Shapiro Spikes and Negative Mobility for Skyrmion Motion on Quasi-One
Dimensional Periodic Substrates

C. Reichhardt and C. J. Olson Reichhardt
Theoretical Division and Center for Nonlinear Studies,
Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA
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Using a simple numerical model of skyrmions in a two-dimensional system interacting with a
quasi-one dimensional periodic substrate under combined dc and ac drives where the dc drive is
applied perpendicular to the substrate periodicity, we show that a rich variety of novel phase locking
dynamics can occur due to the influence of the Magnus term on the skyrmion dynamics. Instead
of Shapiro steps, the velocity response in the direction of the dc drive exhibits a series of spikes,
including extended dc drive intervals over which the skyrmions move in the direction opposite to
the dc drive, producing negative mobility. There are also specific dc drive values at which the
skyrmions move exactly perpendicular to the dc drive direction, giving a condition of absolute
transverse mobility.

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When an overdamped particle is driven by a combined
dc and ac drive over a periodic substrate, a series of steps,
called Shapiro steps [1], appear in the velocity response
over fixed dc drive intervals due to phase locking between
the ac driving frequency and the oscillatory frequency
of the particle motion induced by the substrate period-
icity. Phase locking of this type occurs for dc plus ac
driven Josephson junction arrays [2], sliding charge densi-
ty waves [3], vortices in type-II superconductors [4–6]
or colloids [7] moving over periodic pinning arrays, frac-
tional systems [8], and numerous other nonlinear systems
in which there are two coupled competing frequencies [9–
11]. In a two dimensional (2D) overdamped system with
a quasi-one-dimensional (q1D) substrate, Shapiro steps
only occur when the dc and ac drives are both applied
parallel to the substrate periodicity direction, since the
pinning does not induce a periodic modulation of the
particle motion for perpendicular driving. In some sys-
tems, additional non-dissipative terms can be relevant
to the particle dynamics, such as a Magnus force which
generates a particle velocity component that is perpen-
dicular to the net applied force on the particle. Magnus
effects are known to be important for skyrmions in chiral
magnets, where the ratio of the Magnus term to the
damping term can be ten or higher [12–17]. Skyrmions
can be set into motion by an applied spin-polarized cur-
rent, and the Magnus term has been shown to strongly
affect the interaction of the moving skyrmions with pin-
ning sites, leading to reduced depinning thresholds [14–
16,18,19], a drive dependent skyrmion Hall angle [19–
22], and skyrmion speed up effects [20,21].

Recent studies of skyrmions driven over a periodic q1D
substrate by a dc drive that is parallel to the substrate
periodicity direction combined with a perpendicular ac
drive showed that a new class of Magnus-induced Shapiro
steps arises due to an effective coupling by the Magnus
term of the perpendicular and parallel particle motion,
whereas in the overdamped limit no Shapiro steps oc-
cur for this drive configuration [23]. Here we examine
skyrmions confined to a 2D plane containing a q1D pe-
riodic substrate and moving under the influence of a dc
drive applied perpendicular to the substrate periodicity
direction along with a parallel or perpendicular ac drive,
and we find that a rich variety of dynamical phases can
occur. Instead of Shapiro steps, the particle velocity re-
sponse in the dc drive direction exhibits what we call
Shapiro spikes where the slope of the velocity-force curve
locks to a constant value over a range of dc driving forces.
One of the most remarkable features of this system is that
there are also a series of extended dc drive regions where
the particle motion is in the direction opposite to the
dc drive, known as negative mobility [24–26]. It is even pos-
sible for the particle motion at some drives to be exactly
perpendicular to the dc drive direction, creating a con-
dition of absolute transverse mobility [27]. Negative mo-
bility effects have been observed in overdamped systems
but generally require more complicated substrates, ther-
amal fluctuations, many-particle collective effects, or the
application of multiple ac drives, whereas in the skyrmion
system, negative mobility arises for a much simpler set of
conditions. We map the evolution of the dynamic phases
as a function of ac drive amplitude and the ratio of the
Magnus to the damping term. In addition to their inter-
est as signatures of a new dynamical system, these results
could be important in providing a new way to precisely
control the direction of motion of skyrmions in order to
realize skyrmion-based memory or logic devices [28].

