On the Veltman Condition, the Hierarchy Problem and High-Scale Supersymmetry

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Abstract

In this paper we have considered the possibility that the Standard Model, and its minimal extension with the addition of singlets, merges with a high-scale supersymmetric theory at a scale satisfying the Veltman condition and therefore with no sensitivity to the cutoff. The matching of the Standard Model is achieved at Planckian scales. In its complex singlet extension the matching scale depends on the strength of the coupling between the singlet and Higgs fields. For order one values of the coupling, still in the perturbative region, the matching scale can be located in the TeV ballpark. Even in the absence of quadratic divergences there remains a finite adjustment of the parameters in the high-energy theory which should guarantee that the Higgs and the singlets in the low-energy theory are kept light. This fine-tuning (unrelated to quadratic divergences) is the entire responsibility of the ultraviolet theory and remains as the missing ingredient to provide a full solution to the hierarchy problem.
I. INTRODUCTION

In view of the recent discovery of a particle consistent with the Standard Model (SM) Higgs boson with a mass $m_H \sim 126$ GeV, announced by the ATLAS [1] and CMS [2] collaborations at CERN, the issue of quadratic divergences in the Standard Model Higgs self-energy gains interest. Indeed quadratic divergences are indicative of the fact that the natural order of magnitude of the Higgs mass is $O(f_L \Lambda)$ where $f_L$ is a loop factor and $\Lambda$ represents the scale of new physics beyond the Standard Model (BSM).

Quadratic divergences in the Standard Model were studied by Veltman [3] in the context of dimensional regularization getting the one-loop condition

$$m_H^2 + 2m_W^2 + m_Z^2 - 4m_t^2 = 0 \quad (1)$$

which is satisfied for a value of the Higgs mass $m_H \sim 314$ GeV in flagrant conflict with experimental data.

Meanwhile BSM theories aiming to solve the problem of quadratic divergences have been postulated. There are essentially two class of such theories:

- Theories where the Higgs is composite in the infrared by some strong dynamics. Therefore at high energies the Higgs dissolves into its constituents, only fermionic matter is present and there are no quadratic divergences. This solution is unrelated to the Veltman condition and we will not be concerned about it.

- Theories with extra fields and an extra symmetry such that the new fields with couplings dictated by the symmetry cancel the quadratic divergences of the Higgs self-energy. The prototype of such theories is supersymmetry and in particular the minimal supersymmetric extension of the Standard Model (MSSM). Unlike the previous solution the theory remains perturbative up to high scales. In a supersymmetric theory the absence of quadratic divergences is automatically satisfied. In this paper we will consider this kind of solutions.

The search of supersymmetric particles is being the subject of an intense experimental activity at LHC [5] although for the moment only negative results have been collected

1 Notice that the result corresponding to Eq. (1) was implicit in the early work of Ref. [4], through a tadpole diagram for the physical Higgs contributing to the quantity $\delta m_i^2/m_i^2$ ($i = q, t, W, Z, H$) computed in the broken phase.

2 Notice that Eq. (1) translates the absence of quadratic divergences only in the Standard Model. This equation is modified in extensions of the Standard Model as we will see later on in this paper. For examples in supersymmetric extensions cancellation of quadratic divergences is automatic as every term in Eq. (1) has a counterpart from its respective supersymmetric partner equal in absolute value and with different sign.
and only lower bounds on the mass of supersymmetric particles can be set. Most likely
supersymmetry, if it exists at all, is only realized at high-scale.

In this article we will link both facts: the non appearance of supersymmetric particles
at the LHC energies and the fact that the Veltman condition is not satisfied for the
measured value of the Higgs mass. We will consider that the Veltman condition, although
it is certainly not satisfied at the electroweak scales, can take place at some high-energy
scale $\mu_V$ at which a supersymmetric theory takes over. Speculations on the possibility
that the Veltman condition is not satisfied at low energy but at high scales have already
been considered in earlier studies [6–8] when the Higgs mass was not known and thus
no firm conclusions could be drawn. More recently some authors have reconsidered the
Veltman condition and pointed out [9–12] that it is indeed satisfied close to the Planck
scale.

The theory below $\mu_V$ is an effective Standard Model (or some minimal extension
thereof) where the Veltman condition implies that there are no quadratic divergences.
Therefore considering the SM as an effective theory valid for scales below $\mu_V$ the Higgs
mass is not sensitive to the cutoff scale. Beyond $\mu_V$ the theory is assumed to be super-
symmetric and thus the sensitivity to scales larger than $\mu_V$ is canceled by supersymmetry.
Still the UV theory requires some fine-tuned relation among its parameters to match both
theories at the scale $\mu_V$. This tuning is the remnant of the old SM hierarchy problem.
Either it can have some environmental origin (e.g. due to the huge number of vacua of the
fundamental theory) or perhaps it is guaranteed by some extra symmetry, or even it is a
fine-tuning we have to live with (as it is the case of the cosmological constant problem).
In any case the fine-tuning should be provided by the UV theory.

The paper is organized as follows. In section II the problem of quadratic divergences
in the Standard Model is reviewed along with the relation between the Veltman condition
and the $\text{Str} M^2$. In section III the one-loop Veltman condition is numerically analyzed
in the Standard Model using the NNLO running (three-loop for the beta functions and
two-loop for the couplings matching) renormalization group equation. We show that
for realistic values of the top quark and Higgs masses the Veltman condition is satisfied
around Planckian scales. In section IV we study the merging of the Standard Model with
the MSSM at the scale at which the Veltman condition is satisfied and with a predicted
value of the parameter $\tan \beta$. Within the experimental errors in the top quark and Higgs
masses and in $\alpha_3(m_Z)$ the value of $\tan \beta$ in the high-scale MSSM lies in the interval
$1 \lesssim \tan \beta \lesssim 2$. The merging of the Standard Model and the MSSM at the Veltman scale
requires a fine-tuning which is the remnant of the hierarchy problem in the absence of
quadratic divergences. In section V we study the Veltman condition in the presence of
a complex singlet coupled to the Standard Model Higgs with coupling $\lambda_{SH}(\mu)$. Actually
the scale at which the Veltman condition is fulfilled depends on the value of $\lambda_{SH}(m_Z)$. In
fact for $\lambda_{SH}(m_Z) \simeq \mathcal{O}(1)$ the Veltman scale is $\mathcal{O}$(TeV). Moreover in order to fulfill
the Veltman condition along the singlet field some massive vector-like fermions, coupled to the singlet, need to be introduced. These fermions can be Standard Model singlets in order to not perturb the running of the Veltman condition along the Higgs field. In section VI the merging of the previous theory with the MSSM with the addition of singlet fields is done at the Veltman scale. We show that the merging with the supersymmetric theory cannot be done for any value of $\lambda_{SH}(m_Z)$. In fact it can only be done either for very small values of $\lambda_{SH}(m_Z)$ (essentially zero, i.e. the Standard Model case) or for $\lambda_{SH}(m_Z) \gtrsim 0.3$. For given values of the parameters in the non-supersymmetric theory $\tan \beta$ in the high-scale supersymmetric theory is a prediction. The addition of the singlet requires an additional fine-tuning in the supersymmetric theory, on top of the MSSM’s one, to insure the lightness of the Standard Model Higgs and a second one to satisfy the Veltman condition along the direction of the singlet. Finally section VII contains our conclusions.

II. QUADRATIC DIVERGENCES IN THE STANDARD MODEL

According to [3] within the framework of dimensional regularization a suitable criterion to address the issue of quadratic divergences is the occurrence of poles in the complex dimensional plane for $D$ less than four. In particular at the $n$-loop level, a quadratic divergence corresponds to a pole at $D = 4 - 2/n$. Naive quadratic divergences at the one-loop level thus correspond to poles for $D = 2$.

