Warp-knitted Fabric Defect Segmentation Based on the Shearlet Transform

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Abstract
The Shearlet transform has been a burgeoning method applied in the area of image processing recently which, differing from the Wavelet transform, has excellent properties in processing singularities for multidimensional signals. Not only is it similar to the performance of the Curvelet transform, it also overcomes the disadvantage of the Curvelet transform with respect to discretization. In this paper, the Shearlet transform with segmented threshold de-nosing is proposed to segment a warp-knitted fabric defect. Firstly a warp-knitted fabric image of size 512*512 is filtered by the Laplacian Pyramid transform and decomposed into low frequency and high frequency coefficients. Secondly the high frequency coefficients are operated with a pseudo-polar grid and then convoluted by the window function. Thirdly the shearlet coefficients will be obtained through redefining the Cartesian coordinates from the pseudo-polar grid coordinates and de-noised by the segmented threshold method. Then the coefficients which have high energy are selected for reconstruction in an inverse way using the previous steps. Finally the iterative threshold method and object operation based on morphology are applied to segment out the defect profile. The experiment’s result states that the Shearlet transform shows excellent performance in segmenting a common warp-knitted fabric defect, indicating that the segment results can be applied for further defect automatic recognition.

Key words: warp-knitted fabric defect, Shearlet transform, Fourier transform, segmented threshold de-nosing.

Introduction
Warp-knitted fabric, being a more and more extensively used material in social production and life, makes the yarn bend in a loop and draw it through the old loop [1]. With fiercer market competition, fabric quality is the pivotal issue to guarantee optimized benefits from fabric production. Fabric defect detection is an important part of quality assurance. In spite of the existence of artificial detection of fabric defects in small and mid-size companies, automated fabric defect detection is a much talked about application in most districts. Traditionally the methods for fabric defect detection are divided into two approaches: the spatial domain method and frequency domain method. But in review [2], they are classified widely into seven types: statistical (Auto-correlation function [3], co-occurrence matrix [4], mathematical morphology [5], the fractal method [6]), the spectral wavelet transform [7], the Fourier transform [8], the Gabor transform [9], the autoregressive model [10], Markov random fields [11], learning neural networks [12], and structural [13], hybrid [14] and motif-based [15].

Most of the methods mentioned above are applied for woven fabric defect detection, but research on warp-knitted fabric defect detection is scarce. In order to meet the demand for warp-knitted fabric quality control, the burgeoning method of image processing is introduced. The method named the Shearlet transform is a flexible operator which combines geometry and multiscale analysis. It has excellent directional sensitivity and overcomes the shortcomings of dealing with distributed discontinuities such as edges in high-dimensional signals. Therefore it will have promising performance in the segmentation of the defect profile and is also widely applied in some applications like the extraction of texture features [27]. By using the Shearlet transform with segmented threshold de-noising, this work segments some common defects in warp-knitted fabric, such as broken warp, oil and holes. The result shown in our work demonstrates that the Shearlet transform is an effective way of warp-knitted fabric defect partition, and the defect profile is distinct enough for further defect recognition.

Shearlet transform
The traditional Wavelet transform has the promising property of processing pointwise singularities for 1-D signals. But recently it is widely acknowledged that this property can be ineffective in distributing discontinuities, such as edges in high-dimensional signals. Therefore many methods for settling this matter have been proposed over the years, such as the complex Wavelet transform [16], Brushlet transform [17], Ridgelet transform [18], Curvelet transform [19], Contourlet transform [20] and Shearlet transform [21-22]. These methods have high direction sensitivity and are excellent for the representation of distributed discontinuities such as edges; however, for some of them it is difficult to implement the discretization of the transforms, which makes them not suitable for high-dimensional signals theoretically. However, the Shearlet transform overcomes this drawback and has more flexible decomposition for multiscale geometric analysis.

The Shearlet transform is constructed for multiscale geometric analysis by affine systems. In the dimension $d = 2$, the affine systems $\zeta$ with composite dilations can be defined as follows.

$$\zeta_{\nu_{\alpha} \beta, x} (\nu, \beta) = [\nu \beta]^{-1/2}$$

(1)

Where $\nu$ is a collection of basis function; $T$ denotes the anisotropy matrix for multi-scale partitions, $S$ is a shear matrix for directional analysis; $l, n$ and $k$ are the scale, direction and translation parameters, respectively; $T$ is used for scale transformation; $S$ relates to geometrical transformation, and $T$ & $S$ are both $2 \times 2$ invertible matrices and $|B| = 1$. For both $a > 0$ and $b \in R$, the matrices of $T$ and $S$ are represented as:

$$T = \begin{bmatrix} a & 0 \\ 0 & \sqrt{\alpha} \end{bmatrix}$$

and

$$S = \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix}$$

(2)

When $a = 2$, $b = 1$, Equation (2) is written as follows:

$$T = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

(3)
On account of equation (6) and (7), the \( \hat{\phi} \) is [23]. This means the frequency element of \( \psi_{i,j,k} \) is supported by a pair of trapezoids as a black region, as shown in Figure 2, and the area of the single trapezoid is of approximate size \( 2^d \times 2^d \). Additionally, the trapezoids are oriented along the lines of the slope \( n^i \).

