technique is required, as this might induce new flaws and/or change the existing ones on the fibres. Most studies on fibre characterization have been limited to small number of samples due to very challenging and time-consuming testing method [2]. A number of studies have highlighted that for a reliable analysis, a large number of single-fibre data is needed [3, 4, 5]. Swolfs et al. reported that several hundred of fibres should be tested to obtain less than 10% of variation in modelling predictions of composite materials [6].

Glass and carbon fibres are the two most industrially important reinforcement fibres for composites [7]. Both of them are brittle in nature and exhibits scatter in strength. Due to their high modulus to weight ratio, they attract wide applications where minimum structural weight is required, e.g. wind turbine blades.
A literature survey on single glass and carbon fibre testing reveals that the analysis of single-fibre tensile test results are mostly restricted to strength distribution using different probabilistic models, where the Weibull model is by far the most widely used [8, 9]. There is a lack of studies analysing the basic mechanical response of single fibres.

In this study, we characterised the mechanical response of glass and carbon fibres using a semi-automated single-fibre testing machine. This equipment enabled us to test large number of fibres with minimized manual handling. The tensile test results were analysed using both engineering and true stress-strain curves, to present a complete picture. In addition, we investigated different modulus determination methods. All the data is available as a Zenodo repository [19].

2. Materials and methods

2.1. Materials
Boron-free R-glass fibres (HiPer-tex™ W 3030, 3B Fiberglass Norway AS) supplied by Owens Corning, Belgium were investigated in this study. The roving had a filament count of 4,000 and a nominal single fibre diameter of 17 μm.

PAN-based carbon fibres (PYROFIL™ TRW40 50L, Mitsubishi Chemical Corporation, Japan) were investigated in this study. The roving had a filament count of 50,000 and a nominal single fibre diameter of 7 μm.

2.2. Single fibre tensile testing
Single-fibre tensile properties of glass and carbon fibres were determined using a semi-automated single-fibre testing machine (TexTechno, Favimat+, Germany). The fibre cross-sectional area \((A, m^2)\) was measured by the built-in vibroscope system. Initially, after a fibre is gripped between the two clamps in the machine, a small tension force is applied to the fibre and it is excited by acoustic waves to measure its resonance frequency. Based on the measured resonance frequency, the fibre linear density \((T, kg/m)\) is determined by using a recently developed iterative analysis method [10]. The method corrects for the fact that the fibre is not perfectly flexible (i.e. the factor \(E/L^2F\) is not neglected, where \(E\) is fibre modulus, \(L\) is fibre length and \(F\) is fibre pre-tension), and this leads to values of linear density that are slightly larger (1-2 %) than the values determined by assuming a simply supported boundary at the clamps. Using the fibre density \((\rho, kg/m^3)\), the fibre cross-sectional area can be calculated \((A = T/\rho)\).

The density of glass and carbon fibres was measured based on gas pycnometry method. A gas pycnometer (Ultrapyc 1200E; Quantachrome Instruments, USA) was used to measure the volume of fibres. The density of glass and carbon fibres were measured to be 2.597 ± 0.013 g/cm³ and 1.774 ± 0.002 g/cm³, respectively. The measured density values are very close to the manufacturer data sheet values of 2.6 g/cm³ and 1.80 g/cm³ for glass and carbon fibres, respectively. The density values based on the manufacturer data sheet were used to determine the fibre cross-sectional area in the vibroscope method described above.

The tensile tests were carried out with a crosshead speed of 1 mm/min. The approximate tensile test-time for each fibre was 1-2 minutes. In order to determine the machine and gripping compliance, glass and carbon fibres were tested at different gauge lengths (initial fibre length) in the range of 40-80 mm, and the data was analysed by a standard method for determination of compliance [11]. In this study, the analysis of the mechanical response of glass and carbon fibres was performed based on 150 fibres tested at a gauge length of 60 mm.