Inquiring after the existence of poles for $D = 2$ in the SM, it was realized by Veltman [3] that such poles exist in vector boson and Higgs self energy diagrams. In particular for the Higgs mass they correspond to the shift, $m^2_H \rightarrow m^2_H + \delta m^2_H$, and the divergence has the form

$$\delta m^2_H = \frac{\Lambda^2}{16\pi^2} C_V , \quad C_V = \sum_{n \geq 1} C_{Vn}, \quad (2)$$

where the contribution $C_{Vn}$ is associated to $n$ loops. In particular at one-loop the Standard Model result is [3]

$$C_{V1} = \frac{3}{v^2} (m^2_H + m^2_Z + 2m^2_W - 4m^2_t). \quad (3)$$

The condition for the absence of the quadratic divergences at one loop stemming from the cancellation between the fermion and boson masses, $C_{V1} = 0$, is known as the Veltman condition at 1-loop (VC1) [3] and was dubbed ”semi-natural” by Veltman himself.

A very simple way to understand the Veltman condition in the Standard Model and generalizations thereof is starting from the one-loop effective potential in the presence of

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3 And also in tadpole diagrams and in connection with the cosmological constant.
4 A similar structure holds for the vector bosons.
the (constant) background Higgs field configuration $\phi$

$$V^{(1)}(\phi) = \frac{1}{64\pi^2} \int \! d^4 k \, Str \left[ \log(k^2 + M^2(\phi)) \right]$$

where

$$Str \left[ \log(k^2 + M^2(\phi)) \right] = \sum_{J=0,\frac{1}{2},1} (-1)^{2J} (2J + 1) \, Tr \left[ \log(k^2 + M^2_J(\phi)) \right]$$

and where $M^2_J(\phi)$ is the matrix of the second derivatives of the Lagrangian at zero momentum $k$ for spin $J$ fields. The mass matrix is thus obtained from $M^2_J(\phi)$ by inserting the vacuum expectation value $\phi = v$, where $v$ is the location of the minimum of the effective potential.

The UV divergences of the one loop effective potential can be displayed by expanding the integrand in powers of large $k$. Writing

$$\log(k^2 + M^2_J) = \log k^2 + \frac{M^2_J}{k^2} - \frac{1}{2} \frac{M^4_J}{k^4} + ...$$

leads to

$$V^{(1)}(\phi) = \frac{1}{64\pi^2} \left[ Str \mathcal{I} \int \frac{d^4 k}{(2\pi)^4} \log k^2 + Str \mathcal{M}^2(\phi) \int \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} + ... \right].$$

If a UV cutoff $\Lambda$ is introduced the first term is a pure (cosmological) constant term with coefficient proportional to $Str \mathcal{I} = n_B - n_F$ which vanishes in theories with equal number of bosonic ($n_B$) and fermionic ($n_F$) degrees of freedom (as e.g. supersymmetric theories). The second term is of order $\Lambda^2$ and determines the presence of quadratic divergences at the one-loop level. Therefore quadratic divergences are absent provided that $Str \mathcal{M}^2 = 0$. More precisely one can even tolerate $Str \mathcal{M}^2 = constant$ since this would correspond to a shift of the zero point energy which remains undetermined in the absence of coupling to gravity. In theories with exact or spontaneously broken supersymmetry the vanishing of $Str \mathcal{M}^2$ is fulfilled whenever the trace-anomaly vanishes. The soft supersymmetry breaking terms are defined as those non supersymmetric terms that can be added to the supersymmetric Lagrangian without spoiling the constancy of $Str \mathcal{M}^2$.

It is important to stress that $Str \mathcal{M}^2$ is a function of $\phi$. In a supersymmetric theory $Str \mathcal{M}^2 = 0$ is true for any value of $\phi$, and thus represents simultaneous satisfaction of three sets of constraints, corresponding to terms which are proportional to $\phi^0$, $\phi$ and $\phi^2$, respectively. In a generic theory the vanishing of $Str \mathcal{M}^2$ will instead occur only for specific values of $\phi$.

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5 Notice that for spin-1/2 fields one should replace $M^2_{1/2}(\phi) \rightarrow M^1_{1/2}(\phi)M_{1/2}(\phi)$
Consider now the SM with Higgs potential

\[ V(\phi) = -\frac{m^2}{2} \phi^2 + \frac{\lambda}{4} \phi^4 \]  

(8)

In fact \( S_{\text{SM}}M^2(\phi) \) can be seen as a function of the renormalization scale \( \mu \sim \phi \), as given by

\[ S_{\text{SM}}M^2(\phi) = H(\mu) + 3G(\mu) + 6W(\mu) + 3Z(\mu) - 12T(\mu) \]  

(9)

where the numerical coefficients in Eq. (9) come from the number of degrees of freedom of the physical Higgs (one), the Goldstone bosons (three), the massive gauge bosons \( Z \) (three) and \( W \) (six) and the top, a Dirac fermion (twelve). In fact

\[ H(\mu) = -m^2(\mu) + 3\lambda(\mu)\phi^2 \]
\[ G(\mu) = -m^2(\mu) + \lambda(\mu)\phi^2 \]
\[ W(\mu) = \frac{1}{4} g^2(\mu) \phi^2 \]
\[ Z(\mu) = \frac{1}{4} (g^2(\mu) + g'^2(\mu)) \phi^2 \]
\[ T(\mu) = \frac{1}{2} y_t^2(\mu) \phi^2 , \]  

(10)

\( y_t \) being the top Yukawa coupling and \( g, g' \) the electroweak gauge couplings. When \( \phi = v \approx 246 \text{ GeV} \), we have that \( H, W, Z, T \) become the physical masses \( m_H^2, m_W^2, m_Z^2, m_t^2 \) while \( G = 0 \). Note that there is no term linear in \( \phi \) in (9), because the SM does not have a cubic scalar invariant term in the Lagrangian. Clearly in the SM it is not possible to have \( S_{\text{SM}}M^2 = 0 \) for general \( \phi \), since the \( m^2 \) terms in (9) do not cancel. The vanishing of \( S_{\text{SM}}M^2 \) will happen only at some specific value of \( \phi \). Since in the RGE we are identifying \( \phi \) with the renormalization scale \( \mu \), for large field values the terms proportional to \( \phi^2 \) in (9) will neatly dominate; in other words the absence of quadratic divergences is provided by the condition \[ \frac{\partial S_{\text{SM}}M^2(\phi)}{\partial \phi^2} = 6\lambda(\mu) + \frac{9}{4} g^2(\mu) + \frac{3}{4} g'^2(\mu) - 6y_t^2(\mu) = 0 , \]  

(11)

that is precisely the Veltman condition at 1-loop since the right hand side of Eq. (3) can be written in terms of running couplings as

\[ C_{V1}(\mu) = 6\lambda(\mu) + \frac{9}{4} g^2(\mu) + \frac{3}{4} g'^2(\mu) - 6y_t^2(\mu) . \]  

(12)

Notice that including two-loop (or higher-loop) corrections will modify the condition (12) by a loop suppressed \( O[1/(4\pi)^2] \) quantity which will translate into a tiny modification of the one-loop Veltman scale \( \mu_{V1} \).
III. THE VELTMAN CONDITION IN THE STANDARD MODEL

In terms of running couplings the VC1 reads as $C_{V1}(\mu) = 0$. An example of the running of the above quantities is provided in Fig. 1. We have performed a NNLO running (three-loop for beta functions [14] and two-loop for the matchings [15]) as discussed in [16]. The plot has been obtained by choosing: $m_H = 126$ GeV for the Higgs mass, $\overline{m}_t(m_t) = 161.5$ GeV for the running top mass in the $\overline{MS}$ scheme evaluated at the top mass, $\alpha_3(m_Z) = 0.1196$ for the strong coupling constant evaluated at the $Z$ mass.