Similarly the collection of shearlets at the vertical cone \( D_0 \) is implied as follows Equation (11).

Finally the Shearlet transform of \( f \in L^2(\mathbb{R}^2) \) can be expressed as Equation (12).

In general, the shearlets construct a tight support flame in different scales and directions. Moreover an optimal representation for the distributed discontinuities like edges in images can be archived as well. Moreover the approximate error of shearlets satisfies Equation (13) to achieve the best approximation. Where \( C \) is normal continuous differentiable function space and \( N \) indicates the \( N \) largest coefficients in the shearlet expression.

\[
\varepsilon_N \leq CN^{-2} \left( \log N \right)^3
\]

Implementation of discrete Shearlet transform in frequency domain

Traditionally the information from the images is discontinuous, and the collection of shearlets presented above is not appropriate to achieve a promising representation of the images. Thus the implementation of discretization for shearlets is essential. Theoretically the implementation of discretization for shearlets can be classified into two methods: the frequency domain and time domain. The frequency domain method is intuitionistic enough for the procedures of discretization and is adopted in this paper to implement the discretization of shearlets.

In order to realize the discretization for shearlets, a window function \( W_{i,j,k}^{(g)} \) for decomposing the frequency plane is defined. \( W_{i,j,k}^{(g)} \) is located on a pair of trapezoids and defined in terms of function \( \psi_2 \), illustrated as Equations (14) and (15).

Where \( \gamma = (\gamma_1, \gamma_2) \in \mathbb{R}^2, \gamma \neq 0, i = 0, 1, 2, \ldots, n, Z \) and \( n = -2^d, \ldots, 2^d - 1 \) is the indicator function of set \( D \). and \( W_{i,j,k}^{(g)} \) satisfies the condition below, illustrated as Equation (16).
\[
\sum_{\gamma_1} \sum_{\gamma_2} \left| W_{\alpha\beta}^{(E)} (\gamma_1, \gamma_2) \right|^2 = 1 \quad (16)
\]

Naturally the Fourier transform of shearlets \(\psi_{\alpha\beta} \) in a discretization form can be obtained by Equation (17), where \( L(\gamma) \) is used for the acquisition of a high frequency signal and \( L(\gamma) = L(\gamma_1, \gamma_2) = \psi_{\beta}(\gamma_1)H_{\alpha}(\gamma_2, \gamma_3) + \psi_{\alpha}(\gamma_2)H_{\beta}(\gamma_1, \gamma_2) \).

Eventually the Shearlet transform of \( f \in L^2 (R^2) \) can be illustrated as Equation (18).

Let the input signal be a 2-D image \( f \) of size \( N \times N \), and the discrete Fourier transform of the image \( f(\gamma_1, \gamma_2) \), the decomposition procedure of the image by Shearlet transform can be implemented by the steps below. The first step of the discrete Shearlet transform is to obtain the high frequency coefficients of the image by using the Laplacian Pyramid transform (LPT).

The LPT decomposes the image into a low frequency coefficient containing essential information of the original image and a high frequency coefficient containing the image details [24]. The decomposition procedure of LPT is illustrated in Figure 3, where \( D \) is the decomposition filter, \( C \) the synthesis filter, and \( M \) the sample matrix. In this decomposition procedure, the low frequency coefficient is obtained by processing the original image \( f \) with the decomposition filter and down-sampled matrix and then processed sequentially by the up-sampled matrix, synthesis filter and a prediction coefficient like the original one acquired. While the high frequency coefficient \( b \) is the D-value between the original image and prediction coefficient. Mathematically the decomposition can be simply expressed as Equation (19). Here the notation is same as that mentioned above.

\[
f_{\lambda}(\gamma_1, \gamma_2) = f(\gamma_1, \gamma_2)L(2^{-2\lambda} \gamma_1, 2^{-2\lambda} \gamma_2) \quad (19)
\]