Simulation—We model a 2D system with periodic
boundary conditions in the x and y directions containing
a q1D substrate and a skyrmion treated with a particle-
based model that has previously been used to examine
driven skyrmion motion in random [18,19], 2D periodic
[21], and 1D periodic substrates [23,24]. The skyrmion
dynamics are determined using the following equation of
motion:

\[ \alpha_d \mathbf{v}_i + \alpha_m \hat{z} \times \mathbf{v}_i = F_{i}^{sp} + F_{dc} + F_{ac}, \]

where the skyrmion velocity is \( \mathbf{v}_i = \frac{d\mathbf{r}_i}{dt} \). On the left hand side, \( \alpha_d \) gives the strength of the damping term, which aligns the skyrmion velocity in the direction of the net external forces, while \( \alpha_m \) is the Magnus term, which rotates the velocity in the direction perpendicular to the net external forces. For varied ratios of \( \alpha_m/\alpha_d \) we impose the constraint \( \alpha_d^2 + \alpha_m^2 = 1 \). The force from the substrate is \( F_{i}^{sp} = \nabla U(x) \hat{x} \) where \( U(x) = U_0 \cos(2\pi x/a) \) and \( a \) is the substrate lattice constant. The substrate strength is defined to be \( A_p = 2\pi U_0/a \). The dc drive \( F_{dc} = F_{dc}^{\perp} \hat{y} \) is applied perpendicular to the substrate periodicity direction, while the ac driving force \( F_{ac} = F_{ac}^{\parallel} \hat{x} \) or \( \hat{y} \) is applied either parallel or perpendicular to the substrate periodicity direction, respectively.

We characterize the system by measuring the velocity response \( V_{||} = 2\pi(V_z)/\omega a \) parallel to the substrate periodicity and \( V_{\perp} = 2\pi(V_y)/\omega a \) perpendicular to the substrate periodicity, so that on a Shapiro step the velocity is integer valued with \( V_{||} = n \) or \( V_{\perp} = n \), allowing us to identify the step number \( n \).

**Results and Discussion** — In Fig. 1(a) we plot \( V_{||} \) and \( V_{\perp} \) versus \( F_{dc}^{\perp} \) for a system with \( A_p = 1.0, \alpha_m/\alpha_d = 9.96, \) and \( F_{ac}^{\perp} = 0.325 \). The dashed lines show the average velocities in the overdamped limit of \( \alpha_m/\alpha_d = 0 \), where the particles simply slide along the \( y \)-direction with an Ohmic response and phase locking does not occur. When the Magnus term is finite, \( V_{||} \) shows a series of phase-locked Shapiro steps, while \( V_{\perp} \) shows a completely different response consisting of spike like features. On each phase-locked step in \( V_{||} \), the slope of \( V_{\perp} \) is constant. The most remarkable feature in \( V_{\perp} \) is that there are four extended intervals of \( F_{dc}^{\perp} \) over which \( V_{\perp} < 0 \), indicating that the particle is moving in the opposite direction to the applied dc drive, a phenomenon known as negative mobility [25, 26]. On a given step, \( V_{||} \) can grow from negative values to positive values, passing through a point at which \( V_{||} \) is finite but \( V_{\perp} = 0 \), indicating that particle is moving exactly perpendicular to the applied dc drive in a phenomenon known as absolute transverse mobility [27]. At higher values of \( F_{dc}^{\perp} \), the negative mobility regions are lost and the minimum value of \( V_{\perp} \) at the bottom of each spike increases with increasing \( F_{dc}^{\perp} \), which is an example of negative differential conductivity. In Fig. 1(b) we show that for \( \alpha_m/\alpha_d = 3.219 \), there are still spikes in \( V_{\perp} \); however, the regions of negative mobility are lost. As \( \alpha_m/\alpha_d \) is further reduced, \( V_{\perp} \) gradually becomes smoother and approaches the dashed line, which indicates the response in the overdamped limit.