In the present work we display all results as a function of $\overline{m}_t(m_t)$, as suggested in [18] and done in [19]. This allows to avoid the theoretical error due to the matching between the pole mass and the running $\overline{MS}$ mass of the top quark. In order to make a link with the pole top mass value $M_t$, we note that: i) According to Ref. [18] the pole top mass value is just about 10.0 GeV larger than the running top mass; ii) According to the most recent analysis of Ref. [19] the top pole mass mean value obtained by combining the latest data from ATLAS, CMS and CDF is $M_t = 173.36 \pm 0.65 \pm 0.3$ GeV at 1$\sigma$, where the last uncertainty is of theoretical origin and it is due to non-perturbative effects of order $\Lambda_{QCD}$. Here we consider it is safe to take the experimental range of the running top mass

![Graph showing the running of the Veltman condition and its various contributions as functions of the renormalization scale $\mu$. The plot has been obtained by choosing: $m_H = 126$ GeV for the Higgs mass, $\overline{m}_t(m_t) = 161.5$ GeV for the running top mass in the $\overline{MS}$ scheme evaluated at the top mass, $\alpha_3(m_Z) = 0.1196$ for the strong coupling constant evaluated at the $Z$ mass.]
to be given by $\overline{m_t}(m_t) = 163.5 \pm 2.0$ GeV at 2σ. The value $\overline{m_t}(m_t) = 161.5$ GeV selected in Fig. 1 is thus the lowest acceptable one at 2σ. Notice that the contribution to the VC1 from the Higgs quartic coupling (dashed line) is smaller than the gauge (dotted line) and Yukawa (dot-dashed line) contributions. Increasing the top mass the Higgs quartic coupling at high scales decreases and, as a consequence, the scale $\mu$ where the VC1 is fulfilled increases. This means that in the SM the VC1 is typically realized at too high a scale, slightly larger than the Planck scale.

FIG. 2: The dependence of $\mu_{V_1}$ on the running top mass $\overline{m_t}(m_t)$. The solid line corresponds to $m_H = 126$ GeV and $\alpha_3(m_Z) = 0.1196$. The shaded band is obtained by varying $m_H$ by ±1 GeV. The dot-dashed lines are obtained by varying $\alpha_3(m_Z)$ in its 2σ range \cite{17}. The shaded grey region emphasizes the range between the Planck mass ($M_{Pl} = 1.2 \times 10^{19}$ GeV) and the reduced Planck mass ($M_{Pl}/\sqrt{8\pi}$).

This is better illustrated in Fig. 2 where we plot the scale $\mu_{V_1}$ such that $C_{V_1}(\mu_{V_1}) = 0$, as a function of the running top mass in the MS scheme. The solid line is obtained for $m_H = 126$ GeV and $\alpha_3(m_Z) = 0.1196$. The shaded region between the dashed lines is obtained by keeping $\alpha_3(m_Z)$ fixed and varying $m_H$ in the range $126 \pm 1$ GeV. The dot-dashed lines are obtained by keeping $m_H = 126$ GeV and varying $\alpha_3(m_Z)$ in its 2σ range \cite{17}. The dots on the lines signal the value of $\overline{m_t}(m_t)$ such that a second vacuum degenerate with the electroweak one appears. For instance focussing on the case $m_H = 126$ GeV and $\alpha_3(m_Z) = 0.1196$ (solid line), this happens for $\overline{m_t}(m_t) \simeq 162$ GeV, and the second degenerate minimum turns out to be located at $\mu = 4.3 \times 10^{17}$ GeV \footnote{Note that the scale at which this happens does not coincide with the scale at which the VC1 is satisfied}.
larger (smaller) values of $m_t(m_t)$ the Higgs potential is thus stable (metastable).

One realizes that the uncertainty on $\mu_{V_1}$ due to the $2\sigma$ range of $\alpha_3(m_Z)$ is larger than the uncertainty due to the range of the Higgs mass. Anyway there is little room in the experimentally allowed SM parameter space for the VC1 to be satisfied at sub-Planckian scales: this happens only for the highest possible values of the Higgs mass and $\alpha_3(m_Z)$ and for the lowest possible values of $m_t(m_t)$.

IV. SM MERGING INTO HIGH-SCALE MSSM

As we pointed out in section I one can interpret the realization of the Veltman condition around the Planck scale as an indication that we have a fine-tuned Standard Model up to the scale $\mu_{V_1}$ which merges into a version of the MSSM which is in turn possibly embedded into a more fundamental theory, as e.g. a superstring theory. This idea of a high-scale MSSM \(^7\) has been put forward recently using ideas based on the stability condition \(^20\)-\(^22\). We will show here that a similar idea can be implemented using as an input the Veltman condition \(^8\).

As the Standard Model is valid to some high-scale it is fine-tuned. At this point we should give up with solving the SM hierarchy (along with the cosmological constant) problem unless a symmetry reason in the UV merging theory, or some kind of environmental selection, is provided as it would be the case of the huge number (landscape) of string vacua. In any way if we denote by $H_1$ and $H_2$ the MSSM Higgs doublets giving mass to the down and up quarks, respectively, with a superpotential $W = \mu H_1 \cdot H_2$, the quadratic terms in the tree-level potential are

$$V_2 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 \cdot H_2 + h.c.)$$ (13)

where $H_1 \cdot H_2 = H_1^a \varepsilon_{ab} H_2^b$ ($\varepsilon_{12} = -1$) and the soft breaking terms $m_{1,2,3}^2 \simeq \mu_{V_1}^2$. The fine-tuning required to have a single light Higgs is provided by the condition

$$m_3^4 \simeq m_1^2 m_2^2$$ (14)

which should be satisfied up to $O[m_\ell^2 (m_1^2 + m_2^2)]$, and the light and heavy eigenstates

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\(^7\) Notice the difference between our construction, i.e. high-scale supersymmetry where the effective theory below $\mu_V$ is just the Standard Model, and the so-called split supersymmetry, where Higgsino and gaugino masses are protected by some symmetry and remain in the low energy theory.

\(^8\) For an early study previous to the Higgs discovery see \(23\).
(i.e. the SM Higgs $H$ and its orthogonal combination $H_{\text{Heavy}}$) are

$$H = \cos \beta H_1 - \sin \beta \tilde{H}_2$$

$$H_{\text{Heavy}} = \sin \beta H_1 + \cos \beta \tilde{H}_2$$  \hspace{1cm} (15)$$

where

$$\tan \beta \simeq |m_1|/|m_2|, \hspace{1cm} (16)$$

$\tilde{H}_2 \equiv \varepsilon H_2^*$ and the mass of the heavy Higgs is $m_{H_{\text{Heavy}}}^2 = m_1^2 + m_2^2$. The projection over the SM-Higgs

$$H_1 \cdot H_2 = -\sin \beta \cos |H|^2, \hspace{1cm} |H_1|^2 + |H_2|^2 = |H|^2$$  \hspace{1cm} (17)$$
yields the light state $H_{S\text{M}}$. At this point we are not going to argue about the origin of this fine-tuning but we will just accept that the SM merges below the scale $\mu_{V_1}$. This means that we are assuming that all soft breaking parameters (soft-breaking masses, gaugino and Higgsino masses and soft trilinear couplings) are $O(\mu_{V_1})$ so that we can match the MSSM and the SM at the scale $\mu_{V_1}$.

