CONSTRAINTS ON SPATIAL VARIATIONS IN THE FINE-STRUCTURE CONSTANT FROM PLANCK

Jon O’Bryan, Joseph Smidt, Francesco De Bernardis, and Asantha Cooray

Department of Physics and Astronomy, University of California, Irvine, CA 92697, USA; jobryan@uci.edu

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ABSTRACT

We use the cosmic microwave background (CMB) anisotropy data from Planck to constrain the spatial fluctuations of the fine-structure constant \( \alpha \) at a redshift of 1100. We use a quadratic estimator to measure the four-point correlation function of the CMB temperature anisotropies and extract the angular power spectrum fine-structure constant spatial variations projected along the line of sight at the last scattering surface. At tens of degree angular scales and above, we constrain the fractional rms fluctuations of the fine-structure constant to be \( (\delta \alpha/\alpha)_{\text{rms}} < 3.4 \times 10^{-3} \) at the 68% confidence level. We find no evidence for a spatially varying \( \alpha \) at a redshift of \( 10^3 \).

Key words: cosmic background radiation – cosmological parameters – inflation

1. INTRODUCTION

One of the key questions of modern physics concerns the possibility that physical constants vary across space and time in the history of the universe. One possible variation that has received recent attention is that of the fine-structure constant, \( \alpha \). The standard value of \( \alpha \) from measurements of the electron magnetic moment anomaly is \( \alpha = 1/137.035999074(44) \) (Mohr et al. 2012). In recent years there has been a great deal of attention given to the possible time and spatial variations of \( \alpha \). From the theory side, such variations are expected from unification (Uzan 2003) and inflation (Bekenstein 2002). From the observational side, contradictory results on the time variability from Webb et al. (1999) and Srianand et al. (2004) regarding absorption line systems have motivated further studies on both the spatial dependence and time variations of \( \alpha \).

Given Thompson scattering of cosmic microwave background (CMB) photons, the CMB anisotropy power spectrum probes the value of \( \alpha \) at the last scattering surface at a redshift \( z \) of 1100 (Nakashima et al. 2008; Martins et al. 2004; Menegoni et al. 2012; Rocha et al. 2004). The constraint comes from the variations to the visibility function, or the probability for a photon to scatter at redshift \( z \), at the last scattering surface. This visibility function is a function of \( \alpha \) and time variations in \( \alpha \) affect the recombination by changing the shape and shifting in time the visibility function, which in turn affect the shape and position of the peaks of the CMB angular power spectrum. The recent Planck analysis (Planck Collaboration et al. 2014) finds time-dependent variations to be constrained to \( \Delta \alpha/\alpha = (3.6 \pm 3.7) \times 10^{-3} \) at the 68% confidence level. They additionally constrain dipolar spatial variations to be \( \delta \alpha/\alpha = (-2.4 \pm 3.7) \times 10^{-2} \) (Planck Collaboration et al. 2014).

Moving beyond the time dependence, it is also useful to consider spatial dependence of \( \alpha \). Spatial variations are expected and present in most theoretical models that also introduce a time variation. We highlight two models of interest here. The first involves a scalar particle coupled to the electromagnetic force leading to loop corrections to \( \alpha \) and spatial variations through spontaneous symmetry breaking (Steinhardt 2004). The second involves a cosmological mechanism typical in axion fields where spatial variations in a coupled scalar field arise quantum mechanically during inflation (Sigurdson et al. 2003).

Observationally, an initial claim for spatially varying \( \alpha \) exists in the literature with quasar absorption line studies using the Keck Telescope and the Very Large Telescope by Webb et al. (1999; King et al. 2012) in the form of a dipole with a statistical significance of 4.2\( \sigma \).

While in recent years the CMB has been used to study the global value of \( \alpha \), CMB anisotropies can also be used to study any spatial variations in \( \alpha \) at the last scattering surface. If there is some underlying physics responsible for variations in \( \alpha \) prior to the last scattering one expects \( \alpha \) variations to be imprinted on the CMB at the horizon scale and larger. Here we present a first study of such a constraint by making use of the Planck CMB maps. We highlight that this measurement we report here is a constraint on the spatial fluctuations and not the mean or globally averaged value of \( \alpha \) that can be studied from the angular power spectrum. Thus our result we report here will not be directly comparable to quoted \( \alpha \) values in the literature from the CMB power spectrum data.