From the dynamics in Fig. 1(a) we define six different regimes for the particle motion. Region I is a phase locked state in which the particle moves in a periodic orbit with \( V_{\perp} > 0 \) and \( V_{||} \geq 0 \). In Fig. 2(a) we show the particle trajectory at \( F_{dc}^{\perp} = 0.015 \), corresponding to the \( n = 0 \) step where the particle orbit translates only along the \( y \)-direction. Figure 2(d) illustrates the \( n = 1 \) step at \( F_{dc}^{\perp} = 0.085 \), where the particle moves in a periodic orbit that translates in both the positive \( x \) and \( y \) directions. Region II is a phase locked state in which the particle moves in the direction opposite to the dc driving force with \( V_{\perp} < 0 \), as shown in Fig. 2(b) for \( F_{dc}^{\perp} = 0.055 \) on the \( n = 1 \) step where the periodic particle orbit translates in the positive \( x \) and negative \( y \) directions. In Region III, which is also phase locked, the particle exhibits absolute transverse mobility and moves strictly in the positive \( x \)-direction with \( V_{||} = 0 \), as shown in Fig. 2(c) for \( F_{dc}^{\perp} = 0.064815 \). This corresponds to a skyrmion Hall angle of \( \theta_{sk} = 90^\circ \). Region IV is a non-phase locked state in which \( V_{||} = 0 \) while \( V_{||} \) is positive. It occurs in the non-step regimes where the particle does not follow a periodic orbit and does not translate along the \( y \)-direction, such as near \( F_{dc}^{\perp} = 0.04 \) in Fig. 1(a). The absolute transverse mobility of Regions III and IV only occurs at specific values of \( F_{dc}^{\perp} \), while the other phases span extended intervals of the dc driving force. Region V is a non-phase locked state where \( V_{||} \) and \( V_{\perp} \) are both positive but the particle does not form a periodic orbit, as illustrated in Fig. 2(e) at \( F_{dc}^{\perp} = 0.095 \). Finally, Region VI is a non-phase locked state in which \( V_{||} < 0 \) and \( V_{||} > 0 \), as shown in Fig. 2(f).
where there is a non-phase locked region with $V_{\perp}$.

For comparison, Fig. 3(c) shows the results of applying a parallel dc drive and a perpendicular ac drive with $F_{dc}^\perp = 0.325$ at $\alpha_m/\alpha_d = 9.96$. Here, phase locking steps are present but the spikes associated with negative mobility are not. The curves for the overdamped case with $\alpha_m/\alpha_d = 0$ show that $V_{\parallel}$ has a finite depinning threshold but no Shapiro steps, while $V_{\perp}$ increases linearly with increasing $F_{dc}^\perp$. In Fig. 3(b), for the same driving configuration at $F_{dc}^\parallel = 2.35$, there are more steps in $V_{\perp}$ as well as regions in which $V_{\perp} < 0$, similar to the perpendicular ac driving case in Fig. 3(a). This shows that it is possible to observe negative mobility and spike features in $V_{\parallel}$ whenever the dc drive is applied perpendicular to the substrate periodicity, regardless of the ac driving direction. The ac drive amplitudes at which the features appear are much lower for perpendicular ac driving than for parallel ac driving.

We next consider the case of a perpendicular dc drive and a parallel ac drive, as shown in Fig. 3(a) where we plot $V_{\perp}$ and $V_{\parallel}$ vs $F_{dc}^\perp$ for a system with the same parameters as in Fig. 3(a) for $F_{dc}^\parallel = 0.325$. Here, for $\alpha_m/\alpha_d = 9.96$, there are still steps in $V_{\parallel}$ and spikes in $V_{\perp}$; however, $V_{\perp} \geq 0$ for all $F_{dc}^\perp$. For the overdamped $\alpha_m/\alpha_d = 0$ case, $V_{\parallel} = 0$ and $V_{\perp}$ increases linearly with increasing $F_{dc}^\perp$. In Fig. 3(b), for the same driving configuration at $F_{ac}^\parallel = 2.35$, there are more steps in $V_{\perp}$ as well as regions in which $V_{\perp} < 0$, similar to the perpendicular ac driving case in Fig. 3(a). This shows that it is possible to observe negative mobility and spike features in $V_{\parallel}$ whenever the dc drive is applied perpendicular to the substrate periodicity, regardless of the ac driving direction. The ac drive amplitudes at which the features appear are much lower for perpendicular ac driving than for parallel ac driving.

For comparison, Fig. 3(c) shows the results of applying a parallel dc drive $F_{dc}^\parallel$ and a perpendicular ac drive with $F_{ac}^\perp = 0.325$ at $\alpha_m/\alpha_d = 9.96$. Here, phase locking steps are present but the spikes associated with negative mobility are not. The curves for the overdamped case with $\alpha_m/\alpha_d = 0$ show that $V_{\parallel}$ has a finite depinning threshold but no Shapiro steps, while $V_{\perp}$ increases linearly with increasing $F_{dc}^\perp$. In Fig. 3(d), both the ac and dc drives are parallel to the substrate periodicity with $F_{ac}^\parallel = 0.325$. At $\alpha_m/\alpha_d = 9.96$, both $V_{\parallel}$ and $V_{\perp}$ exhibit Shapiro steps, while in the overdamped limit with $\alpha_m/\alpha_d = 0$, $V_{\parallel}$ contains Shapiro steps while $V_{\perp} = 0$. This shows that in the overdamped limit, Shapiro steps occur only when both the ac and dc driving are applied parallel to the sub-
substrate periodicity direction.