![Diagram of merging of SM with the MSSM](image)
Recall that the merging with the MSSM at $\mu_{V_1}$ requires
\[
\lambda(\mu_{V_1}) = \frac{1}{8} \left[ g'^2(\mu_{V_1}) + g^2(\mu_{V_1}) \right] \cos^2 2\beta + \frac{3}{16\pi^2} y_t^4(\mu_{V_1}) x_t^2 \left( 1 - \frac{x_t^2}{12} \right),
\] (18)
where the last term is a threshold correction coming from a possible mixing in the stop sector, $x_t = (A_t - \mu/\tan \beta)/m_S$ and we are identifying the common supersymmetric mass $m_S \simeq \mu_{V_1}$. In Fig. 3 we show the running of $C_{V_1}(\mu)$ for particular values of the top and Higgs masses, as well as the running of the different terms in Eq. (18). In particular we can see that the running of the threshold correction $3y_t^4/16\pi^2$ brings it to extremely small values at $\mu_{V_1}$ so that we can, for all purposes, neglect this correction and thus we will assume from here on the case of no mixing, i.e. $x_t = 0$. This means in particular that
\[
0 \leq \lambda(\mu_{V_1}) \leq \frac{1}{8} \left[ g'^2(\mu_{V_1}) + g^2(\mu_{V_1}) \right],
\] (19)
where equality with the left hand side (right hand side) holds for $\tan \beta = 1$ ($\tan \beta \gg 1$).

For instance for the input values adopted in the left panel of Fig. 3 one realizes that it is possible to achieve a viable merging with the MSSM for $\alpha_3(m_Z) \gtrsim 0.1180$. However for a large enough top mass such that $\lambda$ is negative at $\mu_{V_1}$, the merging with the MSSM is not possible for any allowed value of $\alpha_3(m_Z)$.

We can also determine the value of $\tan \beta$ for each point in the plane $[m_H, \overline{m}_t(m_t)]$. Such value becomes an upper bound on $\tan \beta$ as we are neglecting the mixing and we are assuming $x_t \simeq \mathcal{O}(1)$ such that the threshold correction to the quartic coupling is positive. As an example, we consider for definiteness $\alpha_3(m_Z) = 0.1196$ and display in the right panel of Fig. 3 the isocurves of constant $\mu_{V_1}/M_{Pl}$ (solid red) and upper value on $\tan \beta$ (dashed blue). We see that a large $\tan \beta$ is not possible for the Veltman condition to be satisfied below the Planck scale: in fact we are able to fix the upper bound on $\tan \beta$ as $\tan \beta \lesssim 2$. As we can see from the plots in Fig. 3 from the Veltman condition the merging of the SM at low energy into the MSSM at high energy can only be done, for realistic Higgs and top quark masses, at slightly trans-Planckian scales which sheds doubts on the consistency of the whole procedure as gravitational effects are never considered.

Moreover on purely experimental grounds we do not expect the Standard Model to be the effective theory at energies presently explored by LHC as there is no valuable candidate to Dark Matter. From that point of view some SM extension should be favored as a candidate to low-energy effective theory. A quick glance at Fig. 4 shows that the (negative) contribution of the top quark Yukawa coupling in the Veltman condition is responsible for its fulfillment at very high scales. So adding fermions to the SM (e.g. vector like fermions) coupled to the Higgs field will only worsen the situation. It is clear that we need some bosonic contribution to lower the Veltman scale $\mu_V$ and from that point of view one can envisage a generic situation with bosonic Dark Matter. In the next section we will consider the simplest such model where a singlet is coupled to the SM Higgs, a model
already considered in the literature for various purposes \cite{24}, including the improvement of fine-tuning for the Higgs field in the Standard Model \cite{25,27}. We will consider a complex singlet, instead of a real one, to make it simpler the merging with the MSSM extended by a singlet superfield, a supersymmetric theory which has also been extensively studied in the literature \cite{28} and is usually dubbed as the NMSSM \footnote{The name NMSSM is usually reserved to the MSSM plus a singlet chiral superfield, when the scalar component of the singlet acquire a non-zero VEV which makes it possible to give a technical solution to the supersymmetric $\mu$-problem. Independently on whether our singlet does or does not not acquire any VEV, for simplicity and notational convenience we will occasionally keep on calling the model NMSSM by an abuse of language.}.

V. SM EXTENSION WITH SINGLETS

Let us now consider the simplest SM extension with a complex singlet $S$, with a general potential given by

$$V = V_{SM} + \lambda_S |S|^4 + 2\lambda_S |H|^2 |S|^2 + M_S^2 |S|^2 + \cdots \quad (20)$$

where

$$V_{SM} = -m^2 |H|^2 + \lambda |H|^4 \quad (21)$$

We will consider the particular case $\lambda_S(\mu_V) = 0$ which will have a simple merging with the NMSSM as we will see in the next section \footnote{Of course even if we consider $\lambda_S = 0$ at the merging scale it will be generated at lower scales by the renormalization group equations (RGE), although its impact should be tiny as we have checked numerically.}.

As the mass of the complex singlet in the presence of the background field $\phi$ from (20) is given by

$$m_S^2(\phi) = M_S^2(\mu) + \lambda_S(\mu) \phi^2 \quad (22)$$

the Veltman condition for the cancellation of quadratic divergences \footnote{ } reads as

$$6\lambda(\mu) + 2\lambda_S(\mu) + \frac{9}{4} g^2(\mu) + \frac{3}{4} g'^2(\mu) - 6y_t^2(\mu) = 0 \quad (23)$$

where the contribution of the $S$ field to $Str\mathcal{M}^2$ is $2m_S^2(\phi)$ and thus the factor of two in Eq. (23) is the number of degrees of freedom of the complex singlet.

The introduction of a light singlet with Lagrangian given by the potential (20) creates a second hierarchy problem even if the scalar singlet $S$ is heavier (at the TeV scale) than the SM Higgs and independently on whether or not it does acquire a VEV \cite{27}. In particular the coupling $2\lambda_S |S|^2 |H|^2$ creates a quadratic sensitivity on the scale for the
$|S|^2$ term. This quadratic sensitivity can only be canceled by a fermion mass contribution in the presence of the background field $S$. In particular the minimal extension is done by adding to the SM the scalar $S$ and a (singlet) Weyl fermion $f$ with a mass Lagrangian

$$L_f = -\frac{1}{2}(m_f + \lambda_f S)f^\alpha f_\alpha + h.c.$$  \hfill (24)

and thus $M_{1/2}(S) = m_f + \lambda_f S$. The Veltman condition along the $|S|^2$-direction should then be understood as the $S$-dependent part of $Str\mathcal{M}^2(\phi, S)$. It can then be written as

$$8\lambda_{SH}(\mu_V) - 2\lambda_f^2(\mu_V) = 0.$$  \hfill (25)

where the first term comes from the Higgs doublet $H$ (four degrees of freedom) and the second one from the Weyl fermion $f$ (two degrees of freedom). \hfill 11

The one-loop beta functions for the interactions in Eqs. (20) and (24) are given by \[29, 30\]

\begin{align*}
(4\pi)^2 \frac{d\lambda}{dt} &= (12y_t^2 - 3g'^2 - 9g^2)\lambda - 6y_t^4 + \frac{3}{8}[2g^4 + (g'^2 + g^2)^2] + 24\lambda^2 + 4\lambda_{SH}^2, \\
(4\pi)^2 \frac{d\lambda_{SH}}{dt} &= \frac{1}{2}(12y_t^2 - 3g'^2 - 9g^2)\lambda_{SH} + 4\lambda_{SH}(3\lambda + 2\lambda_s) + 8\lambda_{SH}^2 + 4\lambda_{SH}\lambda_f^2, \\
(4\pi)^2 \frac{d\lambda_s}{dt} &= 8\lambda_{SH}^2 + 20\lambda_s^2 + 8\lambda_s\lambda_f^2 - \lambda_f^4, \\
(4\pi)^2 \frac{d\lambda_f}{dt} &= 6\lambda_f^3,
\end{align*}

where $t = \ln \mu/m_Z$.