The high frequency coefficient \( f_{\lambda}(\gamma_1, \gamma_2) \) will be processed on a pseudo-polar grid which describes the samples in the frequency domain as along lines across the origin at different slopes [22]. And the pseudo-polar coordinates \((z, c)\) are defined as follows:

\[
\begin{align*}
(z, c) &= \left( \gamma_1 \frac{2}{\gamma_1} - c, \gamma_2 \right), \text{if } (\gamma_1, \gamma_2) \in D_b \quad (20) \\
(z, c) &= \left( \gamma_2 \frac{2}{\gamma_2} + c, \gamma_1 \right), \text{if } (\gamma_1, \gamma_2) \in D_c \quad (21)
\end{align*}
\]

Therefore the high frequency coefficient \( f_{\lambda}(\gamma_1, \gamma_2) \) can be redefined as \( g_{\lambda}(z, c) = f_{\lambda}(\gamma_1, \gamma_2) \) for \( \gamma_1, \gamma_2 \) and \( \gamma_1, \gamma_2 \).

\[
g_{\lambda}(z, c) \text{ will be filtered by the 1-D band-pass filter along axis } c. \text{ The 1-D band-pass filter here is the discrete Fourier transform of window function } W_{\alpha\beta}^{(E)} \text{ and set as } W(2^2 v - n), \text{ where } n = 2, ..., 2 - 1.
\]

Then the Shearlet coefficients \( f_{\lambda}(\gamma_1, \gamma_2) \) can be simply expressed as below based on Equation (18). It redefines the Cartesian coordinates from the pseudo-polar grid coordinates, and \( k, k_2 \) is the coordinate point of discrete samples of the high frequency coefficient. Similarly Shearlet coefficients at \( D_1 \) can be obtained in this way.

The sketch graph of the decomposition procedure of the Shearlet transform introduced above is also shown in Figure 4, where \( fa \) and \( fd \) are the low frequency coefficient and high frequency coefficient, respectively, and \( W \) is the window function.

### Defect segmentation

Decomposition with segmented threshold de-noising

Figure 5 shows a warp-knitted fabric defect gray image of size \( 512 \times 512 \). With the implementation of the discrete Shearlet transform described in Section “Implementation of discrete Shearlet transform in frequency domain”, the high frequency coefficient at every level can be divided into a number of shearlet coefficients. The key point to decompose the fabric image is the decomposition level.

In order to determine the level of decomposition, the edge information of the image should be taken into consideration. The sum of energy of the shearlet coefficients at the \( l \)-th level is computed and then divided by the last level, as shown in equation (23); if \( R > 1 \), the image should be divided into a number of shearlet coefficients.

\[
\begin{align*}
W_{\lambda}^{(E)}(\gamma) &= \left\{ \begin{array}{ll}
\psi_2 \left( \gamma_1 \frac{2}{\gamma_1} - n \right) H_{\alpha}(\gamma), & \text{if } n = 2^l - 2^l \\
\psi_2 \left( \gamma_1 \frac{2}{\gamma_1} - n \right) H_{\alpha}(\gamma), & \text{if } n = 2^l - 1 \quad (14)
\end{array} \right.
\]

\[
W_{\lambda}^{(L)}(\gamma) &= \left\{ \begin{array}{ll}
\psi_2 \left( \gamma_2 \frac{2}{\gamma_2} + n \right) H_{\alpha}(\gamma), & \text{if } n = 2^l - 2^l \\
\psi_2 \left( \gamma_2 \frac{2}{\gamma_2} + n \right) H_{\alpha}(\gamma), & \text{if } n = 2^l - 1 \quad (15)
\end{array} \right.
\]

\[
\psi_{\lambda}(z, c) = 2^{l/2} L \left( 2^{-2l} \gamma \right) W_{\alpha\beta}^{(E)}(\gamma) e^{2\pi i k_2 \gamma}, \quad E = 1, 2 \\
(17)
\]

\[
W_{\alpha\beta}^{(E)}(\gamma) = \int f(\gamma) L \left( 2^{-2l} \gamma \right) W_{\alpha\beta}^{(E)}(\gamma) e^{2\pi i k_2 \gamma} \, d\gamma, \quad E = 1, 0 \\
(18)
\]

\[
\psi_{\lambda}(z, c) = 2^{l/2} L \left( 2^{-2l} \gamma \right) W_{\alpha\beta}^{(E)}(\gamma) e^{2\pi i k_2 \gamma} \, d\gamma, \quad E = 1, 0 \\
(18)
\]

Equations (14), (15), (17), (18) and (22).
During the decomposition of the image by the Shearlet transform, the shearlet coefficients at every level are free will, enabling the Shearlet transform to provide a more meticulous decomposition for the high frequency coefficient. In our work, the number of shearlet coefficients at every level is set as 10, 10, 18 and 18, respectively. Figures 6 to 9 show the shearlet coefficients of the image at levels 1 to 4. Generally, this meticulous decomposition means that much more directional information can be achieved in this procedure. However, not all the shearlet coefficients contain effective directional information, most of which are distributed with noise and need to be processed for the reconstruction, for which the segmented threshold de-noising method is applied in this paper.