2.3. Stress-strain curve analysis
The mechanical response of the glass and carbon fibres were analysed using engineering and true stress-strain curves. The engineering stress-strain measures of nominal stress \((\sigma_n)\) and engineering strain \((\epsilon_e)\) are determined from the measured load \((P)\), initial cross-sectional area \((A_0)\), initial length \((L_0)\) and current length \((L)\) as follows:
\[ \sigma_n = \frac{P}{A_0} \] (1)

\[ \varepsilon_e = \frac{L - L_0}{L_0} \] (2)

The true stress-strain measures of true stress (\(\sigma_t\)) and logarithmic strain (\(\varepsilon_{ln}\)) are determined from the measured load (P) and current cross-sectional area (A) as follows:

(Assuming constant volume \(dV=0\), \(A_0/A = L/L_0\))

\[ \sigma_t = \frac{P}{A} = \frac{P}{A_0} \cdot \frac{L}{L_0} \] (3)

\[ \sigma_t = \sigma_e(1 + \varepsilon_e) \] (4)

The increment of strain is the incremental increase in displacement (\(dL\)) divided by the current length (\(L\)):

\[ d\varepsilon_{ln} = \frac{dL}{L} \] (5)

Integrating both sides

\[ \int_0^{\varepsilon_{ln}} d\varepsilon_{ln} = \varepsilon_{ln} \] (6)

\[ \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0} \] (7)

Using equation (2)

\[ \ln \frac{L}{L_0} = \ln(1 + \varepsilon_e) \] (8)

\[ \varepsilon_{ln} = \ln(1 + \varepsilon_e) \] (9)

The tangent modulus as a function of strain was determined from the stress-strain data for each tested fibre using the following method:

1) Range of strain was selected. Starting point of the range was the first point of measured strain (%) + 0.1 %. Final point of the range was the failure strain (%) - 0.1 %. This range of strain was divided into 1000 strain values.

2) Linear fitting was performed to calculate the corresponding tangent modulus value at each strain values with a window size of 0.20 % i.e. 0.10 % on each side of the strain value.

2.4. Observation of fibre cross-sectional shape

Samples with a rectangular area of 25x25 mm\(^2\) were cut from unidirectional glass and carbon composites manufactured in another study \[12\] with the same type of glass and carbon fibres used in this study. The samples were cast into cylindrical blocks of epoxy resin. Edges of the blocks were ground and polished. The polished surface of the blocks were sputtered with carbon particles with thickness \(\approx\)10 nanometres.
The cross-sectional shape of glass and carbon fibres was examined using scanning electron microscope (VEGA3 TESCAN, Czech Republic) at 1.38 kx magnification.

3. Results

3.1. Mechanical response of glass fibres
Single-fibre tensile test results of glass fibres have been analysed using engineering and true stress-strain curves as shown in Figure 1. The stress-strain curve for brittle materials like glass fibres is generally believed to be linear over the full range of strain, without showing appreciable plastic deformation. However, as shown in Figure 1a, the engineering stress-strain curves begin to deviate from linearity with increasing strain. In true stress-strain curves, as shown in Figure 1b, the glass fibres can be observed to behave linearly over the full range of strain.

![Figure 1](attachment:image1)

**Figure 1.** Stress-strain curves of single glass fibres: (a) engineering $\sigma_{en}-\varepsilon_{en}$, (b) true $\sigma_{t}-\varepsilon_{ln}$. Dotted line represents a constant modulus line calculated between strain 0.05 % and 0.25 %.

This can also be seen in the determined tangent modulus-strain curves, again comparing engineering and true stress-strain measures, as shown in Figure 2. In Figure 2a, the slope of the linear fit line was calculated to be -3.0 GPa/%. This trend demonstrates that the modulus is decreasing with increasing strain. In Figure 2b, modulus tends to remain almost constant with increasing strain, with the slope of the linear fit line being -0.6 GPa/%

The modulus of glass fibres has been determined using the following five methods, based on the engineering stress-strain curves, the true stress-strain curves and the true tangent modulus-strain curves. The results are summarized in Table 1.

1) Modulus based on using linear regression of the engineering stress-strain curve between strain 0.05 % and 0.25 %. This is the conventional method for determination of materials modulus according to the ISO 527 standard [13].

2) Modulus based on using linear regression of the true stress-strain curve between strain 0.05 % and 0.25 %.

3) Modulus based on the midpoint value of the linear fit line for the engineering tangent modulus-strain curve between strain 0.05 % and 0.25 %.

4) Modulus based on the midpoint value of the linear fit line for the true tangent modulus-strain curve between strain 0.05 % and 0.25 %.
Figure 2. Tangent modulus-strain curves for single glass fibres: (a) engineering $\varepsilon_e$, (b) logarithmic $\ln$. Dotted line (grey) represents a constant modulus line (based on stress-strain curves) calculated between strain 0.05 % and 0.25 %, and dashed line (black) represents a linear fit for the tangent modulus-strain curves.