We plot in the left panel of Fig. 4 the new Higgs VC1 \[23\] and the various terms contributing to it. There are two main differences with respect to the SM case shown in Fig. 1, both of them contributing to lowering the scale at which VC1 is satisfied, $\mu_V \simeq 10^8$ GeV:

- The first difference is that the function $\lambda(\mu)$, the SM quartic coupling, takes larger values than in the pure SM case. The origin of this increase is the contribution of the term $4\lambda_{SH}^2$ in its RGE, the first equation in \[27\]. The largish value of $\lambda(\mu)$ tends then to compensate the negative contribution of $-6y_t^2(\mu)$.

- The second difference is the very existence of the term $2\lambda_{SH}$ in the VC1 \[23\] whose value is sizeable as can be seen from Fig. 4 and tends then to compensate the negative contribution of the top quark Yukawa coupling.

\[11\] As we will see in the next section there is also a quadratically divergent tadpole for the singlet $S + S^*$ from the supertrace term $M_{1/2}(S)M_{1/2}(S)$ which should be consistently cancelled by the matching conditions of the supersymmetric merging theory.
In the right panel of Fig. 4 we also show the VC1 along the $S$-direction, Eq. (25), as well as the running of couplings $\lambda_S(\mu)$ and $\lambda_f(\mu)$ whose boundary conditions are fixed at the scale $\mu_{V_1}$ by $\lambda_S(\mu_{V_1}) = 0$ and Eq. (25). The corresponding initial values of the couplings are indicated in the caption of Fig. 4.

As we can see from Fig. 4 the reduction in the value of $\mu_{V_1}$ is sizable even for a small value of $\lambda_{SH}$. In fact the larger $\lambda_{SH}(m_Z)$ the stronger the reduction of the $\mu_{V_1}$ scale. The obvious question is how much can we lower the $\mu_{V_1}$ scale and in particular whether or not it can be pushed towards the TeV scale. To answer this question we have plotted in Fig. 5 the scale $\mu_{V_1}$ as a function of the running top mass for various values of $\lambda_{SH}(m_Z)$. For each curve we have selected the values of $\lambda_S(m_Z)$ and $\lambda_f(m_Z)$ so that both Veltman conditions, Eqs. (23) and (25), are satisfied along with $\lambda_S(\mu_{V_1}) = 0$. In particular for $\lambda_{SH}(m_Z) = (0.05, 0.1, 0.2, 0.4, 0.8)$ we have chosen respectively $\lambda_S(m_Z) \approx (0.003, 0.007, 0.02, 0.03, 0.05)$ and $\lambda_f(m_Z) \approx (0.41, 0.56, 0.78, 1.1, 1.59)$. We have also shown in Fig. 5, for the sake of comparison, the SM case (where $\lambda_{SH} = 0$) which clearly shows, as it was pointed out in previous sections, that identifying $\mu_{V_1}$ with the Planck scale requires borderline values of the top quark and Higgs masses and of $\alpha_3(m_Z)$. Moreover Fig. 5 shows that, for fixed values of $\overline{m}(m_t)$, $m_H$ and $\alpha_3(m_Z)$, the scale $\mu_{V_1}$ exhibits an exponential sensitivity to the value of $\lambda_{SH}(m_Z)$ and for $\lambda_{SH}(m_Z) \lesssim O(1)$
values of $\mu_{\nu_1} \gtrsim \text{TeV}$, where the theory is perturbative, can be reached. Clearly the UV completion of the theory merging at the scale $\mu_{\nu_1}$, should most probably become non-perturbative at some scale below $M_{Pl}$ when the coupling in the UV theory matching $\lambda_{SH}(\mu_{\nu_1})$ reaches a Landau pole.

In the next section we will study the merging with the simplest supersymmetric theory which consists in the MSSM plus the addition of singlet chiral superfields. In particular we will see that adding to the NMSSM an additional chiral singlet $T$, with coupling and masses satisfying some relations, condition (25) can be satisfied.

VI. MERGING TO THE MSSM WITH SINGLETS

We will assume here that the actual model, the SM plus the complex singlet $S$, merges into a supersymmetric theory. As far as the Veltman condition along the SM Higgs is concerned the simplest such theory is the MSSM plus a chiral supersymmetric singlet $S$
which we are calling NMSSM with a superpotential \( W \) given by

\[
W = \lambda_2 S H_1 \cdot H_2 + \mu_H H_1 \cdot H_2 + \frac{1}{2} \mu_S S^2
\]

(27)

and the potential for the Higgs and singlet sectors, with the addition of soft breaking terms (including a trilinear soft coupling \( A_\lambda \)) is written as

\[
V = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 \cdot H_2 + h.c.) + M_S^2 |S|^2 \\
+ \lambda_2 |H_1 \cdot H_2|^2 + \frac{g^2 + g'^2}{8} (|H_1|^2 - |H_2|^2)^2 \\
+ \lambda_2 \mu_H S(|H_1|^2 + |H_2|^2) + \lambda_2 \mu_S S^\ast (H_1 \cdot H_2) + \lambda_2 A_\lambda S H_1 \cdot H_2 + h.c.
\]

(28)

By making the fine-tuning (14) required to have a light SM Higgs \( H \), and consequently the projections (15)-(17), we can write the potential (28) as

\[
V = V_{SM} + M_S^2 |S|^2 + \lambda_2 |S|^2 |H|^2 + \lambda_2 \left( \mu_H - \frac{\mu_S + A_\lambda}{2} \sin 2\beta \right) (S + S^\ast) |H|^2
\]

(29)

with

\[
\lambda = \frac{1}{8} (g^2 + g'^2) \cos^2 2\beta + \frac{1}{4} \lambda_2^2 \sin^2 2\beta + \frac{3}{16\pi^2} h_t^4 x_t^2 \left( 1 - \frac{x_t^2}{12} \right)
\]

(30)

where we also have introduced the threshold corrections generated by integrating out the heavy stops \( \tilde{Q} \) and \( \tilde{U}^c \) with \( x_t = (A_t - \mu / \tan \beta) / M_S \) and where we are identifying the common supersymmetric mass \( M_S \simeq \mu v_1 \). Then the matching with the potential (20) is done by the condition

\[
\lambda_2^2 (\mu v_1) = 2\lambda_{SH}^2 (\mu v_1)
\]

(31)

and the additional fine-tuning [on top of the MSSM one (14)] at the merging scale \( \mu v_1 \)

\[
\tilde{\mu} \equiv \mu_H - \frac{\mu_S + A_\lambda}{2} \sin 2\beta = \mathcal{O}(\text{TeV}).
\]

(32)

Notice that unless we tune exactly \( \tilde{\mu} = 0 \) the potential (29) differs from the potential (20) by the last term. In fact the last term of potential (29) generates a mixing between the \( R \) field, where

\[
S = \frac{\mathcal{R} + i\mathcal{I}}{\sqrt{2}}
\]