The coefficients acquired are classified into the signal noise coefficient, transition coefficient and signal coefficient based on the energy computed in Equation (24). The four shearlet coefficients that have the least energy at every level are identified as the noise coefficients in our work. The threshold de-noising method used is called as the Sqtwolog rule. The threshold \( T_i \) applied in the Sqtwolog rule is computed as Equation (25), where \( n \) is the number of total Shearlet coefficients at the operated level, and \( \sigma \) is the noise signal deviation. This method has the powerful property of de-noising and can get rid of most noise in shearlet coefficients.

\[
T_i = \sigma \sqrt{2 \ln n} \quad (25)
\]

Then the rest of the shearlet coefficients should be operated by other methods, and the average value \( E_i \) of the four shearlet coefficients mentioned above is computed again, otherwise the current level is the best. In our implementation, the best level is 4.

\[
R_i = \frac{E_1^i + E_2^i + \ldots + E_n^i}{E_{i-1}^i + E_{i-1}^i + \ldots + E_{i-1}^i} \quad (23)
\]

In the above equation, \( E^i_\theta \) is the energy of the \( \theta \)th shearlet coefficient at the \( i \)th level and calculated as the square of the norm of the coefficient matrix \( \text{coef}^i_\theta \), as shown in Equation (24).

\[
E^i_\theta = \text{coef}^i_\theta^2, \quad \theta = 0, 1, 2, 3; \quad n = 1, 2, 3 \ldots \quad (24)
\]
ed. In this work, when any of the remaining shearlet coefficients is larger than $BE_i$ ($B \geq 2$), it will be defined as the signal coefficient and operated with the Rigrsure threshold rule. This method is relatively moderate. The calculation formulas of threshold $T_x$ is shown as follows, where $R_i$ is the $i$th element of risk-vector $R$ and $R_x$ is the smallest risk value.

$$R = \frac{n - 2i - (n - i)E_i + \sum_{i=1}^{n} E_i}{n}$$  \hspace{1cm} (26)

$$T_x = \sigma \sqrt{R_x}$$  \hspace{1cm} (27)

The last step of the segmented threshold de-nosing method is the Minimaxi threshold rule, applied for the remaining transition coefficients. Threshold $T_{\text{MT}}$ is illustrated as Equation (28), for which the notations have been explained before.

$$T_{\text{MT}} = \begin{cases} \sigma (0.3936 + 0.1829 \log_2 n) & \text{if } n \\ 0 & \text{otherwise} \end{cases}$$  \hspace{1cm} (28)

After the steps of the decomposition with the segmented threshold de-noising method is completed, most of the noise in these shearlet coefficients is removed, while keeping most of the edge information needed for reconstruction in the next procedure.

**Reconstruction based on energy**

De-noised shearlet coefficients at every level are obtained after the decomposition of the Shealat transform with the segmented threshold de-noising method. Although these coefficients have been de-noised, it does not mean all of them are useful for the reconstruction of the high frequency at every level; those unnecessary shearlets coefficients should be removed to reconstruct promising...
high frequency coefficients. In this work, valuable shearlet coefficients are selected using an energy method obtained based on Equation (24).

The selection rule for the values is illustrated as Equation (29). Where \( E_{al} \) is the average energy value of the \( l \)th level and \( \text{coeff}_{ij}^l \) is the \( ij \)th shearlet coefficient of the \( l \)th level. The coefficient which is smaller than the average energy value \( E_{al} \) is set as the null coefficient matrix, and the rest coefficient is kept intact.

\[
\text{coeff}_{ij}^l = \begin{cases} 
\text{coeff}_{ij}^l, & E_{ij}^l \geq E_{al} \\
0, & E_{ij}^l < E_{al}
\end{cases} \tag{29}
\]

A schematic graph of the shearlet coefficient selection is shown in Figure 10. The black regions are the coefficients selected, which will be used to recompose the high frequency coefficient of every level. Then the warp-knitted fabric image with a broken warp is reconstructed using the inverse discrete Shearlet transform, which is an inverse procedure of the decomposition. Figure 11 shows the final reconstructed image of Figure 5, which keeps most of the effective information due to the excellent properties of the Shearlet transform.