5) Modulus based on the linear fit line for the true tangent modulus-strain curve at 1.5 % strain.

6) Modulus at 0 % strain determined as the intersection with the y-axis of the linear fit line for the tangent modulus-strain curve.

Table 1. Summary of determined modulus values of glass fibres.

| Modulus determination methods                                      | Modulus (GPa) |
|-------------------------------------------------------------------|---------------|
| Linear regression of the engineering stress-strain curve (0.05 % - 0.25 %) | 86.6 ± 1.5    |
| Linear regression of the true stress-strain curve (0.05 % - 0.25 %)   | 87.0 ± 1.6    |
| Linear fit of the engineering tangent modulus-strain curve (0.05 % - 0.25 %) | 88.5 ± 1.3    |
| Linear fit of the true tangent modulus-strain curve (0.05 % - 0.25 %)   | 89.4 ± 1.6    |
| Linear fit of the true tangent modulus-strain curve (at 1.5 %)       | 88.7 ± 1.6    |
| Linear fit of the true tangent modulus-strain curve (at 0 %)         | 89.5 ± 1.7    |

In Table 1, it can be seen that there, as expected, is a very little difference between the young modulus found performing a linear regression on the stress-strain curve of the engineering or true stress-strain measure. This is because, the strains in the 0.05% to 0.25% strain range is so small that the two stress-strain measure approximate each other. On the other hand, testing thin fibers, it can be a challenge to get a precise measure of the load and strain in this range. During the initial loadings, the influence of an initial alignment of the fibers may affect this precision. Nevertheless, observing a linear relation between the strain and tangent modulus measure, it can be seen that this linearity can be used to make a much more robust stiffness measurement. Making a linear fit of the tangent modulus versus strain in the full tested strain range, a good agreement between the linear fit and the tangent modulus variation is found, see Figure 2a-b. Based on the linear fit of the tangent modulus versus strain curve, modulus values of 88.5 ± 1.3 GPa for the engineering stress-strain case and 89.4 ± 1.6 GPa for the true stress-strain case was determined. Both values significant larger than the 86.6 ± 1.5 GPa and 87.0 ± 1.6 GPa found using a linear regression of the stress-strain curve in the 0.05-0.25% range. As it is not expected
that the glass-fiber material should behave significantly softer for smaller strains, those smaller stiffness values is judge to be influenced by this small measurement precision at the lower strain and load values. Therefore, a modulus of 88.5 ± 1.3 GPa in the engineering stress-strain domain and 89.4 ± 1.6 GPa in the true stress strain domain is found to be more trustworthy.

In addition, a rather constant slope of the stress-strain curve is found using the true stress-strain measure, see Figure 1b and 2b. The modulus values determined at 0 % and 1.5 % are 89.5 ± 1.7 GPa and 88.7 ± 1.6 GPa, respectively. Comparing the average values, modulus of glass fibre decreases by 0.9 % going from 0 % to 1.5 % strain. Considering the standard deviation values, it can be said that the measured modulus value of glass fibre is independent of the applied strain. In the commercial finite element code Abaqus, a linear elastic material will be linear in the true stress-strain measure when including geometrical non-linear effect [14]. It is therefore interesting to see that the glass-fibers, as a good approximation be considered as a linear elastic material using this true stress versus logarithmic strain measure.

3.2. Mechanical response of carbon fibres

Single-fibre tensile test results of carbon fibres have been analysed using engineering and true stress-strain curves as shown in Figure 3. Non-linear behaviour is observed for both engineering and true stress-strain curves. There is an appreciable increase in modulus with increasing strain, resulting in the curves to bend upwards. Curtis et al. [15] first observed this behaviour.

![Stress-strain curves of single carbon fibres](image)

**Figure 3.** Stress-strain curves of single carbon fibres (a) engineering $\sigma_e - \varepsilon_e$, (b) true $\sigma_t - \varepsilon_{ln}$. Dotted line represents a constant modulus line calculated between strain 0.05 % and 0.25 %.

Engineering and true tangent modulus-strain curves are shown in Figure 4. The dashed line represents the linear fit for the tangent modulus-strain curves of all the fibres. In Figure 4a and 4b, the increasing tendency of modulus with increasing strain can be observed, with the slope of the linear fit line being 36.5 GPa/% and 44.3 GPa/% for engineering and true curves, respectively.
Figure 4. Tangent modulus-strain curves for single carbon fibres: (a) engineering $\varepsilon_e$, (b) logarithmic $\varepsilon_{ln}$. Dotted line (grey) represents a constant modulus line (based on stress-strain curves) calculated between strain 0.05 % and 0.25 %, and dashed line (black) represents the linear fit for the tangent modulus-strain curves.