(33)

and the physical Higgs \( h \) through the Lagrangian term \( \propto \tilde{\mu} v R(x) h(x) \). However the latter does not modify the conditions for the vanishing of the Veltman relations, either along the \( H \) and the \( S \) fields worked out in section [V] as it appears in an off-diagonal entry of the squared mass matrix which obviously does not contribute to the quantity \( Str \mathcal{M}^2(H, S) \). However as we will see the term \( \propto \tilde{\mu} R |H|^2 \) is a necessary ingredient in the diagonal Higgs mass so as to cancel the tadpole along the \( R \) field.
In order to also fulfill the Veltman condition along the $S$ field the simplest possibility is adding to the NMSSM an extra singlet superfield $T$ such that $S$ is coupled to it in the superpotential $W_T$ as

$$W_T = \frac{1}{2} \lambda_3 ST^2 + \frac{1}{2} \mu_T T^2. \quad (34)$$

The scalar component of $T$ ($T$) is integrated out at the scale $\mu_{V_1}$ by a soft breaking mass term $m_T^2 T^2$ where $m_T \simeq \mu_{V_1}$, it disappears from the low-energy effective theory and therefore it does not require any extra Veltman condition. The fermionic component of $T$ denoted by $f$ has a mass from the superpotential (34) as

$$\mathcal{L}_f = -\frac{1}{2}(\mu_T + \lambda_S S) f^a f_a + h.c. \quad (35)$$

where we are assuming the supersymmetric parameter $\mu_T = \mathcal{O}(\text{TeV})$ and from the Lagrangian (24) we can identify, at the matching scale $\mu_{V_1}$,

$$\lambda_f(\mu_{V_1}) = \lambda_S(\mu_{V_1}), \quad m_f(\mu_{V_1}) = \mu_T(\mu_{V_1}). \quad (36)$$

Now to cancel all quadratic divergences along the $S$ field direction we need to evaluate the supertrace in the background field $S$. From the potential (29) and the fermion Lagrangian (36) we can write

$$\text{Str} \mathcal{M}^2(S) = 4 \left[ \lambda_2^2 |S|^2 + \lambda_3 \bar{\mu}(S + S^*) \right] - 2 \left[ \lambda_3^2 |S|^2 + \lambda_2 \mu_T (S + S^*) \right] + \cdots \quad (37)$$

where the first term represents the contribution from the Higgs $H$ in $Tr M_0^2$ (four degrees of freedom), the second term the contribution from the Weyl fermion $f$ in $Tr M_1^{1/2} M_1^{1/2}$ (two degrees of freedom) and the ellipsis indicates $S$-independent terms. From Eq. (37) we obtain the absence of quadratic divergences in $|S|^2$ mass terms if

$$2\lambda_2^2(\mu_{V_1}) = \lambda_3^2(\mu_{V_1}) \quad (38)$$

and cancelation of the quadratically divergent tadpole for

$$\bar{\mu}(\mu_{V_1}) = \frac{\mu_T(\mu_{V_1})}{\sqrt{2}}. \quad (39)$$

Finally the low energy effective theory is defined by the potential (29) and the fermion mass and Yukawa Lagrangian (35).

All parameters in the matching condition (30) should be considered at the scale $\mu_{V_1}$ and thus there is no guarantee that the supersymmetric theory could be matched at that scale for realistic values of the top quark and Higgs masses and an arbitrary value of $\lambda_{SH}$. The reason can be understood as follows. Neglecting the mixing in the stop sector \footnote{For values of $\lambda_{SH} \lesssim \mathcal{O}(1)$ the threshold corrections provide a tiny contribution to the Higgs quartic coupling because, as we will see next, the top Yukawa coupling is driven to small values at high energy scales. Therefore from here on we will consider the case of zero mixing.} we
can write the relations

\[
\text{Min} \left[ \frac{1}{2} \lambda_{SH}, \frac{1}{8} (g^2 + g'^2) \right] \leq \lambda \leq \text{Max} \left[ \frac{1}{2} \lambda_{SH}, \frac{1}{8} (g^2 + g'^2) \right]
\]  

(40)

so in the cases where \( \lambda(\mu_{V_1}) \) is outside the above range the merging into the NMSSM is not viable. This happens for small enough values of \( \lambda_{SH}(m_Z) \) as it is shown in Fig. 6 where we plot the Veltman condition for \( \lambda_{SH} = 0.05 \) (left panel) and \( \lambda_{SH} = 0.2 \) (right panel). In each case we have selected the values of \( \lambda_S(m_Z) \) and \( \lambda_f(m_Z) \) so that the Veltman condition in the \( S \) direction, Eq. (25), is also satisfied, along with \( \lambda_S(\mu_{V_1}) = 0 \). In particular we have chosen for the plot in the left panel \( \lambda_S(m_Z) \approx 0.003 \) and \( \lambda_f(m_Z) \approx 0.41 \) and for the plot in the right panel \( \lambda_S(m_Z) \approx 0.02 \) and \( \lambda_f(m_Z) \approx 0.78 \). As one can see the value of \( \lambda(\mu_{V_1}) \) is outside the interval (40) and any reasonable mixing with \( x_t = \mathcal{O}(1) \) could not cope with it.

For larger values of \( \lambda_{SH}(m_Z) \) [\( \lambda_{SH}(m_Z) \gtrsim 0.3 \)] the value of \( \lambda(\mu_{V_1}) \) enters the interval (40) and the merging becomes possible. This is shown in Fig. 7 where two different values of \( \lambda_{SH}(m_Z) \) are used: 0.4 (upper panels) and 0.8 (lower panels). For each curve we again selected, as in Fig. 6, the values of \( \lambda_S(m_Z) \) and \( \lambda_f(m_Z) \) such that the Veltman condition along the \( S \) direction (25) is satisfied, along with \( \lambda_S(\mu_{V_1}) = 0 \). In particular for the upper panels we have chosen \( \lambda_S(m_Z) \approx 0.03 \) and \( \lambda_f(m_Z) \approx 1.1 \), while in the lower panels \( \lambda_S(m_Z) \approx 0.05 \) and \( \lambda_f(m_Z) \approx 1.59 \). Plots in left panels show the RGE running of
FIG. 7: Examples of viable merging with the NMSSM for two values of $\lambda_{SH}(m_Z)$: 0.4 (upper panels) and 0.8 (lower panels). In the left panels we show the running with the scale of the Veltman condition (23) as well as the running of $\lambda(\mu)$ and that of the couplings defining the interval (40): $\frac{1}{2} \lambda_{SH}(\mu)$ and $\frac{1}{8} [g^2(\mu) + g^2(\mu)]$. In the right panels we plot contour lines of constant $\mu_{V1}$ and $\tan \beta$ in the plane $(\mu_{t}(m_{t}), m_{H})$ assuming $\alpha_{3}(m_{Z}) = 0.1196$.

the different terms contributing to the VC1 where we can see that threshold corrections are rather tiny. In particular we can see that for $\lambda_{SH}(m_Z) = 0.8$ the merging happens
at scales $\sim \text{TeV}$. Similarly the matching of $\lambda$ from Eq. (30) provides (in the absence of mixing) the value of $\tan \beta$ at the scale $\mu_{V_1}$. In the right plots of Fig. 7 we show, for the corresponding values of $\lambda_{SH}(m_Z)$, contour plots of the merging scale $\mu_{V_1}$ and $\tan \beta$ for $x_i = 0$.