This paper is organized as follows. In Section 2, we discuss the effects of small spatial variations in \( \alpha \) on the CMB temperature maps, their signature in the four-point correlation function (trispectrum), and derive an estimator to measure these effects. In Section 3, we present our results and discuss constraints on spatial variations in \( \alpha \) as well as future directions.

2. EFFECTS OF PERTURBATIONS IN \( \alpha \) ON THE CMB TEMPERATURE MAP

The signature of spatial variations in \( \alpha \) exist at the four-point function of the CMB anisotropies. Thus an optimal estimator that can measure the trispectrum (Hu 2001), the harmonic or Fourier analog of the four-point correlation function, induced by \( \alpha \) variations is needed to constrain the spatial fluctuations of \( \alpha \). To calculate the observable effects of a spatially dependent \( \alpha \) on the CMB temperature map we follow an approach similar to Sigurdson et al. (2003). We first perform a spherical harmonics expansion of the temperature field \( \theta \):

\[
\tilde{\theta}_{\ell m} \approx \theta_{\ell m} + \int dn \, Y_{\ell m}^* \frac{\partial \theta}{\partial \alpha} \frac{\partial \alpha}{\partial \alpha} + \frac{1}{2} \int dn \, Y_{\ell m}^* (\delta \alpha)^2 \frac{\partial^2 \theta}{\partial \alpha^2} \tag{1}
\]

This allows for a quadratic estimator to measure variations in the fine-structure constant.
respectively. In the above \( \delta \alpha \) captures the line-of-sight projected spatial variations in \( \alpha \) at the last scattering surface. It modifies the temperature field by coupling to the spatial derivatives of the temperature field \( \theta \) with respect to the fine-structure constant. It can be shown that, retaining first-order corrections, no signal from \( \delta \alpha \) is present in the two-point (power spectrum) or three-point (bispectrum) correlation function of the CMB temperature \( \theta \) (This is because we do not see \( \delta \alpha \) terms in correlation functions of \( \theta \) until we go to the fourth order in \( \theta \)). The highest-order corrections related to \( \delta \alpha \) are only visible in the CMB at the four-point level of statistics. We thus focus on its effects on the four-point correlation function or, more naturally in terms of the measurement, on the trispectrum.

Furthermore, hereafter we assume these line-of-sight \( \delta \alpha \) fluctuations in the fine-structure constant are Gaussian about the mean value of \( \alpha \) at \( z = 10^3 \). The line-of-sight projected angular power spectrum can be written as \( \langle \delta \alpha \rangle \langle \delta \alpha \rangle \langle \delta \alpha \rangle \langle \delta \alpha \rangle \rangle \). Our primary goal in this work is a measurement of \( C_\alpha \) from Planck data. A non-zero measurement of \( C_\alpha \) will establish the presence of \( \delta \alpha \) fluctuations at the last scattering surface and the range in \( \ell \) values over which a non-detection is detected will establish the angular scales on the sky over which \( \delta \alpha \) varies from one region of the last scattering surface to another. We assume that the mean value of \( \alpha \), averaged over the last scattering surface, is the standard value and hereafter we fix all other cosmological parameters to the best-fit Planck model (Planck Collaboration et al. 2014).