As can be seen in Figure 4, the increase in modulus of carbon fibres with applied strain appears to follow a two-step increment, and performing a single linear fitting of both these regions will give an incorrect estimate for modulus. As shown in Figure 5, the green dotted line represents the linear regression line from 0 % to 0.4 % strain, having a slope of 69.5 GPa/%. Similarly, the red dotted line represents the linear regression from 0.6 % strain until failure, having a slope of 38.1 GPa/%. The intersection of these two linear fit lines gives a transition point at 0.47 % strain.

Figure 5. True tangent modulus-strain curves for single carbon fibres (same as in Figure 4b). The two linear fit lines “green” and “red” represents the two-step tangent modulus curves.

The modulus of carbon fibres has been determined by the same five methods used for the glass fibres, based on the engineering stress-strain curves, the true stress-strain curves and the true two-step tangent modulus-strain curves. The results are summarized in Table 2.
Table 2. Summary of determined modulus values of carbon fibres.

| Modulus determination methods                                      | Modulus (GPa) |
|--------------------------------------------------------------------|---------------|
| Linear regression of the engineering stress-strain curve (0.05 % - 0.25 %) | 223 ± 6       |
| Linear regression of the true stress-strain curve (0.05 % - 0.25 %)   | 224 ± 6       |
| Linear fit of the engineering tangent modulus-strain curve (0.05 % - 0.25 %) * | 223 ± 7       |
| Linear fit of the true tangent modulus-strain curve (0.05 % - 0.25 %) * | 224 ± 7       |
| Linear fit of the true tangent modulus-strain curve (at 1.5 %) **     | 284 ± 9       |
| Linear fit of the true tangent modulus-strain curve (at 0 %) *       | 213 ± 6       |

* Analysis based on first linear fit line “green”. ** Analysis based on second linear fit line “red”

The modulus values determined at 0 % and 1.5 % are 213 ± 6 GPa and 284 ± 9 GPa, respectively. Comparing the average values, modulus of carbon fibre increases by 33 % going from 0 % to 1.5 % strain. This two-step tangent modulus-strain relation is based upon initial observations. Further in-depth analysis of this trend will be done to verify its existence.

3.3. Relationship between fibre diameter and modulus

In Figure 6, microscopic images of the cross-section of glass and carbon fibres are presented. It can be observed that glass fibres have an almost circular cross-section, while carbon fibres have clearly non-circular kidney-shaped cross-section. The average cross-sectional areas of the fibres were determined with the vibroscopy method using the measured linear density and density of fibres. The average cross-sectional area for glass and carbon fibres are 249 ± 43 μm² and 42 ± 3 μm², respectively. A representative diameter assuming the fibres to be circular in cross-section was calculated, and is only used here as a representative value for the cross-section area. The average diameters of glass and carbon were determined to be 18 ± 1 μm and 7 ± 0.3 μm, respectively.

![Figure 6](image_url)

**Figure 6.** SEM images showing the cross-section shape for (a) glass fibres and (b) carbon fibres.
Figure 7a shows the relation between modulus (linear regression of the true stress-strain curve (0.05 % - 0.25 %)) and diameter for glass fibres. The measured modulus appears to be independent of the fibre diameter. In Figure 7a, the dashed line represents the linear fit for modulus vs fibre diameter. The slope of the linear fit line was calculated to be 0.12 GPa/μm. It implies that a similar modulus was measured irrespective of the difference in the fibre diameter for glass fibres.

Figure 7b shows the relation between modulus and diameter for carbon fibres. The slope of the linear fit line for modulus vs fibre diameter relation was calculated to be -15.2 GPa/μm. It shows a tendency that the fibres with smaller diameters have higher modulus. During the manufacturing of PAN-based carbon fibres, there is a processing step called fibre spinning. The precursor fibres are drawn in a series of stages, in order to increase polymer chain alignment in the fibre axial direction and results in diameter reduction. In general, the precursor fibre tensile modulus will continue to increase as the total draw ratio increases [16]. Then, the conversion of the precursor fibre into carbonized structure requires two heat treatment processes, namely stabilization and carbonization. The fibres are again subjected to an applied force, which is an important processing parameter to influence mechanical properties. The process parameters are optimized to minimize fibre-to-fibre diameter variation and mechanical property variance. Due to the highly anisotropic nature of the graphite elements, the apparent modulus increases as the structural units reorients [17]. Carbon fibres with smaller diameters might have increased alignment of the crystallites, thus showing a higher modulus, as observed in Figure 7b.