### Iterative threshold segmentation and morphological operation

After the reconstructed fabric image is obtained, iterative threshold segmentation is needed for the next step. This
In addition, the miscellaneous points generated by the loop structure of warp-knitted fabric should be eliminated. Figure 5 shows the loop structure graph of the warp-knitted fabric. The wispy loop holes will induce a dramatic gray level change, as does the defect. Because of this, the segmented result is not promising enough, and a great deal of little points are filled up. The morphological operations [25-26] are used to get rid of these points. Those whose area is smaller than the presupposed value will be removed. The miscellaneous points induced by the loop structure is far smaller than this value and will be removed completely. Then the image will be processed by the morphological opening to make the defect visible. Figure 12 shows the loop structure graph of warp-knitted fabric. However, the segmented result should also be processed by follow-up work in order to get a smoother result. In addition, the miscellaneous points generated by the loop structure of warp-knitted fabric should be eliminated. Figure 12 shows the loop structure graph of the warp-knitted fabric of Figure 5. The wispy loop holes will induce a dramatic gray level change, as does the defect. Because of this, the segmented result is not promising enough, and a great deal of little points are filled up. The morphological operations [25-26] are used to get rid of these points. Those whose area is smaller than the presupposed value will be removed. The miscellaneous points induced by the loop structure is far smaller than this value and will be removed completely. Then the image will be processed by the morphological opening to make the defect visible.

The selection rule for the valuables is illustrated as equation (29). Where \( f(i, j) \) is the gray value of the input image \( i \) is the presupposed threshold, object threshold and background threshold respectively, \( f(i, j) \) is the gray value of the input image \( i \) is the presupposed threshold, object threshold and background threshold respectively, \( f(i, j) \) is the gray value of the input image \( f(i, j) \) is the gray value of the input image.

\[
T_o = \frac{\sum_{(i,j)\notin T_o} f(i, j) \times P(i, j)}{\sum_{(i,j)\notin T_o} P(i, j)},
\]

\[
T_a = \frac{\sum_{(i,j)\notin T_a} f(i, j) \times P(i, j)}{\sum_{(i,j)\notin T_a} P(i, j)},
\]

\[
T_{k+1} = \frac{T_o + T_a}{2}, \quad (30)
\]

However, the segmented result should also be processed by follow-up work in order to get a smoother result. In addition, the miscellaneous points generated by the loop structure of warp-knitted fabric should be eliminated. Figure 12 shows the loop structure graph of the warp-knitted fabric of Figure 5. The wispy loop holes will induce a dramatic gray level change, as does the defect. Because of this, the segmented result is not promising enough, and a great deal of little points are filled up. The morphological operations [25-26] are used to get rid of these points. Those whose area is smaller than the presupposed value will be removed. The miscellaneous points induced by the loop structure is far smaller than this value and will be removed completely. Then the image will be processed by the morphological opening to make the defect visible.

The theory of this method can be expressed as Equation (30). Where \( T_o \), \( T_a \) and \( T_b \) are the presupposed threshold, object threshold and background threshold, respectively; \( f(i, j) \) is the gray value of the input image \( f(i, j) \) is the probability of the gray value at point \( i \) is the presupposed threshold, object threshold and background threshold, respectively; \( f(i, j) \) is the gray value of the input image \( f(i, j) \) is the probability of the gray value at point.

\[
T_o = \frac{\sum_{(i,j)\notin T_o} f(i, j) \times P(i, j)}{\sum_{(i,j)\notin T_o} P(i, j)},
\]

\[
T_a = \frac{\sum_{(i,j)\notin T_a} f(i, j) \times P(i, j)}{\sum_{(i,j)\notin T_a} P(i, j)}, \quad (30)
\]

\[
T_{k+1} = \frac{T_o + T_a}{2},
\]

Figure 14. Results of the broken warp at the back bar: a) original, b) reconstructed, c) result.

Figure 15. Results of the oil: a) original, b) reconstructed, c) result.

Figure 16. Results of the oil: a) original, b) reconstructed, c) result.

Figure 17. Results of the oil: a) original, b) reconstructed, c) result.

Figure 18. Results of the oil: a) original, b) reconstructed, c) result.
Then the promising shearlet coefficients segmented threshold de-noising method. Based on the energy and processed by the selection coefficient and signal coefficient specified as the signal noise coefficient, transformed shearlet coefficients, these will be classified  and verisimilar compared with the original fabric defect images. It can be applied in further automatic warp-knitted fabric defect identification.

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