2.1. Analytical Effects in the Trispectrum

The trispectrum can be written as the sum of a Gaussian component and a connected term:

\[
\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle = \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle_G + \langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle_c,
\]

where the \( a_{\ell m} \) are the coefficients of the spherical harmonic expansion. In our study the connected term of the Fourier transform, that is, the term remaining after the Gaussian component is subtracted in Equation (5), represents the trispectrum resulting from non-Gaussian correlations due to \( \delta \alpha \). The Gaussian and connected pieces can be expanded as (Hu 2001)

\[
\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} a_{\ell_4 m_4} \rangle_G = \sum_{LM} (-1)^L T^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) \left( l_1 m_1 l_2 m_2 l_3 m_3 l_4 m_4 - L \right),
\]

where the quantities in parentheses are the Wigner-3j symbols. The two functions \( G^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) \) and \( T^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) \) for the Gaussian and connected components, respectively, can be derived analytically. Proceeding from the expansion in Equation (1), after some tedious but straightforward algebra, we arrive at

\[
C^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) = (-1)^{\ell_1 + \ell_2} \sqrt{(2\ell_1 + 1)(2\ell_2 + 1)} \times C_{\ell_1} C_{\ell_2} \delta_{\ell_1 \ell_3} \delta_{\ell_2 \ell_4} + (2\ell_1 + 1)C_{\ell_1} C_{\ell_2} \times \left( \delta_{\ell_1 \ell_3} \delta_{\ell_2 \ell_4} + \delta_{\ell_1 \ell_4} \delta_{\ell_2 \ell_3} \right).
\]

and

\[
T^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) = C_{\ell_1}^\alpha C_{\ell_2}^\alpha F_{\ell_1 \ell_2}^\alpha F_{\ell_3 \ell_3}^\alpha \times \left( C_{\ell_1}^{0\beta_1/\beta_2} + C_{\ell_2}^{0\beta_1/\beta_2} \right) \left( C_{\ell_3}^{0\beta_3/\beta_4} + C_{\ell_4}^{0\beta_3/\beta_4} \right),
\]

where

\[
F_{\ell_1 \ell_2 \ell_3} = \left( \frac{(2\ell_1 + 1)(2\ell_2 + 1)(2\ell_3 + 3)}{4\pi} \right)^{1/2} \left( l_1 m_1 l_2 m_2 l_3 m_3 0 0 0 \right).
\]

2.2. Measuring Effects in the Wilkinson Microwave Anisotropy Probe with the Trispectrum Estimator

Expanding Equation (9), we have

\[
T^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) = C_{\ell_1}^\alpha C_{\ell_2}^\alpha F_{\ell_1 \ell_2}^\alpha F_{\ell_3 \ell_3}^\alpha \times \left( C_{\ell_1}^{0\beta_1/\beta_2} + C_{\ell_2}^{0\beta_1/\beta_2} \right) \left( C_{\ell_3}^{0\beta_3/\beta_4} + C_{\ell_4}^{0\beta_3/\beta_4} \right),
\]

For simplicity, we rewrite this as

\[
T^{\ell_1 \ell_2}_{\ell_3 \ell_4}(L) = F_{\ell_1 \ell_2} F_{\ell_3 \ell_3} \left( \alpha_{\ell_4} \right) \left( \beta_{\ell_4} \right) \left( \gamma_{\ell_4} \right) \left( \delta_{\ell_4} \right),
\]

where the functions \( \alpha_{\ell_4}, \beta_{\ell_4}, \gamma_{\ell_4}, \delta_{\ell_4} \) are given in Table 1.

In terms of measuring this trispectrum from the Planck map, we need an estimator. Following Smidt et al. (2011), the trispectrum estimator is written as

\[
F_{\ell_1 \ell_2 \ell_3 \ell_4}(L) = \frac{1}{(2\ell_1 + 1)} \sum_m \left[ A^{(\ell_1)} B^{(\ell_2)} C_{\ell_3} D^{(\ell_4)} \right]_{\ell m},
\]
where the functions in the square brackets in Equation (13) are

\[ A^{(x)}_{\ell m} = \frac{a^{(x)}_\ell}{\tilde{C}^x_\ell} b_\ell a_{\ell m}, \quad B^{(x)}_{\ell m} = \frac{b^{(x)}_\ell}{\tilde{C}^x_\ell} b_\ell a_{\ell m}, \]