Figure 7. Relationship between modulus and fibre diameter for (a) glass fibres and (b) carbon fibres.

4. Discussion
By implying a second-order polynomial relation between stress (σ) and strain (ε), a linear relation between tangent modulus and strain can be established as [18]:

\[ E_t = \frac{d\sigma}{d\varepsilon} = \alpha \cdot \varepsilon + E_0 \]  (10)

where \( E_t \) is the tangent modulus at a given value of \( \varepsilon \), \( E_0 \) is the initial stiffness (\( \varepsilon = 0 \)), and \( \alpha \) is a so-called stress-strain curvature coefficient, respectively.

Integrating the equation (10) with respect to \( \varepsilon \) (for intercept at origin) the second-order polynomial relation between stress (σ) and strain (ε) can be established:

\[ \sigma = \frac{1}{2} \cdot \alpha \cdot \varepsilon^2 + E_0 \cdot \varepsilon \]  (11)
Equation (11) can be directly used for glass fibres. Whereas for carbon fibres, based on the two-step modulus curves, Equation (11) can be written as:

\[
\sigma = \begin{cases} 
\frac{1}{2} \cdot \alpha_1 \cdot \varepsilon^2 + E_{0(1)} \cdot \varepsilon & \text{for } \varepsilon < \varepsilon_{tr} = 0.47 \% \\
\frac{1}{2} \cdot \alpha_2 \cdot \varepsilon^2 + E_{0(2)} \cdot \varepsilon & \text{for } \varepsilon > \varepsilon_{tr} = 0.47 \%
\end{cases}
\] (12)

Based on the true tangent modulus-strain curves for glass and carbon fibres in the present study, values for initial modulus $E_0$ at 0 % strain and $\alpha$ (= slope) can be found from the linear fitting lines, and they are summarised in Table 3.

**Table 3. Summary of initial modulus and curvature coefficient of glass and carbon fibres.**

|       | $E_0(1)$ (GPa) | $\alpha_1$ (GPa) | $E_0(2)$ (GPa) | $\alpha_2$ (GPa) |
|-------|----------------|------------------|----------------|------------------|
| Glass | 89.5 ± 1.7     | -57 ± 30         | -              | -                |
| Carbon| 213 ± 6        | +6950 ± 360      | 228 ± 7        | +3810 ± 340      |

Based on the initial modulus $E_0$ and curvature coefficient $\alpha$, stress can be calculated at any particular value of strain. Using these values, the whole stress-strain curve can be modelled.

5. Conclusions

1) The semi-automated single-fibre testing machine (TexTechno, Favimat+) made it possible to test large sample size of glass and carbon fibres, with reduced manual handling and inferences, thus enhancing the accuracy and reliability of the obtained fibre properties.

2) The true stress-strain curve of glass fibres exhibits linear behaviour, with constant modulus over the range of applied strain. The modulus values determined at 0 % and 1.5 % were 89.5 ± 1.7 GPa and 88.7 ± 1.6 GPa, respectively. This linear behaviour makes it possible to determine a reliable fibre modulus based on a larger strain range.

3) The measured modulus of the glass fibres in the 0.05 %-0.25 % strain range was found to be significantly lower compared with the modulus based on a larger strain range. The measurement based on 0.05 %-0.25 % strain range may include some larger measurements error due to smaller displacement and force values and an influence of the initial aligning of the fiber during the initial loading and initial deformation of the rubber grips.

4) The stress-strain curve of carbon fibres exhibits non-linear behaviour, with increasing modulus with applied strain. The modulus values determined at 0 % and 1.5 % are 213 ± 6 GPa and 284 ± 9 GPa, respectively.

5) The study of relationship between fibre diameter and modulus revealed a similar modulus for glass fibres irrespective of the difference in the diameter. For carbon fibres, the fibres with smaller diameters showed a higher modulus.

6) The second-order polynomial relation between stress ($\sigma$) and strain ($\varepsilon$) based on the initial modulus $E_0$ and curvature coefficient $\alpha$ is useful tool to calculated stress at any particular value of strain and to model the whole stress-strain curve.

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