Finally notice that a spin-off of the model is that there are candidates to Dark Matter. In fact the scalar potential (29) has the discrete symmetry $I \to -I$ which opens up the possibility of the real scalar $I$ as a Dark Matter candidate. In fact the Lagrangian term $\lambda_2 |S|^2 |H|^2$ in (29) yields the contact interaction $\frac{1}{4}\lambda_2^2 |I|^2 h^2$ and the tri-linear coupling $\frac{1}{2}\lambda_2 v I h$ with the SM Higgs $h$ which provide annihilation amplitudes into the SM channels: $hh$, $WW$, $ZZ$, $tt$, $\ldots$. This possibility was widely explored in the literature [31] and the requirement of correct thermal cosmological abundance leads, for $\lambda_2 = O(1)$, to mass values $m_S = O(1)$ TeV. More precisely we plot in the left panel of Fig. 8 the contour levels in the plane $(m_S, \lambda_{SH}(m_Z))$, including the contour corresponding to the thermal density $\Omega_{DM} \approx 0.25$, as given by WMAP [17].

If we cross the plot on the left panel of Fig. 8 with the information contained in Fig. 5 we can relate $\mu_V$ to $m_S$ as it is shown in the right panel of Fig. 8. We can see that for $\mu_V \simeq M_{Pl}$ the correct thermal density is obtained for $m_S \simeq 1$ TeV and $\lambda_{SH}(m_Z) \simeq 0.05$, while for lower values of $\mu_V$ we obtain larger values of $m_S$. In particular for $\mu_V \sim 10^5$ GeV the correct thermal density is obtained for $m_S \simeq 7$ TeV and $\lambda_{SH}(m_Z) \simeq 0.4$. Note that in the considered range of $m_S$ values the channels $h \to I I, R R$ is kinematically forbidden.
and there are no constraints from the invisible Higgs width. Of course one has to prevent
the decay $I \to ff$ from the fermion Lagrangian term in $\mathcal{L}^I \psi^T i \gamma_5 \psi$ [where $\psi^T = (f, \bar{f})$]
is a four-dimensional Majorana fermion] which implies the condition $m_I < 2m_f$.

Notice that, after considering the constraints on the invisible Higgs decay, the region
on the left of the $\Omega_{DM} = 0.25$ curve in the left panel of Fig. 8 is allowed but the produced
thermal density is too small and one would need another DM candidate. In particular the
Majorana fermion $\psi$ itself is a candidate to Dark Matter through the coupling $\mathcal{R} \psi^T \psi$ as
it annihilates into the Higgs field through the mixing of $\mathcal{R}$ with the physical Higgs $h$ [32].
On the other hand the region on the right of the $\Omega_{DM} = 0.25$ curve in the left panel of
Fig. 8 is excluded as it would overclose the Universe.

VII. CONCLUSIONS

The hierarchy problem for the Standard Model as an effective theory below a given
cutoff $\Lambda$ is twofold:

- On the one hand the presence of quadratic divergences makes the Higgs mass
  quadratically sensitive to the cutoff scale $\Lambda$. This is a purely Standard Model
  problem which is generated by the existence of quadratic divergences.

- On the other hand the Standard Model must be UV completed at the scale $\Lambda$ by a
  theory without quadratic divergences. Then:

  - Either there is no Higgs in the UV theory because the low energy Higgs is
    composite as a consequence of some infrared strong dynamics and dissolves at
    high energy scales.

  - Or quadratic divergences identically cancel in the UV theory, as it is the case
    of a supersymmetric theory even in the presence of soft breaking terms.

Still the matching of the parameters of the high energy and the low energy theories
should guarantee light Higgs mass parameters at the merging scale. This requires
a fine-tuning on the UV theory parameters whose responsibility entirely lies in the
high-energy theory. Of course this fine-tuning could be avoided if the cutoff $\Lambda$ is
only a loop factor larger than the electroweak scale.

As it was already pointed out the absence of quadratic divergences – dubbed Veltman
condition – is automatically satisfied in supersymmetric theories. However the absence
of experimental hints of supersymmetric partners [5] seems to imply that supersymmetry,
if it exists at all, might be realized at high enough scales $\Lambda$ in which case requiring the
absence of quadratic sensitivity on the cutoff of the low energy effective theory implies
that the matching between the supersymmetric and non-supersymmetric theories should be done at the scale at which the Veltman condition is satisfied. In this case the low-energy effective theory should not exhibit any quadratic sensitivity on the cutoff scale.

In this paper we have explored the general consequences of imposing the absence of quadratic divergences in the Standard Model and extensions thereof, a condition first imposed by Veltman in the context of the Standard Model. This should provide a solution to the above first step of the hierarchy problem. Does this imply a full solution to the hierarchy problem? The answer is clearly no, as the matching at the Veltman scale requires a fine-tuned relationship between parameters (step two above) to have a light Higgs squared mass parameter at the merging scale. However this opens up the possibility that this relation be explained within the high-energy theory: either we admit that we do not have any explanation for this fine-tuning, or it might have an environmental selection origin or perhaps it is due to some symmetries or properties of the UV theory. In this way the fulfillment of the Veltman condition allows us to postpone the solution of the hierarchy problem on the knowledge of the UV theory. In other words, our ignorance on the latter, or the absence of BSM experimental signatures, would prevent us from solving the whole hierarchy problem: a patently obvious truth.

We have considered two models for the low energy effective theory: the Standard Model and its extension with a light complex scalar and a light fermion. The results within the Standard Model point towards a merging with the MSSM around the Planck scale and a value of $\tan \beta$ in the range $1 < \tan \beta < 2$. However for the central values of the top quark and Higgs masses and strong coupling $\alpha_3(m_Z)$ the merging happens at slightly trans-Planckian scales putting doubts on the consistency of the theory as gravitational effects are not considered.

Furthermore we have shown that extending the Standard Model with light bosonic and fermionic degrees of freedom one could decrease the matching scale. In particular we have studied the Standard Model extended by a complex singlet, coupled to the Higgs field, with a coupling $\lambda_{SH}$, and to a singlet fermion, merging at the Veltman scale with its supersymmetric extension. In this case depending on the value of the coupling $\lambda_{SH}$ we can lower the matching scale towards the TeV scale and get a prediction for $\tan \beta$ in the supersymmetric merging theory. In particular for values of $\lambda_{SH}$ such that the Veltman scale is $\mathcal{O}$(few) TeV we can predict that in the supersymmetric theory $\tan \beta \simeq 4 - 5$.

Finally we have observed that a spin-off of the model is that it contains possible candidates to Dark Matter: in particular the real scalar $I \equiv \text{Im} S$, which has the discrete symmetry $I \to -I$, and the Majorana fermion $\psi$ where $\psi^T = (f, \bar{f})$, $f$ being the Weyl fermion in the supermultiplet $T$. We postpone a more detailed analysis of the Dark Matter capabilities of the model to future work.
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[1] G. Aad et al. [ATLAS Collaboration], “Observation of a new particle in the search for the Standard Model Higgs boson with the ATLAS detector at the LHC,” Phys. Lett. B arXiv:1207.7214 [hep-ex].
[2] S. Chatrchyan et al. [CMS Collaboration], “Observation of a new boson at a mass of 125 GeV with the CMS experiment at the LHC,” Phys. Lett. B arXiv:1207.7235 [hep-ex].
[3] M. J. G. Veltman, “The Infrared - Ultraviolet Connection,” Acta Phys. Polon. B 12 (1981) 437.
[4] R. Decker and J. Pestieau, “Lepton self-mass, Higgs scalar and heavy quark masses”, Preprint UCL-IPT-79-19, University of Leuven (1979), presented at DESY Workshop, October 1979. Reprinted in hep-ph/0512126.
[5] For an update of LHC and Tevatron searches, see: O. Buchmueller, ”Direct searches for supersymmetry - where are we today?,” plenary talk at HEP 2013, Stockholm, 18-24 July 2013.
[6] I. Jack and D. R. T. Jones, “Naturalness Without Supersymmetry?,” Phys. Lett. B 234 (1990) 321.
[7] M. S. Al-sarhi, I. Jack and D. R. T. Jones, “Quadratic divergences in gauge theories,” Z. Phys. C 55 (1992) 283.
[8] M. Chaichian, R. Gonzalez Felipe and K. Huitu, “On quadratic divergences and the Higgs mass,” Phys. Lett. B 363 (1995) 101 [hep-ph/9509223].
[9] M. Holthausen, K. S. Lim and M. Lindner, “Planck scale Boundary Conditions and the Higgs Mass,” JHEP 1202 (2012) 037 arXiv:1112.2415 [hep-ph].
[10] Y. Hamada, H. Kawai and K. -y. Oda, “Bare Higgs mass at Planck scale,” Phys. Rev. D
24

[11] F. Jegerlehner, “The Standard model as a low-energy effective theory: what is triggering the Higgs mechanism?,” arXiv:1304.7813 [hep-ph].