\[ G^{(x)}_{\ell m} = \frac{\delta^{(x)}_\ell}{\tilde{C}^x_\ell} b_\ell a_{\ell m}, \quad D^{(x)}_{\ell m} = \frac{\gamma^{(x)}_\ell}{\tilde{C}^x_\ell} b_\ell a_{\ell m}, \]

where \(a_{\ell m}\) are the Fourier coefficients from the Planck map, \(b_\ell\) is the beam transfer function of Planck, and \(\tilde{C}^x_\ell\) is the total power spectrum accounting for noise and beam effects. We write this total power spectrum as \(\tilde{C}^x_\ell = C^x_\ell b_\ell^2 + N_\ell\), with the noise power spectrum given by \(N_\ell\). In addition to the Planck data map, the team has publicly released \(b_\ell\) and the noise map allowing the noise construction to be done exactly. The above estimator for the trispectrum could simply be understood as the power spectrum of squared temperature map. A second estimator for the trispectrum could be designed by taking the power spectrum of the cubic temperature map correlated with the temperature map, \(K_{\ell,\text{data}}\). We do not pursue this three-to-one correlation here as we found it to have a lower signal-to-noise ratio than the two-to-two correlations.

The analogous analytical form of the trispectrum estimator can be obtained by expanding the data estimator with the above weighted maps:

\[ k_{\ell,\text{ana}}^{(2,2)} = \frac{1}{(2\ell + 1)^4} \sum_m \left[ A^{(x)}_{\ell m} B^{(x)}_{\ell m} G^{(x)}_m D^{(x)}_m \right]_{\ell m} \]

\[ = \frac{1}{(2\ell + 1)^4} \sum_m \left[ a^{(x)}_\ell a_{\ell m_1} b^{(x)}_\ell a_{\ell m_2} b^{(x)}_\ell a_{\ell m_3} b^{(x)}_\ell a_{\ell m_4} \right]_{\ell m} \]

\[ = \frac{1}{(2\ell + 1)^4} \sum_m \left[ \left( \frac{c^{(x)\delta}_0}{C^x_0} + C^{(x)\delta}_0 \right) \left( C^{(x)\delta}_0 + C^{(x)\delta}_0 \right) \right]_{\ell m} \]

\[ \times \left( a_{\ell m_1} a_{\ell m_2} a_{\ell m_3} a_{\ell m_4} \right) \]

\[ = \frac{C^{(x)\delta}_0}{(2\ell + 1)^4} \sum_\ell \left[ \frac{F^2_\ell}{C^x_\ell} \frac{F^2_{\ell'}}{C^x_{\ell'}} \right] \times \left( C^{(x)\delta}_0 + C^{(x)\delta}_0 \right)^2. \]

In the above derivation, we have used the connected piece of the trispectrum and would simply replace this with the Gaussian piece to determine the Gaussian estimator. Note that \(k_{\ell,\text{comm}}^{(2,2)} \propto C^{(x)\delta}_0\) and a direct comparison of \(k_{\ell,\text{ana}}^{(2,2)}\) to \(k_{\ell,\text{data}}^{(2,2)}\) under the assumption of \(C^{(x)\delta}_0 = 1\) in the analytical calculation results in a measurement of \(C^{(x)\delta}_0\) from the data. Before this comparison can be made, we note that \(k_{\ell,\text{data}}\) in Equation (13) also includes a Gaussian contribution. This has to be removed from the data through numerical simulations and is equivalent to the removal of the noise bias from angular power spectrum measurements from the data.

In the analytical calculations and to define the four \(\alpha, \beta, \gamma, \delta\) functions in the estimator, we used a modified version of camb (Lewis et al. 2000). To handle a varied \(\alpha\) in the camb (which is used in the derivative power spectrum calculations), we must take into account its effects on the photon visibility function. Replication of these modifications can be achieved by taking into account the effects of \(\alpha\) upon the Thompson scattering cross section, the hydrogen binding energy, the ionization coefficient, the recombination coefficient, and the recombination rates. (For a detailed discussion of this dependence, see Sigurdson et al. 2003.) Figure 1 shows the derivative power spectra \(C^{(x)\delta\beta}_0\) and \(C^{(x)\delta\gamma}_0\) and \(C^{(x)\delta\delta}_0\). For the analysis presented here the noise power spectrum for Planck was obtained from the publicly available SMICA (Planck Collaboration et al. 2014) noise map. In addition to beam and noise effects, corrections to the power spectrum must also be made to account for the masking of the Galactic plane and point sources, among others, with the mask \(W(n)\). The masking results in mode-coupling and can be corrected again through simulations. It was shown by Hivon et al. (2001) that the masking effects on the temperature maps can be removed in the resulting power spectrum by correcting \(C_\ell\) as