The negative mobility for perpendicular dc driving arises due to the combination of the Magnus term and the skyrmion-pinning interactions. Under a finite Magnus term, the dc drive generates an x direction force \( F_x = F_{dc}^x \sin(\theta_{sk}) \) on the skyrmion, where \( \theta_{sk} = \tan^{-1}(\alpha_m/\alpha_d) \). In response, the substrate exerts an x direction force on the skyrmion that the Magnus term transforms into a y velocity component in the range \( V_y = \pm A_p \sin(\theta_{sk}) \). For certain intervals of \( F_{dc}^x \), the \( -y \) portion of the ac driving cycle synchronizes with the time at which the pinning force generates a \( -y \) velocity component, resulting in a net negative value of \( V_y \). Conversely, in other \( F_{dc}^x \) intervals the \( +y \) portion of the ac driving cycle synchronizes with the time at which the substrate generates a \( +y \) velocity component, producing a speed up effect with enhanced positive \( V_y \). Somewhere between these two intervals, \( V_y = 0 \) and absolute transverse mobility occurs. All of these effects become stronger for higher ac amplitude and larger ratios of \( \alpha_m/\alpha_d \). A similar argument can be made for ac driving in the x-direction; however, the ac amplitude must be larger by a factor of approximately \( \alpha_m/\alpha_d \), such as shown in Fig. 3(b), for effects of the same magnitude to occur, since for a parallel ac drive the y-velocity component is multiplied by a factor of \( \cos(\theta_{sk}) \) instead of \( \sin(\theta_{sk}) \).

In Fig. 4(a) we plot the evolution of the different regimes for the system in Fig. 3(a) as a function of \( F_{dc}^x \) and \( F_{ac}^x \), focusing only on the \( n = 0, 1 \), and 2 phase-locked regions. The width of the \( n \)-th phase locked step has the same \( J_n \) or Bessel function oscillating behavior predicted to occur for Shapiro steps [30]. The green shading denotes phase locked regimes with \( V_y > 0 \). White indicates unlocked regions with \( V_y > 0 \). The orange shading indicates phase locked and unlocked regions of negative mobility with \( V_y < 0 \), which form a series of triangles that overlap with the \( n = 1 \) and 2 steps. At the edges of these triangles, absolute transverse mobility with \( V_y = 0 \) and \( V_y \) occurs. We observe similar dynamic phases for steps with higher values of \( n \). This result indicates that the direction of the skyrmion motion can be tuned by varying either the dc or ac perpendicular drives. In Fig. 4(b) we plot a dynamic phase diagram as a function of \( F_{dc}^x \) and \( \alpha_m/\alpha_d \) at \( F_{ac}^x = 0.325 \). Here, for small \( \alpha_m/\alpha_d \), the skyrmion motion is locked in the perpendicular direction. Negative mobility occurs only for \( \alpha_m/\alpha_d > 3.2 \), and higher order steps emerge as \( \alpha_m/\alpha_d \) increases. Similar phase diagrams can be created for parallel ac driving; however, in this case, negative mobility does not occur until much higher ac driving amplitudes are applied. We also find that these effects are robust for multiple interacting skyrmions when the skyrmion-skyrmion interactions are modeled as a repulsive force.

**Summary**—We have shown that when a skyrmion obeying dynamics that are governed by both a Magnus and a dissipative term moves under combined ac and dc drives on a quasi-1D periodic substrate, a rich variety of phase locking phenomena can occur that are absent in the overdamped limit. When the dc drive is applied perpendicular to the substrate periodicity direction, for either parallel or perpendicular ac driving the perpendicular velocity response develops Shapiro steps, while the parallel velocity exhibits Shapiro spikes. We also observe extended dc drive intervals over which the skyrmion moves in the opposite direction to the dc drive, known as negative mobility, while for specific dc drive values we find absolute transverse mobility in which the skyrmion moves exactly transverse to the dc drive. When the dc drive is applied parallel to the substrate periodicity direction, the Shapiro spikes and negative mobility are absent, while in the overdamped limit Shapiro steps only occur when the dc and ac drives are both applied parallel to the substrate periodicity direction. The dynamics we observe should be realizable for skyrmions in chiral magnets interacting with quasi-1D substrates created using 1D thickness modulations or line pinning arrays, and open a new way to control skyrmion motion.

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