[12] F. Jegerlehner, “The hierarchy problem of the electroweak Standard Model revisited,” arXiv:1305.6652 [hep-ph].

[13] M. B. Einhorn and D. R. T. Jones, “The Effective potential and quadratic divergences,” Phys. Rev. D 46 (1992) 5206.

[14] L. N. Mihaila, J. Salomon and M. Steinhauser, “Gauge Coupling Beta Functions in the Standard Model to Three Loops,” Phys. Rev. Lett. 108 (2012) 151602 arXiv:1201.5868 [hep-ph]; K. G. Chetyrkin and M. F. Zoller, “Three-loop $\beta$-functions for top-Yukawa and the Higgs self-interaction in the Standard Model,” JHEP 1206 (2012) 033 arXiv:1205.2892 [hep-ph].

[15] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl and M. Shaposhnikov, “Higgs Boson Mass and New Physics,” JHEP 1210 (2012) 140 arXiv:1205.2893 [hep-ph]; G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, “Higgs mass and vacuum stability in the Standard Model at NNLO,” JHEP 1208 (2012) 098 arXiv:1205.6497 [hep-ph].

[16] I. Masina, “The Higgs boson and Top quark masses as tests of Electroweak Vacuum Stability,” Phys. Rev. D 87 (2013) 053001 arXiv:1209.0393 [hep-ph].

[17] J. Beringer et al. (Particle Data Group), “Review of particle physics”, Phys. Rev. D86, 010001 (2012).

[18] S. Alekhin, A. Djouadi and S. Moch, “The top quark and Higgs boson masses and the stability of the electroweak vacuum,” Phys. Lett. B 716 (2012) 214 arXiv:1207.0980 [hep-ph].

[19] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, “Investigating the near-criticality of the Higgs boson,” arXiv:1307.3536 [hep-ph].

[20] L. J. Hall and Y. Nomura, “A Finely-Predicted Higgs Boson Mass from A Finely-Tuned Weak Scale,” JHEP 1003 (2010) 076 arXiv:0910.2235 [hep-ph].

[21] A. Hebecker, A. K. Knochel and T. Weigand, “A Shift Symmetry in the Higgs Sector: Experimental Hints and Stringy Realizations,” JHEP 1206 (2012) 093 arXiv:1204.2551
[hep-th]; A. Hebecker, A. K. Knochel and T. Weigand, “The Higgs mass from a String-Theoretic Perspective,” Nucl. Phys. B 874 (2013) 1 [arXiv:1304.2767 [hep-th]].

[22] L. E. Ibanez and I. Valenzuela, “The Higgs Mass as a Signature of Heavy SUSY,” JHEP 1305 (2013) 064 [arXiv:1301.5167 [hep-ph]].

[23] J. A. Casas, J. R. Espinosa and I. Hidalgo, “Implications for new physics from fine-tuning arguments. 1. Application to SUSY and seesaw cases,” JHEP 0411 (2004) 057 [hep-ph/0410298].

[24] J. R. Espinosa and M. Quiros, “Novel Effects in Electroweak Breaking from a Hidden Sector,” Phys. Rev. D 76 (2007) 076004 [hep-ph/0701145]; S. Profumo, M. J. Ramsey-Musolf and G. Shaughnessy, “Singlet Higgs phenomenology and the electroweak phase transition,” JHEP 0708 (2007) 010 [arXiv:0705.2425 [hep-ph]]; V. Barger, P. Langacker, M. McCaskey, M. J. Ramsey-Musolf and G. Shaughnessy, “LHC Phenomenology of an Extended Standard Model with a Real Scalar Singlet,” Phys. Rev. D 77 (2008) 035005 [arXiv:0706.4311 [hep-ph]]; V. Barger, P. Langacker, M. McCaskey, M. Ramsey-Musolf and G. Shaughnessy, “Complex Singlet Extension of the Standard Model,” Phys. Rev. D 79 (2009) 015018 [arXiv:0811.0393 [hep-ph]].

[25] A. Kundu and S. Raychaudhuri, “Taming the scalar mass problem with a singlet higgs boson,” Phys. Rev. D 53 (1996) 4042 [hep-ph/9410291].

[26] B. Grzadkowski and J. Wudka, “Pragmatic approach to the little hierarchy problem: the case for Dark Matter and neutrino physics,” Phys. Rev. Lett. 103 (2009) 091802 [arXiv:0902.0628 [hep-ph]]; A. Drozd, B. Grzadkowski and J. Wudka, “Multi-Scalar-Singlet Extension of the Standard Model - the Case for Dark Matter and an Invisible Higgs Boson,” JHEP 1204 (2012) 006 [arXiv:1112.2582 [hep-ph]].

[27] I. Chakraborty and A. Kundu, “Controlling the fine-tuning problem with singlet scalar dark matter,” Phys. Rev. D 87 (2013) 055015 [arXiv:1212.0394 [hep-ph]].

[28] U. Ellwanger, C. Hugonie and A. M. Teixeira, “The Next-to-Minimal Supersymmetric Standard Model,” Phys. Rept. 496 (2010) 1 [arXiv:0910.1785 [hep-ph]].

[29] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, H. M. Lee and A. Strumia, “Stabilization of the Electroweak Vacuum by a Scalar Threshold Effect,” JHEP 1206 (2012) 031 [arXiv:1203.0237 [hep-ph]].
[30] E. J. Chun, S. Jung and H. M. Lee, “Radiative generation of the Higgs potential,” arXiv:1304.5815 [hep-ph].

[31] J. McDonald, “Gauge singlet scalars as cold dark matter,” Phys. Rev. D 50 (1994) 3637 [hep-ph/0702143 [HEP-PH]]; J. R. Espinosa, T. Konstandin, J. M. No and M. Quiros, “Some Cosmological Implications of Hidden Sectors,” Phys. Rev. D 78 (2008) 123528 [arXiv:0809.3215 [hep-ph]]; Y. Mambrini, “Higgs searches and singlet scalar dark matter: Combined constraints from XENON 100 and the LHC,” Phys. Rev. D 84 (2011) 115017 [arXiv:1108.0671 [hep-ph]]; F. Bazzocchi and M. Fabbrichesi, “A simple inert model solves the little hierarchy problem and provides a dark matter candidate,” Eur. Phys. J. C 73 (2013) 2303 [arXiv:1207.0951 [hep-ph]].

[32] K. Cheung, P. -Y. Tseng, Y. -L. S. Tsai and T. -C. Yuan, “Global Constraints on Effective Dark Matter Interactions: Relic Density, Direct Detection, Indirect Detection, and Collider,” JCAP 1205 (2012) 001 [arXiv:1201.3402 [hep-ph]].