\[ \tilde{C}_\ell = \sum_\ell M_{\ell'} C_{\ell'}, \]

where \(M_{\ell'}\) is defined as

\[ M_{\ell'} = \frac{2\ell' + 1}{4\pi} \sum_{\ell''} (2\ell'' + 1) W_{\ell''} \left( \ell, \ell', \ell'' \right)^2, \]

where \(W_{\ell'}\) is the power spectrum of the mask \(W(N)\).

First in order to establish the Gaussian noise bias to the connected two-to-two power spectrum and to account for effects of masking, we created Gaussian simulations using the publicly available healpix software (Gorski et al. 2005) and Equation (13) with the \(a_{\ell m}'s\) obtained from Gaussian realizations of the Planck map, including detector noise as established by the SMICA map. By averaging over the Gaussian simulations, where there are no effects due to \(\delta\) fluctuations, we establish the Gaussian noise term. This is then subtracted from the full trispectrum estimator \(k_{\ell,\text{data}}^{(2,2)}\) (Equation (13)) to obtain only the connected term generated by any non-Gaussian signals in the data, in this case primarily due to \(\delta\) fluctuations.

The full estimator and Gaussian piece are shown in Figure 2. After calculating \(k_{\ell,\text{ana}}^{(2,2)}\) (Equation (16) analytically assuming
We find $\frac{\delta \alpha}{\alpha}$ corresponding to angular scales above $10^{\pi}$ at the 68% confidence level and over the range of $2 < \ell < 500$, respectively, at $z = 10^{\pi}$ for SMICA and SMICA with noise removed, respectively, due to the mask, and for the uncertainty correction, to correct for the noise. We use a total of 250 simulated maps here for the noise-bias correction, to correct for the mask, and for the uncertainty estimates, with the number of simulations restricted by the computational resources to perform this measurement over three weeks. Figure 3 shows the angular power spectrum for spatial variations of $\alpha$, $C_\ell^{\alpha\alpha}$.

3. DISCUSSION

As can be seen in Figure 3 the measured $C_\ell^{\alpha\alpha}$ is consistent with zero, showing no evidence for spatial variations of $\alpha$ when projected at the last scattering surface at a redshift of $10^{\pi}$. The most significant fluctuations are observed for the very low multipoles ($\ell < 5$). However, the value of $C_\ell^{\alpha\alpha}$ is always consistent with zero at the 2σ confidence level. We also repeated the analysis described above by keeping the detector noise signal in the original Planck CMB data map to highlight the possible biasing effects due to the noise. The results are shown in Figure 3. We find that the noise bias is not substantially affecting the analysis at multipoles less than 300 but is a concern at higher multipoles where noise begins to dominate. From the measured $C_\ell^{\alpha\alpha}$, we obtain the line-of-sight projected rms fluctuation of $\delta \alpha$, properly normalized to the value of $\alpha$ today, using $(\delta \alpha/\alpha)^2_{\text{rms}} = (1/4\pi) \sum_\ell (2\ell + 1)C_\ell^{\alpha\alpha}$. From our measurements, we find $(\delta \alpha/\alpha)^2_{\text{rms}}(z = 10^{\pi}) < 6.7 \times 10^{-3}$ and $< 3.4 \times 10^{-3}$ for SMICA and SMICA with noise removed, respectively, at the 68% confidence level and over the range of $2 < \ell < 20$, corresponding to angular scales above $10^5$ or super-horizon scales of the CMB. If we assume $C_\ell^{\alpha\alpha} = A$, a white noise-like power spectrum, then we find $A = (-5.7 \pm 9.4) \times 10^{-5}$ and $(-1.6 \pm 4.8) \times 10^{-6}$ when $2 < \ell < 20$ and $20 < \ell < 500$, respectively. Assuming $C_\ell^{\alpha\alpha}$ is a constant independent of $\ell$ the reduced $\chi^2$ values of the fit are 1.55 and 0.508 for SMICA and SMICA with noise removed, respectively.

Our overall constraint on the spatial fluctuations of $\alpha$ is from the trispectrum and other non-Gaussian mechanisms that also generate a signal in the trispectrum could easily contaminate or confuse the $\delta \alpha$ variations. The largest signal in the CMB trispectrum is expected from gravitational lensing of CMB photons. The lensing perturbations couple to the spatial gradient or super-horizon expansion the amplitude of the fluctuations will subsequently decay as the inverse of time, $t^{-2}$. Our upper limit on the $\delta \alpha$ fluctuations, assuming this model description is correct, then would imply $\delta \alpha$ fluctuations at the level of $10^{-22}$ eV, the resulting $\delta \alpha$ fluctuations will be frozen at the time of last scattering. With cosmological expansion the amplitude of the fluctuations will subsequently decay as the inverse of time, $t^{-3}$. Our upper limit on the $\delta \alpha$ fluctuations, assuming this model description is correct, then would imply $\delta \alpha$ fluctuations at the level of $10^{-27}$ eV, the resulting $\delta \alpha$ fluctuations will be frozen at the time of last scattering. With cosmological expansion the amplitude of the fluctuations will subsequently decay as the inverse of time, $t^{-3}$. Our upper limit on the $\delta \alpha$ fluctuations, assuming this model description is correct, then would imply $\delta \alpha$ fluctuations at the level of $10^{-27}$ eV, the resulting $\delta \alpha$ fluctuations will be frozen at the time of last scattering. With cosmological expansion the amplitude of the fluctuations will subsequently decay as the inverse of time, $t^{-3}$. Our upper limit on the $\delta \alpha$ fluctuations, assuming this model description is correct, then would imply $\delta \alpha$ fluctuations at the level of $10^{-27}$ eV, the resulting $\delta \alpha$ fluctuations will be frozen at the time of last scattering. With cosmological expansion the amplitude of the fluctuations will subsequently decay as the inverse of time, $t^{-3}$. Our upper limit on the $\delta \alpha$ fluctuations, assuming this model description is correct, then would imply $\delta \alpha$ fluctuations at the level of $10^{-27}$ eV, the resulting $\delta \alpha$ fluctuations will be frozen at the time of last scattering. With cosmological expansion the amplitude of the fluctuations will subsequently decay as the inverse of time, $t^{-3}$. Our upper limit on the $\delta \alpha$ fluctuations, assuming this model description is correct, then would imply $\delta \alpha$ fluctuations at the level of $10^{-27}$ eV, the resulting $\delta \alpha$ fluctuations will be frozen at the time of last scattering.
a light scalar field coupled to photons to generate $\delta \alpha$ fluctuations. In the future we expect another one to two orders of magnitude improvement in the $(\delta \alpha/\alpha)_{\text{rms}}(z = 10^3)$ constraint with high sensitivity CMB polarization maps and their trispectra with the cosmic variance limit for rms fluctuation detection at the level of $5 \times 10^{-5}$ with CMB.

4. CONCLUSIONS

We have used CMB anisotropies as measured by Planck to place limits on the amount by which $\alpha$ may vary spatially. This was done by using an estimator constructed for the trispectrum to determine $C_{\ell}^{\alpha \alpha}$ from which we are able to obtain information about the spatial variations in $\alpha$. Considering the region with maximal signal, we obtained constraints of $(\delta \alpha/\alpha)_{\text{rms}} < 3.4 \times 10^{-3}$ at the $1\sigma$ confidence level. At a redshift of $z \sim 1100$, we find no sign of spatial variations in the fine-structure constant.

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